Euclid's Elements Book IV

Philosophy (nature) is written in that great book which ever is before our eyes -- I mean the universe -- but we cannot understand it if we do not first learn the language and grasp the symbols in which it is written. The book is written in mathematical language, and the symbols are triangles, circles and other geometrical figures, without whose help it is impossible to comprehend a single word of it - without which one wanders in vain through a dark labyrinth.

Galileo Galilei



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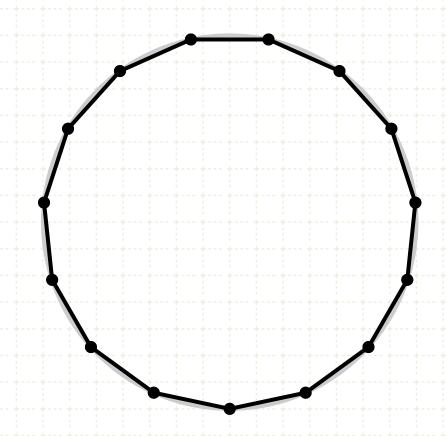
- 1 Fit a given straight line into a given circle, if the line is less than the diameter
- In a given circle to inscribe a triangle equiangular with a given triangle
- 3 About a given circle to circumscribe a triangle equiangular with a given triangle
- 4 In a given triangle, to inscribe a circle
- 5 About a given triangle to circumscribe a circle
- 6 In a given circle to inscribe a square
- 7 About a given circle to circumscribe a square
- 8 In a given square, to inscribe a circle
- 9 About a given square, to circumscribe a circle
- 10 To construct an isosceles triangle having each of the angles at the base double of the remaining one

- 11 In a given circle to inscribe an equilateral and equiangular pentagon
- 12 About a given circle to circumscribe an equilateral and equiangular pentagon
- 13 In a given pentagon, which is equilateral and equiangular, to inscribe a circle
- 14 About a given pentagon, which is equilateral and equiangular, to circumscribe a circle
- 15 In a given circle to inscribe an equilateral and equiangular hexagon
- 16 In a given circle to inscribe a fifteen angled figure which shall be both equilateral and equiangular



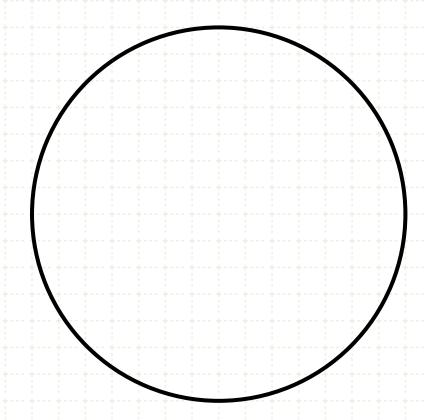


In a given circle to inscribe a fifteen angled figure which shall be both equilateral and equiangular.

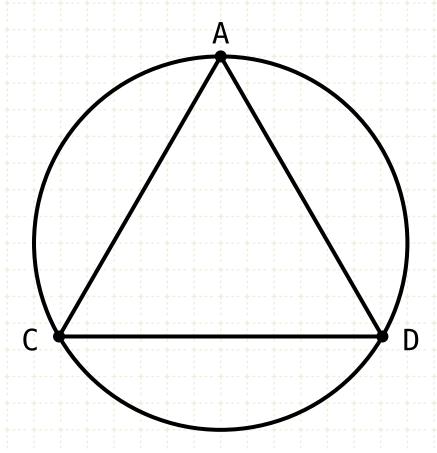


In other words

Given a circle, draw an fifteen sided polygon with equal sides and equal angles on the inside of the circle.



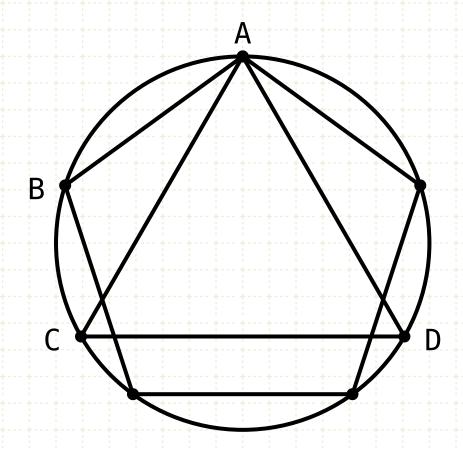
Construction



Construction

Draw an equilateral triangle (I·1) and copy an equiangular version into the circle (IV·2)

In a given circle to inscribe a fifteen angled figure which shall be both equilateral and equiangular.

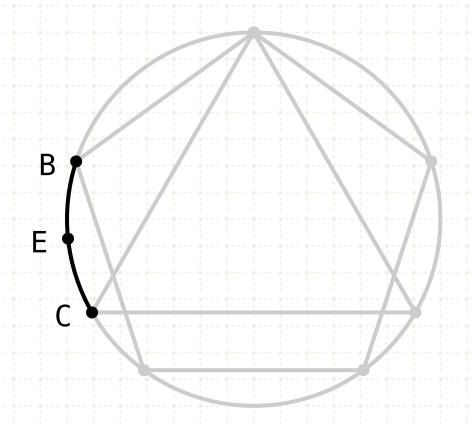


Construction

Draw an equilateral triangle (I·1) and copy an equiangular version into the circle (IV·2)

Inscribe a pentagon in the circle (IV-11)

In a given circle to inscribe a fifteen angled figure which shall be both equilateral and equiangular.



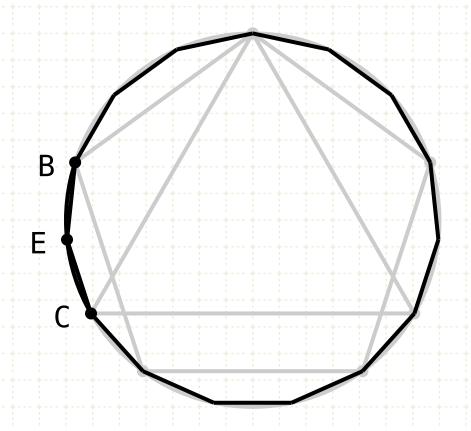
Construction

Draw an equilateral triangle (I·1) and copy an equiangular version into the circle (IV·2)

Inscribe a pentagon in the circle (IV-11)

Bisect the arc BC at point E

In a given circle to inscribe a fifteen angled figure which shall be both equilateral and equiangular.



Construction

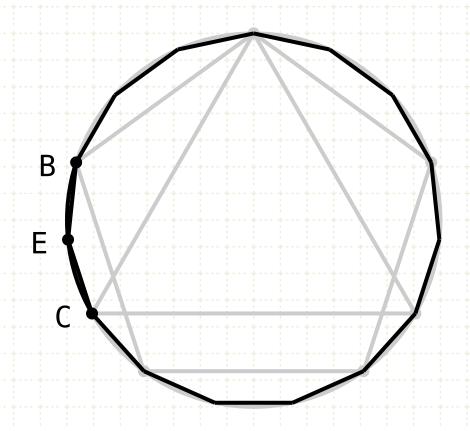
Draw an equilateral triangle (I·1) and copy an equiangular version into the circle (IV·2)

Inscribe a pentagon in the circle (IV-11)

Bisect the arc BC at point E

Join the line CE, and copy this line into the circle, always contiguous to the previous line.

In a given circle to inscribe a fifteen angled figure which shall be both equilateral and equiangular.



Construction

Draw an equilateral triangle (I·1) and copy an equiangular version into the circle (IV·2)

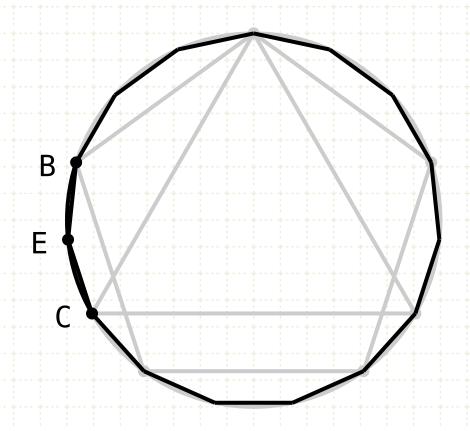
Inscribe a pentagon in the circle (IV-11)

Bisect the arc BC at point E

Join the line CE, and copy this line into the circle, always contiguous to the previous line.

The resulting figure is a 15 sided equiangular, equilateral polygon

In a given circle to inscribe a fifteen angled figure which shall be both equilateral and equiangular.



Construction

Draw an equilateral triangle (I·1) and copy an equiangular version into the circle (IV·2)

Inscribe a pentagon in the circle (IV-11)

Bisect the arc BC at point E

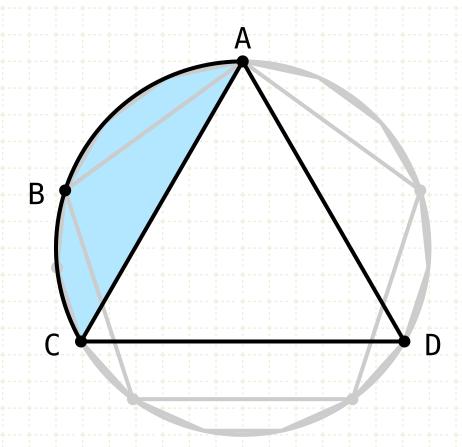
Join the line CE, and copy this line into the circle, always contiguous to the previous line.

The resulting figure is a 15 sided equiangular, equilateral polygon

Proof



In a given circle to inscribe a fifteen angled figure which shall be both equilateral and equiangular.



$$AC = 1/3 \times 15 = 5/15$$

Construction

Draw an equilateral triangle (I·1) and copy an equiangular version into the circle (IV·2)

Inscribe a pentagon in the circle (IV-11)

Bisect the arc BC at point E

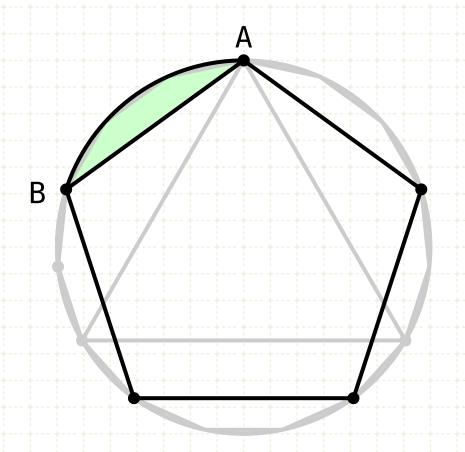
Join the line CE, and copy this line into the circle, always contiguous to the previous line.

The resulting figure is a 15 sided equiangular, equilateral polygon

Proof

Circumference AC is one third of the circle, so it will contain five segments of a fifteen sided equilateral figure

In a given circle to inscribe a fifteen angled figure which shall be both equilateral and equiangular.



$$AC = 1/3 \times 15 = 5/15$$

 $AB = 1/5 \times 15 = 3/15$

Construction

Draw an equilateral triangle (I·1) and copy an equiangular version into the circle (IV·2)

Inscribe a pentagon in the circle (IV-11)

Bisect the arc BC at point E

Join the line CE, and copy this line into the circle, always contiguous to the previous line.

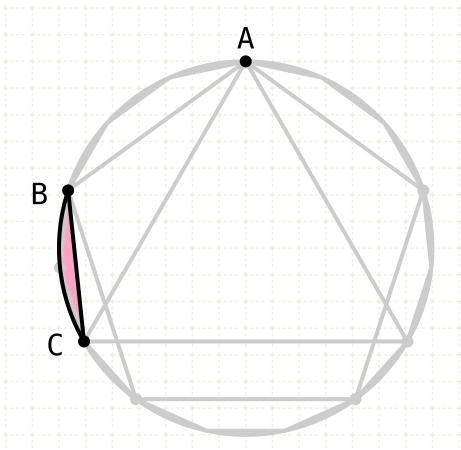
The resulting figure is a 15 sided equiangular, equilateral polygon

Proof

Circumference AC is one third of the circle, so it will contain five segments of a fifteen sided equilateral figure

Circumference AB is one fifth of the circle, so it will contain three segments of a fifteen sided equilateral figure

In a given circle to inscribe a fifteen angled figure which shall be both equilateral and equiangular.



$$AC = 1/3 \times 15 = 5/15$$

$$AB = 1/5 \times 15 = 3/15$$

$$BC = AC - BC = 2/15$$

Construction

Draw an equilateral triangle (I·1) and copy an equiangular version into the circle (IV·2)

Inscribe a pentagon in the circle (IV-11)

Bisect the arc BC at point E

Join the line CE, and copy this line into the circle, always contiguous to the previous line.

The resulting figure is a 15 sided equiangular, equilateral polygon

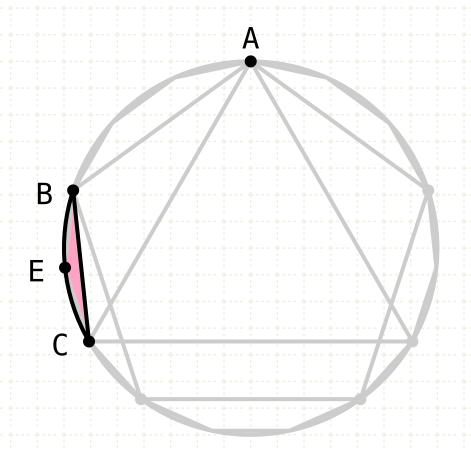
Proof

Circumference AC is one third of the circle, so it will contain five segments of a fifteen sided equilateral figure

Circumference AB is one fifth of the circle, so it will contain three segments of a fifteen sided equilateral figure

Therefore, the remainder of AB substracted from AC (BC) will contain two segments of a fifteen sided equilateral figure

In a given circle to inscribe a fifteen angled figure which shall be both equilateral and equiangular.



$$AC = 1/3 \times 15 = 5/15$$

 $AB = 1/5 \times 15 = 3/15$

Construction

Draw an equilateral triangle (I·1) and copy an equiangular version into the circle (IV·2)

Inscribe a pentagon in the circle (IV-11)

Bisect the arc BC at point E

Join the line CE, and copy this line into the circle, always contiguous to the previous line.

The resulting figure is a 15 sided equiangular, equilateral polygon

Proof

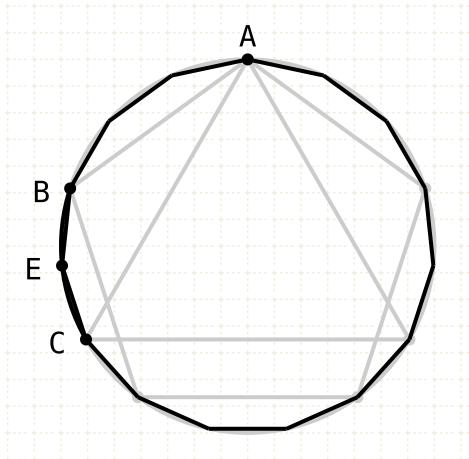
Circumference AC is one third of the circle, so it will contain five segments of a fifteen sided equilateral figure

Circumference AB is one fifth of the circle, so it will contain three segments of a fifteen sided equilateral figure

Therefore, the remainder of AB substracted from AC (BC) will contain two segments of a fifteen sided equilateral figure

Therefore, since E bisects BC, EC is one-fifteenth of a circle

In a given circle to inscribe a fifteen angled figure which shall be both equilateral and equiangular.



$$AC = 1/3 \times 15 = 5/15$$

 $AB = 1/5 \times 15 = 3/15$

Construction

Draw an equilateral triangle (I·1) and copy an equiangular version into the circle (IV·2)

Inscribe a pentagon in the circle (IV-11)

Bisect the arc BC at point E

Join the line CE, and copy this line into the circle, always contiguous to the previous line.

The resulting figure is a 15 sided equiangular, equilateral polygon

Proof

Circumference AC is one third of the circle, so it will contain five segments of a fifteen sided equilateral figure

Circumference AB is one fifth of the circle, so it will contain three segments of a fifteen sided equilateral figure

Therefore, the remainder of AB substracted from AC (BC) will contain two segments of a fifteen sided equilateral figure

Therefore, since E bisects BC, EC is one-fifteenth of a circle

Using the same arguments as for pentagons and hexagons, the points used to divide a circle into equal segments will create an equiangular, equilateral polygon

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