

Euclid's Elements

Book I

*If Euclid did not kindle your youthful enthusiasm, you
were not born to be a scientific thinker.*

Albert Einstein



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Proposition 38 of Book I

Triangles which are on equal bases and in the same parallels equal one another.



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In other words

Triangles with equal base and height have the same area



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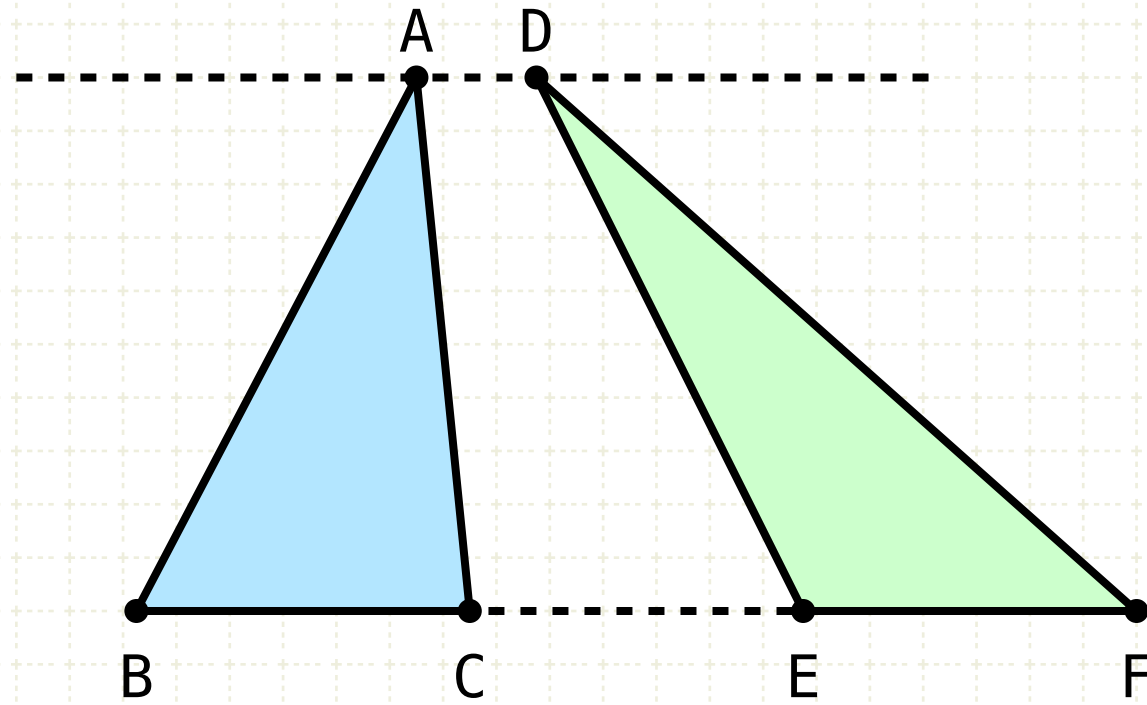
In other words

Given two parallel lines



Proposition 38 of Book I

Triangles which are on equal bases and in the same parallels equal one another.



$AD \parallel BF$
 $BC = EF$

In other words

Triangles with equal base and height have the same area

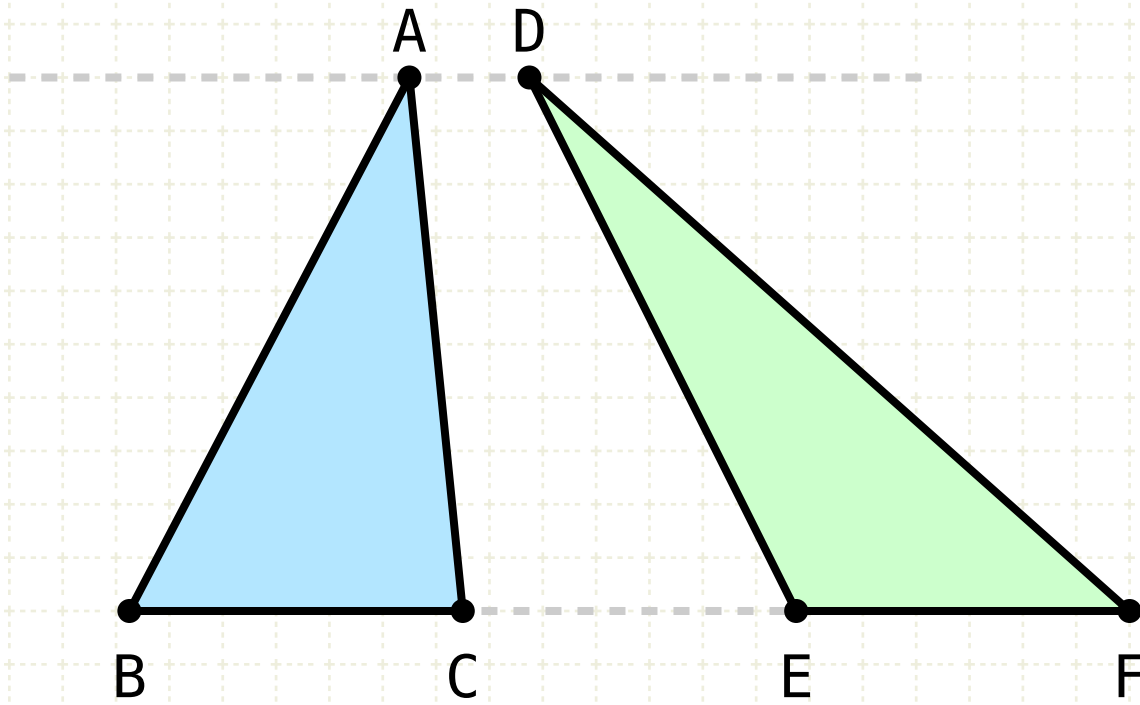
In other words

Given two parallel lines

Let ABC and DEF be triangles with equal bases, and on the same parallels

Proposition 38 of Book I

Triangles which are on equal bases and in the same parallels equal one another.



$$\begin{aligned} AD &\parallel BF \\ BC &= EF \\ \Delta ABC &= \Delta DEF \end{aligned}$$

In other words

Triangles with equal base and height have the same area

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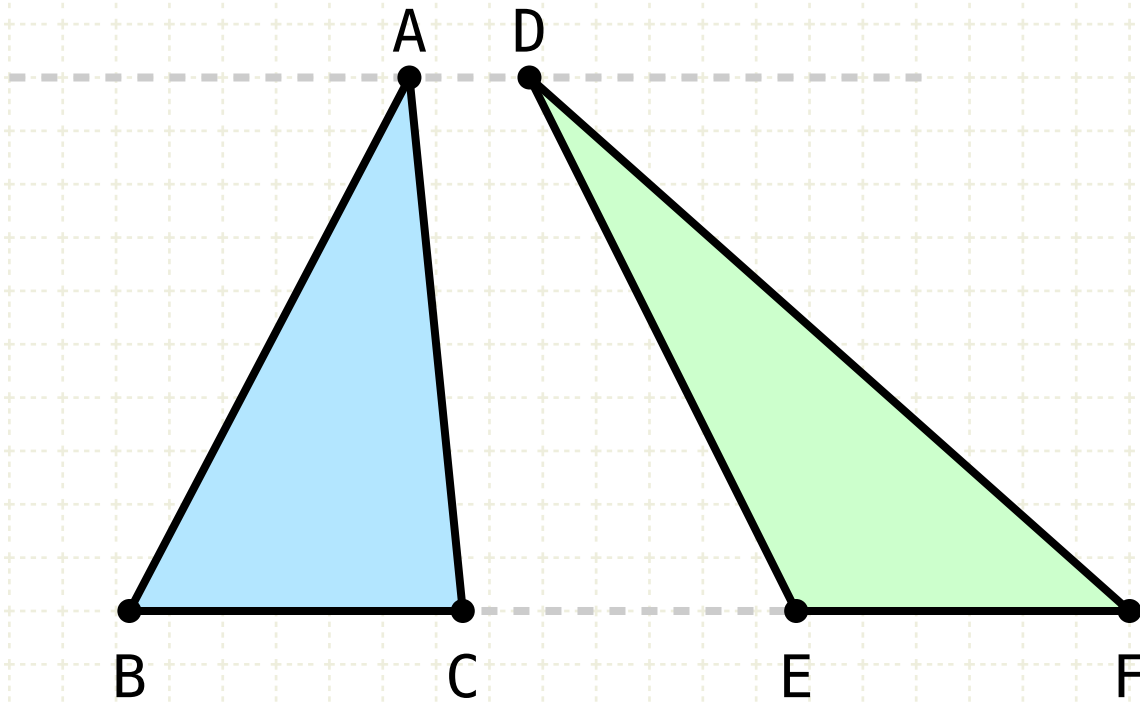
Given two parallel lines

Let ABC and DEF be triangles with equal bases, and on the same parallels

The area ABC is equal to DEF

Proposition 38 of Book I

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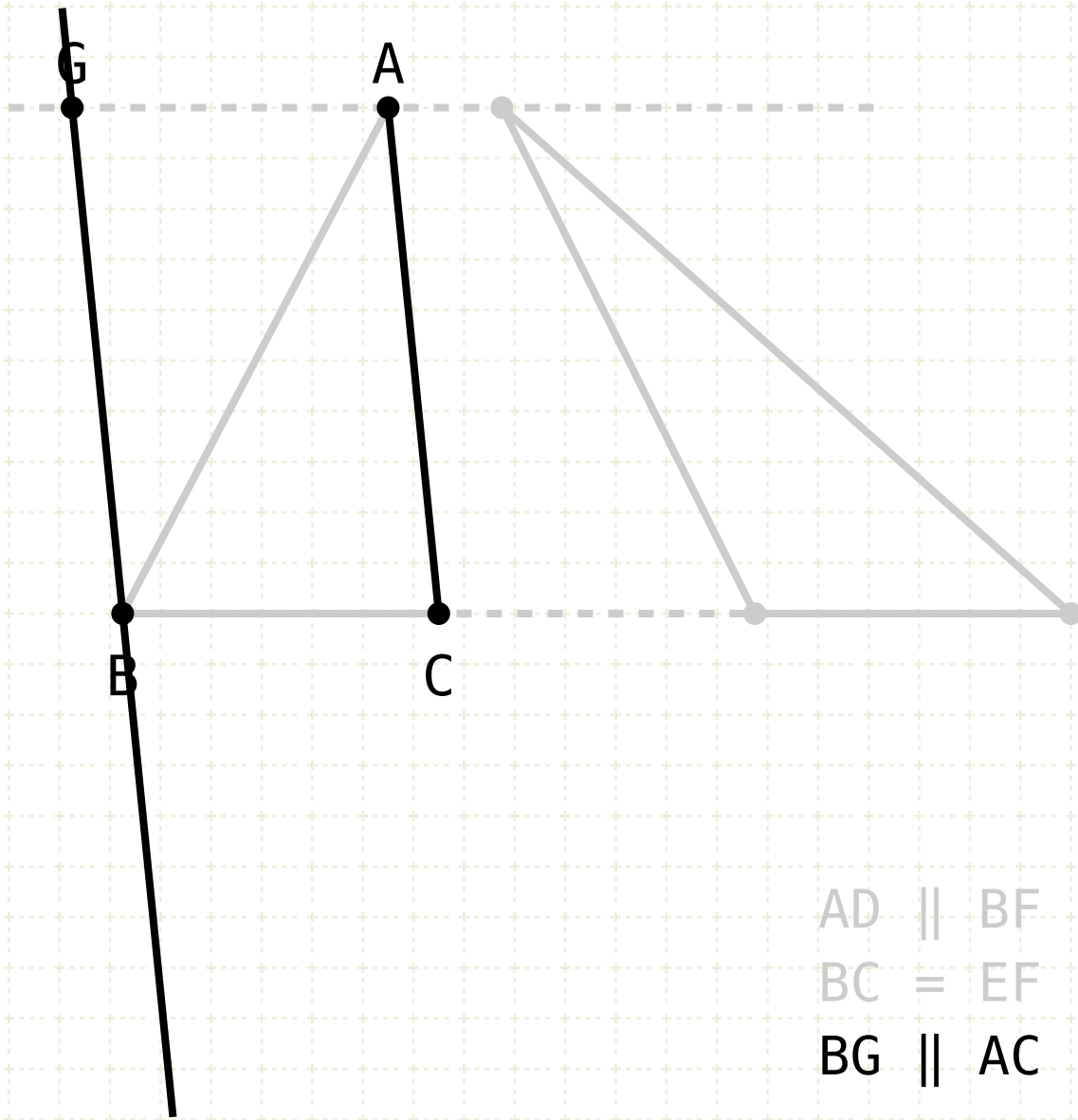
The area ABC is equal to DEF

Proof



Proposition 38 of Book I

Triangles which are on equal bases and in the same parallels equal one another.



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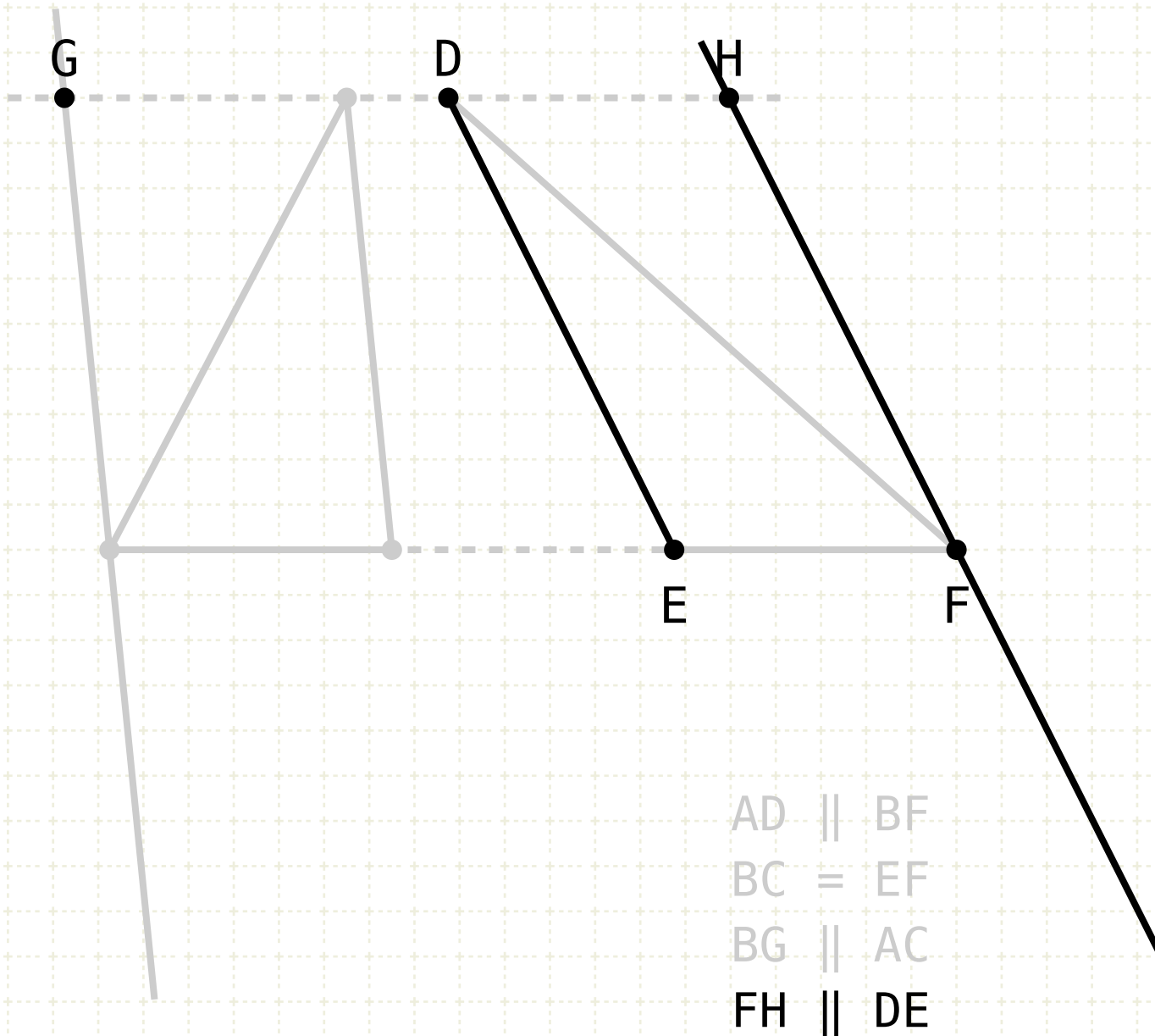
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Draw BG parallel to AC (I-31)

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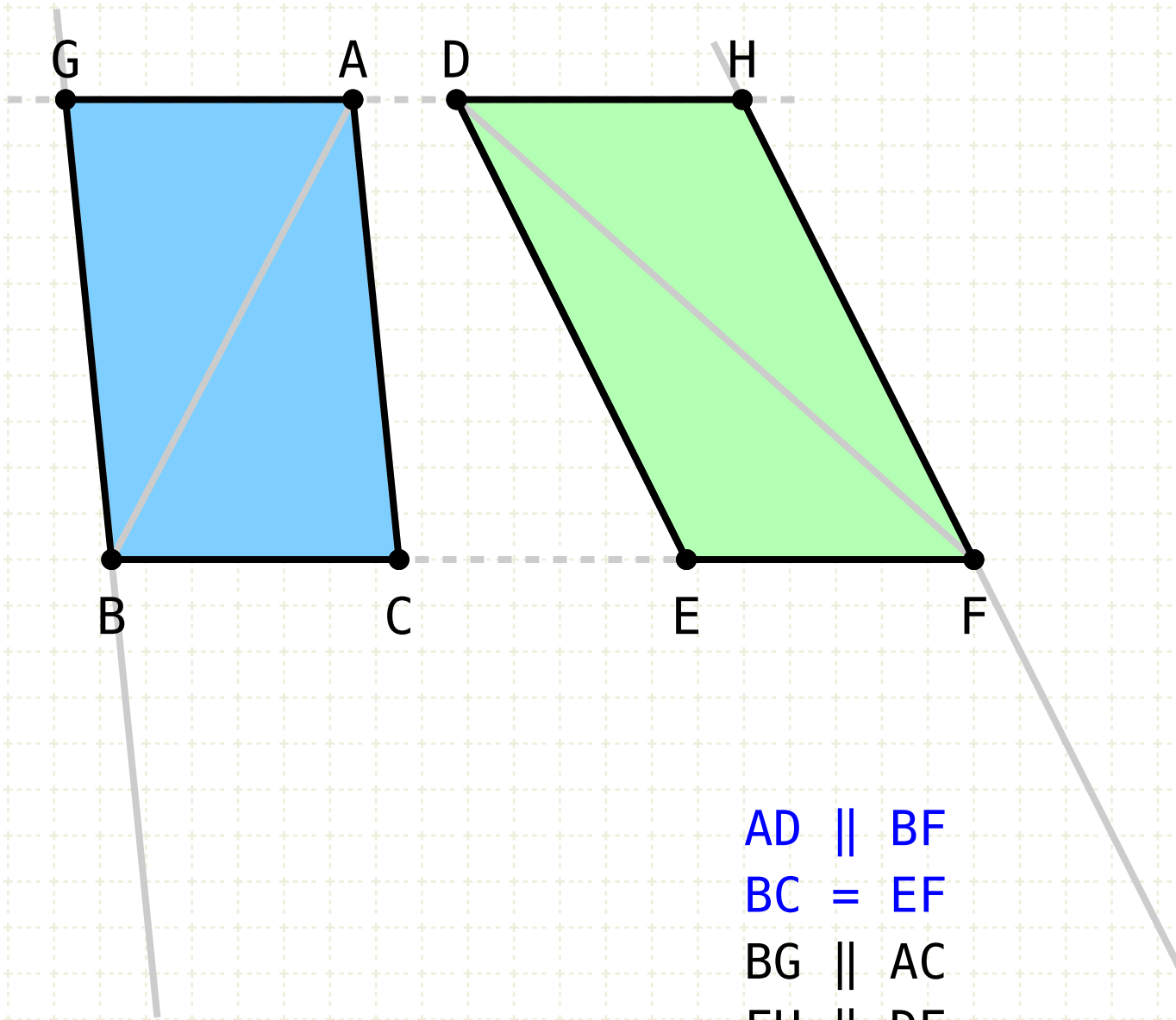
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Draw BG parallel to AC (I-31)

Draw FH parallel to DE (I-31)

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$AD \parallel BF$
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 $GBCA = DEFH$

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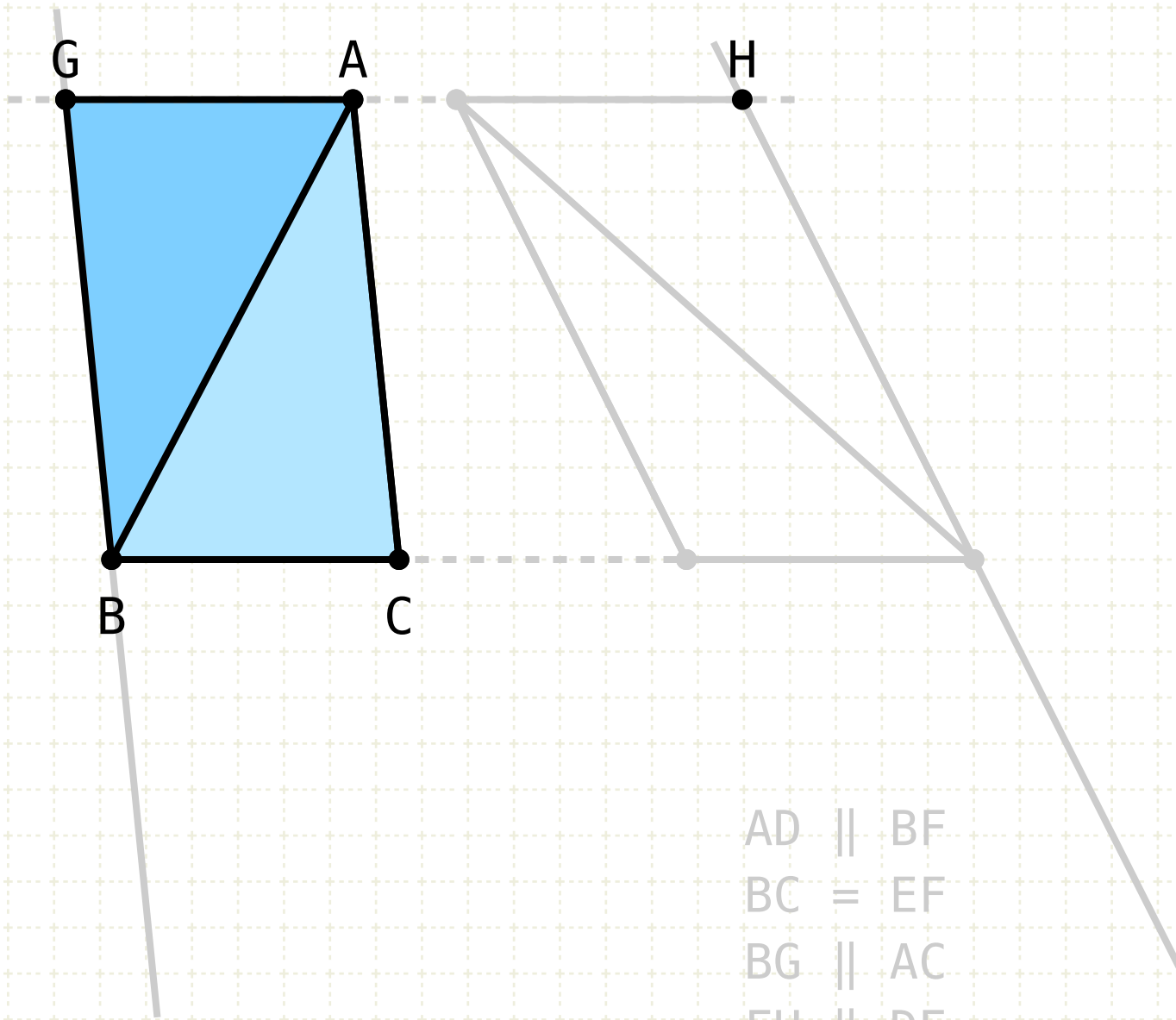
Draw BG parallel to AC (I-31)

Draw FH parallel to DE (I-31)

The parallelograms GBCA and DEFH are equal since they have equal bases and are on two parallel lines. (I-36)

Proposition 38 of Book I

Triangles which are on equal bases and in the same parallels equal one another.



$$\begin{aligned} AD &\parallel BF \\ BC &= EF \\ BG &\parallel AC \\ FH &\parallel DE \\ GBCA &= DEFH \\ \Delta ABC &= \frac{1}{2} GBCA \end{aligned}$$

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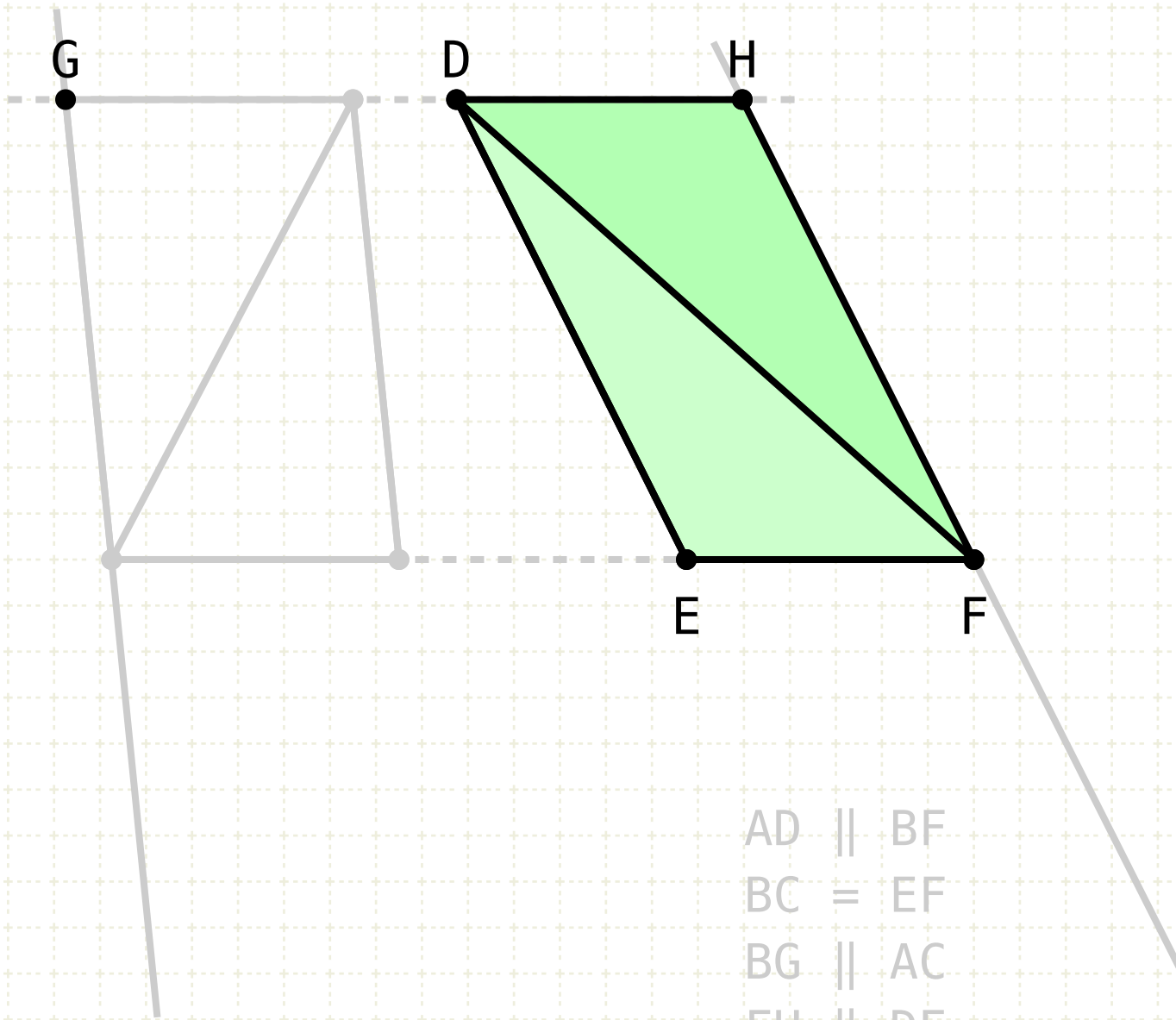
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The parallelograms $GBCA$ and $DEFH$ are equal since they have equal bases and are on two parallel lines. (I-36)

The triangle ABC is half the area of $GBCA$ since line AB bisects the parallelogram (I-34)

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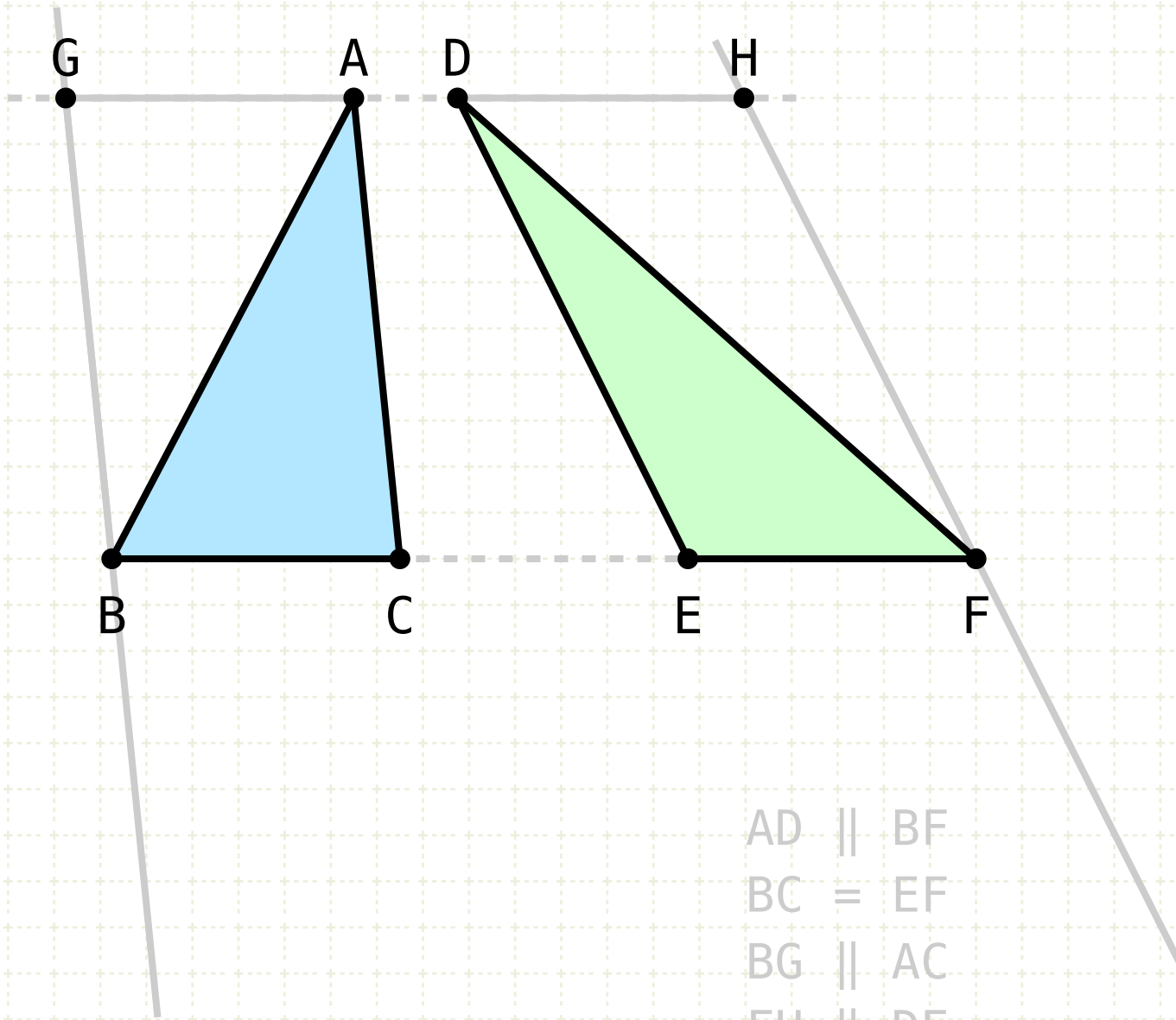
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The triangle DEF is half the area of DEFH since line DE bisects the parallelogram (I·34)



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$BC = EF$

$BG \parallel AC$

$FH \parallel DE$

$GBCA = DEFH$

$\triangle ABC = \frac{1}{2} GBCA$

$\triangle DEF = \frac{1}{2} DEFH$

$\triangle ABC = \triangle DEF$

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Triangles with equal base and height have the same area

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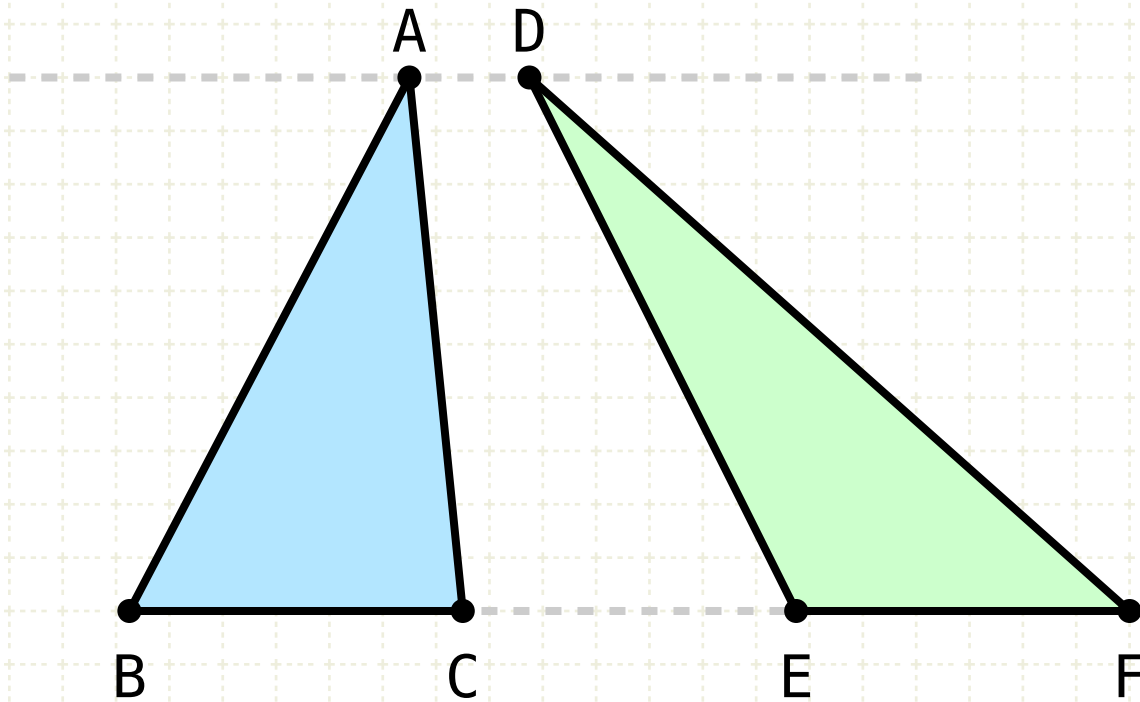
The triangle DEF is half the area of $DEFH$ since line DE bisects the parallelogram (I-34)

Half of equals are equal, so ABC equals DEF



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