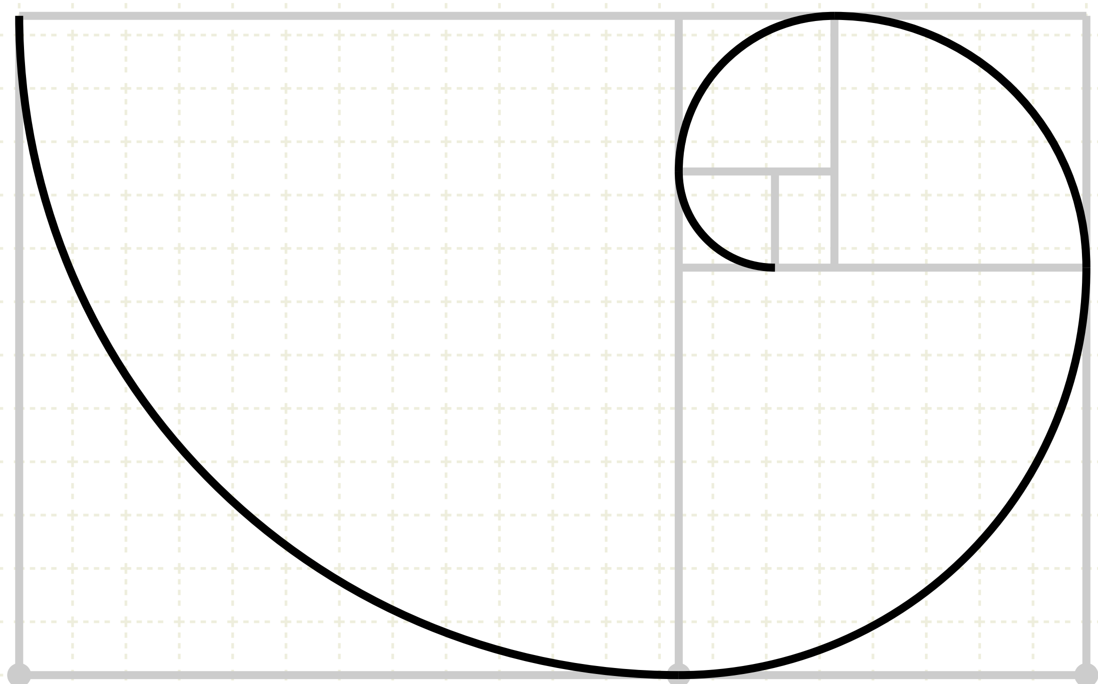
Euclid's Elements

Book VI



One can state, without exaggeration, that the observation of and the search for similarities and differences are the basis of all human knowledge.

Alfred Nobel

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Proposition 5 of Book VI

It two triangles have their sides proportional, the triangles will be equiangular and will have those angles equal which the corresponding sides subtend

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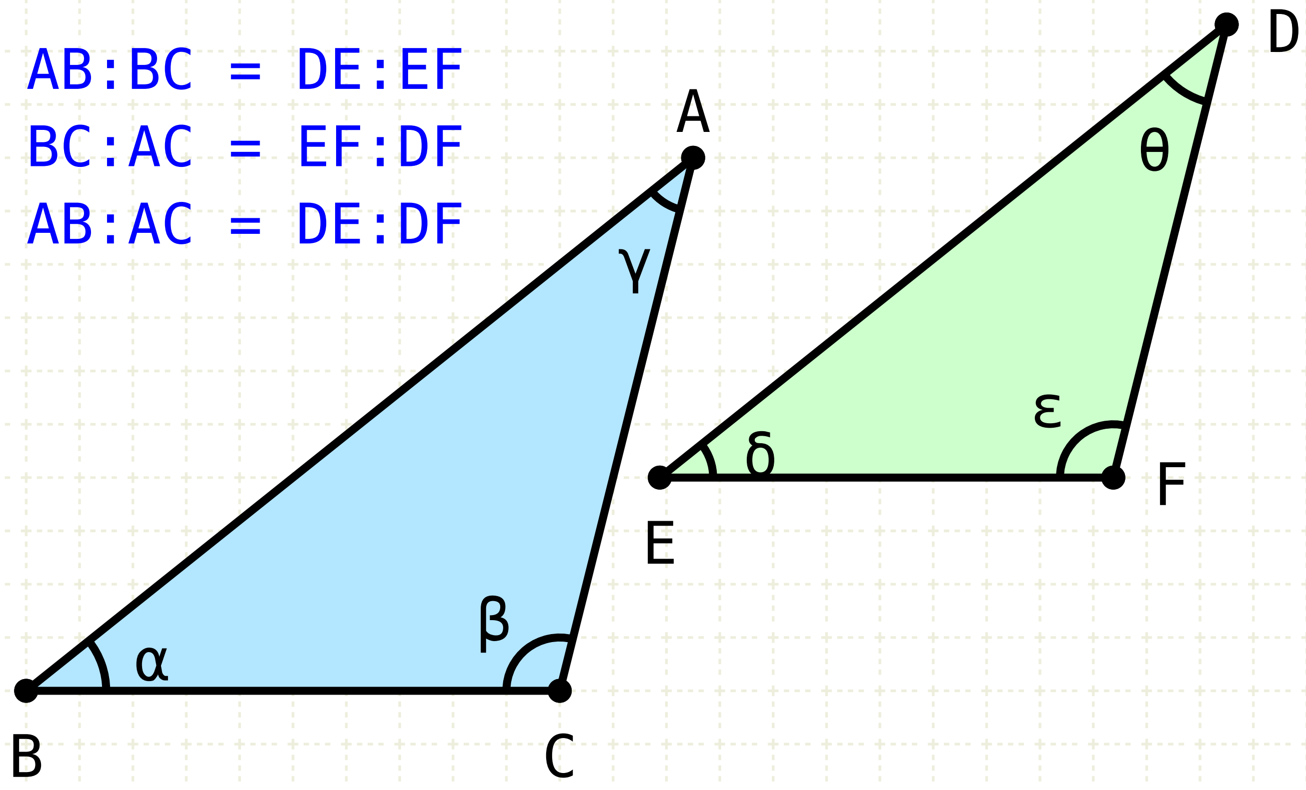
It two triangles have their sides proportional, the triangles will be equiangular and will have those angles equal which the corresponding sides subtend

$AB:BC = DE:EF$
$BC:AC = EF:DF$
$AB:AC = DE:DF$



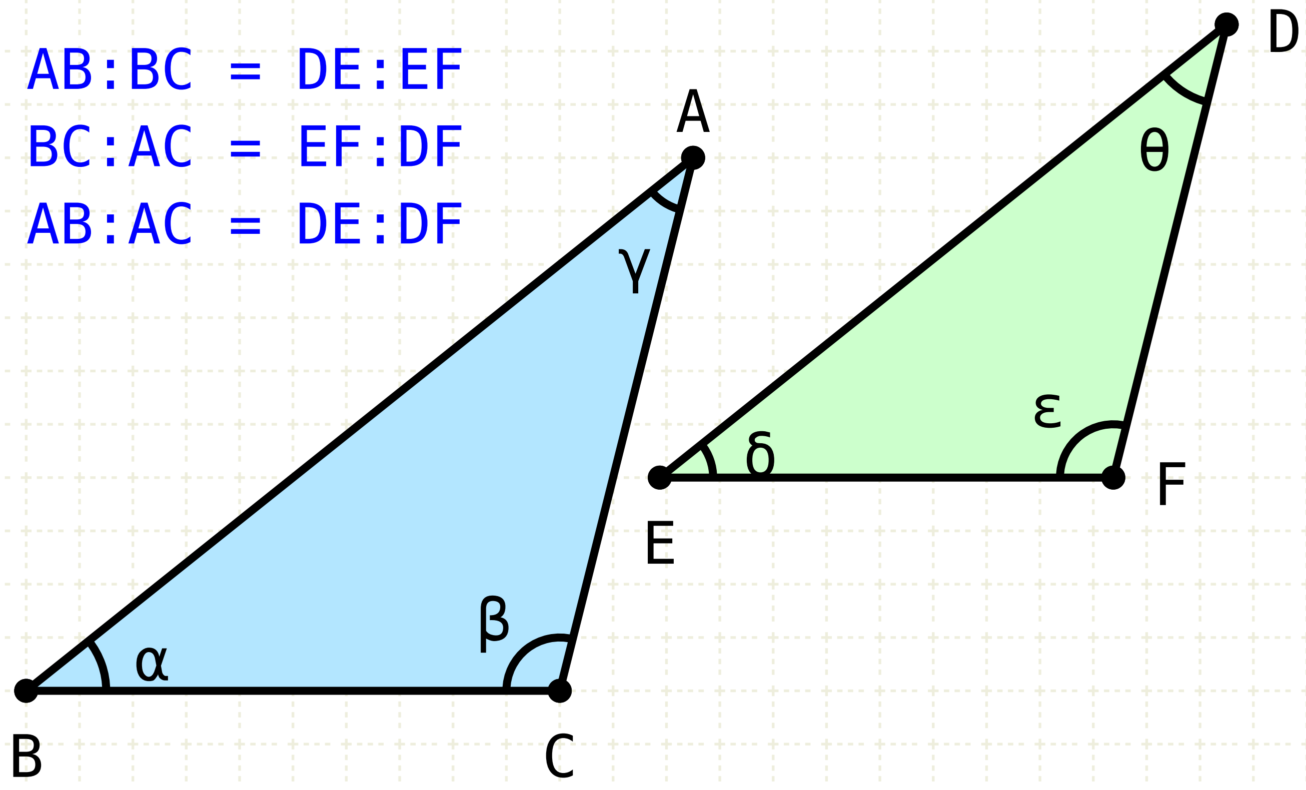
$\alpha = \delta,\ \beta = \varepsilon,\ \gamma = \theta$

In other words

If side 'a' is to side 'b' of one triangle, and is equal to side 'd' to 'e' of another, and similarly for all sides, then the angle between 'a' and 'b' will be equal to the angles between 'd' and 'e'

Proposition 5 of Book VI

It two triangles have their sides proportional, the triangles will be equiangular and will have those angles equal which the corresponding sides subtend



Proof

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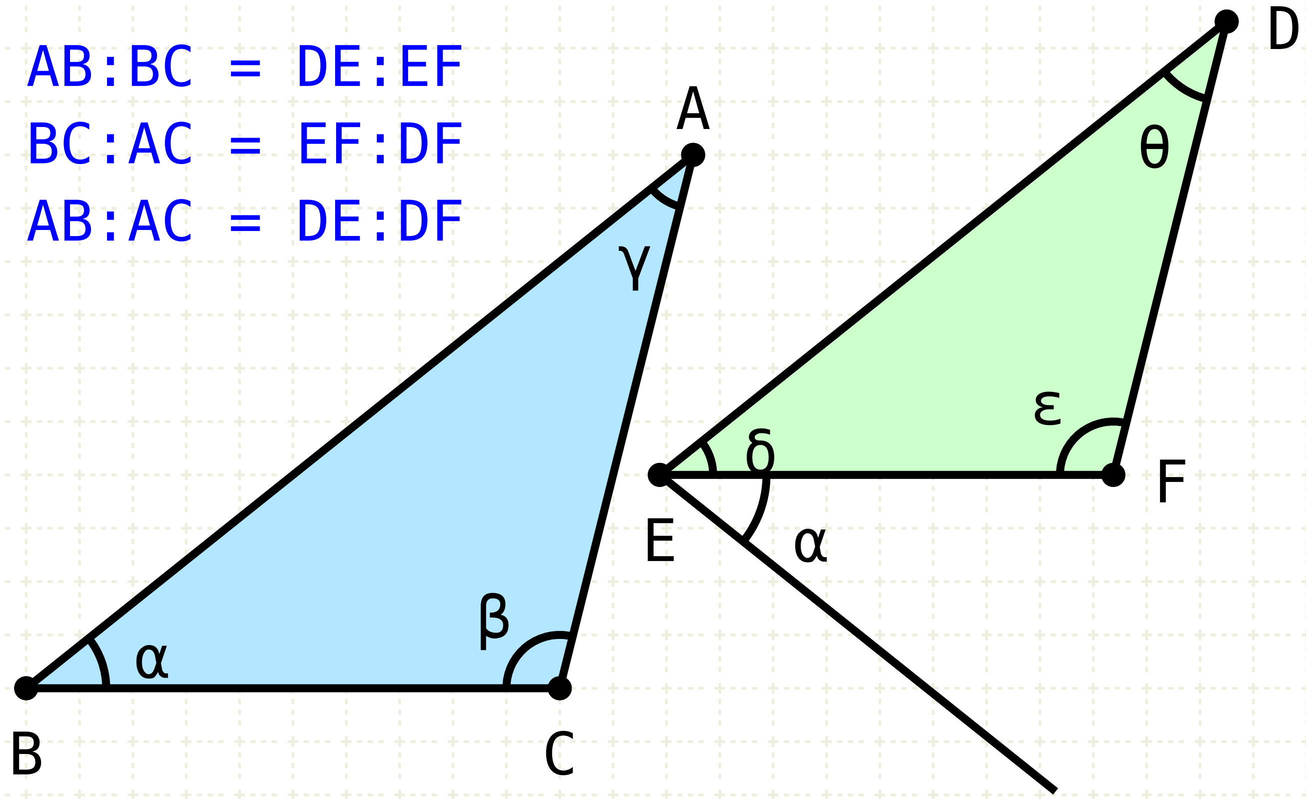

Proof

On the point E, construct an angle FEG on the line EF equal to the angle α (I·23)

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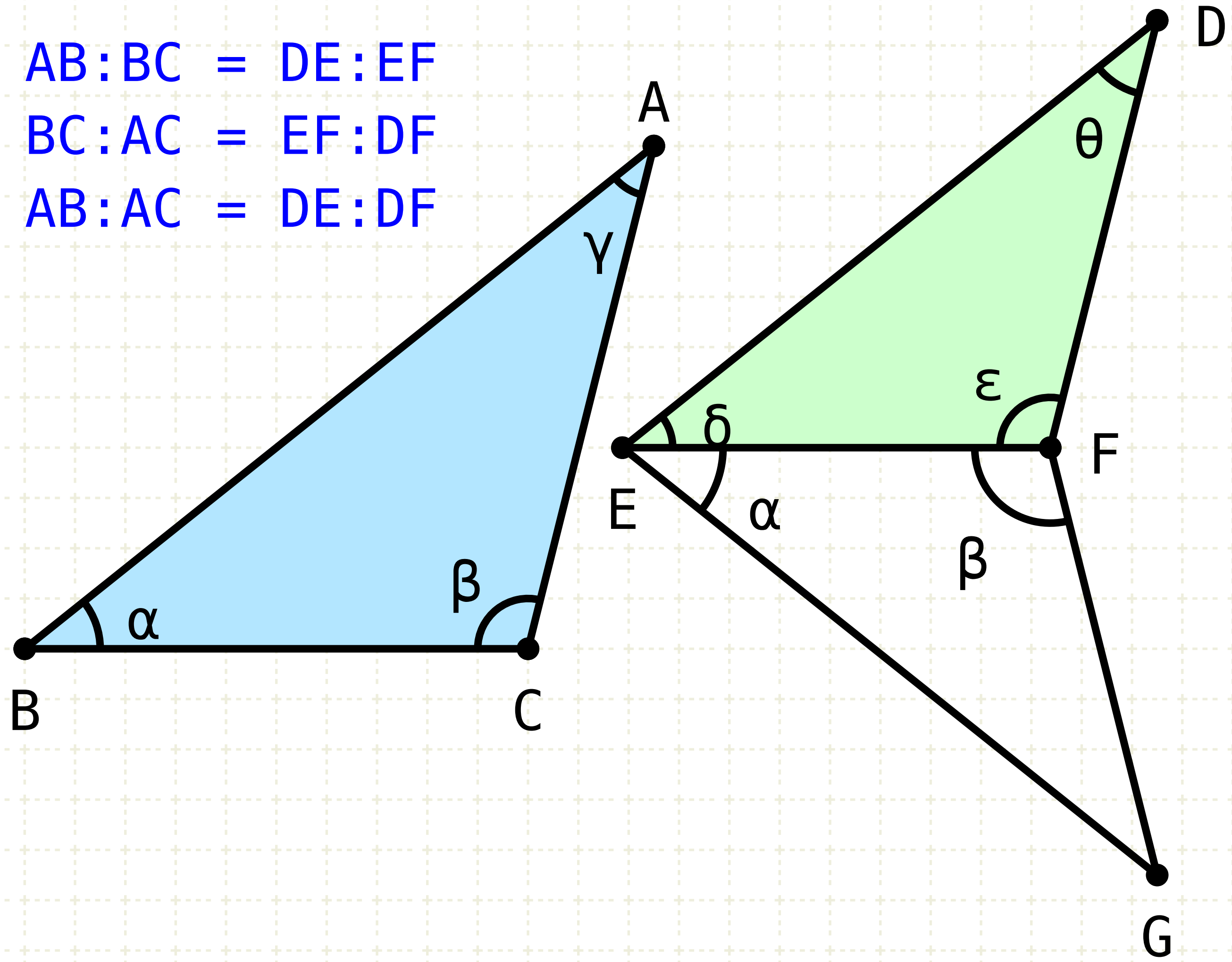

Proof

On the point E, construct an angle FEG on the line EF equal to the angle α (I·23)

On the point F, construct an angle EFG on the line EF equal to the angle β (I·23)

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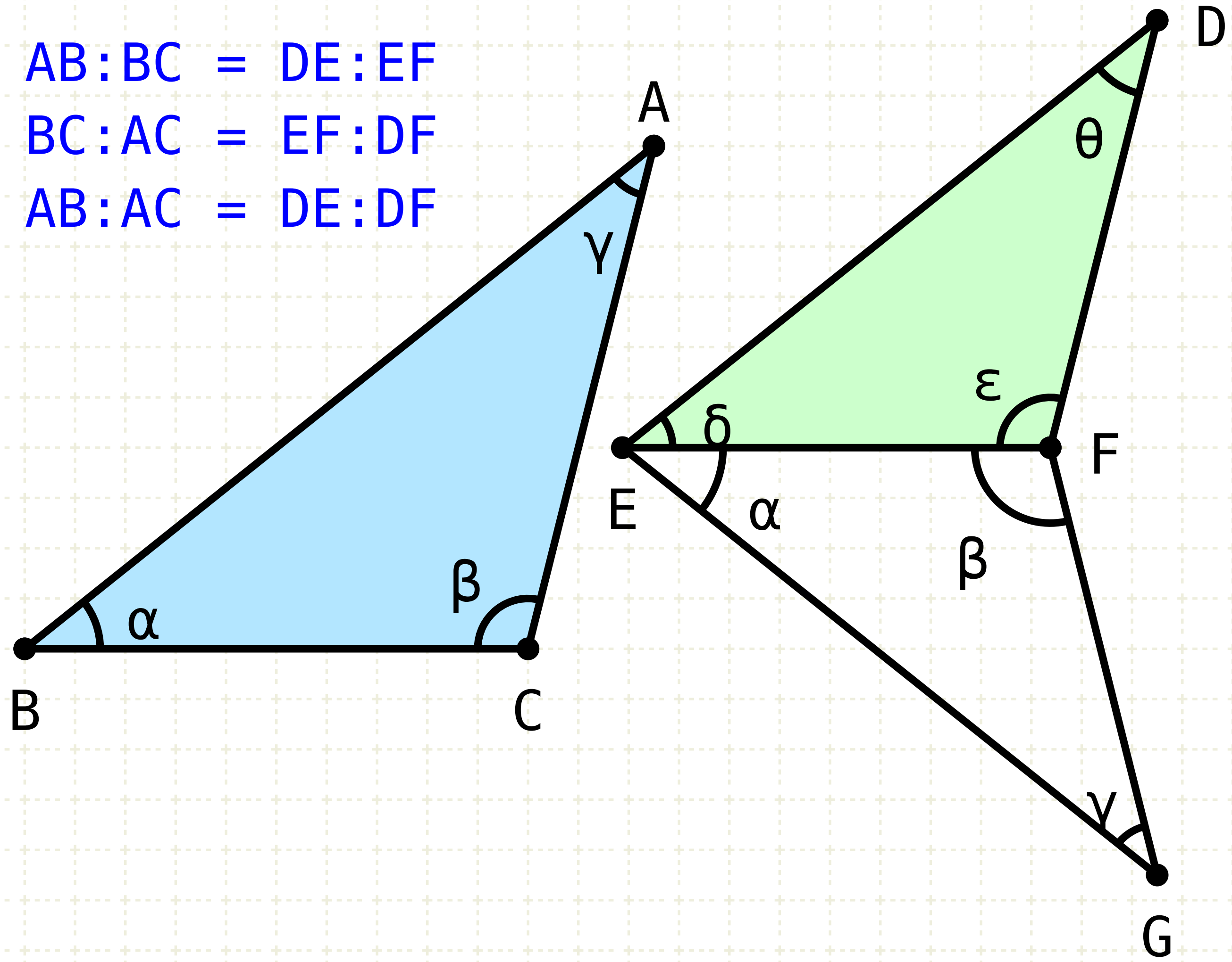

Proof

On the point E, construct an angle FEG on the line EF equal to the angle α (I·23)

On the point F, construct an angle EFG on the line EF equal to the angle β (I·23)

And thus, the angle at G will also be the angle at A (I·32)

Proposition 5 of Book VI

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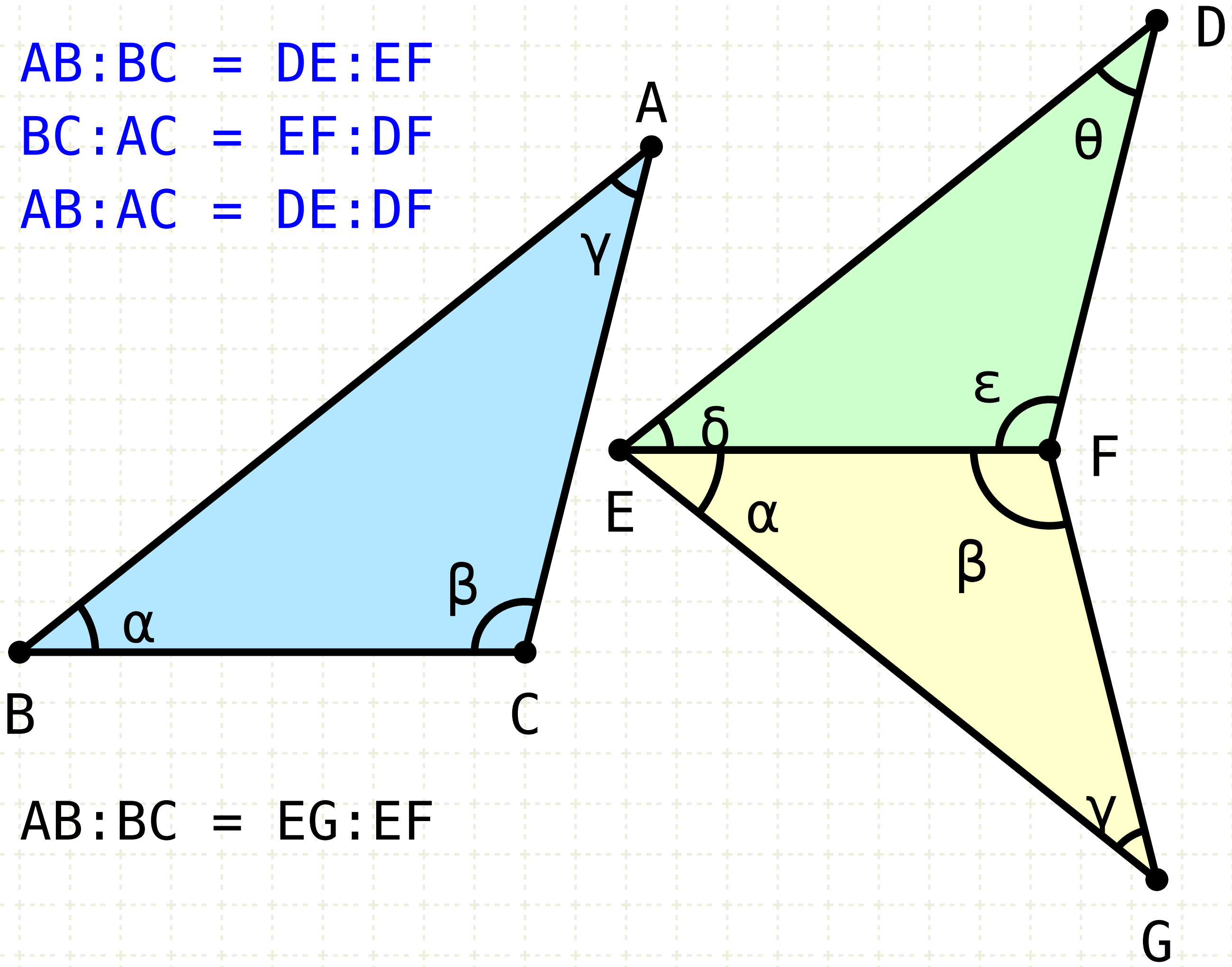

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On the point F, construct an angle EFG on the line EF equal to the angle β (I·23)

And thus, the angle at G will also be the angle at A (I·32)

Therefore the triangle ABC is equiangular to EFG, and as such, the edges surrounding the equal angles will be in proportion, i.e. AB is to BC as EG to EF (VI·4)

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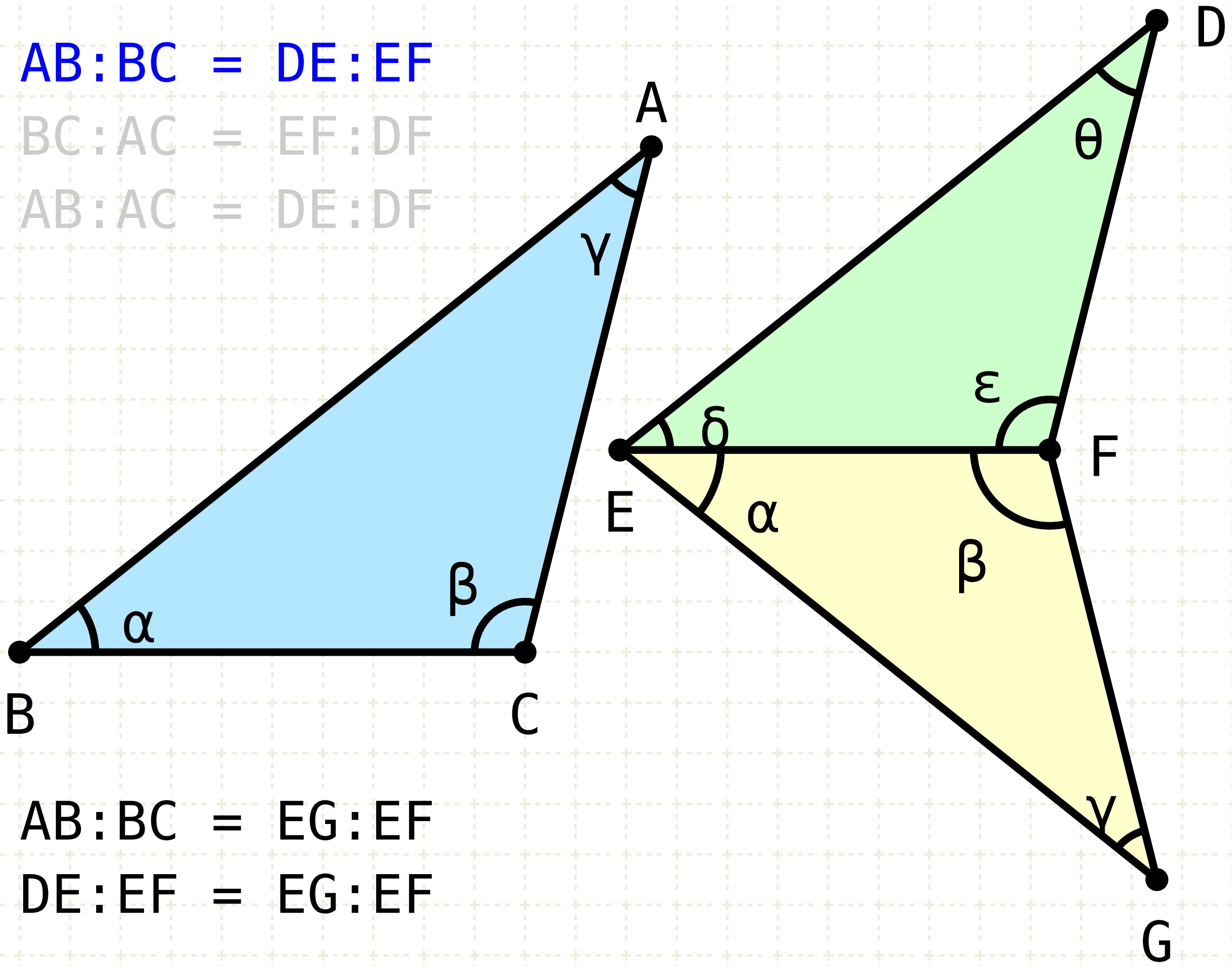

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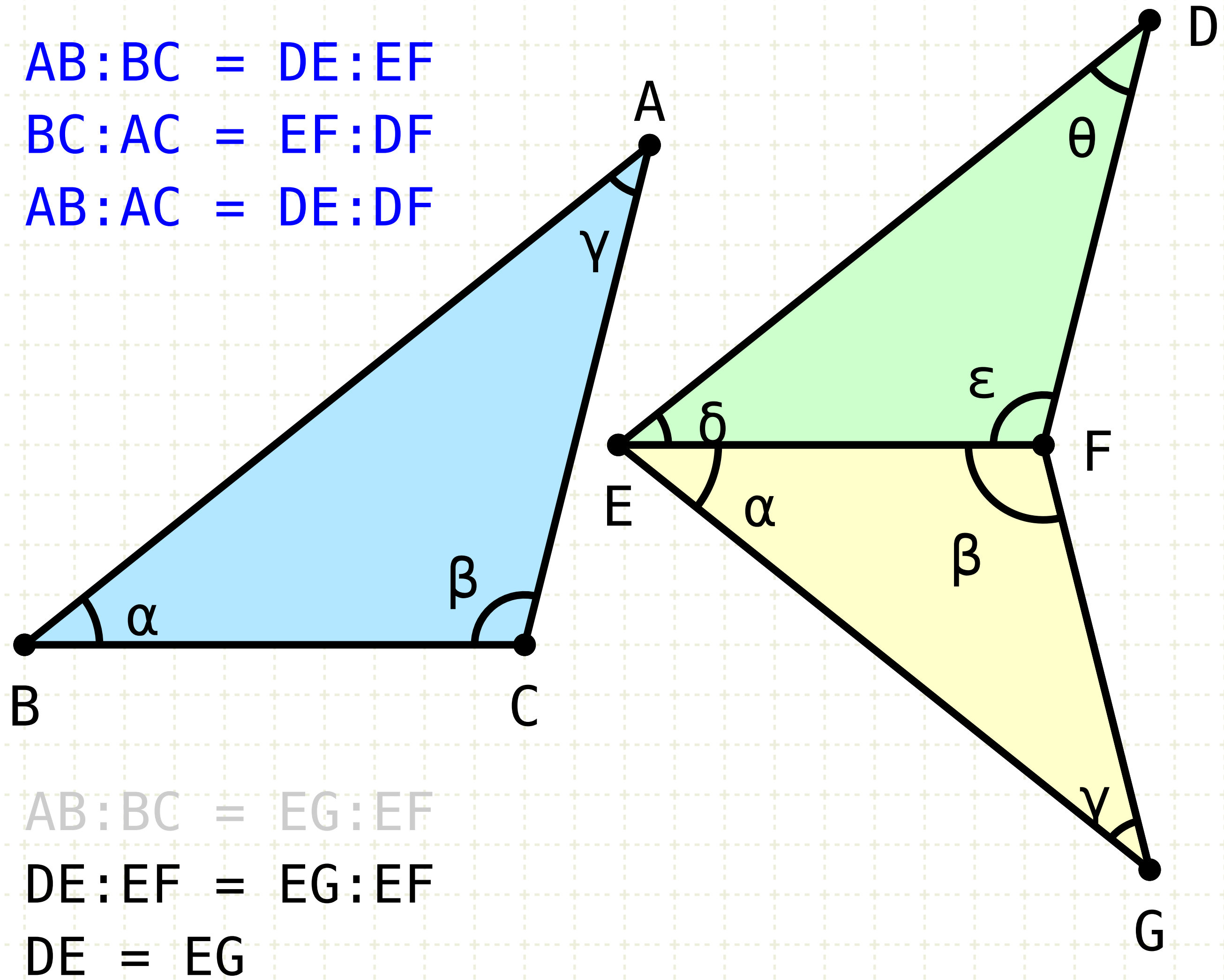
Therefore the triangle ABC is equiangular to EFG, and as such, the edges surrounding the equal angles will be in proportion, i.e. AB is to BC as EG to EF (VI·4)

But the ratio AB to BC is equal to DE to EF, therefore the ratio DE to EF equals EG to EF (V·11)

Proposition 5 of Book VI

It two triangles have their sides proportional, the triangles will be equiangular and will have those angles equal which the corresponding sides subtend

AB:BC = DE:EF
BC:AC = EF:DF
AB:AC = DE:DF



AB:BC = EG:EF
DE:EF = EG:EF
DE = EG

Proof

On the point E, construct an angle FEG on the line EF equal to the angle α (I·23)

On the point F, construct an angle EFG on the line EF equal to the angle β (I·23)

And thus, the angle at G will also be the angle at A (I·32)

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But the ratio AB to BC is equal to DE to EF, therefore the ratio DE to EF equals EG to EF (V·11)

Since DE and EG have the same ratio to EF, DE and EG are equal (V·9),

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AB:BC = DE:EF
BC:AC = EF:DF
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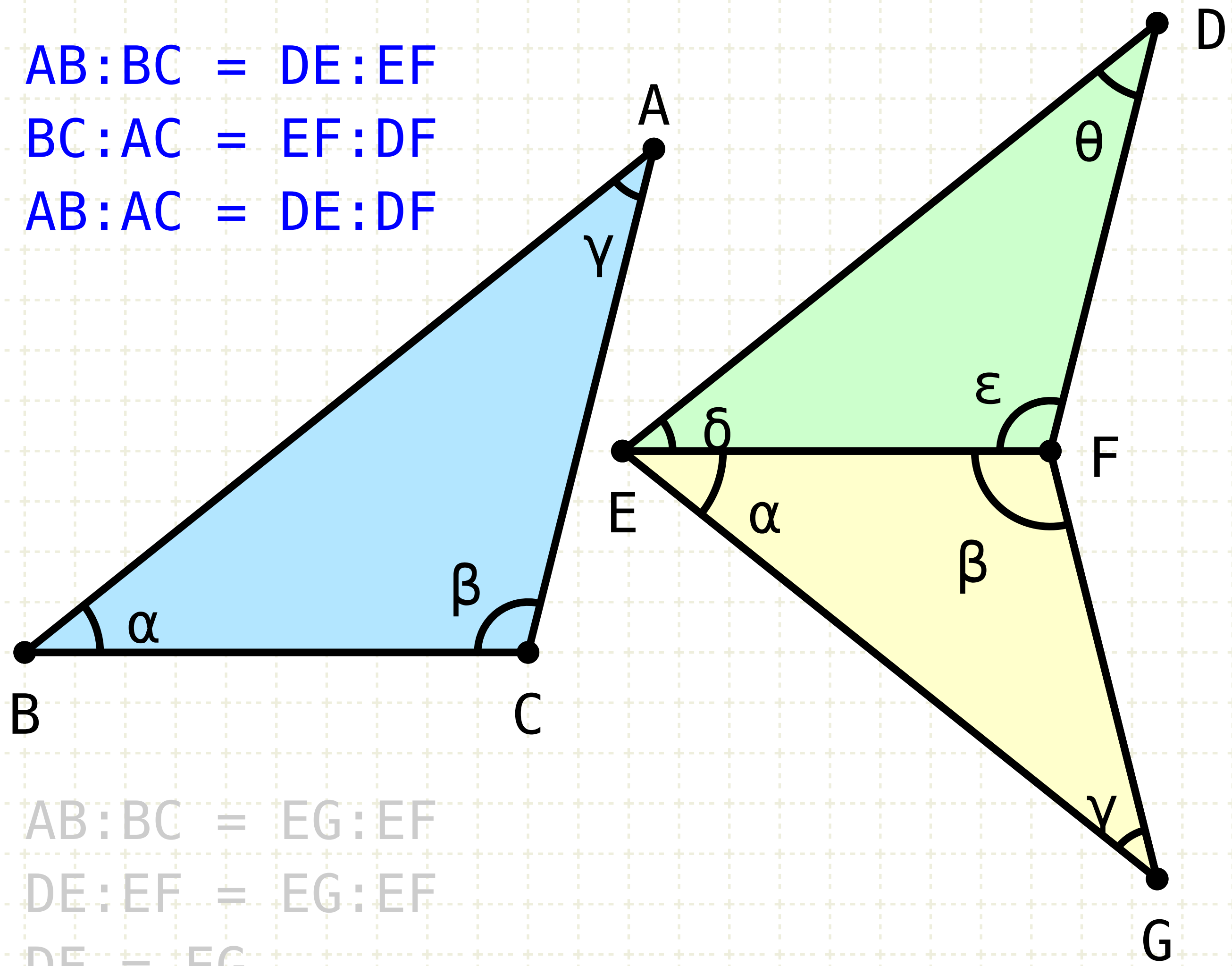

AB:BC = EG:EF
DE:EF = EG:EF
DE = EG
DF = FG

Proof

On the point E, construct an angle FEG on the line EF equal to the angle α (I·23)

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Since DE and EG have the same ratio to EF, DE and EG are equal (V·9),

and for the same reason DF is also equal to FG

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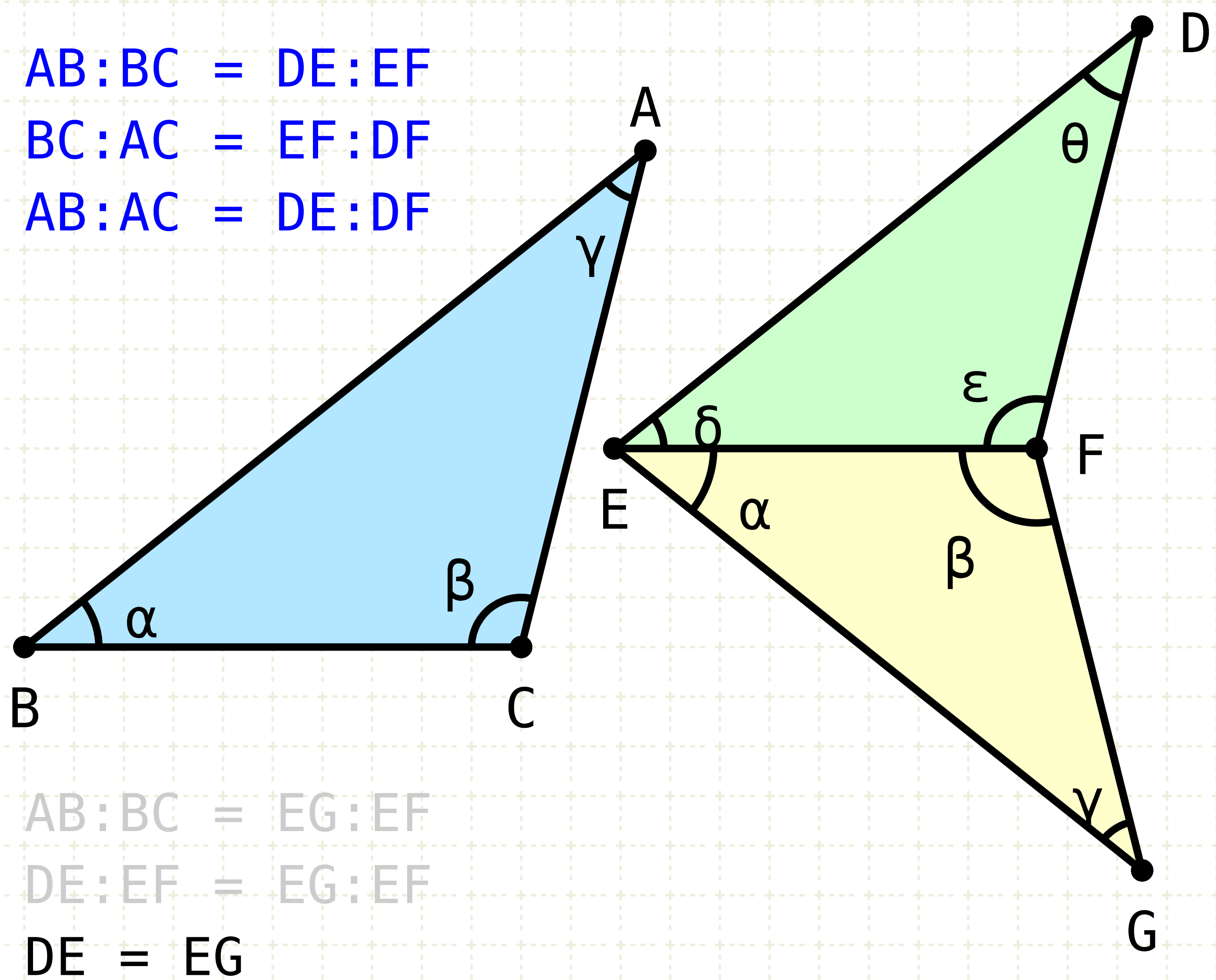

AB:BC = EG:EF

DE:EF = EG:EF

DE = EG

DF = FG

α = δ

Proof

On the point E, construct an angle FEG on the line EF equal to the angle α (I·23)

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Since DE and EG have the same ratio to EF, DE and EG are equal (V·9),

and for the same reason DF is also equal to FG

Since DE is equal to EG, and DF equals FG, and there is a common base EF (three sides equal) then the angle DEF is equal to GEF (I·8),

Proposition 5 of Book VI

It two triangles have their sides proportional, the triangles will be equiangular and will have those angles equal which the corresponding sides subtend

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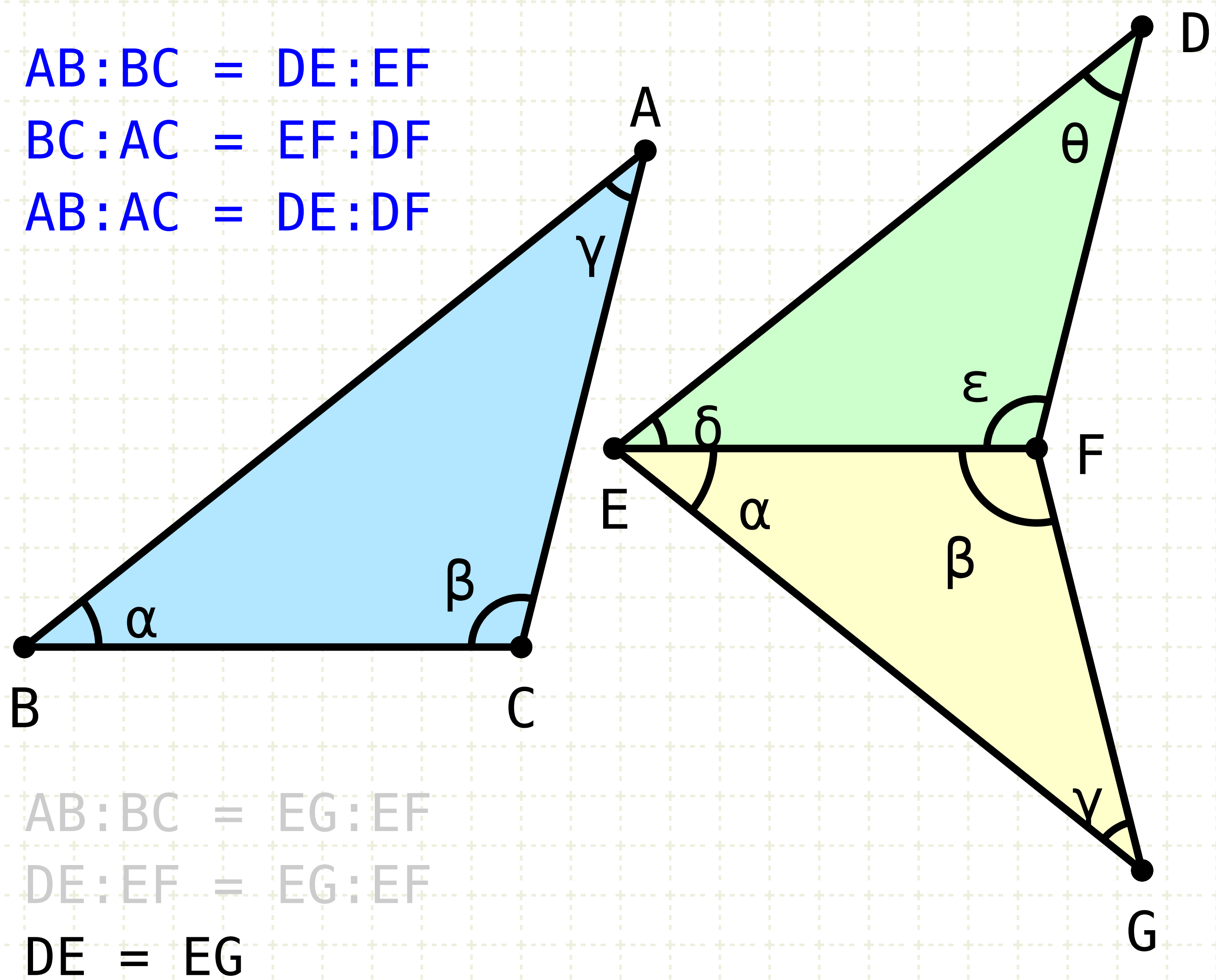

$AB:BC = EG:EF$

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$DF = FG$

$\alpha = \delta$

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Since DE and EG have the same ratio to EF, DE and EG are equal (V·9),

and for the same reason DF is also equal to FG

Since DE is equal to EG, and DF equals FG, and there is a common base EF (three sides equal) then the angle DEF is equal to GEF (I·8),

and finally, since there are two equal sides subtending an equal angle, both triangles DEF and EFG are equal (I·4)

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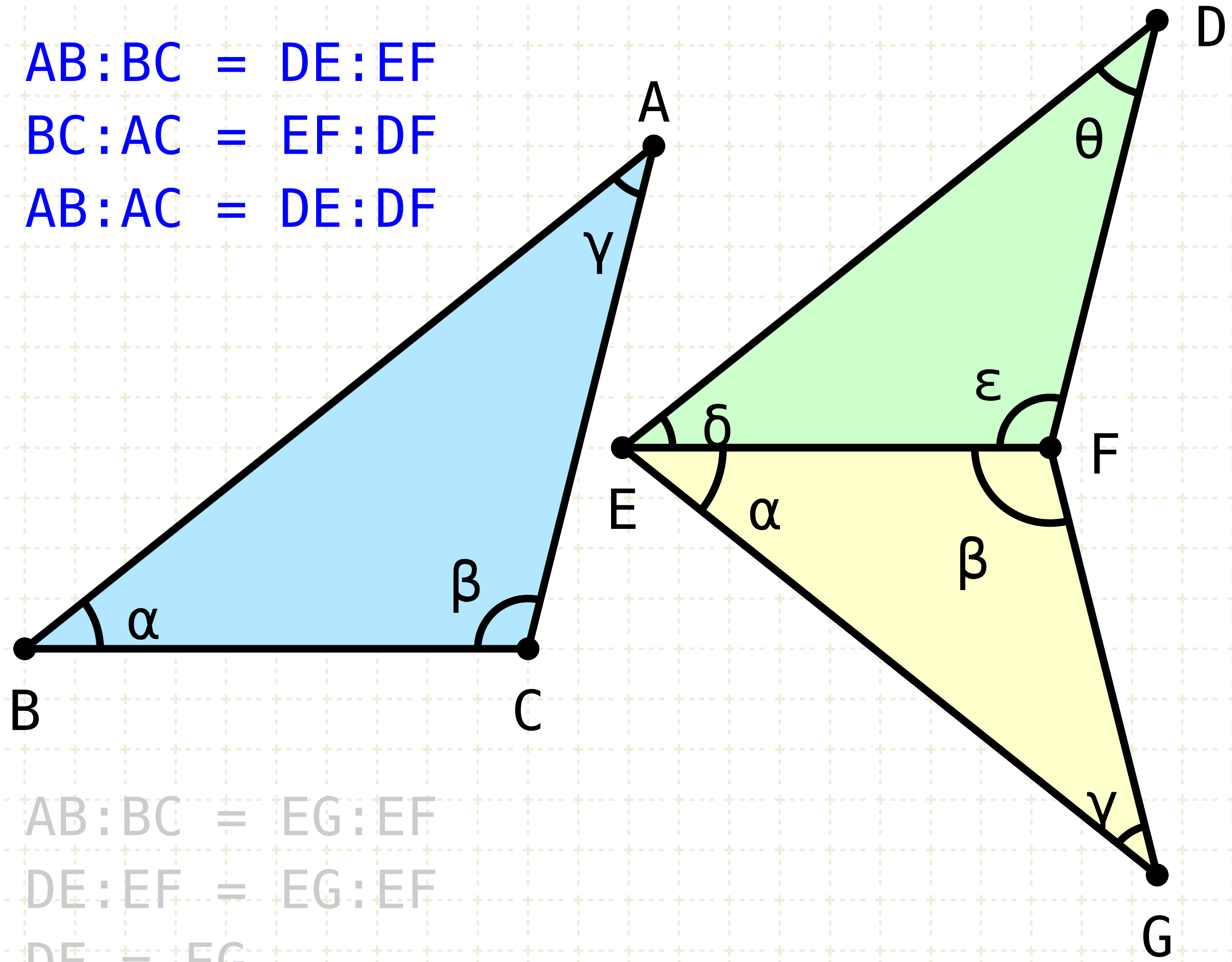

$AB:BC = EG:EF$

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Therefore the triangle ABC is equiangular to EFG, and as such, the edges surrounding the equal angles will be in proportion, i.e. AB is to BC as EG to EF (VI·4)

But the ratio AB to BC is equal to DE to EF, therefore the ratio DE to EF equals EG to EF (V·11)

Since DE and EG have the same ratio to EF, DE and EG are equal (V·9),

and for the same reason DF is also equal to FG

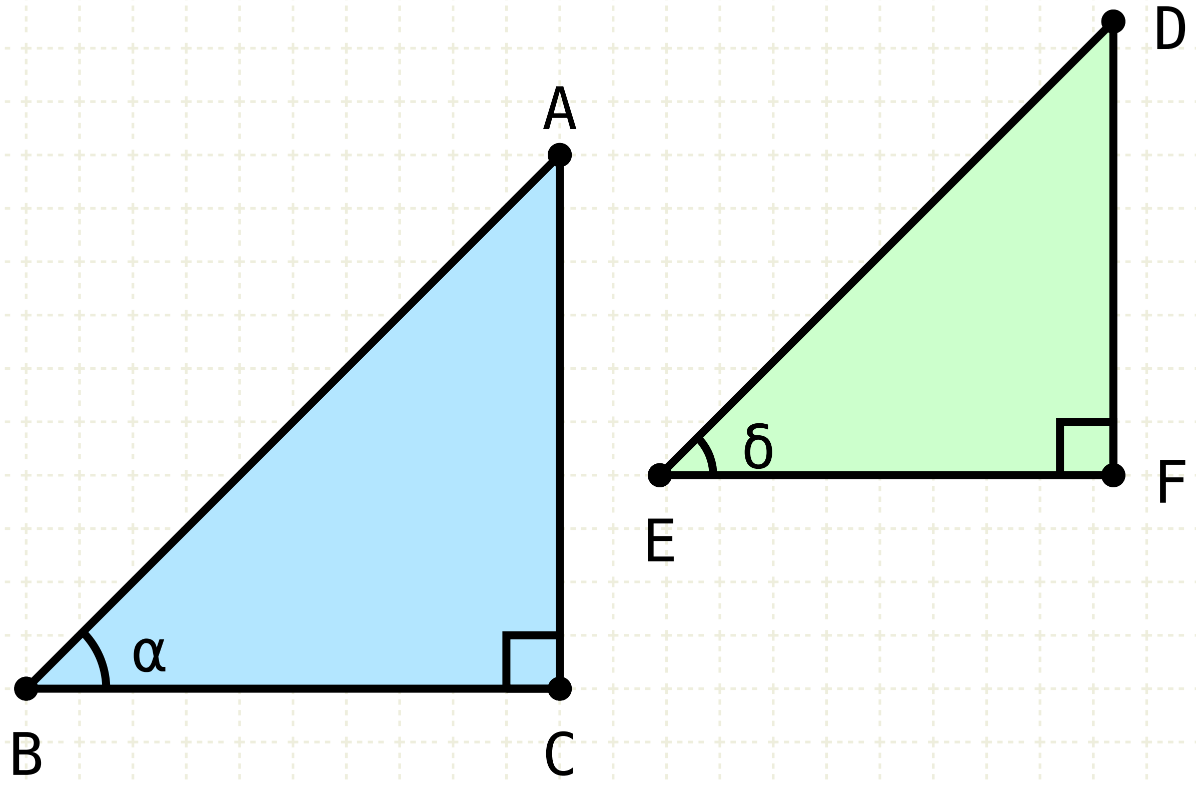
Since DE is equal to EG, and DF equals FG, and there is a common base EF (three sides equal) then the angle DEF is equal to GEF (I·8),

and finally, since there are two equal sides subtending an equal angle, both triangles DEF and EFG are equal (I·4)

So finally, the triangle DEF is equiangular to triangle ABC

Proposition 5 of Book VI

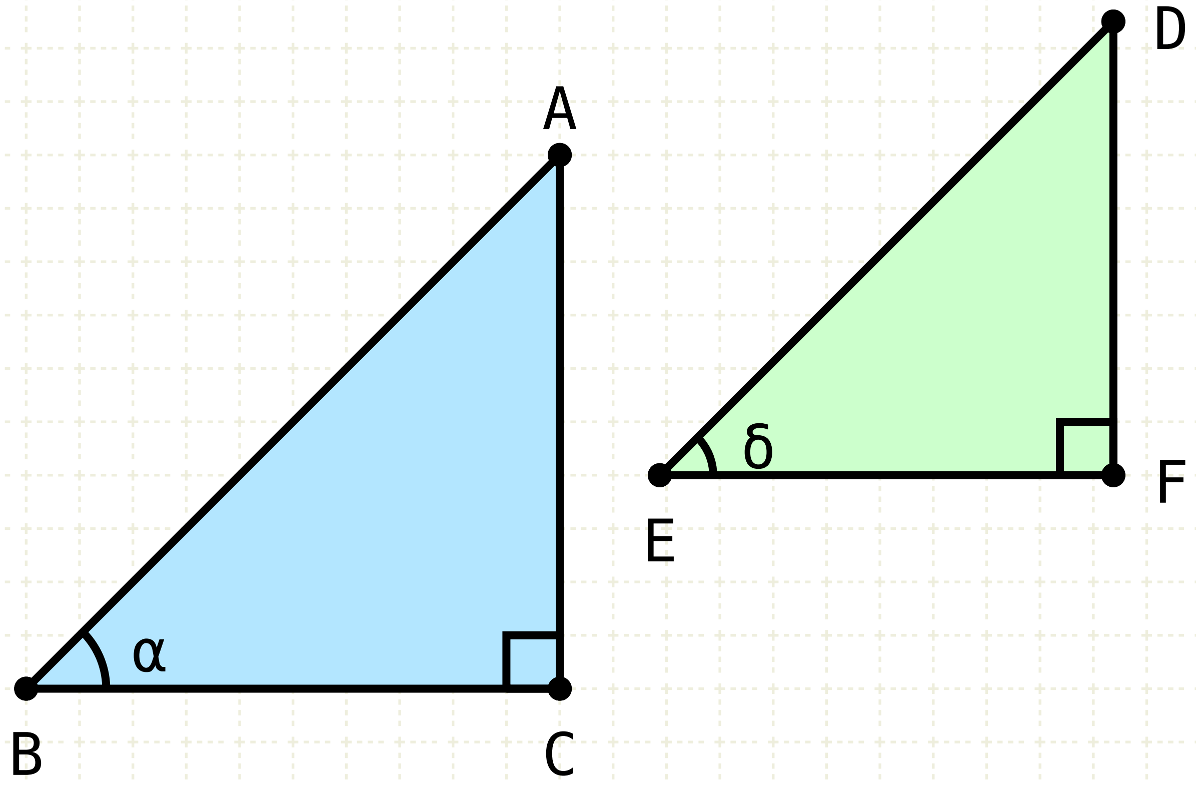
It two triangles have their sides proportional, the triangles will be equiangular and will have those angles equal which the corresponding sides subtend



Aside - Trigonometry

Proposition 5 of Book VI

It two triangles have their sides proportional, the triangles will be equiangular and will have those angles equal which the corresponding sides subtend



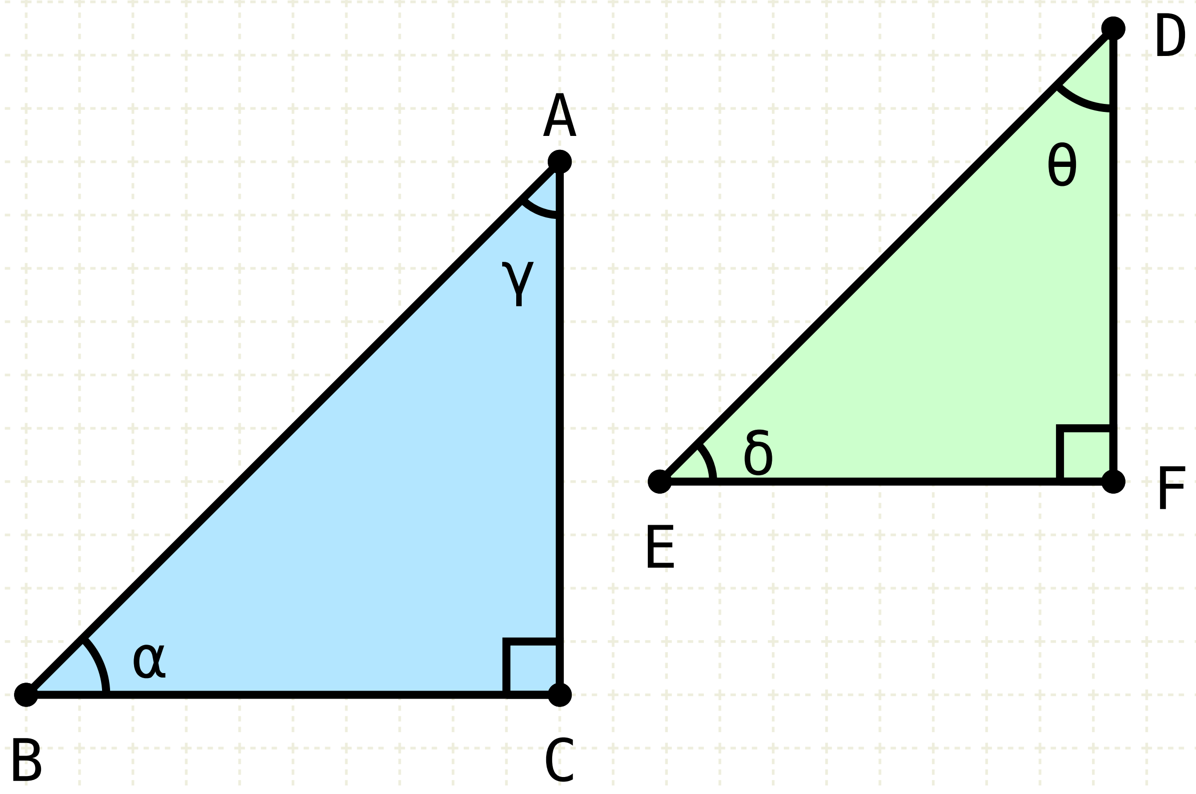
$\alpha = \delta$

Aside - Trigonometry

Consider two right angle triangles where angle ABC equals angle DEF

Proposition 5 of Book VI

It two triangles have their sides proportional, the triangles will be equiangular and will have those angles equal which the corresponding sides subtend



$\alpha = \delta$

$\gamma = \theta$

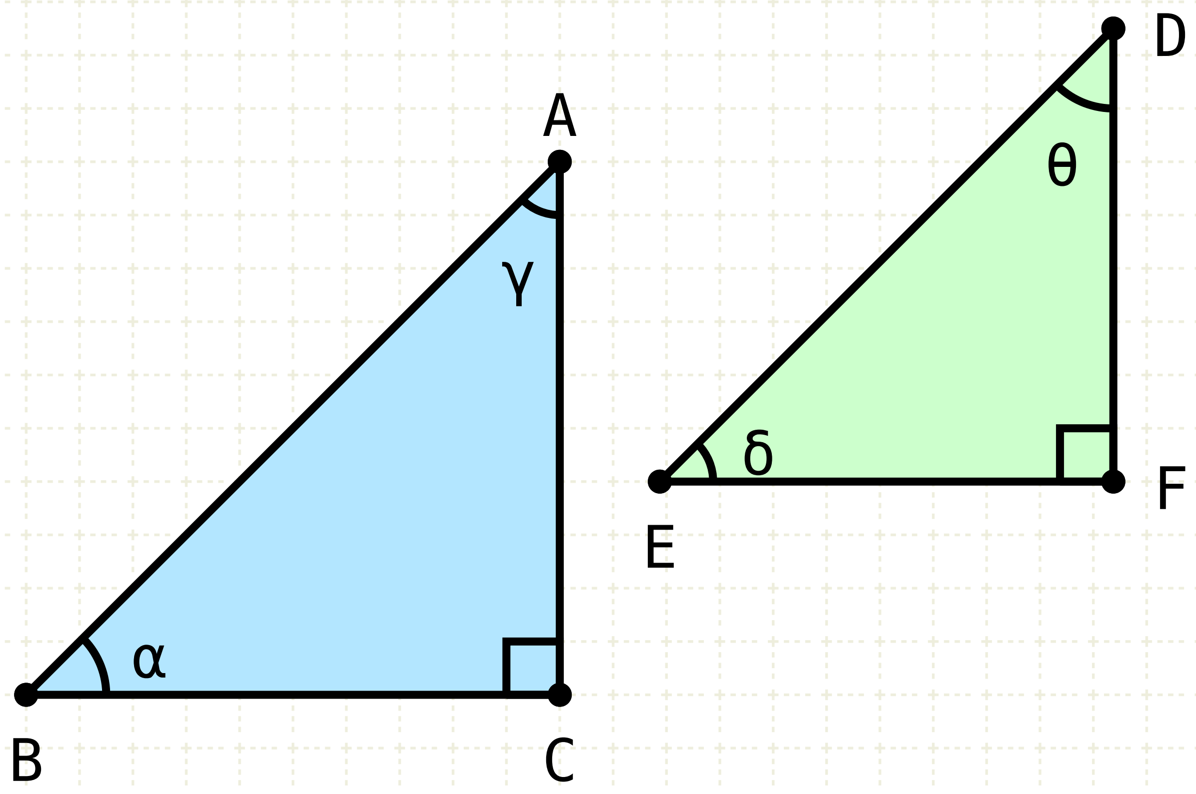
Aside - Trigonometry

Consider two right angle triangles where angle ABC equals angle DEF

Since two of the angles are equal in both triangles, the third must also be equal, hence angle BAC equals EDF

Proposition 5 of Book VI

It two triangles have their sides proportional, the triangles will be equiangular and will have those angles equal which the corresponding sides subtend



$\alpha = \delta$

$\gamma = \theta$

AC:AB = DF:DE

Aside - Trigonometry

Consider two right angle triangles where angle ABC equals angle DEF

Since two of the angles are equal in both triangles, the third must also be equal, hence angle BAC equals EDF

Then, according to (VI·4), the ratio of the sides will be equal, in other words, AC to AB equals DF to DE, etc

Proposition 5 of Book VI

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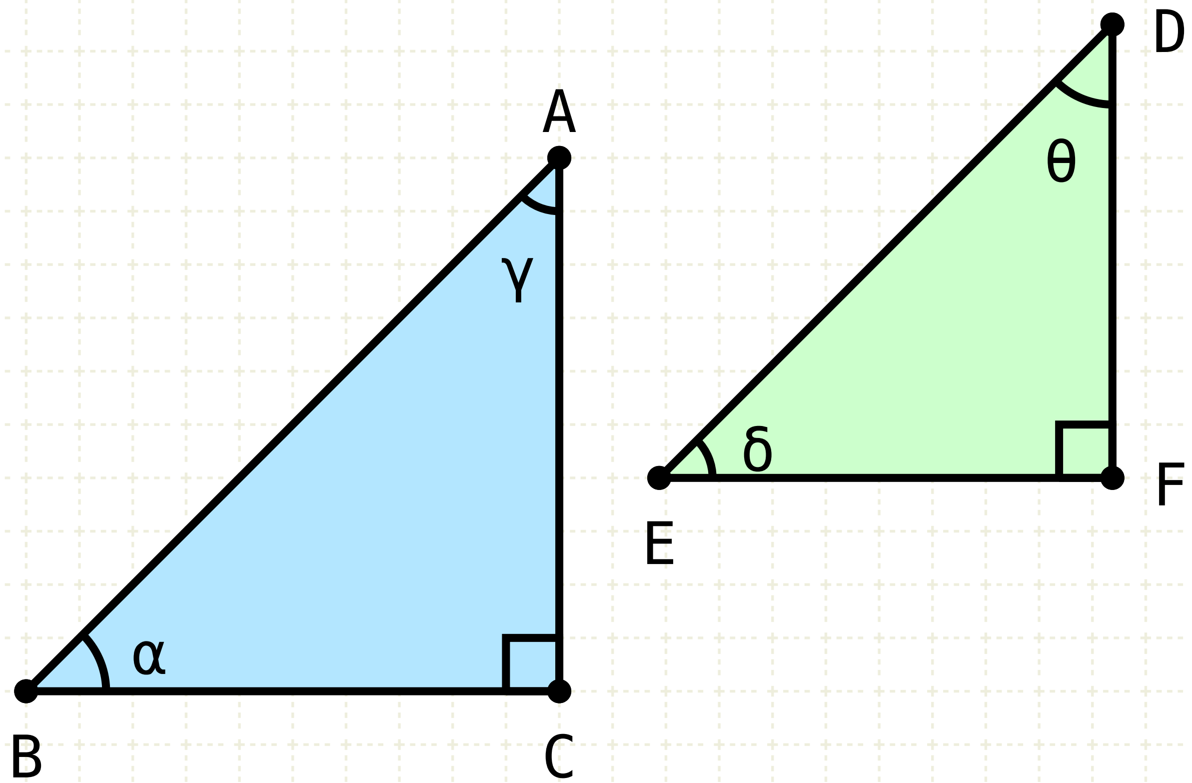

AC:AB = DF:DE

Aside - Trigonometry

Conversly, consider two right triangles where AC to AB equals DF to DE

Proposition 5 of Book VI

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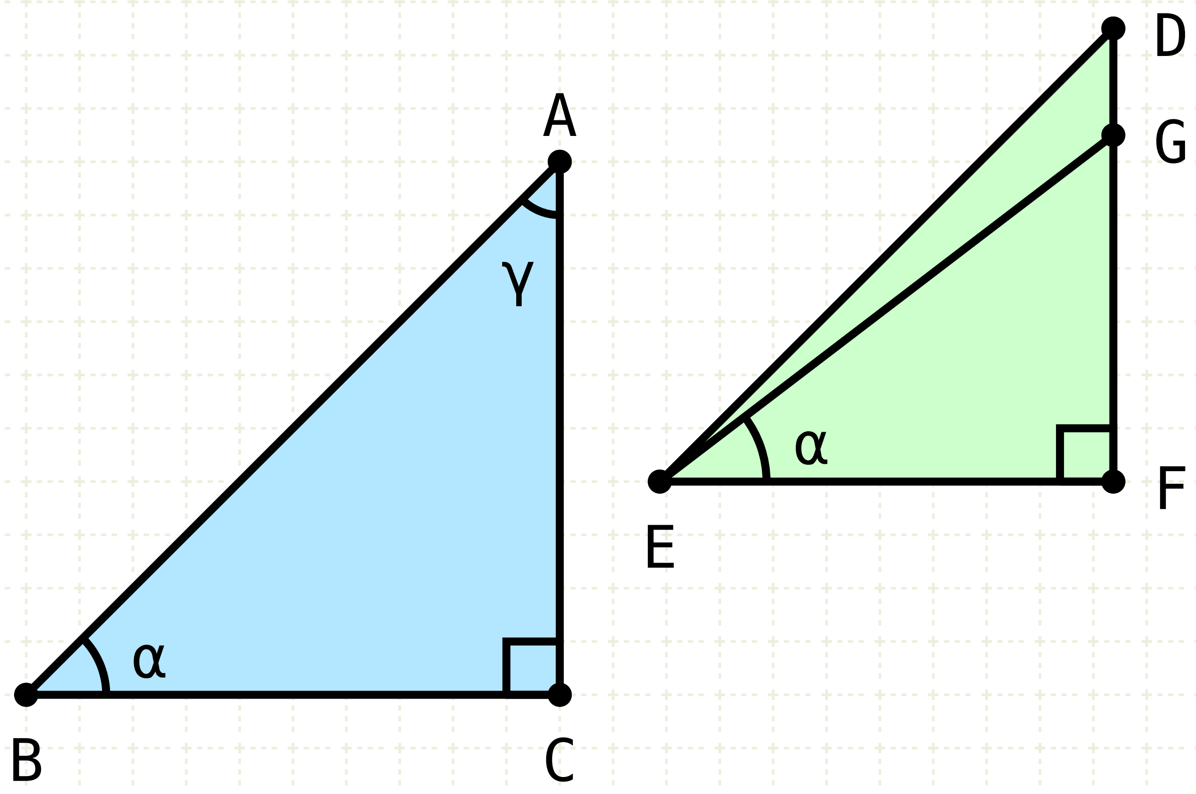


AC:AB = DF:DE

Aside - Trigonometry

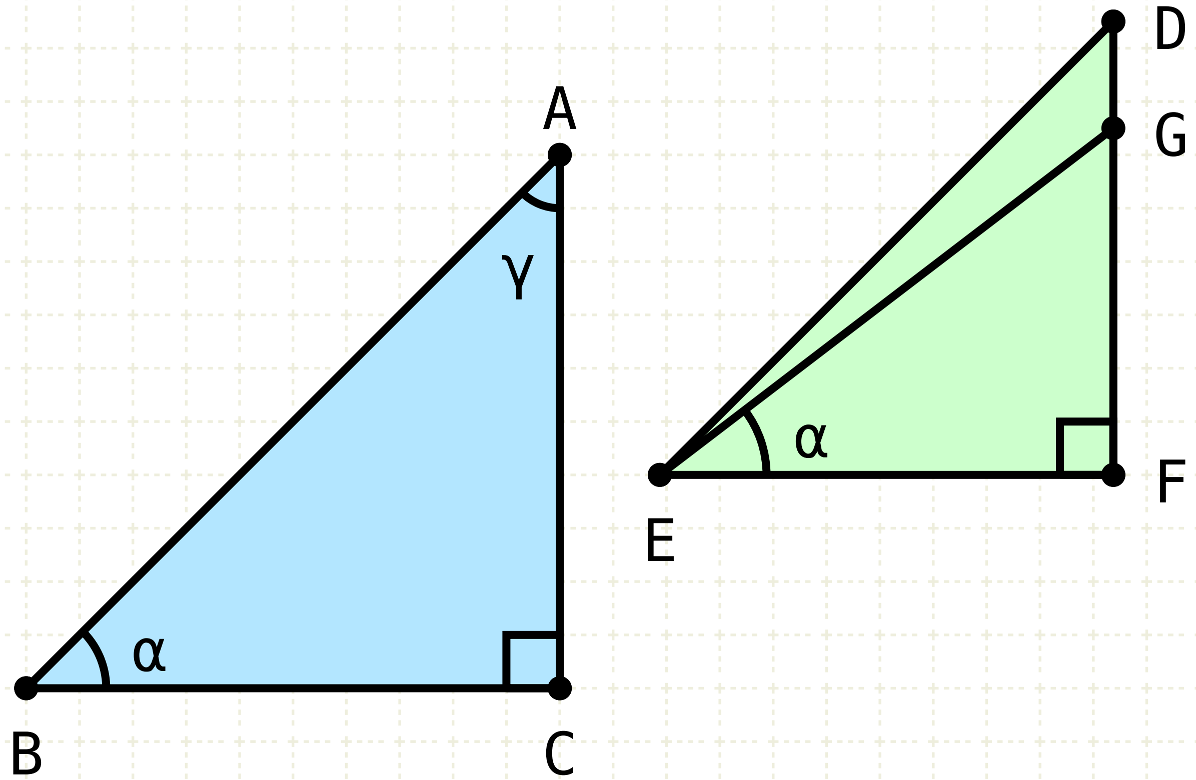
Conversly, consider two right triangles where AC to AB equals DF to DE

Assume that α is not equal to angle δ…

Draw line EG, such that angle GEF equals α

Proposition 5 of Book VI

It two triangles have their sides proportional, the triangles will be equiangular and will have those angles equal which the corresponding sides subtend



AC:AB = DF:DE

FG:EG = AC:AB = DF:DE

Aside - Trigonometry

Conversly, consider two right triangles where AC to AB equals DF to DE

Assume that α is not equal to angle δ…

Draw line EG, such that angle GEF equals α

Triangle GEF is equiangular to ABC, so therefore AC to BE equals FG to EG

Proposition 5 of Book VI

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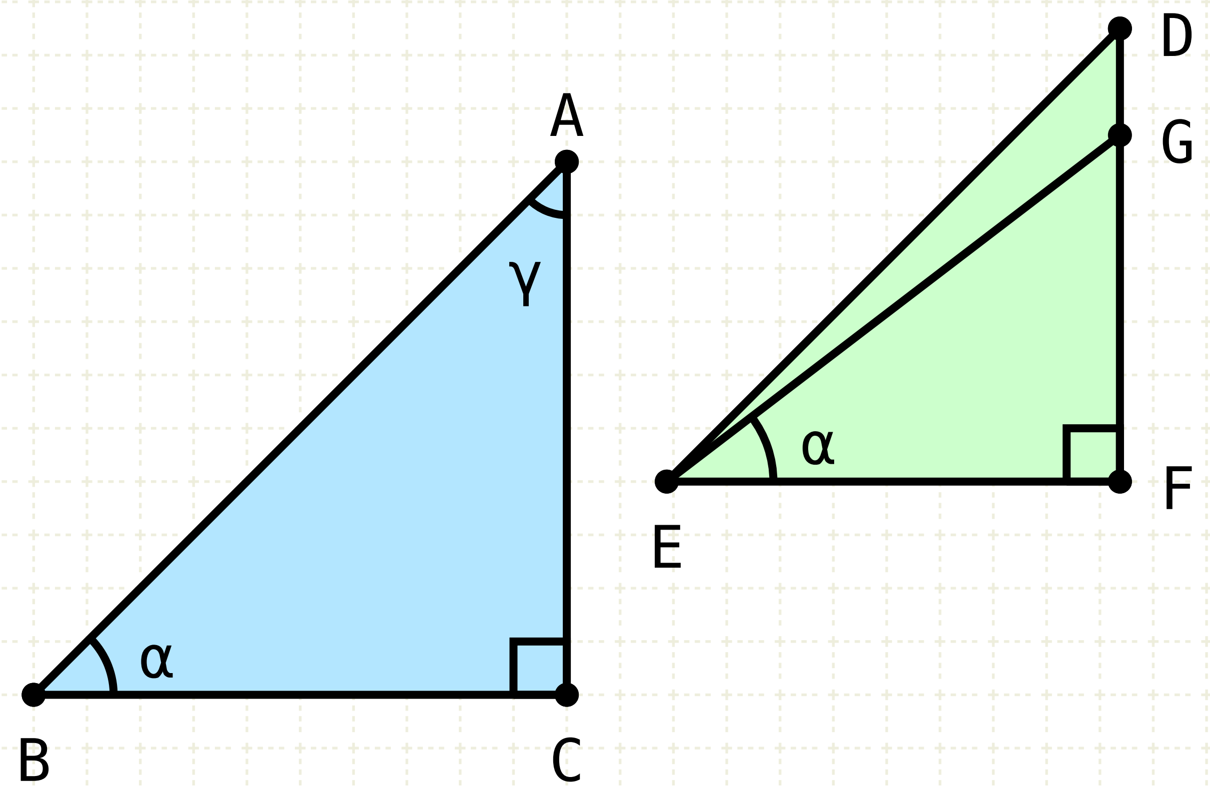

$AC:AB = DF:DE$

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Conversly, consider two right triangles where AC to AB equals DF to DE

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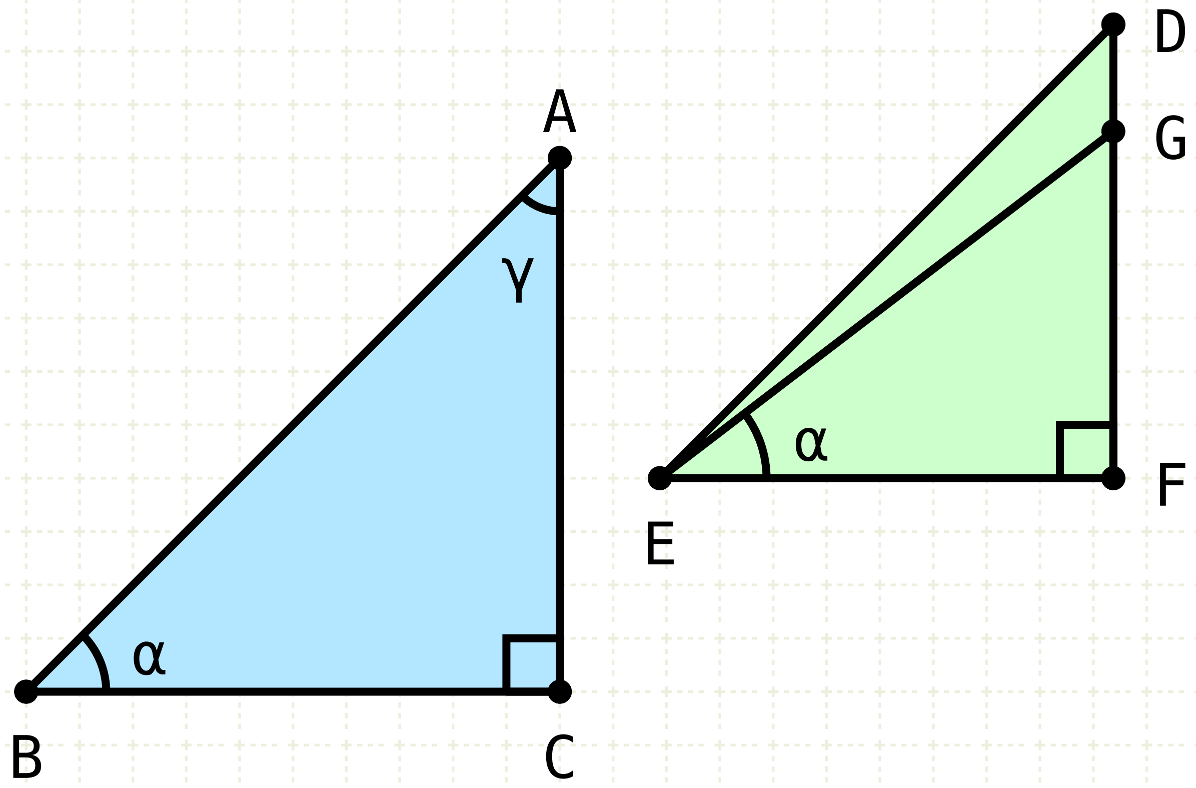
Triangle GEF is equiangular to ABC, so therefore AC to BE equals FG to EG

With a bit of math (pythagoras' theorem), it can be shown that the point G must be the same point as D

Proposition 5 of Book VI

It two triangles have their sides proportional, the triangles will be equiangular and will have those angles equal which the corresponding sides subtend



$c = ED,\ b = DF,\ a = EF,\ d = FG,\ e = EG$

$FG:EG = AC:AB = DF:DE \rightarrow d/e = b/c$

$a^2+b^2 = c^2;\ a^2+d^2 = e^2$

$a^2+(b\cdot(e/c))^2 = e^2$

$a^2 = e^2\cdot(1-(b^2/c^2))$

$a^2 = e^2\cdot((c^2-b^2)/c^2)$

$a^2 = e^2\cdot(a^2/c^2)$

$a^2 = a^2\cdot(e/c)^2$

$\therefore\ e = c$

Aside - Trigonometry

Conversly, consider two right triangles where AC to AB equals DF to DE

Assume that α is not equal to angle δ…

Draw line EG, such that angle GEF equals α

Triangle GEF is equiangular to ABC, so therefore AC to BE equals FG to EG

With a bit of math (pythagoras' theorem), it can be shown that the point G must be the same point as D

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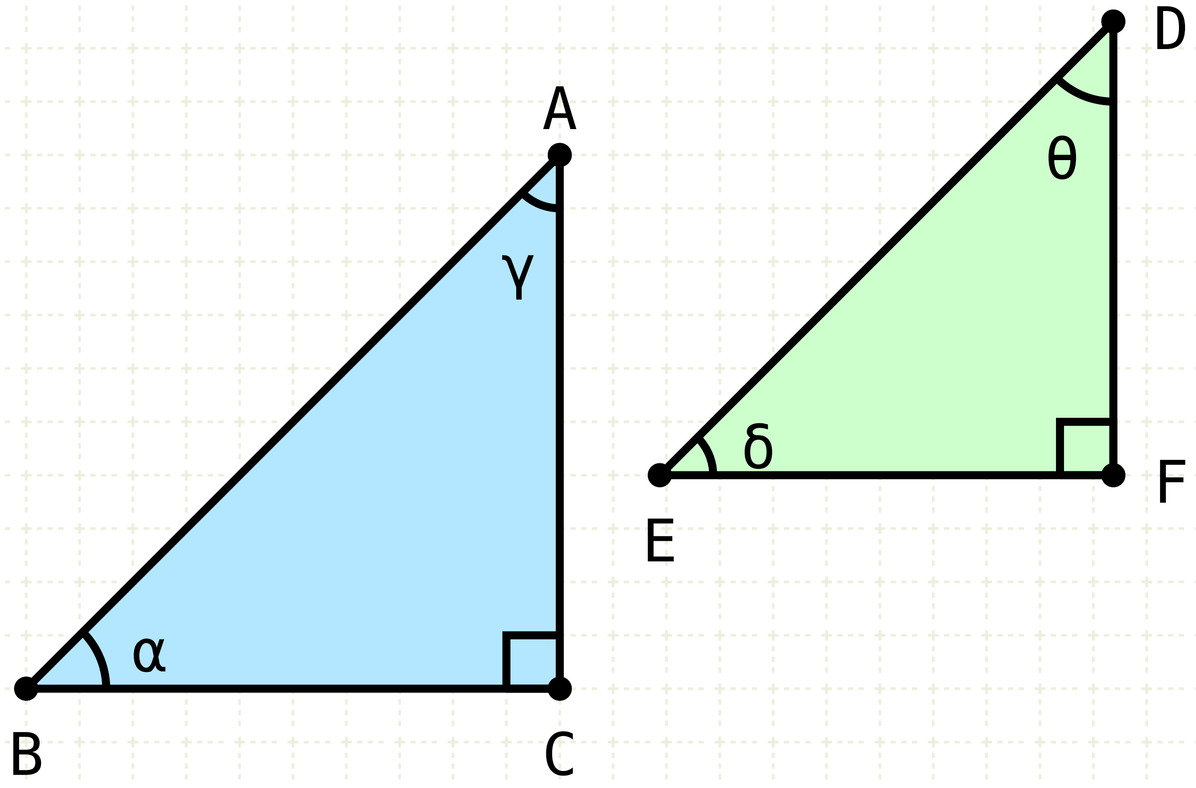


AC:AB = DF:DE

δ = α

Aside - Trigonometry

Conversly, consider two right triangles where AC to AB equals DF to DE

Assume that α is not equal to angle δ…

Draw line EG, such that angle GEF equals α

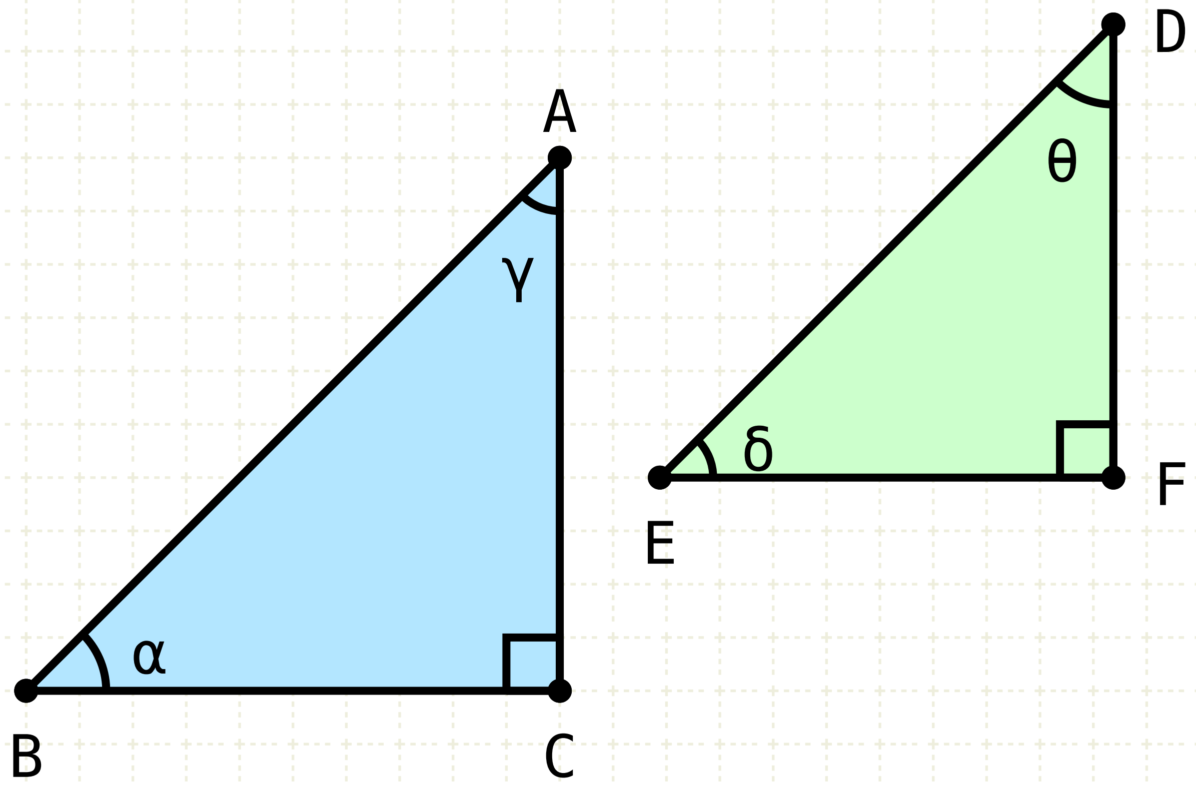
Triangle GEF is equiangular to ABC, so therefore AC to BE equals FG to EG

With a bit of math (pythagoras' theorem), it can be shown that the point G must be the same point as D

So the angle α equals the angle δ

Proposition 5 of Book VI

It two triangles have their sides proportional, the triangles will be equiangular and will have those angles equal which the corresponding sides subtend



$AC:AB = DF:DE \rightarrow \alpha = \delta$

$\alpha = \delta \rightarrow AC:AB = DF:DE$

Aside - Trigonometry

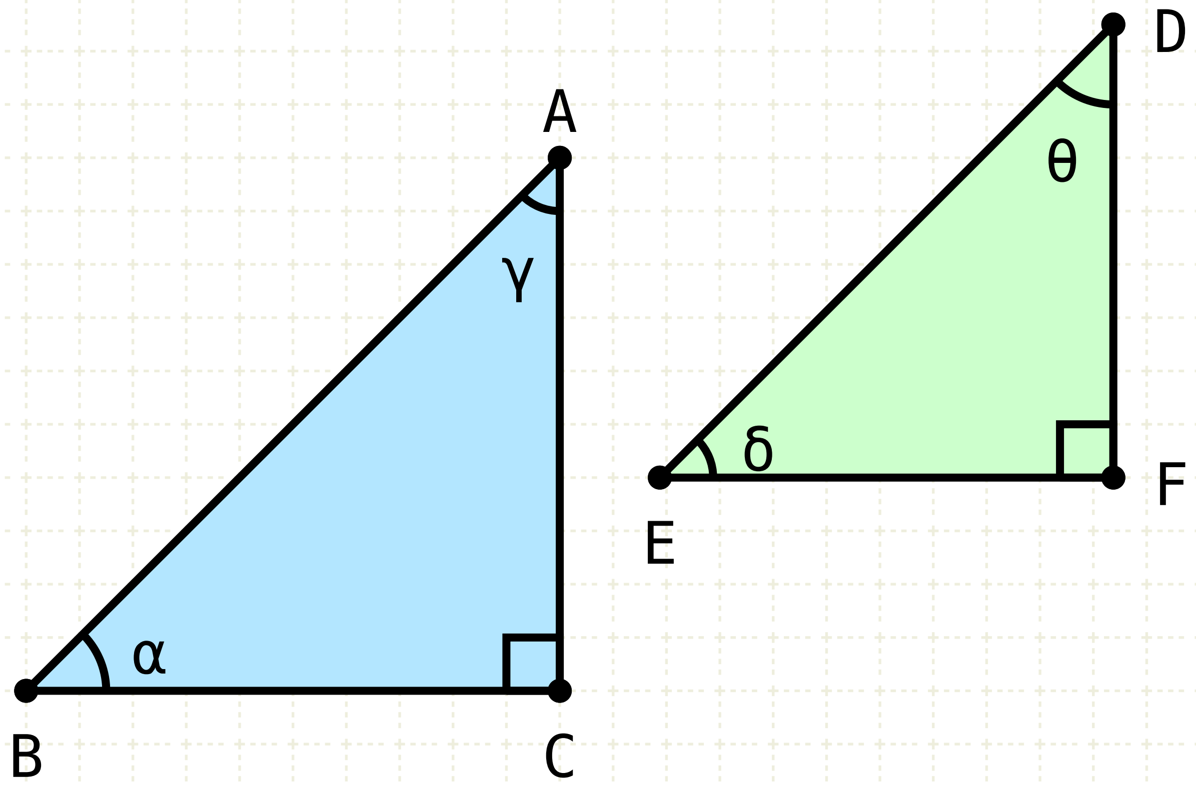
Conclusion:

Given two right triangles ABC and DEF

* If the ratio of AC to AB equals DF to DE, then the angle ABC is equal to the angle DEF
* If the angle ABC is equal to the angle DEF, then the ratio of AC to AB equals DF to DE

Proposition 5 of Book VI

It two triangles have their sides proportional, the triangles will be equiangular and will have those angles equal which the corresponding sides subtend



$$AC:AB = DF:DE \rightarrow \alpha = \delta$$
$$\alpha = \delta \rightarrow AC:AB = DF:DE$$

Definition:

$$\sin(\alpha) = AC:AB$$
$$\sin(\delta) = DF:DE$$

Aside - Trigonometry

Conclusion:

Given two right triangles ABC and DEF

* If the ratio of AC to AB equals DF to DE, then the angle ABC is equal to the angle DEF
* If the angle ABC is equal to the angle DEF, then the ratio of AC to AB equals DF to DE

So for every right angle triangle, the ratio of the sides (opposite over hypotenuse) is unique for every angle

Lets call this ratio, as a function of the angle, 'sine'

Proposition 5 of Book VI

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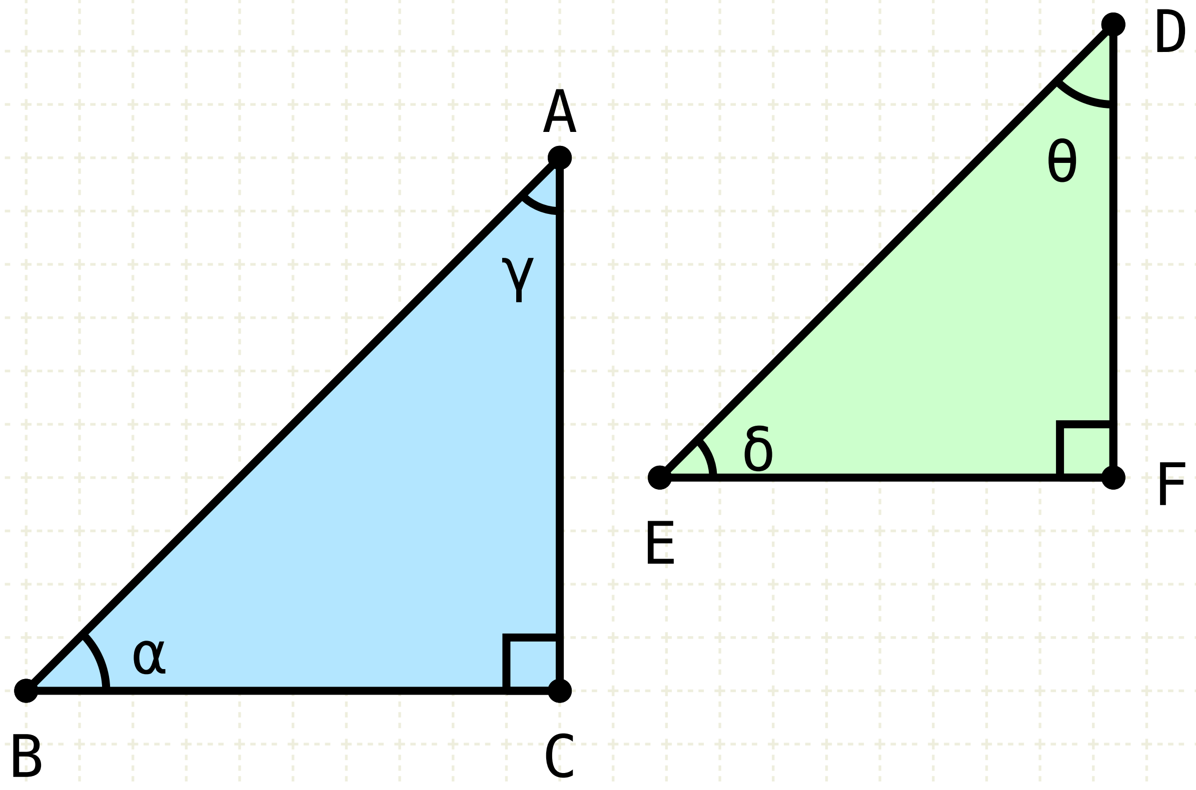

$$AC:AB = DF:DE \rightarrow \alpha = \delta$$
$$\alpha = \delta \rightarrow AC:AB = DF:DE$$

Definition:

$\sin(\alpha) = AC:AB$ $\cos(\alpha) = BC:AB$

$\sin(\delta) = DF:DE$ $\cos(a) = EF:DE$

Aside - Trigonometry

Conclusion:

Given two right triangles ABC and DEF

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We can use the same arguments to define the 'cosine' of an angle as the ratio of BC to AB

Proposition 5 of Book VI

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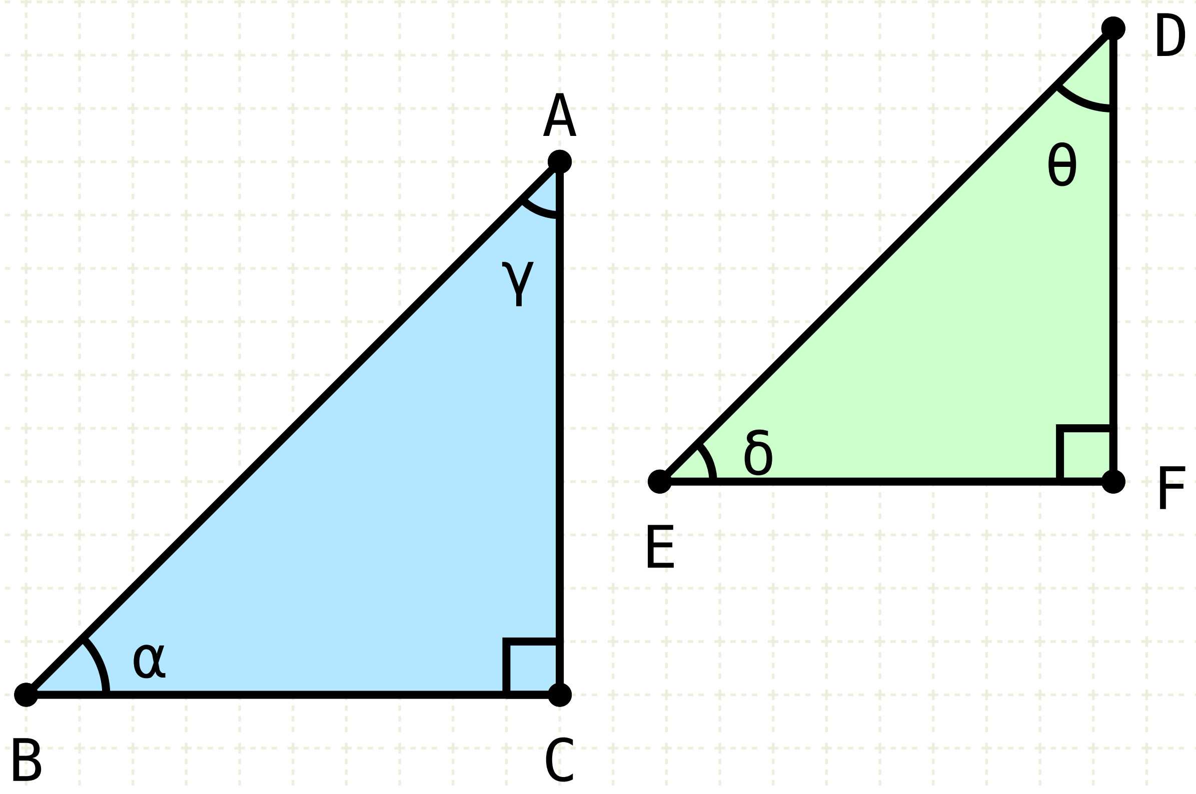

$AC:AB = DF:DE \rightarrow \alpha = \delta$
$\alpha = \delta \rightarrow AC:AB = DF:DE$

Definition:

$\sin(\alpha) = AC:AB \quad \cos(\alpha) = BC:AB$
$\sin(\delta) = DF:DE \quad \cos(a) = EF:DE$

$$\sin^2(\alpha) + \cos^2(\alpha) = (AC)^2/(AB)^2 + (BC)^2/(AB)^2$$
$$= ((AC)^2 + (BC)^2)/(AB)^2$$
$$= (AB)^2/(AB)^2 = 1$$

Aside - Trigonometry

Conclusion:

Given two right triangles ABC and DEF

* If the ratio of AC to AB equals DF to DE, then the angle ABC is equal to the angle DEF
* If the angle ABC is equal to the angle DEF, then the ratio of AC to AB equals DF to DE

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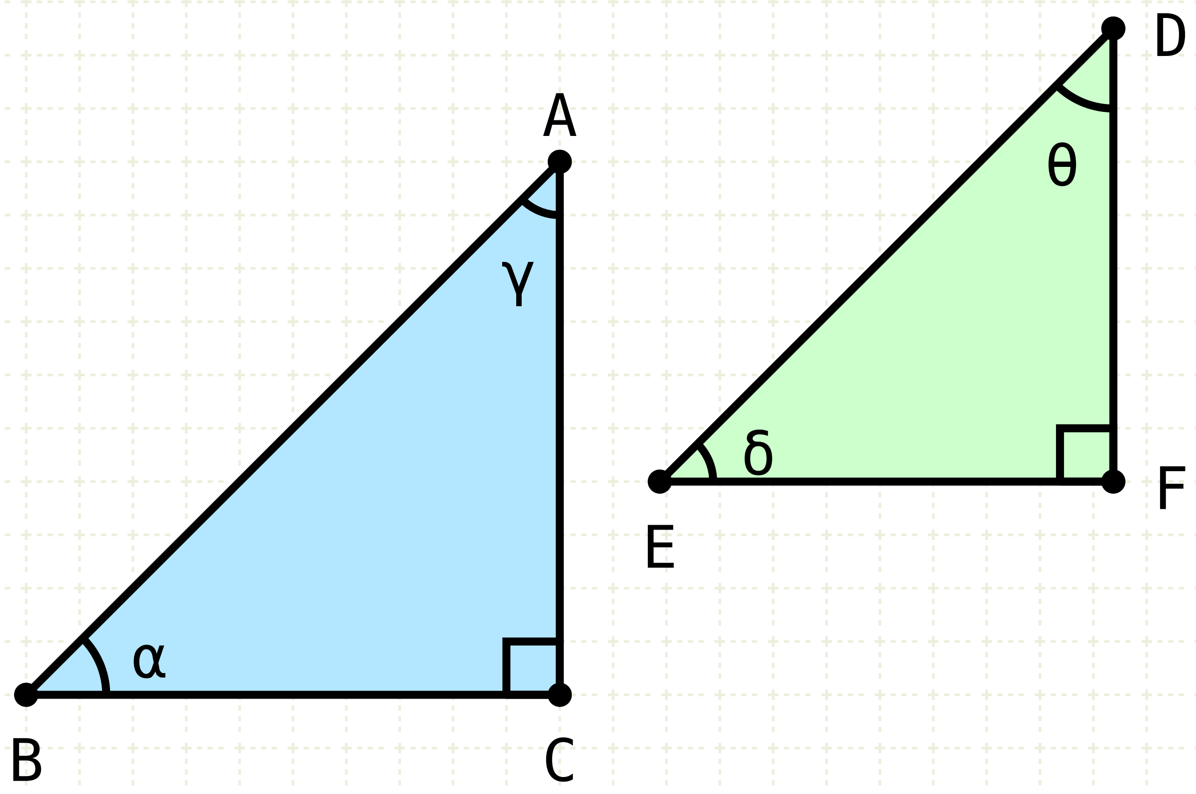

$$AC:AB = DF:DE \rightarrow \alpha = \delta$$
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Definition:

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$$\sin(\delta) = DF:DE \qquad \cos(a) = EF:DE$$

$$\begin{aligned}\sin^2(\alpha) + \cos^2(\alpha) &= (AC)^2/(AB)^2 + (BC)^2/(AB)^2 \\ &= ((AC)^2 + (BC)^2)/(AB)^2 \\ &= (AB)^2/(AB)^2 = 1\end{aligned}$$

Aside - Trigonometry

Conclusion:

Given two right triangles ABC and DEF

* If the ratio of AC to AB equals DF to DE, then the angle ABC is equal to the angle DEF
* If the angle ABC is equal to the angle DEF, then the ratio of AC to AB equals DF to DE

So for every right angle triangle, the ratio of the sides (opposite over hypotenuse) is unique for every angle

Lets call this ratio, as a function of the angle, 'sine'

We can use the same arguments to define the 'cosine' of an angle as the ratio of BC to AB

Sine and cosine have been expanded to include definitions for angles larger than a right angle, and even negative angles, but these ratios shown above are the roots of trigonometry

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