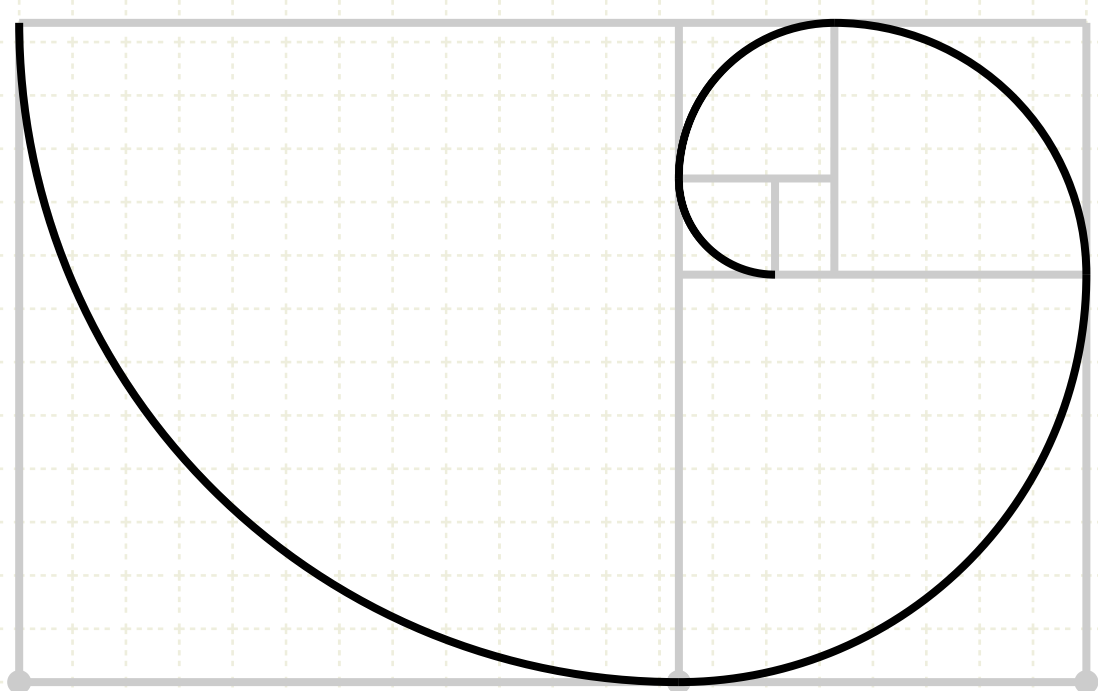
Euclid's Elements

Book VI



One can state, without exaggeration, that the observation of and the search for similarities and differences are the basis of all human knowledge.

Alfred Nobel

Table of Contents, Chapter 6

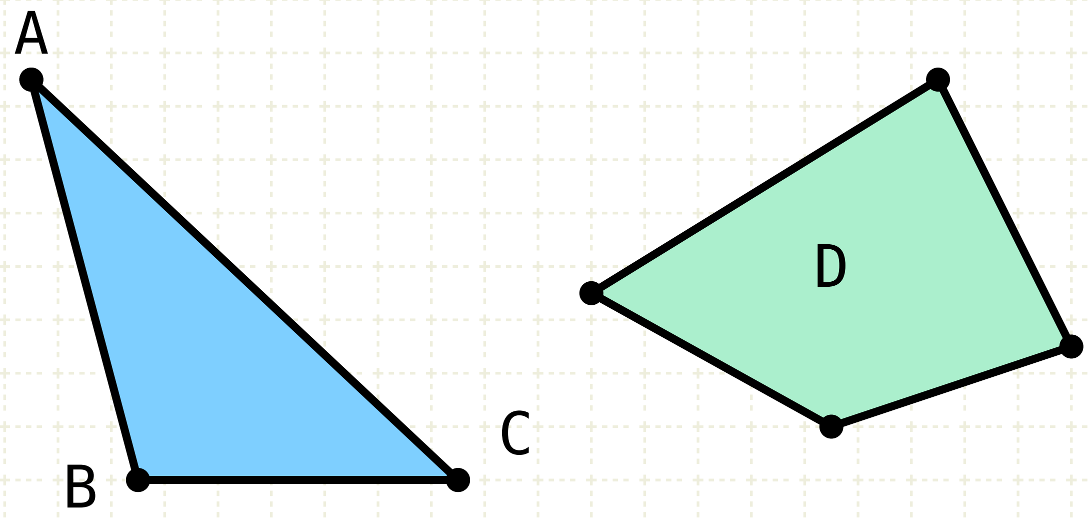
Table of Contents, Chapter 3

Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure

Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure

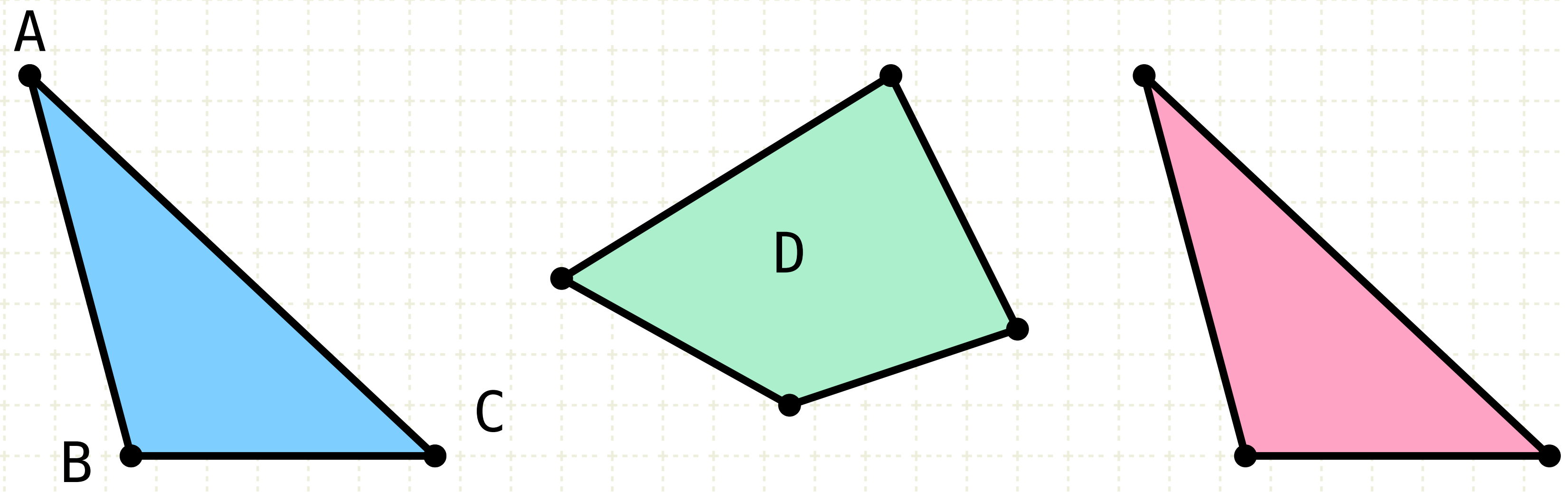


In other words

Given two rectilineal figures (ABC and D for example)

Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure



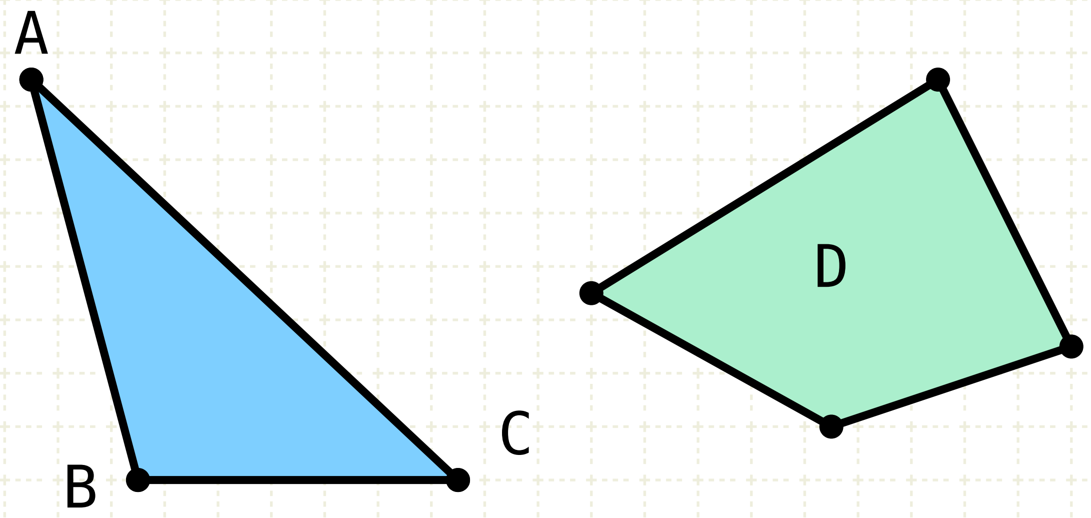
In other words

Given two rectilineal figures (ABC and D for example)

Construct a third figure that is similar to the first (ABC), and equal in area to the second (D)

Proposition 25 of Book VI

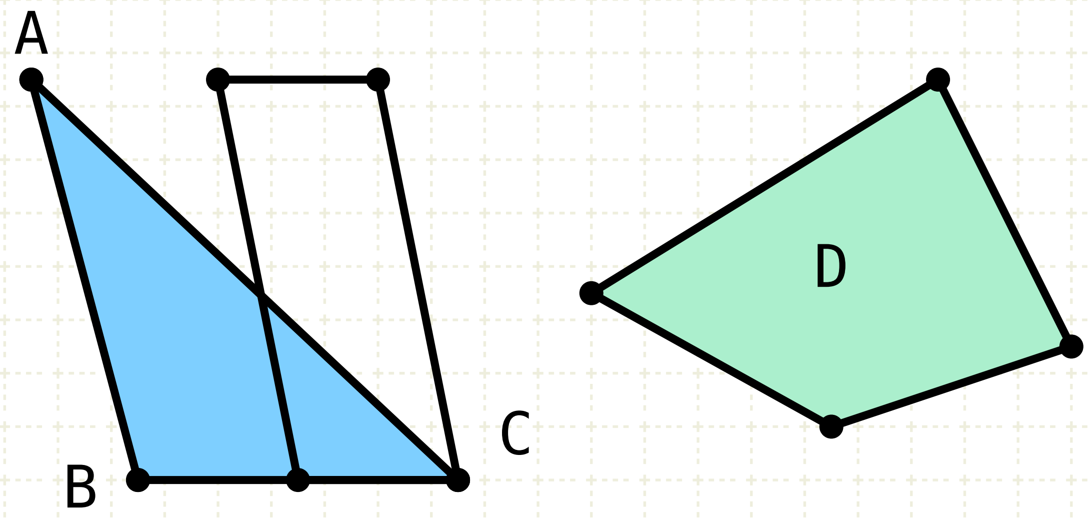
To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure



Construction

Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure



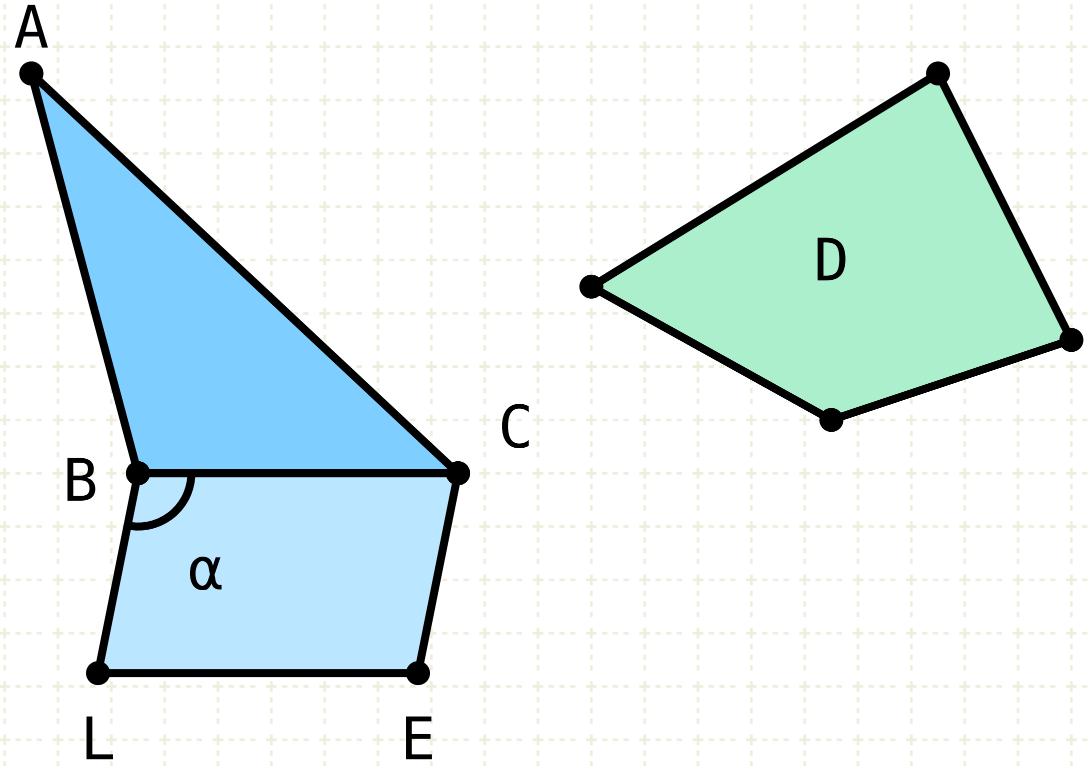
Construction

Construct a parallelogram to the base BC such that it is equal in area to the triangle ABC (I·44)

□BE = ΔABC

Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure



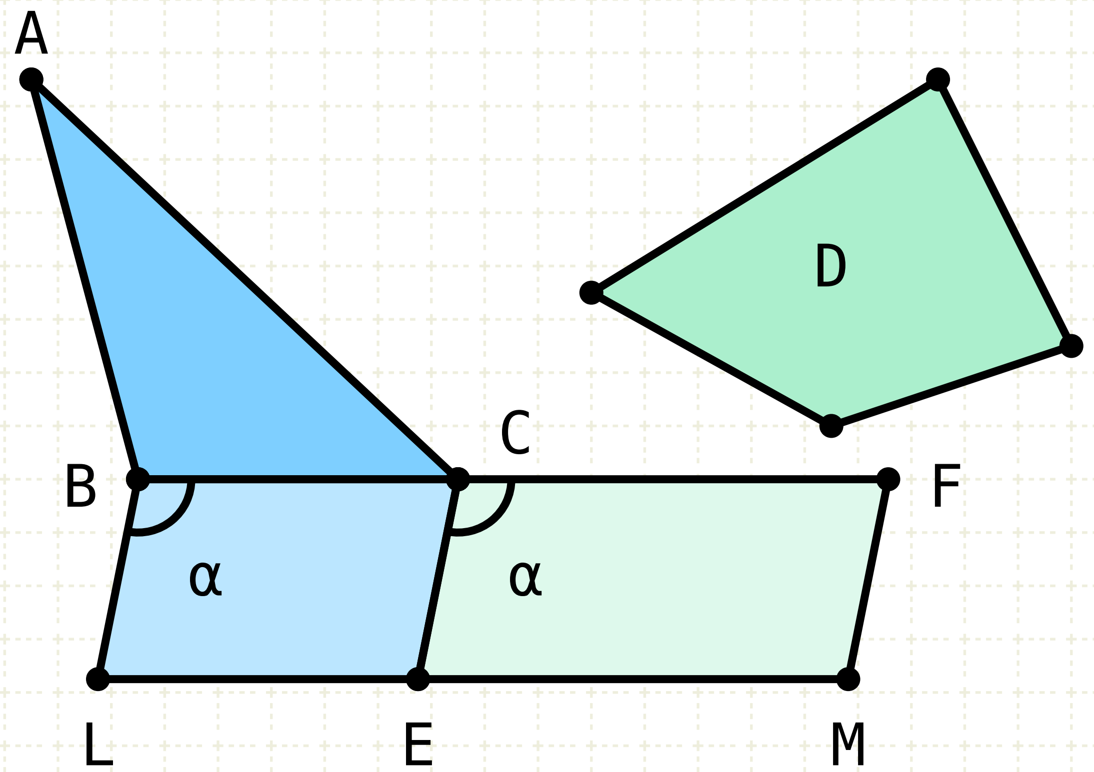
□BE = ΔABC

Construction

Construct a parallelogram to the base BC such that it is equal in area to the triangle ABC (I·44)

Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure



□BE = ΔABC
□EF = □D

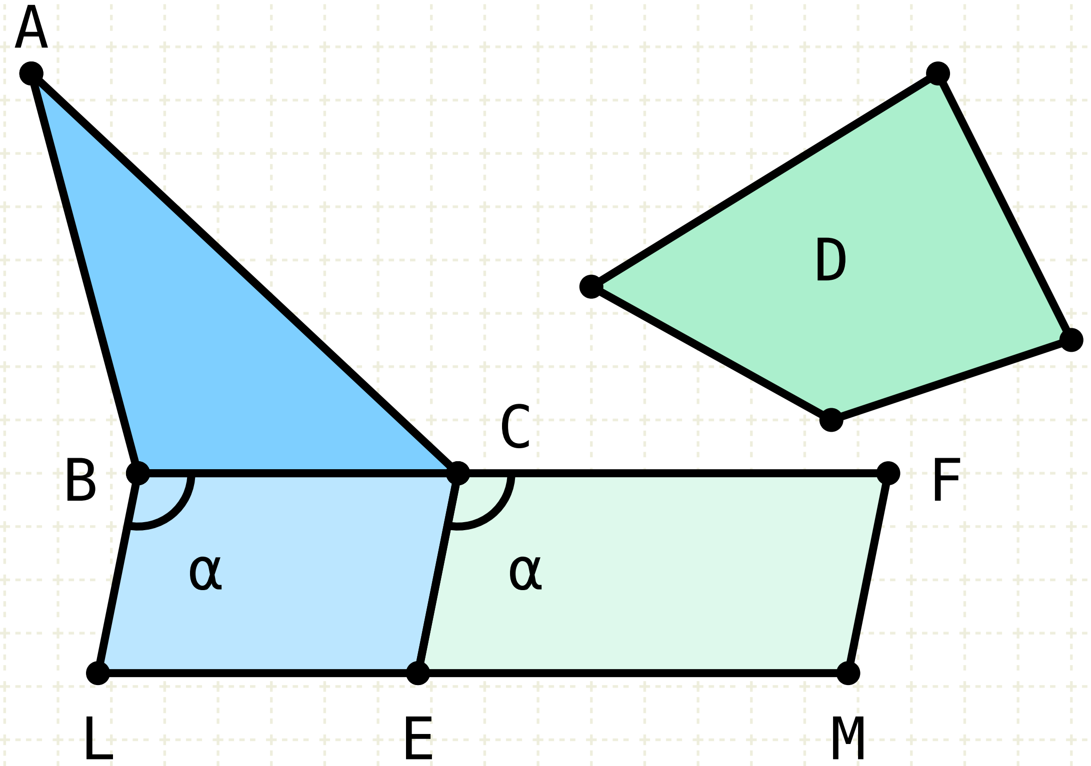
Construction

Construct a parallelogram to the base BC such that it is equal in area to the triangle ABC (I·44)

Construct a parallelogram to the line CE equal in area to D, and with an angle equal to CBL (I·45)

Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure



□BE = ΔABC
□EF = □D

Construction

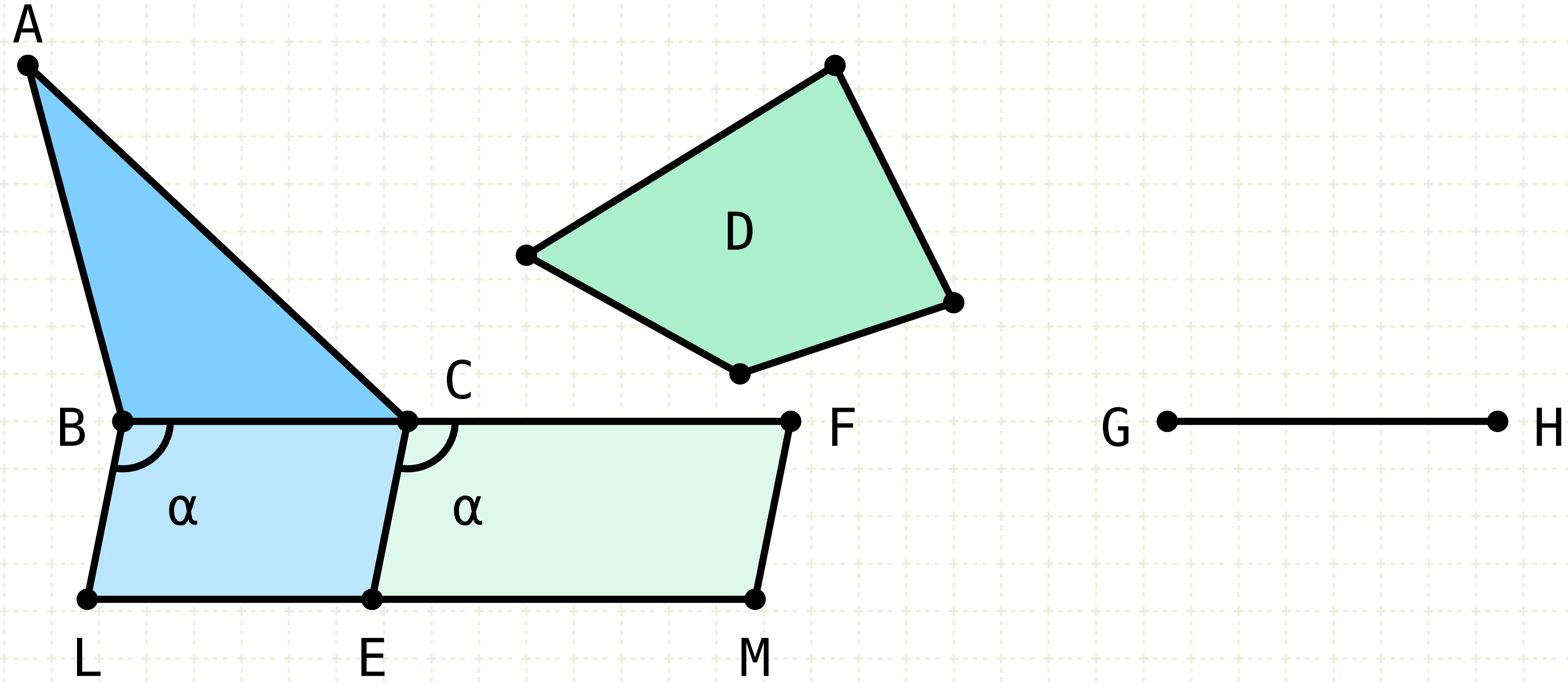
Construct a parallelogram to the base BC such that it is equal in area to the triangle ABC (I·44)

Construct a parallelogram to the line CE equal in area to D, and with an angle equal to CBL (I·45)

Since the angles CBL and FCE are equal, the lines BC,CF are in a straight line as are LE and EM

Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure



□BE = ΔABC
□EF = □D
BC:GH = GH:CF

Construction

Construct a parallelogram to the base BC such that it is equal in area to the triangle ABC (I·44)

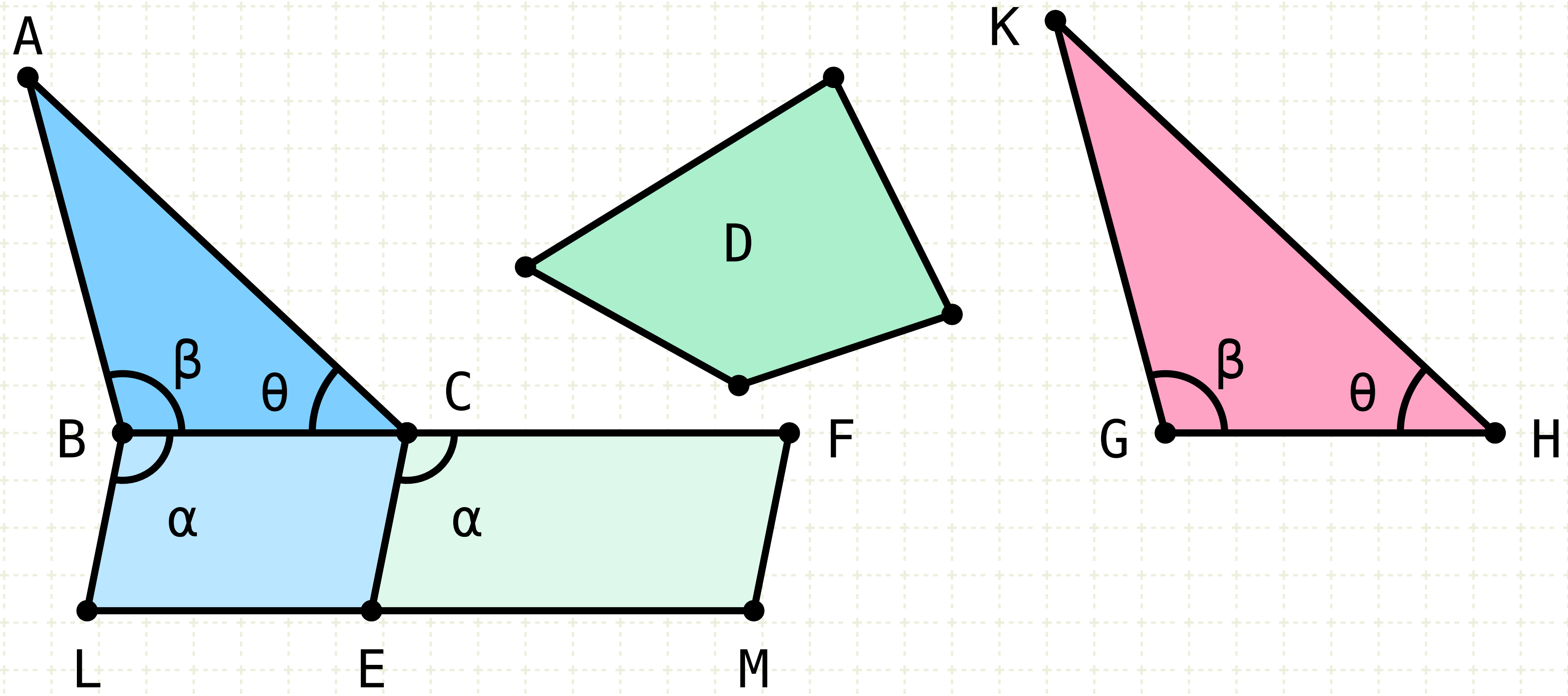
Construct a parallelogram to the line CE equal in area to D, and with an angle equal to CBL (I·45)

Since the angles CBL and FCE are equal, the lines BC,CF are in a straight line as are LE and EM

Draw a line GH which is in a mean proportion to BC,CF (VI·13)

Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure



$\square BE = \Delta ABC$

$\square EF = \square D$

$BC : GH = GH : CF$

$\Delta ABC \sim \Delta KGH$

Construction

Construct a parallelogram to the base BC such that it is equal in area to the triangle ABC (I·44)

Construct a parallelogram to the line CE equal in area to D, and with an angle equal to CBL (I·45)

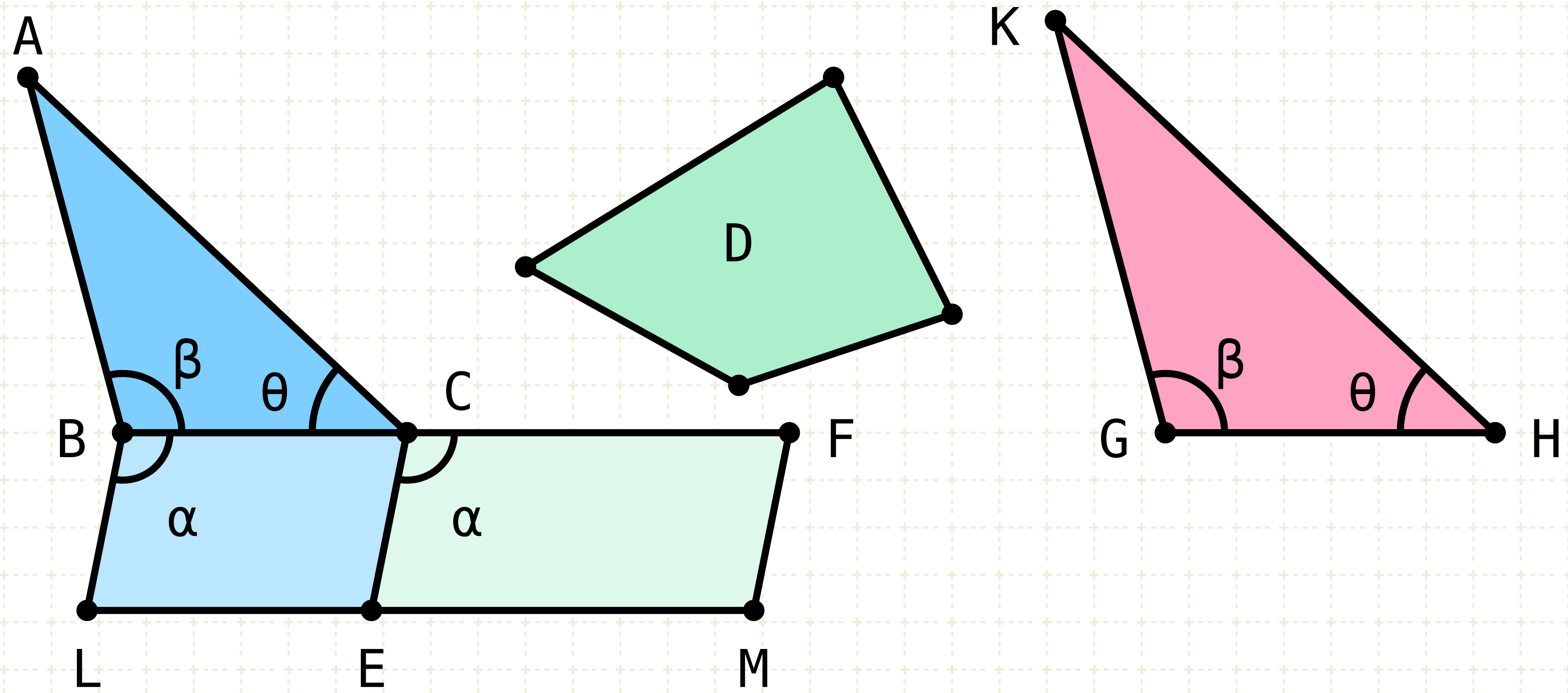
Since the angles CBL and FCE are equal, the lines BC,CF are in a straight line as are LE and EM

Draw a line GH which is in a mean proportion to BC,CF (VI·13)

Draw a figure similar to ABC on the line GH (VI·18)

Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure



□BE = ΔABC

□EF = □D

BC:GH = GH:CF

ΔABC ~ ΔKGH

Construction

Construct a parallelogram to the base BC such that it is equal in area to the triangle ABC (I·44)

Construct a parallelogram to the line CE equal in area to D, and with an angle equal to CBL (I·45)

Since the angles CBL and FCE are equal, the lines BC,CF are in a straight line as are LE and EM

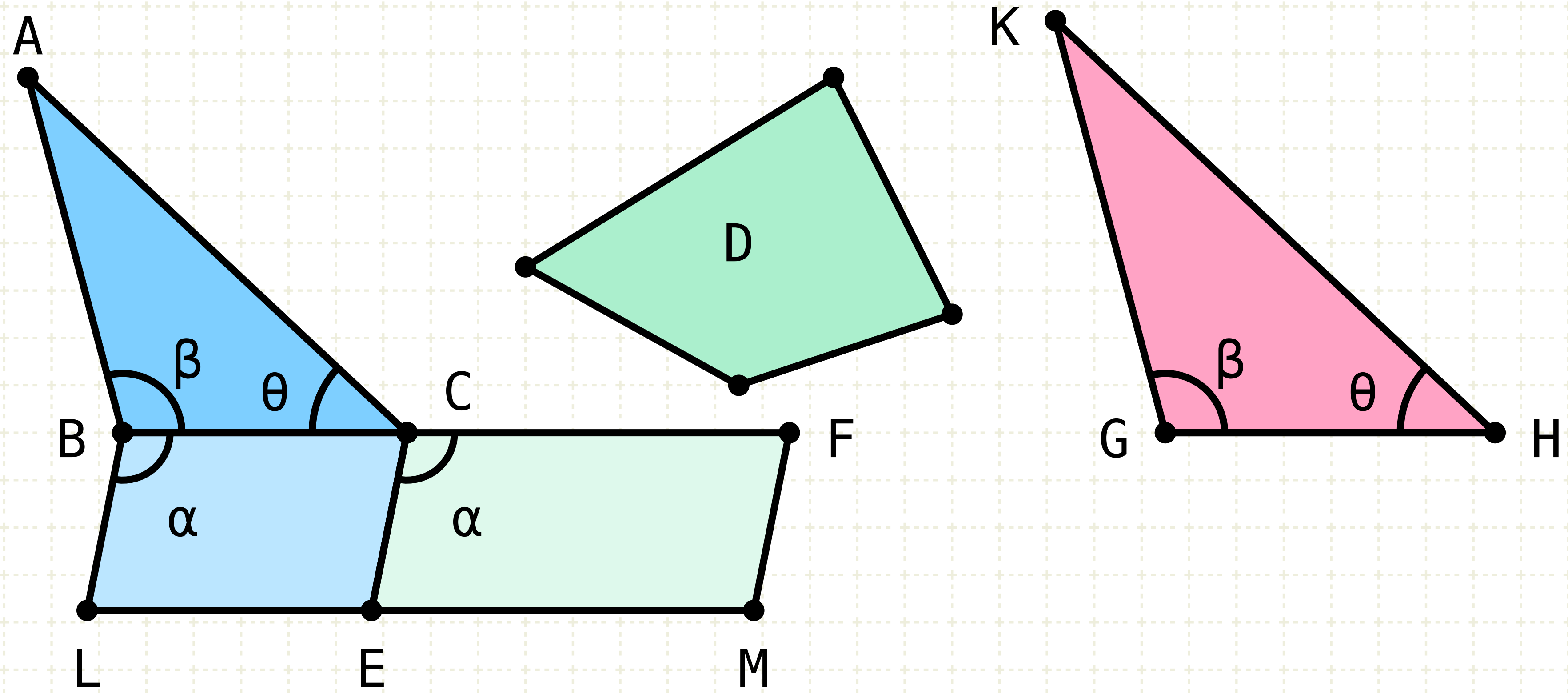
Draw a line GH which is in a mean proportion to BC,CF (VI·13)

Draw a figure similar to ABC on the line GH (VI·18)

Now, the triangle KGH is equal in area to the polygon D

Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure



Proof

□BE = ΔABC

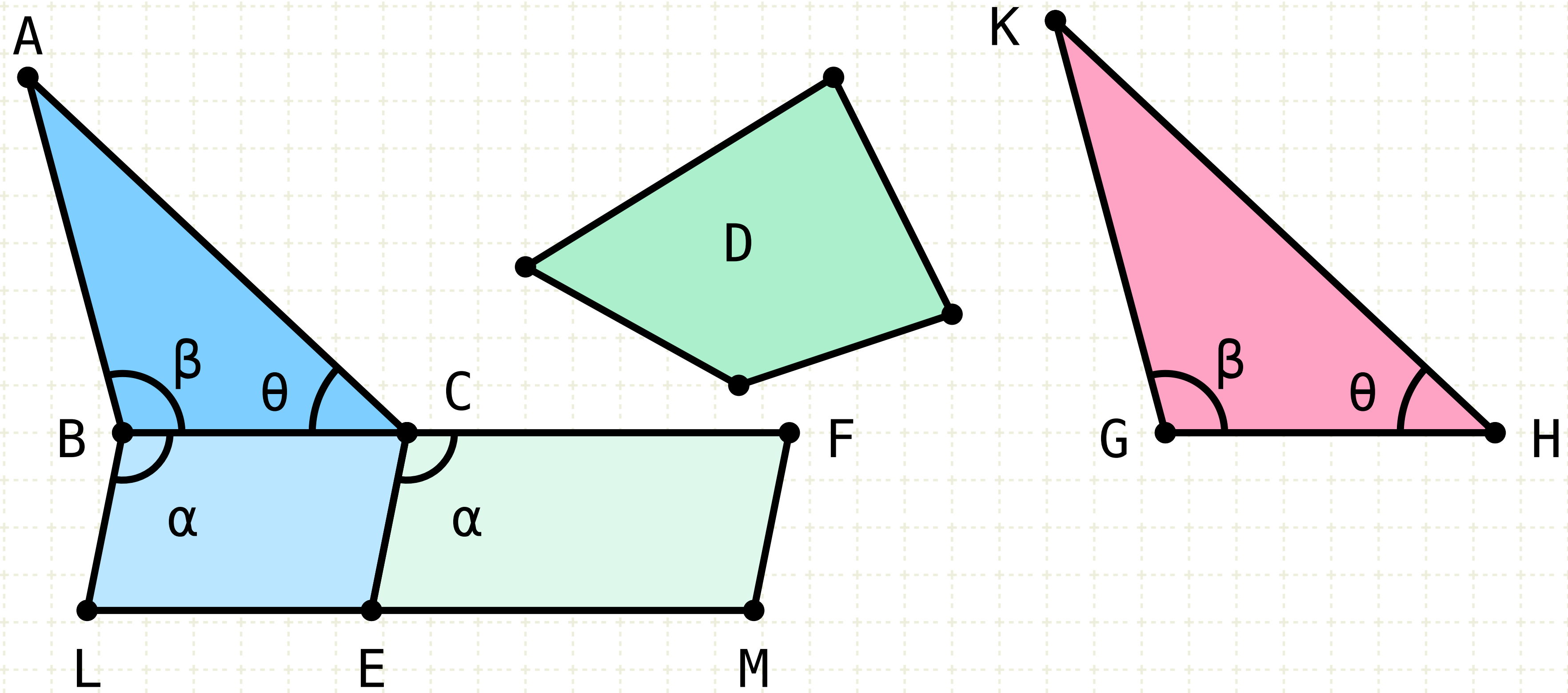
□EF = □D

BC:GH = GH:CF

ΔABC ~ ΔKGH

Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure



□BE = ΔABC
□EF = □D
BC:GH = GH:CF
ΔABC ~ ΔKGH

Proof

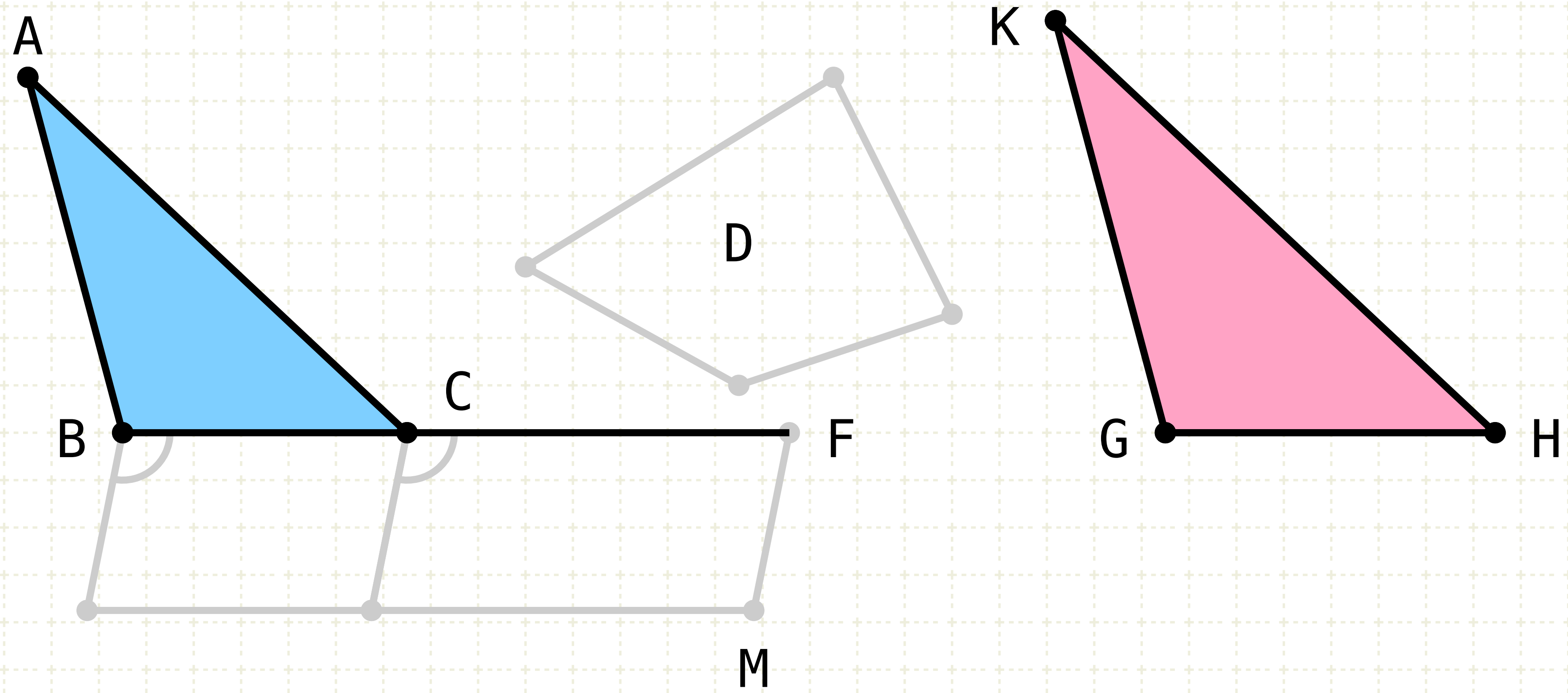
If there are two lines A,B, and if A is to B as B is to C, …

… and two similar figures are drawn on A and B, …

… then the ratio of the areas of the two figures (being the duplicate ratio of A,B) is the ratio A:C (VI·19.Por)

Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure



Proof

If there are two lines A,B, and if A is to B as B is to C, …

… and two similar figures are drawn on A and B, …

… then the ratio of the areas of the two figures (being the duplicate ratio of A,B) is the ratio A:C (VI·19.Por)

Thus the ratio of BC to CF is the ratio of the two triangles ABC to KGH

□BE = ΔABC

□EF = □D

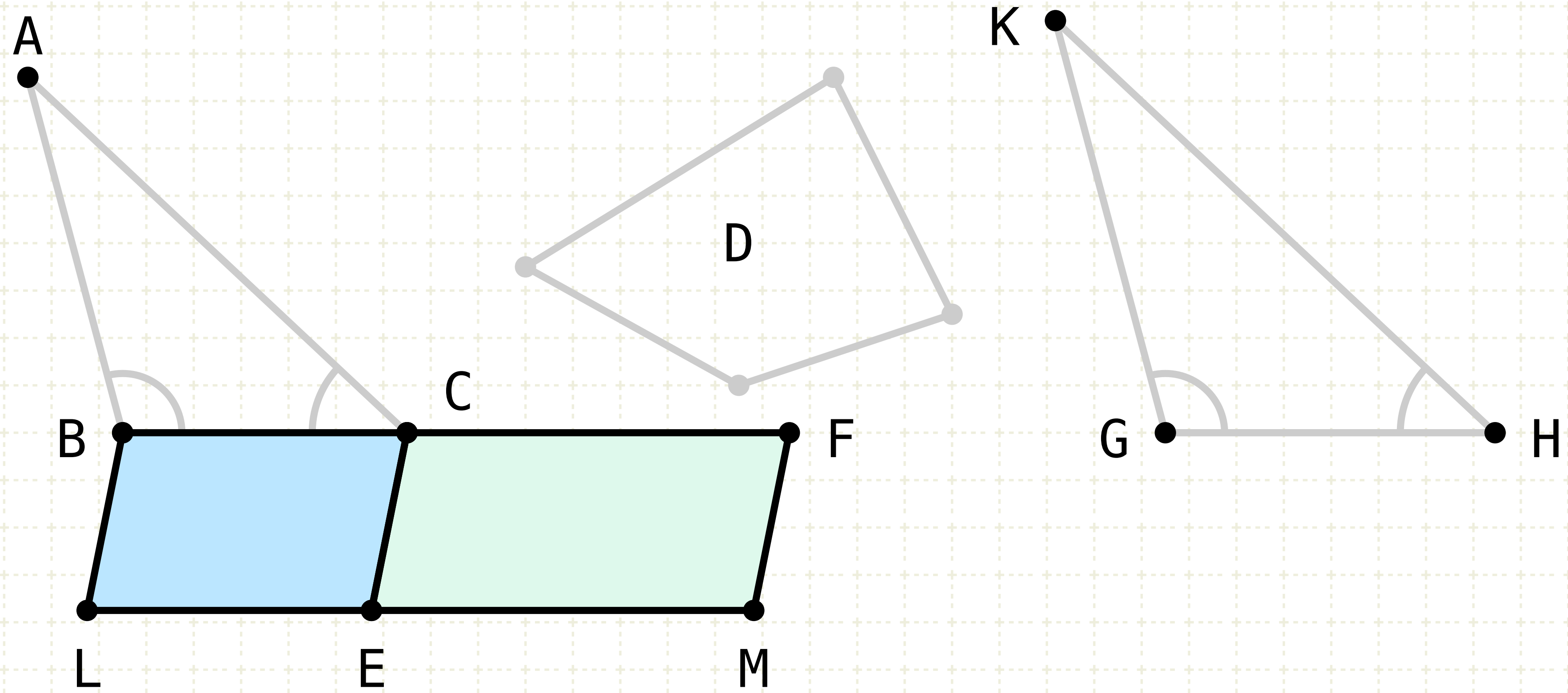
BC:GH = GH:CF

ΔABC ~ ΔKGH

ΔABC:ΔKGH = BC:CF

Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure



□BE = ΔABC

□EF = □D

BC:GH = GH:CF

ΔABC ~ ΔKGH

ΔABC:ΔKGH = BC:CF

BC:CF = □BE:□EF

Proof

If there are two lines A,B, and if A is to B as B is to C, …

… and two similar figures are drawn on A and B, …

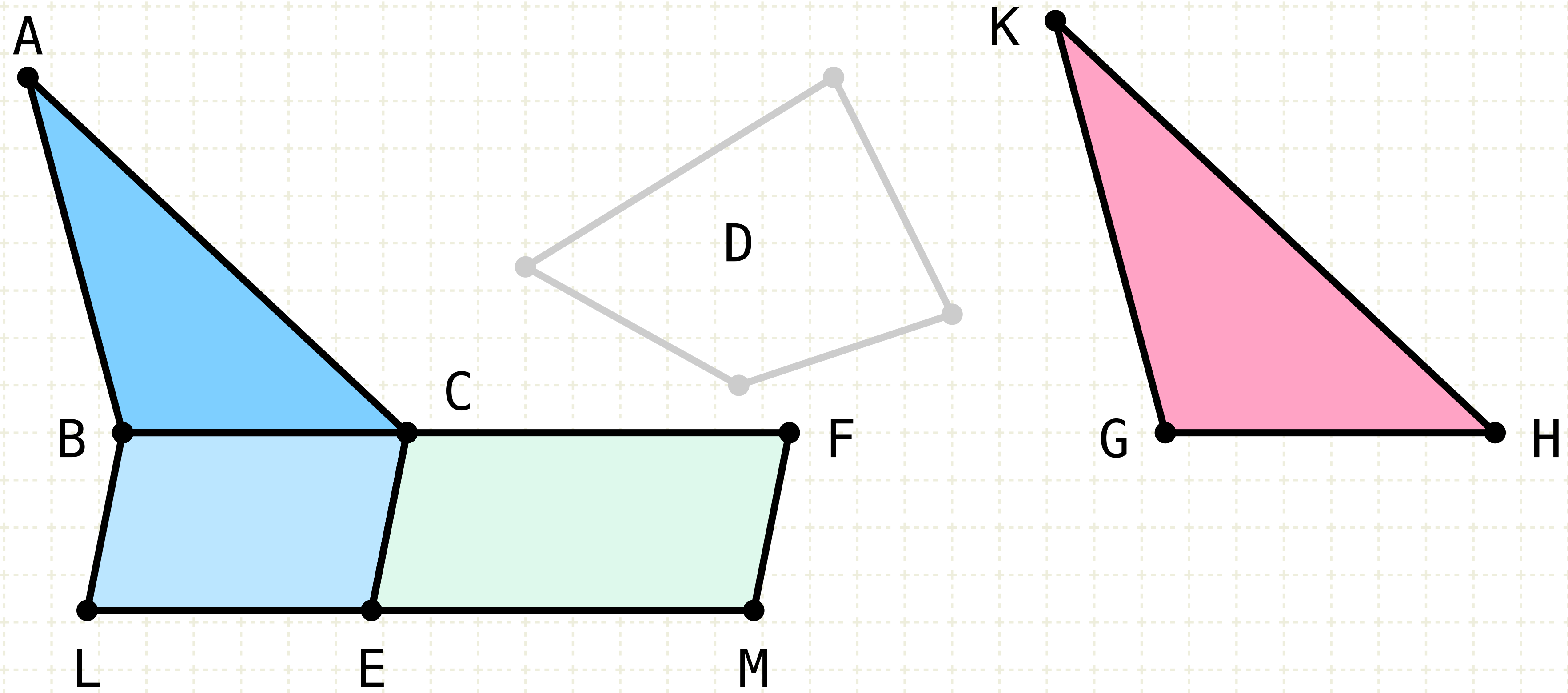
… then the ratio of the areas of the two figures (being the duplicate ratio of A,B) is the ratio A:C (VI·19.Por)

Thus the ratio of BC to CF is the ratio of the two triangles ABC to KGH

But the ratio of the parallelograms BE to EF is equal to the ratio of their bases, BC to CF (VI·1)

Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure



$\square BE = \Delta ABC$

$\square EF = \square D$

$BC:GH = GH:CF$

$\Delta ABC \sim \Delta KGH$

$\Delta ABC:\Delta KGH = BC:CF$

$BC:CF = \square BE:\square EF$

$\Delta ABC:\Delta KGH = \square BE:\square EF$

Proof

If there are two lines A,B, and if A is to B as B is to C, …

… and two similar figures are drawn on A and B, …

… then the ratio of the areas of the two figures (being the duplicate ratio of A,B) is the ratio A:C (VI·19.Por)

Thus the ratio of BC to CF is the ratio of the two triangles ABC to KGH

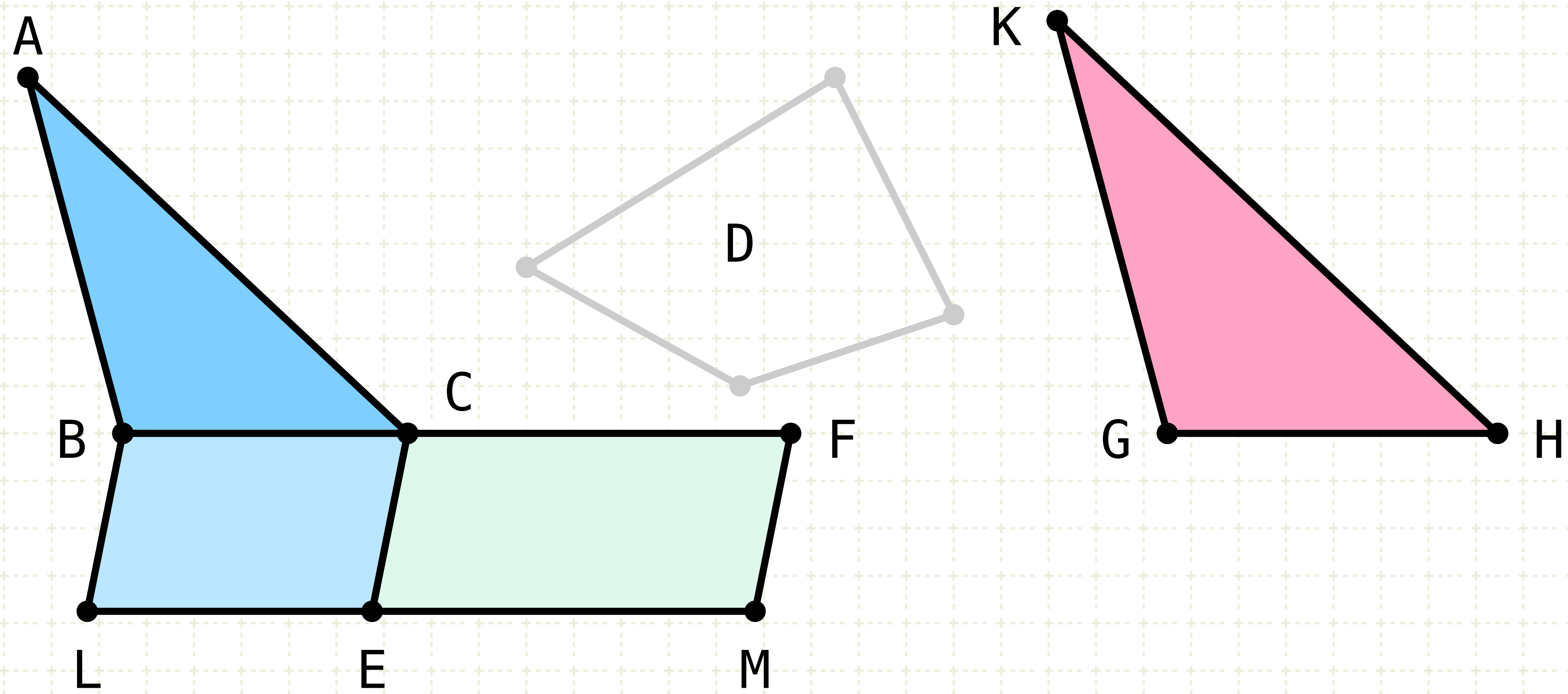
But the ratio of the parallelograms BE to EF is equal to the ratio of their bases, BC to CF (VI·1)

The ratio of BC to CF is equal to both the ratio of the triangles ABC and KGH, and to the ratio of the parallelograms BE and EF

Therefore the ratio of the parallelograms is equal to the ratio of the triangles

Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure



$\square BE = \Delta ABC$

$\square EF = \square D$

$BC:GH = GH:CF$

$\Delta ABC \sim \Delta KGH$

$\Delta ABC:\Delta KGH = BC:CF$

$BC:CF = \square BE:\square EF$

$\Delta ABC:\Delta KGH = \square BE:\square EF$

$\Delta ABC:\square BE = \Delta KGH:\square EF$

Proof

If there are two lines A,B, and if A is to B as B is to C, …

… and two similar figures are drawn on A and B, …

… then the ratio of the areas of the two figures (being the duplicate ratio of A,B) is the ratio A:C (VI·19.Por)

Thus the ratio of BC to CF is the ratio of the two triangles ABC to KGH

But the ratio of the parallelograms BE to EF is equal to the ratio of their bases, BC to CF (VI·1)

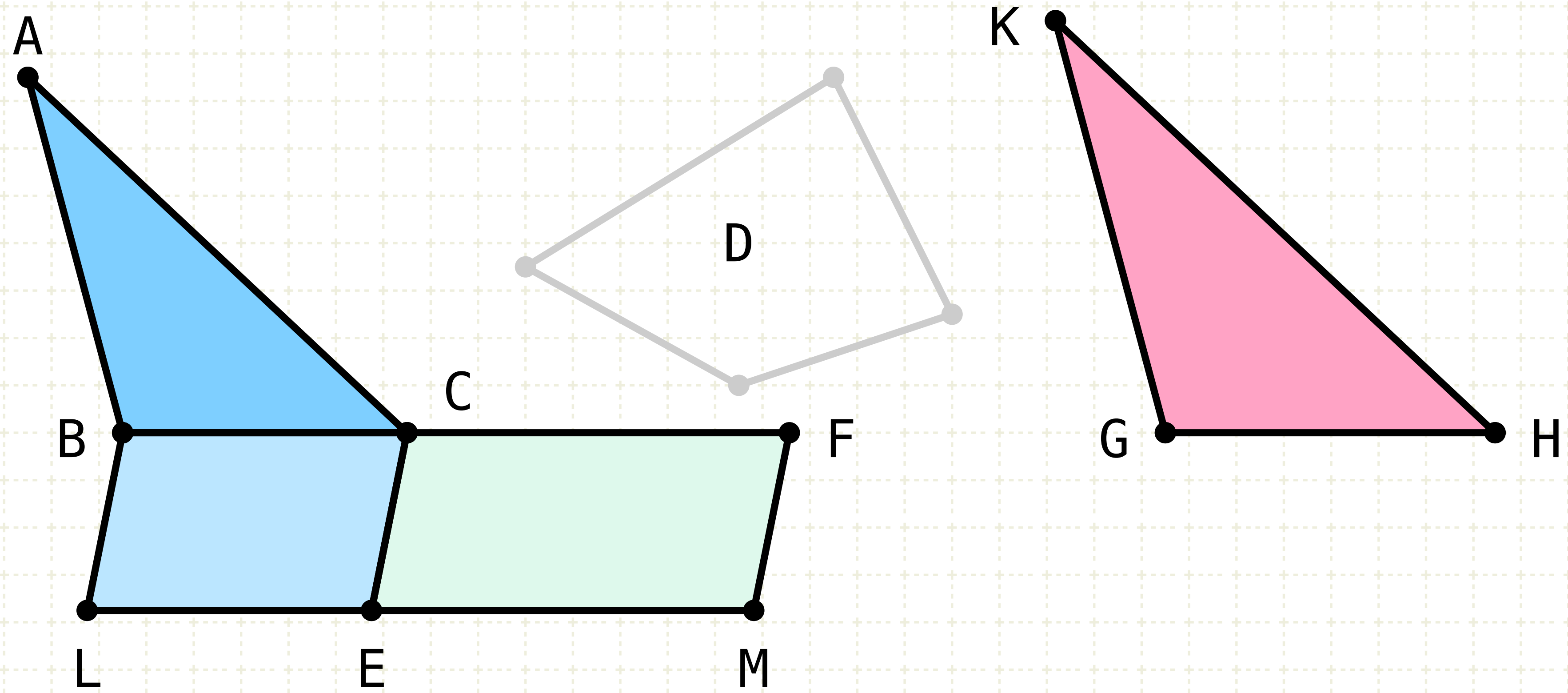
The ratio of BC to CF is equal to both the ratio of the triangles ABC and KGH, and to the ratio of the parallelograms BE and EF

Therefore the ratio of the parallelograms is equal to the ratio of the triangles

Alternately, the triangle ABC to the parallelogram BE is equal to the triangle KGH to the square EF (V·16)

Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure



$\square BE = \Delta ABC$

$\square EF = \square D$

$BC:GH = GH:CF$

$\Delta ABC \sim \Delta KGH$

$\Delta ABC:\Delta KGH = BC:CF$

$BC:CF = \square BE:\square EF$

$\Delta ABC:\Delta KGH = \square BE:\square EF$

$\Delta ABC:\square BE = \Delta KGH:\square EF$

$\square BE:\square BE = \Delta KGH:\square EF$

$\Delta KGH = \square EF$

Proof

If there are two lines A,B, and if A is to B as B is to C, …

… and two similar figures are drawn on A and B, …

… then the ratio of the areas of the two figures (being the duplicate ratio of A,B) is the ratio A:C (VI·19.Por)

Thus the ratio of BC to CF is the ratio of the two triangles ABC to KGH

But the ratio of the parallelograms BE to EF is equal to the ratio of their bases, BC to CF (VI·1)

The ratio of BC to CF is equal to both the ratio of the triangles ABC and KGH, and to the ratio of the parallelograms BE and EF

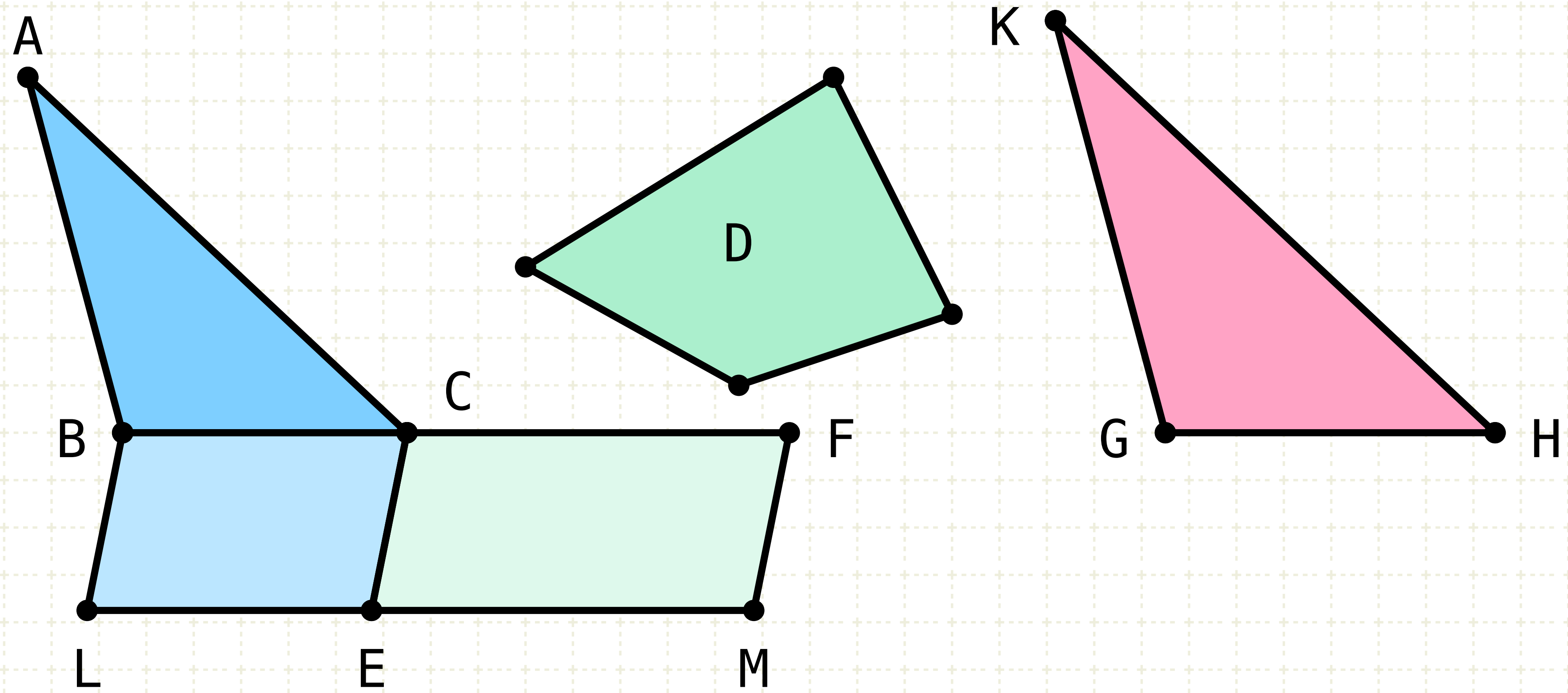
Therefore the ratio of the parallelograms is equal to the ratio of the triangles

Alternately, the triangle ABC to the parallelogram BE is equal to the triangle KGH to the square EF (V·16)

But the triangle ABC is equal to the parallelogram BE, so therefore the triangle KGH is equal to EF

Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure



□BE = ΔABC
□EF = □D
BC:GH = GH:CF
ΔABC ~ ΔKGH

ΔABC:ΔKGH = BC:CF
BC:CF = □BE:□EF
ΔABC:ΔKGH = □BE:□EF
ΔABC:□BE = ΔKGH:□EF

□BE:□BE = ΔKGH:□EF
ΔKGH = □EF
ΔKGH = D

Proof

If there are two lines A,B, and if A is to B as B is to C, …

… and two similar figures are drawn on A and B, …

… then the ratio of the areas of the two figures (being the duplicate ratio of A,B) is the ratio A:C (VI·19.Por)

Thus the ratio of BC to CF is the ratio of the two triangles ABC to KGH

But the ratio of the parallelograms BE to EF is equal to the ratio of their bases, BC to CF (VI·1)

The ratio of BC to CF is equal to both the ratio of the triangles ABC and KGH, and to the ratio of the parallelograms BE and EF

Therefore the ratio of the parallelograms is equal to the ratio of the triangles

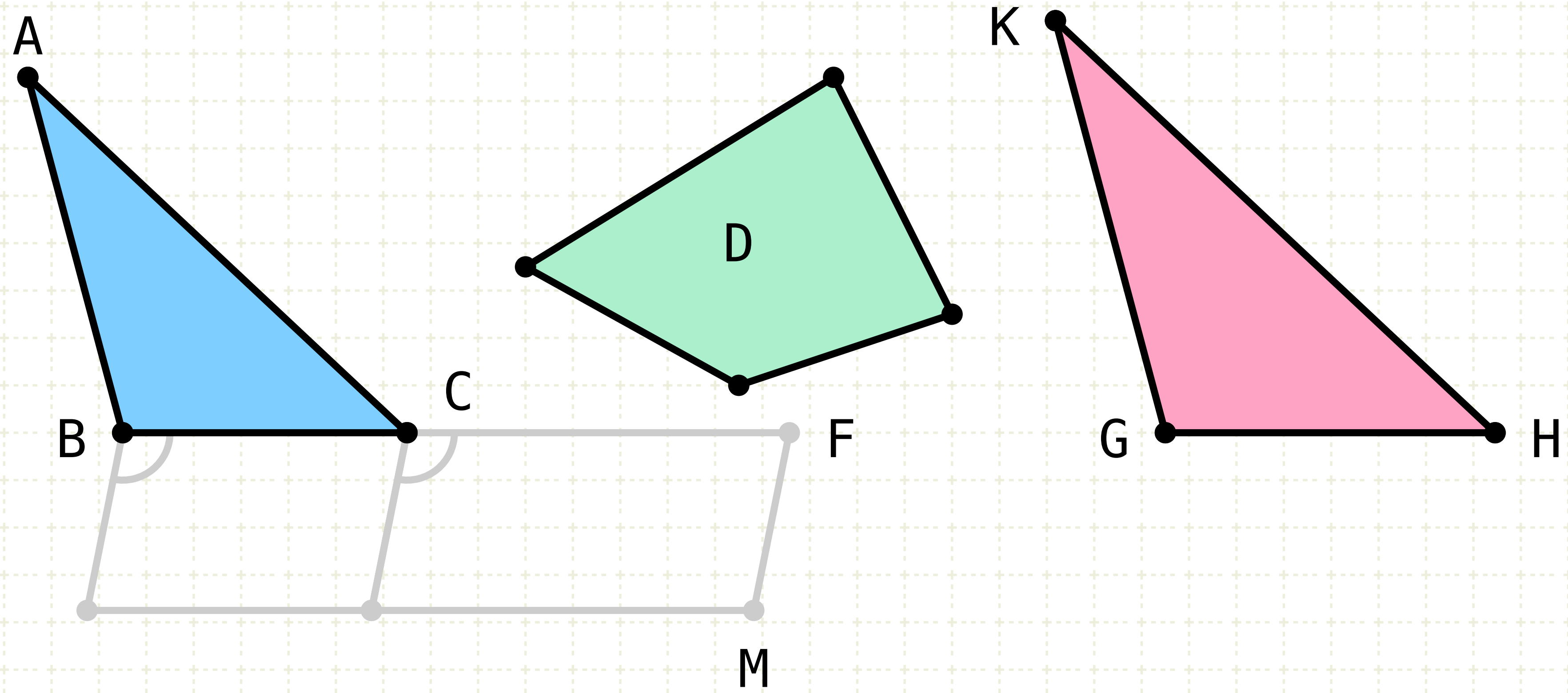
Alternately, the triangle ABC to the parallelogram BE is equal to the triangle KGH to the square EF (V·16)

But the triangle ABC is equal to the parallelogram BE, so therefore the triangle KGH is equal to EF

But the polygon D is equal to the parallelogram EF, so therefore the triangle KGH is equal to D

Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure



$\square BE = \Delta ABC$

$\square EF = \square D$

$BC : GH = GH : CF$

$\Delta ABC \sim \Delta KGH$

$\square BE : \square BE = \Delta KGH : \square EF$

$\Delta KGH = \square EF$

$\Delta KGH = D$

$\Delta ABC : \Delta KGH = BC : CF$

$BC : CF = \square BE : \square EF$

$\Delta ABC : \Delta KGH = \square BE : \square EF$

$\Delta ABC : \square BE = \Delta KGH : \square EF$

Proof

If there are two lines A,B, and if A is to B as B is to C, …

… and two similar figures are drawn on A and B, …

… then the ratio of the areas of the two figures (being the duplicate ratio of A,B) is the ratio A:C (VI·19.Por)

Thus the ratio of BC to CF is the ratio of the two triangles ABC to KGH

But the ratio of the parallelograms BE to EF is equal to the ratio of their bases, BC to CF (VI·1)

The ratio of BC to CF is equal to both the ratio of the triangles ABC and KGH, and to the ratio of the parallelograms BE and EF

Therefore the ratio of the parallelograms is equal to the ratio of the triangles

Alternately, the triangle ABC to the parallelogram BE is equal to the triangle KGH to the square EF (V·16)

But the triangle ABC is equal to the parallelogram BE, so therefore the triangle KGH is equal to EF

But the polygon D is equal to the parallelogram EF, so therefore the triangle KGH is equal to D

Thus KGH is similar to ABC, and equal in area to D

Youtube Videos

https://www.youtube.com/c/SandyBultena