Euclid's Elements

Book VII

Definitions:

1 A unit is that by virtue of which each of the things that exist is called one

2 A number is a multitude composed of units. (not one)

3 A number is part of a number, the less of the greater, when it measures the greater

11 A prime number is that which is measured by a unit alone.

12 Numbers prime to one another are those which are measured by a unit alone as a common measure

15 A number is said to multiply a number when that which is multiplied is added to itself as many times as there are units in the other, and thus some number is produced.

20 Numbers are proportional when the first is the same multiple, or the same part, or the same parts, of the second that the third is of the fourth.

As long as algebra and geometry have been separated, their progress have been slow and their uses limited; but when these two sciences have been united, they have lent each mutual forces, and have marched together towards perfection.

Joseph-Louis Lagrange
(1736 to 1813)

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Proposition 4 of Book VII

Any number is either a part or parts of any number, the less of the greater

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Any number is either a part or parts of any number, the less of the greater

Definitions

3. A number is a 'part' of a number, the less of the greater, when it measures the greater

$A = 10,\ B = 2,$

B is part of A

$A = B + B + B + B + B$

4. but 'parts' when it does not measure it

$A=10,\ B=6$

Let the part of A be 2

$p = 2,\ A = p + p + p + p + p$

B is a multiple of the part of A (B is parts of A)

$B = p + p + p$

Proposition 4 of Book VII

Any number is either a part or parts of any number, the less of the greater

$(A,B) \in \mathbb{N}$

$\exists(p,m,n) \in \mathbb{N}$ such that

$A = m \cdot p$

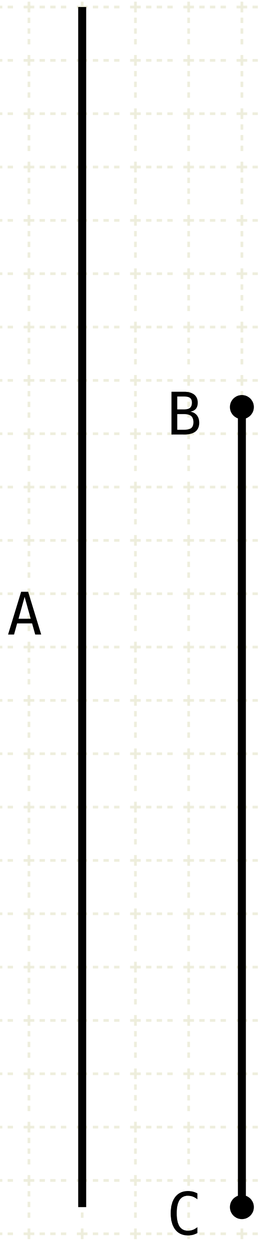
$B = n \cdot p$

In other words

Given two natural numbers, A and B, either B is part of A, or there exists a natural number (a part) that can measure both A and B

Proposition 4 of Book VII

Any number is either a part or parts of any number, the less of the greater



In other words

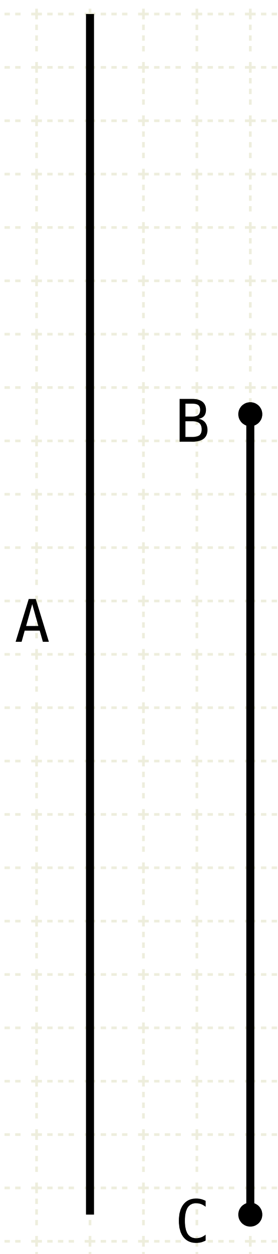
Given two natural numbers, A and B, either B is part of A, or there exists a natural number (a part) that can measure both A and B

Proof

Either A, BC are co-prime, or not

Proposition 4 of Book VII

Any number is either a part or parts of any number, the less of the greater



gcd(A,BC) = 1

In other words

Given two natural numbers, A and B, either B is part of A, or there exists a natural number (a part) that can measure both A and B

Proof

Either A, BC are co-prime, or not

Assume A, BC are co-prime

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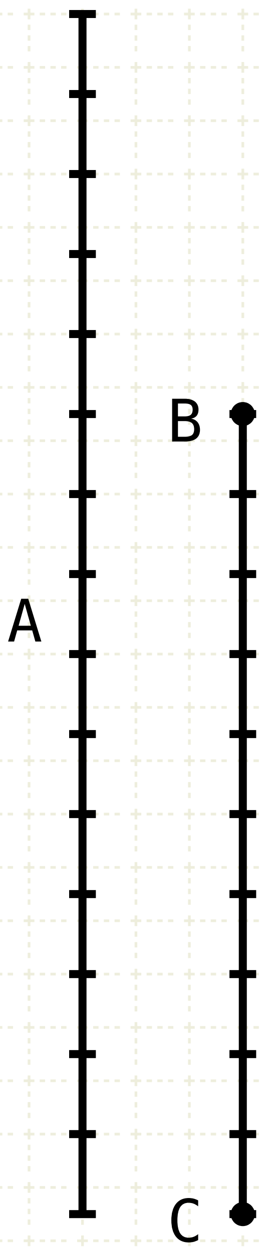

$\gcd(A,BC) = 1$

$u = 1$

$A = q \cdot u$

$BC = p \cdot u$

In other words

Given two natural numbers, A and B, either B is part of A, or there exists a natural number (a part) that can measure both A and B

Proof

Either A, BC are co-prime, or not

Assume A, BC are co-prime

Divide BC into individual units (the number 1)

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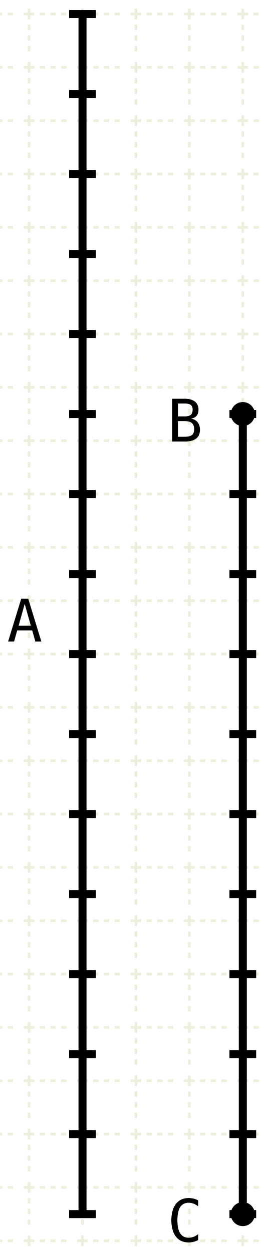

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Proof

Either A, BC are co-prime, or not

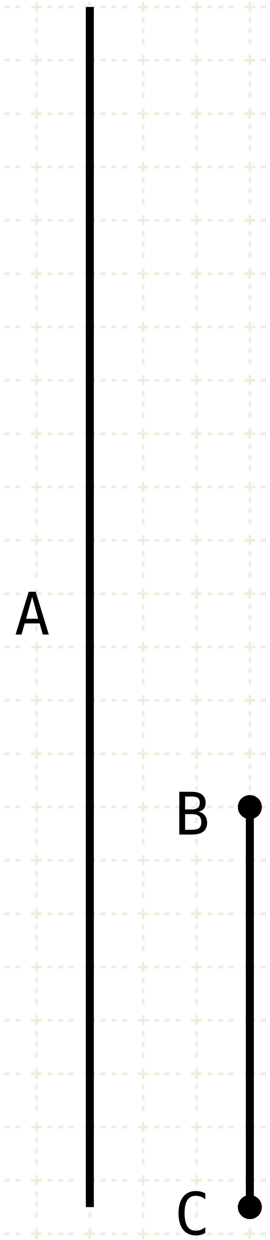
Assume A, BC are co-prime

Divide BC into individual units (the number 1)

Each unit in B will be some part of A, since '1' measures A, so BC will be some parts of A

Proposition 4 of Book VII

Any number is either a part or parts of any number, the less of the greater



gcd(A,BC) = BC

A = q·BC

In other words

Given two natural numbers, A and B, either B is part of A, or there exists a natural number (a part) that can measure both A and B

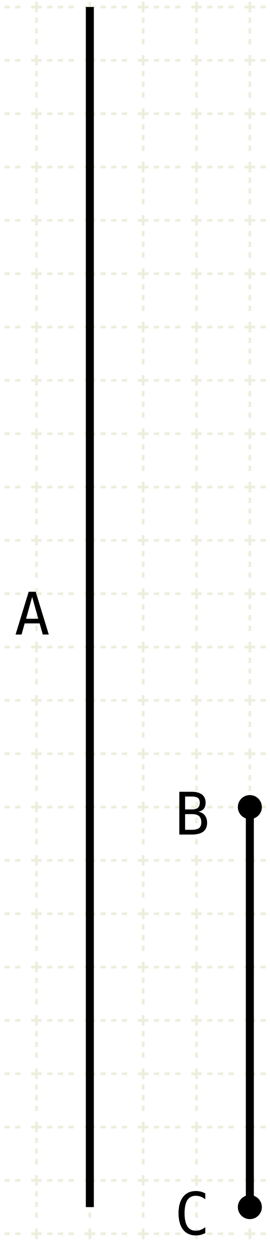
Proof

Either A, BC are co-prime, or not

Assume A, BC are not co-prime and BC measures A

Proposition 4 of Book VII

Any number is either a part or parts of any number, the less of the greater



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If BC measures A, BC is a part of A

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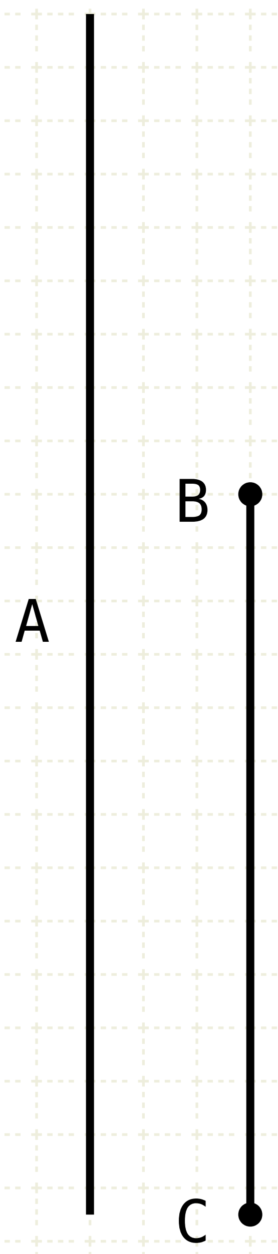

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Proof

Either A, BC are co-prime, or not

Assume A, BC are not co-prime and BC does not measure A

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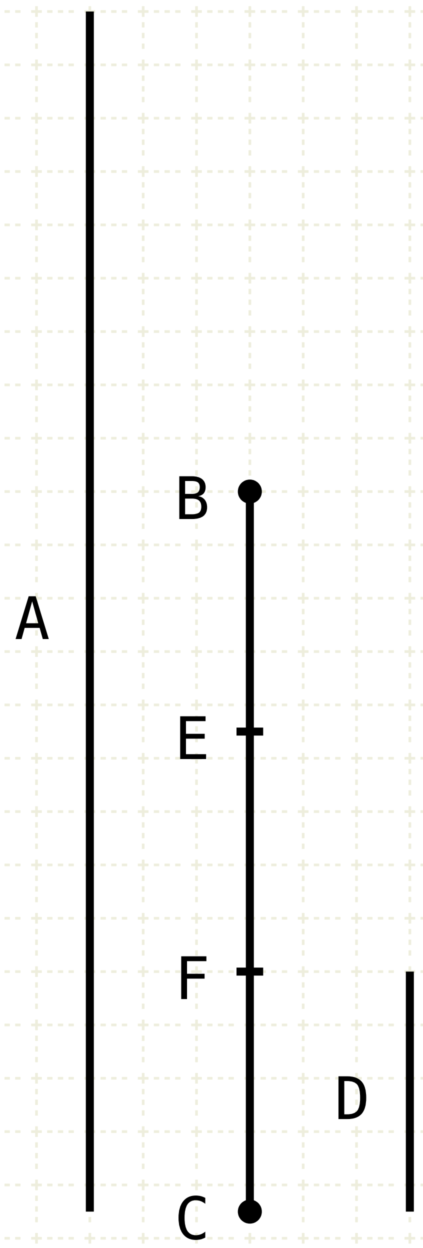

gcd(A,BC) = D

BE = EF = FC = D

BC = BE + EF + FC

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Given two natural numbers, A and B, either B is part of A, or there exists a natural number (a part) that can measure both A and B

Proof

Either A, BC are co-prime, or not

Assume A, BC are not co-prime and BC does not measure A

Find the largest common divisor D (VII·2), and divide BC into the numbers equal to D, namely BE, EF, FC

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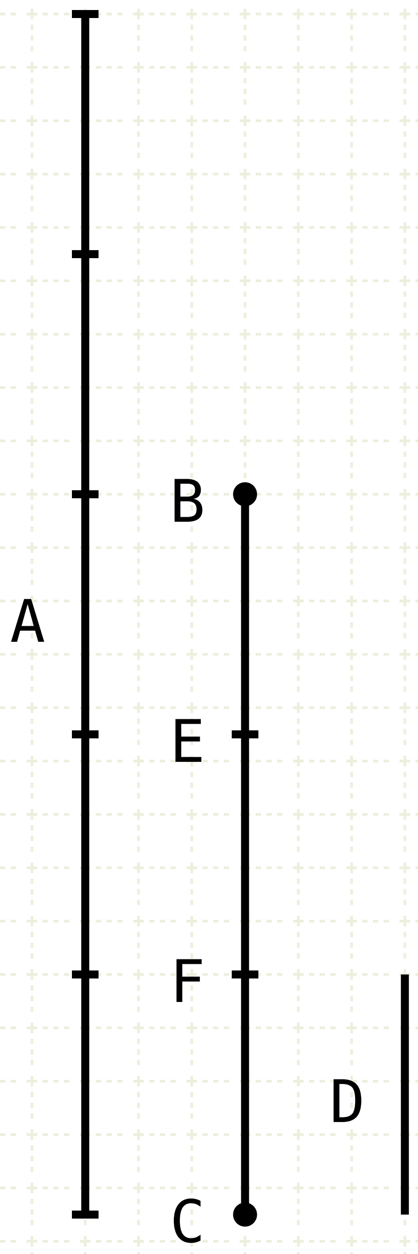

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Since D measures A, D is a part of A

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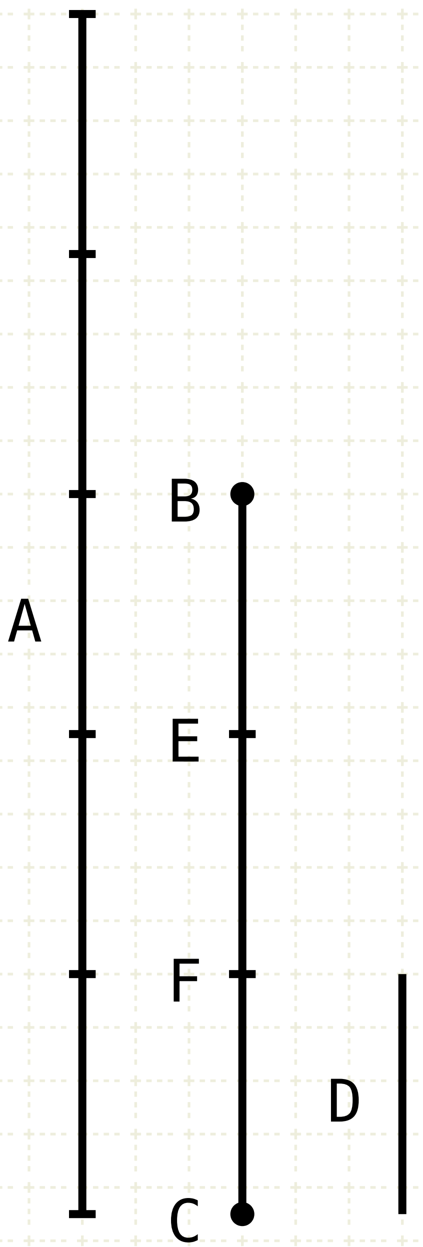

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Since D measures A, D is a part of A

But BE,EF,FC also equal D

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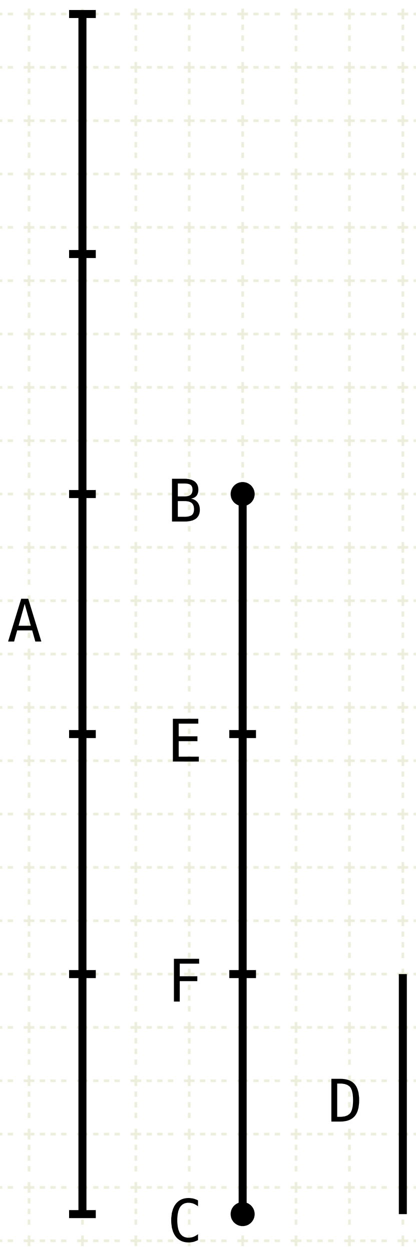

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D is a part of A, so BC is a sum of the parts of A

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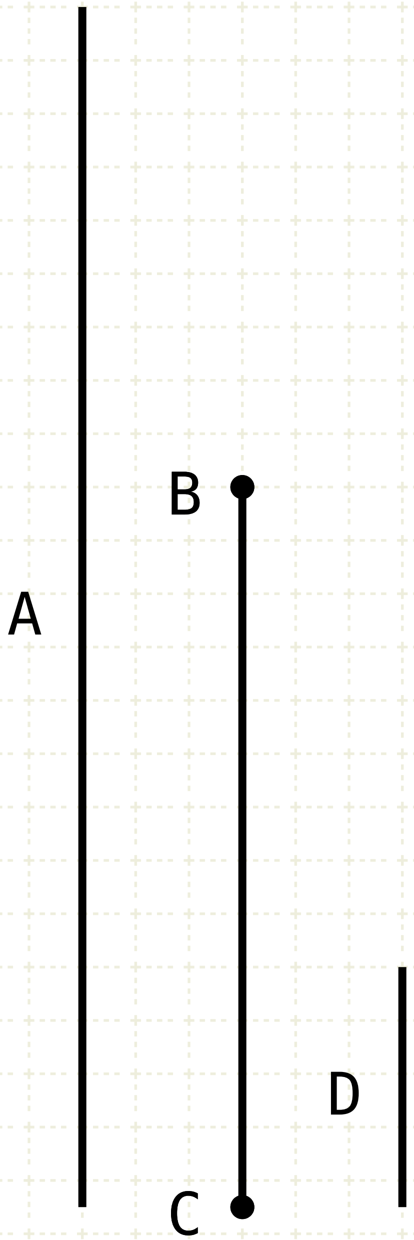

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