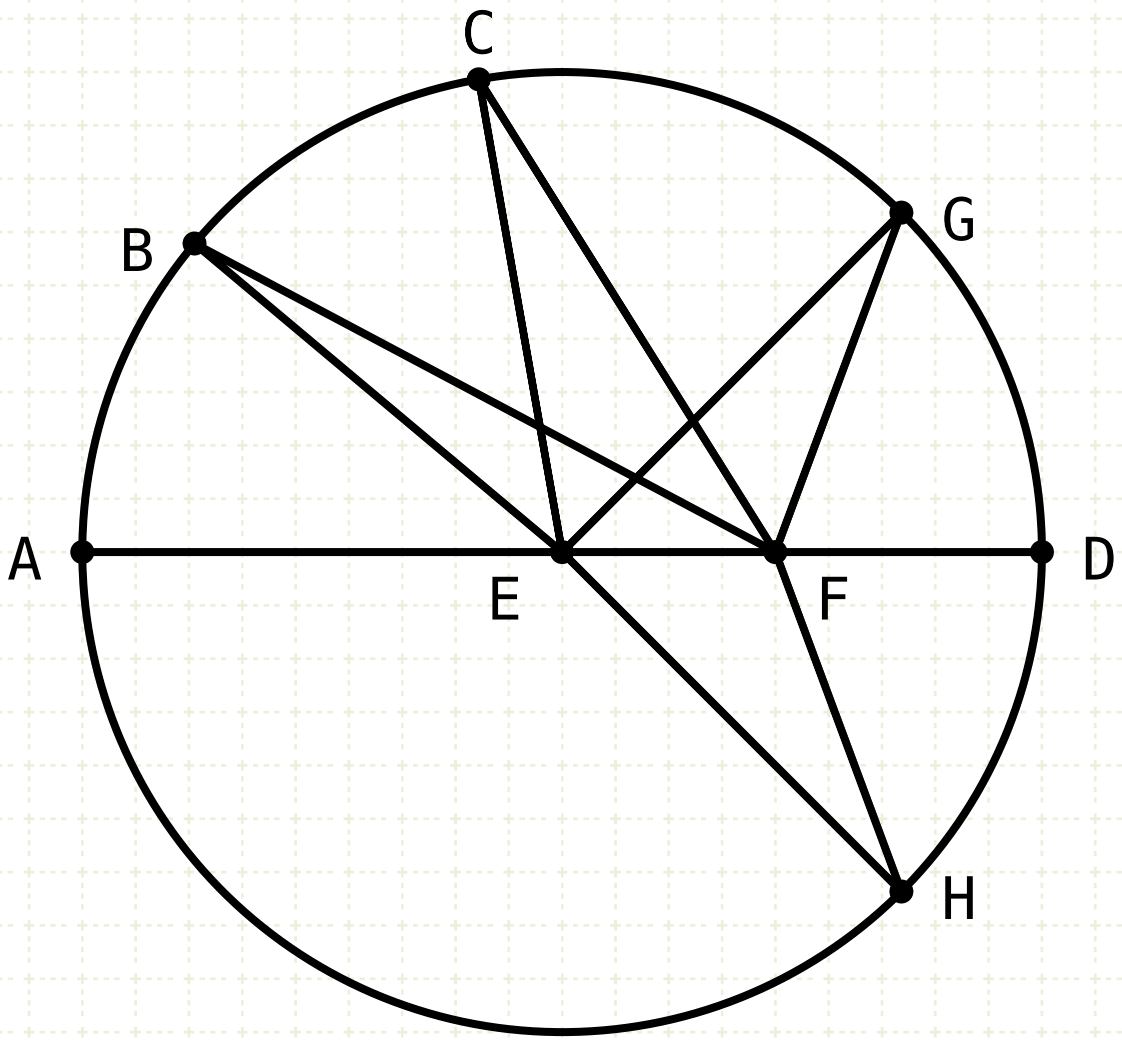

Euclid's Elements

Book III

A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)

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Proposition 3 of Book III

If in a circle a straight line through the center bisect a straight line not through the center, it also cuts it a right angles; and if it cut it at right angles it also bisects it.

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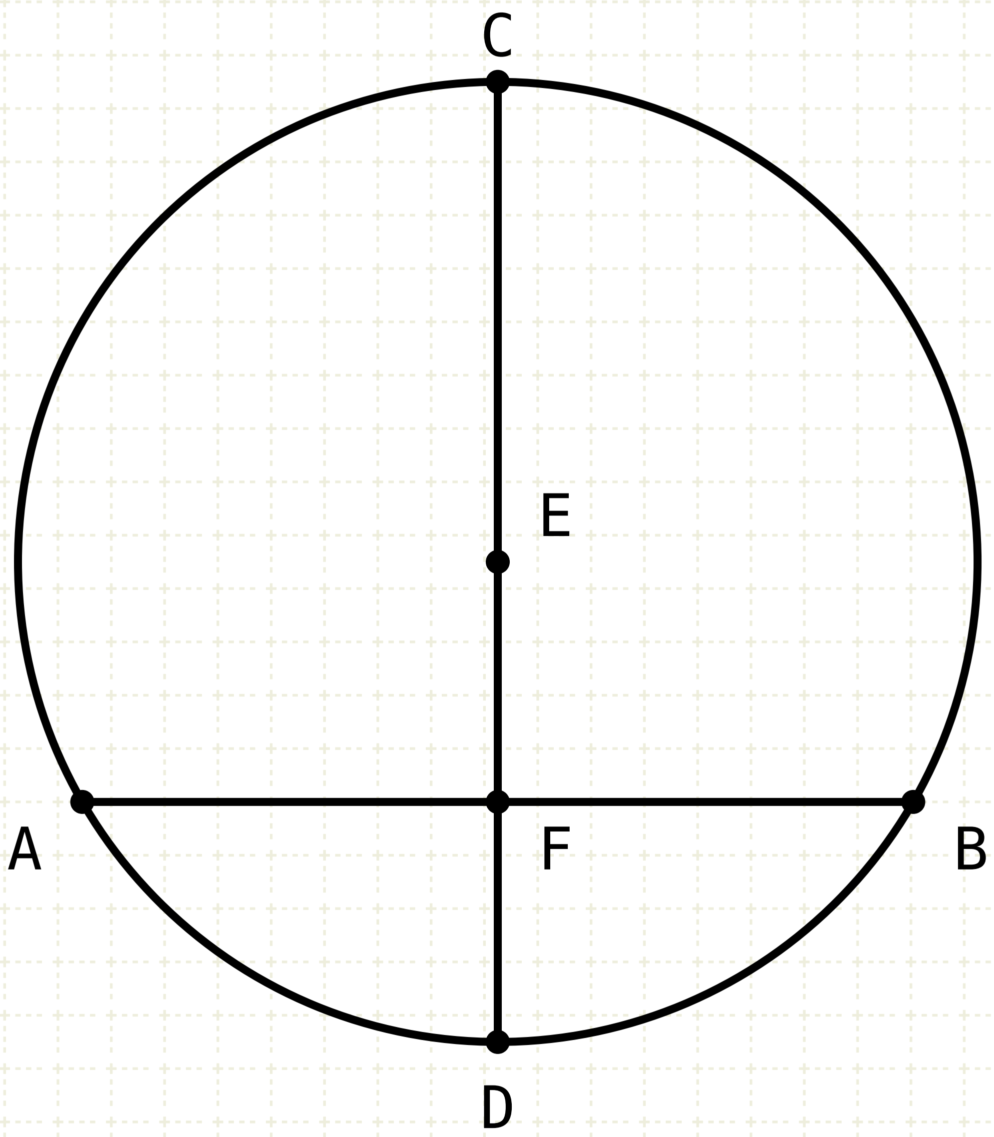

$AF = B \rightarrow \angle AFE = \llcorner$

$\angle AFE = \llcorner \rightarrow AF = FB$

In other words

Let a line CD pass through the center of the circle (E), and cut line (AB) not through the center intersecting at point F.

- If F bisects AB, then CD cuts AB at right angles
- If CD cuts AB at right angles, then F bisects AB

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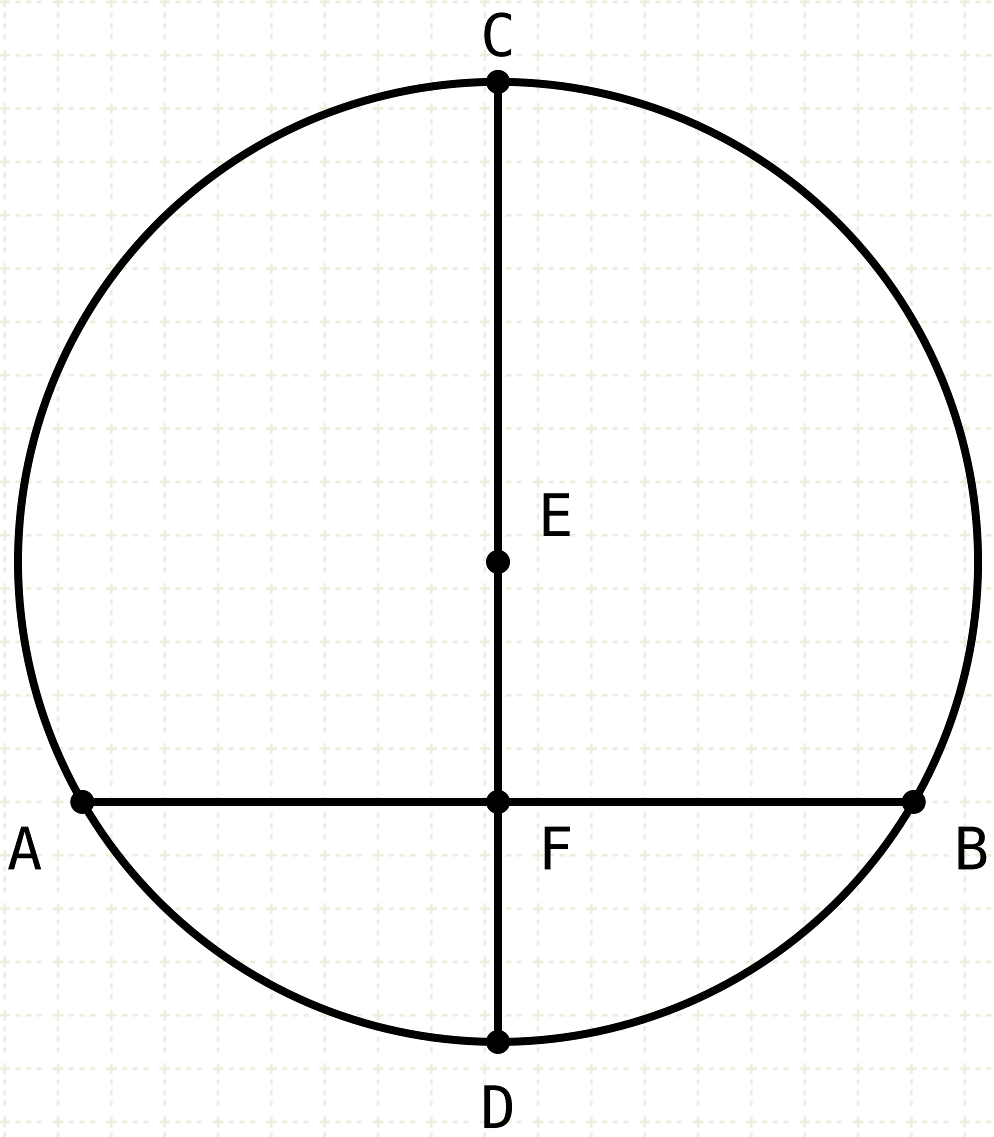

Proof

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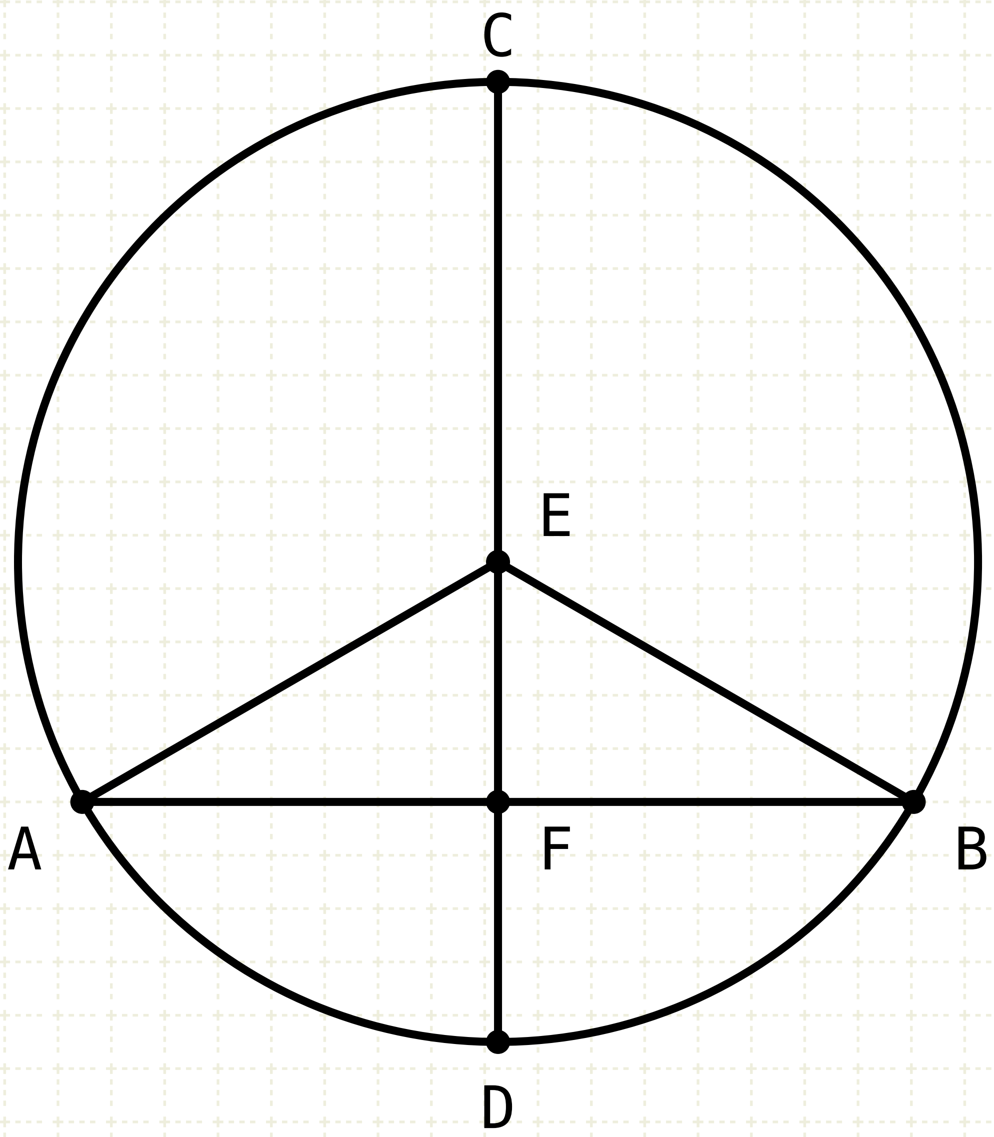

EA = EB

Proof

Join the points EA and EB. Because EA and EB are radii of the same circle they are equal.

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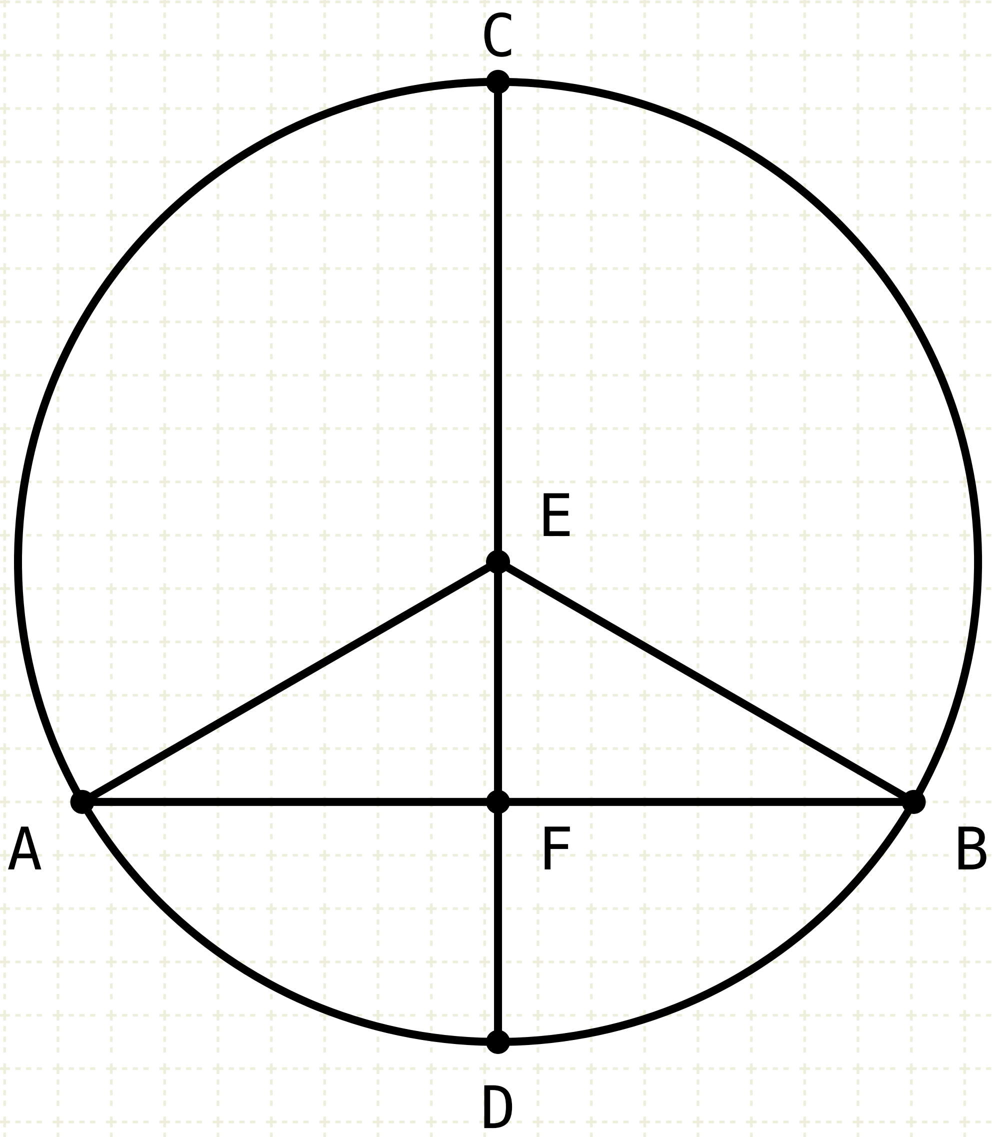

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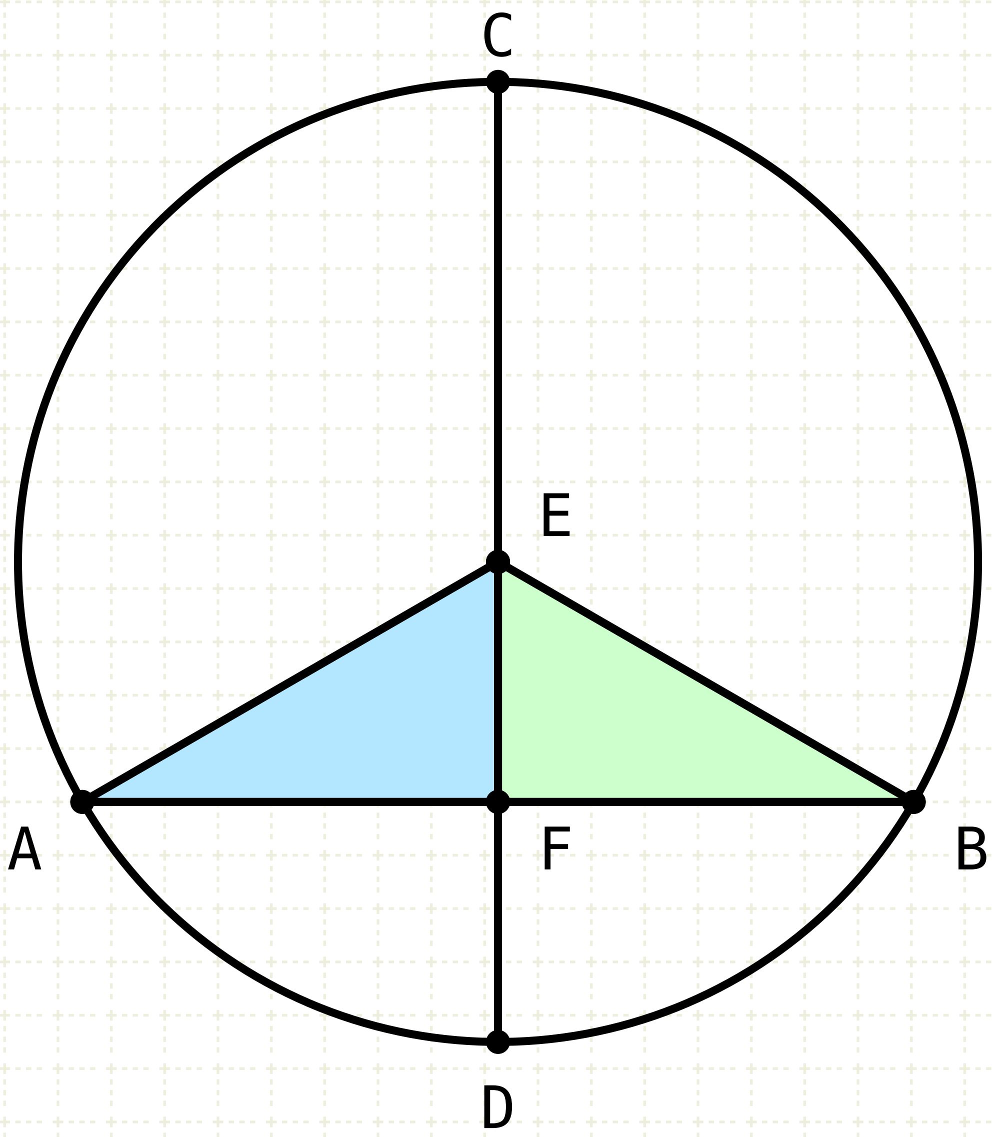

$EA = EB$

If F bisects AB

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$\Delta EAF \equiv \Delta EFB$

Proof

Join the points EA and EB. Because EA and EB are radii of the same circle they are equal.

If F bisects AB, AF and FB are equal

Since AF,FB are equal, EA,EB are equal, and EF is common, we have a two triangles with three equal sides, thus the two triangles are equal (I·8)

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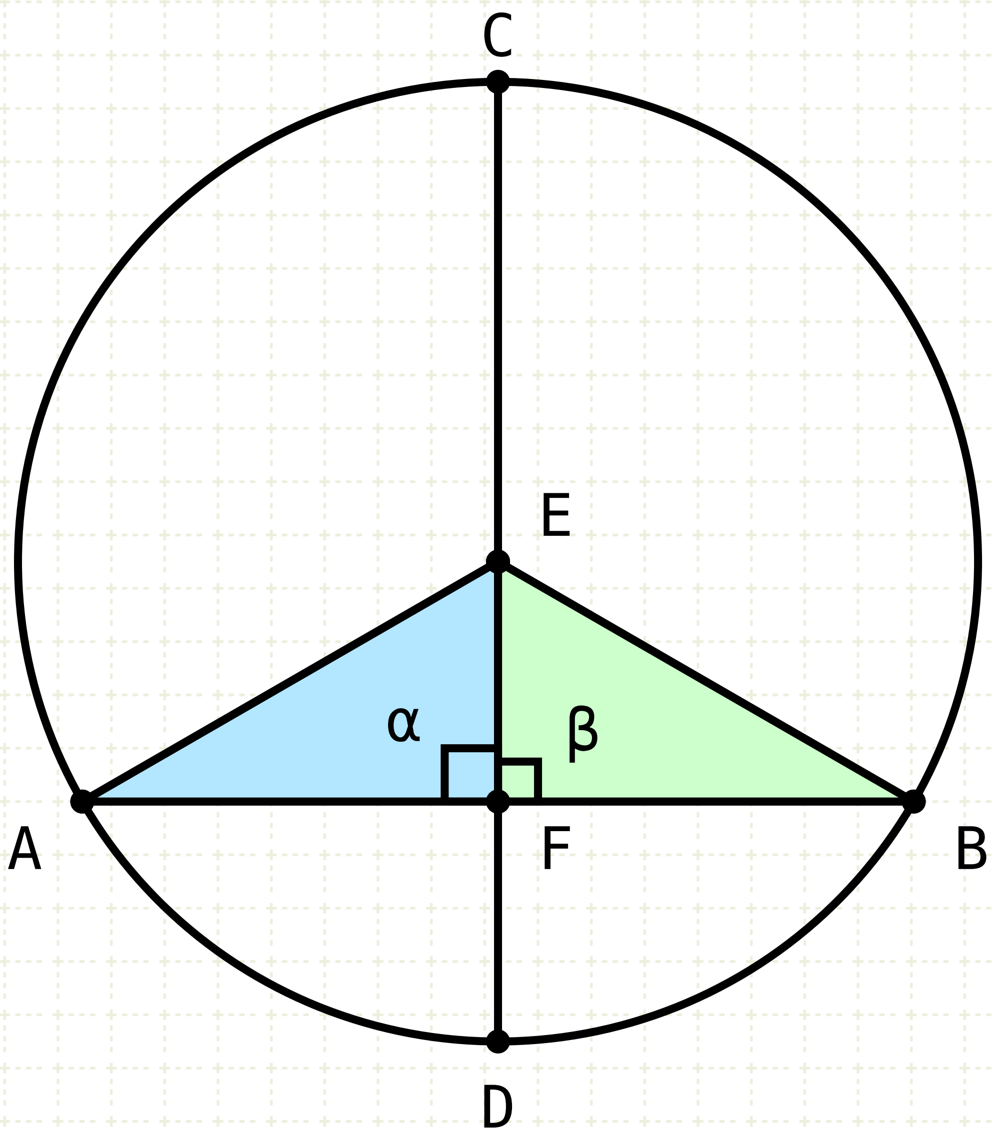

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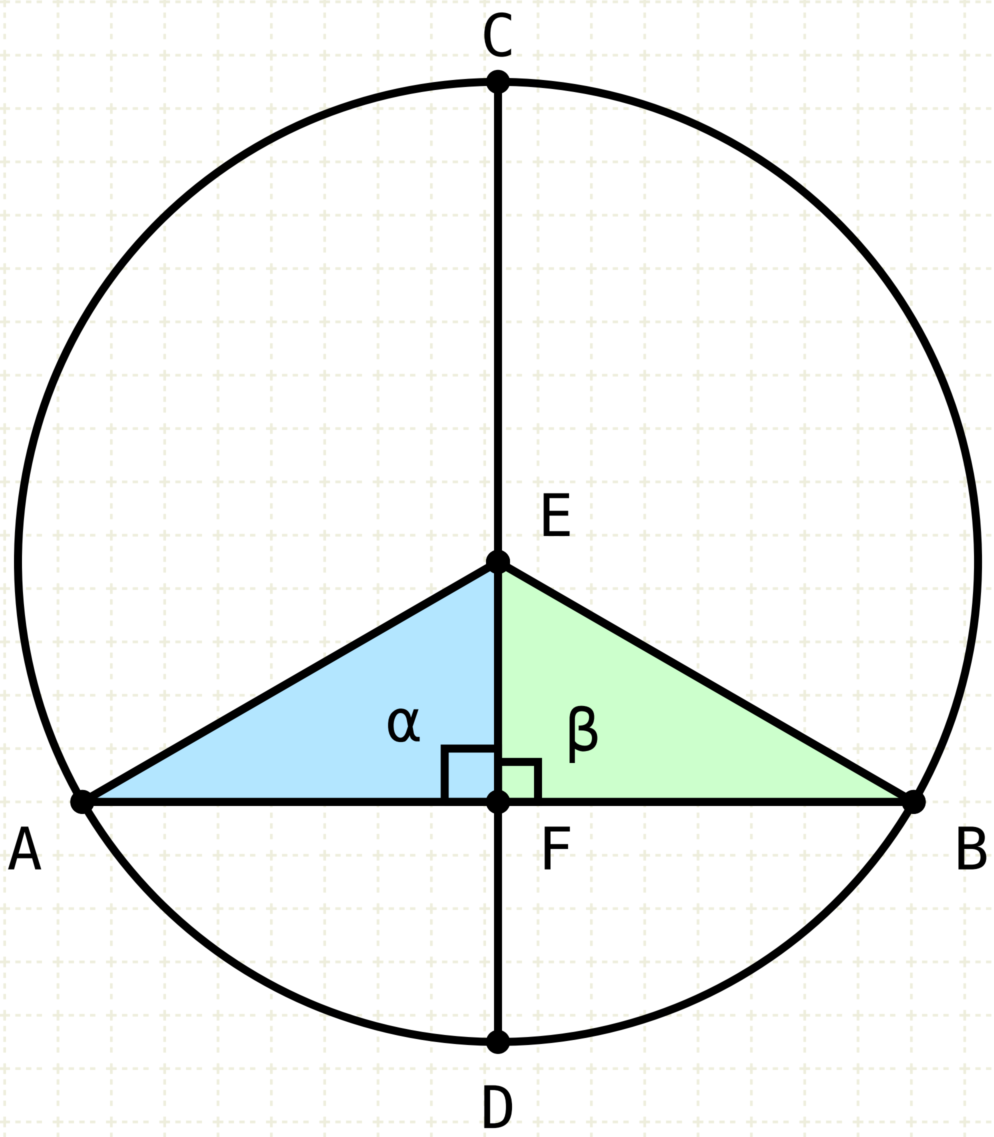

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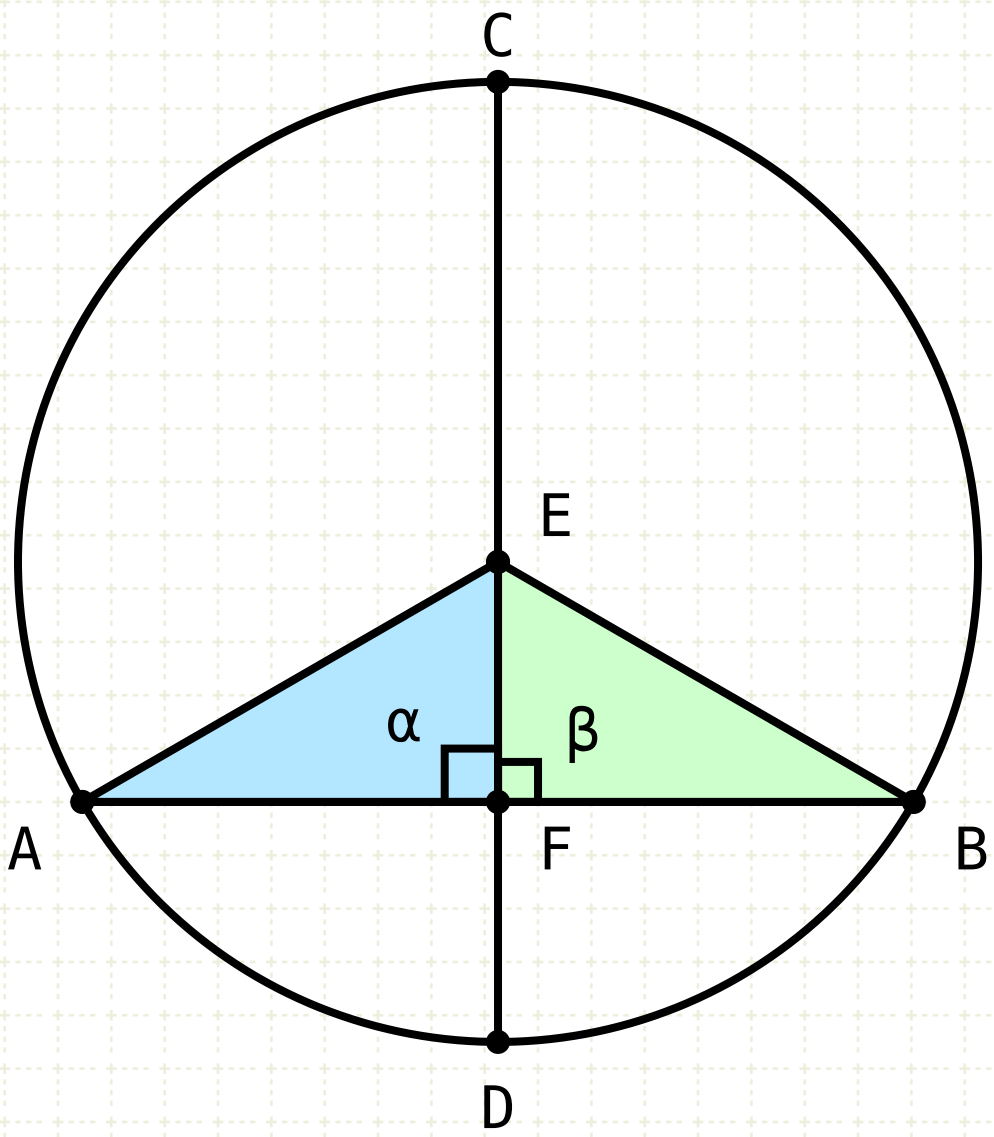

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ΔEAF ≡ ΔEFB

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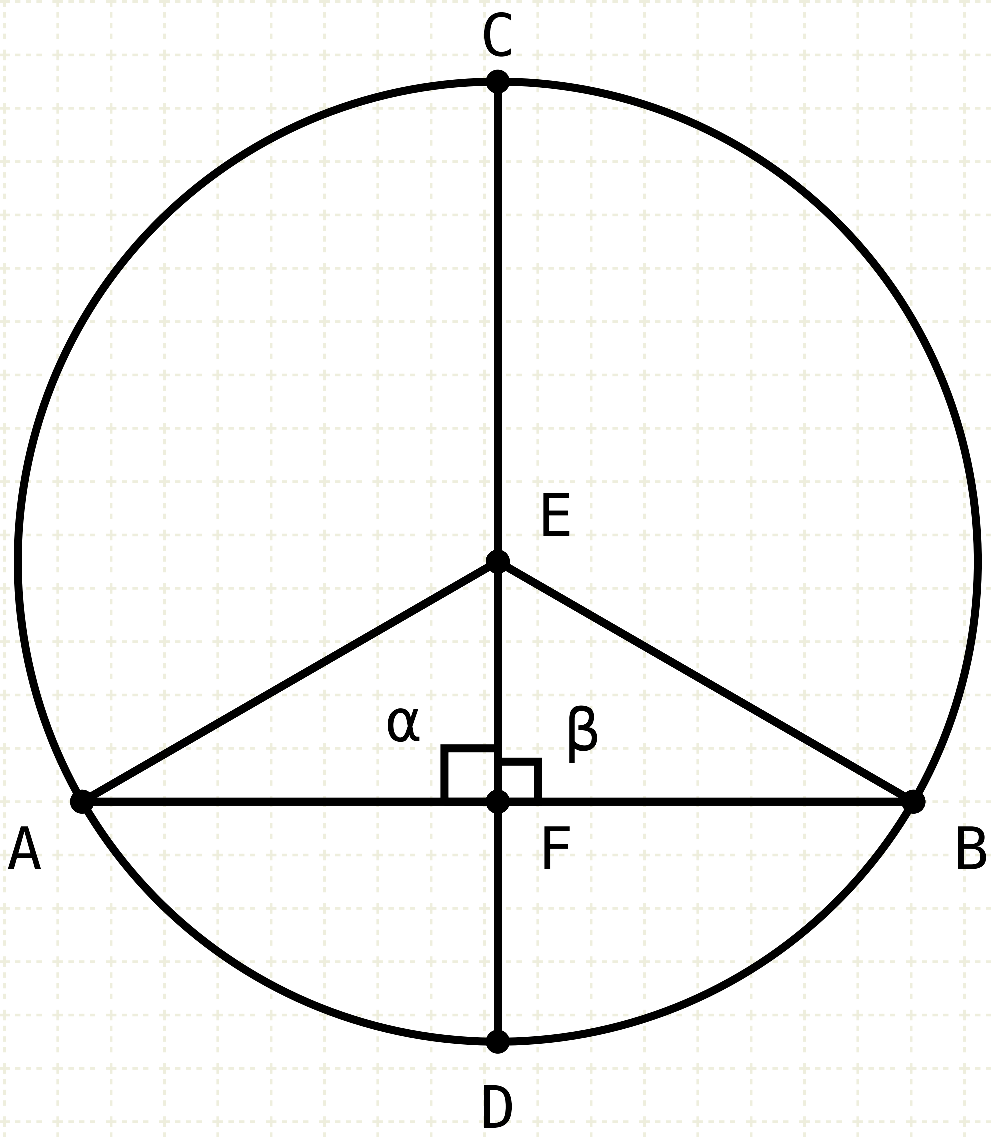

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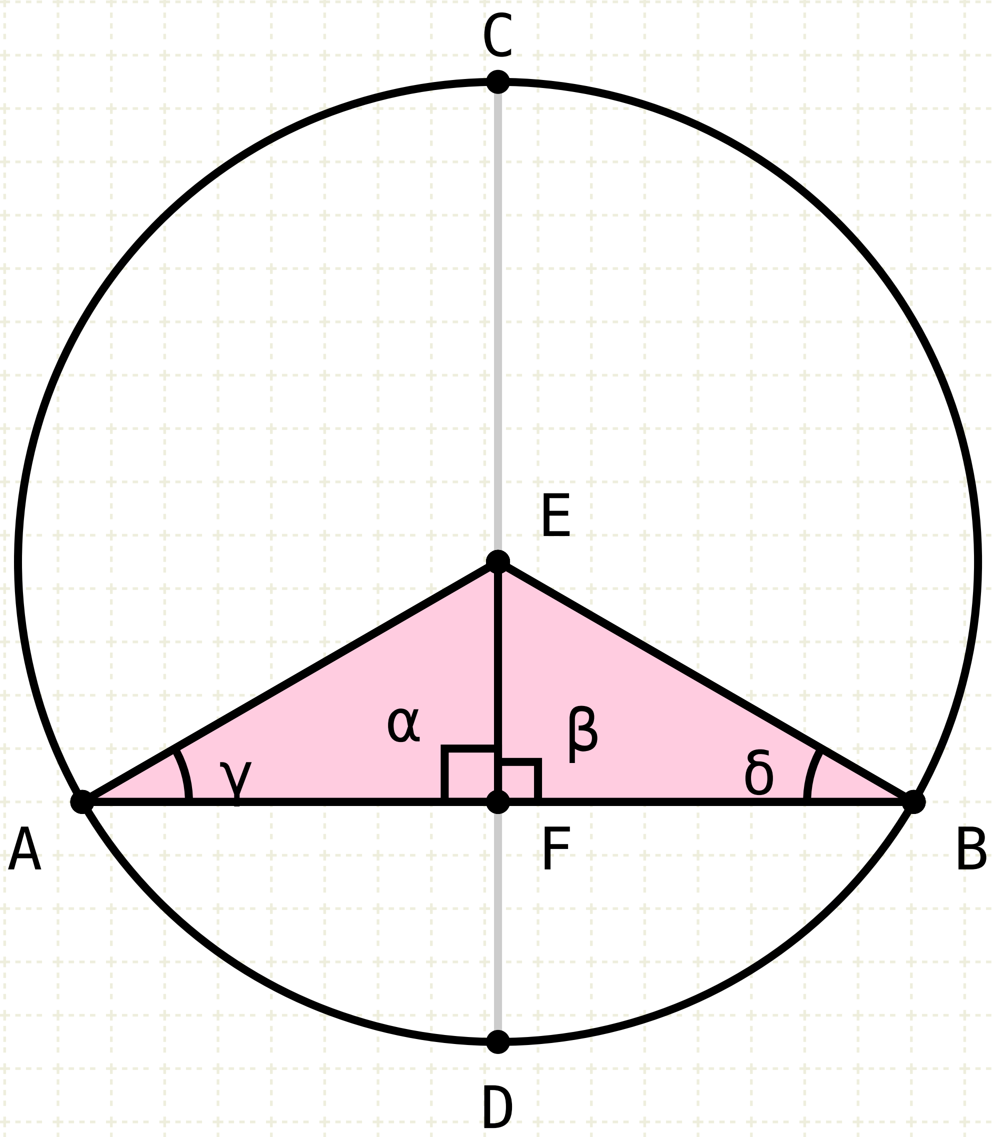

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$\gamma = \delta$

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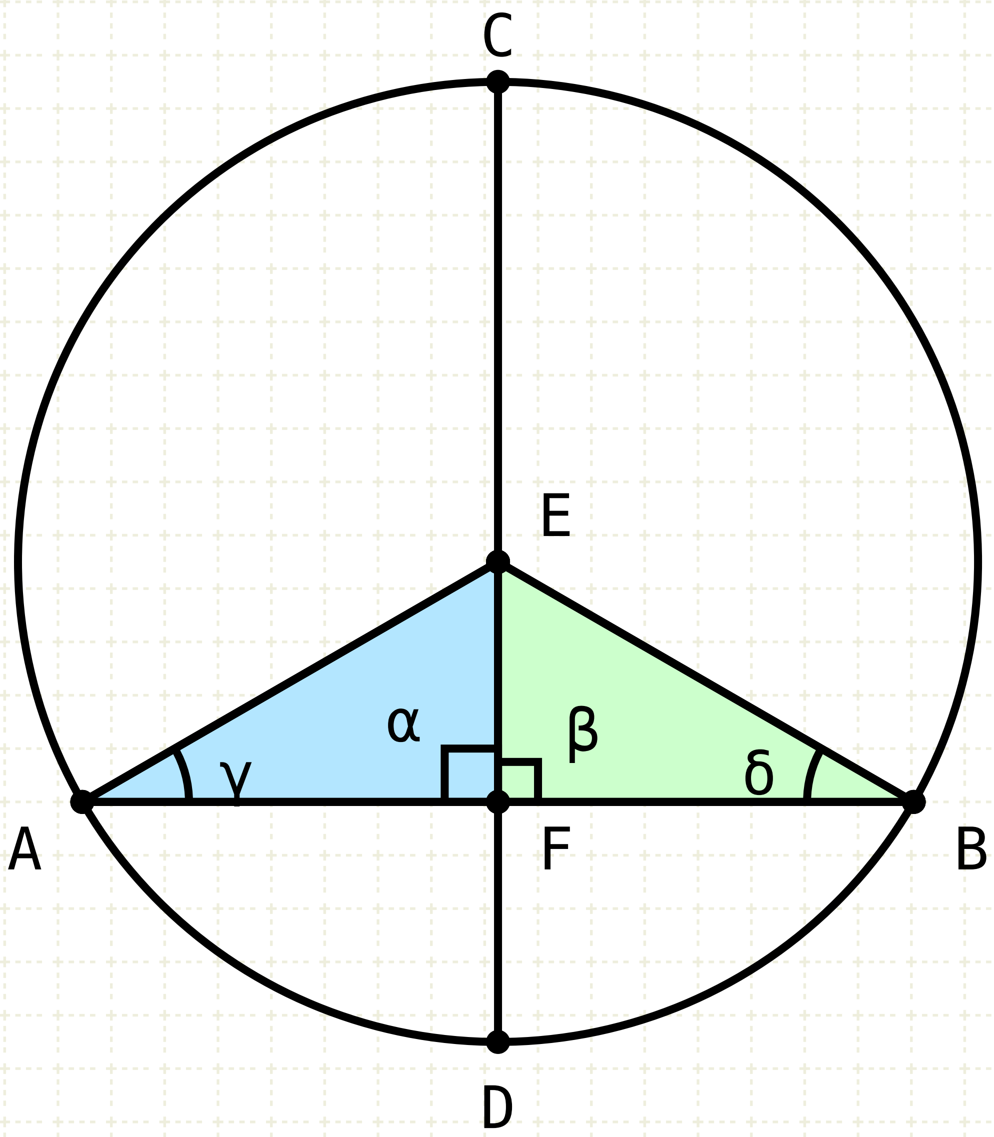

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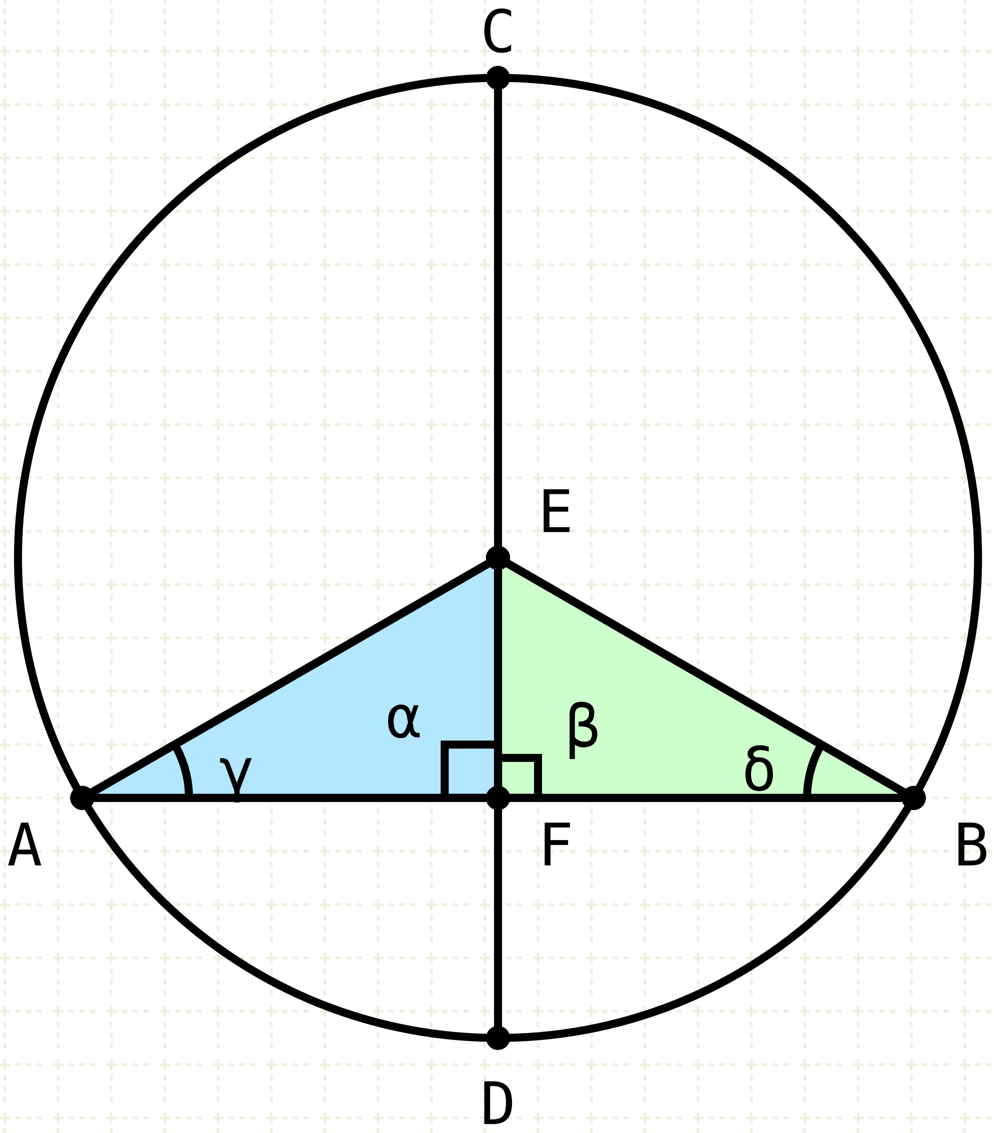

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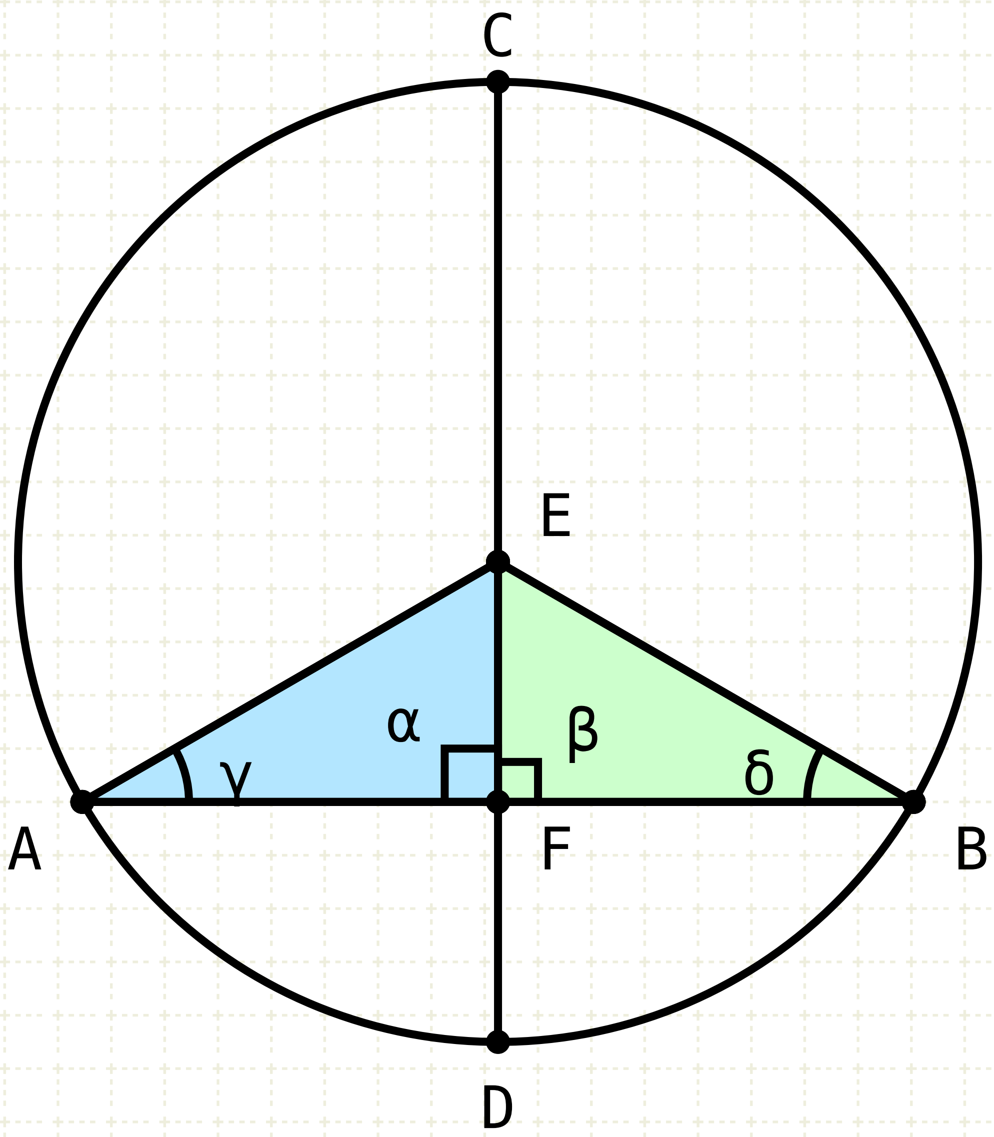

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