# B G G D H

# Euclid's Elements

# Book III

A circle is a round straight line with a hole in the middle.

### **Mark Twain**

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)



### **Table of Contents, Chapter 3**

- 1 To find the centre of a circle
- 2 A chord of a circle always lies inside the circle
- A line through the centre of a circle bisects a chord, and vice versa
- 4 A line not through the centre of a circle does not bisect a chord
- 5 If two circles cut one another, they will not have the same center
- 6 If two circles touch one another, they will not have the same center
- 7 Consider two lines from a point inside a circle to the edge, the longer one will be the one closest to the longest part of the diameter passing through the original point
- 8 Consider two lines from a point outside a circle to the edge, the line closest to the centre will be longer on the concave side and shorter on the convex side

- 9 If three lines, starting at a point 'A' and touching the circle, are all equal, then 'A' is the centre of the circle
- 10 A circle does not cut a circle at more points than two
- 11 Point of contact between two internal circles, and their centres, are collinear
- 12 Point of contact between two external circles, and their centres, are collinear
- 13 A circle does not touch a circle at more points than one, whether it touch it internally or externally.
- In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another.
- The longest line in a circle is its diameter, shorter the farther away from the diameter
- 16 A line on the circle, perpendicular to the diameter, lies outside the circle

- 17 From a given point to draw a straight line touching a given circle
- 18 If line touches a circle, then it is perpendicular to the diameter that touches that point
- 19 If line touches a circle, then the centre of the circle lies on a line perpendicular to the original
- The angle at the centre of a circle is twice that from an angle from the circumference
- In a circle the angles in the same segment are equal to one another
- The opposite angles of quadrilaterals in circles are equal to two right angles
- On the same straight line there cannot be constructed two similar and unequal segments of circles on the same side
- 24 Similar segments of circles on equal straight lines are equal to one another



### **Table of Contents, Chapter 3**

- 25 Given a segment of a circle, to describe the complete circle of which it is a segment.
- 26 In equal circles equal angles stand on equal circumferences
- 27 In equal circles angles standing on equal circumferences are equal to one another
- 28 In equal circles equal straight lines cut off equal circumferences
- 29 In equal circles equal circumferences are subtended by equal straight lines
- 30 To bisect a given circumference
- In a circle the angle in the semicircle is right ...
- The angle between a tangent and a straight line cutting a circle is equal to the angle in the alternate segment
- 33 Construct a circle segment on a given line, such that the angle within the segment is equal to a given angle

- 34 Construct a circle segment on a given circle, such that the angle within the segment is equal to a given angle
- 35 If two circle chords intersect, the segments on one multiplied together equals the segments of the other multiplied together
- 36 Secant-tangent law
- 37 Converse of the secant-tangent law



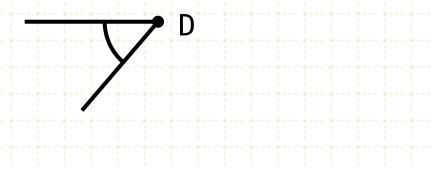
Proposition 34 of Book III

From a given circle to cut off a segment admitting an angle equal to a given rectilineal angle.



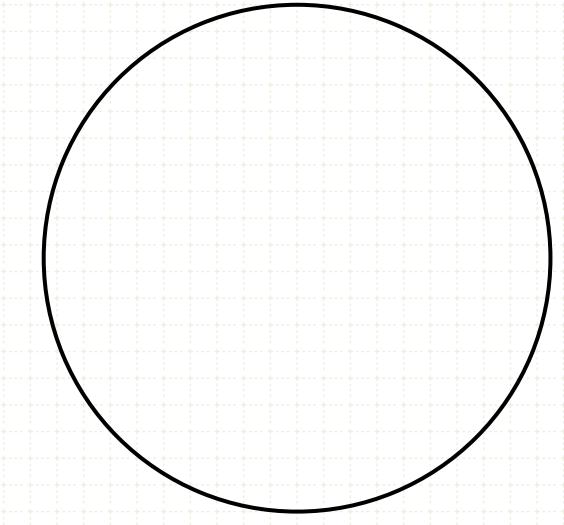
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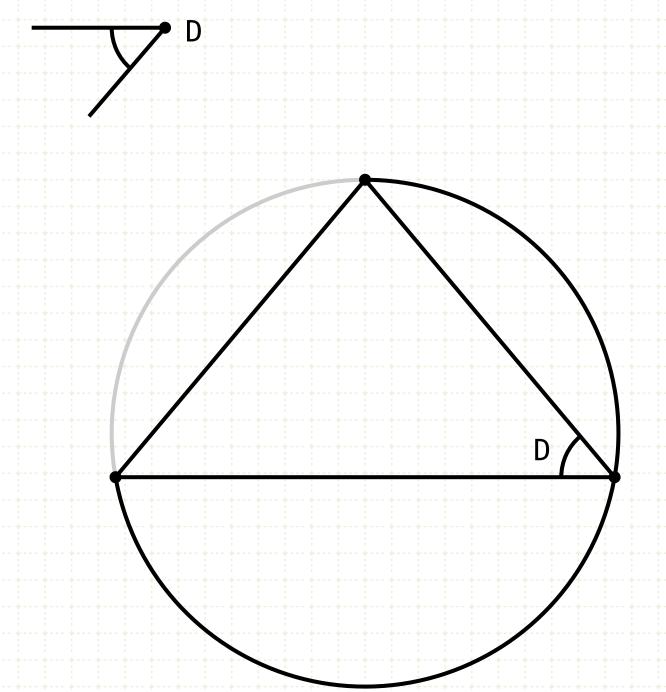


## In other words

Given an angle D, and a circle...



From a given circle to cut off a segment admitting an angle equal to a given rectilineal angle.



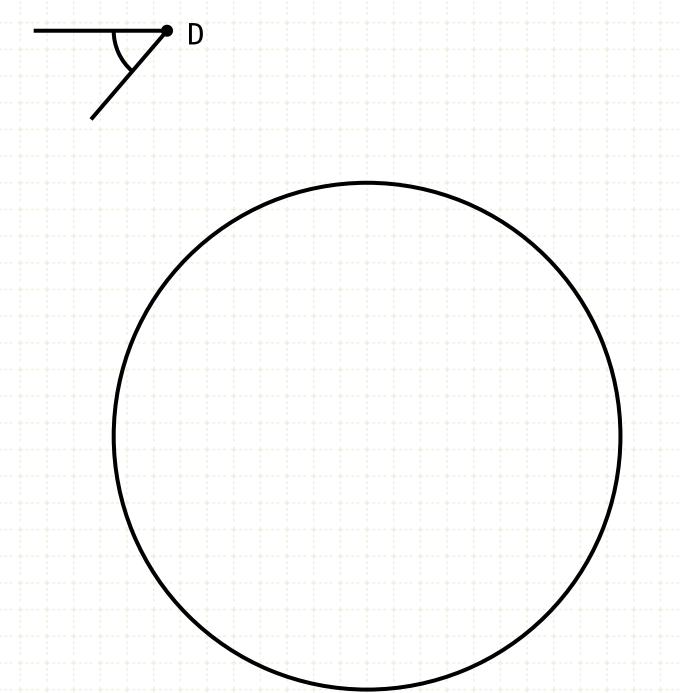
### In other words

Given an angle D, and a circle...

... construct a circle segment such that the angle within the segment is equal to D

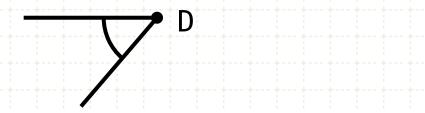
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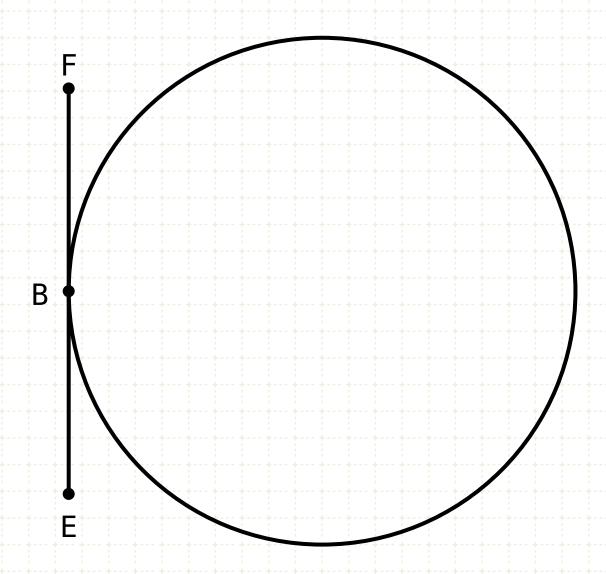
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### Construction

From a given circle to cut off a segment admitting an angle equal to a given rectilineal angle.

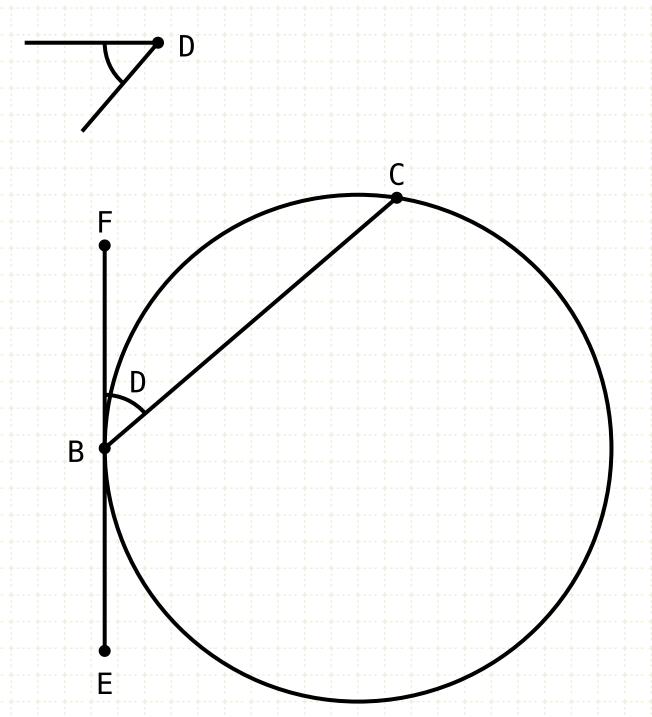




### Construction

Draw a line FE touching the circle at point B (III·17)

From a given circle to cut off a segment admitting an angle equal to a given rectilineal angle.

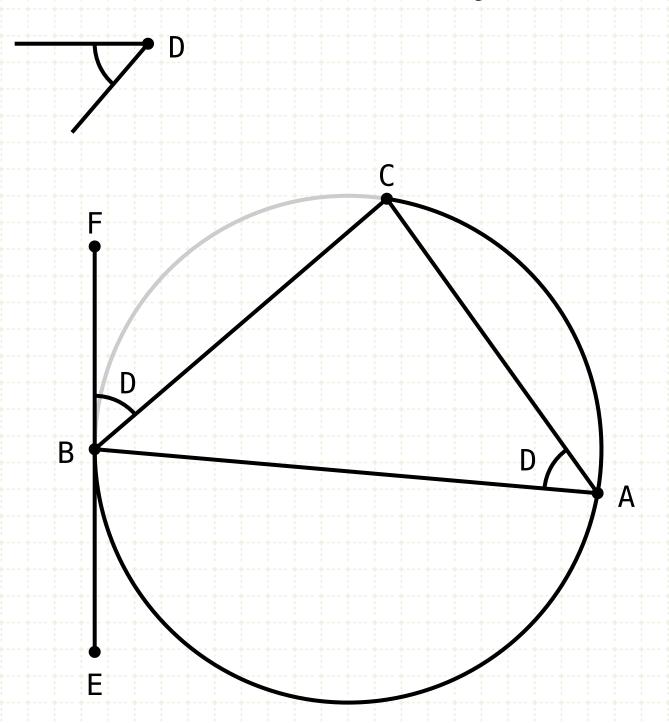


### Construction

Draw a line FE touching the circle at point B (III-17)

Copy the angle D to line FB, such that angle FBC equals D (I-23)

From a given circle to cut off a segment admitting an angle equal to a given rectilineal angle.



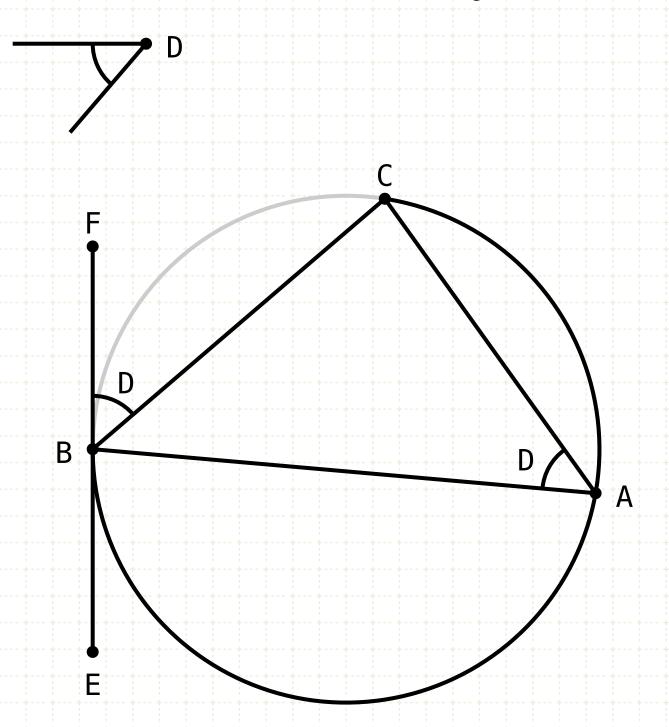
### Construction

Draw a line FE touching the circle at point B (III-17)

Copy the angle D to line FB, such that angle FBC equals D (I-23)

The circle segment BCA contains the angle D

From a given circle to cut off a segment admitting an angle equal to a given rectilineal angle.



### Construction

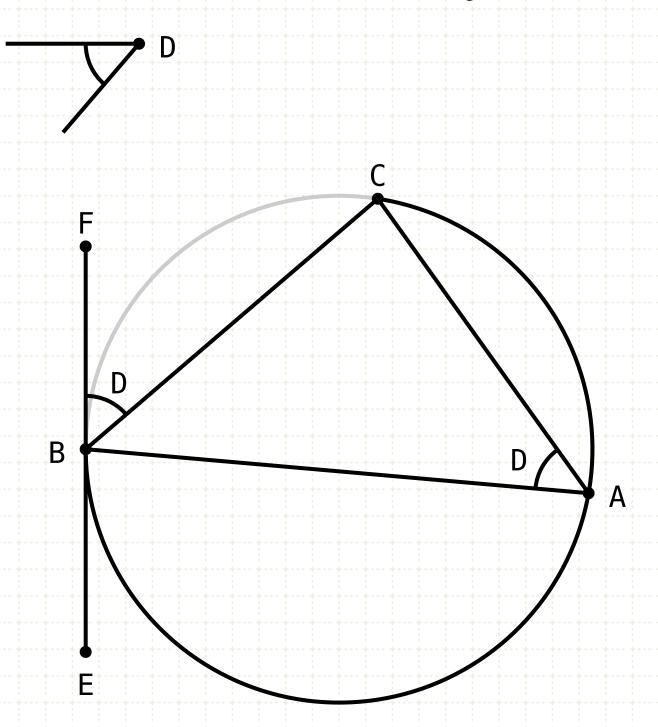
Draw a line FE touching the circle at point B (III-17)

Copy the angle D to line FB, such that angle FBC equals D (I-23)

The circle segment BCA contains the angle D

### **Proof**

From a given circle to cut off a segment admitting an angle equal to a given rectilineal angle.



### Construction

Draw a line FE touching the circle at point B (III-17)
Copy the angle D to line FB, such that angle FBC equals D (I-23)

The circle segment BCA contains the angle D

### **Proof**

Since EF touches the circle, the angle FBC equals the angle in the opposite circle segment (III-32)

The angle FBC is equal to D by construction, so thus the angle in the segment BCA equals D

### **Youtube Videos**

https://www.youtube.com/c/SandyBultena











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