Introduction to computational thinking and programming for CFD (13251)

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Sheet 5

Goals

- Application of quadrature rules (Newton–Cotes formulas)
- Algorithm for the integral
- Order of accuracy, numerical errors, and convergence test
- Linear and double-logarithmic plots

Tasks

1. We consider the following definite integral

$$I = \int_0^1 \frac{\mathrm{d}x}{1+x}.$$

   (a) Analytically compute I. This will serve as reference.

   (b) Numerically compute the approximation I_h for equispaced nodal values using the lower sum and mesh size $h = 0.01$.

   (c) Compute the numerical error $\varepsilon_h = |I_h - I|$.

   (d) Repeat (c) for *feasible* mesh sizes $h \in [10^{-16}, 10^{-1}]$. Can you confirm the theoretically estimated optimal mesh size $h_{\mathrm{opt}} \simeq 10^{-8}$ from the lecture?

   (e) The lower sum is expected to exhibit linear convergence $\varepsilon_h \sim h$. Can you confirm this?

   (f) ($*$) Develop an algorithm that varies the step size (reduction by factor 10 in each step) and plots ε_h versus h. *Hint:* Generate a linear and a double-log plot using `pl.loglog(x,y)`.

   (g) ($*$) Repeat (b–f) for the upper sum and the trapezoidal rule. For the latter, can you confirm the optimal mesh size $h_{\mathrm{opt}} \simeq 10^{-5}$ and quadratic convergence $\varepsilon_h \sim h^2$?