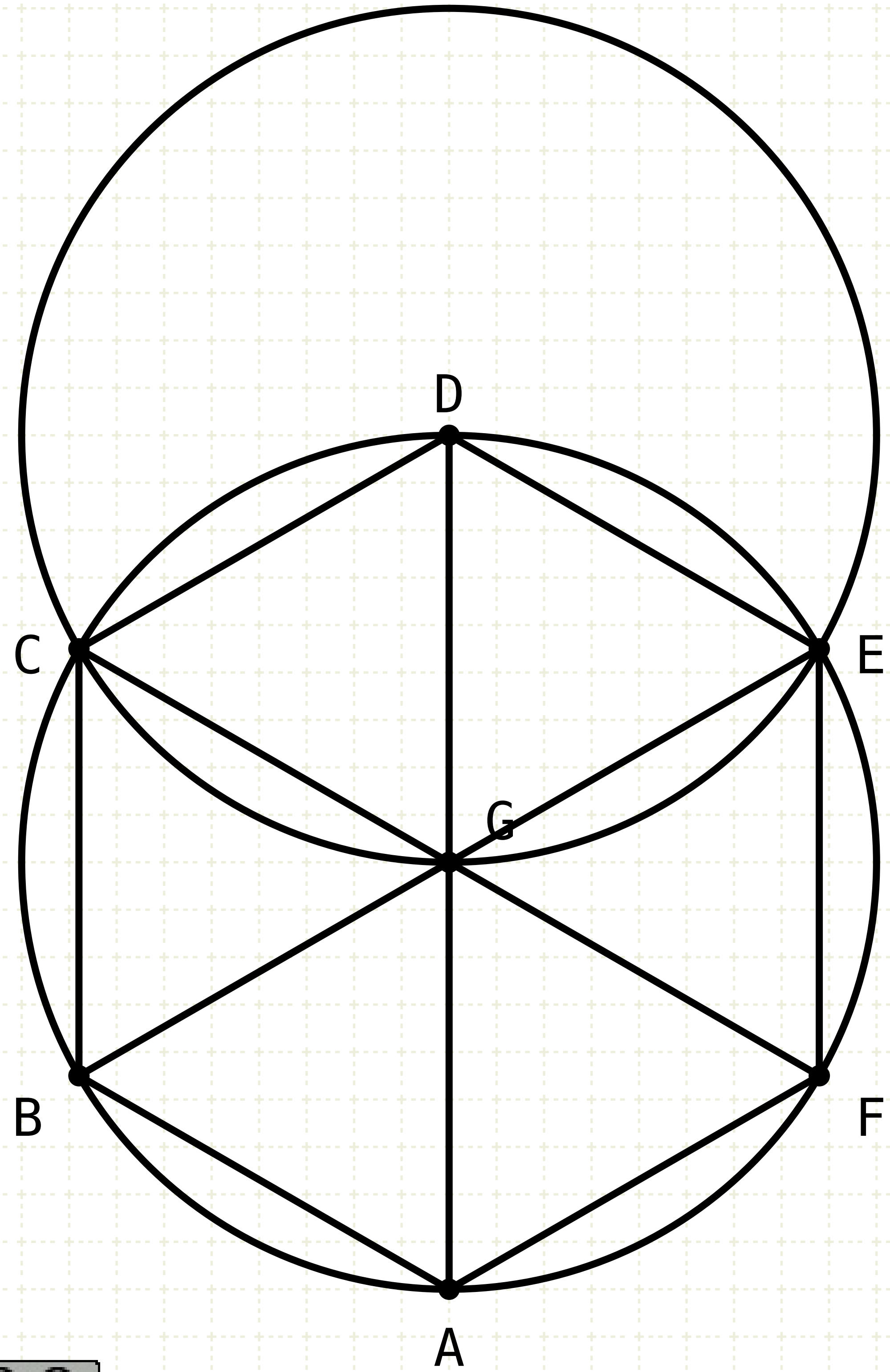
Euclid's Elements

Book IV



Philosophy (nature) is written in that great book which ever is before our eyes -- I mean the universe -- but we cannot understand it if we do not first learn the language and grasp the symbols in which it is written. The book is written in mathematical language, and the symbols are triangles, circles and other geometrical figures, without whose help it is impossible to comprehend a single word of it - without which one wanders in vain through a dark labyrinth.

Galileo Galilei

Proposition 5 of Book IV

About a given triangle to circumscribe a circle.

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About a given triangle to circumscribe a circle.

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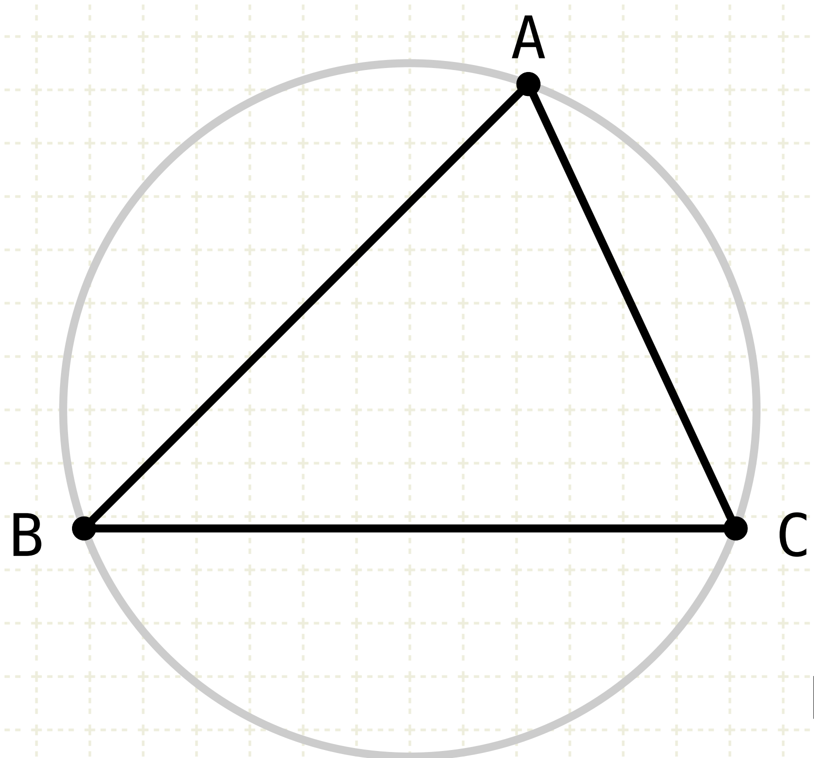
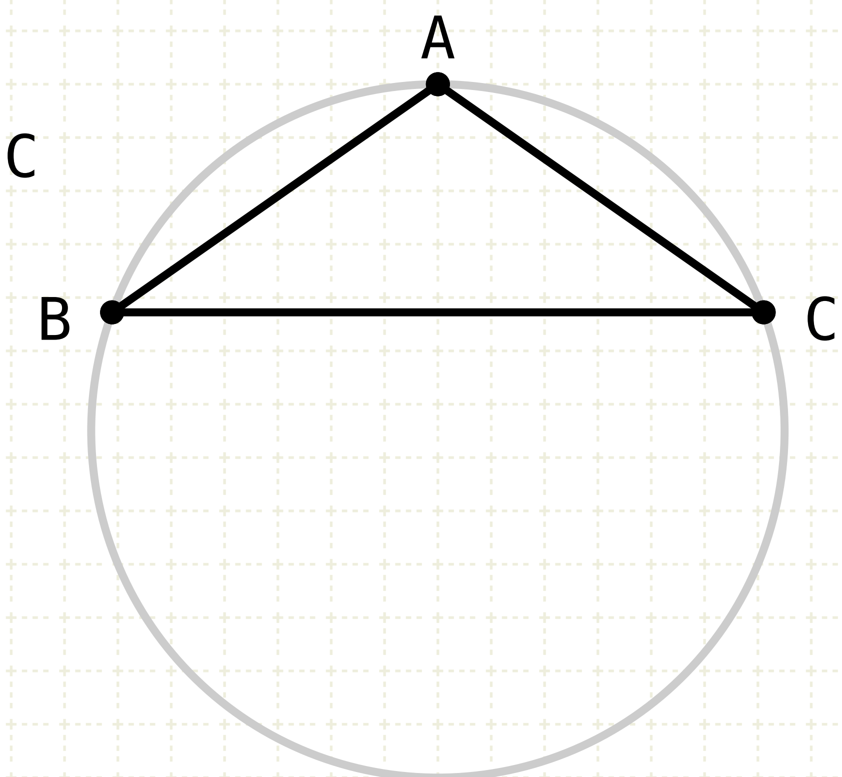
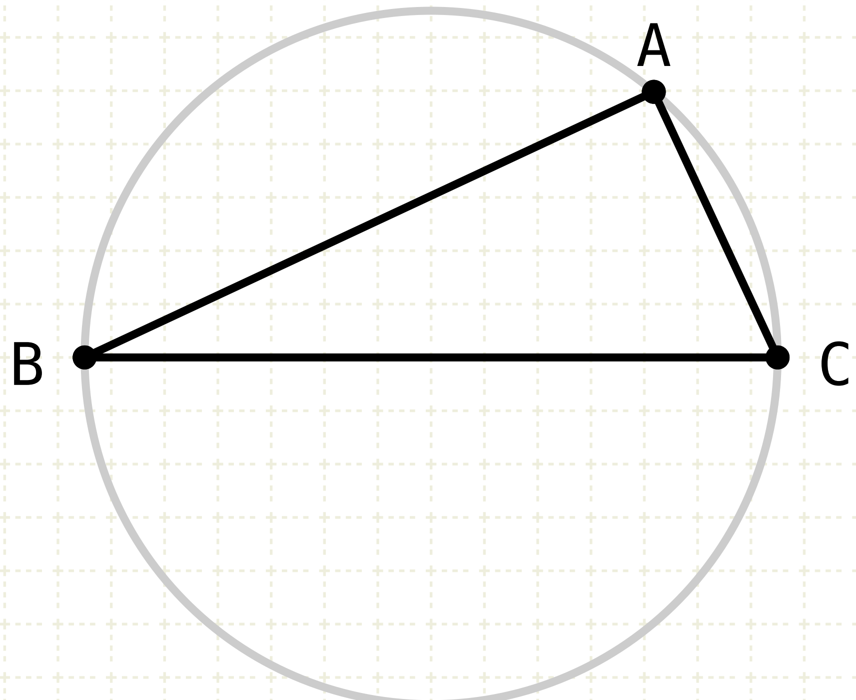
About a given triangle to circumscribe a circle.







In other words

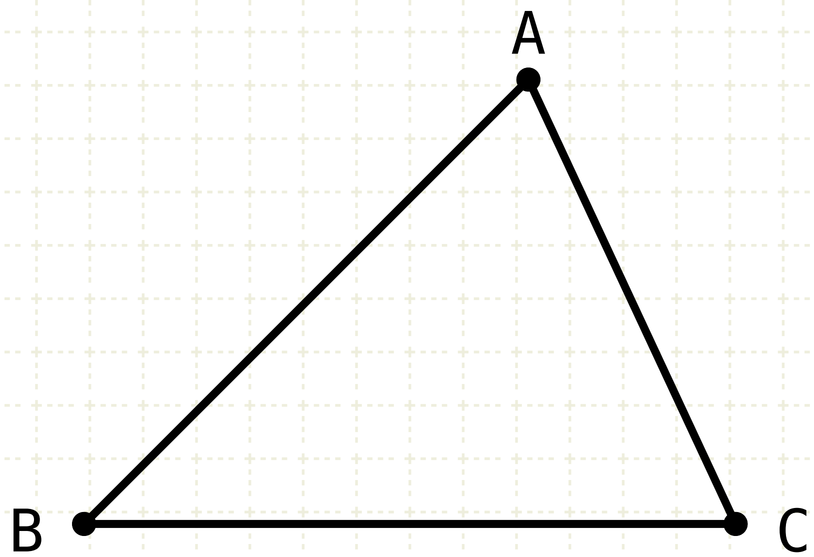
Given a triangle ABC

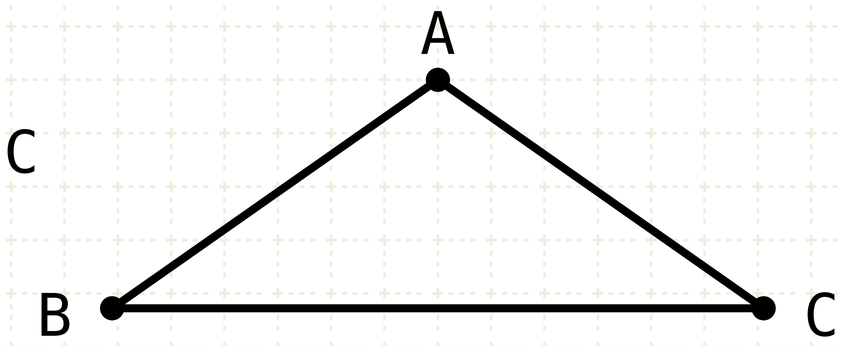
Draw a circle so that it passes through each vertex of the triangle

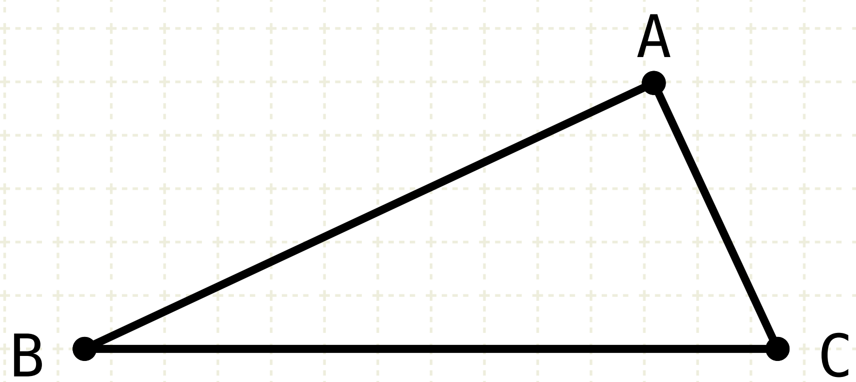
Proposition 5 of Book IV

About a given triangle to circumscribe a circle.

Construction







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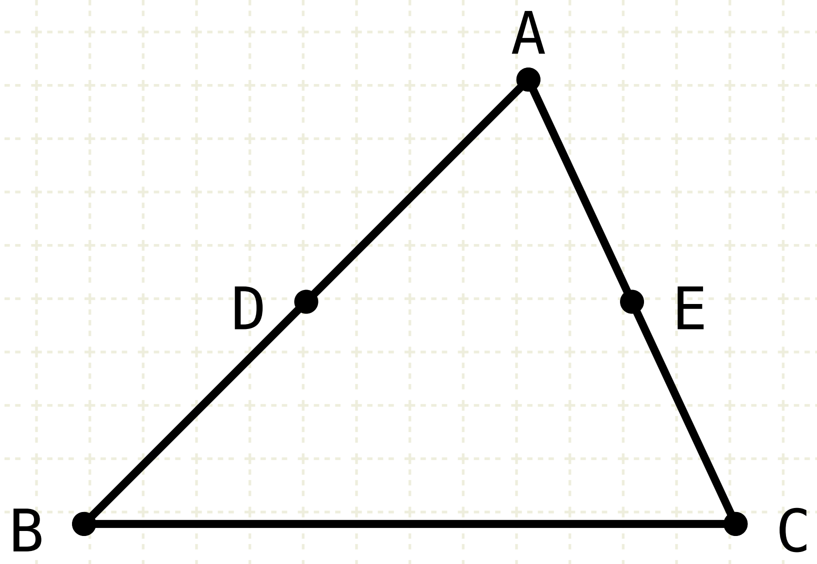
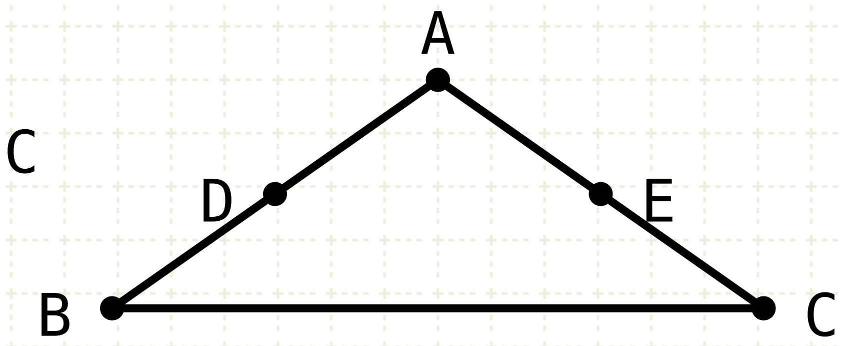
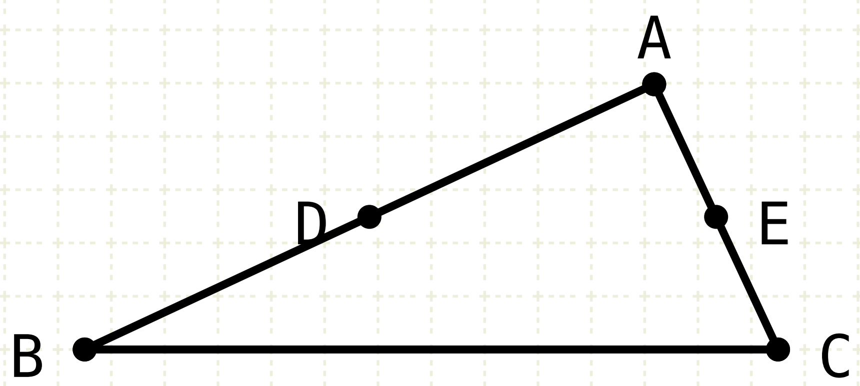




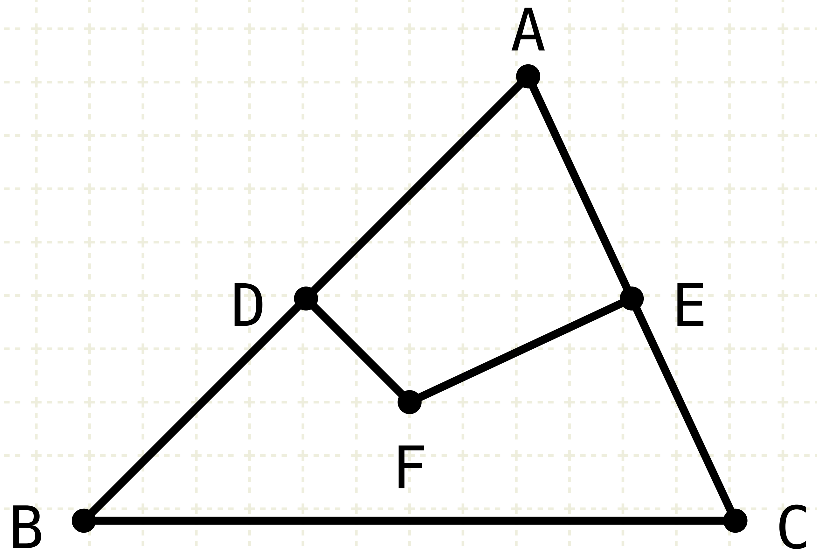


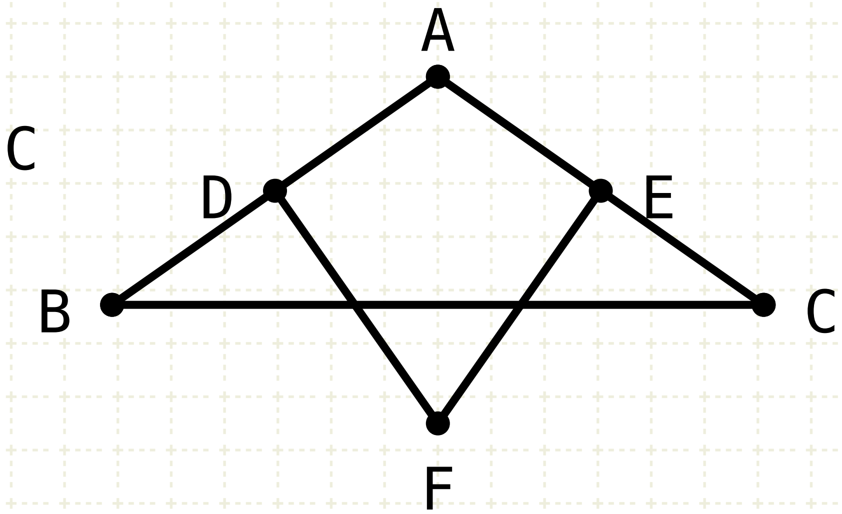
Construction

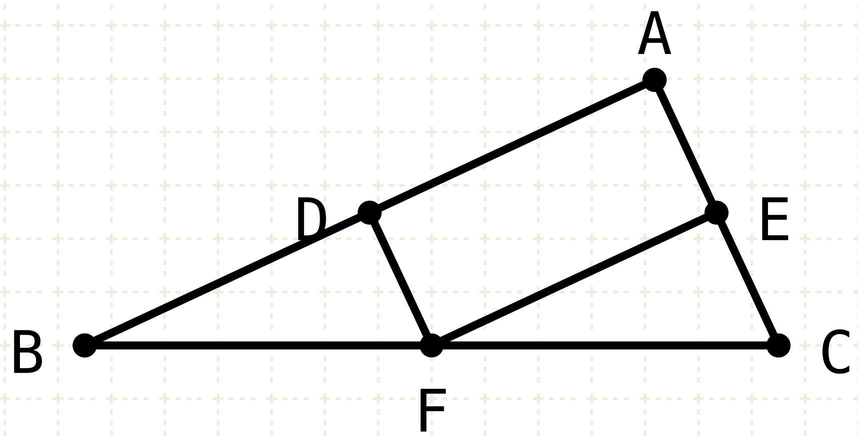
Bisect line AB at point D and line AC at point E (I·10)

Proposition 5 of Book IV

About a given triangle to circumscribe a circle.







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Draw lines from the points D,E perpendicular to their respective sides of the triangle, intersecting at point F (I·11)

Proposition 5 of Book IV

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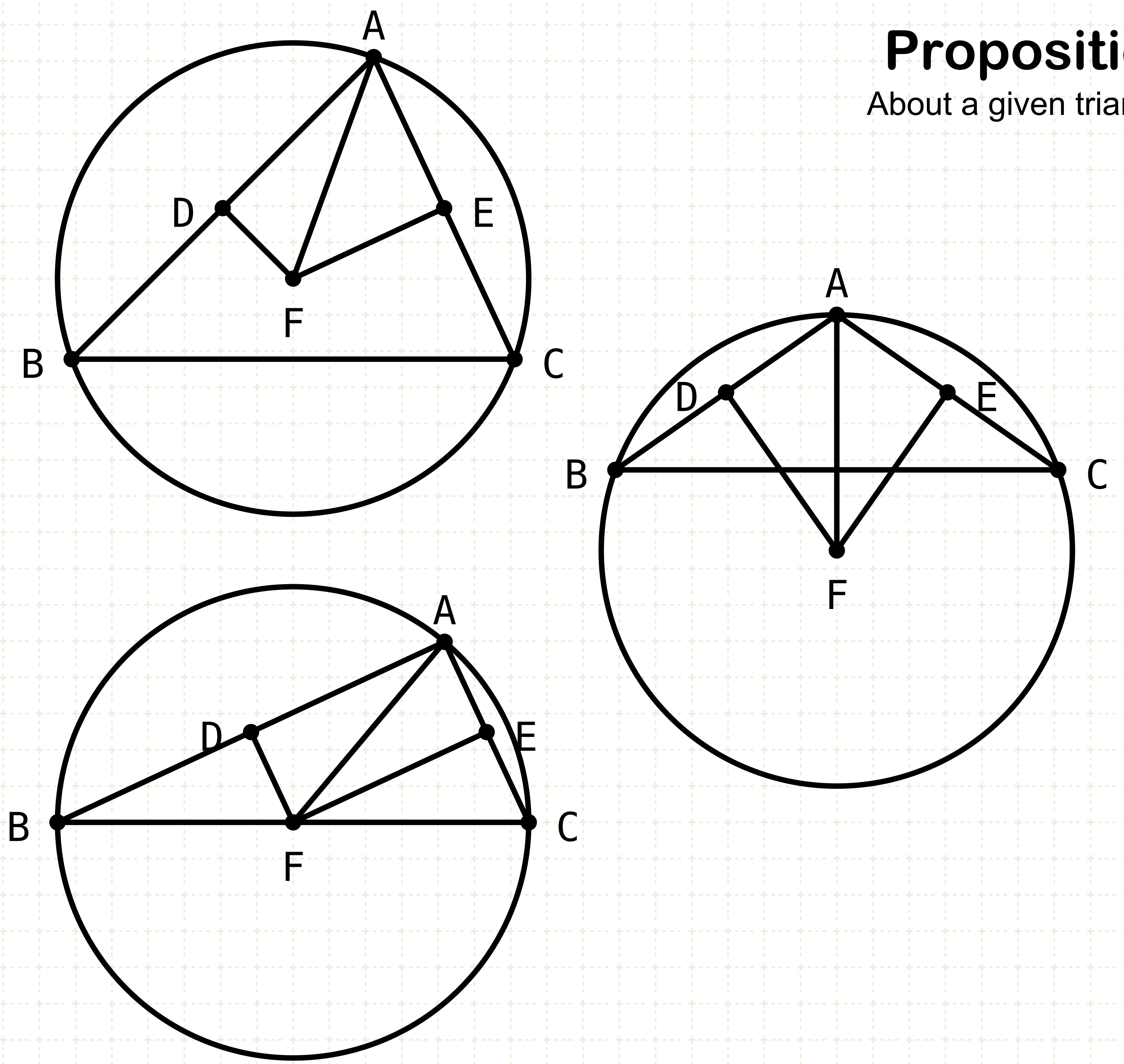

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Bisect line AB at point D and line AC at point E (I·10)

Draw lines from the points D,E perpendicular to their respective sides of the triangle, intersecting at point F (I·11)

With F as the centre, and AF as the radius, it is possible to draw a circle that passes through each point A, B and C

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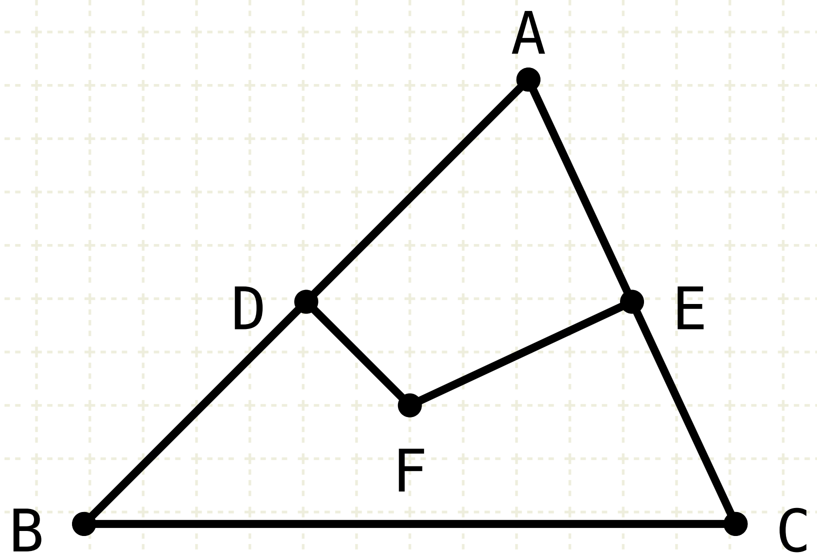

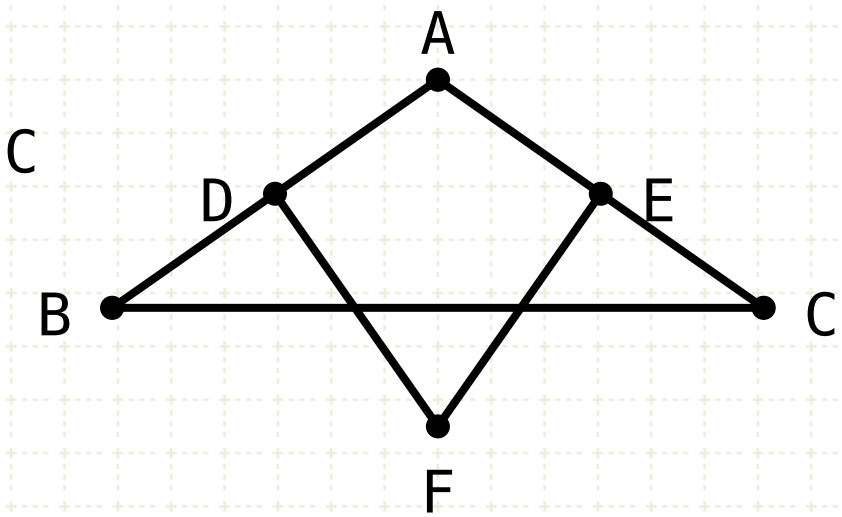

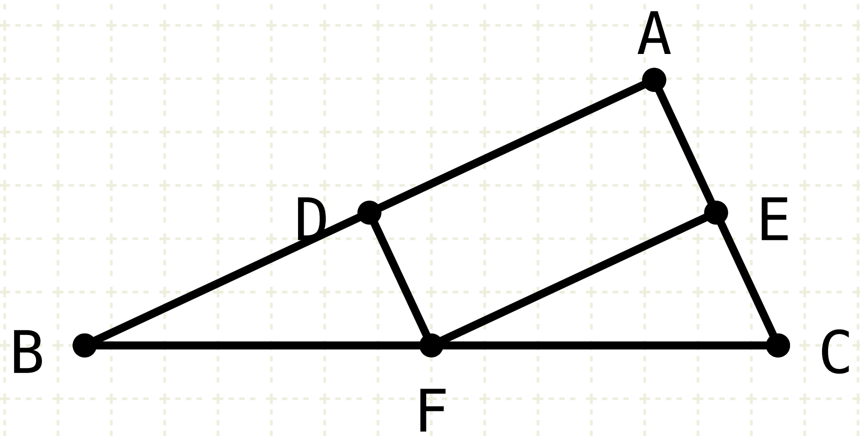

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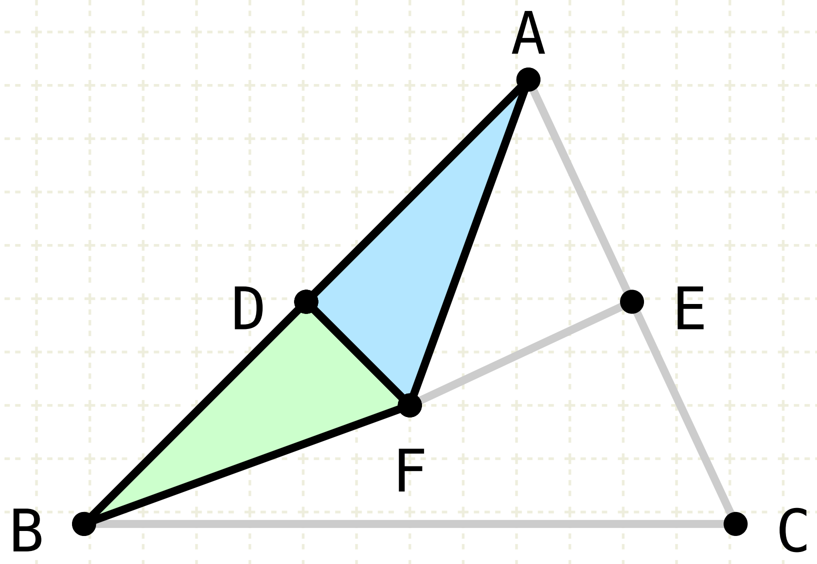

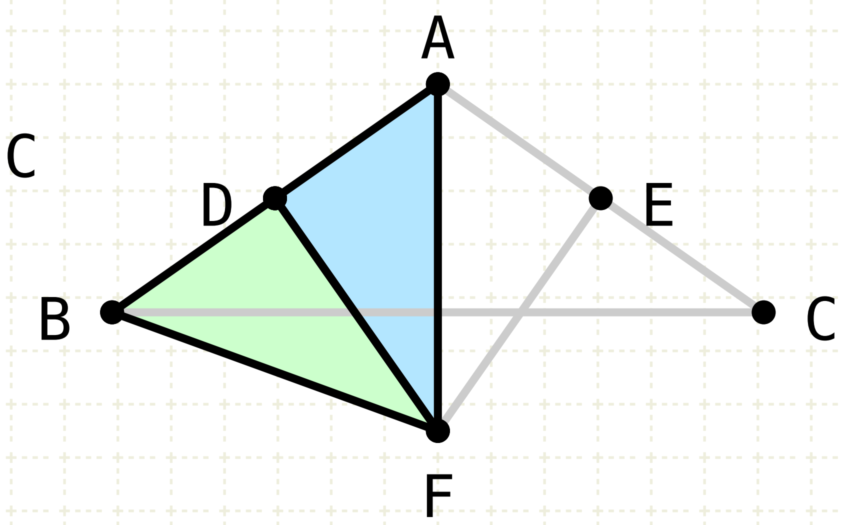

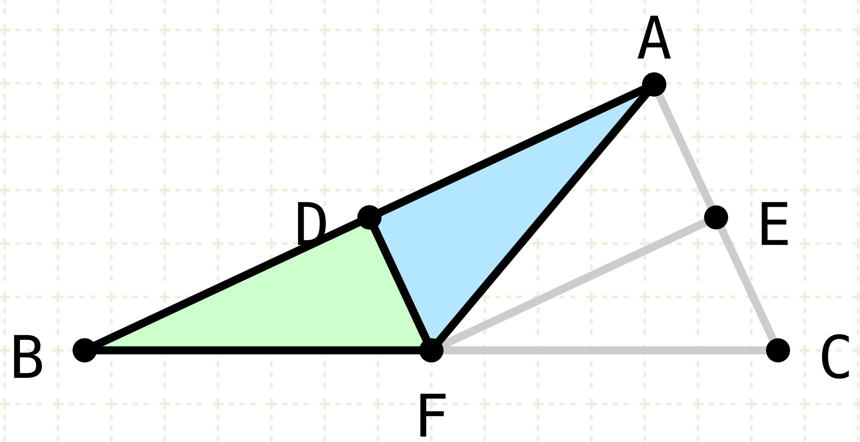

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Proof

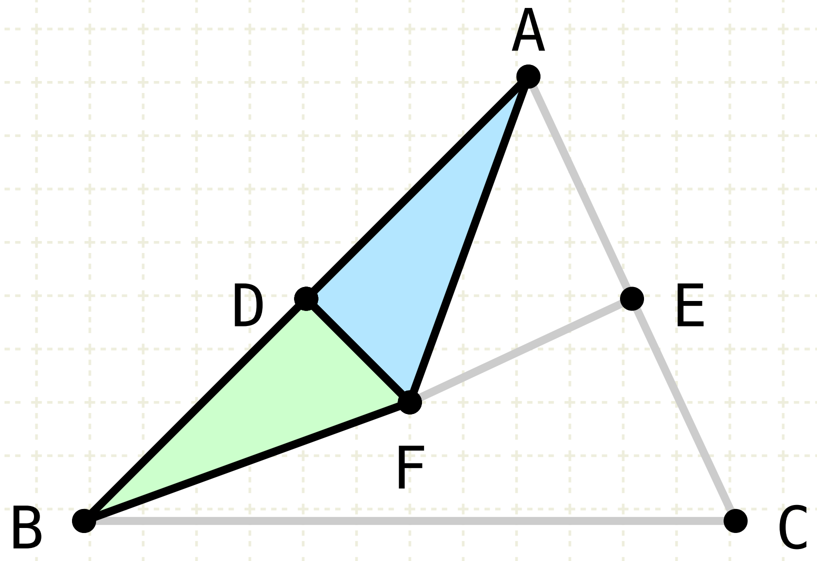
Draw lines BF and AF

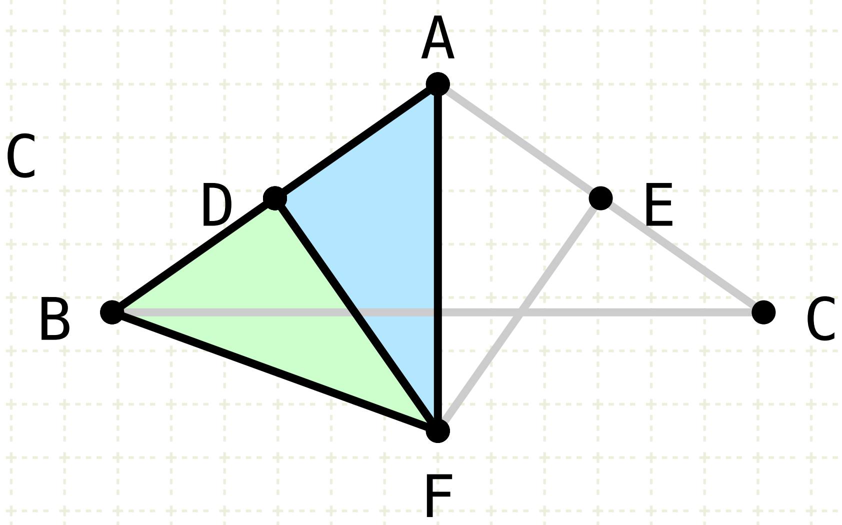
The two triangles ADF and BDF are equal in all respects, since they have a side (BD,AD), angle (ADF = BDF = ∟), side (DF) equal (I·4)

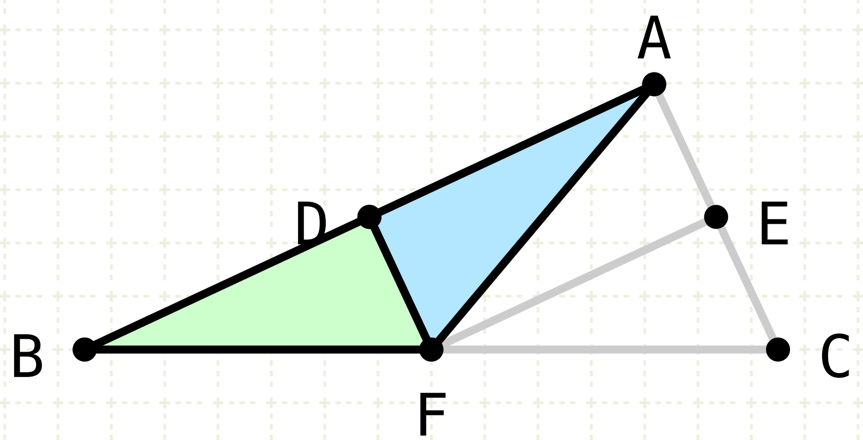
BD = AD
$\angle ADF = \angle BDF = \llcorner$
DF is common

Proposition 5 of Book IV

About a given triangle to circumscribe a circle.







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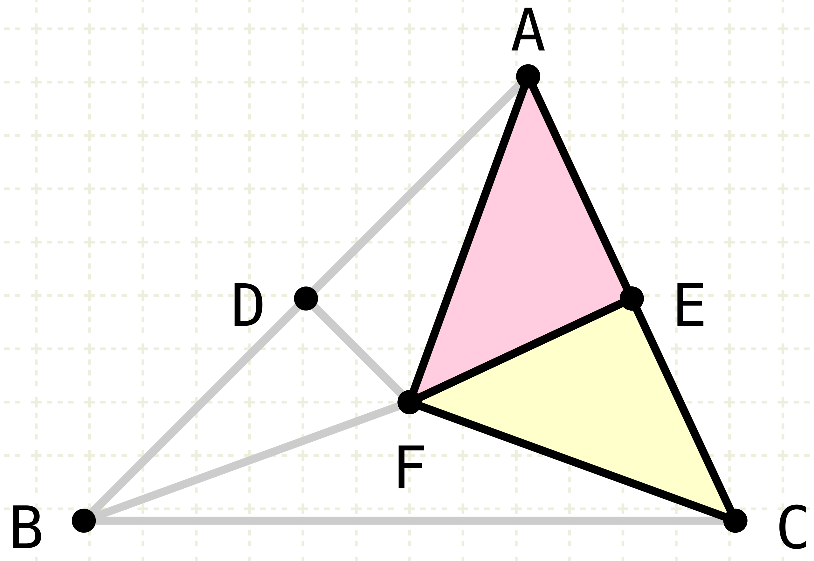

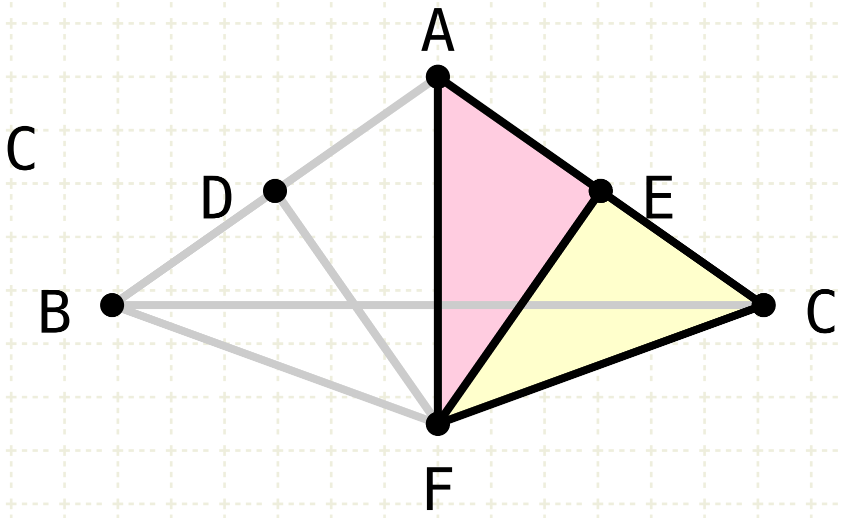

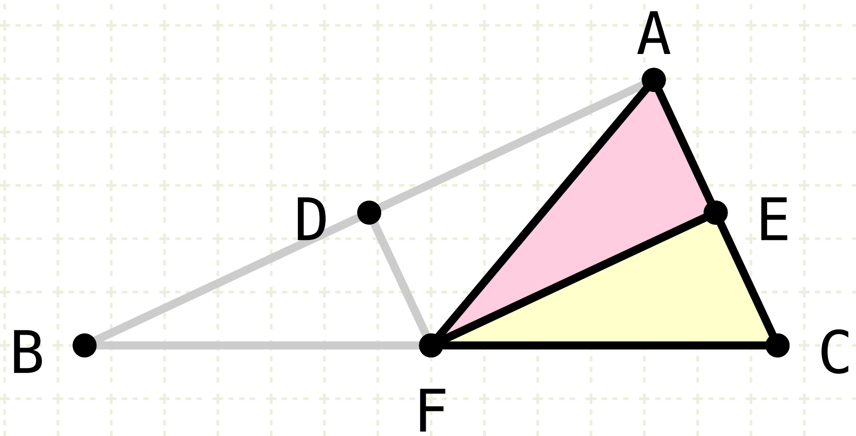

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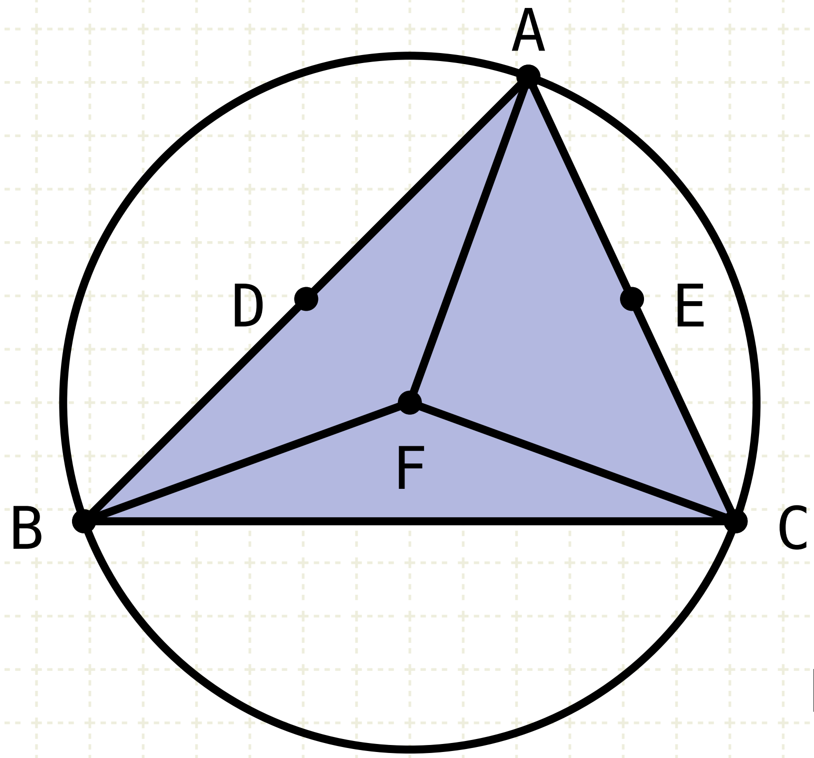

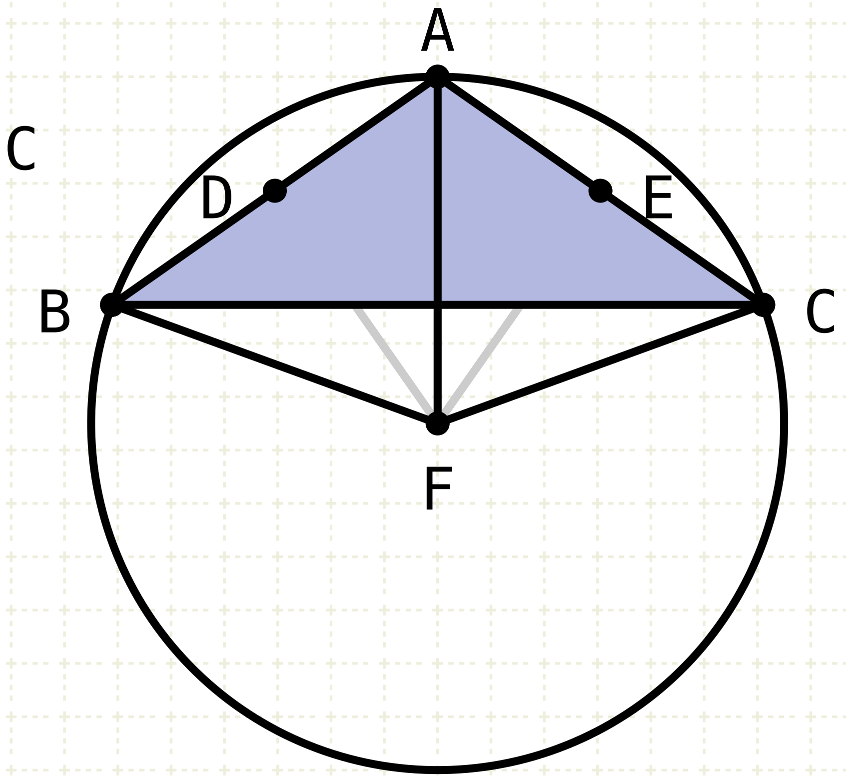

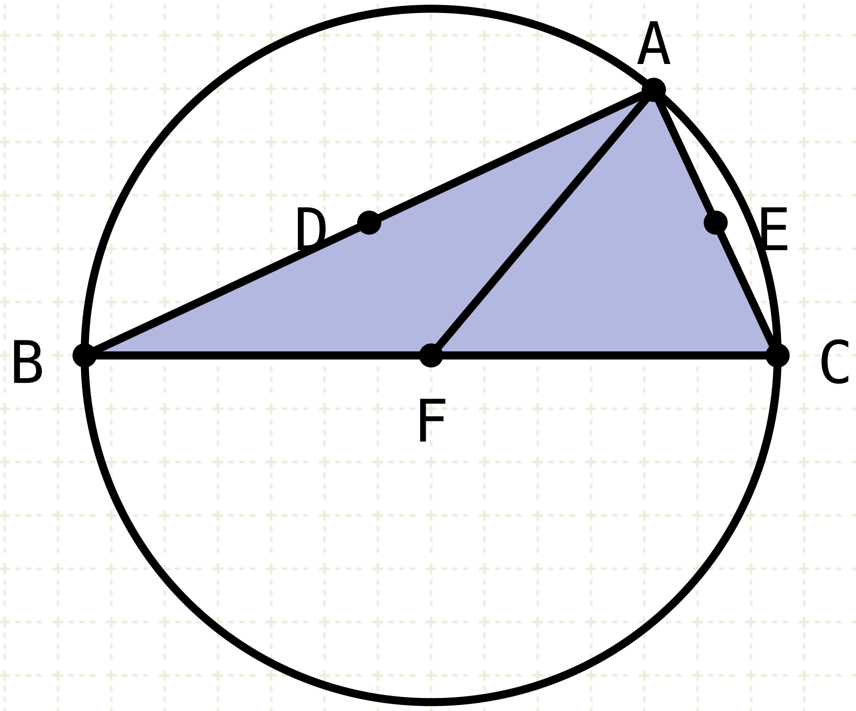

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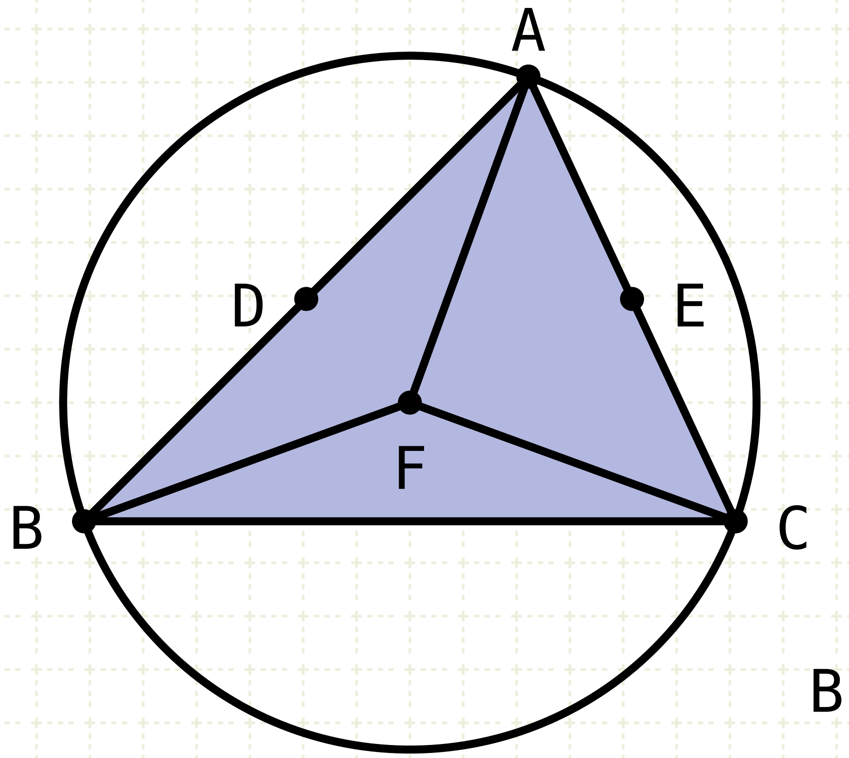

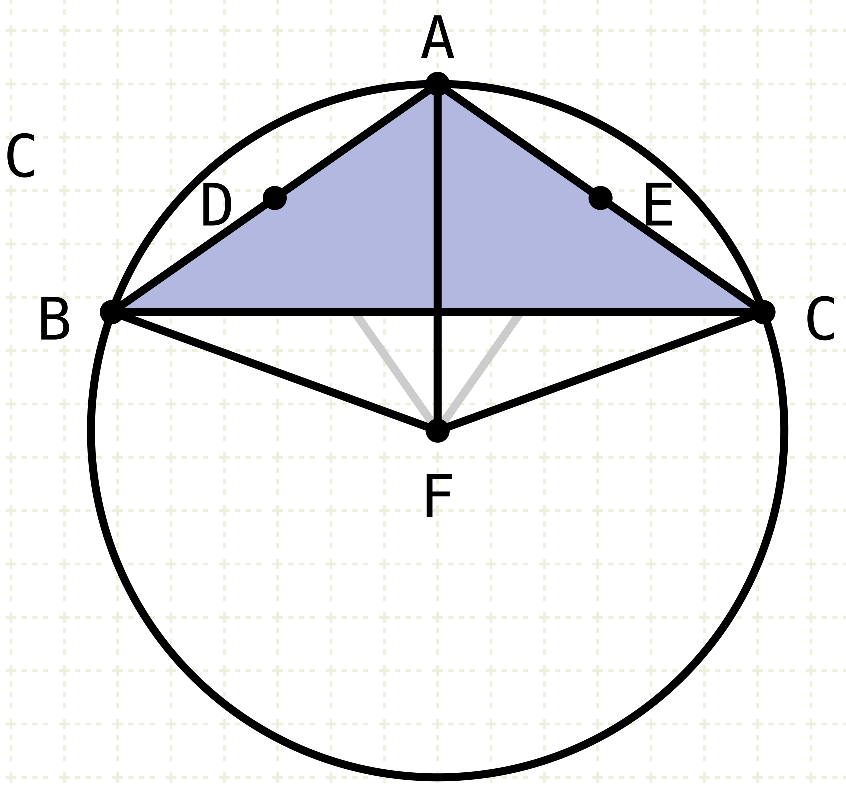

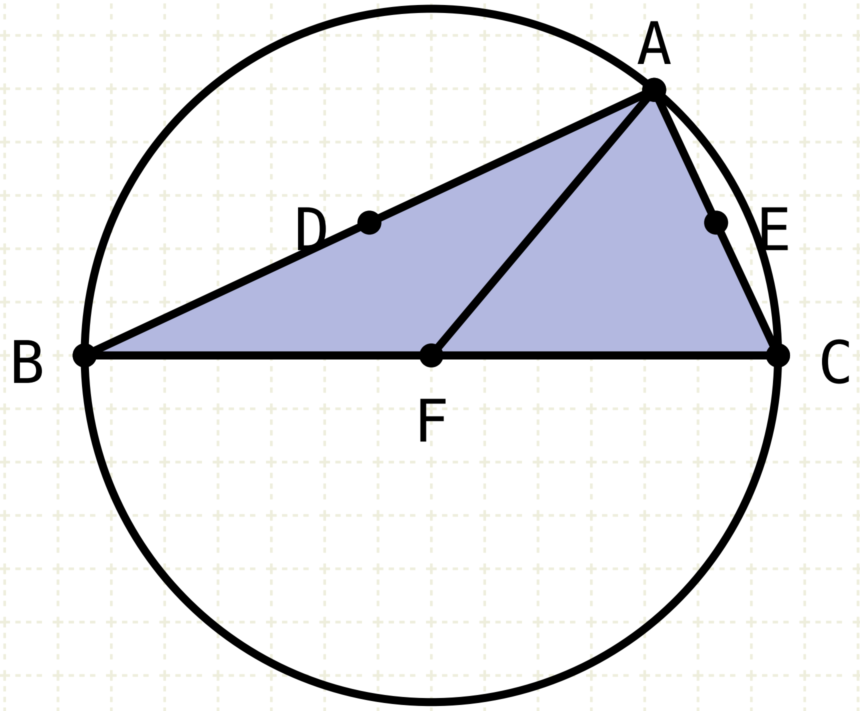

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Note: (III·31)

If the centre of the circle falls within the triangle, then the angle BAC is less than a right angle

If the centre of the circle falls on the line AC, then the angle BAC is less than a right angle

If the centre of the circle falls outside the triangle, then the angle BAC is greater than a right angle

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