

Euclid's Elements

Book VI

One can state, without exaggeration, that the observation of and the search for similarities and differences are the basis of all human knowledge.

Alfred Nobel



Table of Contents, Chapter 6

1	If the height of two triangles are equal, then the ratio of the areas is equal to the ratio of the bases	7	If two triangles have one angle equal to one angle, and the sides about other angles are proportional, and the remaining angles either both less or both not less than a right angle, then triangles will be equiangular	14	In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and vice versa
2	If a line cuts a triangle, parallel to its base, it will cut the sides of the triangle proportionally	8	If in a right-angled triangle a perpendicular be drawn from the right angle to the base, the triangles adjoining the perpendicular are similar both to the whole and to one another	15	In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and vice versa
3	If an angle of a triangle is bisected and the straight line cutting the angle also cuts the base, the segments of the base will have the same ratio as the remaining sides of the triangle	9	From a given straight line to cut off a given fraction	16	If four straight lines are proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means, and vice versa
4	If two triangles have equal angles, then the sides opposite the equal angles are proportional, as well, the sides of the triangles on either side of the equal angles are also proportional	10	To cut a given uncut straight line similarly to a given cut straight line	17	If three straight lines are proportional, the rectangle contained by the extremes is equal to the square on the mean; and vice versa
5	If two triangles have proportional sides, the triangles will be equiangular	11	To two given straight lines to find a third proportional	18	On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure
6	If two triangles have one angle equal to one angle and the sides about the equal angles are proportional, then the triangles will be equiangular	12	To three given straight lines to find a fourth proportional	19	Similar triangles are to one another in the duplicate ratio of the corresponding sides
		13	To two given straight lines to find a mean proportional		



Table of Contents, Chapter 3

20	Similar polygons are divided into the same number of similar triangles, which have the same ratio as the wholes, and the polygons have duplicate ratios to their corresponding sides	26	If from a parallelogram a similar parallelogram with a common angle is subtracted, it is about the same diameter as the original	31	In right-angled triangles the figure on the side subtending the right angle is equal to the similar and similarly described figures on the sides containing the right angle
21	Figures which are are similar to the same rectilineal figure are also similar to one another	27	Of all the parallelograms applied to the same straight line and deficient by parallelogrammic figures similar to a parallelogram drawn on half the said line, the largest will be one that is drawn on half of the straight line and is similar to the defect		
22	If four straight lines are proportional, similar rectilineal figures will also be proportional; and vice versa	28	To a given straight line, apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one		
23	Equiangular parallelograms have to one another the ratio compounded of the ratios of their sides	29	To a given straight line, apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one		
24	In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another	30	To cut a finite straight line in extreme ratio		
25	To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure				



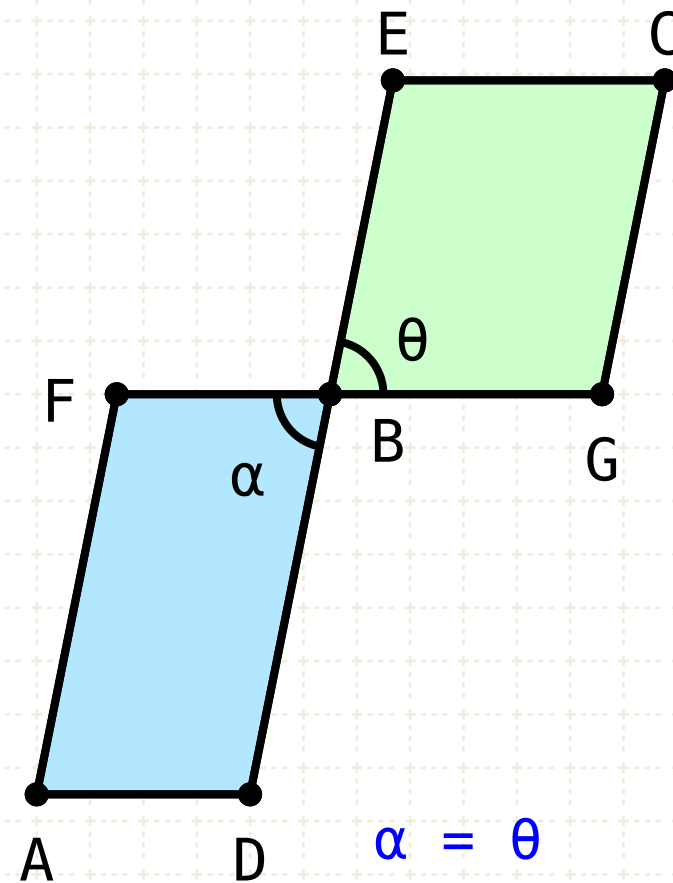
Proposition 14 of Book VI

In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal.



Proposition 14 of Book VI

In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal.



$$\alpha = \theta$$

$$\square AB = \square BC$$

$$DB:BE = GB:BF \rightarrow DB \times BF = GB \times BE$$

$$\alpha = \theta$$

$$DB:BE = GB:BF$$

$$\square AB = \square BC$$

In other words

Given two equiangular parallelograms, where the areas are equal, then the ratios of the sides around the equal angle FB,BD and EB,BG are reciprocally proportional

... or ... the multiplication of the two sides of the parallelogram remains constant as long as the area and the angles remain the same

And the inverse

Note:

Assume two objects 'x' and 'y', both with properties '1' and '2'

Proportional:

$$x_1:y_1 = x_2:y_2$$

Reciprocally Proportional:

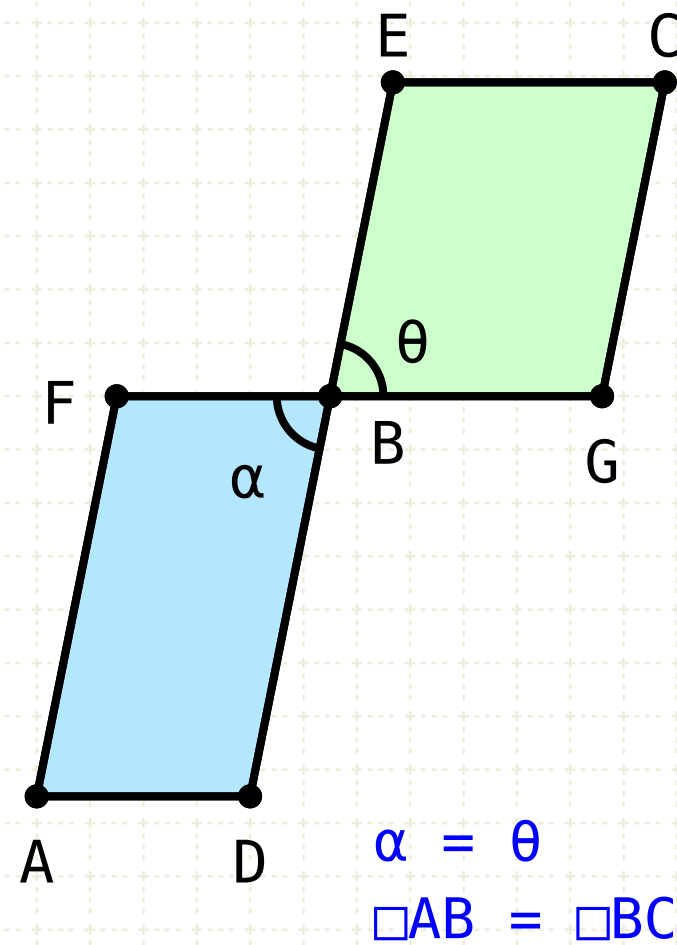
$$x_1:y_1 = y_2:x_2, \quad x_1 \cdot x_2 = y_1 \cdot y_2$$



Proposition 14 of Book VI

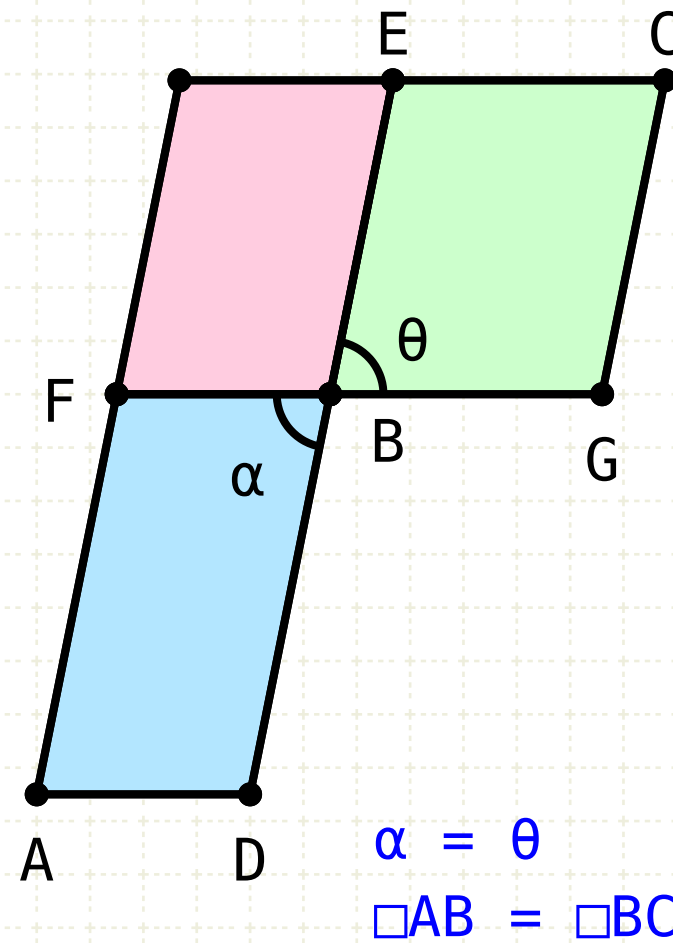
In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal.

Proof (Part 1)



Proposition 14 of Book VI

In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal.



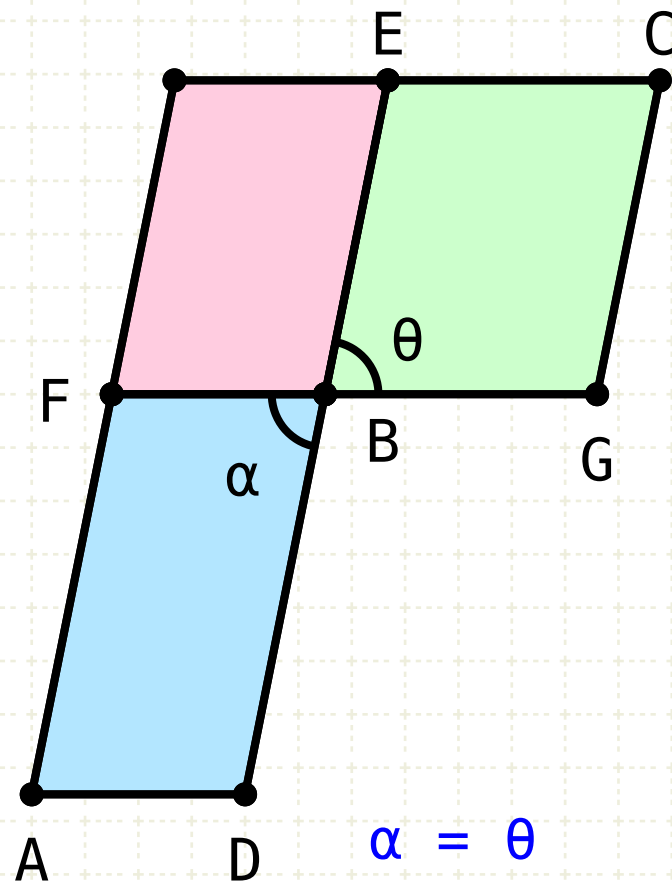
Proof (Part 1)

Let DB, BE be placed in a straight line, therefore FB, GG are also in a straight line (I·14)

Create the parallelogram FE

Proposition 14 of Book VI

In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal.



$$\alpha = \theta$$
$$\square AB = \square BC$$

$$\square AB : \square FE = \square BC : \square EF$$

Proof (Part 1)

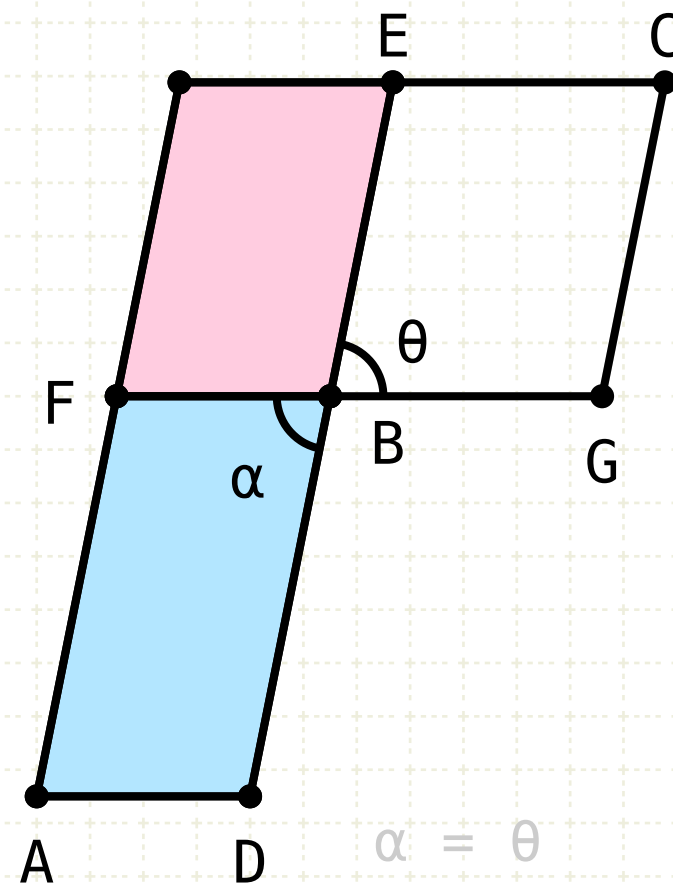
Let DB, BE be placed in a straight line, therefore FB, BG are also in a straight line (I·14)

Create the parallelogram FE

Since parallelograms AB and BC are equal, then the ratios of these to the parallelogram FE will also be equal (V·7)

Proposition 14 of Book VI

In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal.



$$\alpha = \theta$$

$$\square AB = \square BC$$

$$\square AB : \square FE = \square BC : \square EF$$

$$\square AB : \square FE = DB : BE$$

Proof (Part 1)

Let DB, BE be placed in a straight line, therefore FB, GC are also in a straight line (I·14)

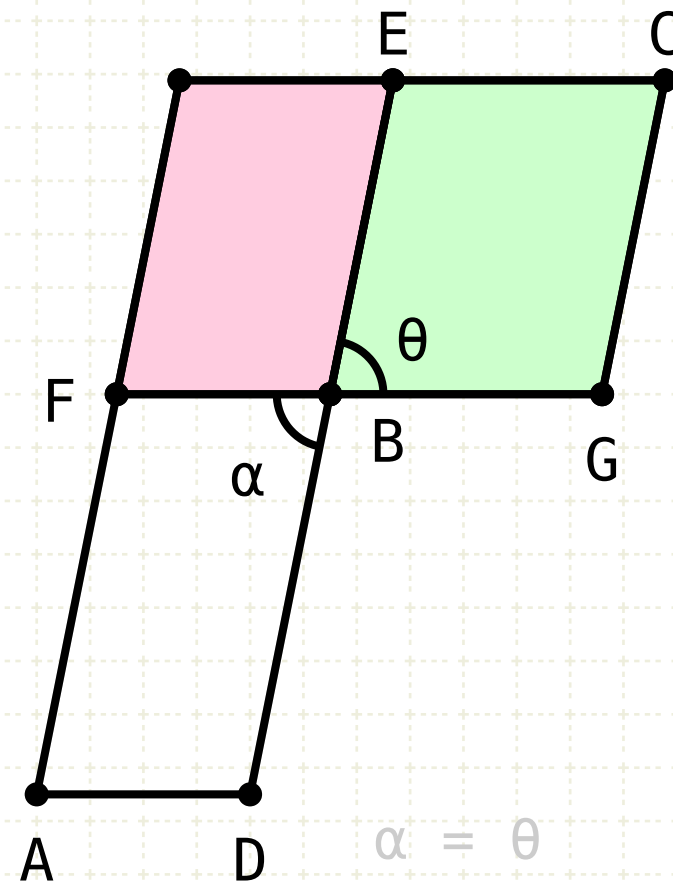
Create the parallelogram FE

Since parallelograms AB and BC are equal, then the ratios of these to the parallelogram FE will also be equal (V·7)

But, as AB is to FE, so is DB to BE (VI·1)

Proposition 14 of Book VI

In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal.



$$\alpha = \theta$$

$$\square AB = \square BC$$

$$\square AB : \square FE = \square BC : \square EF$$

$$\square AB : \square FE = DB : BE$$

$$\square BC : \square FE = BG : BF$$

Proof (Part 1)

Let DB, BE be placed in a straight line, therefore FB, GG are also in a straight line (I·14)

Create the parallelogram FE

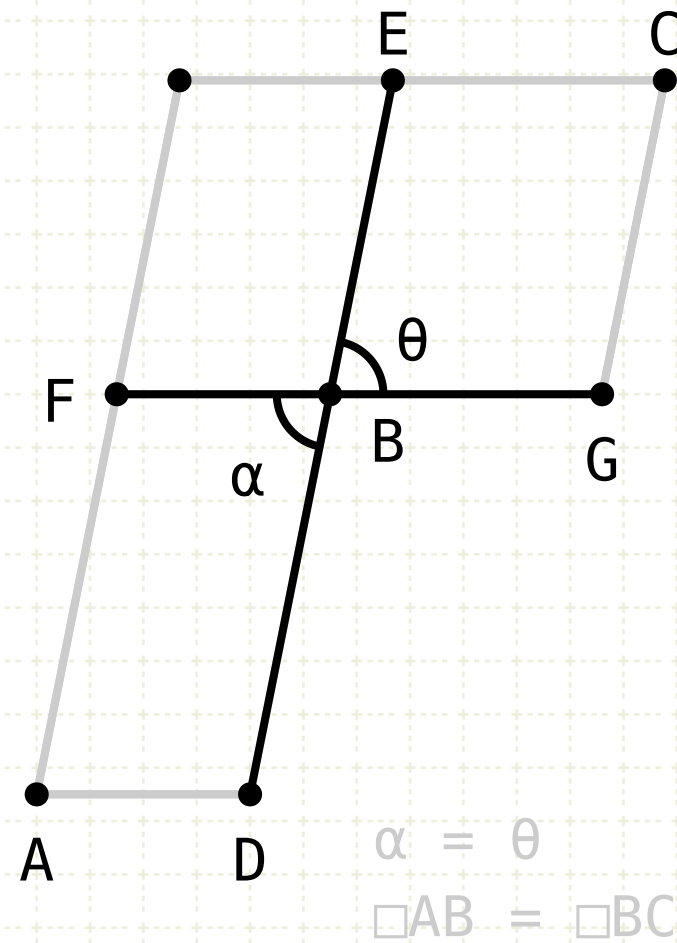
Since parallelograms AB and BC are equal, then the ratios of these to the parallelogram FE will also be equal (V·7)

But, as AB is to FE, so is DB to BE (VI·1)

and as BC is to FE, so is BG to BF (VI·1)

Proposition 14 of Book VI

In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal.



$$\square AB : \square FE = \square BC : \square EF$$

$$\square AB : \square FE = DB : BE$$

$$\square BC : \square FE = BG : BF$$

$$DB : BE = BG : BF$$

Proof (Part 1)

Let DB, BE be placed in a straight line, therefore FB, GG are also in a straight line (I·14)

Create the parallelogram FE

Since parallelograms AB and BC are equal, then the ratios of these to the parallelogram FE will also be equal (V·7)

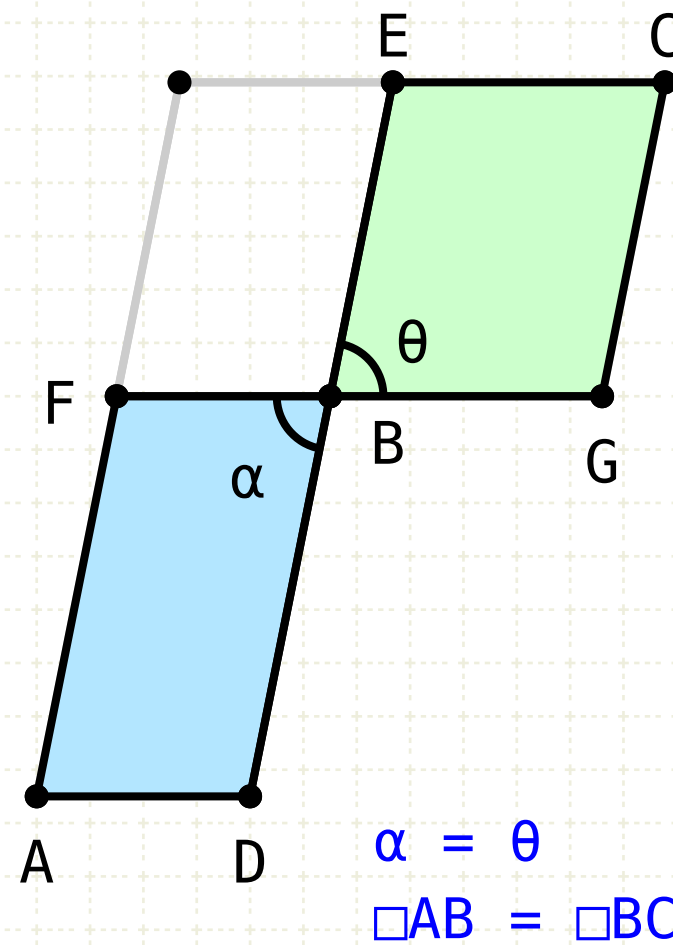
But, as AB is to FE, so is DB to BE (VI·1)

and as BC is to FE, so is BG to BF (VI·1)

Therefore, as DB is to BE, so is BG to BF (V·11)

Proposition 14 of Book VI

In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal.



$$\square AB : \square FE = \square BC : \square EF$$

$$\square AB : \square FE = DB : BE$$

$$\square BC : \square FE = BG : BF$$

$$DB : BE = BG : BF$$

$$\rightarrow DB \times BF = GB \times BE$$

Proof (Part 1)

Let DB, BE be placed in a straight line, therefore FB, GG are also in a straight line (I·14)

Create the parallelogram FE

Since parallelograms AB and BC are equal, then the ratios of these to the parallelogram FE will also be equal (V·7)

But, as AB is to FE, so is DB to BE (VI·1)

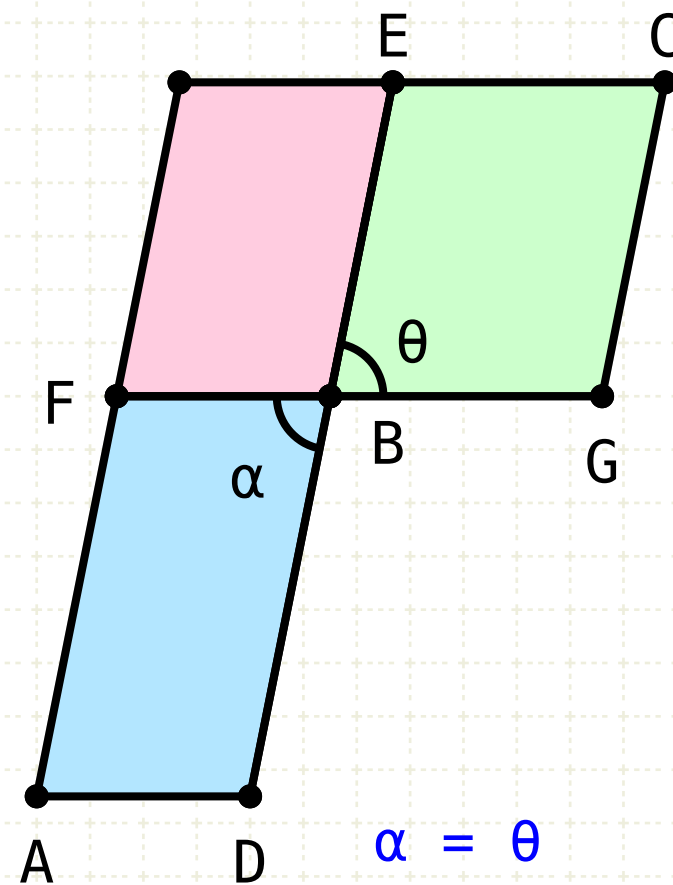
and as BC is to FE, so is BG to BF (VI·1)

Therefore, as DB is to BE, so is BG to BF (V·11)

Thus, in two equal parallelograms, the sides about the equal angles are reciprocally proportional

Proposition 14 of Book VI

In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal.



$$\alpha = \theta$$
$$DB:BE = GB:BF$$

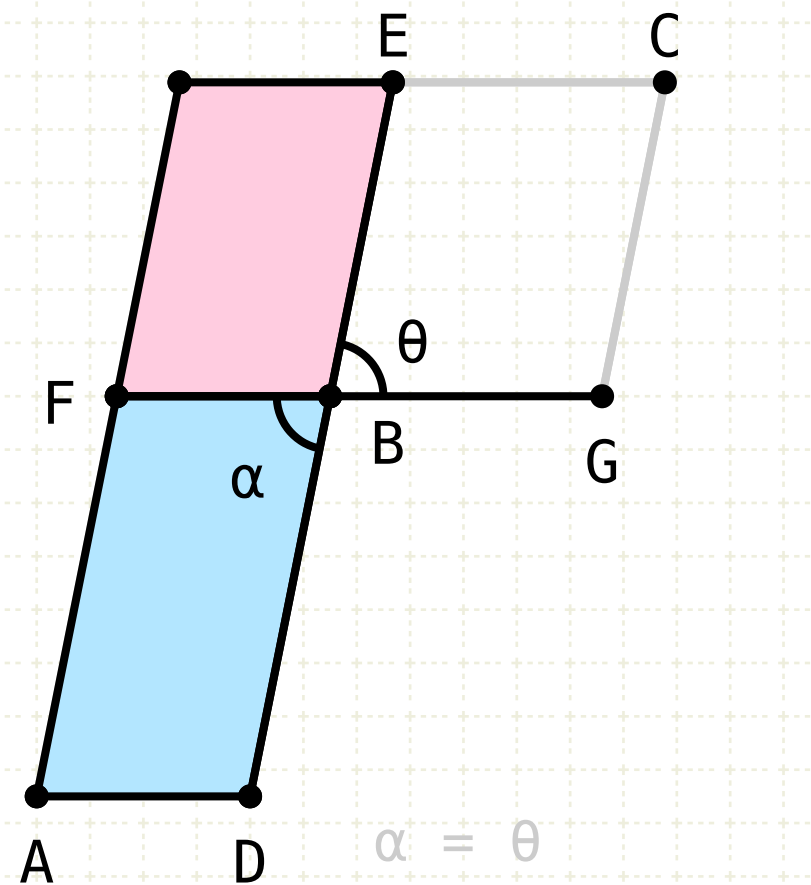
Proof (Part 2)

Let DB, BE be placed in a straight line, therefore FB, GC are also in a straight line (I-14)

Create the parallelogram FE

Proposition 14 of Book VI

In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal.



$$\alpha = \theta$$

$$DB:BE = GB:BF$$

$$DB:BE = \square AB:\square EF$$

Proof (Part 2)

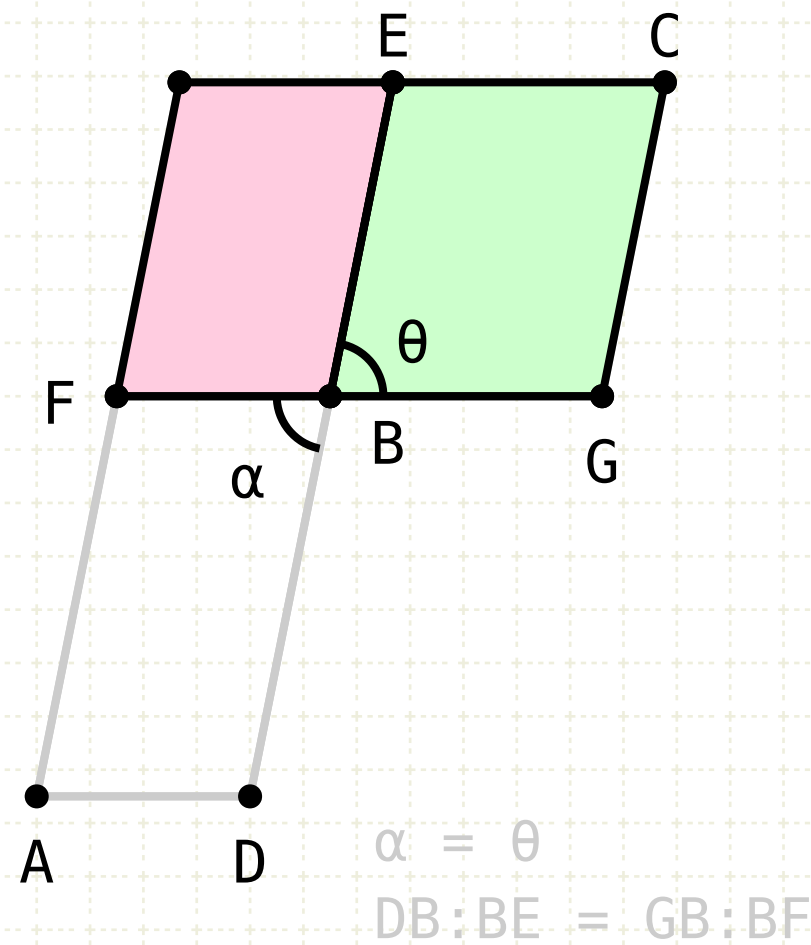
Let DB, BE be place in in a straight line, therefore FB, GG are also in a straight line (I·14)

Create the parallelogram FE

The ratio of DB to BE is equal to the ratio of the parallelograms AB to EF (VI·1)

Proposition 14 of Book VI

In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal.



$$DB:BE = \square AB:\square EF$$

$$BG:BF = \square BC:\square EF$$

Proof (Part 2)

Let DB, BE be place in in a straight line, therefore FB, GG are also in a straight line (I·14)

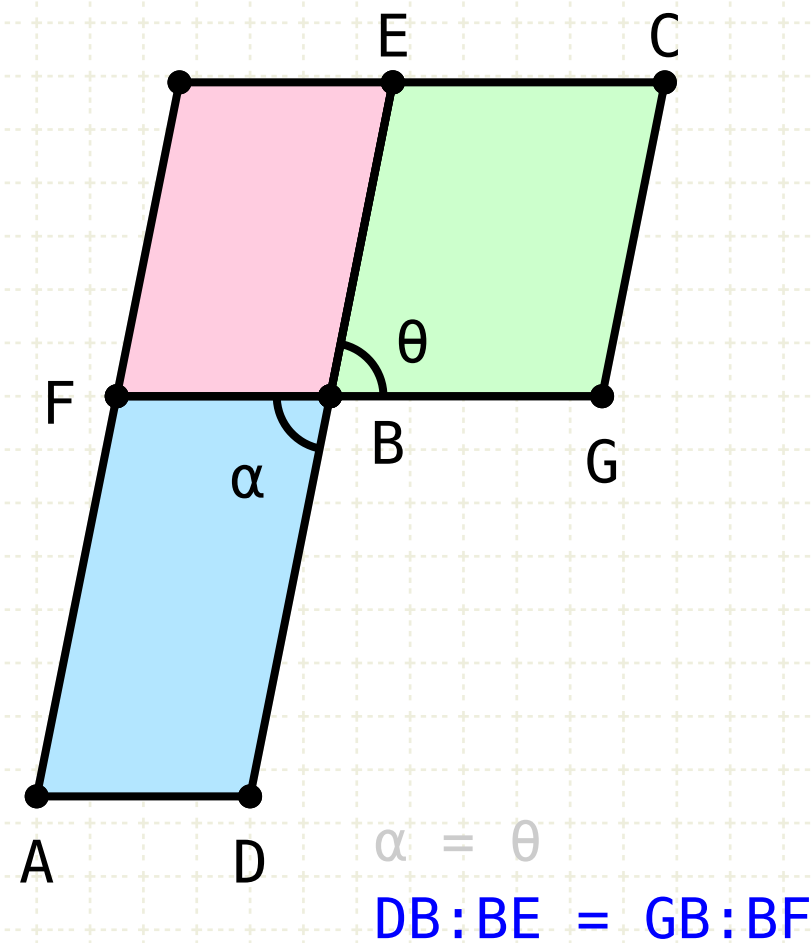
Create the parallelogram FE

The ratio of DB to BE is equal to the ratio of the parallelograms AB to EF (VI·1)

The ratio of BG to BF is equal to the ratio of the parallelograms BC to EF (VI·1)

Proposition 14 of Book VI

In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal.



$$\begin{aligned} DB:BE &= \square AB:\square EF \\ BG:BF &= \square BC:\square EF \\ \square AB:\square EF &= \square BC:\square EF \end{aligned}$$

Proof (Part 2)

Let DB, BE be placed in a straight line, therefore FB, GC are also in a straight line (I·14)

Create the parallelogram FE

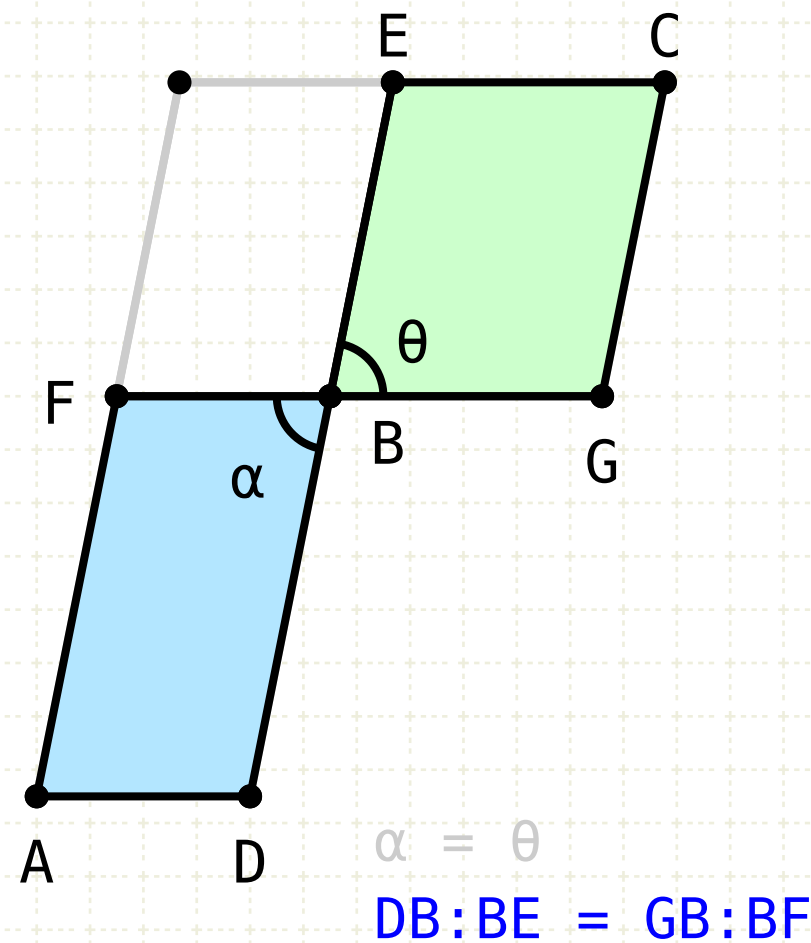
The ratio of DB to BE is equal to the ratio of the parallelograms AB to EF (VI·1)

The ratio of BG to BF is equal to the ratio of the parallelograms BC to EF (VI·1)

Therefore the ratio of the parallelograms AB to FE is equal to BC to FE (V·11)

Proposition 14 of Book VI

In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal.



$$\begin{aligned} DB:BE &= \square AB:\square EF \\ BG:BF &= \square BC:\square EF \\ \square AB:\square EF &= \square BC:\square EF \end{aligned}$$

$$\square AB = \square BC$$

Proof (Part 2)

Let DB, BE be placed in a straight line, therefore FB, GC are also in a straight line (I·14)

Create the parallelogram FE

The ratio of DB to BE is equal to the ratio of the parallelograms AB to EF (VI·1)

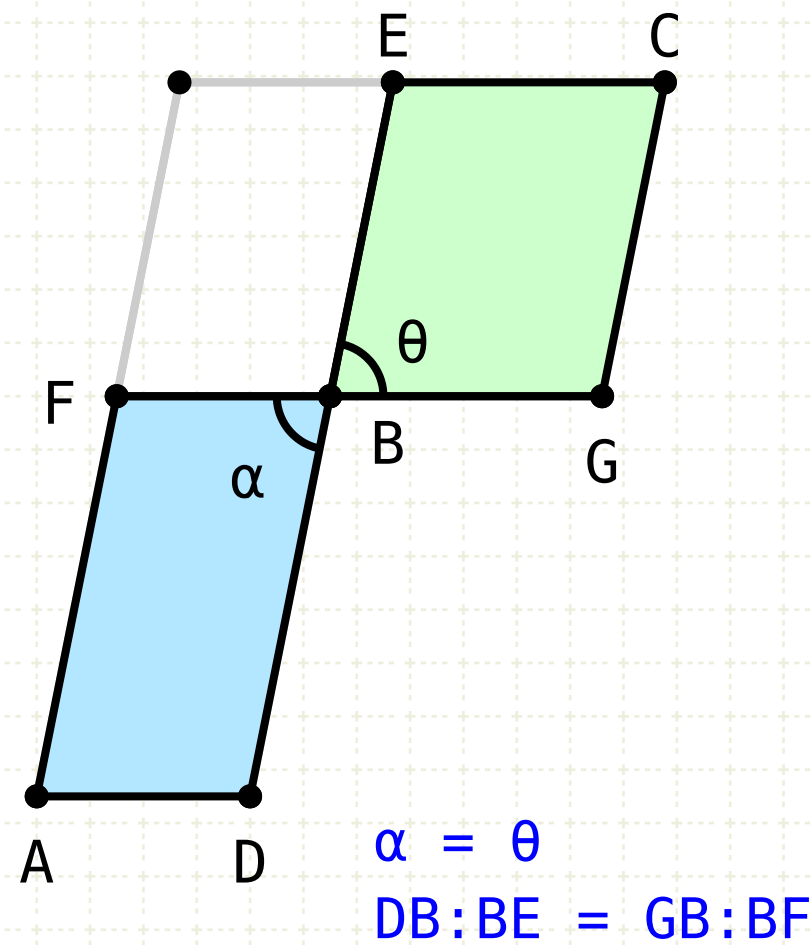
The ratio of BG to BF is equal to the ratio of the parallelograms BC to EF (VI·1)

Therefore the ratio of the parallelograms AB to FE is equal to BC to FE (V·11)

And thus the parallelograms AB and EF are equal (V·9)

Proposition 14 of Book VI

In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal.



$$\begin{aligned} DB:BE &= \square AB:\square EF \\ BG:BF &= \square BC:\square EF \\ \square AB:\square EF &= \square BC:\square EF \end{aligned}$$

$$\square AB = \square BC$$

Proof (Part 2)

Let DB, BE be placed in a straight line, therefore FB, BG are also in a straight line (I·14)

Create the parallelogram FE

The ratio of DB to BE is equal to the ratio of the parallelograms AB to EF (VI·1)

The ratio of BG to BF is equal to the ratio of the parallelograms BC to EF (VI·1)

Therefore the ratio of the parallelograms AB to FE is equal to BC to FE (V·11)

And thus the parallelograms AB and EF are equal (V·9)

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