

Euclid's Elements

Book VI

One can state, without exaggeration, that the observation of and the search for similarities and differences are the basis of all human knowledge.

Alfred Nobel



Table of Contents, Chapter 6

1	If the height of two triangles are equal, then the ratio of the areas is equal to the ratio of the bases	7	If two triangles have one angle equal to one angle, and the sides about other angles are proportional, and the remaining angles either both less or both not less than a right angle, then triangles will be equiangular	14	In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and vice versa
2	If a line cuts a triangle, parallel to its base, it will cut the sides of the triangle proportionally			15	In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and vice versa
3	If an angle of a triangle is bisected and the straight line cutting the angle also cuts the base, the segments of the base will have the same ratio as the remaining sides of the triangle	8	If in a right-angled triangle a perpendicular be drawn from the right angle to the base, the triangles adjoining the perpendicular are similar both to the whole and to one another	16	If four straight lines are proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means, and vice versa
4	If two triangles have equal angles, then the sides opposite the equal angles are proportional, as well, the sides of the triangles on either side of the equal angles are also proportional	9	From a given straight line to cut off a given fraction	17	If three straight lines are proportional, the rectangle contained by the extremes is equal to the square on the mean; and vice versa
5	If two triangles have proportional sides, the triangles will be equiangular	10	To cut a given uncut straight line similarly to a given cut straight line	18	On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure
6	If two triangles have one angle equal to one angle and the sides about the equal angles are proportional, then the triangles will be equiangular	11	To two given straight lines to find a third proportional	19	Similar triangles are to one another in the duplicate ratio of the corresponding sides
		12	To three given straight lines to find a fourth proportional		
		13	To two given straight lines to find a mean proportional		



Table of Contents, Chapter 3

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|----|--|----|---|----|---|
| 20 | Similar polygons are divided into the same number of similar triangles, which have the same ratio as the wholes, and the polygons have duplicate ratios to their corresponding sides | 26 | If from a parallelogram a similar parallelogram with a common angle is subtracted, it is about the same diameter as the original | 31 | In right-angled triangles the figure on the side subtending the right angle is equal to the similar and similarly described figures on the sides containing the right angle |
| 21 | Figures which are similar to the same rectilineal figure are also similar to one another | 27 | Of all the parallelograms applied to the same straight line and deficient by parallelogrammic figures similar to a parallelogram drawn on half the said line, the largest will be one that is drawn on half of the straight line and is similar to the defect | | |
| 22 | If four straight lines are proportional, similar rectilineal figures will also be proportional; and vice versa | 28 | To a given straight line, apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one | | |
| 23 | Equiangular parallelograms have to one another the ratio compounded of the ratios of their sides | 29 | To a given straight line, apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one | | |
| 24 | In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another | 30 | To cut a finite straight line in extreme ratio | | |
| 25 | To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure | | | | |



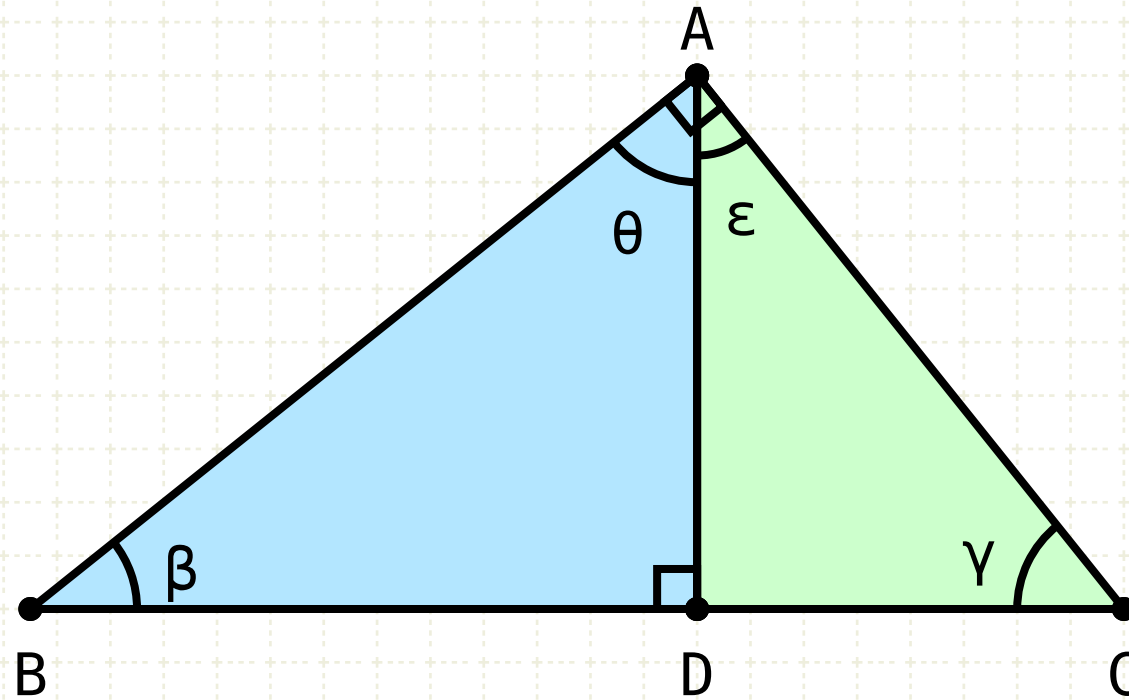
Proposition 8 of Book VI

If in a right-angled triangle a perpendicular be drawn from the right angle to the base, the triangles adjoining the perpendicular are similar both to the whole and to one another.



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$$\gamma = \theta, \quad \beta = \epsilon$$

$$AD:AB = AC:BC = CD:AC$$

$$AB:BD = BC:AB = AC:AD$$

$$AD:BD = AC:AB = CD:AD$$

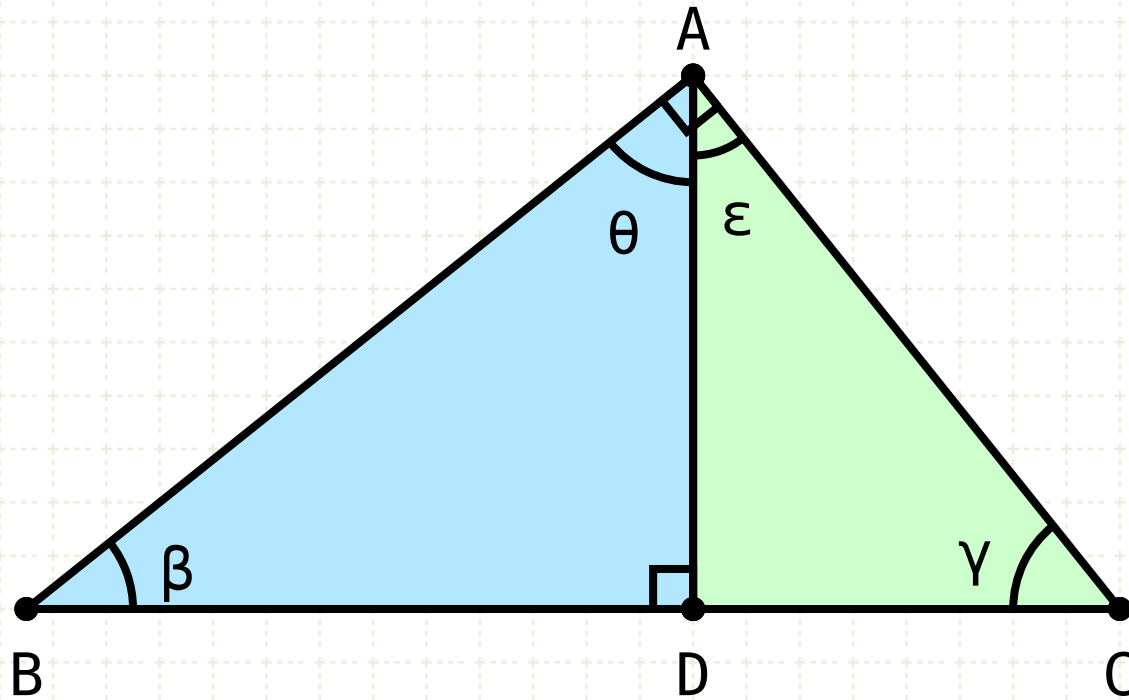
In other words

If we take a right angle triangle (ABC), where the angle BAC is a right angle, and we drop a perpendicular AD to the side BC , then...

The triangles BAD and ADC will be similar to the original triangle ABC , AND they will be similar to each other

Proposition 8 of Book VI

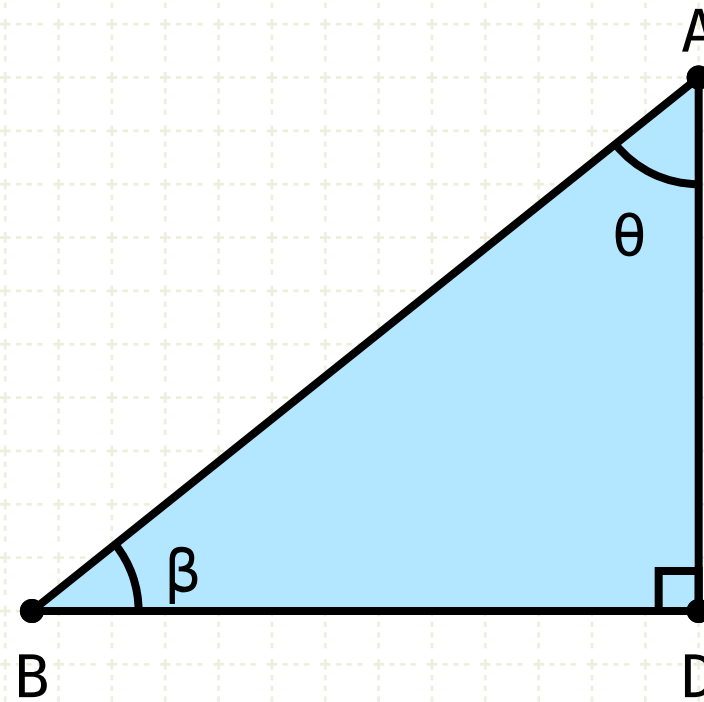
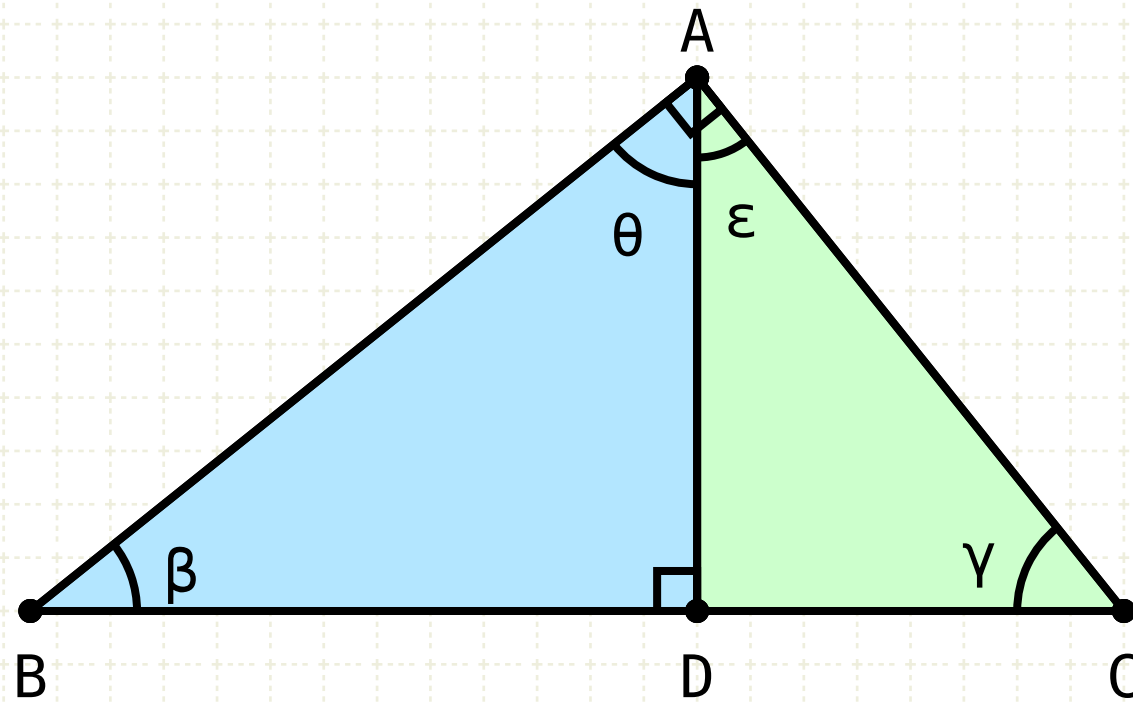
If in a right-angled triangle a perpendicular be drawn from the right angle to the base, the triangles adjoining the perpendicular are similar both to the whole and to one another.



Proof

Proposition 8 of Book VI

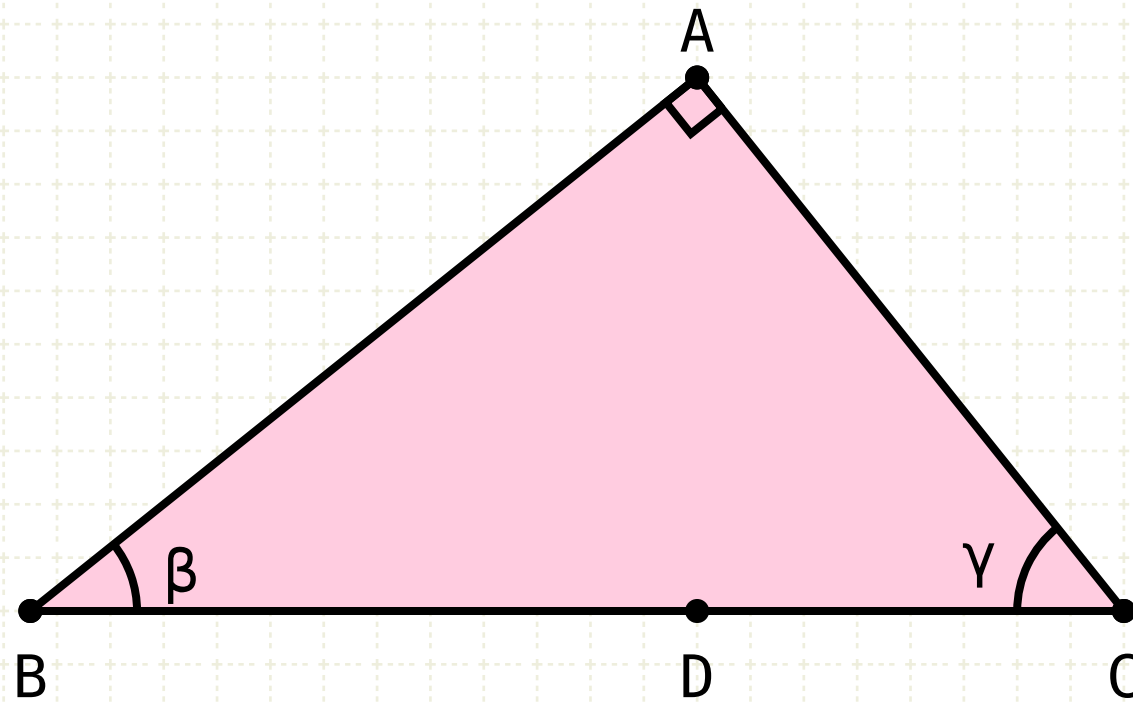
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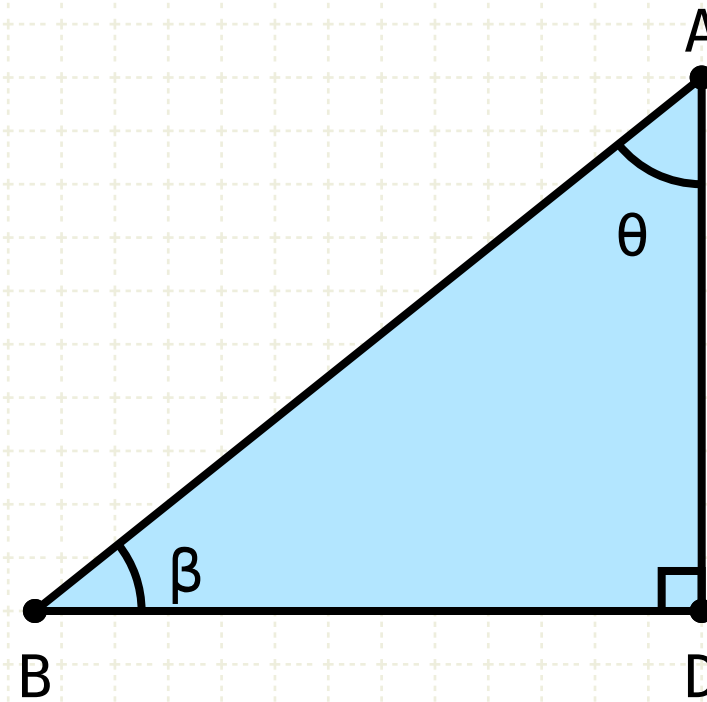
Proposition 8 of Book VI

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$$\angle BAC = \angle BDA = L$$

$$\theta = \gamma$$



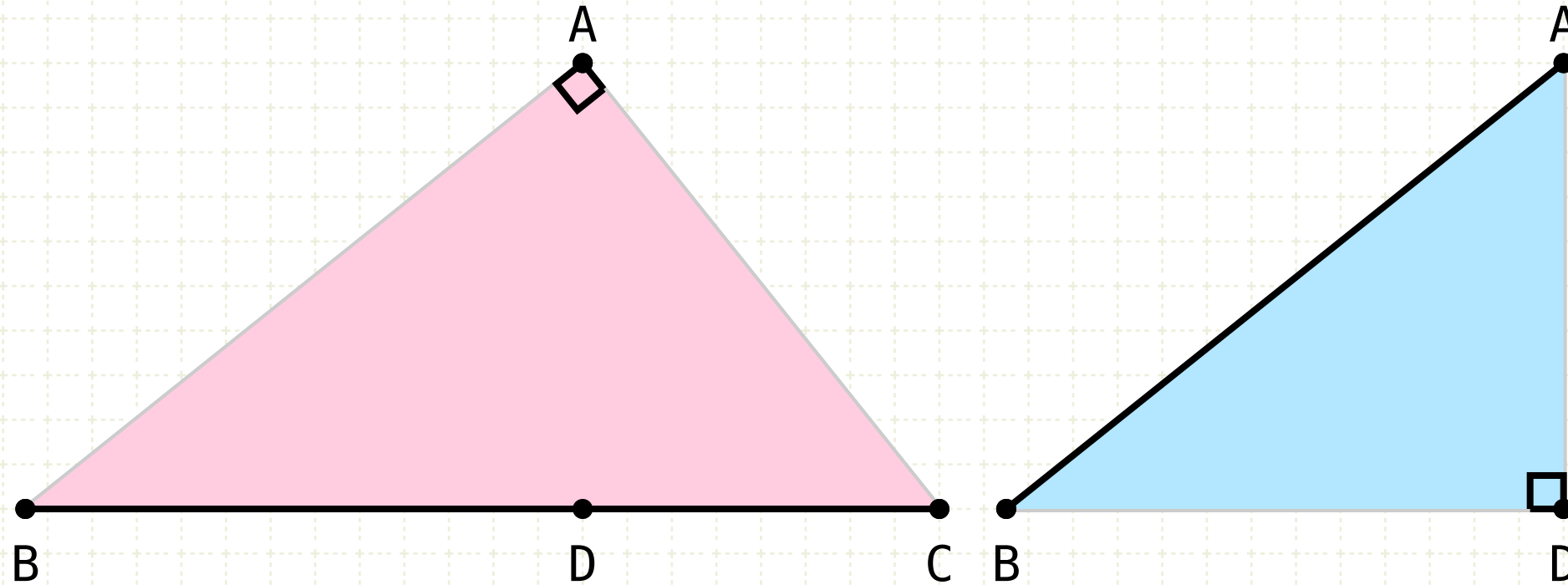
Proof

Angle BAC equals angle ADB (both are right angles), and the angle β is common to the two triangles ABD and ABC

Thus the remaining angles (BCA and BAD) must be equal (I.32)

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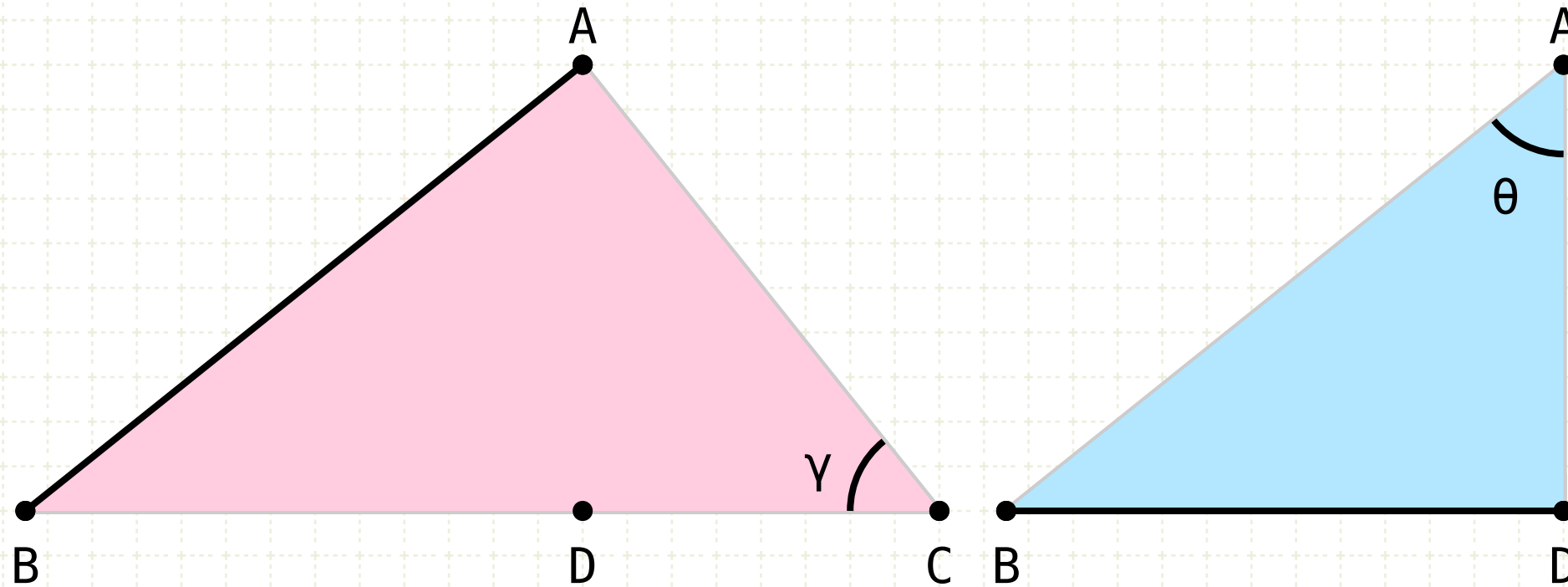
Thus the remaining angles (BCA and BAD) must be equal (I.32)

Therefore triangle ABC is equiangular with triangle ABD

Therefore, as BC (subtends the right angle in ABC) is to BA (subtends the right angle in the triangle ABD),

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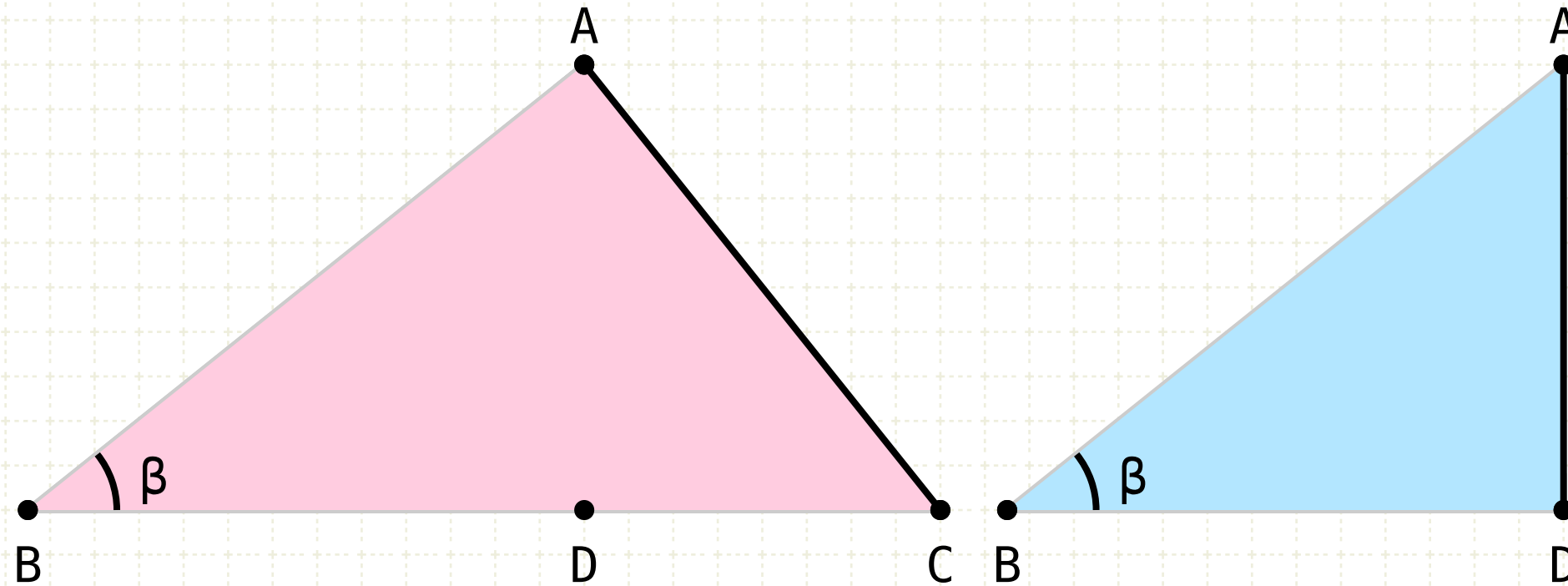
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so is AB (subtends angle γ in ABC) to BD (subtends the angle θ in ABD)

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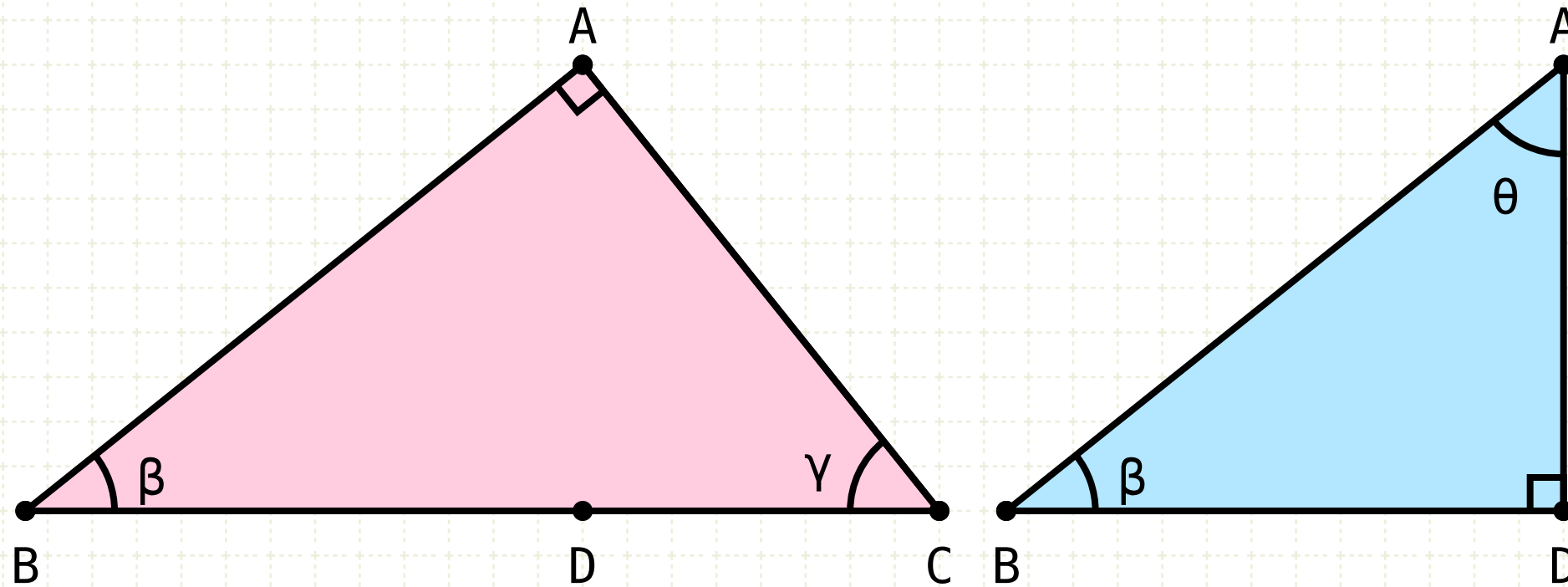
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and so is AC (subtends angle β in ABC) to AD (subtends the angle β in ABD) (VI.4)

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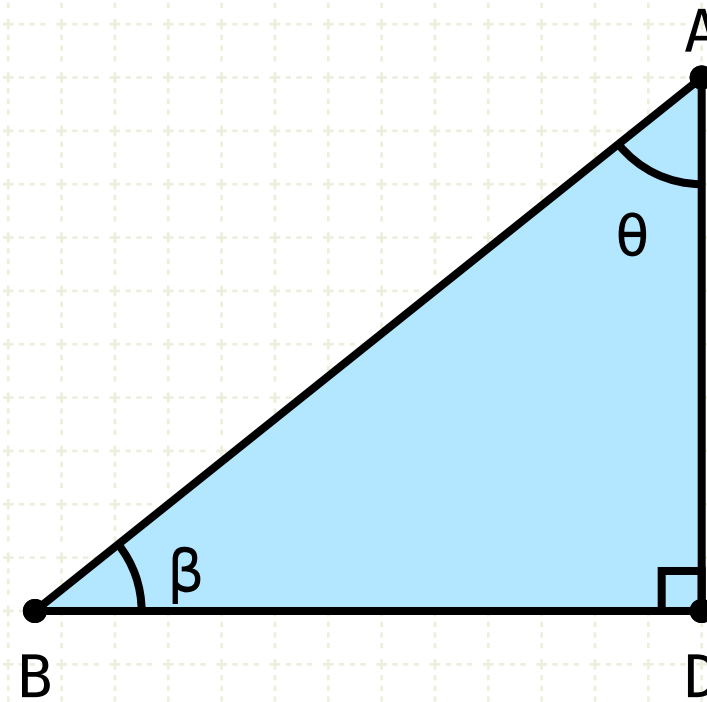
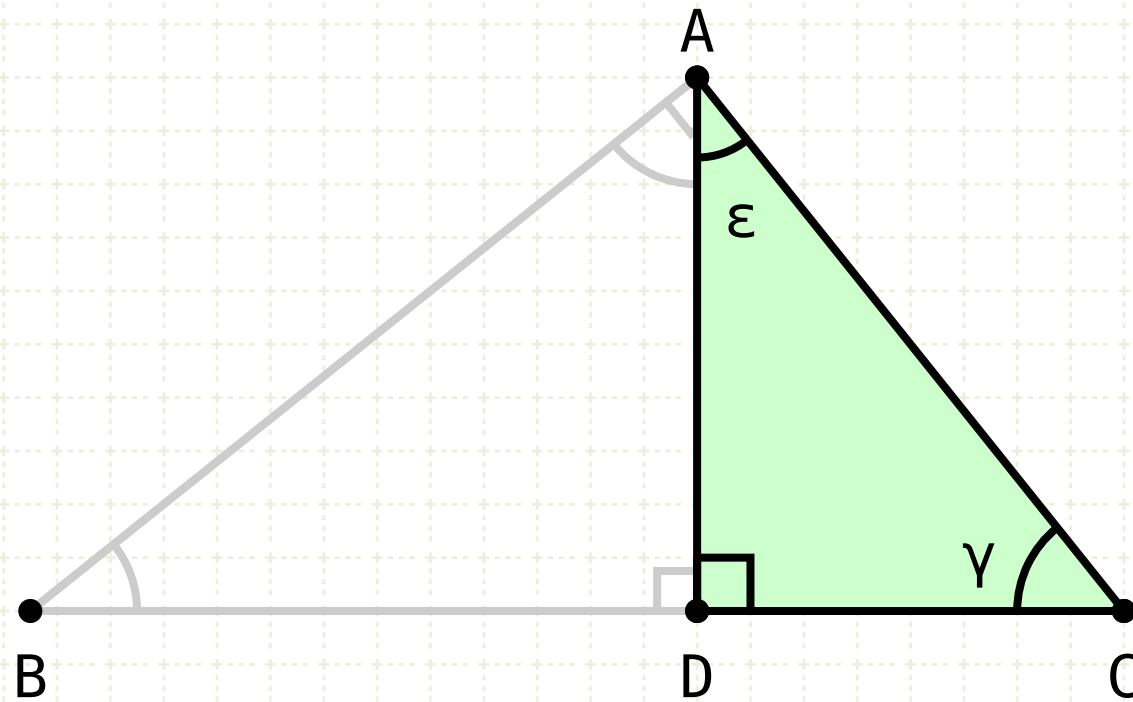
so is AB (subtends angle γ in ABC) to BD (subtends the angle θ in ABD)

and so is AC (subtends angle β in ABC) to AD (subtends the angle β in ABD) (VI.4)

So, ABC and ABD are equiangular, and their sides are proportional, which, by definition (VI.Def.1), means they are similar triangles

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If in a right-angled triangle a perpendicular be drawn from the right angle to the base, the triangles adjoining the perpendicular are similar both to the whole and to one another.



Proof (cont)

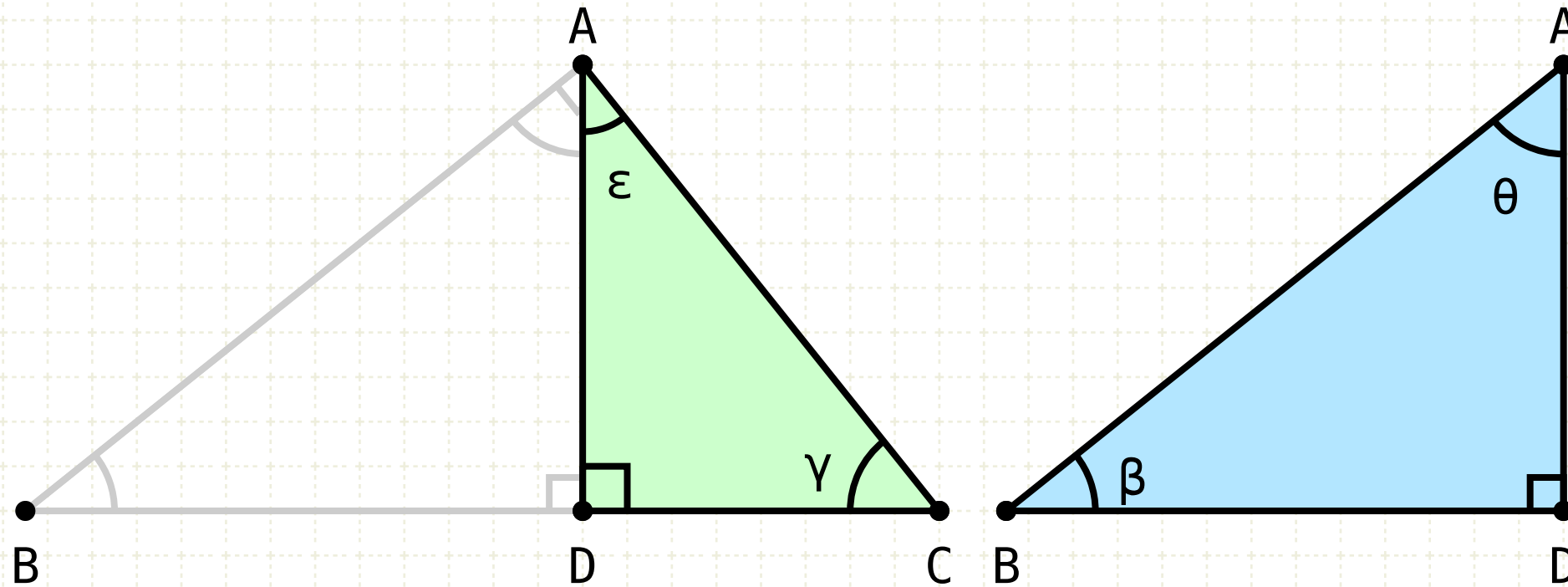
$$\angle BAC = \angle BDA = L$$

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Proposition 8 of Book VI

If in a right-angled triangle a perpendicular be drawn from the right angle to the base, the triangles adjoining the perpendicular are similar both to the whole and to one another.



Proof (cont)

Angle BDA equals angle ADC (both are right angles), and the angle θ is already been proven equal to γ

Thus the remaining angles (β and ε) must be equal (I·32)

$$\angle BAC = \angle BDA = L$$

$$\theta = \gamma$$

$$BC:BA = AB:BD = AC:AD$$

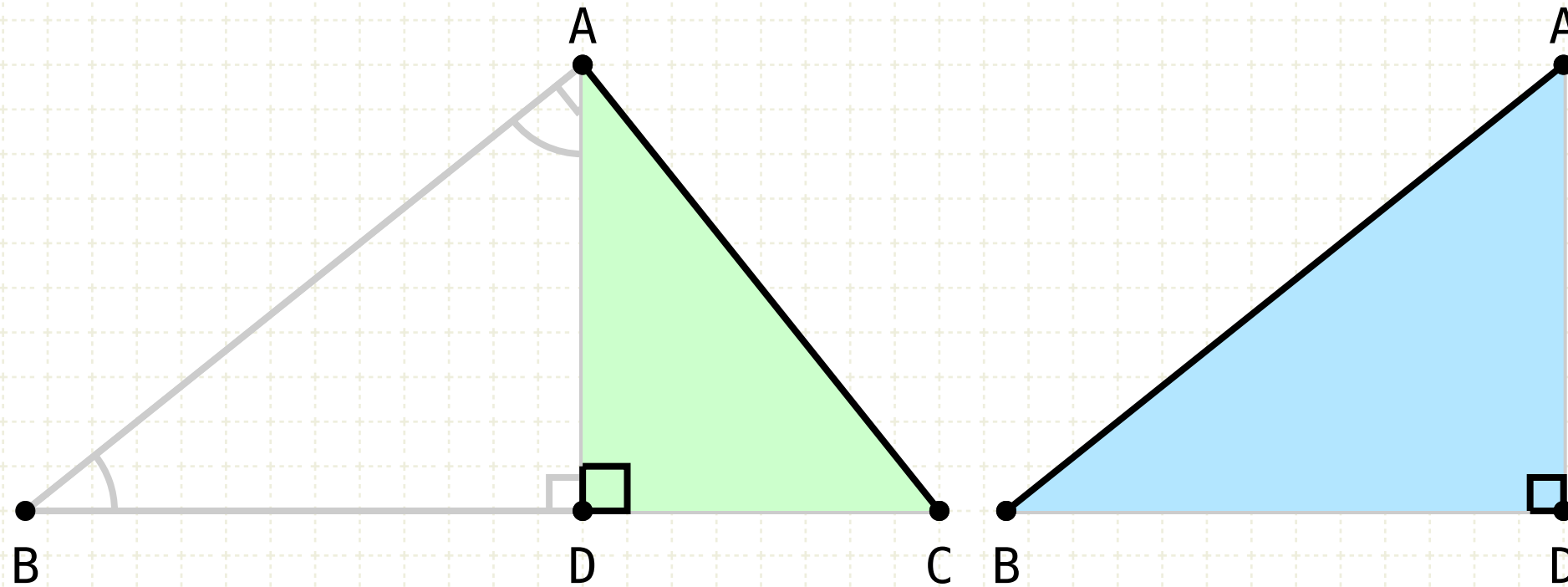
$$\angle BDC = \angle BDA = L$$

$$\varepsilon = \beta$$



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If in a right-angled triangle a perpendicular be drawn from the right angle to the base, the triangles adjoining the perpendicular are similar both to the whole and to one another.



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$$\epsilon = \beta$$

$$AB : AC$$

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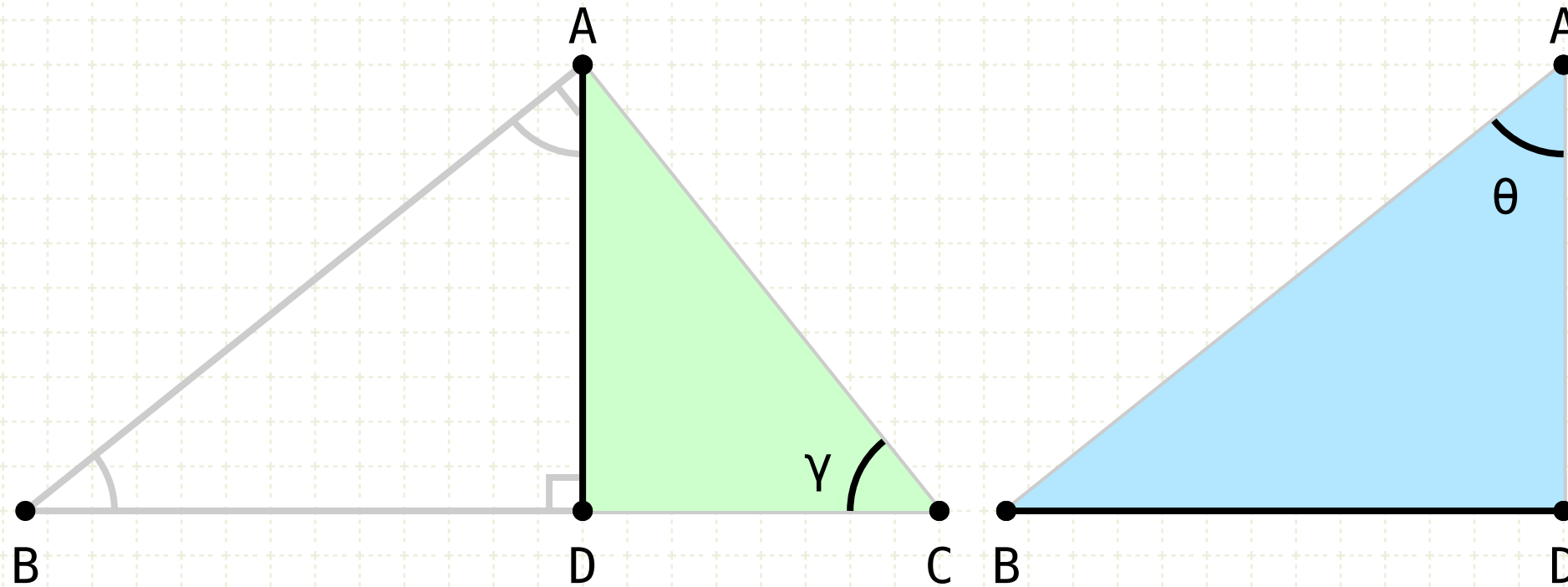
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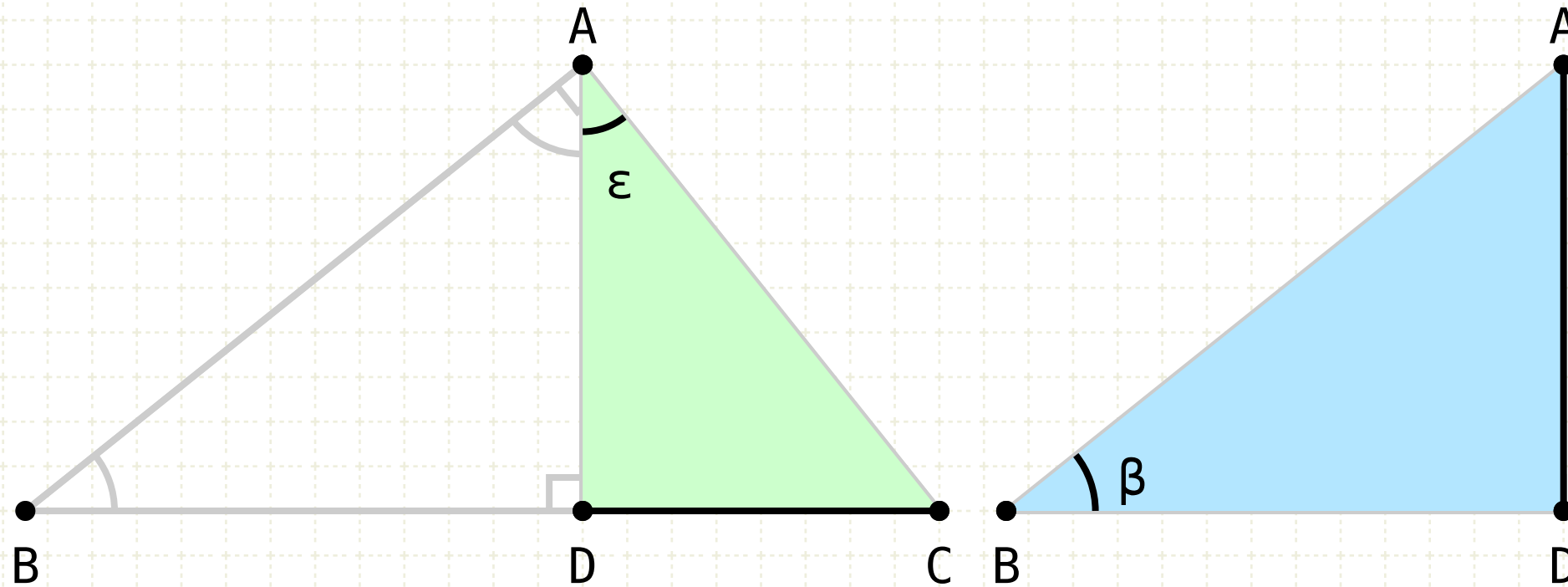
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$$AB : AC = BD : AD = AD : DC$$

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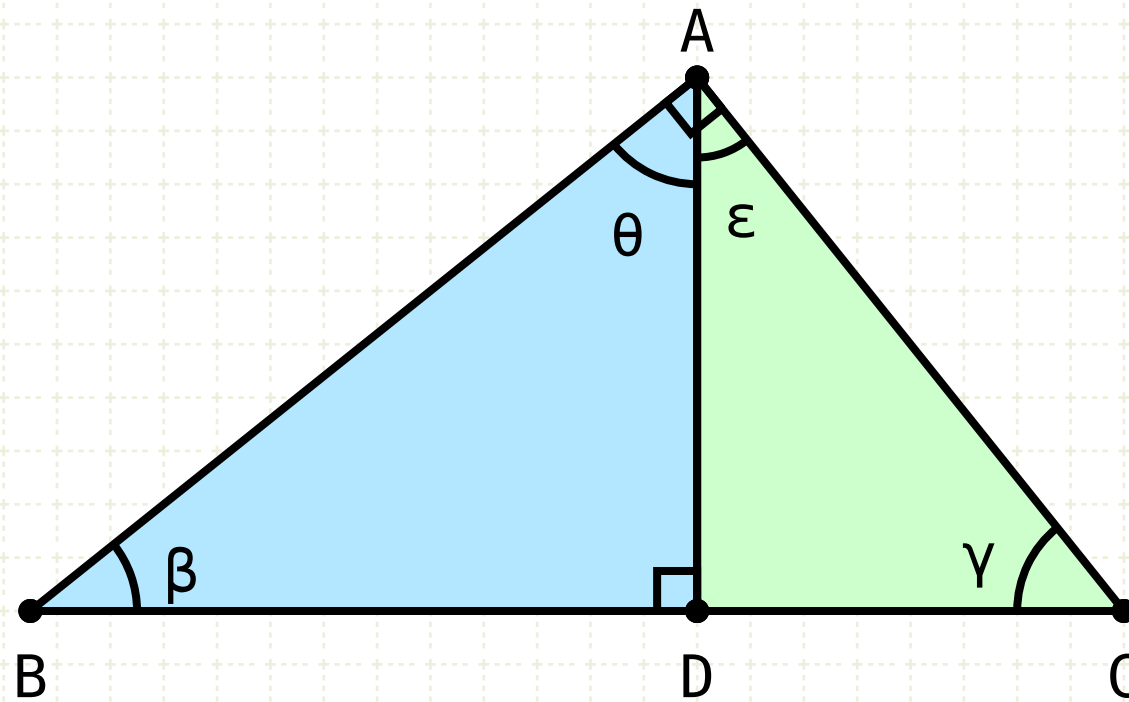
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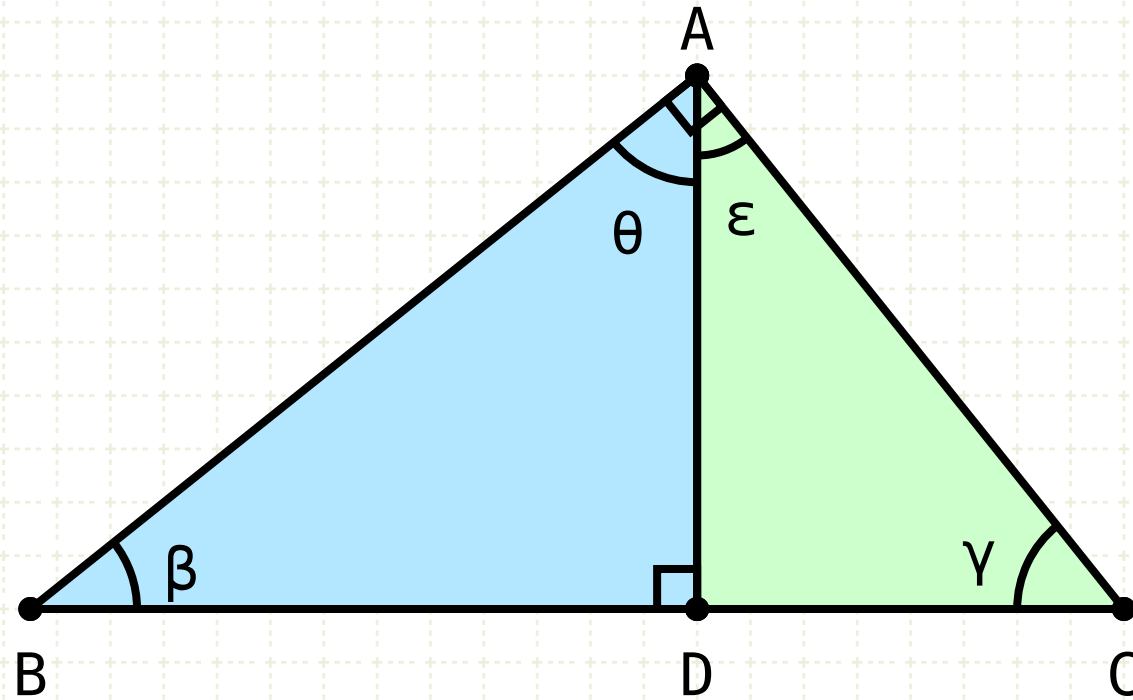
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Porism

In a right angle triangle, if a line be drawn from the right angle, perpendicular to the base, then this line will be the mean proportional between the segments of the base.



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