

# Euclid's Elements

## Book III



*A circle is a round straight line with a hole in the middle.*

**Mark Twain**

quoting a schoolchild in "-English as She Is Taught-"

*If people stand in a circle long enough, they'll eventually begin to dance.*

**George Carlin, Napalm and Silly Putty (2001)**



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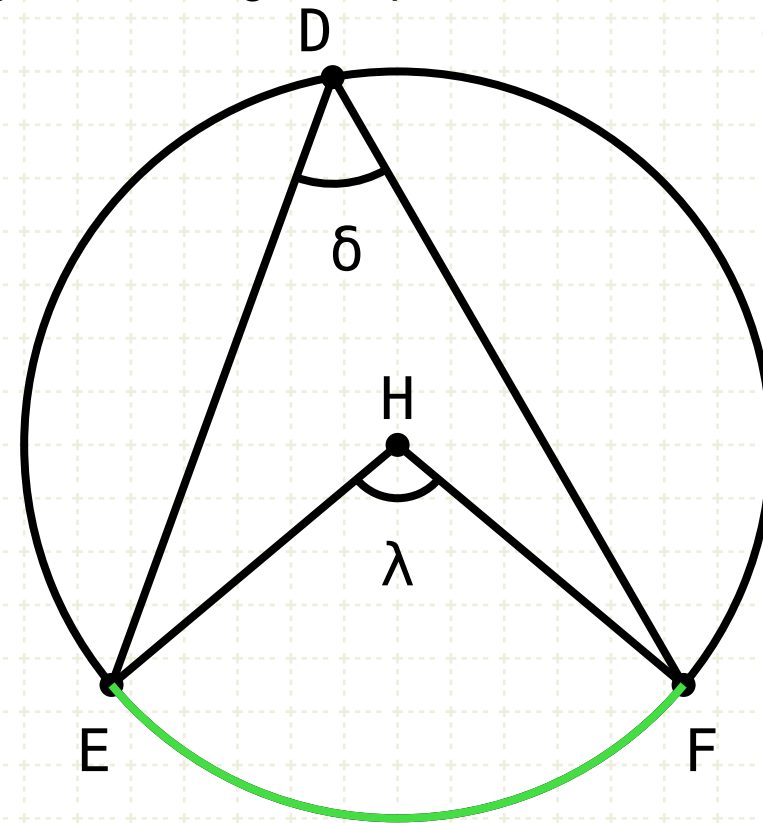
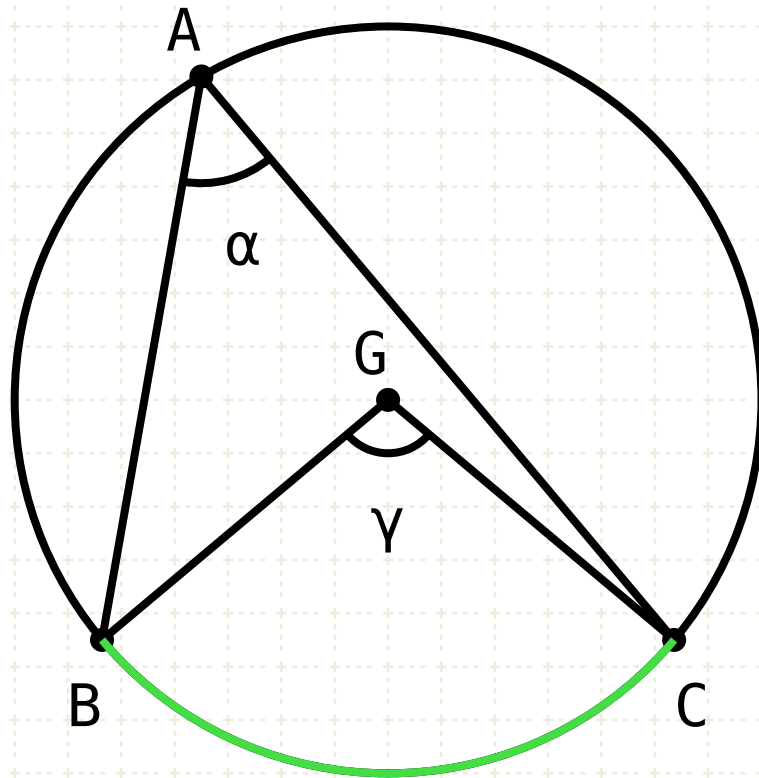
# Proposition 27 of Book III

In equal circles angles standing on equal circumferences are equal to one another, whether they stand at the centres or at the circumferences.



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## In other words

Given two equal circles (as shown)

If the circumference BC equals the circumference EF, then

$\alpha$  equals  $\delta$  and  $\gamma$  equals  $\lambda$

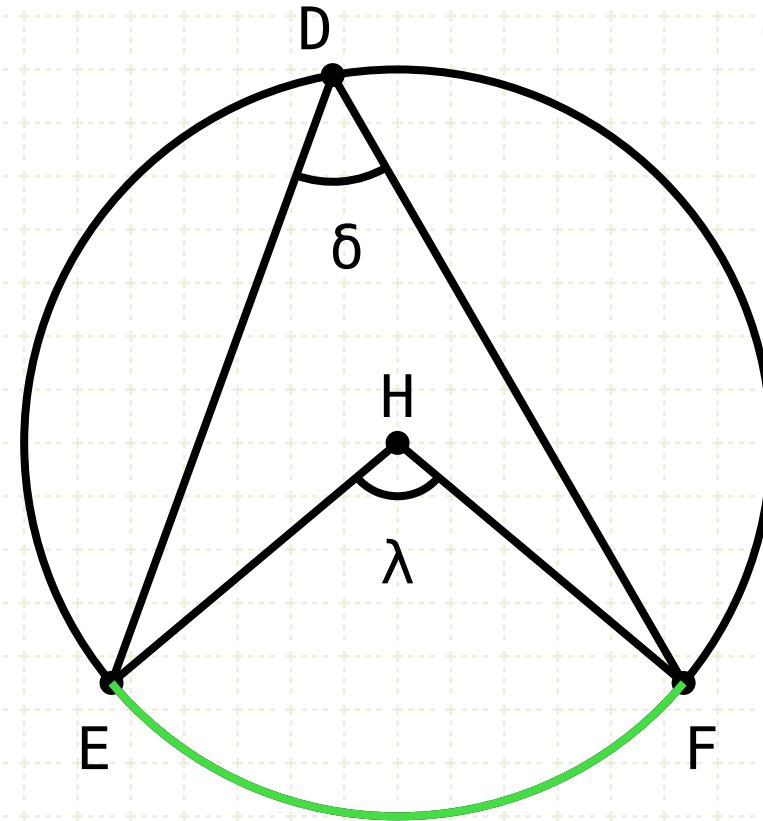
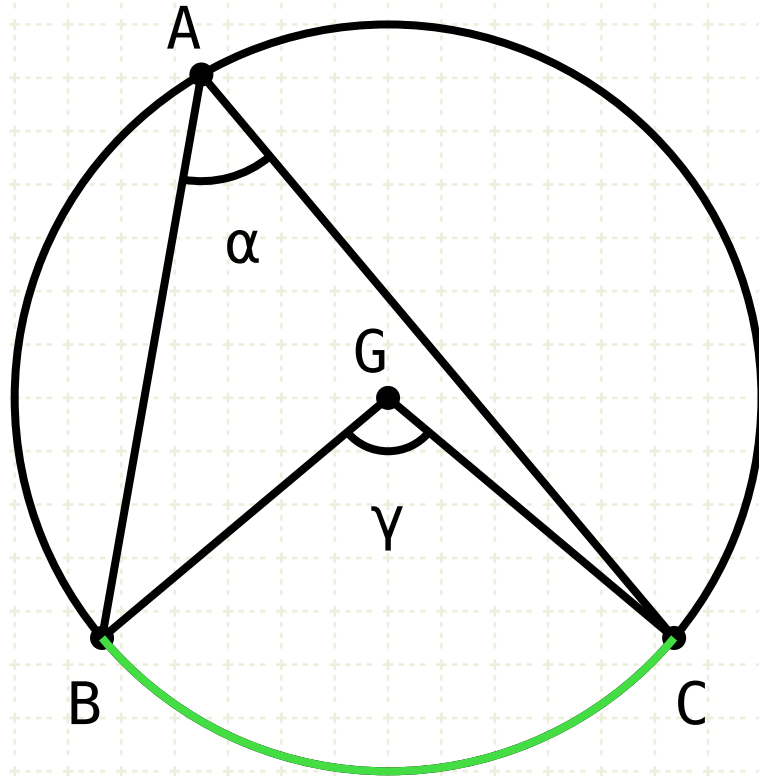
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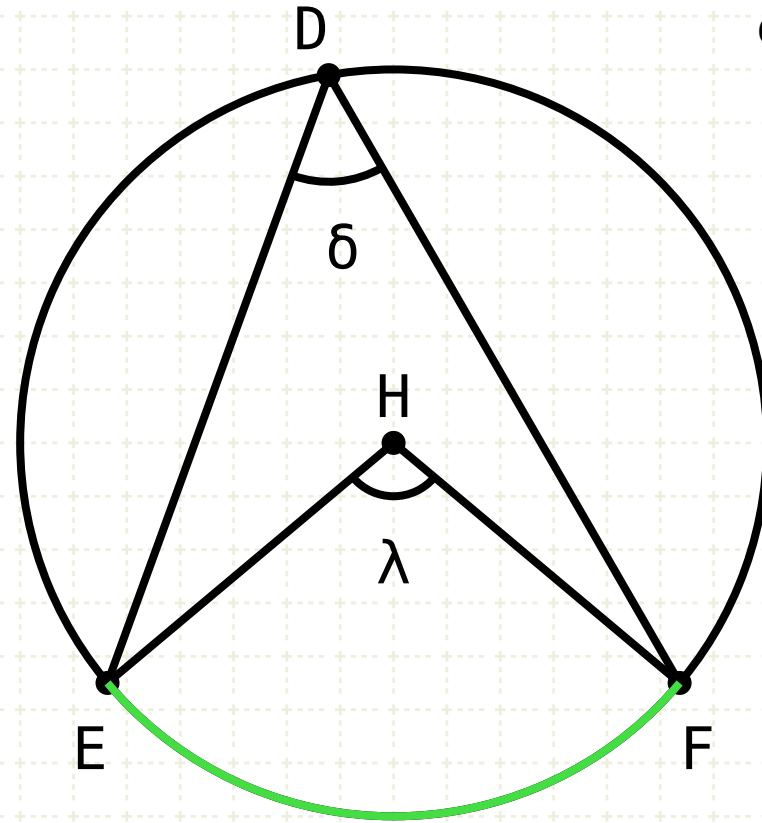
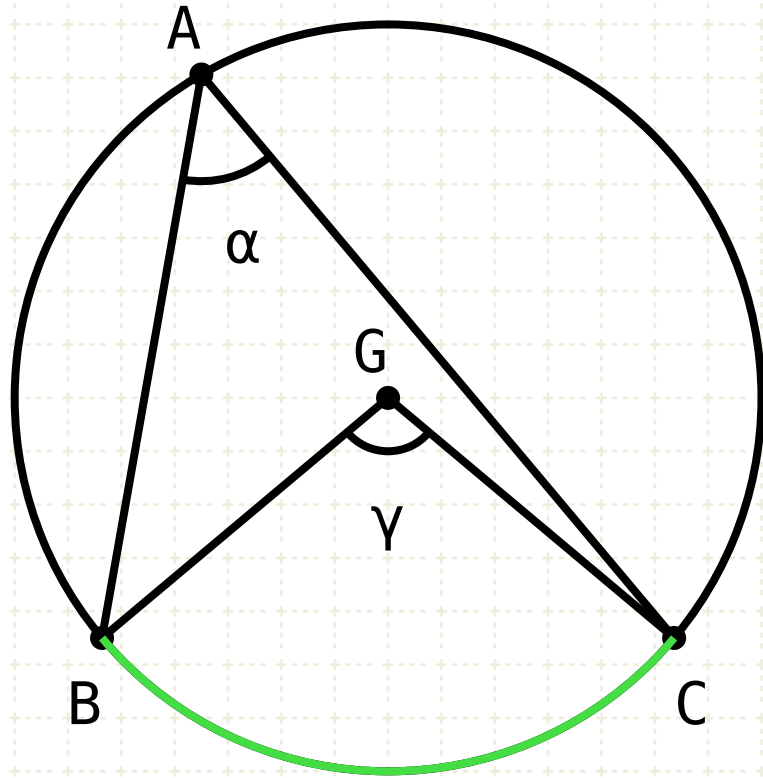
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### Proof by Contradiction

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$$\sphericalangle ABC = \sphericalangle EDF$$
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$$\gamma > \lambda$$

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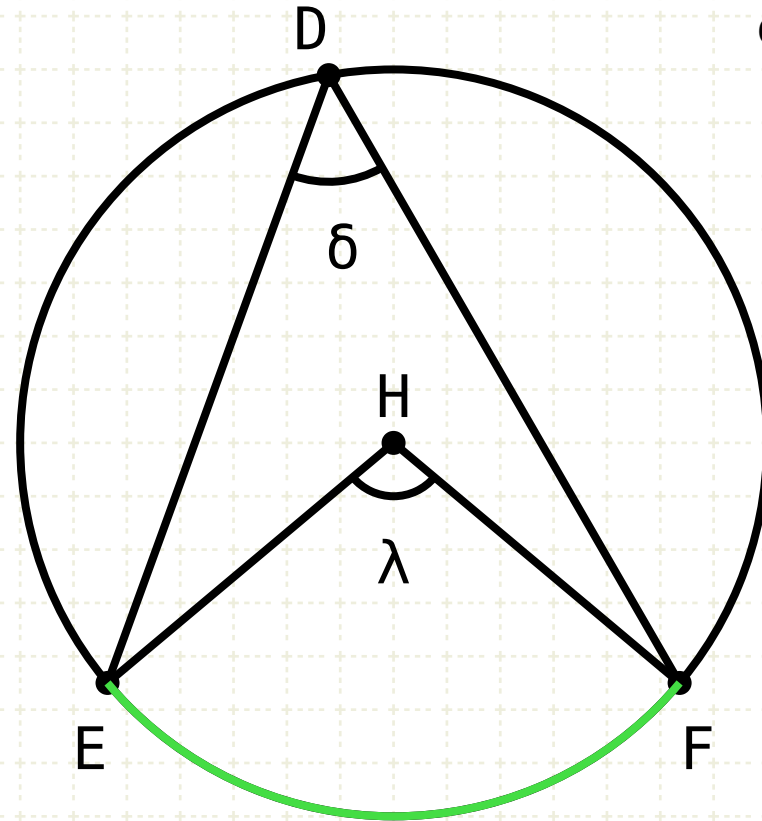
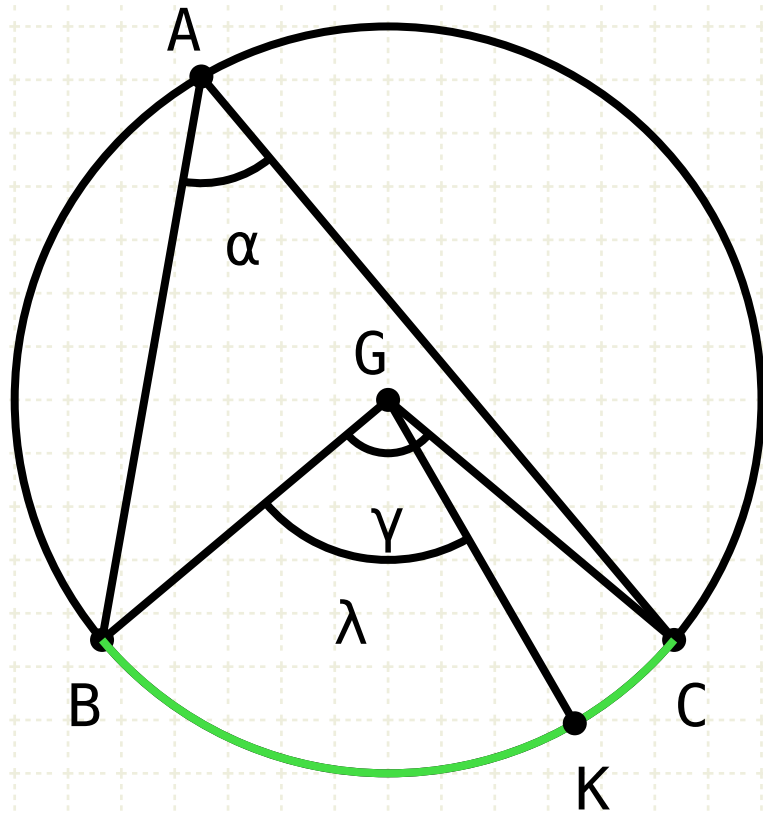
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Assume that  $\gamma$  is larger than  $\lambda$

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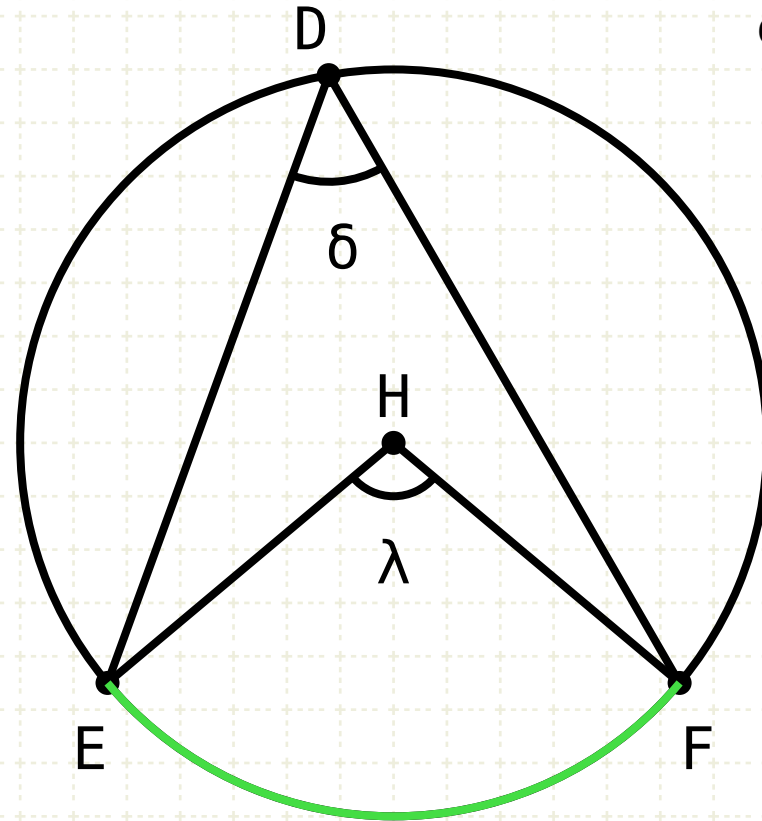
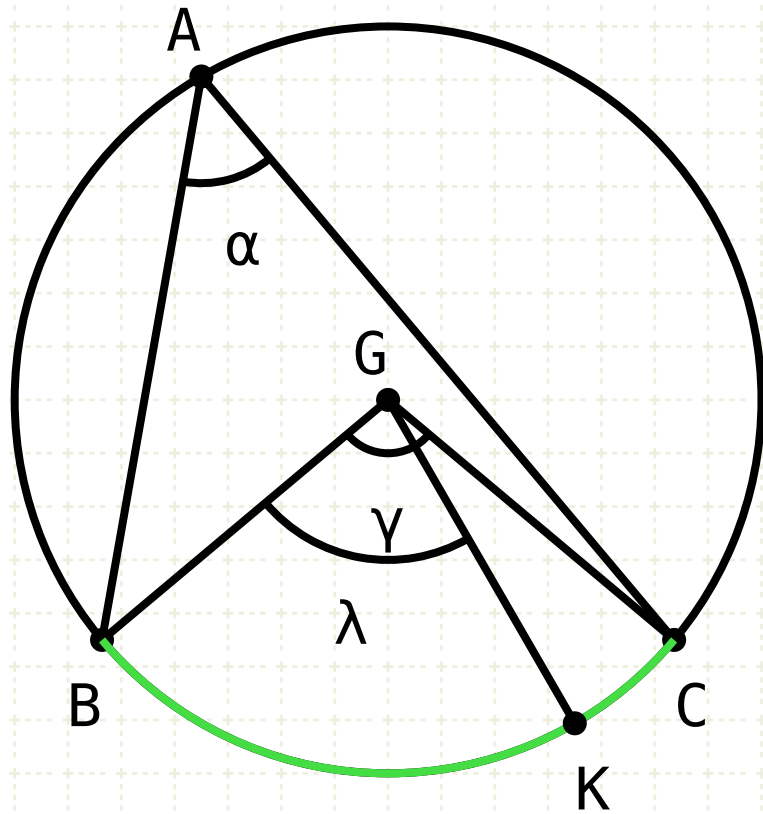
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If angle BGK is equal to EHF, then the circumferences subtended by these angles are also equal (III·26), in other words BK equals EF

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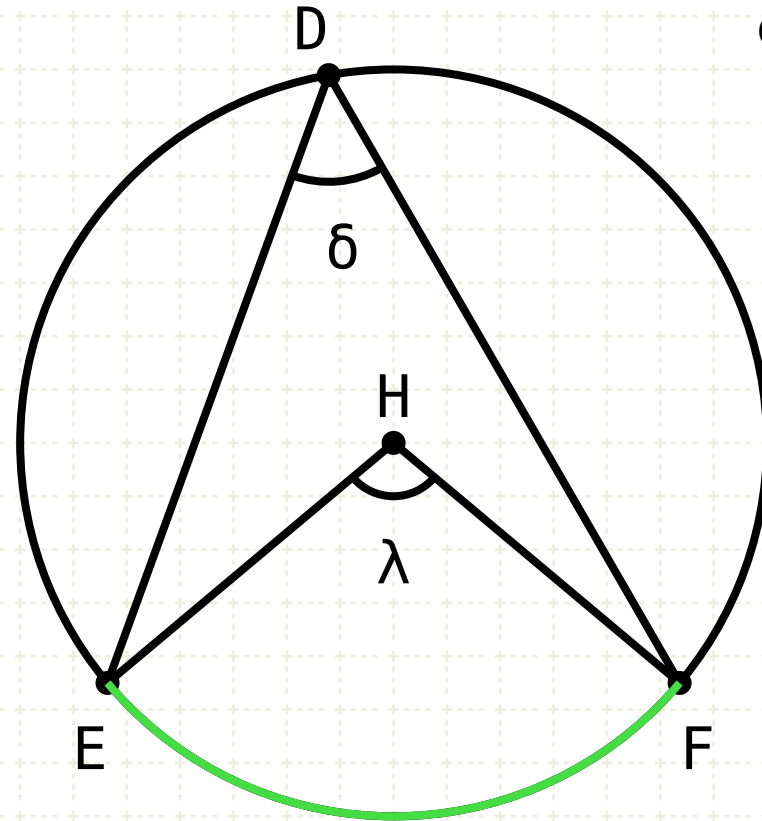
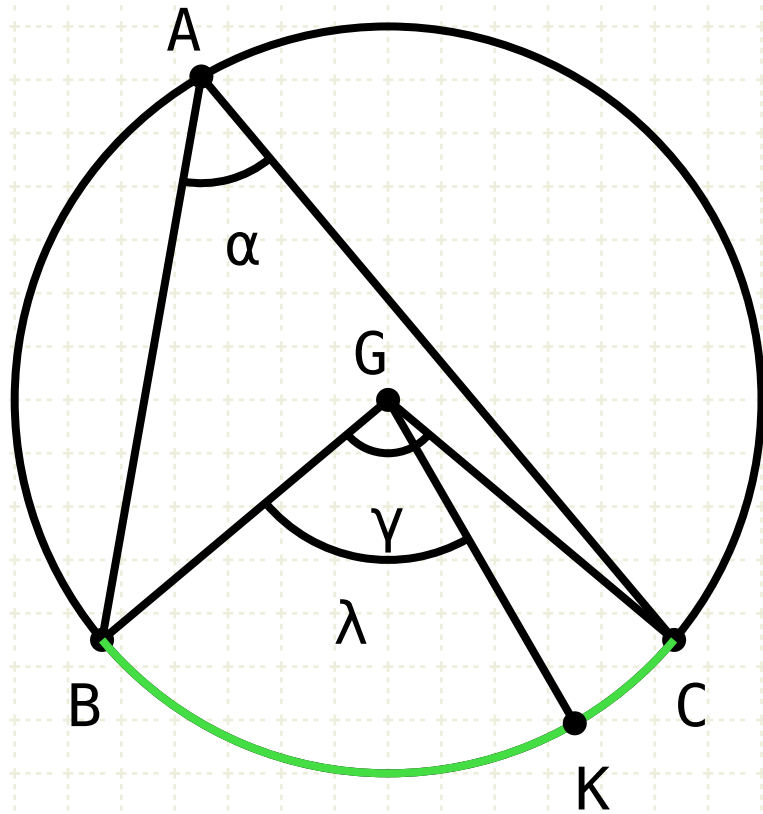
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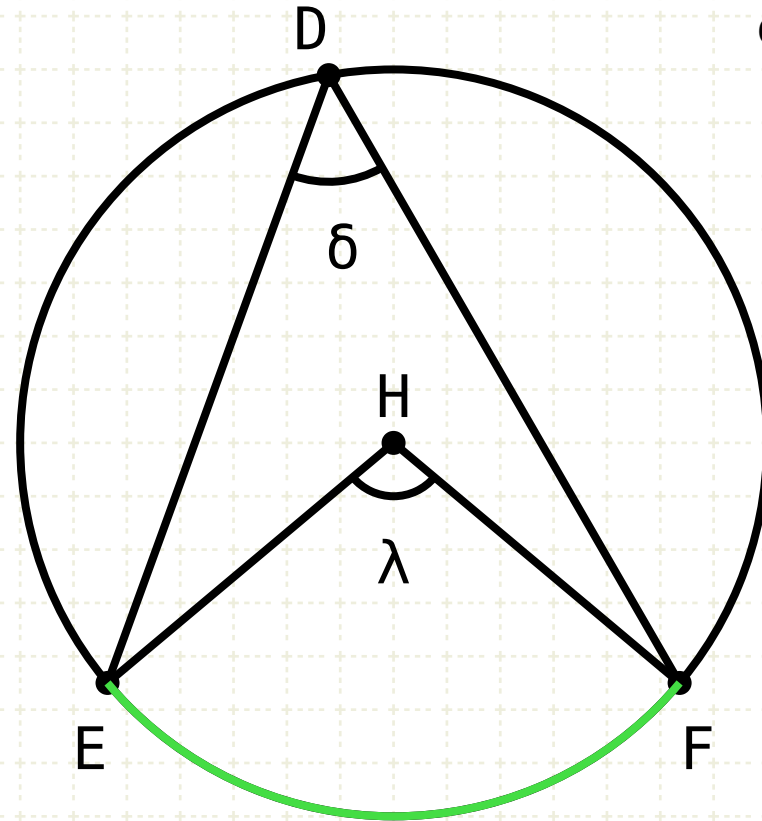
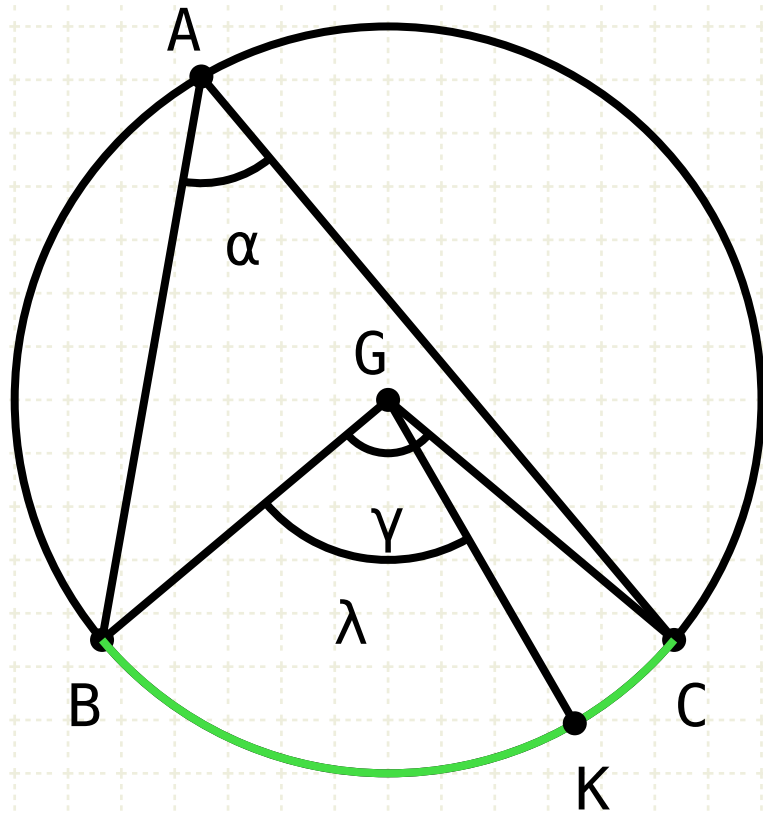
Construct angle BGK such that it equals  $\lambda$  (I-23)

If angle BGK is equal to EHF, then the circumferences subtended by these angles are also equal (III-26), in other words BK equals EF

But EF equals BC, therefore BK equals BC, which is impossible

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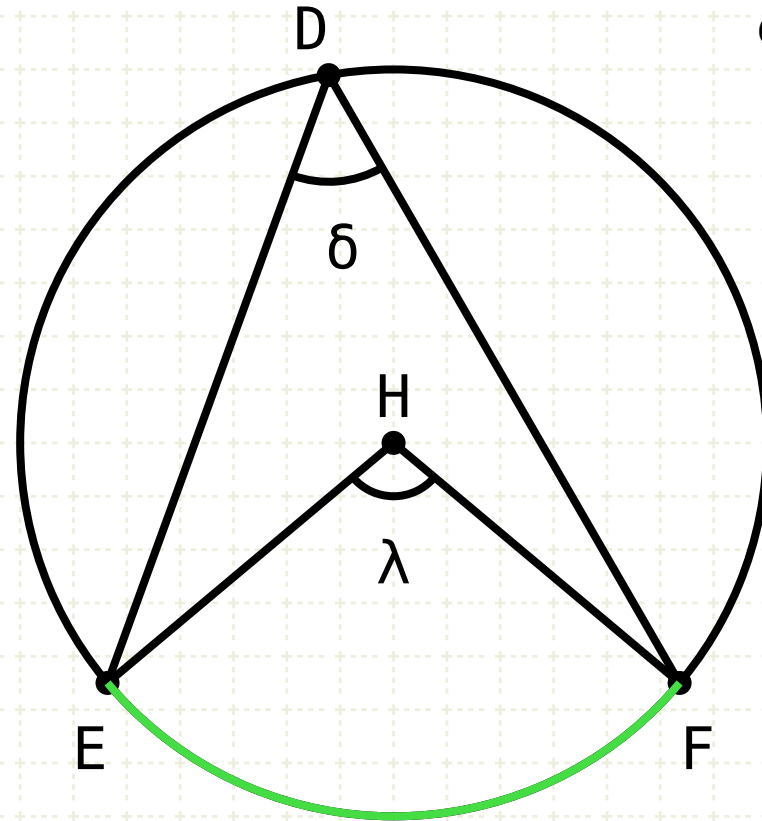
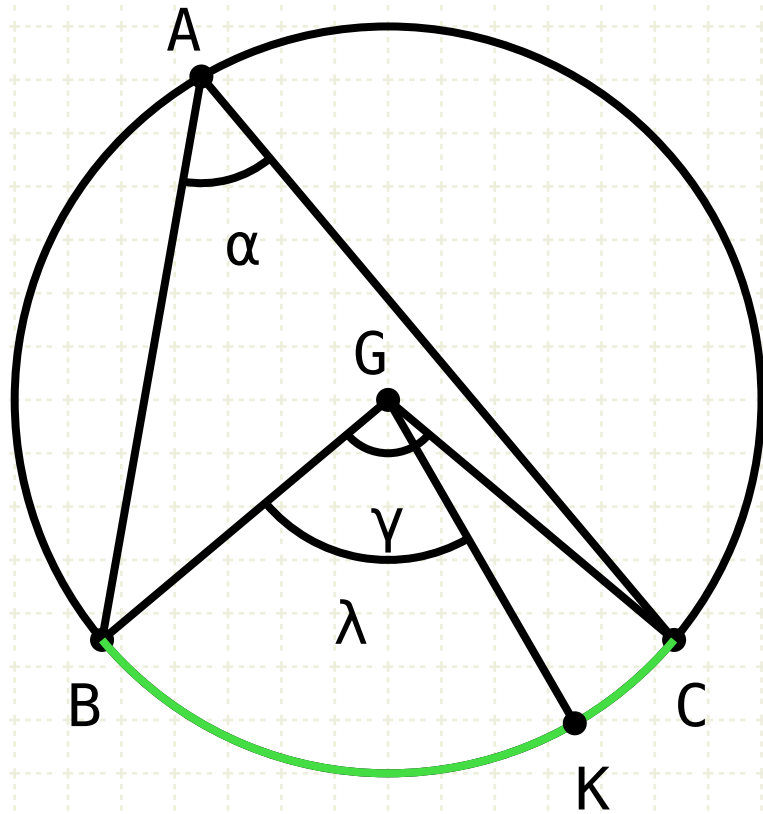
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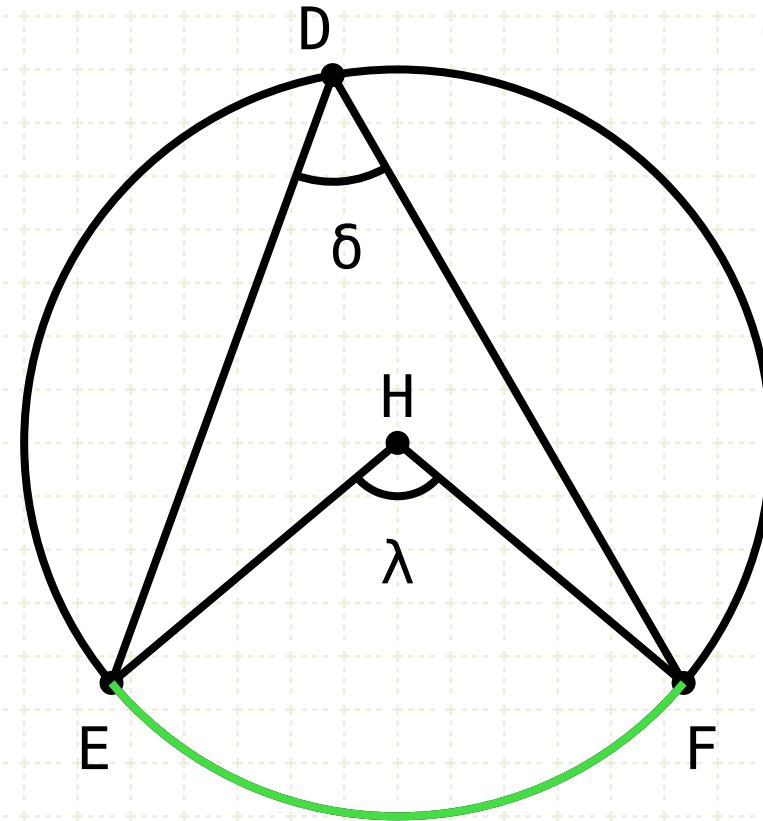
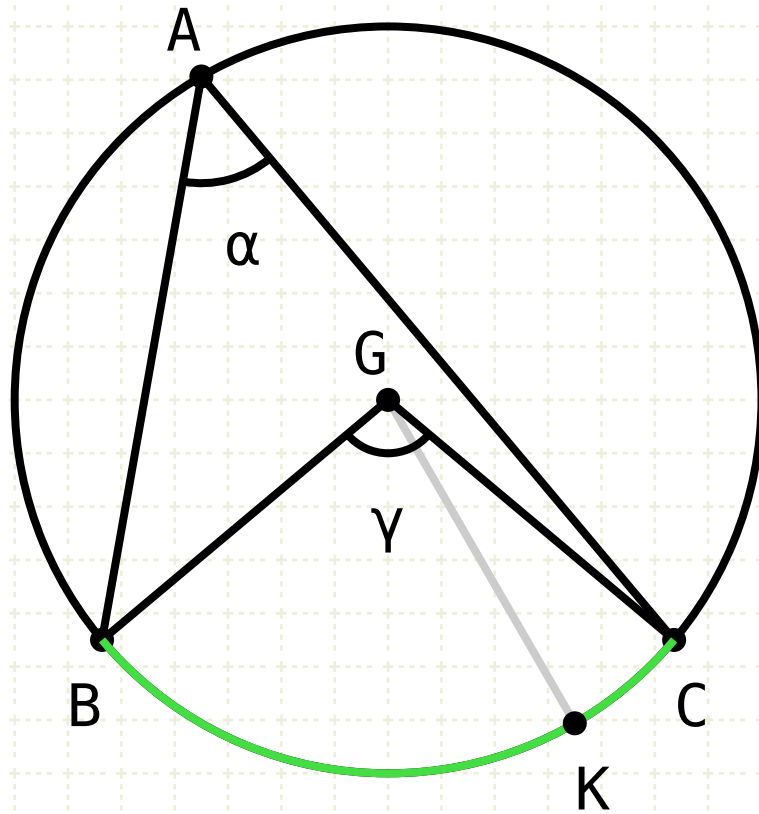
But EF equals BC, therefore BK equals BC, which is impossible

Therefore the original assumption is wrong



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$$\begin{aligned}\odot ABC &= \odot EDF \\ \frown BC &= \frown EF\end{aligned}$$

$$\begin{aligned}\gamma &> \lambda \\ \frown BC &> \frown BK \\ \frown BK &= \frown EF \\ \frown BK &= \frown BC \\ \therefore \gamma &= \lambda\end{aligned}$$

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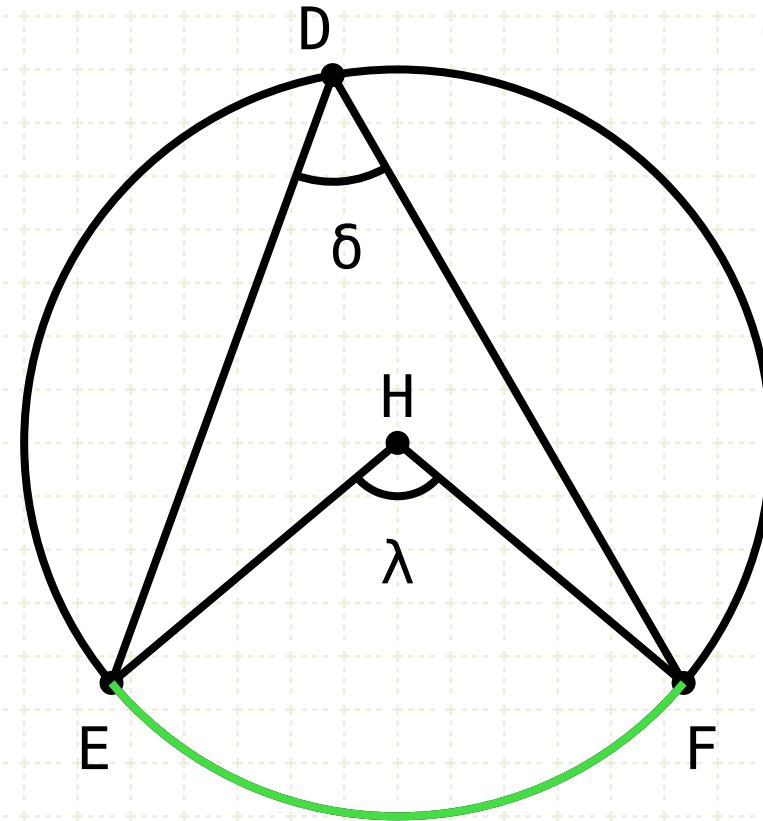
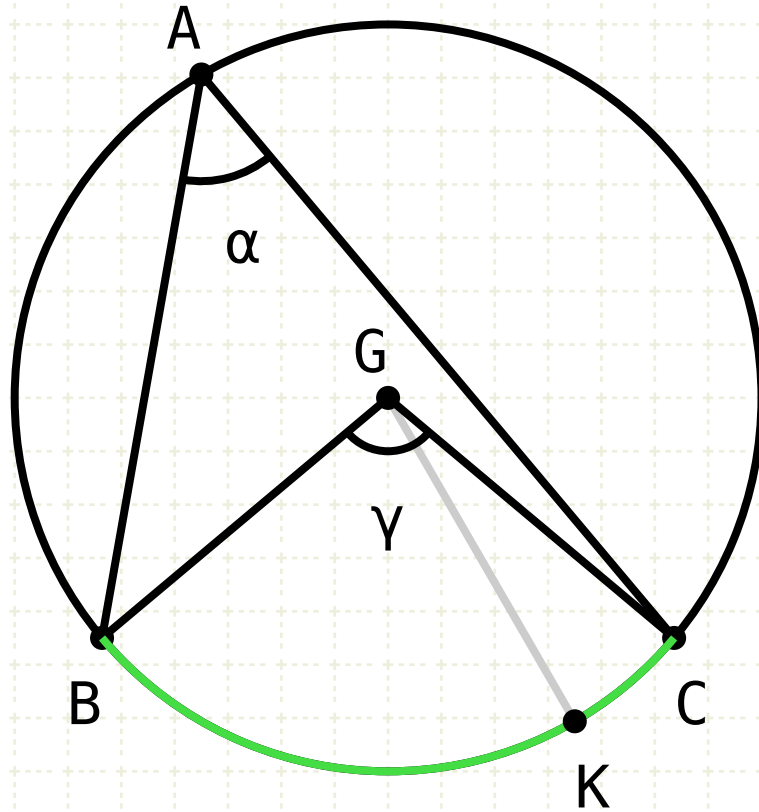
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Therefore the angles BGC equals EHF



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$$\begin{aligned}\therefore \gamma &= \lambda \\ \alpha &= \frac{1}{2} \gamma \\ \delta &= \frac{1}{2} \lambda\end{aligned}$$

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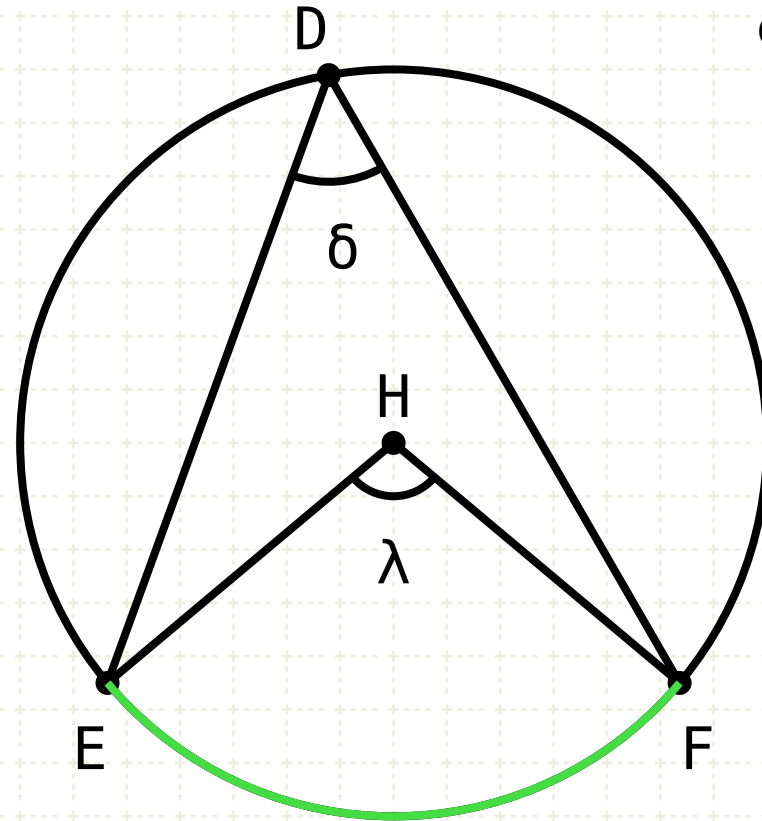
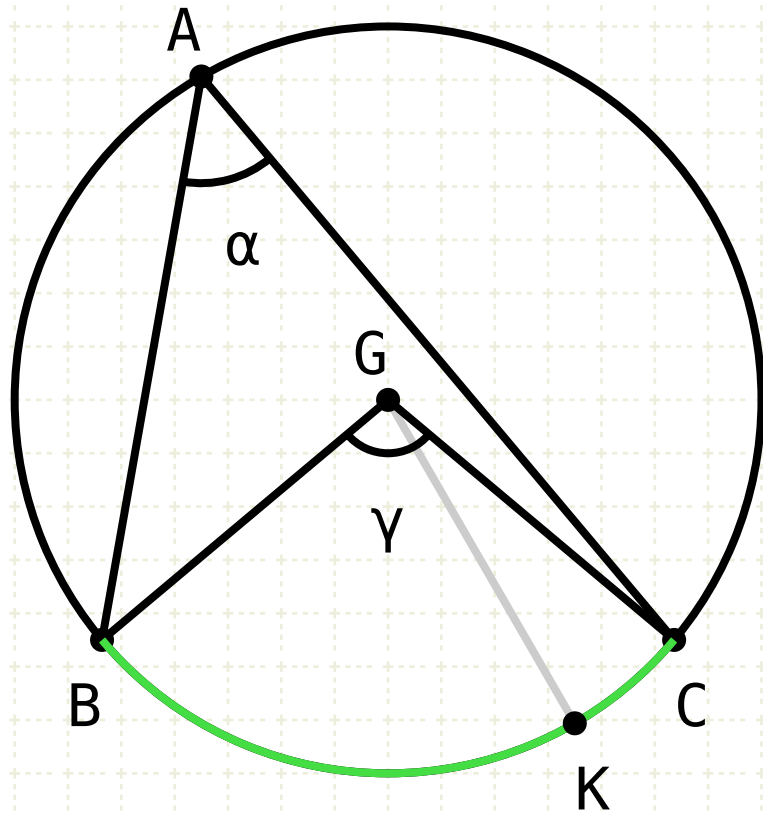
Therefore the angles BGC equals EHF

The angle at the circumference is half the angle at the centre of a circle if the base is the same (III·20) therefore  $\alpha$  is half  $\gamma$  and  $\delta$  is half  $\lambda$



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$$\gamma > \lambda$$

$$\frown BC > \frown BK$$

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$$\frown BK = \frown BC$$

$$\therefore \gamma = \lambda$$

$$\alpha = \frac{1}{2} \gamma$$

$$\delta = \frac{1}{2} \lambda$$

$$\alpha = \delta$$

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Therefore the angles BGC equals EHF

The angle at the circumference is half the angle at the centre of a circle if the base is the same (III-20) therefore  $\alpha$  is half  $\gamma$  and  $\delta$  is half  $\lambda$

Then since  $\gamma$  is equal to  $\lambda$ ,  $\alpha$  is equal to  $\delta$



# Youtube Videos

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