

Euclid's Elements

Book VII

Definitions:

- 1 A unit is that by virtue of which each of the things that exist is called one
- 2 A number is a multitude composed of units. (not one)
- 3 A number is part of a number, the less of the greater, when it measures the greater
- 11 A prime number is that which is measured by a unit alone.
- 12 Numbers prime to one another are those which are measured by a unit alone as a common measure
- 15 A number is said to multiply a number when that which is multiplied is added to itself as many times as there are units in the other, and thus some number is produced.
- 20 Numbers are proportional when the first is the same multiple, or the same part, or the same parts, of the second that the third is of the fourth.

As long as algebra and geometry have been separated, their progress have been slow and their uses limited; but when these two sciences have been united, they have lent each mutual forces, and have marched together towards perfection.

Joseph-Louis Lagrange
(1736 to 1813)



Table of Contents, Chapter 7

1	Determine if two numbers are relatively prime	10	If $B = (p/q) \cdot A$ and $D = (p/q) \cdot C$, and If $B = (r/s) \cdot D$, then $A = (r/s) \cdot C$	21	If A,B are relatively prime, then A,B are the smallest whole numbers that can be used to describe the ratio A:B
2	Find the greatest common divisor for two numbers	11	If $A:B = C:D$, then $(A-C):(B-D) = A:B$	22	If A,B are the smallest whole numbers that can be used to describe the ratio A:B, then A,B are relatively prime
3	Find the largest common divisor for three numbers	12	If $A:B = C:D$, then $(A+C):(B+C) = A:B$	23	If A,B are relatively prime and if $A = n \cdot C$, then B,C are relatively prime
4	Given two natural numbers, A and B, either B is part of A, or there exists a natural number (a part) that can measure both A and B	13	If $A:B = C:D$, then $A:C = B:D$	24	If A,C are relatively prime and B,C are relatively prime then the $A \times B$ is relatively prime to C
5	If $B = (1/q) \cdot A$ and $D = (1/q) \cdot C$, then $(B+D) = (1/q) \cdot (A+C)$	14	If $A:B = D:E$ and $B:C = E:F$, then $A:C = D:F$	25	If A,B are relatively prime then A^2, B are relatively prime
6	If $B = (p/q) \cdot A$ and $D = (p/q) \cdot C$, then $(B+D) = (p/q) \cdot (A+C)$	15	If $B = i \cdot 1$ and $E = i \cdot D$, and if $D = j \cdot 1$ then $E = j \cdot B$	26	If A is relatively prime to C and D, and if B is also relatively prime to C and D, then $A \times B$ is relatively prime to $C \times D$
7	If $B = A/q$ and $D = C/q$, $B > D$, then $(B-D) = (A-C)/q$	16	$A \times B = B \times A$	27	If A,B are relatively prime, then A^2, B^2 are relatively prime, and A^3, B^3 are relatively prime, and so on
8	If $B = (p/q) \cdot A$ and $D = (p/q) \cdot C$, $B > D$, then $(B-D) = (p/q) \cdot (A-C)$	17	If $D = A \times B$ and $E = A \times C$ then $D:E = B:C$		
9	If $B = (1/q) \cdot A$ and $D = (1/q) \cdot C$, and If $B = (r/s) \cdot D$, then $A = (r/s) \cdot C$	18	If $D = B \times A$ and $E = C \times A$ then $D:E = B:C$		
		19	If $A:B = C:D$ then $A \times D = B \times C$ If $A \times D = B \times C$ then $A:B = C:D$		
		20	Given the ratio A:B and C,D are the smallest numbers such that $A:B = C:D$ then $A = n \cdot C$ and $B = n \cdot D$		



Table of Contents, Chapter 7

- | | | | |
|----|--|----|---|
| 28 | If A,B are relatively prime, then A,(A+B) are relatively prime | 37 | If $A = p \cdot B$, then $A = q \cdot C$ where $C = p \cdot 1$ |
| 29 | If A is prime, and $B \neq n \cdot A$, then A,B are relatively prime | 38 | If $A = (1/c) \cdot B$ and $C = c \cdot 1$ then $A = n \cdot C$ |
| 30 | If $C = A \times B$ and $C = i \cdot D$ where D is prime, then either $A = j \cdot D$ or $B = j \cdot D$ | 39 | Find the smallest number that has the fractions $1/a$, $1/b$, $1/c$ |
| 31 | If $A = B \times C$, then $A = j \cdot D$ where D is prime | | |
| 32 | If A is a number then it is either prime, or $A = j \cdot D$ where D is prime | | |
| 33 | Find the smallest numbers X,Y,Z where the ratio X:Y:Z is equal to the given ratio A:B:C | | |
| 34 | Find the lowest common denominator of 2 numbers | | |
| 35 | If E is the lowest common denominator of A,B, and if $C = n \cdot A = m \cdot B$, then $C = i \cdot E$ | | |
| 36 | Find the least common multiple of 3 numbers | | |



Proposition 5 of Book VII

If a number be a part of a number, and another be the same part of another, the sum will also be the same part of the sum that the one is of the one.



Proposition 5 of Book VII

If a number be a part of a number, and another be the same part of another, the sum will also be the same part of the sum that the one is of the one.

Definitions

3. A number is a 'part' of a number, the less of the greater, when it measures the greater

$$A = 10, B = 2,$$

B is part of A

$$A = B + B + B + B + B$$

4. but 'parts' when it does not measure it

$$A=10, B=6$$

Let the part of A be 2

$$p = 2, A = p + p + p + p + p$$

B is a multiple of the part of A (B is parts of A)

$$B = p + p + p$$

A part of one number is the same as the part of another number if it is the same fraction

$$A = 10, B = 4$$

$$p_A = (1/2)A = 5$$

$$p_B = (1/2)B = 2$$

$$p_A \text{ same as } p_B$$



Proposition 5 of Book VII

If a number be a part of a number, and another be the same part of another, the sum will also be the same part of the sum that the one is of the one.

$$b = (1/q)a$$

$$d = (1/q)c$$

$$\rightarrow (b+d) = (1/q)(a+c)$$

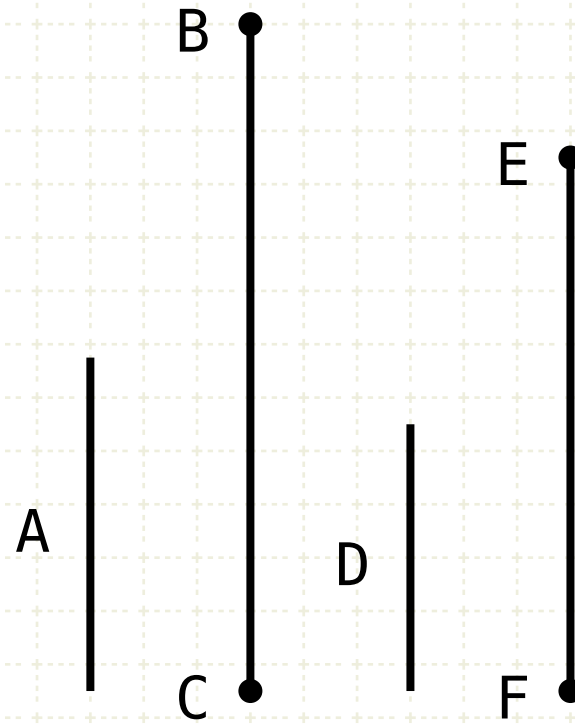
In other words

If b is the same fraction of a as d is to c, then the sum b,d will also be the same fraction of the sum a,c



Proposition 5 of Book VII

If a number be a part of a number, and another be the same part of another, the sum will also be the same part of the sum that the one is of the one.



$$BC = q \cdot A, \quad A = BC/q$$

$$EF = q \cdot D, \quad D = EF/q$$

In other words

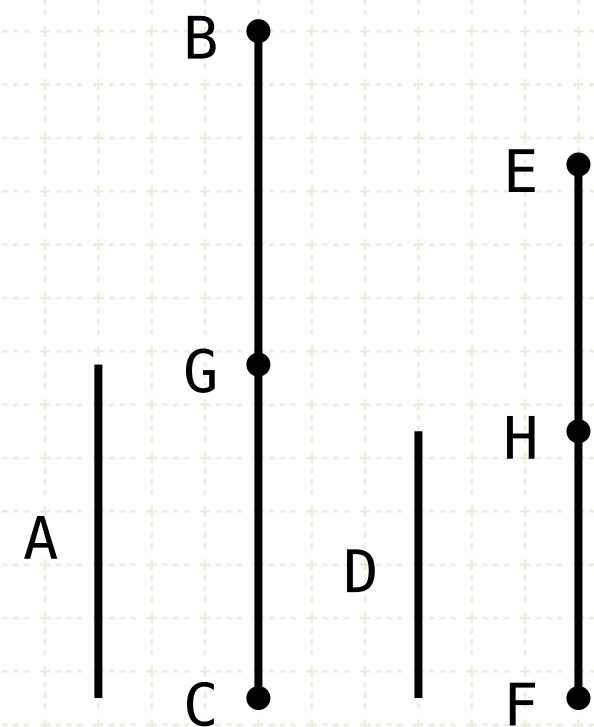
If b is the same fraction of a as d is to c , then the sum b, d will also be the same fraction of the sum a, c

Proof

Let the number A be part (fraction) of BC , and D be the same part (fraction) of EF

Proposition 5 of Book VII

If a number be a part of a number, and another be the same part of another, the sum will also be the same part of the sum that the one is of the one.



$$BC = q \cdot A, \quad A = BC/q$$

$$EF = q \cdot D, \quad D = EF/q$$

$$A = BG = GC$$

$$D = EH = HF$$

In other words

If b is the same fraction of a as d is to c, then the sum b,d will also be the same fraction of the sum a,c

Proof

Let the number A be part (fraction) of BC, and D be the same part (fraction) of EF

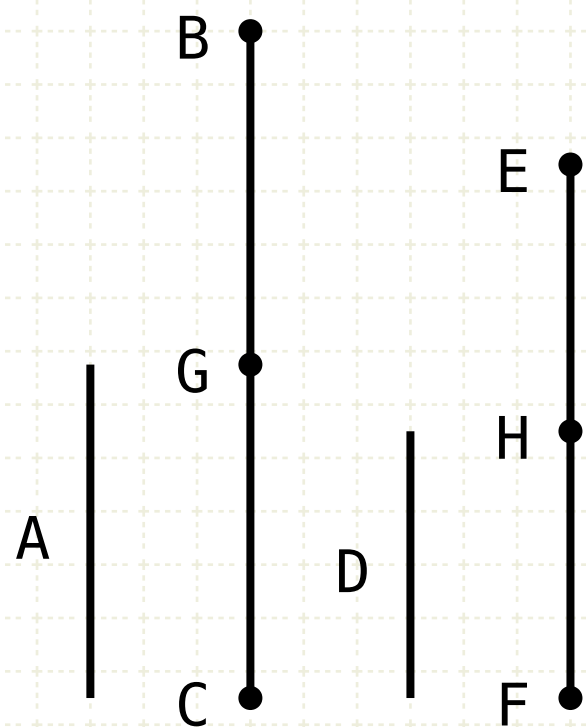
Let BC be divided into the numbers equal to A, namely BG, GC

Let EF be divided into the numbers equal to D, namely EH, HF



Proposition 5 of Book VII

If a number be a part of a number, and another be the same part of another, the sum will also be the same part of the sum that the one is of the one.



$$BC = q \cdot A, \quad A = BC/q$$

$$EF = q \cdot D, \quad D = EF/q$$

$$A = BG = GC$$

$$D = EH = HF$$

$$A + D = BG + EH$$

In other words

If b is the same fraction of a as d is to c, then the sum b,d will also be the same fraction of the sum a,c

Proof

Let the number A be part (fraction) of BC, and D be the same part (fraction) of EF

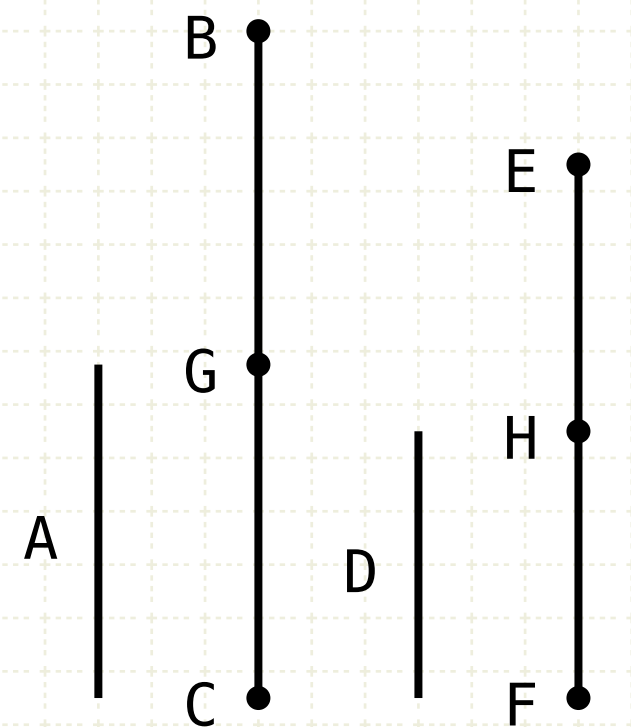
Let BC be divided into the numbers equal to A, namely BG, GC

Let EF be divided into the numbers equal to D, namely EH, HF

The sum of BG,EH equals the sum of A,D, since A equals BG, and D equals EH

Proposition 5 of Book VII

If a number be a part of a number, and another be the same part of another, the sum will also be the same part of the sum that the one is of the one.



$$BC = q \cdot A, \quad A = BC/q$$
$$EF = q \cdot D, \quad D = EF/q$$

$$A = BG = GC$$
$$D = EH = HF$$

$$A + D = BG + EH$$
$$A + D = GC + HF$$

In other words

If b is the same fraction of a as d is to c, then the sum b,d will also be the same fraction of the sum a,c

Proof

Let the number A be part (fraction) of BC, and D be the same part (fraction) of EF

Let BC be divided into the numbers equal to A, namely BG, GC

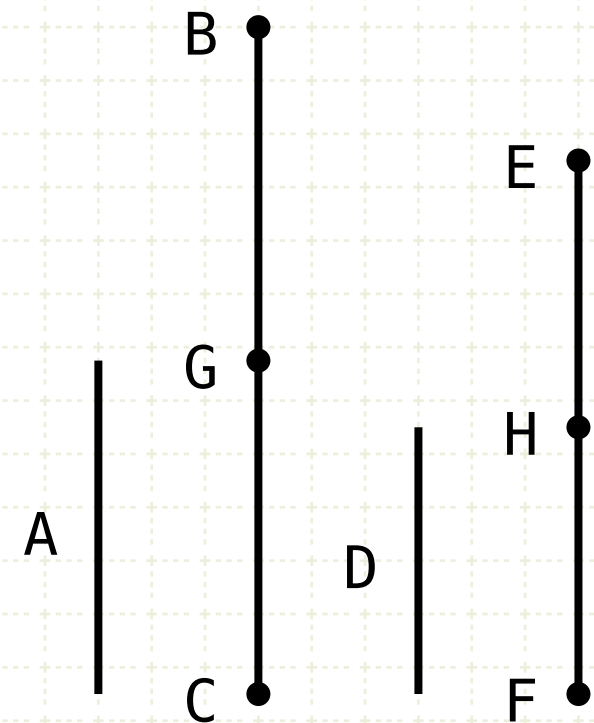
Let EF be divided into the numbers equal to D, namely EH, HF

The sum of BG,EH equals the sum of A,D, since A equals BG, and D equals EH

Likewise, the sum of GC,HF equals the sum of A,D, since A equals GC, and D equals HF

Proposition 5 of Book VII

If a number be a part of a number, and another be the same part of another, the sum will also be the same part of the sum that the one is of the one.



$$BC = q \cdot A, \quad A = BC/q$$
$$EF = q \cdot D, \quad D = EF/q$$

$$A = BG = GC$$
$$D = EH = HF$$

$$A + D = BG + EH$$
$$A + D = GC + HF$$
$$BC = A + A + \dots + A$$
$$EF = D + D + \dots + D$$
$$BC+EF = A+D + A+D + \dots + A+D$$

In other words

If b is the same fraction of a as d is to c, then the sum b,d will also be the same fraction of the sum a,c

Proof

Let the number A be part (fraction) of BC, and D be the same part (fraction) of EF

Let BC be divided into the numbers equal to A, namely BG, GC

Let EF be divided into the numbers equal to D, namely EH, HF

The sum of BG,EH equals the sum of A,D, since A equals BG, and D equals EH

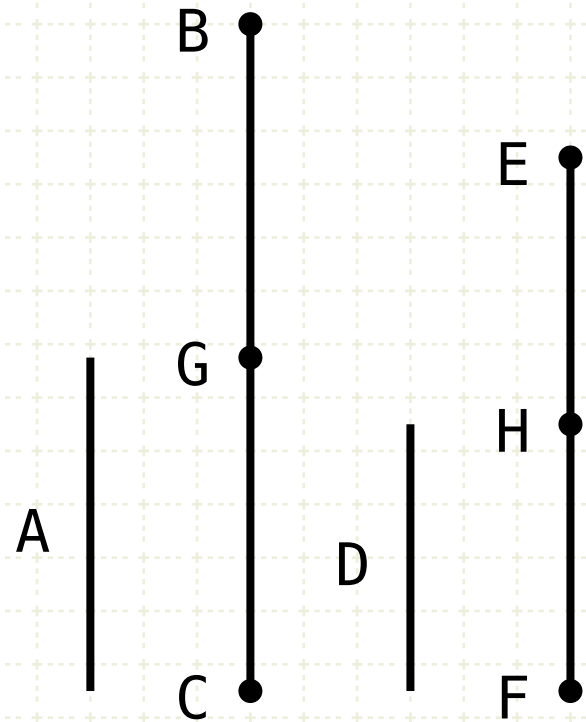
Likewise, the sum of GC,HF equals the sum of A,D, since A equals GC, and D equals HF

Given that BC and EF have the same number of parts, the previous process can be repeated for every part in BC and EF, repeatedly adding A,D



Proposition 5 of Book VII

If a number be a part of a number, and another be the same part of another, the sum will also be the same part of the sum that the one is of the one.



$$BC = q \cdot A, \quad A = BC/q$$

$$EF = q \cdot D, \quad D = EF/q$$

$$A = BG = GC$$

$$D = EH = HF$$

$$A + D = BG + EH$$

$$A + D = GC + HF$$

$$BC = A + A + \dots + A$$

$$EF = D + D + \dots + D$$

$$BC + EF = A + D + A + D + \dots + A + D$$

$$BC + EF = q \cdot (A + D),$$

In other words

If b is the same fraction of a as d is to c, then the sum b,d will also be the same fraction of the sum a,c

Proof

Let the number A be part (fraction) of BC, and D be the same part (fraction) of EF

Let BC be divided into the numbers equal to A, namely BG, GC

Let EF be divided into the numbers equal to D, namely EH, HF

The sum of BG,EH equals the sum of A,D, since A equals BG, and D equals EH

Likewise, the sum of GC,HF equals the sum of A,D, since A equals GC, and D equals HF

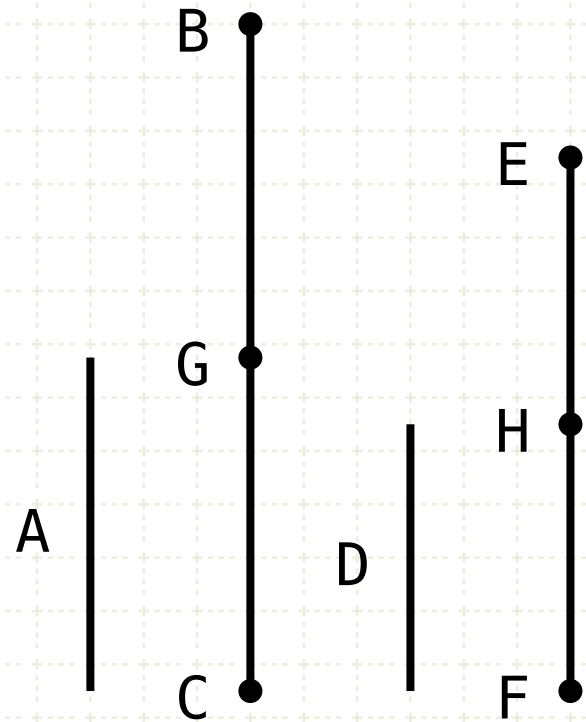
Given that BC and EF have the same number of parts, the previous process can be repeated for every part in BC and EF, repeatedly adding A,D

Thus the sum of BC and EF will be the the sum of A and D, repeated as many times as there are parts D in EF



Proposition 5 of Book VII

If a number be a part of a number, and another be the same part of another, the sum will also be the same part of the sum that the one is of the one.



$$BC = q \cdot A, A = BC/q$$

$$EF = q \cdot D, D = EF/q$$

$$A = BG = GC$$

$$D = EH = HF$$

$$A + D = BG + EH$$

$$A + D = GC + HF$$

$$BC = A + A + \dots + A$$

$$EF = D + D + \dots + D$$

$$BC + EF = A + D + A + D + \dots + A + D$$

$$BC + EF = q \cdot (A + D),$$

$$A + D = (BC + EF)/q$$

In other words

If b is the same fraction of a as d is to c, then the sum b,d will also be the same fraction of the sum a,c

Proof

Let the number A be part (fraction) of BC, and D be the same part (fraction) of EF

Let BC be divided into the numbers equal to A, namely BG, GC

Let EF be divided into the numbers equal to D, namely EH, HF

The sum of BG,EH equals the sum of A,D, since A equals BG, and D equals EH

Likewise, the sum of GC,HF equals the sum of A,D, since A equals GC, and D equals HF

Given that BC and EF have the same number of parts, the previous process can be repeated for every part in BC and EF, repeatedly adding A,D

Thus the sum of BC and EF will be the the sum of A and D, repeated as many times as there are parts D in EF

Thus the sum of A,D will be the same part (fraction) of BC,EF as A is to BC D is to EF



Youtube Videos

<https://www.youtube.com/c/SandyBultena>

Copyright © 2019 by Sandy Bultena.



Except where otherwise noted, this work is licensed under
<http://creativecommons.org/licenses/by-nc/3.0>