$$\boxed{h = x_0 - x_1}$$

↳ difference b/w input point

forward ↑

starting x x_0 x

back ↓

$x = x_0 + ph$

$$\boxed{p = \frac{x - x_0}{h}}$$

$h = x_{i+1} - x_i$

Newton forward formula

$$y_p = y_0 + \Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \ldots\ldots \frac{p(p-1)\cdots(p-(n-1))}{n!}\Delta^n y_0 \quad \text{--- (A)}$$

Newton Backward formula

Numerical Differenciation (Forward).

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0 + \cdots$$

$$y_p = y_0 + p\Delta y_0 + \frac{p^2 - p}{2!}\Delta^2 y_0 + \frac{p^3 - 2p^2 + 2p}{3!}\Delta^3 y_0 + \frac{p^4 - 6p^3 + 11p^2 - 6p}{4!} + \cdots\cdots$$

$x = x_0 + ph$

$P = \frac{x - x_0}{h}$

$\frac{dp}{dx} = \frac{1}{h}$

$\frac{dy}{dx} = \frac{dy}{dp} \times \frac{dp}{dx}$

$$\frac{dy_p}{dx} = y'_p = \frac{d}{dx}\left(\text{---------} y \text{---------} \right)$$

$$= \frac{1}{h}\frac{d}{dp}\left(y_0 + \Delta y_0 p + \frac{p^2 - p}{2!}\Delta^2 y_0 + \frac{p^3 - 3p^2 + 2p}{3!}\Delta^3 y_0 + \cdots\cdots \right)$$

$$y'_p = \frac{1}{h}\left(0 + \Delta y_0 + \frac{2p-1}{2!}\Delta^2 y_0 + \frac{3p^2 - 6p + 2}{3!}\Delta^3 y_0 + \frac{4p^3 - 18p^2 + 22p - 6}{4!}\Delta^4 y_0 \right.$$

$$\boxed{\frac{dy}{dx} = \frac{1}{h}\frac{dy}{dp}}.$$

$$y''_p = \frac{1}{h^2}\left(\frac{2}{2!}\Delta^2 y_0 + \frac{6p - 6}{3!}\Delta^3 y_0 + \frac{12p^2 - 36p + 22}{4!}\Delta^4 y_0 \right)$$

$$y'''_p = \frac{1}{h^3}\left(0 + \frac{6}{3!}\Delta^3 y_0 + \frac{24p - 36}{4!}\Delta^4 y_0 + \cdots\cdots \right).$$

$$y''''_p = \frac{1}{4}\left(0 + 0 + \frac{24}{4!}\Delta^4 y_0 + \cdots \right).$$

⋮

Scanned with CamScanner

$p = x_0 - x_0$

At Tabulated point $x - x_0 = \text{val} \rightarrow p = 0.$

put $p = 0$,

$$y'p = \frac{1}{h}\left(\Delta y_0 - \tfrac{1}{2}\Delta^2 y_0 + \tfrac{2}{3!}\Delta^3 y_0 - \tfrac{6}{4!}\Delta^4 y_0\right).$$

(Backward).

$$y_p \approx y_0 + P\nabla y_0 + \frac{P(P+1)}{2!}\nabla^2 y_0 + \frac{P(P+1)(P+2)}{3!}\nabla^3 y_0 + \frac{P(P+1)(P+2)(P+3)}{4!}\nabla^4 y_0 + \cdots$$

$$y_p = y_0 + P\nabla y_0 + \frac{P^2+P}{2!}\nabla^2 y_0 + \frac{P^3+3p^2+2p}{3!}\nabla^3 y_0 + \frac{P^4+6p^3+11p^2+6p}{4!}\nabla^4 y_0 + \cdots$$

$$y'p = \frac{1}{h}\left(\nabla y_0 + \frac{2p+1}{2!}\nabla^2 y_0 + \frac{3p^2+6p+2}{3!}\nabla^3 y_0 + \frac{4p^3+18p^2+22p+6}{4!}\nabla^4 y_0 + \cdots\right)$$

$y''p$

At Tabulated point

$$y'p = \frac{1}{h}\left(\nabla y_0 + \frac{1}{2}\nabla^2 y_0 + \frac{2}{3!}\nabla^3 y_0 + \frac{6}{4!}\nabla^4 y_0 \cdots\right).$$

$$y''p = \frac{1}{h^2}\left(\nabla^2 y_0 + \frac{6p+6}{3!}\nabla^3 y_0 + \frac{12p^2+36p+22}{4!}\nabla^4 y_0 \cdots\right).$$

At p Tabulated point.

$$y''p = \frac{1}{h^2}\left(\nabla^2 y_0 + \frac{6\nabla^3 y_0}{3!} + \frac{22\nabla^4 y_0}{4!} + \cdots\right).$$

NEWTON

Book
pg #167

CENTERAL INTERPOLATION:

$$y_p = y_0 + p\,\mu\delta y_0 + \frac{p^2}{2!}\delta^2 y_0 + \frac{p(p^2-1)}{3!}\mu\delta^3 y_0 + \frac{p^2(p^2-1)}{4!}\delta^4 y_0 \cdots$$

By taking derivatives:-

$$y'_p = \frac{1}{h}\left[0 + \mu\delta y_0 + p\,\delta^2 y_0 + \frac{3p^2-1}{6}\mu\delta^3 y_0 + \frac{4p^3-2p}{24}\delta^4 y_0 \cdots\right].$$

$$y''_p = \frac{1}{h^2}\left[0+0+\delta^2 y_0 + p\,\mu\delta^3 y_0 + \frac{12p^2-2}{24}\delta^4 y_0 \cdots\right].$$

$$y'''_p = \frac{1}{h^3}\left[0+0+0+\mu\delta^3 y_0 + p\,\delta^4 y_0 + \cdots\right].$$

$$y''''_p = 1/h^4\left[0+0+0+0+\delta^4 y_0 + \cdots\right].$$

At tabulated point $p=0$:-

$$y'_p = \frac{1}{h}\left[\mu\delta y_0 - \frac{1}{6}\mu\delta^3 y_0 + \cdots\right].$$

$$y''_p = 1/h^2\left[\delta y_0 - 1/12\,\delta^4 y_0 + \cdots\right.$$

$$y'''_p = 1/h^3\left[\mu\delta^3 y_0 + \cdots\right].$$

Deravative → show Behavior (continuous).

INTEGRATION →

pg # 52.
example = 4.1

→ ON-EQUAL DATA Distributed →

Tripzoid rule:
$$\int_a^b f(x)\,dx = \frac{h}{2}\left[f(x_0) + f(x_1)\right]$$
↳ for finding area.

LAGRANGE'S ~~LIGRANTION~~ INTERPOLATION →

x	y
$x_0 = 1$	$y_0 = 6$
$x_1 = 2$	$y_1 = 5$
$x_2 = 7$	$y_2 = 10$
$x_3 = 9$	$y_3 = 11$

$$L_0(x) = \frac{(x-2)(x-7)(x-9)}{(1-2)(1-7)(1-9)}.$$

$$y = L_0(x)y_0 + L_1(x_1)y_1 + L_2(x)y_2.$$

$$L_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}$$

$$L_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}.$$

$$L_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}.$$

Tripzoid rule with error Term in.

$$\int_{x_0}^{x_1} f(x)\,dx = \frac{h}{2}\left[f(x_0) + f(x_1)\right] - \frac{h^3}{12} f''(\xi_e)$$

(exi)

* if fon is linear Than error will be zero.

$f(x) = x^2$

$f'(x) = 2x$

$f''(x) = 2.$

$f''(\xi_e) = 2.$

$f(x) = x^3$

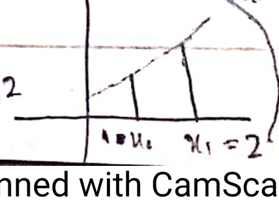
$f'(x) = 3x^2$

$f''(x) = 6x.$

$f(\xi_e) = 6x$

$f(2) = 6(2) = 12$

ξ_e = put maximum value to get maximum error.



Example

$I = \int_0^1 (x+1)\,dx$

By direct integration

$$= \int_0^1 x\,dx + \int_0^1 1\,dx \Rightarrow \frac{x^2}{2}\Big|_0^1 + x\Big|_0^1 \Rightarrow \left(\tfrac{1}{2} - \tfrac{0}{2}\right) + (1-0) = \frac{1}{2} + 1 = \frac{3}{2}$$

ANS

By numeric integration

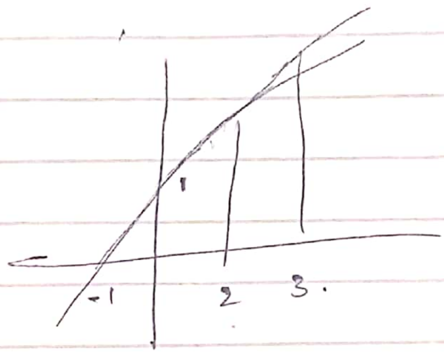
$f(x) = x+1$ $\qquad h = x_1 - x_0 =$

$f(x_0) = 0+1 = 1$ $\qquad h = 1-0 = 1$

$f(x_1) = 1+1 = 2.$

$$\int_{x_0}^{x_1} f(x)\,dx = \tfrac{1}{2}[1+2].$$

$$= 3/2.$$



$I = \int_0^1 3x^2\,dx.$

By direct. $\qquad \frac{3x^3}{3}\Big|_0^1 = 1$ ANS.

By numeric integration

$f(x_0) = 3x^2$ $\qquad h=1$

$f(x_0) = f(0) = 0.$

$f(x_1) = f(1) = 3.$

$$\int_0^1 f(x)\,dx = \frac{1}{2}[0+3].$$

$$= 3/2$$

$$= 1.5$$

This Method if exact value is not possible.

$|error| = |exact - Appropriate|$

$= |1 - 1.5|$

$= |0.5|$

$= 0.5$

$\frac{h^3}{12} f''(\xi)$

$f(x) = 3x^2$

$f'(x) = 6x$

$f''(x) = 6$

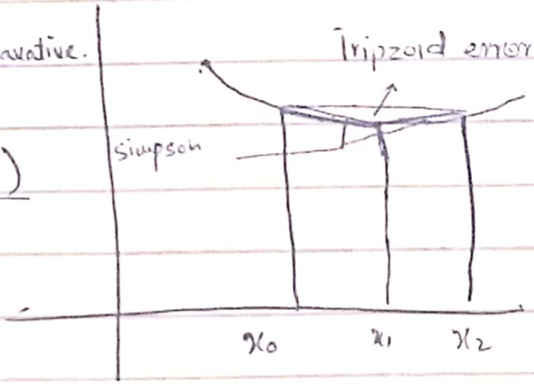
$\frac{1}{12} \times 6$

$= \frac{1}{2} = 0.5$

Simpson 1/3 rule:-

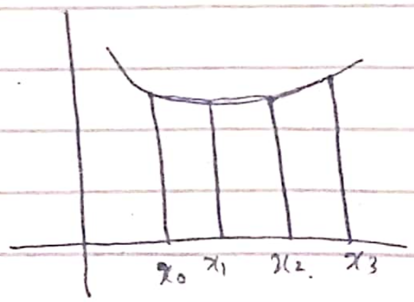
fourth derivative.

$$\int_{x_0}^{x_2} f(x)dx = \frac{h}{3}\left[f(x_0) + 4f(x_1) + f(x_2)\right] - \frac{h^5 f^{(4)}(\xi)}{90}$$



Simpon 3/8 Rule:

$$\int_{x_0}^{x_3} f(x)dx = \frac{3}{8}h\left[f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)\right] - \frac{3}{80} h^5 f^{(4)}(\xi)$$



Example:

$$\int_1^3 f(x)dx = \int_1^3 (x^3 - 2x^2 + 7x - 5)dx.$$

pg # 192 – 194.

By direct

$$= \frac{x^4}{4} - \frac{2x^3}{3} + \frac{7x^2}{2} - 5x. \Rightarrow \left(\frac{81}{4} - \frac{54}{3} + \frac{63}{2} - 15\right) - \left(\frac{1}{4} - \frac{2}{3} + \frac{7}{2} - 5\right)$$

$$= 20.666 = 20 \times \frac{2}{3}$$

By Trap.

$h = 3 - 1 = 2.$

$f(x_0) = 1$

$f(x_1) = f(3) = 25$

$\frac{2}{2}[1 + 25]$

$= 25$

$h = b - a.$

error Term:

$\frac{h^3}{12} f''(\xi).$

$= \frac{8}{12} \times 14$

$9.333.$

$f'(x) = 3x^2 - 4x + 7$

$f''(x) = 6x - 4$

$f''(\xi) = 18 - 4 = 14.$

By simpson 1/3 r

$x_1 = x_0 + h$

$x_2 = x_1 + h$

$h = \frac{b-a}{2}$ $h = \frac{3-1}{2} = 1.$

$f(x_0) = f(1) = 1.$

$f(x_1) = 9,\ f(x_2) = 25$

$= \frac{1}{3}[1 + (9)(4) + 25]$

$= \frac{1}{3}[1 + 36 + 25].$

$= 20.6666 \Rightarrow 20.\frac{2}{3}$

error Term:

$f(x)' = 3x^2 - 4x + 7$

$f(x)'' = 6x - 4$

$f(x)''' = 6$

$f(x)^{(4)} = 0.$

so it gives actual solution

By simpson 3/8 r

$h = \frac{b-a}{3} = \frac{3-1}{3} = 2/3.$

$$= \frac{3}{8} \cdot \frac{2}{3}\left[1 + (3)\left(\frac{155}{27}\right) + 3\left(\frac{355}{27}\right) + 25\right].$$

$= 20.666 = 20 \times \frac{2}{3}$

↳ exact solution.

↳ error Term is zero

$x_0 = 1$

$x_1 = 1 + 2/3 = 5/3.$

$x_2 = 5/3 + 2/3 = 7/3$

$x_3 = 7/3 + 2/3 = 9/3 = 3.$

$f(x_1) = f(5/3) = (5/3)^3 - 2(5/3)^2 + 7(5/3) + 5$

$= 4/27 - 2(4/9) + 14/3 + 5$

$= 4/27 - 4/9 + 14/3 + 5$

$= \frac{4 - 12 + 9(14) + 27(5)}{27}$