

Euclid's Elements

Book VI

One can state, without exaggeration, that the observation of and the search for similarities and differences are the basis of all human knowledge.

Alfred Nobel



Table of Contents, Chapter 6

1	If the height of two triangles are equal, then the ratio of the areas is equal to the ratio of the bases	7	If two triangles have one angle equal to one angle, and the sides about other angles are proportional, and the remaining angles either both less or both not less than a right angle, then triangles will be equiangular	14	In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and vice versa
2	If a line cuts a triangle, parallel to its base, it will cut the sides of the triangle proportionally			15	In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and vice versa
3	If an angle of a triangle is bisected and the straight line cutting the angle also cuts the base, the segments of the base will have the same ratio as the remaining sides of the triangle	8	If in a right-angled triangle a perpendicular be drawn from the right angle to the base, the triangles adjoining the perpendicular are similar both to the whole and to one another	16	If four straight lines are proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means, and vice versa
4	If two triangles have equal angles, then the sides opposite the equal angles are proportional, as well, the sides of the triangles on either side of the equal angles are also proportional	9	From a given straight line to cut off a given fraction	17	If three straight lines are proportional, the rectangle contained by the extremes is equal to the square on the mean; and vice versa
5	If two triangles have proportional sides, the triangles will be equiangular	10	To cut a given uncut straight line similarly to a given cut straight line	18	On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure
6	If two triangles have one angle equal to one angle and the sides about the equal angles are proportional, then the triangles will be equiangular	11	To two given straight lines to find a third proportional	19	Similar triangles are to one another in the duplicate ratio of the corresponding sides
		12	To three given straight lines to find a fourth proportional		
		13	To two given straight lines to find a mean proportional		



Table of Contents, Chapter 3

20	Similar polygons are divided into the same number of similar triangles, which have the same ratio as the wholes, and the polygons have duplicate ratios to their corresponding sides	26	If from a parallelogram a similar parallelogram with a common angle is subtracted, it is about the same diameter as the original	31	In right-angled triangles the figure on the side subtending the right angle is equal to the similar and similarly described figures on the sides containing the right angle
21	Figures which are are similar to the same rectilineal figure are also similar to one another	27	Of all the parallelograms applied to the same straight line and deficient by parallelogrammic figures similar to a parallelogram drawn on half the said line, the largest will be one that is drawn on half of the straight line and is similar to the defect		
22	If four straight lines are proportional, similar rectilineal figures will also be proportional; and vice versa	28	To a given straight line, apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one		
23	Equiangular parallelograms have to one another the ratio compounded of the ratios of their sides	29	To a given straight line, apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one		
24	In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another	30	To cut a finite straight line in extreme ratio		
25	To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure				



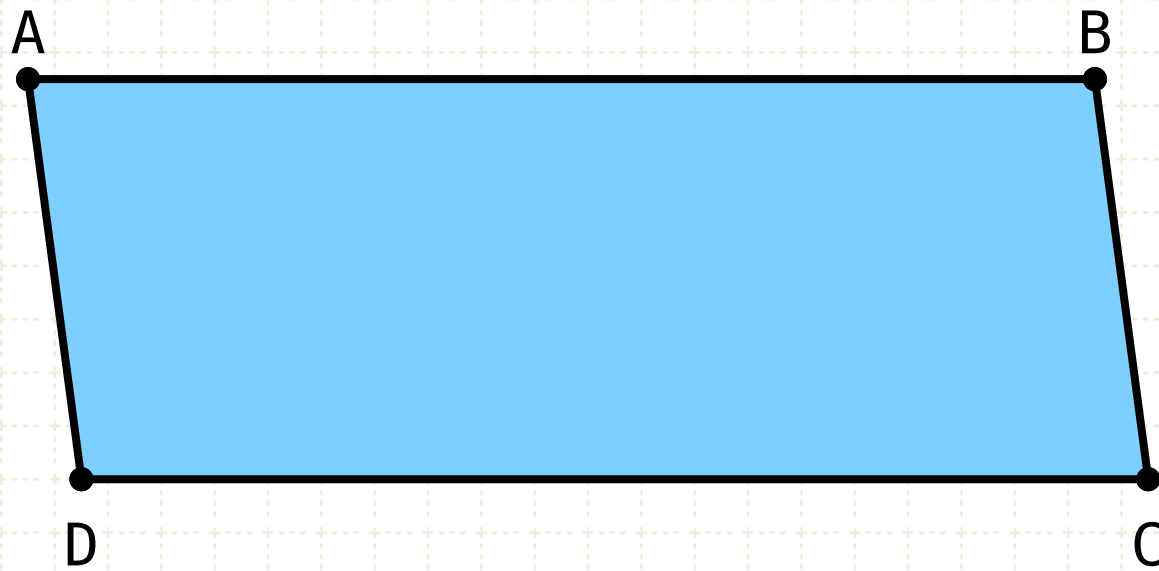
Proposition 24 of Book VI

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.



Proposition 24 of Book VI

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.

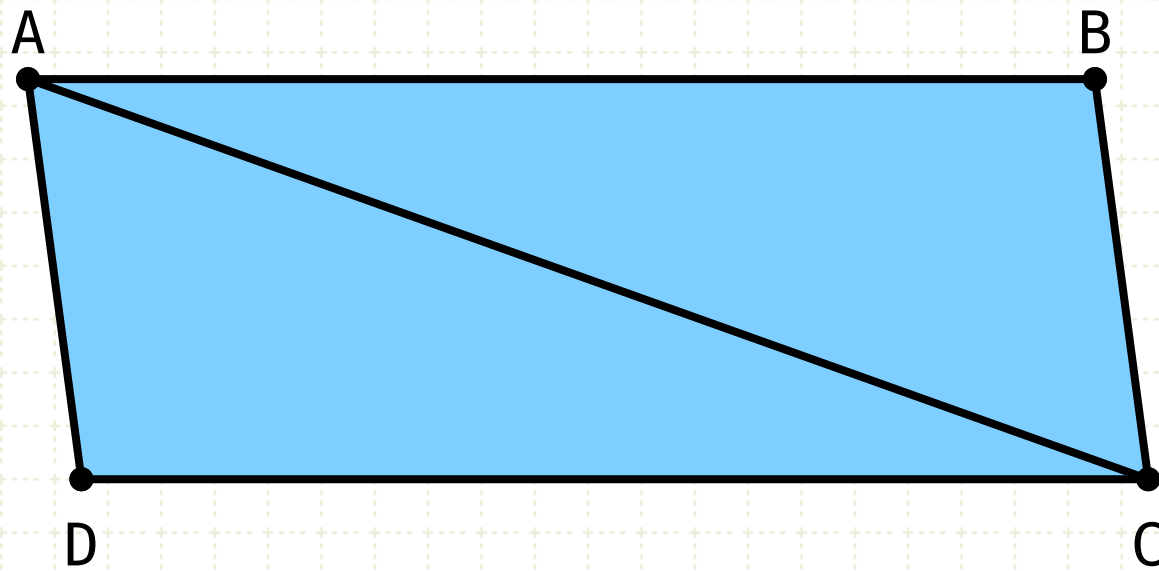


In other words

Start with a parallelogram

Proposition 24 of Book VI

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.



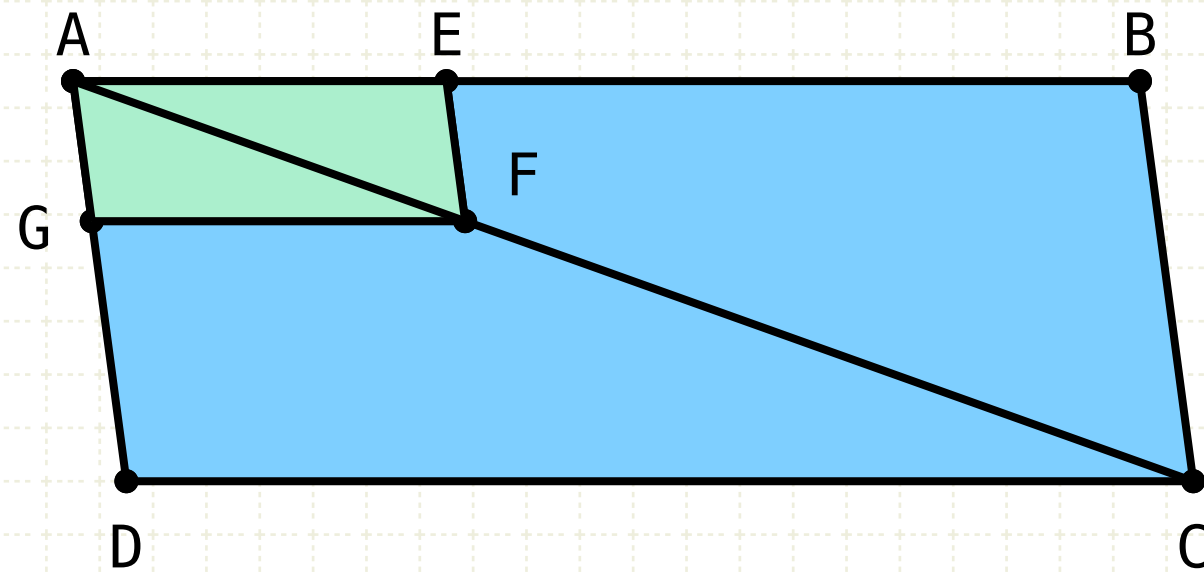
In other words

Start with a parallelogram

Draw the diameter AC

Proposition 24 of Book VI

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.



In other words

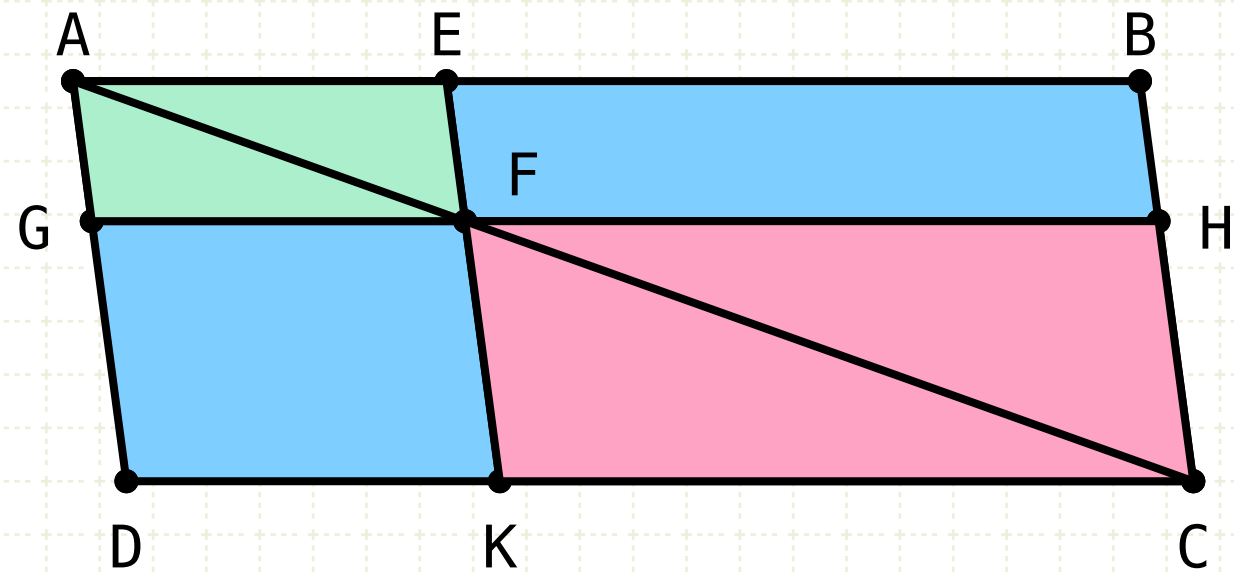
Start with a parallelogram

Draw the diameter AC

Construct a parallelogram EG on the diameter AC

Proposition 24 of Book VI

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.



In other words

Start with a parallelogram

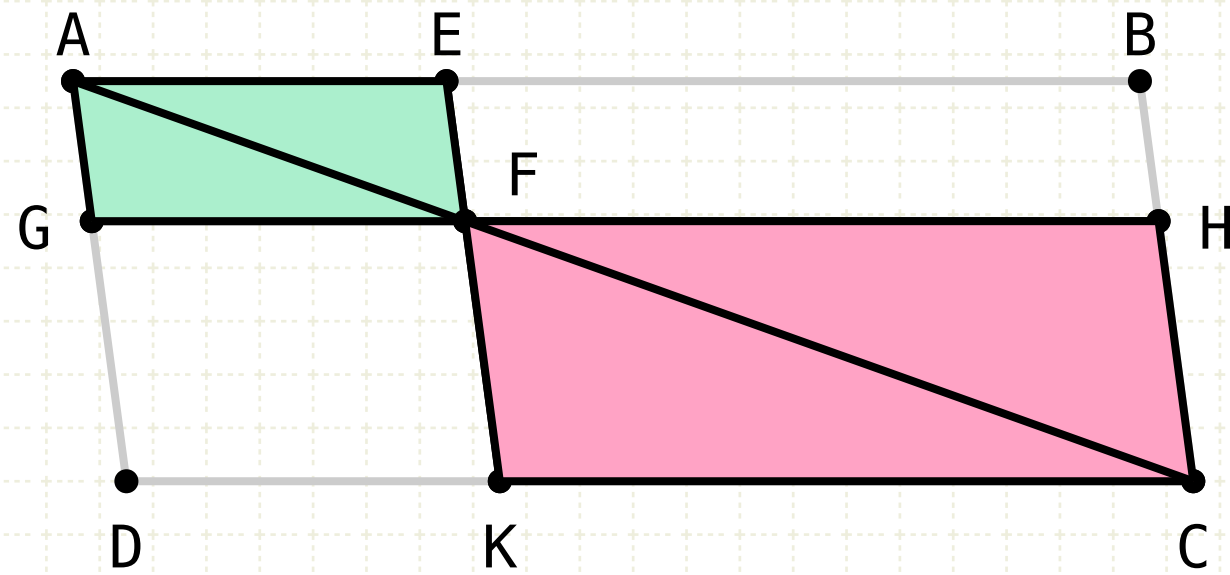
Draw the diameter AC

Construct a parallelogram EG on the diameter AC

Construct a parallelogram HK on the diameter AC

Proposition 24 of Book VI

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.



$$\square AC \sim \square EG \sim \square HK$$

In other words

Start with a parallelogram

Draw the diameter AC

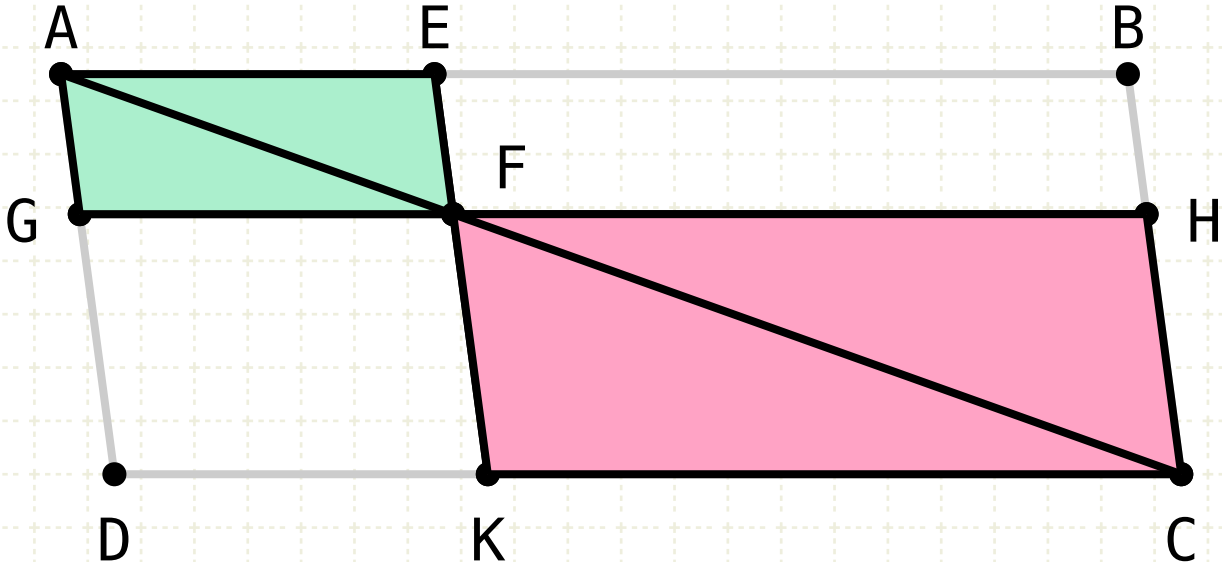
Construct a parallelogram EG on the diameter AC

Construct a parallelogram HK on the diameter AC

The resulting parallelograms will all be similar to one another

Proposition 24 of Book VI

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.

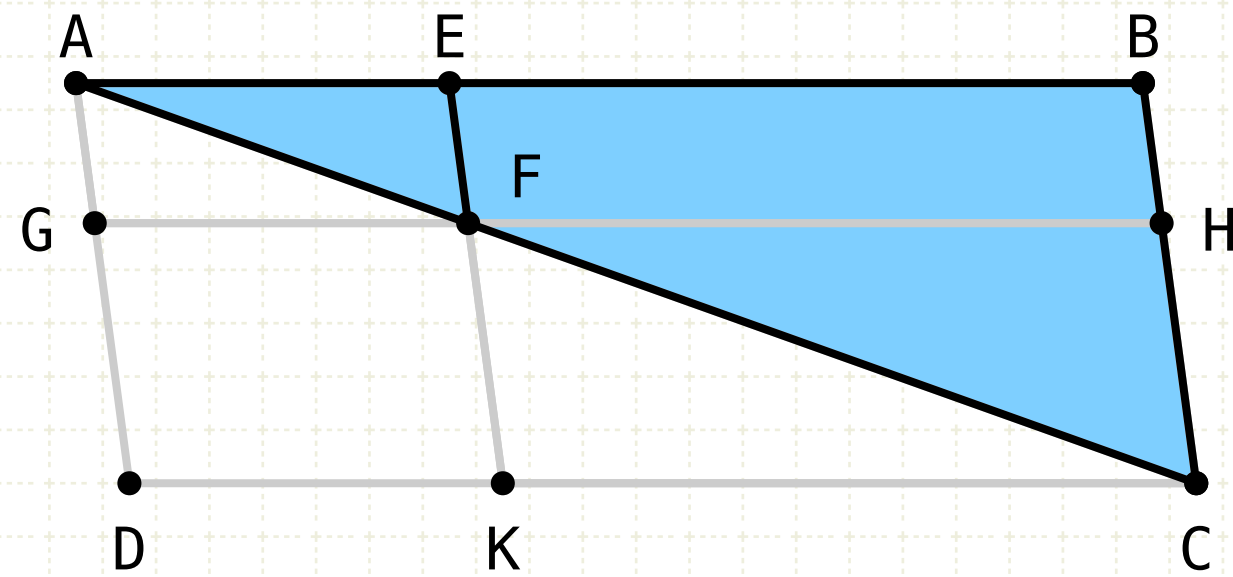


Proof



Proposition 24 of Book VI

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.



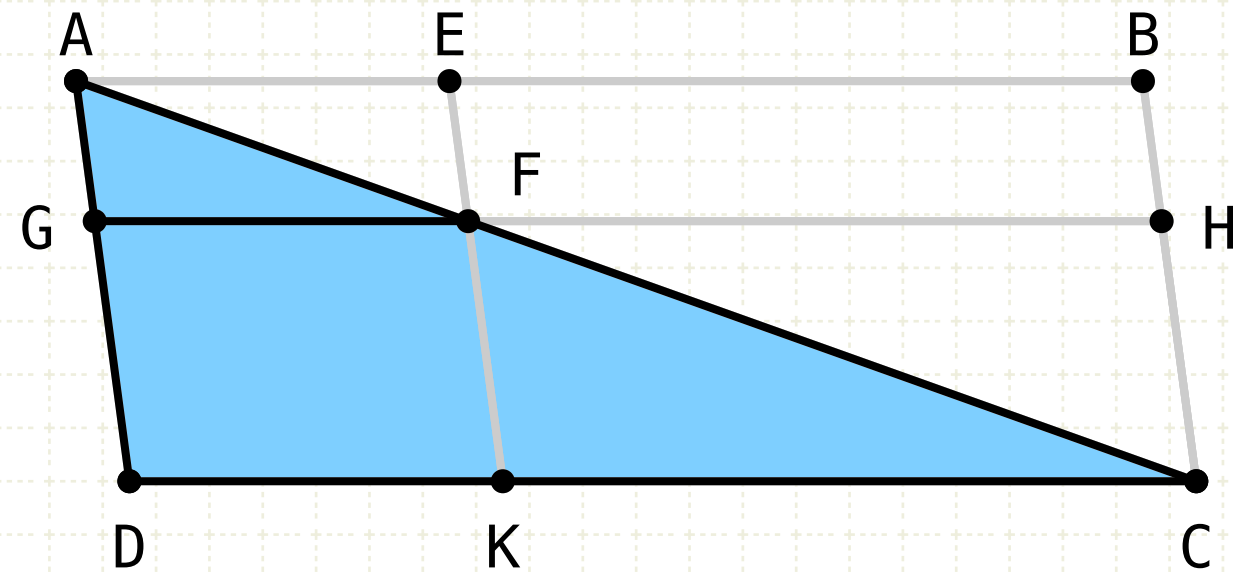
$$BE : EA = CF : FA$$

Proof

In the triangle ABC, EF and BC are parallel, thus BE is to EA as CF is to FA (VI-2)

Proposition 24 of Book VI

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.



$$BE : EA = CF : FA$$

$$CF : FA = DG : GA$$

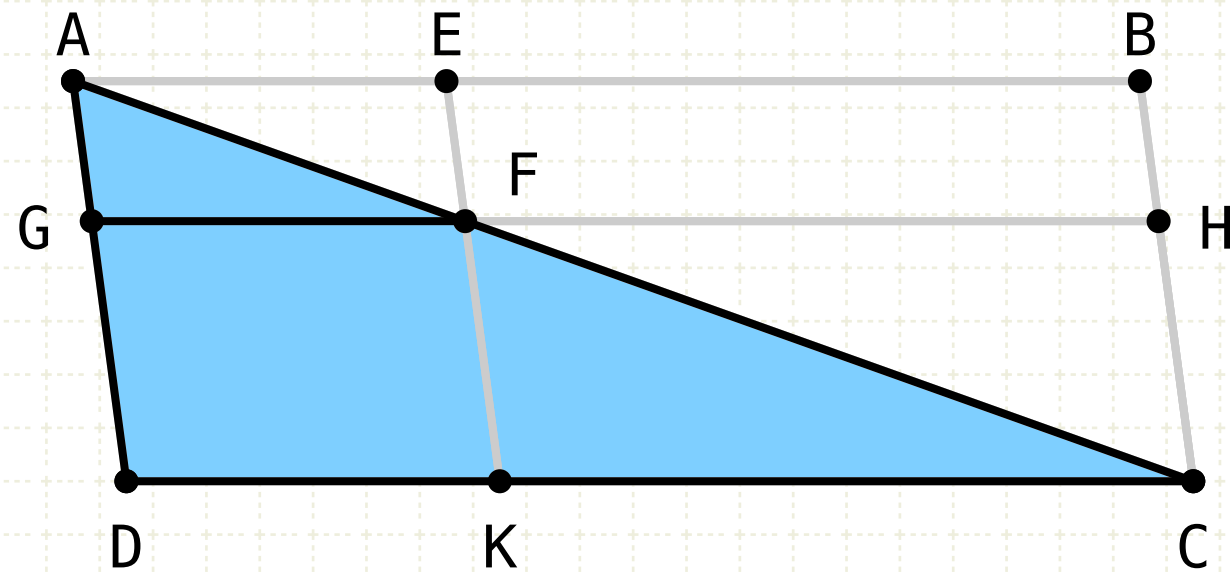
Proof

In the triangle ABC, EF and BC are parallel, thus BE is to EA as CF is to FA (VI·2)

In the triangle ACD, FG and CD are parallel, thus CF is to FA as DG is to GA (VI·2)

Proposition 24 of Book VI

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.



$$BE : EA = CF : FA$$

$$CF : FA = DG : GA$$

$$BE : EA = DG : GA$$

Proof

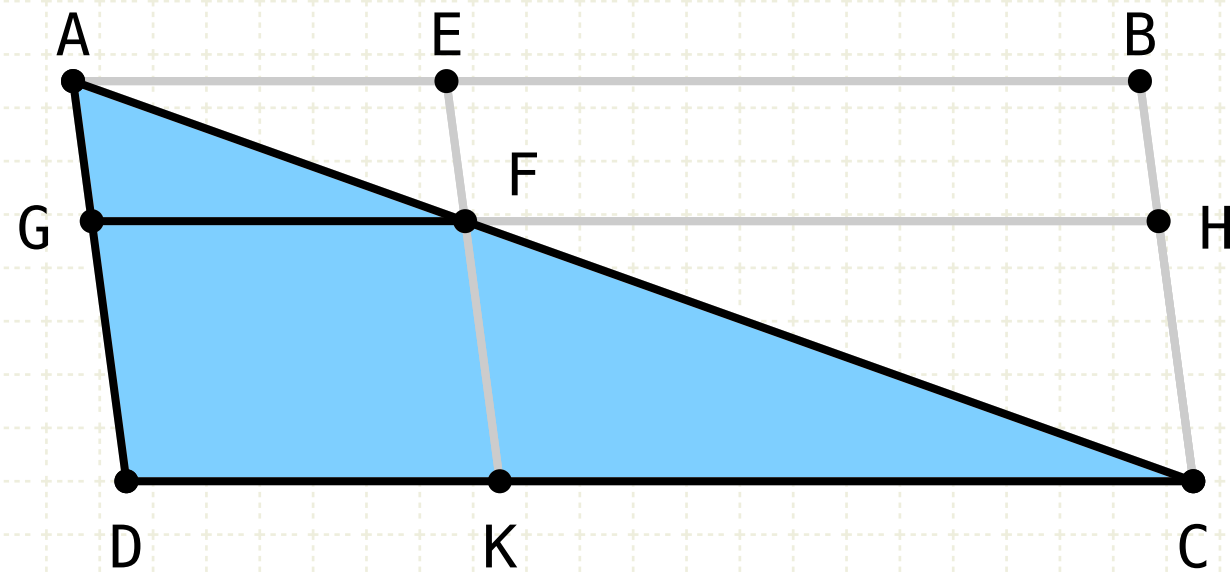
In the triangle ABC, EF and BC are parallel, thus BE is to EA as CF is to FA (VI·2)

In the triangle ACD, FG and CD are parallel, thus CF is to FA as DG is to GA (VI·2)

Thus, BE is to EA as DG is to GA

Proposition 24 of Book VI

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.



$$BE:EA = CF:FA$$

$$CF:FA = DG:GA$$

$$BE:EA = DG:GA$$

$$(BE+EA):EA = (DG+GA):GA$$

$$BA:EA = DA:GA$$

Proof

In the triangle ABC, EF and BC are parallel, thus BE is to EA as CF is to FA (VI·2)

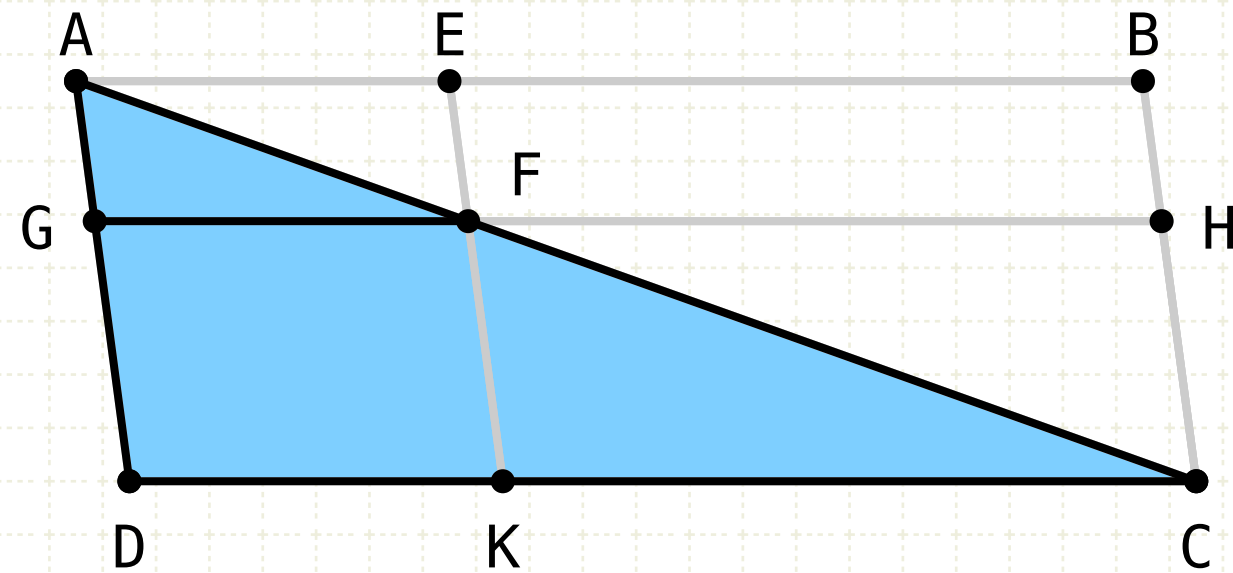
In the triangle ACD, FG and CD are parallel, thus CF is to FA as DG is to GA (VI·2)

Thus, BE is to EA as DG is to GA

Therefore (componendo) BA is to EA as DA is to AG (V·18)

Proposition 24 of Book VI

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.



$$BE:EA = CF:FA$$

$$CF:FA = DG:GA$$

$$BE:EA = DG:GA$$

$$(BE+EA):EA = (DG+GA):GA$$

$$BA:EA = DA:GA$$

$$BA:DA = EA:GA$$

Proof

In the triangle ABC, EF and BC are parallel, thus BE is to EA as CF is to FA (VI·2)

In the triangle ACD, FG and CD are parallel, thus CF is to FA as DG is to GA (VI·2)

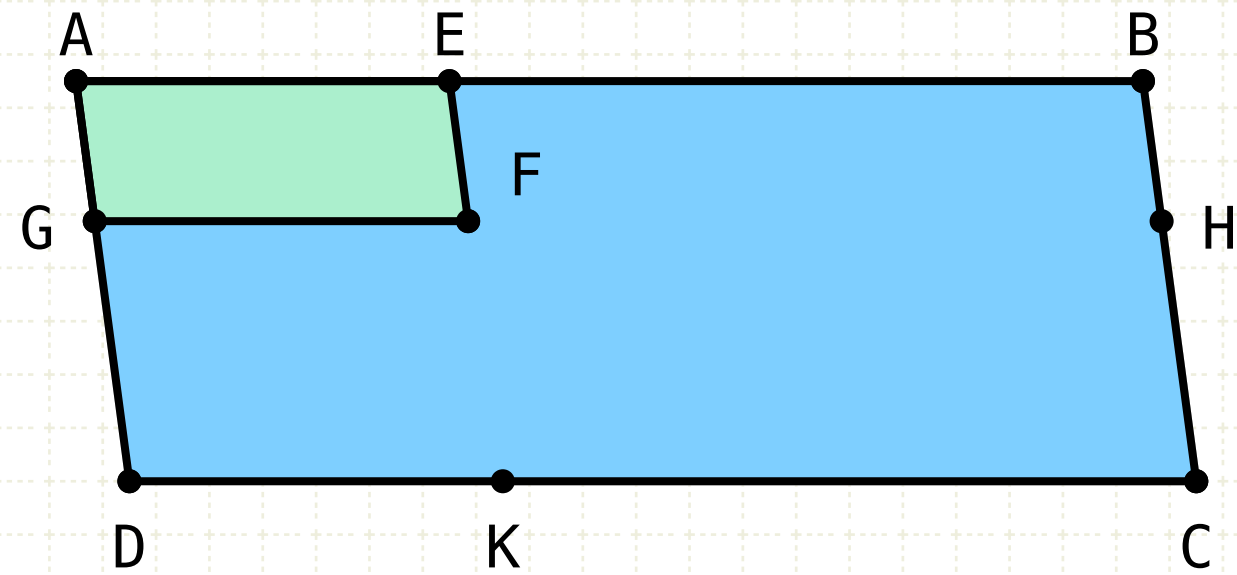
Thus, BE is to EA as DG is to GA

Therefore (componendo) BA is to EA as DA is to AG (V·18)

and alternately BA is to DA so is AE to AG (V·16)

Proposition 24 of Book VI

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.



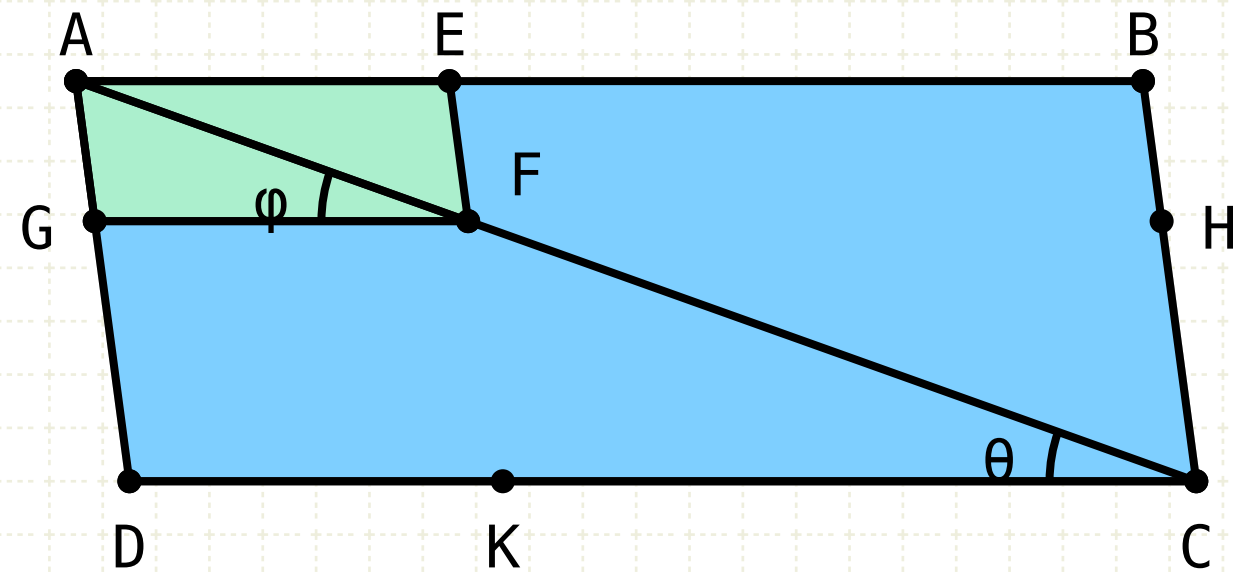
$$BA:DA = EA:GA$$

Proof (cont)

For the parallelograms ABCD, EG, the sides about the common angle BAD are proportional

Proposition 24 of Book VI

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.



$$BA:DA = EA:GA$$

$$\varphi = \theta$$

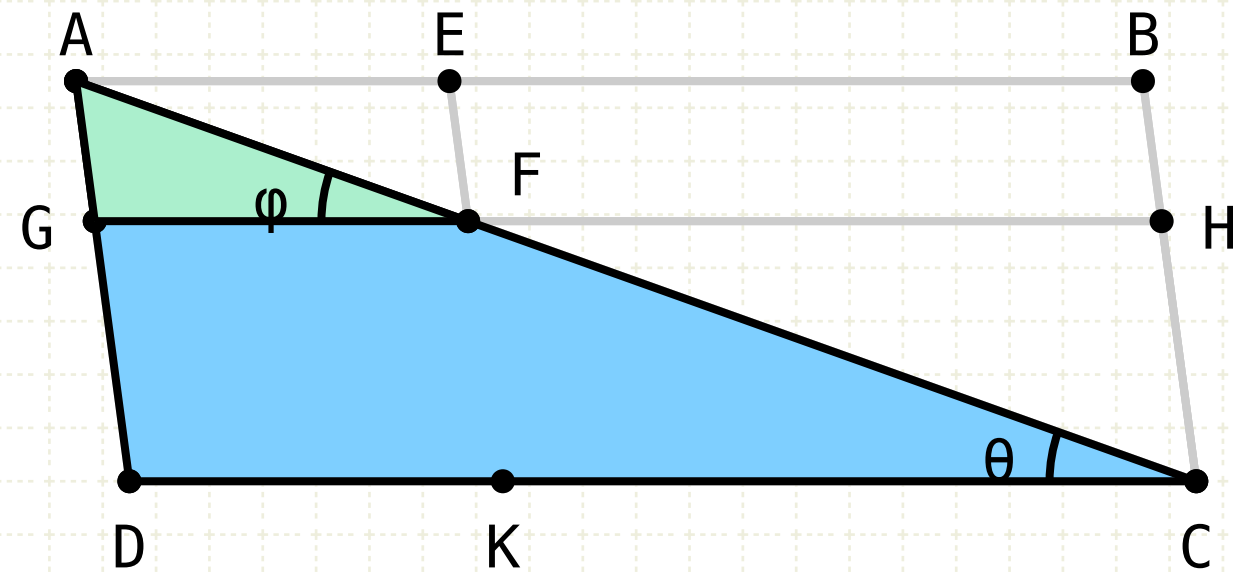
Proof (cont)

For the parallelograms ABCD, EG, the sides about the common angle BAD are proportional

Since GF is parallel to DC, then angles AFG, DCA are equal

Proposition 24 of Book VI

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.



$$BA:DA = EA:GA$$

$$\varphi = \theta$$

$\triangle AGF$ equiangular to $\triangle ADC$

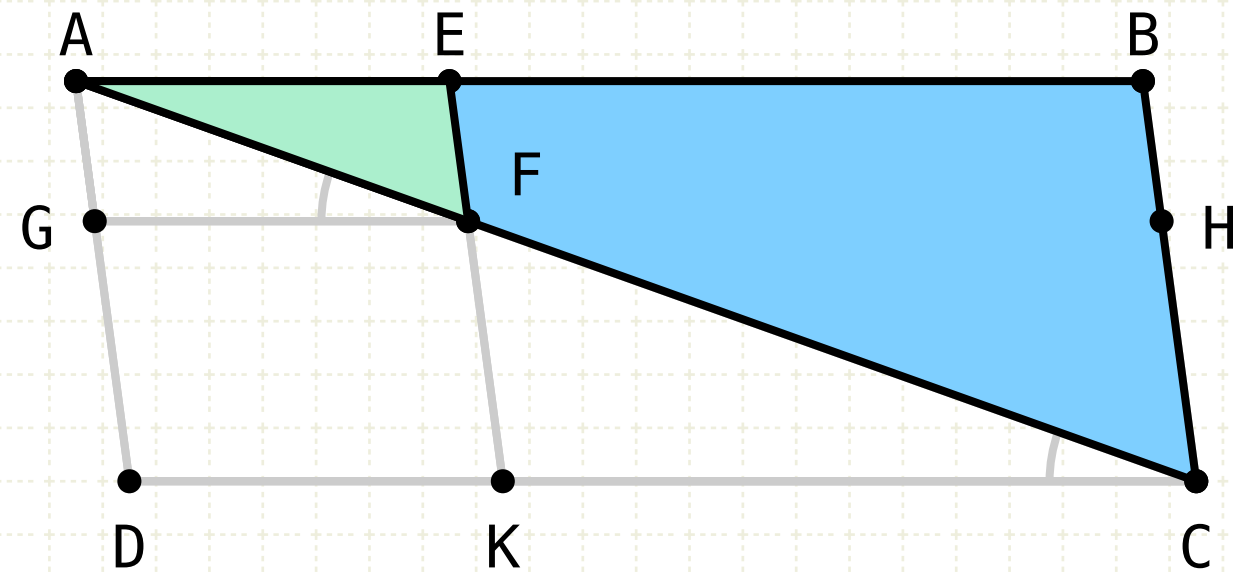
Proof (cont)

For the parallelograms ABCD, EG, the sides about the common angle BAD are proportional

Since GF is parallel to DC, then angles AFG, DCA are equal and angle GAF is common to both triangles AGF and ADC, therefore these two triangles are equiangular

Proposition 24 of Book VI

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.



$$BA:DA = EA:GA$$

$$\varphi = \theta$$

$\triangle AGF$ equiangular to $\triangle ADC$

$\triangle AFE$ equiangular to $\triangle ACB$

Proof (cont)

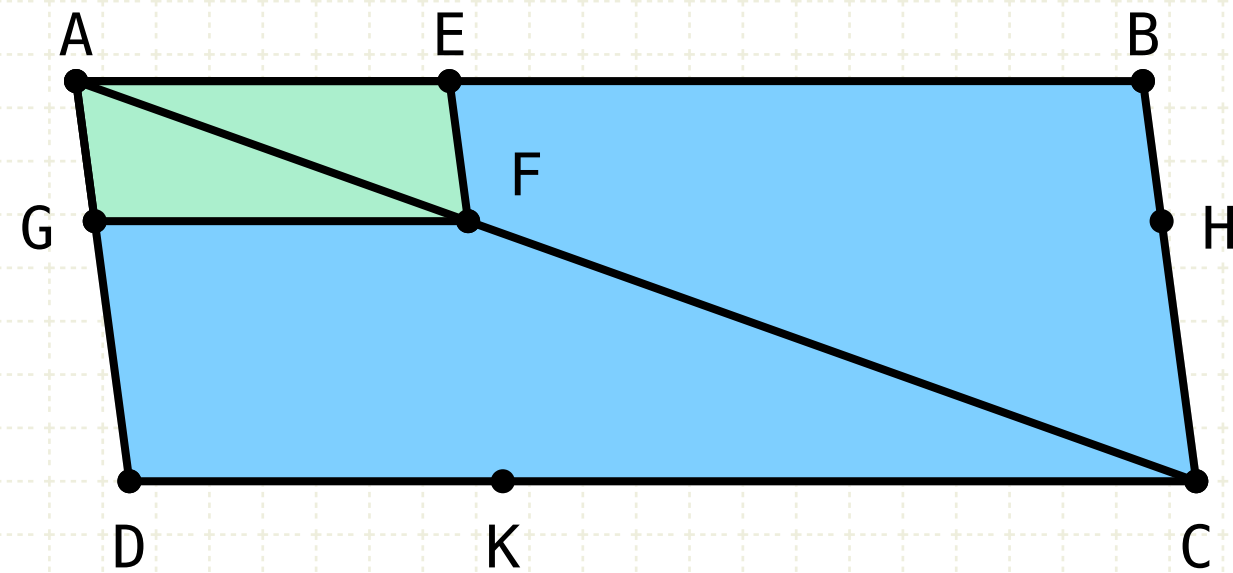
For the parallelograms ABCD, EG, the sides about the common angle BAD are proportional

Since GF is parallel to DC, then angles AFG, DCA are equal and angle GAF is common to both triangles AGF and ADC, therefore these two triangles are equiangular

For the same reason triangle ACB is equiangular with triangle AFE

Proposition 24 of Book VI

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.



$$BA:DA = EA:GA$$

$$\varphi = \theta$$

$\triangle AGF$ equiangular to $\triangle ADC$

$\triangle AFE$ equiangular to $\triangle ACB$

$\square ABCD$ equiangular to $\square EG$

Proof (cont)

For the parallelograms $ABCD$, EG , the sides about the common angle BAD are proportional

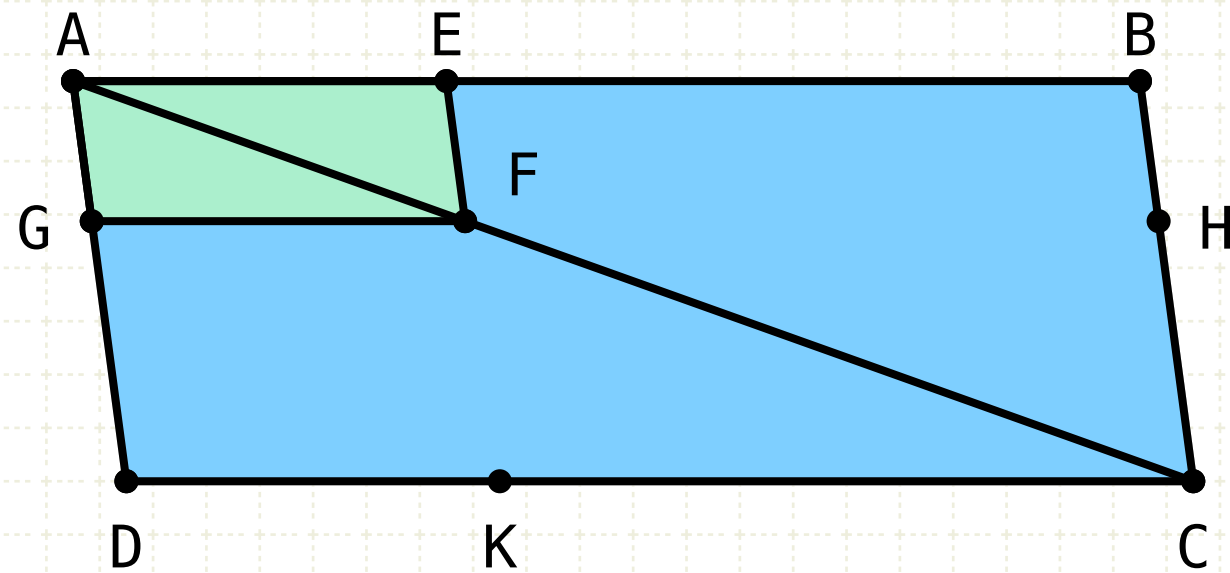
Since GF is parallel to DC , then angles AFG , DCA are equal and angle GAF is common to both triangles AGF and ADC , therefore these two triangles are equiangular

For the same reason triangle ACB is equiangular with triangle AFE

So the whole parallelogram AF is equiangular to the parallelogram $ABCD$

Proposition 24 of Book VI

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.



$$BA:DA = EA:GA$$

$$\varphi = \theta$$

$\triangle AGF$ equiangular to $\triangle ADC$

$\triangle AFE$ equiangular to $\triangle ACB$

$\square ABCD$ equiangular to $\square EG$

$$AD:DC = AG:GF$$

$$DC:CA = GF:FA$$

$$AC:CB = AF:FE$$

$$CB:BA = FE:EA$$

Proof (cont)

For the parallelograms ABCD, EG, the sides about the common angle BAD are proportional

Since GF is parallel to DC, then angles AFG, DCA are equal and angle GAF is common to both triangles AGF and ADC, therefore these two triangles are equiangular

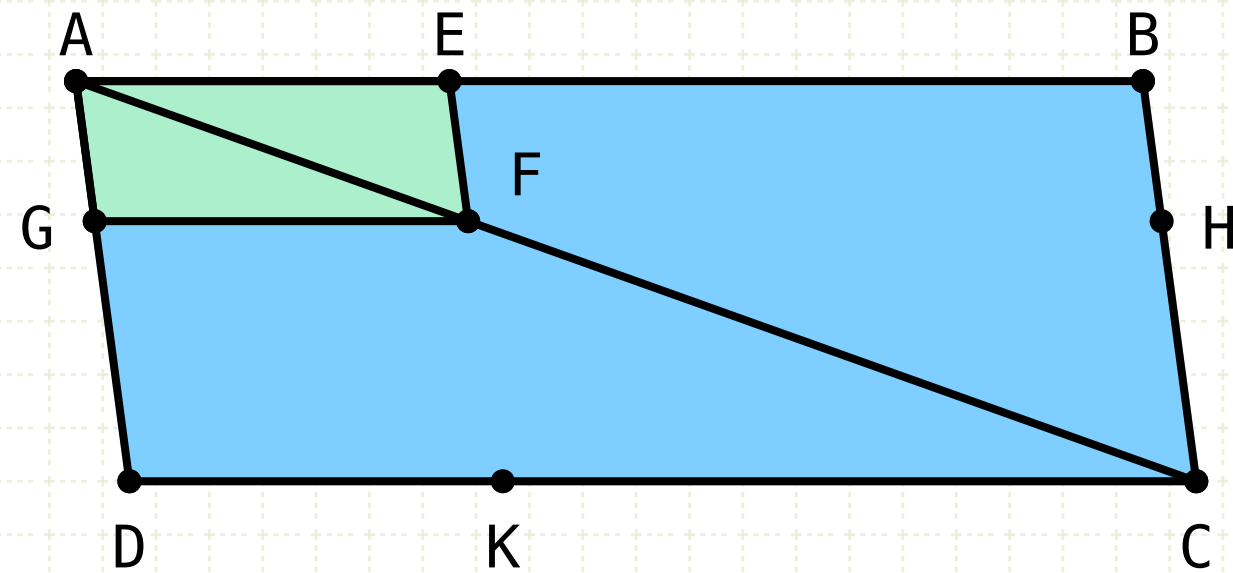
For the same reason triangle ACB is equiangular with triangle AFE

So the whole parallelogram AF is equiangular to the parallelogram ABCD

And the sides of the equiangular triangles are in proportion

Proposition 24 of Book VI

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.



$$BA:DA = EA:GA$$

$$\varphi = \theta$$

$\triangle AGF$ equiangular to $\triangle ADC$

$\triangle AFE$ equiangular to $\triangle ACB$

$\square ABCD$ equiangular to $\square EG$

$$AD:DC = AG:GF$$

$$DC:CA = GF:FA$$

$$AC:CB = AF:FE$$

$$CB:BA = FE:EA$$

$$DC:CB = GF:FE$$

Proof (cont)

For the parallelograms ABCD, EG, the sides about the common angle BAD are proportional

Since GF is parallel to DC, then angles AFG, DCA are equal and angle GAF is common to both triangles AGF and ADC, therefore these two triangles are equiangular

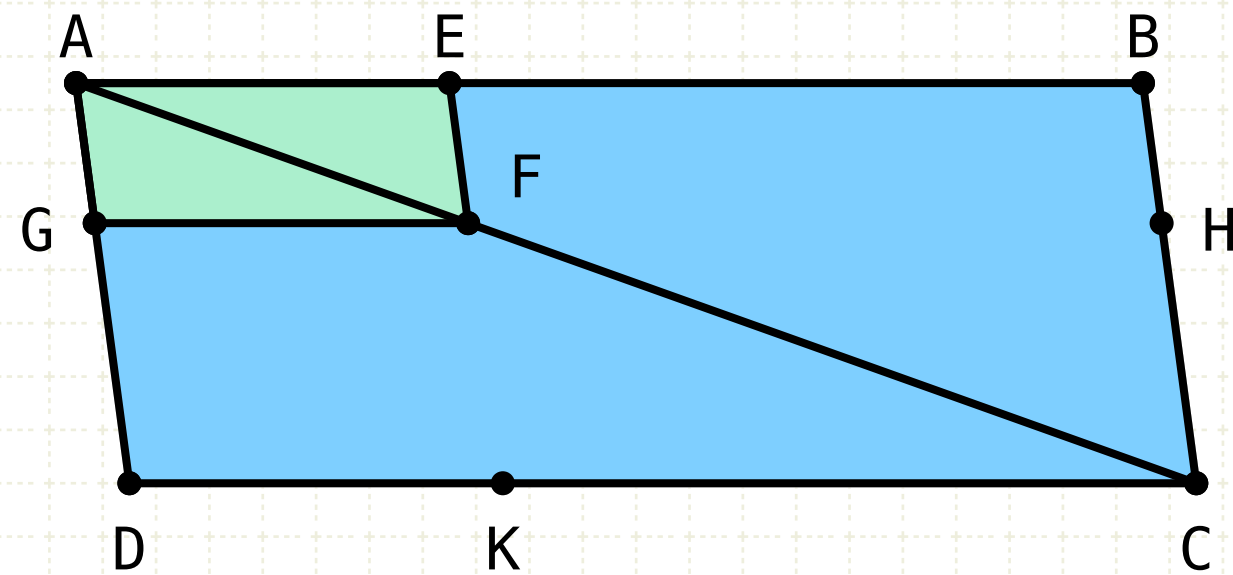
For the same reason triangle ACB is equiangular with triangle AFE

So the whole parallelogram AF is equiangular to the parallelogram ABCD

And the sides of the equiangular triangles are in proportion therefore (ex aequali) DC is to CB as GF is to FE (V.22)

Proposition 24 of Book VI

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.



$$BA:DA = EA:GA$$

$$\varphi = \theta$$

$\triangle AGF$ equiangular to $\triangle ADC$

$\triangle AFE$ equiangular to $\triangle ACB$

$\square ABCD$ equiangular to $\square EG$

$$AD:DC = AG:GF$$

$$DC:CA = GF:FA$$

$$AC:CB = AF:FE$$

$$CB:BA = FE:EA$$

$$DC:CB = GF:FE$$

Proof (cont)

For the parallelograms ABCD, EG, the sides about the common angle BAD are proportional

Since GF is parallel to DC, then angles AFG, DCA are equal and angle GAF is common to both triangles AGF and ADC, therefore these two triangles are equiangular

For the same reason triangle ACB is equiangular with triangle AFE

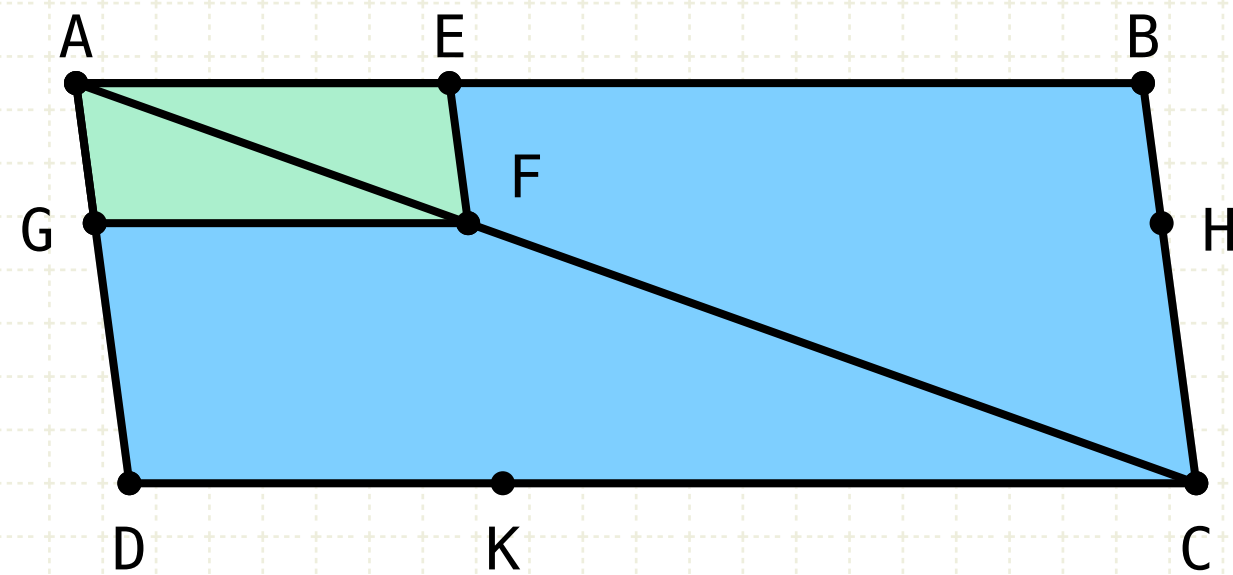
So the whole parallelogram AF is equiangular to the parallelogram ABCD

And the sides of the equiangular triangles are in proportion therefore (ex aequali) DC is to CB as GF is to FE (V.22)

Thus, the sides about equal angles in the parallelograms ABCD, EG are proportional

Proposition 24 of Book VI

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.



$$BA:DA = EA:GA$$

$$\varphi = \theta$$

$\triangle AGF$ equiangular to $\triangle ADC$

$\triangle AFE$ equiangular to $\triangle ACB$

$\square ABCD$ equiangular to $\square EG$

$$AD:DC = AG:GF$$

$$DC:CA = GF:FA$$

$$AC:CB = AF:FE$$

$$CB:BA = FE:EA$$

$$DC:CB = GF:FE$$

$$\square ABCD \sim \square GE$$

Proof (cont)

For the parallelograms ABCD, EG, the sides about the common angle BAD are proportional

Since GF is parallel to DC, then angles AFG, DCA are equal and angle GAF is common to both triangles AGF and ADC, therefore these two triangles are equiangular

For the same reason triangle ACB is equiangular with triangle AFE

So the whole parallelogram AF is equiangular to the parallelogram ABCD

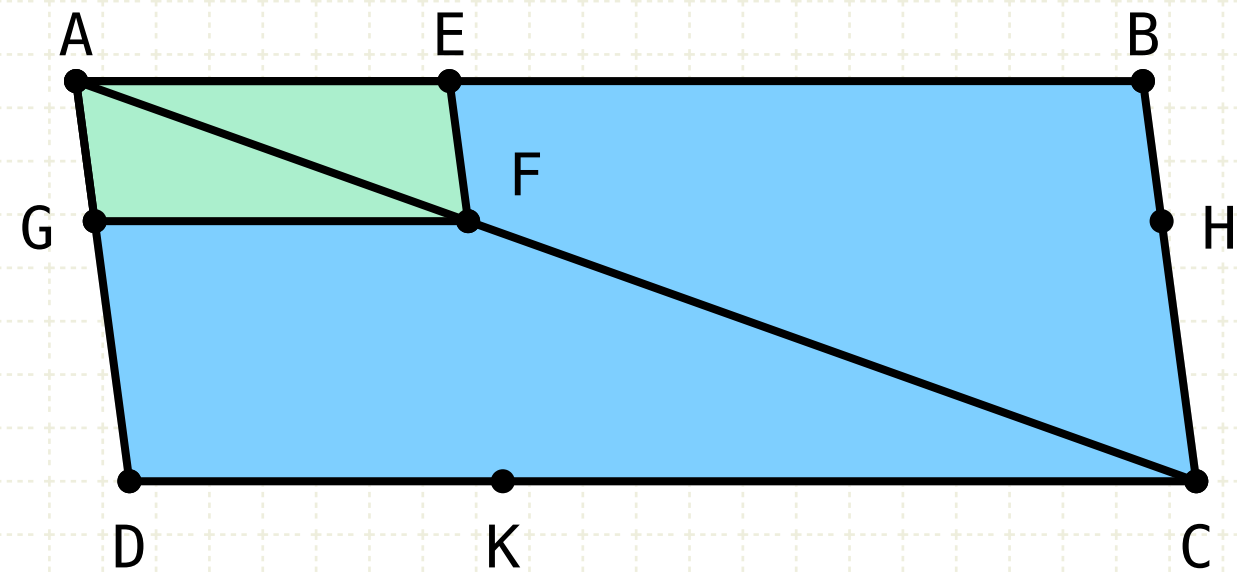
And the sides of the equiangular triangles are in proportion therefore (ex aequali) DC is to CB as GF is to FE (V.22)

Thus, the sides about equal angles in the parallelograms ABCD, EG are proportional

Therefore, the parallelograms ABCD and GE are similar (VI.Def.1)

Proposition 24 of Book VI

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.



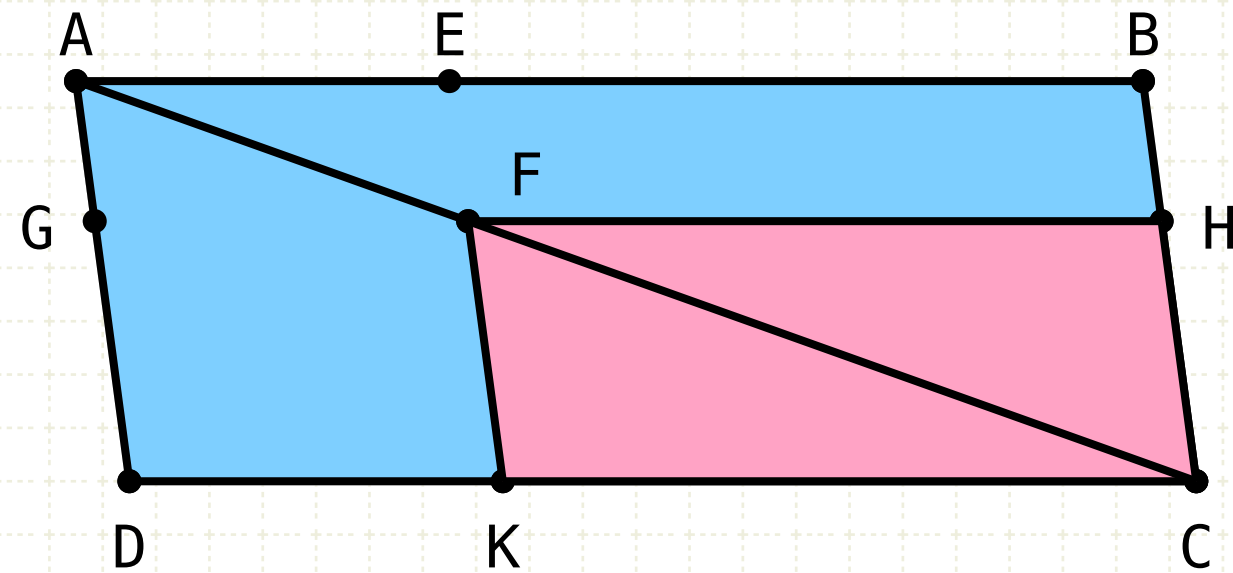
$$\square ABCD \sim \square GE$$

Proof (cont)

Therefore, the parallelograms ABCD and GE are similar
(VI.Def.1)

Proposition 24 of Book VI

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.



$$\square ABCD \sim \square GE$$

$$\square ABCD \sim \square KH$$

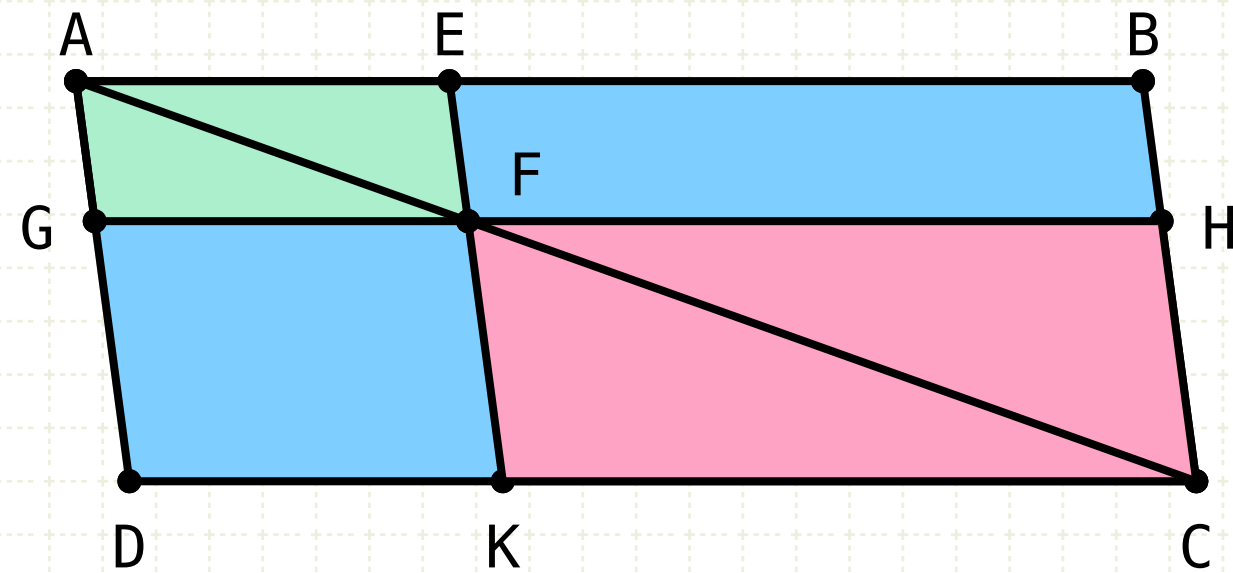
Proof (cont)

Therefore, the parallelograms ABCD and GE are similar (VI.Def.1)

Using the same logic, it can be shown that KH is similar to ABCD

Proposition 24 of Book VI

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.



$$\square ABCD \sim \square GE$$

$$\square ABCD \sim \square KH$$

$$\square GE \sim \square KH$$

Proof (cont)

Therefore, the parallelograms ABCD and GE are similar (VI.Def.1)

Using the same logic, it can be shown that KH is similar to ABCD

But if two figures are similar to a third, they are also similar to each other (VI-21)

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