Rubik's Mathematics: A Twist on Numbers

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Abstract

The Rubik's Cube, an iconic 3D puzzle, has captured the imagination of enthusiasts and mathematicians alike for decades. This poster delves into the mathematical intricacies of the Rubik's Cube. We investigate the cube's symmetry, its staggering number of possible permutations, and the use of computer aided proof-assistants to calculate the algorithms to solve the Rubik's Cube. Whether you're a puzzle enthusiast or a math fanatic, this poster invites you to embark on a fascinating journey into the world of Rubik's Cube mathematics.

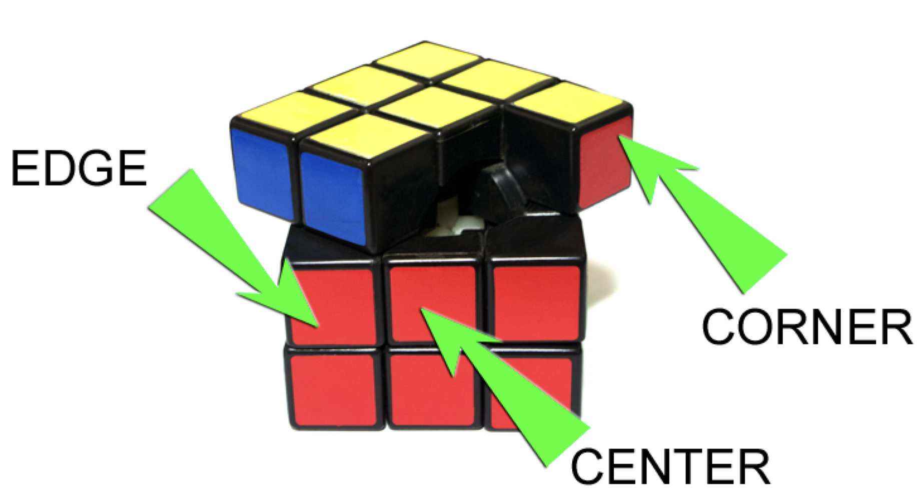
Combinations of a Rubik's Cube

There are three (four if you count the not-visible core) types of pieces:

- **Corners:** 8 of these on each corner of the cube
- **Edges:** 12 of these connecting adjacent corners
- **Centers:** 6 of these on each face of the cube

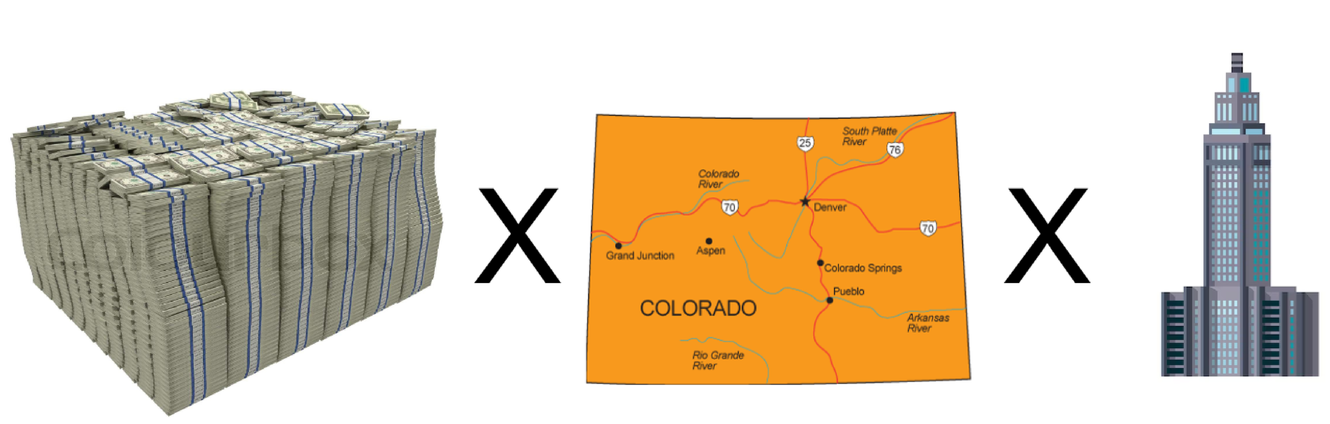
There are 8! ways we can permute the corner pieces to the corners of a cube, and 12! ways we can permute the edge pieces into the 12 edge slots of a cube. There are 3^8 ways we can orient the corner pieces and 2^{12} ways we can the edge pieces. We also have to divide by 12, since some states are impossible. This yields the final formula:

$$\frac{8! \cdot 12! \cdot 3^8 \cdot 2^{12}}{12} = 43252003274489856000$$



How big is 43 Quintillion?

Imagine we had a dollar for each rubik's cube combination there was. If were to lay one layer of one dollar bills in Colorado, it would take 25 trillion dollars. We would have to stack that another 2 million times to use all of our money.
The stack would be about the same height of the tallest building in Denver. In other words, the money would cover all of Colorado in a layer as tall as a skyscraper.



Solving a Rubik's Cube

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God's Algorithm and Number

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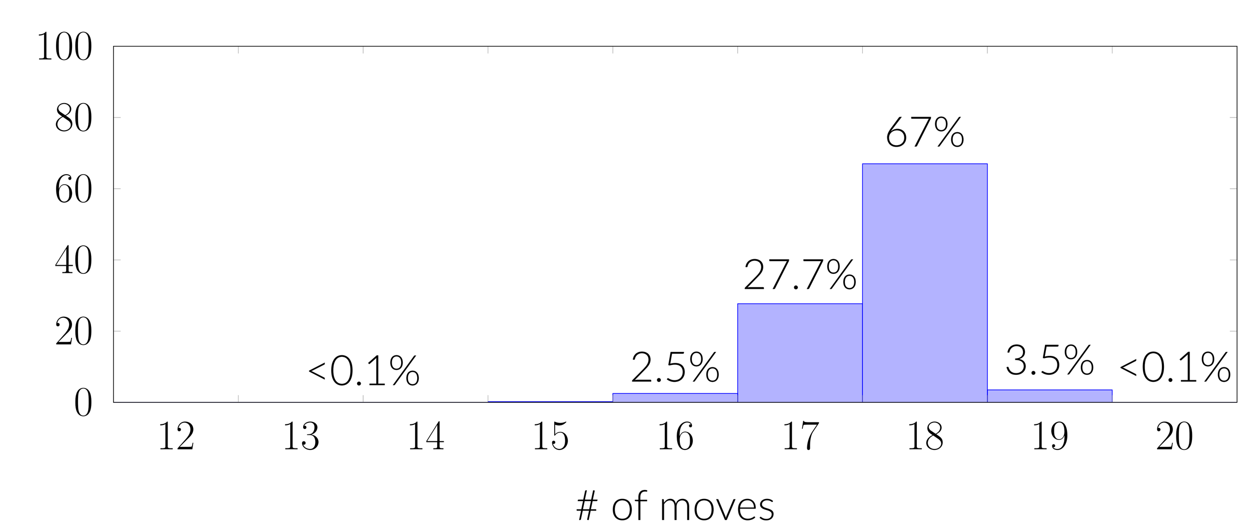

Proof Assistants/Computer Aided proof

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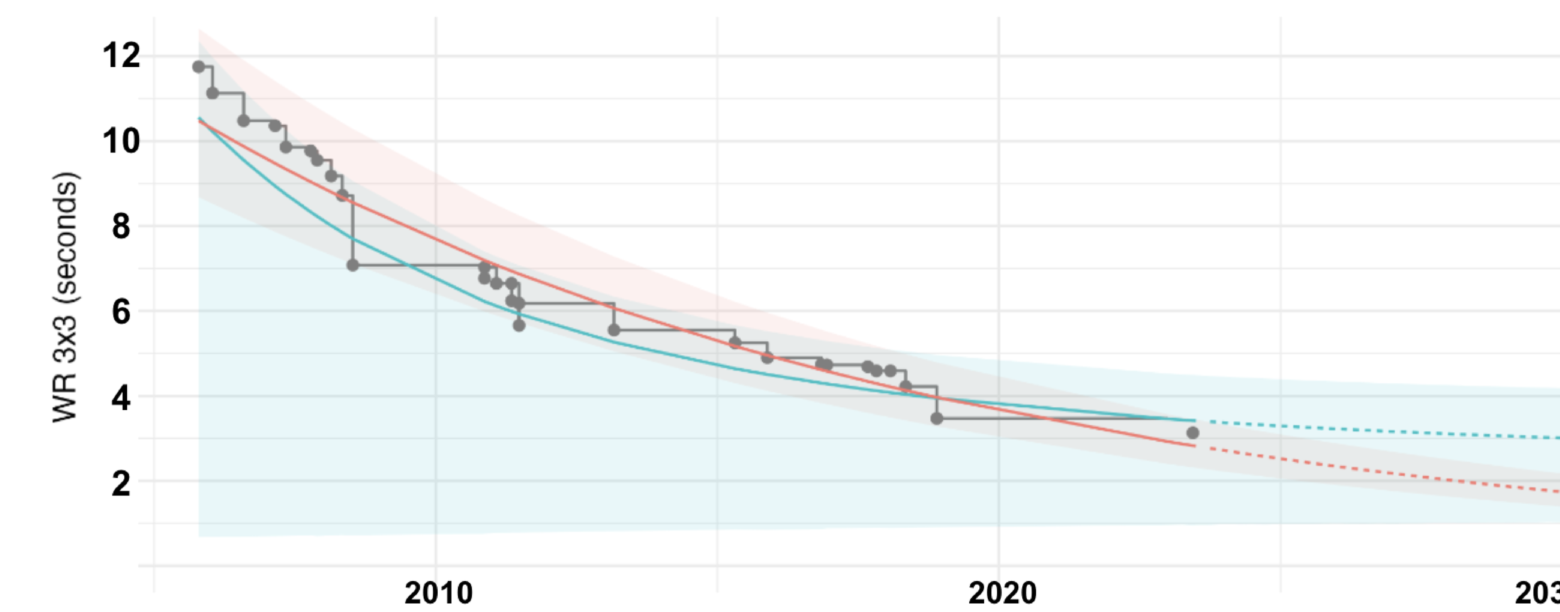
Applications to Speed Cubing

There are many different methods for Speed Cubing, all aimed at Solving the cube as fast as possible. This involves coming up with ways to use less moves and using easily executed algorithms. Here are a couple:

Method	# Turns	# Algorithms	Average Times (s)
Beginner	80-100	15	30-120
CFOP	55-60	78	5-30
Roux	45-50	100+	5-20
ZZ	45-55	493	5-15

As more methods and algorithms get developed with the aid of computers, speed-cube times have been getting lower considerably:



What about larger Rubik's Cubes?

Unsurprisingly, the number of possible combinations of larger Rubik's Cube scale exponentially

- **4x4:** 7.4 quattuordecillion ($7.5 \cdot 10^{45}$) - 20s solve time
- **5x5:** 283 trevigintillion ($283 \cdot 10^{72}$) - 38s solve time
- **6x6:** Big number with 117 digits - 75s solve time
- **7x7:** Big number with 165 digits - 110s solve time

The general formula for the combinations on an $n \times n$ cube is:

$$7! \cdot 3^6 \left(24 \cdot 2^{10} \cdot 12!\right)^{n \bmod 2} (24!)^{\lfloor \frac{n-2}{2} \rfloor} \left(\frac{24!}{4!^6}\right)^{\lfloor \left(\frac{n-2}{2}\right)^2 \rfloor}$$

References