

Euclid's Elements

Book III



A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)



Table of Contents, Chapter 3

1	To find the centre of a circle	9	If three lines, starting at a point 'A' and touching the circle, are all equal, then 'A' is the centre of the circle	17	From a given point to draw a straight line touching a given circle
2	A chord of a circle always lies inside the circle	10	A circle does not cut a circle at more points than two	18	If line touches a circle, then it is perpendicular to the diameter that touches that point
3	A line through the centre of a circle bisects a chord, and vice versa	11	Point of contact between two internal circles, and their centres, are collinear	19	If line touches a circle, then the centre of the circle lies on a line perpendicular to the original
4	A line not through the centre of a circle does not bisect a chord	12	Point of contact between two external circles, and their centres, are collinear	20	The angle at the centre of a circle is twice that from an angle from the circumference
5	If two circles cut one another, they will not have the same center	13	A circle does not touch a circle at more points than one, whether it touch it internally or externally.	21	In a circle the angles in the same segment are equal to one another
6	If two circles touch one another, they will not have the same center	14	In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another.	22	The opposite angles of quadrilaterals in circles are equal to two right angles
7	Consider two lines from a point inside a circle to the edge, the longer one will be the one closest to the longest part of the diameter passing through the original point	15	The longest line in a circle is its diameter, shorter the farther away from the diameter	23	On the same straight line there cannot be constructed two similar and unequal segments of circles on the same side
8	Consider two lines from a point outside a circle to the edge, the line closest to the centre will be longer on the concave side and shorter on the convex side	16	A line on the circle, perpendicular to the diameter, lies outside the circle	24	Similar segments of circles on equal straight lines are equal to one another



Table of Contents, Chapter 3

- | | | | |
|----|---|----|--|
| 25 | Given a segment of a circle, to describe the complete circle of which it is a segment. | 34 | Construct a circle segment on a given circle, such that the angle within the segment is equal to a given angle |
| 26 | In equal circles equal angles stand on equal circumferences | 35 | If two circle chords intersect, the segments on one multiplied together equals the segments of the other multiplied together |
| 27 | In equal circles angles standing on equal circumferences are equal to one another | | |
| 28 | In equal circles equal straight lines cut off equal circumferences | 36 | Secant-tangent law |
| 29 | In equal circles equal circumferences are subtended by equal straight lines | 37 | Converse of the secant-tangent law |
| 30 | To bisect a given circumference | | |
| 31 | In a circle the angle in the semicircle is right ... | | |
| 32 | The angle between a tangent and a straight line cutting a circle is equal to the angle in the alternate segment | | |
| 33 | Construct a circle segment on a given line, such that the angle within the segment is equal to a given angle | | |



Proposition 14 of Book III

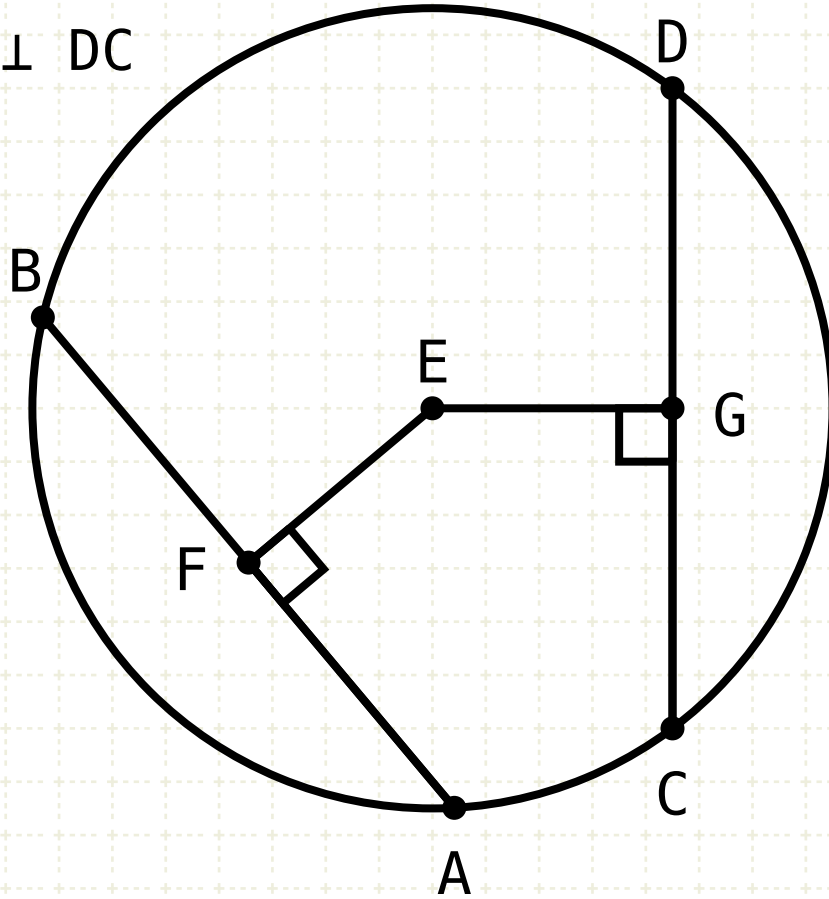
In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another.



Proposition 14 of Book III

In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another.

$EF \perp BA$
 $EG \perp DC$



In other words

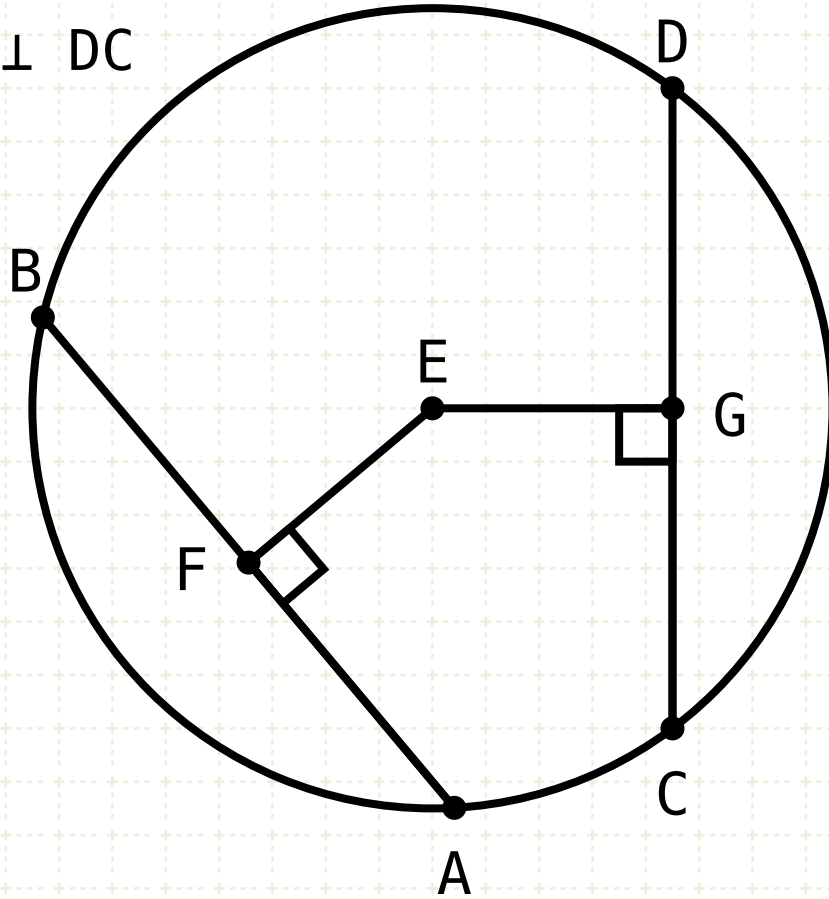
If E is the centre of the circle, EF perpendicular to BA, and EG perpendicular to DC, then

- (1) If AB equals CD, then EG equals EF
- (2) If EF equals EG, then AB equals CD

Proposition 14 of Book III

In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another.

$EF \perp BA$
 $EG \perp DC$



In other words

If E is the centre of the circle, EF perpendicular to BA, and EG perpendicular to DC, then

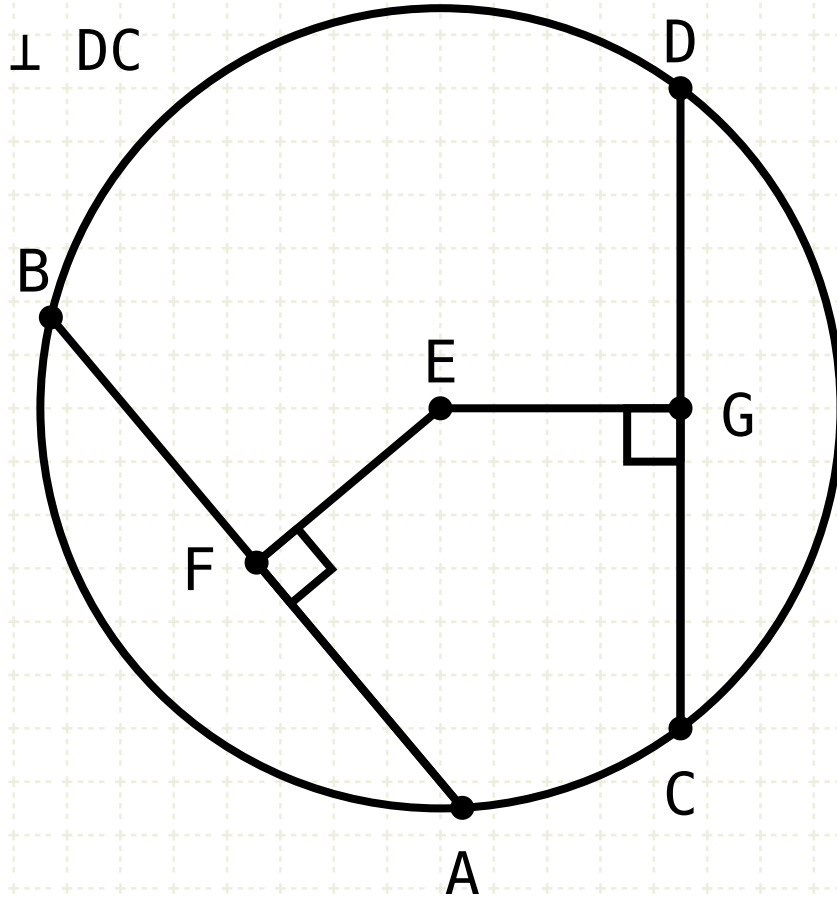
- (1) If AB equals CD, then EG equals EF
- (2) If EF equals EG, then AB equals CD

Proof

Proposition 14 of Book III

In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another.

$EF \perp BA$
 $EG \perp DC$



$$AF = FB, \quad AB = 2 \cdot AF$$
$$DG = GC, \quad DC = 2 \cdot GC$$

In other words

If E is the centre of the circle, EF perpendicular to BA, and EG perpendicular to DC, then

- (1) If AB equals CD, then EG equals EF
- (2) If EF equals EG, then AB equals CD

Proof

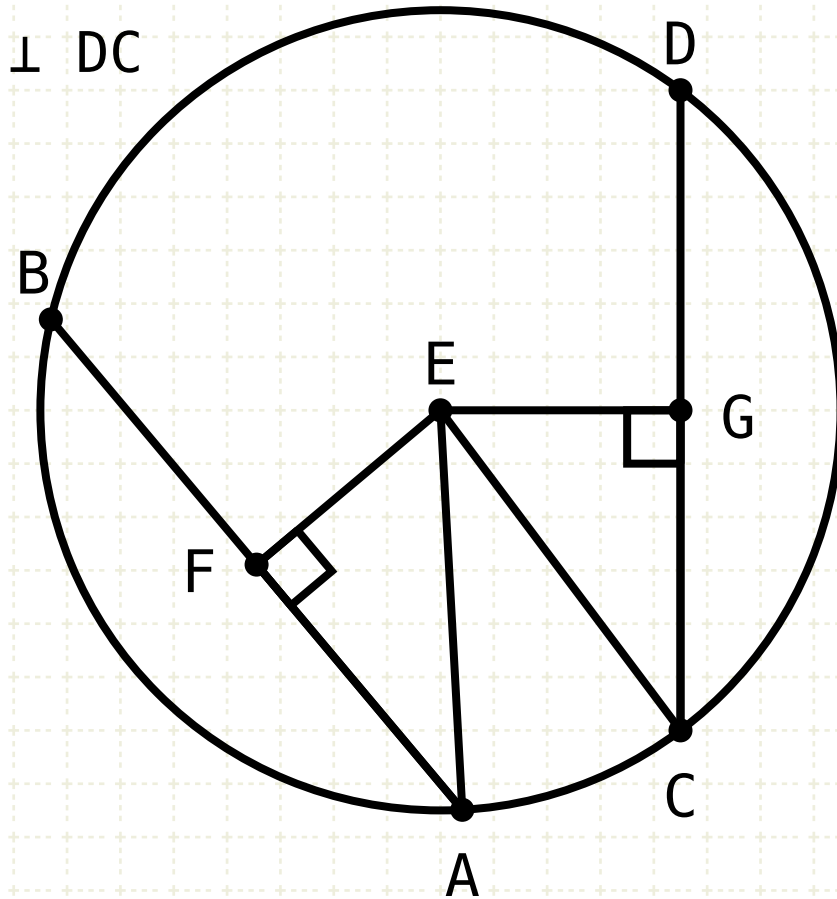
The straight line EF bisects AB, and EG bisects DC (III·3), therefore AB is twice AF, and DC is twice GC

Proposition 14 of Book III

In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another.

$EF \perp BA$
 $EG \perp DC$

$$AF = FB, \quad AB = 2 \cdot AF$$
$$DG = GC, \quad DC = 2 \cdot GC$$



In other words

If E is the centre of the circle, EF perpendicular to BA, and EG perpendicular to DC, then

- (1) If AB equals CD, then EG equals EF
- (2) If EF equals EG, then AB equals CD

Proof

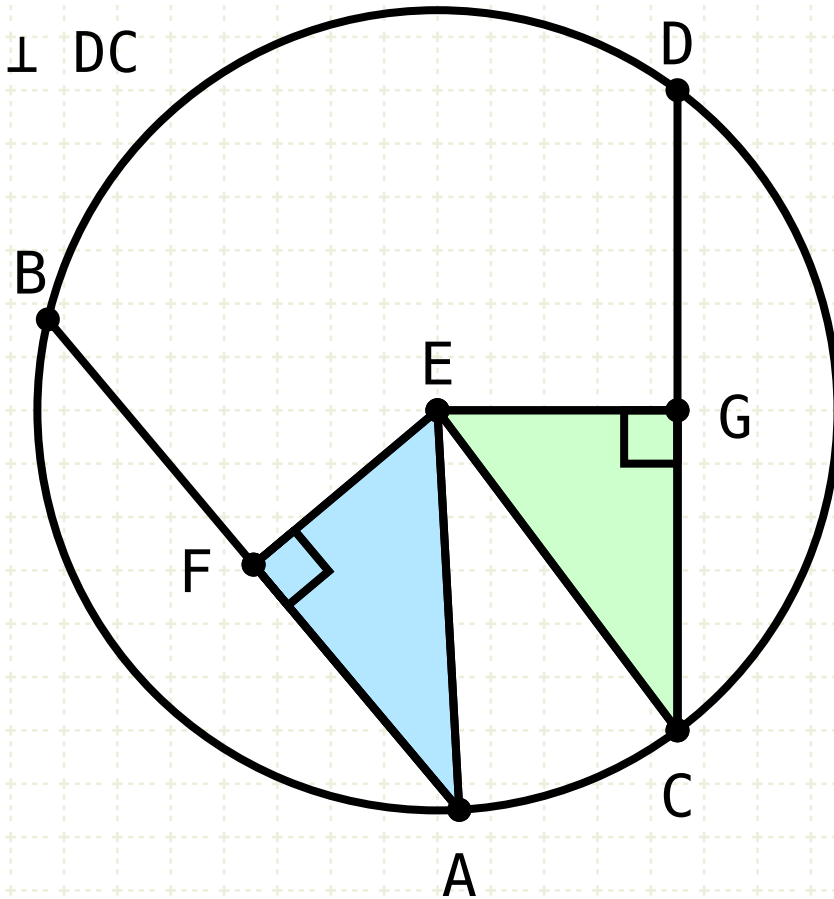
The straight line EF bisects AB, and EG bisects DC (III·3), therefore AB is twice AF, and DC is twice GC

Join AE and CE

Proposition 14 of Book III

In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another.

$EF \perp BA$
 $EG \perp DC$



$$AF = FB, \quad AB = 2 \cdot AF$$

$$DG = GC, \quad DC = 2 \cdot GC$$

$$EF^2 + AF^2 = AE^2$$

$$EG^2 + GC^2 = CE^2$$

In other words

If E is the centre of the circle, EF perpendicular to BA, and EG perpendicular to DC, then

- (1) If AB equals CD, then EG equals EF
- (2) If EF equals EG, then AB equals CD

Proof

The straight line EF bisects AB, and EG bisects DC (III·3), therefore AB is twice AF, and DC is twice GC

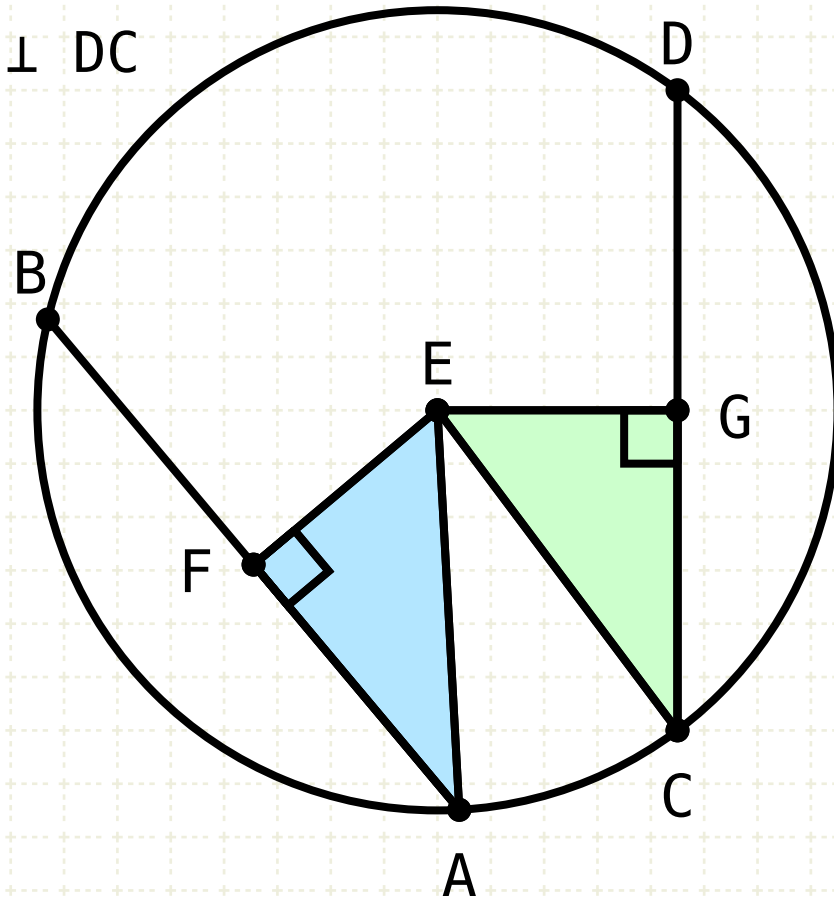
Join AE and CE

Triangles EFA and ECG are right angle triangles, and therefore the sum of the squares on the right angle equals the square of the line opposite (I·47)

Proposition 14 of Book III

In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another.

$EF \perp BA$
 $EG \perp DC$



$$AF = FB, \quad AB = 2 \cdot AF$$

$$DG = GC, \quad DC = 2 \cdot GC$$

$$EF^2 + AF^2 = AE^2$$

$$EG^2 + GC^2 = CE^2$$

$$AE = CE$$

In other words

If E is the centre of the circle, EF perpendicular to BA, and EG perpendicular to DC, then

- (1) If AB equals CD, then EG equals EF
- (2) If EF equals EG, then AB equals CD

Proof

The straight line EF bisects AB, and EG bisects DC (III·3), therefore AB is twice AF, and DC is twice GC

Join AE and CE

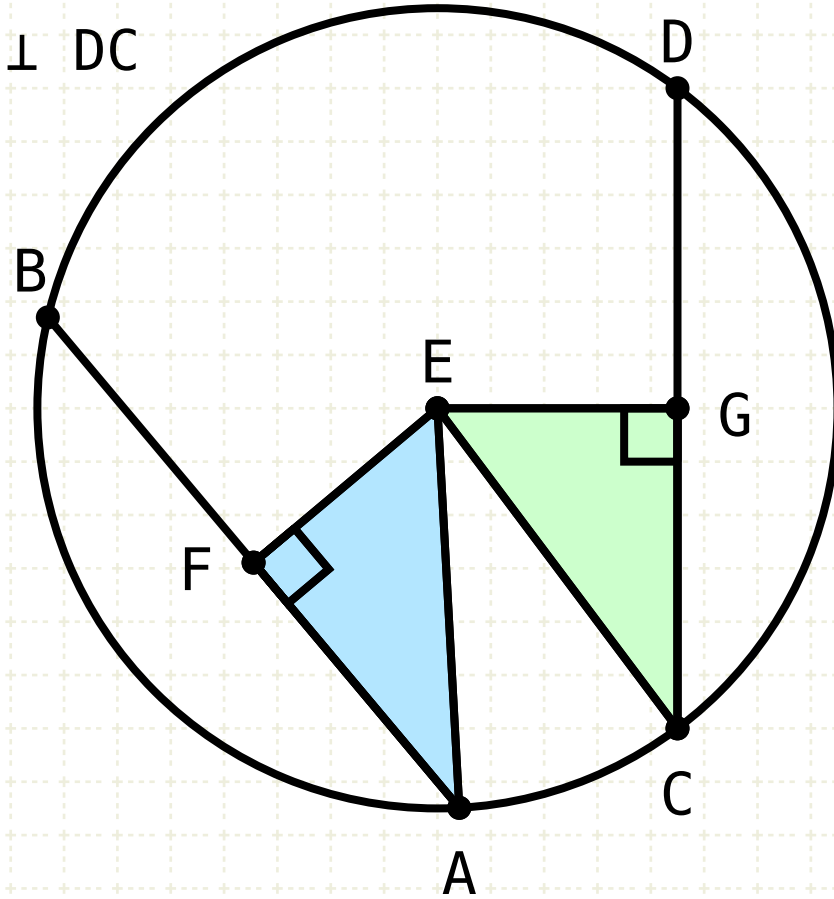
Triangles EFA and ECG are right angle triangles, and therefore the sum of the squares on the right angle equals the square of the line opposite (I·47)

AE and CE are equal since they are the radii of the circle

Proposition 14 of Book III

In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another.

$EF \perp BA$
 $EG \perp DC$



$$AF = FB, \quad AB = 2 \cdot AF$$

$$DG = GC, \quad DC = 2 \cdot GC$$

$$EF^2 + AF^2 = AE^2$$

$$EG^2 + GC^2 = CE^2$$

$$AE = CE$$

$$EF^2 + AF^2 = EG^2 + GC^2$$

In other words

If E is the centre of the circle, EF perpendicular to BA, and EG perpendicular to DC, then

- (1) If AB equals CD, then EG equals EF
- (2) If EF equals EG, then AB equals CD

Proof

The straight line EF bisects AB, and EG bisects DC (III·3), therefore AB is twice AF, and DC is twice GC

Join AE and CE

Triangles EFA and ECG are right angle triangles, and therefore the sum of the squares on the right angle equals the square of the line opposite (I·47)

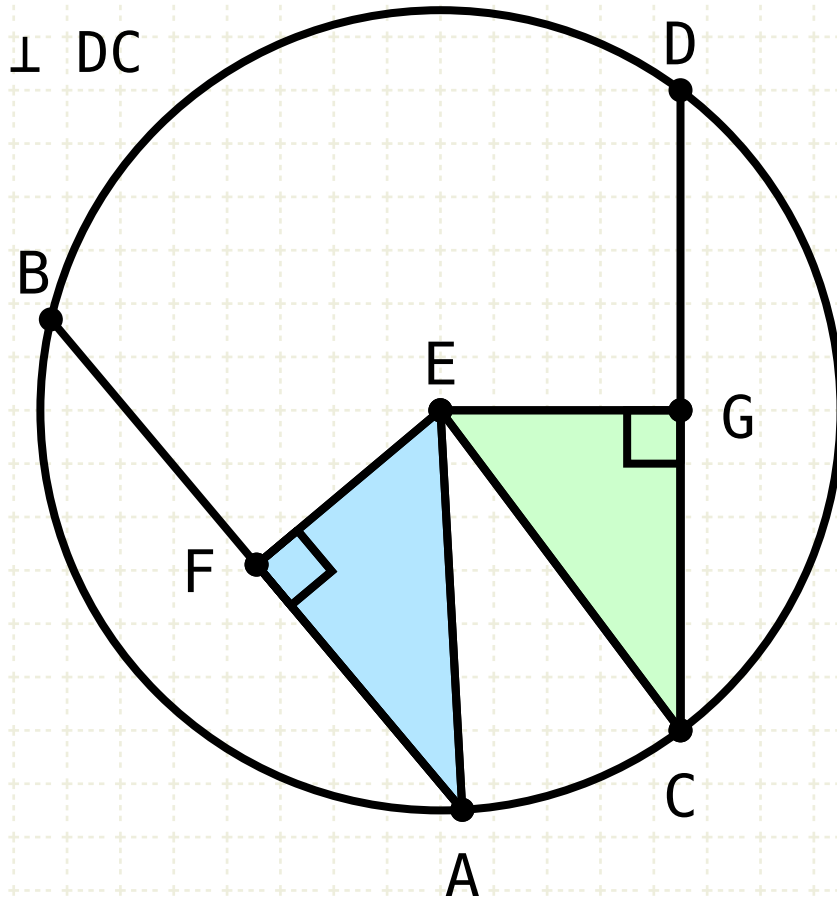
AE and CE are equal since they are the radii of the circle

Therefore the sum of the squares EF, AF equals the sum of the squares EG, GC

Proposition 14 of Book III

In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another.

$EF \perp BA$
 $EG \perp DC$



$$AF = FB, \quad AB = 2 \cdot AF$$

$$DG = GC, \quad DC = 2 \cdot GC$$

$$EF^2 + AF^2 = AE^2$$

$$EG^2 + GC^2 = CE^2$$

$$AE = CE$$

$$EF^2 + AF^2 = EG^2 + GC^2$$

$$(1) \text{ if } AB = CD$$

$$2 \cdot AF = 2 \cdot GC$$

$$AF = GC$$

$$AF^2 = GC^2$$

In other words

If E is the centre of the circle, EF perpendicular to BA, and EG perpendicular to DC, then

(1) If AB equals CD, then EG equals EF

(2) If EF equals EG, then AB equals CD

Proof

The straight line EF bisects AB, and EG bisects DC (III·3), therefore AB is twice AF, and DC is twice GC

Join AE and CE

Triangles EFA and ECG are right angle triangles, and therefore the sum of the squares on the right angle equals the square of the line opposite (I·47)

AE and CE are equal since they are the radii of the circle

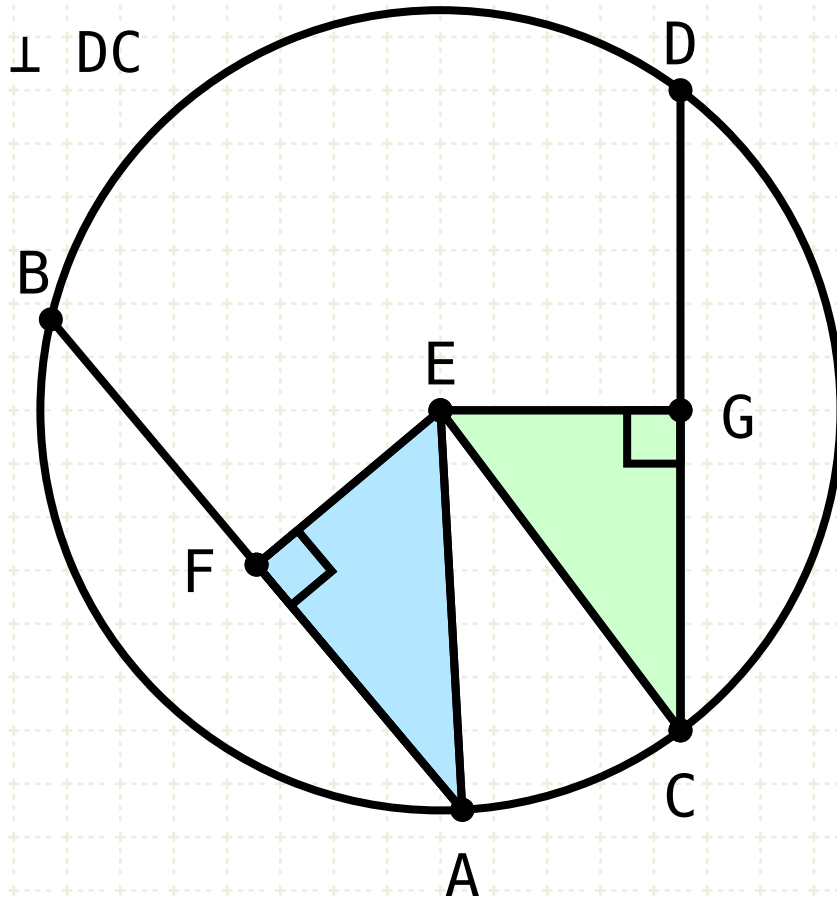
Therefore the sum of the squares EF, AF equals the sum of the squares EG, GC

(1) If AB equals CD, then AF equals GC, and AF squared equals GC squared

Proposition 14 of Book III

In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another.

$EF \perp BA$
 $EG \perp DC$



$$AF = FB, \quad AB = 2 \cdot AF$$

$$DG = GC, \quad DC = 2 \cdot GC$$

$$EF^2 + AF^2 = AE^2$$

$$EG^2 + GC^2 = CE^2$$

$$AE = CE$$

$$EF^2 + AF^2 = EG^2 + GC^2$$

$$(1) \text{ if } AB = DC$$

$$2 \cdot AF = 2 \cdot GC$$

$$AF = GC$$

$$AF^2 = GC^2$$

$$EF^2 = EG^2$$

In other words

If E is the centre of the circle, EF perpendicular to BA, and EG perpendicular to DC, then

(1) If AB equals DC, then EG equals EF

(2) If EF equals EG, then AB equals DC

Proof

The straight line EF bisects AB, and EG bisects DC (III·3), therefore AB is twice AF, and DC is twice GC

Join AE and CE

Triangles EFA and ECG are right angle triangles, and therefore the sum of the squares on the right angle equals the square of the line opposite (I·47)

AE and CE are equal since they are the radii of the circle

Therefore the sum of the squares EF, AF equals the sum of the squares EG, GC

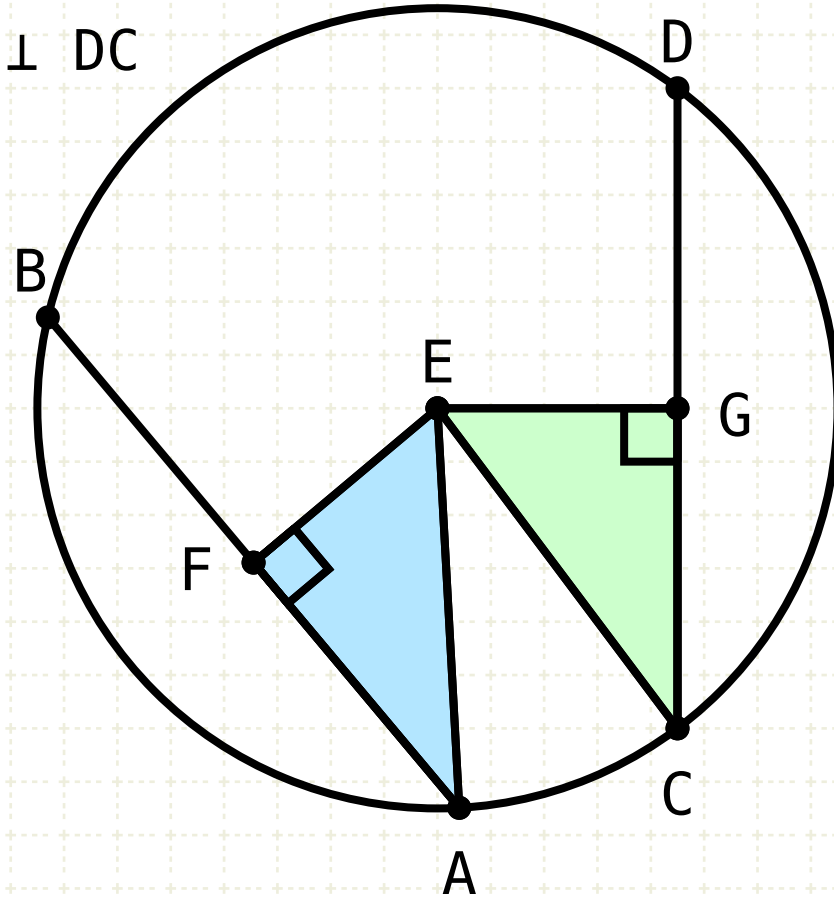
(1) If AB equals DC, then AF equals GC, and AF squared equals GC squared

Then EF squared equals EG squared, or EF equals EG

Proposition 14 of Book III

In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another.

$EF \perp BA$
 $EG \perp DC$



$$AF = FB, \quad AB = 2 \cdot AF$$

$$DG = GC, \quad DC = 2 \cdot GC$$

$$EF^2 + AF^2 = AE^2$$

$$EG^2 + GC^2 = CE^2$$

$$AE = CE$$

$$EF^2 + AF^2 = EG^2 + GC^2$$

$$(1) \text{ if } AB = DC$$

$$2 \cdot AF = 2 \cdot GC$$

$$AF = GC$$

$$AF^2 = GC^2$$

$$EF^2 = EG^2$$

$$EF = EG$$

In other words

If E is the centre of the circle, EF perpendicular to BA, and EG perpendicular to DC, then

(1) If AB equals DC, then EG equals EF

(2) If EF equals EG, then AB equals DC

Proof

The straight line EF bisects AB, and EG bisects DC (III·3), therefore AB is twice AF, and DC is twice GC

Join AE and CE

Triangles EFA and ECG are right angle triangles, and therefore the sum of the squares on the right angle equals the square of the line opposite (I·47)

AE and CE are equal since they are the radii of the circle

Therefore the sum of the squares EF, AF equals the sum of the squares EG, GC

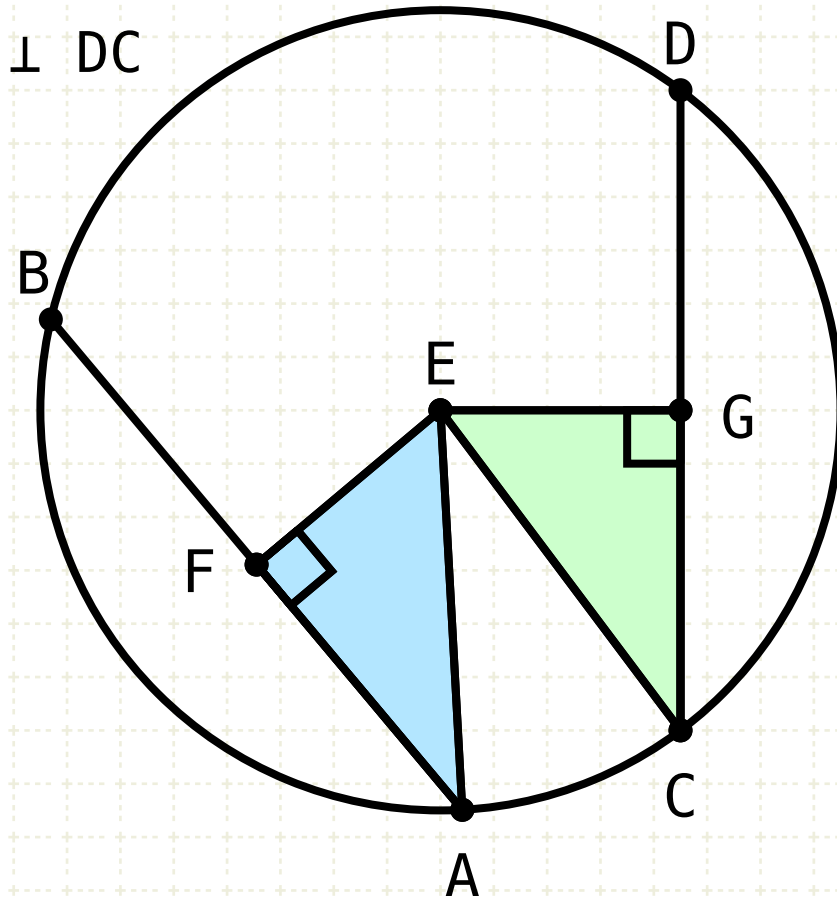
(1) If AB equals DC, then AF equals GC, and AF squared equals GC squared

Then EF squared equals EG squared, or EF equals EG

Proposition 14 of Book III

In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another.

$EF \perp BA$
 $EG \perp DC$



$$AF = FB, \quad AB = 2 \cdot AF$$

$$DG = GC, \quad DC = 2 \cdot GC$$

$$EF^2 + AF^2 = AE^2$$

$$EG^2 + GC^2 = CE^2$$

$$AE = CE$$

$$EF^2 + AF^2 = EG^2 + GC^2$$

$$(1) \text{ if } AB = DC$$

$$2 \cdot AF = 2 \cdot GC$$

$$AF = GC$$

$$AF^2 = GC^2$$

$$EF^2 = EG^2$$

$$EF = EG$$

$$(2) \text{ if } EF = EG$$

$$EF^2 = EG^2$$

In other words

If E is the centre of the circle, EF perpendicular to BA, and EG perpendicular to DC, then

(1) If AB equals DC, then EG equals EF

(2) If EF equals EG, then AB equals DC

Proof

The straight line EF bisects AB, and EG bisects DC (III·3), therefore AB is twice AF, and DC is twice GC

Join AE and CE

Triangles EFA and ECG are right angle triangles, and therefore the sum of the squares on the right angle equals the square of the line opposite (I·47)

AE and CE are equal since they are the radii of the circle

Therefore the sum of the squares EF,AF equals the sum of the squares EG,GC

(1) If AB equals DC, then AF equals GC, and AF squared equals GC squared

Then EF squared equals EG squared, or EF equals EG

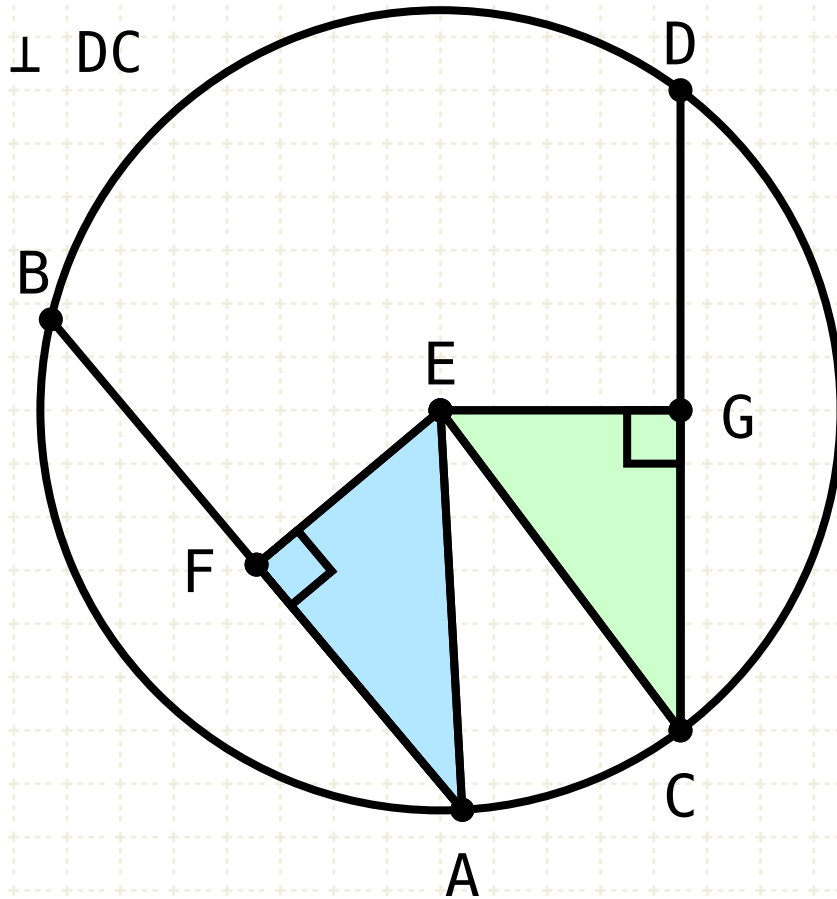
(2) Similarly, if EF equals EG, then AF squared equals CG squared



Proposition 14 of Book III

In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another.

$EF \perp BA$
 $EG \perp DC$



$$AF = FB, \quad AB = 2 \cdot AF$$

$$DG = GC, \quad DC = 2 \cdot GC$$

$$EF^2 + AF^2 = AE^2$$

$$EG^2 + GC^2 = CE^2$$

$$AE = CE$$

$$EF^2 + AF^2 = EG^2 + GC^2$$

$$(1) \text{ if } AB = DC$$

$$2 \cdot AF = 2 \cdot GC$$

$$AF = GC$$

$$AF^2 = GC^2$$

$$EF^2 = EG^2$$

$$EF = EG$$

$$(2) \text{ if } EF = EG$$

$$EF^2 = EG^2$$

$$AF^2 = GC^2$$

In other words

If E is the centre of the circle, EF perpendicular to BA, and EG perpendicular to DC, then

(1) If AB equals DC, then EG equals EF

(2) If EF equals EG, then AB equals DC

Proof

The straight line EF bisects AB, and EG bisects DC (III·3), therefore AB is twice AF, and DC is twice GC

Join AE and CE

Triangles EFA and ECG are right angle triangles, and therefore the sum of the squares on the right angle equals the square of the line opposite (I·47)

AE and CE are equal since they are the radii of the circle

Therefore the sum of the squares EF,AF equals the sum of the squares EG,GC

(1) If AB equals DC, then AF equals GC, and AF squared equals GC squared

Then EF squared equals EG squared, or EF equals EG

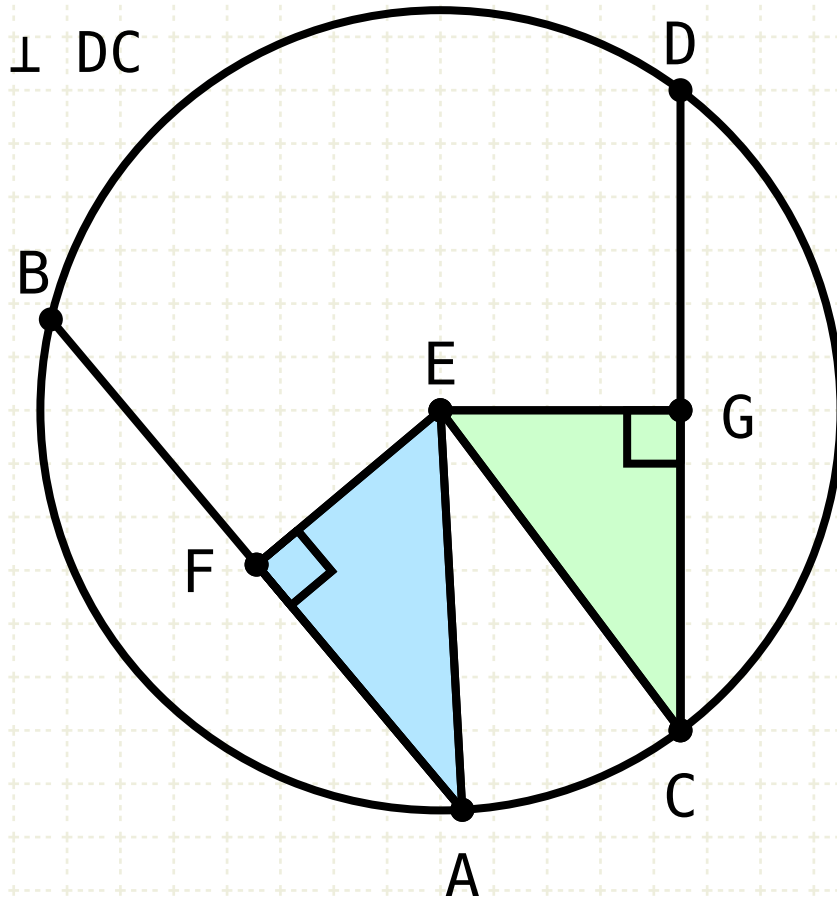
(2) Similarly, if EF equals EG, then AF squared equals CG squared



Proposition 14 of Book III

In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another.

$EF \perp BA$
 $EG \perp DC$



$$AF = FB, \quad AB = 2 \cdot AF$$

$$DG = GC, \quad DC = 2 \cdot GC$$

$$EF^2 + AF^2 = AE^2$$

$$EG^2 + GC^2 = CE^2$$

$$AE = CE$$

$$EF^2 + AF^2 = EG^2 + GC^2$$

$$(1) \text{ if } AB = CD$$

$$2 \cdot AF = 2 \cdot GC$$

$$AF = GC$$

$$AF^2 = GC^2$$

$$EF^2 = EG^2$$

$$EF = EG$$

$$(2) \text{ if } EF = EG$$

$$EF^2 = EG^2$$

$$AF^2 = GC^2$$

$$AF = GC$$

$$AB = CD$$

In other words

If E is the centre of the circle, EF perpendicular to BA, and EG perpendicular to DC, then

(1) If AB equals CD, then EG equals EF

(2) If EF equals EG, then AB equals CD

Proof

The straight line EF bisects AB, and EG bisects DC (III·3), therefore AB is twice AF, and DC is twice GC

Join AE and CE

Triangles EFA and ECG are right angle triangles, and therefore the sum of the squares on the right angle equals the square of the line opposite (I·47)

AE and CE are equal since they are the radii of the circle

Therefore the sum of the squares EF, AF equals the sum of the squares EG, GC

(1) If AB equals CD, then AF equals GC, and AF squared equals GC squared

Then EF squared equals EG squared, or EF equals EG

(2) Similarly, if EF equals EG, then AF squared equals CG squared

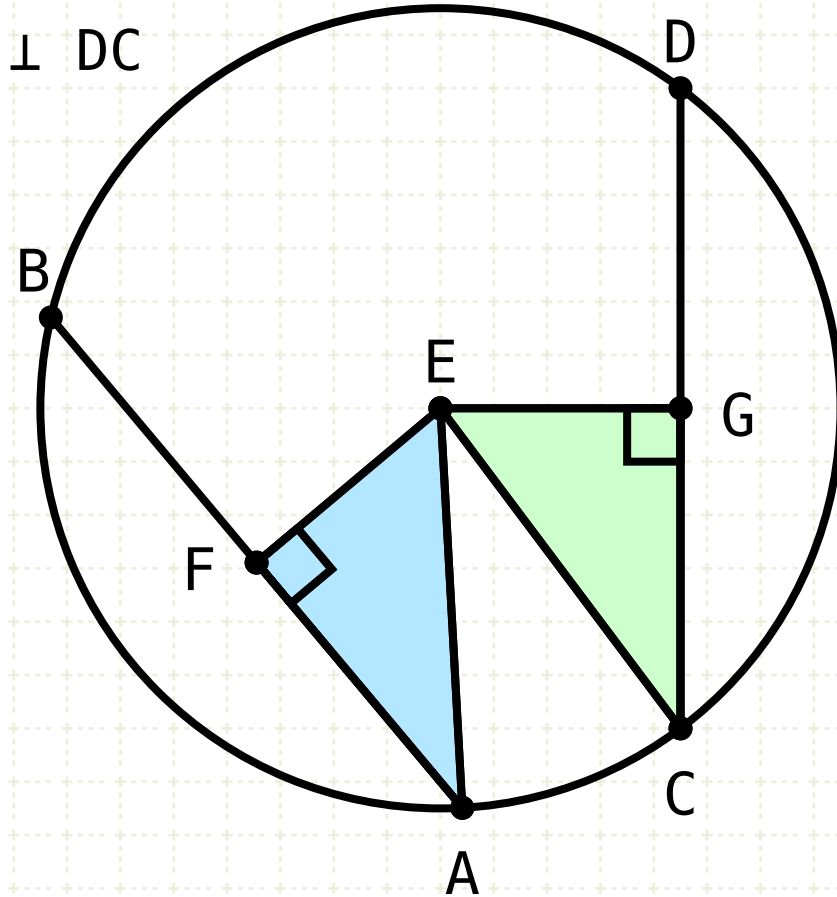
AF equals CG, and AB equals CD



Proposition 14 of Book III

In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another.

$EF \perp BA$
 $EG \perp DC$



$$AF = FB, \quad AB = 2 \cdot AF$$

$$DG = GC, \quad DC = 2 \cdot GC$$

$$EF^2 + AF^2 = AE^2$$

$$EG^2 + GC^2 = CE^2$$

$$AE = CE$$

$$EF^2 + AF^2 = EG^2 + GC^2$$

$$(1) \text{ if } AB = CD$$

$$2 \cdot AF = 2 \cdot GC$$

$$AF = GC$$

$$AF^2 = GC^2$$

$$EF^2 = EG^2$$

$$EF = EG$$

$$(2) \text{ if } EF = EG$$

$$EF^2 = EG^2$$

$$AF^2 = GC^2$$

$$AF = GC$$

$$AB = CD$$

In other words

If E is the centre of the circle, EF perpendicular to BA, and EG perpendicular to DC, then

(1) If AB equals CD, then EG equals EF

(2) If EF equals EG, then AB equals CD

Proof

The straight line EF bisects AB, and EG bisects DC (III·3), therefore AB is twice AF, and DC is twice GC

Join AE and CE

Triangles EFA and ECG are right angle triangles, and therefore the sum of the squares on the right angle equals the square of the line opposite (I·47)

AE and CE are equal since they are the radii of the circle

Therefore the sum of the squares EF, AF equals the sum of the squares EG, GC

(1) If AB equals CD, then AF equals GC, and AF squared equals GC squared

Then EF squared equals EG squared, or EF equals EG

(2) Similarly, if EF equals EG, then AF squared equals CG squared

AF equals CG, and AB equals CD

Youtube Videos

<https://www.youtube.com/c/SandyBultena>

Copyright © 2019 by Sandy Bultena.



Except where otherwise noted, this work is licensed under
<http://creativecommons.org/licenses/by-nc/3.0>