

# Euclid's Elements

## Book VI

*One can state, without exaggeration, that the observation of and the search for similarities and differences are the basis of all human knowledge.*

**Alfred Nobel**



# Table of Contents, Chapter 6

1	If the height of two triangles are equal, then the ratio of the areas is equal to the ratio of the bases	7	If two triangles have one angle equal to one angle, and the sides about other angles are proportional, and the remaining angles either both less or both not less than a right angle, then triangles will be equiangular	14	In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and vice versa
2	If a line cuts a triangle, parallel to its base, it will cut the sides of the triangle proportionally			15	In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and vice versa
3	If an angle of a triangle is bisected and the straight line cutting the angle also cuts the base, the segments of the base will have the same ratio as the remaining sides of the triangle	8	If in a right-angled triangle a perpendicular be drawn from the right angle to the base, the triangles adjoining the perpendicular are similar both to the whole and to one another	16	If four straight lines are proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means, and vice versa
4	If two triangles have equal angles, then the sides opposite the equal angles are proportional, as well, the sides of the triangles on either side of the equal angles are also proportional	9	From a given straight line to cut off a given fraction	17	If three straight lines are proportional, the rectangle contained by the extremes is equal to the square on the mean; and vice versa
5	If two triangles have proportional sides, the triangles will be equiangular	10	To cut a given uncut straight line similarly to a given cut straight line	18	On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure
6	If two triangles have one angle equal to one angle and the sides about the equal angles are proportional, then the triangles will be equiangular	11	To two given straight lines to find a third proportional	19	<b>Similar triangles are to one another in the duplicate ratio of the corresponding sides</b>
		12	To three given straight lines to find a fourth proportional		
		13	To two given straight lines to find a mean proportional		



# Table of Contents, Chapter 3

20	Similar polygons are divided into the same number of similar triangles, which have the same ratio as the wholes, and the polygons have duplicate ratios to their corresponding sides	26	If from a parallelogram a similar parallelogram with a common angle is subtracted, it is about the same diameter as the original	31	In right-angled triangles the figure on the side subtending the right angle is equal to the similar and similarly described figures on the sides containing the right angle
21	Figures which are are similar to the same rectilineal figure are also similar to one another	27	Of all the parallelograms applied to the same straight line and deficient by parallelogrammic figures similar to a parallelogram drawn on half the said line, the largest will be one that is drawn on half of the straight line and is similar to the defect		
22	If four straight lines are proportional, similar rectilineal figures will also be proportional; and vice versa	28	To a given straight line, apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one		
23	Equiangular parallelograms have to one another the ratio compounded of the ratios of their sides	29	To a given straight line, apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one		
24	In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another	30	To cut a finite straight line in extreme ratio		
25	To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure				



# Proposition 19 of Book VI

Similar triangles are to one another in the duplicate ratio of the corresponding sides





# Proposition 19 of Book VI

Similar triangles are to one another in the duplicate ratio of the corresponding sides

## Definition - Duplicate Ratio (V.Def.9)

If A is to B as B is to C, then the ratio of A to C is the duplicate ratio of A to B

$A:B=B:C \rightarrow A:C$  duplicate ratio of  $A:B$

*Examples:*

$4:6 = 6:9$  (both equal 2:3)

$\therefore 4:9$  is duplicate ratio of 2:3

$4:10 = 10:25$  (both equal 2:5)

$\therefore 4:25$  is duplicate ratio of 2:5

*Fractions:*

$\frac{a}{b} = \frac{b}{c}, \quad \frac{a}{c} = ?$

$\frac{a}{b} \times \frac{a}{b} = \frac{b}{c} \times \frac{a}{b} \quad \frac{a \times a}{b \times b} = \frac{1}{c} \times \frac{a}{1} \quad \frac{a^2}{b^2} = \frac{a}{c}$

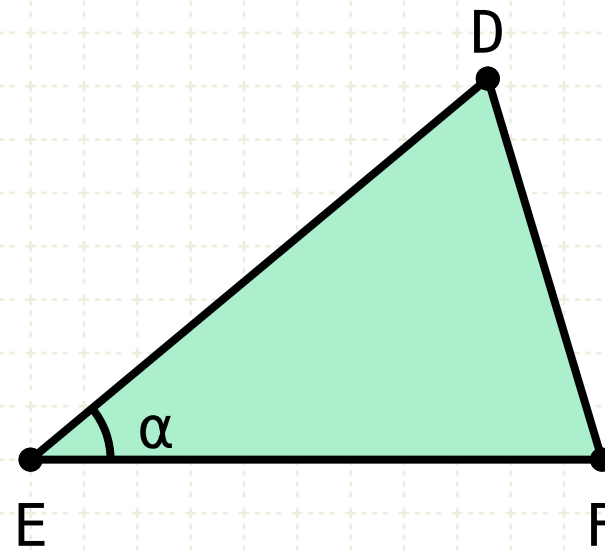
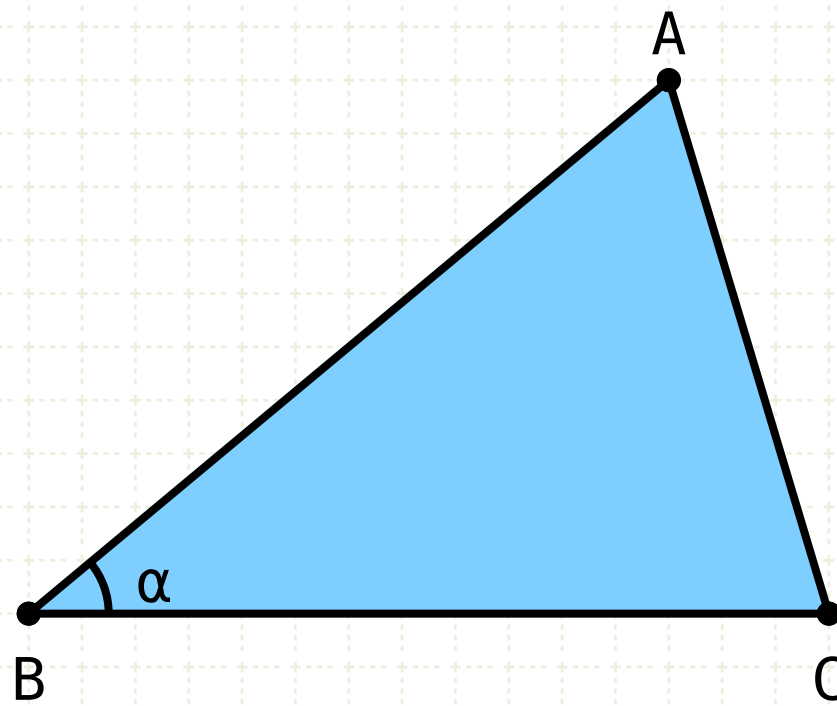
Thus the duplicate ratio can be written as:

$A:B = B:C \rightarrow A:C = (A:B)^2$



# Proposition 19 of Book VI

Similar triangles are to one another in the duplicate ratio of the corresponding sides



## In other words

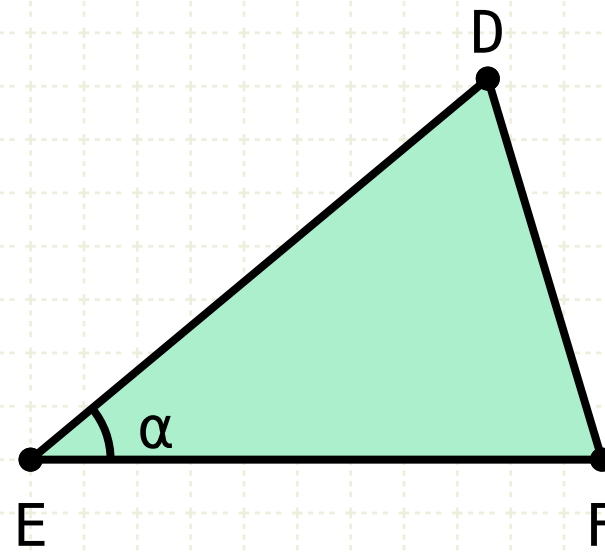
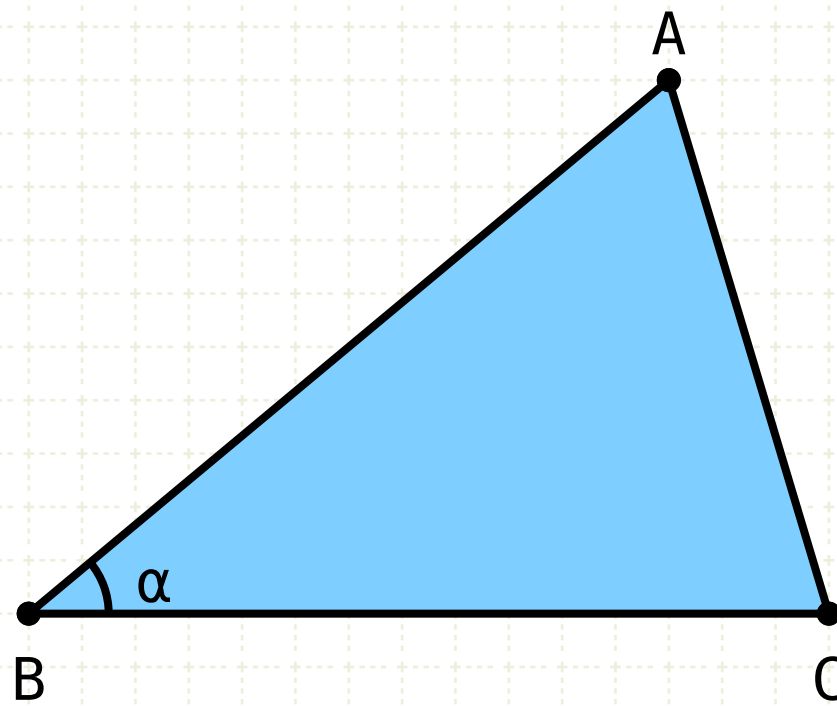
If there are two triangles which are similar (equal angles, sides proportional), then the ratio of the areas of the triangles is the square of the ratio of the sides

$$\triangle ABC \sim \triangle DEF$$

$$\rightarrow \triangle ABC : \triangle DEF = (AB : DE)^2$$

# Proposition 19 of Book VI

Similar triangles are to one another in the duplicate ratio of the corresponding sides

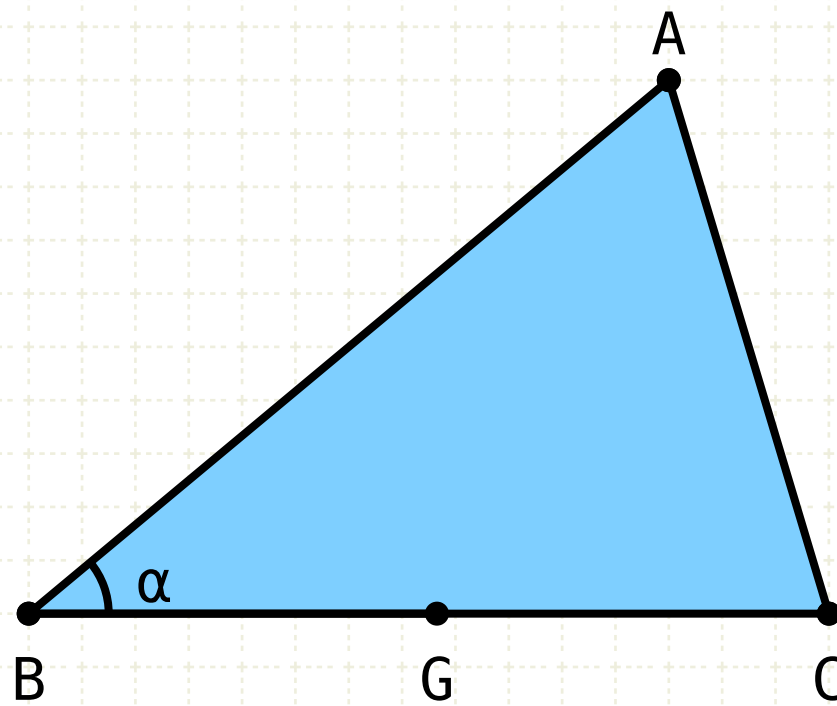


Proof

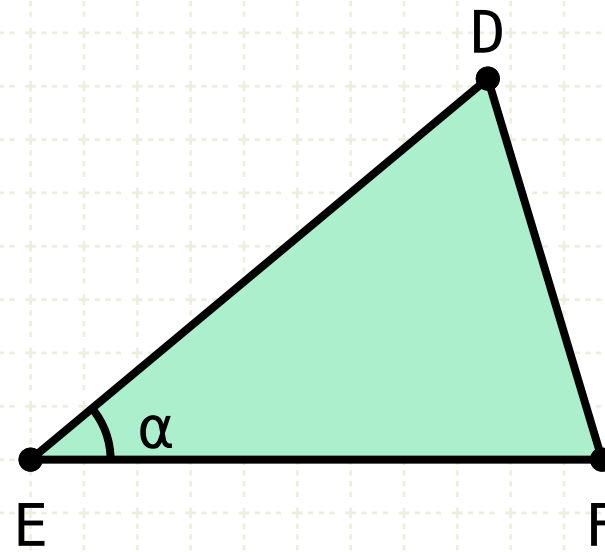
$$\triangle ABC \sim \triangle DEF$$

# Proposition 19 of Book VI

Similar triangles are to one another in the duplicate ratio of the corresponding sides



$$\triangle ABC \sim \triangle DEF$$
$$BC : EF = EF : BG$$



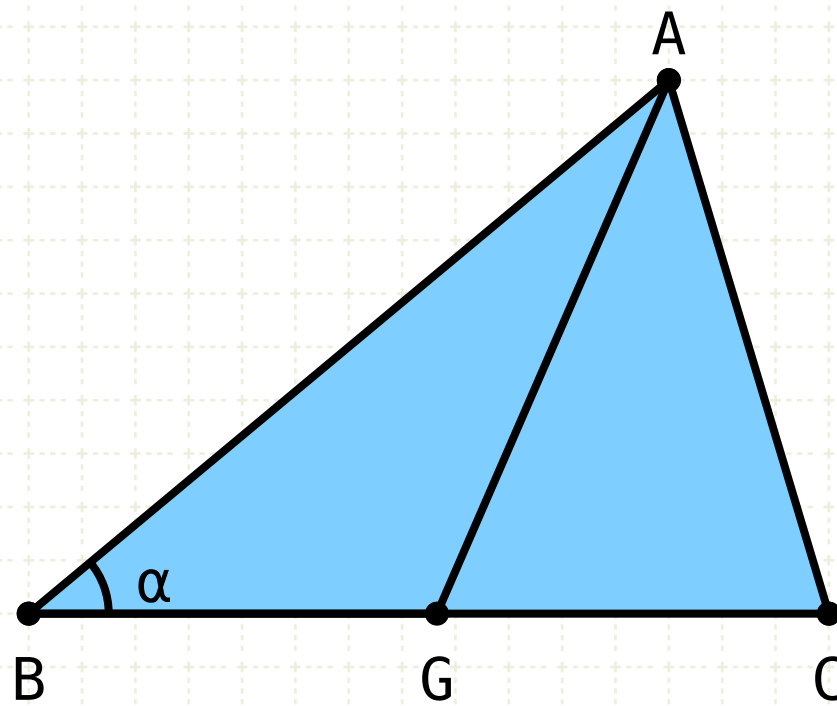
## Proof

Construct a third proportional BG such that BC to EF is EF to BG (VI·11)

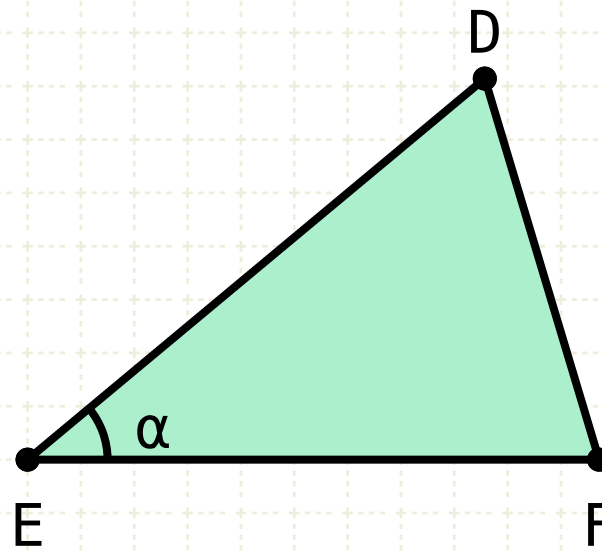


# Proposition 19 of Book VI

Similar triangles are to one another in the duplicate ratio of the corresponding sides



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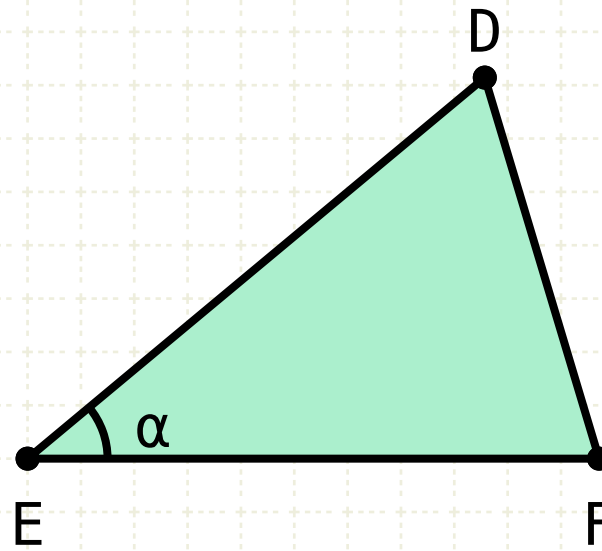
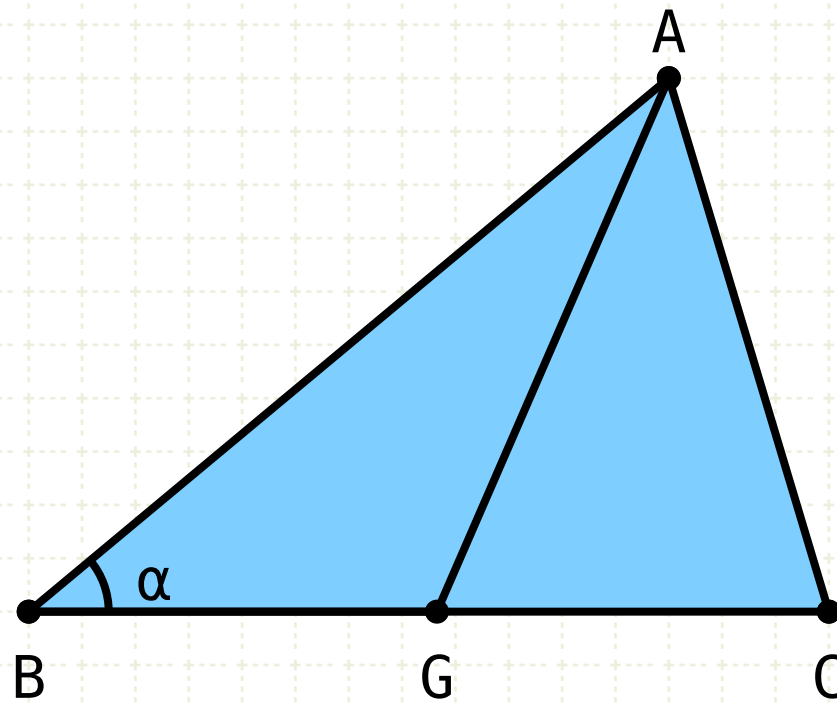
## Proof

Construct a third proportional BG such that BC to EF is EF to BG (VI·11)

Draw the line AG

# Proposition 19 of Book VI

Similar triangles are to one another in the duplicate ratio of the corresponding sides



$$\triangle ABC \sim \triangle DEF$$

$$BC : EF = EF : BG$$

$$AB : BC = DE : EF$$

$$AB : DE = BC : EF$$

## Proof

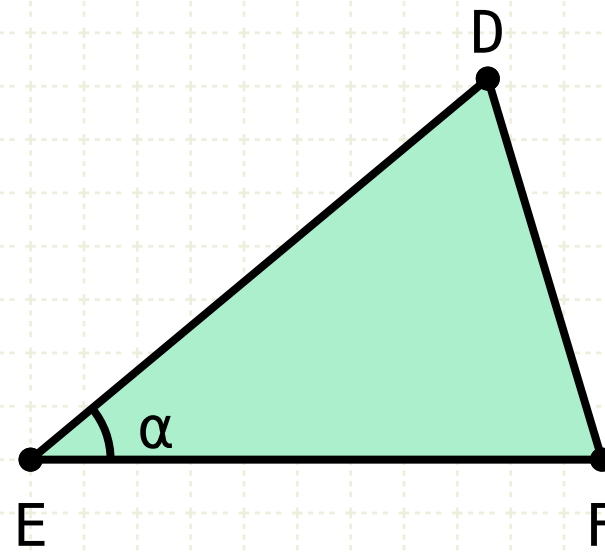
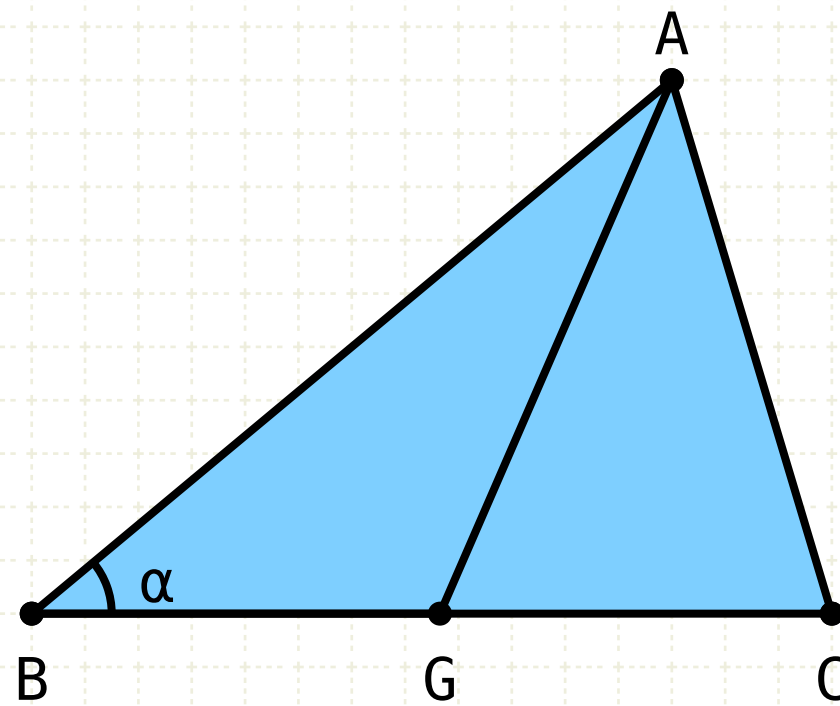
Construct a third proportional BG such that BC to EF is EF to BG (VI·11)

Draw the line AG

As AB is to BC, so is DE to EF, or alternately, AB is to DE as BC is to EF (V·16)

# Proposition 19 of Book VI

Similar triangles are to one another in the duplicate ratio of the corresponding sides



$$\triangle ABC \sim \triangle DEF$$

$$BC : EF = EF : BG$$

$$AB : BC = DE : EF$$

$$AB : DE = BC : EF$$

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## Proof

Construct a third proportional BG such that BC to EF is EF to BG (VI·11)

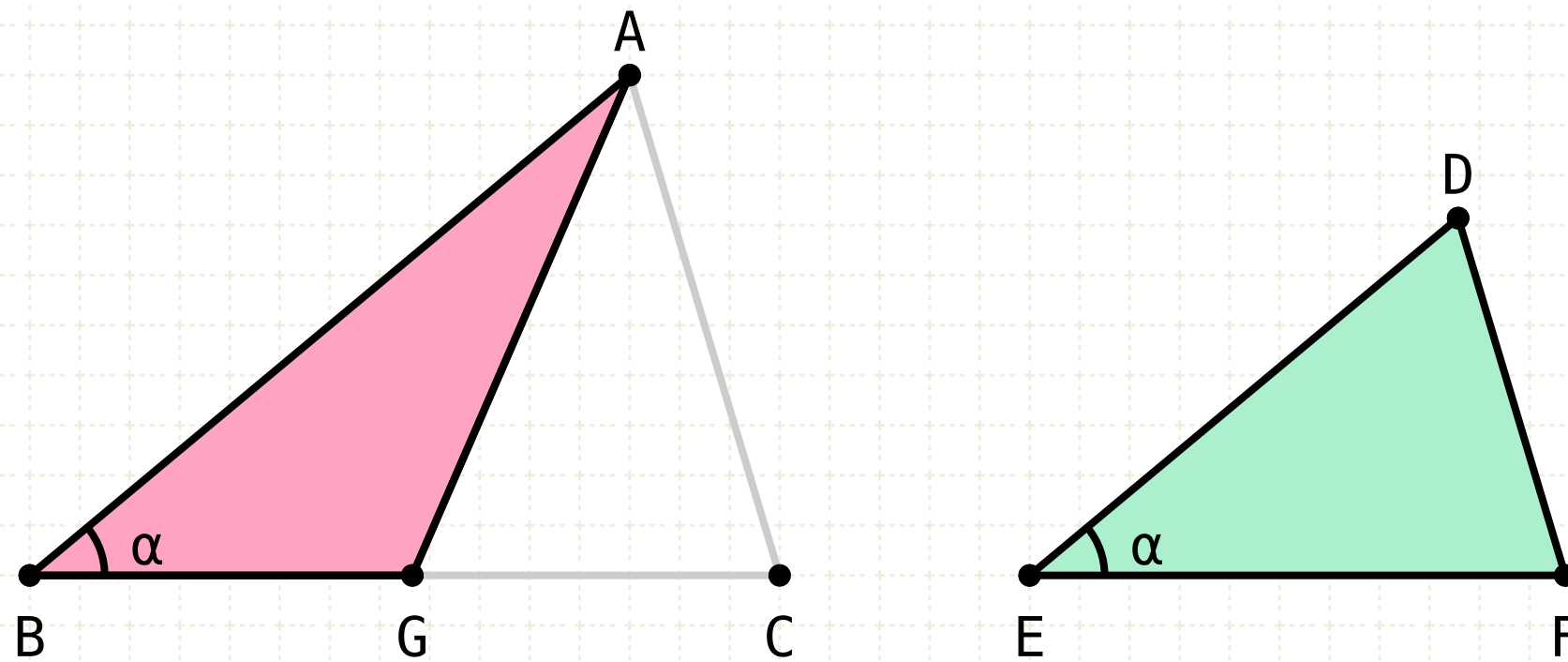
Draw the line AG

As AB is to BC, so is DE to EF, or alternately, AB is to DE as BC is to EF (V·16)

Therefore, AB is to DE as EF is to BG (V·11)

# Proposition 19 of Book VI

Similar triangles are to one another in the duplicate ratio of the corresponding sides



$$\triangle ABC \sim \triangle DEF$$

$$BC : EF = EF : BG$$

$$AB : BC = DE : EF$$

$$AB : DE = BC : EF$$

$$AB : DE = EF : BG$$

## Proof

Construct a third proportional  $BG$  such that  $BC$  to  $EF$  is  $EF$  to  $BG$  (VI·11)

Draw the line  $AG$

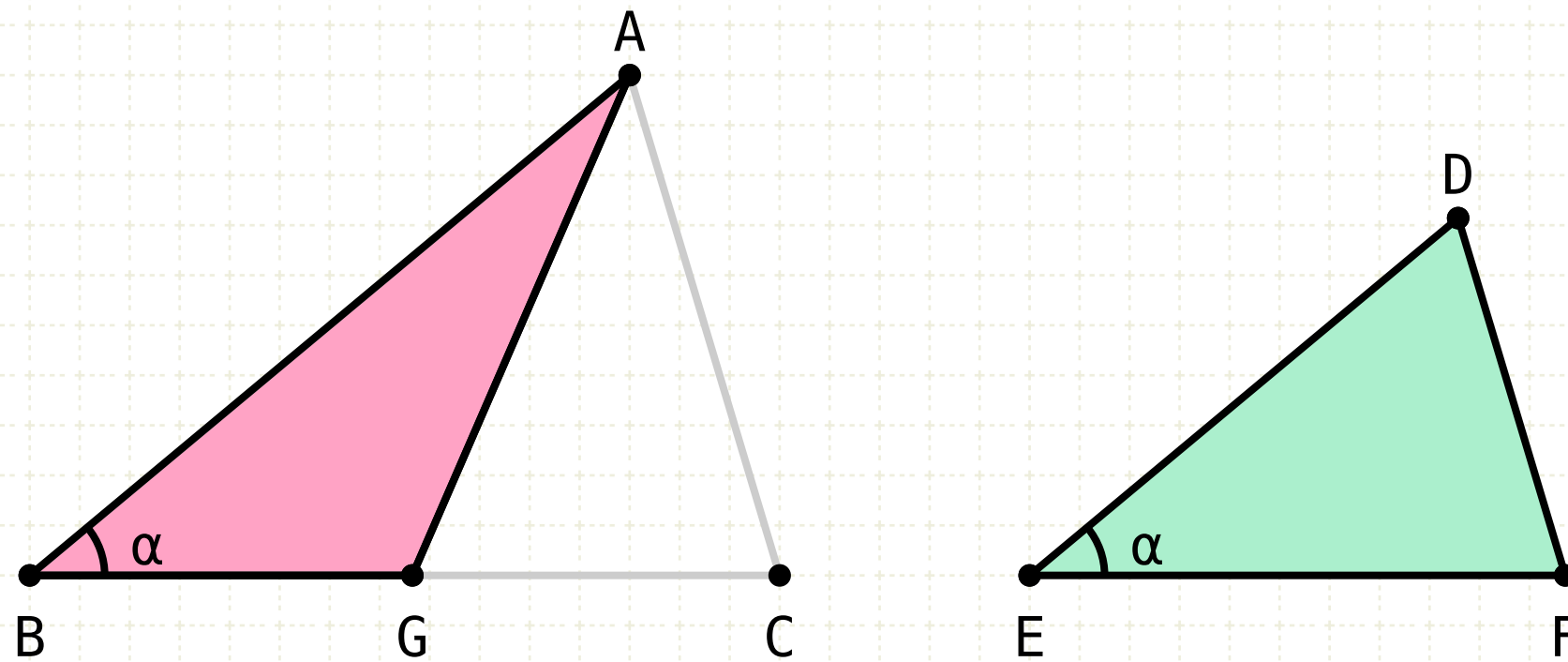
As  $AB$  is to  $BC$ , so is  $DE$  to  $EF$ , or alternately,  $AB$  is to  $DE$  as  $BC$  is to  $EF$  (V·16)

Therefore,  $AB$  is to  $DE$  as  $EF$  is to  $BG$  (V·11)

The sides about the the equal angle in triangles  $ABG$  and  $DEF$  are reciprocally proportional

# Proposition 19 of Book VI

Similar triangles are to one another in the duplicate ratio of the corresponding sides



$$\begin{aligned}\triangle ABC &\sim \triangle DEF \\ BC:EF &= EF:BG \\ AB:BC &= DE:EF \\ AB:DE &= BC:EF \\ AB:DE &= EF:BG \\ \triangle ABG &= \triangle DEF\end{aligned}$$

## Proof

Construct a third proportional BG such that BC to EF is EF to BG (VI·11)

Draw the line AG

As AB is to BC, so is DE to EF, or alternately, AB is to DE as BC is to EF (V·16)

Therefore, AB is to DE as EF is to BG (V·11)

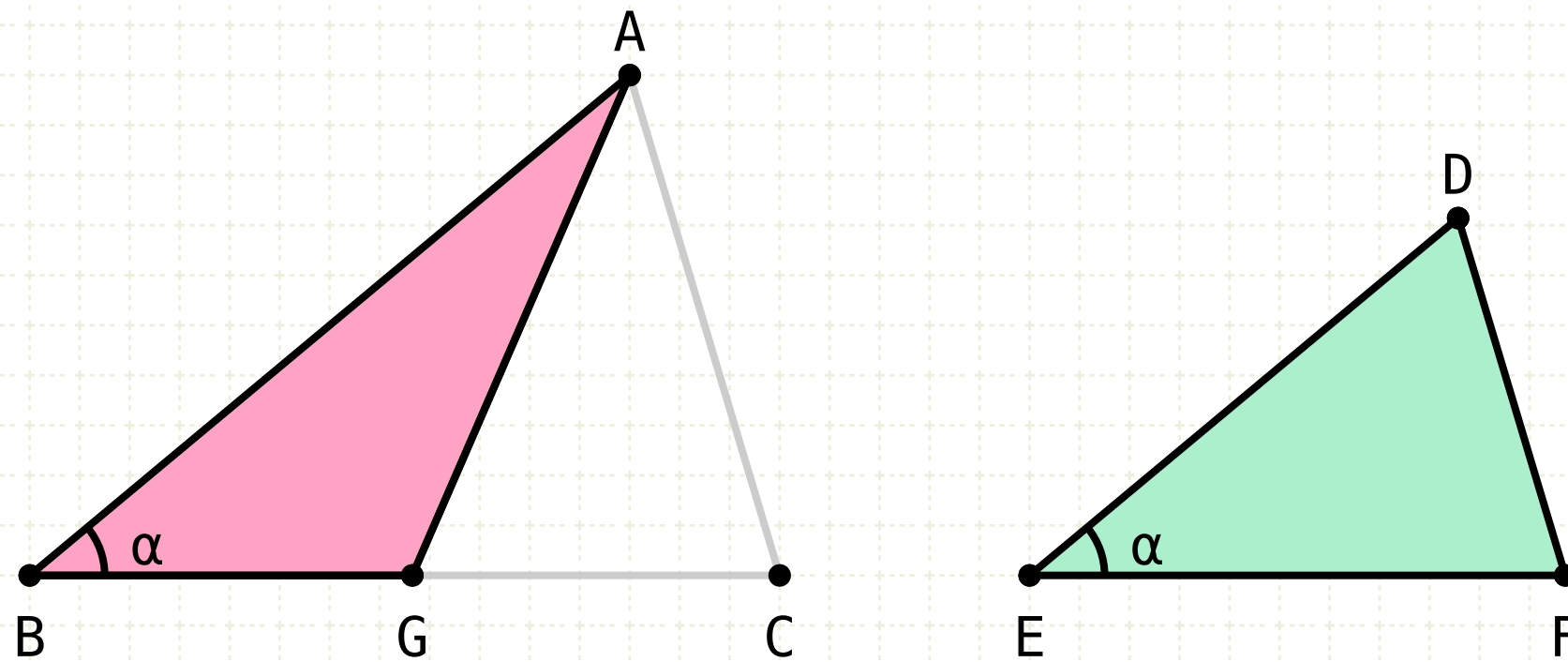
The sides about the the equal angle in triangles ABG and DEF are reciprocally proportional

And triangles whose sides are reciprocally proportional about an equal angle, are also equal (VI·15)



# Proposition 19 of Book VI

Similar triangles are to one another in the duplicate ratio of the corresponding sides



$$\begin{aligned}\triangle ABC &\sim \triangle DEF \\ BC:EF &= EF:BG \\ AB:BC &= DE:EF \\ AB:DE &= BC:EF \\ AB:DE &= EF:BG \\ \triangle ABG &= \triangle DEF \\ BC:BG &= (BC:EF)^2\end{aligned}$$

## Proof

Construct a third proportional  $BG$  such that  $BC$  to  $EF$  is  $EF$  to  $BG$  (VI·11)

Draw the line  $AG$

As  $AB$  is to  $BC$ , so is  $DE$  to  $EF$ , or alternately,  $AB$  is to  $DE$  as  $BC$  is to  $EF$  (V·16)

Therefore,  $AB$  is to  $DE$  as  $EF$  is to  $BG$  (V·11)

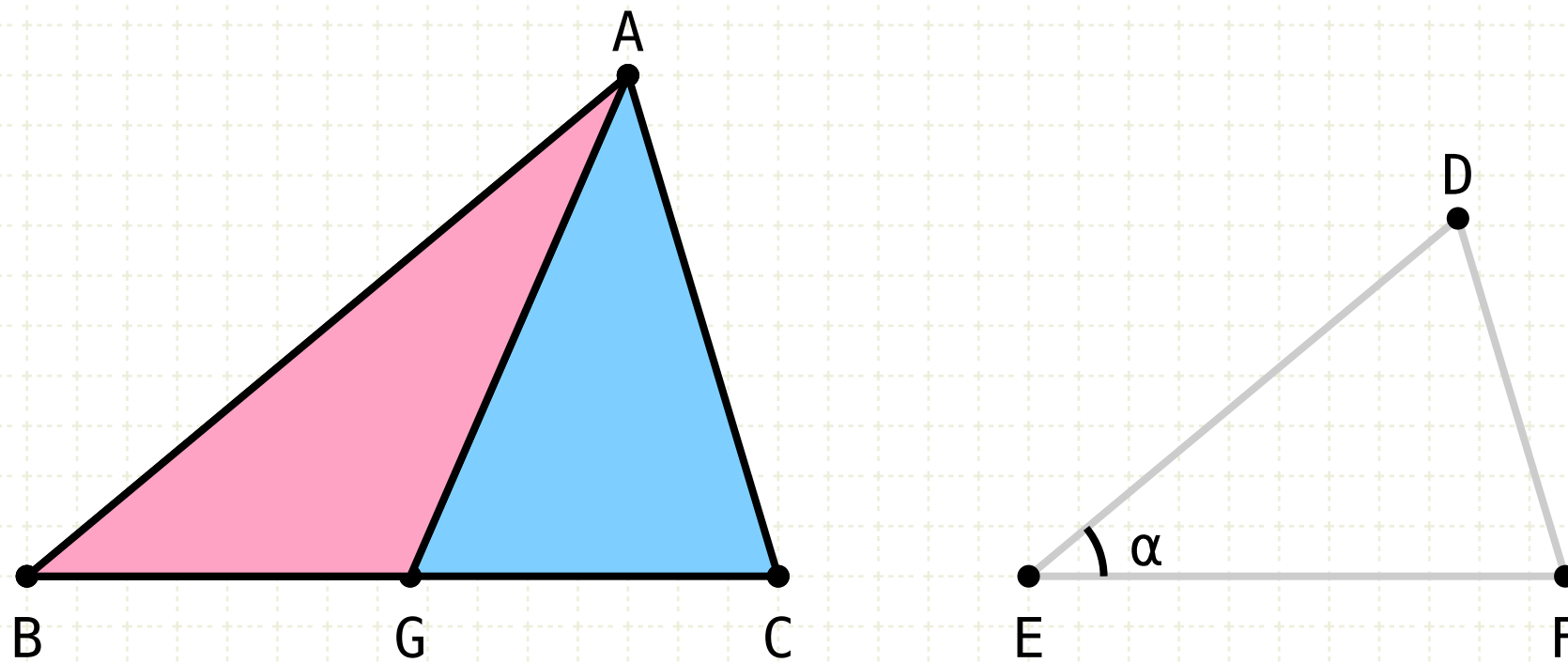
The sides about the the equal angle in triangles  $ABG$  and  $DEF$  are reciprocally proportional

And triangles whose sides are reciprocally proportional about an equal angle, are also equal (VI·15)

Since  $BC$  is to  $EF$  is as  $EF$  is to  $BG$ , the ratio  $BC$  to  $BG$  is the duplicate ratio of  $BC$  to  $EF$  (V.Def.9)

# Proposition 19 of Book VI

Similar triangles are to one another in the duplicate ratio of the corresponding sides



$$\begin{aligned}\triangle ABC &\sim \triangle DEF \\ BC:EF &= EF:BG \\ AB:BC &= DE:EF \\ AB:DE &= BC:EF \\ AB:DE &= EF:BG \\ \triangle ABG &= \triangle DEF \\ BC:BG &= (BC:EF)^2\end{aligned}$$

$$\triangle ABC:\triangle ABG = BC:BG$$

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Construct a third proportional  $BG$  such that  $BC$  to  $EF$  is  $EF$  to  $BG$  (VI·11)

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The sides about the the equal angle in triangles  $ABG$  and  $DEF$  are reciprocally proportional

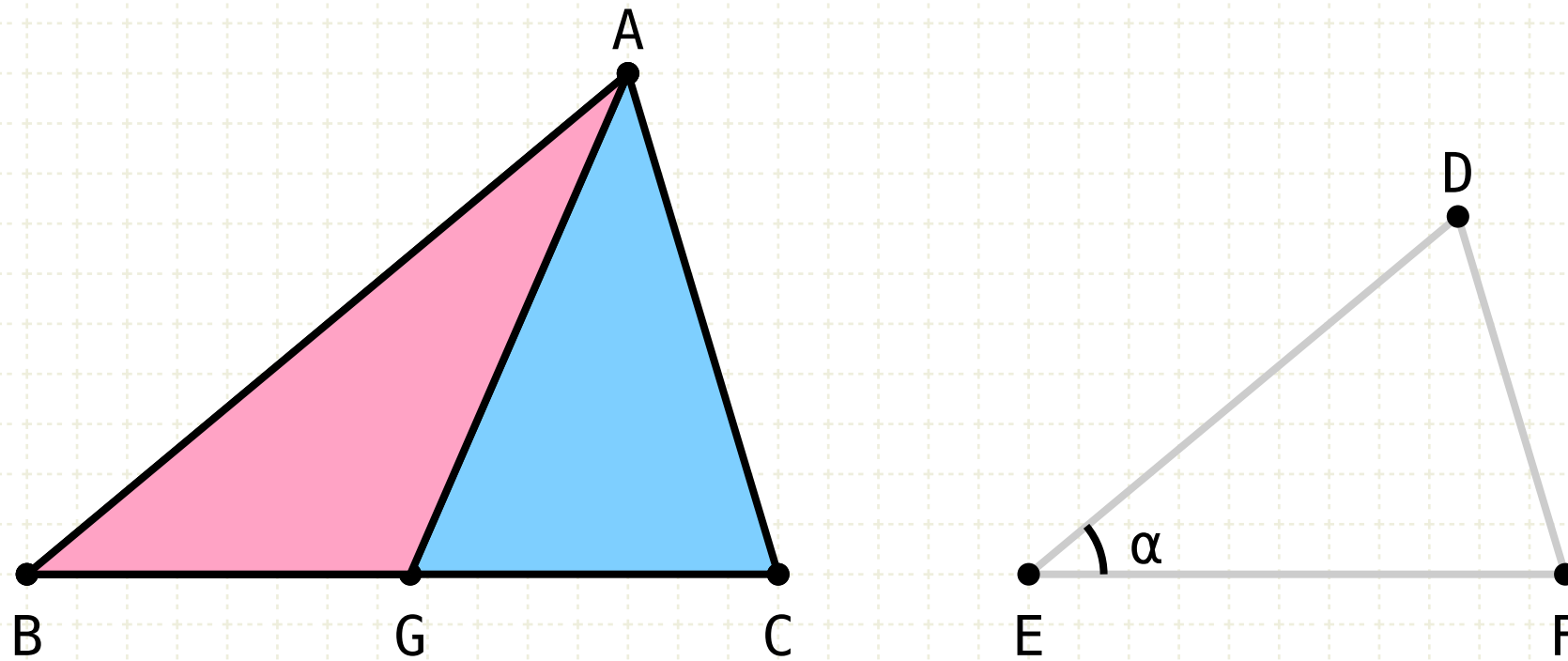
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Since  $BC$  is to  $EF$  is as  $EF$  is to  $BG$ , the ratio  $BC$  to  $BG$  is the duplicate ratio of  $BC$  to  $EF$  (V.Def.9)

The ratio of the triangles  $ABC$  and  $ABG$  is proportional to the bases,  $BC$  and  $BG$  (VI·1)

# Proposition 19 of Book VI

Similar triangles are to one another in the duplicate ratio of the corresponding sides



$$\begin{aligned}\triangle ABC &\sim \triangle DEF \\ BC:EF &= EF:BG \\ AB:BC &= DE:EF \\ AB:DE &= BC:EF \\ AB:DE &= EF:BG \\ \triangle ABG &= \triangle DEF \\ BC:BG &= (BC:EF)^2\end{aligned}$$

$$\begin{aligned}\triangle ABC:\triangle ABG &= BC:BG \\ \triangle ABC:\triangle ABG &= (BC:EF)^2\end{aligned}$$

## Proof

Construct a third proportional BG such that BC to EF is EF to BG (VI·11)

Draw the line AG

As AB is to BC, so is DE to EF, or alternately, AB is to DE as BC is to EF (V·16)

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The sides about the the equal angle in triangles ABG and DEF are reciprocally proportional

And triangles whose sides are reciprocally proportional about an equal angle, are also equal (VI·15)

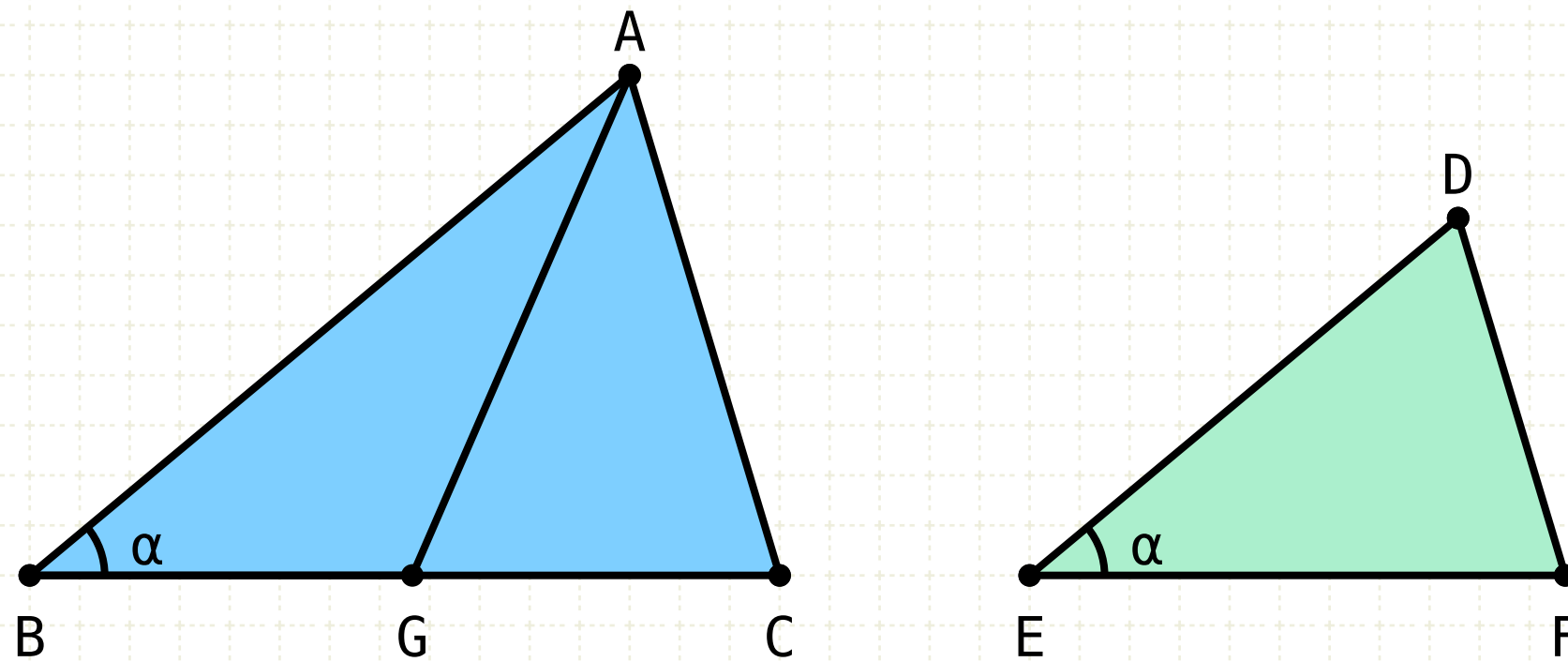
Since BC is to EF is as EF is to BG, the ratio BC to BG is the duplicate ratio of BC to EF (V.Def.9)

The ratio of the triangles ABC and ABG is proportional to the bases, BC and BG (VI·1)

Thus the ratio of BC to BG is the duplicate ratio of BC to EF

# Proposition 19 of Book VI

Similar triangles are to one another in the duplicate ratio of the corresponding sides



$$\begin{aligned}\triangle ABC &\sim \triangle DEF \\ BC:EF &= EF:BG \\ AB:BC &= DE:EF \\ AB:DE &= BC:EF \\ AB:DE &= EF:BG \\ \triangle ABG &= \triangle DEF \\ BC:BG &= (BC:EF)^2\end{aligned}$$

$$\begin{aligned}\triangle ABC:\triangle ABG &= BC:BG \\ \triangle ABC:\triangle ABG &= (BC:EF)^2 \\ \triangle ABC:\triangle DEF &= (BC:EF)^2\end{aligned}$$

## Proof

Construct a third proportional BG such that BC to EF is EF to BG (VI·11)

Draw the line AG

As AB is to BC, so is DE to EF, or alternately, AB is to DE as BC is to EF (V·16)

Therefore, AB is to DE as EF is to BG (V·11)

The sides about the the equal angle in triangles ABG and DEF are reciprocally proportional

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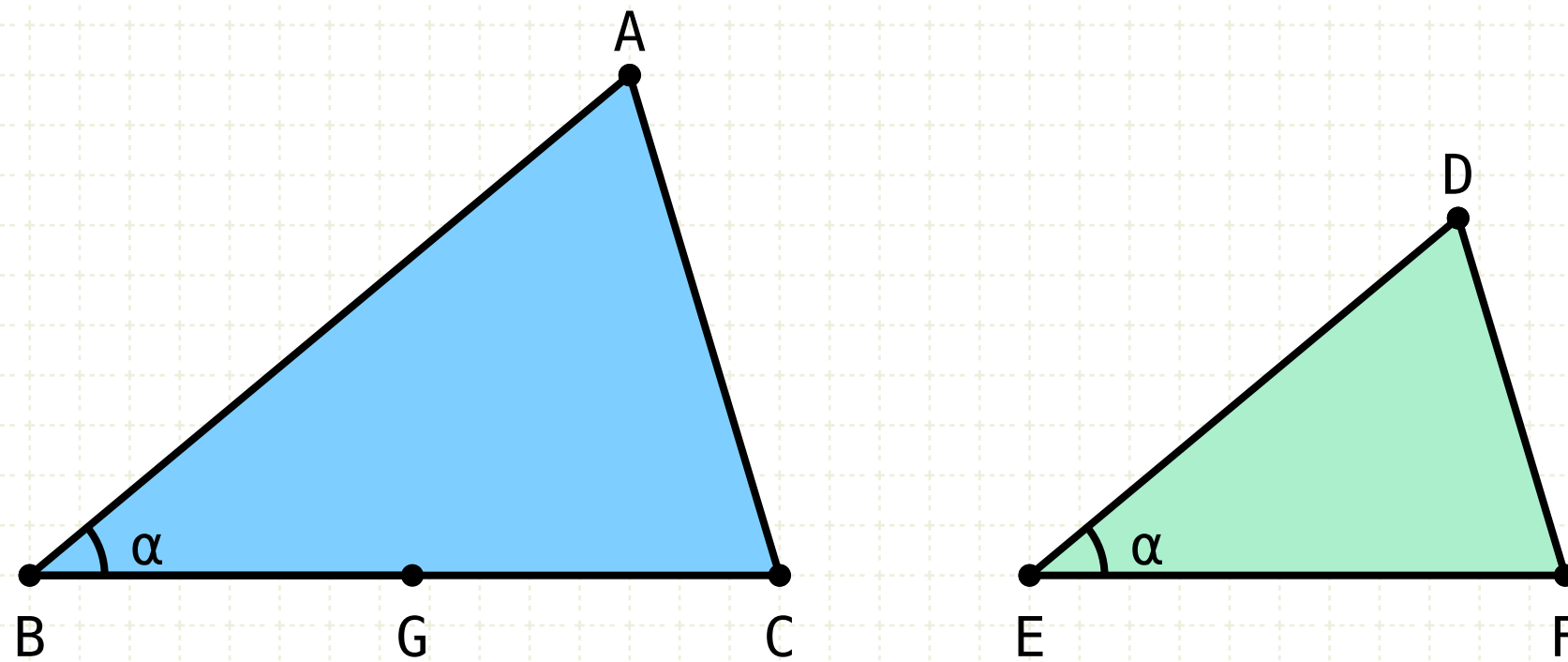
Thus the ratio of BC to BG is the duplicate ratio of BC to EF

But ABG equals DEF, therefore the ratio of ABC to DEF is the duplicate ratio of their sides



# Proposition 19 of Book VI

Similar triangles are to one another in the duplicate ratio of the corresponding sides



$$\triangle ABC \sim \triangle DEF$$

$$BC:EF = EF:BG$$

$$AB:BC = DE:EF$$

$$AB:DE = BC:EF$$

$$AB:DE = EF:BG$$

$$\triangle ABG = \triangle DEF$$

$$BC:BG = (BC:EF)^2$$

$$\triangle ABC:\triangle ABG = BC:BG$$

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## Proof

Construct a third proportional BG such that BC to EF is EF to BG (VI·11)

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As AB is to BC, so is DE to EF, or alternately, AB is to DE as BC is to EF (V·16)

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And triangles whose sides are reciprocally proportional about an equal angle, are also equal (VI·15)

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The ratio of the triangles ABC and ABG is proportional to the bases, BC and BG (VI·1)

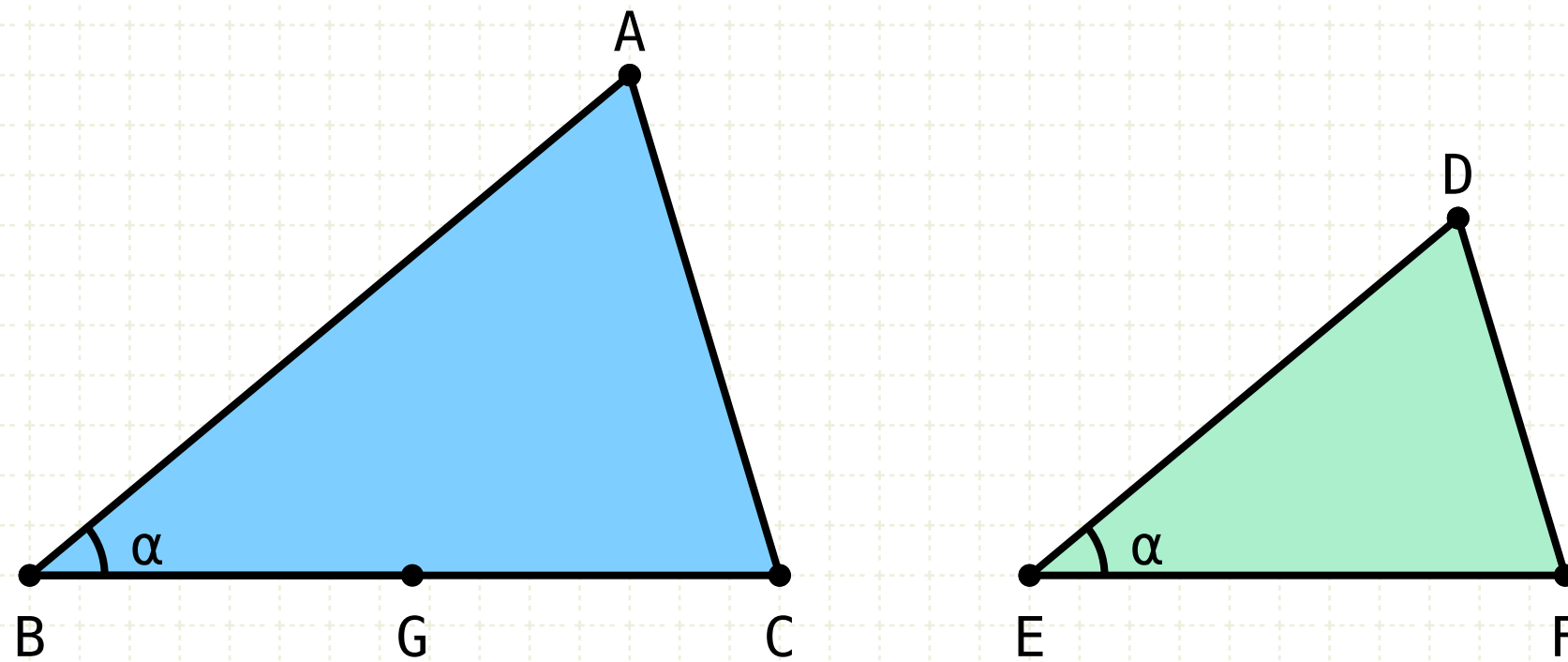
Thus the ratio of BC to BG is the duplicate ratio of BC to EF

But ABG equals DEF, therefore the ratio of ABC to DEF is the duplicate ratio of their sides



# Proposition 19 of Book VI

Similar triangles are to one another in the duplicate ratio of the corresponding sides



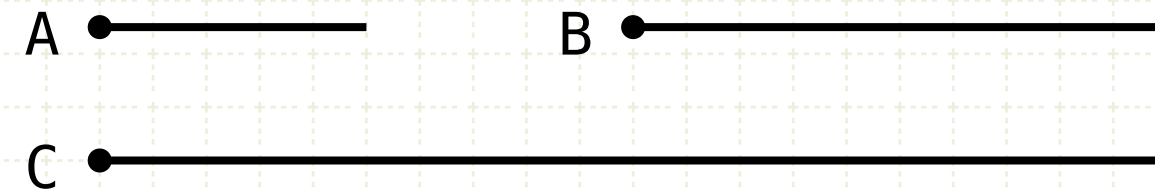
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$$\begin{aligned}\triangle ABC:\triangle ABG &= BC:BG \\ \triangle ABC:\triangle ABG &= (BC:EF)^2 \\ \triangle ABC:\triangle DEF &= (BC:EF)^2\end{aligned}$$

## Porism

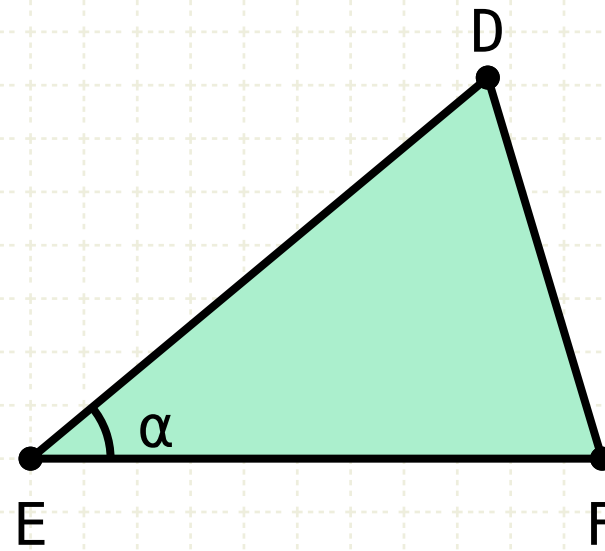
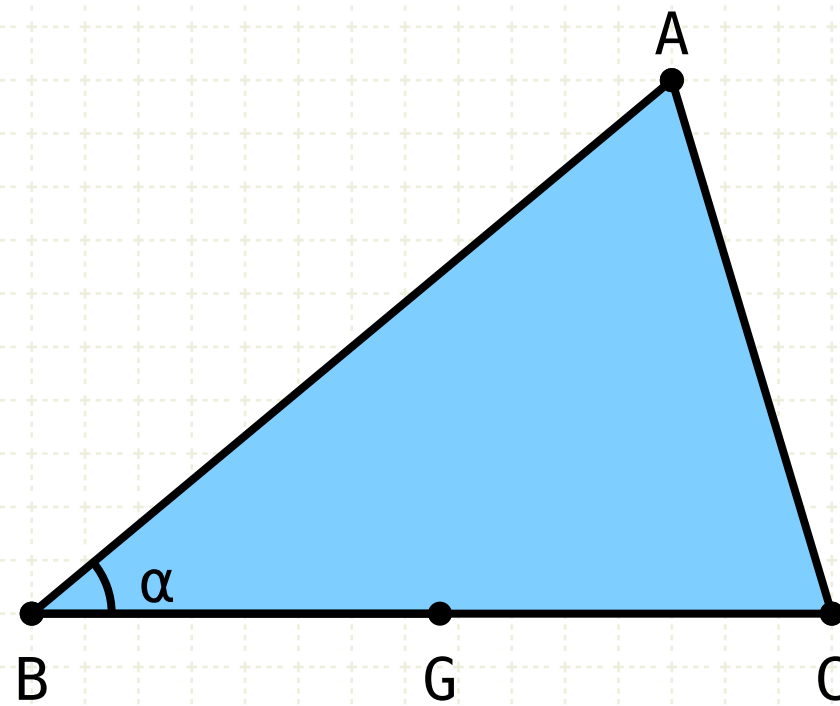
If three straight lines are proportional (A is to B as B is to C)  
If a two similar figures are drawn on A and B  
Then the ratio of these two figures is equal to the ratio of A to C

$$A:B = B:C$$



# Proposition 19 of Book VI

Similar triangles are to one another in the duplicate ratio of the corresponding sides



## Porism

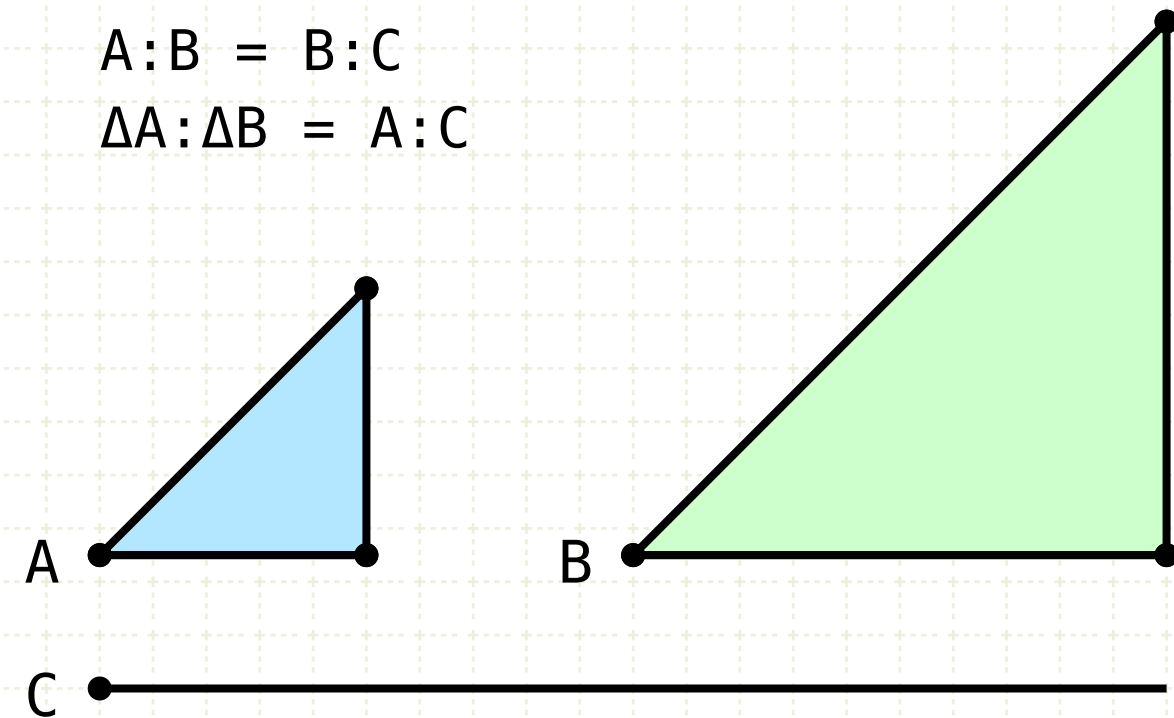
If three straight lines are proportional (A is to B as B is to C)

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Then the ratio of these two figures is equal to the ratio of A to C

$$A:B = B:C$$

$$\Delta A:\Delta B = A:C$$



$$\begin{aligned}\Delta ABC &\sim \Delta DEF \\ BC:EF &= EF:BG \\ AB:BC &= DE:EF \\ AB:DE &= BC:EF \\ AB:DE &= EF:BG \\ \Delta ABG &= \Delta DEF \\ BC:BG &= (BC:EF)^2\end{aligned}$$

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<https://www.youtube.com/c/SandyBultena>

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