Follow it on your own risk
I am not responsible for any error.
=KING

Final.

x	0	1	2	3	4	5	6
y	2	3	10	29	66	127	218

a) Make a difference table.

b) Estimate $f(1.3)$

c) Estimate $f(4.9)$

d) Estimate $f(3.1)$.

e) $f'(1.3)$

f) $f''(4.9)$

g) $f'''(3.1)$.

a)

x	y_0	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$
0	2				
		1			
1	3		6		
		7		6	
2	10		12		0
		19		6	
3	29		18		0
		37		6	
4	66		24		0
		61		6	
5	127		30		
		91			
6	218				

b).

By Newton forward interpolatio

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y$$

$$+ \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y$$

Scanned with CamScanner

where $p = \frac{x - x_0}{h}$

& $h = 1$

$p = \frac{1.3 - 1}{1}$

$p = 0.3$

putting values.

$$y_p = 3 + (0.3)(7) + \frac{(0.3)(0.3-1)\,12}{2!}$$

$$+ \frac{(0.3)(0.3-1)(0.3-2)(6)}{6}$$

$$+ \frac{(0.3)(0.3-1)(0.3-2)(0.3-3)(0)}{24}$$

$y_p = 3 + 2.1 - 1.26 + 0.357 + 0$

$\boxed{y_p = 4.197}$

Scanned with CamScanner

c) By Newton Backward

$$y_p = y_0 + p\Delta y_0 + \frac{p(p+1)}{2!}\Delta^2 y_0$$

$$+ \frac{p(p+1)(p+2)}{3!}\Delta^3 y_0$$

where

$$p = \frac{x - x_0}{h}$$

$$p = \frac{4.9 - 4}{1}$$

$$p = 0.9$$

putting values.

$$y_p = 66 + (0.9)(37) + \frac{(0.9)(0.9+1)}{2}(18)$$

$$+ \frac{(0.9)(0.9+1)(0.9+2)}{6}(6)$$

$$y_p = 66 + 33.3 + 15.39 + 4.959$$

$$\boxed{y_p = 119.649}$$

1) By Stirling / Center Interpolation

$$y_p = y_0 + p\mu\delta y_0 + \frac{p^2}{2!}\delta^2 y_0$$

$$+ \frac{p(p^2-1)}{3!}\mu\delta^3 y_0 + \frac{p^2(p^2-1)}{4!}\delta^4 y_0$$

~~putting valu~~

where $p = \frac{3.1-3}{1} = 0.1$

$$p\mu\delta y_0 = (0.1)\left(\frac{19+37}{2}\right) = 2.8$$

$$\frac{p^2}{2}\delta^2 y_0 = \frac{(0.1)^2 18}{2} = 0.09$$

$$\frac{p(p^2-1)}{6}\mu\delta^3 y_0 = \frac{(0.1)(0.1^2-1)(6)}{6} = -0.099$$

$$\frac{p^2(p^2-1)}{4!} = 0 \text{ becaus } \delta^4 y_0 = 0.$$

putting values

~~$y_p = 2$~~

$y_p = 29 + 2.8 + 0.09 - 0.099$

$$\boxed{y_p = 31.791}$$

e) Taking derivative of N- farward

$$y'_p = \frac{1}{h}\left[0 + \Delta y_0 + \frac{2p-1}{2!}\Delta^2 y_0 + \frac{(3p^2 - 6p + 2)\Delta^3 y_0}{6}\right].$$

putting values.

where.

~~$p =$~~ $h = 1$

$p = 0.3$.

$$y'_p = \left[7 + \frac{2(0.3-1)\,12}{2!} + \frac{(3(0.3)^2 - 6(0.3) \ldots}{6}\right.$$

$y'p = 7 - 8.4 + \cancel{0.9}\ 2.1$

$\boxed{\cancel{y'p = 2.3}}$ $\boxed{y'p = -0.3}$

(ii) Taking 1st derivative of N-backward.

$$y_p = y_0 + p\Delta y_0 + \frac{p(p+1)}{2!}\Delta^2 y_0 + \frac{p(p+1)(p+2)}{3!}\Delta^3 y_0$$

$$y'p = \frac{1}{h}\left[0 + \Delta y_0 + \frac{(2p+1)}{2!}\Delta^2 y_0 + \frac{(3p^2+6p+2)}{6}\Delta^3 y_0\right]$$

Taking second derivative

$$y''p = \frac{1}{h^2}\left[0 + \frac{(2+0)}{2!}\Delta^2 y_0 + \frac{(6p+6+0)}{6}\Delta^3 y_0\right.$$

$$y''p = \frac{1}{h^2}\left[\frac{\cancel{2}\,\Delta^2 y_0}{\cancel{2}} + (p+1)\Delta^3 y_0 \right.$$

where $h = 1$
$p = 0.9.$

$$y''p = \frac{1}{1}\left[18 + (0.9+1)(6)\right].$$

$$\boxed{y''p = 29.4}$$

g) Taking First derivative of stirling

$$y_p = y_0 + p\,\mu\delta y_0 + \frac{p^2}{2!}\delta^2 y_0 + \frac{p(p^2-1)}{3!}\mu\delta^3 y_0$$

$$y'p = \frac{1}{h}\left[0 + \mu\delta y_0 + \frac{2p}{2!}\delta^2 y_0 + \frac{(3p^2-1)}{3!}\mu\delta^3 y_0\right].$$

Taking second derivative.

$$y''p = \frac{1}{h^2}\left[0 + \delta^2 y_0 + \frac{6p}{3!}\mu\delta^3 y_0\right]$$

Taking third derivative.

$$y''' = \frac{1}{h^3}\left[0 + \mu\delta^3 y_0\right].$$

putting values

$$y''' = \frac{1}{1}\left[6\right].$$

$$\boxed{y''' = 6}$$

Quest

use tripzoid & simpson to estimate integral with error term

$$\int_1^3 f(x)\,dx = \int_1^3 (x^3 - 2x^2 + 7x - 5)\,dx$$

a) Tripzoidal Rule.

$h = b - a$.

$h = 3 - 1$

$h = 2$.

$\Rightarrow \frac{h}{2}\left[f(x_0) + f(x_1) \right]$.

$\Rightarrow \frac{h}{2}\left[f(0) + f(1) \right]$.

$\Rightarrow \frac{2}{2}\left[(0 - 2(0) + 7(0) - 5) + (1 + 2(1) + 7(1) \right.$

$\Rightarrow [-5 + 5]$

$= 0$ -

$$\Rightarrow \frac{h}{2}\left[f(x_0)+f(x_1)\right]-\frac{h^3}{12}f''(x).$$

$f(x_0)=f(1)=1-2+7-5=1.$

$f(x_1)=f(3)=27-18+21-5=25.$

$$\Rightarrow \frac{\cancel{2}}{\cancel{2}}\left[1+25\right]-\frac{(2)^3}{12}f''(x).$$

$$=26-\frac{8}{12}f''(x).$$

$\therefore f'(x)=3x^2-4x+7-0$

$=f''(x)=6x-4$

$$\Rightarrow 26-\frac{8}{12}(6x-4).$$

let $x=1.$

$$\Rightarrow 26-\frac{8}{12}(2).\ \boxed{=24.66}$$

Now let $x=3.$

$$\Rightarrow 26-\frac{8}{12}(14)\ \boxed{=16.666}$$

b) Simpso $\frac{1}{3}$ rule.

$$h = \frac{b-a}{2} = \frac{2}{2} = 1.$$

$x_0 = 1$
$x_1 = 2$
$x_2 = 3.$

$f(x_0) = f(1) = 1.$
$f(x_1) = f(2) = \cancel{8} - \cancel{8} + 14 - 5 = 9.$
$f(x_2) = f(3) = 25.$

$$\Rightarrow \frac{h}{3}\left[f(x_0) + f(x_2) + 4f(x_1)\right] - \frac{h^5}{90} f^4(x)$$

$$\Rightarrow \frac{1}{3}\left[1 + 25 + 36\right] - \frac{1}{90} f^4(x)$$

$$\Rightarrow \frac{1}{3}(62) - \frac{1}{90} f^4(x).$$

$\Rightarrow 20.6666 - \frac{1}{90} f^4(x).$

$\because f'(x) = 3x^2 - 4x + 7$

$f''(x) = 6x - 4.$

$f'''(x) = 6$

$f^4(x) = 0.$

$\Rightarrow 20.6666 - \frac{1}{90}(0)$

$\Rightarrow 20.6666.$

c) Simpson 3/8

$h = \frac{b-a}{3}$

$h = \frac{2}{3}$ ~~0.6666~~.

~~$f(x_0)$~~ ~~$x_0 = 0.6666$~~ ~~2/3~~

~~$x_1 = 1.33326$~~ ~~4/3~~

~~$x_2 = 1.9998$~~ ~~6/3~~

~~$x_3 = 2.6666$~~ ~~8/3~~

First value lower interval last upper interval.

$x_0 = 1$ $x_3 = 3$

$x_1 = 1.6666$

$x_2 = 2.3332$

values ghalat put ki hai
else method sahi hai.

$f(x_0) = (0.6666)^3 + 2(0.6666)^2 + 7(0.6666) - 5$

$f(x_0) = 0.2962 + 0.8887 + 4.6662 - 5$

$f(x_0) = 0.8511$

$f(x_1) = (1.33326)^3 + 2(1.33326)^2 + 7(1.33326) - 5$

$f(x_1) = 10.257.$

$f(x_2) = (1.9998)^3 + 2(1.9998)^2 + 7(1.9998) - 5.$

$f(x_2) = 24.994.$

$f(x_3) = (2.6666)^3 + 2(2.6666)^2 + 7(2.6666) - 5$

$f(x_3) = 46.849.$

$\Rightarrow \frac{3}{8} h \left[f(x_0) + f(x_3) + 3(f(x_1) + f(x_2)) \right]$ [illegible]

$= \frac{3}{8}(0.6666)\left[0.8511 + 46.849 + 3(10.257 + 24.994)\right]$

$= 0.249975(153.4531)$

$\Rightarrow 38.359.$

Question ~~Apr~~

Apply Simpson composite to the integral.

$$\int_1^{1.30} \sqrt{x}\, dx \quad \text{taking } 2m = 6.$$

Sol:
$$h = \frac{b-a}{2m} = \frac{1.30 - 1}{6} = \frac{0.30}{6}$$

$$\boxed{h = 0.05}$$

$x_0 = 1.$
$x_1 = 1.05$
$x_2 = 1.1$
$x_3 = 1.15$
$x_4 = 1.2$
$x_5 = 1.25$
$x_6 = 1.3$

$f(x_0) = \sqrt{1} = 1.$ $\quad f(x_5) = \sqrt{1.25} = 1.1180$
$f(x_1) = \sqrt{1.05} = 1.0246$
$f(x_2) = \sqrt{1.1} = 1.0488$ $\quad f(x_6) = \sqrt{1.3} = 1.1401$
$f(x_3) = \sqrt{1.15} = 1.0723$
$f(x_4) = \sqrt{1.2} = 1.0954$

$-1/2 \quad 1/2 - \frac{2}{2}$ $\quad 1/2 - 1 \quad -1/2 - 1$

error term for Simpson 1/3

$$-\frac{h^5}{90} \, n \, f^4(x)$$

$$f(x) = x^{1/2}.$$

$$f'(x) = \frac{1}{2} x^{-1/2}.$$

$$f''(x) = -\frac{1}{4} x^{-3/2}.$$

$$f'''(x) = \frac{3}{8} x^{-5/2}$$

$$f^4(x) = -\frac{15}{16} x^{-7/2}$$

let $x = 1$ (beause we need maximum value & here it is negative so we choose the lower limit).

$$f^4(x) = -0.9375$$

so.

$$\frac{h^5}{90} \, n \, f^4(x) = -(0.05)^5 (3)(-0.9375)$$

error term = 0.000000 8789.

Now By Simpson 1/3.

$$\Rightarrow \frac{h}{3}\left[f(x_0) + f(x_6) + 4(f(x_1) + f(x_3) + f(x_5)) + 2(f(x_2) + f(x_4))\right]$$

$$\Rightarrow \frac{0.05}{3}\left[1 + 1.1402 + 4(1.0246 + 1.0723 + 1.1180) + 2(1.0488 + 1.0954)\right]$$

$$\Rightarrow \frac{0.05}{3}(19.2881).$$

$\Rightarrow 0.32146833$ - error term.

$$\boxed{\Rightarrow 0.32146745}$$

Solve with simpson 3/8 rule with 6 intervals.

$$I = \int_1^2 \frac{dx}{5+3x}$$

Solution:

So here $3m = 6$.

$m = 2$

So $h = \frac{b-a}{3m} = \frac{2-1}{6} = \frac{1}{6} = 0.1666$

$x_0 = 1$
$x_1 = 1.1666$
$x_2 = 1.3332$
$x_3 = 1.4998$
$x_4 = 1.6664$
$x_5 = 1.833$
$x_6 = 2$.

$$f(x) = \frac{1}{5+3x}.$$

$$f(x_0) = \frac{1}{5+3} = 0.125$$

$$f(x_1) = \frac{1}{8.4998} = 0.1176$$

$$f(x_2) = \frac{1}{8.996} = 0.1111$$

$$f(x_3) = \frac{1}{9.4994} = 0.1052$$

$$f(x_4) = \frac{1}{9.9992} = 0.1000$$

$$f(x_5) = \frac{1}{10.499} = 0.0952$$

$$f(x_6) = \frac{1}{11} = 0.0909.$$

error term:

$$- \frac{3h^5}{80} m f^4(x) \quad \therefore f(x) = \frac{1944}{(3x+5)^5}$$

$$- 3(0.1666)^5 (2) \left(\frac{1944}{(11)^5} \right)$$

$$(0.00077)$$

$$- \cancel{0.0996} \left(16.0661 \right)$$

error term = ~~16.0596~~ 0.01237

Now By Simpson 3/8 rule.

$$\Rightarrow \frac{3}{8}h\left[f(x_0)+f(x_6)+2(f(x_3))+3(f(x_1)\right. \\ \left. f(x_2)+f(x_4)+f(x_5))\right].$$

$$\Rightarrow \frac{3}{8}(0.1666)\left[0.125+0.0909+2(0.1052)\right. \\ \left.+3(0.1176+0.1111+0.2000 \right. \\ \left. +0.0952)\right]$$

$$\Rightarrow 0.0624[1.7916]$$

$$\Rightarrow 0.11179 - \text{error term}.$$

$$\boxed{\Rightarrow 0.09942}$$

Euler Method

Formula = $Y_{n+1} = Y_n + h f(x_n, y_n)$

Example

Solve $\frac{dy}{dx} = x + y$ with

the boundary condition
$y = 1$ at $x = 0$. Find approximal
value of y at $x = 0.1$.

Sol:-
Let the step size / interval = 5

So
$$h = \frac{b-a}{5} = \frac{0.1}{5} = 0.02$$

So
$x_0 = 0$ ~~$y_0 = 1$~~
$x_1 = 0.02$
$x_2 = 0.04$
$x_3 = 0.06$
$x_4 = 0.08$
$x_5 = 0.1$

~~y~~ Now we need to find ~~y_5~~ so ~~to~~
~~for~~ ~~y_1~~ for y_1.

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$n = 0$

$$y_1 = y_0 + h(x_0 + y_0)$$

$$y_1 = 1 + 0.02(0 + 1)$$

$$y_1 = 1 + 0.02$$

$$y_1 = 1.02$$

For y_2

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$n = 1$

$$y_2 = y_1 + h(x_1 + y_1)$$

$$y_2 = 1.02 + 0.02(0.02 + 1.02)$$

$$y_2 = 1.0408$$

For y_3

$n = 2$

$y_3 = \cancel{y}\ 1.0408 + 0.02(0.04 + 1.0408)$

$y_3 = 1.0624$

For y_4

$n = 3$

$y_4 = 1.0624 + 0.02(0.06 + 1.0624)$

$y_4 = 1.084$

For y_5

$n = 4$

$y_5 = \cancel{y_4}\ 1.084 + 0.02(0.08 + 1.084)$

$\boxed{y_5 = 1.107}$

Milne Predictor/corrector method

Predictor.

$$y_{n+1} = y_{n-3} + \frac{4}{3}h\left[2f_n - f_{n-1} + 2f_{n-2}\right]$$

Corrector

$$y_{n+1} = y_{n-1} + \frac{h}{3}\left(f_{n+1} + 4f_n + f_{n-1}\right).$$

Example.

$$y' = \frac{x+y}{2}$$

x_i	0	0.5	1	1.5
y_i	2	2.636	3.595	4.968

Find $y(2)$.

Sol: Here $h = 0.5$

we need to find

$y_4^p = ?$

$y_4^c = ?$

So $f_1 = \frac{x_1 + y_1}{2} = \frac{0.5 + 2.636}{2}$

$f_1 = 1.568$

$f_2 = \frac{x_2 + y_2}{2} = \frac{1 + 3.595}{2}$

$f_2 = 2.2975$

$f_3 = \frac{x_3 + y_3}{2} = 3.234$

Now lets take $n = 3$

$y_{n+1} = y_{n-3} + \frac{4}{3} h \left[2f_n - f_{n-1} + 2f_{n-2} \right]$

$$y_4' = y_0 + \frac{4}{3}h\left[2f_3 - f_2 + 2f_1\right]$$

putting values

$$y_4' = 2 + \frac{4}{3}h\left[2(3.234) - 2.2975 + 2(1.518)\right]$$

$$y_4' = 2 + 0.666[7.3065]$$

$$\boxed{y_4'^{p} = 6.866} \Rightarrow f_4 = 4.433$$

Now Milne corrector

$$y_{n+1}' = y_{n-1} + \frac{h}{3}\left(f_{n+1} + 4f_n + f_{n-1}\right)$$

$$y_4' = y_2 + \frac{h}{3}\left(f_4 + 4f_3 + f_2\right)$$

$$y_4' = 3.595 + \frac{0.5}{3}\left(\overset{4.433}{\cancel{6.866}} + 4(3.234) + 2.\right.$$

$$\boxed{y_4' = \overset{6.8714}{\cancel{6.8\quad 7.2635}}}$$

Example 2.

10 Using Milne corrector method find y when $x = 0.8$

Given $\frac{dy}{dx} = x - y^2$

$y(0) = 0$, $y(0.2) = 0.02$, $y(0.4) = 0.0795$

$y(0.6) = 0.1762$

Sol:

x	y	f_n
$x_0 = 0$	$y_0 = 0$	$f_0 = 0 - 0^2 = 0$
$x_1 = 0.2$	$y_1 = 0.02$	$f_1 = 0.2 - (0.02)^2 = 0.1996$
$x_2 = 0.4$	$y_2 = 0.0795$	$f_2 = 0.4 - (0.0795)^2 = 0.3936$
$x_3 = 0.6$	$y_3 = 0.1762$	$f_3 = 0.6 - (0.1762) = 0.5689$
$x_4 = 0.8$	$y_4^p = ?$ $y_4^c = ?$	

0.2666

here $h = 0.2$

let $n = 3$.

~~for~~ for corrector first we need to compute predictor

$$y_{n+1}^{p} = y_{n-3} + \frac{4}{3} h \left(2f_n - f_{n-1} + 2f_{n-2} \right)$$

$$y_4^{p} = y_0 + \frac{4}{3}(0.2)\left(2f_3 - f_2 + 2f_1 \right)$$

putting values.

$$y_4^{p} = 0 + \frac{4}{3}(0.2)\left(2(0.5689) - 0.3936 + 2(0.1992) \right)$$

$$\boxed{y_4^{p} = 0.3048} \Rightarrow f_4 = 0.8 - y_4^{p}$$

$$f_4 = 0.707$$

Now corrector.

$$y_{n+1} = y_{n-1} + \frac{h}{3}\left(f_{n+1} + 4f_n + f_{n-1} \right)$$

$$y_4 = y_2 + \frac{0.2}{3}\left(f_4 + 4f_3 + f_2 \right).$$

$$y_4^c = 0.0795 + \cancel{0.166}\,0.066 \left(\cancel{0.3048}\,0.701 + 4(0.5689) + 0.3936 \right)$$

$$\cancel{y_4^c = 0.5731}$$

$$\boxed{y_4^c = 0.3023}$$

Adam Moulton predictor/corrector

predictor:

$$y_{n+1} = y_n + \frac{h}{24}\left[55f_n - 59f_{n-1} + 37f_{n-2} - 9f_{n-3}\right]$$

corrector:

$$y_{n+1} = y_n + \frac{h}{24}\left[9f_{n+1} + 19f_n - 5f_{n-1} + f_{n-2}\right]$$

Example:

use Adam's - Moulto predictor, corrector formula to find $y(0.4)$ correct to 4 decimal places

$$\frac{dy}{dx} = \frac{y-x}{y+x}.$$

x	0	0.1	0.2	0.3
y	1.0000	1.0911	1.1678	1.2335

Sol:-

$h = 0.1$

x_n	y_n	f_n
0	1.0000	$f_0 = \frac{1-0}{1+0} = 1.0000$
0.1	1.0911	$f_1 = \frac{0.9911}{1.1911} = 0.8320$
0.2	1.1678	$f_2 = \frac{0.9678}{1.3678} = 0.7075$
0.3	1.2335	$f_3 = \frac{0.9335}{1.5335} = 0.6087$

By Adam predictor

$$y_{n+1} = y_n + \frac{h}{2}\left[55 f_n - 59 f_{n-1} + 37 f_{n-2} - 9 f_{n-3}\right]$$

Put $n = 3$.

0.0041

$$y_4 = y_3 + \frac{h}{24}\left[55f_3 - 59f_2 + 37f_1 - 9f_0\right]$$

putting values.

$$y_4 = 1.2335 + \frac{0.1}{24}\left[55(0.6087) - 59(0.7075) + 37(0.8320) - 9(1.0000)\right]$$

$$\boxed{y_4^p = 1.2889}$$

$$f_4 = \frac{0.8889}{1.6889} = 0.5263.$$

Now By Adam Moulton corrector.

$$y_{n+1} = y_n + \frac{h}{24}\left[9f_{n+1} + 19f_n - 5f_{n-1} + f_{n-2}\right].$$

$n=3$

$$y_4^c = y_3 + \frac{h}{24}\left[9f_4 + 19f_3 - 5f_2 + f_1\right].$$

$$y_4^c = 1.2335 + \frac{0.1}{24}\left[9(0.5263) + 19(0.6087) - 5(0.7075) + 0.8320\right]$$

$$\boxed{y_4^c = 1.2892}$$

Runga Kutta 2nd Order

Formula

$$y_{n+1} = y_n + \frac{h}{2}(k_1 + k_2)$$

where

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf(x_n + h, y_n + k_1)$$

Example.

Given that $\frac{dy}{dx} = y - x$

where $y(0) = 2$,

~~where~~ Find $y(0.1)$ & $y(0.2)$.

Sol:

we have

$h = 0.1$.

$x_0 = 0$ $\quad y_0 = 2$

$x_1 = 0.1$ $\quad y_1 = ?$

$x_2 = 0.2$ $\quad y_2 = ?$

For $y(0.2)$

$$y_{n+1} = y_n + \frac{h}{2}(k_1 + k_2) \longrightarrow 1$$

Put $n=0$

$$y_1 = y_0 + \frac{h}{2}(k_1 + k_2). \text{ — } 1$$

First we need to find k_1.

$k_1 = hf(x_n, y_n)$

$k_1 = hf(x_0, y_0)$ $\quad \therefore f(x_0, y_0) = y_0 - x_0$

$k_1 = (0.1)(2)$

$k_1 = 0.2$

Now

$k_2 = hf(x_0 + h, y_0 + k_1)$

$k_2 = (0.1)f(0.1, 2.2)$.

$k_2 = (0.1)(2.2 - 0.1)$.

$k_2 = 0.21$

putting values in eq(1).

$$y_1 = 2 + \frac{h}{2}\left(0.2 + 0.21\right).$$

$$\boxed{y_1 = \cancel{2.020}}\ 2.020.$$

Now

For $y(0.2)$.

$$y_{n+1} = y_n + \frac{h}{2}\left(k_1 + k_2\right) \text{ ——}$$

$n = 1$.

$$y_2 = y_1 + \frac{h}{2}\left(k_1 + k_2\right). \text{ —— } (2)$$

$$k_1 = hf(x_n, y_n)$$

$$k_1 = hf(x_1, y_1).$$

$$k_1 = (0.1)(2.205 - 0.1).$$

$$k_1 = 0.2105.$$

$k_2 = hf(x_n + h, y_n + k_1)$

$k_2 = (0.1) f(x_1 + h, y_1 + k_1)$

$k_2 = (0.1) f(0.2, 2.4155)$

$k_2 = 0.22155.$

putting in eq(2)

$y_2 = 2.205 + \frac{h}{2}(0.2105 + 0.22155)$

$\boxed{y_2 = \cancel{2.427}}$ 2.2266.

Rang kutta 4th order

Formula.

$$y_{n+1} = y_n + \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$$

where.

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right).$$

$$k_4 = hf(x_n + h, y_n + k_3).$$

Example

Given that

$\frac{dy}{dx} = x+y^2$

$y(0) = 1$.

$h = 0.1$.

Find $y(0.2)$.

Sol:

$x_0 = 0$ $\quad$ $y_0 = 1$
$x_1 = 0.1$ $\quad$ $y_1 = ?$
$x_2 = 0.2$ $\quad$ $y_2 = ?$

$$y_{n+1} = y_n + \frac{1}{6}\left[K_1 + 2K_2 + 2K_3 + K_4\right].$$

let $n=0$

$$y_1 = y_0 + \frac{1}{6}\left[K_1 + 2K_2 + 2K_3 + K_4\right] \text{ — (1)}$$

Now.

$k_1 = hf(x_0, y_0).$

$k_1 = (0.1)(0 + 1^2)$

$k_1 = 0.1.$

$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$

$k_2 = (0.1)(0 + \frac{0.1}{2}, 1 + \frac{0.1}{0.1}$

$k_2 = (0.1)(0.05 + (2)^2)$

$k_2 = 0.405.$

$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$

$k_3 = (0.1)(0.05 + 1.2025)$

$k_3 = 0.125$

$k_4 = hf(x_0 + h, y_0 + k_3.)$

$k_4 = (0.1)(0.1 + 1.125).$

$k_4 = 0.122$

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putting in eq(1).

$$y_1 = 1 + \frac{1}{6}\left(0.2 + 2(0.405) + 2(0.125) + 0.122\right)$$

$$\boxed{y_1 = 1.213}$$

Ab issi tarah y_2 nikalo mere dil ka zoor nahi.