FIN4104/4911
Quantitative Analysis for Financial Decisions

Chapter 10: Time-Series Analysis

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Course Outline

- Time Series data
- Autocorrelation
- Models

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Time Series data

A Time series is made up by dynamic data collected **over time**!

LinkedIn daily stock market closing price

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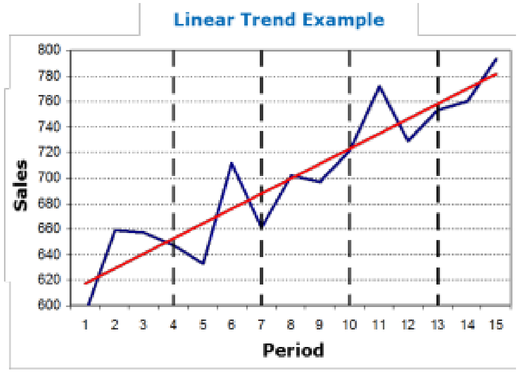
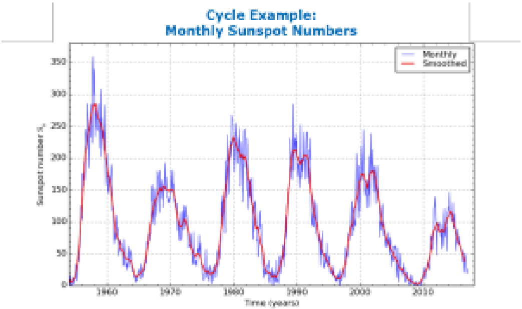
Time Series Properties:

- TREND
  The general direction in which the series is running during a long period
  A **TREND** exists when there is a long-term increase or decrease in the data.
  It does not have to be necessarily linear (could be exponential or others functional form).



- CYCLE
  Long-term fluctuations that occur regularly in the series A CYCLE is an oscillatory component (i.e. Upward or Downward swings) which is repeated after a certain number of years, so:
  - May vary in length and usually lasts several years (from 2 up to 20/30)
  - Difficult to detect, because it is often confused with the trend component

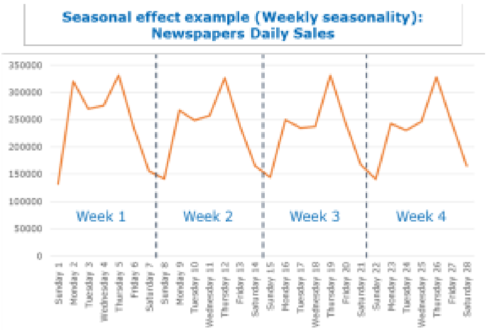


Time Series Properties:

- ## SEASONAL EFFECTS

  Short-term fluctuations that occur regularly – often associated with months or quarters

  A **SEASONAL PATTERN** exists when a series is influenced by seasonal factors (e.g., the quarter of the year, the month, day of the week). Seasonality is always of a fixed and known period.

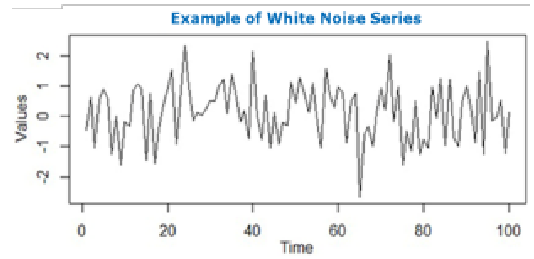


- ## RESIDUAL

  Whatever remains after the other components have been taken into account

  The residual/error component is everything that is not considered in previous components

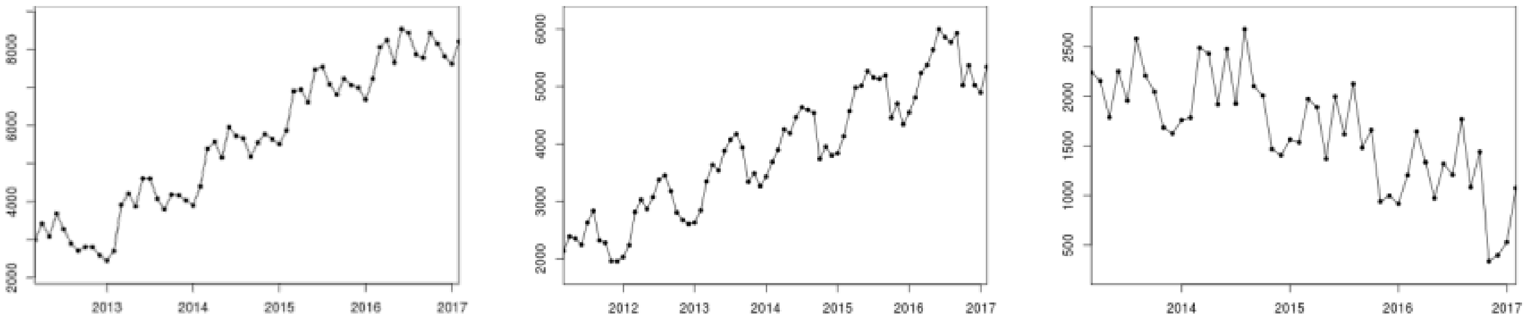
  Typically, it is assumed to be the sum of a set of random factors (e.g. a **white noise series**) not relevant for describing the dynamics of the series



Seasonal effect: additive seasonality

- When the seasonality in Additive, the dynamics of the components are **independents from each other**; for instance, an increase in the trend-cycle will not cause an increase in the magnitude of seasonal dips
- The difference of the trend and the raw data is **roughly constant in similar periods of time** (months, quarters) irrespectively of the tendency of the trend

EXAMPLES OF ADDITIVE SEASONALITY

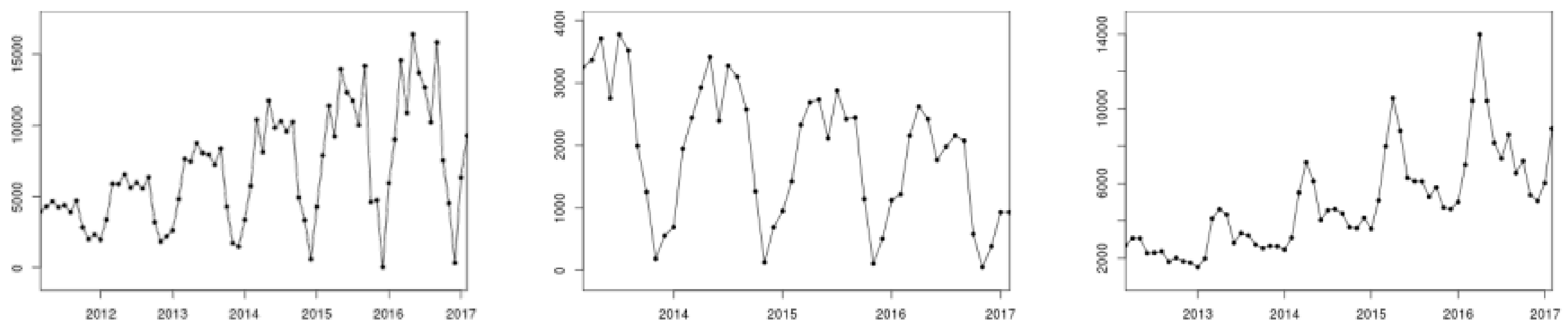


Time series (Y) = Trend effect + Seasonal effect

Seasonal effect: multiplicative seasonality

- In the multiplicative model the amplitude of the seasonality increase (decrease) with an increasing (decreasing) trend, therefore, on the contrary to the additive case, the **components are not independent from each other**
- When the variation in the seasonal pattern (or the variation around the trend-cycle) **appears to be proportional** to the level of the time series, then a multiplicative model is more appropriate.

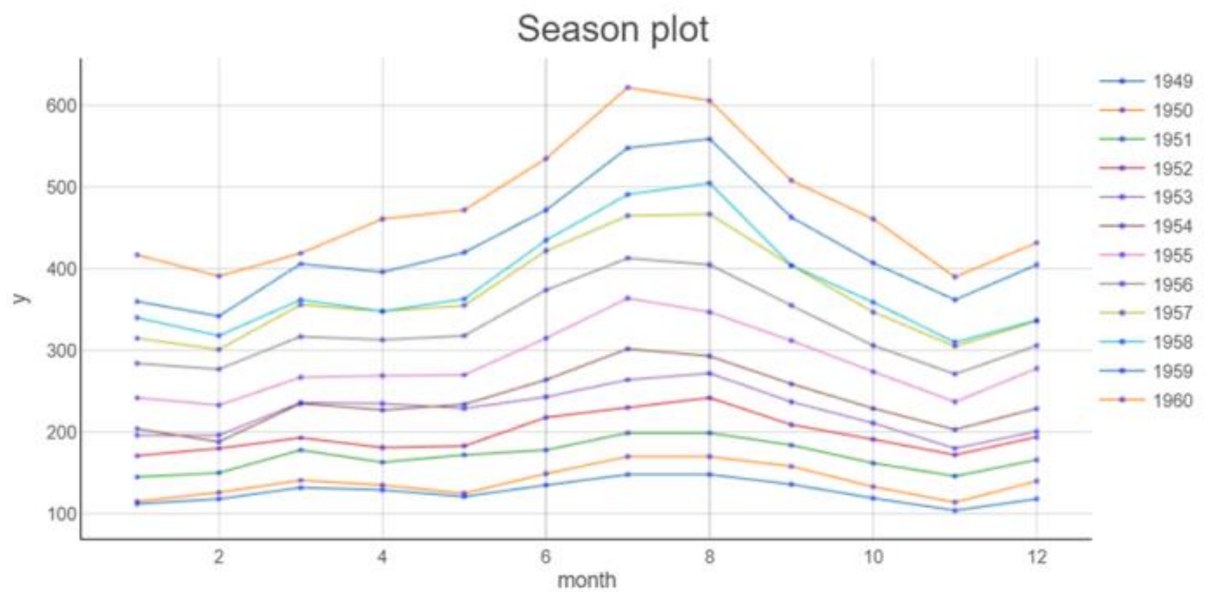
EXAMPLES OF MULTIPLICATIVE SEASONALITY



Time series (Y) = Trend effect x Seasonal effect

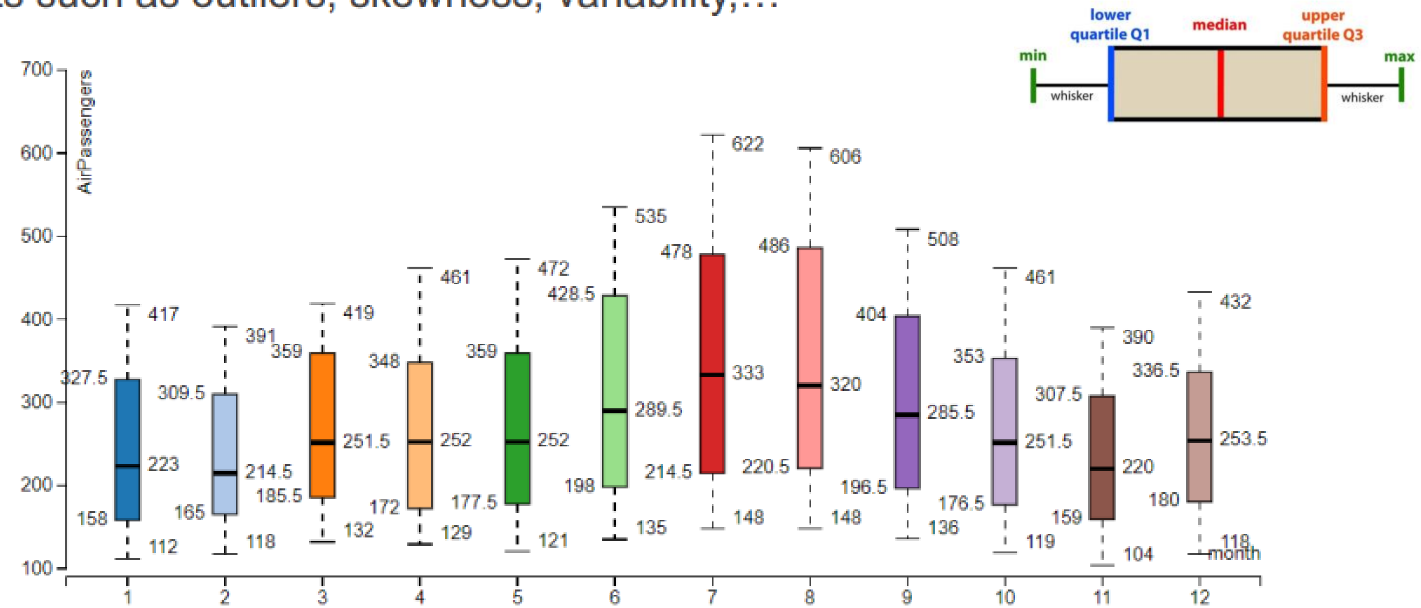
Graphical Analysis: Seasonal Plot

- Produce the **Seasonal plot** of the Time series in order to analyze more in detail the seasonal component (and possible changes in seasonality over time)



Graphical Analysis: Box Plot

Create the **conditional Box plot** of the Time series in order to deeply understand the distribution of data in the same period of each seasons and focusing on specific aspects such as outliers, skewness, variability,…



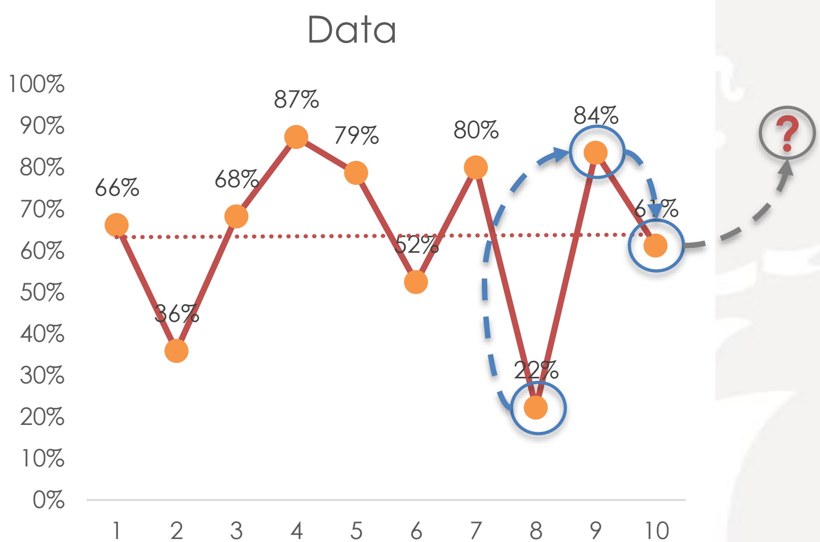
Course Outline

- Time Series data
- Autocorrelation
- Models

What is Autocorrelation?

- Correlation of a time series with a lagged copy of itself

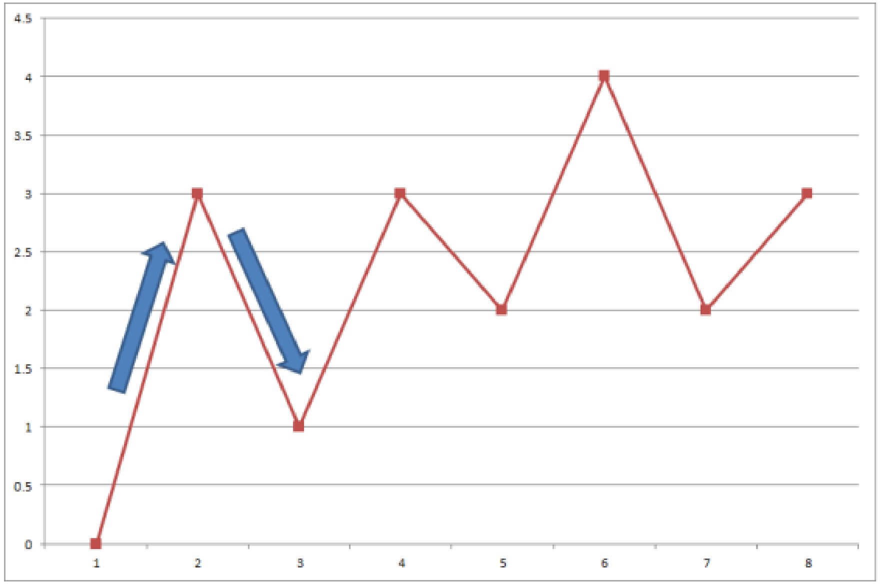
Series	Lagged Series
5	
10	5
15	10
20	15
25	20
⋮	⋮



- Also called **serial correlation**
- Lag-one autocorrelation

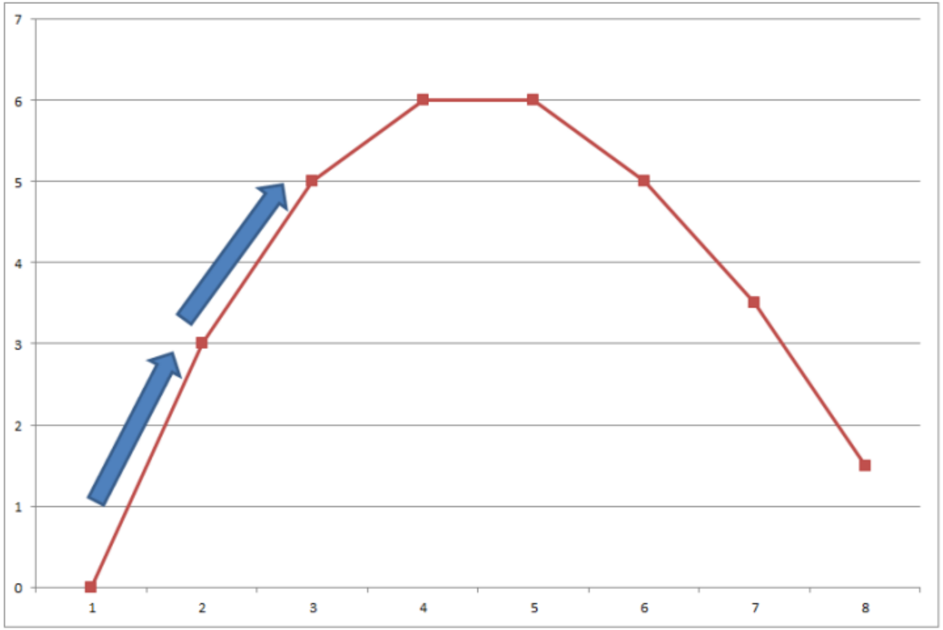
Interpretation of Autocorrelation

- **Mean Reversion** - Negative autocorrelation



Interpretation of Autocorrelation

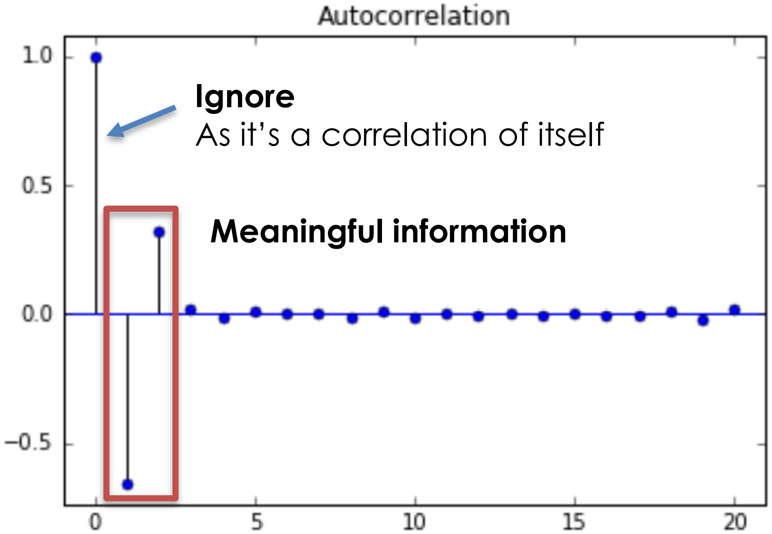
- **Momentum**, or **Trend Following** - Positive autocorrelation



Autocorrelation Function

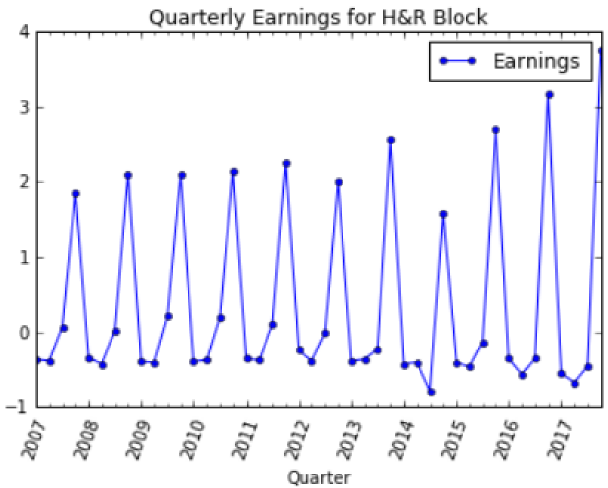
- Autocorrelation Function (ACF): The autocorrelation as a function of the lag
- Equals one at lag-zero
- Interesting information beyond lag-one

- Can use last two values in series for forecasting

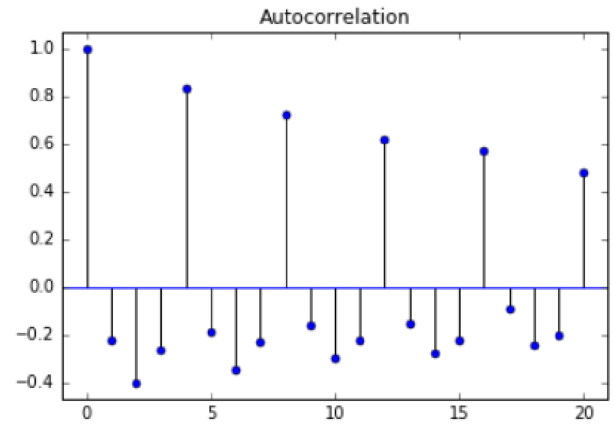


Example: Seasonal Earnings

- Earnings for H&R Block

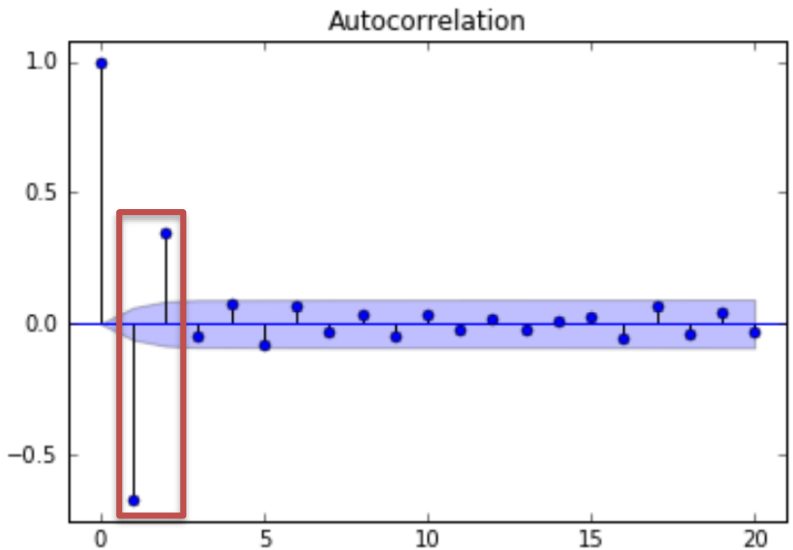


- ACF for H&R Block



From the graphs above we can see the seasonal of earnings of H&B Block in Q4

Confidence Interval of ACF



- Example: `alpha=0.05`
  - 5% chance that if true autocorrelation is zero, it will fall outside blue band
- Confidence bands are wider if:
  - Alpha lower
  - Fewer observations

Less chance that lag will be significant

- Under some simplifying assumptions, 95% confidence bands are $\pm 2/\sqrt{N}$
- If you want no bands on plot, set `alpha=1`

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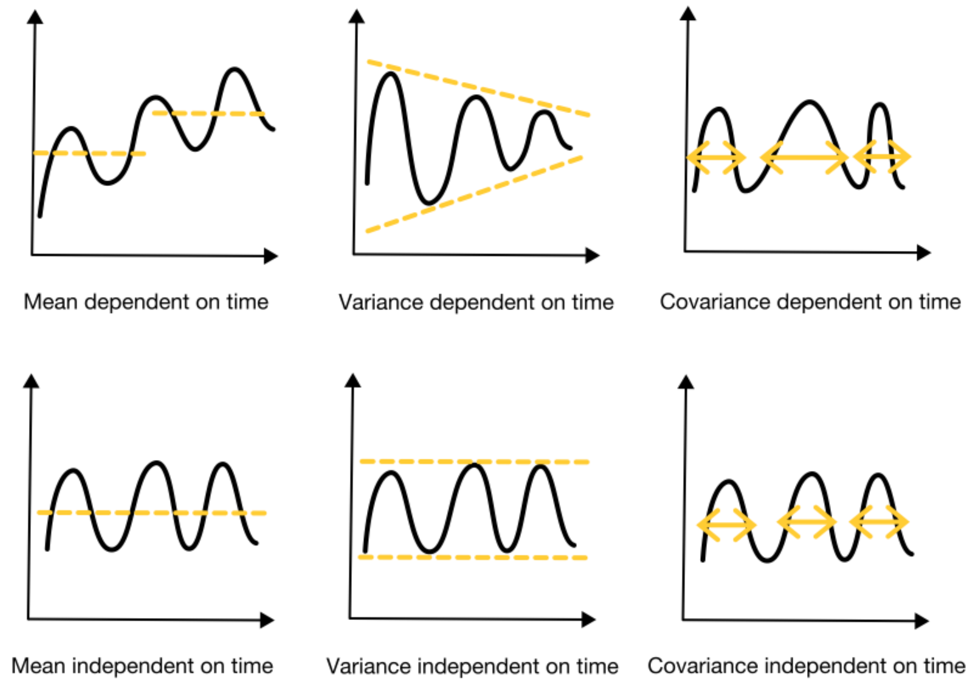
Stationarity

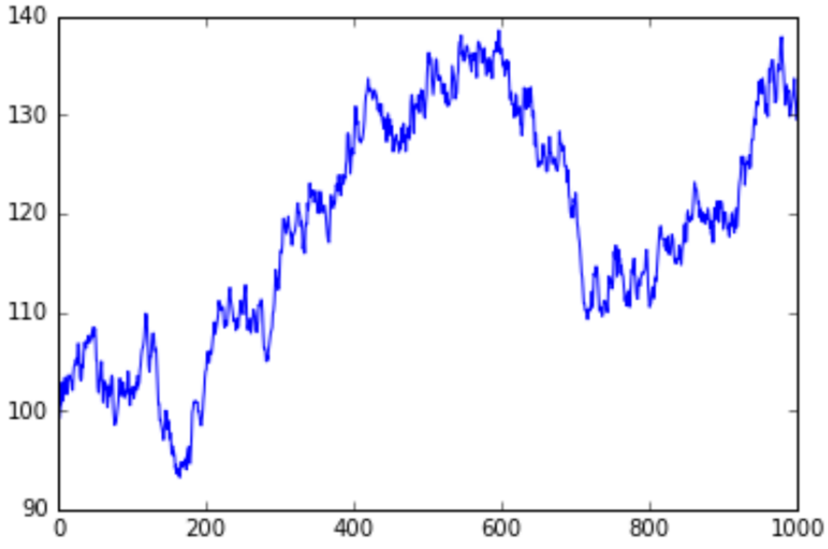
- **Strong stationarity**: entire distribution of data is time-invariant
- **Weak stationarity:** mean, variance and autocorrelation are time-invariant (i.e., for autocorrelation, corr$(X_t, X_{t-\tau})$ is only a function of τ)
- If parameters vary with time, too many parameters to estimate
- Can only estimate a parsimonious model with a few parameters

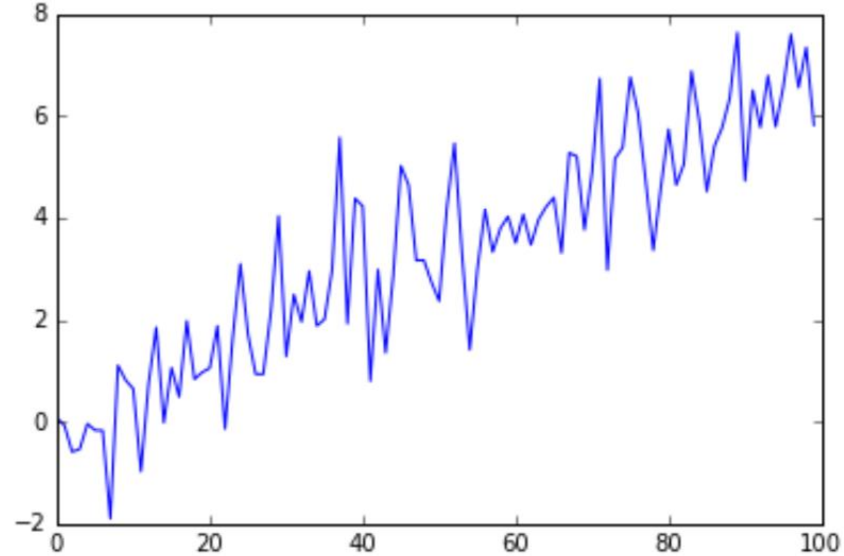
Stationarity

- A stationary time series is one whose properties do not depend on the time at which the series is observed. Thus, time series with trends, or with seasonality, are not stationary — the trend and seasonality will affect the value of the time series at different times.
- On the other hand, a white noise series is stationary — it does not matter when you observe it, it should look much the same at any point in time.



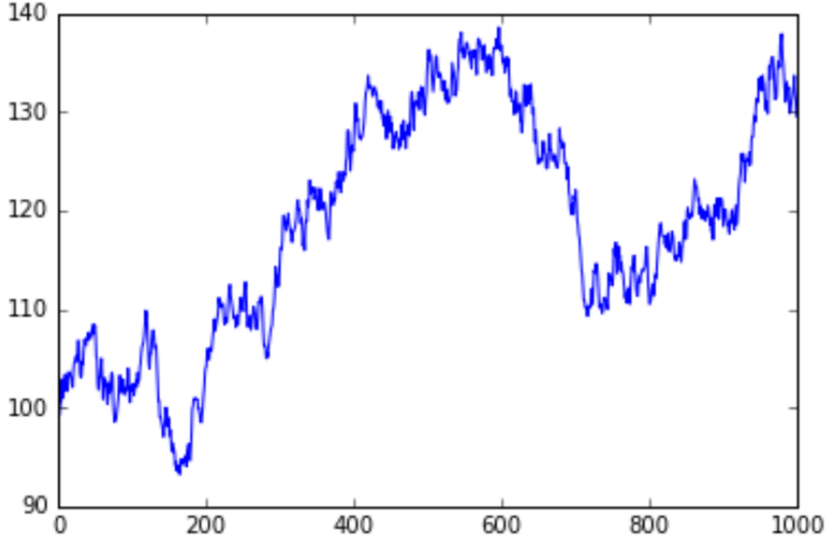
Stationary vs Non-stationary



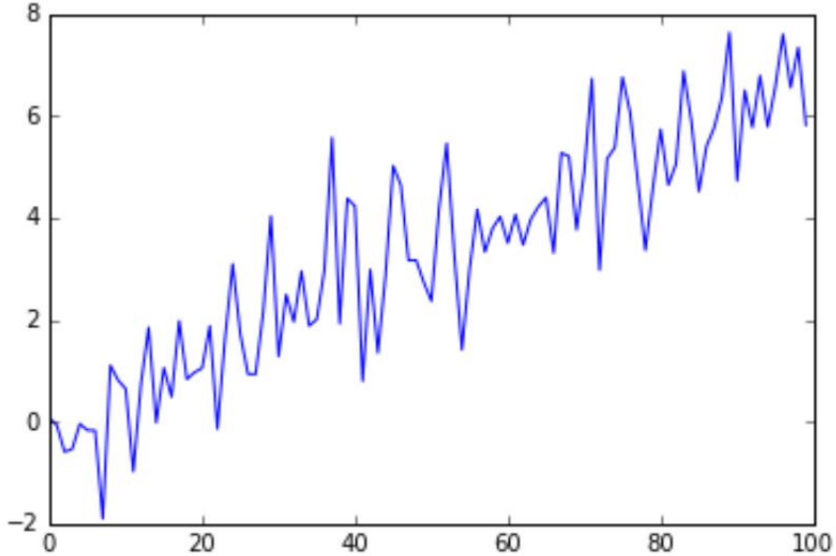


Stationary vs Non-stationary

Random walk

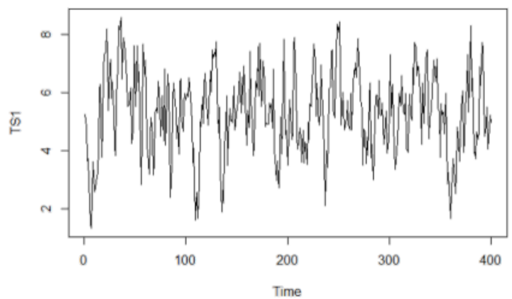


Change in mean or sd overtime

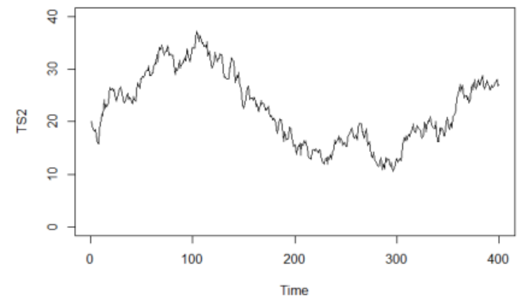


Stationary vs Non-stationary

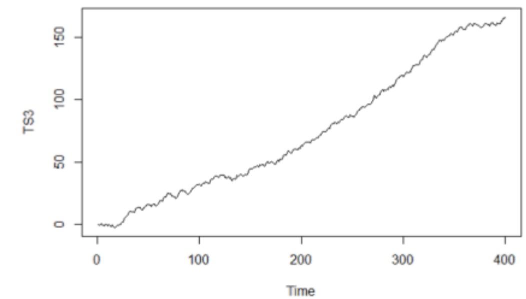
Stationary Time Series example



Non-Stationary Time Series example 1



Non-Stationary Time Series example 2

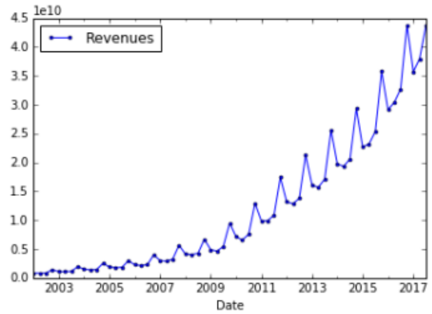


- Use ACF plots to inspect seasonality from energy consumption data
- Remove seasonality and check again the ACF plot

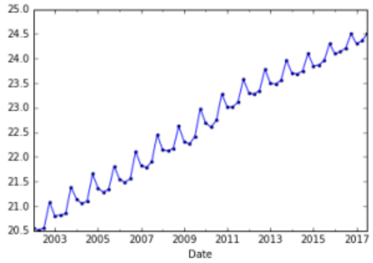
Transforming Nonstationary to Stationary

- AMZN Quarterly Revenues

```
plt.plot(AMZN)
```

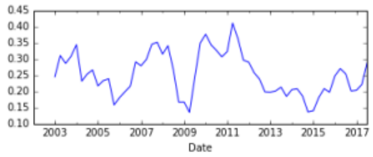



```
# Log of AMZN Revenues
plt.plot(np.log(AMZN))
```



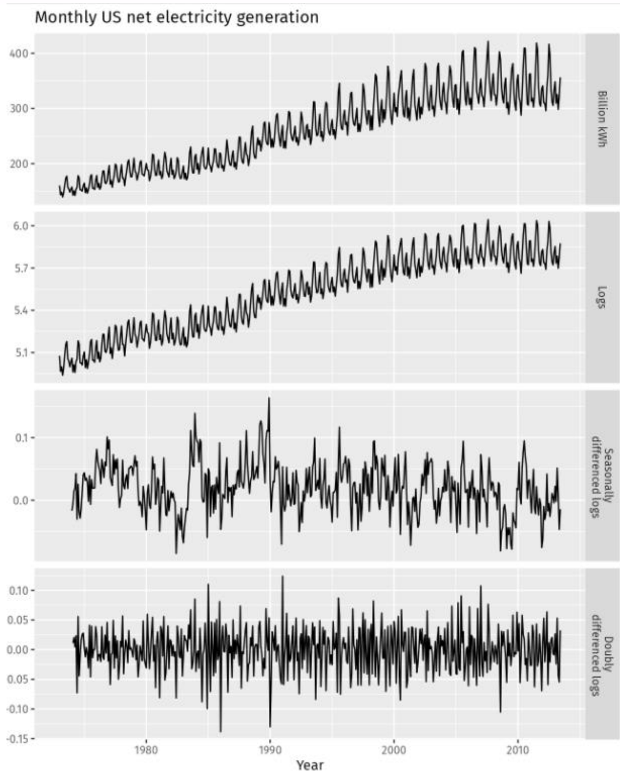
Make its variance constant

```
# Log, then seasonal difference
plt.plot(np.log(AMZN).diff(4))
```



Make its mean constant

Transforming Nonstationary to Stationary



Original

Make its variance constant

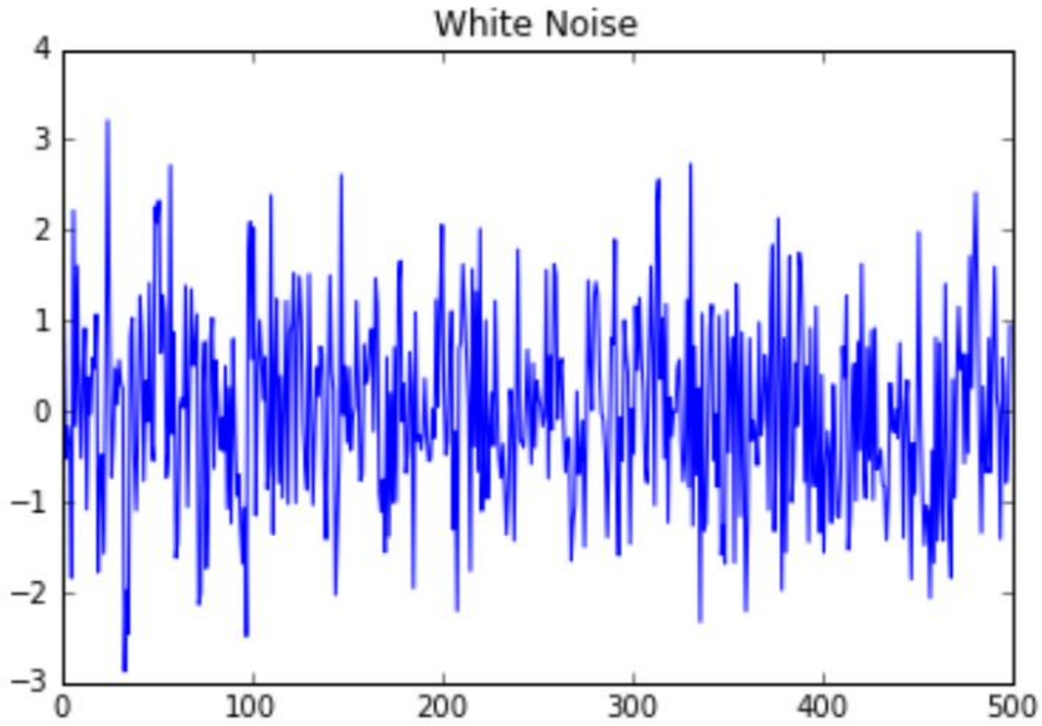
Make its mean constant

Second-order differencing - change in the changes

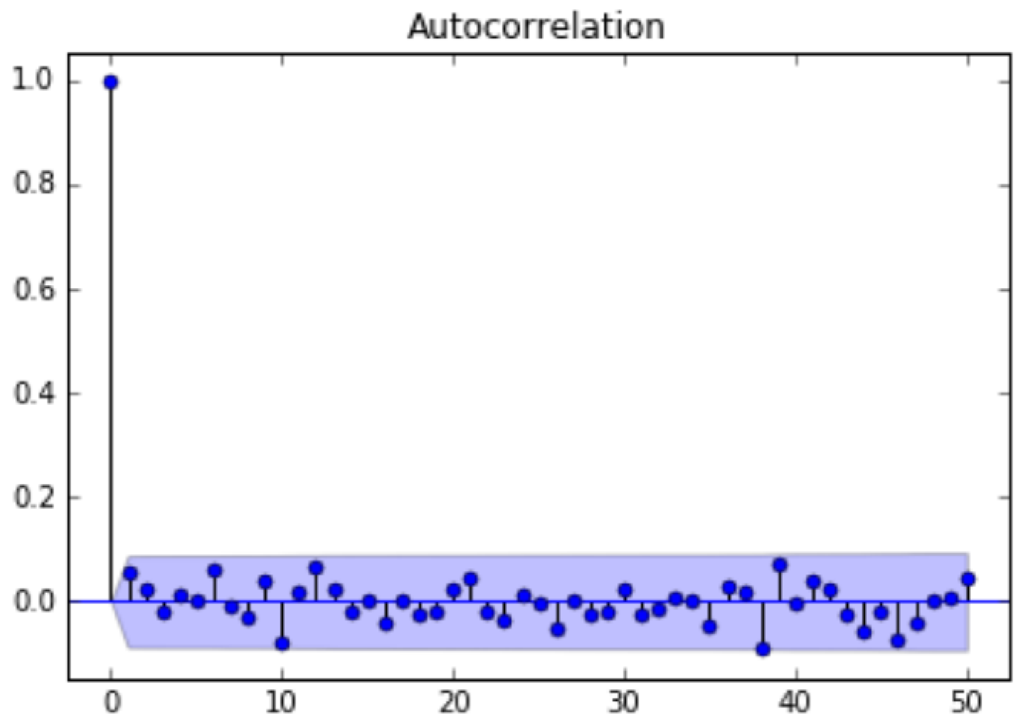
What is White Noise?

- White Noise is a series with:
  - Constant mean
  - Constant variance
  - Zero autocorrelations at all lags
- Special Case: if data has normal distribution, then *Gaussian White Noise*

What does white noise look like?



White noise autocorrelation



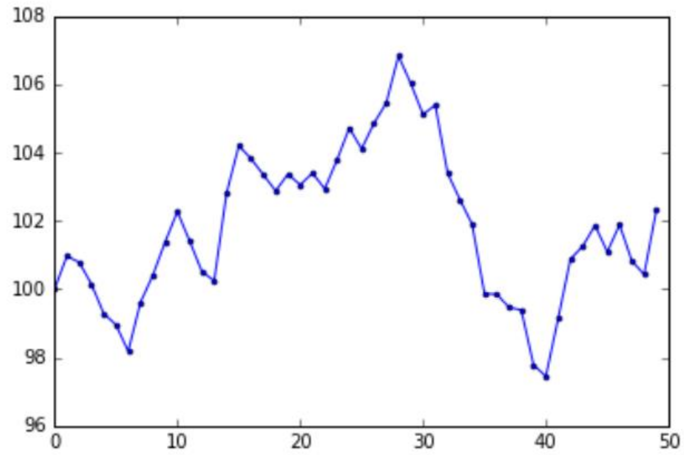
Pull data from yahoo API and plot autocorrelation of S&P500 whether it resembles white noise process

Random Walk

- Today's Price = Yesterday's Price + Noise

$$P_t = P_{t-1} + \epsilon_t$$

- Plot of simulated data



Random Walk

- Today's Price = Yesterday's Price + Noise

$$P_t = P_{t-1} + \epsilon_t$$

- Change in price is white noise

$$P_t - P_{t-1} = \epsilon_t$$

- Can't forecast a random walk
- Best forecast for tomorrow's price is today's price

Random Walk

- Today's Price = Yesterday's Price + Noise

$$P_t = P_{t-1} + \epsilon_t$$

- Random walk with drift:

$$P_t = \mu + P_{t-1} + \epsilon_t$$

- Change in price is white noise with non-zero mean:

$$P_t - P_{t-1} = \mu + \epsilon_t$$

Statistical test for Random Walk

- Random walk with drift

$$P_t = \mu + P_{t-1} + \epsilon_t$$

- Regression test for random walk

$$P_t = \alpha + \beta P_{t-1} + \epsilon_t$$

- Test:

$H_0 : \beta = 1$ (random walk)

$H_1 : \beta < 1$ (not random walk)

Statistical test for Random Walk

- Regression test for random walk

$$P_t = \alpha + \beta P_{t-1} + \epsilon_t$$

- Equivalent to

$$P_t - P_{t-1} = \alpha + \beta P_{t-1} + \epsilon_t$$

- Test:

$$H_0 : \beta = 0 \text{ (random walk)}$$

$$H_1 : \beta < 0 \text{ (not random walk)}$$

- This test is called the **Dickey-Fuller** test
- If you add more lagged changes on the right hand side, it's the **Augmented Dickey-Fuller** test

ADF test in python

White Noise VS Random Walk

Key Differences

Feature	White Noise	Random Walk
Correlation	No correlation	Positive autocorrelation
Stationarity	Stationary	Non-stationary
Visualization	Scattered dots	Connected dots
Applications	Benchmarking, ARMA models	Modeling random trends

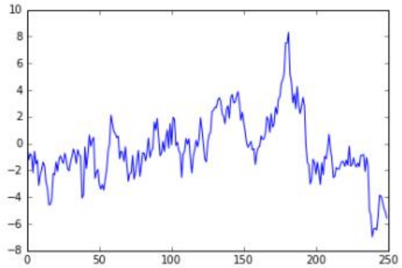
AR(1) model

$$R_t = \mu + \phi R_{t-1} + \epsilon_t$$

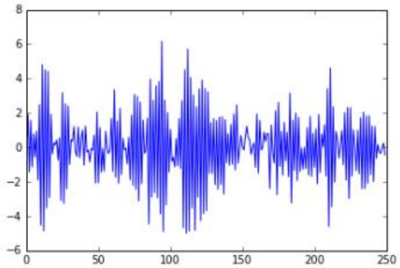
- Since only one lagged value on right hand side, this is called:
  - AR model of order 1, or
  - AR(1) model
- AR parameter is ϕ
- For stationarity, $-1 < \phi < 1$
- Negative ϕ: Mean Reversion
- Positive ϕ: Momentum

Comparison of AR(1) Time Series

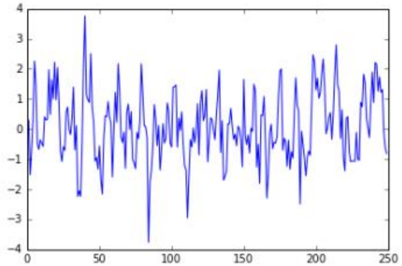
- $\phi = 0.9$



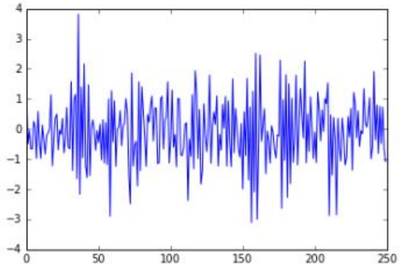
- $\phi = -0.9$



- $\phi = 0.5$



- $\phi = -0.5$



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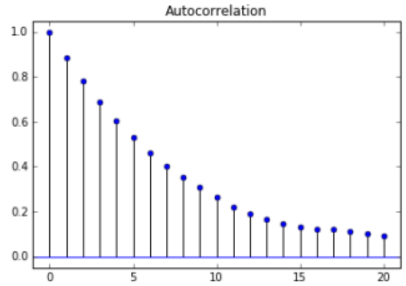
AR(1) Autocorrelation

- $\phi = 0.9$



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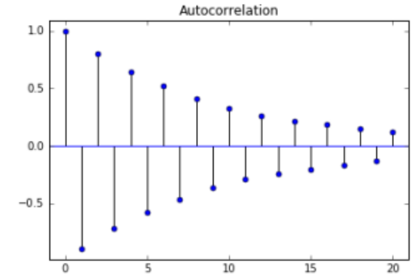

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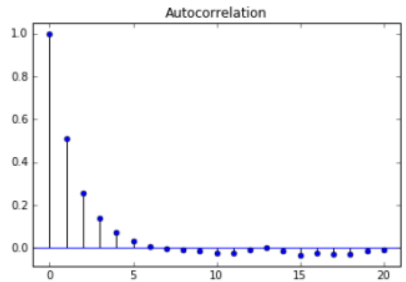
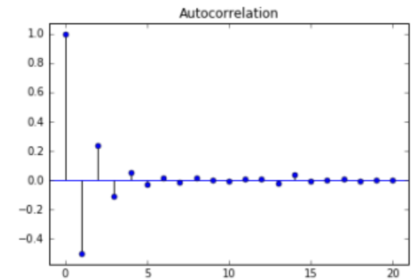


- $\phi = -0.5$



Higher Order AR Models

- AR(1)

$$R_t = \mu + \phi_1 R_{t-1} + \epsilon_t$$

- AR(2)

$$R_t = \mu + \phi_1 R_{t-1} + \phi_2 R_{t-2} + \epsilon_t$$

- AR(3)

$$R_t = \mu + \phi_1 R_{t-1} + \phi_2 R_{t-2} + \phi_3 R_{t-3} + \epsilon_t$$

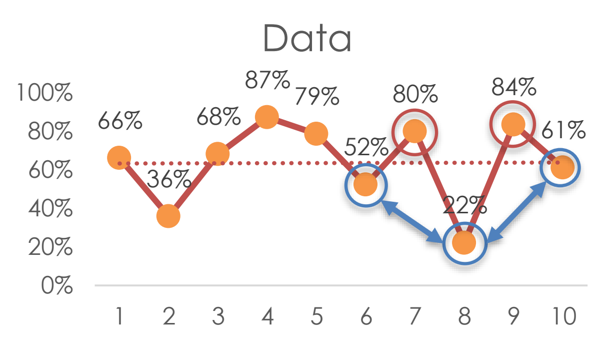
- ...

Identifying the order of AR model

- The order of an AR(p) model will usually be unknown
- Two techniques to determine order
  - Partial Autocorrelation Function
  - Information criteria

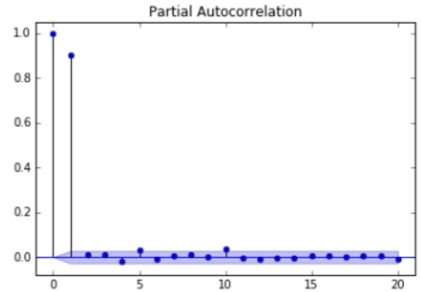
Partial Autocorrelation Function (PACF)

$$R_t = \phi_{0,1} + \boxed{\phi_{1,1}} R_{t-1} + \epsilon_{1t}$$
$$R_t = \phi_{0,2} + \phi_{1,2} R_{t-1} + \boxed{\phi_{2,2}} R_{t-2} + \epsilon_{2t}$$
$$R_t = \phi_{0,3} + \phi_{1,3} R_{t-1} + \phi_{2,3} R_{t-2} + \boxed{\phi_{3,3}} R_{t-3} + \epsilon_{3t}$$
$$R_t = \phi_{0,4} + \phi_{1,4} R_{t-1} + \phi_{2,4} R_{t-2} + \phi_{3,4} R_{t-3} + \boxed{\phi_{4,4}} R_{t-4} + \epsilon_{4t}$$
$$\vdots$$

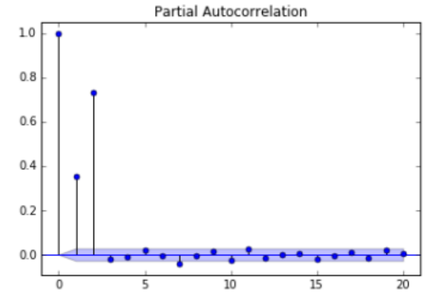


PACF for different AR Models

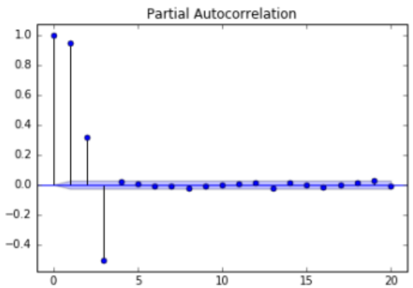
- AR(1)



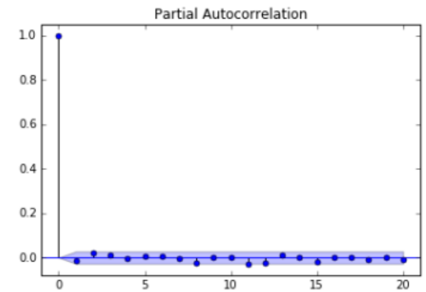
- AR(2)



- AR(3)

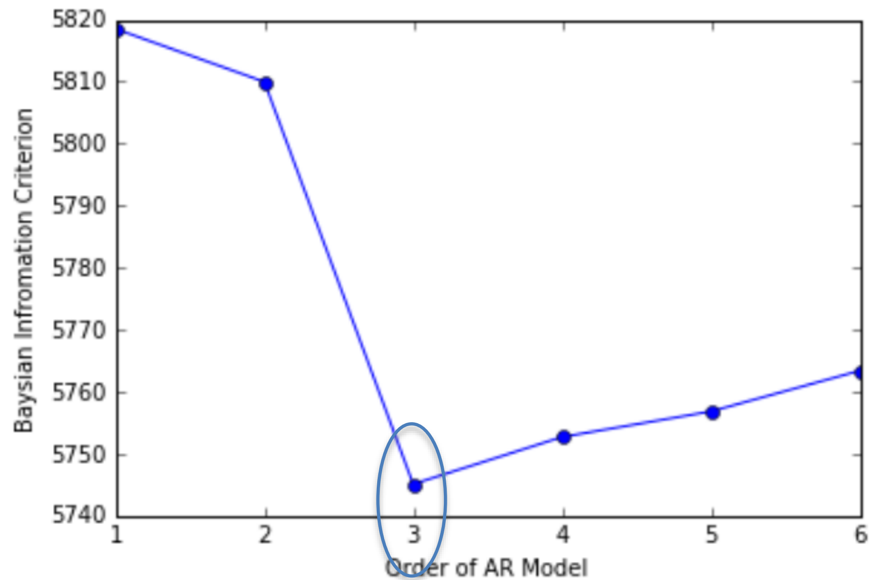


- White Noise



Information Criteria

- Fit a simulated AR(3) to different AR(p) models
- Choose p with the lowest BIC



MA(1) Model

$$R_t = \mu + \epsilon_t + \theta \, \epsilon_{t-1}$$

- Since only one lagged error on right hand side, this is called:
  - MA model of order 1, or
  - MA(1) model
- MA parameter is θ
- Stationary for all values of θ

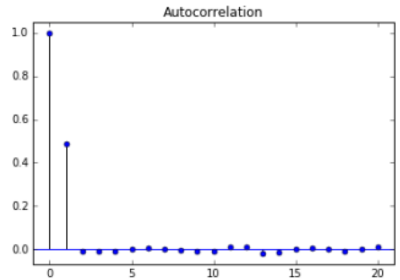
Interpretation of MA(1)

$$R_t = \mu + \epsilon_t + \theta\,\epsilon_{t-1}$$

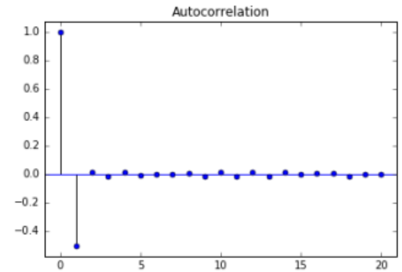
- Negative θ: One-Period Mean Reversion
- Positive θ: One-Period Momentum
- Note: One-period autocorrelation is $\theta/(1+\theta^2)$, not θ

MA(1) - ACF

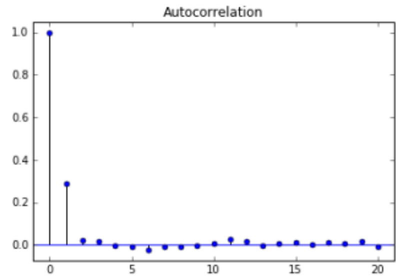
- $\theta = 0.9$



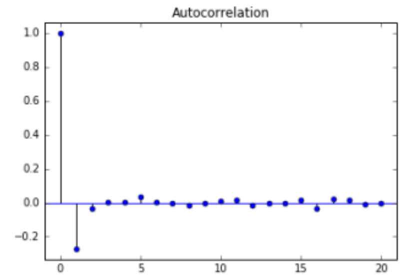
- $\theta = -0.9$



- $\theta = 0.5$



- $\theta = -0.5$



MSME BUSINESS SCHOOL
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Higher Order MA Models

- MA(1)

$$R_t = \mu + \epsilon_t - \theta_1\,\epsilon_{t-1}$$

- MA(2)

$$R_t = \mu + \epsilon_t - \theta_1\,\epsilon_{t-1} - \theta_2\,\epsilon_{t-2}$$

- MA(3)

$$R_t = \mu + \epsilon_t - \theta_1\,\epsilon_{t-1} - \theta_2\,\epsilon_{t-2} - \theta_3\,\epsilon_{t-3}$$

- ...

ARMA Model

- ARMA(1,1) model:

$$R_t = \mu + \phi R_{t-1} + \epsilon_t + \theta \epsilon_{t-1}$$