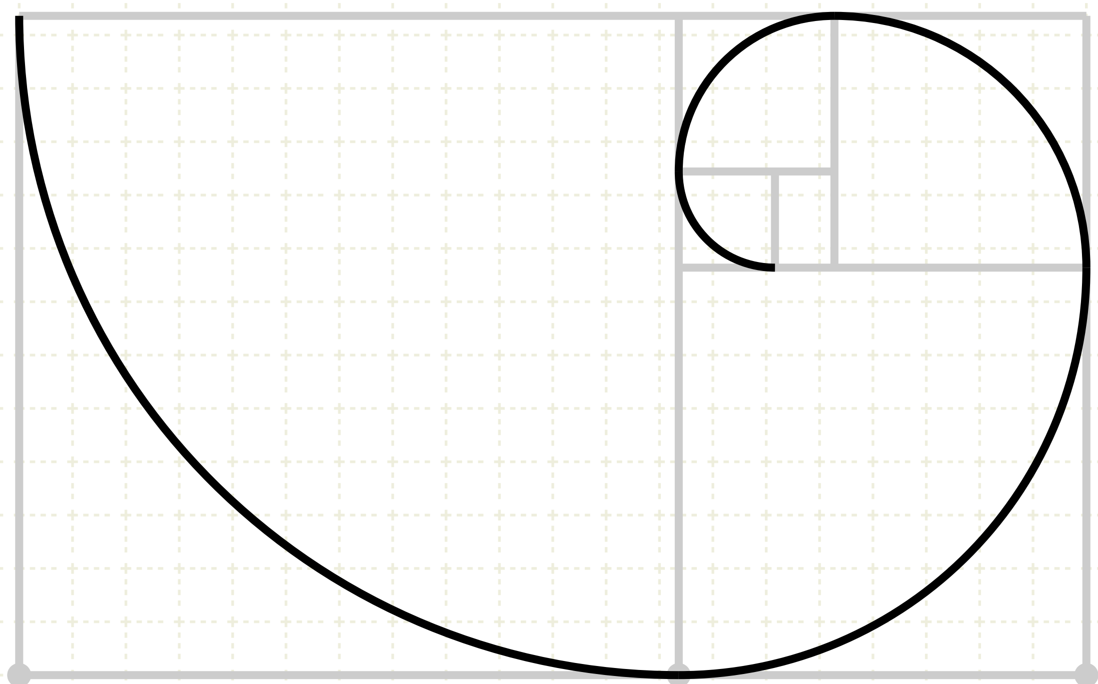
Euclid's Elements

Book VI



One can state, without exaggeration, that the observation of and the search for similarities and differences are the basis of all human knowledge.

Alfred Nobel

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Proposition 19 of Book VI

Similar triangles are to one another in the duplicate ratio of the corresponding sides

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Similar triangles are to one another in the duplicate ratio of the corresponding sides

Definition - Duplicate Ratio (V.Def.9)

If A is to B as B is to C, then the ratio of A to C is the duplicate ratio of A to B

A:B=B:C → A:C duplicate ratio of A:B

Examples:

4:6 = 6:9 (both equal 2:3)

∴ 4:9 is duplicate ratio of 2:3

4:10 = 10:25 (both equal 2:5)

∴ 4:25 is duplicate ratio of 2:5

Fractions:

Using fractions, if 'a' over 'b' equals 'b' over 'c', what is 'a' over 'c' ?

$$\frac{a}{b} = \frac{b}{c}, \quad \frac{a}{c} = ?$$

Multiply both sides of the equation by 'a' over 'b'

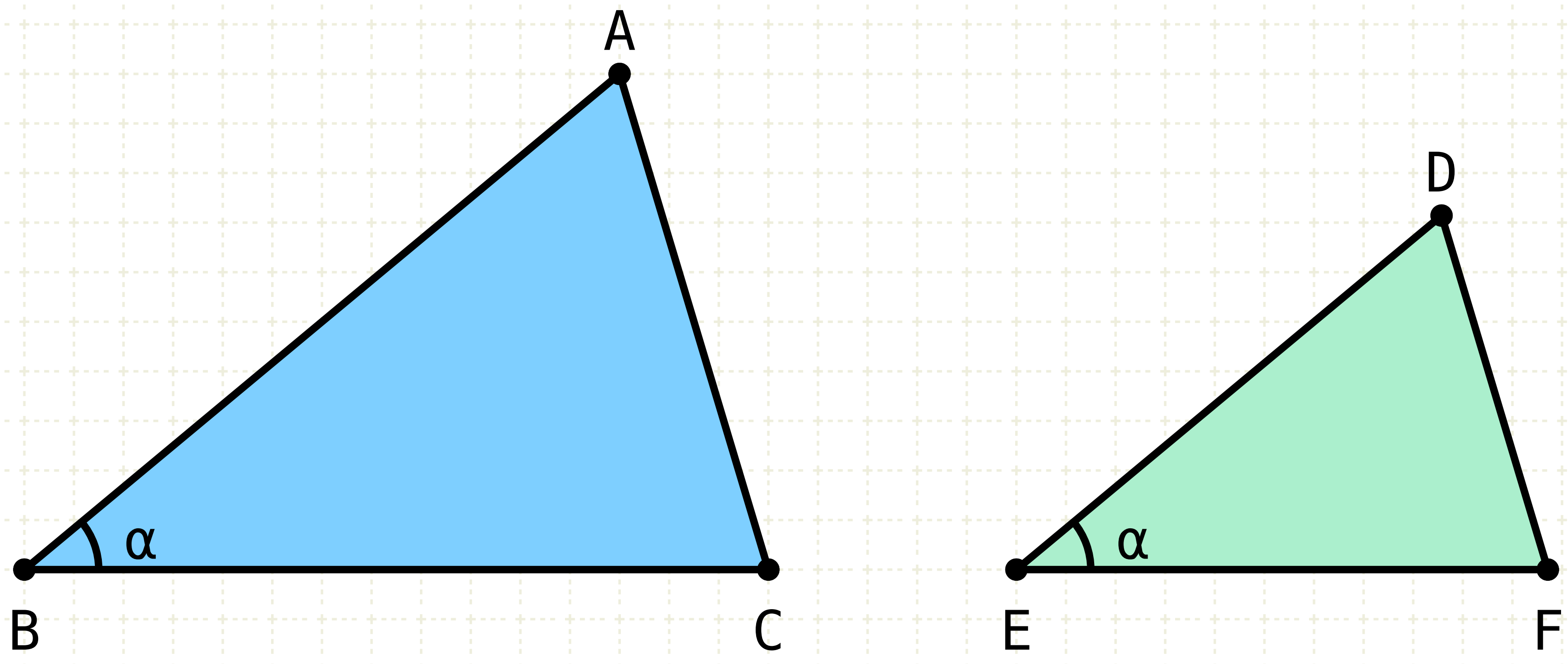
$$\frac{a}{b}\times\frac{a}{b} = \frac{b}{c}\times\frac{a}{b} \qquad \frac{a\times a}{b\times b} = \frac{1}{c}\times\frac{a}{1} \qquad \frac{a^2}{b^2} = \frac{a}{c}$$

Thus the duplicate ratio can be written as:

A:B = B:C → A:C = $(A:B)^2$

Proposition 19 of Book VI

Similar triangles are to one another in the duplicate ratio of the corresponding sides



In other words

If there are two triangles which are similar (equal angles, sides proportional), then the ratio of the areas of the triangles is the square of the ratio of the sides

$\Delta ABC \sim \Delta DEF$

$\rightarrow \Delta ABC : \Delta DEF = (AB:DE)^2$

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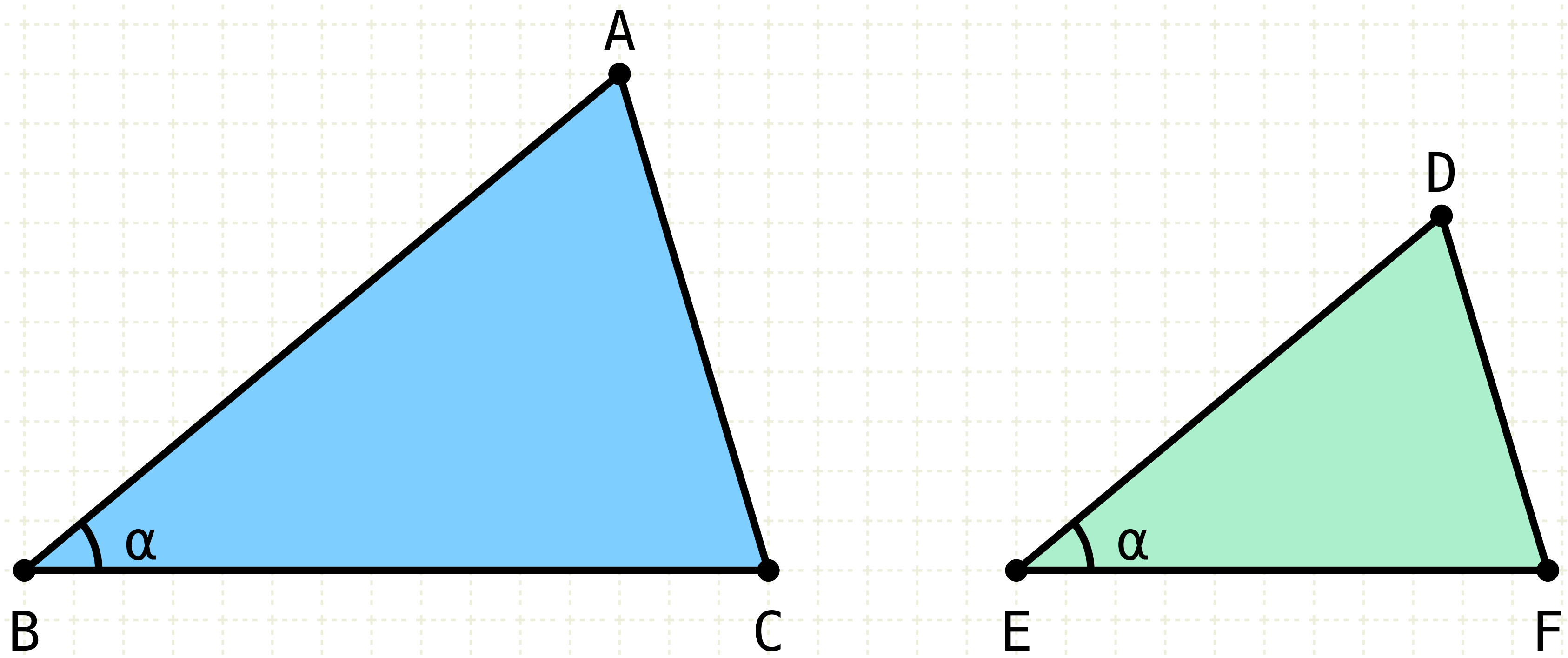

Proof

ΔABC ~ ΔDEF

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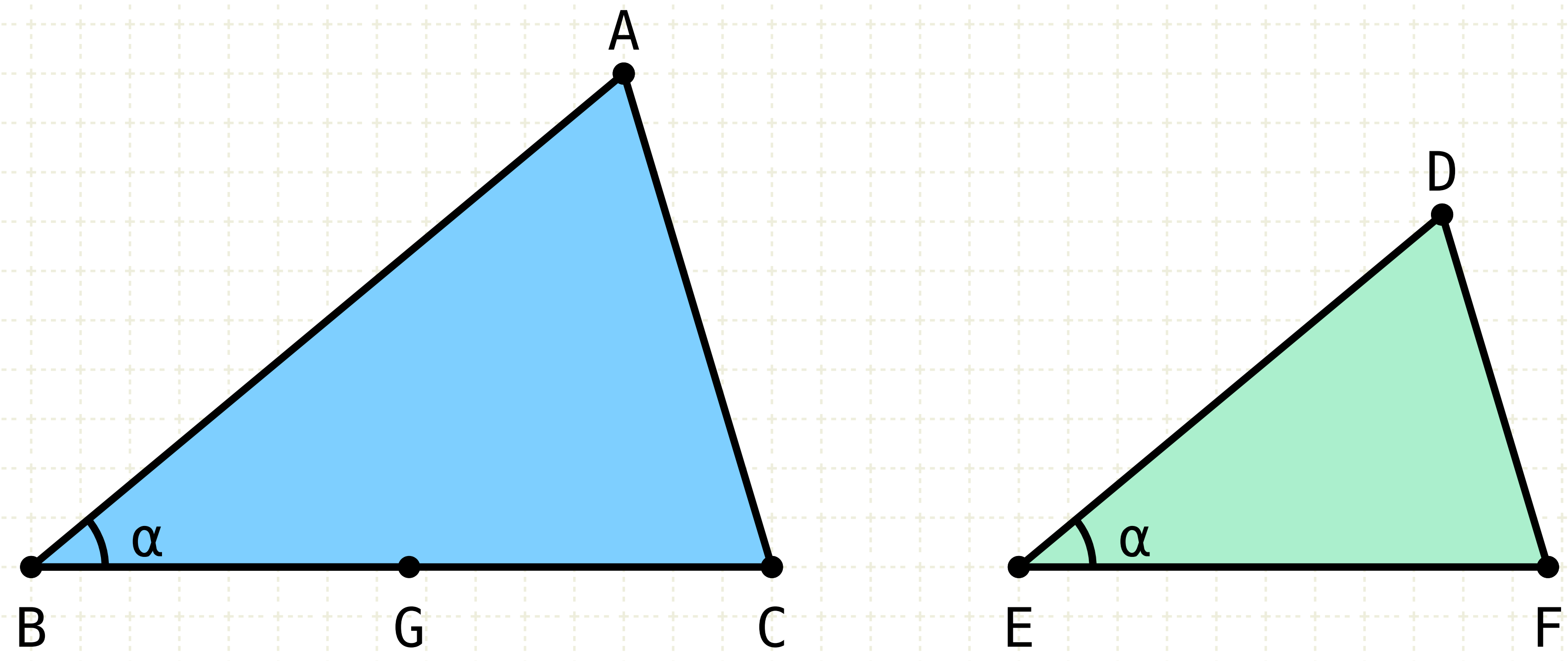

Proof

Construct a third proportional BG such that BC to EF is EF to BG (VI·11)

ΔABC ~ ΔDEF

BC:EF = EF:BG

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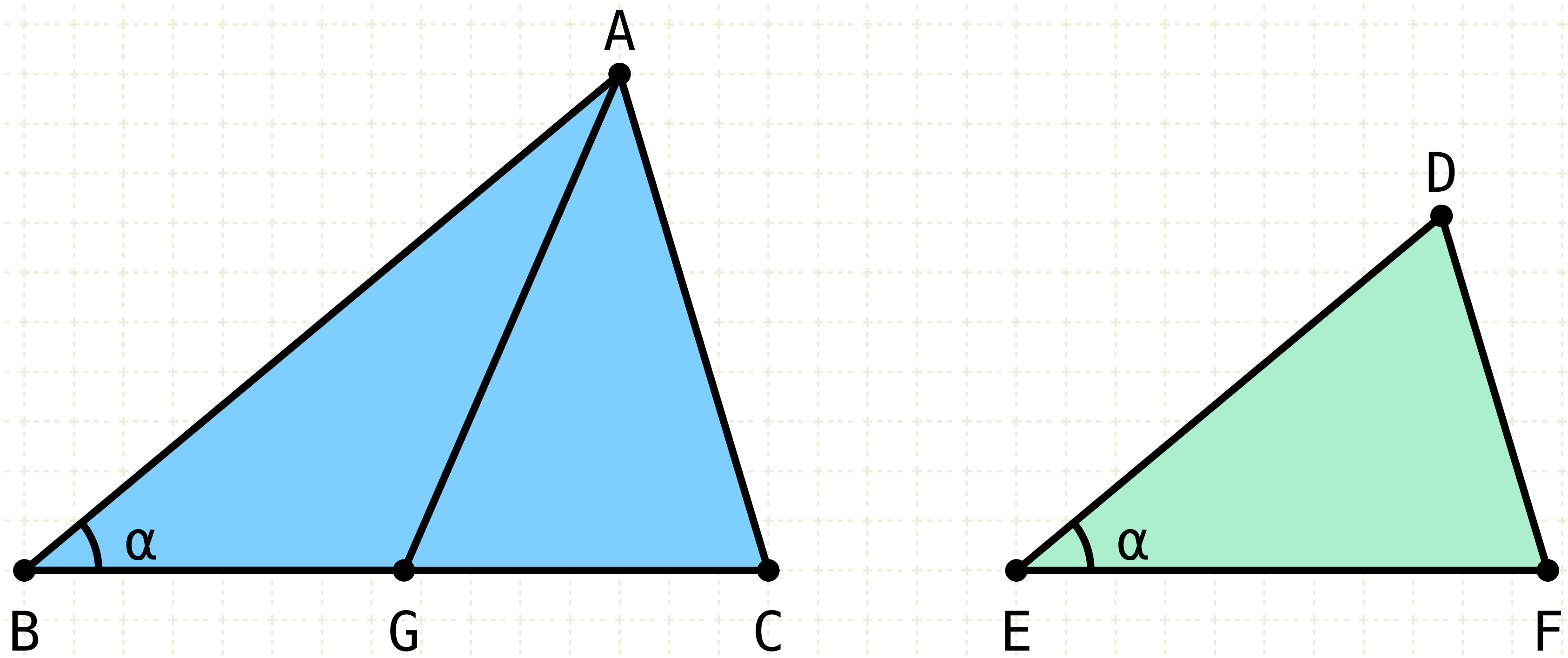

$\Delta ABC \sim \Delta DEF$

$BC:EF = EF:BG$

Proof

Construct a third proportional BG such that BC to EF is EF to BG (VI·11)

Draw the line AG

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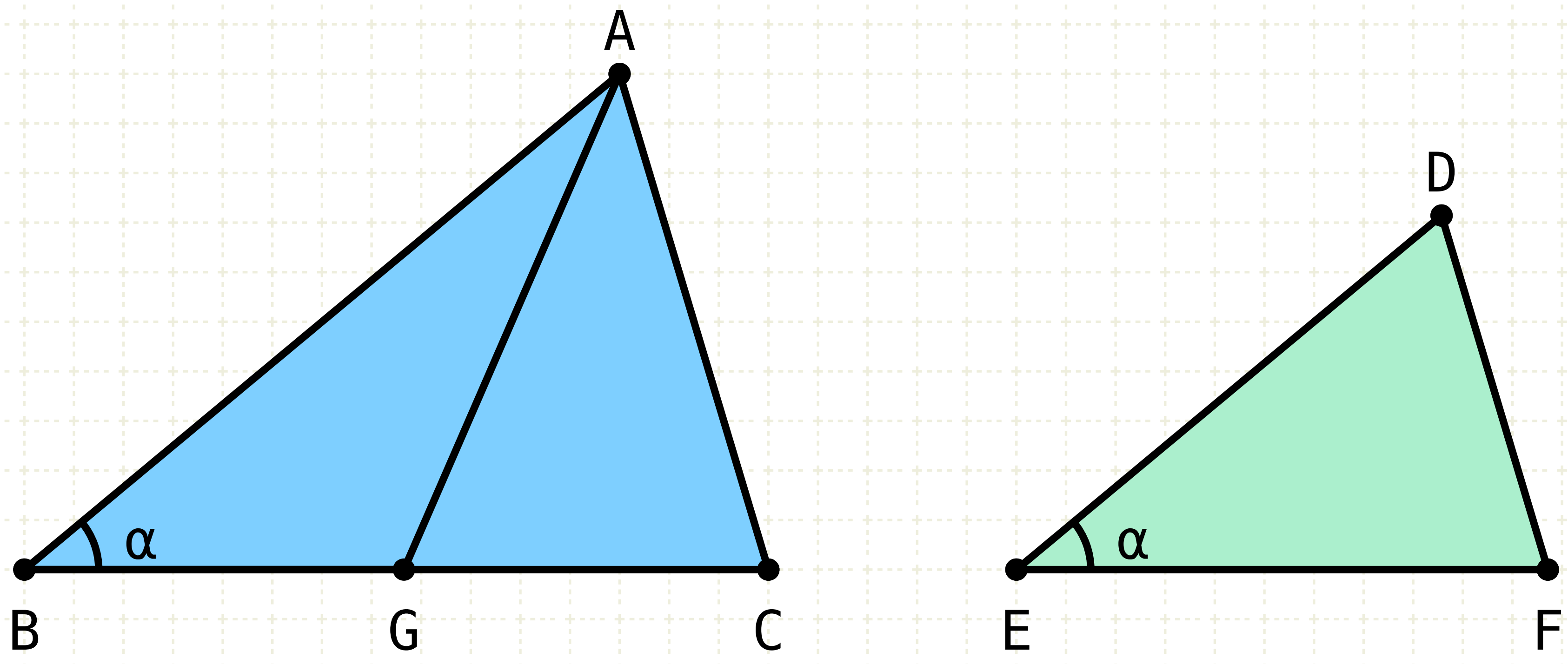

Proof

Construct a third proportional BG such that BC to EF is EF to BG (VI·11)

Draw the line AG

As AB is to BC, so is DE to EF, or alternately, AB is to DE as BC is to EF (V·16)

$\Delta ABC \sim \Delta DEF$

$BC:EF = EF:BG$

$AB:BC = DE:EF$

$AB:DE = BC:EF$

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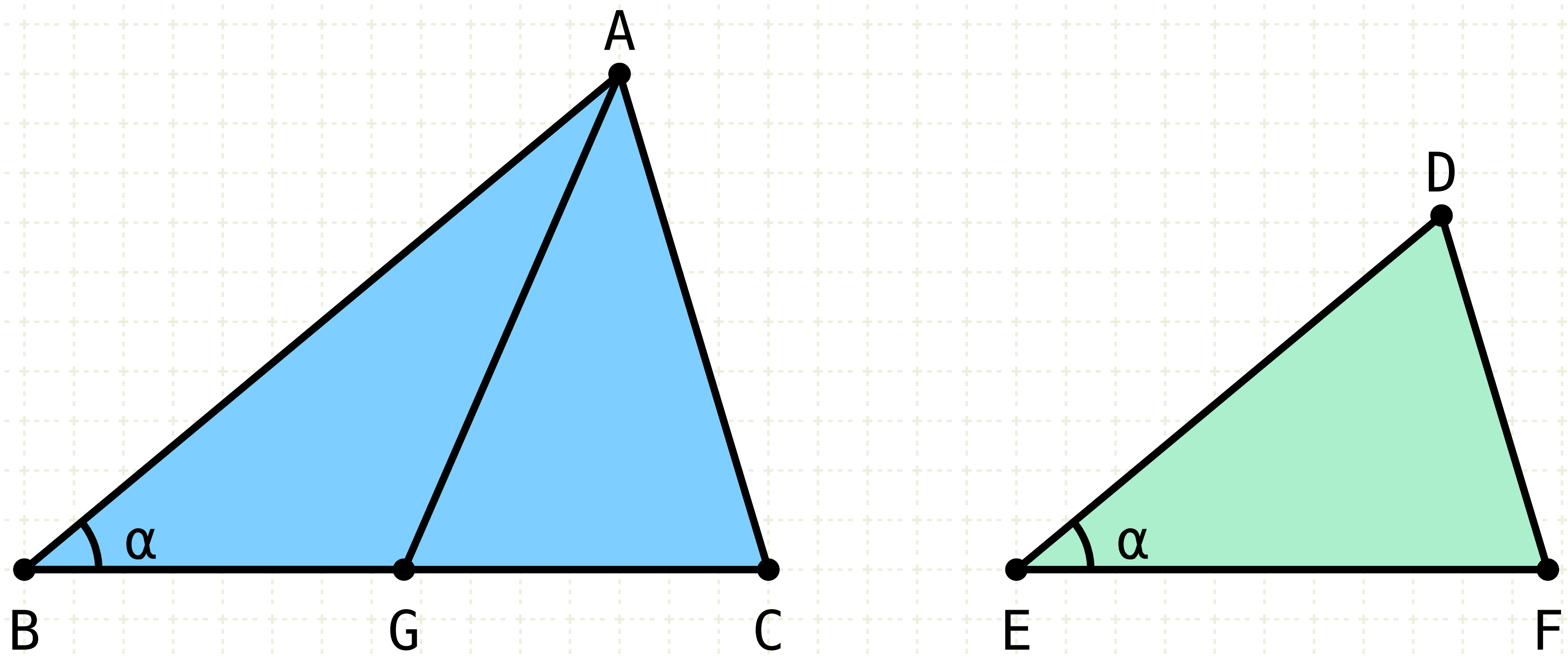

$\Delta ABC \sim \Delta DEF$

$BC:EF = EF:BG$

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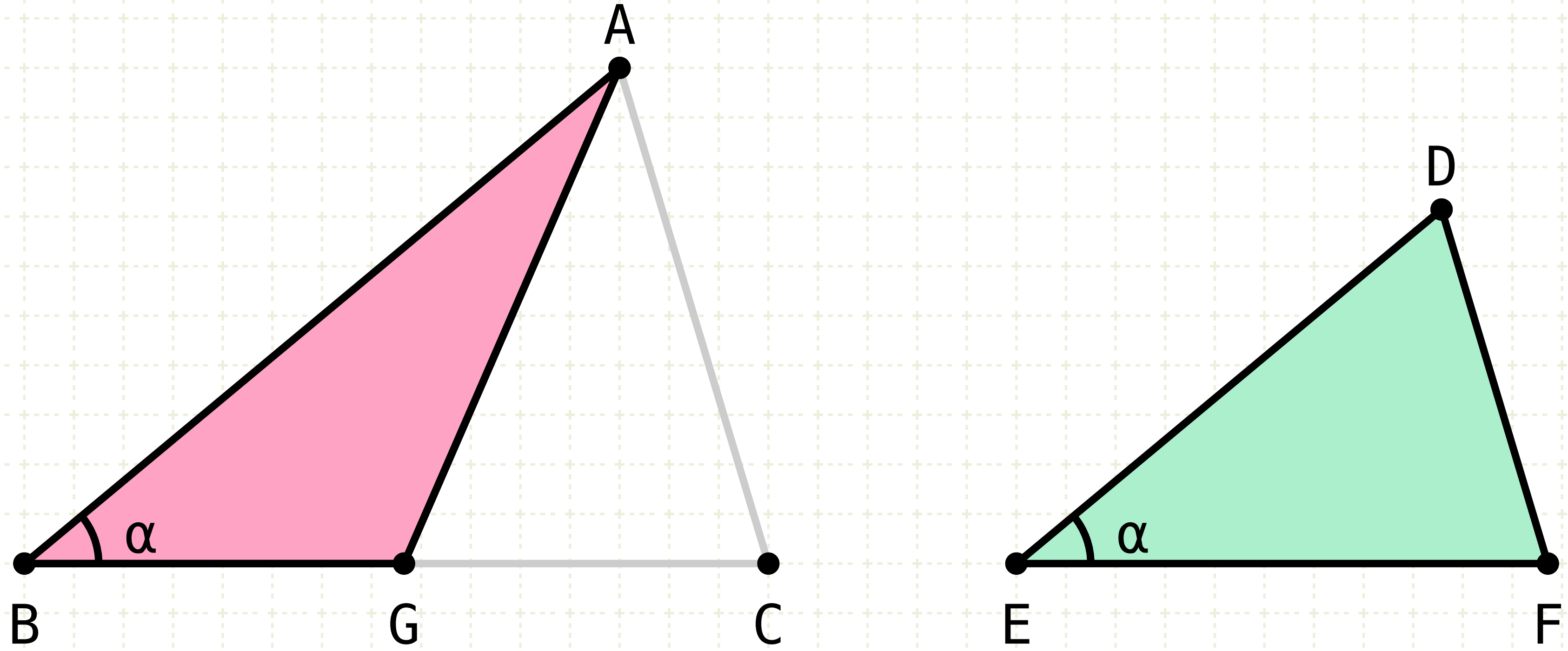

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The sides about the the equal angle in triangles ABG and DEF are reciprocally proportional

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$BC:EF = EF:BG$

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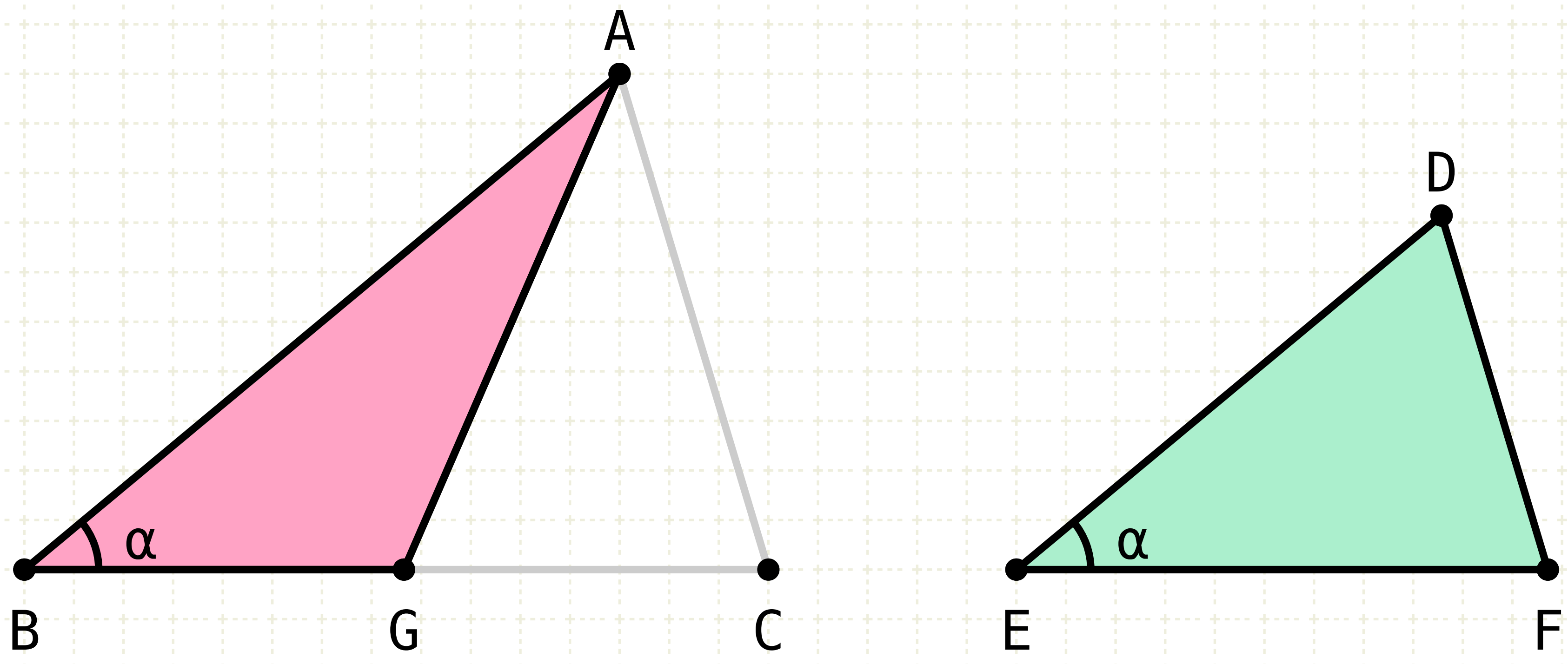

$\Delta ABC \sim \Delta DEF$

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The sides about the the equal angle in triangles ABG and DEF are reciprocally proportional

And triangles whose sides are reciprocally proportional about an equal angle, are also equal (VI·15)

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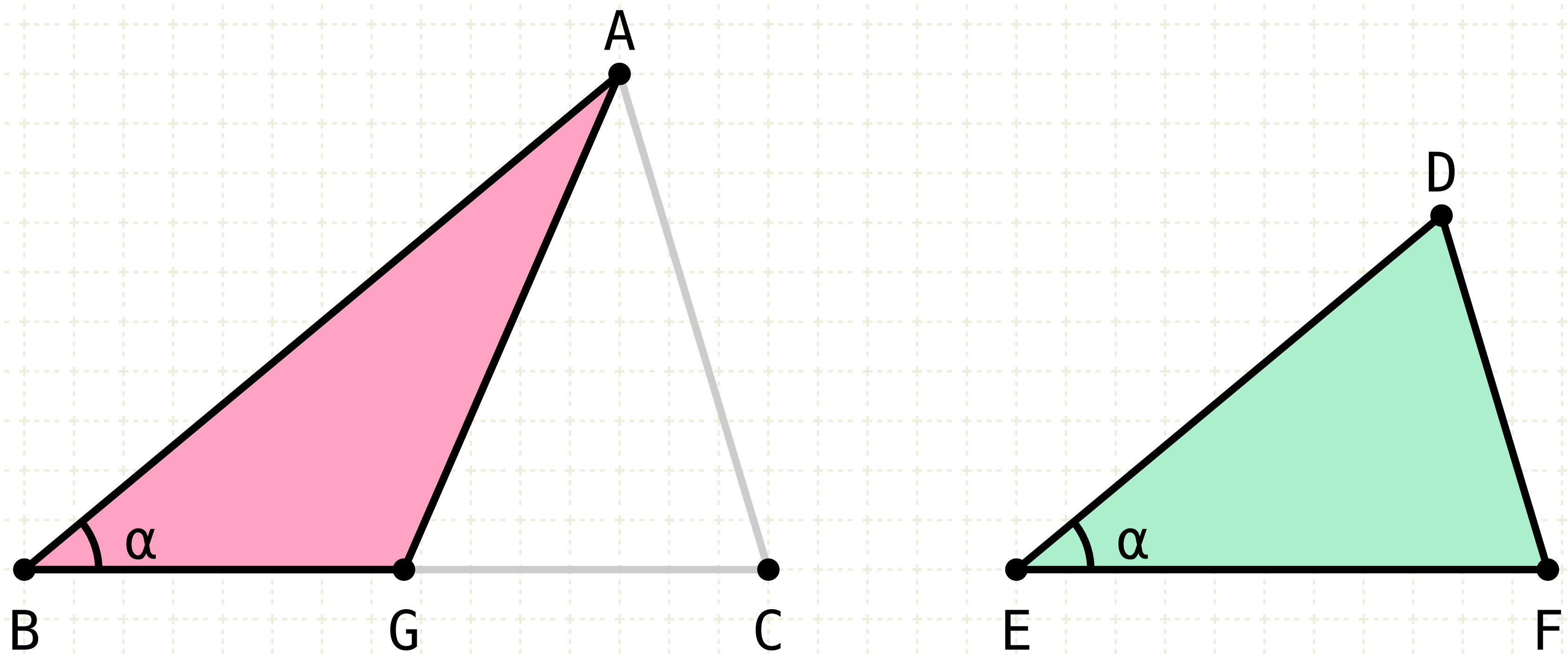

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$BC:BG = (BC:EF)^2$

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And triangles whose sides are reciprocally proportional about an equal angle, are also equal (VI·15)

Since BC is to EF is as EF is to BG, the ratio BC to BG is the duplicate ratio of BC to EF (V.Def.9)

Proposition 19 of Book VI

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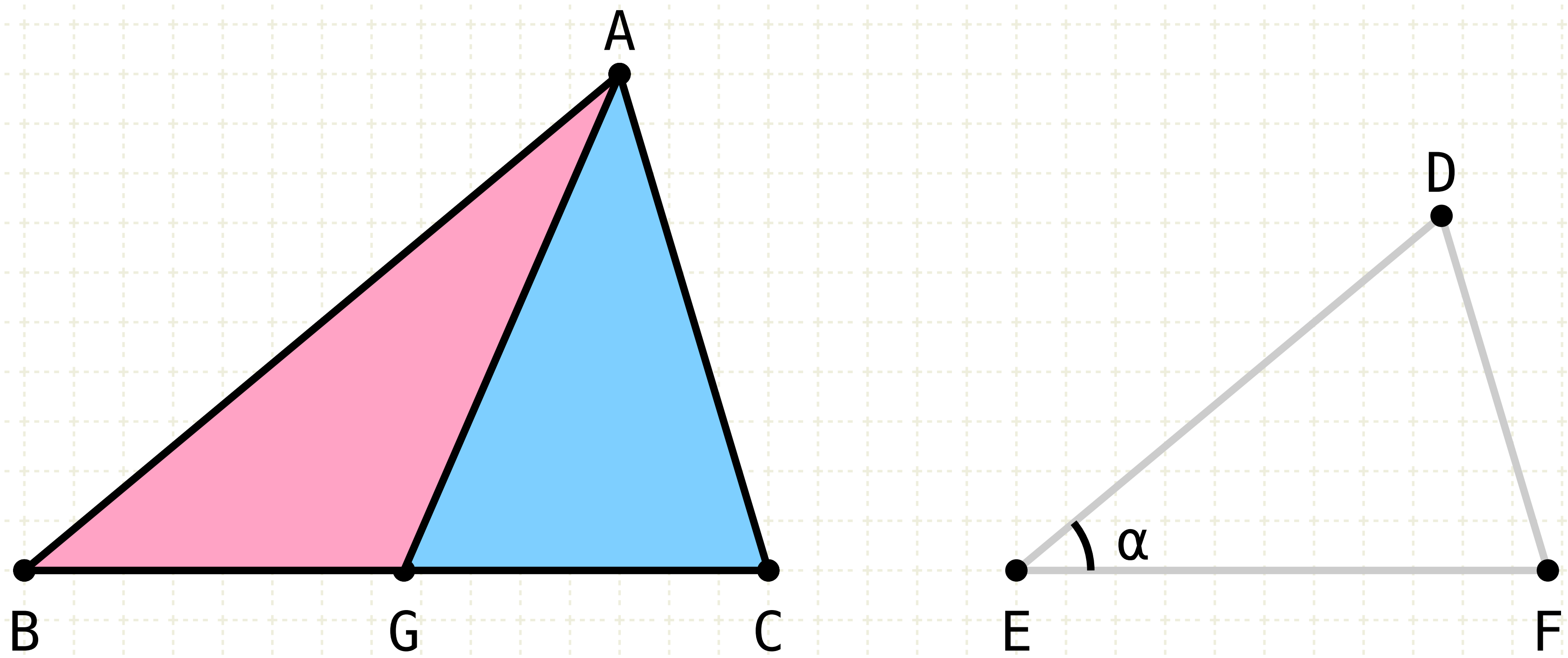

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The ratio of the triangles ABC and ABG is proportional to the bases, BC and BG (VI·1)

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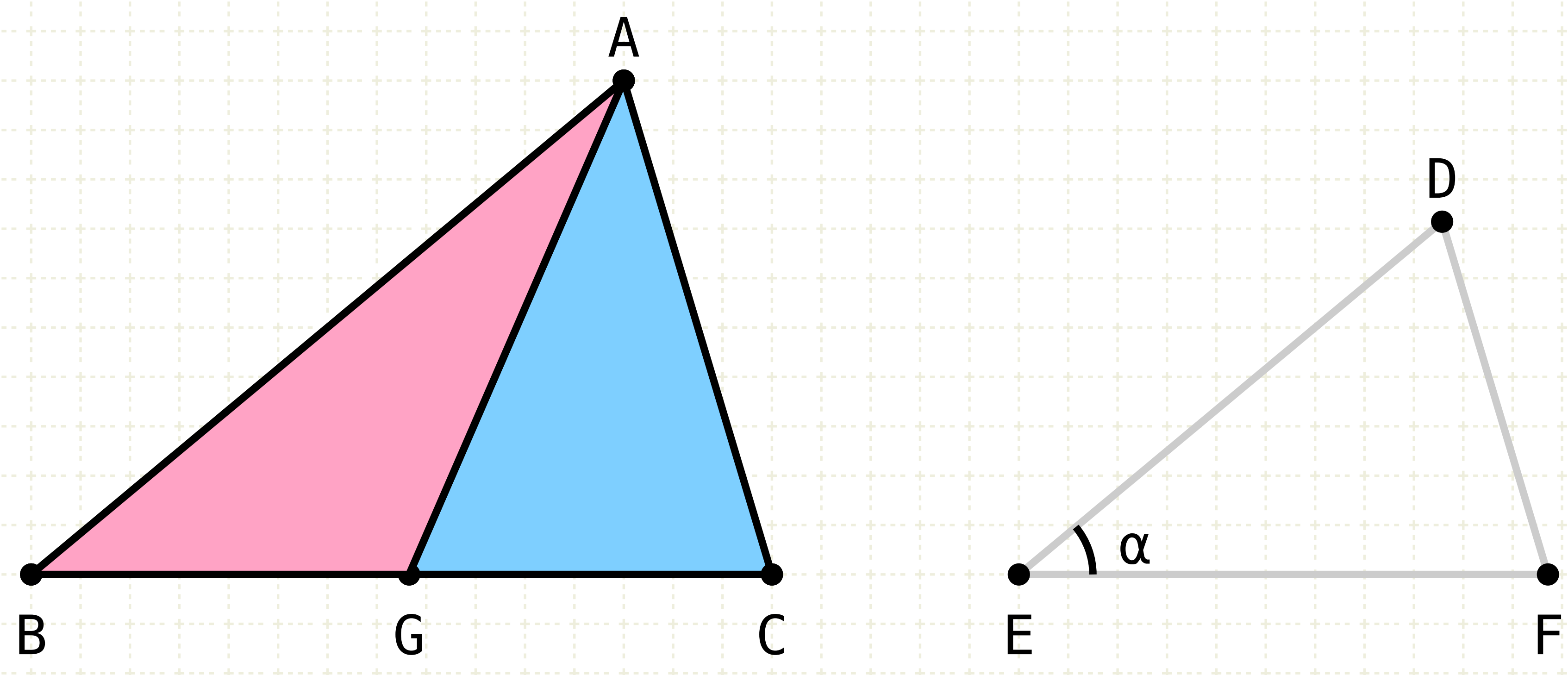

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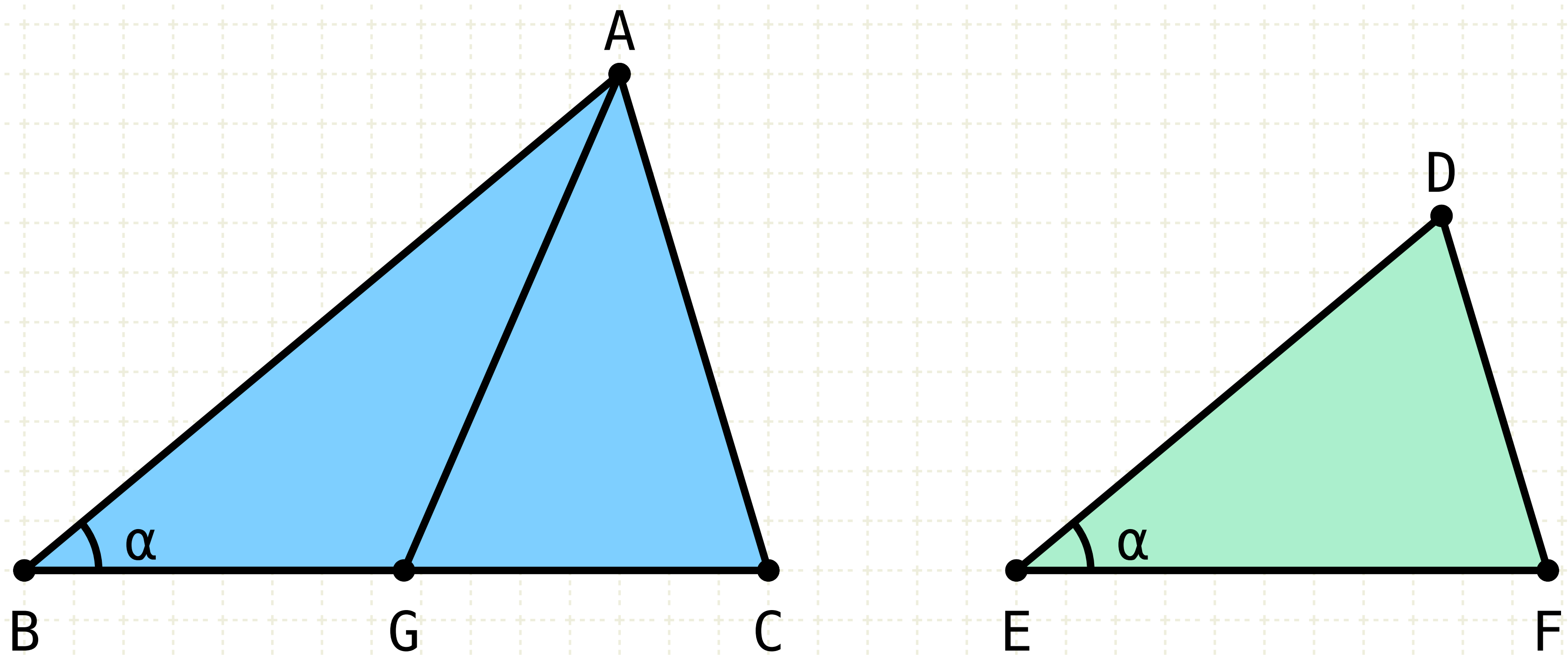

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Thus the ratio of BC to BG is the duplicate ratio of BC to EF

But ABG equals DEF, therefore the ratio of ABC to DEF is the duplicate ratio of their sides

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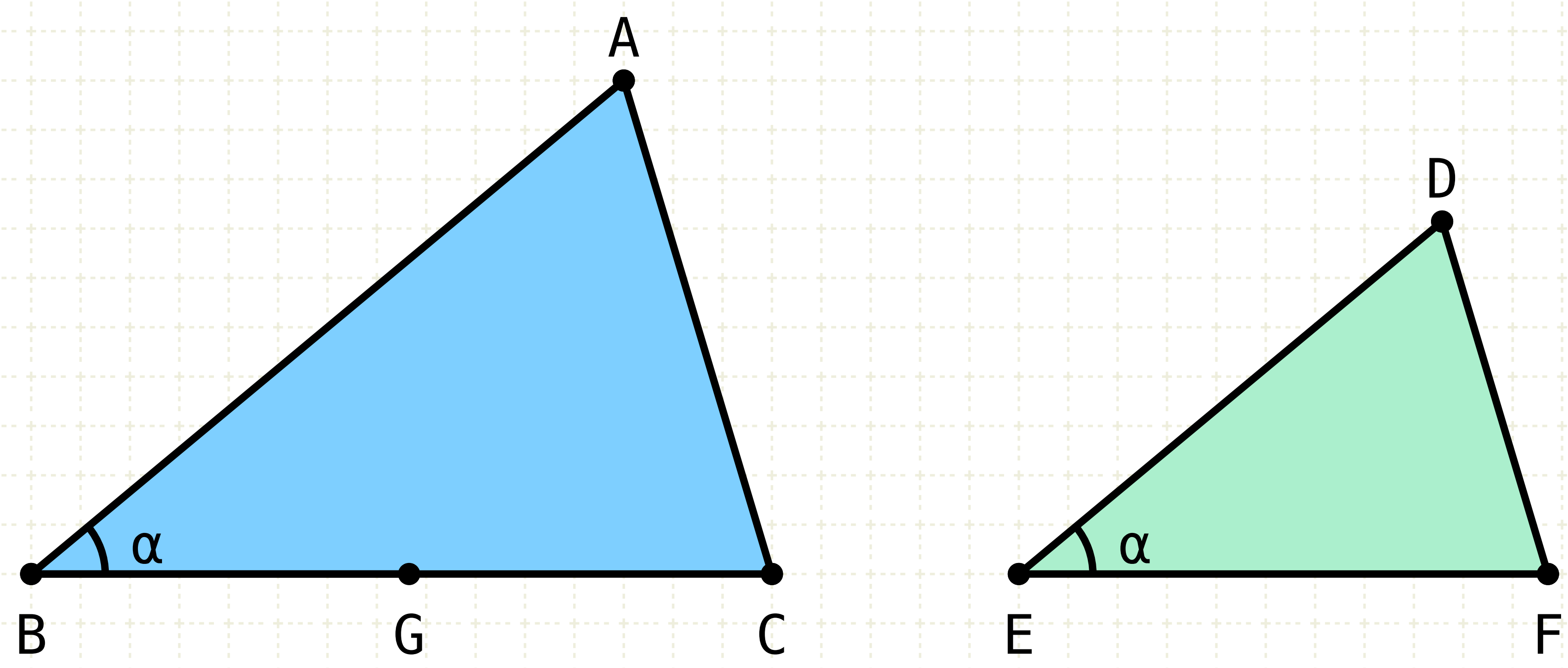

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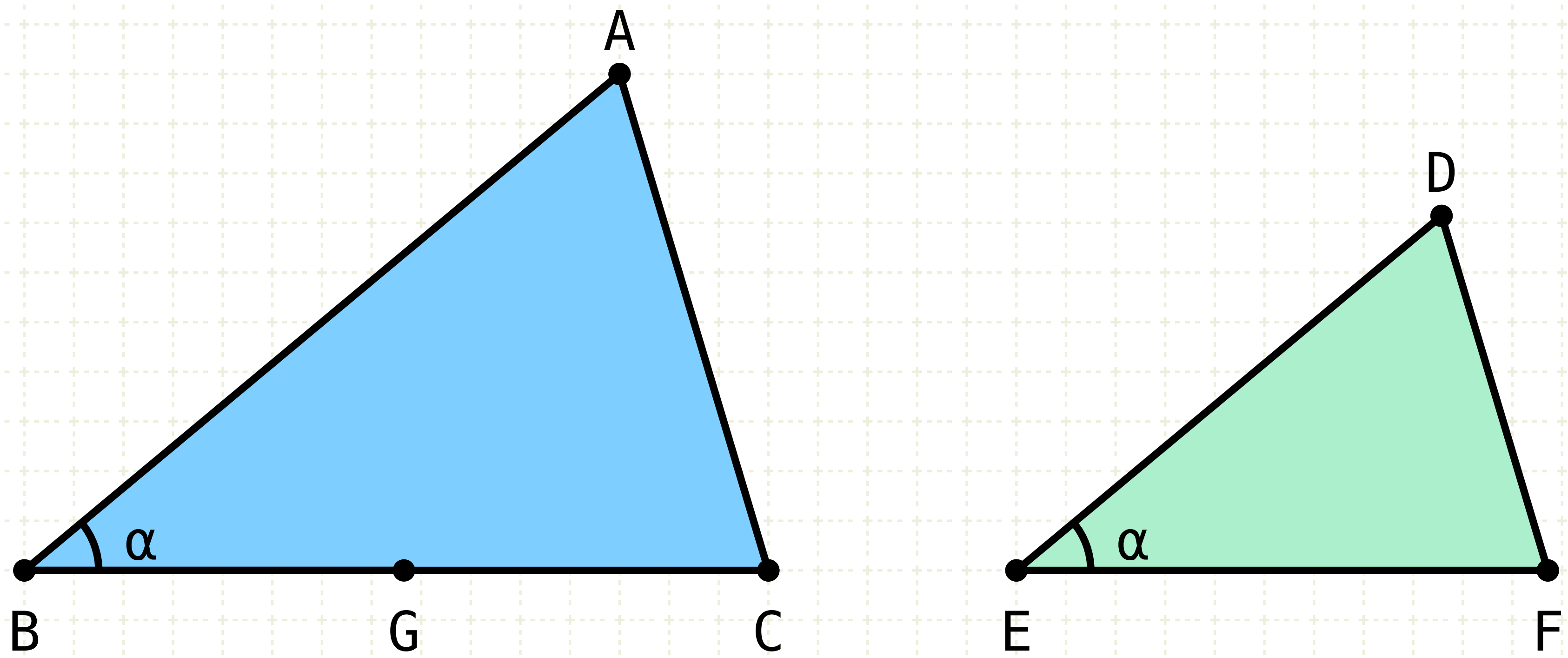

Porism

If three straight lines are proportional (A is to B as B is to C)

If a two similar figures are drawn on A and B

Then the ratio of these two figures is equal to the ratio of A to C

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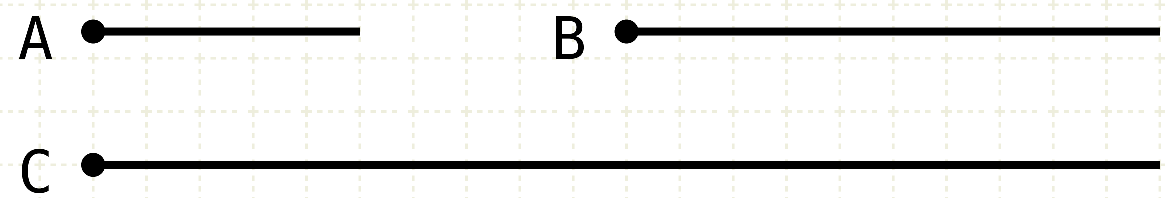

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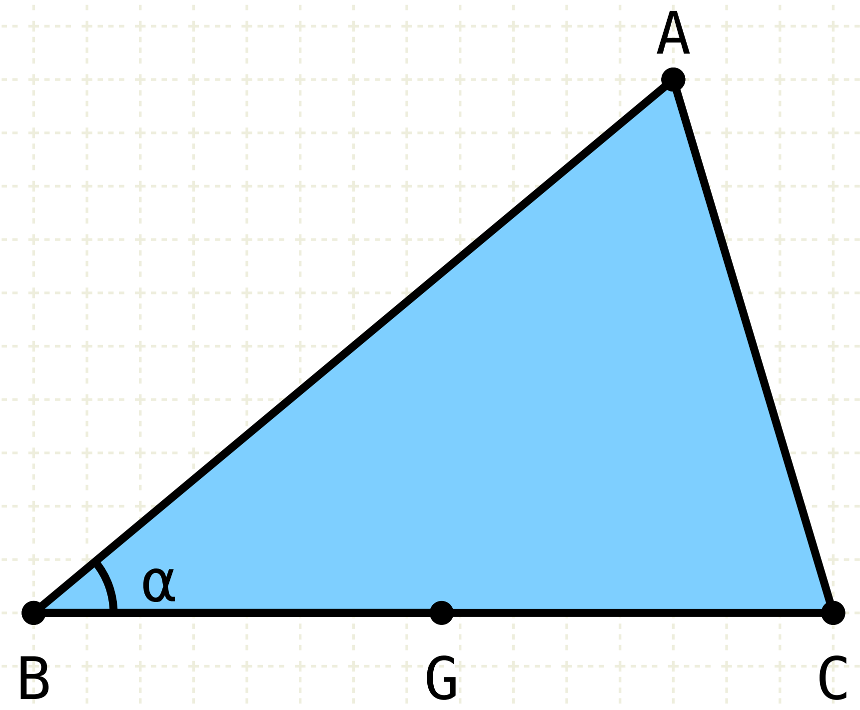

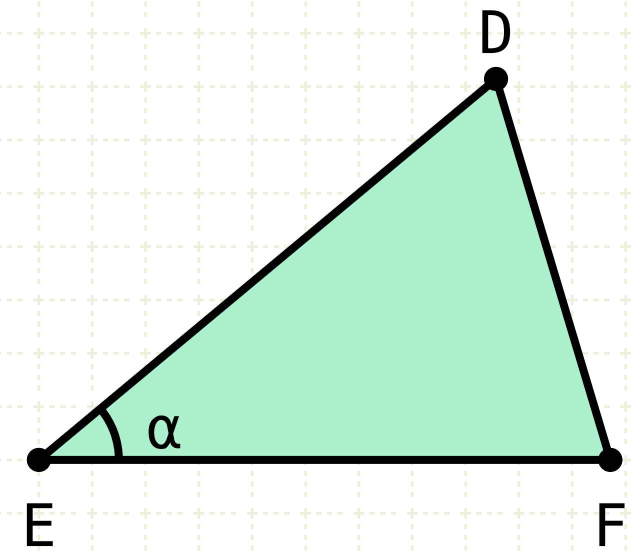

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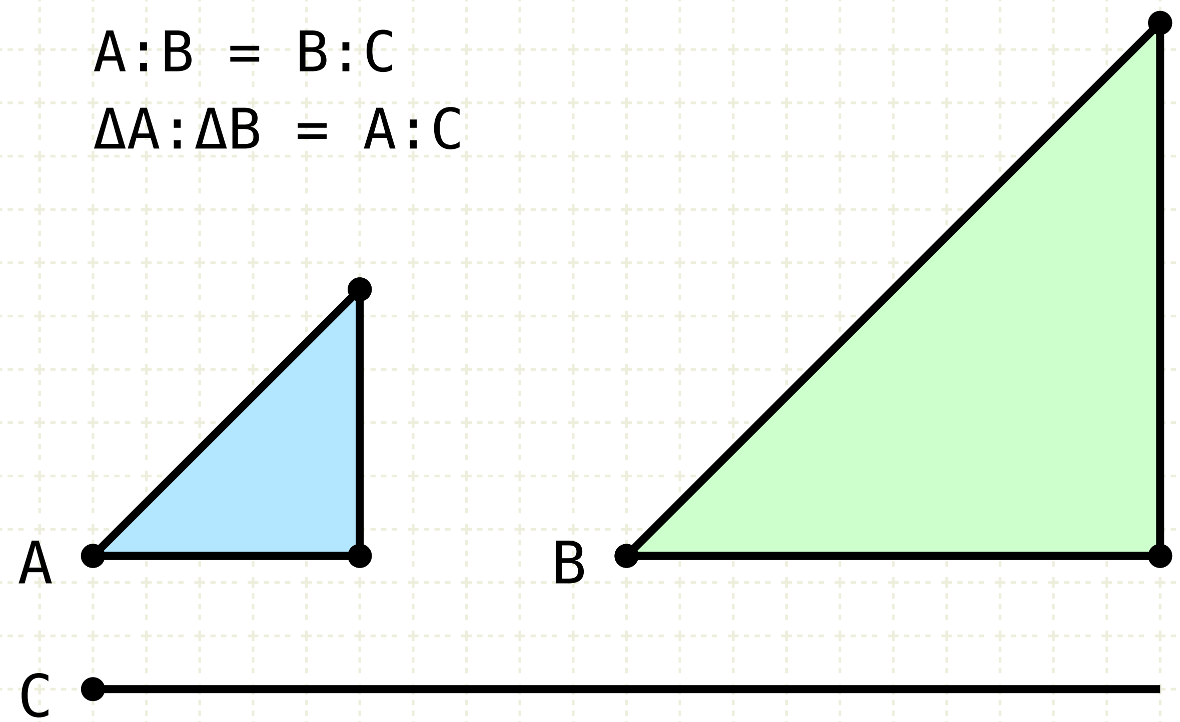

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