

Euclid's Elements

Book VI

One can state, without exaggeration, that the observation of and the search for similarities and differences are the basis of all human knowledge.

Alfred Nobel



Table of Contents, Chapter 6

1	If the height of two triangles are equal, then the ratio of the areas is equal to the ratio of the bases	7	If two triangles have one angle equal to one angle, and the sides about other angles are proportional, and the remaining angles either both less or both not less than a right angle, then triangles will be equiangular	14	In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and vice versa
2	If a line cuts a triangle, parallel to its base, it will cut the sides of the triangle proportionally			15	In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and vice versa
3	If an angle of a triangle is bisected and the straight line cutting the angle also cuts the base, the segments of the base will have the same ratio as the remaining sides of the triangle	8	If in a right-angled triangle a perpendicular be drawn from the right angle to the base, the triangles adjoining the perpendicular are similar both to the whole and to one another	16	If four straight lines are proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means, and vice versa
4	If two triangles have equal angles, then the sides opposite the equal angles are proportional, as well, the sides of the triangles on either side of the equal angles are also proportional	9	From a given straight line to cut off a given fraction	17	If three straight lines are proportional, the rectangle contained by the extremes is equal to the square on the mean; and vice versa
5	If two triangles have proportional sides, the triangles will be equiangular	10	To cut a given uncut straight line similarly to a given cut straight line	18	On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure
6	If two triangles have one angle equal to one angle and the sides about the equal angles are proportional, then the triangles will be equiangular	11	To two given straight lines to find a third proportional	19	Similar triangles are to one another in the duplicate ratio of the corresponding sides
		12	To three given straight lines to find a fourth proportional		
		13	To two given straight lines to find a mean proportional		



Table of Contents, Chapter 3

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|----|--|----|---|----|---|
| 20 | Similar polygons are divided into the same number of similar triangles, which have the same ratio as the wholes, and the polygons have duplicate ratios to their corresponding sides | 26 | If from a parallelogram a similar parallelogram with a common angle is subtracted, it is about the same diameter as the original | 31 | In right-angled triangles the figure on the side subtending the right angle is equal to the similar and similarly described figures on the sides containing the right angle |
| 21 | Figures which are similar to the same rectilineal figure are also similar to one another | 27 | Of all the parallelograms applied to the same straight line and deficient by parallelogrammic figures similar to a parallelogram drawn on half the said line, the largest will be one that is drawn on half of the straight line and is similar to the defect | | |
| 22 | If four straight lines are proportional, similar rectilineal figures will also be proportional; and vice versa | 28 | To a given straight line, apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one | | |
| 23 | Equiangular parallelograms have to one another the ratio compounded of the ratios of their sides | 29 | To a given straight line, apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one | | |
| 24 | In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another | 30 | To cut a finite straight line in extreme ratio | | |
| 25 | To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure | | | | |



Proposition 29 of Book VI

To a given straight line to apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one



Proposition 29 of Book VI

To a given straight line to apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one

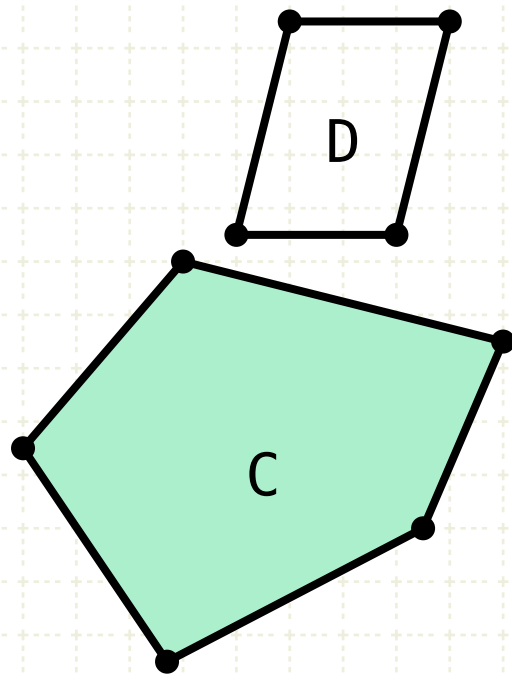
In other words

Given a straight line AB and



Proposition 29 of Book VI

To a given straight line to apply a parallelogram equal to a given rectilinear figure and exceeding by a parallelogrammic figure similar to a given one



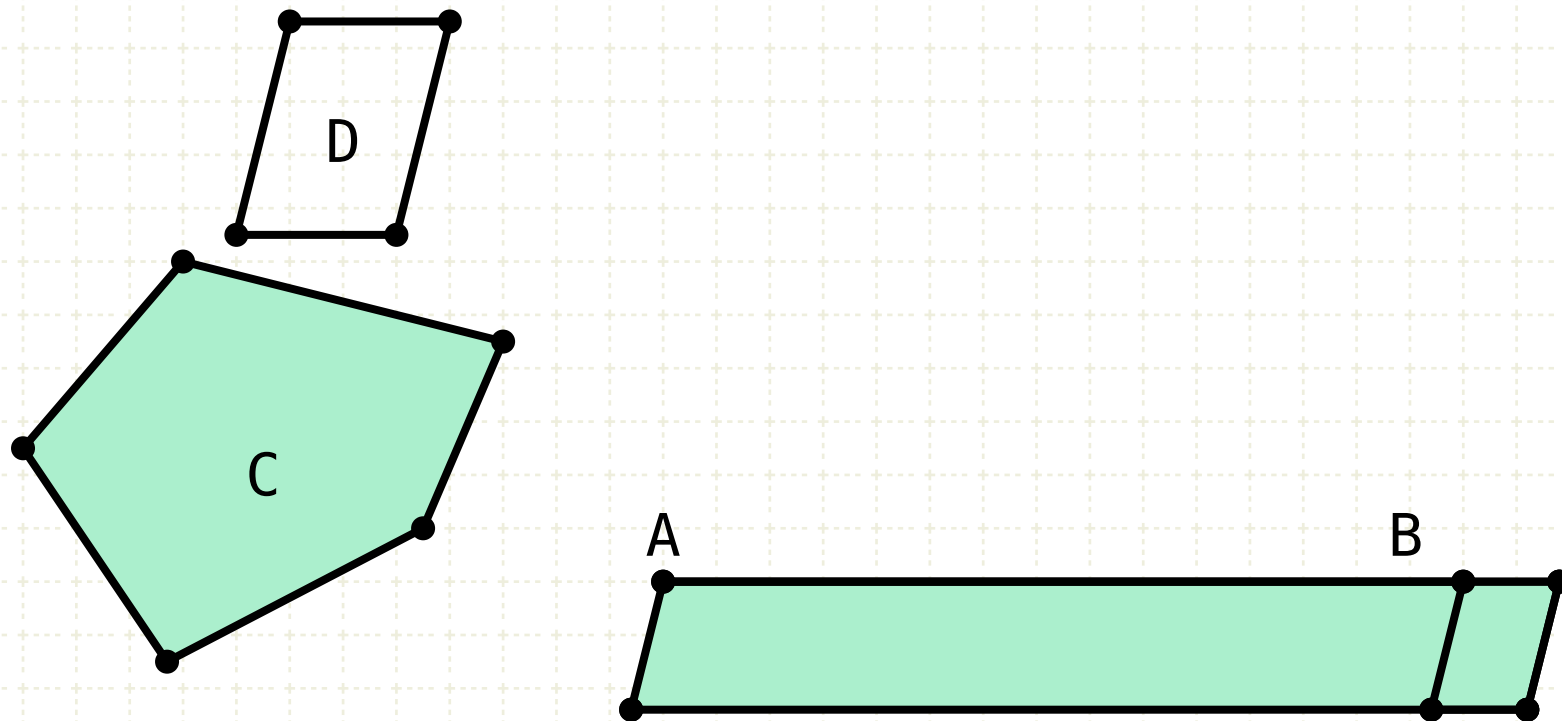
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Given a straight line AB and

Let C be a rectilinear figure and D be a parallelogram

Proposition 29 of Book VI

To a given straight line to apply a parallelogram equal to a given rectilinear figure and exceeding by a parallelogrammic figure similar to a given one



In other words

Given a straight line AB and

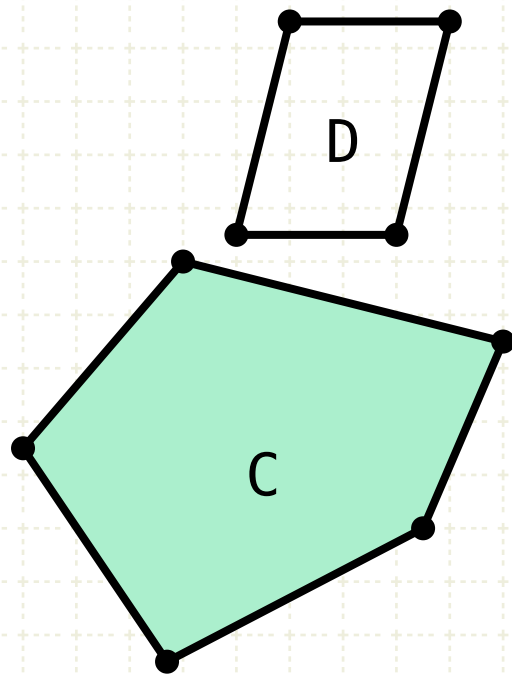
Let C be a rectilinear figure and D be a parallelogram

We want to draw a parallelogram on AB such that ...

- * If a parallelogram similar to parallelogram D is added, then...
- * the sum is equal in area to C

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To a given straight line to apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one

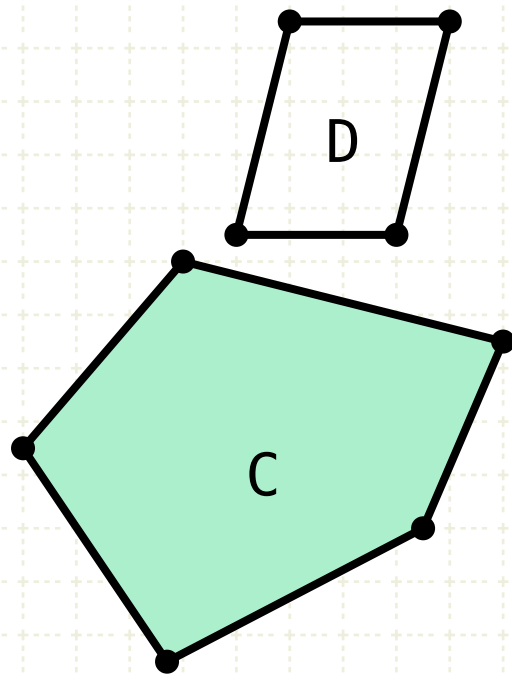


Construction



Proposition 29 of Book VI

To a given straight line to apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one



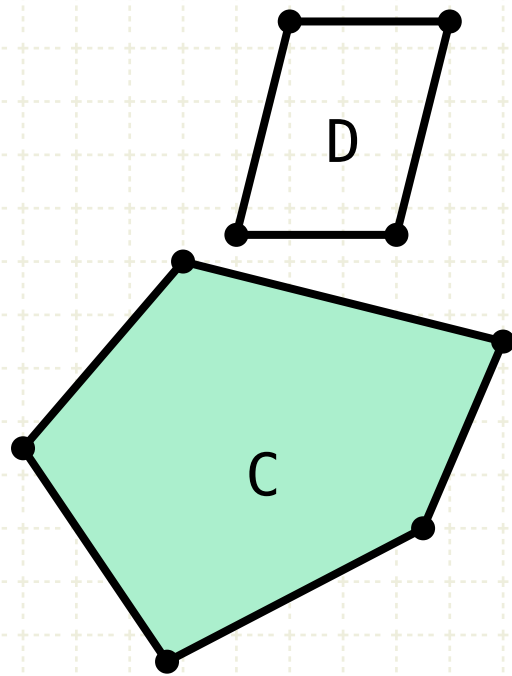
Construction

Bisect the line AB at point E

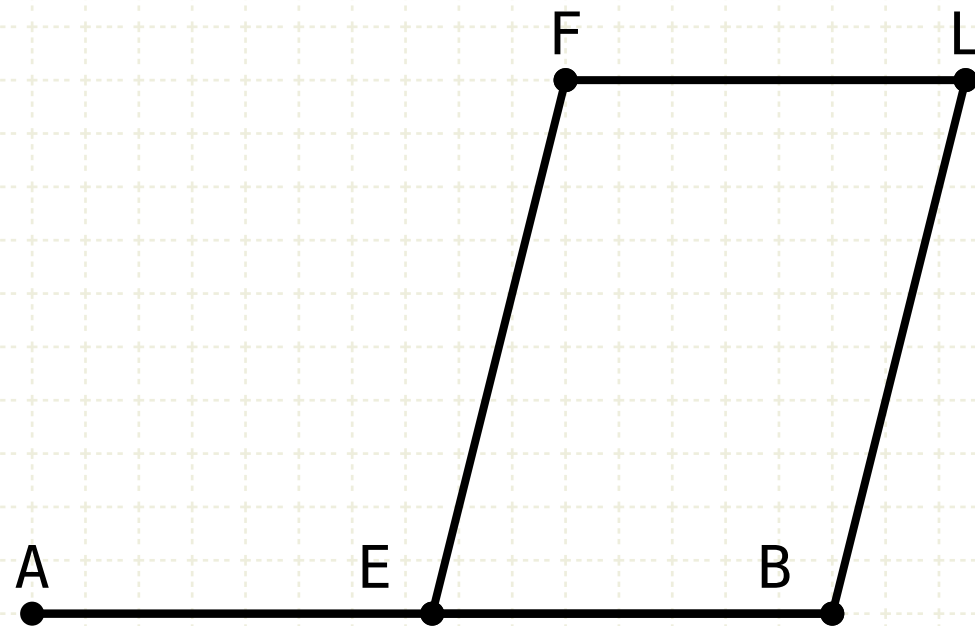


Proposition 29 of Book VI

To a given straight line to apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one



$FB \sim D$



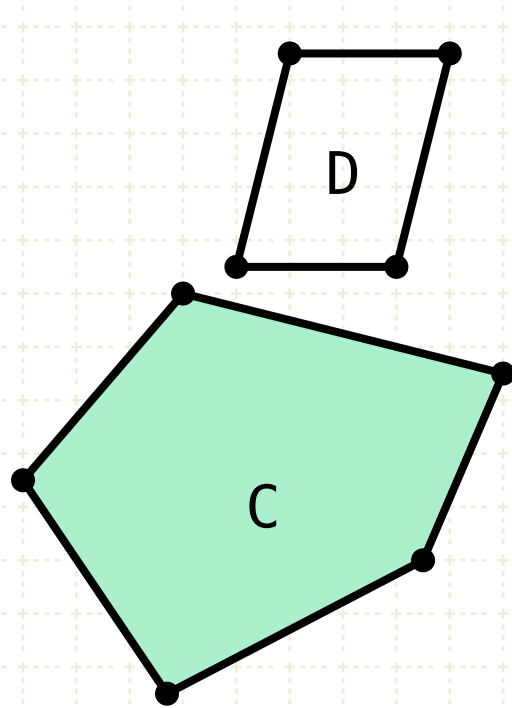
Construction

Bisect the line AB at point E

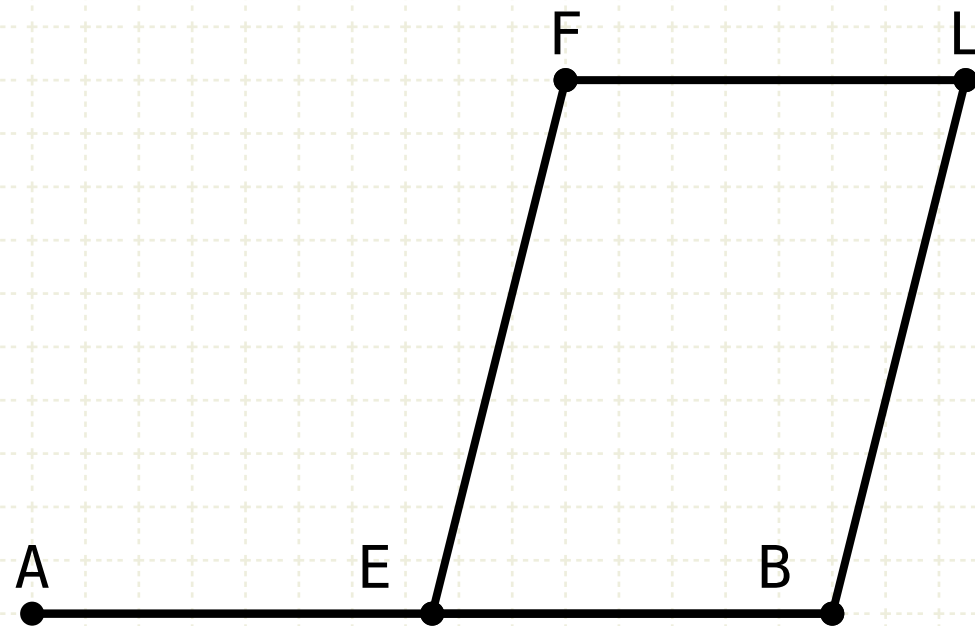
Create a parallelogram similar to D on line EB (VI·18)

Proposition 29 of Book VI

To a given straight line to apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one



$FB \sim D$



Construction

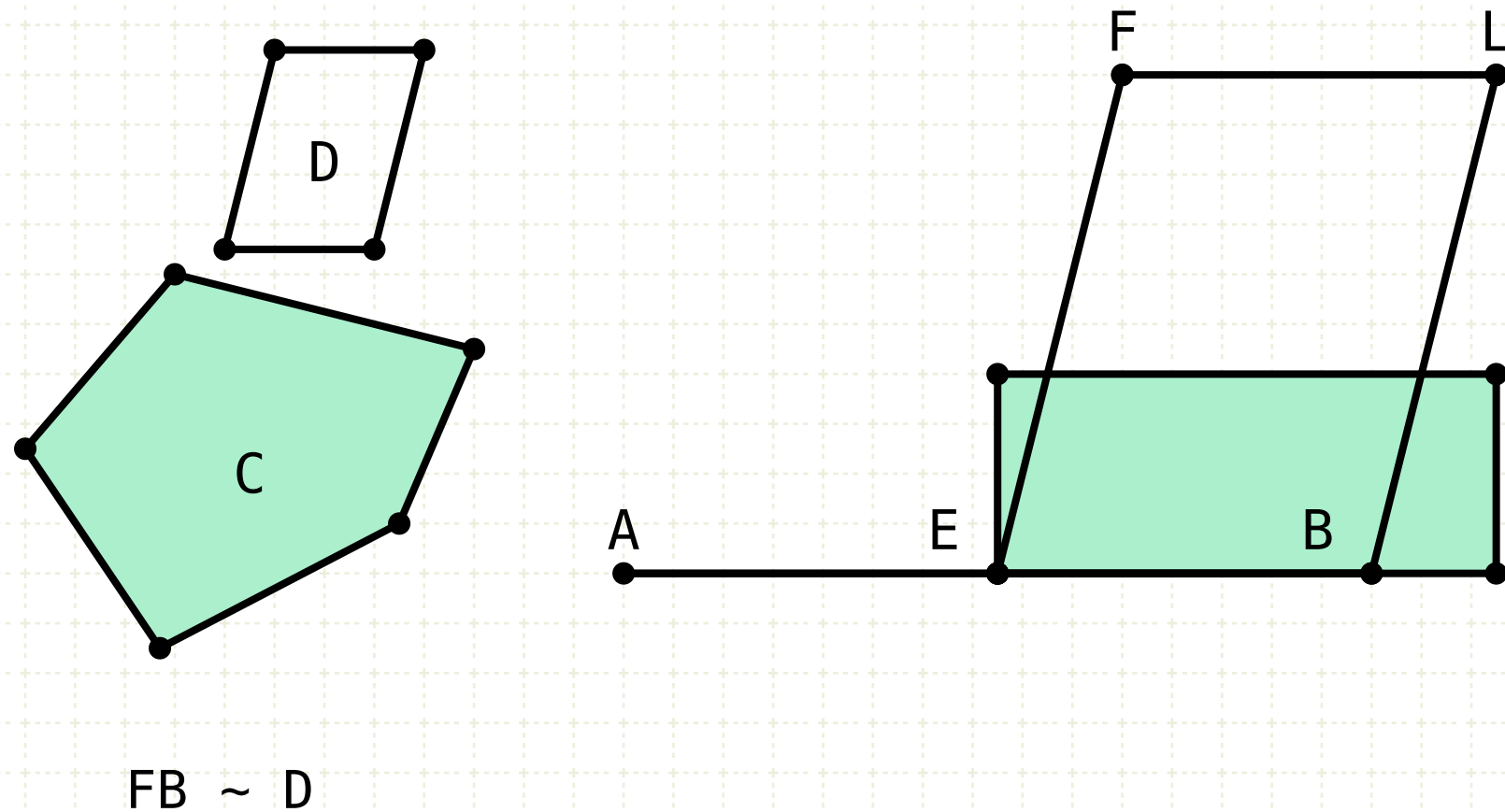
Bisect the line AB at point E

Create a parallelogram similar to D on line EB (VI·18)

Let GH be constructed such that it is equal to the the area of FB plus the area of C, and is similar to D (VI·25)

Proposition 29 of Book VI

To a given straight line to apply a parallelogram equal to a given rectilinear figure and exceeding by a parallelogrammic figure similar to a given one



Construction

Bisect the line AB at point E

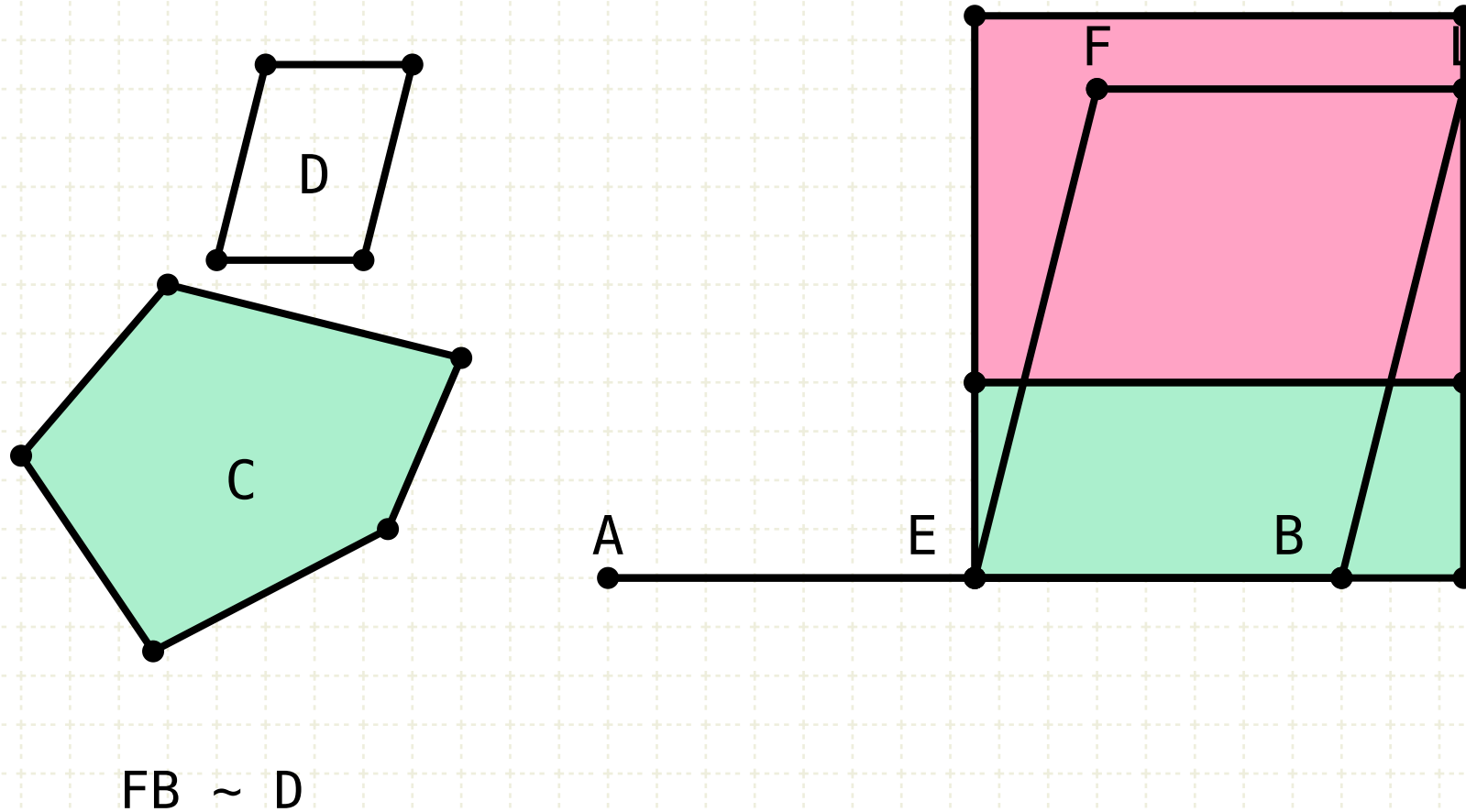
Create a parallelogram similar to D on line EB (VI·18)

Let GH be constructed such that it is equal to the the area of FB plus the area of C, and is similar to D (VI·25)

* Copy the rectilinear figure C to a rectangle

Proposition 29 of Book VI

To a given straight line to apply a parallelogram equal to a given rectilinear figure and exceeding by a parallelogrammic figure similar to a given one



Construction

Bisect the line AB at point E

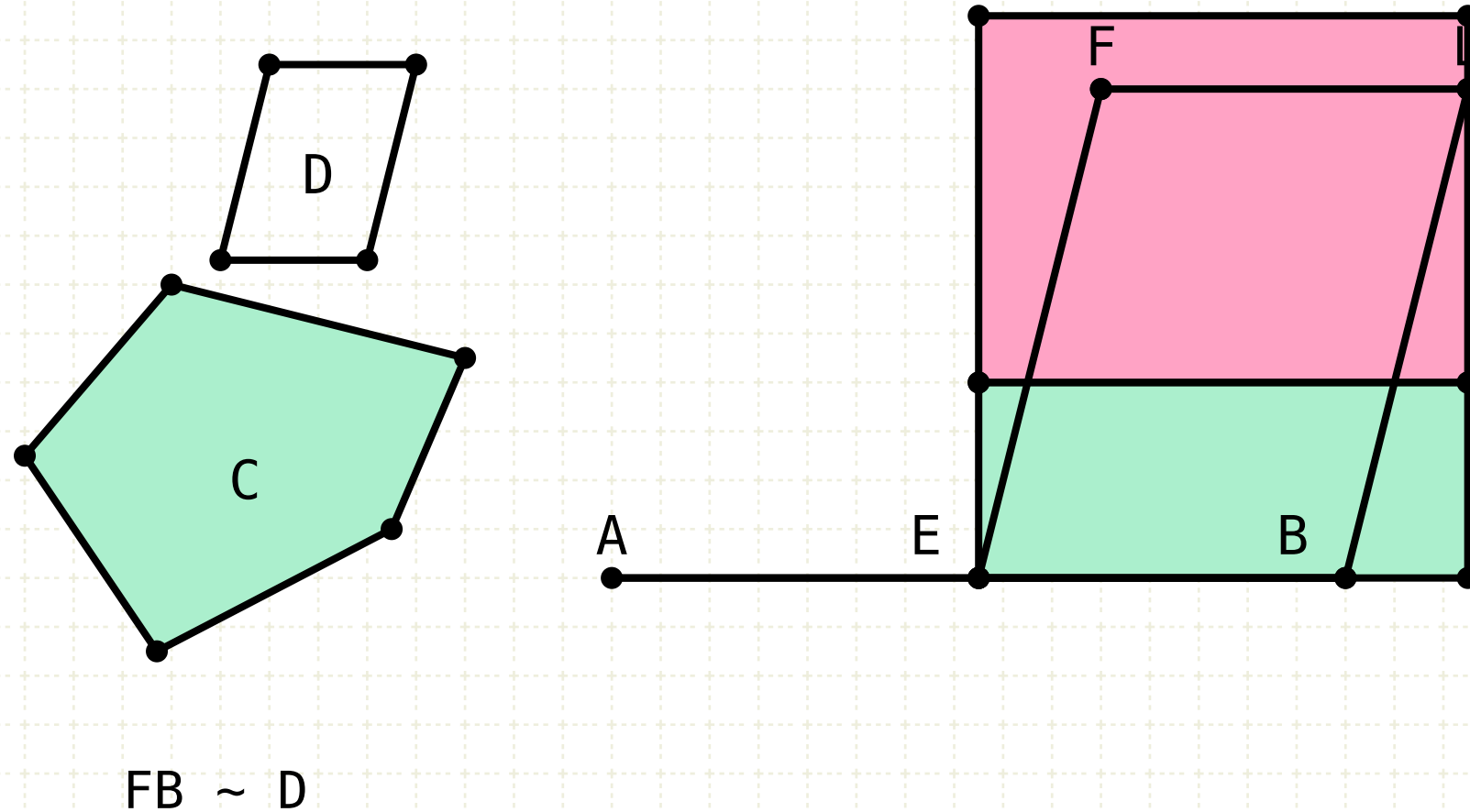
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Let GH be constructed such that it is equal to the the area of FB plus the area of C, and is similar to D (VI·25)

- * Copy the rectilinear figure C to a rectangle
- * Construct another rectangle on the top of the previous rectangle such that it is equal in area to the parallelogram FB (I·44)

Proposition 29 of Book VI

To a given straight line to apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one



Construction

Bisect the line AB at point E

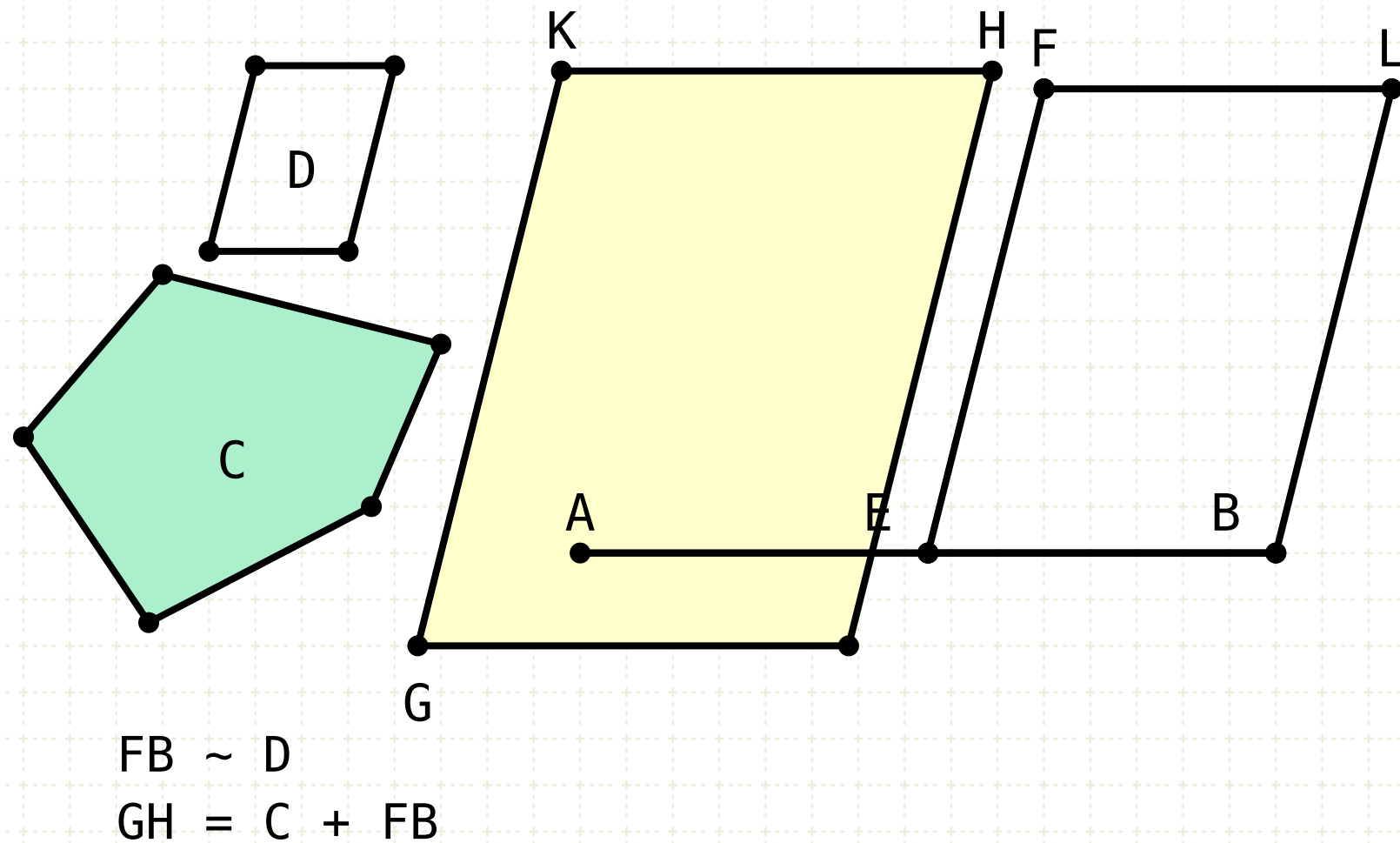
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- * Copy the sum of these two rectangles to a polygon similar to D

Proposition 29 of Book VI

To a given straight line to apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one



Construction

Bisect the line AB at point E

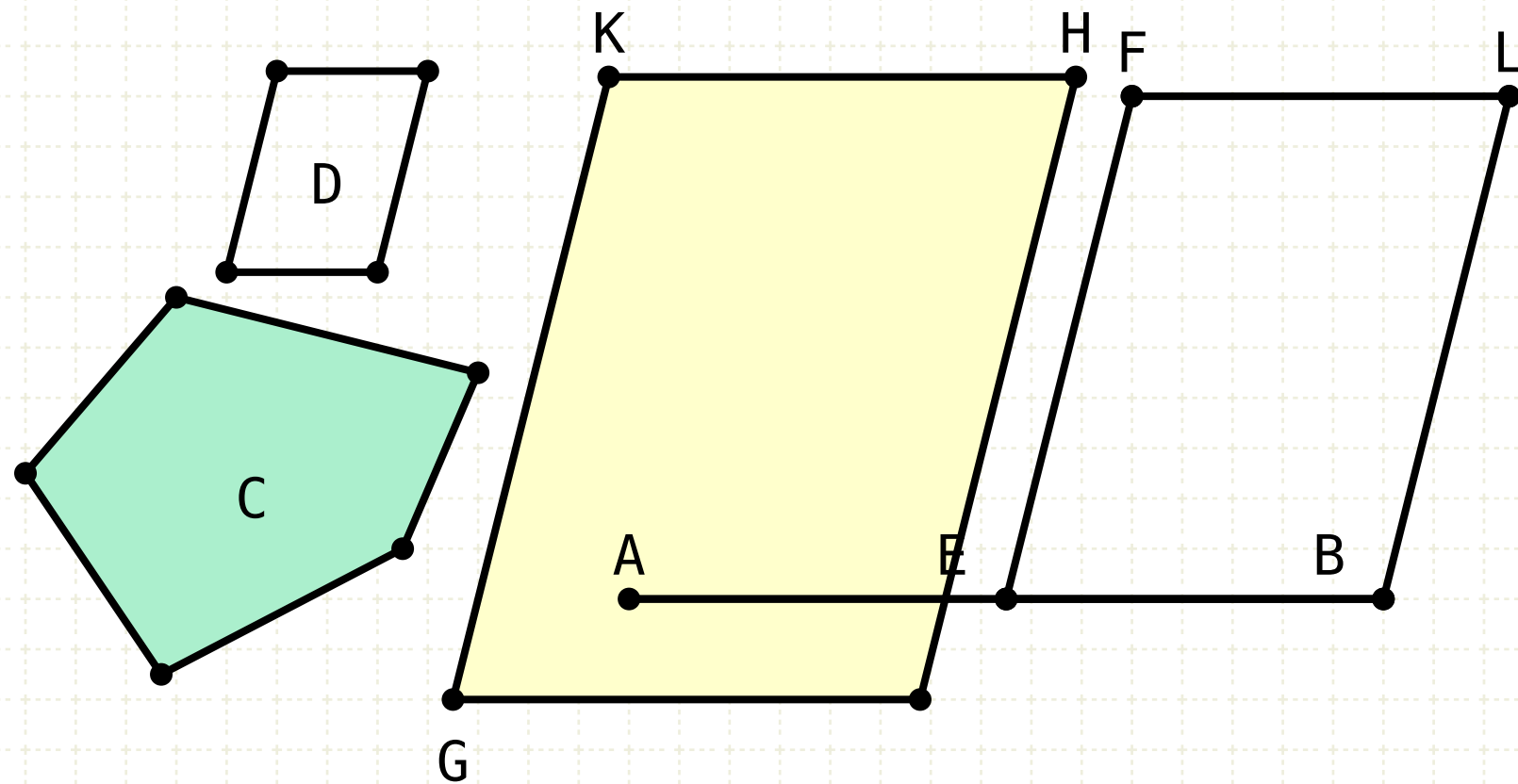
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Proposition 29 of Book VI

To a given straight line to apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one



$FB \sim D$

$GH = C + FB$

$GH \sim D$

$\therefore GH \sim FB$

Construction

Bisect the line AB at point E

Create a parallelogram similar to D on line EB (VI-18)

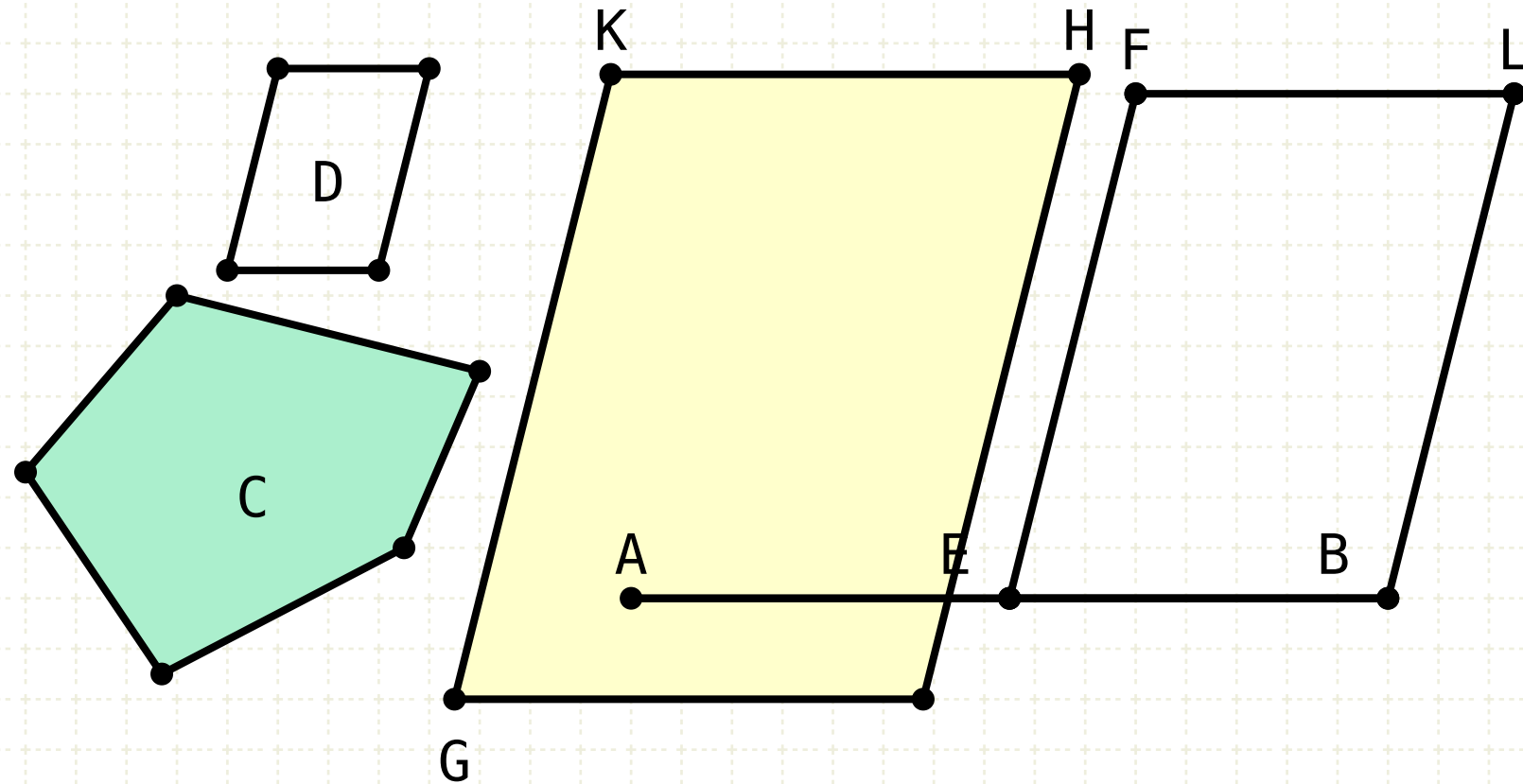
Let GH be constructed such that it is equal to the the area of FB plus the area of C, and is similar to D (VI-25)

- * Copy the rectilineal figure C to a rectangle
- * Construct another rectangle on the top of the previous rectangle such that it is equal in area to the parallelogram FB (I-44)
- * Copy the sum of these two rectangles to a polygon similar to D

GH is similar to D, which is also similar to FB, therefore GH is similar to FB (VI-21)

Proposition 29 of Book VI

To a given straight line to apply a parallelogram equal to a given rectilinear figure and exceeding by a parallelogrammic figure similar to a given one



$FB \sim D$
 $GH = C + FB$
 $GH \sim D$
 $\therefore GH \sim FB$

Construction

Bisect the line AB at point E

Create a parallelogram similar to D on line EB (VI-18)

Let GH be constructed such that it is equal to the the area of FB plus the area of C, and is similar to D (VI-25)

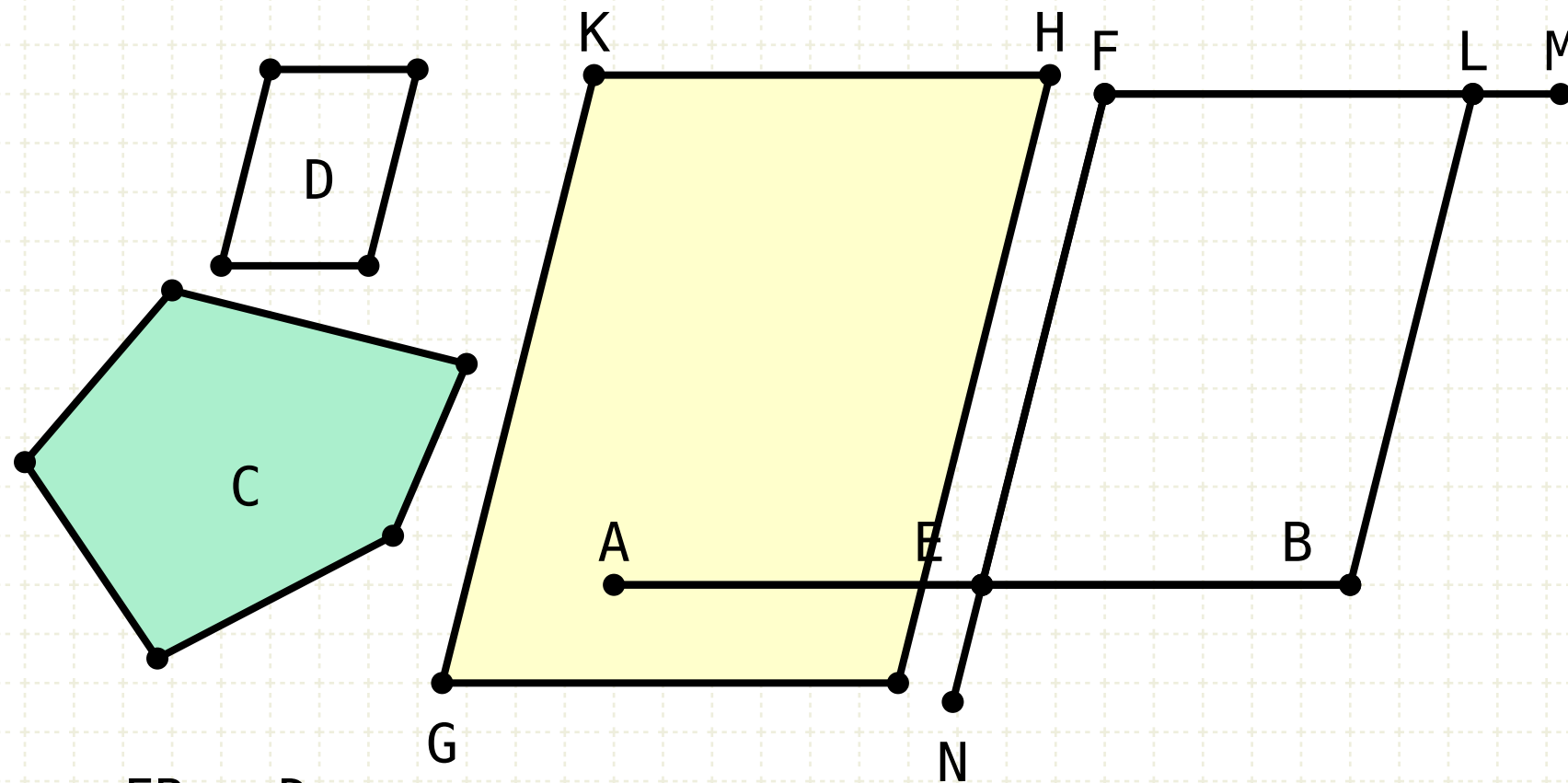
- * Copy the rectilinear figure C to a rectangle
- * Construct another rectangle on the top of the previous rectangle such that it is equal in area to the parallelogram FB (I-44)
- * Copy the sum of these two rectangles to a polygon similar to D

GH is similar to D, which is also similar to FB, therefore GH is similar to FB (VI-21)

GH is larger than FB, and since they are similar, GK is larger than FE and KH is larger than FL

Proposition 29 of Book VI

To a given straight line to apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one


$$FB \sim D$$
$$GH = C + FB$$
$$GH \sim D$$
$$\therefore GH \sim FB$$

Construction

Bisect the line AB at point E

Create a parallelogram similar to D on line EB (VI-18)

Let GH be constructed such that it is equal to the the area of FB plus the area of C, and is similar to D (VI.25)

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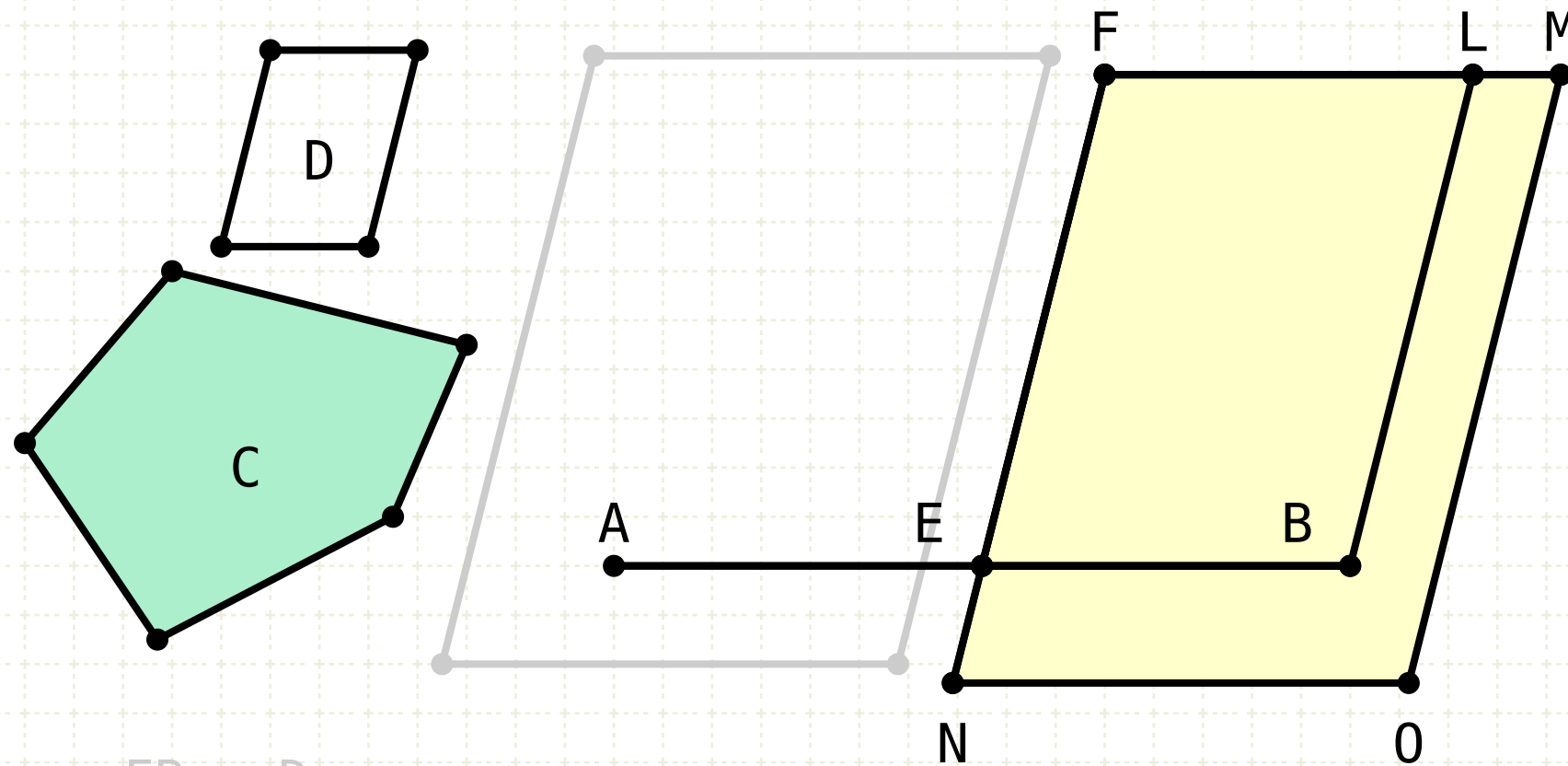
GH is larger than FB, and since they are similar, GK is larger than FE and KH is larger than FL

Extend the line FL and FE such that FLM is equal to KH and FEN is equal to KG



Proposition 29 of Book VI

To a given straight line to apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one



$FB \sim D$
 $GH = C + FB$
 $GH \sim D$
 $\therefore GH \sim FB$
 $MN = GH$

Construction

Bisect the line AB at point E

Create a parallelogram similar to D on line EB (VI-18)

Let GH be constructed such that it is equal to the the area of FB plus the area of C, and is similar to D (VI-25)

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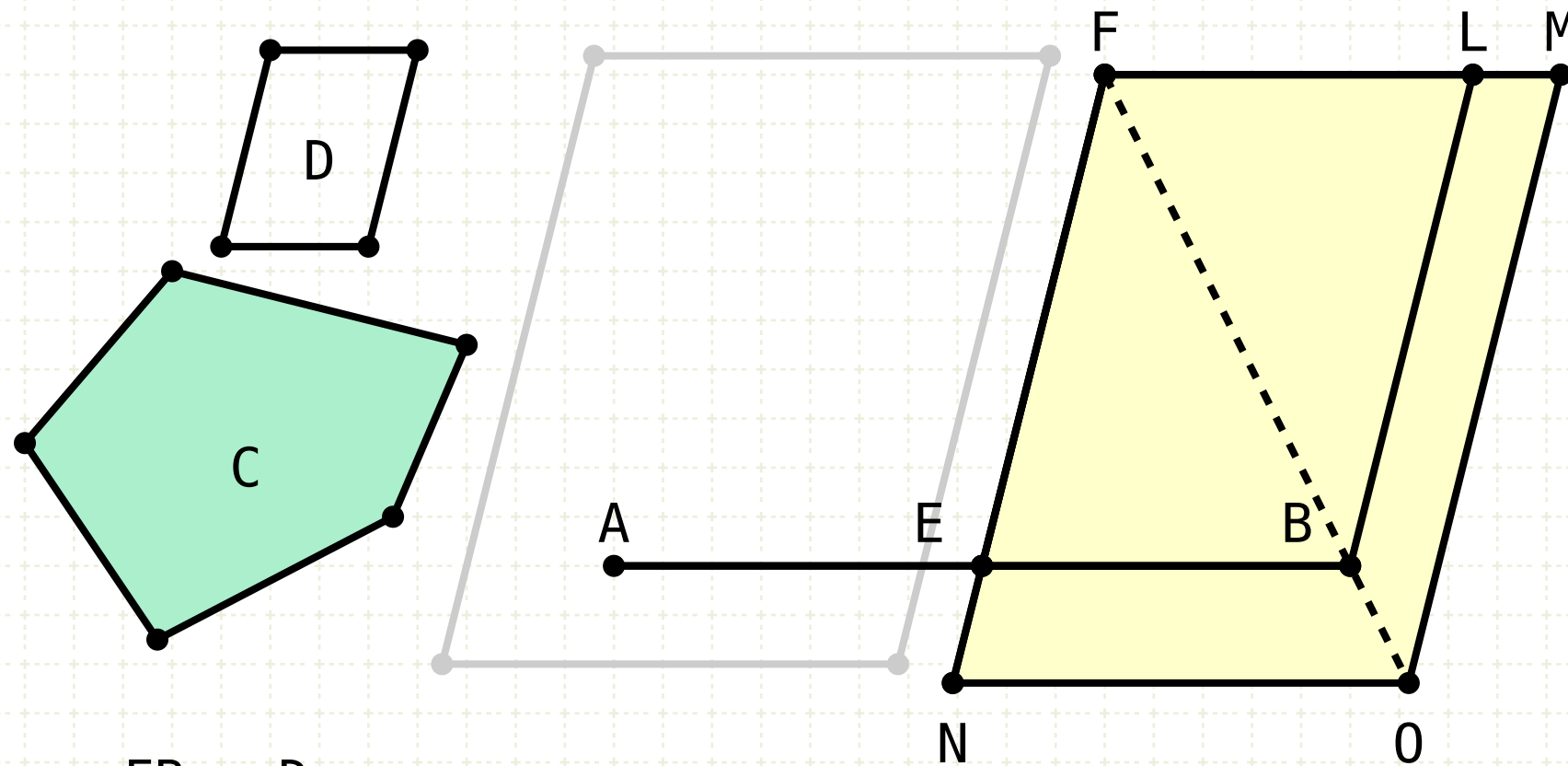
GH is larger than FB, and since they are similar, GK is larger than FE and KH is larger than FL

Extend the line FL and FE such that FLM is equal to KH and FEN is equal to KG

Complete the parallelogram MN

Proposition 29 of Book VI

To a given straight line to apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one



$$FB \sim D$$

$$GH = C + FB$$

$$GH \sim D$$

$$\therefore GH \sim FB$$

$$MN = GH$$

$$MN \sim GH \sim D \sim FB$$

Construction

Bisect the line AB at point E

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GH is similar to D, which is also similar to FB, therefore GH is similar to FB (VI-21)

GH is larger than FB, and since they are similar, GK is larger than FE and KH is larger than FL

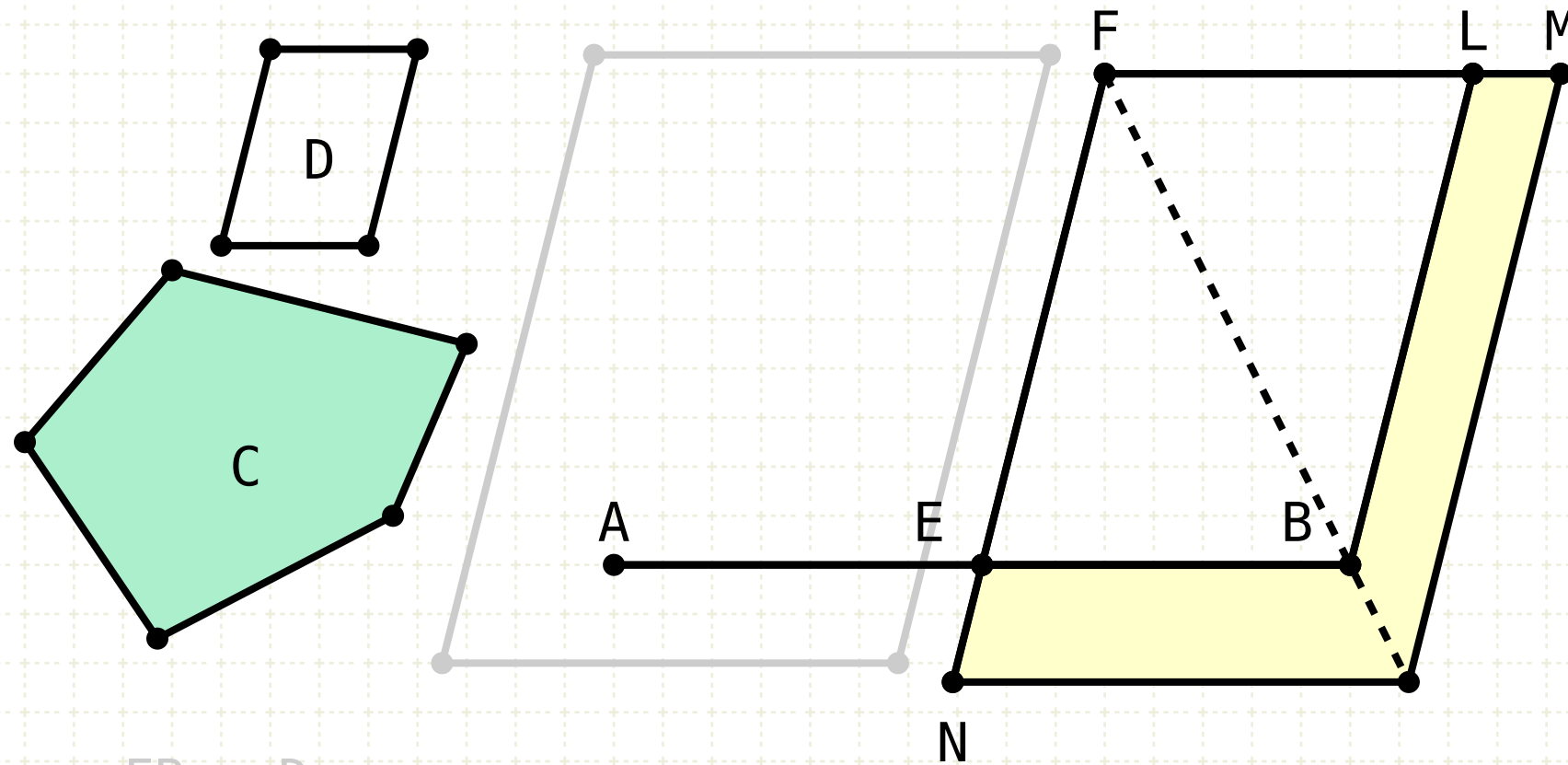
Extend the line FL and FE such that FLM is equal to KH and FEN is equal to KG

Complete the parallelogram MN

Since MN is similar to GH, so is MN similar to FB (VI-21), thus the points F,B, and O lie on the same diagonal (VI-26)

Proposition 29 of Book VI

To a given straight line to apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one



Construction (cont.)

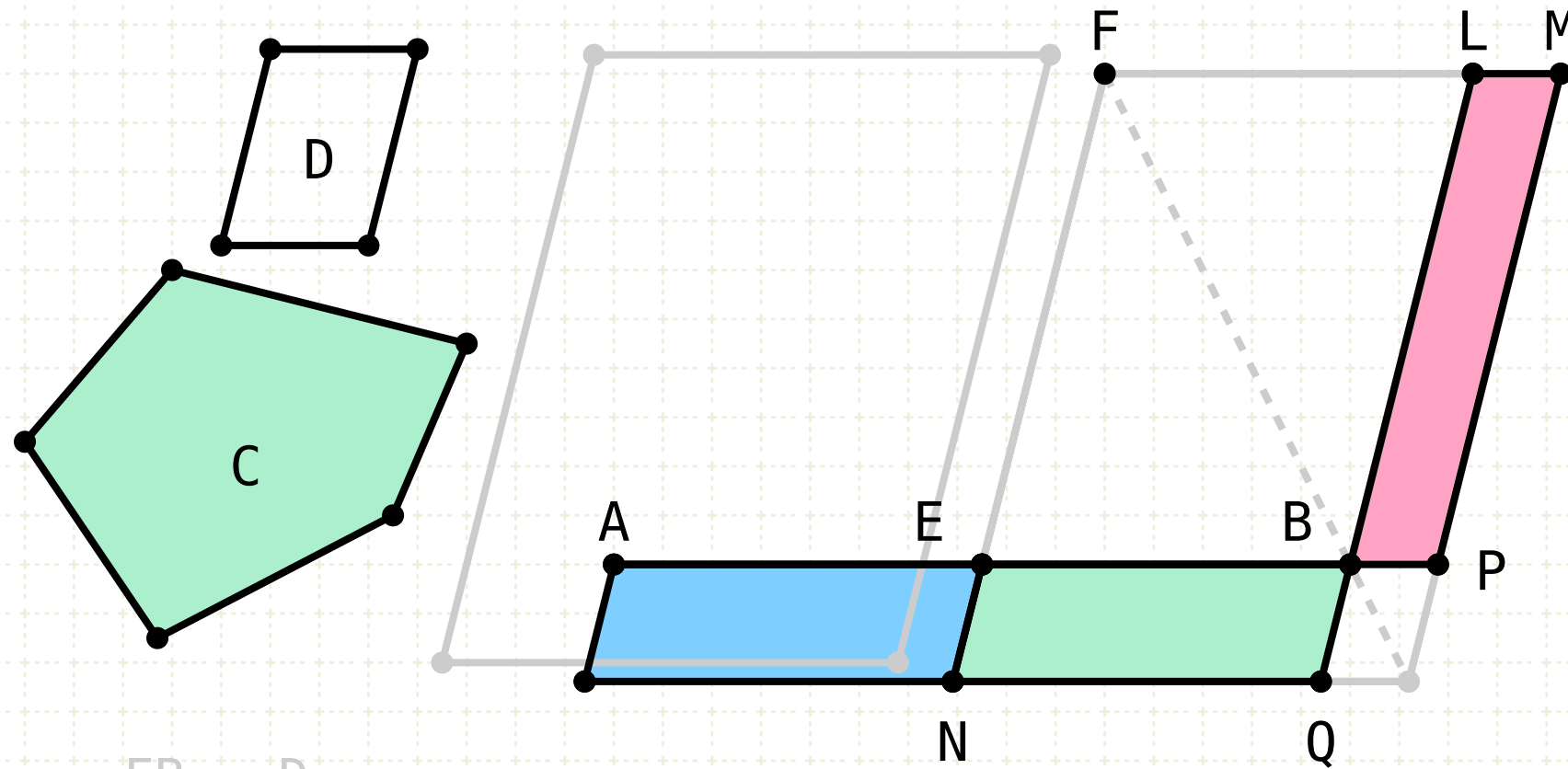
GH is equal to sum of FB and C, and since MN is equal to GH, the remaining gnomon (NOMLBE) is equal in area to C

$$\begin{aligned} FB &\sim D \\ GH &= C + FB \\ GH &\sim D \\ \therefore GH &\sim FB \\ MN &= GH \end{aligned}$$

$$\begin{aligned} MN &\sim GH \sim D \sim FB \\ NOMLBE &= C \end{aligned}$$

Proposition 29 of Book VI

To a given straight line to apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one



Construction (cont.)

GH is equal to sum of FB and C, and since MN is equal to GH, the remaining gnomon (NOMLBE) is equal in area to C

Since AE equals EB, the parallelogram AN equals NB (VI·21) and LP (VI·26)

$$FB \sim D$$

$$GH = C + FB$$

$$GH \sim D$$

GH ~ FB

$$MN = GH$$

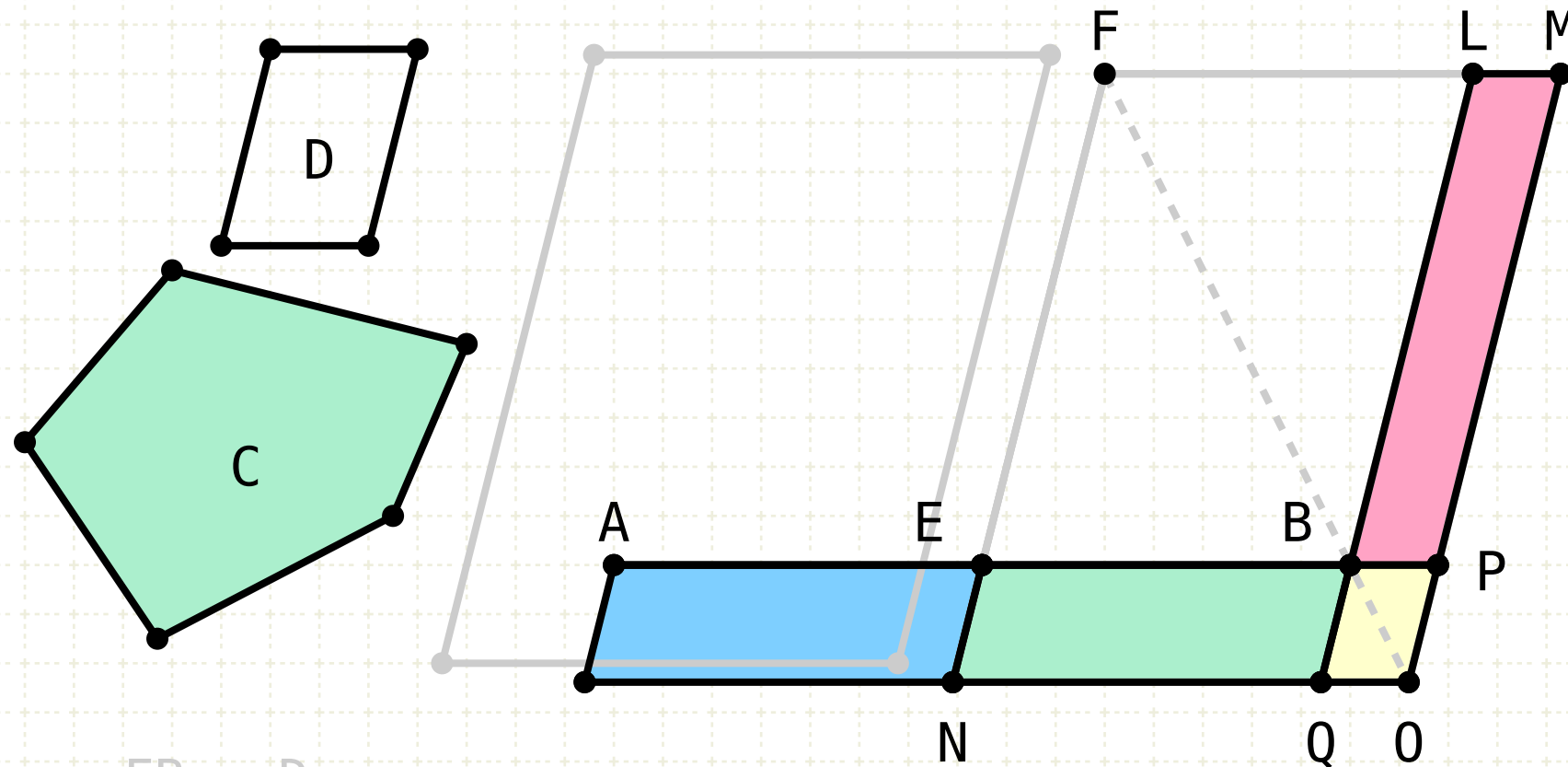
MN ~ GH ~ D ~ FB

$$\text{NOMLBE} = C$$

$$AN = NB = LP$$

Proposition 29 of Book VI

To a given straight line to apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one



Construction (cont.)

GH is equal to sum of FB and C, and since MN is equal to GH, the remaining gnomon (NOMLBE) is equal in area to C

Since AE equals EB, the parallelogram AN equals NB (VI·21) and LP (VI·26)

Add EO to AN and LP, thus AO is equal to the gnomon (NOMLBE)

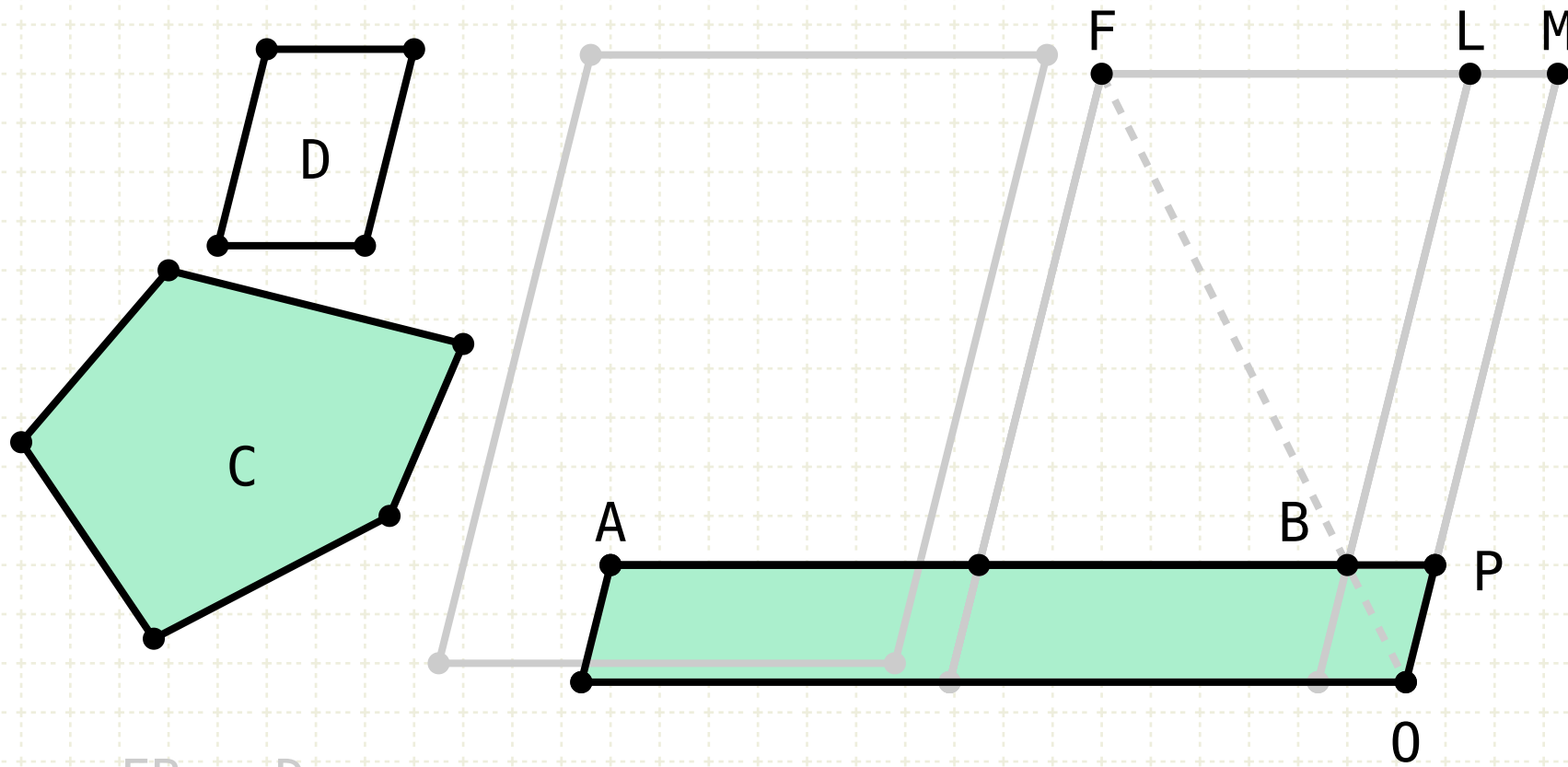
$$\begin{aligned} \text{FB} &\sim \text{D} \\ \text{GH} &= \text{C} + \text{FB} \\ \text{GH} &\sim \text{D} \\ \therefore \text{GH} &\sim \text{FB} \\ \text{MN} &= \text{GH} \end{aligned}$$

$$\begin{aligned} \text{MN} &\sim \text{GH} \sim \text{D} \sim \text{FB} \\ \text{NOMLBE} &= \text{C} \\ \text{AN} &= \text{NB} = \text{LP} \\ \text{AO} &= \text{AN} + \text{EO} \\ \text{AO} &= \text{LP} + \text{EO} = \text{NOMLBE} \end{aligned}$$



Proposition 29 of Book VI

To a given straight line to apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one


$$\begin{aligned} \text{FB} &\sim \text{D} \\ \text{GH} &= \text{C} + \text{FB} \\ \text{GH} &\sim \text{D} \\ \therefore \text{GH} &\sim \text{FB} \\ \text{MN} &= \text{GH} \end{aligned}$$
$$\begin{aligned} \text{MN} &\sim \text{GH} \sim \text{D} \sim \text{FB} \\ \text{NOMLBE} &= \text{C} \\ \text{AN} &= \text{NB} = \text{LP} \\ \text{AO} &= \text{AN} + \text{EO} \\ \text{AO} &= \text{LP} + \text{EO} = \text{NOMLBE} \\ \text{AO} &= \text{C} \end{aligned}$$

Construction (cont.)

GH is equal to sum of FB and C, and since MN is equal to GH, the remaining gnomon (NOMLBE) is equal in area to C

Since AE equals EB, the parallelogram AN equals NB (VI-21) and LP (VI-26)

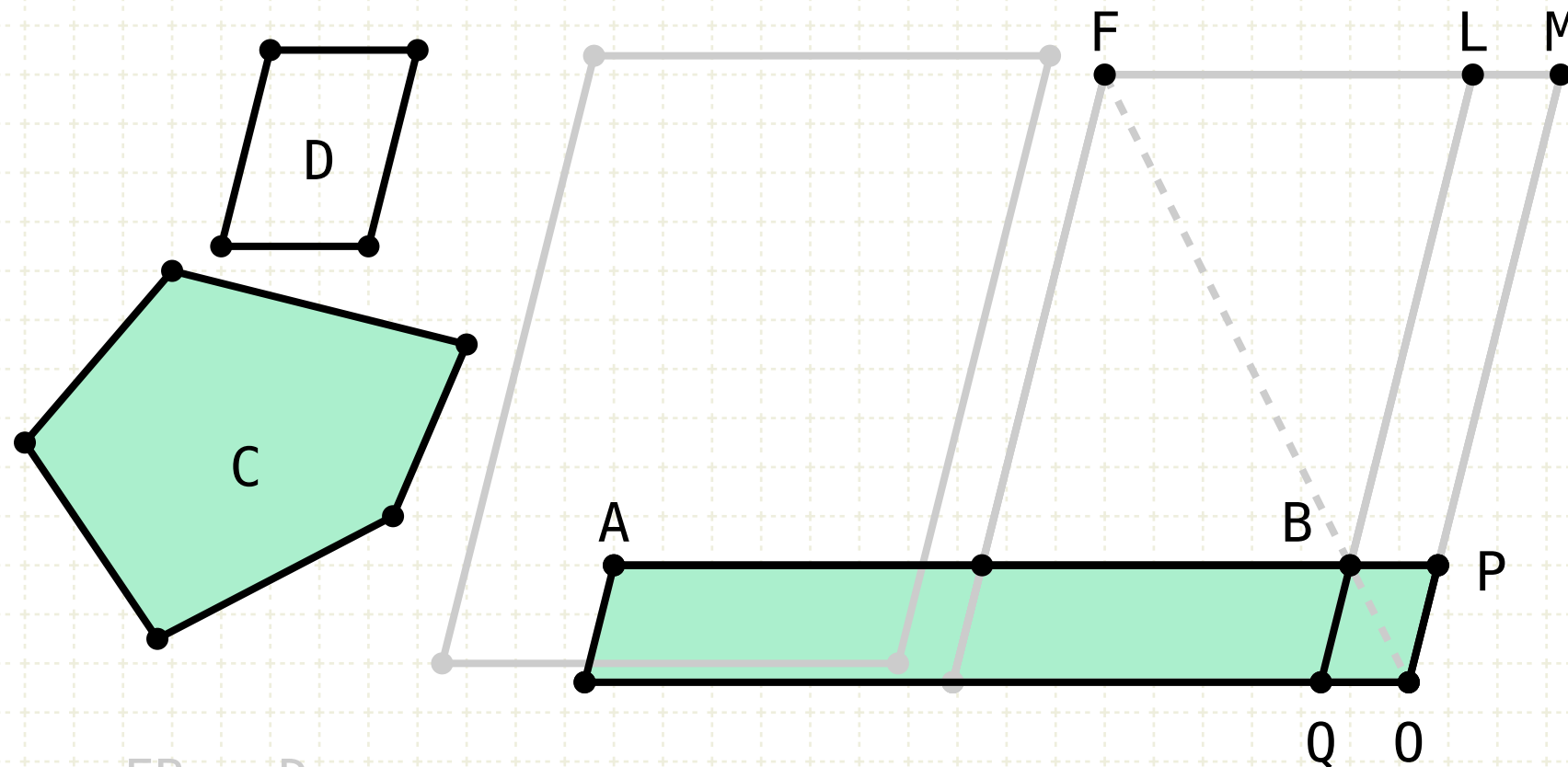
Add EO to AN and LP, thus AO is equal to the gnomon (NOMLBE)

It has already been shown that the gnomon is equal to C , hence AO is equal to C



Proposition 29 of Book VI

To a given straight line to apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one



$$\begin{aligned} FB &\sim D \\ GH &= C + FB \\ GH &\sim D \\ \therefore GH &\sim FB \\ MN &= GH \end{aligned}$$

$$\begin{aligned} MN &\sim GH \sim D \sim FB \\ \text{NOMLBE} &= C \\ AN &= NB = LP \\ A0 &= AN + E0 \\ A0 &= LP + E0 = \text{NOMLBE} \\ A0 &= C \end{aligned}$$

Construction (cont.)

GH is equal to sum of FB and C, and since MN is equal to GH, the remaining gnomon (NOMLBE) is equal in area to C

Since AE equals EB, the parallelogram AN equals NB (VI·21) and LP (VI·26)

Add EO to AN and LP, thus AO is equal to the gnomon (NOMLBE)

It has already been shown that the gnomon is equal to C, hence AO is equal to C

Thus, AO is a parallelogram drawn on AB, equal in size to C, exceeding AB by a parallelogram figure (QP) similar to D

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