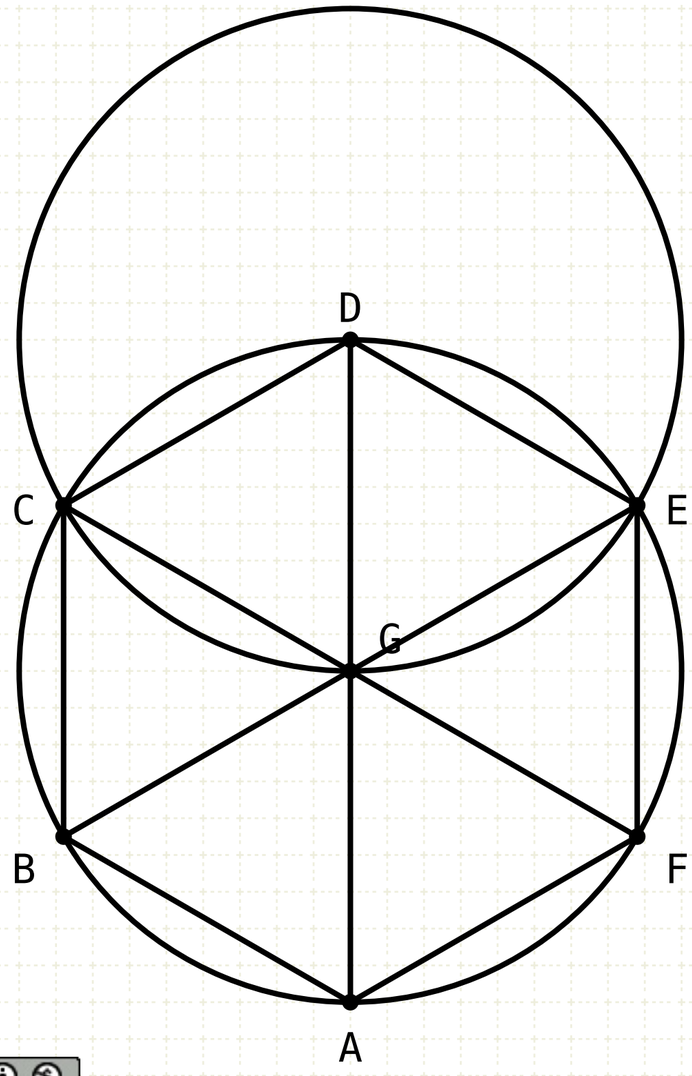
Euclid's Elements

Book IV



Philosophy (nature) is written in that great book which ever is before our eyes -- I mean the universe -- but we cannot understand it if we do not first learn the language and grasp the symbols in which it is written. The book is written in mathematical language, and the symbols are triangles, circles and other geometrical figures, without whose help it is impossible to comprehend a single word of it - without which one wanders in vain through a dark labyrinth.

Galileo Galilei

Proposition 4 of Book IV

In a given triangle, to inscribe a circle.

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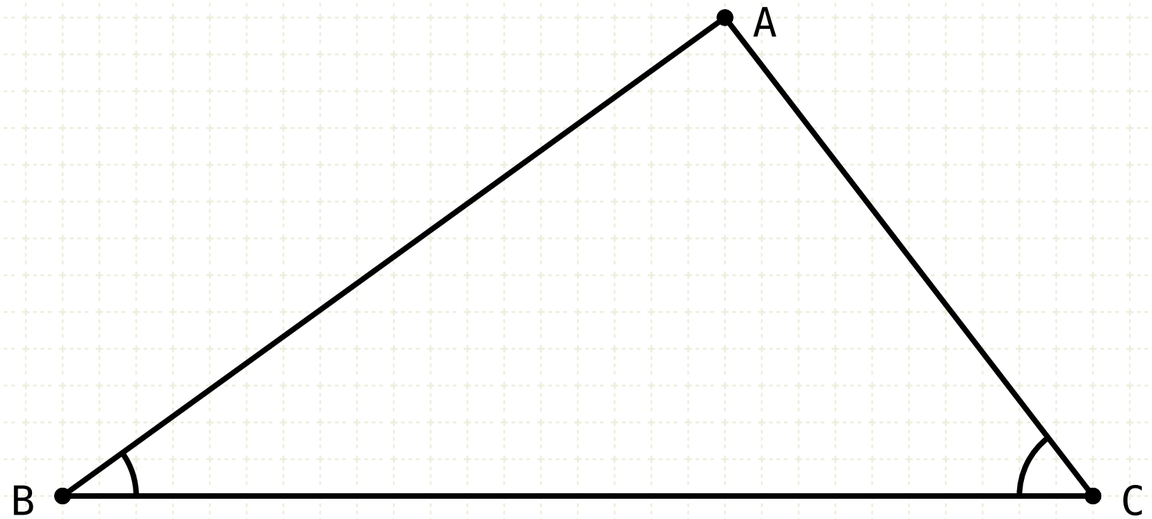

In other words

Given a triangle ABC

Draw a circle so that it touches all three sides of the triangle

Proposition 4 of Book IV

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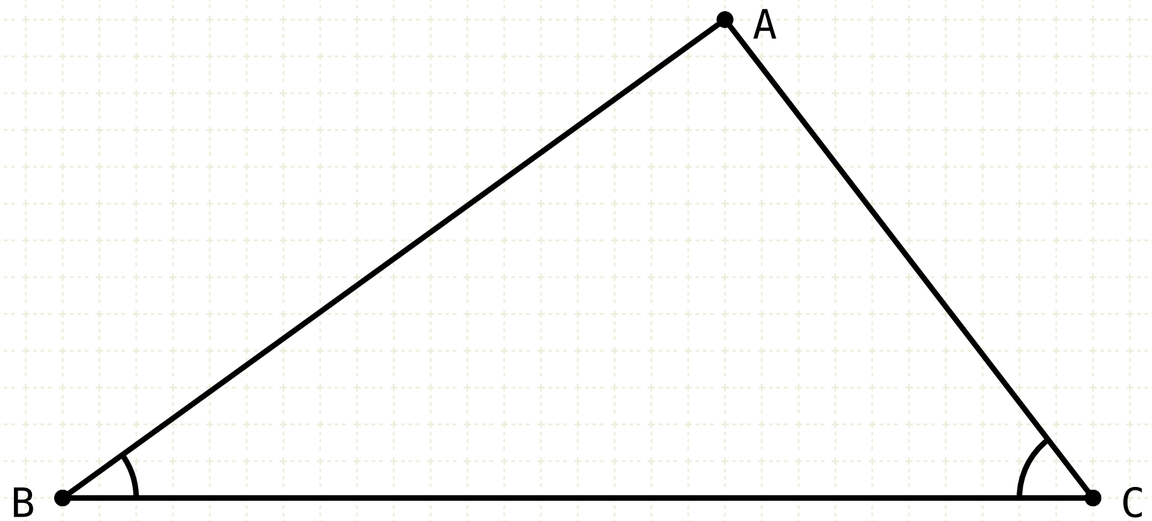

Construction

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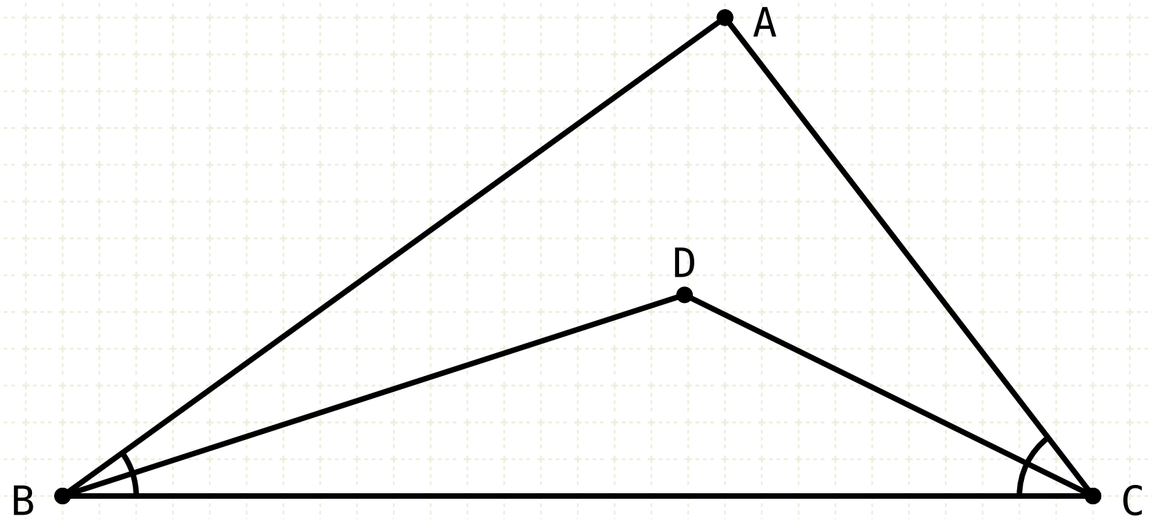

Construction

Bisect the angles at points B and C with lines BD and CD, intersecting at point D

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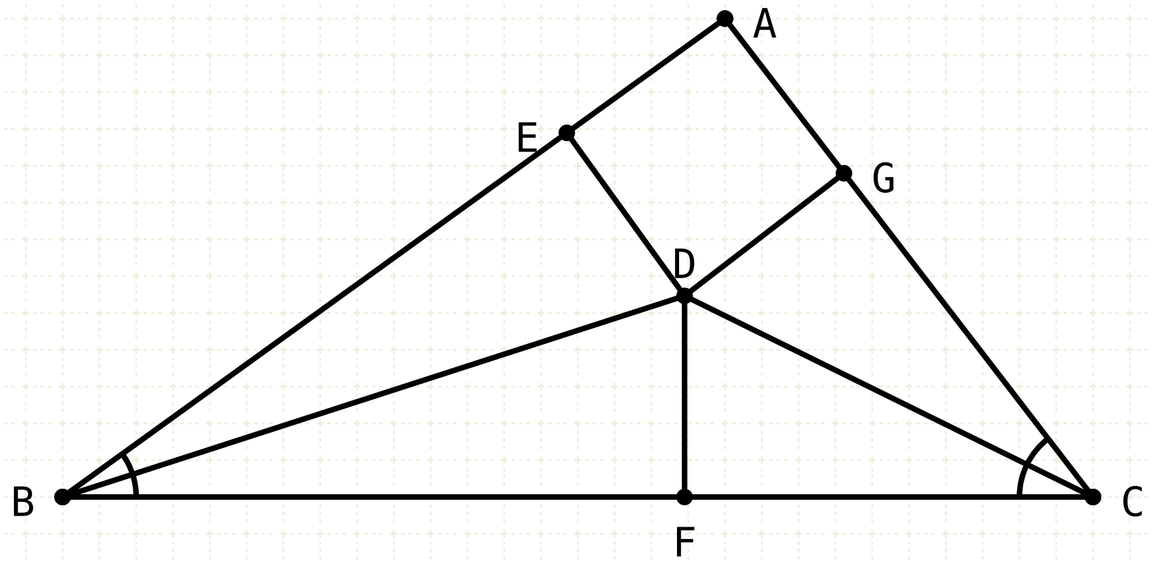

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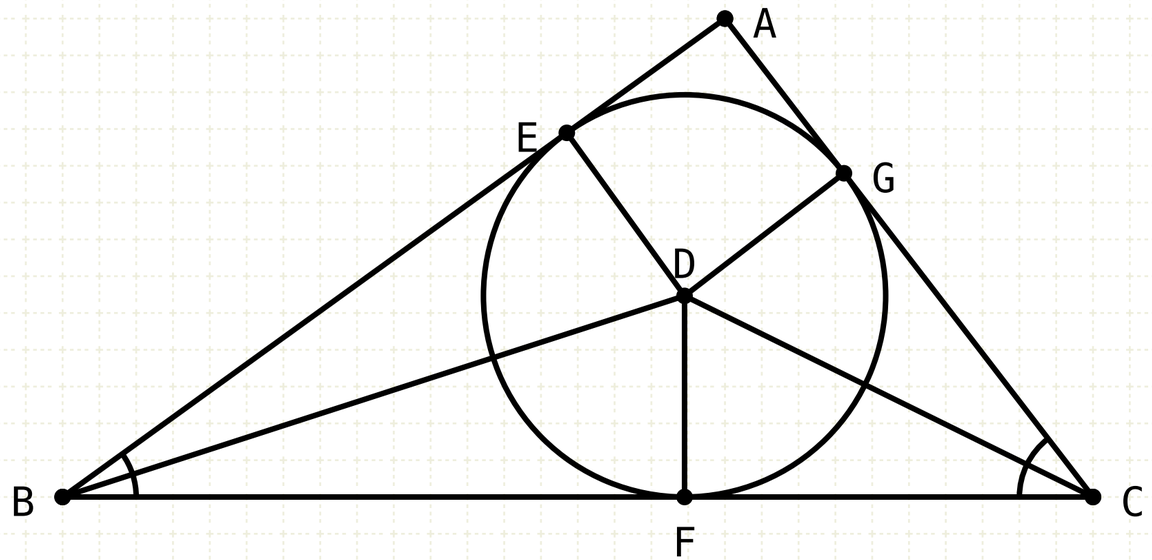

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Bisect the angles at points B and C with lines BD and CD, intersecting at point D

Draw perpendicular lines from the point D to all three sides of the triangle

With D as the centre, and DF as the radius, it is possible to draw a circle that touches all three sides of the triangle

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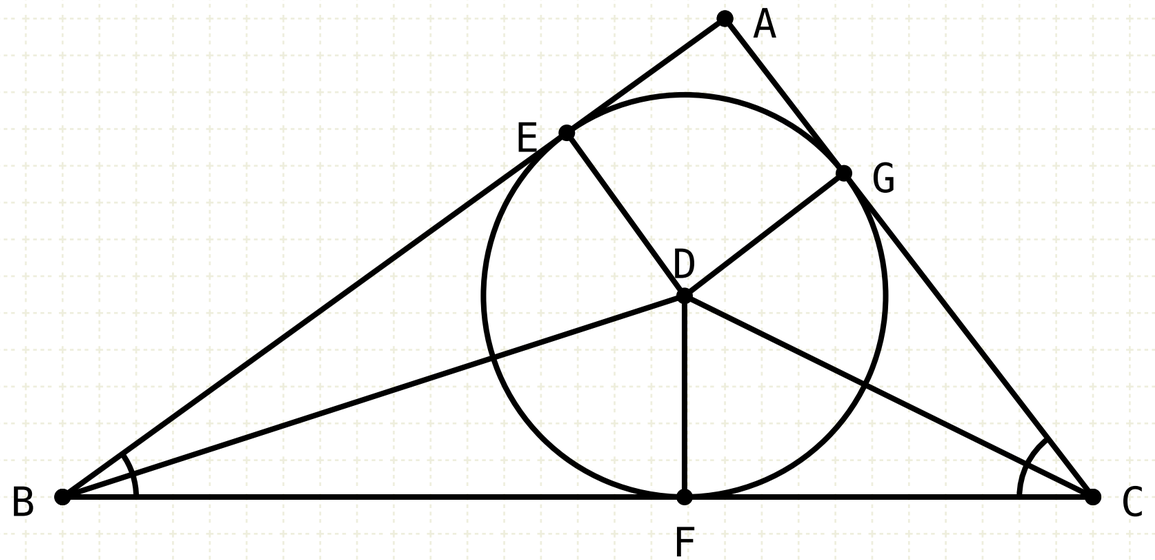

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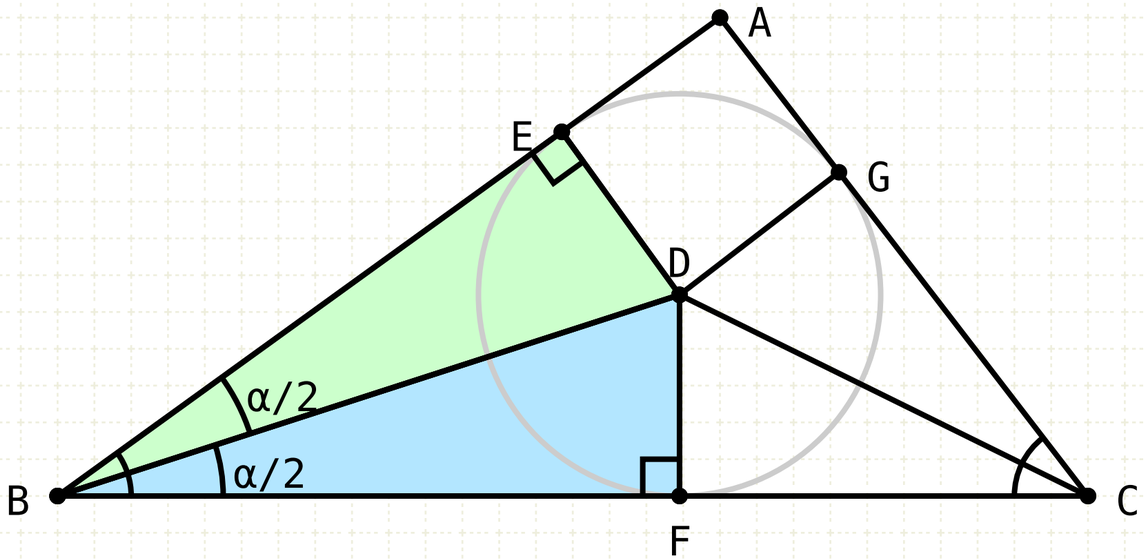

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The two triangles DEB and DBF are equal in all respects, since they have an angle (½α), side (BD) and angle (DFB and DEB) equal (I·26)

∠EBD = ∠DBF = α/2
∠DFE = ∠DEB = ∟
BD is common

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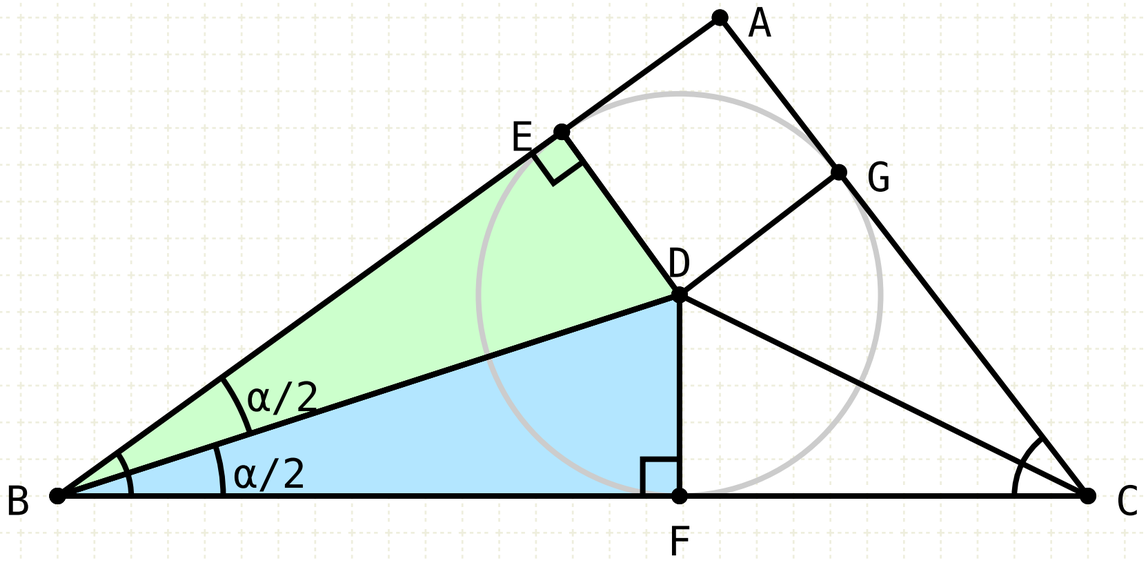

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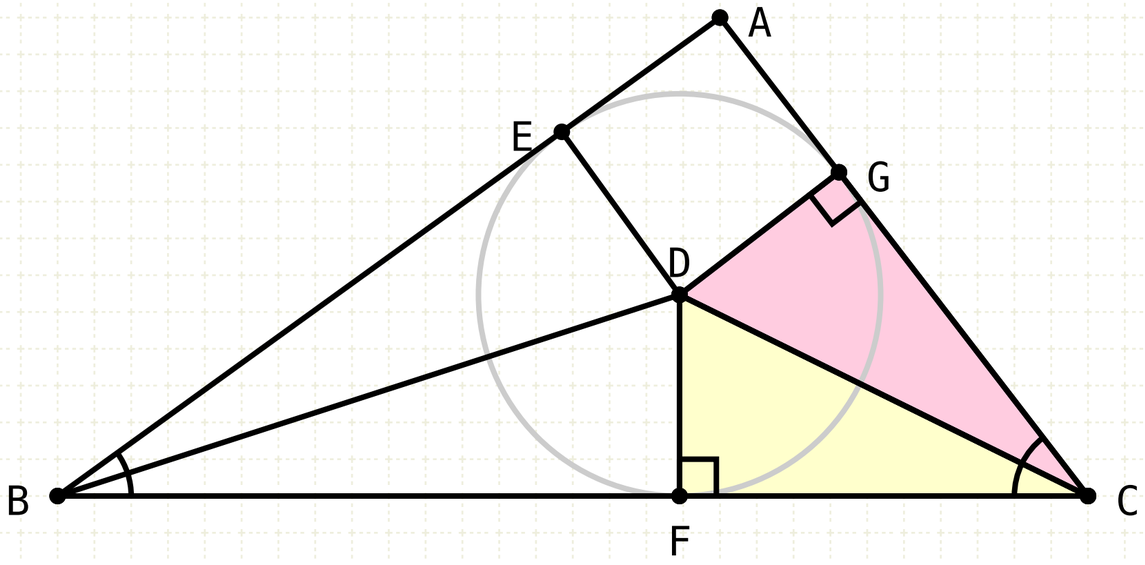

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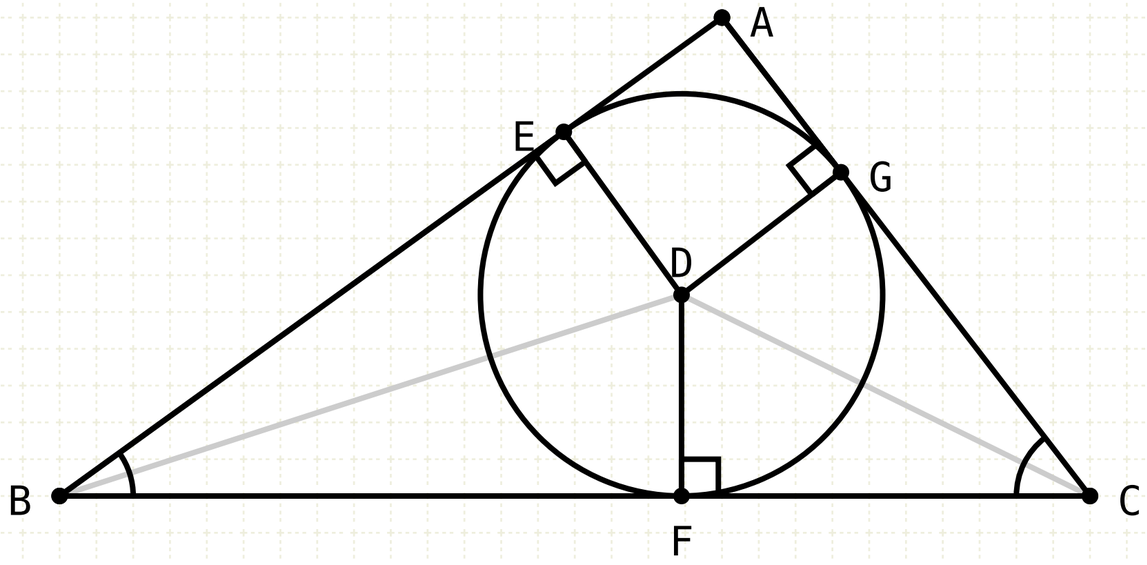

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Since the sides of the triangles are at right angles to the radii of the circle, and are at the extremities of the radii, then they touch the circle (III·16)

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