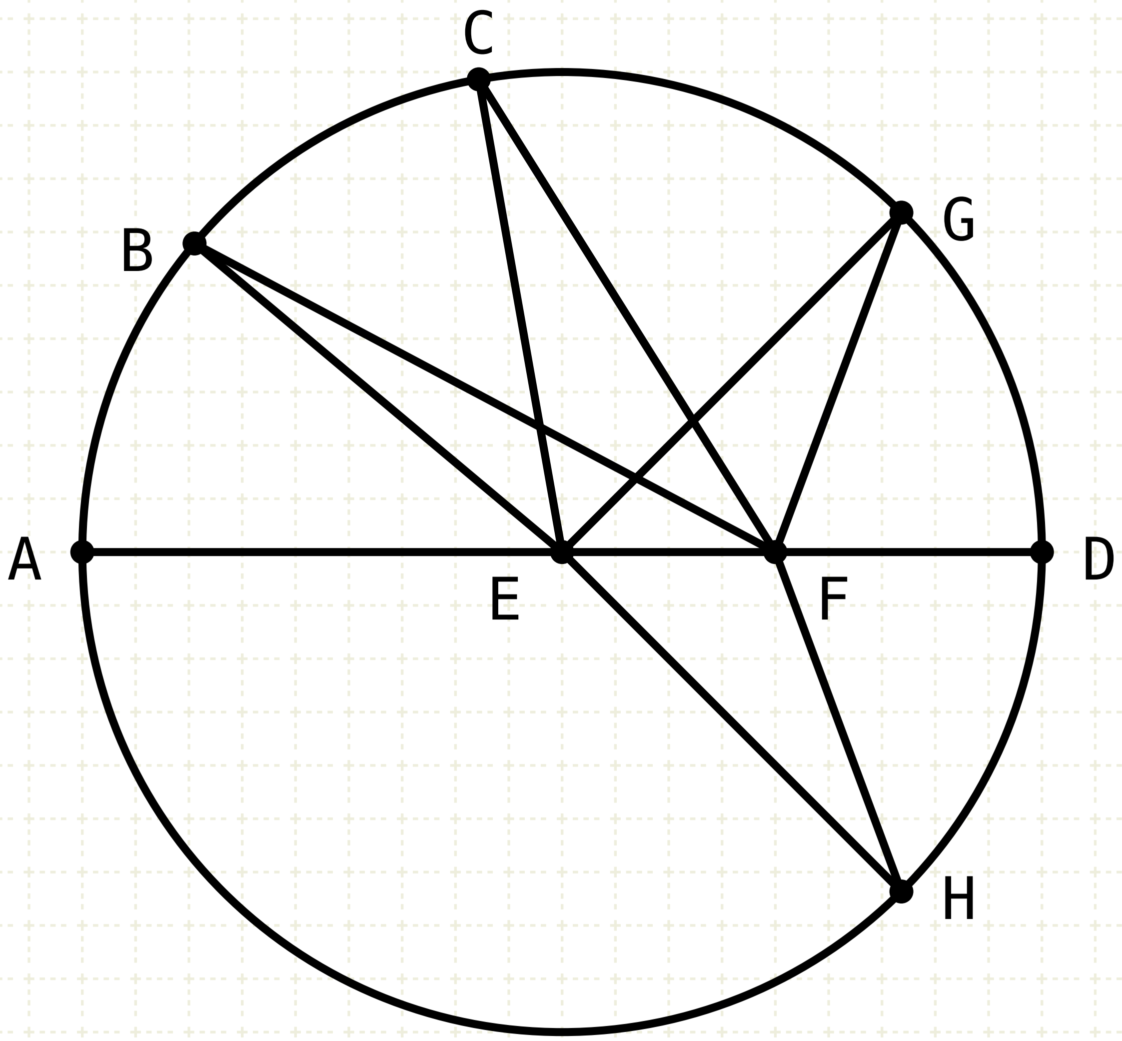
Euclid's Elements

Book III



A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)

Table of Contents, Chapter 3

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Proposition 18 of Book III

If a straight line touch a circle, and a straight line be joined from the centre to the point of contact, the straight line so joined will be perpendicular to the tangent.

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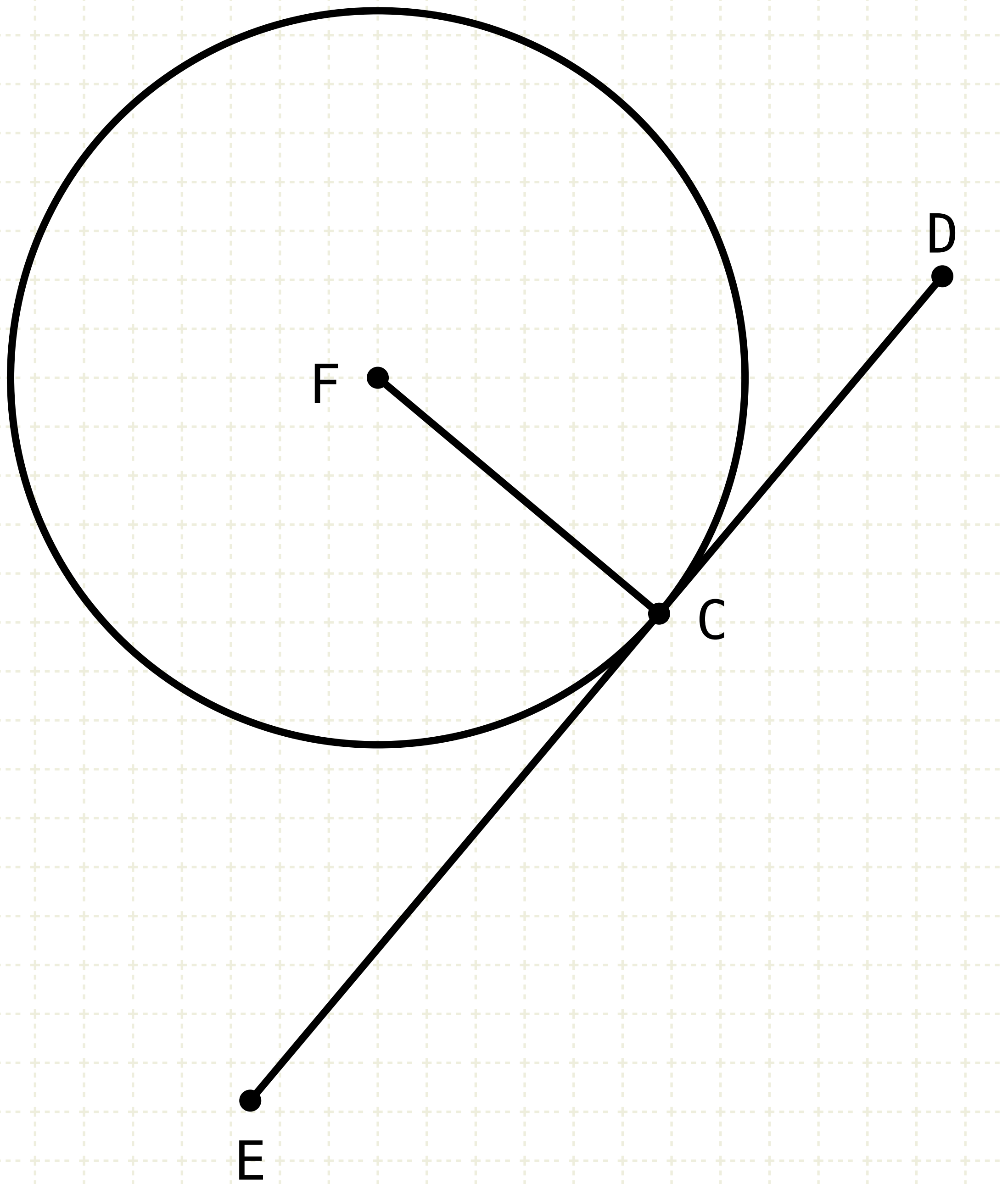

In other words

If line ED touches a circle at point C, then it is perpendicular to the radius FC

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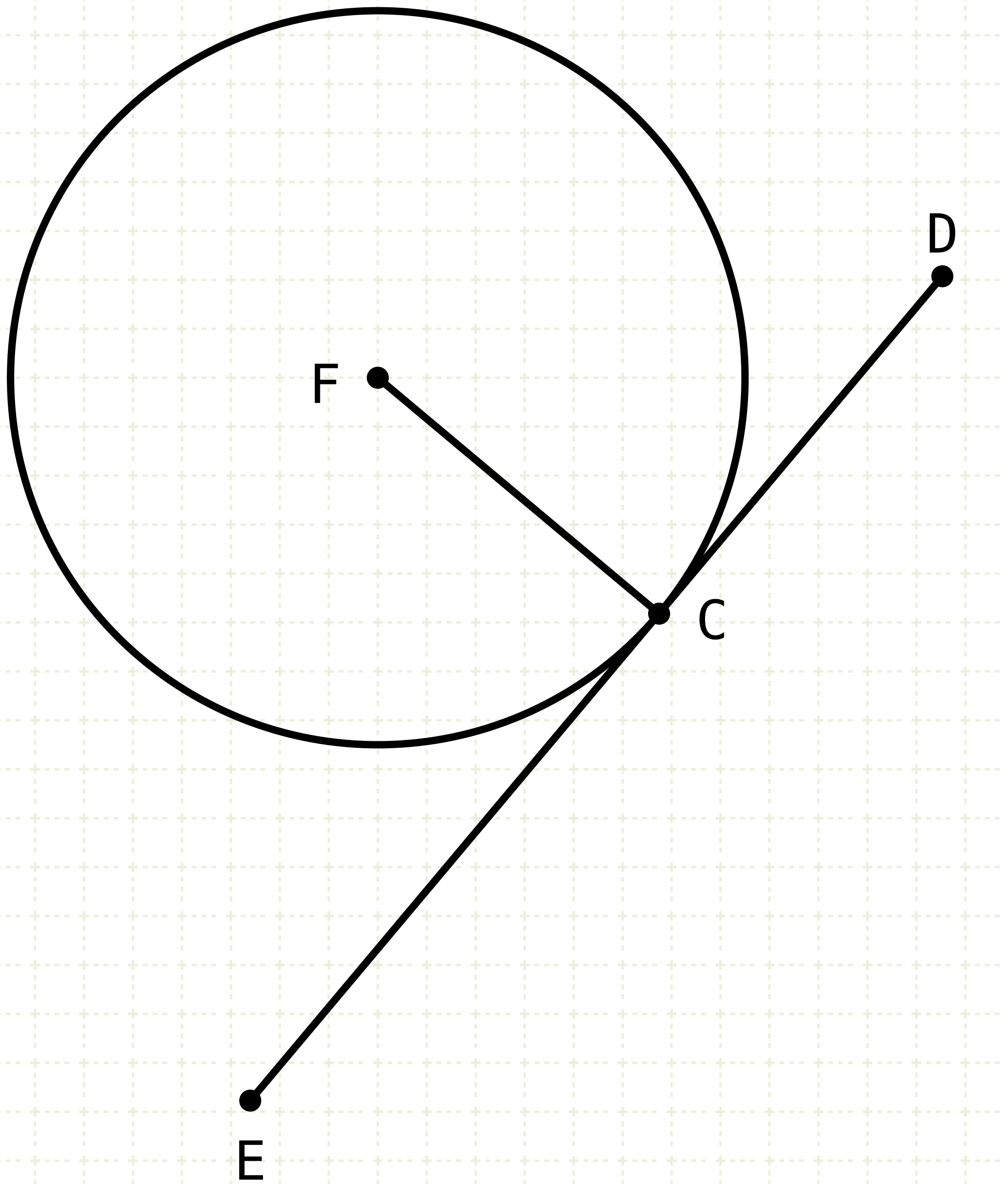

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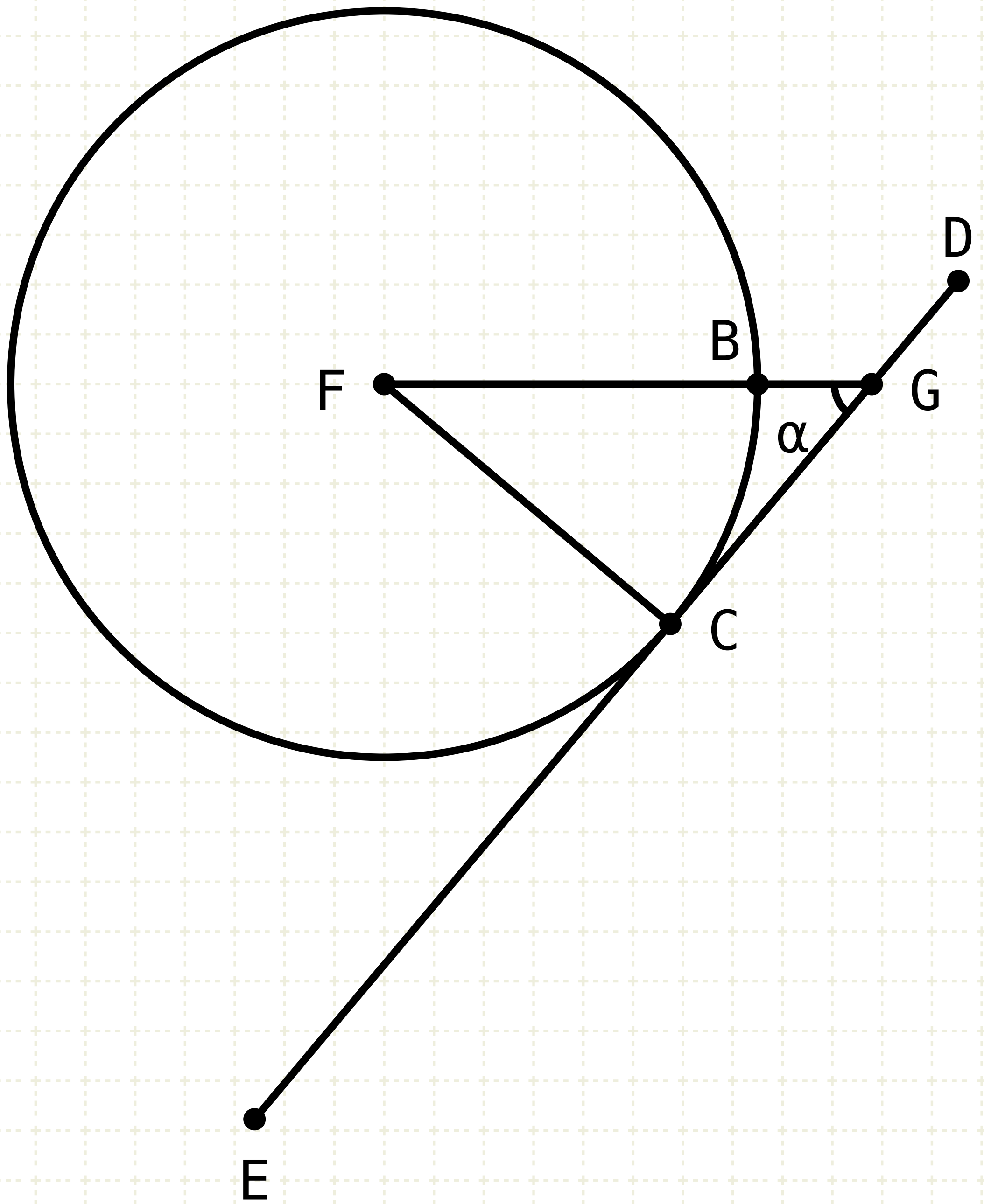

$\beta \neq ∟$
$\alpha = ∟$

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If FCD is not a right angle, draw a line FG perpendicular to ED

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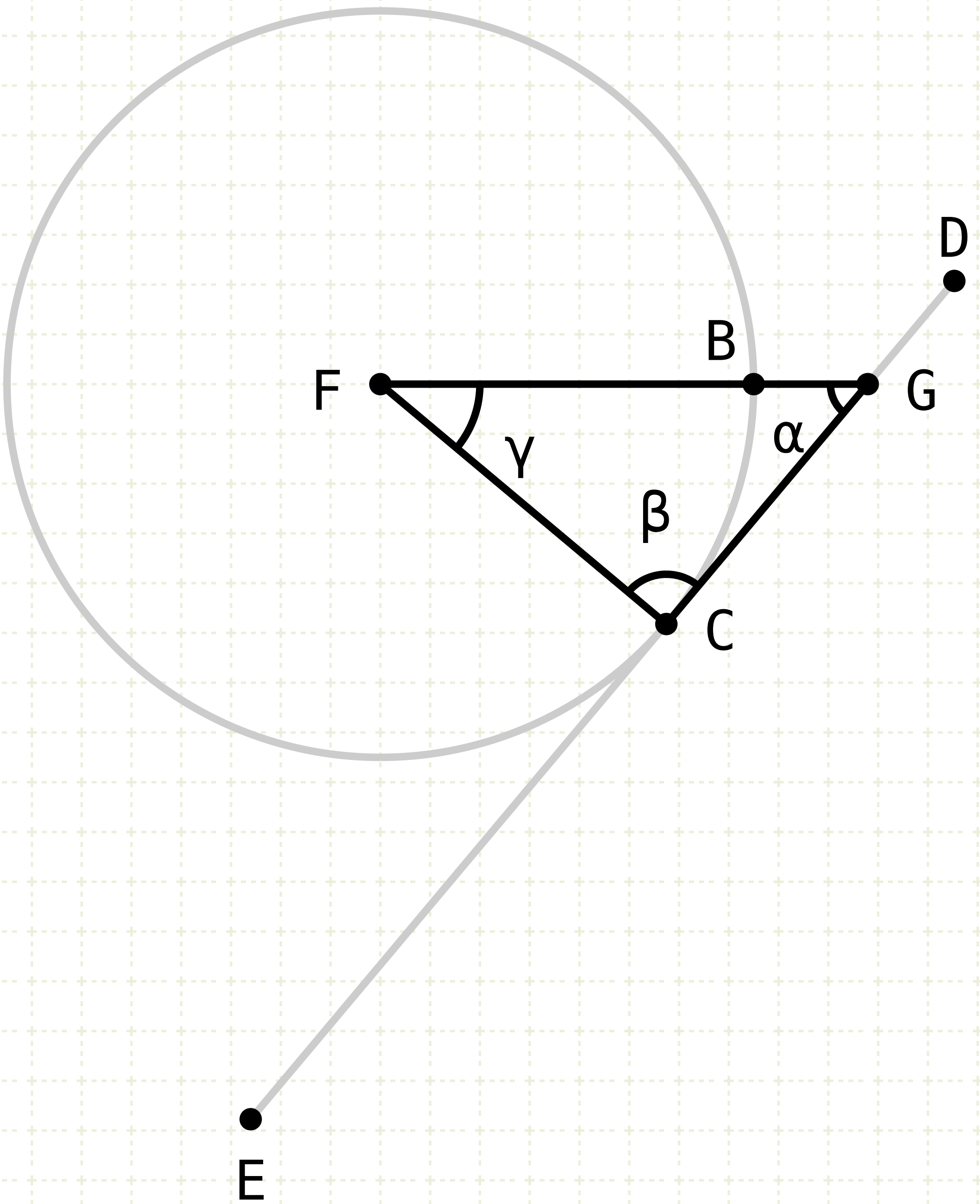

$\beta \neq \llcorner$

$\alpha = \llcorner$

$\alpha + \beta + \gamma = 2 \cdot \llcorner$

$\llcorner + \beta + \gamma = 2 \cdot \llcorner$

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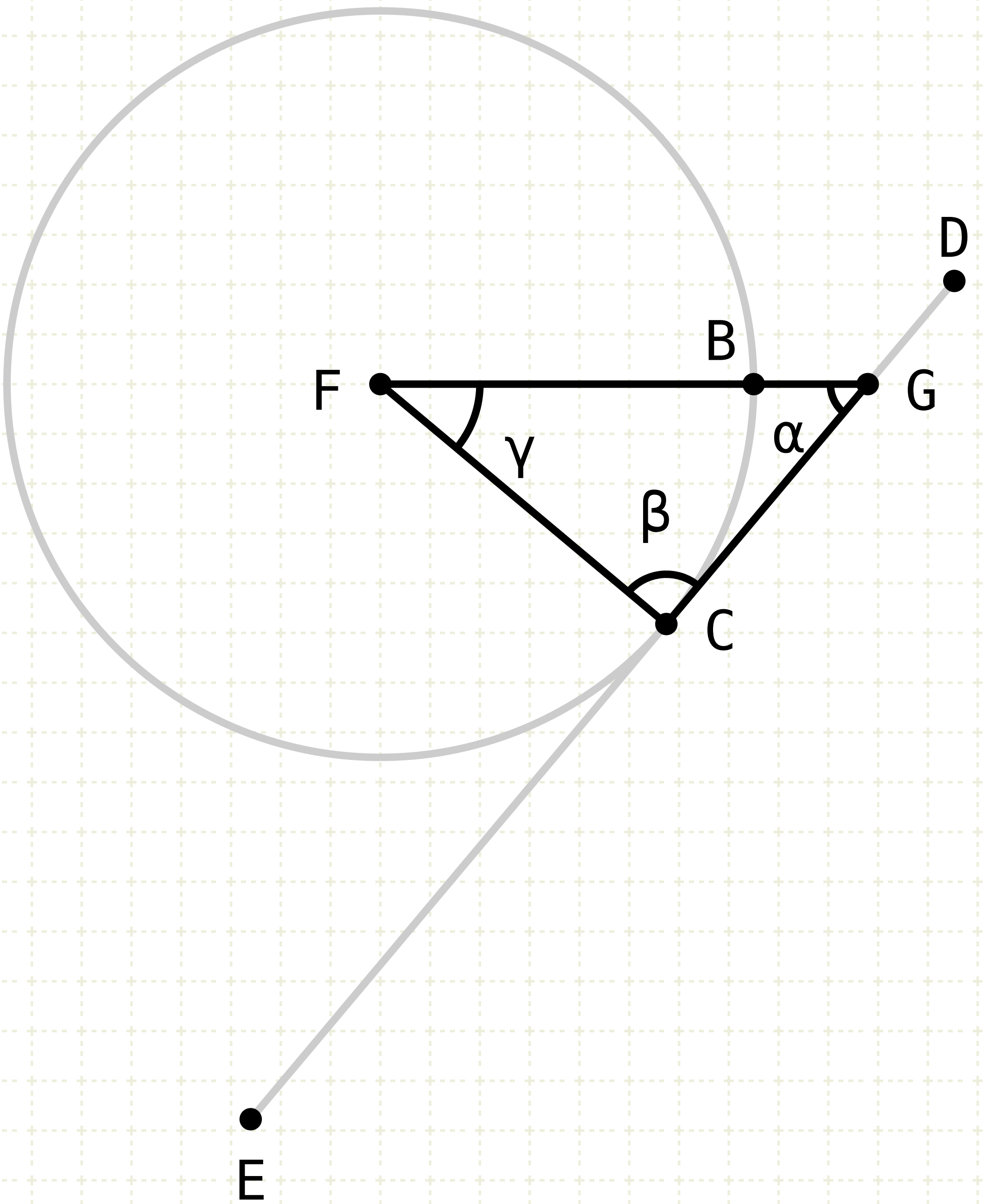

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$$FC > FG$$

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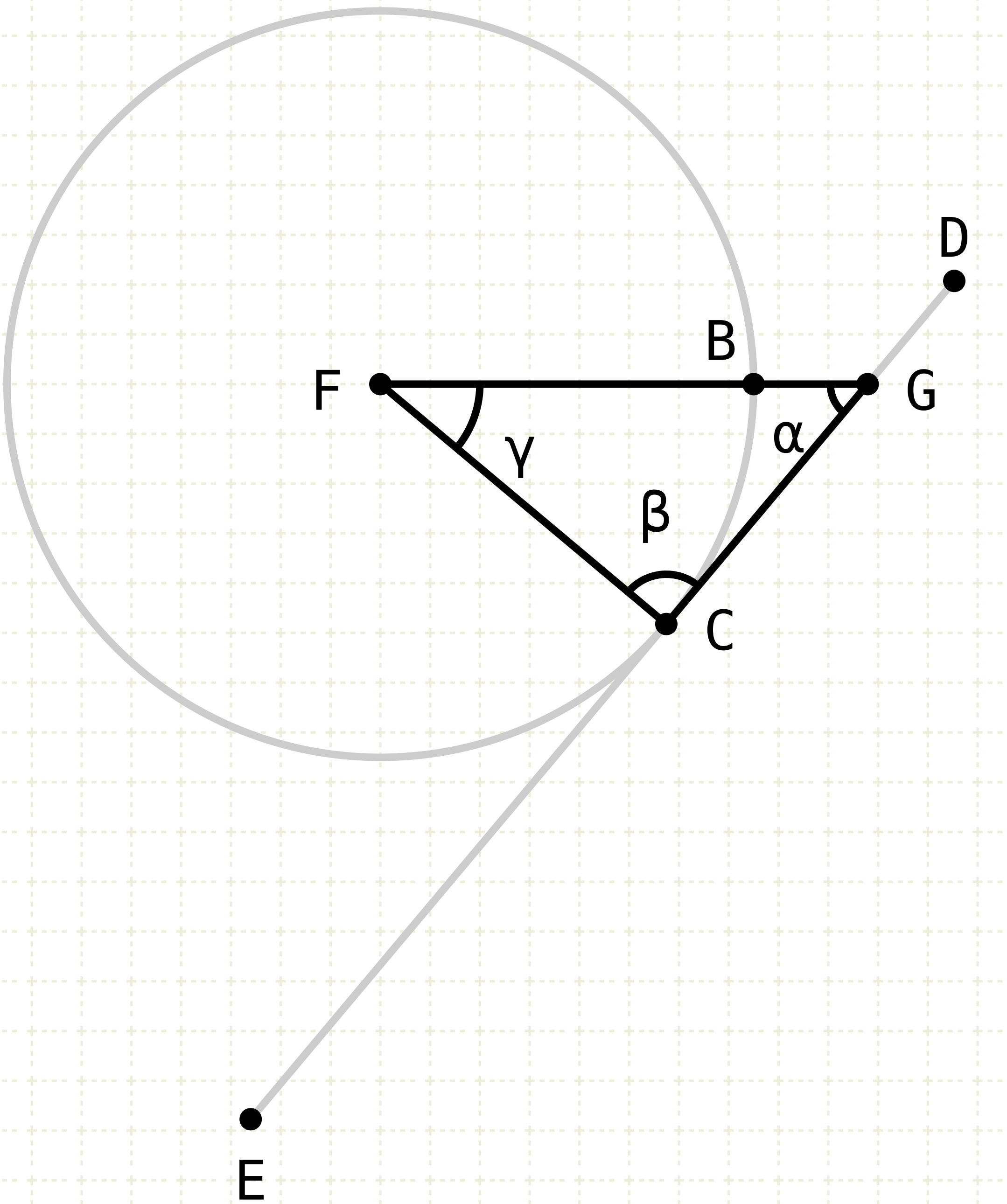

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$FB = FC$

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But FB is equal to FC (radii), and FG is larger than FB, so FG is greater than FC

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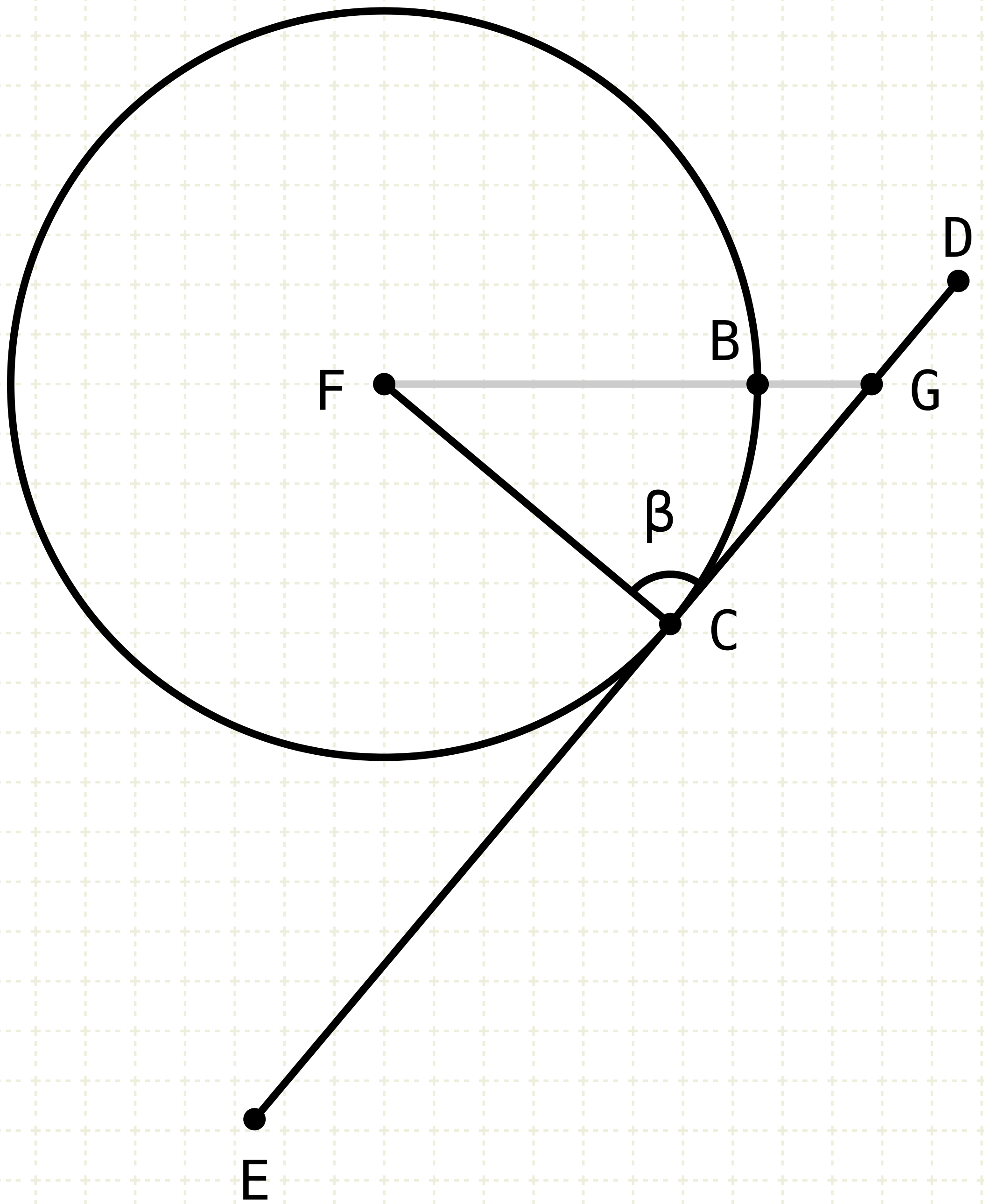

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FG cannot be both larger and smaller than FC, so we have a logical inconsistency

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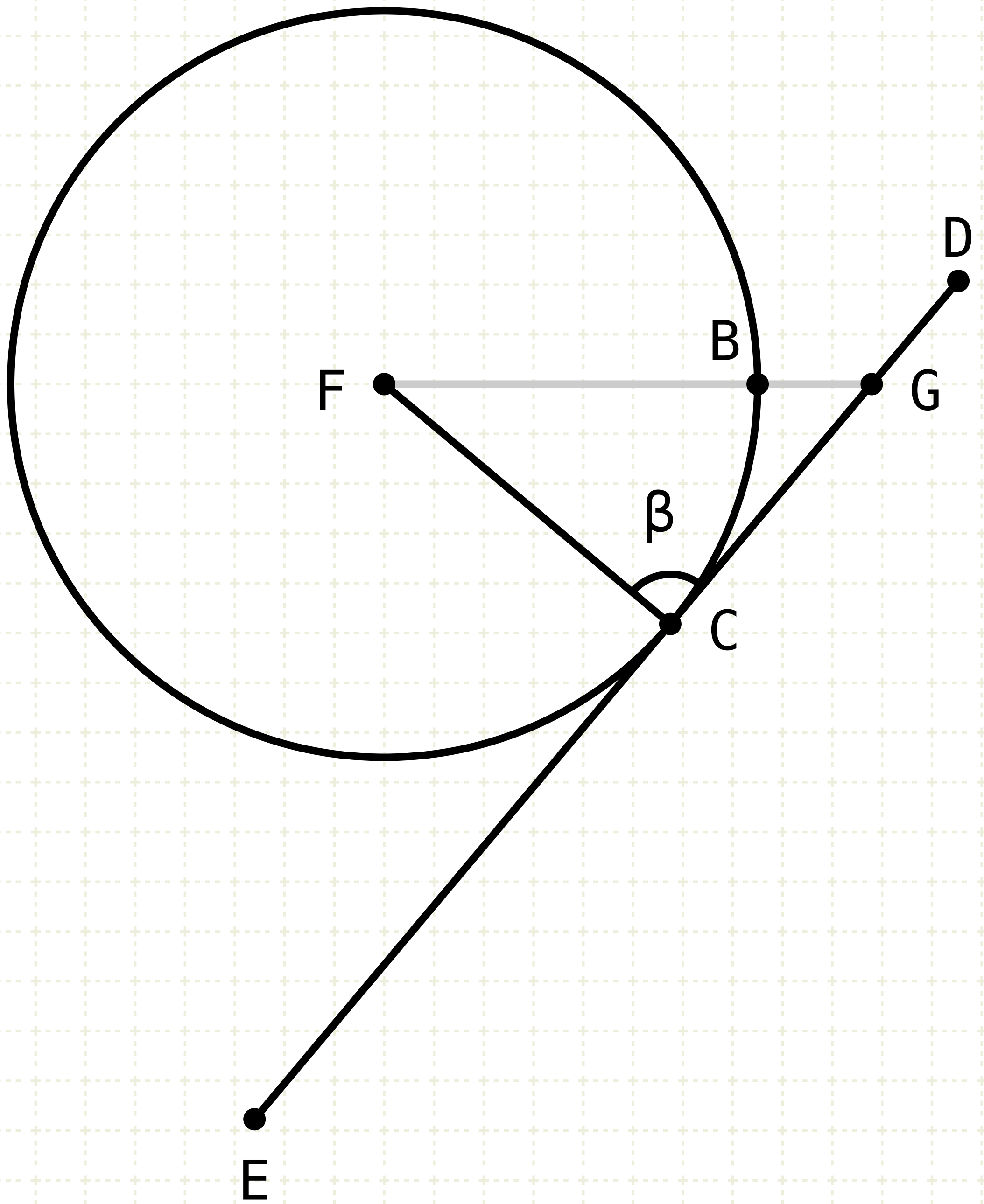

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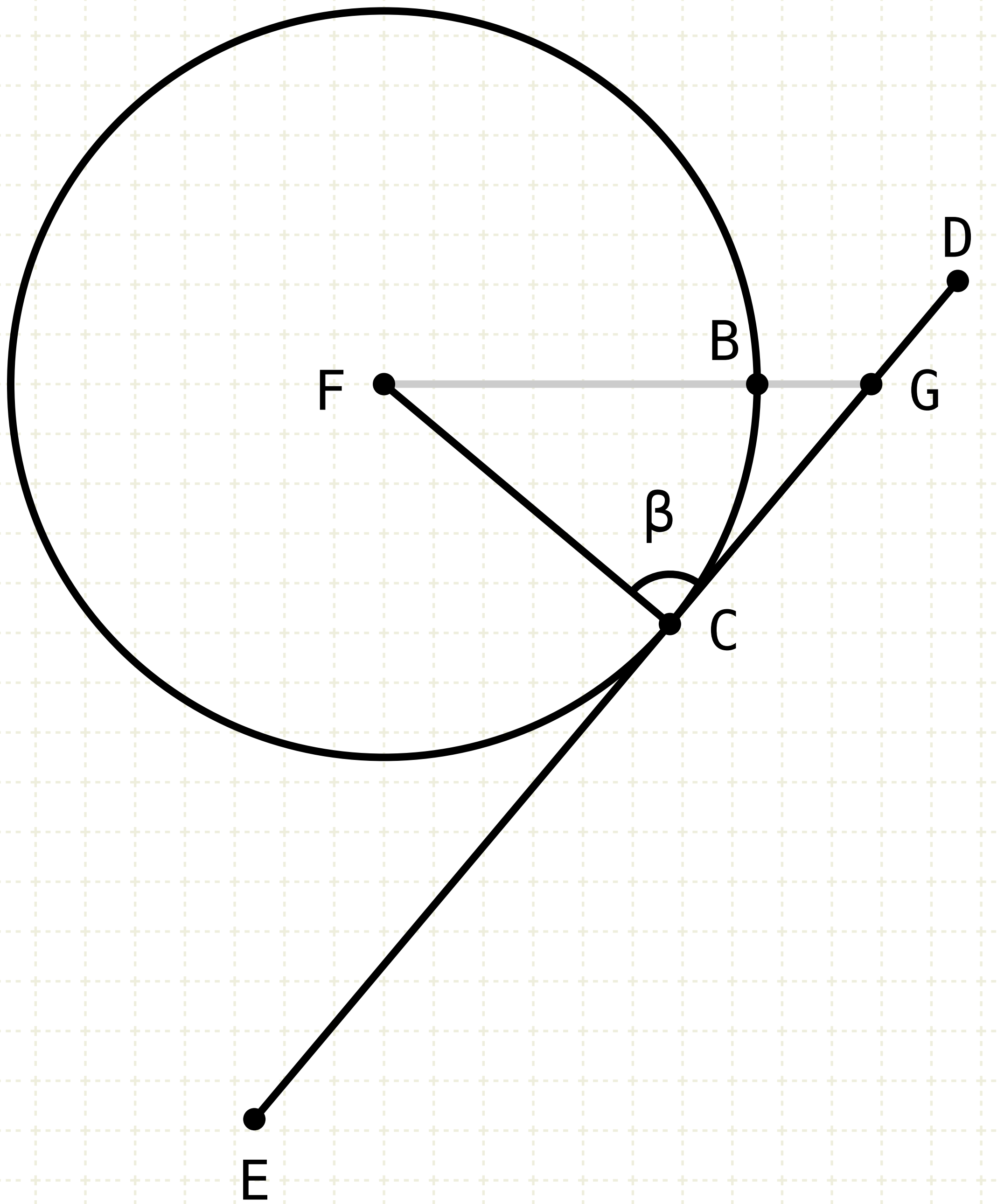

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$\therefore \beta = \llcorner$

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Therefore the angle β must be equal to a right angle

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