

# Euclid's Elements

## Book VI

*One can state, without exaggeration, that the observation of and the search for similarities and differences are the basis of all human knowledge.*

**Alfred Nobel**



# Table of Contents, Chapter 6

1	If the height of two triangles are equal, then the ratio of the areas is equal to the ratio of the bases	7	If two triangles have one angle equal to one angle, and the sides about other angles are proportional, and the remaining angles either both less or both not less than a right angle, then triangles will be equiangular	14	In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and vice versa
2	If a line cuts a triangle, parallel to its base, it will cut the sides of the triangle proportionally			15	In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and vice versa
3	If an angle of a triangle is bisected and the straight line cutting the angle also cuts the base, the segments of the base will have the same ratio as the remaining sides of the triangle	8	If in a right-angled triangle a perpendicular be drawn from the right angle to the base, the triangles adjoining the perpendicular are similar both to the whole and to one another	<b>16</b>	<b>If four straight lines are proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means, and vice versa</b>
4	If two triangles have equal angles, then the sides opposite the equal angles are proportional, as well, the sides of the triangles on either side of the equal angles are also proportional	9	From a given straight line to cut off a given fraction	17	If three straight lines are proportional, the rectangle contained by the extremes is equal to the square on the mean; and vice versa
5	If two triangles have proportional sides, the triangles will be equiangular	10	To cut a given uncut straight line similarly to a given cut straight line	18	On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure
6	If two triangles have one angle equal to one angle and the sides about the equal angles are proportional, then the triangles will be equiangular	11	To two given straight lines to find a third proportional	19	Similar triangles are to one another in the duplicate ratio of the corresponding sides
		12	To three given straight lines to find a fourth proportional		
		13	To two given straight lines to find a mean proportional		



## Table of Contents, Chapter 3

- |    |  |    |   |    |   |
|----|--|----|---|----|---|
| 20 | Similar polygons are divided into the same number of similar triangles, which have the same ratio as the wholes, and the polygons have duplicate ratios to their corresponding sides | 26 | If from a parallelogram a similar parallelogram with a common angle is subtracted, it is about the same diameter as the original  | 31 | In right-angled triangles the figure on the side subtending the right angle is equal to the similar and similarly described figures on the sides containing the right angle |
| 21 | Figures which are similar to the same rectilineal figure are also similar to one another   | 27 | Of all the parallelograms applied to the same straight line and deficient by parallelogrammic figures similar to a parallelogram drawn on half the said line, the largest will be one that is drawn on half of the straight line and is similar to the defect |    |   |
| 22 | If four straight lines are proportional, similar rectilineal figures will also be proportional; and vice versa   | 28 | To a given straight line, apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one   |    |   |
| 23 | Equiangular parallelograms have to one another the ratio compounded of the ratios of their sides   | 29 | To a given straight line, apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one   |    |   |
| 24 | In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another  | 30 | To cut a finite straight line in extreme ratio  |    |   |
| 25 | To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure   |    |   |    |   |



## Proposition 16 of Book VI

If four straight lines be proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means;  
and, if the rectangle contained by the extremes be equal to the rectangle contained by the means, the four straight lines will be  
proportional.





# Proposition 16 of Book VI

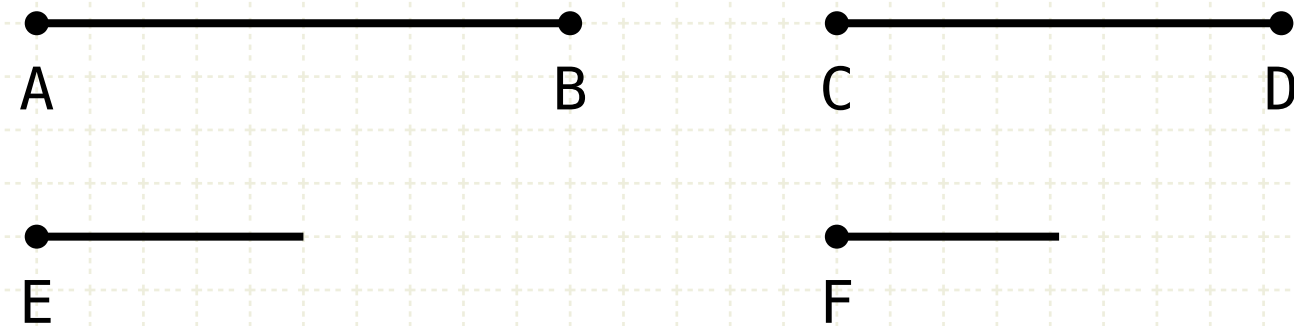
If four straight lines be proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means;  
and, if the rectangle contained by the extremes be equal to the rectangle contained by the means, the four straight lines will be  
proportional.

## In other words

Given four lines AB, CD, E and F that are proportional, AB is to CD as E is to F

Then the product of AB, F is equal to the product CD, E

And the inverse



$$AB:CD = E:F$$

$$\rightarrow AB \cdot F = CD \cdot E$$

$$AB \cdot F = E \cdot CD$$

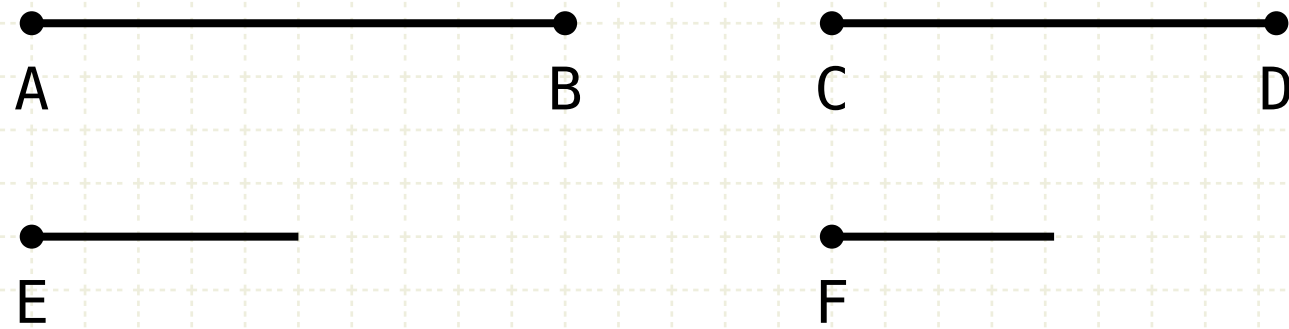
$$\rightarrow AB:CD = E:F$$



## Proposition 16 of Book VI

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and, if the rectangle contained by the extremes be equal to the rectangle contained by the means, the four straight lines will be  
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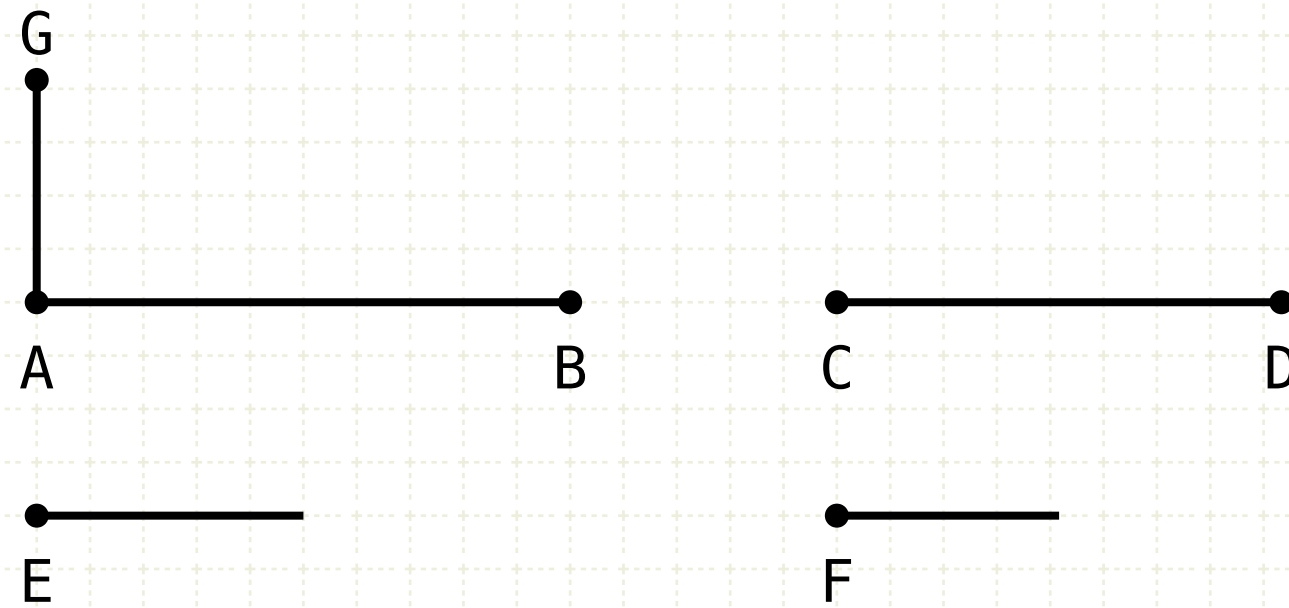
### Proof (Part 1)



$$AB:CD = E:F$$

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If four straight lines be proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means;  
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$$AB:CD = E:F$$

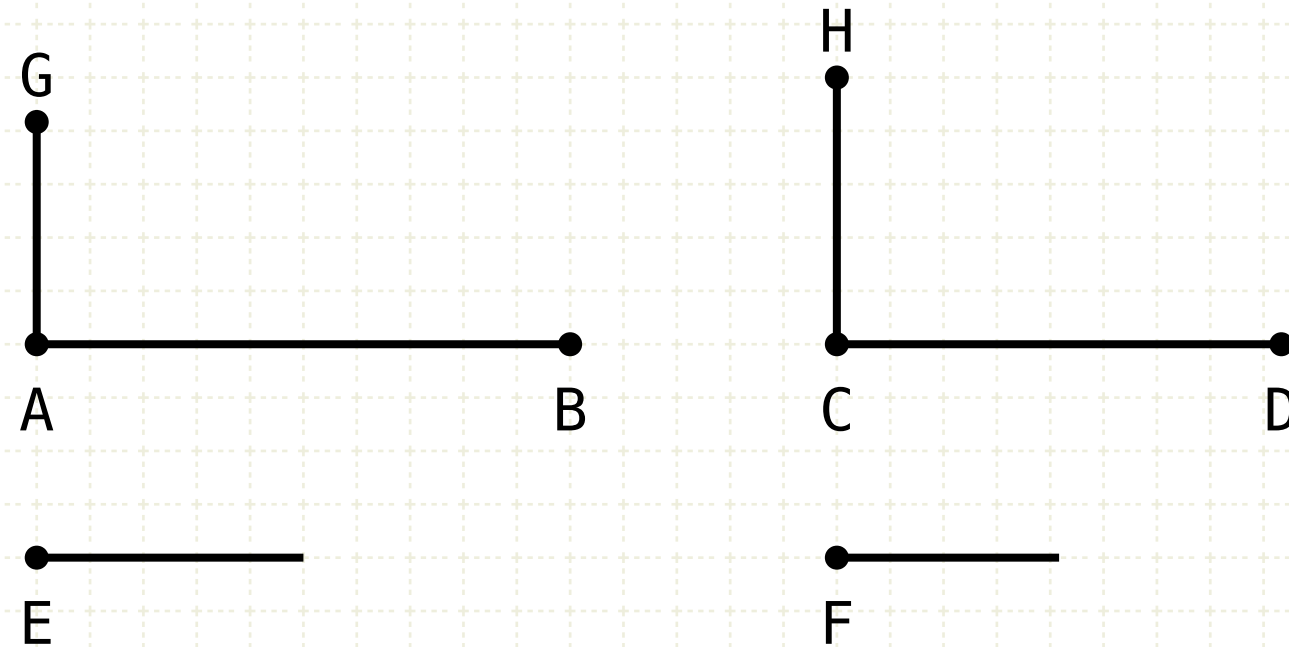
$$AG = F, \quad AG \perp AB$$

## Proof (Part 1)

Copy the line F to line AB, perpendicular to line AB

# Proposition 16 of Book VI

If four straight lines be proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means;  
and, if the rectangle contained by the extremes be equal to the rectangle contained by the means, the four straight lines will be  
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$$AB:CD = E:F$$

$$AG = F, \quad AG \perp AB$$

$$CH = E, \quad CH \perp CD$$

## Proof (Part 1)

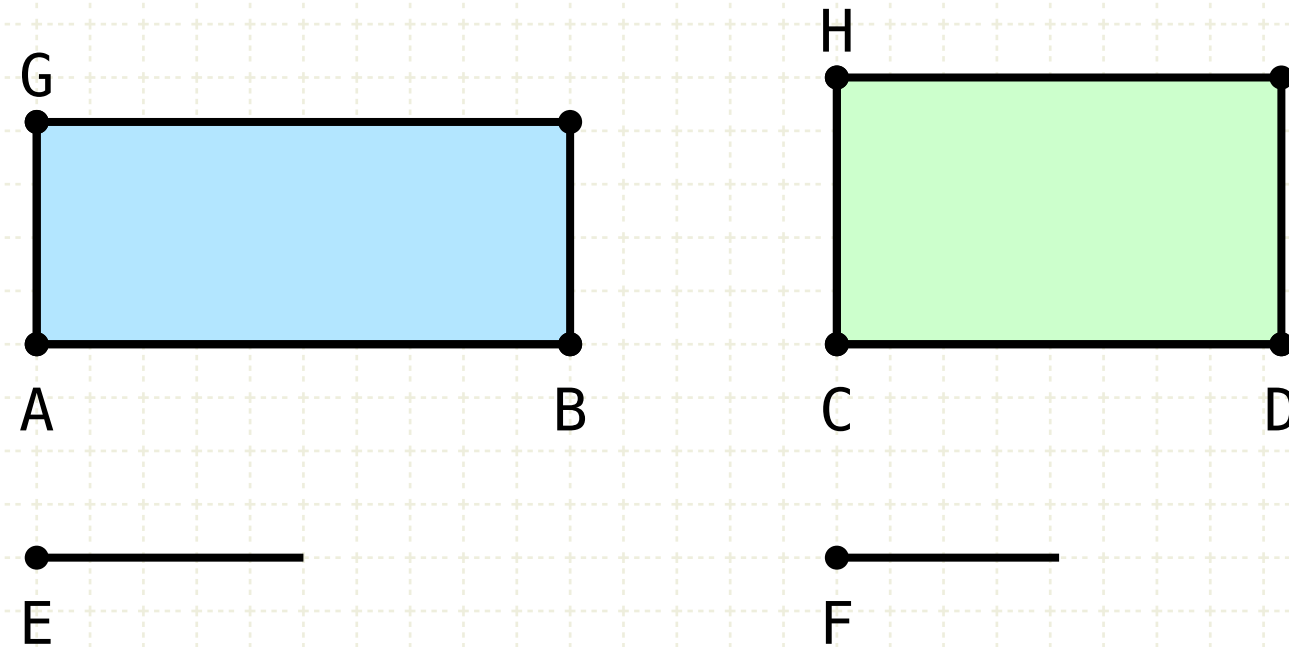
Copy the line F to line AB, perpendicular to line AB

Copy the line E to line CD, perpendicular to line CD



## Proposition 16 of Book VI

If four straight lines be proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means;  
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### Proof (Part 1)

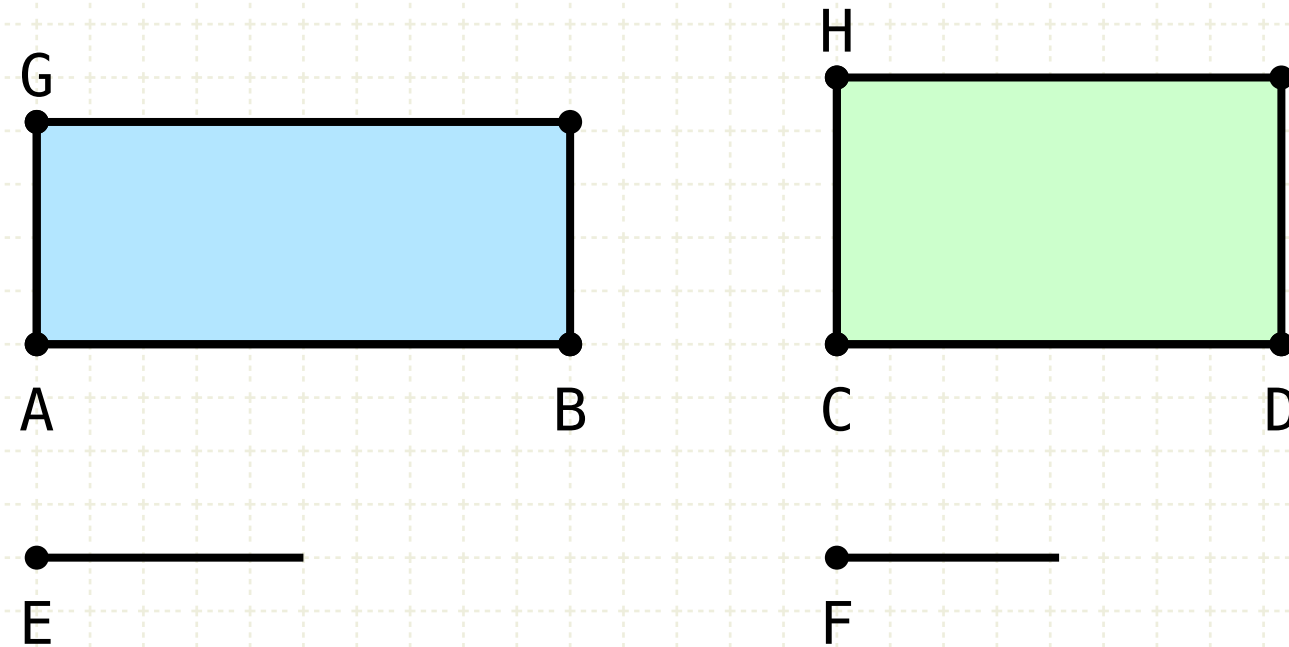
Copy the line F to line AB, perpendicular to line AB

Copy the line E to line CD, perpendicular to line CD

Finish the rectangles

## Proposition 16 of Book VI

If four straight lines be proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means;  
and, if the rectangle contained by the extremes be equal to the rectangle contained by the means, the four straight lines will be  
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$$AB:CD = E:F$$

$$AG = F, \quad AG \perp AB$$

$$CH = E, \quad CH \perp CD$$

$$AB:CD = CH:AG$$

### Proof (Part 1)

Copy the line F to line AB, perpendicular to line AB

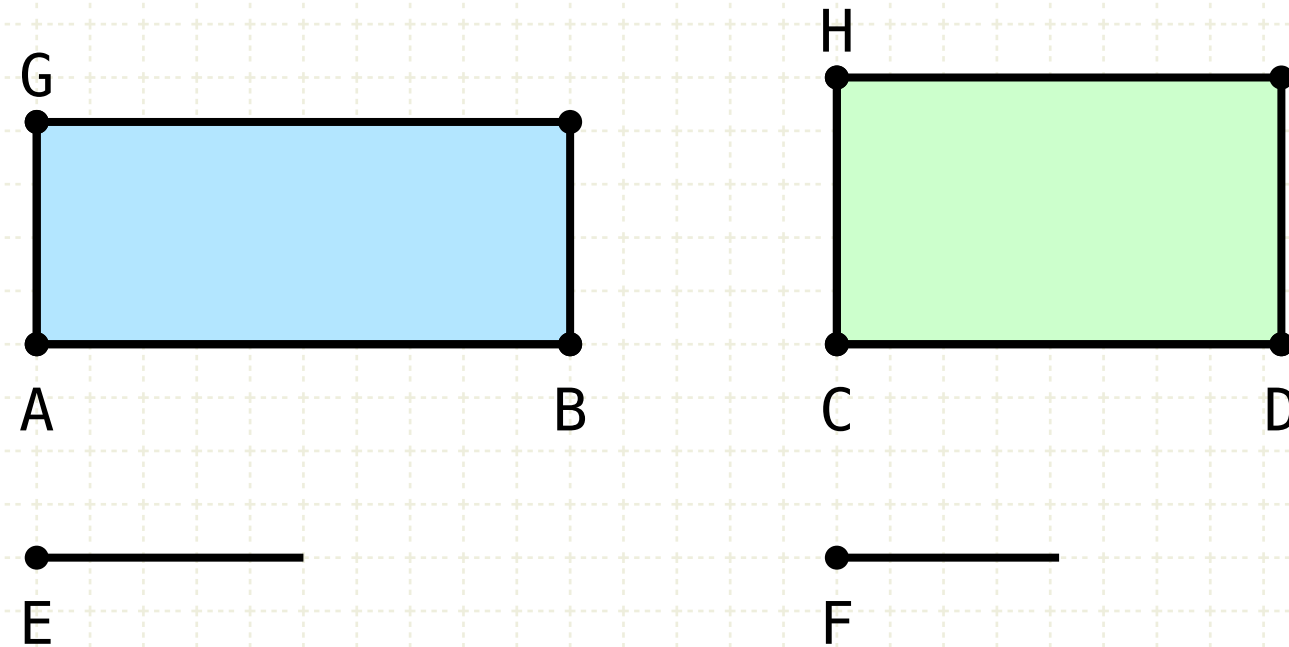
Copy the line E to line CD, perpendicular to line CD

Finish the rectangles

Thus the sides of the rectangles (parallelograms) are inversely  
proportional

## Proposition 16 of Book VI

If four straight lines be proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means;  
and, if the rectangle contained by the extremes be equal to the rectangle contained by the means, the four straight lines will be proportional.



$$AB:CD = E:F$$

$$AG = F, AG \perp AB$$

$$CH = E, CH \perp CD$$

$$AB:CD = CH:AG$$

$$\square BG = \square DH$$

### Proof (Part 1)

Copy the line F to line AB, perpendicular to line AB

Copy the line E to line CD, perpendicular to line CD

Finish the rectangles

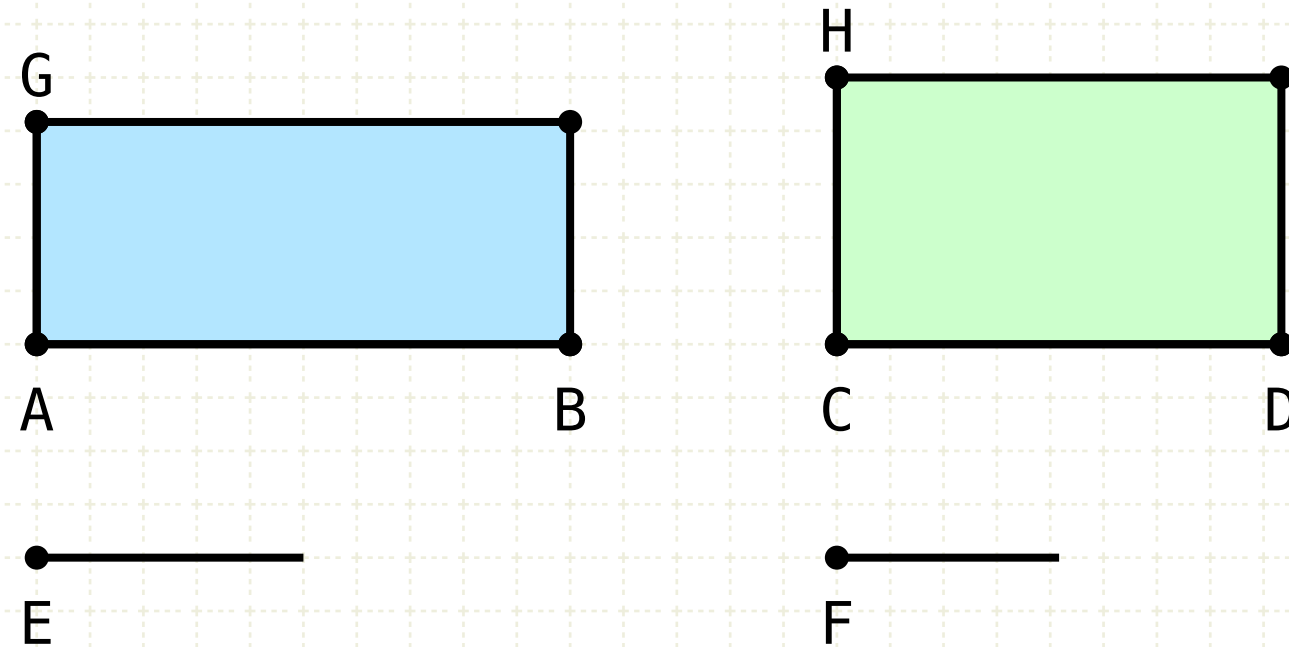
Thus the sides of the rectangles (parallelograms) are inversely proportional

Rectangles BG and DH are equiangular parallelograms where the sides are reciprocally proportional around the equal angle

Therefore, the rectangles BG and DH are equal (VI·14)

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If four straight lines be proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means;  
and, if the rectangle contained by the extremes be equal to the rectangle contained by the means, the four straight lines will be proportional.



$$AB:CD = E:F$$

$$AG = F, \quad AG \perp AB$$

$$CH = E, \quad CH \perp CD$$

$$AB:CD = CH:AG$$

$$\square BG = \square DH$$

$$\square BG = AB \times F$$

$$\square DH = CD \times E$$

### Proof (Part 1)

Copy the line F to line AB, perpendicular to line AB

Copy the line E to line CD, perpendicular to line CD

Finish the rectangles

Thus the sides of the rectangles (parallelograms) are inversely proportional

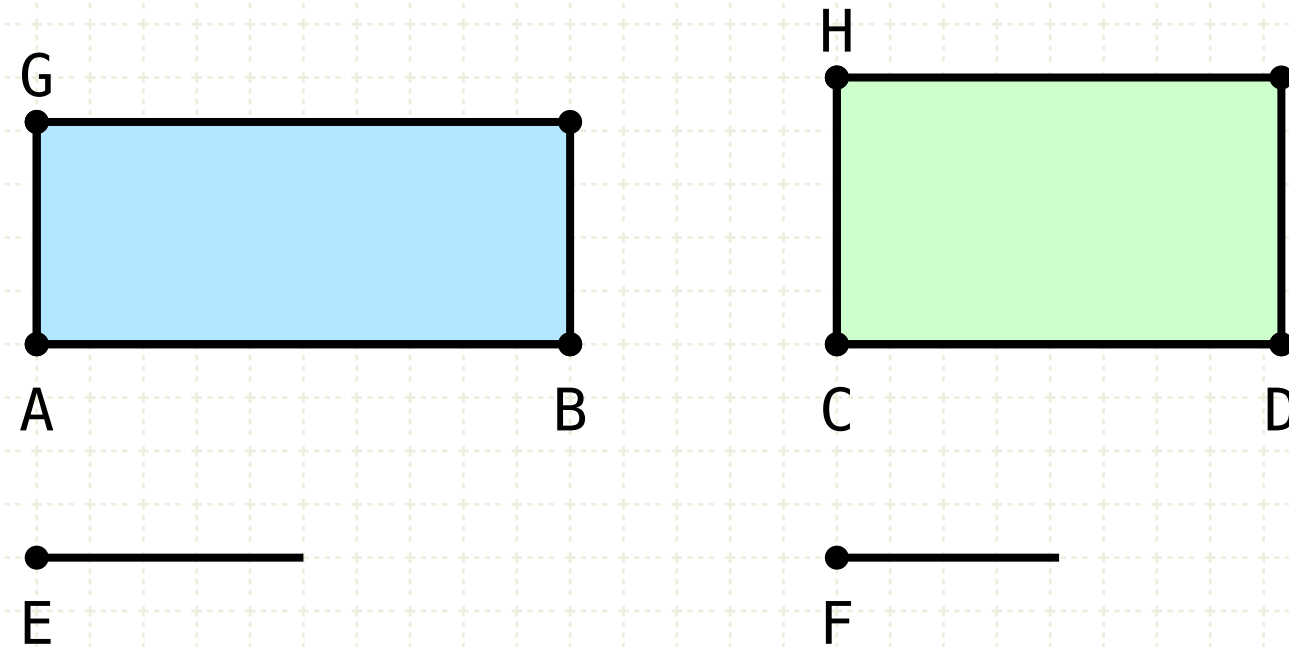
Rectangles BG and DH are equiangular parallelograms where the sides are reciprocally proportional around the equal angle

Therefore, the rectangles BG and DH are equal (VI·14)

And since AG equals F, and CH equals E, then the rectangles BG and DH are equal to the rectangles formed from AB, F and CD, E

## Proposition 16 of Book VI

If four straight lines be proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means;  
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$$AB:CD = E:F$$

$$AG = F, \quad AG \perp AB$$

$$CH = E, \quad CH \perp CD$$

$$AB:CD = CH:AG$$

$$\square BG = \square DH$$

$$\square BG = AB \times F$$

$$\square DH = CD \times E$$

$$AB \times F = CD \times E$$

### Proof (Part 1)

Copy the line F to line AB, perpendicular to line AB

Copy the line E to line CD, perpendicular to line CD

Finish the rectangles

Thus the sides of the rectangles (parallelograms) are inversely proportional

Rectangles BG and DH are equiangular parallelograms where the sides are reciprocally proportional around the equal angle

Therefore, the rectangles BG and DH are equal (VI·14)

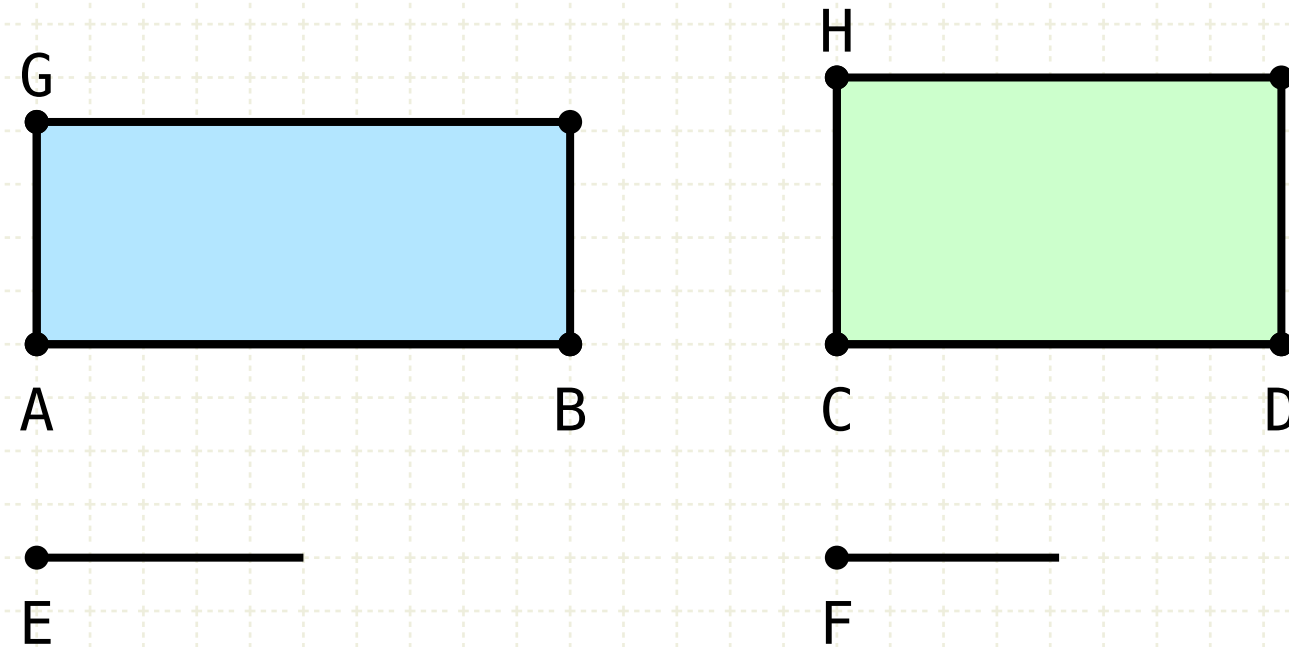
And since AG equals F, and CH equals E, then the rectangles BG and DH are equal to the rectangles formed from AB, F and CD, E

Thus, the rectangle formed by AB, F is equal the rectangle formed by CD, E



## Proposition 16 of Book VI

If four straight lines be proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means;  
and, if the rectangle contained by the extremes be equal to the rectangle contained by the means, the four straight lines will be  
proportional.



$$AB \times F = CD \times E$$

$$AG = F, \quad AG \perp AB$$

$$CH = E, \quad CH \perp CD$$

### Proof (Part 2)

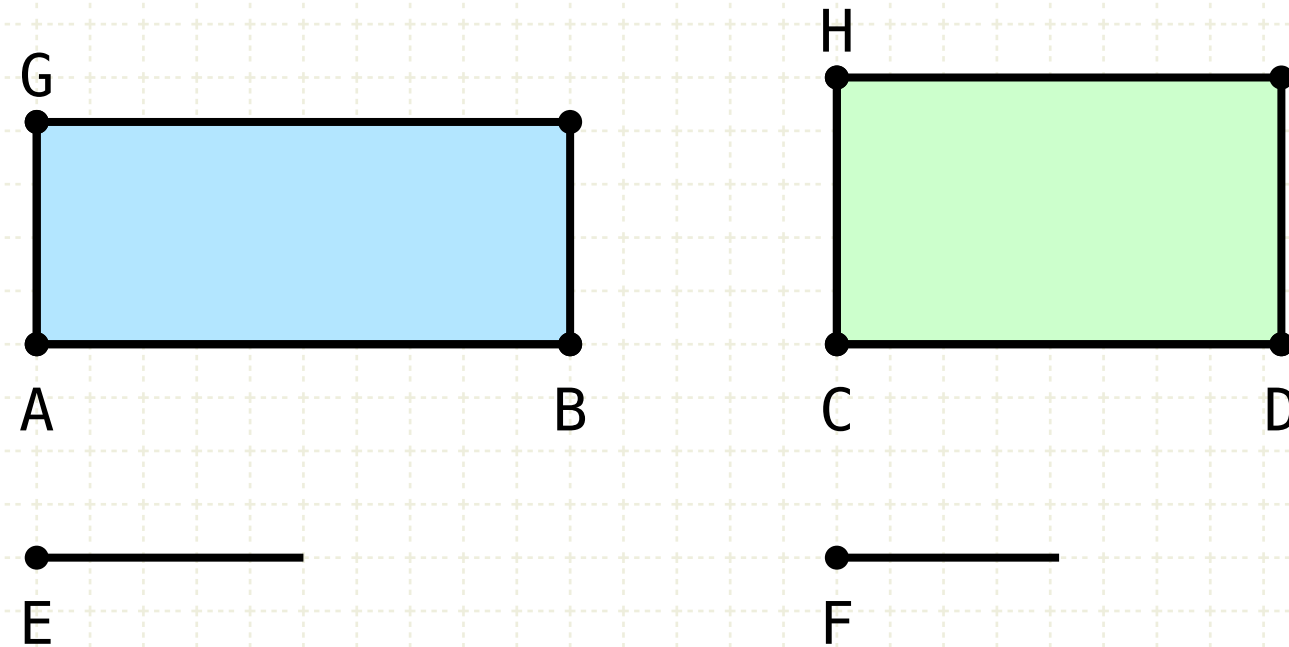
Copy the line F to line AB, perpendicular to line AB

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Finish the rectangles

## Proposition 16 of Book VI

If four straight lines be proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means;  
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$$AB \times F = CD \times E$$

$$AG = F, \quad AG \perp AB$$

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$$\square BG = AB \times F$$

$$\square DH = CD \times E$$

$$\square BG = \square DH$$

### Proof (Part 2)

Copy the line F to line AB, perpendicular to line AB

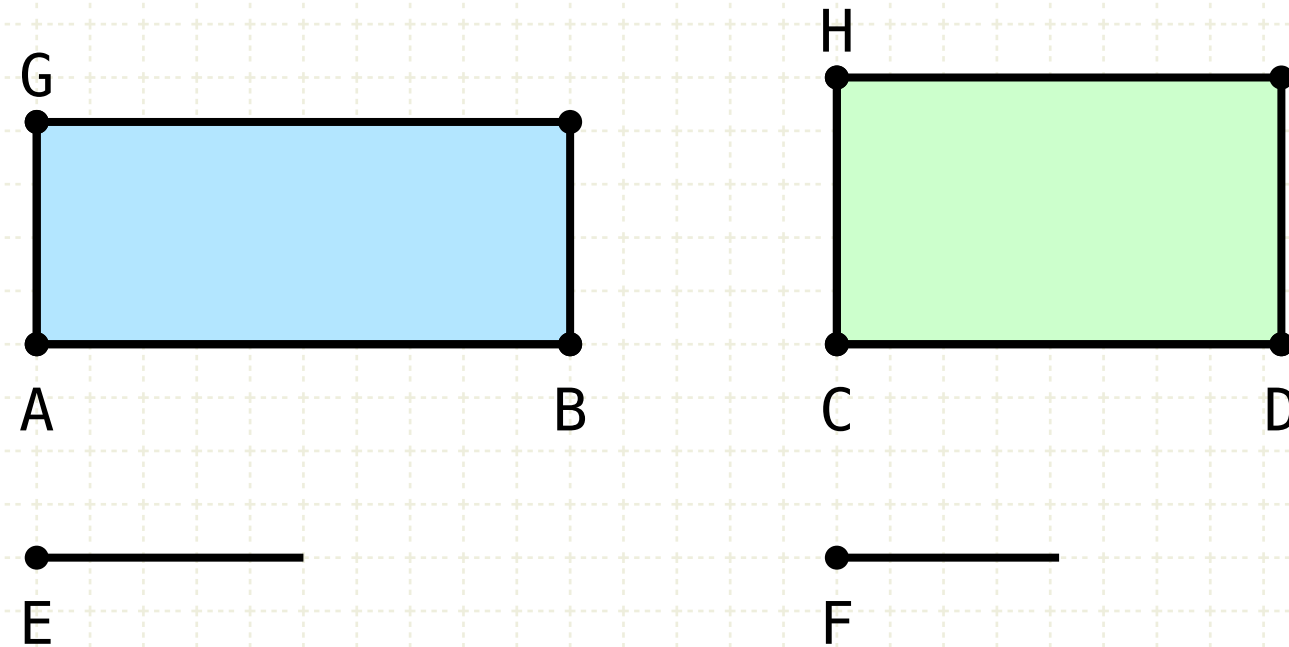
Copy the line E to line CD, perpendicular to line CD

Finish the rectangles

Since AG, F are equal, and CH, E are equal, the rectangles BG and DH are equiangular, the areas are also equal

## Proposition 16 of Book VI

If four straight lines be proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means;  
and, if the rectangle contained by the extremes be equal to the rectangle contained by the means, the four straight lines will be proportional.



$$AB \times F = CD \times E$$

$$AG = F, \quad AG \perp AB$$

$$CH = E, \quad CH \perp CD$$

$$\square BG = AB \times F$$

$$\square DH = CD \times E$$

$$\square BG = \square DH$$

$$AB : CD = CH : AG$$

### Proof (Part 2)

Copy the line F to line AB, perpendicular to line AB

Copy the line E to line CD, perpendicular to line CD

Finish the rectangles

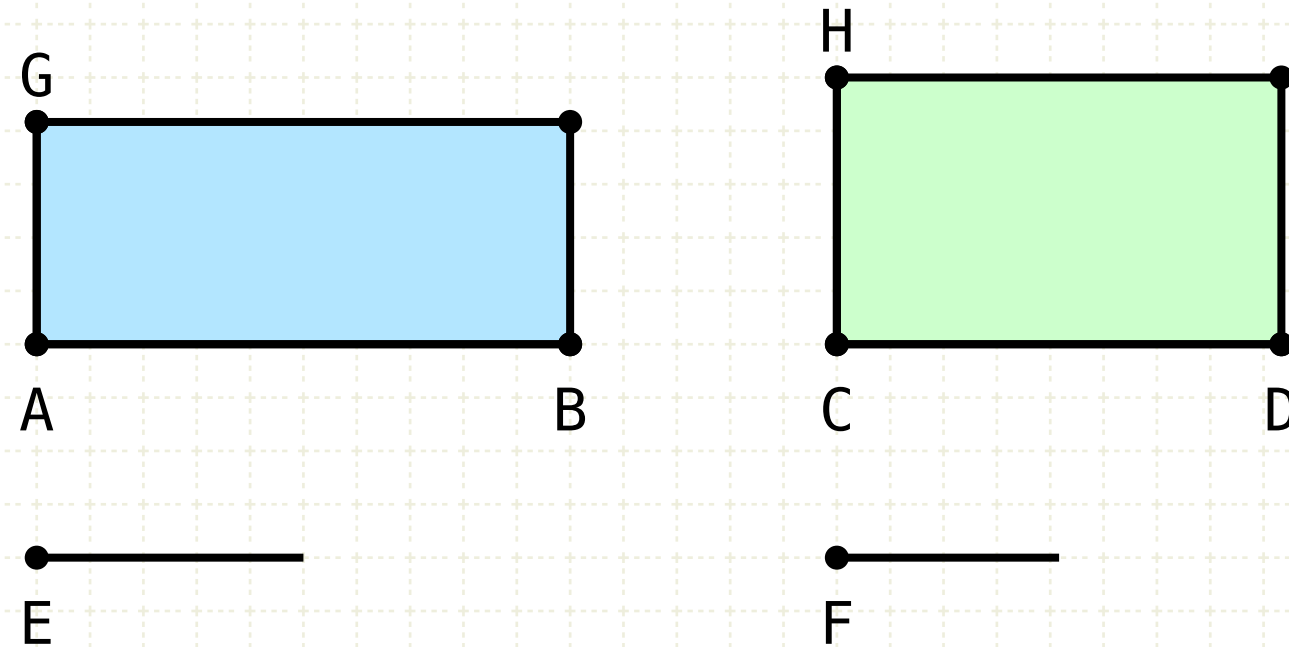
Since AG, F are equal, and CH, E are equal, the rectangles BG and DH are equiangular, the areas are also equal

Equal and equiangular parallelograms the sides about the equal are reciprocally proportional (VI·14)

Therefore AB is to CD as CH is to AG

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If four straight lines be proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means;  
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$$AG = F, \quad AG \perp AB$$

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$$\square BG = AB \times F$$

$$\square DH = CD \times E$$

$$\square BG = \square DH$$

$$AB : CD = CH : AG$$

$$AB : CD = E : F$$

## Proof (Part 2)

Copy the line F to line AB, perpendicular to line AB

Copy the line E to line CD, perpendicular to line CD

Finish the rectangles

Since AG, F are equal, and CH, E are equal, the rectangles BG and DH are equiangular, the areas are also equal

Equal and equiangular parallelograms the sides about the equal are reciprocally proportional (VI·14)

Therefore AB is to CD as CH is to AG

But CH, E are equal, and AG, F are equal, therefore AB is to CD as E is to F

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