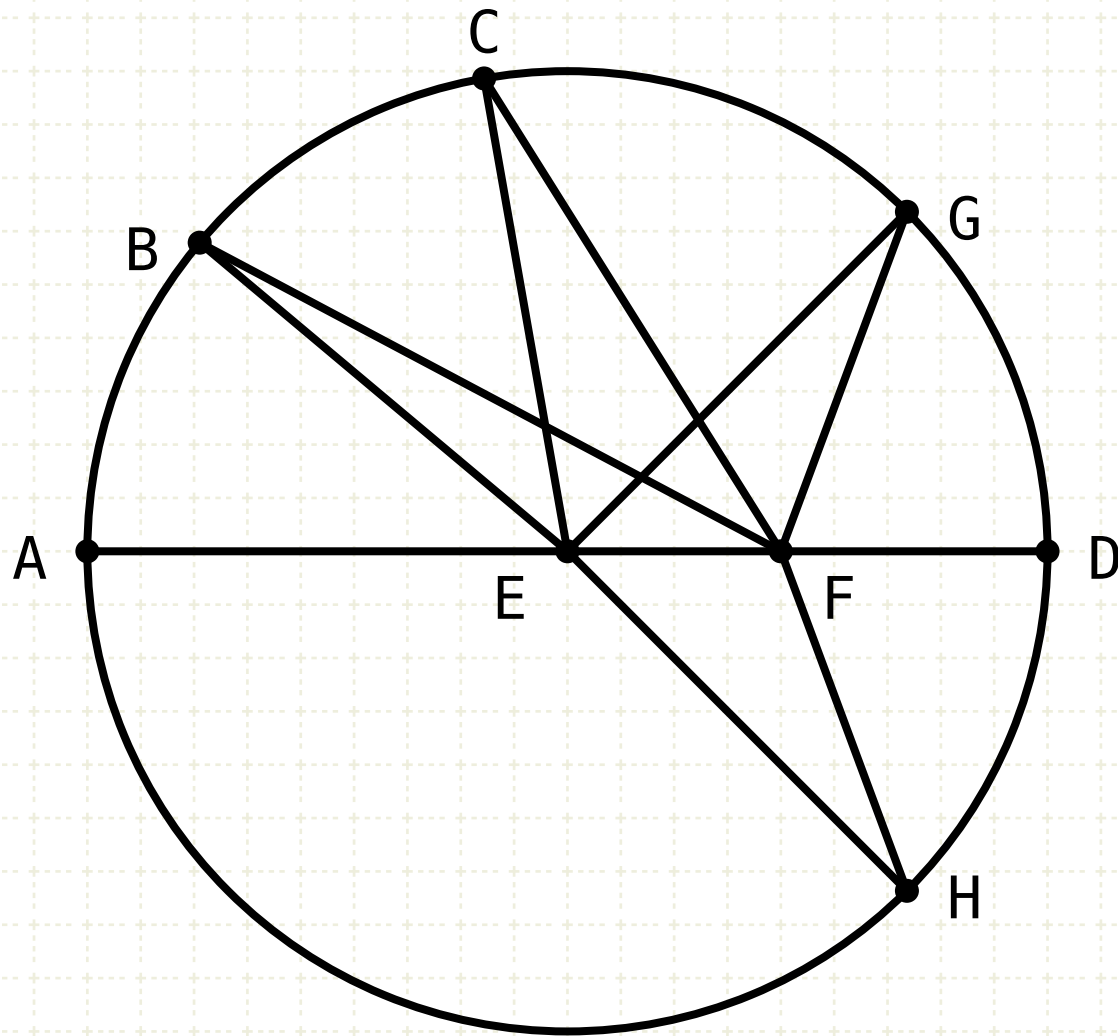


Euclid's Elements

Book III



A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)



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3	A line through the centre of a circle bisects a chord, and vice versa	11	Point of contact between two internal circles, and their centres, are collinear	19	If line touches a circle, then the centre of the circle lies on a line perpendicular to the original
4	A line not through the centre of a circle does not bisect a chord	12	Point of contact between two external circles, and their centres, are collinear	20	The angle at the centre of a circle is twice that from an angle from the circumference
5	If two circles cut one another, they will not have the same center	13	A circle does not touch a circle at more points than one, whether it touch it internally or externally.	21	In a circle the angles in the same segment are equal to one another
6	If two circles touch one another, they will not have the same center	14	In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another.	22	The opposite angles of quadrilaterals in circles are equal to two right angles
7	Consider two lines from a point inside a circle to the edge, the longer one will be the one closest to the longest part of the diameter passing through the original point	15	The longest line in a circle is its diameter, shorter the farther away from the diameter	23	On the same straight line there cannot be constructed two similar and unequal segments of circles on the same side
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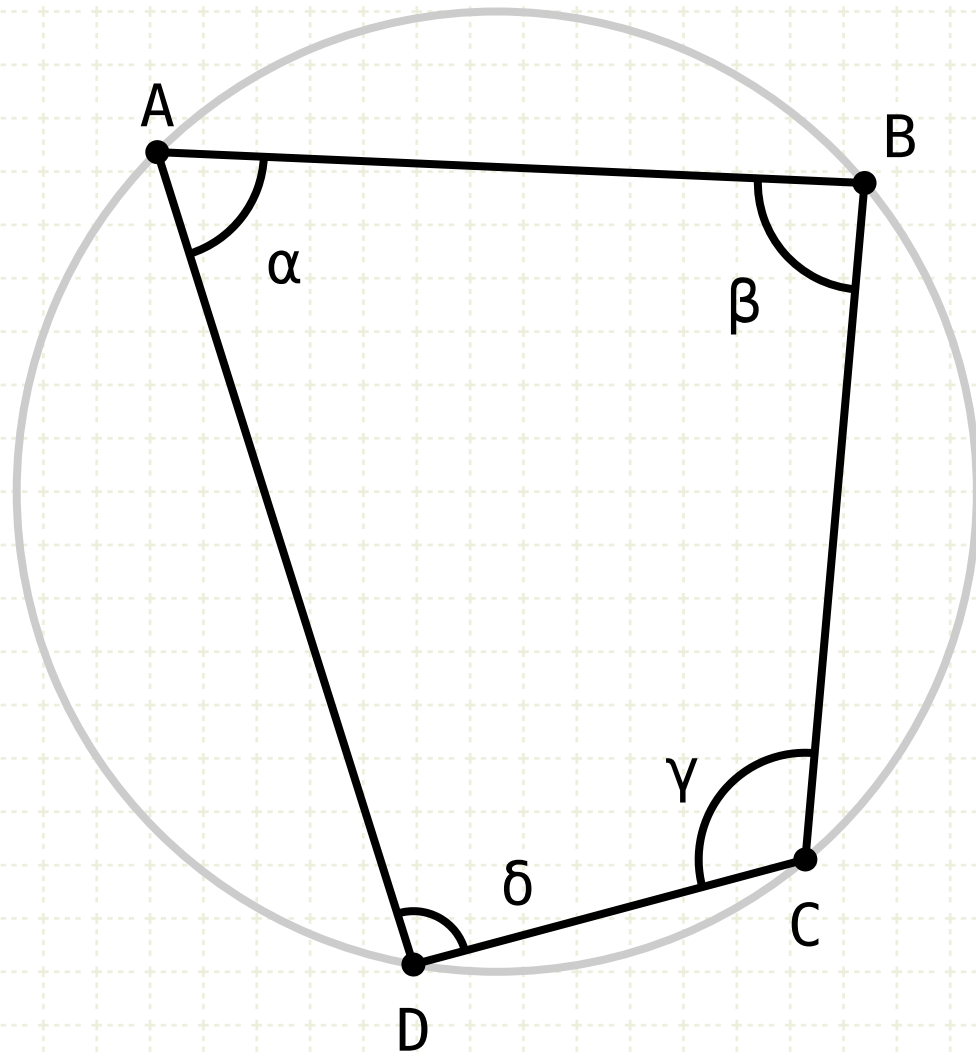
Proposition 22 of Book III

The opposite angles of quadrilaterals in circles are equal to two right angles.



Proposition 22 of Book III

The opposite angles of quadrilaterals in circles are equal to two right angles.



$$\alpha + \gamma = 2\text{R}$$

$$\delta + \beta = 2\text{R}$$

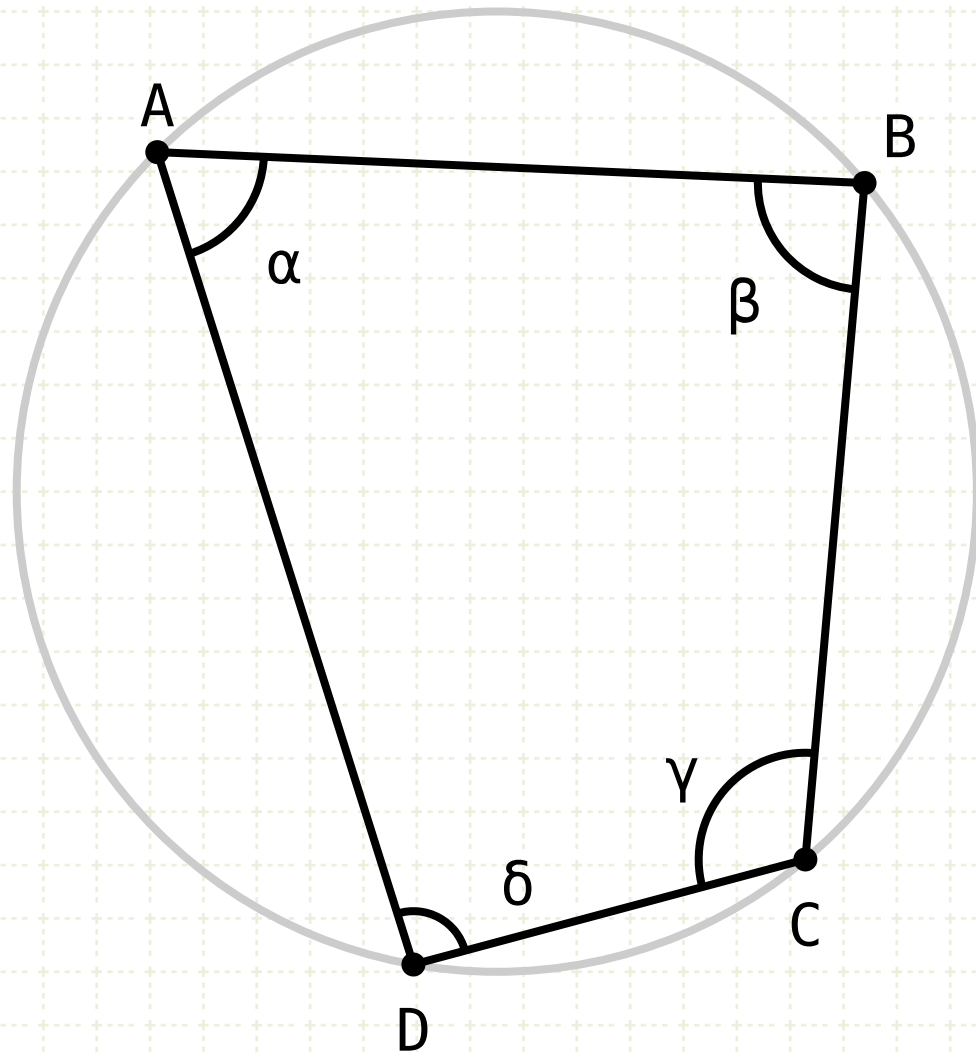
In other words

If a quadrilateral ABCD is drawn within a circle, then the sum of the angles at A and C (α and γ) equals two right angles, similarly, the angles at B and D (β and δ) sum to two right angles

Proposition 22 of Book III

The opposite angles of quadrilaterals in circles are equal to two right angles.

$$\begin{aligned}\angle A &= \alpha, & \angle B &= \beta, \\ \angle C &= \gamma, & \angle D &= \delta\end{aligned}$$



In other words

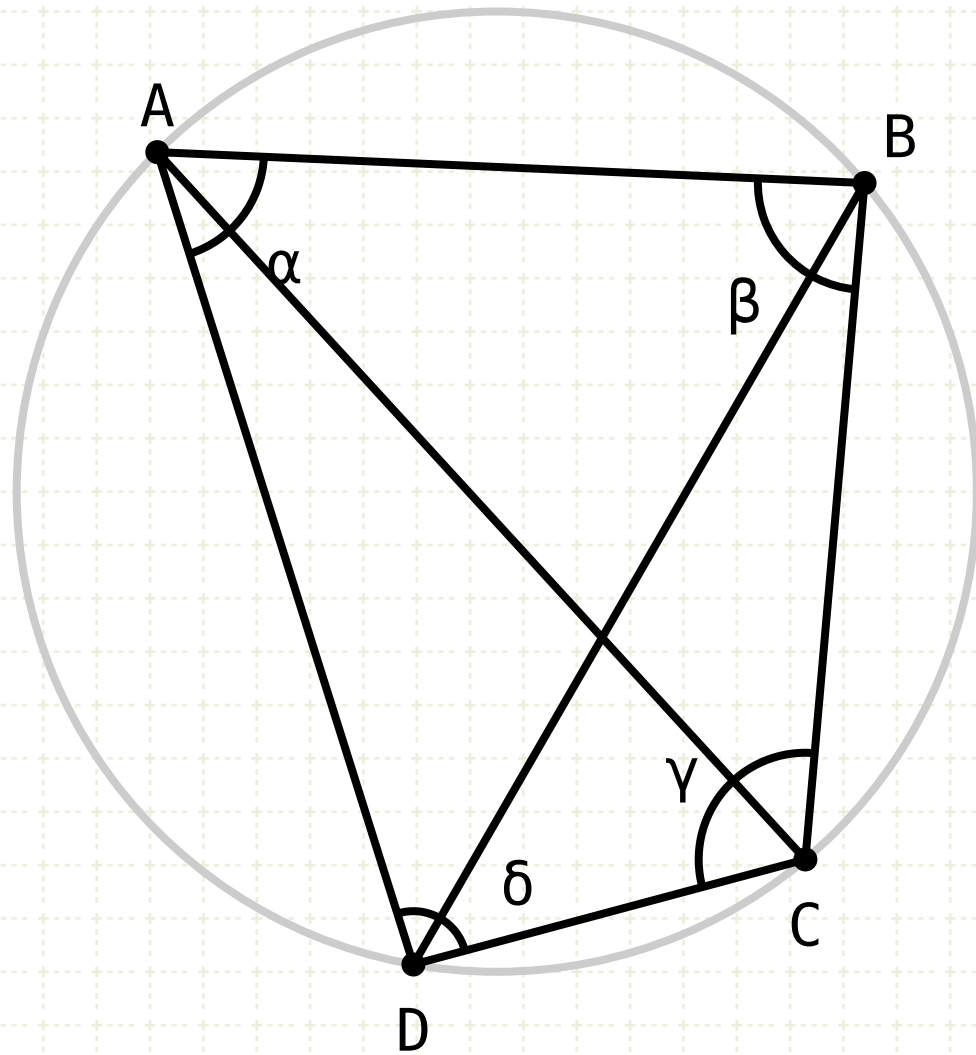
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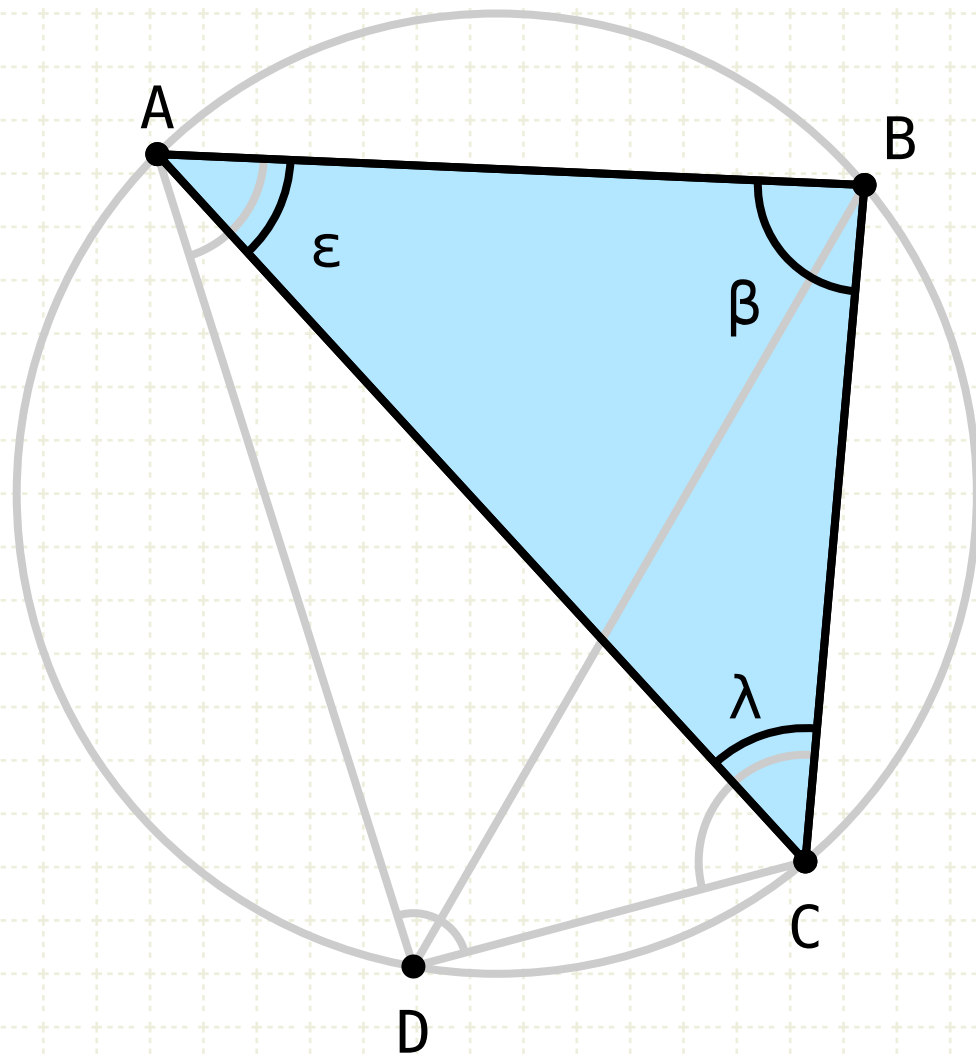
Draw the lines AC and BD

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The opposite angles of quadrilaterals in circles are equal to two right angles.

$$\begin{aligned}\angle A &= \alpha, & \angle B &= \beta, \\ \angle C &= \gamma, & \angle D &= \delta\end{aligned}$$

$$\varepsilon + \lambda + \beta = 2\text{R}$$



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If a quadrilateral ABCD is drawn within a circle, then the sum of the angles at A and C (α and γ) equals two right angles, similarly, the angles at B and D (β and δ) sum to two right angles

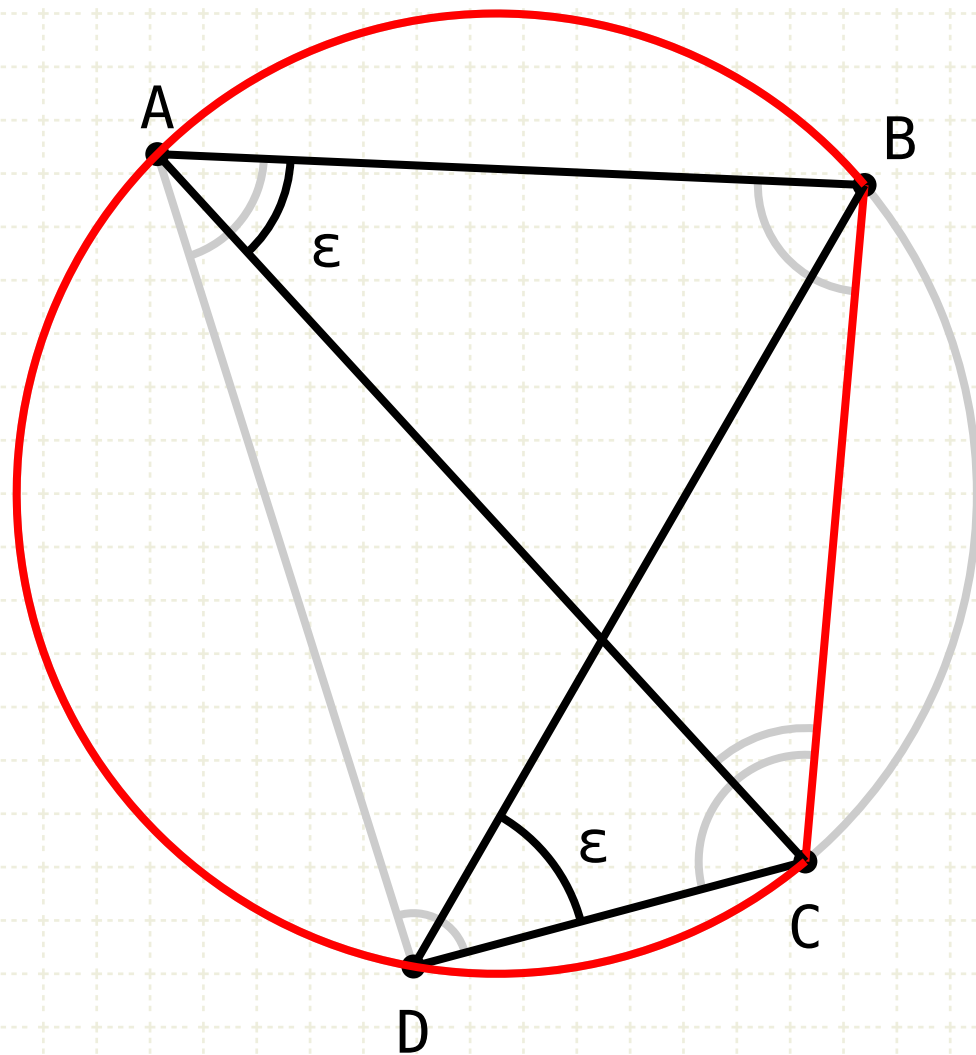
Proof

Draw the lines AC and BD

The sum of the angles inside the triangle ABC equals two right angles (I.32)

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The opposite angles of quadrilaterals in circles are equal to two right angles.



$$\angle A = \alpha, \angle B = \beta, \\ \angle C = \gamma, \angle D = \delta$$

$$\varepsilon + \lambda + \beta = 2\text{R} \\ \angle CAB = \angle BDC = \varepsilon$$

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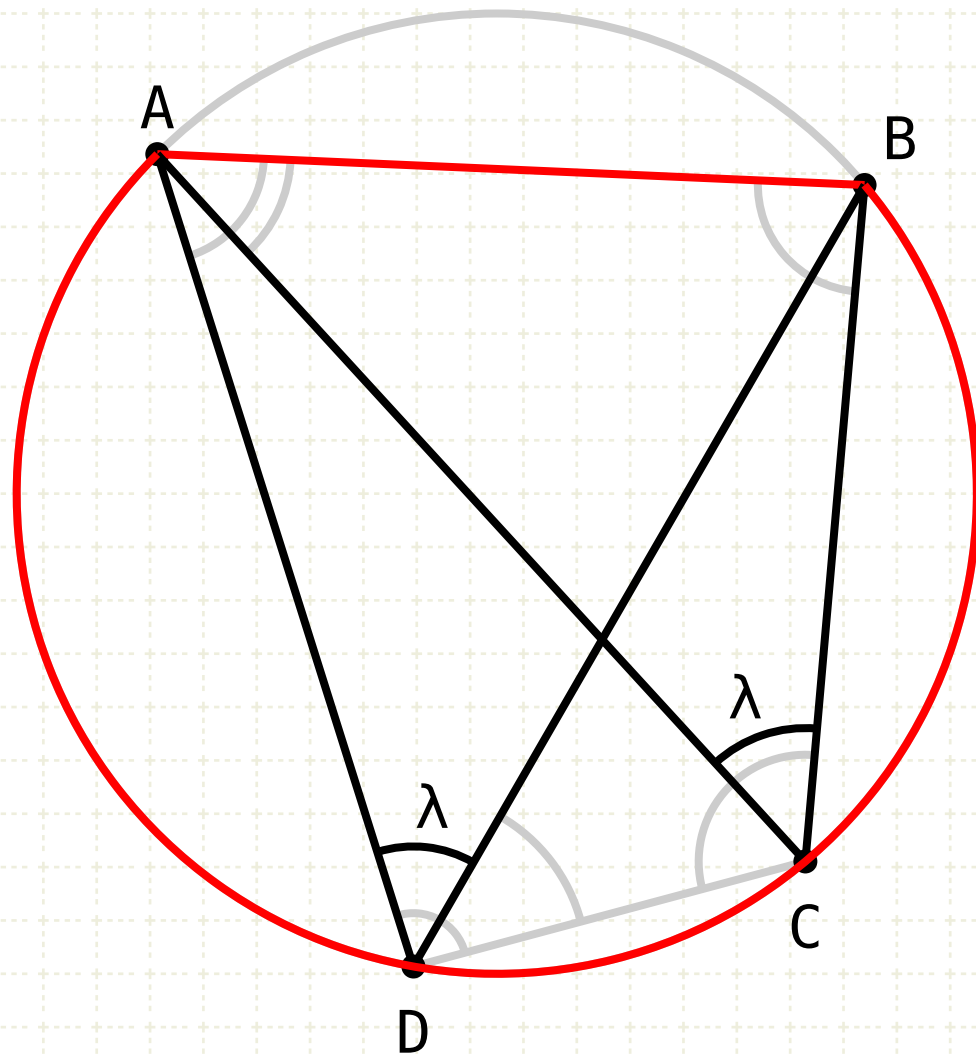
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The sum of the angles inside the triangle ABC equals two right angles (I·32)

Angle CAB is equal to angle BDC because they are in the same circular segment (III·21)

Proposition 22 of Book III

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$$\angle A = \alpha, \angle B = \beta, \\ \angle C = \gamma, \angle D = \delta$$

$$\varepsilon + \lambda + \beta = 2\text{R} \\ \angle CAB = \angle BDC = \varepsilon \\ \angle ACB = \angle ADB = \lambda$$

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If a quadrilateral ABCD is drawn within a circle, then the sum of the angles at A and C (α and γ) equals two right angles, similarly, the angles at B and D (β and δ) sum to two right angles

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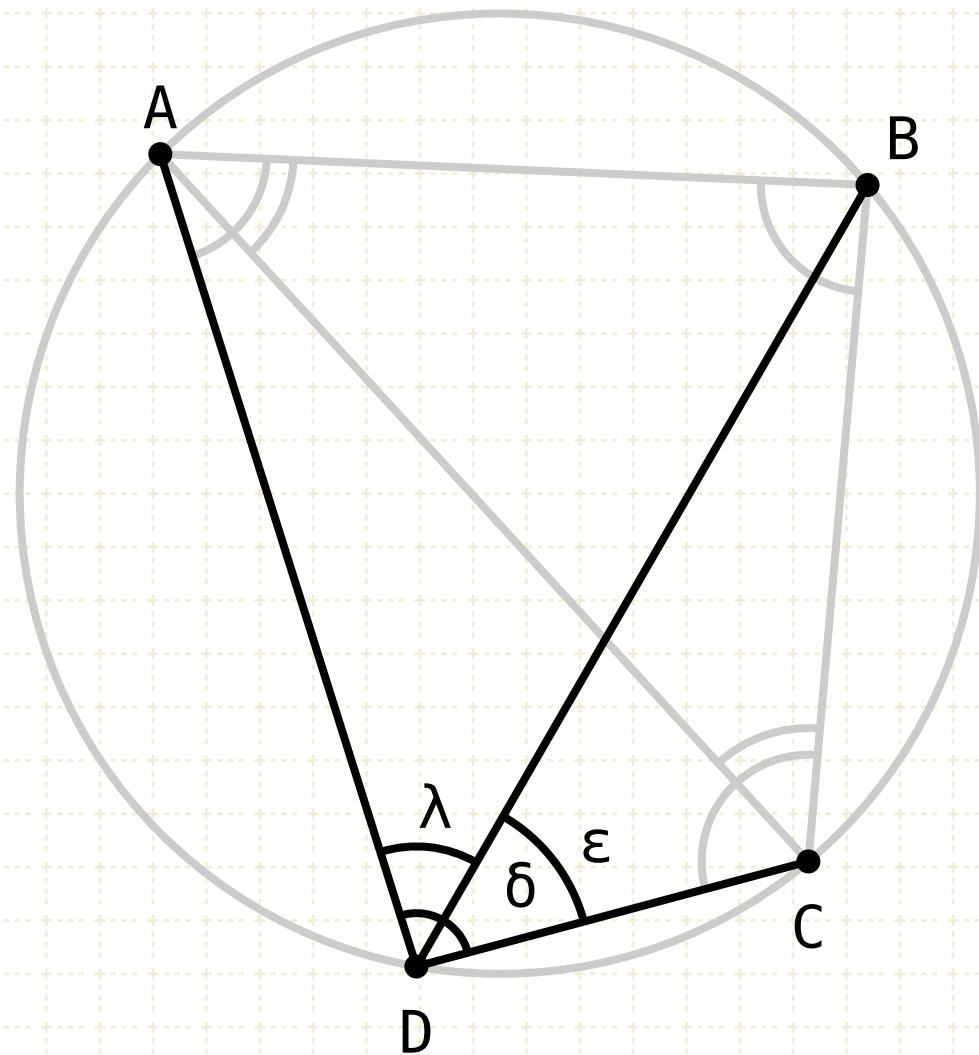
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$$\begin{aligned}\epsilon + \lambda + \beta &= 2\text{R} \\ \angle CAB &= \angle BDC = \epsilon \\ \angle ACB &= \angle ADB = \lambda \\ \delta &= \epsilon + \lambda\end{aligned}$$

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If a quadrilateral ABCD is drawn within a circle, then the sum of the angles at A and C (α and γ) equals two right angles, similarly, the angles at B and D (β and δ) sum to two right angles

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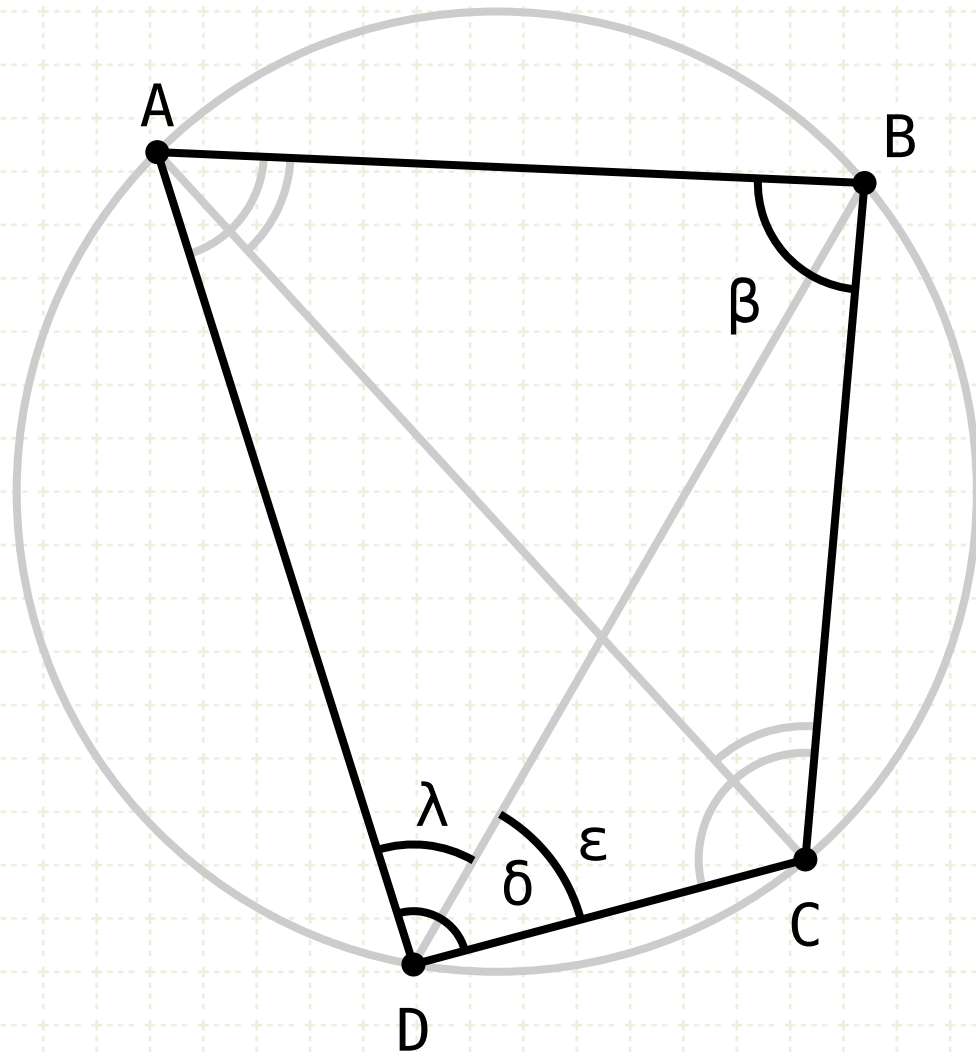
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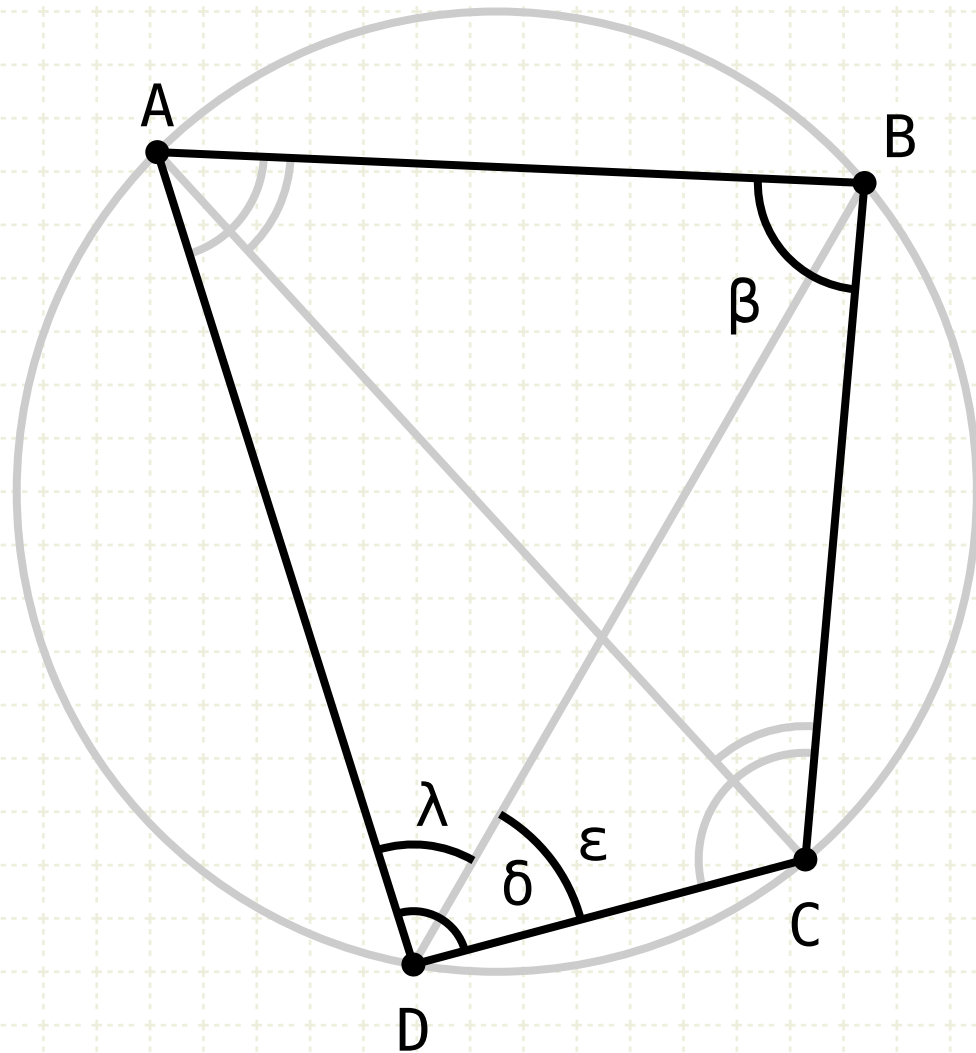
Angle ACB is equal to angle ADB because they are in the same circular segment (III·21)

The angle at D is equal to the sum of the angles ADC and BCD

So the sum of the angles at B and D is equal to the sum of the angles at B (β), ADB (λ), and BDC (ϵ)

Proposition 22 of Book III

The opposite angles of quadrilaterals in circles are equal to two right angles.



$$\begin{aligned}\angle A &= \alpha, & \angle B &= \beta, \\ \angle C &= \gamma, & \angle D &= \delta\end{aligned}$$

$$\begin{aligned}\epsilon + \lambda + \beta &= 2\text{L} \\ \angle CAB &= \angle BDC = \epsilon \\ \angle ACB &= \angle ADB = \lambda \\ \delta &= \epsilon + \lambda\end{aligned}$$

$$\begin{aligned}\angle B + \angle D &= \beta + \delta \\ &= \beta + \epsilon + \lambda \\ &= 2\text{L}\end{aligned}$$

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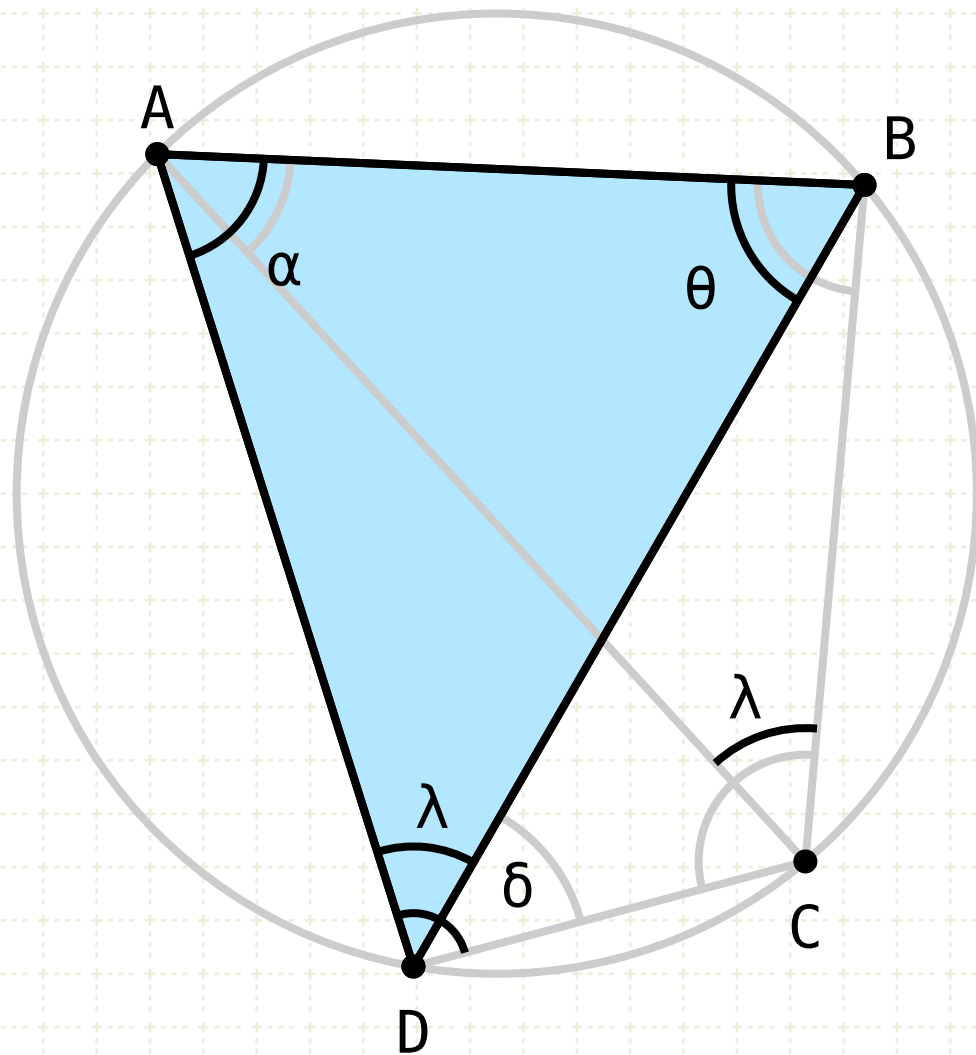
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Which is equal to two right angles

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The opposite angles of quadrilaterals in circles are equal to two right angles.



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$$\alpha + \theta + \lambda = 2\text{L}$$

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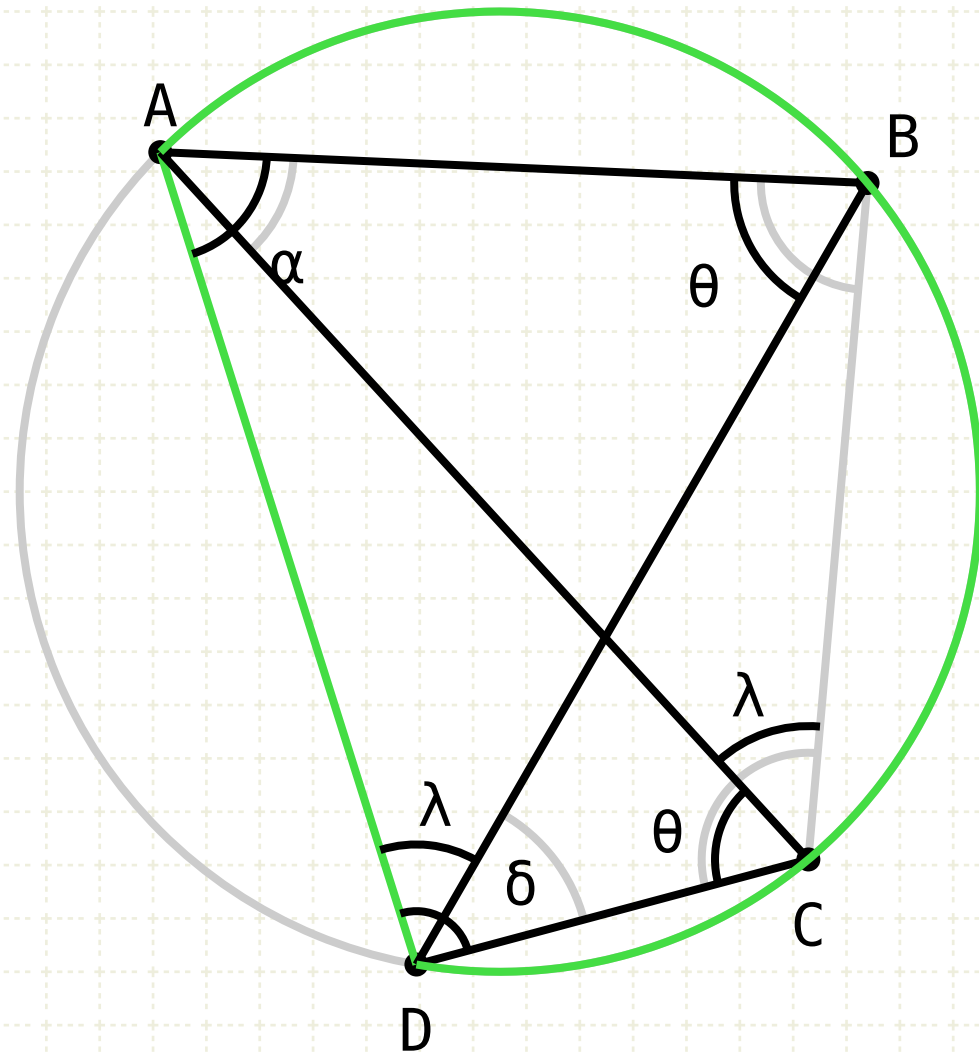
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$$\begin{aligned} \angle B + \angle D \\ &= \beta + \delta \\ &= \beta + \epsilon + \lambda \\ &= 2\text{L} \end{aligned}$$

$$\begin{aligned} \alpha + \theta + \lambda &= 2\text{L} \\ \angle DBA &= \angle ACD = \theta \end{aligned}$$

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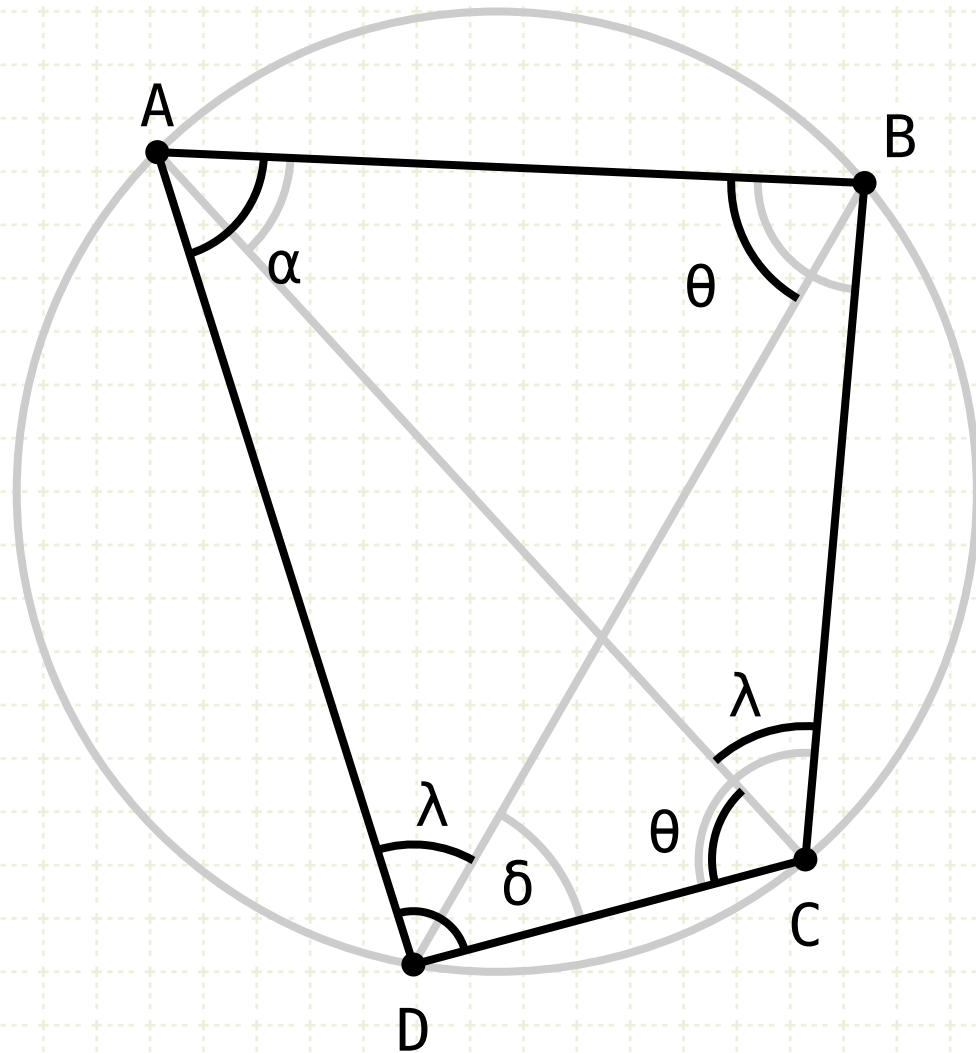
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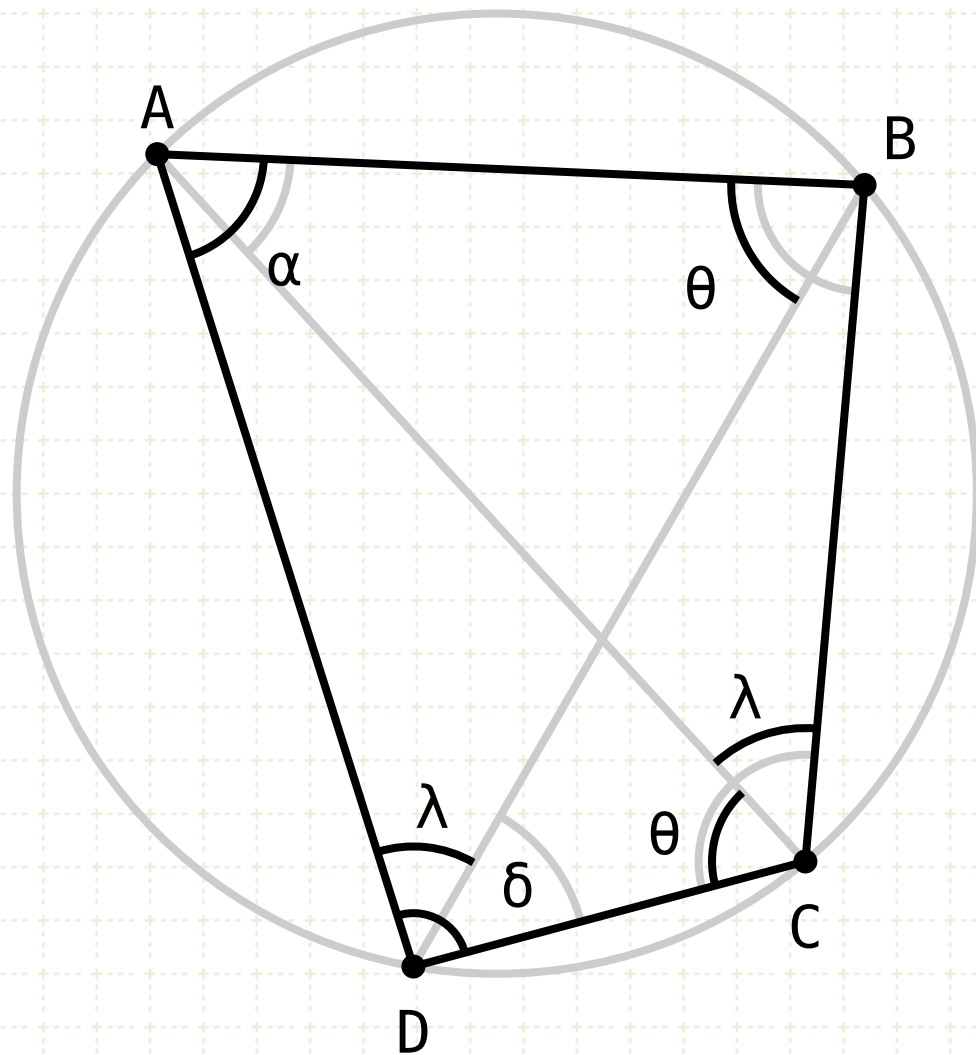
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$$\angle A = \alpha, \angle B = \beta, \\ \angle C = \gamma, \angle D = \delta$$

$$\epsilon + \lambda + \beta = 2L \\ \angle CAB = \angle BDC = \epsilon \\ \angle ACB = \angle ADB = \lambda \\ \delta = \epsilon + \lambda$$

$$\angle B + \angle D \\ = \beta + \delta \\ = \beta + \epsilon + \lambda \\ = 2L$$

$$\alpha + \theta + \lambda = 2L \\ \angle DBA = \angle ACD = \theta \\ \angle A + \angle C \\ = \alpha + \theta + \lambda \\ = 2L$$

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