

# Euclid's Elements

## Book I

*If Euclid did not kindle your youthful enthusiasm, you were not born to be a scientific thinker.*

Albert Einstein



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# Proposition 40 of Book I

Equal triangles which are on equal bases and on the same side are also in the same parallels.

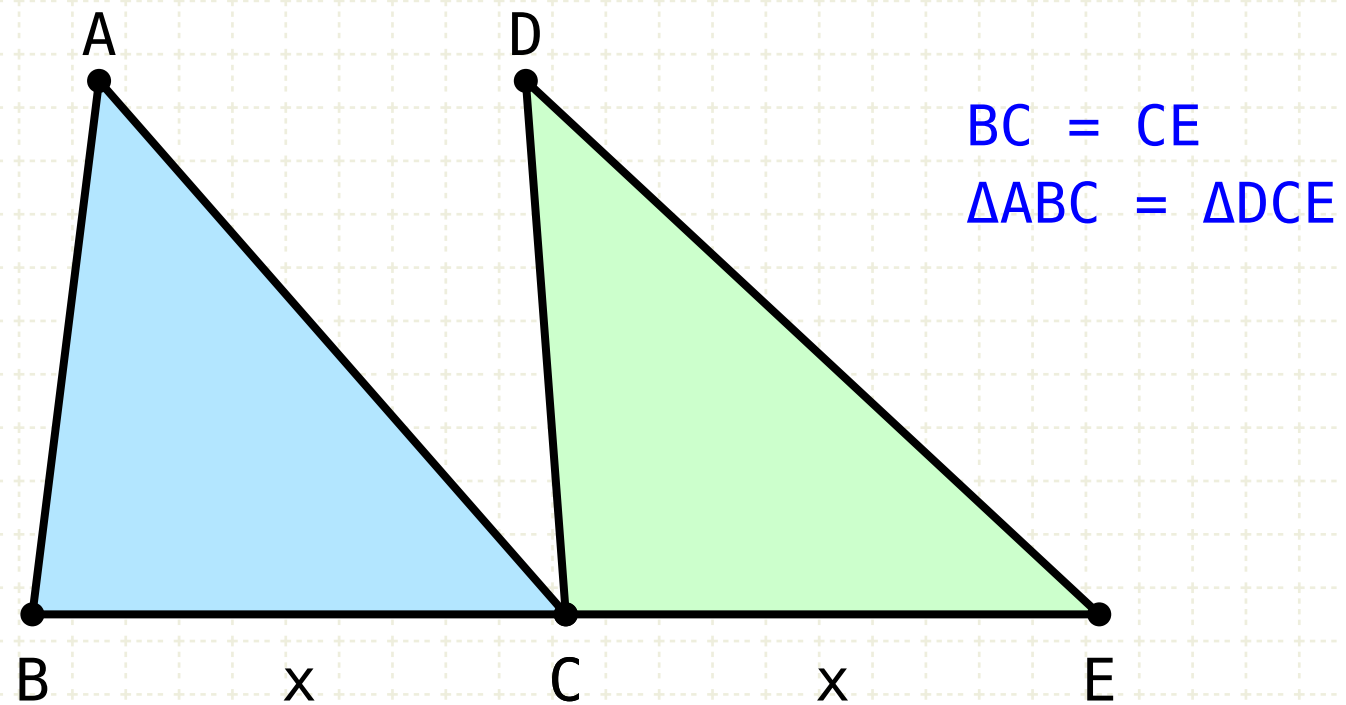


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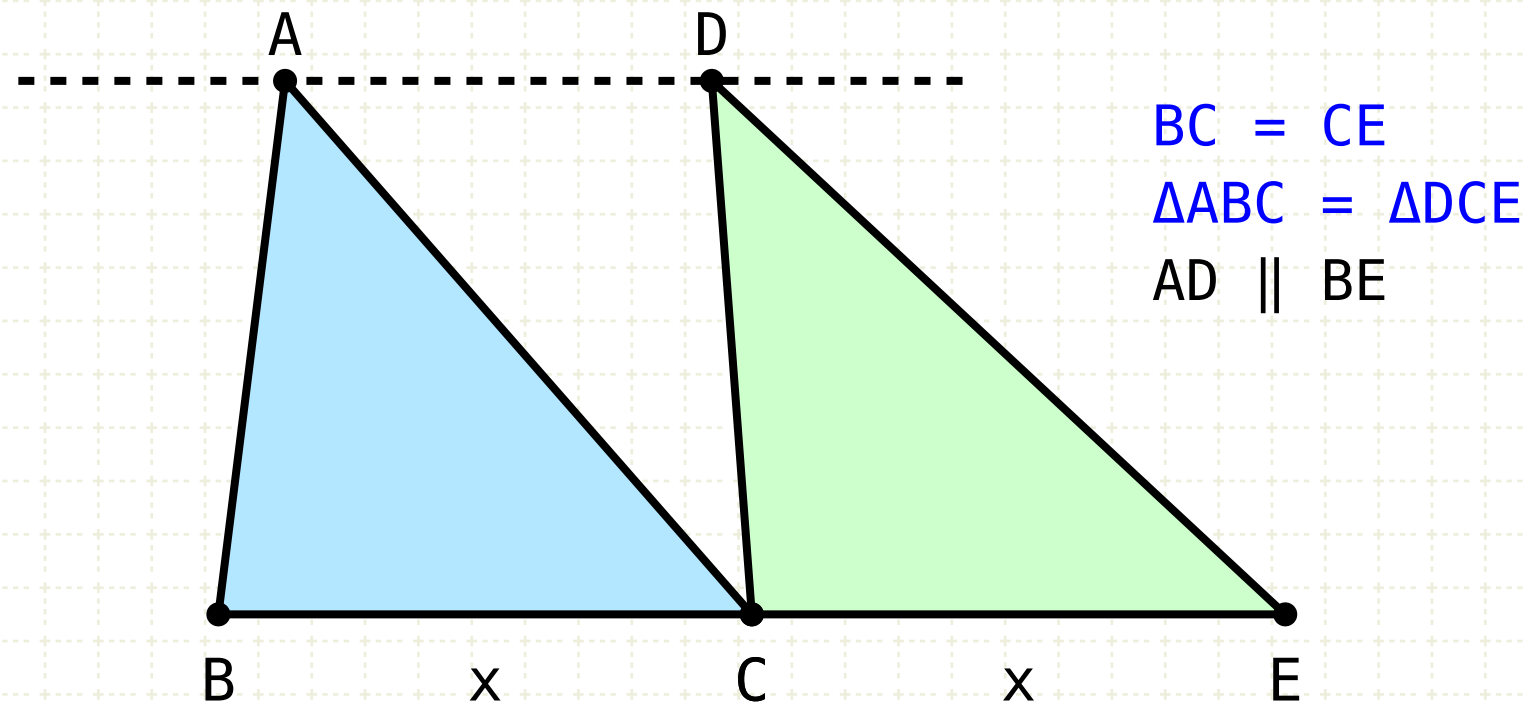
## In other words

Let ABC and DCE be triangles, lying on the same line, with equal bases and equal areas



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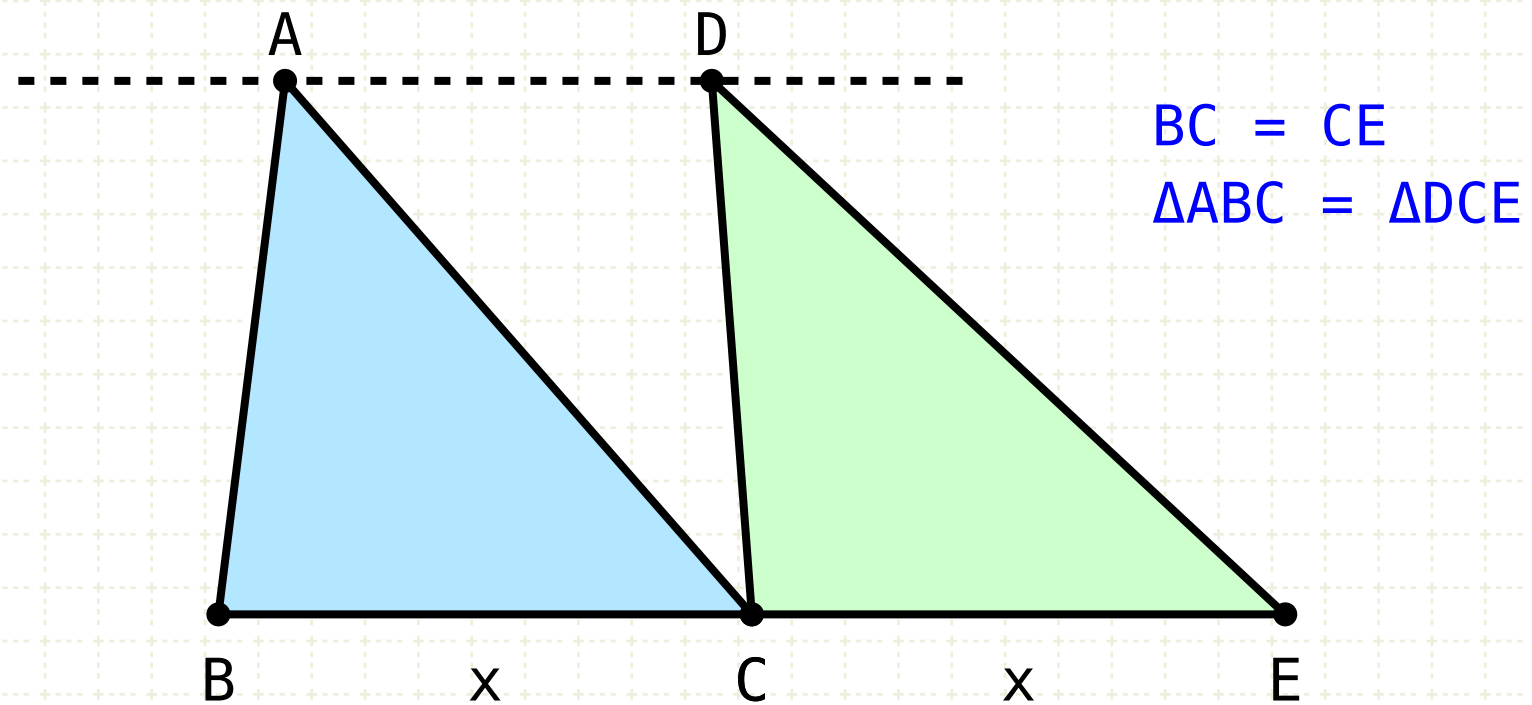
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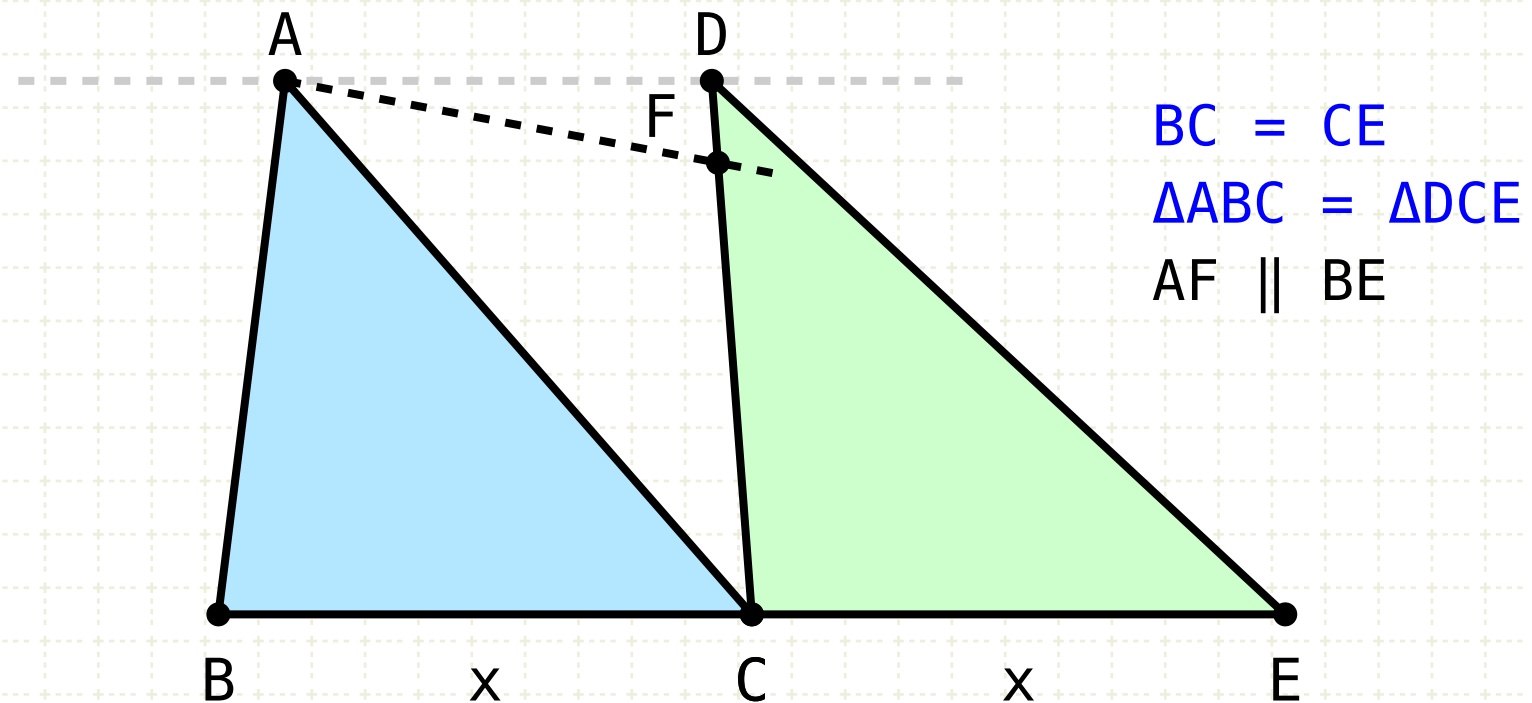
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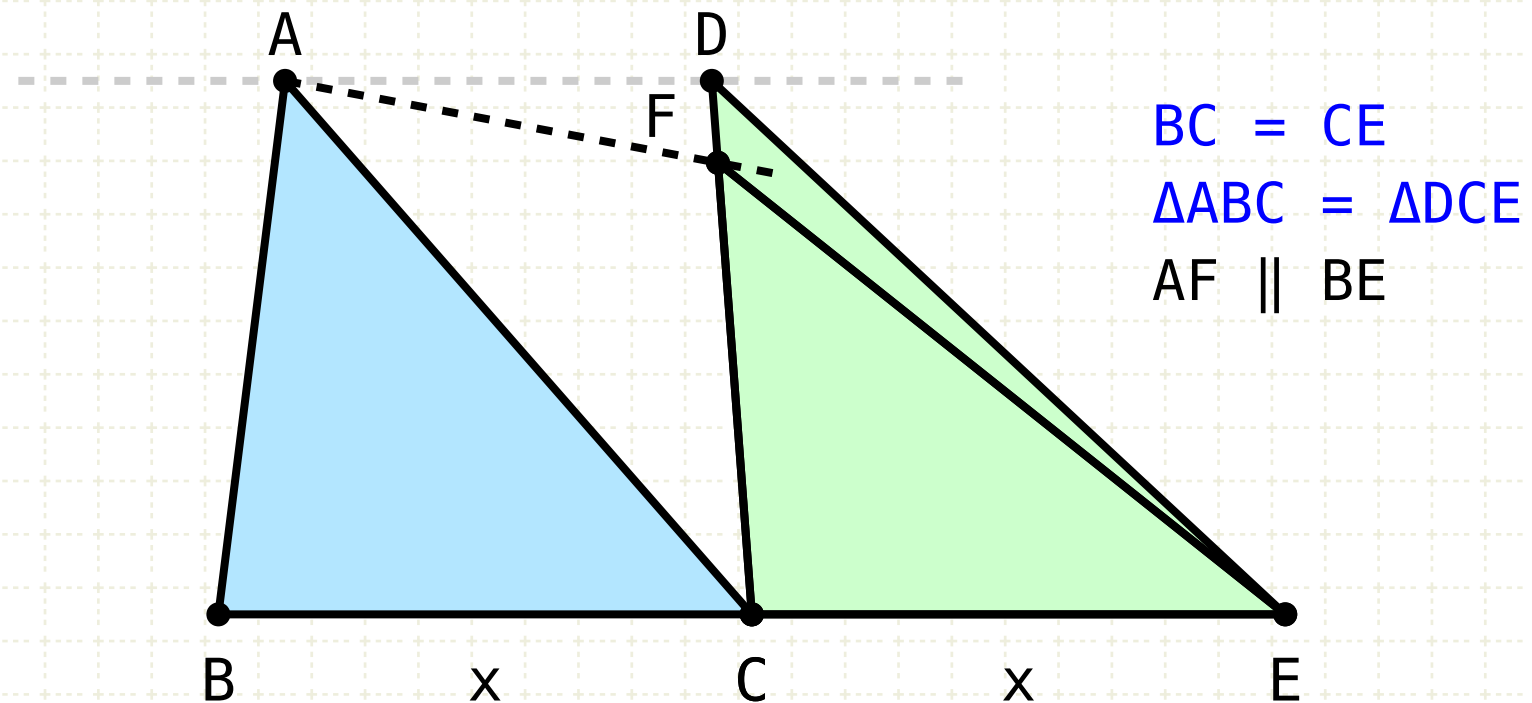
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Draw line AF, parallel to BE and passing point A and line DC



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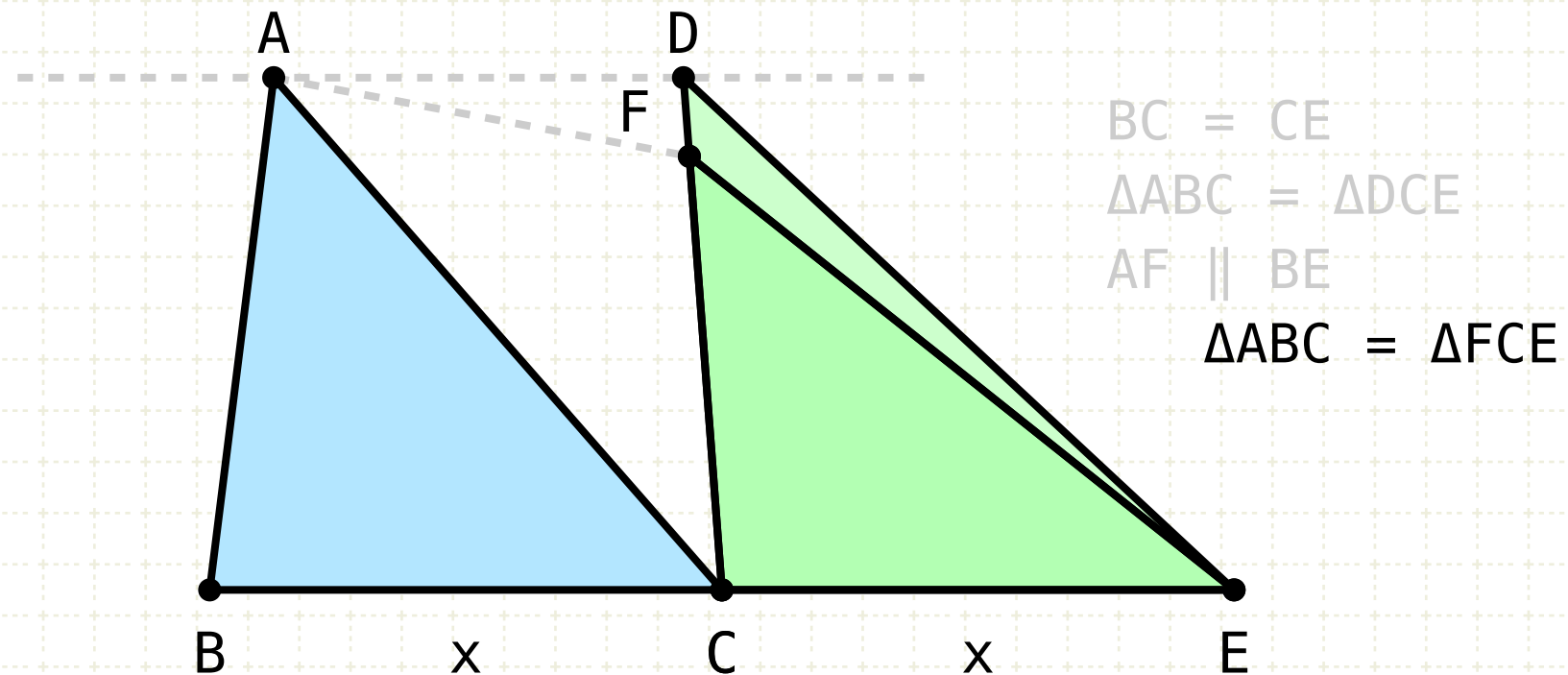
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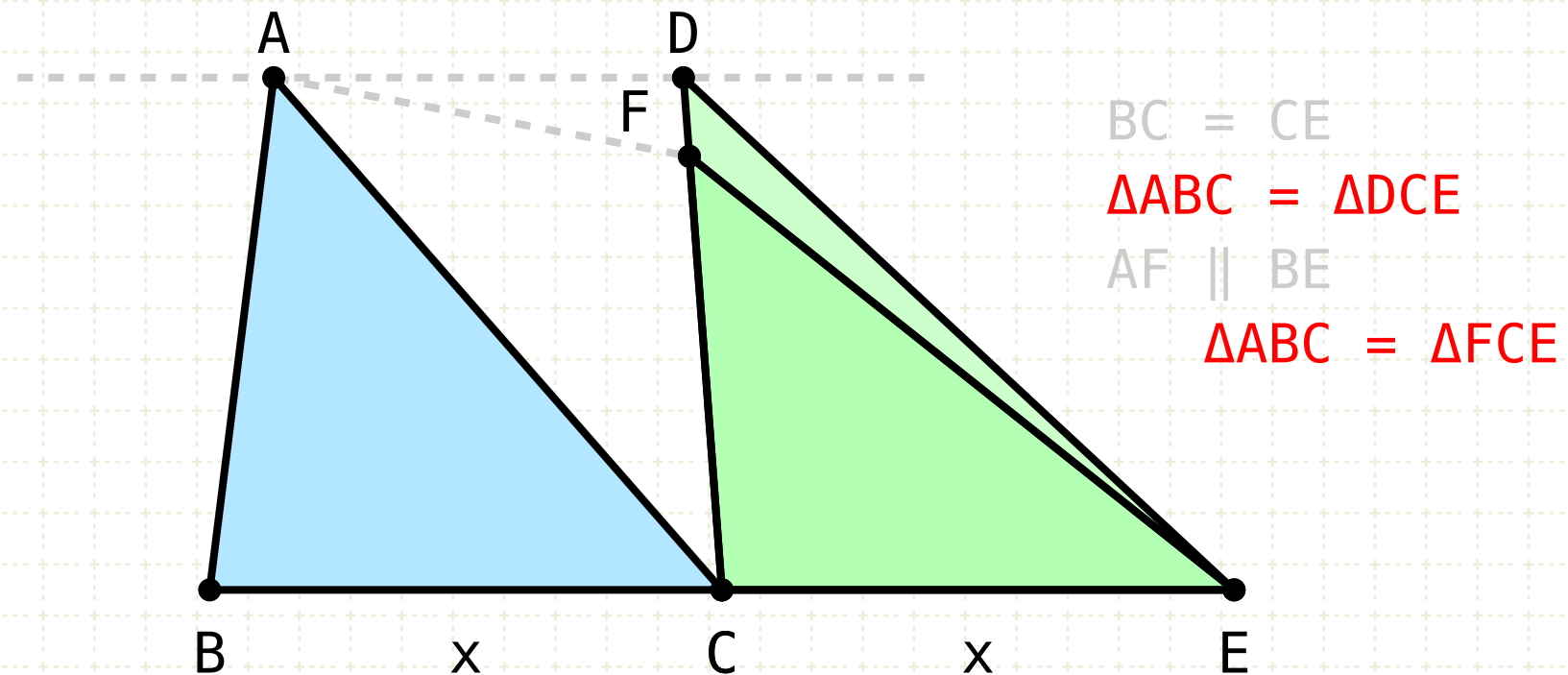
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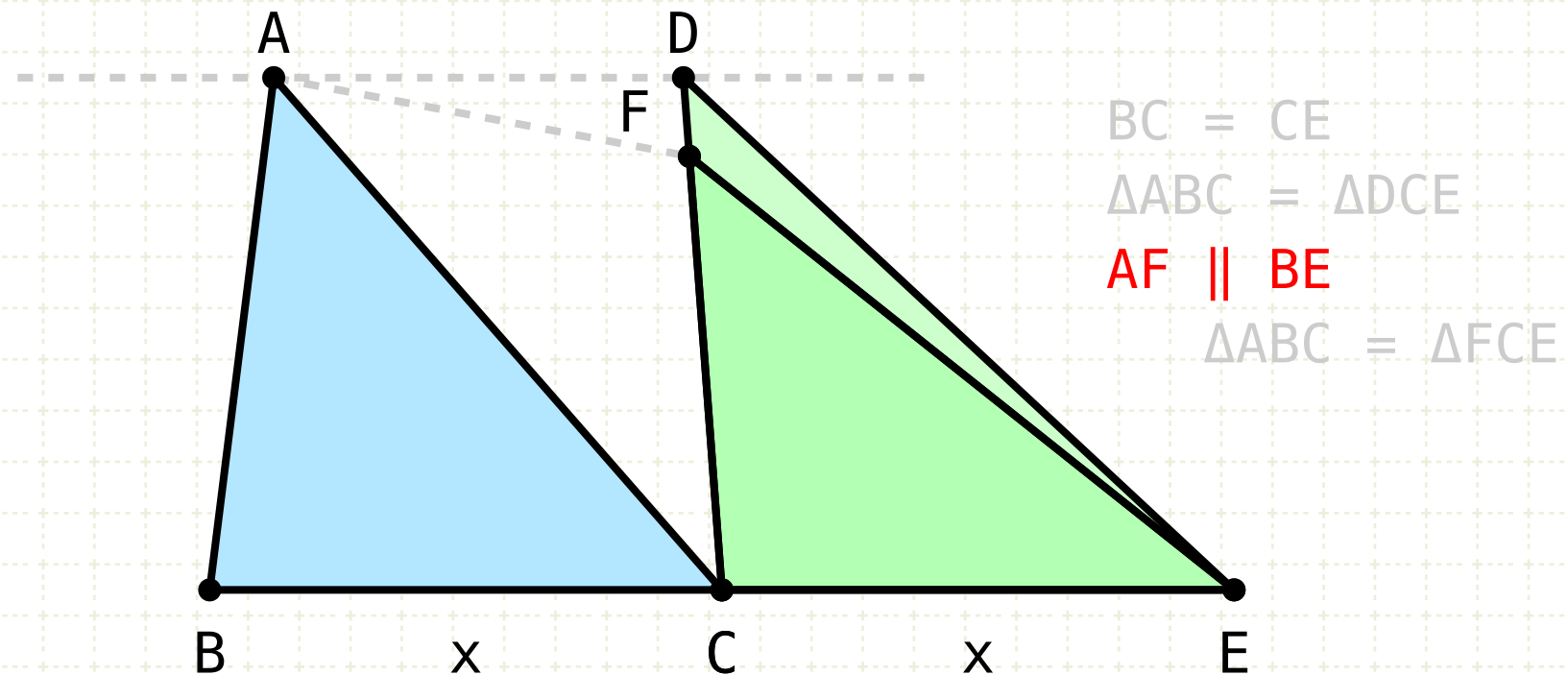
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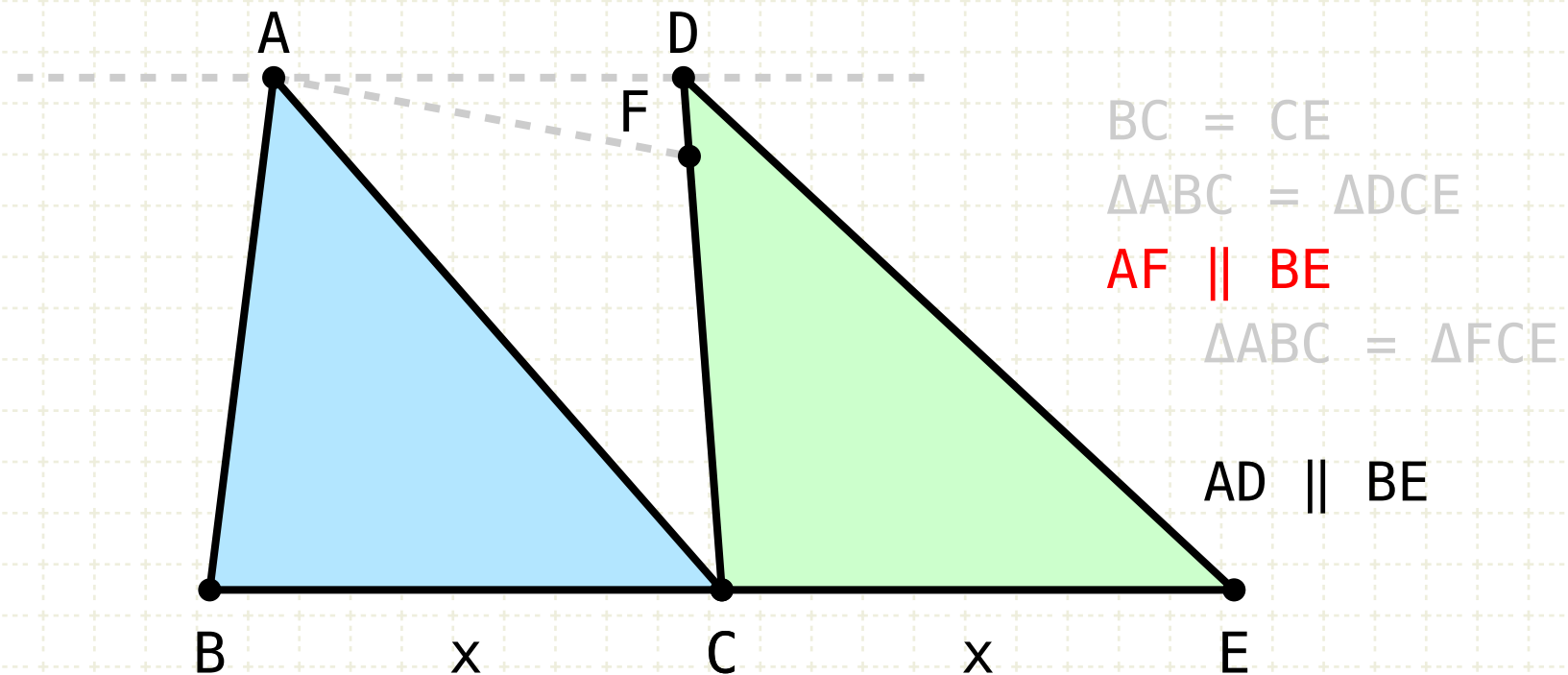
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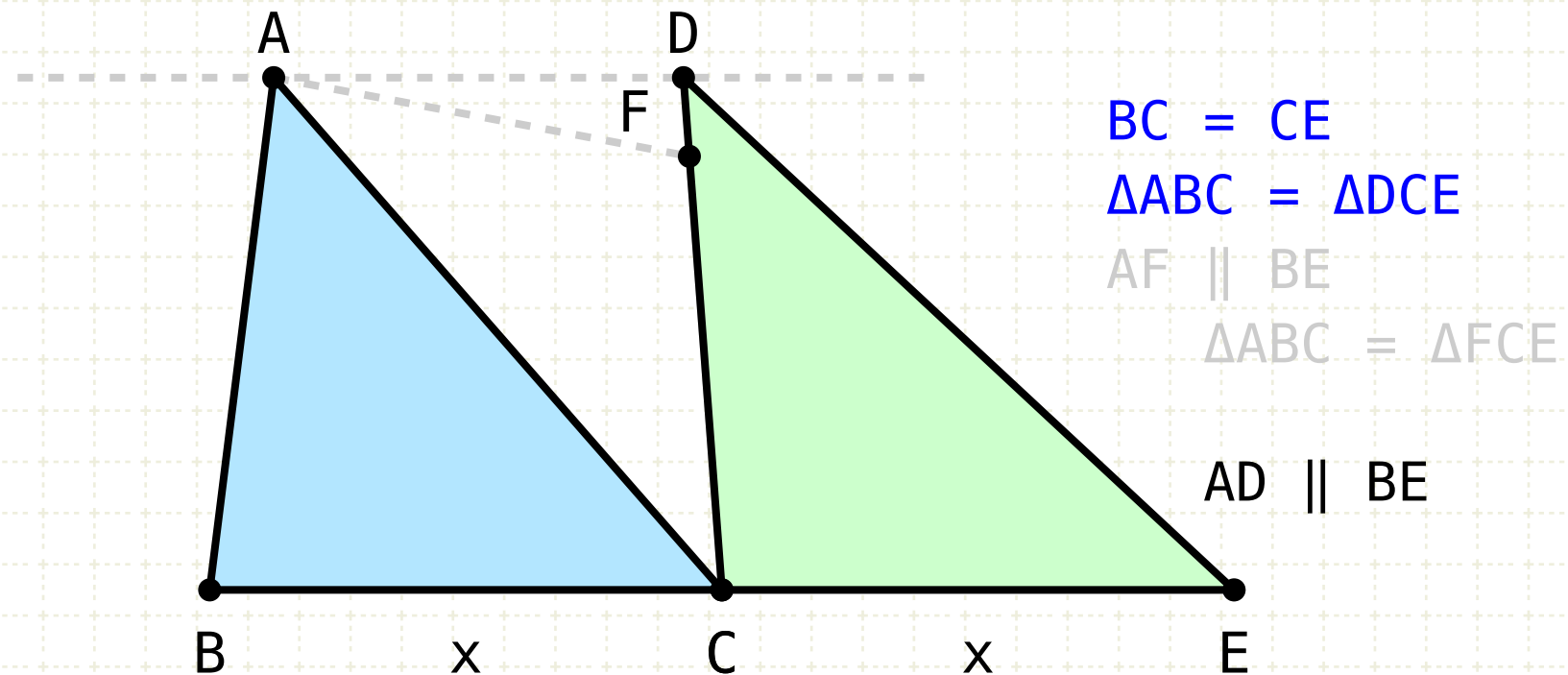
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