Euclid's Elements

Book VI



One can state, without exaggeration, that the observation of and the search for similarities and differences are the basis of all human knowledge.

Alfred Nobel



Table of Contents, Chapter 6

- If the height of two triangles are equal, then the ratio of the areas is equal to the ratio of the bases
- If a line cuts a triangle, parallel to its base, it will cut the sides of the triangle proportionally
- If an angle of a triangle is bisected and the straight line cutting the angle also cuts the base, the segments of the base will have the same ratio as the remaining sides of the triangle
- If two triangles have equal angles, then the sides opposite the equal angles are proportional, as well, the sides of the triangles on either side of the equal angles are also proportional
- 5 It two triangles have proportional sides, the triangles will be equiangular
- If two triangles have one angle equal to one angle and the sides about the equal angles are proportional, then the triangles will be equiangular

- 7 If two triangles have one angle equal to one angle, and the sides about other angles are proportional, and the remaining angles either both less or both not less than a right angle, then triangles will be equiangular
- If in a right-angled triangle a perpendicular be drawn from the right angle to the base, the triangles adjoining the perpendicular are similar both to the whole and to one another
- 9 From a given straight line to cut off a given fraction
- 10 To cut a given uncut straight line similarly to a given cut straight line
- 11 To two given straight lines to find a third proportional
- 12 To three given straight lines to find a fourth proportional
- 13 To two given straight lines to find a mean proportional

- 14 In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and vice versa
- 15 In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and vice versa
- 16 If four straight lines are proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means, and vice versa
- 17 If three straight lines are proportional, the rectangle contained by the extremes is equal to the square on the mean; and vice versa
- 18 On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure
- 19 Similar triangles are to one another in the duplicate ratio of the corresponding sides



Table of Contents, Chapter 3

- 20 Similar polygons are divided into the same number of similar triangles, which have the same ratio as the wholes, and the polygons have duplicate ratios to their corresponding sides
- 21 Figures which are are similar to the same rectilineal figure are also similar to one another
- 22 If four straight lines are proportional, similar rectilineal figures will also be proportional; and vice versa
- 23 Equiangular parallelograms have to one another the ratio compounded of the ratios of their sides
- 24 In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another
- 25 To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure

- 26 If from a parallelogram a similar parallelogram with a common angle is subtracted, it is about the same diameter as the original
- 27 Of all the parallelograms applied to the same straight line and deficient by parallelogrammic figures similar to a parallelogram drawn on half the said line, the largest will be one that is drawn on half of the straight line and is similar to the defect
- 28 To a given straight line, apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one
- 29 To a given straight line, apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one
- 30 To cut a finite straight line in extreme ratio

In right-angled triangles the figure on the side subtending the right angle is equal to the similar and similarly described figures on the sides containing the right angle



Proposition 13 of Book VI To two given straight lines to find a mean proportional.



To two given straight lines to find a mean proportional.





$$A:D = D:B$$

In other words

Given two lines A and B, find a third D, such that the ratio of A to D is equal to the ratio D to B

To two given straight lines to find a mean proportional.



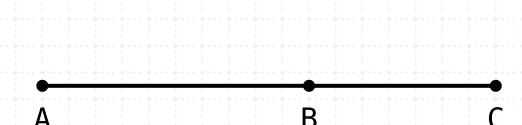
Given two lines A and B, find a third D, such that the ratio of A to D is equal to the ratio D to B

Construction





To two given straight lines to find a mean proportional.



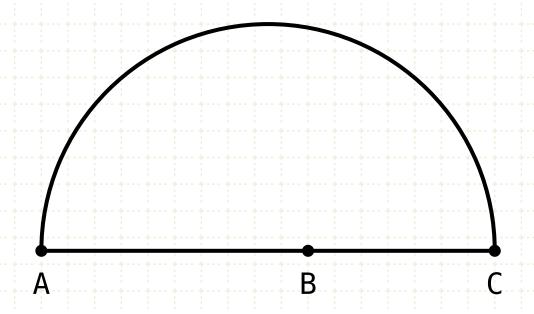
In other words

Given two lines A and B, find a third D, such that the ratio of A to D is equal to the ratio D to B

Construction

Position the two lines so that they form a straight line

To two given straight lines to find a mean proportional.



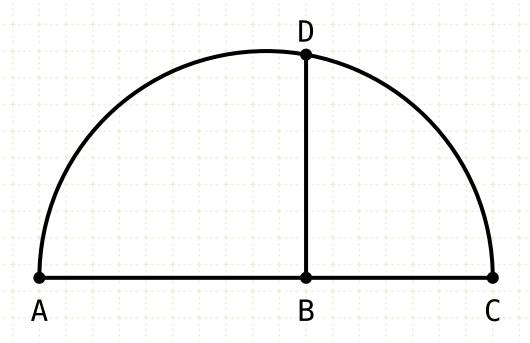
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Construction

Position the two lines so that they form a straight line Draw a semi-circle on line AC

To two given straight lines to find a mean proportional.



In other words

Given two lines A and B, find a third D, such that the ratio of A to D is equal to the ratio D to B

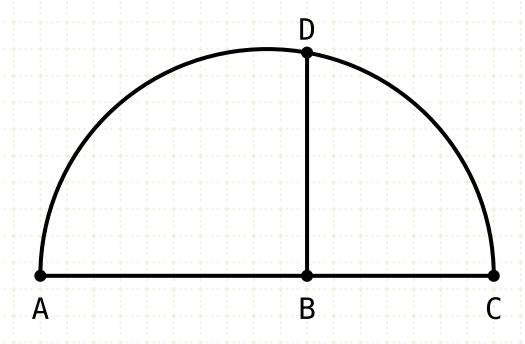
Construction

Position the two lines so that they form a straight line

Draw a semi-circle on line AC

Draw a line perpendicular to AC, from point B, intersecting the semi-circle at point D

To two given straight lines to find a mean proportional.



AB:BD = BD:BC

In other words

Given two lines A and B, find a third D, such that the ratio of A to D is equal to the ratio D to B

Construction

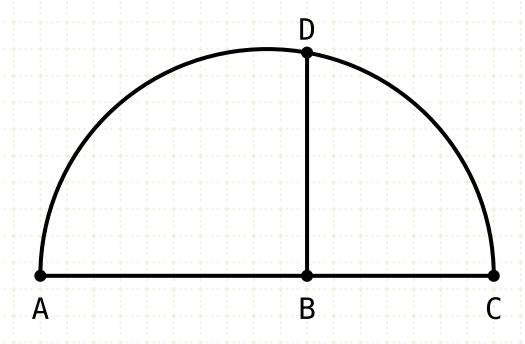
Position the two lines so that they form a straight line

Draw a semi-circle on line AC

Draw a line perpendicular to AC, from point B, intersecting the semi-circle at point D

BD is the mean proportional to AB, BC

To two given straight lines to find a mean proportional.



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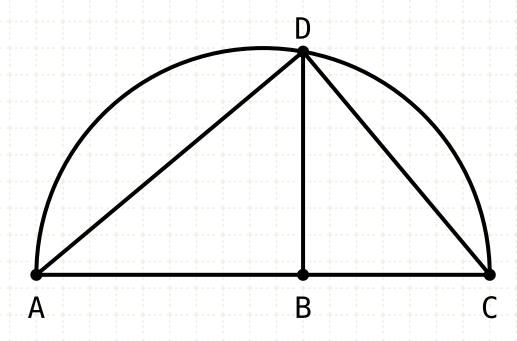
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Proof

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Construction

Position the two lines so that they form a straight line

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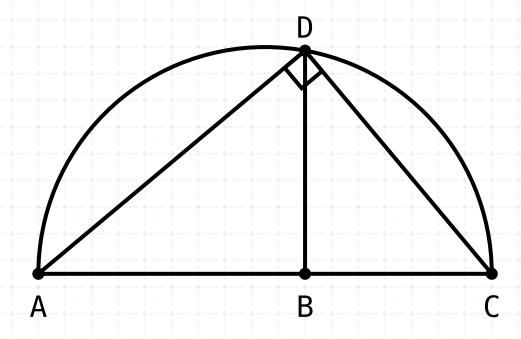
Draw a line perpendicular to AC, from point B, intersecting the semi-circle at point D

BD is the mean proportional to AB, BC

Proof

Join points AD, and CD

To two given straight lines to find a mean proportional.



AB:BD = BD:BC

In other words

Given two lines A and B, find a third D, such that the ratio of A to D is equal to the ratio D to B

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Draw a semi-circle on line AC

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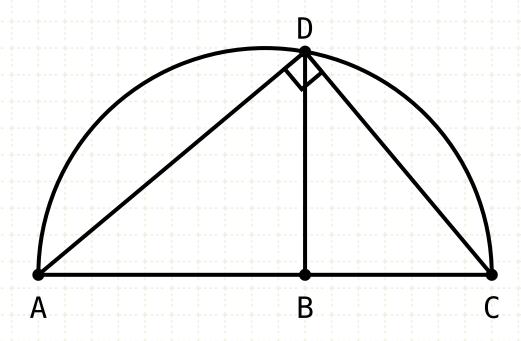
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Proof

Join points AD, and CD

The angle ADC is a right angle (III-31)

To two given straight lines to find a mean proportional.



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Given two lines A and B, find a third D, such that the ratio of A to D is equal to the ratio D to B

Construction

Position the two lines so that they form a straight line

Draw a semi-circle on line AC

Draw a line perpendicular to AC, from point B, intersecting the semi-circle at point D

BD is the mean proportional to AB, BC

Proof

Join points AD, and CD

The angle ADC is a right angle (III-31)

Since DB has been drawn from a right angle triangle to its base, DB is the mean proportional to AB, BC (VI·8.Por)

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