

Euclid's Elements

Book I

If Euclid did not kindle your youthful enthusiasm, you were not born to be a scientific thinker.

Albert Einstein



Table of Contents, Chapter 1

1	Construct an equilateral triangle	15	Vertical angles equal one another	29	Lines parallel, alternate angles are equal
2	Copy a line	16	Exterior angle larger than interior angle	30	Lines parallel to same line are parallel to themselves
3	Subtract one line from another	17	Sum of two interior angles less than 180	31	Construct one line parallel to another
4	Equal triangles if equal side-angle-side	18	Greater side opposite of greater angle	32	Sum of interior angles of a triangle = 180
5	Isosceles triangle gives equal base angles	19	Greater angle opposite of greater side	33	Lines joining ends of equal parallels are parallel
6	Equal base angles gives isosceles triangle	20	Sum of two angles greater than third	34	Opposite sides-angles equal in parallelogram
7	Two sides of triangle meet at unique point	21	Triangle within triangle has smaller sides	35	Parallelograms, same base-height have equal area
8	Equal triangles if equal side-side-side	22	Construct triangle from given lines	36	Parallelograms, equal base-height have equal area
9	How to bisect an angle	23	Copy an angle	37	Triangles, same base-height have equal area
10	Bisect a line	24	Larger angle gives larger base	38	Triangles, equal base-height have equal area
11	Construct right angle, point on line	25	Larger base gives larger angle		
12	Construct perpendicular, point to line	26	Equal triangles if equal angle-side-angle		
13	Sum of angles on straight line = 180	27	Alternate angles equal then lines parallel		
14	Two lines form a single line if angle = 180	28	Sum of interior angles = 180 , lines parallel		



Table of Contents, Chapter 1

- 39 Equal triangles on same base, have equal height
- 40 Equal triangles on equal base, have equal height
- 41 Triangle is half parallelogram with same base and height
- 42 Construct parallelogram with equal area as triangle
- 43 Parallelogram complements are equal
- 44 Construct parallelogram on line, equal to triangle
- 45 Construct parallelogram equal to polygon
- 46 Construct a square
- 47 Pythagoras' theorem
- 48 Inverse Pythagoras' theorem



Proposition 33 of Book I

Straight lines which join the ends of equal and parallel straight lines in the same directions are themselves equal and parallel.



Proposition 33 of Book I

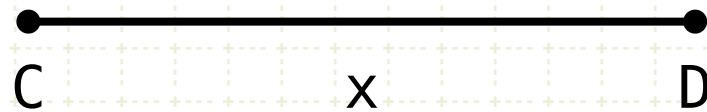
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$$\begin{array}{l} AB = CD \\ AB \parallel CD \end{array}$$

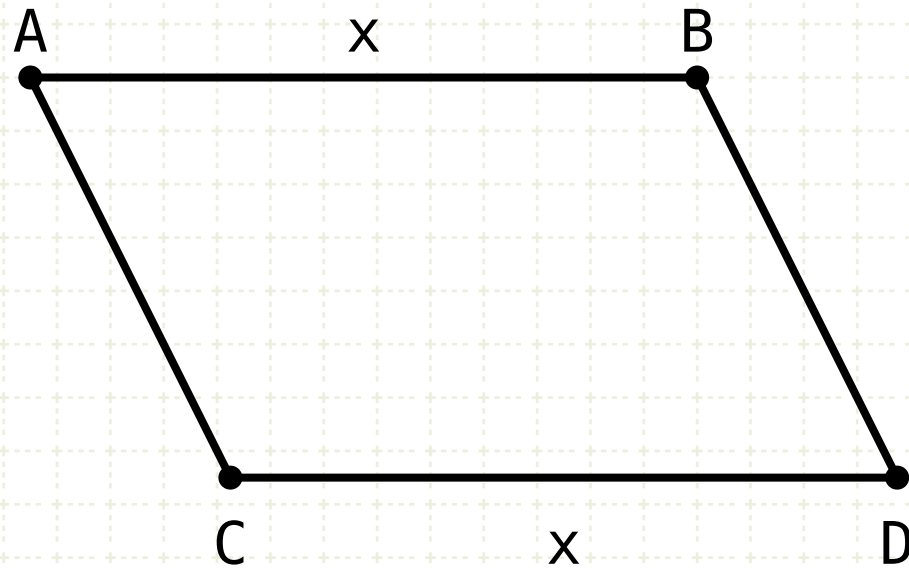
In other words

Let lines AB and CD be parallel and equal



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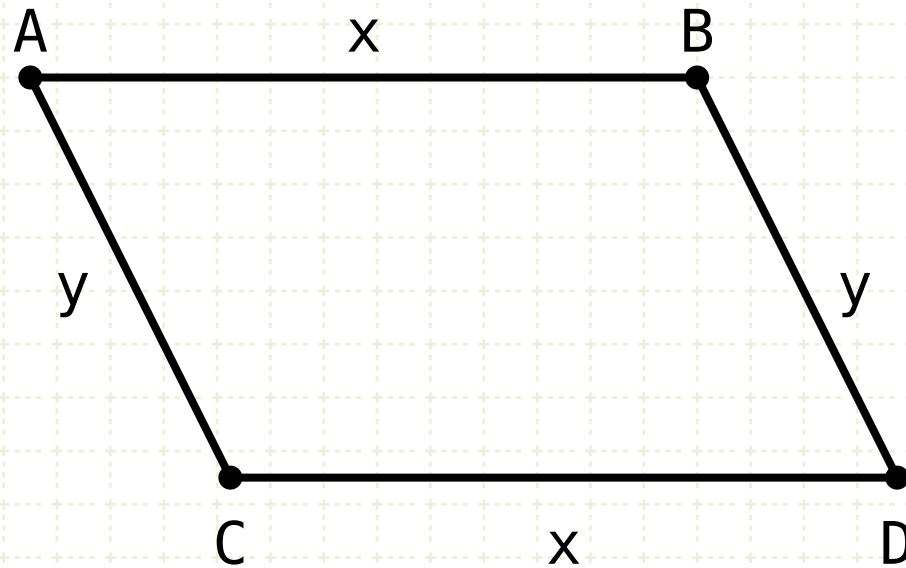
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Let lines AB and CD be parallel and equal
Create lines AC and BD



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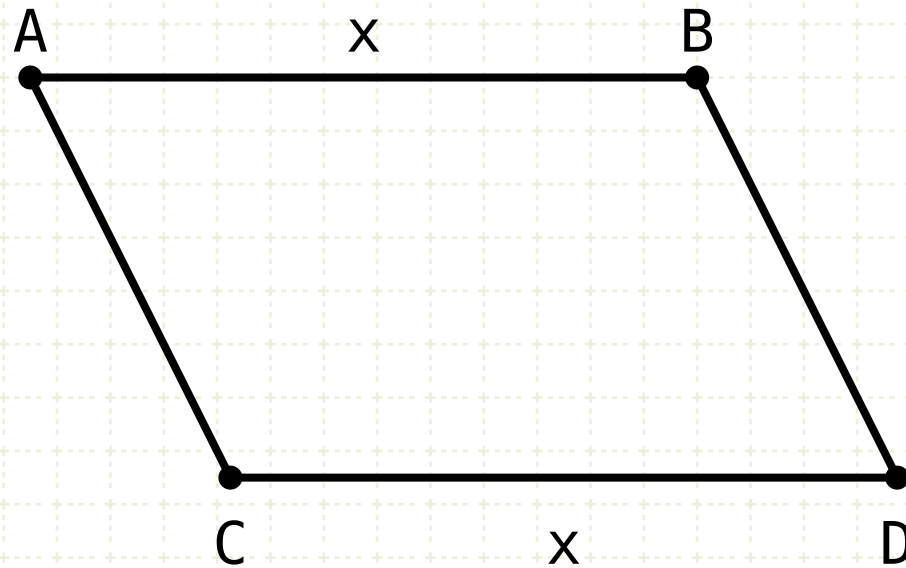
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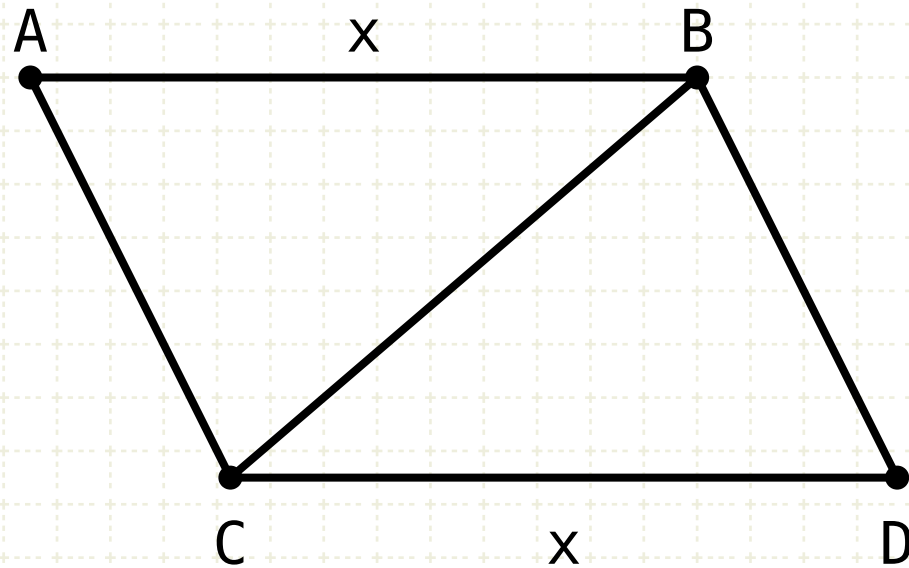
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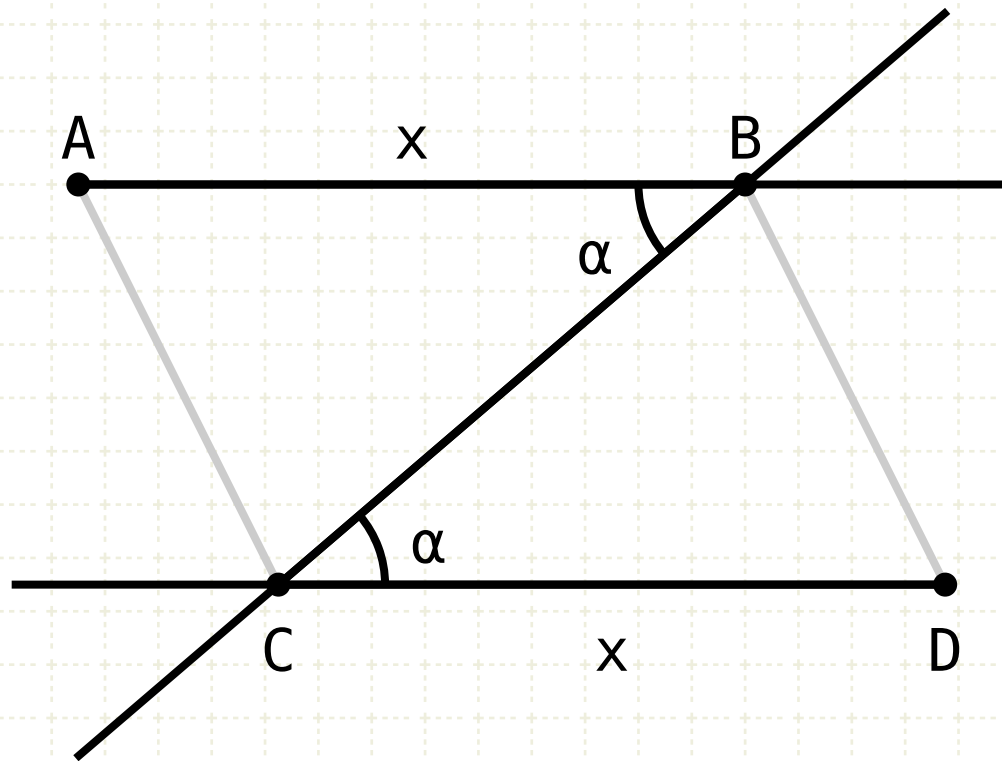
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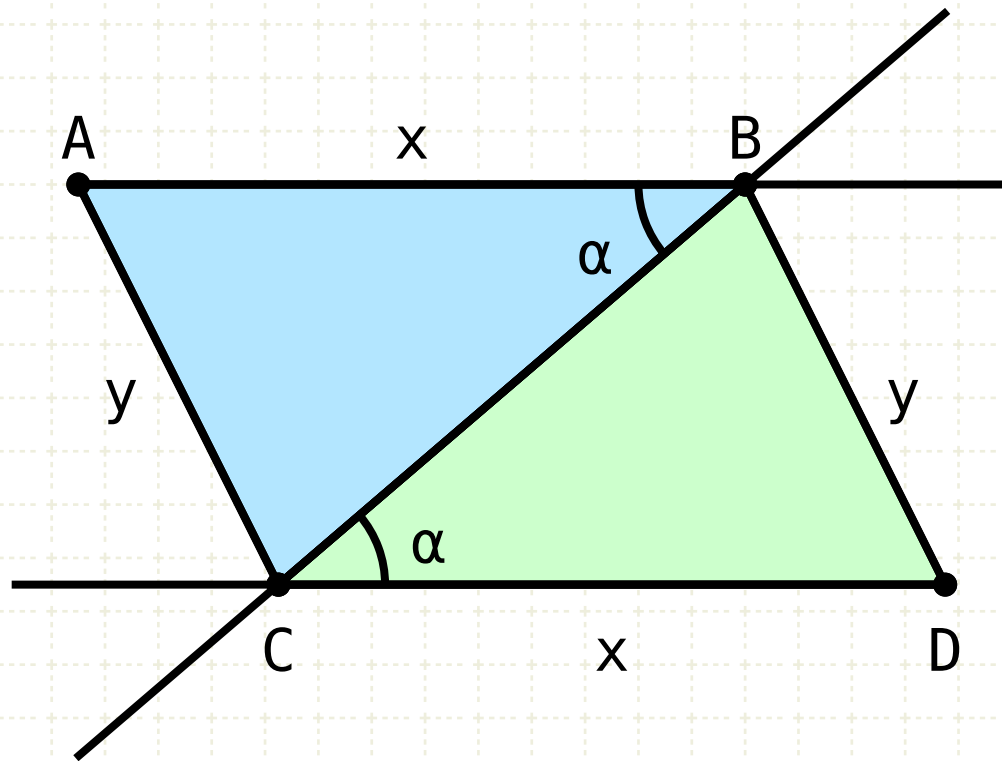
Join points BC

Since line BC intersects two parallel lines (AB and CD), angles ABC and BCD are equal (I·29)



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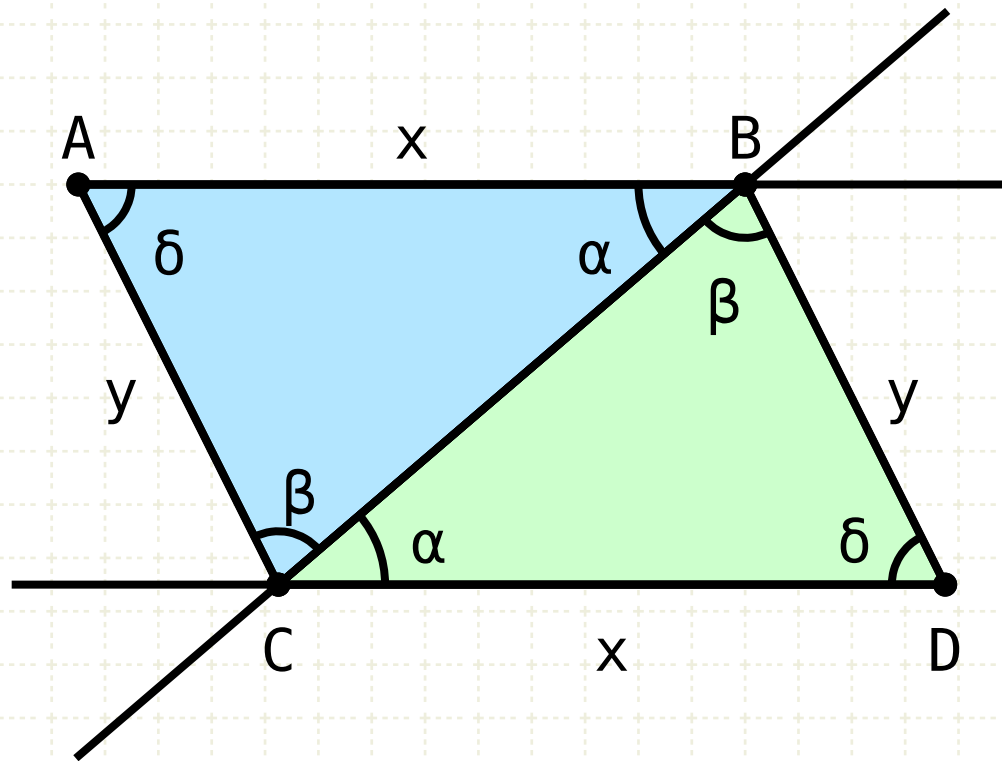
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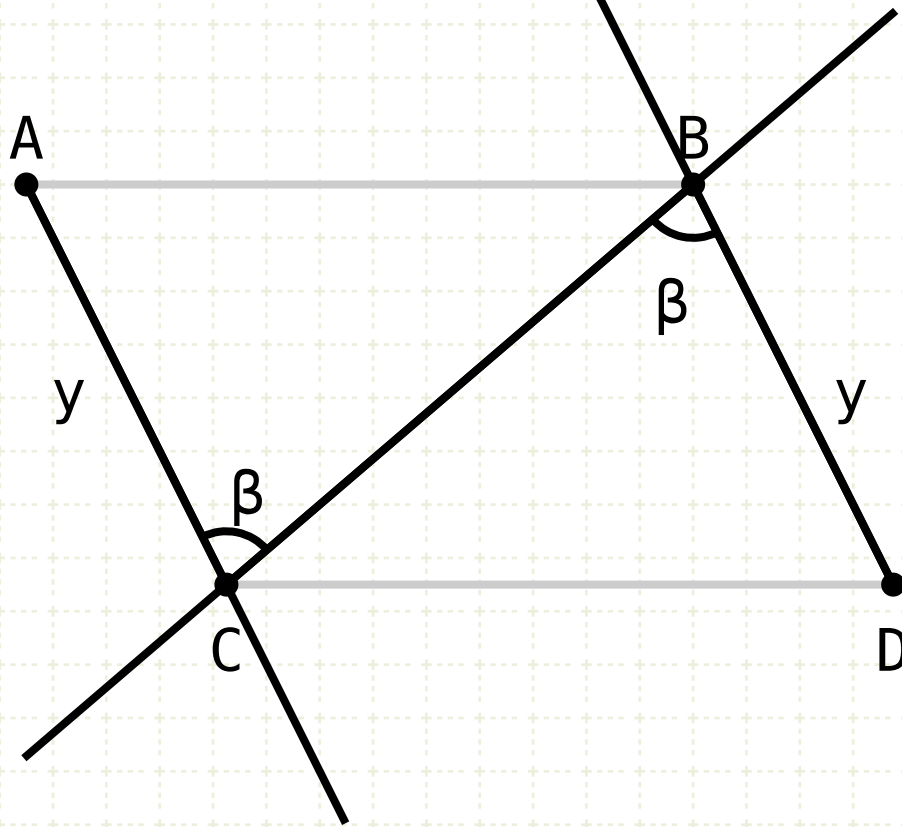
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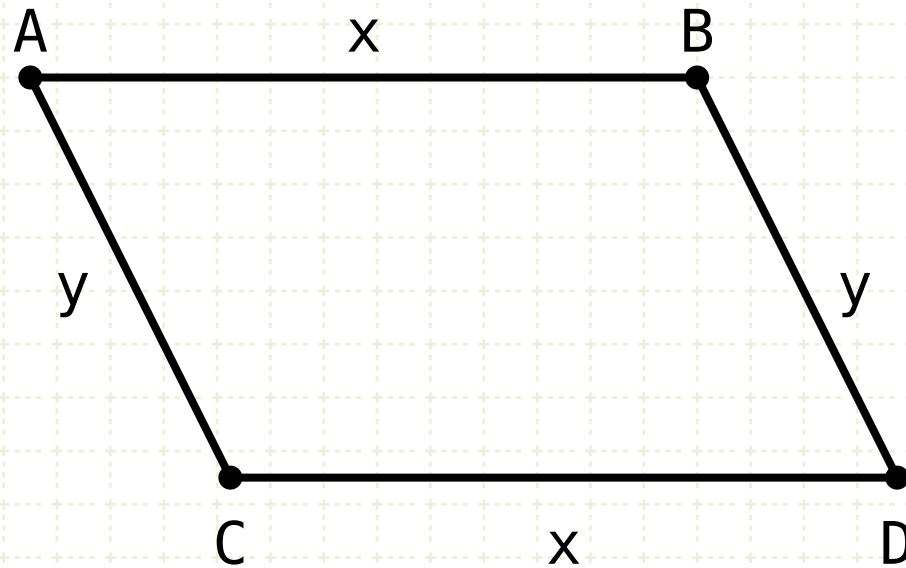
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