

# Euclid's Elements

## Book III



*A circle is a round straight line with a hole in the middle.*

**Mark Twain**

quoting a schoolchild in "-English as She Is Taught-"

*If people stand in a circle long enough, they'll eventually begin to dance.*

**George Carlin, Napalm and Silly Putty (2001)**



# Table of Contents, Chapter 3

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2	A chord of a circle always lies inside the circle	10	A circle does not cut a circle at more points than two	18	If line touches a circle, then it is perpendicular to the diameter that touches that point
3	A line through the centre of a circle bisects a chord, and vice versa	11	Point of contact between two internal circles, and their centres, are collinear	19	If line touches a circle, then the centre of the circle lies on a line perpendicular to the original
4	A line not through the centre of a circle does not bisect a chord	12	Point of contact between two external circles, and their centres, are collinear	20	The angle at the centre of a circle is twice that from an angle from the circumference
5	If two circles cut one another, they will not have the same center	13	A circle does not touch a circle at more points than one, whether it touch it internally or externally.	21	In a circle the angles in the same segment are equal to one another
6	If two circles touch one another, they will not have the same center	14	In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another.	22	The opposite angles of quadrilaterals in circles are equal to two right angles
7	Consider two lines from a point inside a circle to the edge, the longer one will be the one closest to the longest part of the diameter passing through the original point	15	The longest line in a circle is its diameter, shorter the farther away from the diameter	23	<b>On the same straight line there cannot be constructed two similar and unequal segments of circles on the same side</b>
8	Consider two lines from a point outside a circle to the edge, the line closest to the centre will be longer on the concave side and shorter on the convex side	16	A line on the circle, perpendicular to the diameter, lies outside the circle	24	Similar segments of circles on equal straight lines are equal to one another



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| 26 | In equal circles equal angles stand on equal circumferences   | 35 | If two circle chords intersect, the segments on one multiplied together equals the segments of the other multiplied together |
| 27 | In equal circles angles standing on equal circumferences are equal to one another                               |    |  |
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| 32 | The angle between a tangent and a straight line cutting a circle is equal to the angle in the alternate segment |    |  |
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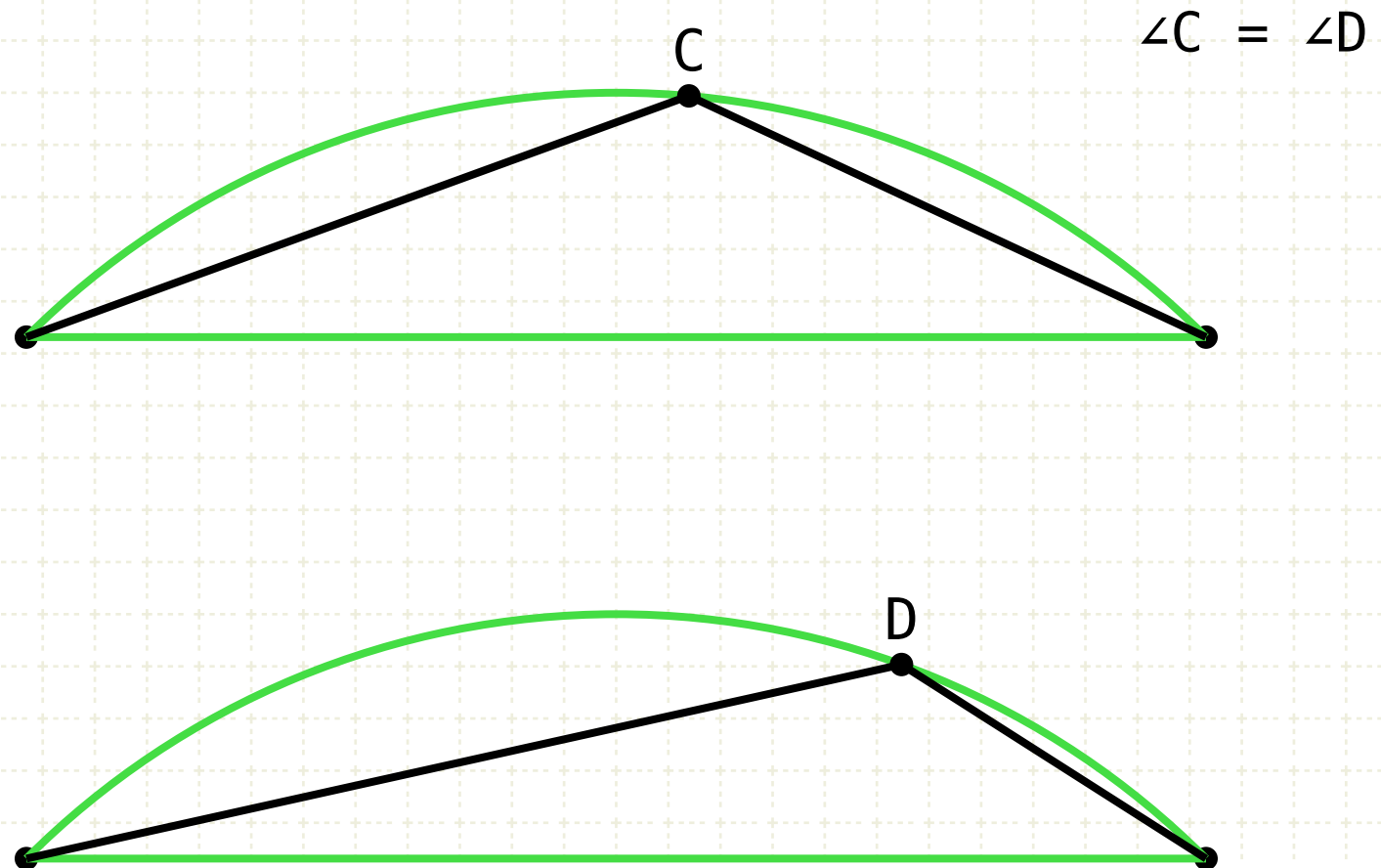
# Proposition 24 of Book III

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## Definition - Similar segments of circles

'Similar segments of circles' are those which admit equal angles, or in which the angles are equal to one another

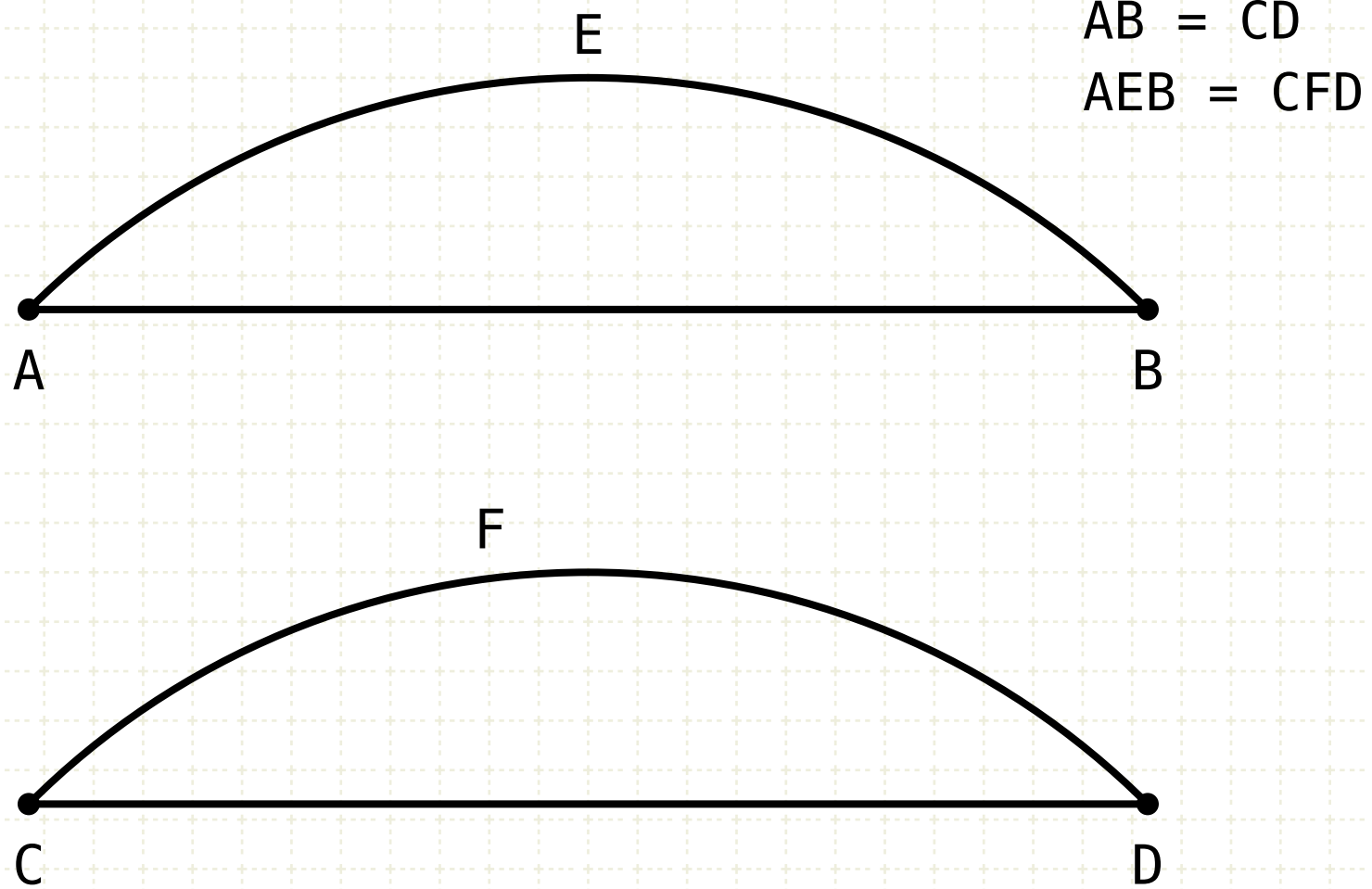


# Proposition 24 of Book III

Similar segments of circles on equal straight lines are equal to one another.

## In other words

If two similar segments are placed on one another, they will coincide with one another



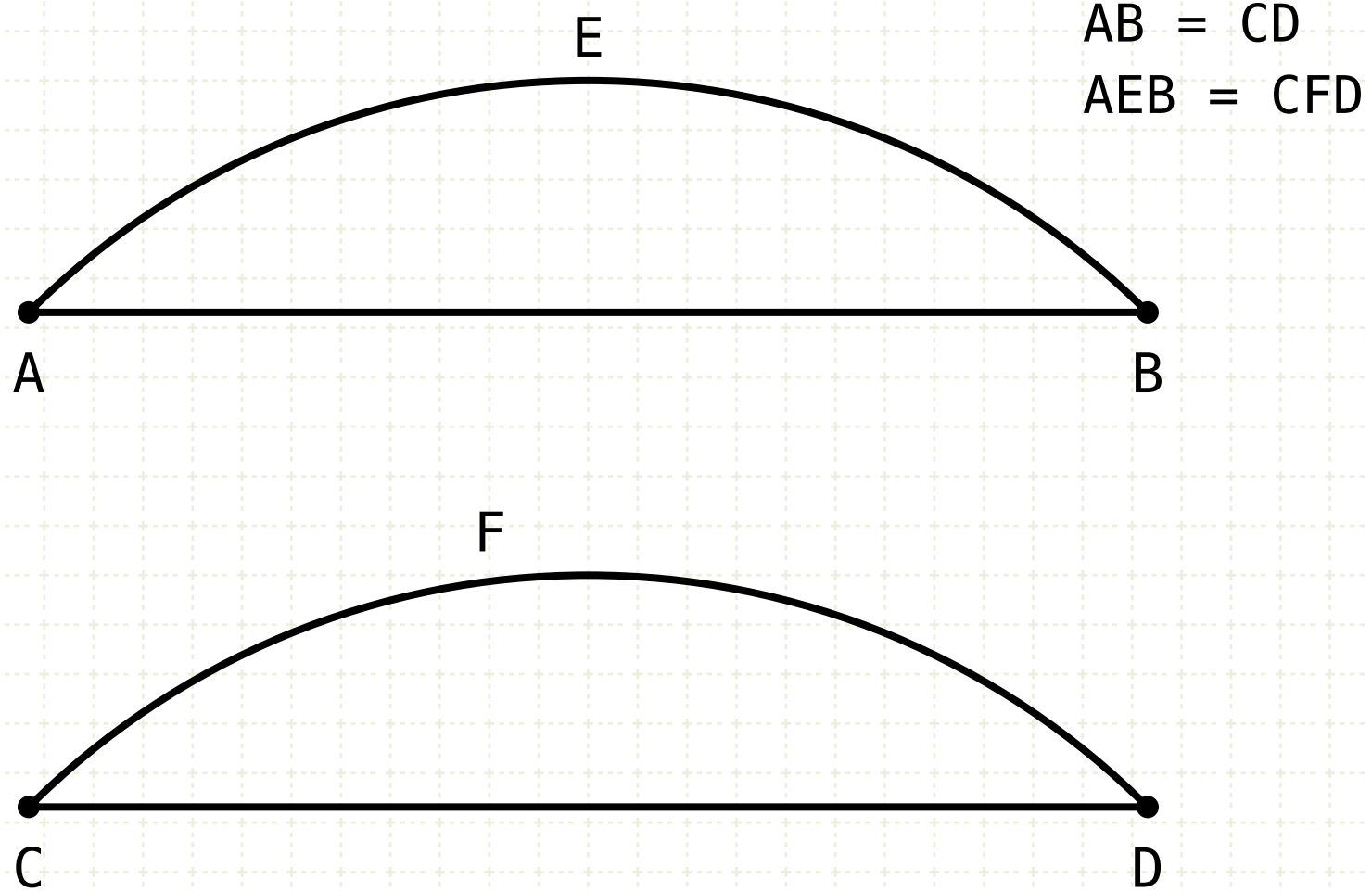
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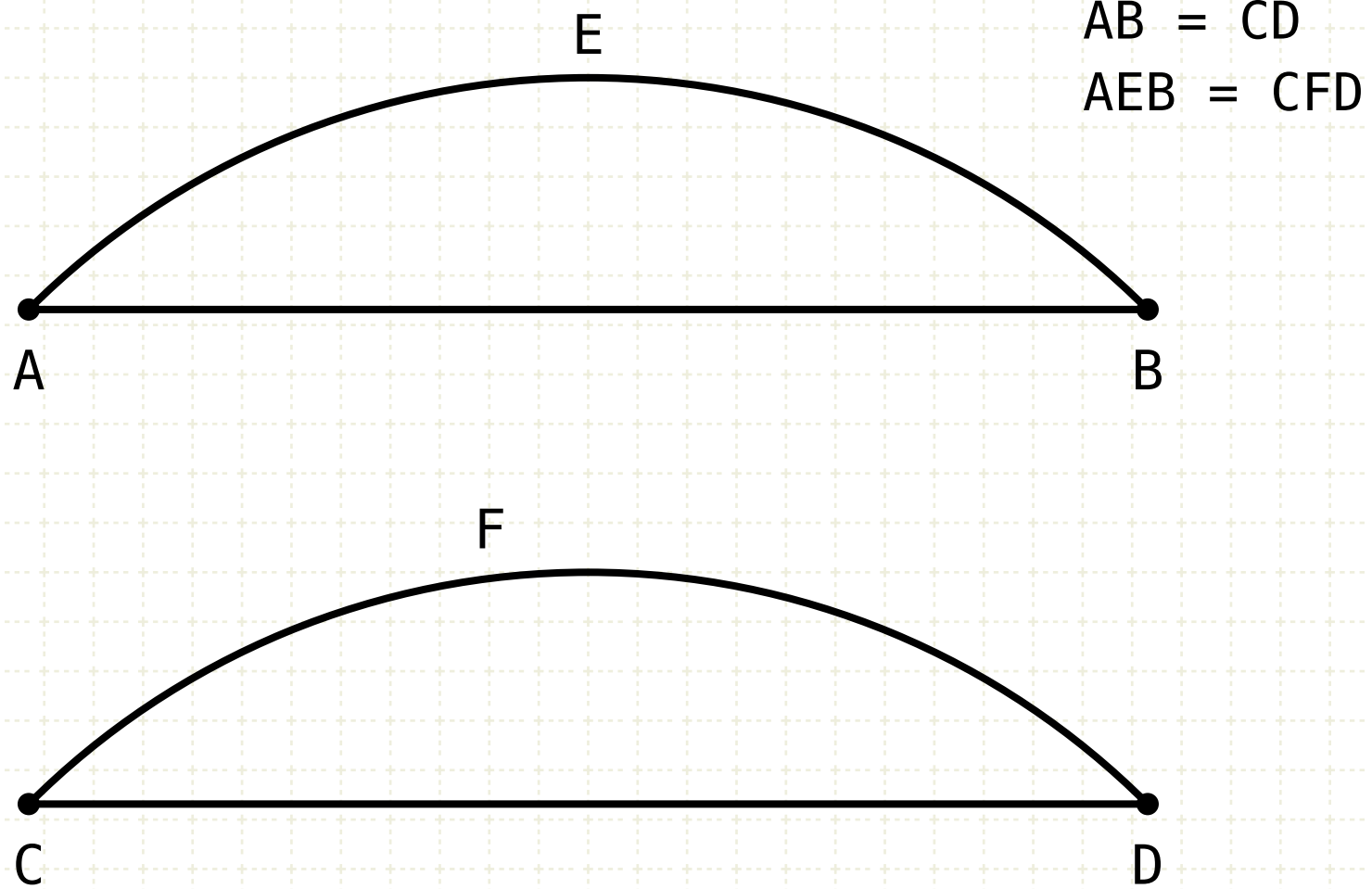
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## Proof by Contradiction



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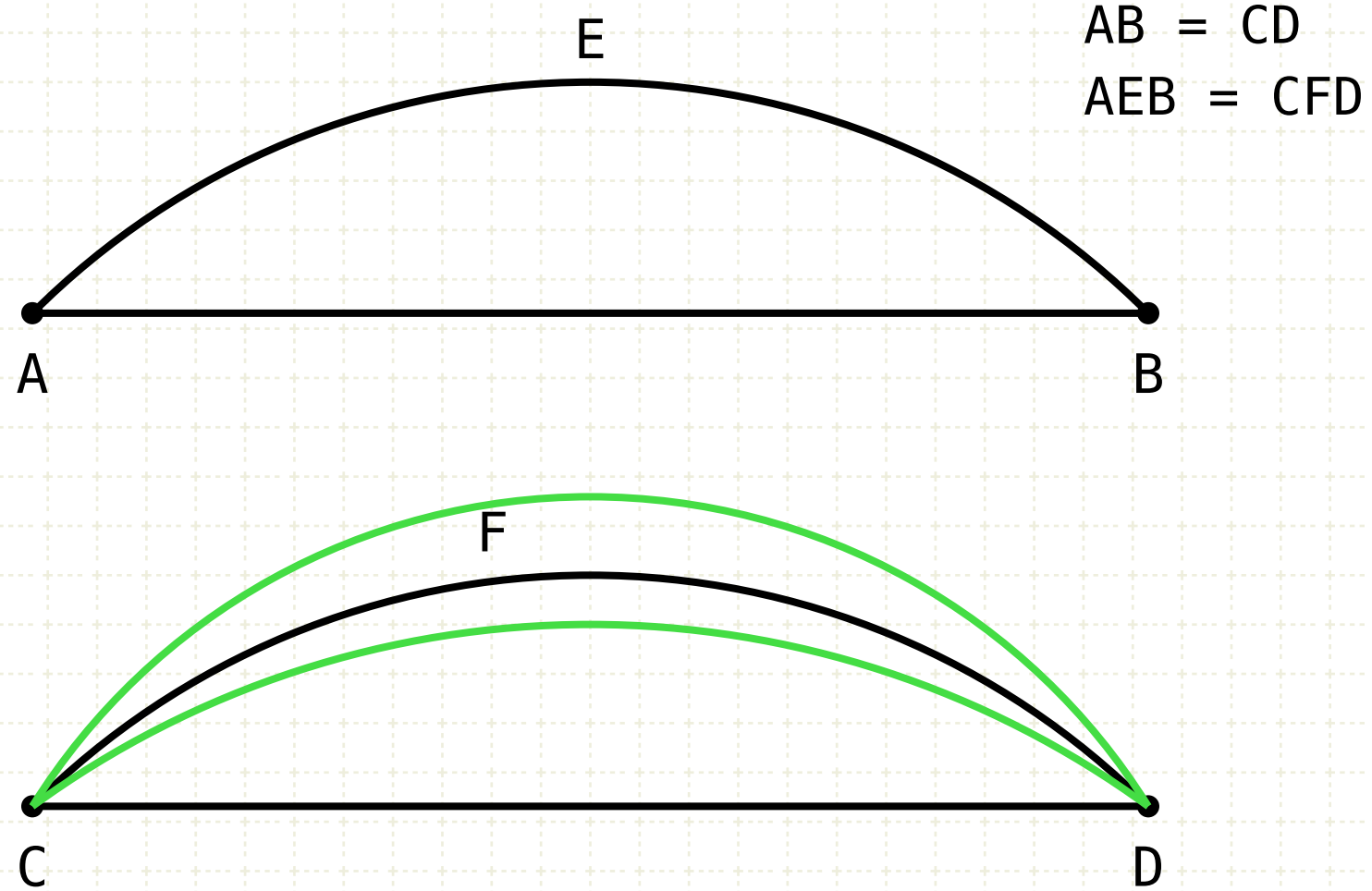
Assume that when the segment AEB is placed on CED, the circumference AB will not coincide with the circumference CD

Then it will either



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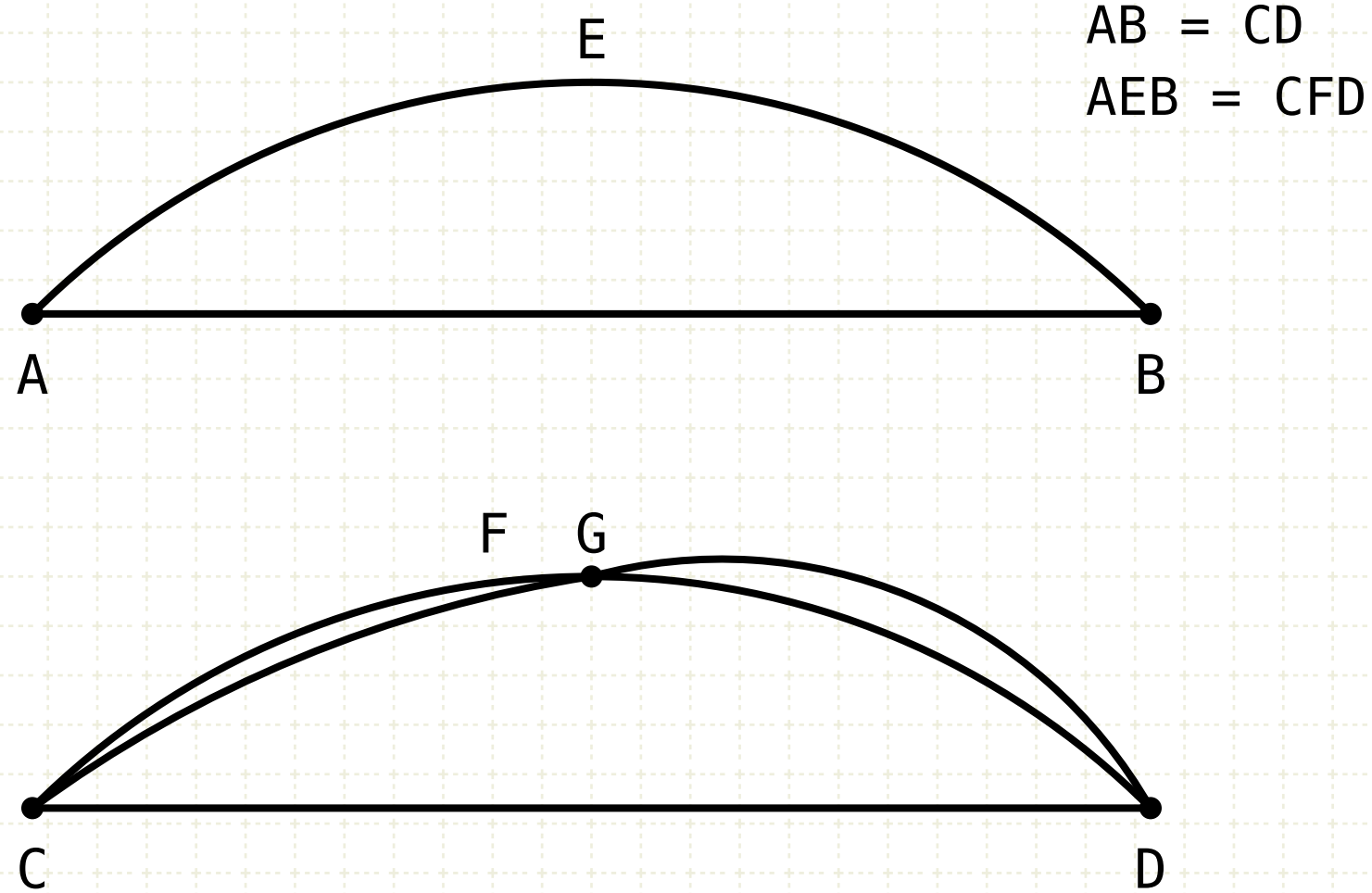
Assume that when the segment AEB is placed on CED, the circumference AB will not coincide with the circumference CD

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- fall on the inside or the outside  
which is impossible (III·23)

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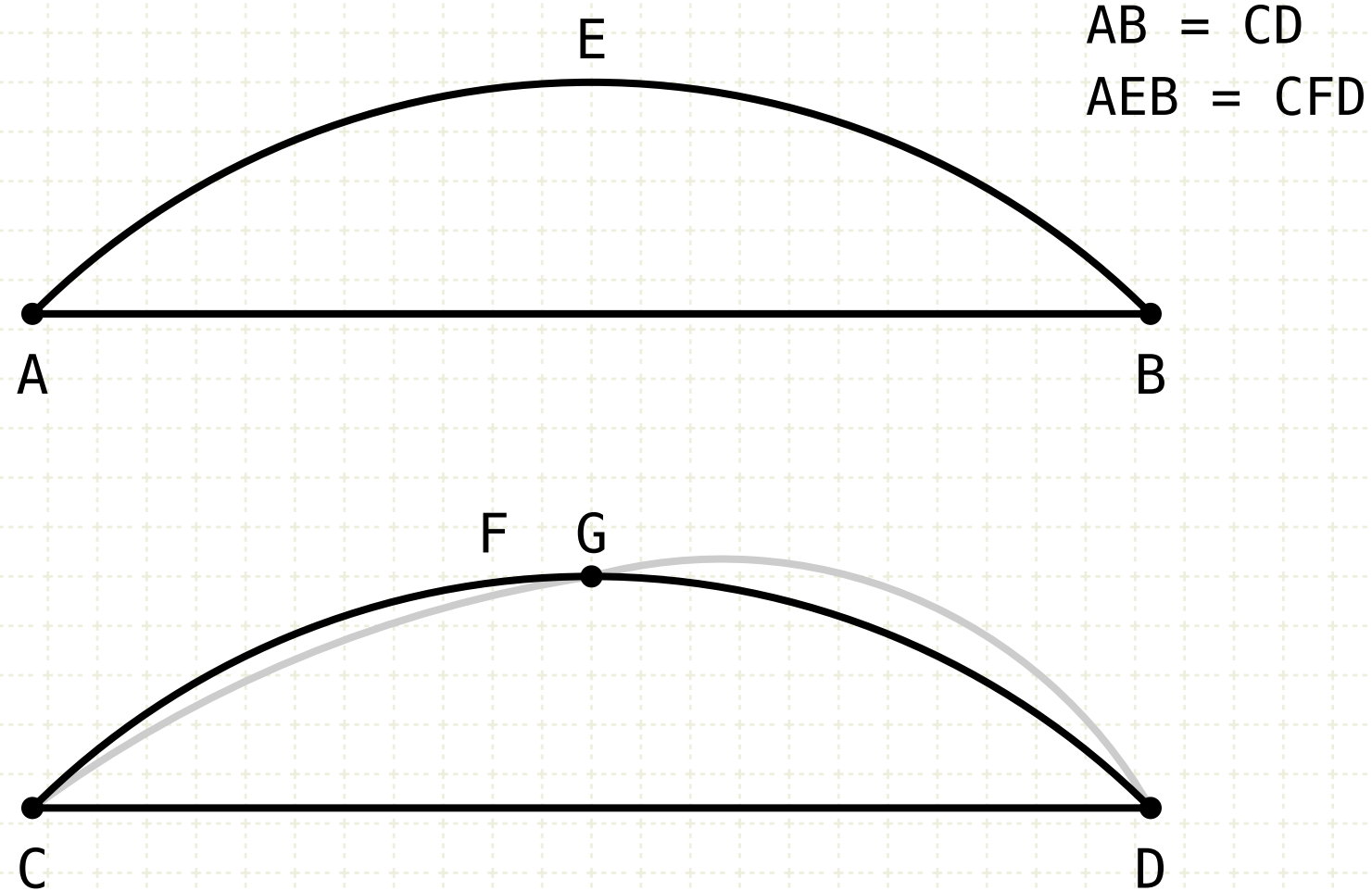
- fall on the inside or the outside  
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or

- fall awry, such as CGD  
which is impossible, because then the circles would cut each other in more than two places, which is impossible (III·10)

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Therefore, segment AEB would coincide with CFD, and will be equal to it

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