Euclid's Elements

Book VII

Definitions:

1 A unit is that by virtue of which each of the things that exist is called one

2 A number is a multitude composed of units. (not one)

3 A number is part of a number, the less of the greater, when it measures the greater

11 A prime number is that which is measured by a unit alone.

12 Numbers prime to one another are those which are measured by a unit alone as a common measure

15 A number is said to multiply a number when that which is multiplied is added to itself as many times as there are units in the other, and thus some number is produced.

20 Numbers are proportional when the first is the same multiple, or the same part, or the same parts, of the second that the third is of the fourth.

As long as algebra and geometry have been separated, their progress have been slow and their uses limited; but when these two sciences have been united, they have lent each mutual forces, and have marched together towards perfection.

Joseph-Louis Lagrange
(1736 to 1813)

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Proposition 24 of Book VII

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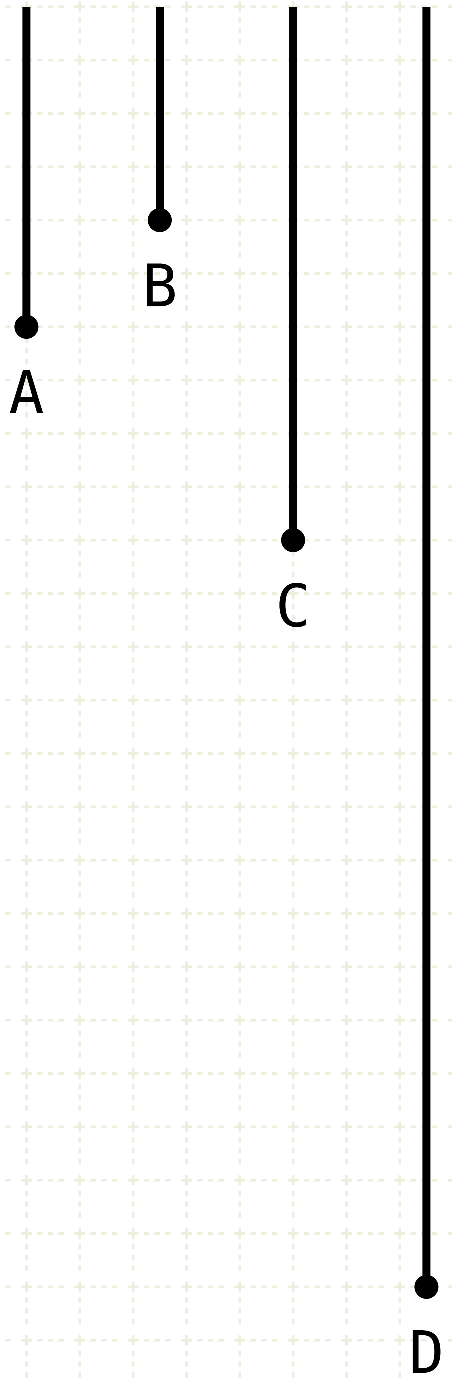

$$\gcd(A,C) = 1$$
$$\gcd(B,C) = 1$$
$$D = A \times B$$

In other words

If A and B are both prime to C, and D is the product of A,B

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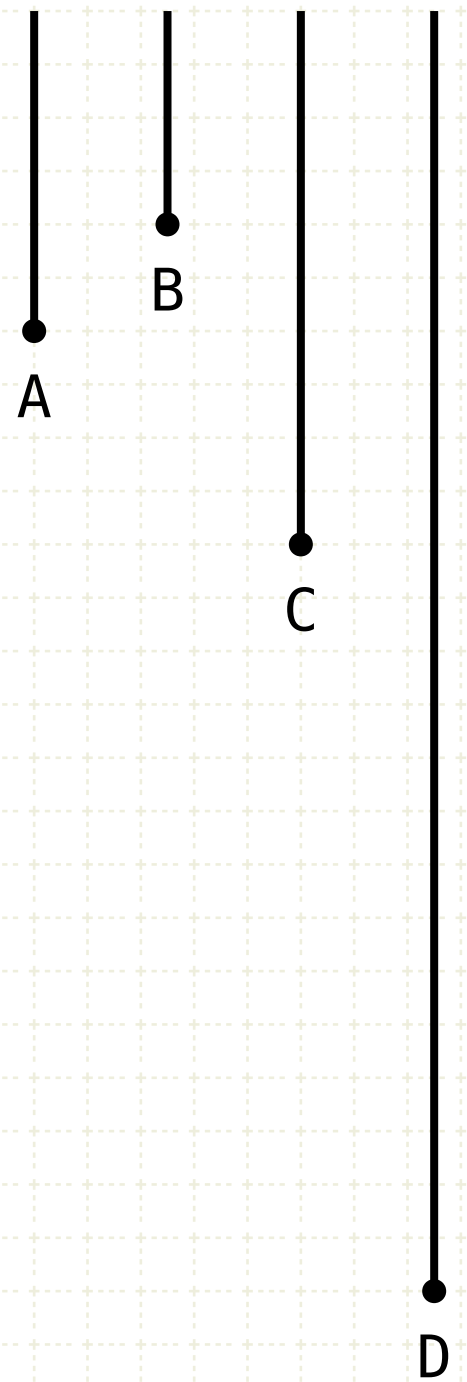

$\gcd(A,C) = 1$

$\gcd(B,C) = 1$

$D = A \times B$

$\gcd(C,D) = 1$

In other words

If A and B are both prime to C, and D is the product of A,B

Then C and D are prime to one another

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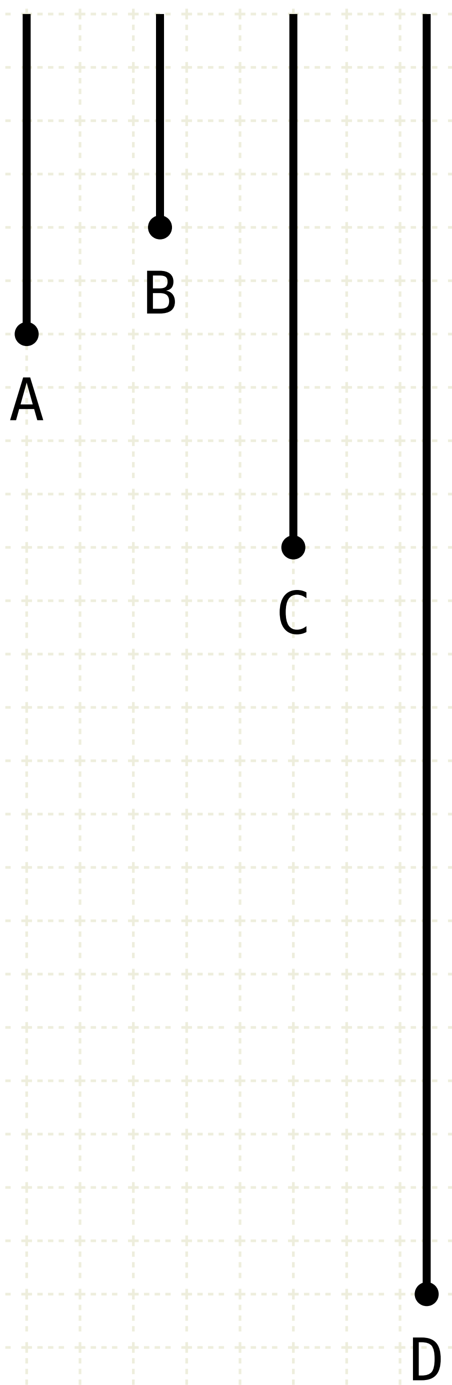

Proof by Contradiction

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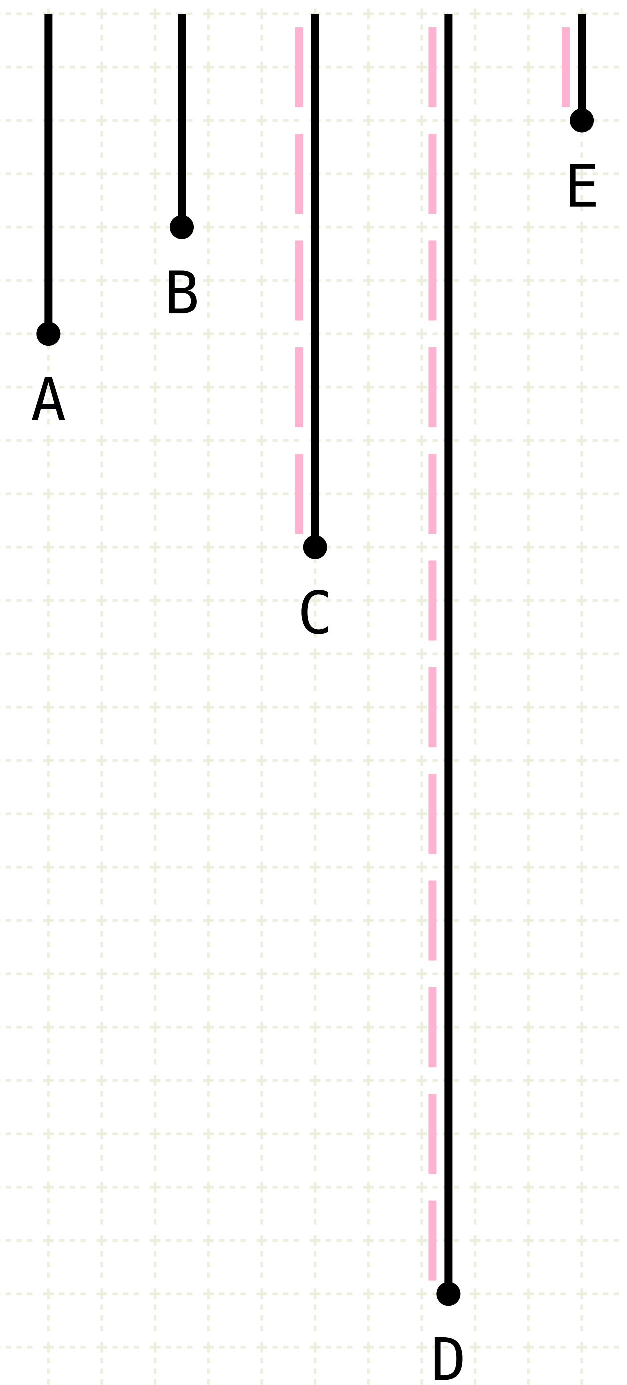

$\gcd(A,C) = 1$

$\gcd(B,C) = 1$

$D = A \times B$

$C = k \cdot E$

$D = f \cdot E$

Proof by Contradiction

Assume that C and D are not prime to one another, and E measures both of them

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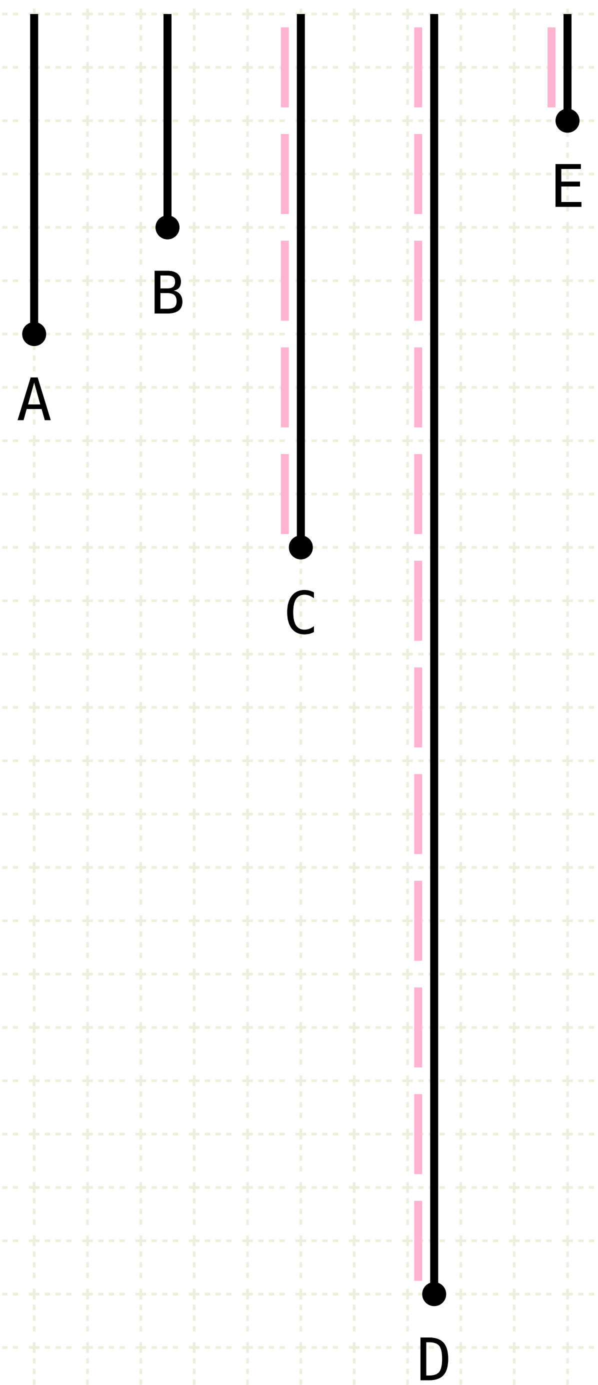

gcd(A,C) = 1

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gcd(A,E) = 1

Proof by Contradiction

Assume that C and D are not prime to one another, and E measures both of them

Since A and C are relatively prime, and E measures C, then A and E are also relatively prime (VII·23)

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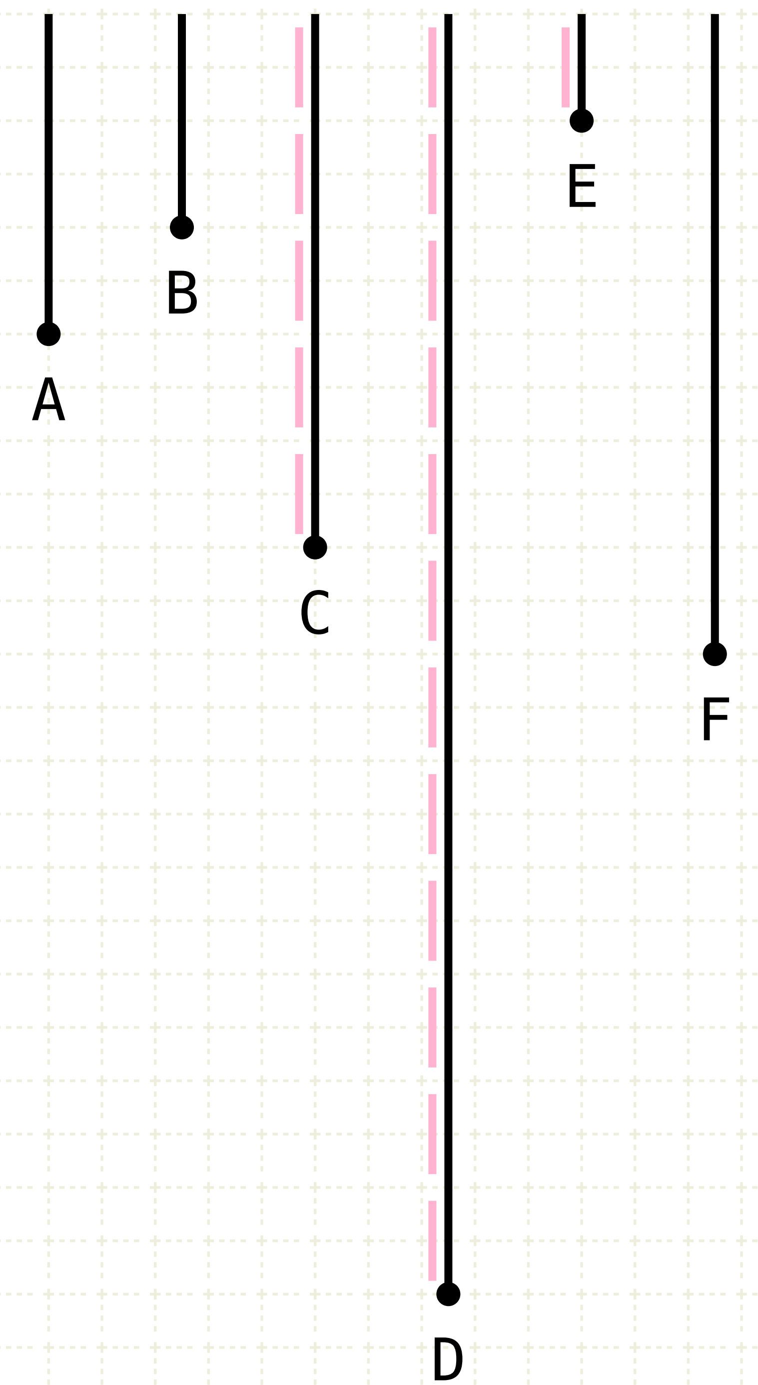

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$\gcd(A,E) = 1$

$F = f \cdot 1$

Proof by Contradiction

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Let F be equal to the number of times that E measures D

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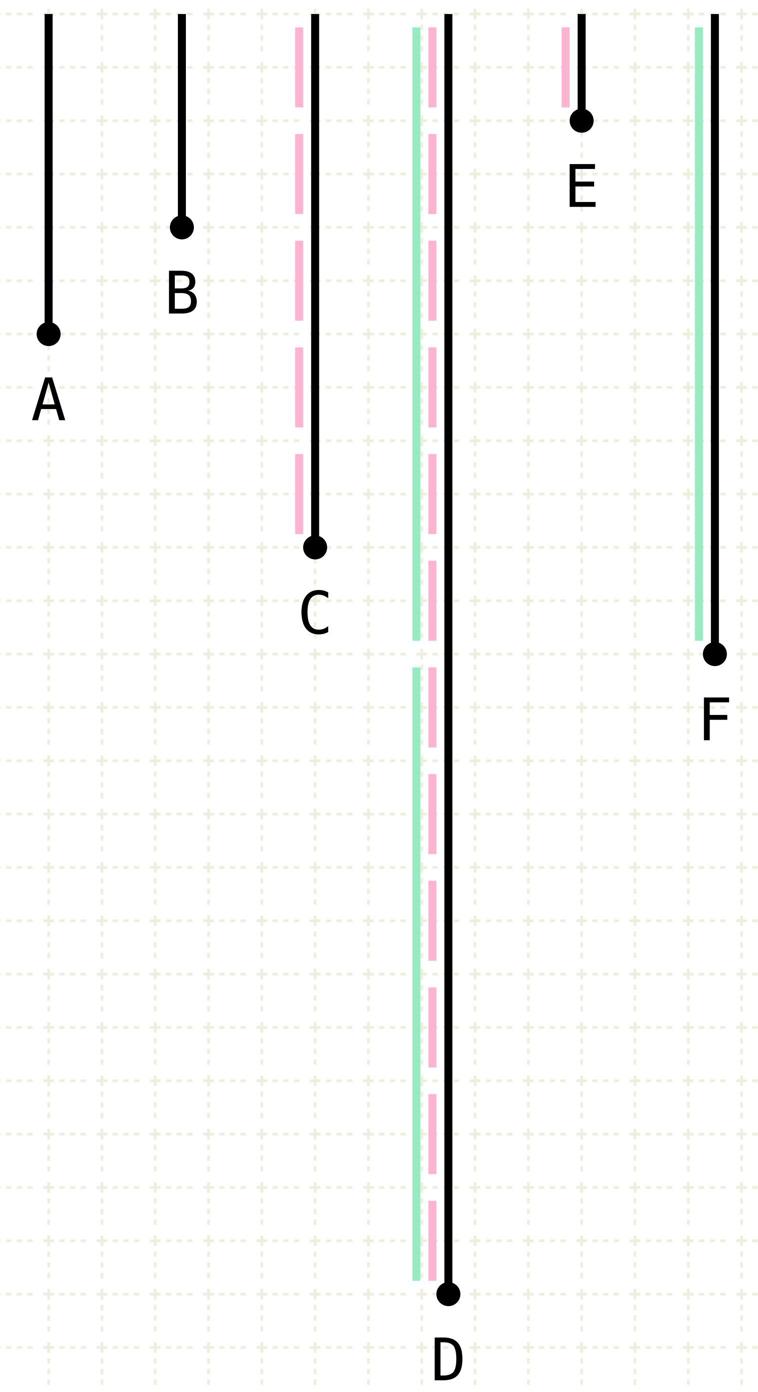

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$\gcd(A,E) = 1$

$F = f \cdot 1$

$E = e \cdot 1$

$D = e \cdot F$

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Therefore F also measures D according to the units in E (VII·16)

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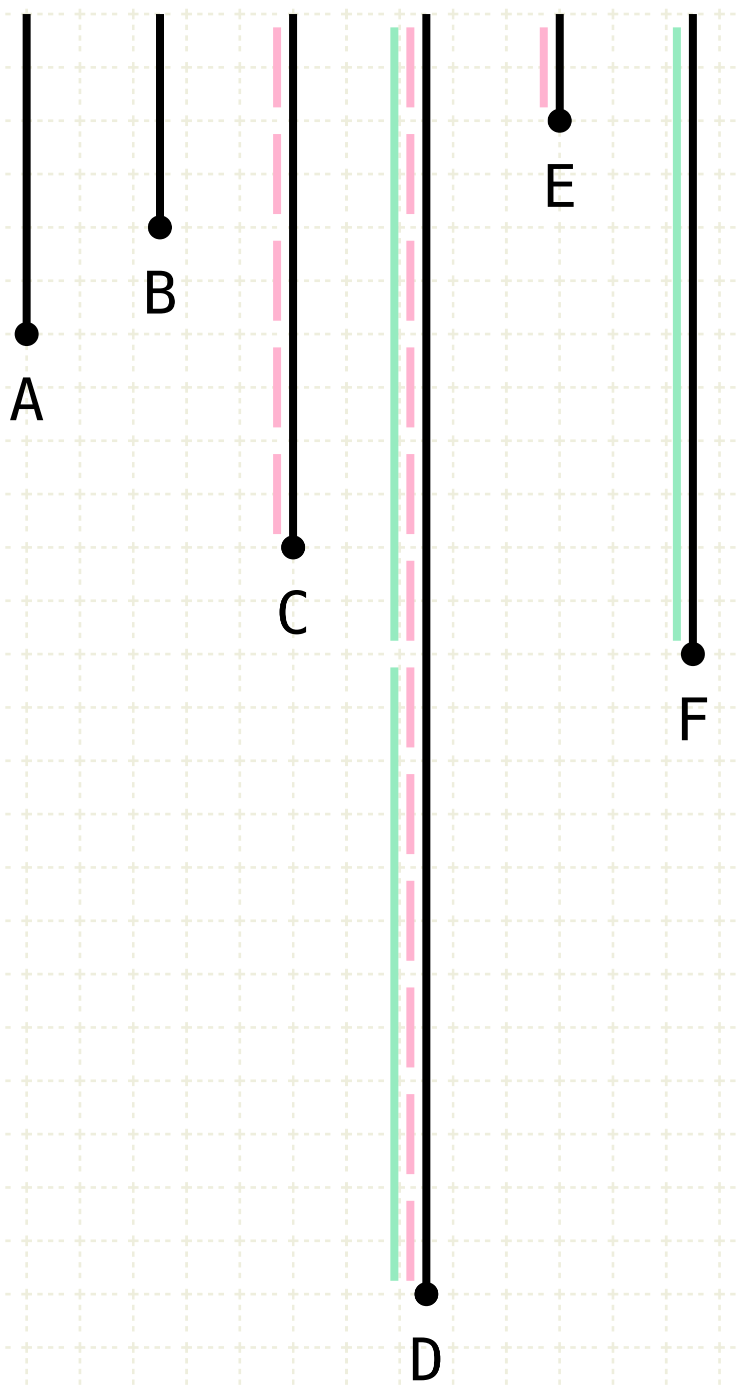

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Therefore D equals F times E (VII.Def.15)

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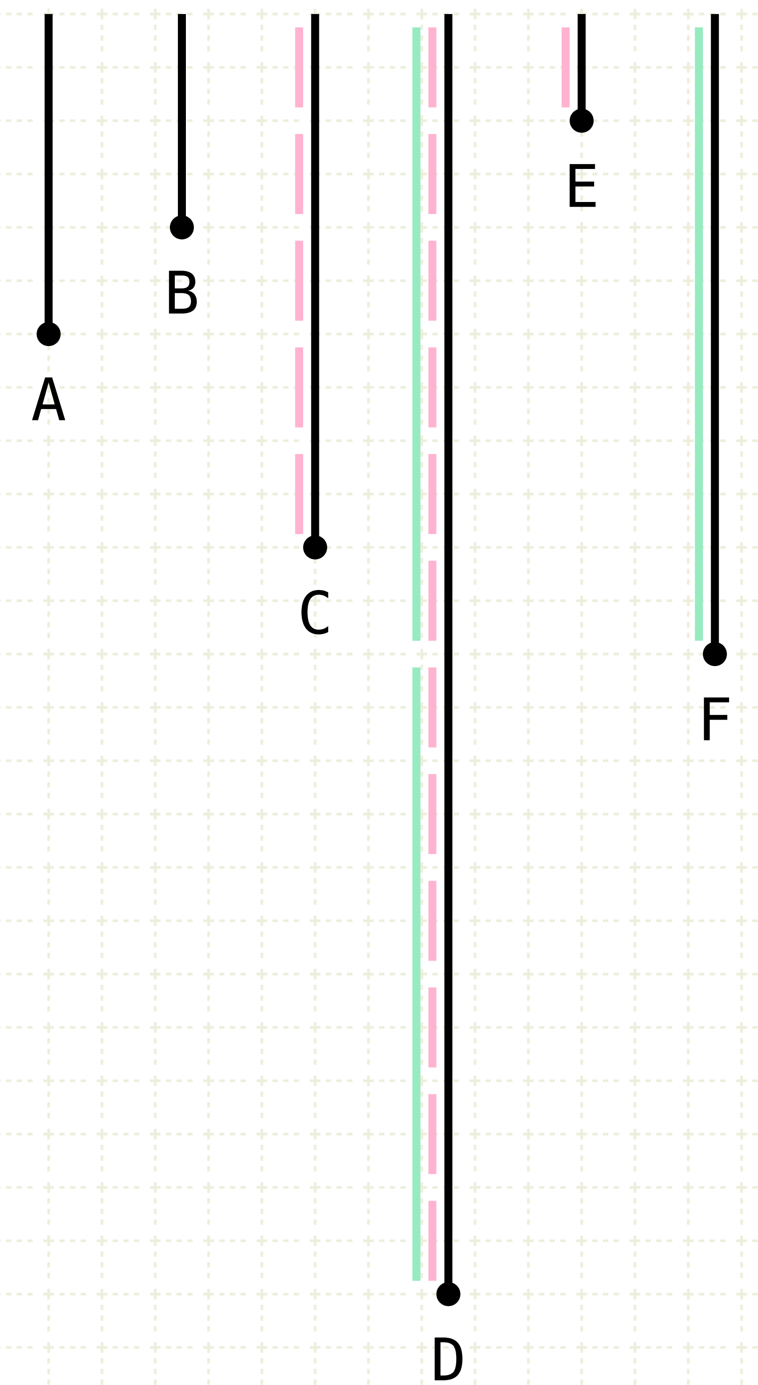

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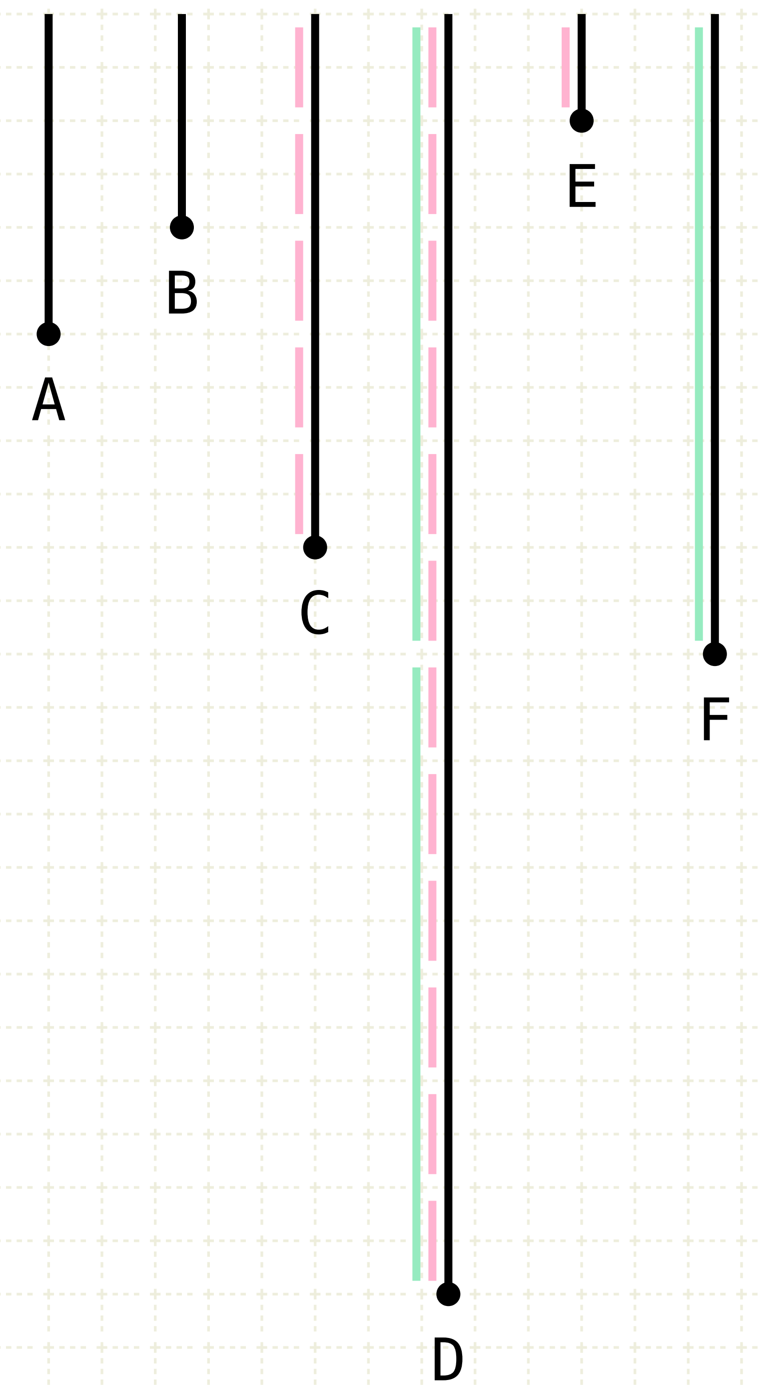

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But D is also equal to A times B, therefore F times E equals A times B

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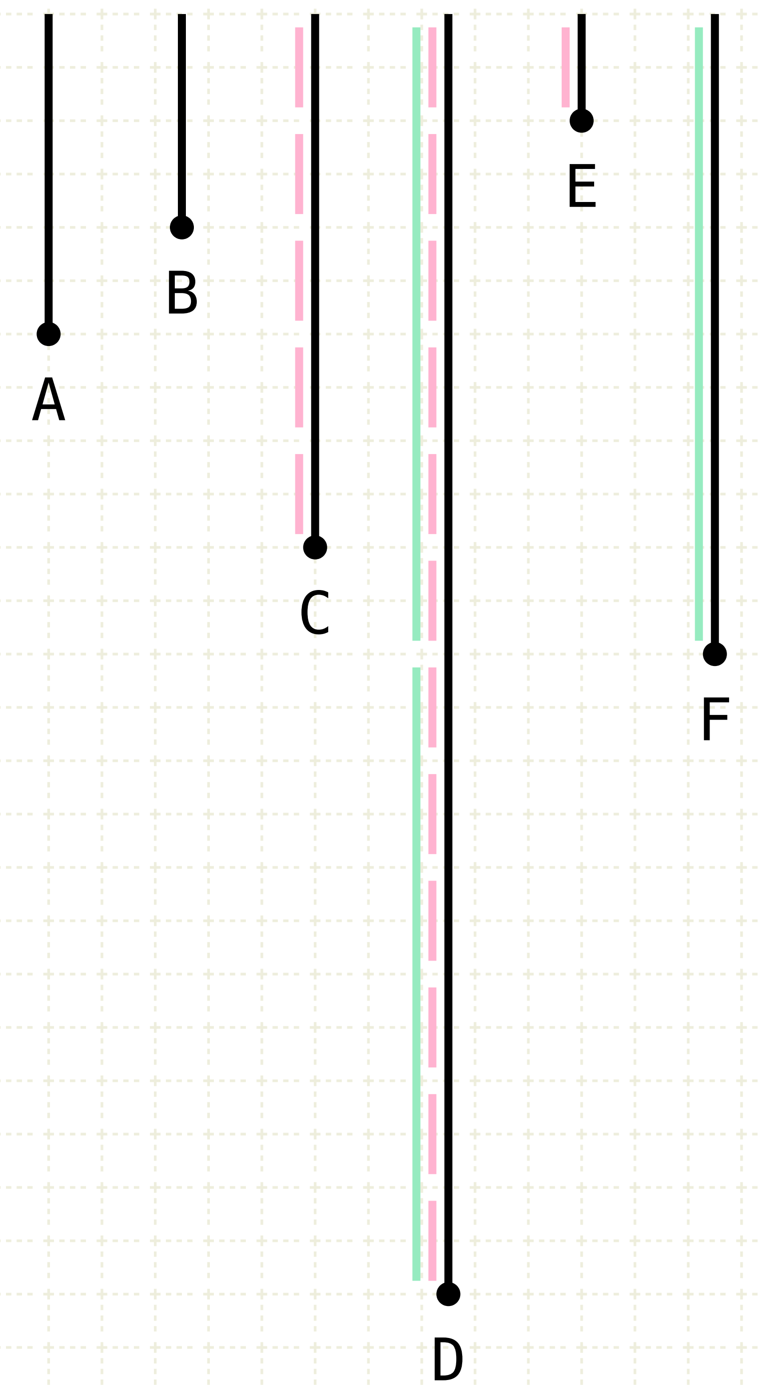

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$E : A = B : F$

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If the product of the extremes are equal to that of the means, the four numbers are proportional, in other words E is to A as B is to F (VII·19)

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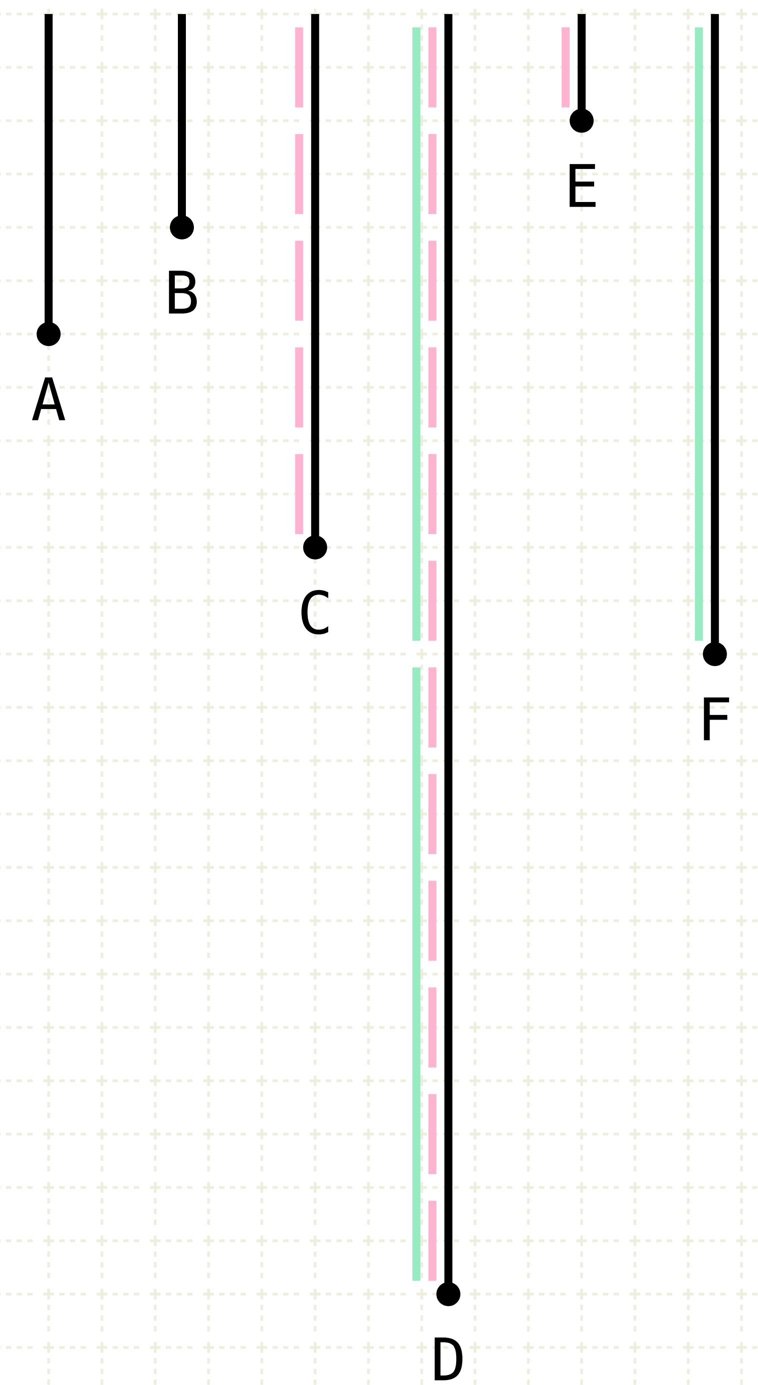

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$F \times E = A \times B$

$E:A = B:F$

$S = \{ (x,y) \mid x \in \mathbb{N},\ y \in \mathbb{N},\ x:y = E:A \}$

$(E:A) \in S$ such that $E \leq x,\ A \leq y,\ \forall (x,y) \in S$

Proof by Contradiction

Assume that C and D are not prime to one another, and E measures both of them

Since A and C are relatively prime, and E measures C, then A and E are also relatively prime (VII·23)

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But D is also equal to A times B, therefore F times E equals A times B

If the product of the extremes are equal to that of the means, the four numbers are proportional, in other words E is to A as B is to F (VII·19)

A and E are relatively prime, so therefore they are the smallest numbers which can represent the ratio E to A (VII·21)

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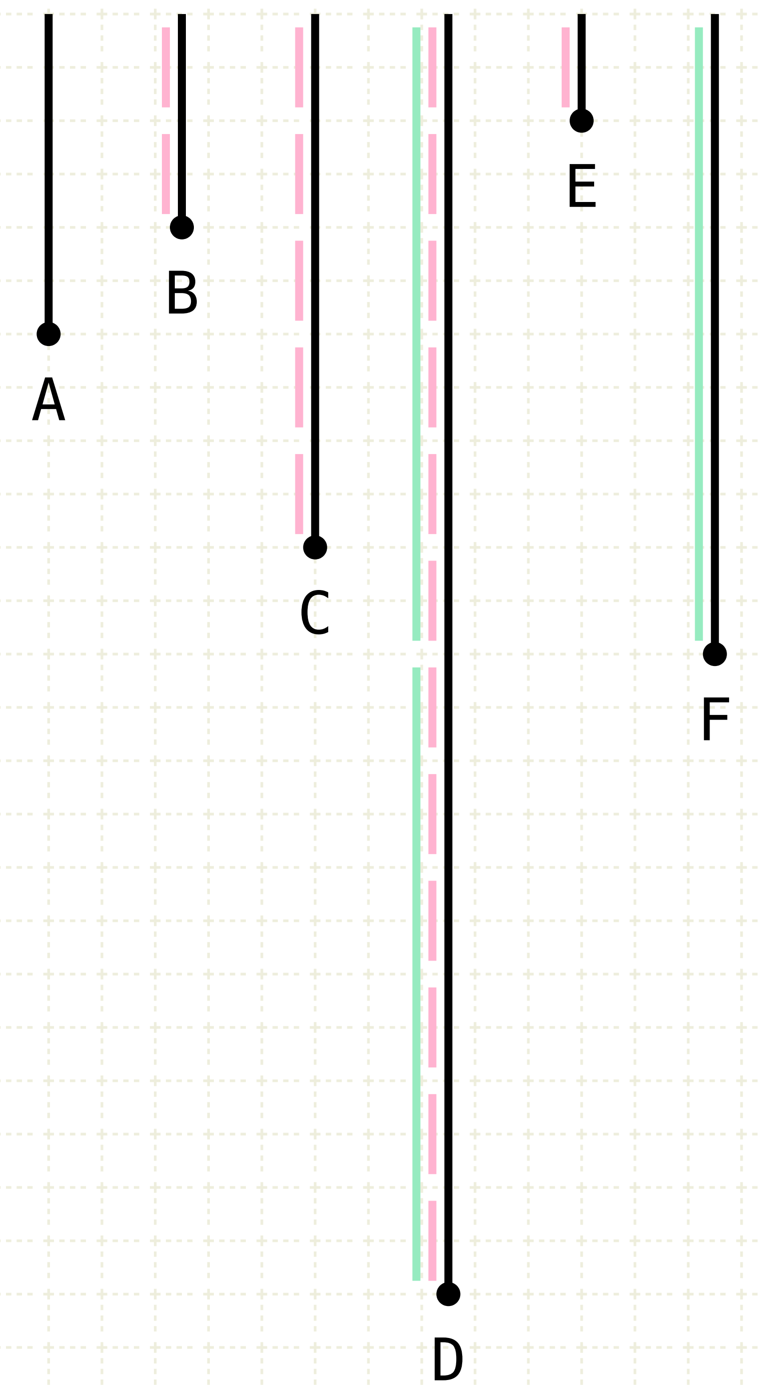

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A and E are relatively prime, so therefore they are the smallest numbers which can represent the ratio E to A (VII·21)

Since A and E are the smallest numbers in the ratio of B and F, E will measure B (and A will measure F) (VII·20)

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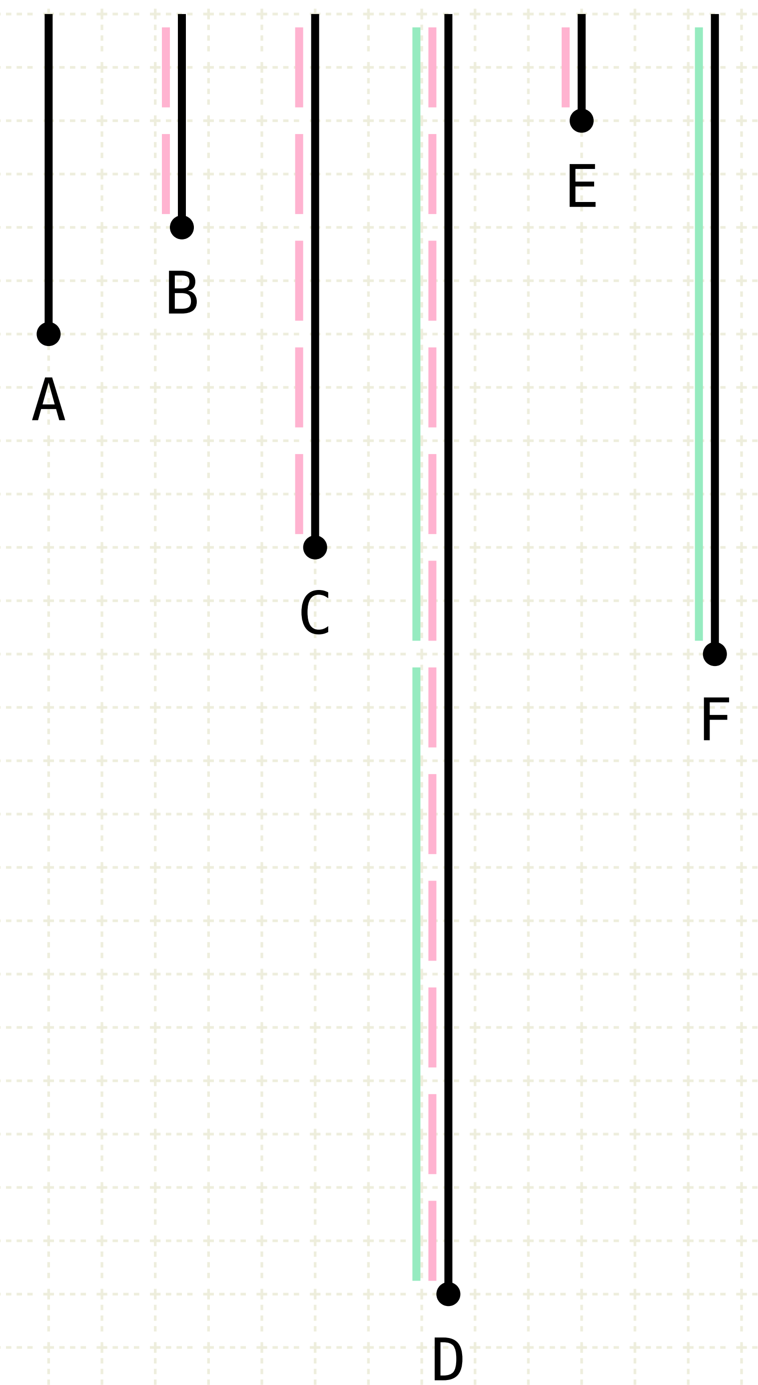

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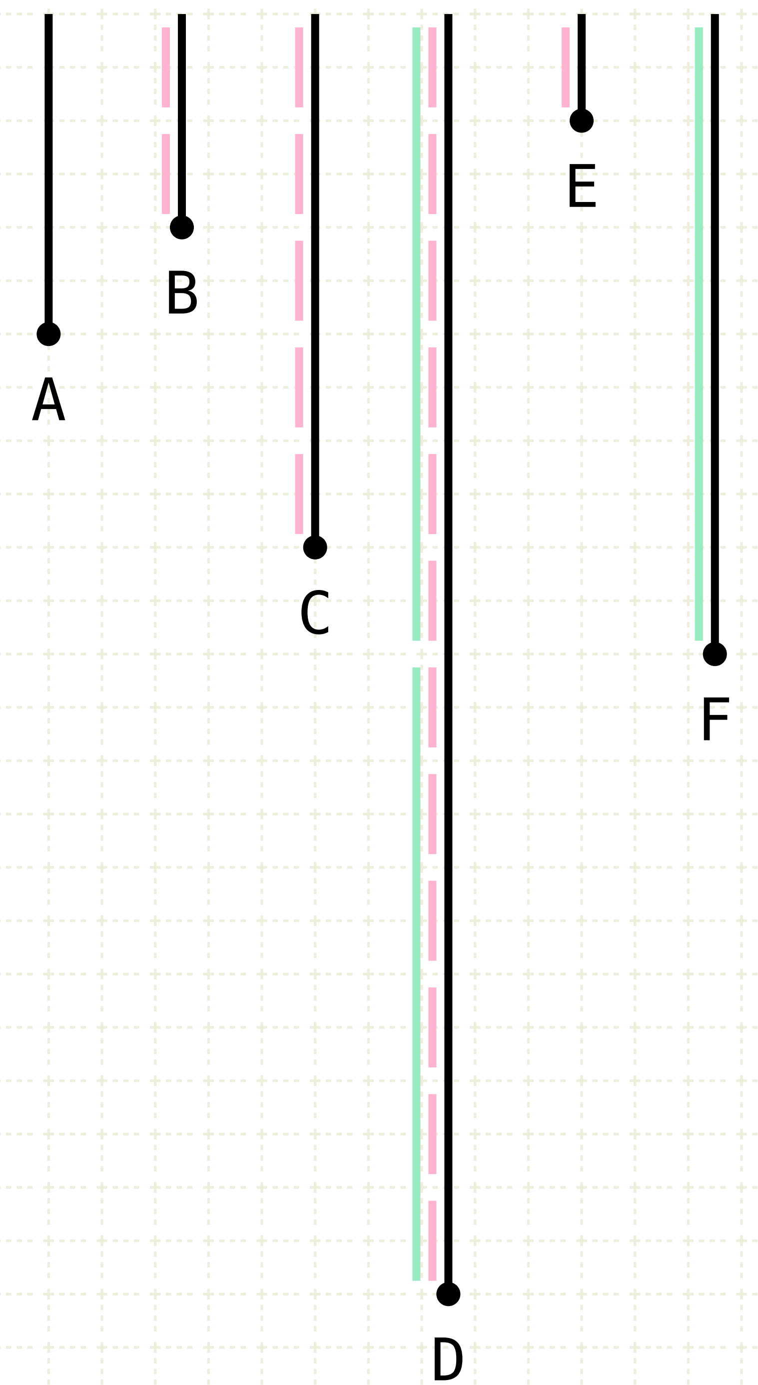

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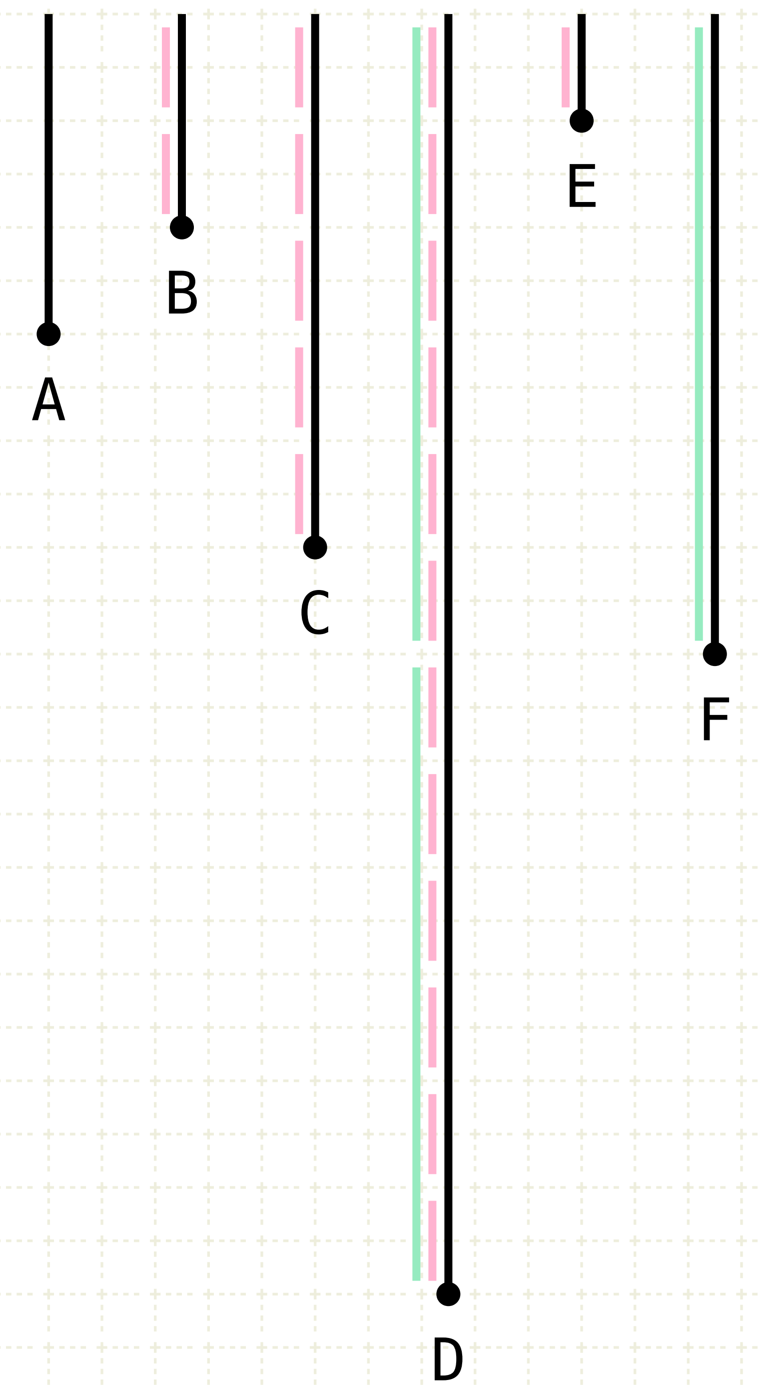

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(E:A)∈S such that E≤x, A≤y, ∀(x,y)∈S
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