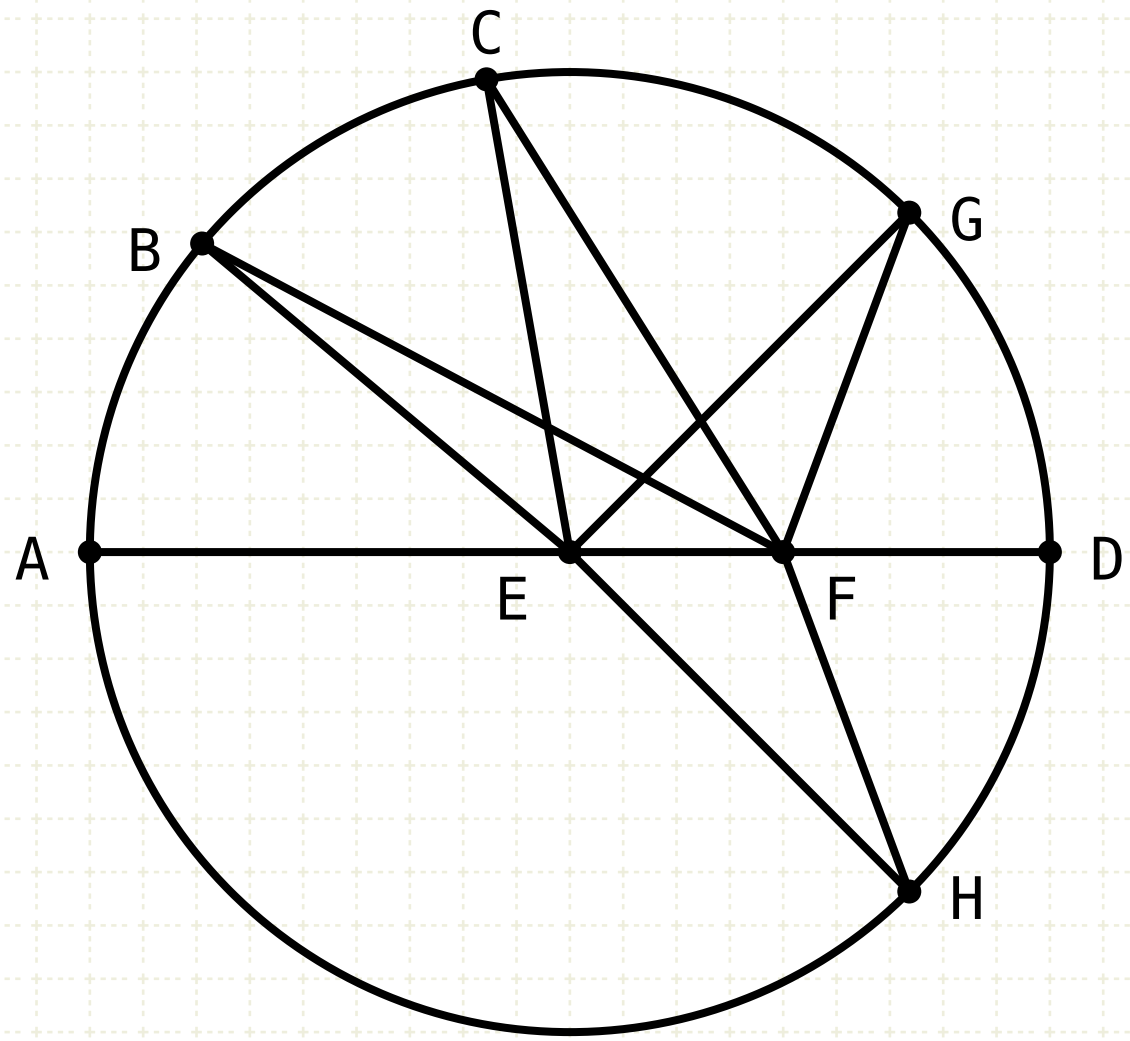
Euclid's Elements

Book III



A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)

Table of Contents, Chapter 3

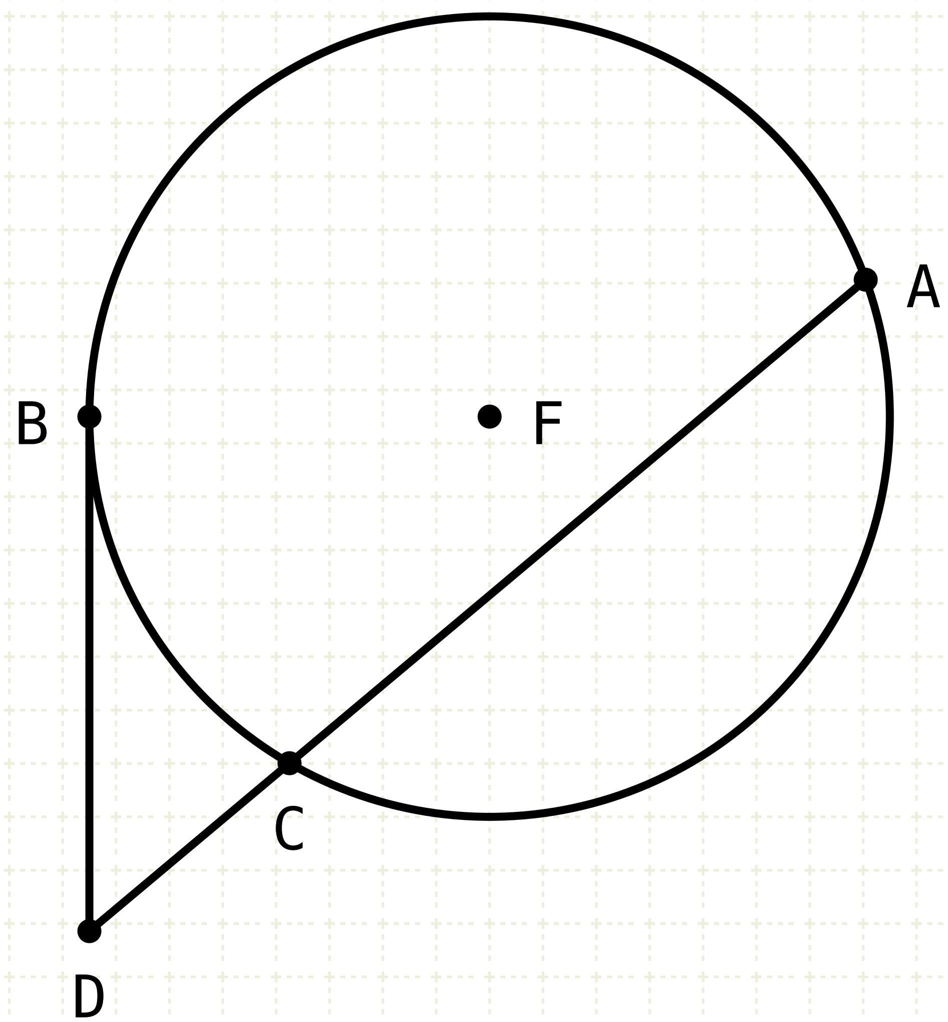
Table of Contents, Chapter 3

Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.

Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.



$AD \cdot CD = BD^2$

In other words

Let line AD cut a circle in two places, where A,C are the intersection points on the circumference of the circle

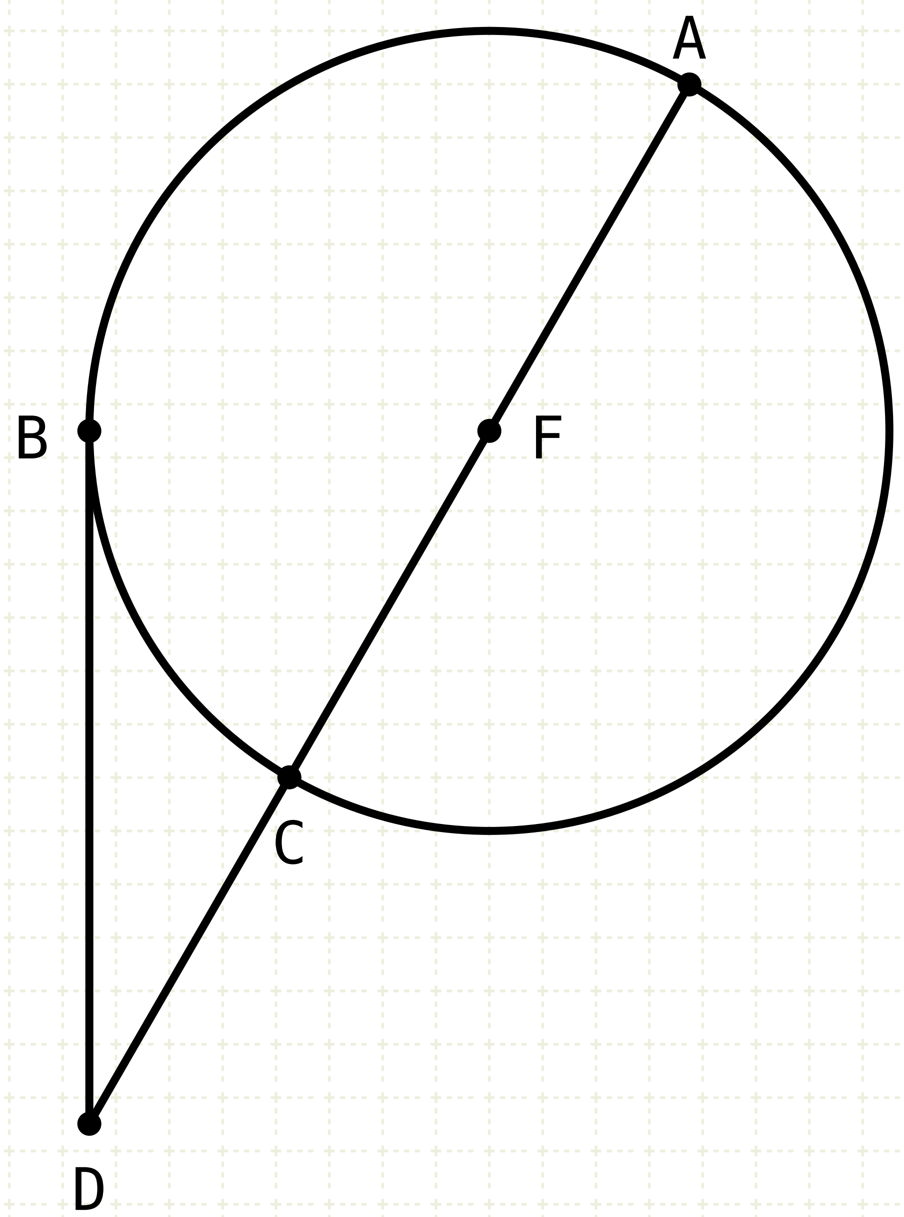
Definition of secant: a line that cuts a circle in two points

Let a line drawn from D touch the circle at point B

Then the product AD,CD equals BD squared

Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.

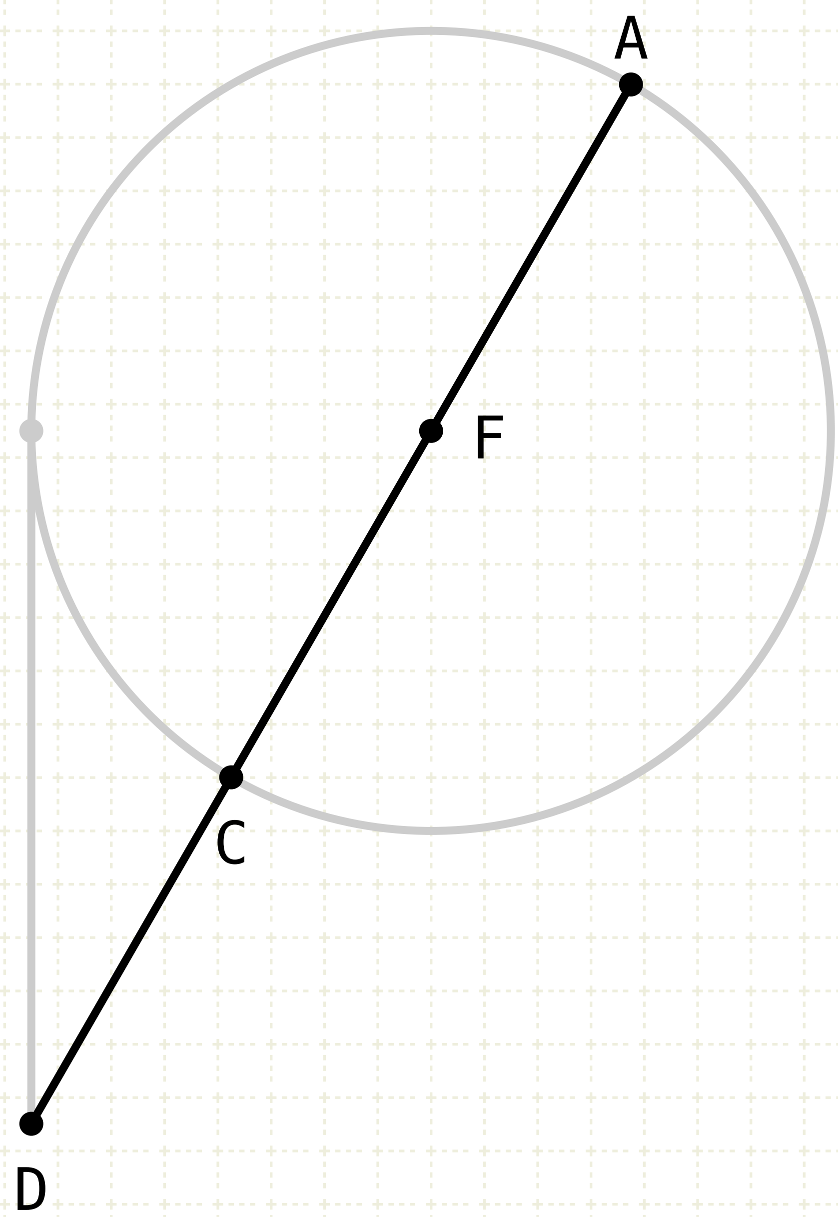


AF = FC

Proof - AD passes through centre of circle

Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.



$AF = FC$

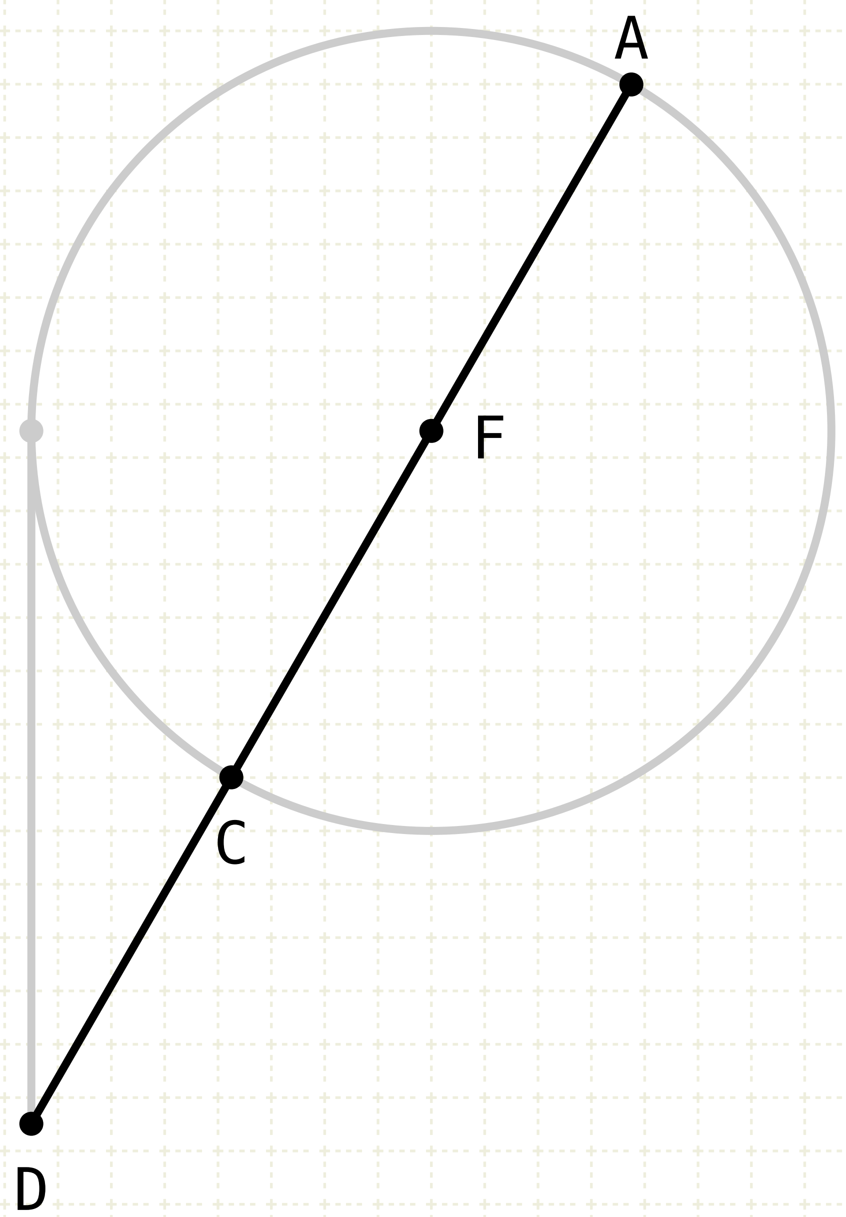
$AD \cdot CD + FC^2 = FD^2$

Proof - AD passes through centre of circle

If a line AC is bisected at point F, and extended from C to point D, the product AD,CD plus the square of FC equals the square of FD (II·6)

Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.

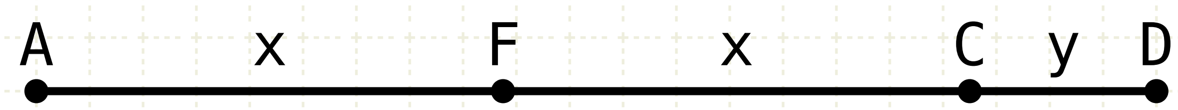


$AF = FC$

$AD \cdot CD + FC^2 = FD^2$

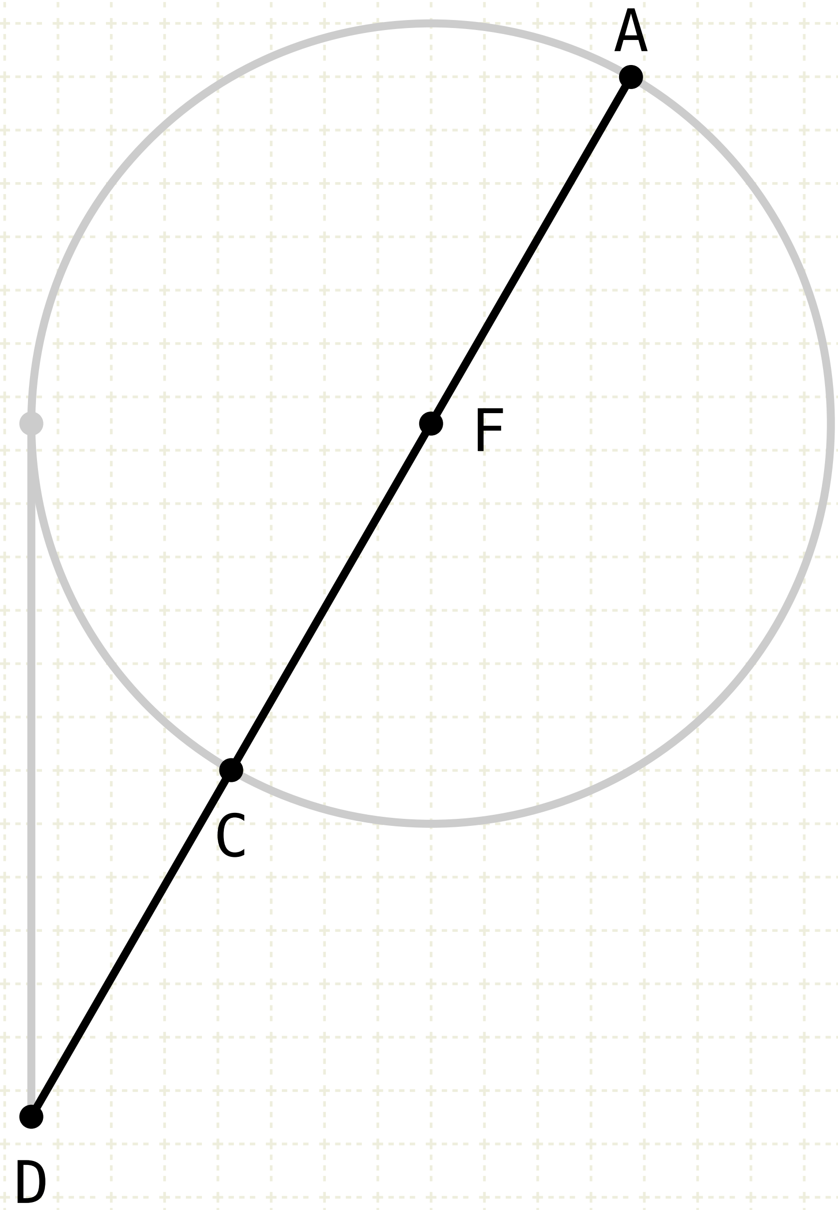
Proof - AD passes through centre of circle

If a line AC is bisected at point F, and extended from C to point D, the product AD,CD plus the square of FC equals the square of FD (II·6)



Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.

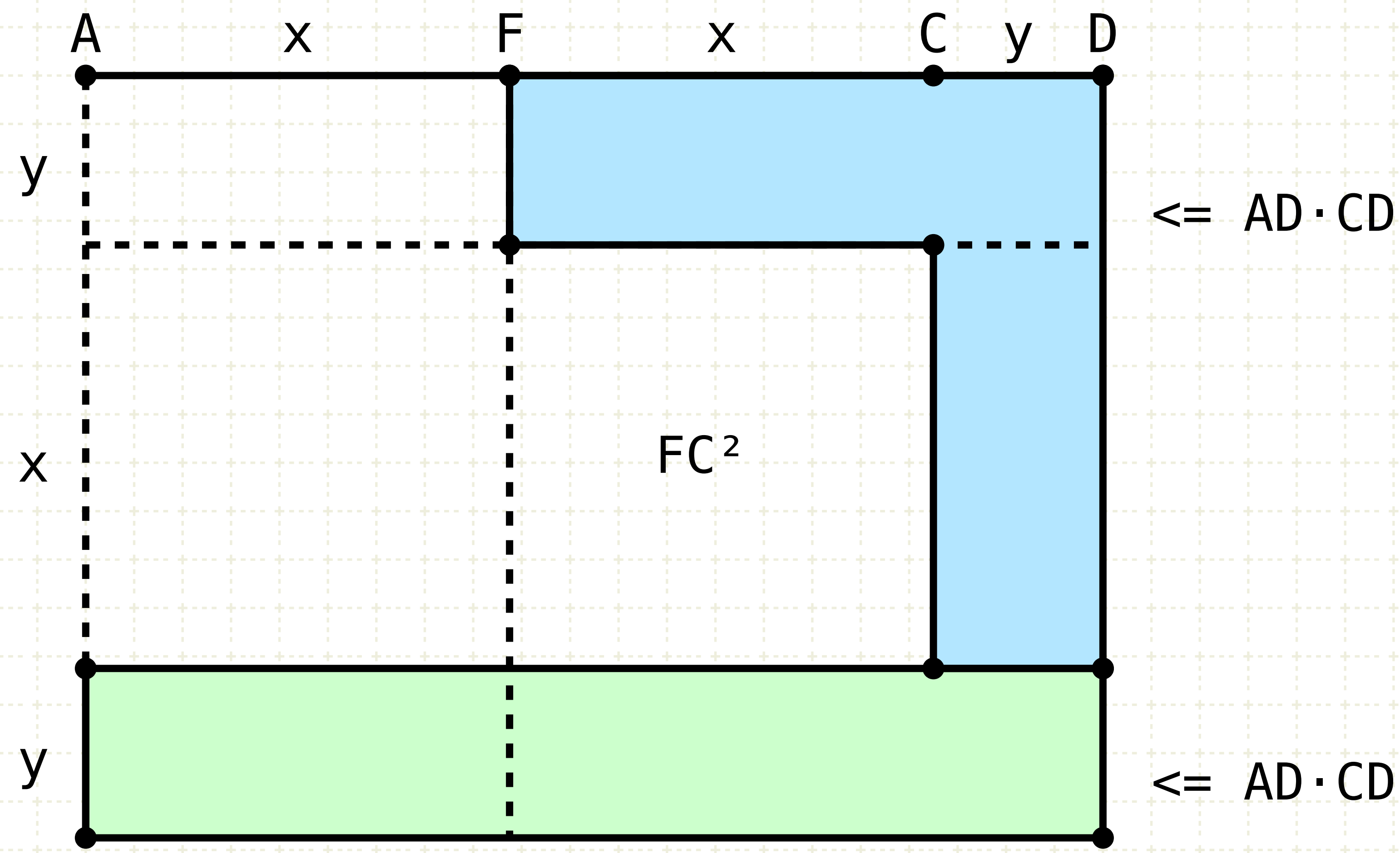


$AF = FC$

$AD \cdot CD + FC^2 = FD^2$

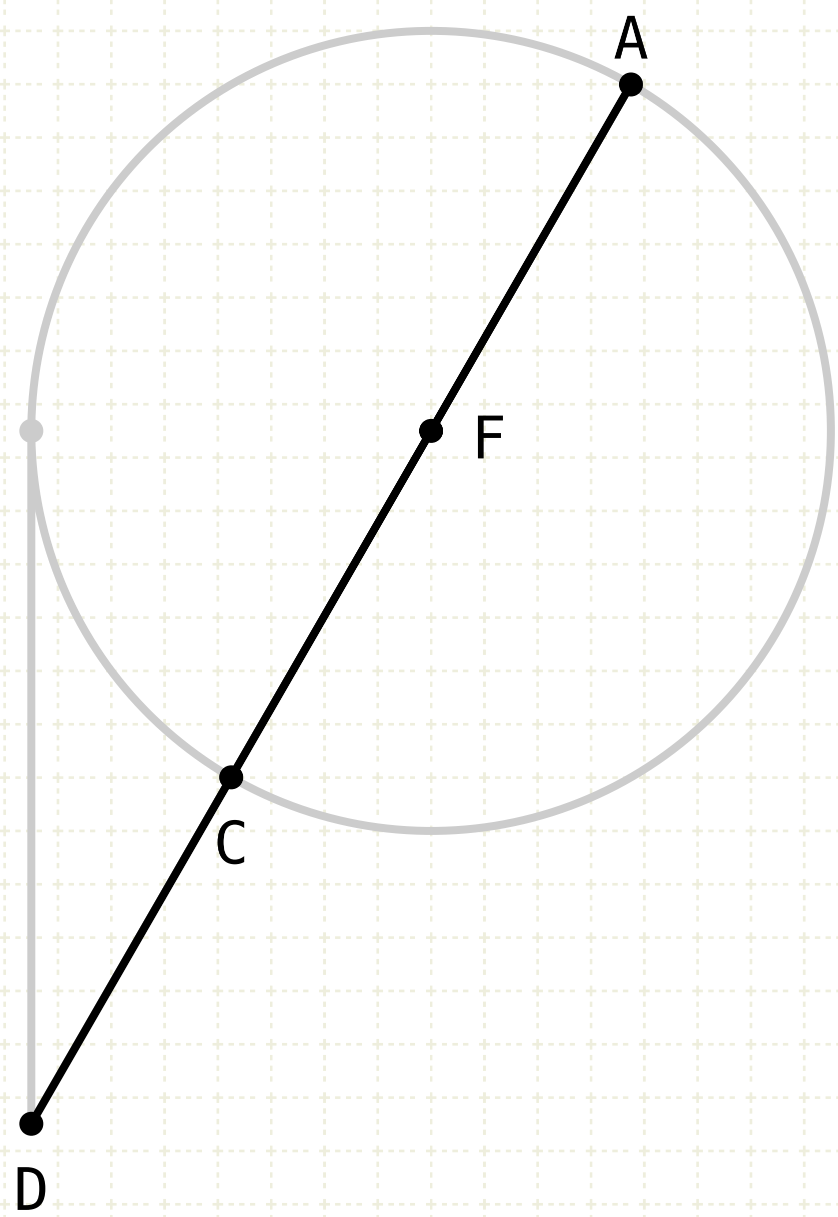
Proof - AD passes through centre of circle

If a line AC is bisected at point F, and extended from C to point D, the product AD,CD plus the square of FC equals the square of FD (II·6)



Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.



$AF = FC$

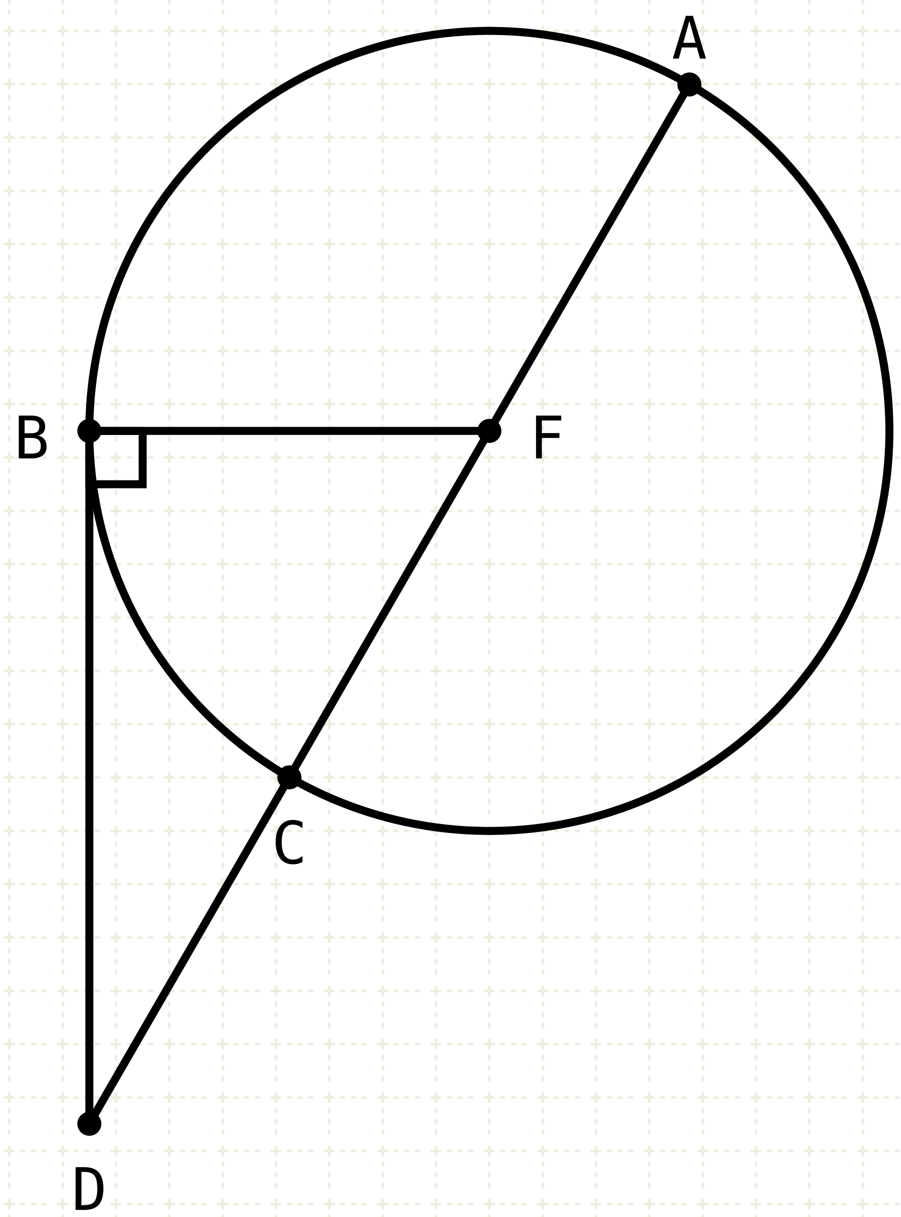
$AD \cdot CD + FC^2 = FD^2$

Proof - AD passes through centre of circle

If a line AC is bisected at point F, and extended from C to point D, the product AD,CD plus the square of FC equals the square of FD (II·6)

Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.



AF = FC

$AD \cdot CD + FC^2 = FD^2$

BF = FC

Proof - AD passes through centre of circle

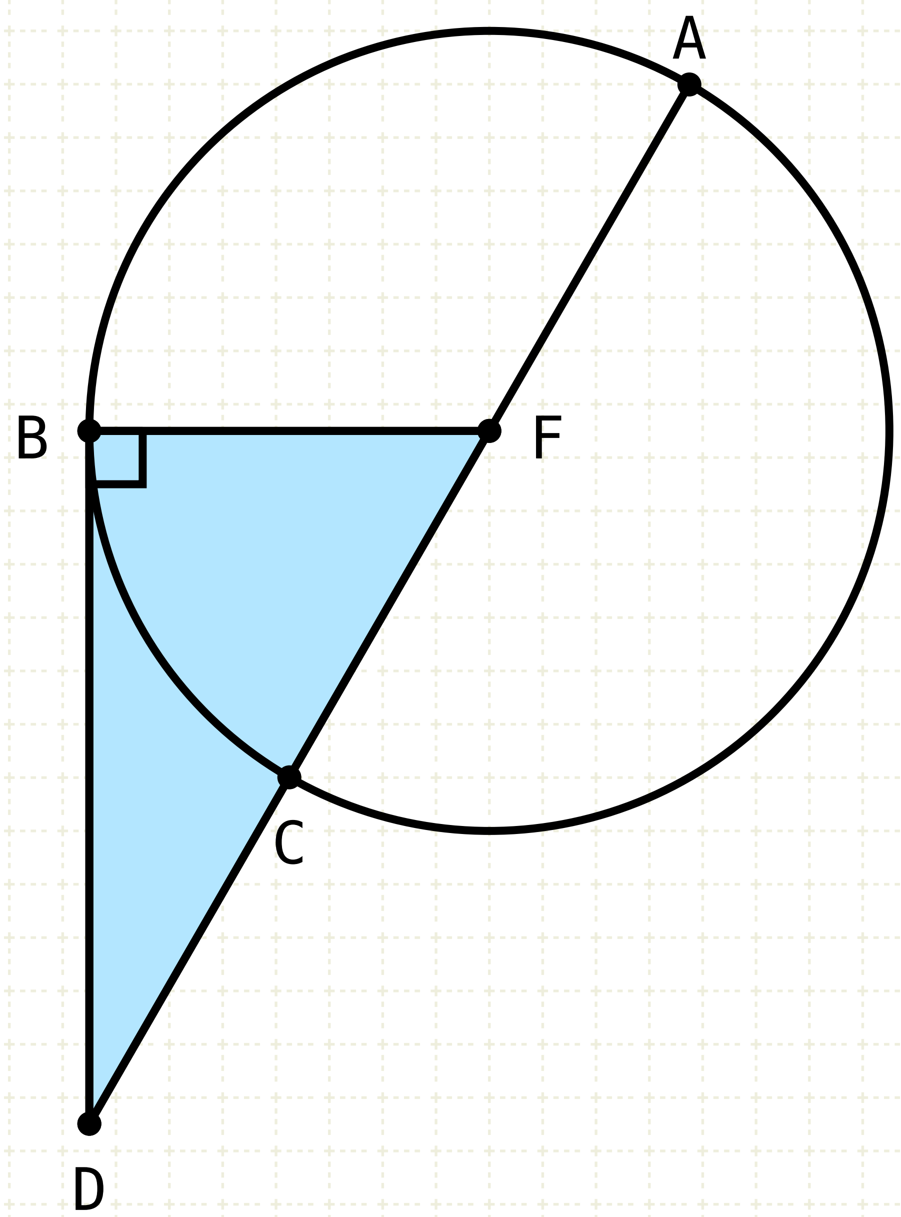
If a line AC is bisected at point F, and extended from C to point D, the product AD,CD plus the square of FC equals the square of FD (II·6)

Draw line FB

Angle FBD is right (III·18)

Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.



$AF = FC$

$AD \cdot CD + FC^2 = FD^2$

$BF = FC$

$BD^2 + BF^2 = FD^2$

Proof - AD passes through centre of circle

If a line AC is bisected at point F, and extended from C to point D, the product AD,CD plus the square of FC equals the square of FD (II·6)

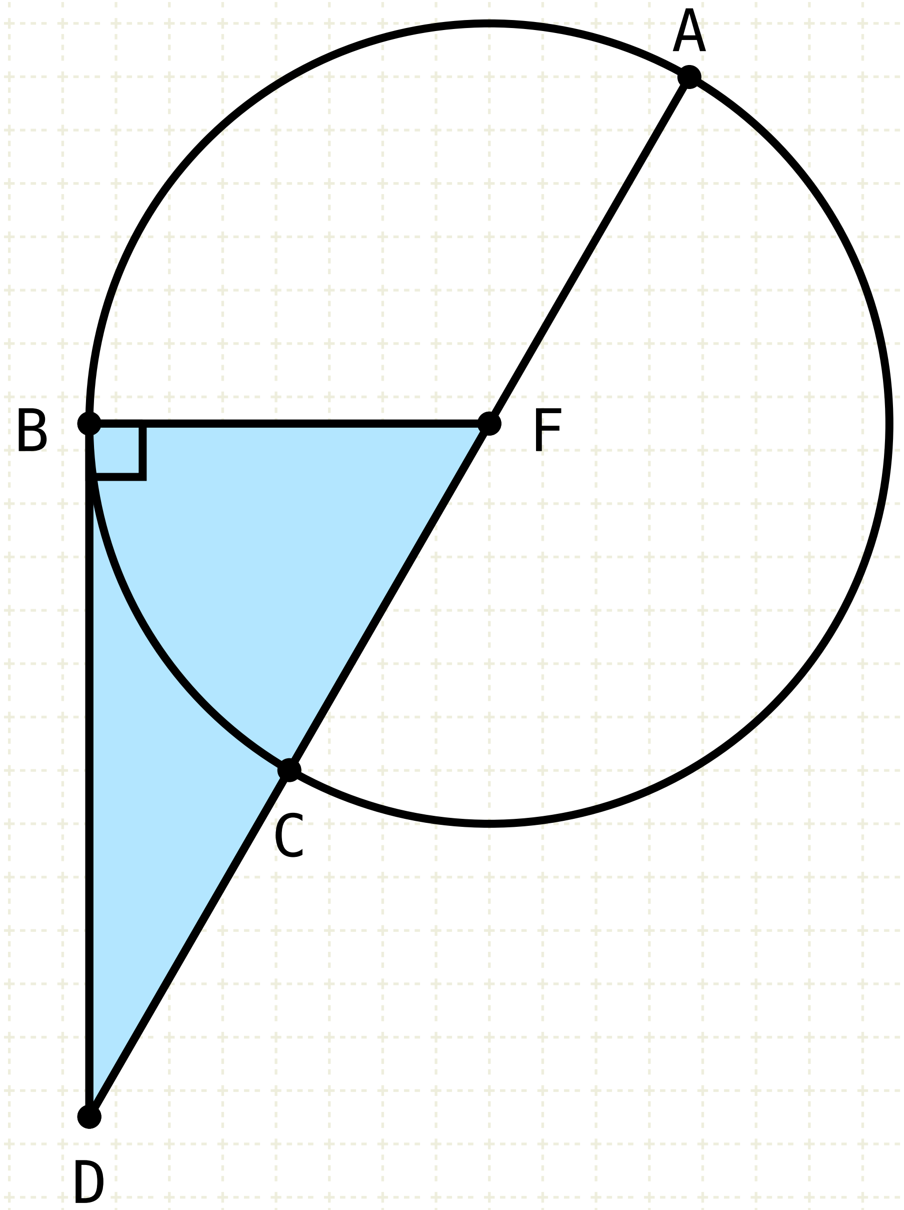
Draw line FB

Angle FBD is right (III·18)

The triangle FBD is a right triangle, and thus follows Pythagoras' rule (I·47), thus the sum of the squares BF,BD equals the square of FD

Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.



$AF = FC$

$AD \cdot CD + FC^2 = FD^2$

$BF = FC$

$BD^2 + BF^2 = FD^2$

$BD^2 + FC^2 = FD^2$

Proof - AD passes through centre of circle

If a line AC is bisected at point F, and extended from C to point D, the product AD,CD plus the square of FC equals the square of FD (II·6)

Draw line FB

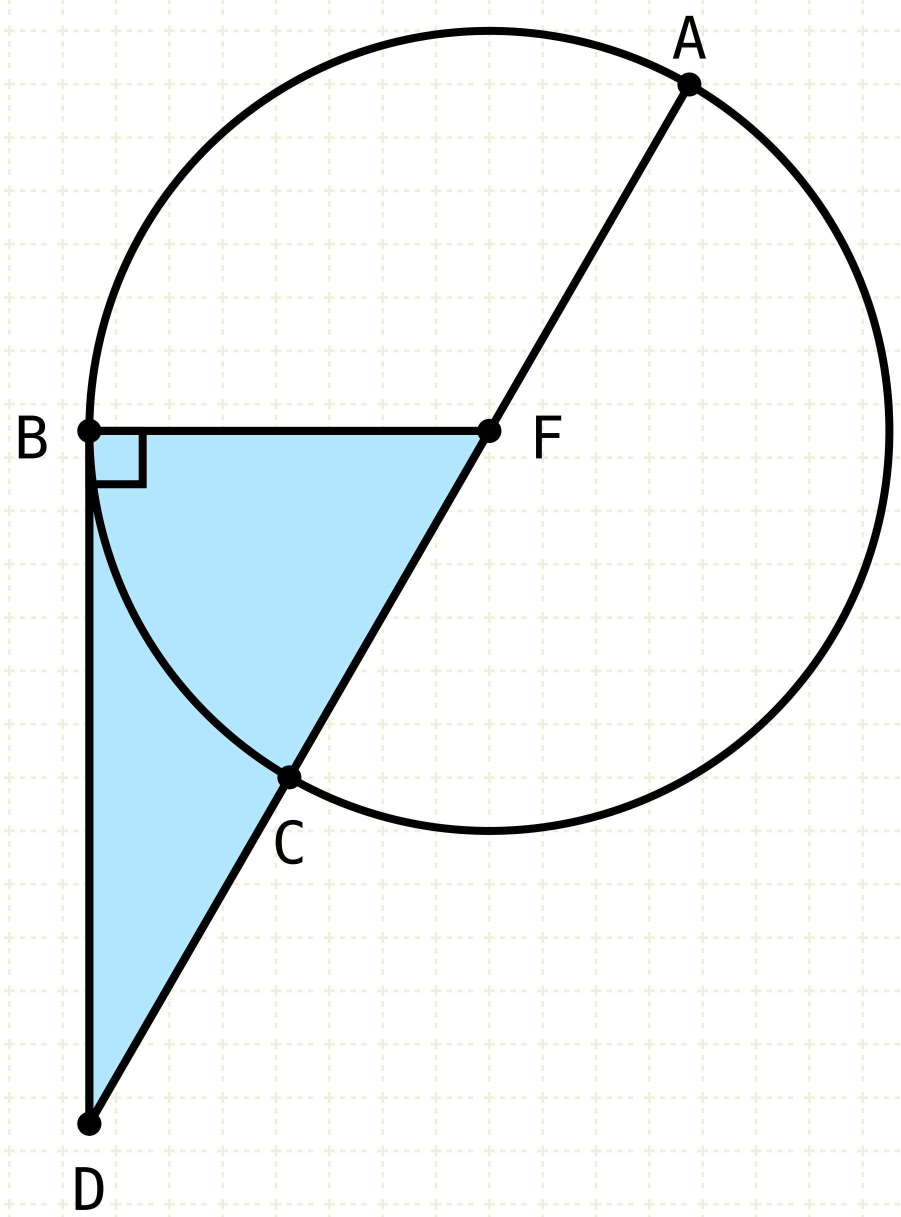
Angle FBD is right (III·18)

The triangle FBD is a right triangle, and thus follows Pythagoras' rule (I·47), thus the sum of the squares BF,BD equals the square of FD

Since BF equals FC, then the square of BF equals the square of FC

Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.



$AF = FC$

$AD \cdot CD + FC^2 = FD^2$

$BF = FC$

$BD^2 + BF^2 = FD^2$

$BD^2 + FC^2 = FD^2$

$BD^2 = FD^2 - FC^2$

Proof - AD passes through centre of circle

If a line AC is bisected at point F, and extended from C to point D, the product AD,CD plus the square of FC equals the square of FD (II·6)

Draw line FB

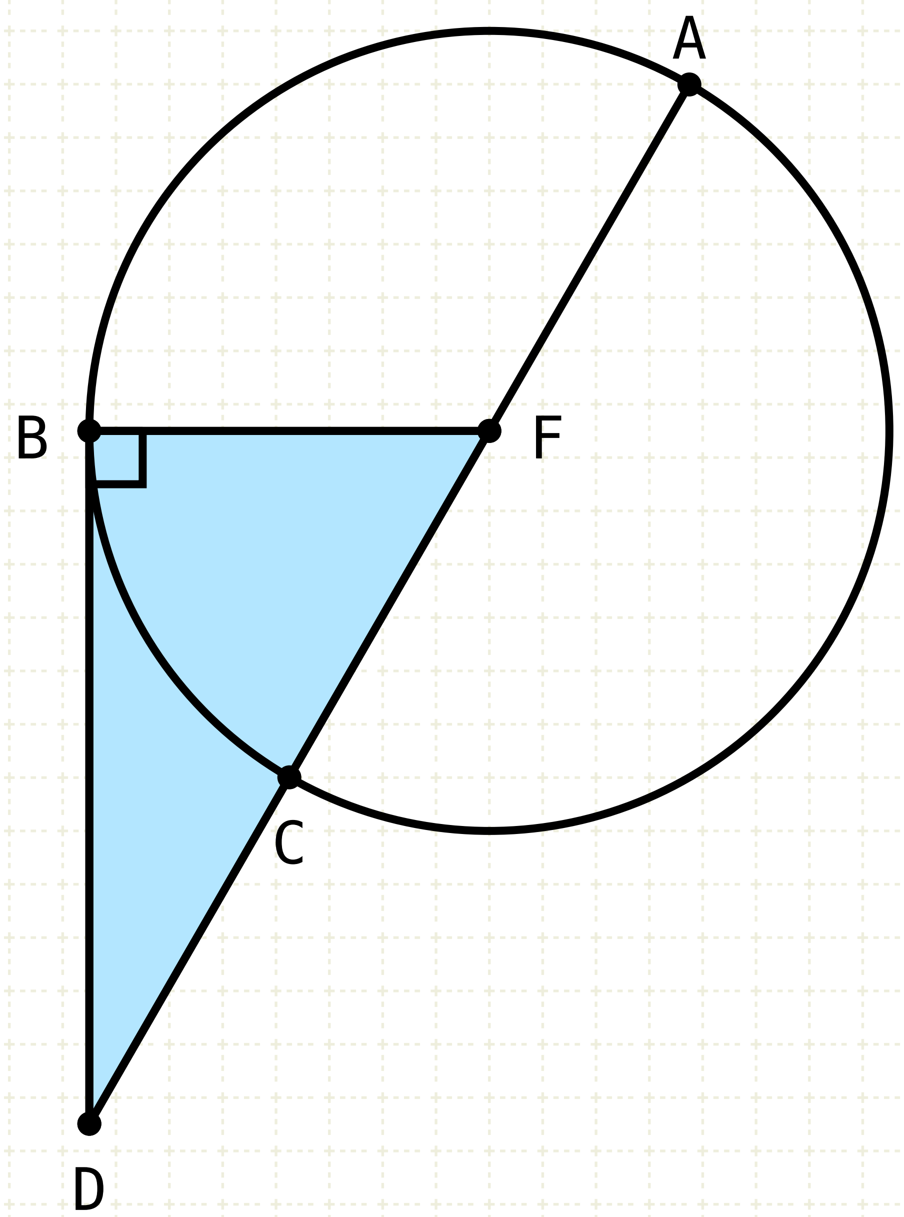
Angle FBD is right (III·18)

The triangle FBD is a right triangle, and thus follows Pythagoras' rule (I·47), thus the sum of the squares BF,BD equals the square of FD

Since BF equals FC, then the square of BF equals the square of FC

Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.



$AF = FC$

$AD \cdot CD + FC^2 = FD^2$

$BF = FC$

$BD^2 + BF^2 = FD^2$

$BD^2 + FC^2 = FD^2$

$BD^2 = FD^2 - FC^2$

$AD \cdot CD = FD^2 - FC^2$

Proof - AD passes through centre of circle

If a line AC is bisected at point F, and extended from C to point D, the product AD,CD plus the square of FC equals the square of FD (II·6)

Draw line FB

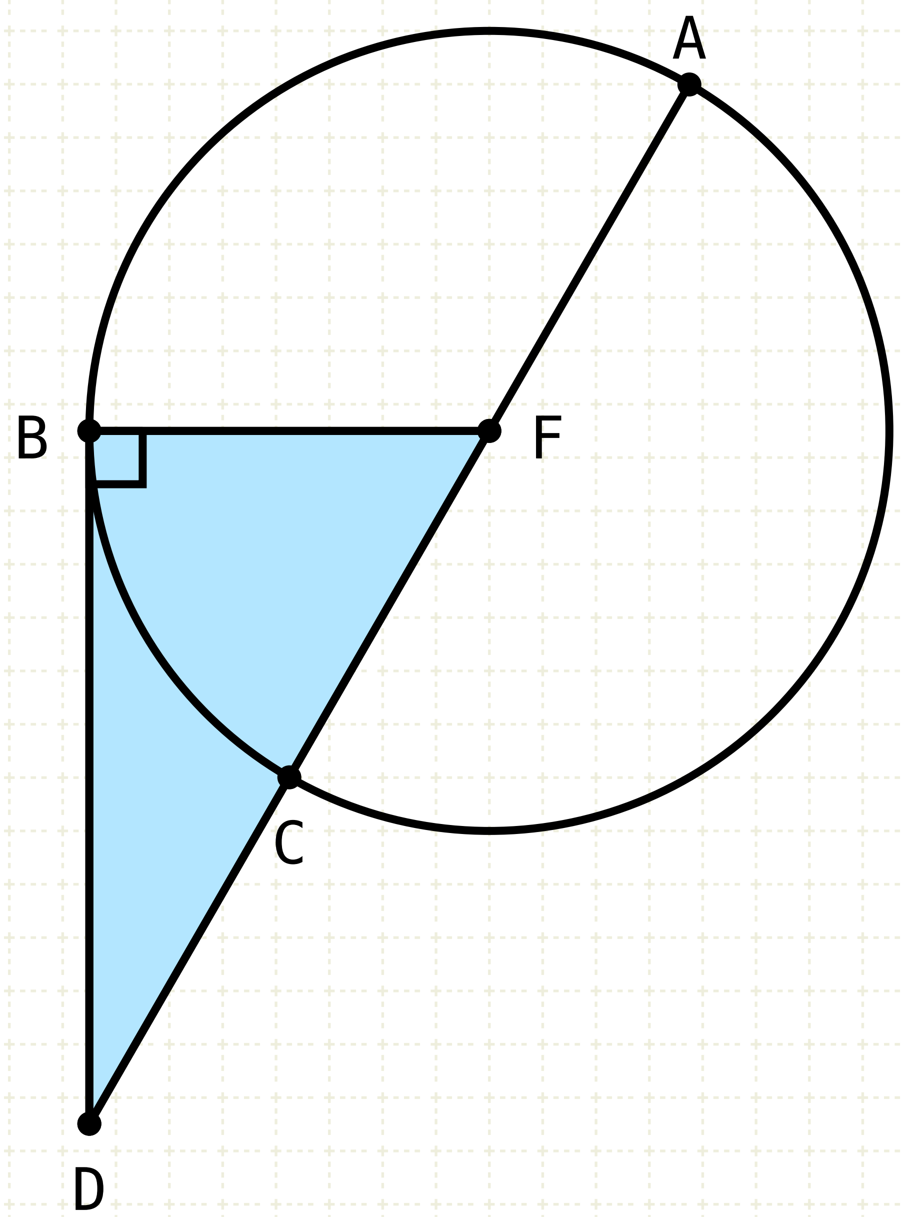
Angle FBD is right (III·18)

The triangle FBD is a right triangle, and thus follows Pythagoras' rule (I·47), thus the sum of the squares BF,BD equals the square of FD

Since BF equals FC, then the square of BF equals the square of FC

Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.



$AF = FC$

$AD \cdot CD + FC^2 = FD^2$

$BF = FC$

$BD^2 + BF^2 = FD^2$

$BD^2 + FC^2 = FD^2$

$BD^2 = FD^2 - FC^2$

$AD \cdot CD = FD^2 - FC^2$

$AD \cdot CD = BD^2$

Proof - AD passes through centre of circle

If a line AC is bisected at point F, and extended from C to point D, the product AD,CD plus the square of FC equals the square of FD (II·6)

Draw line FB

Angle FBD is right (III·18)

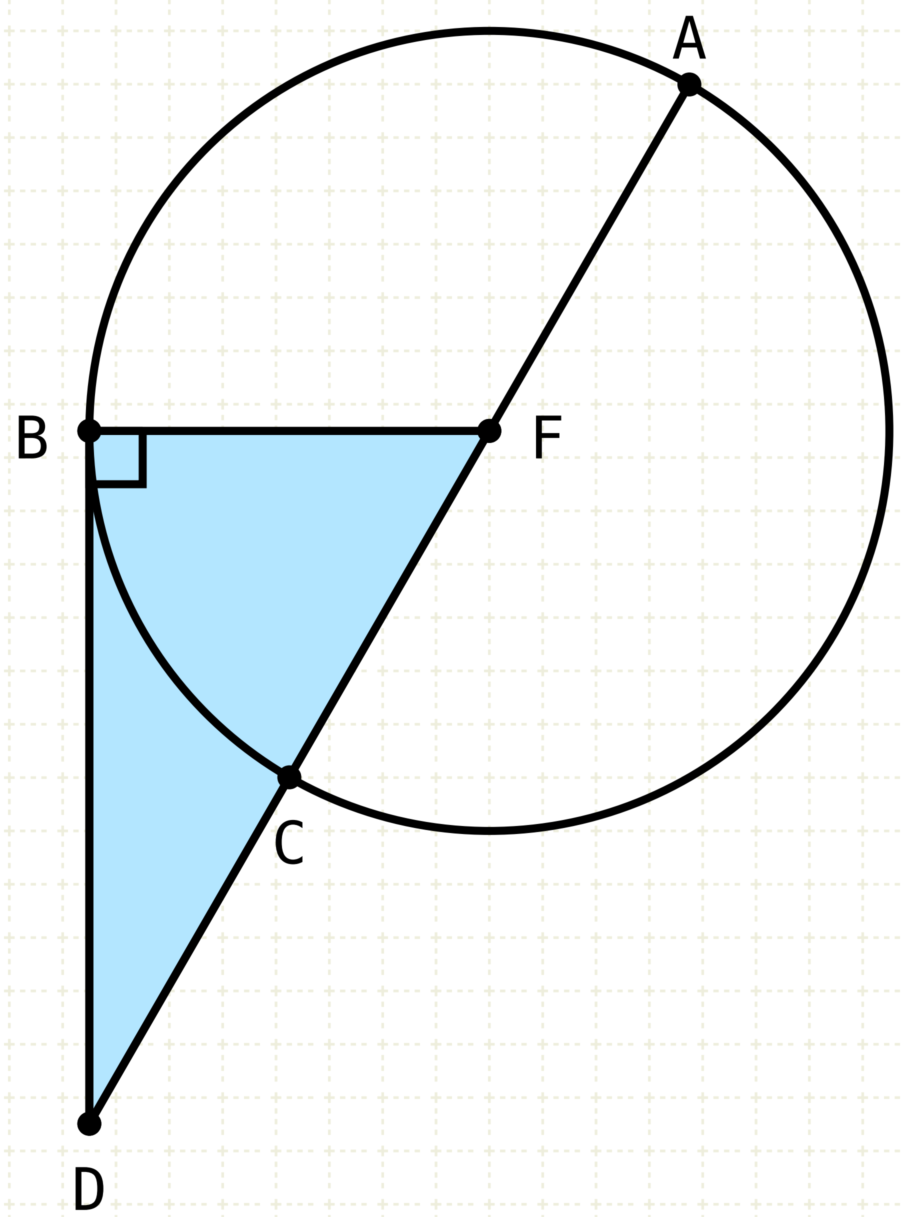
The triangle FBD is a right triangle, and thus follows Pythagoras' rule (I·47), thus the sum of the squares BF,BD equals the square of FD

Since BF equals FC, then the square of BF equals the square of FC

Thus, the product of AD,CD equals the square of BD

Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.



$AF = FC$

$AD \cdot CD + FC^2 = FD^2$

$BF = FC$

$BD^2 + BF^2 = FD^2$

$BD^2 + FC^2 = FD^2$

$BD^2 = FD^2 - FC^2$

$AD \cdot CD = FD^2 - FC^2$

$AD \cdot CD = BD^2$

Proof - AD passes through centre of circle

If a line AC is bisected at point F, and extended from C to point D, the product AD,CD plus the square of FC equals the square of FD (II·6)

Draw line FB

Angle FBD is right (III·18)

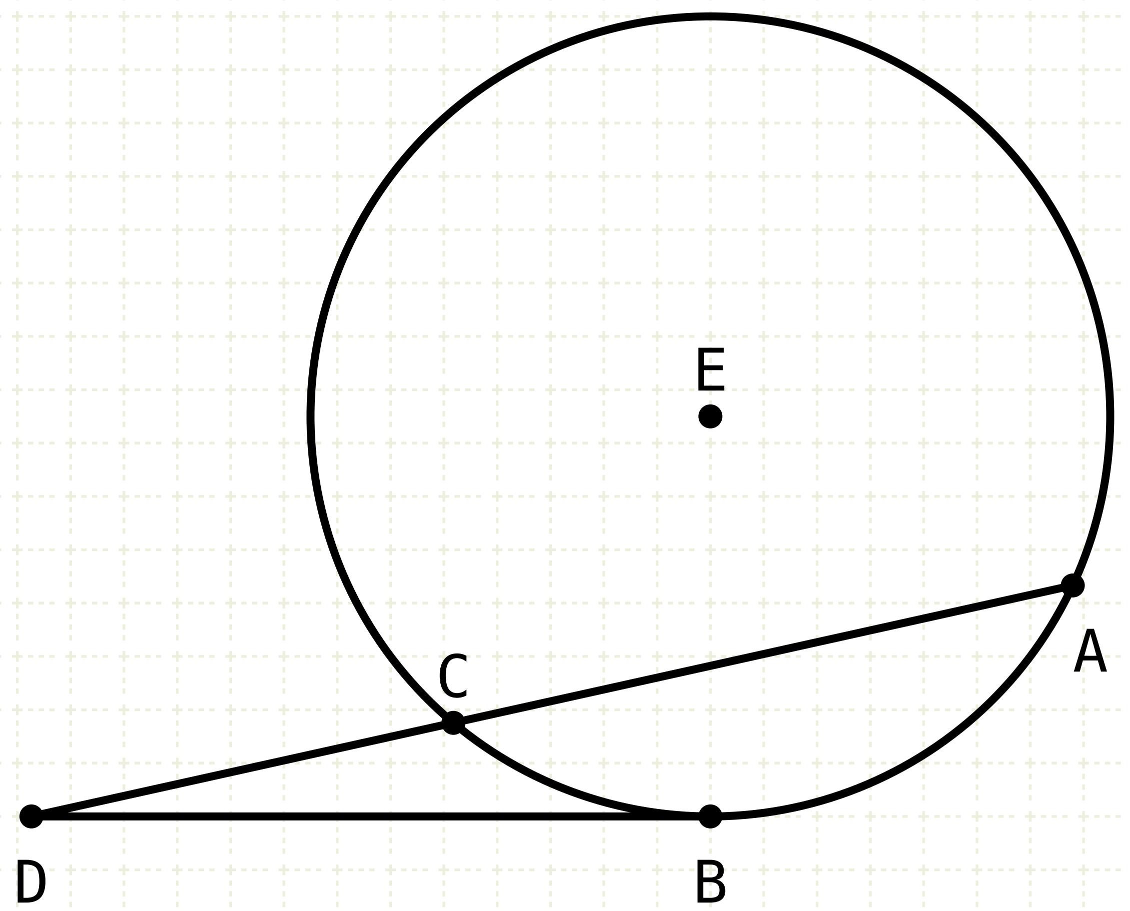
The triangle FBD is a right triangle, and thus follows Pythagoras' rule (I·47), thus the sum of the squares BF,BD equals the square of FD

Since BF equals FC, then the square of BF equals the square of FC

Thus, the product of AD,CD equals the square of BD

Proposition 36 of Book III

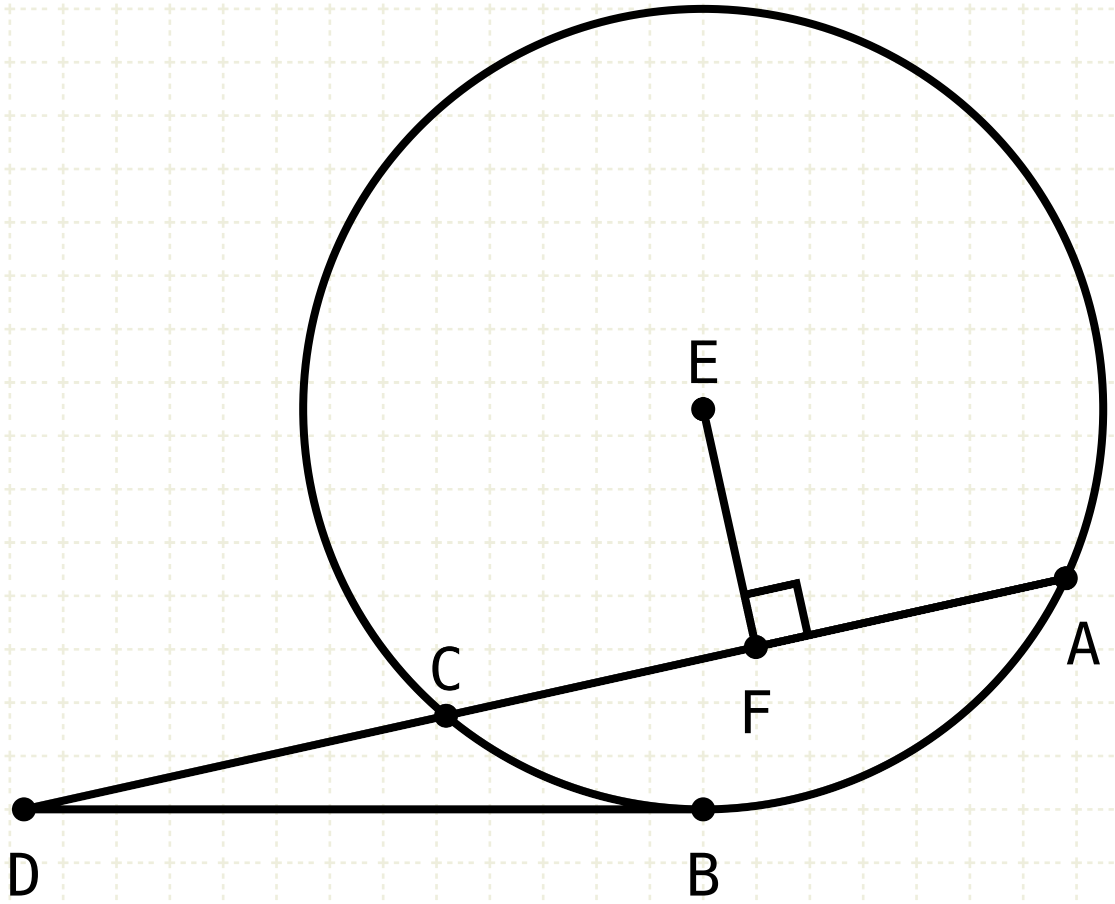
If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.



Proof - AD does not pass through centre of circle

Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.

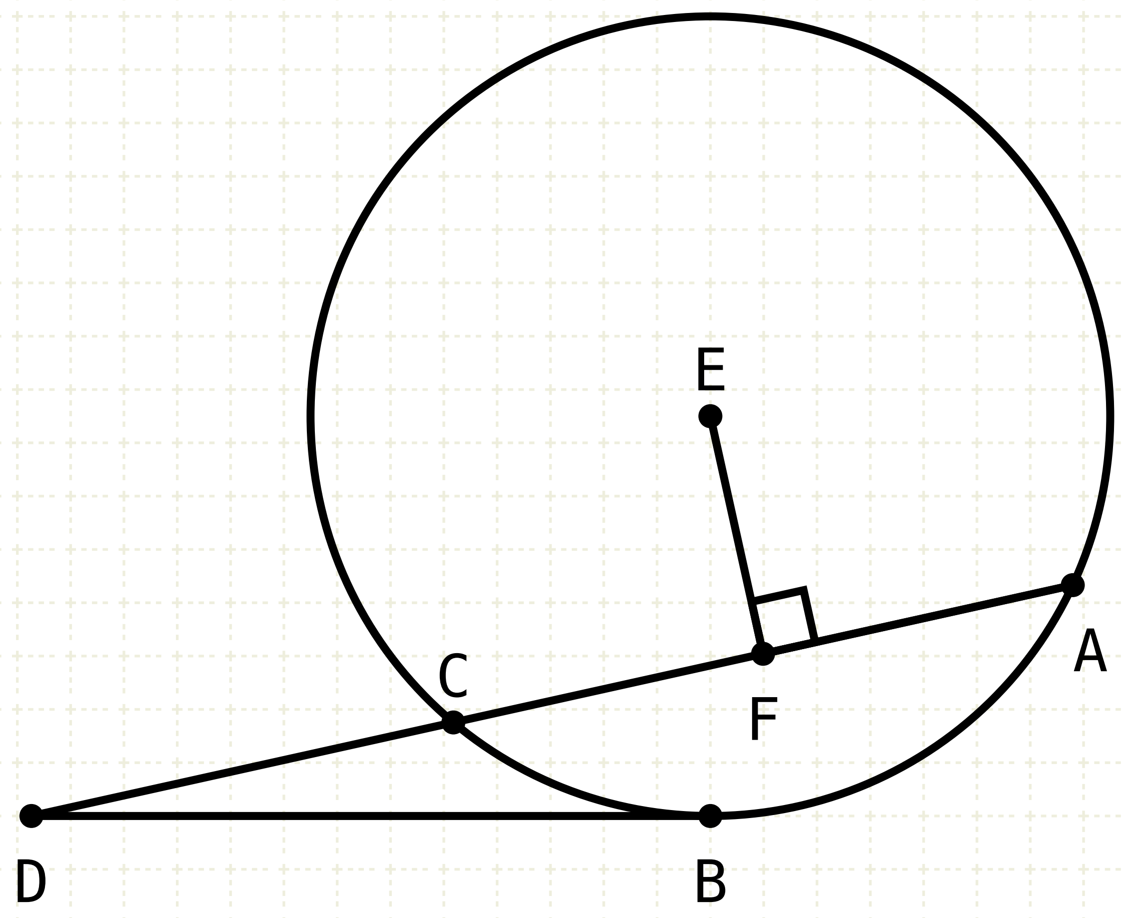


Proof - AD does not pass through centre of circle

Draw a line EF from the centre of the circle E, perpendicular to the line DA

Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.



CF = FA

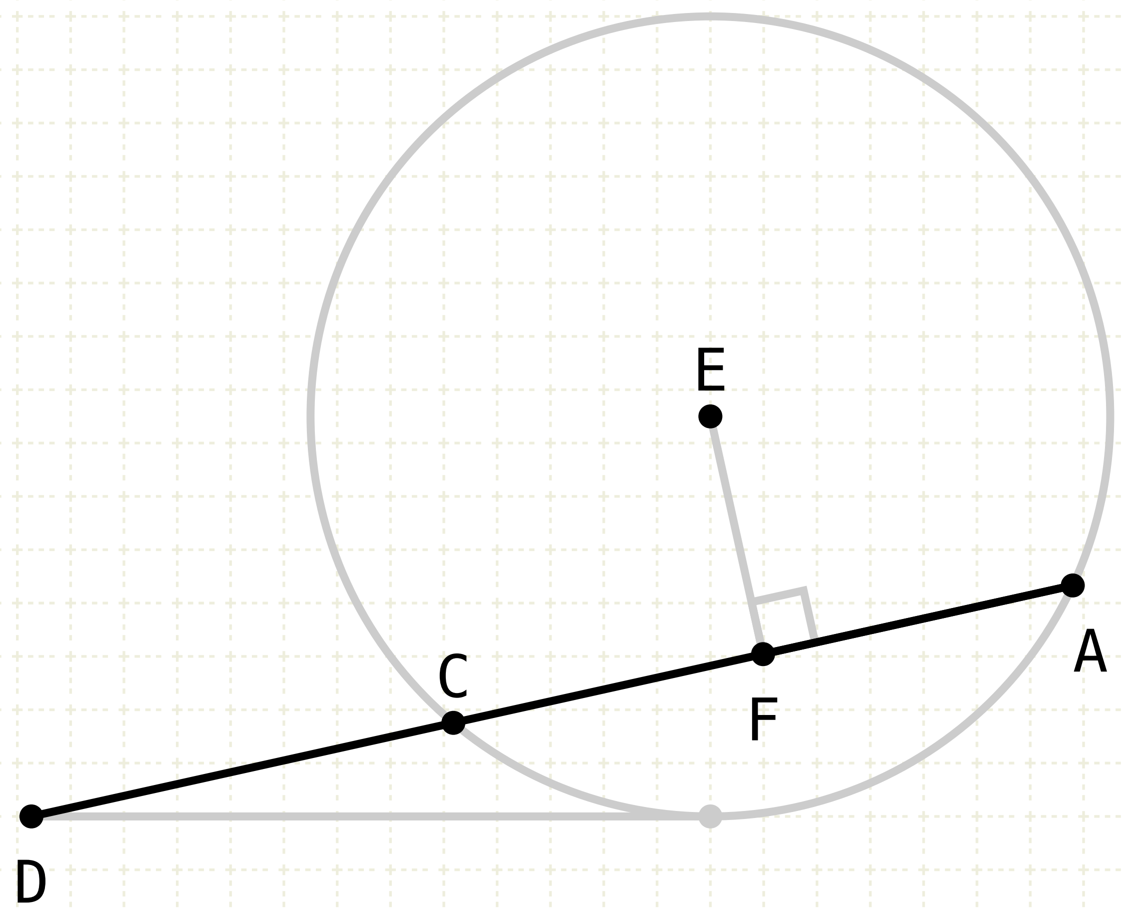
Proof - AD does not pass through centre of circle

Draw a line EF from the centre of the circle E, perpendicular to the line DA

Lines CF and FA are equal (III·18)

Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.



CF = FA

$AD \cdot CD + FC^2 = FD^2$

Proof - AD does not pass through centre of circle

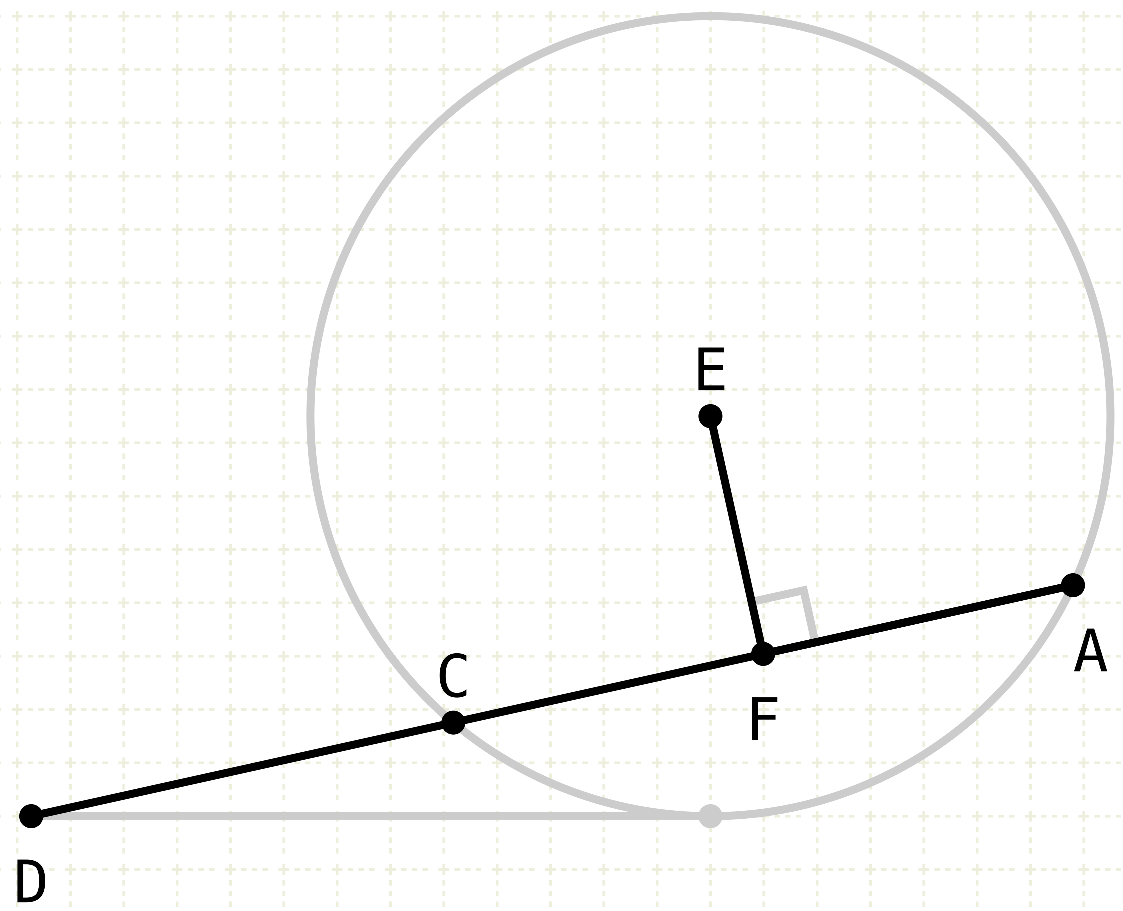
Draw a line EF from the centre of the circle E, perpendicular to the line DA

Lines CF and FA are equal (III·18)

If a line AC is bisected at point F, and extended from C to point D, the product AD,CD plus the square of FC equals the square of FD (II·6)

Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.



$CF = FA$

$AD \cdot CD + FC^2 = FD^2$

$AD \cdot CD + FC^2 + EF^2$
$= FD^2 + EF^2$

Proof - AD does not pass through centre of circle

Draw a line EF from the centre of the circle E, perpendicular to the line DA

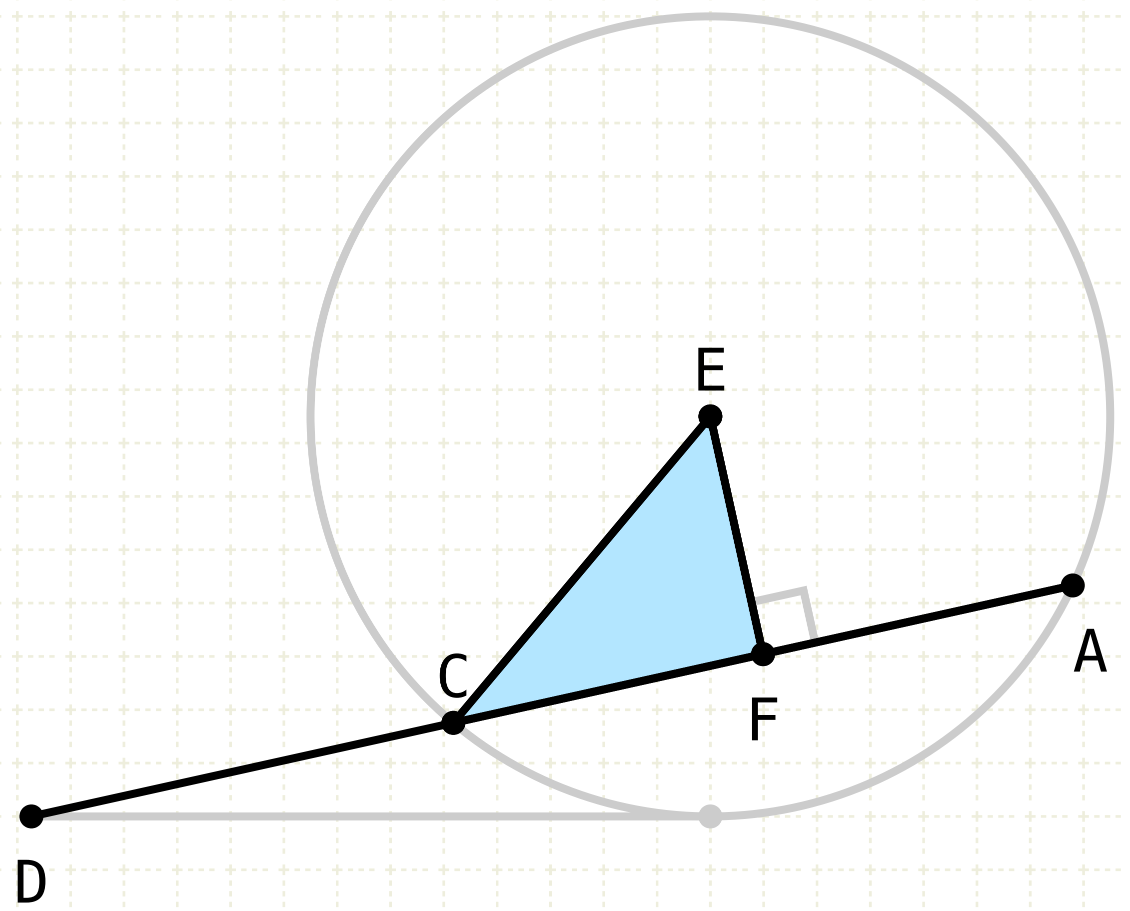
Lines CF and FA are equal (III·18)

If a line AC is bisected at point F, and extended from C to point D, the product AD,CD plus the square of FC equals the square of FD (II·6)

Add the square of EF to both sides of the equality

Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.



$CF = FA$

$AD \cdot CD + FC^2 = FD^2$

$AD \cdot CD + FC^2 + EF^2$

$= FD^2 + EF^2$

$FC^2 + EF^2 = EC^2$

Proof - AD does not pass through centre of circle

Draw a line EF from the centre of the circle E, perpendicular to the line DA

Lines CF and FA are equal (III·18)

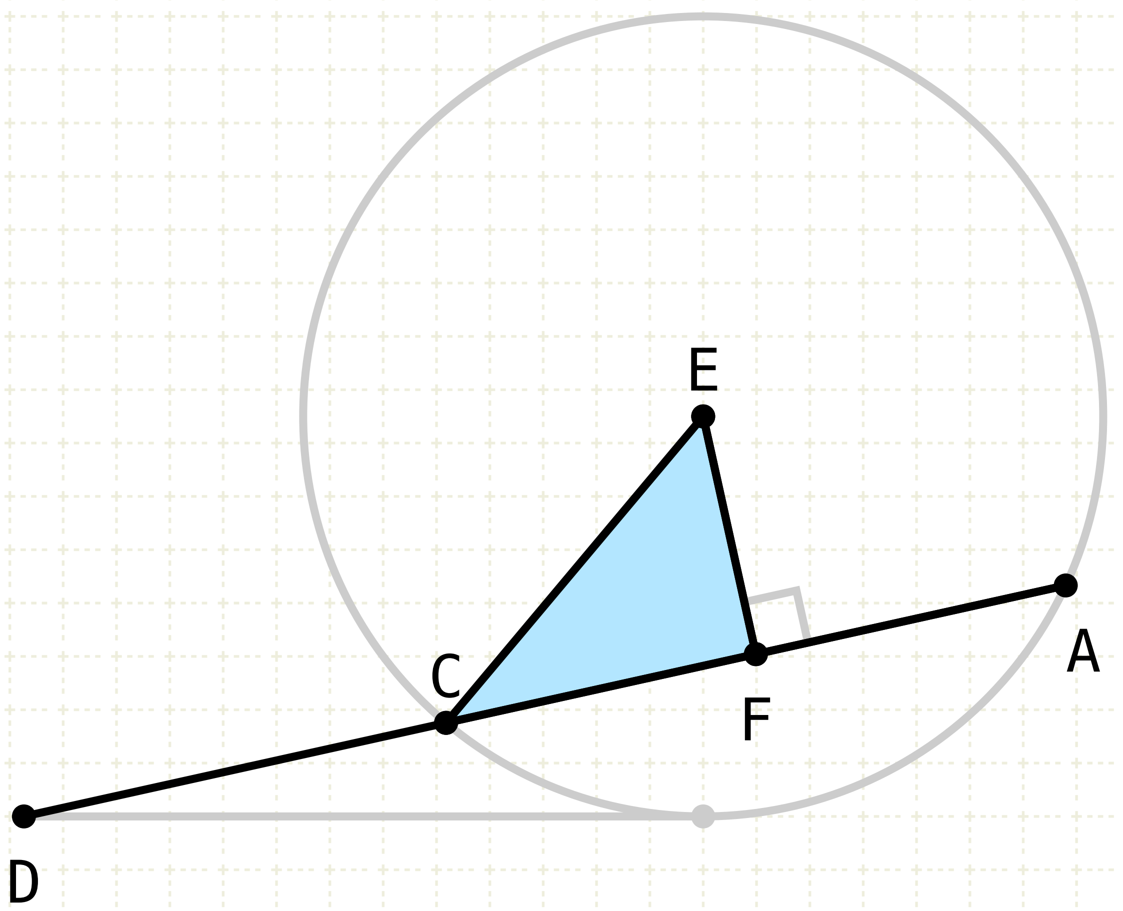
If a line AC is bisected at point F, and extended from C to point D, the product AD,CD plus the square of FC equals the square of FD (II·6)

Add the square of EF to both sides of the equality

In the triangle ECF, the sum of the squares EF,FC equal the square of EC (I·47)

Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.



$CF = FA$

$AD \cdot CD + FC^2 = FD^2$

$AD \cdot CD + FC^2 + EF^2$
$= FD^2 + EF^2$

$FC^2 + EF^2 = EC^2$

$AD \cdot CD + EC^2$
$= FD^2 + EF^2$

Proof - AD does not pass through centre of circle

Draw a line EF from the centre of the circle E, perpendicular to the line DA

Lines CF and FA are equal (III·18)

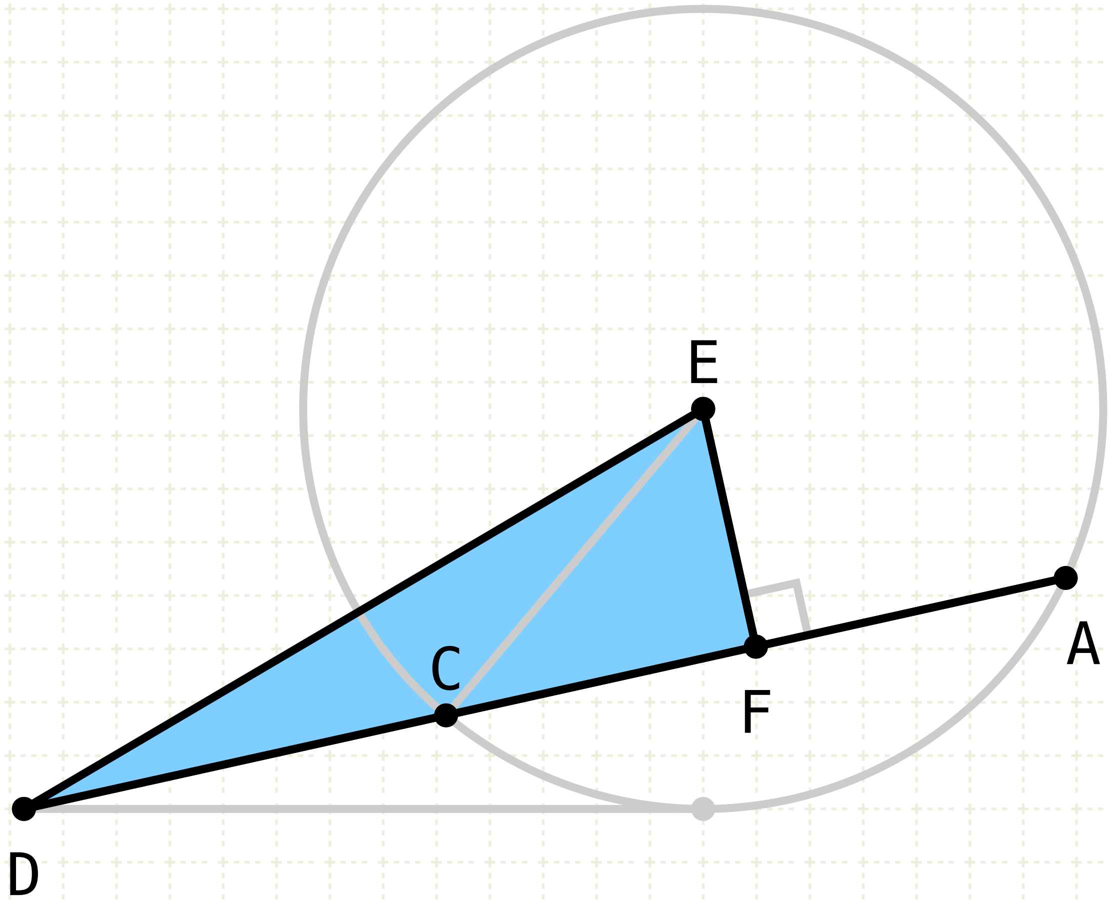
If a line AC is bisected at point F, and extended from C to point D, the product AD,CD plus the square of FC equals the square of FD (II·6)

Add the square of EF to both sides of the equality

In the triangle ECF, the sum of the squares EF,FC equal the square of EC (I·47)

Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.



$CF = FA$

$AD \cdot CD + FC^2 = FD^2$

$AD \cdot CD + FC^2 + EF^2 = FD^2 + EF^2$

$FC^2 + EF^2 = EC^2$

$AD \cdot CD + EC^2 = FD^2 + EF^2$

$FD^2 + EF^2 = ED^2$

Proof - AD does not pass through centre of circle

Draw a line EF from the centre of the circle E, perpendicular to the line DA

Lines CF and FA are equal (III·18)

If a line AC is bisected at point F, and extended from C to point D, the product AD,CD plus the square of FC equals the square of FD (II·6)

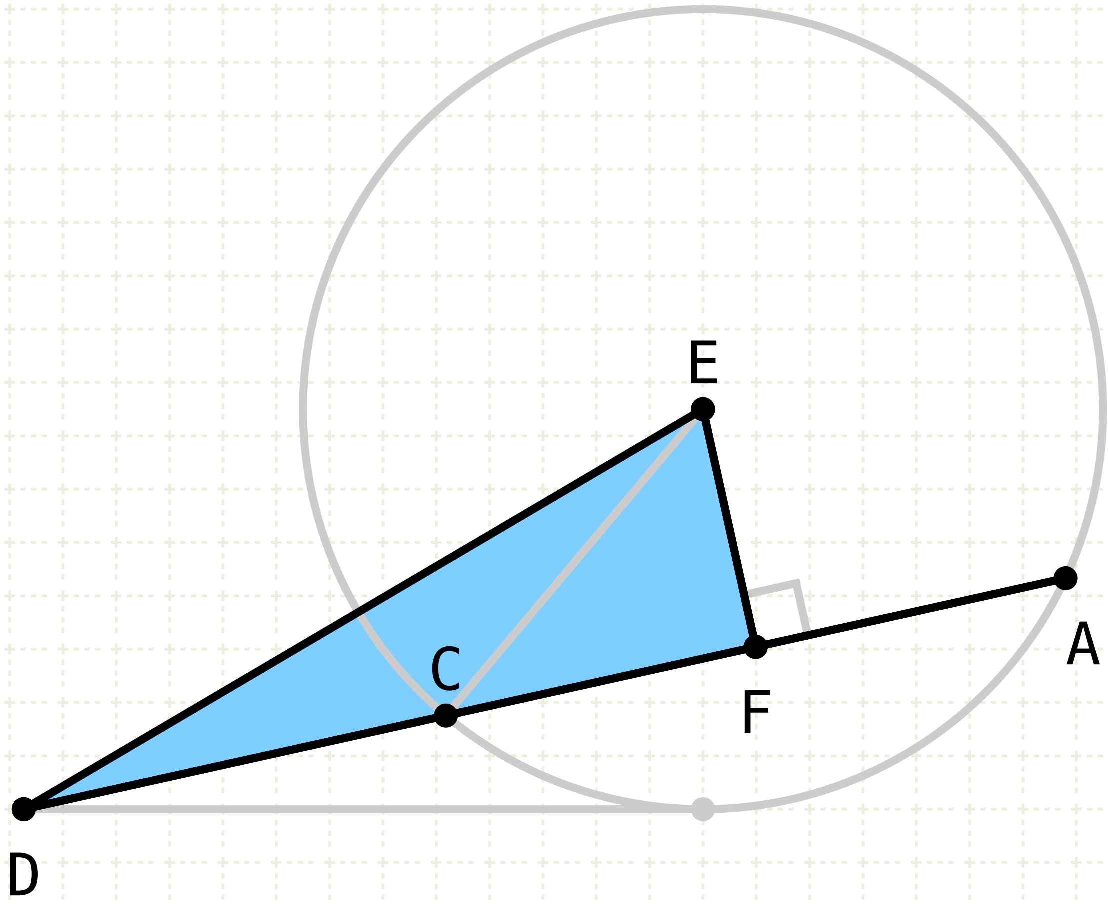
Add the square of EF to both sides of the equality

In the triangle ECF, the sum of the squares EF,FC equal the square of EC (I·47)

Similarly, the sum of the squares FD,EF equal the square of ED (I·47)

Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.



$CF = FA$

$AD \cdot CD + FC^2 = FD^2$

$AD \cdot CD + FC^2 + EF^2 = FD^2 + EF^2$

$FC^2 + EF^2 = EC^2$

$AD \cdot CD + EC^2 = FD^2 + EF^2$

$FD^2 + EF^2 = ED^2$

$AD \cdot CD + EC^2 = ED^2$

Proof - AD does not pass through centre of circle

Draw a line EF from the centre of the circle E, perpendicular to the line DA

Lines CF and FA are equal (III·18)

If a line AC is bisected at point F, and extended from C to point D, the product AD,CD plus the square of FC equals the square of FD (II·6)

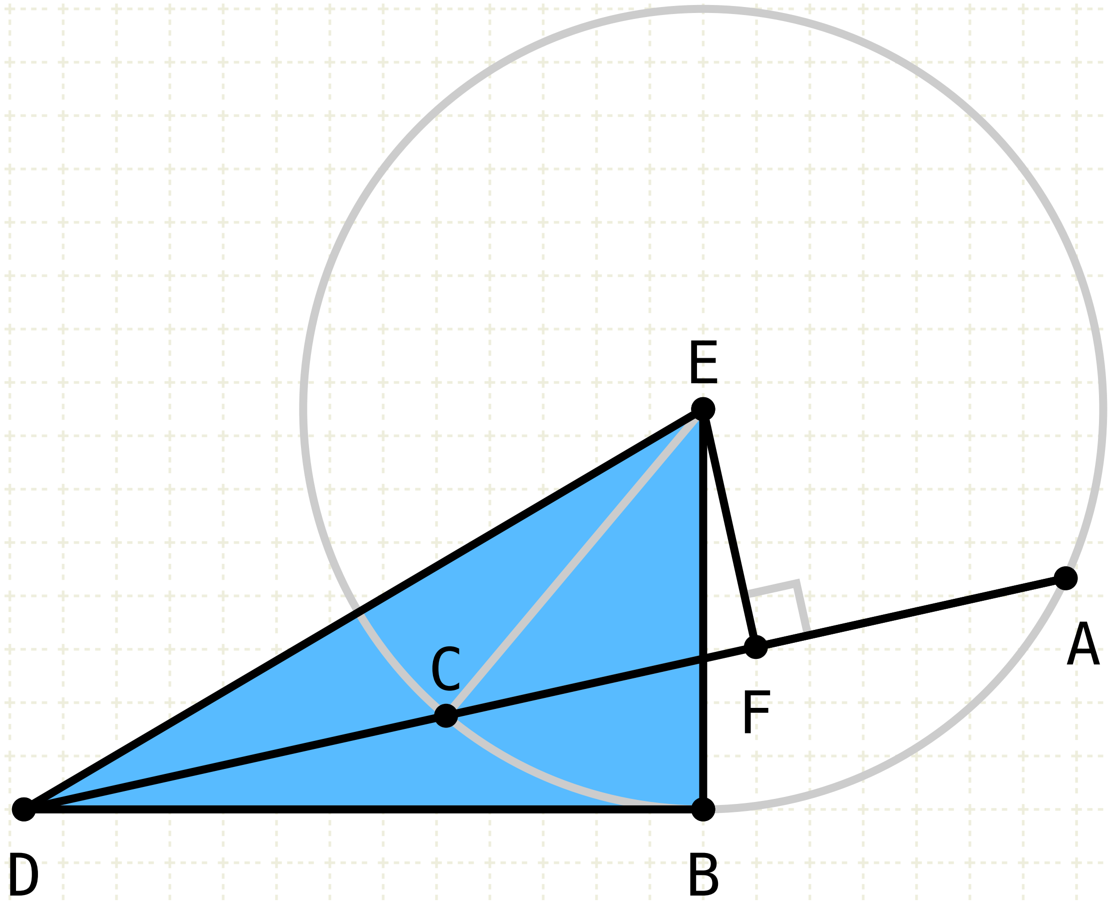
Add the square of EF to both sides of the equality

In the triangle ECF, the sum of the squares EF,FC equal the square of EC (I·47)

Similarly, the sum of the squares FD,EF equal the square of ED (I·47)

Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.



$CF = FA$

$AD \cdot CD + FC^2 = FD^2$

$AD \cdot CD + FC^2 + EF^2 = FD^2 + EF^2$

$FC^2 + EF^2 = EC^2$

$AD \cdot CD + EC^2 = FD^2 + EF^2$

$FD^2 + EF^2 = ED^2$

$AD \cdot CD + EC^2 = ED^2$

$DB^2 + EB^2 = ED^2$

Proof - AD does not pass through centre of circle

Draw a line EF from the centre of the circle E, perpendicular to the line DA

Lines CF and FA are equal (III·18)

If a line AC is bisected at point F, and extended from C to point D, the product AD,CD plus the square of FC equals the square of FD (II·6)

Add the square of EF to both sides of the equality

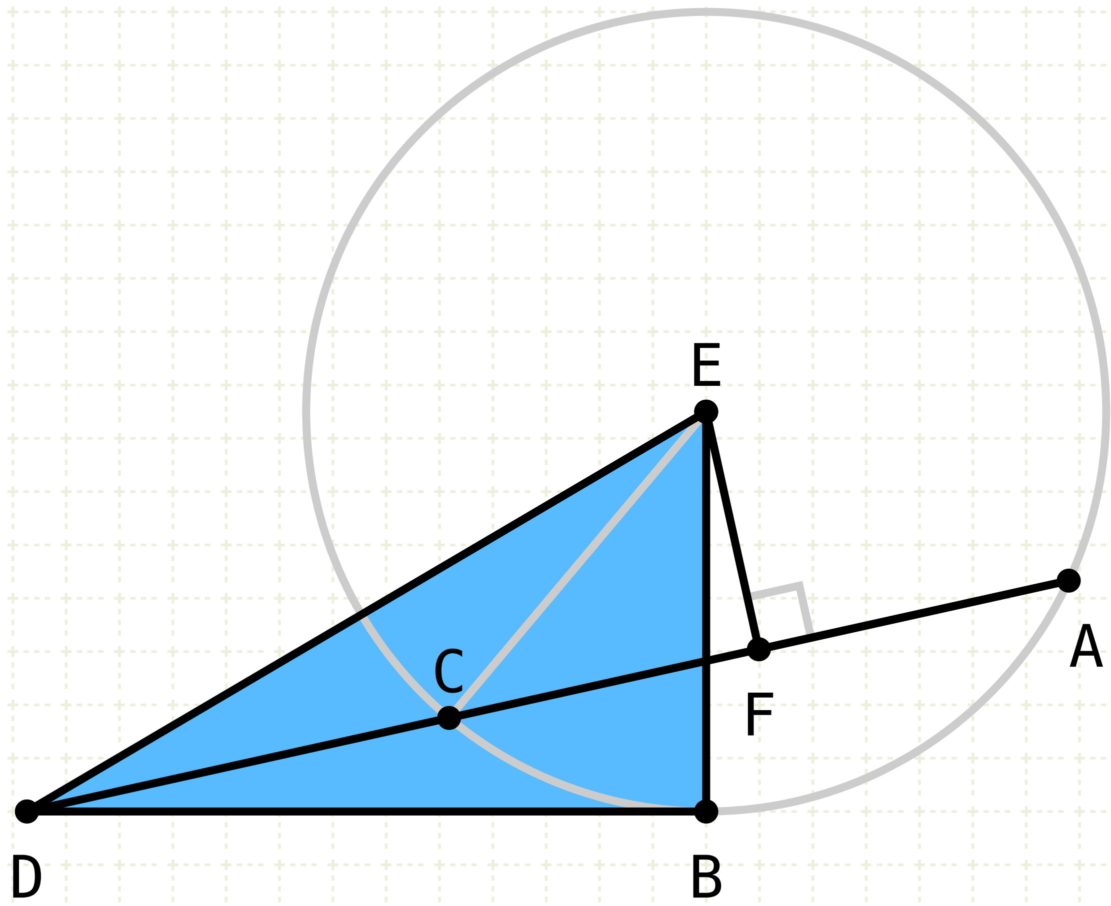
In the triangle ECF, the sum of the squares EF,FC equal the square of EC (I·47)

Similarly, the sum of the squares FD,EF equal the square of ED (I·47)

But the square of ED is just the sum of the squares of DB,EB

Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.



$CF = FA$

$AD \cdot CD + FC^2 = FD^2$

$AD \cdot CD + FC^2 + EF^2$
$= FD^2 + EF^2$

$FC^2 + EF^2 = EC^2$

$AD \cdot CD + EC^2$
$= FD^2 + EF^2$

$FD^2 + EF^2 = ED^2$

$AD \cdot CD + EC^2 = ED^2$

$DB^2 + EB^2 = ED^2$

$EB = EC$

$DB^2 + EC^2 = ED^2$

Proof - AD does not pass through centre of circle

Draw a line EF from the centre of the circle E, perpendicular to the line DA

Lines CF and FA are equal (III·18)

If a line AC is bisected at point F, and extended from C to point D, the product AD,CD plus the square of FC equals the square of FD (II·6)

Add the square of EF to both sides of the equality

In the triangle ECF, the sum of the squares EF,FC equal the square of EC (I·47)

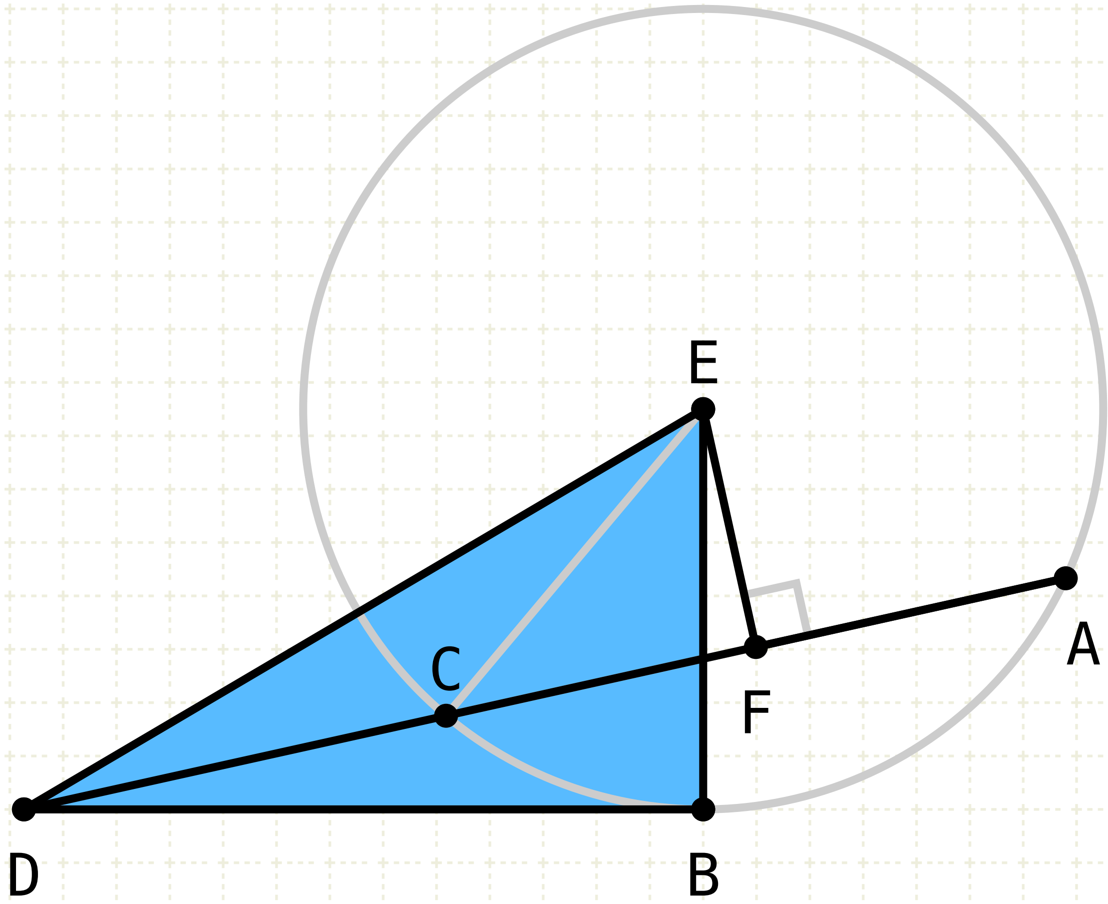
Similarly, the sum of the squares FD,EF equal the square of ED (I·47)

But the square of ED is just the sum of the squares of DB,EB

And since EB equals EC, the square of ED is the sum of the squares DB,EC

Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.



$CF = FA$
$AD \cdot CD + FC^2 = FD^2$
$AD \cdot CD + FC^2 + EF^2 = FD^2 + EF^2$
$FC^2 + EF^2 = EC^2$
$AD \cdot CD + EC^2 = FD^2 + EF^2$
$FD^2 + EF^2 = ED^2$

$AD \cdot CD + EC^2 = ED^2$

$DB^2 + EB^2 = ED^2$
$EB = EC$
$DB^2 + EC^2 = ED^2$

$AD \cdot CD = ED^2 - EC^2$
$DB^2 = ED^2 - EC^2$

Proof - AD does not pass through centre of circle

Draw a line EF from the centre of the circle E, perpendicular to the line DA

Lines CF and FA are equal (III·18)

If a line AC is bisected at point F, and extended from C to point D, the product AD,CD plus the square of FC equals the square of FD (II·6)

Add the square of EF to both sides of the equality

In the triangle ECF, the sum of the squares EF,FC equal the square of EC (I·47)

Similarly, the sum of the squares FD,EF equal the square of ED (I·47)

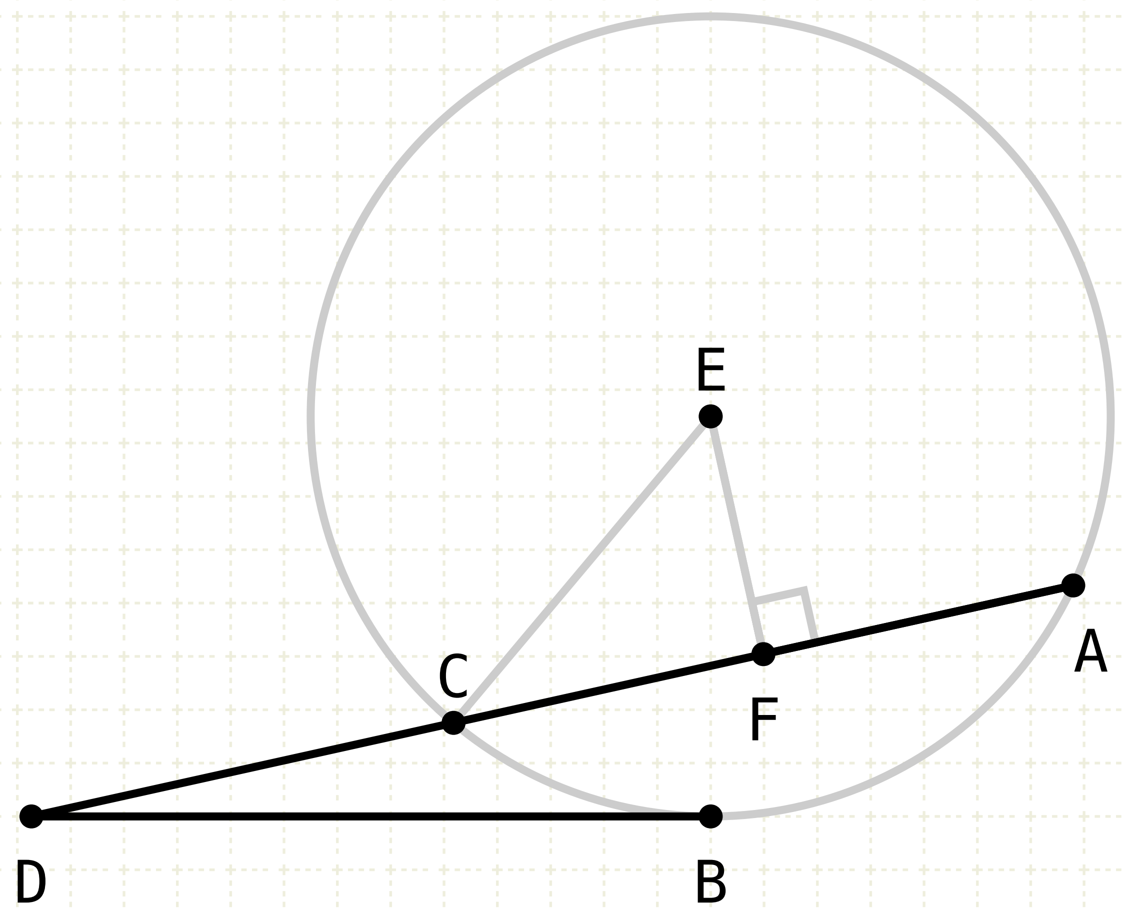
But the square of ED is just the sum of the squares of DB,EB

And since EB equals EC, the square of ED is the sum of the squares DB,EC

Subtract EC from both sides of both equations

Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.



$CF = FA$

$AD \cdot CD + FC^2 = FD^2$

$AD \cdot CD + FC^2 + EF^2 = FD^2 + EF^2$

$FC^2 + EF^2 = EC^2$

$AD \cdot CD + EC^2 = FD^2 + EF^2$

$FD^2 + EF^2 = ED^2$

$AD \cdot CD + EC^2 = ED^2$

$DB^2 + EB^2 = ED^2$

$EB = EC$

$DB^2 + EC^2 = ED^2$

$AD \cdot CD = ED^2 - EC^2$

$DB^2 = ED^2 - EC^2$

$AD \cdot CD = DB^2$

Proof - AD does not pass through centre of circle

Draw a line EF from the centre of the circle E, perpendicular to the line DA

Lines CF and FA are equal (III·18)

If a line AC is bisected at point F, and extended from C to point D, the product AD,CD plus the square of FC equals the square of FD (II·6)

Add the square of EF to both sides of the equality

In the triangle ECF, the sum of the squares EF,FC equal the square of EC (I·47)

Similarly, the sum of the squares FD,EF equal the square of ED (I·47)

But the square of ED is just the sum of the squares of DB,EB

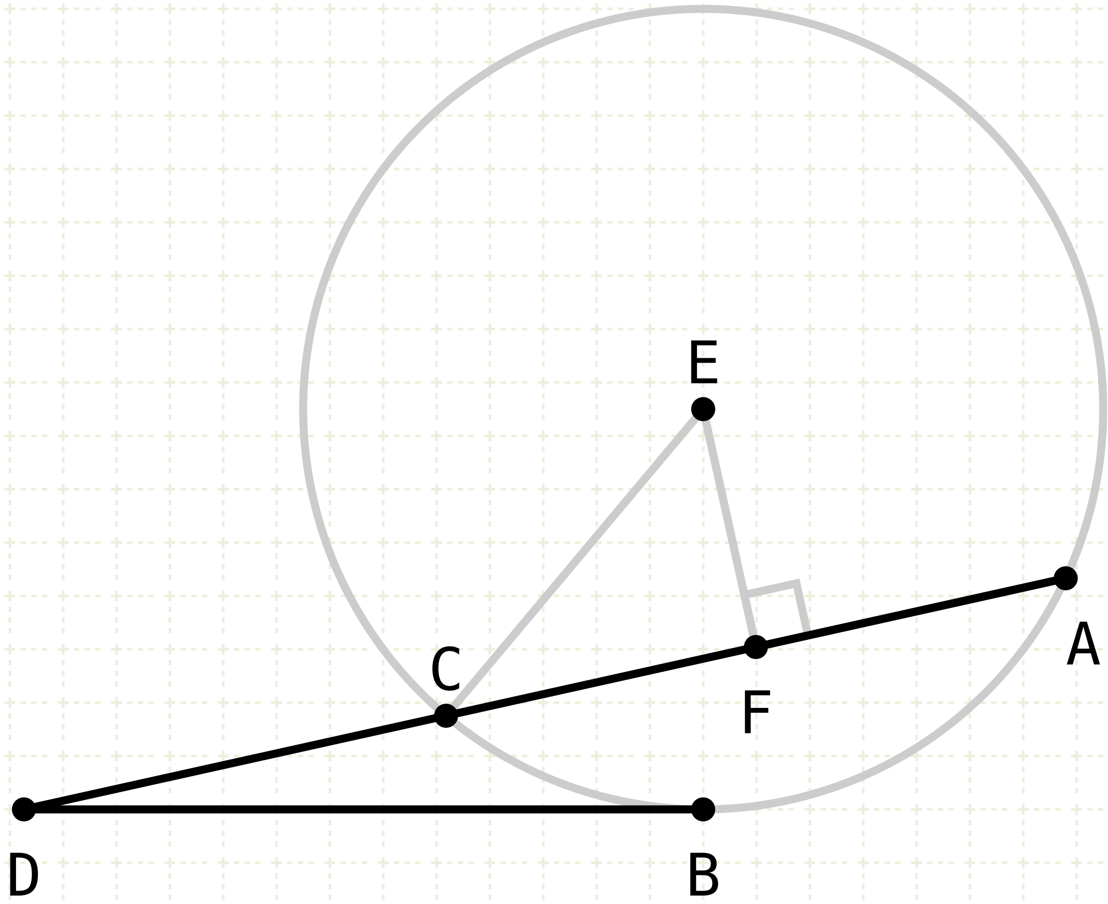
And since EB equals EC, the square of ED is the sum of the squares DB,EC

Subtract EC from both sides of both equations

Since equals are equal to equals, we have proven this proposition

Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.



$$CF = FA$$
$$AD \cdot CD + FC^2 = FD^2$$
$$AD \cdot CD + FC^2 + EF^2 = FD^2 + EF^2$$
$$FC^2 + EF^2 = EC^2$$
$$AD \cdot CD + EC^2 = FD^2 + EF^2$$
$$FD^2 + EF^2 = ED^2$$
$$AD \cdot CD + EC^2 = ED^2$$
$$DB^2 + EB^2 = ED^2$$
$$EB = EC$$
$$DB^2 + EC^2 = ED^2$$
$$AD \cdot CD = ED^2 - EC^2$$
$$DB^2 = ED^2 - EC^2$$
$$AD \cdot CD = DB^2$$

Proof - AD does not pass through centre of circle

Draw a line EF from the centre of the circle E, perpendicular to the line DA

Lines CF and FA are equal (III·18)

If a line AC is bisected at point F, and extended from C to point D, the product AD,CD plus the square of FC equals the square of FD (II·6)

Add the square of EF to both sides of the equality

In the triangle ECF, the sum of the squares EF,FC equal the square of EC (I·47)

Similarly, the sum of the squares FD,EF equal the square of ED (I·47)

But the square of ED is just the sum of the squares of DB,EB

And since EB equals EC, the square of ED is the sum of the squares DB,EC

Subtract EC from both sides of both equations

Since equals are equal to equals, we have proven this proposition

Youtube Videos

https://www.youtube.com/c/SandyBultena