Euclid's Elements

Book VI



One can state, without exaggeration, that the observation of and the search for similarities and differences are the basis of all human knowledge.

Alfred Nobel



Table of Contents, Chapter 6

- If the height of two triangles are equal, then the ratio of the areas is equal to the ratio of the bases
- If a line cuts a triangle, parallel to its base, it will cut the sides of the triangle proportionally
- If an angle of a triangle is bisected and the straight line cutting the angle also cuts the base, the segments of the base will have the same ratio as the remaining sides of the triangle
- If two triangles have equal angles, then the sides opposite the equal angles are proportional, as well, the sides of the triangles on either side of the equal angles are also proportional
- 5 It two triangles have proportional sides, the triangles will be equiangular
- 6 If two triangles have one angle equal to one angle and the sides about the equal angles are proportional, then the triangles will be equiangular

- If two triangles have one angle equal to one angle, and the sides about other angles are proportional, and the remaining angles either both less or both not less than a right angle, then triangles will be equiangular
- If in a right-angled triangle a perpendicular be drawn from the right angle to the base, the triangles adjoining the perpendicular are similar both to the whole and to one another
- 9 From a given straight line to cut off a given fraction
- 10 To cut a given uncut straight line similarly to a given cut straight line
- 11 To two given straight lines to find a third proportional
- 12 To three given straight lines to find a fourth proportional
- 13 To two given straight lines to find a mean proportional

- 14 In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and vice versa
- In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and vice versa
- 16 If four straight lines are proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means, and vice versa
- 17 If three straight lines are proportional, the rectangle contained by the extremes is equal to the square on the mean; and vice versa
- 18 On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure
- 19 Similar triangles are to one another in the duplicate ratio of the corresponding sides



Table of Contents, Chapter 3

- 20 Similar polygons are divided into the same number of similar triangles, which have the same ratio as the wholes, and the polygons have duplicate ratios to their corresponding sides
- 21 Figures which are are similar to the same rectilineal figure are also similar to one another
- 22 If four straight lines are proportional, similar rectilineal figures will also be proportional; and vice versa
- 23 Equiangular parallelograms have to one another the ratio compounded of the ratios of their sides
- 24 In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another
- 25 To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure

- 26 If from a parallelogram a similar parallelogram with a common angle is subtracted, it is about the same diameter as the original
- 27 Of all the parallelograms applied to the same straight line and deficient by parallelogrammic figures similar to a parallelogram drawn on half the said line, the largest will be one that is drawn on half of the straight line and is similar to the defect
- 28 To a given straight line, apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one
- 29 To a given straight line, apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one

- 30 To cut a finite straight line in extreme ratio
- In right-angled triangles the figure on the side subtending the right angle is equal to the similar and similarly described figures on the sides containing the right angle



To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect

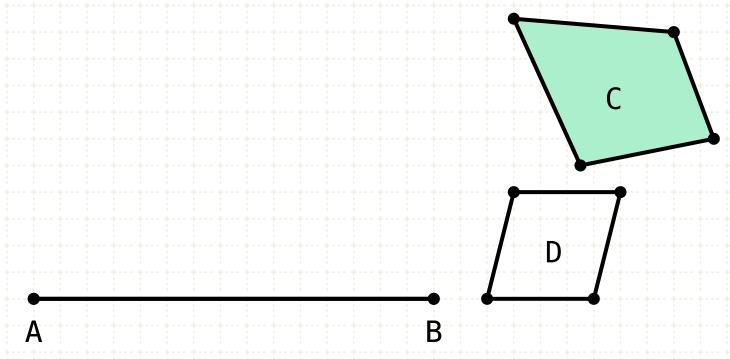
In other words

Given a straight line AB and





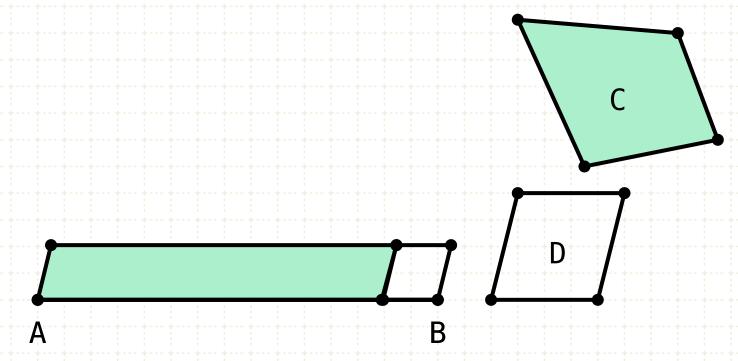
To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



In other words

Given a straight line AB and Let C be a rectilineal figure and D be a parallelogram

To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



In other words

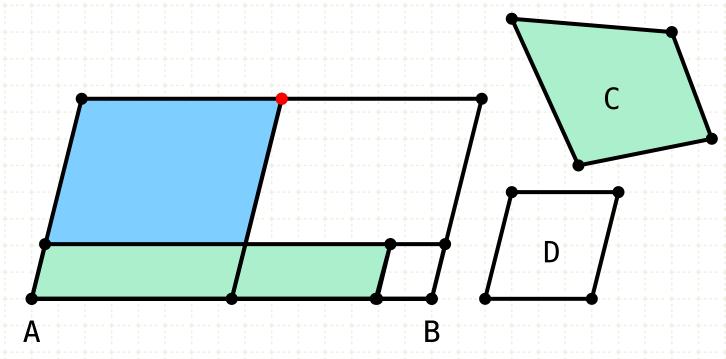
Given a straight line AB and

Let C be a rectilineal figure and D be a parallelogram

We want to draw a parallelogram on AB such that ...

- If a parallelogram similar to parallelogram D is removed, then...
- * the remainder is equal in area to C

To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



In other words

Given a straight line AB and

Let C be a rectilineal figure and D be a parallelogram

We want to draw a parallelogram on AB such that ...

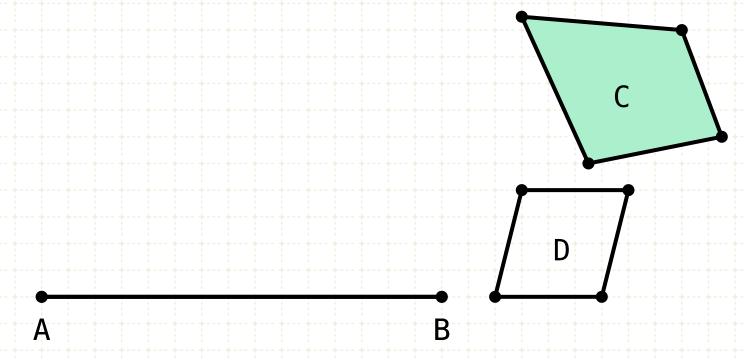
- If a parallelogram similar to parallelogram D is removed, then...
- * the remainder is equal in area to C

Note that:

the area of C cannot be greater than the area of half the parallelogram on the line AB

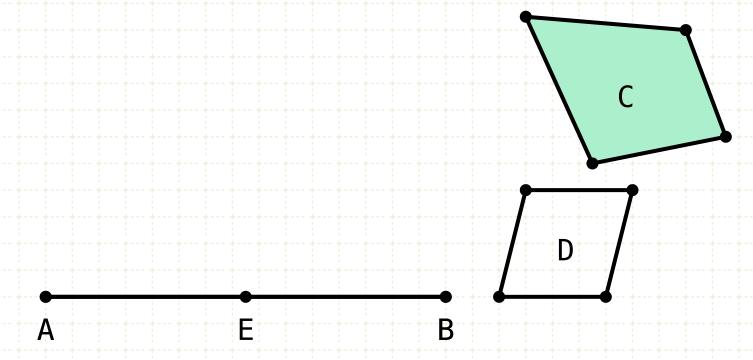


To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



Construction

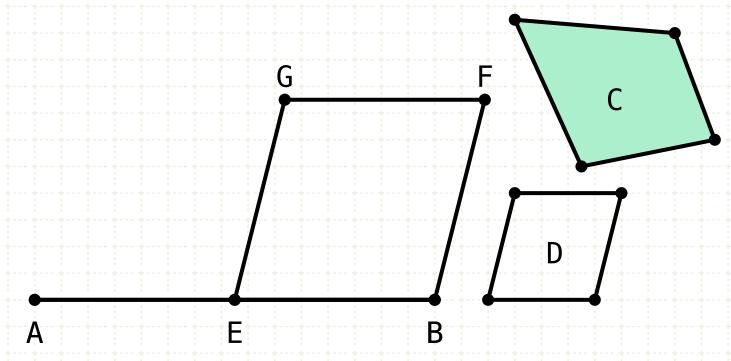
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Construction

Bisect the line AB at point E

To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



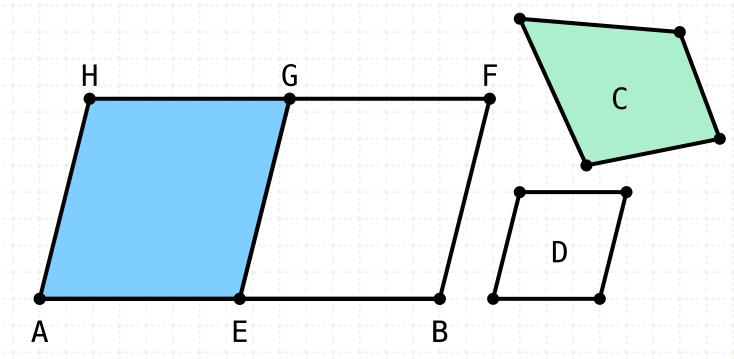
GB ~ D

Construction

Bisect the line AB at point E

Create a parallelogram similar to D on line EB (VI·18)

To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



GB ~ D

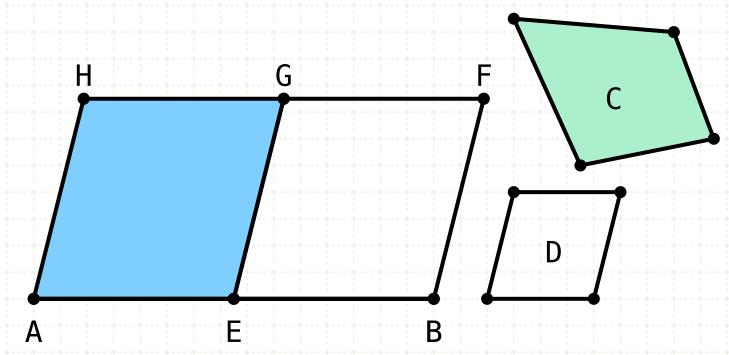
Construction

Bisect the line AB at point E

Create a parallelogram similar to D on line EB (VI·18)

Let the parallelogram AG be completed

To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



Construction

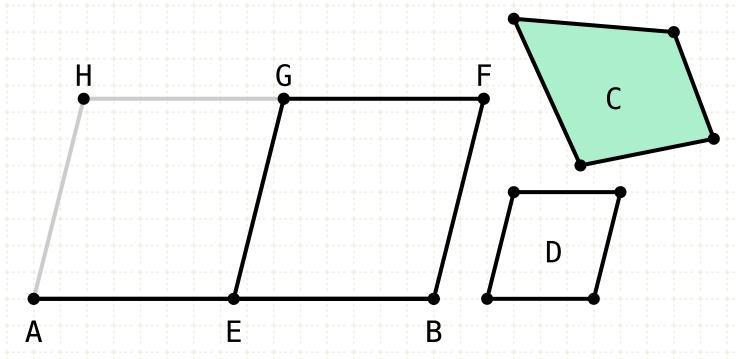
Bisect the line AB at point E

Create a parallelogram similar to D on line EB (VI·18)

Let the parallelogram AG be completed

If AG is equal in size to C, then we are finished, otherwise ... HE is greater than C

To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



$$HE = GB$$

Construction

Bisect the line AB at point E

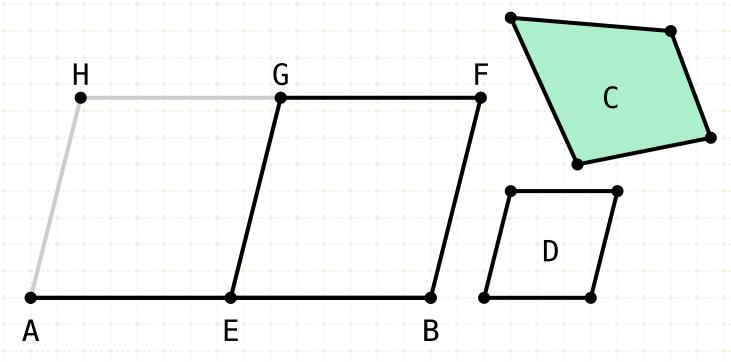
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If AG is equal in size to C, then we are finished, otherwise ... HE is greater than C

HE is equal to GB, therefore GB is also greater than C

To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



If HE > C

HE = GB

GB > C

Construction

Bisect the line AB at point E

Create a parallelogram similar to D on line EB (VI·18)

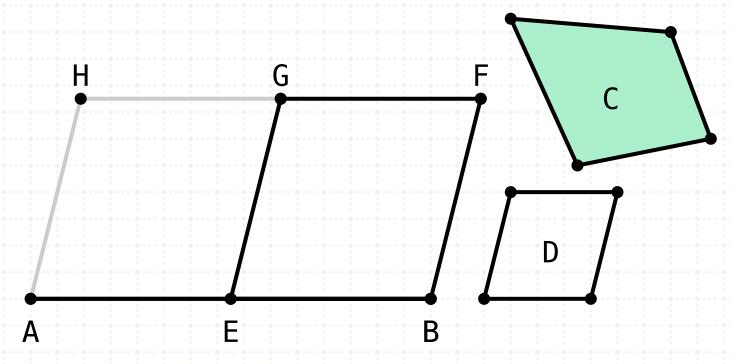
Let the parallelogram AG be completed

If AG is equal in size to C, then we are finished, otherwise ... HE is greater than C

HE is equal to GB, therefore GB is also greater than C

Let KLMN be constructed such that it is equal to the area of GB minus the area of C, and is similar to D (VI·25)

To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



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Bisect the line AB at point E

Create a parallelogram similar to D on line EB (VI-18)

Let the parallelogram AG be completed

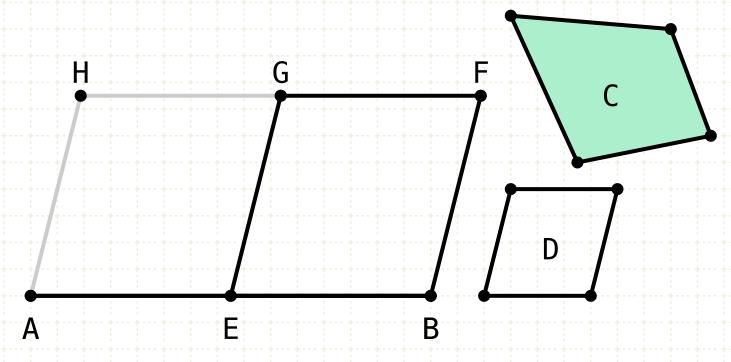
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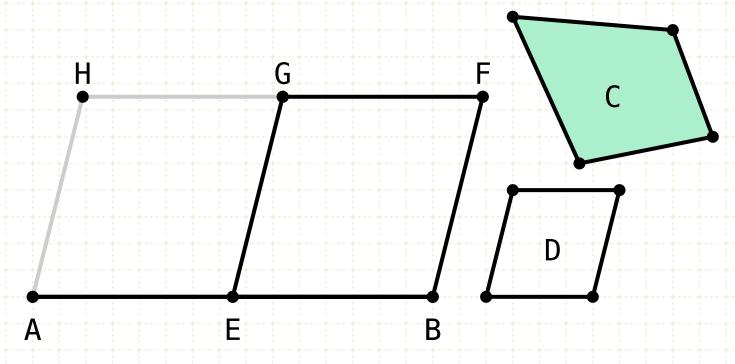
To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



Let KLMN be constructed...

... such that it is equal to the area of GB minus the area of C, and is similar to D

To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect

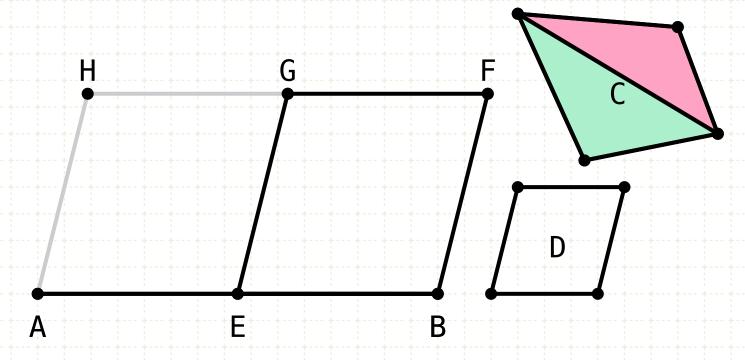


Let KLMN be constructed...

... such that it is equal to the area of GB minus the area of C, and is similar to D

Copy the rectilineal figure C to a parallelogram on line EB, with an inner angle of GEB

To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



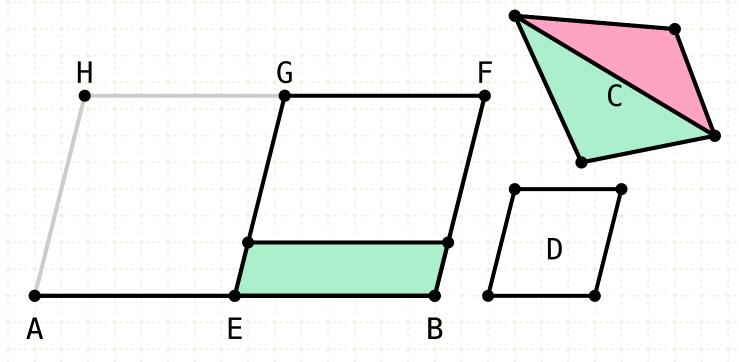
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Copy the rectilineal figure C to a parallelogram on line EB, with an inner angle of GEB

* Split C into two triangles C1 and C2

To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



Let KLMN be constructed...

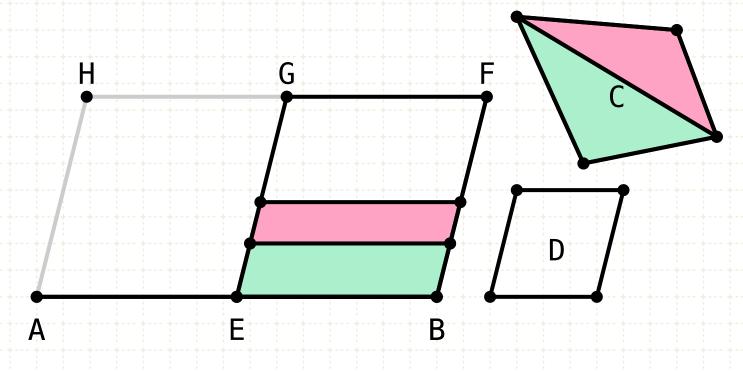
... such that it is equal to the area of GB minus the area of C, and is similar to D

Copy the rectilineal figure C to a parallelogram on line EB, with an inner angle of GEB

- * Split C into two triangles C1 and C2
- * Construct a parallelogram on the base EB such that it is equal in area to the triangle C1 (I·44)



To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



Let KLMN be constructed...

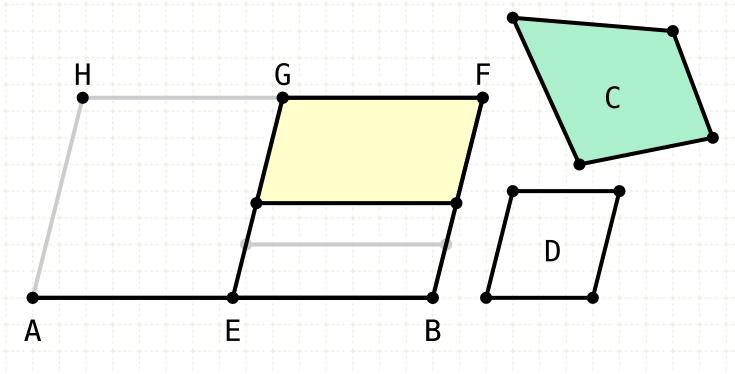
... such that it is equal to the area of GB minus the area of C, and is similar to D

Copy the rectilineal figure C to a parallelogram on line EB, with an inner angle of GEB

- * Split C into two triangles C1 and C2
- * Construct a parallelogram on the base EB such that it is equal in area to the triangle C1 (I·44)
- Construct a parallelogram on the top of the previous parallelogram such that it is equal in area to the triangle C2 (I·44)



To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



Let KLMN be constructed...

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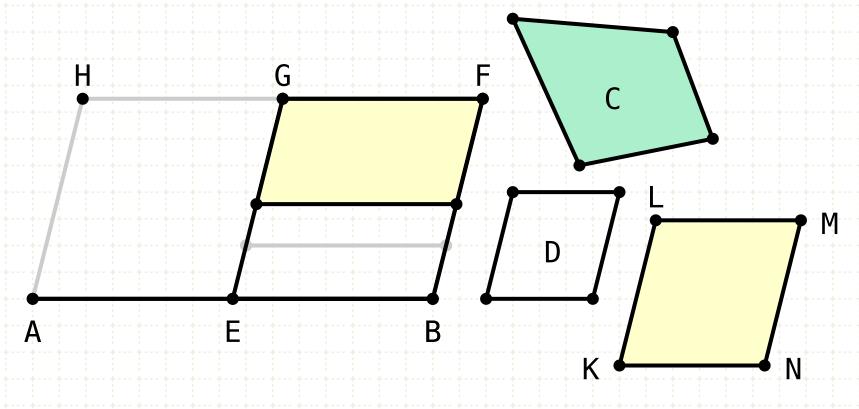
Copy the rectilineal figure C to a parallelogram on line EB, with an inner angle of GEB

- * Split C into two triangles C1 and C2
- * Construct a parallelogram on the base EB such that it is equal in area to the triangle C1 (I·44)
- * Construct a parallelogram on the top of the previous parallelogram such that it is equal in area to the triangle C2 (I·44)

What is left over in the parallelogram EF is now equal to the area of EF minus the area of C



To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



Let KLMN be constructed...

... such that it is equal to the area of GB minus the area of C, and is similar to D

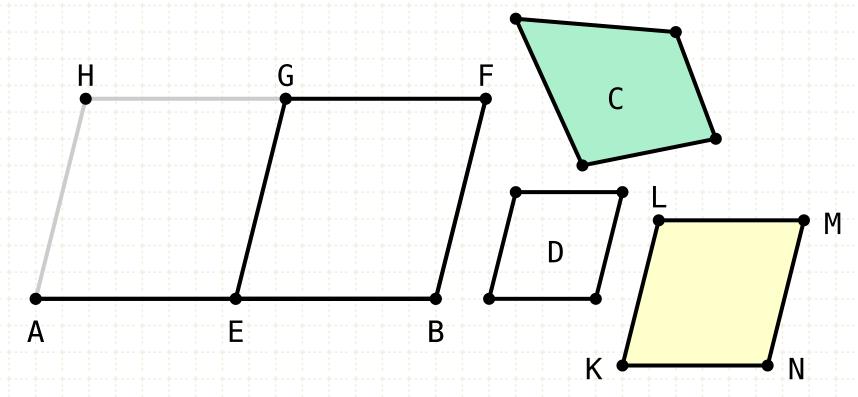
Copy the rectilineal figure C to a parallelogram on line EB, with an inner angle of GEB

- Split C into two triangles C1 and C2
- * Construct a parallelogram on the base EB such that it is equal in area to the triangle C1 (I·44)
- * Construct a parallelogram on the top of the previous parallelogram such that it is equal in area to the triangle C2 (I·44)

What is left over in the parallelogram EF is now equal to the area of EF minus the area of C

Now, copy this new polygon to KLMN, which is similar to the polygon D (VI-25)

To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



Construction

Bisect the line AB at point E

Create a parallelogram similar to D on line EB (VI-18)

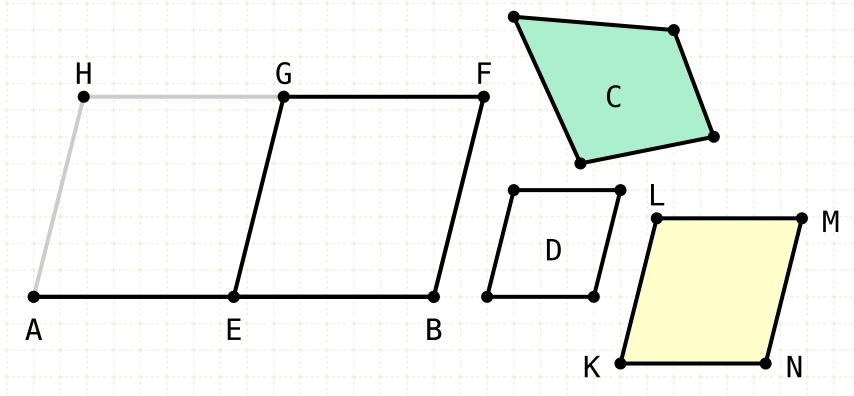
Let the parallelogram AG be completed

If AG is equal in size to C, then we are finished, otherwise ... HE is greater than C

HE is equal to GB, therefore GB is also greater than C

Let KLMN be constructed such that it is equal to the the area of GB minus the area of C, and is similar to D (VI-25)

To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



$$HE = GB$$

$$KM + C = GB$$

$$KM \sim D$$

Construction

Bisect the line AB at point E

Create a parallelogram similar to D on line EB (VI-18)

Let the parallelogram AG be completed

If AG is equal in size to C, then we are finished, otherwise ... HE is greater than C

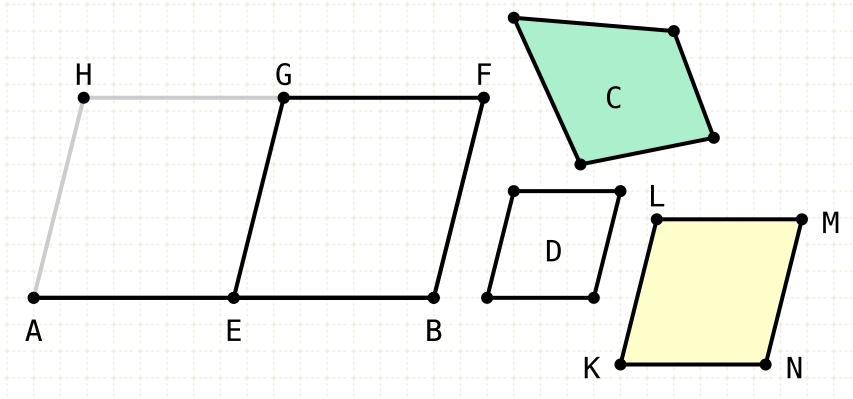
HE is equal to GB, therefore GB is also greater than C

Let KLMN be constructed such that it is equal to the area of GB minus the area of C, and is similar to D (VI·25)

KLMN is similar to D, which is also similar to GB, therefore KLMN is similar to GB (VI·21)



To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



$$HE = GB$$

$$KM + C = GB$$

$$KM \sim D$$

Construction

Bisect the line AB at point E

Create a parallelogram similar to D on line EB (VI-18)

Let the parallelogram AG be completed

If AG is equal in size to C, then we are finished, otherwise ... HE is greater than C

HE is equal to GB, therefore GB is also greater than C

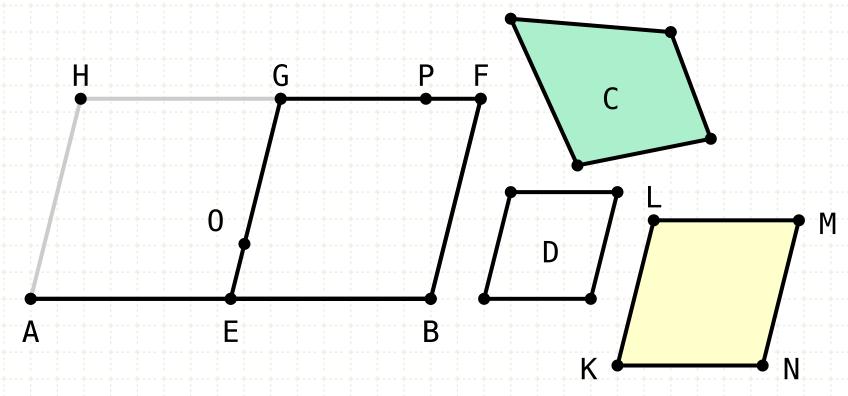
Let KLMN be constructed such that it is equal to the the area of GB minus the area of C, and is similar to D (VI·25)

KLMN is similar to D, which is also similar to GB, therefore KLMN is similar to GB (VI-21)

Since KM is smaller than GB, and since they are similar, LM is less than GF and LK is less than GE



To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



$$KM + C = GB$$

$$KM \sim D$$

Construction

Bisect the line AB at point E

Create a parallelogram similar to D on line EB (VI-18)

Let the parallelogram AG be completed

If AG is equal in size to C, then we are finished, otherwise ... HE is greater than C

HE is equal to GB, therefore GB is also greater than C

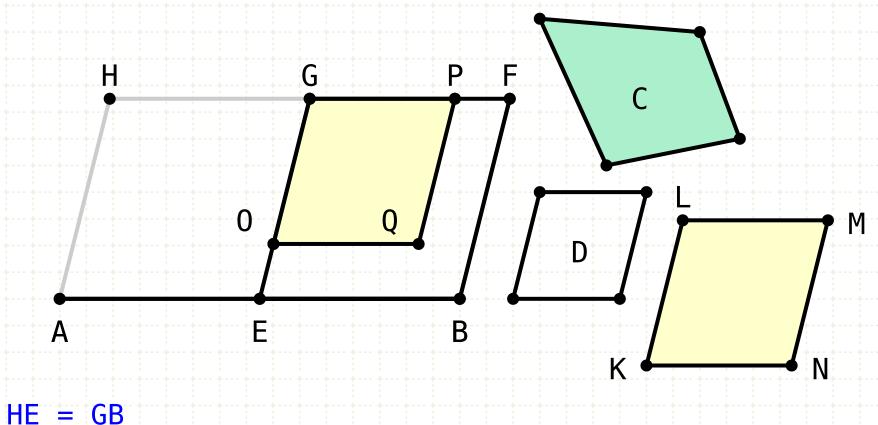
Let KLMN be constructed such that it is equal to the the area of GB minus the area of C, and is similar to D (VI·25)

KLMN is similar to D, which is also similar to GB, therefore KLMN is similar to GB (VI-21)

Since KM is smaller than GB, and since they are similar, LM is less than GF and LK is less than GE

Copy line KL to GE, and LM to GF

To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



D ~ GB

∴ KM ~ GB

GQ = KM

Construction

Bisect the line AB at point E

Create a parallelogram similar to D on line EB (VI-18)

Let the parallelogram AG be completed

If AG is equal in size to C, then we are finished, otherwise ... HE is greater than C

HE is equal to GB, therefore GB is also greater than C

Let KLMN be constructed such that it is equal to the the area of GB minus the area of C, and is similar to D (VI·25)

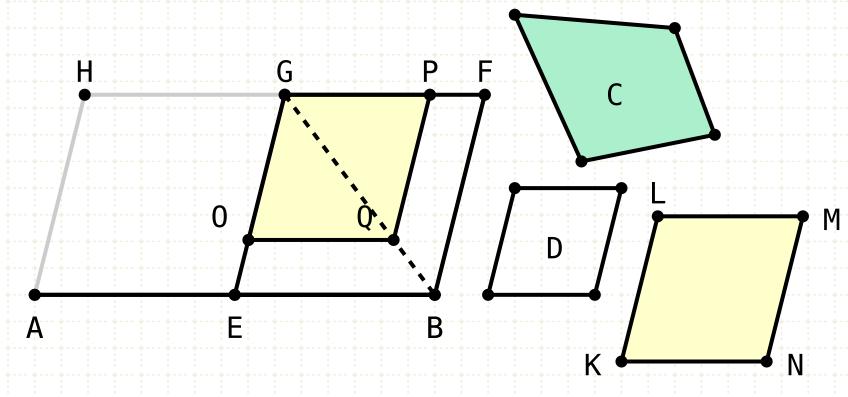
KLMN is similar to D, which is also similar to GB, therefore KLMN is similar to GB (VI·21)

Since KM is smaller than GB, and since they are similar, LM is less than GF and LK is less than GE

Copy line KL to GE, and LM to GF

Complete the parallelogram OGPQ

To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



 $GO \sim KM \sim D \sim GB$

$$HE = GB$$

$$KM + C = GB$$

GO = KM

Construction

Bisect the line AB at point E

Create a parallelogram similar to D on line EB (VI-18)

Let the parallelogram AG be completed

If AG is equal in size to C, then we are finished, otherwise ... HE is greater than C

HE is equal to GB, therefore GB is also greater than C

Let KLMN be constructed such that it is equal to the the area of GB minus the area of C, and is similar to D (VI·25)

KLMN is similar to D, which is also similar to GB, therefore KLMN is similar to GB (VI·21)

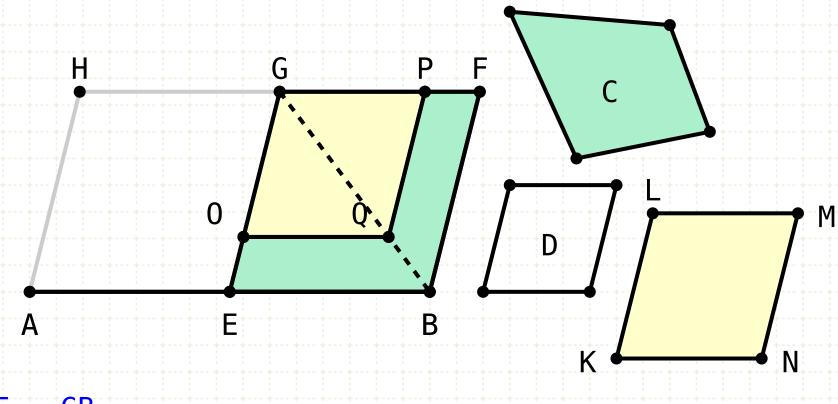
Since KM is smaller than GB, and since they are similar, LM is less than GF and LK is less than GE

Copy line KL to GE, and LM to GF

Complete the parallelogram OGPQ

Since GB is similar to KM, so is GQ similar to GB (VI·21), thus the points Q and B lie on the same diagonal (VI·26)

To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



 $GO \sim KM \sim D \sim GB$

OEBFQP = C

$$HE = GB$$

$$KM + C = GB$$

$$\underline{GQ} = KM$$



Construction

Bisect the line AB at point E

Create a parallelogram similar to D on line EB (VI-18)

Let the parallelogram AG be completed

If AG is equal in size to C, then we are finished, otherwise ... HE is greater than C

HE is equal to GB, therefore GB is also greater than C

Let KLMN be constructed such that it is equal to the the area of GB minus the area of C, and is similar to D (VI·25)

KLMN is similar to D, which is also similar to GB, therefore KLMN is similar to GB (VI-21)

Since KM is smaller than GB, and since they are similar, LM is less than GF and LK is less than GE

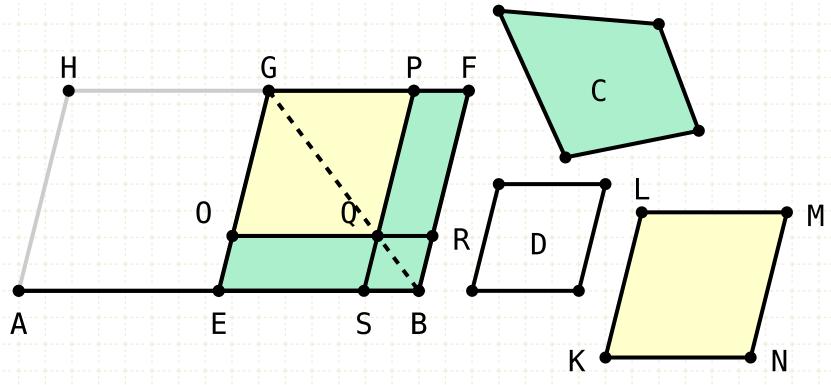
Copy line KL to GE, and LM to GF

Complete the parallelogram OGPQ

Since GB is similar to KM, so is GQ similar to GB (VI·21), thus the points Q and B lie on the same diagonal (VI·26)

GB is equal to sum of KM and C, and since GQ is equal to KM, the remaining gnomon (OEBFPQ) is equal in area to C

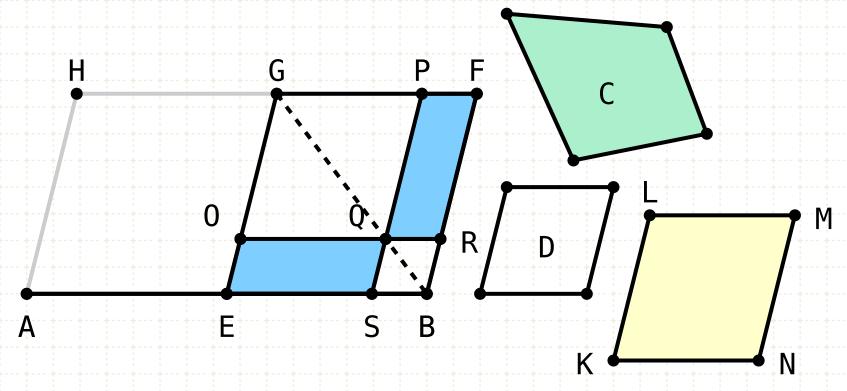
To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



Construction (cont.)

extend OQ to R, and PQ to S

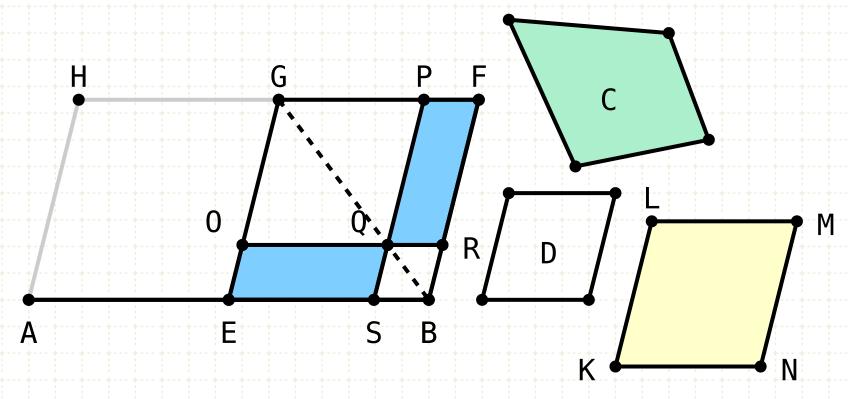
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Construction (cont.)

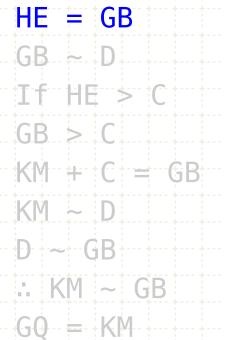
extend OQ to R, and PQ to S The parallelograms PR and OS are equal

To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect

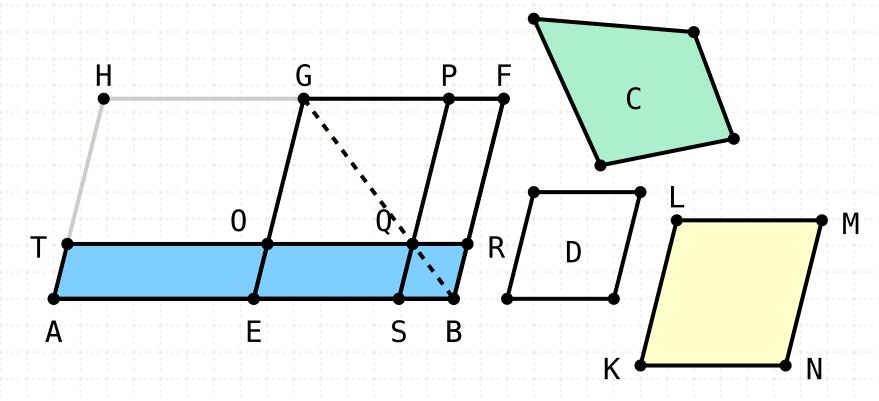


Construction (cont.)

extend OQ to R, and PQ to S The parallelograms PR and OS are equal If QB is added to PR and OS, then OB and PB are equal



To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect

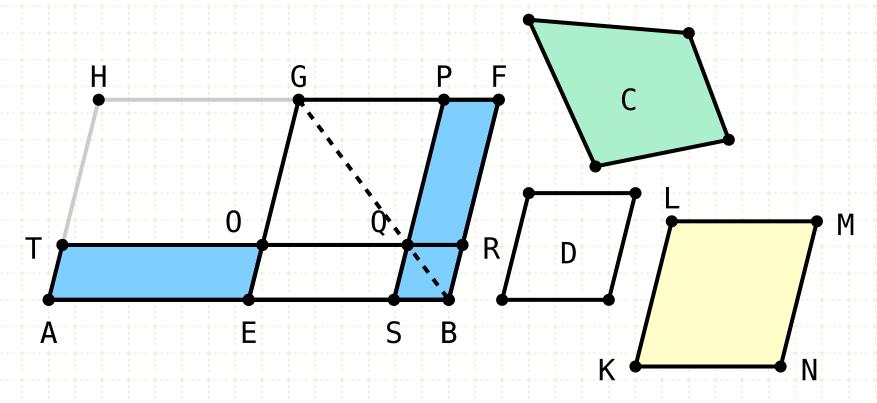


Construction (cont.)

extend OQ to R, and PQ to S
The parallelograms PR and OS are equal
If QB is added to PR and OS, then OB and PB are equal
OB is equal to TE, since AE and EB are equal (I·36)



To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect

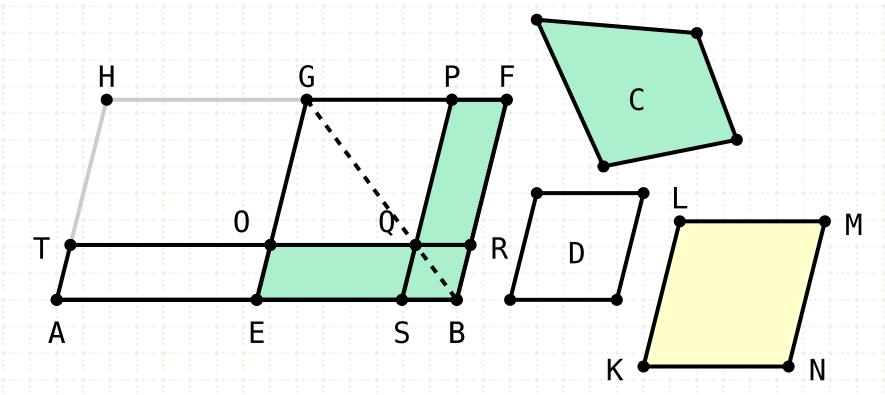


Construction (cont.)

extend OQ to R, and PQ to S The parallelograms PR and OS are equal If QB is added to PR and OS, then OB and PB are equal OB is equal to TE, since AE and EB are equal (I·36) PB equals OB, TE is equal to PB



To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



Construction (cont.)

extend OQ to R, and PQ to S

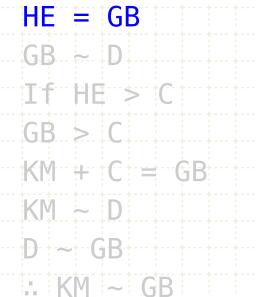
The parallelograms PR and OS are equal

If QB is added to PR and OS, then OB and PB are equal

OB is equal to TE, since AE and EB are equal (I-36)

PB equals OB, TE is equal to PB

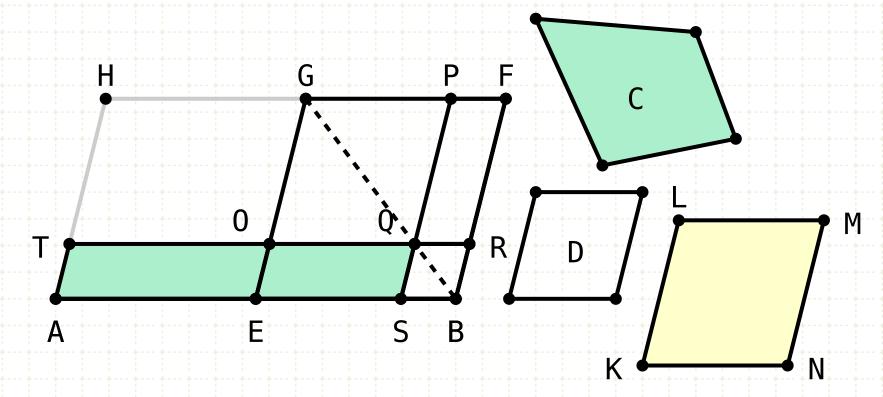
But PB, or TE, added together with OS is the gnomon that is equal to the area of C





GO = KM

To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



Construction (cont.)

extend OQ to R, and PQ to S

The parallelograms PR and OS are equal

If QB is added to PR and OS, then OB and PB are equal

OB is equal to TE, since AE and EB are equal (I-36)

PB equals OB, TE is equal to PB

But PB, or TE, added together with OS is the gnomon that is equal to the area of C

HE = GB

$$KM + C = GB$$

$$GQ = KM$$

$$GQ \sim KM \sim D \sim GB$$

$$OEBFQP = C$$

$$PR = 0S$$

$$PB = 0B$$

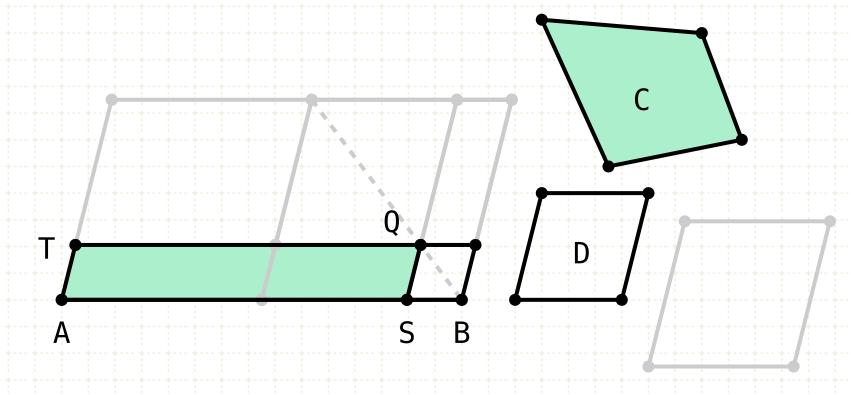
$$TE = OB$$

$$TE = PB$$

$$OS + PB = C$$

$$OS + TE = C$$

To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



$$HE = GB$$

$$KM + C = GB$$

$$GO = KM$$

$$\mathbf{G} \mathbf{Q} = \mathbf{K} \mathbf{M}$$

$$\mathbf{G} \mathbf{Q} \mathbf{S}$$

$$\mathbf{F} \mathbf{N} \mathbf{S}$$

 $GO \sim KM \sim D \sim GB$

Construction (cont.)

extend OQ to R, and PQ to S

The parallelograms PR and OS are equal

If QB is added to PR and OS, then OB and PB are equal

OB is equal to TE, since AE and EB are equal (I-36)

PB equals OB, TE is equal to PB

But PB, or TE, added together with OS is the gnomon that is equal to the area of C

Thus TS is a parallelogram, drawn on AB, minus the parallelogram QB (which is similar to D), whose area equals the polygon C

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