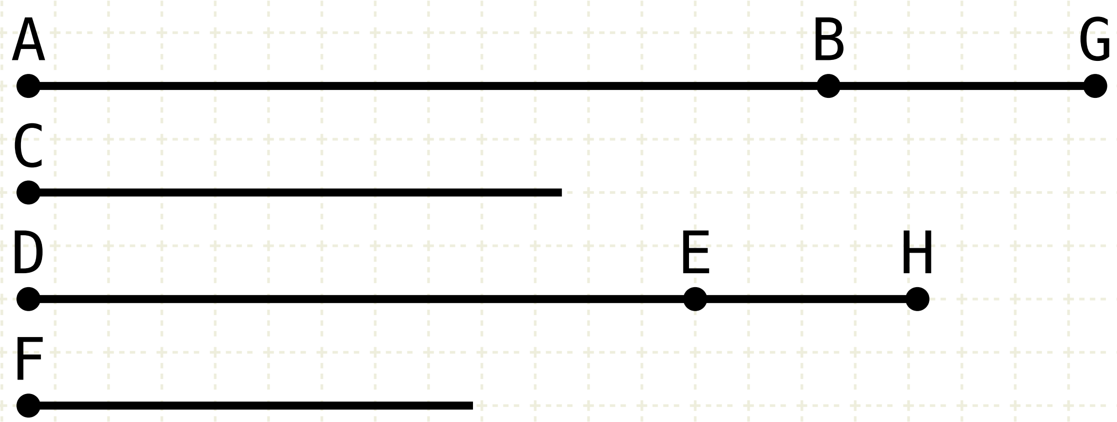
Euclid's Elements

Book V



$AB:C = DE:F$

$BG:C = EH:F$

$AG:C = DH:F$

Proportions are what makes the old Greek temples classic in their beauty. They are like huge blocks, from which the air has been literally hewn out between the columns.

Arne Jacobsen

Table of Contents, Chapter 5

Proposition 17 of Book V

If magnitudes be proportional COMPONENDO, they will also be proportional SEPARANDO

Proposition 17 of Book V

If magnitudes be proportional COMPONENDO, they will also be proportional SEPARANDO

the componendo (composition) ratio of A:B is (A+B):B

the separando (separated) ratio of (A+B):B is A:B

the convertendo (in conversion) ratio of (A+B):B is (A+B):A

Definitions

14. COMPOSITION OF A RATIO means taking the antecedent together with the consequent as one in relation to the consequent by itself
15. SEPARATION OF A RATIO means taking the excess by which the antecedent exceeds the consequent in relation to the consequent by itself
16. CONVERSION OF A RATIO means taking the antecedent in relation to the excess by which the antecedent exceeds the consequent.

Proposition 17 of Book V

If magnitudes be proportional COMPONENDO, they will also be proportional SEPARANDO

Proposition 17 of Book V

If magnitudes be proportional COMPONENDO, they will also be proportional SEPARANDO

In other words

If AB, BE, CD, DF be magnitudes proportional COMPONENDO (V·def·14)

AB:EB = CD:FD

(AE+EB):EB = (CF+FD):FD

Proposition 17 of Book V

If magnitudes be proportional COMPONENDO, they will also be proportional SEPARANDO

In other words

If AB, BE, CD, DF be magnitudes proportional COMPONENDO (V·def·14)

… then they will also be proportional SEPARANDO (V·def·15)

AB:EB = CD:FD
(AE+EB):EB = (CF+FD):FD

AE:EB = CF:FD

Proposition 17 of Book V

If magnitudes be proportional COMPONENDO, they will also be proportional SEPARANDO

In other words

If AB, BE, CD, DF be magnitudes proportional COMPONENDO (V·def·14)

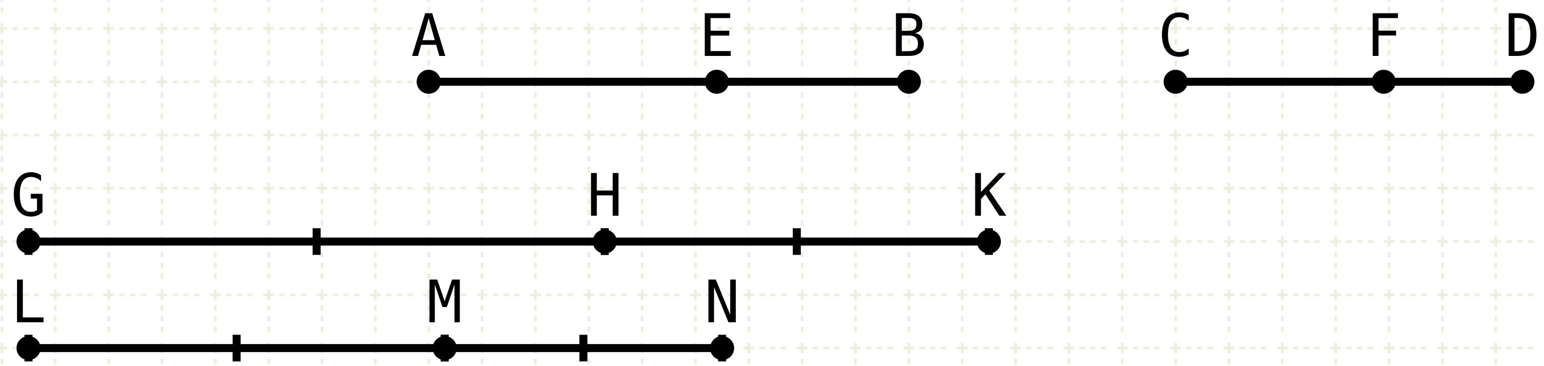
… then they will also be proportional SEPARANDO (V·def·15)

Proof

(AE+EB):EB = (CF+FD):FD

Proposition 17 of Book V

If magnitudes be proportional COMPONENDO, they will also be proportional SEPARANDO



In other words

If AB, BE, CD, DF be magnitudes proportional COMPONENDO (V·def·14)

… then they will also be proportional SEPARANDO (V·def·15)

Proof

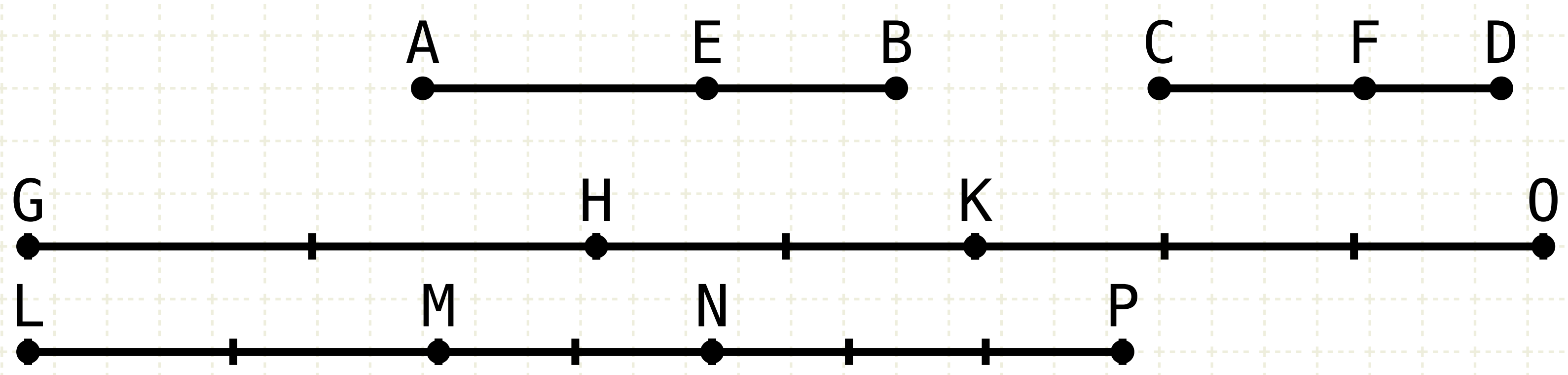
Let GH, HK, LM, MN be equimultiples of AE, EB, CF, FD

$$(AE+EB):EB = (CF+FD):FD$$

$$GH = m \cdot AE$$
$$HK = m \cdot EB$$
$$LM = m \cdot CF$$
$$MN = m \cdot FD$$

Proposition 17 of Book V

If magnitudes be proportional COMPONENDO, they will also be proportional SEPARANDO



In other words

If AB, BE, CD, DF be magnitudes proportional COMPONENDO (V·def·14)

… then they will also be proportional SEPARANDO (V·def·15)

Proof

Let GH, HK, LM, MN be equimultiples of AE, EB, CF, FD

Let KO, NP be chance equimultiples of EB, FD

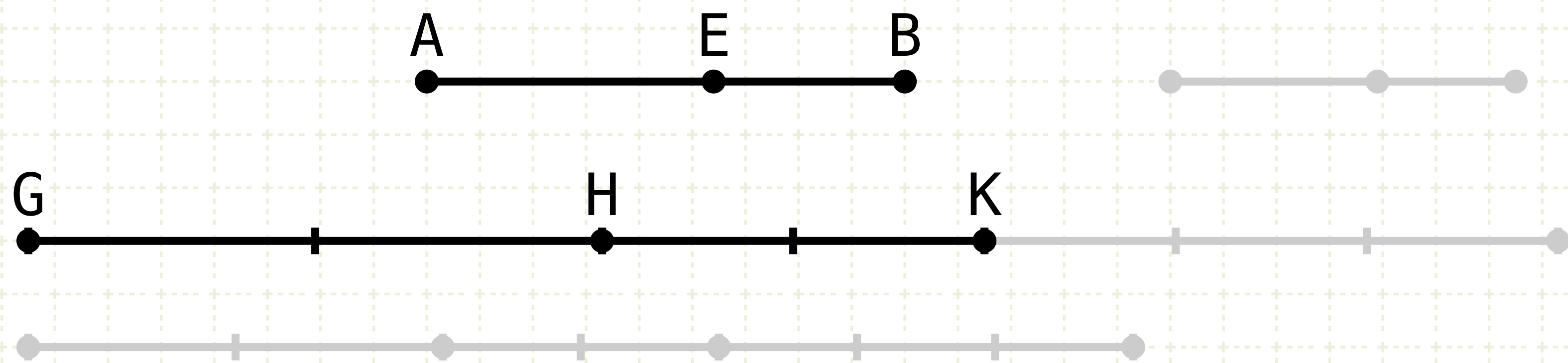
$$(AE+EB):EB = (CF+FD):FD$$

$$GH = m \cdot AE$$
$$HK = m \cdot EB$$
$$LM = m \cdot CF$$
$$MN = m \cdot FD$$

$$KO = n \cdot EB$$
$$NP = n \cdot FD$$

Proposition 17 of Book V

If magnitudes be proportional COMPONENDO, they will also be proportional SEPARANDO



$(AE+EB):EB = (CF+FD):FD$

$GH = m \cdot AE$ $\quad GK = m \cdot AB$

$HK = m \cdot EB$

$LM = m \cdot CF$

$MN = m \cdot FD$

$KO = n \cdot EB$

$NP = n \cdot FD$

In other words

If AB, BE, CD, DF be magnitudes proportional COMPONENDO (V·def·14)

… then they will also be proportional SEPARANDO (V·def·15)

Proof

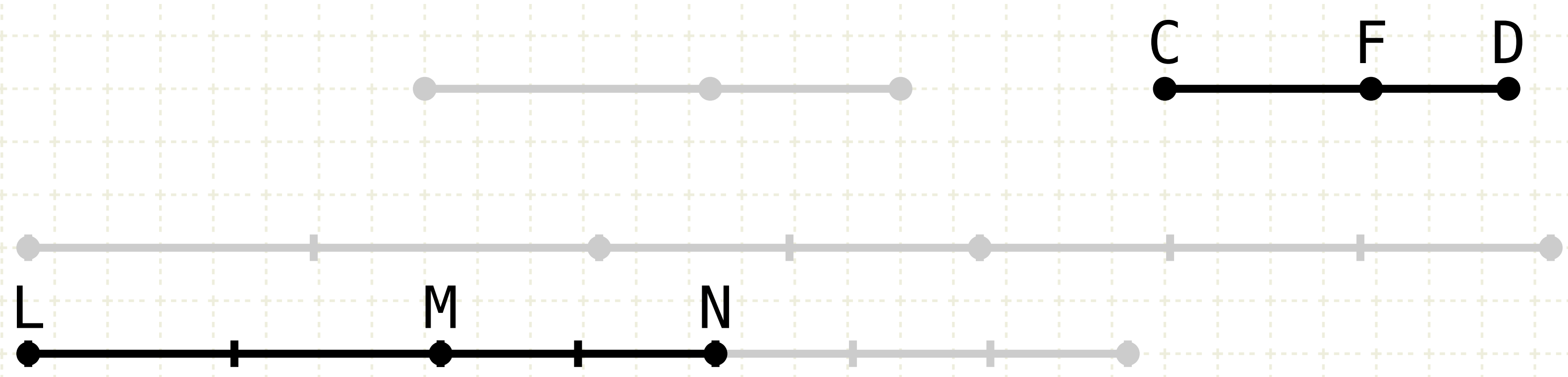
Let GH, HK, LM, MN be equimultiples of AE, EB, CF, FD

Let KO, NP be chance equimultiples of EB, FD

Since GH and HK are equimultiples of AE and EB, then the sum of GH,HK is the same equimultiple of the sum of AE,EB (V·1)

Proposition 17 of Book V

If magnitudes be proportional COMPONENDO, they will also be proportional SEPARANDO



(AE+EB):EB = (CF+FD):FD

GH = m·AE
HK = m·EB
LM = m·CF
MN = m·FD

KO = n·EB
NP = n·FD

GK = m·AB
LN = m·CD

In other words

If AB, BE, CD, DF be magnitudes proportional COMPONENDO (V·def·14)

… then they will also be proportional SEPARANDO (V·def·15)

Proof

Let GH, HK, LM, MN be equimultiples of AE, EB, CF, FD

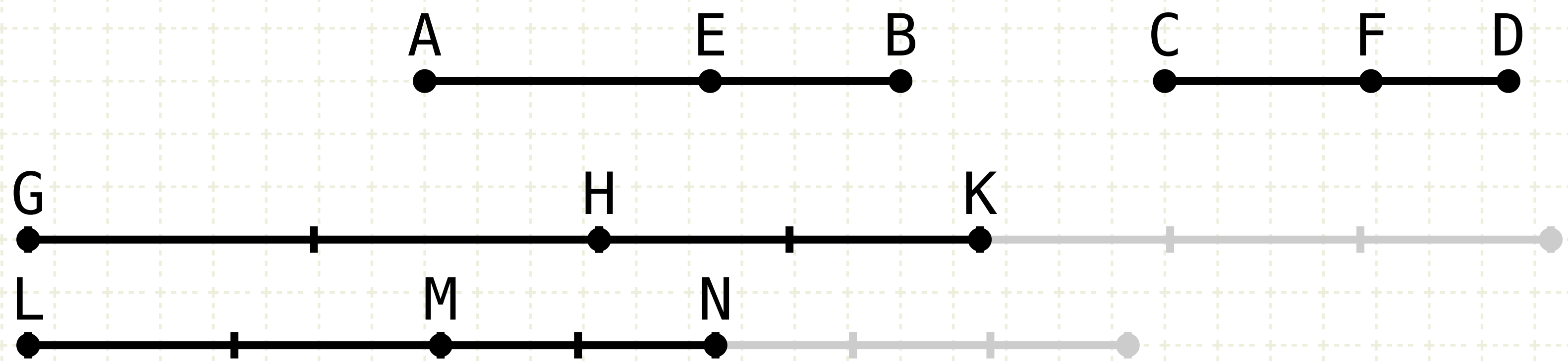
Let KO, NP be chance equimultiples of EB, FD

Since GH and HK are equimultiples of AE and EB, then the sum of GH,HK is the same equimultiple of the sum of AE,EB (V·1)

Also, since LM,MN are equimultiples of CF,FD, then the sum of LM,MN is the same equimultiple of the sum of CF,FD (V·1)

Proposition 17 of Book V

If magnitudes be proportional COMPONENDO, they will also be proportional SEPARANDO



$(AE+EB):EB = (CF+FD):FD$

$GH = m \cdot AE$
$HK = m \cdot EB$
$LM = m \cdot CF$
$MN = m \cdot FD$

$KO = n \cdot EB$
$NP = n \cdot FD$

$GK = m \cdot AB$
$LN = m \cdot CD$

In other words

If AB, BE, CD, DF be magnitudes proportional COMPONENDO (V·def·14)

… then they will also be proportional SEPARANDO (V·def·15)

Proof

Let GH, HK, LM, MN be equimultiples of AE, EB, CF, FD

Let KO, NP be chance equimultiples of EB, FD

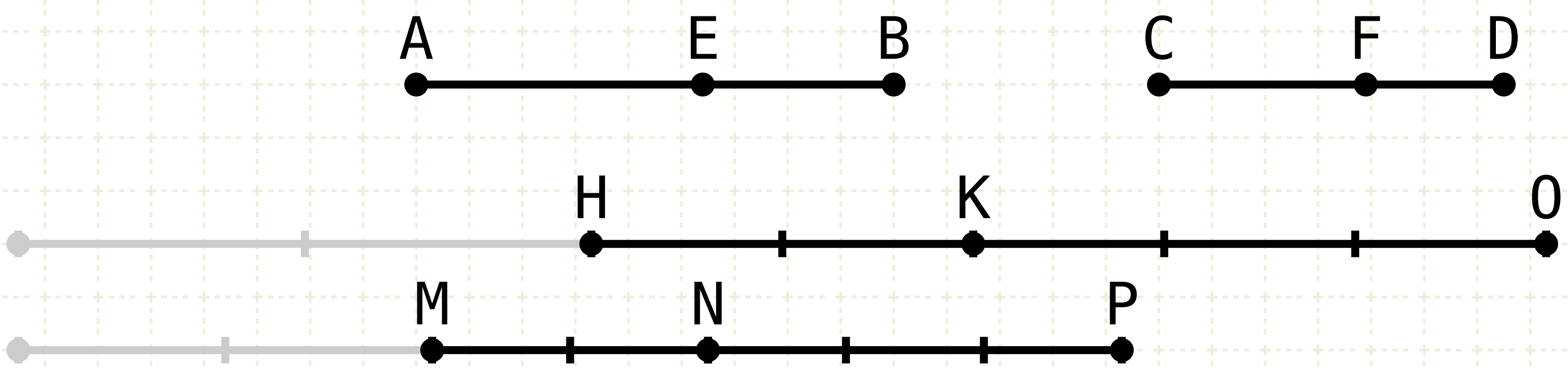
Since GH and HK are equimultiples of AE and EB, then the sum of GH,HK is the same equimultiple of the sum of AE,EB (V·1)

Also, since LM,MN are equimultiples of CF,FD, then the sum of LM,MN is the same equimultiple of the sum of CF,FD (V·1)

Because everything is equimultiple to everything else, GK and LN are equimultiples of AB and CD

Proposition 17 of Book V

If magnitudes be proportional COMPONENDO, they will also be proportional SEPARANDO



$(AE+EB):EB = (CF+FD):FD$

$GH = m \cdot AE$

$HK = m \cdot EB$

$LM = m \cdot CF$

$MN = m \cdot FD$

$KO = n \cdot EB$

$NP = n \cdot FD$

$GK = m \cdot AB$

$LN = m \cdot CD$

$HO = (m+n) \cdot EB = k \cdot EB$

$MP = (m+n) \cdot FD = k \cdot FD$

In other words

If AB, BE, CD, DF be magnitudes proportional COMPONENDO (V·def·14)

… then they will also be proportional SEPARANDO (V·def·15)

Proof

Let GH, HK, LM, MN be equimultiples of AE, EB, CF, FD

Let KO, NP be chance equimultiples of EB, FD

Since GH and HK are equimultiples of AE and EB, then the sum of GH,HK is the same equimultiple of the sum of AE,EB (V·1)

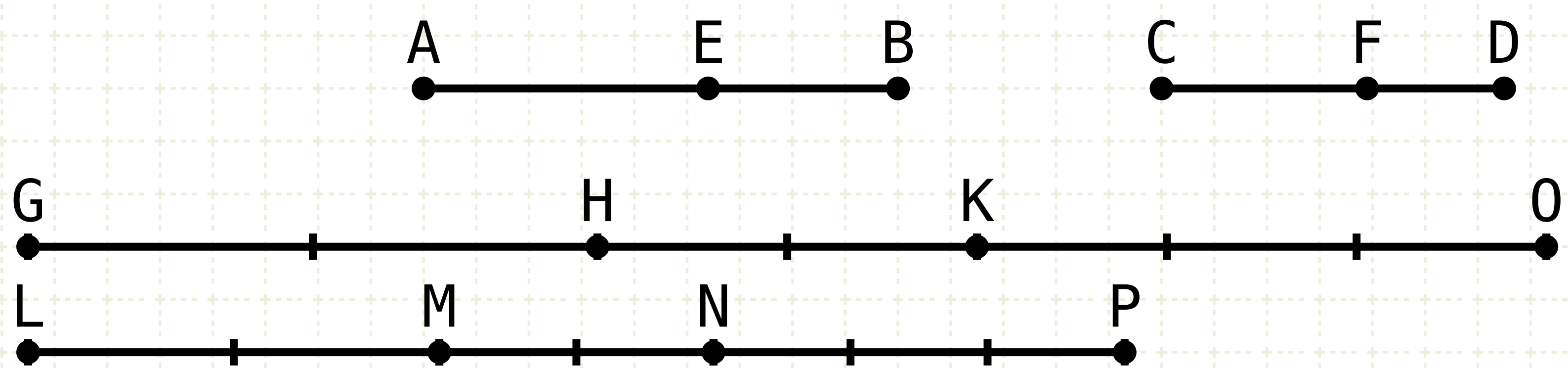
Also, since LM,MN are equimultiples of CF,FD, then the sum of LM,MN is the same equimultiple of the sum of CF,FD (V·1)

Because everything is equimultiple to everything else, GK and LN are equimultiples of AB and CD

HK,MN are equimultiples of EB,FD and KO,NP are also equimultiples of EB,FD, then HO,MP are also equimultiples of EB,FD (V·2)

Proposition 17 of Book V

If magnitudes be proportional COMPONENDO, they will also be proportional SEPARANDO



(AE+EB):EB = (CF+FD):FD

GH = m·AE
HK = m·EB
LM = m·CF
MN = m·FD

KO = n·EB
NP = n·FD

GK = m·AB
LN = m·CD
HO = (m+n)·EB = k·EB
MP = (m+n)·FD = k·FD

m·AB >=< k·EB → m·CD >=< k·FD
GK >=< HO → LN >=< MP

In other words

If AB, BE, CD, DF be magnitudes proportional COMPONENDO (V·def·14)

… then they will also be proportional SEPARANDO (V·def·15)

Proof

Let GH, HK, LM, MN be equimultiples of AE, EB, CF, FD

Let KO, NP be chance equimultiples of EB, FD

Since GH and HK are equimultiples of AE and EB, then the sum of GH,HK is the same equimultiple of the sum of AE,EB (V·1)

Also, since LM,MN are equimultiples of CF,FD, then the sum of LM,MN is the same equimultiple of the sum of CF,FD (V·1)

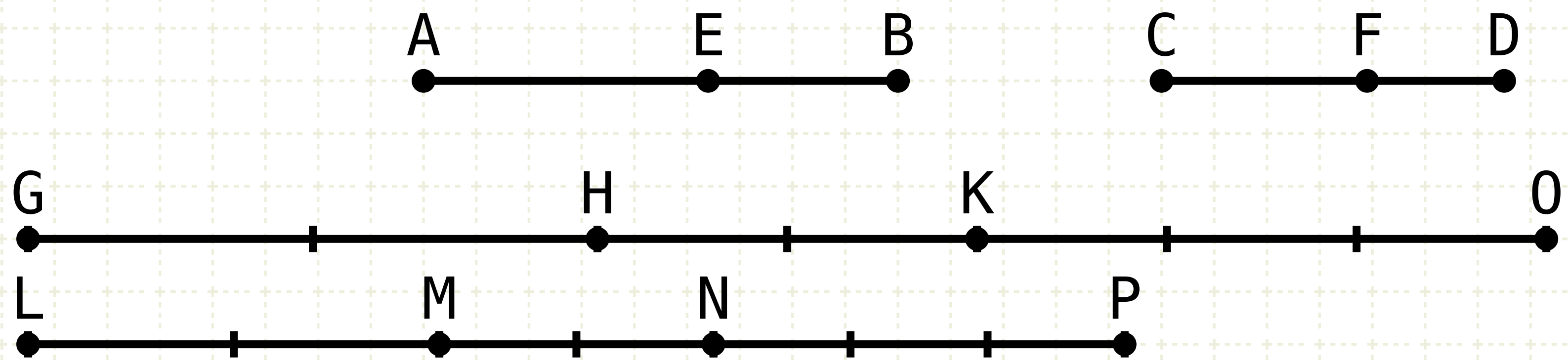
Because everything is equimultiple to everything else, GK and LN are equimultiples of AB and CD

HK,MN are equimultiples of EB,FD and KO,NP are also equimultiples of EB,FD, then HO,MP are also equimultiples of EB,FD (V·2)

Since AB is to EB as CD to FD, and chance equimultiples have been taken of AB,CD and EB,FD, then if GK is greater than HO, so is LN greater than MP, etc. (V·def·5)

Proposition 17 of Book V

If magnitudes be proportional COMPONENDO, they will also be proportional SEPARANDO



$(AE+EB):EB = (CF+FD):FD$

$GH = m \cdot AE$
$HK = m \cdot EB$
$LM = m \cdot CF$
$MN = m \cdot FD$

$KO = n \cdot EB$
$NP = n \cdot FD$

$GK = m \cdot AB$
$LN = m \cdot CD$
$HO = (m+n) \cdot EB = k \cdot EB$
$MP = (m+n) \cdot FD = k \cdot FD$

$m \cdot AB >=< k \cdot EB \rightarrow m \cdot CD >=< k \cdot FD$

$GK >=< HO \rightarrow LN >=< MP$

In other words

If AB, BE, CD, DF be magnitudes proportional COMPONENDO (V·def·14)

… then they will also be proportional SEPARANDO (V·def·15)

Proof

Let GH, HK, LM, MN be equimultiples of AE, EB, CF, FD

Let KO, NP be chance equimultiples of EB, FD

Since GH and HK are equimultiples of AE and EB, then the sum of GH,HK is the same equimultiple of the sum of AE,EB (V·1)

Also, since LM,MN are equimultiples of CF,FD, then the sum of LM,MN is the same equimultiple of the sum of CF,FD (V·1)

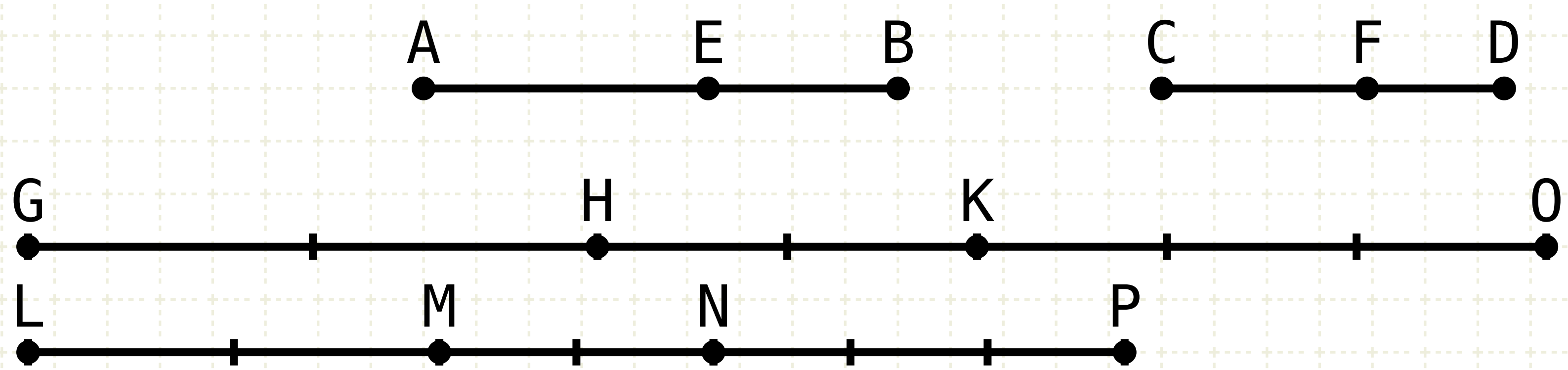
Because everything is equimultiple to everything else, GK and LN are equimultiples of AB and CD

HK,MN are equimultiples of EB,FD and KO,NP are also equimultiples of EB,FD, then HO,MP are also equimultiples of EB,FD (V·2)

Since AB is to EB as CD to FD, and chance equimultiples have been taken of AB,CD and EB,FD, then if GK is greater than HO, so is LN greater than MP, etc. (V·def·5)

Proposition 17 of Book V

If magnitudes be proportional COMPONENDO, they will also be proportional SEPARANDO



In other words

If AB, BE, CD, DF be magnitudes proportional COMPONENDO (V·def·14)

… then they will also be proportional SEPARANDO (V·def·15)

Proof (cont)

(AE+EB):EB = (CF+FD):FD

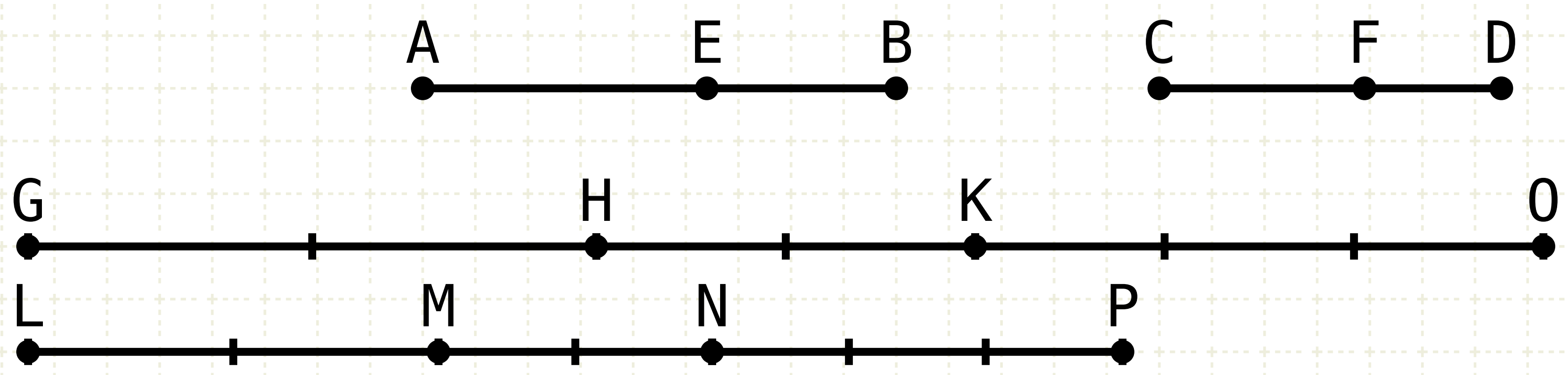
GH = m·AE
HK = m·EB
LM = m·CF
MN = m·FD

KO = n·EB
NP = n·FD

GK >=< HO → LN >=< MP

Proposition 17 of Book V

If magnitudes be proportional COMPONENDO, they will also be proportional SEPARANDO



In other words

If AB, BE, CD, DF be magnitudes proportional COMPONENDO (V·def·14)

… then they will also be proportional SEPARANDO (V·def·15)

Proof (cont)

Take the case where GK is greater than HO

(AE+EB):EB = (CF+FD):FD

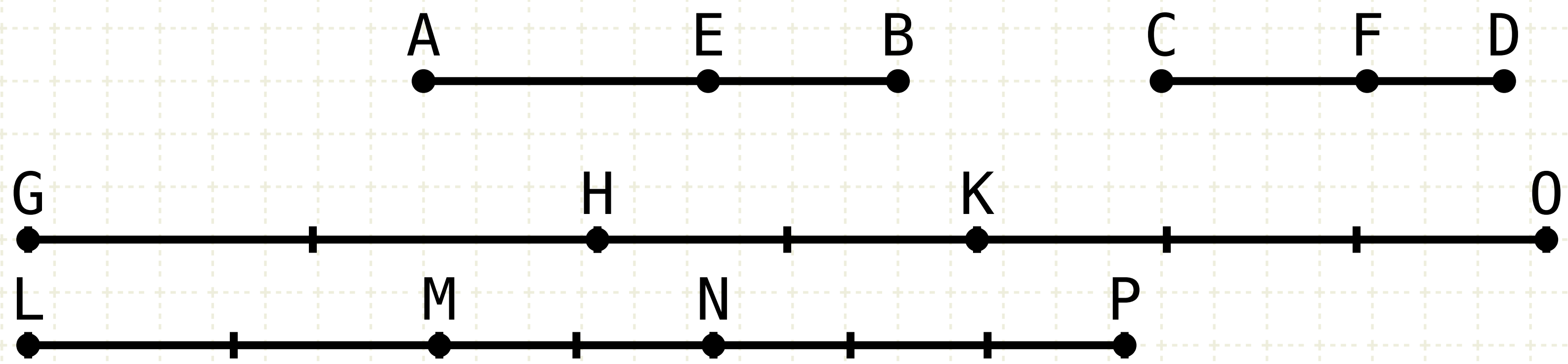
GH = m·AE
HK = m·EB
LM = m·CF
MN = m·FD

KO = n·EB
NP = n·FD

GK >=< HO → LN >=< MP
GK > HO → LN > MP

Proposition 17 of Book V

If magnitudes be proportional COMPONENDO, they will also be proportional SEPARANDO



(AE+EB):EB = (CF+FD):FD

GH = m·AE
HK = m·EB
LM = m·CF
MN = m·FD

KO = n·EB
NP = n·FD

GK >=< HO → LN >=< MP
GK > HO → LN > MP
GK - HK > HO - HK
GH > KO

In other words

If AB, BE, CD, DF be magnitudes proportional COMPONENDO (V·def·14)

… then they will also be proportional SEPARANDO (V·def·15)

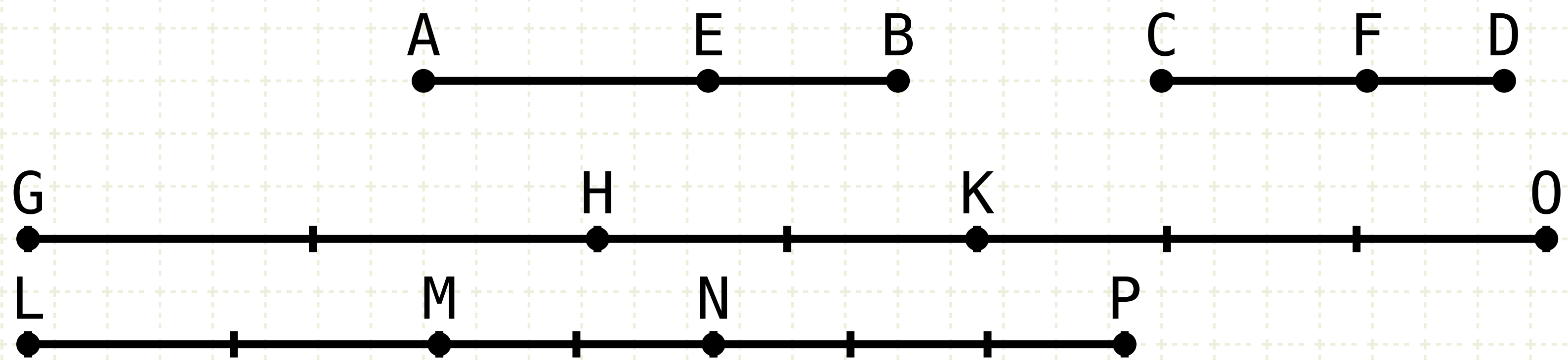
Proof (cont)

Take the case where GK is greater than HO

Subtract HK from both, then GH is also in excess of KO

Proposition 17 of Book V

If magnitudes be proportional COMPONENDO, they will also be proportional SEPARANDO



$(AE+EB):EB = (CF+FD):FD$

$GH = m \cdot AE$
$HK = m \cdot EB$
$LM = m \cdot CF$
$MN = m \cdot FD$

$KO = n \cdot EB$
$NP = n \cdot FD$

GK >=< HO → LN >=< MP
$GK > HO \rightarrow LN > MP$
$GK - HK > HO - HK$
$GH > KO$
$LN - MN > MP - MN$
$LM > NP$

In other words

If AB, BE, CD, DF be magnitudes proportional COMPONENDO (V·def·14)

… then they will also be proportional SEPARANDO (V·def·15)

Proof (cont)

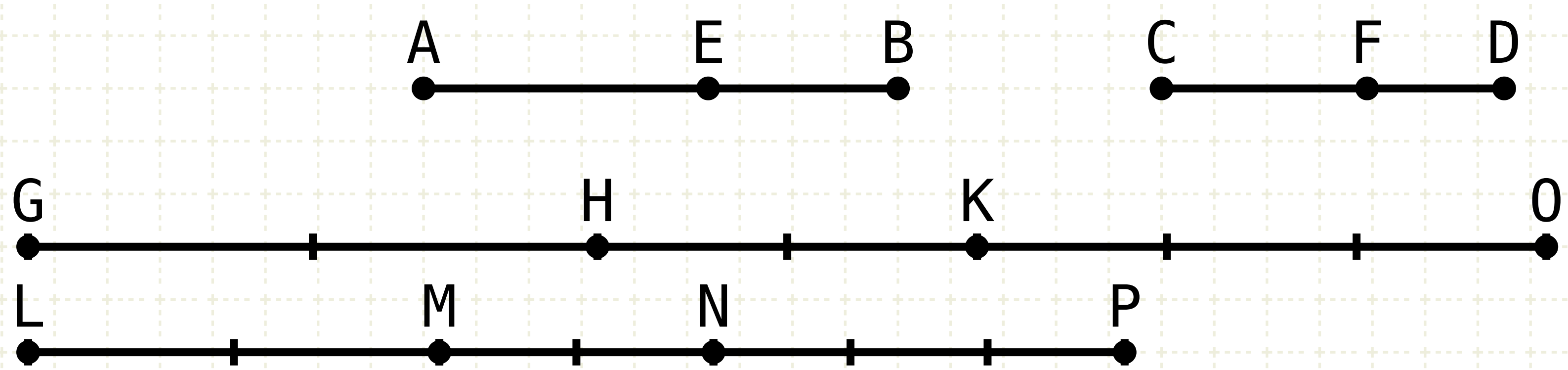
Take the case where GK is greater than HO

Subtract HK from both, then GH is also in excess of KO

If GK is greater than HO, then LN is also greater than MP, subtract MN from both, giving LM is greater than NP

Proposition 17 of Book V

If magnitudes be proportional COMPONENDO, they will also be proportional SEPARANDO



$$(AE+EB):EB = (CF+FD):FD$$

$GH = m \cdot AE$

$HK = m \cdot EB$

$LM = m \cdot CF$

$MN = m \cdot FD$

$KO = n \cdot EB$

$NP = n \cdot FD$

$GK \gtreqless HO \rightarrow LN \gtreqless MP$

$GK > HO \rightarrow LN > MP$

$GK - HK > HO - HK$

$GH > KO$

$LN - MN > MP - MN$

$LM > NP$

$GH \gtreqless KO \rightarrow LM \gtreqless NP$

In other words

If AB, BE, CD, DF be magnitudes proportional COMPONENDO (V·def·14)

… then they will also be proportional SEPARANDO (V·def·15)

Proof (cont)

Take the case where GK is greater than HO

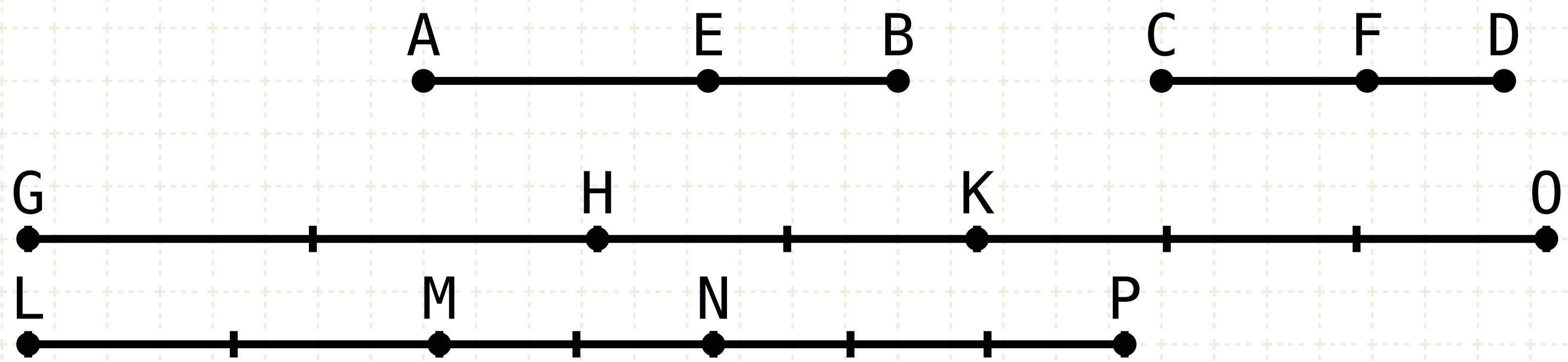
Subtract HK from both, then GH is also in excess of KO

If GK is greater than HO, then LN is also greater than MP, subtract MN from both, giving LM is greater than NP

Similarly, we can show that if GH is equal to HO, then LM is equal to NP, if equal, equal

Proposition 17 of Book V

If magnitudes be proportional COMPONENDO, they will also be proportional SEPARANDO



$(AE+EB):EB = (CF+FD):FD$

$GH = m \cdot AE$

$HK = m \cdot EB$

$LM = m \cdot CF$

$MN = m \cdot FD$

$KO = n \cdot EB$

$NP = n \cdot FD$

$GK \ {>}{=}{<}\ HO \rightarrow LN \ {>}{=}{<}\ MP$

$GK > HO \rightarrow LN > MP$

$GK - HK > HO - HK$

$GH > KO$

$LN - MN > MP - MN$

$LM > NP$

$GH \ {>}{=}{<}\ KO \rightarrow LM \ {>}{=}{<}\ NP$

$m \cdot AE \ {>}{=}{<}\ n \cdot EB \rightarrow m \cdot CF \ {>}{=}{<}\ n \cdot FD$

$AE:EB = CF:FD$

In other words

If AB, BE, CD, DF be magnitudes proportional COMPONENDO (V·def·14)

… then they will also be proportional SEPARANDO (V·def·15)

Proof (cont)

Take the case where GK is greater than HO

Subtract HK from both, then GH is also in excess of KO

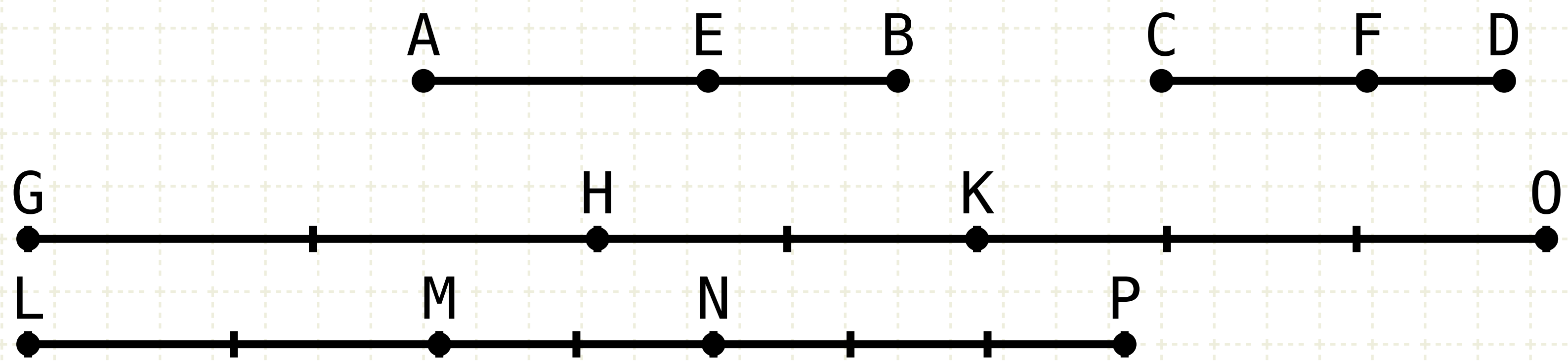
If GK is greater than HO, then LN is also greater than MP, subtract MN from both, giving LM is greater than NP

Similarly, we can show that if GH is equal to HO, then LM is equal to NP, if equal, equal

But GH,LM are equimultiples of AE,CF and KO,NP are equimultiples of EB,FD, therefore AE is to EB as CF is to FD (V·def·5)

Proposition 17 of Book V

If magnitudes be proportional COMPONENDO, they will also be proportional SEPARANDO



$(AE+EB):EB = (CF+FD):FD$

$GH = m \cdot AE$
$HK = m \cdot EB$
$LM = m \cdot CF$
$MN = m \cdot FD$

$KO = n \cdot EB$
$NP = n \cdot FD$

$GK \gtreqless HO \rightarrow LN \gtreqless MP$
$GK > HO \rightarrow LN > MP$
$GK - HK > HO - HK$
$GH > KO$
$LN - MN > MP - MN$
$LM > NP$

$GH \gtreqless KO \rightarrow LM \gtreqless NP$
$m \cdot AE \gtreqless n \cdot EB \rightarrow m \cdot CF \gtreqless n \cdot FD$
$AE:EB = CF:FD$

In other words

If AB, BE, CD, DF be magnitudes proportional COMPONENDO (V·def·14)

… then they will also be proportional SEPARANDO (V·def·15)

Proof (cont)

Take the case where GK is greater than HO

Subtract HK from both, then GH is also in excess of KO

If GK is greater than HO, then LN is also greater than MP, subtract MN from both, giving LM is greater than NP

Similarly, we can show that if GH is equal to HO, then LM is equal to NP, if equal, equal

But GH,LM are equimultiples of AE,CF and KO,NP are equimultiples of EB,FD, therefore AE is to EB as CF is to FD (V·def·5)

Youtube Videos

https://www.youtube.com/c/SandyBultena