Math

Date: / /20 (Interpolation). Day:

General

The difference table

x	$F(x)$	Δ	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6
x_0	y_0						
		Δy_0					
x_1	y_1		$\Delta^2 y_0$				
		Δy_1		$\Delta^3 y_0$			
x_2	y_2		$\Delta^2 y_1$		$\Delta^4 y_0$		
		Δy_2		$\Delta^3 y_1$		$\Delta^5 y_0$	
x_3	y_3		$\Delta^2 y_2$		$\Delta^4 y_1$		$\Delta^6 y_0$
		Δy_3		$\Delta^3 y_2$		$\Delta^5 y_1$	
x_4	y_4		$\Delta^2 y_3$		$\Delta^4 y_2$		
		Δy_4		$\Delta^3 y_3$			
x_5	y_5		$\Delta^2 y_4$				
		Δy_5					
x_6	y_6						

Example 5.3 :- In the following table values $y = x + \sin x^2$ are tabulated.

x	y
1.0	1.84147
1.1	2.03562
1.2	2.19146
1.3	2.29290
1.4	2.32571
1.5	2.27807
1.6	2.149355

(861)

Construct a difference table and estimate F(1.04), F(1.57), F(1.28), F(1.384)

(861)

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why N of the Δ^6 ???

y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1.8417					
	0.19415				
2.03568		-0.03831			
	0.15584		-0.01609		
2.19146		-0.05440		0.0136	
	0.10144		-0.01473		0.00305
2.29290		-0.06913		0.00441	
	0.03831		-0.01032		0.003785
2.33501		-0.07945		0.008195	
	-0.04714		-0.008195		
2.27807		-0.81575			
	-0.128715				
2.149355					

(a)

To estimate $F(1.04)$, we use Newton forward formula:

$$y_P = y_0 + P\Delta y_0 + \frac{P(P-1)}{2!}\Delta^2 y_0 + \frac{P(P-1)(P-2)}{3!}\Delta^3 y_0$$

Here

$\boxed{y_0 = 1.8417}$

and

"first value of y_0"

$$p = \frac{x - x_0}{h}$$

h = difference of x value

if $x = (1.04)$ ~~equal value of x~~

$x_0 = 1.00 \rightarrow$ first value of x

$$P = \frac{(1.04 - 1.00)}{0.1} = \frac{0.04}{0.1}$$

$$\boxed{P = 0.4}$$

put the values in formula.

$$y_P = y_0 + P\Delta y_0 + \frac{P(P-1)}{2!}\Delta^2 y_0 + \frac{P(P-1)(P-2)}{3!}\Delta^3 y_0$$

$$\Rightarrow 1.8147 + 0.4(0.19415) + \frac{0.4(0.4-1)}{2!}(-0.0383)$$

$$+ \frac{0.4(0.4-1)(0.4-2)}{3!}(-0.01609)$$

$1.89936 + 0.0035698536 - 0.008059$

$$\boxed{yp = 1.893864}$$

(b)

To estimate $F(1.57)$, we use Newton's backward difference.

$$yp = y_0 + p\nabla y_0 + \frac{p(p+1)}{2!}\nabla^2 y_0 + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_0$$

$+ \cdots\cdots$

Here

$y_0 = 2.149355$
last vale of "y"

$p = \frac{x - x_0}{h}$, $x_0 \rightarrow$ last value of "x"

$x = 1.57$ given val

$p = 1.57 - 1.6$

$\boxed{p = -0.03}$

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$$y_p = y_0 + p\nabla y_0 + \frac{p(p+1)}{2!}\nabla^2 y_0 + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_0$$

$$y_p = 2.149355 + (-0.03)(-0.12875) + \cancel{(-0.03)}$$

$$+ \frac{(-0.03)(-0.03+1)}{2!}(-0.081575) +$$

$$+ \frac{-0.03(-0.03+1)(-0.03+2)}{3!}(-0.002125)$$

$$\boxed{y_p = 2.14666}$$

(Numerical Differentiation!)

Newton forward interpolation formula!

$$y(x) = y_0 + P\Delta y_0 + \frac{P(P-1)}{2!}\Delta^2 y_0 + \frac{P(P-1)(P-2)}{3!}\Delta^3 y_0$$

Here $P = \dfrac{x - x_0}{h}$.

So $\boxed{x = Ph + x_0}$

Put in Newton forward!

$$y(x_0+Ph) = y_0 + P\Delta y_0 + \frac{(P^2-P)}{2!}\Delta^2 y_0 + \frac{P^3-3p^2+2p}{3!}\Delta^3 y_0$$

Differentiate with respect to "p" "so"

$$y'(x)\,h = \Delta y_0 + \frac{(2P-1)}{2!}\Delta^2 y_0 + \frac{3p^2-6p+2}{3!} + \cdots$$

$$\boxed{y'(x) = \frac{1}{h}\left[\Delta y_0 + \frac{(2P-1)}{2!}\Delta^2 y_0 + \frac{3p^2-6p+2}{3!}\Delta^3 y_0\right]}$$

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Again differentiating with respect to "p"

$$\boxed{y''(x) = \frac{1}{h^2}\left[\Delta^2 y_0 + (P-1)\,\Delta^3 y_0 + \cdots\right]}$$

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Exp 10.2 :: Find the first and second derivative of the functions tabulated below at the points $x = 2.31$ and 3.6

x	0	1	2	3	4	5	6
y	2	3	10	29	66	127	218

(801)

First draw the difference table!

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	2				
		1			
1	3		6		
		7		6	
2	10		12		0
		19		6	
3	29		18		0
		37		6	
4	66		24		0
		61		6	
5	127		30		
		91			
6	218				

For "3", 29 and 69 → ~~...~~ 37 → Δy_0

For "3", 37 and 61 → ~~...~~ 24 → $\Delta^2 y_0$

For "3" 24 and 30 → 6 $\Delta^3 y_0$ ~~...~~

$87 - 19 = \boxed{23}$

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when $x = 3$, $x_0 = 3$

$$p = \frac{x - x_0}{h} \Rightarrow \frac{3-3}{1} = 0$$

$$\boxed{p = 0}$$

~~Using first differentiation Formula~~

→ using numerical differentiation first derivative formula using newton Forward.

$$y'(x) = \frac{1}{h}\left[\Delta y_0 + \frac{(2p-1)}{2!}\Delta^2 y_0 + \frac{3p^2-6p+2}{3!}\Delta^3 y_0 + \dots\right.$$

$$y'(x) = \frac{1}{1}\left(37 + \frac{(2(0)-1)}{2!}24 + \frac{3(0)-6(0)+2}{3!}6 + \right.$$

$$y'(x) = 37 - \frac{24}{2} + \frac{2 \cdot 6}{6} +$$

$$y'(x) = 37 - 12 + 2$$

$$\boxed{y'(x) = 27}$$

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→ Using numerical differentiation and derivative.

$$y''(x) = \frac{1}{h^2}\left[\Delta^2 y_0 + (p-1)\,\Delta^3 y_0 + \cdots\right]$$

(Sol)

$h = 1,\quad \Delta^2 y_0 = 24,\quad \Delta^3 y_0 = 6$

$p = 0$

$$y''(x) = \frac{1}{1}\left[24 + (0-1)\,6 + \cdots\right]$$

$$y''(x) = [24 - 6]$$

$$\boxed{y''(x) = 18}$$

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Numerical Differentiation using Newton Backward

"First Derivative"

$$y(x) = y_0 + p\nabla_0 + \frac{p(p+1)}{2!}\nabla^2 y_0 + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_0 + \cdots$$

$$p = \frac{x - x_0}{h}$$

$$x - x_0 = ph$$

$$\boxed{x = ph + x_0}$$

$$y(ph + x_0) = y_0 + p\nabla y_0 + \frac{(p^2+p)}{2!}\nabla^2 y_0 + \frac{p^3+3p^2+2p}{3!}\nabla^3 y_0 + \cdots$$

differentiate w.r.t to "p"

$$y'(x)\,h = 0 + \nabla y_0 + \frac{(2p+1)}{2!}\nabla^2 y_0 + \frac{3p^2+6p+2}{3!}\nabla^3 y_0 + \cdots$$

$$\boxed{y'(x) = \frac{1}{h}\left[\nabla y_0 + \frac{(2p+1)}{2!}\nabla^2 y_0 + \frac{3p^2+6p+2}{3!}\nabla^3 y_0 + \cdots\right]}$$

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2nd Derivative

$$y''(x) = \frac{1}{h^2}\left[0 + \frac{2}{2}\nabla^2 y_0 + \frac{(6p+6+0)}{6}\nabla^3 y_0\right.$$

$$y''(x) = \frac{1}{h^2}\left[\nabla^2 y_0 + \frac{6(p+1)}{6}\nabla^3 y_0\right.$$

$$\boxed{y''(x) = \frac{1}{h^2}\left[\nabla^2 y_0 + (p+1)\nabla^3 y_0\right.}$$

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Find the first derivative and second derivative of the function tabulated Exp 10.1 at the points. $x = 5$, and 5.7

(Use Diff table of 10.1)

$x = 5$, $x_0 = 5$

$p = \frac{x - x_0}{h} \Rightarrow \frac{5-5}{1} = 0$ $\boxed{p = 0}$

~~Numerical~~

(first derivative using newton backward)

$$y'(x) = \frac{1}{h}\left[\nabla y_0 + \frac{(2p+1)}{2!}\nabla^2 y_0 + \frac{(3p^2+6p+2)}{3!}\nabla^3 y_0 + \dots\right.$$

$$y'(x) = \frac{1}{1}\left[61 + \frac{(2(0)+1)}{2} 24 + \frac{(3(0)^2+6(0)+2)}{3!} 6\right]$$

$$y'(x) = 61 + 12 + 2$$

$$\boxed{y'(x) = 75}$$

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$$y''(x) = \frac{1}{h^2}\left[\nabla^2 y_0 + (p+1)\nabla^3 y_0\right]$$

$$y''(x) = \frac{1}{1}\left[84 + (0+1)\,6\right]$$

$$y''(x) = [84 + 6]$$

$$\boxed{y''(x) = 30}$$

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Now

$x = 5.7$, $x_0 = 6$

$p = \dfrac{x - x_0}{h} \Rightarrow \dfrac{5.7 - 6}{1}$

$\boxed{p = -0.3}$

$y'(x) = \dfrac{1}{h}\left[\nabla y_0 + \dfrac{(2p+1)}{2!}\nabla^2 y_0 + \dfrac{(3p^2 + 6p + 2)}{3!}\nabla^3 y_0\right.$

$y'(x) = \dfrac{1}{1}\left[91 + \dfrac{(2(-0.3)+1)}{2} 30 + \dfrac{(3(0.3)^2 + 6(0.3) + 2)}{6} 6\right.$

$y'(x) = 91 + \dfrac{(-0.6+1)}{2} 30 + \dfrac{(3(0.09) - 1.8 + 2)}{6} 6$

$y'(x) = 91 + 6 + 0.47$

$\boxed{y'(x) = 97.47}$

18

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$$y''(x) = \frac{1}{h^2}\left[\nabla^2 y_0 + (p+1)\nabla^3 y_0\right]$$

$$y''(x) = \frac{1}{1}\left[30 + (-0.3+1)\,6\right]$$

$$y''(x) = \left[30 + 4.8\right]$$

$$\boxed{y''(x) = 34.8}$$

→ Two points forward Formula!

$$y'_{x_0} = \frac{1}{h}\left[y_1 - y_0\right]$$

→ Three-point forward Formula!

$$\frac{1}{2h}\left[-y_2 + 4y_1 - 3y_0\right]$$

→ Four point forward Formula'

$$\frac{1}{6h}\left[2y_3 - 9y_2 + 18y_1 - 11y_0\right]$$