Euclid's Elements

Book VII

Definitions:

1 A unit is that by virtue of which each of the things that exist is called one

2 A number is a multitude composed of units. (not one)

3 A number is part of a number, the less of the greater, when it measures the greater

11 A prime number is that which is measured by a unit alone.

12 Numbers prime to one another are those which are measured by a unit alone as a common measure

15 A number is said to multiply a number when that which is multiplied is added to itself as many times as there are units in the other, and thus some number is produced.

20 Numbers are proportional when the first is the same multiple, or the same part, or the same parts, of the second that the third is of the fourth.

As long as algebra and geometry have been separated, their progress have been slow and their uses limited; but when these two sciences have been united, they have lent each mutual forces, and have marched together towards perfection.

Joseph-Louis Lagrange
(1736 to 1813)

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Proposition 29 of Book VII

Any prime number is prime to any number which it does not measure.

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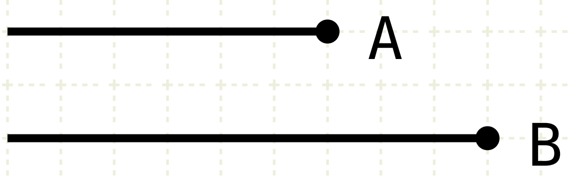


A is prime

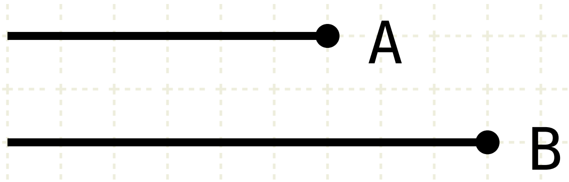
$B \neq q \cdot A,\ q>1$

In other words

Let A be prime, and not measure B

Proposition 29 of Book VII

Any prime number is prime to any number which it does not measure.

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A is prime
B ≠ q·A, q>1
gcd(A,B) = 1
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In other words

Let A be prime, and not measure B

then A and B are prime to each other

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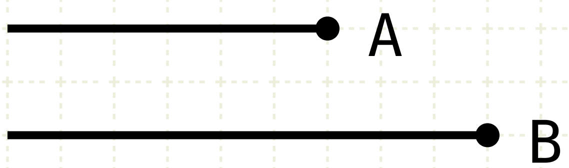

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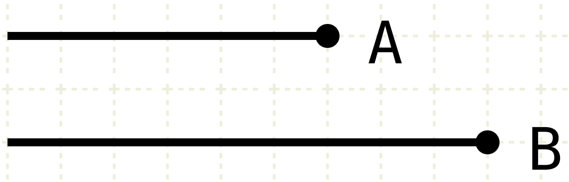

A is prime

$B \neq q \cdot A,\ q>1$

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Proof by Contradiction

Assume B and A are not prime to one another

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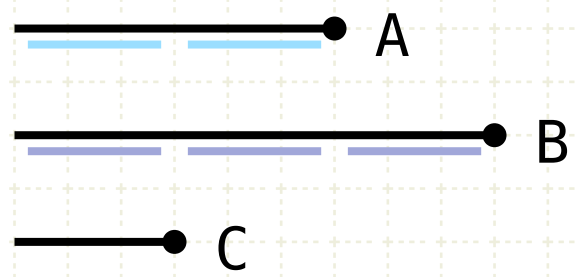

A is prime

$B \neq q \cdot A,\ q>1$

$\gcd(A,B) \neq 1$

$A = s \cdot C$

$B = r \cdot C$

In other words

Let A be prime, and not measure B

then A and B are prime to each other

Proof by Contradiction

Assume B and A are not prime to one another

Then some number, C, will measure them

Let that number be C

Proposition 29 of Book VII

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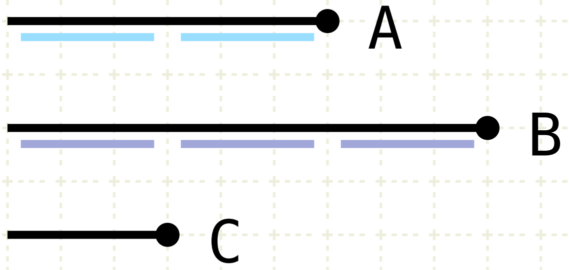

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Proof by Contradiction

Assume B and A are not prime to one another

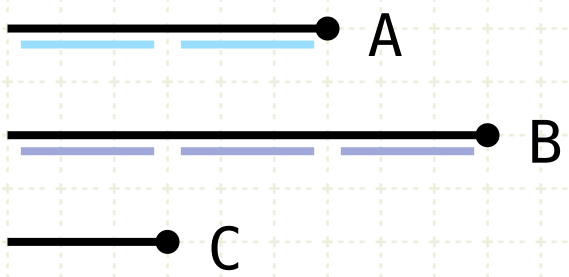
Then some number, C, will measure them

Let that number be C

Since C measures B, and A does not measure B, then A and C are not equal

Proposition 29 of Book VII

Any prime number is prime to any number which it does not measure.



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Since C measures B, and A does not measure B, then A and C are not equal

But C measures A as well as B

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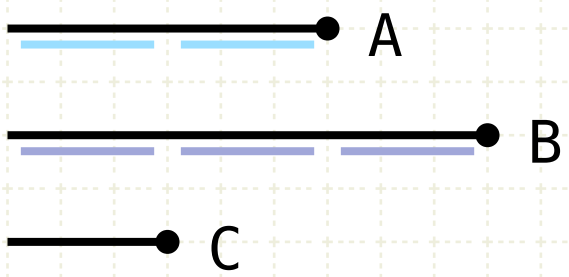

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But A is prime and it cannot be measured by C unless it is equal to C, which it is not

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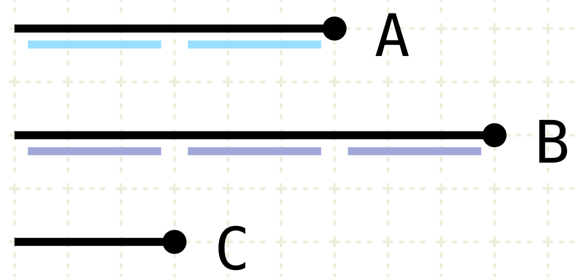

A is prime

$B \neq q \cdot A, \; q > 1$

$\gcd(A,B) \neq 1$ ✗

$A = s \cdot C$

$B = r \cdot C$

$A \neq C$

In other words

Let A be prime, and not measure B

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Proof by Contradiction

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Let that number be C

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But A is prime and it cannot be measured by C unless it is equal to C, which it is not

So the original assumption that A and B are not co-prime is invalid

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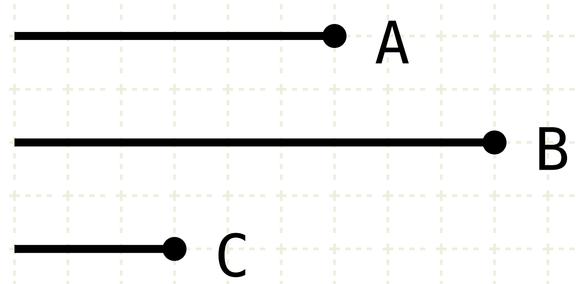

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A and B are co-prime

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