

Euclid's Elements

Book I

*If Euclid did not kindle your youthful enthusiasm, you
were not born to be a scientific thinker.*

Albert Einstein

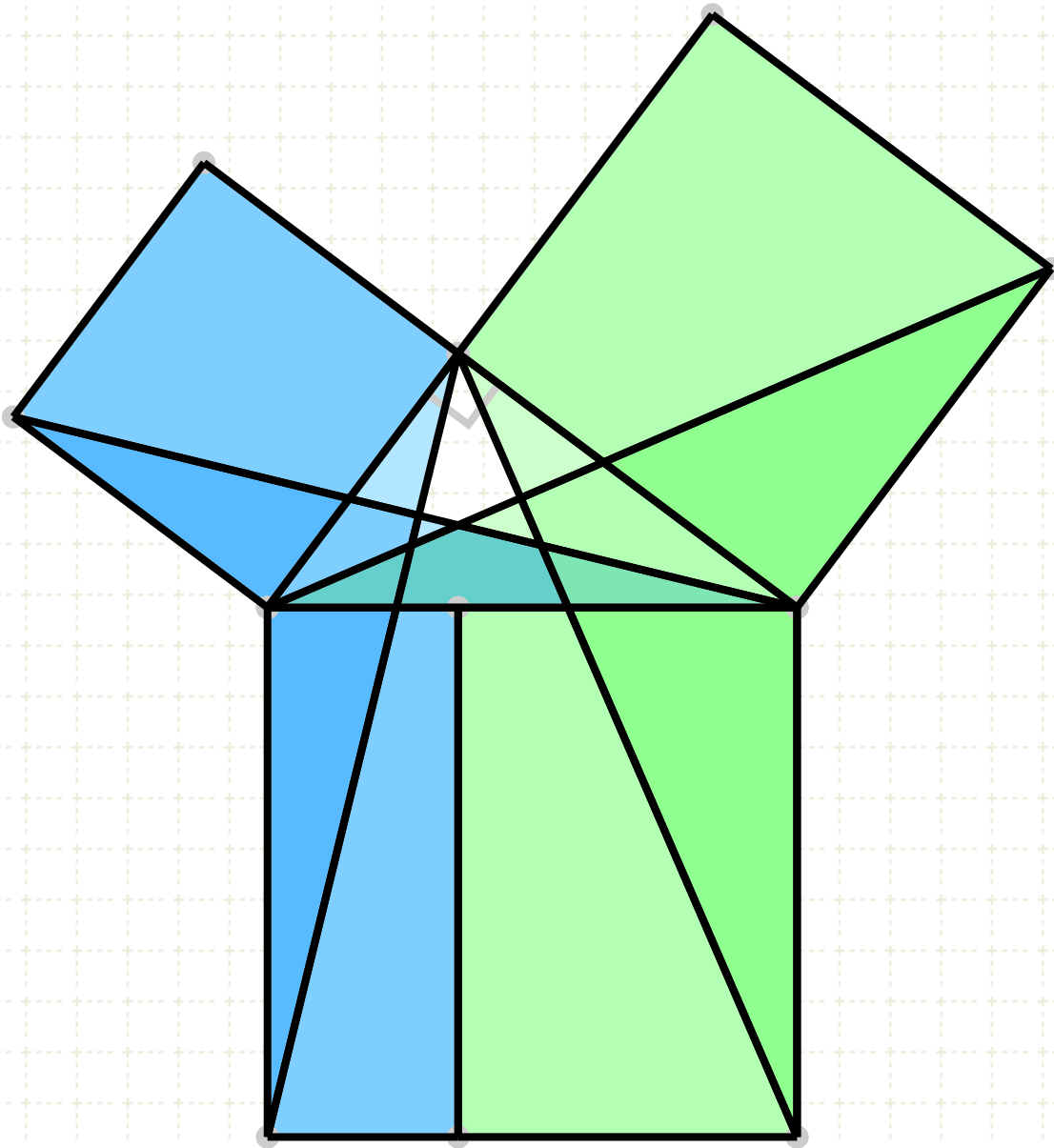


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Proposition 26 of Book I

If two triangles have two angles equal to two angles respectively, and one side equal to one side, namely, either the side adjoining the equal angles, or that opposite one of the equal angles, then the remaining sides equal the remaining sides and the remaining angle equals the remaining angle.



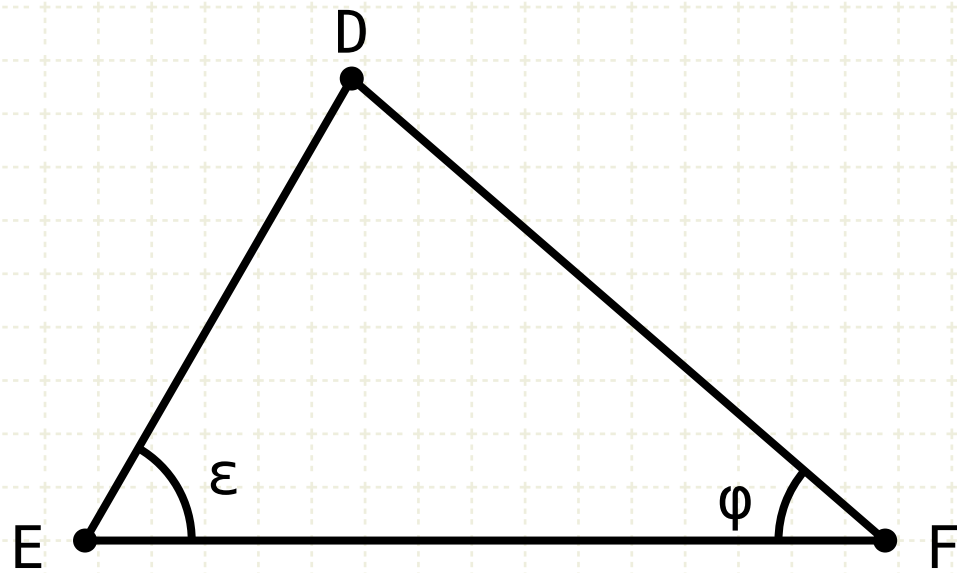
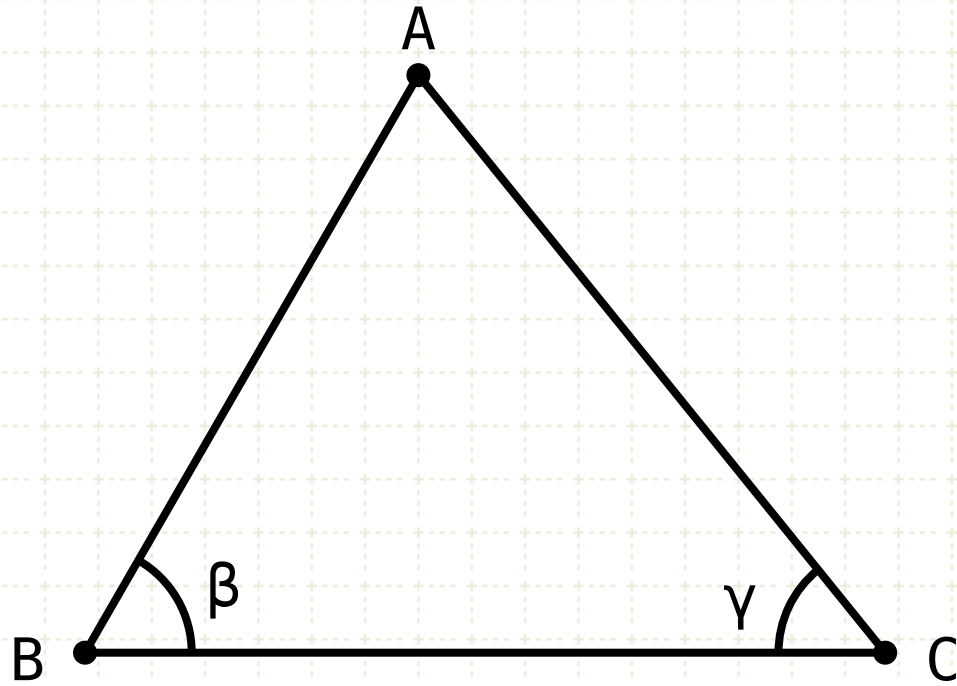
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$$\begin{aligned}\varepsilon &= \beta \\ \varphi &= \gamma \\ EF &= BC\end{aligned}$$

In other words

Given two triangles ABC and DEF, where BC equals EF, and angles ABC and DEF are equal, and angles BCA and EFD are equal (ASA)



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$$\varepsilon = \beta$$

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$$\triangle ABC \equiv \triangle DEF$$

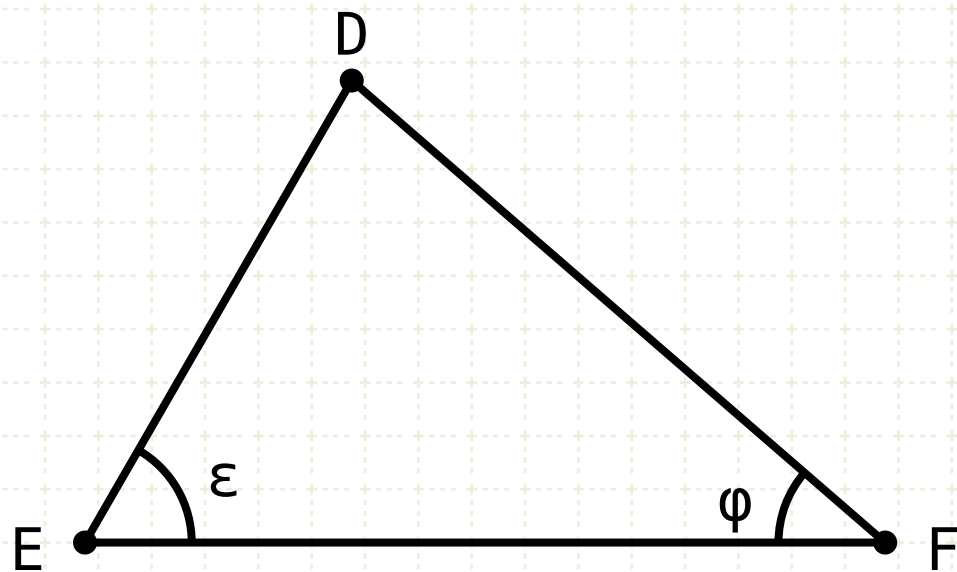
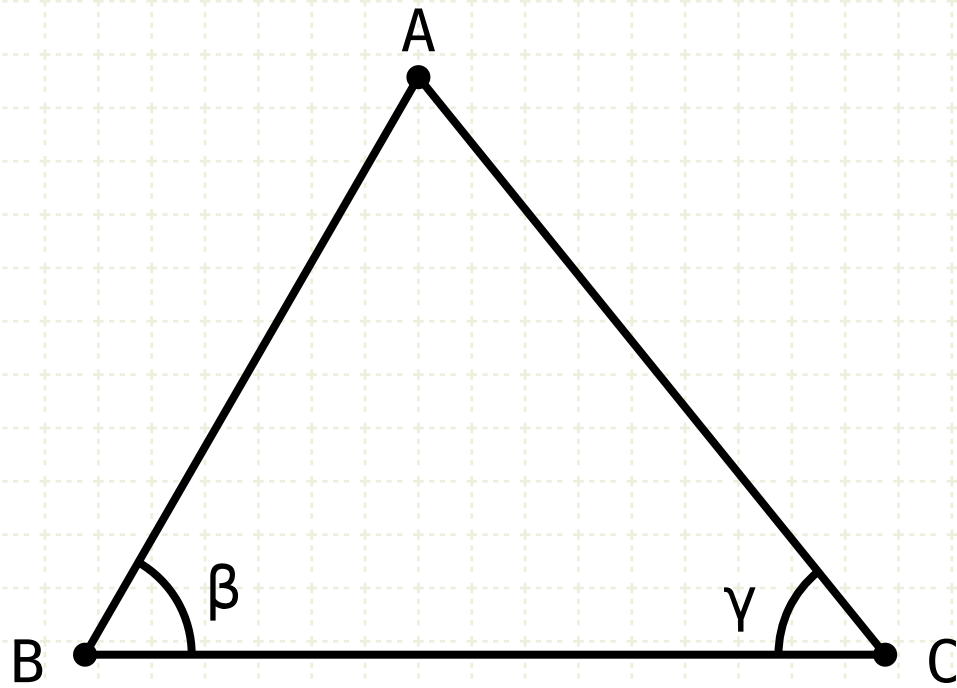
$$DE = AB$$

$$DF = AC$$

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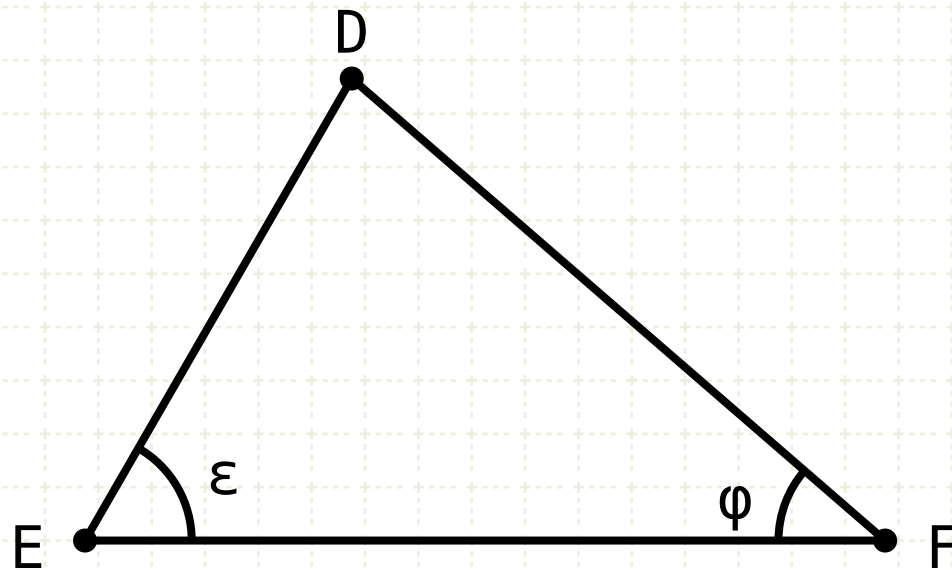
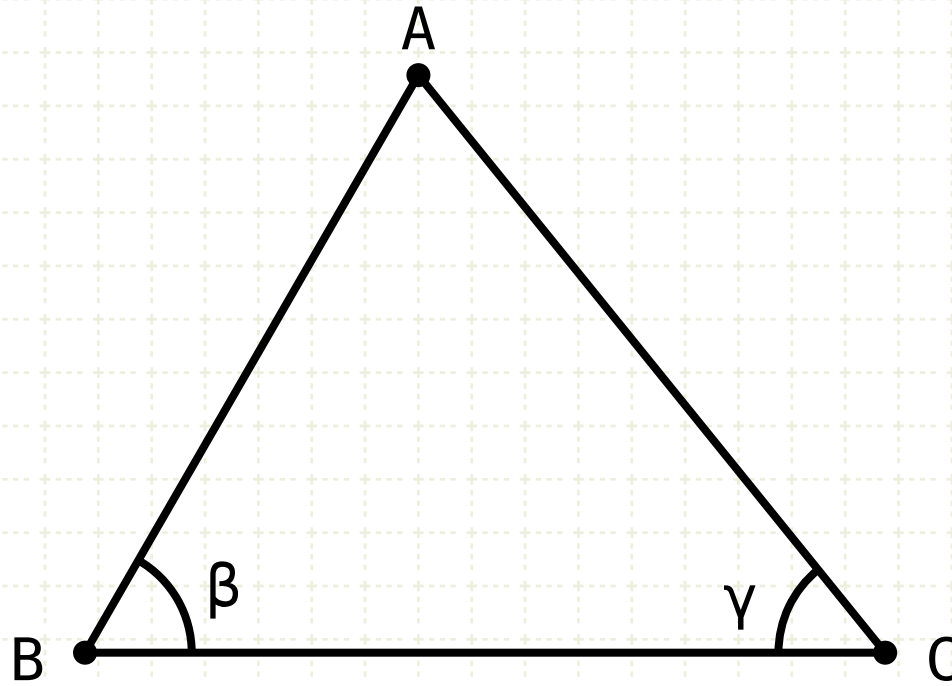
Then the two triangles are equivalent



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$$\varepsilon = \beta$$

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$$EF = BC$$

$$AB > DE$$

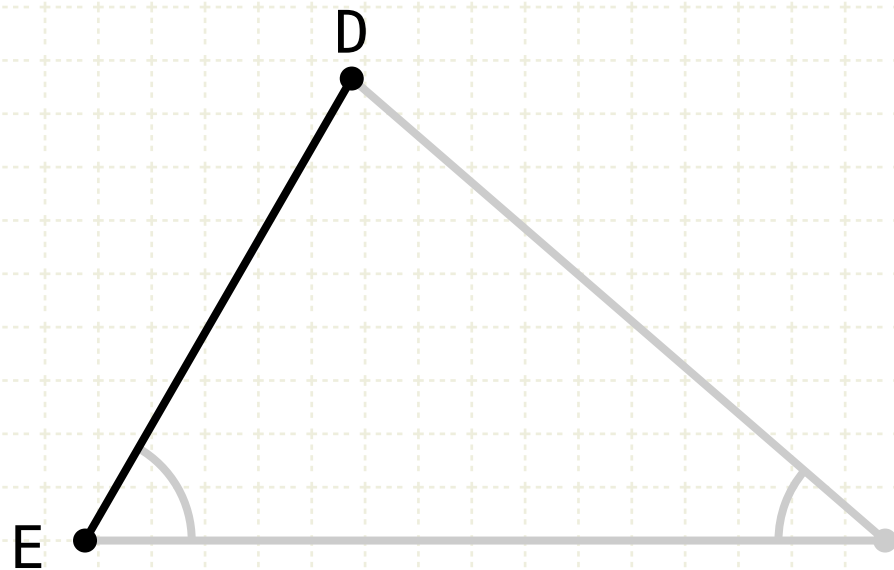
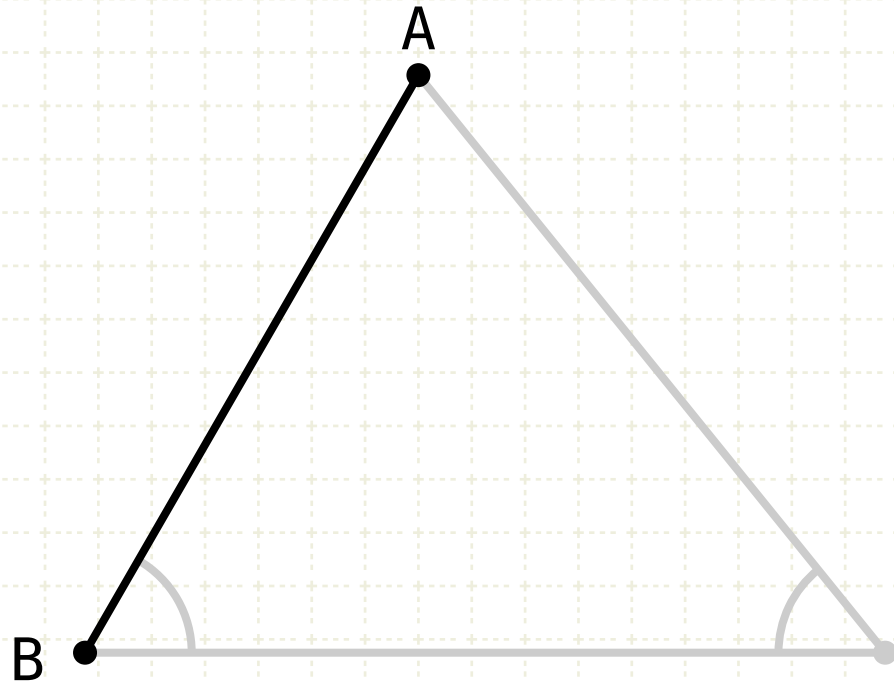
In other words

Given two triangles ABC and DEF, where BC equals EF, and angles ABC and DEF are equal, and angles BCA and EFD are equal (ASA)

Then the two triangles are equivalent

Proof by Contradiction

Assume that AB is greater than DE



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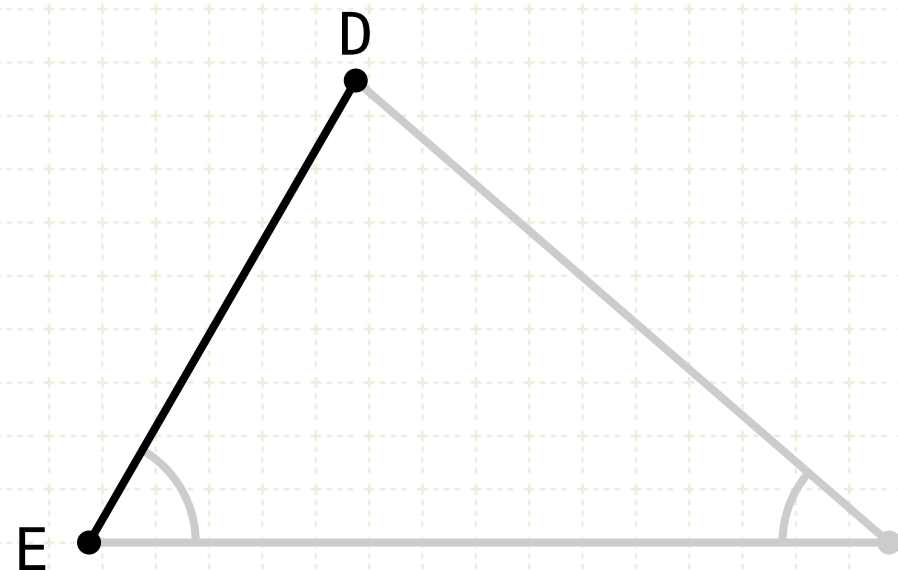
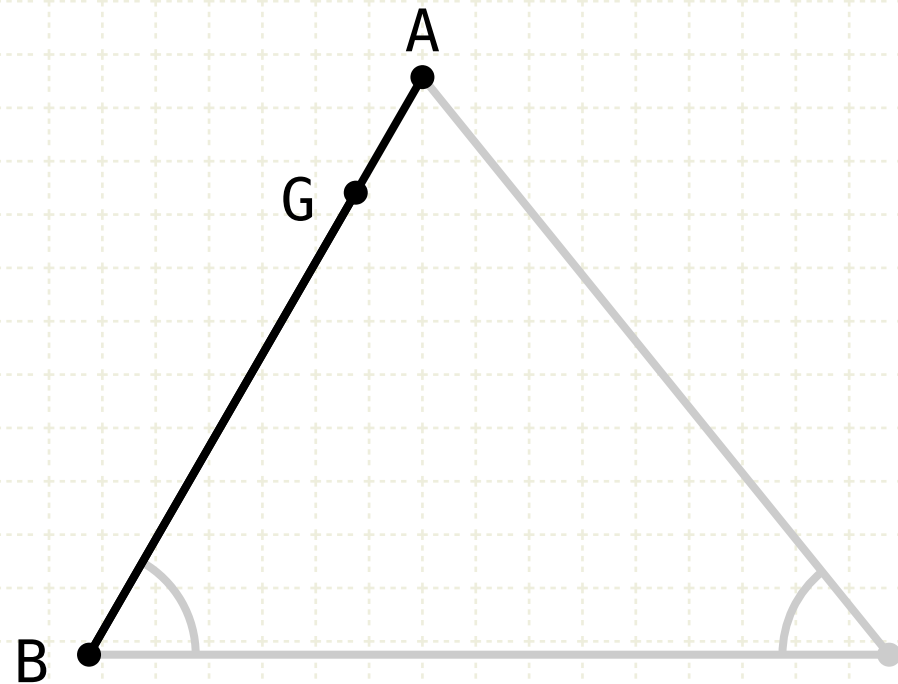
$$\varepsilon = \beta$$

$$\varphi = \gamma$$

$$EF = BC$$

$$AB > DE$$

$$BG = DE$$



In other words

Given two triangles ABC and DEF, where BC equals EF, and angles ABC and DEF are equal, and angles BCA and EFD are equal (ASA)

Then the two triangles are equivalent

Proof by Contradiction

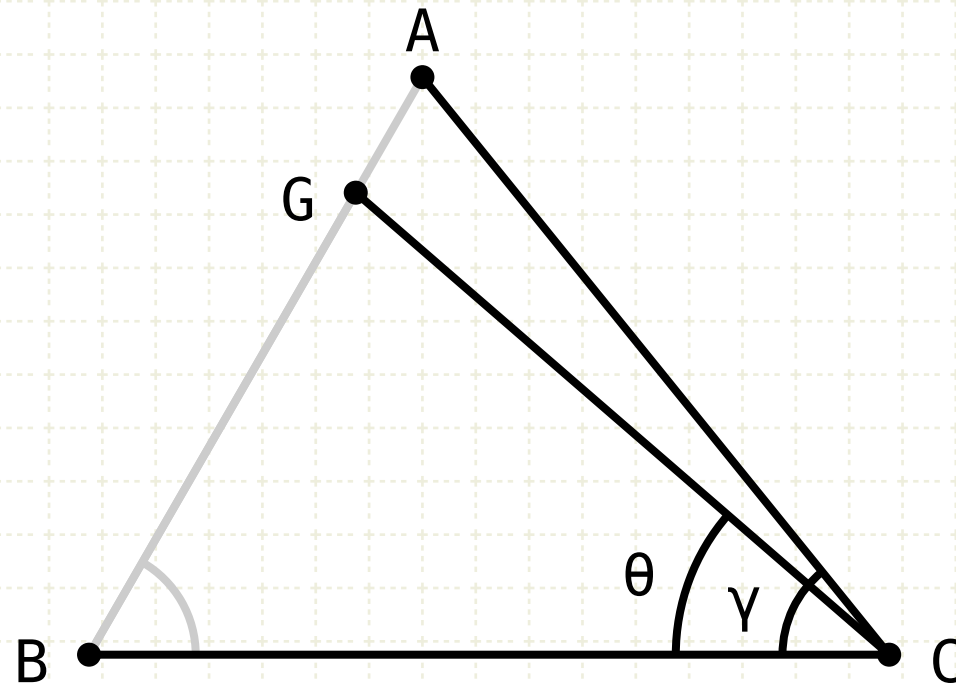
Assume that AB is greater than DE

Create a point G such that BG equals DE



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If two triangles have two angles equal to two angles respectively, and one side equal to one side, namely, either the side adjoining the equal angles, or that opposite one of the equal angles, then the remaining sides equal the remaining sides and the remaining angle equals the remaining angle.



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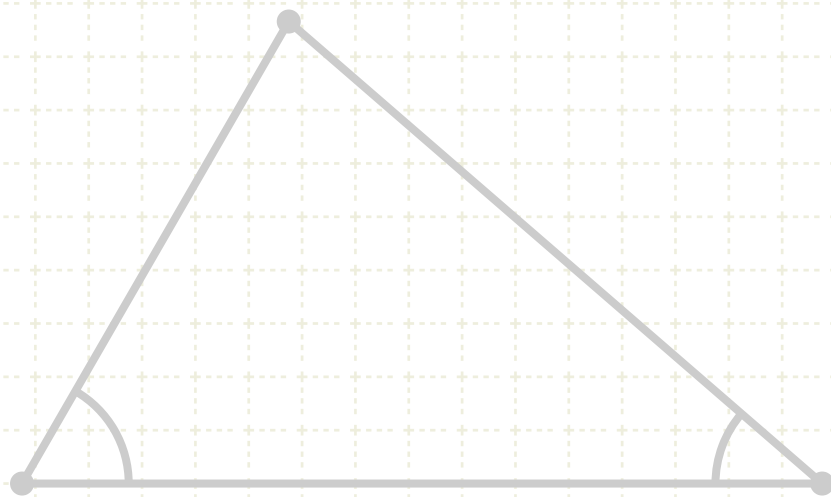
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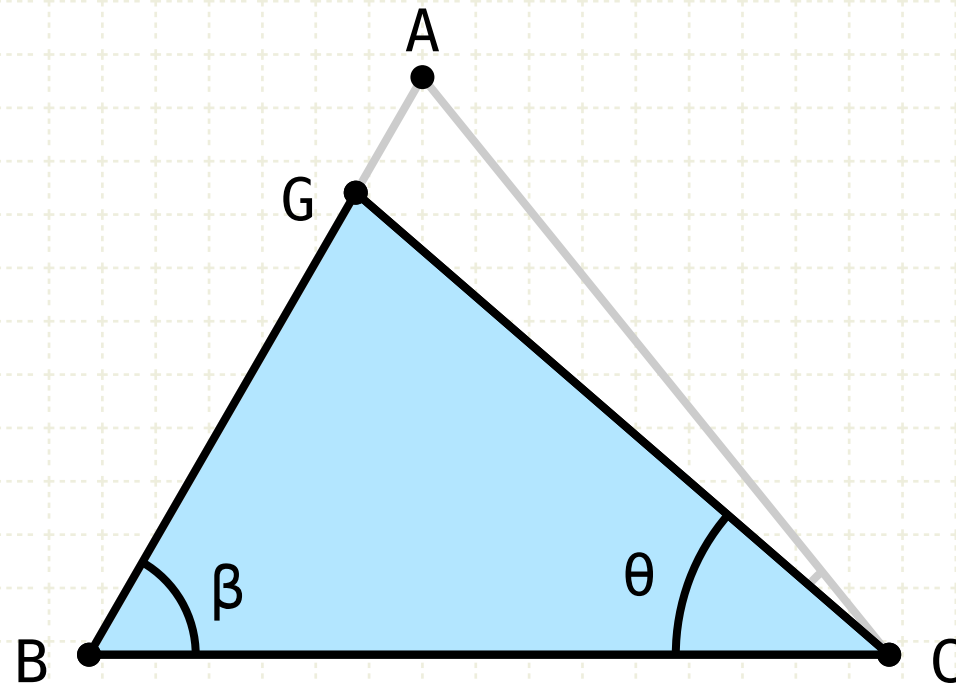
Create a point G such that BG equals DE

Create line GC, angle BCA is greater than BCG



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If two triangles have two angles equal to two angles respectively, and one side equal to one side, namely, either the side adjoining the equal angles, or that opposite one of the equal angles, then the remaining sides equal the remaining sides and the remaining angle equals the remaining angle.



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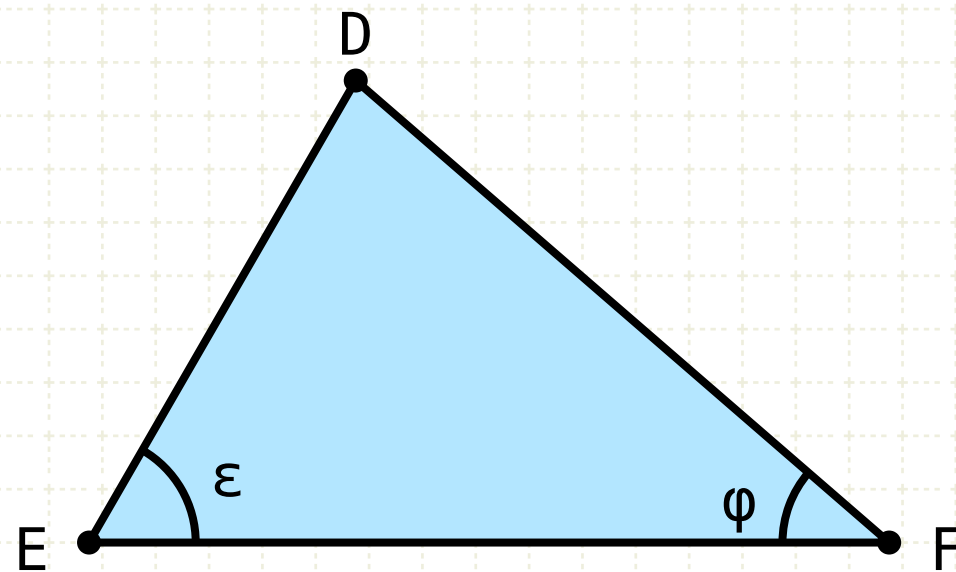
$$EF = BC$$

$$AB > DE$$

$$BG = DE$$

$$\theta < \gamma$$

$$\triangle GBC \equiv \triangle DEF$$



In other words

Given two triangles ABC and DEF, where BC equals EF, and angles ABC and DEF are equal, and angles BCA and EFD are equal (ASA)

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Proof by Contradiction

Assume that AB is greater than DE

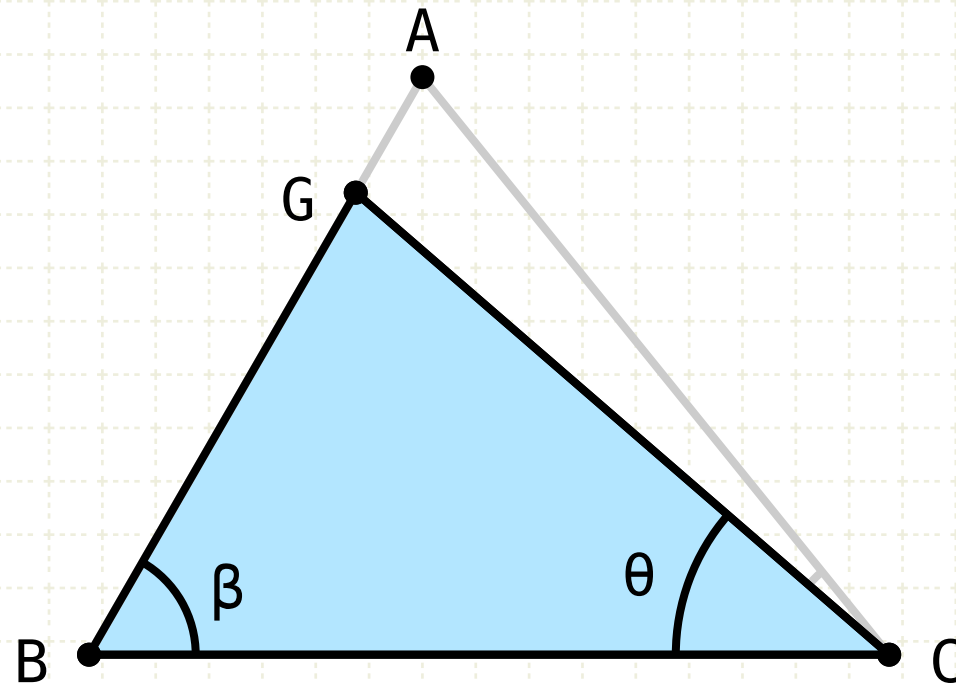
Create a point G such that BG equals DE

Create line GC, angle BCA is greater than BCG

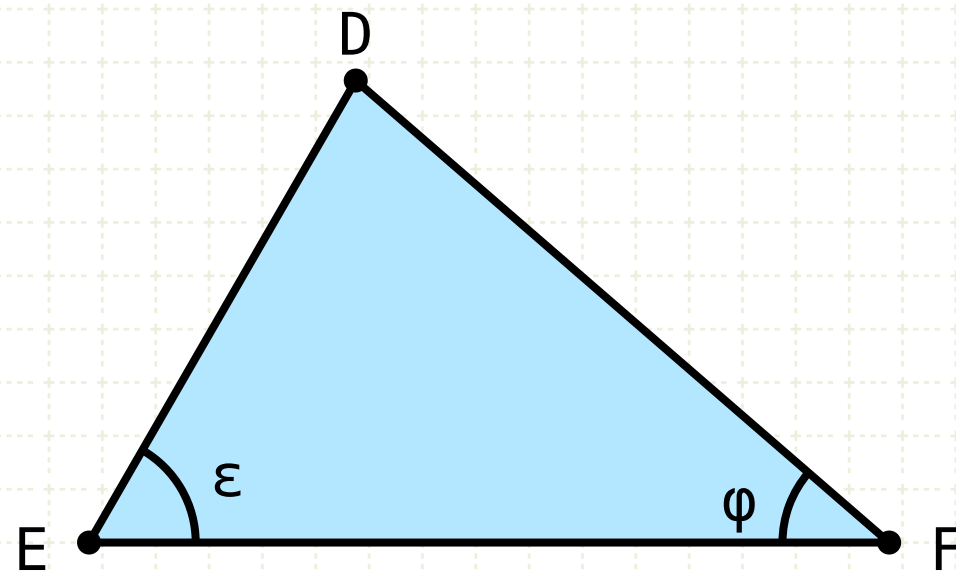
Triangle GBC has two sides and an angle that is equivalent in triangle DEF, hence they are equal in all respects (I-4)

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$$\begin{aligned}\varepsilon &= \beta \\ \varphi &= \gamma \\ EF &= BC \\ AB &> DE \\ BG &= DE \\ \theta &< \gamma \\ \triangle GBC &\equiv \triangle DEF \\ \theta &= \varphi\end{aligned}$$



In other words

Given two triangles ABC and DEF, where BC equals EF, and angles ABC and DEF are equal, and angles BCA and EFD are equal (ASA)

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Proof by Contradiction

Assume that AB is greater than DE

Create a point G such that BG equals DE

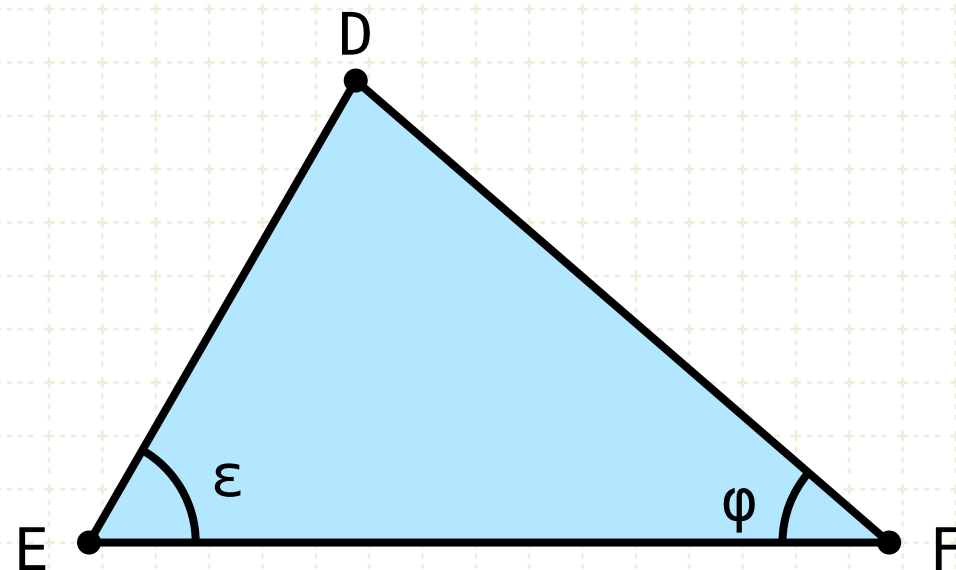
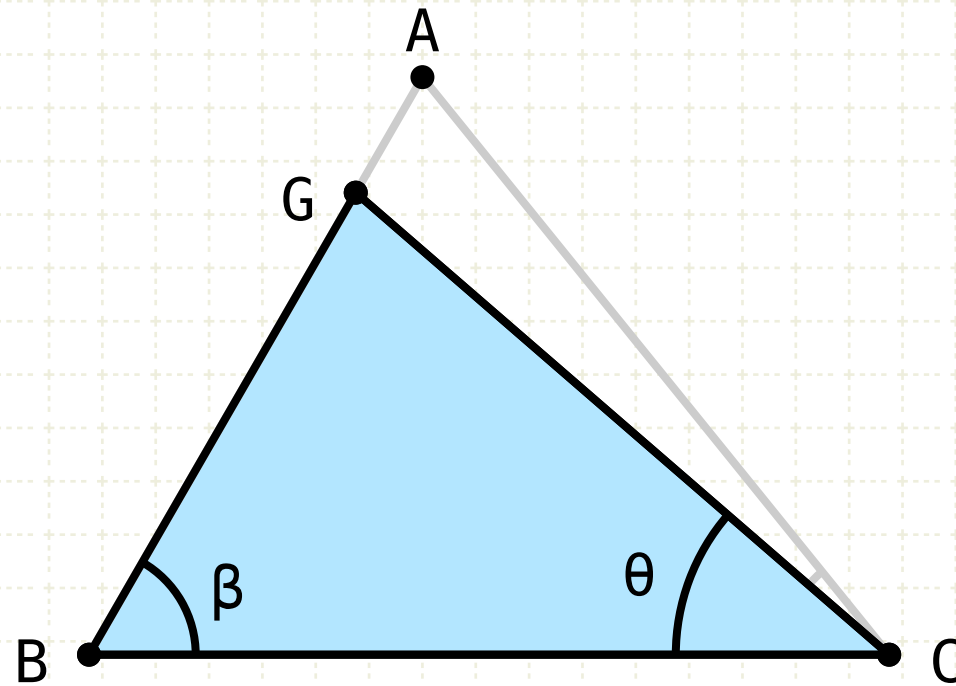
Create line GC, angle BCA is greater than BCG

Triangle GBC has two sides and an angle that is equivalent in triangle DEF, hence they are equal in all respects (I-4)

Thus, angle BCG is equal to angle DFE, which is defined as equal to angle BCA

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Given two triangles ABC and DEF, where BC equals EF, and angles ABC and DEF are equal, and angles BCA and EFD are equal (ASA)

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Proof by Contradiction

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Create a point G such that BG equals DE

Create line GC, angle BCA is greater than BCG

Triangle GBC has two sides and an angle that is equivalent in triangle DEF, hence they are equal in all respects (I-4)

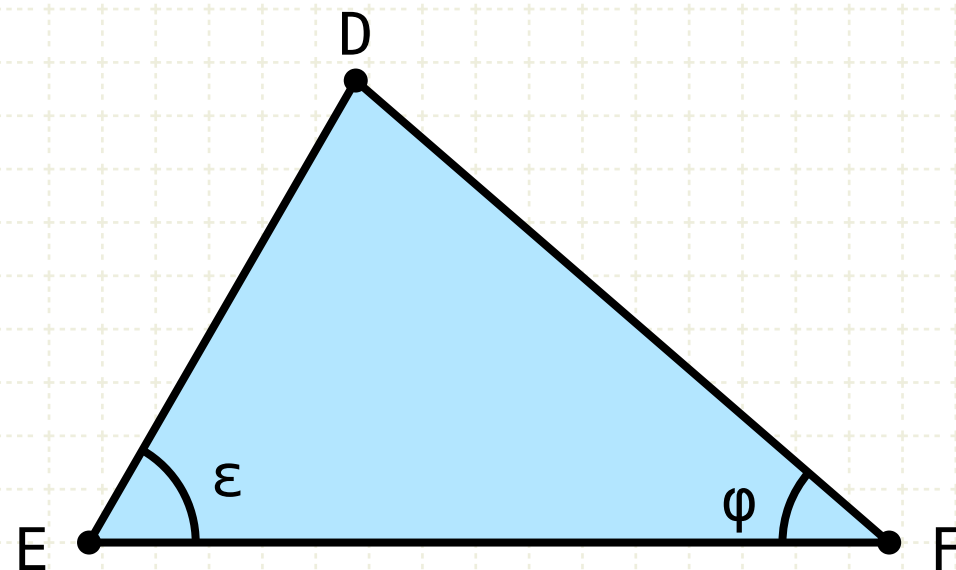
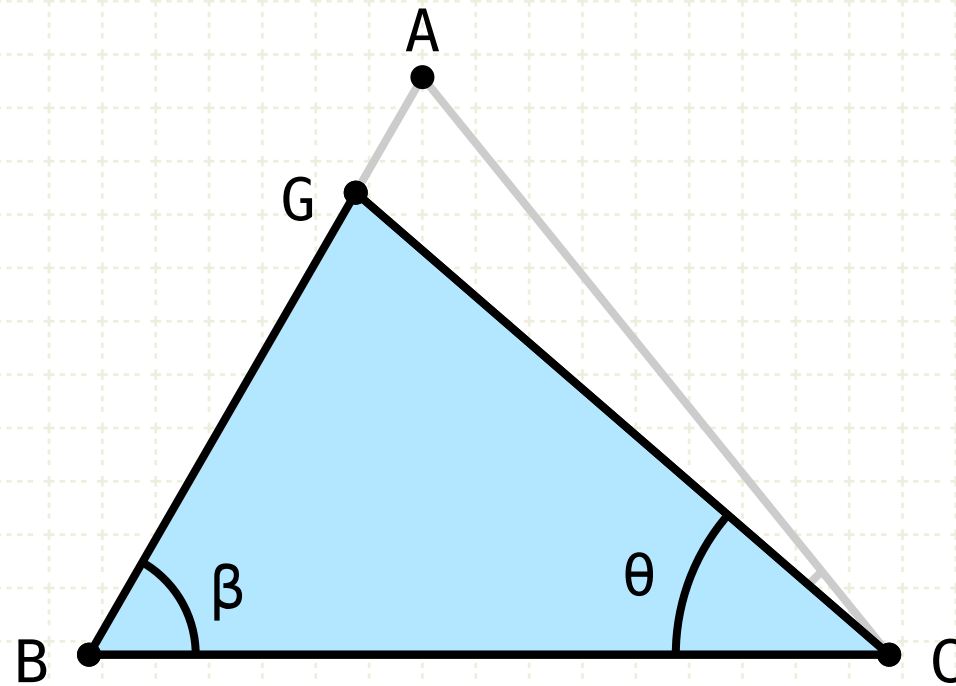
Thus, angle BCG is equal to angle DFE, which is defined as equal to angle BCA

Angle BCG cannot be both less than AND equal to BCA



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$$AB > DE \quad x$$

$$BG = DE$$

$$\theta < \gamma$$

$$\triangle GBC \equiv \triangle DEF$$

$$\theta = \varphi$$

In other words

Given two triangles ABC and DEF, where BC equals EF, and angles ABC and DEF are equal, and angles BCA and EFD are equal (ASA)

Then the two triangles are equivalent

Proof by Contradiction

Assume that AB is greater than DE

Create a point G such that BG equals DE

Create line GC, angle BCA is greater than BCG

Triangle GBC has two sides and an angle that is equivalent in triangle DEF, hence they are equal in all respects (I-4)

Thus, angle BCG is equal to angle DFE, which is defined as equal to angle BCA

Angle BCG cannot be both less than AND equal to BCA

Thus the original assumption must be incorrect



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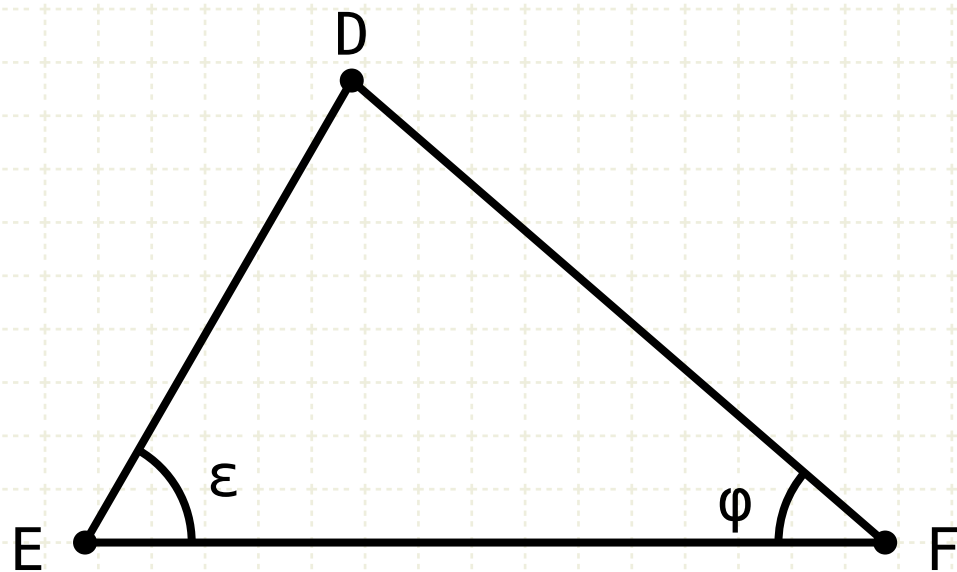
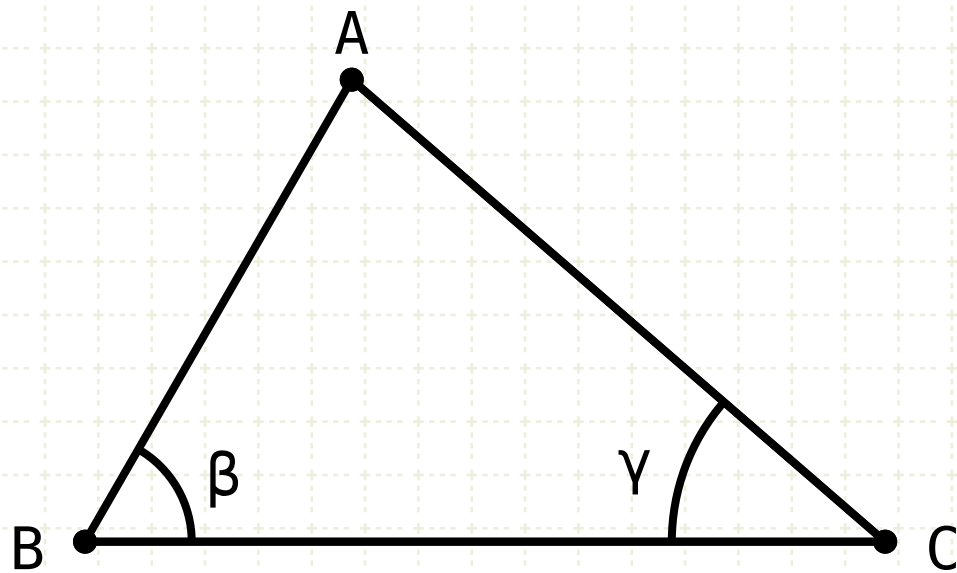
$$BG = DE$$

$$\theta < \gamma$$

$$\triangle GBC \equiv \triangle DEF$$

$$\theta = \varphi$$

$$AB = DE$$



In other words

Given two triangles ABC and DEF, where BC equals EF, and angles ABC and DEF are equal, and angles BCA and EFD are equal (ASA)

Then the two triangles are equivalent

Proof by Contradiction

Assume that AB is greater than DE

Create a point G such that BG equals DE

Create line GC, angle BCA is greater than BCG

Triangle GBC has two sides and an angle that is equivalent in triangle DEF, hence they are equal in all respects (I-4)

Thus, angle BCG is equal to angle DFE, which is defined as equal to angle BCA

Angle BCG cannot be both less than AND equal to BCA

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Therefore AB equals DE



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If two triangles have two angles equal to two angles respectively, and one side equal to one side, namely, either the side adjoining the equal angles, or that opposite one of the equal angles, then the remaining sides equal the remaining sides and the remaining angle equals the remaining angle.

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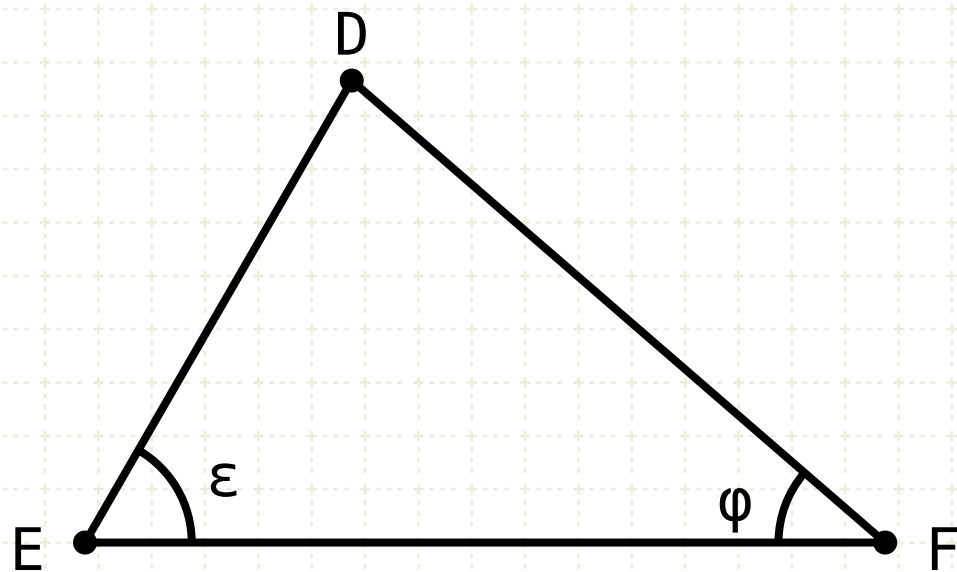
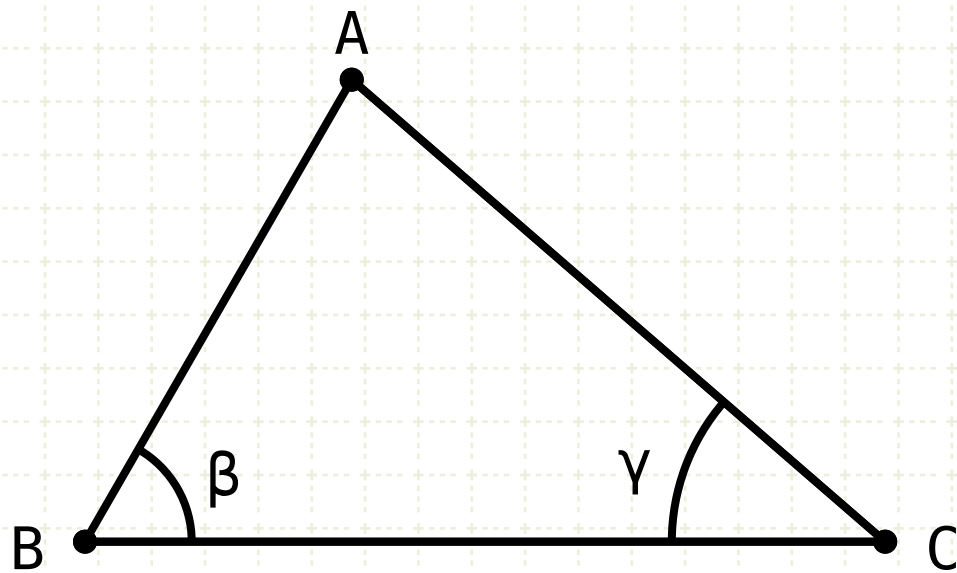
$$\theta < \gamma$$

$$\triangle GBC \equiv \triangle DEF$$

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$$AB = DE$$

$$\triangle ABC \equiv \triangle DEF$$



In other words

Given two triangles ABC and DEF, where BC equals EF, and angles ABC and DEF are equal, and angles BCA and EFD are equal (ASA)

Then the two triangles are equivalent

Proof by Contradiction

Assume that AB is greater than DE

Create a point G such that BG equals DE

Create line GC, angle BCA is greater than BCG

Triangle GBC has two sides and an angle that is equivalent in triangle DEF, hence they are equal in all respects (I-4)

Thus, angle BCG is equal to angle DFE, which is defined as equal to angle BCA

Angle BCG cannot be both less than AND equal to BCA

Thus the original assumption must be incorrect

Therefore AB equals DE

Since we have two triangles, with two equal sides, with equivalent angles, then the two triangles are equal in all respects (I-4)



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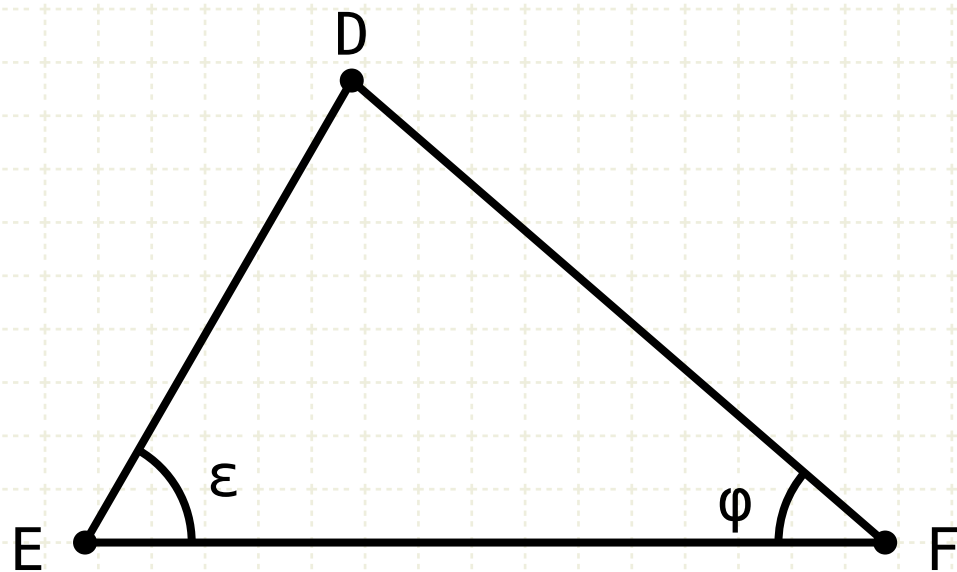
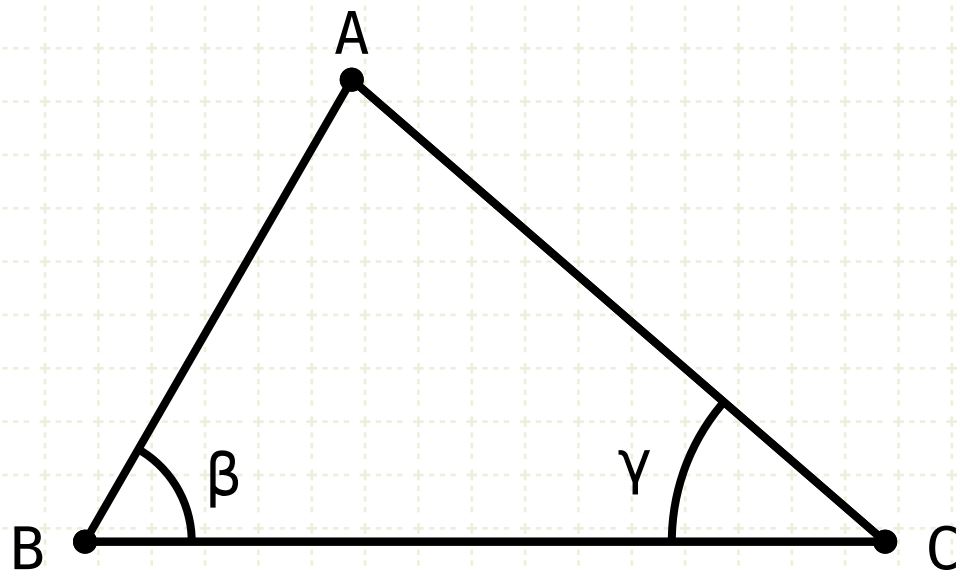
$$\theta < \gamma$$

$$\triangle GBC \equiv \triangle DEF$$

$$\theta = \varphi$$

$$AB = DE$$

$$\triangle ABC \equiv \triangle DEF$$



In other words

Given two triangles ABC and DEF, where BC equals EF, and angles ABC and DEF are equal, and angles BCA and EFD are equal (ASA)

Then the two triangles are equivalent

Proof by Contradiction

Assume that AB is greater than DE

Create a point G such that BG equals DE

Create line GC, angle BCA is greater than BCG

Triangle GBC has two sides and an angle that is equivalent in triangle DEF, hence they are equal in all respects (I-4)

Thus, angle BCG is equal to angle DFE, which is defined as equal to angle BCA

Angle BCG cannot be both less than AND equal to BCA

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Since we have two triangles, with two equal sides, with equivalent angles, then the two triangles are equal in all respects (I-4)



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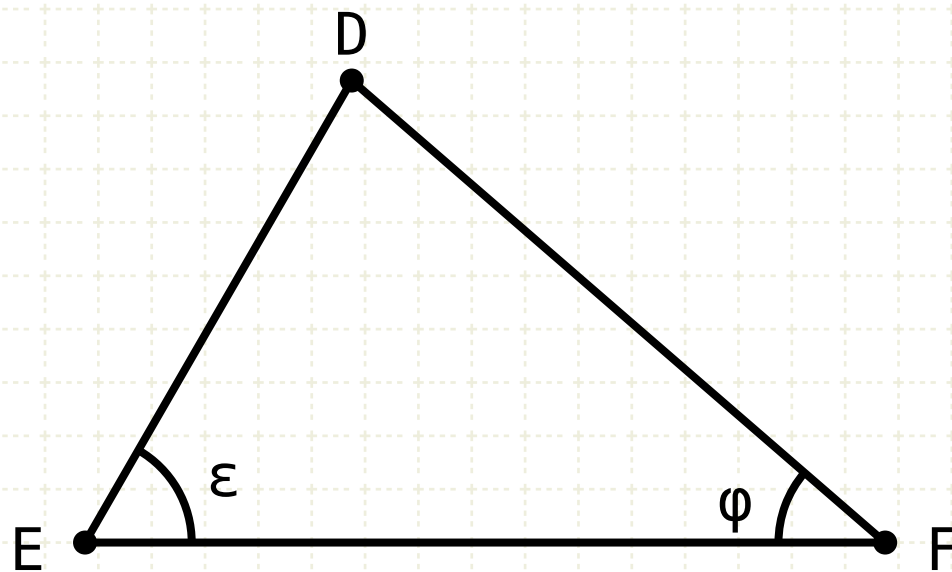
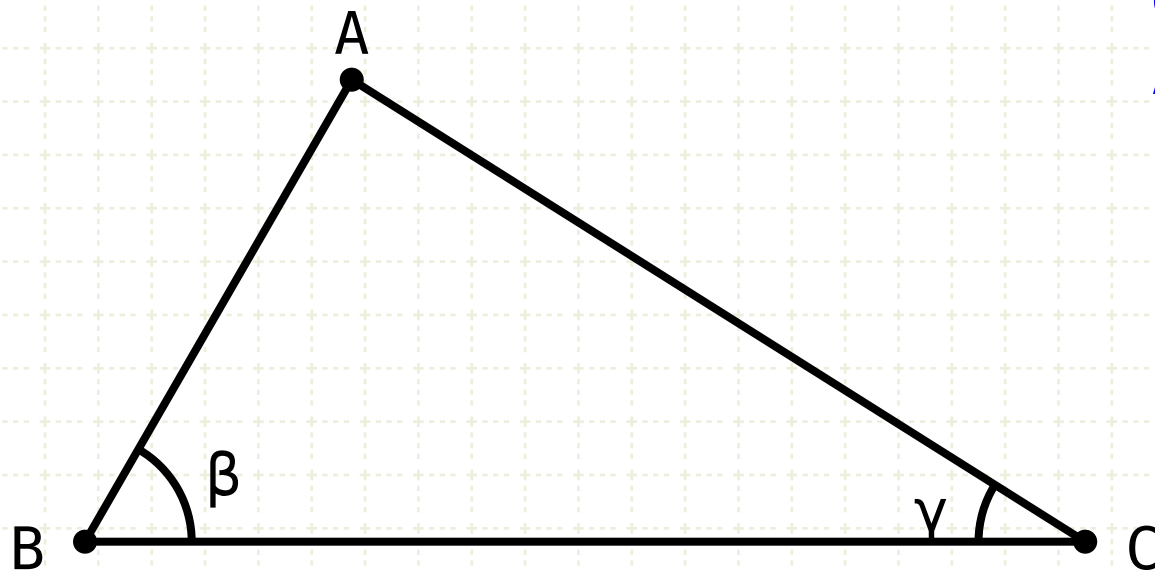
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$$\delta = \alpha$$

$$AC = DF$$

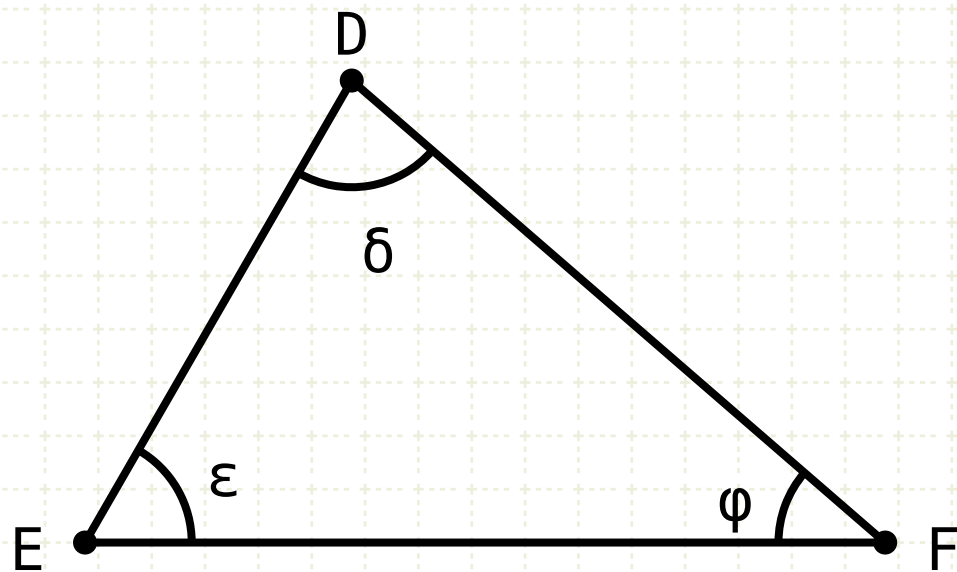
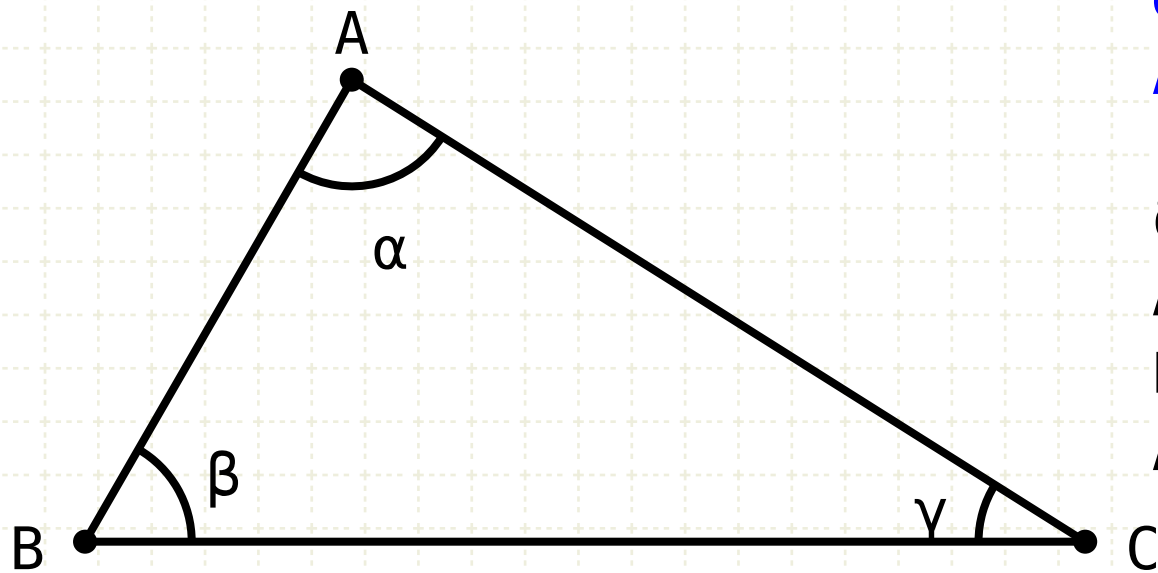
$$BC = EF$$

$$\triangle ABC \equiv \triangle DEF$$

In other words

Given two triangles ABC and DEF, where AB equals DE, and angles ABC and DEF are equal, and angles BCA and EFD are equal (SSA)

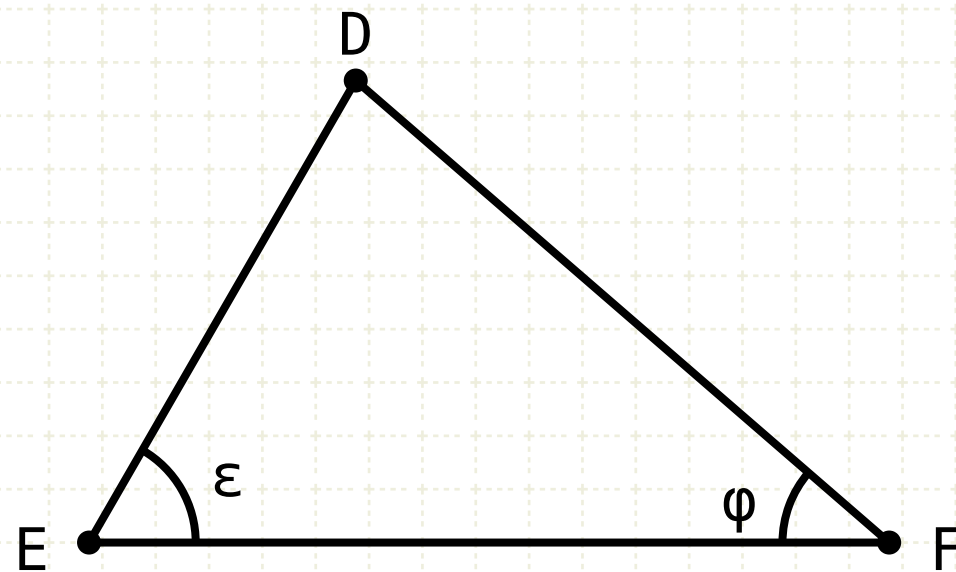
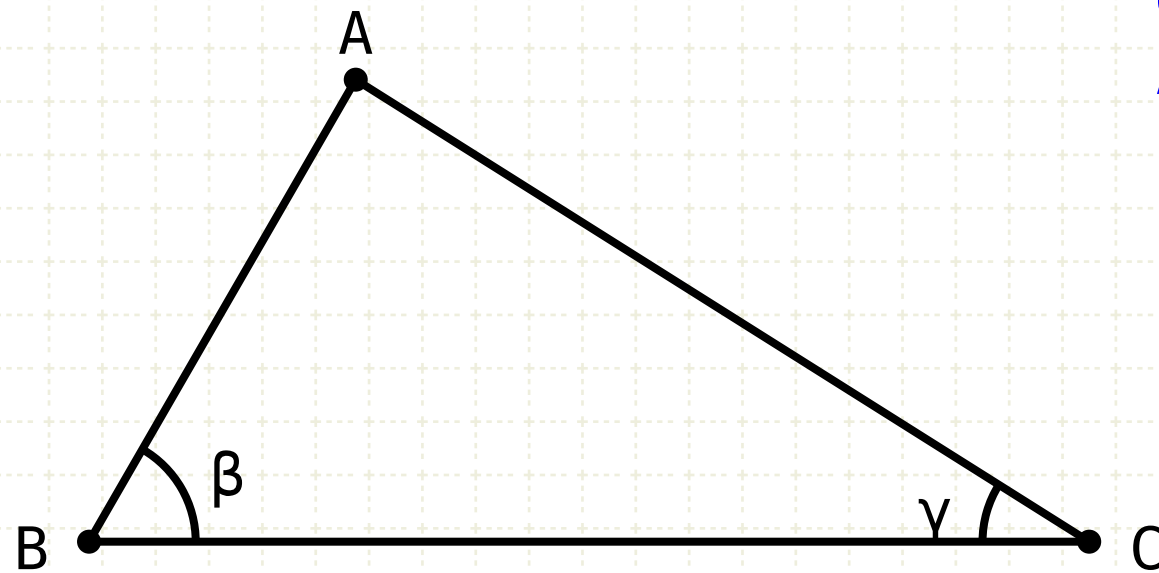
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Then the two triangles are equal in all respects

Proof by Contradiction



Proposition 26 of Book I

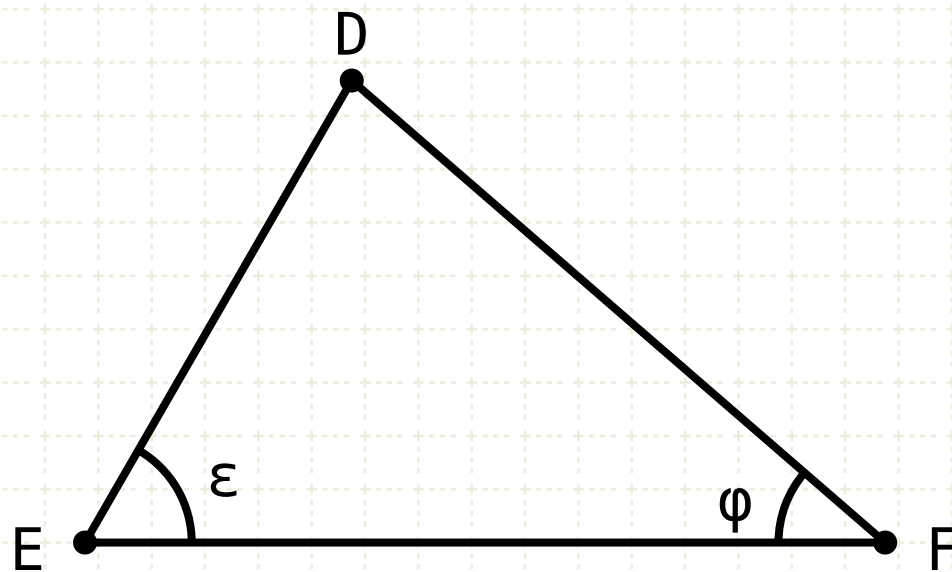
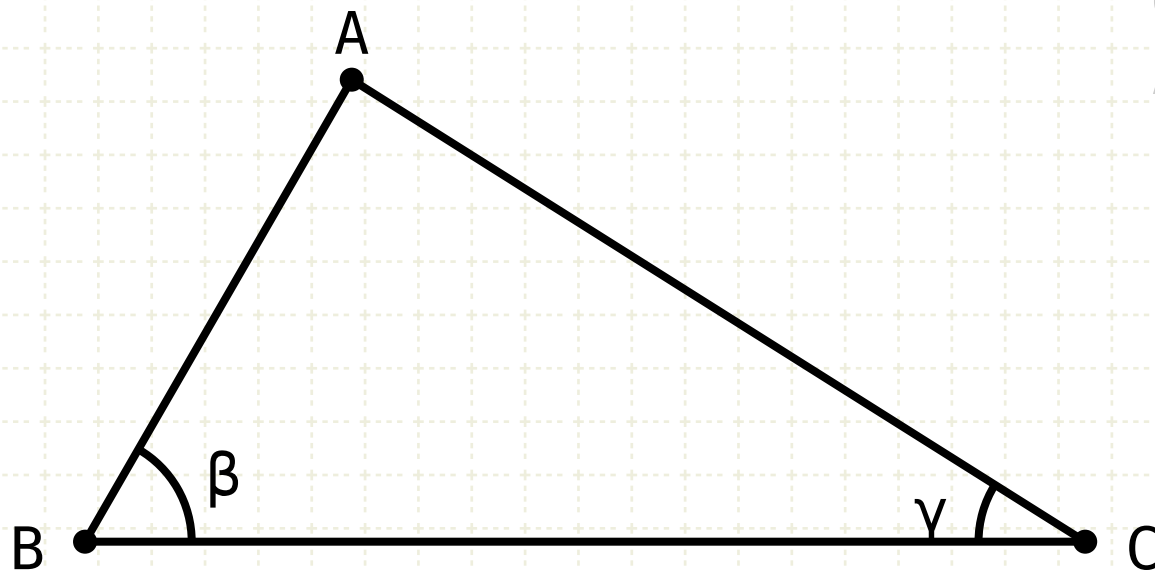
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$$\varepsilon = \beta$$

$$\varphi = \gamma$$

$$AB = DE$$

$$BC > EF$$



In other words

Given two triangles ABC and DEF, where AB equals DE, and angles ABC and DEF are equal, and angles BCA and EFD are equal (SSA)

Then the two triangles are equal in all respects

Proof by Contradiction

Assume that BC is greater than EF



Proposition 26 of Book I

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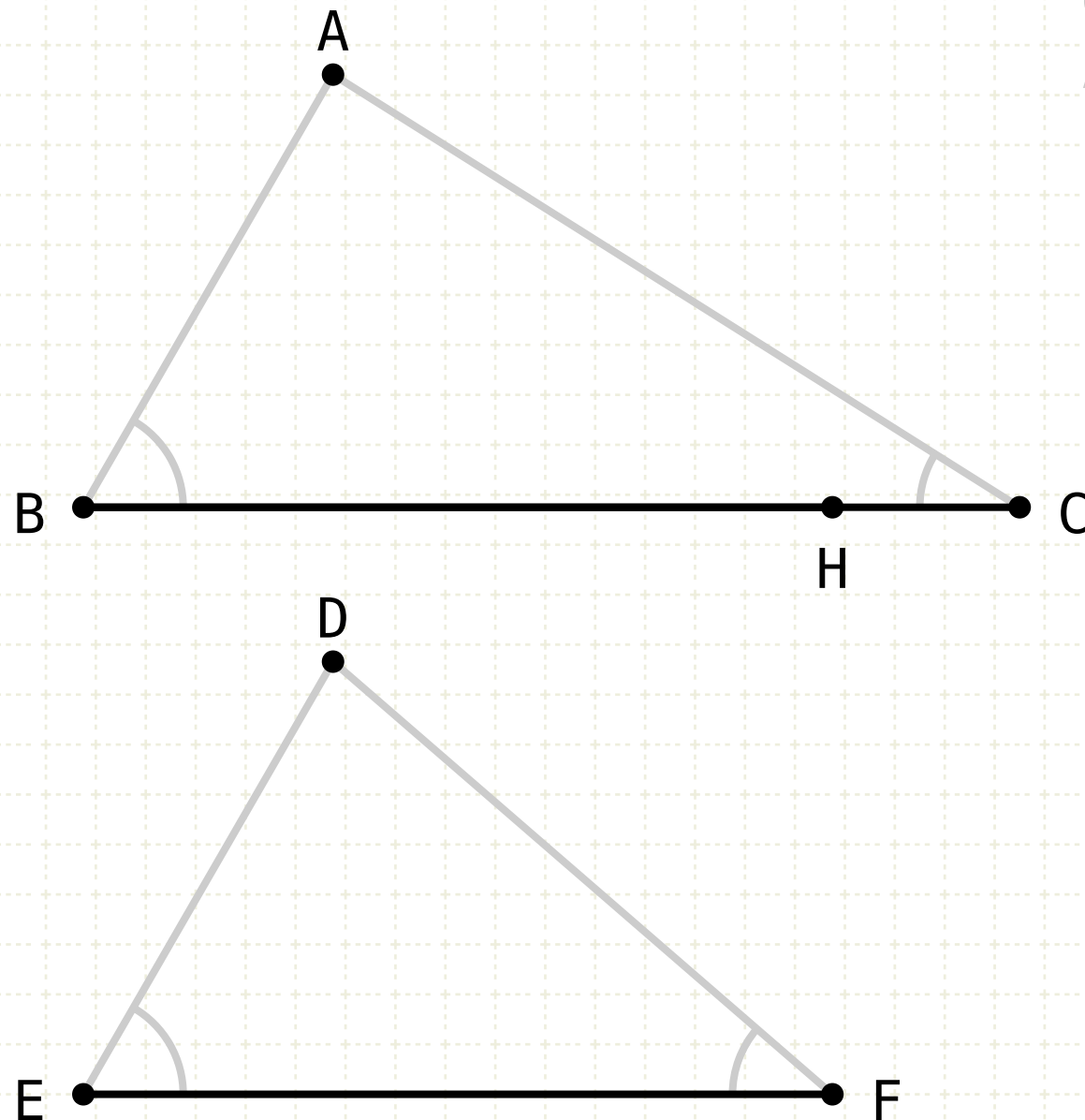
$$\varepsilon = \beta$$

$$\varphi = \gamma$$

$$AB = DE$$

$$BC > EF$$

$$BH = EF$$



In other words

Given two triangles ABC and DEF, where AB equals DE, and angles ABC and DEF are equal, and angles BCA and EFD are equal (SSA)

Then the two triangles are equal in all respects

Proof by Contradiction

Assume that BC is greater than EF

Create a point H such that BH equals EF (I.2)



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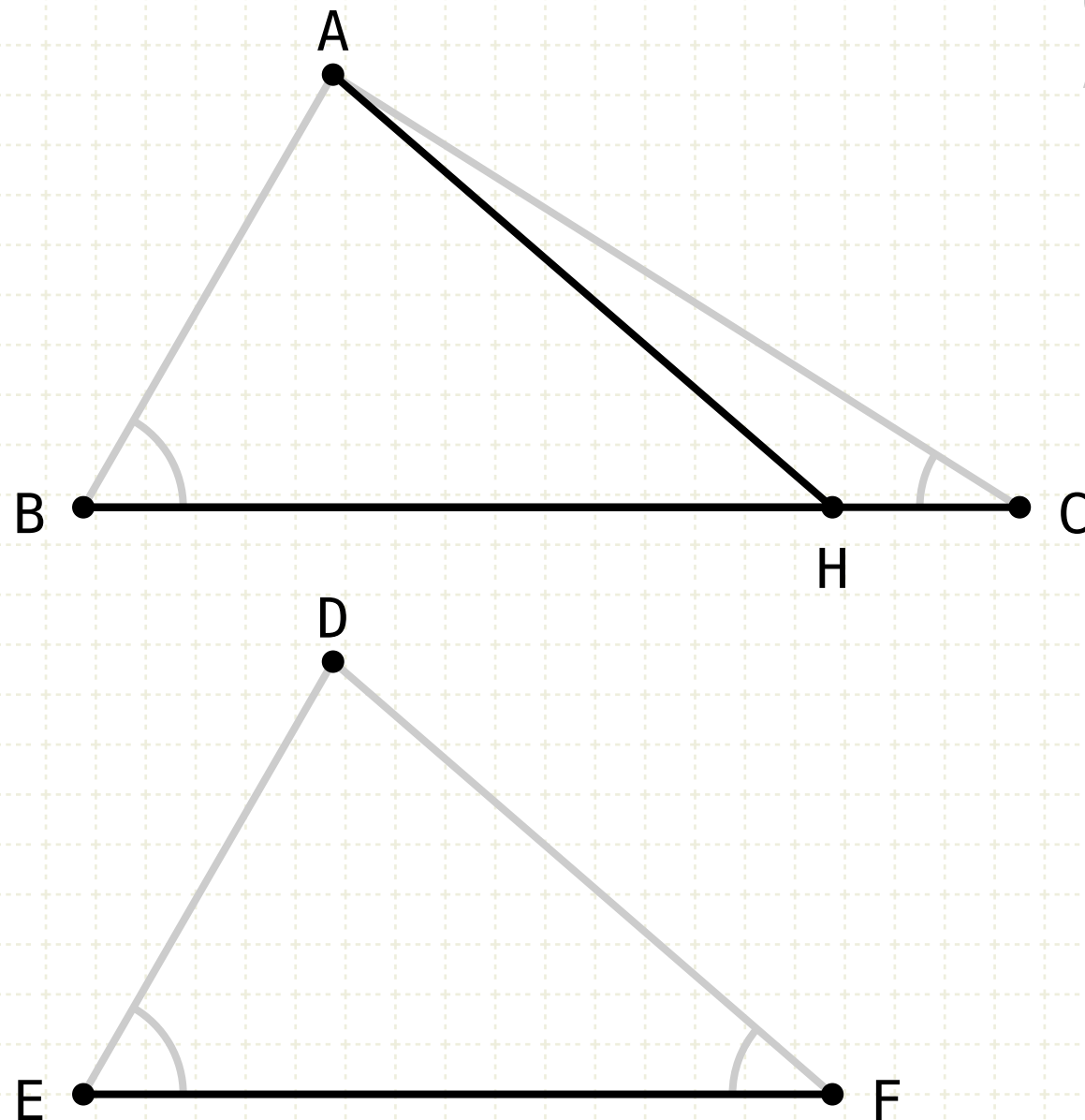
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Given two triangles ABC and DEF, where AB equals DE, and angles ABC and DEF are equal, and angles BCA and EFD are equal (SSA)

Then the two triangles are equal in all respects

Proof by Contradiction

Assume that BC is greater than EF

Create a point H such that BH equals EF (I.2)

Create line HA



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$$\varepsilon = \beta$$

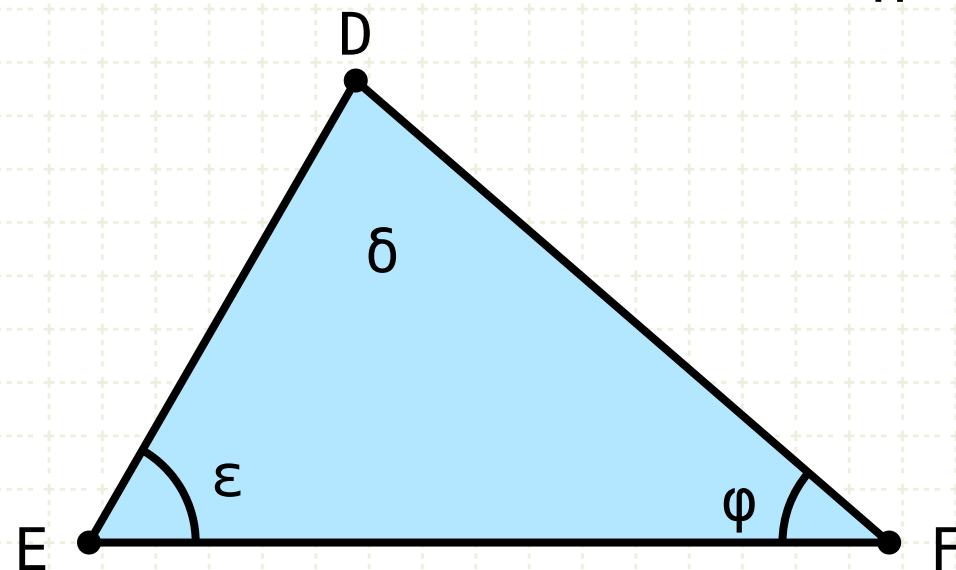
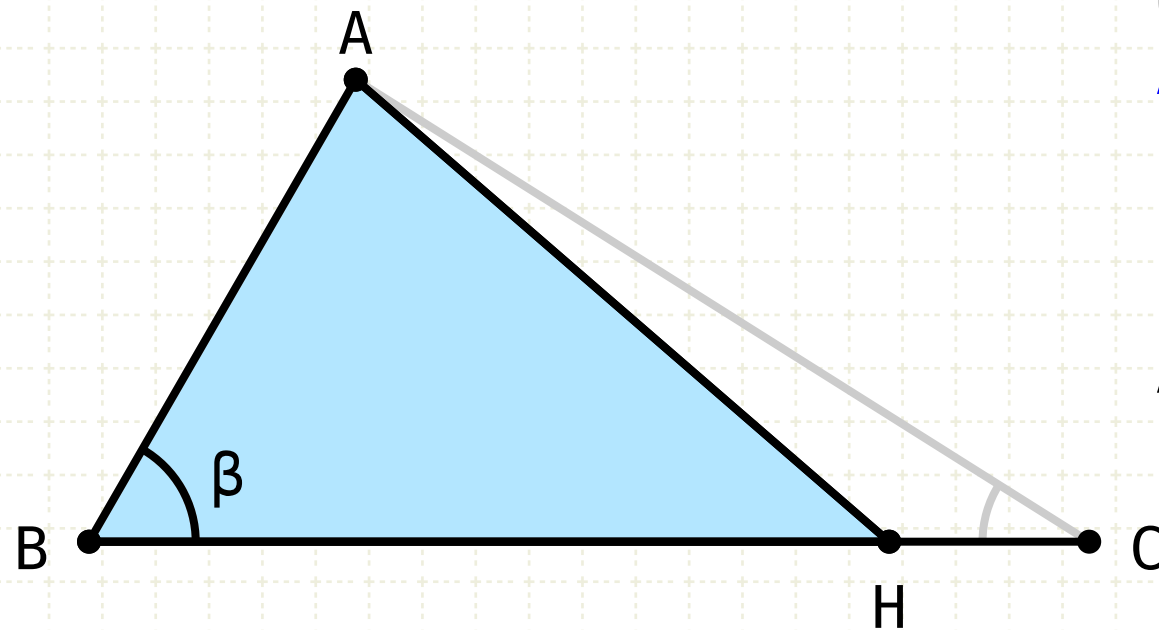
$$\varphi = \gamma$$

$$AB = DE$$

$$BC > EF$$

$$BH = EF$$

$$\triangle ABH \equiv \triangle DEF$$



In other words

Given two triangles ABC and DEF, where AB equals DE, and angles ABC and DEF are equal, and angles BCA and EFD are equal (SSA)

Then the two triangles are equal in all respects

Proof by Contradiction

Assume that BC is greater than EF

Create a point H such that BH equals EF (I-2)

Create line HA

Triangle ABH has two sides and an angle that is equivalent in triangle DEF, hence they are equal in all respects (I-4)

Proposition 26 of Book I

If two triangles have two angles equal to two angles respectively, and one side equal to one side, namely, either the side adjoining the equal angles, or that opposite one of the equal angles, then the remaining sides equal the remaining sides and the remaining angle equals the remaining angle.

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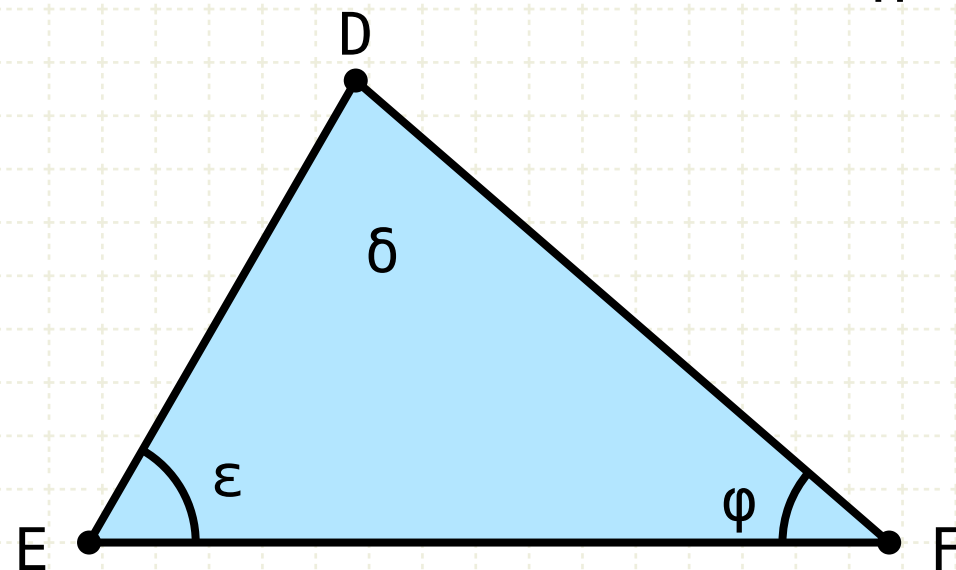
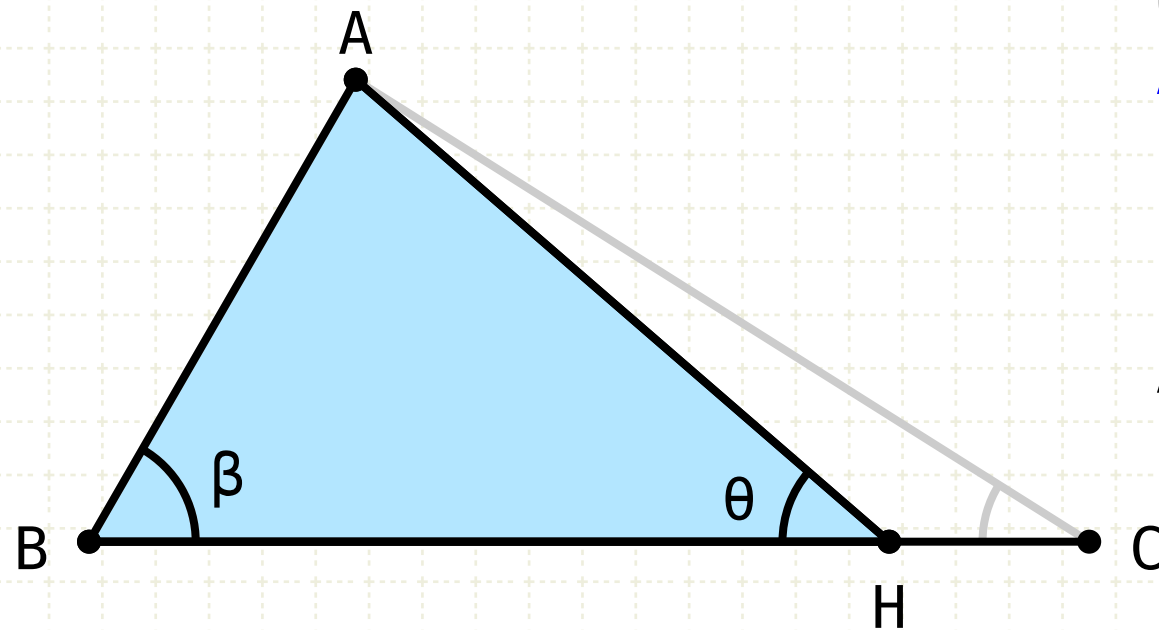
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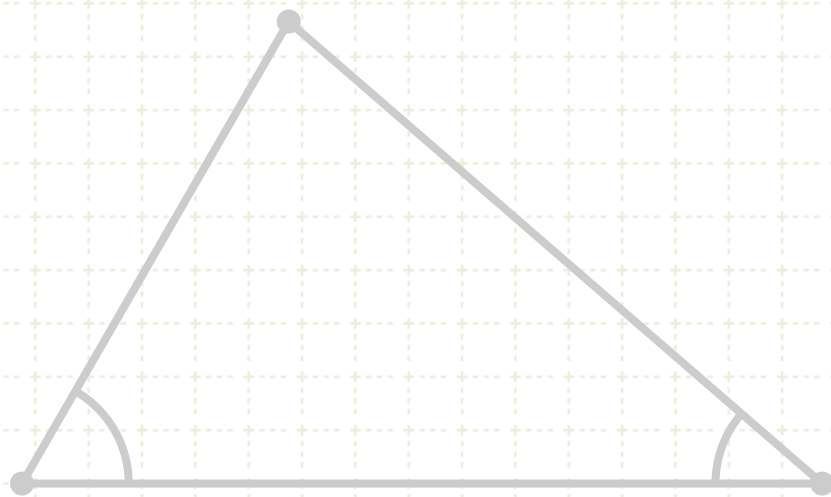
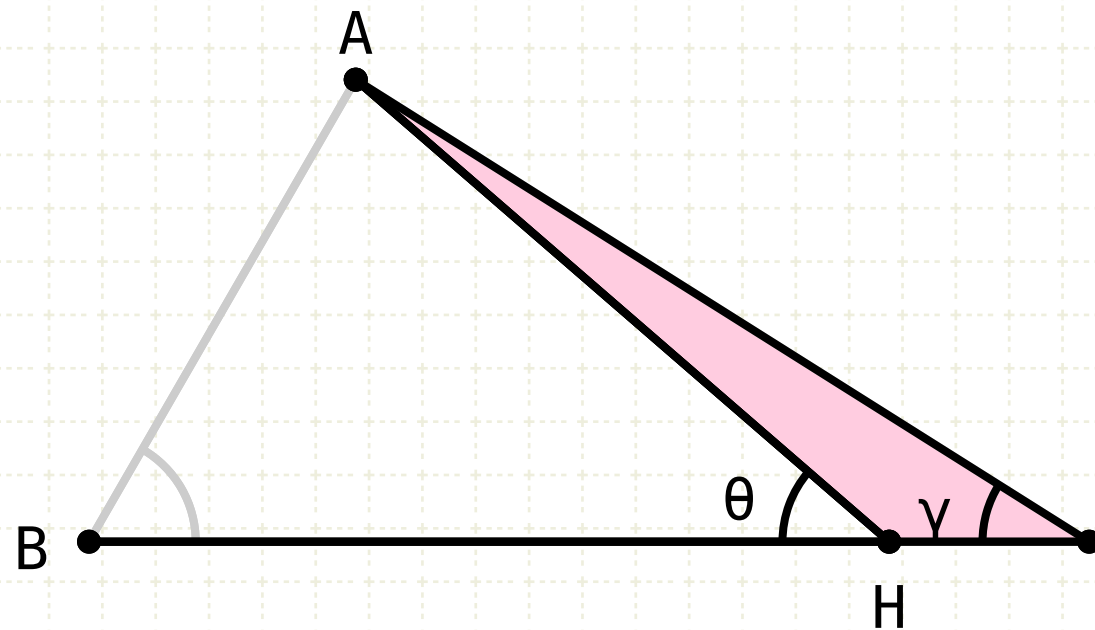
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$$\begin{aligned}\epsilon &= \beta \\ \varphi &= \gamma \\ AB &= DE \\ BC &> EF \\ BH &= EF \\ \triangle ABH &\equiv \triangle DEF \\ \theta &= \varphi \\ \theta &> \gamma\end{aligned}$$



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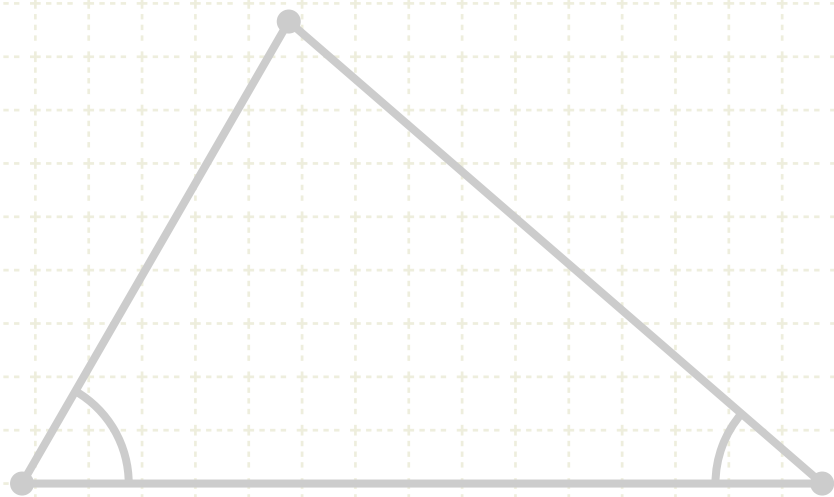
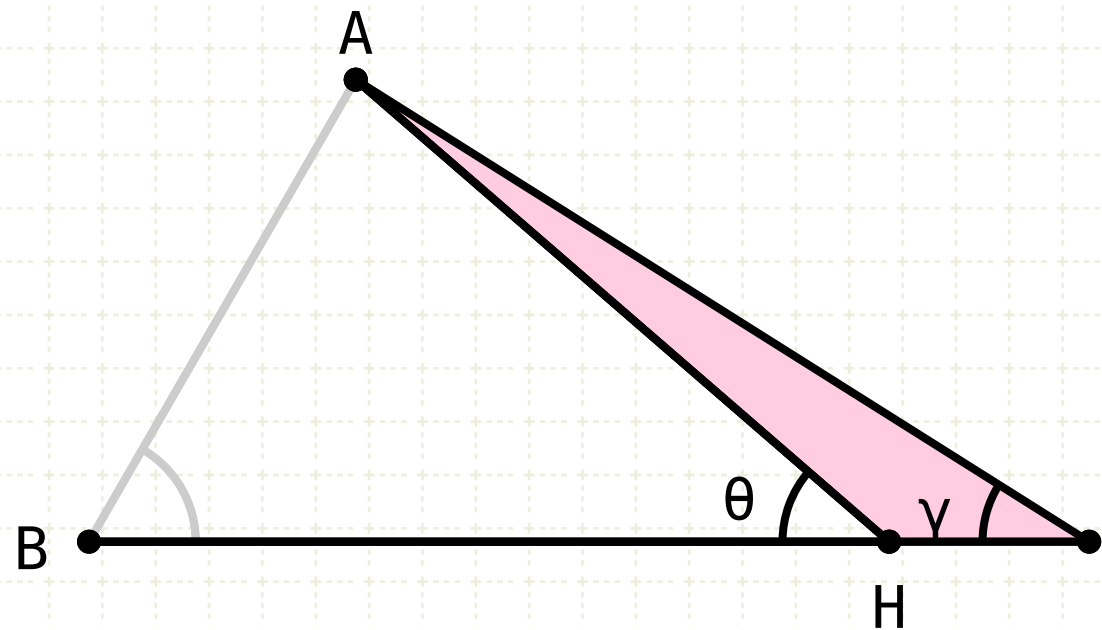
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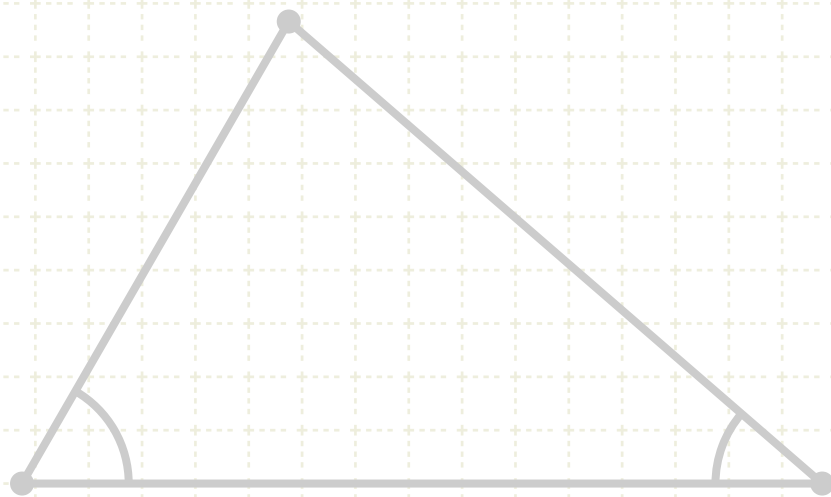
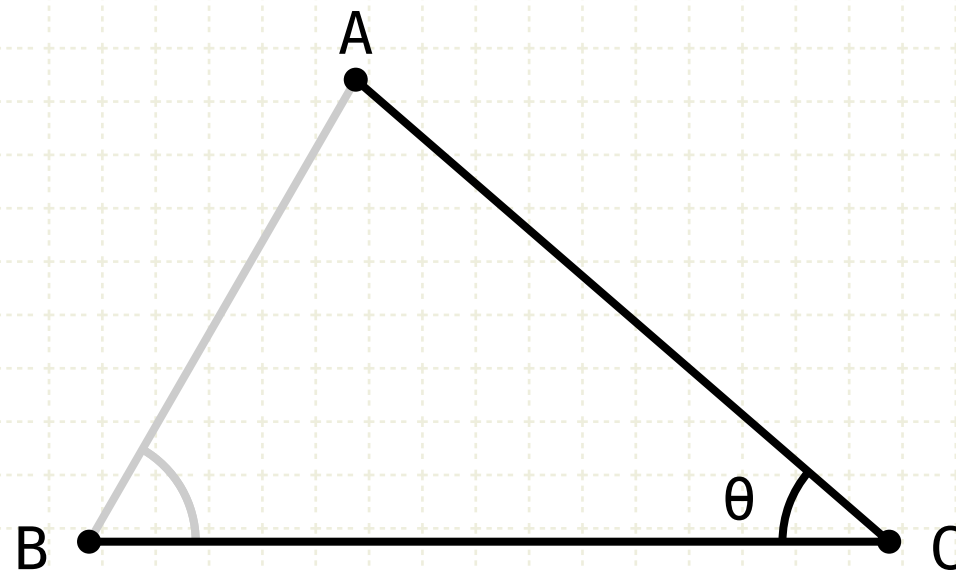
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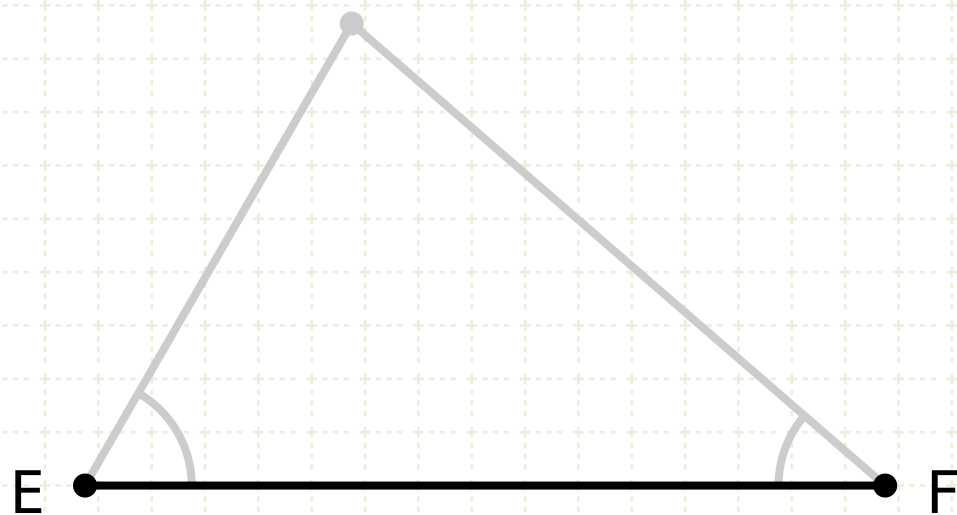
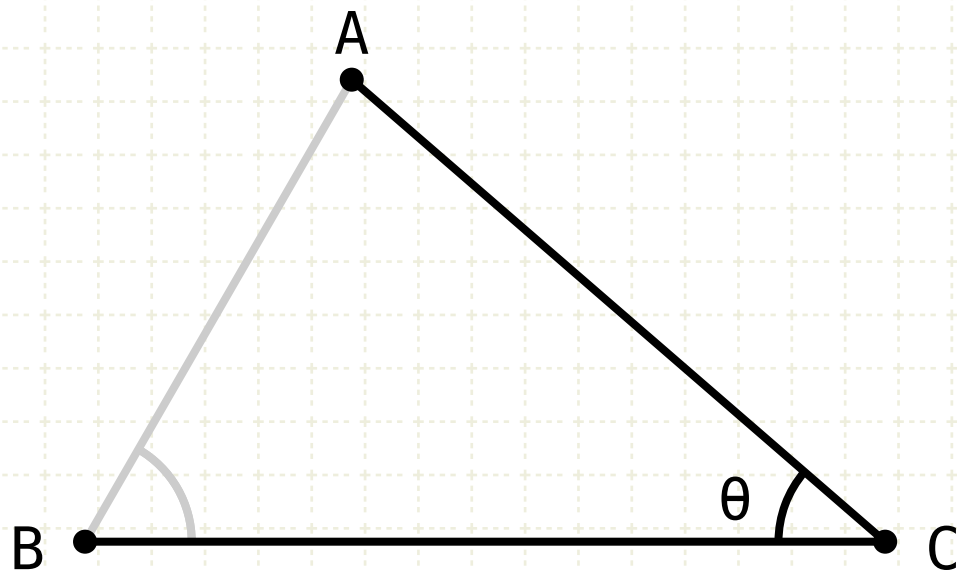
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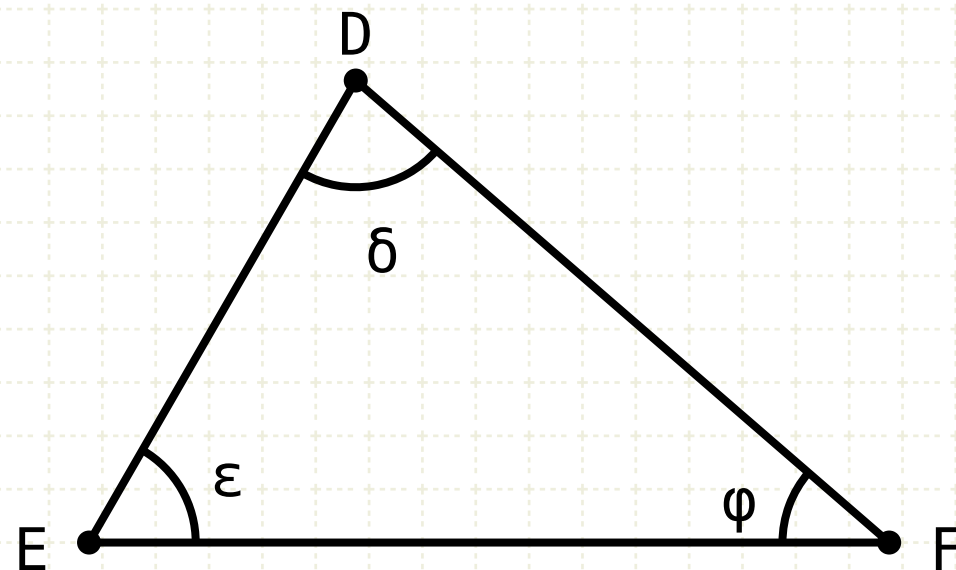
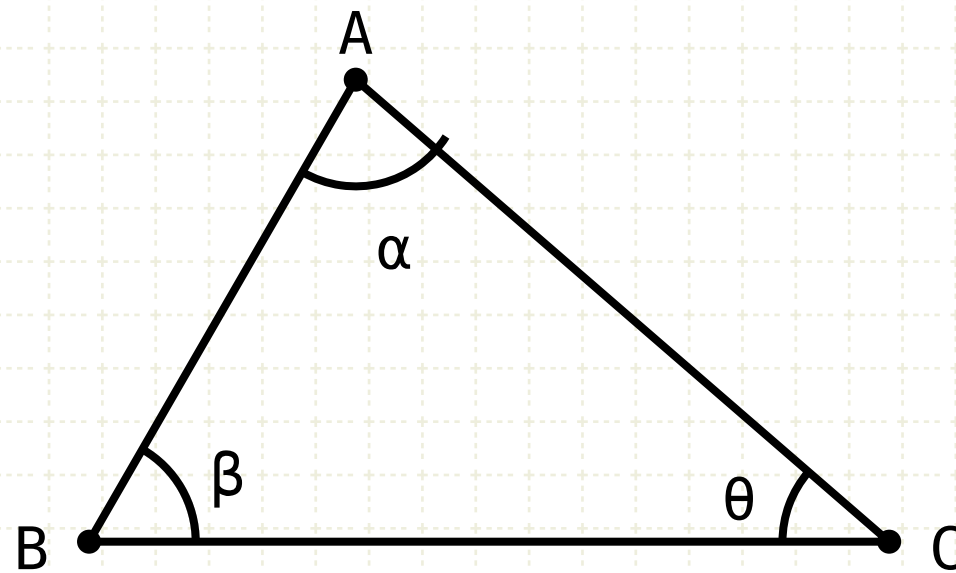
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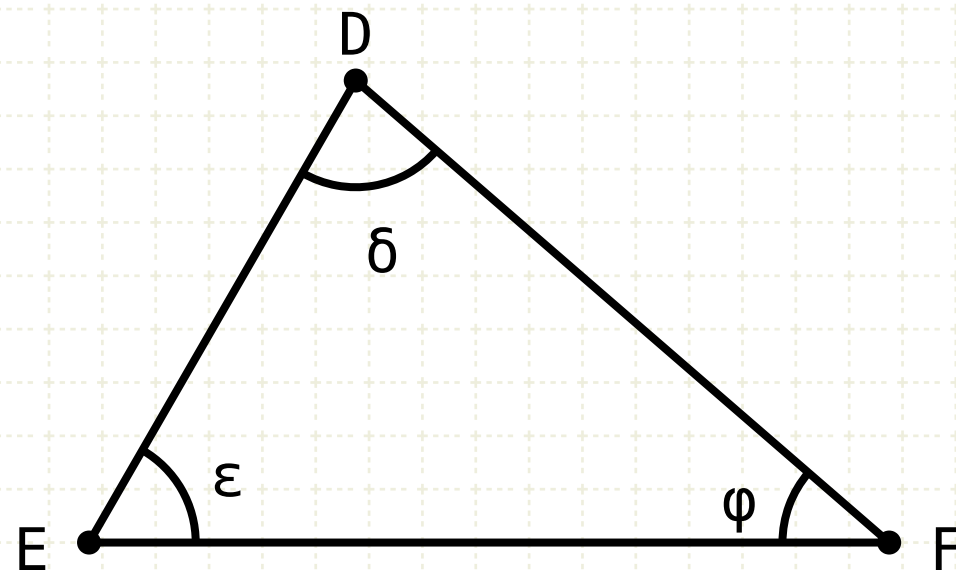
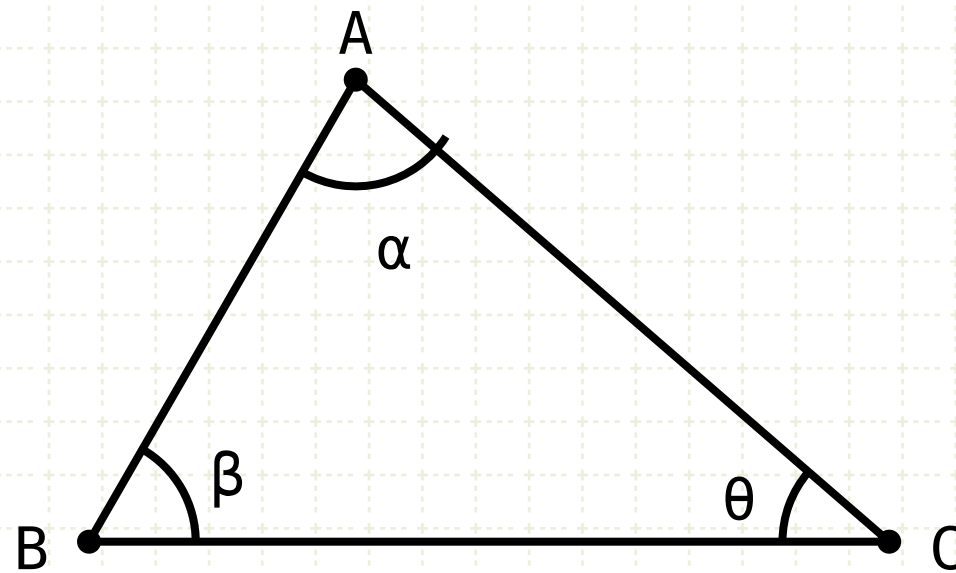
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