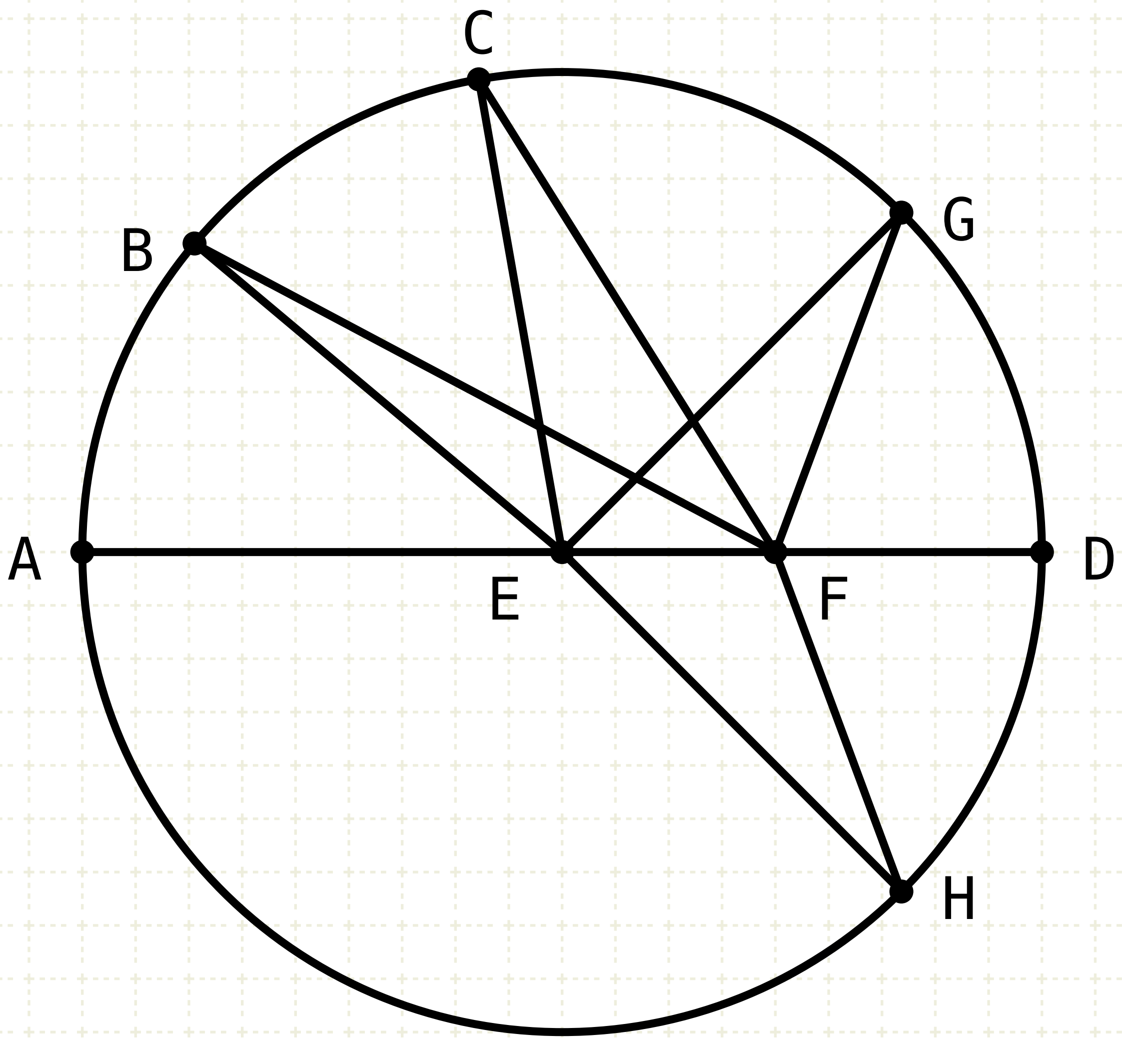

Euclid's Elements

Book III

A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)

Table of Contents, Chapter 3

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Proposition 27 of Book III

In equal circles angles standing on equal circumferences are equal to one another, whether they stand at the centres or at the circumferences.

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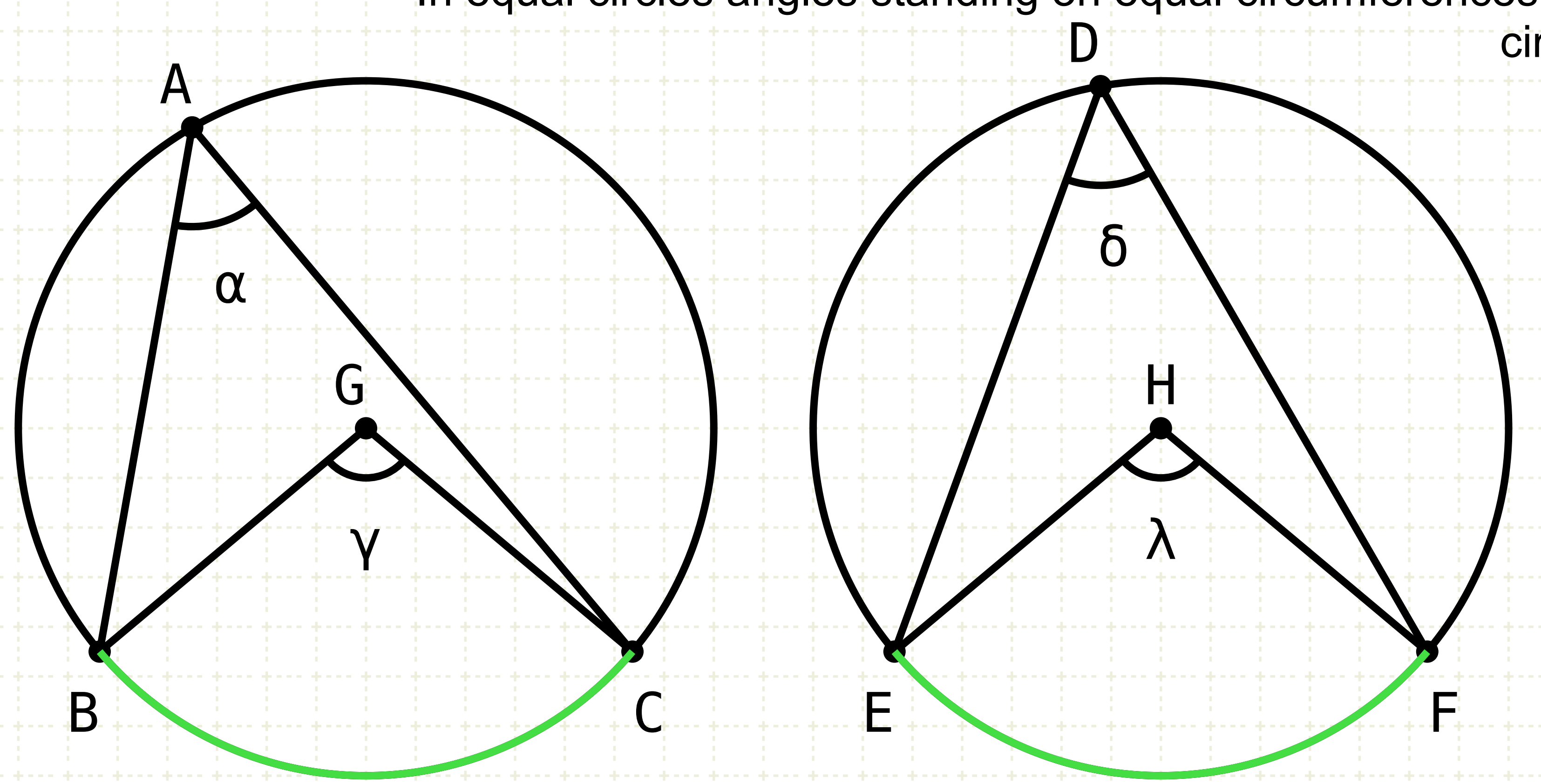

In other words

Given two equal circles (as shown)

If the circumference BC equals the circumference BF, then

α equals δ and γ equals λ

⊙ABC = ⊙EDF

↺BC = ↺EF

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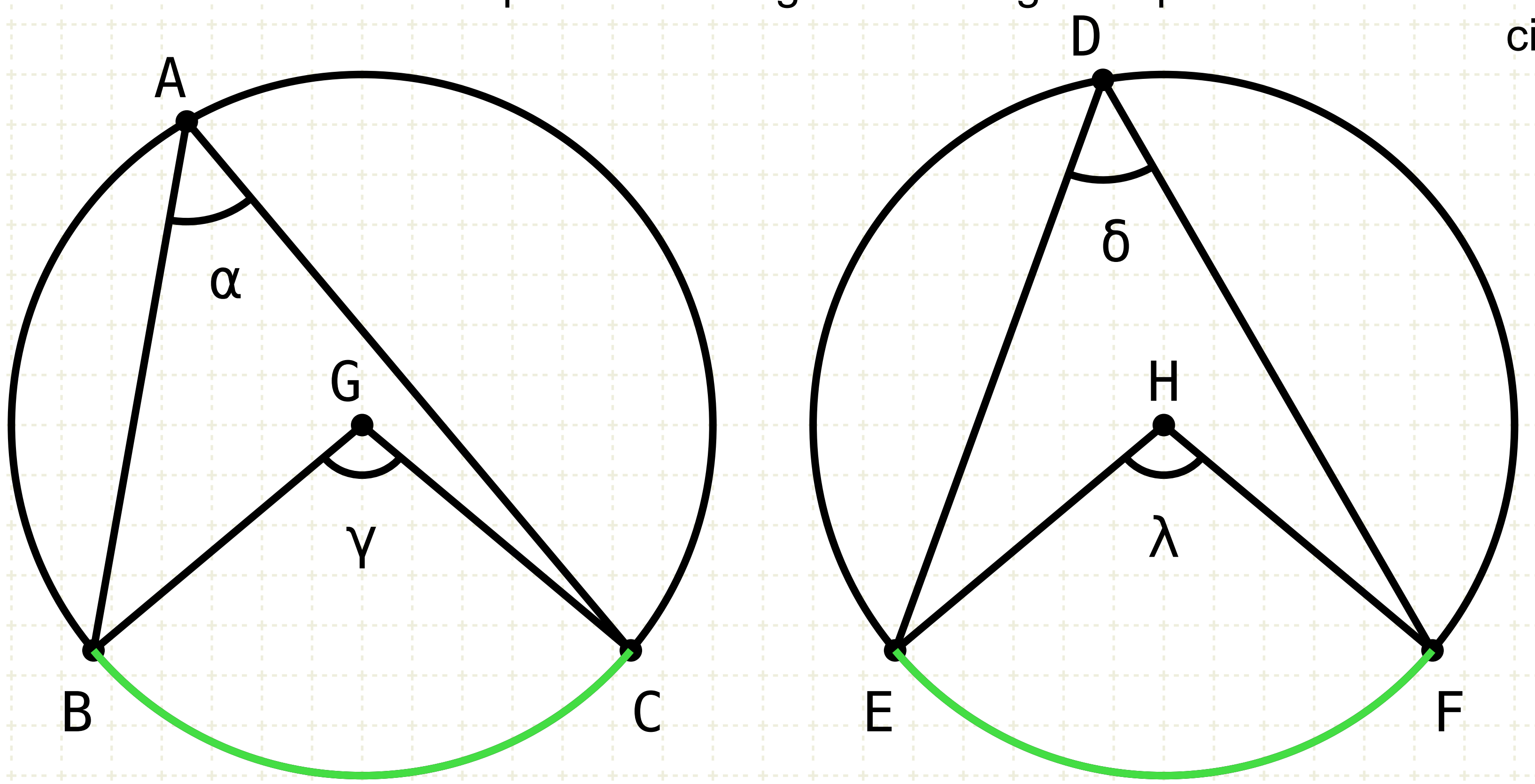

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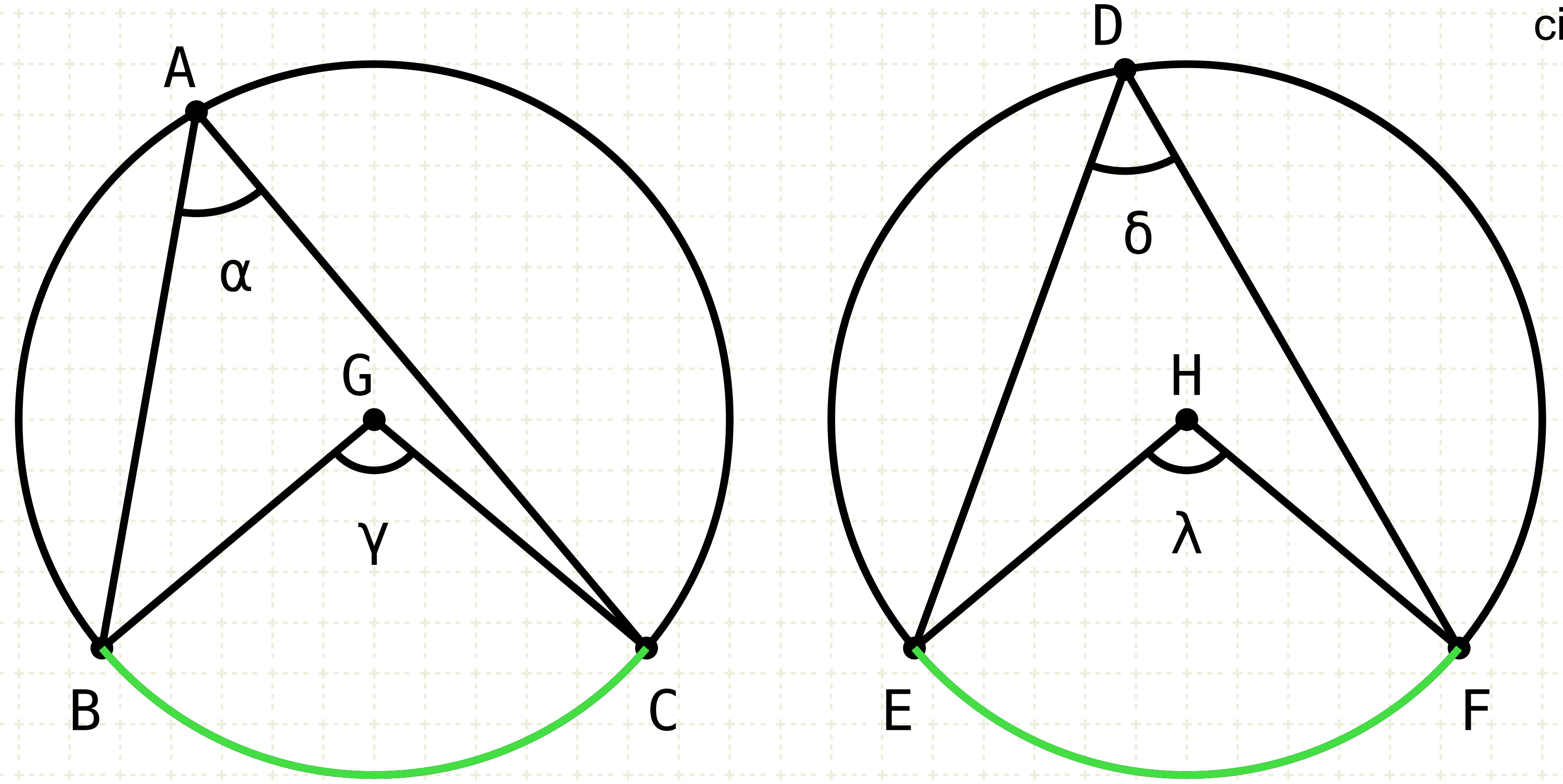

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Assume that γ is larger than λ

⊙ABC = ⊙EDF
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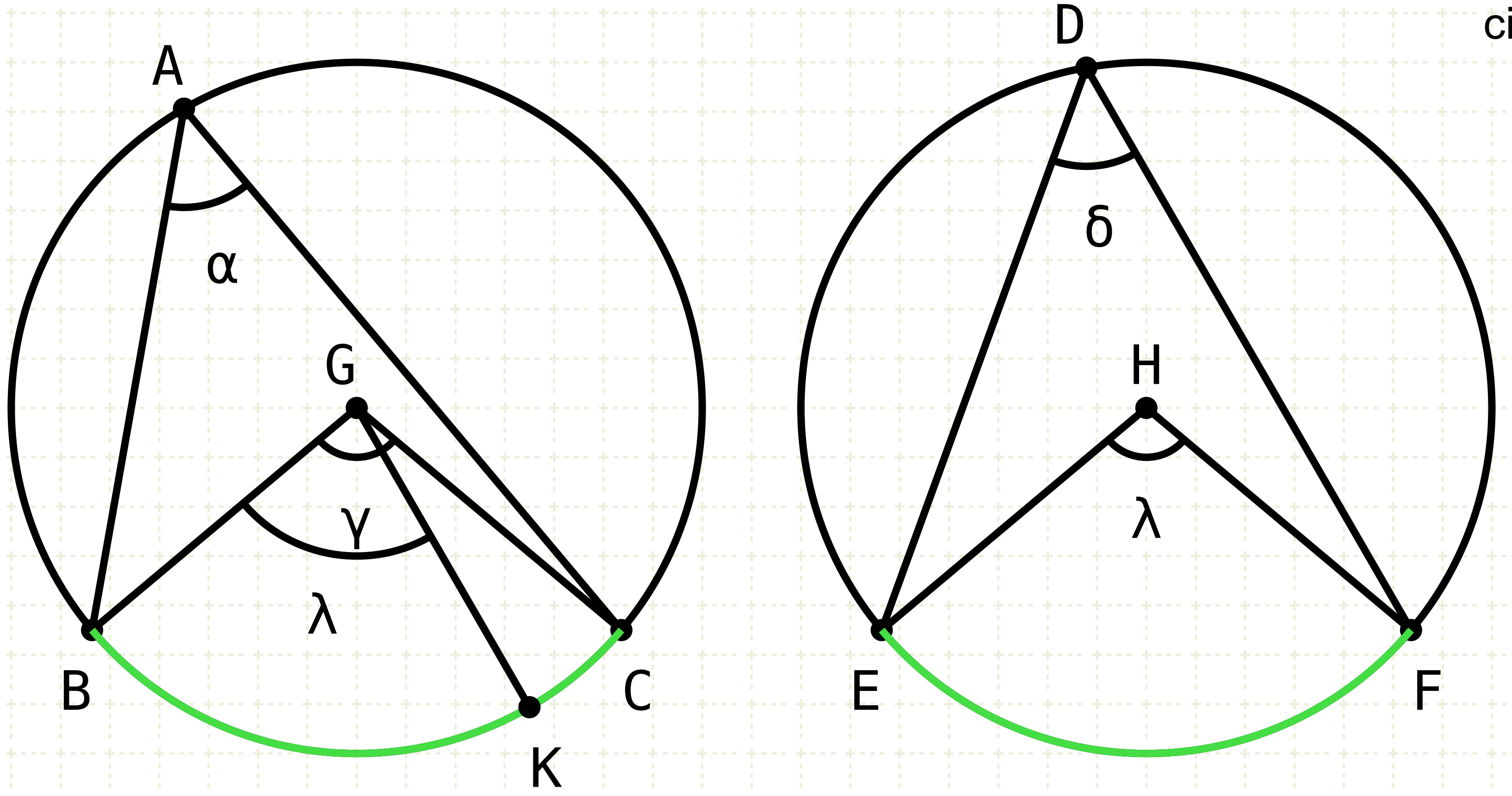

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Construct angle BGK such that it equals λ (I·23)

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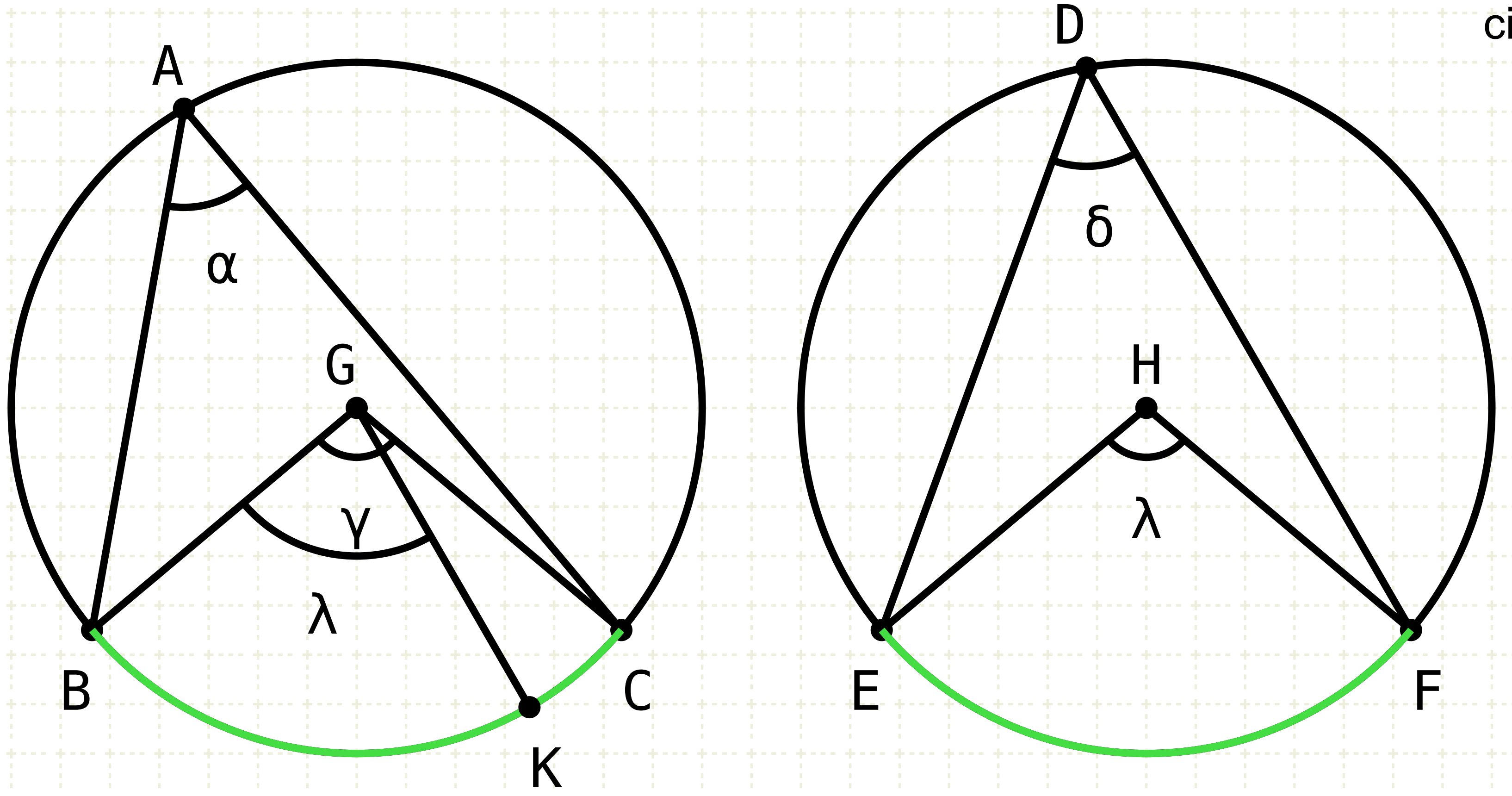

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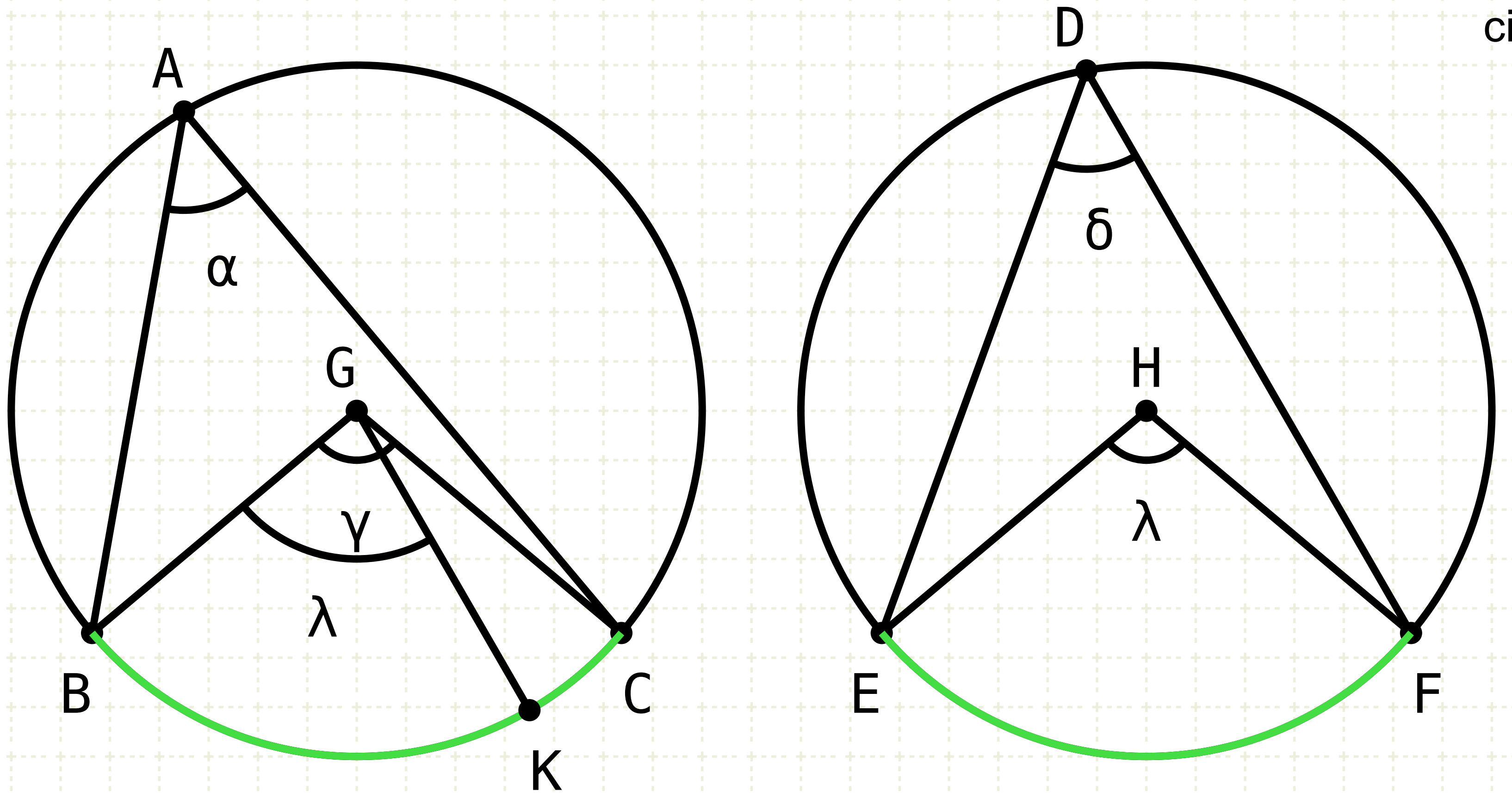

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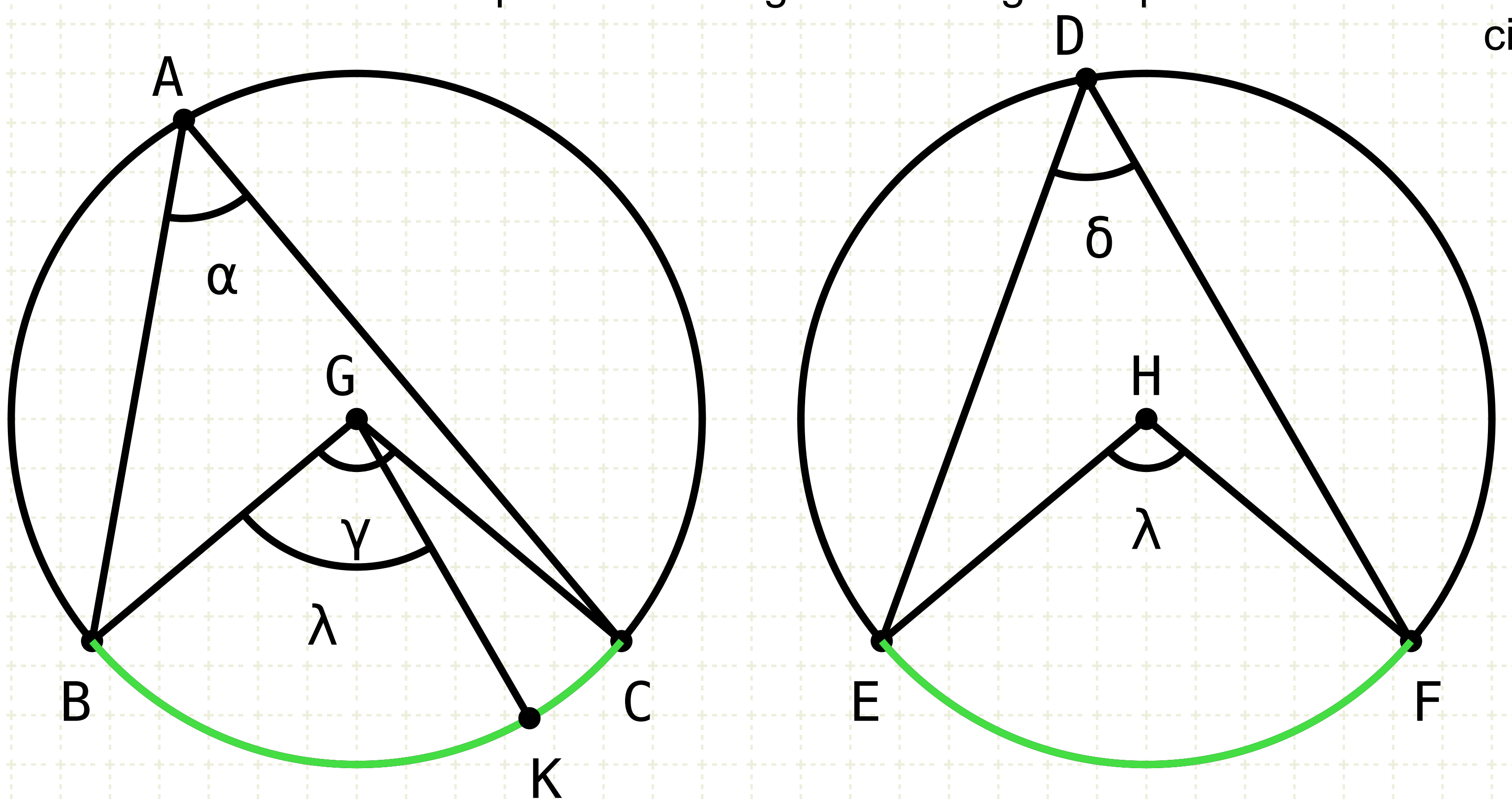

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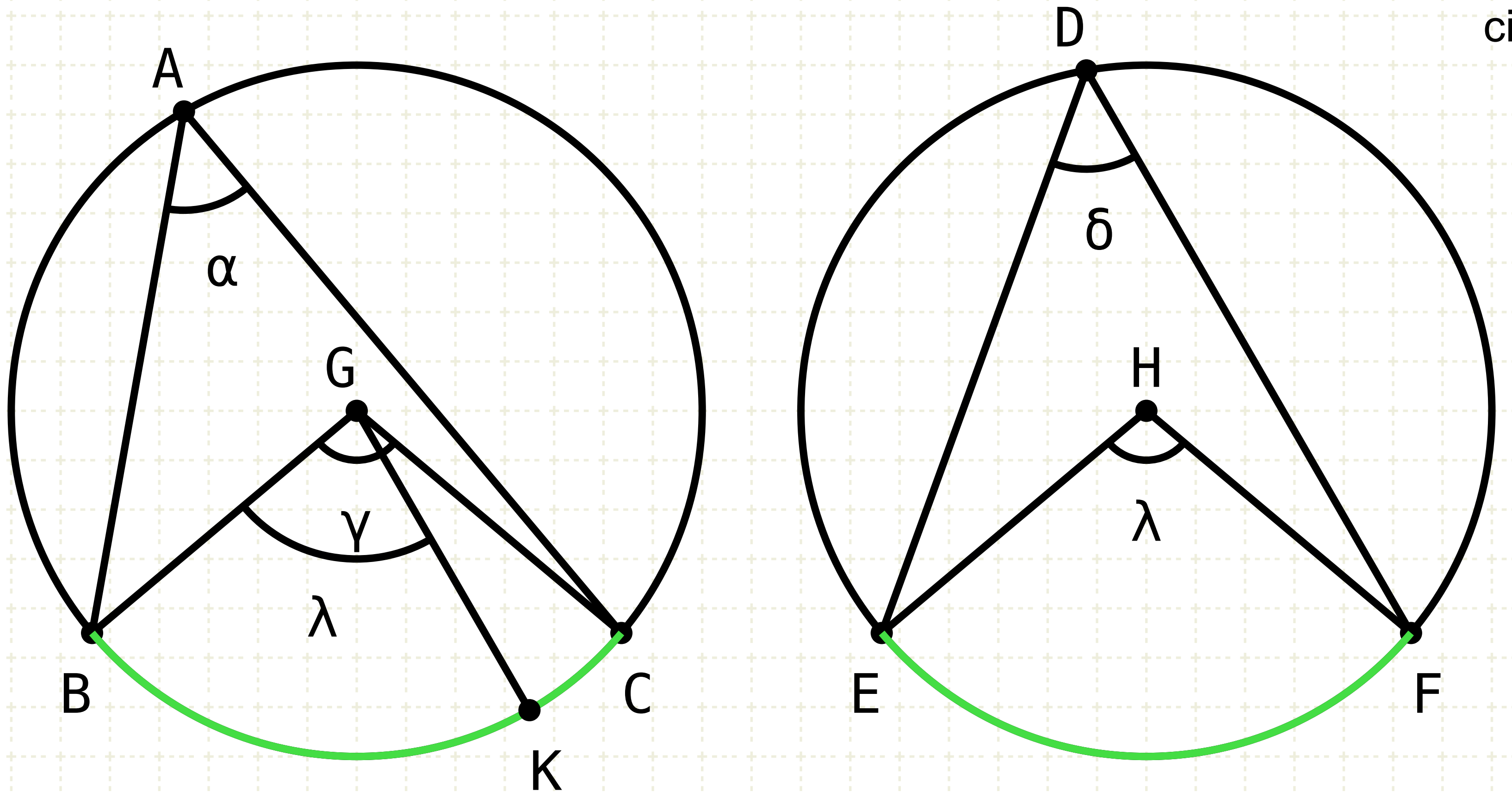

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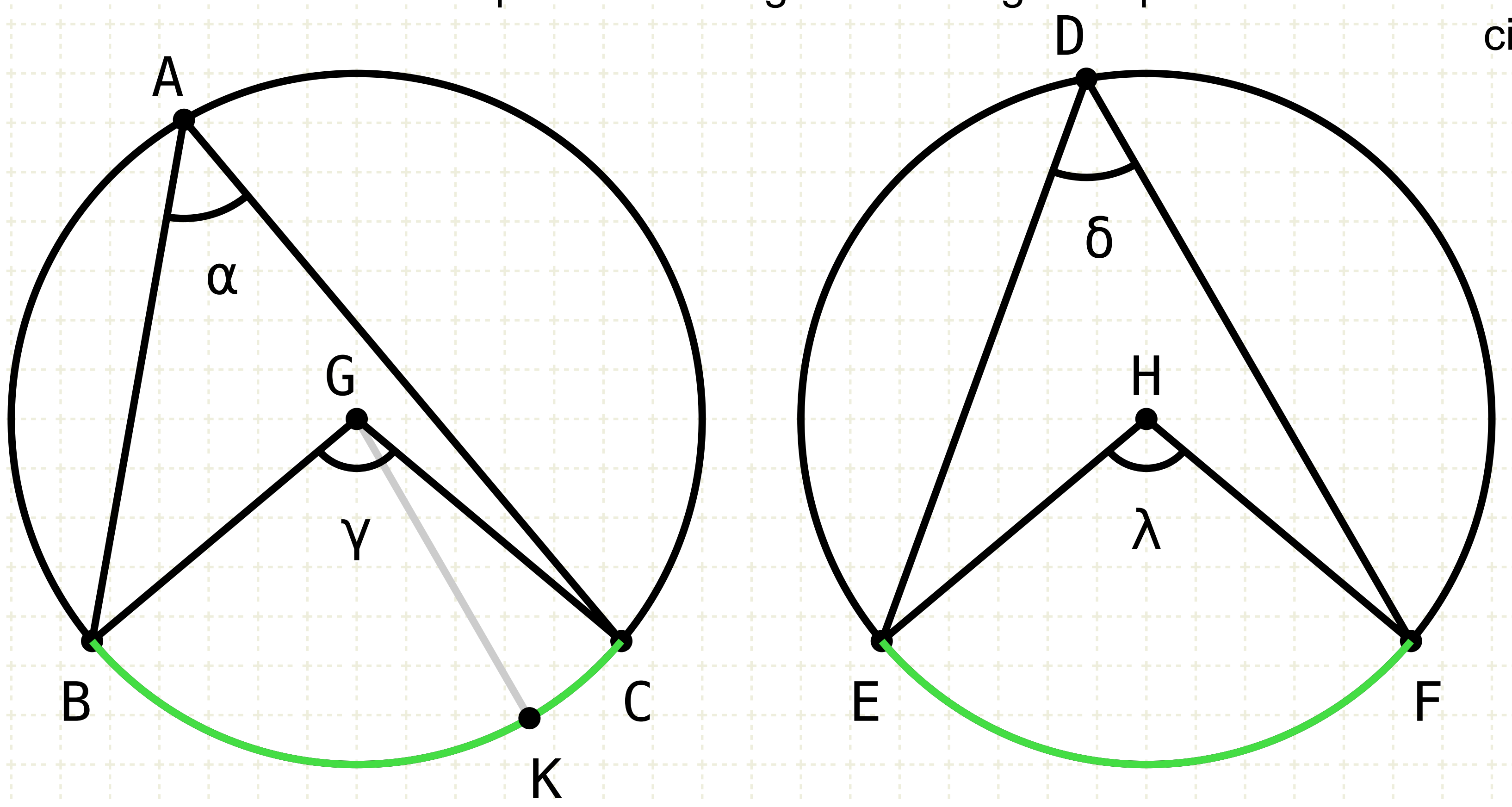

$\odot ABC = \odot EDF$

$\circlearrowleft BC = \circlearrowleft EF$

$\gamma > \lambda$

$\circlearrowleft BC > \circlearrowleft BK$

$\circlearrowleft BK = \circlearrowleft EF$

$\circlearrowleft BK = \circlearrowleft BC$

$\therefore \gamma = \lambda$

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Therefore the angles BGC equals EHF

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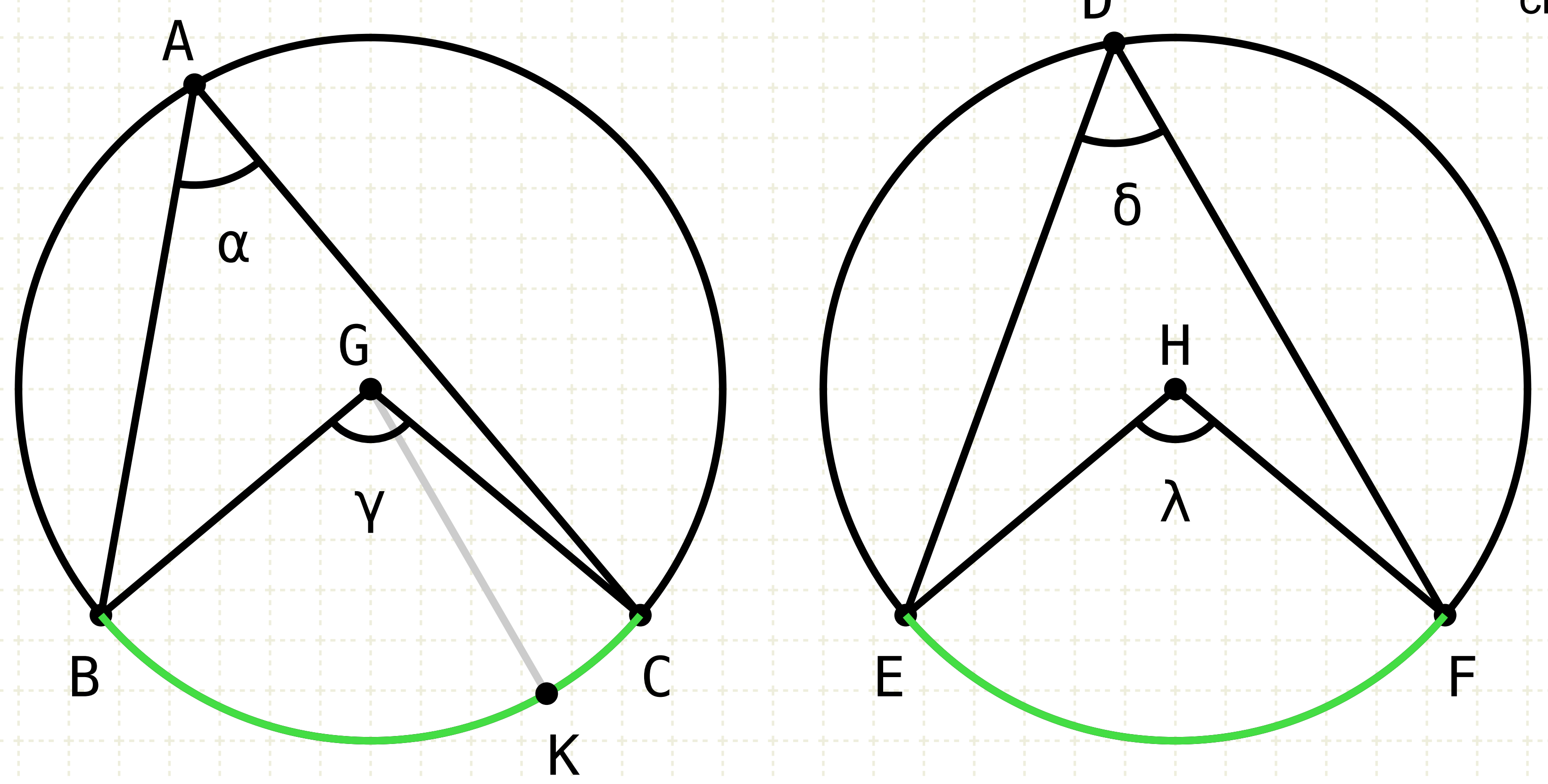

$\odot ABC = \odot EDF$
$\circlearrowleft BC = \circlearrowleft EF$

$\gamma > \lambda$
$\circlearrowleft BC > \circlearrowleft BK$
$\circlearrowleft BK = \circlearrowleft EF$
$\circlearrowleft BK = \circlearrowleft BC$

$\therefore \gamma = \lambda$
$\alpha = \frac{1}{2} \gamma$
$\delta = \frac{1}{2} \lambda$

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The angle at the circumference is half the angle at the centre of a circle if the base is the same (III·20) therefore α is half γ and δ is half λ

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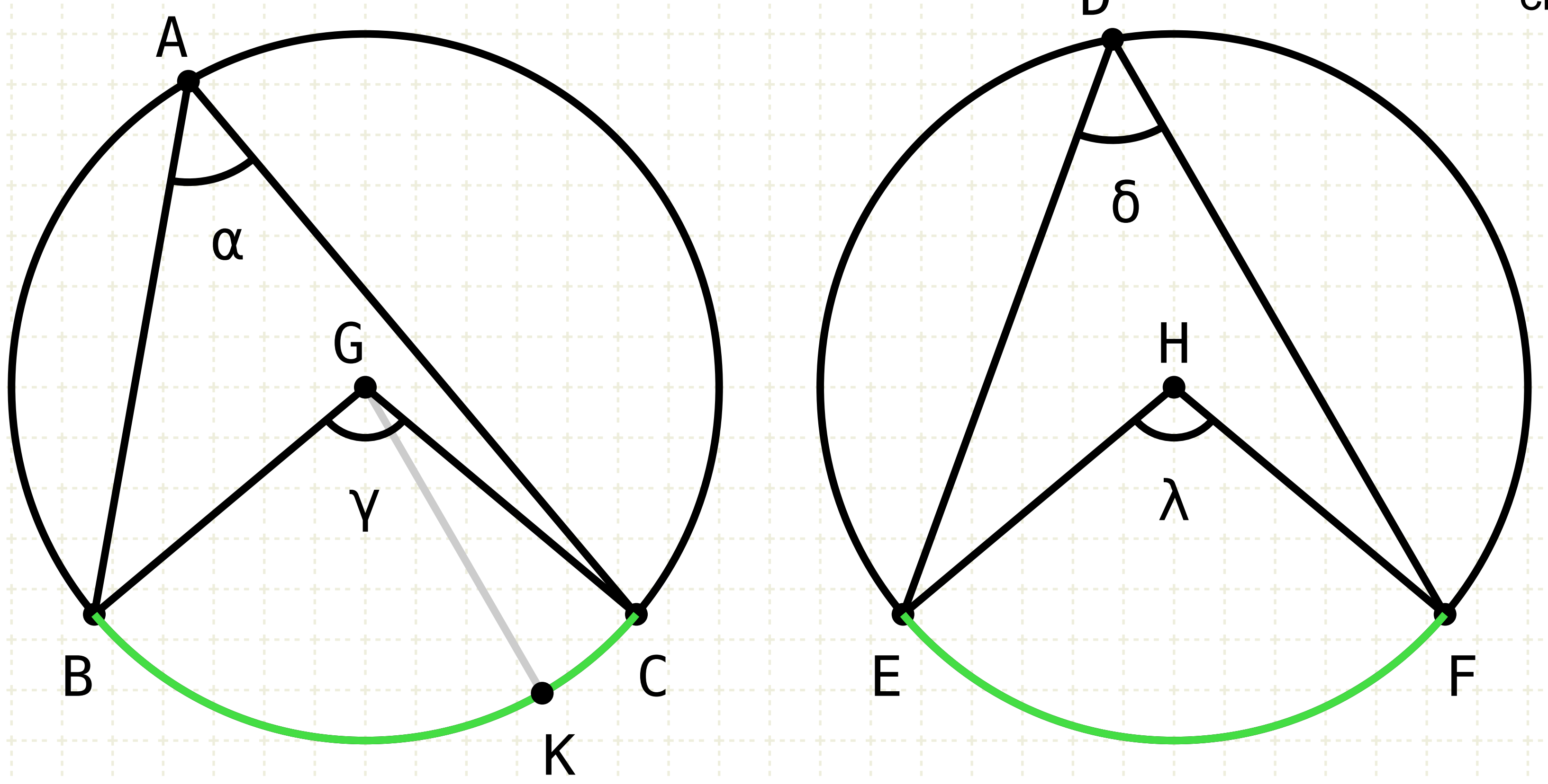

⊙ABC = ⊙EDF
↺BC = ↺EF

γ > λ
↺BC > ↺BK
↺BK = ↺EF
↺BK = ↺BC

∴ γ = λ
$\alpha = \frac{1}{2}\gamma$
$\delta = \frac{1}{2}\lambda$
α = δ

In other words

Given two equal circles (as shown)

If the circumference BC equals the circumference BF, then

α equals δ and γ equals λ

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The angle at the circumference is half the angle at the centre of a circle if the base is the same (III·20) therefore α is half γ and δ is half λ

Then since γ is equal to λ, α is equal to δ

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