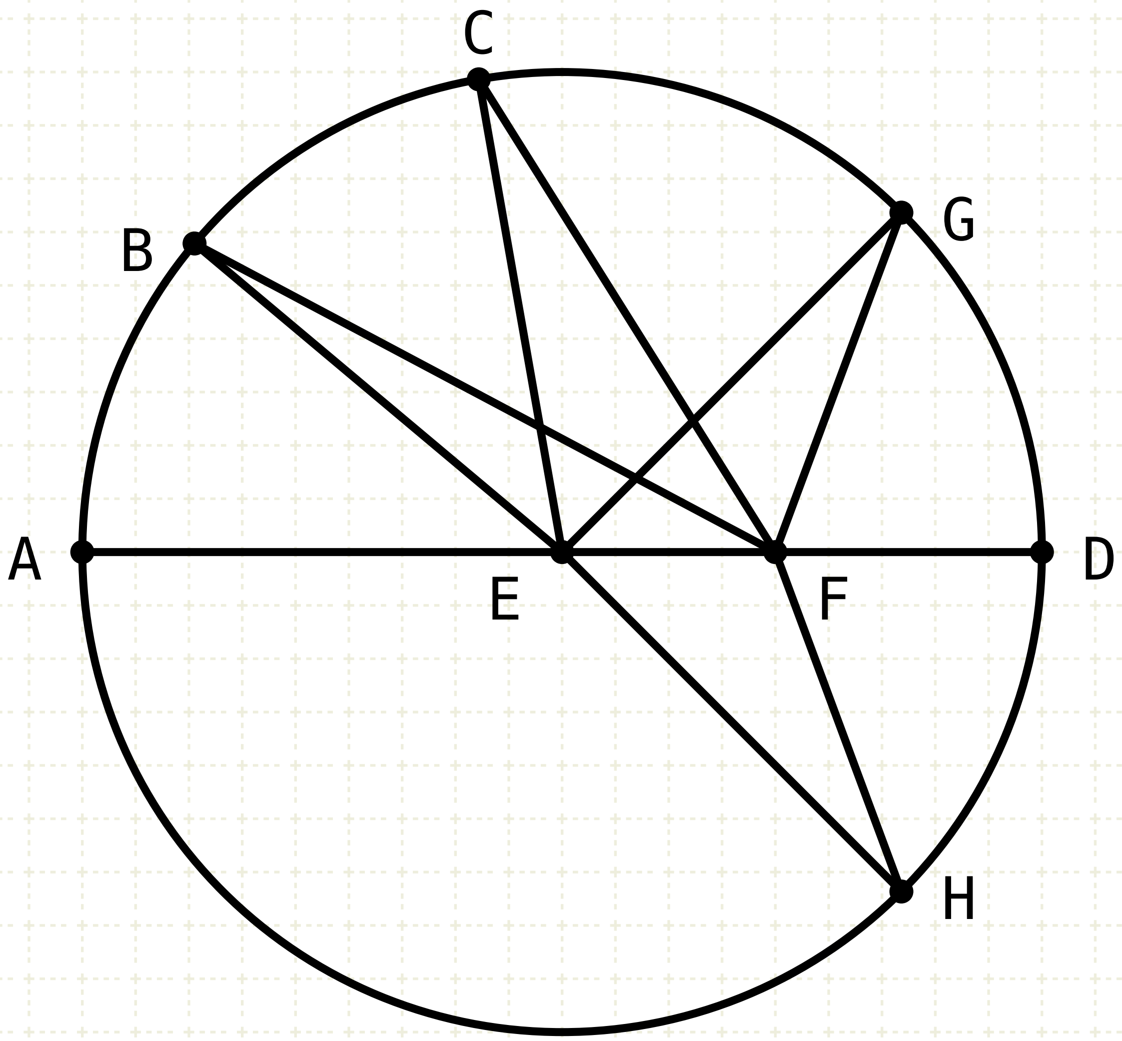

Euclid's Elements

Book III

A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)

Table of Contents, Chapter 3

1 To find the centre of a circle

2 A chord of a circle always lies inside the circle

3 A line through the centre of a circle bisects a chord, and vice versa

4 A line not through the centre of a circle does not bisect a chord

5 If two circles cut one another, they will not have the same center

6 If two circles touch one another, they will not have the same center

7 Consider two lines from a point inside a circle to the edge, the longer one will be the one closest to the longest part of the diameter passing through the original point

8 Consider two lines from a point outside a circle to the edge, the line closest to the centre will be longer on the concave side and shorter on the convex side

9 If three lines, starting at a point 'A' and touching the circle, are all equal, then 'A' is the centre of the circle

10 A circle does not cut a circle at more points than two

11 Point of contact between two internal circles, and their centres, are collinear

12 Point of contact between two external circles, and their centres, are collinear

13 A circle does not touch a circle at more points than one, whether it touch it internally or externally.

14 In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another.

15 The longest line in a circle is its diameter, shorter the farther away from the diameter

16 A line on the circle, perpendicular to the diameter, lies outside the circle

17 From a given point to draw a straight line touching a given circle

18 If line touches a circle, then it is perpendicular to the diameter that touches that point

19 If line touches a circle, then the centre of the circle lies on a line perpendicular to the original

20 The angle at the centre of a circle is twice that from an angle from the circumference

21 In a circle the angles in the same segment are equal to one another

22 The opposite angles of quadrilaterals in circles are equal to two right angles

23 On the same straight line there cannot be constructed two similar and unequal segments of circles on the same side

24 Similar segments of circles on equal straight lines are equal to one another

Table of Contents, Chapter 3

25 Given a segment of a circle, to describe the complete circle of which it is a segment.

26 In equal circles equal angles stand on equal circumferences

27 In equal circles angles standing on equal circumferences are equal to one another

28 In equal circles equal straight lines cut off equal circumferences

29 In equal circles equal circumferences are subtended by equal straight lines

30 To bisect a given circumference

31 In a circle the angle in the semicircle is right …

32 The angle between a tangent and a straight line cutting a circle is equal to the angle in the alternate segment

33 Construct a circle segment on a given line, such that the angle within the segment is equal to a given angle

34 Construct a circle segment on a given circle, such that the angle within the segment is equal to a given angle

35 If two circle chords intersect, the segments on one multiplied together equals the segments of the other multiplied together

36 Secant-tangent law

37 Converse of the secant-tangent law

Proposition 2 of Book III

If on the circumference on a circle two points be taken at random, the straight line joining the points will fall within the circle.

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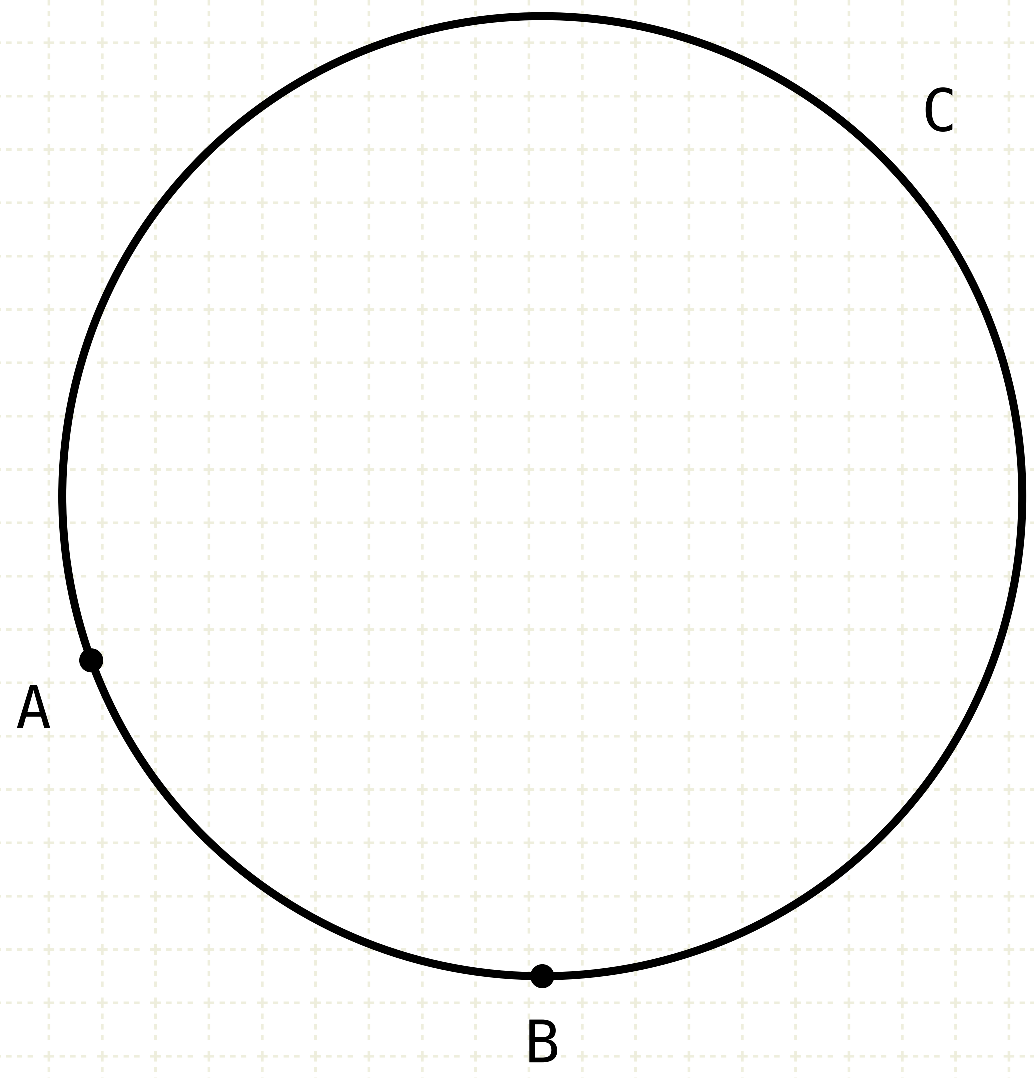

In other words

Let there be two points, A and B, randomly placed on the circumference of the circle

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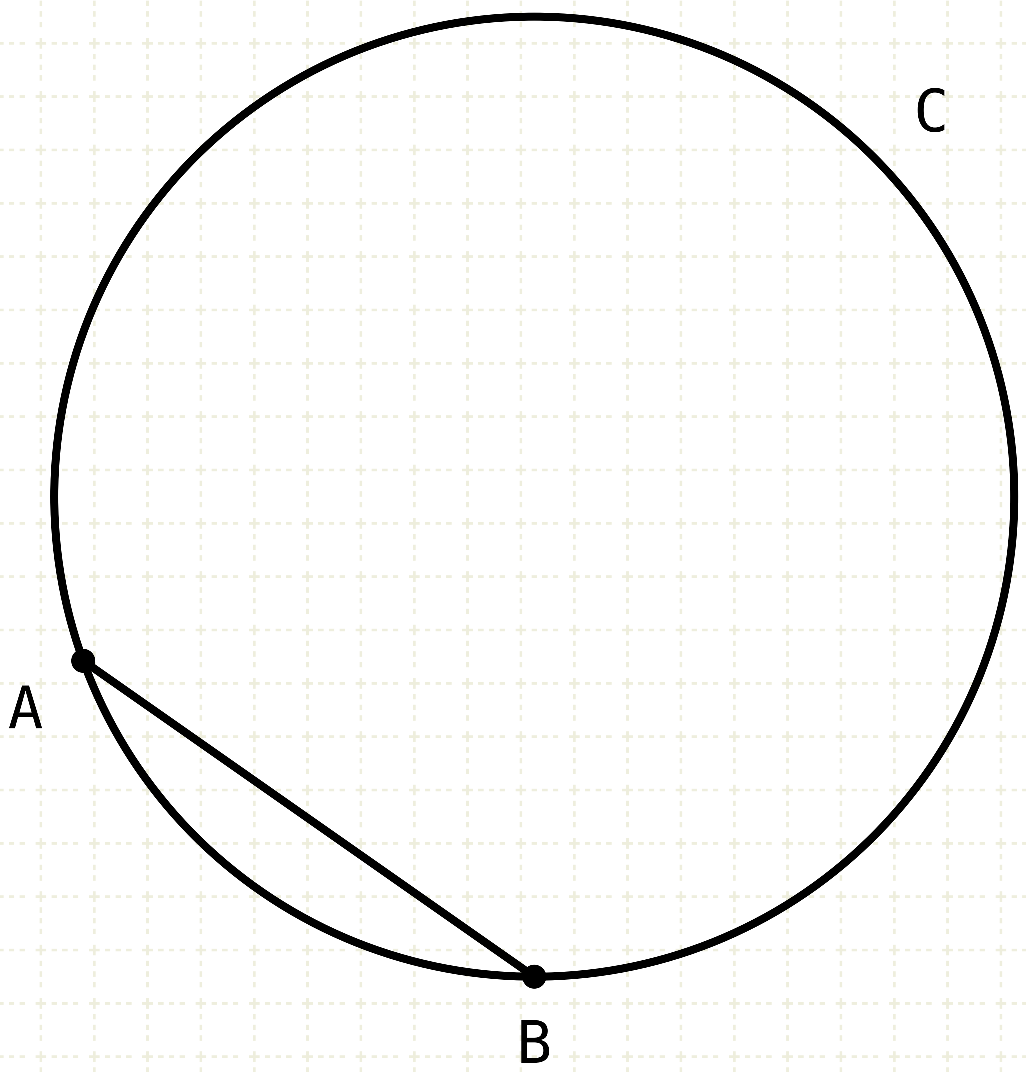

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Let there be two points, A and B, randomly placed on the circumference of the circle

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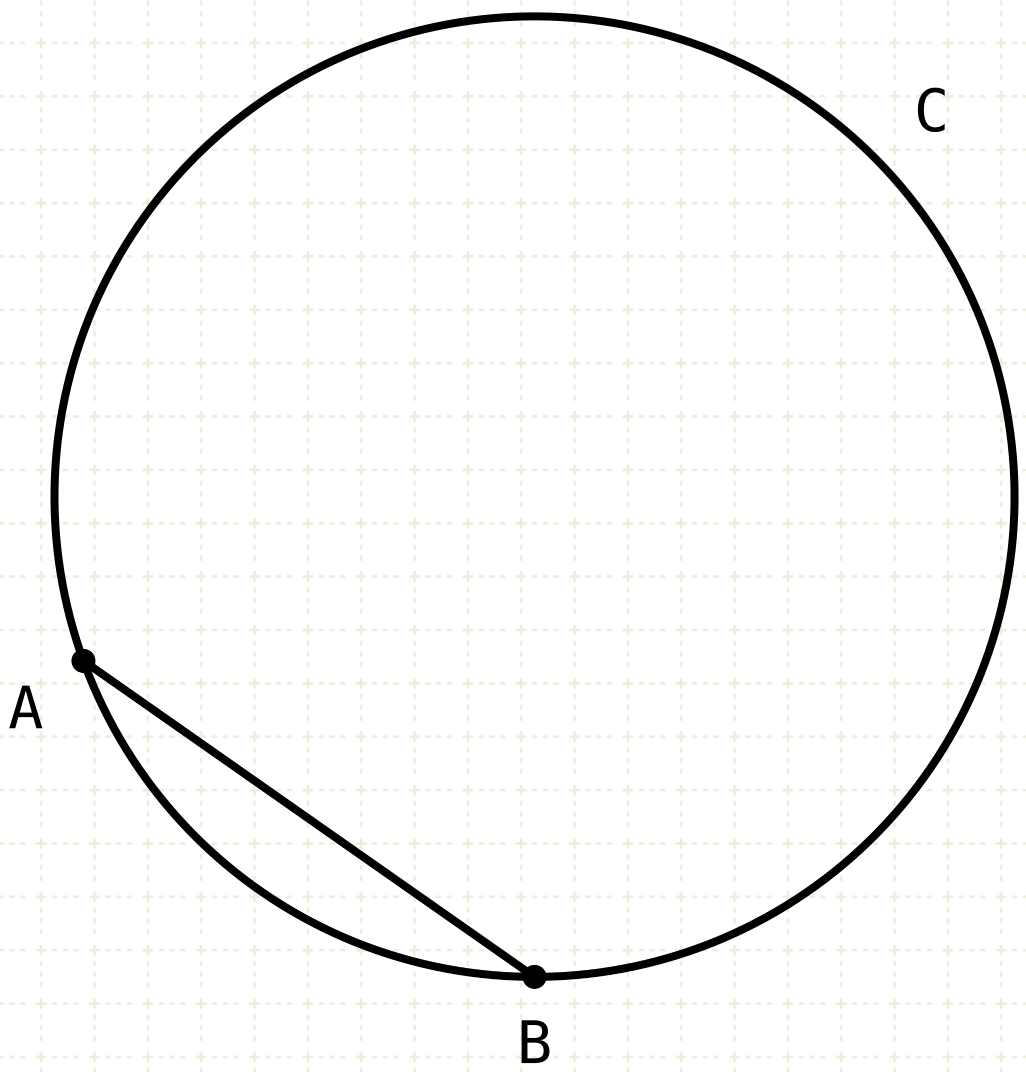

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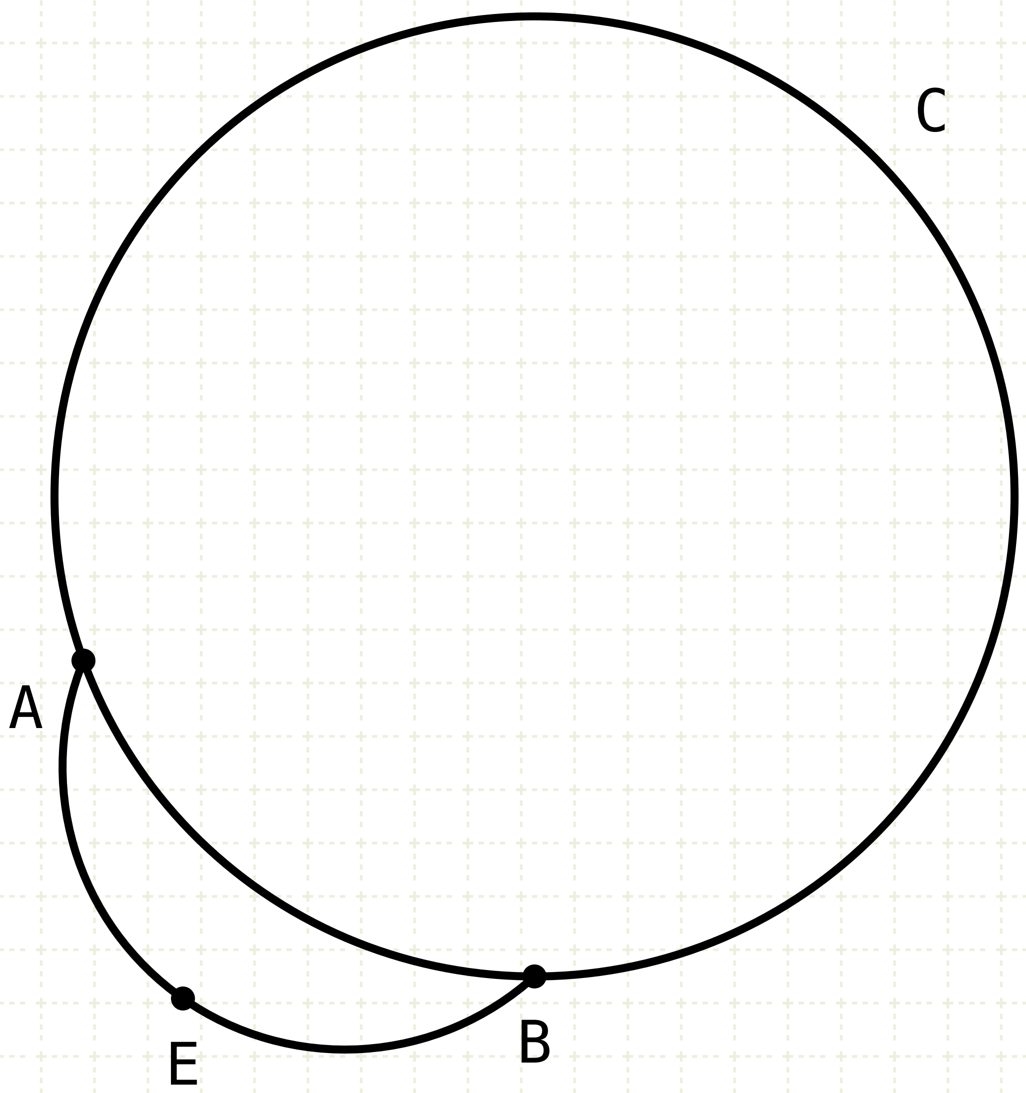

Pretend AB is a straight line

If AB is a straight line

and E is outside the circle…

DE > DF

In other words

Let there be two points, A and B, randomly placed on the circumference of the circle

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Proof by contradiction

Let E be a point on the straight line AB, and let it be outside of the circle

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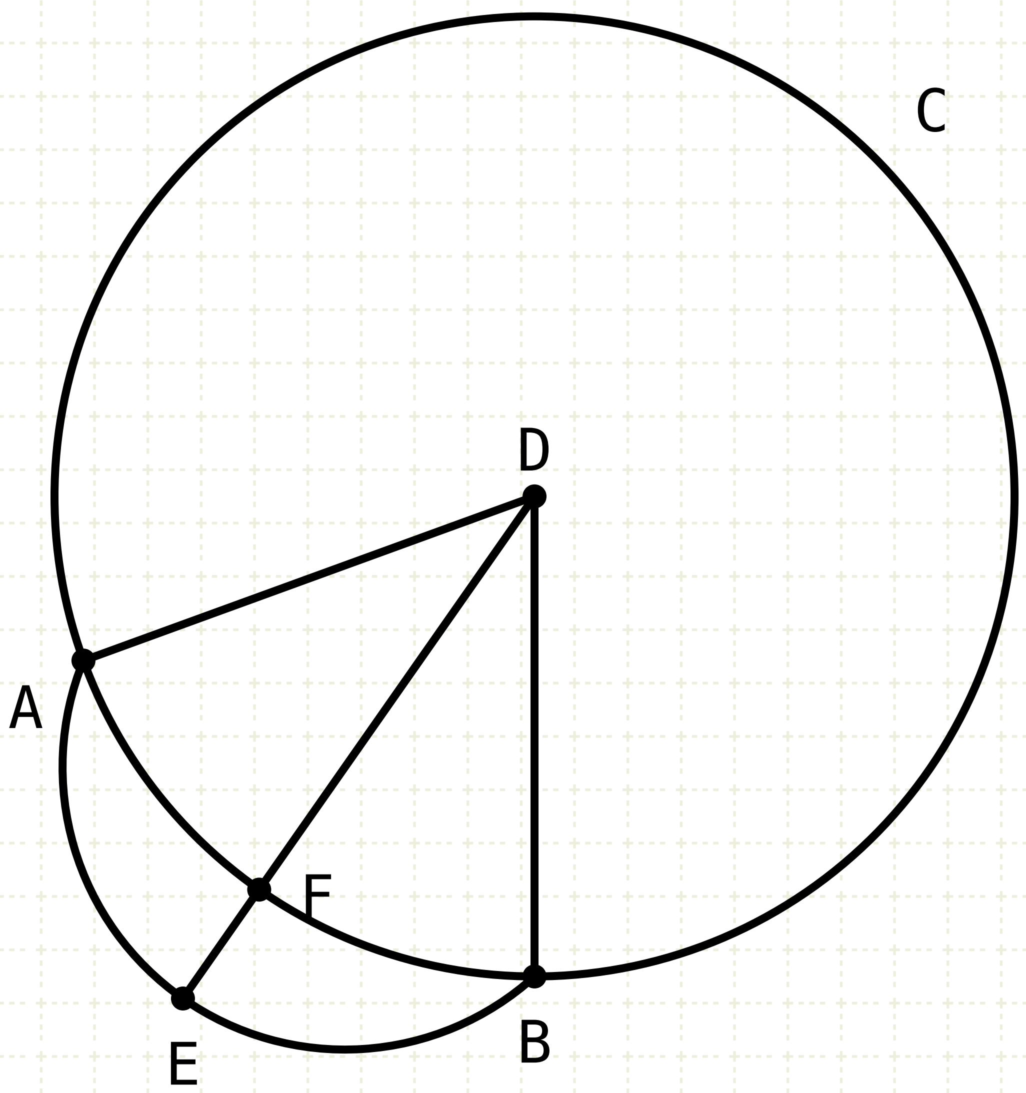

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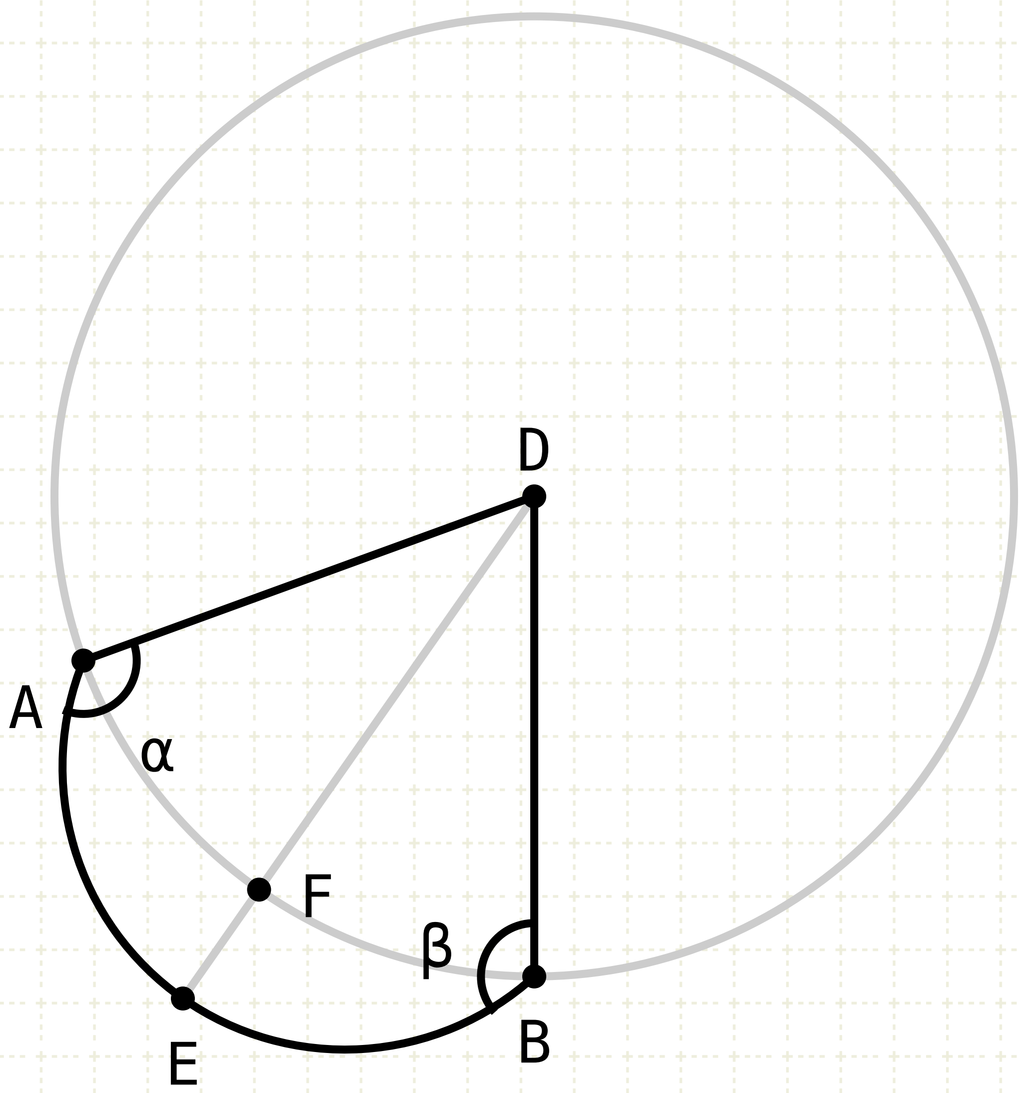

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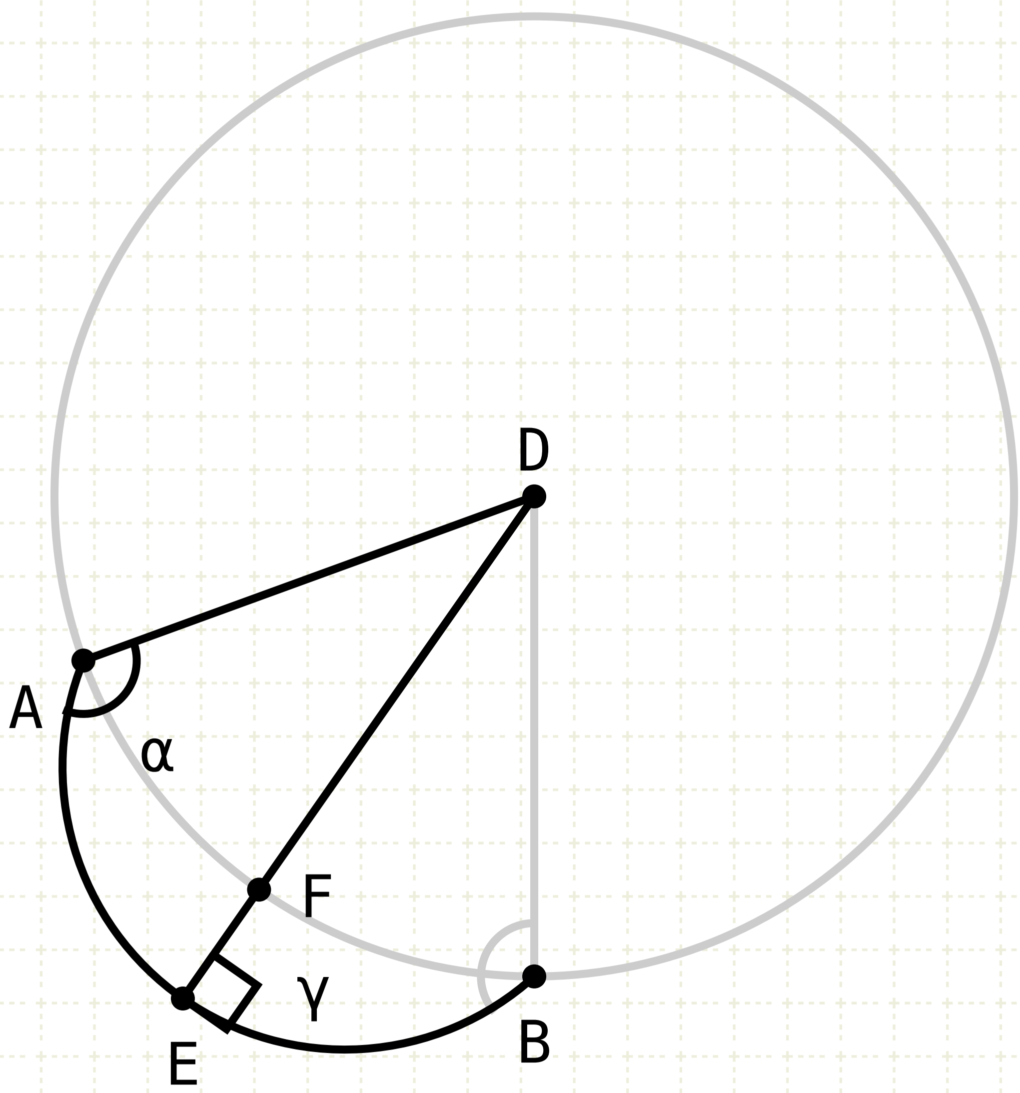

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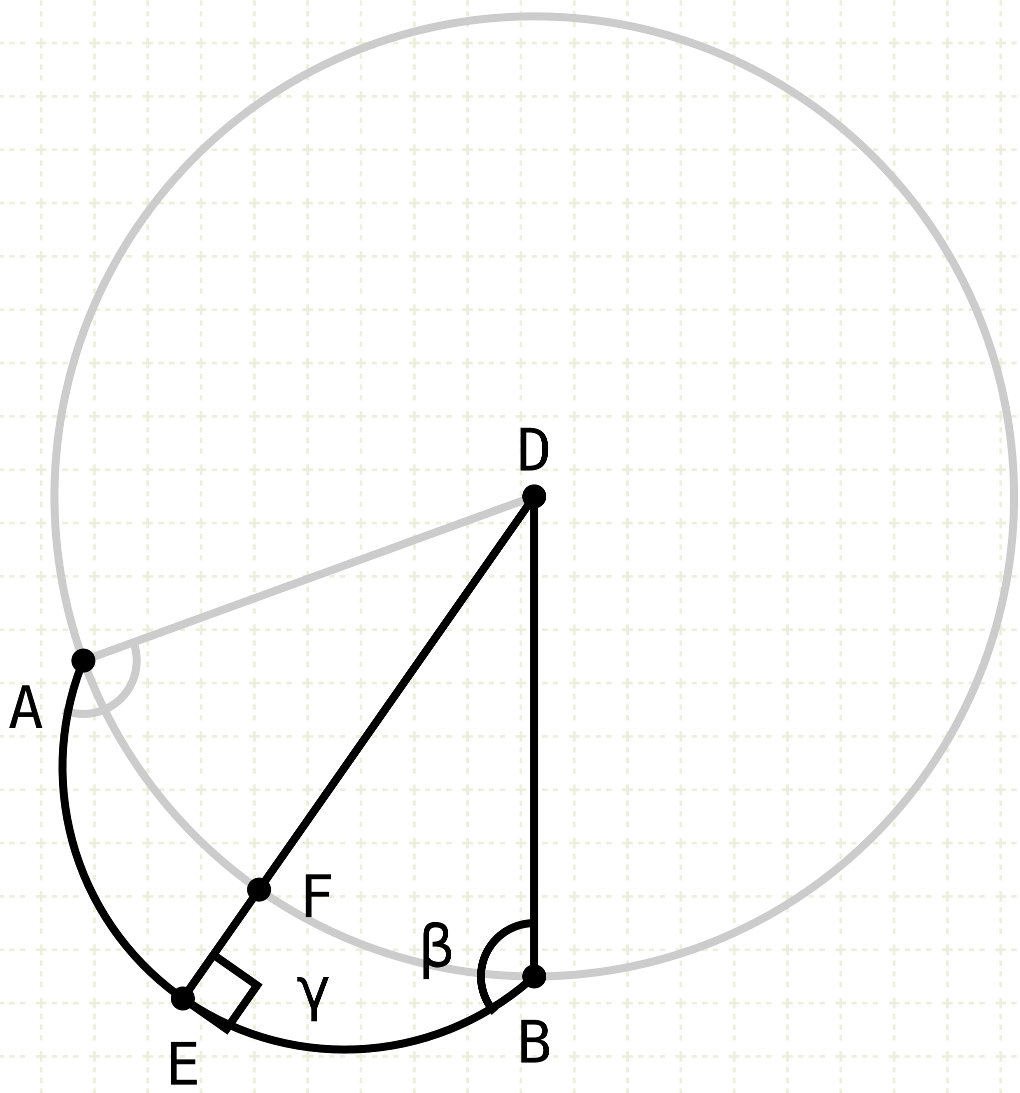

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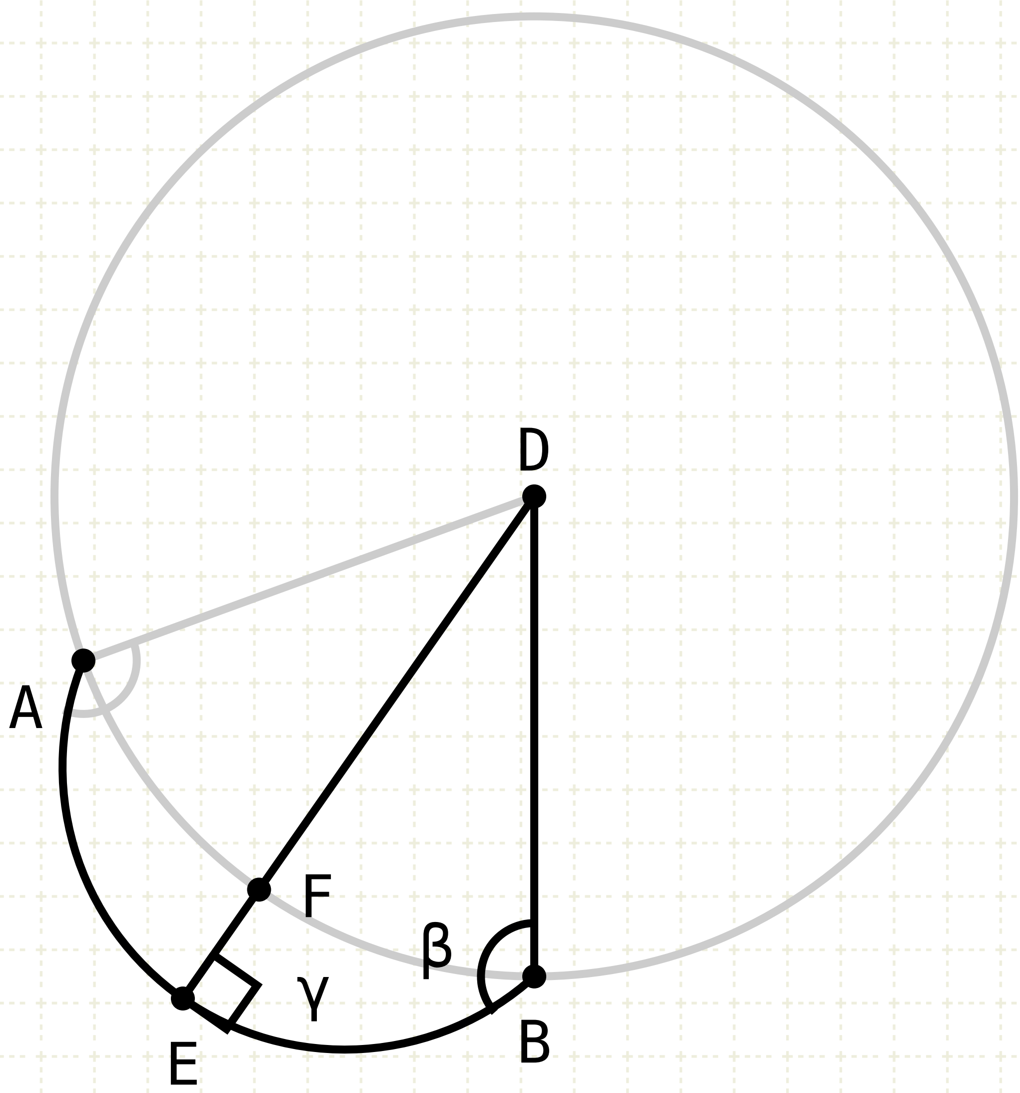

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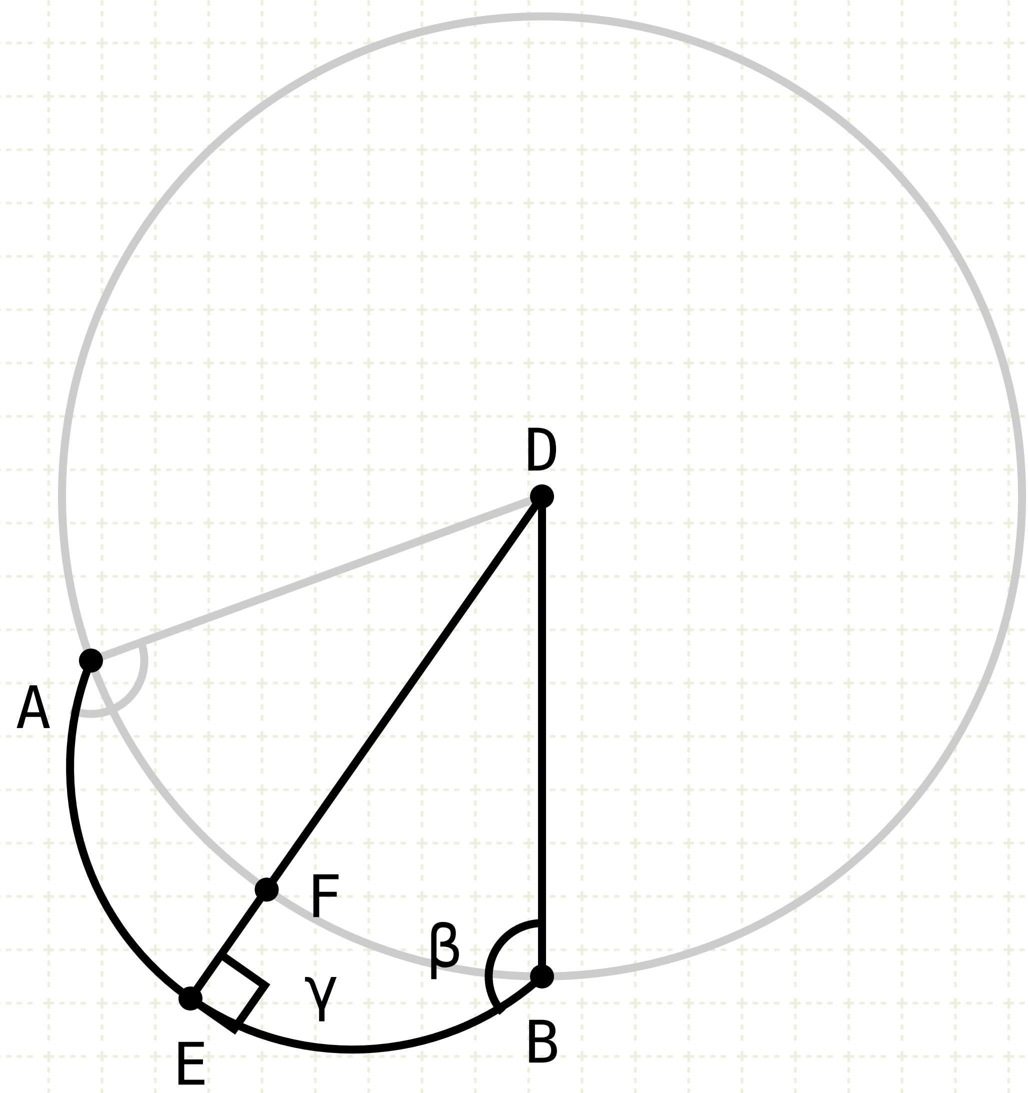

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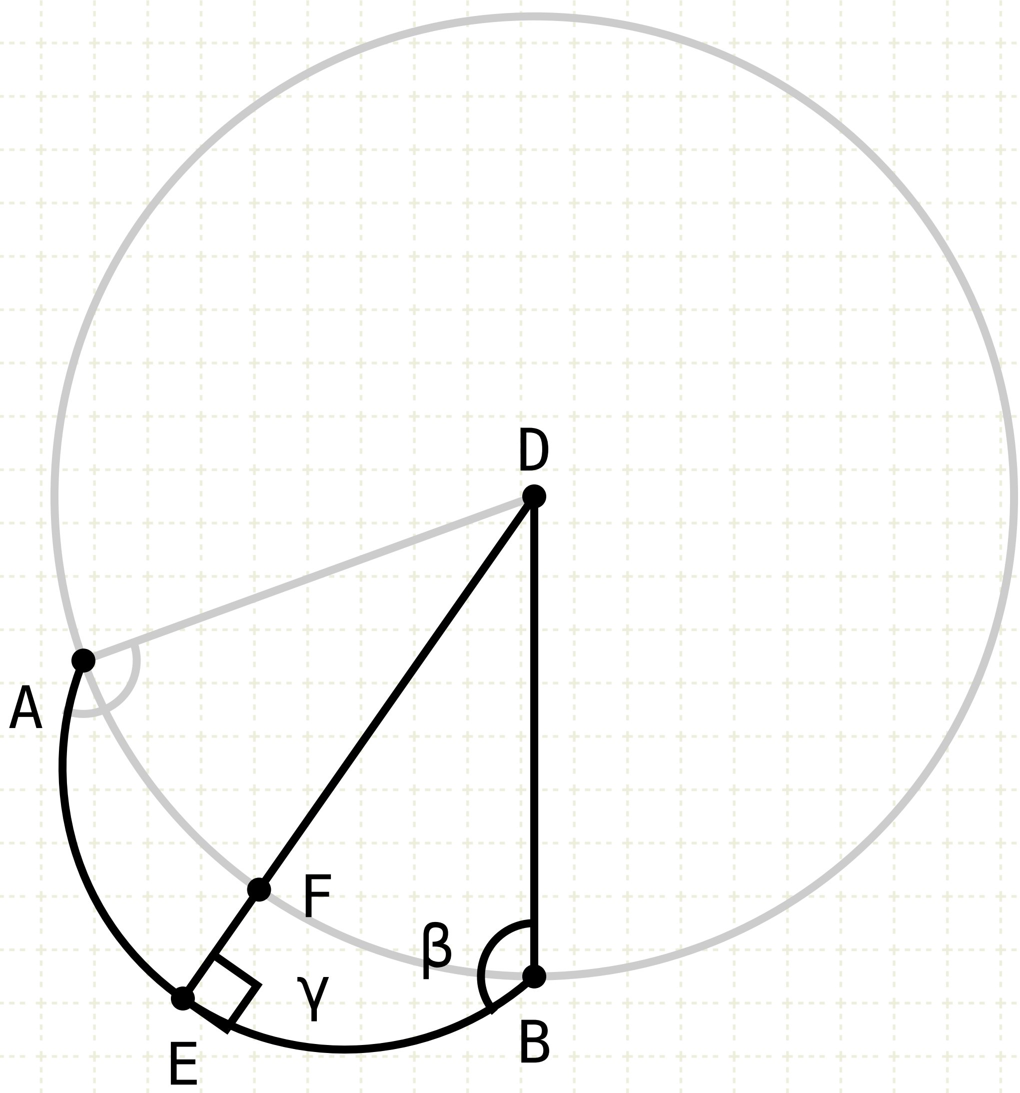

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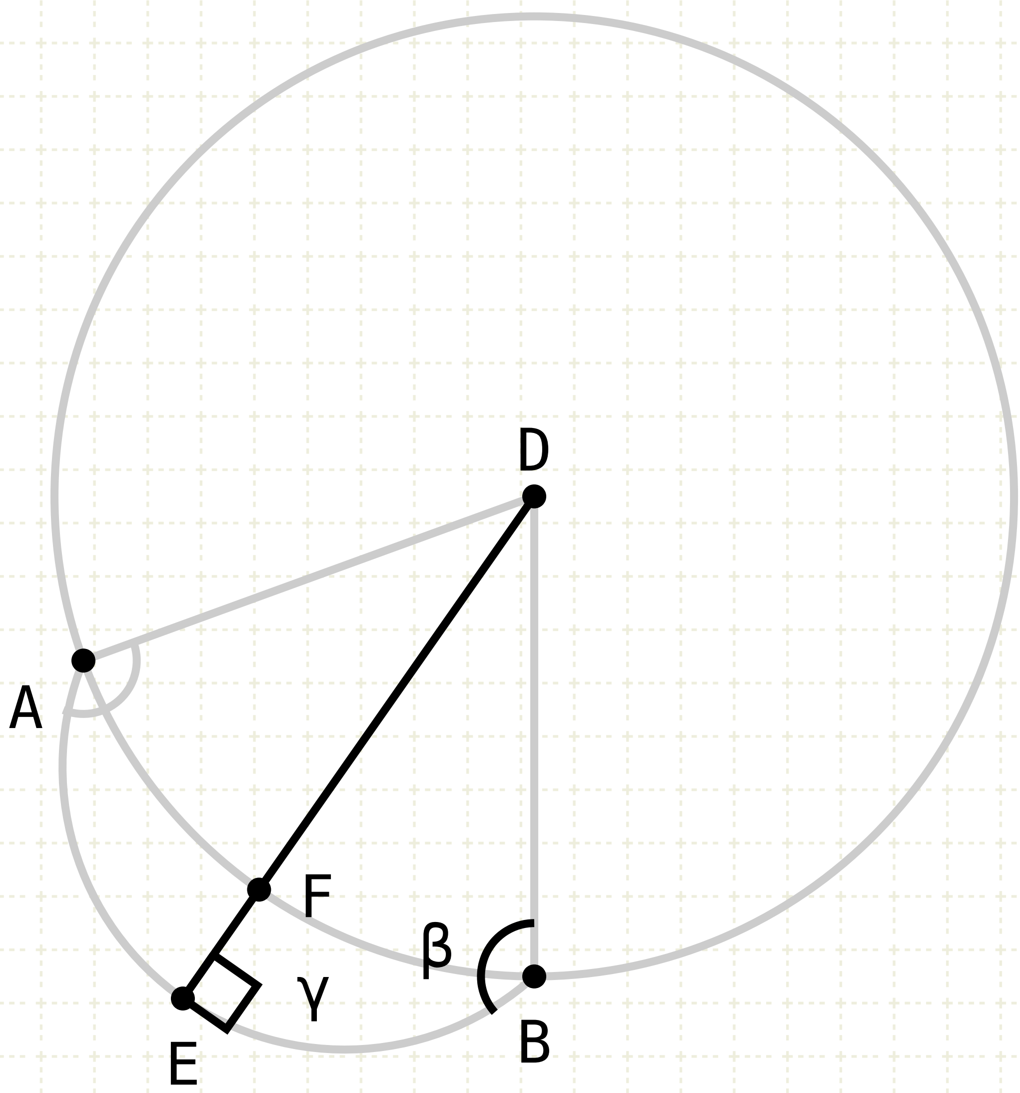

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But DE is larger than DF (by definition), so we have a logical inconsistency

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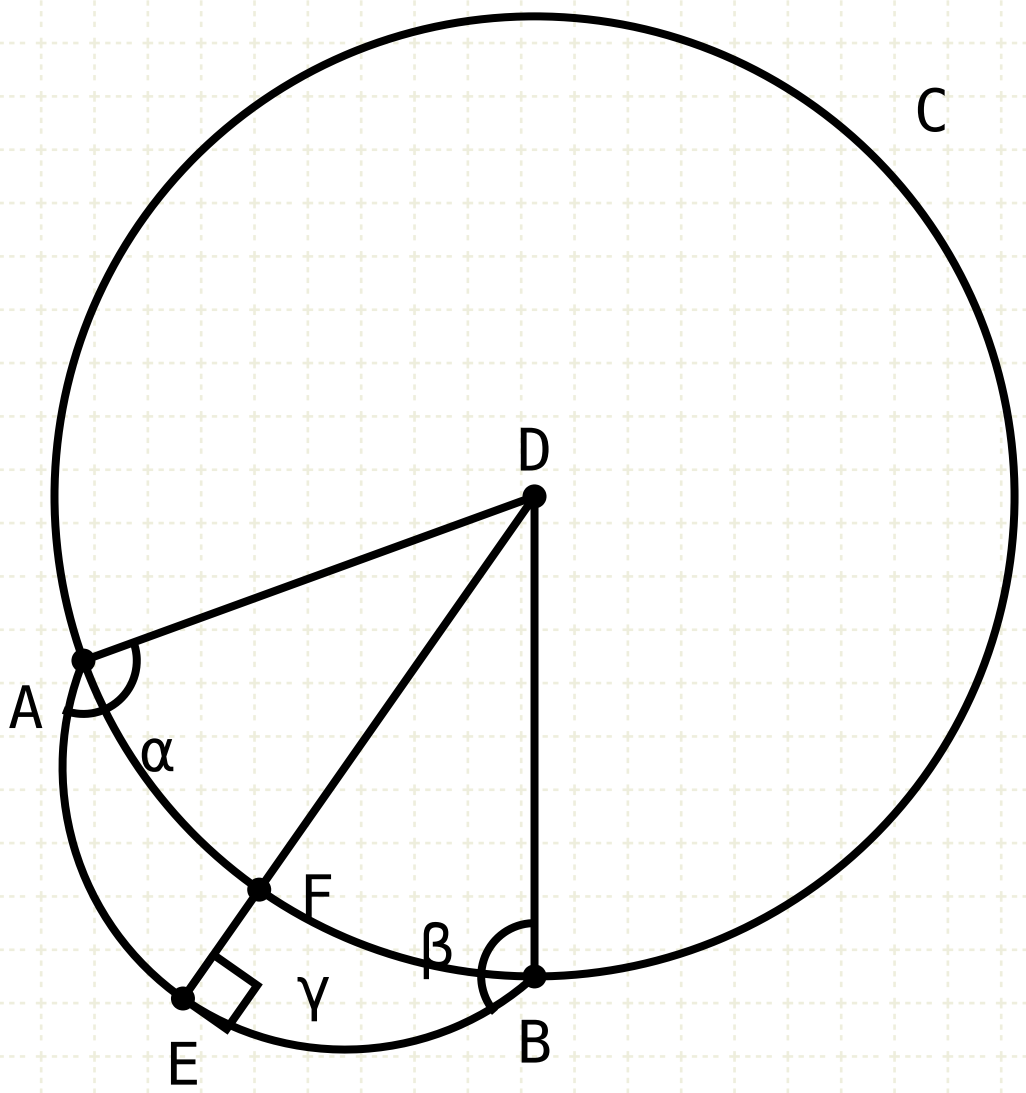

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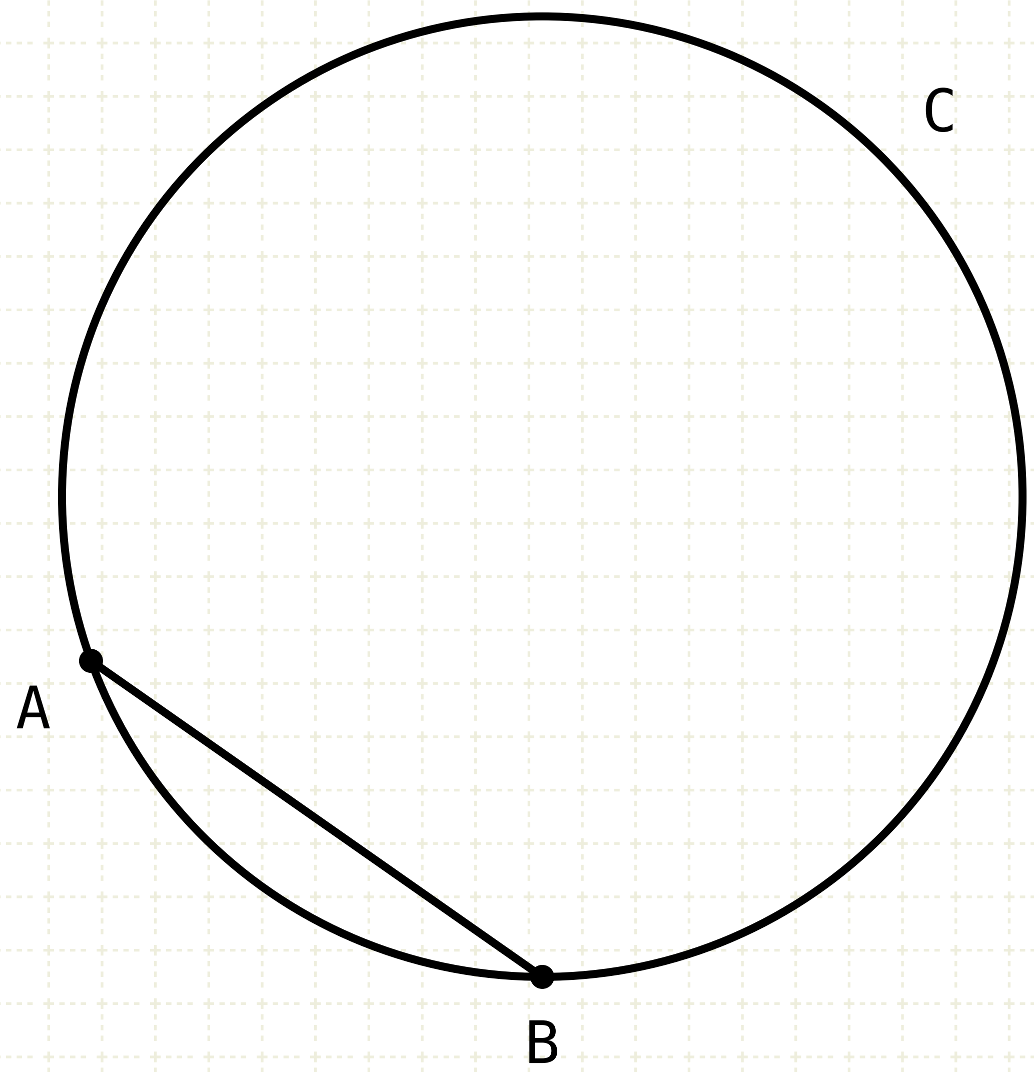

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AB is a straight line and inside the circle

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