

# Euclid's Elements

## Book I

*If Euclid did not kindle your youthful enthusiasm, you were not born to be a scientific thinker.*

Albert Einstein



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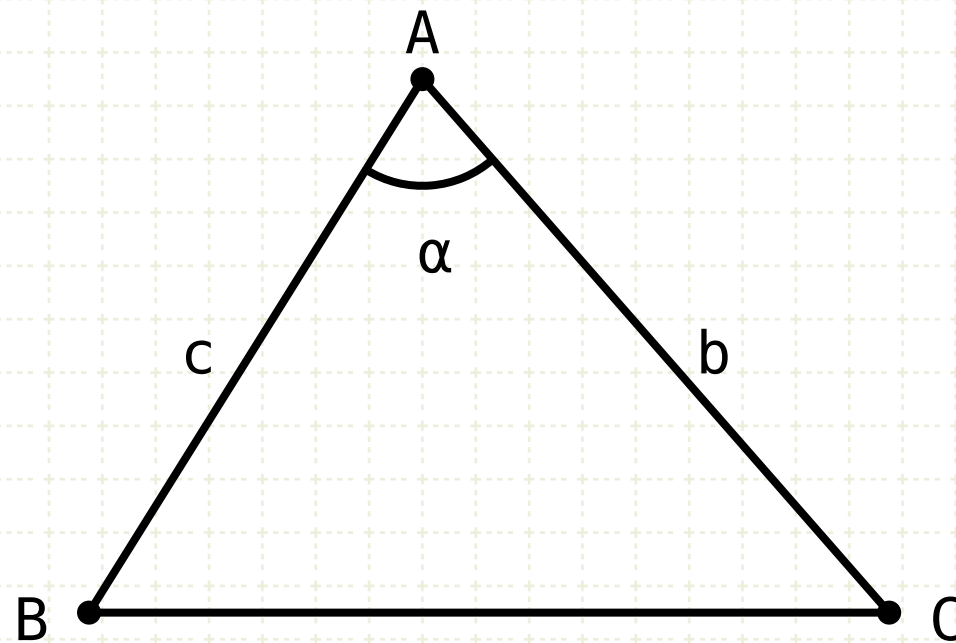
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If two triangles have two sides equal to two sides respectively, but have one of the angles contained by the equal straight lines greater than the other, then they also have the base greater than the base.



# Proposition 24 of Book I

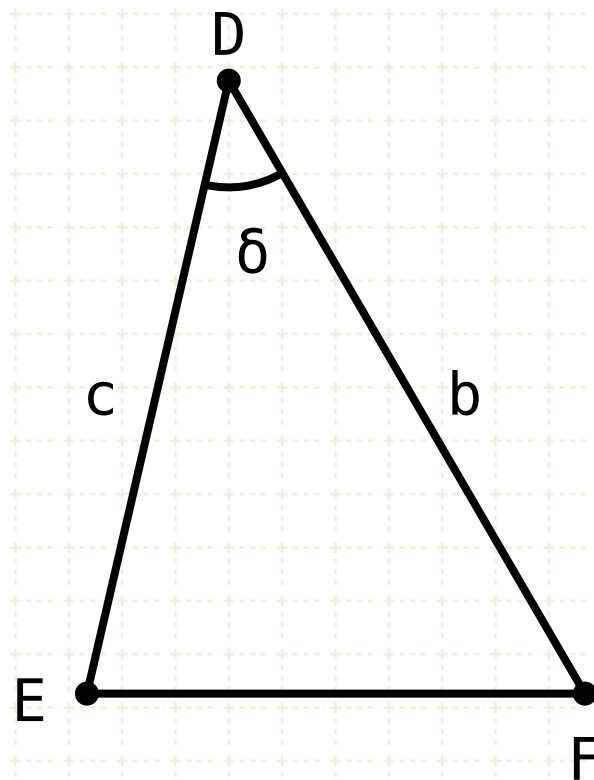
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$$\alpha > \delta$$
$$AB = DE = c$$
$$AC = DF = b$$

## In other words

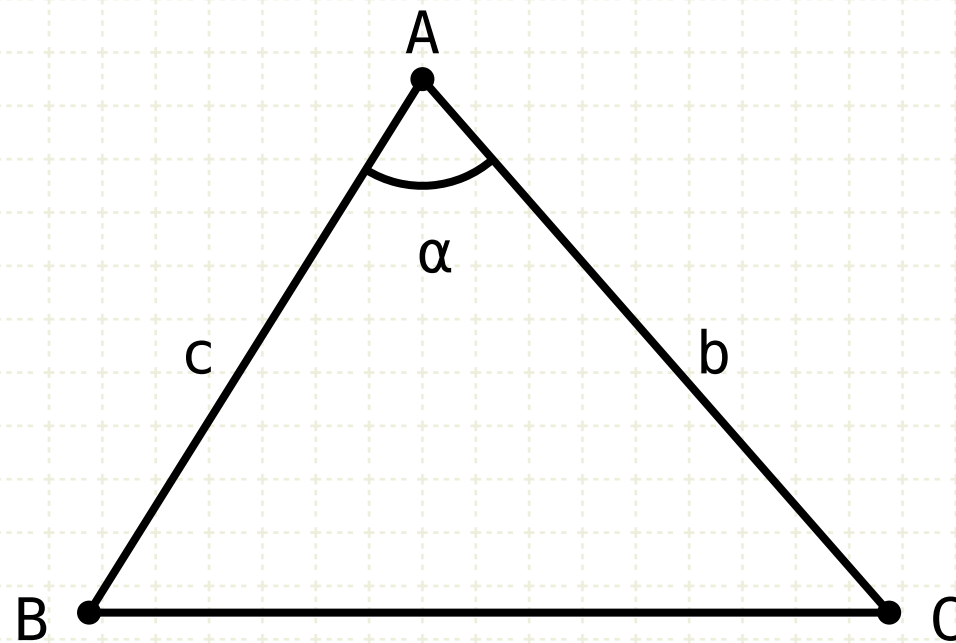
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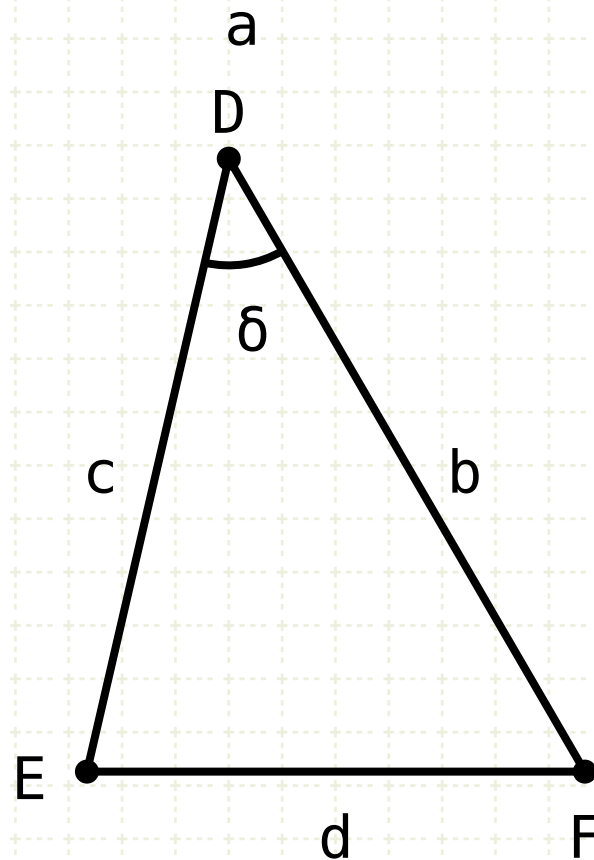


$$\alpha > \delta$$

$$AB = DE = c$$

$$AC = DF = b$$

$$BC > EF, a > d$$



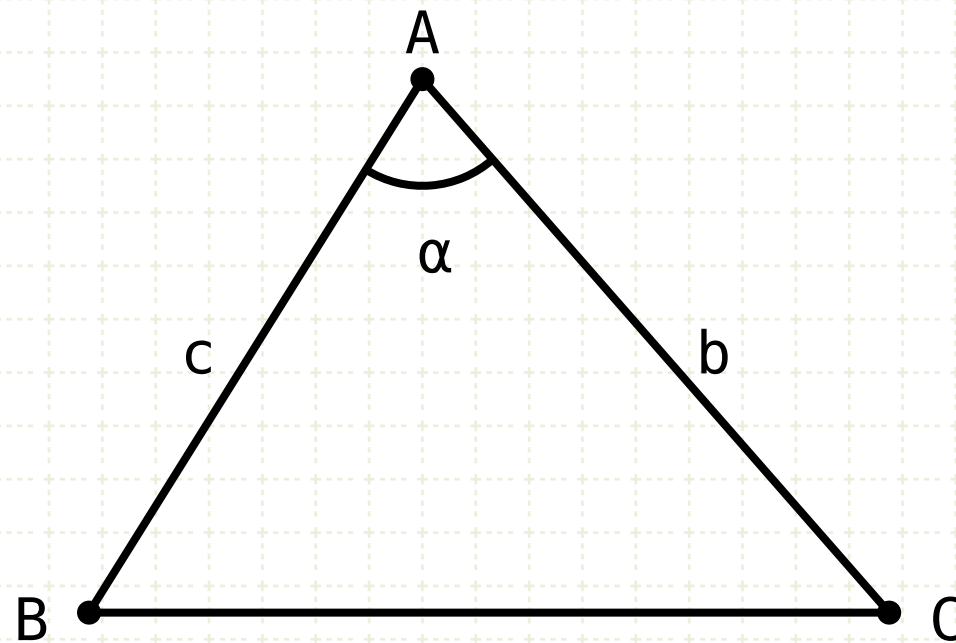
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Given two triangles ABC and DEF, where lengths AB equals DE and AC equals DF, and angle BAC is greater than DEF

Then length BC is greater than length EF

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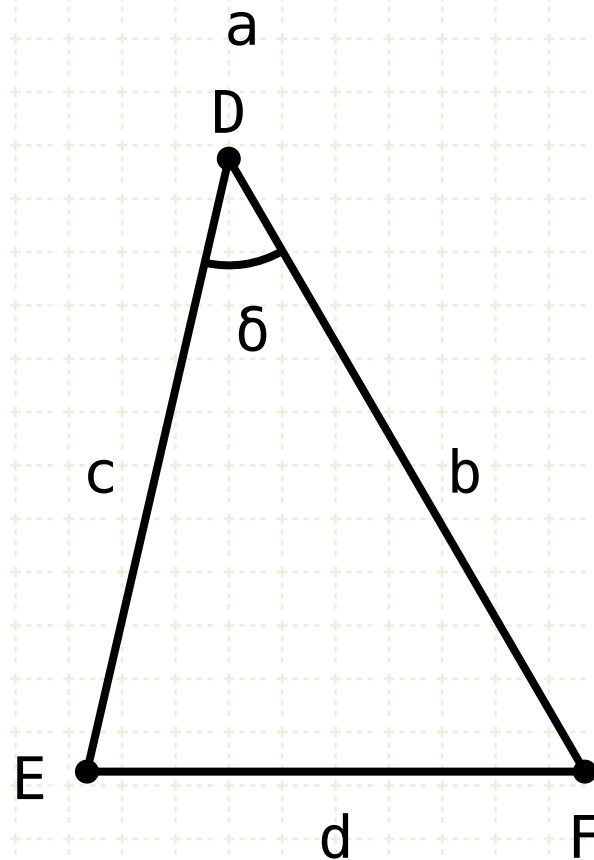
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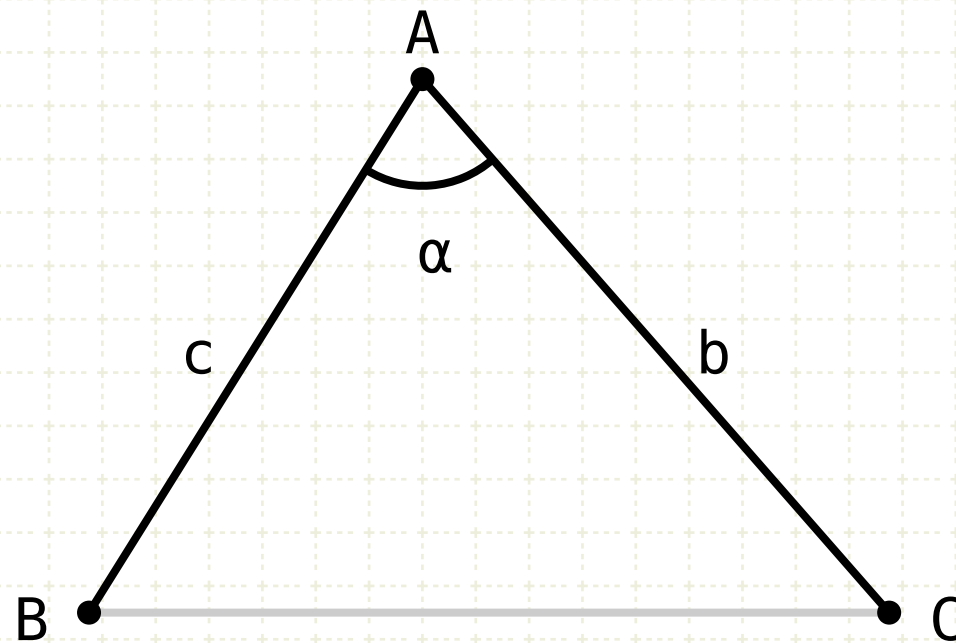
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## Proof



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If two triangles have two sides equal to two sides respectively, but have one of the angles contained by the equal straight lines greater than the other, then they also have the base greater than the base.



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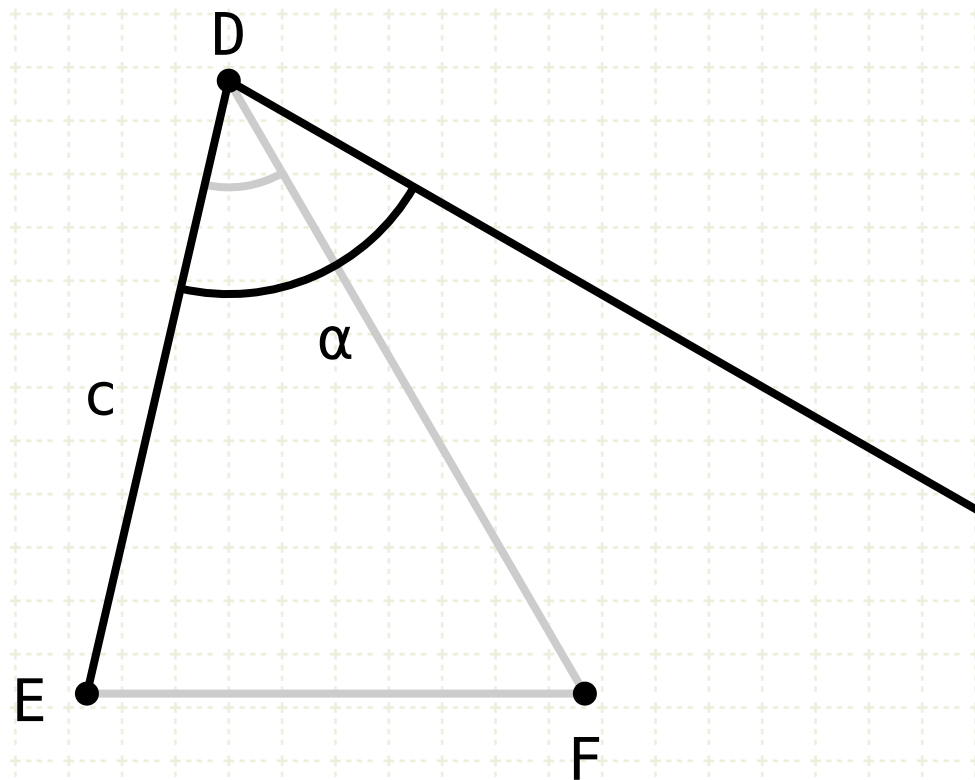
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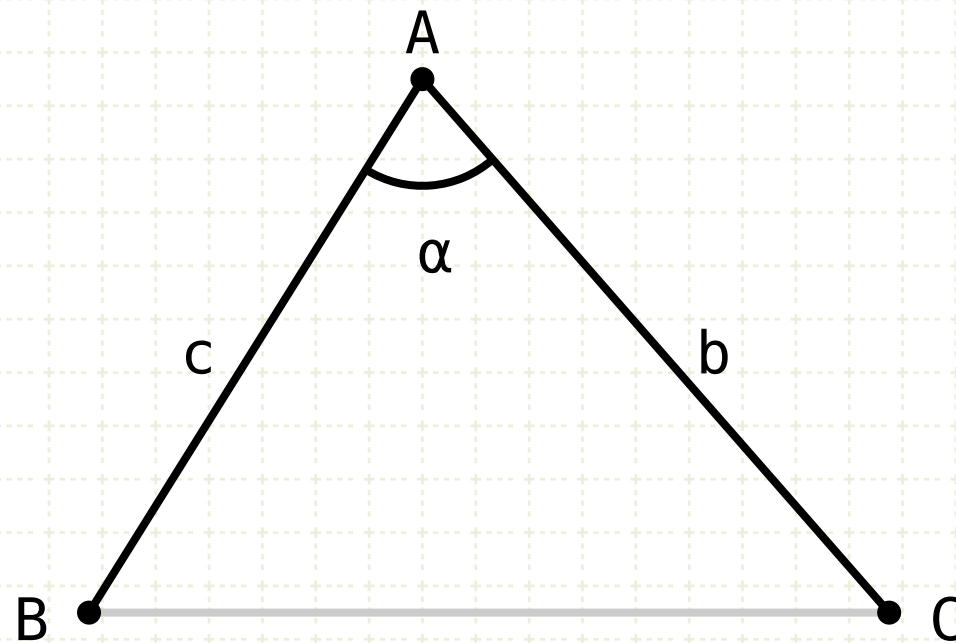
Copy the angle BAC onto line ED at point D (I-23)





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If two triangles have two sides equal to two sides respectively, but have one of the angles contained by the equal straight lines greater than the other, then they also have the base greater than the base.



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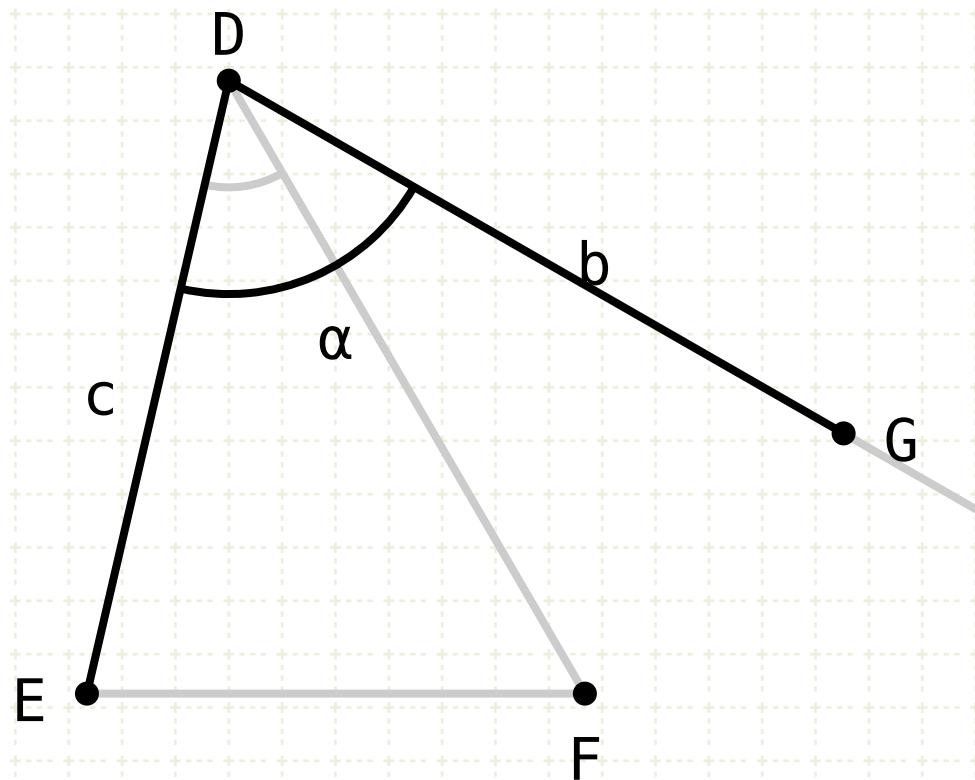
Given two triangles ABC and DEF, where lengths AB equals DE and AC equals DF, and angle BAC is greater than DEF

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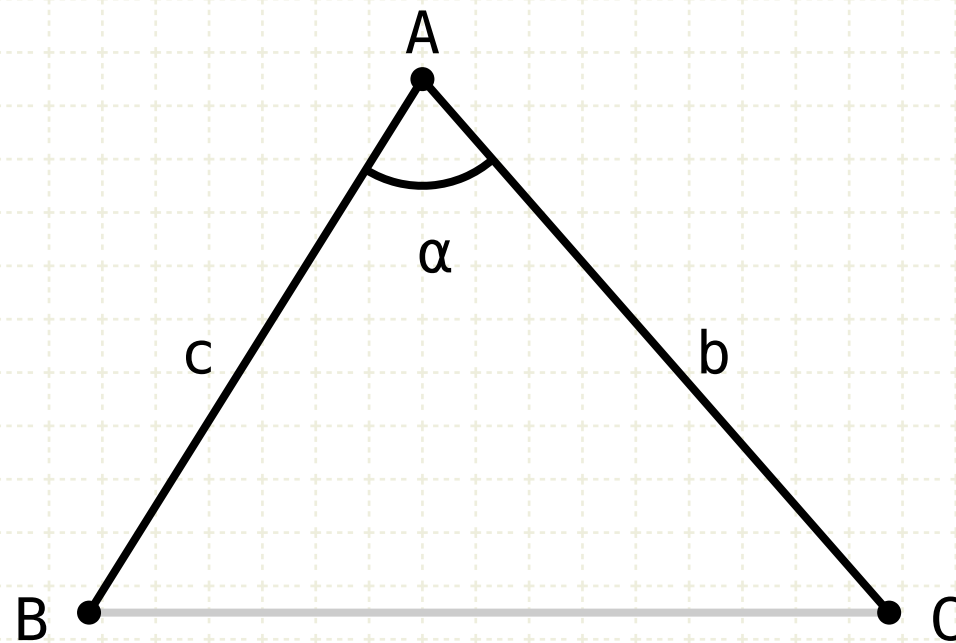
Copy the angle BAC onto line ED at point D (I-23)

Define point G on the copied angle such that DG equals DF



# Proposition 24 of Book I

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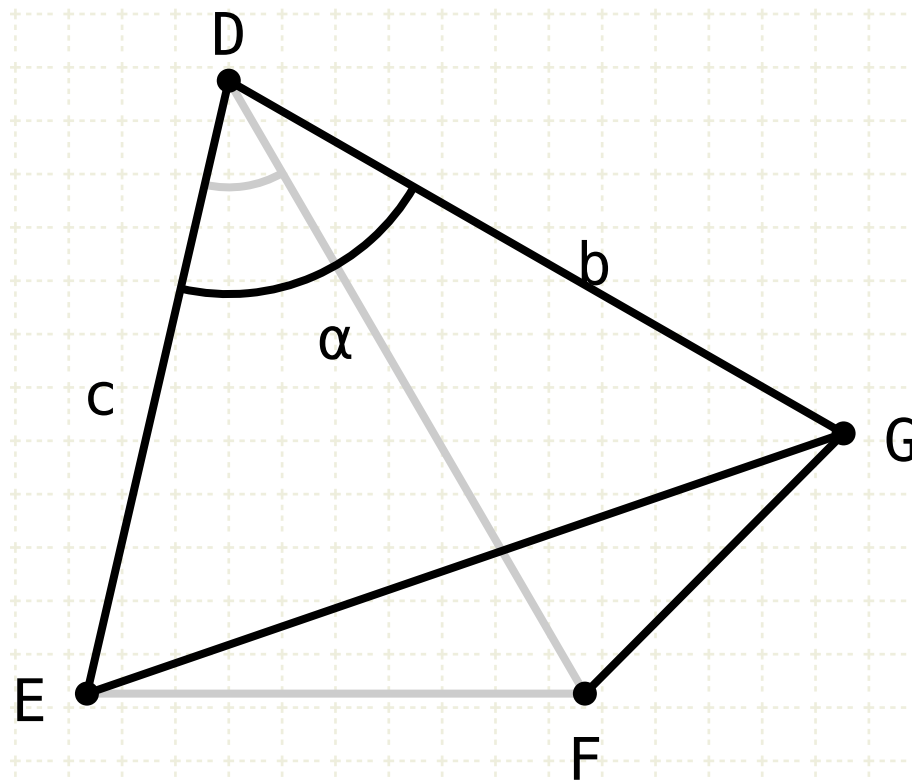
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Copy the angle BAC onto line ED at point D (I-23)

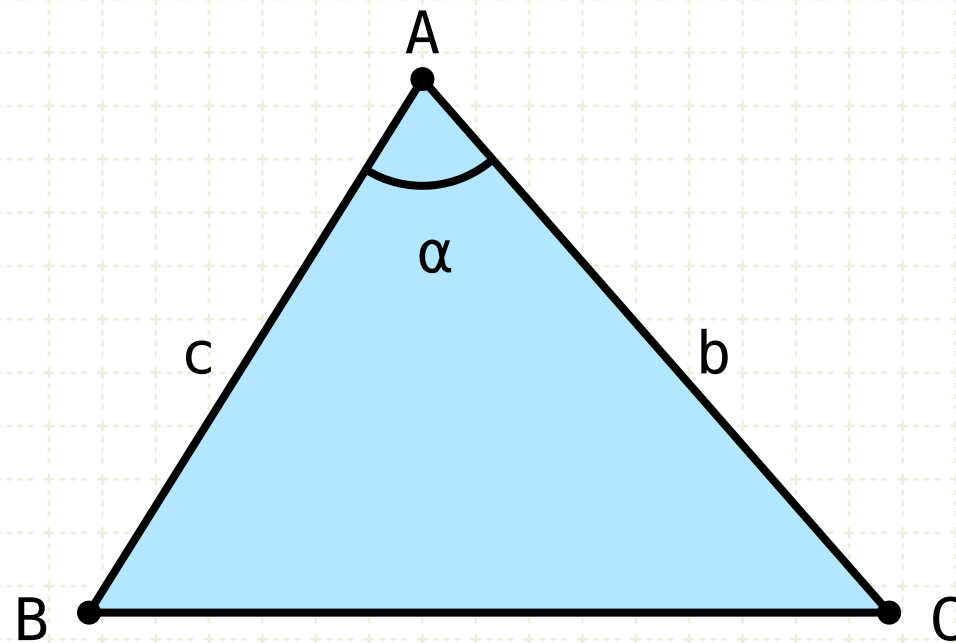
Define point G on the copied angle such that DG equals DF

Construct line EG and FG

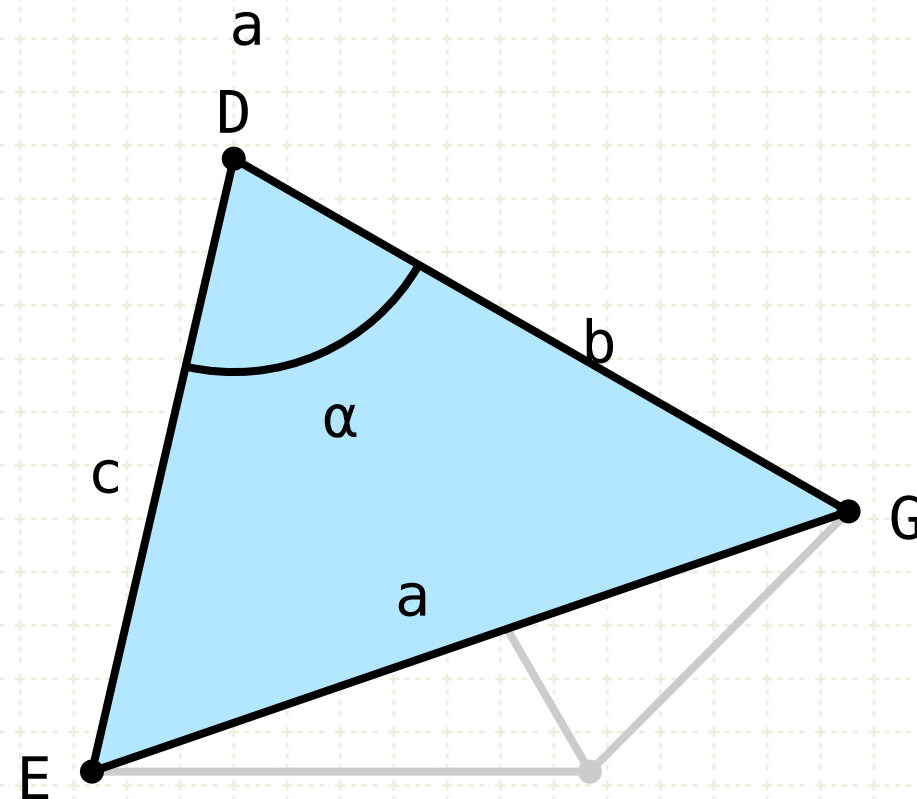


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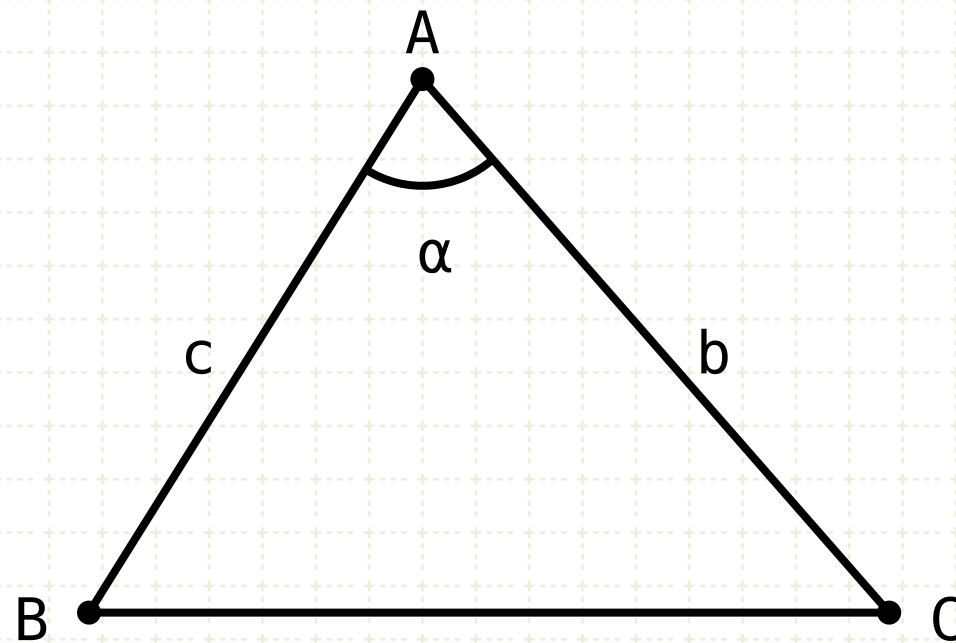
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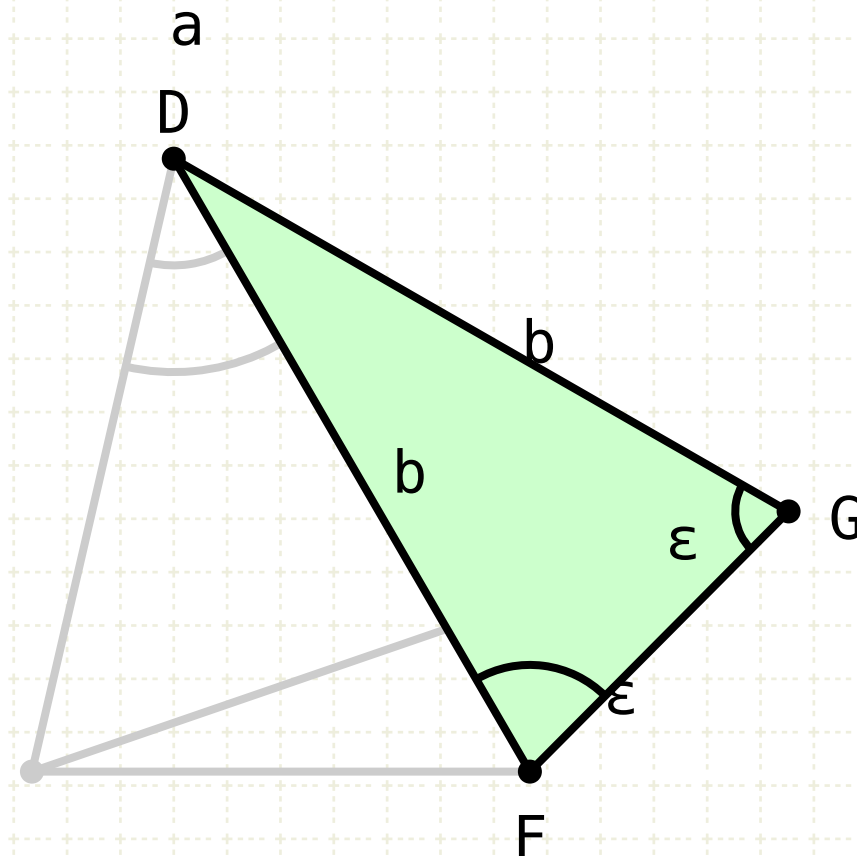
Triangle ABC and DEG have two equal sides with an equal angle between them, hence they are equal, and the line BC equals EG (I-4)

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Copy the angle BAC onto line ED at point D (I·23)

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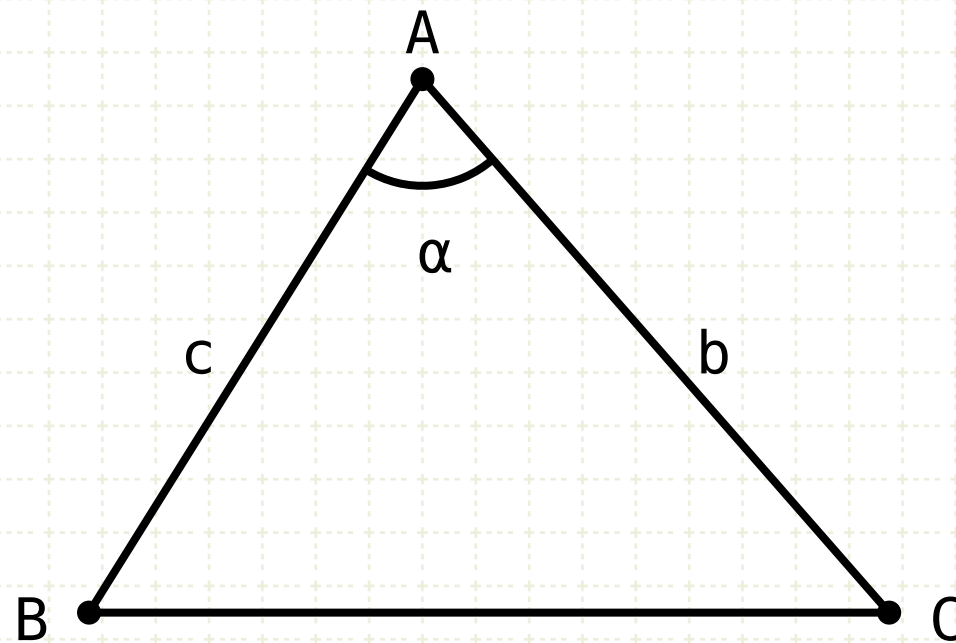
Triangle ABC and DEG have two equal sides with an equal angle between them, hence they are equal, and the line BC equals EG (I·4)

Consider triangle FDG

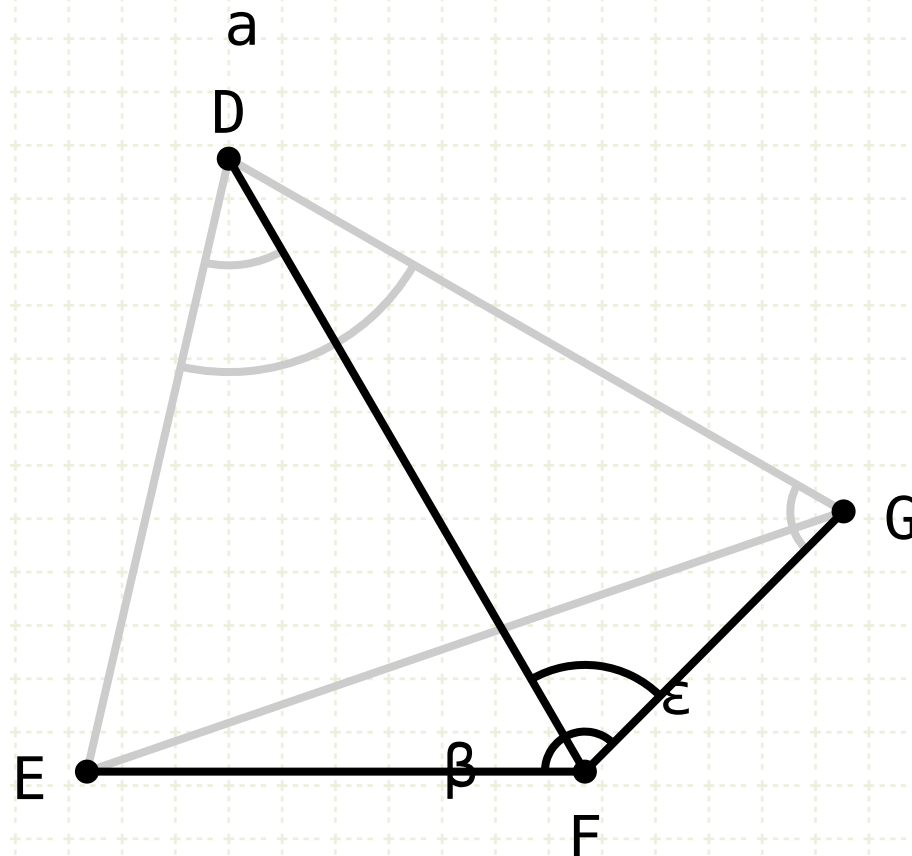
Angles DFG and DGF are equal since the triangle is an isosceles triangle (I·5)

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## In other words

Given two triangles ABC and DEF, where lengths AB equals DE and AC equals DF, and angle BAC is greater than DEF

Then length BC is greater than length EF

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Copy the angle BAC onto line ED at point D (I·23)

Define point G on the copied angle such that DG equals DF

Construct line EG and FG

Triangle ABC and DEG have two equal sides with an equal angle between them, hence they are equal, and the line BC equals EG (I·4)

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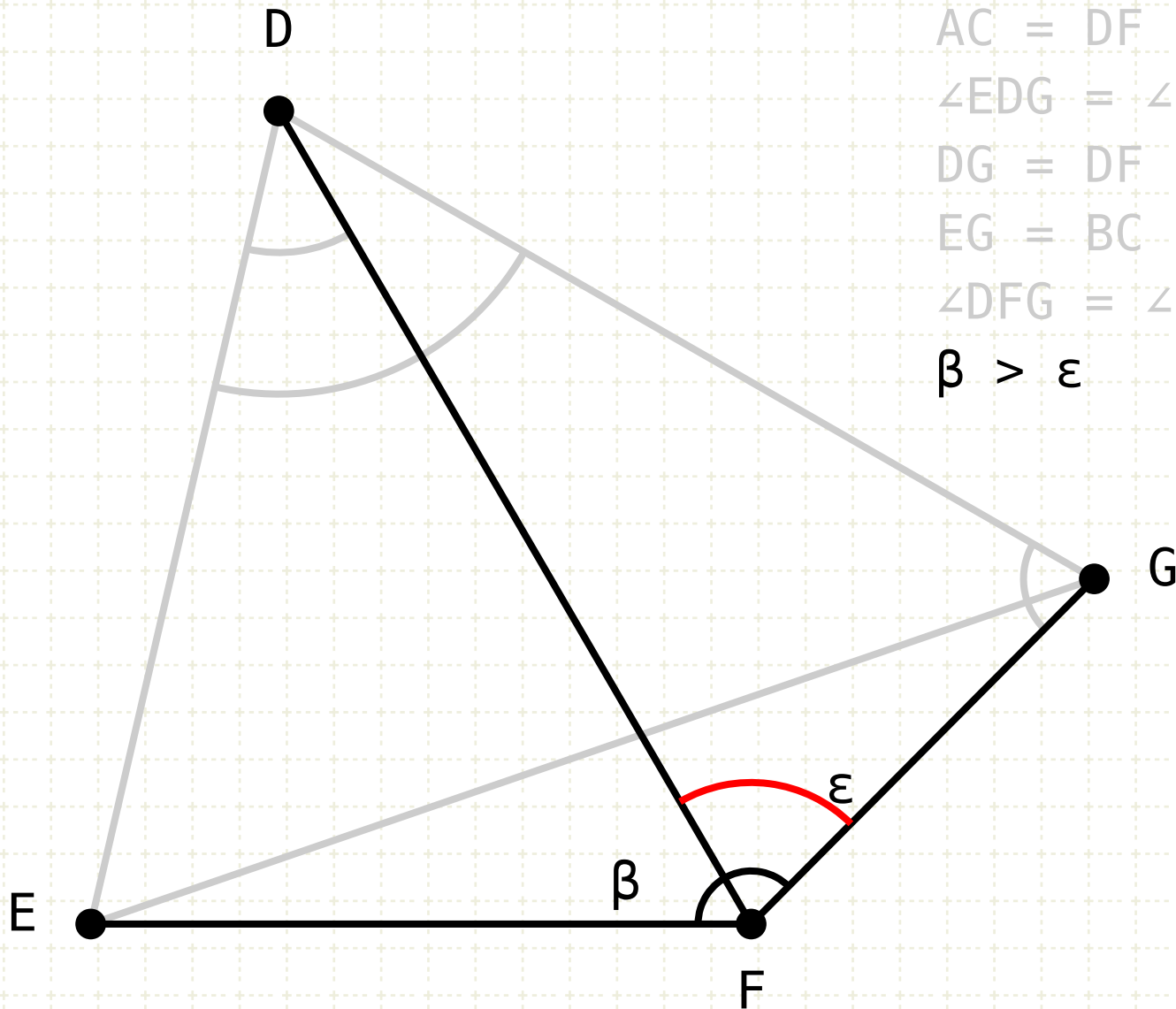
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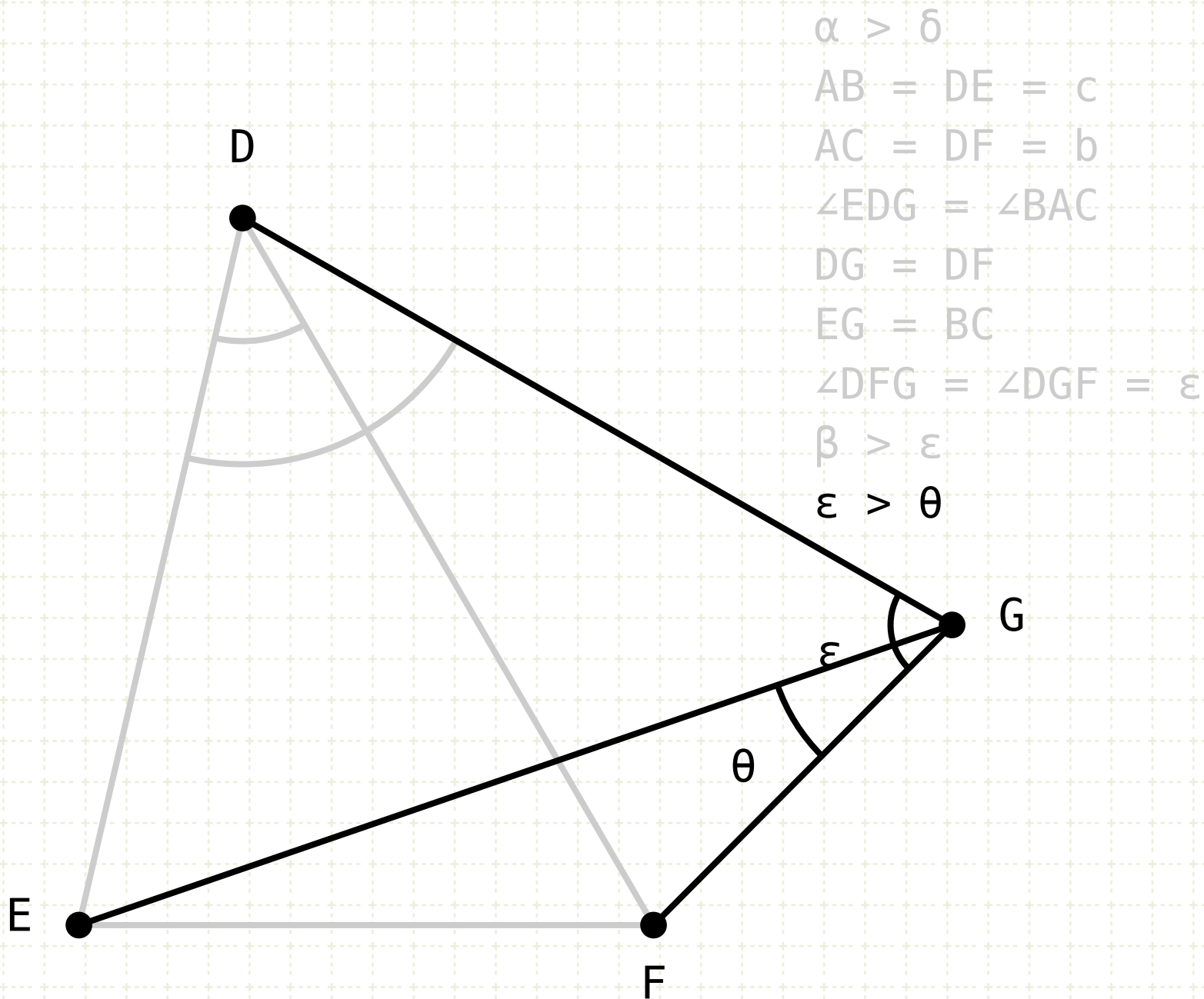
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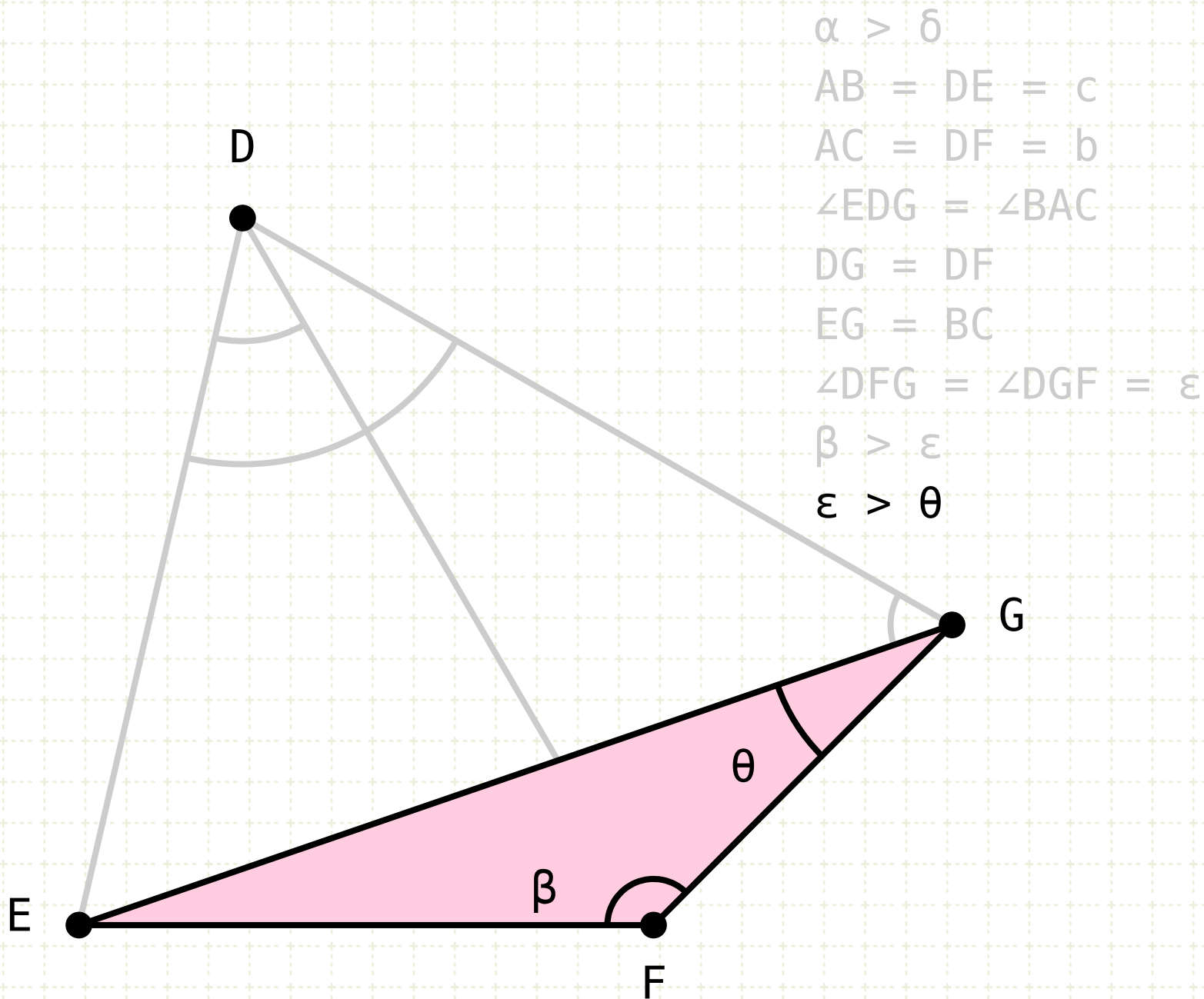
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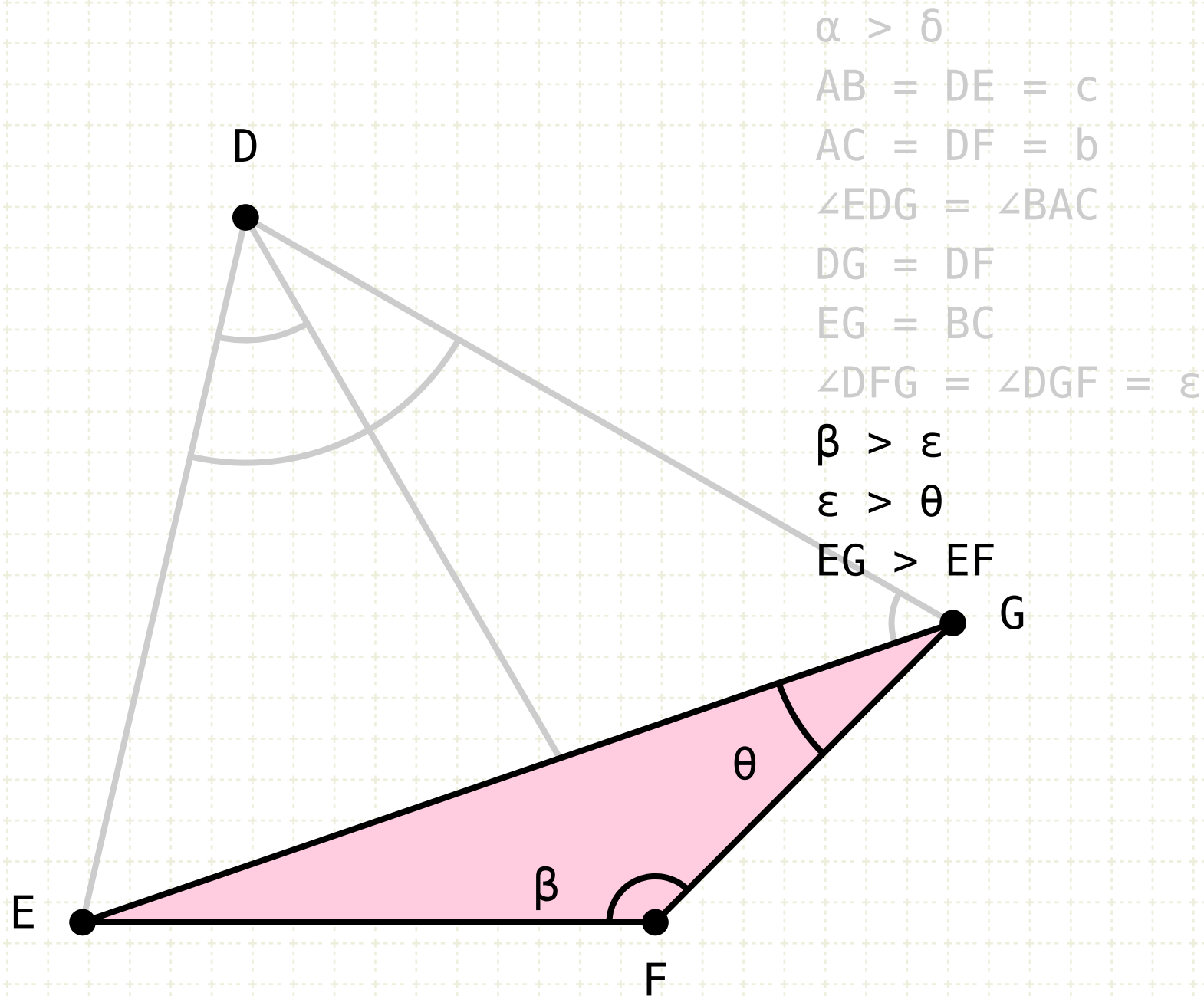
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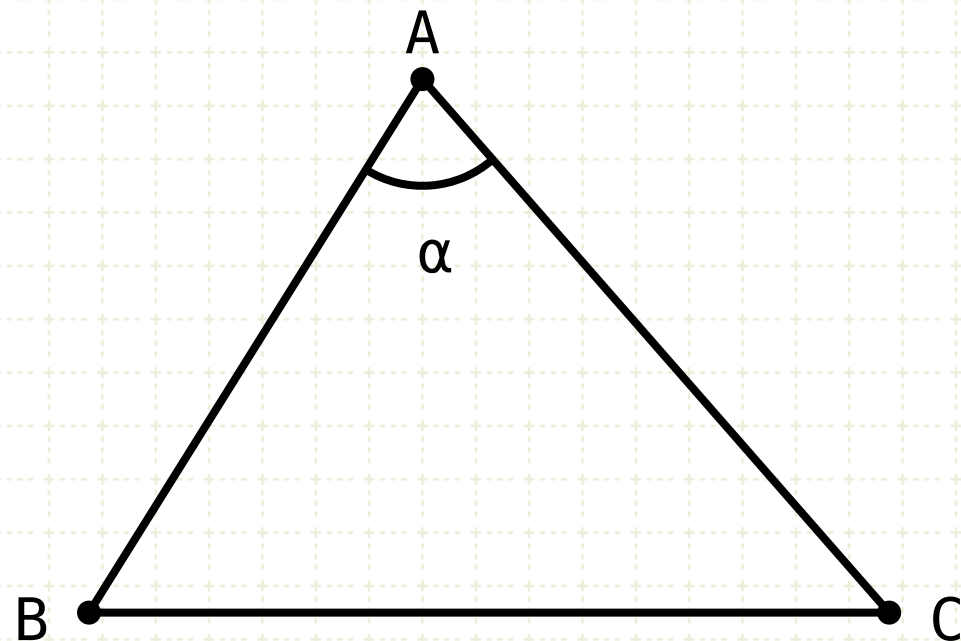
The angle EFG is greater than EGF, hence line EG is greater than EF (I·19)



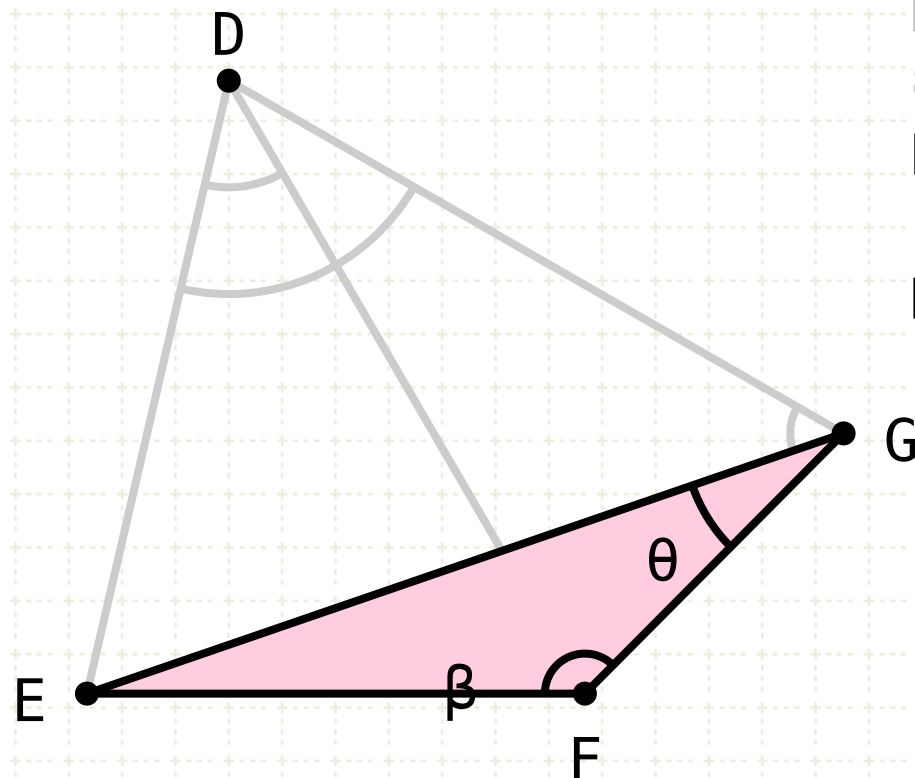


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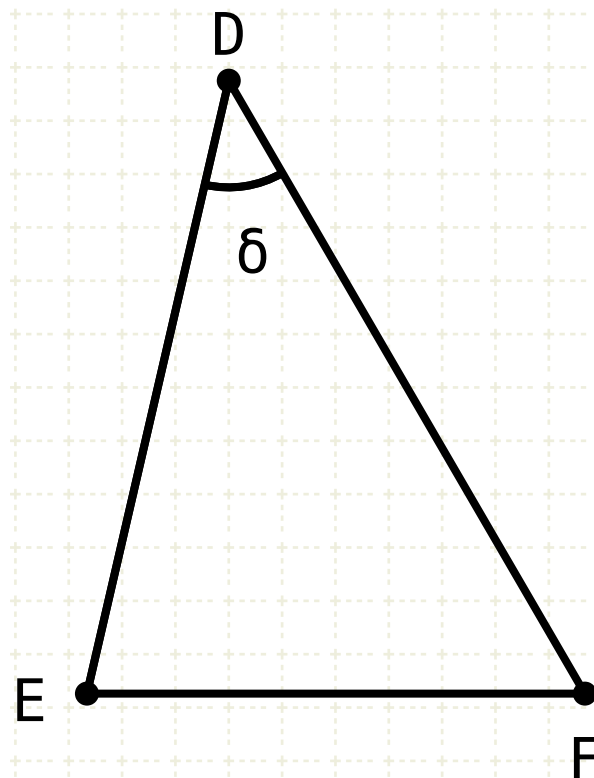
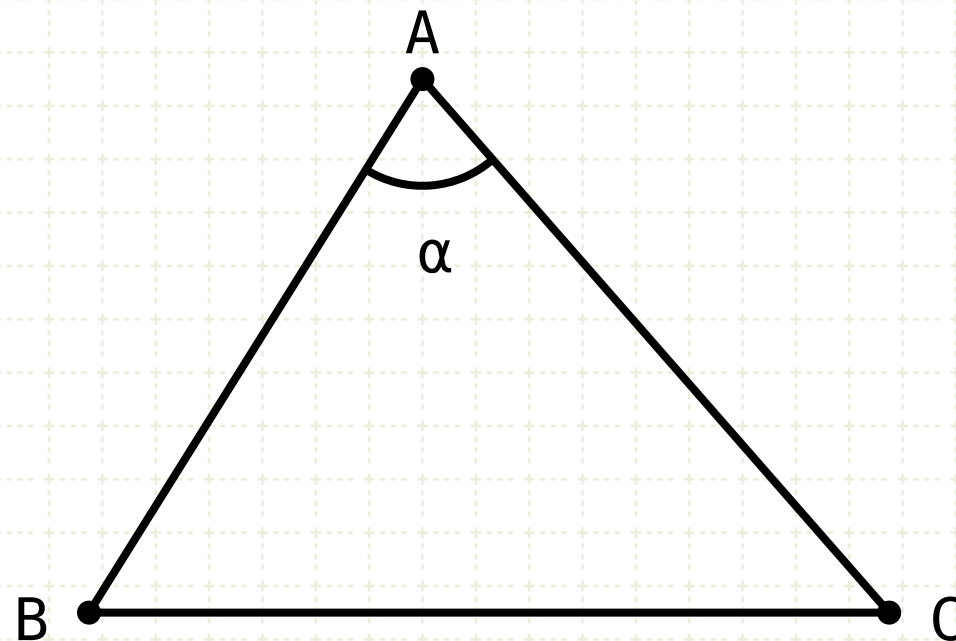
Since EG is equal to BC, BC is greater than EF





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Angle EFG is greater than DFG

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The angle EFG is greater than EGF, hence line EG is greater than EF (I·19)

Since EG is equal to BC, BC is greater than EF



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