

# Euclid's Elements

## Book III



*A circle is a round straight line with a hole in the middle.*

**Mark Twain**

quoting a schoolchild in "-English as She Is Taught-"

*If people stand in a circle long enough, they'll eventually begin to dance.*

**George Carlin, Napalm and Silly Putty (2001)**



## Table of Contents, Chapter 3

|   |  |    |   |    |  |
|---|--|----|---|----|--|
| 1 | To find the centre of a circle   | 9  | If three lines, starting at a point 'A' and touching the circle, are all equal, then 'A' is the centre of the circle                                | 17 | From a given point to draw a straight line touching a given circle   |
| 2 | A chord of a circle always lies inside the circle  | 10 | A circle does not cut a circle at more points than two  | 18 | If line touches a circle, then it is perpendicular to the diameter that touches that point                         |
| 3 | A line through the centre of a circle bisects a chord, and vice versa  | 11 | Point of contact between two internal circles, and their centres, are collinear   | 19 | If line touches a circle, then the centre of the circle lies on a line perpendicular to the original               |
| 4 | A line not through the centre of a circle does not bisect a chord  | 12 | Point of contact between two external circles, and their centres, are collinear   | 20 | The angle at the centre of a circle is twice that from an angle from the circumference                             |
| 5 | If two circles cut one another, they will not have the same center   | 13 | A circle does not touch a circle at more points than one, whether it touch it internally or externally.   | 21 | In a circle the angles in the same segment are equal to one another  |
| 6 | If two circles touch one another, they will not have the same center   | 14 | In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another. | 22 | The opposite angles of quadrilaterals in circles are equal to two right angles                                     |
| 7 | Consider two lines from a point inside a circle to the edge, the longer one will be the one closest to the longest part of the diameter passing through the original point | 15 | The longest line in a circle is its diameter, shorter the farther away from the diameter  | 23 | On the same straight line there cannot be constructed two similar and unequal segments of circles on the same side |
| 8 | Consider two lines from a point outside a circle to the edge, the line closest to the centre will be longer on the concave side and shorter on the convex side             | 16 | A line on the circle, perpendicular to the diameter, lies outside the circle  | 24 | Similar segments of circles on equal straight lines are equal to one another                                       |



## Table of Contents, Chapter 3

- |           |   |    |  |
|-----------|---|----|--|
| 25        | Given a segment of a circle, to describe the complete circle of which it is a segment.                          | 34 | Construct a circle segment on a given circle, such that the angle within the segment is equal to a given angle               |
| 26        | In equal circles equal angles stand on equal circumferences   | 35 | If two circle chords intersect, the segments on one multiplied together equals the segments of the other multiplied together |
| 27        | In equal circles angles standing on equal circumferences are equal to one another                               |    |  |
| 28        | In equal circles equal straight lines cut off equal circumferences  | 36 | Secant-tangent law   |
| <b>29</b> | <b>In equal circles equal circumferences are subtended by equal straight lines</b>                              | 37 | Converse of the secant-tangent law   |
| 30        | To bisect a given circumference   |    |  |
| 31        | In a circle the angle in the semicircle is right ...  |    |  |
| 32        | The angle between a tangent and a straight line cutting a circle is equal to the angle in the alternate segment |    |  |
| 33        | Construct a circle segment on a given line, such that the angle within the segment is equal to a given angle    |    |  |



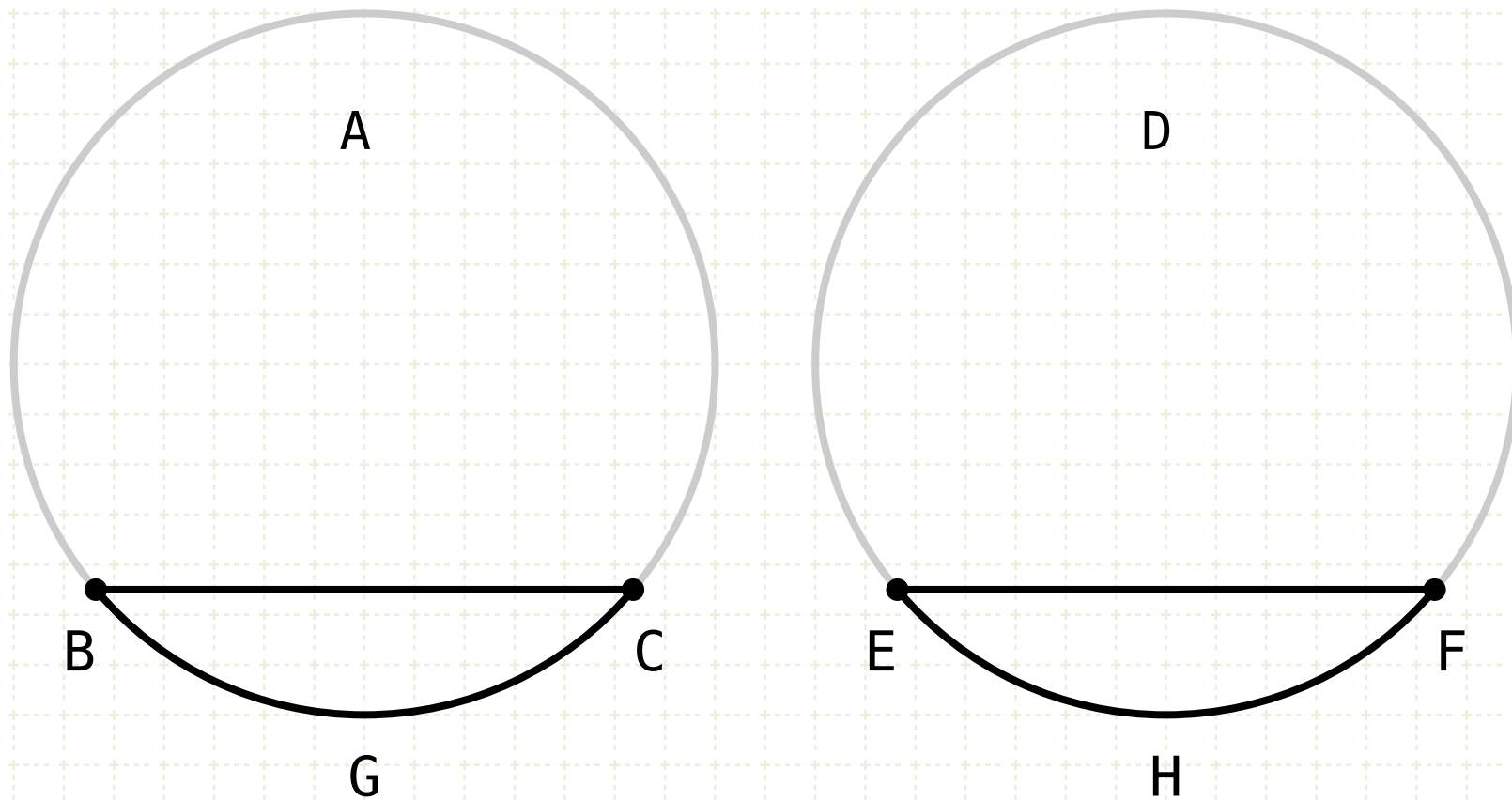
# Proposition 29 of Book III

In equal circles equal circumferences are subtended by equal straight lines.



# Proposition 29 of Book III

In equal circles equal circumferences are subtended by equal straight lines.



$$\begin{aligned} \odot A &= \odot D \\ \frown BGC &= \frown EHF \\ \rightarrow BC &= EF \end{aligned}$$

## In other words

Given two equal circles (as shown)

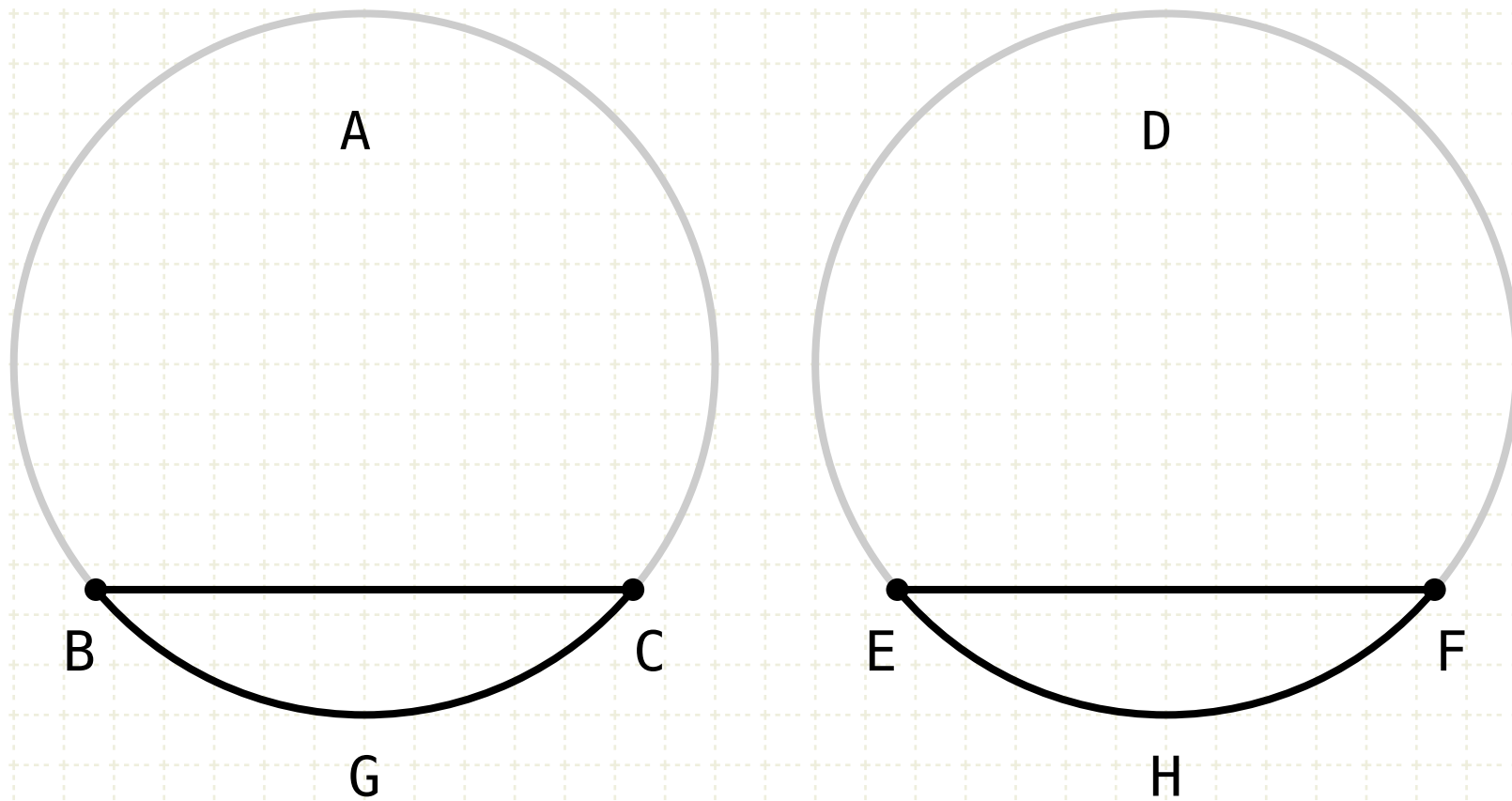
If the circumference BGC equals EHF, then line BC equals line EF





# Proposition 29 of Book III

In equal circles equal circumferences are subtended by equal straight lines.



$$\odot A = \odot D$$
$$\frown BGC = \frown EHF$$

## In other words

Given two equal circles (as shown)

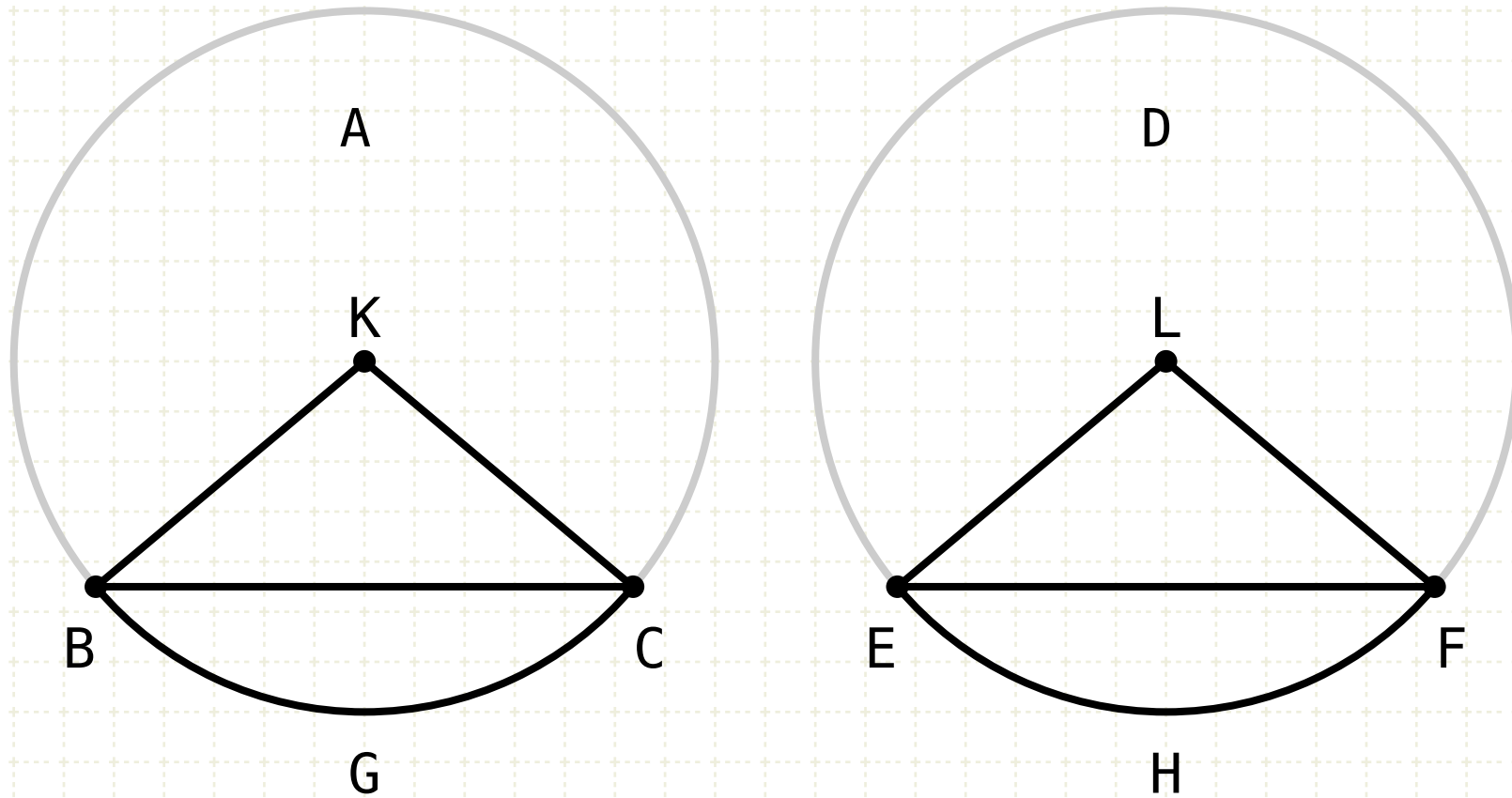
If the circumference BGC equals EHF, then line BC equals line EF

## Proof



# Proposition 29 of Book III

In equal circles equal circumferences are subtended by equal straight lines.



$$\odot A = \odot D$$
$$\frown BGC = \frown EHF$$

## In other words

Given two equal circles (as shown)

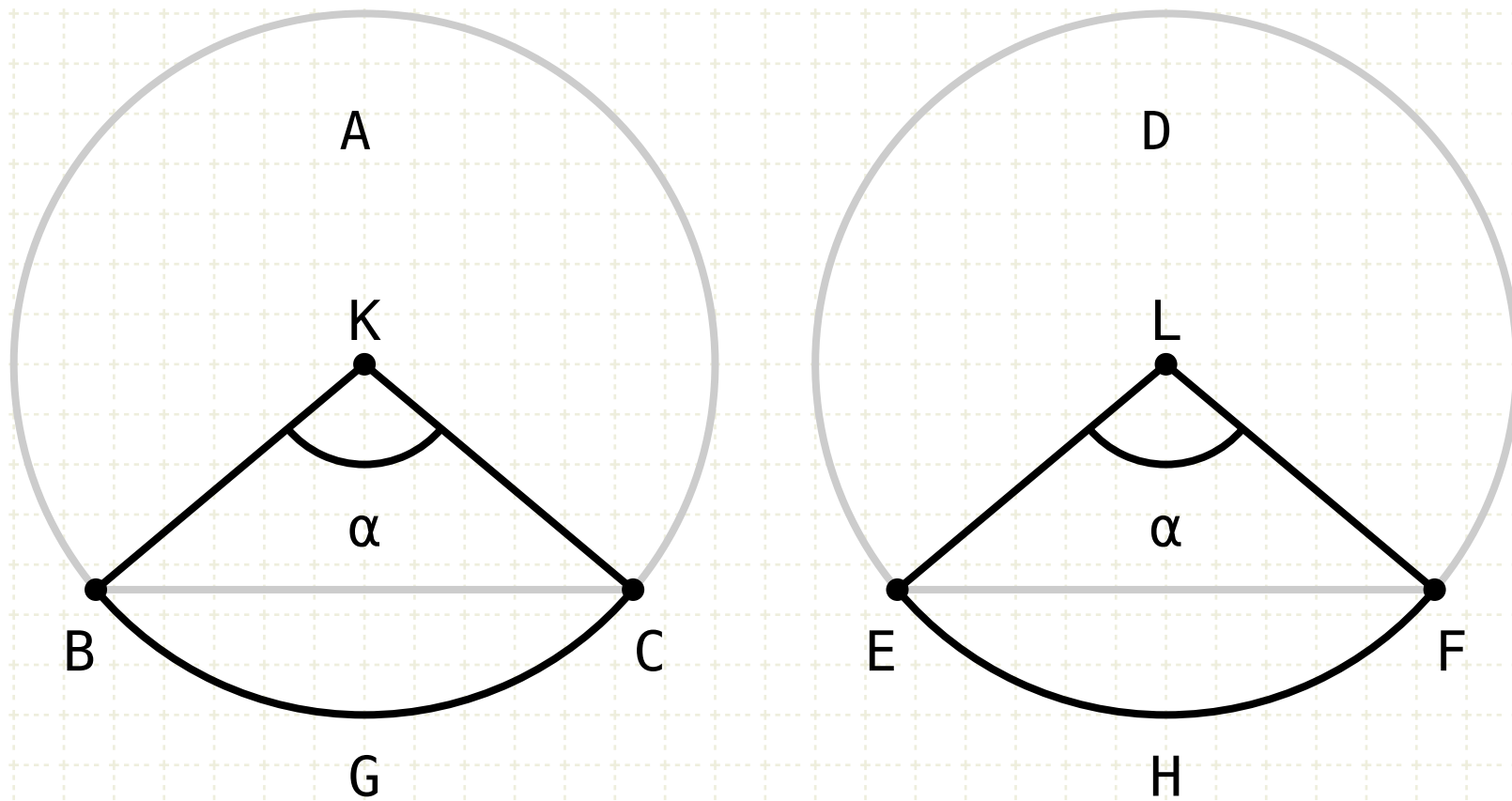
If the circumference BGC equals EHF, then line BC equals line EF

## Proof

Take the centre (K,L) of the circles A and D and draw the radii KB, KC, LE and LF

## Proposition 29 of Book III

In equal circles equal circumferences are subtended by equal straight lines.



$$\odot A = \odot D$$

$$\frown BGC = \frown EHF$$

$$\angle BKC = \angle ELF$$

### In other words

Given two equal circles (as shown)

If the circumference BGC equals EHF, then line BC equals line EF

### Proof

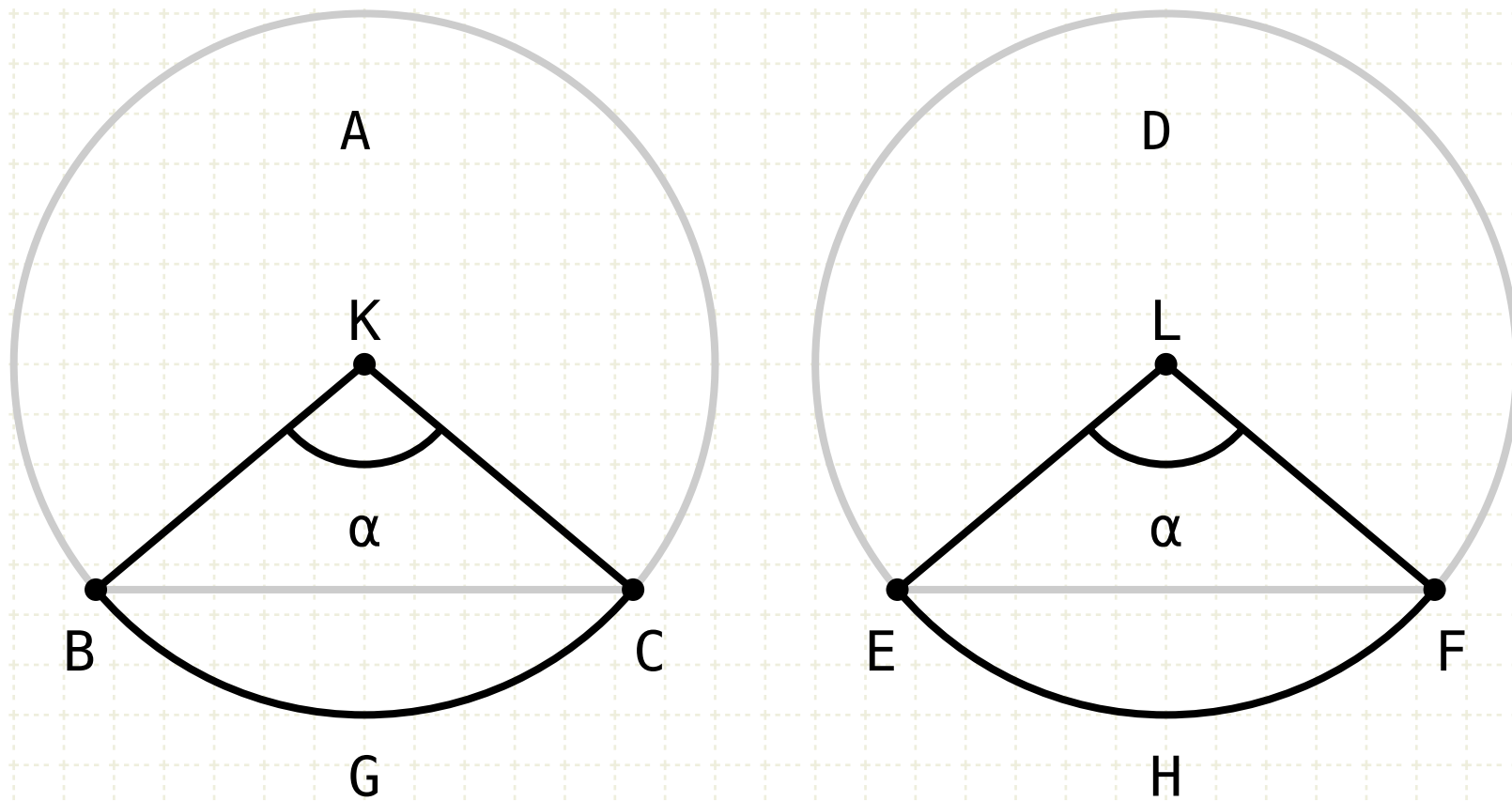
Take the centre (K,L) of the circles A and D and draw the radii KB, KC, LE and LF

Since the circumferences BGC and EHF are equal, then the angles from the centre are also equal (III·27)



## Proposition 29 of Book III

In equal circles equal circumferences are subtended by equal straight lines.



$$\odot A = \odot D$$

$$\frown BGC = \frown EHF$$

$$\angle BKC = \angle ELF$$

$$KB = KC = LE = LF$$

### In other words

Given two equal circles (as shown)

If the circumference BGC equals EHF, then line BC equals line EF

### Proof

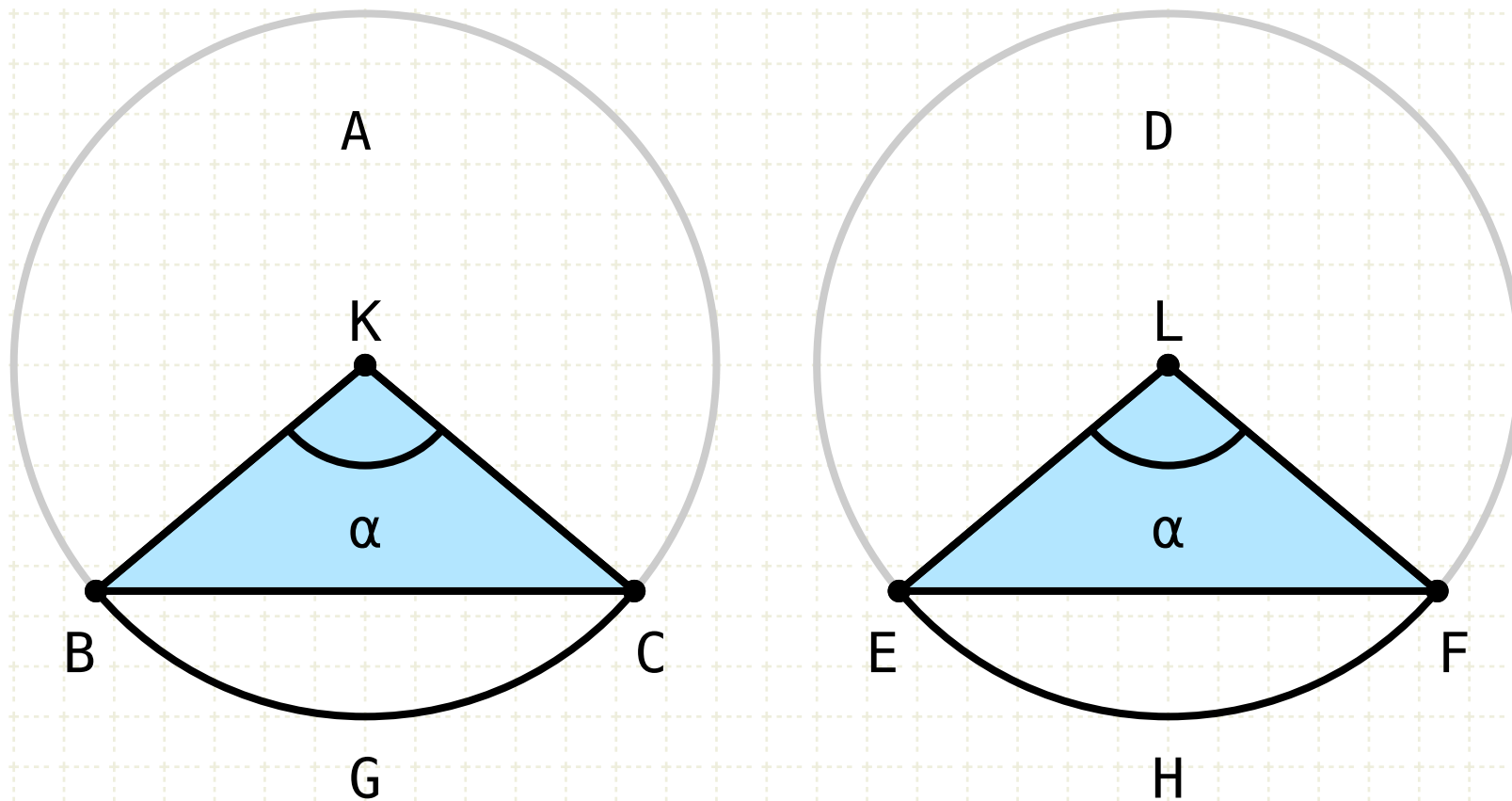
Take the centre (K,L) of the circles A and D and draw the radii KB, KC, LE and LF

Since the circumferences BGC and EHF are equal, then the angles from the centre are also equal (III·27)

Since KB, KC, LE, LF are all radii of equal circles, they are all equal

## Proposition 29 of Book III

In equal circles equal circumferences are subtended by equal straight lines.



$$\begin{aligned}\odot A &= \odot D \\ \frown BGC &= \frown EHF \\ \angle BKC &= \angle ELF \\ KB &= KC = LE = LF \\ \triangle BCK &\equiv \triangle EFL\end{aligned}$$

### In other words

Given two equal circles (as shown)

If the circumference BGC equals EHF, then line BC equals line EF

### Proof

Take the centre (K,L) of the circles A and D and draw the radii KB, KC, LE and LF

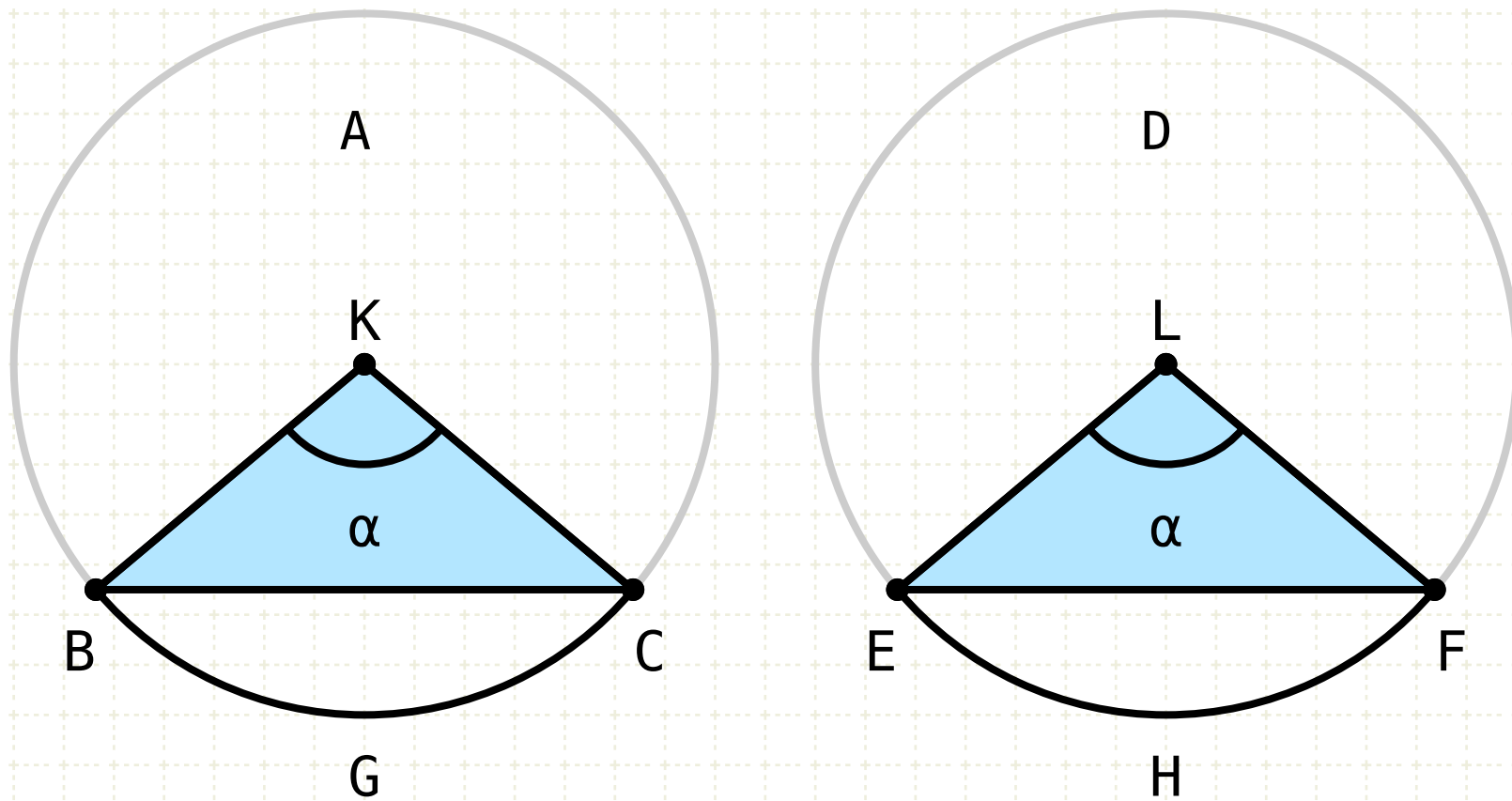
Since the circumferences BGC and EHF are equal, then the angles from the centre are also equal (III·27)

Since KB, KC, LE, LF are all radii of equal circles, they are all equal

Since each triangle has two lines equal to two lines respectively, with an equal angle between (side-angle-side), then the triangles are equal in all respects (I·4)

## Proposition 29 of Book III

In equal circles equal circumferences are subtended by equal straight lines.



$$\odot A = \odot D$$

$$\frown BGC = \frown EHF$$

$$\angle BKC = \angle ELF$$

$$KB = KC = LE = LF$$

$$\triangle BCK \equiv \triangle EFL$$

$$BC = EF$$

### In other words

Given two equal circles (as shown)

If the circumference BGC equals EHF, then line BC equals line EF

### Proof

Take the centre (K,L) of the circles A and D and draw the radii KB, KC, LE and LF

Since the circumferences BGC and EHF are equal, then the angles from the centre are also equal (III·27)

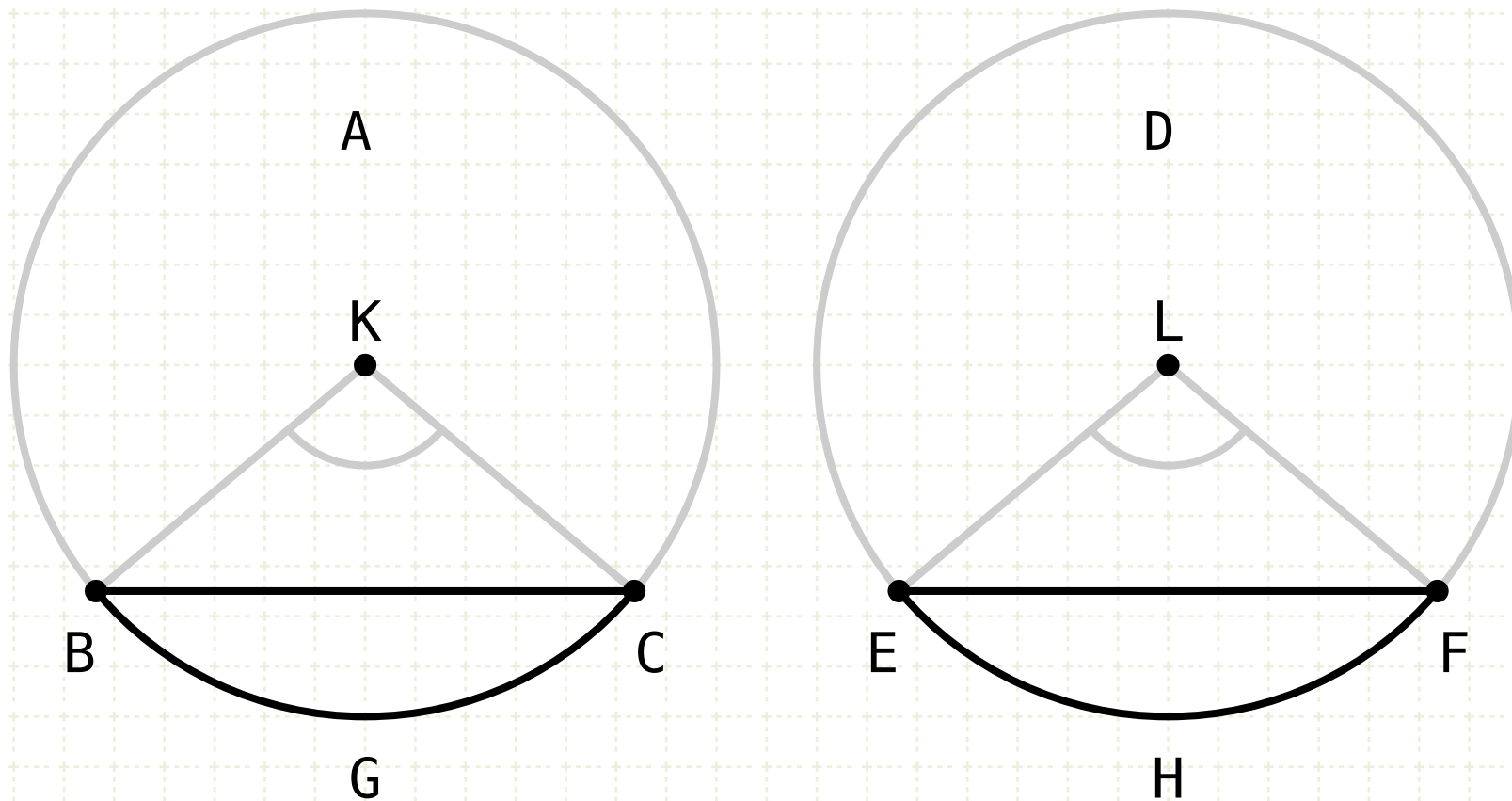
Since KB, KC, LE, LF are all radii of equal circles, they are all equal

Since each triangle has two lines equal to two lines respectively, with an equal angle between (side-angle-side), then the triangles are equal in all respects (I·4)

Therefore the base BC equals EF

## Proposition 29 of Book III

In equal circles equal circumferences are subtended by equal straight lines.



$$\odot A = \odot D$$

$$\frown BGC = \frown EHF$$

$$\angle BKC = \angle ELF$$

$$KB = KC = LE = LF$$

$$\triangle BCK \equiv \triangle EFL$$

$$BC = EF$$

### In other words

Given two equal circles (as shown)

If the circumference BGC equals EHF, then line BC equals line EF

### Proof

Take the centre (K,L) of the circles A and D and draw the radii KB, KC, LE and LF

Since the circumferences BGC and EHF are equal, then the angles from the centre are also equal (III·27)

Since KB, KC, LE, LF are all radii of equal circles, they are all equal

Since each triangle has two lines equal to two lines respectively, with an equal angle between (side-angle-side), then the triangles are equal in all respects (I·4)

Therefore the base BC equals EF

# Youtube Videos

<https://www.youtube.com/c/SandyBultena>

*Copyright © 2019 by Sandy Bultena.*



Except where otherwise noted, this work is licensed under  
<http://creativecommons.org/licenses/by-nc/3.0>