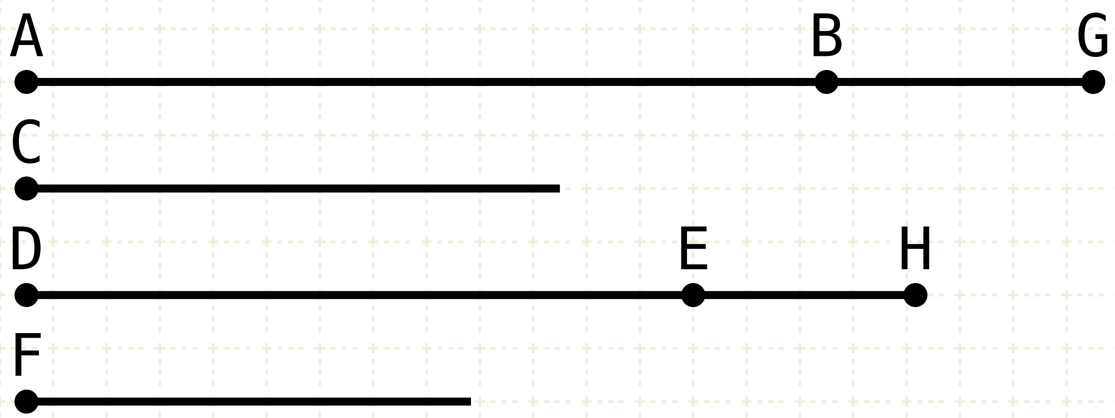
Euclid's Elements

Book V



AB:C = DE:F
BG:C = EH:F

AG:C = DH:F

Proportions are what makes the old Greek temples classic in their beauty. They are like huge blocks, from which the air has been literally hewn out between the columns.

Arne Jacobsen

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Proposition 16 of Book V

If our magnitudes be proportional, they will also be proportional alternately

Proposition 16 of Book V

If our magnitudes be proportional, they will also be proportional alternately

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A:B = C:D → A,B,C,D are proportional

A,C → antecedents
B,D → consequents

given two ratios
 A:B and C:D
then the alternate ratios are:
 A:C and B:D
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Definitions

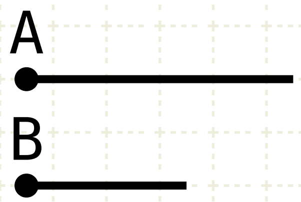
6. Let magnitudes which have the same ratio be called PROPORTIONAL
12. ALTERNATE RATIO means taking the antecedent in relation to the antecedent and the consequent in relation to the consequent

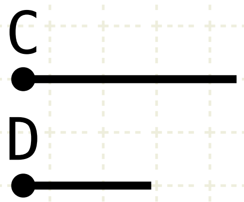
Proposition 16 of Book V

If our magnitudes be proportional, they will also be proportional alternately

Proposition 16 of Book V

If our magnitudes be proportional, they will also be proportional alternately





In other words

If A,B,C,D are proportional,

… then they will also be alternately proportional

$A:B = C:D$

$A:C = B:D$

Proposition 16 of Book V

If our magnitudes be proportional, they will also be proportional alternately

In other words

If A,B,C,D are proportional,

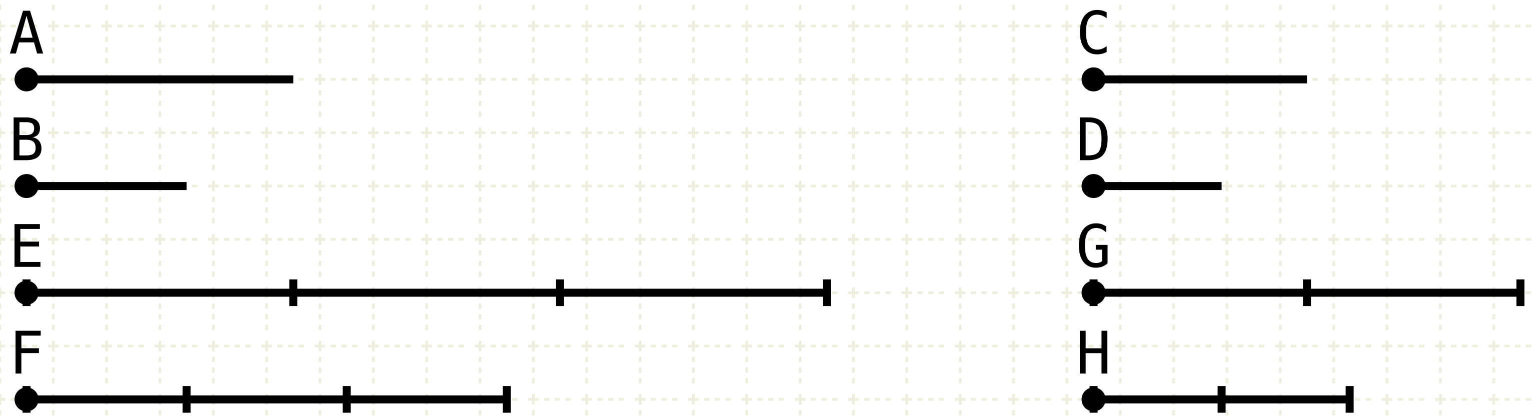
… then they will also be alternately proportional

Proof

`A:B = C:D`

Proposition 16 of Book V

If our magnitudes be proportional, they will also be proportional alternately



A:B = C:D
E = m·A
F = m·B
G = n·C
H = n·D

In other words

If A,B,C,D are proportional,

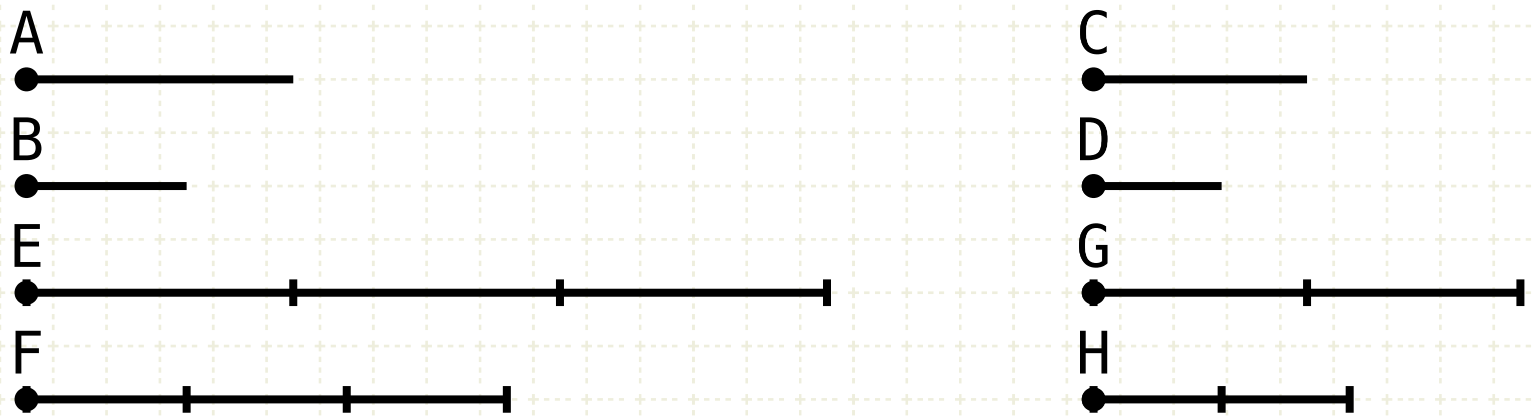
… then they will also be alternately proportional

Proof

Let E,F be equimultiples of A and B, and G,H other chance equimultiple of C,D

Proposition 16 of Book V

If our magnitudes be proportional, they will also be proportional alternately



$A:B = C:D$

$E = m \cdot A$

$F = m \cdot B$

$G = n \cdot C$

$H = n \cdot D$

$E:F = A:B$

In other words

If A,B,C,D are proportional,

… then they will also be alternately proportional

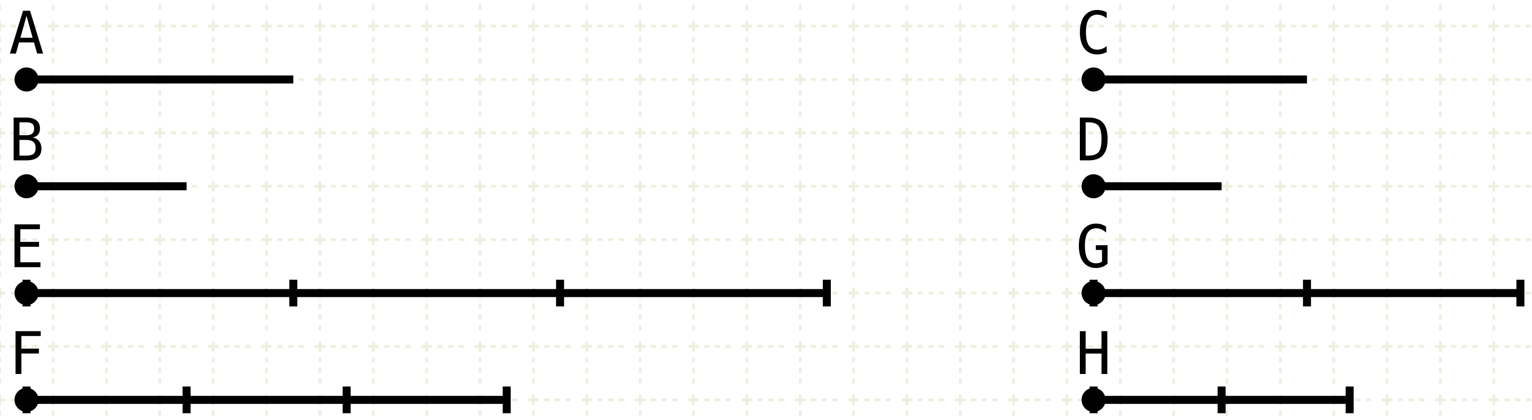
Proof

Let E,F be equimultiples of A and B, and G,H other chance equimultiple of C,D

Since E,F are equimultiples of A,B, their ratios are equal (V·15)

Proposition 16 of Book V

If our magnitudes be proportional, they will also be proportional alternately



$A:B = C:D$

$E = m \cdot A$

$F = m \cdot B$

$G = n \cdot C$

$H = n \cdot D$

$E:F = A:B$

$E:F = C:D$

In other words

If A,B,C,D are proportional,

… then they will also be alternately proportional

Proof

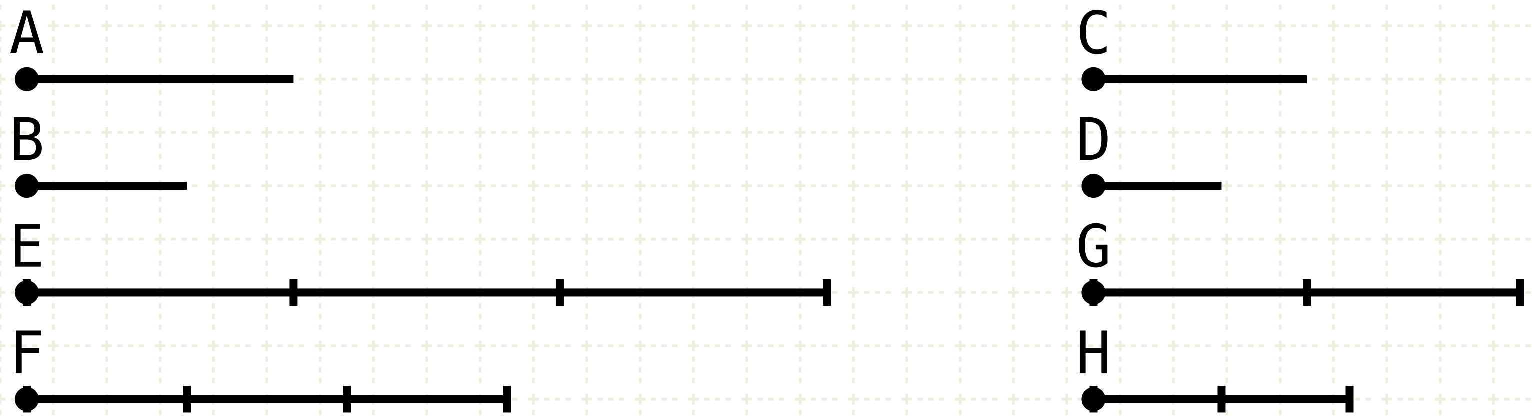
Let E,F be equimultiples of A and B, and G,H other chance equimultiple of C,D

Since E,F are equimultiples of A,B, their ratios are equal (V·15)

But A is to B as C is to D, therefore E is to F as C is to D (V·11)

Proposition 16 of Book V

If our magnitudes be proportional, they will also be proportional alternately



A:B = C:D

E = m·A

F = m·B

G = n·C

H = n·D

E:F = A:B

E:F = C:D

G:H = C:D

In other words

If A,B,C,D are proportional,

… then they will also be alternately proportional

Proof

Let E,F be equimultiples of A and B, and G,H other chance equimultiple of C,D

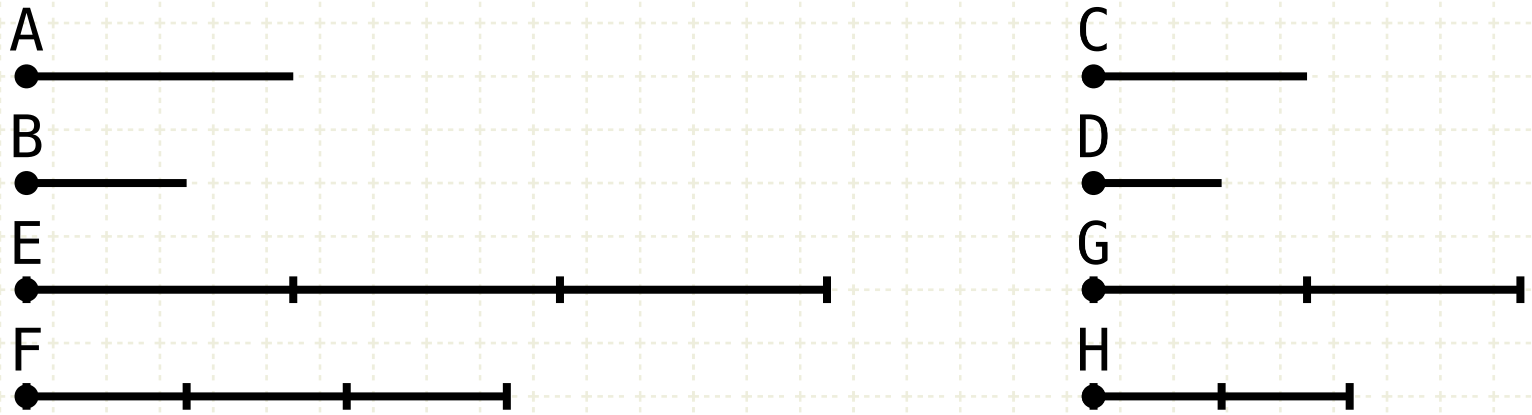
Since E,F are equimultiples of A,B, their ratios are equal (V·15)

But A is to B as C is to D, therefore E is to F as C is to D (V·11)

Again, since G, H are equimultiples of C,D, therefore as C is to D, so is G to H (V·15)

Proposition 16 of Book V

If our magnitudes be proportional, they will also be proportional alternately



$A:B = C:D$
$E = m \cdot A$
$F = m \cdot B$
$G = n \cdot C$
$H = n \cdot D$

$E:F = A:B$
$E:F = C:D$
$G:H = C:D$
$G:H = E:F$

In other words

If A,B,C,D are proportional,

… then they will also be alternately proportional

Proof

Let E,F be equimultiples of A and B, and G,H other chance equimultiple of C,D

Since E,F are equimultiples of A,B, their ratios are equal (V·15)

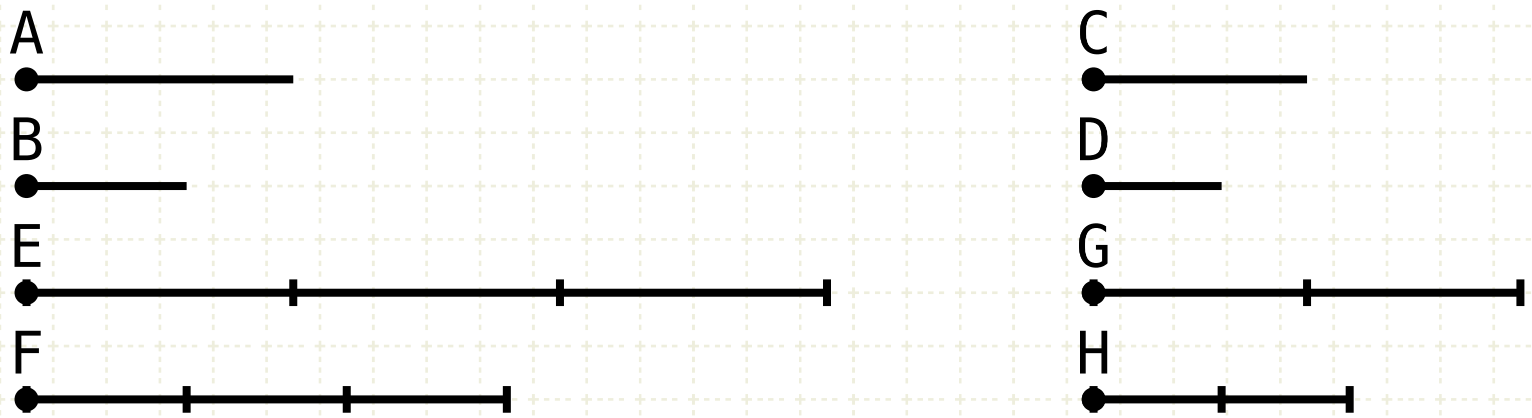
But A is to B as C is to D, therefore E is to F as C is to D (V·11)

Again, since G, H are equimultiples of C,D, therefore as C is to D, so is G to H (V·15)

But C is to D as E is to F, therefore G is to H as E is to F (V·11)

Proposition 16 of Book V

If our magnitudes be proportional, they will also be proportional alternately



A:B = C:D
E = m·A
F = m·B
G = n·C
H = n·D

E:F = A:B
E:F = C:D
G:H = C:D
G:H = E:F
E >=< G → F >=< H

In other words

If A,B,C,D are proportional,

… then they will also be alternately proportional

Proof

Let E,F be equimultiples of A and B, and G,H other chance equimultiple of C,D

Since E,F are equimultiples of A,B, their ratios are equal (V·15)

But A is to B as C is to D, therefore E is to F as C is to D (V·11)

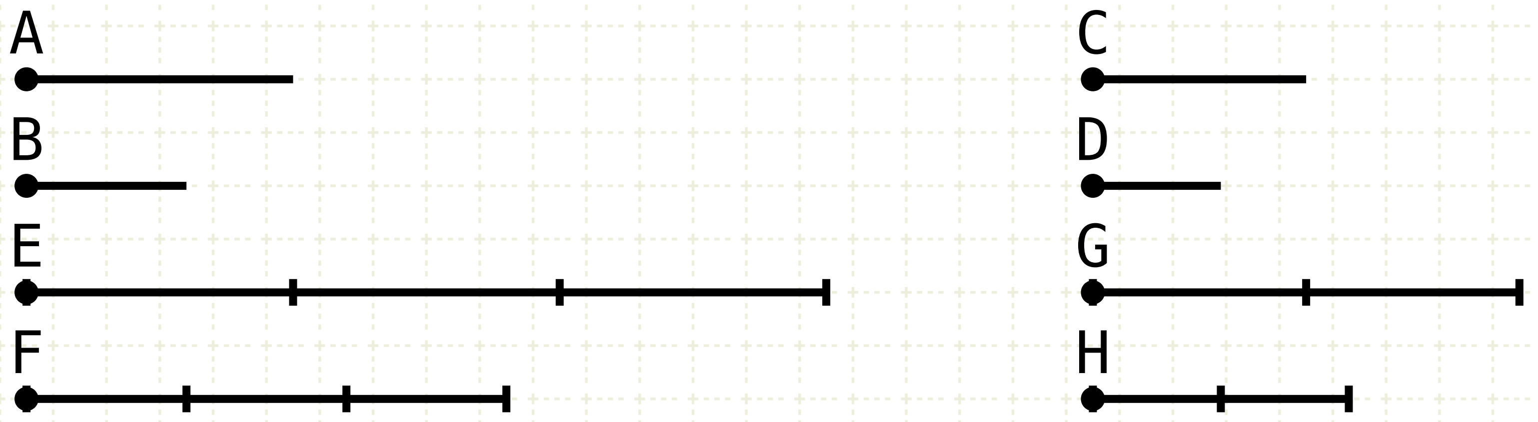
Again, since G, H are equimultiples of C,D, therefore as C is to D, so is G to H (V·15)

But C is to D as E is to F, therefore G is to H as E is to F (V·11)

Because G,H,E,F are proportional, then if E is greater than G, then F is greater than H, etc (V·14)

Proposition 16 of Book V

If our magnitudes be proportional, they will also be proportional alternately



A:B = C:D

E = m·A

F = m·B

G = n·C

H = n·D

E:F = A:B

E:F = C:D

G:H = C:D

G:H = E:F

E >=< G → F >=< H

m·A >=< n·C → m·B >=< n·D

In other words

If A,B,C,D are proportional,

… then they will also be alternately proportional

Proof

Let E,F be equimultiples of A and B, and G,H other chance equimultiple of C,D

Since E,F are equimultiples of A,B, their ratios are equal (V·15)

But A is to B as C is to D, therefore E is to F as C is to D (V·11)

Again, since G, H are equimultiples of C,D, therefore as C is to D, so is G to H (V·15)

But C is to D as E is to F, therefore G is to H as E is to F (V·11)

Because G,H,E,F are proportional, then if E is greater than G, then F is greater than H, etc (V·14)

Now E,F are equimultiples of A,B and G,H equimultiples of C,D so …

Proposition 16 of Book V

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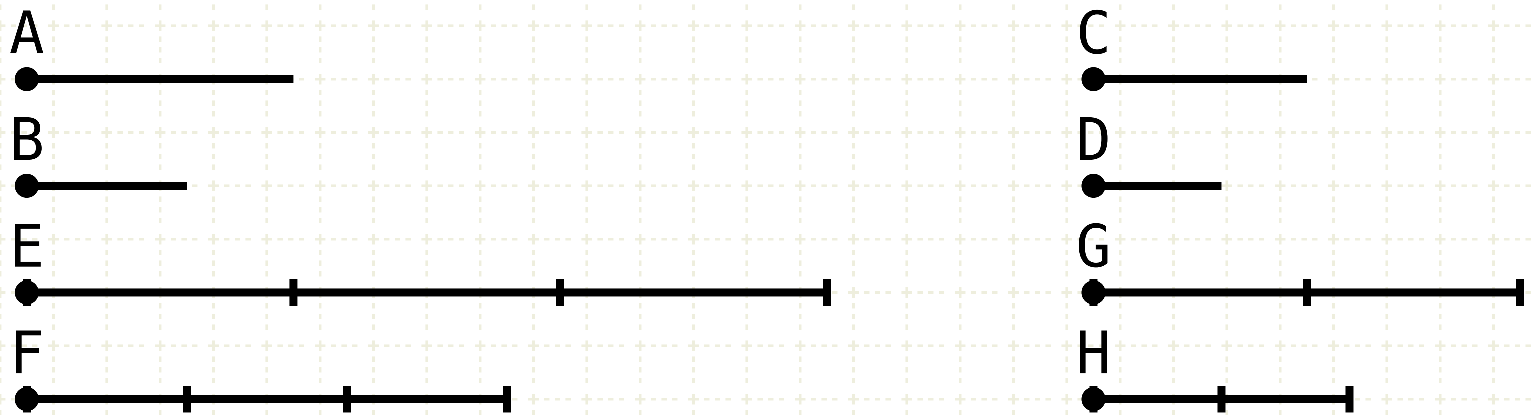

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E = m·A
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E:F = A:B
E:F = C:D
G:H = C:D
G:H = E:F
E >=< G → F >=< H
m·A >=< n·C → m·B >=< n·D
A:C = B:D

In other words

If A,B,C,D are proportional,

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Proof

Let E,F be equimultiples of A and B, and G,H other chance equimultiple of C,D

Since E,F are equimultiples of A,B, their ratios are equal (V·15)

But A is to B as C is to D, therefore E is to F as C is to D (V·11)

Again, since G, H are equimultiples of C,D, therefore as C is to D, so is G to H (V·15)

But C is to D as E is to F, therefore G is to H as E is to F (V·11)

Because G,H,E,F are proportional, then if E is greater than G, then F is greater than H, etc (V·14)

Now E,F are equimultiples of A,B and G,H equimultiples of C,D so …

… A is to C as B is to D (V·def·5)

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