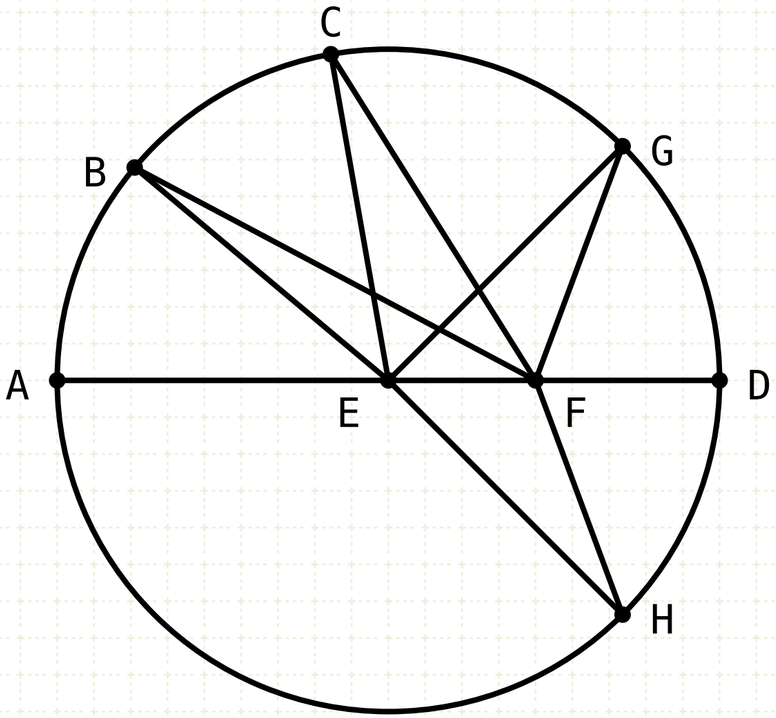
Euclid's Elements

Book III



A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)

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Proposition 21 of Book III

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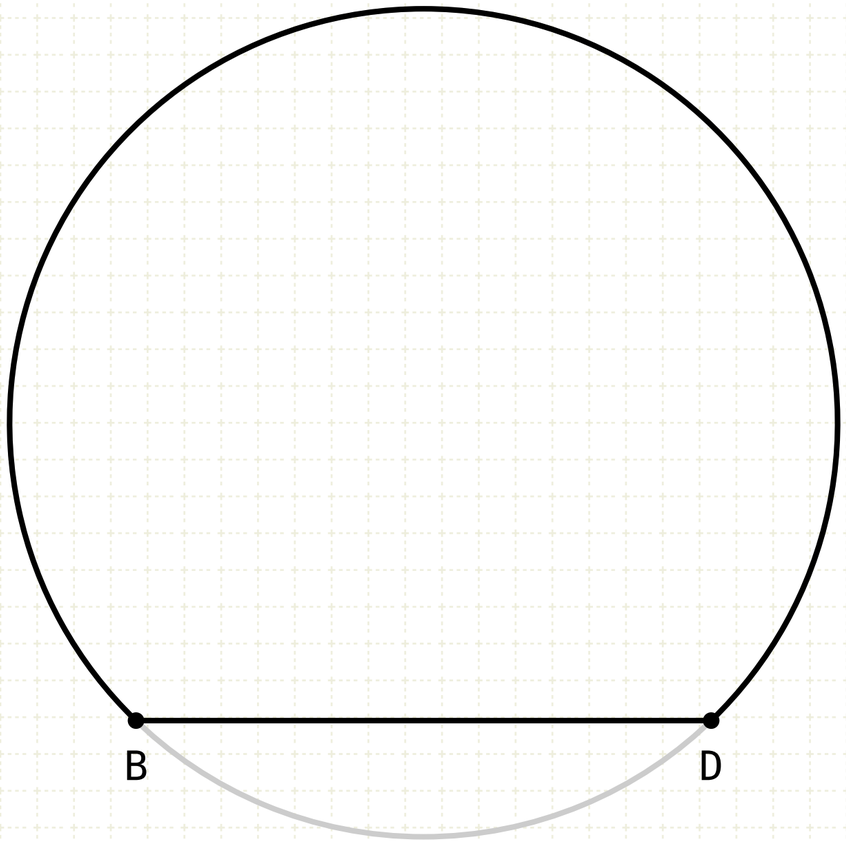

Definition - Segment of a Circle

A 'segment of a circle' is the figure contained by a straight line and a circumference of a circle

Proposition 21 of Book III

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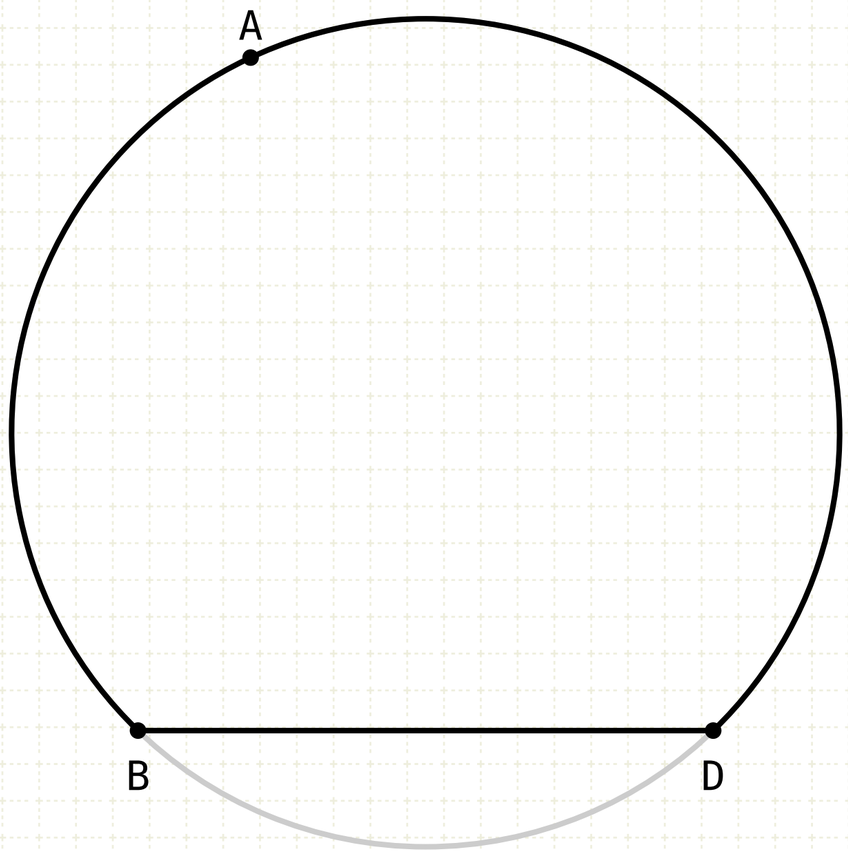

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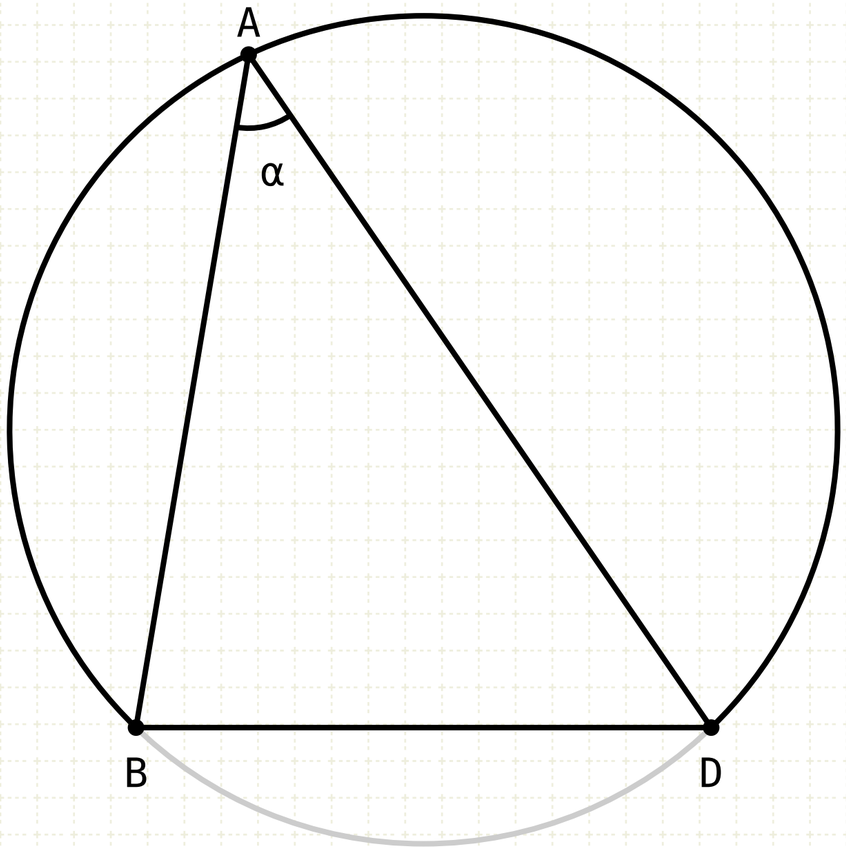

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Definition - Angles in a Segment

An 'angle in a segment' is the angle which,

- when a point is taken on the circumference of the segment (point A) and
- straight lines are joined from it to the extremities of the straight line (BD) which is the 'base of the segment',

is contained by the straight lines so joined (angle α)

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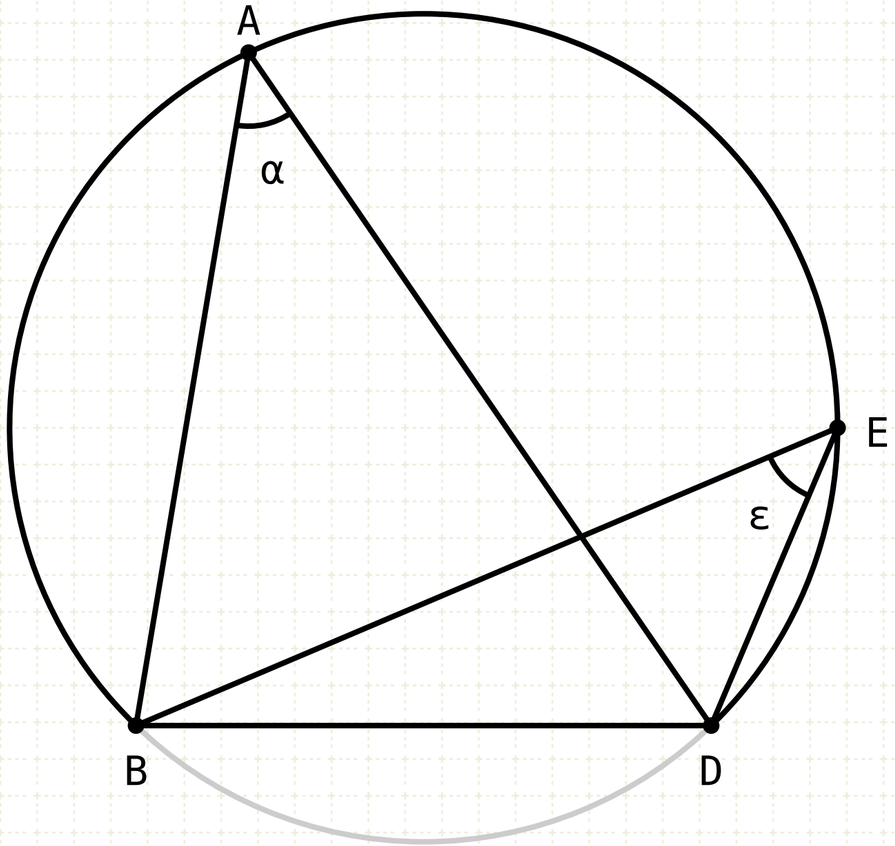

$$\varepsilon = \alpha$$

In other words

If we have a circle segment BAD, any angle formed by lines from a given point on the circumference of the circle to the points B and D will be equal to any other angle formed in the same fashion

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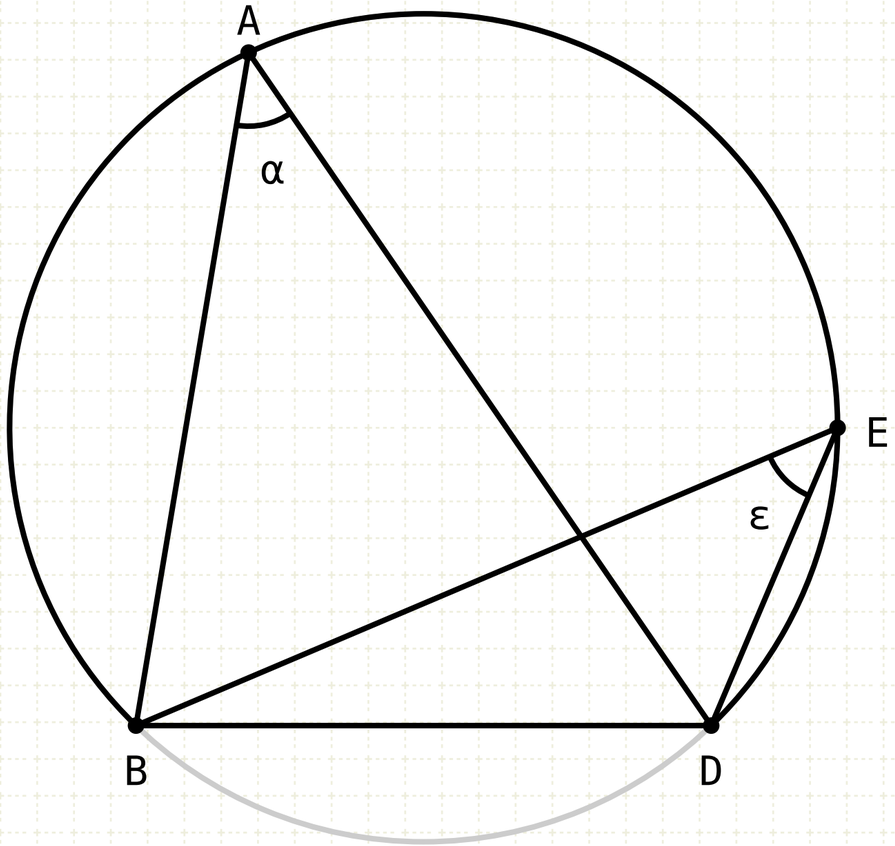

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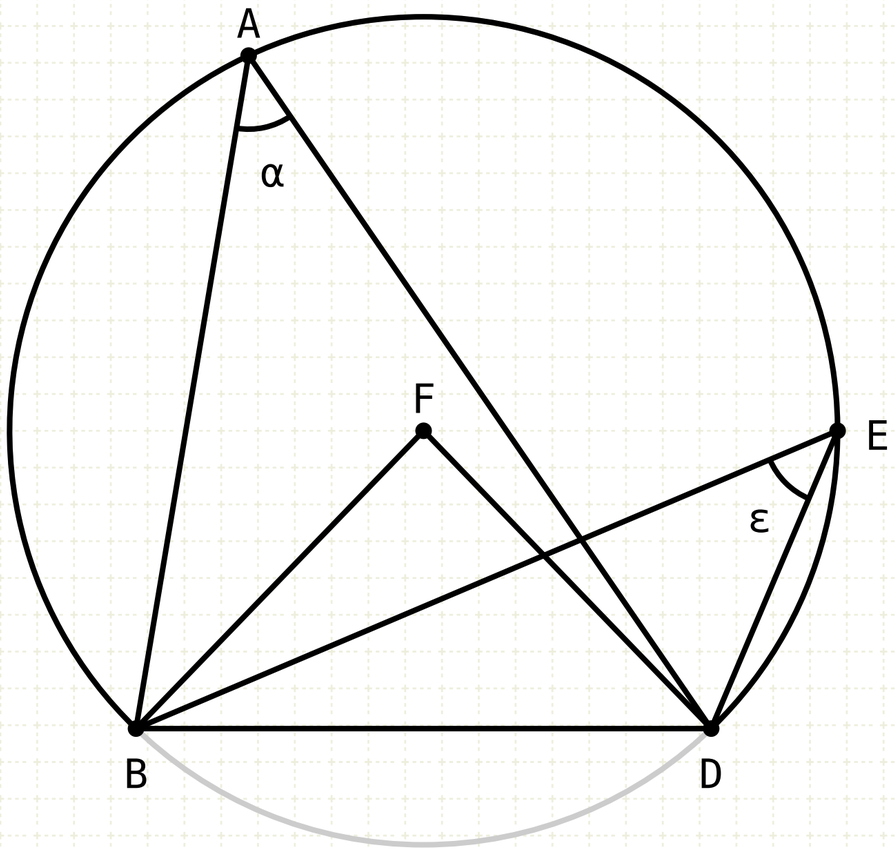

F is the centre of the circle

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Let F be the centre of the circle, and join lines FB and FD

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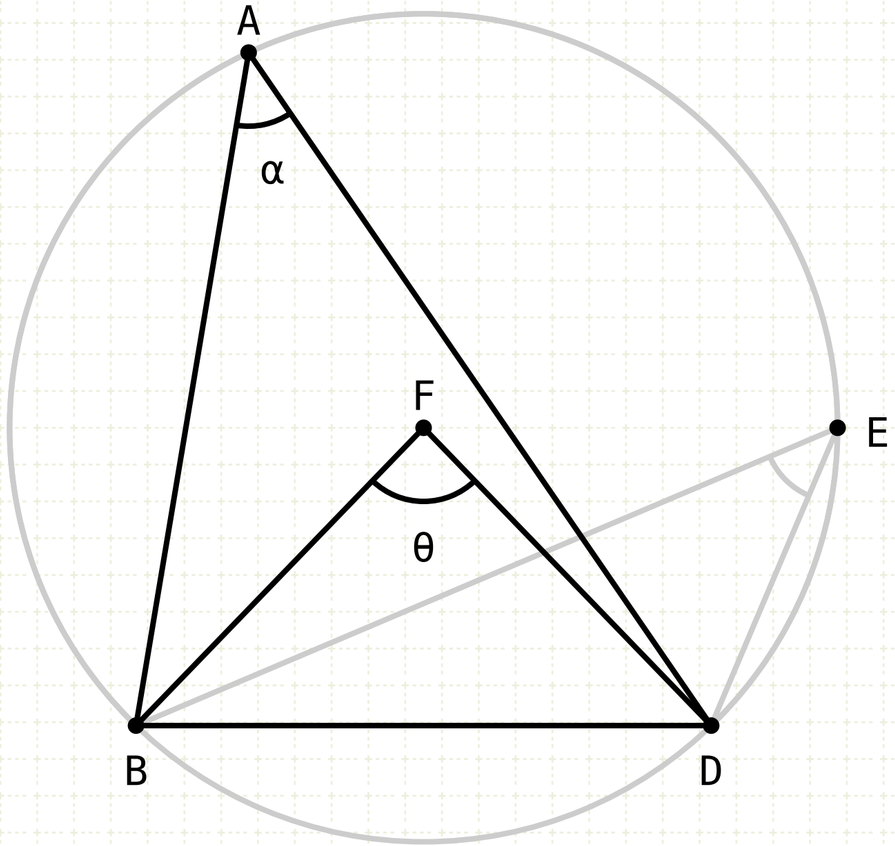

F is the centre of the circle

$\theta = 2\alpha$

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Let F be the centre of the circle, and join lines FB and FD

The angle BFD (θ) formed from the centre has the same base as BAD (α) formed from the circumference of the circle,

Therefore BFD equals twice BAD (III·20)

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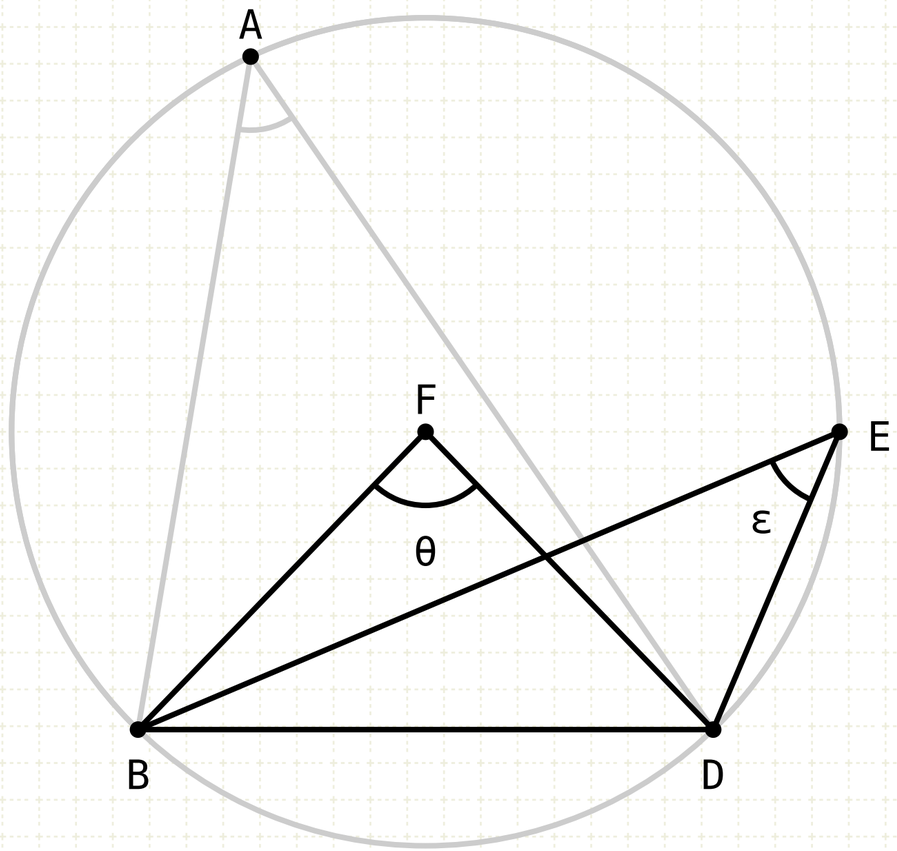

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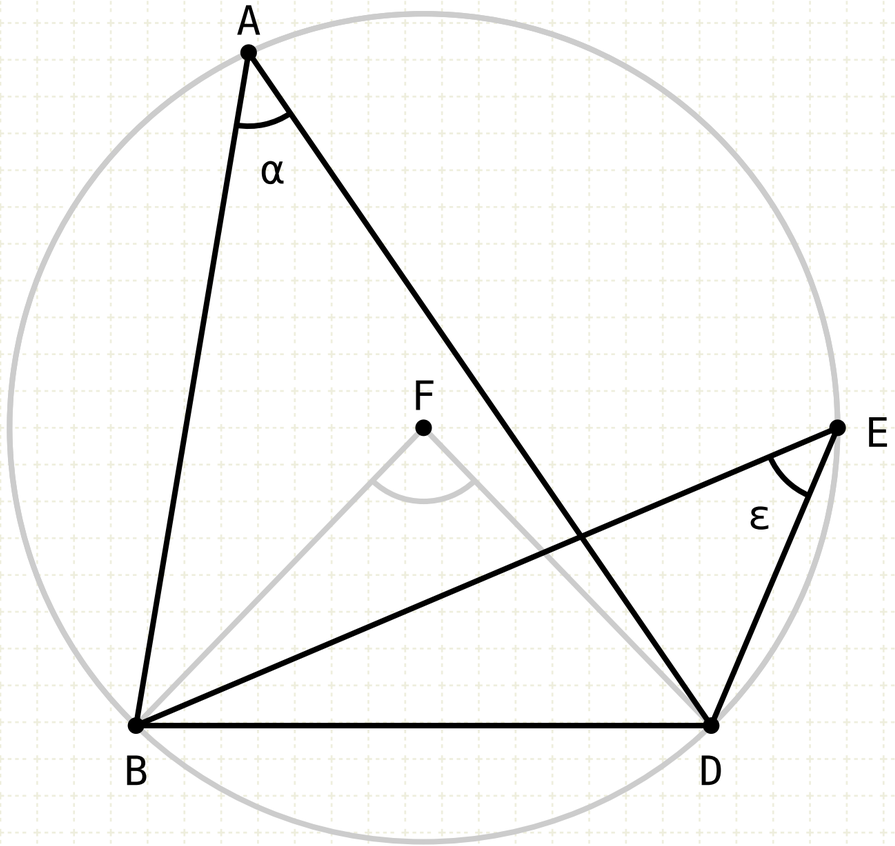

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Similarly, the angle BFD equals twice BED

Thus, BAD (α) and BED (ε) are equal

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