

# Euclid's Elements

## Book I

*If Euclid did not kindle your youthful enthusiasm, you were not born to be a scientific thinker.*

Albert Einstein



# Table of Contents, Chapter 1

1	Construct an equilateral triangle	15	Vertical angles equal one another	29	Lines parallel, alternate angles are equal
2	Copy a line	16	Exterior angle larger than interior angle	30	Lines parallel to same line are parallel to themselves
3	Subtract one line from another	17	Sum of two interior angles less than 180	31	Construct one line parallel to another
4	Equal triangles if equal side-angle-side	18	Greater side opposite of greater angle	32	Sum of interior angles of a triangle = 180
5	Isosceles triangle gives equal base angles	19	Greater angle opposite of greater side	33	Lines joining ends of equal parallels are parallel
6	Equal base angles gives isosceles triangle	20	Sum of two angles greater than third	34	Opposite sides-angles equal in parallelogram
7	Two sides of triangle meet at unique point	21	Triangle within triangle has smaller sides	35	Parallelograms, same base-height have equal area
8	Equal triangles if equal side-side-side	22	Construct triangle from given lines	36	Parallelograms, equal base-height have equal area
9	How to bisect an angle	23	Copy an angle	37	Triangles, same base-height have equal area
10	Bisect a line	24	Larger angle gives larger base	38	Triangles, equal base-height have equal area
11	Construct right angle, point on line	25	Larger base gives larger angle		
12	Construct perpendicular, point to line	26	Equal triangles if equal angle-side-angle		
13	Sum of angles on straight line = 180	27	Alternate angles equal then lines parallel		
14	Two lines form a single line if angle = 180	28	Sum of interior angles = 180 , lines parallel		



# Table of Contents, Chapter 1

39	Equal triangles on same base, have equal height
40	Equal triangles on equal base, have equal height
41	Triangle is half parallelogram with same base and height
42	Construct parallelogram with equal area as triangle
43	Parallelogram complements are equal
44	Construct parallelogram on line, equal to triangle
45	Construct parallelogram equal to polygon
46	Construct a square
<b>47</b>	<b>Pythagoras' theorem</b>
48	Inverse Pythagoras' theorem



# Proposition 47 of Book I

In right-angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle.

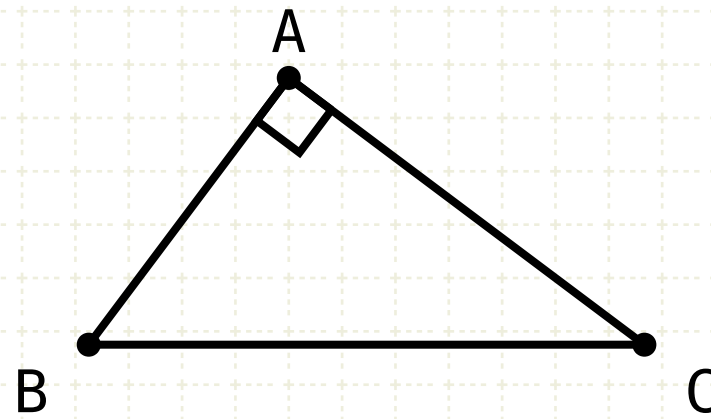


# Proposition 47 of Book I

In right-angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle.

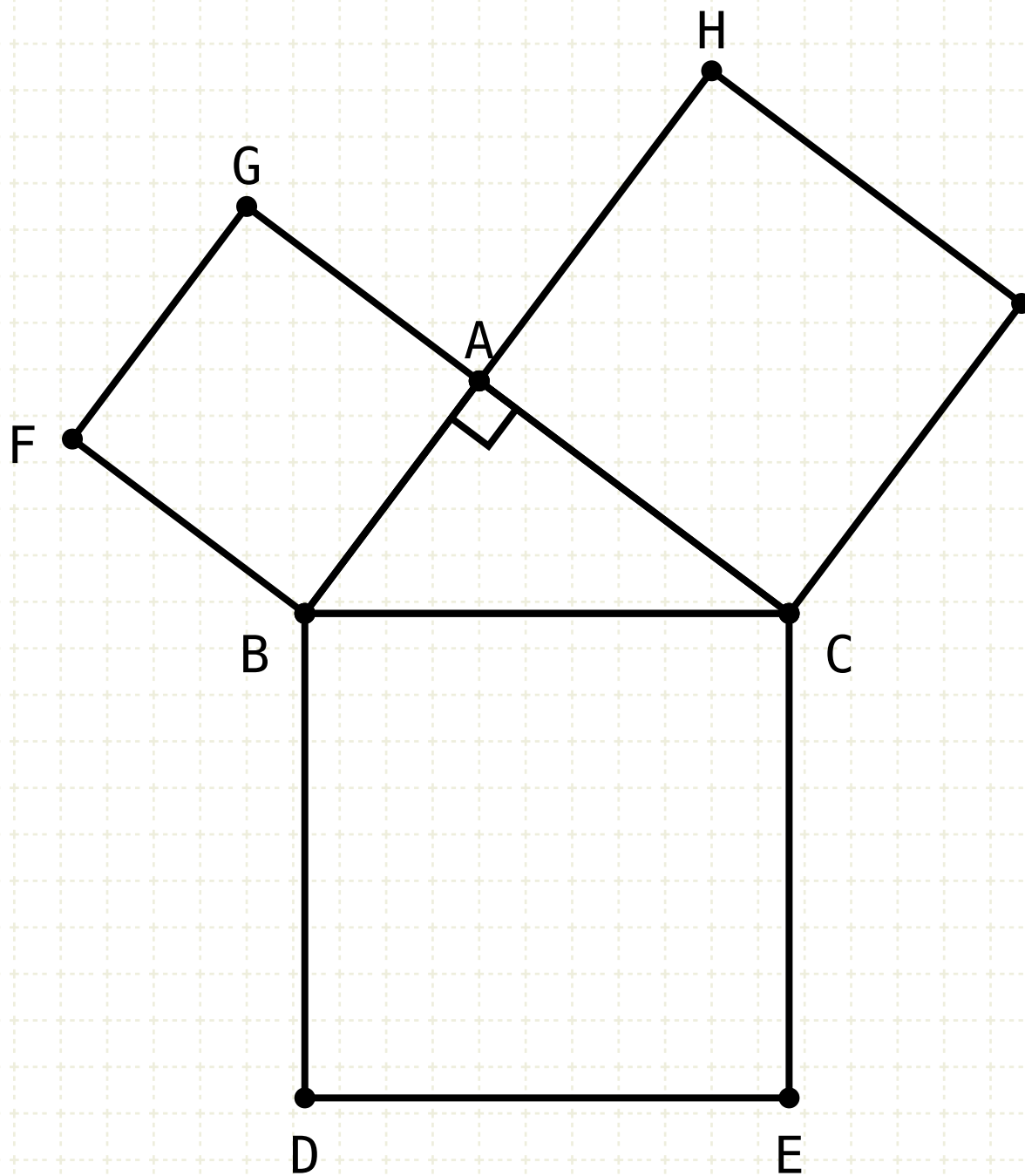
**In other words...**

Given a right angle triangle ABC



# Proposition 47 of Book I

In right-angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle.



ABFG is a square

ACKH is a square

BCED is a square

## In other words...

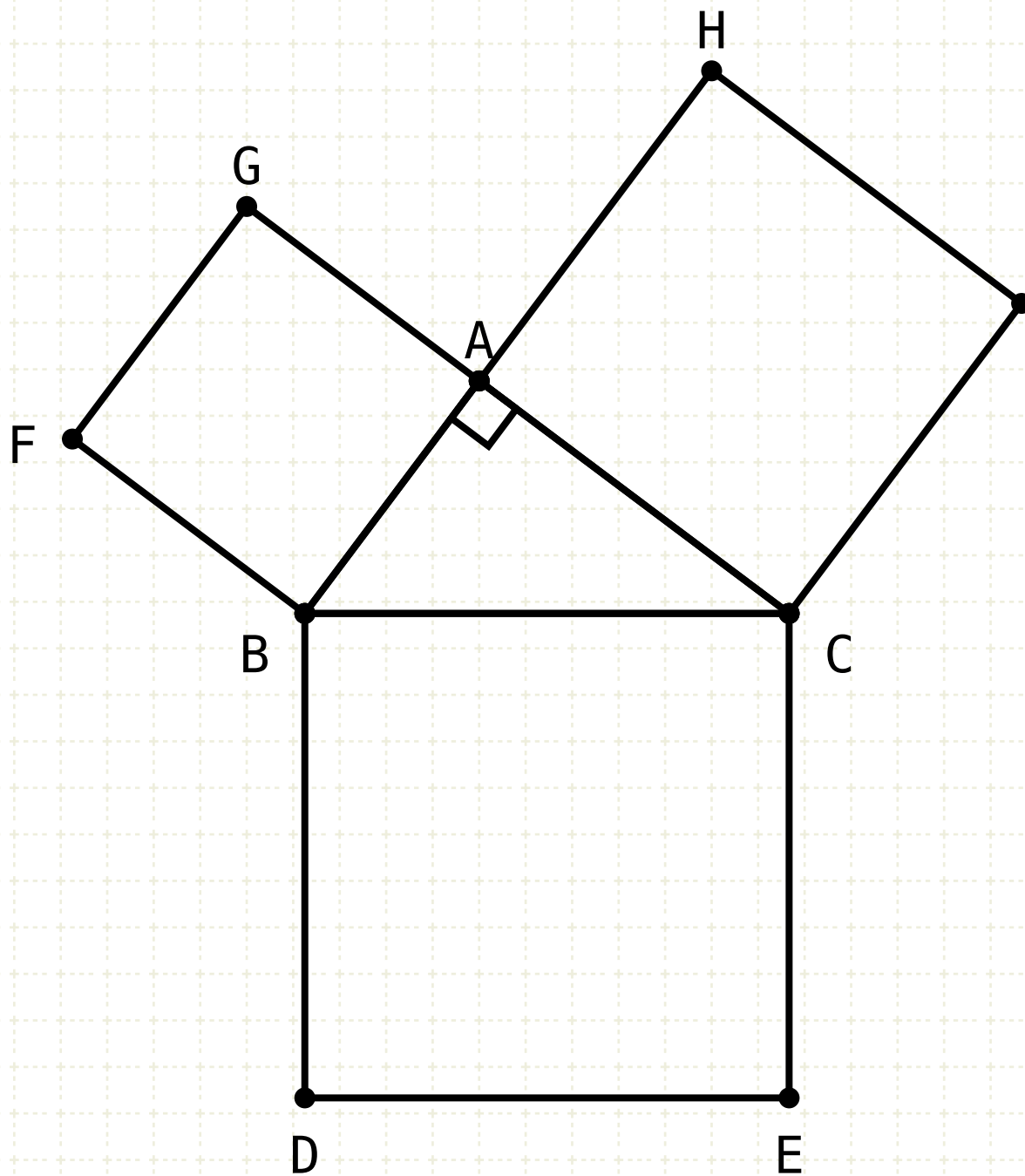
Given a right angle triangle ABC

where squares have been construct squares on all sides (I-46)



# Proposition 47 of Book I

In right-angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle.



ABFG is a square

ACKH is a square

BCED is a square

K

$$\square AB + \square AC = \square BC$$

## In other words...

Given a right angle triangle ABC

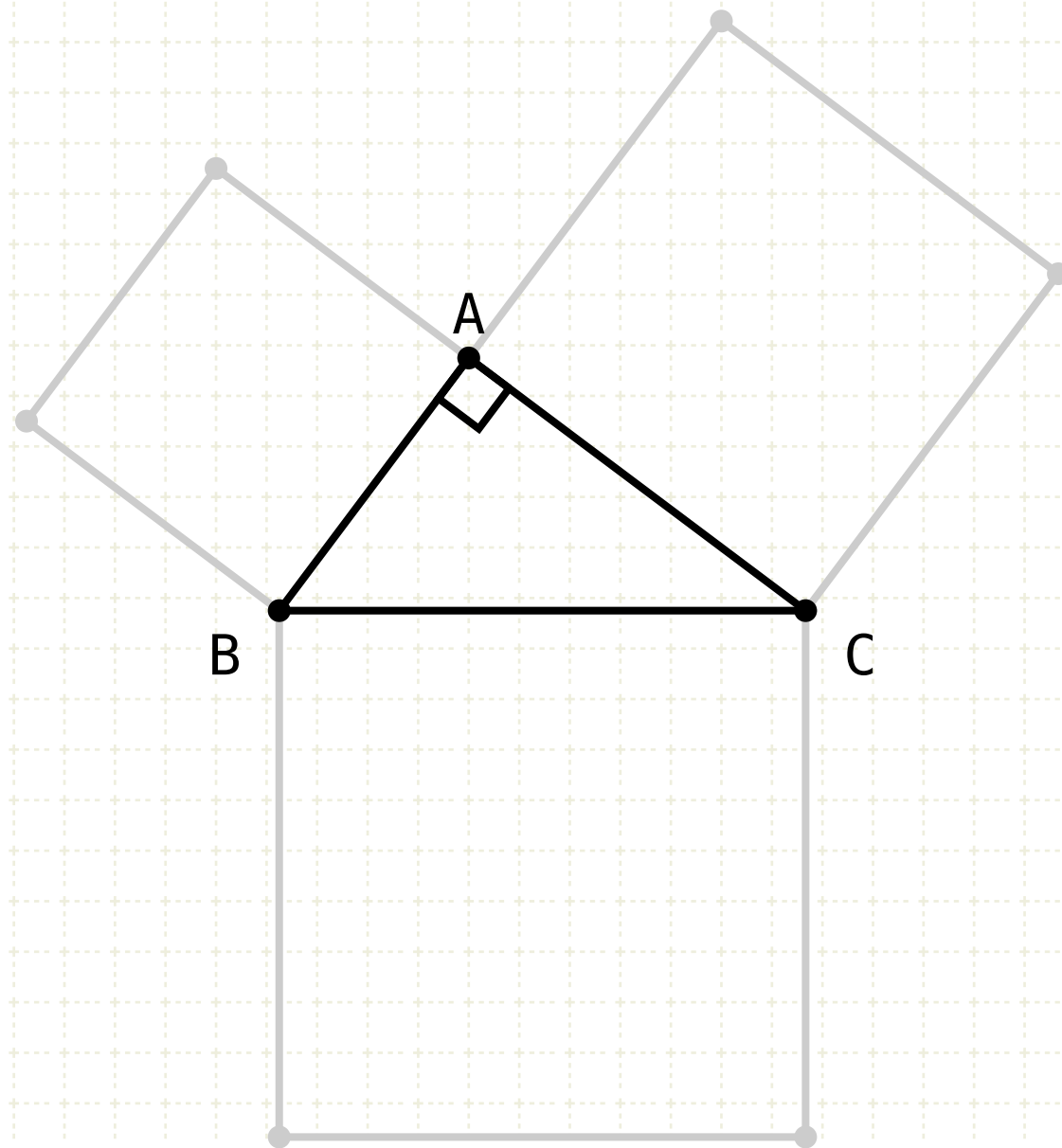
where squares have been construct squares on all sides (I-46)

Then the sum of the squares of lines AB and AC equals the square of BC

# Proposition 47 of Book I

In right-angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle.

**Proof:**

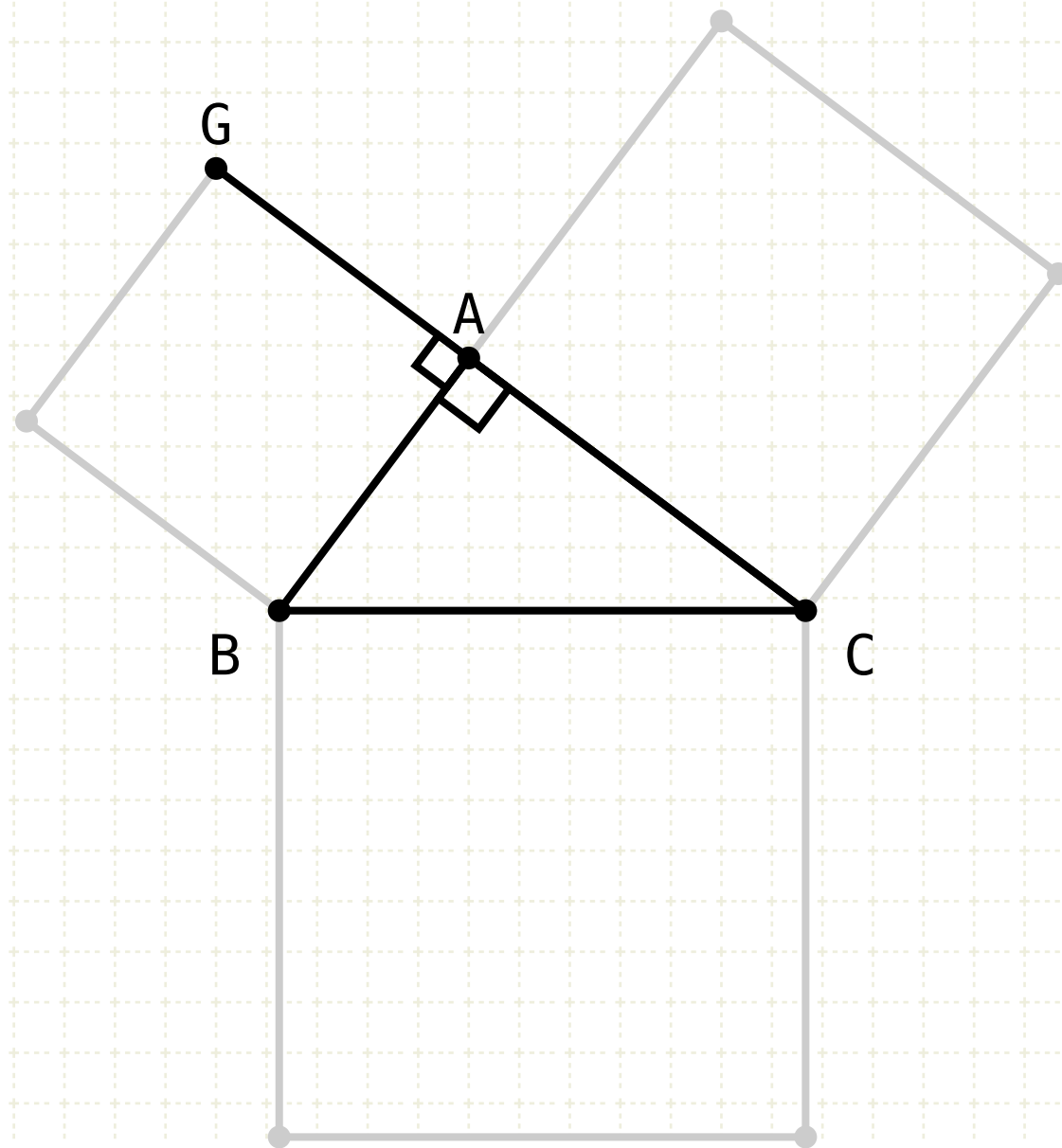


ABFG is a square  
ACKH is a square  
BCED is a square



# Proposition 47 of Book I

In right-angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle.



ABFG is a square

ACKH is a square

BCED is a square

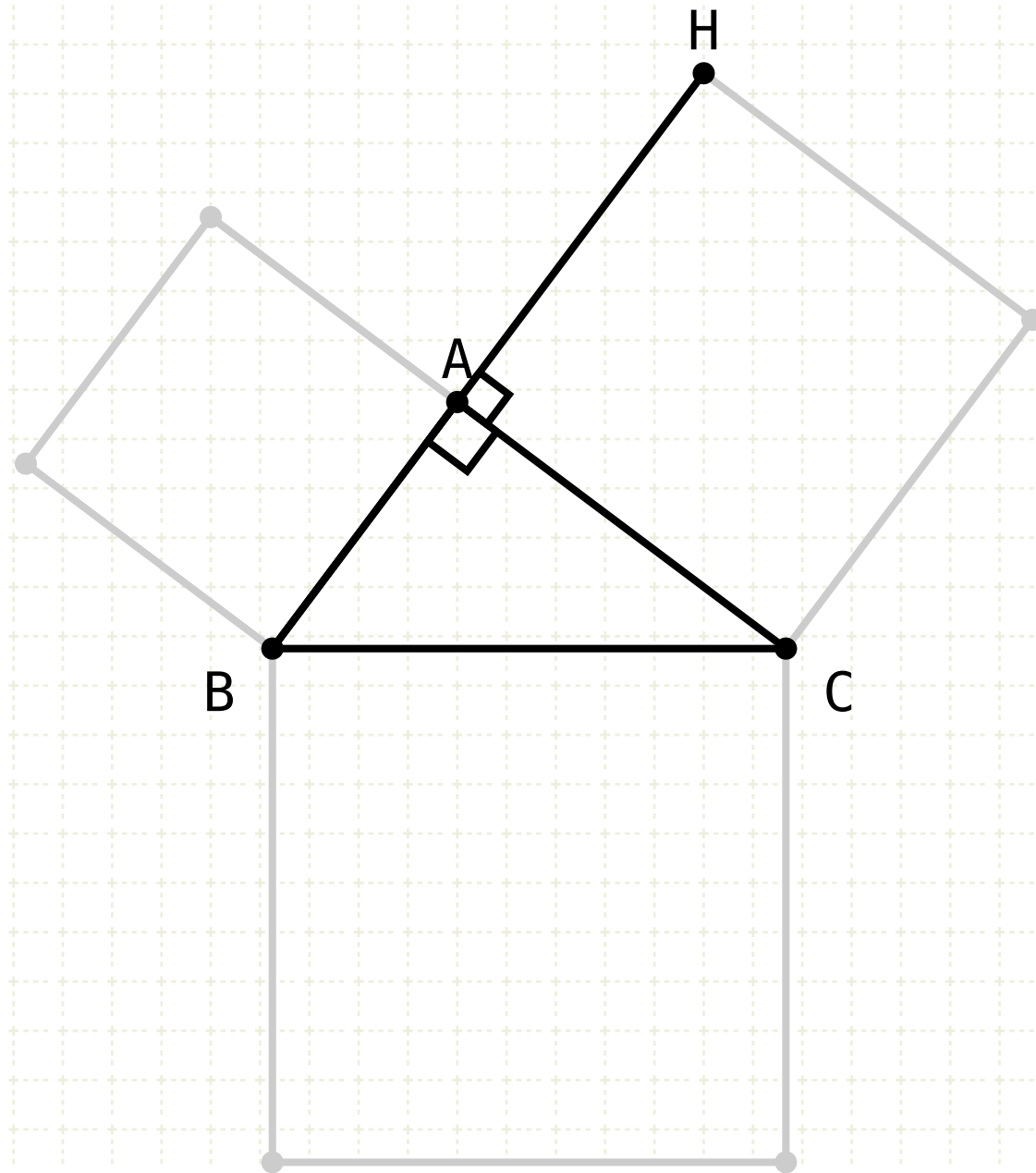
$GA, AC = GC$

## Proof:

By construction, angle GAB is a right angle, as is BAC, therefore lines GA and AC form a single line GC (I-14)

# Proposition 47 of Book I

In right-angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle.



ABFG is a square

ACKH is a square

BCED is a square

GA, AC = GC

BA, AH = BH

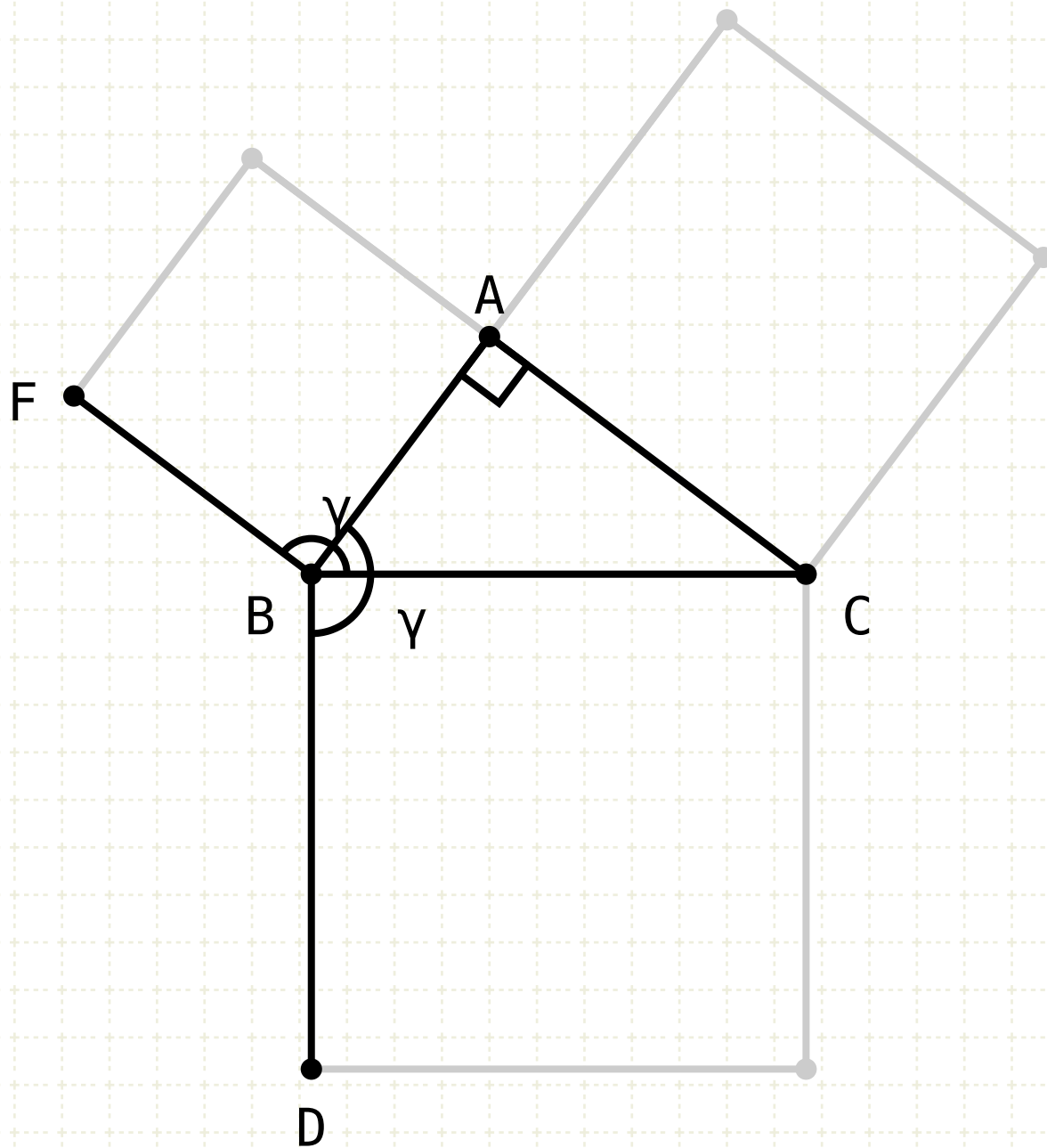
## Proof:

By construction, angle GAB is a right angle, as is BAC, therefore lines GA and AC form a single line GC (I·14)

Similarly for line BH (I·14)

# Proposition 47 of Book I

In right-angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle.



ABFG is a square

ACKH is a square

BCED is a square

GA, AC = GC

BA, AH = BH

$\angle FBC = \gamma + \angle ABC = \angle ABD$

## Proof:

By construction, angle GAB is a right angle, as is BAC, therefore lines GA and AC form a single line GC (I·14)

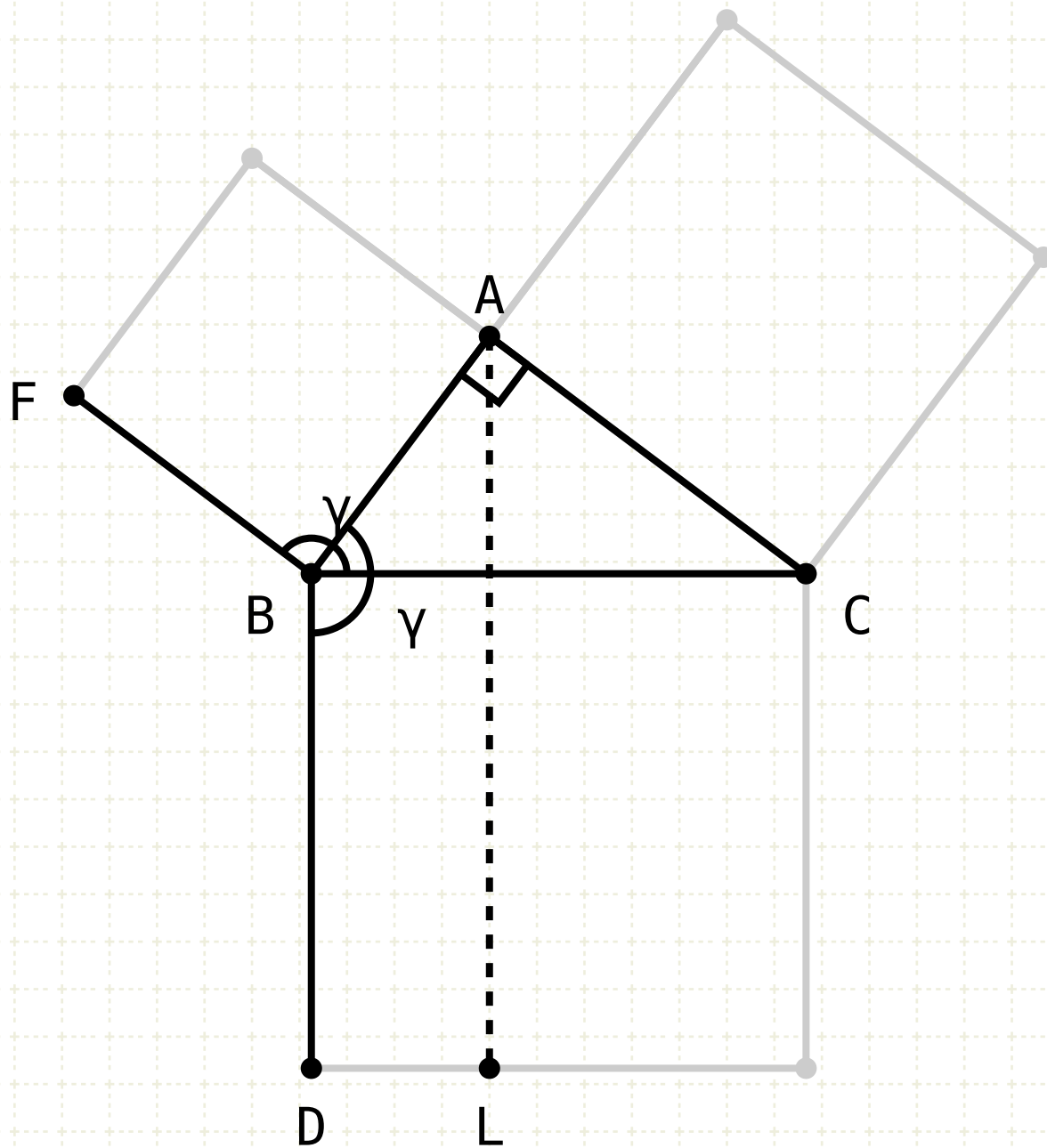
Similarly for line BH (I·14)

Angles FBA and CBD are both right angles

Adding angle ABC to both demonstrates that angles FBC and ABD are also equal

# Proposition 47 of Book I

In right-angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle.



ABFG is a square

ACKH is a square

BCED is a square

GA, AC = GC

BA, AH = BH

$\angle FBC = \angle L + \angle ABC = \angle ABD$

AL  $\parallel$  BD

## Proof:

By construction, angle GAB is a right angle, as is BAC, therefore lines GA and AC form a single line GC (I·14)

Similarly for line BH (I·14)

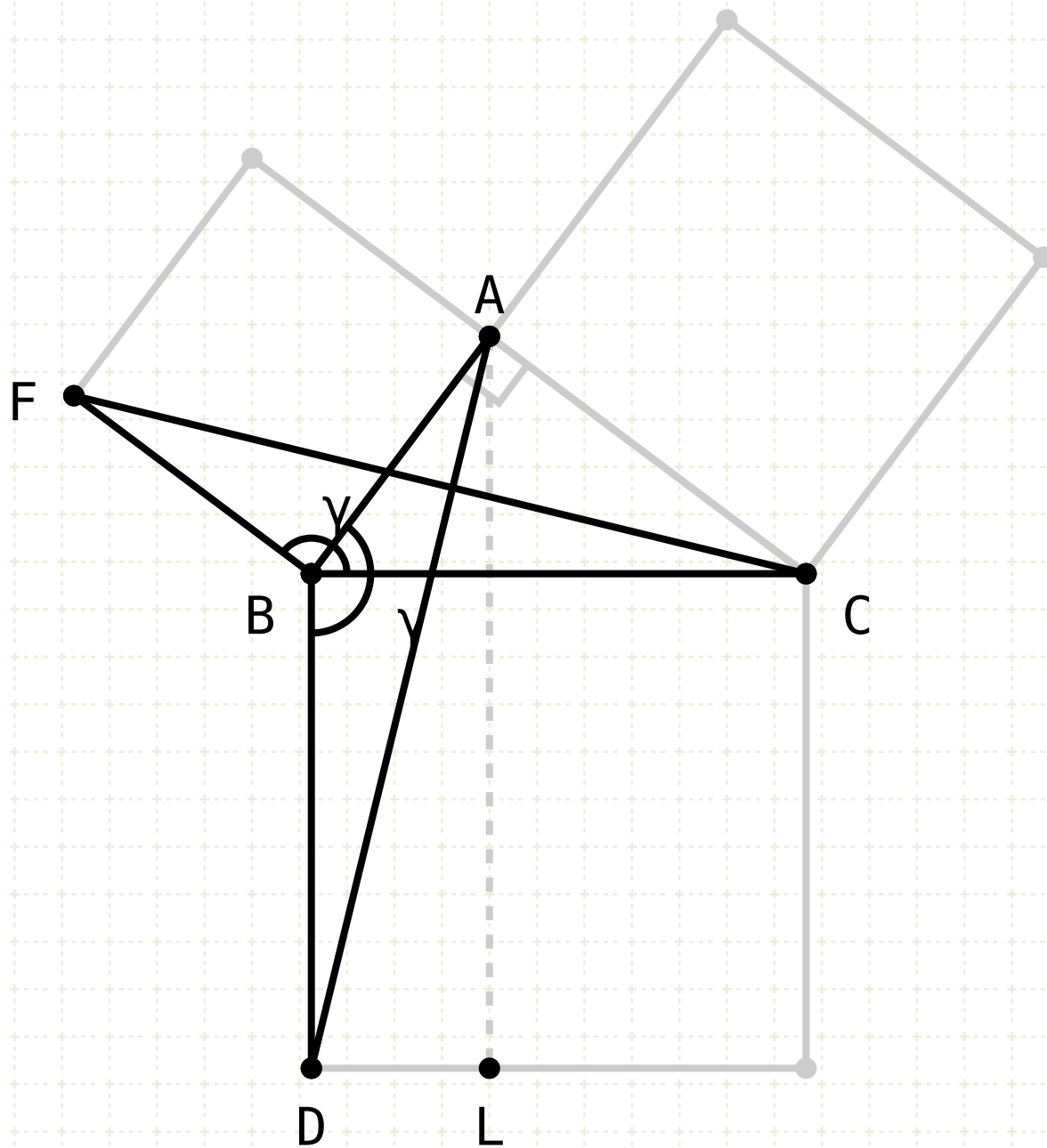
Angles FBA and CBD are both right angles

Adding angle ABC to both demonstrates that angles FBC and ABD are also equal

Draw a line from A, parallel to BD

# Proposition 47 of Book I

In right-angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle.



ABFG is a square

ACKH is a square

BCED is a square

GA, AC = GC

BA, AH = BH

$\angle FBC = \angle + \angle ABC = \angle ABD$

AL  $\parallel$  BD

## Proof:

By construction, angle GAB is a right angle, as is BAC, therefore lines GA and AC form a single line GC (I·14)

Similarly for line BH (I·14)

Angles FBA and CBD are both right angles

Adding angle ABC to both demonstrates that angles FBC and ABD are also equal

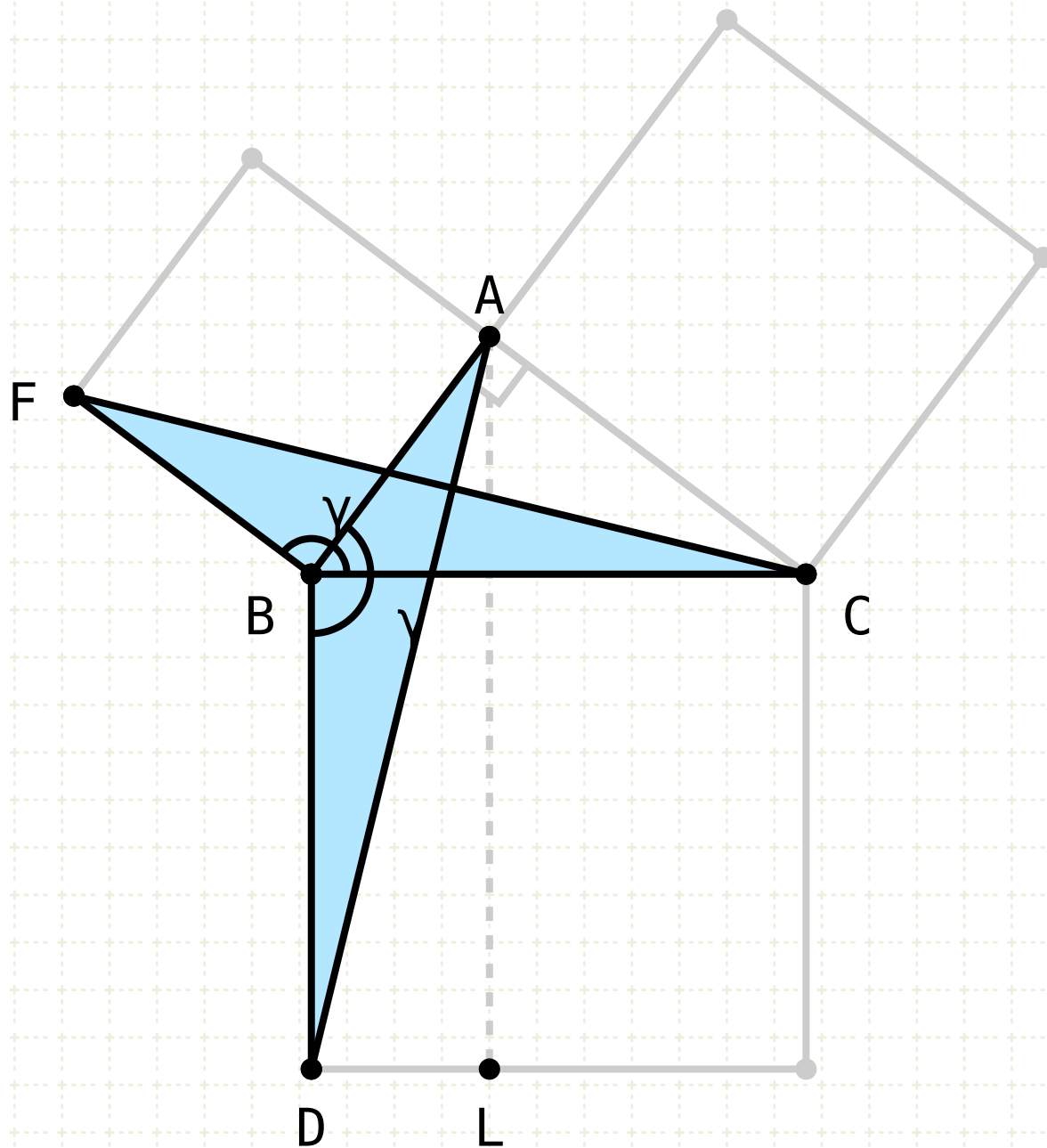
Draw a line from A, parallel to BD

Draw lines AD and FC, and consider triangles FBC and ABD



# Proposition 47 of Book I

In right-angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle.



ABFG is a square

ACKH is a square

BCED is a square

GA, AC = GC

BA, AH = BH

$\angle FBC = L + \angle ABC = \angle ABD$

AL  $\parallel$  BD

$\triangle FBC = \triangle ABD$

## Proof:

By construction, angle GAB is a right angle, as is BAC, therefore lines GA and AC form a single line GC (I·14)

Similarly for line BH (I·14)

Angles FBA and CBD are both right angles

Adding angle ABC to both demonstrates that angles FBC and ABD are also equal

Draw a line from A, parallel to BD

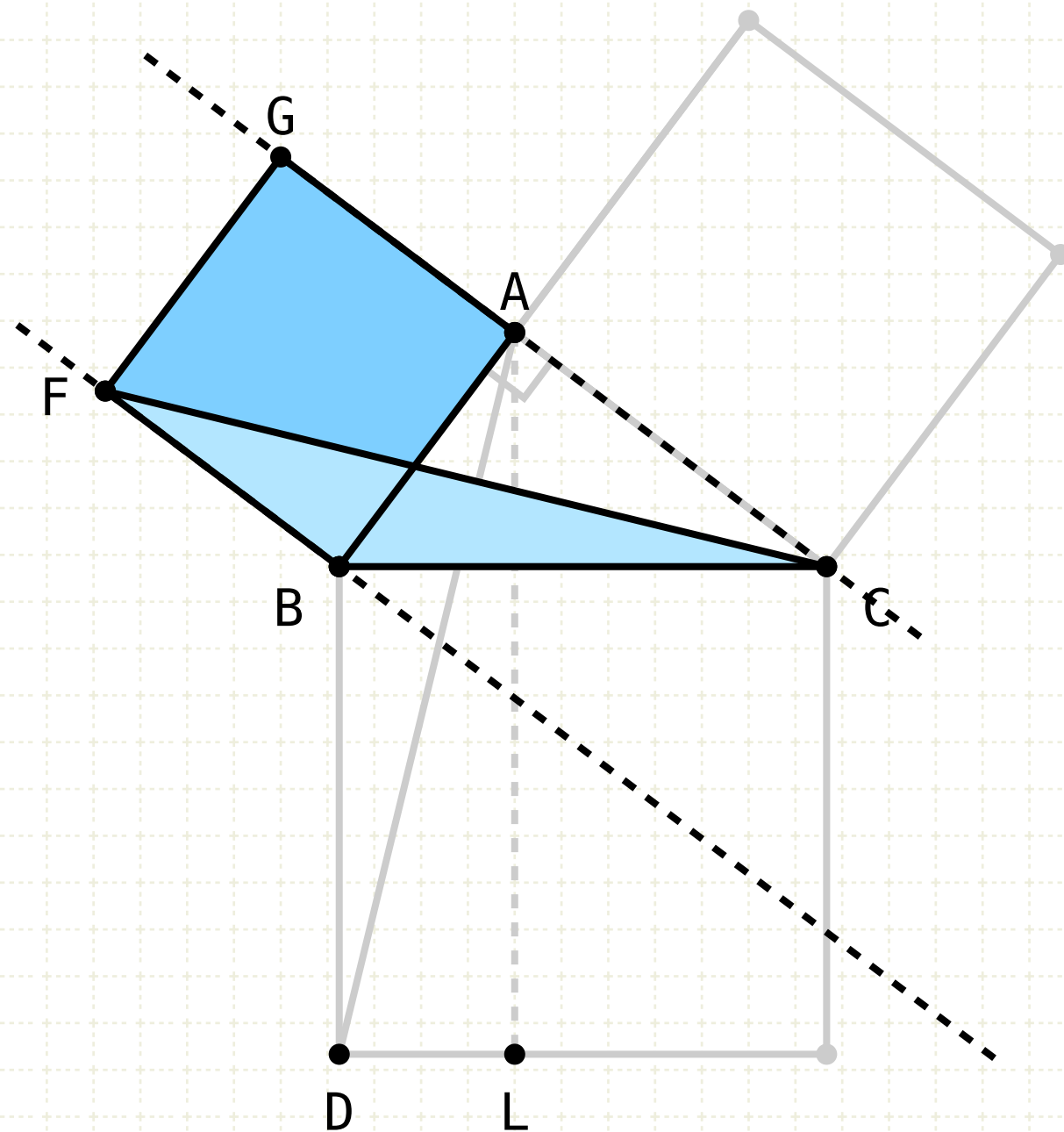
Draw lines AD and FC, and consider triangles FBC and ABD

The two triangles are equal, FB equals AB, BC equals BD, with a common angle  $\gamma$  (I·4)



# Proposition 47 of Book I

In right-angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle.



ABFG is a square

ACKH is a square

BCED is a square

GA, AC = GC

BA, AH = BH

$\angle FBC = \angle ABC = \angle ABD$

AL  $\parallel$  BD

$\triangle FBC = \triangle ABD$

$\triangle FBC = \frac{1}{2} \square AB$

## Proof:

By construction, angle GAB is a right angle, as is BAC, therefore lines GA and AC form a single line GC (I·14)

Similarly for line BH (I·14)

Angles FBA and CBD are both right angles

Adding angle ABC to both demonstrates that angles FBC and ABD are also equal

Draw a line from A, parallel to BD

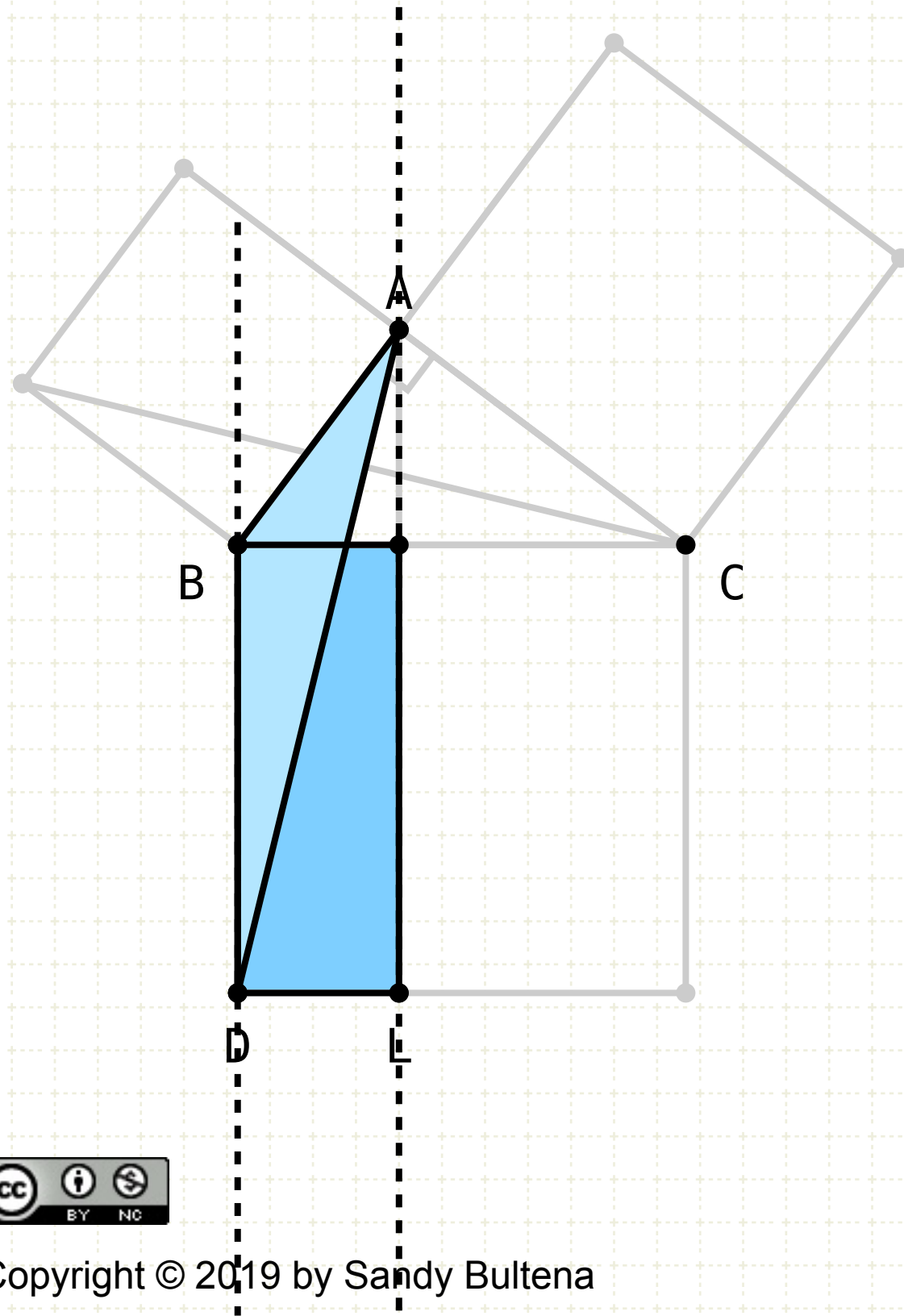
Draw lines AD and FC, and consider triangles FBC and ABD

The two triangles are equal, FB equals AB, BC equals BD, with a common angle  $\gamma$  (I·4)

The square AB and the triangle FBC share the same base, and are enclosed by the same parallel lines GC,FB thus FBC is one half ABFG (I·41)

# Proposition 47 of Book I

In right-angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle.



ABFG is a square

ACKH is a square

BCED is a square

GA, AC = GC

BA, AH = BH

$\angle FBC = \angle ABC = \angle ABD$

AL || BD

$\triangle FBC = \triangle ABD$

$\triangle FBC = \frac{1}{2} \square AB$

$\triangle ABD = \frac{1}{2} \square BDL$

## Proof:

By construction, angle GAB is a right angle, as is BAC, therefore lines GA and AC form a single line GC (I·14)

Similarly for line BH (I·14)

Angles FBA and CBD are both right angles

Adding angle ABC to both demonstrates that angles FBC and ABD are also equal

Draw a line from A, parallel to BD

Draw lines AD and FC, and consider triangles FBC and ABD

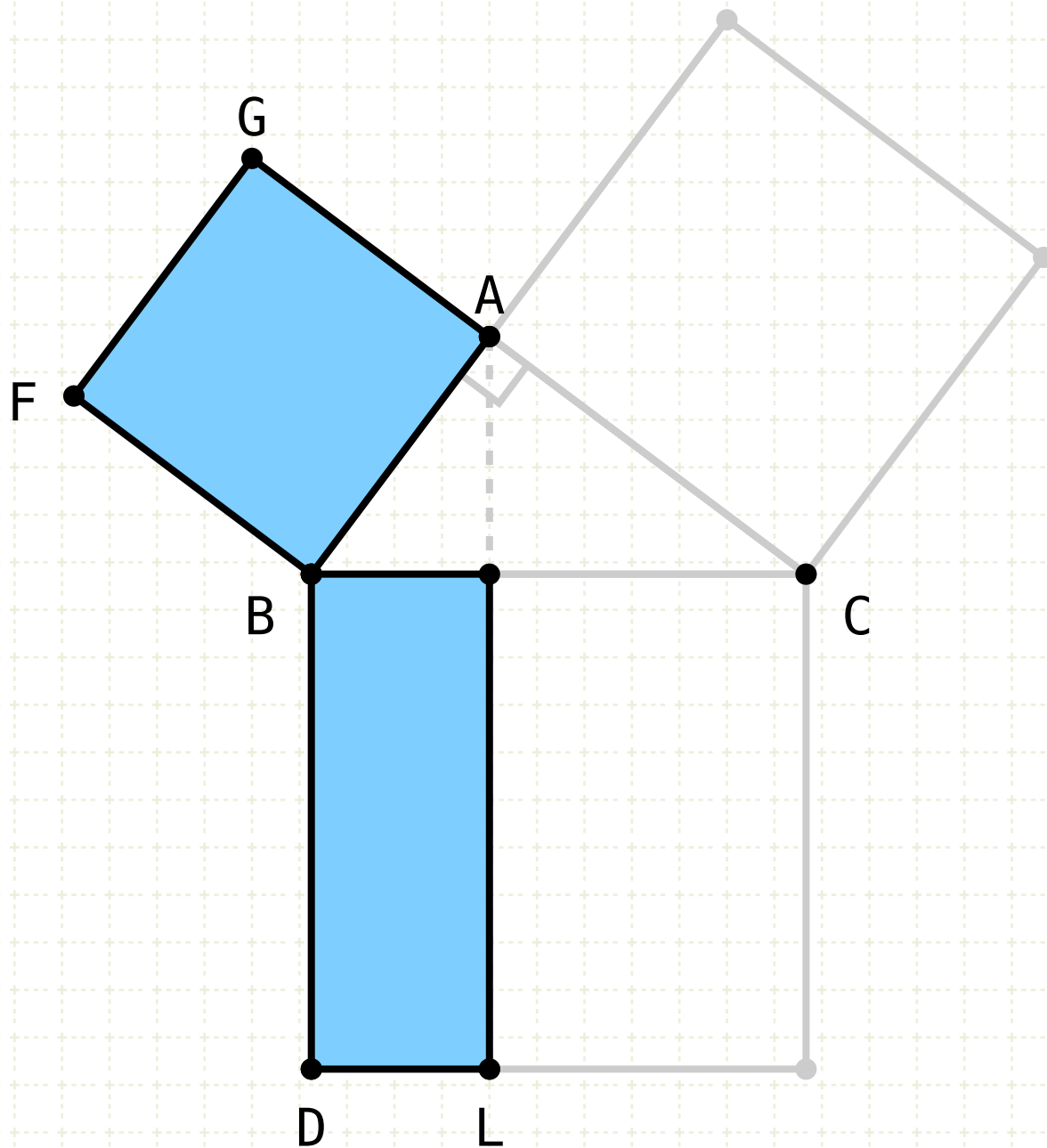
The two triangles are equal, FB equals AB, BC equals BD, with a common angle  $\gamma$  (I·4)

The square AB and the triangle FBC share the same base, and are enclosed by the same parallel lines GC,FB thus FBC is one half ABFG (I·41)

The triangle ABD equals half the parallelogram BDL (I·41)

# Proposition 47 of Book I

In right-angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle.



ABFG is a square

ACKH is a square

BCED is a square

GA, AC = GC

BA, AH = BH

$\angle FBC = \angle + \angle ABC = \angle ABD$

AL  $\parallel$  BD

$\triangle FBC = \triangle ABD$

$\triangle FBC = \frac{1}{2} \square AB$

$\triangle ABD = \frac{1}{2} \square BDL$

$\square AB = \square BDL$

## Proof:

By construction, angle GAB is a right angle, as is BAC, therefore lines GA and AC form a single line GC (I·14)

Similarly for line BH (I·14)

Angles FBA and CBD are both right angles

Adding angle ABC to both demonstrates that angles FBC and ABD are also equal

Draw a line from A, parallel to BD

Draw lines AD and FC, and consider triangles FBC and ABD

The two triangles are equal, FB equals AB, BC equals BD, with a common angle  $\gamma$  (I·4)

The square AB and the triangle FBC share the same base, and are enclosed by the same parallel lines GC,FB thus FBC is one half ABFG (I·41)

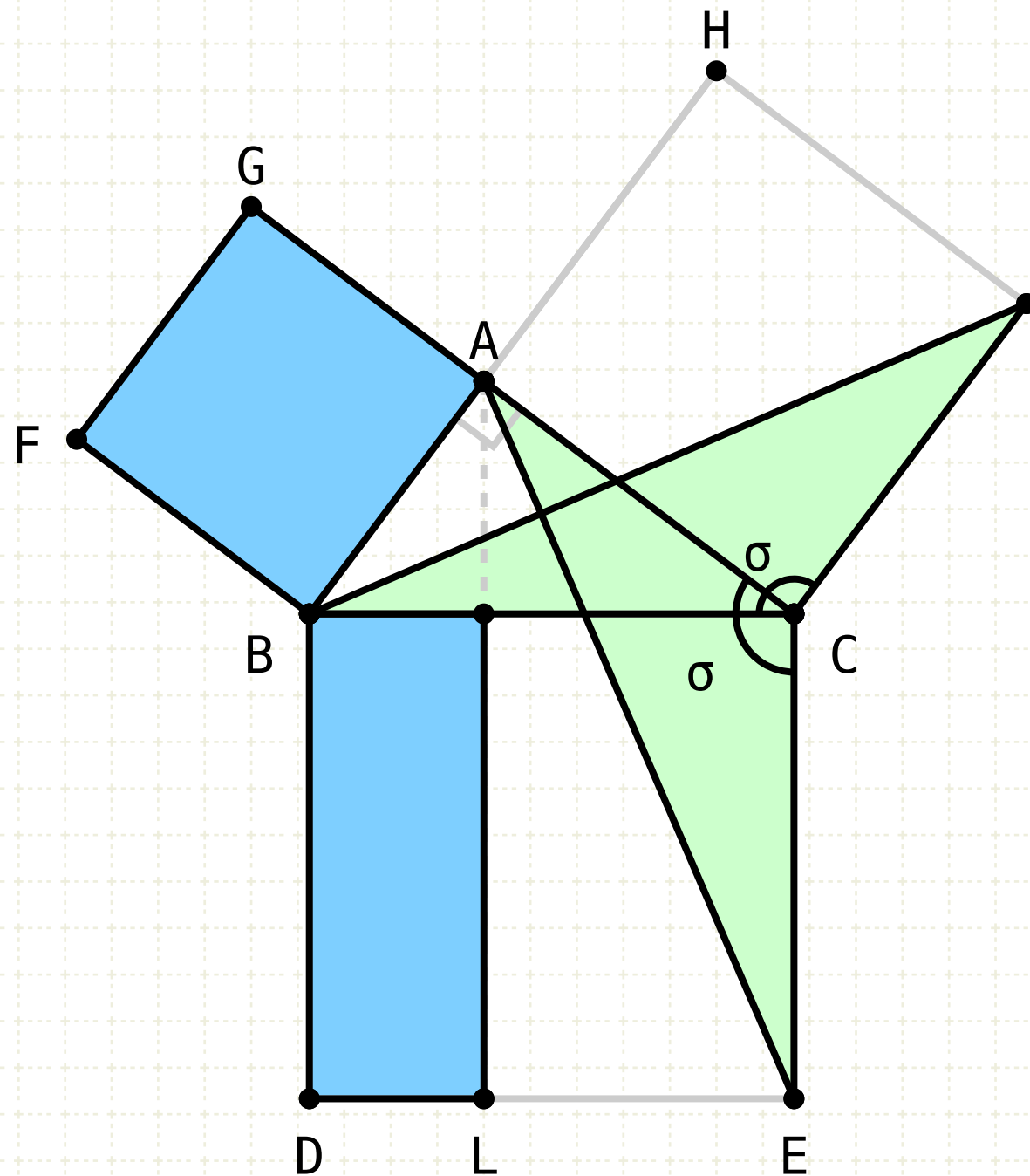
The triangle ABD equals half the parallelogram BDL (I·41)

Therefore, the square of AB equals the polygon BDL



# Proposition 47 of Book I

In right-angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle.



ABFG is a square

ACKH is a square

BCED is a square

K

GA, AC = GC

BA, AH = BH

$\angle FBC = \angle L + \angle ABC = \angle ABD$

AL || BD

$\triangle FBC = \triangle ABD$

$\triangle FBC = \frac{1}{2} \square AB$

$\triangle ABD = \frac{1}{2} \square BDL$

$\square AB = \square BDL$

$\angle BCK = \angle ACE$

$\triangle ECA = \triangle BCK$

## Proof:

By construction, angle GAB is a right angle, as is BAC, therefore lines GA and AC form a single line GC (I·14)

Similarly for line BH (I·14)

Angles FBA and CBD are both right angles

Adding angle ABC to both demonstrates that angles FBC and ABD are also equal

Draw a line from A, parallel to BD

Draw lines AD and FC, and consider triangles FBC and ABD

The two triangles are equal, FB equals AB, BC equals BD, with a common angle  $\gamma$  (I·4)

The square AB and the triangle FBC share the same base, and are enclosed by the same parallel lines GC,FB thus FBC is one half ABFG (I·41)

The triangle ABD equals half the parallelogram BDL (I·41)

Therefore, the square of AB equals the polygon BDL

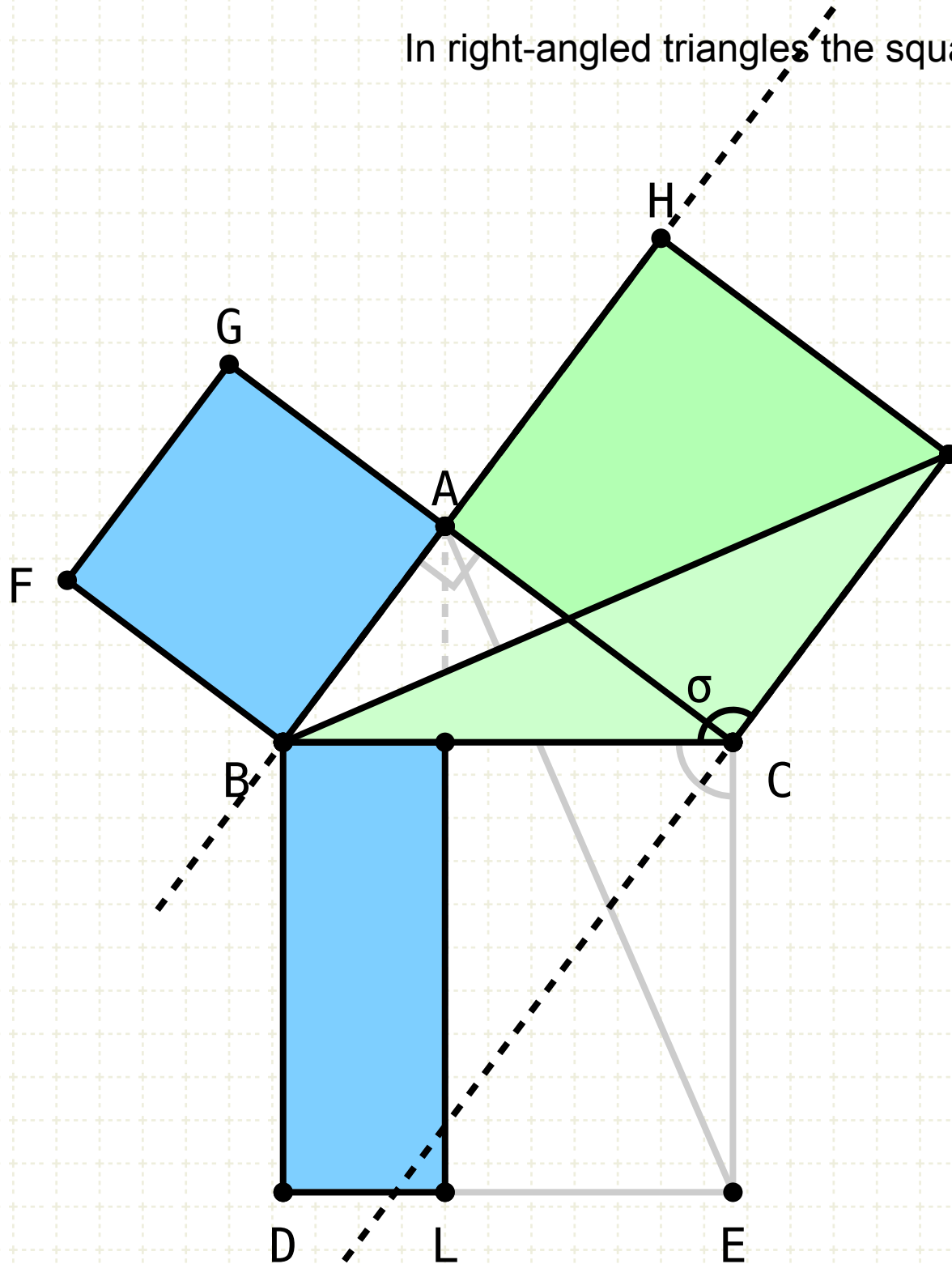
Applying the same logic as before, triangles BCK and AEC are equal (I·4)





# Proposition 47 of Book I

In right-angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle.



ABFG is a square

ACKH is a square

BCED is a square

GA, AC = GC

BA, AH = BH

$\angle FBC = \angle ABC = \angle ABD$

AL || BD

$\triangle FBC = \triangle ABD$

$\triangle FBC = \frac{1}{2} \square AB$

$\triangle ABD = \frac{1}{2} \square BDL$

$\square AB = \square BDL$

$\angle BCK = \angle ACE$

$\triangle ECA = \triangle BCK$

$\triangle BCK = \frac{1}{2} \square AC$

## Proof:

By construction, angle GAB is a right angle, as is BAC, therefore lines GA and AC form a single line GC (I·14)

Similarly for line BH (I·14)

Angles FBA and CBD are both right angles

Adding angle ABC to both demonstrates that angles FBC and ABD are also equal

Draw a line from A, parallel to BD

Draw lines AD and FC, and consider triangles FBC and ABD

The two triangles are equal, FB equals AB, BC equals BD, with a common angle  $\gamma$  (I·4)

The square AB and the triangle FBC share the same base, and are enclosed by the same parallel lines GC,FB thus FBC is one half ABFG (I·41)

The triangle ABD equals half the parallelogram BDL (I·41)

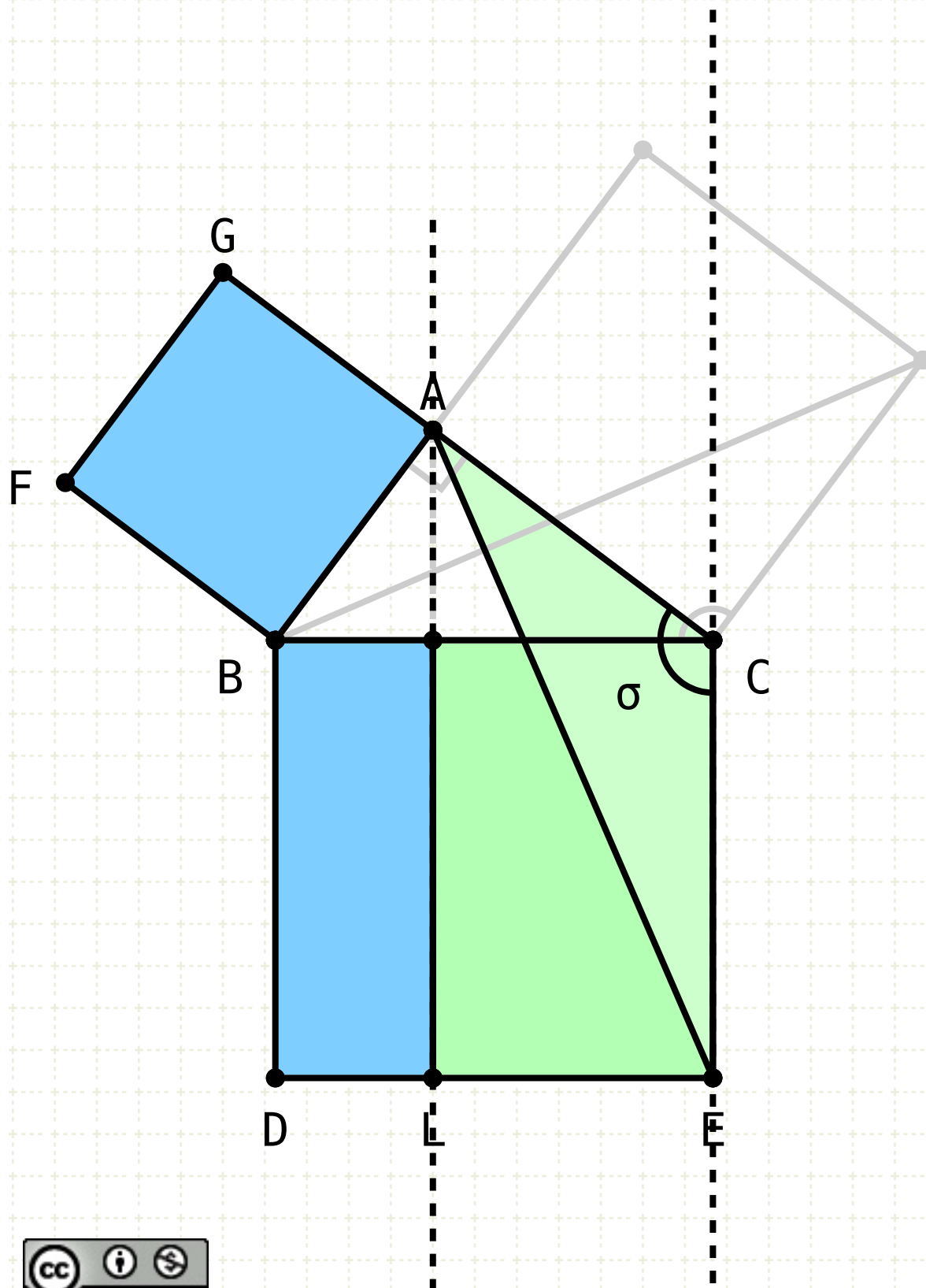
Therefore, the square of AB equals the polygon BDL

Applying the same logic as before, triangles BCK and AEC are equal (I·4)

Triangle BCK is half the square AC

## Proposition 47 of Book I

In right-angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle.



ABFG is a square

ACKH is a square

BCED is a square

$$GA, AC = GC$$
$$BA, AH = BH$$
$$\angle FBC = L + \angle ABC = \angle ABD$$
$$AL \parallel BD$$
$$\Delta FBC = \Delta ABD$$
$$\Delta FBC = \frac{1}{2} \square ABC$$
$$\Delta ABD = \frac{1}{2} \square BDL$$
$$\square AB = \square BDL$$
$$\angle BCK = \angle ACE$$

ΔΕCΑ = ΔBCK

$$\Delta BCK = \frac{1}{5} \square AC$$
$$\Delta ECA = \frac{1}{2} \square CEL$$

## Proof:

By construction, angle GAB is a right angle, as is BAC, therefore lines GA and AC form a single line GC (I.14)

Similarly for line BH (I-14)

Angles FBA and CBD are both right angles

Adding angle ABC to both demonstrates that angles FBC and ABD are also equal

Draw a line from A, parallel to BD

Draw lines AD and FC, and consider triangles FBC and ABD

The two triangles are equal, FB equals AB, BC equals BD, with a common angle  $\gamma$  (I.4)

The square AB and the triangle FBC share the same base, and are enclosed by the same parallel lines GC,FB thus FBC is one half ABFG (I-41)

The triangle ABD equals half the parallelogram BDL (I.41)

Therefore, the square of AB equals the polygon BDL

Applying the same logic as before, triangles BCK and AEC are equal (I.4)

Triangle BCK is half the square AC

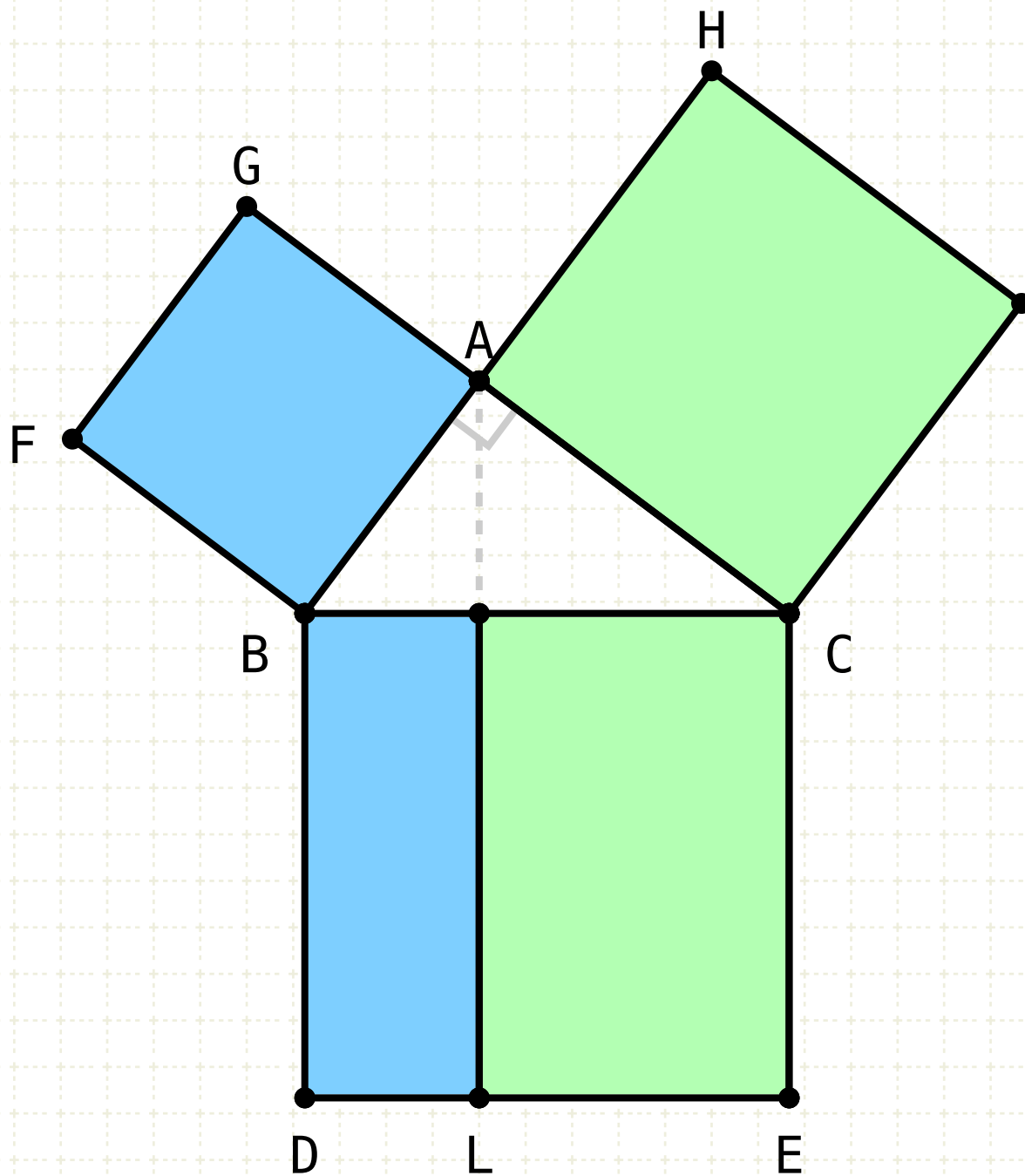
Triangle ECA is half the parallelogram CEL (I-41)





# Proposition 47 of Book I

In right-angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle.



ABFG is a square  
 ACKH is a square  
 BCED is a square  
 K  
 GA, AC = GC  
 BA, AH = BH  
 $\angle FBC = \angle + \angle ABC = \angle ABD$   
 $AL \parallel BD$   
 $\triangle FBC = \triangle ABD$   
 $\triangle FBC = \frac{1}{2} \square BDL$   
 $\triangle ABD = \frac{1}{2} \square BDL$   
 $\square AB = \square BDL$   
 $\angle BCK = \angle ACE$   
 $\triangle ECA = \triangle BCK$   
 $\triangle BCK = \frac{1}{2} \square AC$   
 $\triangle ECA = \frac{1}{2} \square CEL$   
 $\square AC = \square CEL$

## Proof:

By construction, angle GAB is a right angle, as is BAC, therefore lines GA and AC form a single line GC (I·14)

Similarly for line BH (I·14)

Angles FBA and CBD are both right angles

Adding angle ABC to both demonstrates that angles FBC and ABD are also equal

Draw a line from A, parallel to BD

Draw lines AD and FC, and consider triangles FBC and ABD

The two triangles are equal, FB equals AB, BC equals BD, with a common angle  $\gamma$  (I·4)

The square AB and the triangle FBC share the same base, and are enclosed by the same parallel lines GC,FB thus FBC is one half ABFG (I·41)

The triangle ABD equals half the parallelogram BDL (I·41)

Therefore, the square of AB equals the polygon BDL

Applying the same logic as before, triangles BCK and AEC are equal (I·4)

Triangle BCK is half the square AC

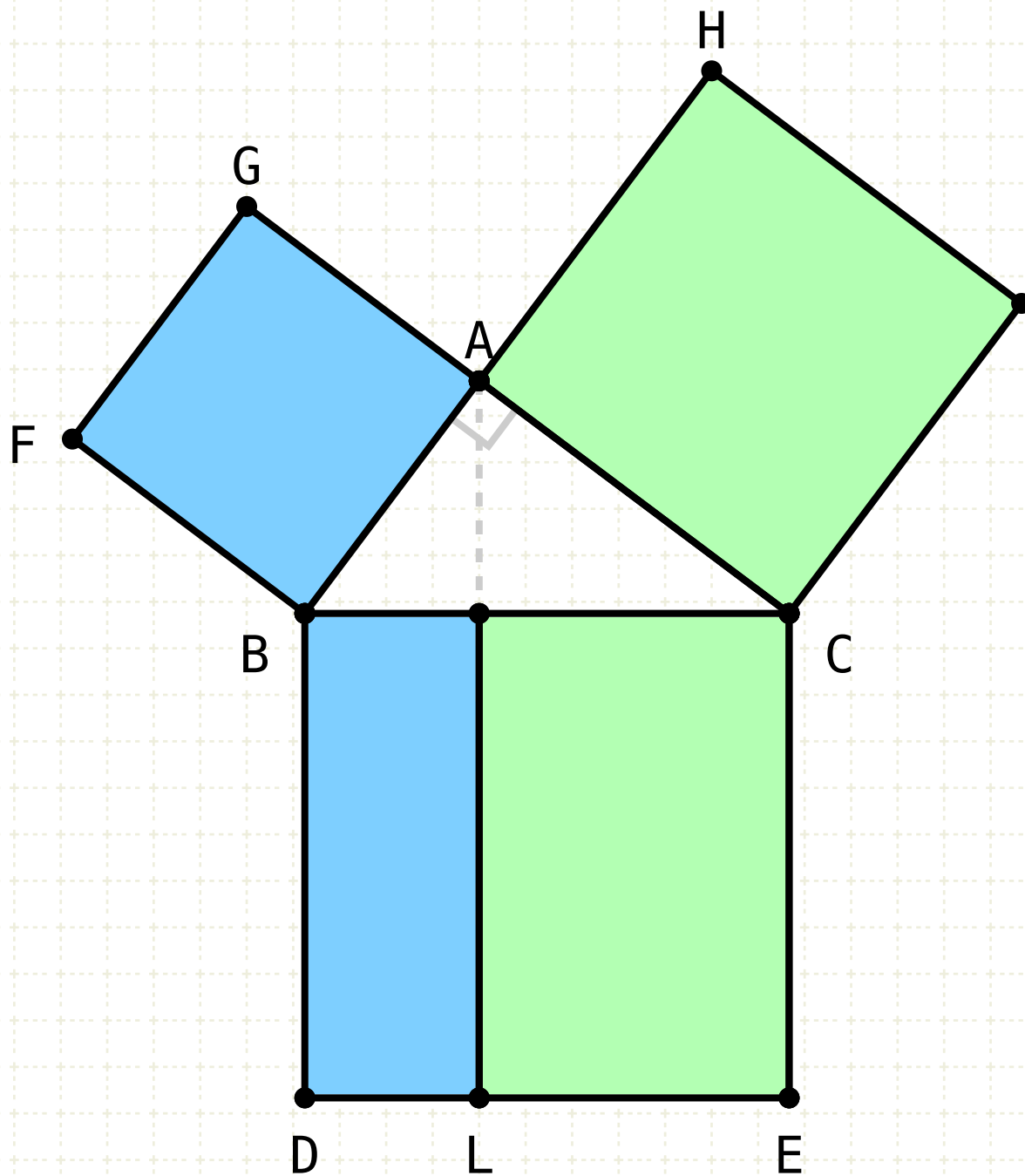
Triangle ECA is half the parallelogram CEL (I·41)

Therefore the square of AC equals the parallelogram CEL



# Proposition 47 of Book I

In right-angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle.



ABFG is a square  
 ACKH is a square  
 BCED is a square  
 K  
 GA, AC = GC  
 BA, AH = BH  
 $\angle FBC = \angle + \angle ABC = \angle ABD$   
 AL  $\parallel$  BD  
 $\triangle FBC = \triangle ABD$   
 $\triangle FBC = \frac{1}{2} \square BDL$   
 $\triangle ABD = \frac{1}{2} \square BDL$   
 $\square AB = \square BDL$   
 $\angle BCK = \angle ACE$   
 $\triangle ECA = \triangle BCK$   
 $\triangle BCK = \frac{1}{2} \square CEL$   
 $\triangle ECA = \frac{1}{2} \square CEL$   
 $\square AC = \square CEL$   
 $\square BC = \square BDL + \square CEL$

## Proof:

By construction, angle GAB is a right angle, as is BAC, therefore lines GA and AC form a single line GC (I·14)

Similarly for line BH (I·14)

Angles FBA and CBD are both right angles

Adding angle ABC to both demonstrates that angles FBC and ABD are also equal

Draw a line from A, parallel to BD

Draw lines AD and FC, and consider triangles FBC and ABD

The two triangles are equal, FB equals AB, BC equals BD, with a common angle  $\gamma$  (I·4)

The square AB and the triangle FBC share the same base, and are enclosed by the same parallel lines GC,FB thus FBC is one half ABFG (I·41)

The triangle ABD equals half the parallelogram BDL (I·41)

Therefore, the square of AB equals the polygon BDL

Applying the same logic as before, triangles BCK and AEC are equal (I·4)

Triangle BCK is half the square AC

Triangle ECA is half the parallelogram CEL (I·41)

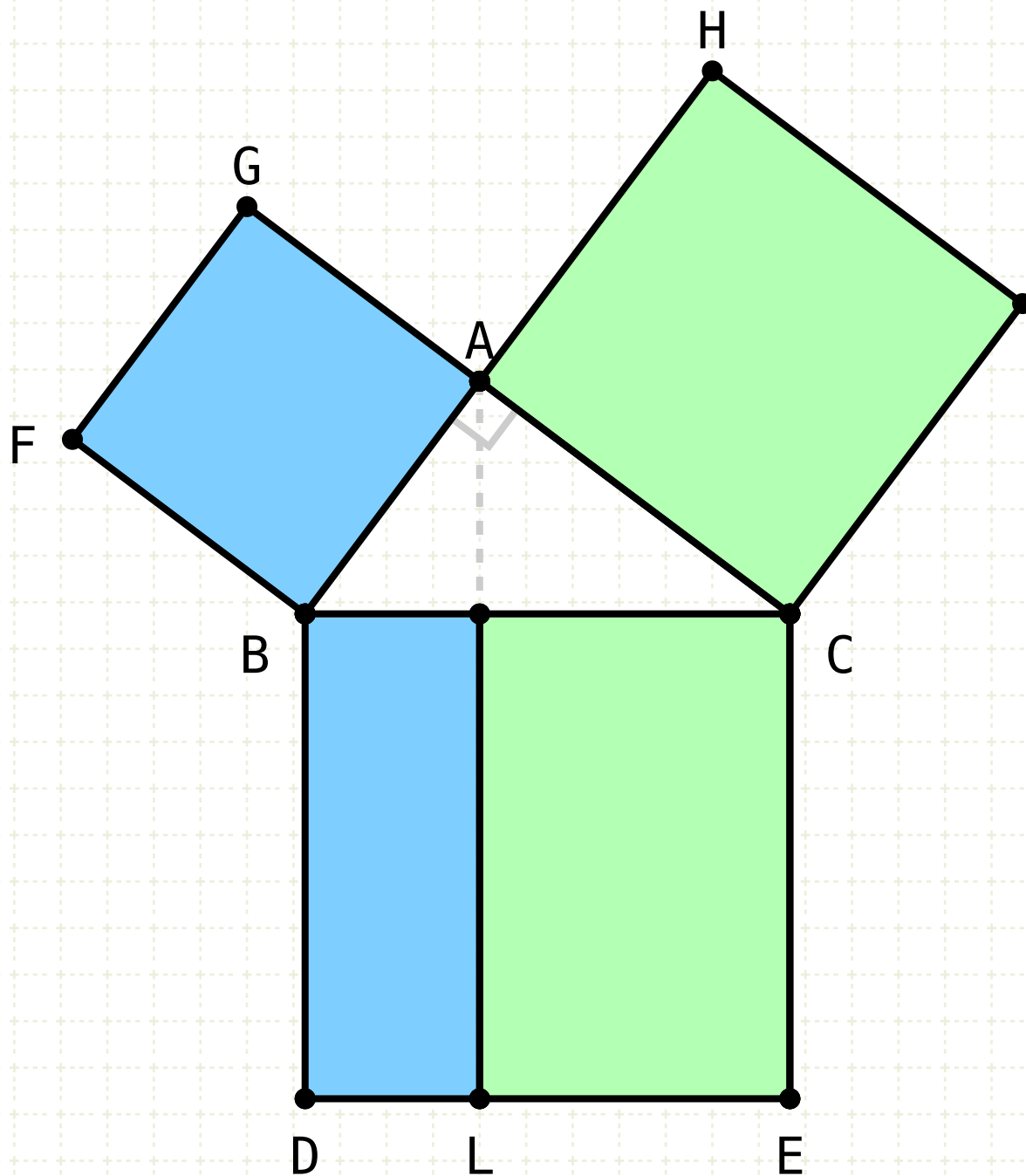
Therefore the square of AC equals the parallelogram CEL

The square of line BC equals the sum of BDL and CEL



# Proposition 47 of Book I

In right-angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle.



ABFG is a square  
 ACKH is a square  
 BCED is a square  
 K  
 GA, AC = GC  
 BA, AH = BH  
 $\angle FBC = \angle + \angle ABC = \angle ABD$   
 AL  $\parallel$  BD  
 $\triangle FBC = \triangle ABD$   
 $\triangle FBC = \frac{1}{2} \square BDL$   
 $\triangle ABD = \frac{1}{2} \square BDL$   
 $\square AB = \square BDL$   
 $\angle BCK = \angle ACE$   
 $\triangle ECA = \triangle BCK$   
 $\triangle BCK = \frac{1}{2} \square CEL$   
 $\triangle ECA = \frac{1}{2} \square CEL$   
 $\square AC = \square CEL$   
 $\square BC = \square BDL + \square CEL$   
 $\square BC = \square AB + \square AC$

## Proof:

By construction, angle GAB is a right angle, as is BAC, therefore lines GA and AC form a single line GC (I·14)

Similarly for line BH (I·14)

Angles FBA and CBD are both right angles

Adding angle ABC to both demonstrates that angles FBC and ABD are also equal

Draw a line from A, parallel to BD

Draw lines AD and FC, and consider triangles FBC and ABD

The two triangles are equal, FB equals AB, BC equals BD, with a common angle  $\gamma$  (I·4)

The square AB and the triangle FBC share the same base, and are enclosed by the same parallel lines GC,FB thus FBC is one half ABFG (I·41)

The triangle ABD equals half the parallelogram BDL (I·41)

Therefore, the square of AB equals the polygon BDL

Applying the same logic as before, triangles BCK and AEC are equal (I·4)

Triangle BCK is half the square AC

Triangle ECA is half the parallelogram CEL (I·41)

Therefore the square of AC equals the parallelogram CEL

The square of line BC equals the sum of BDL and CEL

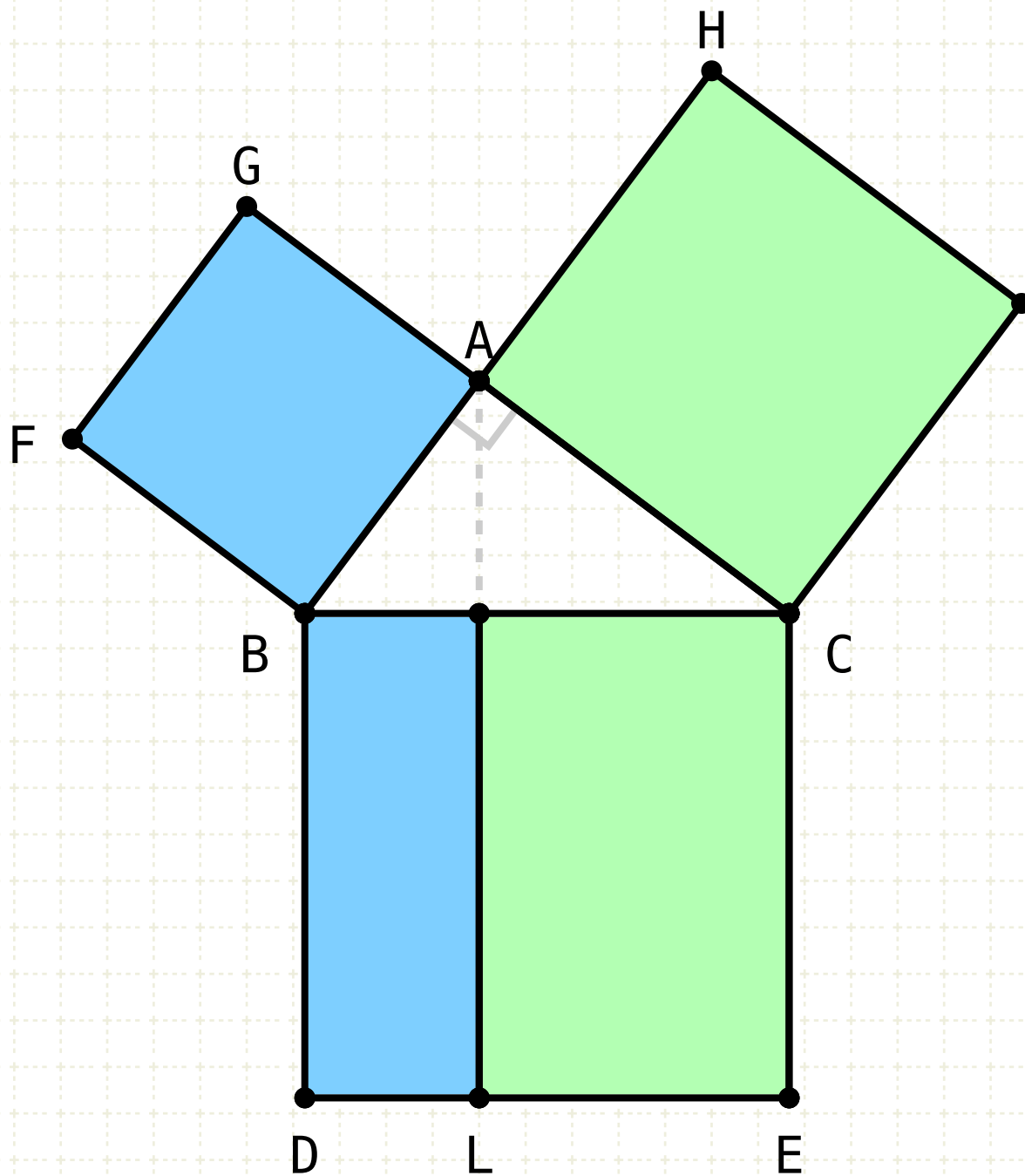
The sum of the squares of lines AB and AC equals the square of BC





# Proposition 47 of Book I

In right-angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle.



ABFG is a square

ACKH is a square

BCED is a square

K

$GA, AC = GC$

$BA, AH = BH$

$\angle FBC = \angle + \angle ABC = \angle ABD$

$AL \parallel BD$

$\triangle FBC = \triangle ABD$

$\triangle FBC = \frac{1}{2} \square AB$

$\triangle ABD = \frac{1}{2} \square BDL$

$\square AB = \square BDL$

$\angle BCK = \angle ACE$

$\triangle ECA = \triangle BCK$

$\triangle BCK = \frac{1}{2} \square AC$

$\triangle ECA = \frac{1}{2} \square CEL$

$\square AC = \square CEL$

$\square BC = \square BDL + \square CEL$

$\square BC = \square AB + \square AC$

## Proof:

By construction, angle GAB is a right angle, as is BAC, therefore lines GA and AC form a single line GC (I·14)

Similarly for line BH (I·14)

Angles FBA and CBD are both right angles

Adding angle ABC to both demonstrates that angles FBC and ABD are also equal

Draw a line from A, parallel to BD

Draw lines AD and FC, and consider triangles FBC and ABD

The two triangles are equal, FB equals AB, BC equals BD, with a common angle  $\gamma$  (I·4)

The square AB and the triangle FBC share the same base, and are enclosed by the same parallel lines GC,FB thus FBC is one half ABFG (I·41)

The triangle ABD equals half the parallelogram BDL (I·41)

Therefore, the square of AB equals the polygon BDL

Applying the same logic as before, triangles BCK and AEC are equal (I·4)

Triangle BCK is half the square AC

Triangle ECA is half the parallelogram CEL (I·41)

Therefore the square of AC equals the parallelogram CEL

The square of line BC equals the sum of BDL and CEL

The sum of the squares of lines AB and AC equals the square of BC



# Youtube Videos

<https://www.youtube.com/c/SandyBultena>

*Copyright © 2019 by Sandy Bultena.*



Except where otherwise noted, this work is licensed under  
<http://creativecommons.org/licenses/by-nc/3.0>