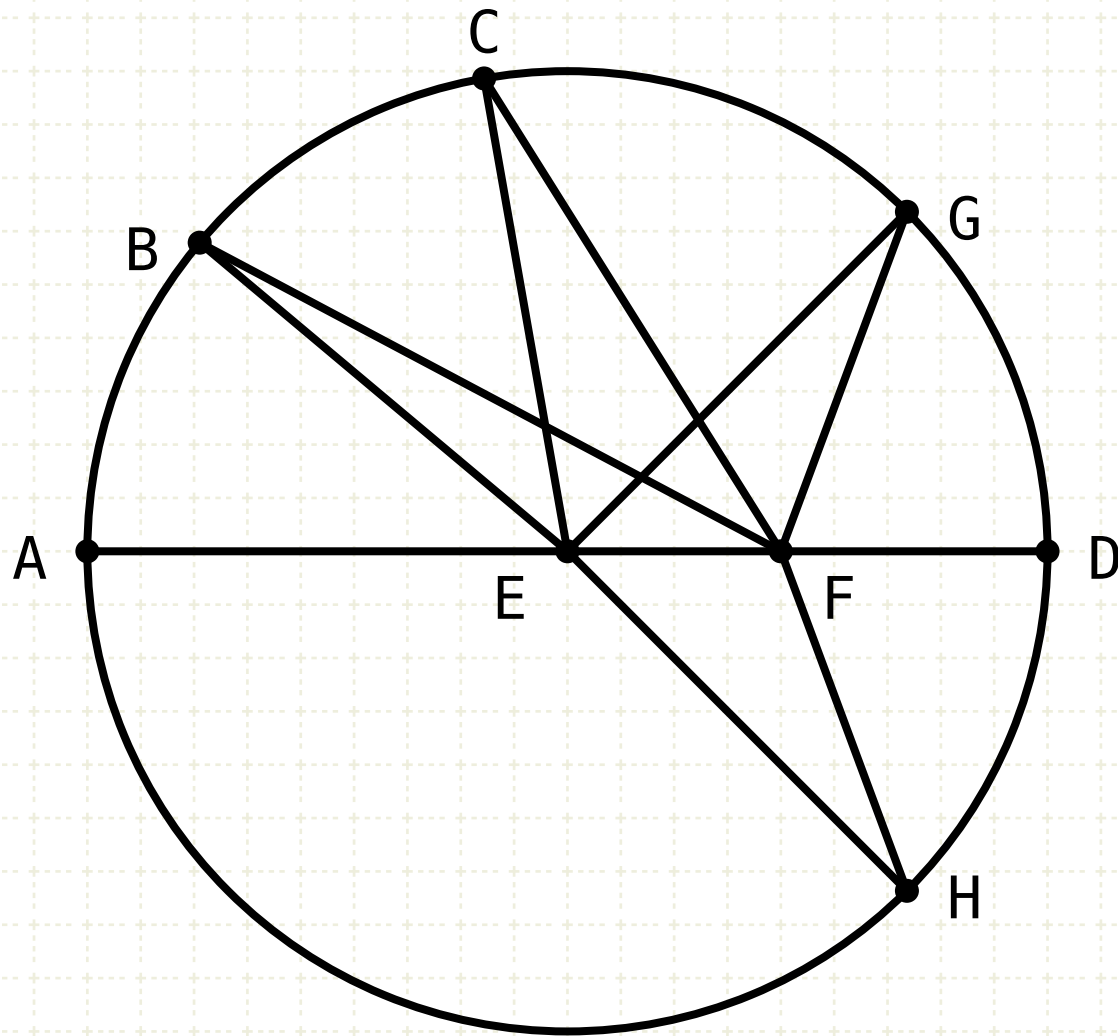# Euclid's Elements

Book III

A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)

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F center of AB

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In other words

Given two circles, one inside the other, touching at point A, where F and G are the centres of the circles…

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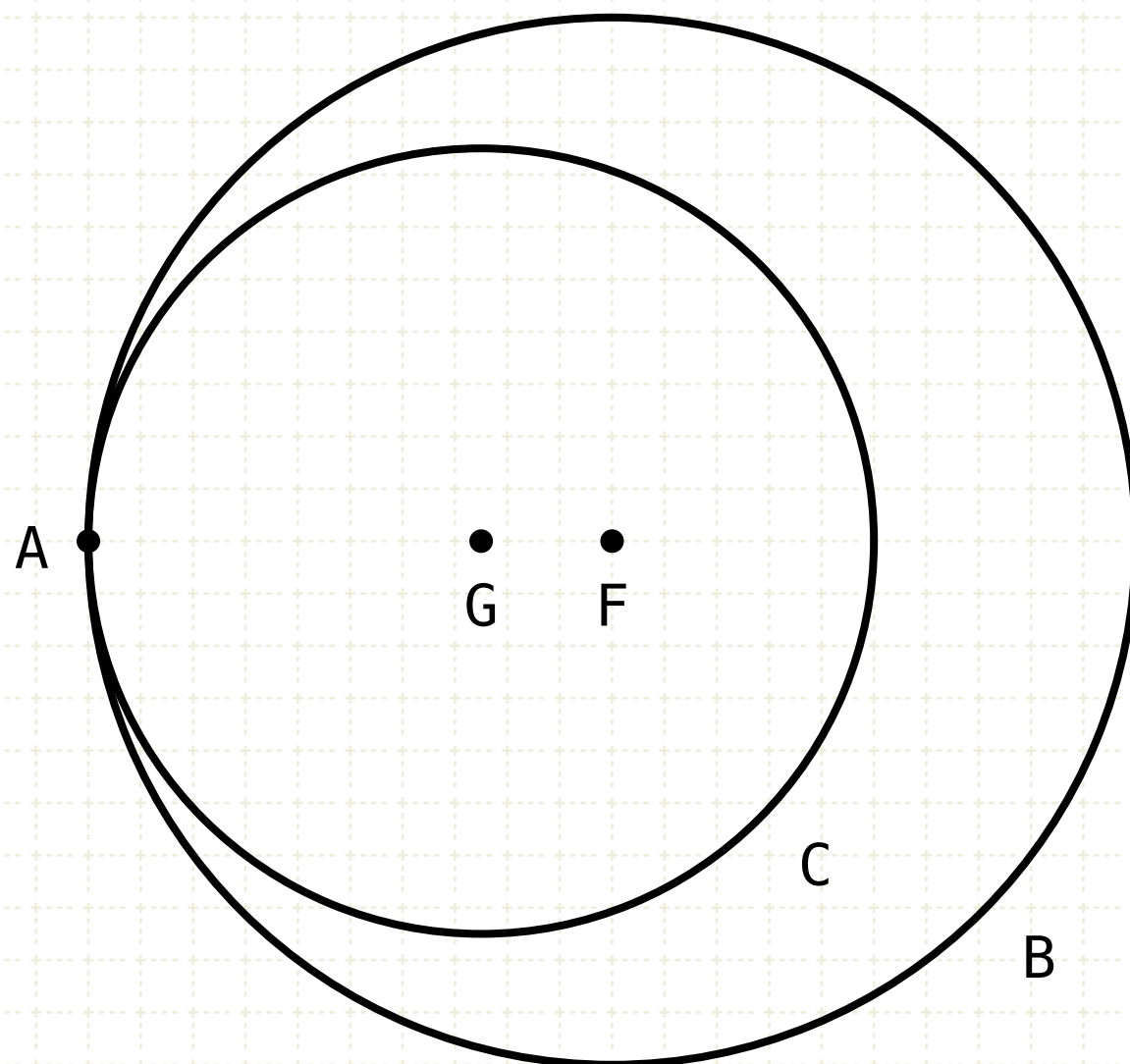
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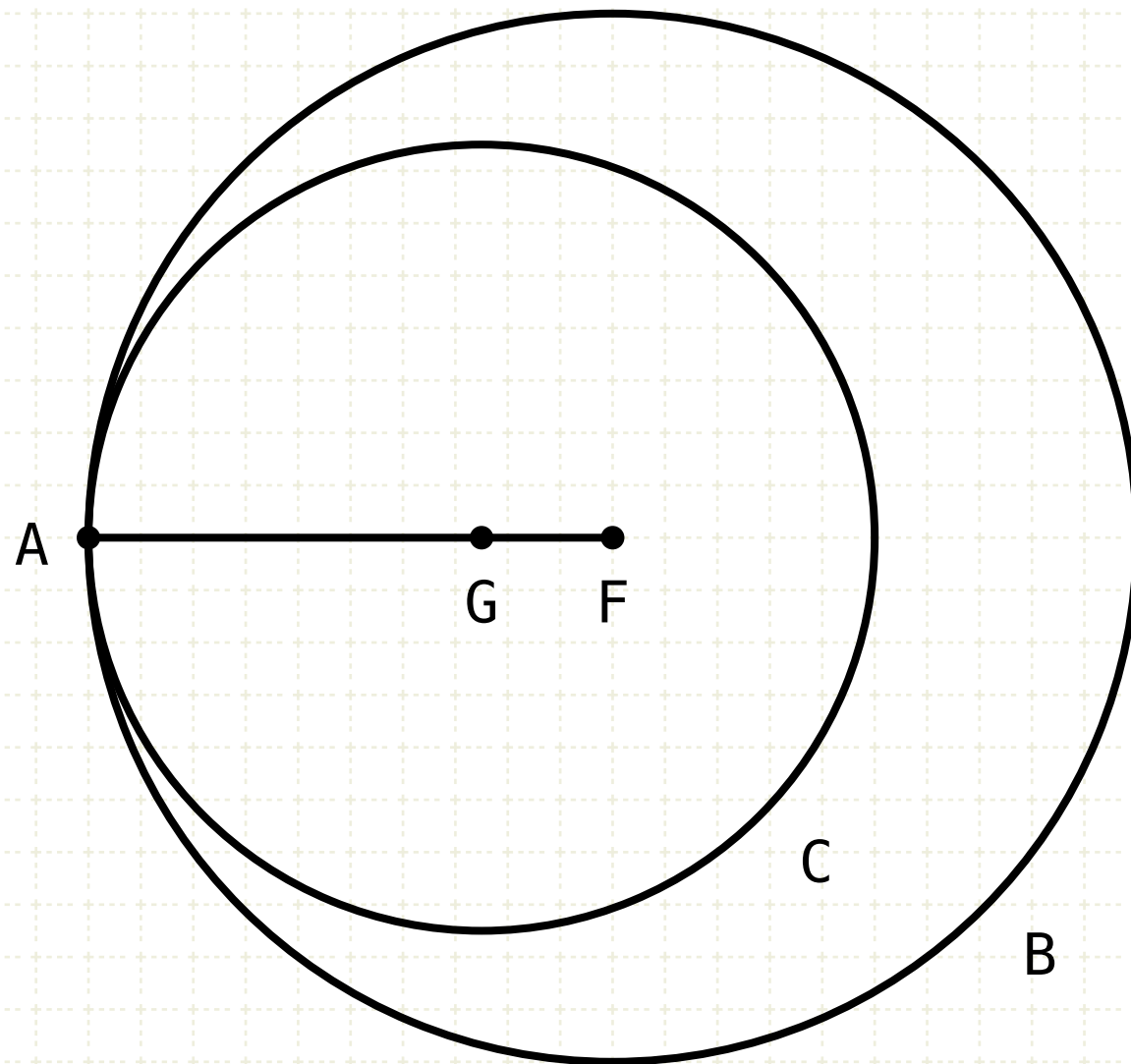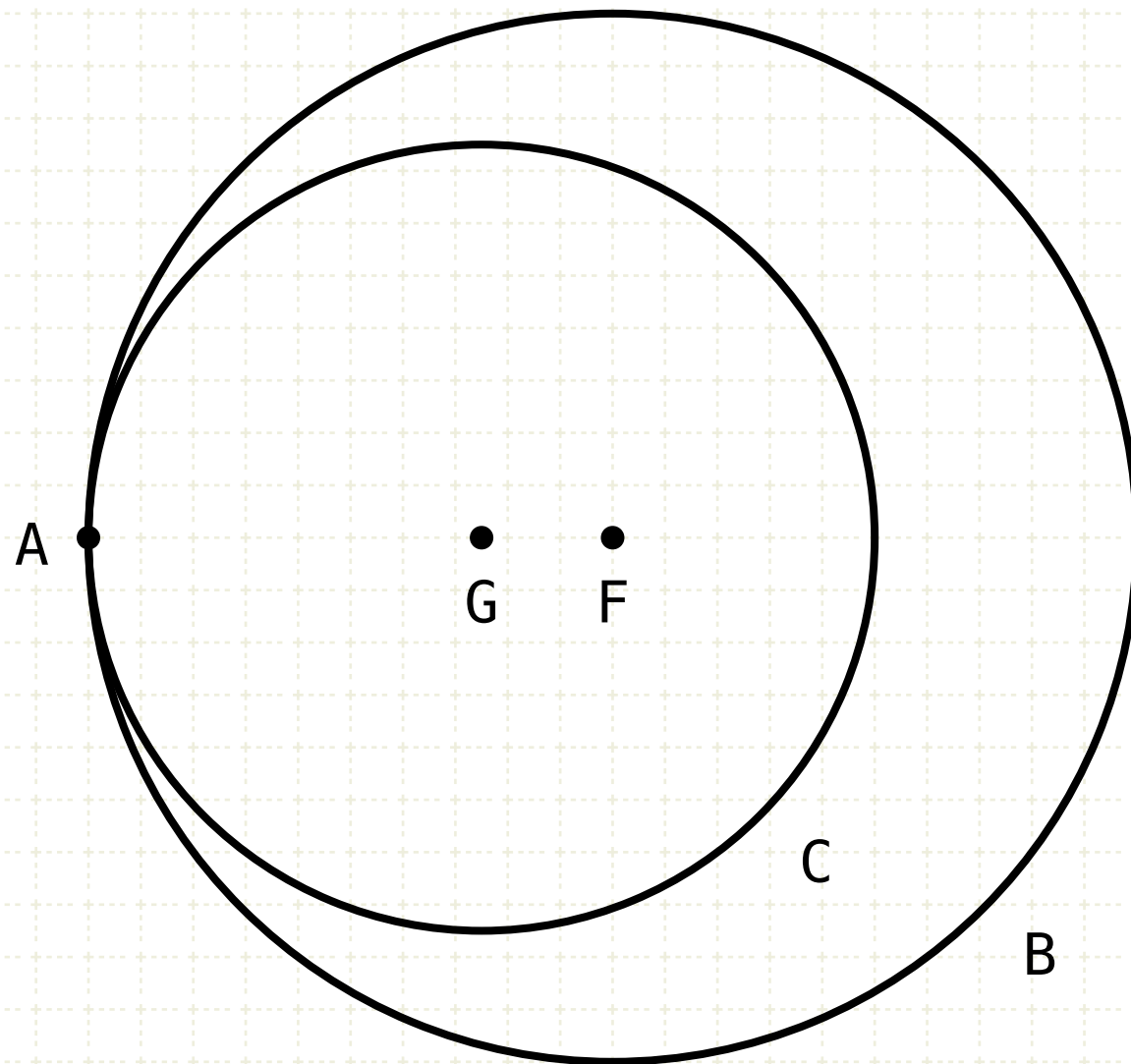

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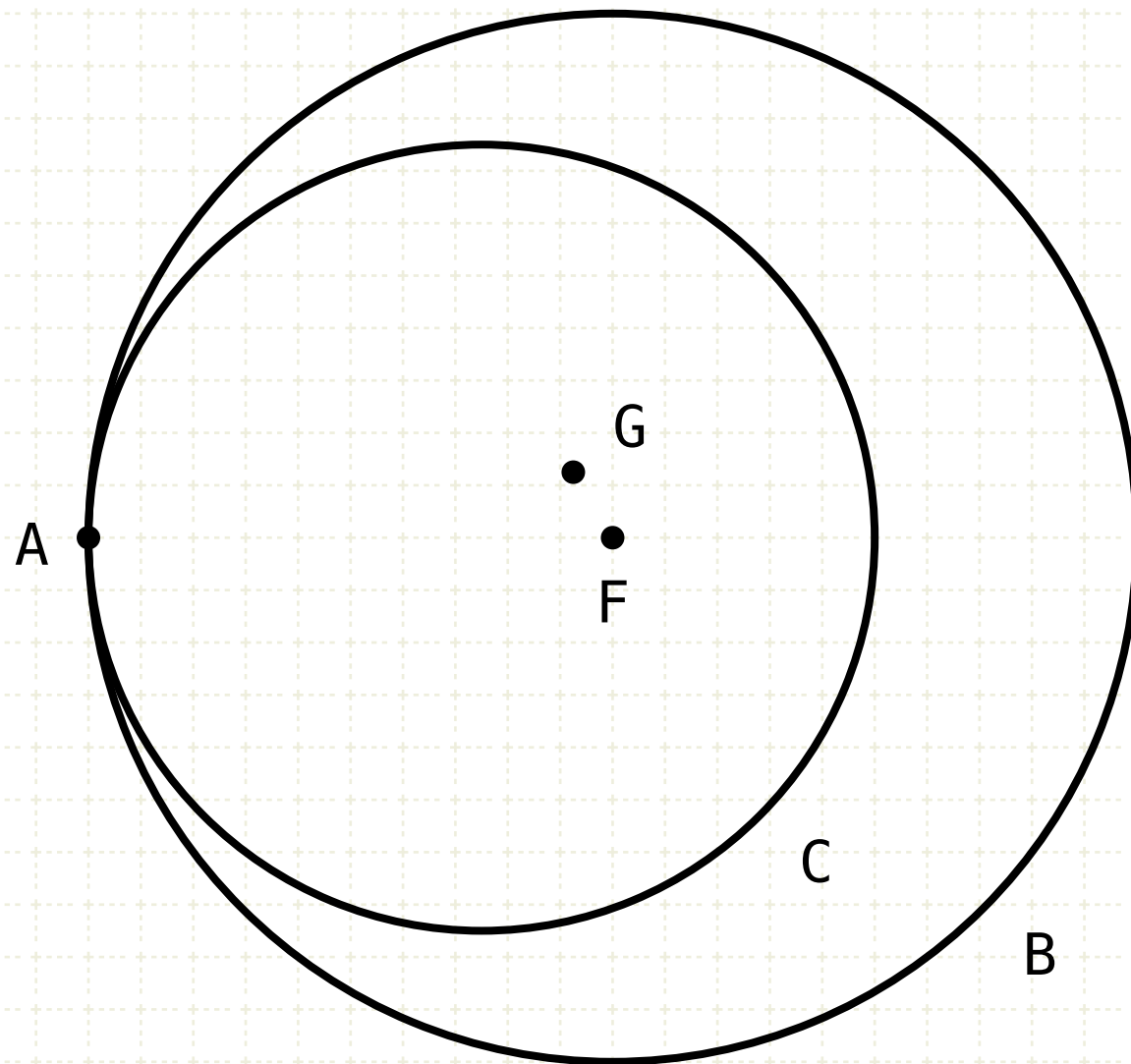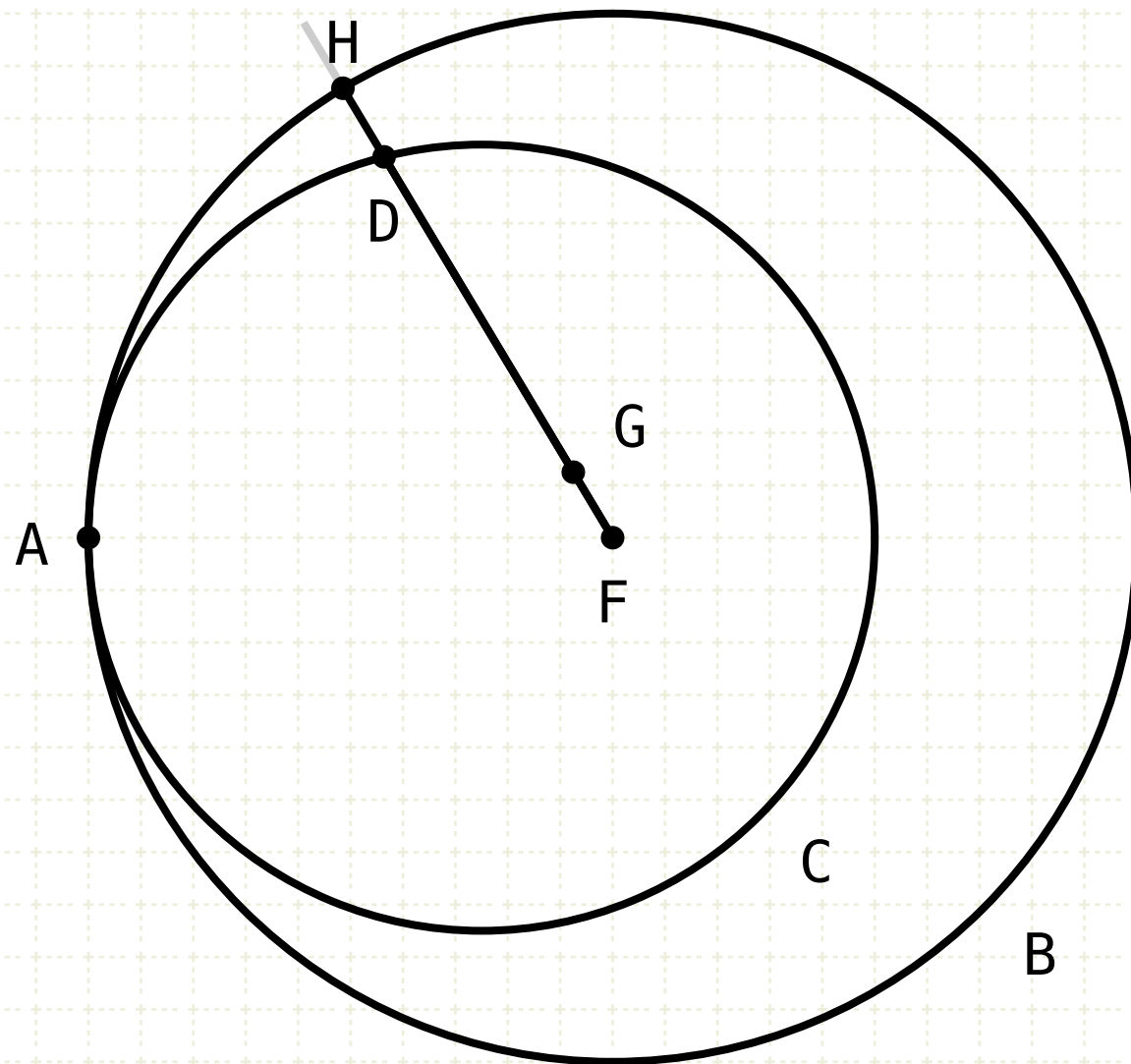Given two circles, one inside the other, touching at point A, where F and G are the centres of the circles…

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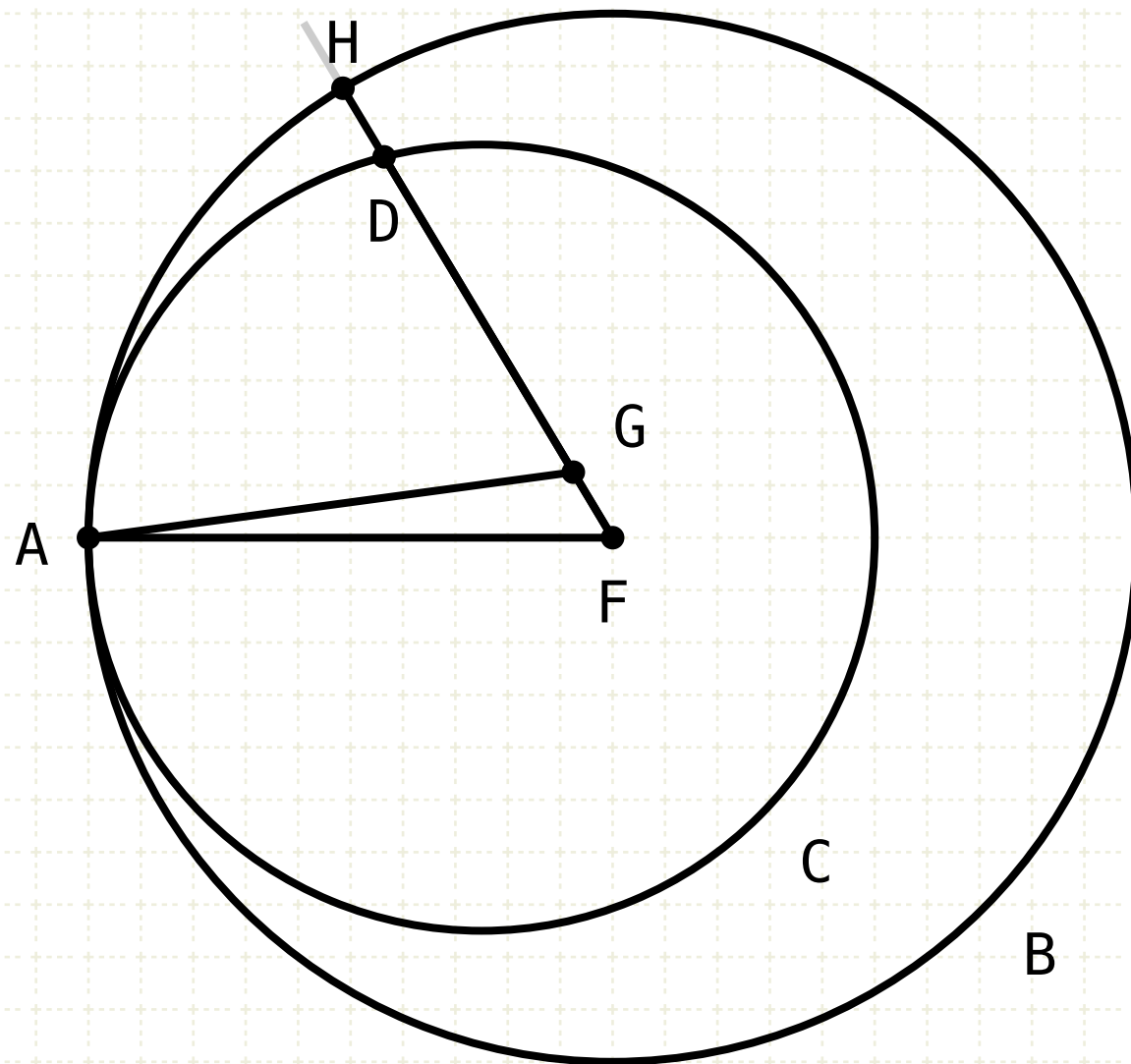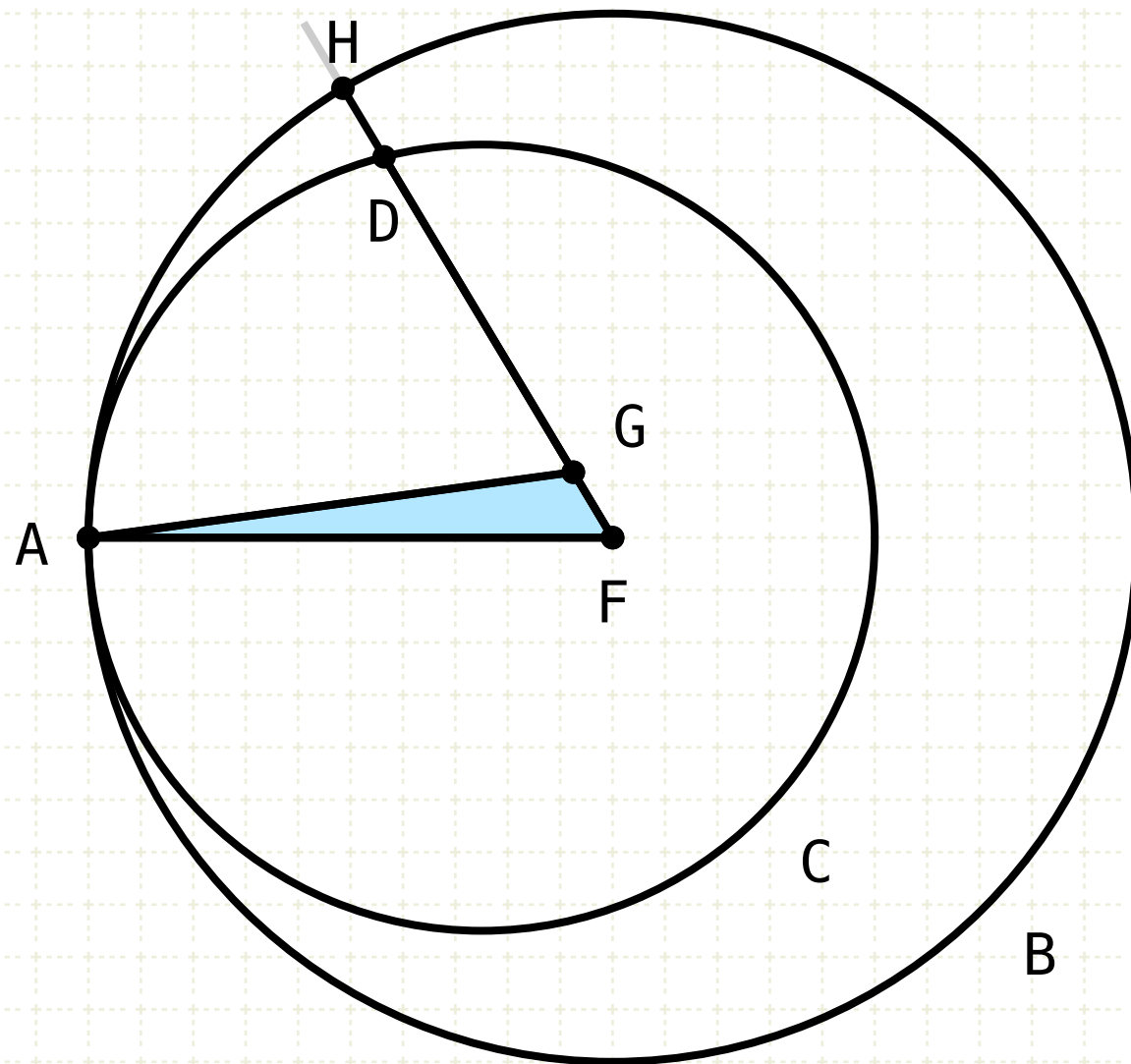
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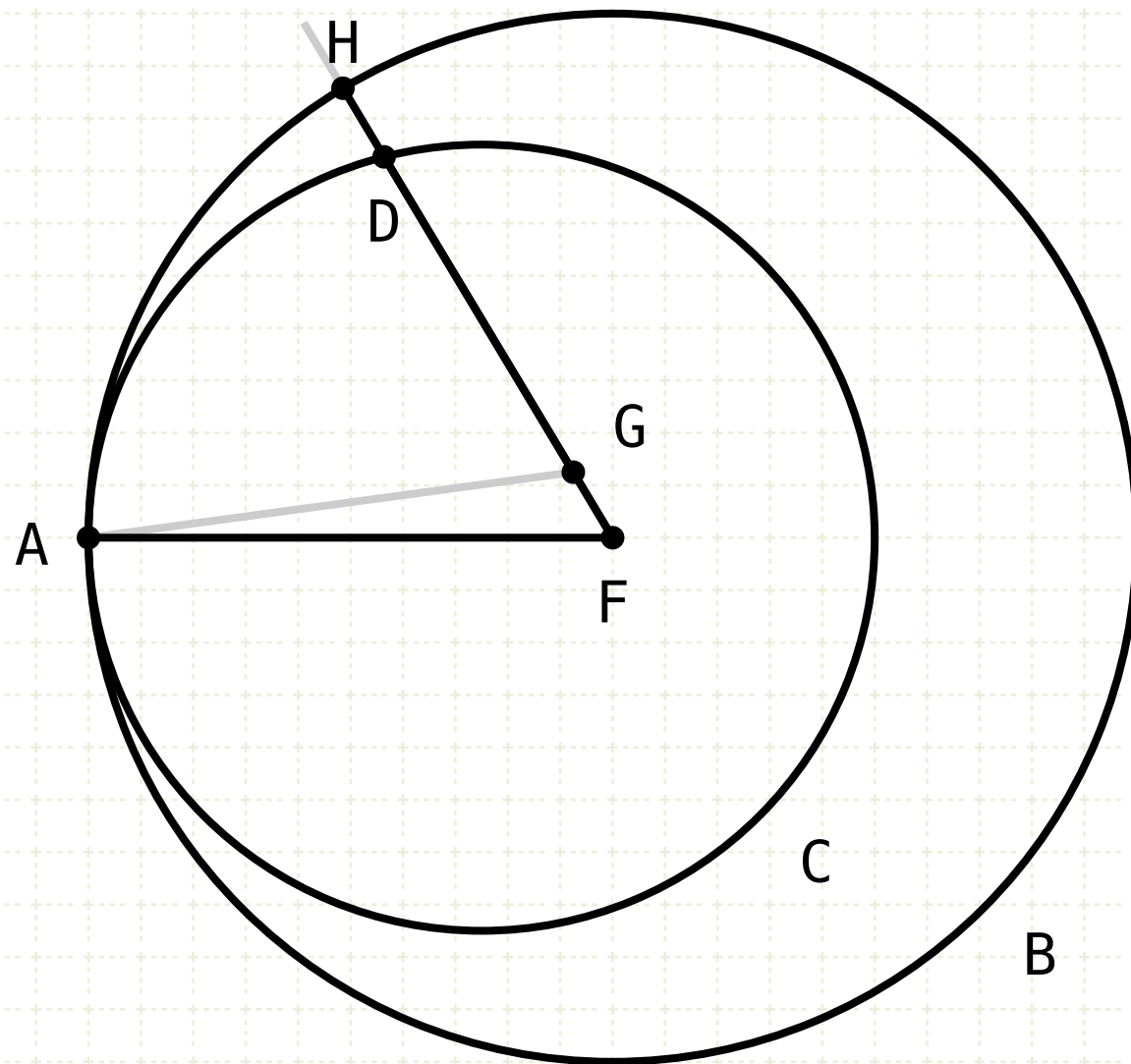
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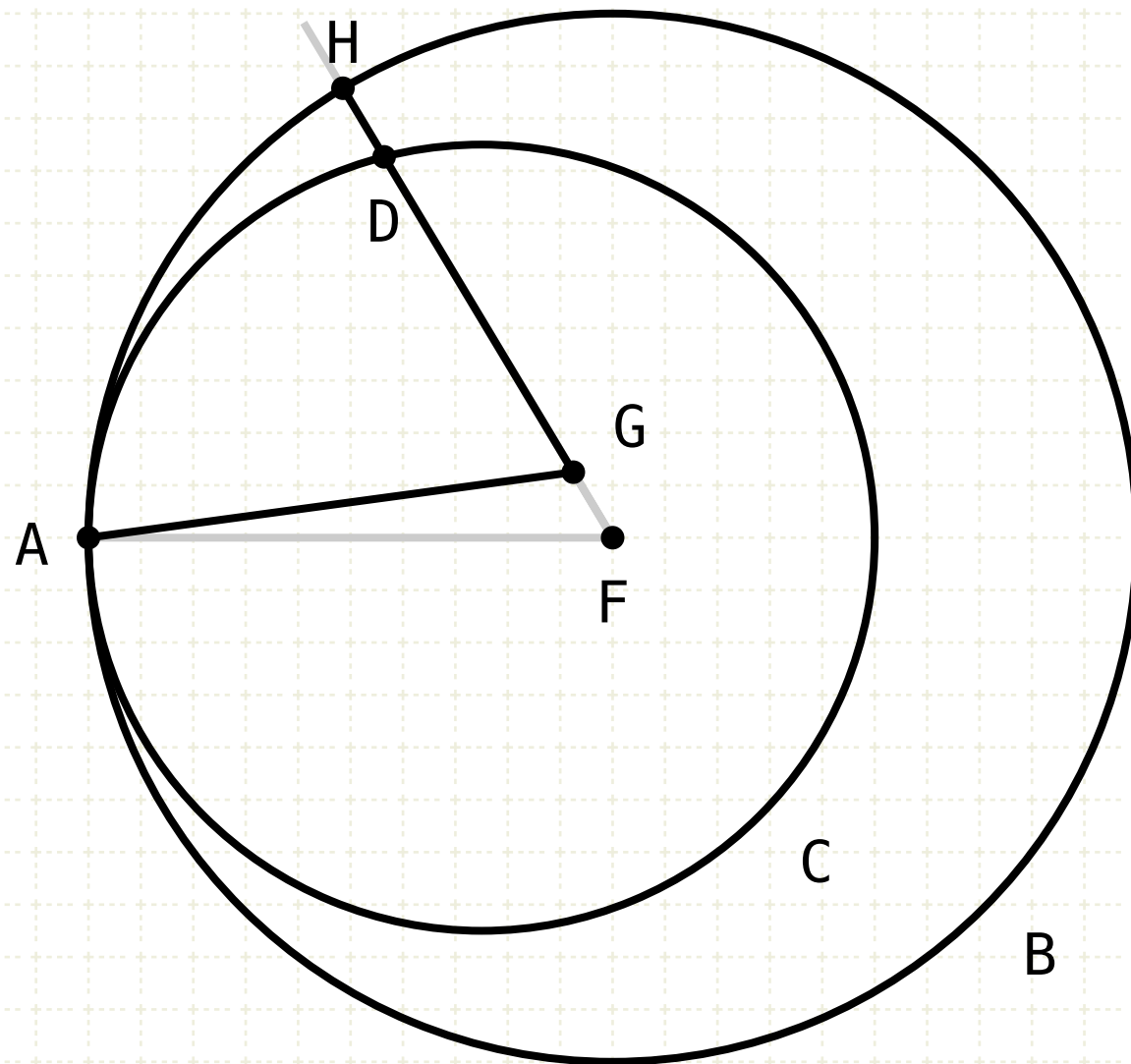Let the line GF be extended such that it intersects the circles at D and H

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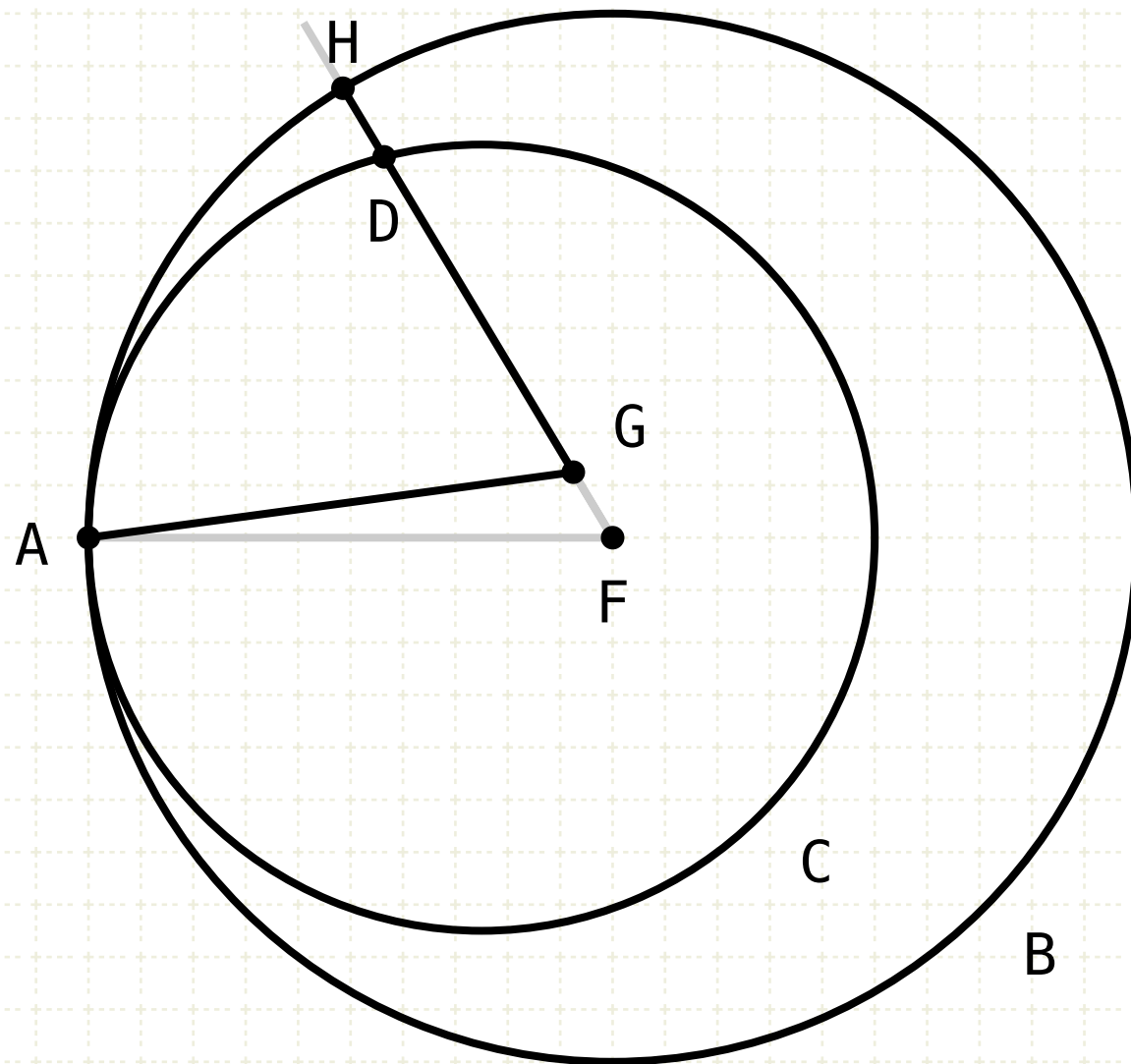The sum of two sides of a triangle is larger than the third (I·20)

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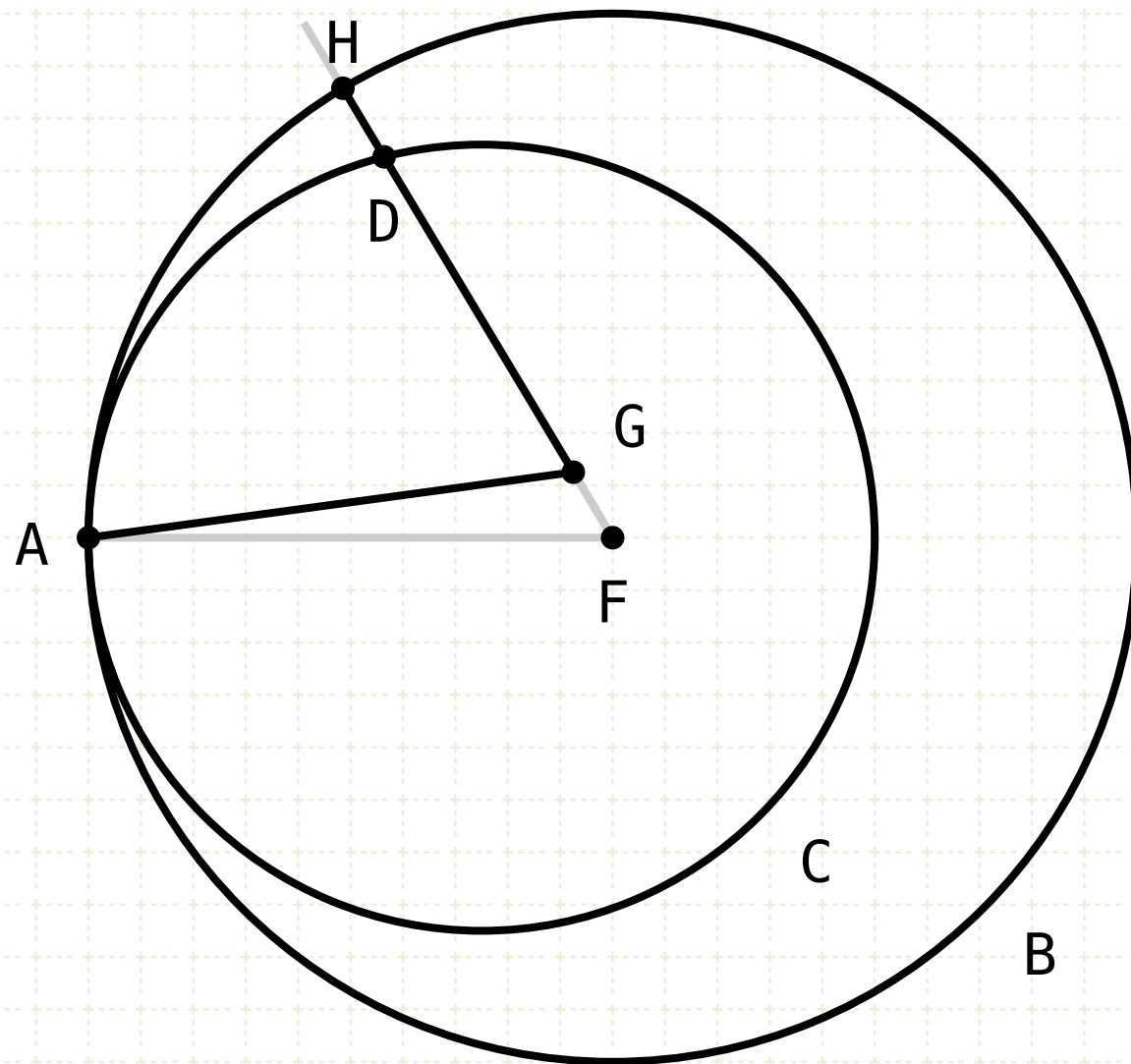The sum of two sides of a triangle is larger than the third (I·20)

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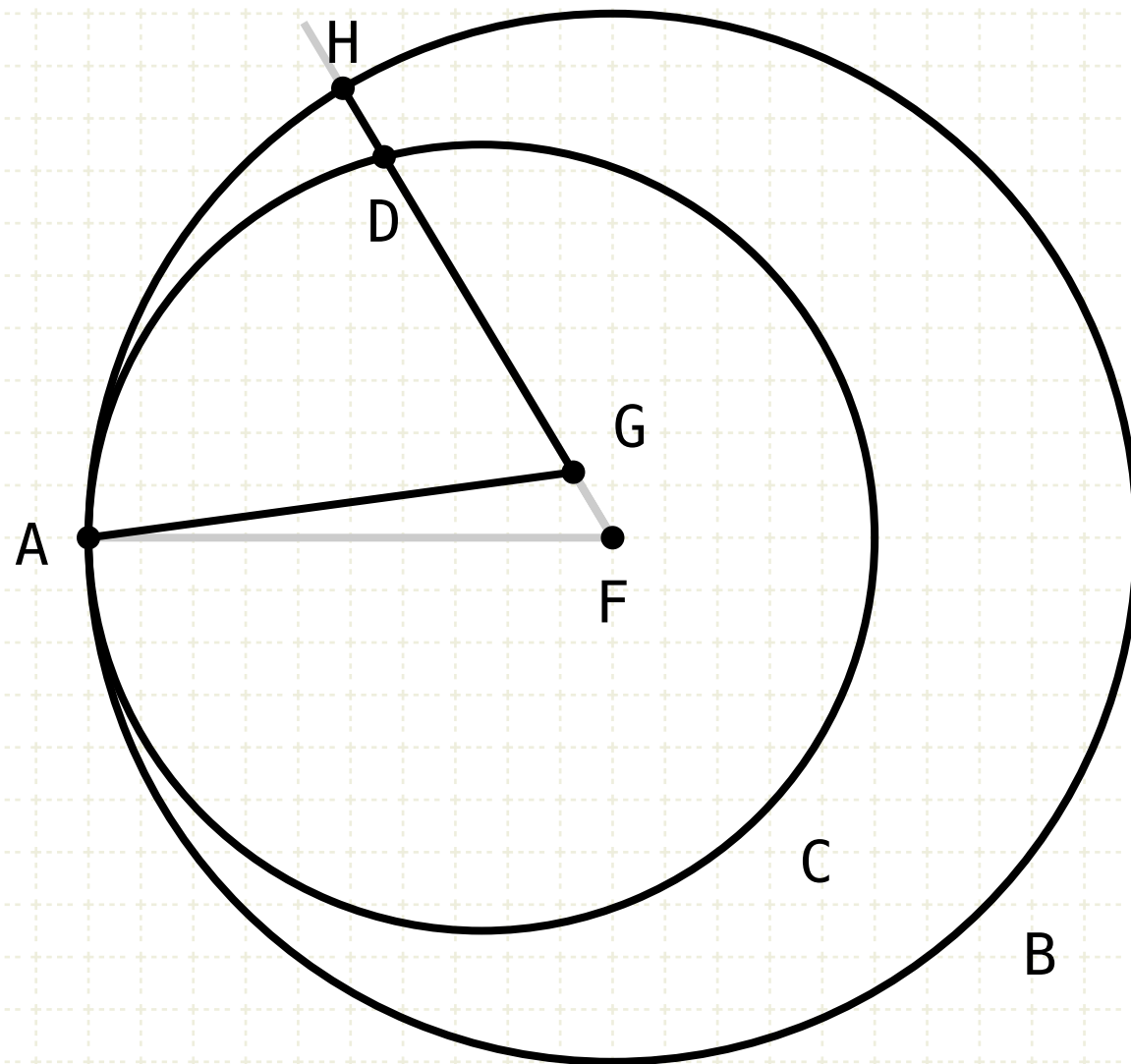Subtract GF from both sides, we have AG is greater than GH

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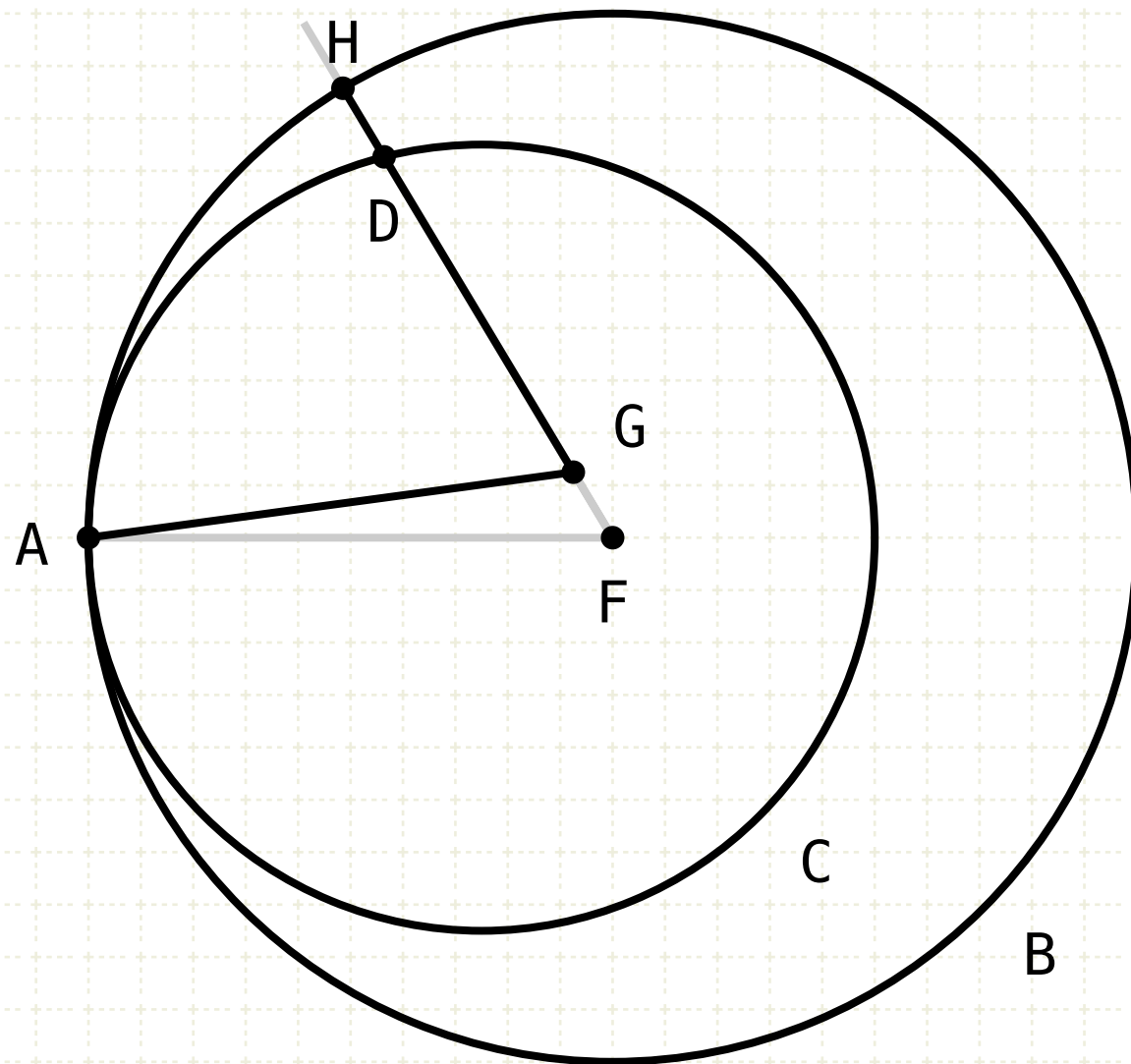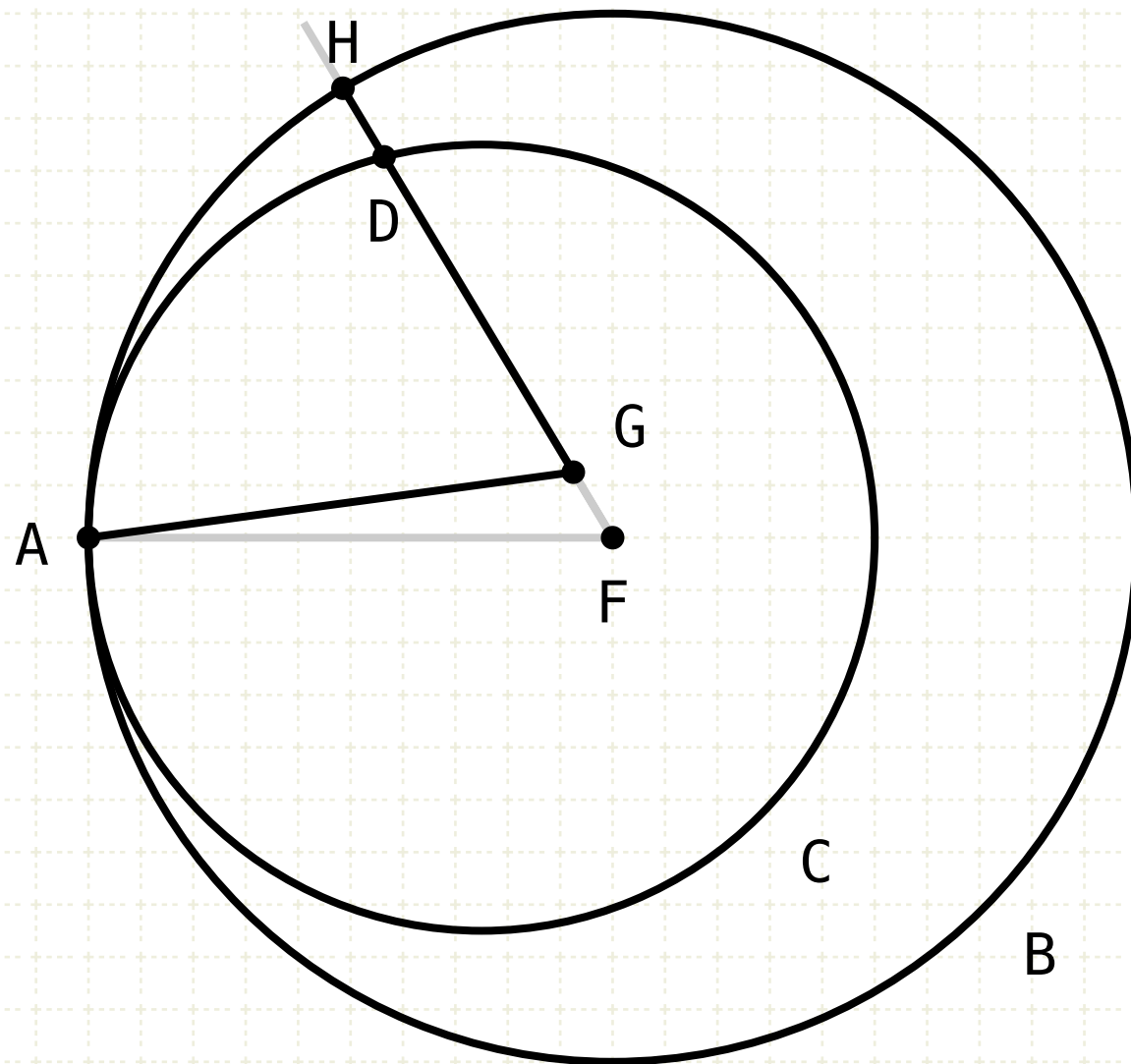AG is equal to GD since they are radii of AC, thus GD is greater than GH

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And thus the straight line from G,F will fall on the intersection between the two circles

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