

blacklitterman

November 23, 2025

0.1 Black-Litterman Model Summary

0.1.1 Methodology

1. **Reverse Optimization:** Derived implied equilibrium returns ($\hat{\mu}$) from market cap weights using the formula $\hat{\mu} = \Sigma w$
 - Calculated risk aversion coefficient (ρ) from market portfolio characteristics
 - These represent what the market “believes” about future returns
2. **Investor Views:** Defined subjective views about future performance
3. **Black-Litterman Blending:** Combined equilibrium returns with investor views
 - Views weighted by confidence levels
 - Higher confidence = greater influence on final returns
 - λ parameter controls uncertainty in prior beliefs
4. **Portfolio Optimization:** Used BL expected returns to find optimal allocation
 - Maximized Sharpe ratio subject to long-only constraints; deviation bands; and minimum allocations
 - Compared to market cap weighted portfolio

0.1.2 Key Parameters

- **Risk-free rate:** UST 10Y
- **(prior uncertainty):** 0.05
- **(risk aversion):** Derived from market portfolio
- **Historical data:** 20 years of daily returns

0.1.3 Interpretation

The resulting optimal weights reflect a balance between: - Market equilibrium (what everyone collectively believes) - Your subjective views (what you think will happen) - Statistical uncertainty (confidence in predictions)

1 Market Implied Equilibrium Weights

	Ticker	Market Cap	Weight (%)
0	R1KG	40583000000000	28.206679
1	R1KV	36517000000000	25.380659
2	R2K	27920000000000	1.940543
3	RMCG	48190000000000	3.349382
4	RMCV	10248000000000	7.122737
5	INT	19312000000000	20.604638
6	EM	12555000000000	13.395362

2 Historical Returns and Volatility

	Column	First Valid Date
0	EM	2003-04-14
1	INT	2003-04-14
2	R1KV	2003-04-14
3	R1KG	2003-04-14
4	R2K	2003-04-14
5	RMCG	2003-04-14
6	RMCV	2003-04-14

2.1 Returns

Return Statistics:

	Ticker	Total Compound Return (%)	Annualized Return (CAGR) (%)	\
0	EM	215.80	5.23	
1	INT	232.97	5.47	
2	R1KV	403.66	7.43	
3	R1KG	919.78	10.84	
4	R2K	344.15	6.83	
5	RMCG	610.11	9.07	
6	RMCV	498.08	8.25	

	Average Annual Return (%)	Min Annual Return (%)	\
0	8.80	-60.25	
1	7.49	-47.19	
2	8.99	-41.97	
3	12.21	-42.41	
4	9.47	-40.77	
5	10.99	-49.41	
6	10.02	-43.44	

	25th Percentile Annual Return (%)	Median Annual Return (%)	\
0	-5.22	10.11	

1	-1.38	10.53
2	-2.47	13.15
3	4.80	11.95
4	-0.39	11.81
5	4.57	10.87
6	-3.25	12.77

	75th Percentile Annual Return (%)	Max Annual Return (%)	Years of Data
0	26.34	63.34	22.57
1	20.35	46.46	22.57
2	16.30	32.80	22.57
3	29.68	39.94	22.57
4	18.00	49.42	22.57
5	23.97	40.33	22.57
6	21.87	41.21	22.57

2.2 Volatility

Volatility Statistics:

	Ticker	Total Volatility (%)	Average Rolling 3Y Volatility (%)	\
0	EM	27.22	26.35	
1	INT	20.73	20.33	
2	R1KV	19.04	18.55	
3	R1KG	19.56	19.03	
4	R2K	23.88	23.44	
5	RMCG	21.39	20.89	
6	RMCV	20.38	19.87	
		Period 1 (2003-04 to 2007-10)	Period 2 (2007-10 to 2012-04)	\
0		22.94	44.40	
1		14.40	33.08	
2		12.03	29.28	
3		12.42	25.48	
4		17.90	33.88	
5		14.36	29.72	
6		12.24	30.69	
		Period 3 (2012-04 to 2016-10)	Period 4 (2016-10 to 2021-05)	\
0		19.44	22.34	
1		15.99	18.04	
2		13.23	20.17	
3		13.38	21.33	
4		16.42	24.35	
5		14.48	21.72	
6		13.82	21.98	

Period 5 (2021-05 to 2025-11)	
0	18.16
1	16.30
2	15.13
3	21.82
4	22.83
5	22.73
6	17.68

2.3 Sharpe

Sharpe Ratio Statistics:

Ticker	Total Sharpe Ratio	Period 1 (2003-04 to 2007-10) \
0 EM	0.18	1.37
1 INT	0.17	1.34
2 R1KV	0.26	1.08
3 R1KG	0.42	0.71
4 R2K	0.23	0.83
5 RMCG	0.33	1.03
6 RMCV	0.30	1.32

	Period 2 (2007-10 to 2012-04)	Period 3 (2012-04 to 2016-10) \
0	-0.16	-0.23
1	-0.34	0.08
2	-0.23	0.58
3	-0.08	0.58
4	-0.10	0.39
5	-0.08	0.44
6	-0.11	0.61

	Period 4 (2016-10 to 2021-05)	Period 5 (2021-05 to 2025-11)
0	0.28	-0.12
1	0.32	0.14
2	0.41	0.19
3	0.79	0.44
4	0.45	-0.06
5	0.65	0.09
6	0.37	0.05

3 Covariance & Correlation

Annual Expected Returns:

	Avg. Annual Return
ticker	
EM	0.088022
INT	0.074877

R1KV	0.089854
R1KG	0.122106
R2K	0.094748
RMCG	0.109859
RMCV	0.100194

Covariance Matrix:

ticker	EM	INT	R1KV	R1KG	R2K	RMCG	RMCV
ticker							
EM	0.074098	0.048820	0.041505	0.041111	0.049181	0.045529	0.043639
INT	0.048820	0.042964	0.034524	0.033313	0.039945	0.036465	0.036055
R1KV	0.041505	0.034524	0.036245	0.032419	0.040861	0.035966	0.037645
R1KG	0.041111	0.033313	0.032419	0.038256	0.039878	0.039483	0.034343
R2K	0.049181	0.039945	0.040861	0.039878	0.057016	0.046438	0.045363
RMCG	0.045529	0.036465	0.035966	0.039483	0.046438	0.045740	0.039478
RMCV	0.043639	0.036055	0.037645	0.034343	0.045363	0.039478	0.041552

Correlation Matrix:

ticker	EM	INT	R1KV	R1KG	R2K	RMCG	RMCV
ticker							
EM	1.000000	0.865255	0.800884	0.772152	0.756656	0.782059	0.786448
INT	0.865255	1.000000	0.874862	0.821689	0.807064	0.822576	0.853315
R1KV	0.800884	0.874862	1.000000	0.870616	0.898840	0.883310	0.970025
R1KG	0.772152	0.821689	0.870616	1.000000	0.853859	0.943870	0.861366
R2K	0.756656	0.807064	0.898840	0.853859	1.000000	0.909342	0.931965
RMCG	0.782059	0.822576	0.883310	0.943870	0.909342	1.000000	0.905554
RMCV	0.786448	0.853315	0.970025	0.861366	0.931965	0.905554	1.000000

Asset Correlation Analysis:

Interpretation:

- Avg Correlation: Mean correlation with other assets (higher = more correlated)
 - Mkt-Weighted Avg Corr: Correlation weighted by market cap (more relevant measure)
 - Corr with Portfolio: Correlation with the market portfolio
 - Diversification Ratio: Higher = better diversification benefit; ratio of standalone vol to marginal vol
-
-

Ticker	Avg Correlation	Mkt-Weighted Avg Corr	Corr with Portfolio	\
0 EM	0.7939	0.8098	0.9367	
1 INT	0.8408	0.8441	0.9452	
2 R1KV	0.8831	0.8700	0.9296	

3	R1KG	0.8539	0.8421	0.9028
4	R2K	0.8596	0.8361	0.9032
5	RMCG	0.8745	0.8366	0.9268
6	RMCV	0.8848	0.8534	0.9294

	Diversification Ratio	Standalone Volatility
0	1.0676	0.2722
1	1.0580	0.2073
2	1.0757	0.1904
3	1.1077	0.1956
4	1.1072	0.2388
5	1.0789	0.2139
6	1.0760	0.2038

Diversification Ratio = Individual Volatility / Marginal Contribution to Portfolio Risk

R1KG has the highest diversification ratio because:

1. **High Individual Volatility (numerator):** R1KG has high standalone volatility, which increases the ratio
2. **High Correlation with Market Portfolio (denominator effect):** Because R1KG is highly correlated with the market portfolio AND has a large weight in the market portfolio (~33%), its marginal contribution to portfolio risk is actually LOWER than you'd expect. When an asset is highly correlated with the portfolio, adding more of it doesn't increase portfolio risk proportionally to its own volatility. The formula for marginal contribution to risk (MCR) is:

$$MCR = \frac{\text{Covariance with Portfolio}}{\text{Portfolio Volatility}}$$

Since R1KG is already heavily weighted in the market portfolio and highly correlated with it, the portfolio is essentially "already exposed" to R1KG's risk. So increasing R1KG's weight adds less marginal risk than you'd expect from its high individual volatility.

Result: High ratio, but NOT because it's actually diversifying - rather because the portfolio is already so concentrated in it **This is actually a warning sign**, not a positive feature. A high diversification ratio for R1KG indicates the portfolio is over-concentrated in R1KG's risk factors. True diversifiers (like EM or INT if they had lower correlation) would reduce portfolio risk more effectively per unit of their own volatility.

Risk Aversion Coefficient (): 1.1611

Market Return: 9.2472%

Market Variance: 0.0452

Market Stdev: 0.2126

Implied Equilibrium Returns (from Reverse Optimization):

Ticker	Market Weight (%)	Implied Return (%)
0 R1KG	28.206679	6.293828

1	R1KV	25.380659	4.836078
2	R2K	1.940543	4.368512
3	RMCG	3.349382	4.358681
4	RMCV	7.122737	5.323316
5	INT	20.604638	4.892870
6	EM	13.395362	4.676196

Individual Asset Sharpe Ratios (Implied Returns):

	Ticker	Implied Return (%)	Volatility (%)	Sharpe Ratio	Market Weight (%)
0	R1KG	6.2938	27.2209	0.0843	28.2067
4	RMCV	5.3233	23.8781	0.0554	7.1227
5	INT	4.8929	21.3869	0.0417	20.6046
1	R1KV	4.8361	20.7278	0.0403	25.3807
6	EM	4.6762	20.3844	0.0332	13.3954
2	R2K	4.3685	19.0382	0.0194	1.9405
3	RMCG	4.3587	19.5592	0.0183	3.3494

Highest Sharpe: R1KG = 0.0843

Lowest Sharpe: RMCG = 0.0183

Sharpe Gap: 0.0659

Unconstrained mean-variance optimization allocates 100% to the highest Sharpe asset when diversification benefits don't outweigh the risk-adjusted return advantage.

Weight Verification Test:

	Ticker	Original Market Weight	Calculated Weight	Difference	\
0	IWF	0.282067	0.282066	-0.000001	
1	IWD	0.253807	0.253809	0.000002	
2	IWM	0.019405	0.019404	-0.000001	
3	IWP	0.033494	0.033495	0.000001	
4	IWS	0.071227	0.071225	-0.000002	
5	EFA	0.206046	0.206045	-0.000001	
6	EEM	0.133954	0.133956	0.000002	

	Absolute Error
0	0.000001
1	0.000002
2	0.000001
3	0.000001
4	0.000002
5	0.000001
6	0.000002

```
Maximum Absolute Error: 0.000002
Mean Absolute Error: 0.000002
Test PASSED (threshold: 0.001)
```

3.1 Scalar vs. Asset-Specific Risk Aversion

The choice between scalar vs. array risk aversion involves important tradeoffs:

3.1.1 Pro:

- Captures heterogeneous investor risk preferences across asset classes (e.g., institutions may be more risk-averse to emerging markets than domestic equities)
- Can reflect market segmentation where different investor bases dominate different assets
- Allows modeling of risk budgeting frameworks where risk tolerance varies by asset category

3.1.2 Cons:

- No clear market observable to estimate individual values - you'd need to make subjective assumptions
- Loss of interpretability: a single has clear economic meaning (market price of risk), but asset-specific values are harder to justify
- Violates the assumption of a representative investor with consistent preferences across all assets
- In standard CAPM/equilibrium theory, all investors face the same efficient frontier, implying a single market-wide risk aversion
- Can lead to arbitrage opportunities in theory (why would rational investors have different risk aversion for the same dollar of risk?)

The Black-Litterman model already allows you to express asset-specific views through the P and Q matrices. If you believe certain assets deserve different treatment, you can express that through higher-confidence views rather than modifying the equilibrium framework.

The power of Black-Litterman is separating equilibrium (market consensus) from your subjective views. Using asset-specific risk aversion muddies this distinction and essentially bakes your views into the “neutral” equilibrium, which defeats the model’s elegance.

4 Investor Views

```
{'View 1: R1KG outperforms R1KV by 5%': {'assets': ['R1KG', 'R1KV'],
  'weights': [1, -1],
  'return': 0.05,
  'confidence': 0.75},
'View 2: 60/40 R1KG/R1KV outperforms INT by 4%': {'assets': ['R1KG',
  'R1KV',
  'INT'],
  'weights': [0.6, 0.4, -1],
  'return': 0.04,
```

```

'confidence': 0.75},
'View 3: 60/40 R1KG/R1KV outperforms R2k by 5%': {'assets': ['R1KG',
'R1KV',
'R2K'],
'weights': [0.6, 0.4, -1],
'return': 0.05,
'confidence': 0.75},
'View 4: 60/40 RMCG/RMCV outperforms R2K by 2.5%': {'assets': ['RMCG',
'RMCV',
'R2K'],
'weights': [0.6, 0.4, -1],
'return': 0.025,
'confidence': 0.75},
'View 5: EM outperforms INT by 3%': {'assets': ['EM', 'INT'],
'weights': [1, -1],
'return': 0.03,
'confidence': 0.75}]

```

P Matrix (View Picks):

	R1KG	R1KV	R2K	RMCG	RMCV	INT	EM
0	1.0	-1.0	0.0	0.0	0.0	0.0	0.0
1	0.6	0.4	0.0	0.0	0.0	-1.0	0.0
2	0.6	0.4	-1.0	0.0	0.0	0.0	0.0
3	0.0	0.0	-1.0	0.6	0.4	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0	-1.0	1.0

Q Vector (View Returns):

[0.05 0.04 0.05 0.025 0.03]

Omega Matrix (View Uncertainties):

```

[[0.02589538 0.          0.          0.          0.        ],
 [0.          0.02522139 0.          0.          0.        ],
 [0.          0.          0.02107162 0.          0.        ],
 [0.          0.          0.          0.00891987 0.        ],
 [0.          0.          0.          0.          0.01111388]]
```

5 Black-Litterman Expected Returns

Combining Equilibrium Returns with Investor Views

5.0.1 The Black-Litterman Formula

$$E[R] = [(\tau\Sigma)^{-1} + P'\Omega^{-1}P]^{-1}[(\tau\Sigma)^{-1}\pi + P'\Omega^{-1}Q]$$

Where: - τ (tau): Scalar representing uncertainty in the prior equilibrium (typically 0.025-0.05) - Σ (Sigma): Covariance matrix - P : Matrix picking out assets in each view - Ω (Omega): Diagonal matrix of view uncertainties - π : Implied equilibrium returns from reverse optimization - Q : Vector of expected returns according to investor views

5.0.2 The Parameter

$\tau = 0.05$ controls how much you trust the market equilibrium returns ():

- **Smaller** (0.01-0.025): High confidence in equilibrium → views have **less** influence
- **Larger** (0.04-0.1): Low confidence in equilibrium → views have **more** influence

Mathematical effect: Larger → less precision in (Σ)¹ → equilibrium estimates treated as more uncertain → more weight given to investor views (Q) relative to equilibrium ()

5.0.3 Interpretation

The posterior expected returns are a **precision-weighted average** of: 1. Market equilibrium beliefs () - what the market collectively thinks 2. Your subjective views (Q) - what you believe will happen

The weighting is determined by both and the confidence levels in your views.

Black-Litterman Expected Returns:

	Ticker	Market Weight (%)	Implied Return (%)	BL Expected Return (%)	\
0	R1KG	28.206679	6.293828	6.748555	
1	R1KV	25.380659	4.836078	5.038519	
2	R2K	1.940543	4.368512	4.467144	
3	RMCG	3.349382	4.358681	4.448212	
4	RMCV	7.122737	5.323316	5.468053	
5	INT	20.604638	4.892870	4.961760	
6	EM	13.395362	4.676196	4.797351	

	Return Adjustment (%)
0	0.454727
1	0.202441
2	0.098632
3	0.089531
4	0.144737
5	0.068890
6	0.121155

5.0.4 Column Descriptions

- **Market Weight (%)**: Current market capitalization weight
 - Derived from free float-adjusted market caps; Reflects the “consensus” allocation across all investors
- **Implied Return (%)**: Equilibrium returns from reverse optimization
 - Calculated using $= \Sigma w$; What the market “believes” returns should be to justify current weights; Represents the **prior** (before incorporating your views)
- **BL Expected Return (%)**: Black-Litterman posterior returns

- Precision-weighted blend of implied returns () and your views (Q); Represents the **posterior** (after incorporating your views)
- **Return Adjustment (%)**: BL Expected Return - Implied Return
 - Shows how much your views moved each asset's expected return
 - Positive = your views increased expected return vs. equilibrium; Negative = your views decreased expected return vs. equilibrium; Magnitude depends on view confidence and parameter

6 Mean Variance Optimization with BL Expected Returns

Optimization Configuration:

Using Option 3: Market cap constraints ($\pm 15.0\%$) + minimum 2.5% allocation
(maximize Sharpe ratio)

Optimization successful: True

Market Portfolio (BL Returns):

Return: 5.47%
Volatility: 21.26%
Sharpe Ratio: 0.0693

Optimal BL Portfolio:

Return: 5.83%
Volatility: 22.60%
Sharpe Ratio: 0.0811

Portfolio Allocation Comparison:

	Ticker	Market Weight (%)	BL Optimal Weight (%)	Weight Change (%)	\
0	R1KG	28.2067	43.2067	15.0000	
1	R1KV	25.3807	21.5659	-3.8147	
2	R2K	1.9405	2.5000	0.5595	
3	RMCG	3.3494	2.5000	-0.8494	
4	RMCV	7.1227	22.1227	15.0000	
5	INT	20.6046	5.6046	-15.0000	
6	EM	13.3954	2.5000	-10.8954	
		Implied Return (%)	BL Expected Return (%)		
0		6.2938	6.7486		
1		4.8361	5.0385		
2		4.3685	4.4671		
3		4.3587	4.4482		
4		5.3233	5.4681		
5		4.8929	4.9618		
6		4.6762	4.7974		

7 Unconstrained MVO

Optimization Objective: The unconstrained portfolio maximizes the Sharpe ratio using **implied equilibrium returns** () without any investor views. This shows the theoretical optimal portfolio based purely on what the market believes and excludes the constraints defined in the optimization above.

Where: - \mathbf{R} = Portfolio Return = $\mathbf{w}^T \times \mu$ - Weighted average of implied equilibrium returns - comes from reverse optimization: $\mu = \Sigma w_{\text{market}}$

- σ = Portfolio Volatility = $\sqrt{(\mathbf{w}^T \times \Sigma \times \mathbf{w})}$
 - Standard deviation of portfolio returns
 - Σ is the covariance matrix from historical data
- R_f = Risk-Free Rate = 4%
 - The return of a risk-free asset (e.g., T-bills)

Key Points: - Uses **(implied returns)**, NOT BL_mean (which includes your views) - No limits on deviation from market weights - Shows theoretical optimum, often concentrated in a single asset - Useful for understanding what pure mean-variance optimization would do without practical constraints

Maximum Sharpe Ratio Portfolio (Unconstrained Long-Only, No Views):

Uses implied equilibrium returns (π_i) - not adjusted by investor views

Return: 0.0629 (6.29%)

Volatility: 0.2722 (27.22%)

Sharpe Ratio: 0.0843

Ticker	Max Sharpe Weight (%)	Market Weight (%)	BL Optimal Weight (%)	\
0 R1KG	100.0	28.21	43.21	
1 R1KV	0.0	25.38	21.57	
2 R2K	0.0	1.94	2.50	
3 RMCG	0.0	3.35	2.50	
4 RMCV	0.0	7.12	22.12	
5 INT	0.0	20.60	5.60	
6 EM	0.0	13.40	2.50	

	Difference from Market (%)	Difference from BL Optimal (%)
0	71.79	56.79
1	-25.38	-21.57
2	-1.94	-2.50
3	-3.35	-2.50
4	-7.12	-22.12
5	-20.60	-5.60
6	-13.40	-2.50

TODO: need to determine why the unconstrained optimization vol for a 100% R1KG

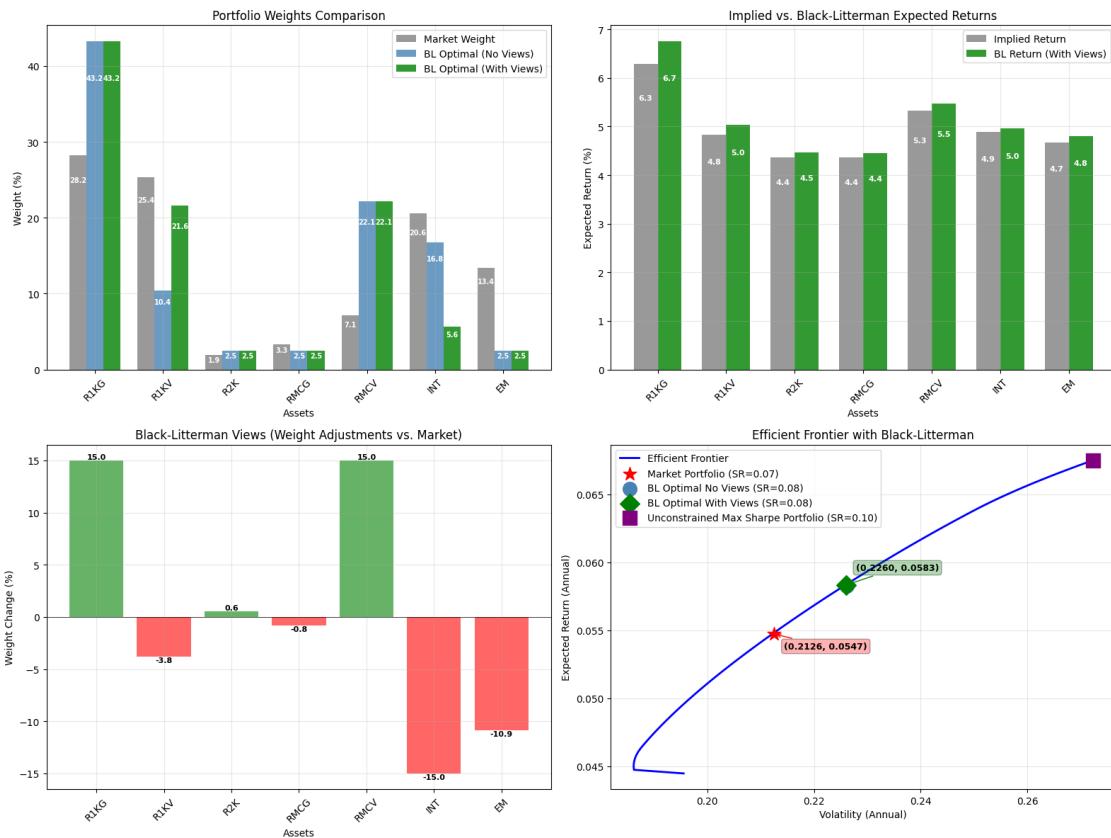
portfolio is not matching the calculated vol from the returns data

```

ticker
EM      0.272209
INT     0.207278
R1KV    0.190382
R1KG    0.195592
R2K     0.238781
RMCG    0.213869
RMCV    0.203844
dtype: float64

```

8 BL Plot



9 Risk Analytics

Comprehensive risk analysis of the optimal Black-Litterman portfolio including:

- **Risk Contribution:** Marginal and component risk by asset
- **Tracking Error:** Deviation from market portfolio
- **Downside Risk:** Semi-deviation and tail risk metrics
- **Concentration Risk:** Portfolio diversification measures

Risk Contribution Analysis:

Portfolio Volatility: 0.2260

Tracking Error vs Market: 0.0263

Information Ratio: 0.1373

	Ticker	Weight (%)	Marginal Risk	Component Risk	Risk Contribution (%)	\
0	R1KG	43.2067	0.2616	0.1130	50.0152	
1	R1KV	21.5659	0.1940	0.0418	18.5071	
2	R2K	2.5000	0.1730	0.0043	1.9131	
3	RMCG	2.5000	0.1708	0.0043	1.8895	
4	RMCV	22.1227	0.2134	0.0472	20.8886	
5	INT	5.6046	0.1913	0.0107	4.7448	
6	EM	2.5000	0.1846	0.0046	2.0416	

	Return Contribution (%)	Risk/Return Ratio	Active Weight (%)	\
0	49.9883	3.8768	15.0000	
1	18.6285	3.8495	-3.8147	
2	1.9146	3.8718	0.5595	
3	1.9065	3.8402	-0.8494	
4	20.7385	3.9028	15.0000	
5	4.7675	3.8563	-15.0000	
6	2.0561	3.8475	-10.8954	

	TE Contribution (%)
0	107.2115
1	-12.1040
2	1.1917
3	-1.3177
4	51.9379
5	-24.0936
6	-22.8259

Verification: Sum of Component Risks = 0.226014 (should equal portfolio vol = 0.226014)

Verification: Sum of Risk Contributions = 100.00% (should equal 100%)

Downside Risk Analysis:

	Metric	BL Optimal Portfolio	Market Portfolio	Difference
0	Semi-Deviation (Annual)	0.1810	0.1730	0.0081
1	Sortino Ratio	0.1012	0.0851	0.0161
2	VaR 95% (Annual)	-0.3229	-0.3032	-0.0196
3	VaR 99% (Annual)	-0.6307	-0.6068	-0.0239

4	CVaR 95% (Annual)	-0.5432	-0.5147	-0.0284
5	CVaR 99% (Annual)	-0.9899	-0.9412	-0.0487
6	Maximum Drawdown	-0.6644	-0.6514	-0.0130
7	Downside Capture Ratio	1.0675	1.0000	0.0675

Interpretation:

- Lower semi-deviation = less downside volatility
- Higher Sortino ratio = better risk-adjusted returns considering only downside risk
- VaR/CVaR = potential losses at different confidence levels
- Lower maximum drawdown = smaller peak-to-trough decline
- Downside capture < 1.0 = portfolio declines less than market on down days

Concentration Risk Analysis:

	Metric	BL Optimal Portfolio	Market Portfolio	\
0	Herfindahl-Hirschman Index	0.2871	0.2109	
1	Effective Number of Assets	3.4825	4.7405	
2	Gini Coefficient	0.5155	0.3896	
3	Diversification Ratio	1.0702	1.0726	
4	Maximum Single Weight	0.4321	0.2821	
5	Top 3 Assets Concentration	0.8690	0.7419	
6	Number of Assets	7.0000	7.0000	

	Difference
0	0.0762
1	-1.2579
2	0.1260
3	-0.0025
4	0.1500
5	0.1270
6	0.0000

Interpretation:

- Lower HHI = more diversified portfolio
- Higher effective N = more evenly distributed weights
- Lower Gini = more equal weight distribution
- Higher diversification ratio = better diversification benefits
- Lower top 3 concentration = less reliance on few positions

