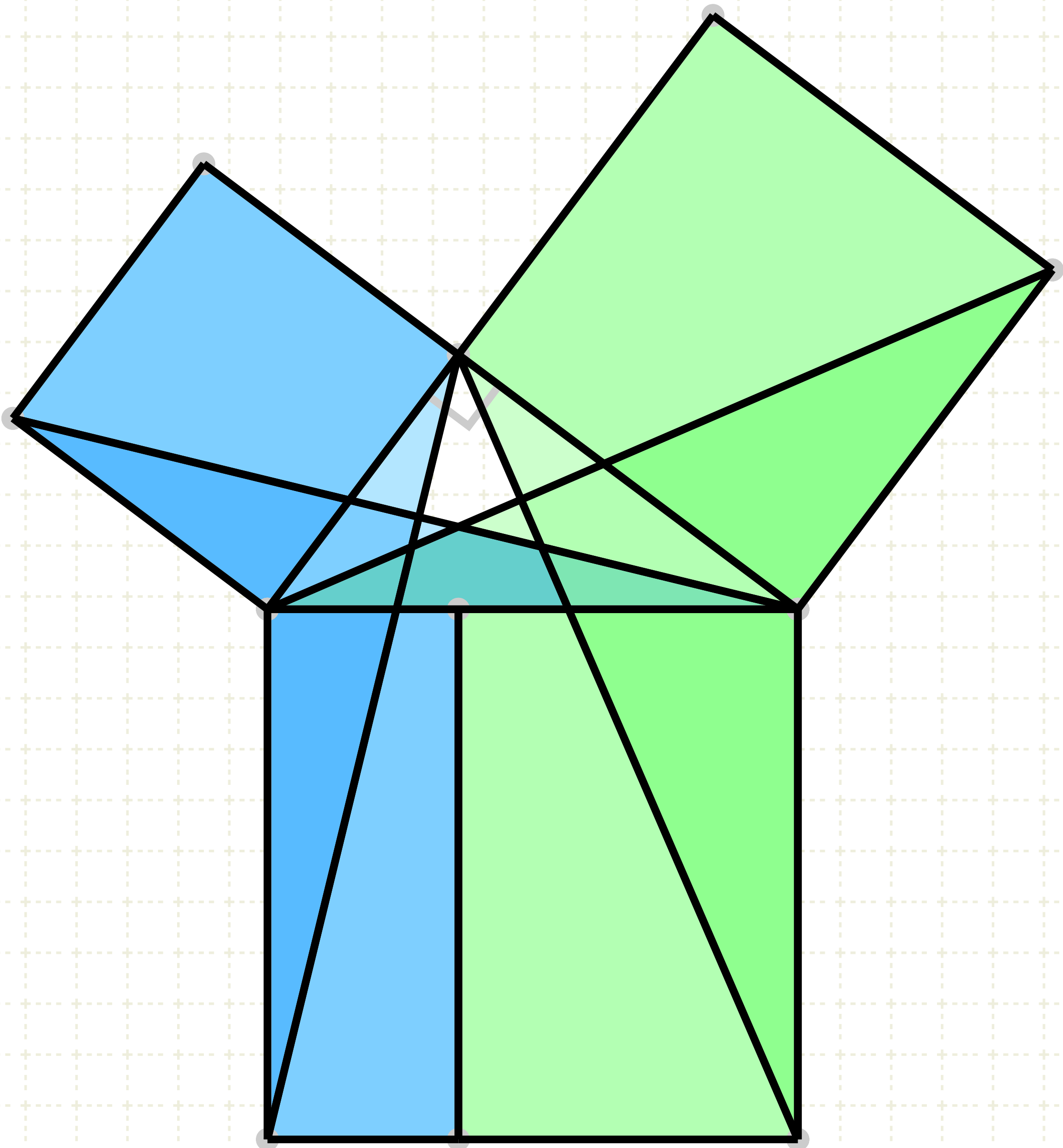
Euclid's Elements

Book I



If Euclid did not kindle your youthful enthusiasm, you were not born to be a scientific thinker.

Albert Einstein

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Proposition 7 of Book I

Given two straight lines constructed from the ends of a straight line and meeting in a point, there cannot be constructed from the ends of the same straight line, and on the same side of it, two other straight lines meeting in another point and equal to the former two respectively, namely each equal to that from the same end.

Proposition 7 of Book I

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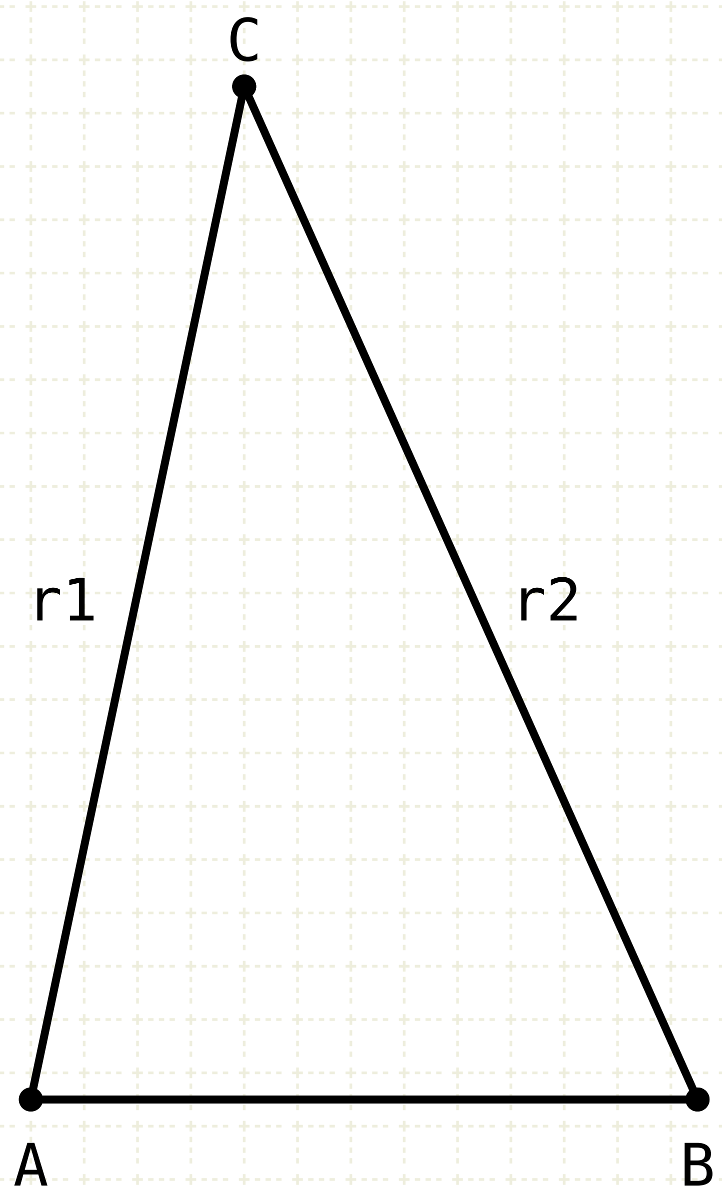

In other words

Given a triangle ABC

There is a unique point C where the sides of the triangle, AC and BC, meet

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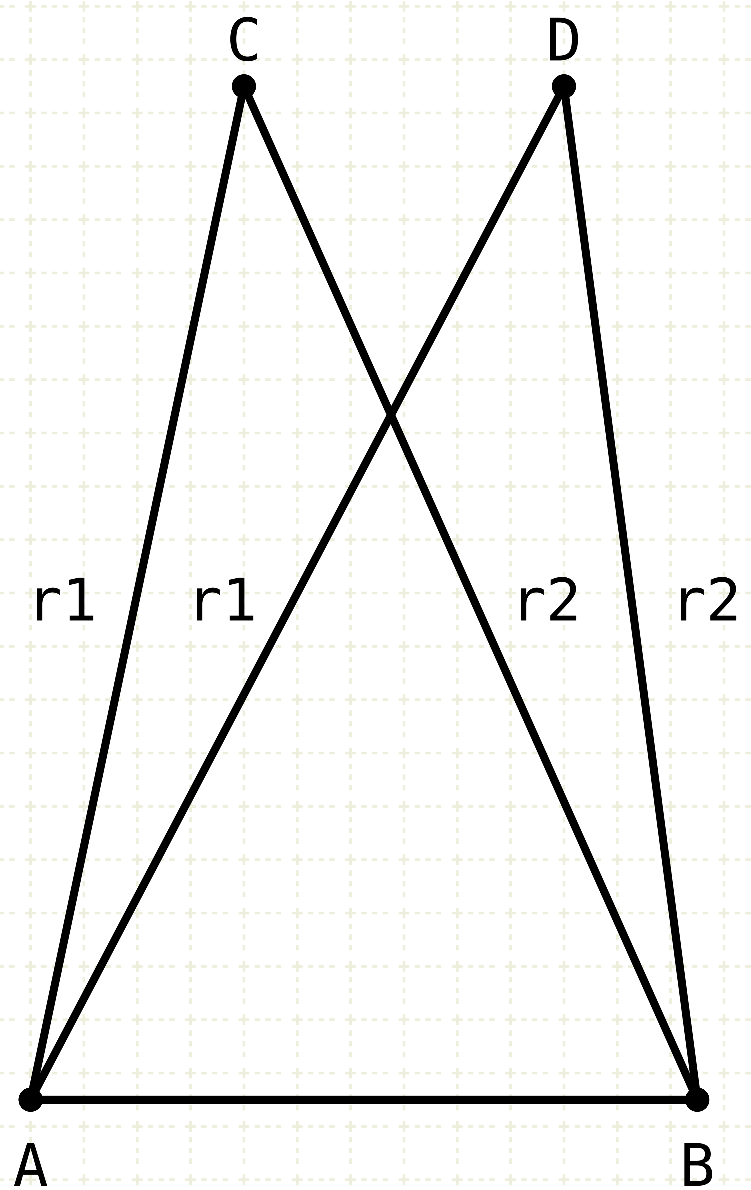

AC = AD = r1

BC = BD = r2

In other words

Given a triangle ABC

There is a unique point C where the sides of the triangle, AC and BC, meet

Proof by Contradiction

Assume there is a point D where AD is equal in length to AC and BD is equal in length to BC

Create triangle ABD

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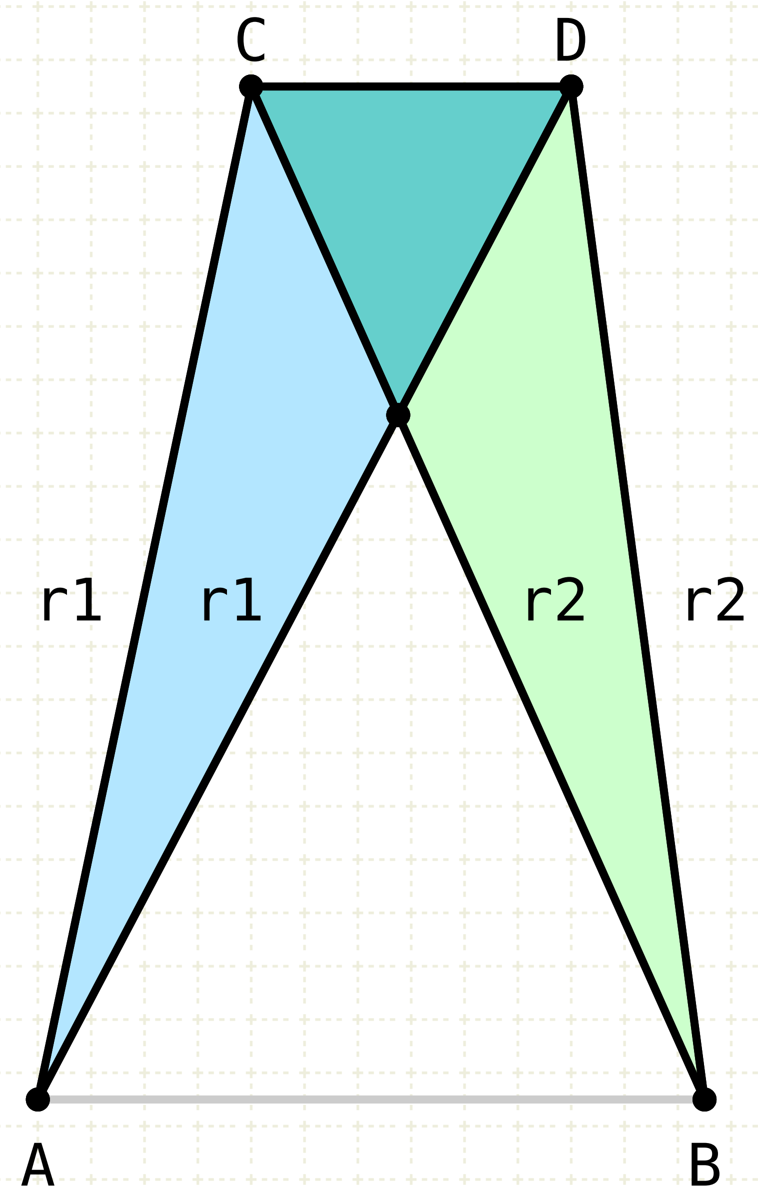

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Create triangle ABD

Construct line CD, thus creating triangles ACD and BCD

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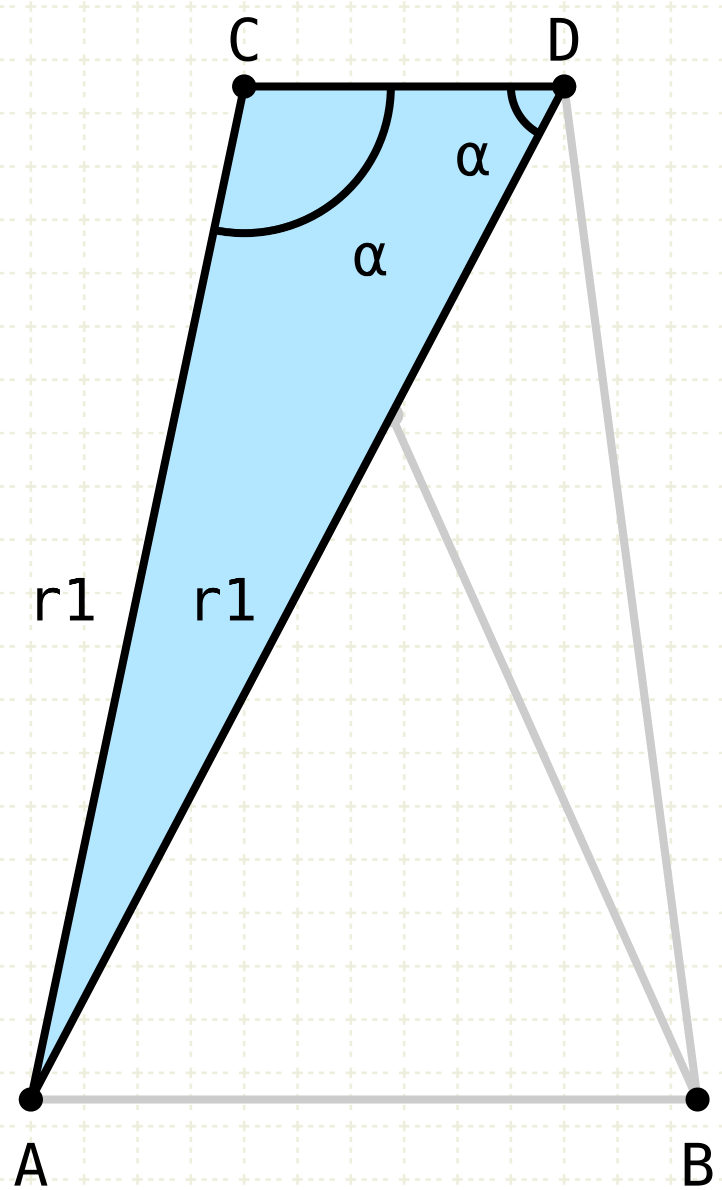

$AC = AD = r1$

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$\angle ACD = \angle CDA = \alpha$

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Triangle ACD is an isosceles triangle by definition, so the base angles must be equal (I·6)

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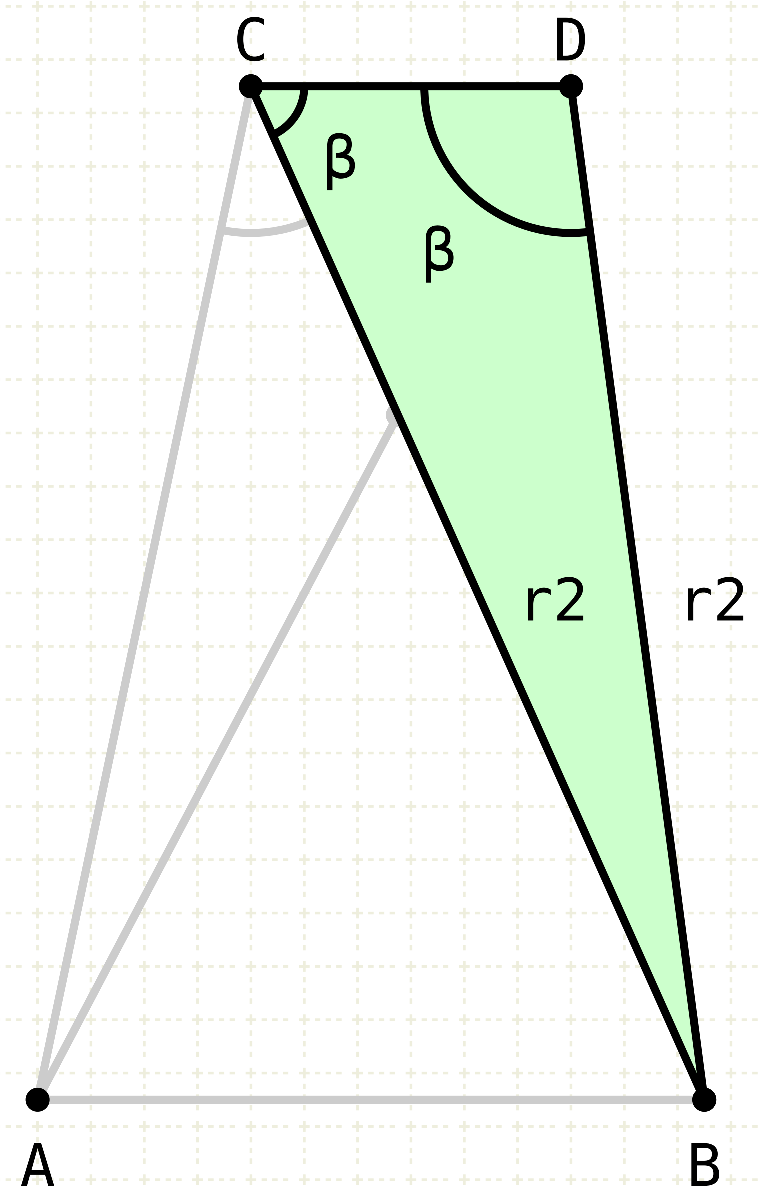

AC = AD = r1

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∠ACD = ∠CDA = α

∠BCD = ∠CDB = β

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Assume there is a point D where AD is equal in length to AC and BD is equal in length to BC

Create triangle ABD

Construct line CD, thus creating triangles ACD and BCD

Triangle ACD is an isosceles triangle by definition, so the base angles must be equal (I·6)

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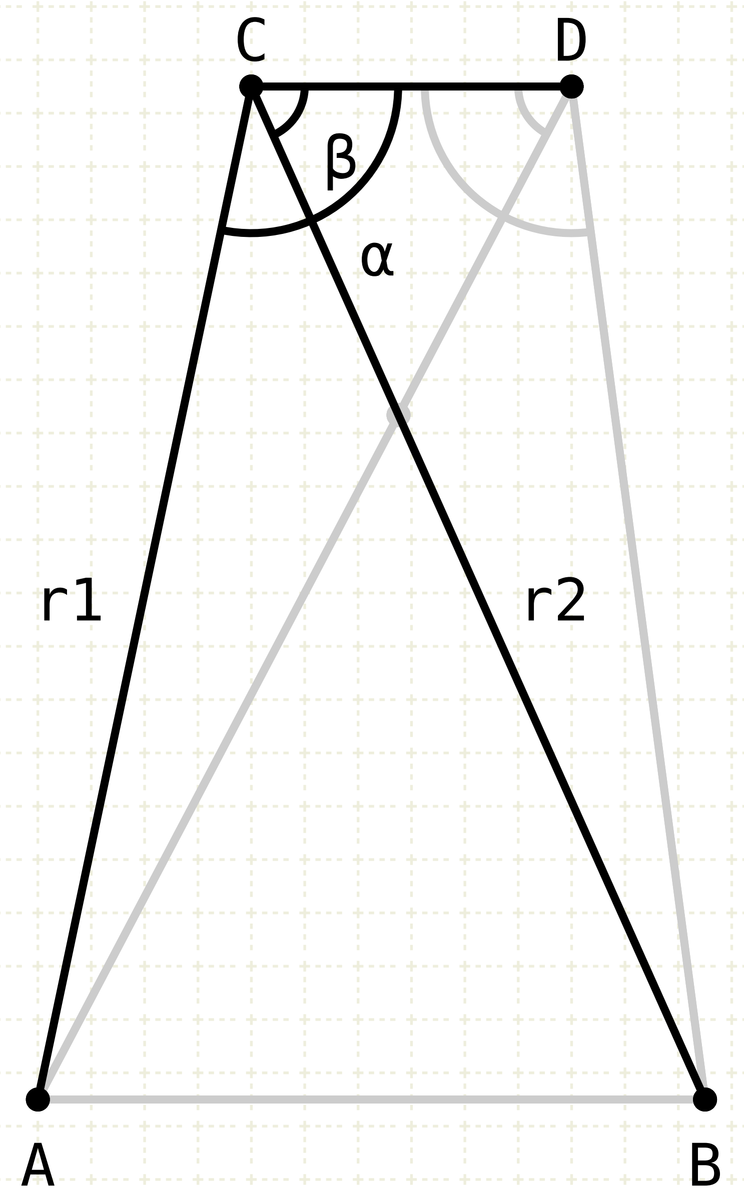

$$AC = AD = r1$$
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$$\angle ACD = \angle CDA = \alpha$$
$$\angle BCD = \angle CDB = \beta$$
$$\angle ACD > \angle BCD$$
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Looking at the figure, we can see that angle DCA (α) is larger that angle DCB (β)

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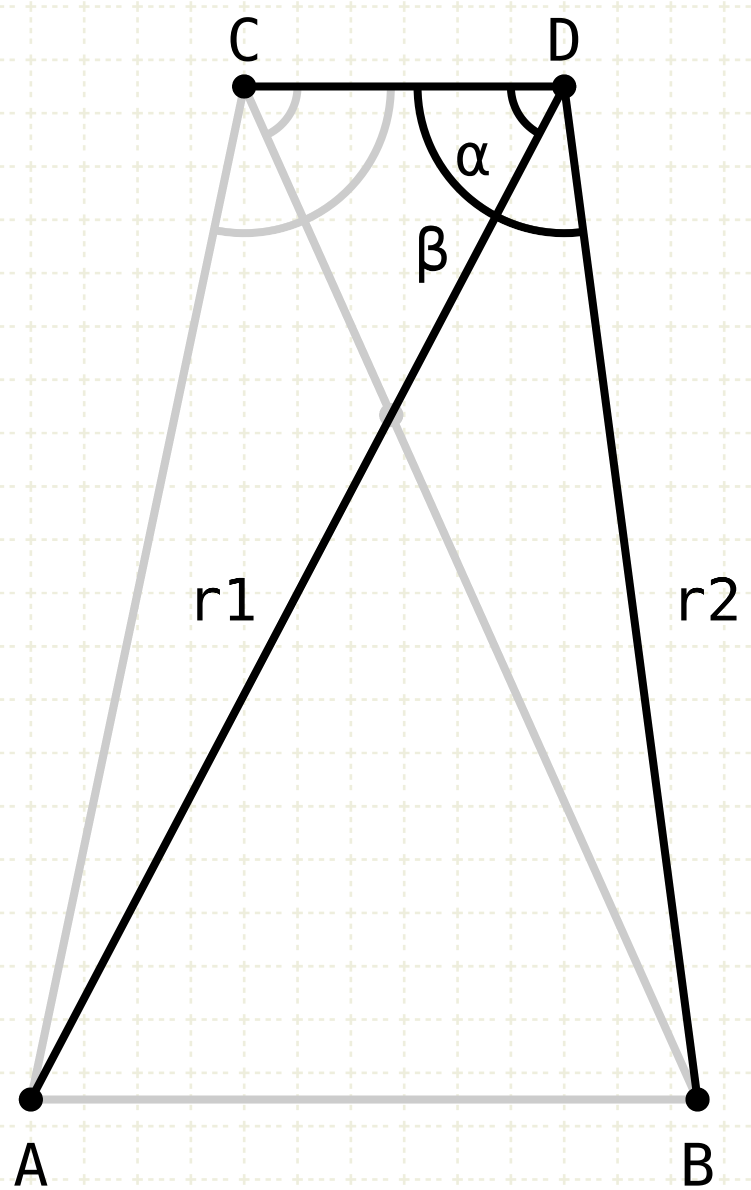

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Similarly, angle CDA (α) is less than angle CDB (β)

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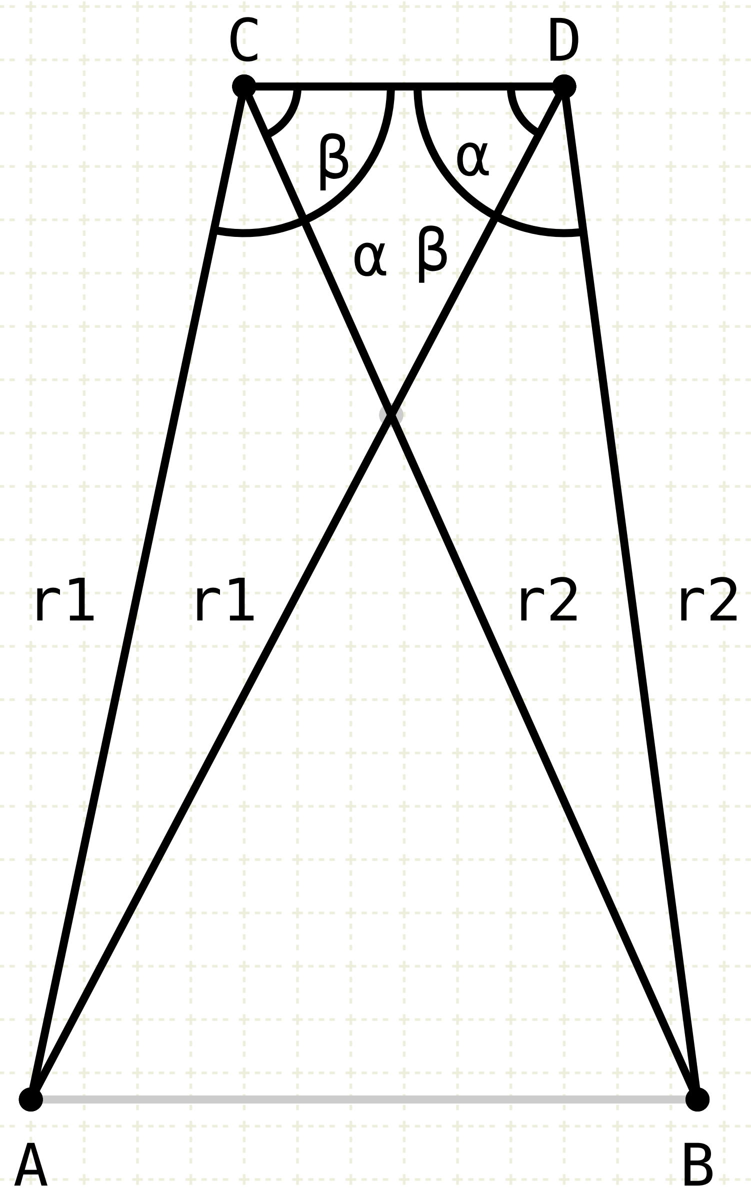

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Looking at the figure, we can see that angle DCA (α) is larger that angle DCB (β)

Similarly, angle CDA (α) is less than angle CDB (β)

But, since angle CDA is equal to DCA, and angle DCB is equal to angle CDB we have CDA simultaneously bigger and less than DBC

This is impossible

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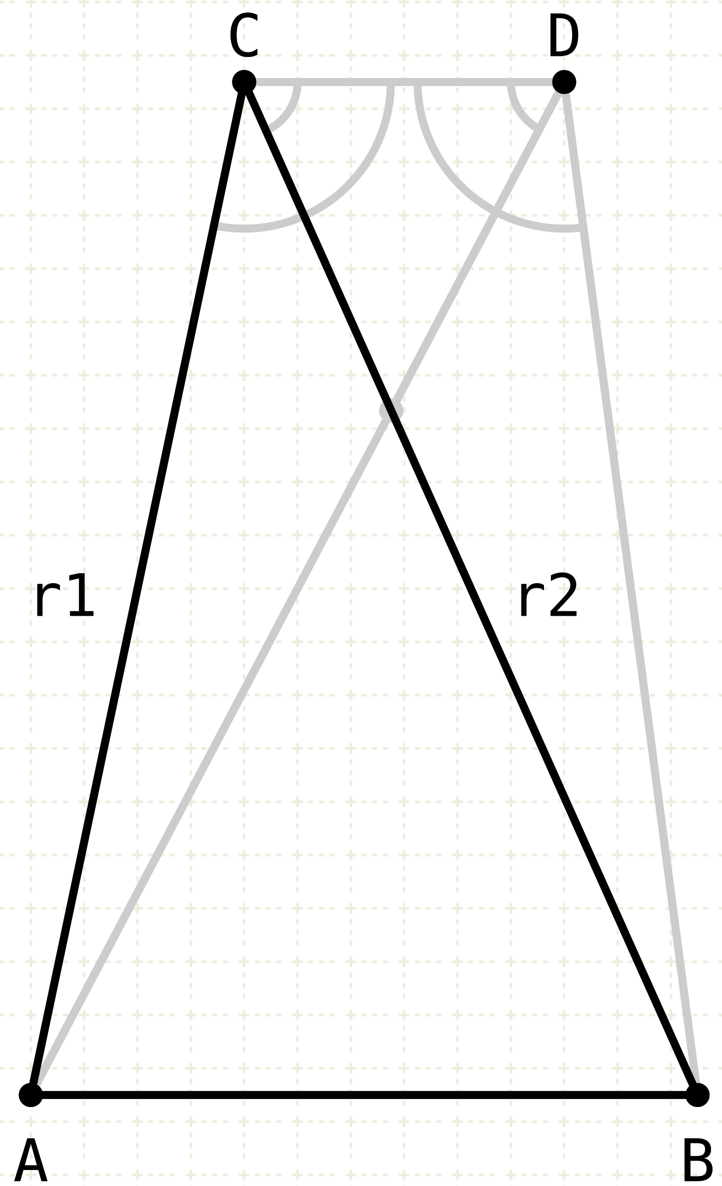

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This is impossible

Thus, we have demonstrated that point D cannot exist, and the point C is unique.

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