blacklitterman

November 23, 2025

0.1 Black-Litterman Model Summary

0.1.1 Methodology

1. **Reverse Optimization**: Derived implied equilibrium returns ( ) from market cap weights using the formula `= Σw`
   - Calculated risk aversion coefficient ( ) from market portfolio characteristics
   - These represent what the market "believes" about future returns
2. **Investor Views**: Defined subjective views about future performance
   - View 1: US Large Cap Growth outperforms Value by 3%
   - View 2: Emerging Markets absolute return of 8%
   - View 3: US Growth outperforms International by 4%
3. **Black-Litterman Blending**: Combined equilibrium returns with investor views
   - Views weighted by confidence levels
   - Higher confidence = greater influence on final returns
   - parameter controls uncertainty in prior beliefs
4. **Portfolio Optimization**: Used BL expected returns to find optimal allocation
   - Maximized Sharpe ratio subject to long-only constraints
   - Compared to market cap weighted portfolio

0.1.2 Key Parameters

- **Risk-free rate**: 4%
- **(prior uncertainty)**: 0.025
- **(risk aversion)**: Derived from market portfolio
- **Historical data**: 5 years of daily returns

0.1.3 Interpretation

The resulting optimal weights reflect a balance between: - Market equilibrium (what everyone collectively believes) - Your subjective views (what you think will happen) - Statistical uncertainty (confidence in predictions)

Adjust the views, confidence levels, and parameter to reflect your market outlook.

1 Market Implied Equilibrium Weights

	Ticker	Market Cap	Weight (%)
0	R1KG	21500000000000	26.615087
1	R1KV	21500000000000	26.615087
2	R2K	2200000000000	2.723404
3	RMCG	3250000000000	4.023211
4	RMCV	3250000000000	4.023211
5	INT	25000000000000	29.032258
6	EM	6000000000000	6.967742

2 Historical Returns and Volatility

	Column	First Valid Date
0	EM	2003-04-14
1	INT	2003-04-14
2	R1KV	2003-04-14
3	R1KG	2003-04-14
4	R2K	2003-04-14
5	RMCG	2003-04-14
6	RMCV	2003-04-14

2.1 Returns

	Ticker	Total Compound Return (%)	Annualized Return (CAGR) (%)	\
0	EM	215.80	5.23	
1	INT	232.97	5.47	
2	R1KV	403.66	7.43	
3	R1KG	919.78	10.84	
4	R2K	344.15	6.83	
5	RMCG	610.11	9.07	
6	RMCV	498.08	8.25	

	Average Annual Return (%)	Min Annual Return (%)	\
0	8.80	-60.25	
1	7.49	-47.19	
2	8.99	-41.97	
3	12.21	-42.41	
4	9.47	-40.77	
5	10.99	-49.41	
6	10.02	-43.44	

	25th Percentile Annual Return (%)	Median Annual Return (%)	\
0	-5.22	10.11	
1	-1.38	10.53	
2	-2.47	13.15	

3	4.80	11.95
4	-0.39	11.81
5	4.57	10.87
6	-3.25	12.77

	75th Percentile Annual Return (%)	Max Annual Return (%)	Years of Data
0	26.34	63.34	22.57
1	20.35	46.46	22.57
2	16.30	32.80	22.57
3	29.68	39.94	22.57
4	18.00	49.42	22.57
5	23.97	40.33	22.57
6	21.87	41.21	22.57

2.2 Volatility

Volatility Statistics:

	Ticker	Total Volatility (%)	Average Rolling Volatility (%) \
0	EM	27.22	24.63
1	INT	20.73	18.94
2	R1KV	19.04	17.06
3	R1KG	19.56	18.04
4	R2K	23.88	22.41
5	RMCG	21.39	19.75
6	RMCV	20.38	18.38

	Period 1 (2003-04 to 2007-10)	Period 2 (2007-10 to 2012-04) \
0	22.94	44.40
1	14.40	33.08
2	12.03	29.28
3	12.42	25.48
4	17.90	33.88
5	14.36	29.72
6	12.24	30.69

	Period 3 (2012-04 to 2016-10)	Period 4 (2016-10 to 2021-05) \
0	19.44	22.34
1	15.99	18.04
2	13.23	20.17
3	13.38	21.33
4	16.42	24.35
5	14.48	21.72
6	13.82	21.98

	Period 5 (2021-05 to 2025-11)
0	18.16

1	16.30
2	15.13
3	21.82
4	22.83
5	22.73
6	17.68

2.3 Sharpe

Sharpe Ratio Statistics:

	Ticker	Total Sharpe Ratio	Period 1 (2003-04 to 2007-10)	Period 2 (2007-10 to 2012-04)	Period 3 (2012-04 to 2016-10)	Period 4 (2016-10 to 2021-05)	Period 5 (2021-05 to 2025-11)
0	EM	0.18	1.37	-0.16	-0.23	0.28	-0.12
1	INT	0.17	1.34	-0.34	0.08	0.32	0.14
2	R1KV	0.26	1.08	-0.23	0.58	0.41	0.19
3	R1KG	0.42	0.71	-0.08	0.58	0.79	0.44
4	R2K	0.23	0.83	-0.10	0.39	0.45	-0.06
5	RMCG	0.33	1.03	-0.08	0.44	0.65	0.09
6	RMCV	0.30	1.32	-0.11	0.61	0.37	0.05

3 Covariance & Correlation

Annual Expected Returns:

ticker	
EM	0.088022
INT	0.074877
R1KV	0.089854
R1KG	0.122106
R2K	0.094748

```
RMCG    0.109859
RMCV    0.100194
dtype: float64

Covariance Matrix:
ticker        EM       INT      R1KV      R1KG       R2K      RMCG      RMCV
ticker
EM      0.074098  0.048820  0.041505  0.041111  0.049181  0.045529  0.043639
INT     0.048820  0.042964  0.034524  0.033313  0.039945  0.036465  0.036055
R1KV    0.041505  0.034524  0.036245  0.032419  0.040861  0.035966  0.037645
R1KG    0.041111  0.033313  0.032419  0.038256  0.039878  0.039483  0.034343
R2K     0.049181  0.039945  0.040861  0.039878  0.057016  0.046438  0.045363
RMCG    0.045529  0.036465  0.035966  0.039483  0.046438  0.045740  0.039478
RMCV    0.043639  0.036055  0.037645  0.034343  0.045363  0.039478  0.041552

Correlation Matrix:
ticker        EM       INT      R1KV      R1KG       R2K      RMCG      RMCV
ticker
EM      1.000000  0.865255  0.800884  0.772152  0.756656  0.782059  0.786448
INT     0.865255  1.000000  0.874862  0.821689  0.807064  0.822576  0.853315
R1KV    0.800884  0.874862  1.000000  0.870616  0.898840  0.883310  0.970025
R1KG    0.772152  0.821689  0.870616  1.000000  0.853859  0.943870  0.861366
R2K     0.756656  0.807064  0.898840  0.853859  1.000000  0.909342  0.931965
RMCG    0.782059  0.822576  0.883310  0.943870  0.909342  1.000000  0.905554
RMCV    0.786448  0.853315  0.970025  0.861366  0.931965  0.905554  1.000000
Asset Correlation Analysis:
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Interpretation:
- Avg Correlation: Mean correlation with other assets (higher = more correlated)
- Mkt-Weighted Avg Corr: Correlation weighted by market cap (more relevant
measure)
- Corr with Portfolio: Correlation with the market portfolio
- Diversification Ratio: Higher = better diversification benefit
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  Ticker  Avg Correlation  Mkt-Weighted Avg Corr  Corr with Portfolio  \
0     EM           0.7939                 0.8114               0.9334
1    INT           0.8408                 0.8420               0.9447
2   R1KV           0.8831                 0.8648               0.9247
3   R1KG           0.8539                 0.8505               0.9111
4    R2K           0.8596                 0.8377               0.8985
5   RMCG           0.8745                 0.8297               0.9349
```

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6   RMCV            0.8848                     0.8577                    0.9231

   Diversification Ratio  Standalone Volatility
0                 1.0714                 0.2722
1                 1.0585                 0.2073
2                 1.0814                 0.1904
3                 1.0976                 0.1956
4                 1.1130                 0.2388
5                 1.0696                 0.2139
6                 1.0833                 0.2038
```

4 Reverse Optimization

How = Σw Works

4.0.1 The Calculation Steps

1. **Σ @ w__mkt** → Multiplies the 7×7 covariance matrix by the 7×1 weight vector → produces a 7×1 vector representing the **marginal contribution to portfolio risk** for each asset
2. **lambda__risk__aversion** × → Scales this by the risk aversion coefficient → converts risk contributions into **expected return premiums**

4.0.2 Equilibrium Intuition

In equilibrium, assets with higher contributions to portfolio risk must offer higher expected returns to compensate investors. The formula says:

- **Assets that are more volatile** (high variance) → higher implied returns
- **Assets that correlate positively with the portfolio** → higher implied returns
- **Assets with larger market weights** → already reflect equilibrium pricing

Note on market weights: Assets with larger market cap weights don't necessarily need higher expected returns - their current weights already represent the market's equilibrium assessment of their risk/return profile. If an asset has a large market cap weight, there are two possible reasons:

1. It offers high expected returns (investors want to hold a lot of it)
2. It has low risk or diversification benefits (it's "safe" to hold a lot of it)

The market weight itself is the **outcome** of equilibrium pricing - it already incorporates all the information about risk and return that investors collectively believe.

4.0.3 Example

If emerging markets (EM) has high volatility and positive correlation with the rest of the portfolio, the formula will assign it a higher implied return to justify investors holding it at its current market weight.

```
Risk Aversion Coefficient ( ): 1.1939
Market Return: 9.3403%
Market Variance: 0.0447
Market Stdev: 0.2115

Implied Equilibrium Returns (from Reverse Optimization):

  Ticker  Market Weight (%)  Implied Return (%)
0   R1KG          26.615087            6.415429
1   R1KV          26.615087            4.944381
2    R2K           2.723404            4.445371
3   RMCG           4.023211            4.499714
4   RMCV           4.023211            5.417163
5    INT          29.032258            5.048777
6     EM           6.967742            4.751485

Individual Asset Sharpe Ratios (Implied Returns):
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  Ticker  Implied Return (%)  Volatility (%)  Sharpe Ratio  Market Weight (%)
0   R1KG              6.4154         27.2209        0.0887            26.6151
4   RMCV              5.4172         23.8781        0.0593             4.0232
5    INT              5.0488         21.3869        0.0490            29.0323
1   R1KV              4.9444         20.7278        0.0456            26.6151
6     EM              4.7515         20.3844        0.0369             6.9677
3   RMCG              4.4997         19.5592        0.0255             4.0232
2    R2K              4.4454         19.0382        0.0234             2.7234

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Highest Sharpe: R1KG = 0.0887
Lowest Sharpe: R2K = 0.0234
Sharpe Gap: 0.0653

Unconstrained mean-variance optimization allocates 100% to the highest Sharpe
asset
when diversification benefits don't outweigh the risk-adjusted return advantage.
```

4.1 Scalar vs. Asset-Specific Risk Aversion

The choice between scalar vs. array risk aversion involves important tradeoffs:

4.1.1 Pro:

- Captures heterogeneous investor risk preferences across asset classes (e.g., institutions may be more risk-averse to emerging markets than domestic equities)
- Can reflect market segmentation where different investor bases dominate different assets
- Allows modeling of risk budgeting frameworks where risk tolerance varies by asset category

4.1.2 Cons:

- No clear market observable to estimate individual values - you'd need to make subjective assumptions
- Loss of interpretability: a single has clear economic meaning (market price of risk), but asset-specific values are harder to justify
- Violates the assumption of a representative investor with consistent preferences across all assets
- In standard CAPM/equilibrium theory, all investors face the same efficient frontier, implying a single market-wide risk aversion
- Can lead to arbitrage opportunities in theory (why would rational investors have different risk aversion for the same dollar of risk?)

The Black-Litterman model already allows you to express asset-specific views through the P and Q matrices. If you believe certain assets deserve different treatment, you can express that through higher-confidence views rather than modifying the equilibrium framework.

The power of Black-Litterman is separating equilibrium (market consensus) from your subjective views. Using asset-specific risk aversion muddies this distinction and essentially bakes your views into the "neutral" equilibrium, which defeats the model's elegance.

```
Weight Verification Test:


  Ticker  Original Market Weight  Calculated Weight    Difference  \
0    IWF                0.266151           0.266151  1.039022e-07
1    IWD                0.266151           0.266150 -5.031852e-07
2    IWM                0.027234           0.027240  5.628593e-06
3    IWP                0.040232           0.040230 -1.928512e-06
4    IWS                0.040232           0.040234  2.196299e-06
5    EFA                0.290323           0.290325  2.041681e-06
6    EEM                0.069677           0.069670 -7.538778e-06

   Absolute Error
0    1.039022e-07
1    5.031852e-07
2    5.628593e-06
3    1.928512e-06
4    2.196299e-06
5    2.041681e-06
6    7.538778e-06


Maximum Absolute Error: 0.000008
Mean Absolute Error: 0.000003
Test PASSED (threshold: 0.001)
```

5 Investor Views

```
P Matrix (View Picks):
   R1KG  R1KV  R2K  RMCG  RMCV  INT   EM
0   1.0  -1.0  0.0   0.0   0.0  0.0  0.0
1   0.6   0.4  0.0   0.0   0.0 -1.0  0.0
2   0.6   0.4 -1.0   0.0   0.0  0.0  0.0
3   0.0   0.0 -1.0   0.6   0.4  0.0  0.0
4   0.0   0.0  0.0   0.0   0.0 -1.0  1.0

Q Vector (View Returns):
[0.05  0.04  0.05  0.025 0.03 ]

Omega Matrix (View Uncertainties):
[[0.02589538 0.         0.         0.         0.        ]
 [0.         0.02522139 0.         0.         0.        ]
 [0.         0.         0.02107162 0.         0.        ]
 [0.         0.         0.         0.00891987 0.        ]
 [0.         0.         0.         0.         0.01111388]]
```

6 Black-Litterman Expected Returns

Combining Equilibrium Returns with Investor Views

6.0.1 The Black-Litterman Formula

$$E[R] = [(\tau\Sigma)^{-1} + P'\Omega^{-1}P]^{-1}[(\tau\Sigma)^{-1}\pi + P'\Omega^{-1}Q]$$

Where: - (tau): Scalar representing uncertainty in the prior equilibrium (typically 0.025-0.05) - **Σ** (Sigma): Covariance matrix - **P**: Matrix picking out assets in each view - **Ω** (Omega): Diagonal matrix of view uncertainties - (pi): Implied equilibrium returns from reverse optimization - **Q**: Vector of expected returns according to investor views

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6.0.2 The Parameter

`tau = 0.025` controls how much you trust the market equilibrium returns ( ):

- **Smaller** (0.01-0.025): High confidence in equilibrium → views have **less** influence
- **Larger** (0.04-0.1): Low confidence in equilibrium → views have **more** influence

Mathematical effect: Larger → less precision in (Σ) 1 → equilibrium estimates treated as more uncertain → more weight given to investor views (Q) relative to equilibrium ( )

6.0.3 Interpretation

The posterior expected returns are a **precision-weighted average** of: 1. Market equilibrium beliefs ( ) - what the market collectively thinks 2. Your subjective views (Q) - what you believe will happen

The weighting is determined by both and the confidence levels in your views.

Black-Litterman Expected Returns:

	Ticker	Market Weight (%)	Implied Return (%)	BL Expected Return (%) \
0	R1KG	26.615087	6.415429	6.867654
1	R1KV	26.615087	4.944381	5.145896
2	R2K	2.723404	4.445371	4.544507
3	RMCG	4.023211	4.499714	4.585208
4	RMCV	4.023211	5.417163	5.559484
5	INT	29.032258	5.048777	5.113059
6	EM	6.967742	4.751485	4.872743

	Return Adjustment (%)
0	0.452225
1	0.201515
2	0.099136
3	0.085494
4	0.142321
5	0.064282
6	0.121257

6.0.4 Column Descriptions

- **Market Weight (%)**: Current market capitalization weight
  - Derived from free float-adjusted market caps; Reflects the "consensus" allocation across all investors
- **Implied Return (%)**: Equilibrium returns from reverse optimization
  - Calculated using = Σw; What the market "believes" returns should be to justify current weights; Represents the **prior** (before incorporating your views)
- **BL Expected Return (%)**: Black-Litterman posterior returns
  - Precision-weighted blend of implied returns ( ) and your views (Q); Represents the **posterior** (after incorporating your views)
- **Return Adjustment (%)**: BL Expected Return - Implied Return
  - Shows how much your views moved each asset's expected return
  - Positive = your views increased expected return vs. equilibrium; Negative = your views decreased expected return vs. equilibrium; Magnitude depends on view confidence and parameter

7 Mean Variance Optimization with BL Expected Returns

```
Using Option 3: Market cap constraints (±10%) + minimum 2.5% allocation
(maximize Sharpe ratio)
Optimization successful: True

Market Portfolio (BL Returns):
  Return: 5.55%
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  Volatility: 21.15%
  Sharpe Ratio: 0.0734

Optimal BL Portfolio:
  Return: 5.90%
  Volatility: 22.42%
  Sharpe Ratio: 0.0848

Portfolio Allocation Comparison:

  Ticker  Market Weight (%)  BL Optimal Weight (%)  Weight Change (%)  \
0   R1KG            26.6151                41.6151            15.0000
1   R1KV            26.6151                17.8294            -8.7856
2    R2K             2.7234                 2.5000            -0.2234
3   RMCG             4.0232                 2.5000            -1.5232
4   RMCV             4.0232                19.0232            15.0000
5    INT            29.0323                14.0323           -15.0000
6     EM             6.9677                 2.5000            -4.4677

   Implied Return (%)  BL Expected Return (%)
0             6.4154                  6.8677
1             4.9444                  5.1459
2             4.4454                  4.5445
3             4.4997                  4.5852
4             5.4172                  5.5595
5             5.0488                  5.1131
6             4.7515                  4.8727

Maximum Sharpe Ratio Portfolio (Unconstrained Long-Only):
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Return: 0.0687 (6.87%)
Volatility: 0.2722 (27.22%)
Sharpe Ratio: 0.1053
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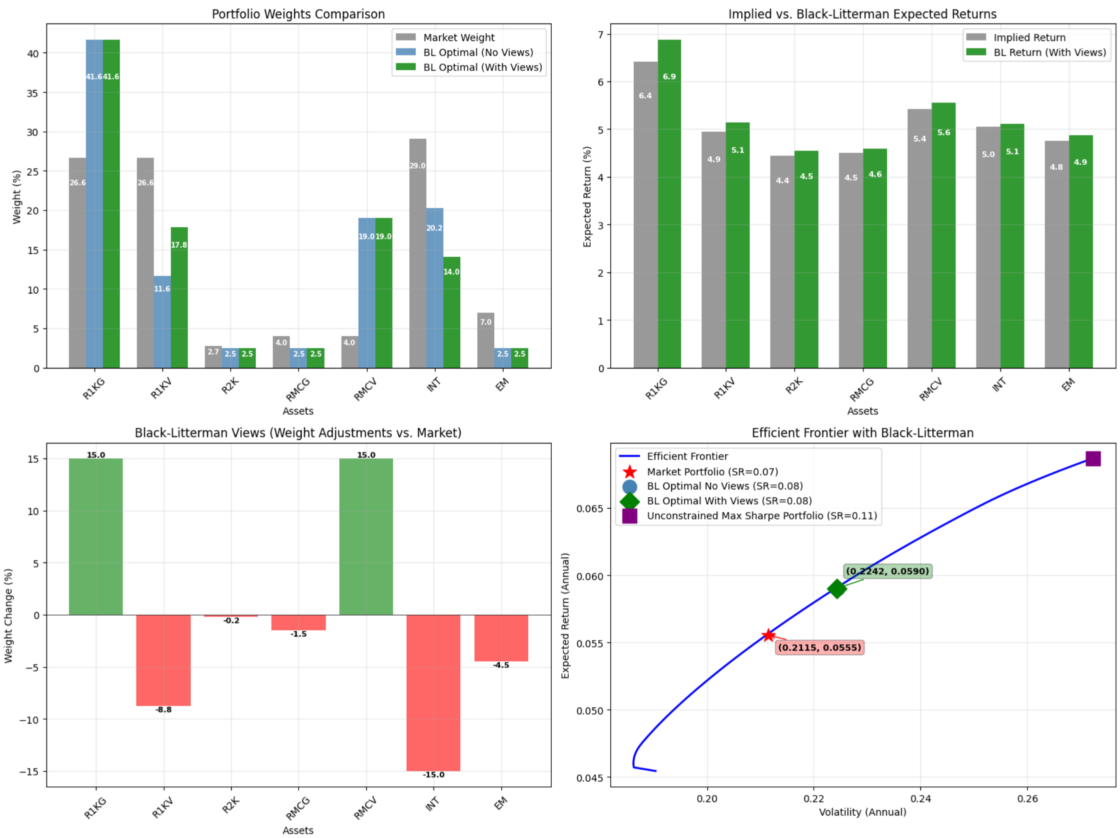
  Ticker  Max Sharpe Weight (%)  Market Weight (%)  BL Optimal Weight (%)  \
0   R1KG                  100.0              26.62                  41.62
1   R1KV                    0.0              26.62                  17.83
2    R2K                    0.0               2.72                   2.50
3   RMCG                    0.0               4.02                   2.50
4   RMCV                    0.0               4.02                  19.02
5    INT                    0.0              29.03                  14.03
6     EM                    0.0               6.97                   2.50

   Difference from Market (%)  Difference from BL Optimal (%)
0                       73.38                           58.38
```

1	-26.62	-17.83
2	-2.72	-2.50
3	-4.02	-2.50
4	-4.02	-19.02
5	-29.03	-14.03
6	-6.97	-2.50

8 BL Plot



9 Risk Analytics

Comprehensive risk analysis of the optimal Black-Litterman portfolio including: - **Risk Contribution**: Marginal and component risk by asset - **Tracking Error**: Deviation from market portfolio - **Downside Risk**: Semi-deviation and tail risk metrics - **Concentration Risk**: Portfolio diversification measures

```
Risk Contribution Analysis:
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Portfolio Volatility: 0.2242
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Tracking Error vs Market: 0.0256
Information Ratio: 0.1355
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  Ticker  Weight (%)  Marginal Risk  Component Risk  Risk Contribution (%)  \
0   R1KG     41.6151         0.2606          0.1085                48.3635
1   R1KV     17.8294         0.1930          0.0344                15.3487
2    R2K      2.5000         0.1735          0.0043                 1.9342
3   RMCG      2.5000         0.1730          0.0043                 1.9289
4   RMCV     19.0232         0.2145          0.0408                18.1976
5    INT     14.0323         0.1943          0.0273                12.1592
6     EM      2.5000         0.1855          0.0046                 2.0680

   Return Contribution (%)  Risk/Return Ratio  Active Weight (%)  \
0                  48.4354             3.7948            15.0000
1                  15.5490             3.7514            -8.7856
2                   1.9254             3.8176            -0.2234
3                   1.9427             3.7735            -1.5232
4                  17.9235             3.8585            15.0000
5                  12.1594             3.8003           -15.0000
6                   2.0645             3.8067            -4.4677

   TE Contribution (%)
0             107.4630
1             -25.0838
2              -0.5682
3              -2.5739
4              62.3371
5             -29.3480
6             -12.2262

Verification: Sum of Component Risks = 0.224247 (should equal portfolio vol =
0.224247)
Verification: Sum of Risk Contributions = 100.00% (should equal 100%)

Downside Risk Analysis:
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                  Metric  BL Optimal Portfolio  Market Portfolio  Difference
0  Semi-Deviation (Annual)               0.1799            0.1721      0.0078
1           Sortino Ratio               0.1057            0.0903      0.0154
2        VaR 95% (Annual)              -0.3211           -0.3040     -0.0172
3        VaR 99% (Annual)              -0.6319           -0.6149     -0.0170
4       CVaR 95% (Annual)              -0.5390           -0.5123     -0.0267
5       CVaR 99% (Annual)              -0.9806           -0.9326     -0.0480
6        Maximum Drawdown              -0.6606           -0.6481     -0.0124
7   Downside Capture Ratio               1.0632            1.0000      0.0632
```

```
Interpretation:
- Lower semi-deviation = less downside volatility
- Higher Sortino ratio = better risk-adjusted returns considering only downside
risk
- VaR/CVaR = potential losses at different confidence levels
- Lower maximum drawdown = smaller peak-to-trough decline
- Downside capture < 1.0 = portfolio declines less than market on down days

Concentration Risk Analysis:
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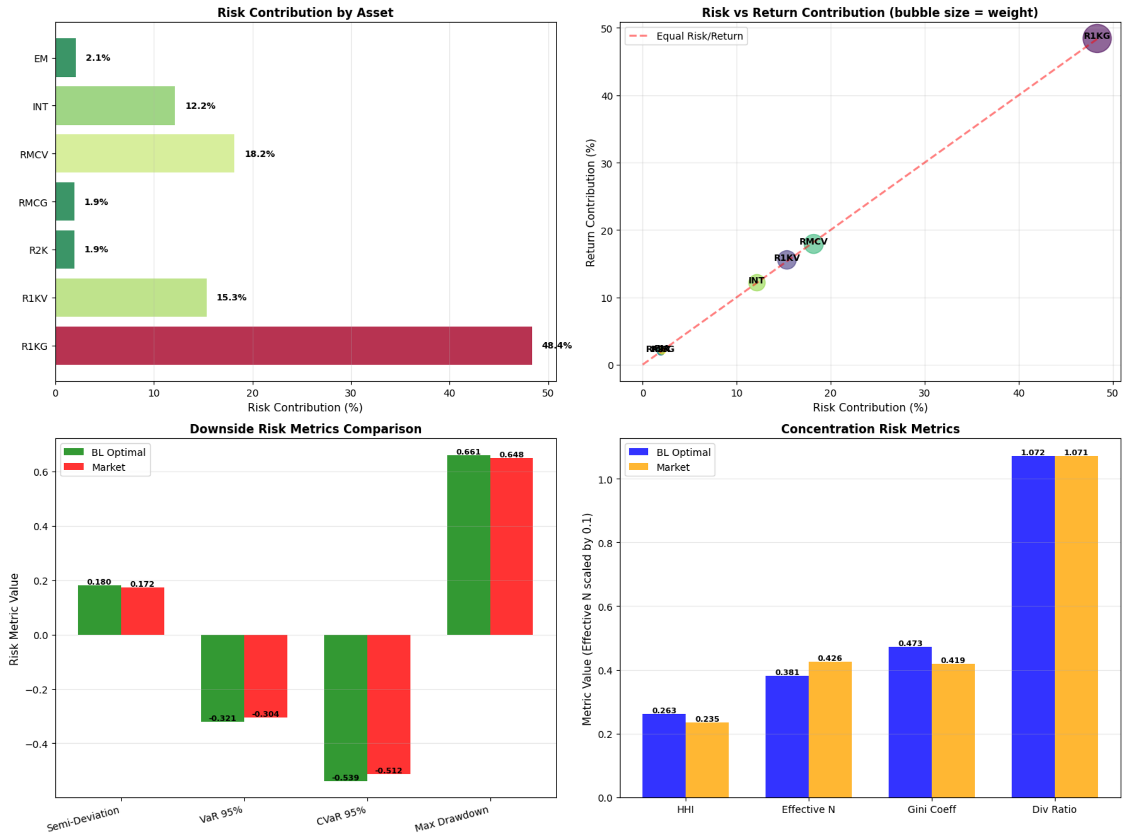
                       Metric  BL Optimal Portfolio  Market Portfolio  \
0  Herfindahl-Hirschman Index                0.2627            0.2348
1  Effective Number of Assets                3.8063            4.2591
2            Gini Coefficient                0.4735            0.4191
3       Diversification Ratio                1.0721            1.0713
4       Maximum Single Weight                0.4162            0.2903
5  Top 3 Assets Concentration                0.7847            0.8226
6            Number of Assets                7.0000            7.0000

   Difference
0      0.0279
1     -0.4528
2      0.0543
3      0.0008
4      0.1258
5     -0.0379
6      0.0000


Interpretation:
- Lower HHI = more diversified portfolio
- Higher effective N = more evenly distributed weights
- Lower Gini = more equal weight distribution
- Higher diversification ratio = better diversification benefits
- Lower top 3 concentration = less reliance on few positions
```


Risk Contribution by Asset
EM
INT
RMCV
RMCG
R2K
R1KV
R1KG
2.1%
12.2%
18.2%
1.9%
1.9%
15.3%
48.4%
Risk Contribution (%)
Risk vs Return Contribution (bubble size = weight)
Equal Risk/Return
R1KG
RMCV
R1KV
INT
Return Contribution (%)
Risk Contribution (%)
Downside Risk Metrics Comparison
BL Optimal
Market
Risk Metric Value
0.180
0.172
-0.321
-0.304
-0.539
-0.512
0.661
0.648
Semi-Deviation
VaR 95%
CVaR 95%
Max Drawdown
Concentration Risk Metrics
BL Optimal
Market
Metric Value (Effective N scaled by 0.1)
0.263
0.235
0.381
0.426
0.473
0.419
1.072
1.071
HHI
Effective N
Gini Coeff
Div Ratio