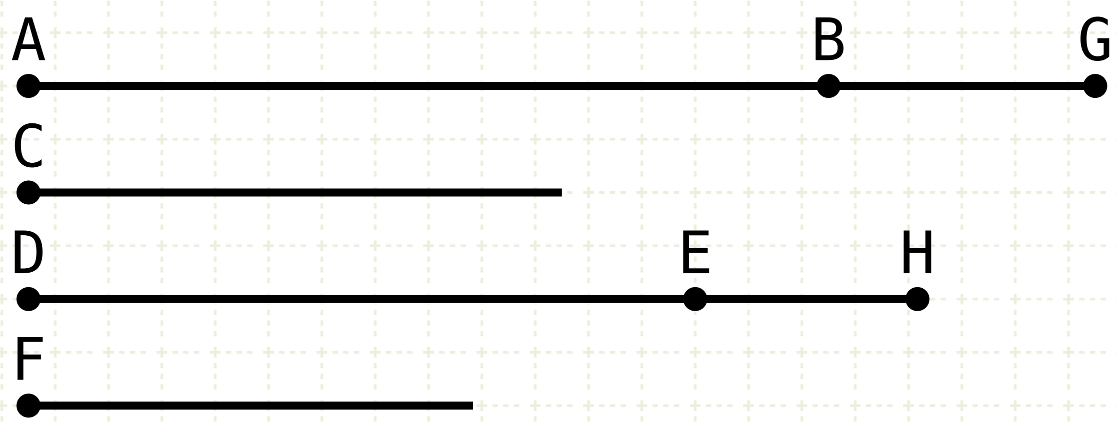
Euclid's Elements

Book V



$AB:C = DE:F$

$BG:C = EH:F$

$AG:C = DH:F$

Proportions are what makes the old Greek temples classic in their beauty. They are like huge blocks, from which the air has been literally hewn out between the columns.

Arne Jacobsen

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Proposition 12 of Book V

If any number of magnitudes be proportional, as one of the antecedents is to one of the consequents, so will all the antecedents be to all the consequents.

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If any number of magnitudes be proportional, as one of the antecedents is to one of the consequents, so will all the antecedents be to all the consequents.

$A:B = C:D \rightarrow A,B,C,D$ are proportional

$A,C \rightarrow$ antecedents
$B,D \rightarrow$ consequents

Definitions

6. Let magnitudes which have the same ratio be called PROPORTIONAL

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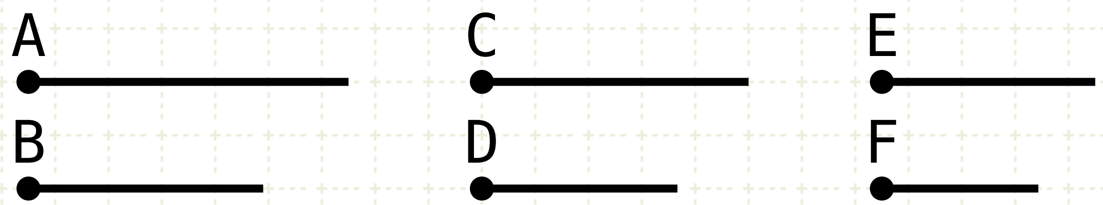

In other words

If A is to B as C is to D, and C is to D as E is to F then …

… the ratio of the sum of A,C,E to the sum of B,D,F is also the ratio of A to B

A:B = C:D = E:F

→ (A+C+E):(B+D+F) = A:B

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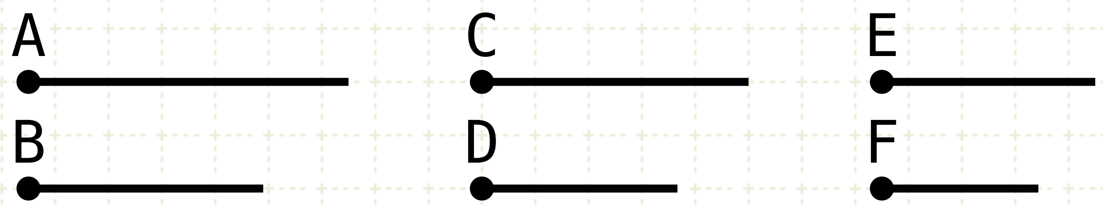

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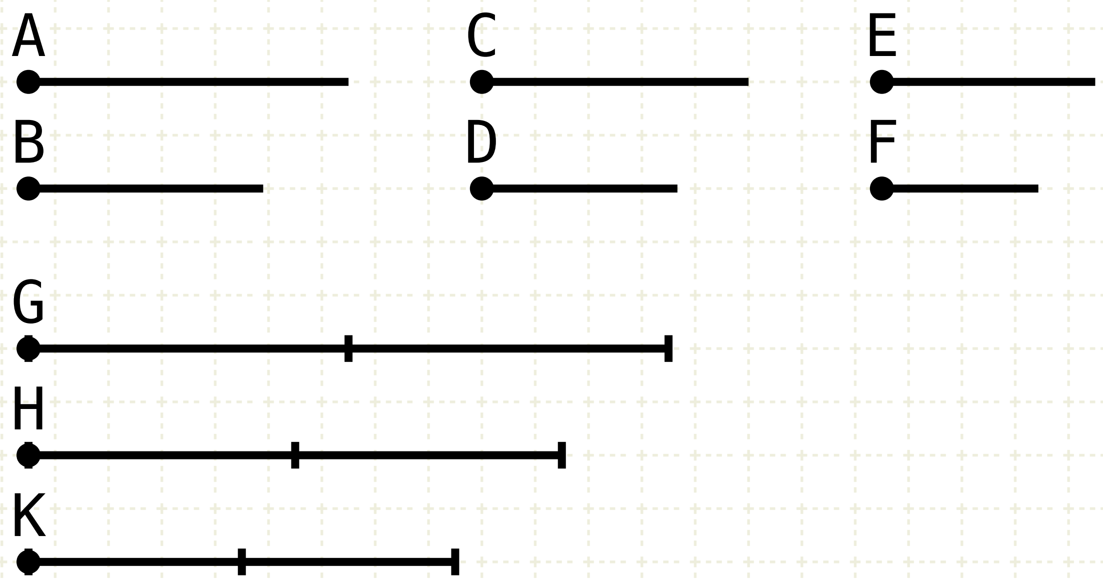

$$A:B = C:D = E:F$$

$$G = p \cdot A$$

$$H = p \cdot C$$

$$K = p \cdot E$$

In other words

If A is to B as C is to D, and C is to D as E is to F then …

… the ratio of the sum of A,C,E to the sum of B,D,F is also the ratio of A to B

Proof

Let G,H,K be equimultiples of A,C and E

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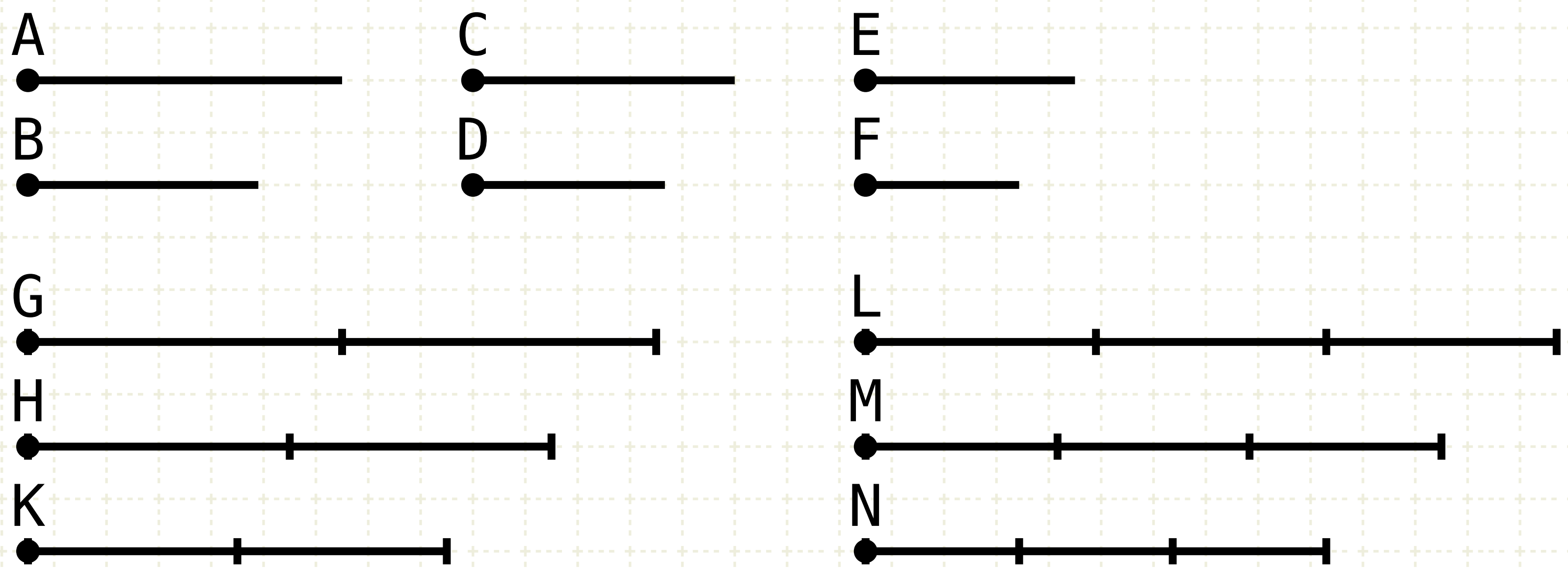

$A:B = C:D = E:F$
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$L = q \cdot B$
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In other words

If A is to B as C is to D, and C is to D as E is to F then …

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Let G,H,K be equimultiples of A,C and E

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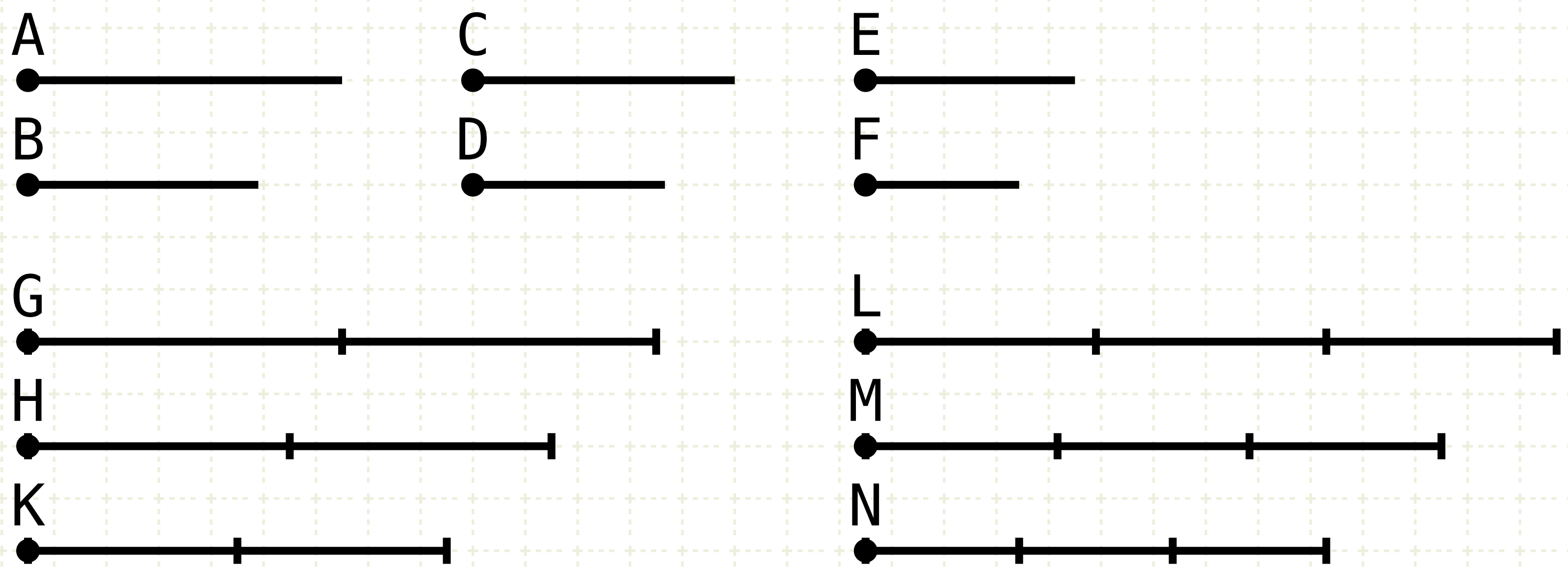

A:B = C:D = E:F
G = p·A
H = p·C
K = p·E
L = q·B
M = q·D
N = q·F

p·A >=< q·B
→ p·C >=< q·D
→ p·E >=< q·F
G >=< L
→ H >=< M
→ K >=< N

In other words

If A is to B as C is to D, and C is to D as E is to F then …

… the ratio of the sum of A,C,E to the sum of B,D,F is also the ratio of A to B

Proof

Let G,H,K be equimultiples of A,C and E

Let L,M,N be equimultiples of B,D and F

Since the ratios A to B, C to D, E to F are equal, and since equimultiples G,H,K and L,M,N have been taken…

… so if G is less than L, then H is less than M and K is less than N, etc (V.def 5)

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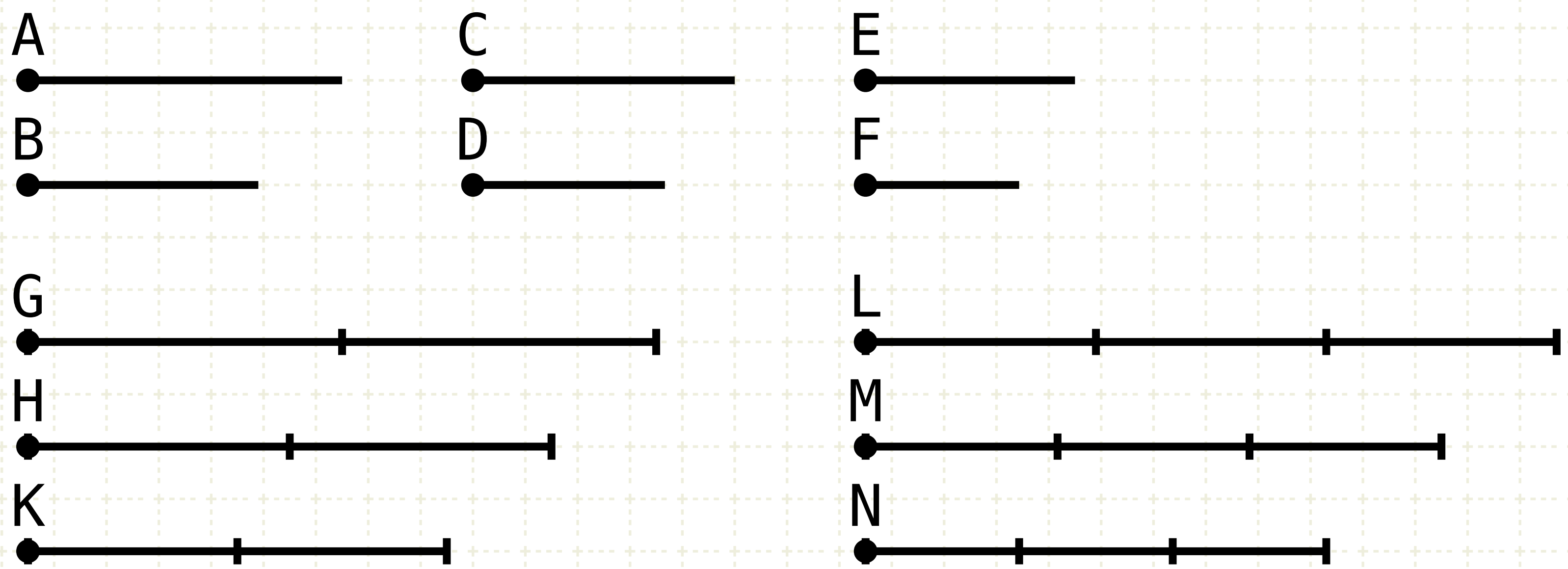

$A:B = C:D = E:F$

$G = p \cdot A$

$H = p \cdot C$

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$L = q \cdot B$

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$p \cdot A \gtreqless q \cdot B$

$\rightarrow p \cdot C \gtreqless q \cdot D$

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$G \gtreqless L$

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$\rightarrow G + H + K \gtreqless L + M + N$

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Proof

Let G,H,K be equimultiples of A,C and E

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Since the ratios A to B, C to D, E to F are equal, and since equimultiples G,H,K and L,M,N have been taken…

… so if G is less than L, then H is less than M and K is less than N, etc (V.def 5)

If G is less than L, and H is less than M and K is less than N then the sum of G,H,K will be less than L,M,N

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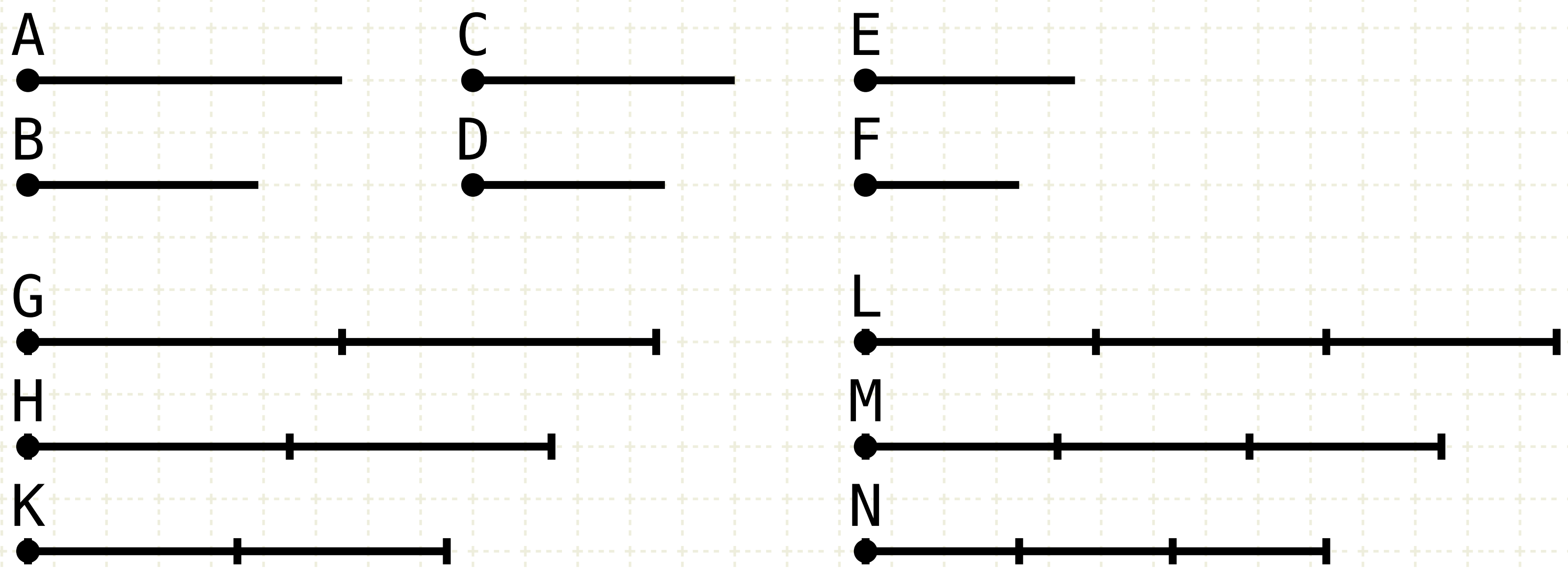

$$A:B = C:D = E:F$$
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$$p\cdot A \gtreqless q\cdot B$$
$$\rightarrow p\cdot C \gtreqless q\cdot D$$
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$$G \gtreqless L$$
$$\rightarrow H \gtreqless M$$
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$$\rightarrow G + H + K \gtreqless L + M + N$$
$$G + H + K = p\cdot(A + C + E)$$

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If A is to B as C is to D, and C is to D as E is to F then …

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Let G,H,K be equimultiples of A,C and E

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But G,H,K are equimultiples of A,C,E, so the sum G,H,K is the same multiple of the sum A,C,E (V·1)

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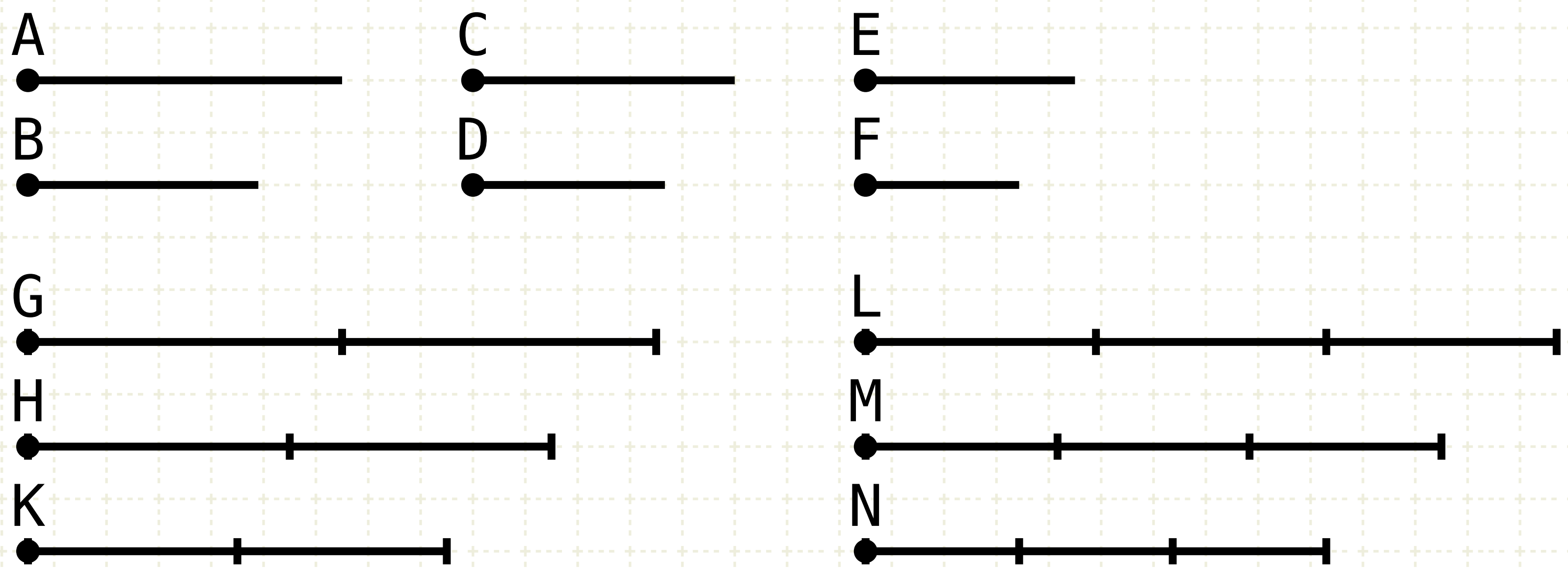

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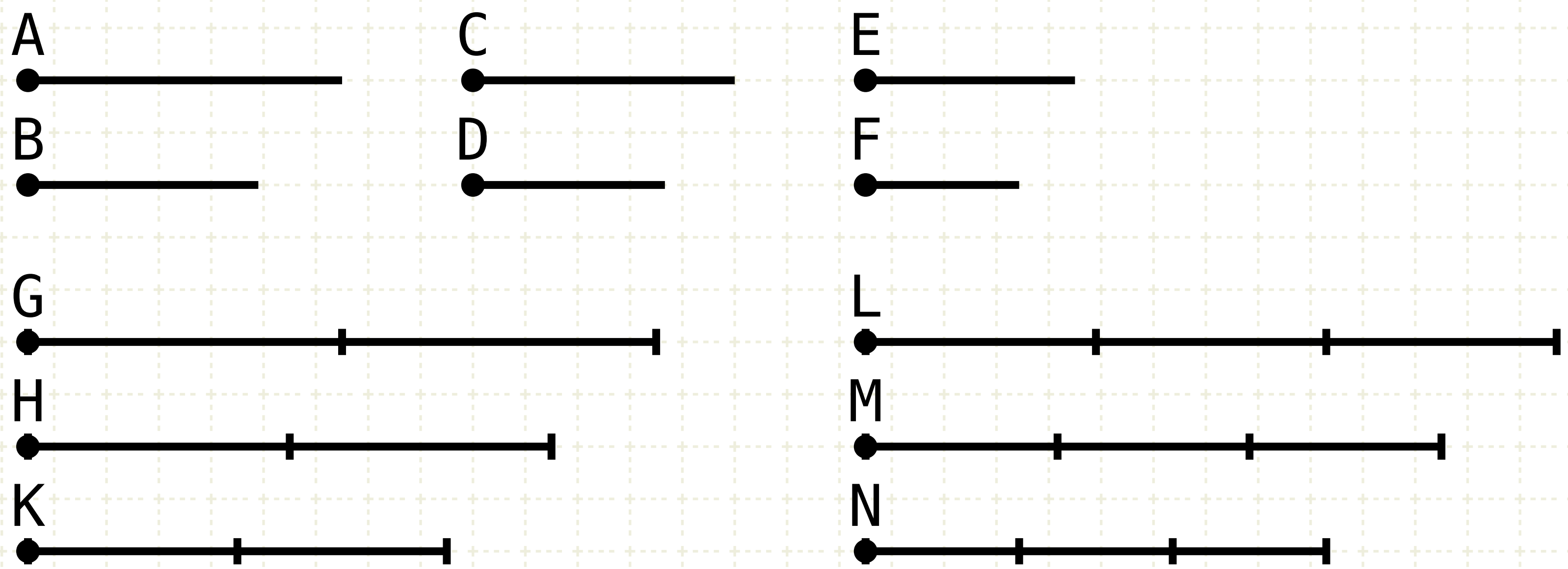

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$\rightarrow G + H + K >=< L + M + N$

$G + H + K = p \cdot (A + C + E)$

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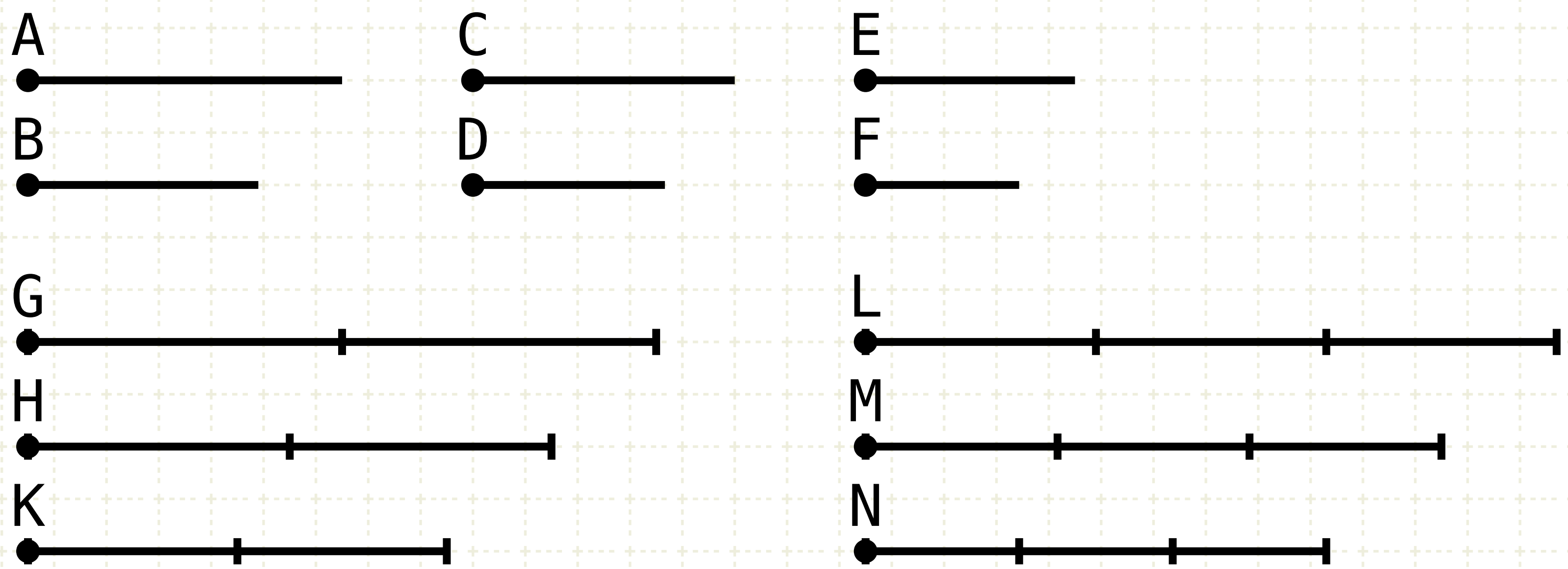

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Thus, the sum of A,C,E is to the sum of B,D,F is equal in ratio to A to B (V def.5)

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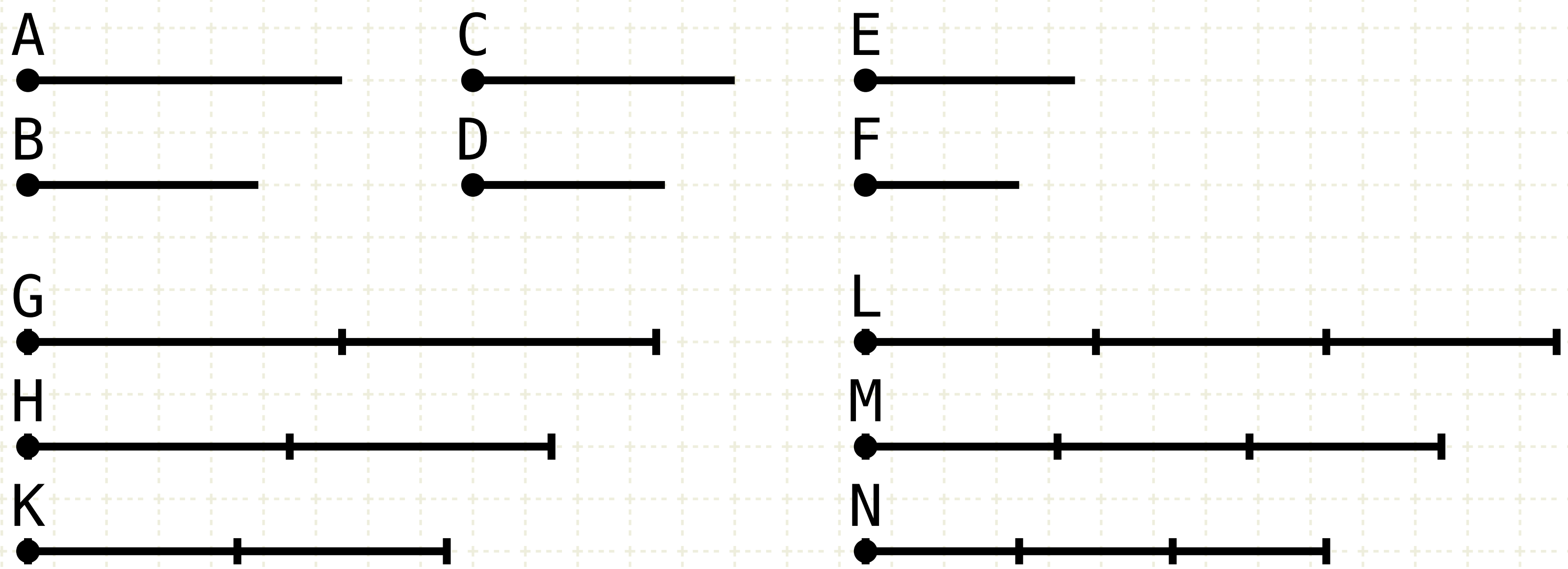

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