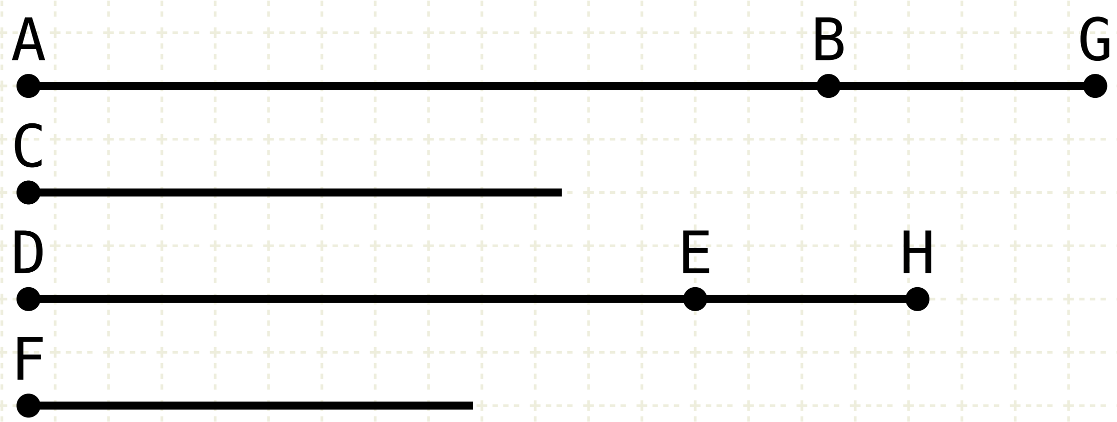
Euclid's Elements

Book V



AB:C = DE:F
BG:C = EH:F

AG:C = DH:F

Proportions are what makes the old Greek temples classic in their beauty. They are like huge blocks, from which the air has been literally hewn out between the columns.

Arne Jacobsen

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Proposition 23 of Book V

If there be three magnitudes, and others equal to them in multitude, which taken two and two together are in the same ratio, and the proportion of them be perturbed, they will also be in the same ratio EX AEQUALI

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If there be three magnitudes, and others equal to them in multitude, which taken two and two together are in the same ratio, and the proportion of them be perturbed, they will also be in the same ratio EX AEQUALI

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ratio EX AEQUALI
  a:b = d:e
  b:c = e:f
→ a:c = d:f
ratio PERTURBED PROPORTION
  a:b = e:f
  b:c = d:e
→ a:c = d:f
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Definitions

17. A ratio EX AEQUALI arises when, there being several magnitudes and another set equal to them in multitude which taken two and two are in the same proportion, as the first is to the last among the the first magnitudes, so is the first is to the last among the second magnitudes
18. A PERTURBED PROPORTION arises when, there being three magnitudes and another set equal to them in multitude, as antecedent is to consequent among the first magnitudes, so is antecedent to consequent among the second magnitudes, while, as the consequent is to a third among the first magnitudes, so is a third to the antecedent among the second magnitudes

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If there be three magnitudes, and others equal to them in multitude, which taken two and two together are in the same ratio, and the proportion of them be perturbed, they will also be in the same ratio EX AEQUALI

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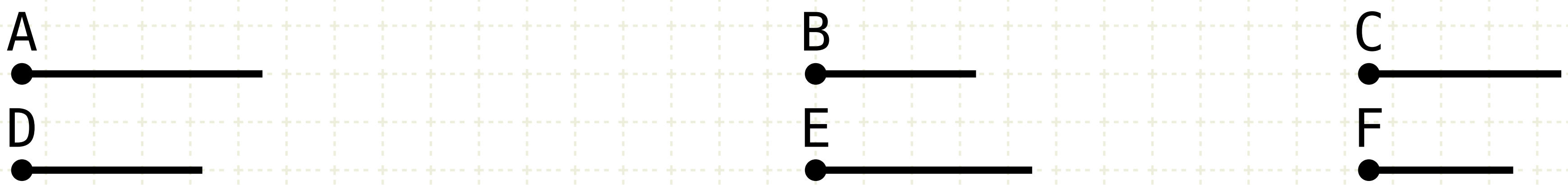

In other words

Given two sets of numbers A,B,C and D,E,F in perturbed proportion, where A is to B as E is to F, and where B is to C as D is to E

Then A is to C as D is to F

$$A:B = E:F$$
$$B:C = D:E$$

$$\rightarrow A:C = D:F$$

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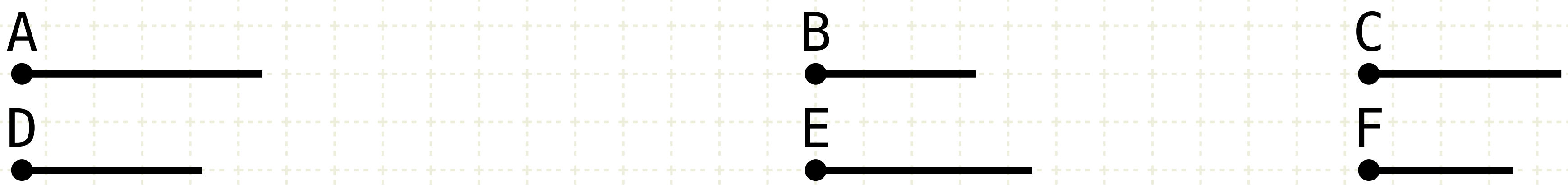

In other words

Given two sets of numbers A,B,C and D,E,F in perturbed proportion, where A is to B as E is to F, and where B is to C as D is to E

Then A is to C as D is to F

Proof

A:B = E:F
B:C = D:E

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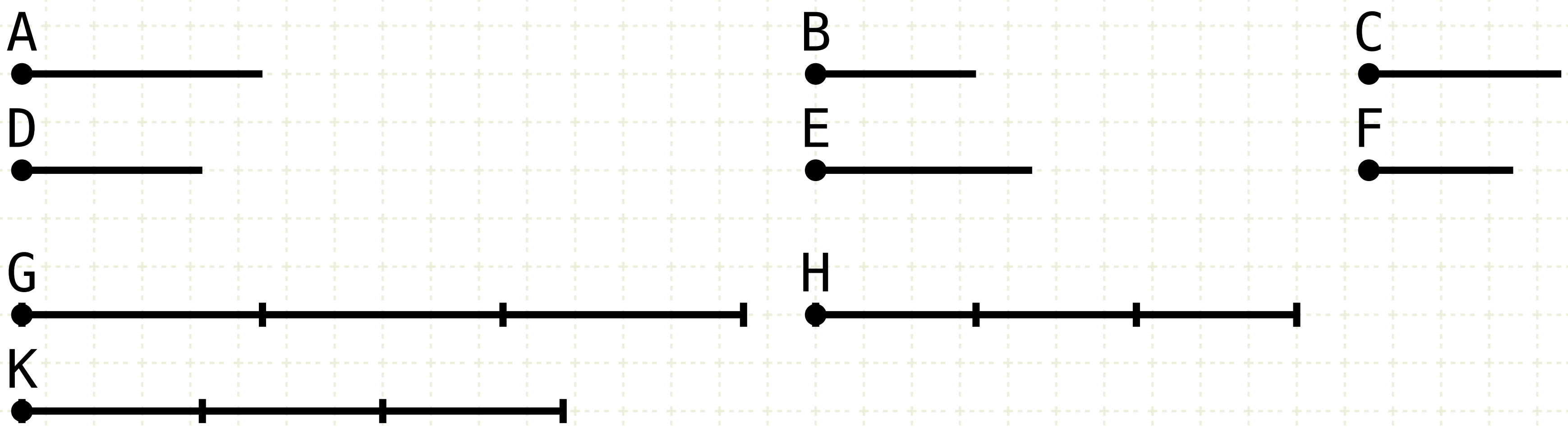

$$A:B = E:F$$
$$B:C = D:E$$

$$G = m \cdot A$$
$$H = m \cdot B$$
$$K = m \cdot D$$

In other words

Given two sets of numbers A,B,C and D,E,F in perturbed proportion, where A is to B as E is to F, and where B is to C as D is to E

Then A is to C as D is to F

Proof

Let G,H,K be equimultiples of A, B and D

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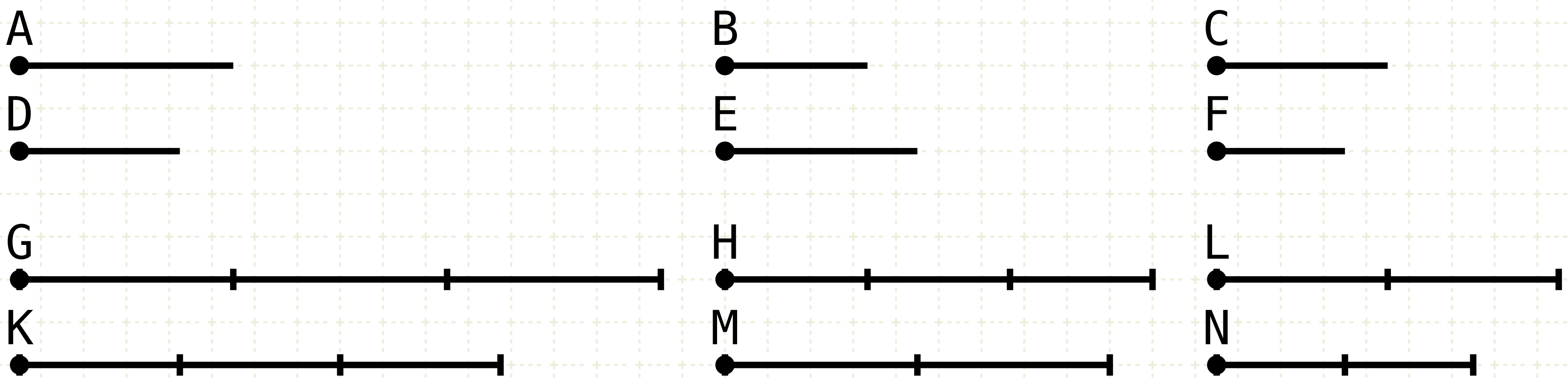

In other words

Given two sets of numbers A,B,C and D,E,F in perturbed proportion, where A is to B as E is to F, and where B is to C as D is to E

Then A is to C as D is to F

Proof

Let G,H,K be equimultiples of A, B and D

Let L,M,N be equimultiples of C, E and F

$A:B = E:F$

$B:C = D:E$

$G = m \cdot A$

$H = m \cdot B$

$K = m \cdot D$

$L = n \cdot C$

$M = n \cdot E$

$N = n \cdot F$

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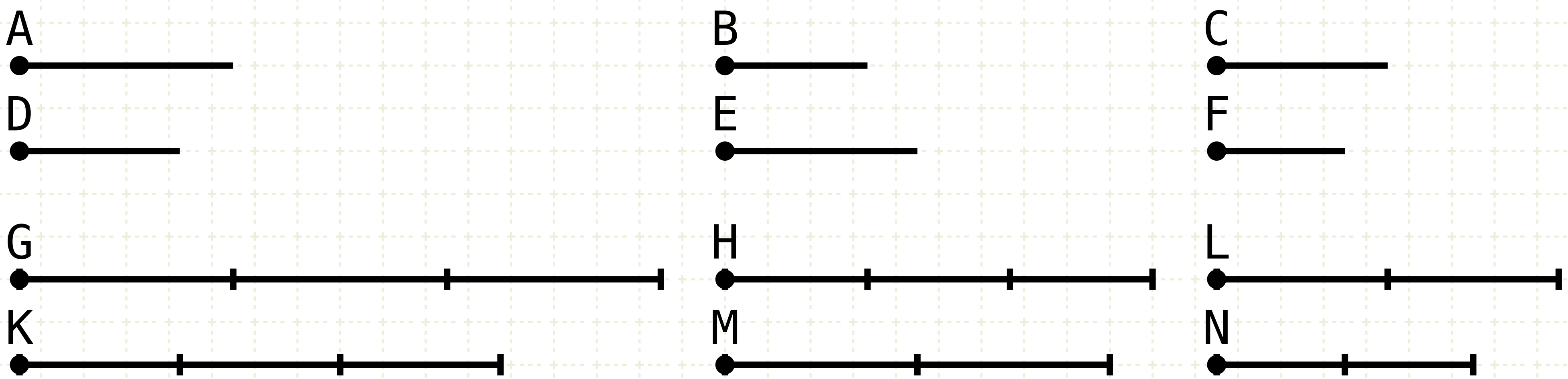

A:B = E:F

B:C = D:E

A:B = G:H

$G = m \cdot A$

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In other words

Given two sets of numbers A,B,C and D,E,F in perturbed proportion, where A is to B as E is to F, and where B is to C as D is to E

Then A is to C as D is to F

Proof

Let G,H,K be equimultiples of A, B and D

Let L,M,N be equimultiples of C, E and F

Since G,H are equimultiples of A,B then A is to B as G is to H (V·15)

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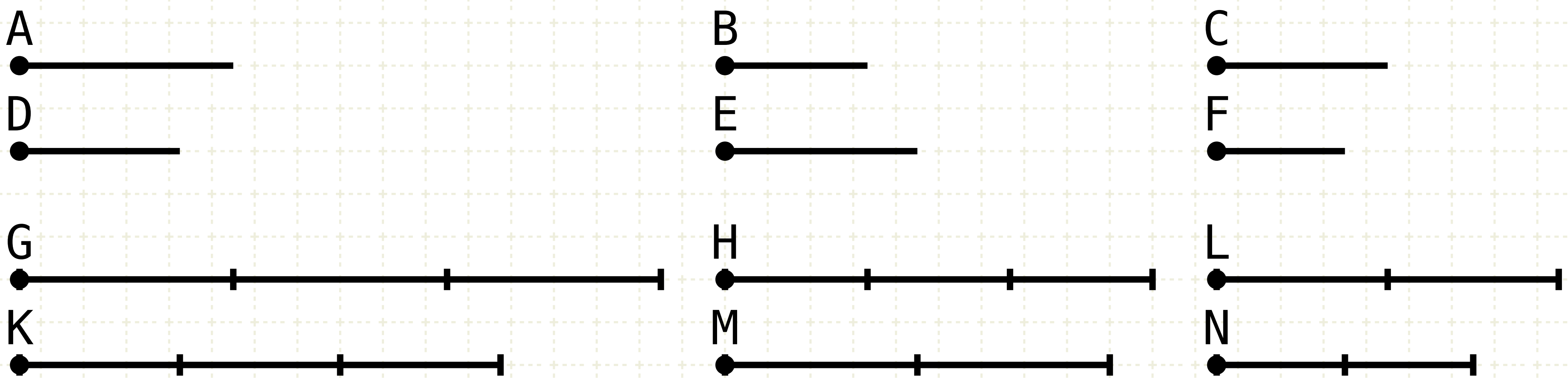

$A:B = E:F$

$B:C = D:E$

$A:B = G:H$

$E:F = M:N$

$G = m \cdot A$

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Given two sets of numbers A,B,C and D,E,F in perturbed proportion, where A is to B as E is to F, and where B is to C as D is to E

Then A is to C as D is to F

Proof

Let G,H,K be equimultiples of A, B and D

Let L,M,N be equimultiples of C, E and F

Since G,H are equimultiples of A,B then A is to B as G is to H (V·15)

Similarly E is to F as M is to N

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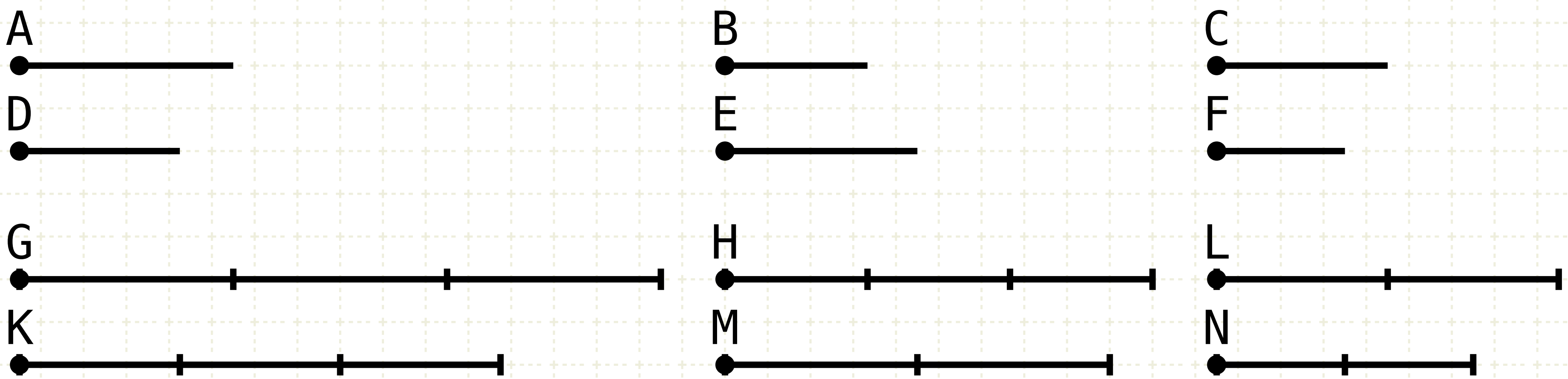

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$M = n \cdot E$

$N = n \cdot F$

$A:B = G:H$

$E:F = M:N$

$G:H = M:N$

In other words

Given two sets of numbers A,B,C and D,E,F in perturbed proportion, where A is to B as E is to F, and where B is to C as D is to E

Then A is to C as D is to F

Proof

Let G,H,K be equimultiples of A, B and D

Let L,M,N be equimultiples of C, E and F

Since G,H are equimultiples of A,B then A is to B as G is to H (V·15)

Similarly E is to F as M is to N

And, as A is to B, so is E to F; hence as G is to H, so is M to N (V·11)

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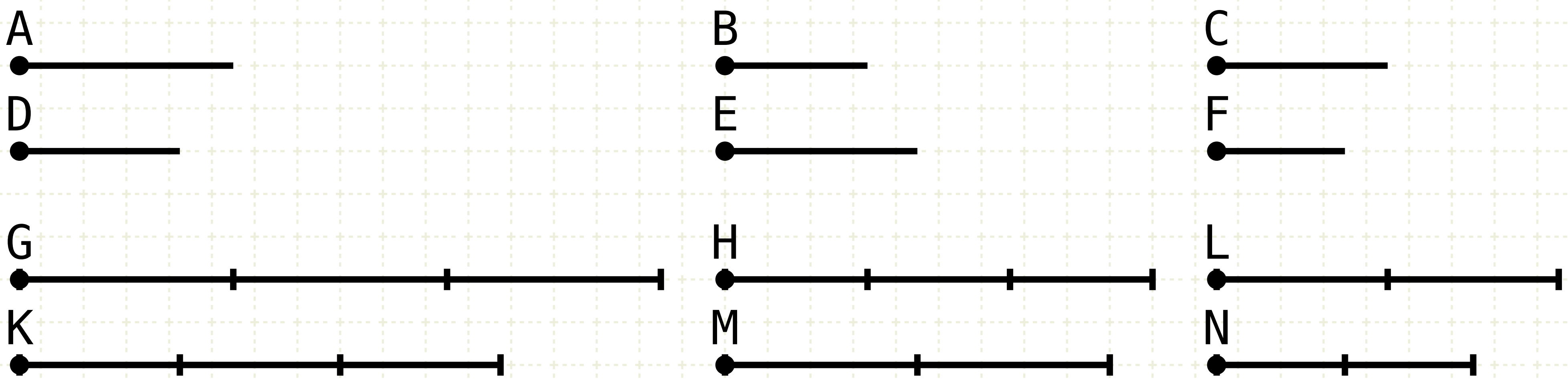

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$H = m \cdot B$

$K = m \cdot D$

$L = n \cdot C$

$M = n \cdot E$

$N = n \cdot F$

$A:B = G:H$

$E:F = M:N$

$G:H = M:N$

$B:D = C:E$

In other words

Given two sets of numbers A,B,C and D,E,F in perturbed proportion, where A is to B as E is to F, and where B is to C as D is to E

Then A is to C as D is to F

Proof

Let G,H,K be equimultiples of A, B and D

Let L,M,N be equimultiples of C, E and F

Since G,H are equimultiples of A,B then A is to B as G is to H (V·15)

Similarly E is to F as M is to N

And, as A is to B, so is E to F; hence as G is to H, so is M to N (V·11)

Since B is to C as D is to E, alternately B is to D as C is to E (V·16)

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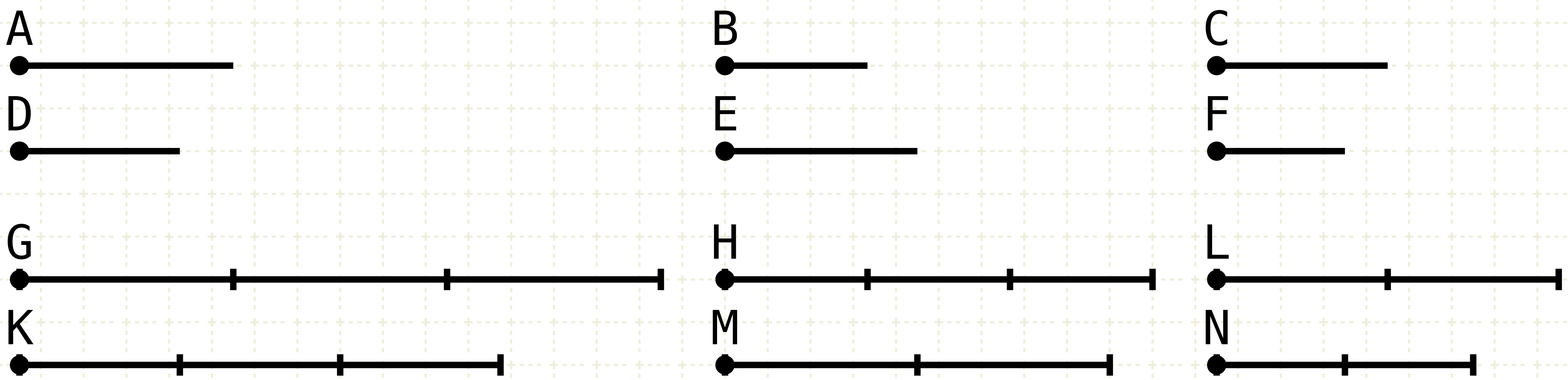

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$K = m \cdot D$

$L = n \cdot C$

$M = n \cdot E$

$N = n \cdot F$

$A:B = G:H$

$E:F = M:N$

$G:H = M:N$

$B:D = C:E$

$B:D = H:K$

In other words

Given two sets of numbers A,B,C and D,E,F in perturbed proportion, where A is to B as E is to F, and where B is to C as D is to E

Then A is to C as D is to F

Proof

Let G,H,K be equimultiples of A, B and D

Let L,M,N be equimultiples of C, E and F

Since G,H are equimultiples of A,B then A is to B as G is to H (V·15)

Similarly E is to F as M is to N

And, as A is to B, so is E to F; hence as G is to H, so is M to N (V·11)

Since B is to C as D is to E, alternately B is to D as C is to E (V·16)

Since H,K are equimultiples of B,D, B is to D as H is to K (V·15)

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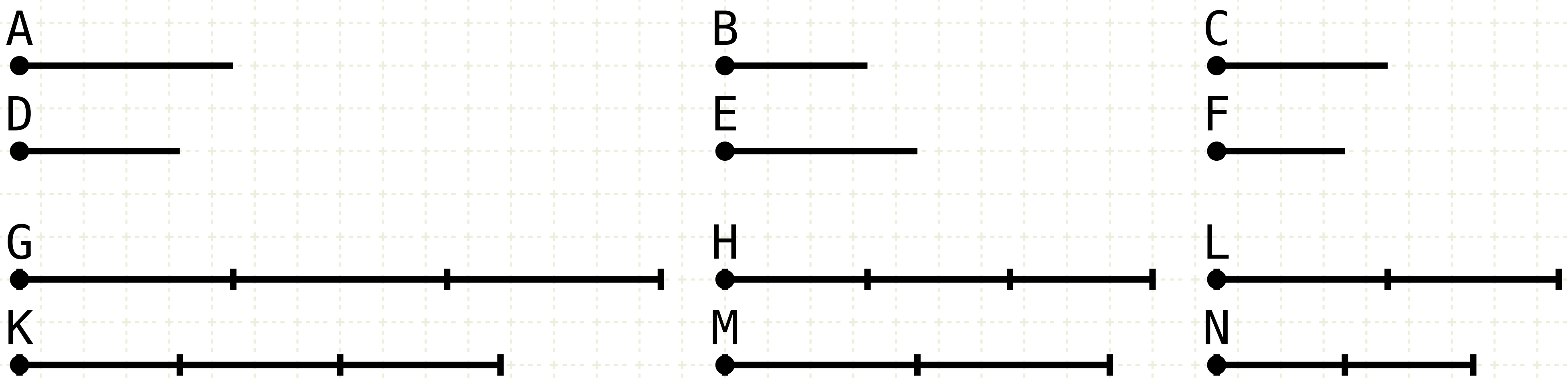

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$N = n \cdot F$

$A:B = G:H$

$E:F = M:N$

$G:H = M:N$

$B:D = C:E$

$B:D = H:K$

$H:K = C:E$

In other words

Given two sets of numbers A,B,C and D,E,F in perturbed proportion, where A is to B as E is to F, and where B is to C as D is to E

Then A is to C as D is to F

Proof

Let G,H,K be equimultiples of A, B and D

Let L,M,N be equimultiples of C, E and F

Since G,H are equimultiples of A,B then A is to B as G is to H (V·15)

Similarly E is to F as M is to N

And, as A is to B, so is E to F; hence as G is to H, so is M to N (V·11)

Since B is to C as D is to E, alternately B is to D as C is to E (V·16)

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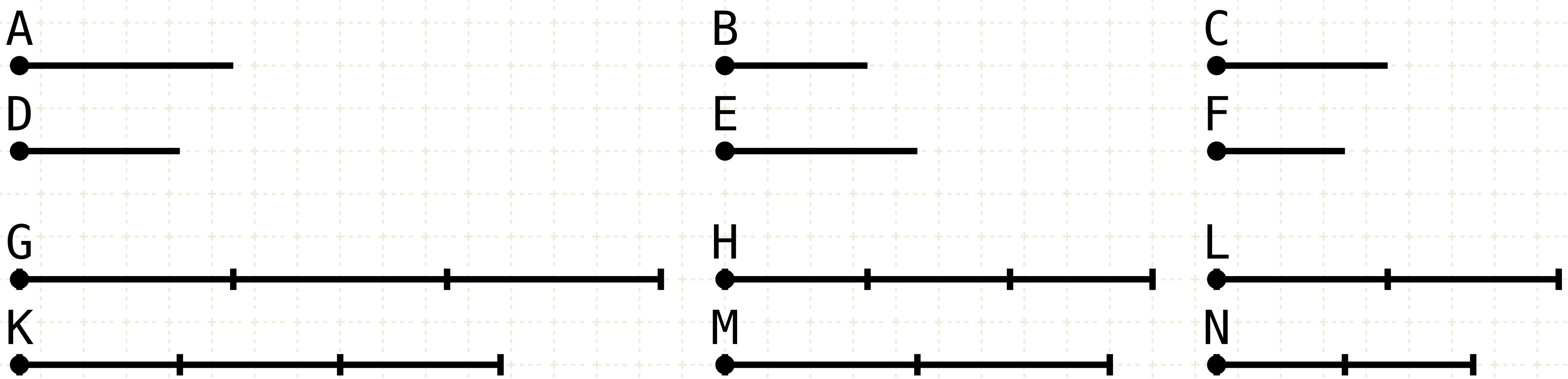

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$M = n \cdot E$
$N = n \cdot F$

$A:B = G:H$
$E:F = M:N$
$G:H = M:N$
$B:D = C:E$
$B:D = H:K$
$H:K = C:E$
$C:E = L:M$

In other words

Given two sets of numbers A,B,C and D,E,F in perturbed proportion, where A is to B as E is to F, and where B is to C as D is to E

Then A is to C as D is to F

Proof

Let G,H,K be equimultiples of A, B and D

Let L,M,N be equimultiples of C, E and F

Since G,H are equimultiples of A,B then A is to B as G is to H (V·15)

Similarly E is to F as M is to N

And, as A is to B, so is E to F; hence as G is to H, so is M to N (V·11)

Since B is to C as D is to E, alternately B is to D as C is to E (V·16)

Since H,K are equimultiples of B,D, B is to D as H is to K (V·15)

As B is to D, so is C to E; hence as H is to K, so is C to E (V·11)

L,M are equimultiples of C,E, so C is to E as L is to M (V·15)

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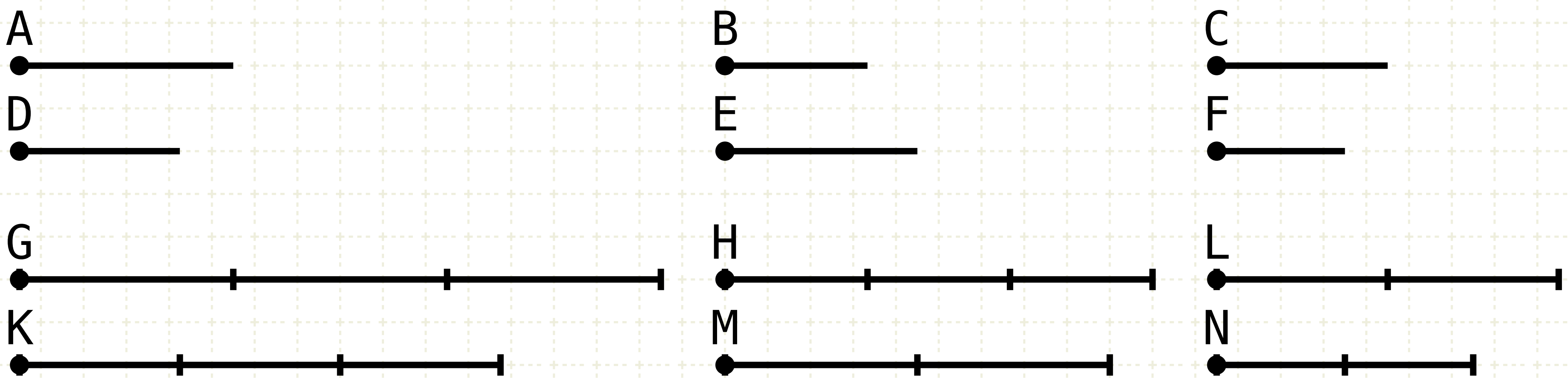

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$M = n \cdot E$

$N = n \cdot F$

$A:B = G:H$

$E:F = M:N$

$G:H = M:N$

$B:D = C:E$

$B:D = H:K$

$H:K = C:E$

$C:E = L:M$

$H:K = L:M$

In other words

Given two sets of numbers A,B,C and D,E,F in perturbed proportion, where A is to B as E is to F, and where B is to C as D is to E

Then A is to C as D is to F

Proof

Let G,H,K be equimultiples of A, B and D

Let L,M,N be equimultiples of C, E and F

Since G,H are equimultiples of A,B then A is to B as G is to H (V·15)

Similarly E is to F as M is to N

And, as A is to B, so is E to F; hence as G is to H, so is M to N (V·11)

Since B is to C as D is to E, alternately B is to D as C is to E (V·16)

Since H,K are equimultiples of B,D, B is to D as H is to K (V·15)

As B is to D, so is C to E; hence as H is to K, so is C to E (V·11)

L,M are equimultiples of C,E, so C is to E as L is to M (V·15)

Since C is to E as H is to K, H is to K as L is to M (V·11)

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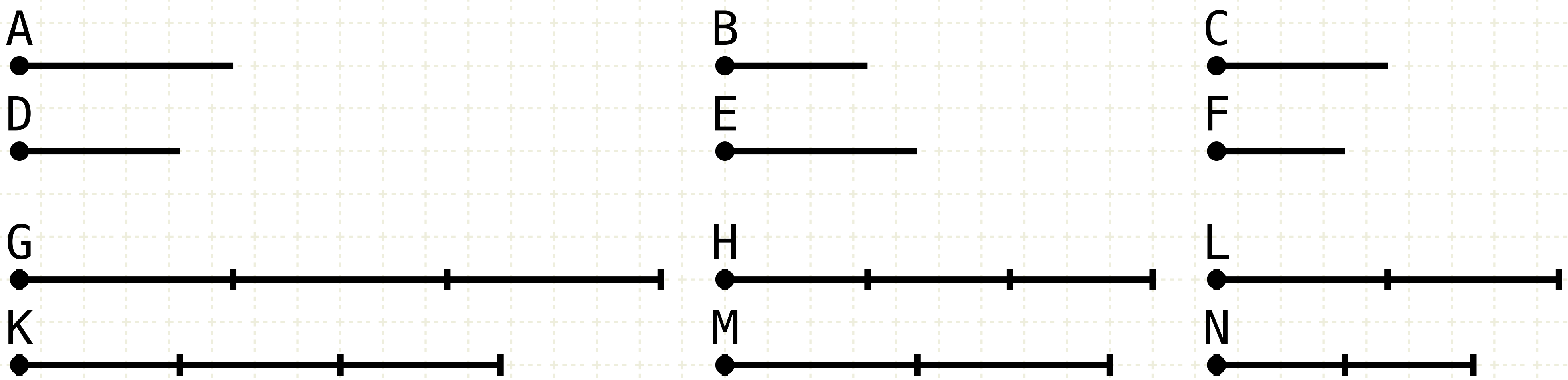
If there be three magnitudes, and others equal to them in multitude, which taken two and two together are in the same ratio, and the proportion of them be perturbed, they will also be in the same ratio EX AEQUALI



A:B = E:F
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G = m·A
H = m·B
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L = n·C
M = n·E
N = n·F

A:B = G:H
E:F = M:N
G:H = M:N
B:D = C:E
B:D = H:K
H:K = C:E
C:E = L:M
H:K = L:M
H:L = K:M

In other words

Given two sets of numbers A,B,C and D,E,F in perturbed proportion, where A is to B as E is to F, and where B is to C as D is to E

Then A is to C as D is to F

Proof

Let G,H,K be equimultiples of A, B and D

Let L,M,N be equimultiples of C, E and F

Since G,H are equimultiples of A,B then A is to B as G is to H (V·15)

Similarly E is to F as M is to N

And, as A is to B, so is E to F; hence as G is to H, so is M to N (V·11)

Since B is to C as D is to E, alternately B is to D as C is to E (V·16)

Since H,K are equimultiples of B,D, B is to D as H is to K (V·15)

As B is to D, so is C to E; hence as H is to K, so is C to E (V·11)

L,M are equimultiples of C,E, so C is to E as L is to M (V·15)

Since C is to E as H is to K, H is to K as L is to M (V·11)

And alternately, as H is to L, so is K to M (V·16)

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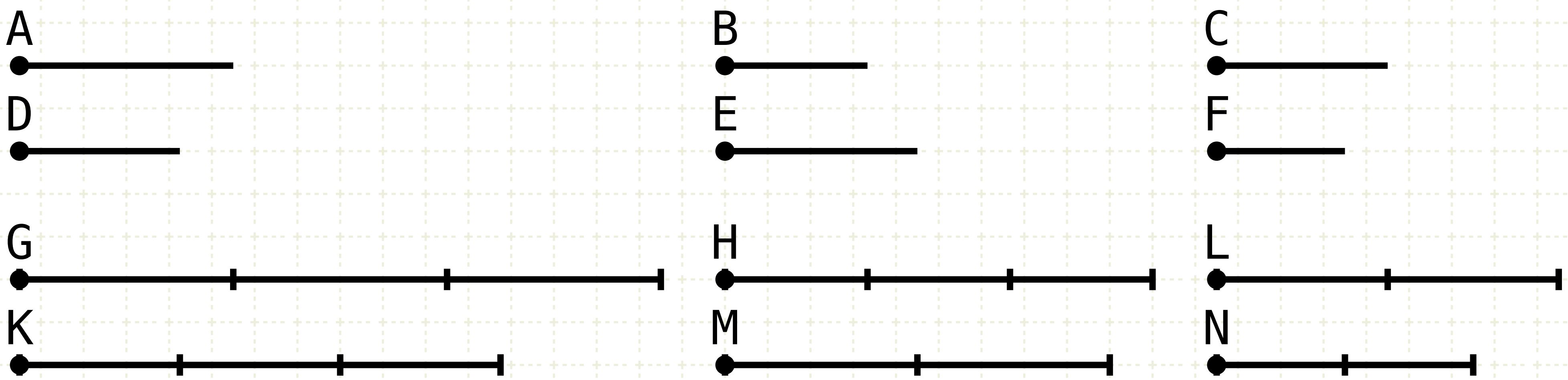

$A:B = E:F$

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$H = m \cdot B$
$K = m \cdot D$
$L = n \cdot C$
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$N = n \cdot F$

$A:B = G:H$
$E:F = M:N$
$G:H = M:N$
$B:D = C:E$
$B:D = H:K$
$H:K = C:E$
$C:E = L:M$
$H:K = L:M$
$H:L = K:M$

In other words

Given two sets of numbers A,B,C and D,E,F in perturbed proportion, where A is to B as E is to F, and where B is to C as D is to E

Then A is to C as D is to F

Proof

Let G,H,K be equimultiples of A, B and D

Let L,M,N be equimultiples of C, E and F

Since G,H are equimultiples of A,B then A is to B as G is to H (V·15)

Similarly E is to F as M is to N

And, as A is to B, so is E to F; hence as G is to H, so is M to N (V·11)

Since B is to C as D is to E, alternately B is to D as C is to E (V·16)

Since H,K are equimultiples of B,D, B is to D as H is to K (V·15)

As B is to D, so is C to E; hence as H is to K, so is C to E (V·11)

L,M are equimultiples of C,E, so C is to E as L is to M (V·15)

Since C is to E as H is to K, H is to K as L is to M (V·11)

And alternately, as H is to L, so is K to M (V·16)

Therefore G,H,L and K,M,N are in a perturbed ratio

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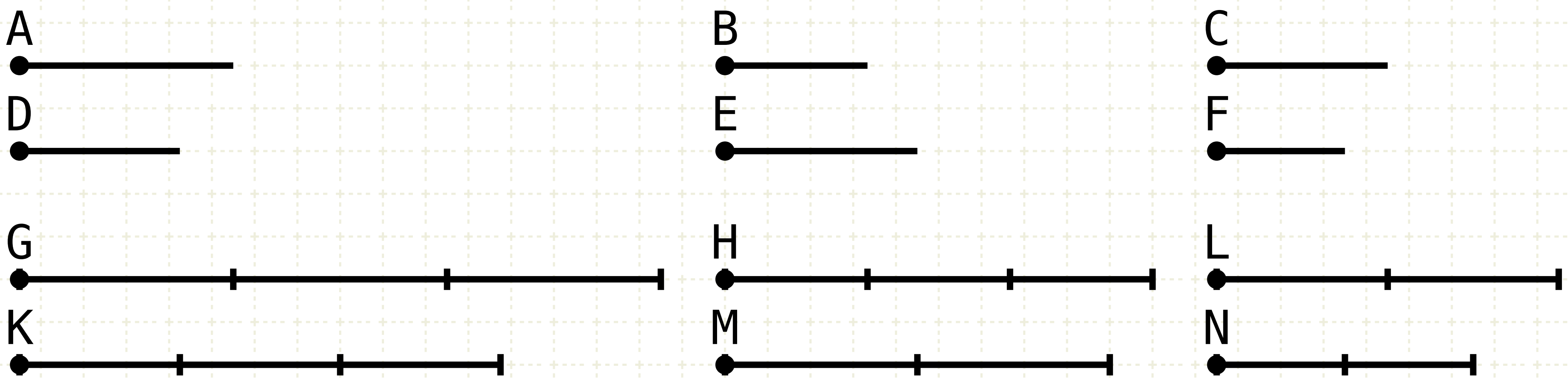
If there be three magnitudes, and others equal to them in multitude, which taken two and two together are in the same ratio, and the proportion of them be perturbed, they will also be in the same ratio EX AEQUALI



A:B = E:F
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M = n·E
N = n·F

A:B = G:H
E:F = M:N
G:H = M:N
B:D = C:E
B:D = H:K
H:K = C:E
C:E = L:M
H:K = L:M
H:L = K:M
G >=< L → K >=<N

In other words

Given two sets of numbers A,B,C and D,E,F in perturbed proportion, where A is to B as E is to F, and where B is to C as D is to E

Then A is to C as D is to F

Proof

Let G,H,K be equimultiples of A, B and D

Let L,M,N be equimultiples of C, E and F

Since G,H are equimultiples of A,B then A is to B as G is to H (V·15)

Similarly E is to F as M is to N

And, as A is to B, so is E to F; hence as G is to H, so is M to N (V·11)

Since B is to C as D is to E, alternately B is to D as C is to E (V·16)

Since H,K are equimultiples of B,D, B is to D as H is to K (V·15)

As B is to D, so is C to E; hence as H is to K, so is C to E (V·11)

L,M are equimultiples of C,E, so C is to E as L is to M (V·15)

Since C is to E as H is to K, H is to K as L is to M (V·11)

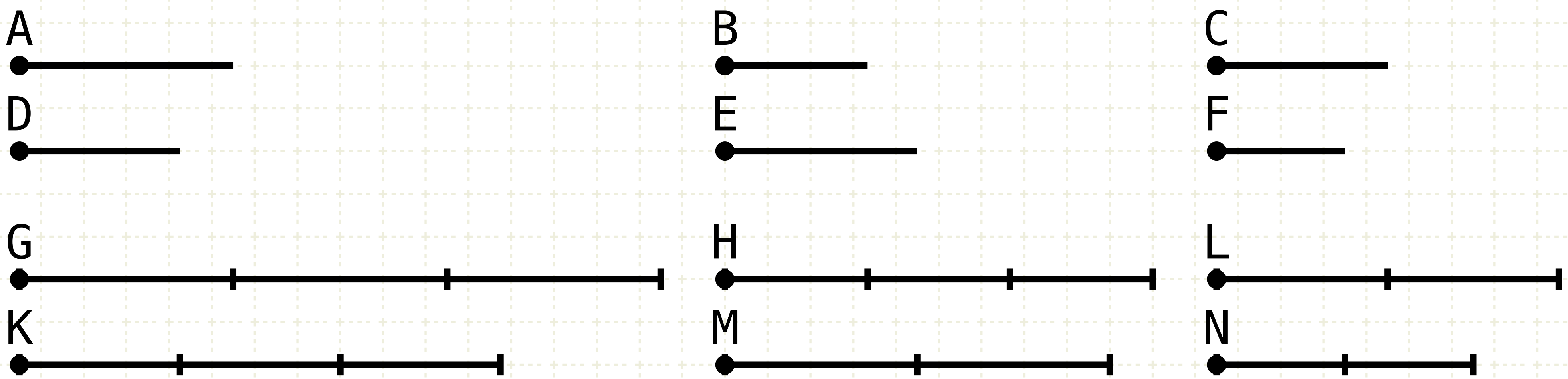
And alternately, as H is to L, so is K to M (V·16)

Therefore G,H,L and K,M,N are in a perturbed ratio

Thus if G is greater than L, K is also greater than N, etc (V·21)

Proposition 23 of Book V

If there be three magnitudes, and others equal to them in multitude, which taken two and two together are in the same ratio, and the proportion of them be perturbed, they will also be in the same ratio EX AEQUALI



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A:B = G:H
E:F = M:N
G:H = M:N
B:D = C:E
B:D = H:K
H:K = C:E
C:E = L:M
H:K = L:M
H:L = K:M

$G >=< L \rightarrow K >=< N$

$m \cdot A >=< n \cdot C \rightarrow m \cdot D >=< n \cdot F$

In other words

Given two sets of numbers A,B,C and D,E,F in perturbed proportion, where A is to B as E is to F, and where B is to C as D is to E

Then A is to C as D is to F

Proof

Let G,H,K be equimultiples of A, B and D

Let L,M,N be equimultiples of C, E and F

Since G,H are equimultiples of A,B then A is to B as G is to H (V·15)

Similarly E is to F as M is to N

And, as A is to B, so is E to F; hence as G is to H, so is M to N (V·11)

Since B is to C as D is to E, alternately B is to D as C is to E (V·16)

Since H,K are equimultiples of B,D, B is to D as H is to K (V·15)

As B is to D, so is C to E; hence as H is to K, so is C to E (V·11)

L,M are equimultiples of C,E, so C is to E as L is to M (V·15)

Since C is to E as H is to K, H is to K as L is to M (V·11)

And alternately, as H is to L, so is K to M (V·16)

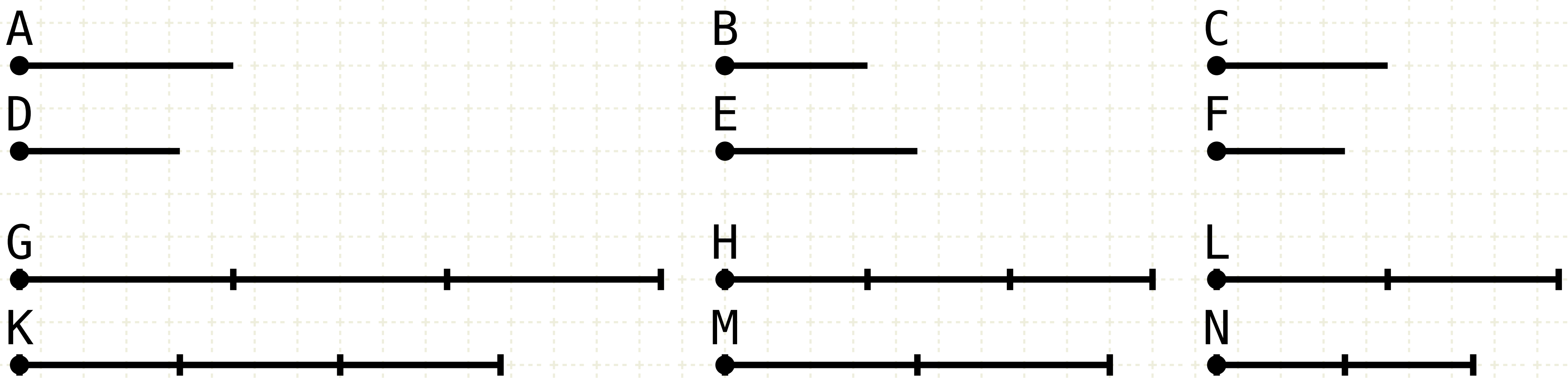
Therefore G,H,L and K,M,N are in a perturbed ratio

Thus if G is greater than L, K is also greater than N, etc (V·21)

G,K are equimultiples of A,D and L,N of C,F

Proposition 23 of Book V

If there be three magnitudes, and others equal to them in multitude, which taken two and two together are in the same ratio, and the proportion of them be perturbed, they will also be in the same ratio EX AEQUALI



$A:B = E:F$

$B:C = D:E$

$G = m \cdot A$

$H = m \cdot B$

$K = m \cdot D$

$L = n \cdot C$

$M = n \cdot E$

$N = n \cdot F$

$A:B = G:H$

$E:F = M:N$

$G:H = M:N$

$B:D = C:E$

$B:D = H:K$

$H:K = C:E$

$C:E = L:M$

$H:K = L:M$

$H:L = K:M$

$G \gtreqless L \rightarrow K \gtreqless N$

$m \cdot A \gtreqless n \cdot C \rightarrow m \cdot D \gtreqless n \cdot F$

$A:C = D:F$

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L,M are equimultiples of C,E, so C is to E as L is to M (V·15)

Since C is to E as H is to K, H is to K as L is to M (V·11)

And alternately, as H is to L, so is K to M (V·16)

Therefore G,H,L and K,M,N are in a perturbed ratio

Thus if G is greater than L, K is also greater than N, etc (V·21)

G,K are equimultiples of A,D and L,N of C,F

Therefore A is to C as D is to F (V·def·5)

Proposition 23 of Book V

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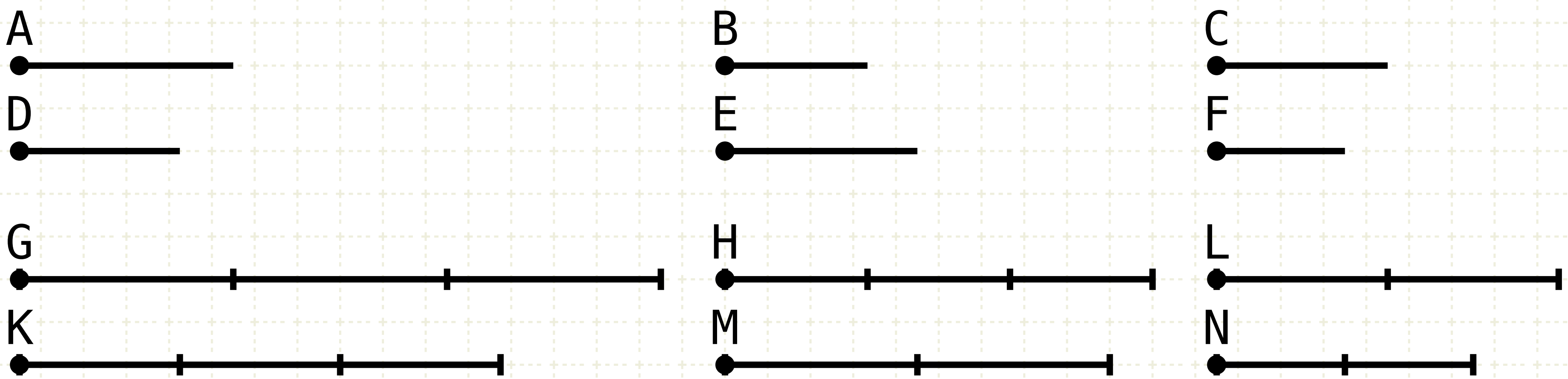

A:B = E:F
B:C = D:E

G = m·A
H = m·B
K = m·D
L = n·C
M = n·E
N = n·F

A:B = G:H
E:F = M:N
G:H = M:N
B:D = C:E
B:D = H:K
H:K = C:E
C:E = L:M
H:K = L:M
H:L = K:M
G >=< L → K >=<N
m·A >=< n·C → m·D >=<n·F

A:C = D:F

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