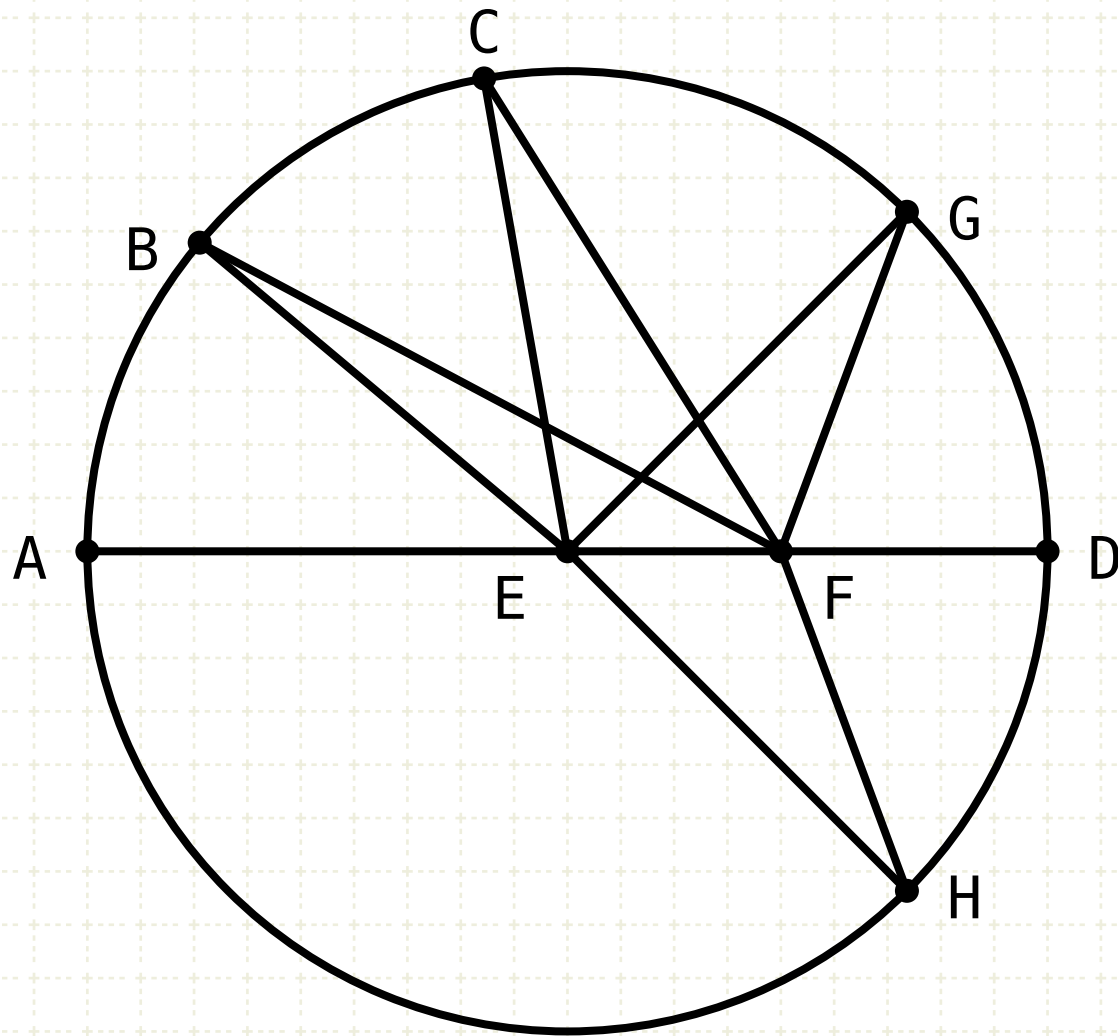


Euclid's Elements

Book III



A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)



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2	A chord of a circle always lies inside the circle	10	A circle does not cut a circle at more points than two	18	If line touches a circle, then it is perpendicular to the diameter that touches that point
3	A line through the centre of a circle bisects a chord, and vice versa	11	Point of contact between two internal circles, and their centres, are collinear	19	If line touches a circle, then the centre of the circle lies on a line perpendicular to the original
4	A line not through the centre of a circle does not bisect a chord	12	Point of contact between two external circles, and their centres, are collinear	20	The angle at the centre of a circle is twice that from an angle from the circumference
5	If two circles cut one another, they will not have the same center	13	A circle does not touch a circle at more points than one, whether it touch it internally or externally.	21	In a circle the angles in the same segment are equal to one another
6	If two circles touch one another, they will not have the same center	14	In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another.	22	The opposite angles of quadrilaterals in circles are equal to two right angles
7	Consider two lines from a point inside a circle to the edge, the longer one will be the one closest to the longest part of the diameter passing through the original point	15	The longest line in a circle is its diameter, shorter the farther away from the diameter	23	On the same straight line there cannot be constructed two similar and unequal segments of circles on the same side
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| 26 | In equal circles equal angles stand on equal circumferences | 35 | If two circle chords intersect, the segments on one multiplied together equals the segments of the other multiplied together |
| 27 | In equal circles angles standing on equal circumferences are equal to one another | | |
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Proposition 8 of Book III

If a point be taken outside a circle and from the point straight lines be drawn through to the circle, one of which is through the centre and the others are drawn at random, then, of the straight lines which fall on the concave circumference, that through the centre is greatest, while of the rest the nearer to that through the centre is always greater than the more remote, but, of the straight lines falling on the convex circumference, that between the point and the diameter is least, while of the rest the nearer to the least is always less than the more remote, and only two equal straight lines will fall on the circle from the point, one on each side of the least.



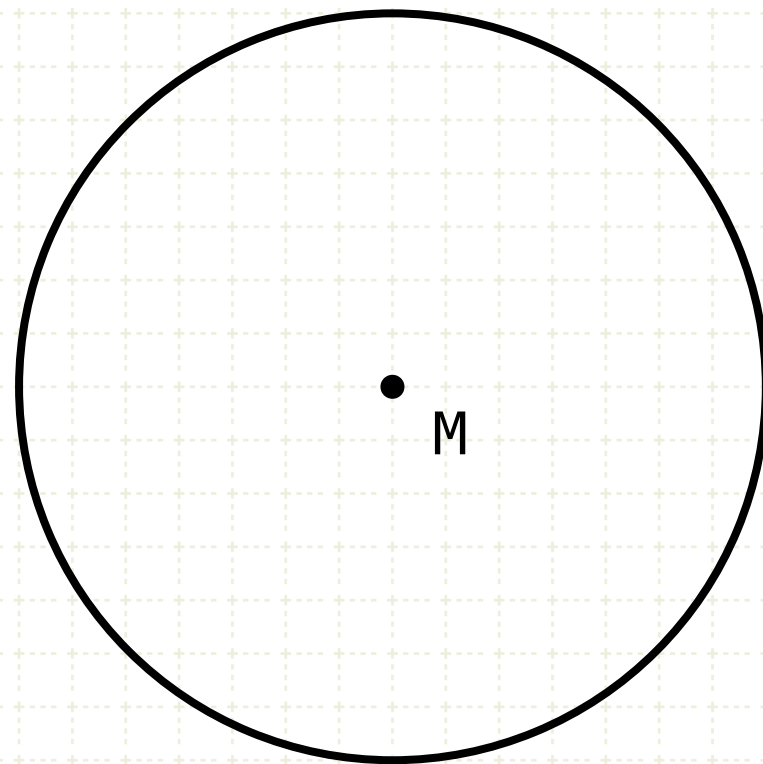
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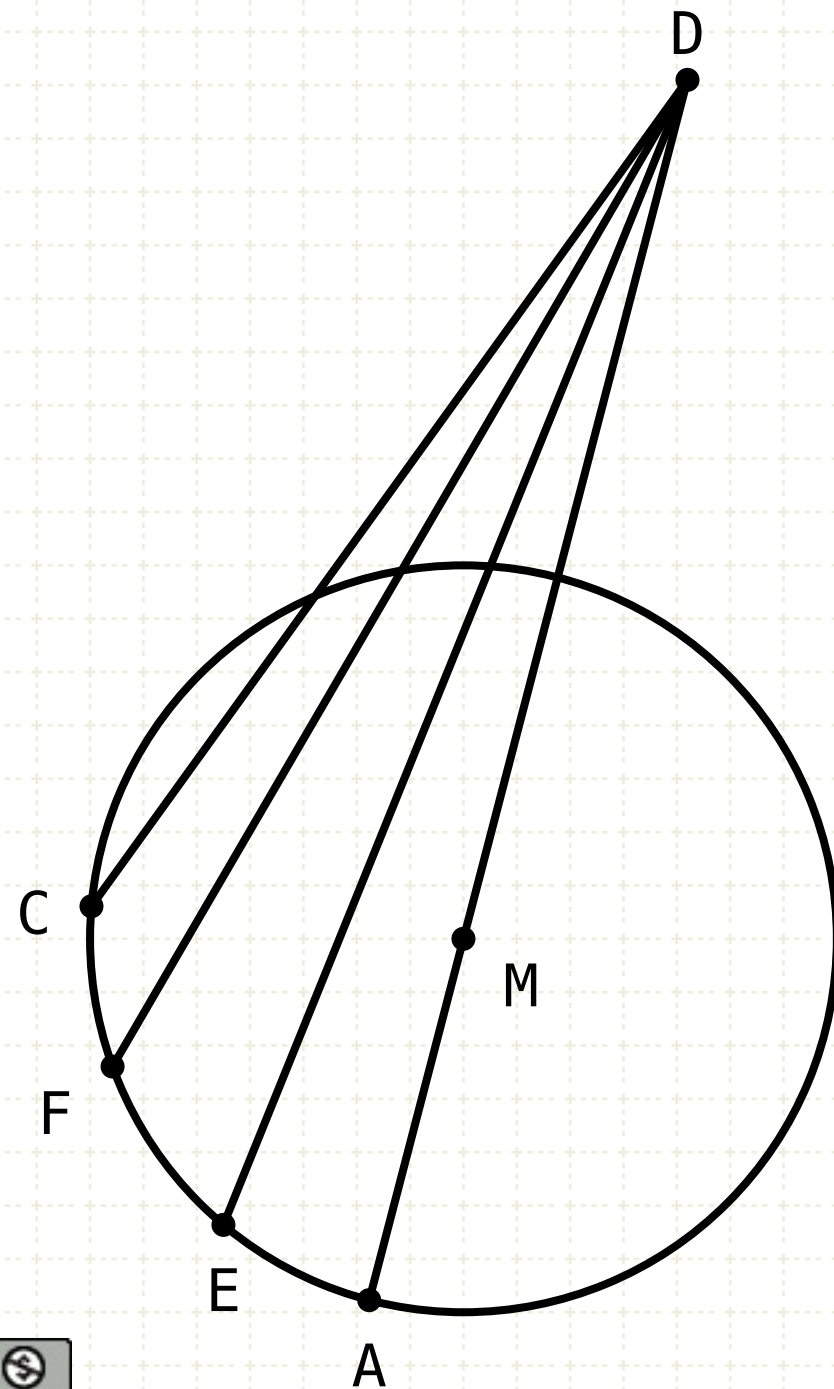
In other words

Let M be the center of a circle, and D be a point outside of the circle



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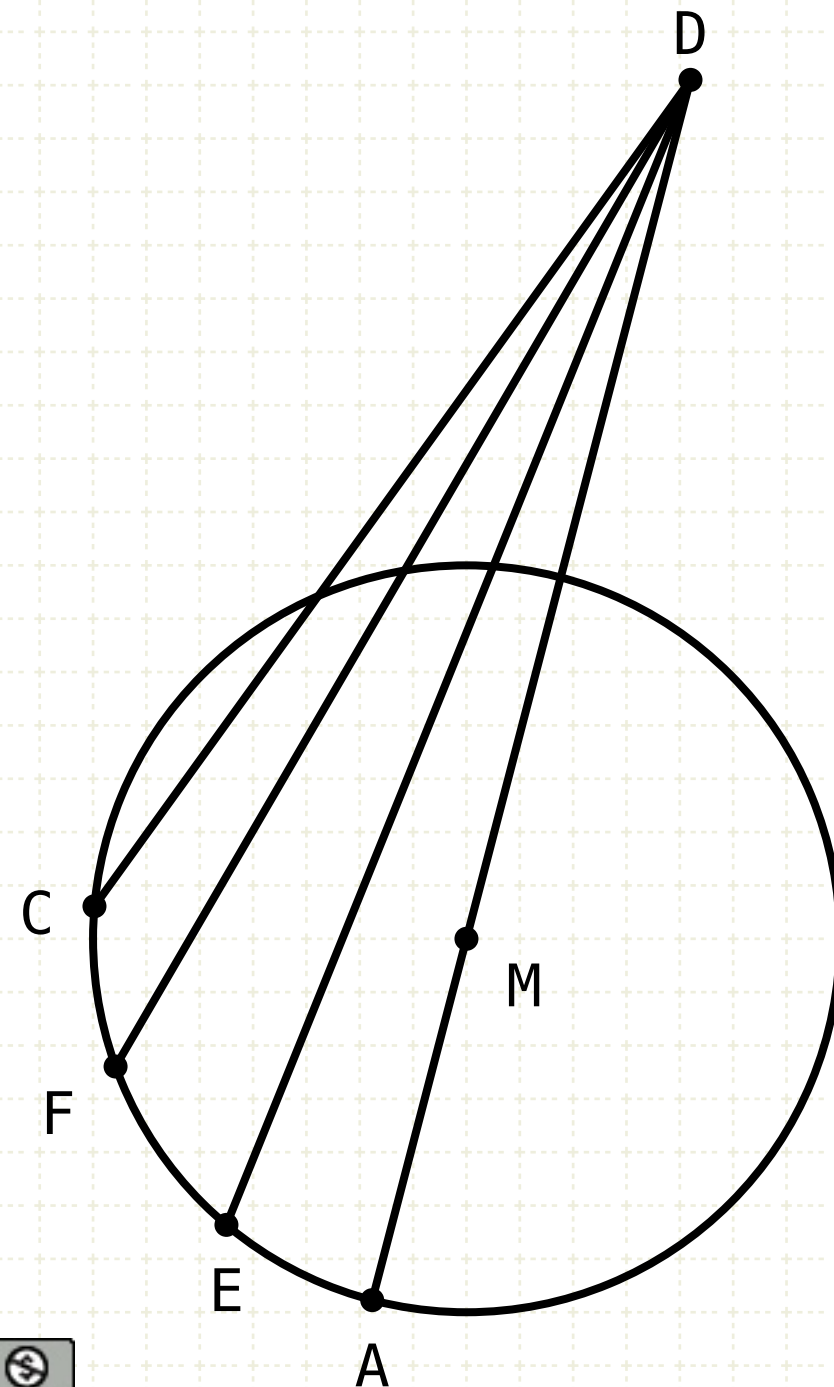
In other words

Let M be the center of a circle, and D be a point outside of the circle

Draw lines from D to points A, E, F, C on the far end of the circle, where DA passes through the center of the circle at point M

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$$DA > DE > DF > DC$$

In other words

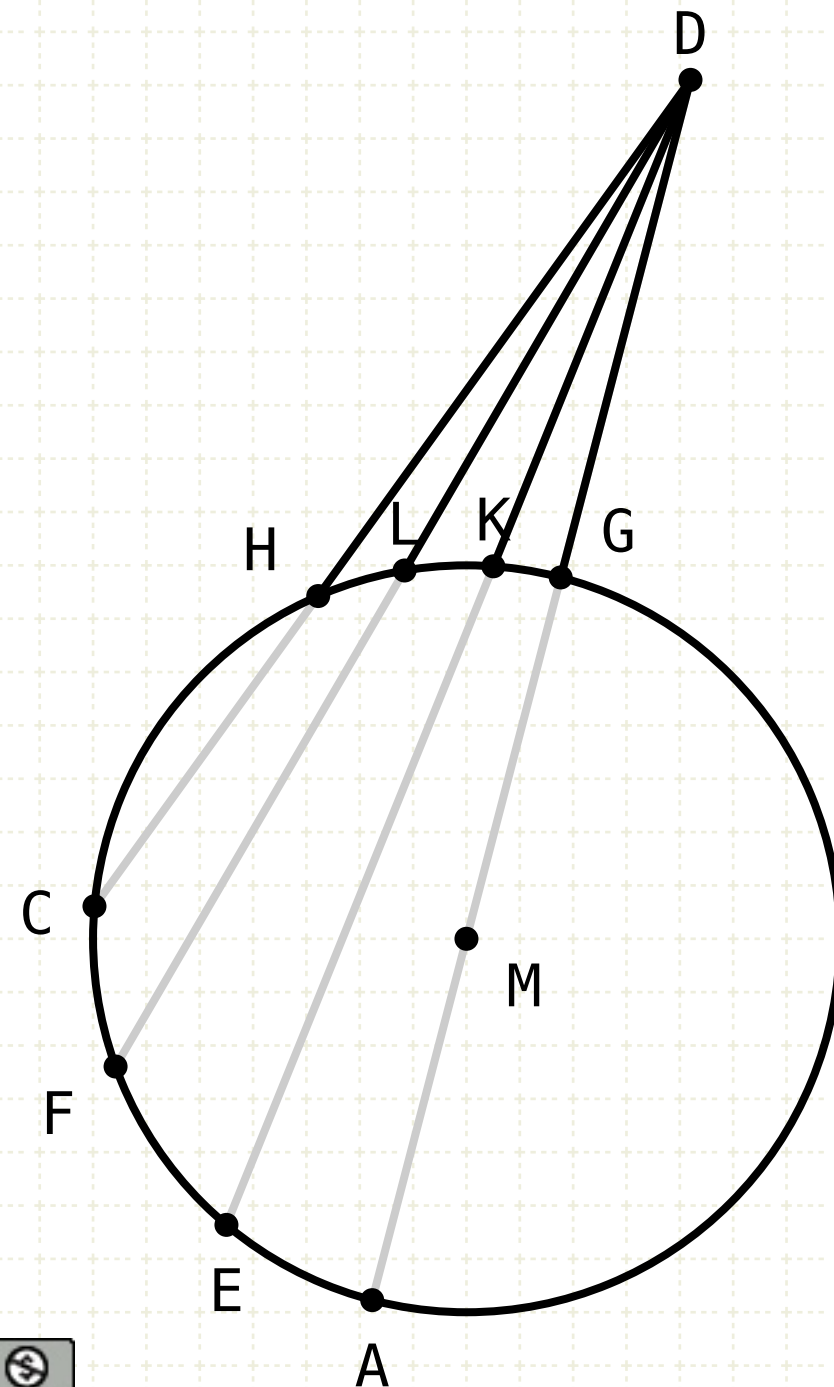
Let M be the center of a circle, and D be a point outside of the circle

Draw lines from D to points A, E, F, C on the far end of the circle, where DA passes through the center of the circle at point M

Of the lines falling on the concave part of the circle, DA is the largest, DE the next, and so on

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$$DA > DE > DF > DC$$

$$DG < DK < DL < DH$$

In other words

Let M be the center of a circle, and D be a point outside of the circle

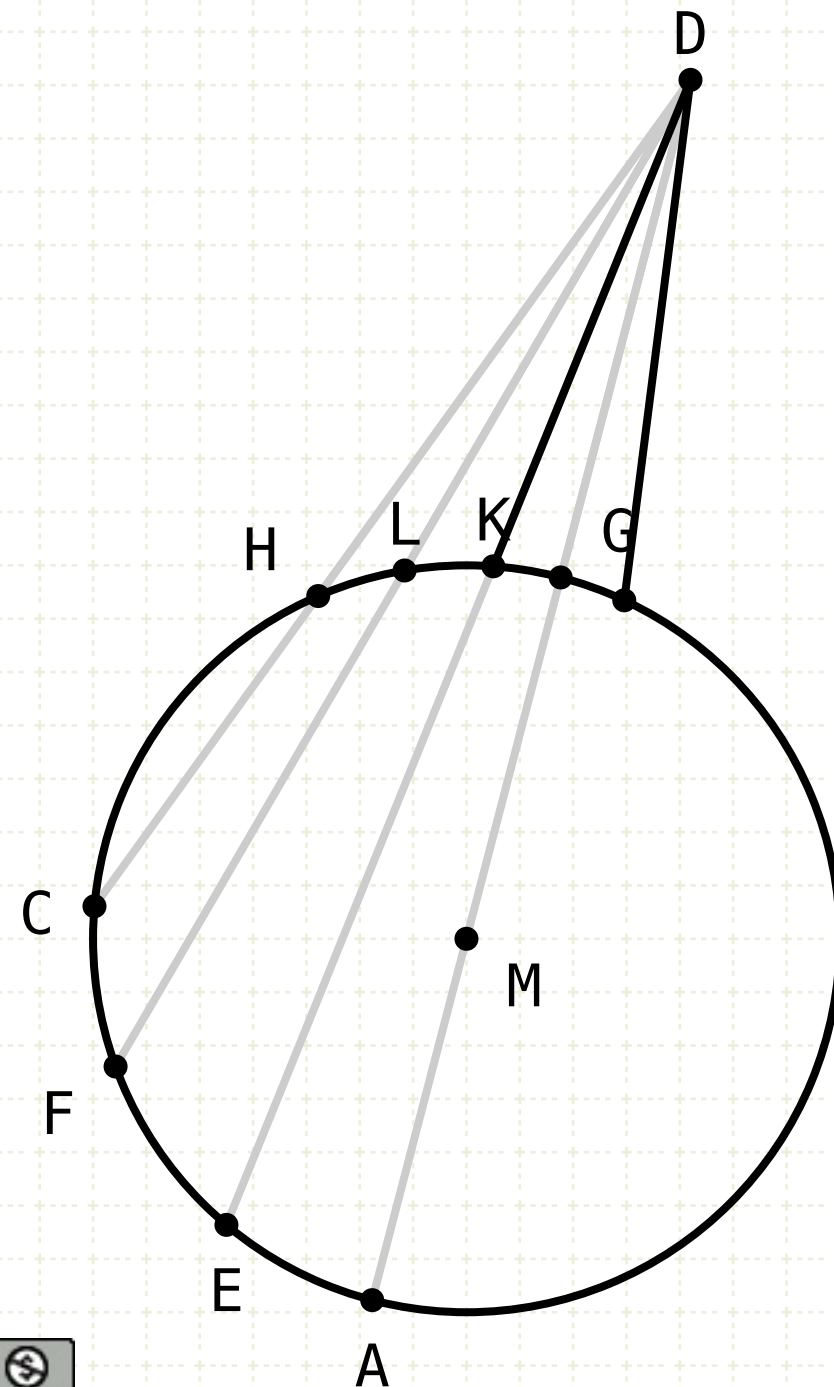
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Of the lines falling on the concave part of the circle, DA is the largest, DE the next, and so on

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$$\begin{aligned} DA &> DE > DF > DC \\ DG &< DK < DL < DH \end{aligned}$$

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Let M be the center of a circle, and D be a point outside of the circle

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Of the lines falling on the concave part of the circle, DA is the largest, DE the next, and so on

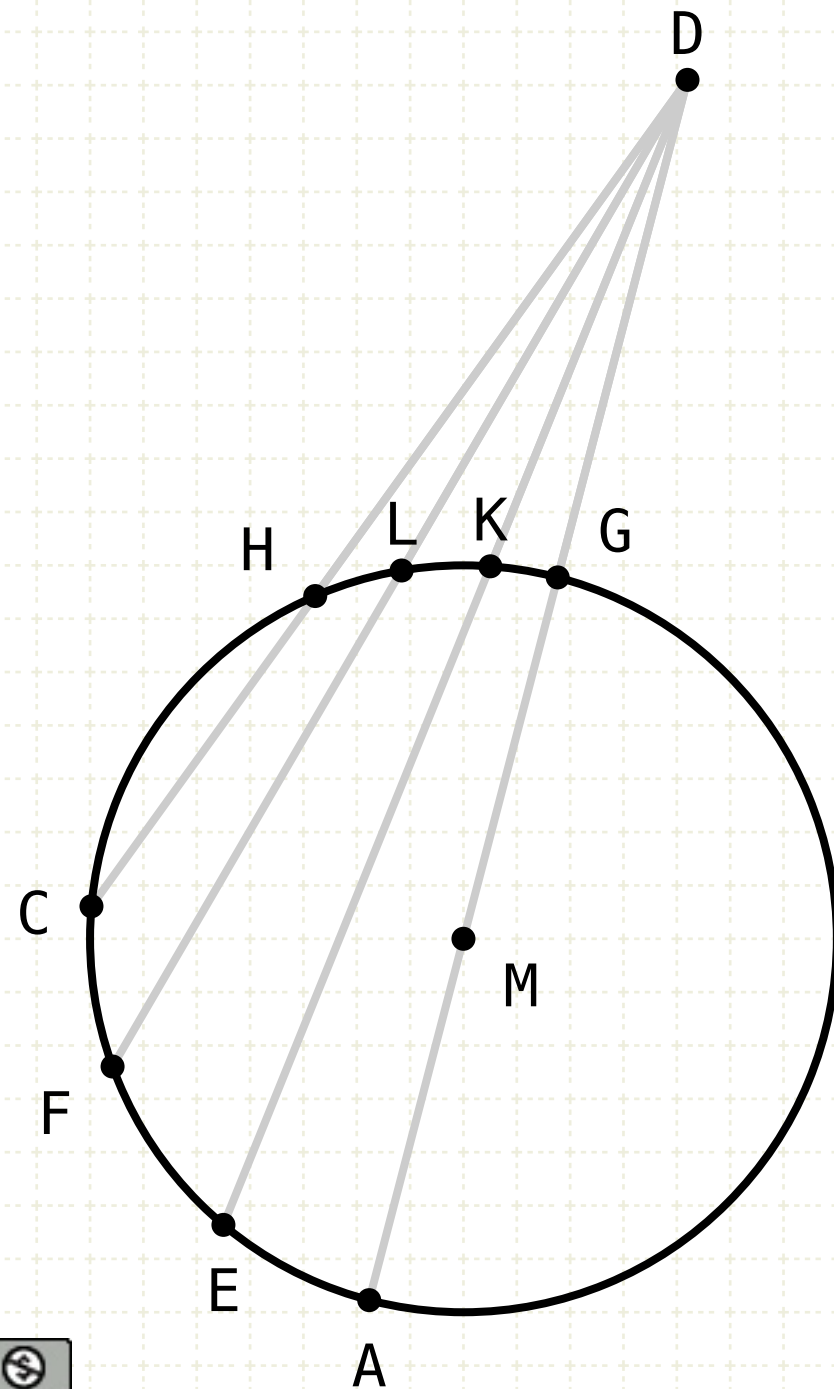
Of the lines falling on the convex part of the circle, DG is the least, DK the next, and so on

Finally, only two straight and equal lines from point D will fall on the circle, one on either side of DG

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Proof (part 1)



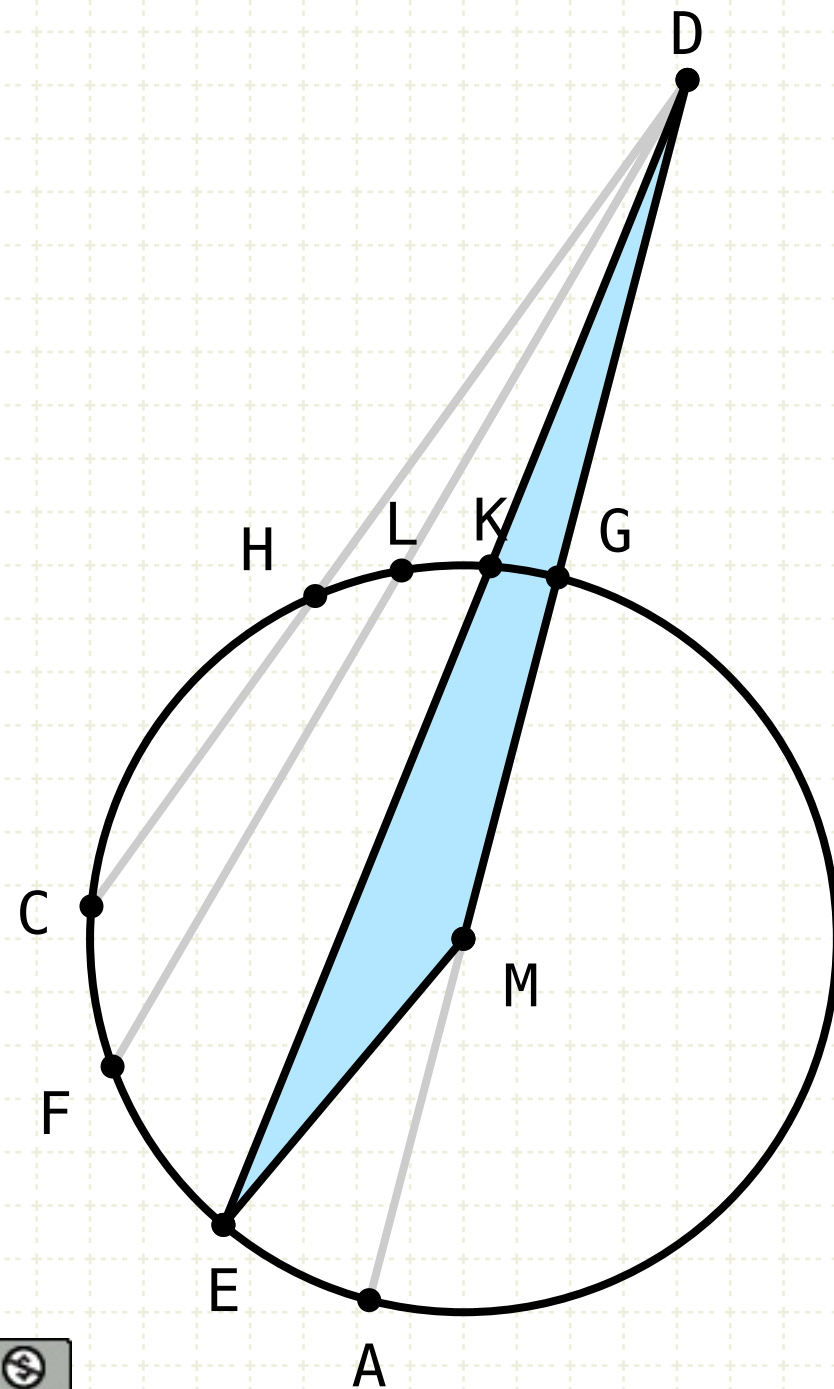
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$$EM + DM > DE$$

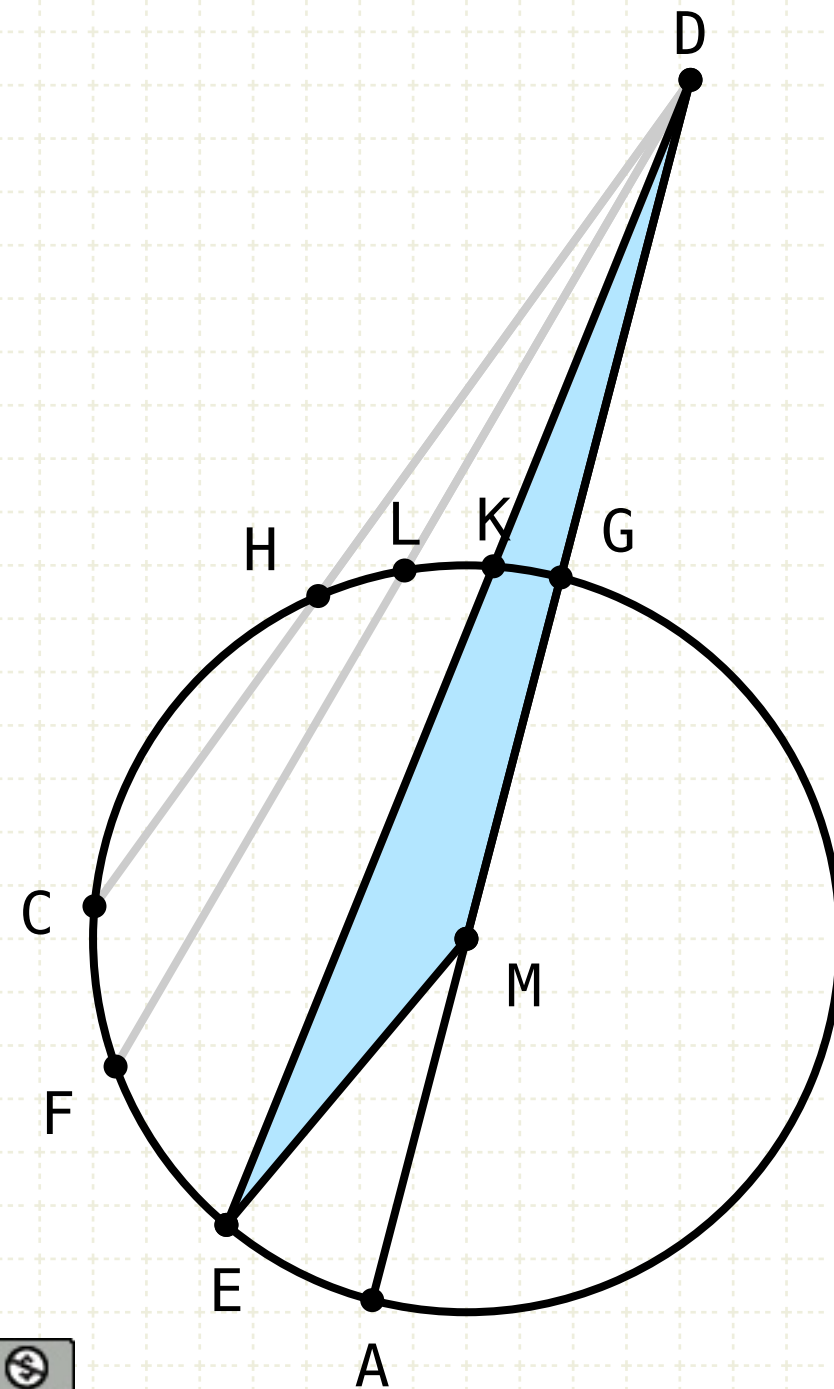
Proof (part 1)

Consider the triangle DEM, the sum of two sides of any triangle is larger than the third (I.20)



Proposition 8 of Book III

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$$EM + DM > DE$$

$$EM = AM$$

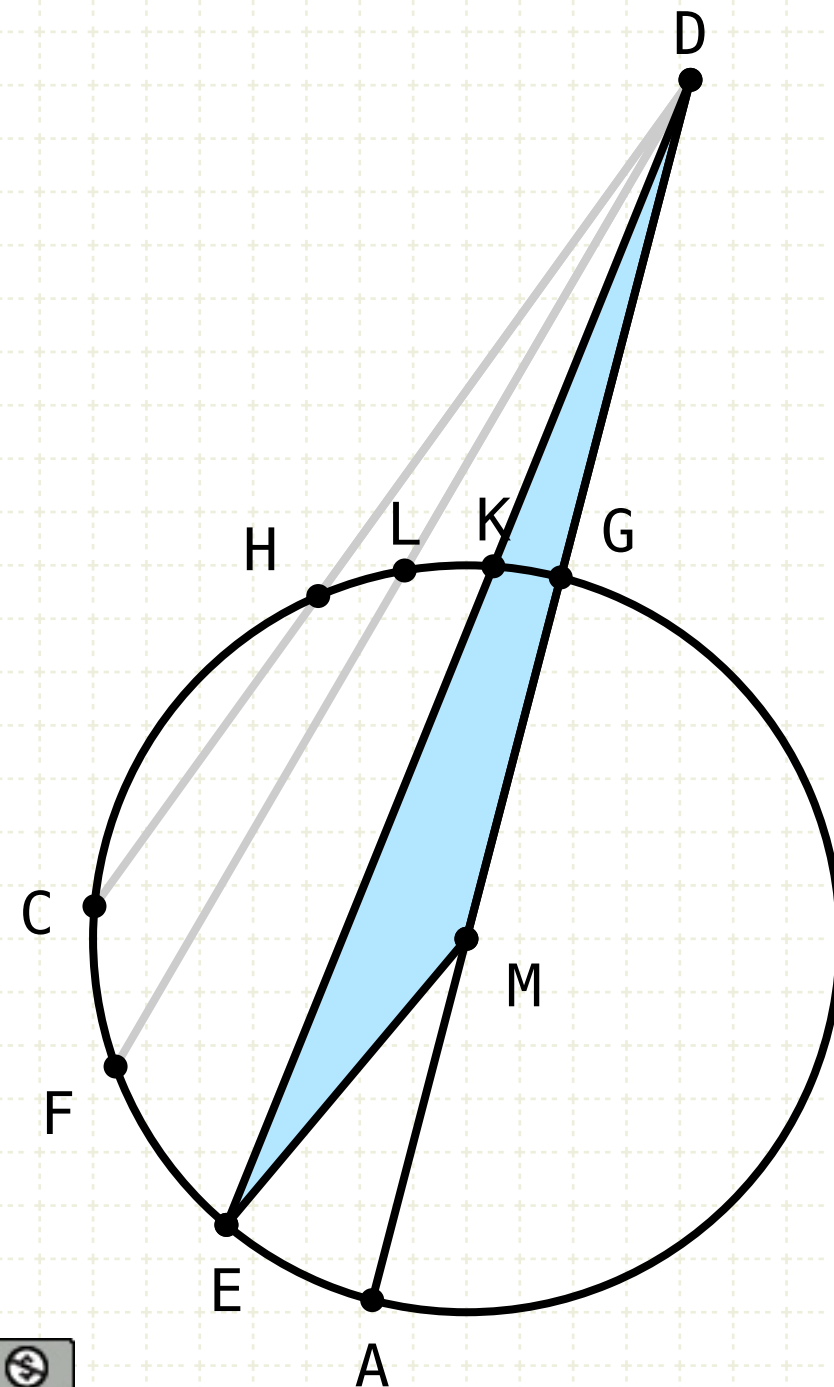
Proof (part 1)

Consider the triangle DEM, the sum of two sides of any triangle is larger than the third (I.20)

The lines EM and AM are radii of the same circle, and thus are equal

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$$EM + DM > DE$$

$$EM = AM$$

$$AM + DM > DE$$

$$DA > DE$$

Proof (part 1)

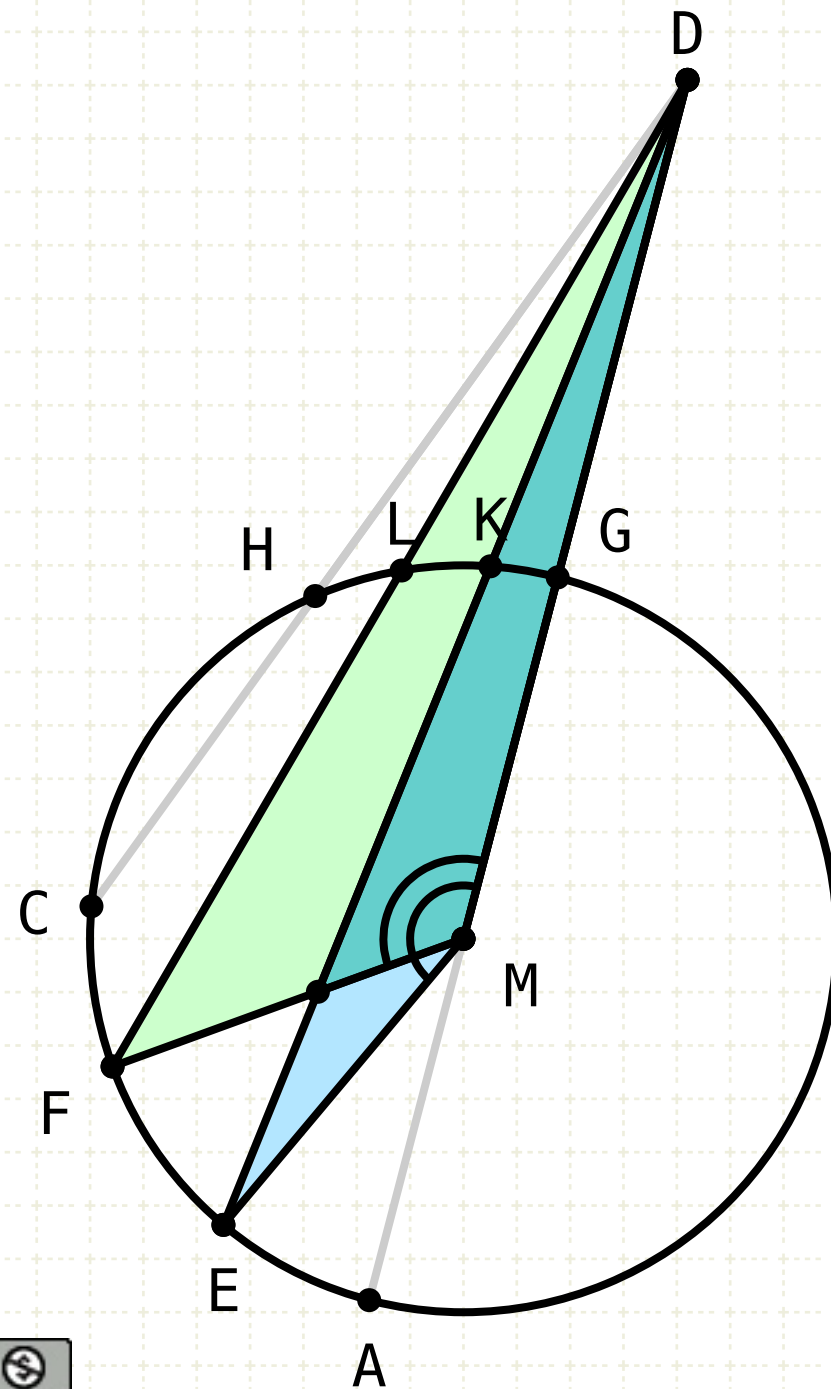
Consider the triangle DEM, the sum of two sides of any triangle is larger than the third (I.20)

The lines EM and AM are radii of the same circle, and thus are equal

Thus, AM plus DM is greater than DE, and since DA equal AM, DM, DA is greater than DE

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$$EM + DM > DE$$

EM = AM

AM + DM > DE

DA > DE

FM = EM

Proof (part 1)

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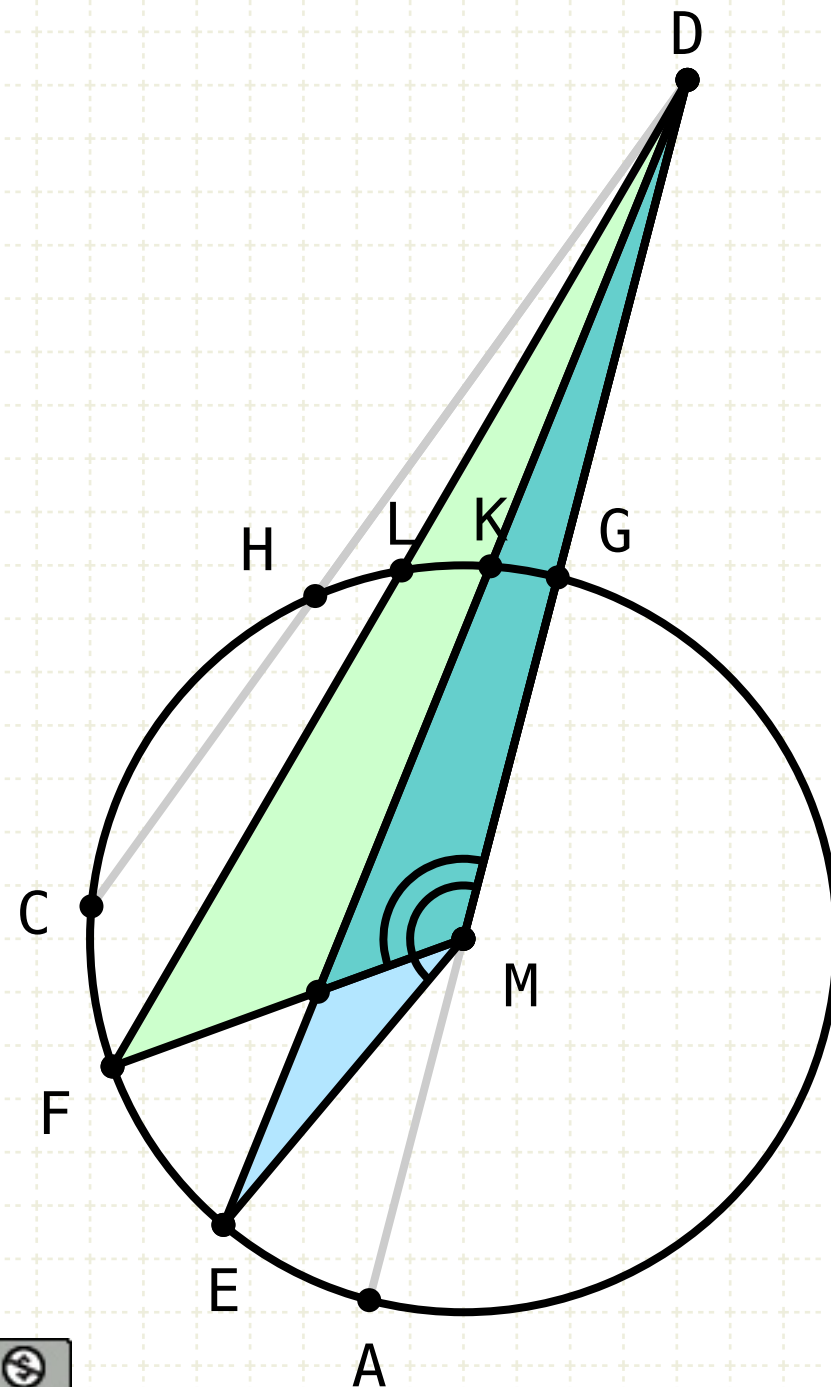
Thus, $AM + DM$ is greater than DE , and since DA equal AM, DM , DA is greater than DE

Compare the triangles DFM and DEM, FM and EM are equal, and DM is common to both, so we have two triangles with two equal sides,



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$$EM + DM > DE$$

$$EM = AM$$

$$AM + DM > DE$$

$$DA > DE$$

$$FM = EM$$

$$DE > DF$$

Proof (part 1)

Consider the triangle DEM, the sum of two sides of any triangle is larger than the third (I·20)

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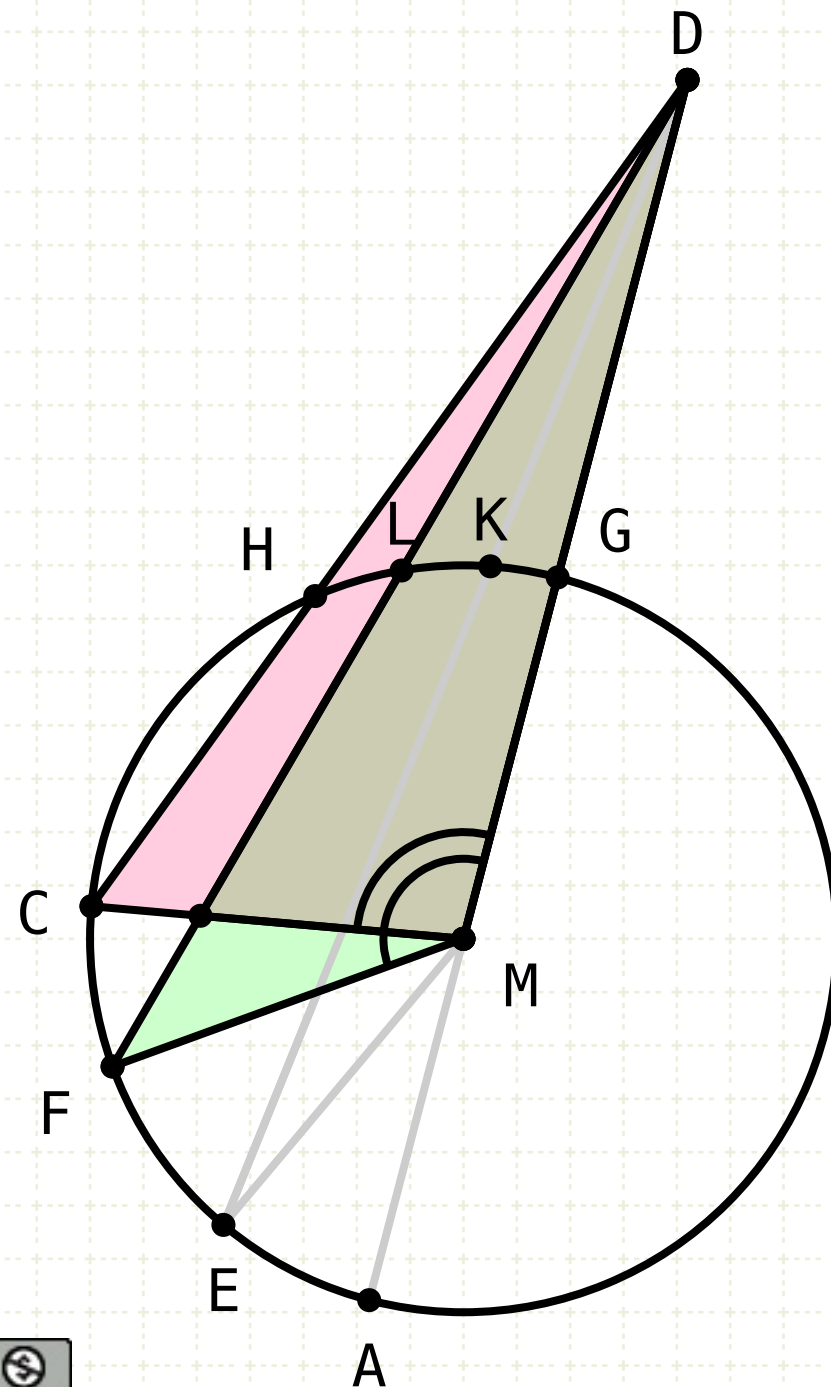
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Since the angle DME is larger than the angle DMF, DE is larger than DF (I·24)

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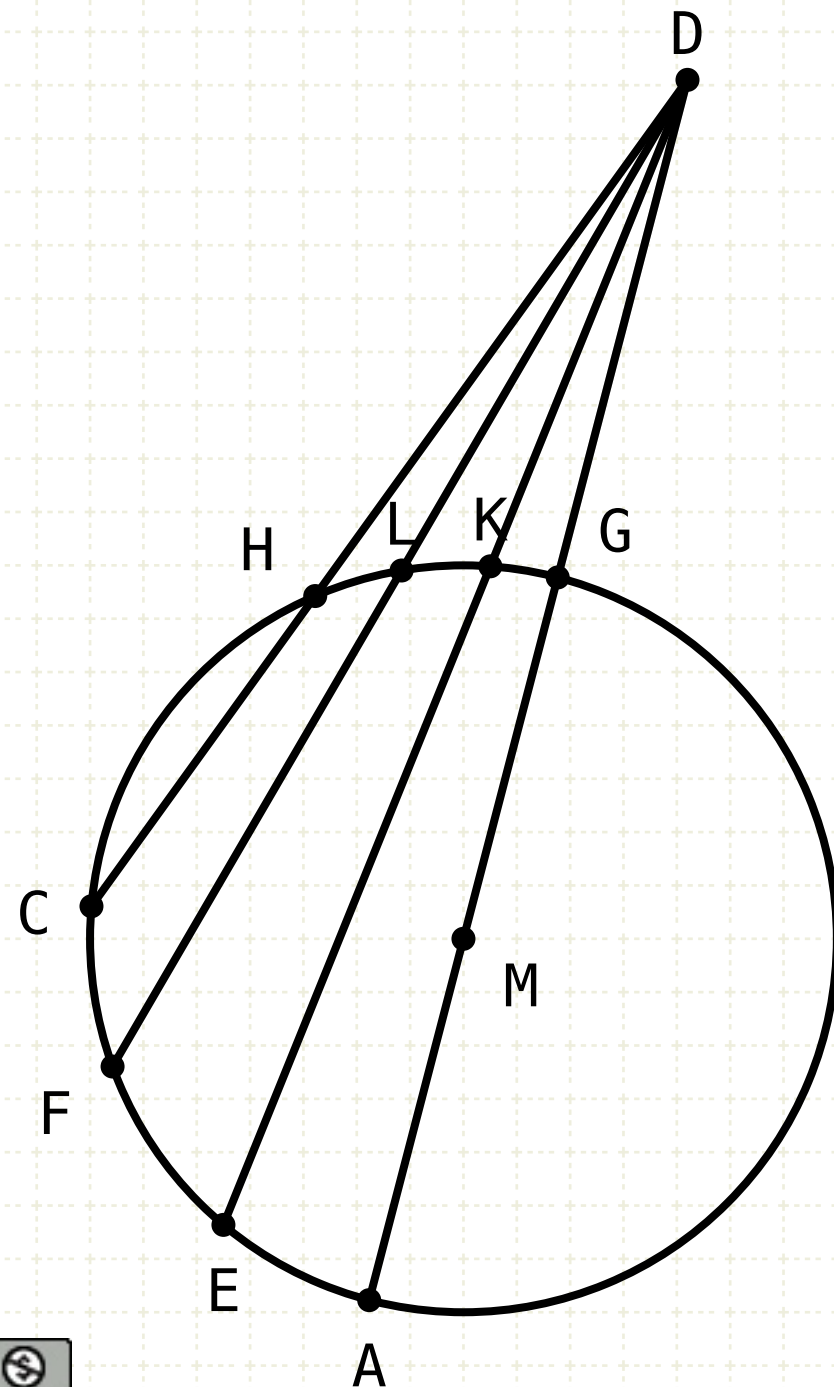
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Similarly, DF is larger than DC

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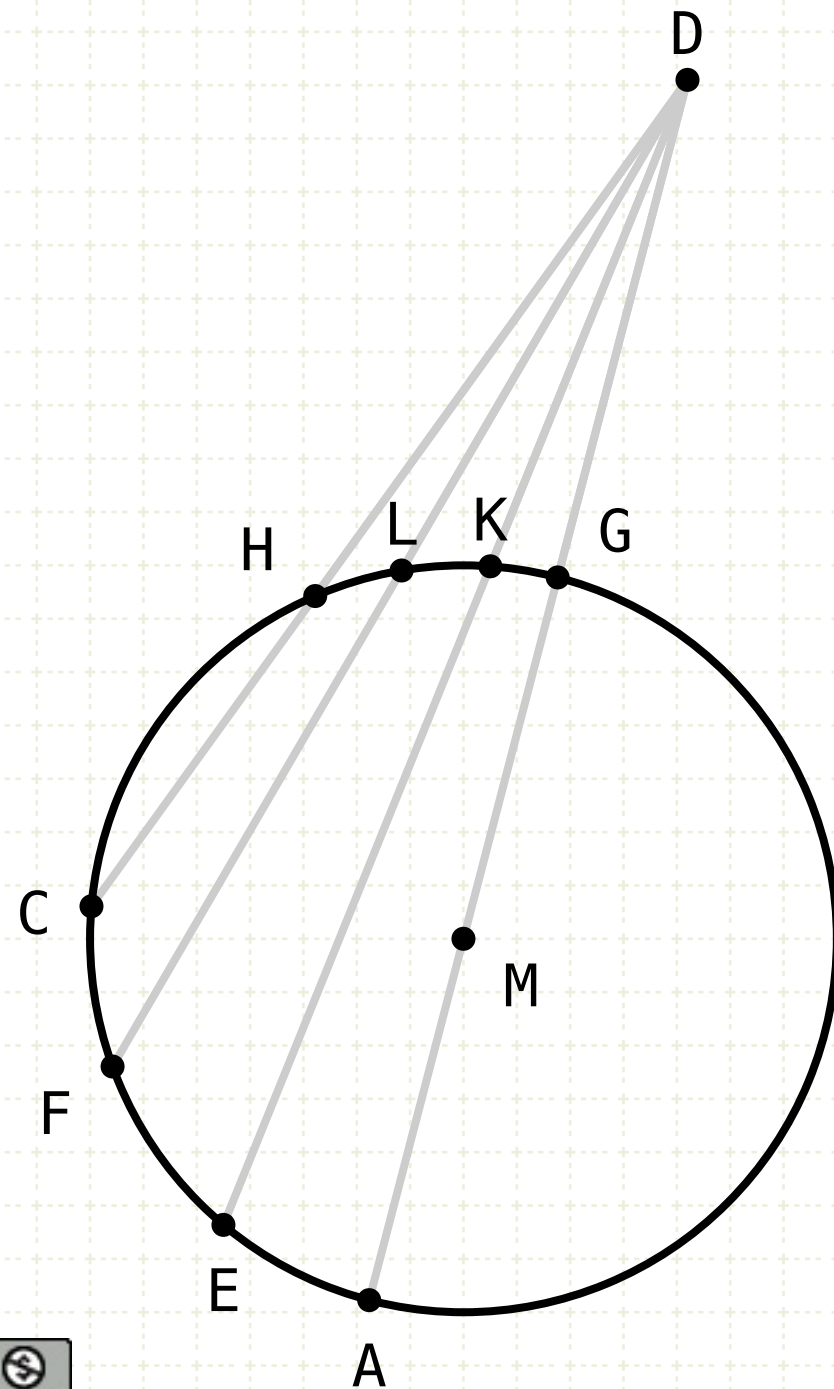
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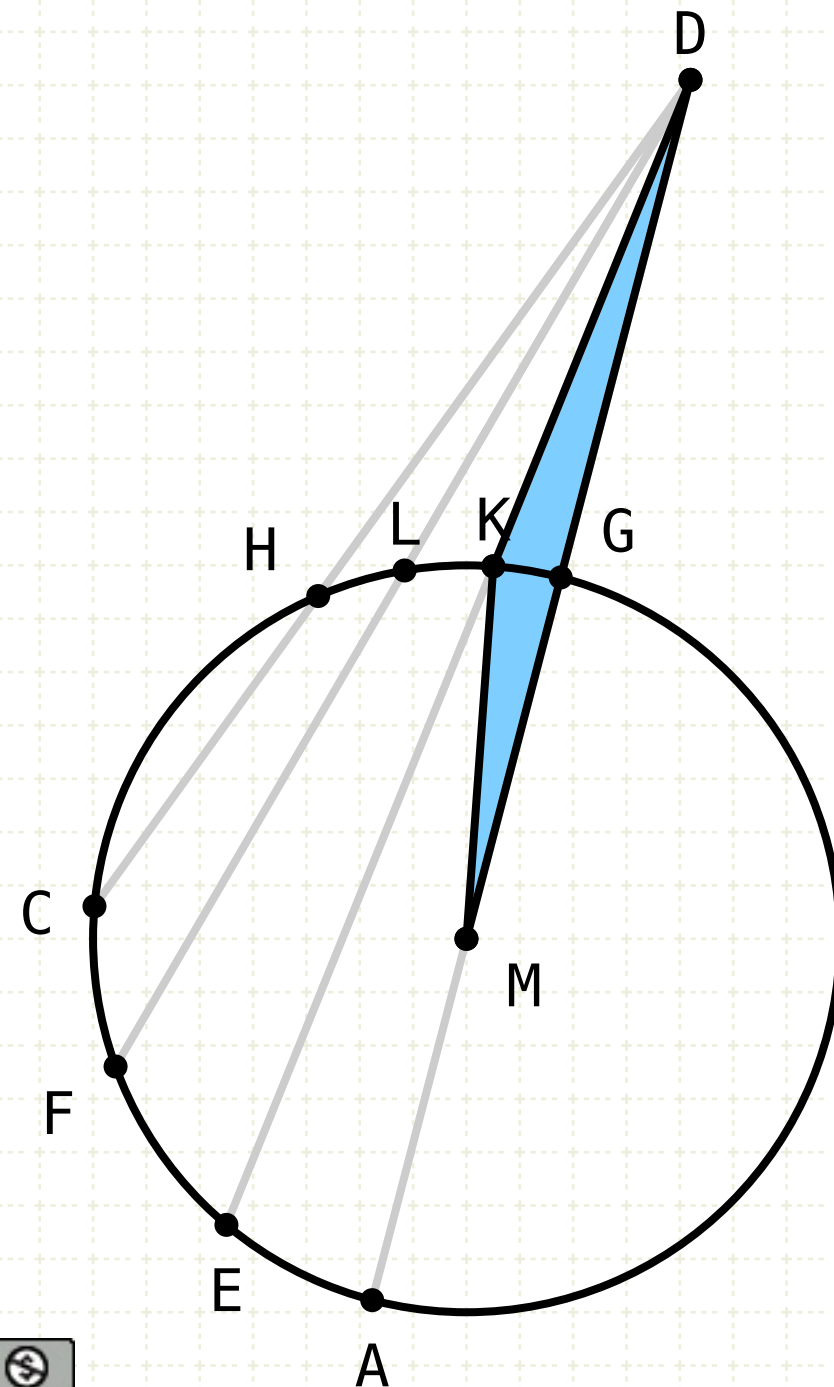
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Proof (part 2)



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$$DK + KM > DM$$

$$DK + KM > DG + GM$$

$$DK > DG$$

Proof (part 2)

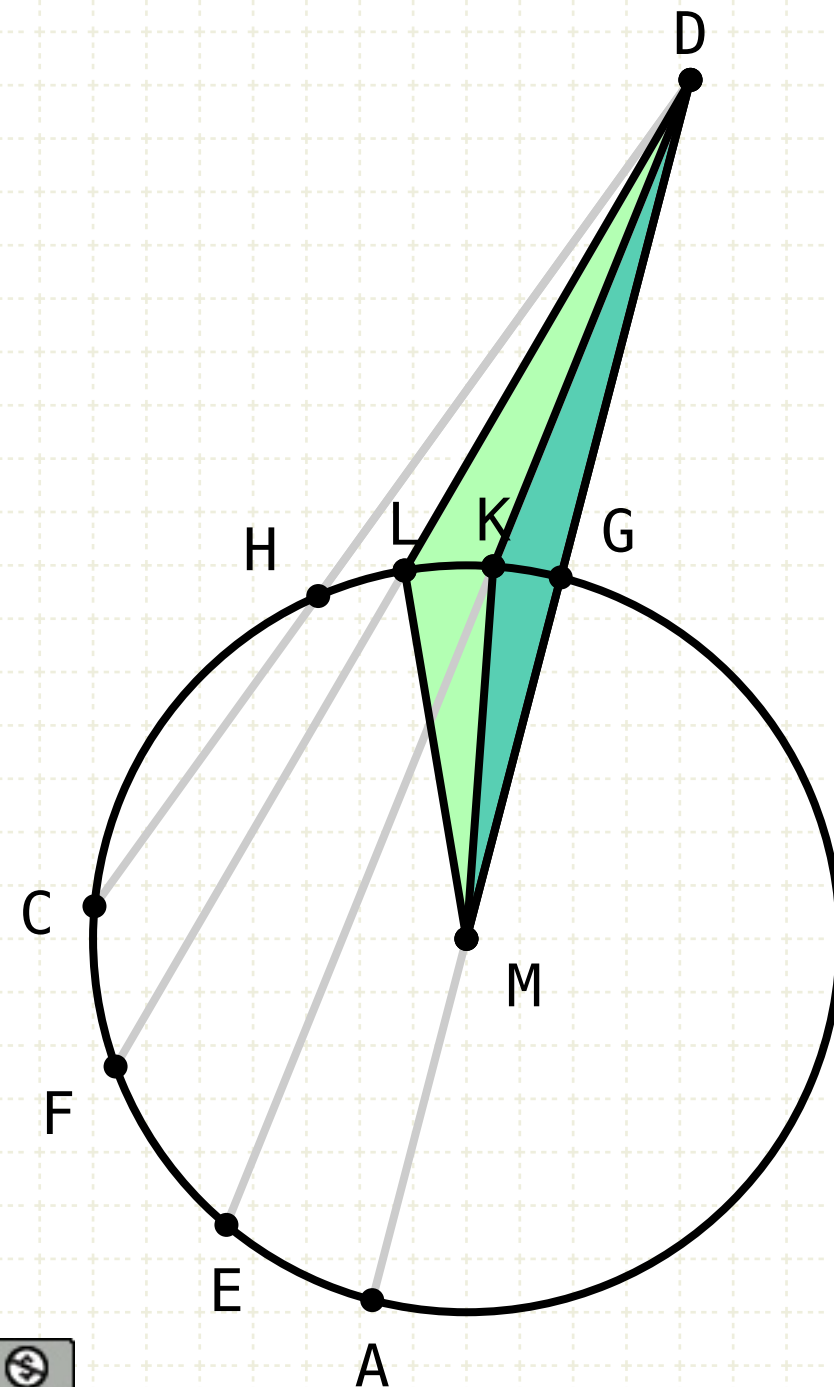
Consider triangle DKM, DK plus KM is greater than DM (I.20)

But KM is equal to GM

Subtract GM from both sides of the inequality gives DK is greater than DG

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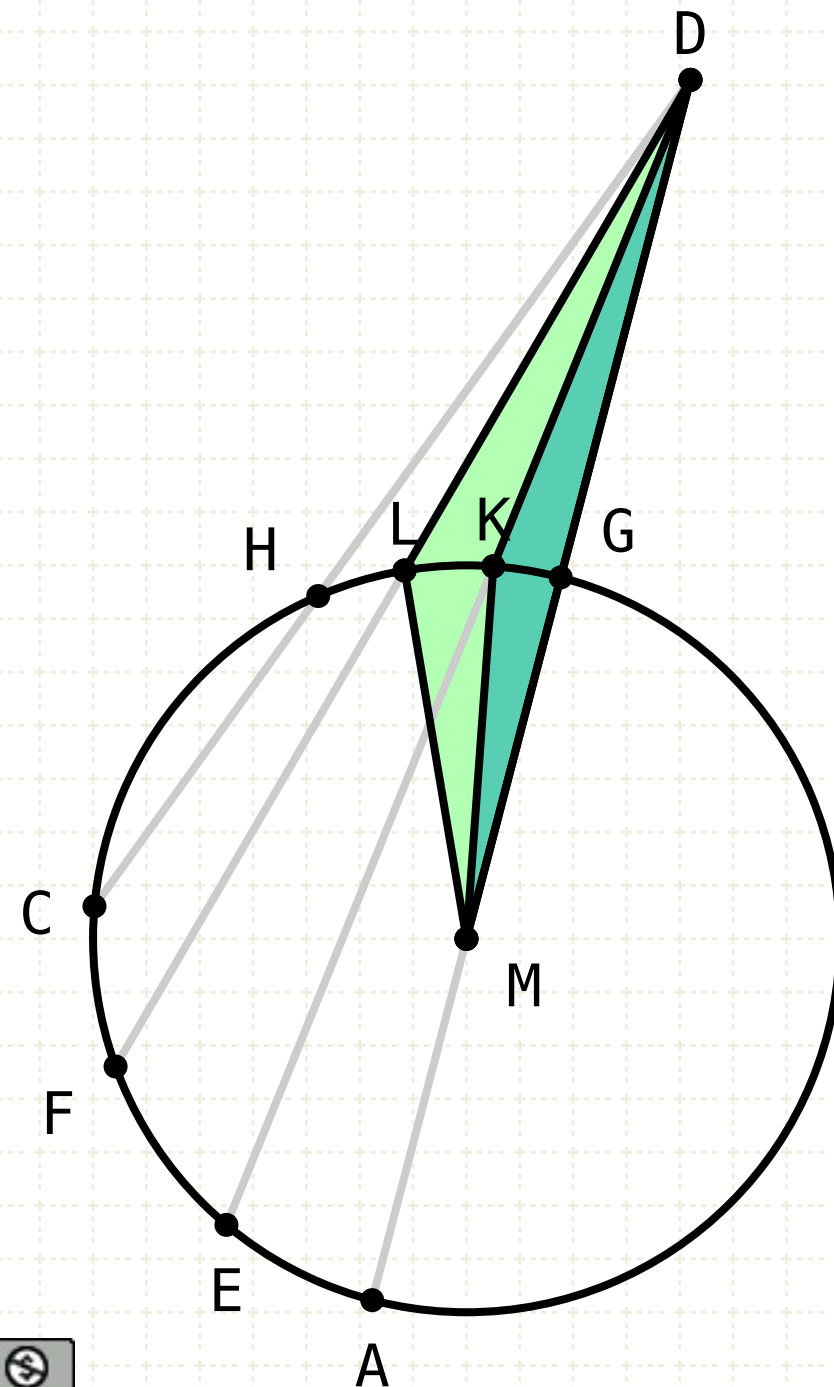
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Consider the two triangles DKM and DLM

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$$DK + KM > DG + GM$$

$$DK > DG$$

$$DL + LM > DK + KM$$

Proof (part 2)

Consider triangle DKM, DK plus KM is greater than DM (I·20)

But KM is equal to GM

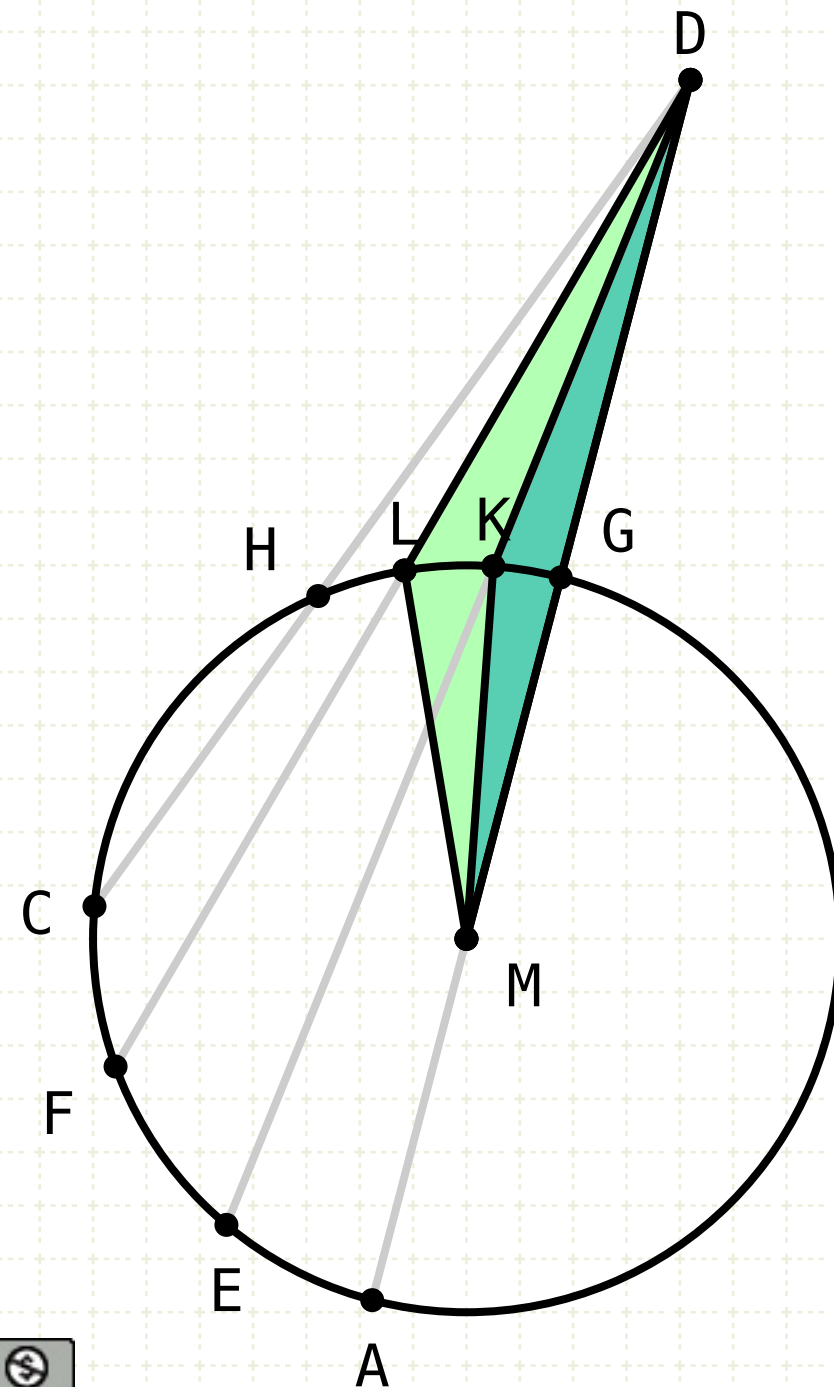
Subtract GM from both sides of the inequality gives DK is greater than DG

Consider the two triangles DKM and DLM

The two lines DK and KM are wholly within the triangle DLM, therefore the sum of DK,KM is less than the sum of DL,LM (I·21)

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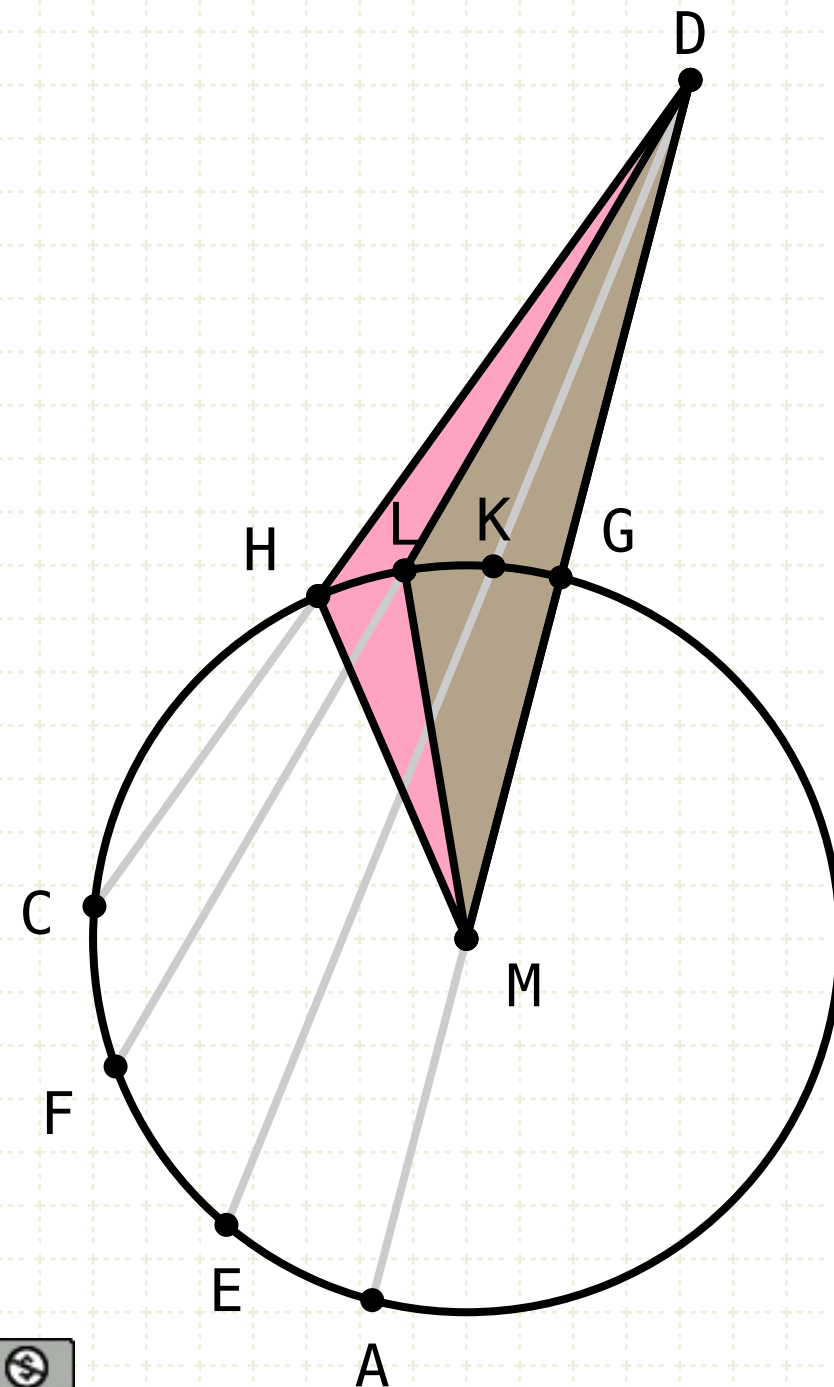
But LM is equal to KM

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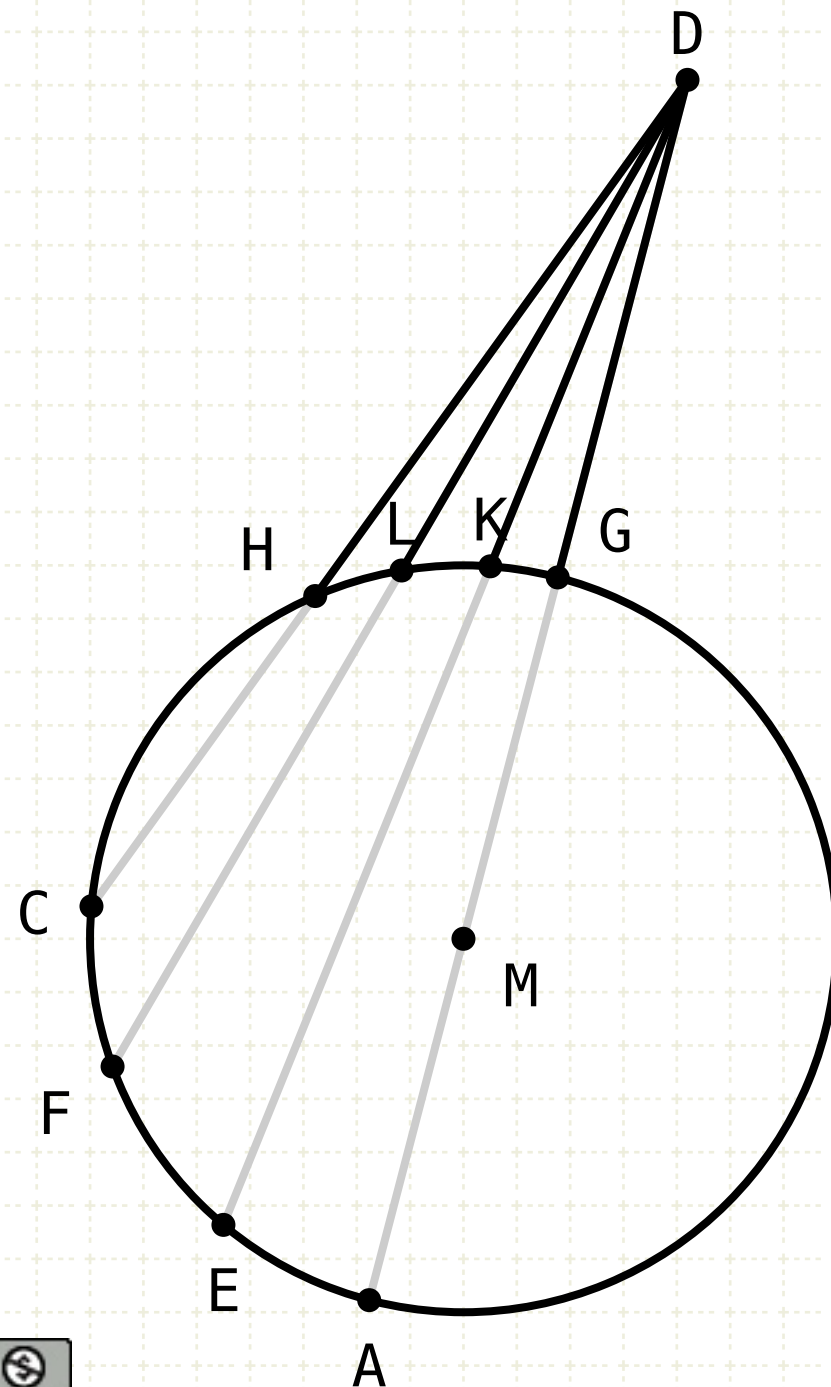
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Subtract KM from both sides of the inequality gives DL is greater than DK

Using the same logic, we have DH greater than DL

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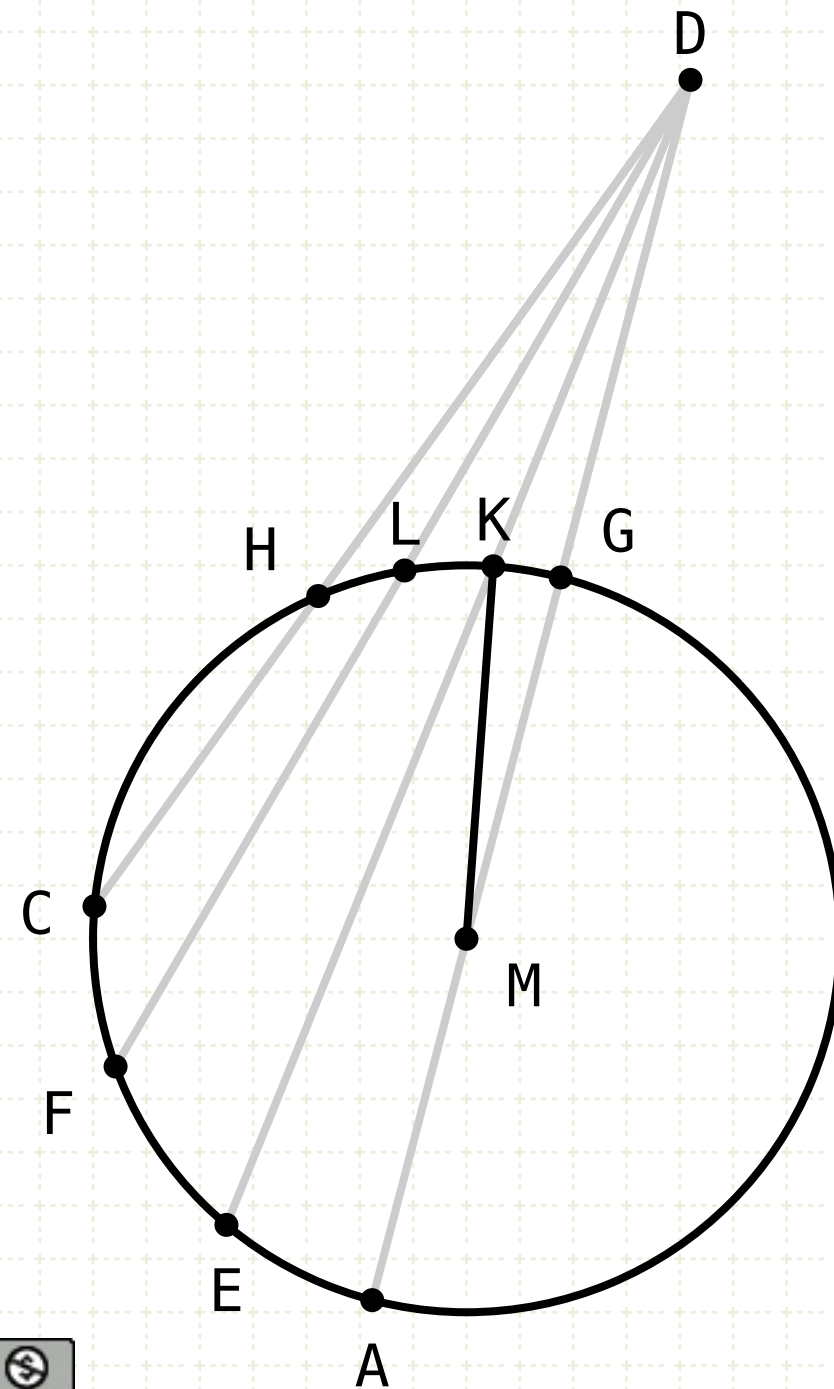
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Using the same logic, we have DH greater than DL

Proposition 8 of Book III

If a point be taken outside a circle and from the point straight lines be drawn through to the circle, one of which is through the centre and the others are drawn at random, then, of the straight lines which fall on the concave circumference, that through the centre is greatest, while of the rest the nearer to that through the centre is always greater than the more remote, but, of the straight lines falling on the convex circumference, that between the point and the diameter is least, while of the rest the nearer to the least is always less than the more remote, and only two equal straight lines will fall on the circle from the point, one on each side of the least.

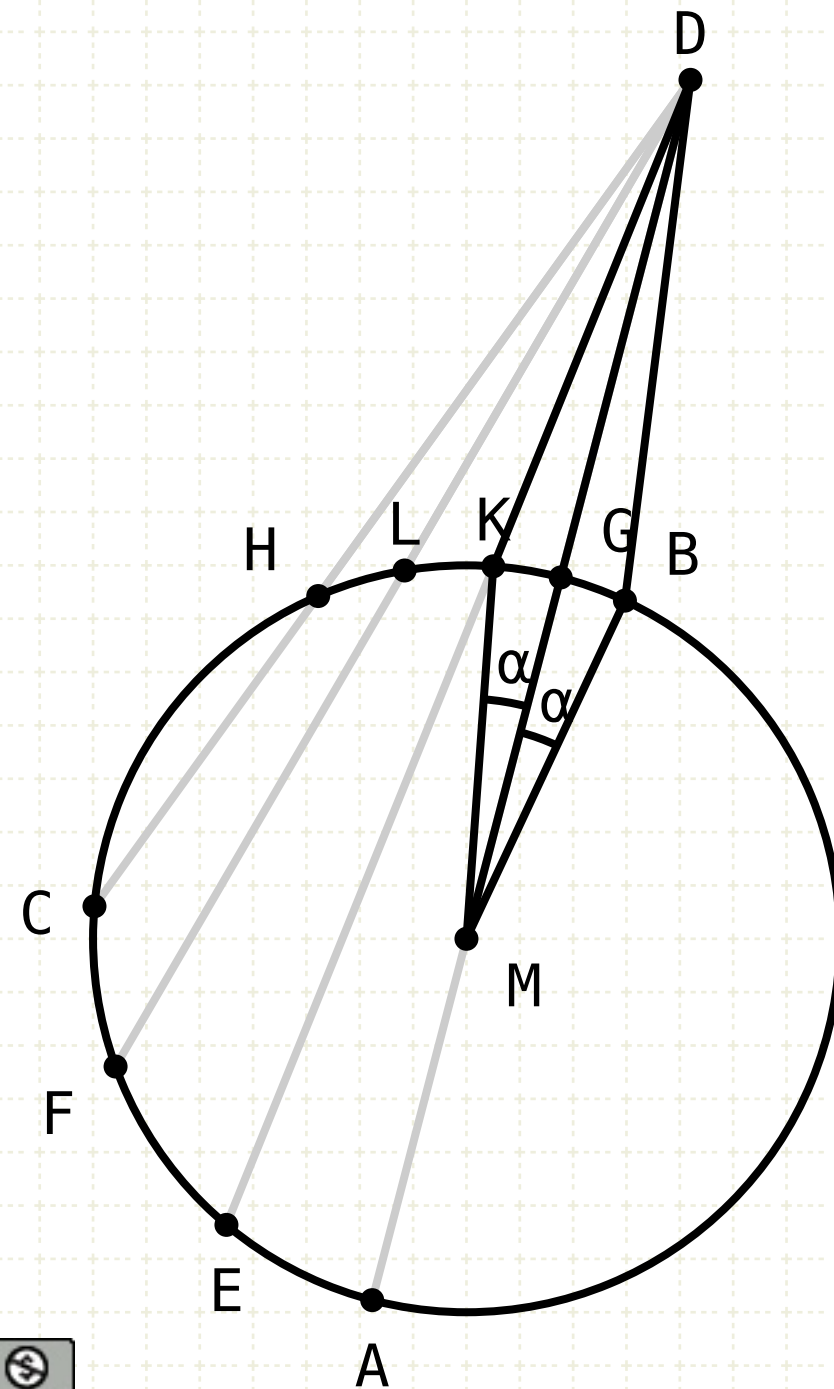


Proof (part 3)

Construct a line MB such that the angle DMK equals DMB, and draw the line DB

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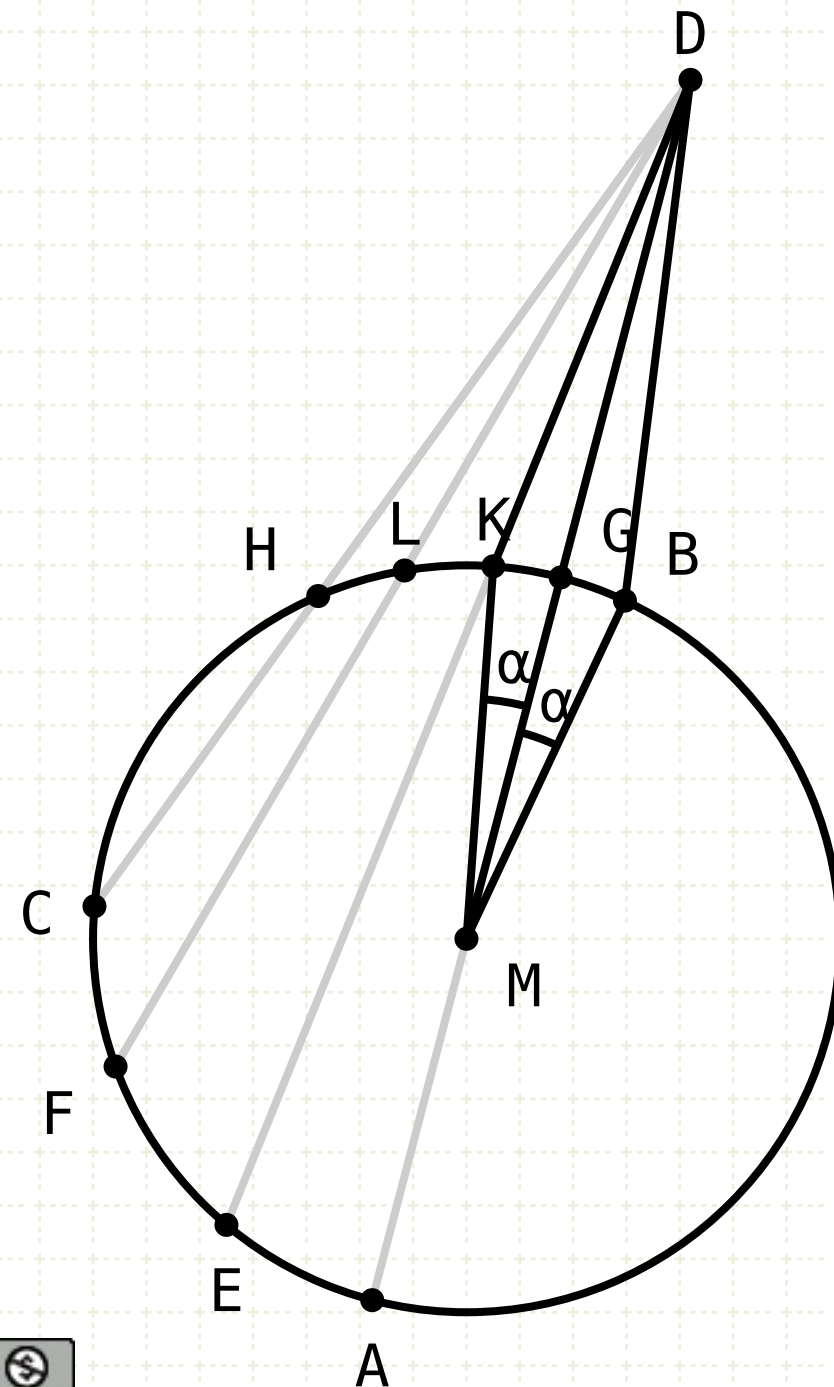


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$$\triangle DKM \equiv \triangle DBM$$

$$DK = DB$$

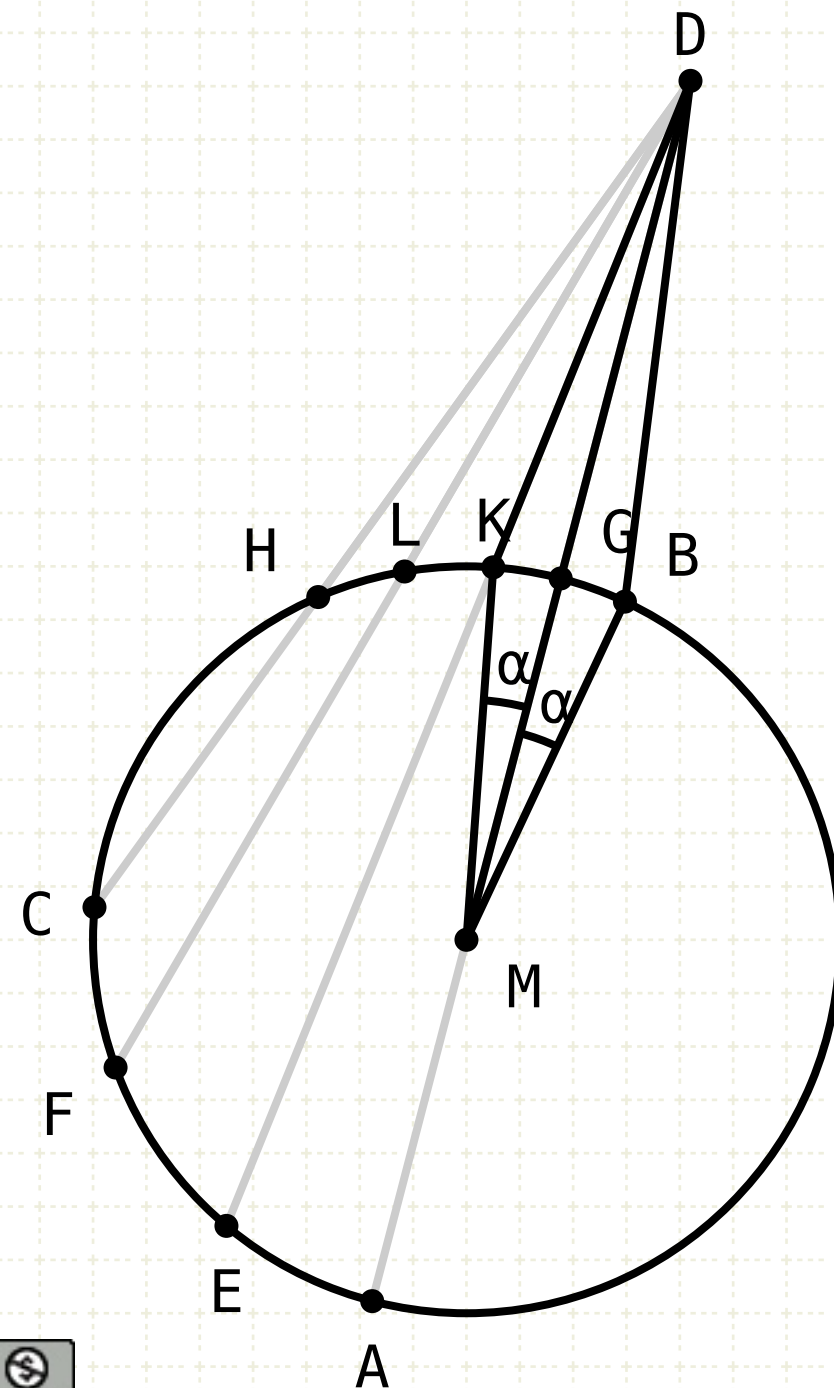
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Construct a line MB such that the angle DMK equals DMB, and draw the line DB

MK equals MB (radii of the same circle) and MD is common to both, so with two triangles with side-angle-side SAS equal, the triangles are equal and therefore KD equals BD (I·4)

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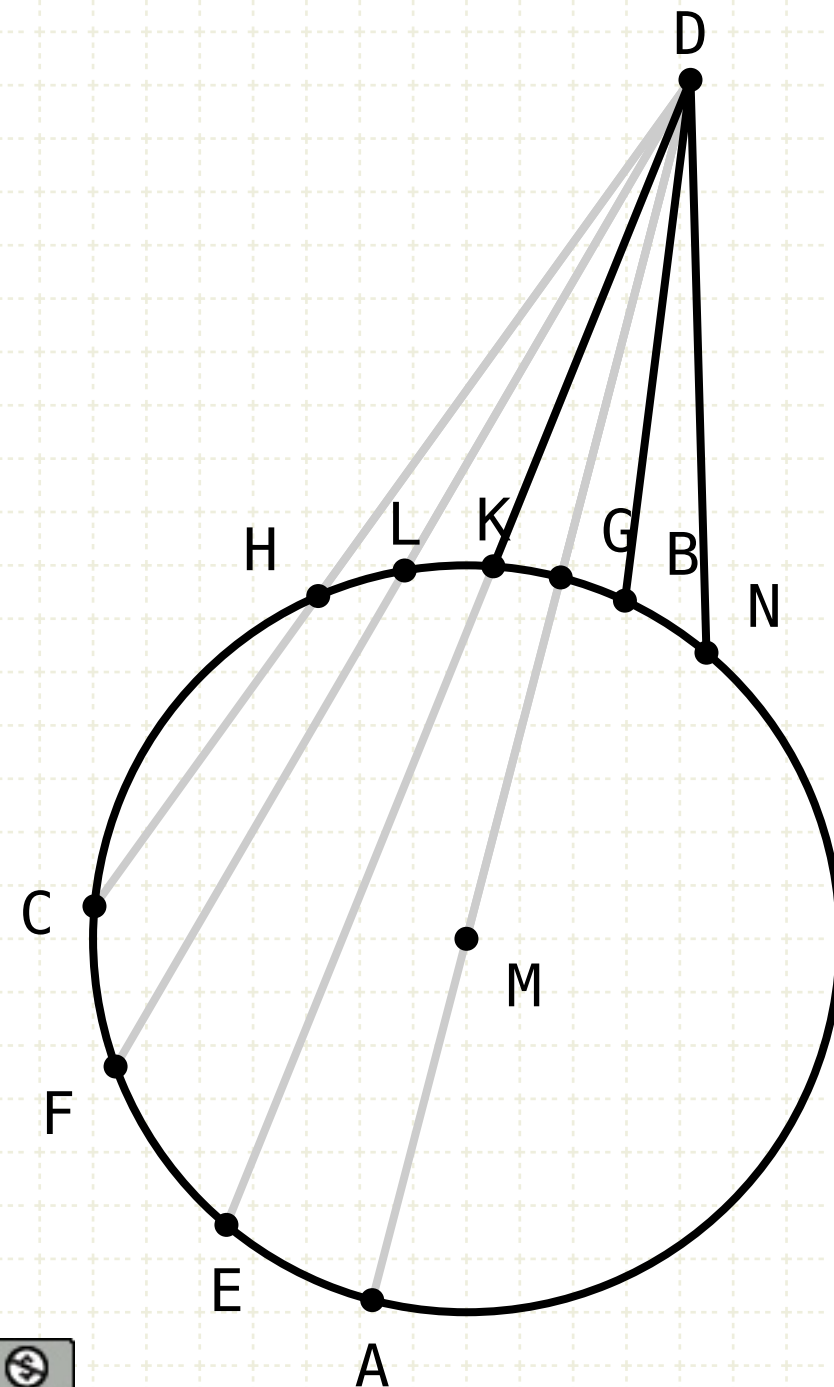
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There is no other line that can fall from D to the circle equal in length to DK and DB

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$$\triangle DKM \equiv \triangle DBM$$

$$DK = DB$$

Assume...

$$DN = DK$$

Proof (part 3)

Construct a line MB such that the angle DMK equals DMB, and draw the line DB

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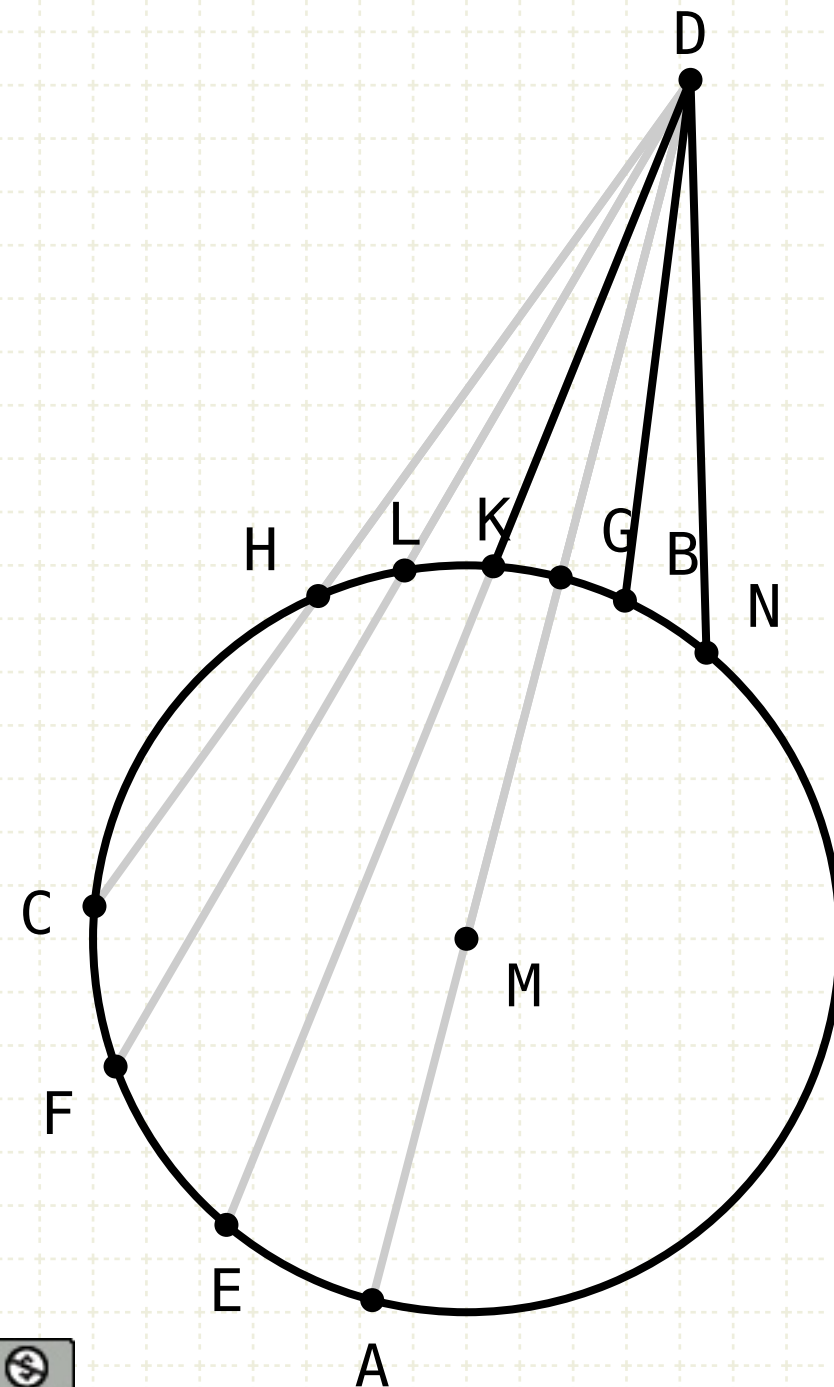
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Proof by contradiction:

Assume a line DN exists, equal in length to DK

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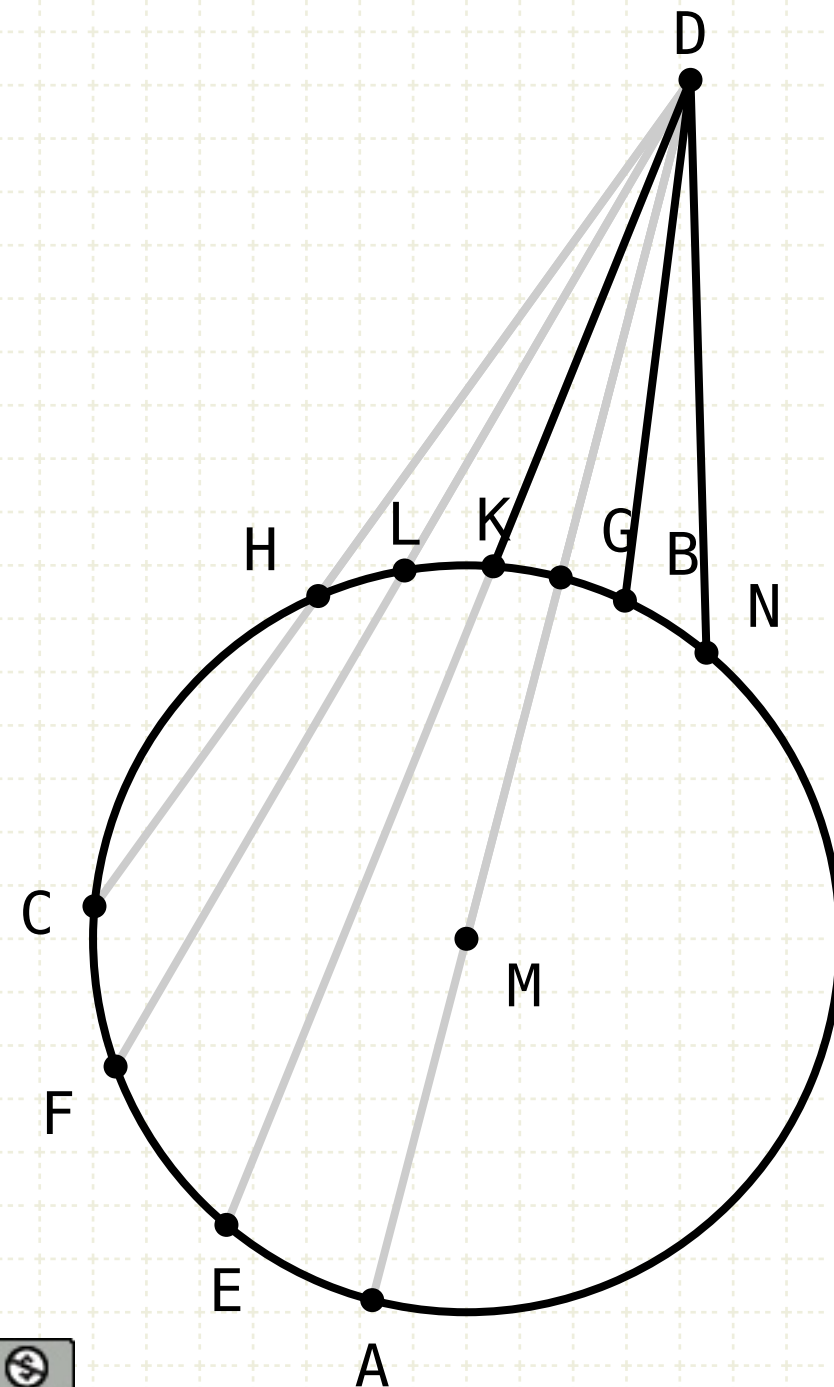
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DK is equal to DB, but DN is equal to DK, therefore DN equals DB

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$$\triangle DKM \equiv \triangle DBM$$

$$DK = DB$$

Assume...

$$DN = DK$$

$$DN = DB \quad \times$$

$$DB < DN$$

Proof (part 3)

Construct a line MB such that the angle DMK equals DMB, and draw the line DB

MK equals MB (radii of the same circle) and MD is common to both, so with two triangles with side-angle-side SAS equal, the triangles are equal and therefore KD equals BD (I-4)

There is no other line that can fall from D to the circle equal in length to DK and DB

Proof by contradiction:

Assume a line DN exists, equal in length to DK

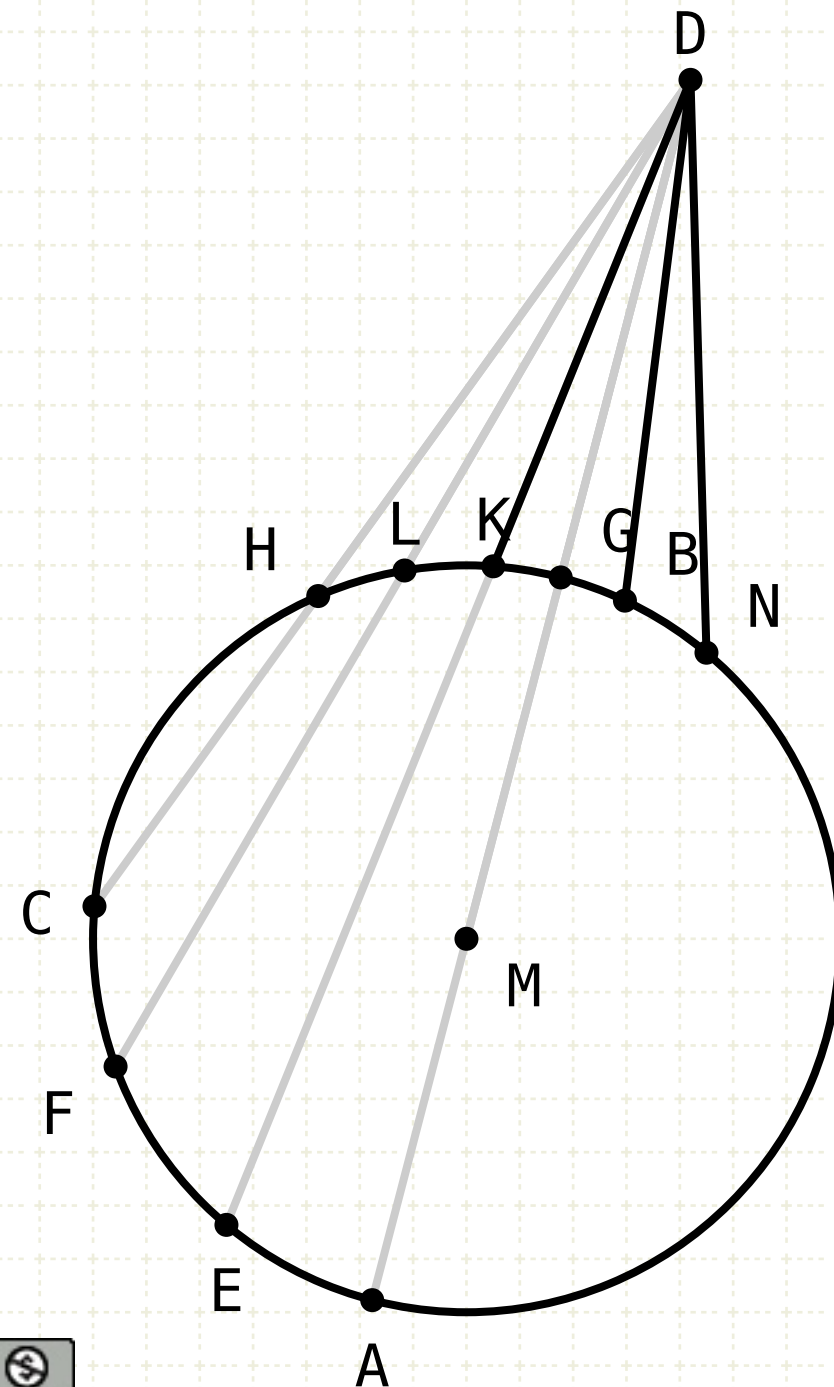
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But, according to the second part of this proposition, DB, being closer to DG, is smaller than DN, which contradicts the original statement



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Assume...

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$$DN = DB \quad x$$

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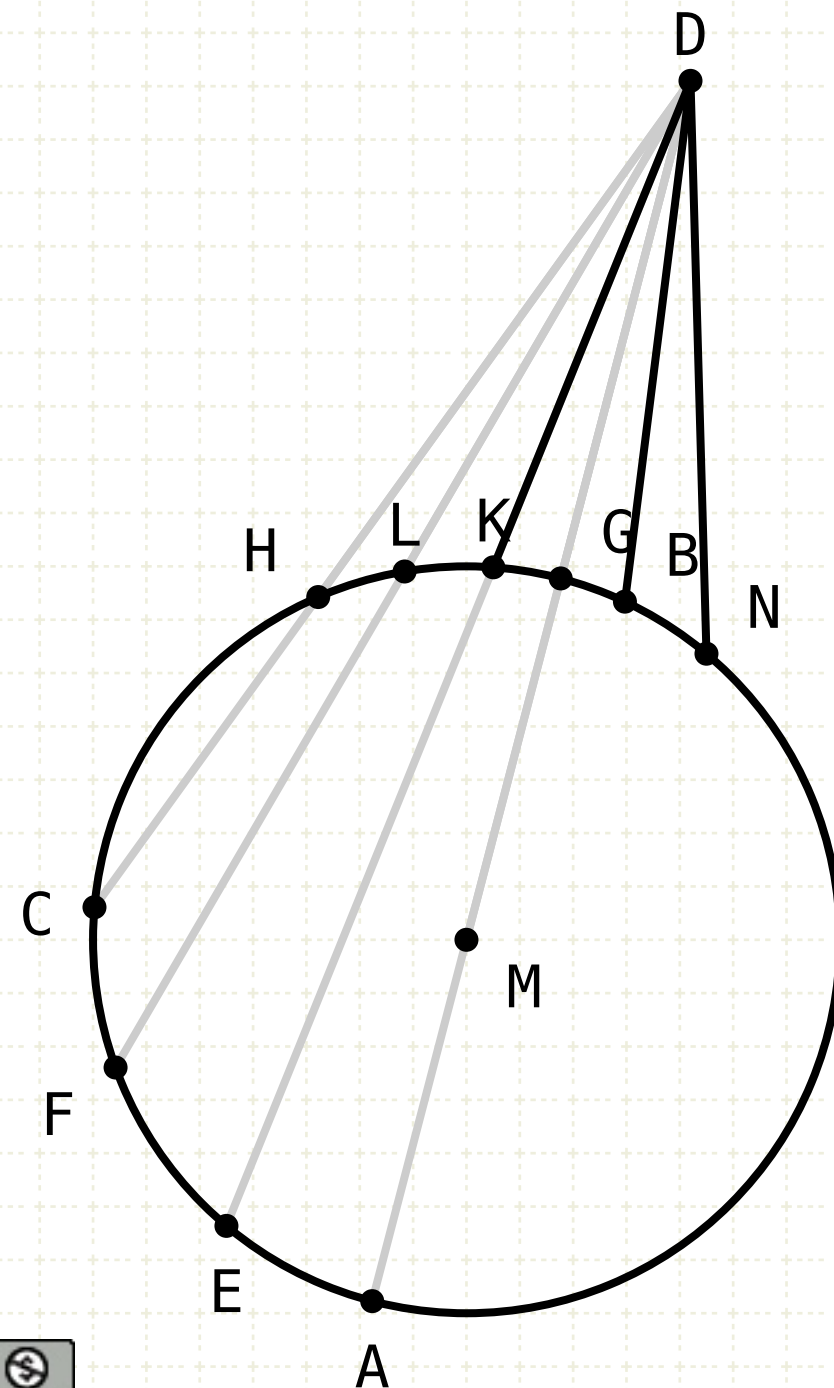
But, according to the second part of this proposition, DB, being closer to DG, is smaller than DN, which contradicts the original statement

Therefore there are only two lines of equal length from D to the circle circumference



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$$\triangle DKM \equiv \triangle DBM$$

$$DK = DB$$

Assume...

$$DN = DK$$

$$DN = DB \quad x$$

$$DB < DN$$

$$DN \neq DK$$

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