ORDINARY DIFFERENTIAL EQUATIONS (ODE)

- D.E
  - ODE
    - approximate solution. (Numerical solutions)
      - pure-numerical solutions
        - single step methods
          - ↳ Euller's Methods
          - ↳ RKs Methods
            - ↳ Runge Kutta
        - Multipsteps Methods
          - ↳ Adam's Methods
          - ↳ Milien's Methods.
      - semi-numerical solution (series solutions)
    - Exact solution
  - P.D.E

All Numerical solution are based on Tailors serious :-

$$y(x+h) = y(x) + hy'(x) + \frac{h^2}{2!}y''x + \dots\dots$$

Normally

$$y = f(x)$$

IN ODE

$$\boxed{\frac{dy}{dx} = f(x,y).}$$

$$\frac{d^2y}{dx} = f(x,y,y')$$

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$$y(x+h) = y(x) + hy'(x) \quad \text{Euller formula.}$$

$$\boxed{y(n+1) = y(n) + hf(x_n, y_n).}$$

same h h Δx

$x_0 \quad x_1 \quad x_2 \quad x_3 \quad x_4 \quad \dots\dots \quad x_{n-1} \quad x_n$

$x \quad x+h \quad x+2h \quad x+3h$

if domain $[0,1]$.

$h = 0.1$

$x_0 = 0$

~~$x_1 = x_0 + h = 0.1$~~

$\vdots$

$x_{10} = x_9 + h = 1.0$.

if $n=0$.

$$y_1 = y_0 + hf(x_0 + y_0).$$

if $n=1$

$$y_2 = y_1 + hf(x_1, y_1)$$

if $n=2$

$$y_3 = y_2 + hf(x_2, y_2).$$

SECOND Method.

$$\frac{\Delta y}{\Delta x} = \tan x$$

$$\Delta y = \Delta x \tan x$$

$$y_2 - y_1 = h\frac{dy}{dx}$$

$$y_2 = y_1 + hf(x_1, y_1).$$

$$y_3 = y_2 + hf(x_2, y_2)$$

Example

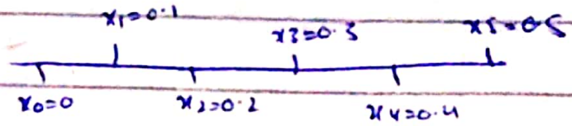
initial condition

$$\frac{dy}{dx} = x - y, \quad \boxed{y(0) = 1}$$

$x_0 = 0 \qquad y_0 = 1$

Sol

let $h = 0.1$ and domain



$$y_{n+1} = y_n + h f(x_n, y_n).$$

$x - y$

$$y_{n+1} = y_n + h(x_n - y_n).$$

if $n = 0$.

$$y_1 = y_0 + h(x_0 - y_0).$$

$$y_1 = 1 + 0.1(0 - 1).$$

$$y_1 = 1 - 0.1$$

$$y_1 = 0.9.$$

if $n = 1$.

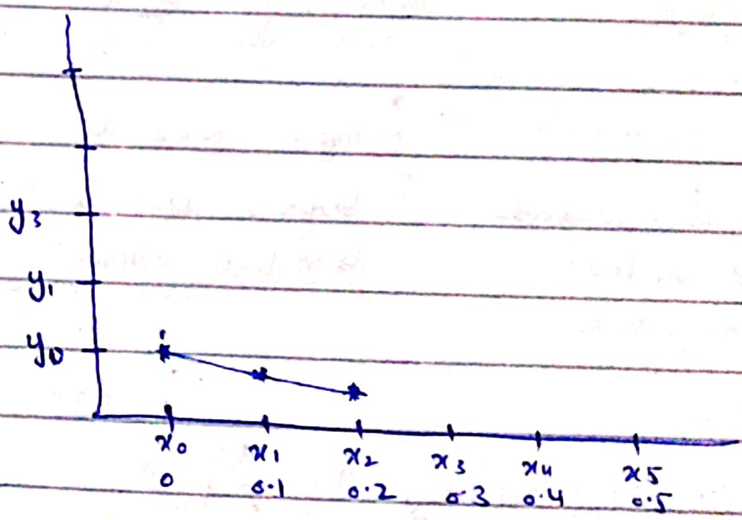
$$y_2 = y_1 + h f(x_1, y_1).$$

$$y_2 = y_1 + h(x_1 - y_1).$$

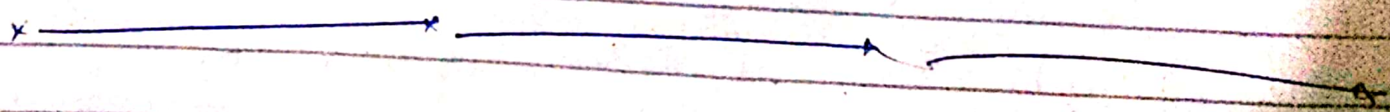
$$y_2 = 0.9 + 0.1(0.1 - 0.9).$$

$$y_2 = 0.9 - 0.08$$

$$y_2 = 0.82$$



find Till y_5.



RK's - METHOD's

The exact solution of $\frac{dy}{dx} = x - y$. $y = 2e^{-x} + x - 1$.

$y(0.1) = 2e^{-0.1} + 0.1 - 1$

$= 0.909.$

RK - 2

$$y_{n+1} = y_n + \frac{h}{2}(k_1 + k_2).$$

where $k_1 = f(x_n, y_n)$

$k_2 = f(x_n + h, y_n + k_1)$.

Example

$$\frac{dy}{dx} = x - y \qquad y(0) = 1 \qquad h = 0.1 \qquad 0 \le x \le 0.5.$$

$n = 0$

$$y_1 = y_0 + \frac{h}{2}(k_1 + k_2)$$

$k_1 = f(x_0, y_0)$

$= (x_0 - y_0)$

$\boxed{k_1 = 0 - 1 = -1.}$

$k_2 = (x_0 + h - (y_0 + k_1))$

$0 + 0.1 - (1 - 1)$

$\boxed{k_2 = 0.1}$

$y_1 = y_0 + \frac{h}{2}(k_1 + k_2)$

$= 1 + \frac{0.1}{2}(-1 + 0.1)$

$= 1 - \frac{0.09}{2} \Rightarrow 1 - 0.045$

$\boxed{y_1 = 0.955}$

example 12.5
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RK-4:-

$y_{n+1} = y_n + \frac{1}{6}(K_1 + 2(K_2 + K_3) + K_4)$.

where $K_1 = h\, f(x_n, y_n)$
$K_2 = h\, f(x_n + h/2, y_n + \frac{K_1}{2})$.
$K_3 = h\, f(x_n + h/2, y_n + K_2/2)$
$K_4 = h\, f(x_n + h, y_n + K_3$

$f(x,y) = y' = x - y$. $\quad y(0) = 1$, $h = 0.1$.

$K_1 = (x_n - y_n)h$
$K_1 = (x_0 - y_0)h$
$K_1 = (0 - 1)h$
$\boxed{K_1 = (-1)}(0.1)$
$\boxed{K_1 = -0.1}$

$K_2 = h((x_0 + h/2) - (y_0 + \frac{K_1}{2}))$.
$K_2 = h((0 + \frac{0.1}{2}) - (1 - 0.1/2))$.
$K_2 = 0.1(\frac{0.1}{2} - \frac{1}{2})$
$\boxed{K_2 \quad -0.09}$

$K_3 = h\left[(x_0 + h/2) - (y_n + \frac{K_2}{2})\right]$

$= h\left[(0 + \frac{0.1}{2}) - (1 - \frac{0.45}{2})\right]$

$= 0.1\left[\frac{0.1}{2} - (\frac{1.55}{2})\right]$

$= 0.1\left(\frac{0.1 - 1.55}{2}\right)$

$\boxed{K_3 = \cancel{-0.725}\ -0.0905}$

$K_4 = [(x_0 + h) - (y_0 + K_3)]h$
$= ((0 + 0.1) - (1 - 0.725))0.1$
$= ((0.1) - 0.275)(0.1$
$\boxed{K_4 = \cancel{-0.175}}$ -0.08095

Now

$y_1 = y_0 + \frac{0.1}{6}(\cancel{1} + 2(\cancel{-0.45 - 0.725}) - 0.175)$

$y_1 = y_0 + \frac{1}{6}(-0.1 + 2(-0.09 - 0.0905) - 0.080$

$\boxed{y_1 = 0.90968}$

$$y_2 = y_1 + \frac{1}{6}\left(K_1 + 2(K_2 + K_3) + K_4\right).$$

$h = 0.1.$

$y_1 = 0.90968$

$x_1 = 0.1.$

$K_1 = h(x_1 - y_1)$

$K_1 = (0.1)(0.1 - 0.90968)$

$\boxed{K_1 = -0.08097}$

$$K_2 = (0.1)\left(\left(0.1 + \frac{0.1}{2}\right) - \left(0.90968 - \frac{0.08097}{2}\right)\right)$$

$\boxed{K_2 = -0.07192}$

$$K_3 = (0.1)\left(\left(0.1 + \frac{0.1}{2}\right) - \left(0.90968 - \frac{0.07192}{2}\right)\right)$$

$\boxed{K_3 = -0.07237}$.

$$K_4 = (0.1)\left((0.1 + 0.1) - (0.90968 - 0.07237)\right)$$

$\boxed{K_4 = -0.063731}$

$$y_2 = 0.90968 + \frac{1}{6}\left(-0.08097 + 2(-0.07192 - 0.07237) - 0.06373\right).$$

$y_2 = 0.83747$

Scanned with CamScanner

y_3 ——

$y_0 \; y_1 \; y_2$

MULTI STEPS METHODS:-

* Require some single steps method for kick up.

MILNE'S METHOD:-

Predictor (Milen's) $\Rightarrow$ ~~y_{n+3}~~ $y_{n+1} = y_{n-3} + \frac{4}{3} h [2(f_n) - (f_{n-1}) + 2(f_{n-2})]$

Corrector $y_{n+1} = y_{n-1} + \frac{h}{3} [f_{n+1} + 4(f_n) + (f_{n-1})]$.

~~$n=3$~~

Example

$f(x,y) = x - y$, $h = 0.1$ $\quad 0 \le x \le 0.5$

$y_0 = 1$, $y_1 = 0.90968$, $y_2 = 0.83747$, $y_3 = 0.78164$.

if $n=3$

so predictor = $y_{p4} = y_0 + \frac{4}{3}(0.1)[2(x_3 - y_3) - (x_2 - y_2) + 2(x_1 - y_1)]$.

$y_{p4} = 1 + \frac{0.4}{3} [2(0.3 - 0.78164) - (0.2 - 0.83747) + 2(0.1 - 0.90968)]$

$\boxed{y_{p4} = 0.740644}$: predicted value of y_4.

correct

$y_{c4} = y_2 + \frac{h}{3} [(x_4 - y_4) + 4(x_3 - y_3) + (x_2 - y_2)]$.

$y_{c4} = 0.83747 [(0.4 - 0.740644) + 4(0.3 - 0.78164) + (0.2 - 0.83747)]$

$\boxed{y_{c4} = 0.74064}$ $\rightarrow$ Approximate value of y_4.

Adams-Moulton Method:

Predictor: $y_{n+1} = y_n + \frac{h}{24}\left[55(f_n) - 59(f_{n-1}) + 37(f_{n-2}) - 9f_{n-3}\right]$.

Corrector: $y_{n+1} = y_n + \frac{h}{24}\left(9(f_{n+1}) + 19(f_n) - 5(f_{n-1}) + f(n-2)\right)$.

Example

i) $n = 3$

Predictor

$y_{p}^{4} = y_3 + \frac{0.1}{24}\left[55(x_3 - y_3) - 59(x_2 - y_2) + 37(x_1 - y_1) - 9(x_0 - y_0)\right]$.

$y_p 4 =$

Corrector $=$ $y_c^4 = y_3 + \frac{0.1}{24}\left[9(0.4 - y_4) + 19(x_3 - y_3) - 5(x_2 - y_2) + (x_1 - y_1)\right]$.