

# Euclid's Elements

## Book V



$$AB:C = DE:F$$

$$BG:C = EH:F$$

$$AG:C = DH:F$$

*Proportions are what makes the old Greek temples classic in their beauty. They are like huge blocks, from which the air has been literally hewn out between the columns.*

**Arne Jacobsen**



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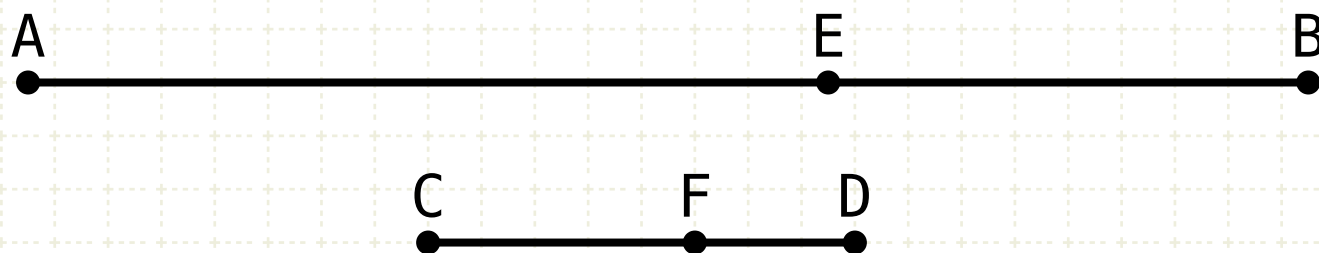
## Proposition 5 of Book V

If a magnitude be the same multiple of a magnitude that a part subtracted is of a part subtracted, the remainder will also be the same multiple of the remainder that the whole is of the whole



## Proposition 5 of Book V

If a magnitude be the same multiple of a magnitude that a part subtracted is of a part subtracted, the remainder will also be the same multiple of the remainder that the whole is of the whole



$$AB = n \cdot CD$$

$$AE = n \cdot CF$$

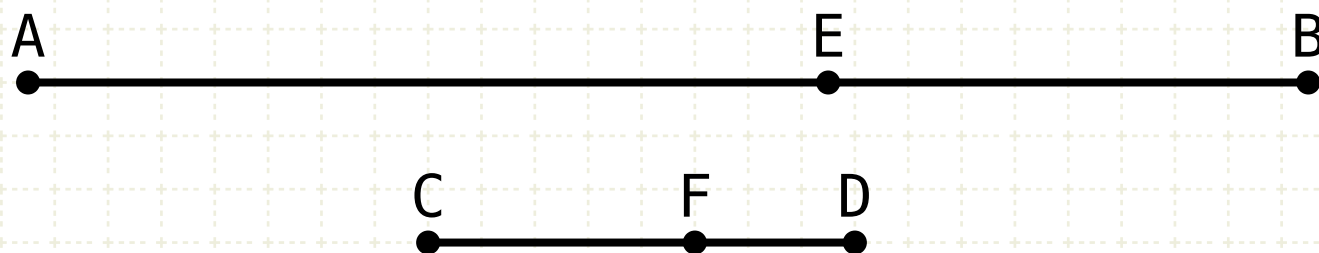
### In other words

Let AB be the same multiple of CD as AE is of CF



## Proposition 5 of Book V

If a magnitude be the same multiple of a magnitude that a part subtracted is of a part subtracted, the remainder will also be the same multiple of the remainder that the whole is of the whole



### In other words

Let AB be the same multiple of CD as AE is of CF

Then the remainder of AB minus the part AE is the same multiple of the remainder of CD minus CF

$$AB = n \cdot CD$$

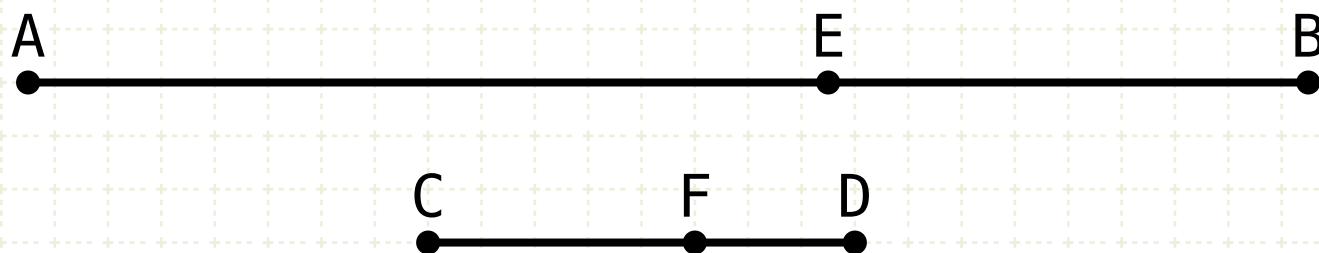
$$AE = n \cdot CF$$

$$EB = n \cdot FD$$

$$AB - AE = n \cdot CD - n \cdot CF = n \cdot (CD - CF)$$

## Proposition 5 of Book V

If a magnitude be the same multiple of a magnitude that a part subtracted is of a part subtracted, the remainder will also be the same multiple of the remainder that the whole is of the whole



$$AB = n \cdot CD$$

$$AE = n \cdot CF$$

### In other words

Let AB be the same multiple of CD as AE is of CF

Then the remainder of AB minus the part AE is the same multiple of the remainder of CD minus CF

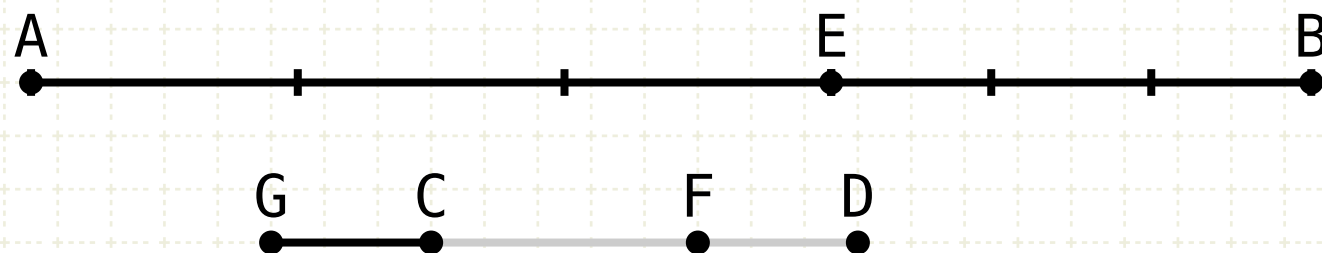
### Proof





## Proposition 5 of Book V

If a magnitude be the same multiple of a magnitude that a part subtracted is of a part subtracted, the remainder will also be the same multiple of the remainder that the whole is of the whole



$$AB = n \cdot CD$$

$$AE = n \cdot CF$$

$$EB = n \cdot GC$$

### In other words

Let AB be the same multiple of CD as AE is of CF

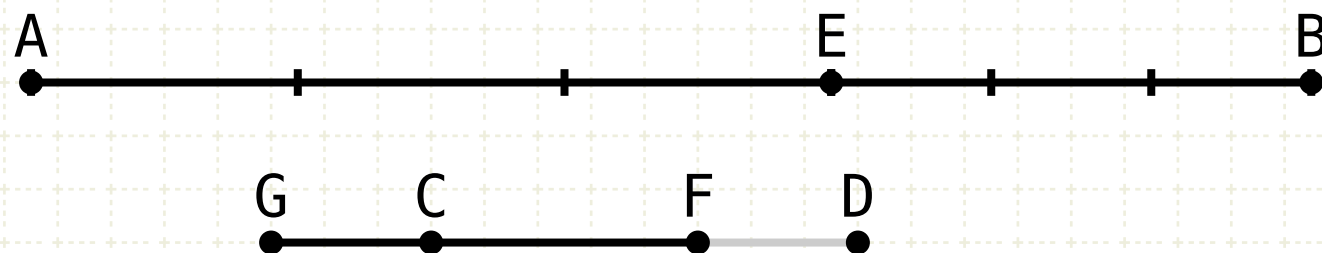
Then the remainder of AB minus the part AE is the same multiple of the remainder of CD minus CF

### Proof

Create a line GC such that EB is the same multiple of GC as AE is to CF

## Proposition 5 of Book V

If a magnitude be the same multiple of a magnitude that a part subtracted is of a part subtracted, the remainder will also be the same multiple of the remainder that the whole is of the whole



$$AB = n \cdot CD$$

$$AE = n \cdot CF$$

$$EB = n \cdot GC$$

$$AE + EB = n \cdot CF + n \cdot GC = n \cdot (CF + GC)$$

$$AB = n \cdot GF$$

### In other words

Let AB be the same multiple of CD as AE is of CF

Then the remainder of AB minus the part AE is the same multiple of the remainder of CD minus CF

### Proof

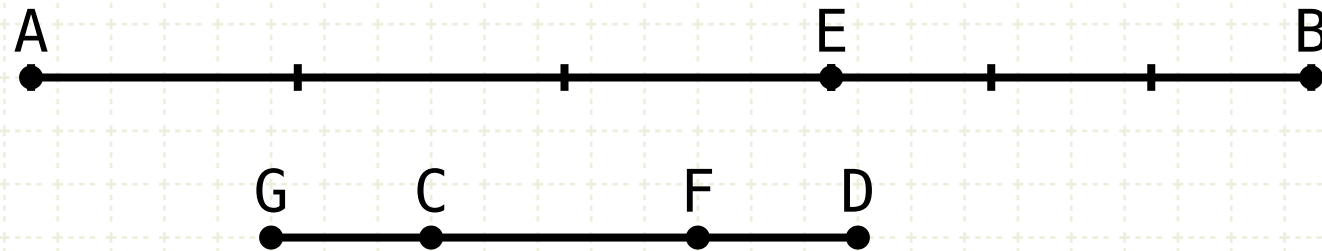
Create a line GC such that EB is the same multiple of GC as AE is to CF

Since AE and EB are equimultiples of CF and GC, then the sum of the lines are also of the same multiple (V.1)



## Proposition 5 of Book V

If a magnitude be the same multiple of a magnitude that a part subtracted is of a part subtracted, the remainder will also be the same multiple of the remainder that the whole is of the whole



$$AB = n \cdot CD$$

$$AE = n \cdot CF$$

$$EB = n \cdot GC$$

$$AE + EB = n \cdot CF + n \cdot GC = n \cdot (CF + GC)$$

$$AB = n \cdot GF$$

$$GF = CD$$

### In other words

Let AB be the same multiple of CD as AE is of CF

Then the remainder of AB minus the part AE is the same multiple of the remainder of CD minus CF

### Proof

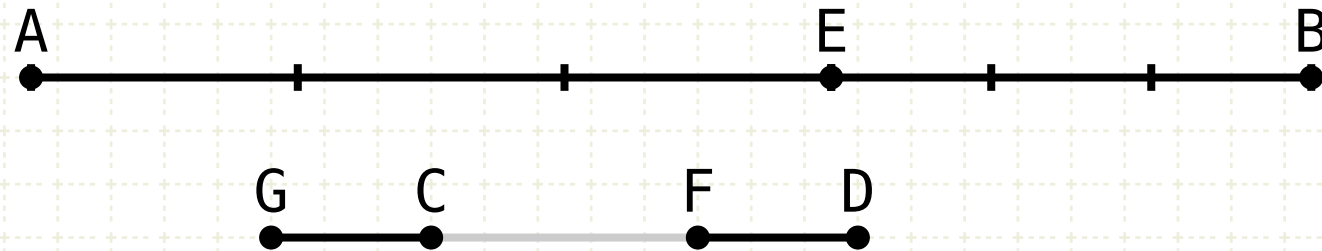
Create a line GC such that EB is the same multiple of GC as AE is to CF

Since AE and EB are equimultiples of CF and GC, then the sum of the lines are also of the same multiple (V.1)

But AB is by definition the same multiple of CD, so CD equals GF

## Proposition 5 of Book V

If a magnitude be the same multiple of a magnitude that a part subtracted is of a part subtracted, the remainder will also be the same multiple of the remainder that the whole is of the whole



$$AB = n \cdot CD$$

$$AE = n \cdot CF$$

$$EB = n \cdot GC$$

$$AE + EB = n \cdot CF + n \cdot GC = n \cdot (CF + GC)$$

$$AB = n \cdot GF$$

$$GF = CD$$

$$GF - CF = CD - CF$$

$$GC = FD$$

### In other words

Let AB be the same multiple of CD as AE is of CF

Then the remainder of AB minus the part AE is the same multiple of the remainder of CD minus CF

### Proof

Create a line GC such that EB is the same multiple of GC as AE is to CF

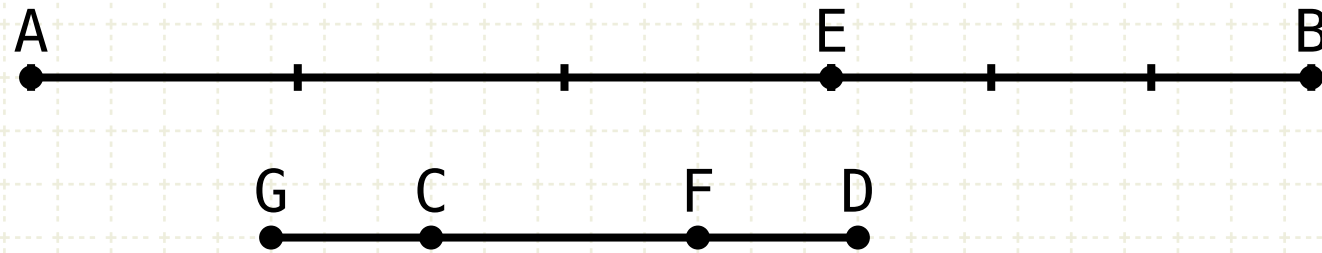
Since AE and EB are equimultiples of CF and GC, then the sum of the lines are also of the same multiple (V.1)

But AB is by definition the same multiple of CD, so CD equals GF

Subtract CF from both, which gives GC equals FD

## Proposition 5 of Book V

If a magnitude be the same multiple of a magnitude that a part subtracted is of a part subtracted, the remainder will also be the same multiple of the remainder that the whole is of the whole



$$AB = n \cdot CD$$

$$AE = n \cdot CF$$

$$EB = n \cdot GC$$

$$AE + EB = n \cdot CF + n \cdot GC = n \cdot (CF + GC)$$

$$AB = n \cdot GF$$

$$GF = CD$$

$$GF - CF = CD - CF$$

$$GC = FD$$

$$EB = n \cdot FD$$

### In other words

Let AB be the same multiple of CD as AE is of CF

Then the remainder of AB minus the part AE is the same multiple of the remainder of CD minus CF

### Proof

Create a line GC such that EB is the same multiple of GC as AE is to CF

Since AE and EB are equimultiples of CF and GC, then the sum of the lines are also of the same multiple (V.1)

But AB is by definition the same multiple of CD, so CD equals GF

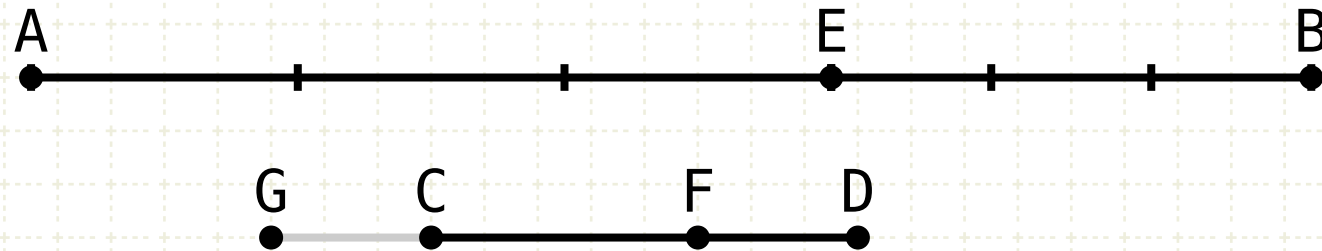
Subtract CF from both, which gives GC equals FD

Since GC equals FD, and since EB is the same multiple of GC as AE is to CF, then EB is the same multiple of FD



## Proposition 5 of Book V

If a magnitude be the same multiple of a magnitude that a part subtracted is of a part subtracted, the remainder will also be the same multiple of the remainder that the whole is of the whole



$$AB = n \cdot CD$$

$$AE = n \cdot CF$$

$$EB = n \cdot GC$$

$$AE + EB = n \cdot CF + n \cdot GC = n \cdot (CF + GC)$$

$$AB = n \cdot GF$$

$$GF = CD$$

$$GF - CF = CD - CF$$

$$GC = FD$$

$$EB = n \cdot FD$$

### In other words

Let AB be the same multiple of CD as AE is of CF

Then the remainder of AB minus the part AE is the same multiple of the remainder of CD minus CF

### Proof

Create a line GC such that EB is the same multiple of GC as AE is to CF

Since AE and EB are equimultiples of CF and GC, then the sum of the lines are also of the same multiple (V.1)

But AB is by definition the same multiple of CD, so CD equals GF

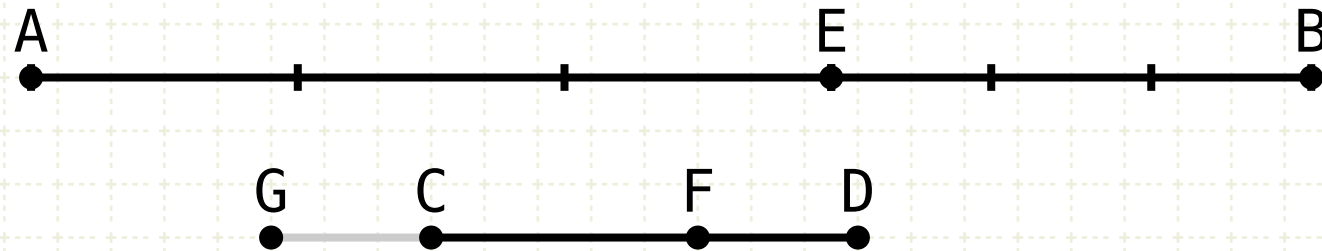
Subtract CF from both, which gives GC equals FD

Since GC equals FD, and since EB is the same multiple of GC as AE is to CF, then EB is the same multiple of FD

But, by definition, AE is the same multiple of CF that AB is of CD therefore EB is the same multiple of FD that AB is of CD

## Proposition 5 of Book V

If a magnitude be the same multiple of a magnitude that a part subtracted is of a part subtracted, the remainder will also be the same multiple of the remainder that the whole is of the whole



$$AB = n \cdot CD$$

$$AE = n \cdot CF$$

$$EB = n \cdot GC$$

$$AE + EB = n \cdot CF + n \cdot GC = n \cdot (CF + GC)$$

$$AB = n \cdot GF$$

$$GF = CD$$

$$GF - CF = CD - CF$$

$$GC = FD$$

$$EB = n \cdot FD$$

$$AB - AE = n \cdot CD - n \cdot CF = n \cdot (CD - CF) = n \cdot FD$$

### In other words

Let AB be the same multiple of CD as AE is of CF

Then the remainder of AB minus the part AE is the same multiple of the remainder of CD minus CF

### Proof

Create a line GC such that EB is the same multiple of GC as AE is to CF

Since AE and EB are equimultiples of CF and GC, then the sum of the lines are also of the same multiple (V.1)

But AB is by definition the same multiple of CD, so CD equals GF

Subtract CF from both, which gives GC equals FD

Since GC equals FD, and since EB is the same multiple of GC as AE is to CF, then EB is the same multiple of FD

But, by definition, AE is the same multiple of CF that AB is of CD therefore EB is the same multiple of FD that AB is of CD



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