

Euclid's Elements

Book I

If Euclid did not kindle your youthful enthusiasm, you were not born to be a scientific thinker.

Albert Einstein



Table of Contents, Chapter 1

1	Construct an equilateral triangle	15	Vertical angles equal one another	29	Lines parallel, alternate angles are equal
2	Copy a line	16	Exterior angle larger than interior angle	30	Lines parallel to same line are parallel to themselves
3	Subtract one line from another	17	Sum of two interior angles less than 180	31	Construct one line parallel to another
4	Equal triangles if equal side-angle-side	18	Greater side opposite of greater angle	32	Sum of interior angles of a triangle = 180
5	Isosceles triangle gives equal base angles	19	Greater angle opposite of greater side	33	Lines joining ends of equal parallels are parallel
6	Equal base angles gives isosceles triangle	20	Sum of two angles greater than third	34	Opposite sides-angles equal in parallelogram
7	Two sides of triangle meet at unique point	21	Triangle within triangle has smaller sides	35	Parallelograms, same base-height have equal area
8	Equal triangles if equal side-side-side	22	Construct triangle from given lines	36	Parallelograms, equal base-height have equal area
9	How to bisect an angle	23	Copy an angle	37	Triangles, same base-height have equal area
10	Bisect a line	24	Larger angle gives larger base	38	Triangles, equal base-height have equal area
11	Construct right angle, point on line	25	Larger base gives larger angle		
12	Construct perpendicular, point to line	26	Equal triangles if equal angle-side-angle		
13	Sum of angles on straight line = 180	27	Alternate angles equal then lines parallel		
14	Two lines form a single line if angle = 180	28	Sum of interior angles = 180 , lines parallel		



Table of Contents, Chapter 1

39	Equal triangles on same base, have equal height
40	Equal triangles on equal base, have equal height
41	Triangle is half parallelogram with same base and height
42	Construct parallelogram with equal area as triangle
43	Parallelogram complements are equal
44	Construct parallelogram on line, equal to triangle
45	Construct parallelogram equal to polygon
46	Construct a square
47	Pythagoras' theorem
48	Inverse Pythagoras' theorem



Proposition 21 of Book I

If from the ends of one of the sides of a triangle two straight lines are constructed meeting within the triangle, then the sum of the straight lines so constructed is less than the sum of the remaining two sides of the triangle, but the constructed straight lines contain a greater angle than the angle contained by the remaining two sides.

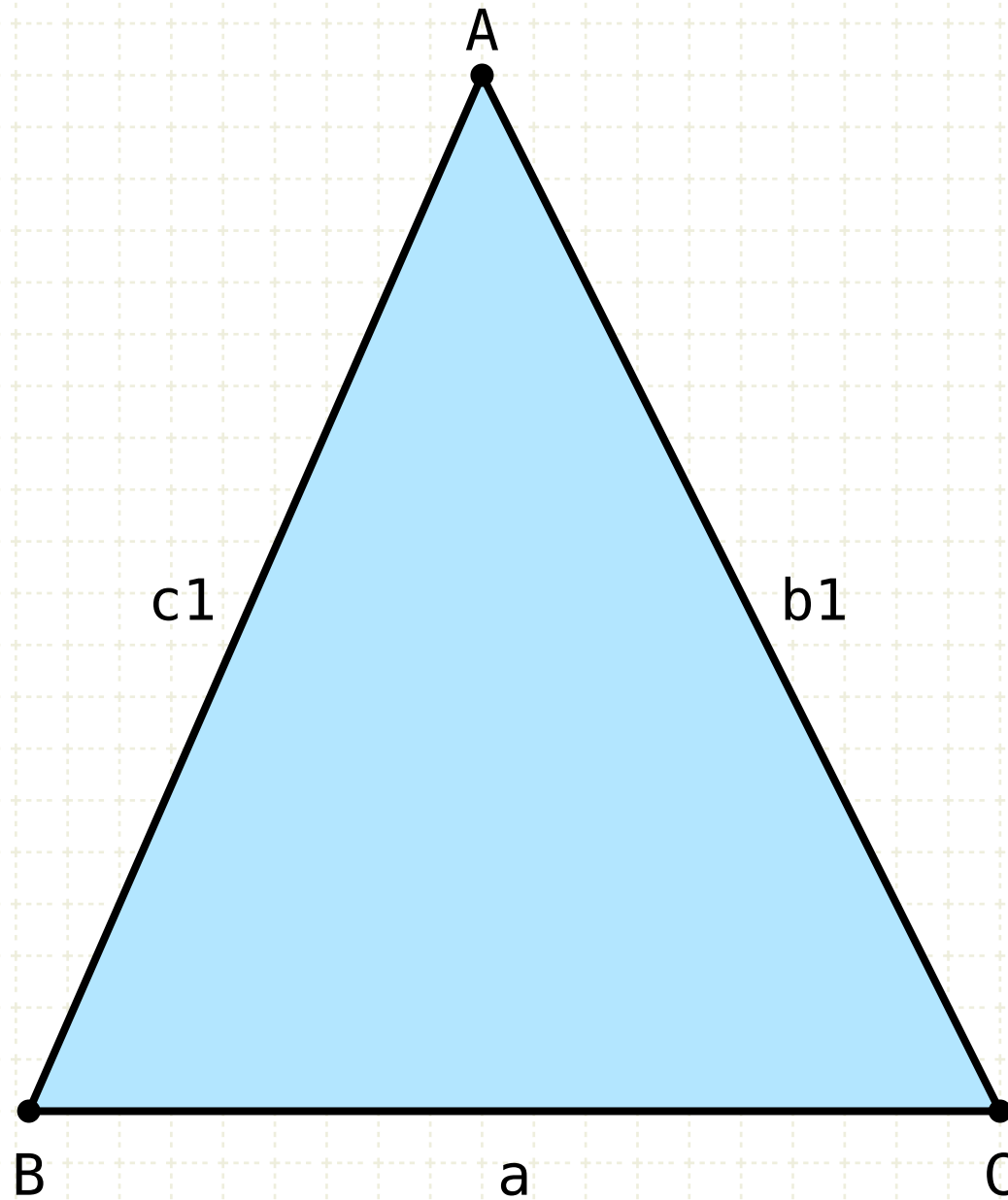


Proposition 21 of Book I

If from the ends of one of the sides of a triangle two straight lines are constructed meeting within the triangle, then the sum of the straight lines so constructed is less than the sum of the remaining two sides of the triangle, but the constructed straight lines contain a greater angle than the angle contained by the remaining two sides.

In other words

Given a triangle ABC



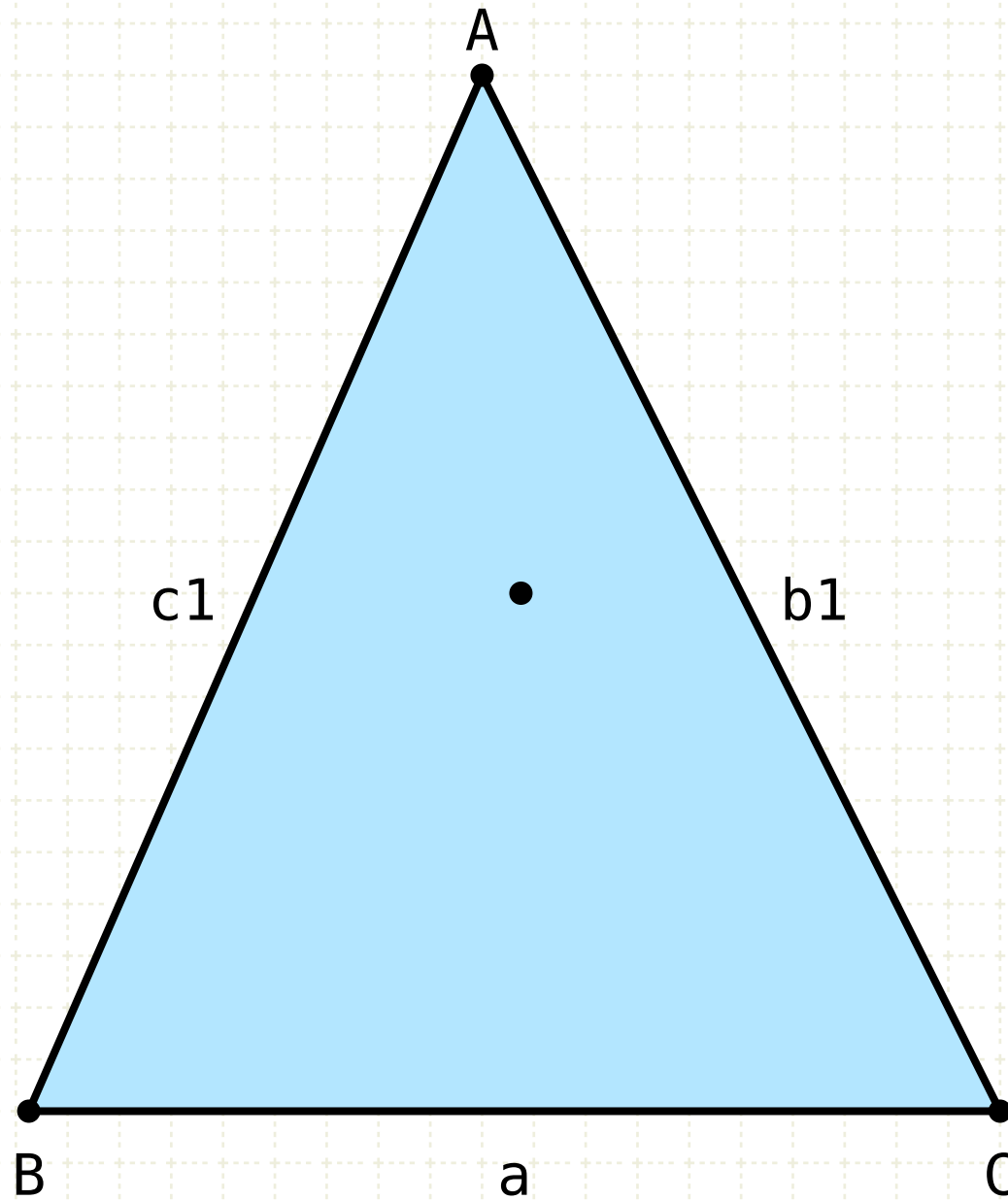
Proposition 21 of Book I

If from the ends of one of the sides of a triangle two straight lines are constructed meeting within the triangle, then the sum of the straight lines so constructed is less than the sum of the remaining two sides of the triangle, but the constructed straight lines contain a greater angle than the angle contained by the remaining two sides.

In other words

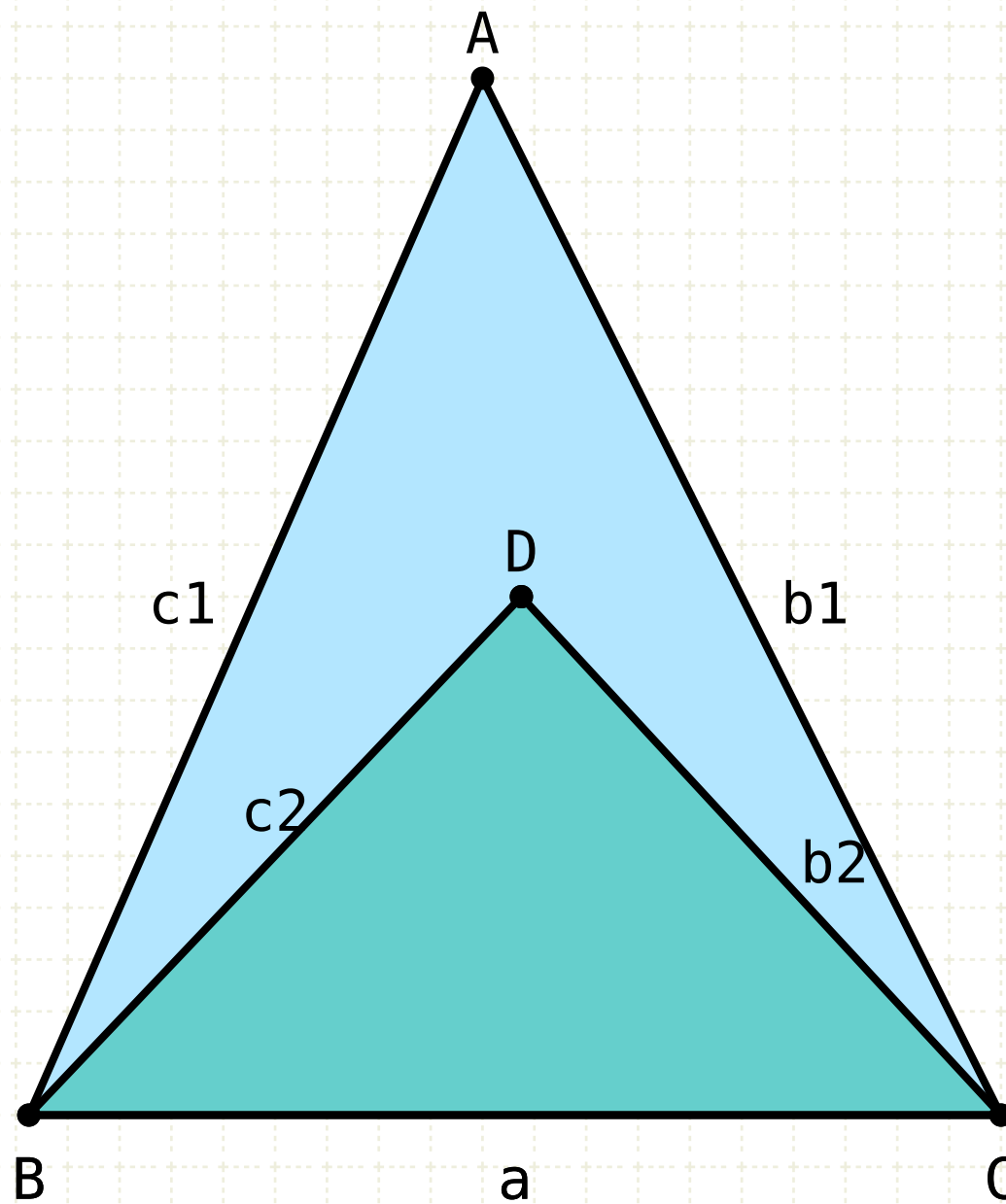
Given a triangle ABC

From a point within the triangle ABC...



Proposition 21 of Book I

If from the ends of one of the sides of a triangle two straight lines are constructed meeting within the triangle, then the sum of the straight lines so constructed is less than the sum of the remaining two sides of the triangle, but the constructed straight lines contain a greater angle than the angle contained by the remaining two sides.



In other words

Given a triangle ABC

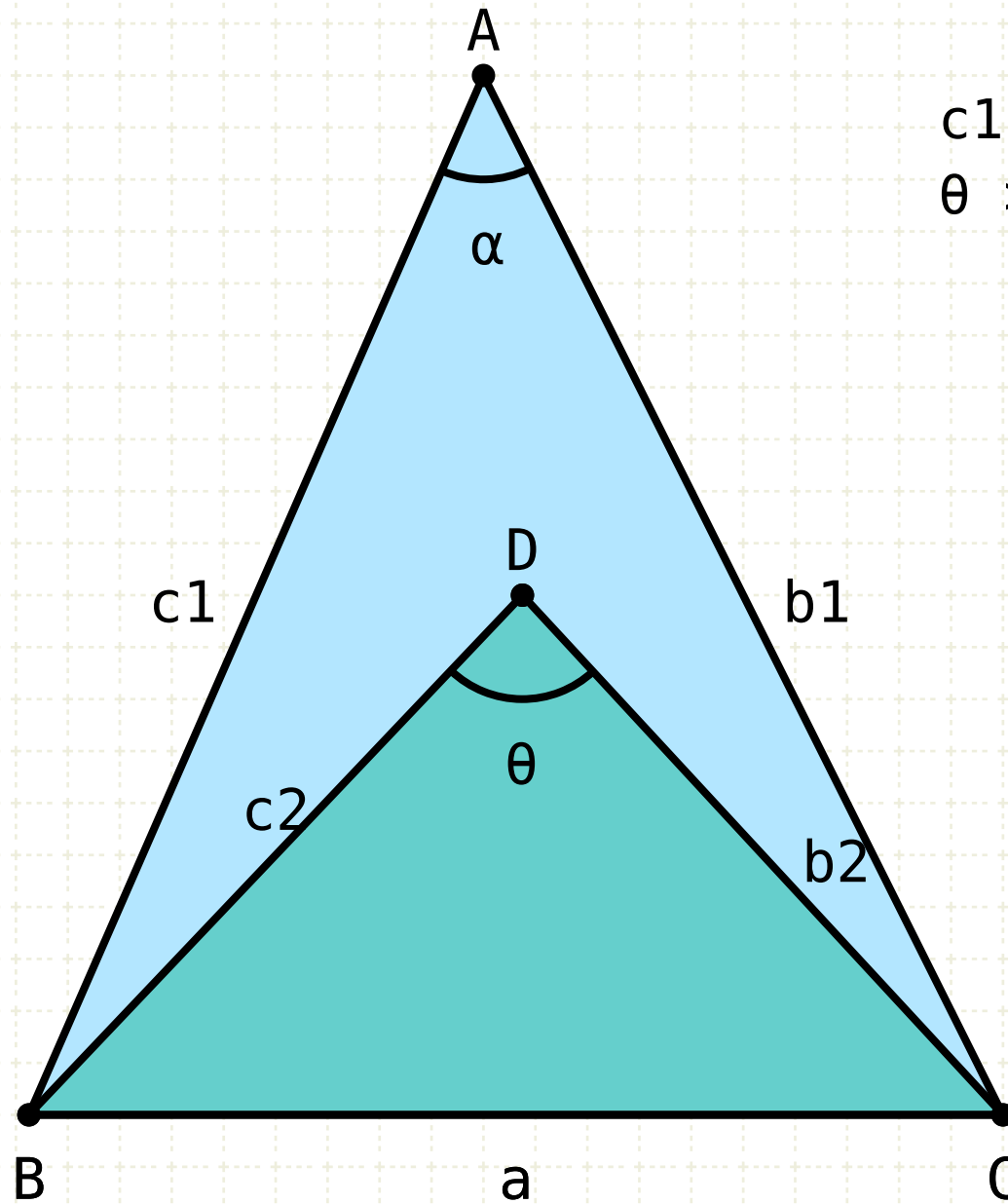
From a point within the triangle ABC...

... construct a second triangle DBC



Proposition 21 of Book I

If from the ends of one of the sides of a triangle two straight lines are constructed meeting within the triangle, then the sum of the straight lines so constructed is less than the sum of the remaining two sides of the triangle, but the constructed straight lines contain a greater angle than the angle contained by the remaining two sides.



$$c1 + b1 > c2 + b2$$
$$\theta > \alpha$$

In other words

Given a triangle ABC

From a point within the triangle ABC...

... construct a second triangle DBC

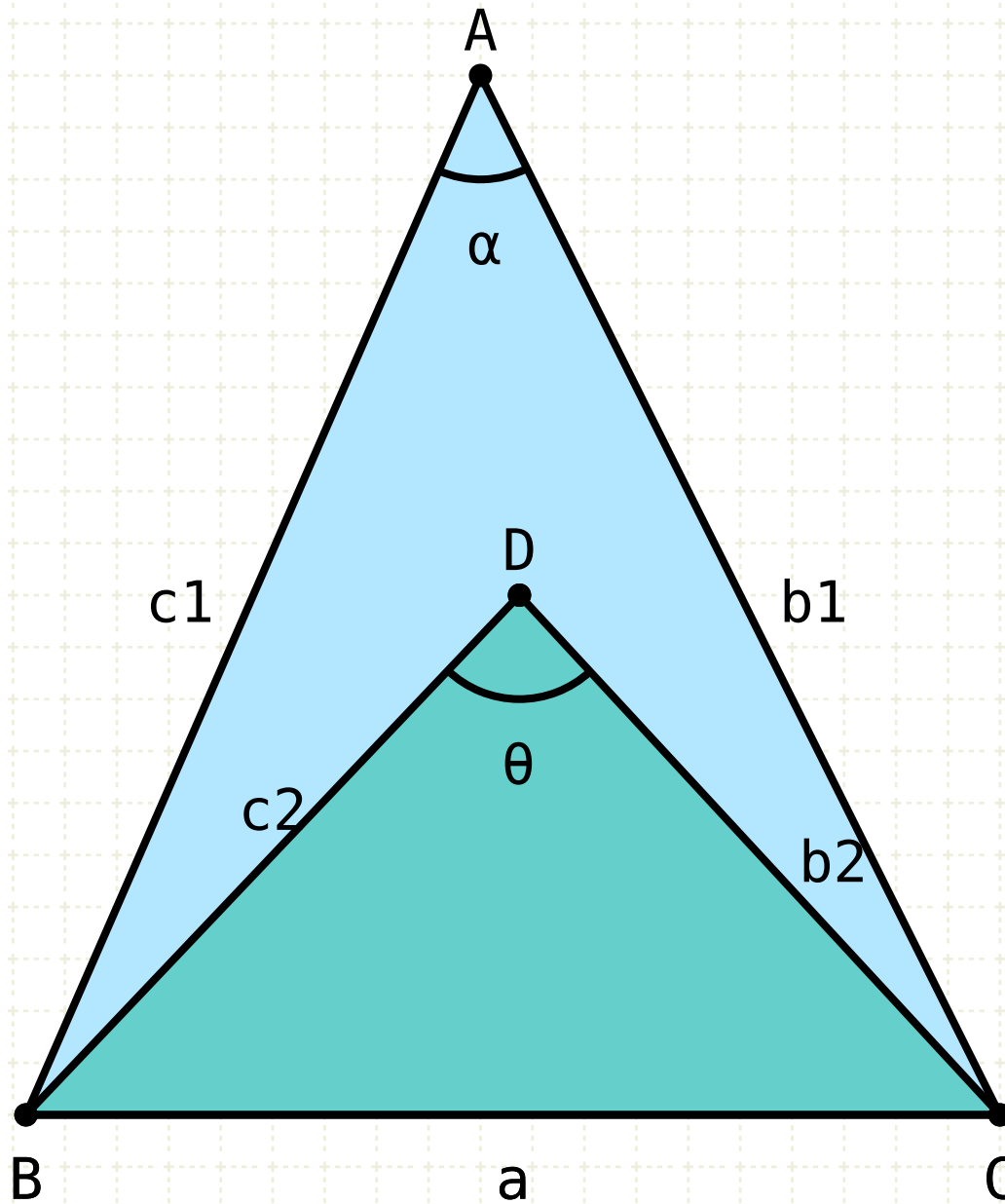
The sum of the lines BD and DC is less than the sum of the lines BA and AC, and the angle BDC is greater than angle BAC



Proposition 21 of Book I

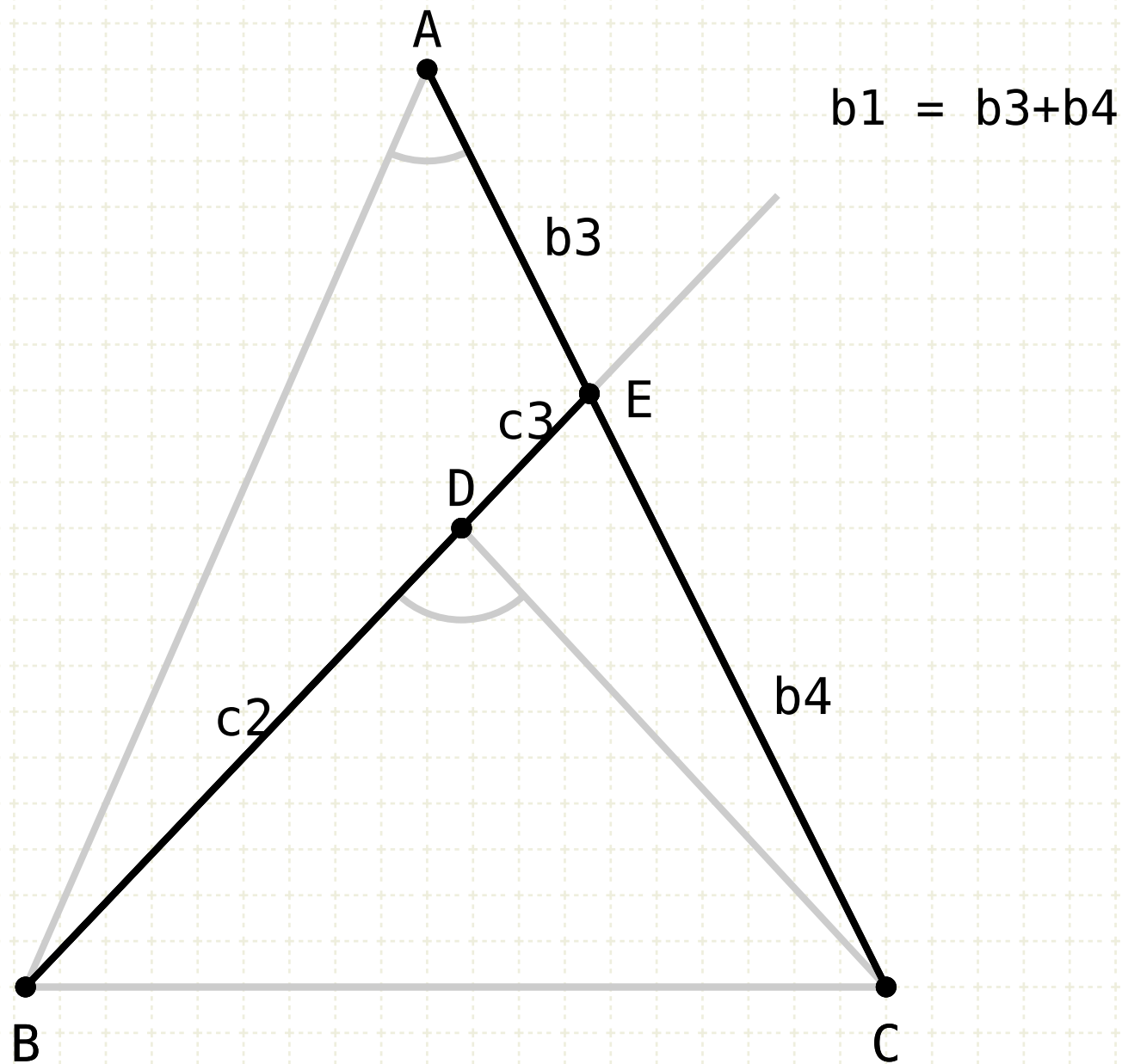
If from the ends of one of the sides of a triangle two straight lines are constructed meeting within the triangle, then the sum of the straight lines so constructed is less than the sum of the remaining two sides of the triangle, but the constructed straight lines contain a greater angle than the angle contained by the remaining two sides.

Proof



Proposition 21 of Book I

If from the ends of one of the sides of a triangle two straight lines are constructed meeting within the triangle, then the sum of the straight lines so constructed is less than the sum of the remaining two sides of the triangle, but the constructed straight lines contain a greater angle than the angle contained by the remaining two sides.



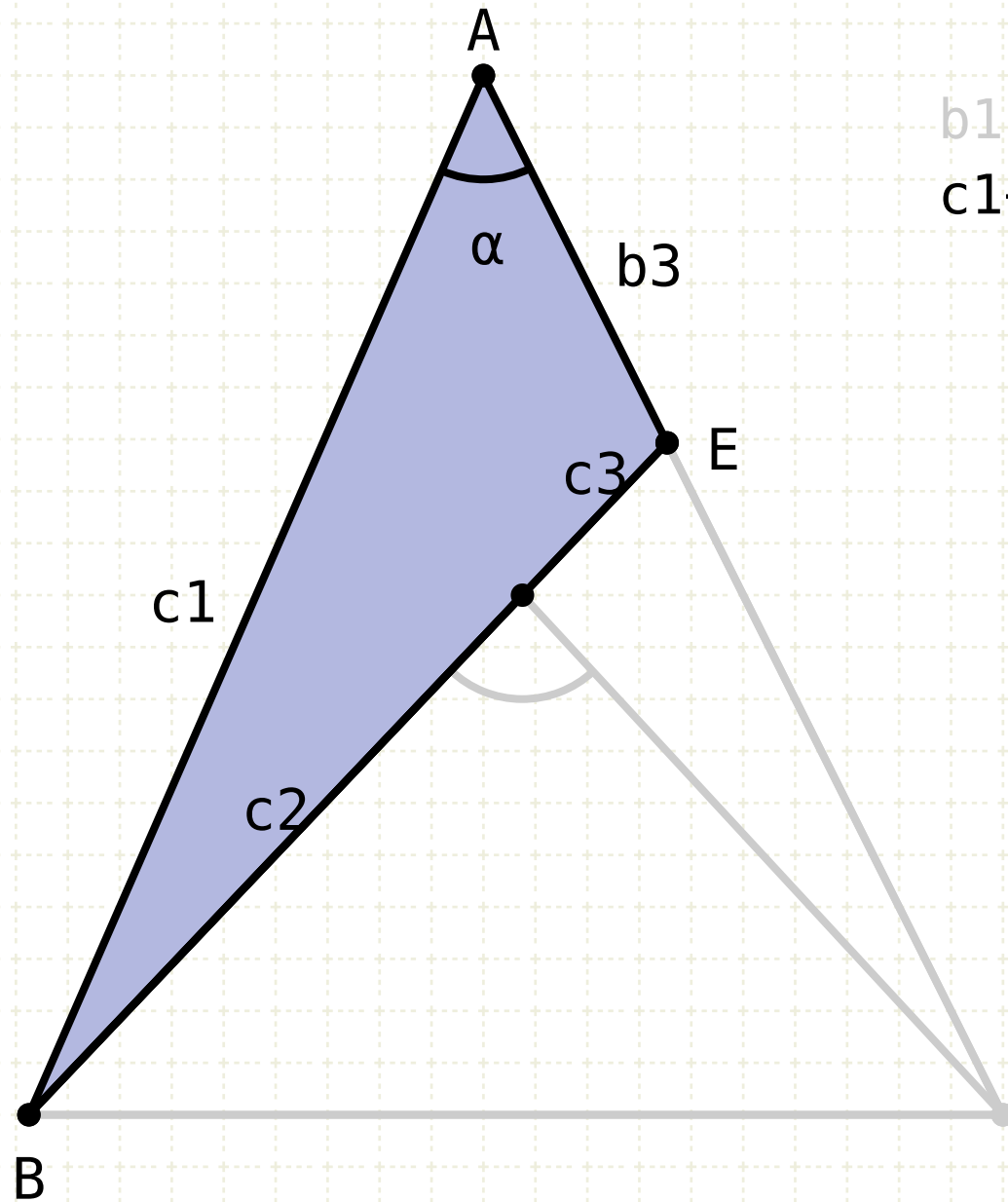
Proof

Extend BD such that it intersects AC at point E



Proposition 21 of Book I

If from the ends of one of the sides of a triangle two straight lines are constructed meeting within the triangle, then the sum of the straight lines so constructed is less than the sum of the remaining two sides of the triangle, but the constructed straight lines contain a greater angle than the angle contained by the remaining two sides.



$$b1 = b3 + b4$$
$$c1 + b3 > c2 + c3$$

Proof

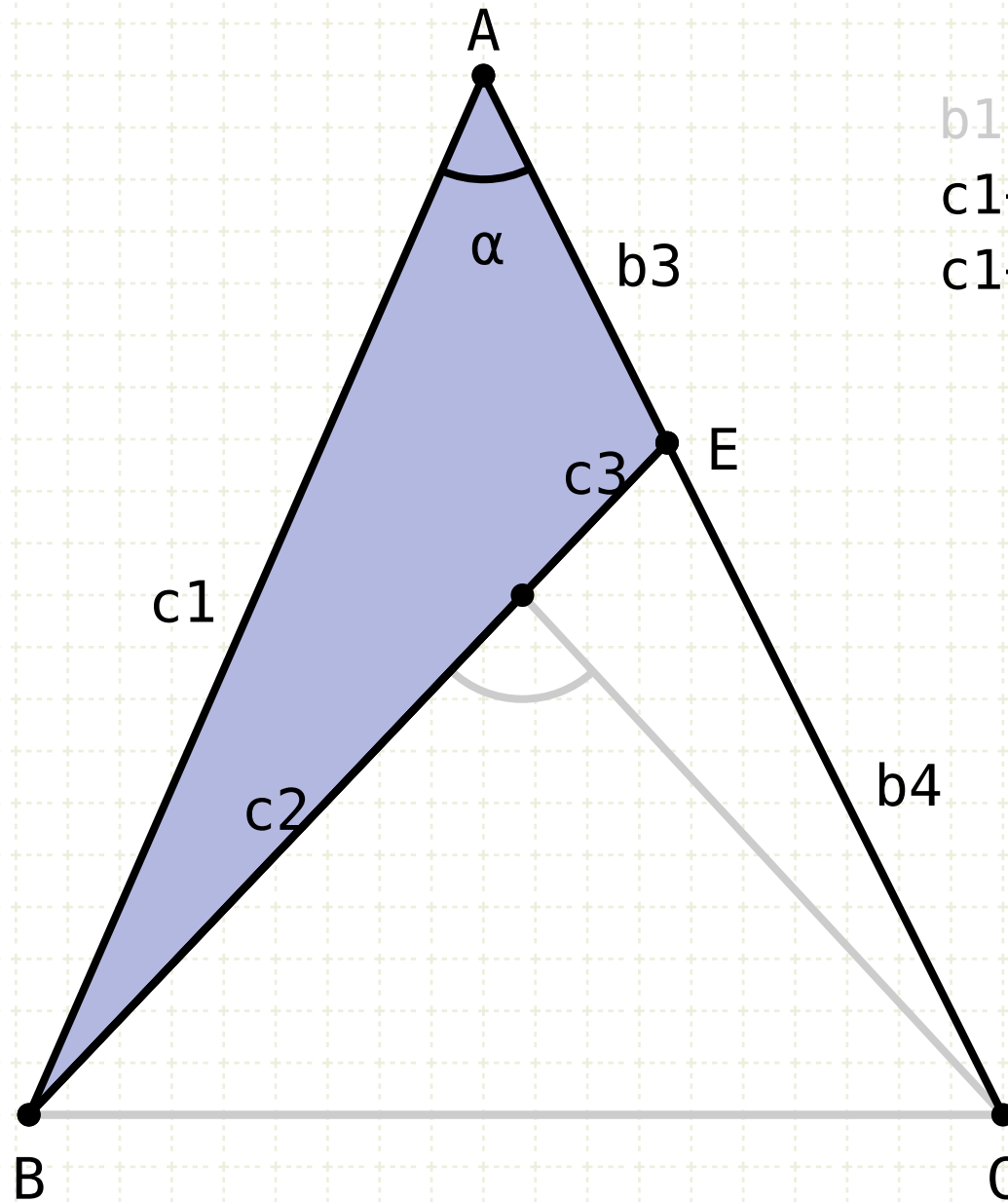
Extend BD such that it intersects AC at point E

Consider triangle ABE

The sum of lines AB and AE is greater than BE (I-18)

Proposition 21 of Book I

If from the ends of one of the sides of a triangle two straight lines are constructed meeting within the triangle, then the sum of the straight lines so constructed is less than the sum of the remaining two sides of the triangle, but the constructed straight lines contain a greater angle than the angle contained by the remaining two sides.



$$b1 = b3 + b4$$

$$c1 + b3 > c2 + c3$$

$$c1 + b3 + b4 > c2 + c3 + b4$$

Proof

Extend BD such that it intersects AC at point E

Consider triangle ABE

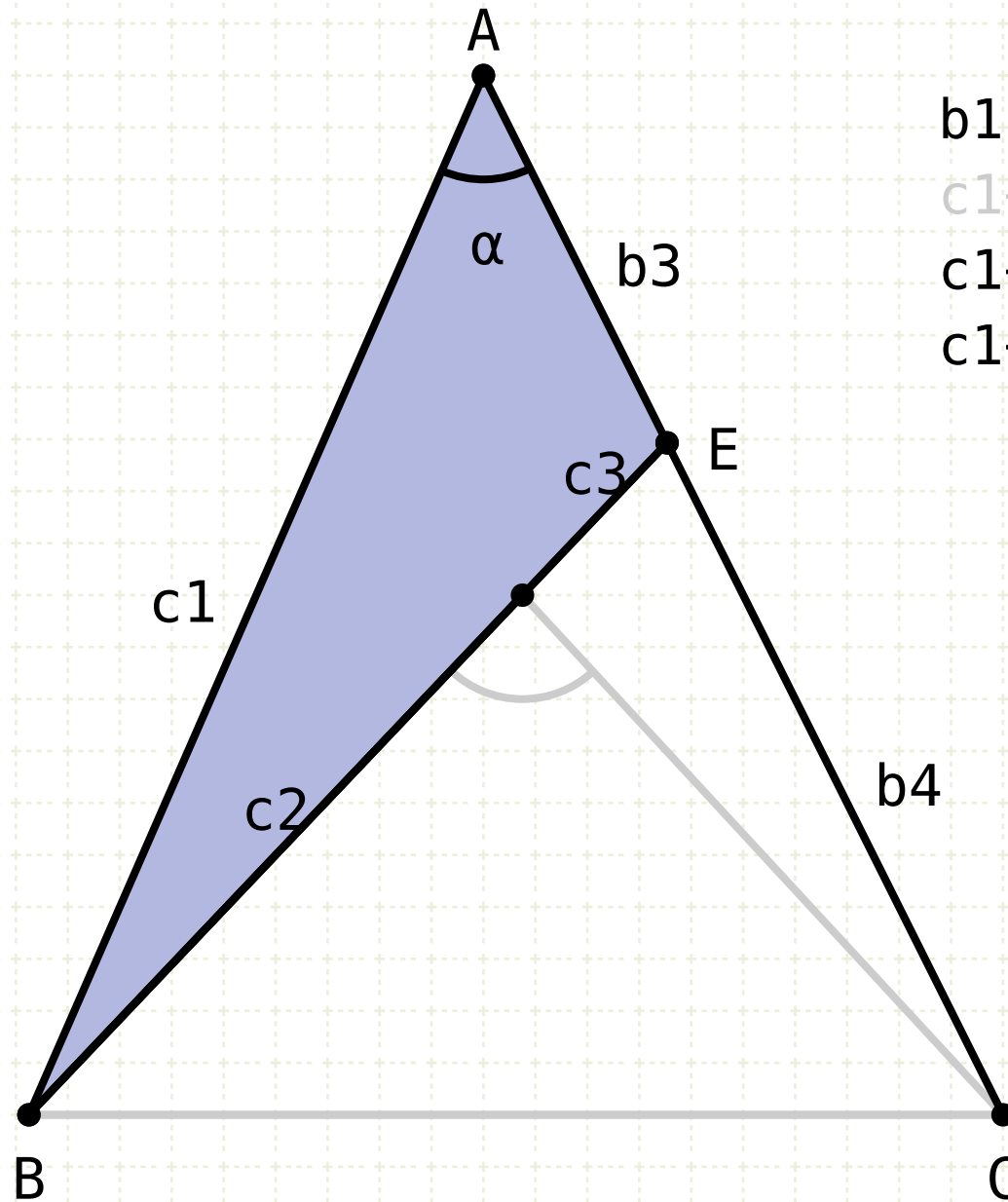
The sum of lines AB and AE is greater than BE (I-18)

Add length EC to both each part of the inequality



Proposition 21 of Book I

If from the ends of one of the sides of a triangle two straight lines are constructed meeting within the triangle, then the sum of the straight lines so constructed is less than the sum of the remaining two sides of the triangle, but the constructed straight lines contain a greater angle than the angle contained by the remaining two sides.



$$b1 = b3 + b4$$

$$c1 + b3 > c2 + c3$$

$$c1 + b3 + b4 > c2 + c3 + b4$$

$$c1 + b1 > c2 + c3 + b4$$

Proof

Extend BD such that it intersects AC at point E

Consider triangle ABE

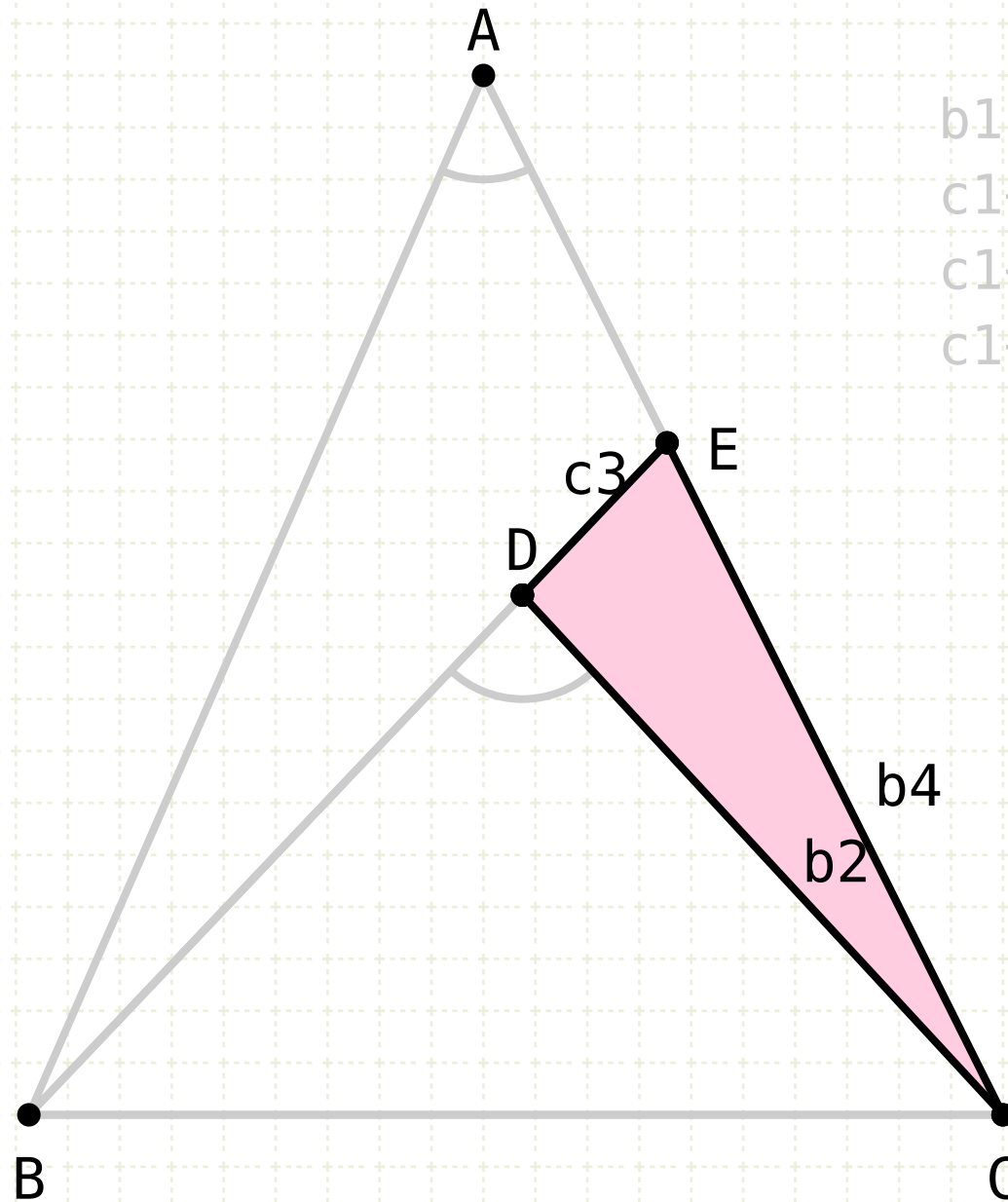
The sum of lines AB and AE is greater than BE (I-18)

Add length EC to both each part of the inequality



Proposition 21 of Book I

If from the ends of one of the sides of a triangle two straight lines are constructed meeting within the triangle, then the sum of the straight lines so constructed is less than the sum of the remaining two sides of the triangle, but the constructed straight lines contain a greater angle than the angle contained by the remaining two sides.



$$b1 = b3 + b4$$

$$c1 + b3 > c2 + c3$$

$$c1 + b3 + b4 > c2 + c3 + b4$$

$$c1 + b1 > c2 + c3 + b4$$

$$c3 + b4 > b2$$

Proof

Extend BD such that it intersects AC at point E

Consider triangle ABE

The sum of lines AB and AE is greater than BE (I·18)

Add length EC to both each part of the inequality

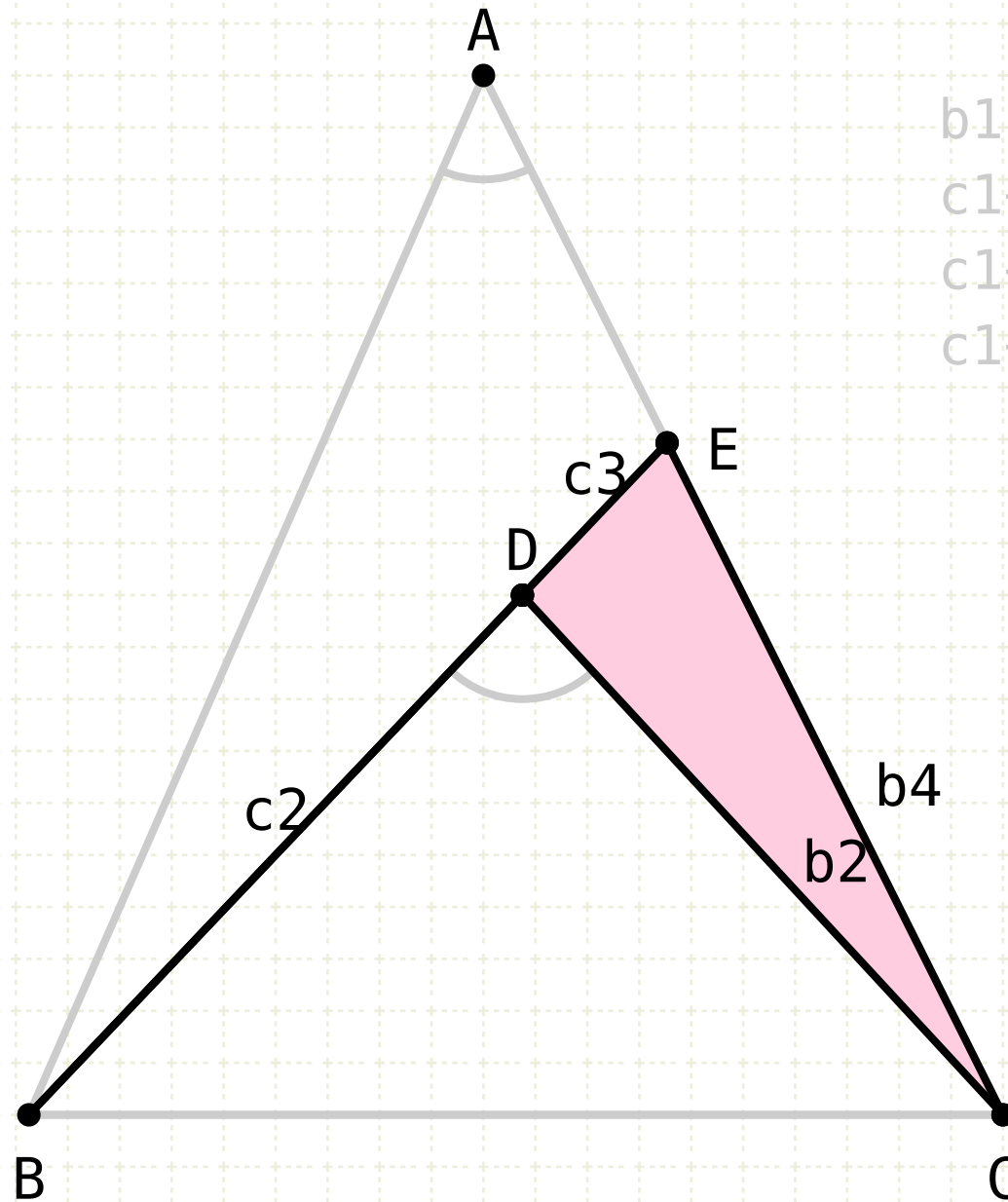
Consider triangle DEC

The sum of lines DE and EC is greater than CD (I·18)



Proposition 21 of Book I

If from the ends of one of the sides of a triangle two straight lines are constructed meeting within the triangle, then the sum of the straight lines so constructed is less than the sum of the remaining two sides of the triangle, but the constructed straight lines contain a greater angle than the angle contained by the remaining two sides.



$$b1 = b3 + b4$$

$$c1 + b3 > c2 + c3$$

$$c1 + b3 + b4 > c2 + c3 + b4$$

$$c1 + b1 > c2 + c3 + b4$$

$$c3 + b4 > b2$$

$$c2 + c3 + b4 > c2 + b2$$

Proof

Extend BD such that it intersects AC at point E

Consider triangle ABE

The sum of lines AB and AE is greater than BE (I·18)

Add length EC to both each part of the inequality

Consider triangle DEC

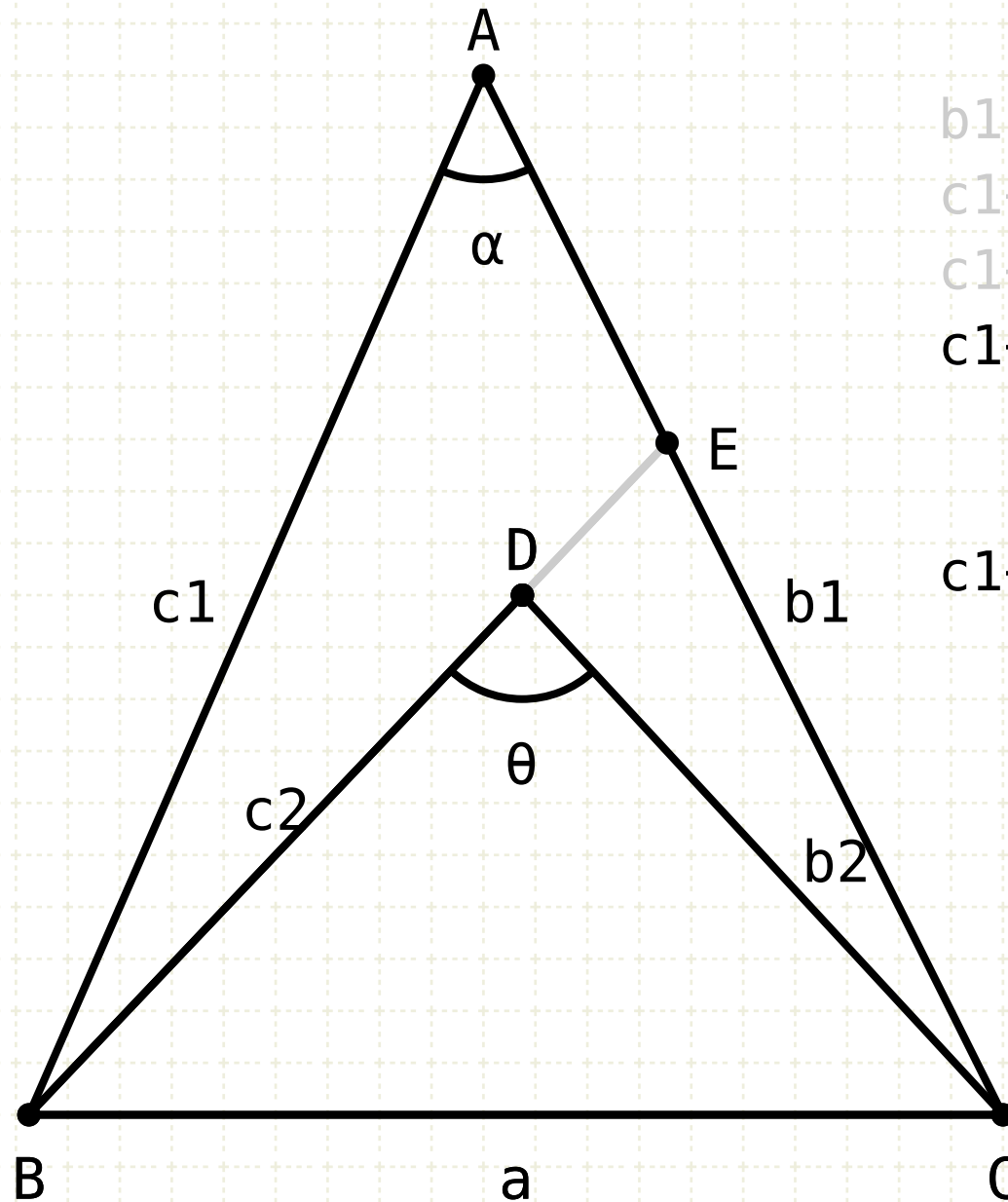
The sum of lines DE and EC is greater than CD (I·18)

Add BD to both sides of the inequality



Proposition 21 of Book I

If from the ends of one of the sides of a triangle two straight lines are constructed meeting within the triangle, then the sum of the straight lines so constructed is less than the sum of the remaining two sides of the triangle, but the constructed straight lines contain a greater angle than the angle contained by the remaining two sides.



$$\begin{aligned}
 b_1 &= b_3 + b_4 \\
 c_1 + b_3 &> c_2 + c_3 \\
 c_1 + b_3 + b_4 &> c_2 + c_3 + b_4 \\
 c_1 + b_1 &> c_2 + c_3 + b_4 \\
 c_3 + b_4 &> b_2 \\
 c_2 + c_3 + b_4 &> c_2 + b_2 \\
 c_1 + b_1 &> c_2 + b_2
 \end{aligned}$$

Proof

Extend BD such that it intersects AC at point E

Consider triangle ABE

The sum of lines AB and AE is greater than BE (I·18)

Add length EC to both each part of the inequality

Consider triangle DEC

The sum of lines DE and EC is greater than CD (I·18)

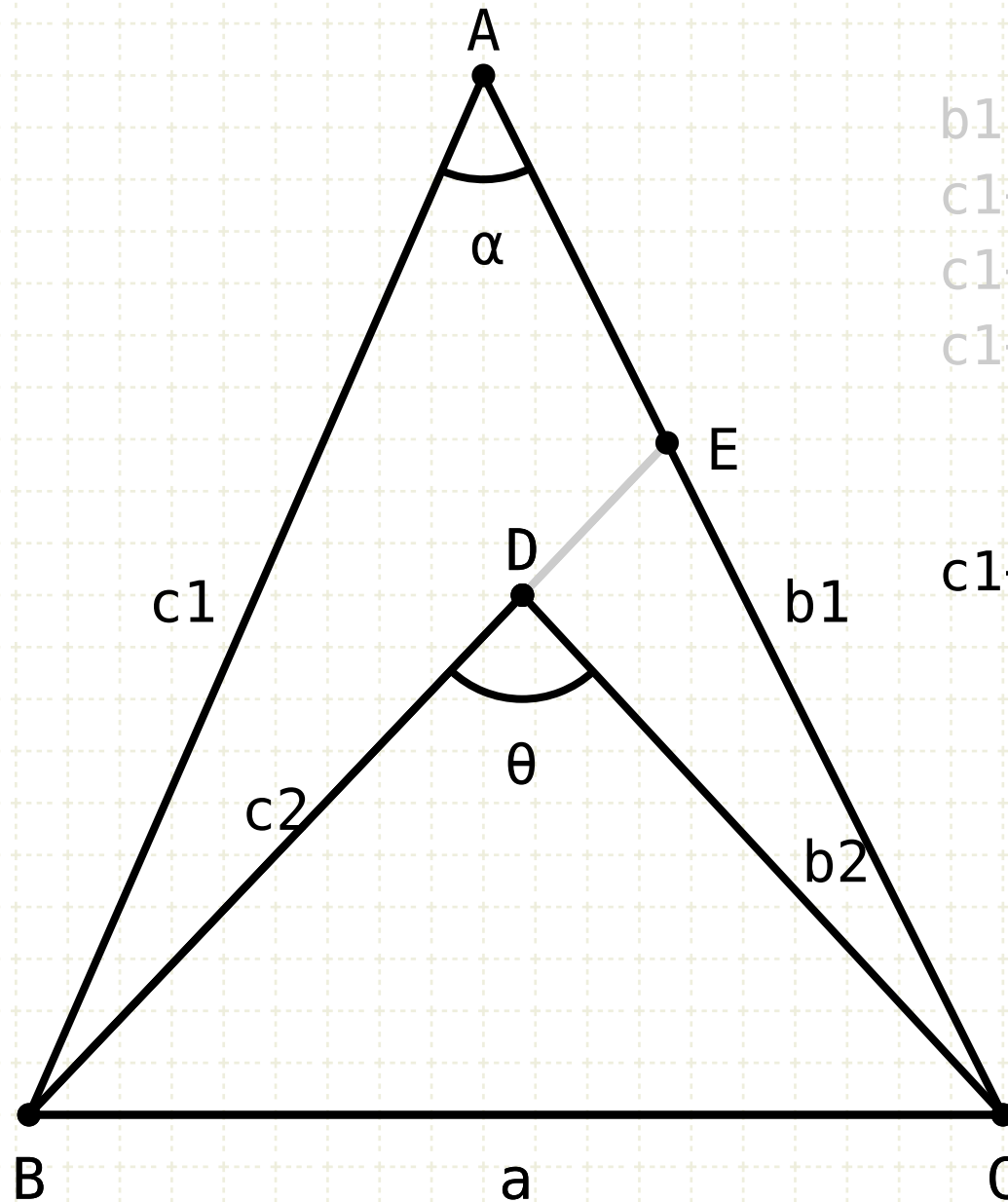
Add BD to both sides of the inequality

Thus, the sum of AB and AC is greater than the sum of DB and DC



Proposition 21 of Book I

If from the ends of one of the sides of a triangle two straight lines are constructed meeting within the triangle, then the sum of the straight lines so constructed is less than the sum of the remaining two sides of the triangle, but the constructed straight lines contain a greater angle than the angle contained by the remaining two sides.



$$\begin{aligned}
 b_1 &= b_3 + b_4 \\
 c_1 + b_3 &> c_2 + c_3 \\
 c_1 + b_3 + b_4 &> c_2 + c_3 + b_4 \\
 c_1 + b_1 &> c_2 + c_3 + b_4 \\
 &\qquad c_3 + b_4 > b_2 \\
 c_2 + c_3 + b_4 &> c_2 + b_2 \\
 c_1 + b_1 &> c_2 + b_2
 \end{aligned}$$

Proof

Extend BD such that it intersects AC at point E

Consider triangle ABE

The sum of lines AB and AE is greater than BE (I·18)

Add length EC to both each part of the inequality

Consider triangle DEC

The sum of lines DE and EC is greater than CD (I·18)

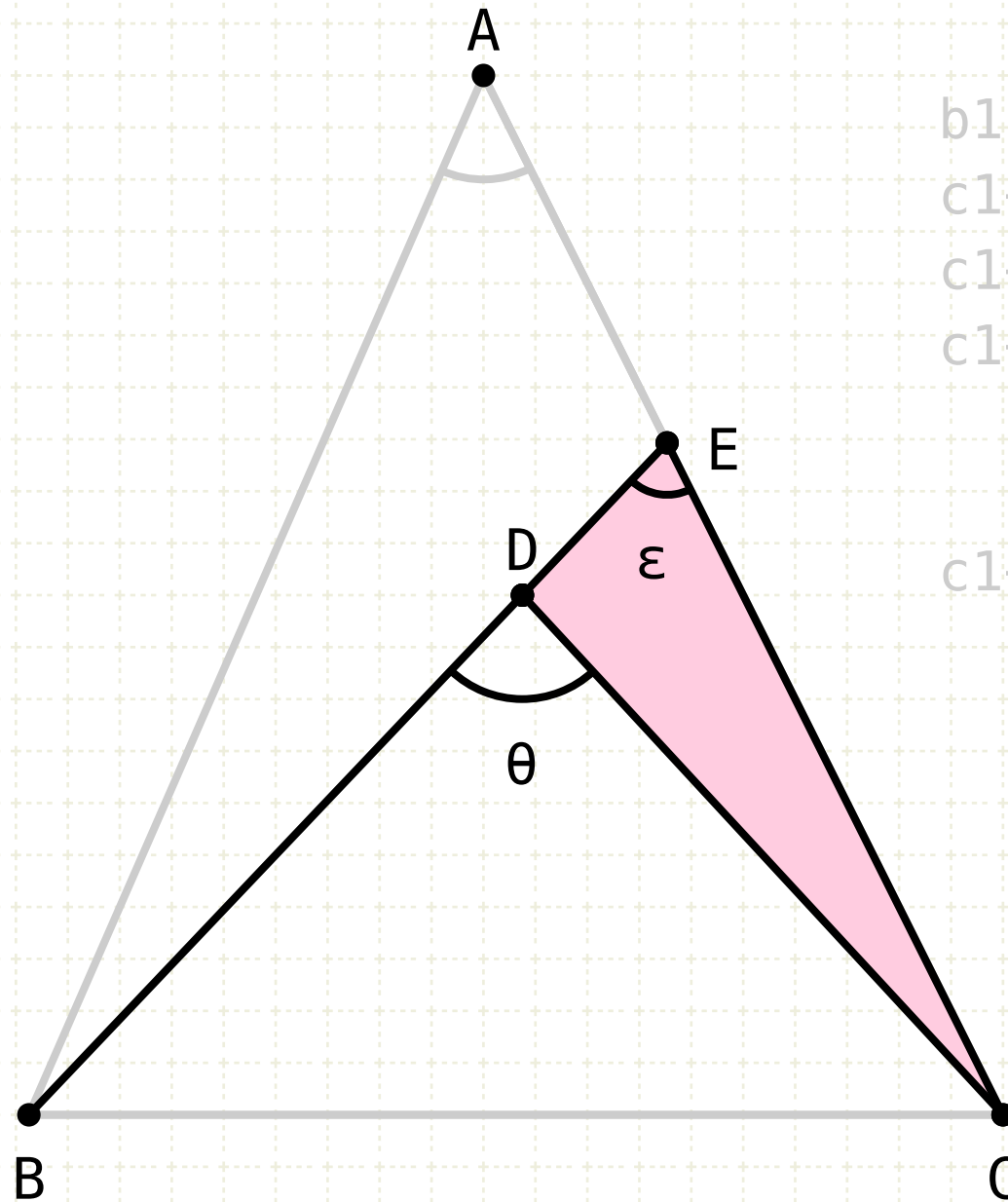
Add BD to both sides of the inequality

Thus, the sum of AB and AC is greater than the sum of DB and DC



Proposition 21 of Book I

If from the ends of one of the sides of a triangle two straight lines are constructed meeting within the triangle, then the sum of the straight lines so constructed is less than the sum of the remaining two sides of the triangle, but the constructed straight lines contain a greater angle than the angle contained by the remaining two sides.



$$\begin{aligned} b_1 &= b_3 + b_4 \\ c_1 + b_3 &> c_2 + c_3 \\ c_1 + b_3 + b_4 &> c_2 + c_3 + b_4 \\ c_1 + b_1 &> c_2 + c_3 + b_4 \\ &\qquad c_3 + b_4 > b_2 \\ c_2 + c_3 + b_4 &> c_2 + b_2 \\ c_1 + b_1 &> c_2 + b_2 \\ \theta &> \epsilon \end{aligned}$$

Proof

Extend BD such that it intersects AC at point E

Consider triangle ABE

The sum of lines AB and AE is greater than BE (I·18)

Add length EC to both each part of the inequality

Consider triangle DEC

The sum of lines DE and EC is greater than CD (I·18)

Add BD to both sides of the inequality

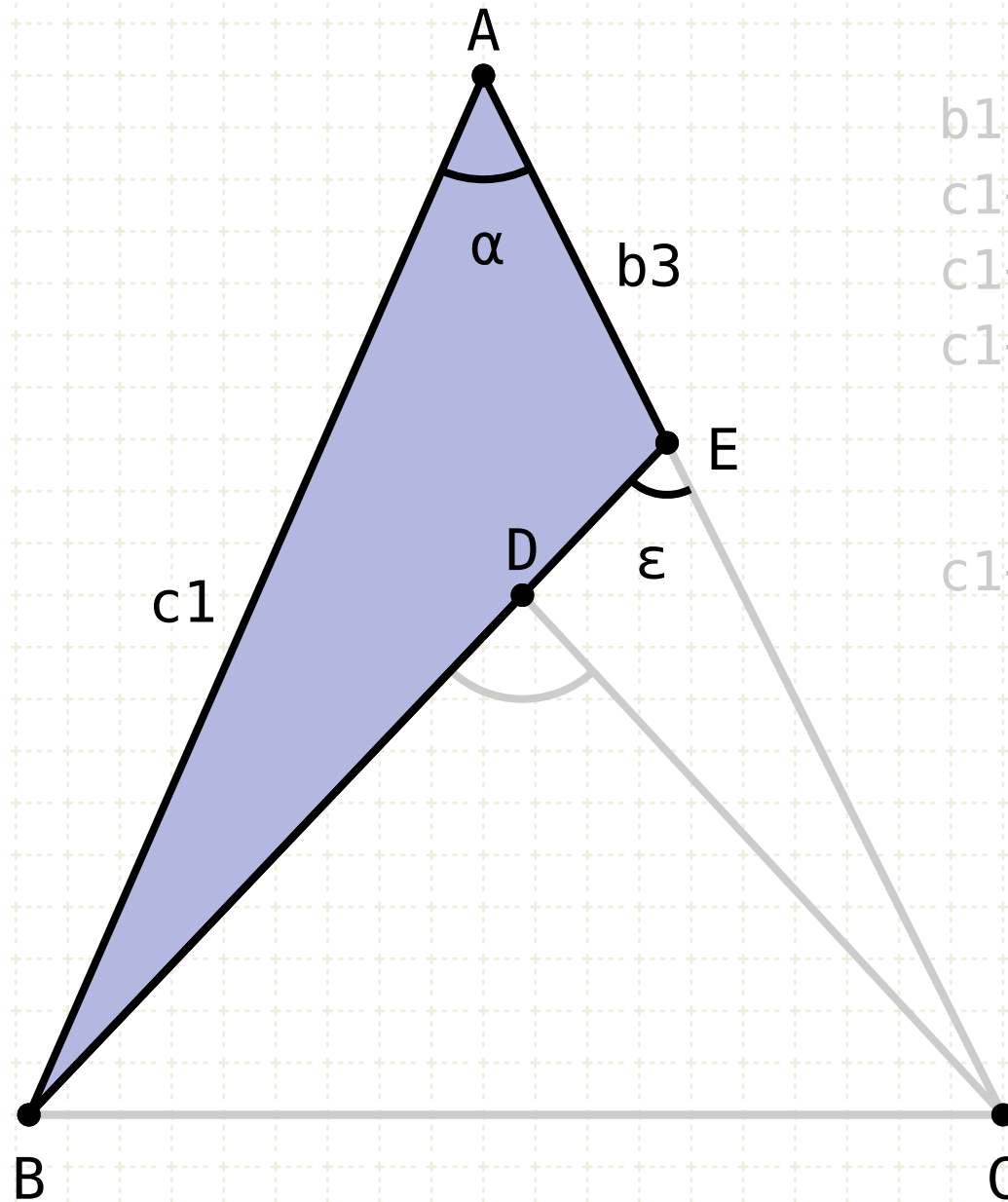
Thus, the sum of AB and AC is greater than the sum of DB and DC

Angle BDC is an exterior angle to triangle DCE, hence it is larger than the angle DEC (I·16)



Proposition 21 of Book I

If from the ends of one of the sides of a triangle two straight lines are constructed meeting within the triangle, then the sum of the straight lines so constructed is less than the sum of the remaining two sides of the triangle, but the constructed straight lines contain a greater angle than the angle contained by the remaining two sides.



$$b1 = b3 + b4$$

$$c1 + b3 > c2 + c3$$

$$c1 + b3 + b4 > c2 + c3 + b4$$

$$c1 + b1 > c2 + c3 + b4$$

$$c3 + b4 > b2$$

$$c2 + c3 + b4 > c2 + b2$$

$$c1 + b1 > c2 + b2$$

$$\theta > \epsilon$$

$$\epsilon > \alpha$$

Proof

Extend BD such that it intersects AC at point E

Consider triangle ABE

The sum of lines AB and AE is greater than BE (I·18)

Add length EC to both each part of the inequality

Consider triangle DEC

The sum of lines DE and EC is greater than CD (I·18)

Add BD to both sides of the inequality

Thus, the sum of AB and AC is greater than the sum of DB and DC

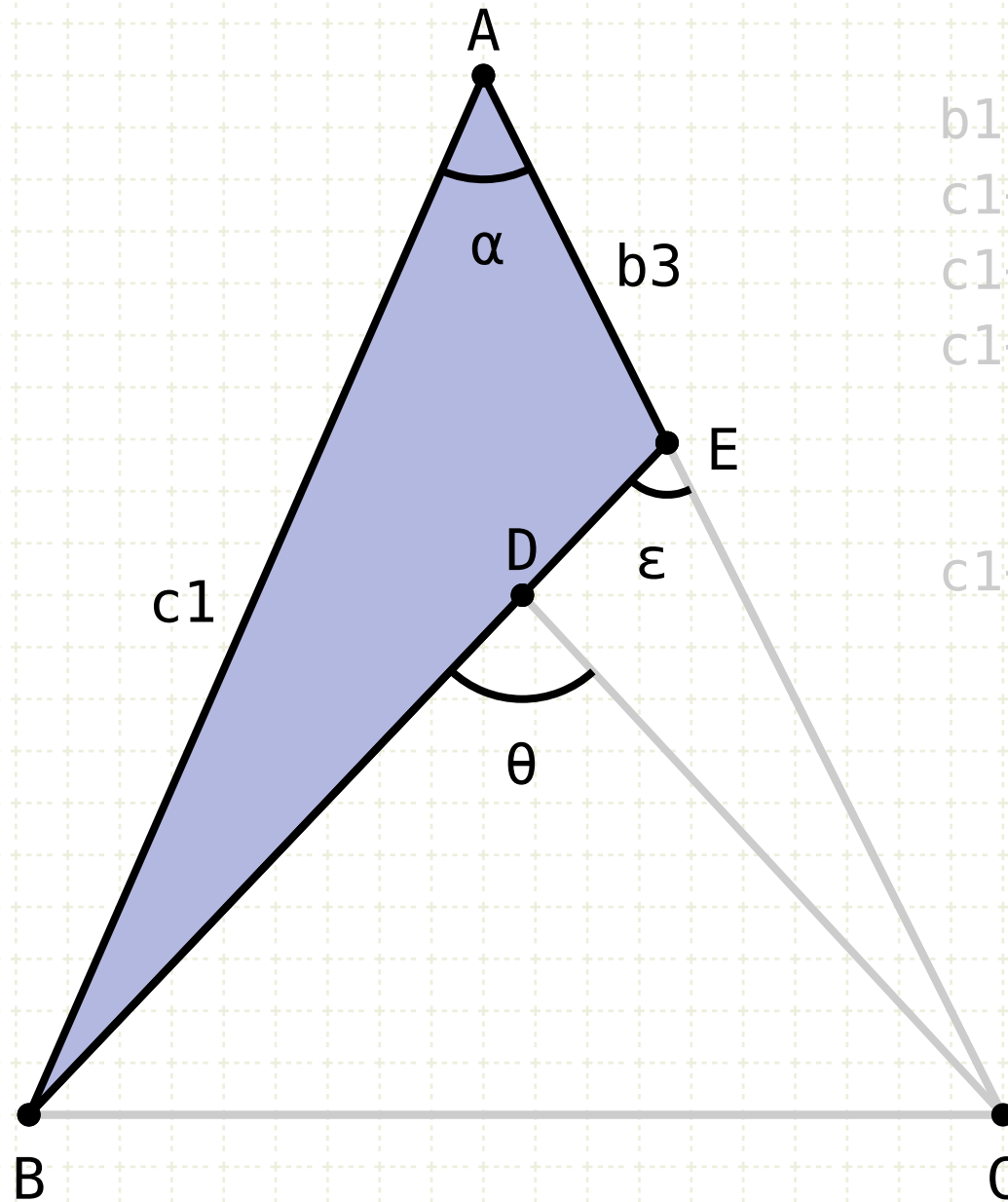
Angle BDC is an exterior angle to triangle DCE, hence it is larger than the angle DEC (I·16)

Angle DEC is an exterior angle to triangle EAB, hence it is larger than the angle EAB (I·16)



Proposition 21 of Book I

If from the ends of one of the sides of a triangle two straight lines are constructed meeting within the triangle, then the sum of the straight lines so constructed is less than the sum of the remaining two sides of the triangle, but the constructed straight lines contain a greater angle than the angle contained by the remaining two sides.



$$\begin{aligned}
 b1 &= b3 + b4 \\
 c1 + b3 &> c2 + c3 \\
 c1 + b3 + b4 &> c2 + c3 + b4 \\
 c1 + b1 &> c2 + c3 + b4 \\
 c3 + b4 &> b2 \\
 c2 + c3 + b4 &> c2 + b2 \\
 c1 + b1 &> c2 + b2 \\
 \theta &> \epsilon \\
 \epsilon &> \alpha \\
 \theta &> \epsilon > \alpha \\
 \theta &> \alpha
 \end{aligned}$$

Proof

Extend BD such that it intersects AC at point E

Consider triangle ABE

The sum of lines AB and AE is greater than BE (I·18)

Add length EC to both each part of the inequality

Consider triangle DEC

The sum of lines DE and EC is greater than CD (I·18)

Add BD to both sides of the inequality

Thus, the sum of AB and AC is greater than the sum of DB and DC

Angle BDC is an exterior angle to triangle DCE, hence it is larger than the angle DEC (I·16)

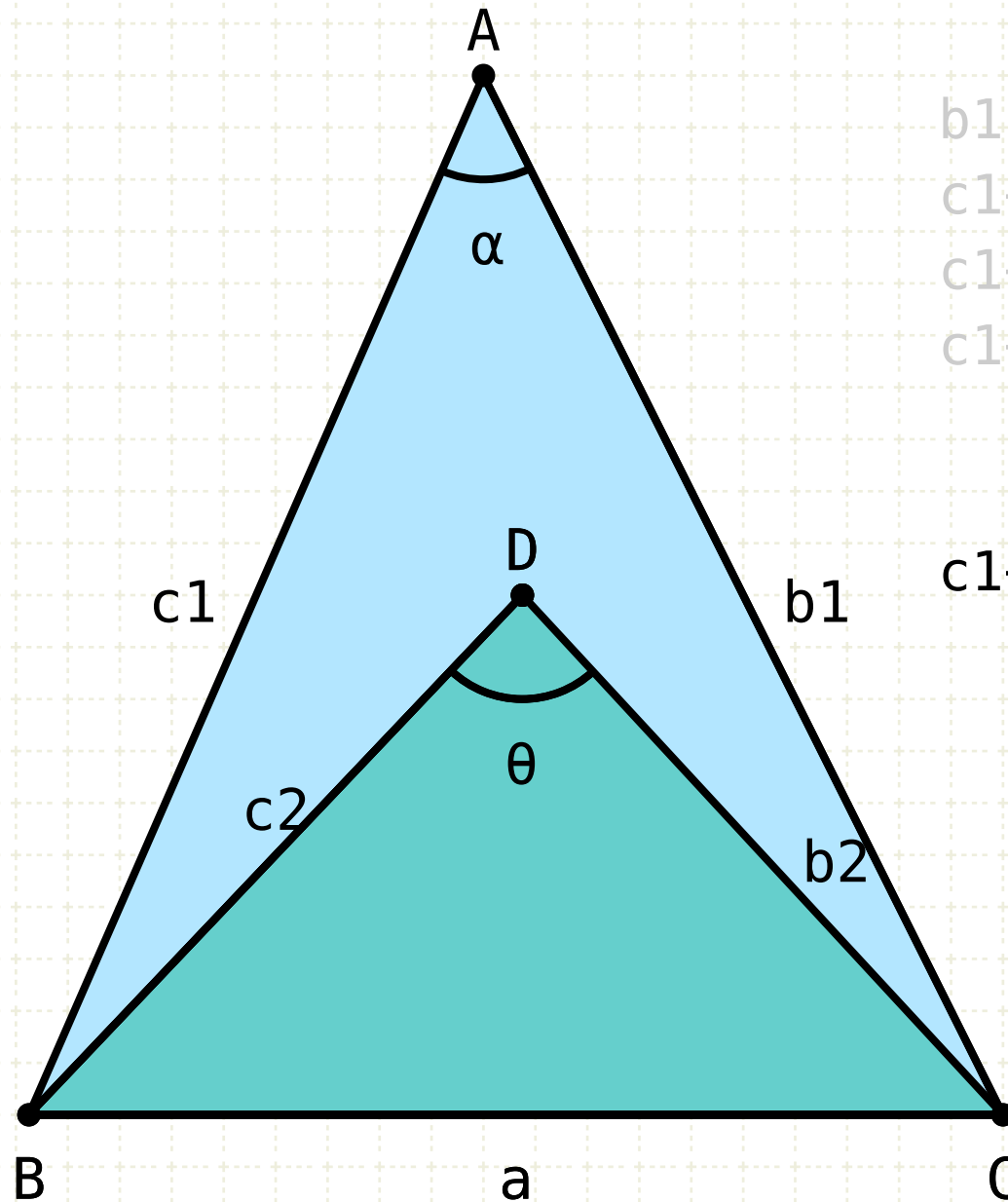
Angle DEC is an exterior angle to triangle EAB, hence it is larger than the angle EAB (I·16)

Thus, angle BDC is greater than angle ABC



Proposition 21 of Book I

If from the ends of one of the sides of a triangle two straight lines are constructed meeting within the triangle, then the sum of the straight lines so constructed is less than the sum of the remaining two sides of the triangle, but the constructed straight lines contain a greater angle than the angle contained by the remaining two sides.



$$\begin{aligned}
 b_1 &= b_3 + b_4 \\
 c_1 + b_3 &> c_2 + c_3 \\
 c_1 + b_3 + b_4 &> c_2 + c_3 + b_4 \\
 c_1 + b_1 &> c_2 + c_3 + b_4 \\
 c_3 + b_4 &> b_2 \\
 c_2 + c_3 + b_4 &> c_2 + b_2 \\
 c_1 + b_1 &> c_2 + b_2 \\
 \theta &> \varepsilon \\
 \varepsilon &> \alpha \\
 \theta &> \varepsilon > \alpha \\
 \theta &> \alpha
 \end{aligned}$$

Proof

Extend BD such that it intersects AC at point E

Consider triangle ABE

The sum of lines AB and AE is greater than BE (I·18)

Add length EC to both each part of the inequality

Consider triangle DEC

The sum of lines DE and EC is greater than CD (I·18)

Add BD to both sides of the inequality

Thus, the sum of AB and AC is greater than the sum of DB and DC

Angle BDC is an exterior angle to triangle DCE, hence it is larger than the angle DEC (I·16)

Angle DEC is an exterior angle to triangle EAB, hence it is larger than the angle EAB (I·16)

Thus, angle BDC is greater than angle ABC



Youtube Videos

<https://www.youtube.com/c/SandyBultena>

Copyright © 2019 by Sandy Bultena.



Except where otherwise noted, this work is licensed under
<http://creativecommons.org/licenses/by-nc/3.0>