

Euclid's Elements

Book I

*If Euclid did not kindle your youthful enthusiasm, you
were not born to be a scientific thinker.*

Albert Einstein

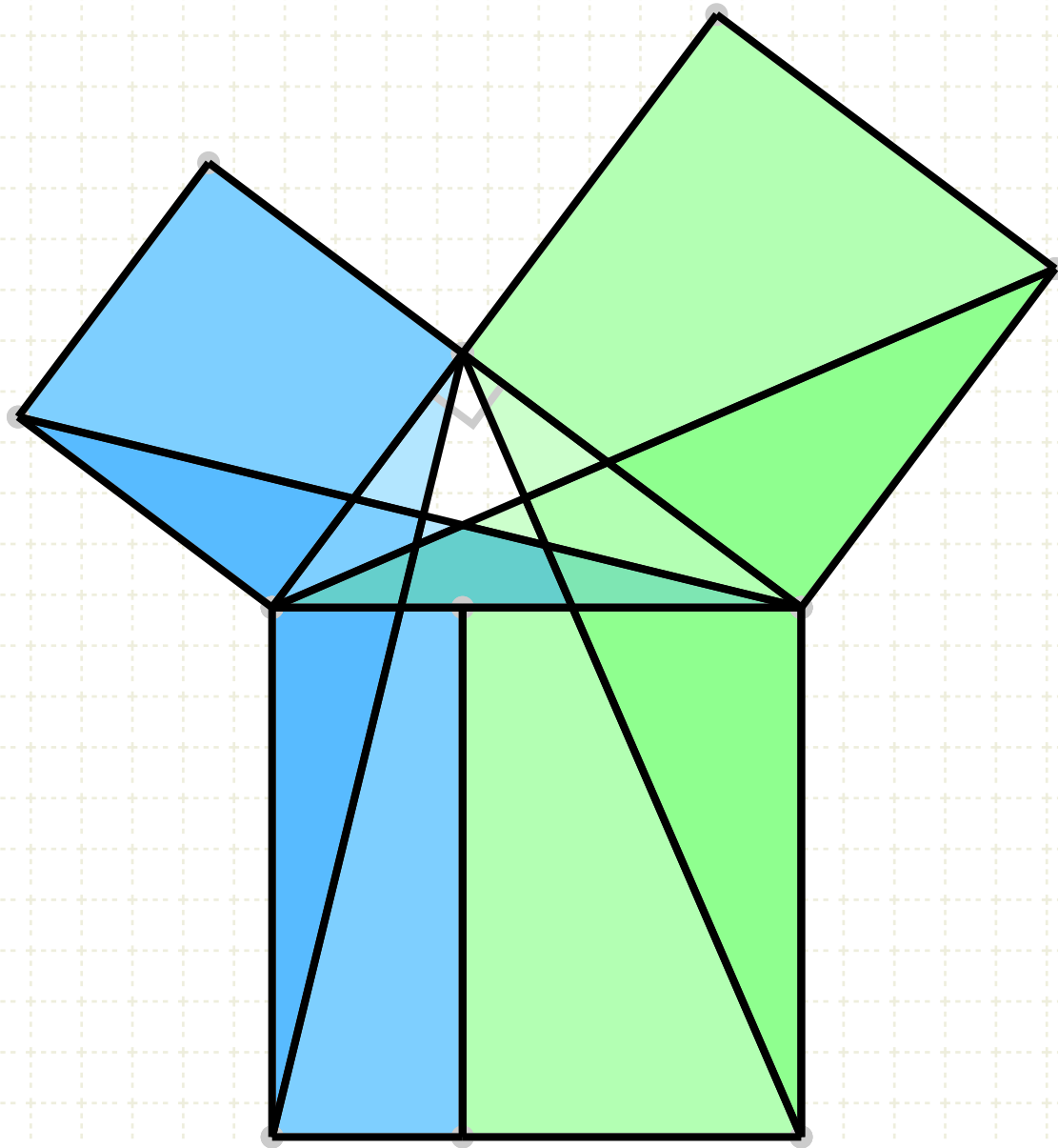


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Proposition 15 of Book I

If two straight lines cut one another, then they make the vertical angles equal to one another.

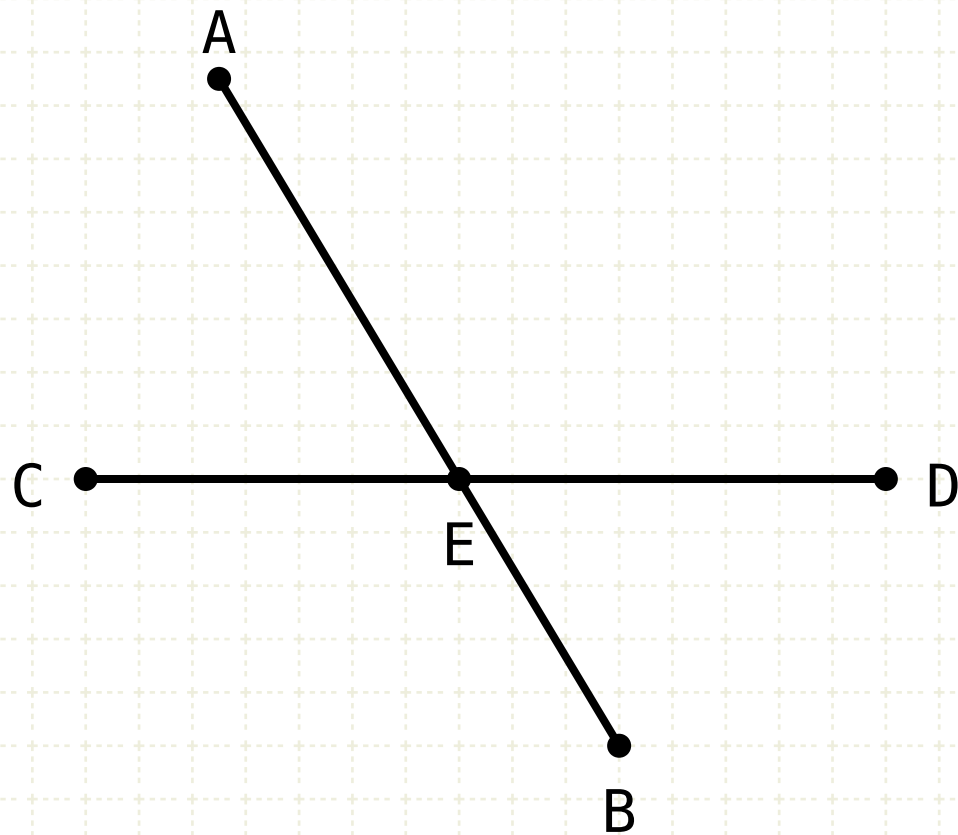


Proposition 15 of Book I

If two straight lines cut one another, then they make the vertical angles equal to one another.

In other words

Given two arbitrary line segments AB and CD which intersect at point E



Proposition 15 of Book I

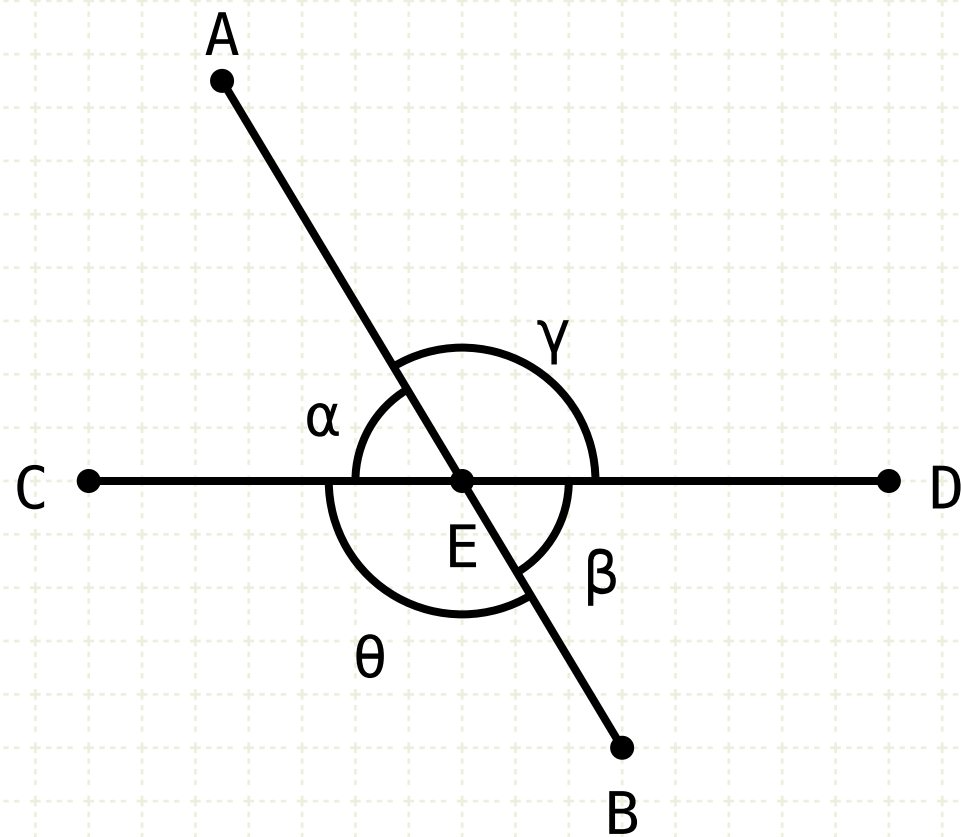
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Given two arbitrary line segments AB and CD which intersect at point E

Angles AEC and DEB are equal

Angles AED and CEB are equal



$$\alpha = \beta$$
$$\gamma = \theta$$

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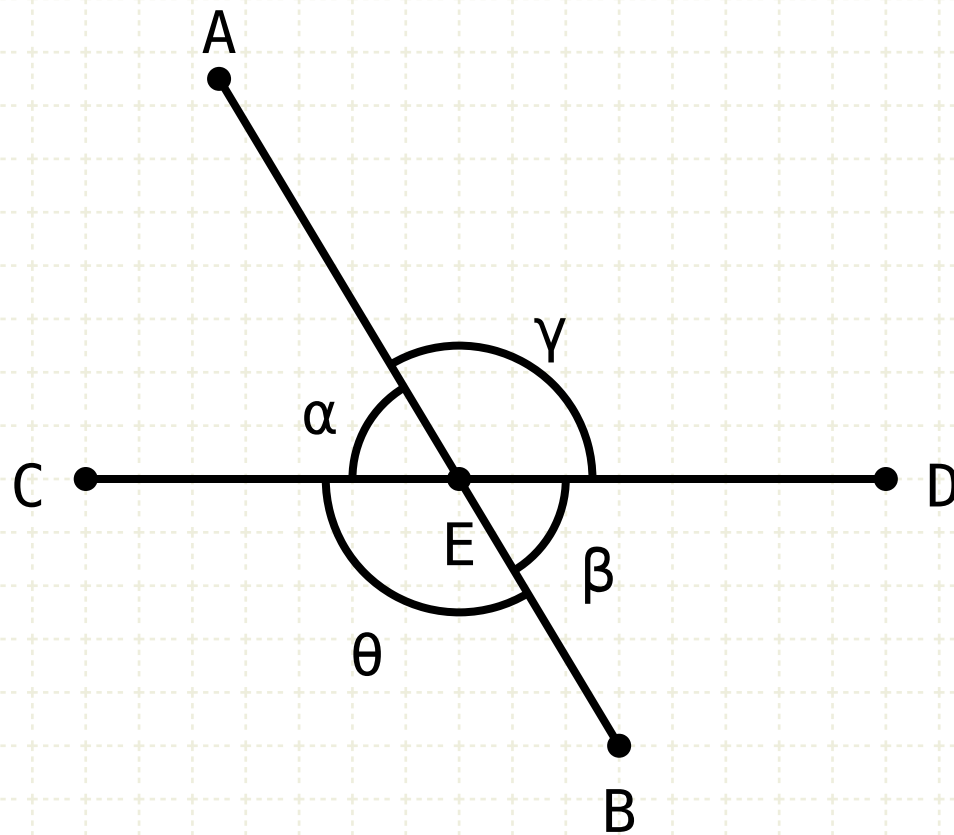
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Proof



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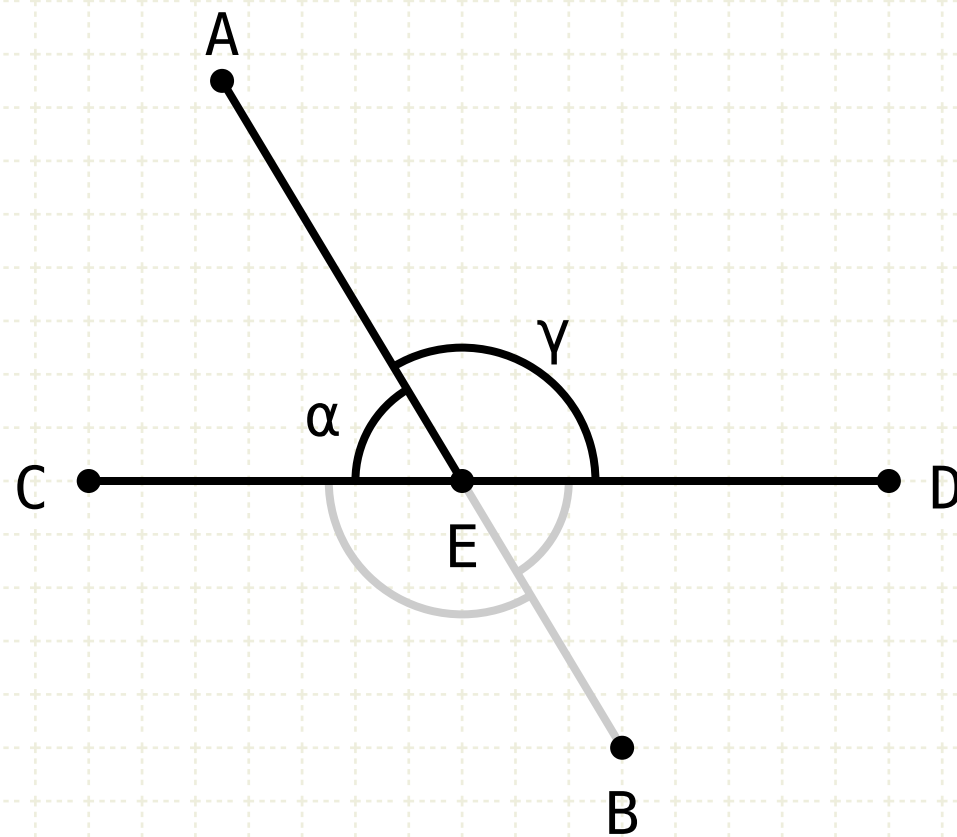
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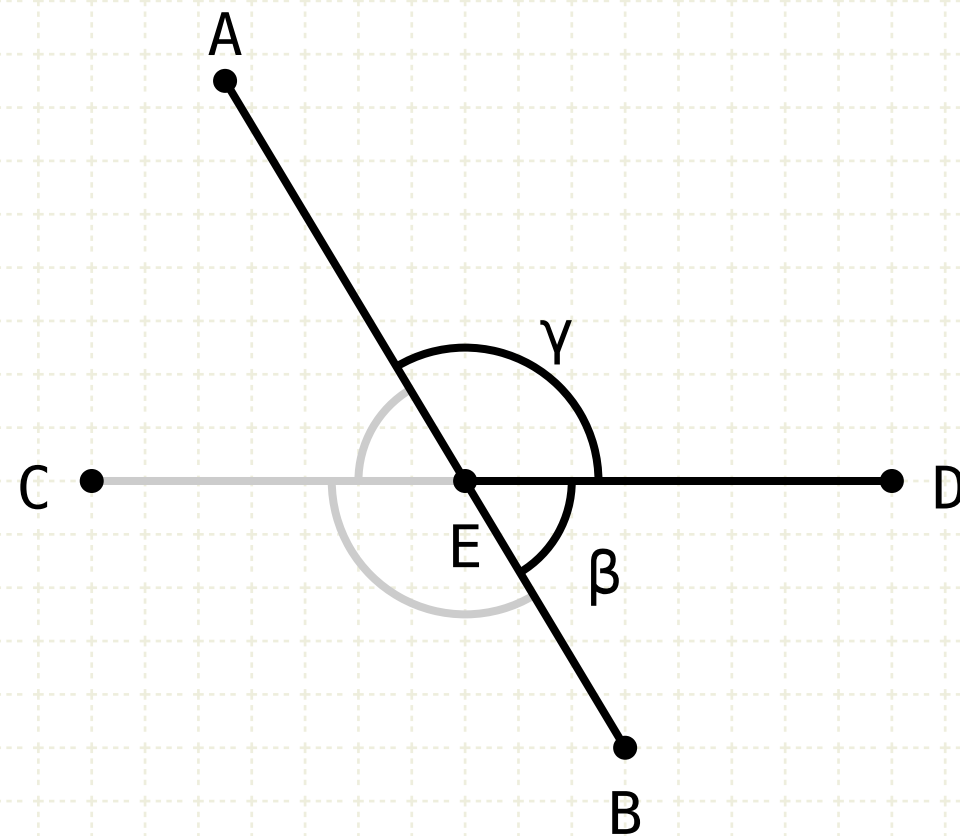
CD is a straight line, so the sum of AEC and AED equals two right angles (I·13)



$$\alpha + \gamma = L + L$$

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$$\begin{array}{rcl} \alpha & + & \gamma = \text{L} + \text{L} \\ \gamma & + & \beta = \text{L} + \text{L} \end{array}$$

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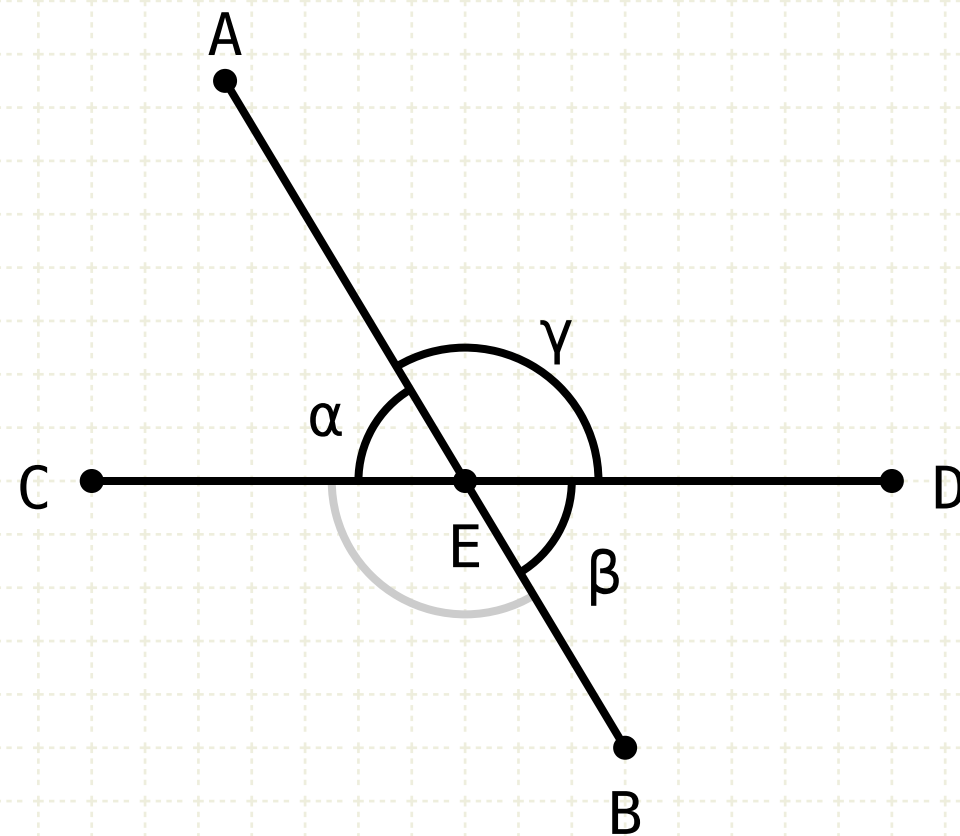
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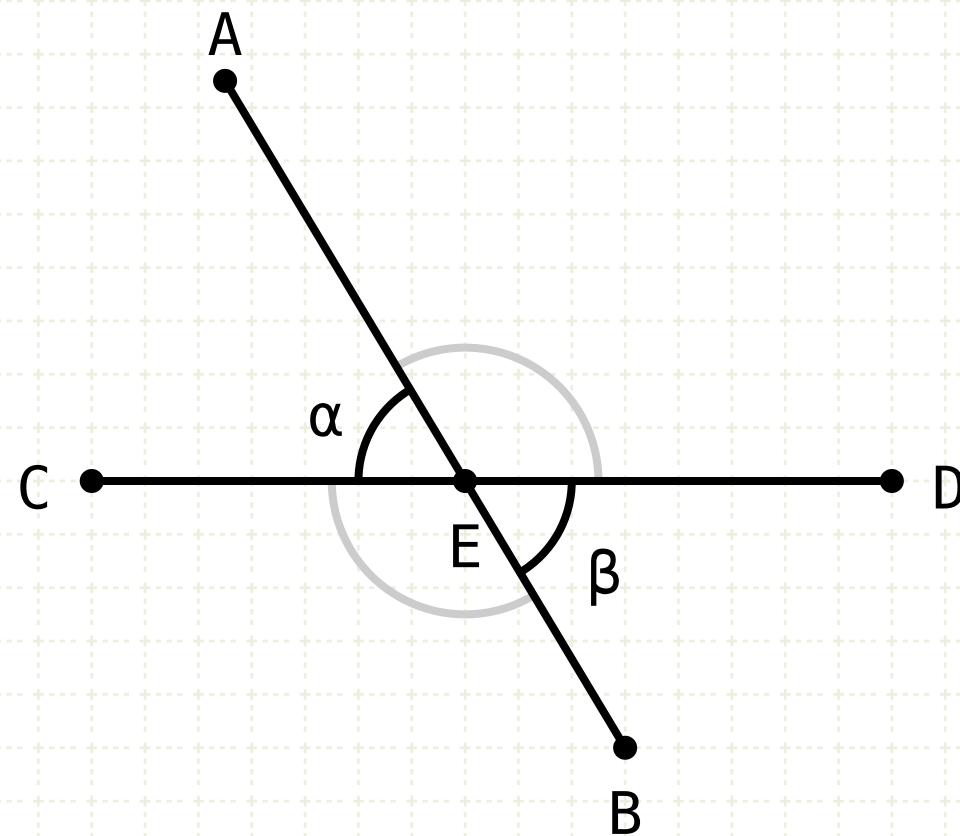
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Proposition 15 of Book I

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$$\begin{aligned}\alpha + \gamma &= \text{L} + \text{L} \\ \gamma + \beta &= \text{L} + \text{L} \\ \alpha + \gamma &= \gamma + \beta \\ \therefore \alpha &= \beta\end{aligned}$$

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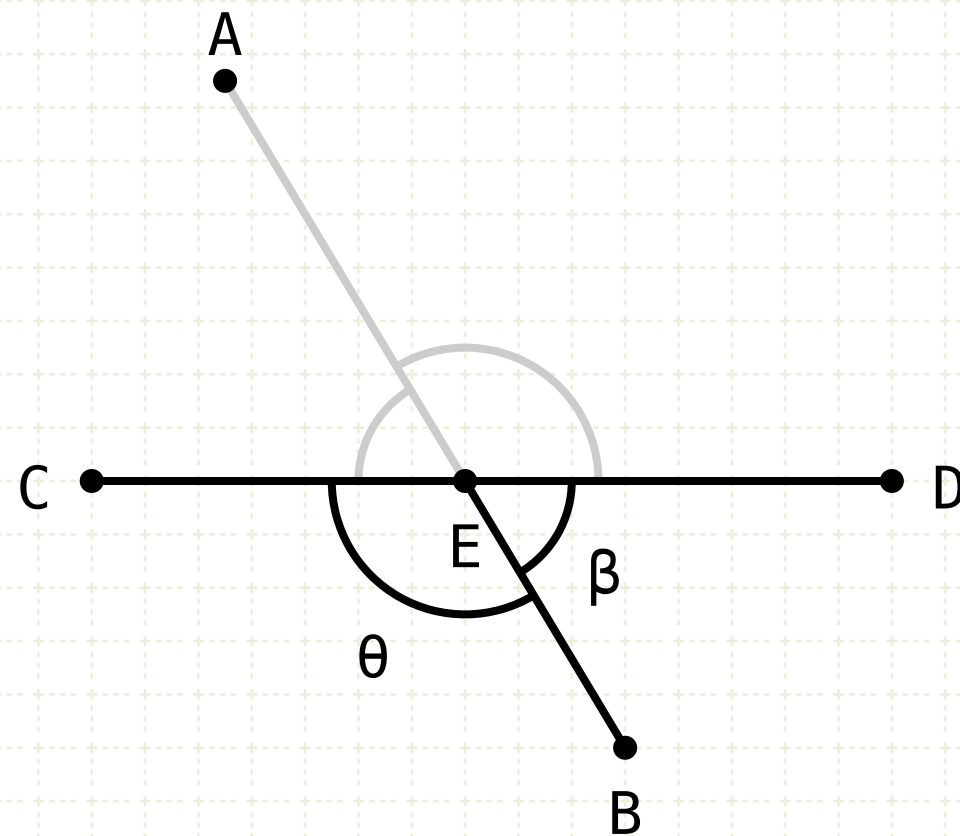
AB is a straight line, so the sum of AED and DEB equals two right angles (I·13)

Since the sums of the angles are equal to the same thing (two right angles), they are equal to each other

Thus angle AEC is equal to angle DEB

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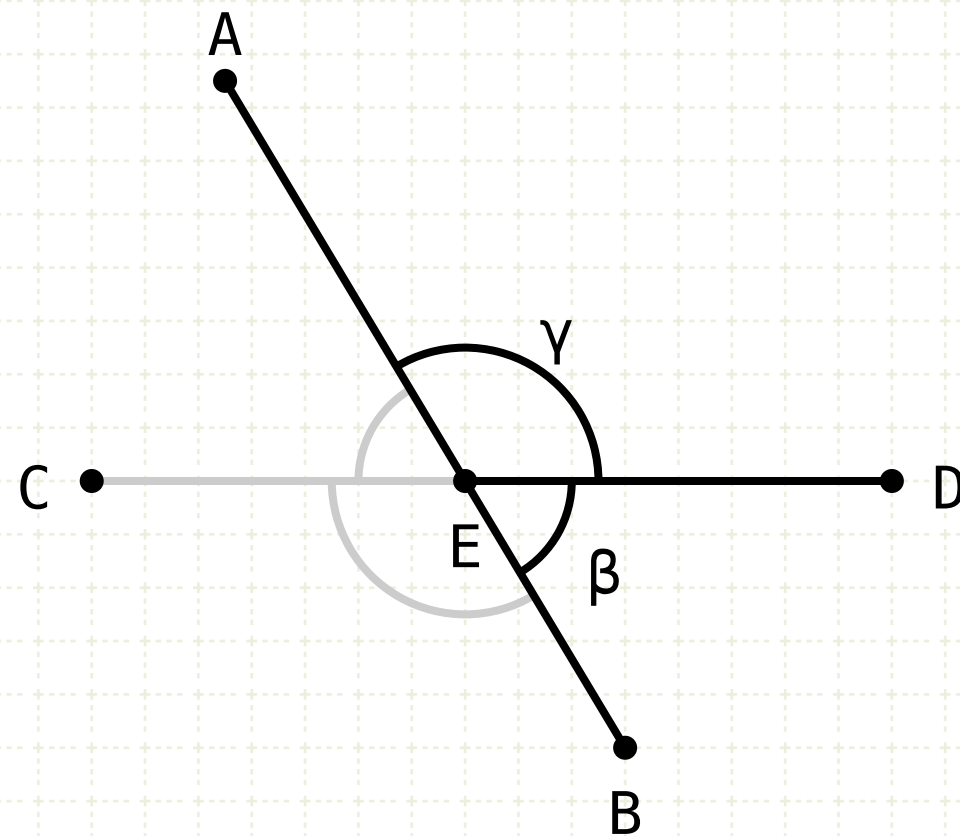
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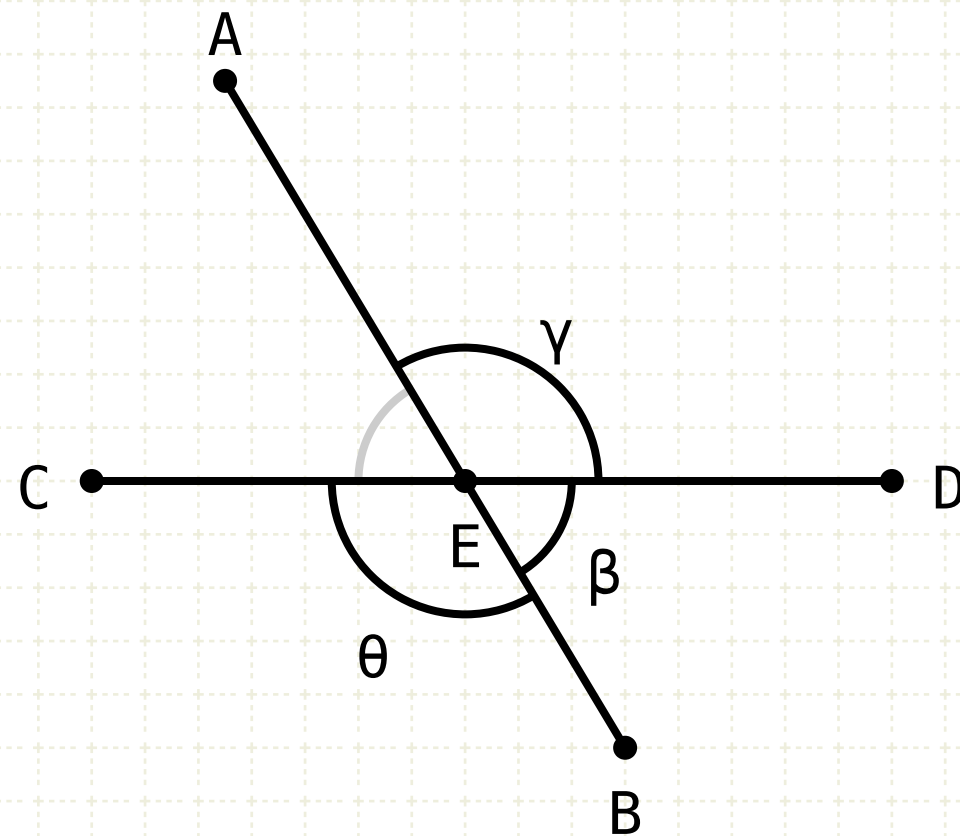
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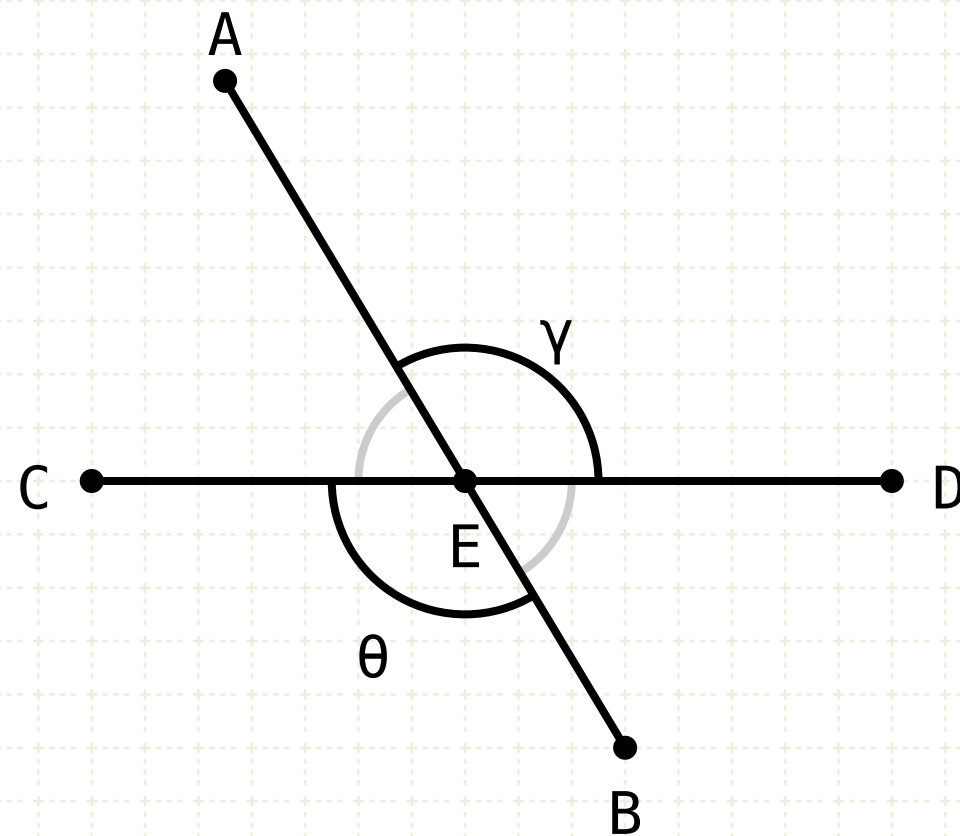
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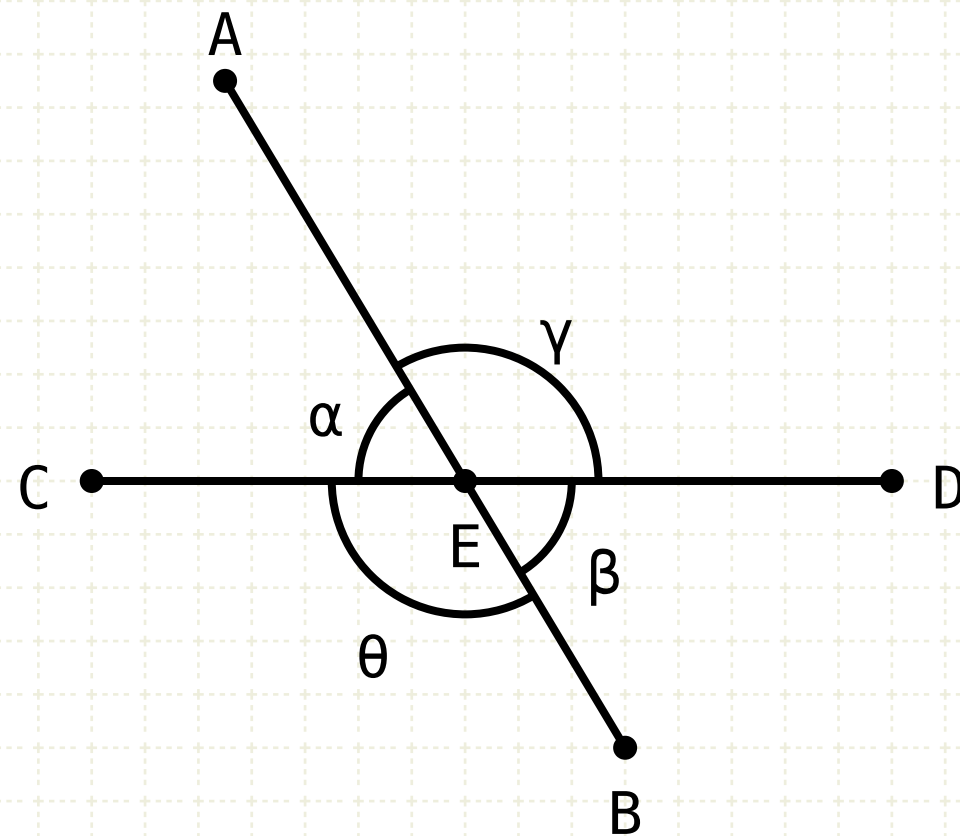
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