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Rubik's Mathematics: A Twist on Numbers

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Abstract

The Rubik's Cube, an iconic 3D puzzle, has captured the imagination of enthusiasts and mathematicians alike for decades. This poster delves into the mathematical intricacies of the Rubik's Cube. We investigate the cube's symmetry, its staggering number of possible permutations, and the use of computer aided proofs to calculate the algorithms to solve the Rubik's Cube.

Combinations of a Rubik's Cube

There are three visible types of pieces:

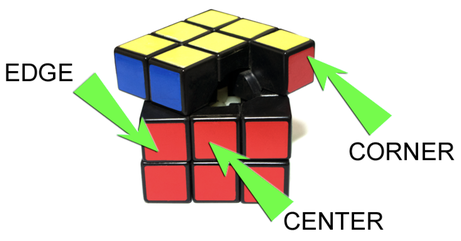
- **Corners:** 8 of these on each corner of the cube
- **Edges:** 12 of these connecting adjacent corners
- **Centers:** 6 of these on each face of the cube

Permutations on the Rubik's Cube:

- **8!** ways to permute corner pieces into their slots
- **12!** ways to permute edge pieces into their slots
- 3^8 ways to orient each corner piece
- 2^{12} ways to orient each edge piece

We also have to divide by 12, since some states are impossible.

$$\frac{8! \cdot 12! \cdot 3^8 \cdot 2^{12}}{12} = 43252003274489856000$$



How big is 43 Quintillion?

Imagine 43 Quintillion Dollar bills:

- One layer of bills over all of Colorado costs $ 25 trillion
- One stack of bills 800 feet tall costs $ 2 million

Combined, it will cost about $ 43 Quintillion

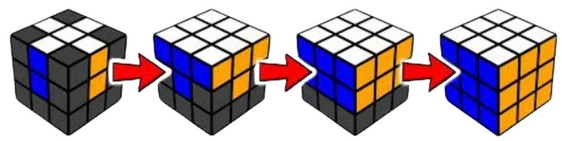
Solving a Rubik's Cube

A popular method for speed-solving is CFOP where you create a Center **Cross**, solve the First two layers(**F2L**), Orient the last layer(**OLL**), and Permute the last layer(**PLL**). Maximum turns for each step:

C Center Cross - 8 moves
F First 2 Layers(F2L) - 28 moves
O Orientation of Last Layer(OLL) - 11 moves
P Permutation of Last Layer(PLL) - 14 moves

Across the four steps in CFOP we have about 60 moves needed to solve a 3x3 Cube.

CFOP solving method:

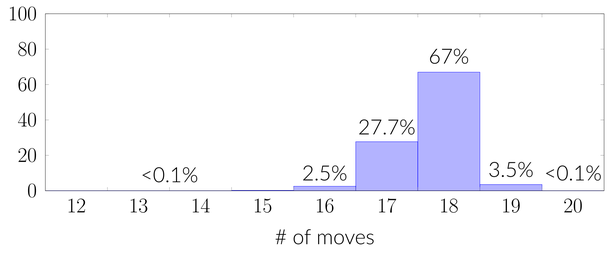


God's Algorithm and Number

God's algorithm - hypothetical perfect solving method for a Rubik's Cube

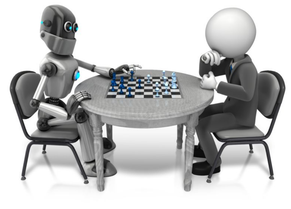
God's number - maximum number of moves required to solve any Rubik's Cube configuration.

- Proven to be 20 after 35 core-years of computation in July 2010



Other Computer Aided Proofs

- **Connect Four:** Always win for first player
- **Sudoku:** You need at least 12 clues to solve a Sudoku Puzzle
- **Chess:** Analyzing Endgames
- **Chopsticks:** Always draw if played perfectly
- **Kepler's Conjecture:** Optimal sphere-packing in 3 Dimensions
- **Four Color Theorem:** Proved the conjecture



Applications to Speed Cubing

Speedcubing: Solving scrambled Rubik's cube as fast as possible

Table of different solving methods

Method	# Turns	# Algorithms	Average Times (s)
Beginner	80-100	15	30-120
CFOP	55-60	78	5-30
Roux	45-50	100+	5-20
ZZ	45-55	493	5-15

As more methods and algorithms get developed with the aid of computers, speed-cube times have been getting lower considerably:

Evolution of 3x3 Rubik's Cube WR

What about larger Rubik's Cubes?

The number of possible combinations of larger Rubik's Cube scale exponentially

- **4x4:** 7.4 quattuordecillion ($7.5 \cdot 10^{45}$) - 20s solve time
- **5x5:** 283 trevigintillion ($283 \cdot 10^{72}$) - 38s solve time
- **6x6:** Big number with 117 digits - 75s solve time
- **7x7:** Big number with 165 digits - 110s solve time

The general formula for the combinations on an $n \times n$ cube is:

$$7! \cdot 3^6 \left(24 \cdot 2^{10} \cdot 12!\right)^{n \bmod 2} (24!)^{\lfloor \frac{n-2}{2} \rfloor} \left(\frac{24!}{4!^6}\right)^{\lfloor \left(\frac{n-2}{2}\right)^2 \rfloor}$$

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