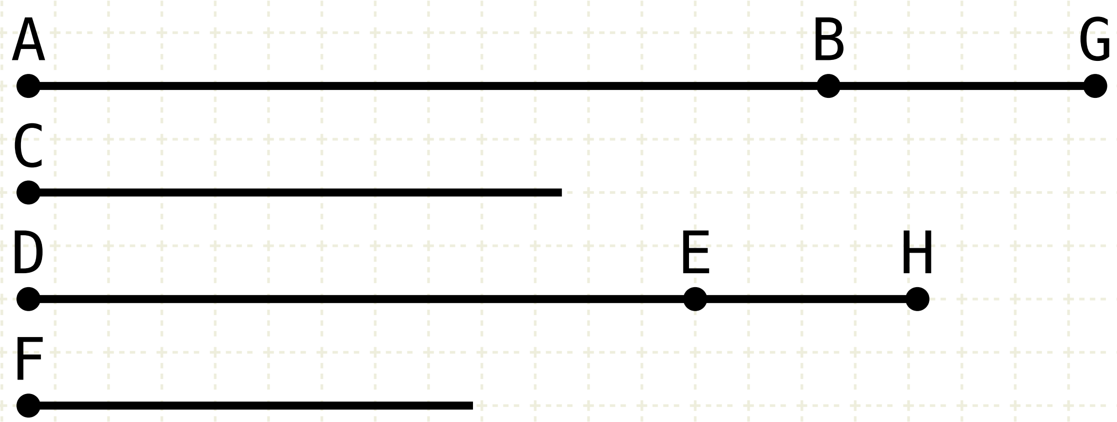
Euclid's Elements

Book V



$AB:C = DE:F$
$BG:C = EH:F$

$AG:C = DH:F$

Proportions are what makes the old Greek temples classic in their beauty. They are like huge blocks, from which the air has been literally hewn out between the columns.

Arne Jacobsen

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Proposition 19 of Book V

If, as a whole is to a whole, so is a part subtracted to a part subtracted, the remainder will also be to the remainder as whole to whole.

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the componendo (composition) ratio of A:B is (A+B):B

the separando (separated) ratio of (A+B):B is A:B

the convertendo (in conversion) ratio of (A+B):B is (A+B):A

Definitions

14. COMPOSITION OF A RATIO means taking the antecedent together with the consequent as one in relation to the consequent by itself
15. SEPARATION OF A RATIO means taking the excess by which the antecedent exceeds the consequent in relation to the consequent by itself
16. CONVERSION OF A RATIO means taking the antecedent in relation to the excess by which the antecedent exceeds the consequent.

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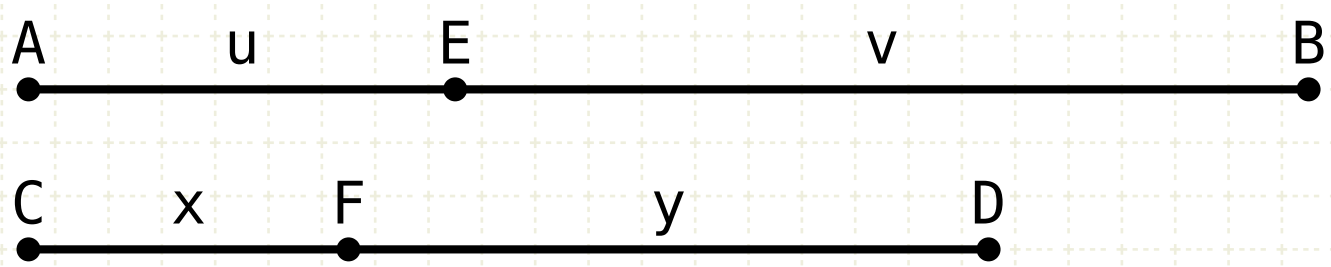

In other words

As the whole AB is to the whole CD, let the part AE be to CF,

… then the remainders (AB less AE, CD less CF) will also be proportional

```
AB:CD = AE:CF                (u+v):(x+y) = u:x

AB:CD = (AB-AE):(CD-CF)      (u+v):(x+y) = ((u+v)-u):((x+y)-x)
AB:CD = EB:FD                (u+v):(x+y) = v:y


a:b = c:d → a:b = (a-c):(b-d)
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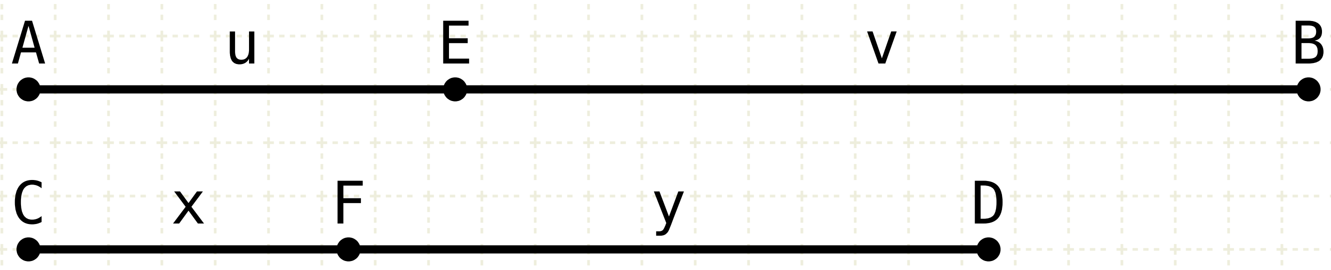

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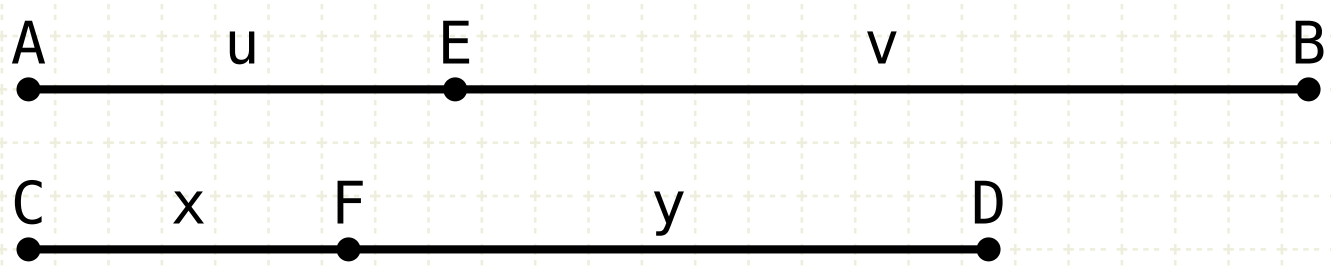

$AB:CD = AE:CF$ $\qquad$ $(u+v):(x+y) = u:x$

$AB:AE = CD:CF$ $\qquad$ $(u+v):u = (x+y):x$

In other words

As the whole AB is to the whole CD, let the part AE be to CF,

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Proof

Since AB is to CD as AE is to CF, the alternative ratios are also equal, AB is to AE as CD is to CF (V·16)

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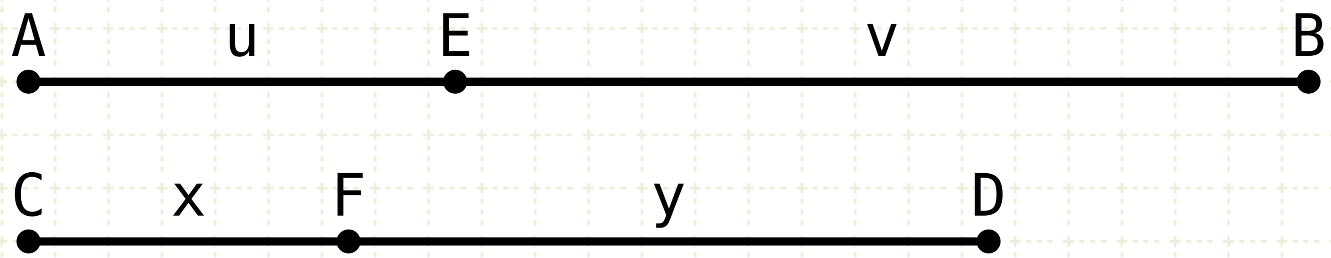

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(AB-AE):AE = (CD-CF):CF	((u+v)-u):u = ((x+y)-x):x
EB:AE = FD:CF	v:u = y:x

In other words

As the whole AB is to the whole CD, let the part AE be to CF,

… then the remainders (AB less AE, CD less CF) will also be proportional

Proof

Since AB is to CD as AE is to CF, the alternative ratios are also equal, AB is to AE as CD is to CF (V·16)

And since the magnitudes are proportional COMPONENDO, they will also be proportional SEPARANDO (V·17)

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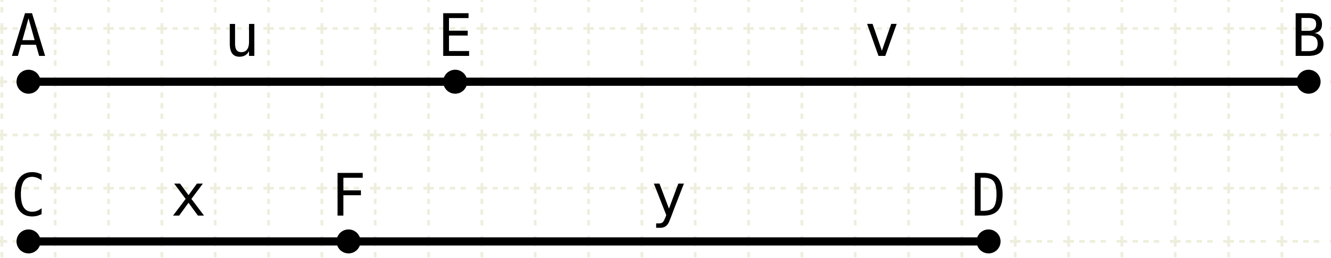


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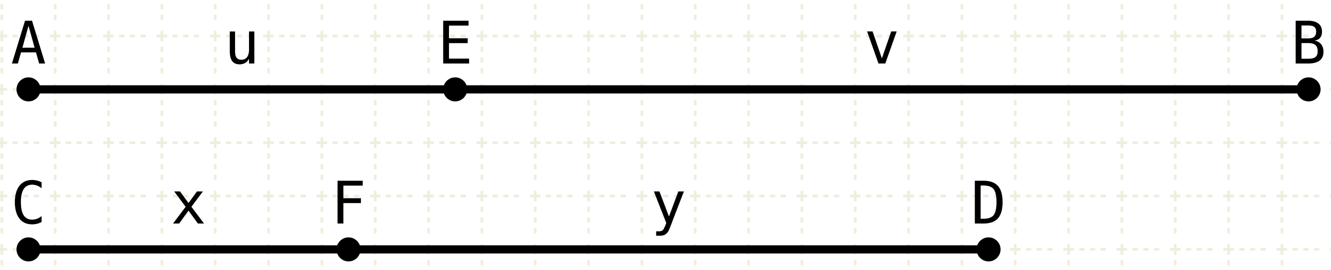

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But AE is to CF as AB is to CD, therefore EB is to FD as AB is to CD (V·11)

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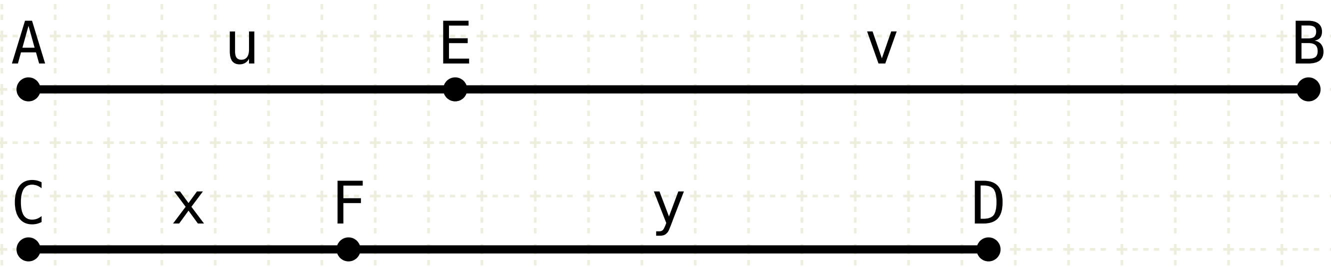

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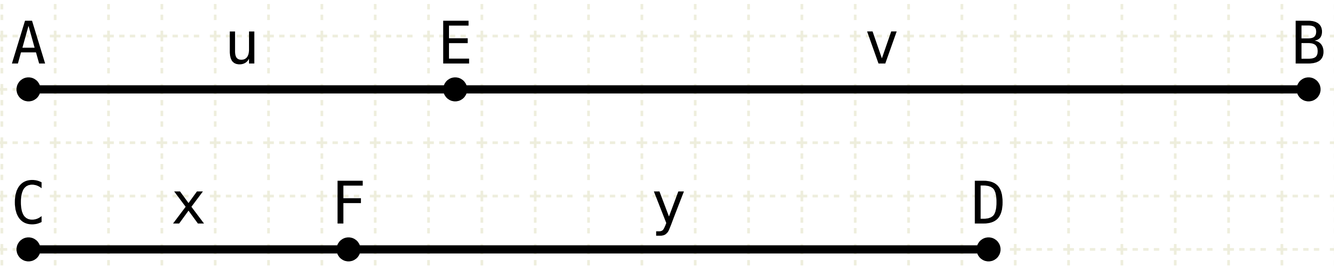

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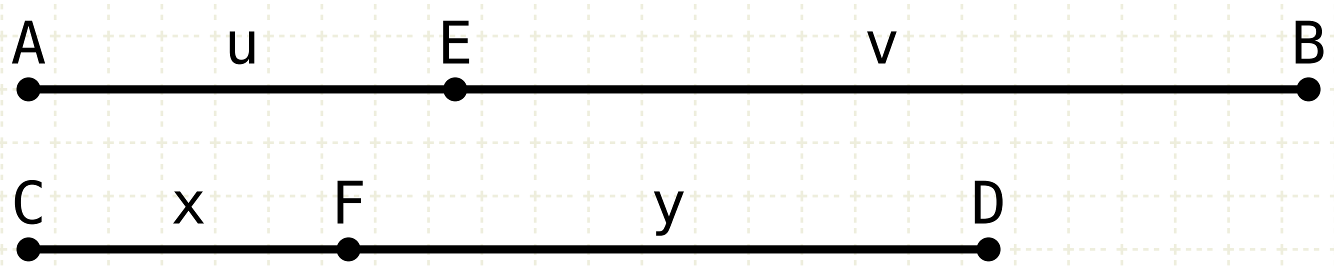

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