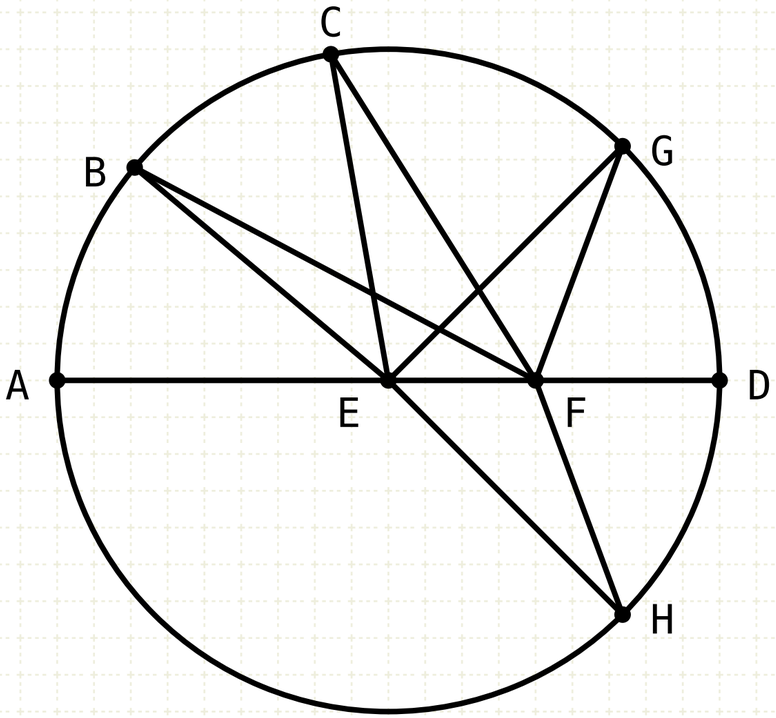
Euclid's Elements

Book III



A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)

Table of Contents, Chapter 3

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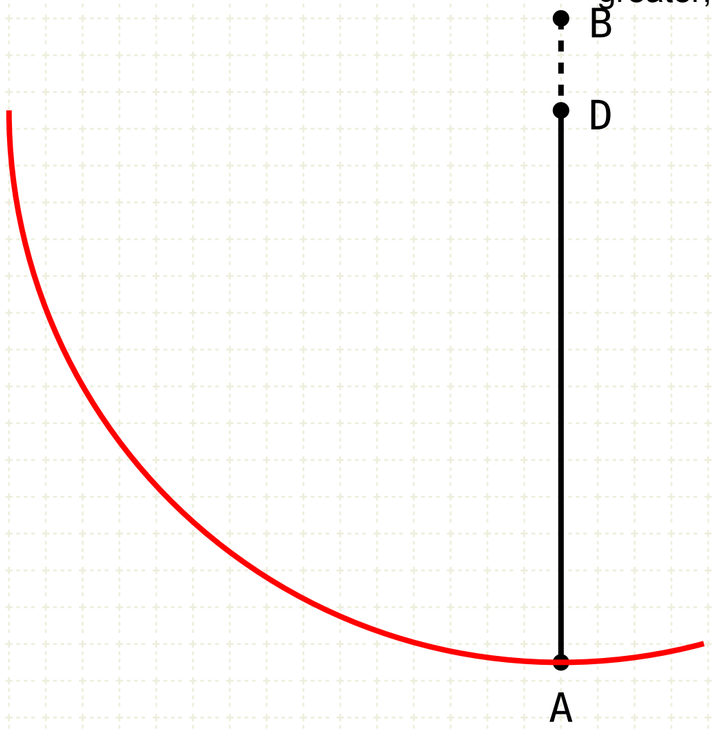
Proposition 16 of Book III

The straight line drawn at right angles to the diameter of a circle from its extremity will fall outside the circle, and into the space between the straight line and the circumference another straight line cannot be interposed; further the angle of the semicircle is greater, and the remaining angle less, than any acute rectilineal angle.

Proposition 16 of Book III

The straight line drawn at right angles to the diameter of a circle from its extremity will fall outside the circle, and into the space between the straight line and the circumference another straight line cannot be interposed; further the angle of the semicircle is greater, and the remaining angle less, than any acute rectilineal angle.

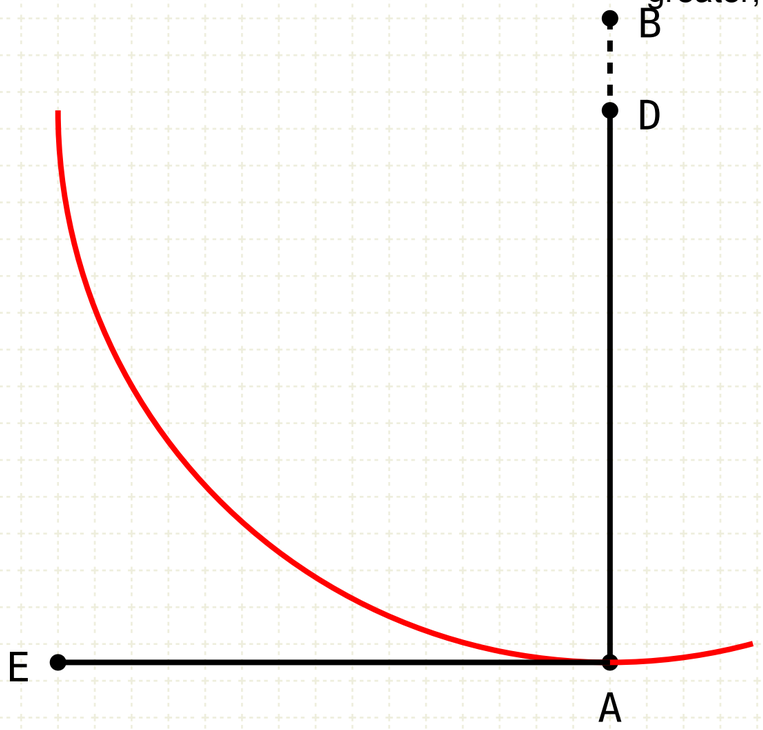


In other words

Given a circle, with AB as a diameter, and D as the centre…

Proposition 16 of Book III

The straight line drawn at right angles to the diameter of a circle from its extremity will fall outside the circle, and into the space between the straight line and the circumference another straight line cannot be interposed; further the angle of the semicircle is greater, and the remaining angle less, than any acute rectilineal angle.



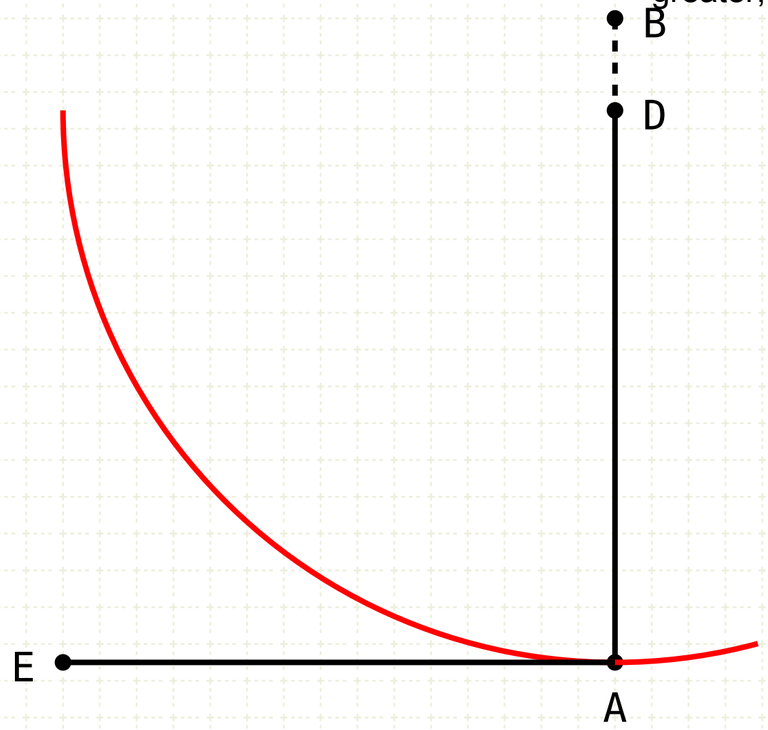
In other words

Given a circle, with AB as a diameter, and D as the centre…

Draw a line AE from point A, perpendicular to AB

Proposition 16 of Book III

The straight line drawn at right angles to the diameter of a circle from its extremity will fall outside the circle, and into the space between the straight line and the circumference another straight line cannot be interposed; further the angle of the semicircle is greater, and the remaining angle less, than any acute rectilineal angle.



In other words

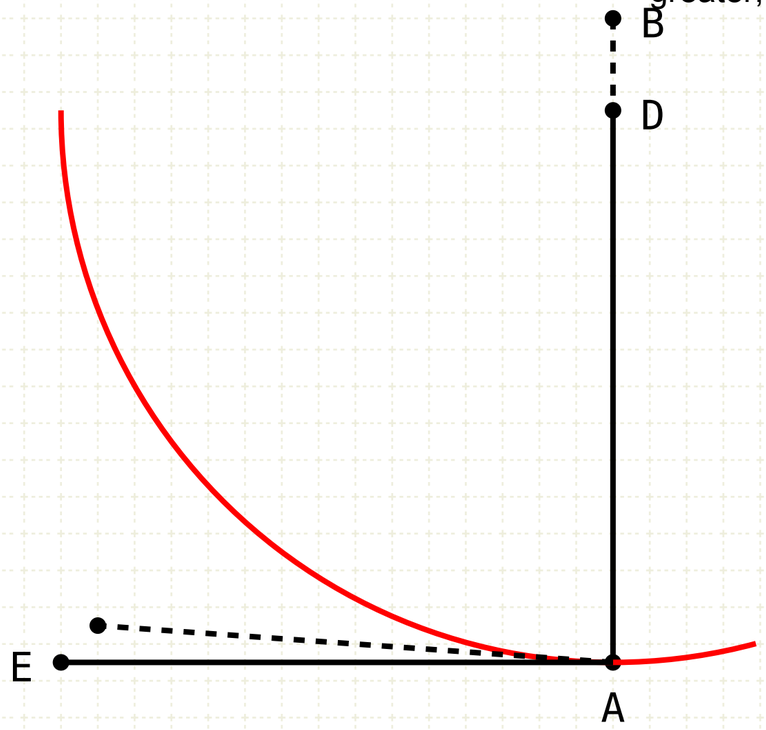
Given a circle, with AB as a diameter, and D as the centre…

Draw a line AE from point A, perpendicular to AB

(1) The Line AE falls outside of the circle

Proposition 16 of Book III

The straight line drawn at right angles to the diameter of a circle from its extremity will fall outside the circle, and into the space between the straight line and the circumference another straight line cannot be interposed; further the angle of the semicircle is greater, and the remaining angle less, than any acute rectilineal angle.



In other words

Given a circle, with AB as a diameter, and D as the centre…

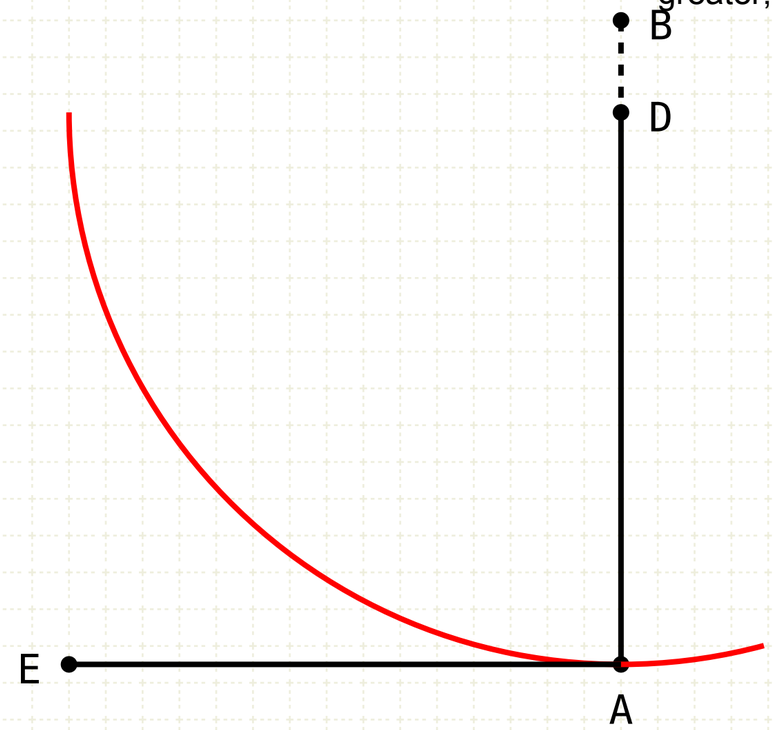
Draw a line AE from point A, perpendicular to AB

(1) The Line AE falls outside of the circle

(2) No other line from point A can squeeze between the circumference, and the line AE

Proposition 16 of Book III

The straight line drawn at right angles to the diameter of a circle from its extremity will fall outside the circle, and into the space between the straight line and the circumference another straight line cannot be interposed; further the angle of the semicircle is greater, and the remaining angle less, than any acute rectilineal angle.



In other words

Given a circle, with AB as a diameter, and D as the centre…

Draw a line AE from point A, perpendicular to AB

(1) The Line AE falls outside of the circle

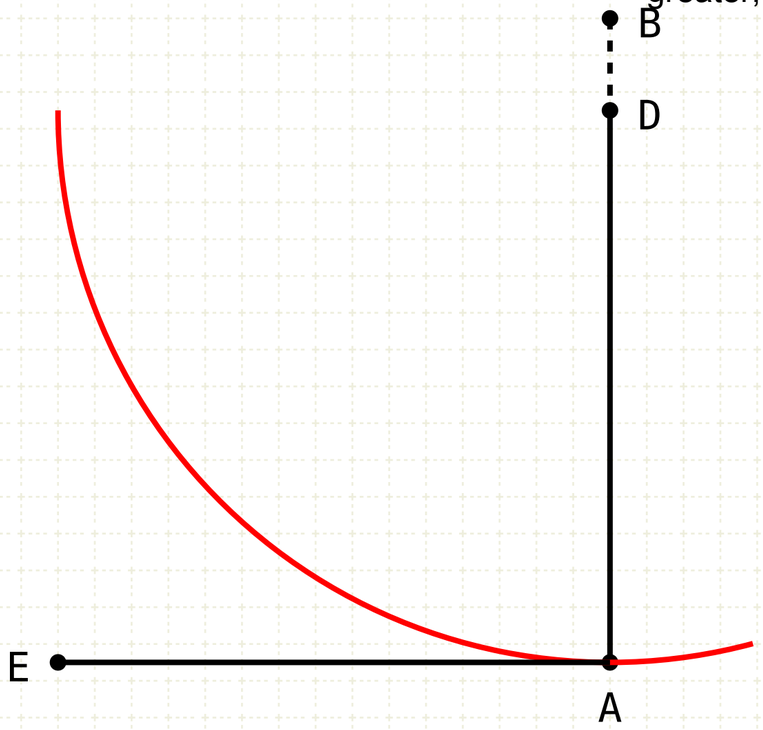
(2) No other line from point A can squeeze between the circumference, and the line AE

(3) - The angle between AB and the circumference of the circle cannot be less than a right angle,

- and the angle between AE and the circumference cannot be greater than zero

Proposition 16 of Book III

The straight line drawn at right angles to the diameter of a circle from its extremity will fall outside the circle, and into the space between the straight line and the circumference another straight line cannot be interposed; further the angle of the semicircle is greater, and the remaining angle less, than any acute rectilineal angle.



In other words

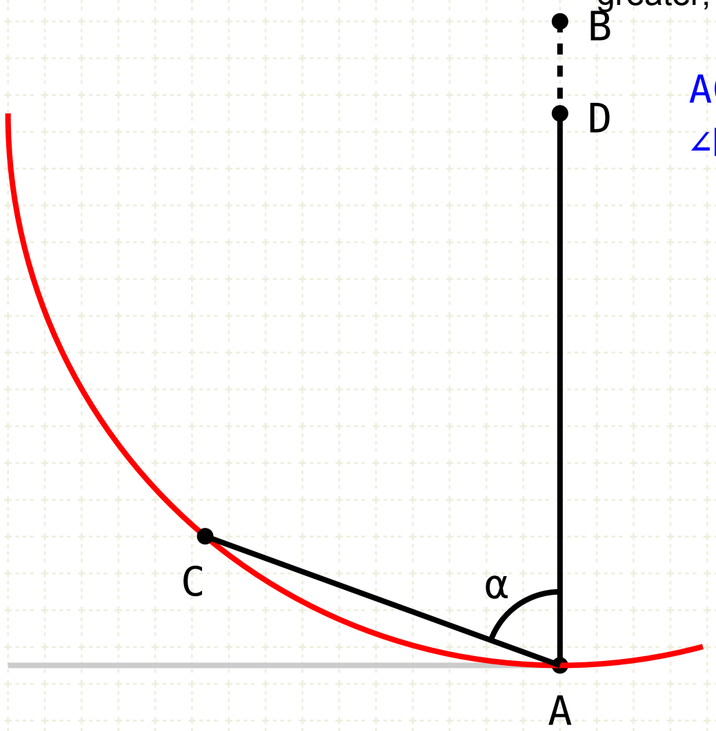
Given a circle, with AB as a diameter, and D as the centre…

(1) The Line AE falls outside of the circle

Proof by Contradiction (1)

Proposition 16 of Book III

The straight line drawn at right angles to the diameter of a circle from its extremity will fall outside the circle, and into the space between the straight line and the circumference another straight line cannot be interposed; further the angle of the semicircle is greater, and the remaining angle less, than any acute rectilineal angle.



$AC \perp AB$

$\angle DAC = \alpha = \llcorner$

In other words

Given a circle, with AB as a diameter, and D as the centre…

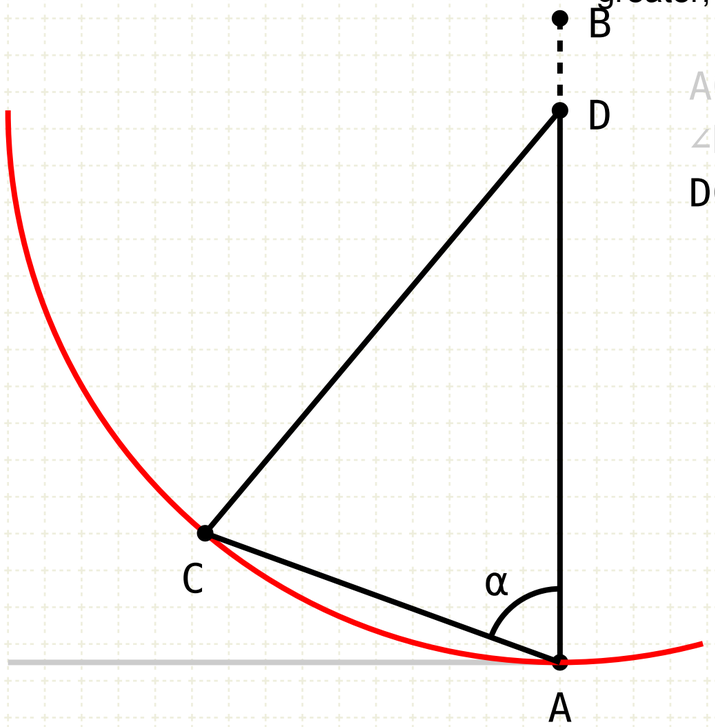
(1) The Line AE falls outside of the circle

Proof by Contradiction (1)

Assume that the line perpendicular to AB lies within the circle, such as line AC

Proposition 16 of Book III

The straight line drawn at right angles to the diameter of a circle from its extremity will fall outside the circle, and into the space between the straight line and the circumference another straight line cannot be interposed; further the angle of the semicircle is greater, and the remaining angle less, than any acute rectilineal angle.



AC ⊥ AB

∠DAC = α = ∟

DC = AC

In other words

Given a circle, with AB as a diameter, and D as the centre…

(1) The Line AE falls outside of the circle

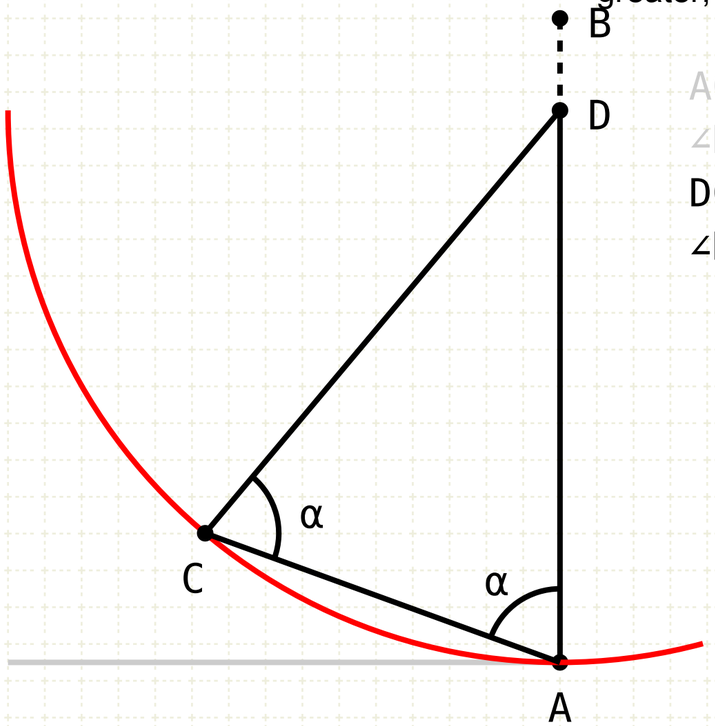
Proof by Contradiction (1)

Assume that the line perpendicular to AB lies within the circle, such as line AC

Draw the line DC

Proposition 16 of Book III

The straight line drawn at right angles to the diameter of a circle from its extremity will fall outside the circle, and into the space between the straight line and the circumference another straight line cannot be interposed; further the angle of the semicircle is greater, and the remaining angle less, than any acute rectilineal angle.



AC ⊥ AB

∠DAC = α = ∟

DC = AC

∠DAC = ∠DCA = α

In other words

Given a circle, with AB as a diameter, and D as the centre…

(1) The Line AE falls outside of the circle

Proof by Contradiction (1)

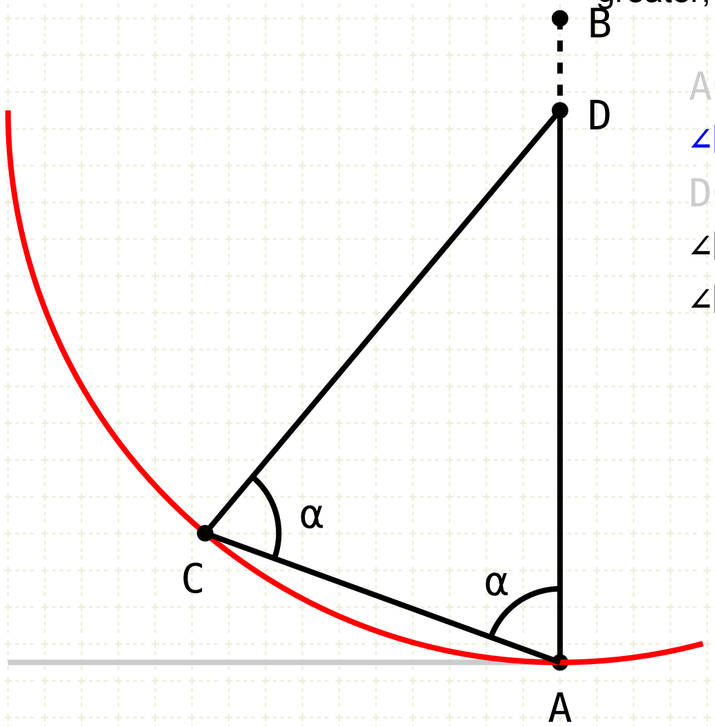
Assume that the line perpendicular to AB lies within the circle, such as line AC

Draw the line DC

DC equals AC, therefore the triangle is an isosceles triangle, and the angles DAC and DCA are equal (I·5)

Proposition 16 of Book III

The straight line drawn at right angles to the diameter of a circle from its extremity will fall outside the circle, and into the space between the straight line and the circumference another straight line cannot be interposed; further the angle of the semicircle is greater, and the remaining angle less, than any acute rectilineal angle.



$AC \perp AB$

$\angle DAC = \alpha = \llcorner$

$DC = AC$

$\angle DAC = \angle DCA = \alpha$

$\angle DAC + \angle DCA = 2 \cdot \llcorner$

In other words

Given a circle, with AB as a diameter, and D as the centre…

(1) The Line AE falls outside of the circle

Proof by Contradiction (1)

Assume that the line perpendicular to AB lies within the circle, such as line AC

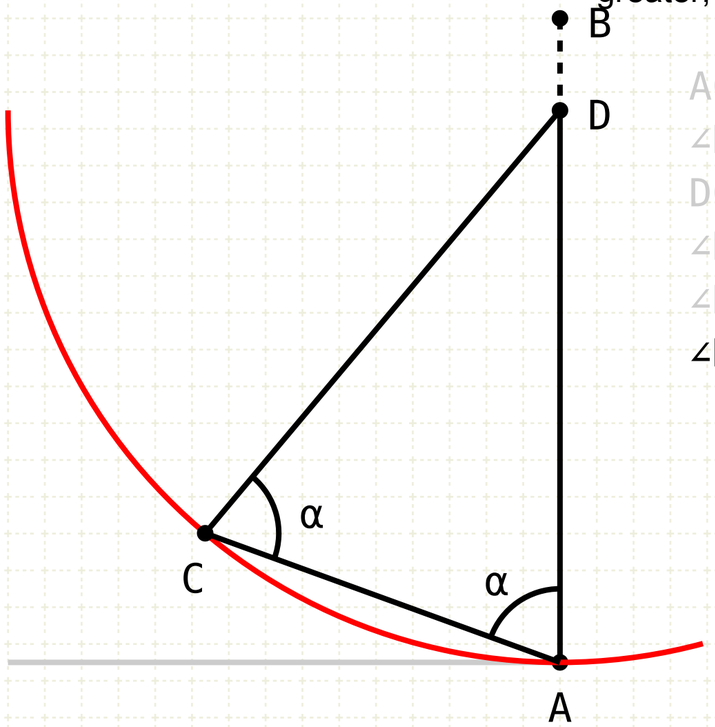
Draw the line DC

DC equals AC, therefore the triangle is an isosceles triangle, and the angles DAC and DCA are equal (I·5)

DAC is right, which means DCA is also right, and their sum is equal to two right angles

Proposition 16 of Book III

The straight line drawn at right angles to the diameter of a circle from its extremity will fall outside the circle, and into the space between the straight line and the circumference another straight line cannot be interposed; further the angle of the semicircle is greater, and the remaining angle less, than any acute rectilineal angle.



AC ⊥ AB
∠DAC = α = ∟
DC = AC
∠DAC = ∠DCA = α
∠DAC + ∠DCA = 2·∟
∠DAC + ∠DCA < 2·∟

In other words

Given a circle, with AB as a diameter, and D as the centre…

(1) The Line AE falls outside of the circle

Proof by Contradiction (1)

Assume that the line perpendicular to AB lies within the circle, such as line AC

Draw the line DC

DC equals AC, therefore the triangle is an isosceles triangle, and the angles DAC and DCA are equal (I·5)

DAC is right, which means DCA is also right, and their sum is equal to two right angles

The sum of any two angles in a triangle must be less than two right angles (I·17)

Proposition 16 of Book III

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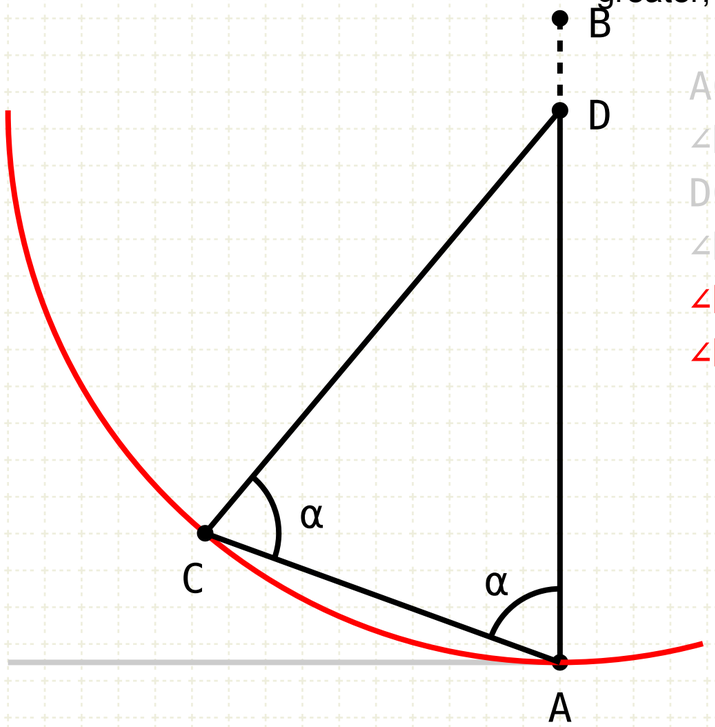

$AC \perp AB$

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$DC = AC$

$\angle DAC = \angle DCA = \alpha$

$\angle DAC + \angle DCA = 2 \cdot \llcorner$

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In other words

Given a circle, with AB as a diameter, and D as the centre…

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Draw the line DC

DC equals AC, therefore the triangle is an isosceles triangle, and the angles DAC and DCA are equal (I·5)

DAC is right, which means DCA is also right, and their sum is equal to two right angles

The sum of any two angles in a triangle must be less than two right angles (I·17)

Thus we have a contradiction

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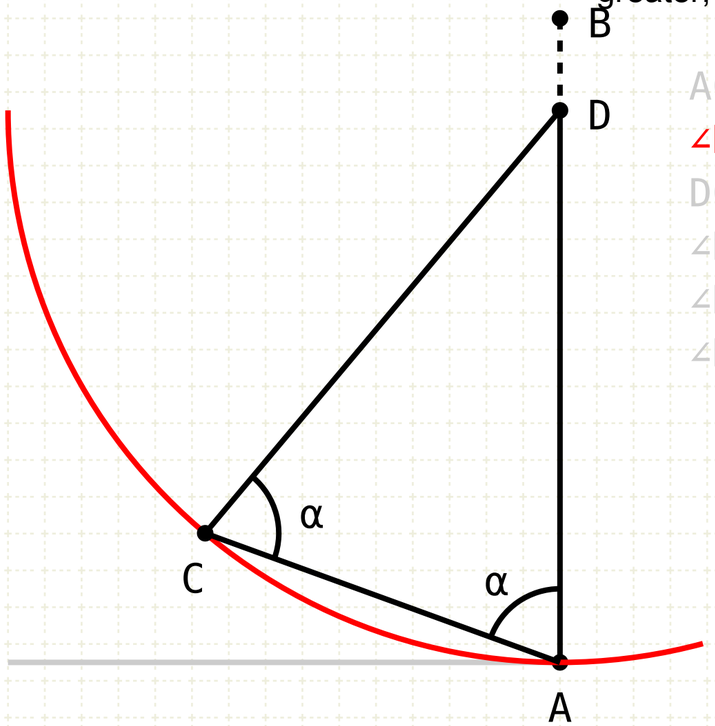
The straight line drawn at right angles to the diameter of a circle from its extremity will fall outside the circle, and into the space between the straight line and the circumference another straight line cannot be interposed; further the angle of the semicircle is greater, and the remaining angle less, than any acute rectilineal angle.



AC ⊥ AB
∠DAC = α = ∟
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In other words

Given a circle, with AB as a diameter, and D as the centre…

(1) The Line AE falls outside of the circle

Proof by Contradiction (1)

Assume that the line perpendicular to AB lies within the circle, such as line AC

Draw the line DC

DC equals AC, therefore the triangle is an isosceles triangle, and the angles DAC and DCA are equal (I·5)

DAC is right, which means DCA is also right, and their sum is equal to two right angles

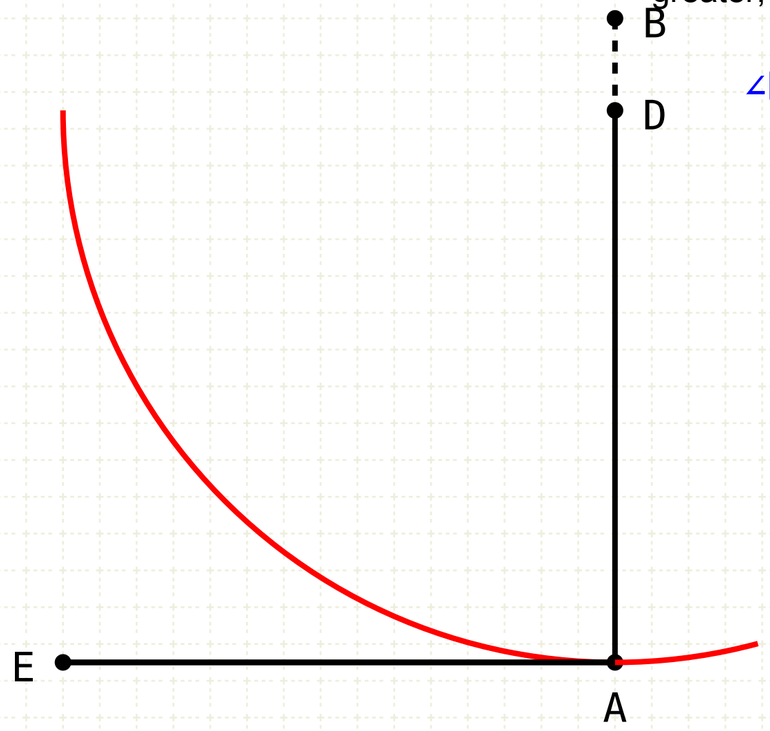
The sum of any two angles in a triangle must be less than two right angles (I·17)

Thus we have a contradiction

The angle DAC cannot be a right angle

Proposition 16 of Book III

The straight line drawn at right angles to the diameter of a circle from its extremity will fall outside the circle, and into the space between the straight line and the circumference another straight line cannot be interposed; further the angle of the semicircle is greater, and the remaining angle less, than any acute rectilineal angle.



In other words

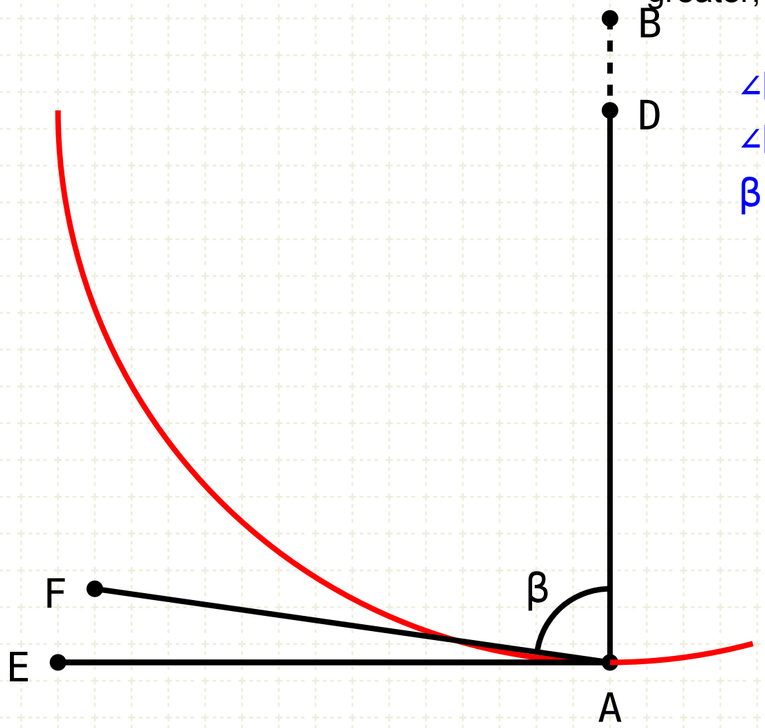
Given a circle, with AB as a diameter, and D as the centre…

(2) No other line from point A can squeeze between the circumference, and the line AE

Proof by Contradiction (2)

Proposition 16 of Book III

The straight line drawn at right angles to the diameter of a circle from its extremity will fall outside the circle, and into the space between the straight line and the circumference another straight line cannot be interposed; further the angle of the semicircle is greater, and the remaining angle less, than any acute rectilineal angle.



∠DAE = ∟

∠DAF = β < ∠DAE

β < ∟

In other words

Given a circle, with AB as a diameter, and D as the centre…

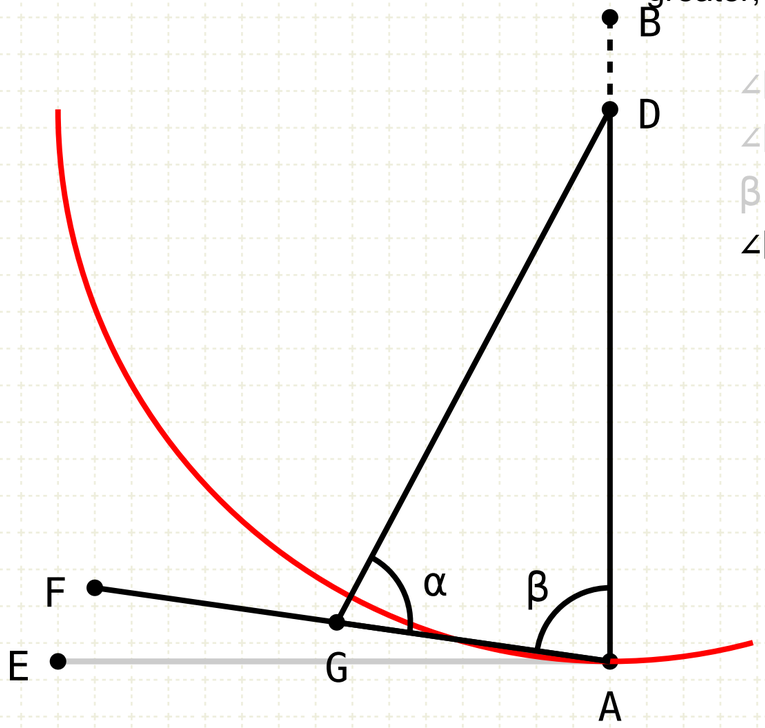
(2) No other line from point A can squeeze between the circumference, and the line AE

Proof by Contradiction (2)

Assume that the line FA lies between the circumference of the circle and the line AE

Proposition 16 of Book III

The straight line drawn at right angles to the diameter of a circle from its extremity will fall outside the circle, and into the space between the straight line and the circumference another straight line cannot be interposed; further the angle of the semicircle is greater, and the remaining angle less, than any acute rectilineal angle.



∠DAE = ∟

∠DAF = β < ∠DAE

β < ∟

∠DGA = α = ∟

In other words

Given a circle, with AB as a diameter, and D as the centre…

(2) No other line from point A can squeeze between the circumference, and the line AE

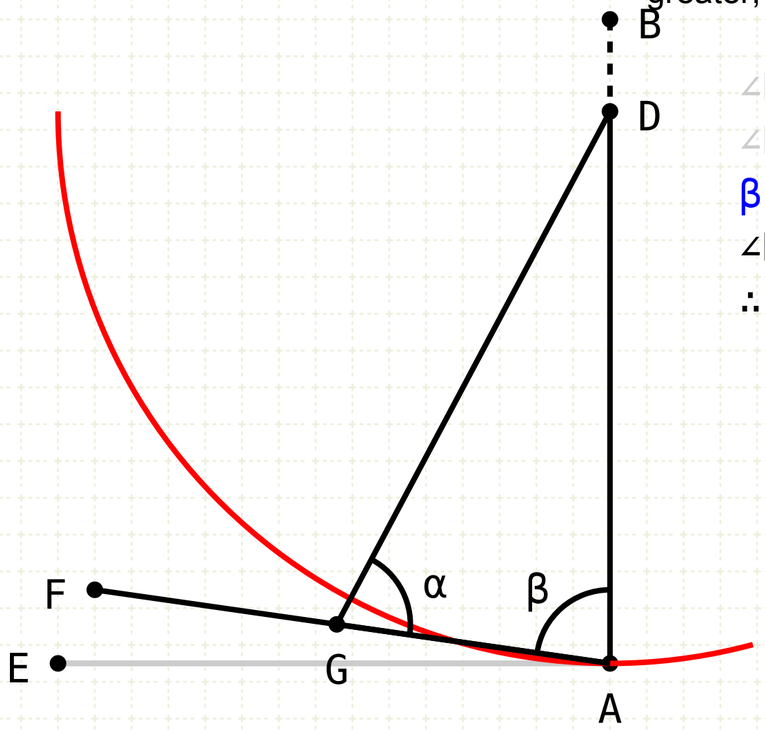
Proof by Contradiction (2)

Assume that the line FA lies between the circumference of the circle and the line AE

Draw a perpendicular line from D to line FA, intersecting at the point G

Proposition 16 of Book III

The straight line drawn at right angles to the diameter of a circle from its extremity will fall outside the circle, and into the space between the straight line and the circumference another straight line cannot be interposed; further the angle of the semicircle is greater, and the remaining angle less, than any acute rectilineal angle.



∠DAE = ∟

∠DAF = β < ∠DAE

β < ∟

∠DGA = α = ∟

∴ α > β

In other words

Given a circle, with AB as a diameter, and D as the centre…

(2) No other line from point A can squeeze between the circumference, and the line AE

Proof by Contradiction (2)

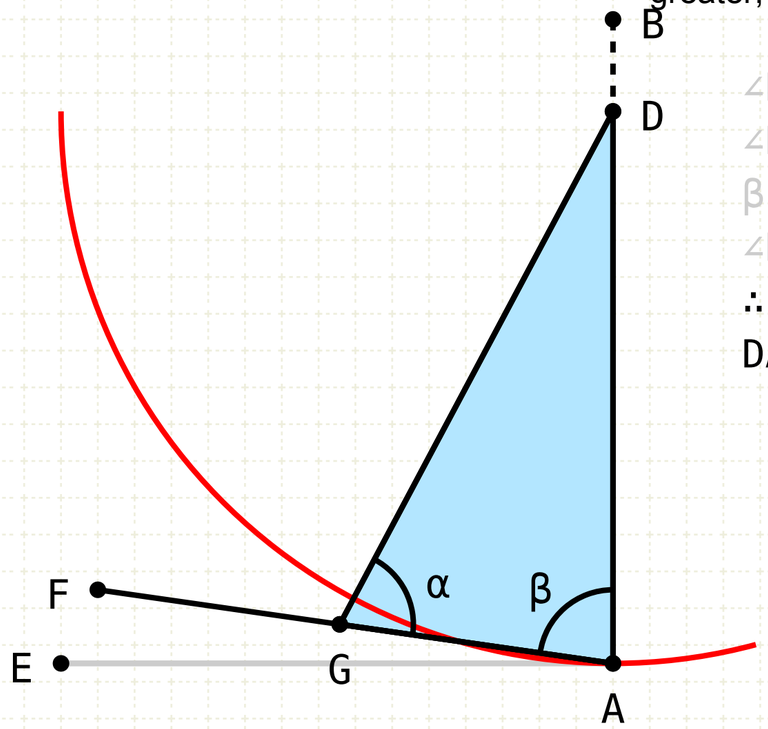
Assume that the line FA lies between the circumference of the circle and the line AE

Draw a perpendicular line from D to line FA, intersecting at the point G

Angle DGA (α) is right, and DAG (β) is less than a right angle, so DGA is greater than DAG

Proposition 16 of Book III

The straight line drawn at right angles to the diameter of a circle from its extremity will fall outside the circle, and into the space between the straight line and the circumference another straight line cannot be interposed; further the angle of the semicircle is greater, and the remaining angle less, than any acute rectilineal angle.



∠DAE = ∟
∠DAF = β < ∠DAE
β < ∟
∠DGA = α = ∟
∴ α > β
DA > DG

In other words

Given a circle, with AB as a diameter, and D as the centre…

(2) No other line from point A can squeeze between the circumference, and the line AE

Proof by Contradiction (2)

Assume that the line FA lies between the circumference of the circle and the line AE

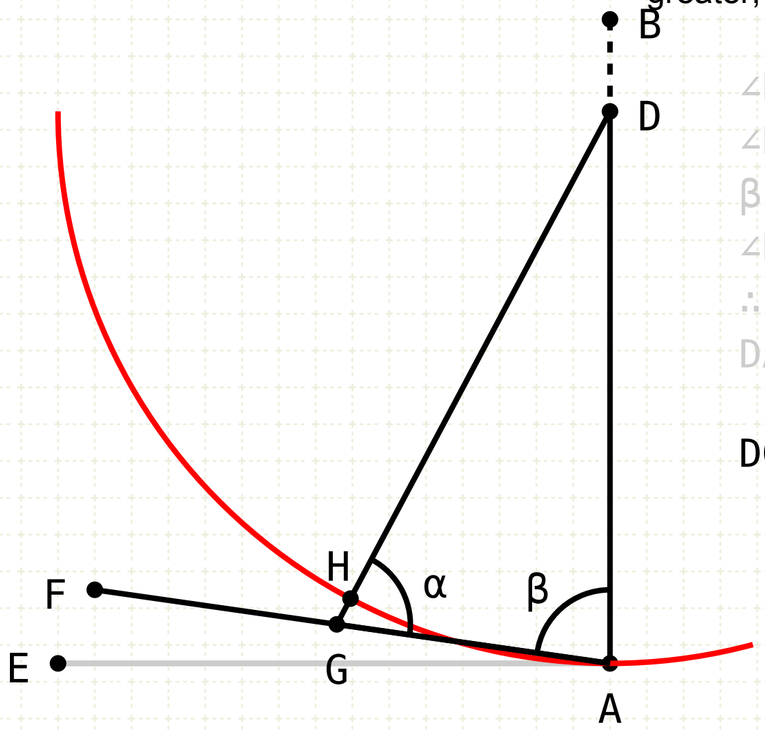
Draw a perpendicular line from D to line FA, intersecting at the point G

Angle DGA (α) is right, and DAG (β) is less than a right angle, so DGA is greater than DAG

In a triangle, the larger line is opposite the larger angle, thus DA is larger than DG (I·19)

Proposition 16 of Book III

The straight line drawn at right angles to the diameter of a circle from its extremity will fall outside the circle, and into the space between the straight line and the circumference another straight line cannot be interposed; further the angle of the semicircle is greater, and the remaining angle less, than any acute rectilineal angle.



∠DAE = ∟
∠DAF = β < ∠DAE
β < ∟
∠DGA = α = ∟
∴ α > β
DA > DG

DG > DH

In other words

Given a circle, with AB as a diameter, and D as the centre…

(2) No other line from point A can squeeze between the circumference, and the line AE

Proof by Contradiction (2)

Assume that the line FA lies between the circumference of the circle and the line AE

Draw a perpendicular line from D to line FA, intersecting at the point G

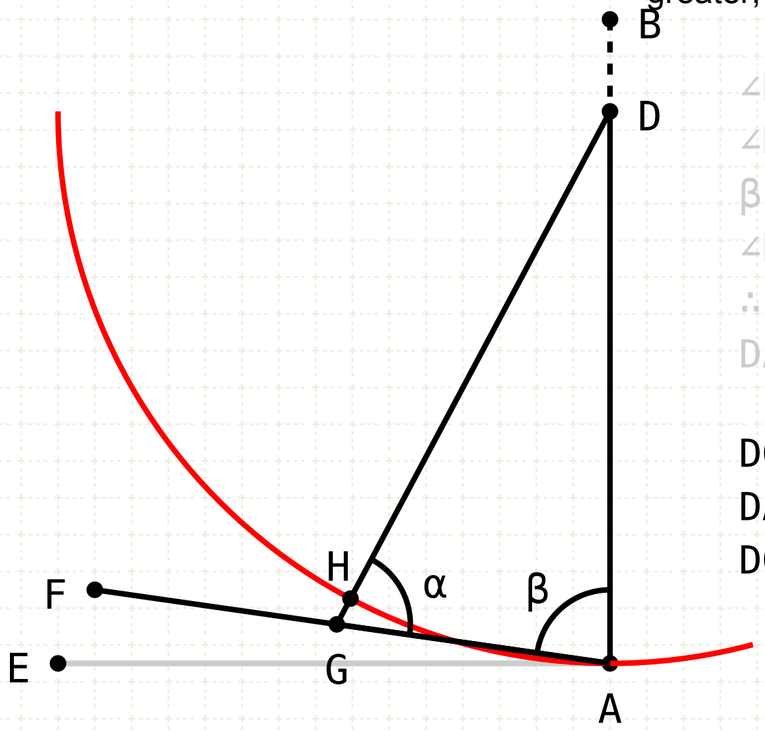
Angle DGA (α) is right, and DAG (β) is less than a right angle, so DGA is greater than DAG

In a triangle, the larger line is opposite the larger angle, thus DA is larger than DG (I·19)

DG is larger than DH

Proposition 16 of Book III

The straight line drawn at right angles to the diameter of a circle from its extremity will fall outside the circle, and into the space between the straight line and the circumference another straight line cannot be interposed; further the angle of the semicircle is greater, and the remaining angle less, than any acute rectilineal angle.



∠DAE = ∟
∠DAF = β < ∠DAE
β < ∟
∠DGA = α = ∟
∴ α > β
DA > DG

DG > DH
DA = DH
DG > DA

In other words

Given a circle, with AB as a diameter, and D as the centre…

(2) No other line from point A can squeeze between the circumference, and the line AE

Proof by Contradiction (2)

Assume that the line FA lies between the circumference of the circle and the line AE

Draw a perpendicular line from D to line FA, intersecting at the point G

Angle DGA (α) is right, and DAG (β) is less than a right angle, so DGA is greater than DAG

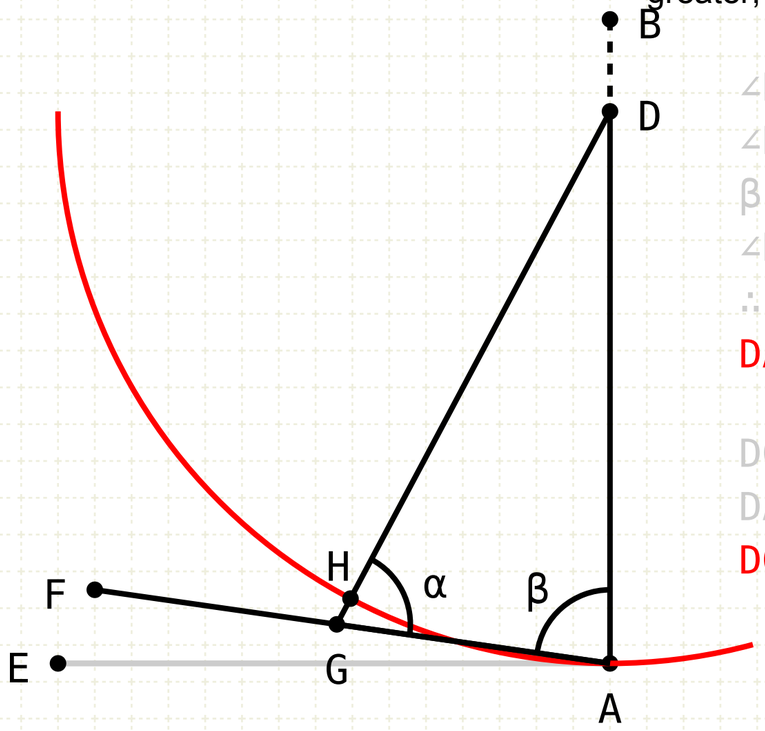
In a triangle, the larger line is opposite the larger angle, thus DA is larger than DG (I·19)

DG is larger than DH

DH and DA are equal (radii of the same circle), thus DG is larger than DA

Proposition 16 of Book III

The straight line drawn at right angles to the diameter of a circle from its extremity will fall outside the circle, and into the space between the straight line and the circumference another straight line cannot be interposed; further the angle of the semicircle is greater, and the remaining angle less, than any acute rectilineal angle.



∠DAE = ∟

∠DAF = β < ∠DAE

β < ∟

∠DGA = α = ∟

∴ α > β

DA > DG

DG > DH

DA = DH

DG > DA

In other words

Given a circle, with AB as a diameter, and D as the centre…

(2) No other line from point A can squeeze between the circumference, and the line AE

Proof by Contradiction (2)

Assume that the line FA lies between the circumference of the circle and the line AE

Draw a perpendicular line from D to line FA, intersecting at the point G

Angle DGA (α) is right, and DAG (β) is less than a right angle, so DGA is greater than DAG

In a triangle, the larger line is opposite the larger angle, thus DA is larger than DG (I·19)

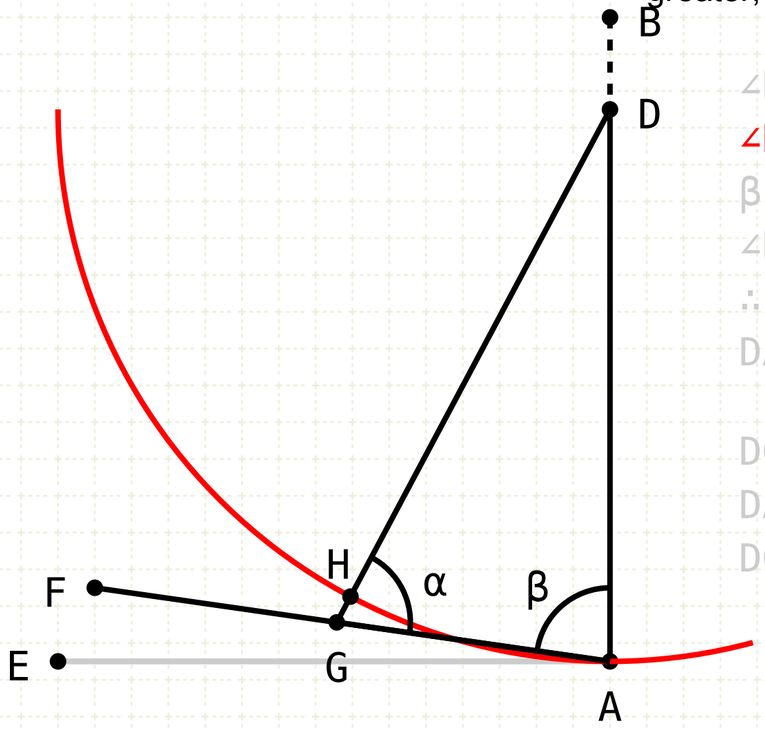
DG is larger than DH

DH and DA are equal (radii of the same circle), thus DG is larger than DA

Thus we have a contradiction

Proposition 16 of Book III

The straight line drawn at right angles to the diameter of a circle from its extremity will fall outside the circle, and into the space between the straight line and the circumference another straight line cannot be interposed; further the angle of the semicircle is greater, and the remaining angle less, than any acute rectilineal angle.



∠DAE = ∟

∠DAF = β < ∠DAE

β < ∟

∠DGA = α = ∟

∴ α > β

DA > DG

DG > DH

DA = DH

DG > DA

In other words

Given a circle, with AB as a diameter, and D as the centre…

(2) No other line from point A can squeeze between the circumference, and the line AE

Proof by Contradiction (2)

Assume that the line FA lies between the circumference of the circle and the line AE

Draw a perpendicular line from D to line FA, intersecting at the point G

Angle DGA (α) is right, and DAG (β) is less than a right angle, so DGA is greater than DAG

In a triangle, the larger line is opposite the larger angle, thus DA is larger than DG (I·19)

DG is larger than DH

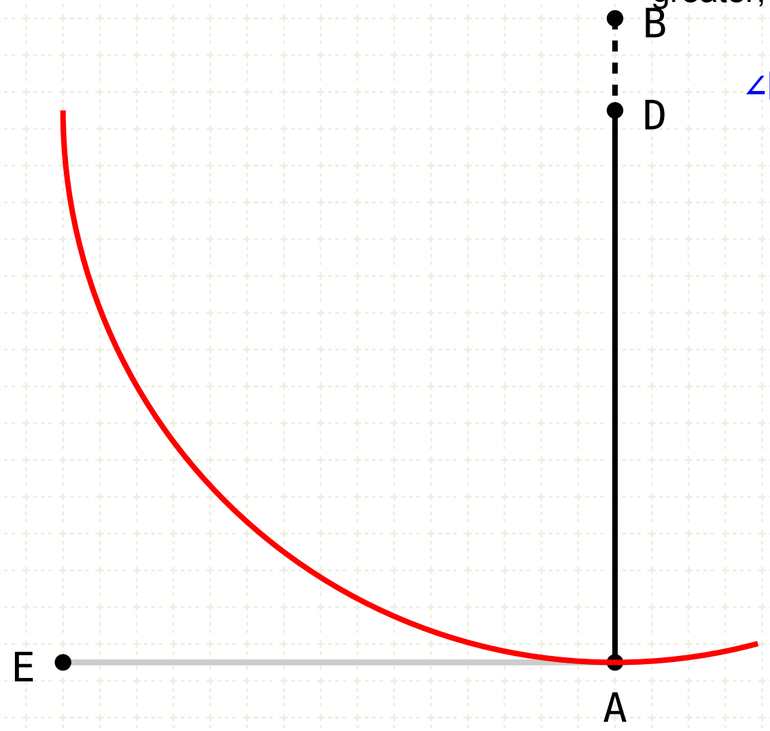
DH and DA are equal (radii of the same circle), thus DG is larger than DA

Thus we have a contradiction

The line FA cannot exist between the line AE and the circumference

Proposition 16 of Book III

The straight line drawn at right angles to the diameter of a circle from its extremity will fall outside the circle, and into the space between the straight line and the circumference another straight line cannot be interposed; further the angle of the semicircle is greater, and the remaining angle less, than any acute rectilineal angle.



In other words

Given a circle, with AB as a diameter, and D as the centre…

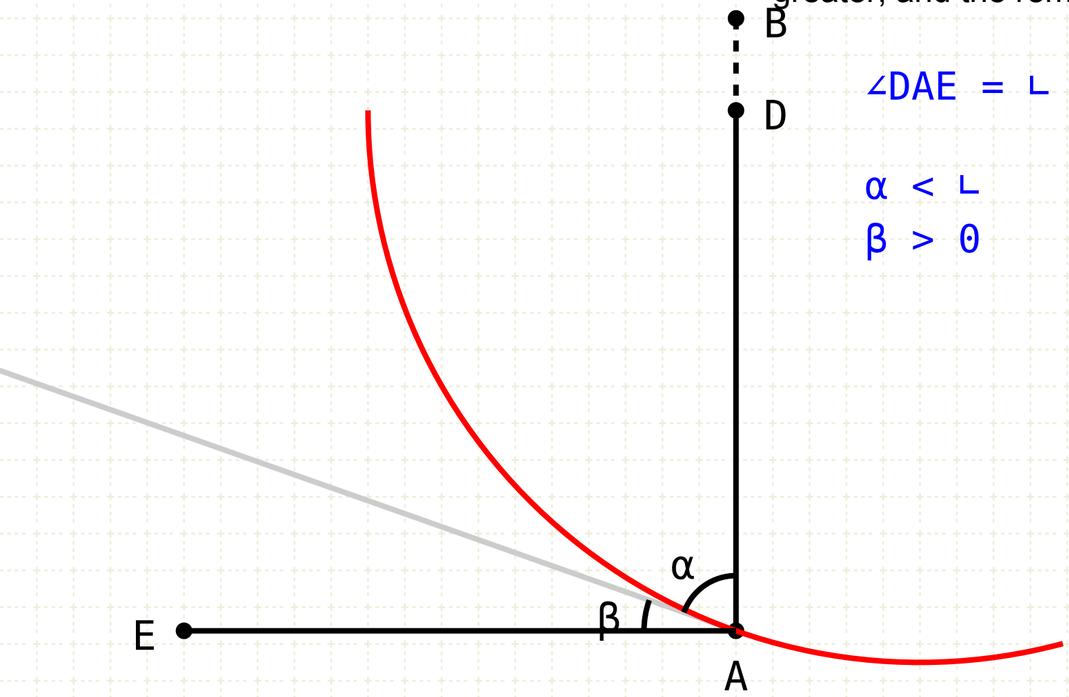
(3) - The angle between AB and the circumference of the circle cannot be less than a right angle,

- and the angle between AE and the circumference cannot be less than zero

Proof by Contradiction (3)

Proposition 16 of Book III

The straight line drawn at right angles to the diameter of a circle from its extremity will fall outside the circle, and into the space between the straight line and the circumference another straight line cannot be interposed; further the angle of the semicircle is greater, and the remaining angle less, than any acute rectilineal angle.



In other words

Given a circle, with AB as a diameter, and D as the centre…

(3) - The angle between AB and the circumference of the circle cannot be less than a right angle,

- and the angle between AE and the circumference cannot be less than zero

Proof by Contradiction (3)

Assume that the angle between AB and the circumference is less than a right angle, and that the angle between AE and the circumference is greater than zero

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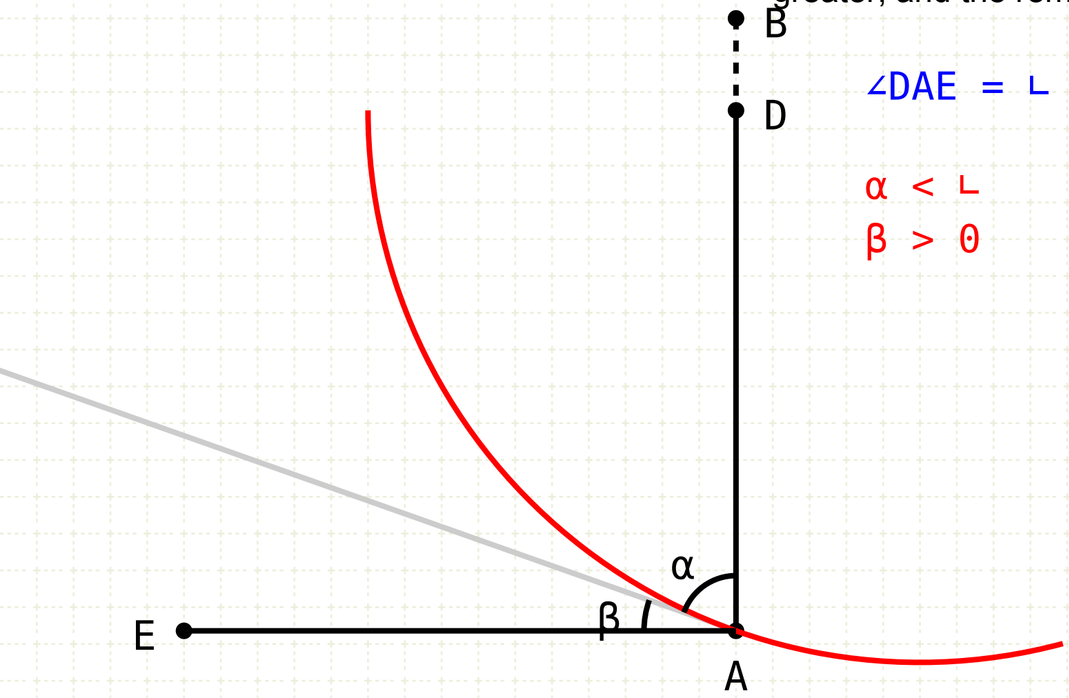

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Then the tangent (grey line) can be inserted between the circumference and the line EA, which was just proven to be impossible

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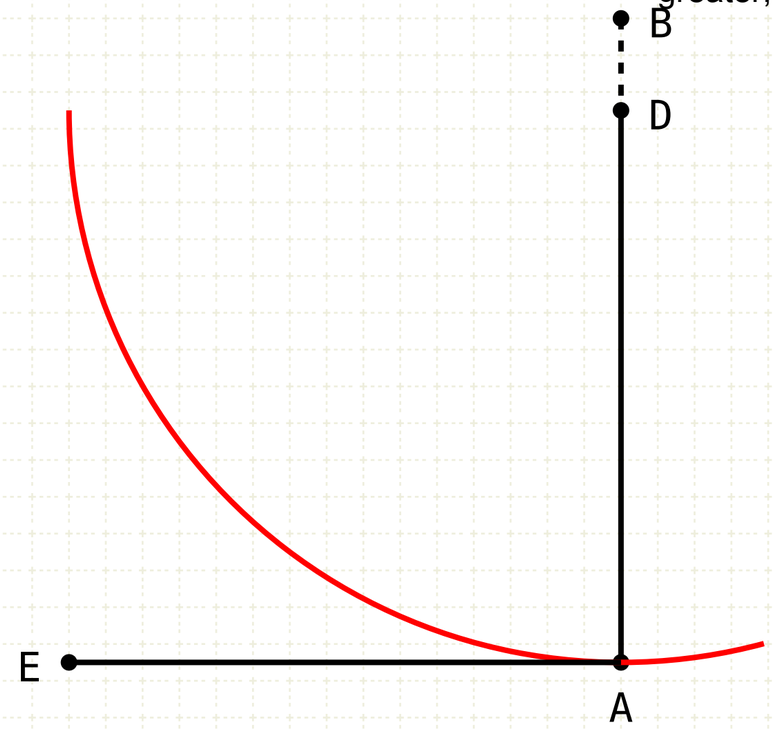

In other words

Given a circle, with AB as a diameter, and D as the centre…

Draw a line AE from point A, perpendicular to AB

(1) The Line AE falls outside of the circle

(2) No other line from point A can squeeze between the circumference, and the line AE

(3) - The angle between AB and the circumference of the circle cannot be less than a right angle,

- and the angle between AE and the circumference cannot be greater than zero

PORISM

A straight line drawn at right angles to the diameter of a circle, from one of the endpoints, touches the circle

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