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Rubik's Mathematics: A Twist on Numbers

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Abstract

The Rubik's Cube, an iconic 3D puzzle, has captured the imagination of enthusiasts and mathematicians alike for decades. This poster delves into the mathematical intricacies of the Rubik's Cube. We investigate the cube's symmetry, its staggering number of possible permutations, and the use of computer aided proof-assistants to calculate the algorithms to solve the Rubik's Cube. Whether you're a puzzle enthusiast or a math fanatic, this poster invites you to embark on a fascinating journey into the world of Rubik's Cube mathematics.

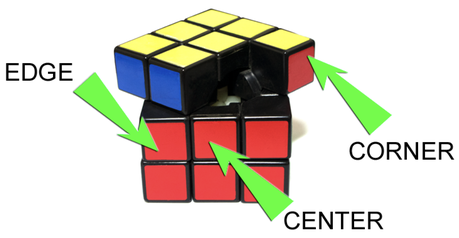
Combinations of a Rubik's Cube

There are three (four if you count the not-visible core) types of pieces:

- **Corners:** 8 of these on each corner of the cube
- **Edges:** 12 of these connecting adjacent corners
- **Centers:** 6 of these on each face of the cube

There are $8!$ ways we can permute the corner pieces to the corners of a cube, and $12!$ ways we can permute the edge pieces into the 12 edge slots of a cube. There are 3^8 ways we can orient the corner pieces and 2^{12} ways we can the edge pieces. We also have to divide by 12, since some states are impossible. This yields the final formula:

$$\frac{8! \cdot 12! \cdot 3^8 \cdot 2^{12}}{12} = 43252003274489856000$$



How big is 43 Quintillion?

Imagine we had a dollar for each rubik's cube combination there was. If were to lay one layer of one dollar bills in Colorado, it would take 25 trillion dollars. We would have to stack that another 2 million times to use all of our money.

The stack would be about the same height of the tallest building in Denver. In other words, the money would cover all of Colorado in a layer as tall as a skyscraper.

Solving a Rubik's Cube

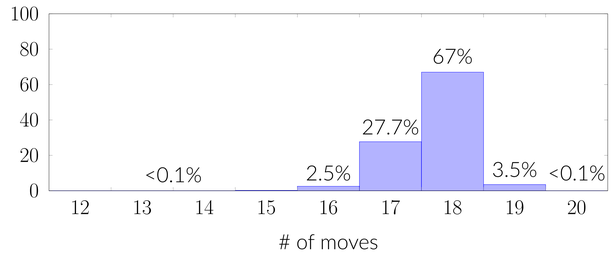
VA popular method for speed-solving is CFOP(used by many world record holders) where you create a Center **Cross**, solve the First two layers(**F2L**), Orient the last layer(**OLL**), and Permute the last layer(**PLL**). We can figure out the maximum number of moves it would take for a solve using CFOP by going adding up the maximum steps of each step:

1. For the center cross, the maximum needed is eight
2. For F2L, it depends but the maximum is around 24-28
3. For OLL, we can get a maximum of 11
4. And finally for PLL, the maximum is 14

Across the four steps in CFOP we have about 60 moves needed to solve a 3x3 Cube.

God's Algorithm and Number

Naturally, people want to know how to solve the know the best ways to solve a Rubik's cube. **God's algorithm** refers to the hypothetical perfect solving method for a Rubik's Cube, a sequence of moves that would guarantee the shortest possible solution for any scrambled configuration. **God's number** represents the maximum number of moves required to solve the most challenging possible Rubik's Cube configuration. This value was proven to be 20 in July 2010 after extensive computer-assisted calculations and mathematical research, providing a fundamental benchmark for the cube's complexity.



Proof Assistants/Computer Aided proof

Obviously, it would be impossible for humans to go through all 43 Quintillion combinations of a Rubik's cube. However, computers have been able to use exhaustive search and other techniques to determine God's number. Other proofs that have used computers include:

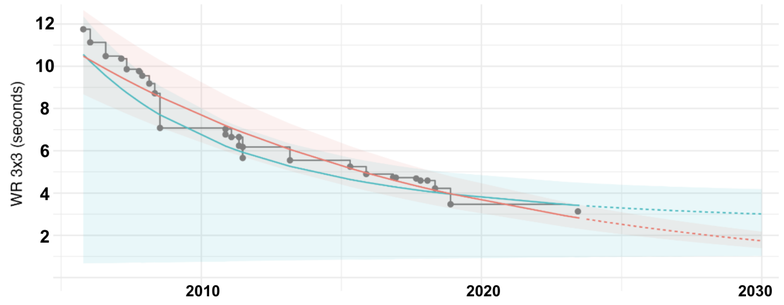
- **Connect Four:** Proved the game is deterministic
- **Kepler's Conjecture:** Optimal sphere-packing in 3 Dimensions
- **Sudoku:** You need at least 12 clues to solve a Sudoku Puzzle

Applications to Speed Cubing

There are many different methods for Speed Cubing, all aimed at Solving the cube as fast as possible. This involves coming up with ways to use less moves and using easily executed algorithms. Here are a couple:

Method	# Turns	# Algorithms	Average Times (s)
Beginner	80-100	15	30-120
CFOP	55-60	78	5-30
Roux	45-50	100+	5-20
ZZ	45-55	493	5-15

As more methods and algorithms get developed with the aid of computers, speed-cube times have been getting lower considerably:



What about larger Rubik's Cubes?

Unsurprisingly, the number of possible combinations of larger Rubik's Cube scale exponentially

- **4x4:** 7.4 quattuordecillion ($7.5 \cdot 10^{45}$) - 20s solve time
- **5x5:** 283 trevigintillion ($283 \cdot 10^{72}$) - 38s solve time
- **6x6:** Big number with 117 digits - 75s solve time
- **7x7:** Big number with 165 digits - 110s solve time

The general formula for the combinations on an $n \times n$ cube is:

$$7! \cdot 3^6 \left(24 \cdot 2^{10} \cdot 12!\right)^{n \bmod 2} (24!)^{\lfloor \frac{n-2}{2} \rfloor} \left(\frac{24!}{4!^6}\right)^{\lfloor \left(\frac{n-2}{2}\right)^2 \rfloor}$$

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