index_style_regressions

November 23, 2025

style box

regressions

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The autoreload extension is already loaded. To reload it, use:
  %reload_ext autoreload
```

1 Data Retrieval

```
ticker           ACWI        EM       INT      R1KV      R1KG       R2K  \
date
2008-03-31 -0.015691  0.005898  0.003622  0.010400  0.000552  0.001464
2008-04-01  0.026609  0.037679  0.031084  0.031494  0.032528  0.035956
2008-04-02  0.010918  0.000787  0.002154 -0.000923 -0.001422  0.002399
2008-04-03  0.005725  0.012031  0.002957  0.003163  0.001956  0.002674
2008-04-04 -0.000621 -0.004043  0.003348 -0.004353  0.004078  0.000139
…               …       …       …       …       …       …
2025-11-17 -0.010766 -0.011897 -0.013102 -0.010850 -0.009473 -0.020076
2025-11-18 -0.008624 -0.004984 -0.013061 -0.001235 -0.012142  0.003046
2025-11-19  0.001382 -0.003522 -0.003022 -0.001236  0.007372 -0.000171
2025-11-20 -0.015234 -0.013837 -0.013931 -0.010646 -0.020279 -0.018680
2025-11-21  0.009841 -0.000188  0.015336  0.015386  0.005808  0.027933

ticker           RMCG       RMC      RMCV       R3K     SP500
date
2008-03-31  0.006533  0.010530  0.012205  0.005528  0.003492
2008-04-01  0.033300  0.031672  0.036046  0.033801  0.034556
2008-04-02  0.005241  0.005677  0.003745 -0.000128  0.000659
2008-04-03  0.003128  0.003189  0.002983  0.002661  0.002483
2008-04-04  0.008203  0.003685  0.001340  0.000759 -0.001095
…               …       …       …       …       …
2025-11-17 -0.018960 -0.016410 -0.016088 -0.010302 -0.009360
2025-11-18 -0.000372  0.001181  0.002209 -0.007168 -0.008433
2025-11-19  0.003867 -0.000859 -0.003094  0.003391  0.003856
2025-11-20 -0.022669 -0.015368 -0.012473 -0.015850 -0.015360
2025-11-21  0.013947  0.019227  0.020923  0.012138  0.009912

[4439 rows x 11 columns]
```

2 Regression Functions

3 Results

3.1 Methodology

This analysis employs **Returns-Based Style Analysis (RBSA)**, a technique introduced by William Sharpe (1992) to decompose the returns of a portfolio or index into the returns of various style factors or asset classes.

3.1.1 Regression Approaches

We implement two complementary regression methodologies:

1. Unconstrained Regression (Standard OLS) The unconstrained approach uses ordinary least squares regression with an intercept:

$$R_t = \alpha + \sum_{i=1}^{n} \beta_i F_{i,t} + \varepsilon_t$$

Characteristics: - Betas represent **factor sensitivities** or exposures - Sum of betas is **not constrained** to equal 1 - Includes an **intercept term** (alpha) - Provides **statistical significance tests** (t-tests, F-test, p-values) - Betas can be positive or negative

Interpretation: This approach measures how sensitive the dependent variable is to movements in each independent variable. A beta of 0.5 means a 1% change in the factor is associated with a 0.5% change in the dependent variable.

2. Constrained Regression (Portfolio Decomposition) The constrained approach uses optimization to find portfolio weights that best replicate the dependent variable:

$$R_t = \sum_{i=1}^{n} \beta_i F_{i,t} + \varepsilon_t$$

Subject to: $\sum_{i=1}^{n} \beta_i = 1$

Optional: $\beta_i \geq 0$ (long-only constraint)

Characteristics: - Betas represent **portfolio weights** or allocation percentages - Sum of betas **equals exactly 1** (100%) - **No intercept** term (forced to zero) - No statistical significance tests (optimization-based, not statistical) - Betas can be negative (long/short) unless `long_only=True`

Interpretation: This approach decomposes the dependent variable into a weighted combination of factors. A beta of 0.6 means 60% of the portfolio/index can be represented by that factor.

3.1.2 Rolling Window Analysis

For each approach, we conduct **rolling regressions** using: - **Window size:** Specified number of years of historical data - **Step size:** Frequency of regression updates (daily, weekly, monthly) - **Output:** Time series of betas, R-squared, and other metrics

This allows us to observe how factor exposures or portfolio composition change over time.

3.1.3 Analytical Framework

Static Regression: - Uses a fixed time period (e.g., last 3 years) - Provides point-in-time estimate of factor exposures - Includes comprehensive statistical tests

Rolling Regression: - Continuously updates as the window moves forward - Reveals temporal dynamics and regime changes - Visualized through four key charts: 1. **Rolling Betas:** Factor exposures over time 2. **R-squared:** Model fit quality over time 3. **Beta Sum:** Validation metric (should equal 1 for constrained) 4. **Cumulative Returns:** Actual performance of all variables

3.1.4 Model Validation

- **R-squared:** Measures proportion of variance explained (0 to 1)
- **Adjusted R-squared:** Penalizes for number of factors
- **F-statistic:** Tests overall model significance (unconstrained only)
- **P-values:** Test individual coefficient significance (unconstrained only)
- **Beta Sum:** Verification that constraint is satisfied (constrained only)

3.2 S&P 500 by Value and Growth

3.2.1 Unconstrained

```
======================================================================
Style Regression Results: SP500
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Model Summary:
```

	Metric	Value
0	Dependent Variable	SP500
1	Model Type	Unconstrained OLS
2	Period Start	2022-11-21
3	Period End	2025-11-21
4	Observations	752
5	R-squared	0.993651
6	Adjusted R-squared	0.993634
7	F-statistic	58611.114634
8	F-statistic p-value	1.110223e-16
9	Intercept	0.000008
10	Intercept p-value	0.784406

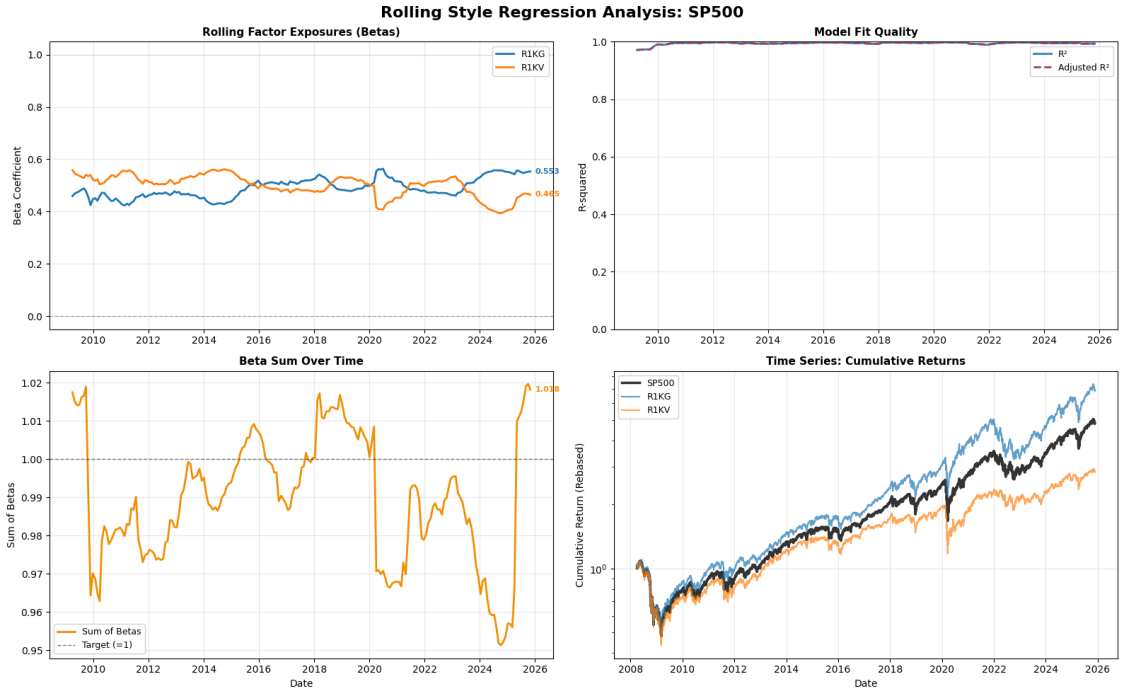
```
Coefficients:
```

	Variable	Beta	P-value	Significance
0	R1KG	0.55119	0.0	***
1	R1KV	0.43860	0.0	***

```
Significance levels: * p<0.05, ** p<0.01, *** p<0.001
======================================================================

            n_obs  rsquared  adj_rsquared  R1KG_beta  R1KG_pval  R1KV_beta  \
date
2025-07-03    251  0.993244      0.993189   0.550067        0.0   0.462399
2025-08-04    250  0.993318      0.993264   0.547503        0.0   0.467928
2025-09-03    251  0.993395      0.993341   0.550108        0.0   0.469001
2025-10-03    251  0.993204      0.993150   0.552840        0.0   0.466821
2025-10-31    251  0.993251      0.993197   0.553398        0.0   0.464770

            R1KV_pval
date
2025-07-03        0.0
2025-08-04        0.0
2025-09-03        0.0
2025-10-03        0.0
2025-10-31        0.0
```



3.2.2 Constrained

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======================================================================
Style Regression Results: SP500
```

(Constrained Regression: Betas sum to 1)

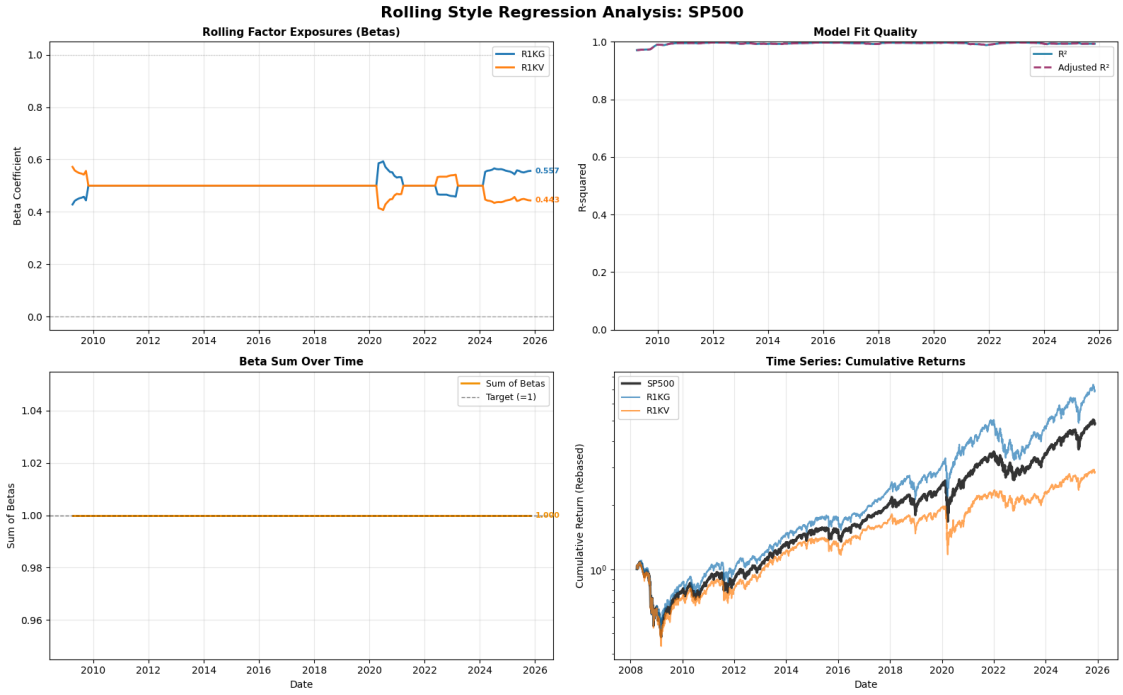
Model Summary:

	Metric	Value
0	Dependent Variable	SP500
1	Model Type	Constrained (Σ =1)
2	Period Start	2022-11-21
3	Period End	2025-11-21
4	Observations	752
5	R-squared	0.993572
6	Adjusted R-squared	0.993563
7	Sum of Betas	1.000000

Coefficients:

	Variable	Beta	P-value	Significance
0	R1KG	0.551008	None	
1	R1KV	0.448992	None	

date	n_obs	rsquared	adj_rsquared	R1KG_beta	R1KV_beta
2025-07-03	251	0.993143	0.993115	0.552142	0.447858
2025-08-04	250	0.993162	0.993134	0.550531	0.449469
2025-09-03	251	0.993152	0.993125	0.553149	0.446851
2025-10-03	251	0.992943	0.992914	0.555648	0.444352
2025-10-31	251	0.993032	0.993004	0.556518	0.443482



3.3 Russell Mid Cap by Russell Mid Cap Value and Russell Mid Cap Growth

3.3.1 Unconstrained

```
======================================================================
Style Regression Results: RMC
======================================================================
```

Model Summary:

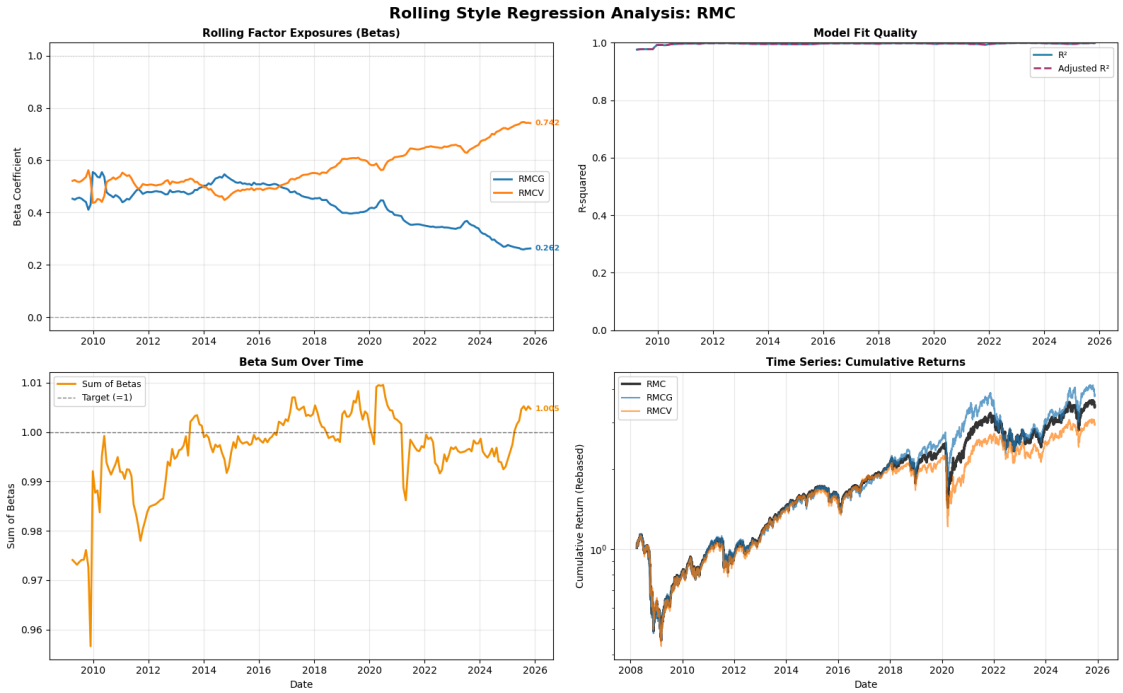
	Metric	Value
0	Dependent Variable	RMC
1	Model Type	Unconstrained OLS
2	Period Start	2022-11-21
3	Period End	2025-11-21
4	Observations	752
5	R-squared	0.997756
6	Adjusted R-squared	0.997750
7	F-statistic	166482.890701
8	F-statistic p-value	1.110223e-16
9	Intercept	0.000012
10	Intercept p-value	0.503511

Coefficients:

```
  Variable      Beta  P-value Significance
0     RMCG  0.282092      0.0          ***
1     RMCV  0.715887      0.0          ***

Significance levels: * p<0.05, ** p<0.01, *** p<0.001
======================================================================
```

	n_obs	rsquared	adj_rsquared	RMCG_beta	RMCG_pval	RMCV_beta	RMCV_pval
date							
2025-07-03	251	0.998429	0.998417	0.259623	0.0	0.745001	0.0
2025-08-04	250	0.998478	0.998465	0.258713	0.0	0.746534	0.0
2025-09-03	251	0.998564	0.998552	0.261288	0.0	0.743172	0.0
2025-10-03	251	0.998658	0.998647	0.261863	0.0	0.743350	0.0
2025-10-31	251	0.998747	0.998737	0.262451	0.0	0.742274	0.0



3.3.2 Constrained

```
======================================================================
Style Regression Results: RMC
 (Constrained Regression: Betas sum to 1)
======================================================================

Model Summary:
```

	Metric	Value
0	Dependent Variable	RMC
1	Model Type	Constrained (Σ =1)
2	Period Start	2022-11-21
3	Period End	2025-11-21
4	Observations	752
5	R-squared	0.997751
6	Adjusted R-squared	0.997748
7	Sum of Betas	1.000000

Coefficients:

	Variable	Beta	P-value	Significance
0	RMCG	0.281796	None	
1	RMCV	0.718204	None	

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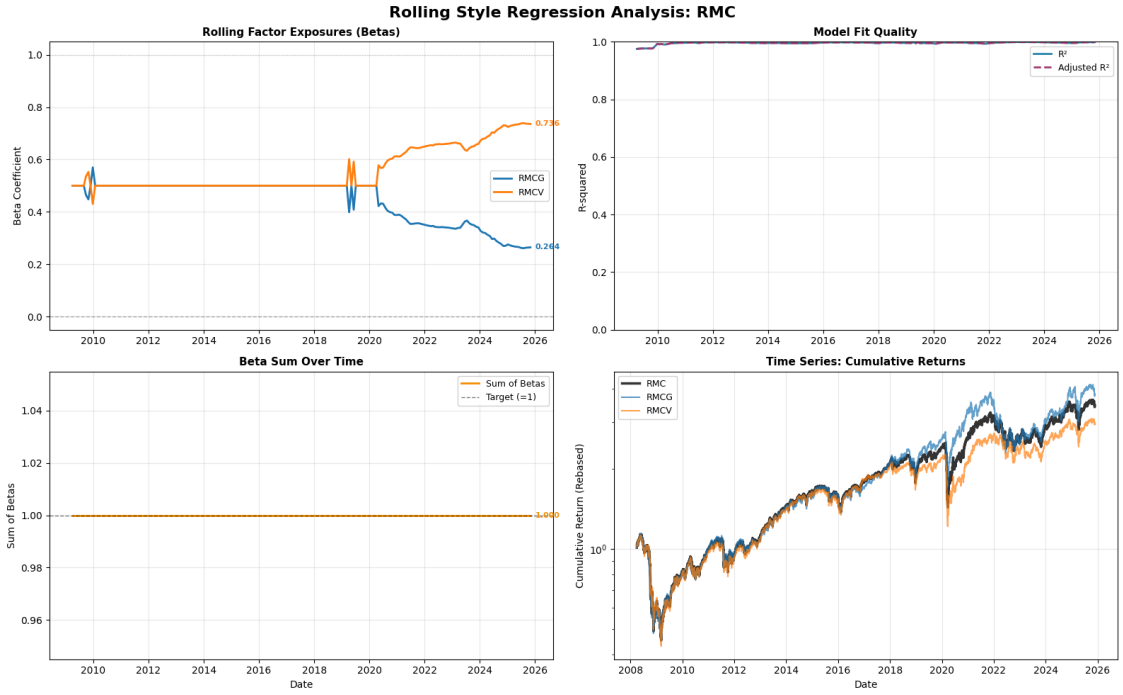
```

date	n_obs	rsquared	adj_rsquared	RMCG_beta	RMCV_beta
2025-07-03	251	0.998412	0.998406	0.261775	0.738225
2025-08-04	250	0.998456	0.998450	0.261037	0.738963
2025-09-03	251	0.998547	0.998542	0.262975	0.737025
2025-10-03	251	0.998636	0.998630	0.263697	0.736303
2025-10-31	251	0.998728	0.998723	0.264219	0.735781



3.4 Russell 3000 by S&P500, Russell Mid Cap, and Russell 2000

3.4.1 Unonstrained

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======================================================================
Style Regression Results: R3K
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Model Summary:
```

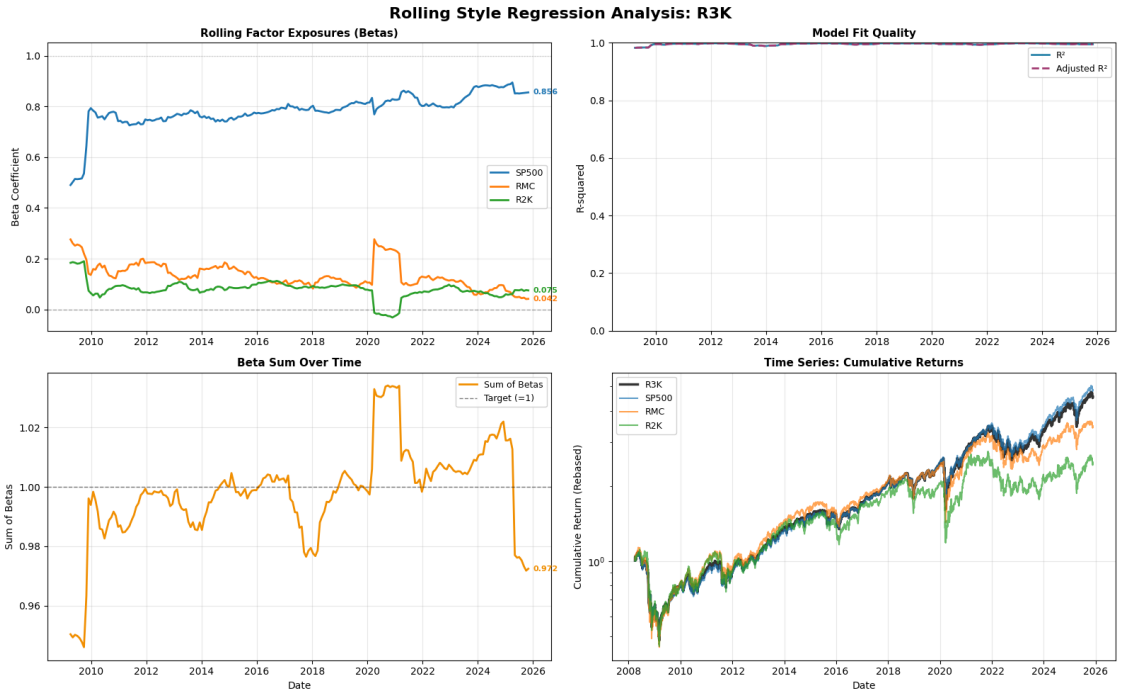
	Metric	Value
0	Dependent Variable	R3K
1	Model Type	Unconstrained OLS
2	Period Start	2022-11-21
3	Period End	2025-11-21
4	Observations	752
5	R-squared	0.995310
6	Adjusted R-squared	0.995291
7	F-statistic	52916.159017
8	F-statistic p-value	1.110223e-16
9	Intercept	0.000017
10	Intercept p-value	0.480418

```
Coefficients:
```

```
  Variable      Beta       P-value Significance
0    SP500  0.861737  0.000000e+00          ***
1      RMC  0.061480  2.659895e-11          ***
2      R2K  0.068139  0.000000e+00          ***

Significance levels: * p<0.05, ** p<0.01, *** p<0.001
======================================================================
```

date	n_obs	rsquared	adj_rsquared	SP500_beta	SP500_pval	RMC_beta	RMC_pval	R2K_beta	R2K_pval
2025-07-03	251	0.994584	0.994518	0.851522	0.0	0.049534	0.013215	0.075302	1.237099e-10
2025-08-04	250	0.994475	0.994408	0.852575	0.0	0.044237	0.028798	0.078452	3.159133e-10
2025-09-03	251	0.994461	0.994394	0.853696	0.0	0.046401	0.015201	0.073171	9.837204e-10
2025-10-03	251	0.994435	0.994367	0.854757	0.0	0.040849	0.030931	0.076181	2.953870e-10
2025-10-31	251	0.994598	0.994533	0.855760	0.0	0.041671	0.022083	0.074984	1.915810e-10



3.4.2 Constrained

```
======================================================================
Style Regression Results: R3K
 (Constrained Regression: Betas sum to 1)
======================================================================

Model Summary:
```

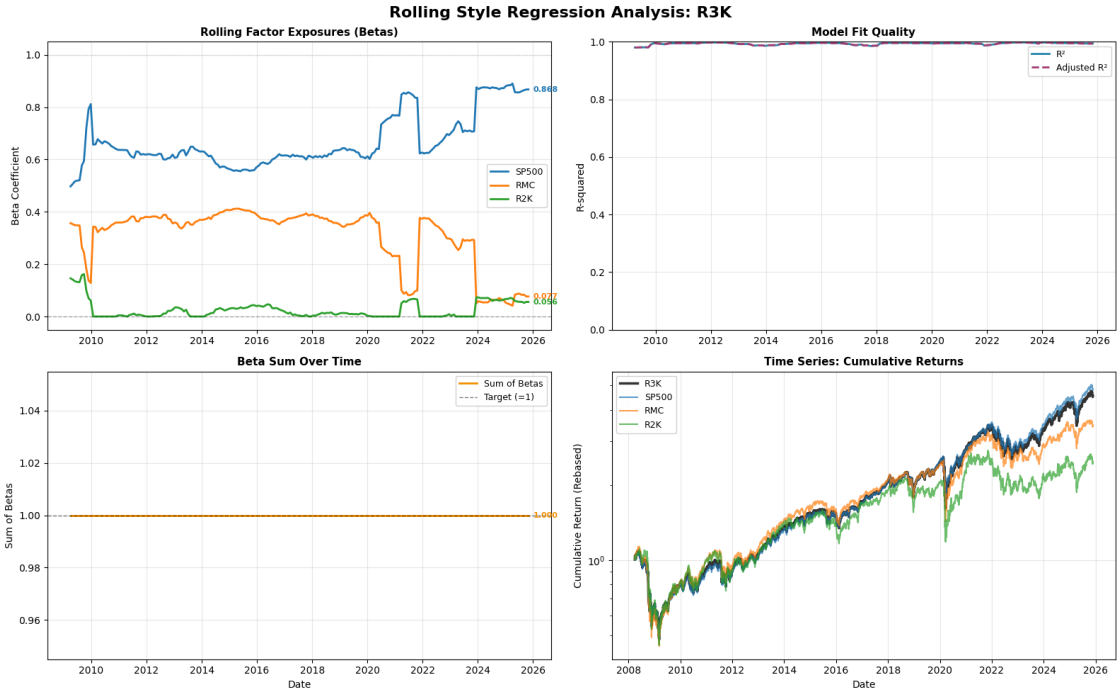
	Metric	Value
0	Dependent Variable	R3K
1	Model Type	Constrained (Σ =1)
2	Period Start	2022-11-21
3	Period End	2025-11-21
4	Observations	752
5	R-squared	0.995242
6	Adjusted R-squared	0.995229
7	Sum of Betas	1.000000

Coefficients:

	Variable	Beta	P-value	Significance
0	SP500	0.869437		None
1	RMC	0.069156		None
2	R2K	0.061407		None

```
======================================================================
```

date	n_obs	rsquared	adj_rsquared	SP500_beta	RMC_beta	R2K_beta
2025-07-03	251	0.994101	0.994054	0.856429	0.087577	0.055995
2025-08-04	250	0.993930	0.993881	0.861139	0.082624	0.056236
2025-09-03	251	0.993811	0.993761	0.864850	0.083165	0.051985
2025-10-03	251	0.993698	0.993647	0.867684	0.076622	0.055694
2025-10-31	251	0.993901	0.993852	0.867885	0.076540	0.055575



3.5 MSCI ACWI by S&P500, Russell Mid Cap, Russell 2000, EM, and EAFE

3.5.1 Unconstrained

```
======================================================================
Style Regression Results: ACWI
======================================================================
```

Model Summary:

	Metric	Value
0	Dependent Variable	ACWI
1	Model Type	Unconstrained OLS
2	Period Start	2022-11-21
3	Period End	2025-11-21
4	Observations	752
5	R-squared	0.996580
6	Adjusted R-squared	0.996557
7	F-statistic	43474.482852
8	F-statistic p-value	1.110223e-16
9	Intercept	0.000014
10	Intercept p-value	0.463290

Coefficients:

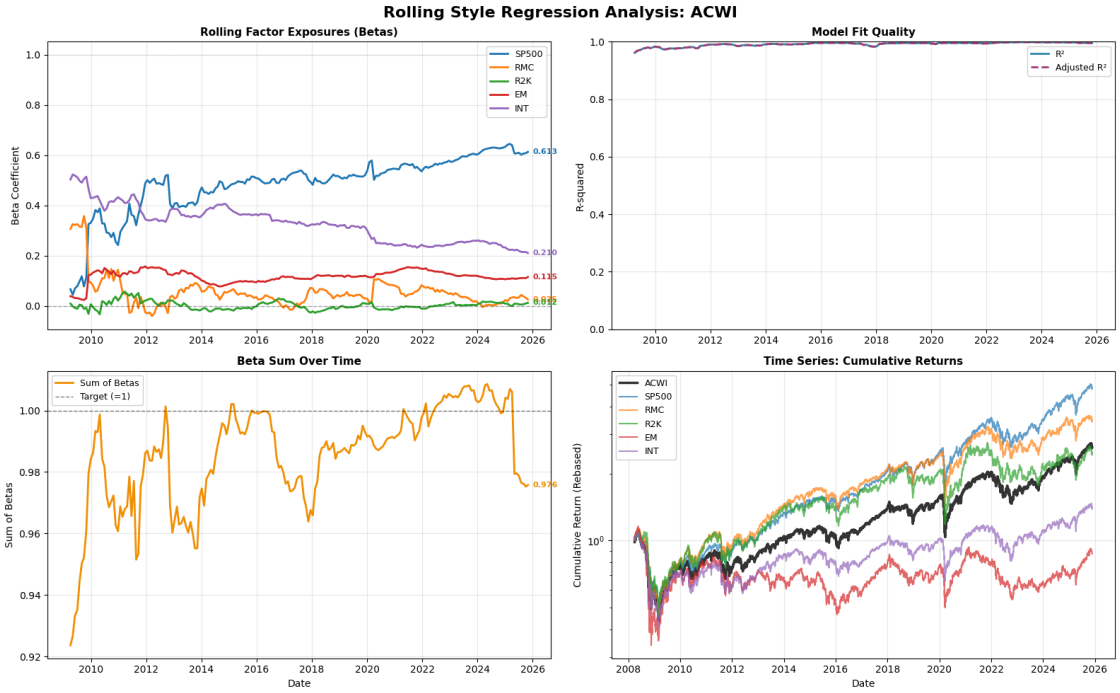
```
  Variable      Beta   P-value Significance
0    SP500  0.614050  0.000000          ***
1      RMC  0.016425  0.024410            *
2      R2K  0.012703  0.002192           **
3       EM  0.115587  0.000000          ***
4      INT  0.231927  0.000000          ***

Significance levels: * p<0.05, ** p<0.01, *** p<0.001
======================================================================


            n_obs  rsquared  adj_rsquared  SP500_beta  SP500_pval  RMC_beta  \
date
2025-07-03    251  0.995930      0.995847    0.609601         0.0  0.033487
2025-08-04    250  0.995802      0.995716    0.602219         0.0  0.043715
2025-09-03    251  0.995575      0.995484    0.606626         0.0  0.037413
2025-10-03    251  0.995297      0.995201    0.608055         0.0  0.032425
2025-10-31    251  0.995409      0.995315    0.613322         0.0  0.025409

            RMC_pval  R2K_beta  R2K_pval   EM_beta  EM_pval  INT_beta  \
date
2025-07-03  0.037015  0.006568  0.461046  0.109943      0.0  0.219215
2025-08-04  0.007513  0.004943  0.602870  0.110481      0.0  0.215083
2025-09-03  0.018107  0.006738  0.474038  0.111012      0.0  0.214415
2025-10-03  0.043837  0.009576  0.324068  0.110442      0.0  0.214787
2025-10-31  0.103157  0.012076  0.205333  0.114811      0.0  0.210139

            INT_pval
date
2025-07-03       0.0
2025-08-04       0.0
2025-09-03       0.0
2025-10-03       0.0
2025-10-31       0.0
```



3.5.2 Constrained

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======================================================================
Style Regression Results: ACWI
 (Constrained Regression: Betas sum to 1)
======================================================================

Model Summary:

                Metric                Value
0  Dependent Variable                 ACWI
1          Model Type  Constrained (Σ =1)
2        Period Start           2022-11-21
3          Period End           2025-11-21
4        Observations                  752
5           R-squared             0.996492
6  Adjusted R-squared             0.996474
7        Sum of Betas             1.000000

Coefficients:

  Variable      Beta P-value Significance
0    SP500  0.615653    None
1      RMC  0.031144    None
```

```
2       R2K  0.002211    None
3        EM  0.119184    None
4       INT  0.231807    None
```

```
==========================================================================
```

```
            n_obs  rsquared  adj_rsquared  SP500_beta  RMC_beta      R2K_beta  \
date
2025-07-03    251  0.995534      0.995461    0.613406  0.044160  0.000000e+00
2025-08-04    250  0.995292      0.995215    0.608195  0.053898  0.000000e+00
2025-09-03    251  0.995058      0.994977    0.615368  0.046497  0.000000e+00
2025-10-03    251  0.994748      0.994662    0.612137  0.049154  3.832787e-19
2025-10-31    251  0.994905      0.994822    0.613310  0.044093  0.000000e+00

             EM_beta  INT_beta
date
2025-07-03  0.109300  0.233134
2025-08-04  0.117165  0.220742
2025-09-03  0.115366  0.222769
2025-10-03  0.112826  0.225883
2025-10-31  0.123617  0.218979
```