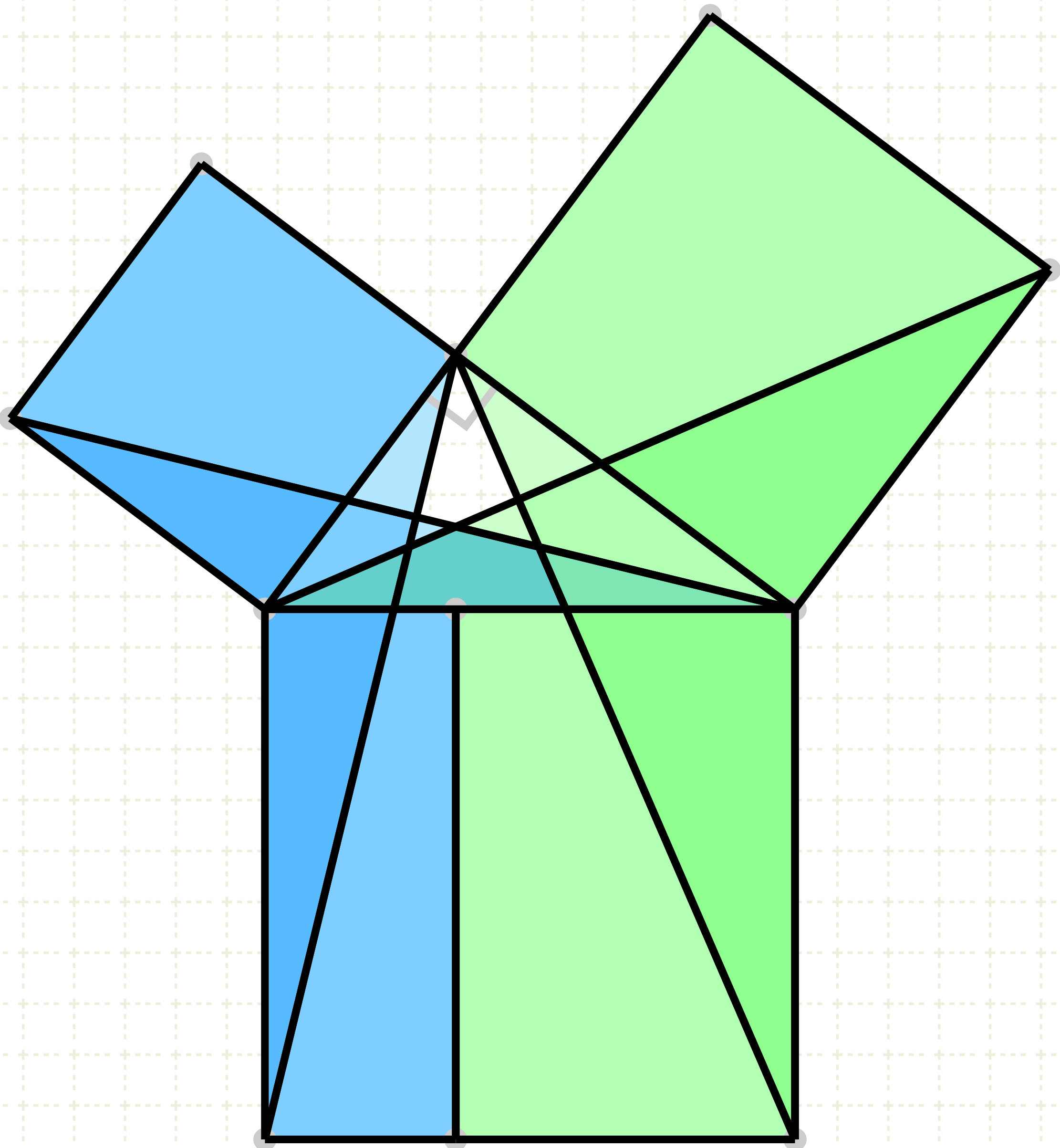
Euclid's Elements

Book I



If Euclid did not kindle your youthful enthusiasm, you were not born to be a scientific thinker.

Albert Einstein

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Proposition 29 of Book I

A straight line falling on parallel straight lines makes the alternate angles equal to one another, the exterior angle equal to the interior and opposite angle, and the sum of the interior angles on the same side equal to two right angles.

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In other words

Given with two parallel lines AB and CD

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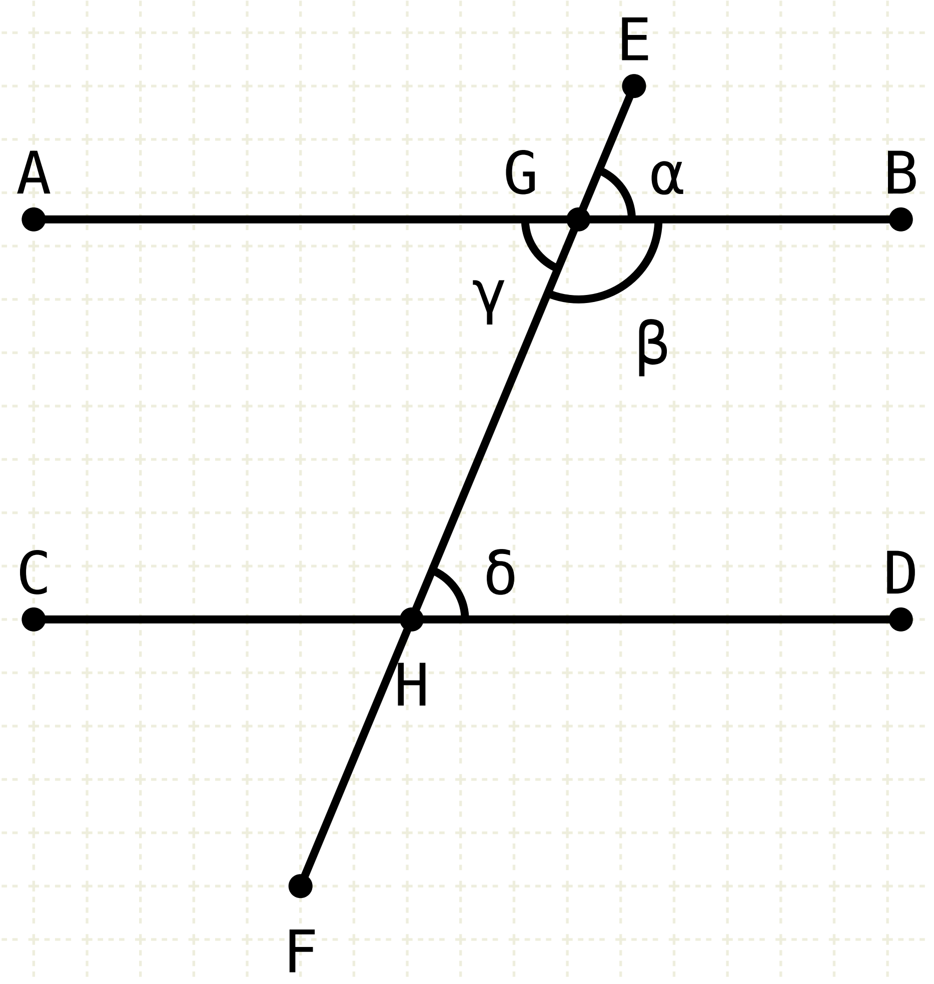

In other words

Given with two parallel lines AB and CD

And a third line EF such that it intersects lines AB and CD at points G and H

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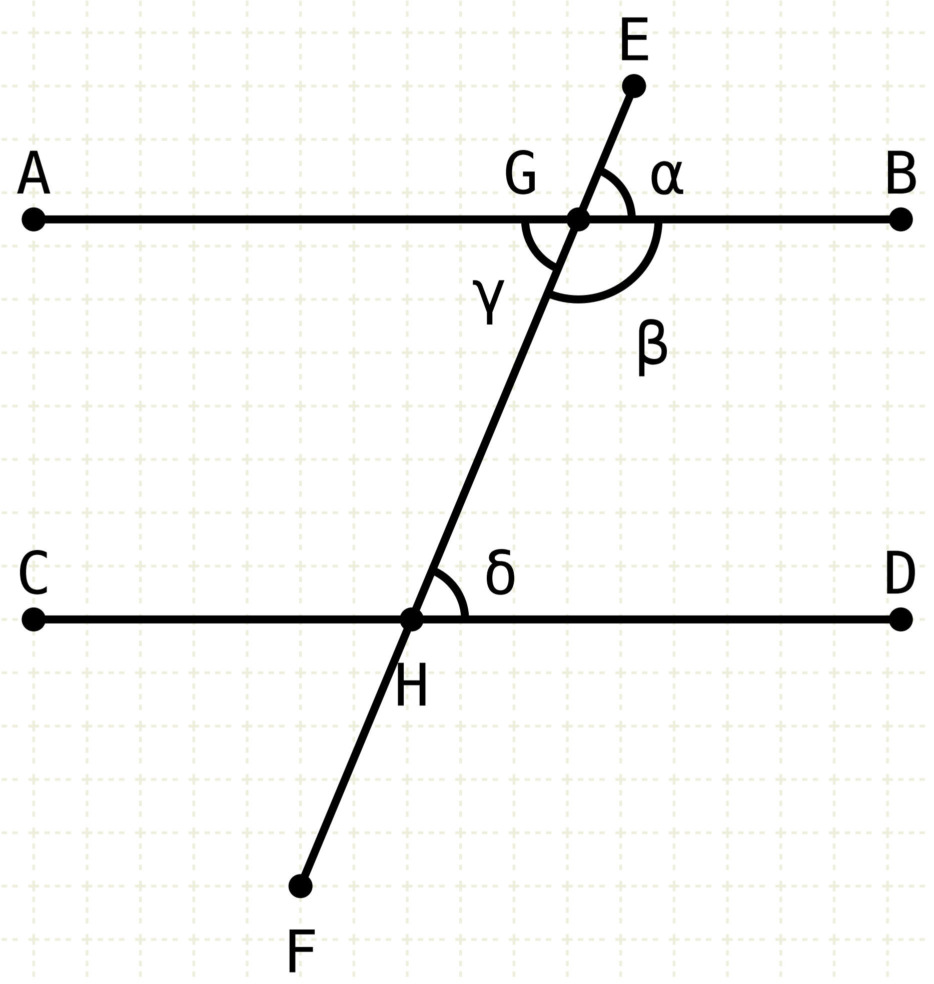


if $AB \parallel CD$

$\Rightarrow \gamma = \delta$

$\Rightarrow \alpha = \delta$

$\Rightarrow \beta + \delta = ∟+∟$

In other words

Given with two parallel lines AB and CD

And a third line EF such that it intersects lines AB and CD at points G and H

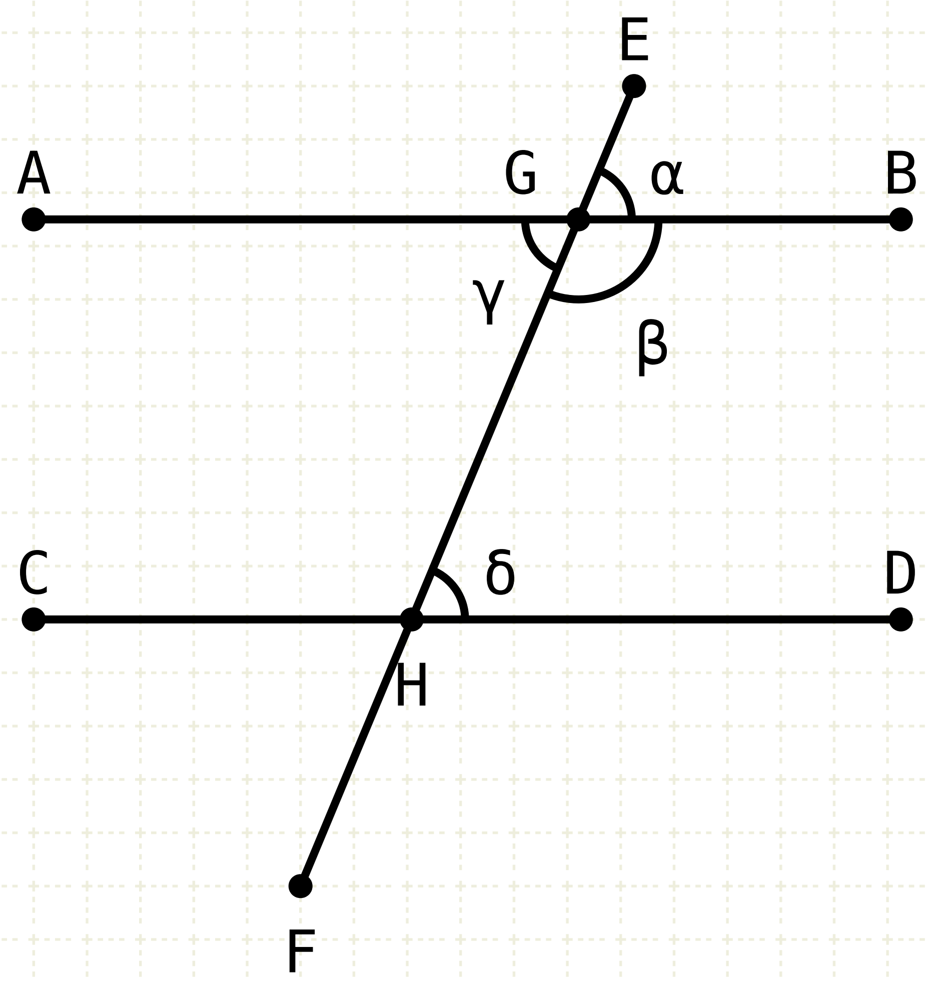
Then angles AGH and GHD are equal, angles EGB and GHD are equal, angles BGH and GHD added together equal two right angles

Proposition 29 of Book I

A straight line falling on parallel straight lines makes the alternate angles equal to one another, the exterior angle equal to the interior and opposite angle, and the sum of the interior angles on the same side equal to two right angles.

AB || CD

Proof by Contradiction



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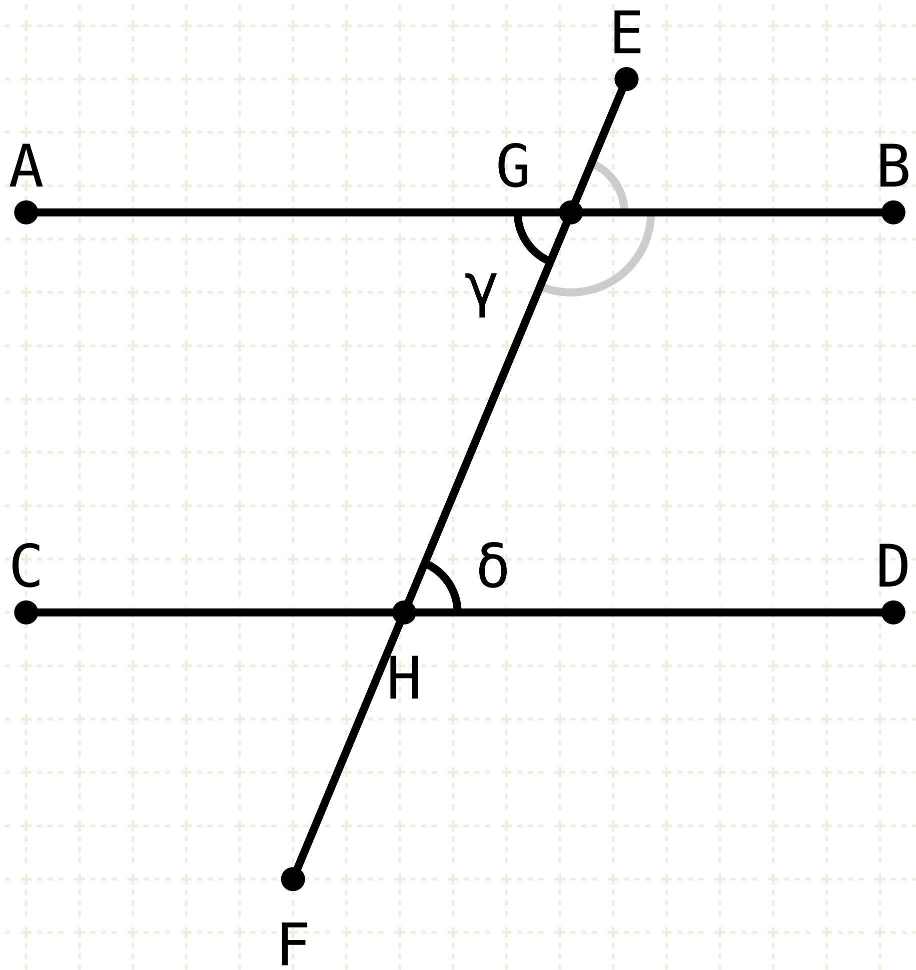
AB || CD

$\gamma > \delta$



Proof by Contradiction

Assume that AGH is greater than GHD

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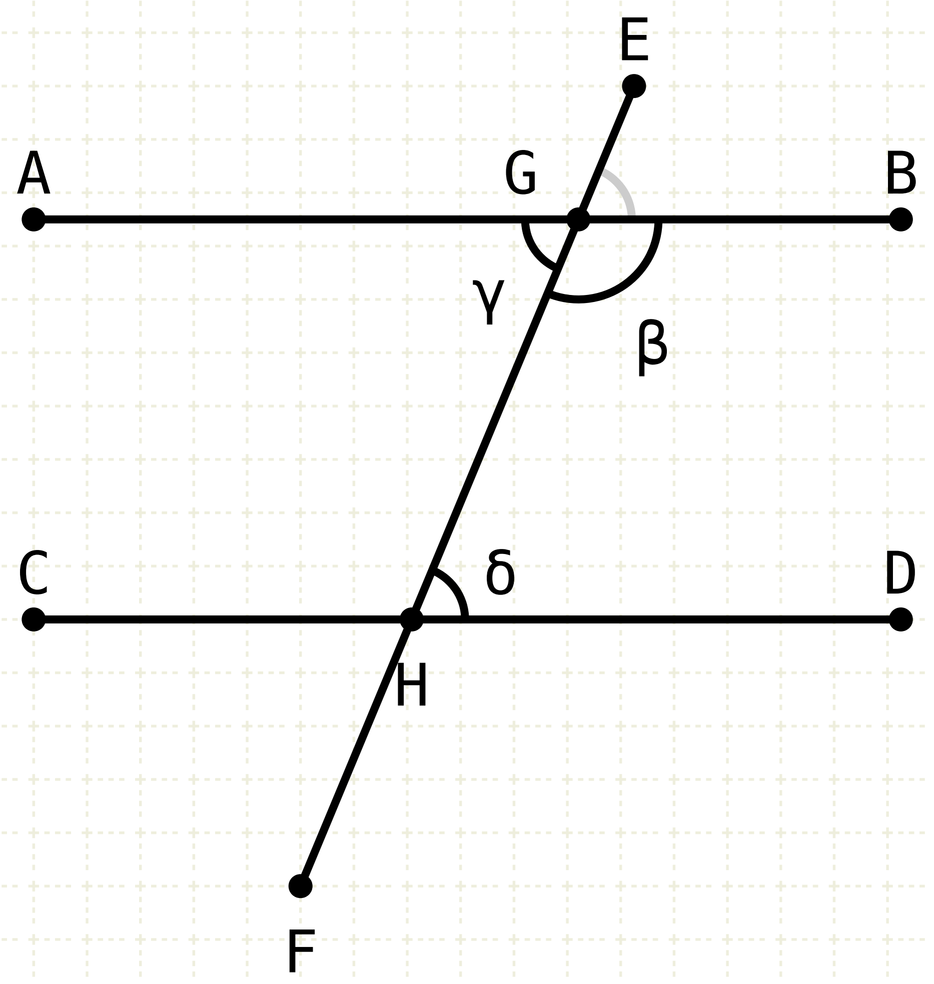

AB || CD

$\gamma > \delta$

$\gamma+\beta > \delta+\beta$

Proof by Contradiction

Assume that AGH is greater than GHD

Add the angle BGH to both

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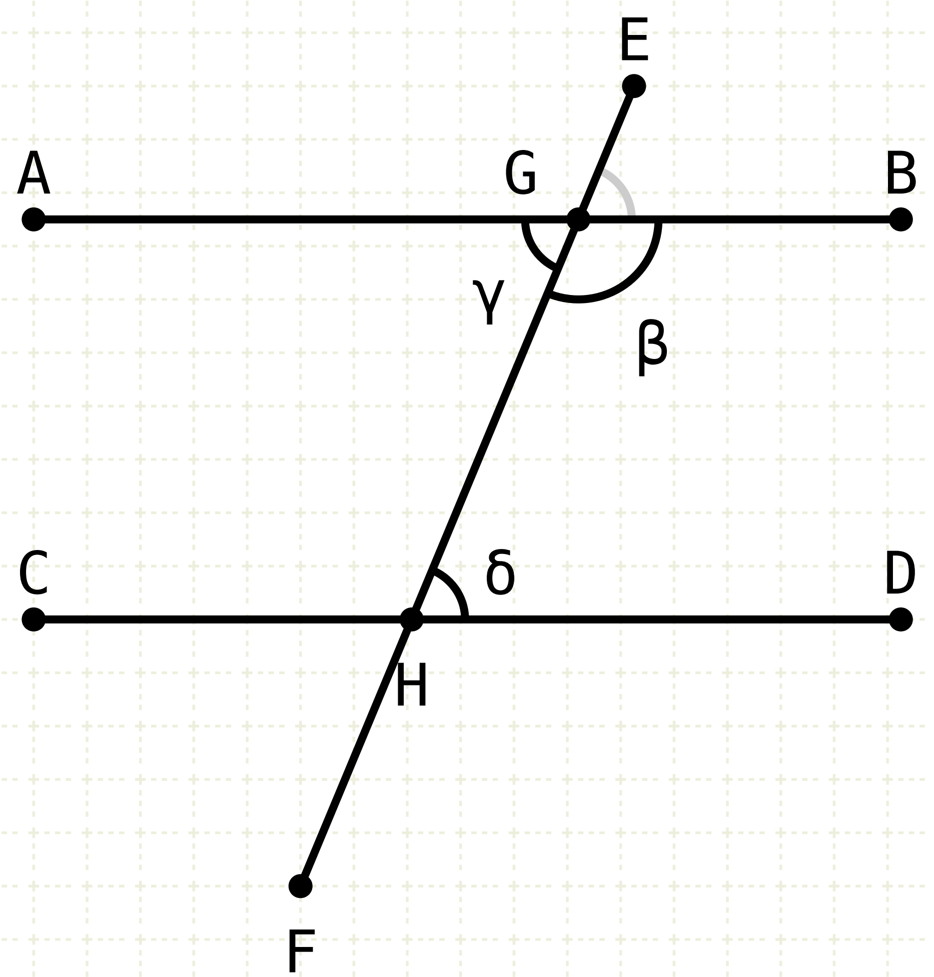

$$AB \parallel CD$$

$$\gamma > \delta$$

$$\gamma+\beta > \delta+\beta$$

$$\gamma+\beta = \llcorner+\llcorner$$

$$\llcorner+\llcorner > \delta+\beta$$

Proof by Contradiction

Assume that AGH is greater than GHD

Add the angle BGH to both

The sum of angles BGH and AGH is equal to two right angles (I·13), thus angles GHD and BGH are less than two right angles

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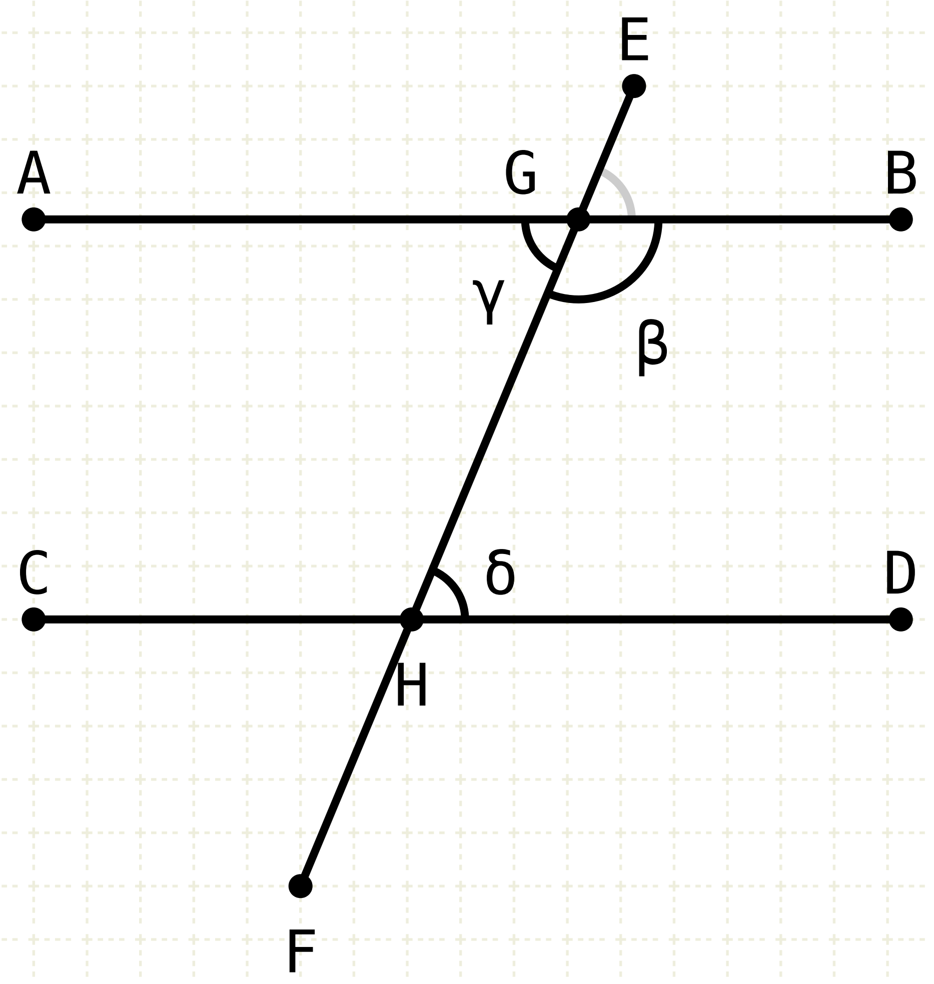

$AB \parallel CD$

$\gamma > \delta$

$\gamma+\beta > \delta+\beta$

$\gamma+\beta = \llcorner+\llcorner$

$\llcorner+\llcorner > \delta+\beta$

$AB \neq\parallel CD$

Proof by Contradiction

Assume that AGH is greater than GHD

Add the angle BGH to both

The sum of angles BGH and AGH is equal to two right angles (I·13), thus angles GHD and BGH are less than two right angles

If the sum of the angles BGH and GHD is less than two right angles, the lines (if extended) will eventually meet (postulate 5), and hence are not parallel

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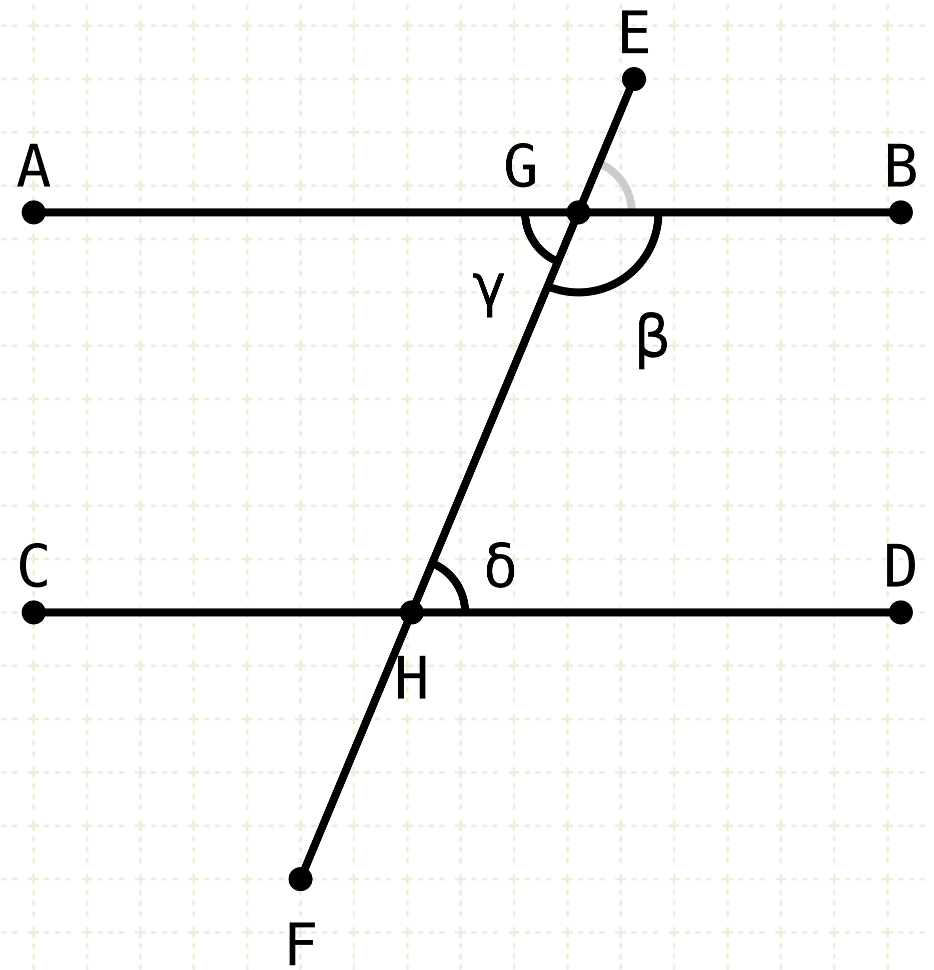

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$\gamma+\beta > \delta+\beta$

$\gamma+\beta = ∟+∟$

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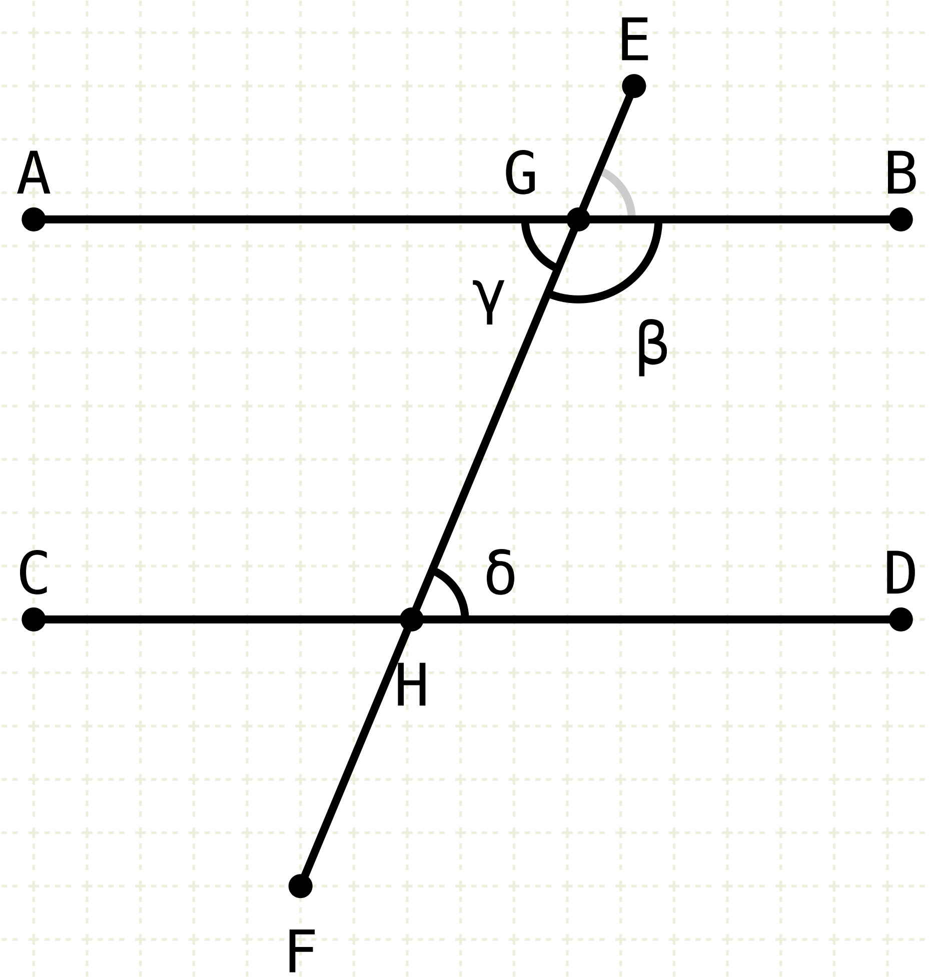

$AB \parallel CD$

$\gamma > \delta$ ✗

$\gamma+\beta > \delta+\beta$

$\gamma+\beta = ∟+∟$

$∟+∟ > \delta+\beta$

$AB \not\parallel CD$

Proof by Contradiction

Assume that AGH is greater than GHD

Add the angle BGH to both

The sum of angles BGH and AGH is equal to two right angles (I·13), thus angles GHD and BGH are less than two right angles

If the sum of the angles BGH and GHD is less than two right angles, the lines (if extended) will eventually meet (postulate 5), and hence are not parallel

This is a contradiction, and hence our original assumption was wrong

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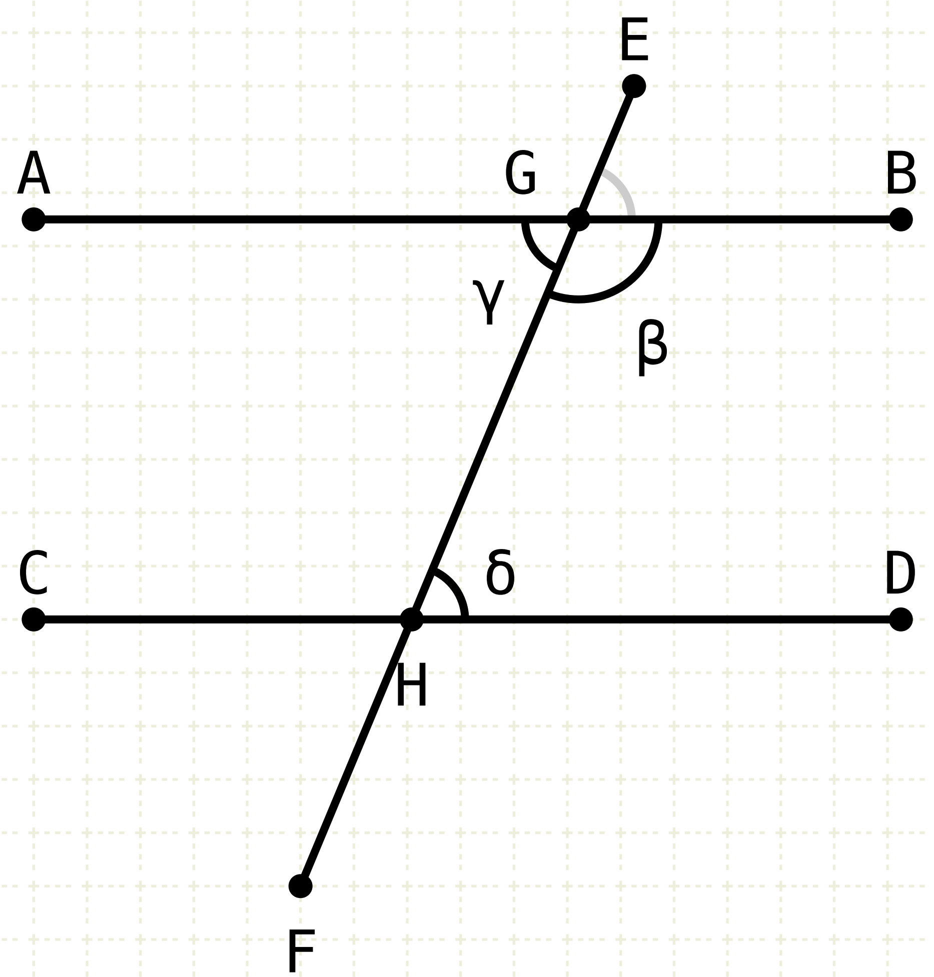

$AB \parallel CD$

$\gamma > \delta$ ✗

$\gamma + \beta > \delta + \beta$

$\gamma + \beta = ∟ + ∟$

$∟ + ∟ > \delta + \beta$

$AB \not\parallel CD$

$\gamma = \delta$

Proof by Contradiction

Assume that AGH is greater than GHD

Add the angle BGH to both

The sum of angles BGH and AGH is equal to two right angles (I·13), thus angles GHD and BGH are less than two right angles

If the sum of the angles BGH and GHD is less than two right angles, the lines (if extended) will eventually meet (postulate 5), and hence are not parallel

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Therefore AGH equals GHD

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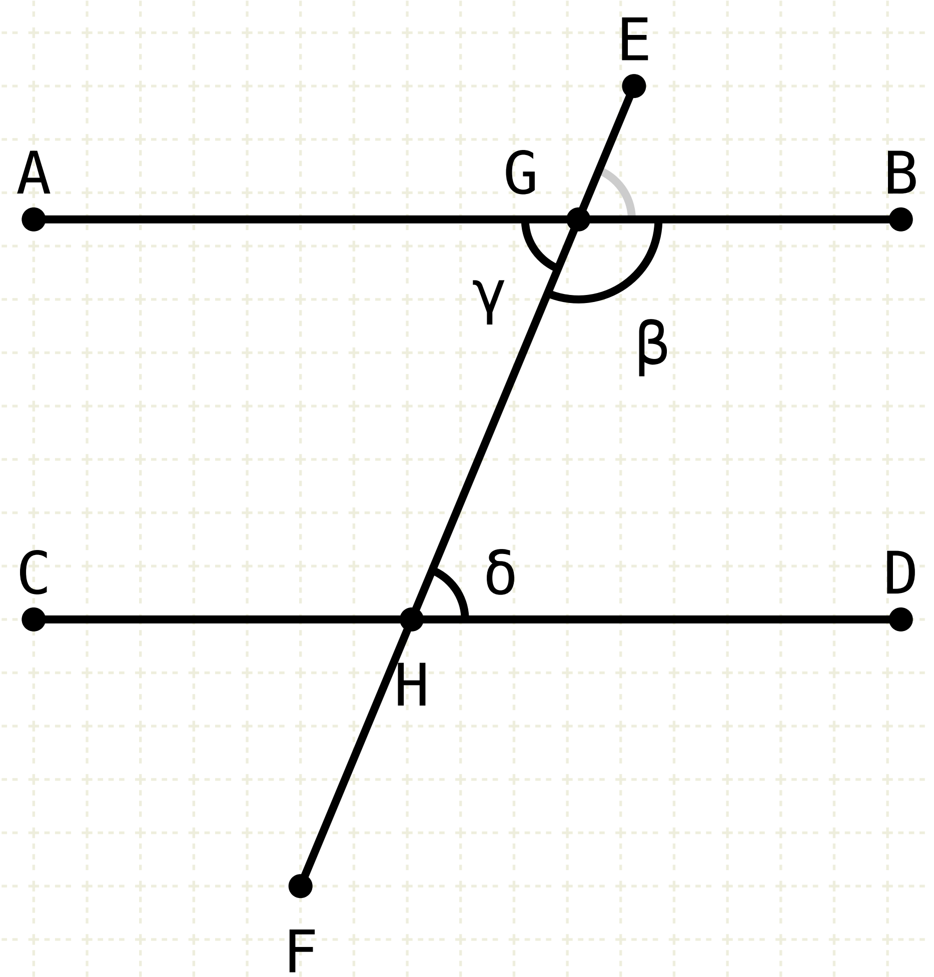

$AB \parallel CD$

$\gamma > \delta$ ✗

$\gamma+\beta > \delta+\beta$

$\gamma+\beta = \llcorner+\llcorner$

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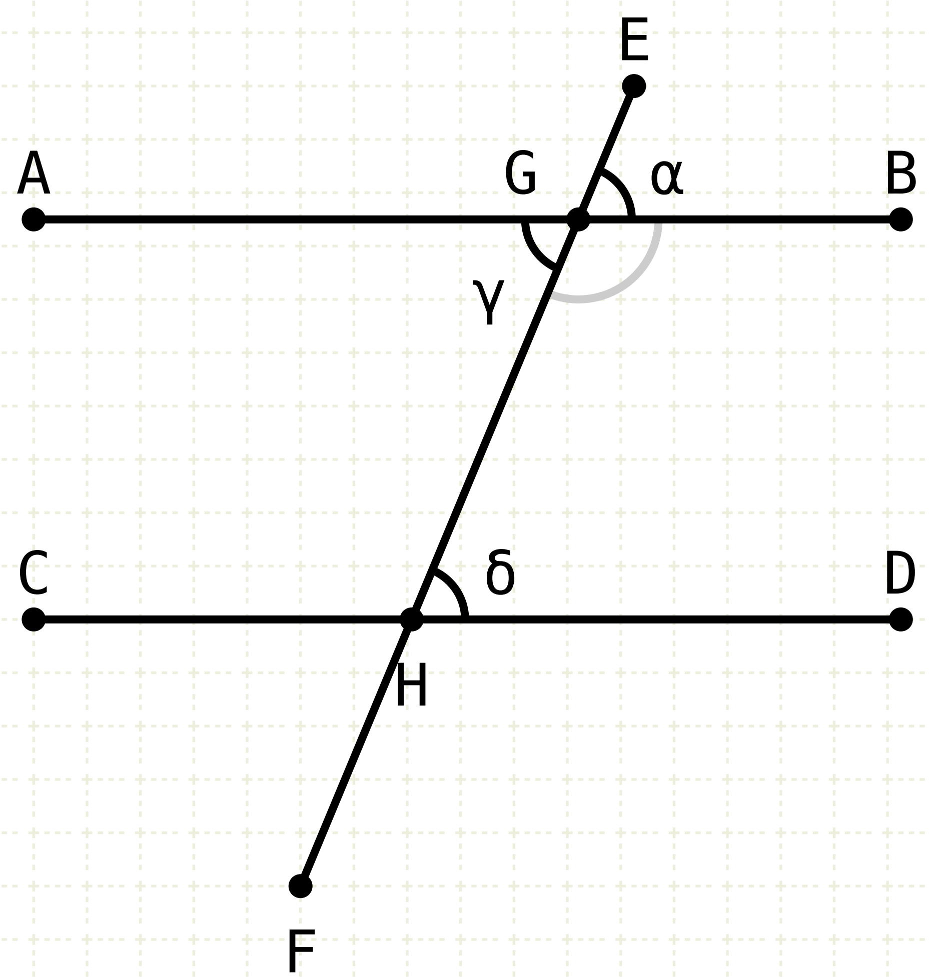

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Proof by Contradiction

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The sum of angles BGH and AGH is equal to two right angles (I·13), thus angles GHD and BGH are less than two right angles

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Therefore AGH equals GHD

Angle EGB equals AGH (I·15) and AGH equals GHD

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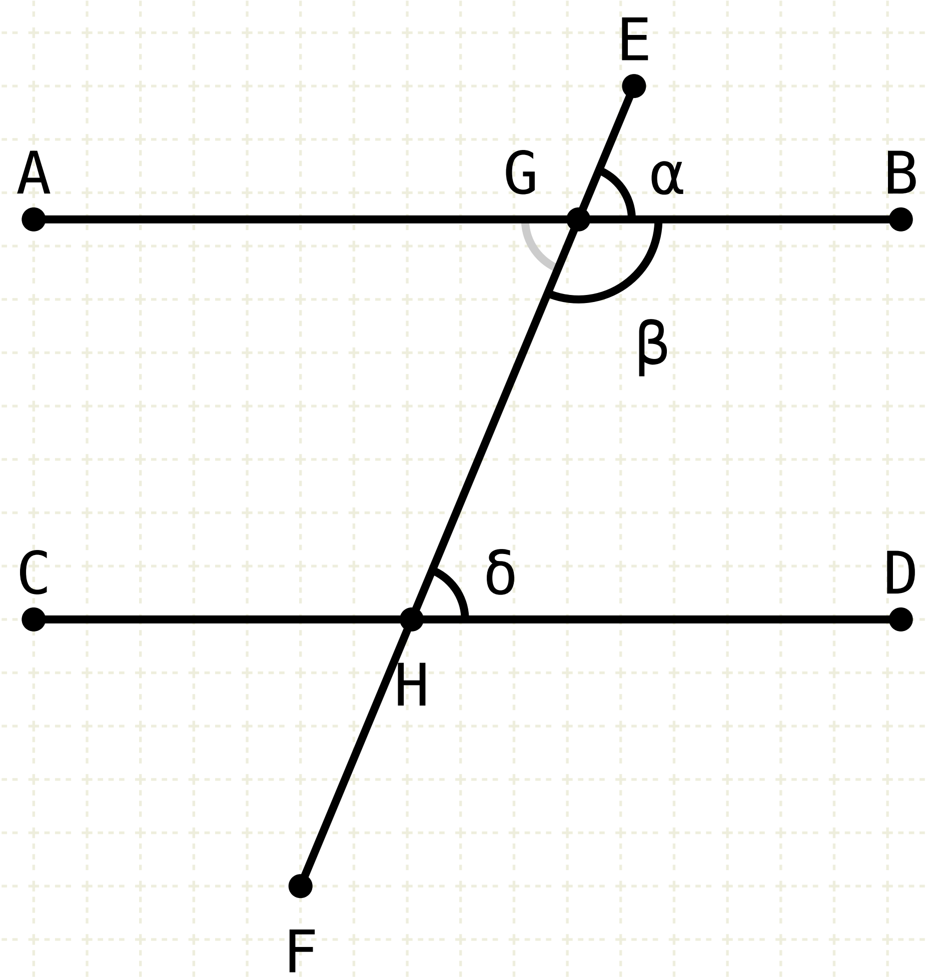

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Proof by Contradiction

Assume that AGH is greater than GHD

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The sum of angles BGH and AGH is equal to two right angles (I·13), thus angles GHD and BGH are less than two right angles

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Therefore AGH equals GHD

Angle EGB equals AGH (I·15) and AGH equals GHD

Add angle BGH to EGB and GHD

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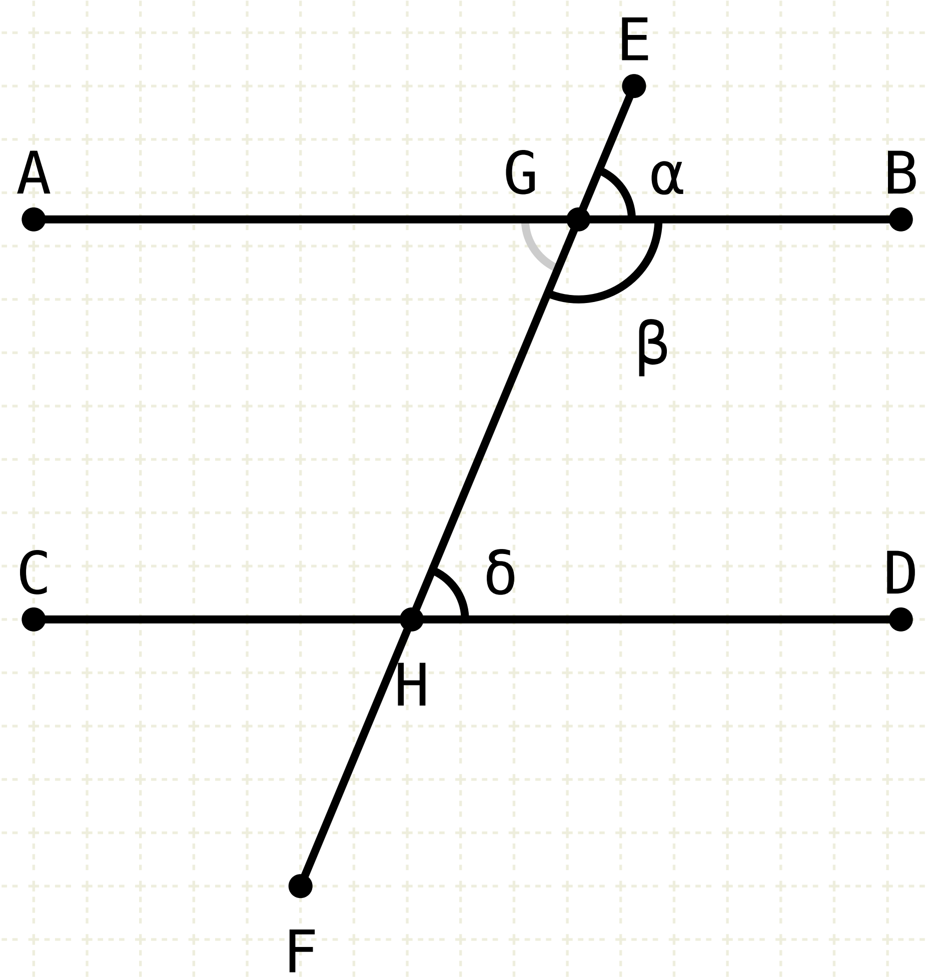

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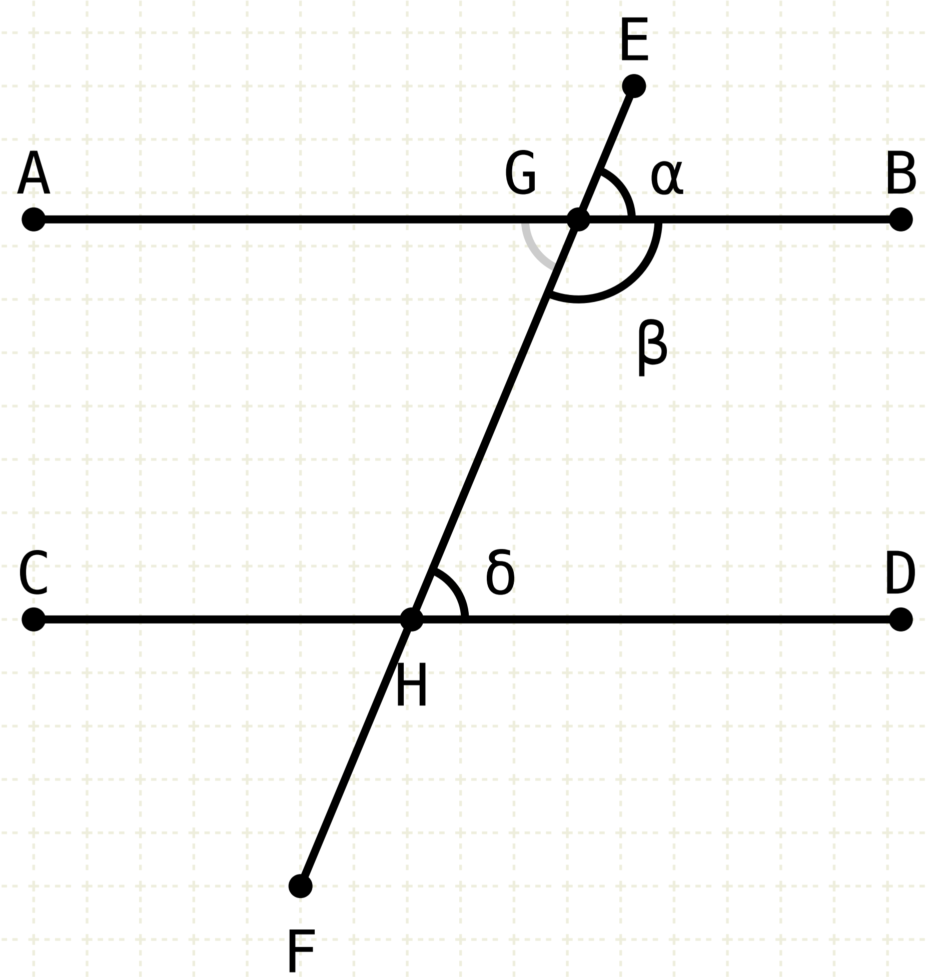

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Angle EGB equals AGH (I·15) and AGH equals GHD

Add angle BGH to EGB and GHD

The sum of EGB and BGH is two right angles,

therefore the sum of BGH and GHD is also two right angles

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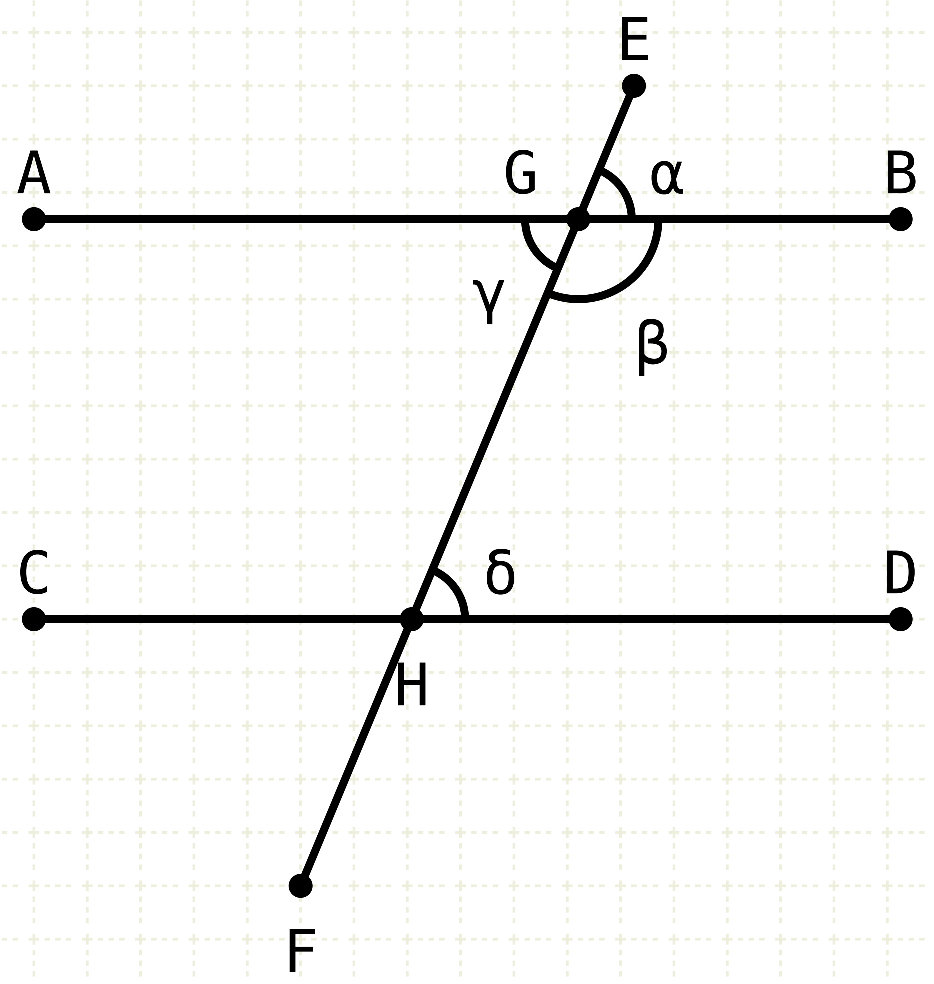

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