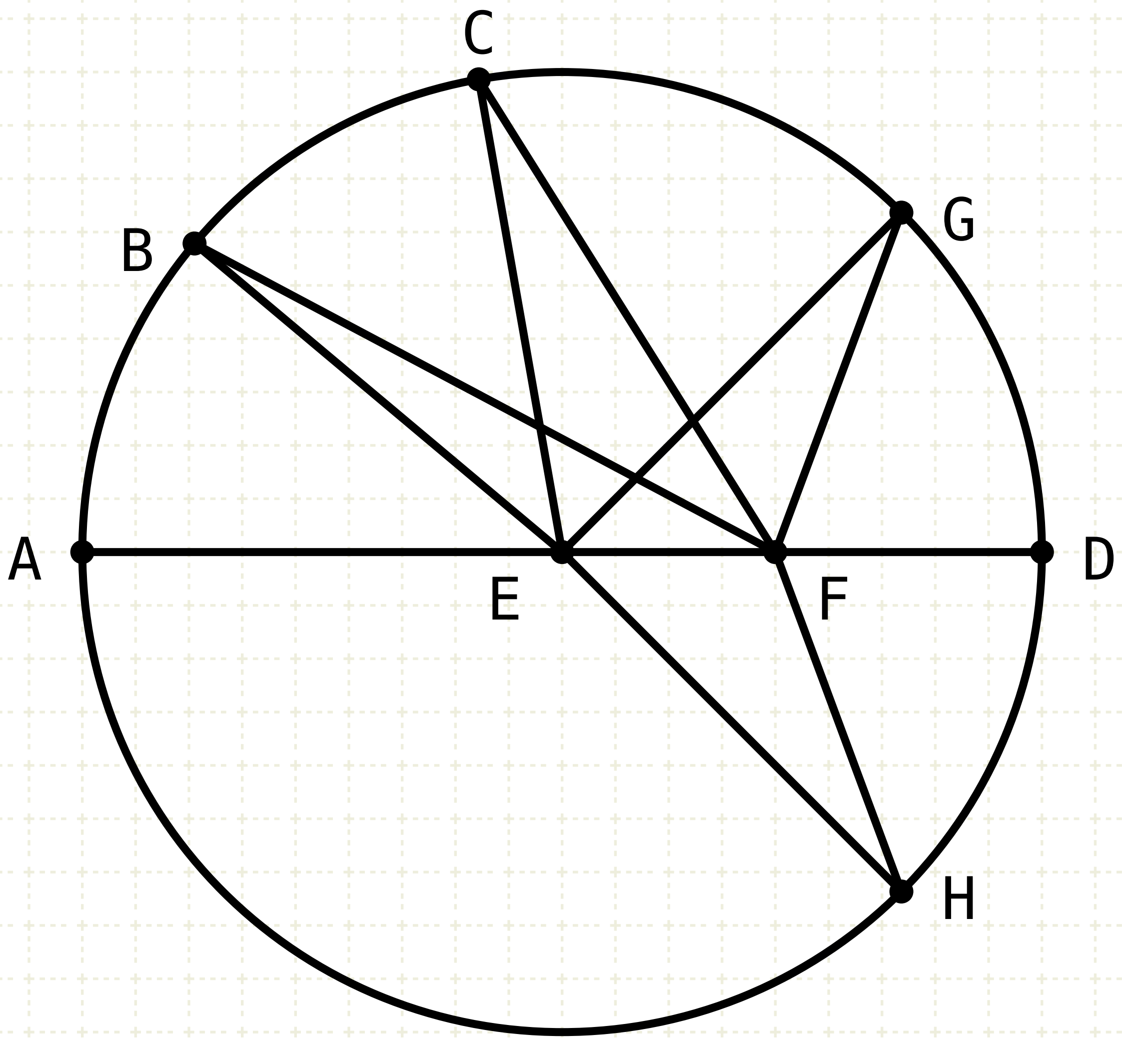
Euclid's Elements

Book III



A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)

Table of Contents, Chapter 3

1 To find the centre of a circle

2 A chord of a circle always lies inside the circle

3 A line through the centre of a circle bisects a chord, and vice versa

4 A line not through the centre of a circle does not bisect a chord

5 If two circles cut one another, they will not have the same center

6 If two circles touch one another, they will not have the same center

7 Consider two lines from a point inside a circle to the edge, the longer one will be the one closest to the longest part of the diameter passing through the original point

8 Consider two lines from a point outside a circle to the edge, the line closest to the centre will be longer on the concave side and shorter on the convex side

9 If three lines, starting at a point 'A' and touching the circle, are all equal, then 'A' is the centre of the circle

10 A circle does not cut a circle at more points than two

11 Point of contact between two internal circles, and their centres, are collinear

12 Point of contact between two external circles, and their centres, are collinear

13 A circle does not touch a circle at more points than one, whether it touch it internally or externally.

14 In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another.

15 The longest line in a circle is its diameter, shorter the farther away from the diameter

16 A line on the circle, perpendicular to the diameter, lies outside the circle

17 From a given point to draw a straight line touching a given circle

18 If line touches a circle, then it is perpendicular to the diameter that touches that point

19 If line touches a circle, then the centre of the circle lies on a line perpendicular to the original

20 The angle at the centre of a circle is twice that from an angle from the circumference

21 In a circle the angles in the same segment are equal to one another

22 The opposite angles of quadrilaterals in circles are equal to two right angles

23 On the same straight line there cannot be constructed two similar and unequal segments of circles on the same side

24 Similar segments of circles on equal straight lines are equal to one another

Table of Contents, Chapter 3

25 Given a segment of a circle, to describe the complete circle of which it is a segment.

26 In equal circles equal angles stand on equal circumferences

27 In equal circles angles standing on equal circumferences are equal to one another

28 In equal circles equal straight lines cut off equal circumferences

29 In equal circles equal circumferences are subtended by equal straight lines

30 To bisect a given circumference

31 In a circle the angle in the semicircle is right …

32 The angle between a tangent and a straight line cutting a circle is equal to the angle in the alternate segment

33 Construct a circle segment on a given line, such that the angle within the segment is equal to a given angle

34 Construct a circle segment on a given circle, such that the angle within the segment is equal to a given angle

35 If two circle chords intersect, the segments on one multiplied together equals the segments of the other multiplied together

36 Secant-tangent law

37 Converse of the secant-tangent law

Proposition 7 of Book III

If on a diameter of a circle a point be taken which is not the center of the circle, and from the point straight lines fall upon the circle, that will be the greatest on which the center is, the remainder on the same diameter will be the least and of the rest the nearer to the straight line through the center is always greater than the more remote, and only two equal straight lines will fall from the point on the circle, one on each side of the least straight line.

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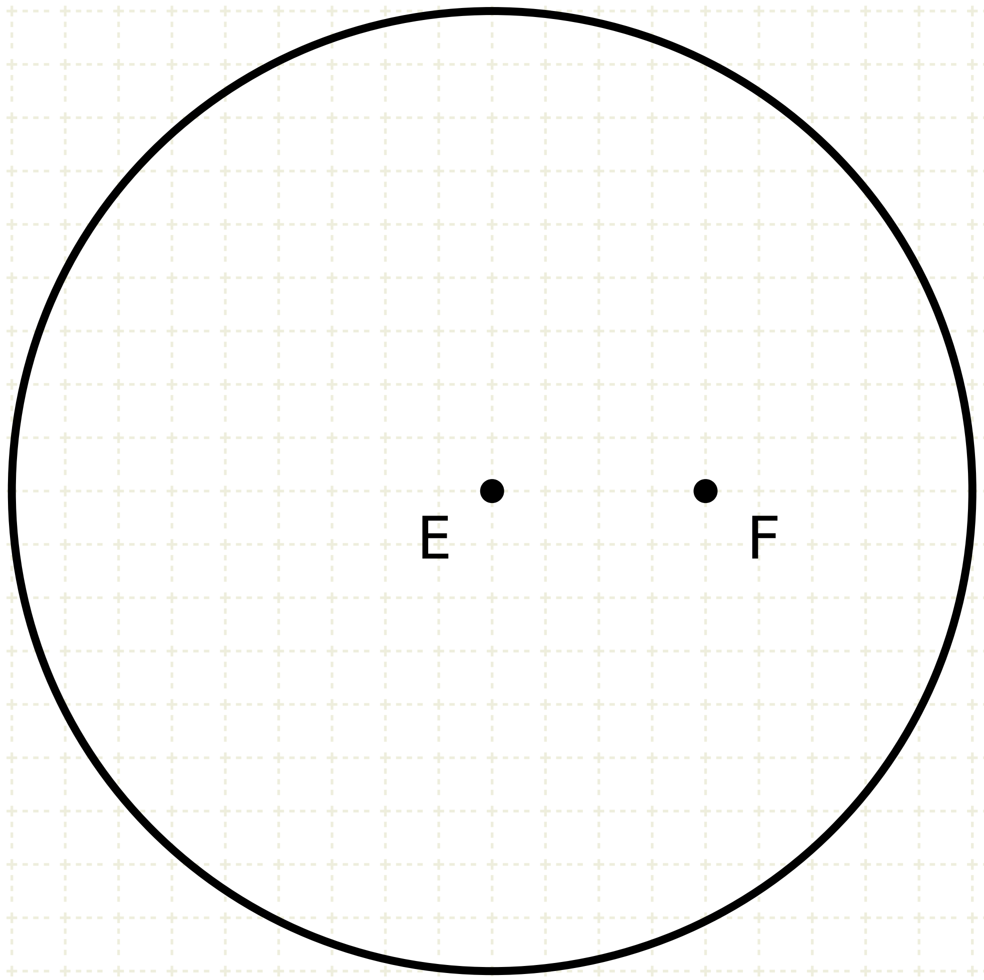
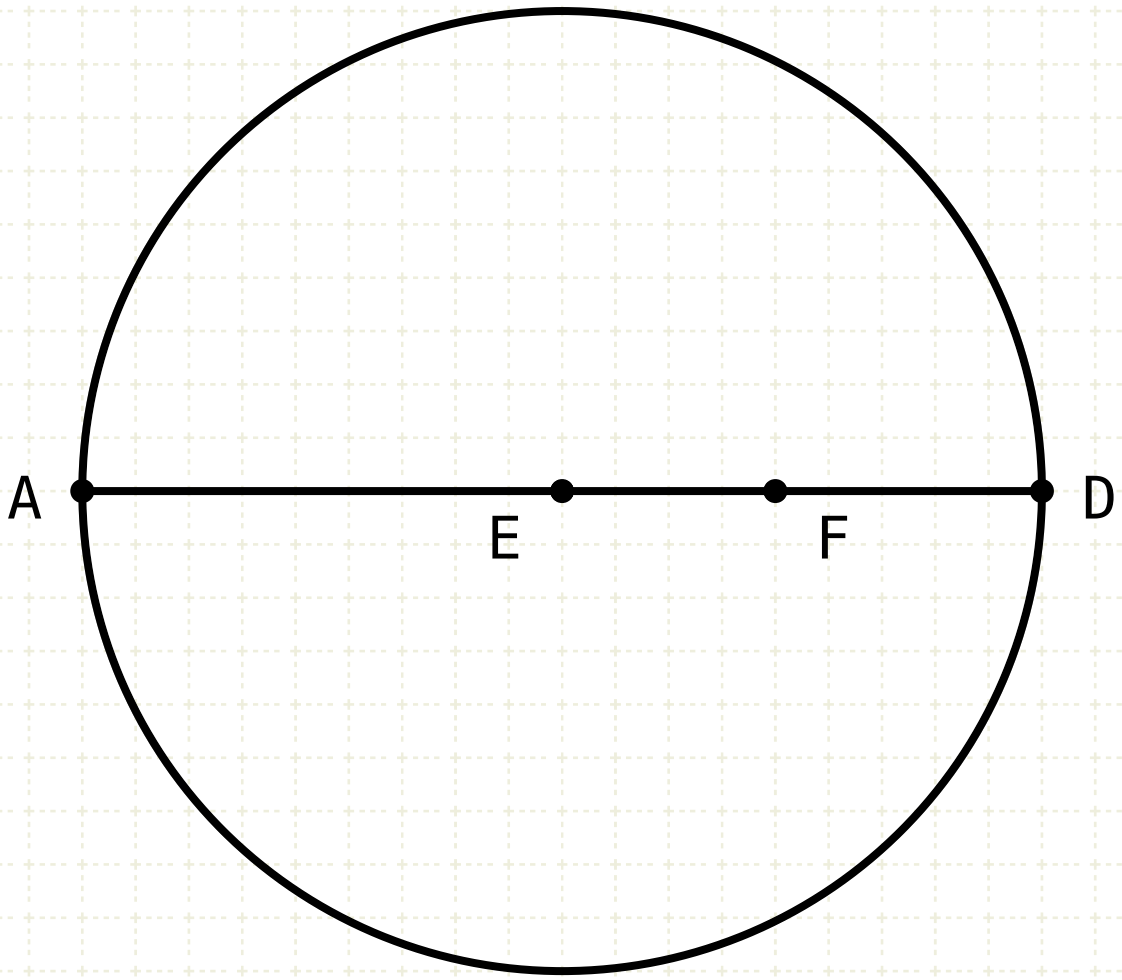


In other words

Let E be the center of a circle, and F be a point not at the center of the circle

Proposition 7 of Book III

If on a diameter of a circle a point be taken which is not the center of the circle, and from the point straight lines fall upon the circle, that will be the greatest on which the center is, the remainder on the same diameter will be the least and of the rest the nearer to the straight line through the center is always greater than the more remote, and only two equal straight lines will fall from the point on the circle, one on each side of the least straight line.



FA > FD

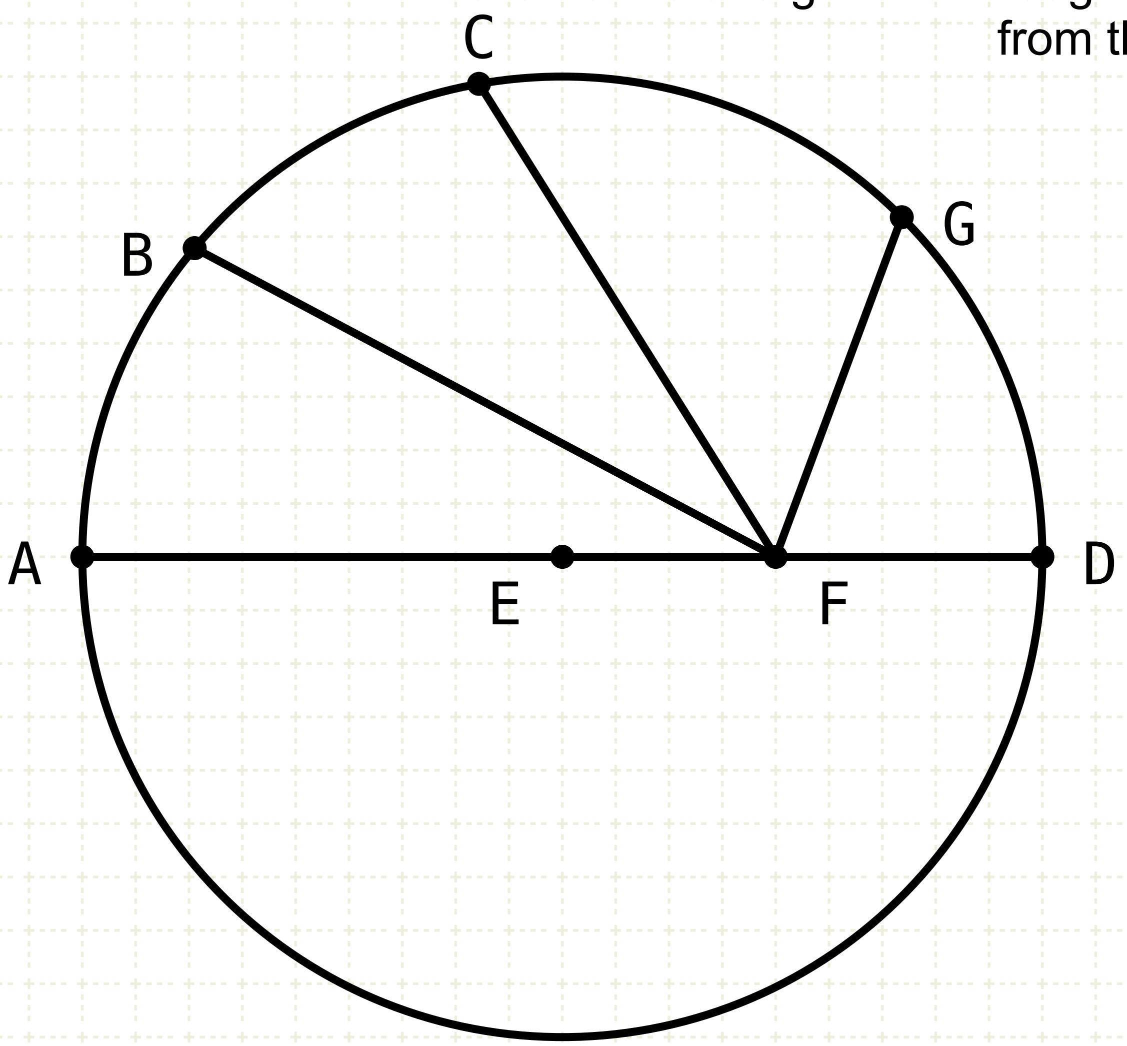
In other words

Let E be the center of a circle, and F be a point not at the center of the circle

The line FA, drawn through the center E, will be larger than the line FD, which is on the same diameter as FA

Proposition 7 of Book III

If on a diameter of a circle a point be taken which is not the center of the circle, and from the point straight lines fall upon the circle, that will be the greatest on which the center is, the remainder on the same diameter will be the least and of the rest the nearer to the straight line through the center is always greater than the more remote, and only two equal straight lines will fall from the point on the circle, one on each side of the least straight line.



FA > FD

FA>FB>FC>FG>FD

In other words

Let E be the center of a circle, and F be a point not at the center of the circle

The line FA, drawn through the center E, will be larger than the line FD, which is on the same diameter as FA

The line FB will be larger than FC because the line FB is closer to FA

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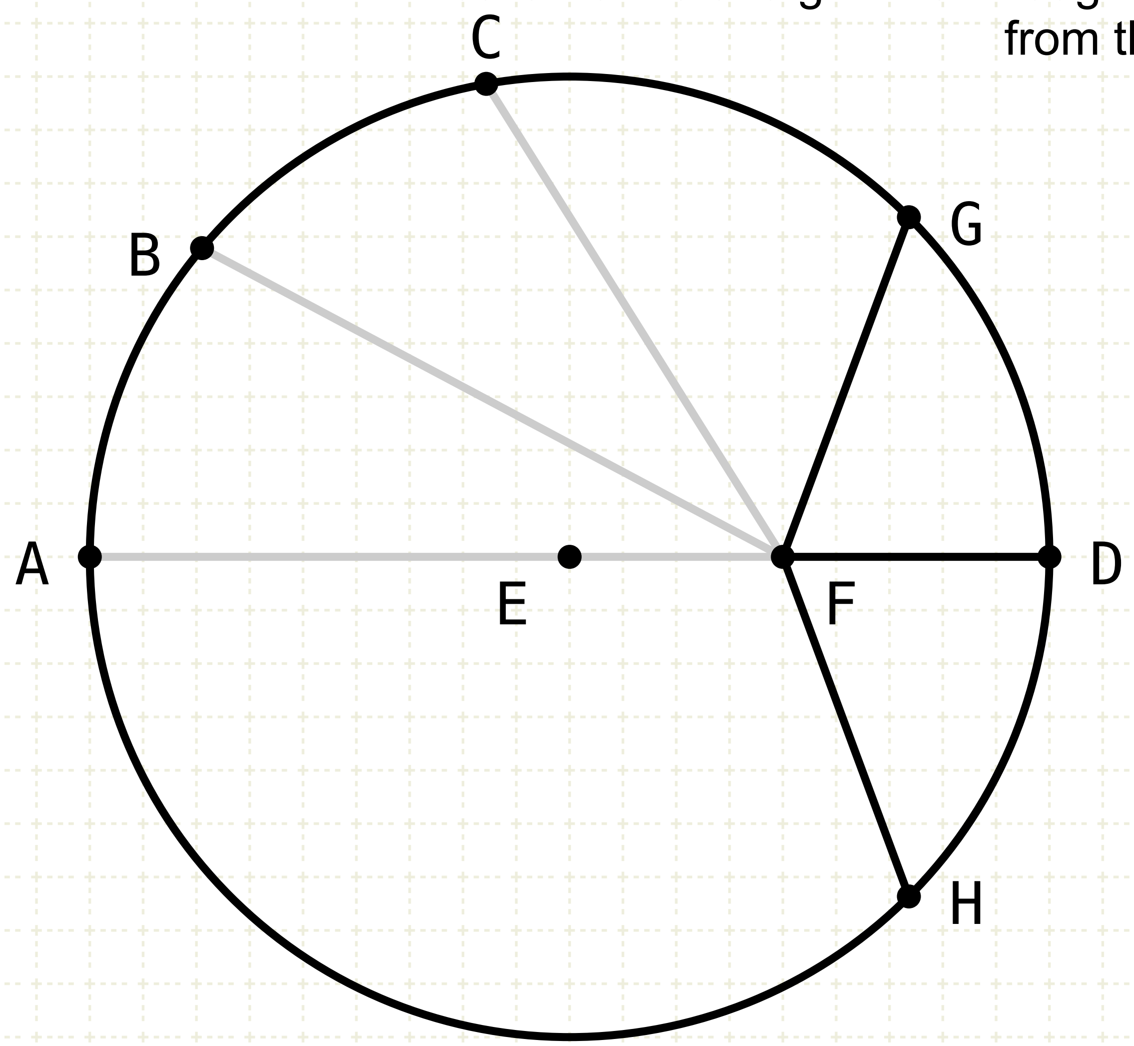
If on a diameter of a circle a point be taken which is not the center of the circle, and from the point straight lines fall upon the circle, that will be the greatest on which the center is, the remainder on the same diameter will be the least and of the rest the nearer to the straight line through the center is always greater than the more remote, and only two equal straight lines will fall from the point on the circle, one on each side of the least straight line.



FA > FD

FA>FB>FC>FG>FD

In other words

Let E be the center of a circle, and F be a point not at the center of the circle

The line FA, drawn through the center E, will be larger than the line FD, which is on the same diameter as FA

The line FB will be larger than FC because the line FB is closer to FA

Also, only two straight and equal lines from point F will fall on the circle, one on either side of FD

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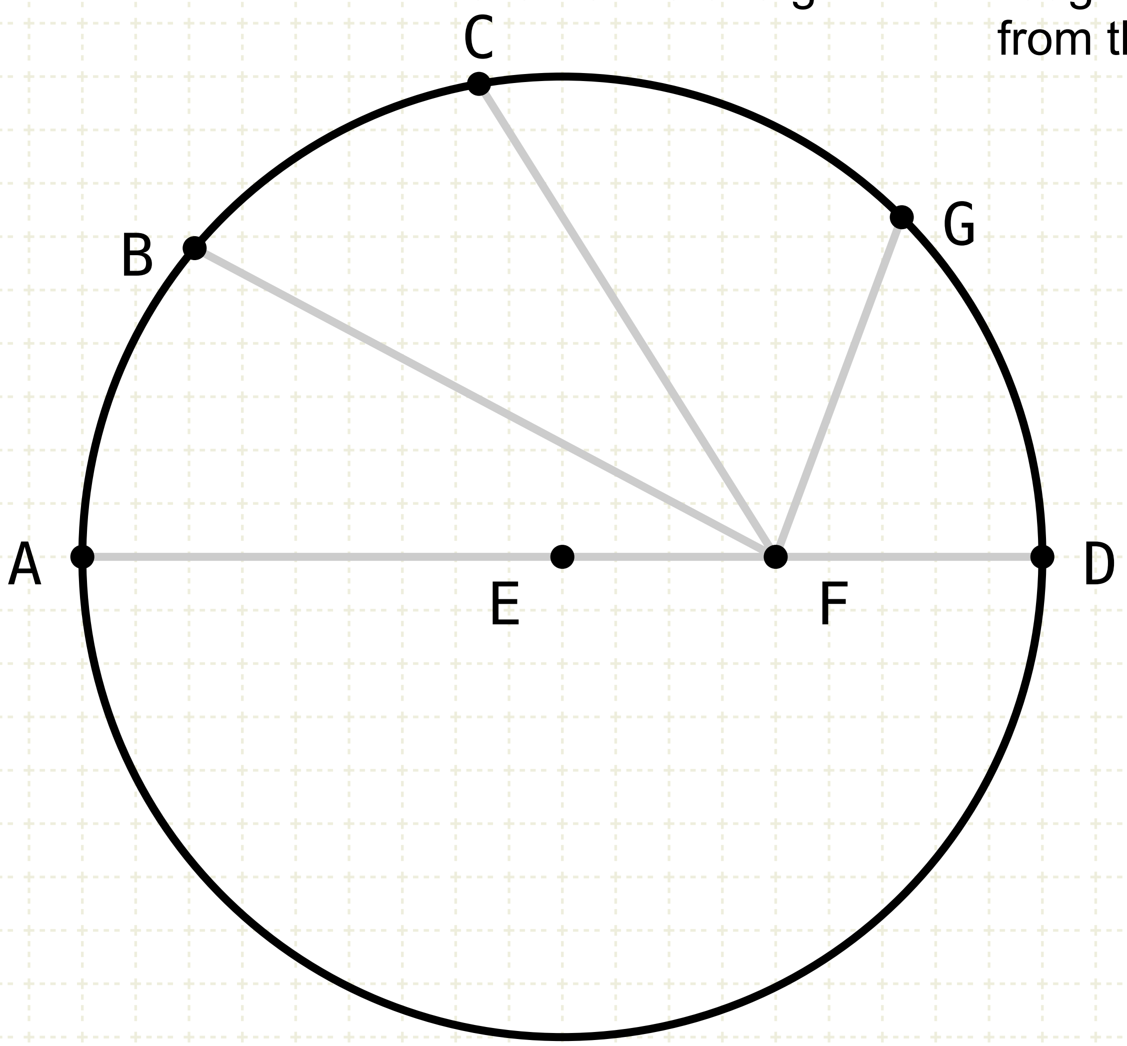
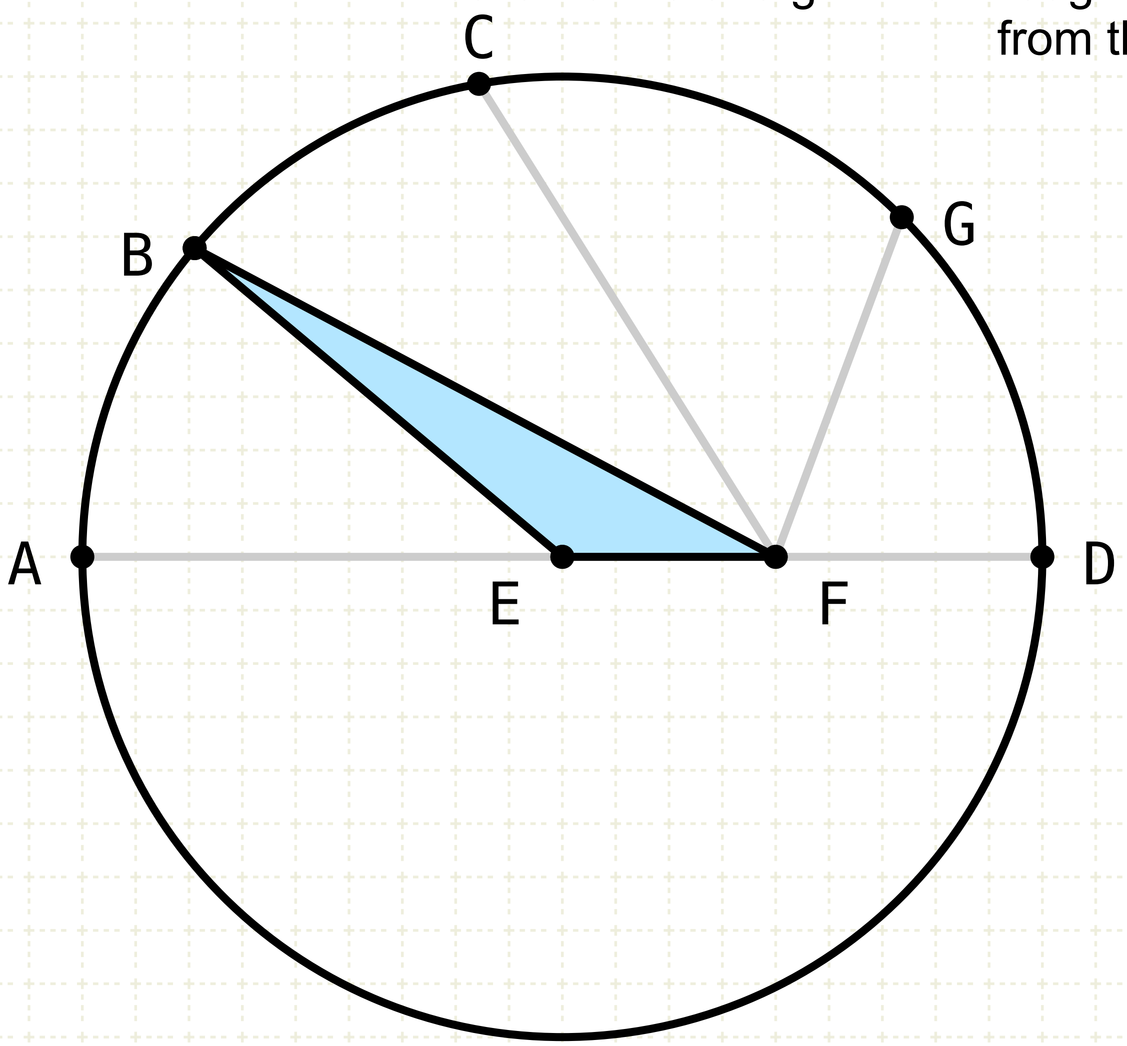
If on a diameter of a circle a point be taken which is not the center of the circle, and from the point straight lines fall upon the circle, that will be the greatest on which the center is, the remainder on the same diameter will be the least and of the rest the nearer to the straight line through the center is always greater than the more remote, and only two equal straight lines will fall from the point on the circle, one on each side of the least straight line.



Proof (part 1)

Proposition 7 of Book III

If on a diameter of a circle a point be taken which is not the center of the circle, and from the point straight lines fall upon the circle, that will be the greatest on which the center is, the remainder on the same diameter will be the least and of the rest the nearer to the straight line through the center is always greater than the more remote, and only two equal straight lines will fall from the point on the circle, one on each side of the least straight line.



EB + FE > FB

Proof (part 1)

Consider the triangle BEF, the sum of two sides of any triangle is larger than the third (I·20)

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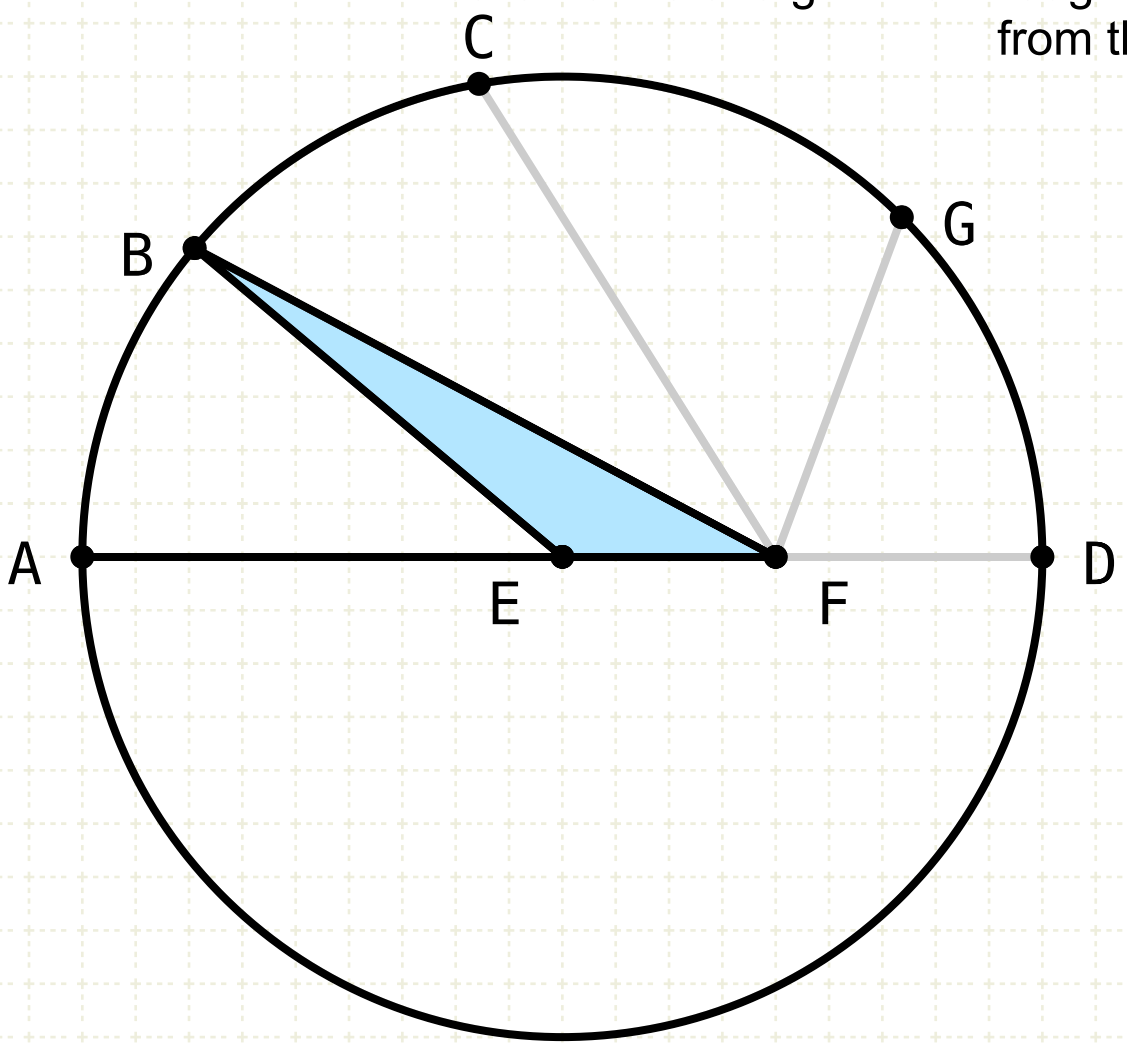
If on a diameter of a circle a point be taken which is not the center of the circle, and from the point straight lines fall upon the circle, that will be the greatest on which the center is, the remainder on the same diameter will be the least and of the rest the nearer to the straight line through the center is always greater than the more remote, and only two equal straight lines will fall from the point on the circle, one on each side of the least straight line.



EB + FE > FB

EA = EB

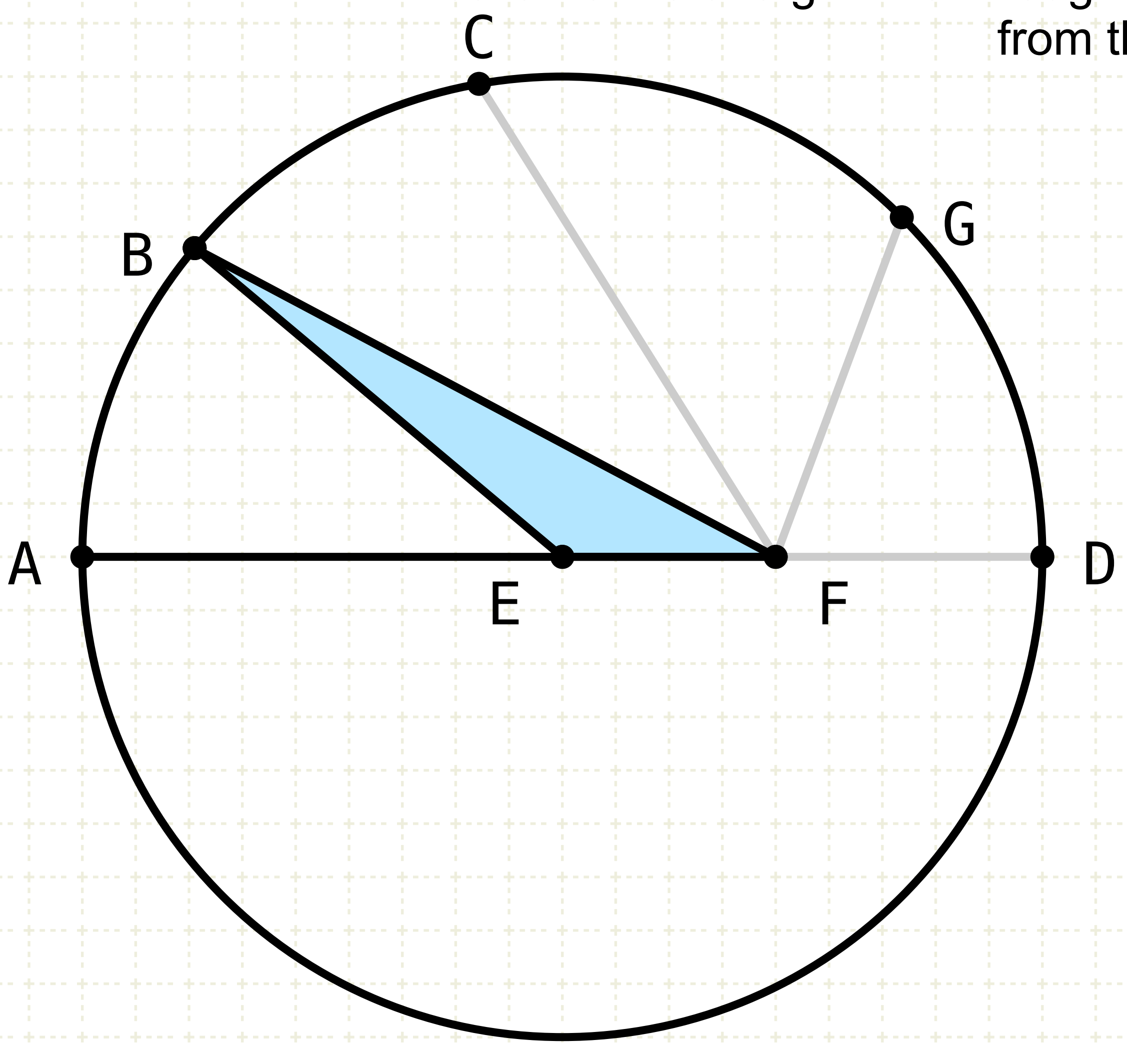
Proof (part 1)

Consider the triangle BEF, the sum of two sides of any triangle is larger than the third (I·20)

The lines EA and EB are radii of the same circle, and thus are equal

Proposition 7 of Book III

If on a diameter of a circle a point be taken which is not the center of the circle, and from the point straight lines fall upon the circle, that will be the greatest on which the center is, the remainder on the same diameter will be the least and of the rest the nearer to the straight line through the center is always greater than the more remote, and only two equal straight lines will fall from the point on the circle, one on each side of the least straight line.



$$EB + FE > FB$$
$$EA = EB$$
$$EA + FE > FB$$
$$FA > FB$$

Proof (part 1)

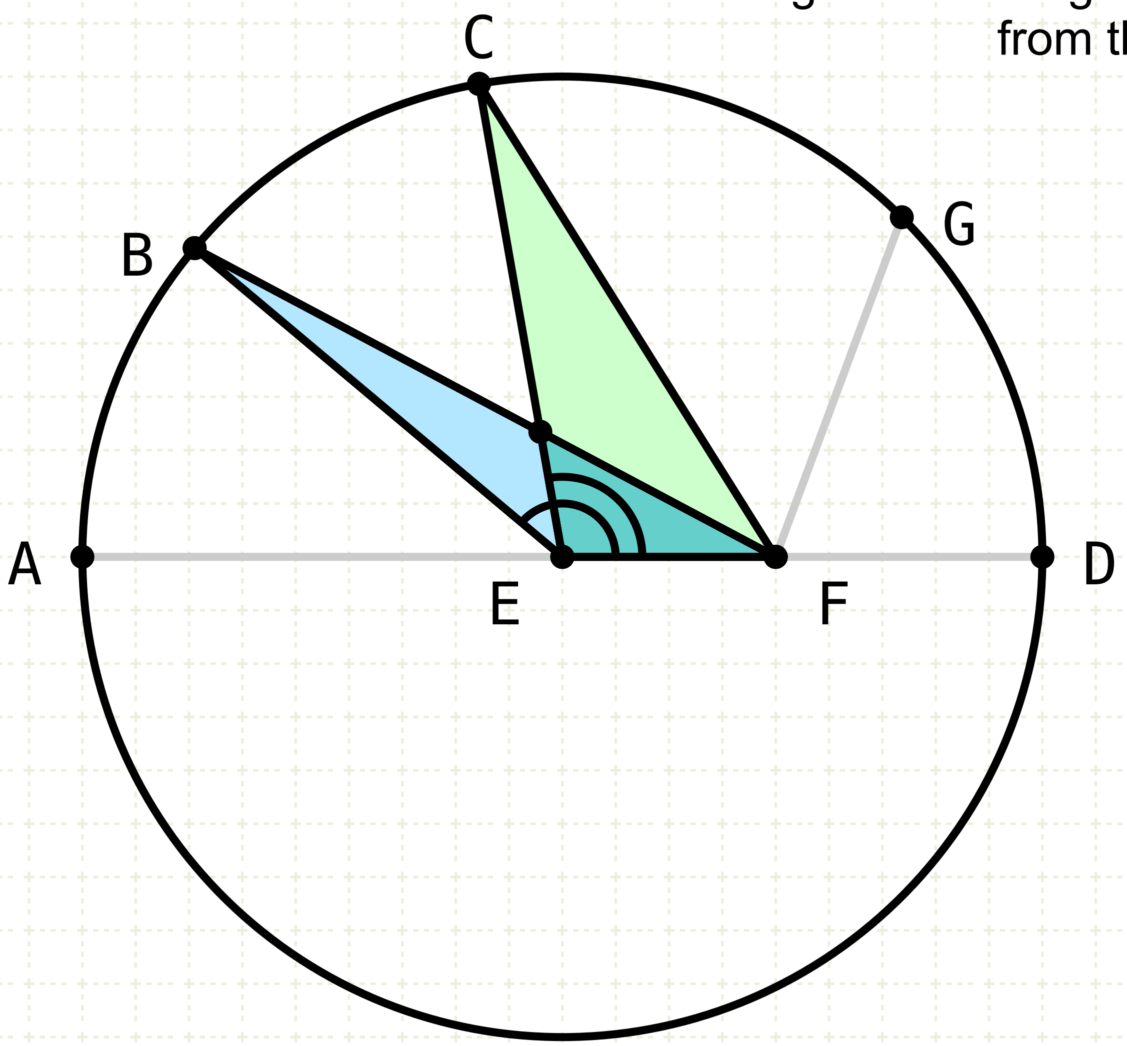
Consider the triangle BEF, the sum of two sides of any triangle is larger than the third (I·20)

The lines EA and EB are radii of the same circle, and thus are equal

Thus, EA plus FE is greater than FB, and since FA equal EA,FE, FA is greater than FB

Proposition 7 of Book III

If on a diameter of a circle a point be taken which is not the center of the circle, and from the point straight lines fall upon the circle, that will be the greatest on which the center is, the remainder on the same diameter will be the least and of the rest the nearer to the straight line through the center is always greater than the more remote, and only two equal straight lines will fall from the point on the circle, one on each side of the least straight line.



$EB + FE > FB$

$EA = EB$

$EA + FE > FB$

$FA > FB$

$BE = CE$

Proof (part 1)

Consider the triangle BEF, the sum of two sides of any triangle is larger than the third (I·20)

The lines EA and EB are radii of the same circle, and thus are equal

Thus, EA plus FE is greater than FB, and since FA equal EA,FE, FA is greater than FB

Compare the triangles BEF and CEF, BE and CE are equal, and FE is common to both, so we have two triangles with two equal sides,

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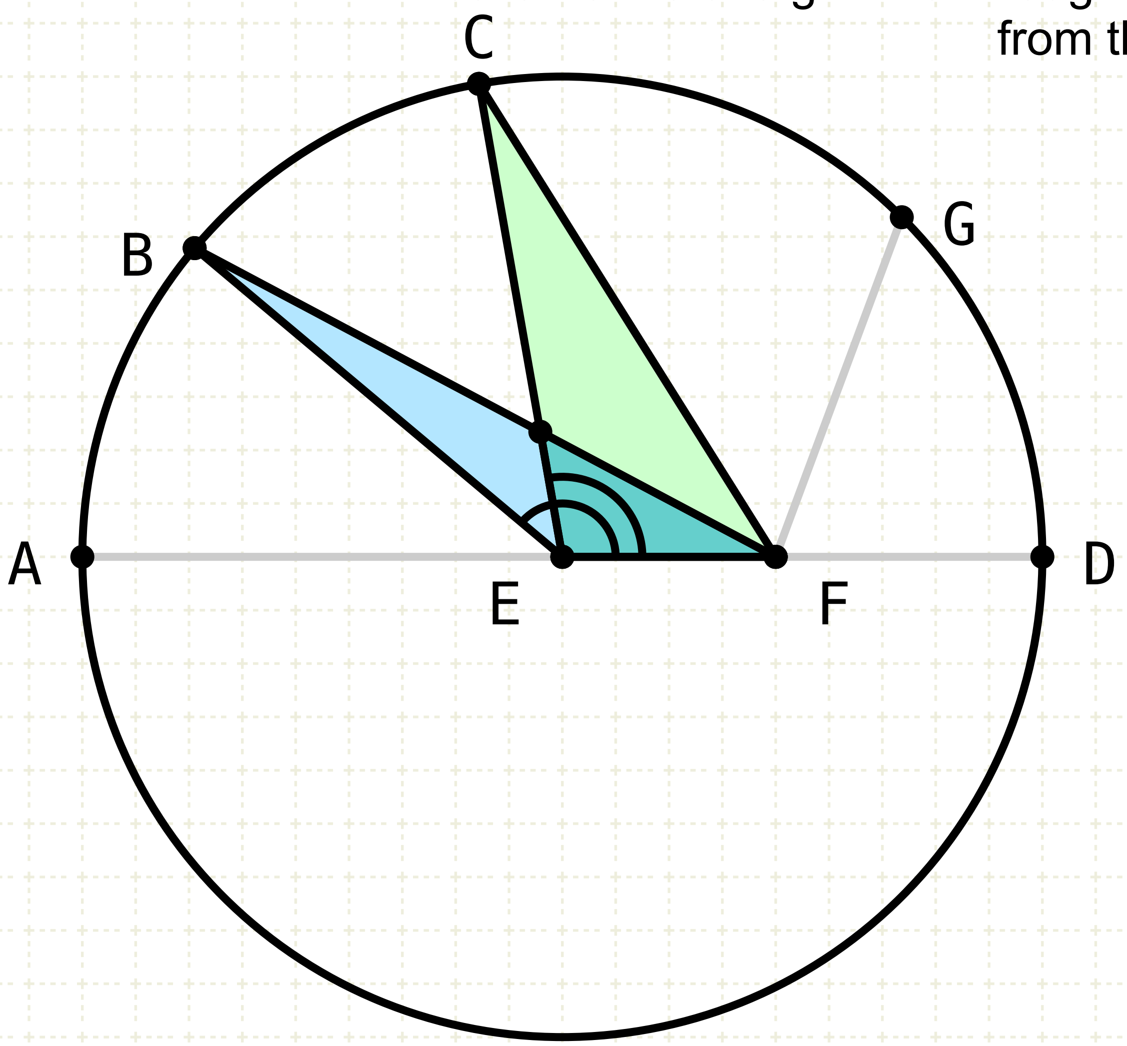
If on a diameter of a circle a point be taken which is not the center of the circle, and from the point straight lines fall upon the circle, that will be the greatest on which the center is, the remainder on the same diameter will be the least and of the rest the nearer to the straight line through the center is always greater than the more remote, and only two equal straight lines will fall from the point on the circle, one on each side of the least straight line.



EB + FE > FB
EA = EB
EA + FE > FB
FA > FB

BE = CE

FB > FC

Proof (part 1)

Consider the triangle BEF, the sum of two sides of any triangle is larger than the third (I·20)

The lines EA and EB are radii of the same circle, and thus are equal

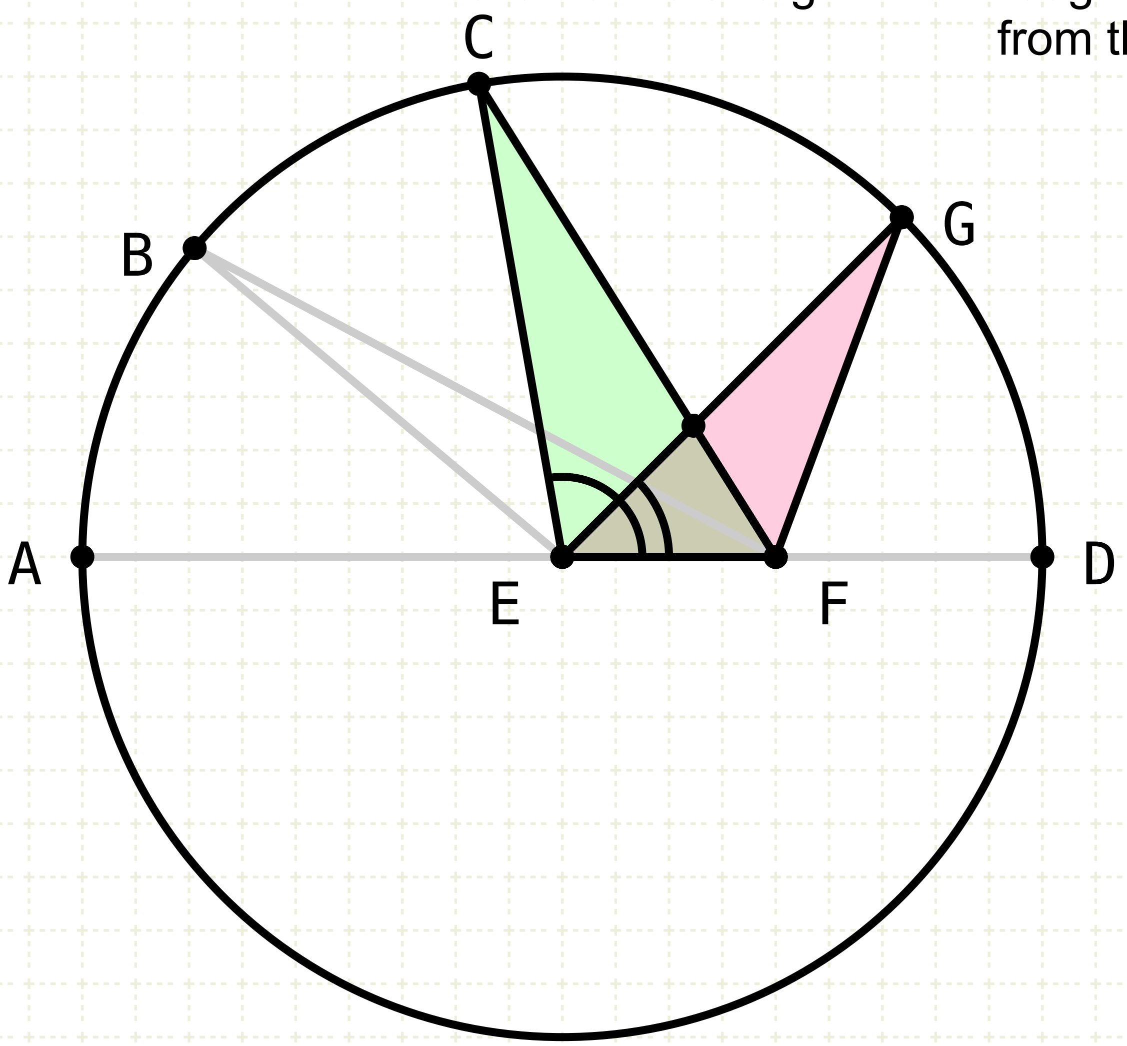
Thus, EA plus FE is greater than FB, and since FA equal EA,FE, FA is greater than FB

Compare the triangles BEF and CEF, BE and CE are equal, and FE is common to both, so we have two triangles with two equal sides,

Since the angle BEF is larger than the angle CEF, FB is larger than FC (I·24)

Proposition 7 of Book III

If on a diameter of a circle a point be taken which is not the center of the circle, and from the point straight lines fall upon the circle, that will be the greatest on which the center is, the remainder on the same diameter will be the least and of the rest the nearer to the straight line through the center is always greater than the more remote, and only two equal straight lines will fall from the point on the circle, one on each side of the least straight line.



EB + FE > FB

EA = EB

EA + FE > FB

FA > FB

BE = CE

FB > FC

FC > FG

Proof (part 1)

Consider the triangle BEF, the sum of two sides of any triangle is larger than the third (I·20)

The lines EA and EB are radii of the same circle, and thus are equal

Thus, EA plus FE is greater than FB, and since FA equal EA,FE, FA is greater than FB

Compare the triangles BEF and CEF, BE and CE are equal, and FE is common to both, so we have two triangles with two equal sides,

Since the angle BEF is larger than the angle CEF, FB is larger than FC (I·24)

Similarly, FC is larger than FG

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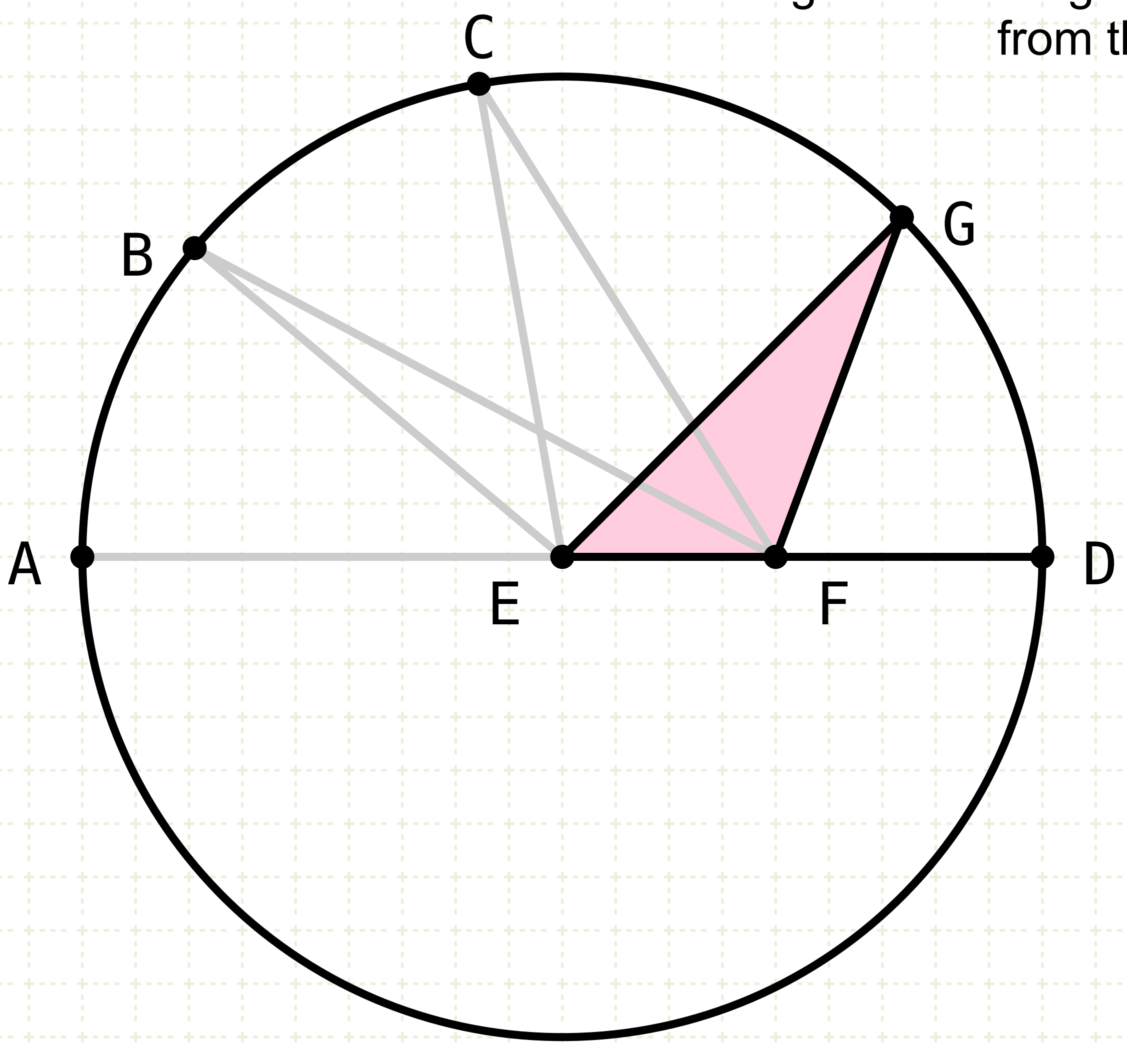

EB + FE > FB
EA = EB
EA + FE > FB
FA > FB

BE = CE

FB > FC

FC > FG

FE + FG > GE

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Similarly, FC is larger than FG

Consider triangle GEF, FE plus FG is greater than GE (I·20)

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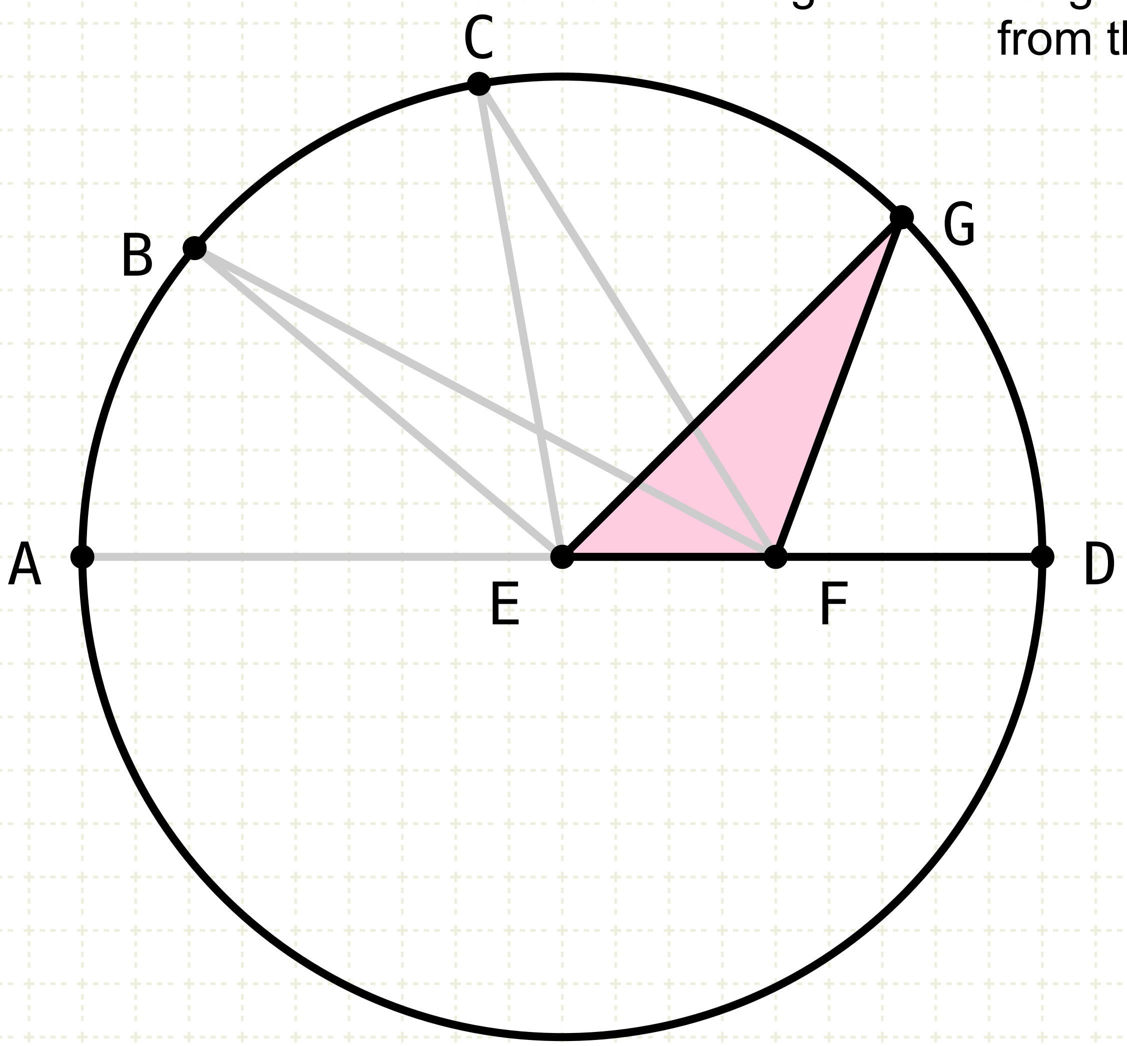

$EB + FE > FB$

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$EA + FE > FB$

$FA > FB$

$BE = CE$

$FB > FC$

$FC > FG$

$FE + FG > GE$

$FE + FG > FE + FD$

Proof (part 1)

Consider the triangle BEF, the sum of two sides of any triangle is larger than the third (I·20)

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Compare the triangles BEF and CEF, BE and CE are equal, and FE is common to both, so we have two triangles with two equal sides,

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Similarly, FC is larger than FG

Consider triangle GEF, FE plus FG is greater than GE (I·20)

But GE is equal to DE, which is equal to the sum of FE, FD

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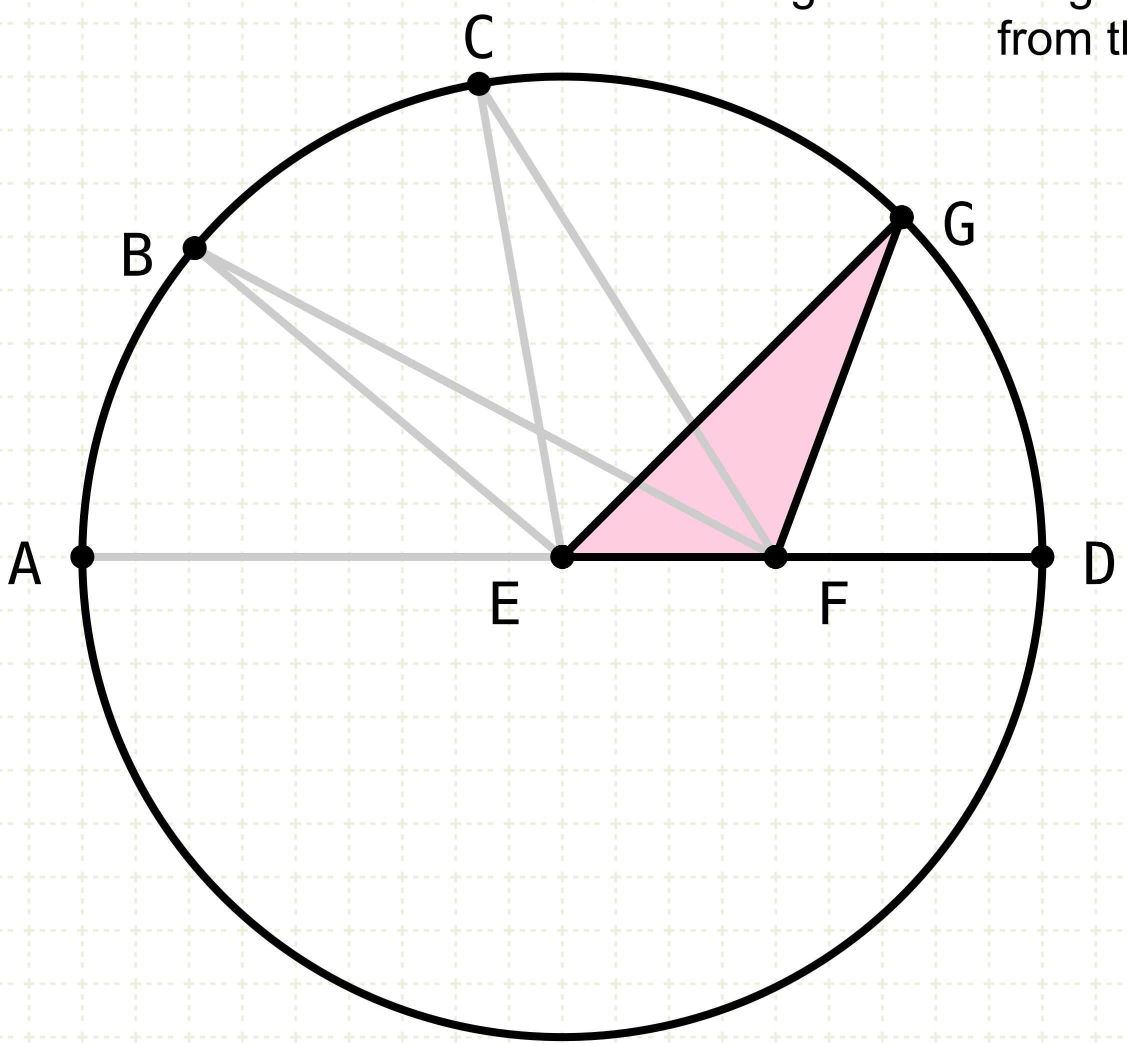
If on a diameter of a circle a point be taken which is not the center of the circle, and from the point straight lines fall upon the circle, that will be the greatest on which the center is, the remainder on the same diameter will be the least and of the rest the nearer to the straight line through the center is always greater than the more remote, and only two equal straight lines will fall from the point on the circle, one on each side of the least straight line.



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$FE + FG > GE$

$FE + FG > FE + FD$

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Proof (part 1)

Consider the triangle BEF, the sum of two sides of any triangle is larger than the third (I·20)

The lines EA and EB are radii of the same circle, and thus are equal

Thus, EA plus FE is greater than FB, and since FA equal EA,FE, FA is greater than FB

Compare the triangles BEF and CEF, BE and CE are equal, and FE is common to both, so we have two triangles with two equal sides,

Since the angle BEF is larger than the angle CEF, FB is larger than FC (I·24)

Similarly, FC is larger than FG

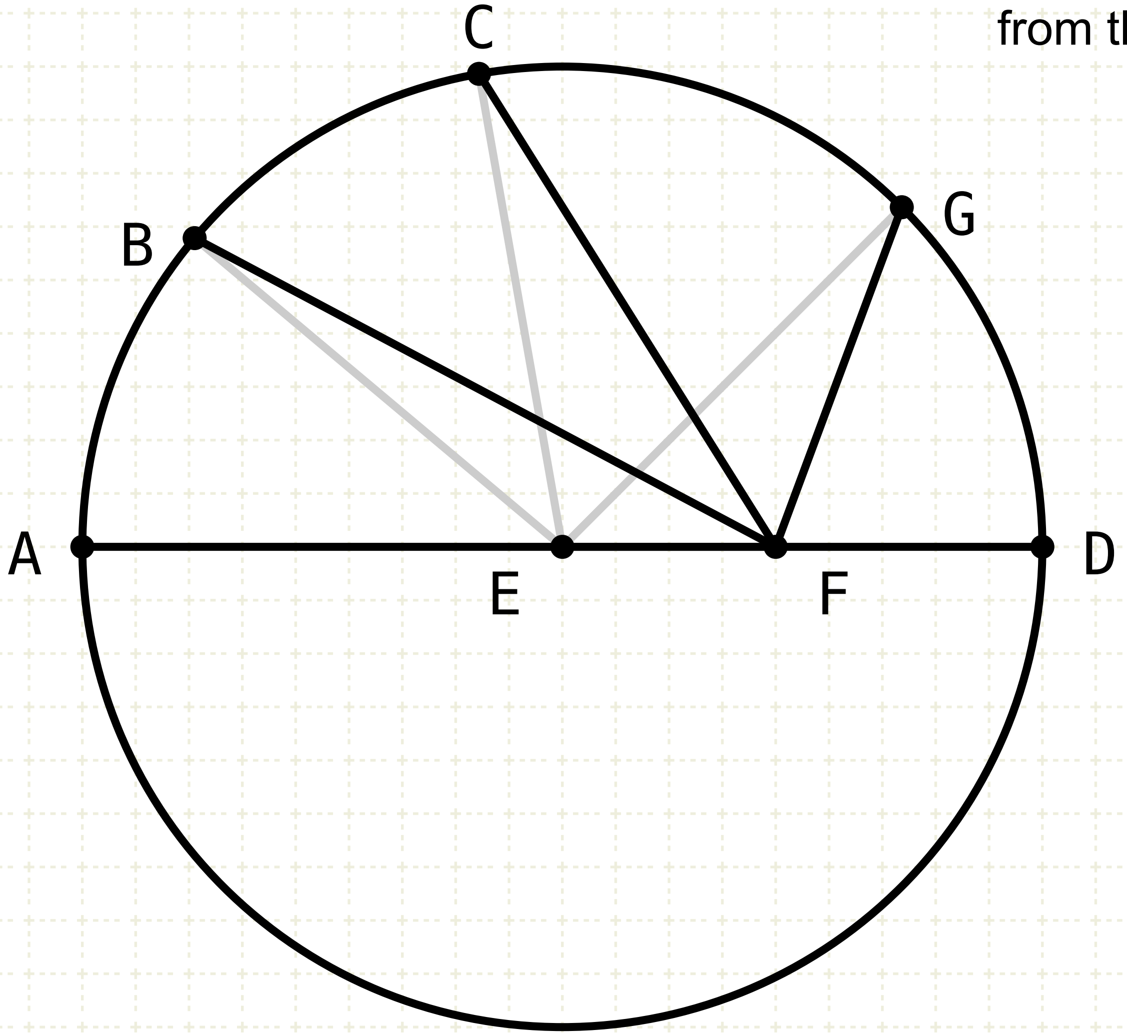
Consider triangle GEF, FE plus FG is greater than GE (I·20)

But GE is equal to DE, which is equal to the sum of FE, FD

Subtract FE from both sides of the inequality gives FG is greater than FD

Proposition 7 of Book III

If on a diameter of a circle a point be taken which is not the center of the circle, and from the point straight lines fall upon the circle, that will be the greatest on which the center is, the remainder on the same diameter will be the least and of the rest the nearer to the straight line through the center is always greater than the more remote, and only two equal straight lines will fall from the point on the circle, one on each side of the least straight line.



EB + FE > FB
EA = EB
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BE = CE

FB > FC

FC > FG

FE + FG > GE
FE + FG > FE + FD
FG > FD

FA>FB>FC>FG>FD

Proof (part 1)

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Compare the triangles BEF and CEF, BE and CE are equal, and FE is common to both, so we have two triangles with two equal sides,

Since the angle BEF is larger than the angle CEF, FB is larger than FC (I·24)

Similarly, FC is larger than FG

Consider triangle GEF, FE plus FG is greater than GE (I·20)

But GE is equal to DE, which is equal to the sum of FE, FD

Subtract FE from both sides of the inequality gives FG is greater than FD

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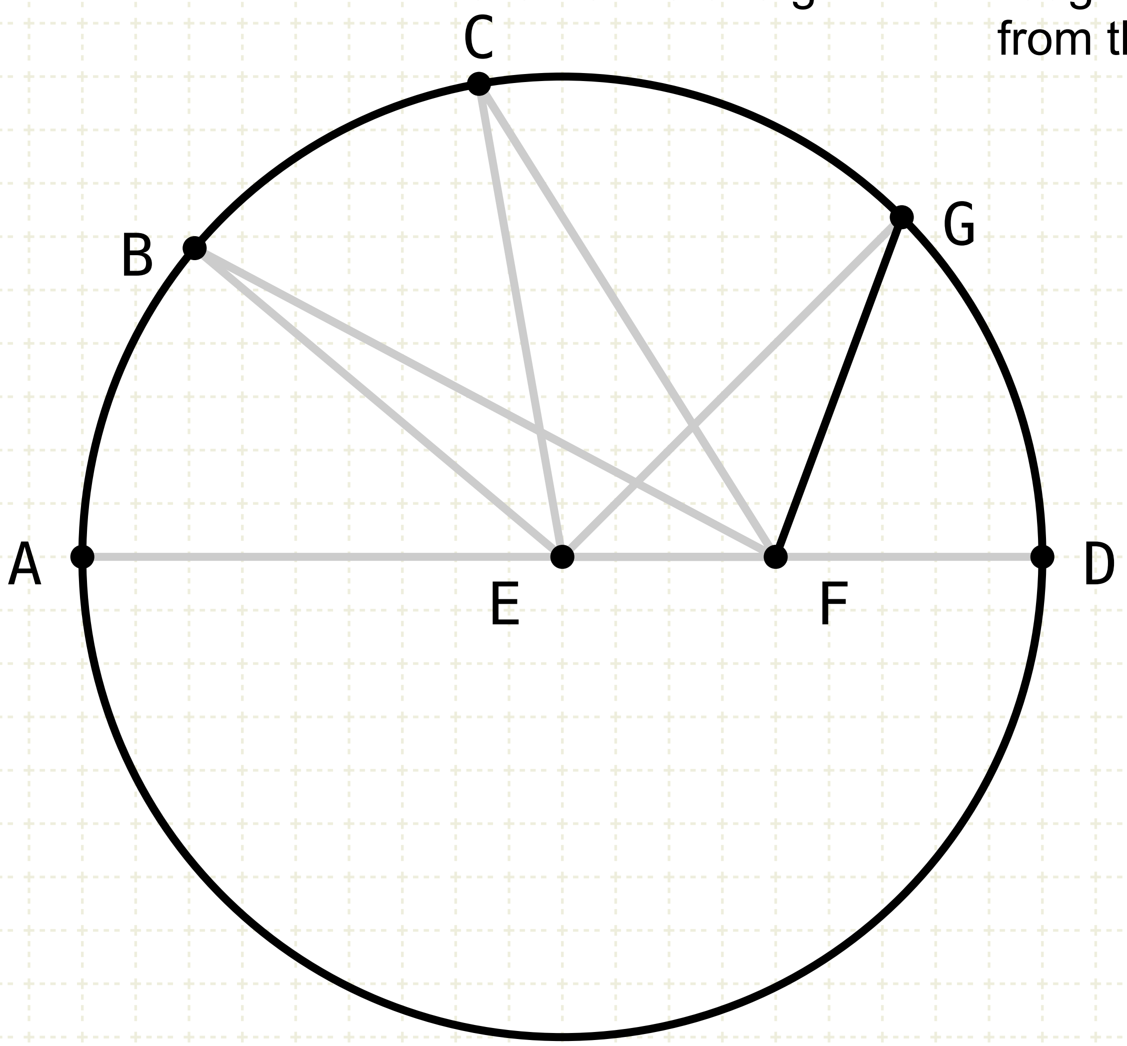

Proof (part 2)

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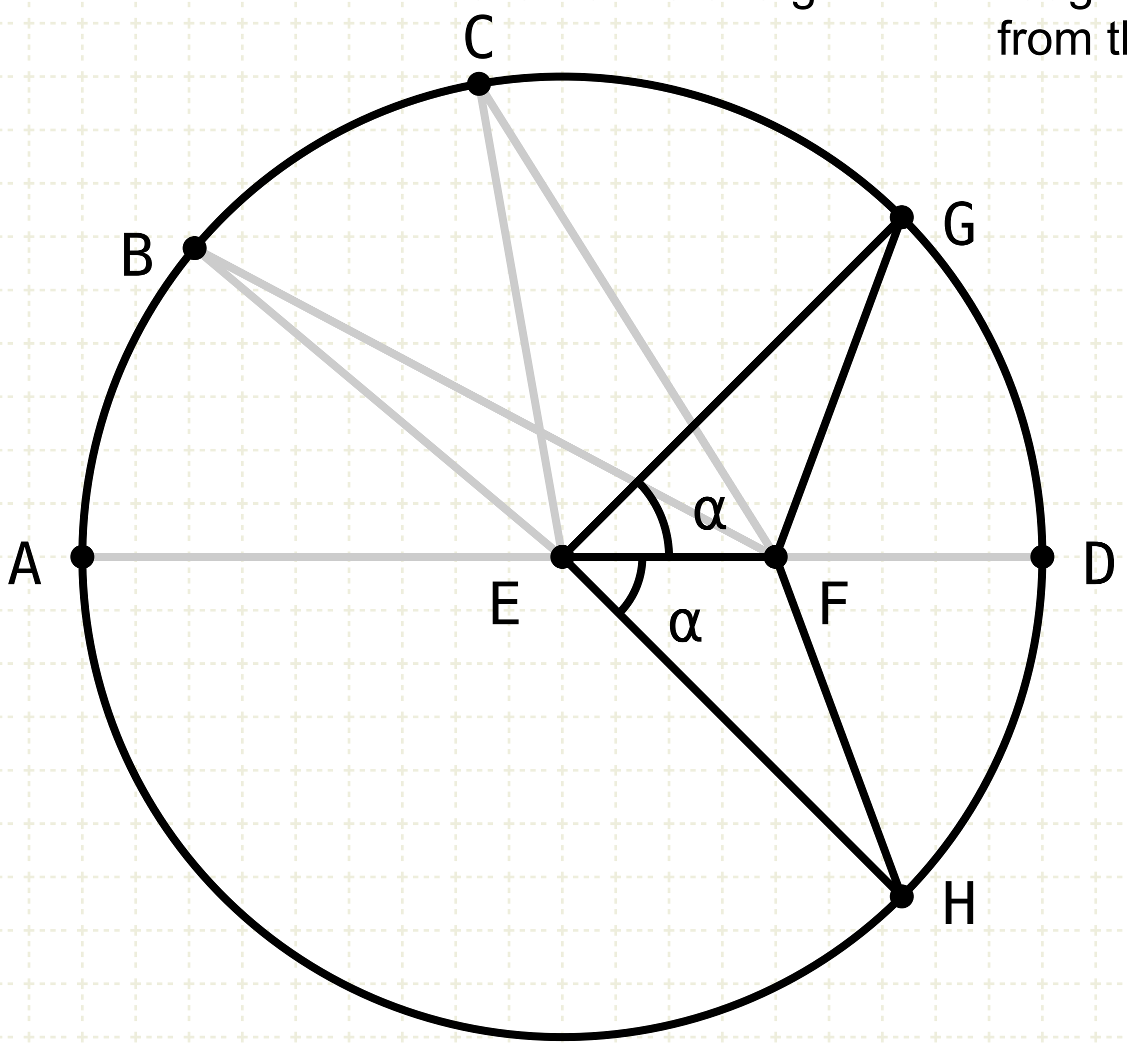

Proof (part 2)

Construct a line EH such that the angle FEH equals FEG, and draw the line FH

Proposition 7 of Book III

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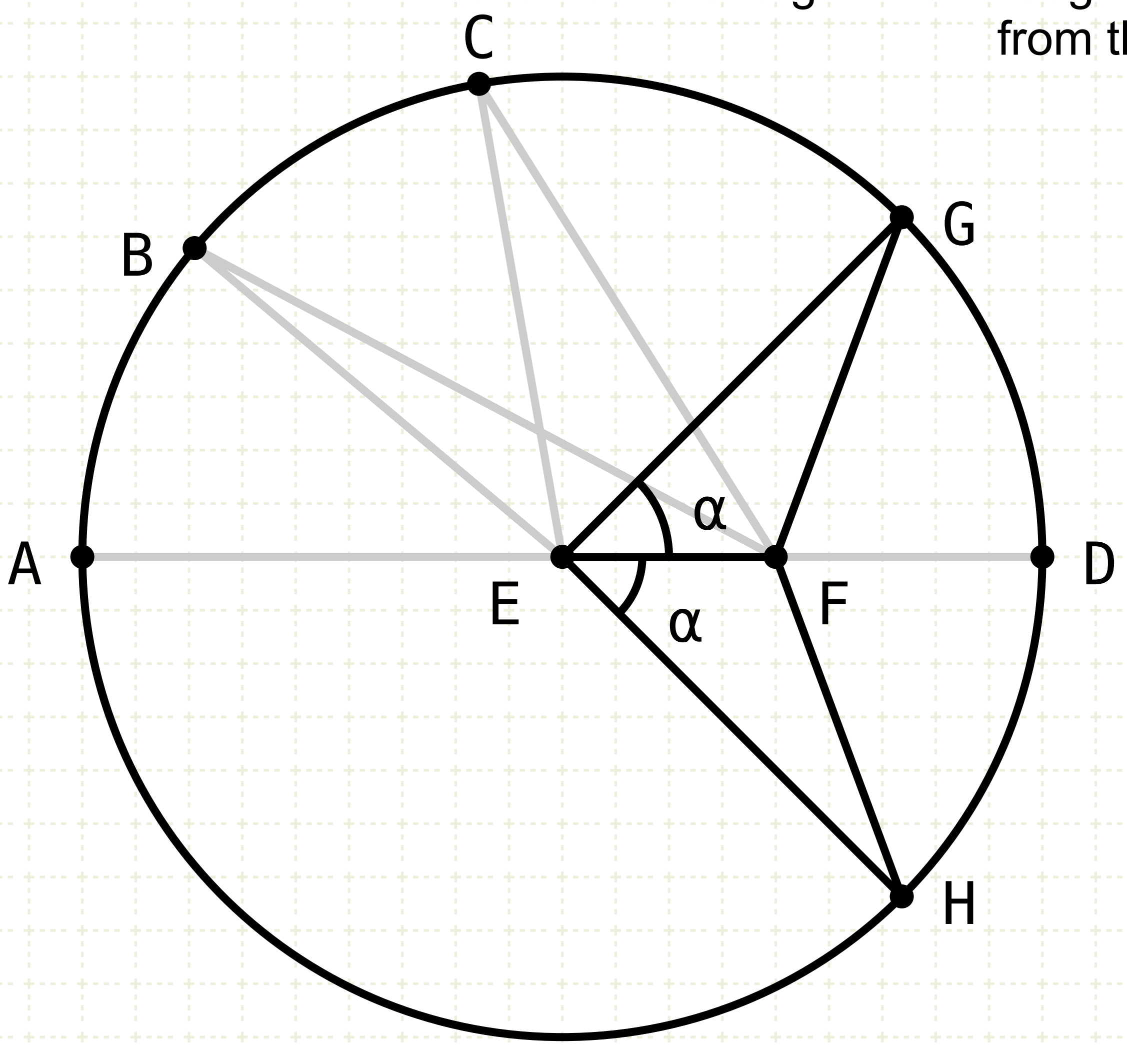


$\Delta EFH \equiv \Delta EGF$

$FG = FH$

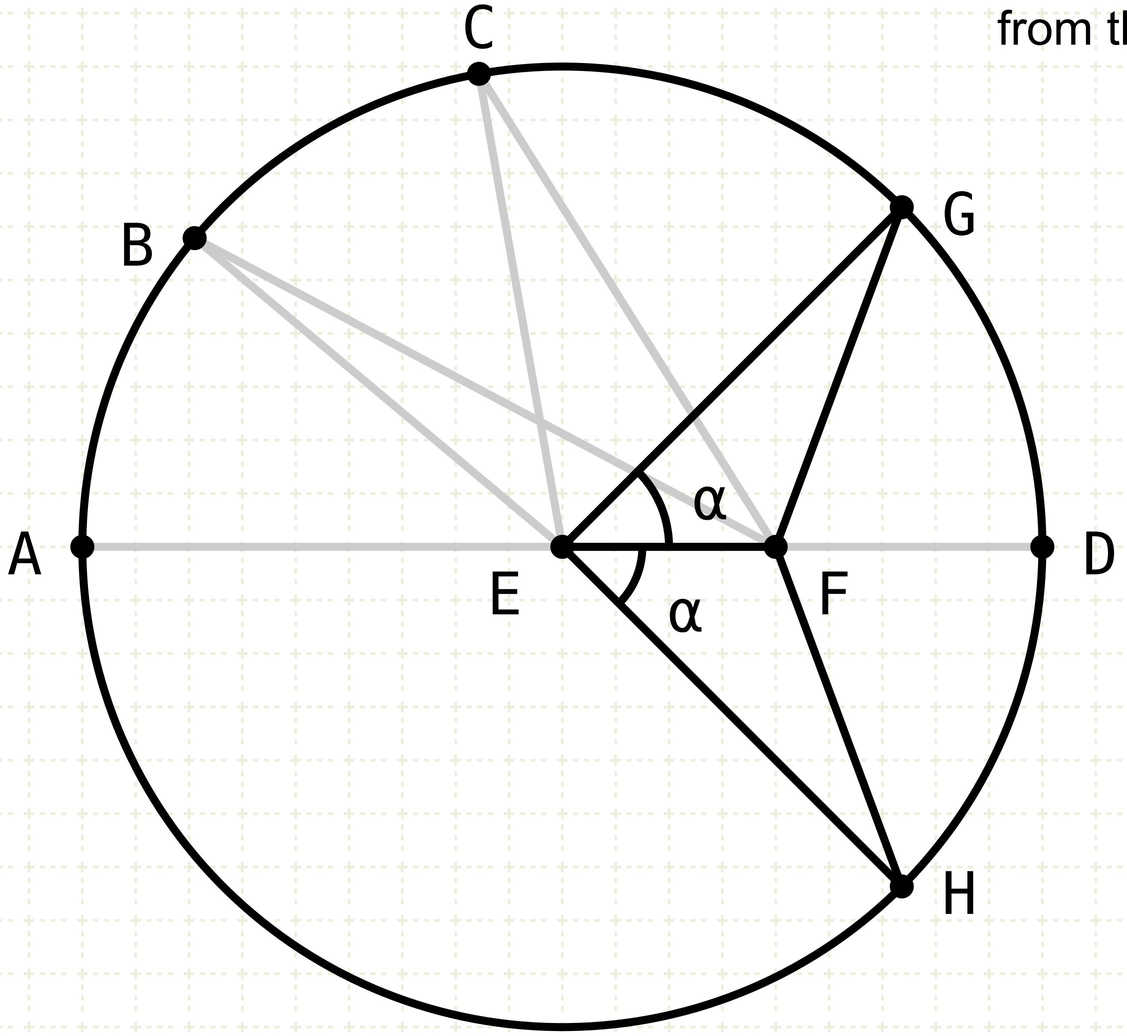
Proof (part 2)

Construct a line EH such that the angle FEH equals FEG, and draw the line FH

EG equals EH (radii of the same circle) and EF is common to both, so with two triangles with side-angle-side SAS equal, the triangles are equal and therefore FH equals FG (I·4)

Proposition 7 of Book III

If on a diameter of a circle a point be taken which is not the center of the circle, and from the point straight lines fall upon the circle, that will be the greatest on which the center is, the remainder on the same diameter will be the least and of the rest the nearer to the straight line through the center is always greater than the more remote, and only two equal straight lines will fall from the point on the circle, one on each side of the least straight line.



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Proof (part 2)

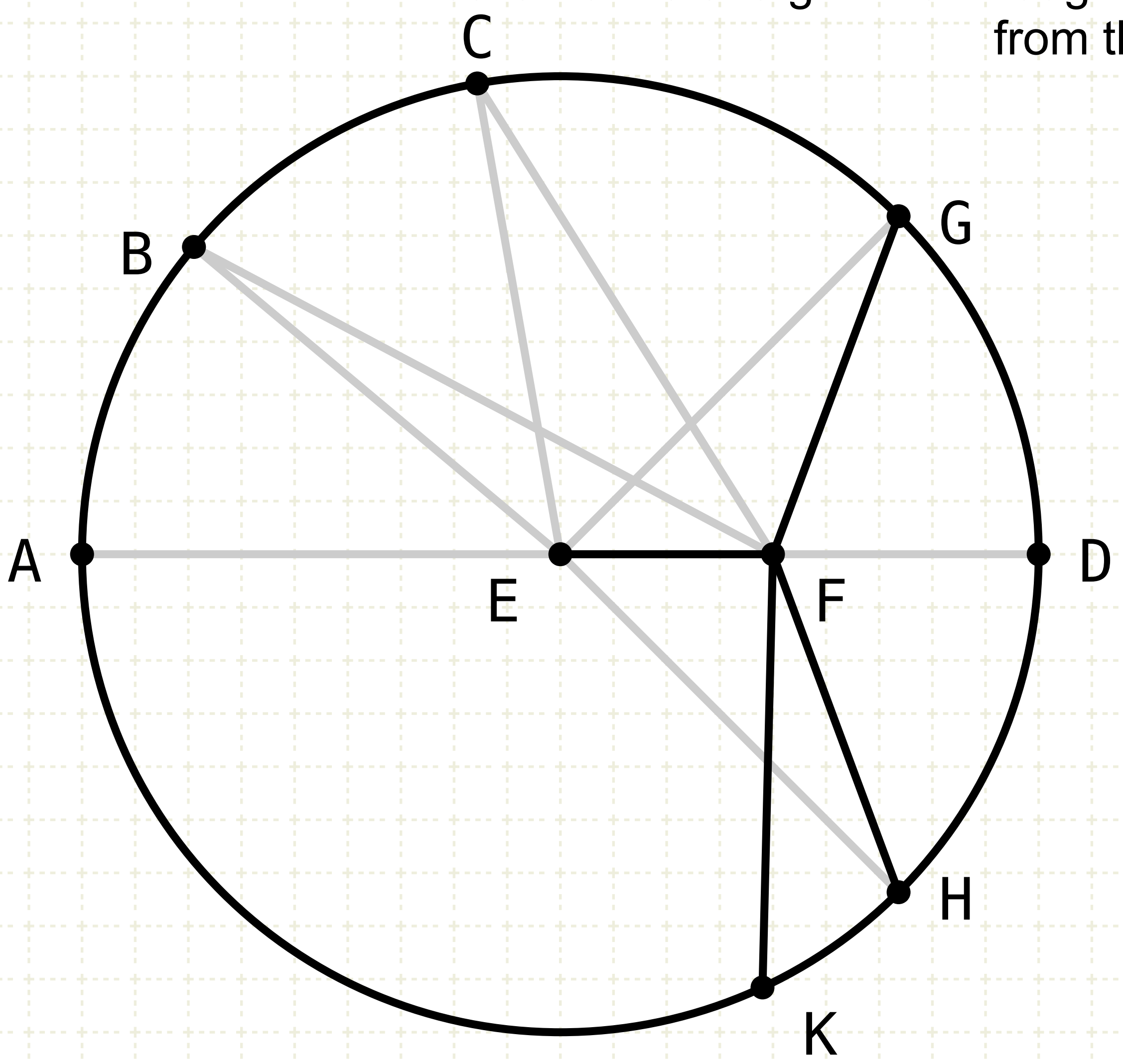
Construct a line EH such that the angle FEH equals FEG, and draw the line FH

EG equals EH (radii of the same circle) and EF is common to both, so with two triangles with side-angle-side SAS equal, the triangles are equal and therefore FH equals FG (I·4)

There is no other line that can fall from F to the circle equal in length to FG and FH

Proposition 7 of Book III

If on a diameter of a circle a point be taken which is not the center of the circle, and from the point straight lines fall upon the circle, that will be the greatest on which the center is, the remainder on the same diameter will be the least and of the rest the nearer to the straight line through the center is always greater than the more remote, and only two equal straight lines will fall from the point on the circle, one on each side of the least straight line.



$\Delta EFH \equiv \Delta EGF$

$FG = FH$

Assume…

$FK = FG$

Proof (part 2)

Construct a line EH such that the angle FEH equals FEG, and draw the line FH

EG equals EH (radii of the same circle) and EF is common to both, so with two triangles with side-angle-side SAS equal, the triangles are equal and therefore FH equals FG (I·4)

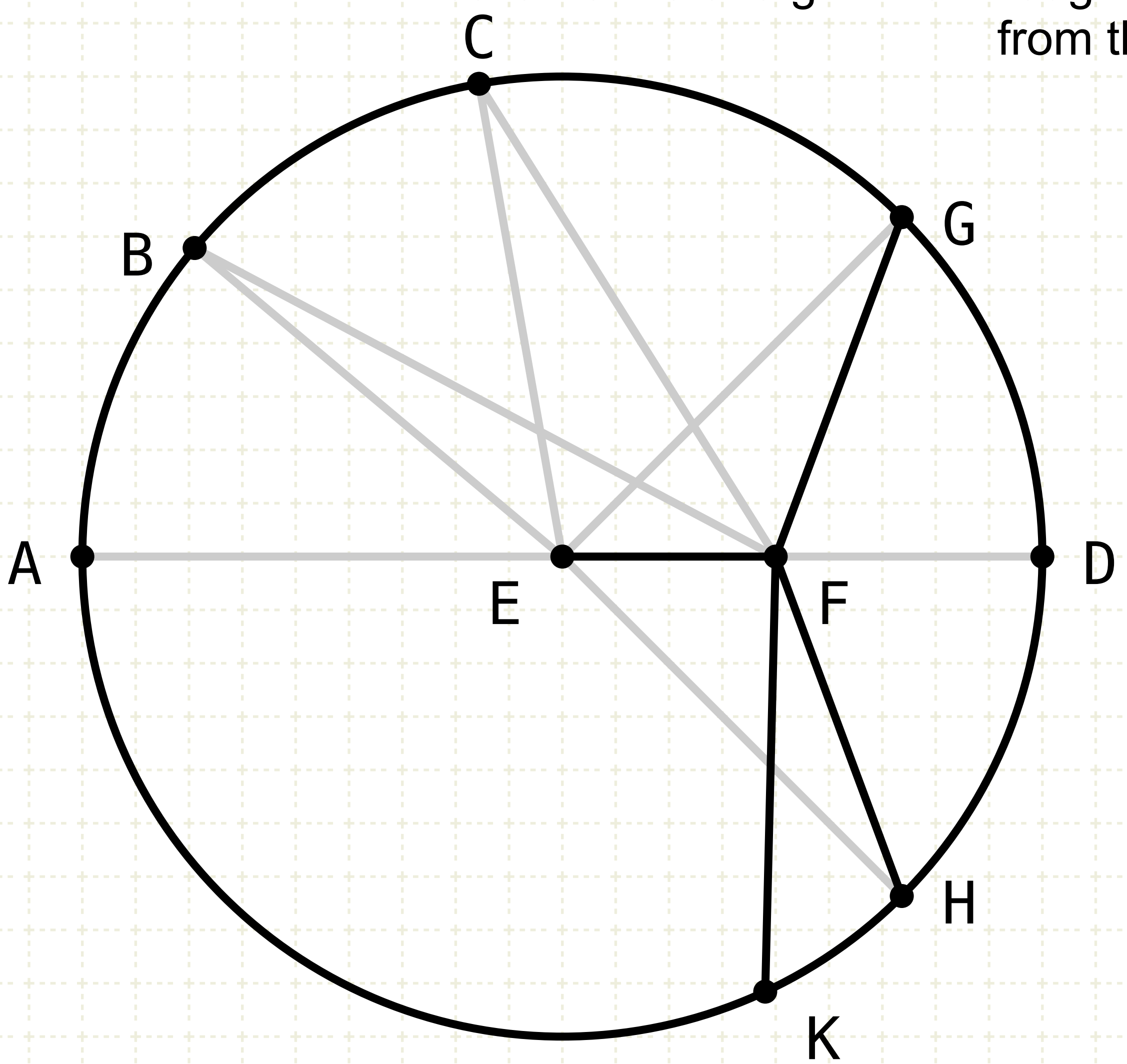
There is no other line that can fall from F to the circle equal in length to FG and FH

Proof by contradiction:

Assume a line FK exists, equal in length to FG

Proposition 7 of Book III

If on a diameter of a circle a point be taken which is not the center of the circle, and from the point straight lines fall upon the circle, that will be the greatest on which the center is, the remainder on the same diameter will be the least and of the rest the nearer to the straight line through the center is always greater than the more remote, and only two equal straight lines will fall from the point on the circle, one on each side of the least straight line.



$\Delta EFH \equiv \Delta EGF$

$FG = FH$

Assume…

$FK = FG$

$FK = FH$

Proof (part 2)

Construct a line EH such that the angle FEH equals FEG, and draw the line FH

EG equals EH (radii of the same circle) and EF is common to both, so with two triangles with side-angle-side SAS equal, the triangles are equal and therefore FH equals FG (I·4)

There is no other line that can fall from F to the circle equal in length to FG and FH

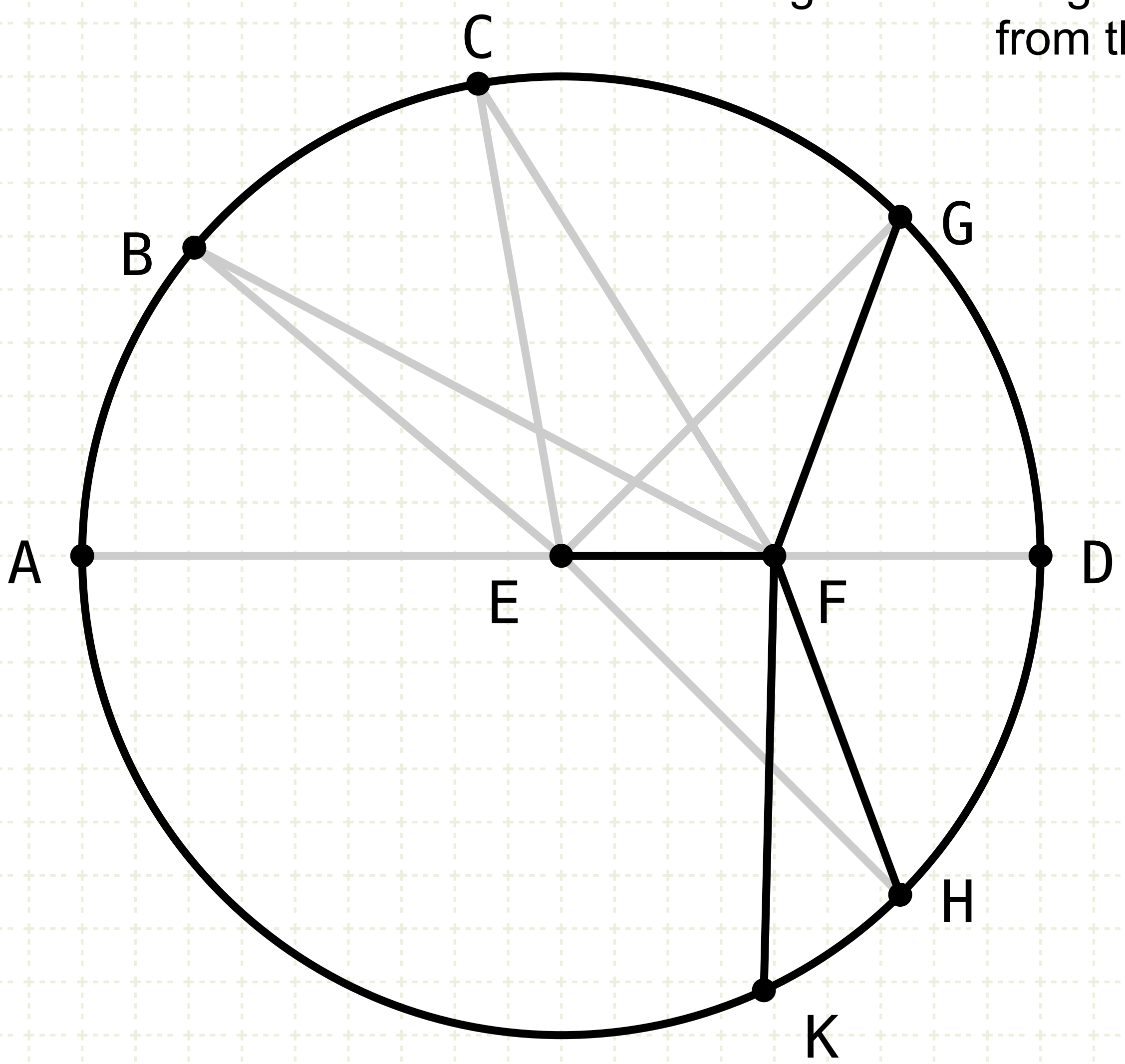
Proof by contradiction:

Assume a line FK exists, equal in length to FG

FK is equal to FG, but FG is equal to FH, therefore FK equals FH

Proposition 7 of Book III

If on a diameter of a circle a point be taken which is not the center of the circle, and from the point straight lines fall upon the circle, that will be the greatest on which the center is, the remainder on the same diameter will be the least and of the rest the nearer to the straight line through the center is always greater than the more remote, and only two equal straight lines will fall from the point on the circle, one on each side of the least straight line.



$\Delta EFH \equiv \Delta EGF$

$FG = FH$

Assume...

$FK = FG$

$FK = FH$ ✗

$FK > FH$

Proof (part 2)

Construct a line EH such that the angle FEH equals FEG, and draw the line FH

EG equals EH (radii of the same circle) and EF is common to both, so with two triangles with side-angle-side SAS equal, the triangles are equal and therefore FH equals FG (I·4)

There is no other line that can fall from F to the circle equal in length to FG and FH

Proof by contradiction:

Assume a line FK exists, equal in length to FG

FK is equal to FG, but FG is equal to FH, therefore FK equals FH

But, according to the first part of this proposition, FK, being closer to FE, is larger than FH, which contradicts the original statement

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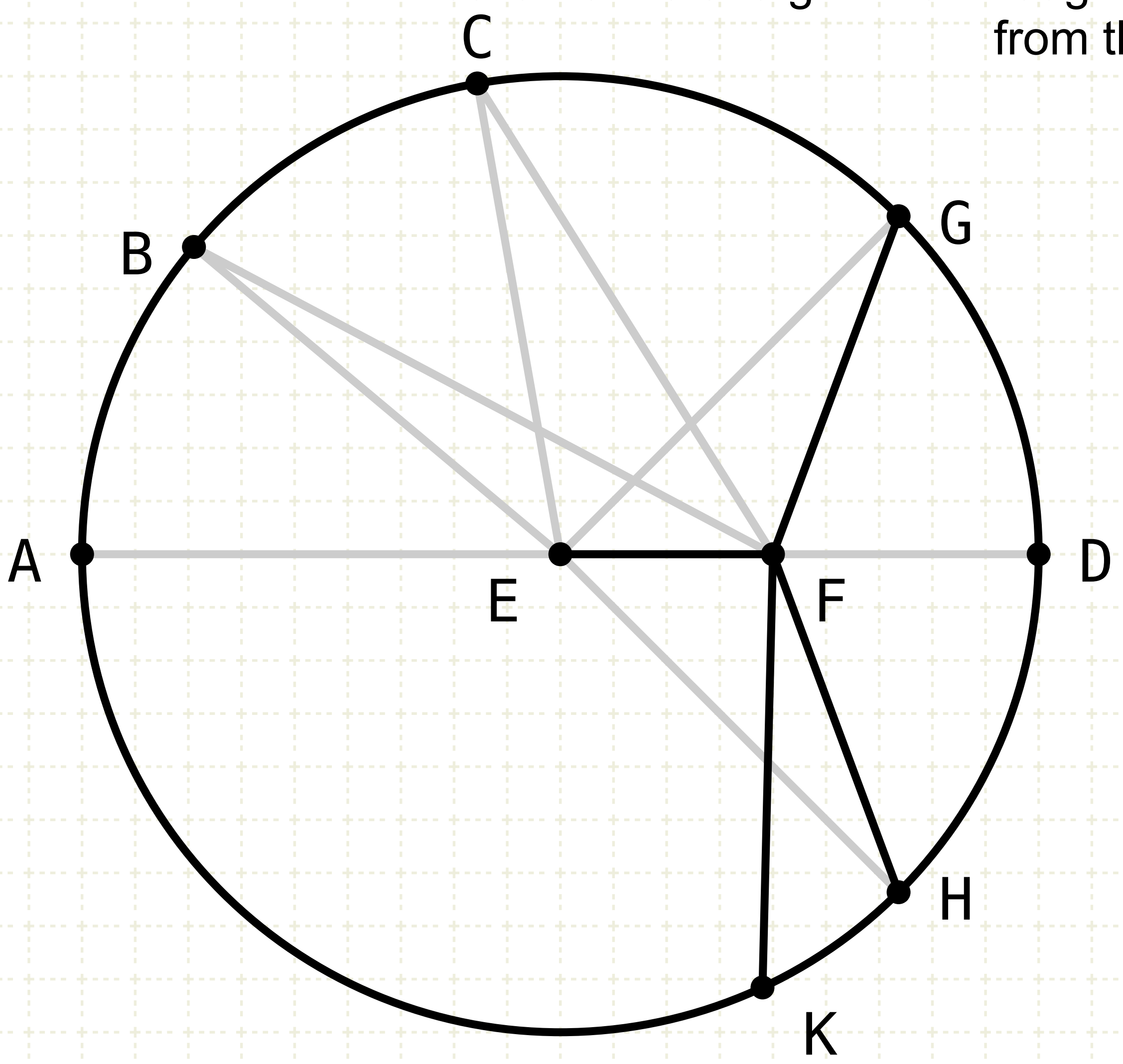

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Therefore there are only two lines of equal length from F to the circle circumference

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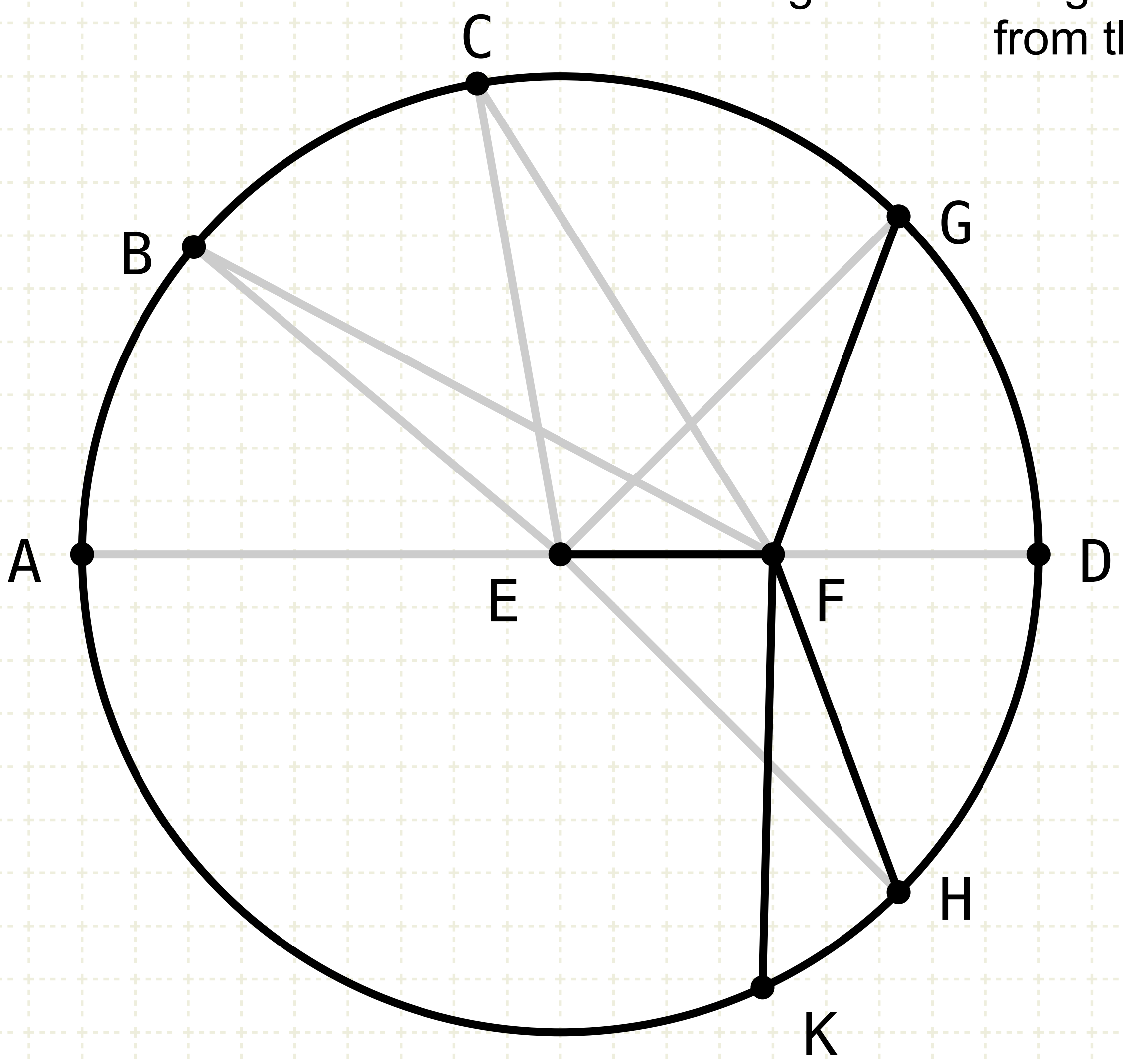

$\Delta EFH \equiv \Delta EGF$

$FG = FH$

Assume...

$FK = FG$

$FK = FH$ ✗

$FK > FH$

$FK \neq FG$

Proof (part 2)

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