

Euclid's Elements

Book VI

One can state, without exaggeration, that the observation of and the search for similarities and differences are the basis of all human knowledge.

Alfred Nobel



Table of Contents, Chapter 6

1	If the height of two triangles are equal, then the ratio of the areas is equal to the ratio of the bases	7	If two triangles have one angle equal to one angle, and the sides about other angles are proportional, and the remaining angles either both less or both not less than a right angle, then triangles will be equiangular	14	In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and vice versa
2	If a line cuts a triangle, parallel to its base, it will cut the sides of the triangle proportionally			15	In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and vice versa
3	If an angle of a triangle is bisected and the straight line cutting the angle also cuts the base, the segments of the base will have the same ratio as the remaining sides of the triangle	8	If in a right-angled triangle a perpendicular be drawn from the right angle to the base, the triangles adjoining the perpendicular are similar both to the whole and to one another	16	If four straight lines are proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means, and vice versa
4	If two triangles have equal angles, then the sides opposite the equal angles are proportional, as well, the sides of the triangles on either side of the equal angles are also proportional	9	From a given straight line to cut off a given fraction	17	If three straight lines are proportional, the rectangle contained by the extremes is equal to the square on the mean; and vice versa
5	If two triangles have proportional sides, the triangles will be equiangular	10	To cut a given uncut straight line similarly to a given cut straight line	18	On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure
6	If two triangles have one angle equal to one angle and the sides about the equal angles are proportional, then the triangles will be equiangular	11	To two given straight lines to find a third proportional	19	Similar triangles are to one another in the duplicate ratio of the corresponding sides
		12	To three given straight lines to find a fourth proportional		
		13	To two given straight lines to find a mean proportional		



Table of Contents, Chapter 3

20	Similar polygons are divided into the same number of similar triangles, which have the same ratio as the wholes, and the polygons have duplicate ratios to their corresponding sides	26	If from a parallelogram a similar parallelogram with a common angle is subtracted, it is about the same diameter as the original	31	In right-angled triangles the figure on the side subtending the right angle is equal to the similar and similarly described figures on the sides containing the right angle
21	Figures which are are similar to the same rectilineal figure are also similar to one another	27	Of all the parallelograms applied to the same straight line and deficient by parallelogrammic figures similar to a parallelogram drawn on half the said line, the largest will be one that is drawn on half of the straight line and is similar to the defect		
22	If four straight lines are proportional, similar rectilineal figures will also be proportional; and vice versa	28	To a given straight line, apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one		
23	Equiangular parallelograms have to one another the ratio compounded of the ratios of their sides	29	To a given straight line, apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one		
24	In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another	30	To cut a finite straight line in extreme ratio		
25	To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure				



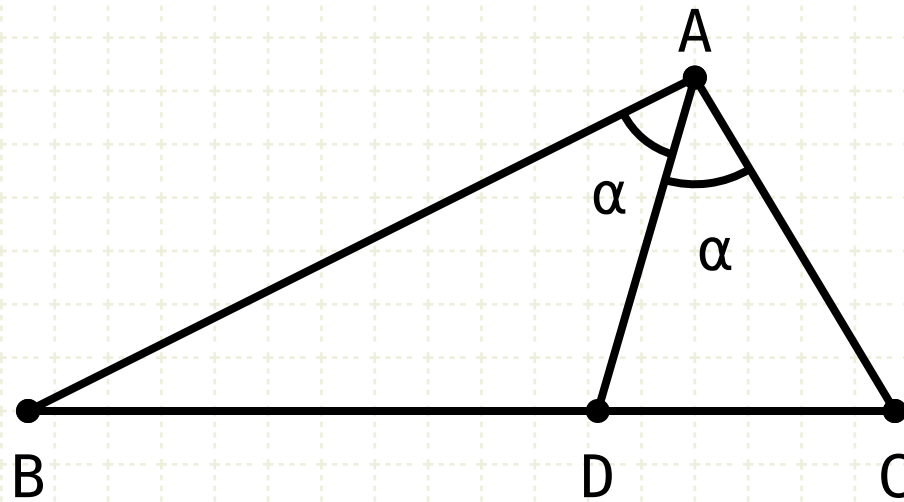
Proposition 3 of Book VI

If an angle of a triangle be bisected and the straight line cutting the angle cut the base also, the segments of the base will have the same ratio as the remaining sides of the triangle; and, if the segments of the base have the same ratio as the remaining sides of the triangle, the straight line joined from the vertex to the point of section will bisect the angle of the triangle.



Proposition 3 of Book VI

If an angle of a triangle be bisected and the straight line cutting the angle cut the base also, the segments of the base will have the same ratio as the remaining sides of the triangle; and, if the segments of the base have the same ratio as the remaining sides of the triangle, the straight line joined from the vertex to the point of section will bisect the angle of the triangle.



$$\begin{aligned}\angle BAD &= \angle DAC \rightarrow BD:DC = BA:AC \\ BD:DC &= BA:AC \rightarrow \angle BAD = \angle DAC\end{aligned}$$

In other words

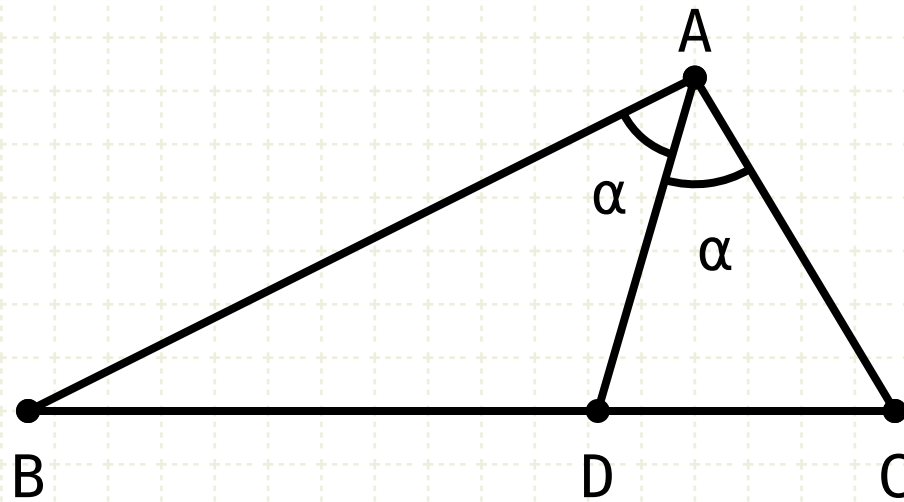
Given a triangle ABC, and let angle BAC be bisected by the straight line BD

Then the ratio of BD to CD is equal to the ratio BA to AC

Proposition 3 of Book VI

If an angle of a triangle be bisected and the straight line cutting the angle cut the base also, the segments of the base will have the same ratio as the remaining sides of the triangle; and, if the segments of the base have the same ratio as the remaining sides of the triangle, the straight line joined from the vertex to the point of section will bisect the angle of the triangle.

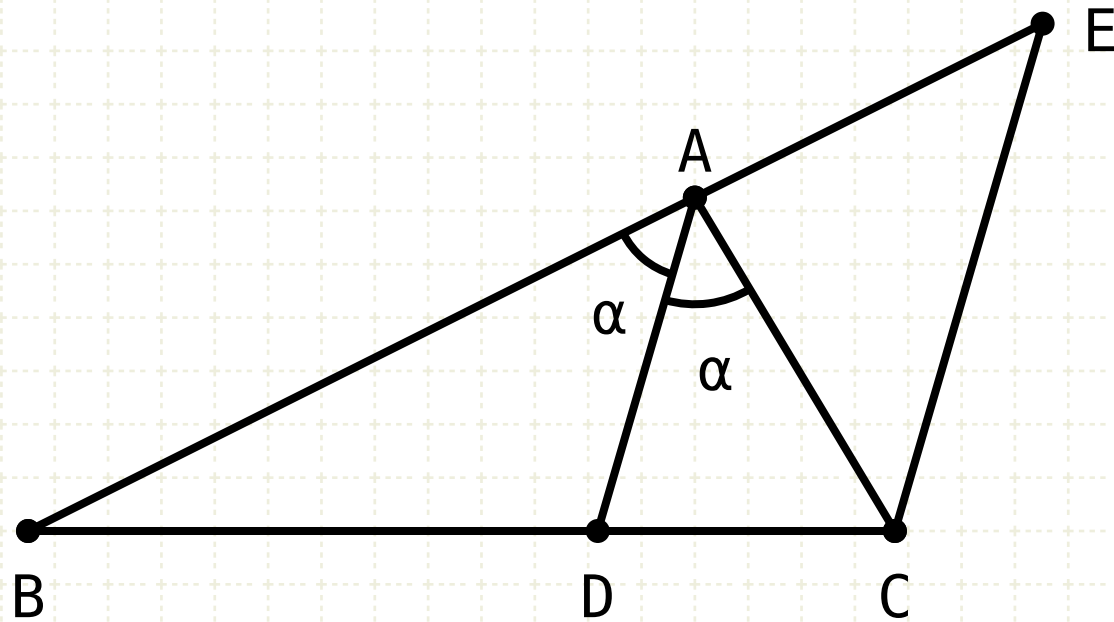
Proof (part 1)



$$\angle BAD = \angle DAC$$

Proposition 3 of Book VI

If an angle of a triangle be bisected and the straight line cutting the angle cut the base also, the segments of the base will have the same ratio as the remaining sides of the triangle; and, if the segments of the base have the same ratio as the remaining sides of the triangle, the straight line joined from the vertex to the point of section will bisect the angle of the triangle.



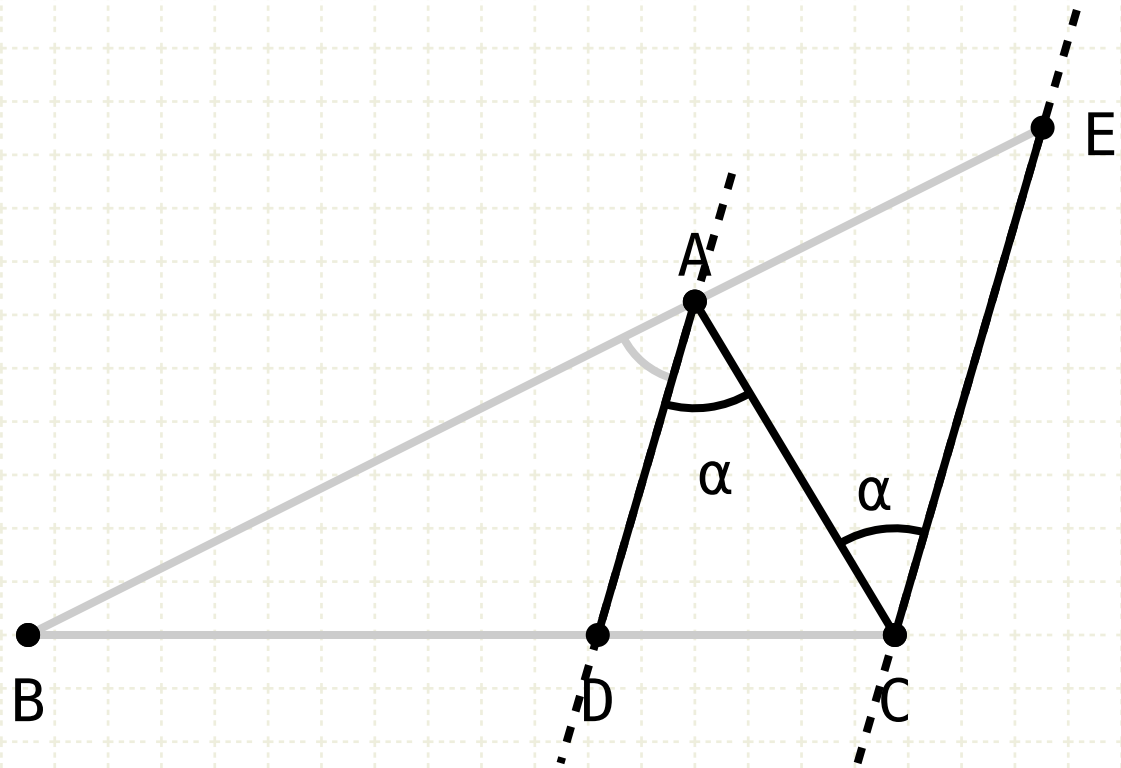
$\angle BAD = \angle DAC$
 $CE \parallel DA$

Proof (part 1)

Draw a line parallel to DA from point C, and let the line BA extend to it at point E

Proposition 3 of Book VI

If an angle of a triangle be bisected and the straight line cutting the angle cut the base also, the segments of the base will have the same ratio as the remaining sides of the triangle; and, if the segments of the base have the same ratio as the remaining sides of the triangle, the straight line joined from the vertex to the point of section will bisect the angle of the triangle.



$$\angle BAD = \angle DAC$$

$$CE \parallel DA$$

$$\angle DAC = \angle ACE$$

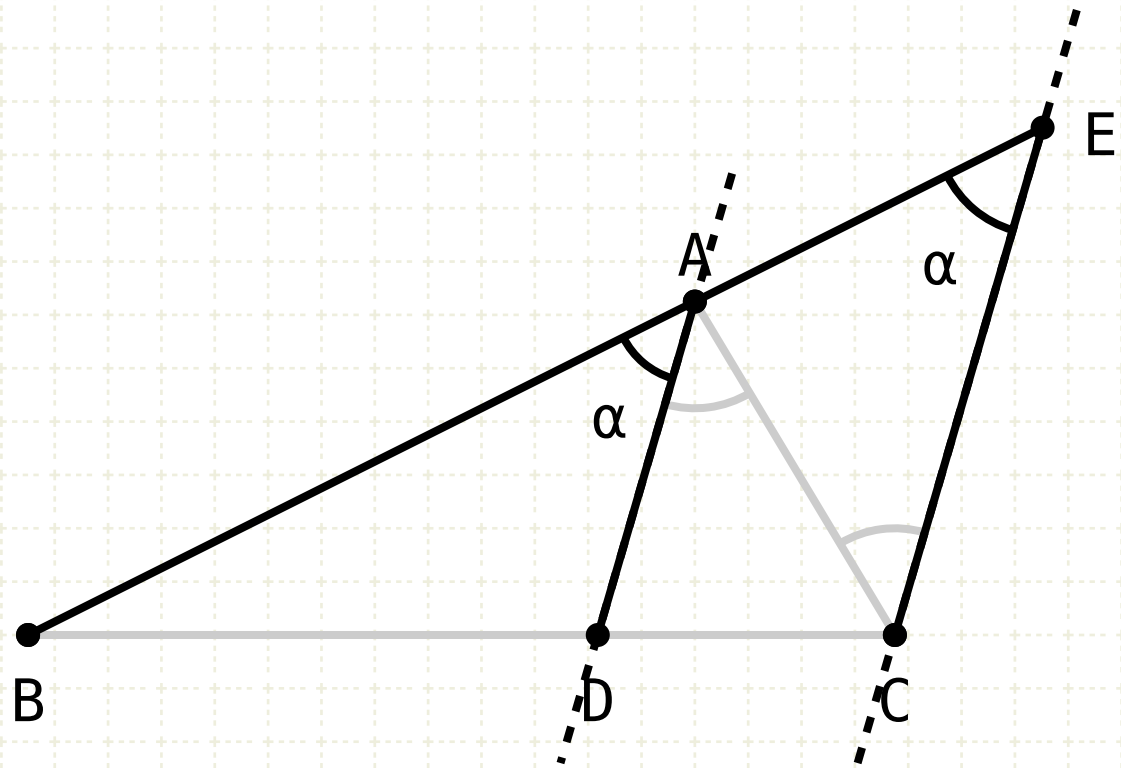
Proof (part 1)

Draw a line parallel to DA from point C, and let the line BA extend to it at point E

AC cuts two parallel lines AD and EC, therefore the opposite interior angle ACE is equal to DAC (I.29)

Proposition 3 of Book VI

If an angle of a triangle be bisected and the straight line cutting the angle cut the base also, the segments of the base will have the same ratio as the remaining sides of the triangle; and, if the segments of the base have the same ratio as the remaining sides of the triangle, the straight line joined from the vertex to the point of section will bisect the angle of the triangle.



$$\angle BAD = \angle DAC$$

$$CE \parallel DA$$

$$\angle DAC = \angle ACE$$

$$\angle BAD = \angle AEC$$

Proof (part 1)

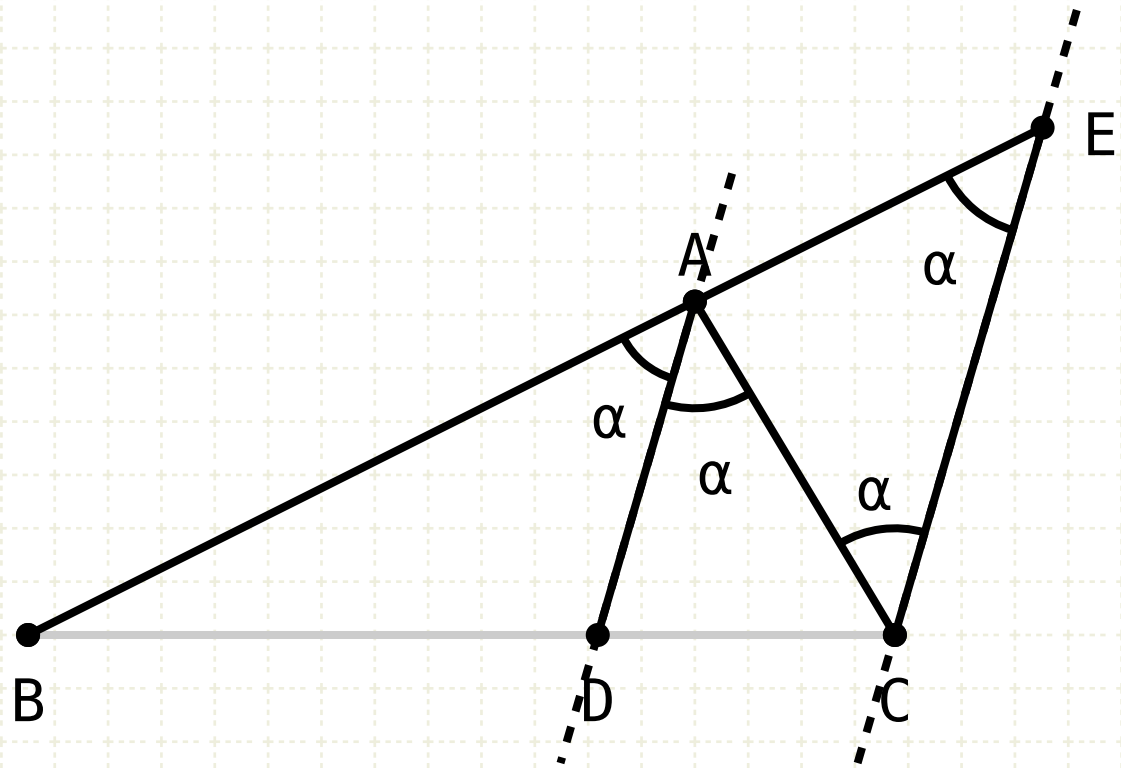
Draw a line parallel to DA from point C, and let the line BA extend to it at point E

AC cuts two parallel lines AD and EC, therefore the opposite interior angle ACE is equal to DAC (I·29)

BE cuts two parallel lines AD and EC, therefore the outside angle BAD is equal to interior angle AEC (I·29)

Proposition 3 of Book VI

If an angle of a triangle be bisected and the straight line cutting the angle cut the base also, the segments of the base will have the same ratio as the remaining sides of the triangle; and, if the segments of the base have the same ratio as the remaining sides of the triangle, the straight line joined from the vertex to the point of section will bisect the angle of the triangle.



$$\angle BAD = \angle DAC$$

$$CE \parallel DA$$

$$\angle DAC = \angle ACE$$

$$\angle BAD = \angle AEC$$

Proof (part 1)

Draw a line parallel to DA from point C, and let the line BA extend to it at point E

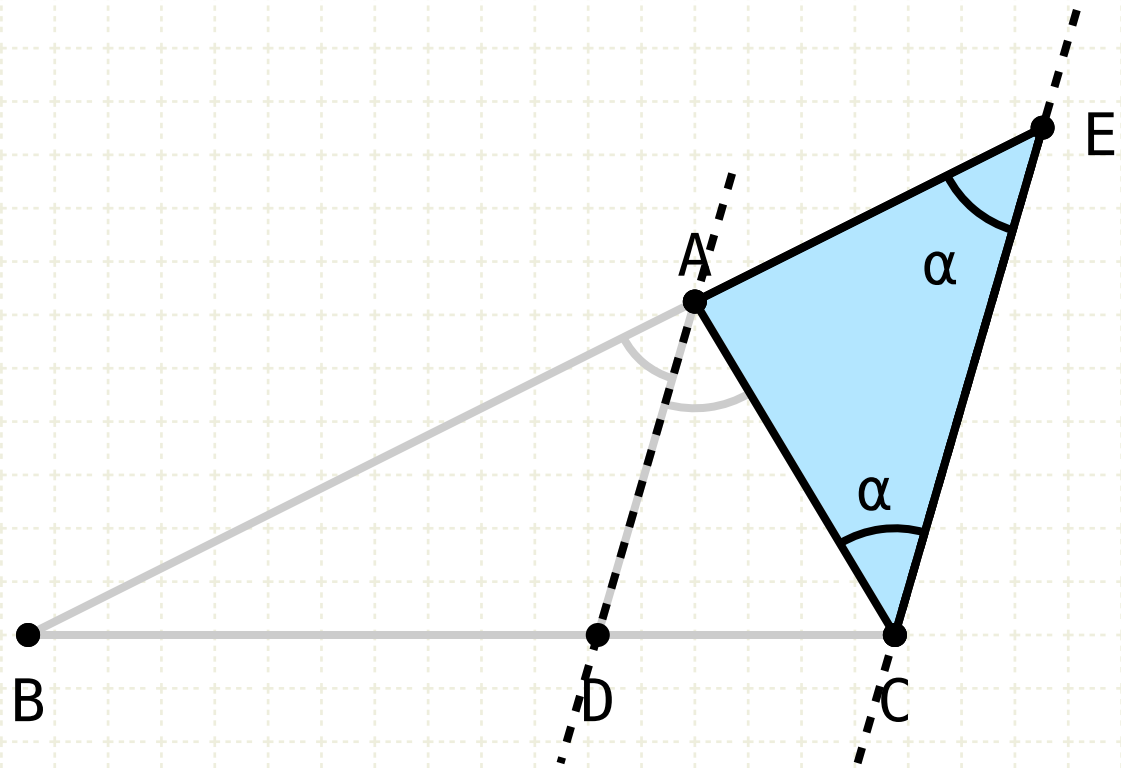
AC cuts two parallel lines AD and EC, therefore the opposite interior angle ACE is equal to DAC (I·29)

BE cuts two parallel lines AD and EC, therefore the outside angle BAD is equal to interior angle AEC (I·29)

Since angles BAD and DAC are equal by construction, angles ACE and AEC are also equal

Proposition 3 of Book VI

If an angle of a triangle be bisected and the straight line cutting the angle cut the base also, the segments of the base will have the same ratio as the remaining sides of the triangle; and, if the segments of the base have the same ratio as the remaining sides of the triangle, the straight line joined from the vertex to the point of section will bisect the angle of the triangle.



$$\angle BAD = \angle DAC$$

$$CE \parallel DA$$

$$\angle DAC = \angle ACE$$

$$\angle BAD = \angle AEC$$

$$AE = AC$$

Proof (part 1)

Draw a line parallel to DA from point C, and let the line BA extend to it at point E

AC cuts two parallel lines AD and EC, therefore the opposite interior angle ACE is equal to DAC (I·29)

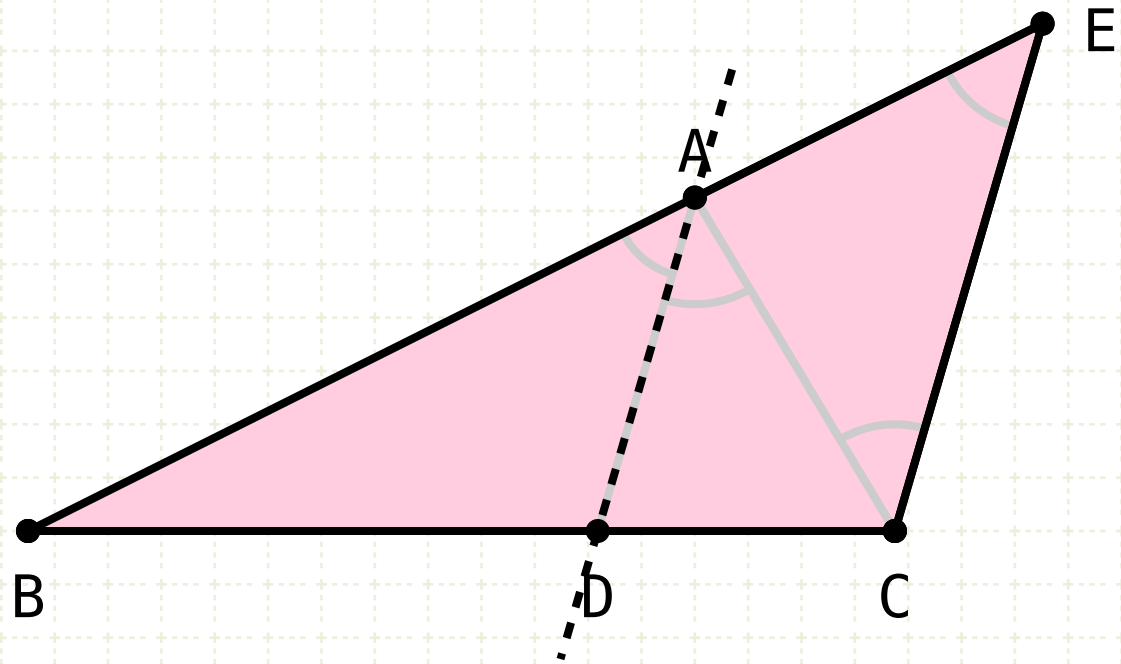
BE cuts two parallel lines AD and EC, therefore the outside angle BAD is equal to interior angle AEC (I·29)

Since angles BAD and DAC are equal by construction, angles ACE and AEC are also equal

Therefore, triangle AEC is an isosceles triangle, and the sides AE and AC are equal

Proposition 3 of Book VI

If an angle of a triangle be bisected and the straight line cutting the angle cut the base also, the segments of the base will have the same ratio as the remaining sides of the triangle; and, if the segments of the base have the same ratio as the remaining sides of the triangle, the straight line joined from the vertex to the point of section will bisect the angle of the triangle.



$$\angle BAD = \angle DAC$$

$$CE \parallel DA$$

$$\angle DAC = \angle ACE$$

$$\angle BAD = \angle AEC$$

$$AE = AC$$

$$BA : AE = BD : DC$$

Proof (part 1)

Draw a line parallel to DA from point C, and let the line BA extend to it at point E

AC cuts two parallel lines AD and EC, therefore the opposite interior angle ACE is equal to DAC (I·29)

BE cuts two parallel lines AD and EC, therefore the outside angle BAD is equal to interior angle AEC (I·29)

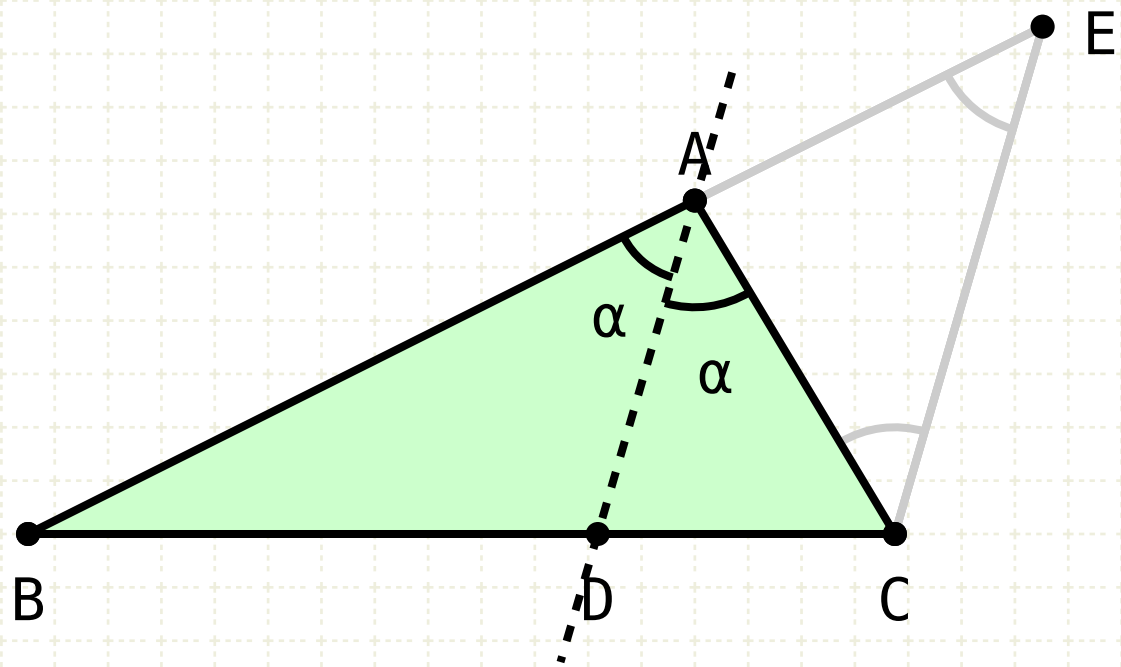
Since angles BAD and DAC are equal by construction, angles ACE and AEC are also equal

Therefore, triangle AEC is an isosceles triangle, and the sides AE and AC are equal

Consider triangle BCE, with a line AD drawn parallel to one of the triangle's sides. The ratio BA to AE is equal to the ratio BD to DC (VI·2)

Proposition 3 of Book VI

If an angle of a triangle be bisected and the straight line cutting the angle cut the base also, the segments of the base will have the same ratio as the remaining sides of the triangle; and, if the segments of the base have the same ratio as the remaining sides of the triangle, the straight line joined from the vertex to the point of section will bisect the angle of the triangle.



$$\angle BAD = \angle DAC$$

$$CE \parallel DA$$

$$\angle DAC = \angle ACE$$

$$\angle BAD = \angle AEC$$

$$AE = AC$$

$$BA : AE = BD : DC$$

$$BA : AC = BD : DC$$

Proof (part 1)

Draw a line parallel to DA from point C, and let the line BA extend to it at point E

AC cuts two parallel lines AD and EC, therefore the opposite interior angle ACE is equal to DAC (I·29)

BE cuts two parallel lines AD and EC, therefore the outside angle BAD is equal to interior angle AEC (I·29)

Since angles BAD and DAC are equal by construction, angles ACE and AEC are also equal

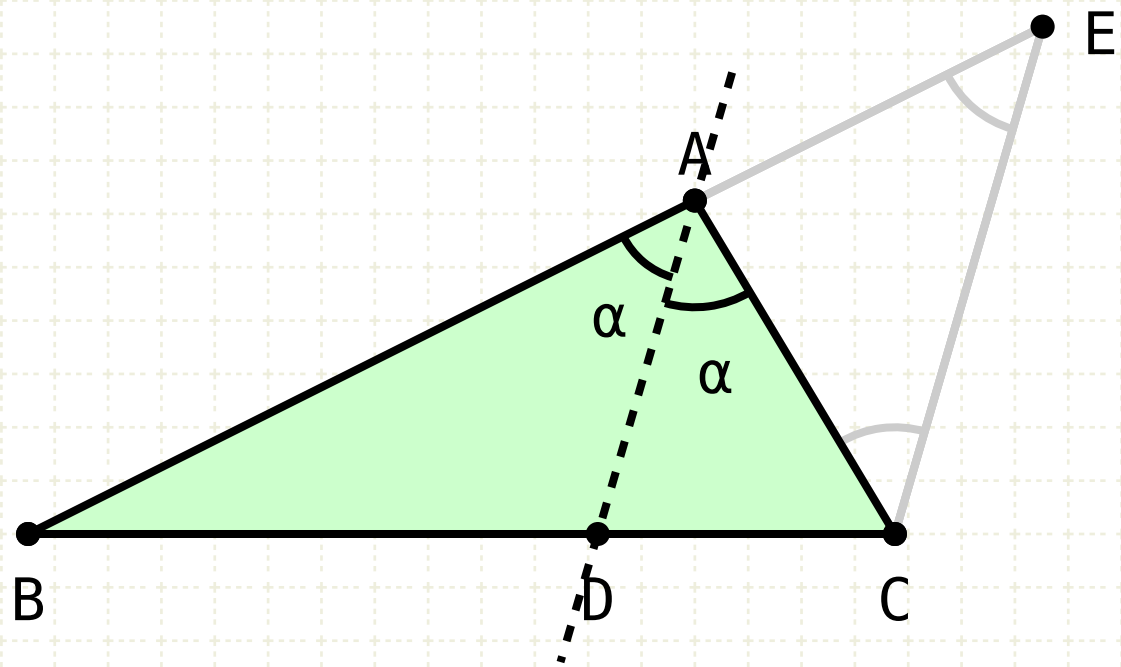
Therefore, triangle AEC is an isosceles triangle, and the sides AE and AC are equal

Consider triangle BCE, with a line AD drawn parallel to one of the triangle's sides. The ratio BA to AE is equal to the ratio BD to DC (VI·2)

And AE is equal to AC, therefore ratio BA to AC is equal to the ratio BD to DC

Proposition 3 of Book VI

If an angle of a triangle be bisected and the straight line cutting the angle cut the base also, the segments of the base will have the same ratio as the remaining sides of the triangle; and, if the segments of the base have the same ratio as the remaining sides of the triangle, the straight line joined from the vertex to the point of section will bisect the angle of the triangle.



$$\angle BAD = \angle DAC$$

$$CE \parallel DA$$

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$$BA : AE = BD : DC$$

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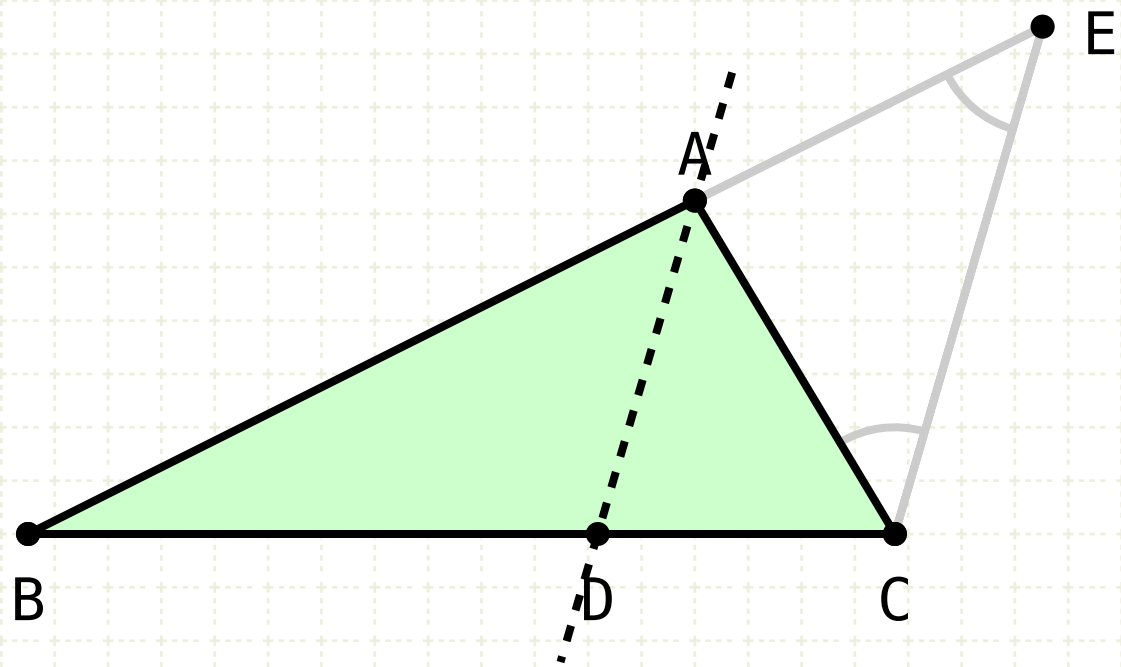
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Proposition 3 of Book VI

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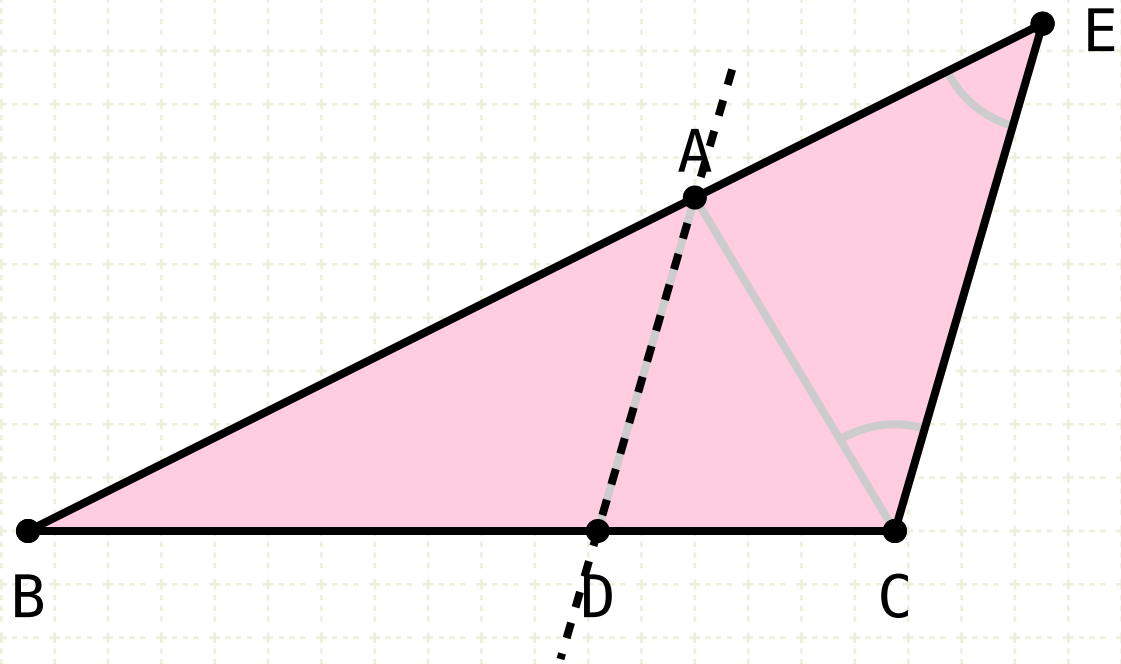
Proof (part 2)



$$\begin{aligned} BA:AC &= BD:DC \\ DA &\parallel CE \end{aligned}$$

Proposition 3 of Book VI

If an angle of a triangle be bisected and the straight line cutting the angle cut the base also, the segments of the base will have the same ratio as the remaining sides of the triangle; and, if the segments of the base have the same ratio as the remaining sides of the triangle, the straight line joined from the vertex to the point of section will bisect the angle of the triangle.



$$BA : AC = BD : DC$$

$$DA \parallel CE$$

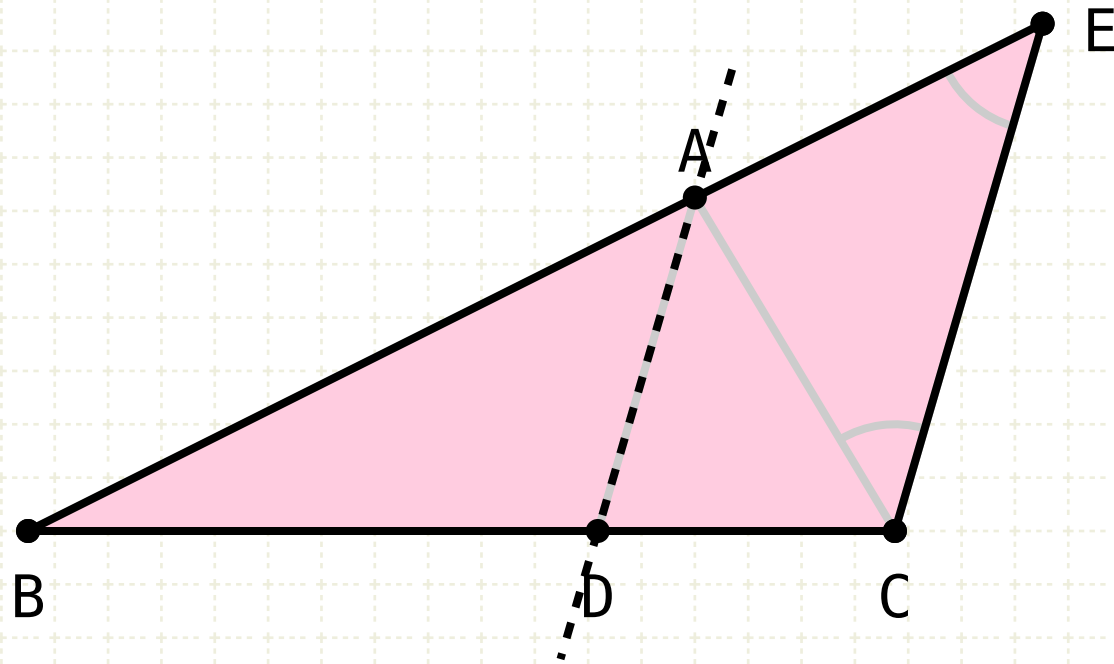
$$BA : AE = BD : DC$$

Proof (part 2)

Since DA is parallel to CE, the ratio AB to AE is equal to the ratio BD to DC (VI-2)

Proposition 3 of Book VI

If an angle of a triangle be bisected and the straight line cutting the angle cut the base also, the segments of the base will have the same ratio as the remaining sides of the triangle; and, if the segments of the base have the same ratio as the remaining sides of the triangle, the straight line joined from the vertex to the point of section will bisect the angle of the triangle.



$$BA:AC = BD:DC$$

$$DA \parallel CE$$

$$BA:AE = BD:DC$$

$$BA:AC = BA:AE$$

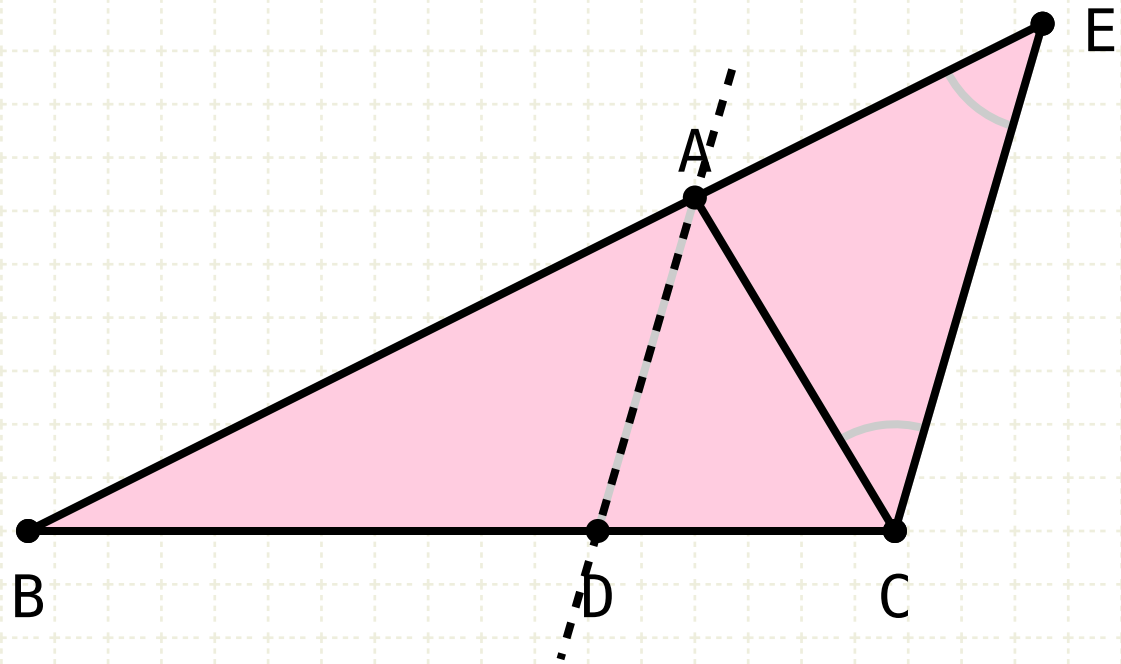
Proof (part 2)

Since DA is parallel to CE, the ratio AB to AE is equal to the ratio BD to DC (VI·2)

If two ratios are equal to the same, they are equal to each other (V·11), so the ratios BA to AC and BA to AE are also equal

Proposition 3 of Book VI

If an angle of a triangle be bisected and the straight line cutting the angle cut the base also, the segments of the base will have the same ratio as the remaining sides of the triangle; and, if the segments of the base have the same ratio as the remaining sides of the triangle, the straight line joined from the vertex to the point of section will bisect the angle of the triangle.



$$BA : AC = BD : DC$$

$$DA \parallel CE$$

$$BA : AE = BD : DC$$

$$BA : AC = BA : AE$$

$$AC = AE$$

Proof (part 2)

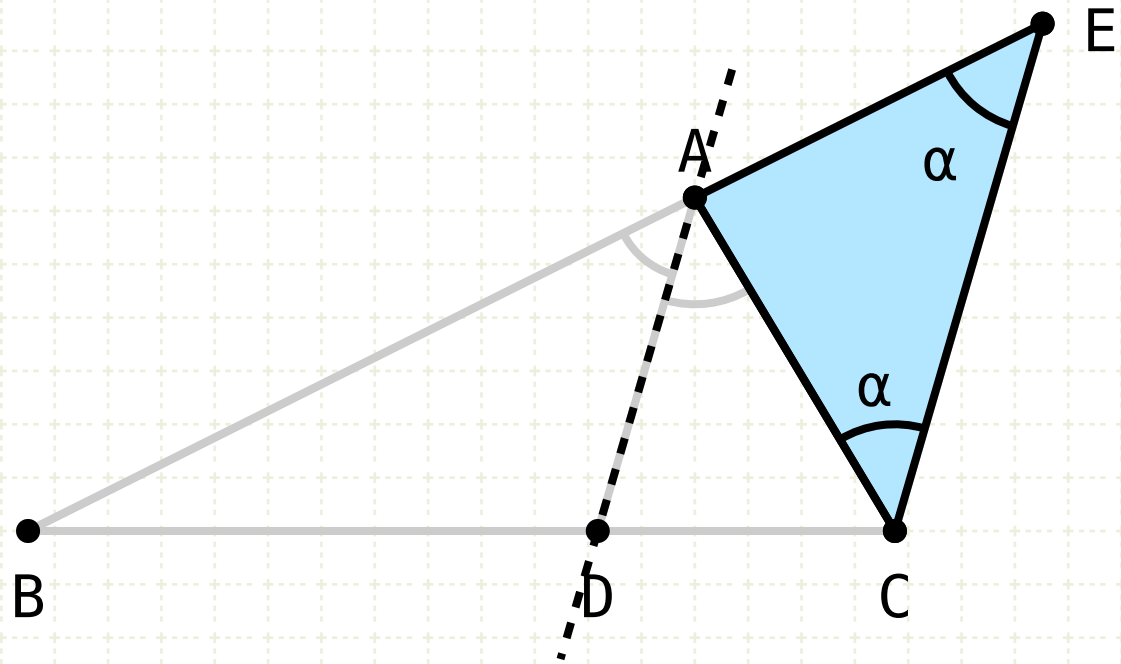
Since DA is parallel to CE, the ratio AB to AE is equal to the ratio BD to DC (VI·2)

If two ratios are equal to the same, they are equal to each other (V·11), so the ratios BA to AC and BA to AE are also equal

Therefore, AC equals AE (V·9)

Proposition 3 of Book VI

If an angle of a triangle be bisected and the straight line cutting the angle cut the base also, the segments of the base will have the same ratio as the remaining sides of the triangle; and, if the segments of the base have the same ratio as the remaining sides of the triangle, the straight line joined from the vertex to the point of section will bisect the angle of the triangle.



$$BA : AC = BD : DC$$

$$DA \parallel CE$$

$$BA : AE = BD : DC$$

$$BA : AC = BA : AE$$

$$AC = AE$$

$$\angle ACE = \angle AEC$$

Proof (part 2)

Since DA is parallel to CE, the ratio AB to AE is equal to the ratio BD to DC (VI·2)

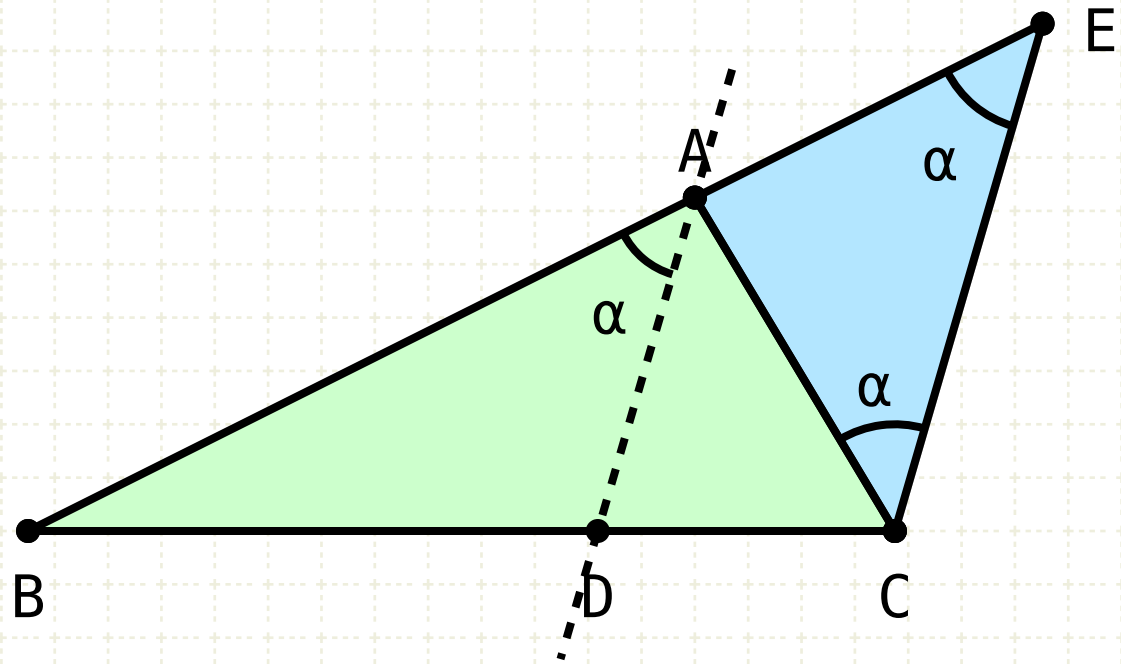
If two ratios are equal to the same, they are equal to each other (V·11), so the ratios BA to AC and BA to AE are also equal

Therefore, AC equals AE (V·9)

If AC and AE are equal, then triangle ACE is an isosceles triangle, and the angles AEC and ACE are equal (I·5)

Proposition 3 of Book VI

If an angle of a triangle be bisected and the straight line cutting the angle cut the base also, the segments of the base will have the same ratio as the remaining sides of the triangle; and, if the segments of the base have the same ratio as the remaining sides of the triangle, the straight line joined from the vertex to the point of section will bisect the angle of the triangle.



$$BA : AC = BD : DC$$

$$DA \parallel CE$$

$$BA : AE = BD : DC$$

$$BA : AC = BA : AE$$

$$AC = AE$$

$$\angle ACE = \angle AEC$$

$$\angle AEC = \angle BAD$$

Proof (part 2)

Since DA is parallel to CE, the ratio AB to AE is equal to the ratio BD to DC (VI·2)

If two ratios are equal to the same, they are equal to each other (V·11), so the ratios BA to AC and BA to AE are also equal

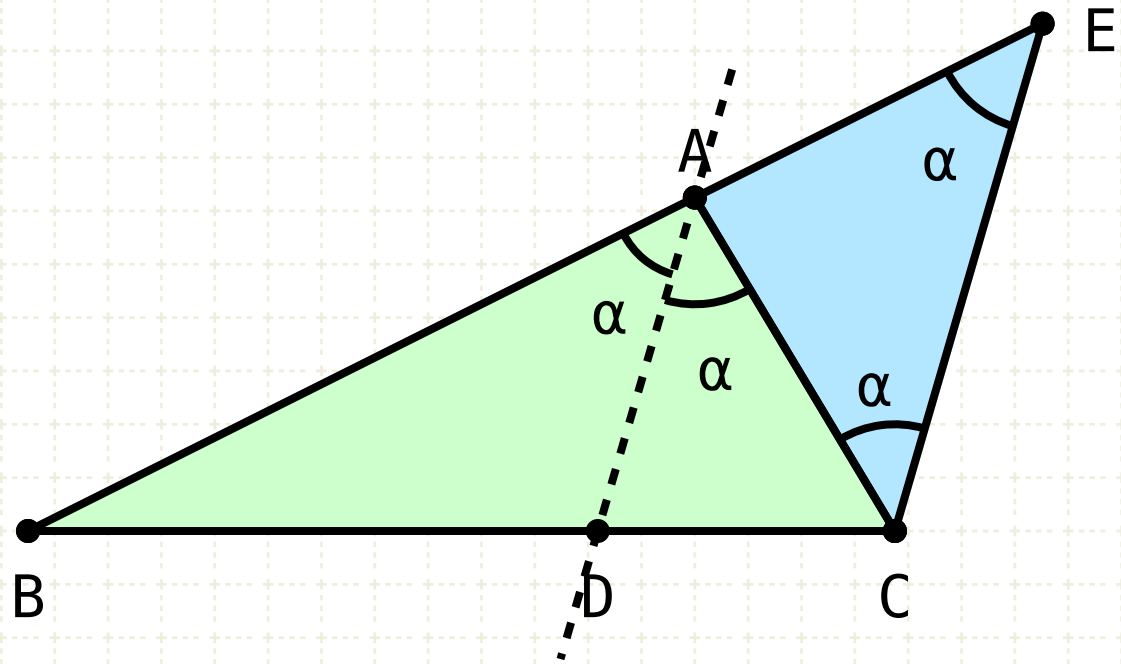
Therefore, AC equals AE (V·9)

If AC and AE are equal, then triangle ACE is an isosceles triangle, and the angles AEC and ACE are equal (I·5)

The angle AEC is equal to the exterior angle BAD of the parallel lines AD and EC (I·29)

Proposition 3 of Book VI

If an angle of a triangle be bisected and the straight line cutting the angle cut the base also, the segments of the base will have the same ratio as the remaining sides of the triangle; and, if the segments of the base have the same ratio as the remaining sides of the triangle, the straight line joined from the vertex to the point of section will bisect the angle of the triangle.



$$BA:AC = BD:DC$$

$$DA \parallel CE$$

$$BA:AE = BD:DC$$

$$BA:AC = BA:AE$$

$$AC = AE$$

$$\angle ACE = \angle AEC$$

$$\angle AEC = \angle BAD$$

$$\angle ACE = \angle CAD$$

Proof (part 2)

Since DA is parallel to CE, the ratio AB to AE is equal to the ratio BD to DC (VI·2)

If two ratios are equal to the same, they are equal to each other (V·11), so the ratios BA to AC and BA to AE are also equal

Therefore, AC equals AE (V·9)

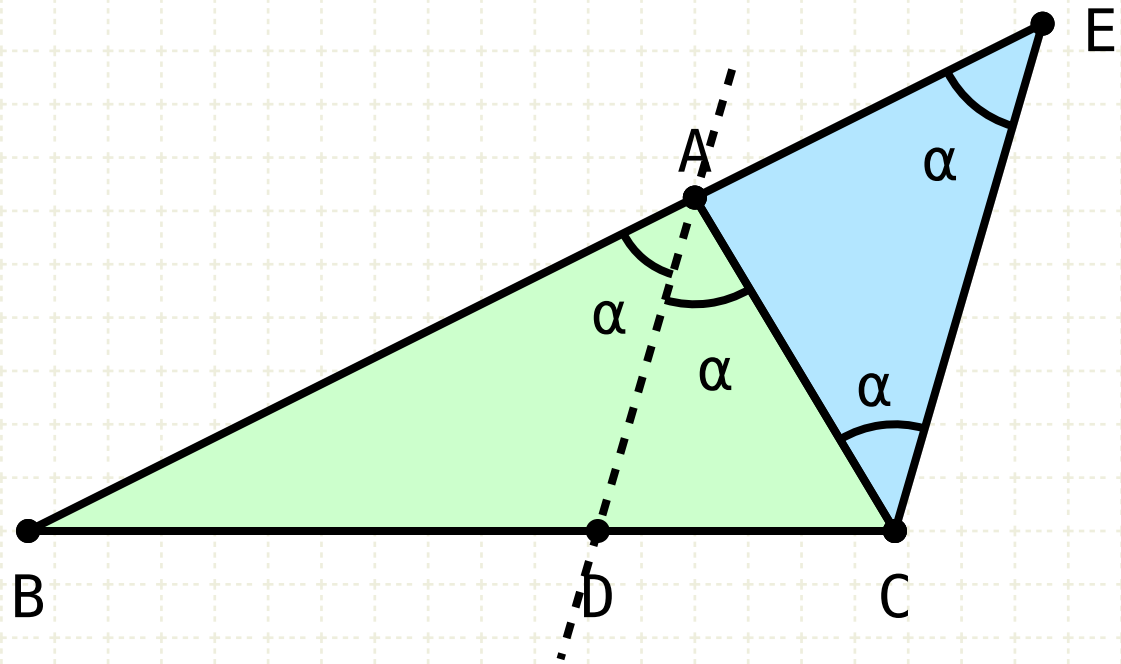
If AC and AE are equal, then triangle ACE is an isosceles triangle, and the angles AEC and ACE are equal (I·5)

The angle AEC is equal to the exterior angle BAD of the parallel lines AD and EC (I·29)

and the angle ACE is equal to the alternate angle CAD (I·29)

Proposition 3 of Book VI

If an angle of a triangle be bisected and the straight line cutting the angle cut the base also, the segments of the base will have the same ratio as the remaining sides of the triangle; and, if the segments of the base have the same ratio as the remaining sides of the triangle, the straight line joined from the vertex to the point of section will bisect the angle of the triangle.



$$BA : AC = BD : DC$$

$$DA \parallel CE$$

$$BA : AE = BD : DC$$

$$BA : AC = BA : AE$$

$$AC = AE$$

$$\angle ACE = \angle AEC$$

$$\angle AEC = \angle BAD$$

$$\angle ACE = \angle CAD$$

$$\angle BAD = \angle CAD$$

Proof (part 2)

Since DA is parallel to CE, the ratio AB to AE is equal to the ratio BD to DC (VI·2)

If two ratios are equal to the same, they are equal to each other (V·11), so the ratios BA to AC and BA to AE are also equal

Therefore, AC equals AE (V·9)

If AC and AE are equal, then triangle ACE is an isosceles triangle, and the angles AEC and ACE are equal (I·5)

The angle AEC is equal to the exterior angle BAD of the parallel lines AD and EC (I·29)

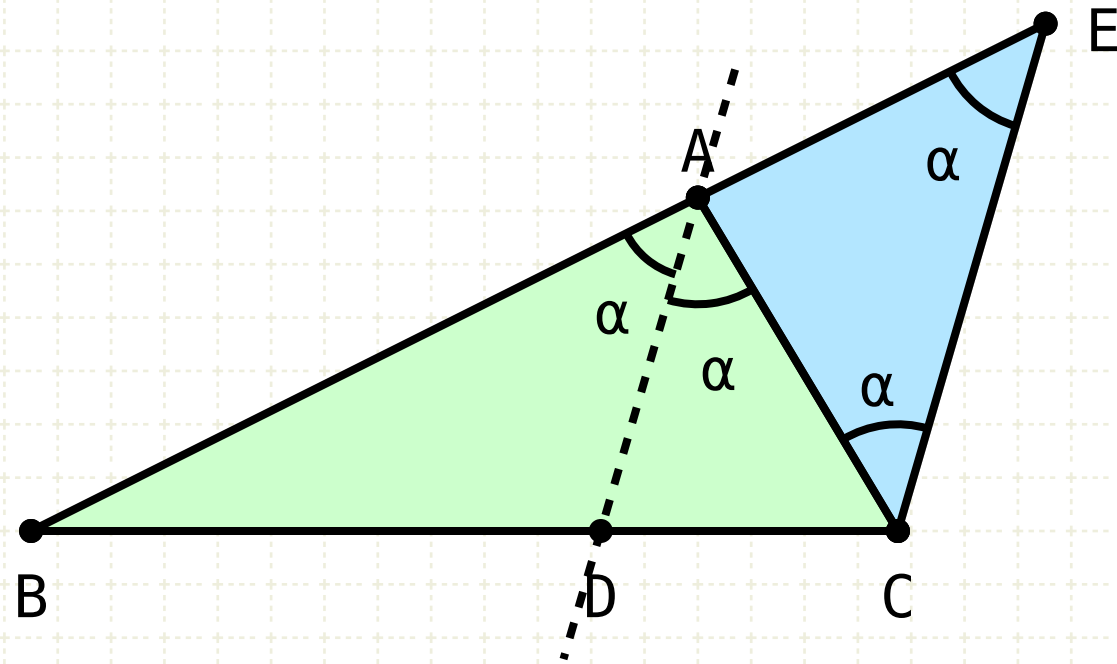
and the angle ACE is equal to the alternate angle CAD (I·29)

Therefore, the line AD bisects the angle BAC



Proposition 3 of Book VI

If an angle of a triangle be bisected and the straight line cutting the angle cut the base also, the segments of the base will have the same ratio as the remaining sides of the triangle; and, if the segments of the base have the same ratio as the remaining sides of the triangle, the straight line joined from the vertex to the point of section will bisect the angle of the triangle.



$$BA:AC = BD:DC$$

$$DA \parallel CE$$

$$BA:AE = BD:DC$$

$$BA:AC = BA:AE$$

$$AC = AE$$

$$\angle ACE = \angle AEC$$

$$\angle AEC = \angle BAD$$

$$\angle ACE = \angle CAD$$

$$\angle BAD = \angle CAD$$

Proof (part 2)

Since DA is parallel to CE, the ratio AB to AE is equal to the ratio BD to DC (VI·2)

If two ratios are equal to the same, they are equal to each other (V·11), so the ratios BA to AC and BA to AE are also equal

Therefore, AC equals AE (V·9)

If AC and AE are equal, then triangle ACE is an isosceles triangle, and the angles AEC and ACE are equal (I·5)

The angle AEC is equal to the exterior angle BAD of the parallel lines AD and EC (I·29)

and the angle ACE is equal to the alternate angle CAD (I·29)

Therefore, the line AD bisects the angle BAC



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