

Introduction to Computational Thinking and Programming for CFD

Module 13251

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Numerical Fluid and Gas Dynamics, BTU Cottbus - Senftenberg

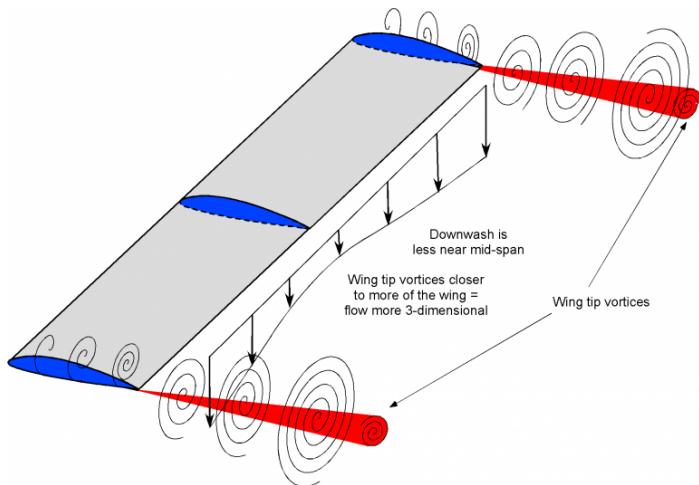


Brandenburg
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3 Meshing, gridded data, and interpolation

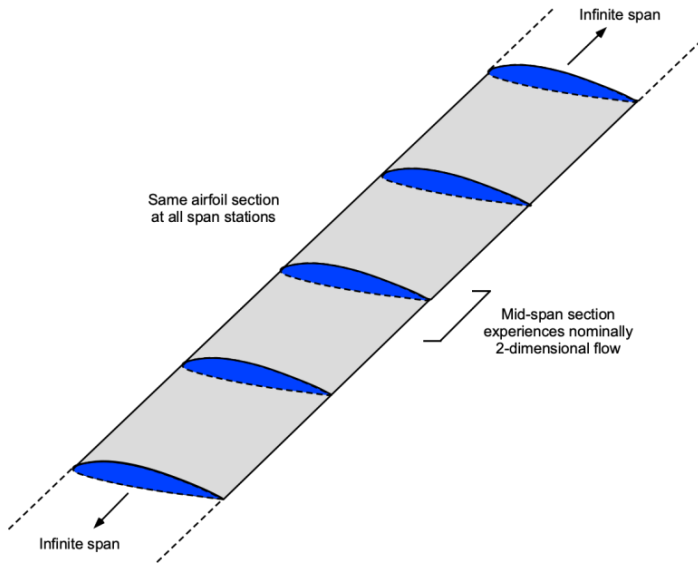
3.1 Overview and definitions

3-D airfoil



Source: <https://eaglepubs.erau.edu/app/uploads/sites/4/2022/07/WingVortices-768x529.png>

2-D airfoil



Source: https://eaglepubs.erau.edu/app/uploads/sites/4/2021/08/Wing_infinite-768x613.png

2-D structured grid (mesh) for the flow around an airfoil I

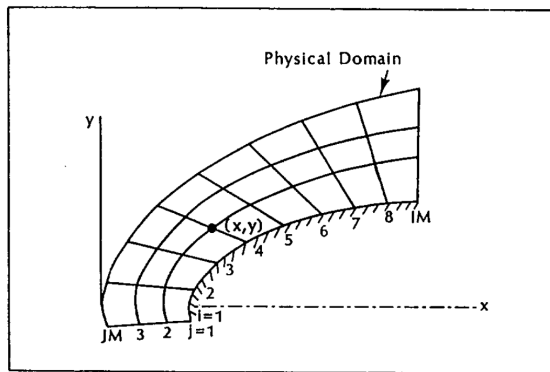


Figure 9-2. A typical 2-D domain for the axisymmetric blunt body problem.

From: K.A. Hoffmann & S.T. Chiang, *Computational Fluid Dynamics: Volume I*, Engineering Education System, 2000.

2-D structured grid (mesh) for the flow around an airfoil II

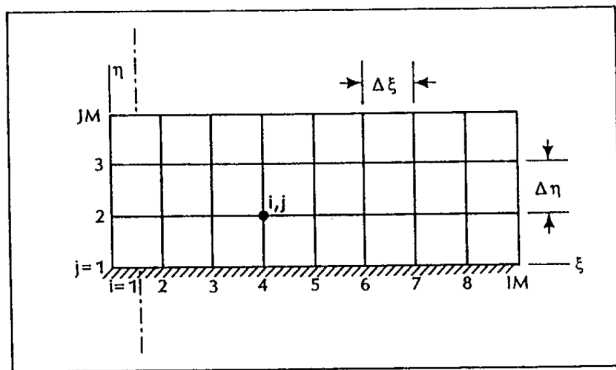


Figure 9-3. Computational domain with constant stepsizes $\Delta\xi$ and $\Delta\eta$.

From: K.A. Hoffmann & S.T. Chiang, *Computational Fluid Dynamics: Volume I*, Engineering Education System, 2000.

2-D structured grid (mesh) for the flow around an airfoil III

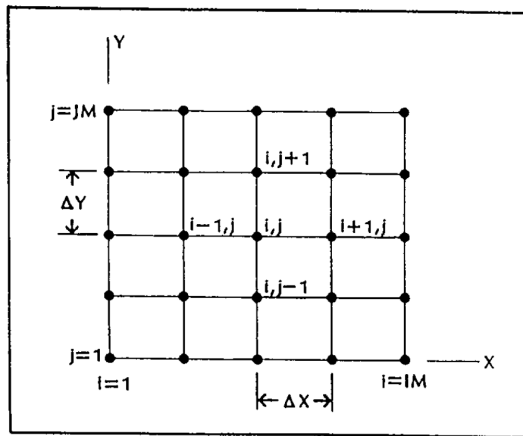


Figure 1-6. Sketch illustrating the nomenclature of computational space.

From: K.A. Hoffmann & S.T. Chiang, *Computational Fluid Dynamics: Volume I*, Engineering Education System, 2000.

Extension to 3-D airfoil geometry II

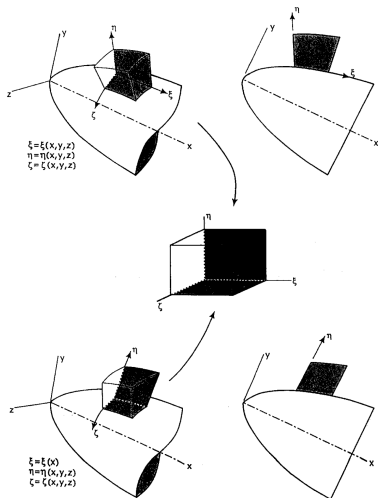


Figure 9-1. Nomenclature for the generalized coordinate system.

From: K.A. Hoffmann & S.T. Chiang, *Computational Fluid Dynamics: Volume I*, Engineering Education System, 2000.

Definitions

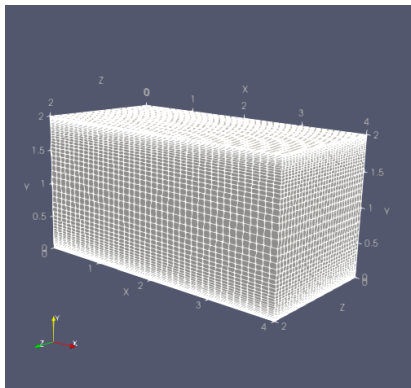
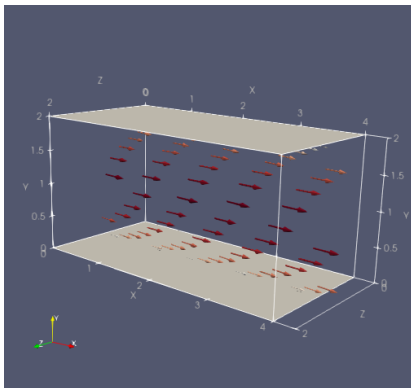
- ▶ A *mesh* (grid) denotes the **discretization of the flow domain** into a finite number, $N_x \cdot N_y \cdot N_z$, of *mesh* (grid) *cells* that store physical properties as **nodal values** at corresponding **nodal points**.
- ▶ Each *mesh cell* (grid cell) is defined by a certain number of **vertices** that are numbered points $\mathbf{x}_{ijk} = (x_{ijk}, y_{ijk}, z_{ijk})^T$, where $i \in [0, N_x]$, $j \in [0, N_y]$, $k \in [0, N_z]$
- ▶ In a Cartesian or **cuboidal grid**, $N_x \cdot N_y \cdot N_z$ mesh cells are built from $(N_x + 1) \cdot (N_y + 1) \cdot (N_z + 1)$ vertices that define the *cell corners* of the *structured grid*.
- ▶ *Mesh generation* (grid generation) denotes the **process** of constructing a mesh (grid). This involves a **numerical algorithm**.
- ▶ *Gridded data* is obtained when some spatial data is **interpolated** to the *nodal points* of the grid, resulting in a set of *nodal values*.

3 Meshing, gridded data, and interpolation

3.2 Algebraic grid generation for channel flow

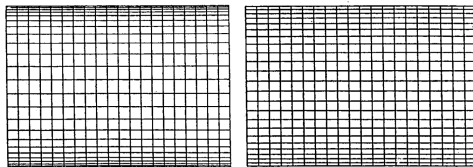
Channel flow configuration and reference grid

- ▶ **Left:** Sketch of the channel flow configuration
- ▶ **Right:** Mesh for the *channel395* OpenFOAM¹ LES tutorial case algebraically generated with the built-in blockMesh application



¹Open-source CFD software *OpenFOAM*: <https://openfoam.org/>

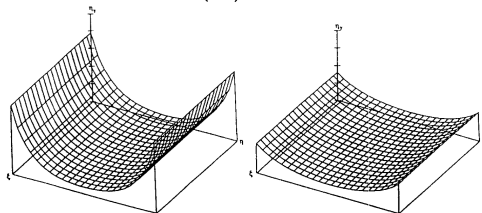
Grid stretching for near-wall refinement (\rightarrow Exercise)



(a) $\beta = 1.05$

(b) $\beta = 1.2$

Figures 9-9a, b. Grid system generated by the transformation function (9-42).



Figures 9-10a, b. Metric distributions for the domains shown in Figure 9-9.

From: K.A. Hoffmann & S.T. Chiang, *Computational Fluid Dynamics: Volume I*, Engineering Education System, 2000.

1-D stretching function for the wall-normal direction

$\xi = \frac{i}{N}$ **equidistant grid coordinate** *auxiliary*

$x = x(\xi)$ **'stretched' spatial coordinate** *physical*

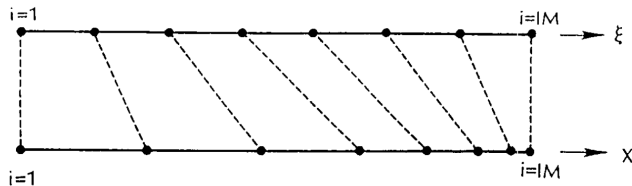


Figure P3-9b. Physical and the corresponding computational grid points distributions.

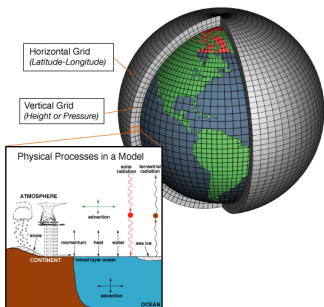
From: K.A. Hoffmann & S.T. Chiang, *Computational Fluid Dynamics: Volume I*, Engineering Education System, 2000.

3 Meshing, gridded data, and interpolation

3.3 Gridded data and interpolation

Gridded data – Application in Earth system science I

- ▶ **Acquire data** from various sources: satellites, balloon soundings, ...
- ▶ **Interpolate data** to various grids as needed by Earth system models
- ▶ **Provide gridded data** interpolated to latitude-longitude grid



Source: NOAA

https:

//celebrating200years.noaa.gov/breakthroughs/
climate_model/modeling_schematic.html

The screenshot shows the NOAA Physical Sciences Laboratory website for Gridded Climate Data. It features a map of the United States with a grid overlay. Below the map is a search bar and a list of datasets.

Gridded Climate Data

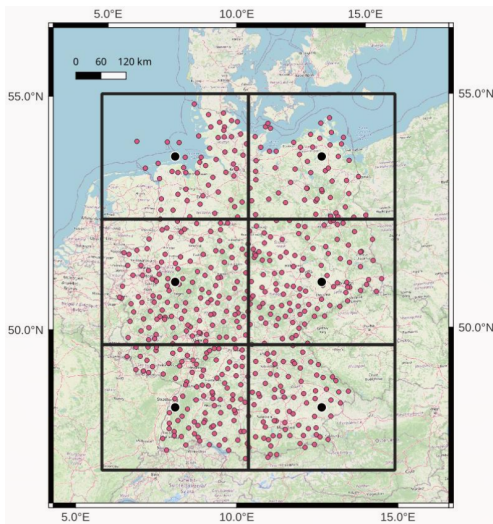
FIND: Clear

Subset By Category:

Alphabetical List

Dataset	Description	Options
Air and Marine Temperature Anomalies: HadCRUT4	Jones (CRU) Combined Air/Marine Temperature Anomalies	
OMAP Precipitation	Monthly and pentad global gridded precipitation means. It includes a standard and enhanced version (with NCEP Reanalysis) from 1979 to near the present.	
COBE Sea Surface Temperature	Monthly 1° x 1° SST dataset from 1891 to present from the JMA. Datasets uses 3D Var to fill gaps.	
COBE SST 2 and Sea Ice	Monthly 1x1 SST dataset from 1850 from the Japanese Meteorological Center (JMA).	
CPC Daily Blended Outgoing Longwave Radiation (OLR) - 1 degree	CBO V1 global OLR data set is constructed at NOAA Climate Prediction Center (CPC) by blending level 2 OLR retrievals from NASA Cloud and Earth Radiant Energy (CERES) broadband measurements, NOAA/NESDIS Hyperspectral measurements, as well as High-resolution Infrared Radiation Sounder (HIRS) measurements.	
CPC Daily Blended Outgoing Longwave Radiation (OLR) - 2.5 degree	CBO V1 global OLR data set is constructed at NOAA Climate Prediction Center (CPC) by blending level 2 OLR retrievals from NASA Cloud and Earth Radiant Energy (CERES) broadband measurements, NOAA/NESDIS Hyperspectral measurements, as well as High-resolution Infrared Radiation Sounder (HIRS) measurements.	
CPC Global Unified Gauge-Based Analysis of Daily Precipitation	CPC 0.5x0.5 Global Daily Unified Gauge-Based Analysis of Precipitation.	
CPC Global Unified Temperature	CPC 0.5x0.5 Global Daily Gridded Temperature	

Gridded data – Application in Earth system science II



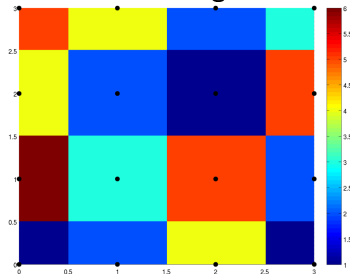
Map showing the definition of the used **grid** (black polygons). The **black circles** (●) mark the *centroids* (nodal points) of the *grid cells*. The amount of data (*station density*) in each cell differs and depends on the location of the grid cell. Corner cells have the lowest coverage of Germany which reduces the number of stations in them.

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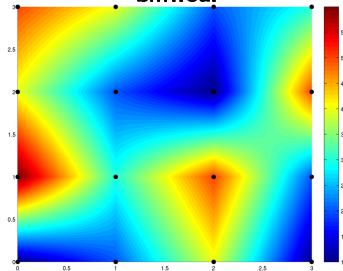
<https://lochbihler.nl/a-gridded-data-set-of-cloud-types-for-germany>

2-D interpolation (for surface data)

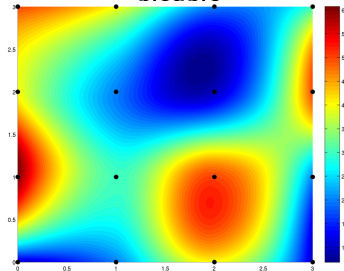
nearest neighbor



bilinear

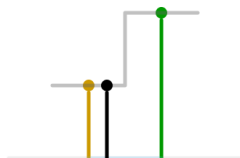


bicubic

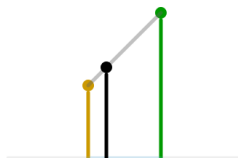


Public domain. Source: Wikipedia, <https://en.wikipedia.org/wiki/Interpolation>

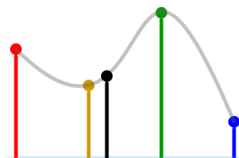
2-D vs. 1-D interpolation



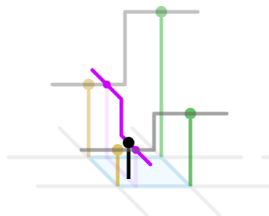
1D nearest-neighbour



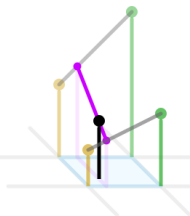
Linear



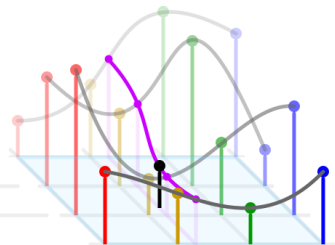
Cubic



2D nearest-neighbour



Bilinear

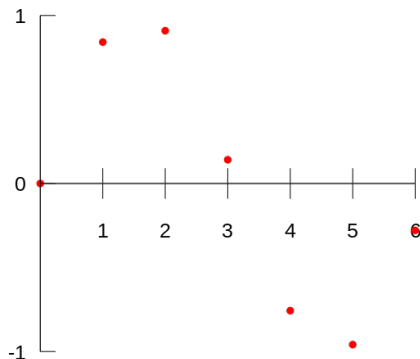


Bicubic

Public domain. Source: Wikipedia, <https://en.wikipedia.org/wiki/Interpolation>

1-D interpolation I

Data

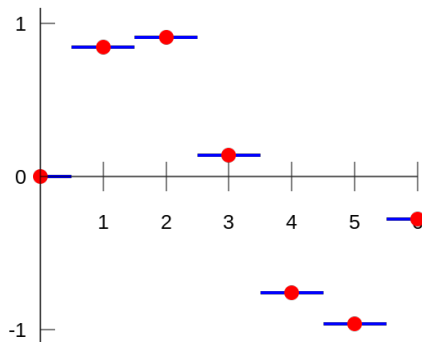


Public domain. Source: Wikipedia
<https://en.wikipedia.org/wiki/Interpolation>

- ▶ **Given:**
gridded data or
unevenly sampled point data
- ▶ **Question:**
How to fill in the gaps?

1-D interpolation II

Piecewise-*constant* interpolation



Public domain. Source: Wikipedia
<https://en.wikipedia.org/wiki/Interpolation>

- ▶ Used in finite-volume-based (FVM) CFD solvers
- ▶ Equivalent name:
nearest neighbor interpolation

$$f(x) = y_i$$

for $x_{i-1/2} \leq x < x_{i+1/2}$
interior points i

1-D interpolation III

Piecewise-*linear* interpolation

- ▶ Used in finite-element-based (FEM) solvers
- ▶ Application of the **two-point equation**

The interpolating property yields a **linear system of equations (LSE)** for the **nodal points**.

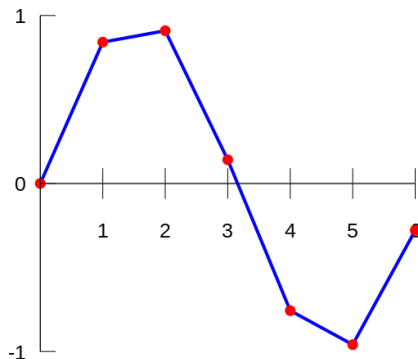
Looking at $i = 0$ and $i + 1 = 1$:

$$f(x) = Ax + B$$

$$y_0 = Ax_0 + B$$

$$y_1 = Ax_1 + B$$

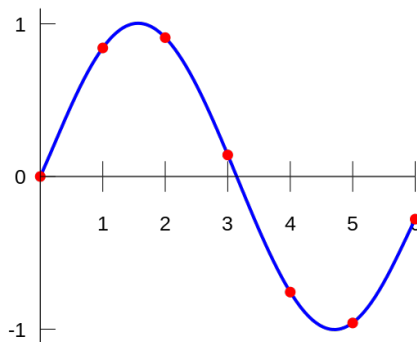
$$\Rightarrow f(x) = \frac{y_1 - y_0}{x_1 - x_0}(x - x_0) + y_0$$



Public domain. Source: Wikipedia
<https://en.wikipedia.org/wiki/Interpolation>

1-D interpolation IV

Higher-order interpolation schemes



Public domain. Source: Wikipedia
<https://en.wikipedia.org/wiki/Interpolation>

- ▶ Used in finite-difference (FDM) and spectral-element (SEM) solvers
- ▶ **Polynomials** yield an extended LSE that couples more nodal points in a non-local manner

The specific C-spline for the left-hand example:

$$f(x) = \begin{cases} -0.152x^3 + 0.994x \\ -0.013x^3 - 0.419x^2 + 1.413x - 0.140 \\ 0.140x^3 - 1.336x^2 + 3.247x - 1.362 \\ 0.158x^3 - 1.495x^2 + 3.723x - 1.838 \\ 0.054x^3 - 0.245x^2 - 1.276x + 4.826 \\ -0.187x^3 + 3.367x^2 - 19.34x + 34.93 \end{cases}$$

Keywords

- ▶ Discretization
- ▶ Mesh (grid) generation
- ▶ Structured grid
- ▶ Gridded data
- ▶ Interpolation