Euclid's Elements

Book VII

Definitions:

1 A unit is that by virtue of which each of the things that exist is called one

2 A number is a multitude composed of units. (not one)

3 A number is part of a number, the less of the greater, when it measures the greater

11 A prime number is that which is measured by a unit alone.

12 Numbers prime to one another are those which are measured by a unit alone as a common measure

15 A number is said to multiply a number when that which is multiplied is added to itself as many times as there are units in the other, and thus some number is produced.

20 Numbers are proportional when the first is the same multiple, or the same part, or the same parts, of the second that the third is of the fourth.

As long as algebra and geometry have been separated, their progress have been slow and their uses limited; but when these two sciences have been united, they have lent each mutual forces, and have marched together towards perfection.

Joseph-Louis Lagrange
(1736 to 1813)

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Table of Contents, Chapter 7

1 Determine if two numbers are relatively prime

2 Find the greatest common divisor for two numbers

3 Find the largest common divisor for three numbers

4 Given two natural numbers, A and B, either B is part of A, or there exists a natural number (a part) that can measure both A and B

5 If $B = (1/q)\cdot A$ and $D = (1/q)\cdot C$, then $(B+D) = (1/q)\cdot(A+C)$

6 If $B = (p/q)\cdot A$ and $D = (p/q)\cdot C$, then $(B+D) = (p/q)\cdot(A+C)$

7 If $B = A/q$ and $D = C/q$, $B>D$, then $(B-D) = (A-C)/q$

8 If $B = (p/q)\cdot A$ and $D = (p/q)\cdot C$, $B>D$, then $(B-D) = (p/q)\cdot(A-C)$

9 If $B = (1/q)\cdot A$ and $D = (1/q)\cdot C$, and If $B = (r/s)\cdot D$, then $A = (r/s)\cdot C$

10 If $B = (p/q)\cdot A$ and $D = (p/q)\cdot C$, and If $B = (r/s)\cdot D$, then $A = (r/s)\cdot C$

11 If $A:B = C:D$, then $(A-C):(B-D) = A:B$

12 If $A:B = C:D$, then $(A+C):(B+C) = A:B$

13 If $A:B = C:D$, then $A:C = B:D$

14 If $A:B = D:E$ and $B:C = E:F$, then $A:C = D:F$

15 If $B = i\cdot 1$ and $E = i\cdot D$, and if $D = j\cdot 1$ then $E = j\cdot B$

16 $A \times B = B \times A$

17 If $D = A \times B$ and $E = A \times C$ then $D:E = B:C$

18 If $D = B \times A$ and $E = C \times A$ then $D:E = B:C$

19 If $A:B = C:D$ then $A \times D = B \times C$
If $A \times D = B \times C$ then $A:B = C:D$

20 Given the ratio A:B and C,D are the smallest numbers such that $A:B = C:D$ then $A = n\cdot C$ and $B = n\cdot D$

21 If A,B are relatively prime, then A,B are the smallest whole numbers that can be used to describe the ratio A:B

22 If A,B are the smallest whole numbers that can be used to describe the ratio A:B, then A,B are relatively prime

23 If A,B are relatively prime and if $A = n\cdot C$, then B,C are relatively prime

24 If A,C are relatively prime and B,C are relatively prime then the $A \times B$ is relatively prime to C

25 If A,B are relatively prime then A^2,B are relatively prime

26 If A is relatively prime to C and D, and if B is also relatively prime to C and D, then $A \times B$ is relatively prime to $C \times D$

27 If A,B are relatively prime, then A^2,B^2 are relatively prime, and A^3,B^3 are relatively prime, and so on

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Table of Contents, Chapter 7

28 If A,B are relatively prime, then A,(A+B) are relatively prime

29 If A is prime, and B ≠ n·A, then A,B are relatively prime

30 If C = A×B and C = i·D where D is prime, then either A = j·D or B = j·D

31 If A = B×C, then A = j·D where D is prime

32 If A is a number then it is either prime, or A = j·D where D is prime

33 Find the smallest numbers X,Y,Z where the ratio X:Y:Z is equal to the given ratio A:B:C

34 Find the lowest common denominator of 2 numbers

35 If E is the lowest common denominator of A,B, and if C = n ·A = m·B, then C = i·E

36 Find the least common multiple of 3 numbers

37 If A = p·B, then A = q·C where C = p·1

38 If A = (1/c)·B and C = c·1 then A = n·C

39 Find the smallest number that has the fractions 1/a, 1/b, 1/c

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Proposition 34 of Book VII

Given two numbers, to find the least number which they measure.

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Proposition 34 of Book VII

Given two numbers, to find the least number which they measure.

Definition: lowest common multiple

$\mathrm{lcm}(A,B) = C$

$S=\{x\}|x\in\mathbb{N},\ x=m\cdot A,\ x=n\cdot B$

$C\in S$ such that $C\leq x,\ \forall(x)\in S$

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Proposition 34 of Book VII

Given two numbers, to find the least number which they measure.

In other words

Given two numbers A,B

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Proposition 34 of Book VII

Given two numbers, to find the least number which they measure.

Find:

`lcm(A,B) = C`

In other words

Given two numbers A,B

Find the lowest common multiple

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Proposition 34 of Book VII

Given two numbers, to find the least number which they measure.

Method 1: A,B are co-prime

gcd(A,B) = 1

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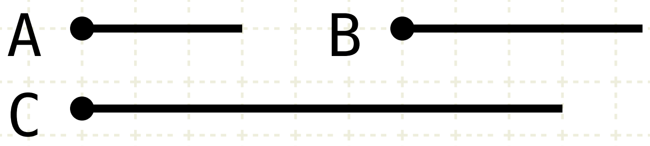
Proposition 34 of Book VII

Given two numbers, to find the least number which they measure.



$$\gcd(A,B) = 1$$

$$C = A \times B$$

$$C = B \times A$$

Method 1: A,B are co-prime

Let C be equal to A times B, which is also equal to B time A

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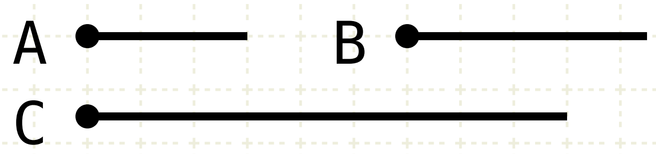
Given two numbers, to find the least number which they measure.



$\gcd(A,B) = 1$

$C = A \times B$

$C = B \times A$

$C = A \cdot B$

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Method 1: A,B are co-prime

Let C be equal to A times B, which is also equal to B time A

Thus B measures C, and A measures C

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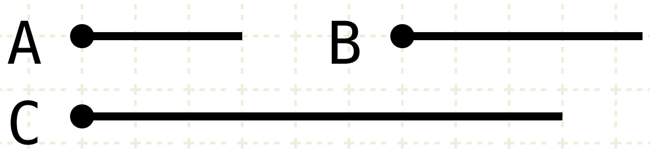

$$\gcd(A,B) = 1$$

$$C = A \times B$$
$$C = B \times A$$
$$C = A \cdot B$$
$$C = B \cdot A$$
$$\operatorname{lcm}(A,B) = C$$

Method 1: A,B are co-prime

Let C be equal to A times B, which is also equal to B time A

Thus B measures C, and A measures C

C is the lowest common multiple of A and B

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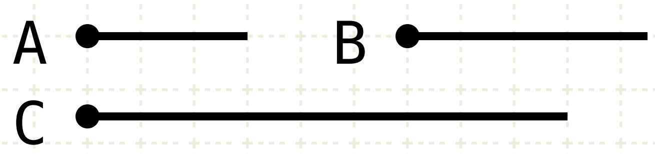

$$\gcd(A,B) = 1$$
$$C = A \times B = B \times A$$
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Method 1: A,B are co-prime

Let C be equal to A times B, which is also equal to B time A

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Proof by Contradiction

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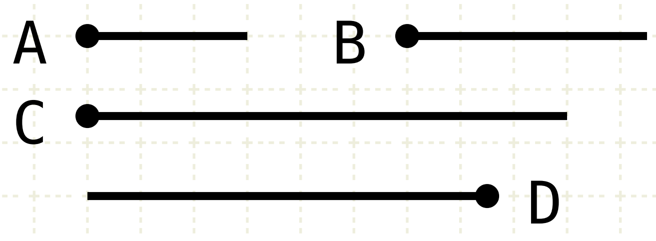

$\gcd(A,B) = 1$

$C = A \times B = B \times A$

$\mathrm{lcm}(A,B) = C$

$D = m \cdot A$

$D = n \cdot B$

$D < C$

Method 1: A,B are co-prime

Let C be equal to A times B, which is also equal to B time A

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C is the lowest common multiple of A and B

Proof by Contradiction

Let A and B measure D, which is less than C

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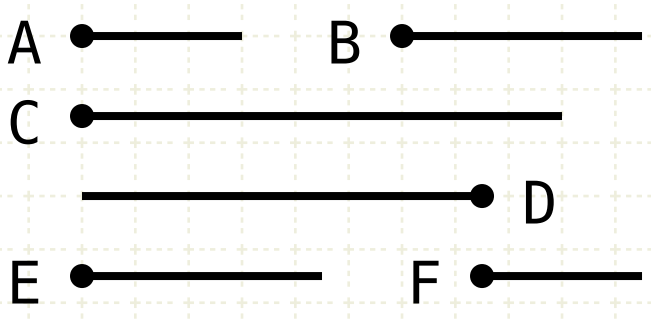

$\gcd(A,B) = 1$

$C = A \times B = B \times A$

$\mathrm{lcm}(A,B) = C$

$D = m \cdot A$

$D = n \cdot B$

$D < C$

$E = m \cdot 1$

$F = n \cdot 1$

Method 1: A,B are co-prime

Let C be equal to A times B, which is also equal to B time A

Thus B measures C, and A measures C

C is the lowest common multiple of A and B

Proof by Contradiction

Let A and B measure D, which is less than C

Then, as many times as A measure D, let there be so many units in E, and as may times and B measures D, let there be so many units in F

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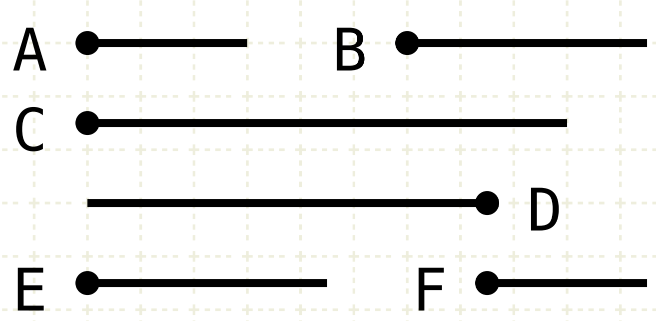
Given two numbers, to find the least number which they measure.



$\gcd(A,B) = 1$

$C = A\times B = B\times A$

$\mathrm{lcm}(A,B) = C$

$D = m\cdot A$

$D = n\cdot B$

$D < C$

$E = m\cdot 1$

$F = n\cdot 1$

$D = E\times A = F\times B$

Method 1: A,B are co-prime

Let C be equal to A times B, which is also equal to B time A

Thus B measures C, and A measures C

C is the lowest common multiple of A and B

Proof by Contradiction

Let A and B measure D, which is less than C

Then, as many times as A measure D, let there be so many units in E, and as may times and B measures D, let there be so many units in F

Thus D is equal to A multiplied by E, and also equal to B multiplied by F (VII.Def.15)

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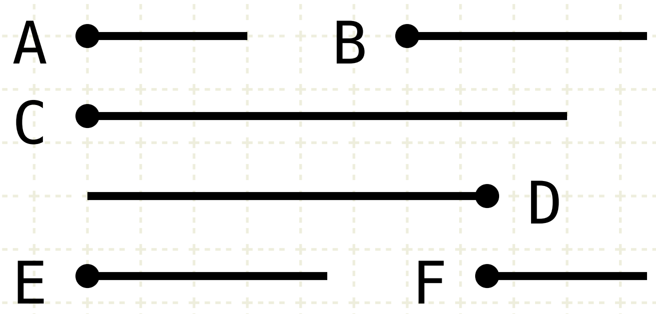
Given two numbers, to find the least number which they measure.



$\gcd(A,B) = 1$

$C = A\times B = B\times A$

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$D = m\cdot A$

$D = n\cdot B$

$D < C$

$E = m\cdot 1$

$F = n\cdot 1$

$D = E\times A = F\times B$

$A:B = F:E$

Method 1: A,B are co-prime

Let C be equal to A times B, which is also equal to B time A

Thus B measures C, and A measures C

C is the lowest common multiple of A and B

Proof by Contradiction

Let A and B measure D, which is less than C

Then, as many times as A measure D, let there be so many units in E, and as may times and B measures D, let there be so many units in F

Thus D is equal to A multiplied by E, and also equal to B multiplied by F (VII.Def.15)

Therefore, as A is to B, so is F to E (VII·19)

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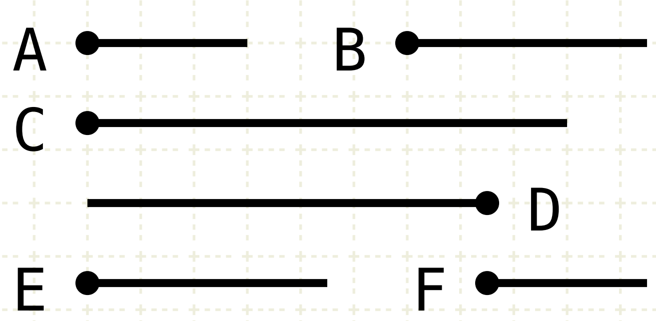
Given two numbers, to find the least number which they measure.



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$D = m\cdot A$

$D = n\cdot B$

$D < C$

$E = m\cdot 1$

$F = n\cdot 1$

$D = E\times A = F\times B$

$A:B = F:E$

$A < F,\ B < E$

Method 1: A,B are co-prime

Let C be equal to A times B, which is also equal to B time A

Thus B measures C, and A measures C

C is the lowest common multiple of A and B

Proof by Contradiction

Let A and B measure D, which is less than C

Then, as many times as A measure D, let there be so many units in E, and as may times and B measures D, let there be so many units in F

Thus D is equal to A multiplied by E, and also equal to B multiplied by F (VII.Def.15)

Therefore, as A is to B, so is F to E (VII·19)

Since A,B are relatively prime, then they are the smallest numbers that can express the ratio A to B (VII·21)

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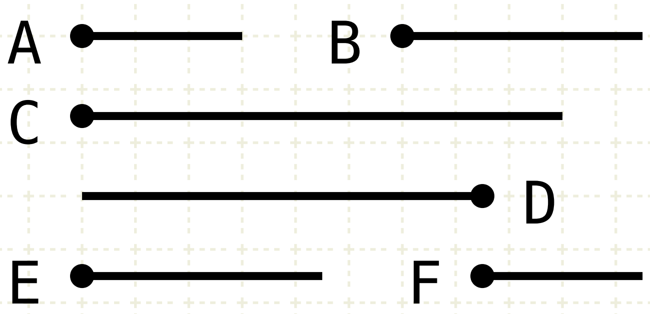
Given two numbers, to find the least number which they measure.



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$$D = m\cdot A$$
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$$D < C$$
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$$F = n\cdot 1$$
$$D = E\times A = F\times B$$
$$A:B = F:E$$
$$A < F,\ B < E$$
$$E = p\cdot B$$

Method 1: A,B are co-prime

Let C be equal to A times B, which is also equal to B time A

Thus B measures C, and A measures C

C is the lowest common multiple of A and B

Proof by Contradiction

Let A and B measure D, which is less than C

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Thus D is equal to A multiplied by E, and also equal to B multiplied by F (VII.Def.15)

Therefore, as A is to B, so is F to E (VII·19)

Since A,B are relatively prime, then they are the smallest numbers that can express the ratio A to B (VII·21)

And if A,B are the smallest numbers that express the ratio of F to E, then A measures F, and B measures E (VI·20)

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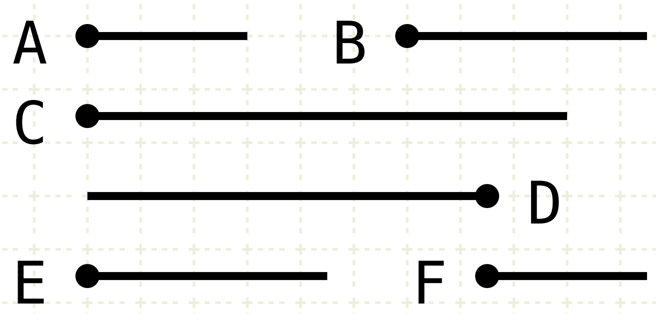
Proposition 34 of Book VII

Given two numbers, to find the least number which they measure.



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$D = m \cdot A$

$D = n \cdot B$

$D < C$

$E = m \cdot 1$

$F = n \cdot 1$

$D = E \times A = F \times B$

$A:B = F:E$

$A < F,\ B < E$

$E = p \cdot B$

$C:D = B:E$

Method 1: A,B are co-prime

Let C be equal to A times B, which is also equal to B time A

Thus B measures C, and A measures C

C is the lowest common multiple of A and B

Proof by Contradiction

Let A and B measure D, which is less than C

Then, as many times as A measure D, let there be so many units in E, and as may times and B measures D, let there be so many units in F

Thus D is equal to A multiplied by E, and also equal to B multiplied by F (VII.Def.15)

Therefore, as A is to B, so is F to E (VII·19)

Since A,B are relatively prime, then they are the smallest numbers that can express the ratio A to B (VII·21)

And if A,B are the smallest numbers that express the ratio of F to E, then A measures F, and B measures E (VI·20)

Since B times A equals C, and E times A equals D, then the ratio B to E is equal to C to D (VI·17)

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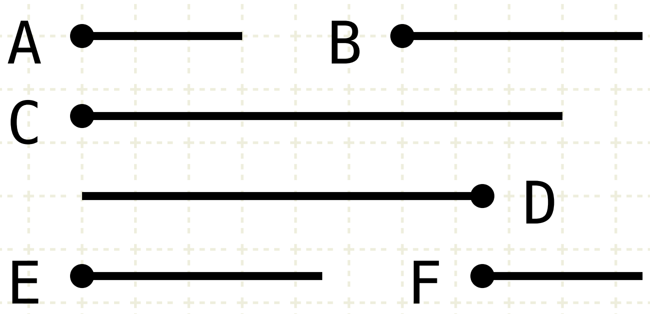
Proposition 34 of Book VII

Given two numbers, to find the least number which they measure.



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$A:B = F:E$

$A < F,\ B < E$

$E = p\cdot B$

$C:D = B:E$

$E < B$

Method 1: A,B are co-prime

Let C be equal to A times B, which is also equal to B time A

Thus B measures C, and A measures C

C is the lowest common multiple of A and B

Proof by Contradiction

Let A and B measure D, which is less than C

Then, as many times as A measure D, let there be so many units in E, and as may times and B measures D, let there be so many units in F

Thus D is equal to A multiplied by E, and also equal to B multiplied by F (VII.Def.15)

Therefore, as A is to B, so is F to E (VII·19)

Since A,B are relatively prime, then they are the smallest numbers that can express the ratio A to B (VII·21)

And if A,B are the smallest numbers that express the ratio of F to E, then A measures F, and B measures E (VI·20)

Since B times A equals C, and E times A equals D, then the ratio B to E is equal to C to D (VI·17)

But D is less than C, so E is less than B

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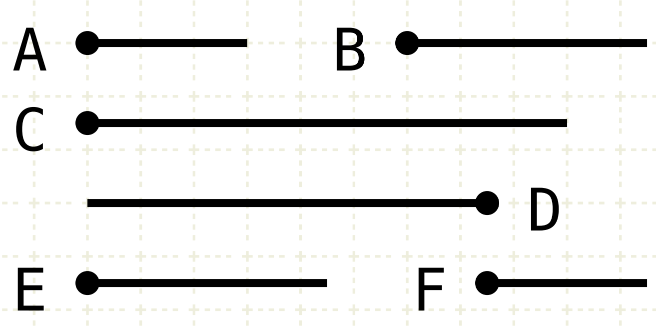
Proposition 34 of Book VII

Given two numbers, to find the least number which they measure.



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$D = m\cdot A$
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$E = m\cdot 1$
$F = n\cdot 1$
$D = E\times A = F\times B$
$A:B = F:E$
$A < F,\ B < E$
$E = p\cdot B$
$C:D = B:E$
$E < B$

Method 1: A,B are co-prime

Let C be equal to A times B, which is also equal to B time A

Thus B measures C, and A measures C

C is the lowest common multiple of A and B

Proof by Contradiction

Let A and B measure D, which is less than C

Then, as many times as A measure D, let there be so many units in E, and as may times and B measures D, let there be so many units in F

Thus D is equal to A multiplied by E, and also equal to B multiplied by F (VII.Def.15)

Therefore, as A is to B, so is F to E (VII·19)

Since A,B are relatively prime, then they are the smallest numbers that can express the ratio A to B (VII·21)

And if A,B are the smallest numbers that express the ratio of F to E, then A measures F, and B measures E (VI·20)

Since B times A equals C, and E times A equals D, then the ratio B to E is equal to C to D (VI·17)

But D is less than C, so E is less than B

But B measures C, so E cannot be less than B

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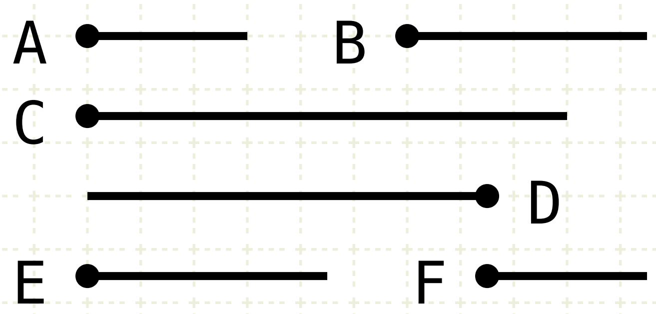
Proposition 34 of Book VII

Given two numbers, to find the least number which they measure.



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Method 1: A,B are co-prime

Let C be equal to A times B, which is also equal to B time A

Thus B measures C, and A measures C

C is the lowest common multiple of A and B

Proof by Contradiction

Let A and B measure D, which is less than C

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Thus D is equal to A multiplied by E, and also equal to B multiplied by F (VII.Def.15)

Therefore, as A is to B, so is F to E (VII·19)

Since A,B are relatively prime, then they are the smallest numbers that can express the ratio A to B (VII·21)

And if A,B are the smallest numbers that express the ratio of F to E, then A measures F, and B measures E (VI·20)

Since B times A equals C, and E times A equals D, then the ratio B to E is equal to C to D (VI·17)

But D is less than C, so E is less than B

But B measures C, so E cannot be less than B

Therefore there is no number D less than C that is measured by A and B, thus C is the lowest common multiple

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Proposition 34 of Book VII

Given two numbers, to find the least number which they measure.

Method 2: A,B are not co-prime

gcd(A,B) ≠ 1

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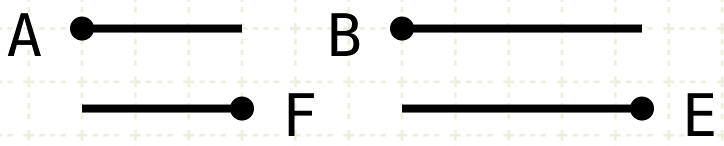
Proposition 34 of Book VII

Given two numbers, to find the least number which they measure.



$\gcd(A,B) \neq 1$

$S=\{(x,y) \mid x:y = A:B \}$

$(F,E)\in S$ such that $F\leq x, E\leq y, \forall (x,y)\in S$

$F:E = A:B$

Method 2: A,B are not co-prime

Find F,E such that F and E are the smallest numbers that have the same ratio of A and B (VII·33)

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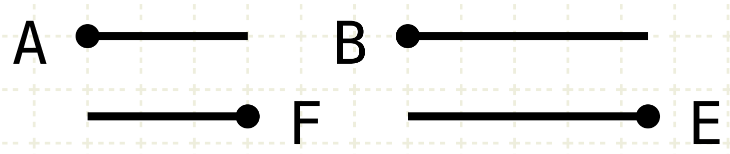
Proposition 34 of Book VII

Given two numbers, to find the least number which they measure.



$\gcd(A,B) \neq 1$

$S=\{(x,y) \mid x:y = A:B \}$

$(F,E) \in S$ such that $F \leq x, E \leq y, \forall (x,y) \in S$

$F:E = A:B$

$E \times A = F \times B$

Method 2: A,B are not co-prime

Find F,E such that F and E are the smallest numbers that have the same ratio of A and B (VII·33)

Thus the product of A and E is equal to the product of B and F (VII·19)

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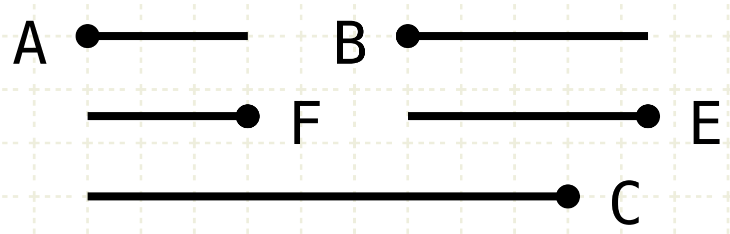
Proposition 34 of Book VII

Given two numbers, to find the least number which they measure.



gcd(A,B) ≠ 1

S={(x,y) | x:y = A:B }

(F,E)∈S such that F≤x,E≤y,∀(x,y)∈S

F:E = A:B

E×A = F×B

C = E×A

C = F×B

Method 2: A,B are not co-prime

Find F,E such that F and E are the smallest numbers that have the same ratio of A and B (VII·33)

Thus the product of A and E is equal to the product of B and F (VII·19)

Let C by equal to A times E and also equal to B times F

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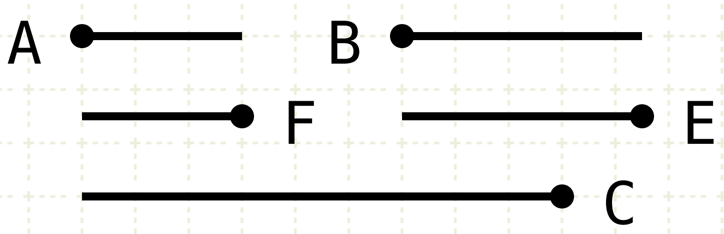
Proposition 34 of Book VII

Given two numbers, to find the least number which they measure.



$\gcd(A,B) \neq 1$

$S=\{(x,y) \mid x:y = A:B \}$

$(F,E) \in S$ such that $F \leq x, E \leq y, \forall (x,y) \in S$

$F:E = A:B$

$E \times A = F \times B$

$C = E \times A$

$C = F \times B$

$C = E \cdot A$

$C = F \cdot B$

Method 2: A,B are not co-prime

Find F,E such that F and E are the smallest numbers that have the same ratio of A and B (VII·33)

Thus the product of A and E is equal to the product of B and F (VII·19)

Let C by equal to A times E and also equal to B times F

Therefore A and B both measure C

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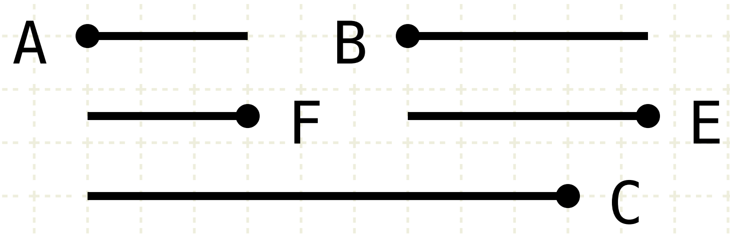
Proposition 34 of Book VII

Given two numbers, to find the least number which they measure.



$\gcd(A,B) \neq 1$

$S=\{(x,y) \mid x:y = A:B \}$

$(F,E)\in S$ such that $F\leq x, E\leq y, \forall (x,y)\in S$

$F:E = A:B$

$E\times A = F\times B$

$C = E\times A$

$C = F\times B$

$C = E\cdot A$

$C = F\cdot B$

$\mathrm{lcm}(A,B) = C$

Method 2: A,B are not co-prime

Find F,E such that F and E are the smallest numbers that have the same ratio of A and B (VII·33)

Thus the product of A and E is equal to the product of B and F (VII·19)

Let C by equal to A times E and also equal to B times F

Therefore A and B both measure C

C is the lowest common multiple of A and B

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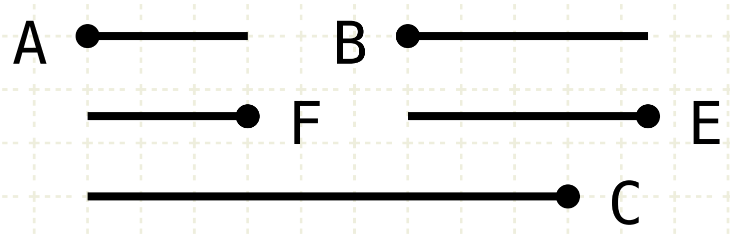
Proposition 34 of Book VII

Given two numbers, to find the least number which they measure.



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$(F,E) \in S$ such that $F \leq x, E \leq y, \forall (x,y) \in S$

$F:E = A:B$

$C = E \times A$

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Method 2: A,B are not co-prime

Find F,E such that F and E are the smallest numbers that have the same ratio of A and B (VII·33)

Let C by equal to A times E and also equal to B times F

C is the lowest common multiple of A and B

Proof by Contradiction

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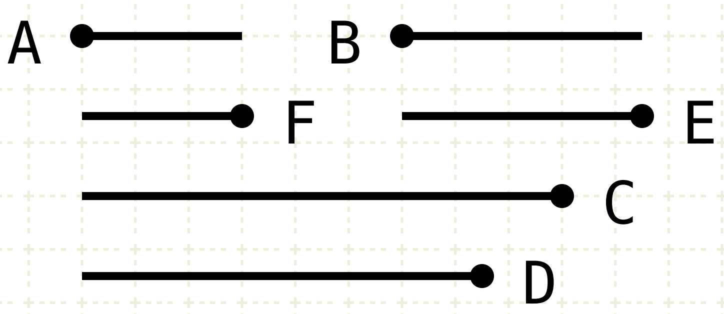
Proposition 34 of Book VII

Given two numbers, to find the least number which they measure.



$\gcd(A,B) \neq 1$

$S=\{(x,y) \mid x:y = A:B\}$

$(F,E)\in S$ such that $F\le x, E\le y, \forall (x,y)\in S$

$F:E = A:B$

$C = E\times A$

$C = F\times B$

$D = m\cdot A$

$D = n\cdot B$

$D < C$

Method 2: A,B are not co-prime

Find F,E such that F and E are the smallest numbers that have the same ratio of A and B (VII·33)

Let C by equal to A times E and also equal to B times F

C is the lowest common multiple of A and B

Proof by Contradiction

Let A and B measure D, which is less than C

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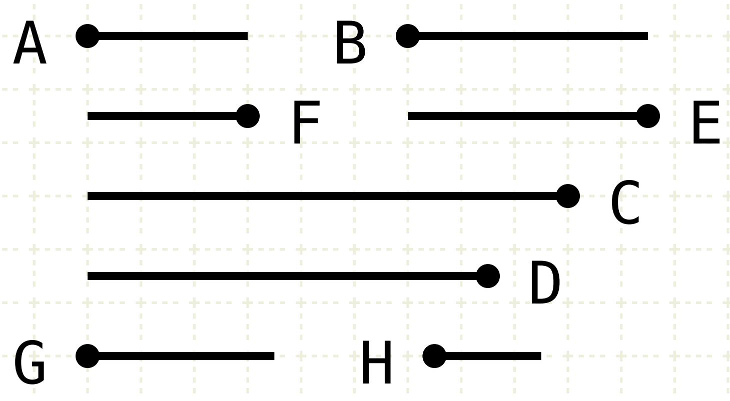
Proposition 34 of Book VII

Given two numbers, to find the least number which they measure.



$$\gcd(A,B) \neq 1$$
$$S=\{(x,y) \mid x:y = A:B\}$$
$$(F,E)\in S \text{ such that } F\le x, E\le y, \forall (x,y)\in S$$
$$F:E = A:B$$
$$C = E\times A$$
$$C = F\times B$$

$$D = m\cdot A$$
$$D = n\cdot B$$
$$D < C$$
$$G = m\cdot 1$$
$$H = n\cdot 1$$

Method 2: A,B are not co-prime

Find F,E such that F and E are the smallest numbers that have the same ratio of A and B (VII·33)

Let C by equal to A times E and also equal to B times F

C is the lowest common multiple of A and B

Proof by Contradiction

Let A and B measure D, which is less than C

Then, as many times as A measure D, let there be so many units in G, and as may times and B measures D, let there be so many units in H

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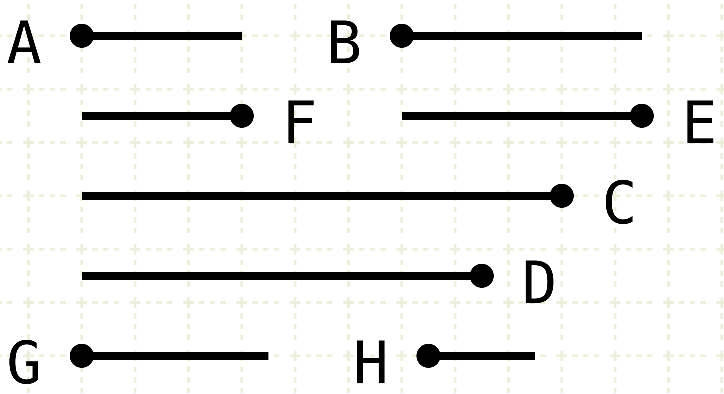
Proposition 34 of Book VII

Given two numbers, to find the least number which they measure.



$\gcd(A,B) \neq 1$

$S=\{(x,y) \mid x:y = A:B\}$

$(F,E)\in S$ such that $F\leq x, E\leq y, \forall(x,y)\in S$

$F:E = A:B$

$C = E\times A$

$C = F\times B$

$D = m\cdot A$

$D = n\cdot B$

$D < C$

$G = m\cdot 1$

$H = n\cdot 1$

$D = G\times A = H\times B$

Method 2: A,B are not co-prime

Find F,E such that F and E are the smallest numbers that have the same ratio of A and B (VII·33)

Let C by equal to A times E and also equal to B times F

C is the lowest common multiple of A and B

Proof by Contradiction

Let A and B measure D, which is less than C

Then, as many times as A measure D, let there be so many units in G, and as may times and B measures D, let there be so many units in H

Thus D is equal to A multiplied by G, and also equal to B multiplied by H

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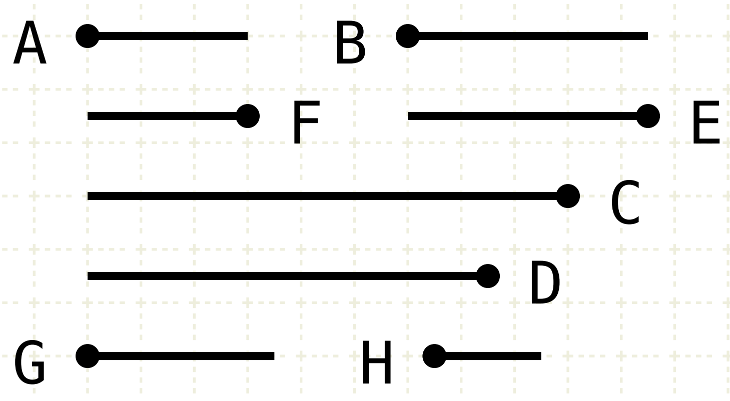
Proposition 34 of Book VII

Given two numbers, to find the least number which they measure.



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$(F,E) \in S$ such that $F \leq x, E \leq y, \forall (x,y) \in S$

$F:E = A:B$

$C = E \times A$

$C = F \times B$

$D = m \cdot A$

$D = n \cdot B$

$D < C$

$G = m \cdot 1$

$H = n \cdot 1$

$D = G \times A = H \times B$

$A:B = H:G$

Method 2: A,B are not co-prime

Find F,E such that F and E are the smallest numbers that have the same ratio of A and B (VII·33)

Let C by equal to A times E and also equal to B times F

C is the lowest common multiple of A and B

Proof by Contradiction

Let A and B measure D, which is less than C

Then, as many times as A measure D, let there be so many units in G, and as may times and B measures D, let there be so many units in H

Thus D is equal to A multiplied by G, and also equal to B multiplied by H

Therefore, as A is to B, so is H to G (VII·19)

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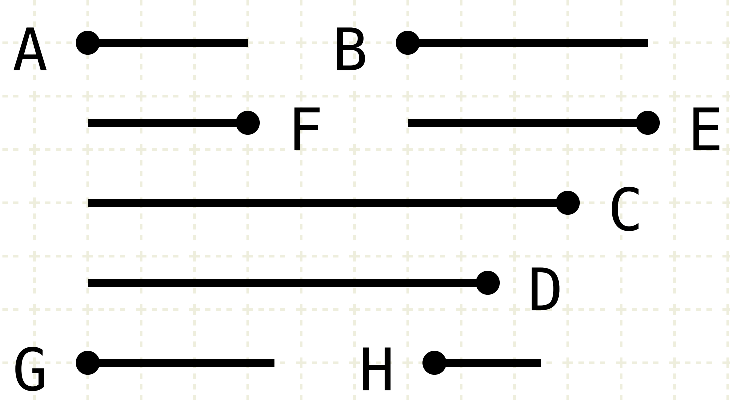
Proposition 34 of Book VII

Given two numbers, to find the least number which they measure.



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$(F,E) \in S$ such that $F \leq x, E \leq y, \forall (x,y) \in S$

$F:E = A:B$

$C = E \times A$

$C = F \times B$

$D = m \cdot A$

$D = n \cdot B$

$D < C$

$G = m \cdot 1$

$H = n \cdot 1$

$D = G \times A = H \times B$

$A:B = H:G$

$F:E = H:G$

Method 2: A,B are not co-prime

Find F,E such that F and E are the smallest numbers that have the same ratio of A and B (VII·33)

Let C by equal to A times E and also equal to B times F

C is the lowest common multiple of A and B

Proof by Contradiction

Let A and B measure D, which is less than C

Then, as many times as A measure D, let there be so many units in G, and as may times and B measures D, let there be so many units in H

Thus D is equal to A multiplied by G, and also equal to B multiplied by H

Therefore, as A is to B, so is H to G (VII·19)

But as A is to B, so is F to E, therefore as F is to E, so is H to G

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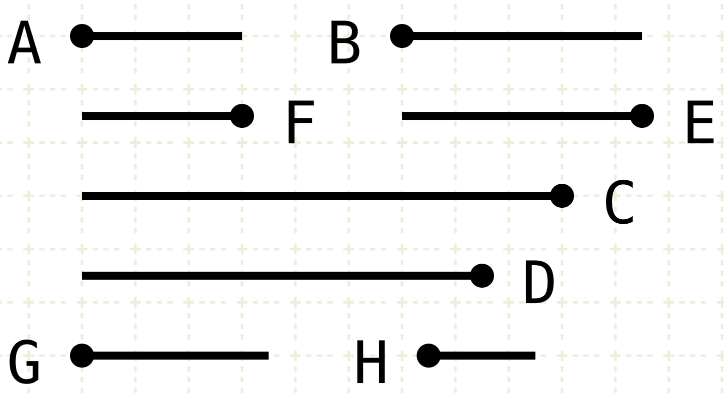
Proposition 34 of Book VII

Given two numbers, to find the least number which they measure.



$\gcd(A,B) \neq 1$

$S=\{(x,y) \mid x:y = A:B \}$

$(F,E)\in S$ such that $F\leq x, E\leq y, \forall (x,y)\in S$

$F:E = A:B$

$C = E\times A$

$C = F\times B$

$D = m\cdot A$

$D = n\cdot B$

$D < C$

$G = m\cdot 1$

$H = n\cdot 1$

$D = G\times A = H\times B$

$A:B = H:G$

$F:E = H:G$

$G = p\cdot E$

Method 2: A,B are not co-prime

Find F,E such that F and E are the smallest numbers that have the same ratio of A and B (VII·33)

Let C by equal to A times E and also equal to B times F

C is the lowest common multiple of A and B

Proof by Contradiction

Let A and B measure D, which is less than C

Then, as many times as A measure D, let there be so many units in G, and as may times and B measures D, let there be so many units in H

Thus D is equal to A multiplied by G, and also equal to B multiplied by H

Therefore, as A is to B, so is H to G (VII·19)

But as A is to B, so is F to E, therefore as F is to E, so is H to G

But F and E are the smallest numbers in the ratio A:B, thus F measures H and E measures G (VII·20)

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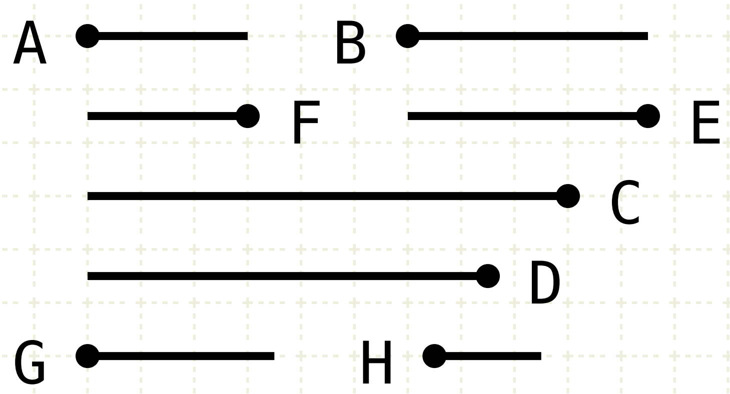
Proposition 34 of Book VII

Given two numbers, to find the least number which they measure.



gcd(A,B) ≠ 1
S={(x,y) | x:y = A:B }
(F,E)∈S such that F≤x,E≤y,∀(x,y)∈S
F:E = A:B
C = E×A
C = F×B

D = m·A
D = n·B
D < C
G = m·1
H = n·1
D = G×A = H×B
A:B = H:G
F:E = H:G
G = p·E
C:D = E:G

Method 2: A,B are not co-prime

Find F,E such that F and E are the smallest numbers that have the same ratio of A and B (VII·33)

Let C by equal to A times E and also equal to B times F

C is the lowest common multiple of A and B

Proof by Contradiction

Let A and B measure D, which is less than C

Then, as many times as A measure D, let there be so many units in G, and as may times and B measures D, let there be so many units in H

Thus D is equal to A multiplied by G, and also equal to B multiplied by H

Therefore, as A is to B, so is H to G (VII·19)

But as A is to B, so is F to E, therefore as F is to E, so is H to G

But F and E are the smallest numbers in the ratio A:B, thus F measures H and E measures G (VII·20)

Since A times E equals C, and G times A equals D, then the ratio E to G is equal to C to D (VI·17)

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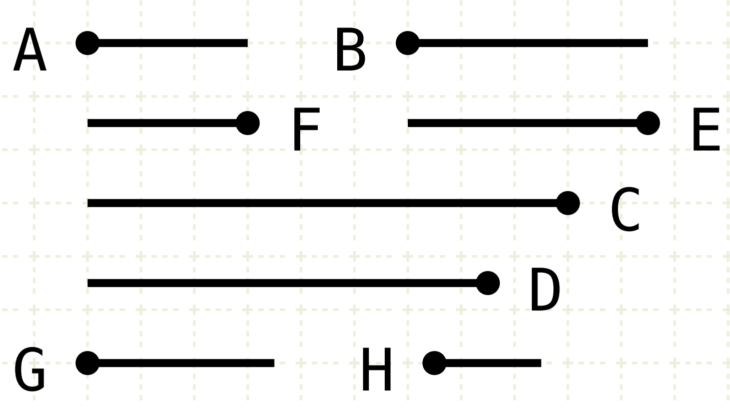
Proposition 34 of Book VII

Given two numbers, to find the least number which they measure.



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$D = m \cdot A$

$D = n \cdot B$

$D < C$

$G = m \cdot 1$

$H = n \cdot 1$

$D = G \times A = H \times B$

$A:B = H:G$

$F:E = H:G$

$G = p \cdot E$

$C:D = E:G$

$G < E$

Method 2: A,B are not co-prime

Find F,E such that F and E are the smallest numbers that have the same ratio of A and B (VII·33)

Let C by equal to A times E and also equal to B times F

C is the lowest common multiple of A and B

Proof by Contradiction

Let A and B measure D, which is less than C

Then, as many times as A measure D, let there be so many units in G, and as may times and B measures D, let there be so many units in H

Thus D is equal to A multiplied by G, and also equal to B multiplied by H

Therefore, as A is to B, so is H to G (VII·19)

But as A is to B, so is F to E, therefore as F is to E, so is H to G

But F and E are the smallest numbers in the ratio A:B, thus F measures H and E measures G (VII·20)

Since A times E equals C, and G times A equals D, then the ratio E to G is equal to C to D (VI·17)

But D is less than C, so G is less than E

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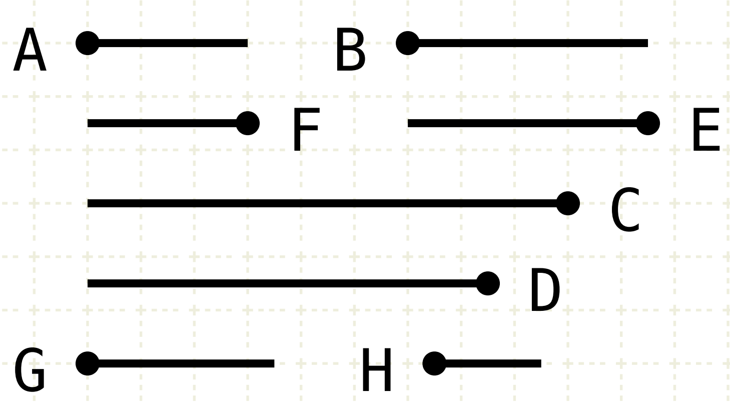
Proposition 34 of Book VII

Given two numbers, to find the least number which they measure.



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$G = m\cdot 1$

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$D = G\times A = H\times B$

$A:B = H:G$

$F:E = H:G$

$G = p\cdot E$

$C:D = E:G$

$G < E$

Method 2: A,B are not co-prime

Find F,E such that F and E are the smallest numbers that have the same ratio of A and B (VII·33)

Let C by equal to A times E and also equal to B times F

C is the lowest common multiple of A and B

Proof by Contradiction

Let A and B measure D, which is less than C

Then, as many times as A measure D, let there be so many units in G, and as may times and B measures D, let there be so many units in H

Thus D is equal to A multiplied by G, and also equal to B multiplied by H

Therefore, as A is to B, so is H to G (VII·19)

But as A is to B, so is F to E, therefore as F is to E, so is H to G

But F and E are the smallest numbers in the ratio A:B, thus F measures H and E measures G (VII·20)

Since A times E equals C, and G times A equals D, then the ratio E to G is equal to C to D (VI·17)

But D is less than C, so G is less than E

But E measures G, so G cannot be less than E

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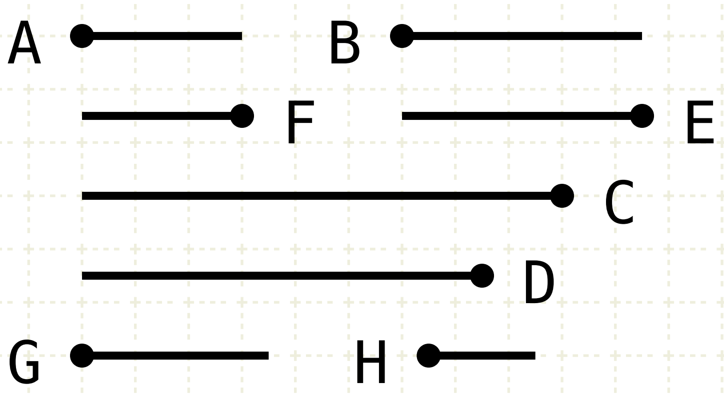
Proposition 34 of Book VII

Given two numbers, to find the least number which they measure.



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$D = m \cdot A$

$D = n \cdot B$

$D < C$

$G = m \cdot 1$

$H = n \cdot 1$

$D = G \times A = H \times B$

$A:B = H:G$

$F:E = H:G$

$G = p \cdot E$

$C:D = E:G$

$G < E$

Method 2: A,B are not co-prime

Find F,E such that F and E are the smallest numbers that have the same ratio of A and B (VII·33)

Let C by equal to A times E and also equal to B times F

C is the lowest common multiple of A and B

Proof by Contradiction

Let A and B measure D, which is less than C

Then, as many times as A measure D, let there be so many units in G, and as may times and B measures D, let there be so many units in H

Thus D is equal to A multiplied by G, and also equal to B multiplied by H

Therefore, as A is to B, so is H to G (VII·19)

But as A is to B, so is F to E, therefore as F is to E, so is H to G

But F and E are the smallest numbers in the ratio A:B, thus F measures H and E measures G (VII·20)

Since A times E equals C, and G times A equals D, then the ratio E to G is equal to C to D (VI·17)

But D is less than C, so G is less than E

But E measures G, so G cannot be less than E

Therefore there is no number D less than C that is measured by A and B, thus C is the lowest common multiple

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