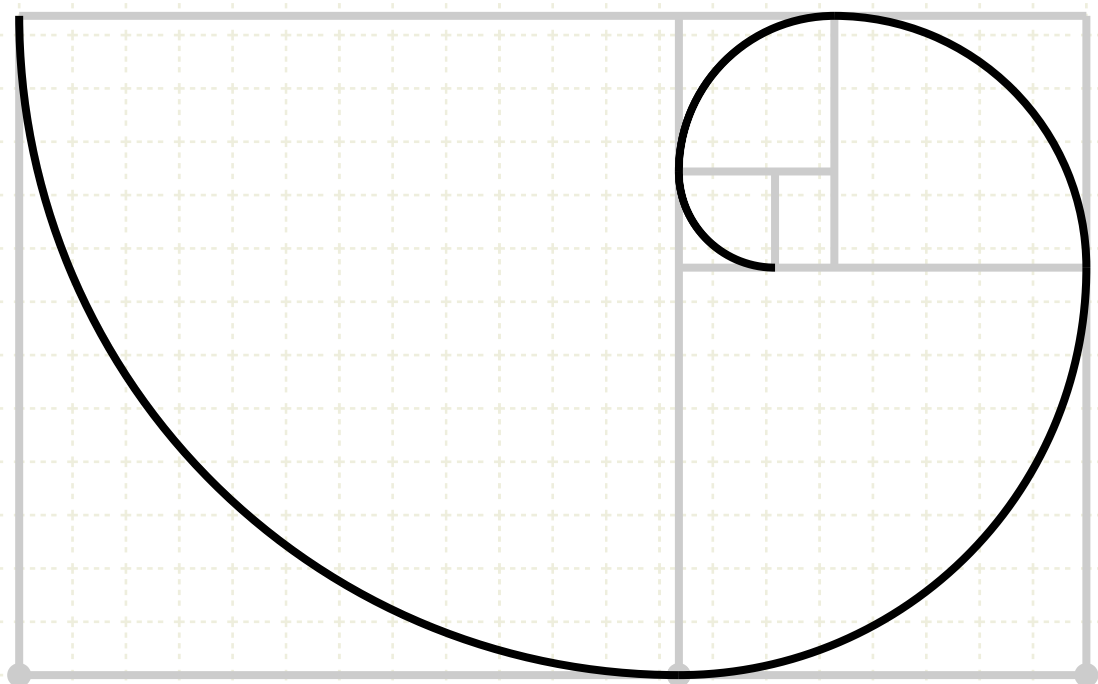
Euclid's Elements

Book VI



One can state, without exaggeration, that the observation of and the search for similarities and differences are the basis of all human knowledge.

Alfred Nobel

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Equiangular parallelograms have to one another the ratio compounded of the ratios of their sides

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Equiangular parallelograms have to one another the ratio compounded of the ratios of their sides

Definition: Compound Ratios

Euclid simply uses 'compound ratios' in this proof without previously defining them, so the definition of compound ratio has been retroactively defined as…[1]

[1] Euclid's Elements - all thirteen books complete in one volume, The Thomas L Heath Translation, Dana Densmore, Editor, Green Lion Press (c) 2013, pg xxix

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…If we have a series of magnitudes, A…D for example

A:D = compound ratio of A:B, B:C, C:D

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If A to B is equal to E to F, B to C equal to G to H, C to D equal J to K, then the ratio of A to D is also equal to the compound ratio of G to H, C to D, and J to K [1]

A:B = E:F

B:C = G:H

C:D = J:K

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A:B = E:F

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C:D = J:K

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For two or more ratios, if we take antecedent as product of antecedents of the ratios and consequent as product of consequents of the ratios, then the ratio thus formed is called mixed or compound ratio. [2]

compound ratio of m:n, p:q = mp:nq

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[2] http://www.math-only-math.com/types-of-ratios.html

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∴ A:D = ABC:BCD

and A:D = EGJ:FHK

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∴ A:D = ABC:BCD
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$$\frac{A}{D} = \frac{A}{D} \times \frac{B \cdot C}{B \cdot C} = \frac{A}{B} \times \frac{B}{C} \times \frac{C}{D}$$

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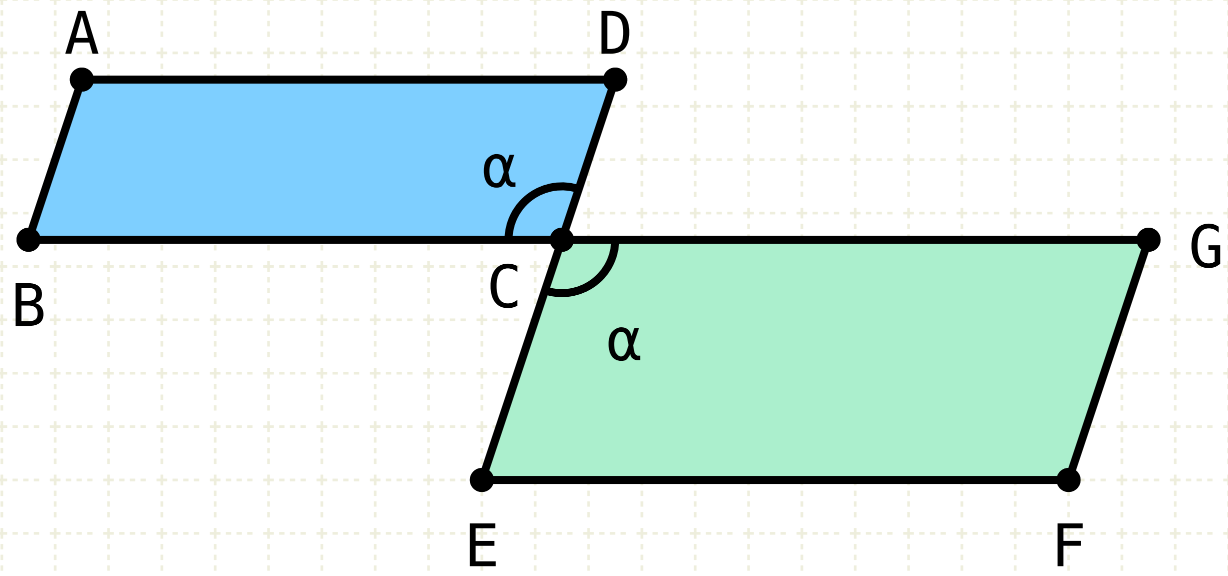
Proposition 23 of Book VI

Equiangular parallelograms have to one another the ratio compounded of the ratios of their sides



In other words

The areas of equiangular parallelograms have ratios that equal the multiplication of the ratios of their sides (compounded ratio)

$$\square AC : \square CF = (BC \cdot CD) : (CG \cdot CE)$$

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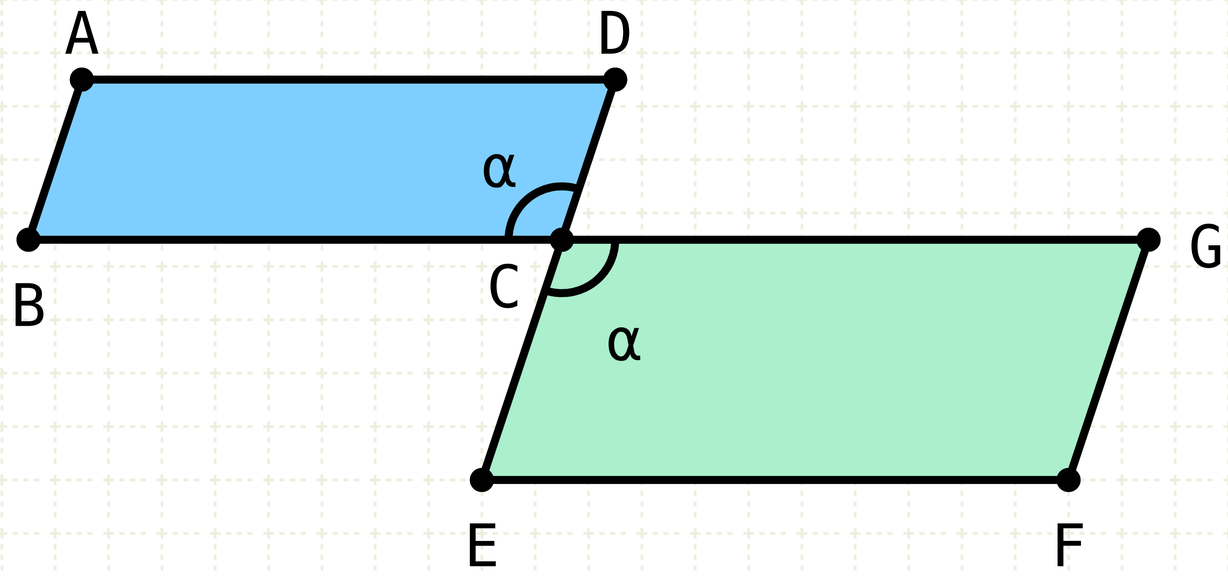

Proof

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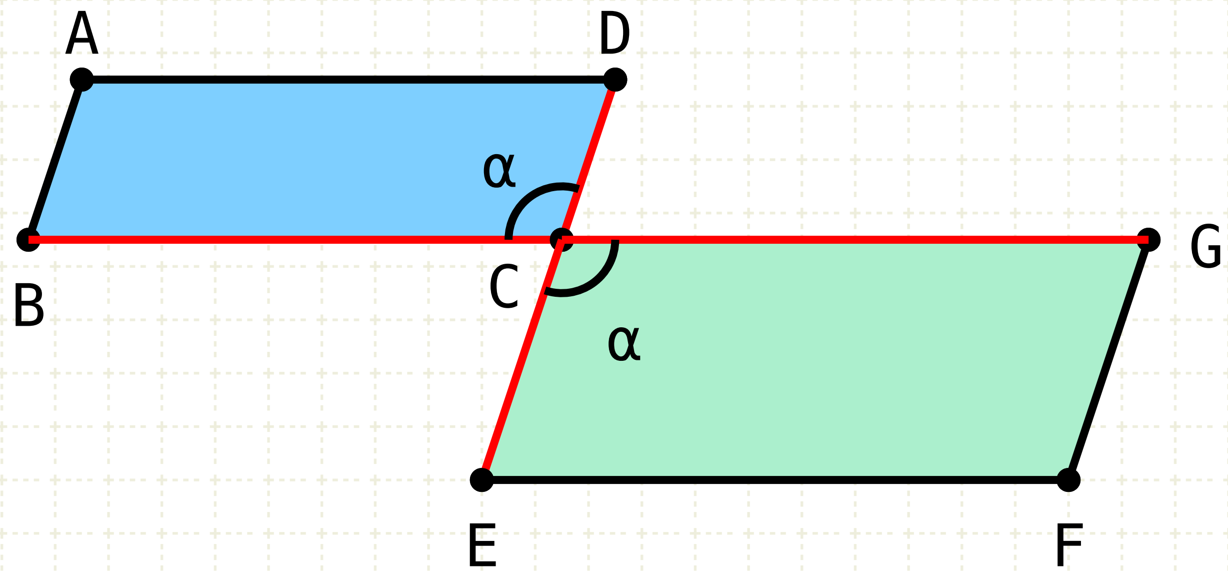

Proof

Let the parallelograms be placed so that BC is in a straight line with CG therefore DC is also in a straight line with CE

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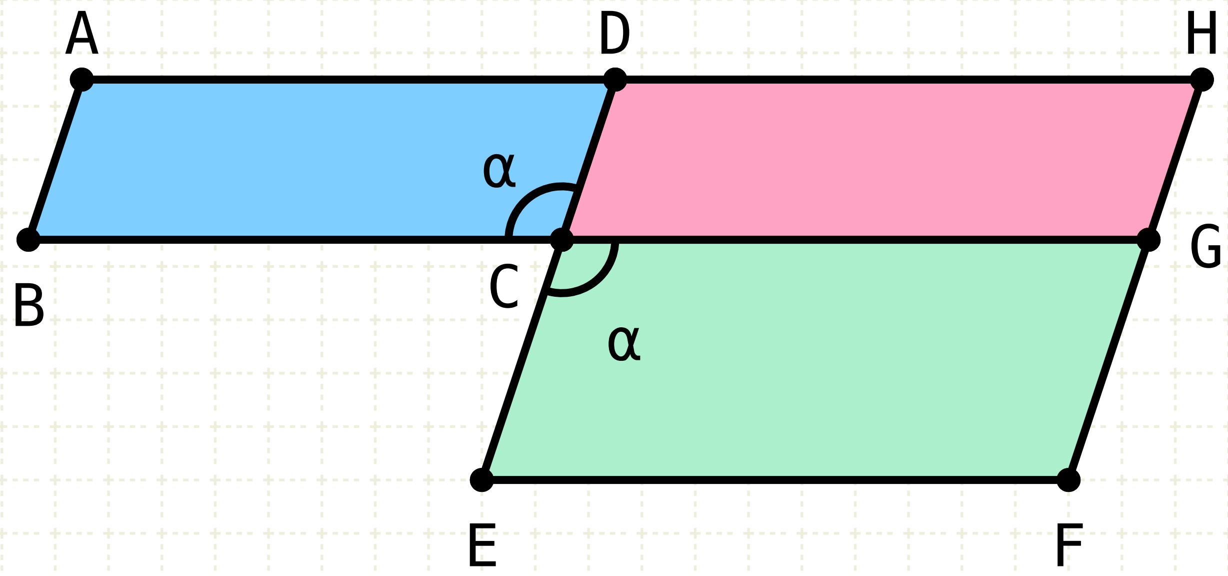

Proof

Let the parallelograms be placed so that BC is in a straight line with CG therefore DC is also in a straight line with CE

Draw the parallelogram DG

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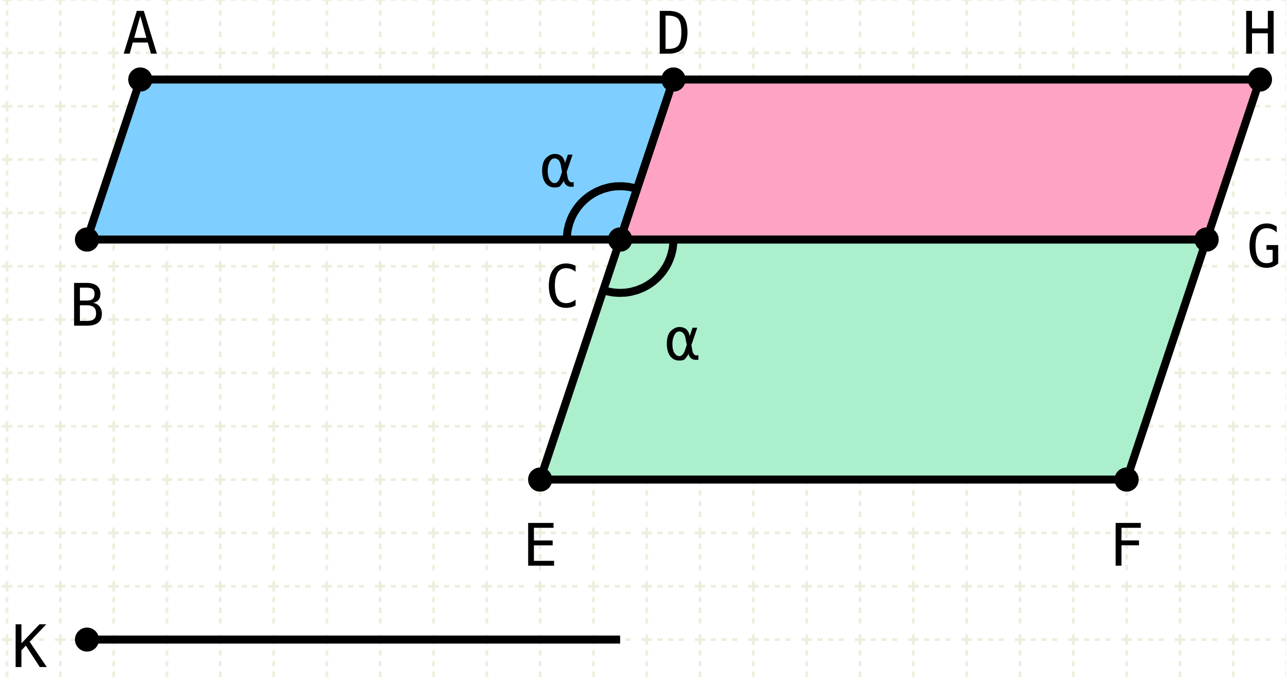

Proof

Let the parallelograms be placed so that BC is in a straight line with CG therefore DC is also in a straight line with CE

Draw the parallelogram DG

Define an arbitrary line K, and draw another line L such that the ratio as BC is to CG so is K to L (VI·12)

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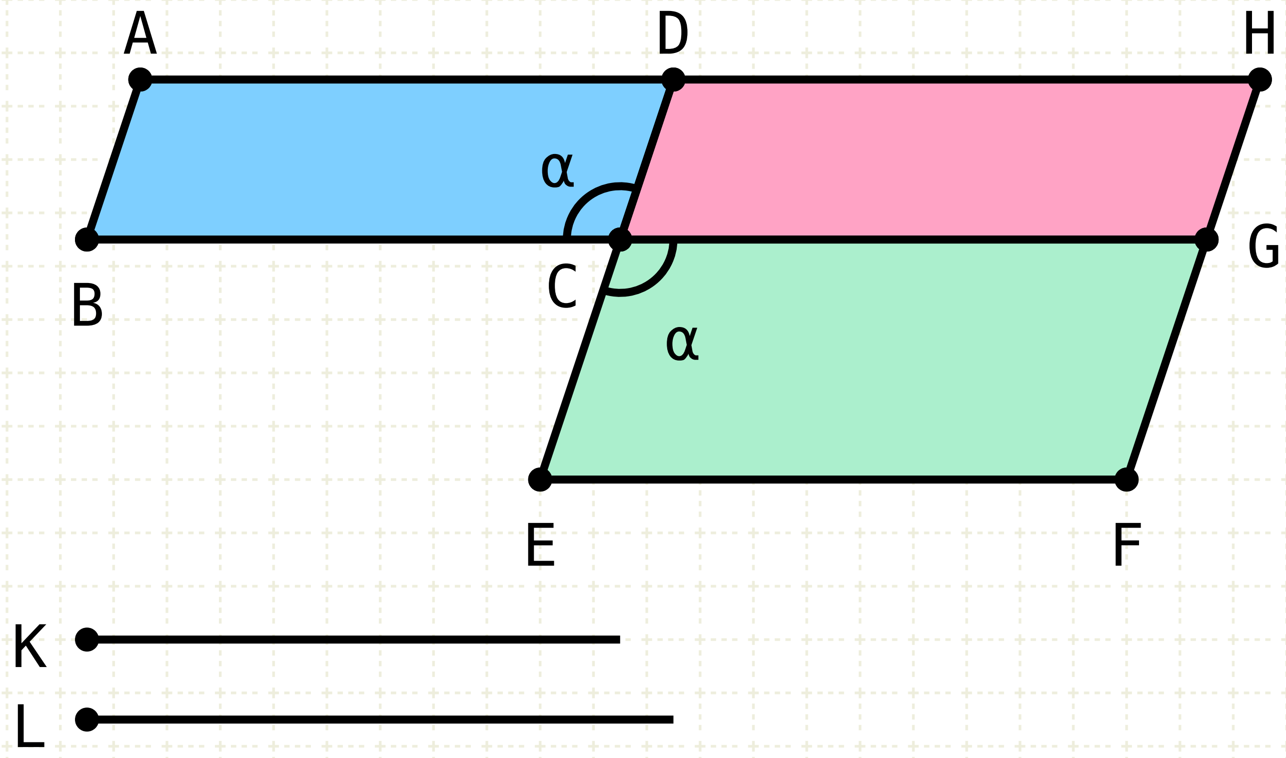

K:L = BC:CG

Proof

Let the parallelograms be placed so that BC is in a straight line with CG therefore DC is also in a straight line with CE

Draw the parallelogram DG

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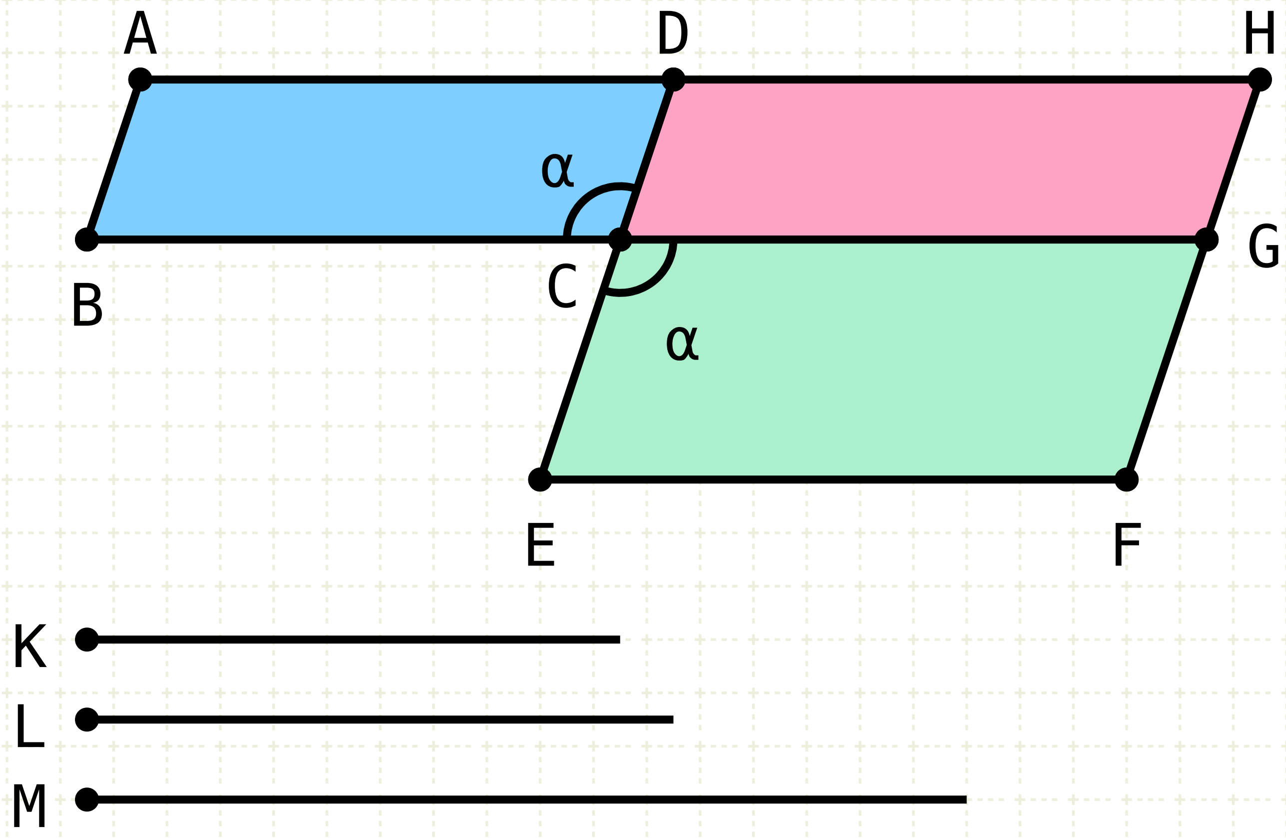

K:L = BC:CG

L:M = CD:CE

Proof

Let the parallelograms be placed so that BC is in a straight line with CG therefore DC is also in a straight line with CE

Draw the parallelogram DG

Define an arbitrary line K, and draw another line L such that the ratio as BC is to CG so is K to L (VI·12)

Draw another line M such that the ratio as DC is to CE so is L to M (VI·12)

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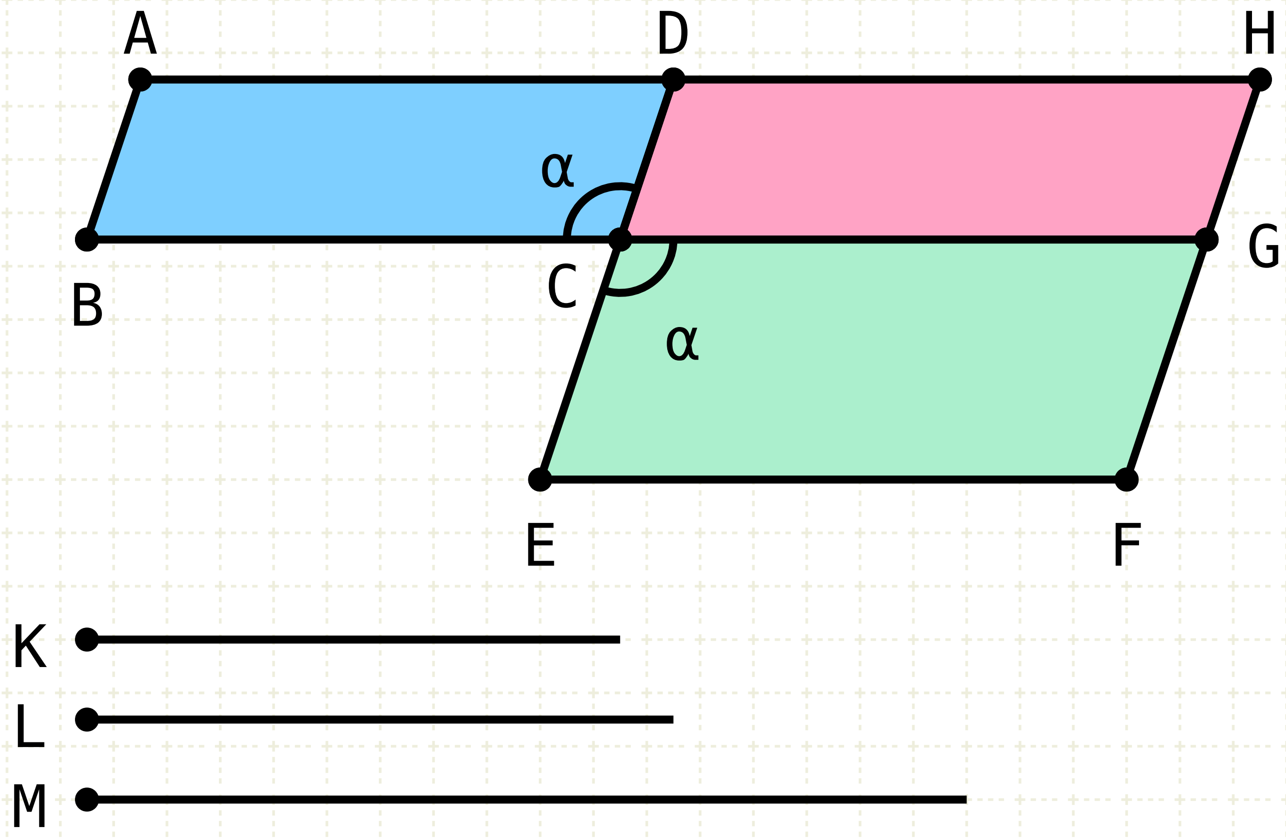

$$K:L = BC:CG$$
$$L:M = CD:CE$$
$$K:M = (BC \cdot CD):(CG \cdot CE)$$

Proof

Let the parallelograms be placed so that BC is in a straight line with CG therefore DC is also in a straight line with CE

Draw the parallelogram DG

Define an arbitrary line K, and draw another line L such that the ratio as BC is to CG so is K to L (VI·12)

Draw another line M such that the ratio as DC is to CE so is L to M (VI·12)

K to M is the compounded ratio of K to L and L to M, therefore it is also equal to the compound ratio of the sides, BC to CG and CD to CE

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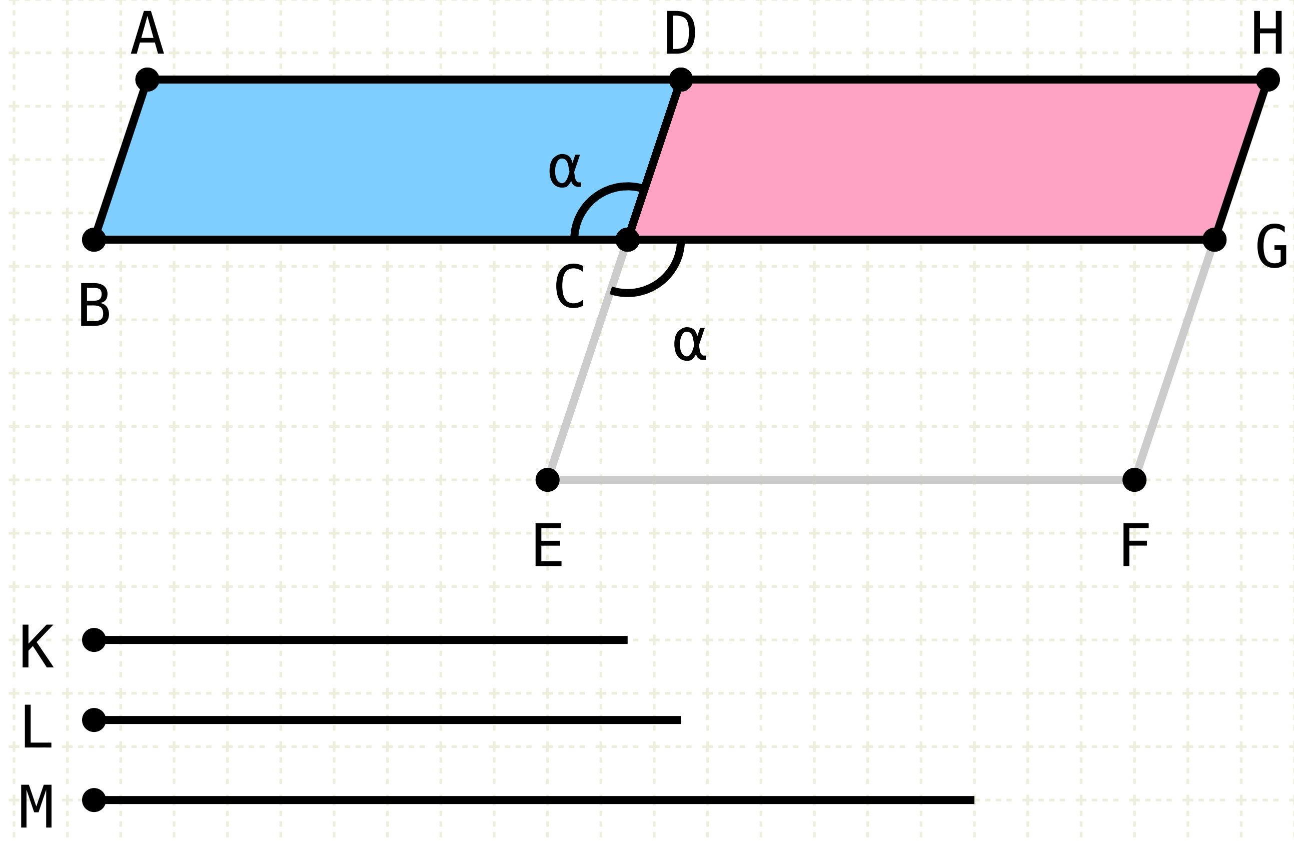

K:L = BC:CG

L:M = CD:CE

K:M = (BC·CD):(CG·CE)

BC:CG = □AC:□CH

Proof

Let the parallelograms be placed so that BC is in a straight line with CG therefore DC is also in a straight line with CE

Draw the parallelogram DG

Define an arbitrary line K, and draw another line L such that the ratio as BC is to CG so is K to L (VI·12)

Draw another line M such that the ratio as DC is to CE so is L to M (VI·12)

K to M is the compounded ratio of K to L and L to M, therefore it is also equal to the compound ratio of the sides, BC to CG and CD to CE

The ratio of the sides BC to CG is equal to the ratio of the parallelograms AC,CH (VI·1)

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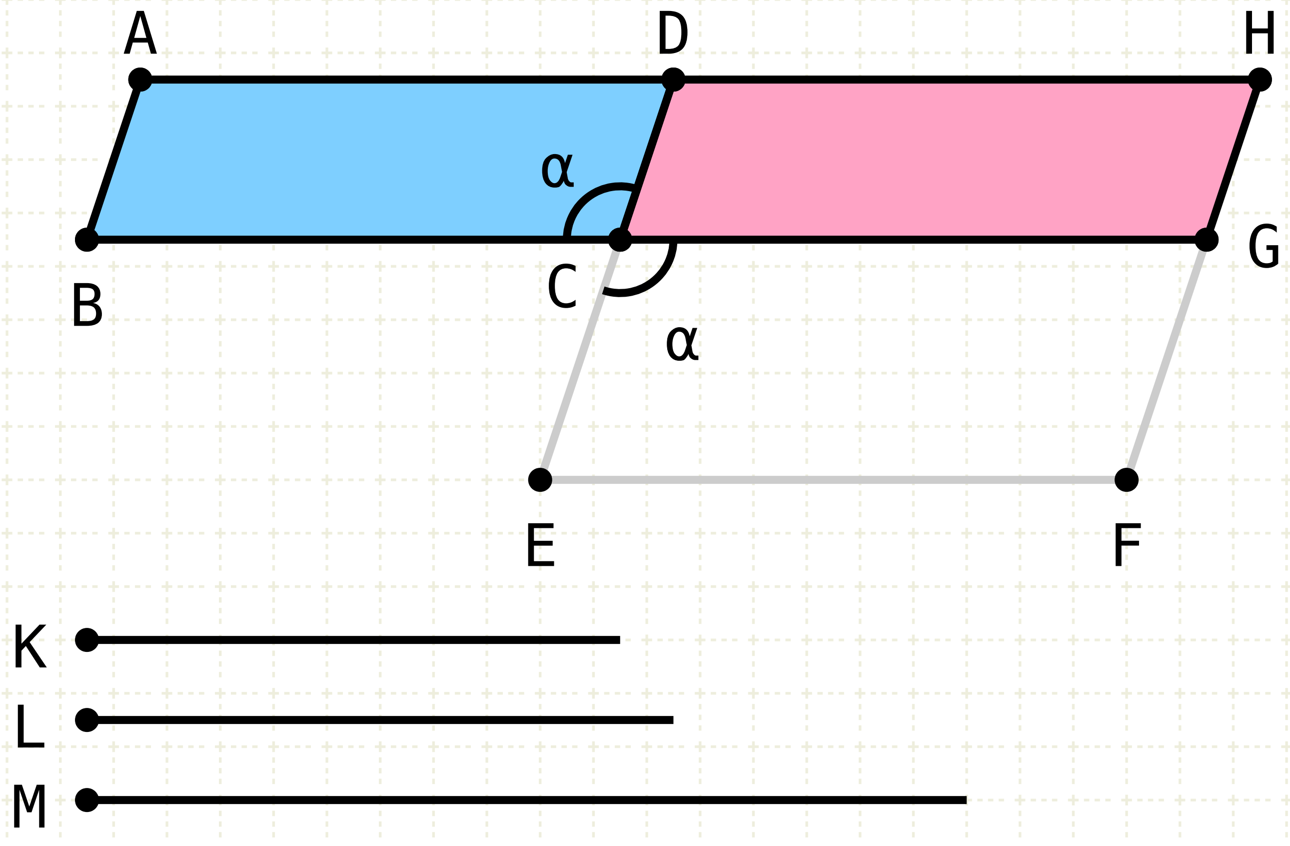

$$K:L = BC:CG$$

$$L:M = CD:CE$$

$$K:M = (BC \cdot CD):(CG \cdot CE)$$

$$BC:CG = \square AC:\square CH$$

$$K:L = \square AC:\square CH$$

Proof

Let the parallelograms be placed so that BC is in a straight line with CG therefore DC is also in a straight line with CE

Draw the parallelogram DG

Define an arbitrary line K, and draw another line L such that the ratio as BC is to CG so is K to L (VI·12)

Draw another line M such that the ratio as DC is to CE so is L to M (VI·12)

K to M is the compounded ratio of K to L and L to M, therefore it is also equal to the compound ratio of the sides, BC to CG and CD to CE

The ratio of the sides BC to CG is equal to the ratio of the parallelograms AC,CH (VI·1)

But K to L is also equal to BC to CG, so K to L is equal to the ratio of the parallelograms AC,CH (V·11)

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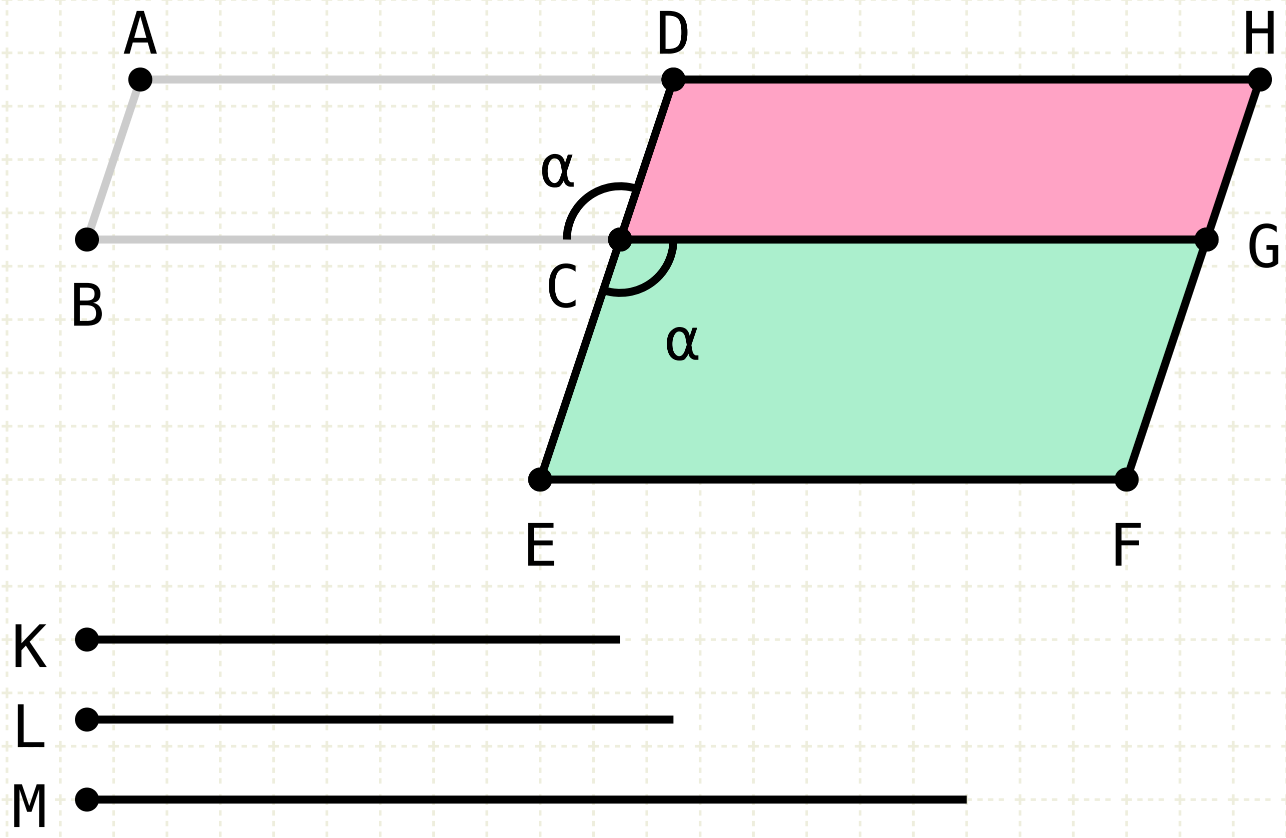

$K:L = BC:CG$

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$BC:CG = \square AC:\square CH$

$K:L = \square AC:\square CH$

$CD:CE = \square CH:\square CF$

Proof

Let the parallelograms be placed so that BC is in a straight line with CG therefore DC is also in a straight line with CE

Draw the parallelogram DG

Define an arbitrary line K, and draw another line L such that the ratio as BC is to CG so is K to L (VI·12)

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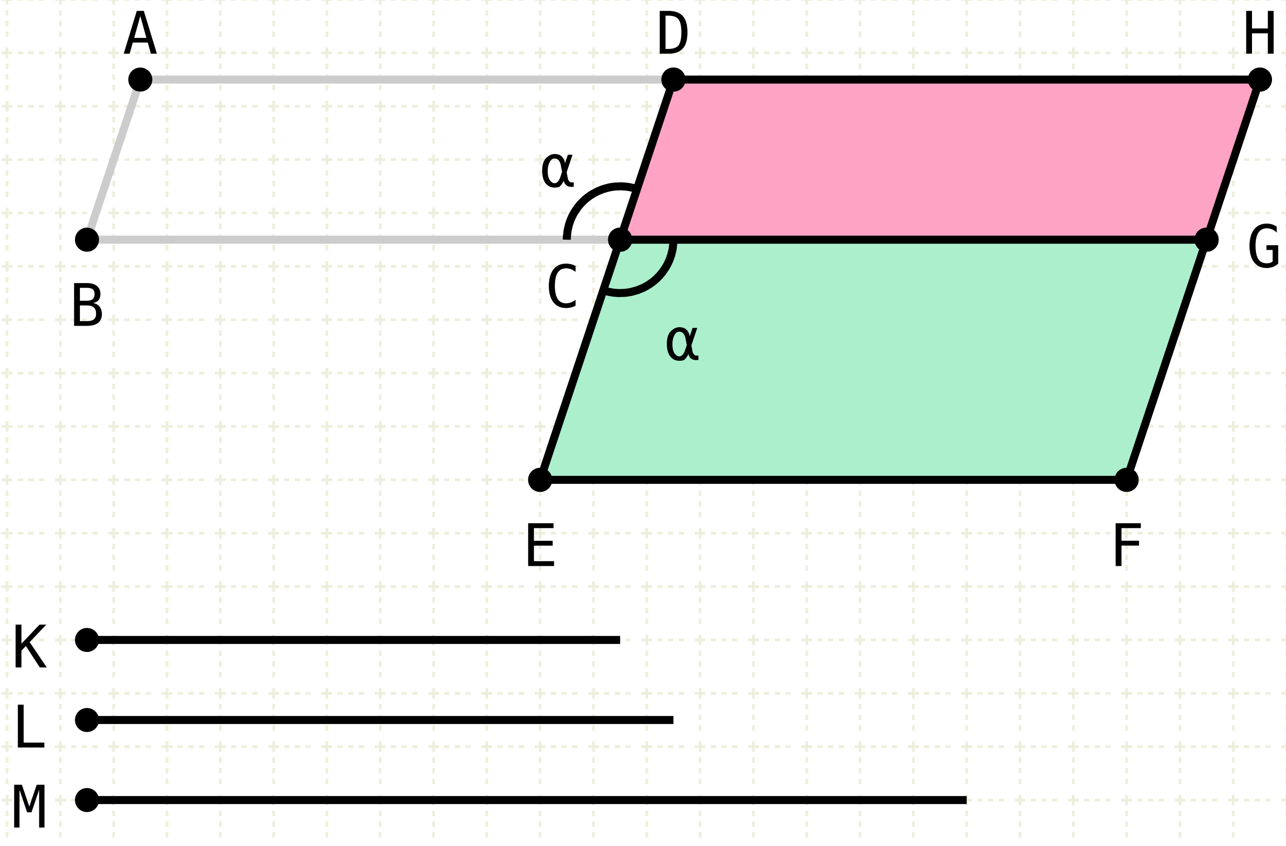

K:L = BC:CG

L:M = CD:CE

K:M = (BC·CD):(CG·CE)

BC:CG = □AC:□CH

K:L = □AC:□CH

CD:CE = □CH:□CF

L:M = □CH:□CF

Proof

Let the parallelograms be placed so that BC is in a straight line with CG therefore DC is also in a straight line with CE

Draw the parallelogram DG

Define an arbitrary line K, and draw another line L such that the ratio as BC is to CG so is K to L (VI·12)

Draw another line M such that the ratio as DC is to CE so is L to M (VI·12)

K to M is the compounded ratio of K to L and L to M, therefore it is also equal to the compound ratio of the sides, BC to CG and CD to CE

The ratio of the sides BC to CG is equal to the ratio of the parallelograms AC,CH (VI·1)

But K to L is also equal to BC to CG, so K to L is equal to the ratio of the parallelograms AC,CH (V·11)

The ratio of the sides DC to CE is equal to the ratio of the parallelograms CH,CF (VI·1)

Therefore L to M is equal to the ratio of the parallelograms CH,CF (V·11)

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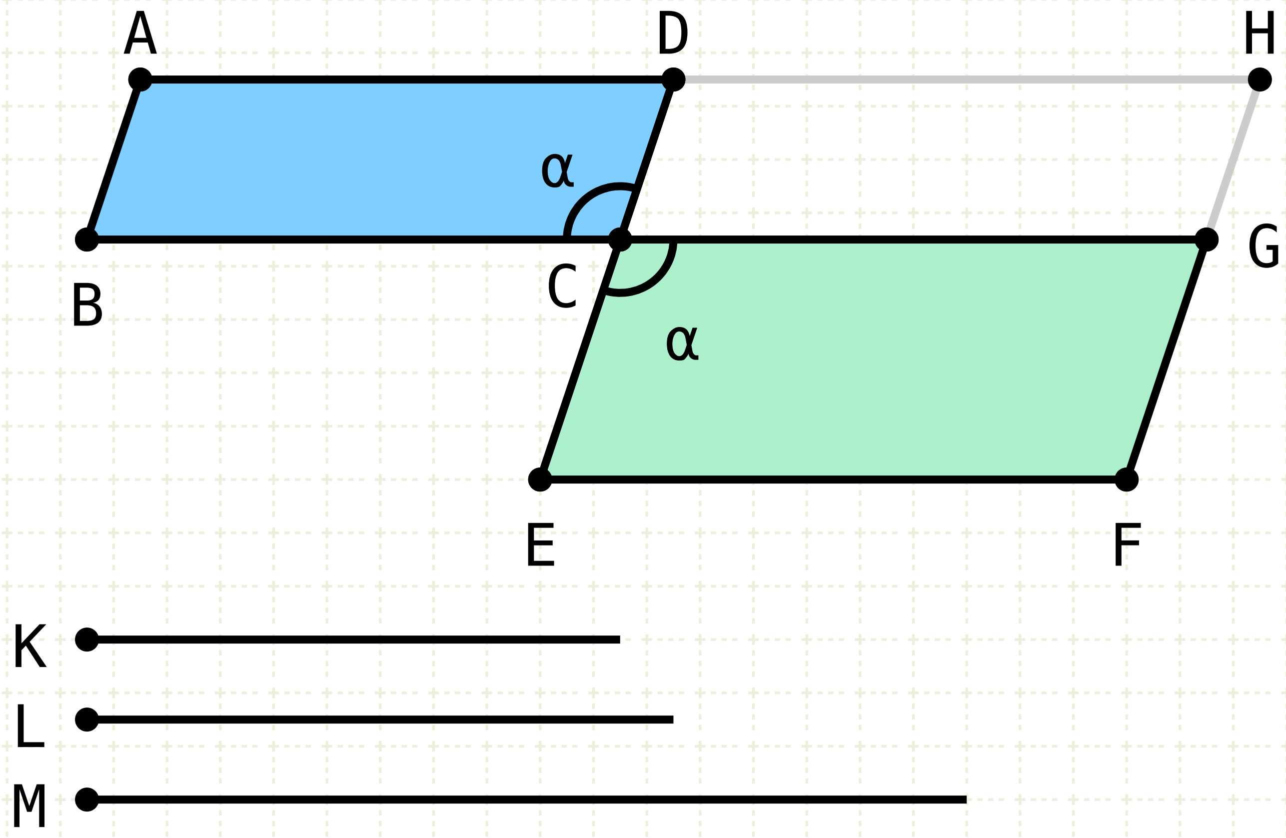

$$K:L = BC:CG$$
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$$BC:CG = \square AC:\square CH$$
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$$CD:CE = \square CH:\square CF$$
$$L:M = \square CH:\square CF$$
$$K:M = \square AC:\square CF$$

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Let the parallelograms be placed so that BC is in a straight line with CG therefore DC is also in a straight line with CE

Draw the parallelogram DG

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Draw another line M such that the ratio as DC is to CE so is L to M (VI·12)

K to M is the compounded ratio of K to L and L to M, therefore it is also equal to the compound ratio of the sides, BC to CG and CD to CE

The ratio of the sides BC to CG is equal to the ratio of the parallelograms AC,CH (VI·1)

But K to L is also equal to BC to CG, so K to L is equal to the ratio of the parallelograms AC,CH (V·11)

The ratio of the sides DC to CE is equal to the ratio of the parallelograms CH,CF (VI·1)

Therefore L to M is equal to the ratio of the parallelograms CH,CF (V·11)

Comparing the ratios of K to L and L to M it can be seen that the ratio K to M is equal (ex aequali) to the ratio of the parallelograms AC to CF (V·22)

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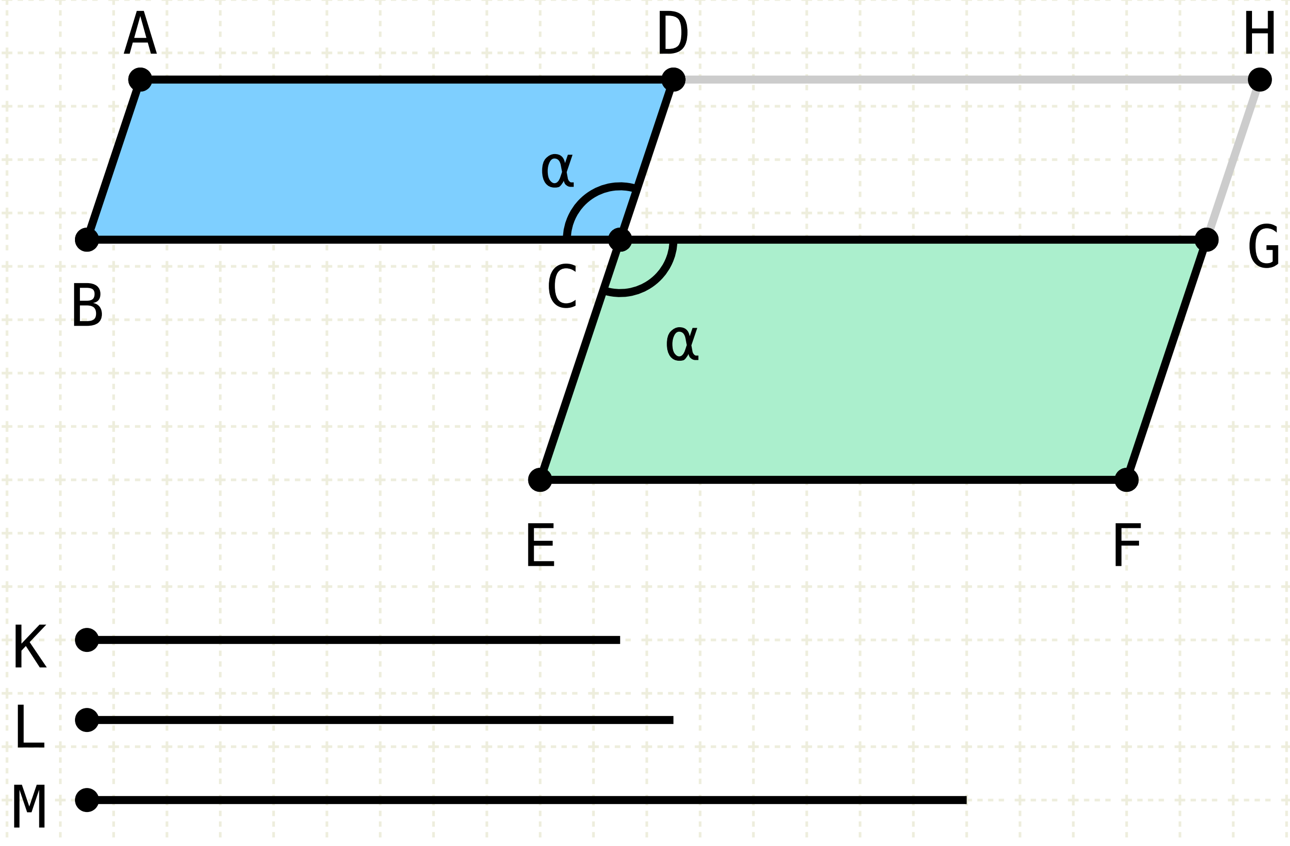

$K:L = BC:CG$

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$L:M = \square CH:\square CF$

$K:M = \square AC:\square CF$

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Proof

Let the parallelograms be placed so that BC is in a straight line with CG therefore DC is also in a straight line with CE

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But K to M is equal to the compound ratio of the sides, therefore the compound ratio of the sides is also equal to the ratio of the parallelograms

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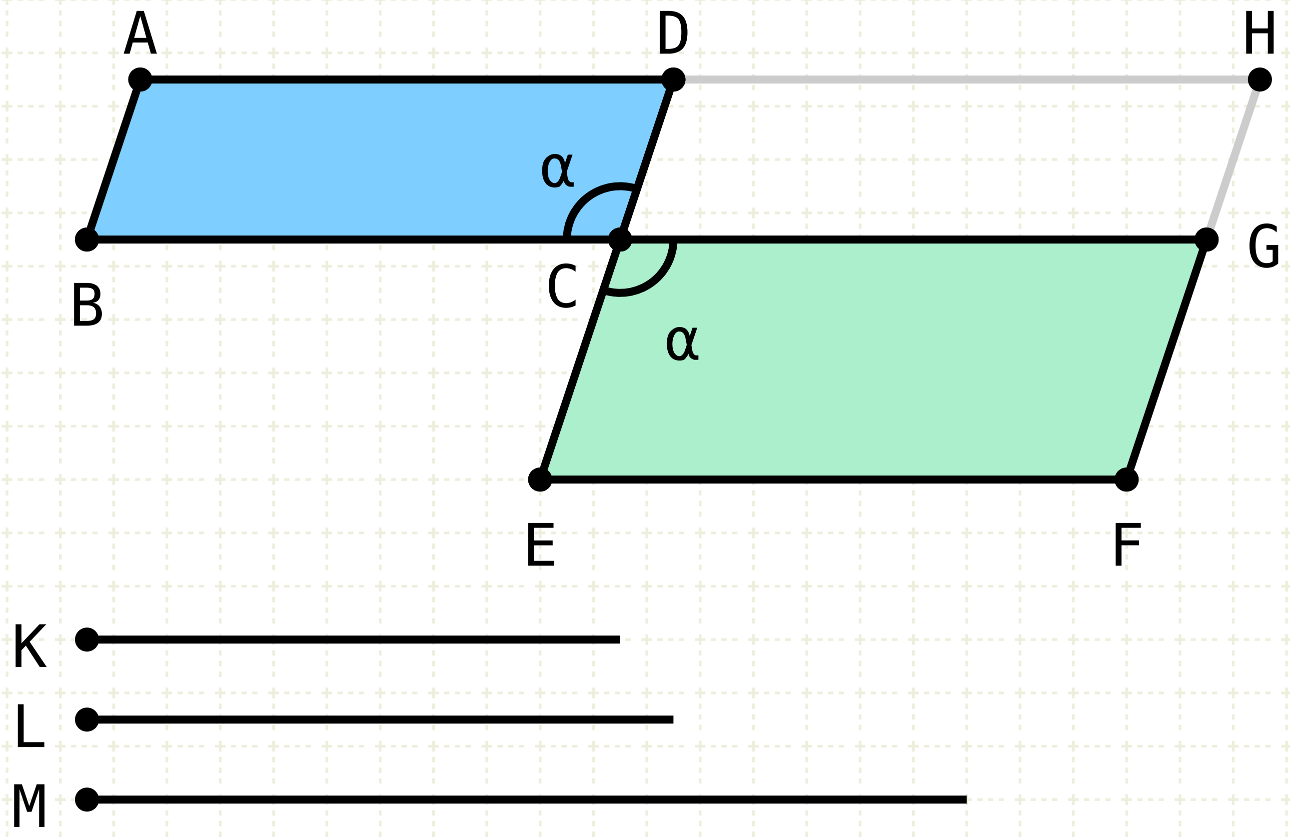

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$$CD:CE = \square CH:\square CF$$
$$L:M = \square CH:\square CF$$
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