Introduction to computational thinking and programming for CFD (13251)

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Sheet 7

Goals

- Statistical analysis of fluctuating data
- Elementary statistics: mean and standard deviation
- Advanced statistics: histogram, PDF, JPDF
- Histogram plots

Problem statement

As an example, we consider the **Lorenz system**

$$\frac{\mathrm{d}x}{\mathrm{d}t} = s\,(y - x), \qquad \frac{\mathrm{d}y}{\mathrm{d}t} = (r - z)\,x - y, \qquad \frac{\mathrm{d}z}{\mathrm{d}t} = xy - bz,$$

with the following control parameter values and initial conditions (superscript 0):

$$b = 8/3 \qquad r = 28 \qquad s = 10$$
$$x^0 = -8 \qquad y^0 = -1 \qquad z^0 = 33$$

The numerical solver for this system of ODEs was developed previously. Take this solver for the following exercises using the time step $\Delta t = 10^{-3}$ and final time $t_{\mathrm{end}} = 40$.

Please turn the page!

Tasks

1. Mean.
   (a) Compute the mean values $\bar{x}$, $\bar{y}$, and $\bar{z}$ of the discrete numerically simulated time series $\{x^i\}$, $\{y^i\}$, and $\{z^i\}$, $i = 0, 1, 2, \ldots, N-1$.
   (b) Encapsulate the implementation of mean in a user-defined function.
   (c) Print out the mean.
   (d) Plot the mean state as black bullet (style `'ko'`) in your existing figures.
2. Standard deviation.
   (a) Compute the standard deviations σ_x, σ_y, and σ_z of the fluctuating data.
   (b) Encapsulate the implementation of the standard deviation in a user-defined function.
   (c) Print out the standard deviation.
3. Histogram and PDF with `pl.hist()`
   (a) Plot the histograms for $\{x^i\}$, $\{y^i\}$, and $\{z^i\}$ using $M = 200$ bins based on the following tutorial: https://matplotlib.org/stable/gallery/statistics/hist.html
   (b) Plot the PDF by normalizing the histogram. *Hint:* Set the Boolean keyword argument `density=True` following this documentation: https://matplotlib.org/stable/api/_as_gen/matplotlib.pyplot.hist.html
4. 2-D histogram and JPDF with `pl.hist2d()`
   (a) Plot the 2-D histogram of $x + y$ and $x - y$ from the discrete samples $\{x^i\}$ an $\{y^i\}$ using $M = 40$ bins for each axis following the above tutorial.
   (b) Plot the JPDF by setting the 'density' keyword argument.
5. $(*)$ Repeat the above tasks, but compute the statistical quantities only for the first and last 5 seconds: $0 \leq t \leq 5$ and $35 \leq t \leq 40$. Does the mean and standard deviation change? *Hint:* In Python, you can use logical expressions as array arguments to extract data. For example, extract all points in $0.5 < x_i < 0.6$ via `xe = x[0.5 < x < 0.6]`.
6. $(*)$ Vary the initial condition x^0, y^0, and z^0. Do the statistics change?
7. $(*)$ Vary the control parameters r, s, and b. Do the statistics change?