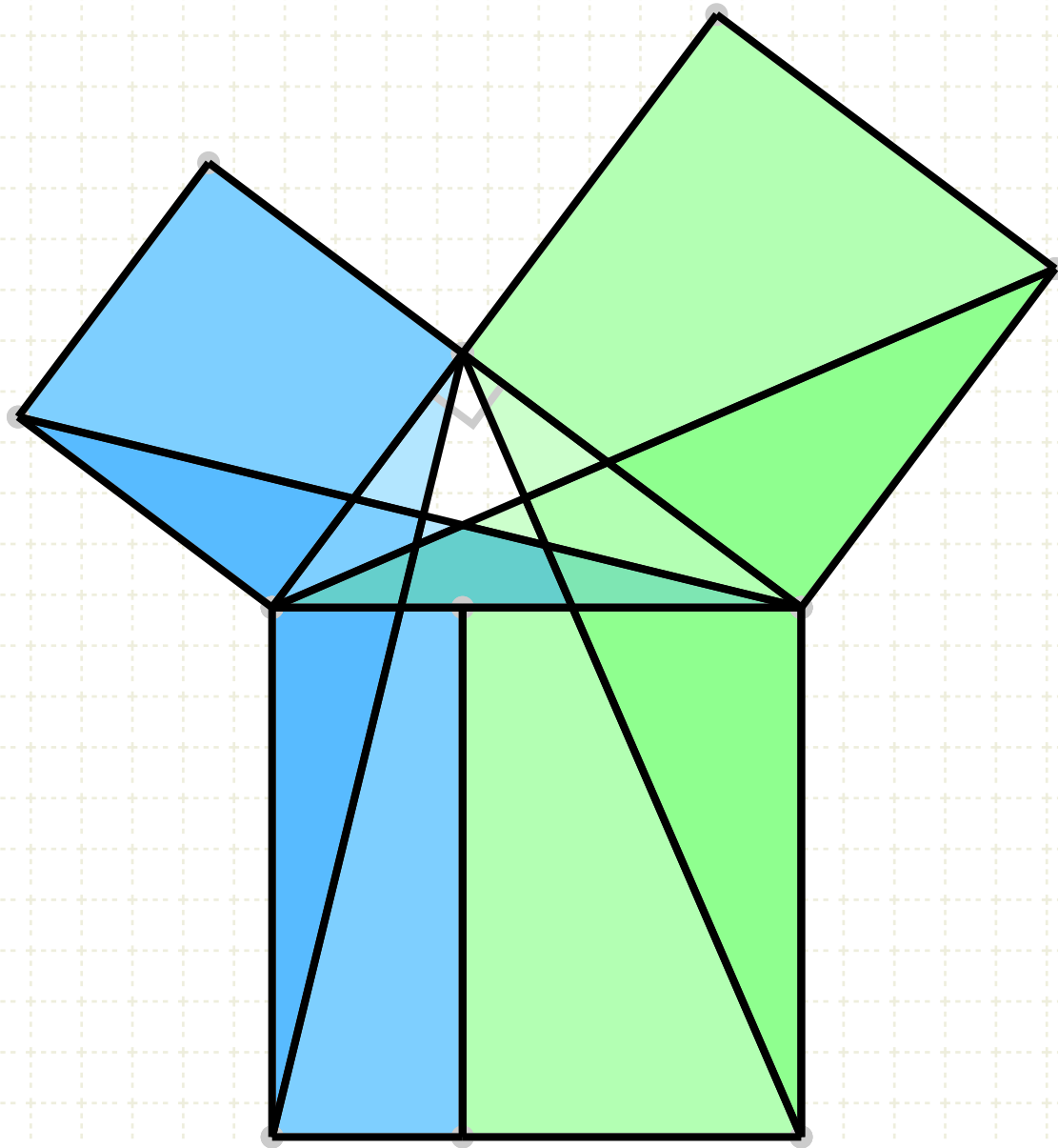


# Euclid's Elements

## Book I

*If Euclid did not kindle your youthful enthusiasm, you  
were not born to be a scientific thinker.*

Albert Einstein



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48	Inverse Pythagoras' theorem



# Proposition 36 of Book I

Parallelograms which are on equal bases and in the same parallels equal one another.



# Proposition 36 of Book I

Parallelograms which are on equal bases and in the same parallels equal one another.

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## In other words

Parallelograms with equal bases and equal heights have equal area

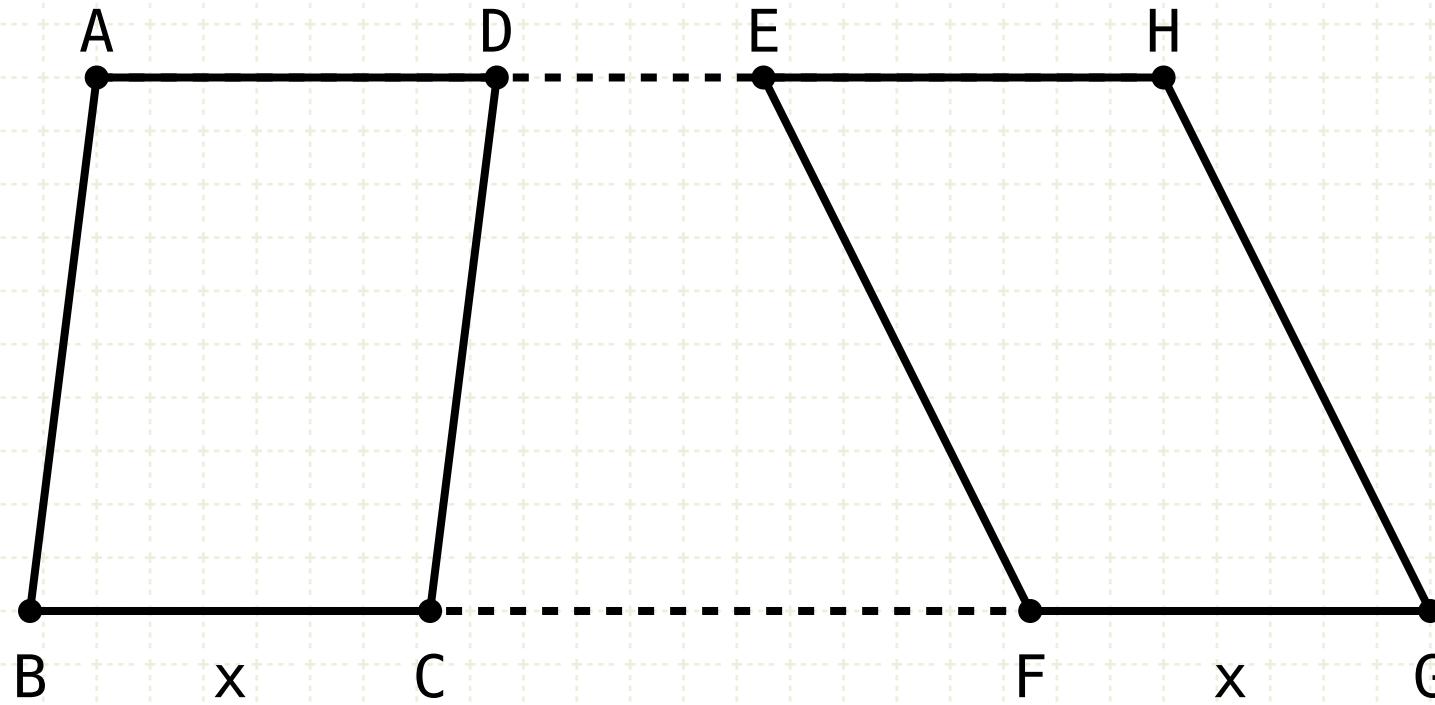
Given two parallel lines

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# Proposition 36 of Book I

Parallelograms which are on equal bases and in the same parallels equal one another.



## In other words

Parallelograms with equal bases and equal heights have equal area

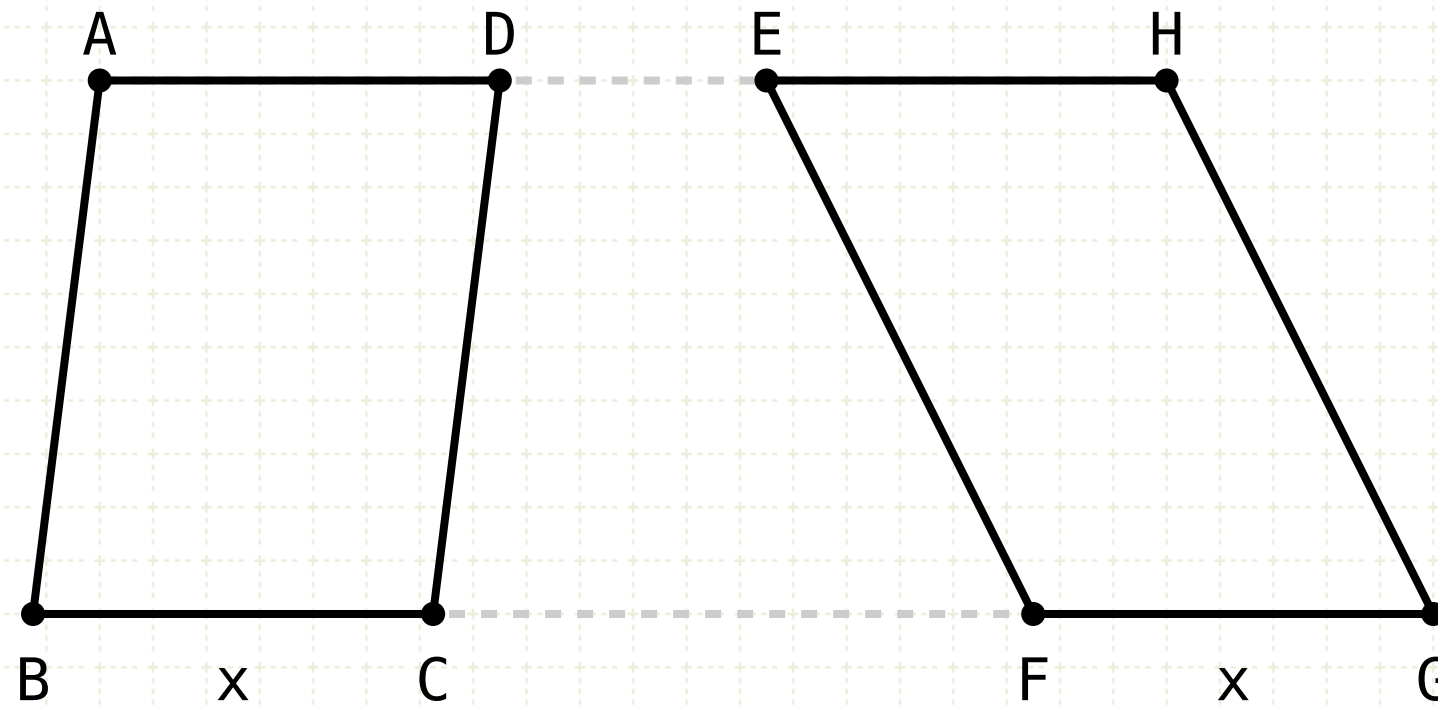
Given two parallel lines

Let ABCD and EFGH be parallelograms with equal bases BC and FG on the same parallels AH and BG



# Proposition 36 of Book I

Parallelograms which are on equal bases and in the same parallels equal one another.



## In other words

Parallelograms with equal bases and equal heights have equal area

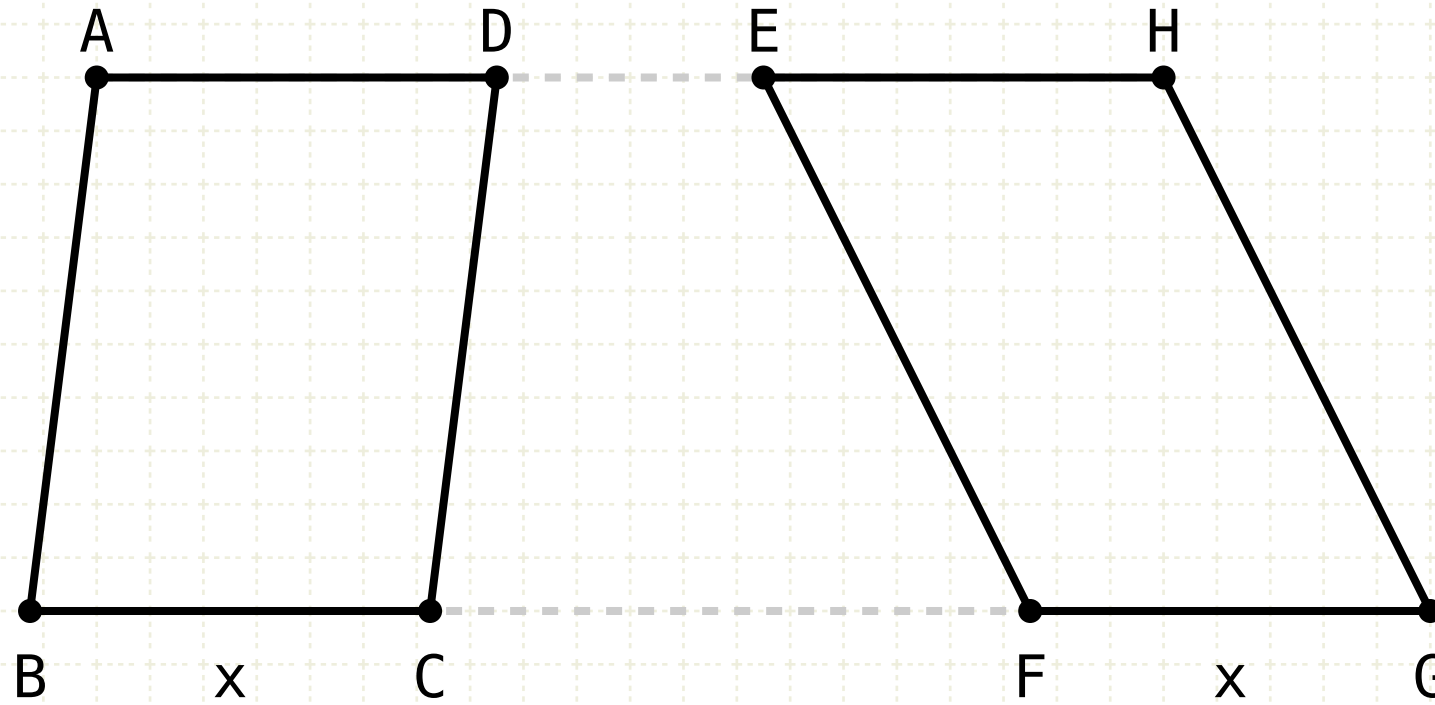
Given two parallel lines

Let ABCD and EFGH be parallelograms with equal bases BC and FG on the same parallels AH and BG

The area ABCD is equal to EFGH

# Proposition 36 of Book I

Parallelograms which are on equal bases and in the same parallels equal one another.



## In other words

Parallelograms with equal bases and equal heights have equal area

Given two parallel lines

Let ABCD and EFGH be parallelograms with equal bases BC and FG on the same parallels AH and BG

The area ABCD is equal to EFGH

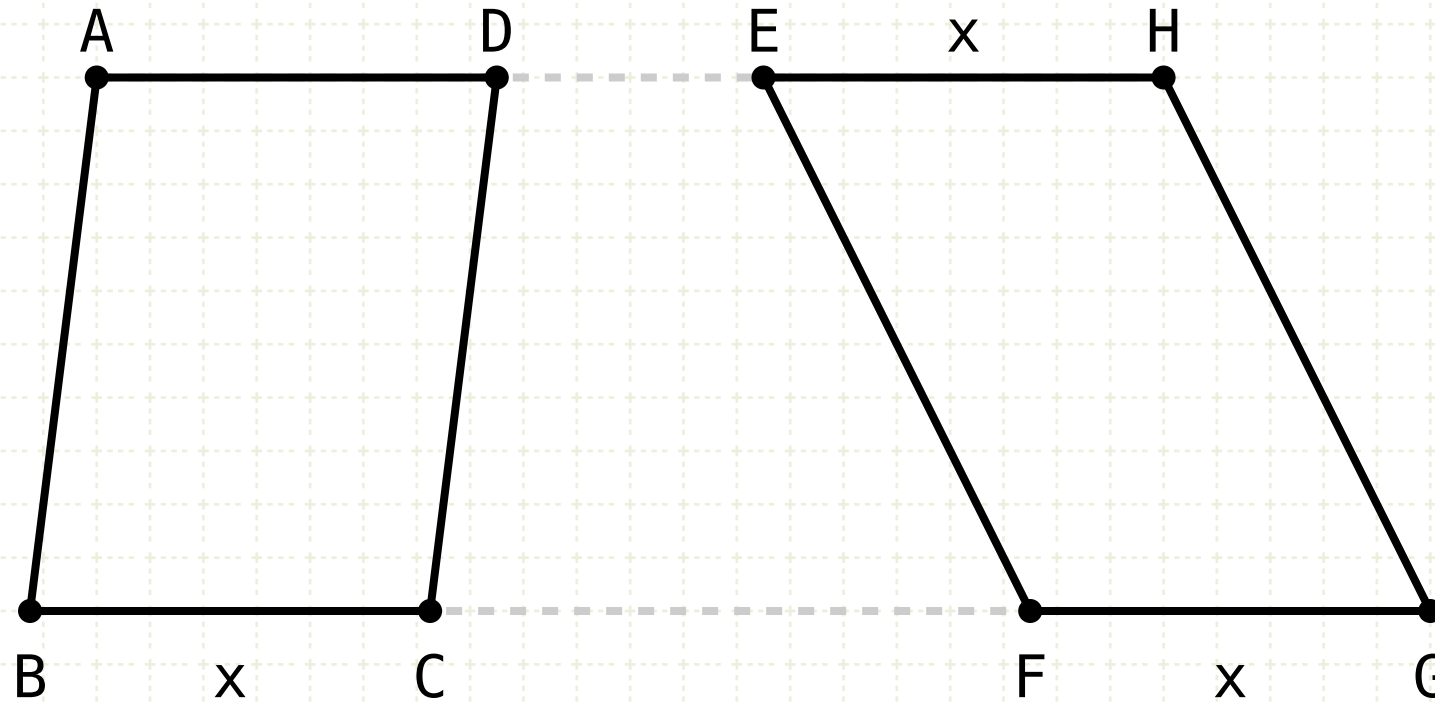
## Proof





# Proposition 36 of Book I

Parallelograms which are on equal bases and in the same parallels equal one another.



$$FG = EH = x$$

## In other words

Parallelograms with equal bases and equal heights have equal area

Given two parallel lines

Let ABCD and EFGH be parallelograms with equal bases BC and FG on the same parallels AH and BG

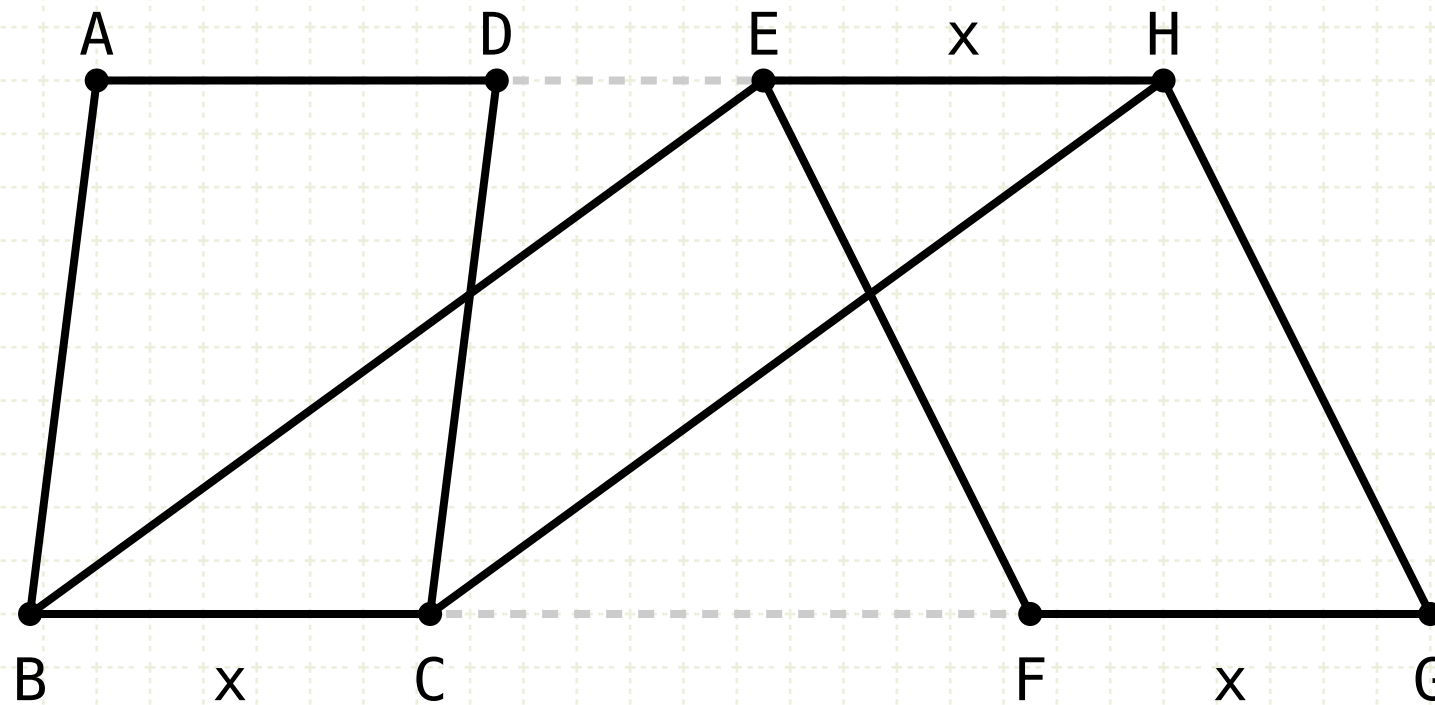
The area ABCD is equal to EFGH

## Proof

FG equals EH since EFGH is a parallelogram (I·34)

# Proposition 36 of Book I

Parallelograms which are on equal bases and in the same parallels equal one another.



$$FG = EH = x$$

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Let ABCD and EFGH be parallelograms with equal bases BC and FG on the same parallels AH and BG

The area ABCD is equal to EFGH

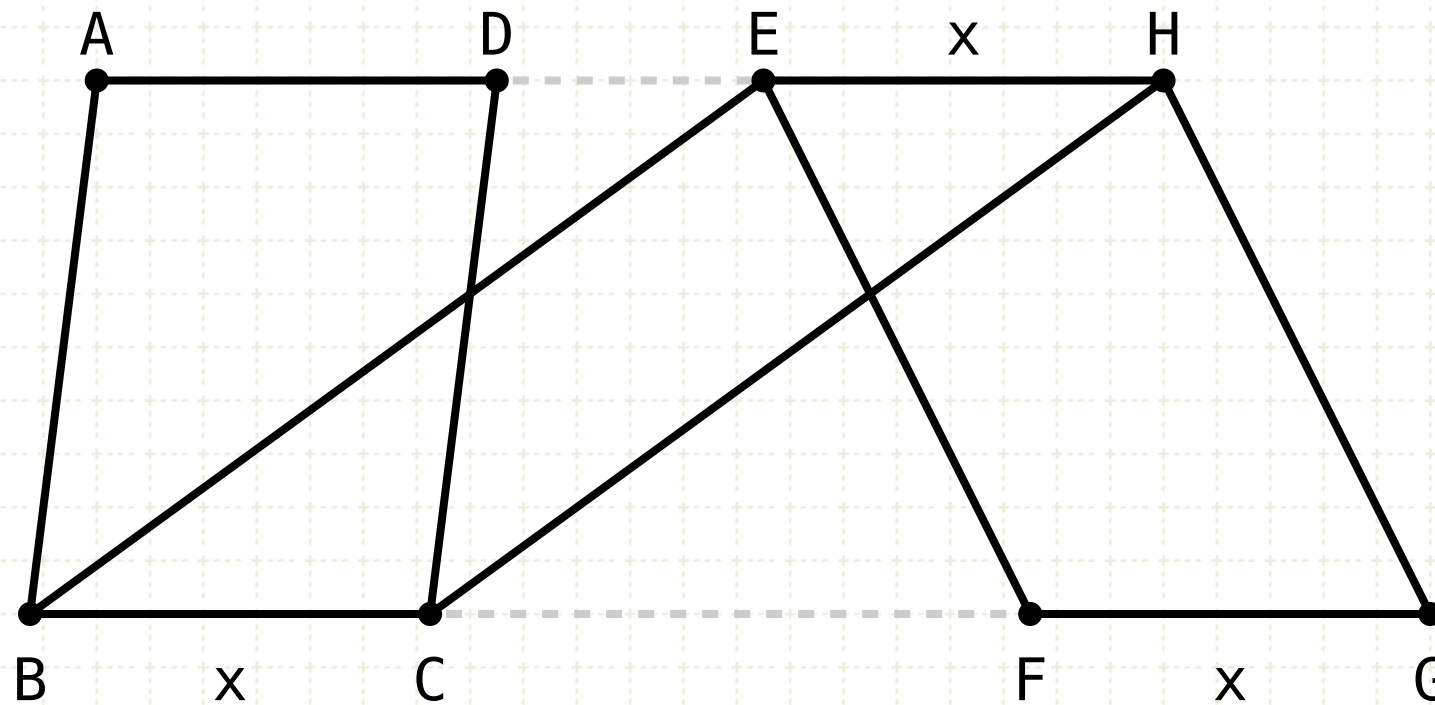
## Proof

FG equals EH since EFGH is a parallelogram (I-34)

Create lines BE and CH

# Proposition 36 of Book I

Parallelograms which are on equal bases and in the same parallels equal one another.



$$\begin{aligned} FG &= EH = x \\ BC &= FG = EH = x \end{aligned}$$

## In other words

Parallelograms with equal bases and equal heights have equal area

Given two parallel lines

Let ABCD and EFGH be parallelograms with equal bases BC and FG on the same parallels AH and BG

The area ABCD is equal to EFGH

## Proof

FG equals EH since EFGH is a parallelogram (I-34)

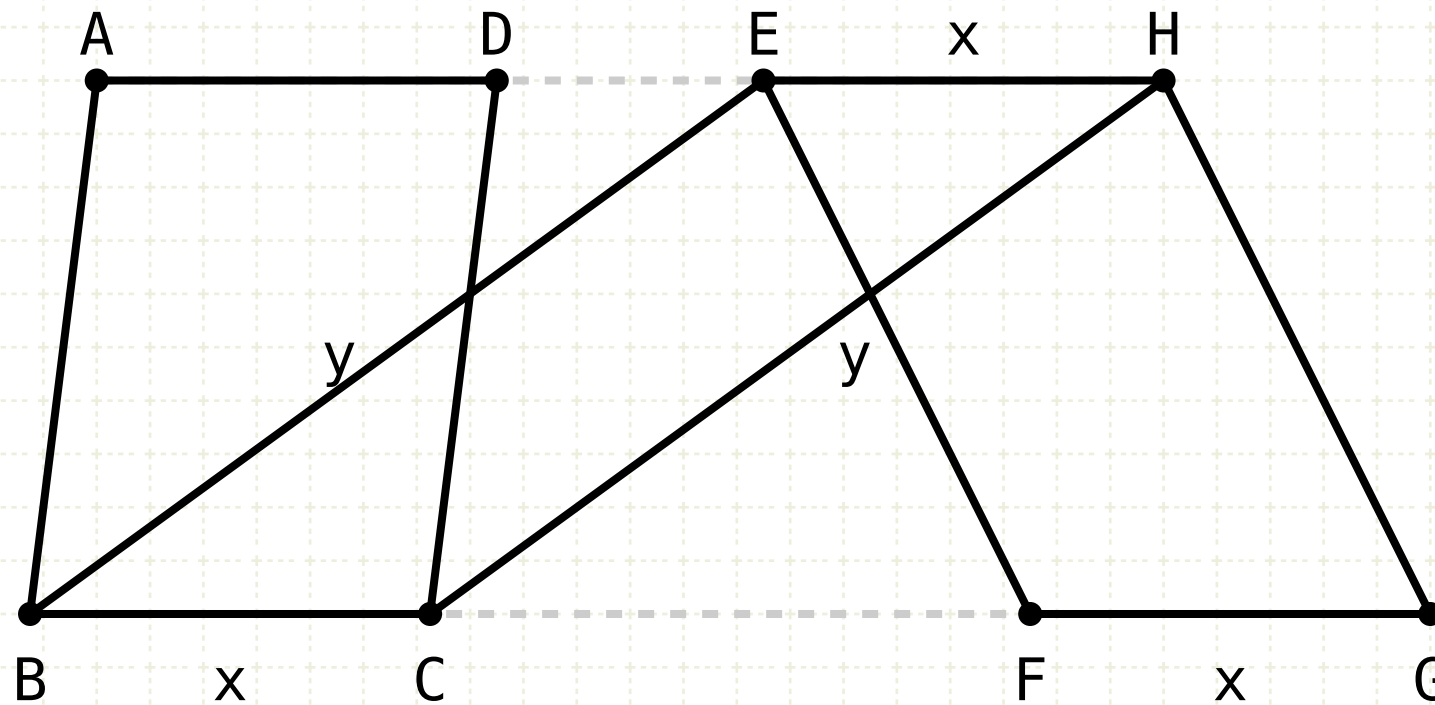
Create lines BE and CH

BC equals FG and FG equals EH, therefore BC equals EH



# Proposition 36 of Book I

Parallelograms which are on equal bases and in the same parallels equal one another.



$$\begin{aligned} FG &= EH = x \\ BC &= FG = EH = x \\ BC &\parallel EH \therefore BE \parallel CH \\ BE &= CH = y \end{aligned}$$

## In other words

Parallelograms with equal bases and equal heights have equal area

Given two parallel lines

Let ABCD and EFGH be parallelograms with equal bases BC and FG on the same parallels AH and BG

The area ABCD is equal to EFGH

## Proof

FG equals EH since EFGH is a parallelogram (I·34)

Create lines BE and CH

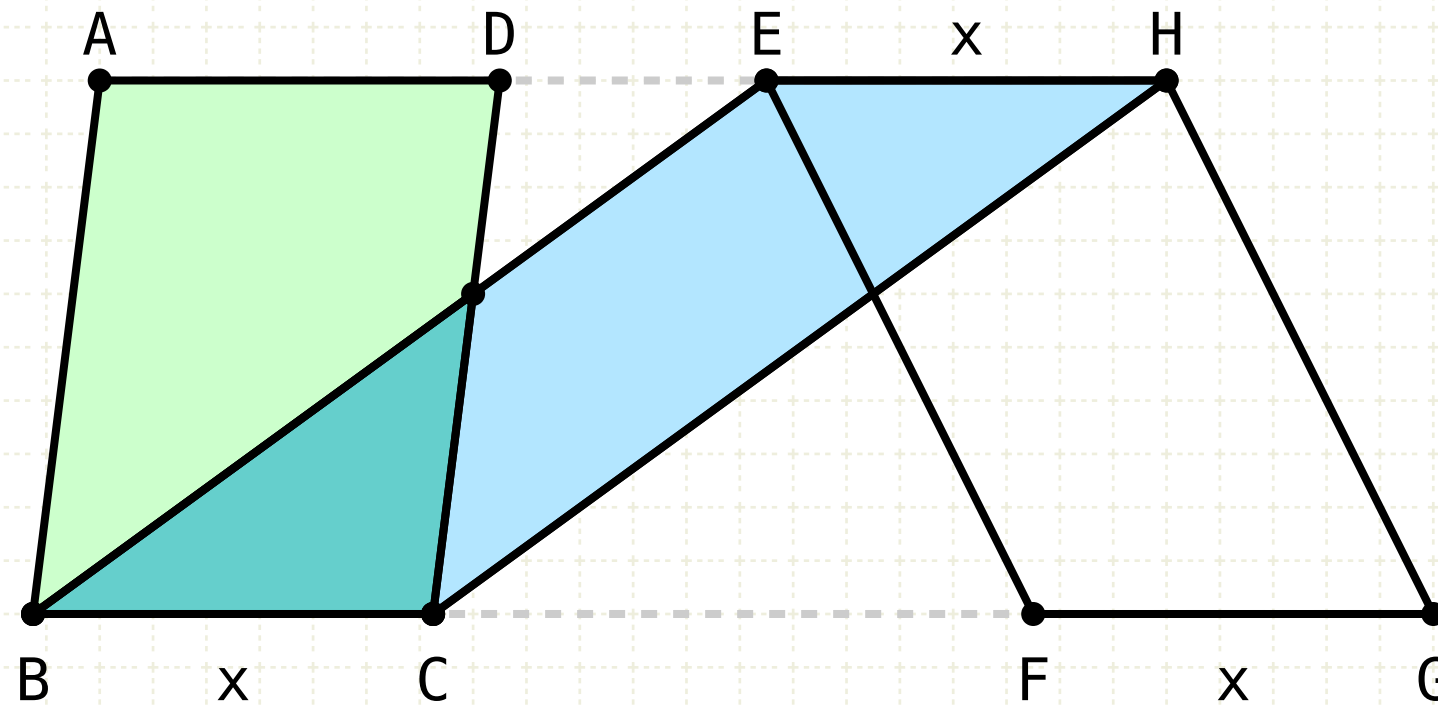
BC equals FG and FG equals EH, therefore BC equals EH

BC and EH are parallel, and equal, therefore the lines joining the endpoints are also equal and parallel (I·33), making EBCH a parallelogram



# Proposition 36 of Book I

Parallelograms which are on equal bases and in the same parallels equal one another.



$$\begin{aligned} FG &= EH = x \\ BC &= FG = EH = x \\ BC &\parallel EH \therefore BE \parallel CH \\ BE &= CH = y \\ EBCH &= ABCD \end{aligned}$$

## In other words

Parallelograms with equal bases and equal heights have equal area

Given two parallel lines

Let ABCD and EFGH be parallelograms with equal bases BC and FG on the same parallels AH and BG

The area ABCD is equal to EFGH

## Proof

FG equals EH since EFGH is a parallelogram (I·34)

Create lines BE and CH

BC equals FG and FG equals EH, therefore BC equals EH

BC and EH are parallel, and equal, therefore the lines joining the endpoints are also equal and parallel (I·33), making EBCH a parallelogram

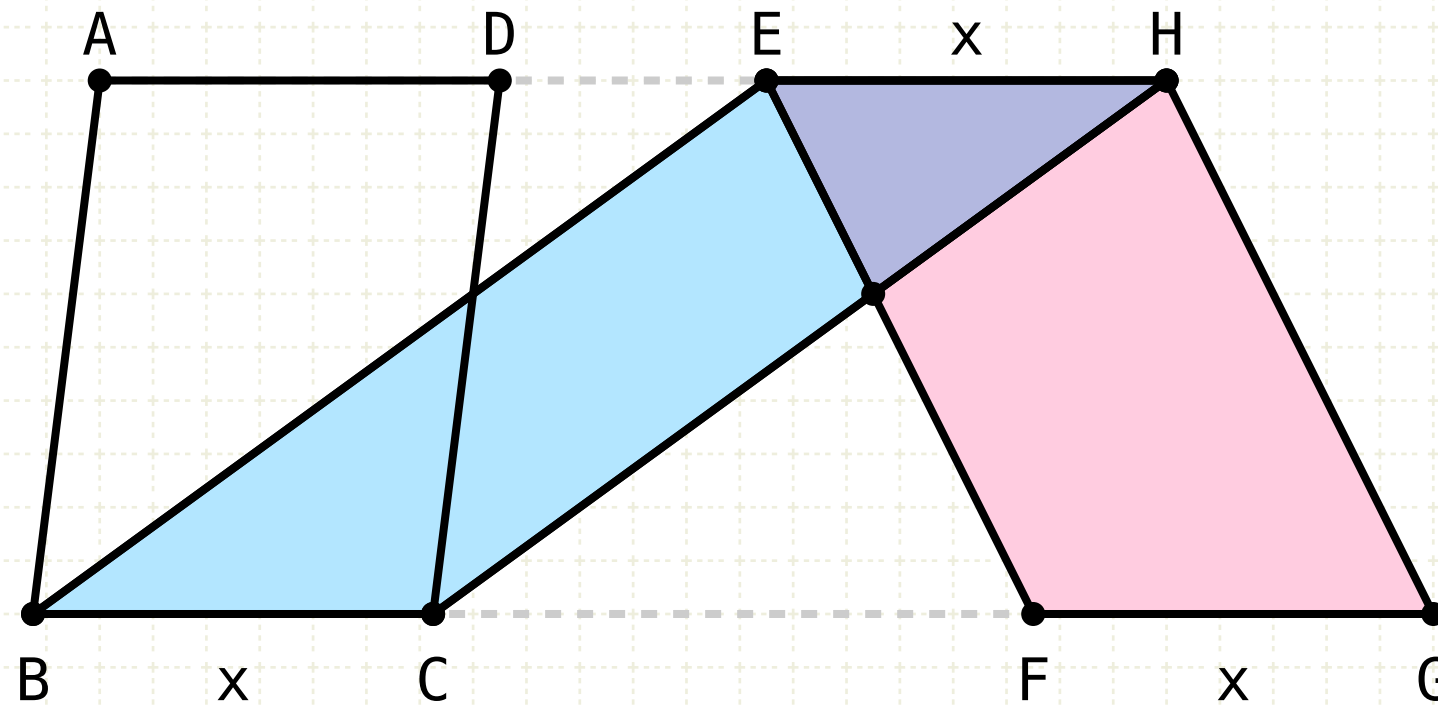
ABCD and EBCH are parallelograms which share the same base and are on the same parallels, so their areas are equal (I·35)





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Parallelograms which are on equal bases and in the same parallels equal one another.



$$\begin{aligned} FG &= EH = x \\ BC &= FG = EH = x \\ BC &\parallel EH \therefore BE \parallel CH \\ BE &= CH = y \\ EBCH &= ABCD \\ EFGH &= EBCH \end{aligned}$$

## In other words

Parallelograms with equal bases and equal heights have equal area

Given two parallel lines

Let ABCD and EFGH be parallelograms with equal bases BC and FG on the same parallels AH and BG

The area ABCD is equal to EFGH

## Proof

FG equals EH since EFGH is a parallelogram (I·34)

Create lines BE and CH

BC equals FG and FG equals EH, therefore BC equals EH

BC and EH are parallel, and equal, therefore the lines joining the endpoints are also equal and parallel (I·33), making EBCH a parallelogram

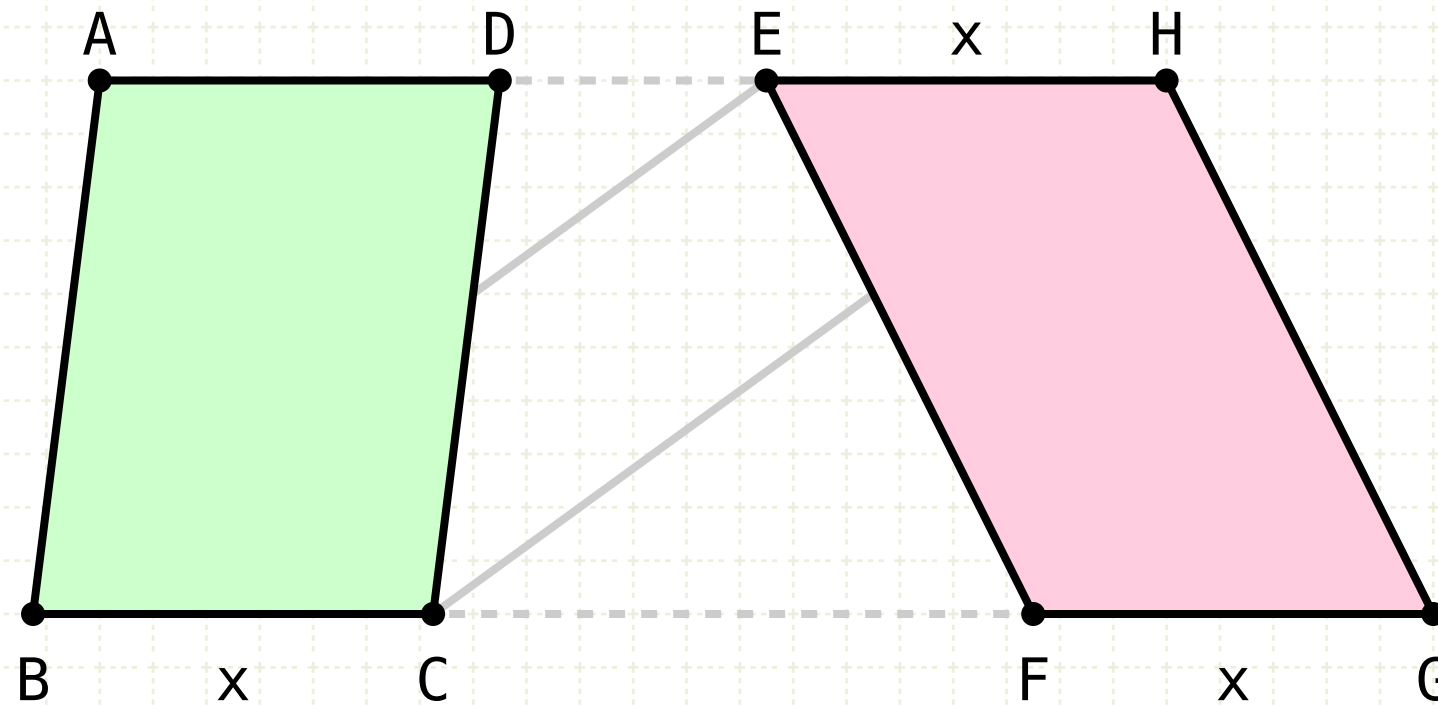
ABCD and EBCH are parallelograms which share the same base and are on the same parallels, so their areas are equal (I·35)

Similarly, EBCH and EFGH are parallelograms which share the same base and are on the same parallels, so their areas are equal (I·35)



# Proposition 36 of Book I

Parallelograms which are on equal bases and in the same parallels equal one another.



$$\begin{aligned} FG &= EH = x \\ BC &= FG = EH = x \\ BC &\parallel EH \therefore BE \parallel CH \\ BE &= CH = y \\ EBCH &= ABCD \\ EFGH &= EBCH \\ ABCD &= EFGH \end{aligned}$$

## In other words

Parallelograms with equal bases and equal heights have equal area

Given two parallel lines

Let ABCD and EFGH be parallelograms with equal bases BC and FG on the same parallels AH and BG

The area ABCD is equal to EFGH

## Proof

FG equals EH since EFGH is a parallelogram (I·34)

Create lines BE and CH

BC equals FG and FG equals EH, therefore BC equals EH

BC and EH are parallel, and equal, therefore the lines joining the endpoints are also equal and parallel (I·33), making EBCH a parallelogram

ABCD and EBCH are parallelograms which share the same base and are on the same parallels, so their areas are equal (I·35)

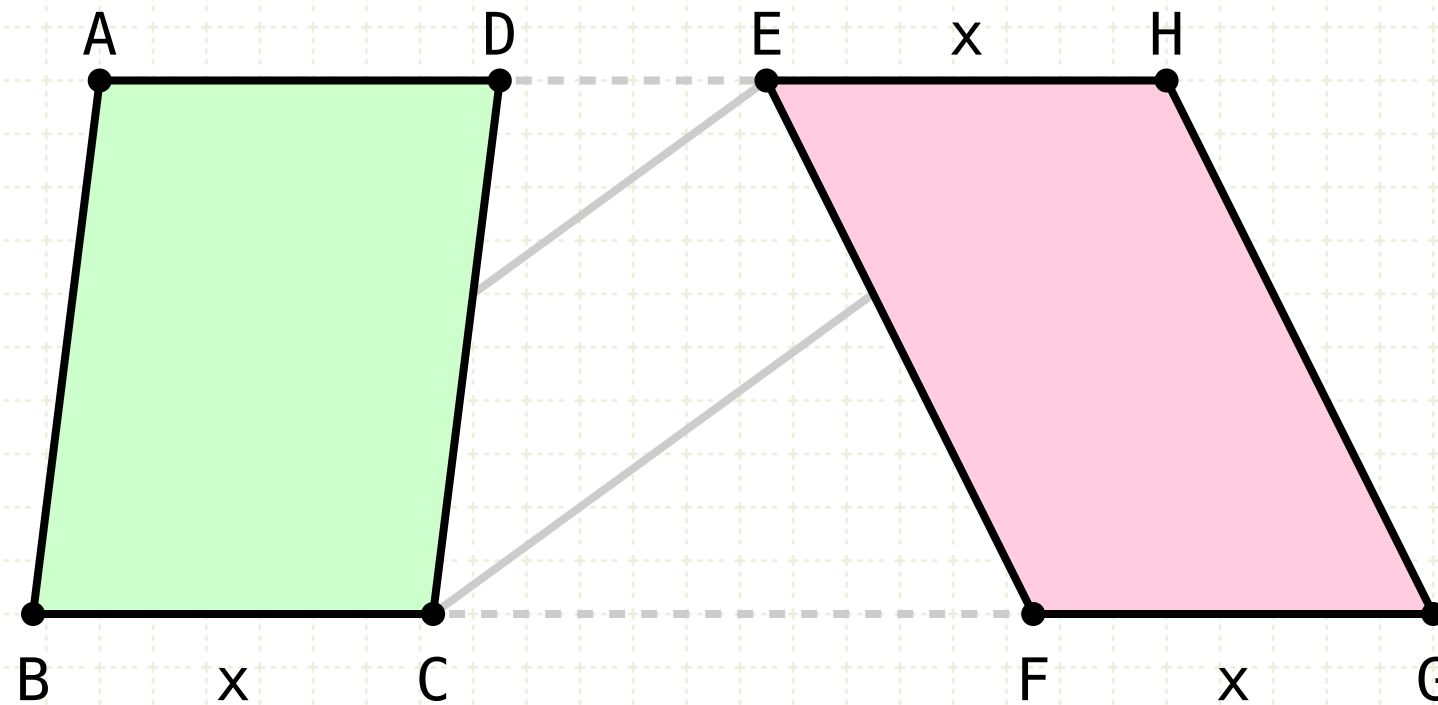
Similarly, EBCH and EFGH are parallelograms which share the same base and are on the same parallels, so their areas are equal (I·35)

Therefore ABCD is equal to EFGH



# Proposition 36 of Book I

Parallelograms which are on equal bases and in the same parallels equal one another.



$$\begin{aligned} FG &= EH = x \\ BC &= FG = EH = x \\ BC &\parallel EH \therefore BE \parallel CH \\ BE &= CH = y \\ EBCH &= ABCD \\ EFGH &= EBCH \end{aligned}$$

$$ABCD = EFGH$$

## In other words

Parallelograms with equal bases and equal heights have equal area

Given two parallel lines

Let ABCD and EFGH be parallelograms with equal bases BC and FG on the same parallels AH and BG

The area ABCD is equal to EFGH

## Proof

FG equals EH since EFGH is a parallelogram (I·34)

Create lines BE and CH

BC equals FG and FG equals EH, therefore BC equals EH

BC and EH are parallel, and equal, therefore the lines joining the endpoints are also equal and parallel (I·33), making EBCH a parallelogram

ABCD and EBCH are parallelograms which share the same base and are on the same parallels, so their areas are equal (I·35)

Similarly, EBCH and EFGH are parallelograms which share the same base and are on the same parallels, so their areas are equal (I·35)

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