

Euclid's Elements

Book VI

One can state, without exaggeration, that the observation of and the search for similarities and differences are the basis of all human knowledge.

Alfred Nobel



Table of Contents, Chapter 6

1	If the height of two triangles are equal, then the ratio of the areas is equal to the ratio of the bases	7	If two triangles have one angle equal to one angle, and the sides about other angles are proportional, and the remaining angles either both less or both not less than a right angle, then triangles will be equiangular	14	In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and vice versa
2	If a line cuts a triangle, parallel to its base, it will cut the sides of the triangle proportionally			15	In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and vice versa
3	If an angle of a triangle is bisected and the straight line cutting the angle also cuts the base, the segments of the base will have the same ratio as the remaining sides of the triangle	8	If in a right-angled triangle a perpendicular be drawn from the right angle to the base, the triangles adjoining the perpendicular are similar both to the whole and to one another	16	If four straight lines are proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means, and vice versa
4	If two triangles have equal angles, then the sides opposite the equal angles are proportional, as well, the sides of the triangles on either side of the equal angles are also proportional	9	From a given straight line to cut off a given fraction	17	If three straight lines are proportional, the rectangle contained by the extremes is equal to the square on the mean; and vice versa
5	If two triangles have proportional sides, the triangles will be equiangular	10	To cut a given uncut straight line similarly to a given cut straight line	18	On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure
6	If two triangles have one angle equal to one angle and the sides about the equal angles are proportional, then the triangles will be equiangular	11	To two given straight lines to find a third proportional	19	Similar triangles are to one another in the duplicate ratio of the corresponding sides
		12	To three given straight lines to find a fourth proportional		
		13	To two given straight lines to find a mean proportional		



Table of Contents, Chapter 3

20	Similar polygons are divided into the same number of similar triangles, which have the same ratio as the wholes, and the polygons have duplicate ratios to their corresponding sides	26	If from a parallelogram a similar parallelogram with a common angle is subtracted, it is about the same diameter as the original	31	In right-angled triangles the figure on the side subtending the right angle is equal to the similar and similarly described figures on the sides containing the right angle
21	Figures which are are similar to the same rectilineal figure are also similar to one another	27	Of all the parallelograms applied to the same straight line and deficient by parallelogrammic figures similar to a parallelogram drawn on half the said line, the largest will be one that is drawn on half of the straight line and is similar to the defect		
22	If four straight lines are proportional, similar rectilineal figures will also be proportional; and vice versa	28	To a given straight line, apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one		
23	Equiangular parallelograms have to one another the ratio compounded of the ratios of their sides	29	To a given straight line, apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one		
24	In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another	30	To cut a finite straight line in extreme ratio		
25	To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure				



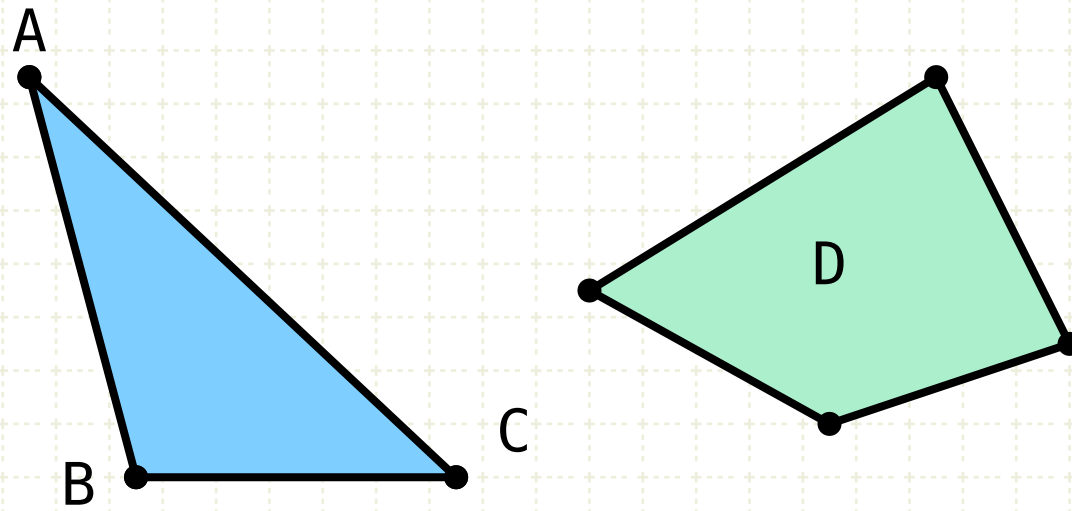
Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure



Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilinear figure and equal to another given rectilinear figure

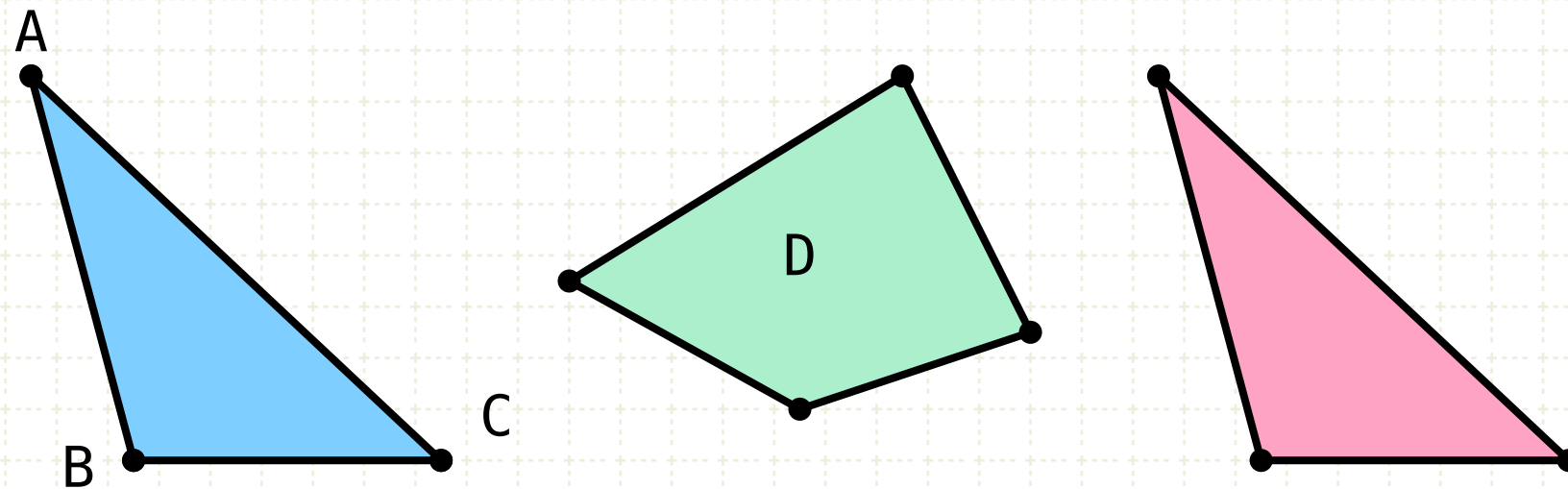


In other words

Given two rectilinear figures (ABC and D for example)

Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure



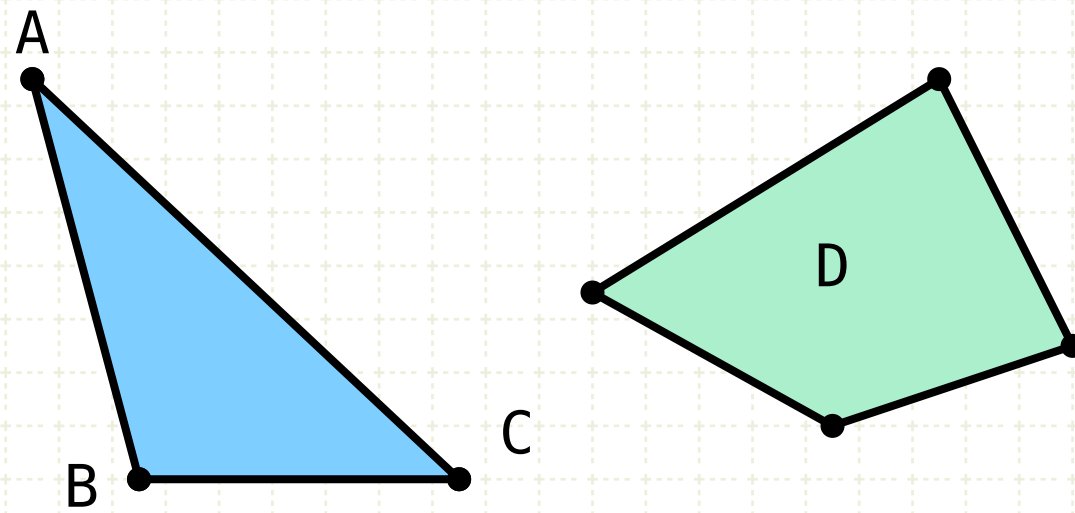
In other words

Given two rectilineal figures (ABC and D for example)

Construct a third figure that is similar to the first (ABC), and equal in area to the second (D)

Proposition 25 of Book VI

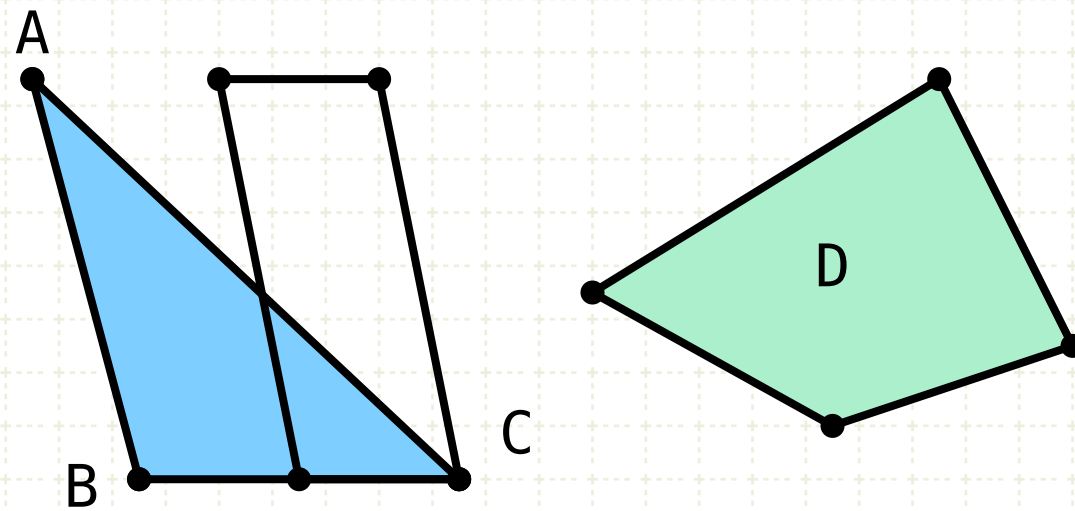
To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure



Construction

Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure



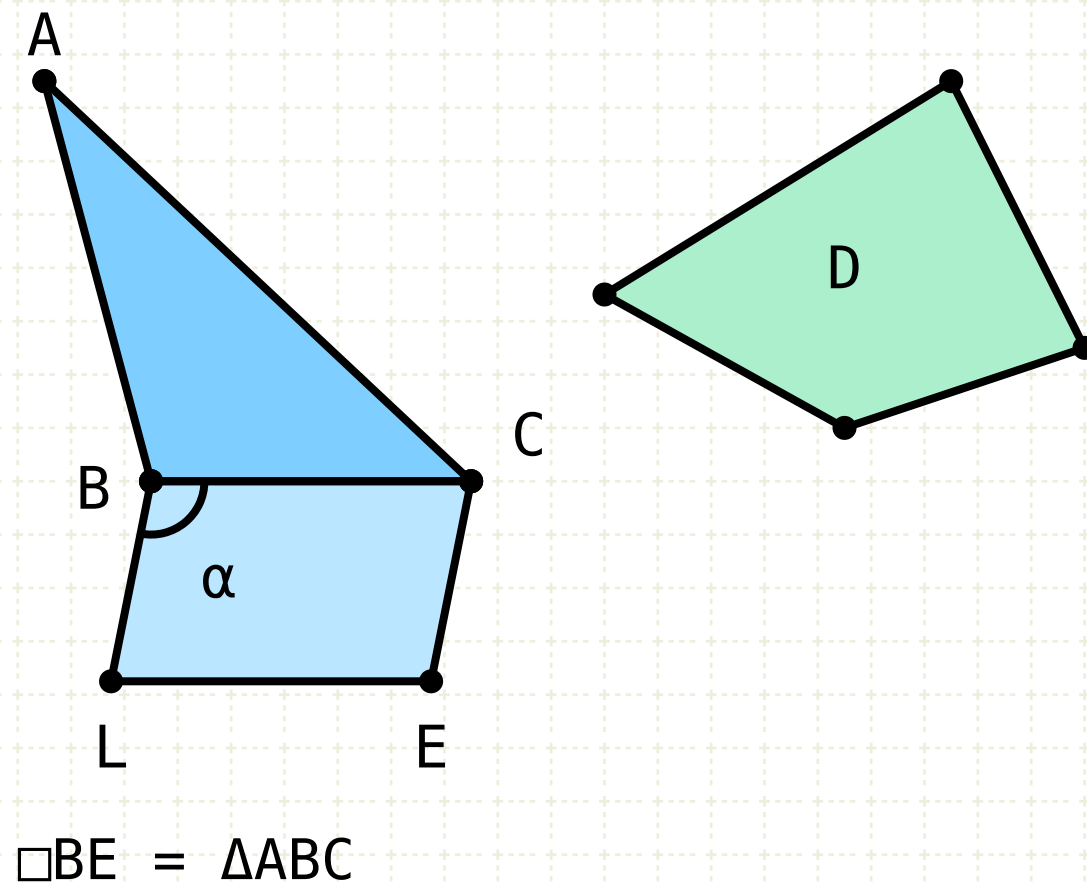
Construction

Construct a parallelogram to the base BC such that it is equal in area to the triangle ABC (I-44)

$$\square BE = \triangle ABC$$

Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure

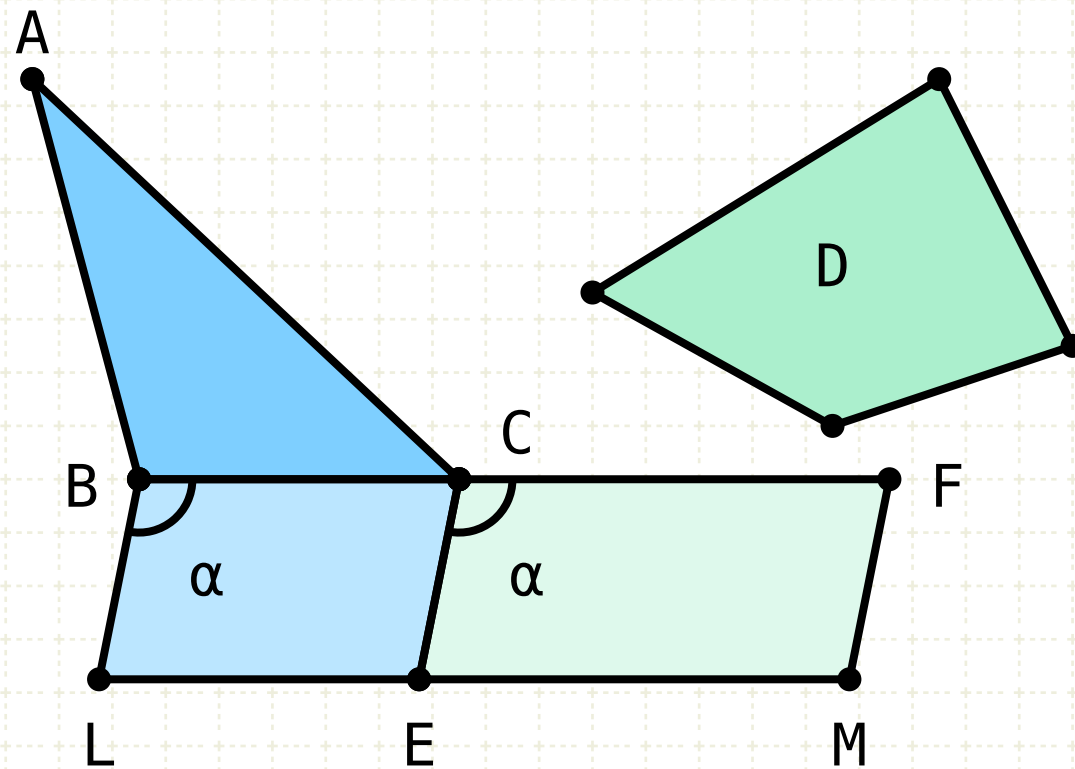


Construction

Construct a parallelogram to the base BC such that it is equal in area to the triangle ABC (I·44)

Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure



$$\square BE = \triangle ABC$$

$$\square EF = \square D$$

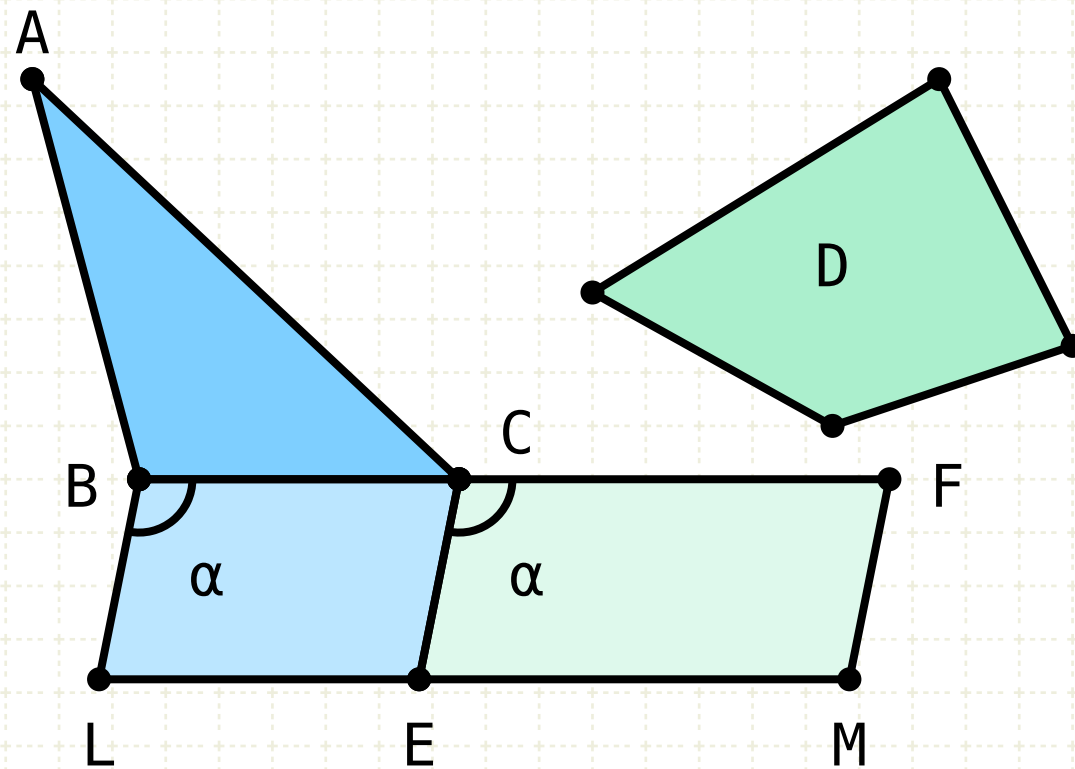
Construction

Construct a parallelogram to the base BC such that it is equal in area to the triangle ABC (I·44)

Construct a parallelogram to the line CE equal in area to D, and with an angle equal to CBL (I·45)

Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure



$$\square BE = \triangle ABC$$

$$\square EF = \square D$$

Construction

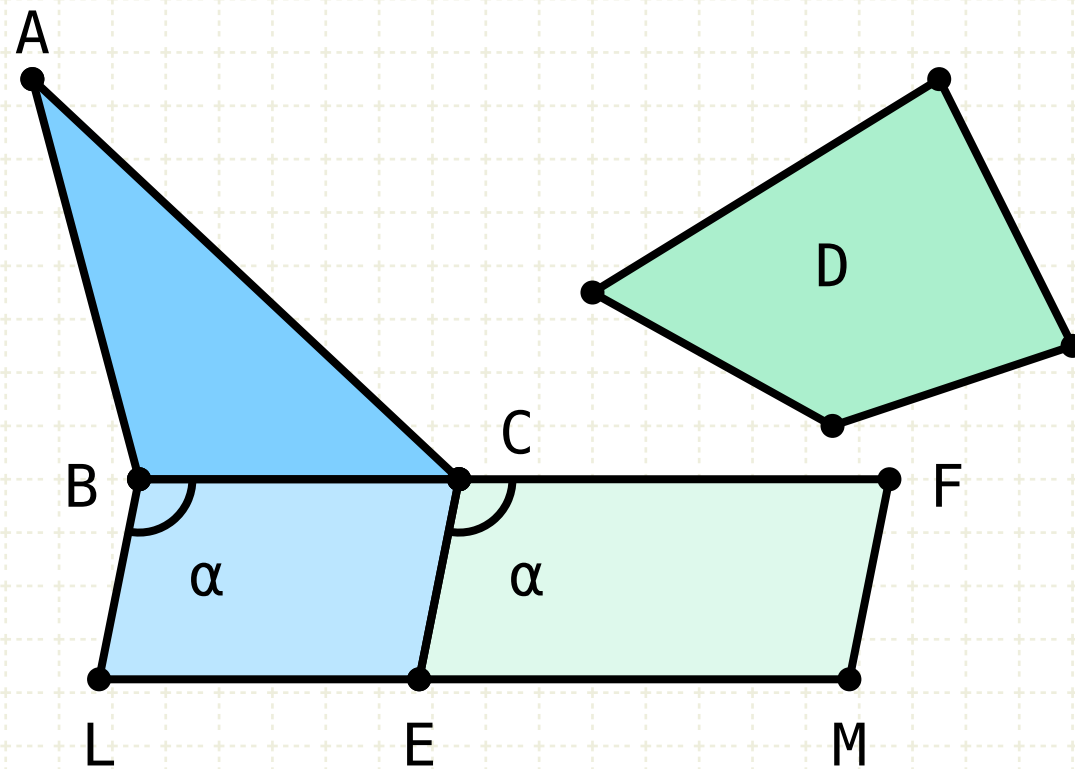
Construct a parallelogram to the base BC such that it is equal in area to the triangle ABC (I·44)

Construct a parallelogram to the line CE equal in area to D, and with an angle equal to CBL (I·45)

Since the angles CBL and FCE are equal, the lines BC, CF are in a straight line as are LE and EM

Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure



$$\square BE = \triangle ABC$$

$$\square EF = \square D$$

$$BC : GH = GH : CF$$



Construction

Construct a parallelogram to the base BC such that it is equal in area to the triangle ABC (I·44)

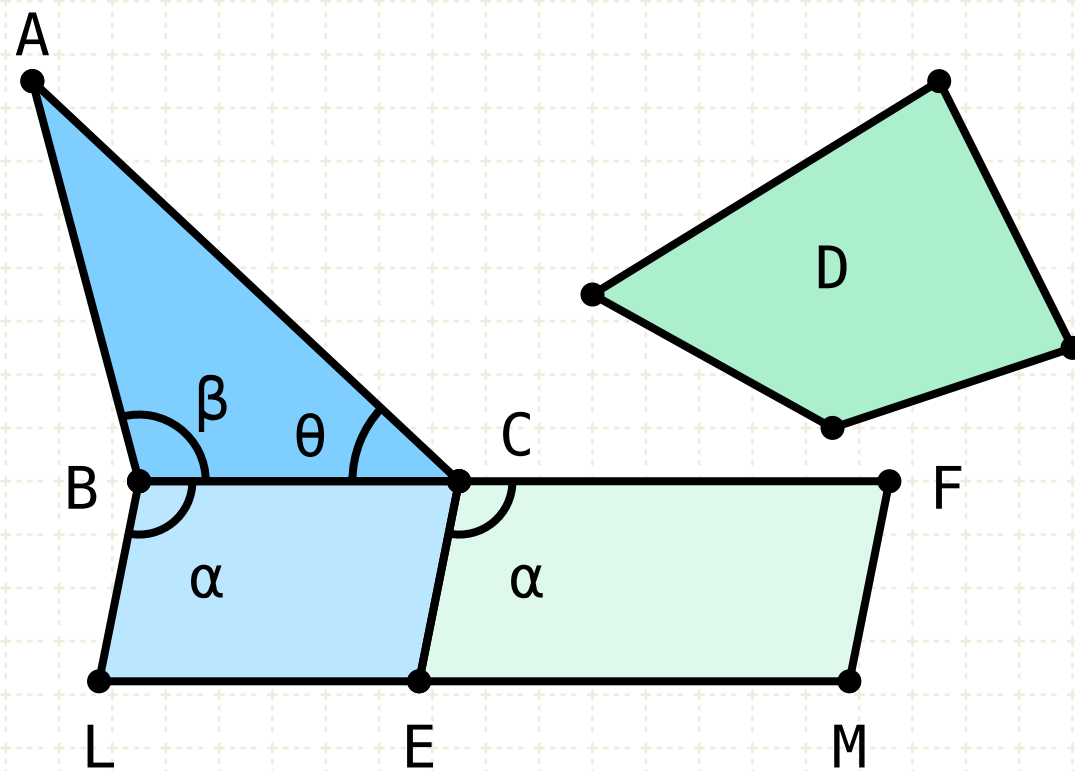
Construct a parallelogram to the line CE equal in area to D, and with an angle equal to CBL (I·45)

Since the angles CBL and FCE are equal, the lines BC, CF are in a straight line as are LE and EM

Draw a line GH which is in a mean proportion to BC, CF (VI·13)

Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure

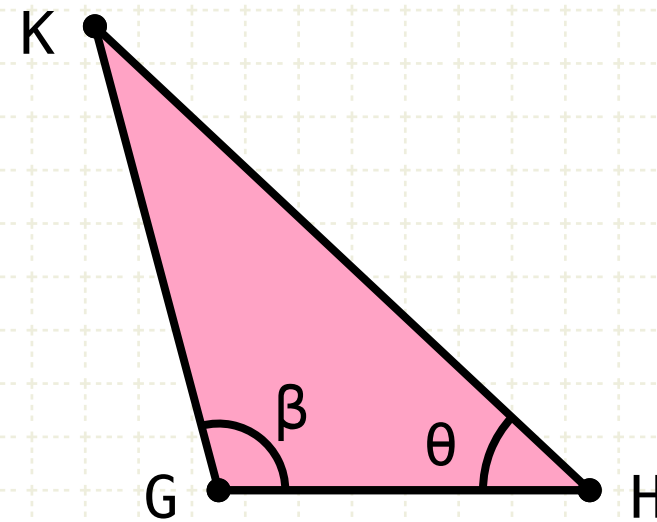


$$\square BE = \triangle ABC$$

$$\square EF = \square D$$

$$BC : GH = GH : CF$$

$$\triangle ABC \sim \triangle KGH$$



Construction

Construct a parallelogram to the base BC such that it is equal in area to the triangle ABC (I·44)

Construct a parallelogram to the line CE equal in area to D, and with an angle equal to CBL (I·45)

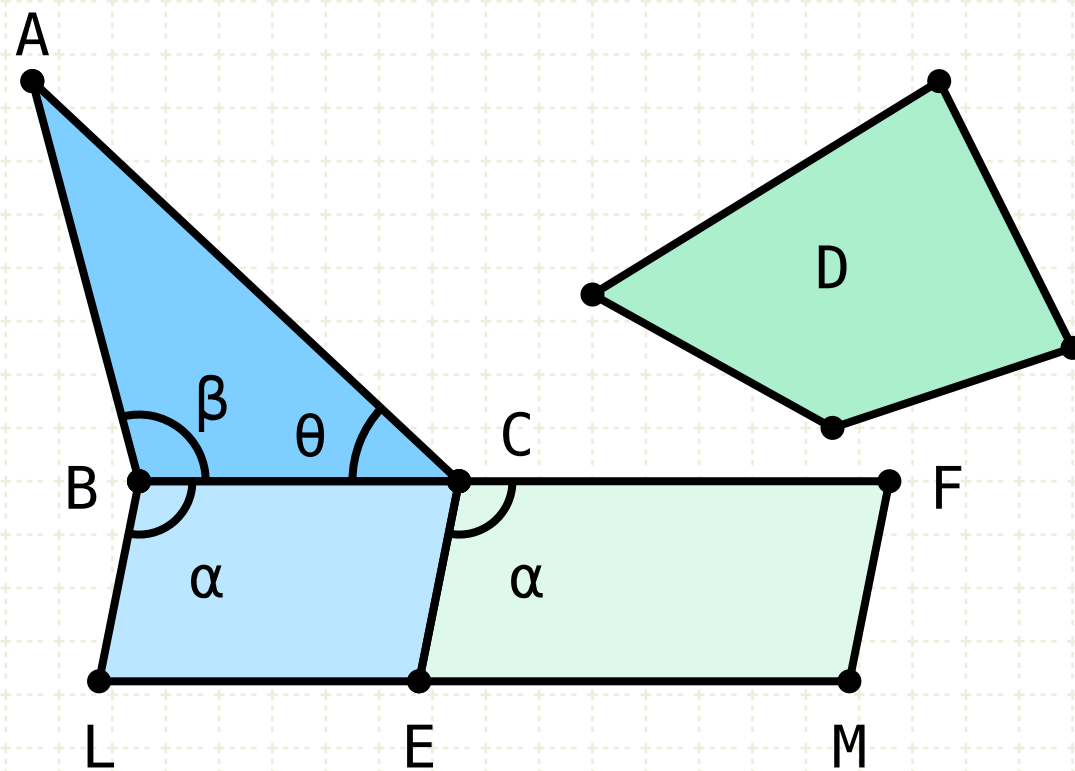
Since the angles CBL and FCE are equal, the lines BC,CF are in a straight line as are LE and EM

Draw a line GH which is in a mean proportion to BC,CF (VI·13)

Draw a figure similar to ABC on the line GH (VI·18)

Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure

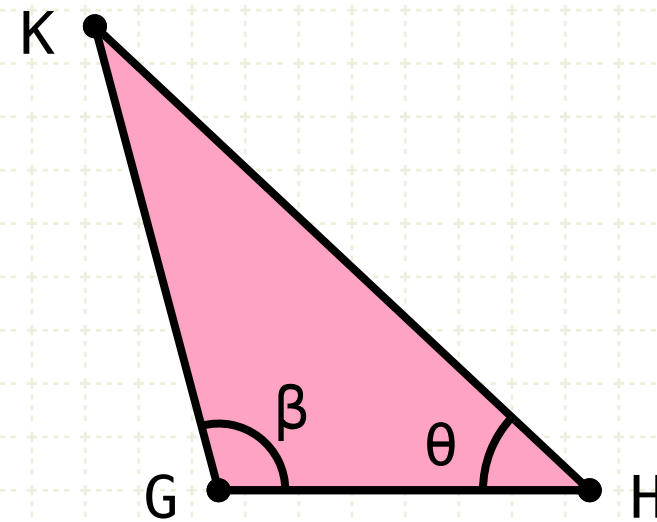


$$\square BE = \Delta ABC$$

$$\square EF = \square D$$

$$BC : GH = GH : CF$$

$$\Delta ABC \sim \Delta KGH$$



Construction

Construct a parallelogram to the base BC such that it is equal in area to the triangle ABC (I·44)

Construct a parallelogram to the line CE equal in area to D, and with an angle equal to CBL (I·45)

Since the angles CBL and FCE are equal, the lines BC,CF are in a straight line as are LE and EM

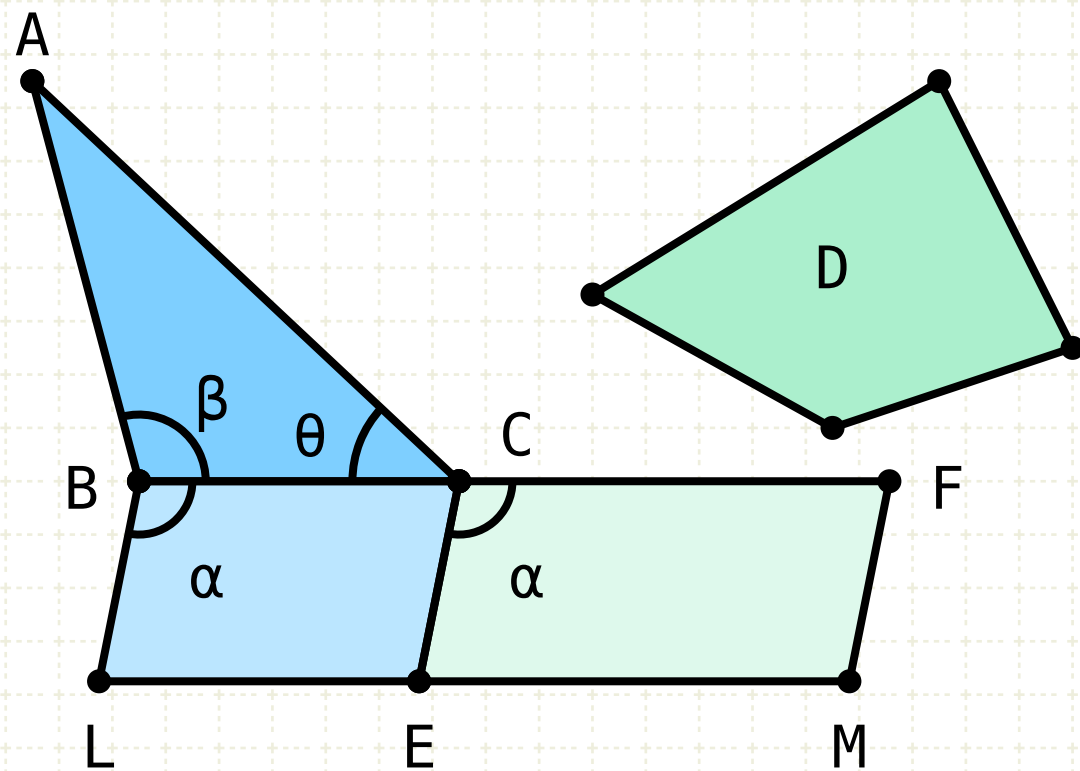
Draw a line GH which is in a mean proportion to BC,CF (VI·13)

Draw a figure similar to ABC on the line GH (VI·18)

Now, the triangle KGH is equal in area to the polygon D

Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure



Proof

$$\square BE = \Delta ABC$$

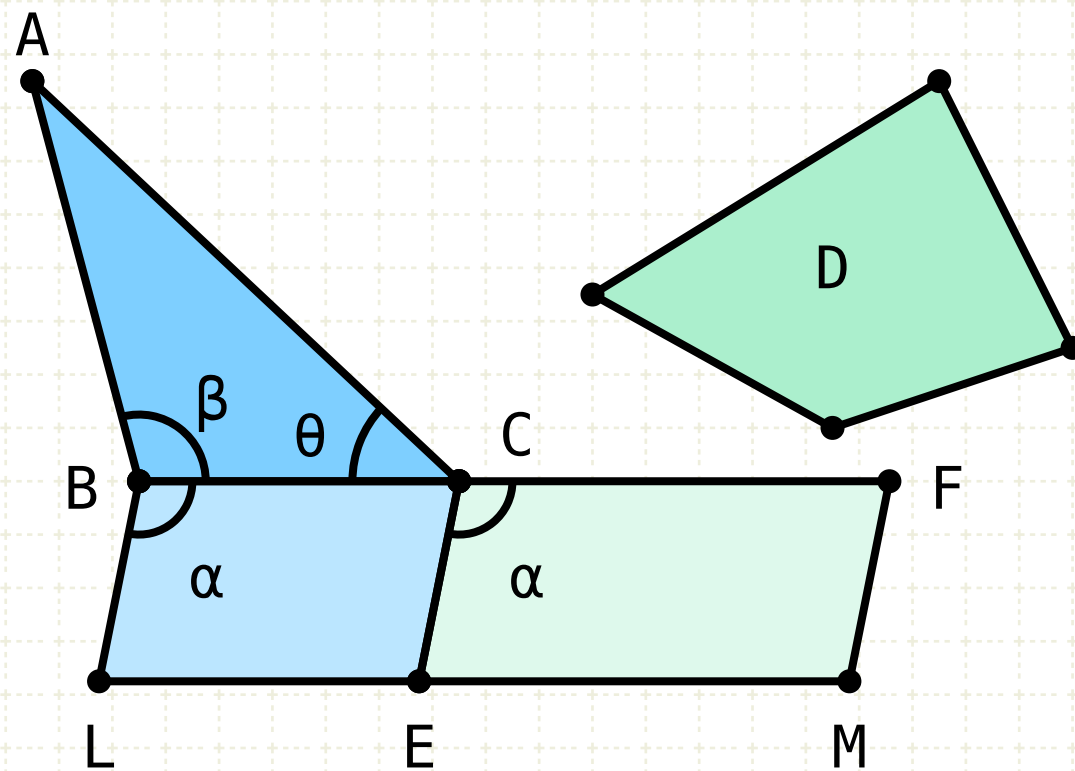
$$\square EF = \square D$$

$$BC:GH = GH:CF$$

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Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure

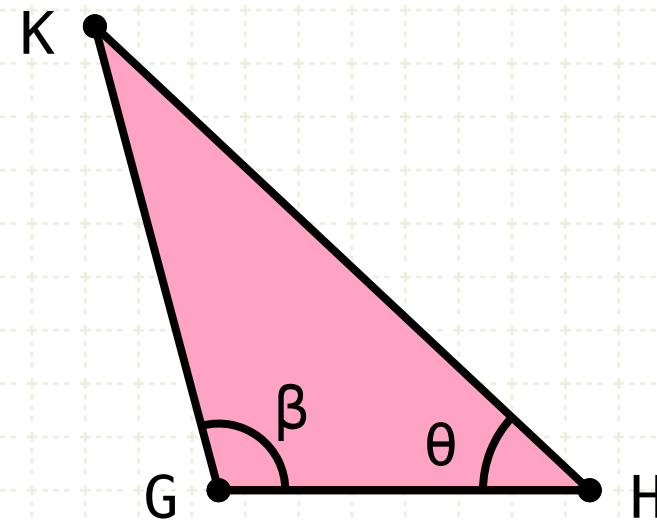


$$\square BE = \triangle ABC$$

$$\square EF = \square D$$

$$BC:GH = GH:CF$$

$$\triangle ABC \sim \triangle KGH$$



Proof

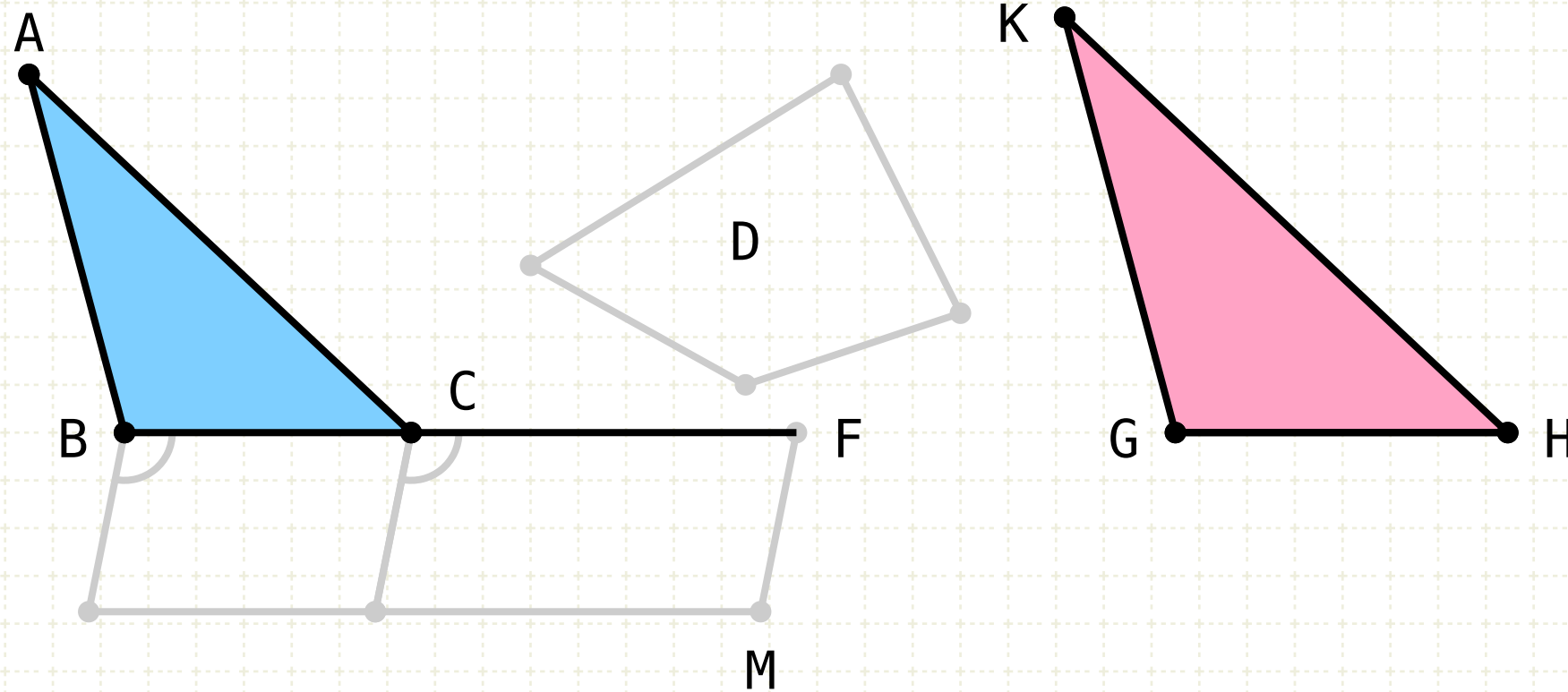
If there are two lines A,B, and if A is to B as B is to C, ...

... and two similar figures are drawn on A and B, ...

... then the ratio of the areas of the two figures (being the duplicate ratio of A,B) is the ratio A:C (VI.19.Por)

Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure



Proof

If there are two lines A,B, and if A is to B as B is to C, ...

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Thus the ratio of BC to CF is the ratio of the two triangles ABC to KGH

$$\square BE = \triangle ABC$$

$$\square EF = \square D$$

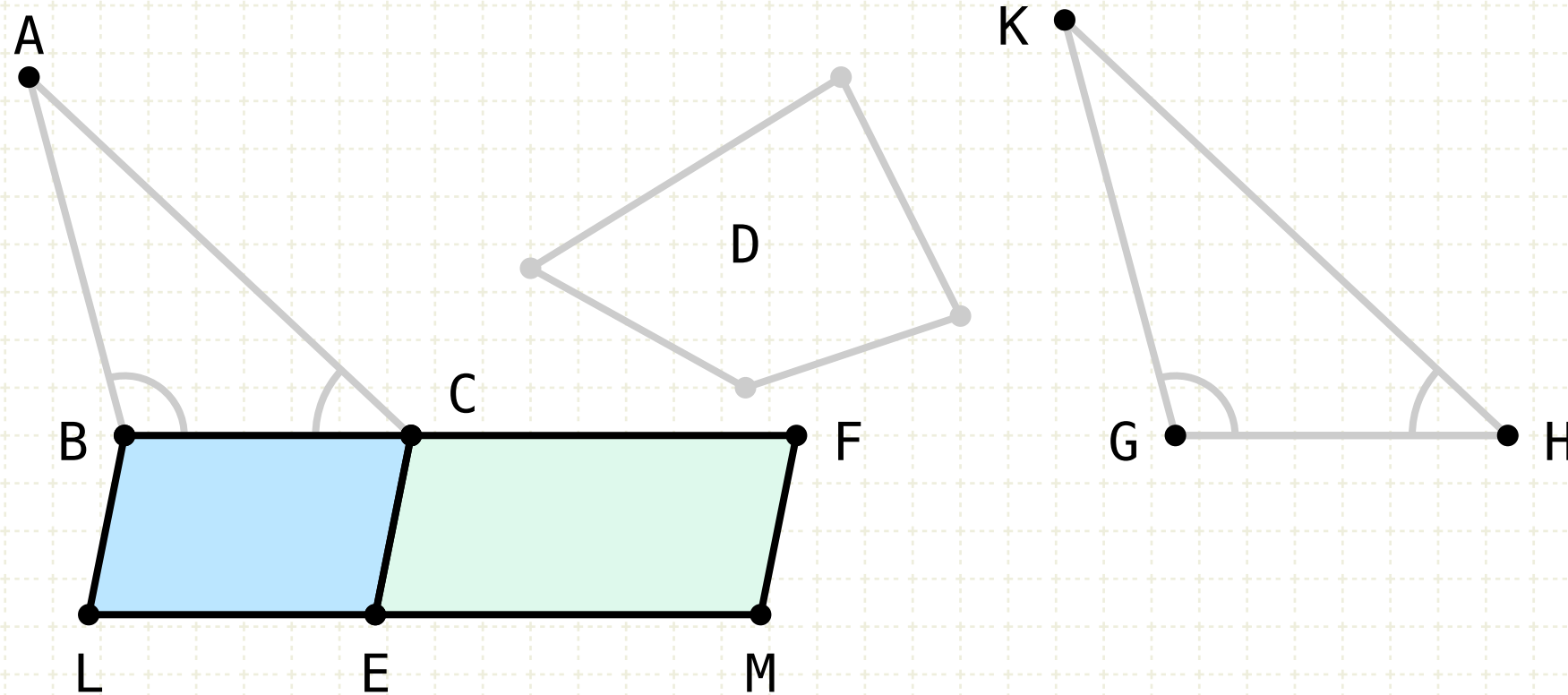
$$BC:GH = GH:CF$$

$$\triangle ABC \sim \triangle KGH$$

$$\triangle ABC:\triangle KGH = BC:CF$$

Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure



Proof

If there are two lines A,B, and if A is to B as B is to C, ...

... and two similar figures are drawn on A and B, ...

... then the ratio of the areas of the two figures (being the duplicate ratio of A,B) is the ratio A:C (VI·19.Por)

Thus the ratio of BC to CF is the ratio of the two triangles ABC to KGH

But the ratio of the parallelograms BE to EF is equal to the ratio of their bases, BC to CF (VI·1)

$$\square BE = \triangle ABC$$

$$\square EF = \square D$$

$$BC : GH = GH : CF$$

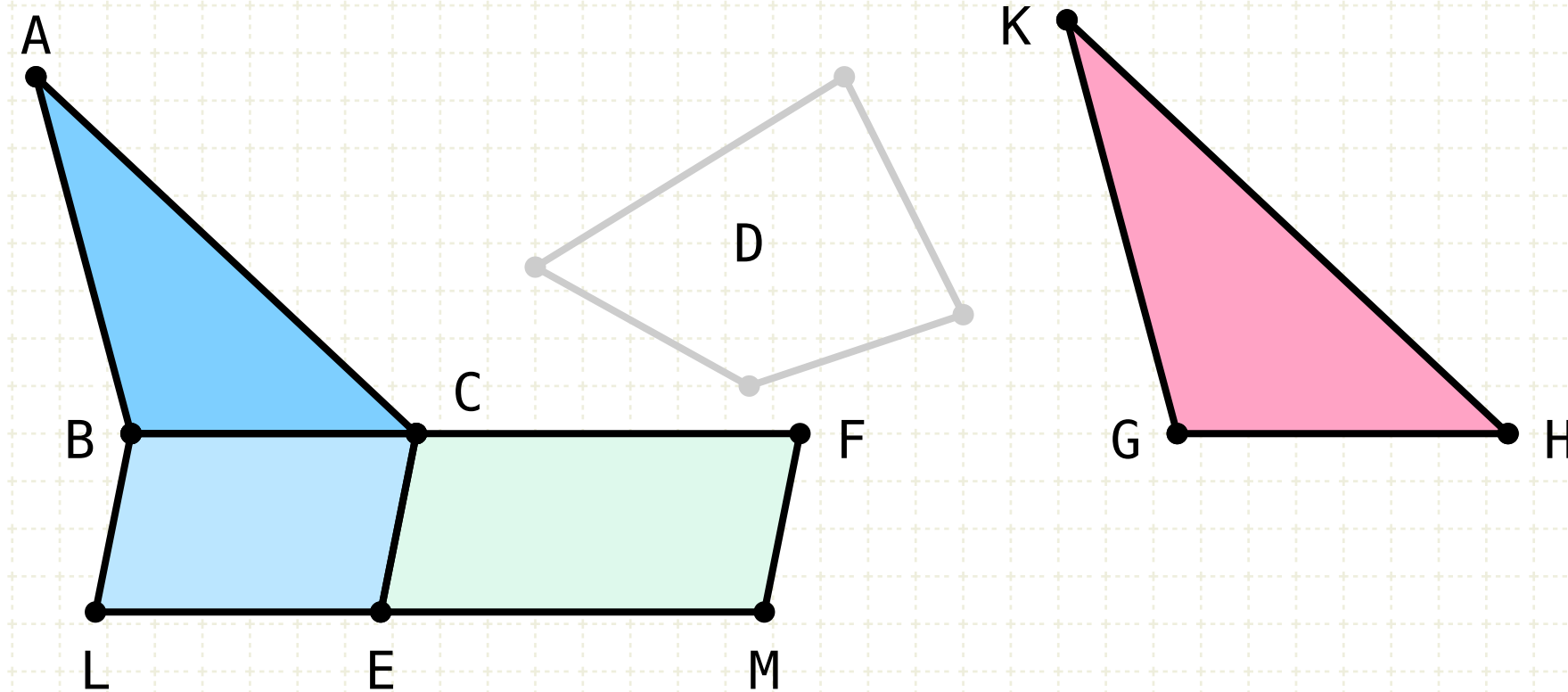
$$\triangle ABC \sim \triangle KGH$$

$$\triangle ABC : \triangle KGH = BC : CF$$

$$BC : CF = \square BE : \square EF$$

Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure



$$\square BE = \Delta ABC$$

$$\square EF = \square D$$

$$BC : GH = GH : CF$$

$$\Delta ABC \sim \Delta KGH$$

$$\Delta ABC : \Delta KGH = BC : CF$$

$$BC : CF = \square BE : \square EF$$

$$\Delta ABC : \Delta KGH = \square BE : \square EF$$

Proof

If there are two lines A,B, and if A is to B as B is to C, ...

... and two similar figures are drawn on A and B, ...

... then the ratio of the areas of the two figures (being the duplicate ratio of A,B) is the ratio A:C (VI·19.Por)

Thus the ratio of BC to CF is the ratio of the two triangles ABC to KGH

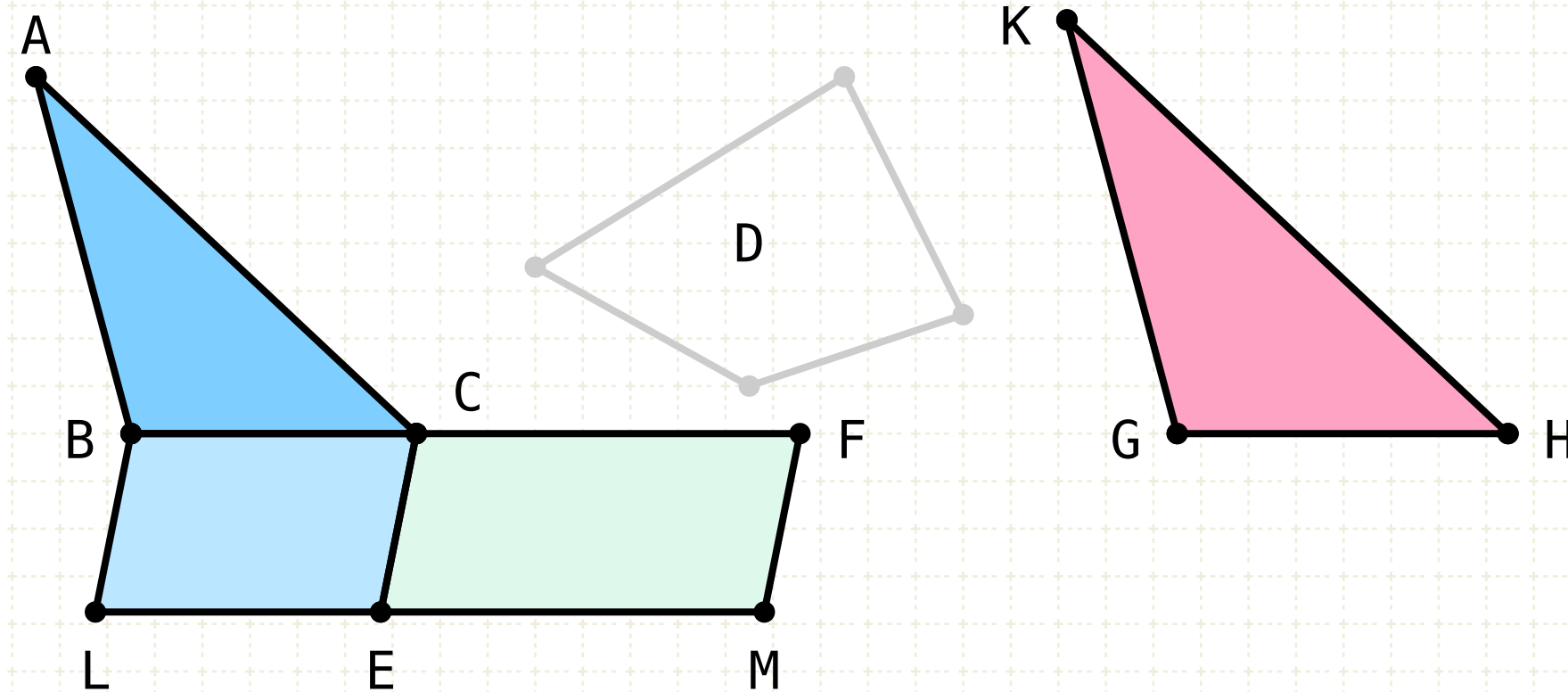
But the ratio of the parallelograms BE to EF is equal to the ratio of their bases, BC to CF (VI·1)

The ratio of BC to CF is equal to both the ratio of the triangles ABC and KGH, and to the ratio of the parallelograms BE and EF

Therefore the ratio of the parallelograms is equal to the ratio of the triangles

Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure



$$\square BE = \Delta ABC$$

$$\square EF = \square D$$

$$BC:GH = GH:CF$$

$$\Delta ABC \sim \Delta KGH$$

$$\Delta ABC:\Delta KGH = BC:CF$$

$$BC:CF = \square BE:\square EF$$

$$\Delta ABC:\Delta KGH = \square BE:\square EF$$

$$\Delta ABC:\square BE = \Delta KGH:\square EF$$

Proof

If there are two lines A,B, and if A is to B as B is to C, ...

... and two similar figures are drawn on A and B, ...

... then the ratio of the areas of the two figures (being the duplicate ratio of A,B) is the ratio A:C (VI·19.Por)

Thus the ratio of BC to CF is the ratio of the two triangles ABC to KGH

But the ratio of the parallelograms BE to EF is equal to the ratio of their bases, BC to CF (VI·1)

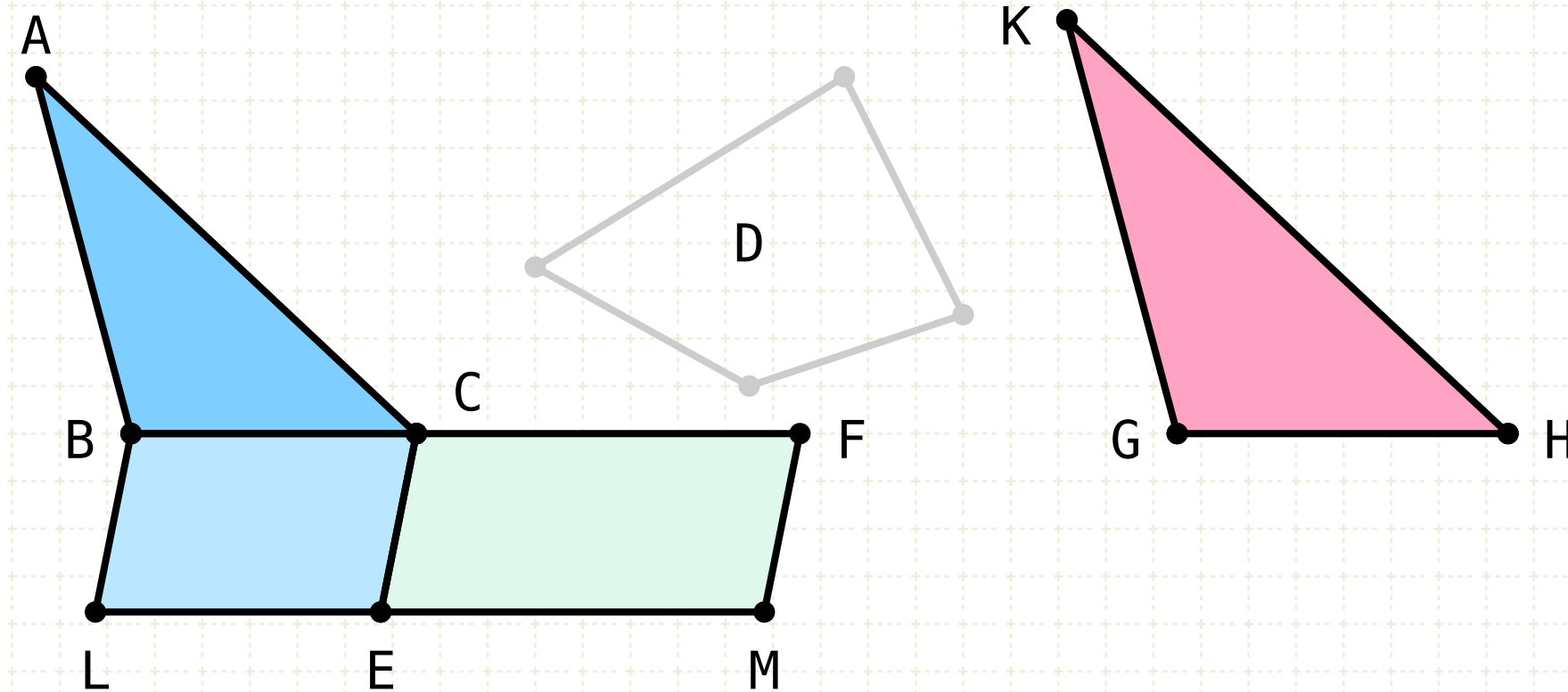
The ratio of BC to CF is equal to both the ratio of the triangles ABC and KGH, and to the ratio of the parallelograms BE and EF

Therefore the ratio of the parallelograms is equal to the ratio of the triangles

Alternately, the triangle ABC to the parallelogram BE is equal to the triangle KGH to the square EF (V·16)

Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure



$$\square BE = \Delta ABC$$

$$\square EF = \square D$$

$$BC:GH = GH:CF$$

$$\Delta ABC \sim \Delta KGH$$

$$\Delta ABC:\Delta KGH = BC:CF$$

$$BC:CF = \square BE:\square EF$$

$$\Delta ABC:\Delta KGH = \square BE:\square EF$$

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Proof

If there are two lines A,B, and if A is to B as B is to C, ...

... and two similar figures are drawn on A and B, ...

... then the ratio of the areas of the two figures (being the duplicate ratio of A,B) is the ratio A:C (VI·19.Por)

Thus the ratio of BC to CF is the ratio of the two triangles ABC to KGH

But the ratio of the parallelograms BE to EF is equal to the ratio of their bases, BC to CF (VI·1)

The ratio of BC to CF is equal to both the ratio of the triangles ABC and KGH, and to the ratio of the parallelograms BE and EF

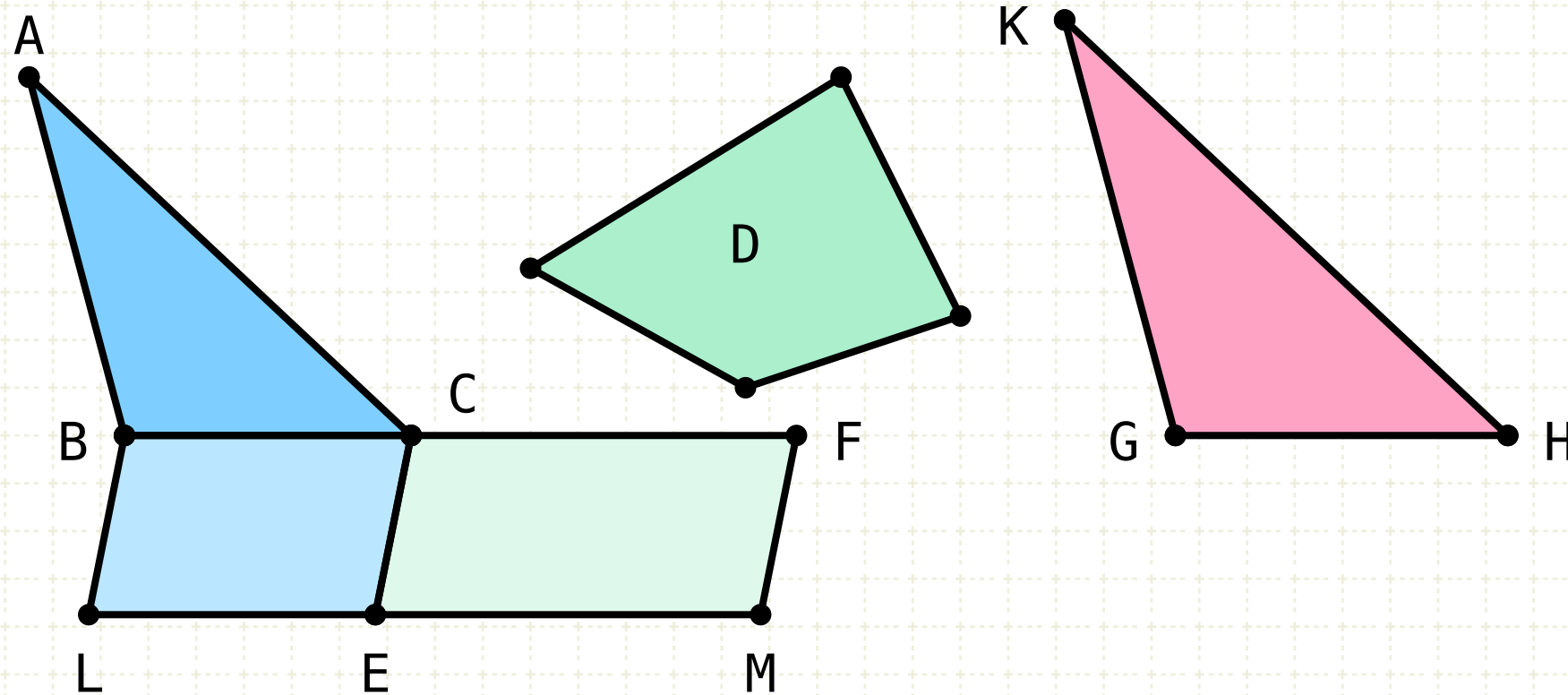
Therefore the ratio of the parallelograms is equal to the ratio of the triangles

Alternately, the triangle ABC to the parallelogram BE is equal to the triangle KGH to the square EF (V·16)

But the triangle ABC is equal to the parallelogram BE, so therefore the triangle KGH is equal to EF

Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure



$$\square BE = \Delta ABC$$

$$\square EF = \square D$$

$$BC:GH = GH:CF$$

$$\Delta ABC \sim \Delta KGH$$

$$\Delta ABC:\Delta KGH = BC:CF$$

$$BC:CF = \square BE:\square EF$$

$$\Delta ABC:\Delta KGH = \square BE:\square EF$$

$$\Delta ABC:\square BE = \Delta KGH:\square EF$$

$$\square BE:\square BE = \Delta KGH:\square EF$$

$$\Delta KGH = \square EF$$

$$\Delta KGH = D$$

Proof

If there are two lines A,B, and if A is to B as B is to C, ...

... and two similar figures are drawn on A and B, ...

... then the ratio of the areas of the two figures (being the duplicate ratio of A,B) is the ratio A:C (VI·19.Por)

Thus the ratio of BC to CF is the ratio of the two triangles ABC to KGH

But the ratio of the parallelograms BE to EF is equal to the ratio of their bases, BC to CF (VI·1)

The ratio of BC to CF is equal to both the ratio of the triangles ABC and KGH, and to the ratio of the parallelograms BE and EF

Therefore the ratio of the parallelograms is equal to the ratio of the triangles

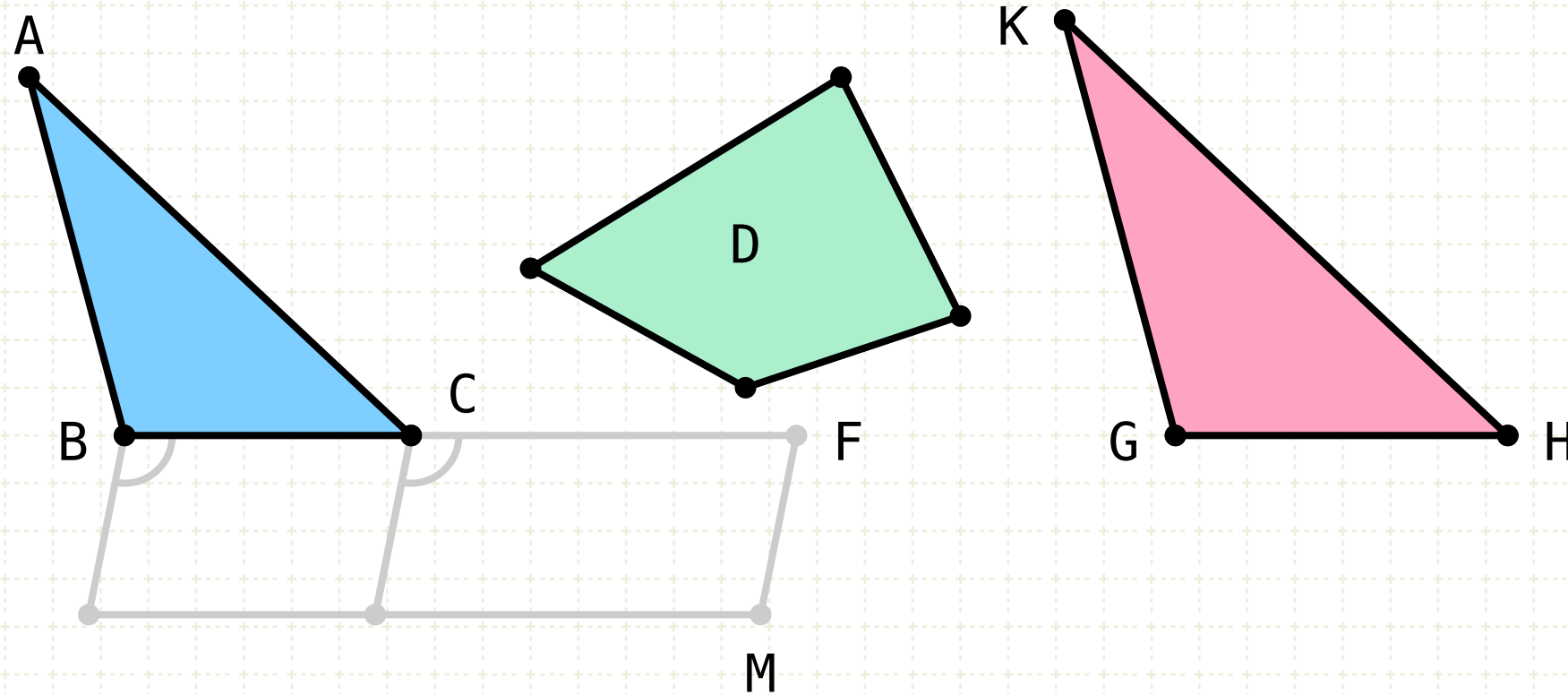
Alternately, the triangle ABC to the parallelogram BE is equal to the triangle KGH to the square EF (V·16)

But the triangle ABC is equal to the parallelogram BE, so therefore the triangle KGH is equal to EF

But the polygon D is equal to the parallelogram EF, so therefore the triangle KGH is equal to D

Proposition 25 of Book VI

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure



$$\square BE = \Delta ABC$$

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$$BC:GH = GH:CF$$

$$\Delta ABC \sim \Delta KGH$$

$$\Delta ABC:\Delta KGH = BC:CF$$

$$BC:CF = \square BE:\square EF$$

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$$\Delta KGH = D$$

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Thus the ratio of BC to CF is the ratio of the two triangles ABC to KGH

But the ratio of the parallelograms BE to EF is equal to the ratio of their bases, BC to CF (VI·1)

The ratio of BC to CF is equal to both the ratio of the triangles ABC and KGH, and to the ratio of the parallelograms BE and EF

Therefore the ratio of the parallelograms is equal to the ratio of the triangles

Alternately, the triangle ABC to the parallelogram BE is equal to the triangle KGH to the square EF (V·16)

But the triangle ABC is equal to the parallelogram BE, so therefore the triangle KGH is equal to EF

But the polygon D is equal to the parallelogram EF, so therefore the triangle KGH is equal to D

Thus KGH is similar to ABC, and equal in area to D



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