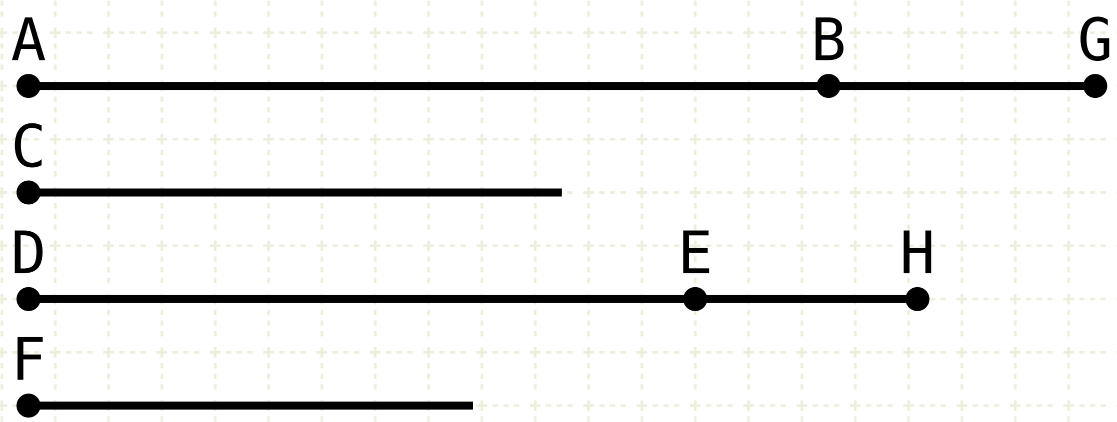
Euclid's Elements

Book V



$AB:C = DE:F$
$BG:C = EH:F$

$AG:C = DH:F$

Proportions are what makes the old Greek temples classic in their beauty. They are like huge blocks, from which the air has been literally hewn out between the columns.

Arne Jacobsen

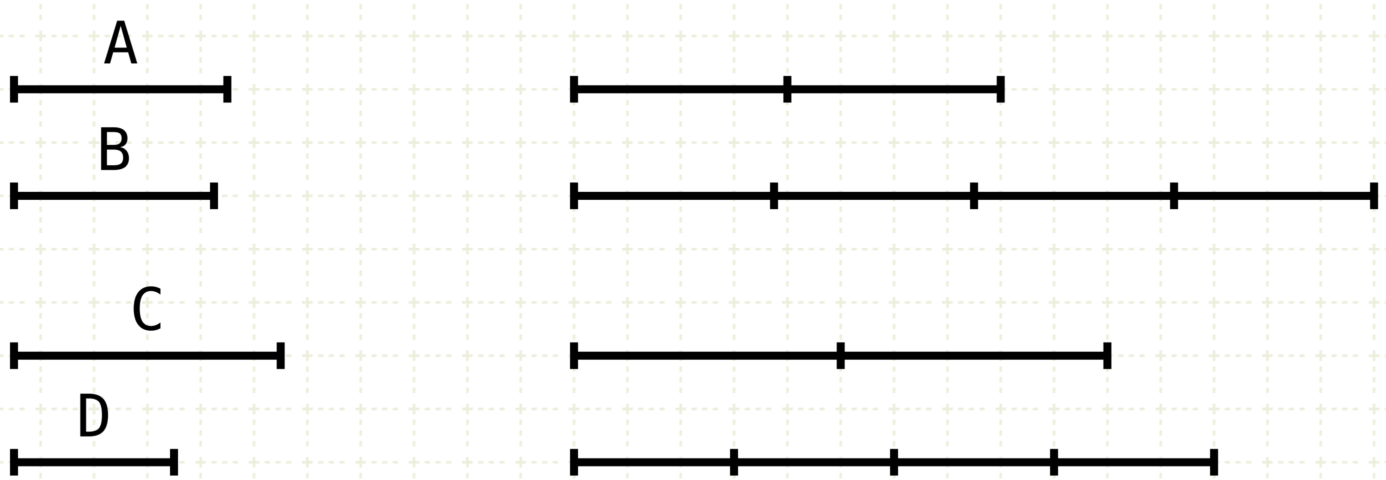
Table of Contents, Chapter 5

Proposition 13 of Book V

If a first magnitude have to a second the same ratio as a third to a fourth, and the third have to the fourth a greater ratio than a fifth has to a sixth, the first will also have to the second a greater ratio than the fifth to the sixth

Proposition 13 of Book V

If a first magnitude have to a second the same ratio as a third to a fourth, and the third have to the fourth a greater ratio than a fifth has to a sixth, the first will also have to the second a greater ratio than the fifth to the sixth




```
If   n·A >  m·B
and  n·C <= m·D
then A:B > C:D
```

Example:

```
compare 80:75 to 100:60
2×80 > 4×75
2×100 < 4×60
∴ 80:75 > 100:60
```

Definitions

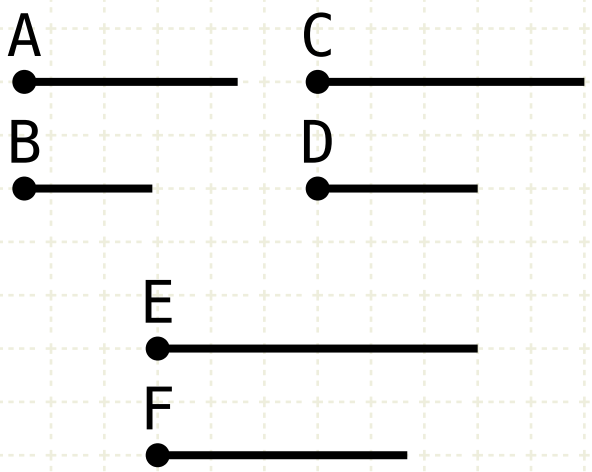
7. When, of the equimultiples, the multiple of the first magnitude exceeds the multiple of the second, but the multiple of the third does not exceed the multiple of the fourth, then the first is said to have a greater ratio to the second than the third has to the fourth.

Proposition 13 of Book V

If a first magnitude have to a second the same ratio as a third to a fourth, and the third have to the fourth a greater ratio than a fifth has to a sixth, the first will also have to the second a greater ratio than the fifth to the sixth

Proposition 13 of Book V

If a first magnitude have to a second the same ratio as a third to a fourth, and the third have to the fourth a greater ratio than a fifth has to a sixth, the first will also have to the second a greater ratio than the fifth to the sixth



$A:B = C:D$

$C:D > E:F$

$\rightarrow A:B > E:F$

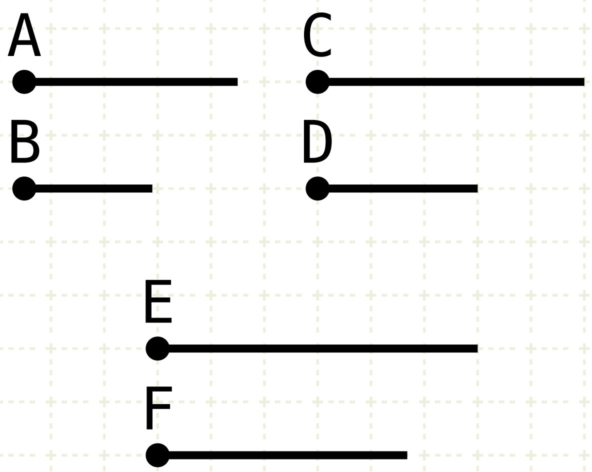
In other words

If A is to B as C is to D, and C is to D greater than E is to F

… then A is to B is also greater than E is to F

Proposition 13 of Book V

If a first magnitude have to a second the same ratio as a third to a fourth, and the third have to the fourth a greater ratio than a fifth has to a sixth, the first will also have to the second a greater ratio than the fifth to the sixth



$A:B = C:D$

$C:D > E:F$

In other words

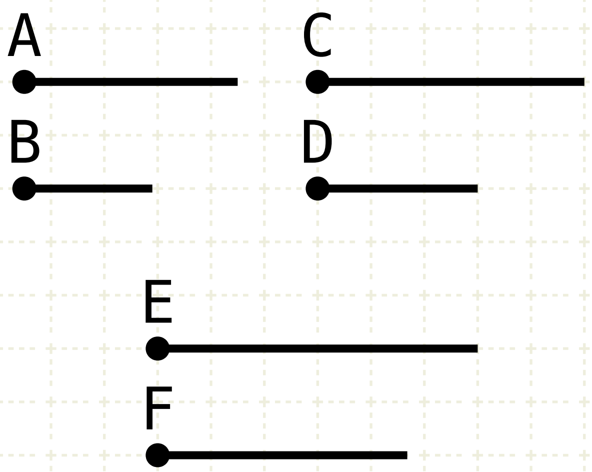
If A is to B as C is to D, and C is to D greater than E is to F

… then A is to B is also greater than E is to F

Proof

Proposition 13 of Book V

If a first magnitude have to a second the same ratio as a third to a fourth, and the third have to the fourth a greater ratio than a fifth has to a sixth, the first will also have to the second a greater ratio than the fifth to the sixth



$$A:B = C:D$$
$$C:D > E:F$$
$$m \cdot C > n \cdot D$$
$$m \cdot E < n \cdot F$$

In other words

If A is to B as C is to D, and C is to D greater than E is to F

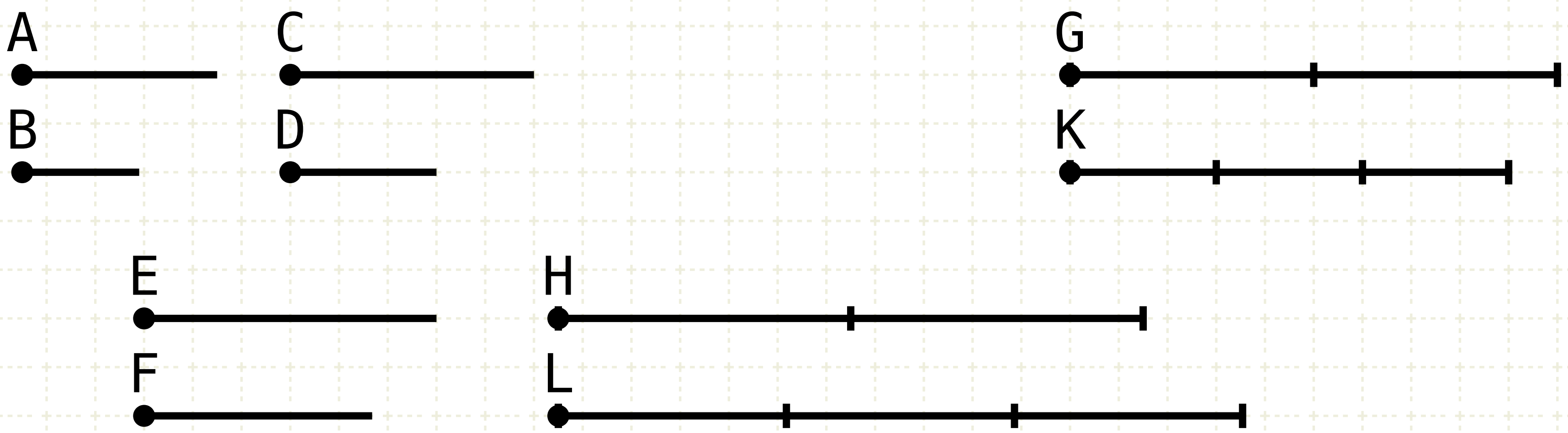
… then A is to B is also greater than E is to F

Proof

Since the ratio of C to D is greater than E to F, there exists equimultiples of C,E and D,F such that the multiple of C is in excess of the multiple of E, whereas the multiple of D is less than the multiple of F (V·def·7)

Proposition 13 of Book V

If a first magnitude have to a second the same ratio as a third to a fourth, and the third have to the fourth a greater ratio than a fifth has to a sixth, the first will also have to the second a greater ratio than the fifth to the sixth



$A:B = C:D$

$C:D > E:F$

$m \cdot C > n \cdot D$

$m \cdot E < n \cdot F$

$G = m \cdot C$

$H = m \cdot E$

$K = n \cdot D$

$L = n \cdot F$

$G > K$

$H < L$

In other words

If A is to B as C is to D, and C is to D greater than E is to F

… then A is to B is also greater than E is to F

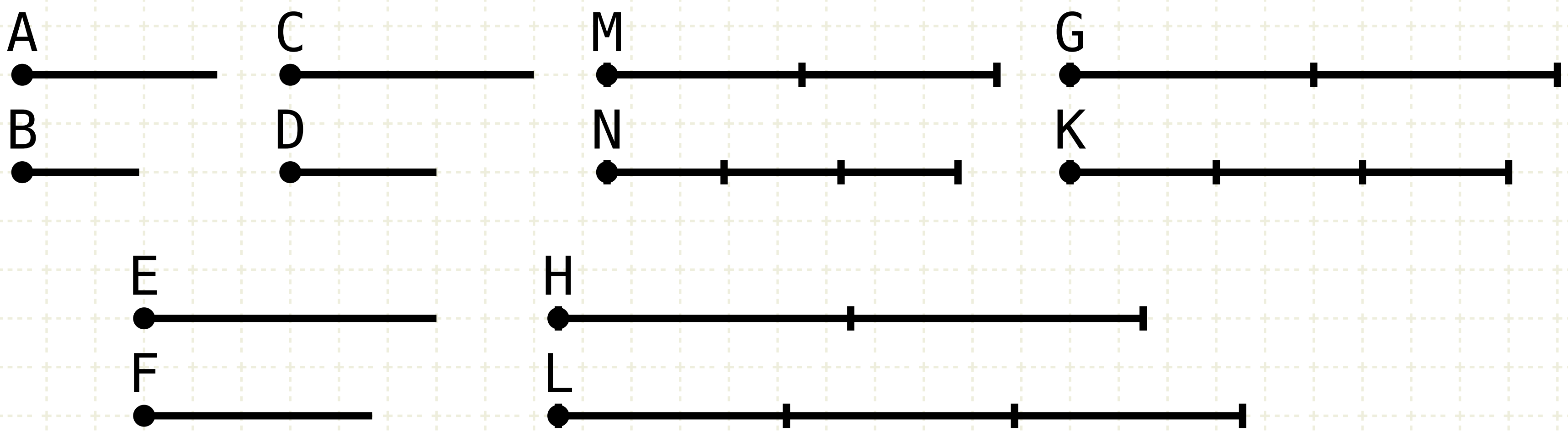
Proof

Since the ratio of C to D is greater than E to F, there exists equimultiples of C,E and D,F such that the multiple of C is in excess of the multiple of E, whereas the multiple of D is less than the multiple of F (V·def·7)

Create equimultiples G,H of C,E and equimultiples K,L of D,F such that G is greater than K, but H is less than L

Proposition 13 of Book V

If a first magnitude have to a second the same ratio as a third to a fourth, and the third have to the fourth a greater ratio than a fifth has to a sixth, the first will also have to the second a greater ratio than the fifth to the sixth



$$A:B = C:D$$
$$C:D > E:F$$
$$m\cdot C > n\cdot D$$
$$m\cdot E < n\cdot F$$
$$G = m\cdot C$$
$$H = m\cdot E$$
$$K = n\cdot D$$
$$L = n\cdot F$$
$$M = m\cdot A$$
$$N = n\cdot B$$

$$G > K$$
$$H < L$$

In other words

If A is to B as C is to D, and C is to D greater than E is to F

… then A is to B is also greater than E is to F

Proof

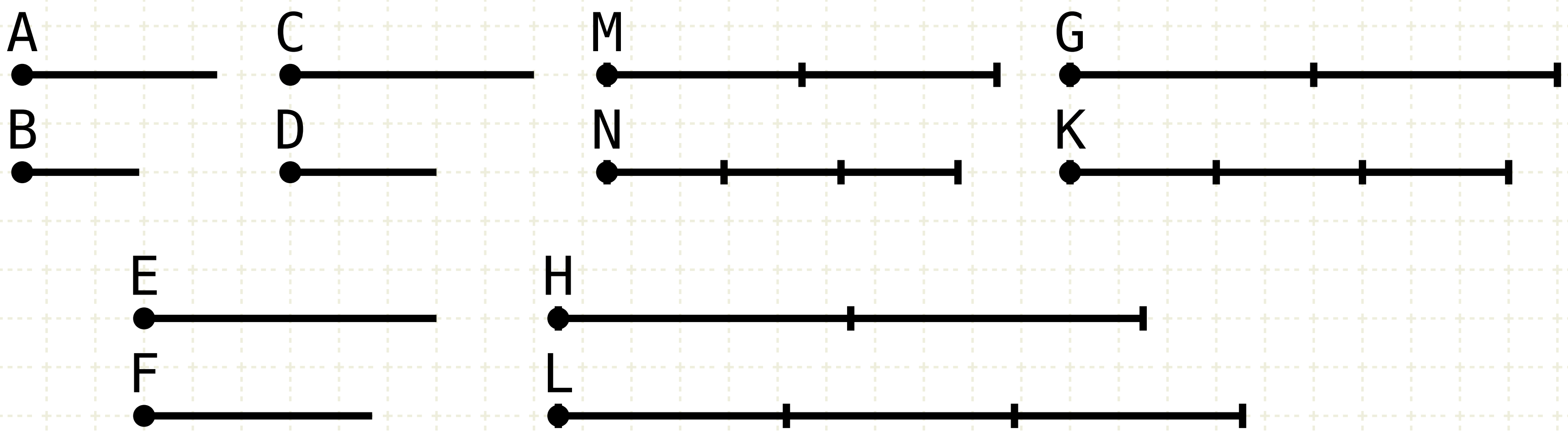
Since the ratio of C to D is greater than E to F, there exists equimultiples of C,E and D,F such that the multiple of C is in excess of the multiple of E, whereas the multiple of D is less than the multiple of F (V·def·7)

Create equimultiples G,H of C,E and equimultiples K,L of D,F such that G is greater than K, but H is less than L

Let M be the same multiple of A as G is to C, and let N be the same multiple of B as K is to D

Proposition 13 of Book V

If a first magnitude have to a second the same ratio as a third to a fourth, and the third have to the fourth a greater ratio than a fifth has to a sixth, the first will also have to the second a greater ratio than the fifth to the sixth



$A:B = C:D$

$C:D > E:F$

$m \cdot C > n \cdot D$

$m \cdot E < n \cdot F$

$G = m \cdot C$

$H = m \cdot E$

$K = n \cdot D$

$L = n \cdot F$

$M = m \cdot A$

$N = n \cdot B$

$G > K$

$H < L$

$m \cdot A >=< n \cdot B \rightarrow m \cdot C >=< n \cdot D$

$M >=< N \rightarrow G >=< K$

In other words

If A is to B as C is to D, and C is to D greater than E is to F

… then A is to B is also greater than E is to F

Proof

Since the ratio of C to D is greater than E to F, there exists equimultiples of C,E and D,F such that the multiple of C is in excess of the multiple of E, whereas the multiple of D is less than the multiple of F (V·def·7)

Create equimultiples G,H of C,E and equimultiples K,L of D,F such that G is greater than K, but H is less than L

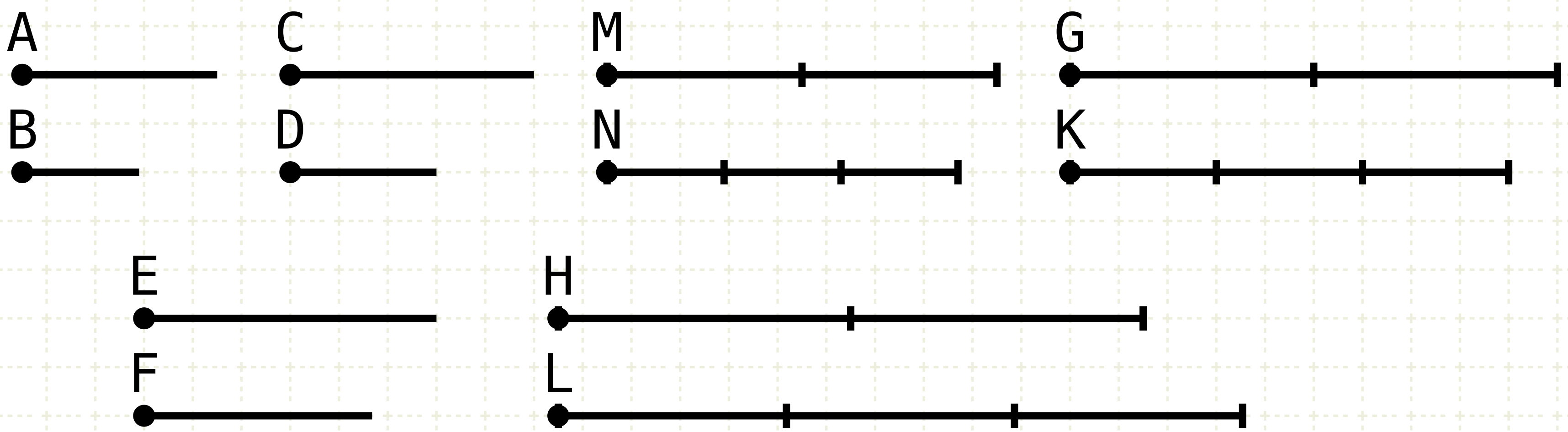
Let M be the same multiple of A as G is to C, and let N be the same multiple of B as K is to D

Since the ratios A to B, C to D are equal and since equimultiples G,H,K and L,M,N have been taken…

… so if M is greater than N, then G is greater than K etc (V.def 5)

Proposition 13 of Book V

If a first magnitude have to a second the same ratio as a third to a fourth, and the third have to the fourth a greater ratio than a fifth has to a sixth, the first will also have to the second a greater ratio than the fifth to the sixth



$A:B = C:D$
$C:D > E:F$
$m\cdot C > n\cdot D$
$m\cdot E < n\cdot F$
$G = m\cdot C$
$H = m\cdot E$
$K = n\cdot D$
$L = n\cdot F$
$M = m\cdot A$
$N = n\cdot B$

$G > K$
$H < L$
$m\cdot A >=< n\cdot B \rightarrow m\cdot C >=< n\cdot D$
$M >=< N \rightarrow G >=< K$
$M > N$

In other words

If A is to B as C is to D, and C is to D greater than E is to F

… then A is to B is also greater than E is to F

Proof

Since the ratio of C to D is greater than E to F, there exists equimultiples of C,E and D,F such that the multiple of C is in excess of the multiple of E, whereas the multiple of D is less than the multiple of F (V·def·7)

Create equimultiples G,H of C,E and equimultiples K,L of D,F such that G is greater than K, but H is less than L

Let M be the same multiple of A as G is to C, and let N be the same multiple of B as K is to D

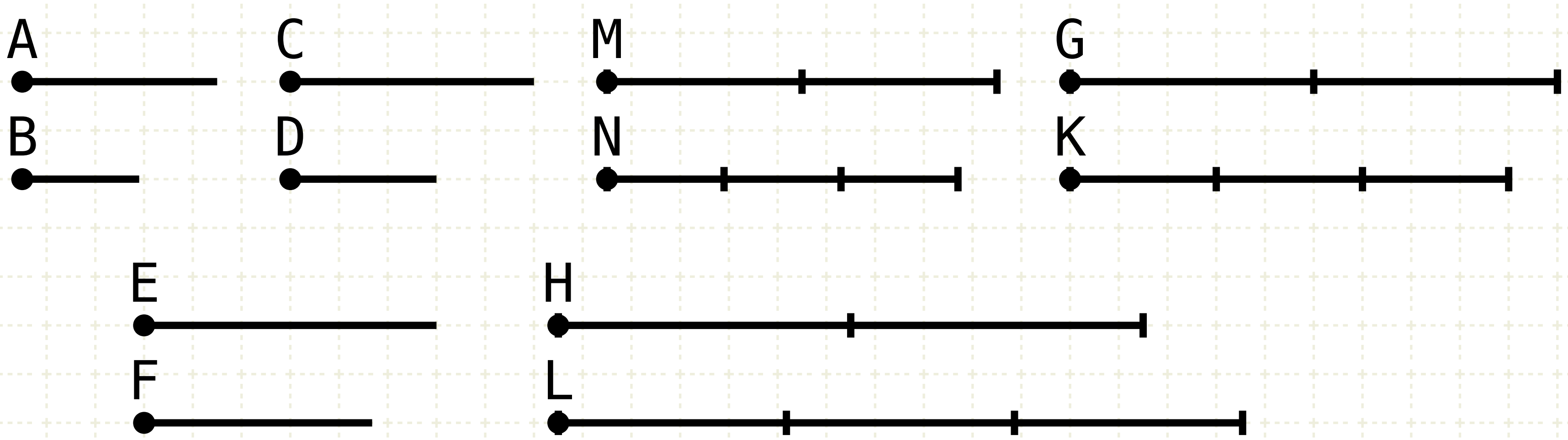
Since the ratios A to B, C to D are equal and since equimultiples G,H,K and L,M,N have been taken…

… so if M is greater than N, then G is greater than K etc (V.def 5)

But G is greater than K, then so is M greater than N

Proposition 13 of Book V

If a first magnitude have to a second the same ratio as a third to a fourth, and the third have to the fourth a greater ratio than a fifth has to a sixth, the first will also have to the second a greater ratio than the fifth to the sixth



$A:B = C:D$

$C:D > E:F$

$m \cdot C > n \cdot D$

$m \cdot E < n \cdot F$

$G = m \cdot C$

$H = m \cdot E$

$K = n \cdot D$

$L = n \cdot F$

$M = m \cdot A$

$N = n \cdot B$

$G > K$

$H < L$

$m \cdot A >=< n \cdot B \rightarrow m \cdot C >=< n \cdot D$

$M >=< N \rightarrow G >=< K$

$M > N$

$m \cdot A > n \cdot B$

$m \cdot E < n \cdot F$

In other words

If A is to B as C is to D, and C is to D greater than E is to F

… then A is to B is also greater than E is to F

Proof

Since the ratio of C to D is greater than E to F, there exists equimultiples of C,E and D,F such that the multiple of C is in excess of the multiple of E, whereas the multiple of D is less than the multiple of F (V·def·7)

Create equimultiples G,H of C,E and equimultiples K,L of D,F such that G is greater than K, but H is less than L

Let M be the same multiple of A as G is to C, and let N be the same multiple of B as K is to D

Since the ratios A to B, C to D are equal and since equimultiples G,H,K and L,M,N have been taken…

… so if M is greater than N, then G is greater than K etc (V.def 5)

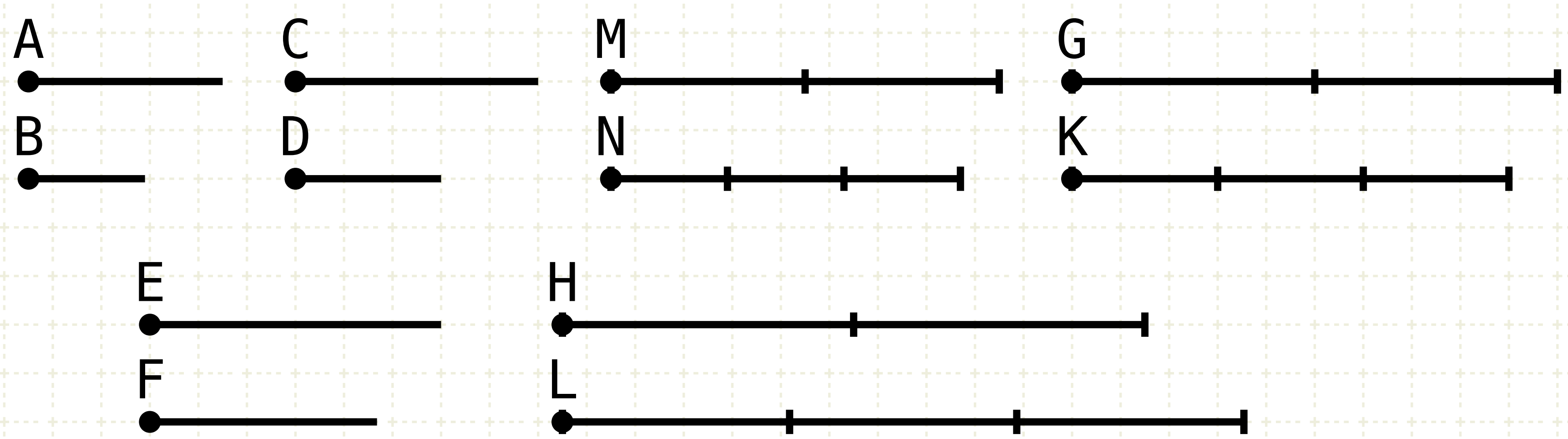
But G is greater than K, then so is M greater than N

M,H are equimultiples of A,E, and N,L are equimultiples of B,F

Since M is greater than N, and H is less than L…

Proposition 13 of Book V

If a first magnitude have to a second the same ratio as a third to a fourth, and the third have to the fourth a greater ratio than a fifth has to a sixth, the first will also have to the second a greater ratio than the fifth to the sixth



$A:B = C:D$

$C:D > E:F$

$m \cdot C > n \cdot D$

$m \cdot E < n \cdot F$

$G = m \cdot C$

$H = m \cdot E$

$K = n \cdot D$

$L = n \cdot F$

$M = m \cdot A$

$N = n \cdot B$

$G > K$

$H < L$

$m \cdot A \gtreqless n \cdot B \rightarrow m \cdot C \gtreqless n \cdot D$

$M \gtreqless N \rightarrow G \gtreqless K$

$M > N$

$m \cdot A > n \cdot B$

$m \cdot E < n \cdot F$

$A:B > E:F$

In other words

If A is to B as C is to D, and C is to D greater than E is to F

… then A is to B is also greater than E is to F

Proof

Since the ratio of C to D is greater than E to F, there exists equimultiples of C,E and D,F such that the multiple of C is in excess of the multiple of E, whereas the multiple of D is less than the multiple of F (V·def·7)

Create equimultiples G,H of C,E and equimultiples K,L of D,F such that G is greater than K, but H is less than L

Let M be the same multiple of A as G is to C, and let N be the same multiple of B as K is to D

Since the ratios A to B, C to D are equal and since equimultiples G,H,K and L,M,N have been taken…

… so if M is greater than N, then G is greater than K etc (V.def 5)

But G is greater than K, then so is M greater than N

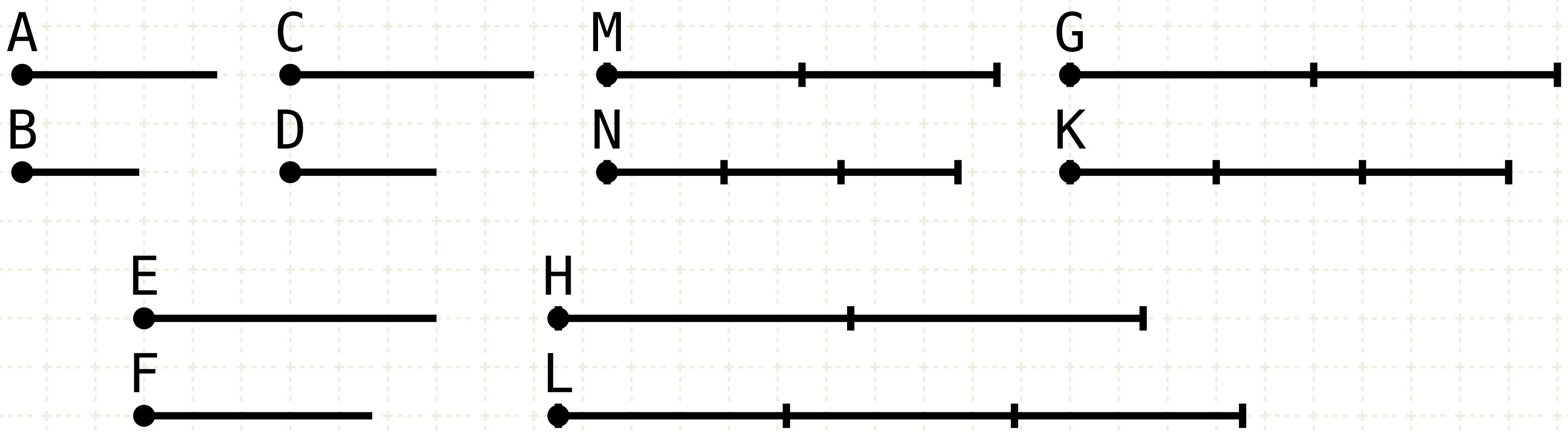
M,H are equimultiples of A,E, and N,L are equimultiples of B,F

Since M is greater than N, and H is less than L…

… the ratio A to B is greater than the ratio E to F (V·def·7)

Proposition 13 of Book V

If a first magnitude have to a second the same ratio as a third to a fourth, and the third have to the fourth a greater ratio than a fifth has to a sixth, the first will also have to the second a greater ratio than the fifth to the sixth



A:B = C:D
C:D > E:F
m·C > n·D
m·E < n·F
G = m·C
H = m·E
K = n·D
L = n·F
M = m·A
N = n·B

G > K
H < L
m·A >=< n·B → m·C >=< n·D
M >=< N → G >=< K
M > N

m·A > n·B
m·E < n·F

A:B > E:F

In other words

If A is to B as C is to D, and C is to D greater than E is to F

… then A is to B is also greater than E is to F

Proof

Since the ratio of C to D is greater than E to F, there exists equimultiples of C,E and D,F such that the multiple of C is in excess of the multiple of E, whereas the multiple of D is less than the multiple of F (V·def·7)

Create equimultiples G,H of C,E and equimultiples K,L of D,F such that G is greater than K, but H is less than L

Let M be the same multiple of A as G is to C, and let N be the same multiple of B as K is to D

Since the ratios A to B, C to D are equal and since equimultiples G,H,K and L,M,N have been taken…

… so if M is greater than N, then G is greater than K etc (V.def 5)

But G is greater than K, then so is M greater than N

M,H are equimultiples of A,E, and N,L are equimultiples of B,F

Since M is greater than N, and H is less than L…

… the ratio A to B is greater than the ratio E to F (V·def·7)

Youtube Videos

https://www.youtube.com/c/SandyBultena