

Euclid's Elements

Book I

*If Euclid did not kindle your youthful enthusiasm, you
were not born to be a scientific thinker.*

Albert Einstein



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Proposition 39 of Book I

Equal triangles which are on the same base and on the same side are also in the same parallels.

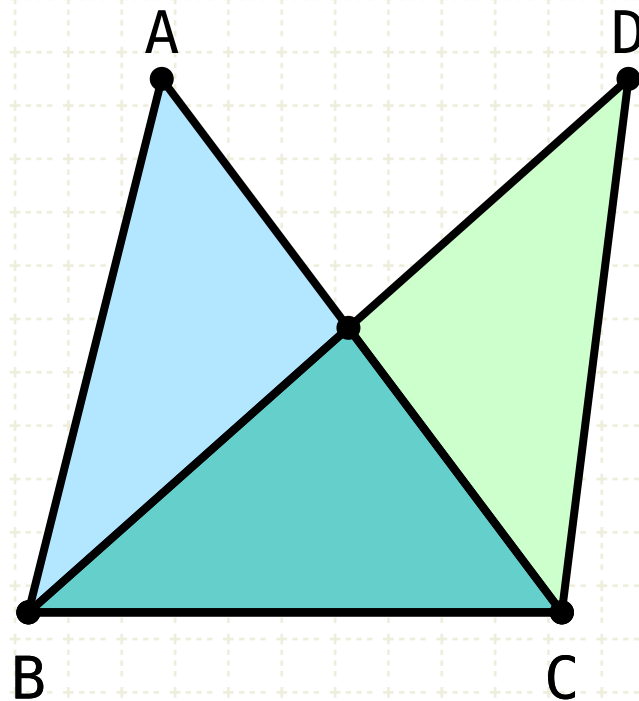


Proposition 39 of Book I

Equal triangles which are on the same base and on the same side are also in the same parallels.

In other words

Let ABC and DBC be triangles with the same base, and equal area



$$\triangle ABC = \triangle DBC$$

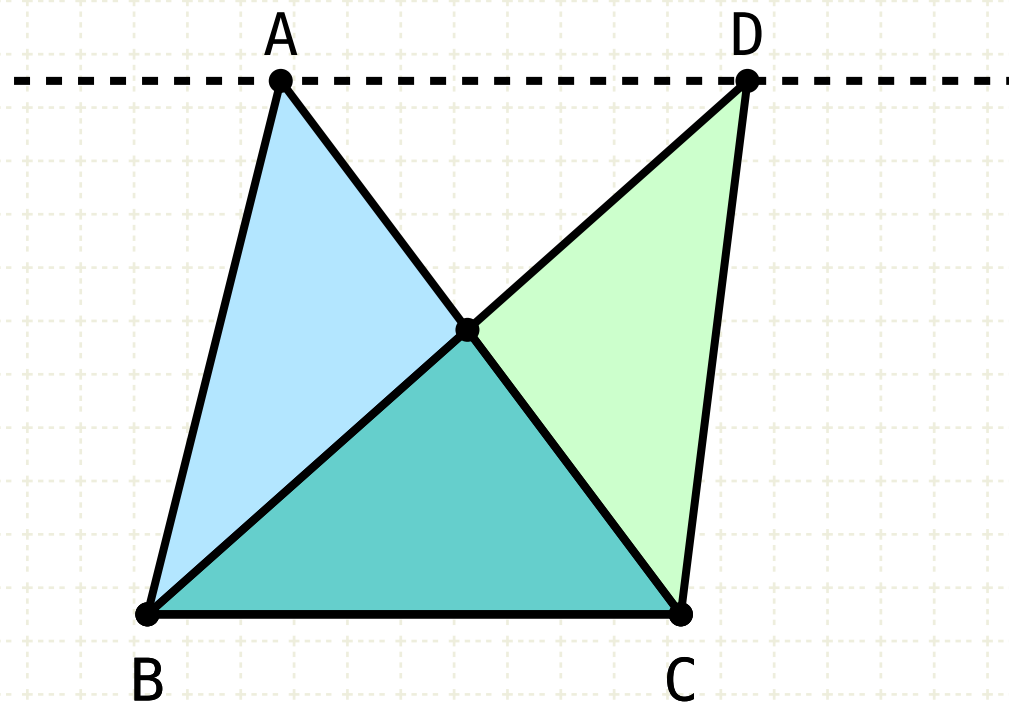
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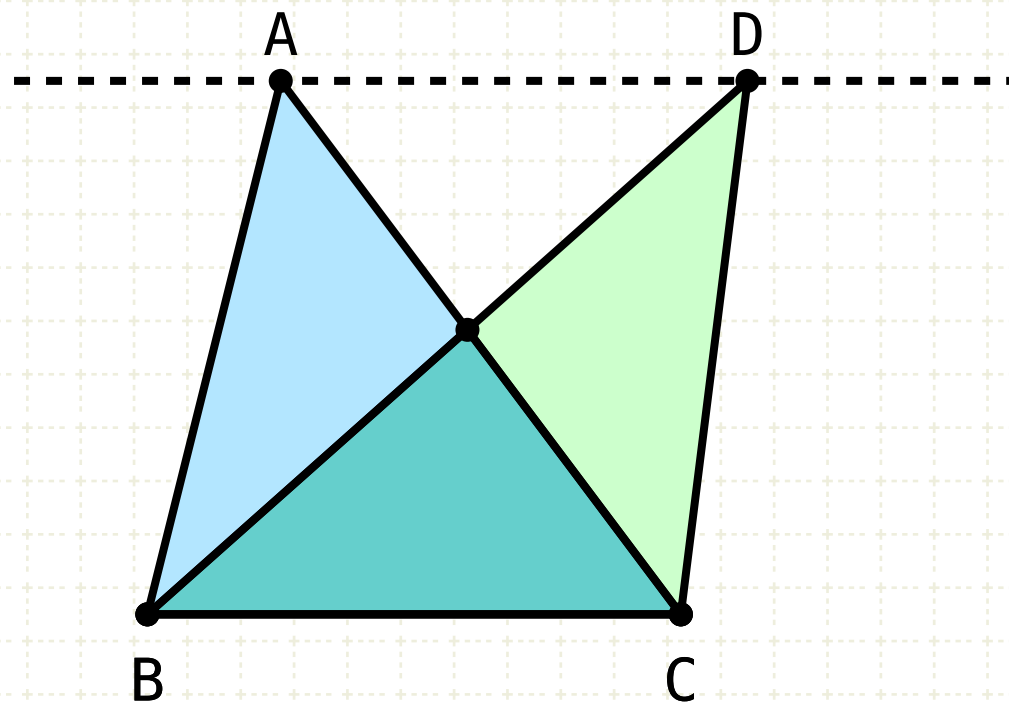
The lines AD and BC are parallel



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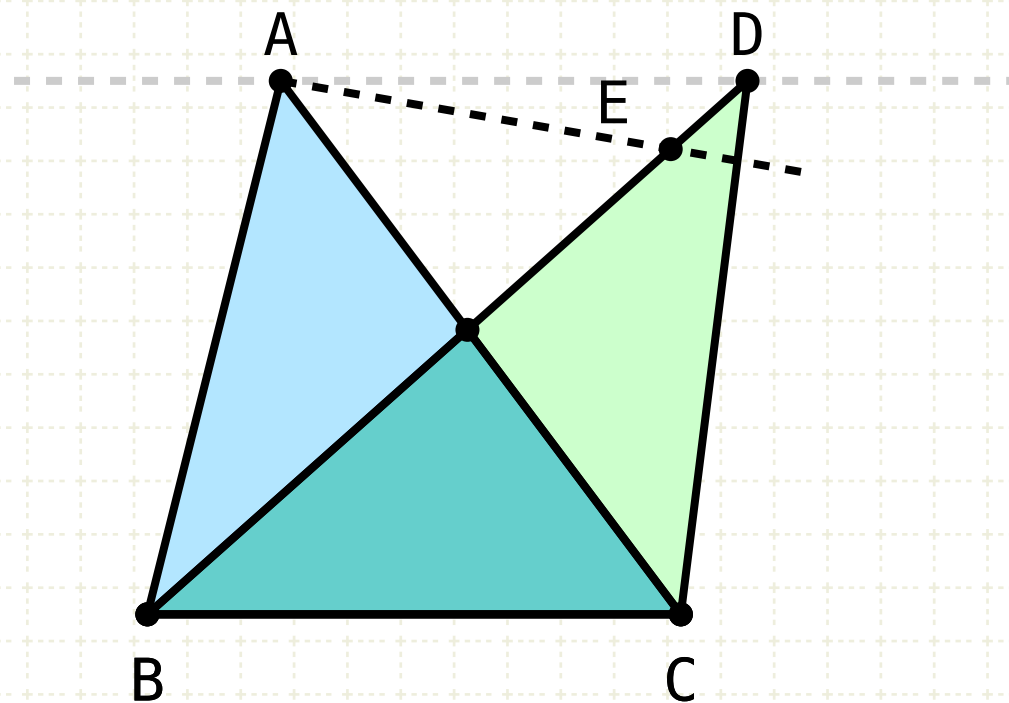
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Proof by Contradiction

Proposition 39 of Book I

Equal triangles which are on the same base and on the same side are also in the same parallels.



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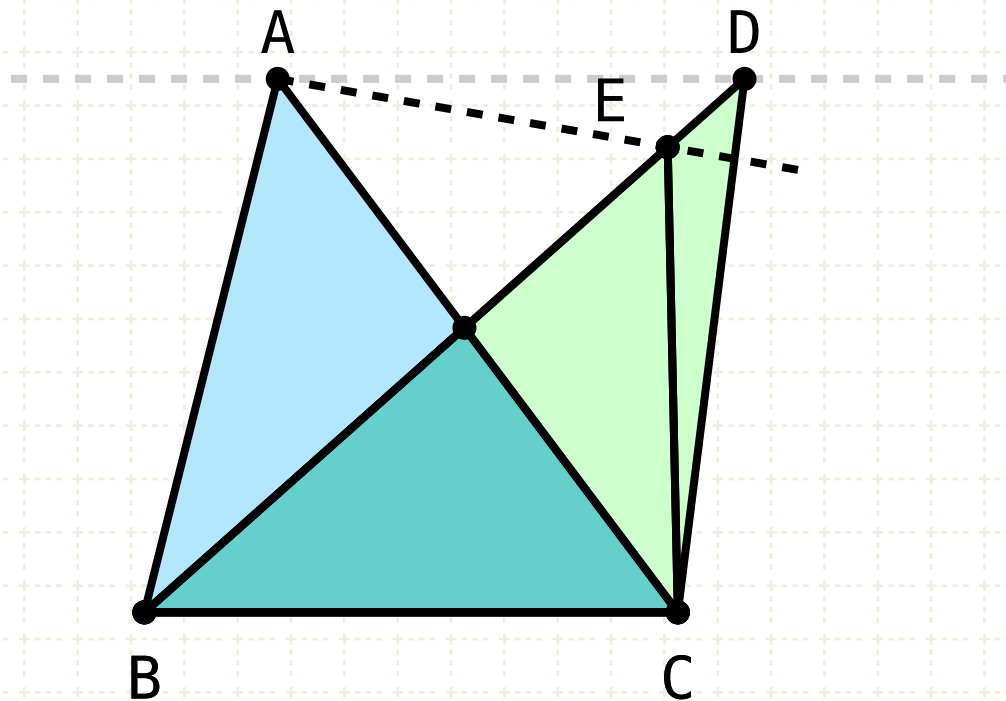
Proof by Contradiction

Assume AD is not parallel to BC

Draw line AE, parallel to BC and passing point A and line DB

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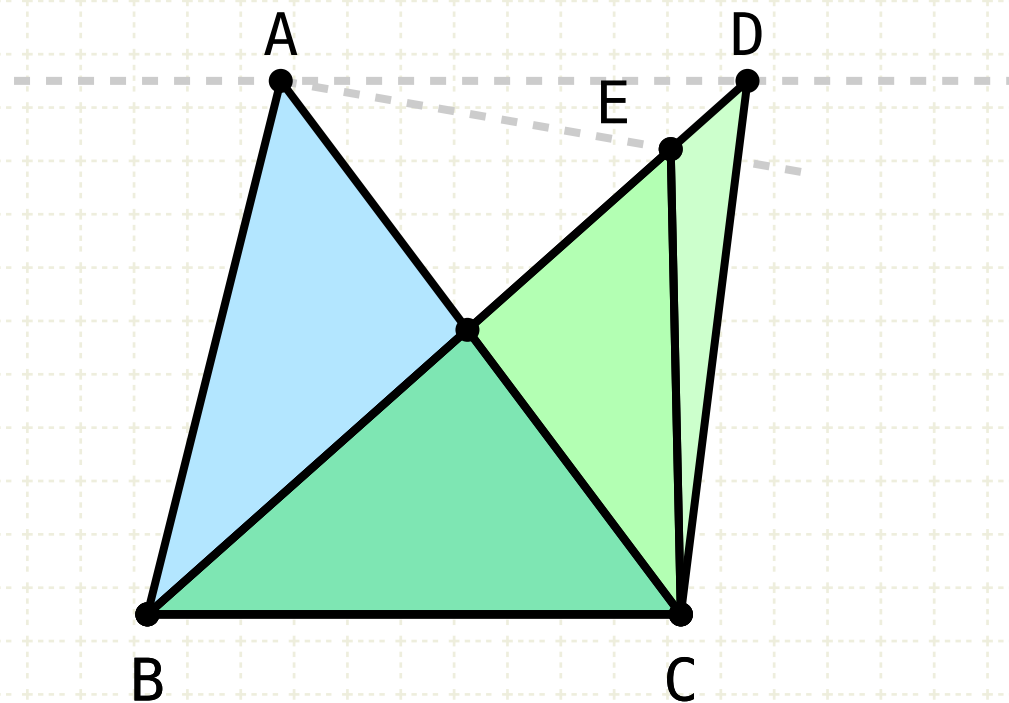
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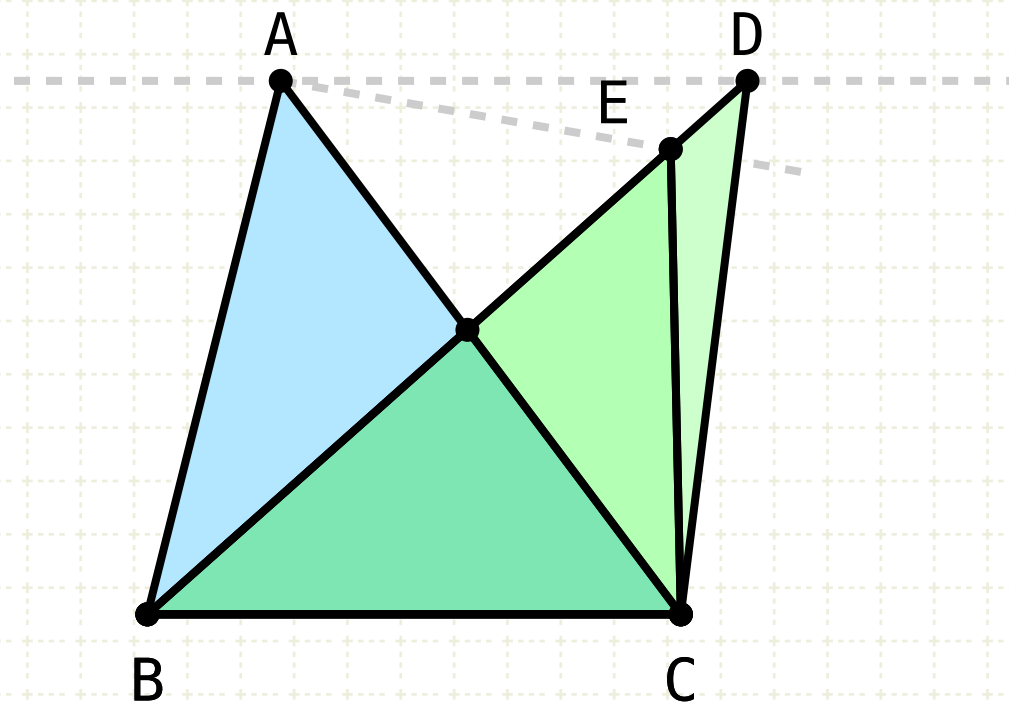
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Triangle ABC equal EBC since AE and BC are parallel (I-37)

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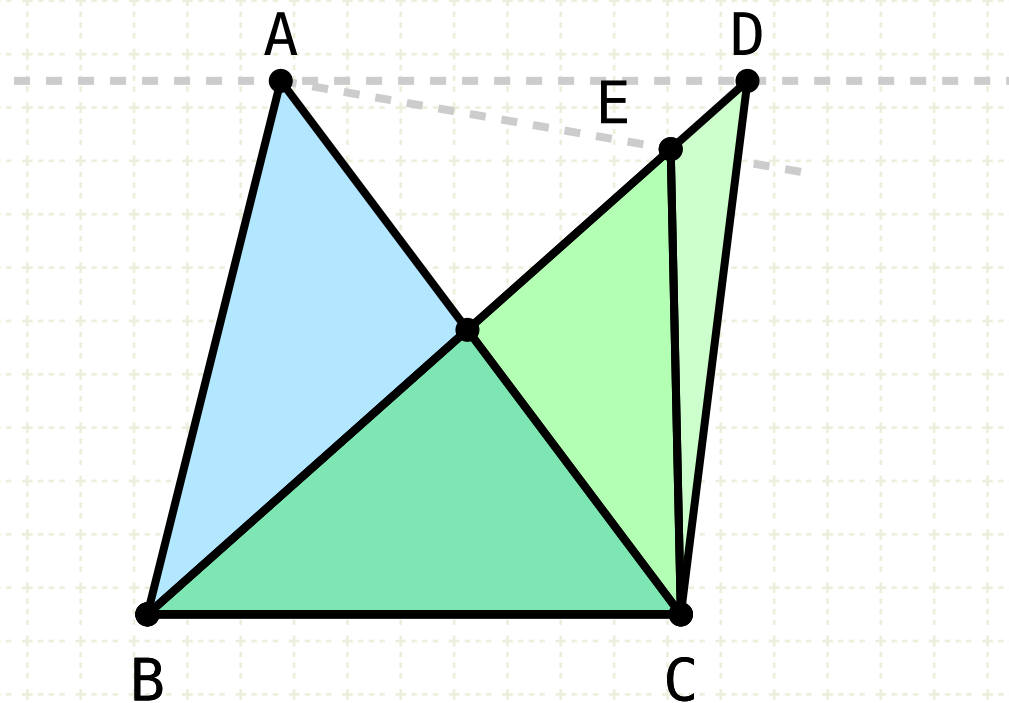
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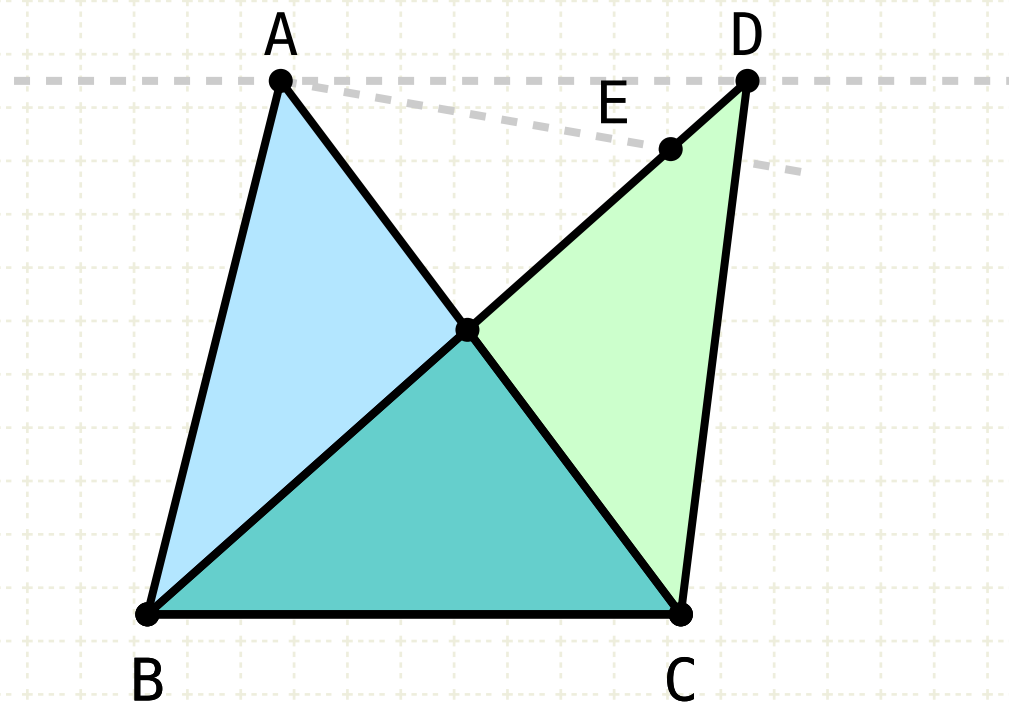
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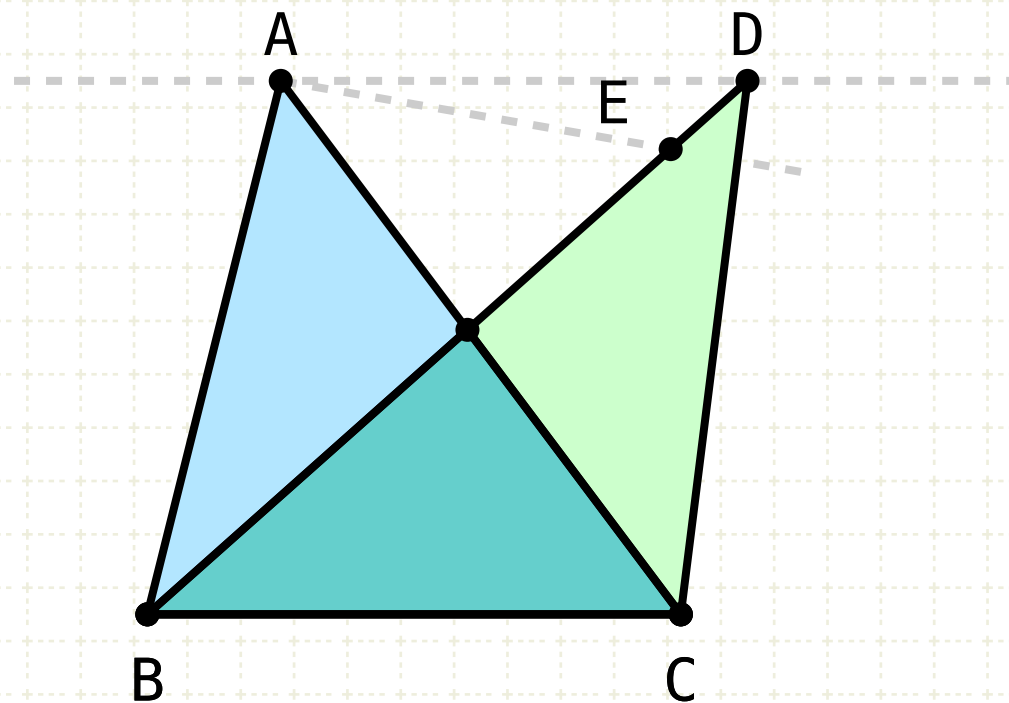
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