

Euclid's Elements

Book III



A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)



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2	A chord of a circle always lies inside the circle	10	A circle does not cut a circle at more points than two	18	If line touches a circle, then it is perpendicular to the diameter that touches that point
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4	A line not through the centre of a circle does not bisect a chord	12	Point of contact between two external circles, and their centres, are collinear	20	The angle at the centre of a circle is twice that from an angle from the circumference
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6	If two circles touch one another, they will not have the same center	14	In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another.	22	The opposite angles of quadrilaterals in circles are equal to two right angles
7	Consider two lines from a point inside a circle to the edge, the longer one will be the one closest to the longest part of the diameter passing through the original point	15	The longest line in a circle is its diameter, shorter the farther away from the diameter	23	On the same straight line there cannot be constructed two similar and unequal segments of circles on the same side
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| 26 | In equal circles equal angles stand on equal circumferences | 35 | If two circle chords intersect, the segments on one multiplied together equals the segments of the other multiplied together |
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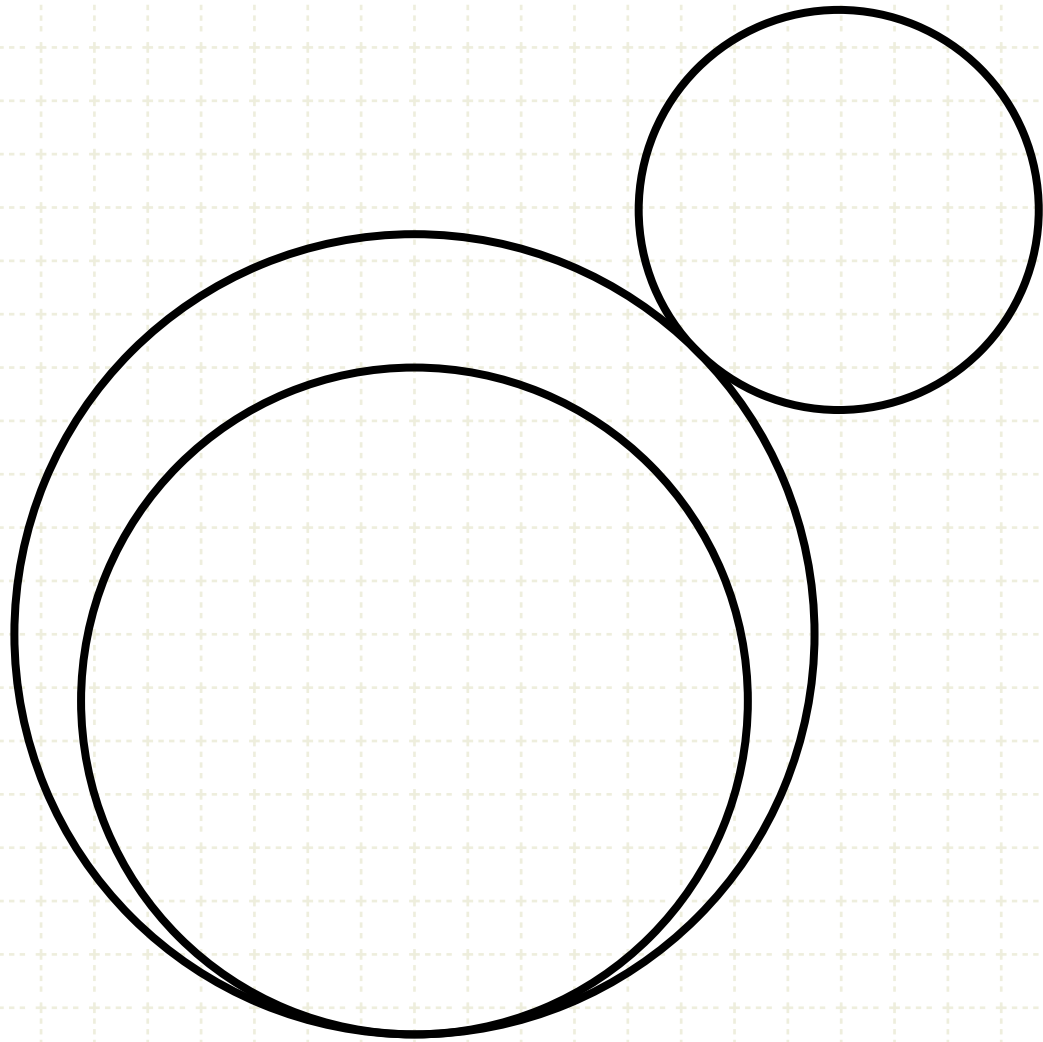
Proposition 13 of Book III

A circle does not touch a circle at more points than one, whether it touch it internally or externally.



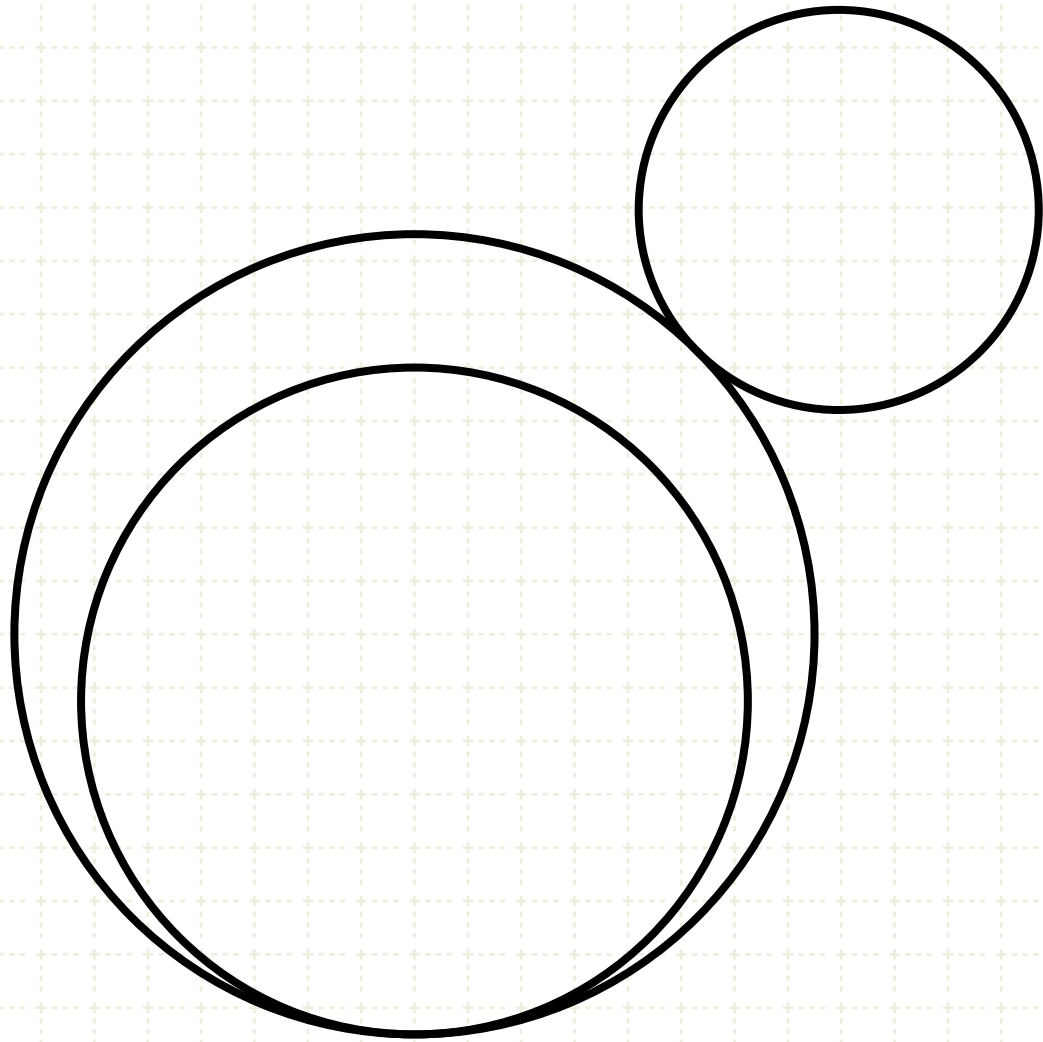
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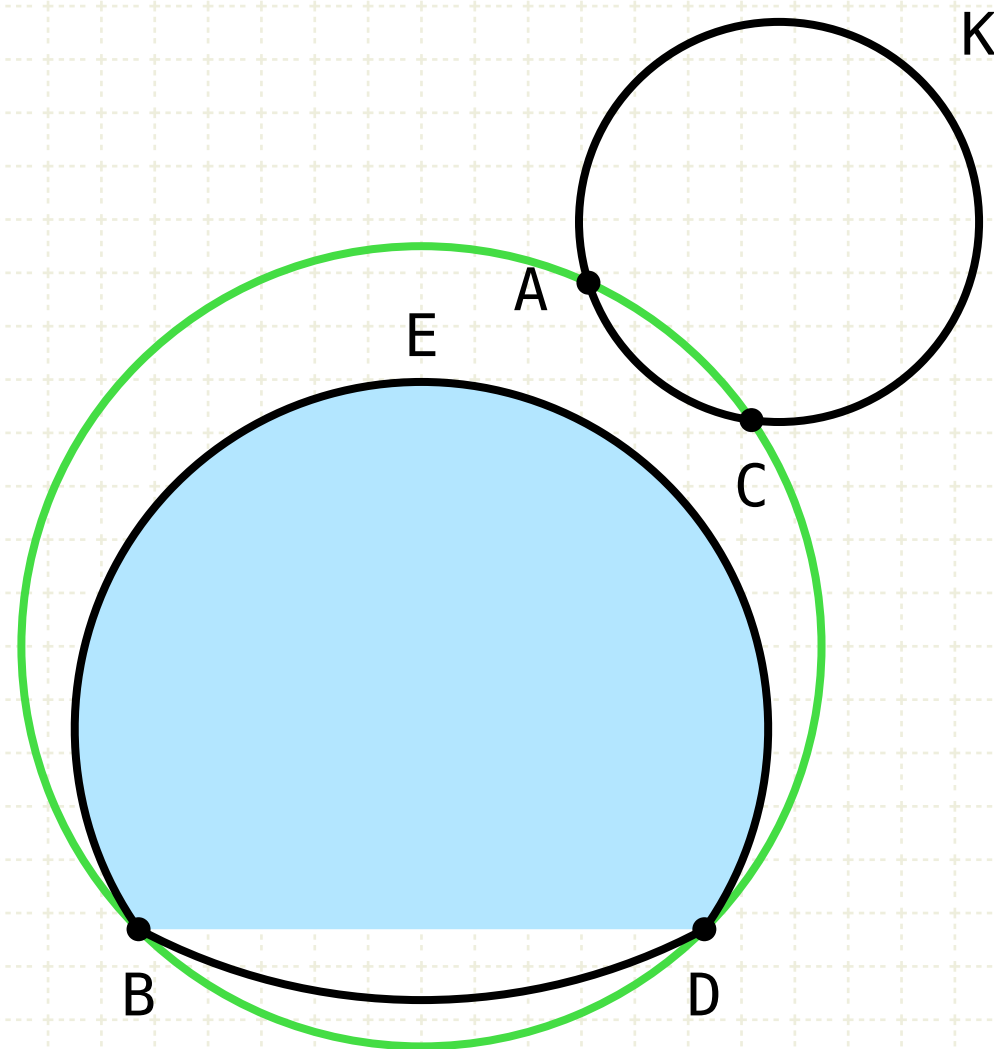
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Proof by Contradiction

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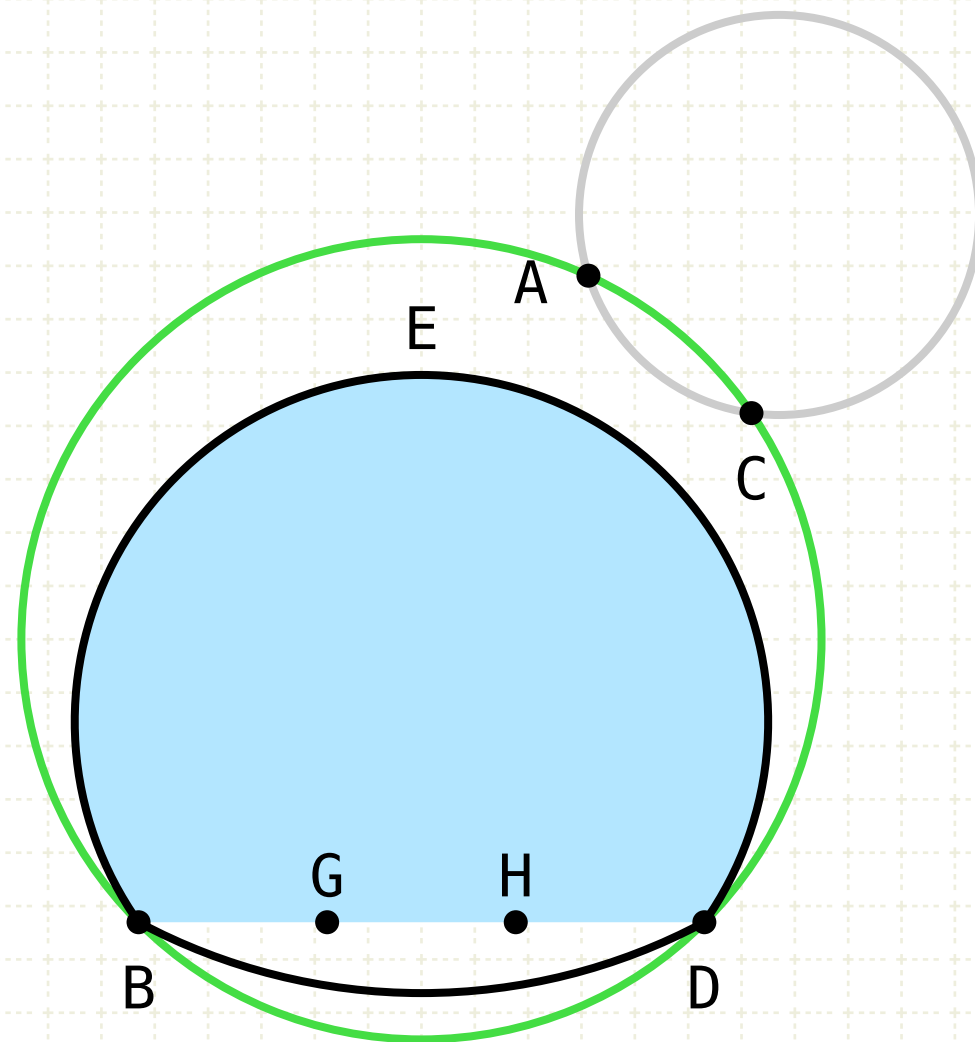
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Assume that the inner circle EBD touches the green circle ABDC at two points B and D

Assume that the outer circle ACK touches the green circle ABDC at two points A and C

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$$BG = GD$$

$$BH = HD$$

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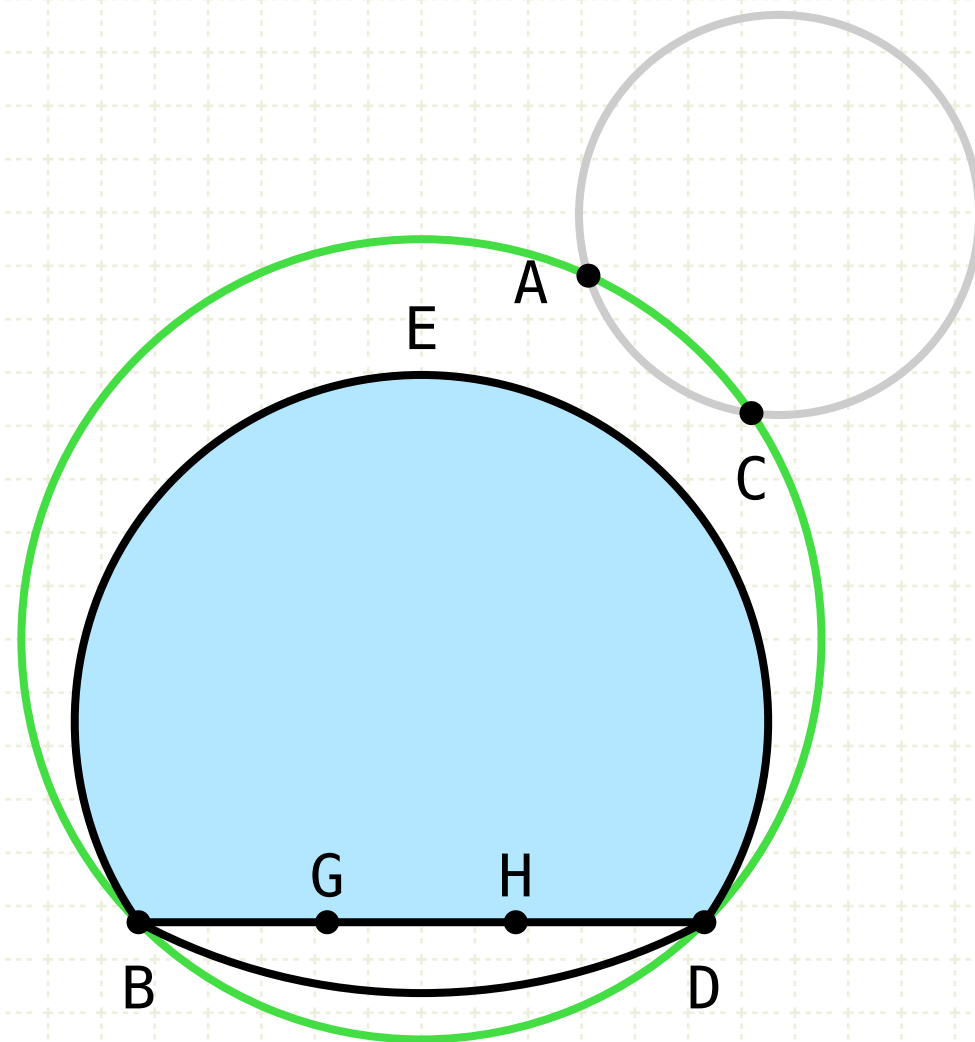
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Let G be the centre of ABDC, and H be the centre of the circle EBD

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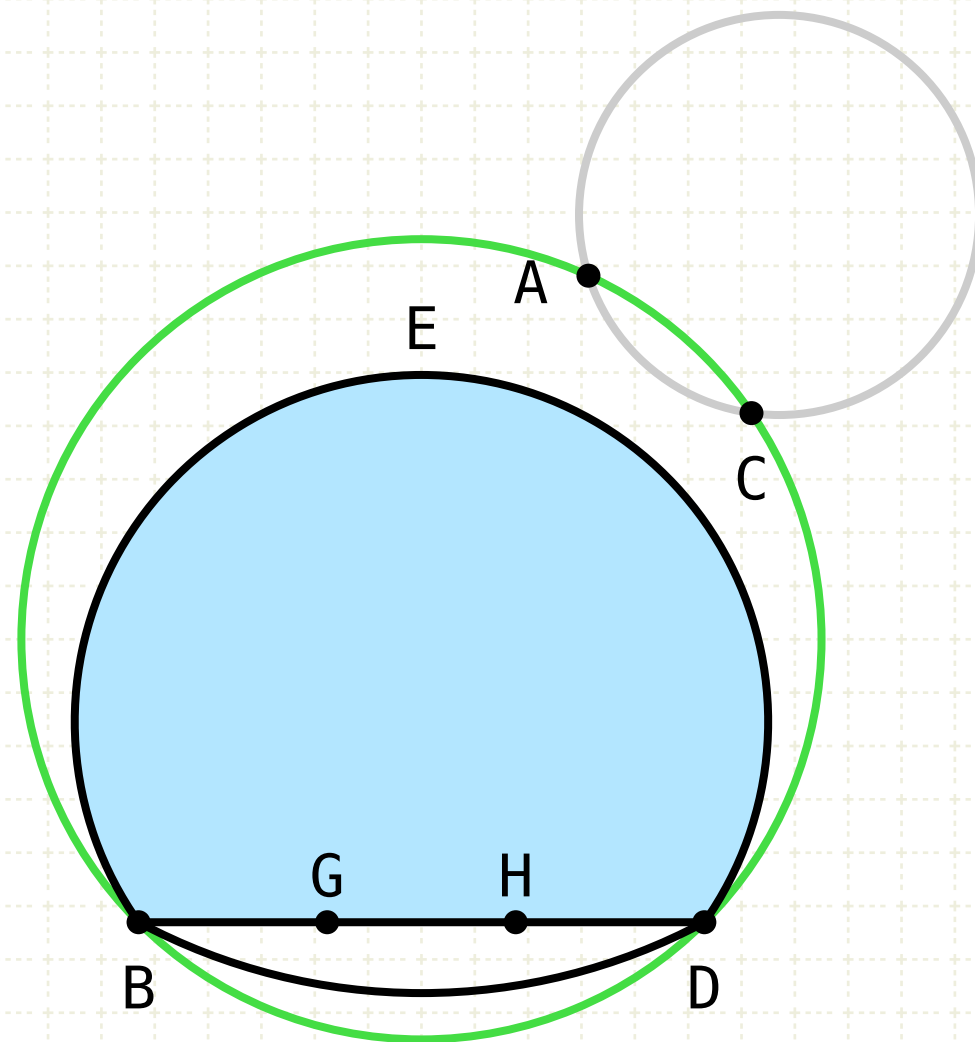
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The straight line GH will intersect the points where the circles touch (III·11)

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$$BG = GD$$

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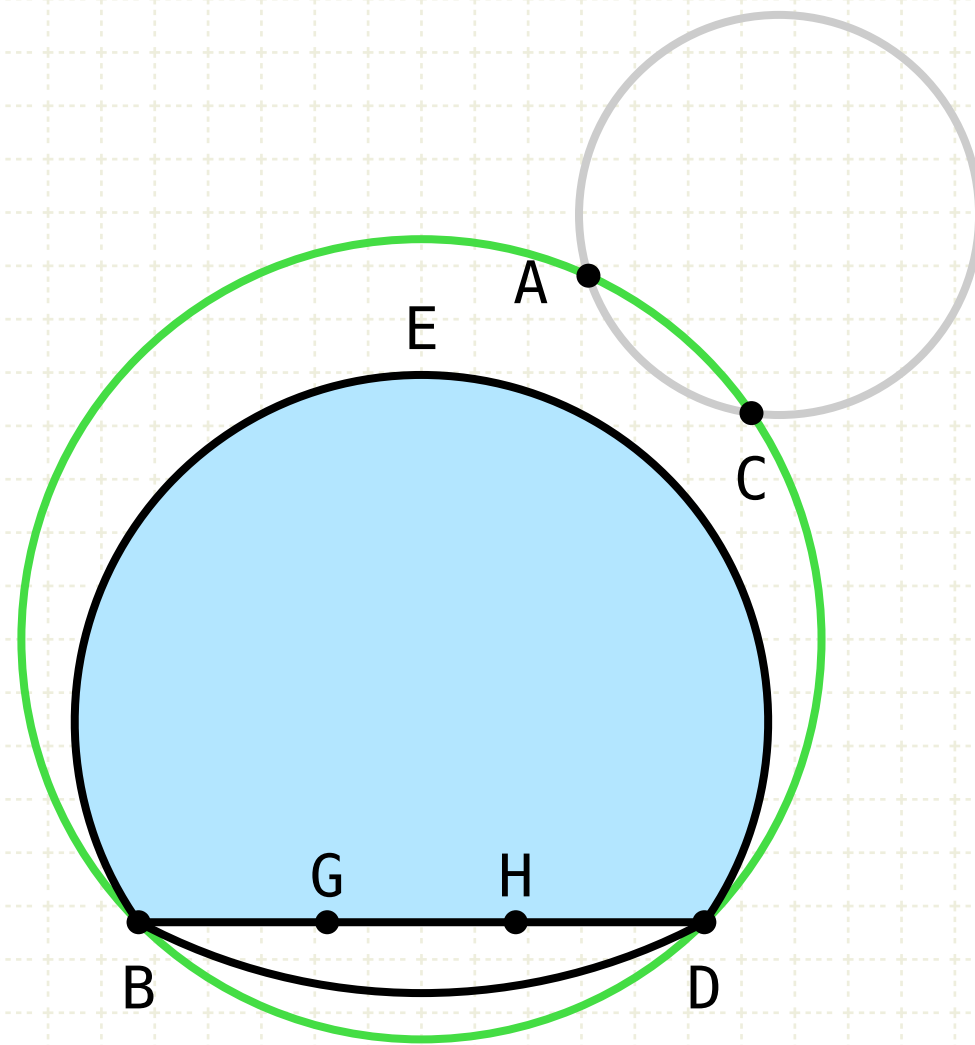
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$$GD > HD$$

$$\therefore BG > HD$$

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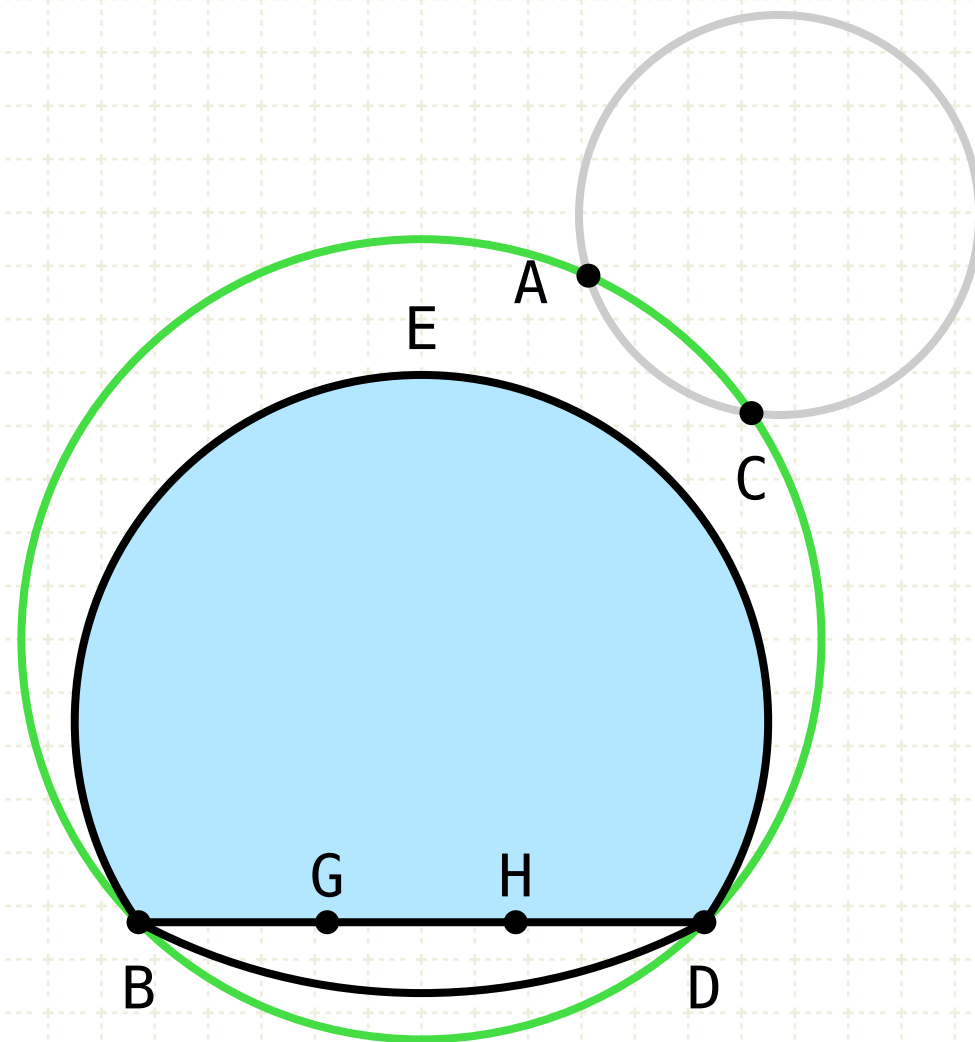
The straight line GH will intersect the points where the circles touch (III·11)

GD is greater than HD

Therefore BG is greater than HD, since BG and GD are equal (radii of the same circle)

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$$\begin{aligned} BG &= GD \\ BH &= HD \\ GD &> HD \\ \therefore BG &> HD \\ BH &> BG > HD \end{aligned}$$

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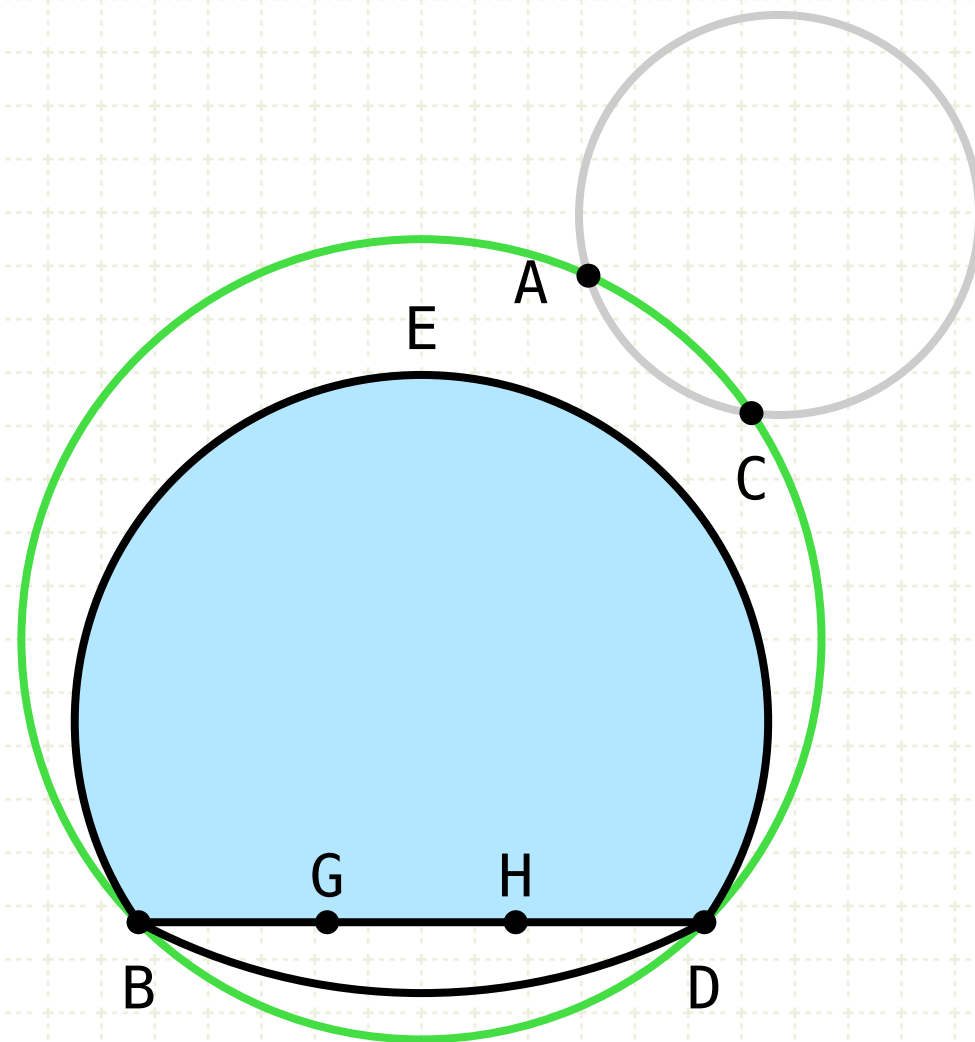
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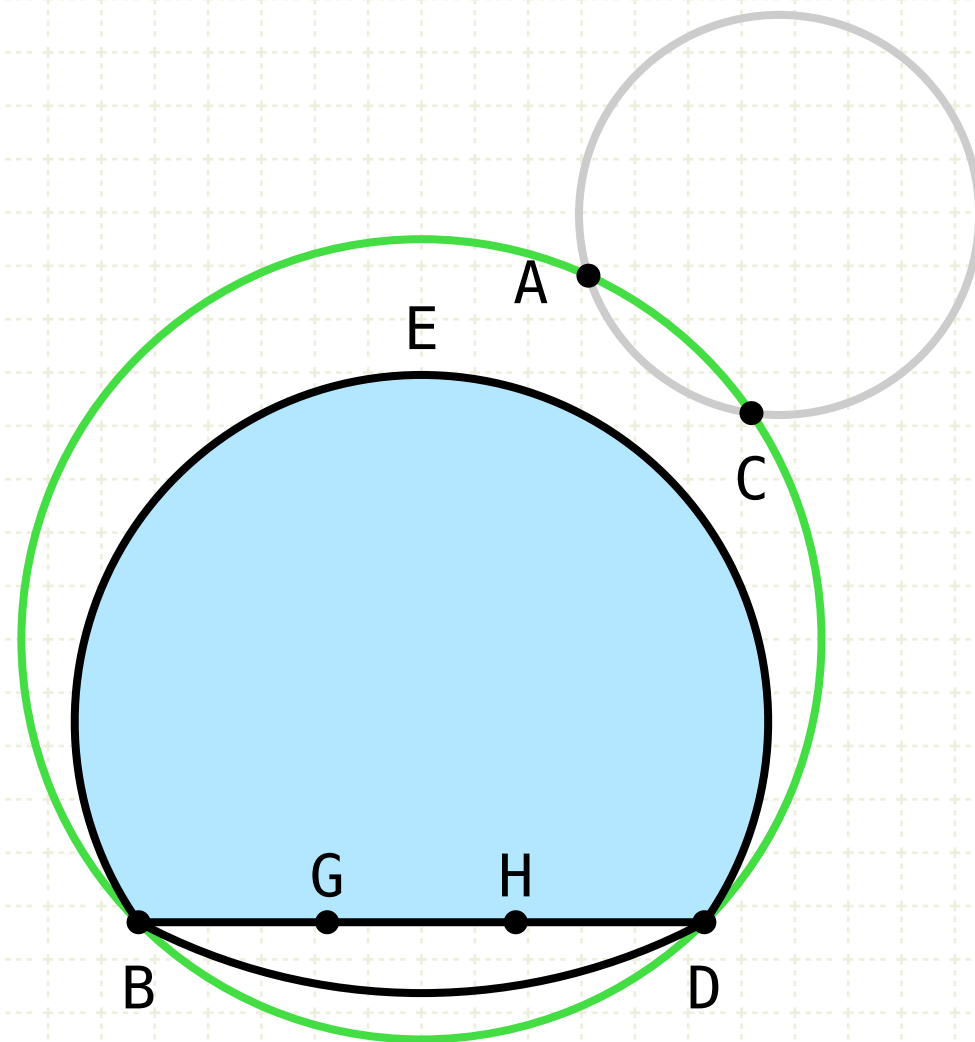
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But H is the centre of the circle, so BH equals HD

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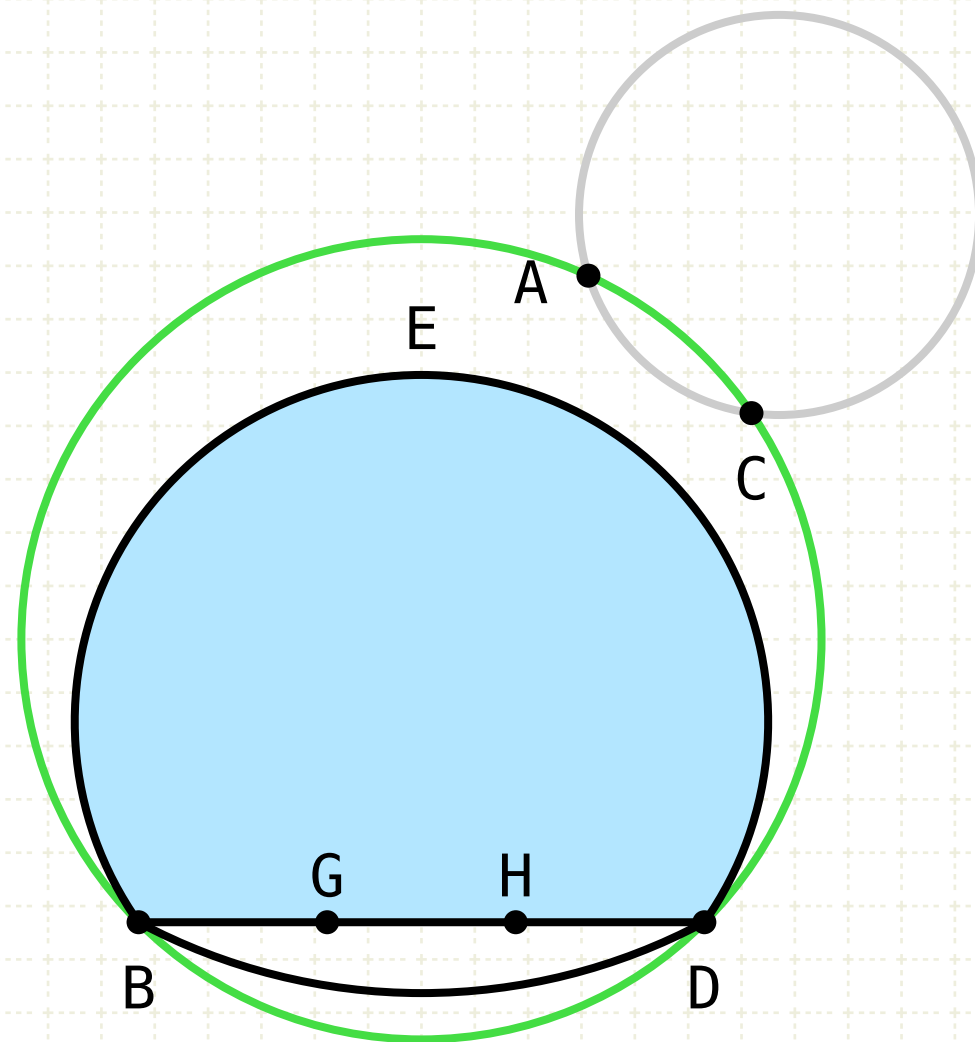
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Which is a contradiction

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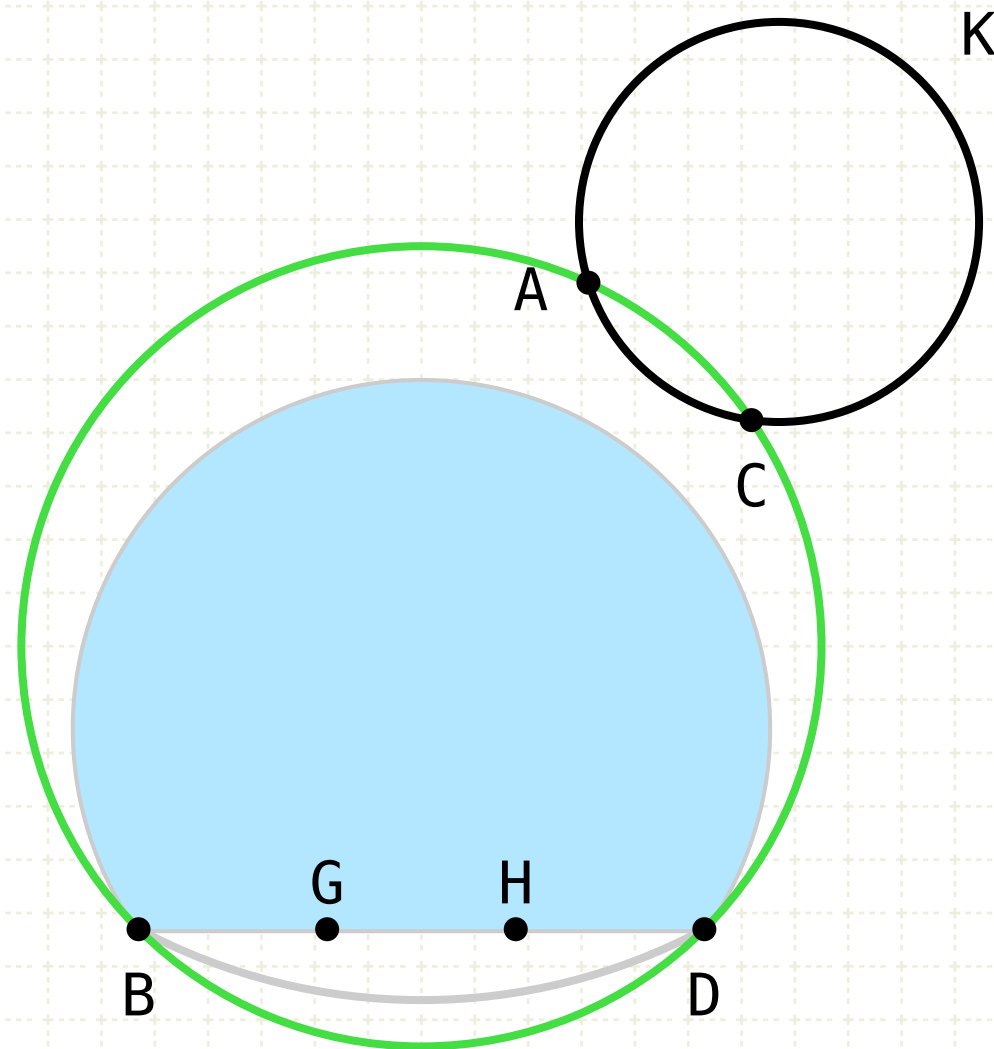
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Thus demonstrating that an inner circle cannot touch an outer circle in more than one place

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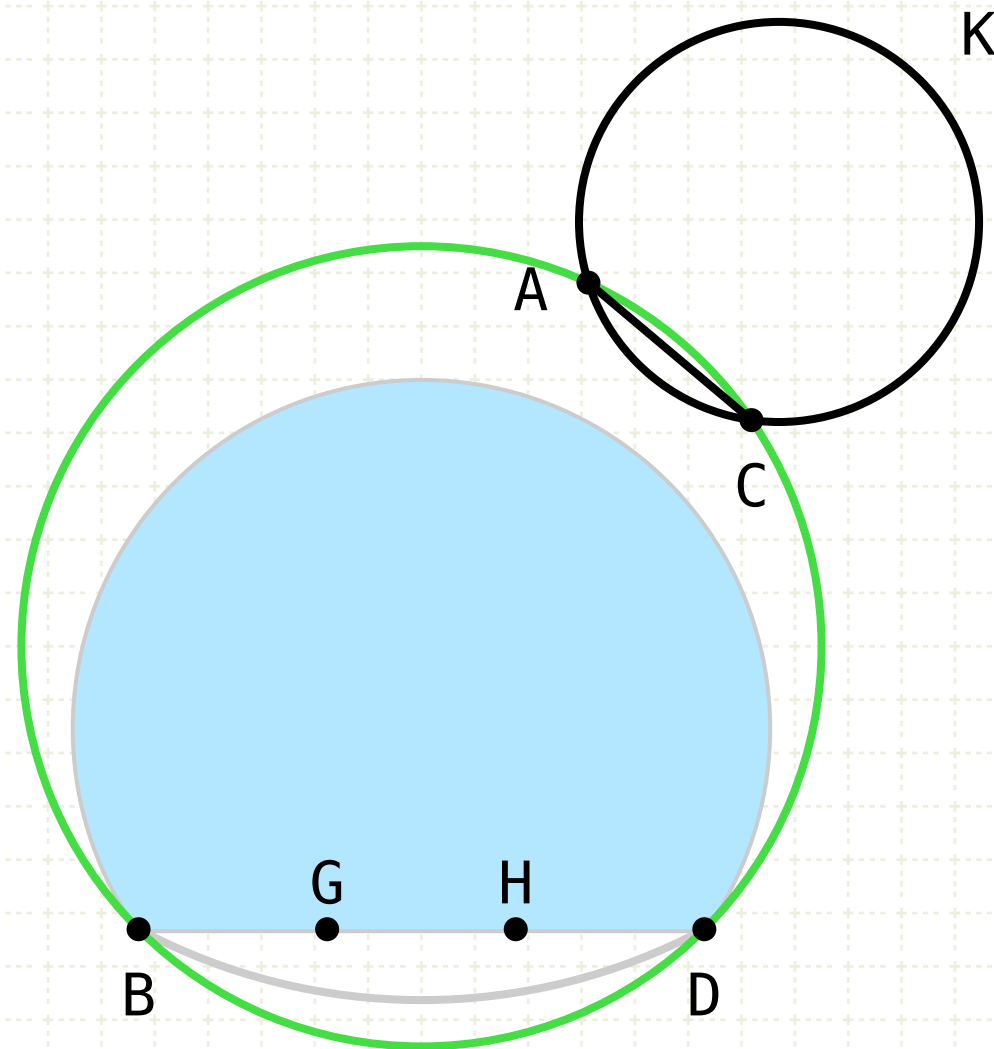
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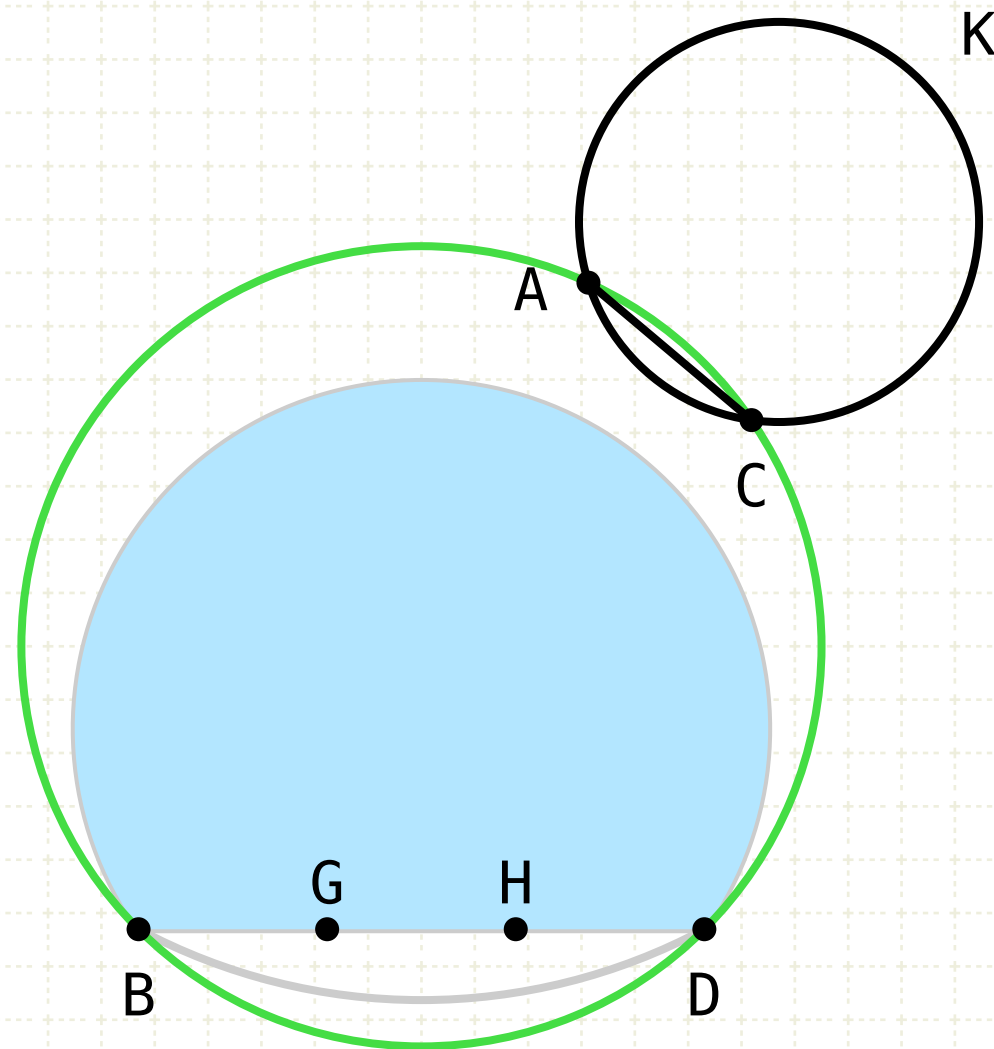
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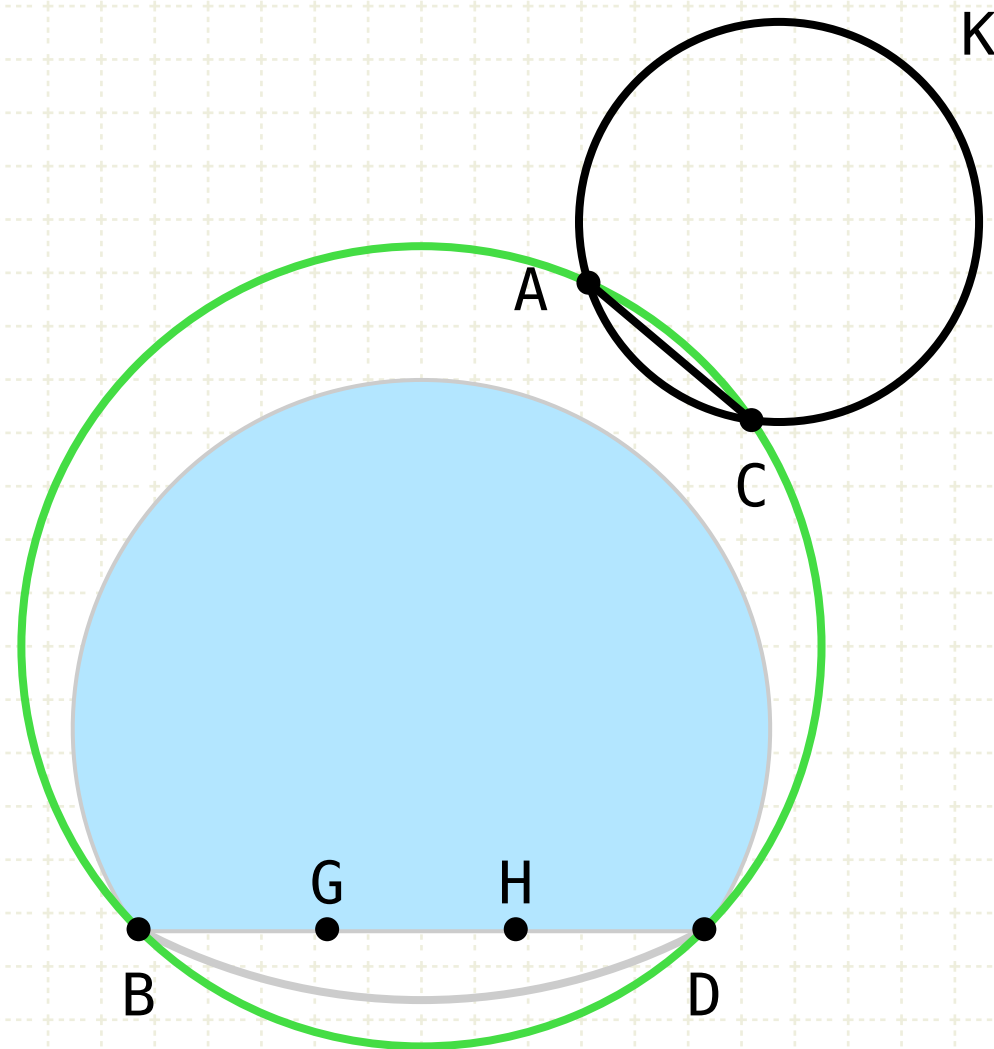
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According to III·2, a line joining two points on a circle lies within the circle

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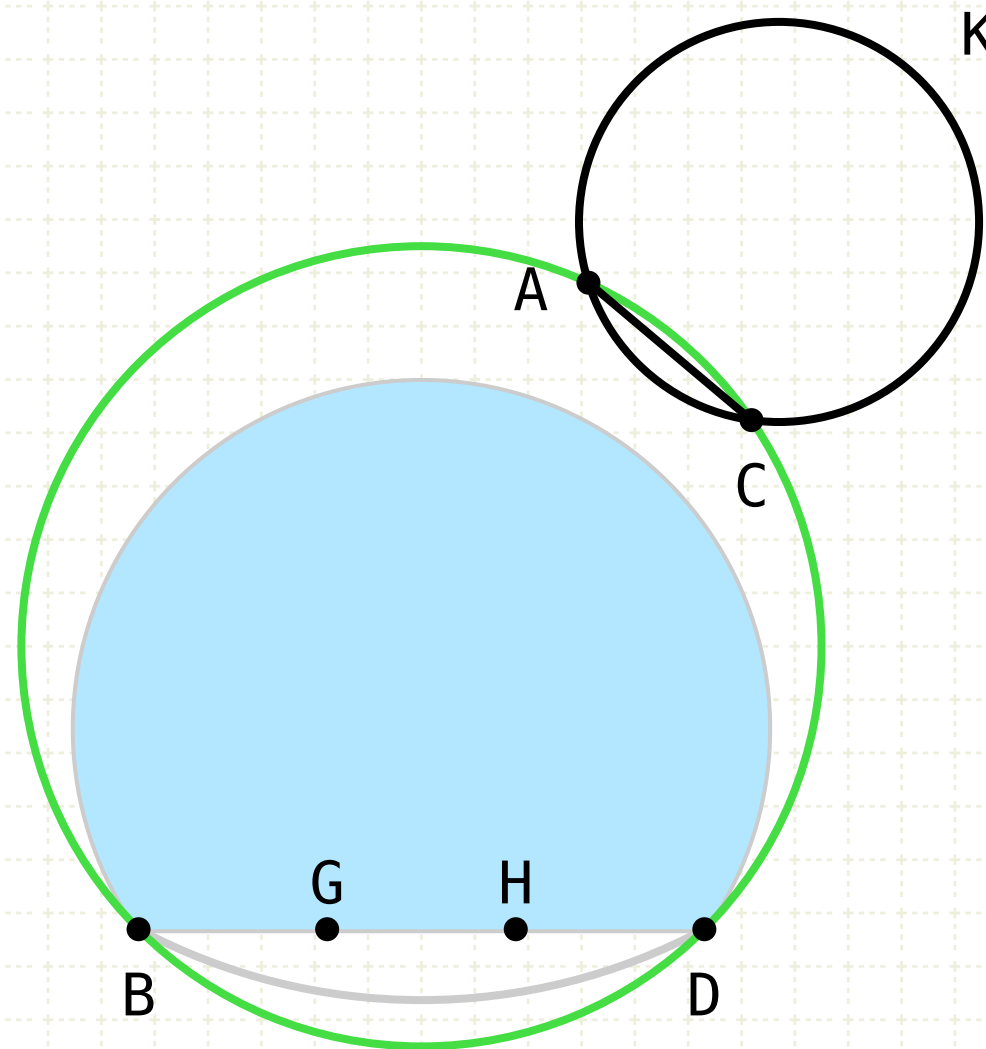
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According to III·2, a line joining two points on a circle lies within the circle

... but if circle ACK only touches circle ABDC, then the line must lie outside of ACK

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This is impossible, so an outer circle cannot touch another circle in more than one place

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