

Euclid's Elements

Book III



A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)



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6	If two circles touch one another, they will not have the same center	14	In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another.	22	The opposite angles of quadrilaterals in circles are equal to two right angles
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Proposition 1 of Book III

To find the center of a given circle.



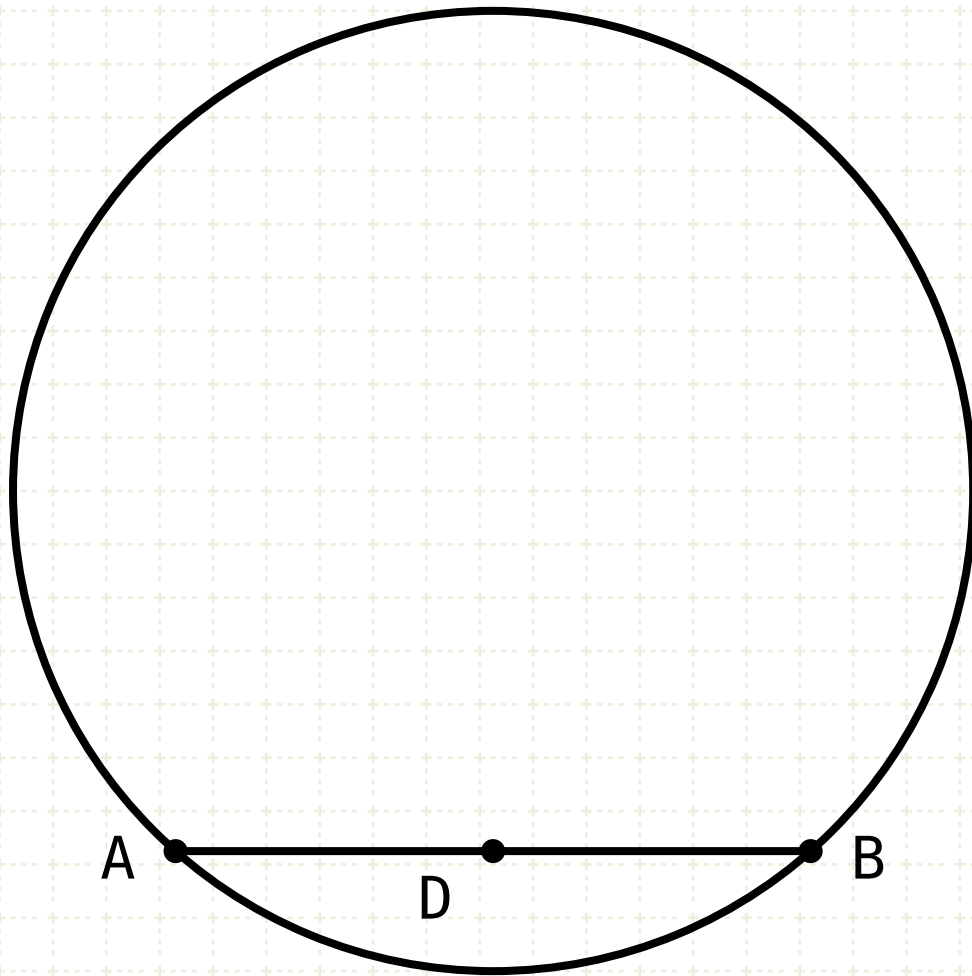
Proposition 1 of Book III

To find the center of a given circle.

Construction

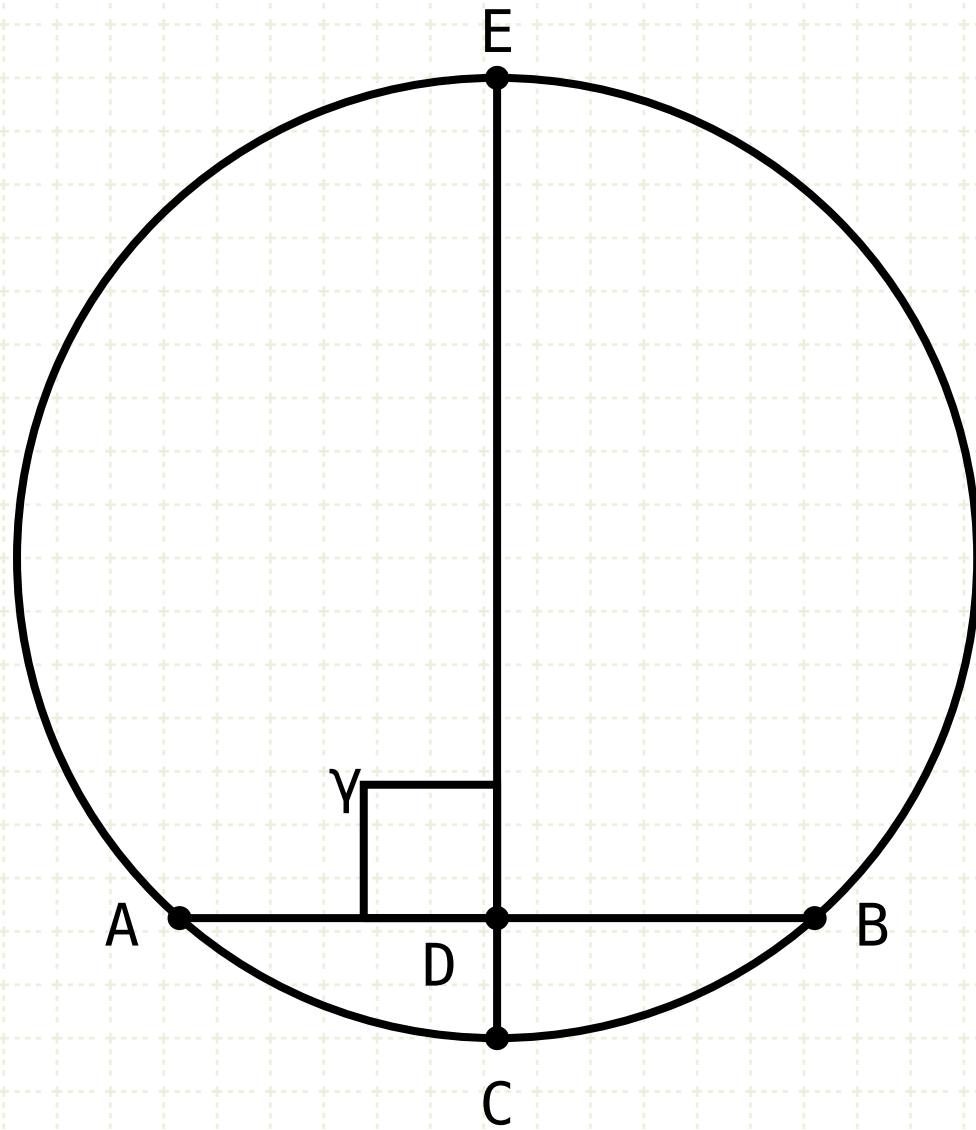
Let a straight line (AB) be drawn through the circle at random,
and be bisected at point D

$$AD = DB$$



Proposition 1 of Book III

To find the center of a given circle.



$$AD = DB$$

$$Y = L$$

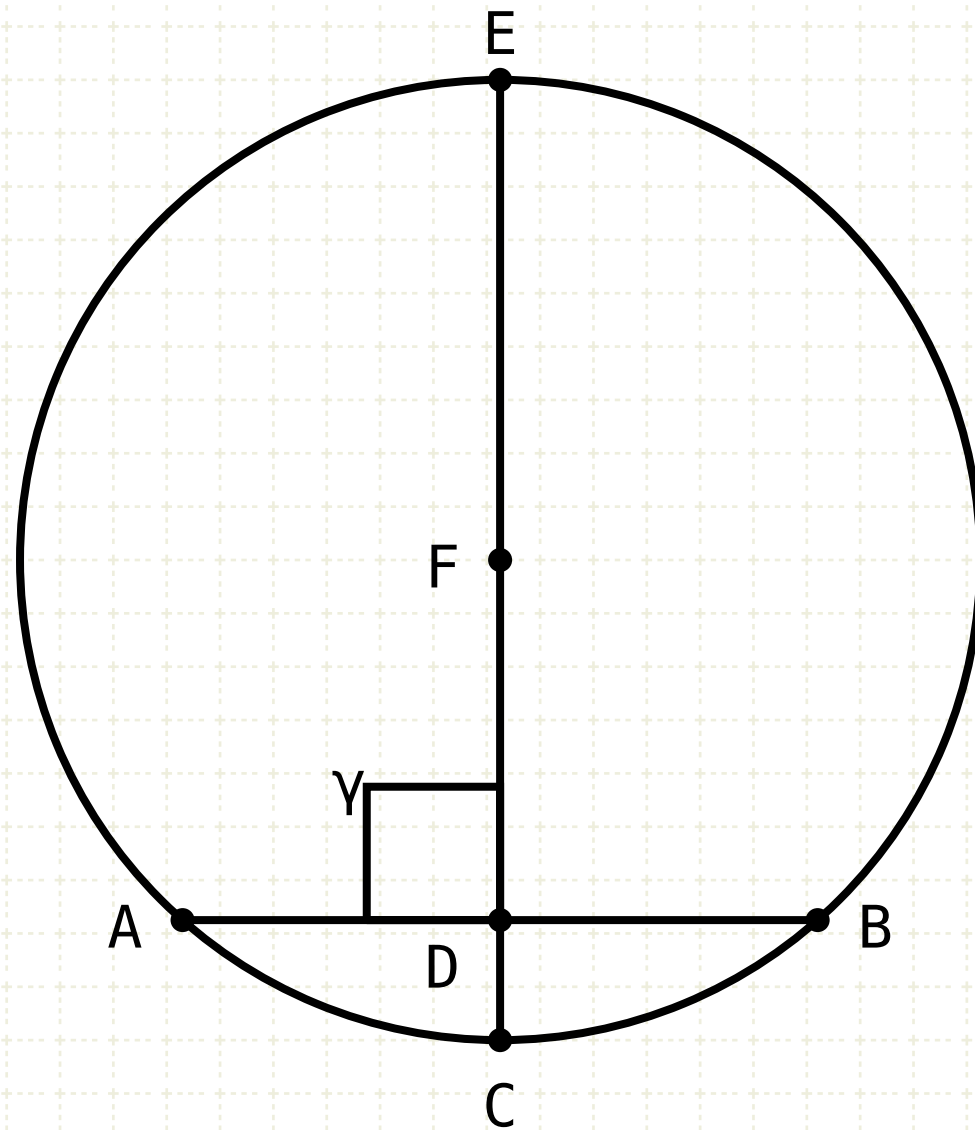
Construction

Let a straight line (AB) be drawn through the circle at random, and be bisected at point D

Draw a line perpendicular to AB through the point D, intersecting the circle at CE

Proposition 1 of Book III

To find the center of a given circle.



$$AD = DB$$

$$\angle Y = \angle L$$

$$CF = FE$$

Construction

Let a straight line (AB) be drawn through the circle at random, and be bisected at point D

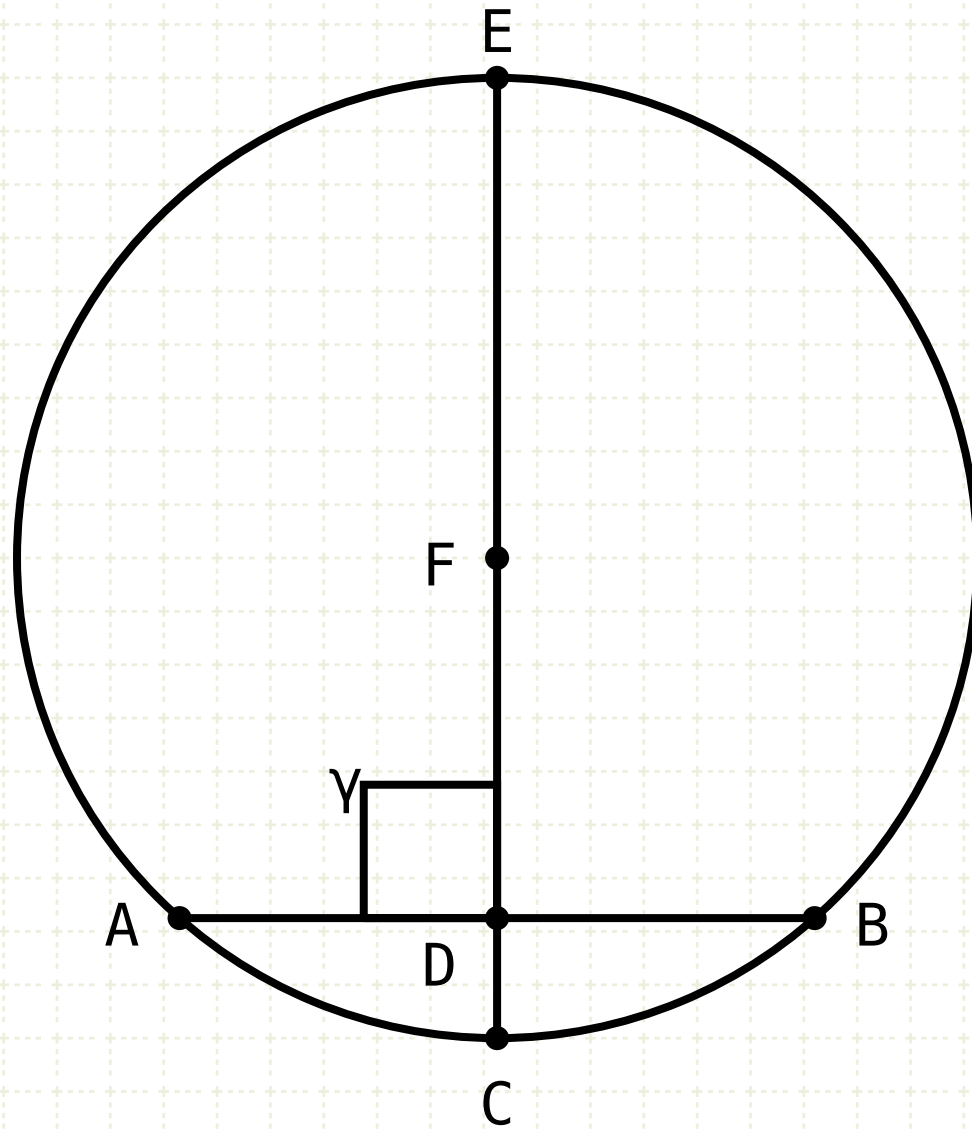
Draw a line perpendicular to AB through the point D, intersecting the circle at CE

Bisect CE at point F. Point F is the center of the circle

Proposition 1 of Book III

To find the center of a given circle.

Proof by Contradiction



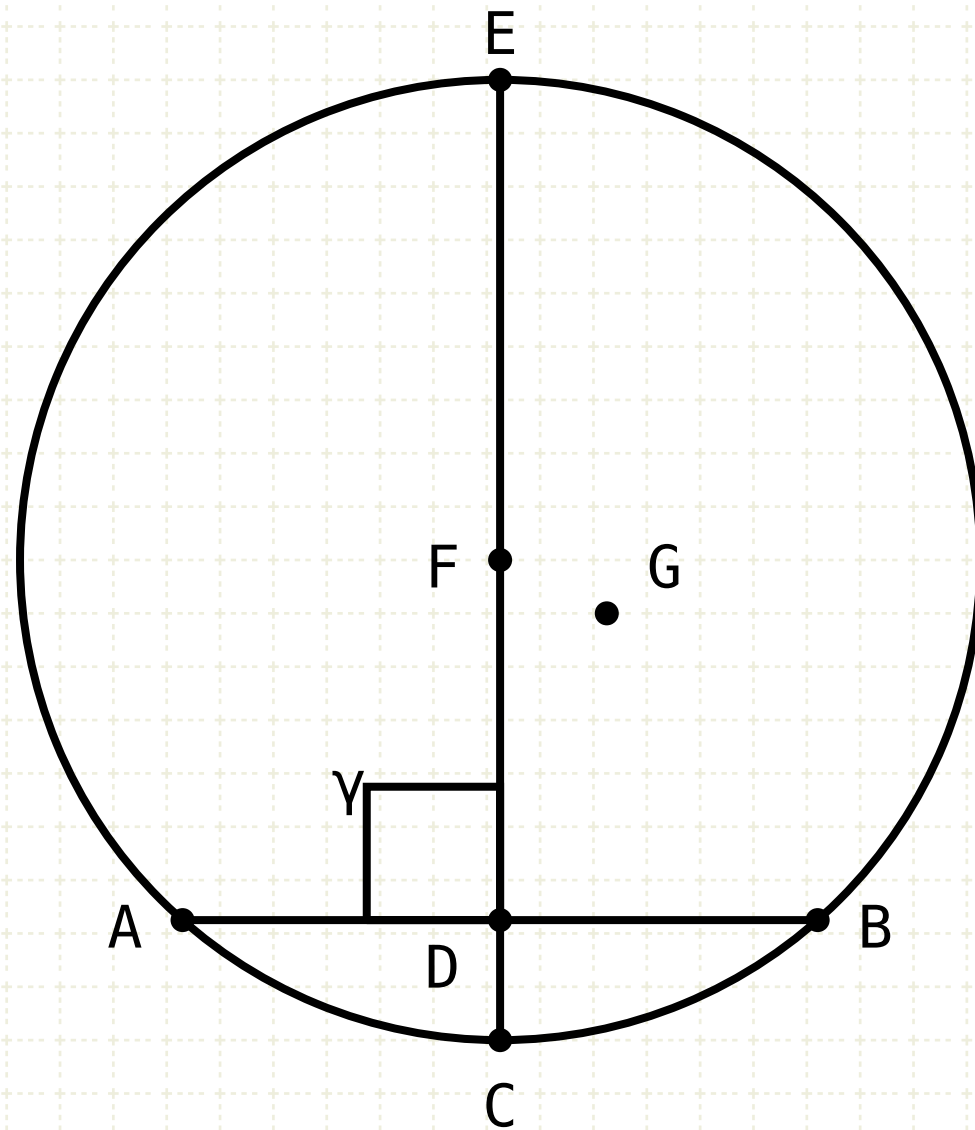
$$AD = DB$$

$$YD = DF$$

$$CF = FE$$

Proposition 1 of Book III

To find the center of a given circle.



$$AD = DB$$

$$CF = FE$$

$$CF = FE$$

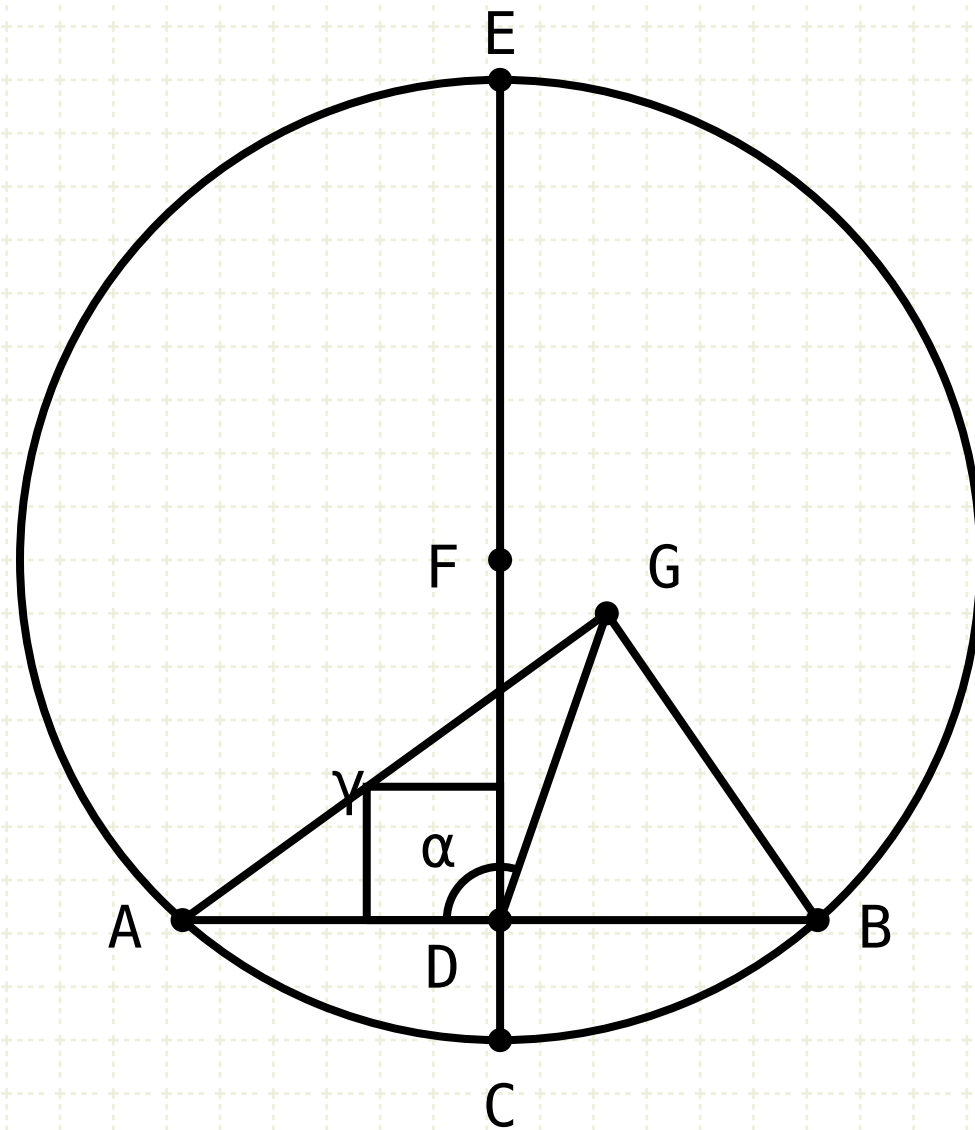
If G is the centre of the circle

Proof by Contradiction

Assume that G is the center of the circle, and that G does not lie on the line CE

Proposition 1 of Book III

To find the center of a given circle.



$$AD = DB$$

$$\gamma = \angle$$

$$CF = FE$$

If G is the centre of the circle

$$\alpha > \gamma$$

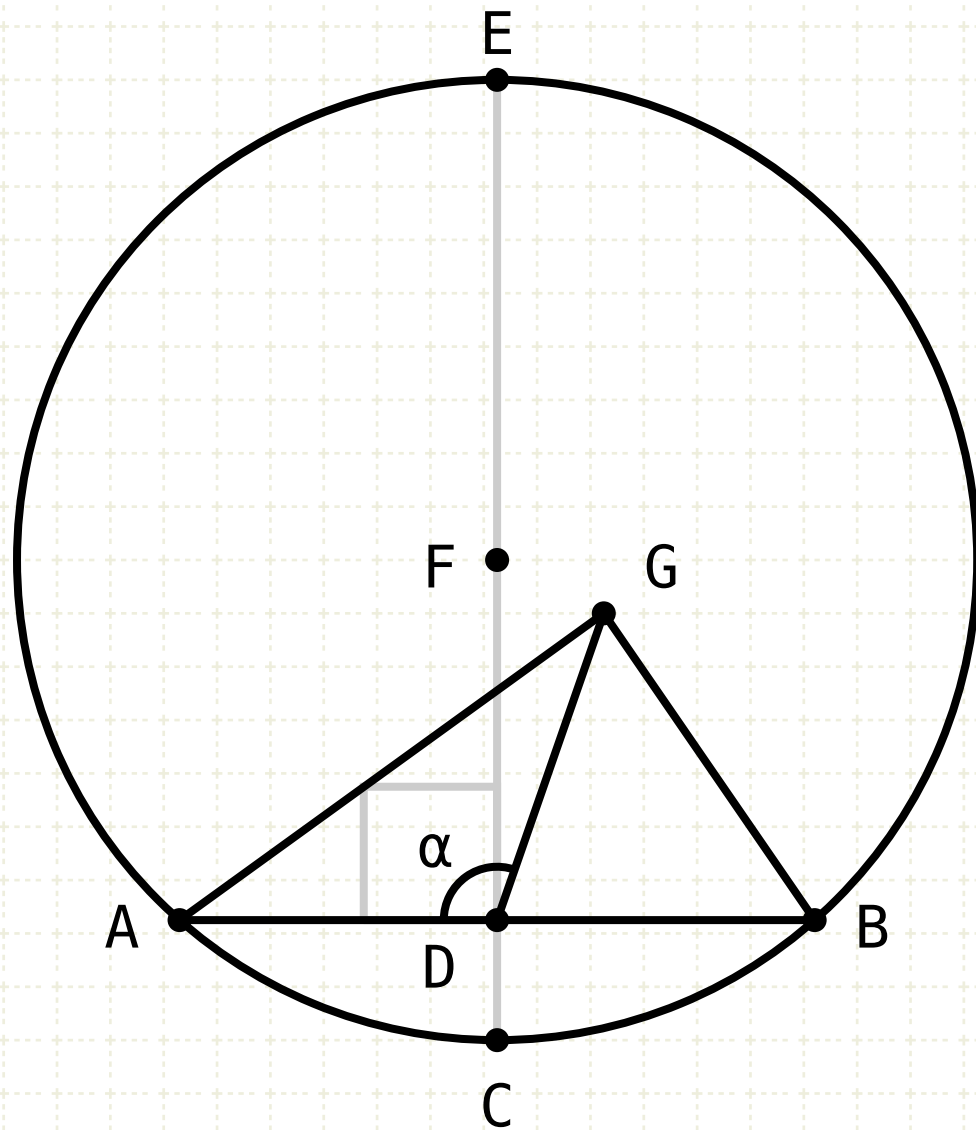
Proof by Contradiction

Assume that G is the center of the circle, and that G does not lie on the line CE

Join GA, GD and GB

Proposition 1 of Book III

To find the center of a given circle.



$$AD = DB$$

$$\gamma = \angle$$

$$CF = FE$$

If G is the centre of the circle

$$\alpha > \gamma$$

$$AG = GB$$

Proof by Contradiction

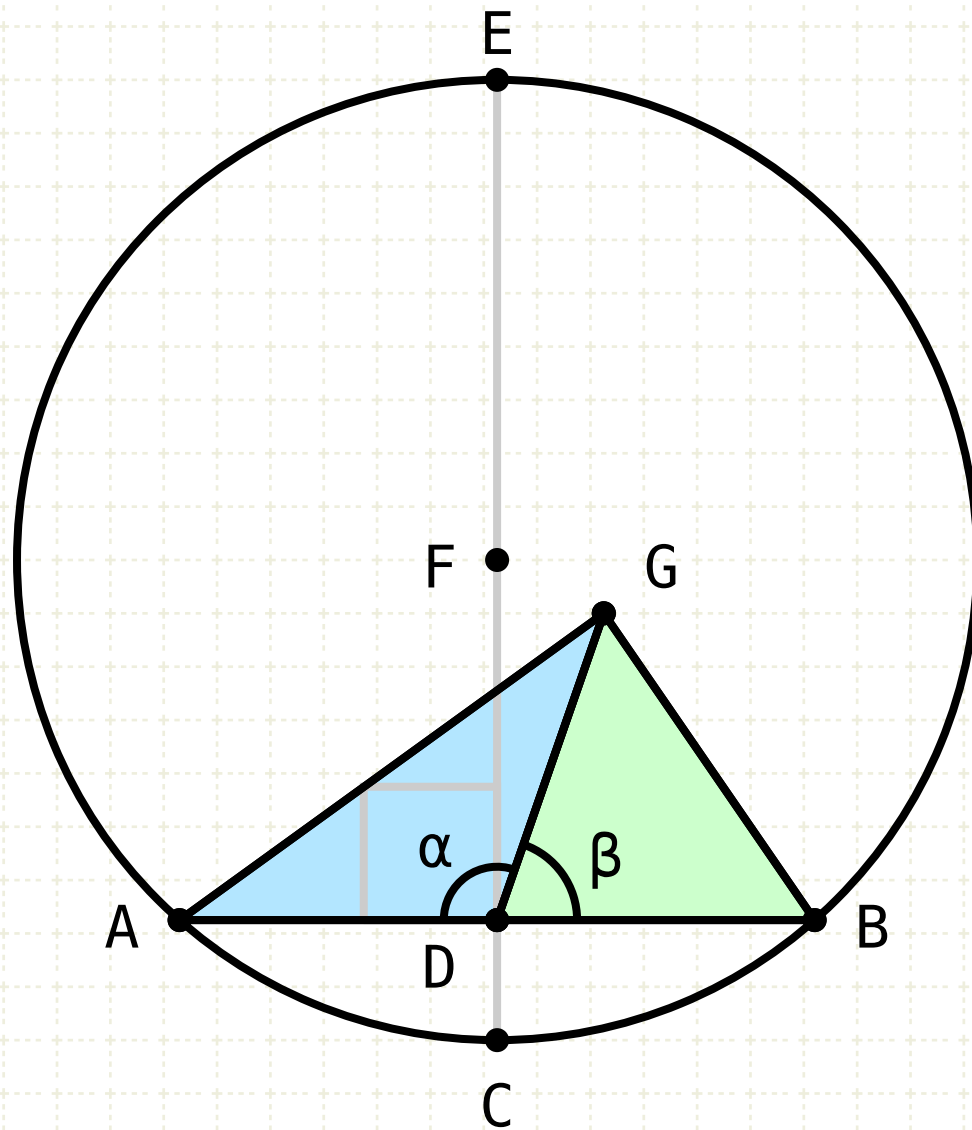
Assume that G is the center of the circle, and that G does not lie on the line CE

Join GA, GD and GB

AG and GB are radii, and thus are equal

Proposition 1 of Book III

To find the center of a given circle.



$$AD = DB$$

$$\angle A = \angle B$$

$$CF = FE$$

If G is the centre of the circle

$$\alpha > \gamma$$

$$AG = GB$$

$$\alpha = \beta$$

Proof by Contradiction

Assume that G is the center of the circle, and that G does not lie on the line CE

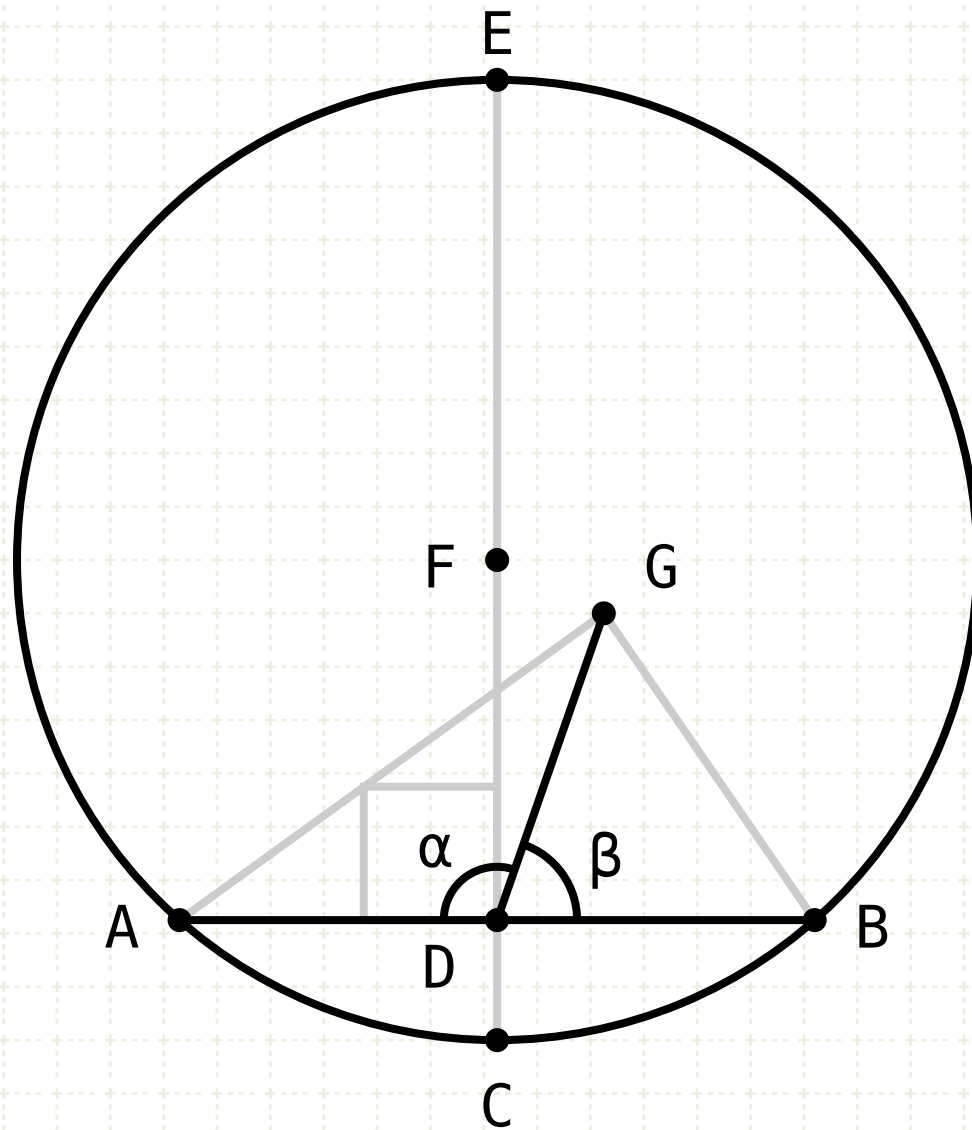
Join GA, GD and GB

AG and GB are radii, and thus are equal

Thus, since the side DG is shared between both triangles ADG and GDB, the triangles have three equal sides, therefore angle ADG (α) equals angle GDB (β) (I.8)

Proposition 1 of Book III

To find the center of a given circle.



$$AD = DB$$

$$\gamma = \angle$$

$$CF = FE$$

If G is the centre of the circle

$$\alpha > \gamma$$

$$AG = GB$$

$$\alpha = \beta$$

$$\alpha = \beta = \angle$$

Proof by Contradiction

Assume that G is the center of the circle, and that G does not lie on the line CE

Join GA, GD and GB

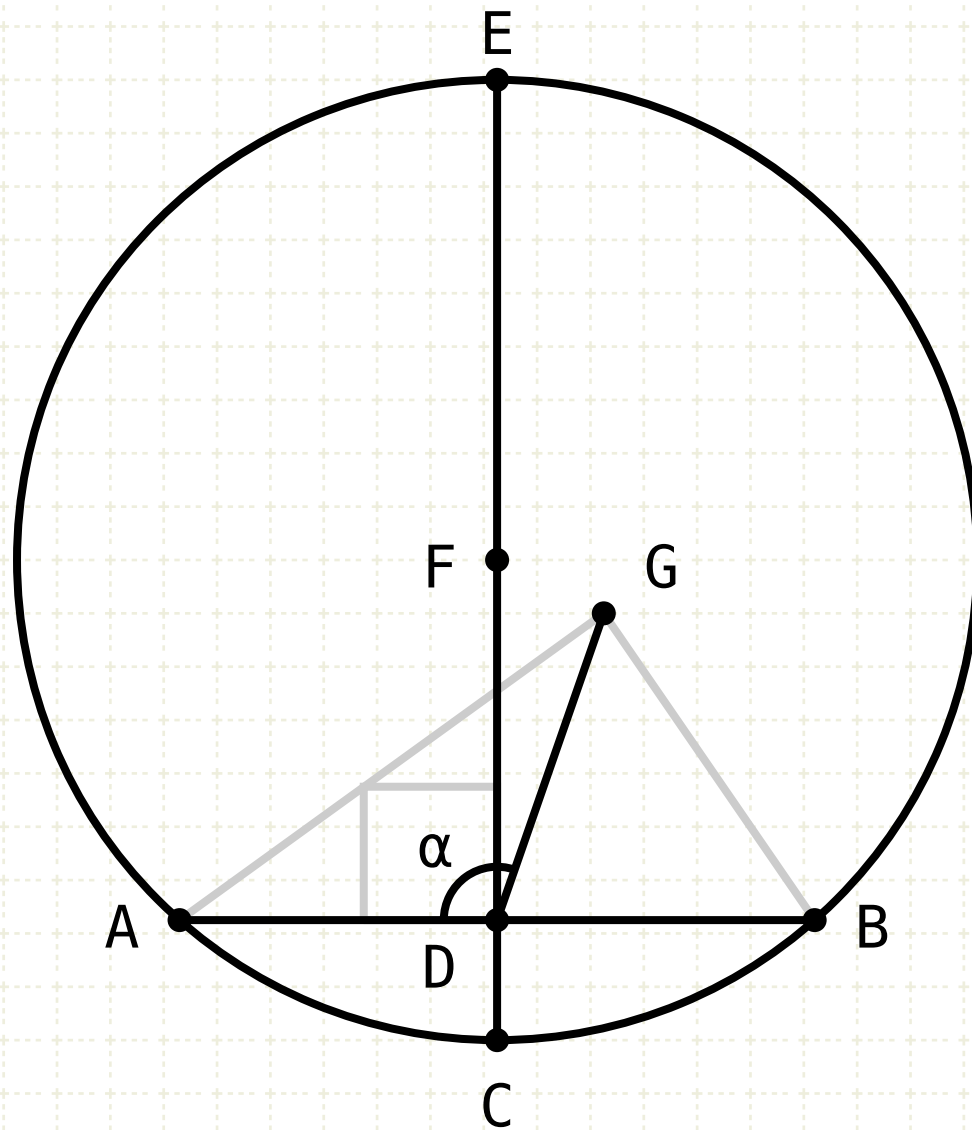
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By definition (I.Def.10), two angles on a straight line are right angles if they are equal to each other, thus α and β are right

Proposition 1 of Book III

To find the center of a given circle.



$$AD = DB$$

$$\gamma = \angle$$

$$CF = FE$$

If G is the centre of the circle

$$\alpha > \gamma$$

$$AG = GB$$

$$\alpha = \beta$$

$$\alpha = \beta = \angle$$

Proof by Contradiction

Assume that G is the center of the circle, and that G does not lie on the line CE

Join GA, GD and GB

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Thus, since the side DG is shared between both triangles ADG and GDB, the triangles have three equal sides, therefore angle ADG (α) equals angle GDB (β) (I.8)

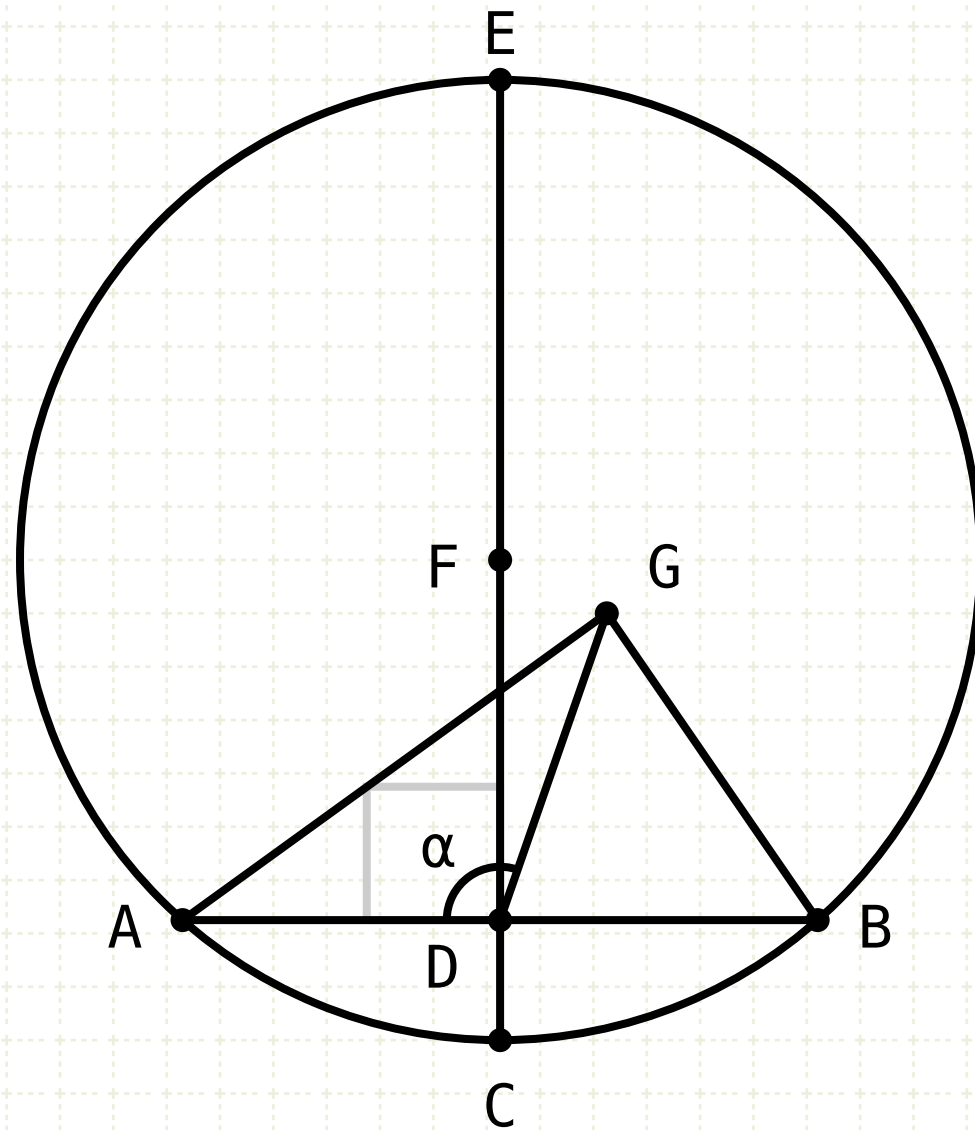
By definition (I.Def.10), two angles on a straight line are right angles if they are equal to each other, thus α and β are right

But GDB (α) is greater than FDA (γ), which is also a right angle (by construction)

The angle α cannot be both greater than and equal to γ

Proposition 1 of Book III

To find the center of a given circle.



$$AD = DB$$

$$\gamma = \angle$$

$$CF = FE$$

If G is the centre of the circle

$$\alpha > \gamma$$

$$AG = GB$$

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Proof by Contradiction

Assume that G is the center of the circle, and that G does not lie on the line CE

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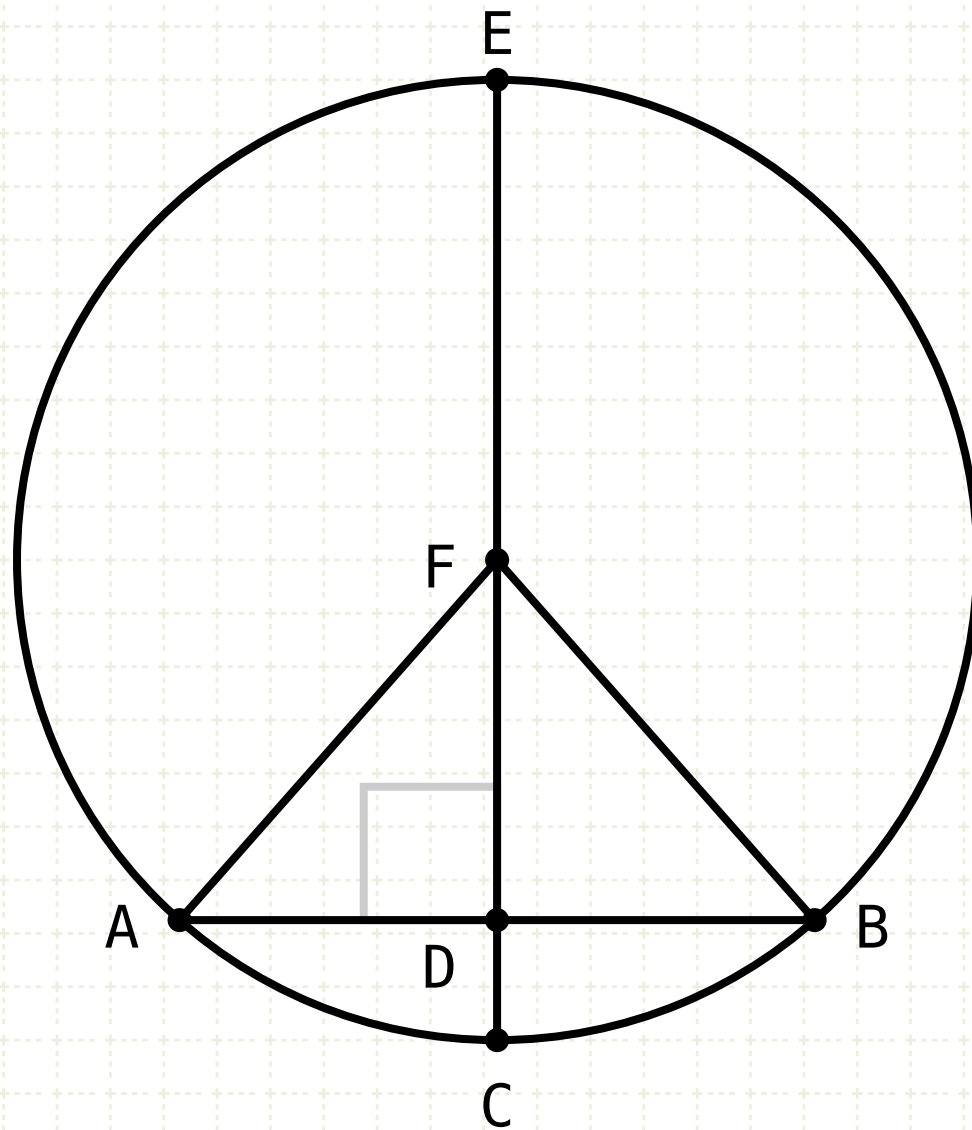
But GDB (α) is greater than FDA (γ), which is also a right angle (by construction)

The angle α cannot be both greater than and equal to γ

So G is not the center of the circle

Proposition 1 of Book III

To find the center of a given circle.



$$AD = DB$$

$$\gamma = \angle$$

$$CF = FE$$

If G is the centre of the circle

$$\alpha > \gamma$$

$$AG = GB$$

$$\alpha = \beta$$

$$\alpha = \beta = \angle$$

Centre of circle lies on EC

Proof by Contradiction

Assume that G is the center of the circle, and that G does not lie on the line CE

Join GA, GD and GB

AG and GB are radii, and thus are equal

Thus, since the side DG is shared between both triangles ADG and GDB, the triangles have three equal sides, therefore angle ADG (α) equals angle GDB (β) (I.8)

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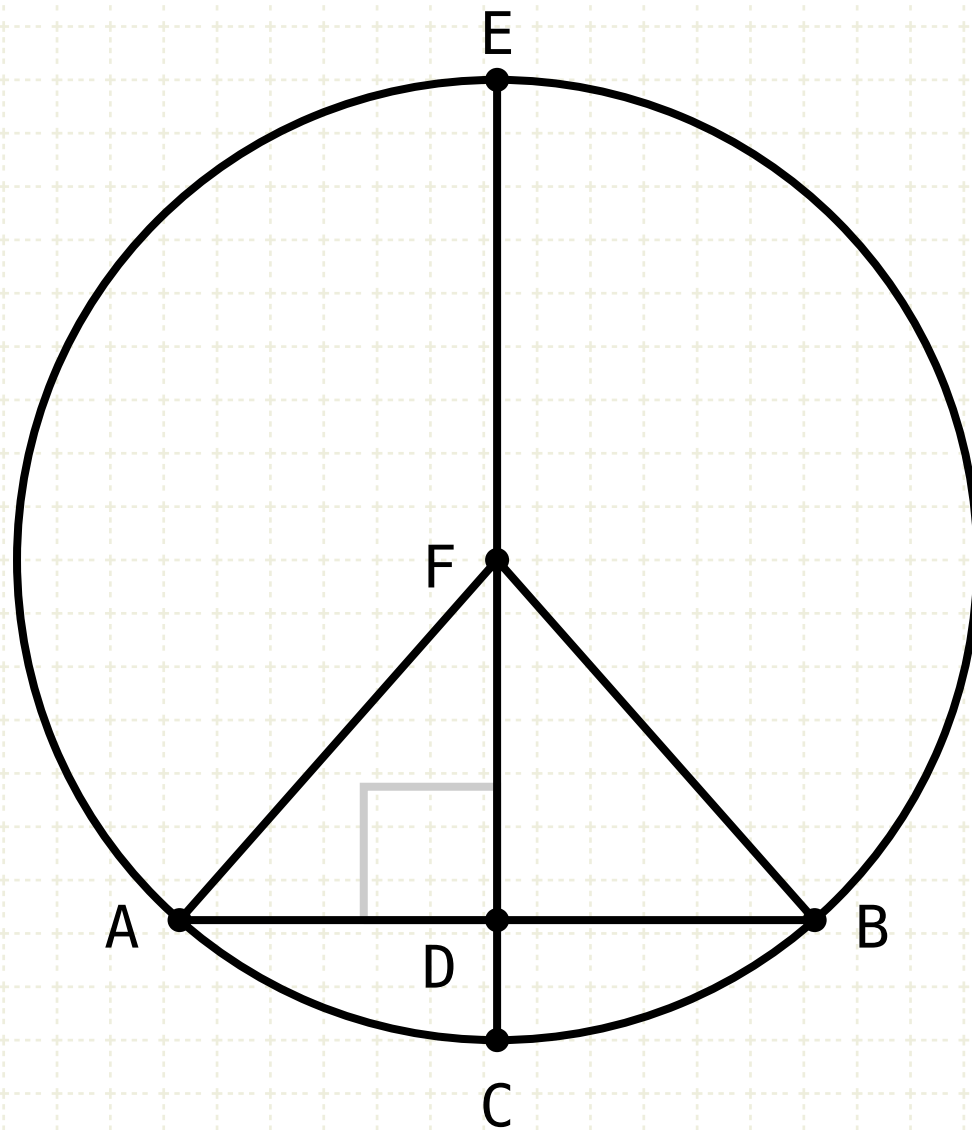
The angle α cannot be both greater than and equal to γ

So G is not the center of the circle

Thus, the centre of the circle must lie on the line CE

Proposition 1 of Book III

To find the center of a given circle.



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$$\gamma = \angle$$

$$CF = FE$$

If G is the centre of the circle

$$\alpha > \gamma$$

$$AG = GB$$

$$\alpha = \beta$$

$$\alpha = \beta = \angle$$

Centre of circle lies on EC

F is the centre

Proof by Contradiction

Assume that G is the center of the circle, and that G does not lie on the line CE

Join GA, GD and GB

AG and GB are radii, and thus are equal

Thus, since the side DG is shared between both triangles ADG and GDB, the triangles have three equal sides, therefore angle ADG (α) equals angle GDB (β) (I.8)

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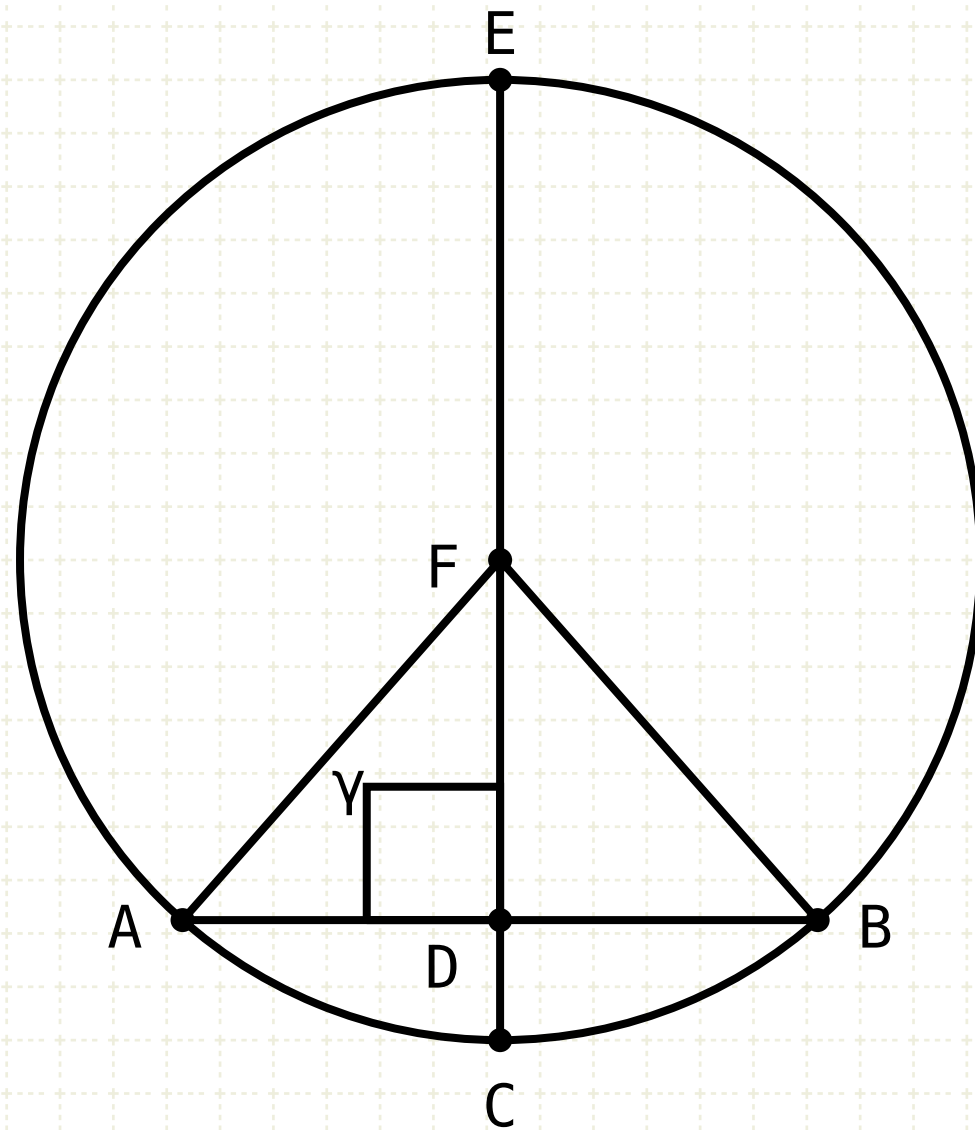
So G is not the center of the circle

Thus, the centre of the circle must lie on the line CE

Note that if G lies on the line CE, and it is the center of the circle, it must coincide with the point F, since the points E and C must be equidistant from the center.

Proposition 1 of Book III

To find the center of a given circle.



$$AD = DB$$

$$\gamma = \angle$$

$$CF = FE$$

If G is the centre of the circle

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Centre of circle lies on EC

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