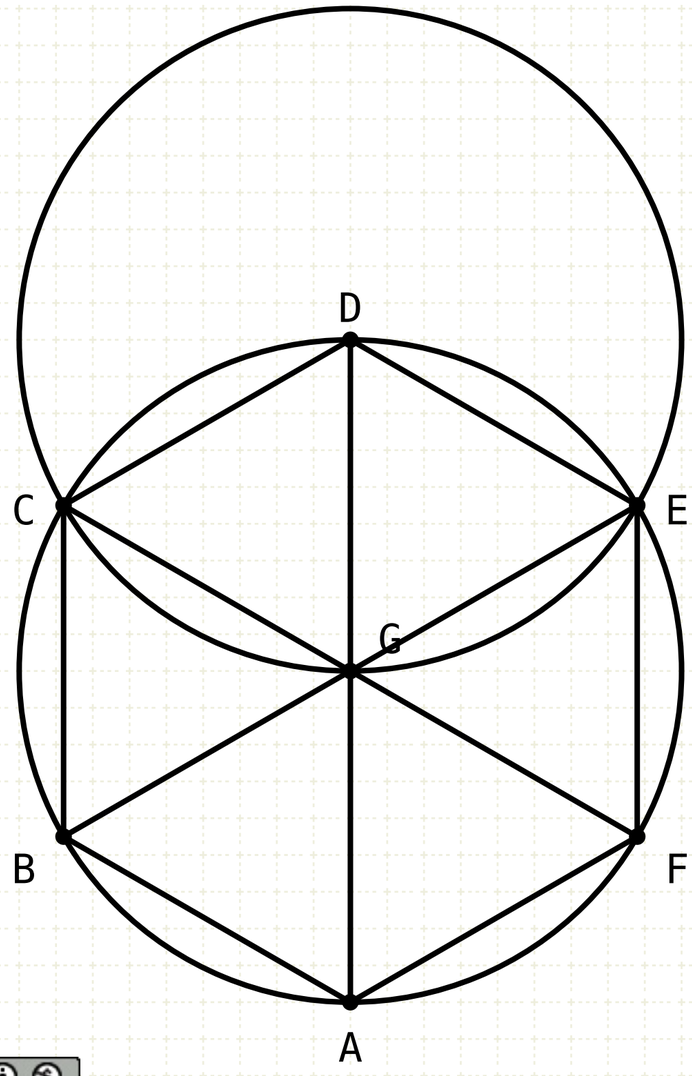
Euclid's Elements

Book IV



Philosophy (nature) is written in that great book which ever is before our eyes -- I mean the universe -- but we cannot understand it if we do not first learn the language and grasp the symbols in which it is written. The book is written in mathematical language, and the symbols are triangles, circles and other geometrical figures, without whose help it is impossible to comprehend a single word of it - without which one wanders in vain through a dark labyrinth.

Galileo Galilei

Proposition 7 of Book IV

About a given circle to circumscribe a square.

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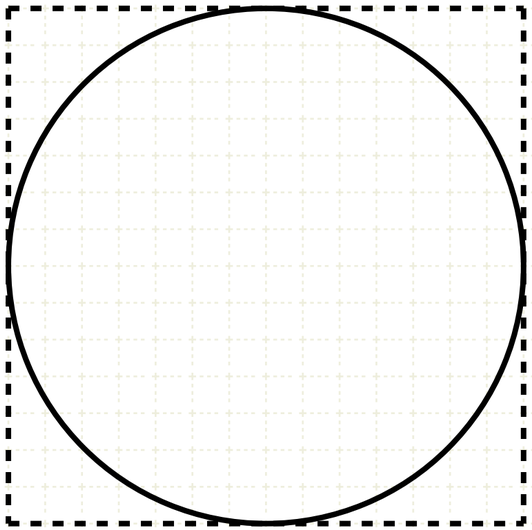
Proposition 7 of Book IV

About a given circle to circumscribe a square.

Proposition 7 of Book IV

About a given circle to circumscribe a square.

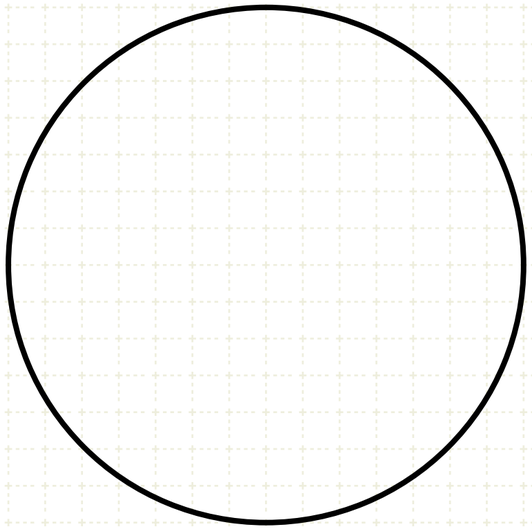


In other words

Given a circle, draw a square ABCD outside the circle

Proposition 7 of Book IV

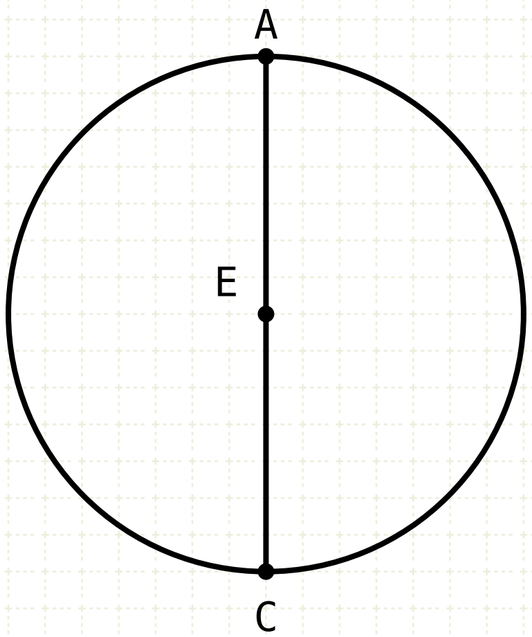
About a given circle to circumscribe a square.



Construction

Proposition 7 of Book IV

About a given circle to circumscribe a square.

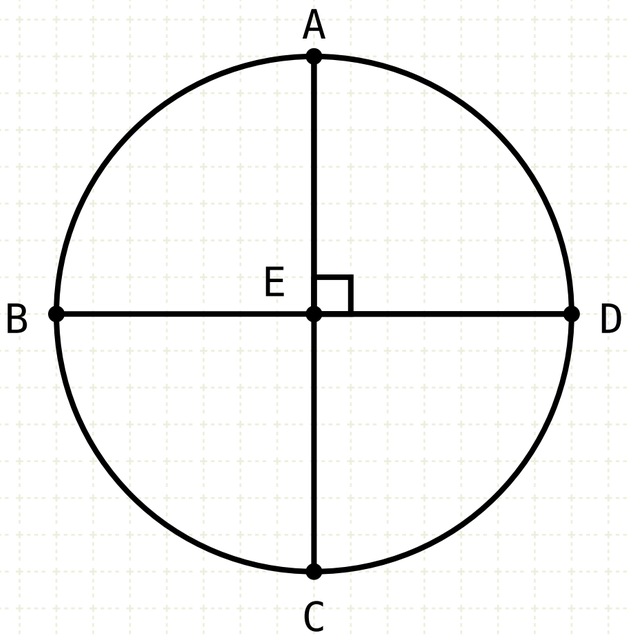


Construction

Draw a diameter AC through the centre of the circle E

Proposition 7 of Book IV

About a given circle to circumscribe a square.



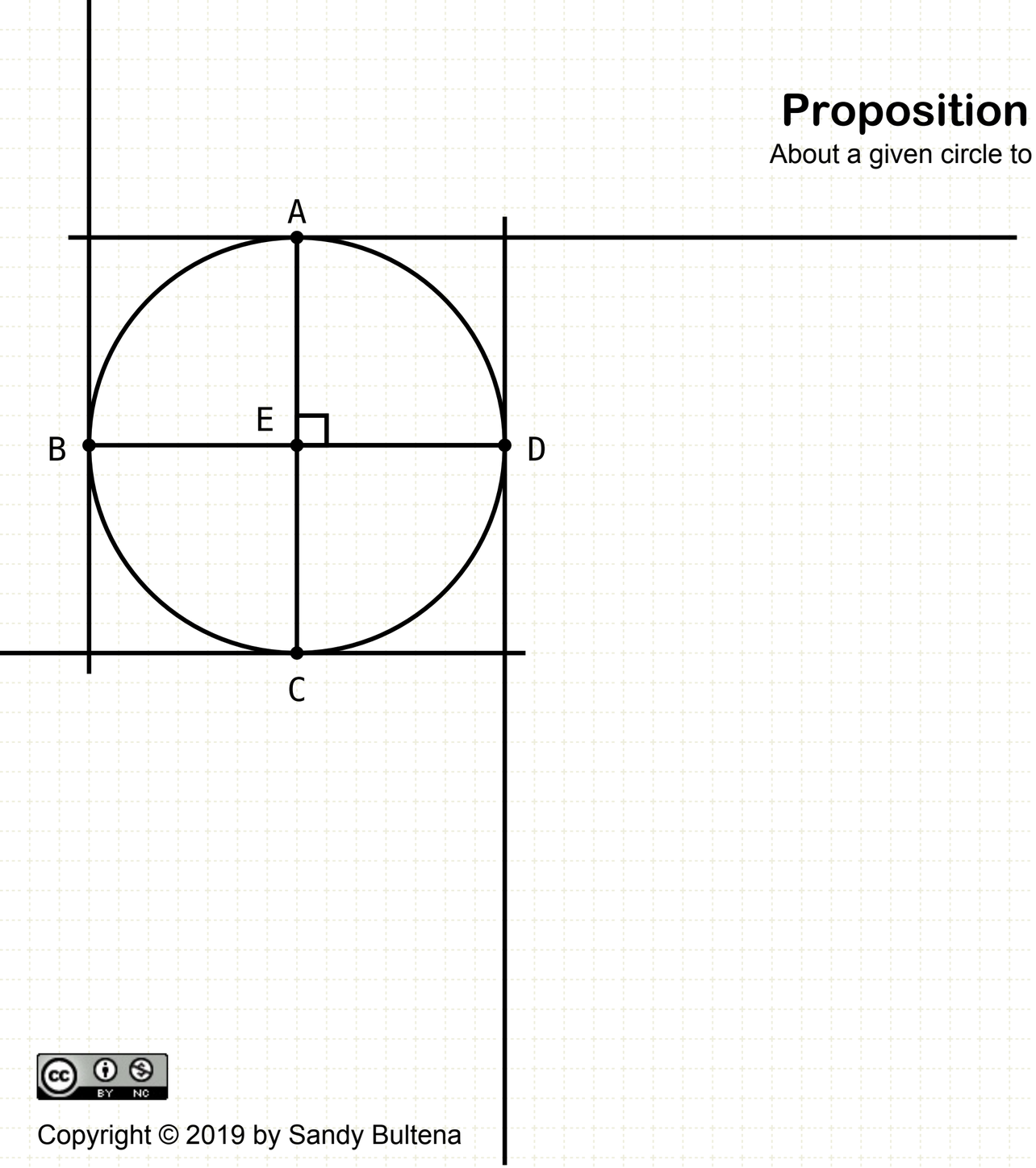
Construction

Draw a diameter AC through the centre of the circle E

Draw a diameter BD, perpendicular to AC, through the centre of the circle E

Proposition 7 of Book IV

About a given circle to circumscribe a square.



Construction

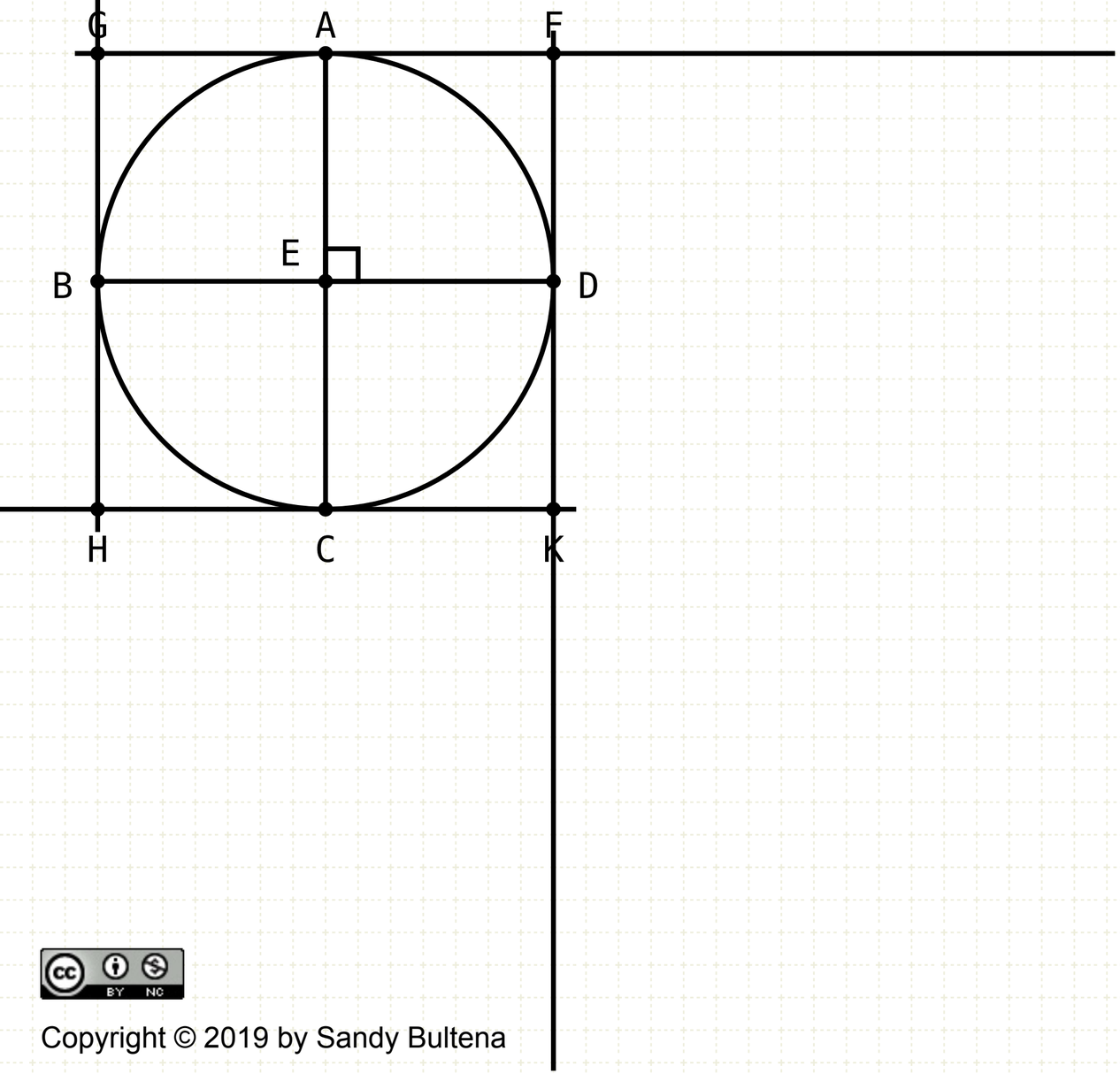
Draw a diameter AC through the centre of the circle E

Draw a diameter BD, perpendicular to AC, through the centre of the circle E

Draw lines at the points A,B,C and D such that they touch the circle (III·16)

Proposition 7 of Book IV

About a given circle to circumscribe a square.



Construction

Draw a diameter AC through the centre of the circle E

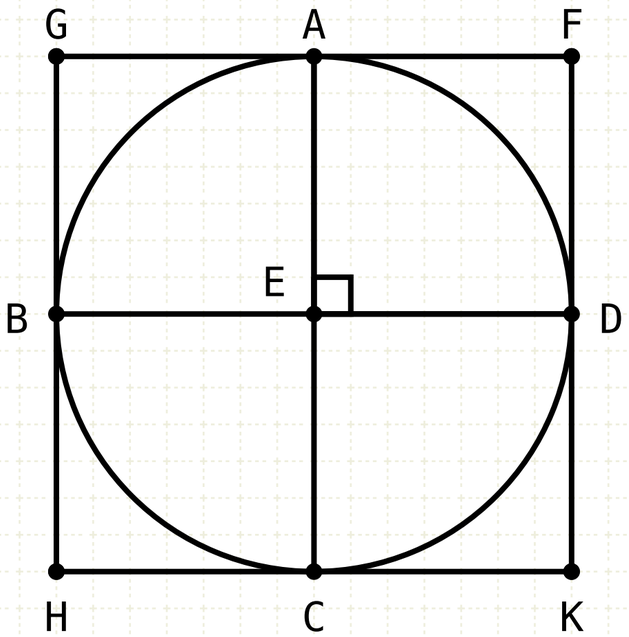
Draw a diameter BD, perpendicular to AC, through the centre of the circle E

Draw lines at the points A,B,C and D such that they touch the circle (III·16)

Let the points where these lines intersect be FGHK

Proposition 7 of Book IV

About a given circle to circumscribe a square.



Construction

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Draw a diameter BD, perpendicular to AC, through the centre of the circle E

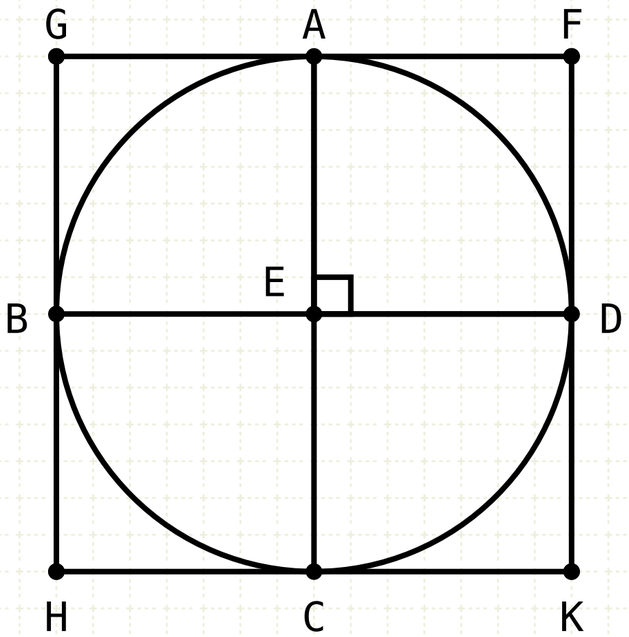
Draw lines at the points A,B,C and D such that they touch the circle (III·16)

Let the points where these lines intersect be FGHK

FGHK is a square

Proposition 7 of Book IV

About a given circle to circumscribe a square.



Proof

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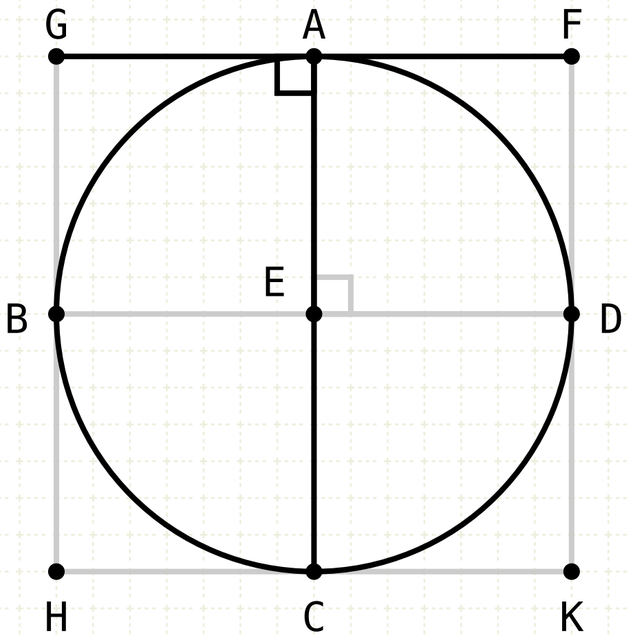
About a given circle to circumscribe a square.

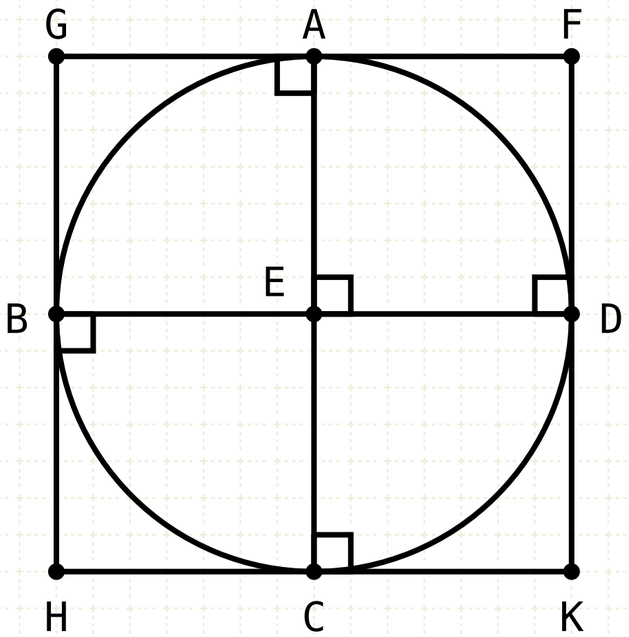


Proof

Since GF touches the circle at point A, and AC passes through the centre of the circle, GAE is a right angle (III·18)

Proposition 7 of Book IV

About a given circle to circumscribe a square.



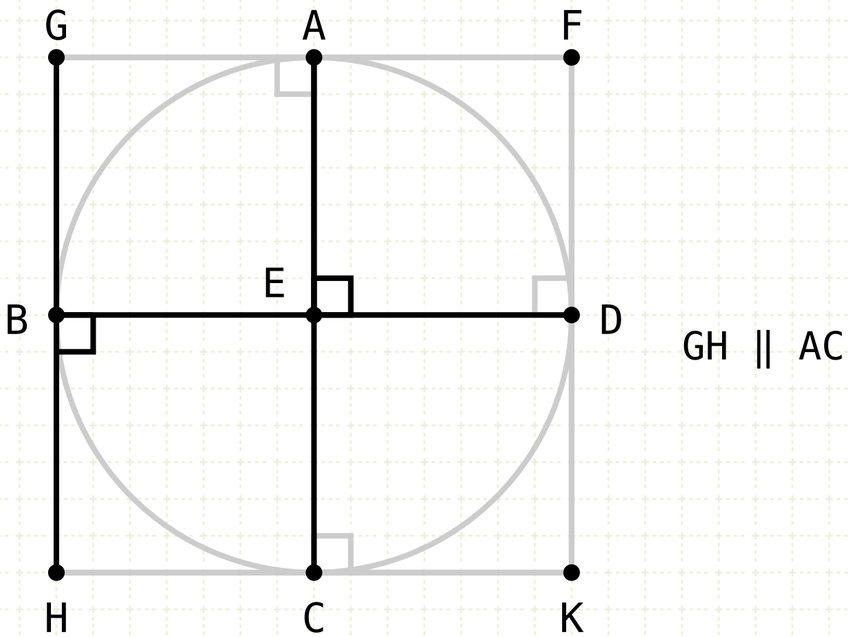
Proof

Since GF touches the circle at point A, and AC passes through the centre of the circle, GAE is a right angle (III·18)

Similarly, so are the angles at B, C and D

Proposition 7 of Book IV

About a given circle to circumscribe a square.



GH || AC

Proof

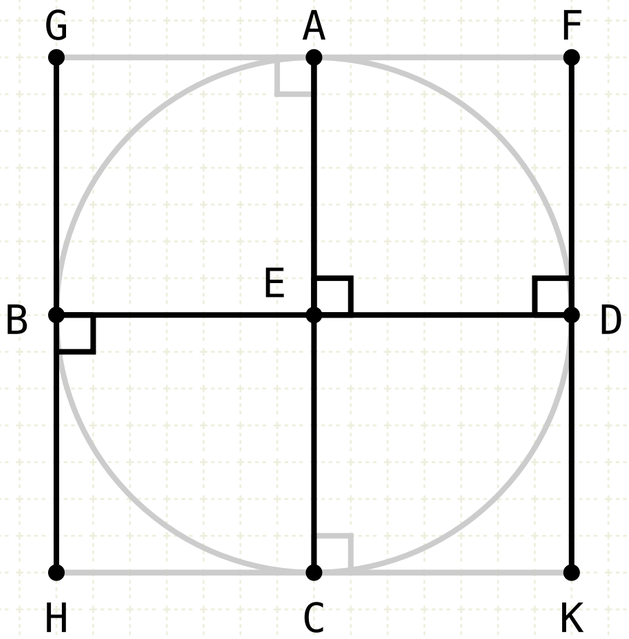
Since GF touches the circle at point A, and AC passes through the centre of the circle, GAE is a right angle (III·18)

Similarly, so are the angles at B, C and D

Angle AEB is a right angle, as is GBE, therefore GH is parallel to AC (I·28)

Proposition 7 of Book IV

About a given circle to circumscribe a square.



GH || AC

AC || FK

Proof

Since GF touches the circle at point A, and AC passes through the centre of the circle, GAE is a right angle (III·18)

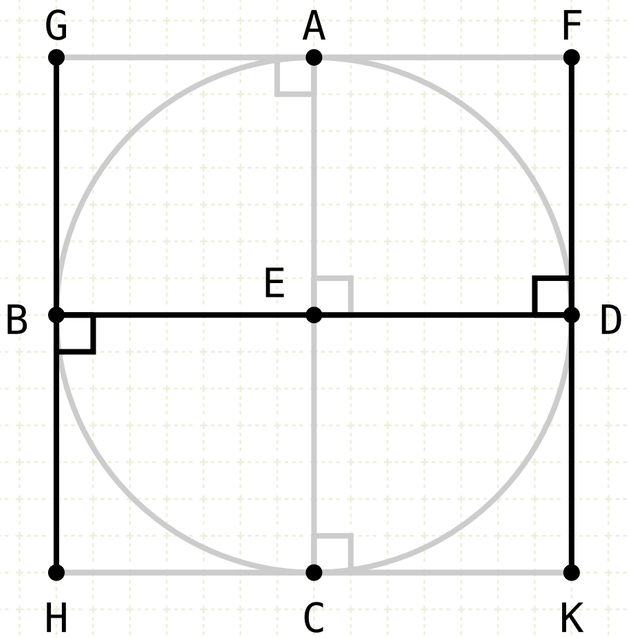
Similarly, so are the angles at B, C and D

Angle AEB is a right angle, as is GBE, therefore GH is parallel to AC (I·28)

By the same reasons, AC is parallel to FK

Proposition 7 of Book IV

About a given circle to circumscribe a square.



GH || AC
AC || FK
FK || GH

Proof

Since GF touches the circle at point A, and AC passes through the centre of the circle, GAE is a right angle (III·18)

Similarly, so are the angles at B, C and D

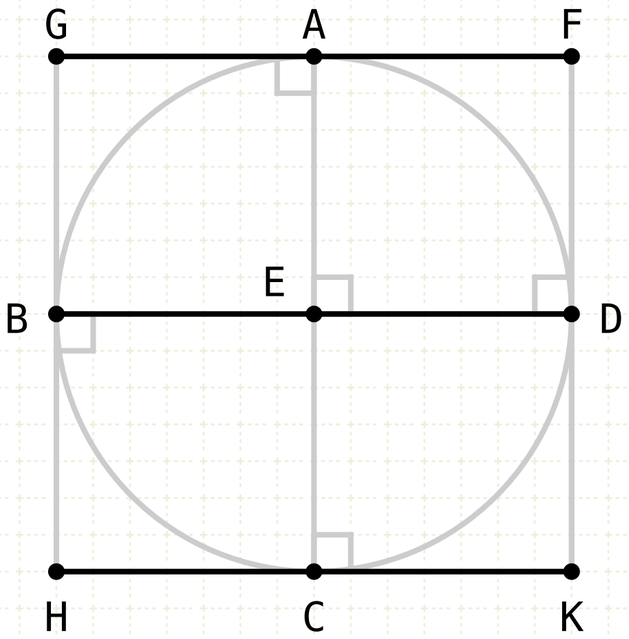
Angle AEB is a right angle, as is GBE, therefore GH is parallel to AC (I·28)

By the same reasons, AC is parallel to FK

Therefore FK is parallel to GH (III·30)

Proposition 7 of Book IV

About a given circle to circumscribe a square.



GH || AC
AC || FK
FK || GH
GF || BD || HK

Proof

Since GF touches the circle at point A, and AC passes through the centre of the circle, GAE is a right angle (III·18)

Similarly, so are the angles at B, C and D

Angle AEB is a right angle, as is GBE, therefore GH is parallel to AC (I·28)

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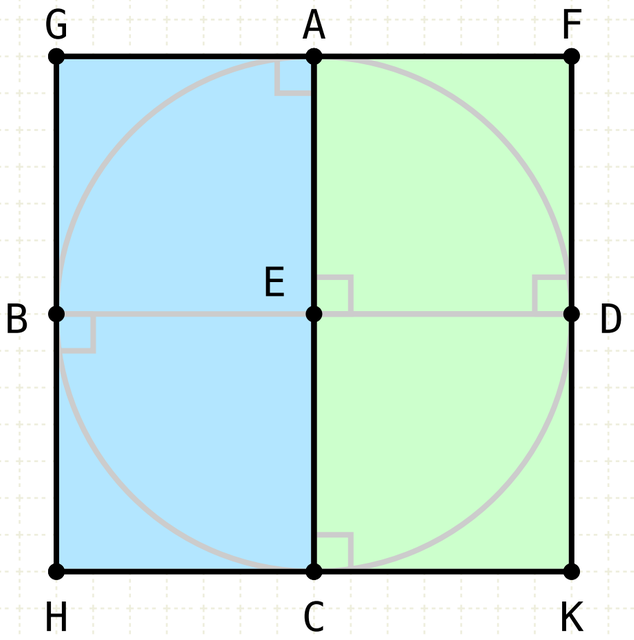
Similarly, GF, BD and HK are all parallel

Proposition 7 of Book IV

About a given circle to circumscribe a square.



GH || AC
AC || FK
FK || GH
GF || BD || HK
GH = AC = FK

Proof

Since GF touches the circle at point A, and AC passes through the centre of the circle, GAE is a right angle (III·18)

Similarly, so are the angles at B, C and D

Angle AEB is a right angle, as is GBE, therefore GH is parallel to AC (I·28)

By the same reasons, AC is parallel to FK

Therefore FK is parallel to GH (III·30)

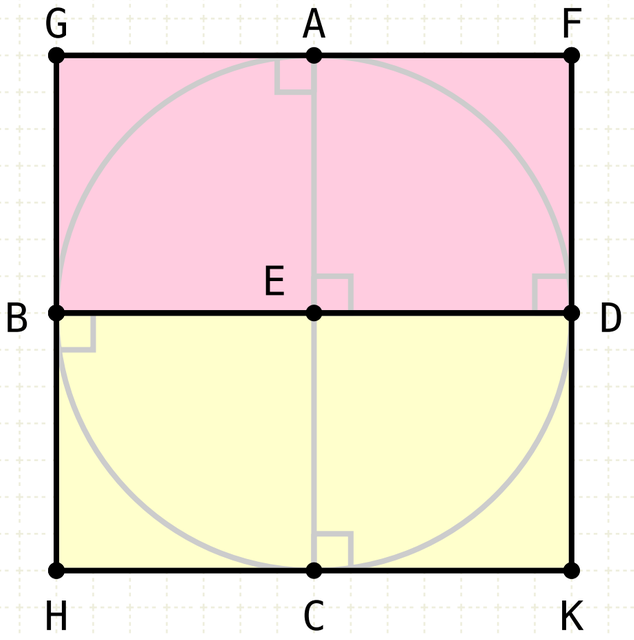
Similarly, GF, BD and HK are all parallel

Since all these lines are parallel to each other, we have the following parallelograms and equalities (i.34)…

The lines GH, AC and FK are all equal

Proposition 7 of Book IV

About a given circle to circumscribe a square.



GH || AC
AC || FK
FK || GH
GF || BD || HK
GH = AC = FK
GF = BD = HK

Proof

Since GF touches the circle at point A, and AC passes through the centre of the circle, GAE is a right angle (III·18)

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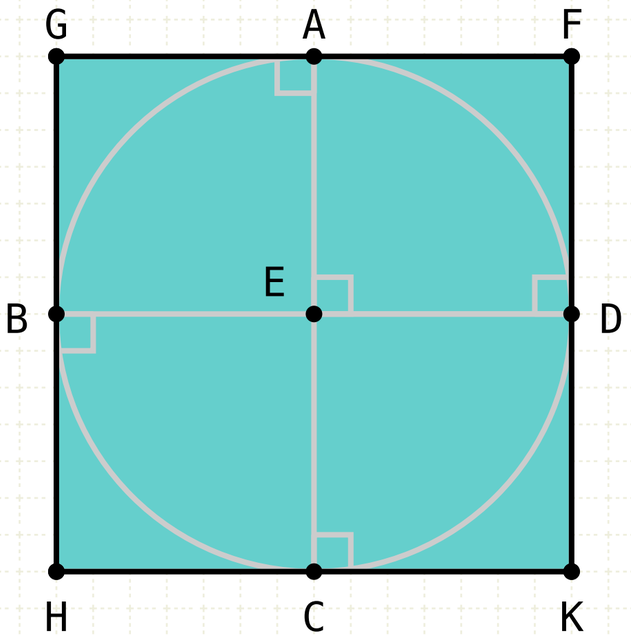


GH || AC
AC || FK
FK || GH
GF || BD || HK
GH = AC = FK
GF = BD = HK
AC = BD
∴ GF = GH

Proof

Since GF touches the circle at point A, and AC passes through the centre of the circle, GAE is a right angle (III·18)

Similarly, so are the angles at B, C and D

Angle AEB is a right angle, as is GBE, therefore GH is parallel to AC (I·28)

By the same reasons, AC is parallel to FK

Therefore FK is parallel to GH (III·30)

Similarly, GF, BD and HK are all parallel

Since all these lines are parallel to each other, we have the following parallelograms and equalities (i.34)…

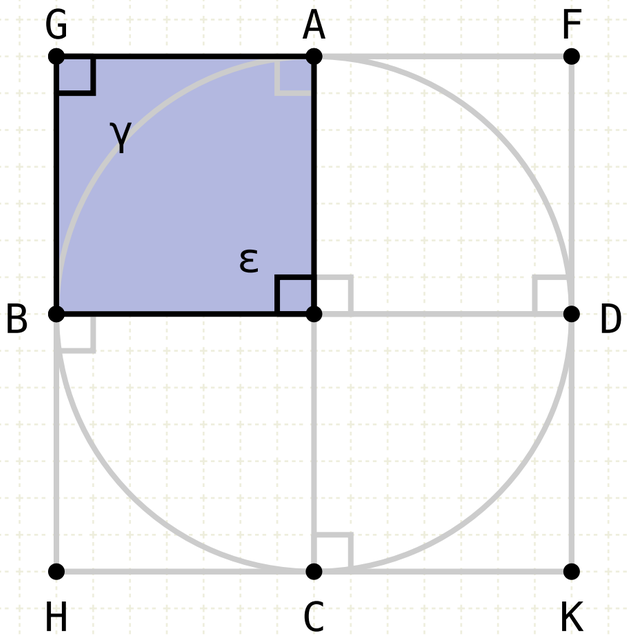
The lines GH, AC and FK are all equal

The lines GF, BD and HK are all equal

But BD equals AC (diameters of the same circle), so GF equals GH, and GFKH is an equilateral

Proposition 7 of Book IV

About a given circle to circumscribe a square.



GH || AC
AC || FK
FK || GH
GF || BD || HK
GH = AC = FK
GF = BD = HK
AC = BD
∴ GF = GH

γ = ε = ∟

Proof

Since GF touches the circle at point A, and AC passes through the centre of the circle, GAE is a right angle (III·18)

Similarly, so are the angles at B, C and D

Angle AEB is a right angle, as is GBE, therefore GH is parallel to AC (I·28)

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The lines GH, AC and FK are all equal

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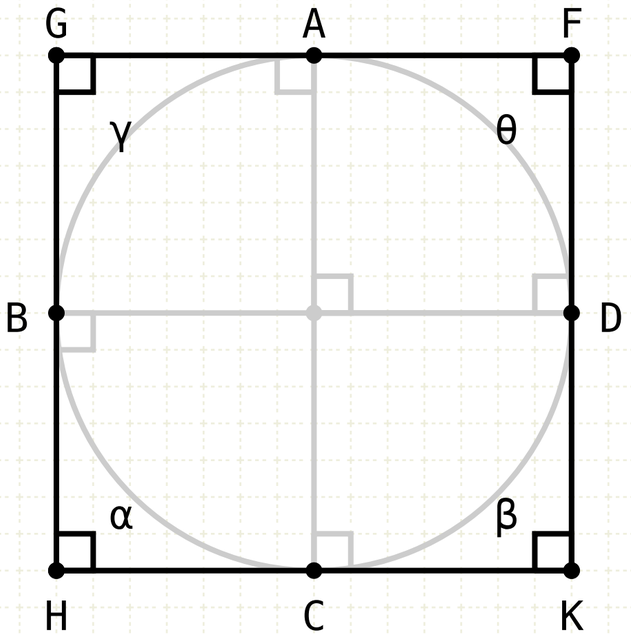
But BD equals AC (diameters of the same circle), so GF equals GH, and GFKH is an equilateral

GBEA is a parallelogram, which means that the angle AGB is equal to the angle in the opposite corner, AEB (I·34)

Proposition 7 of Book IV

About a given circle to circumscribe a square.



GH || AC
AC || FK
FK || GH
GF || BD || HK
GH = AC = FK
GF = BD = HK
AC = BD
∴ GF = GH

γ = ε = ∟
γ = α = β = θ = ∟

Proof

Since GF touches the circle at point A, and AC passes through the centre of the circle, GAE is a right angle (III·18)

Similarly, so are the angles at B, C and D

Angle AEB is a right angle, as is GBE, therefore GH is parallel to AC (I·28)

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GBEA is a parallelogram, which means that the angle AGB is equal to the angle in the opposite corner, AEB (I·34)

Similarly, we can show that all the angles in GFKH are right angled, and thus…

Proposition 7 of Book IV

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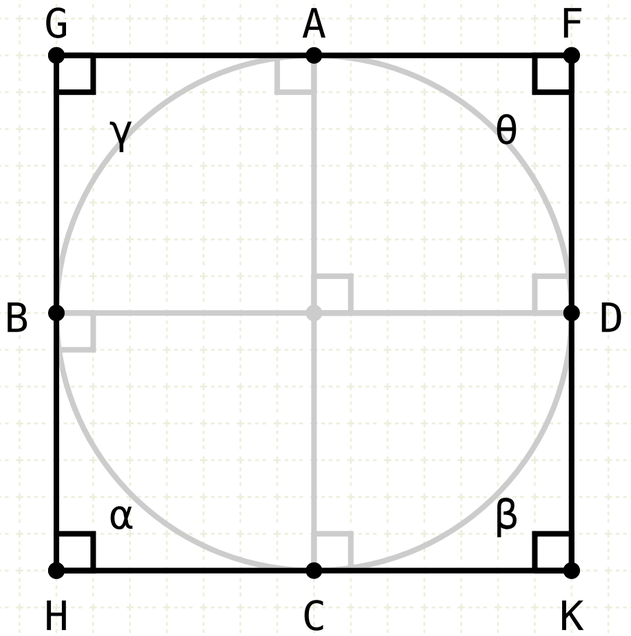

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AC || FK
FK || GH
GF || BD || HK
GH = AC = FK
GF = BD = HK
AC = BD
∴ GF = GH

γ = ε = ∟
γ = α = β = θ = ∟

Proof

Since GF touches the circle at point A, and AC passes through the centre of the circle, GAE is a right angle (III·18)

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Angle AEB is a right angle, as is GBE, therefore GH is parallel to AC (I·28)

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Similarly, we can show that all the angles in GFKH are right angled, and thus…

GFKH is a square

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