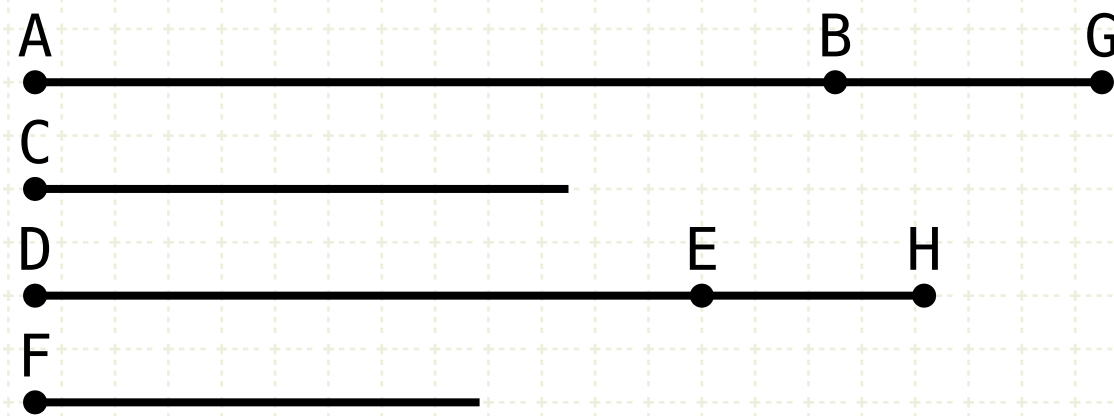


# Euclid's Elements

## Book V



$$AB:C = DE:F$$

$$BG:C = EH:F$$

$$AG:C = DH:F$$

*Proportions are what makes the old Greek temples classic in their beauty. They are like huge blocks, from which the air has been literally hewn out between the columns.*

**Arne Jacobsen**



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## Proposition 20 of Book V

If there be three magnitudes, and others equal to them in multitude, which taken two and two are in the same ratio, and if EX  
AEQUALI the first be greater than the third the fourth will also be greater than the sixth; if equal, equal; and, if less, less.



## Proposition 20 of Book V

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ratio EX AEQUALI

$$a:b = d:e$$

$$b:c = e:f$$

$$\rightarrow a:c = d:f$$

## Definitions

17. A ratio EX AEQUALI arises when, there being several magnitudes and another set equal to them in multitude which taken two and two are in the same proportion, as the first is to the last among the the first magnitudes, so is the first is to the last among the second magnitudes

Or, in other words, it means taking the extreme terms by virtue of the removal of the intermediate terms.



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AEQUALI the first be greater than the third the fourth will also be greater than the sixth; if equal, equal; and, if less, less.



## In other words

Given two sets of numbers A,B,C and D,E,F where A is to B as  
D is to E, and where B is to C as E is to F

Then if A is greater than C, D will be greater than F, and if A is  
equal to C, D will be equal to F, etc

$$A:B = D:E$$

$$B:C = E:F$$

$$A \geq C \rightarrow D \geq F$$





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$$A:B = D:E$$

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## In other words

Given two sets of numbers A,B,C and D,E,F where A is to B as D is to E, and where B is to C as E is to F

Then if A is greater than C, D will be greater than F, and if A is equal to C, D will be equal to F, etc

## Proof



# Proposition 20 of Book V

If there be three magnitudes, and others equal to them in multitude, which taken two and two are in the same ratio, and if EX  
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$$A:B = D:E$$

$$B:C = E:F$$

$$A > C$$

## In other words

Given two sets of numbers A,B,C and D,E,F where A is to B as D is to E, and where B is to C as E is to F

Then if A is greater than C, D will be greater than F, and if A is equal to C, D will be equal to F, etc

## Proof

Let A be greater than C





# Proposition 20 of Book V

If there be three magnitudes, and others equal to them in multitude, which taken two and two are in the same ratio, and if EX  
AEQUALI the first be greater than the third the fourth will also be greater than the sixth; if equal, equal; and, if less, less.



$$A:B = D:E$$

$$B:C = E:F$$

$$A > C$$

$$A:B > C:B$$

## In other words

Given two sets of numbers A,B,C and D,E,F where A is to B as D is to E, and where B is to C as E is to F

Then if A is greater than C, D will be greater than F, and if A is equal to C, D will be equal to F, etc

## Proof

Let A be greater than C

If A is greater than C, when compared to another magnitude, in this case B, A will have a greater ratio to B than C will have to B (V·8)



# Proposition 20 of Book V

If there be three magnitudes, and others equal to them in multitude, which taken two and two are in the same ratio, and if EX  
AEQUALI the first be greater than the third the fourth will also be greater than the sixth; if equal, equal; and, if less, less.



$$A : B = D : E$$

$$B : C = E : F$$

$$A > C$$

$$A : B > C : B$$

$$C : B = F : E$$

## In other words

Given two sets of numbers A,B,C and D,E,F where A is to B as D is to E, and where B is to C as E is to F

Then if A is greater than C, D will be greater than F, and if A is equal to C, D will be equal to F, etc

## Proof

Let A be greater than C

If A is greater than C, when compared to another magnitude, in this case B, A will have a greater ratio to B than C will have to B (V·8)

Since the ratio B to C equals the ratio E to F, then inverse ratios (C to B and F to E) are also equal (V·13)



# Proposition 20 of Book V

If there be three magnitudes, and others equal to them in multitude, which taken two and two are in the same ratio, and if EX  
AEQUALI the first be greater than the third the fourth will also be greater than the sixth; if equal, equal; and, if less, less.



$$A : B = D : E$$

$$B : C = E : F$$

$$A > C$$

$$A : B > C : B$$

$$C : B = F : E$$

$$A : B > F : E$$

## In other words

Given two sets of numbers A,B,C and D,E,F where A is to B as D is to E, and where B is to C as E is to F

Then if A is greater than C, D will be greater than F, and if A is equal to C, D will be equal to F, etc

## Proof

Let A be greater than C

If A is greater than C, when compared to another magnitude, in this case B, A will have a greater ratio to B than C will have to B (V·8)

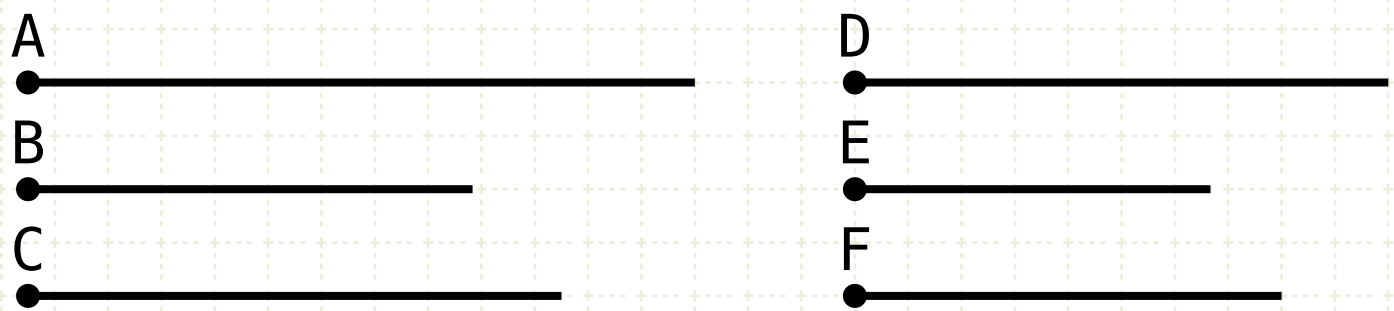
Since the ratio B to C equals the ratio E to F, then inverse ratios (C to B and F to E) are also equal (V·13)

Since A to B is greater than C to B, and C to B equals F to E, then A to B is greater than F to E



# Proposition 20 of Book V

If there be three magnitudes, and others equal to them in multitude, which taken two and two are in the same ratio, and if EX  
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$$A:B = D:E$$

$$B:C = E:F$$

$$A > C$$

$$A:B > C:B$$

$$C:B = F:E$$

$$A:B > F:E$$

$$D:E > F:E$$

## In other words

Given two sets of numbers A,B,C and D,E,F where A is to B as D is to E, and where B is to C as E is to F

Then if A is greater than C, D will be greater than F, and if A is equal to C, D will be equal to F, etc

## Proof

Let A be greater than C

If A is greater than C, when compared to another magnitude, in this case B, A will have a greater ratio to B than C will have to B (V·8)

Since the ratio B to C equals the ratio E to F, then inverse ratios (C to B and F to E) are also equal (V·13)

Since A to B is greater than C to B, and C to B equals F to E, then A to B is greater than F to E

And since A to B equals D to E, then D to E is also greater than F to E



# Proposition 20 of Book V

If there be three magnitudes, and others equal to them in multitude, which taken two and two are in the same ratio, and if EX  
AEQUALI the first be greater than the third the fourth will also be greater than the sixth; if equal, equal; and, if less, less.



$$A : B = D : E$$

$$B : C = E : F$$

$$A > C$$

$$A : B > C : B$$

$$C : B = F : E$$

$$A : B > F : E$$

$$D : E > F : E$$

$$D > F$$

## In other words

Given two sets of numbers A,B,C and D,E,F where A is to B as D is to E, and where B is to C as E is to F

Then if A is greater than C, D will be greater than F, and if A is equal to C, D will be equal to F, etc

## Proof

Let A be greater than C

If A is greater than C, when compared to another magnitude, in this case B, A will have a greater ratio to B than C will have to B (V·8)

Since the ratio B to C equals the ratio E to F, then inverse ratios (C to B and F to E) are also equal (V·13)

Since A to B is greater than C to B, and C to B equals F to E, then A to B is greater than F to E

And since A to B equals D to E, then D to E is also greater than F to E

If D to E is greater than F to E, then D is greater than F (V·10)





# Proposition 20 of Book V

If there be three magnitudes, and others equal to them in multitude, which taken two and two are in the same ratio, and if EXAEQUALI the first be greater than the third the fourth will also be greater than the sixth; if equal, equal; and, if less, less.



$$A:B = D:E$$

$$B:C = E:F$$

$$A > C$$

$$A:B > C:B$$

$$C:B = F:E$$

$$A:B > F:E$$

$$D:E > F:E$$

$$D > F$$

## In other words

Given two sets of numbers A,B,C and D,E,F where A is to B as D is to E, and where B is to C as E is to F

Then if A is greater than C, D will be greater than F, and if A is equal to C, D will be equal to F, etc

## Proof

Let A be greater than C

If A is greater than C, when compared to another magnitude, in this case B, A will have a greater ratio to B than C will have to B (V·8)

Since the ratio B to C equals the ratio E to F, then inverse ratios (C to B and F to E) are also equal (V·13)

Since A to B is greater than C to B, and C to B equals F to E, then A to B is greater than F to E

And since A to B equals D to E, then D to E is also greater than F to E

If D to E is greater than F to E, then D is greater than F (V·10)





# Proposition 20 of Book V

If there be three magnitudes, and others equal to them in multitude, which taken two and two are in the same ratio, and if EX  
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$$A:B = D:E$$

$$B:C = E:F$$

$$A > C$$

$$A:B > C:B$$

$$C:B = F:E$$

$$A:B > F:E$$

$$D:E > F:E$$

$$D > F$$

$$A \geq C \rightarrow D \geq F$$

## In other words

Given two sets of numbers A,B,C and D,E,F where A is to B as D is to E, and where B is to C as E is to F

Then if A is greater than C, D will be greater than F, and if A is equal to C, D will be equal to F, etc

## Proof

Let A be greater than C

If A is greater than C, when compared to another magnitude, in this case B, A will have a greater ratio to B than C will have to B (V·8)

Since the ratio B to C equals the ratio E to F, then inverse ratios (C to B and F to E) are also equal (V·13)

Since A to B is greater than C to B, and C to B equals F to E, then A to B is greater than F to E

And since A to B equals D to E, then D to E is also greater than F to E

If D to E is greater than F to E, then D is greater than F (V·10)

Similarly, we can prove that if A is equal to C, then D is equal to F, and if less, less



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