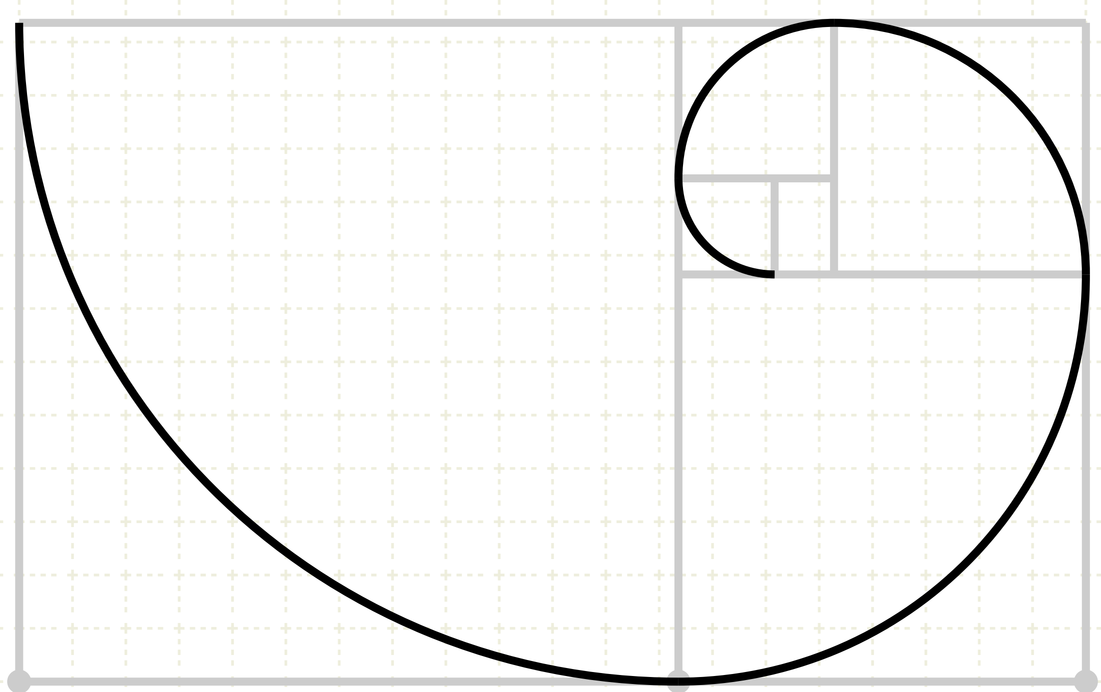
# Euclid's Elements

# Book VI



*One can state, without exaggeration, that the observation of and the search for similarities and differences are the basis of all human knowledge.*

**Alfred Nobel**

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# Table of Contents, Chapter 3

# Proposition 24 of Book VI

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.

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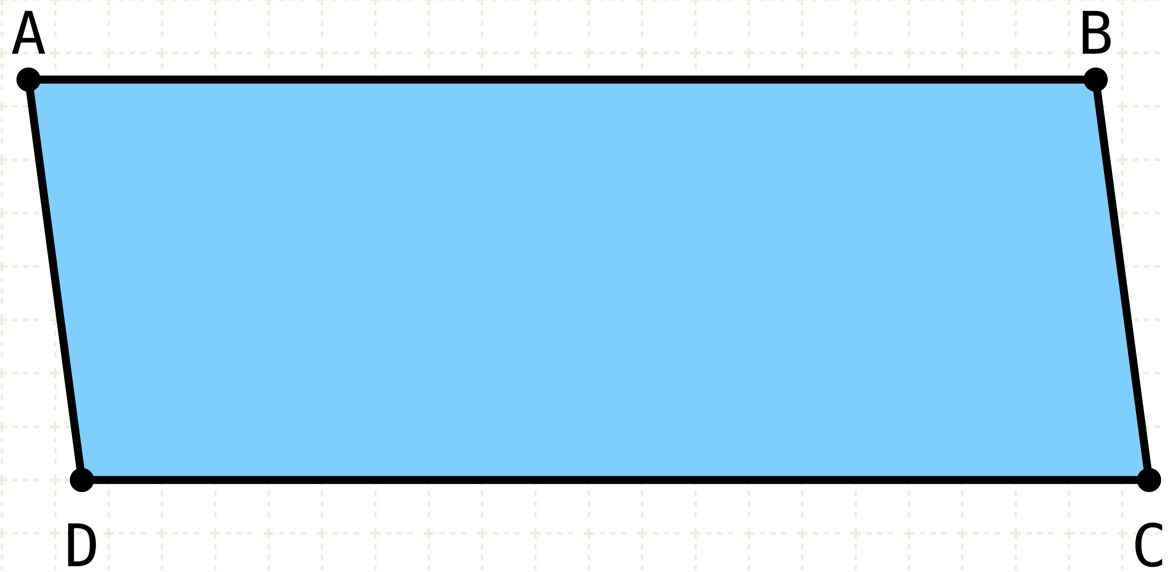
In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.

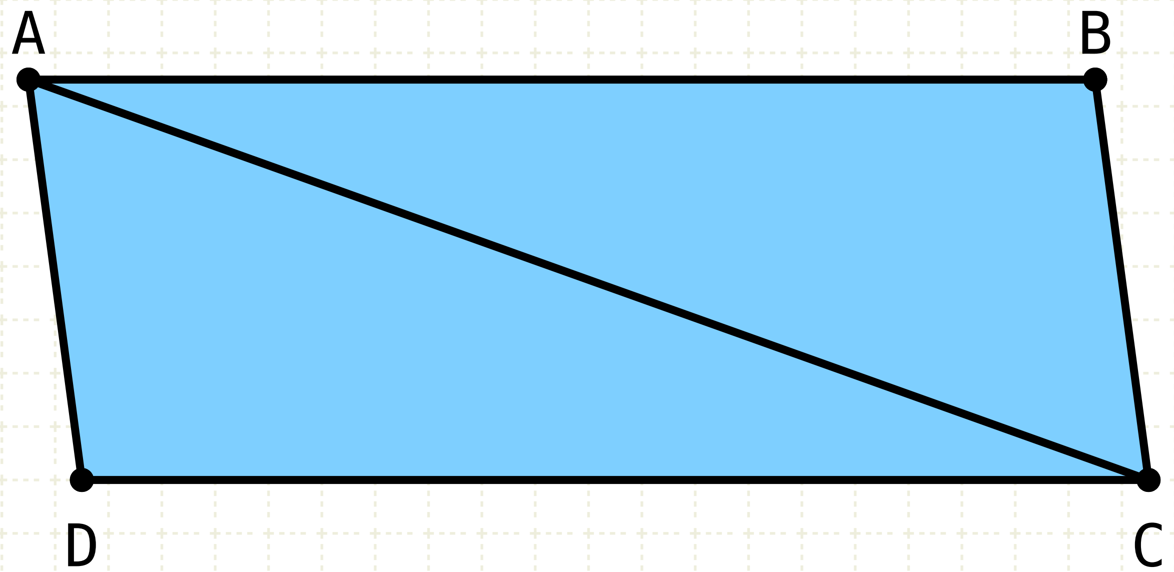



## In other words

Start with a parallelogram

# Proposition 24 of Book VI

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.



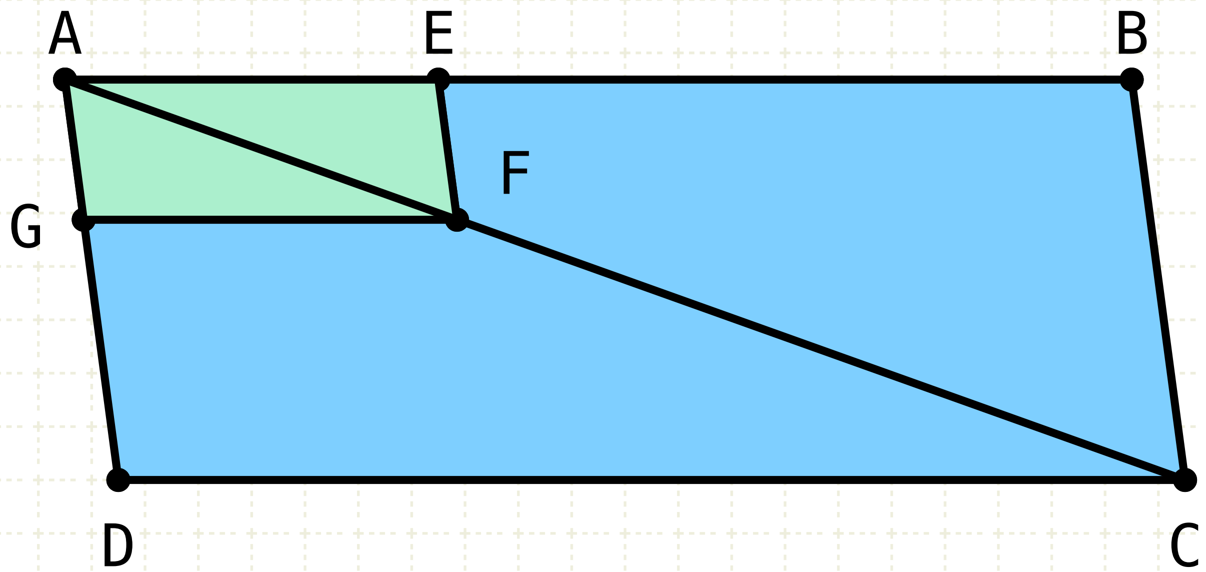

## In other words

Start with a parallelogram

Draw the diameter AC

# Proposition 24 of Book VI

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.




## In other words

Start with a parallelogram

Draw the diameter AC

Construct a parallelogram EG on the diameter AC

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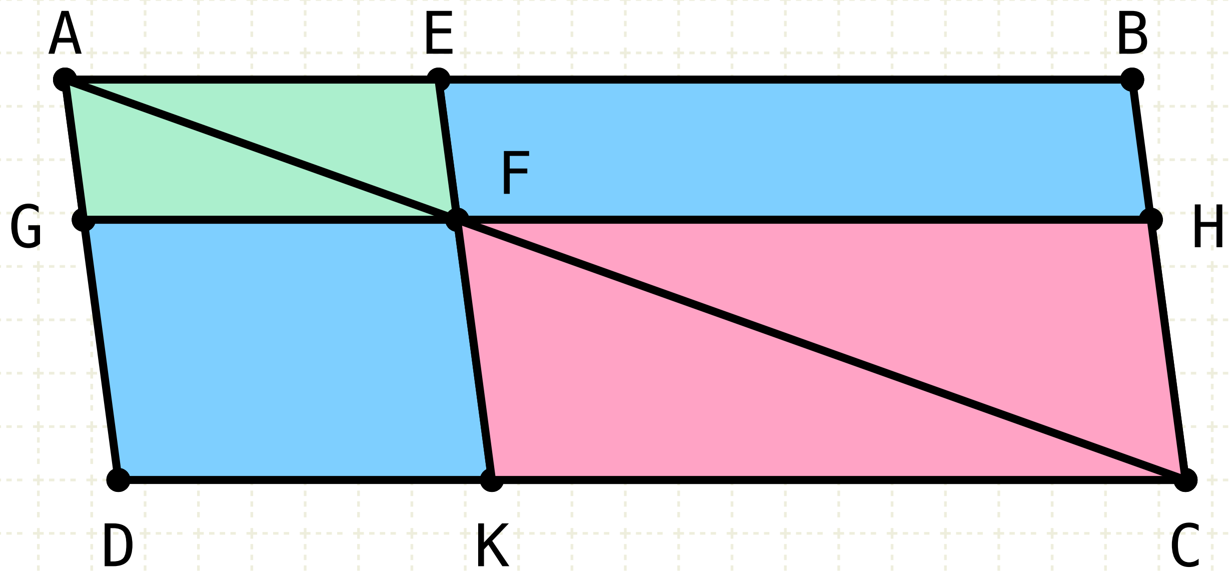



## In other words

Start with a parallelogram

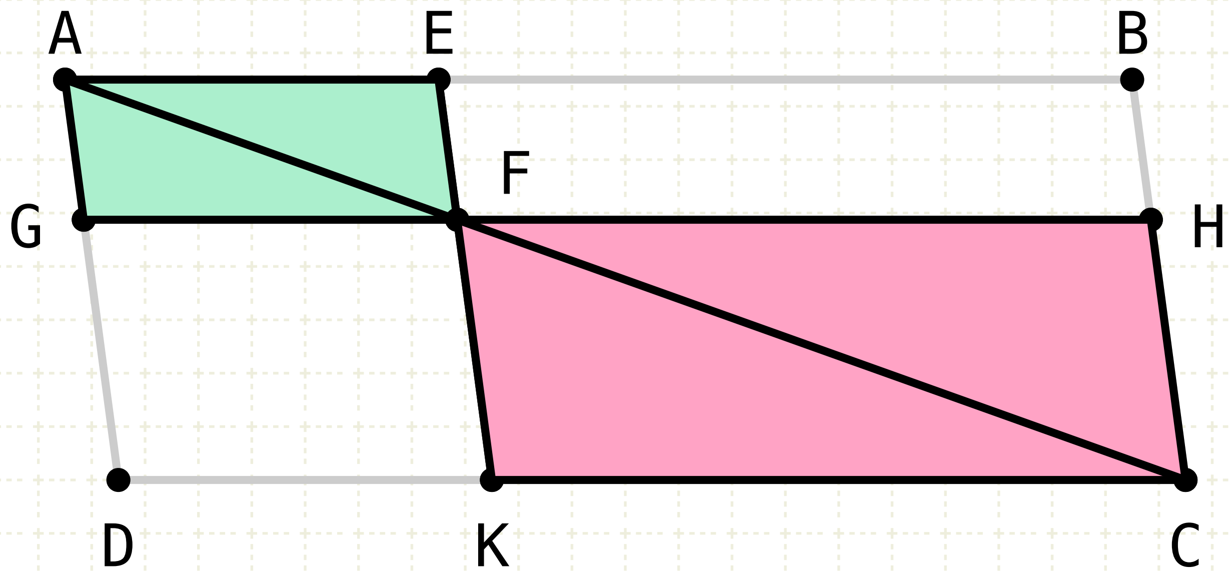
Draw the diameter AC

Construct a parallelogram EG on the diameter AC

Construct a parallelogram HK on the diameter AC

# Proposition 24 of Book VI

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.




□AC ~ □EG ~ □HK

## In other words

Start with a parallelogram

Draw the diameter AC

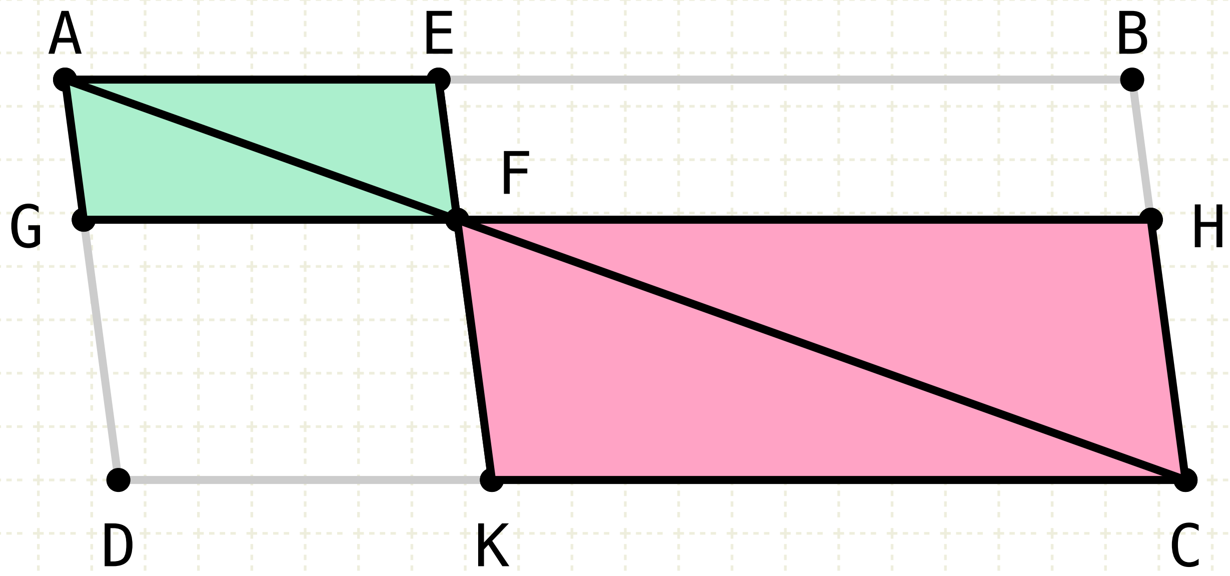
Construct a parallelogram EG on the diameter AC

Construct a parallelogram HK on the diameter AC

The resulting parallelograms will all be similar to one another

# Proposition 24 of Book VI

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.




## Proof

# Proposition 24 of Book VI

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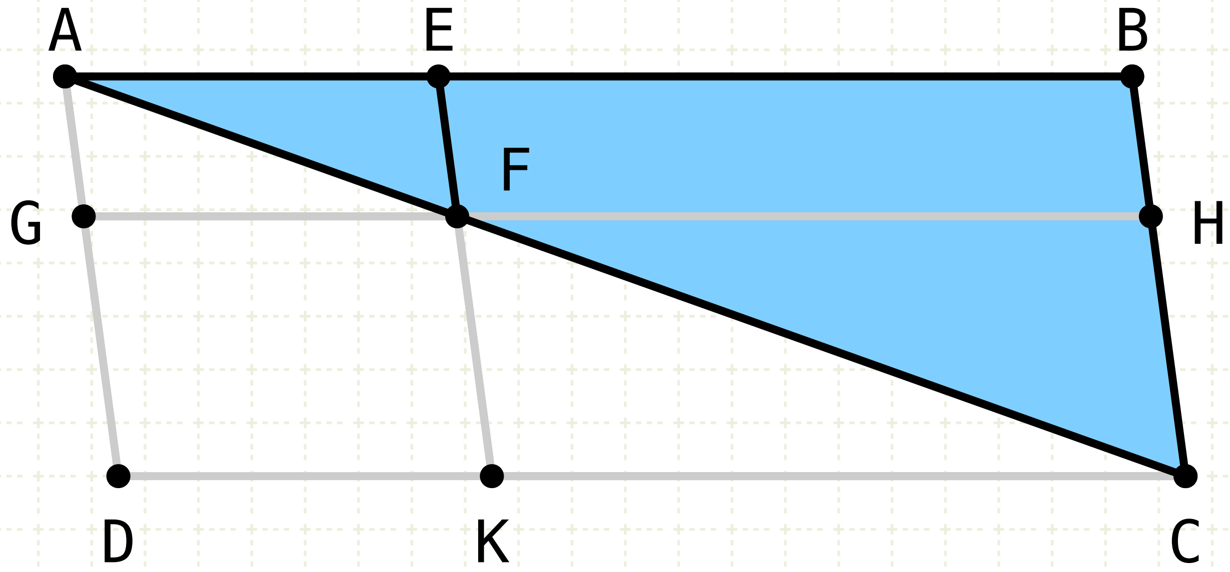


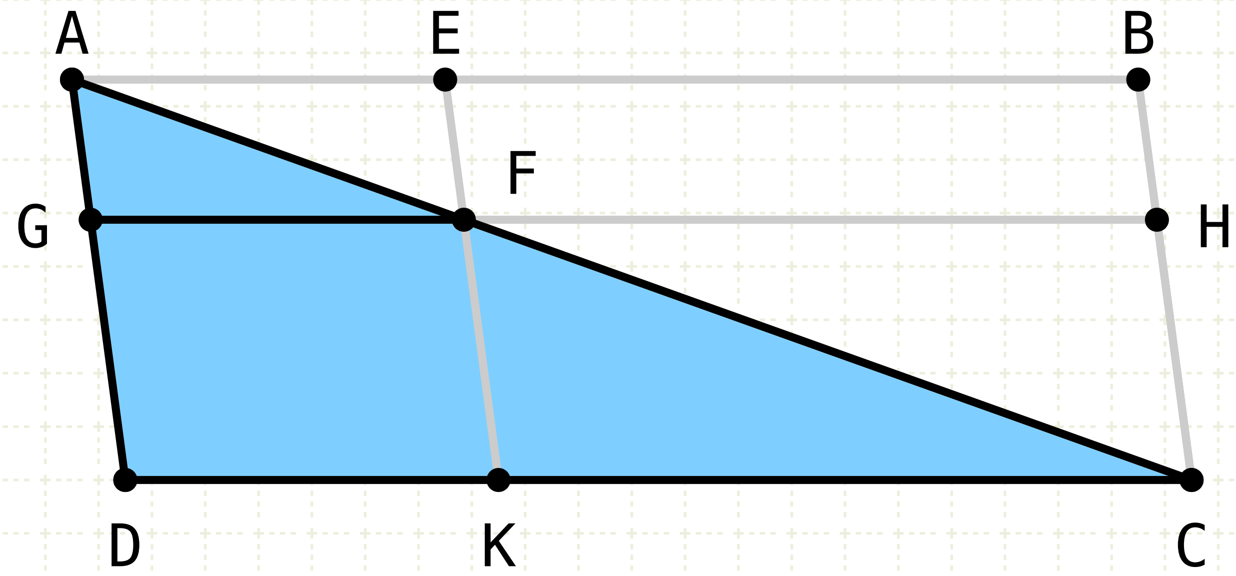

BE:EA = CF:FA

## Proof

In the triangle ABC, EF and BC are parallel, thus BE is to EA as CF is to FA (VI·2)

# Proposition 24 of Book VI

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.




BE:EA = CF:FA

CF:FA = DG:GA

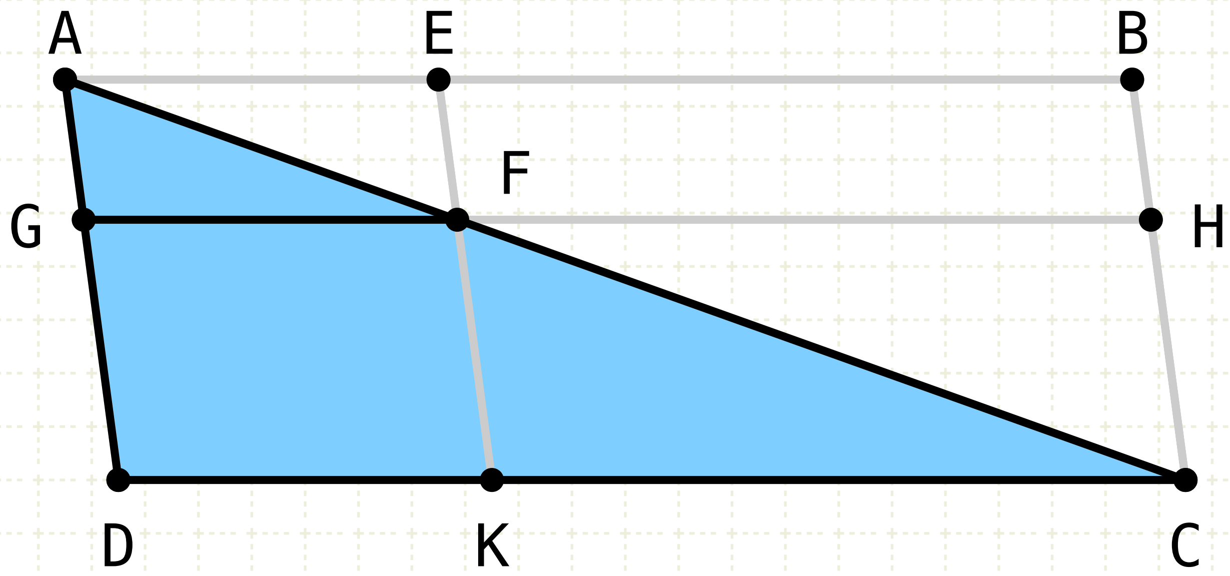
## Proof

In the triangle ABC, EF and BC are parallel, thus BE is to EA as CF is to FA (VI·2)

In the triangle ACD, FG and CD are parallel, thus CF is to FA as DG is to GA (VI·2)

# Proposition 24 of Book VI

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.




BE:EA = CF:FA

CF:FA = DG:GA

BE:EA = DG:GA

## Proof

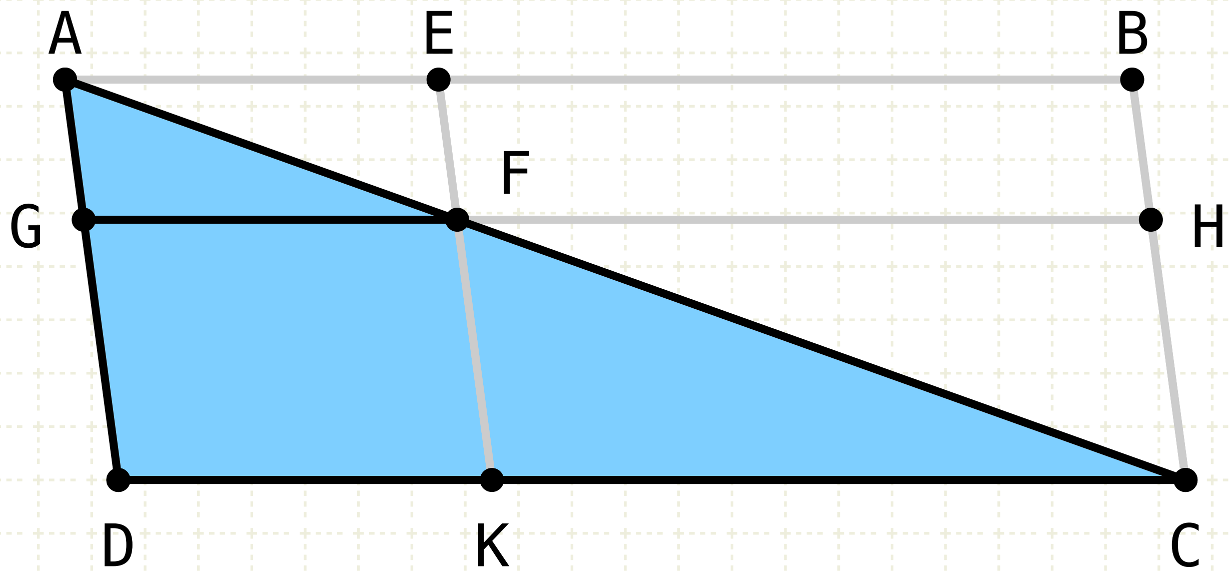
In the triangle ABC, EF and BC are parallel, thus BE is to EA as CF is to FA (VI·2)

In the triangle ACD, FG and CD are parallel, thus CF is to FA as DG is to GA (VI·2)

Thus, BE is to EA as DG is to GA

# Proposition 24 of Book VI

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.




BE:EA = CF:FA

CF:FA = DG:GA

BE:EA = DG:GA

(BE+EA):EA = (DG+GA):GA

BA:EA = DA:GA

## Proof

In the triangle ABC, EF and BC are parallel, thus BE is to EA as CF is to FA (VI·2)

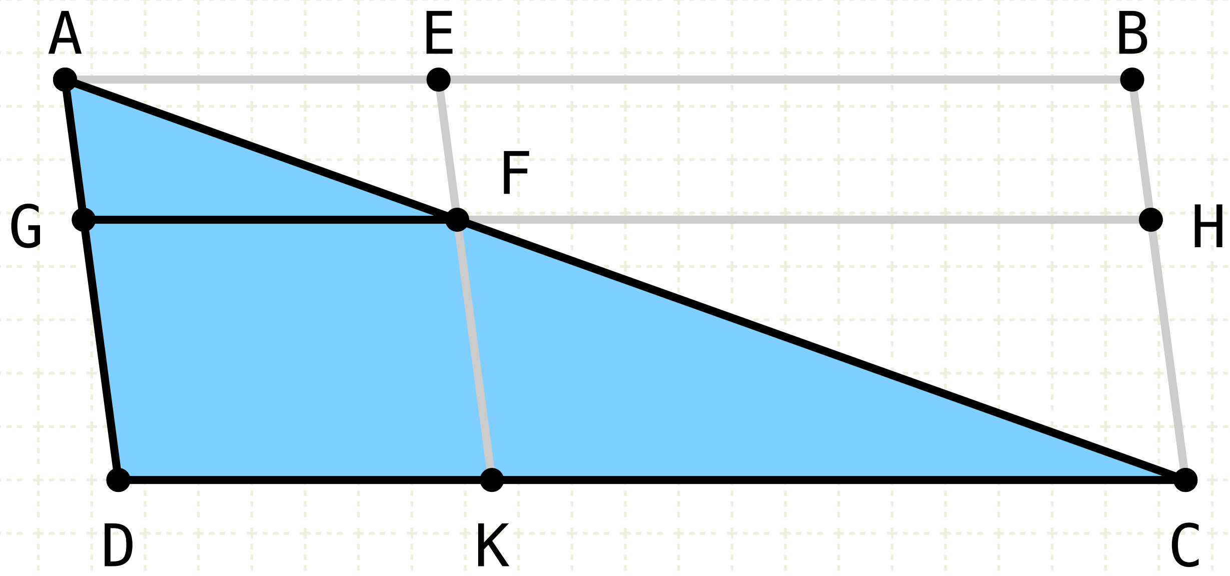
In the triangle ACD, FG and CD are parallel, thus CF is to FA as DG is to GA (VI·2)

Thus, BE is to EA as DG is to GA

Therefore (componendo) BA is to EA as DA is to AG (V·18)

# Proposition 24 of Book VI

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.




BE:EA = CF:FA

CF:FA = DG:GA

BE:EA = DG:GA

(BE+EA):EA = (DG+GA):GA

BA:EA = DA:GA

BA:DA = EA:GA

## Proof

In the triangle ABC, EF and BC are parallel, thus BE is to EA as CF is to FA (VI·2)

In the triangle ACD, FG and CD are parallel, thus CF is to FA as DG is to GA (VI·2)

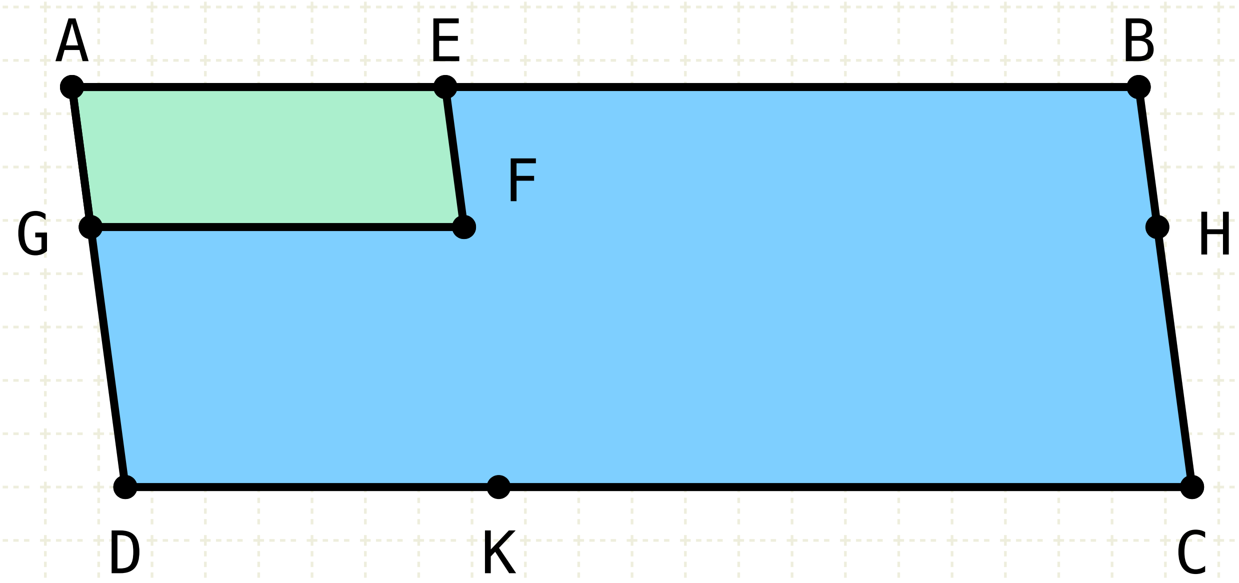
Thus, BE is to EA as DG is to GA

Therefore (componendo) BA is to EA as DA is to AG (V·18)

and alternately BA is to DA so is AE to AG (V·16)

# Proposition 24 of Book VI

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.




BA:DA = EA:GA

## Proof (cont)

For the parallelograms ABCD, EG, the sides about the common angle BAD are proportional

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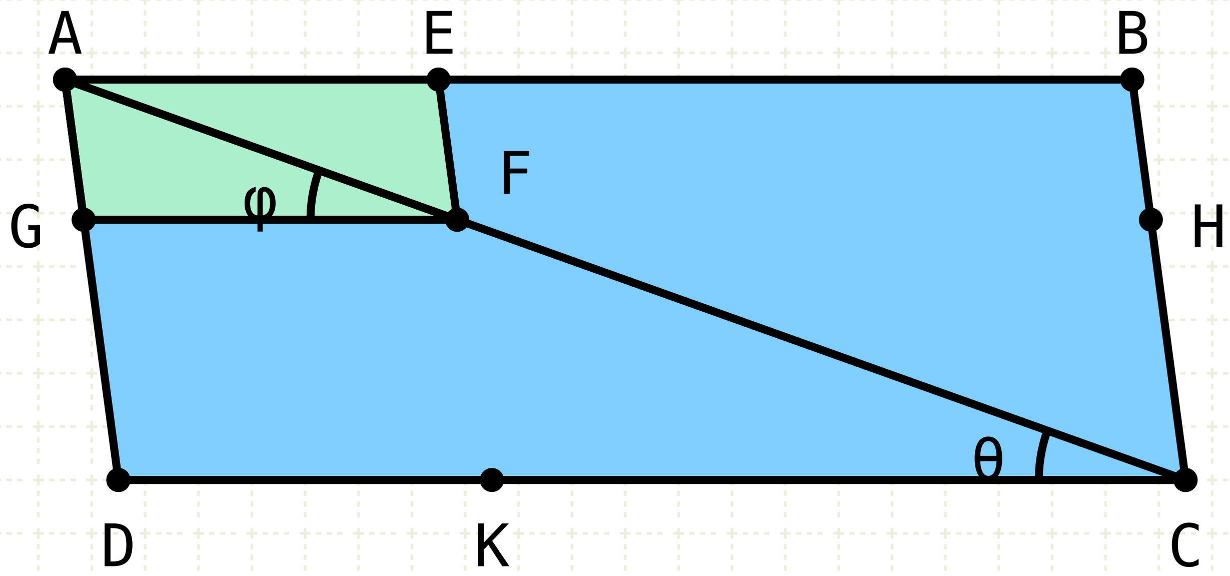
In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.




BA:DA = EA:GA

φ = θ

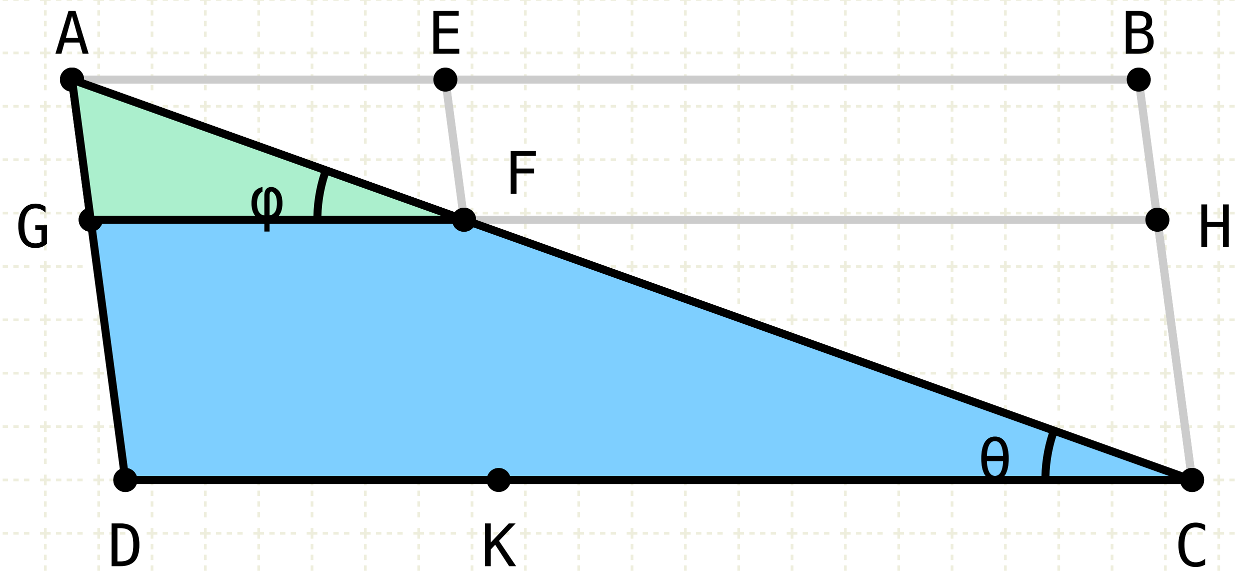
## Proof (cont)

For the parallelograms ABCD, EG, the sides about the common angle BAD are proportional

Since GF is parallel to DC, then angles AFG, DCA are equal

# Proposition 24 of Book VI

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.




BA:DA = EA:GA

$\varphi = \theta$

ΔAGF equiangular to ΔADC

## Proof (cont)

For the parallelograms ABCD, EG, the sides about the common angle BAD are proportional

Since GF is parallel to DC, then angles AFG, DCA are equal

and angle GAF is common to both triangles AGF and ADC, therefore these two triangles are equiangular

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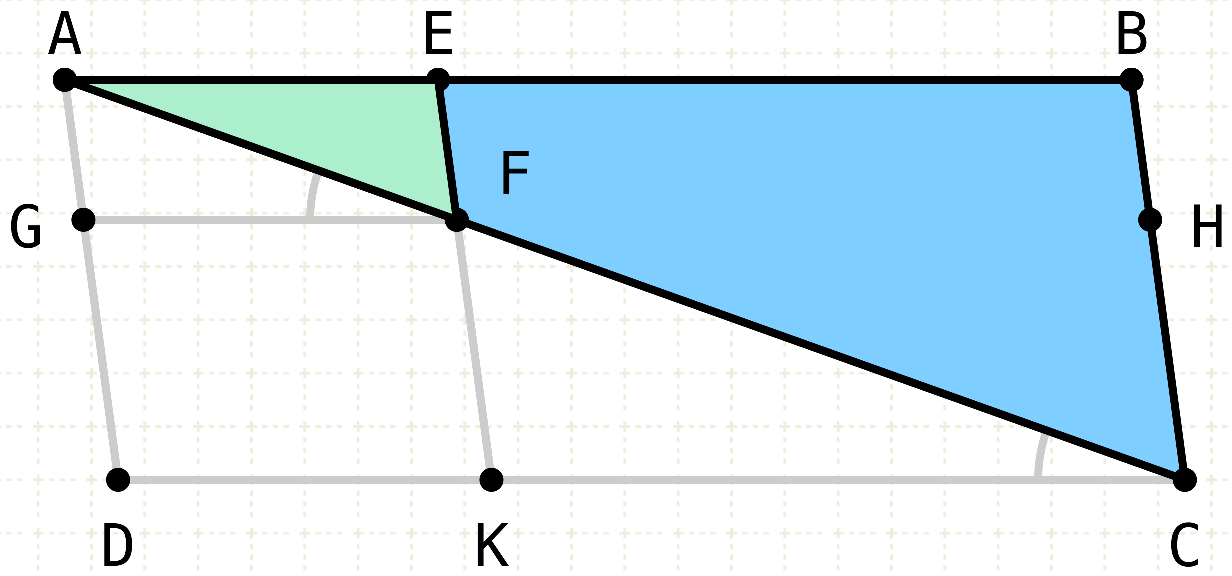


BA:DA = EA:GA

$\varphi = \theta$

ΔAGF equiangular to ΔADC

ΔAFE equiangular to ΔACB

## Proof (cont)

For the parallelograms ABCD, EG, the sides about the common angle BAD are proportional

Since GF is parallel to DC, then angles AFG, DCA are equal

and angle GAF is common to both triangles AGF and ADC, therefore these two triangles are equiangular

For the same reason triangle ACB is equiangular with triangle AFE

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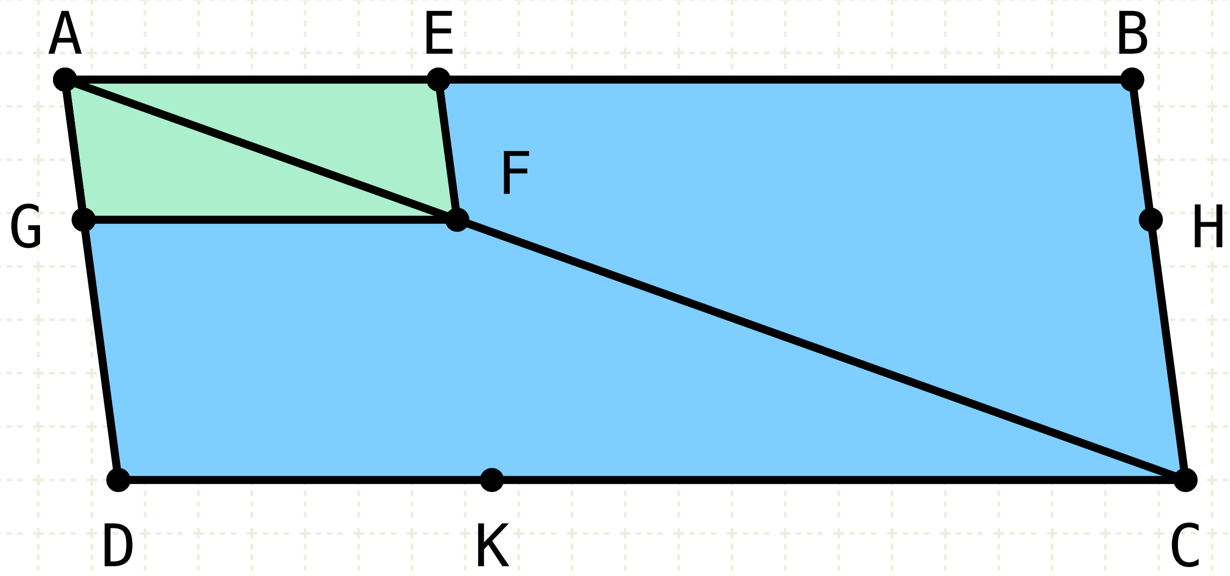


BA:DA = EA:GA

φ = θ

ΔAGF equiangular to ΔADC

ΔAFE equiangular to ΔACB

□ABCD equiangular to □EG

## Proof (cont)

For the parallelograms ABCD, EG, the sides about the common angle BAD are proportional

Since GF is parallel to DC, then angles AFG, DCA are equal

and angle GAF is common to both triangles AGF and ADC, therefore these two triangles are equiangular

For the same reason triangle ACB is equiangular with triangle AFE

So the whole parallelogram AF is equiangular to the parallelogram ABCD

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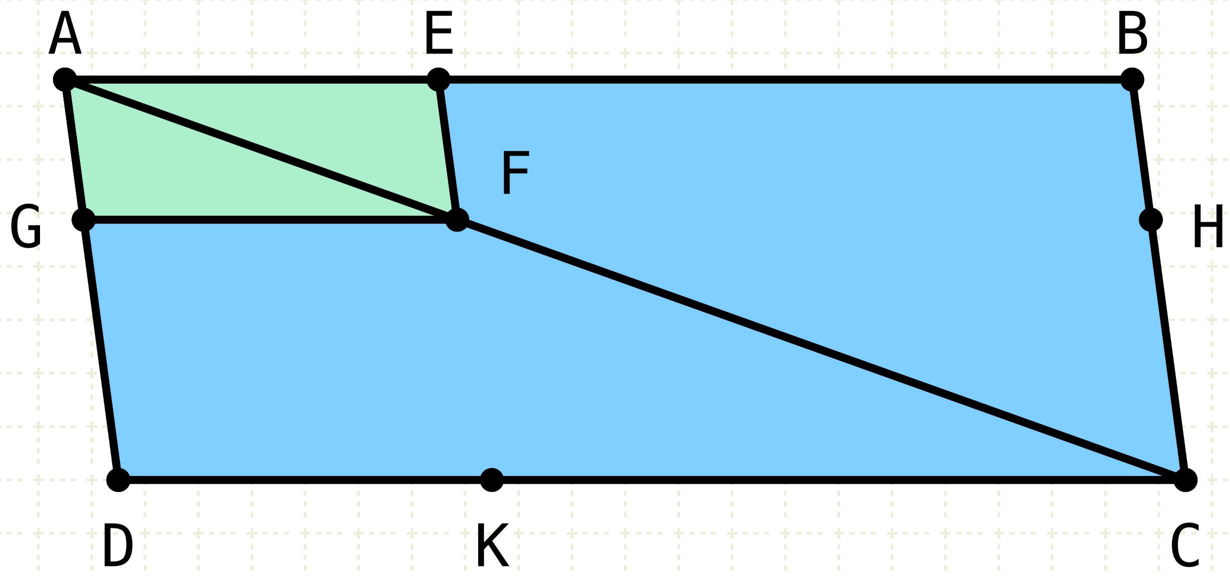
In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.




BA:DA = EA:GA

φ = θ

ΔAGF equiangular to ΔADC

ΔAFE equiangular to ΔACB

□ABCD equiangular to □EG

AD:DC = AG:GF

DC:CA = GF:FA

AC:CB = AF:FE

CB:BA = FE:EA

## Proof (cont)

For the parallelograms ABCD, EG, the sides about the common angle BAD are proportional

Since GF is parallel to DC, then angles AFG, DCA are equal

and angle GAF is common to both triangles AGF and ADC, therefore these two triangles are equiangular

For the same reason triangle ACB is equiangular with triangle AFE

So the whole parallelogram AF is equiangular to the parallelogram ABCD

And the sides of the equiangular triangles are in proportion

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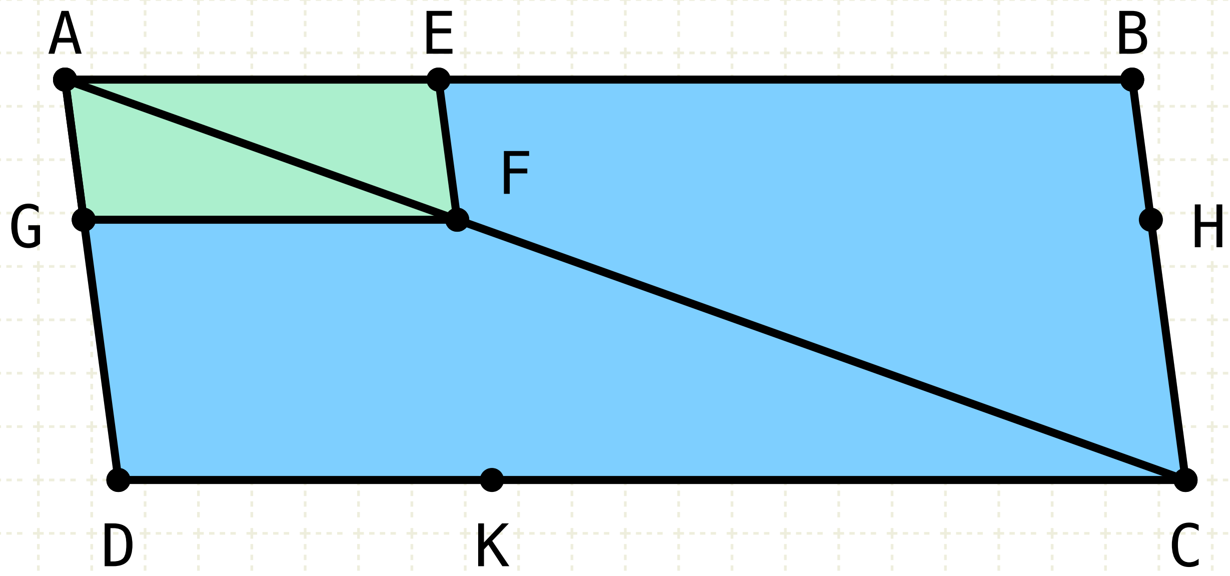
In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.




BA:DA = EA:GA

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ΔAGF equiangular to ΔADC

ΔAFE equiangular to ΔACB

□ABCD equiangular to □EG

AD:DC = AG:GF

DC:CA = GF:FA

AC:CB = AF:FE

CB:BA = FE:EA

DC:CB = GF:FE

## Proof (cont)

For the parallelograms ABCD, EG, the sides about the common angle BAD are proportional

Since GF is parallel to DC, then angles AFG, DCA are equal

and angle GAF is common to both triangles AGF and ADC, therefore these two triangles are equiangular

For the same reason triangle ACB is equiangular with triangle AFE

So the whole parallelogram AF is equiangular to the parallelogram ABCD

And the sides of the equiangular triangles are in proportion

therefore (ex aequali) DC is to CB as GF is to FE (V·22)

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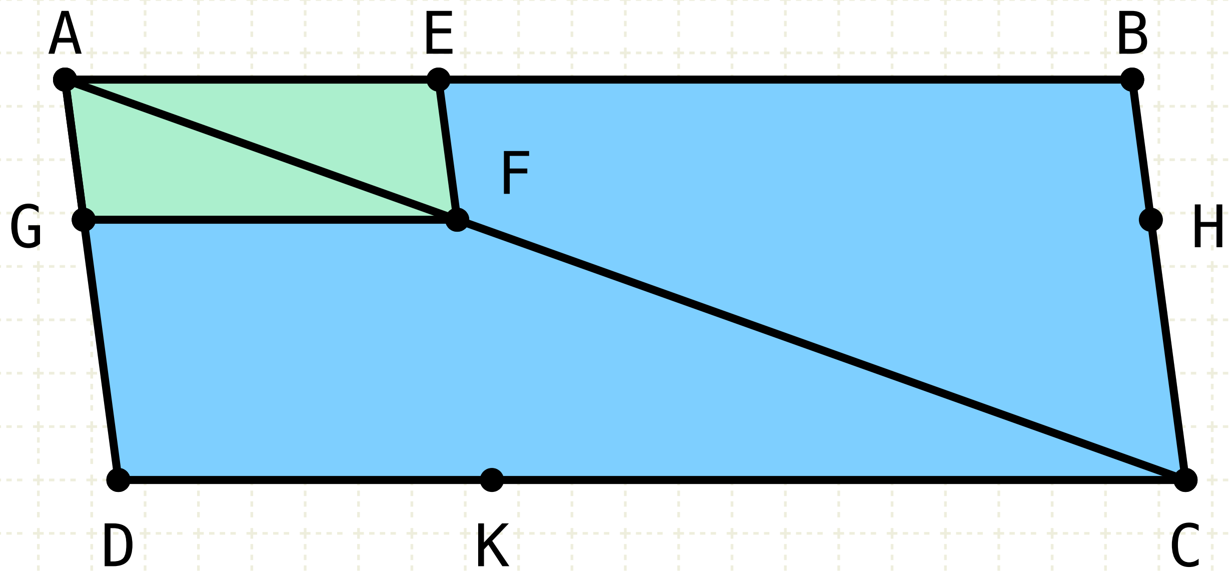
In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.




BA:DA = EA:GA

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ΔAGF equiangular to ΔADC

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□ABCD equiangular to □EG

AD:DC = AG:GF

DC:CA = GF:FA

AC:CB = AF:FE

CB:BA = FE:EA

DC:CB = GF:FE

## Proof (cont)

For the parallelograms ABCD, EG, the sides about the common angle BAD are proportional

Since GF is parallel to DC, then angles AFG, DCA are equal

and angle GAF is common to both triangles AGF and ADC, therefore these two triangles are equiangular

For the same reason triangle ACB is equiangular with triangle AFE

So the whole parallelogram AF is equiangular to the parallelogram ABCD

And the sides of the equiangular triangles are in proportion

therefore (ex aequali) DC is to CB as GF is to FE (V·22)

Thus, the sides about equal angles in the parallelograms ABCD, EG are proportional

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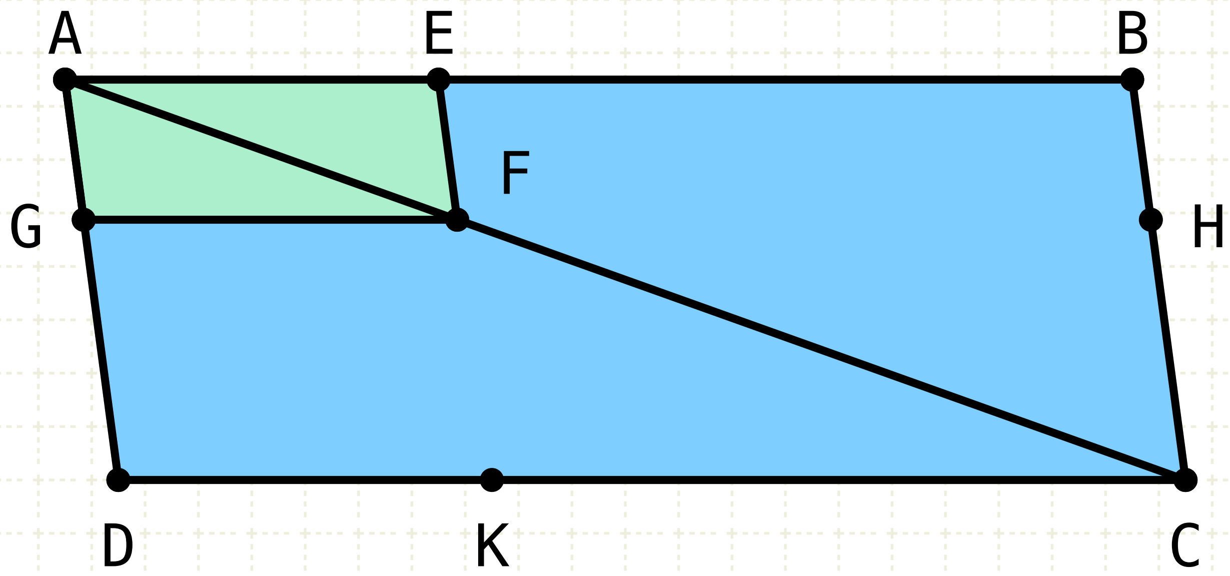
In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.




BA:DA = EA:GA

φ = θ

ΔAGF equiangular to ΔADC

ΔAFE equiangular to ΔACB

□ABCD equiangular to □EG

AD:DC = AG:GF

DC:CA = GF:FA

AC:CB = AF:FE

CB:BA = FE:EA

DC:CB = GF:FE

□ABCD ~ □GE

## Proof (cont)

For the parallelograms ABCD, EG, the sides about the common angle BAD are proportional

Since GF is parallel to DC, then angles AFG, DCA are equal

and angle GAF is common to both triangles AGF and ADC, therefore these two triangles are equiangular

For the same reason triangle ACB is equiangular with triangle AFE

So the whole parallelogram AF is equiangular to the parallelogram ABCD

And the sides of the equiangular triangles are in proportion

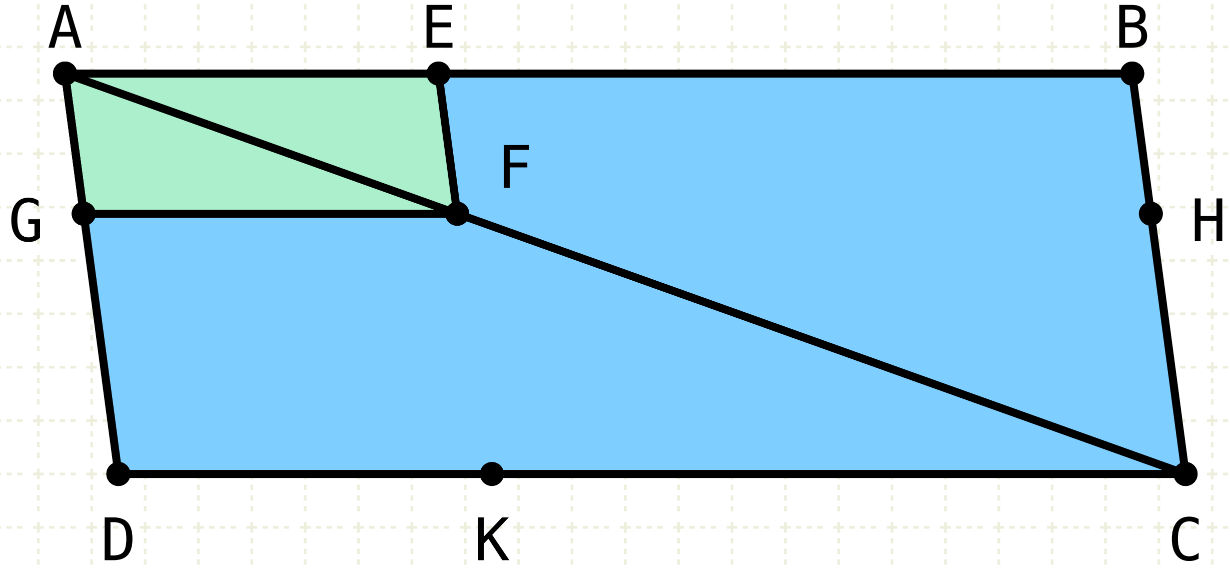
therefore (ex aequali) DC is to CB as GF is to FE (V·22)

Thus, the sides about equal angles in the parallelograms ABCD, EG are proportional

Therefore, the parallelograms ABCD and GE are similar (VI.Def.1)

# Proposition 24 of Book VI

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.




□ABCD ~ □GE

## Proof (cont)

Therefore, the parallelograms ABCD and GE are similar (VI.Def.1)

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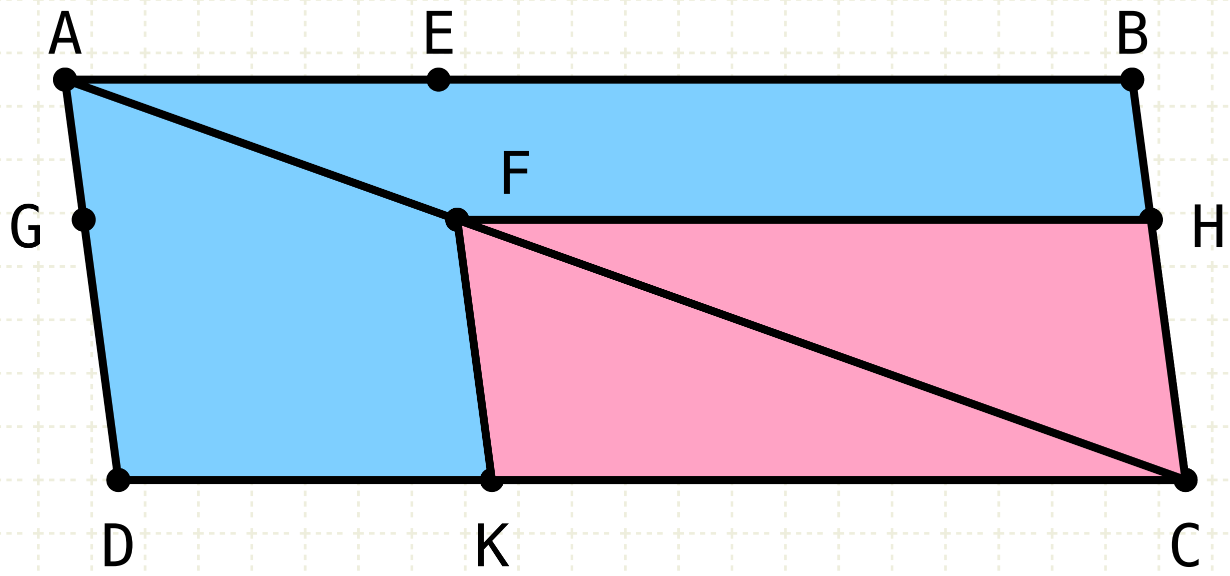


□ABCD ~ □GE

□ABCD ~ □KH

## Proof (cont)

Therefore, the parallelograms ABCD and GE are similar (VI.Def.1)

Using the same logic, it can be shown that KH is similar to ABCD

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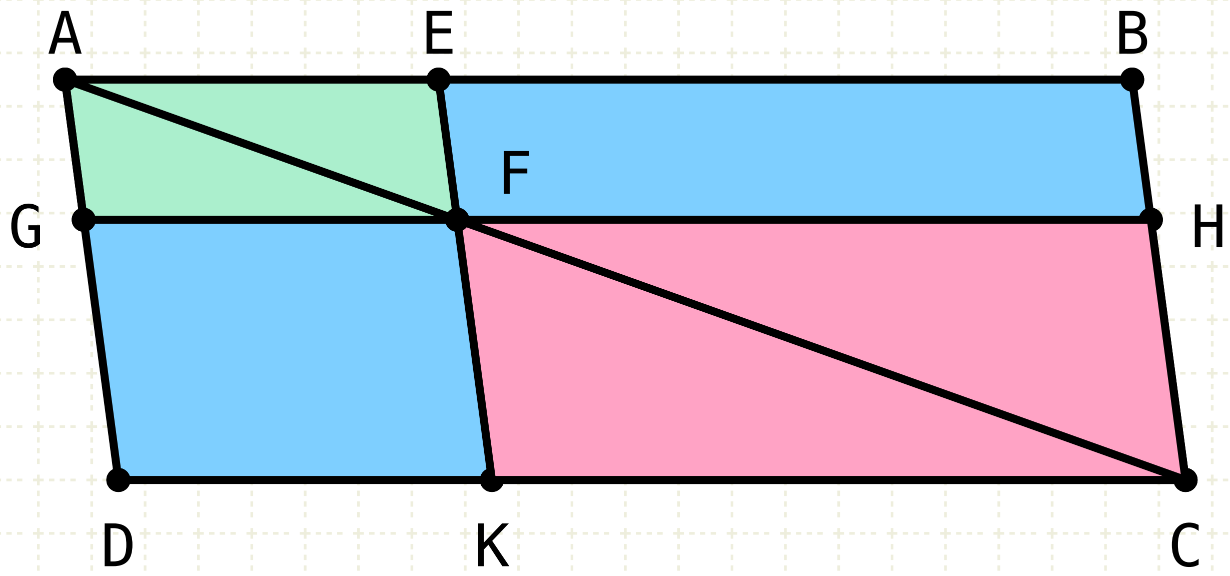
In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.




□ABCD ~ □GE
□ABCD ~ □KH
□GE ~ □KH

## Proof (cont)

Therefore, the parallelograms ABCD and GE are similar (VI.Def.1)

Using the same logic, it can be shown that KH is similar to ABCD

But if two figures are similar to a third, they are also similar to each other (VI·21)

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