

Euclid's Elements

Book III



A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)



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| 2 | A chord of a circle always lies inside the circle | 10 | A circle does not cut a circle at more points than two | 18 | If line touches a circle, then it is perpendicular to the diameter that touches that point |
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| 6 | If two circles touch one another, they will not have the same center | 14 | In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another. | 22 | The opposite angles of quadrilaterals in circles are equal to two right angles |
| 7 | Consider two lines from a point inside a circle to the edge, the longer one will be the one closest to the longest part of the diameter passing through the original point | 15 | The longest line in a circle is its diameter, shorter the farther away from the diameter | 23 | On the same straight line there cannot be constructed two similar and unequal segments of circles on the same side |
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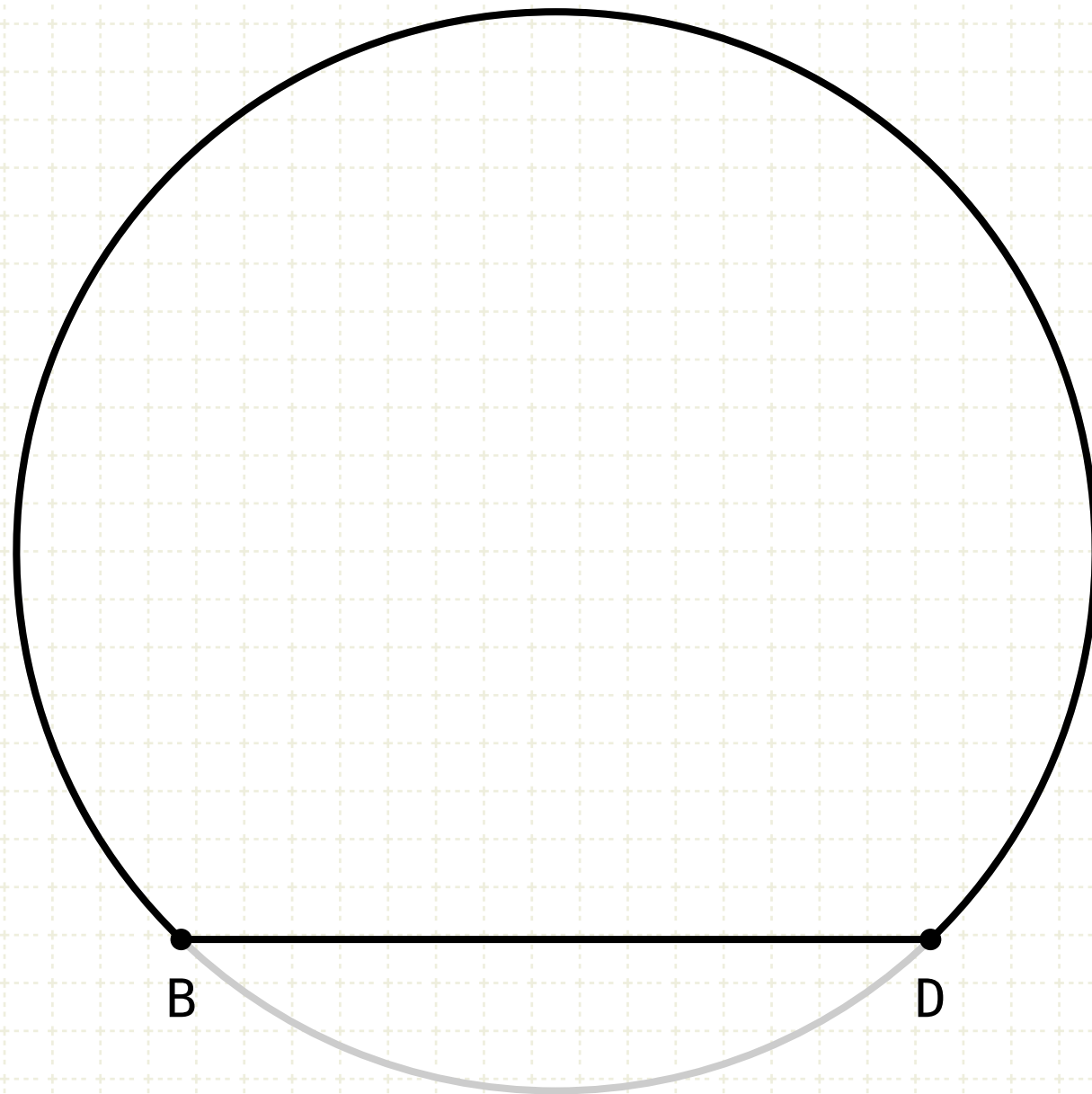
Proposition 21 of Book III

In a circle the angles in the same segment are equal to one another.



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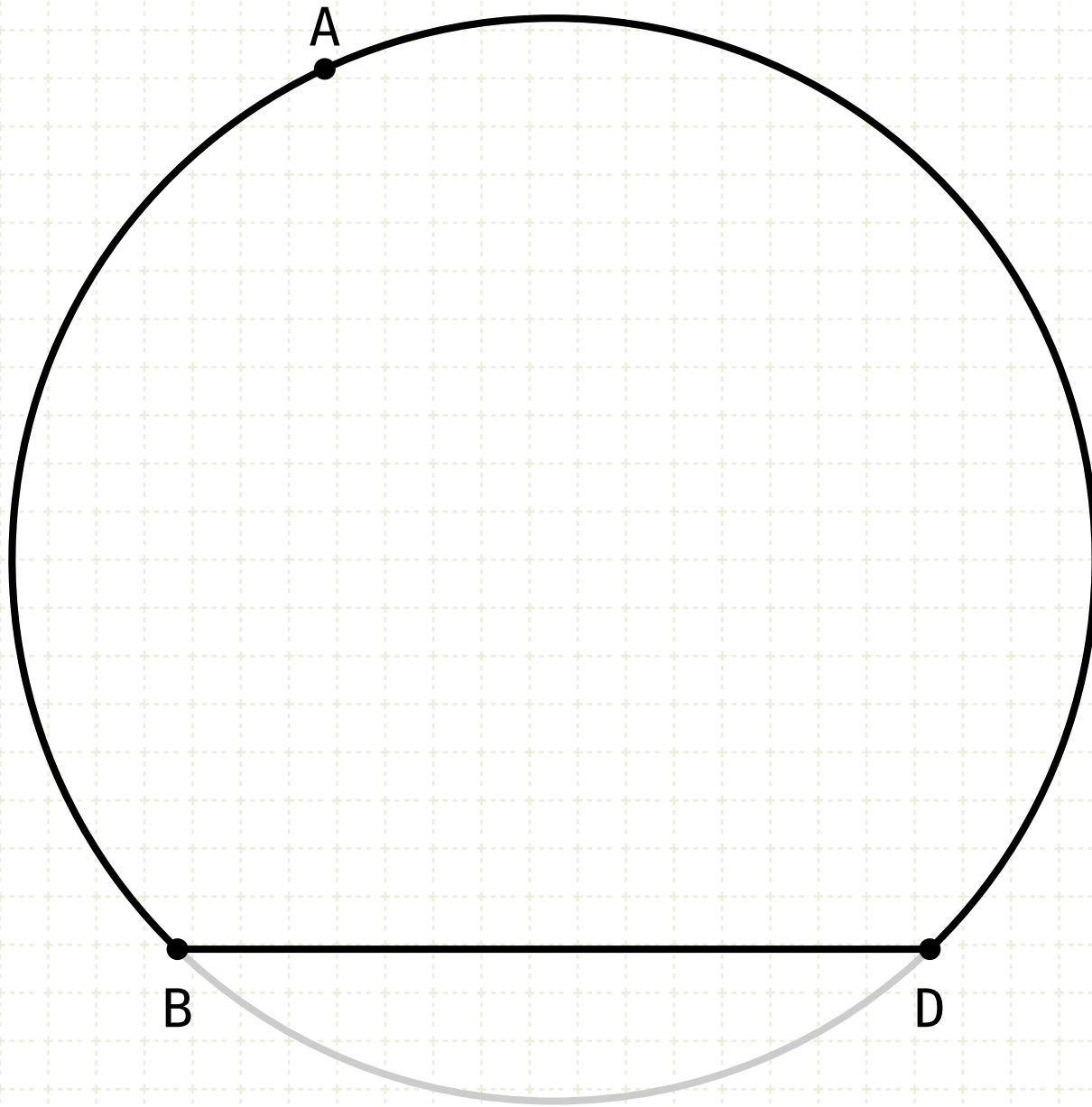


Definition - Segment of a Circle

A 'segment of a circle' is the figure contained by a straight line and a circumference of a circle

Proposition 21 of Book III

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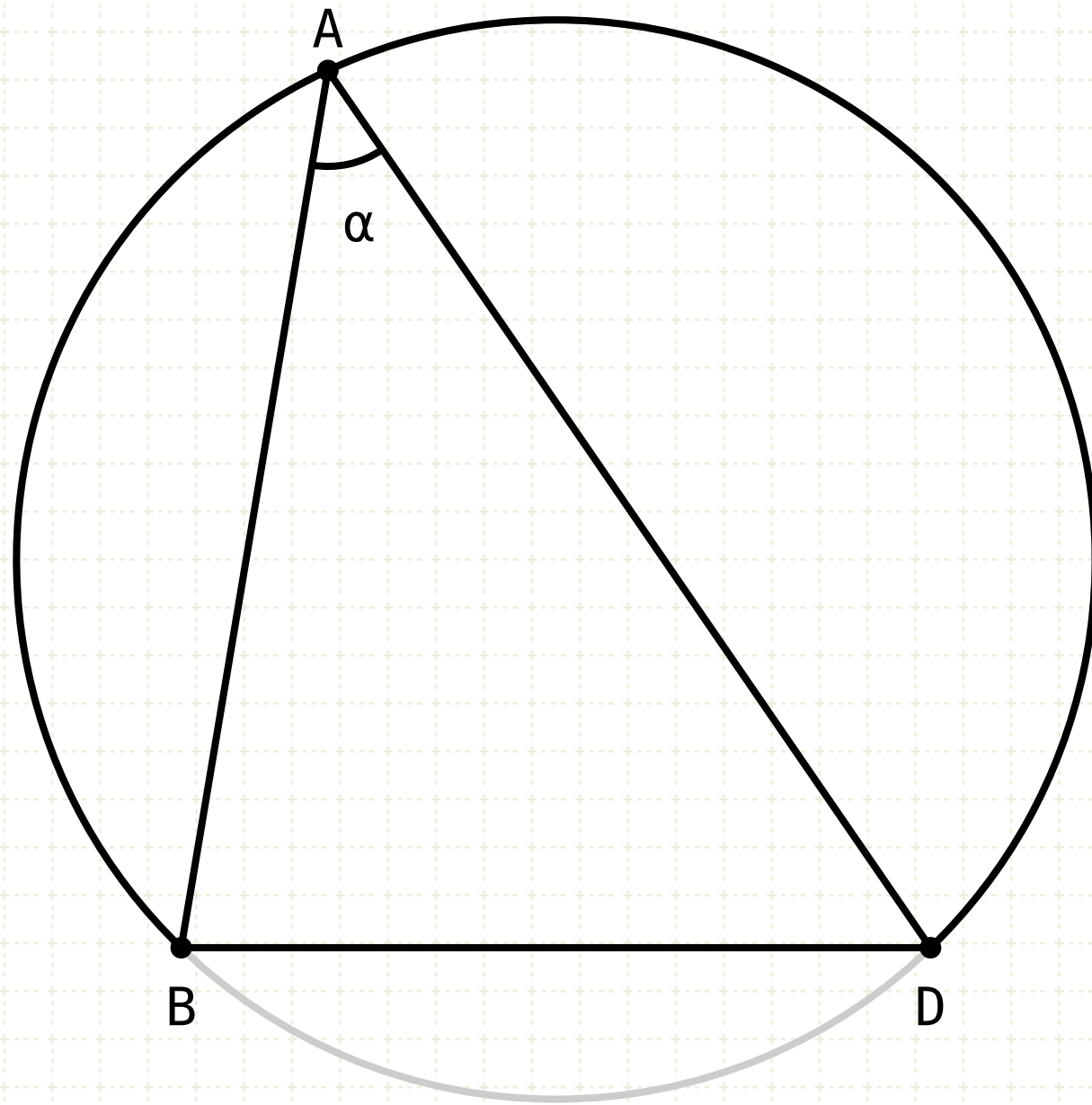
Definition - Angles in a Segment

An 'angle in a segment' is the angle which,

- when a point is taken on the circumference of the segment (point A) and

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Definition - Angles in a Segment

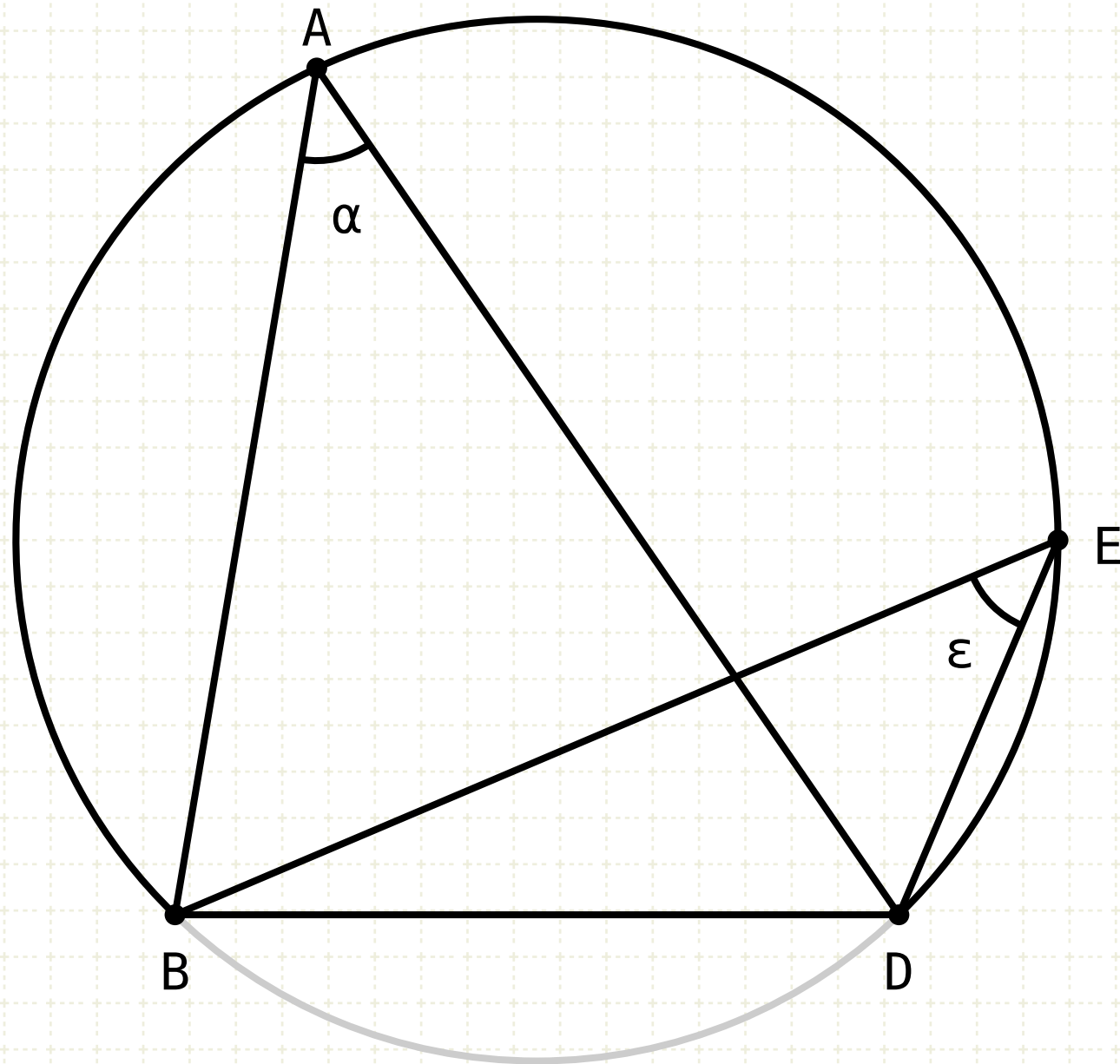
An 'angle in a segment' is the angle which,

- when a point is taken on the circumference of the segment (point A) and
- straight lines are joined from it to the extremities of the straight line (BD) which is the 'base of the segment',

is contained by the straight lines so joined (angle α)

Proposition 21 of Book III

In a circle the angles in the same segment are equal to one another.



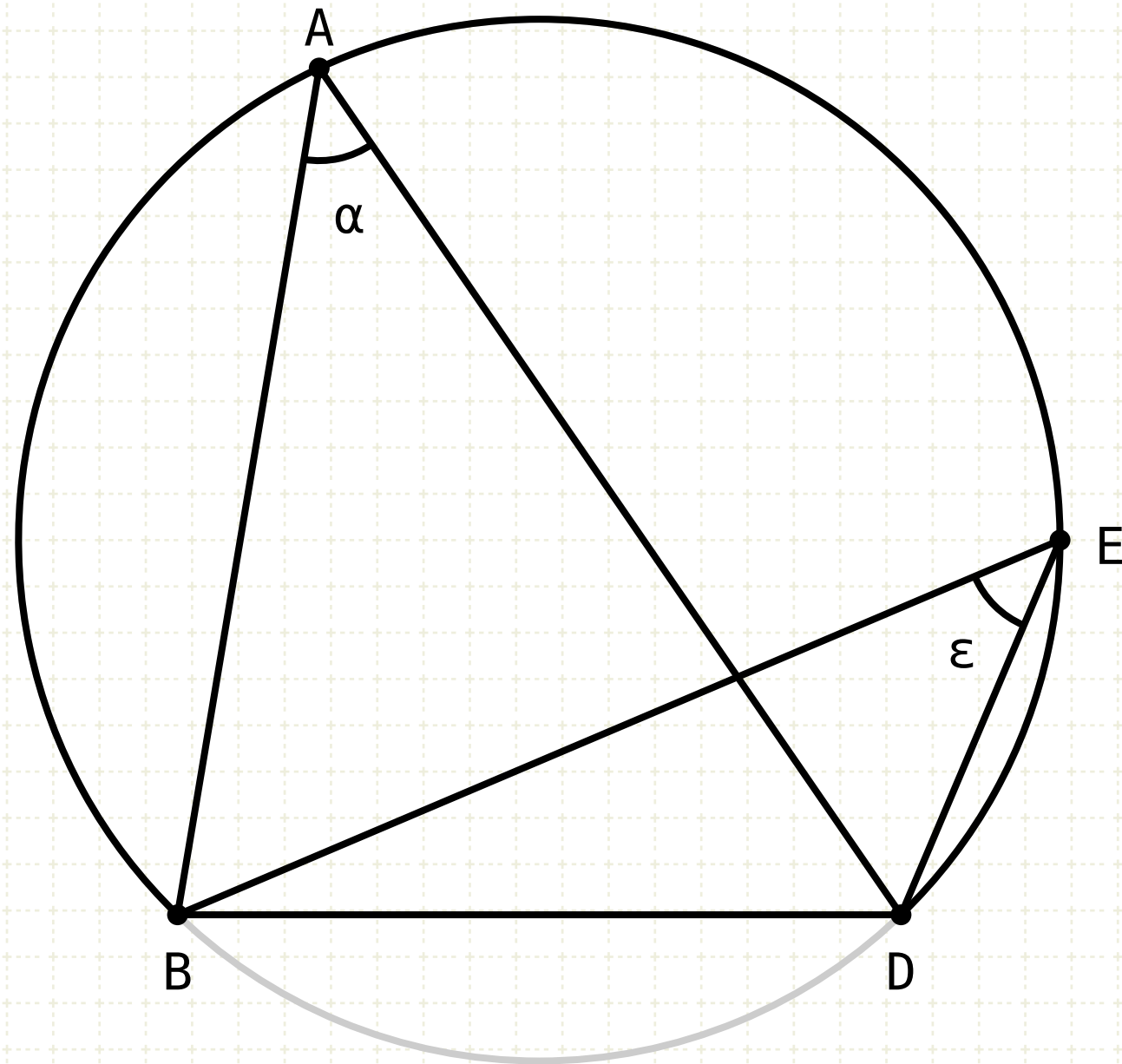
$$\epsilon = \alpha$$

In other words

If we have a circle segment BAD, any angle formed by lines from a given point on the circumference of the circle to the points B and D will be equal to any other angle formed in the same fashion

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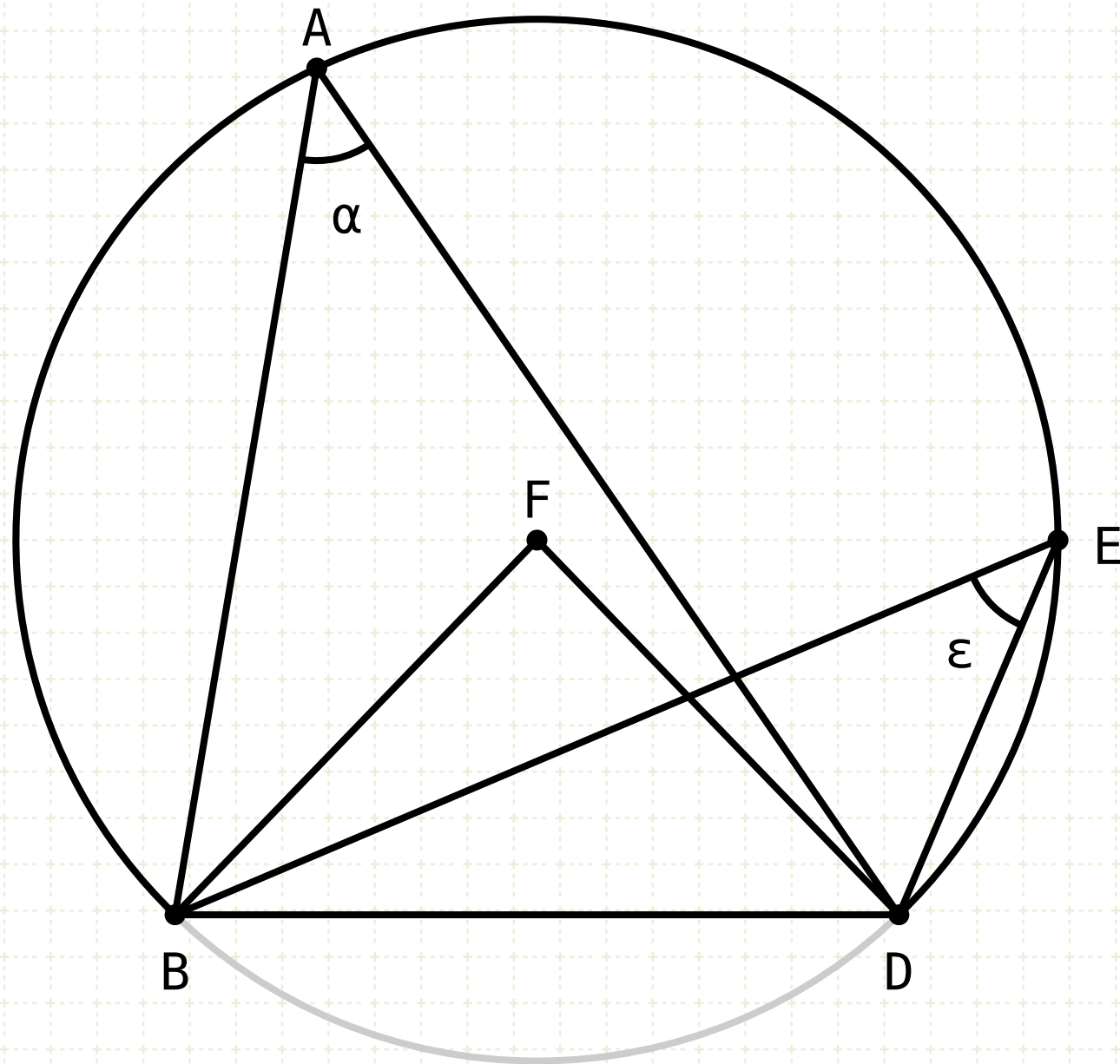
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Proof

Proposition 21 of Book III

In a circle the angles in the same segment are equal to one another.

F is the centre of the circle



In other words

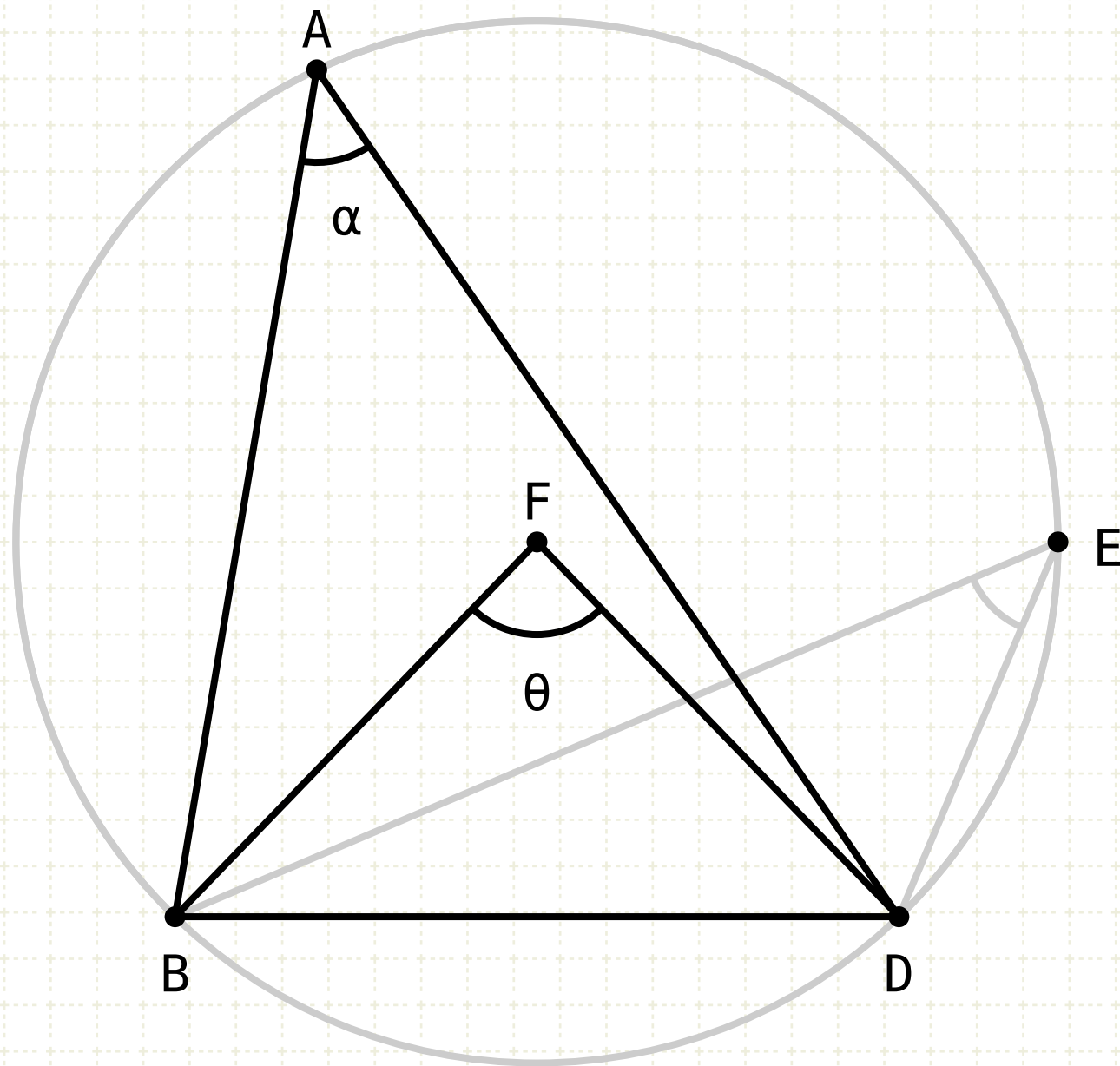
If we have a circle segment BAD, any angle formed by lines from a given point on the circumference of the circle to the points B and D will be equal to any other angle formed in the same fashion

Proof

Let F be the centre of the circle, and join lines FB and FD

Proposition 21 of Book III

In a circle the angles in the same segment are equal to one another.



F is the centre of the circle

$$\theta = 2\alpha$$

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If we have a circle segment BAD, any angle formed by lines from a given point on the circumference of the circle to the points B and D will be equal to any other angle formed in the same fashion

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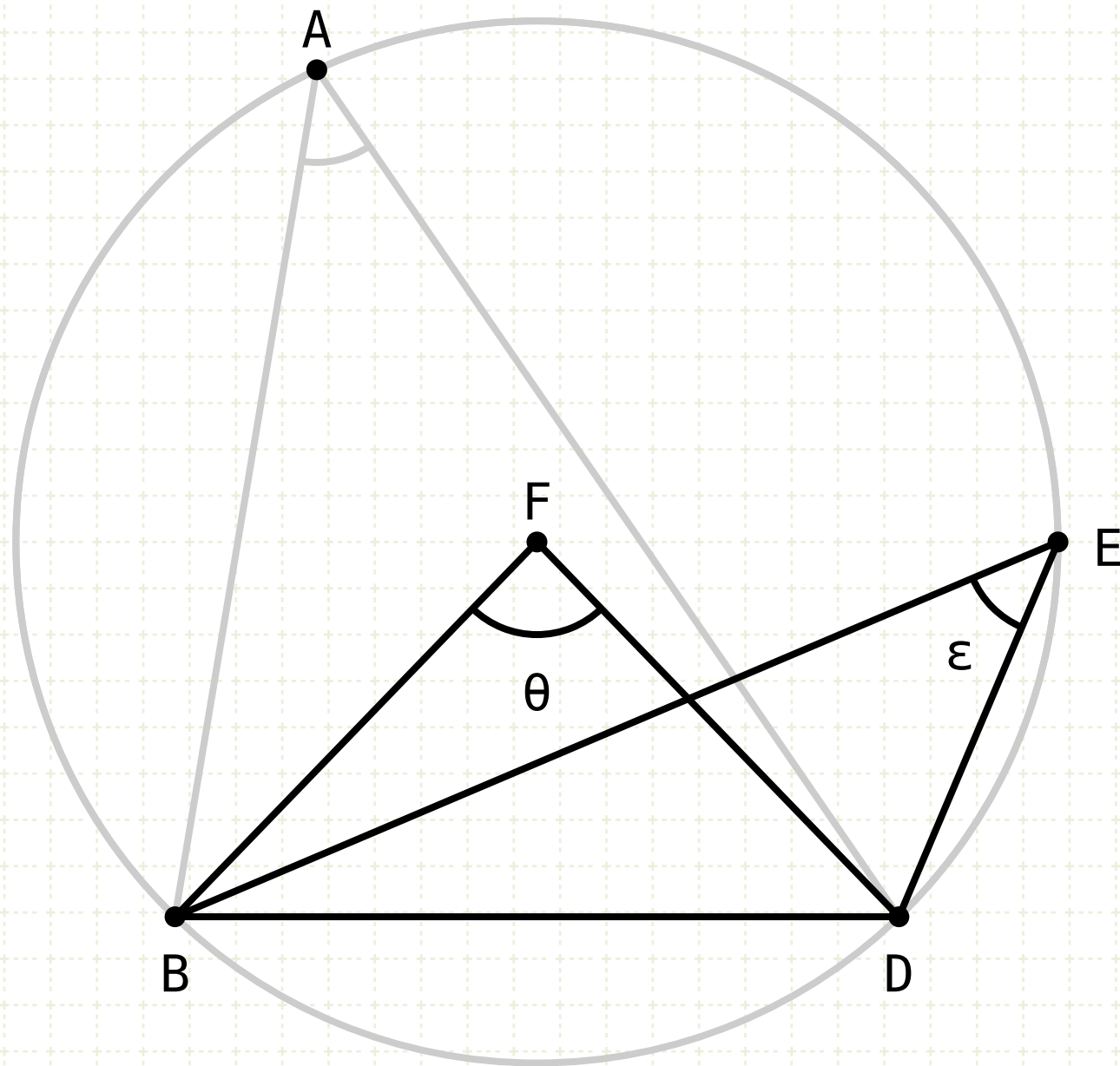
Let F be the centre of the circle, and join lines FB and FD

The angle BFD (θ) formed from the centre has the same base as BAD (α) formed from the circumference of the circle,

Therefore BFD equals twice BAD (III·20)

Proposition 21 of Book III

In a circle the angles in the same segment are equal to one another.



F is the centre of the circle

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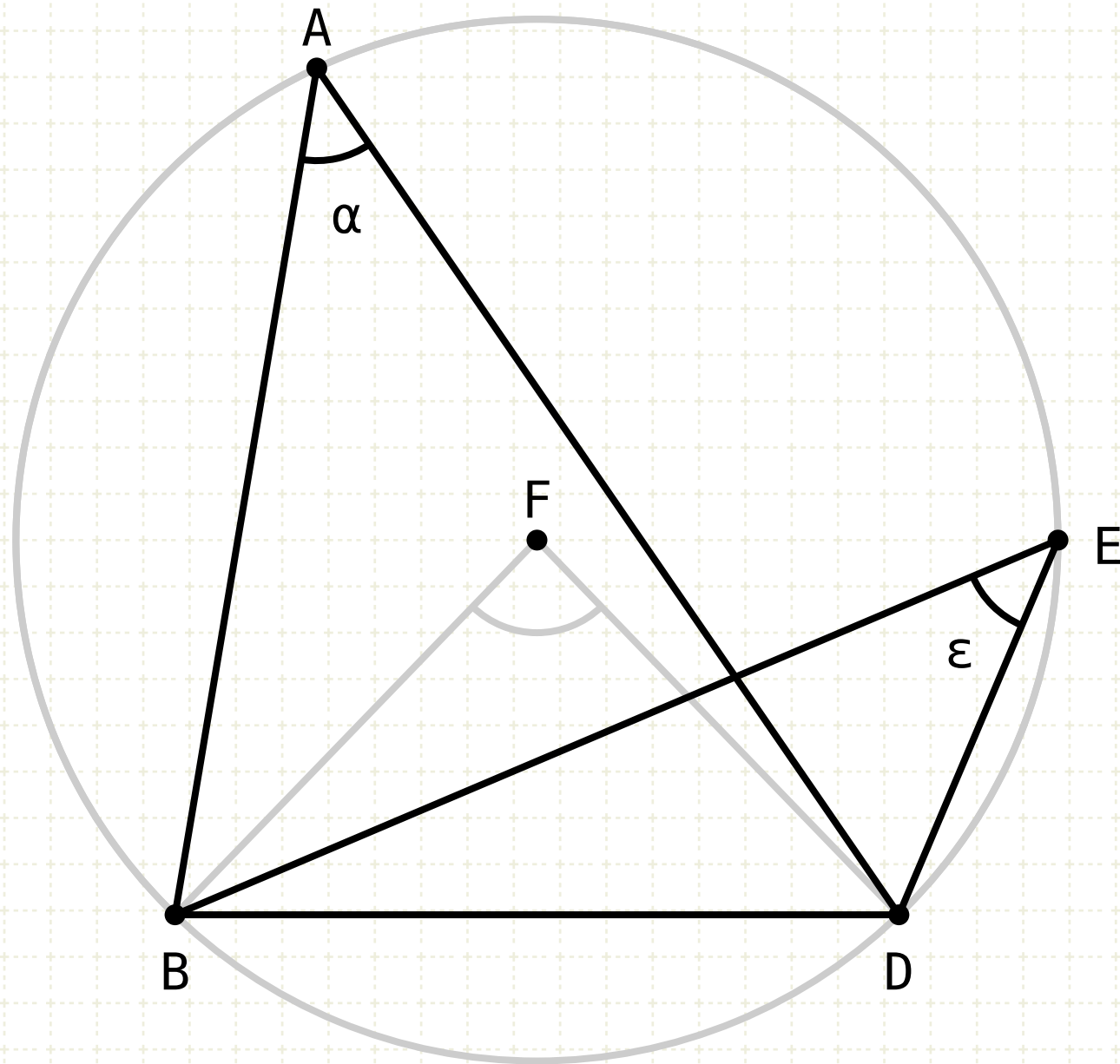
The angle BFD (θ) formed from the centre has the same base as BAD (α) formed from the circumference of the circle,

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Similarly, the angle BFD equals twice BED

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If we have a circle segment BAD, any angle formed by lines from a given point on the circumference of the circle to the points B and D will be equal to any other angle formed in the same fashion

Proof

Let F be the centre of the circle, and join lines FB and FD

The angle BFD (θ) formed from the centre has the same base as BAD (α) formed from the circumference of the circle,

Therefore BFD equals twice BAD (III·20)

Similarly, the angle BFD equals twice BED

Thus, BAD (α) and BED (ϵ) are equal

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