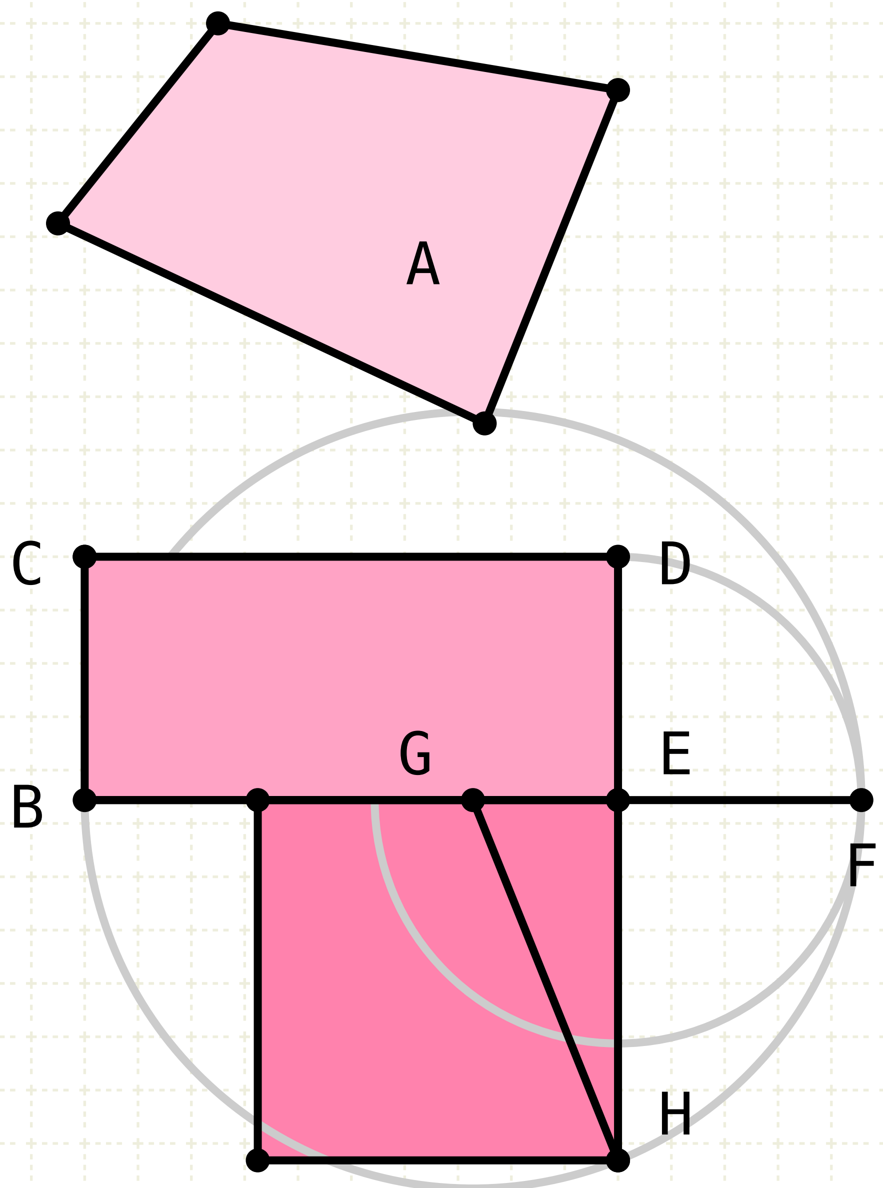
Euclid's Elements

Book II



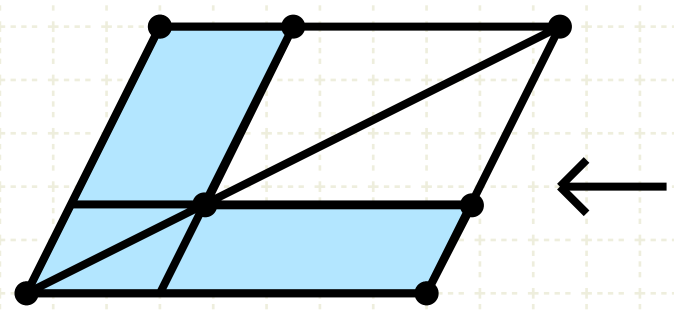
It is a remarkable fact in the history of geometry, that the Elements of Euclid, written two thousand years ago, are still regarded by many as the best introduction to the mathematical sciences.

Florian Cajori,

A History of Mathematics (1893)

Definitions:

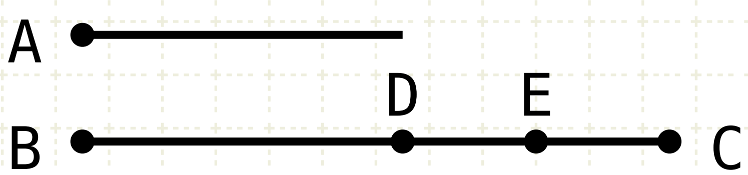
Any rectangular parallelogram is said to be contained by the two straight lines containing the right angle.



And in any parallelogrammic area let any one whatever of the parallelograms about its diameter with the two complements be called a gnomon.

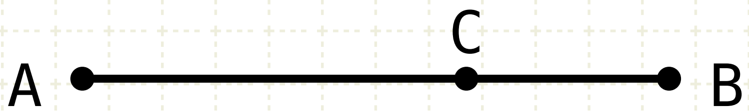
Table of Contents, Chapter 2

1



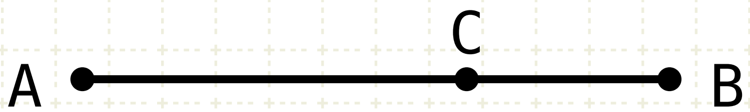
$A \cdot BC = A \cdot BD + A \cdot DE + A \cdot EC$

2



$AB^2 = AB \cdot AC + AB \cdot BC$

3

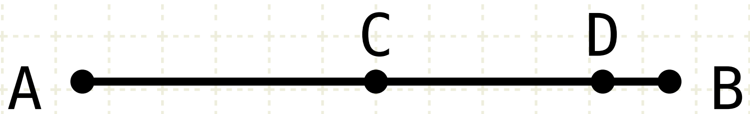


$AB \cdot CB = AC \cdot CB + CB^2$

4

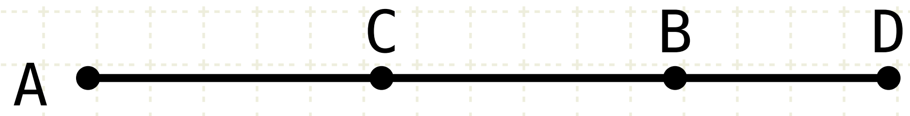
$AB^2 = AC^2 + CB^2 + 2 \cdot AC \cdot CB$

5



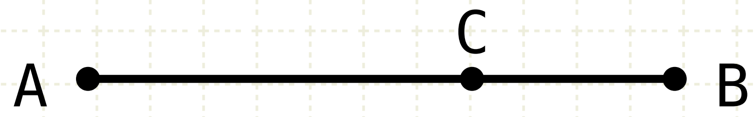
$AD \cdot DB + CD^2 = CB^2$

6



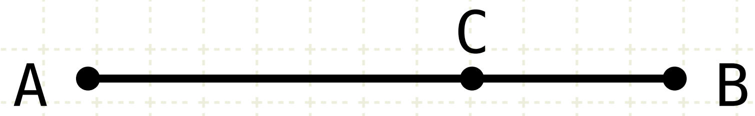
$AD \cdot DB + CB^2 = CD^2$

7



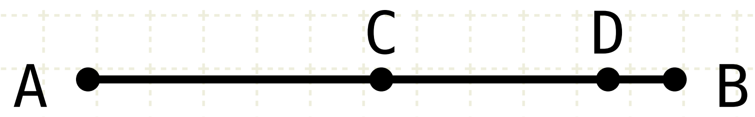
$AB^2 + BC^2 = AC^2 + 2 \cdot AB \cdot BC$

8



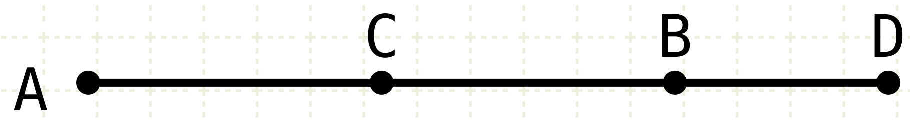
$4 \cdot AB \cdot BC + AC^2 = (AB+BC)^2$

9



$AD^2 + DB^2 = 2 \cdot (AC^2 + CD^2)$

10



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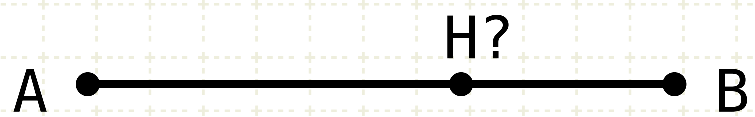
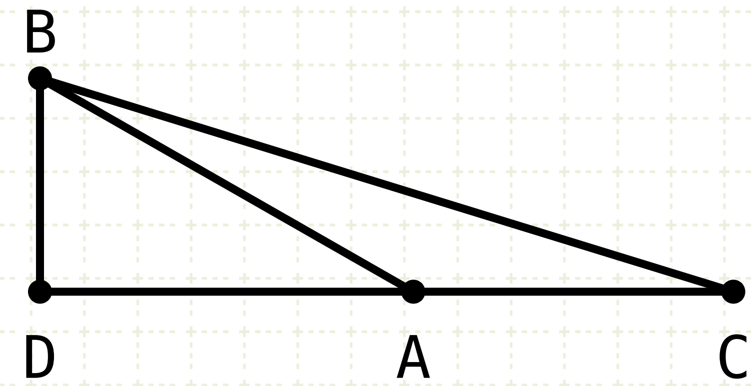


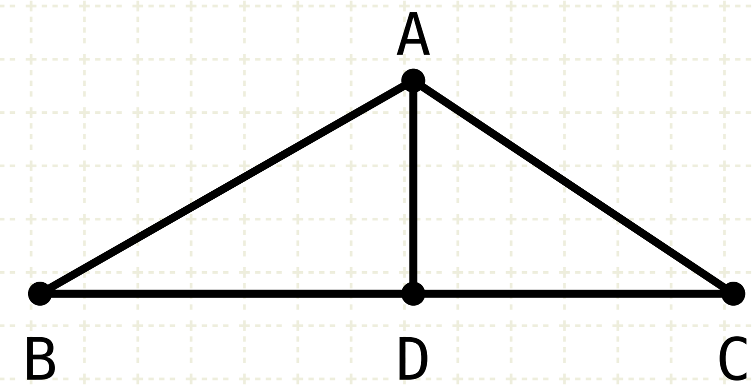
Find H. $AB \cdot BH = AH^2$

12



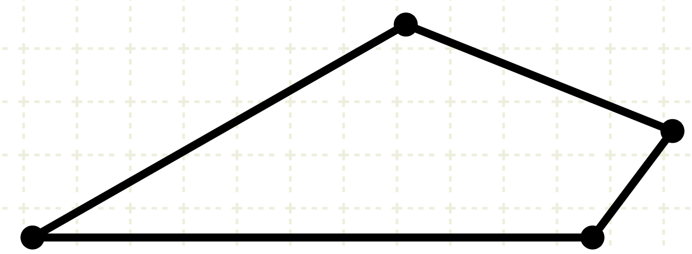
Cosine Law. $BC^2 = AB^2+AC^2+2 \cdot AD \cdot AC$

13



Cosine Law. $AC^2 = AB^2+BC^2-2 \cdot BD \cdot BC$

14



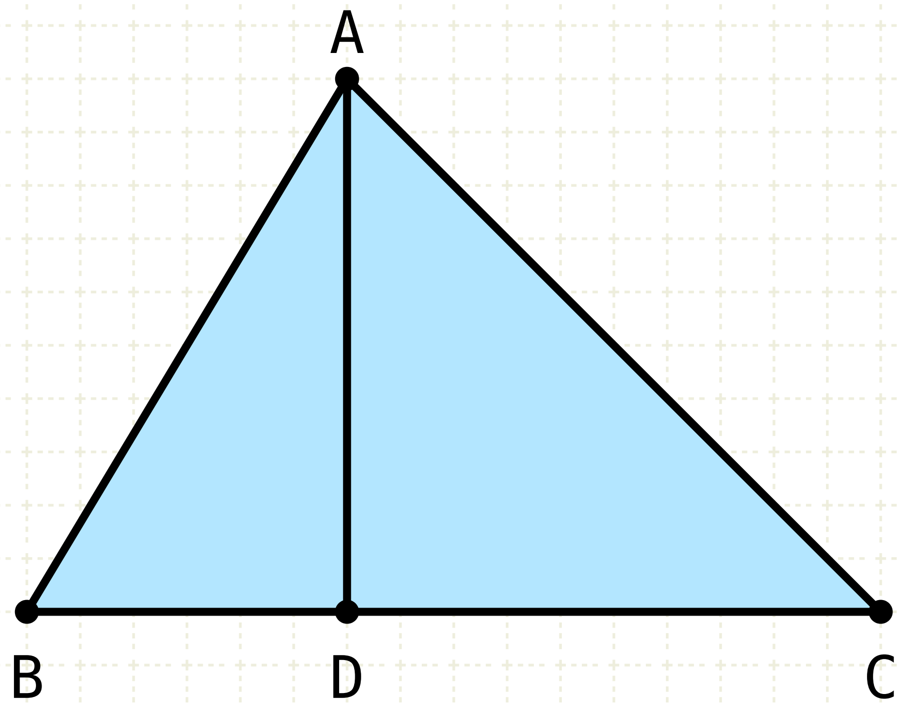
Find square of polygon

Proposition 13 of Book II

In acute-angled triangles the square on the side subtending the acute angle is less than the squares on the sides containing the acute angle by twice the rectangle contained by one of the sides about the acute angle, namely that on which the perpendicular falls, and the straight line cut off within by the perpendicular towards the acute angle.

Proposition 13 of Book II

In acute-angled triangles the square on the side subtending the acute angle is less than the squares on the sides containing the acute angle by twice the rectangle contained by one of the sides about the acute angle, namely that on which the perpendicular falls, and the straight line cut off within by the perpendicular towards the acute angle.



In other words

Given an acute triangle ABC, where the acute angle is opposite of AC. Define point D as the intersection of the perpendicular from point A to the line BC.

The square of AC equals the square of AB and BC plus twice the rectangle formed by BC,BD

$$AC^2 = AB^2 + BC^2 - 2 \cdot BD \cdot BC$$

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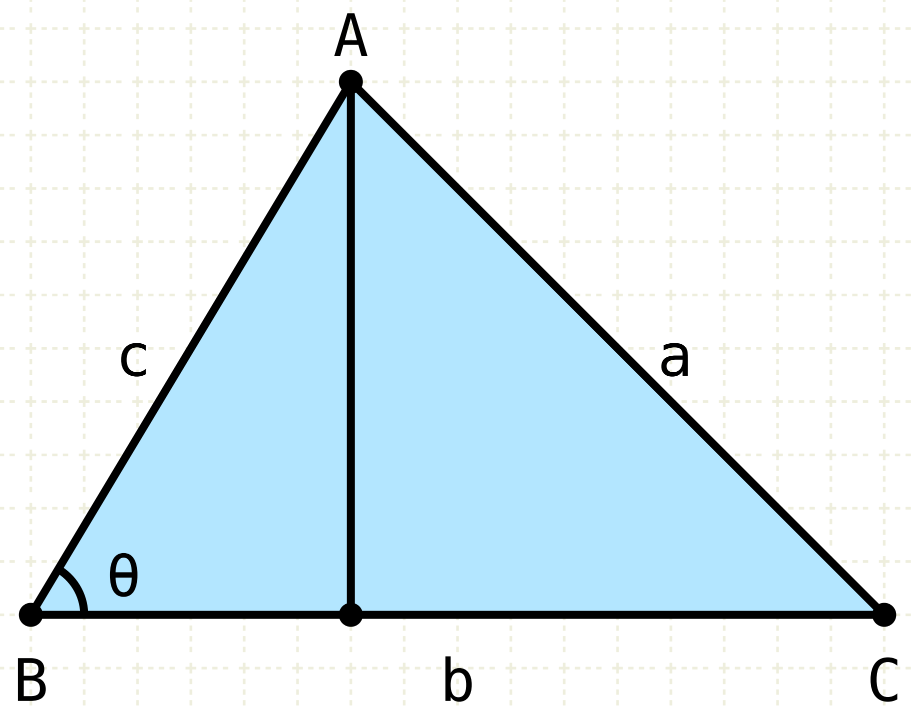

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Or… the cosine law

$$AC=a,\ AB=c,\ BC=b,\ BD = c \cdot \cos(\theta)$$

$$AC^2 = AB^2 + BC^2 - 2 \cdot BC \cdot BD$$
$$a^2 = c^2 + b^2 - 2 \cdot b \cdot c \cdot \cos(\theta)$$

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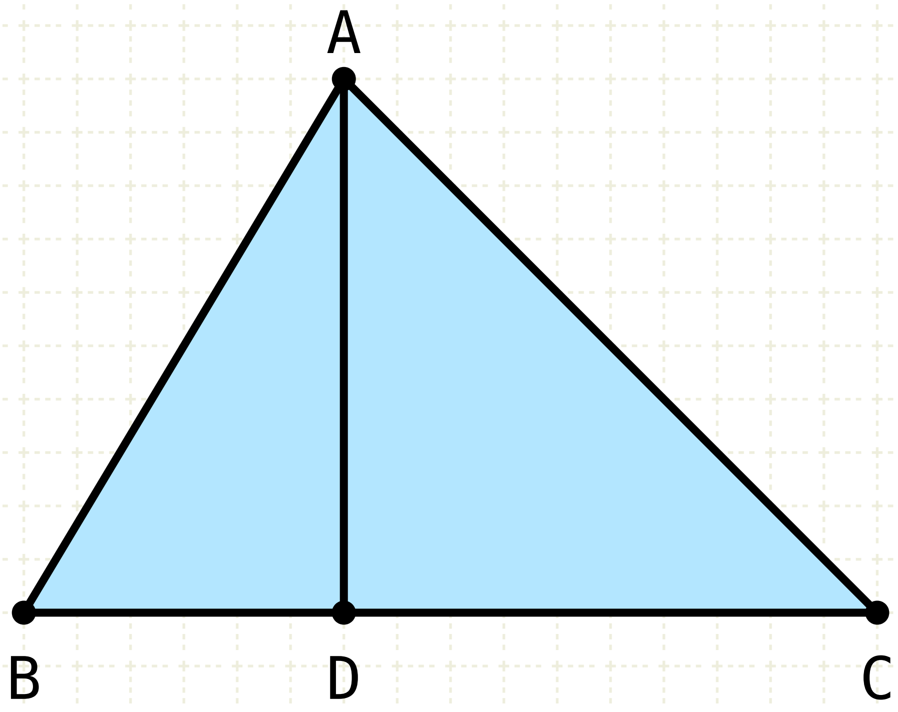

$$BC^2 + BD^2 = DC^2 + 2 \cdot BC \cdot BD$$

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Given an acute triangle ABC, where the acute angle is opposite of AC. Define point D as the intersection of the perpendicular from point A to the line BC.

The square of AC equals the square of AB and BC plus twice the rectangle formed by BC,BD

$$AC^2 = AB^2 + BC^2 - 2 \cdot BD \cdot BC$$

Proof

The line BC is cut at a point D, and thus the sum of the squares of BC and BD is equal to the square of DC plus twice the rectangle formed by BC and BD (II·7)

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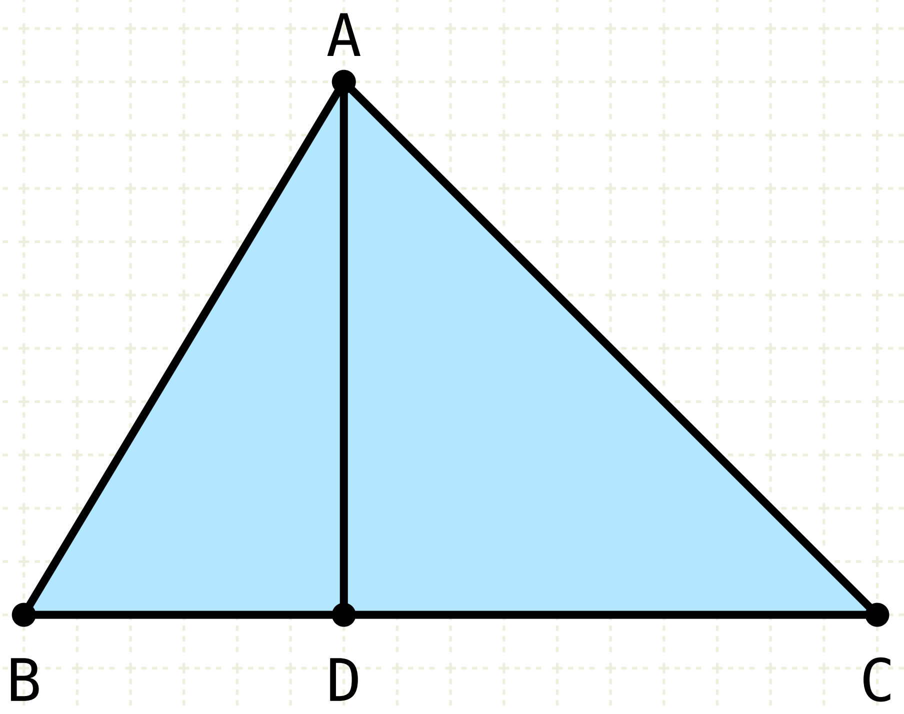

$$BC^2 + BD^2 = DC^2 + 2 \cdot BC \cdot BD$$
$$BC^2 + (BD^2 + AD^2) = (DC^2 + AD^2) + 2 \cdot BC \cdot BD$$

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Given an acute triangle ABC, where the acute angle is opposite of AC. Define point D as the intersection of the perpendicular from point A to the line BC.

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Add the square of AD to both sides of the equality

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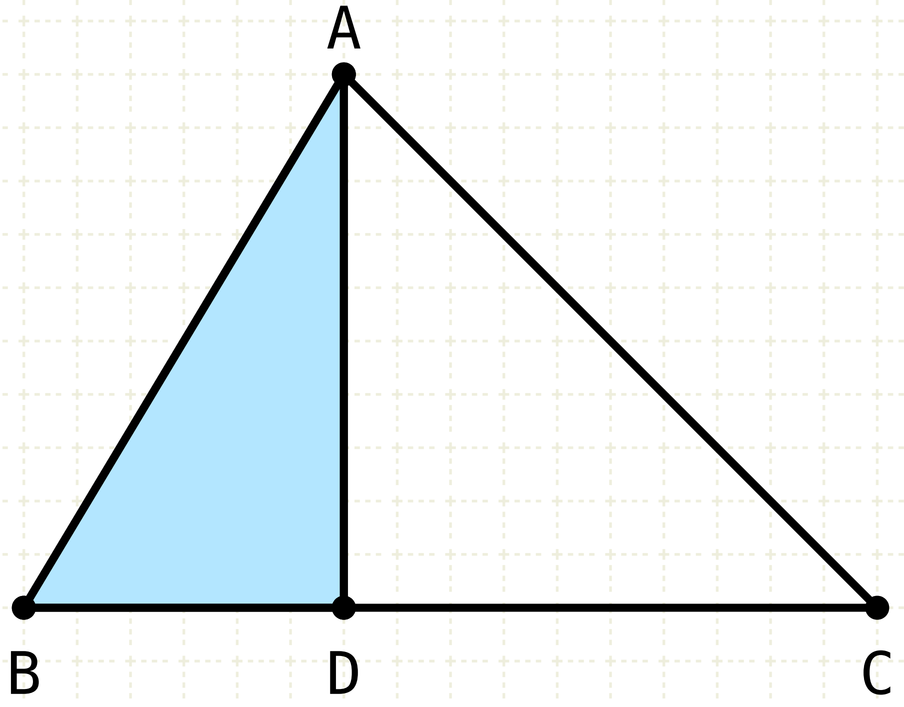

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$$BD^2 + AD^2 = AB^2$$

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The squares of BD and AD equals the square of AB (I·47)

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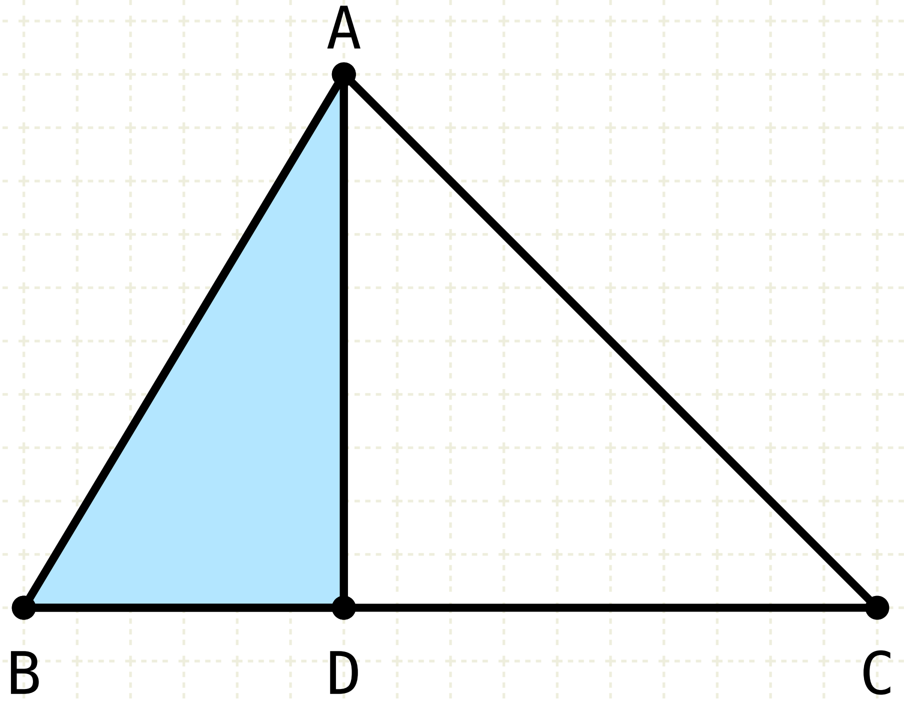

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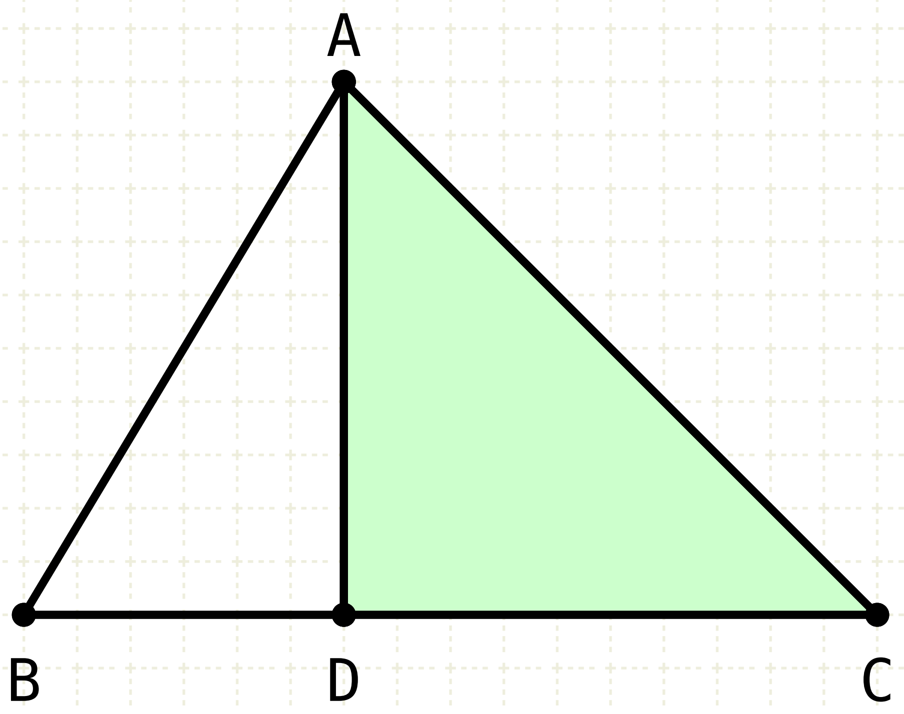

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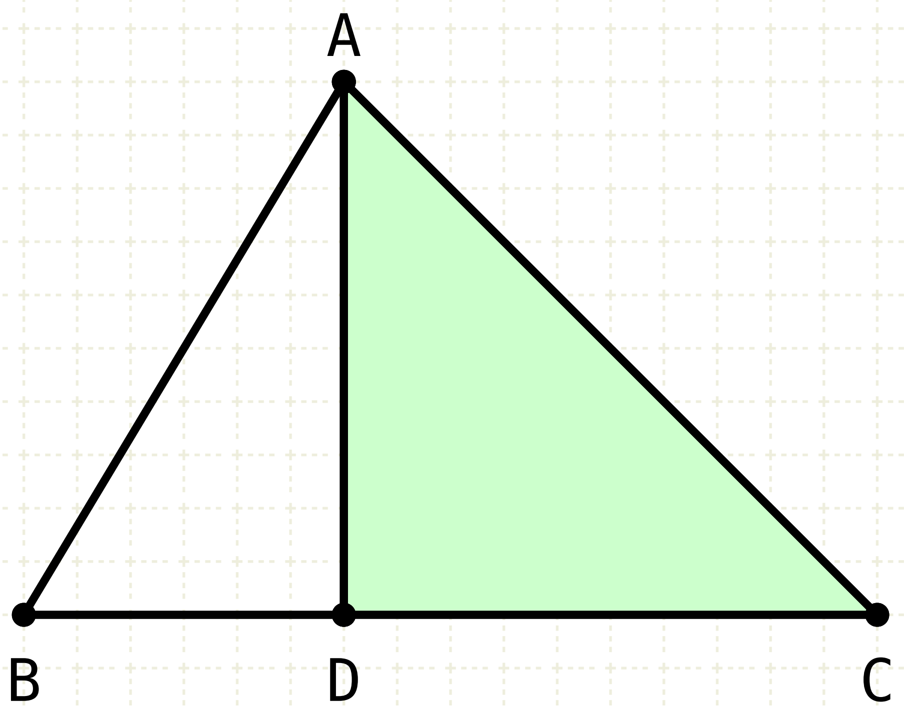

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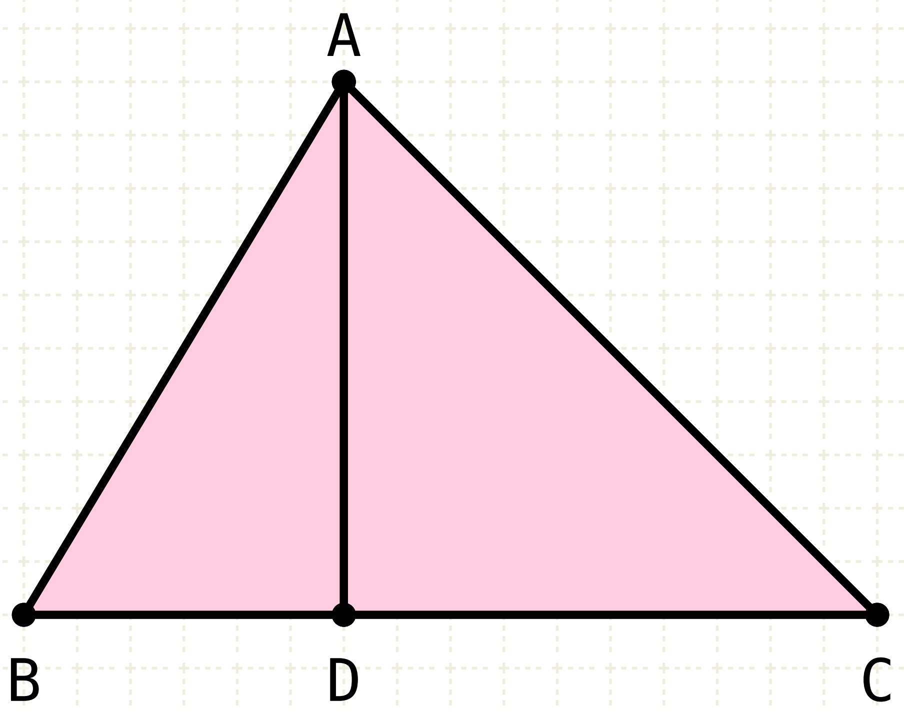

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Add the square of AD to both sides of the equality

The squares of BD and AD equals the square of AB (I·47)

The squares of AD and DC equals the square of AC (I·47)

Thus the square of AC is equal to the sum of the squares of BC and AB, less the rectangle formed by BC,BD

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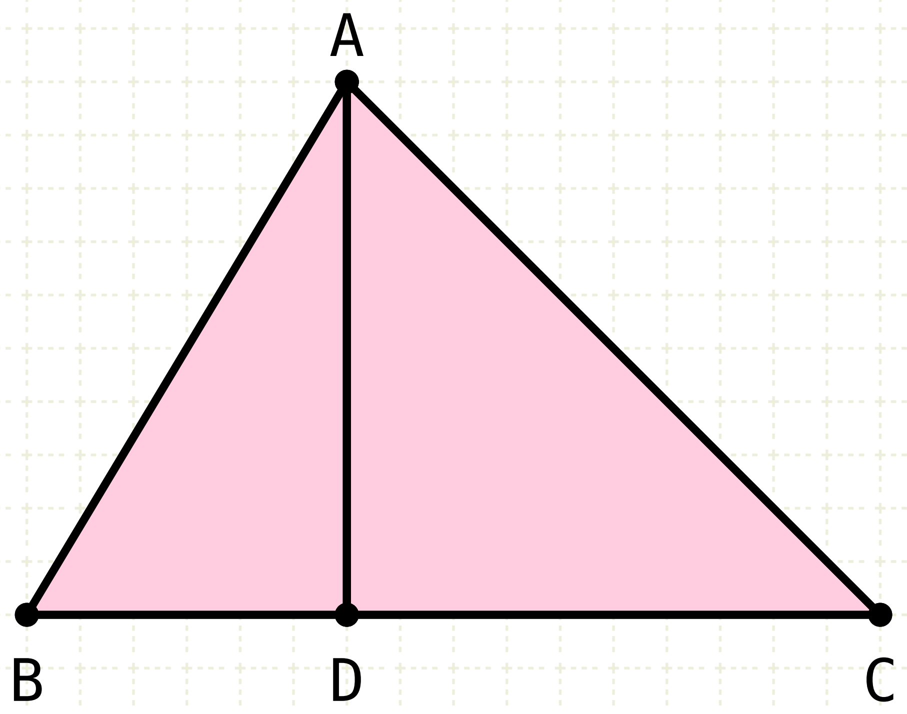

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