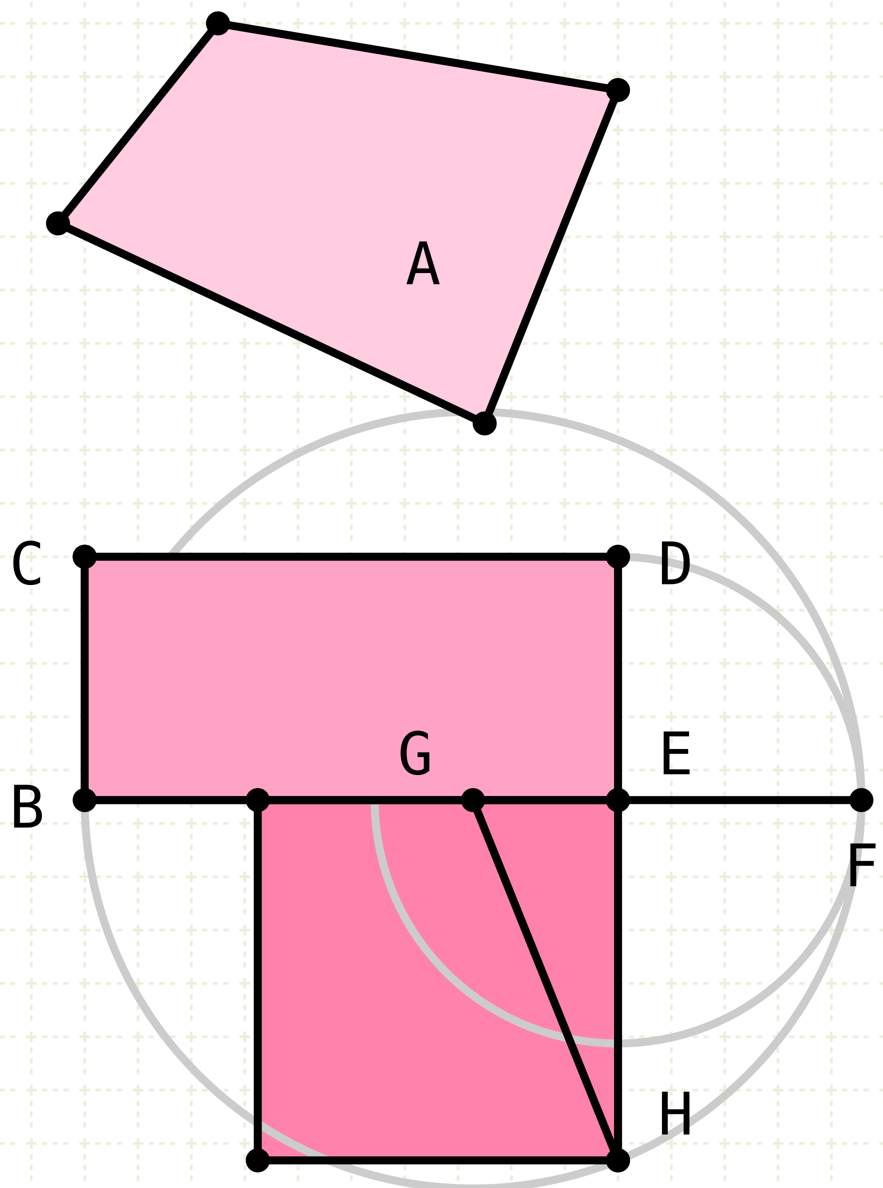
Euclid's Elements

Book II

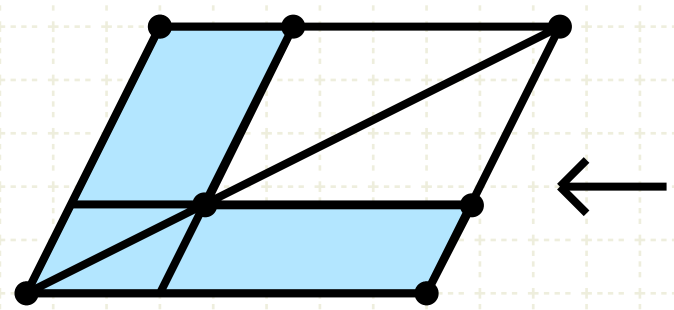


It is a remarkable fact in the history of geometry, that the Elements of Euclid, written two thousand years ago, are still regarded by many as the best introduction to the mathematical sciences.

Florian Cajori,

A History of Mathematics (1893)

Definitions:



Any rectangular parallelogram is said to be contained by the two straight lines containing the right angle.

And in any parallelogrammic area let any one whatever of the parallelograms about its diameter with the two complements be called a gnomon.

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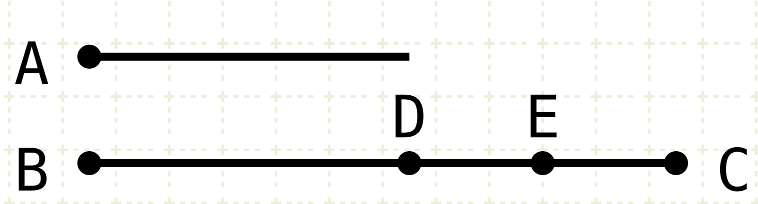
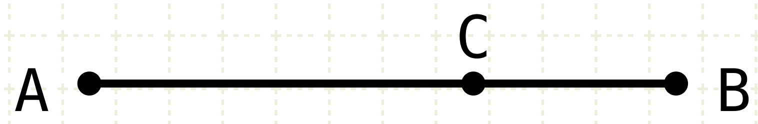


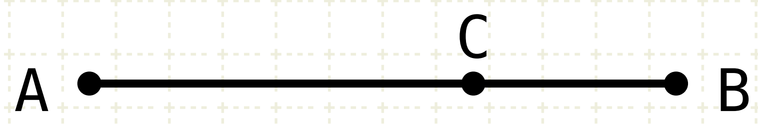
$A \cdot BC = A \cdot BD + A \cdot DE + A \cdot EC$

2



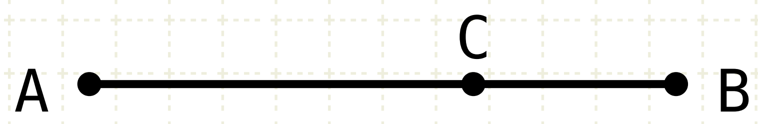
$AB^2 = AB \cdot AC + AB \cdot BC$

3



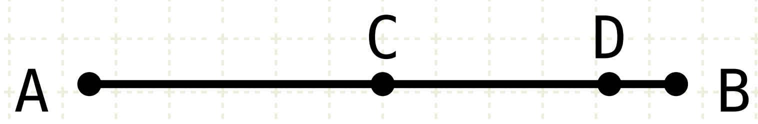
$AB \cdot CB = AC \cdot CB + CB^2$

4



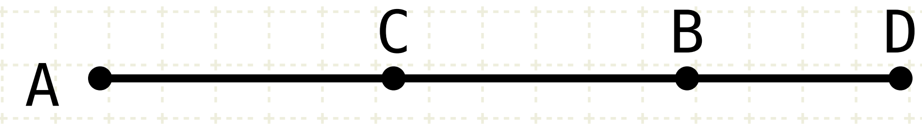
$AB^2 = AC^2 + CB^2 + 2 \cdot AC \cdot CB$

5



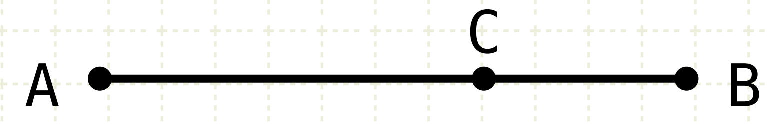
$AD \cdot DB + CD^2 = CB^2$

6



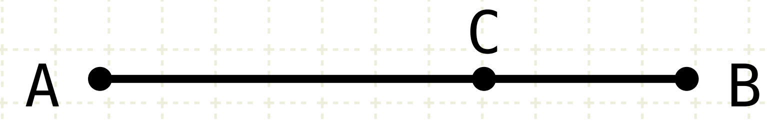
$AD \cdot DB + CB^2 = CD^2$

7



$AB^2 + BC^2 = AC^2 + 2 \cdot AB \cdot BC$

8



$\mathbf{4 \cdot AB \cdot BC + AC^2 = (AB+BC)^2}$

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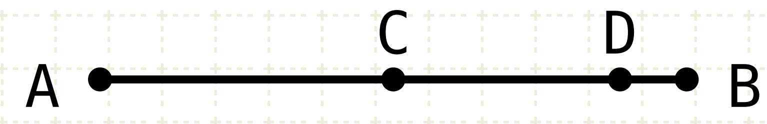
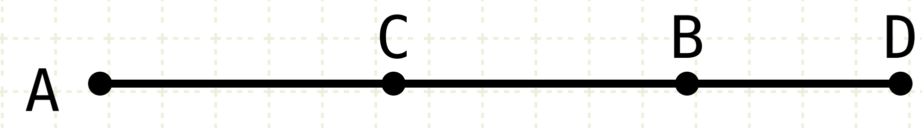


$AD^2 + DB^2 = 2 \cdot (AC^2 + CD^2)$

10



$AD^2 + DB^2 = 2 \cdot (AC^2 + CD^2)$

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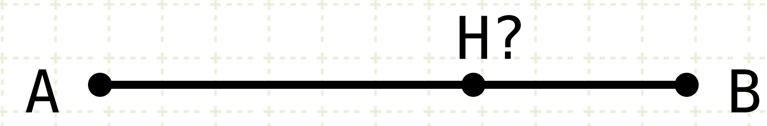
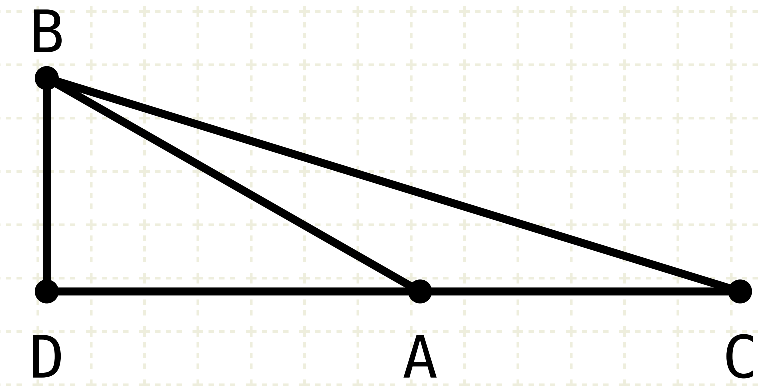


Find H. $AB \cdot BH = AH^2$

12



Cosine Law. $BC^2 = AB^2+AC^2+2 \cdot AD \cdot AC$

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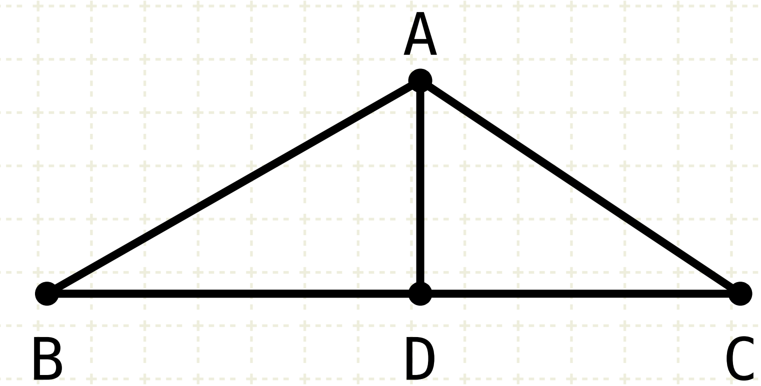

Cosine Law. $AC^2 = AB^2+BC^2-2 \cdot BD \cdot BC$

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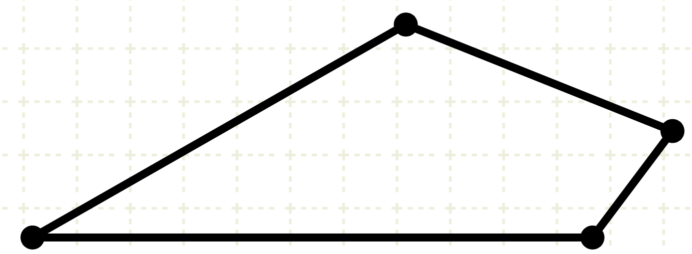

Find square of polygon

Proposition 8 of Book II

If a straight line be cut at random, four times the rectangle contained by the whole and one of the segments together with the square on the remaining segment is equal to the square described on the whole and aforesaid segment as on one straight line.

Proposition 8 of Book II

If a straight line be cut at random, four times the rectangle contained by the whole and one of the segments together with the square on the remaining segment is equal to the square described on the whole and aforesaid segment as on one straight line.

$AB = AC+CB$

In other words

Let AB be a straight line, arbitrarily cut at point C

Proposition 8 of Book II

If a straight line be cut at random, four times the rectangle contained by the whole and one of the segments together with the square on the remaining segment is equal to the square described on the whole and aforesaid segment as on one straight line.

$AB = AC+CB$

$4AB \cdot BC + AC \cdot AC$

$= (AB+BC) \cdot (AB+BC)$

In other words

Let AB be a straight line, arbitrarily cut at point C

Then four times the rectangle formed by lines AB and BC plus the square of AC is equal to the square of AB added to BC

Proposition 8 of Book II

If a straight line be cut at random, four times the rectangle contained by the whole and one of the segments together with the square on the remaining segment is equal to the square described on the whole and aforesaid segment as on one straight line.

AB = AC+CB

In other words

Let AB be a straight line, arbitrarily cut at point C

Then four times the rectangle formed by lines AB and BC plus the square of AC is equal to the square of AB added to BC

Construction:

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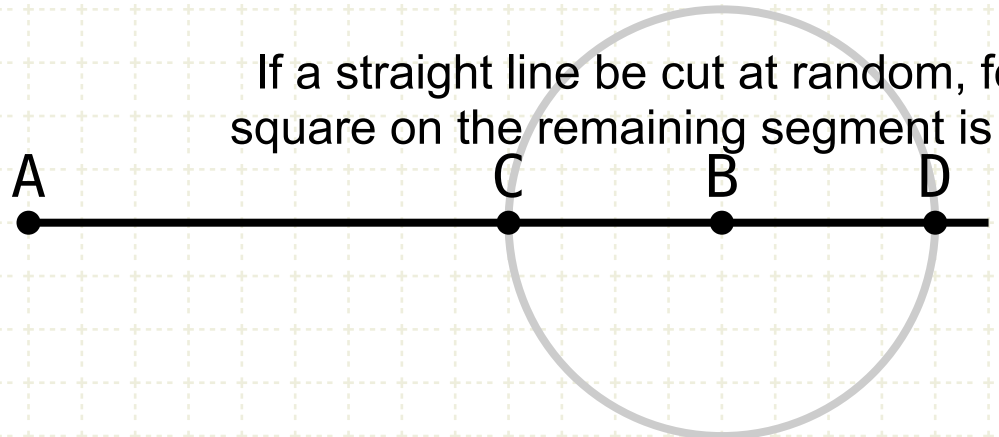
If a straight line be cut at random, four times the rectangle contained by the whole and one of the segments together with the square on the remaining segment is equal to the square described on the whole and aforesaid segment as on one straight line.



AB = AC+CB, CB = BD

In other words

Let AB be a straight line, arbitrarily cut at point C

Then four times the rectangle formed by lines AB and BC plus the square of AC is equal to the square of AB added to BC

Construction:

Extend the line AB to point D such that CB is equal to BD

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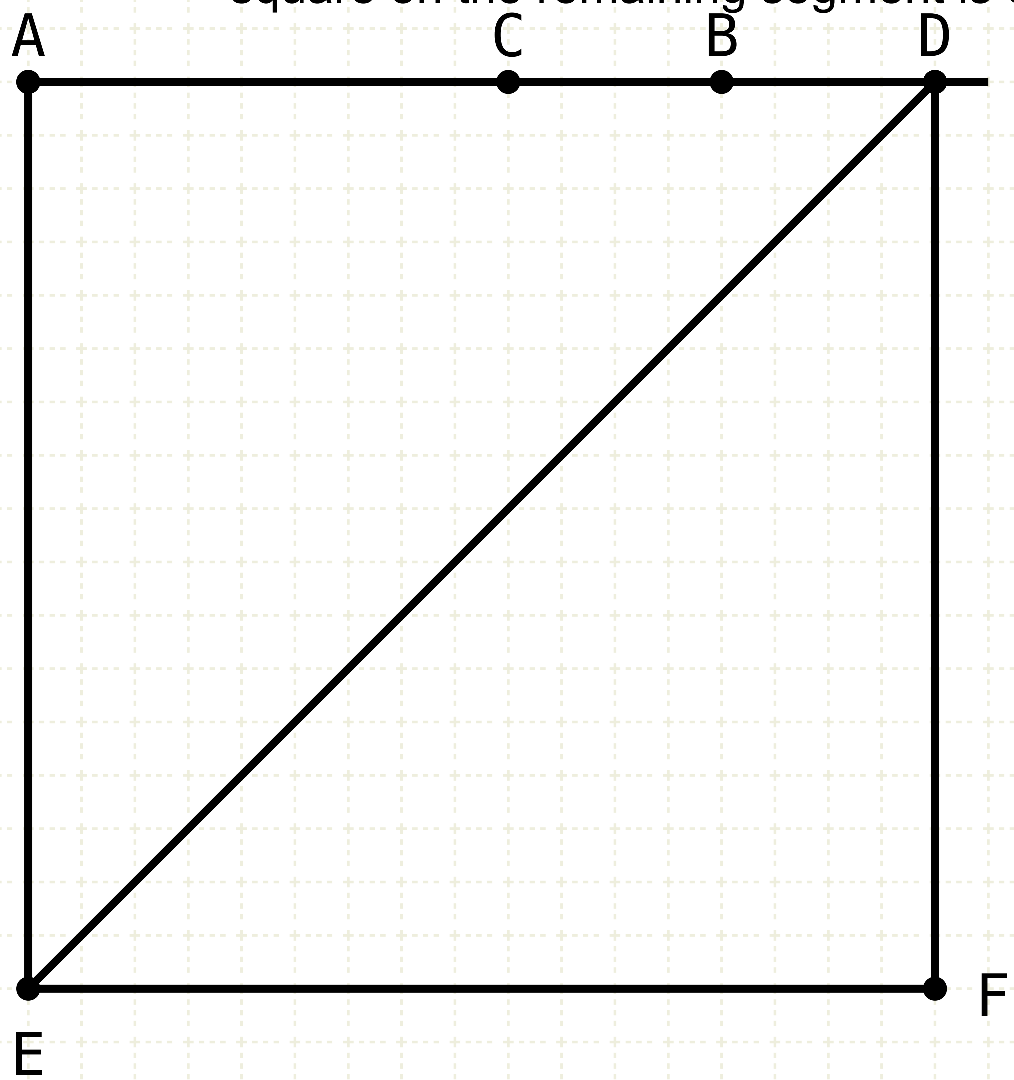

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Construction:

Extend the line AB to point D such that CB is equal to BD

Draw a square AEFD on the line AD (I·46), and draw the diagonal DE

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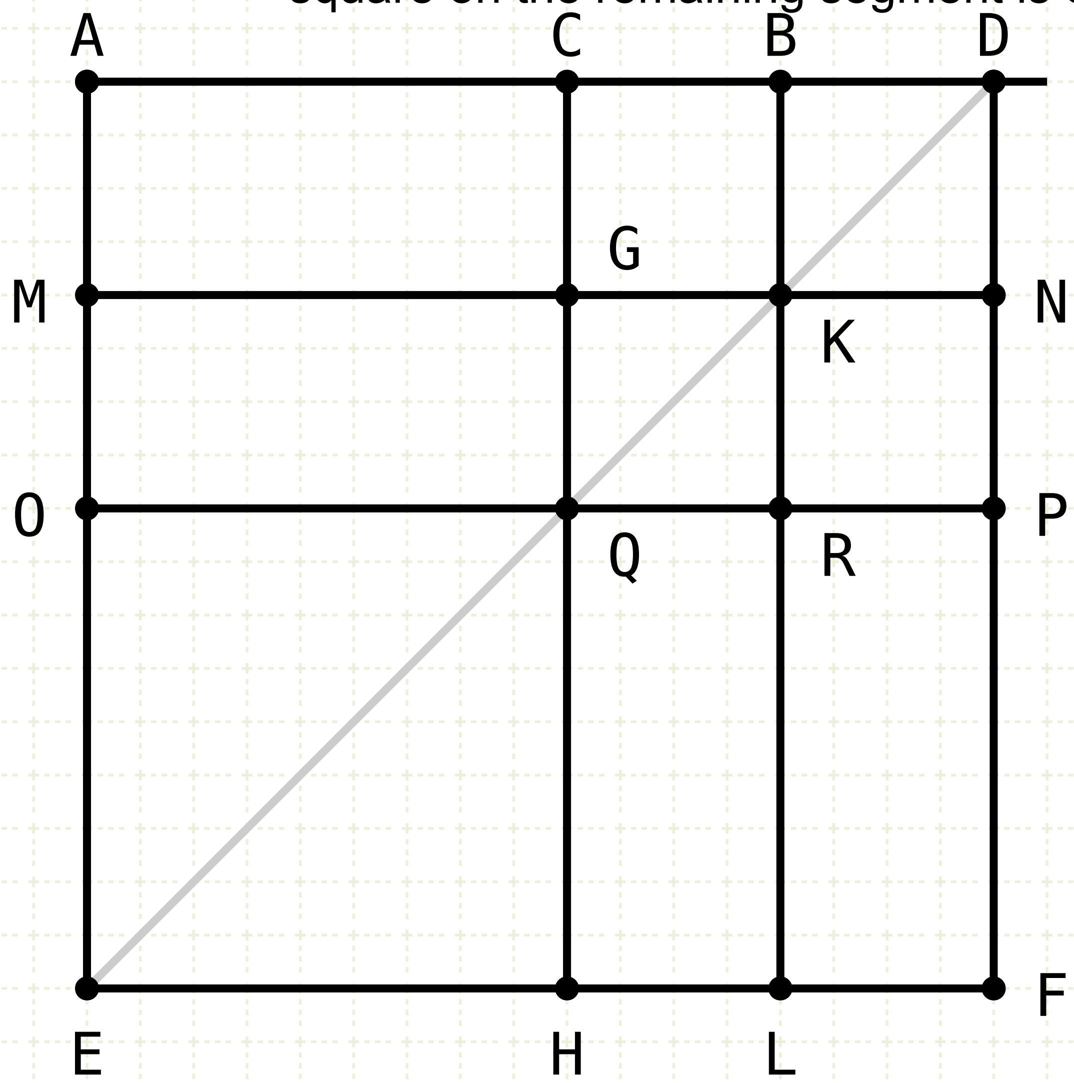

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Construction:

Extend the line AB to point D such that CB is equal to BD

Draw a square AEFD on the line AD (I·46), and draw the diagonal DE

Draw lines CH,BL parallel to AE (I·31)

Draw lines MN,OP parallel to AD (I·31)

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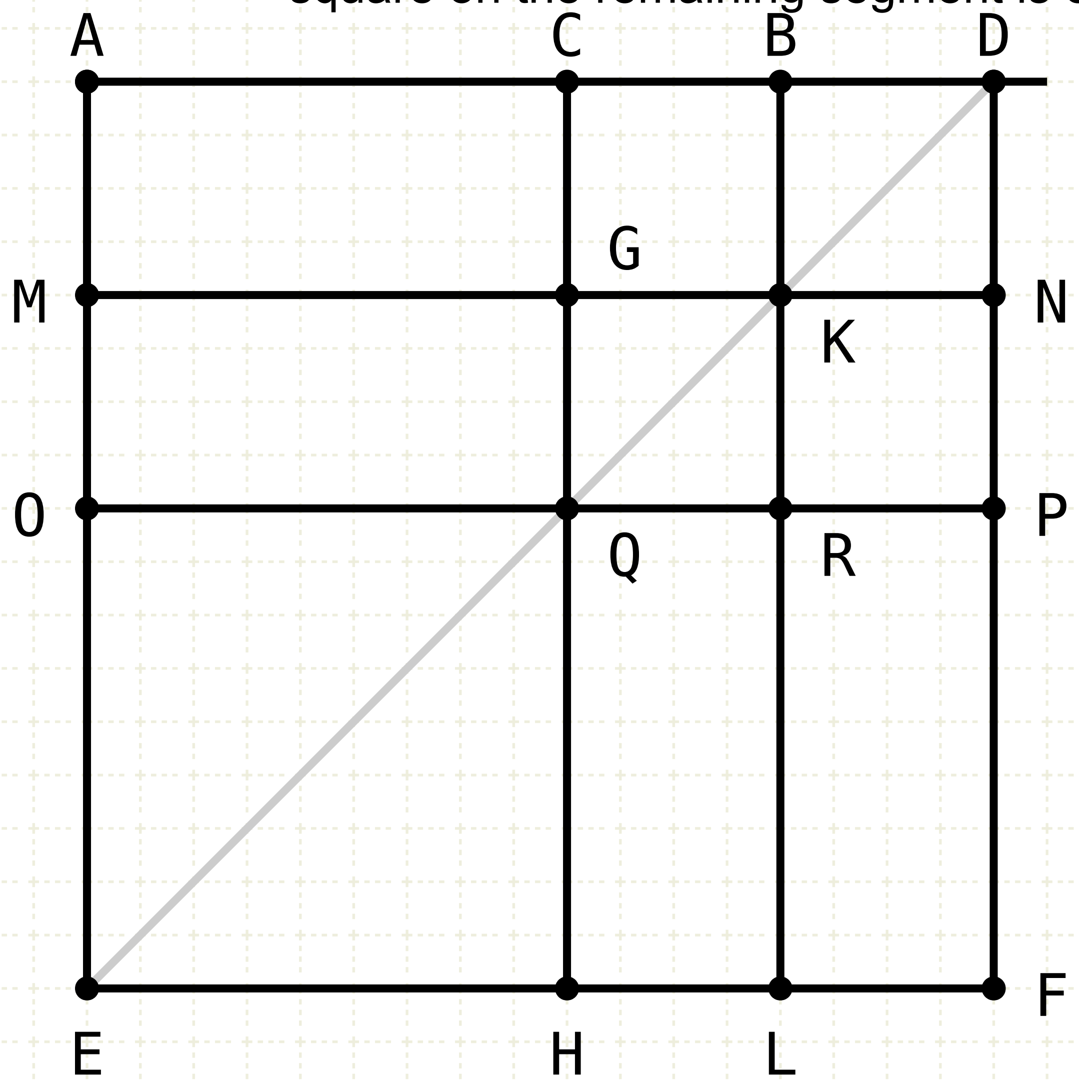

AB = AC+CB, CB = BD

GK = KN

QR = RP

In other words

Let AB be a straight line, arbitrarily cut at point C

Then four times the rectangle formed by lines AB and BC plus the square of AC is equal to the square of AB added to BC

Proof:

Since CB is equal to BD, and CB is also equal to GK (I·34) and BD is equal to KN, then GK is equal to KN

Similarly, QR is equal to RP

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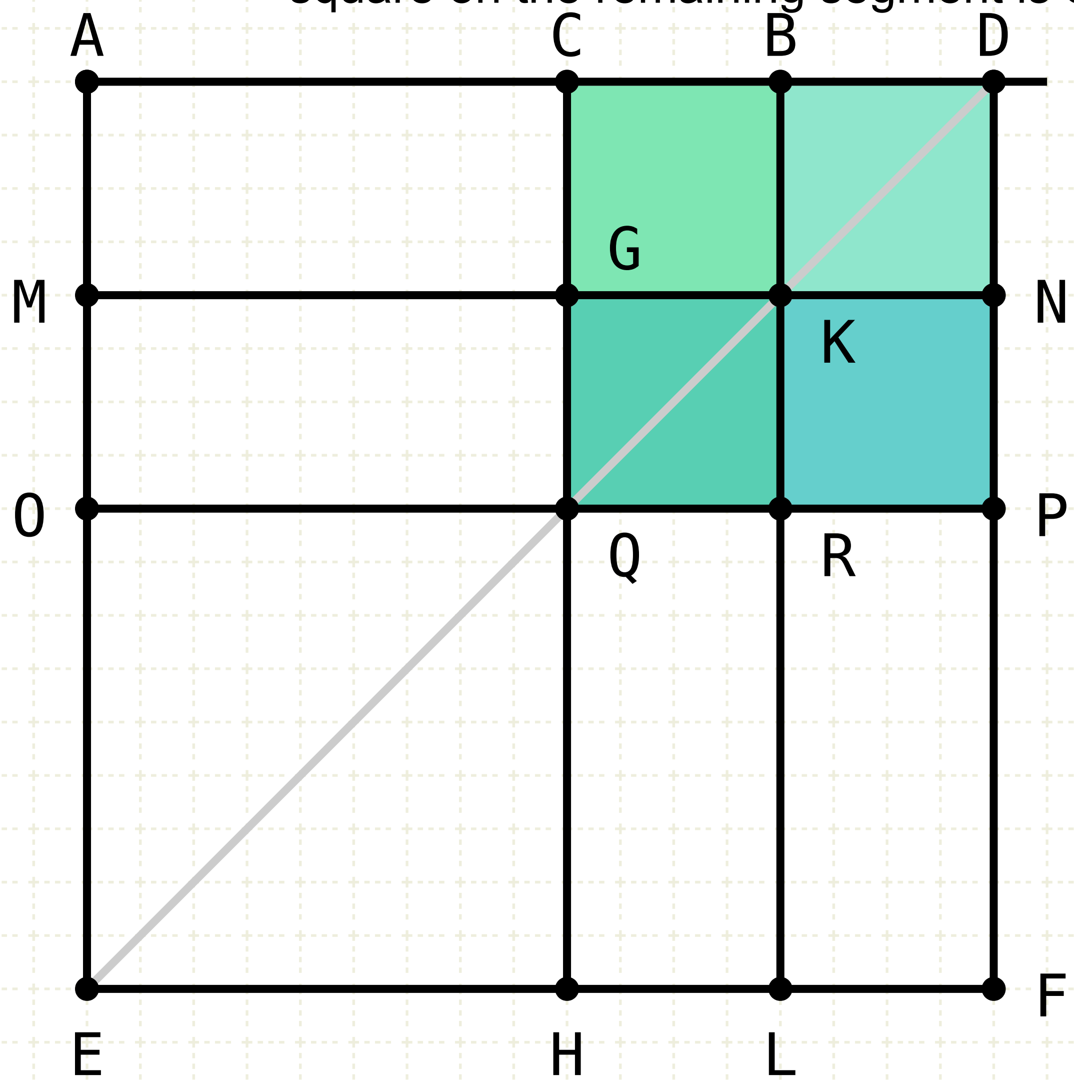
If a straight line be cut at random, four times the rectangle contained by the whole and one of the segments together with the square on the remaining segment is equal to the square described on the whole and aforesaid segment as on one straight line.



AB = AC+CB, CB = BD

GK = KN

QR = RP

□CK = □BN

□GR = □KP

In other words

Let AB be a straight line, arbitrarily cut at point C

Then four times the rectangle formed by lines AB and BC plus the square of AC is equal to the square of AB added to BC

Proof:

Since CB is equal to BD, and CB is also equal to GK (I·34) and BD is equal to KN, then GK is equal to KN

Similarly, QR is equal to RP

Thus CK and BN are equal, as are GR and KP (I·36)

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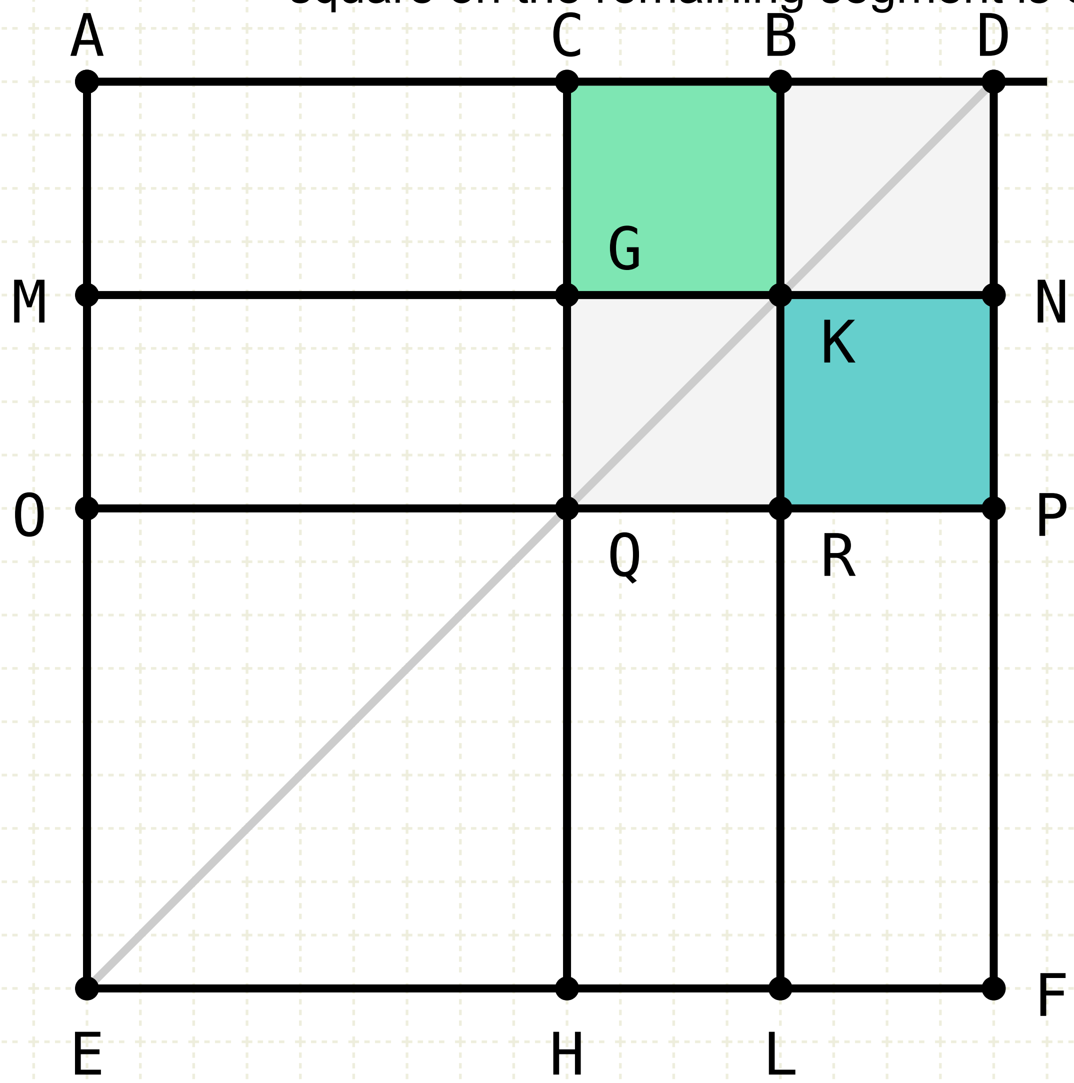

AB = AC+CB, CB = BD
GK = KN
QR = RP
□CK = □BN
□GR = □KP
□CK = □KP

In other words

Let AB be a straight line, arbitrarily cut at point C

Then four times the rectangle formed by lines AB and BC plus the square of AC is equal to the square of AB added to BC

Proof:

Since CB is equal to BD, and CB is also equal to GK (I·34) and BD is equal to KN, then GK is equal to KN

Similarly, QR is equal to RP

Thus CK and BN are equal, as are GR and KP (I·36)

CK and KP are equal (I·43)

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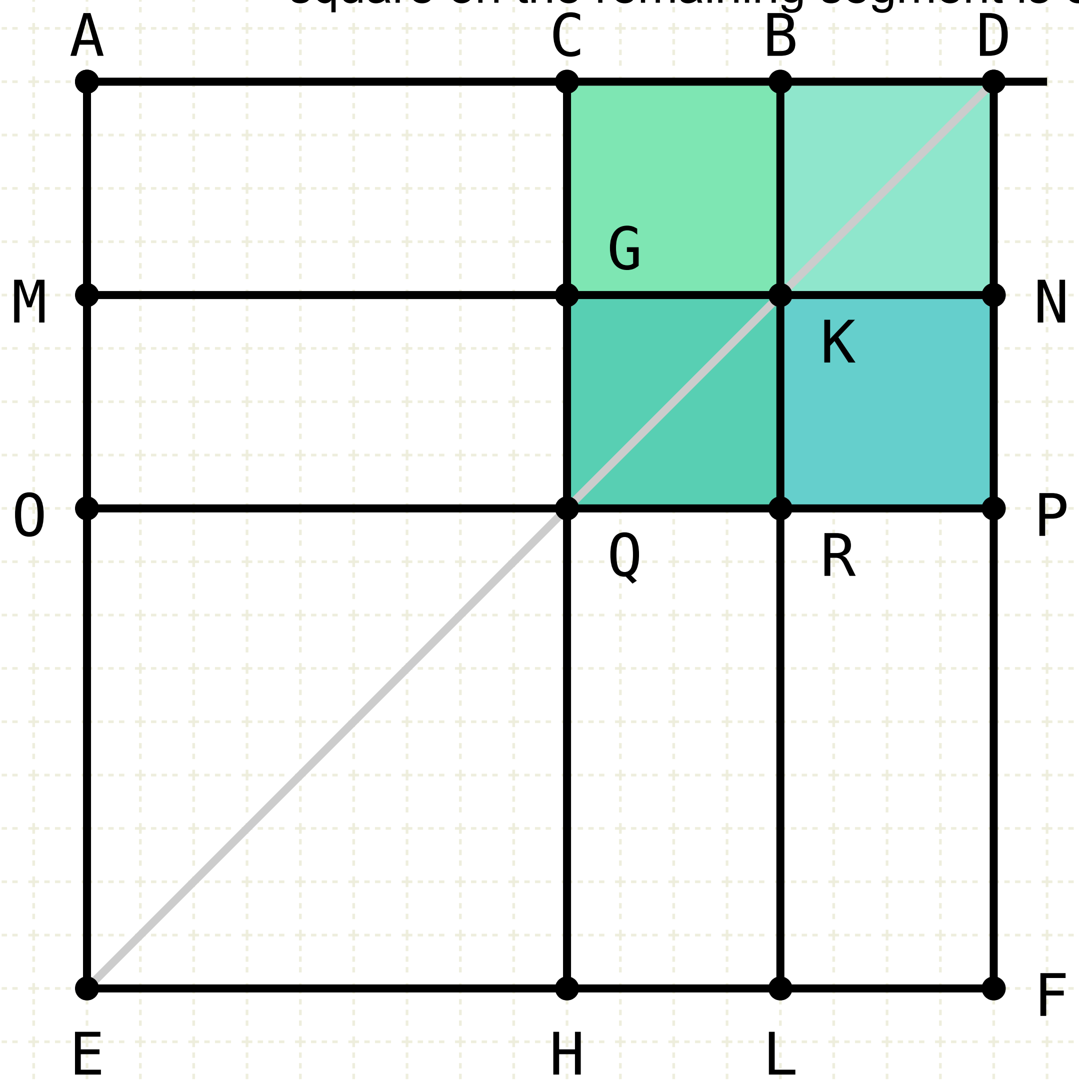

AB = AC+CB, CB = BD

GK = KN

QR = RP

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In other words

Let AB be a straight line, arbitrarily cut at point C

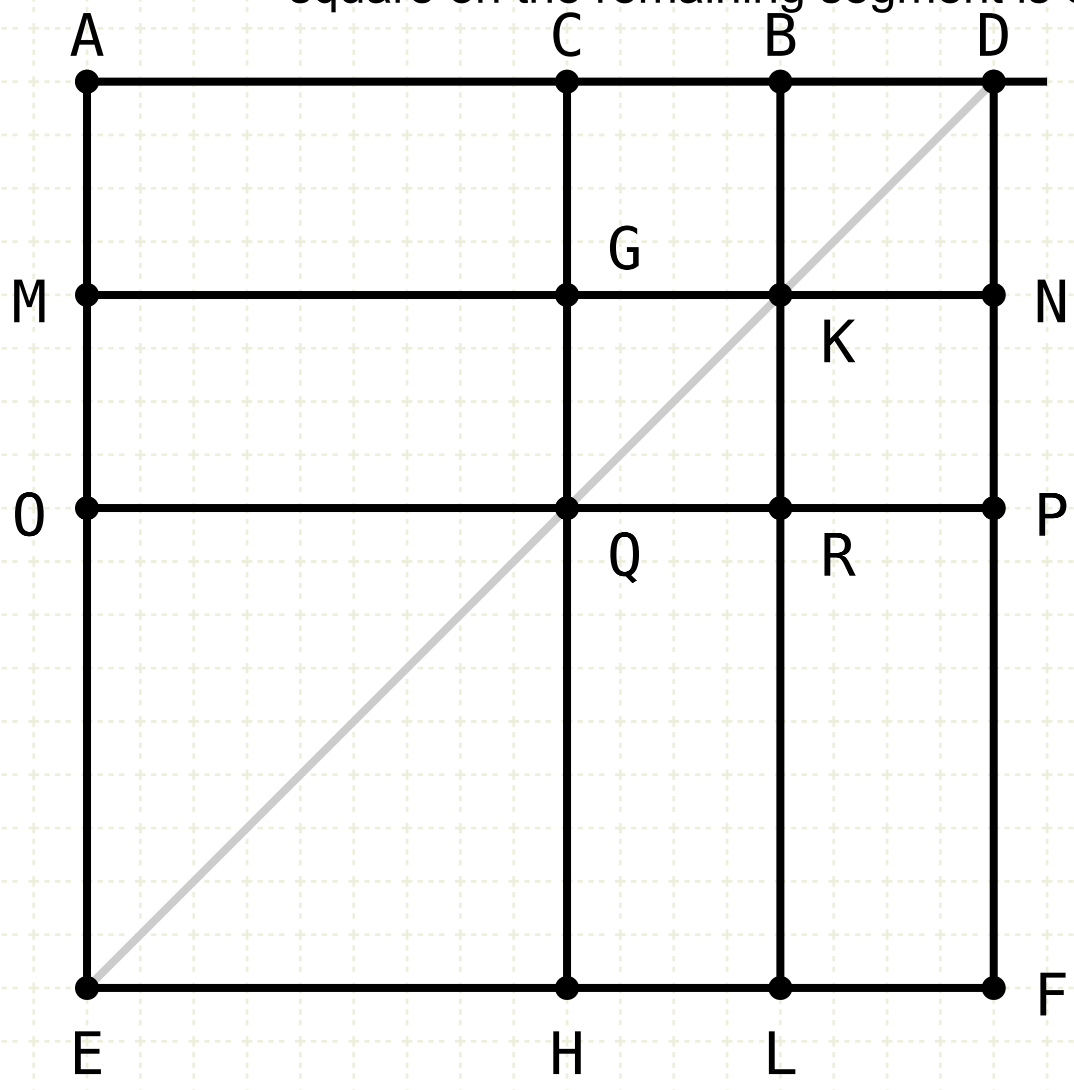
Then four times the rectangle formed by lines AB and BC plus the square of AC is equal to the square of AB added to BC

Proof:

Therefore CK, BN, GR, KP are all equal, and the sum equals four CK

Proposition 8 of Book II

If a straight line be cut at random, four times the rectangle contained by the whole and one of the segments together with the square on the remaining segment is equal to the square described on the whole and aforesaid segment as on one straight line.



AB = AC+CB, CB = BD

GK = KN

QR = RP

□CK = □BN

□GR = □KP

□CK = □KP

CG = GQ

□CK = □BN = □GR = □KP

In other words

Let AB be a straight line, arbitrarily cut at point C

Then four times the rectangle formed by lines AB and BC plus the square of AC is equal to the square of AB added to BC

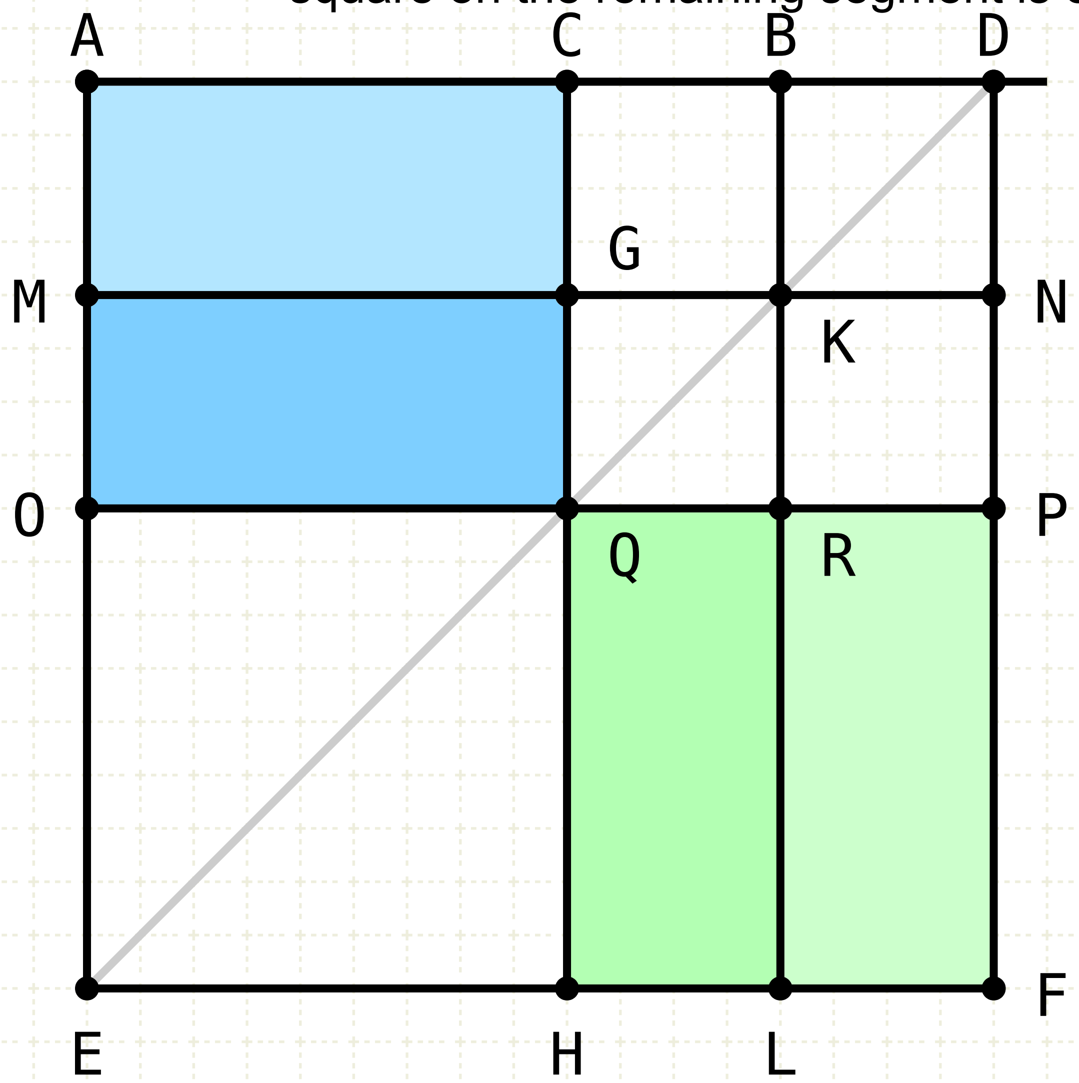
Proof:

Therefore CK, BN, GR, KP are all equal, and the sum equals four CK

Again, since CB is equal to BD, and BD is also equal to BK which is equal to CG, and CB is equal to GK, which is equal to GQ, then CG is equal to GQ

Proposition 8 of Book II

If a straight line be cut at random, four times the rectangle contained by the whole and one of the segments together with the square on the remaining segment is equal to the square described on the whole and aforesaid segment as on one straight line.



AB = AC+CB, CB = BD
GK = KN
QR = RP
□CK = □BN
□GR = □KP
□CK = □KP
CG = GQ
□AG = □MQ
□QL = □RF

□CK = □BN = □GR = □KP

In other words

Let AB be a straight line, arbitrarily cut at point C

Then four times the rectangle formed by lines AB and BC plus the square of AC is equal to the square of AB added to BC

Proof:

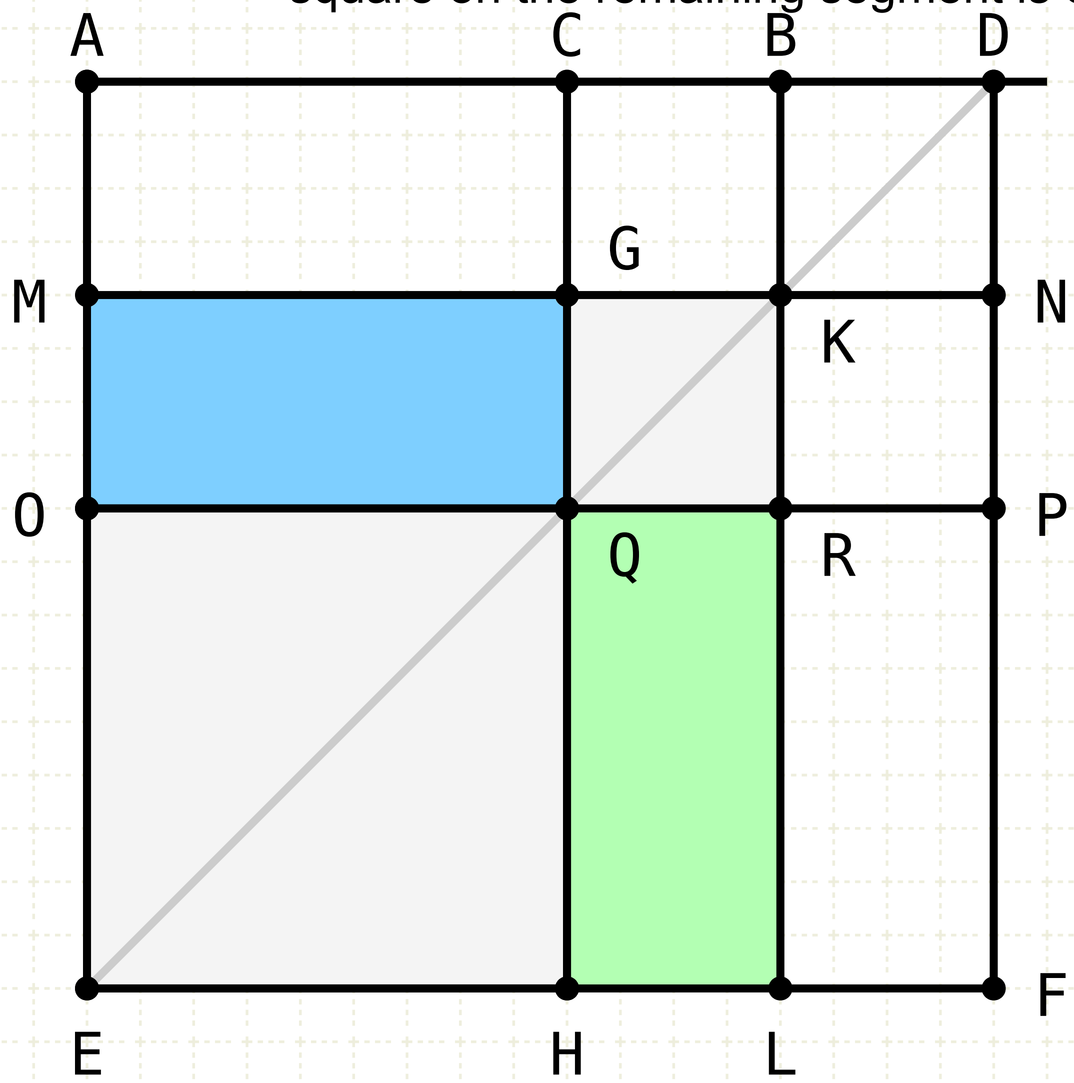
Therefore CK, BN, GR, KP are all equal, and the sum equals four CK

Again, since CB is equal to BD, and BD is also equal to BK which is equal to CG, and CB is equal to GK, which is equal to GQ, then CG is equal to GQ

Thus AG and MQ are equal, as are QL and RF (I·36)

Proposition 8 of Book II

If a straight line be cut at random, four times the rectangle contained by the whole and one of the segments together with the square on the remaining segment is equal to the square described on the whole and aforesaid segment as on one straight line.



AB = AC+CB, CB = BD

GK = KN

QR = RP

□CK = □BN

□GR = □KP

□CK = □KP

CG = GQ

□AG = □MQ

□QL = □RF

□MQ = □QL

□CK = □BN = □GR = □KP

In other words

Let AB be a straight line, arbitrarily cut at point C

Then four times the rectangle formed by lines AB and BC plus the square of AC is equal to the square of AB added to BC

Proof:

Therefore CK, BN, GR, KP are all equal, and the sum equals four CK

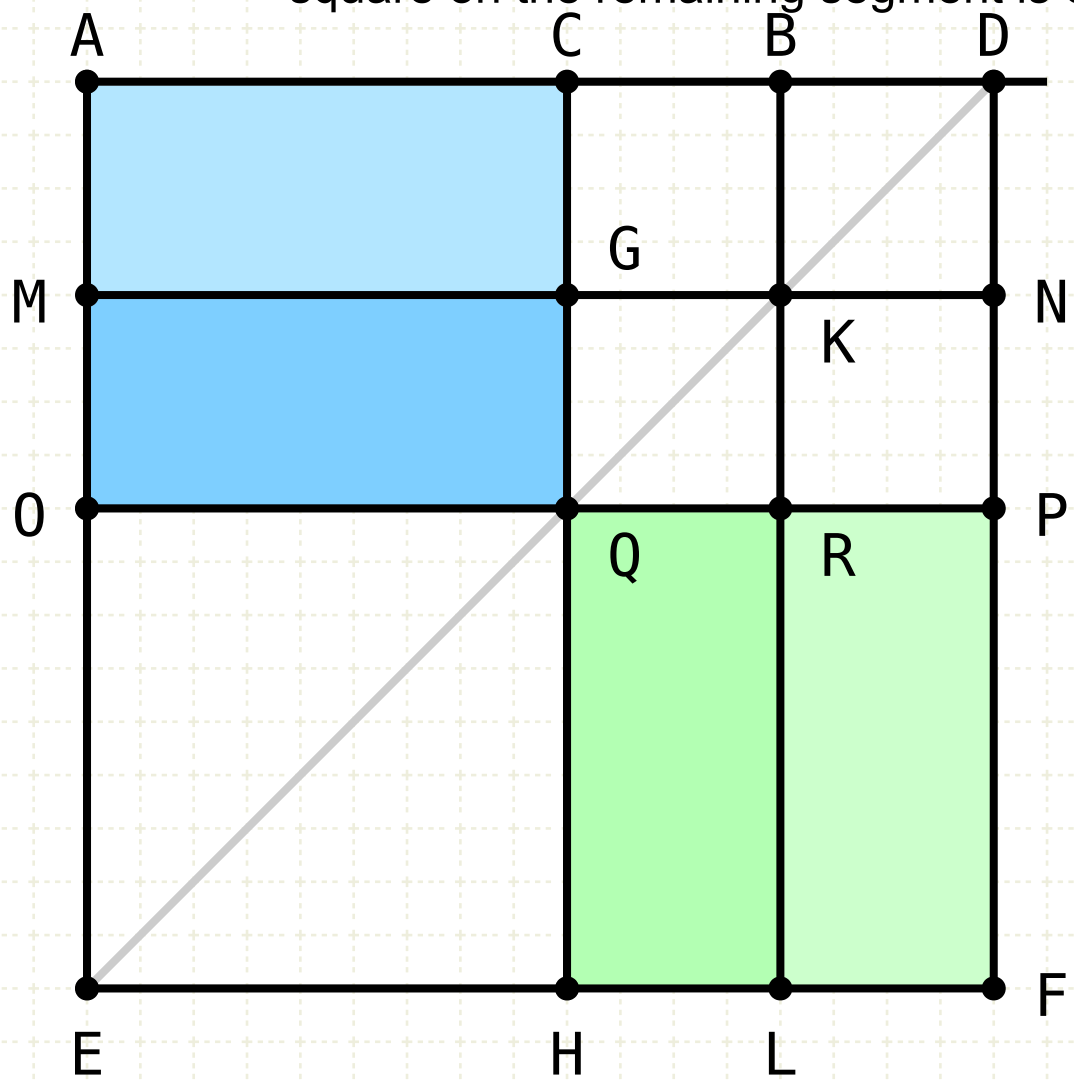
Again, since CB is equal to BD, and BD is also equal to BK which is equal to CG, and CB is equal to GK, which is equal to GQ, then CG is equal to GQ

Thus AG and MQ are equal, as are QL and RF (I·36)

MQ and QL are equal (I·43)

Proposition 8 of Book II

If a straight line be cut at random, four times the rectangle contained by the whole and one of the segments together with the square on the remaining segment is equal to the square described on the whole and aforesaid segment as on one straight line.



AB = AC+CB, CB = BD
GK = KN
QR = RP
□CK = □BN
□GR = □KP
□CK = □KP
CG = GQ
□AG = □MQ
□QL = □RF
□MQ = □QL

□CK = □BN = □GR = □KP
□MQ = □QL = □AG = □RF

In other words

Let AB be a straight line, arbitrarily cut at point C

Then four times the rectangle formed by lines AB and BC plus the square of AC is equal to the square of AB added to BC

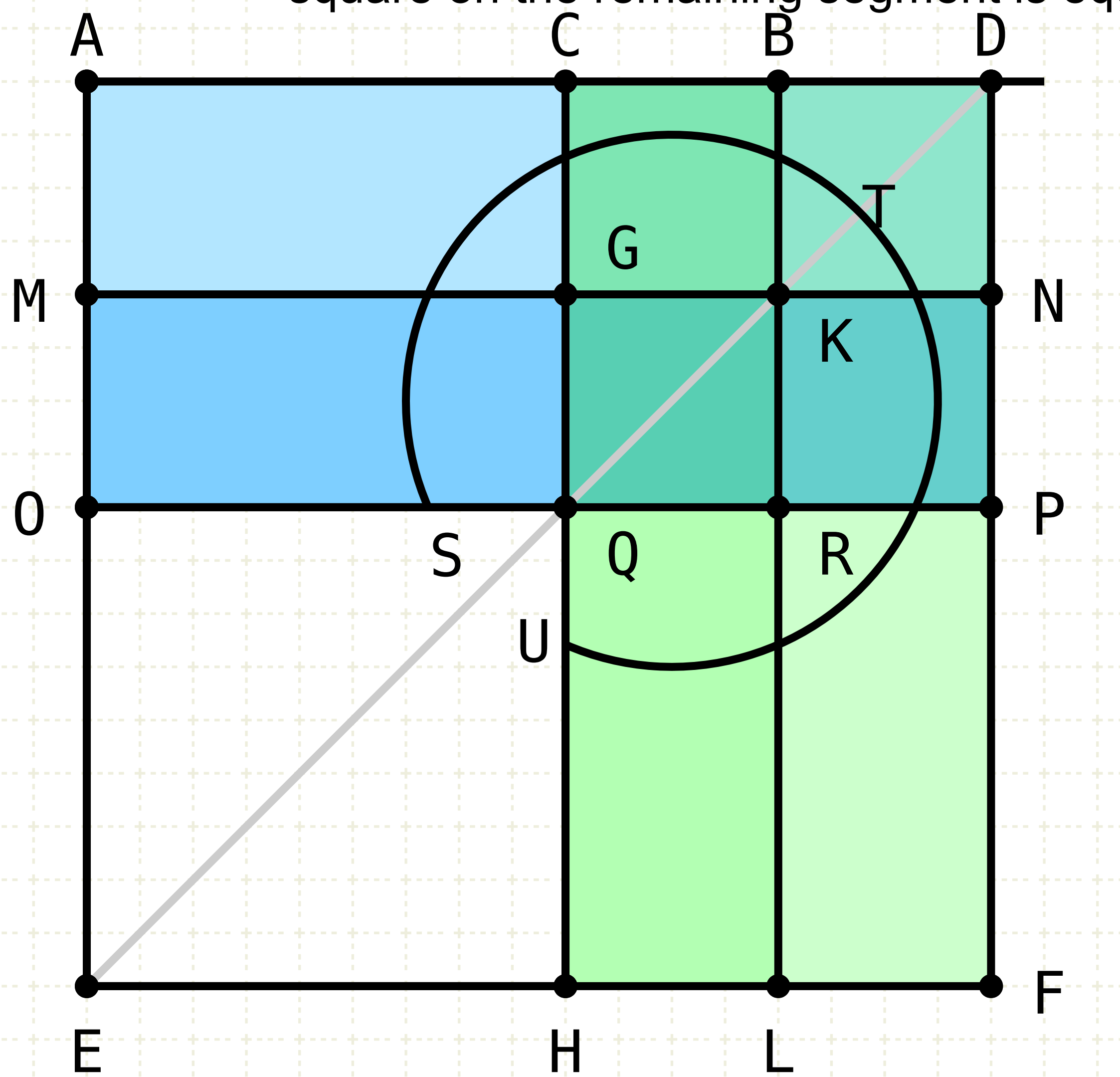
Proof:

Therefore CK, BN, GR, KP are all equal, and the sum equals four CK

Therefore MQ,QL,AG,RF are all equal, and the sum of the areas is four AG

Proposition 8 of Book II

If a straight line be cut at random, four times the rectangle contained by the whole and one of the segments together with the square on the remaining segment is equal to the square described on the whole and aforesaid segment as on one straight line.



AB = AC+CB, CB = BD
GK = KN
QR = RP
□CK = □BN
□GR = □KP
□CK = □KP
CG = GQ
□AG = □MQ
□QL = □RF
□MQ = □QL

□CK = □BN = □GR = □KP
□MQ = □QL = □AG = □RF
STU = 4·(□AG + □CK) = 4□AK

In other words

Let AB be a straight line, arbitrarily cut at point C

Then four times the rectangle formed by lines AB and BC plus the square of AC is equal to the square of AB added to BC

Proof:

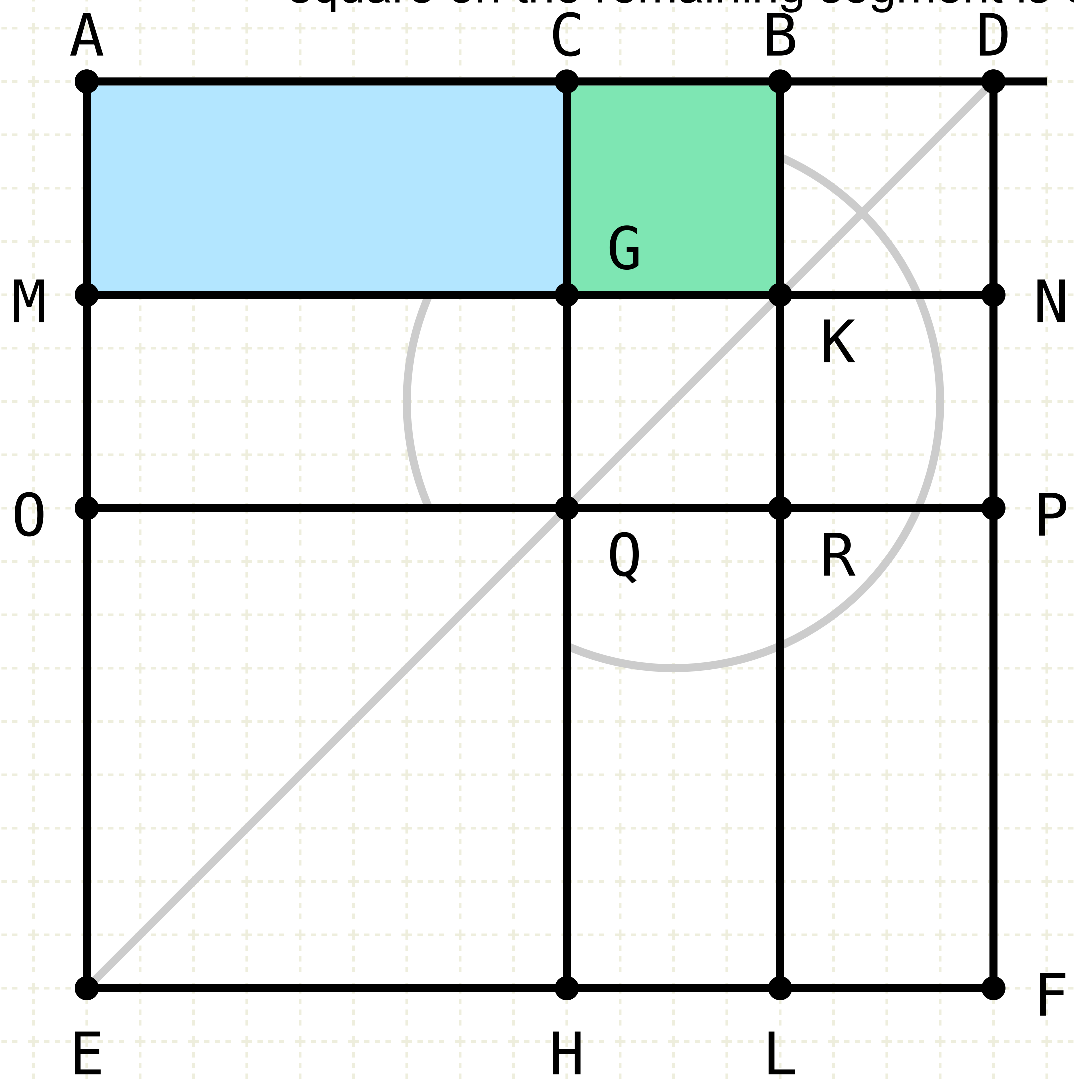
Therefore CK, BN, GR, KP are all equal, and the sum equals four CK

Therefore MQ,QL,AG,RF are all equal, and the sum of the areas is four AG

The gnomon STU is equal to the sum of all eight areas, which is also equal to four times AG plus CK

Proposition 8 of Book II

If a straight line be cut at random, four times the rectangle contained by the whole and one of the segments together with the square on the remaining segment is equal to the square described on the whole and aforesaid segment as on one straight line.



$AB = AC+CB, \ CB = BD$

$GK = KN$

$QR = RP$

$\square CK = \square BN$

$\square GR = \square KP$

$\square CK = \square KP$

$CG = GQ$

$\square AG = \square MQ$

$\square QL = \square RF$

$\square MQ = \square QL$

$\square CK = \square BN = \square GR = \square KP$

$\square MQ = \square QL = \square AG = \square RF$

$STU = 4 \cdot (\square AG + \square CK) = 4 \square AK$

$STU = 4 \square AK = 4 \cdot AB \cdot BD \quad \therefore \ STU = 4 \cdot AB \cdot BD$

In other words

Let AB be a straight line, arbitrarily cut at point C

Then four times the rectangle formed by lines AB and BC plus the square of AC is equal to the square of AB added to BC

Proof:

Therefore CK, BN, GR, KP are all equal, and the sum equals four CK

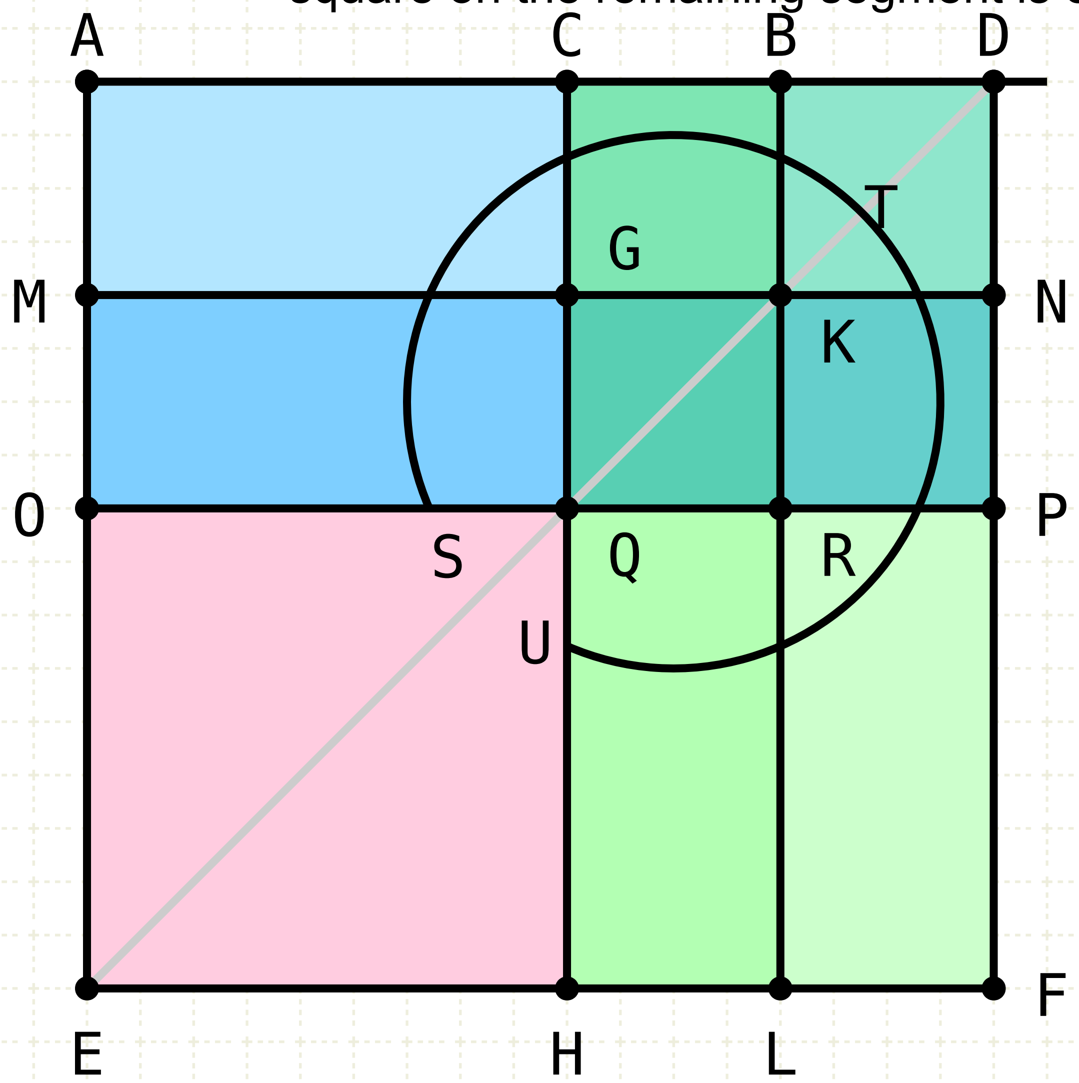
Therefore MQ,QL,AG,RF are all equal, and the sum of the areas is four AG

The gnomon STU is equal to the sum of all eight areas, which is also equal to four times AG plus CK

AK is the rectangle formed by AB, BD (since BK equals BD), hence four AK is equal to four times AB, BD, which is also equal to the gnomon STU

Proposition 8 of Book II

If a straight line be cut at random, four times the rectangle contained by the whole and one of the segments together with the square on the remaining segment is equal to the square described on the whole and aforesaid segment as on one straight line.



$AB = AC+CB,\ CB = BD$

$GK = KN$

$QR = RP$

$\square CK = \square BN$

$\square GR = \square KP$

$\square CK = \square KP$

$CG = GQ$

$\square AG = \square MQ$

$\square QL = \square RF$

$\square MQ = \square QL$

$\square CK = \square BN = \square GR = \square KP$

$\square MQ = \square QL = \square AG = \square RF$

$STU = 4 \cdot (\square AG + \square CK) = 4\square AK$

$STU = 4\square AK = 4 \cdot AB \cdot BD \quad \therefore\ STU = 4 \cdot AB \cdot BD$

$STU + \square OH = AD \cdot AD = 4 \cdot AB \cdot BD + AC \cdot AC$

In other words

Let AB be a straight line, arbitrarily cut at point C

Then four times the rectangle formed by lines AB and BC plus the square of AC is equal to the square of AB added to BC

Proof:

Therefore CK, BN, GR, KP are all equal, and the sum equals four CK

Therefore MQ,QL,AG,RF are all equal, and the sum of the areas is four AG

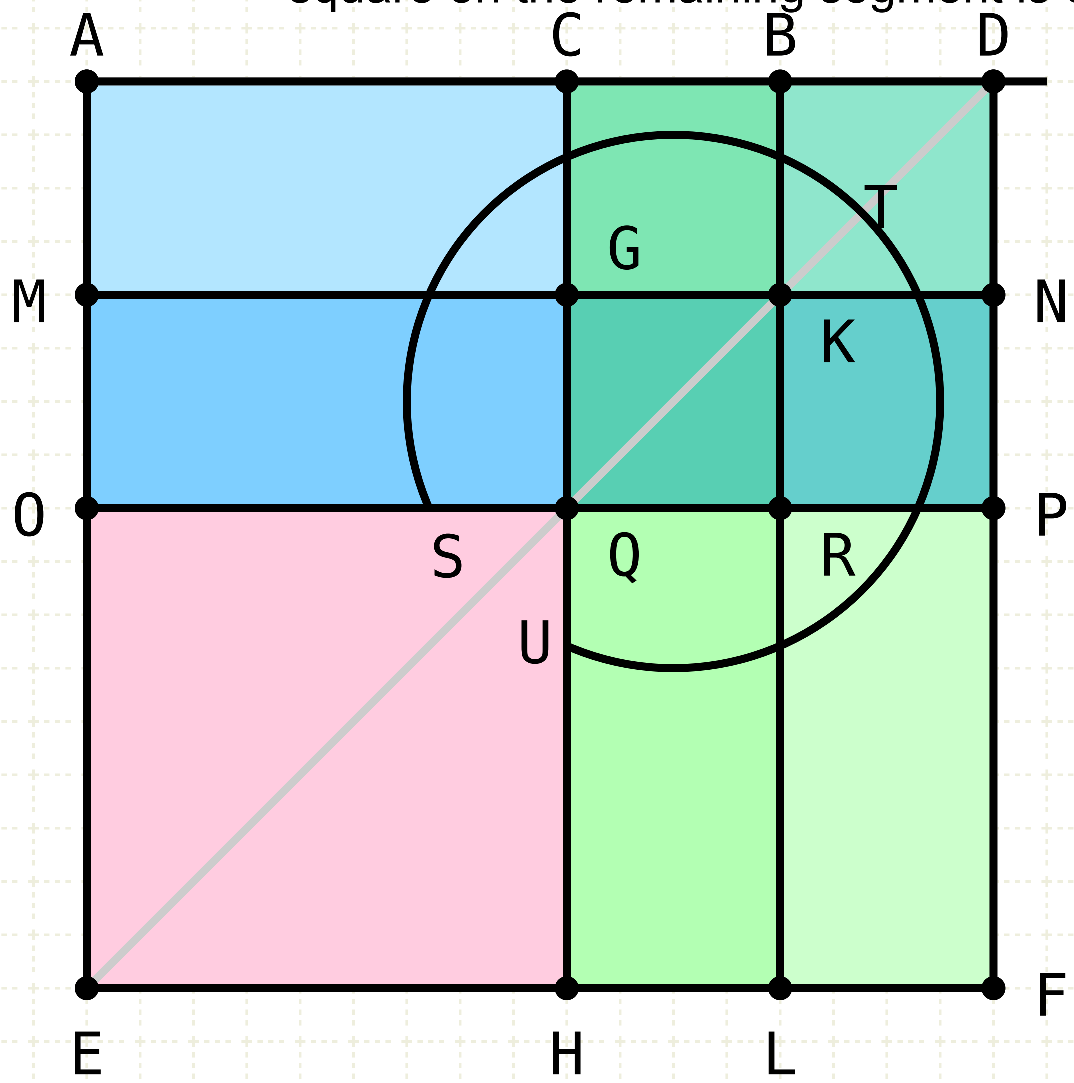
The gnomon STU is equal to the sum of all eight areas, which is also equal to four times AG plus CK

AK is the rectangle formed by AB, BD (since BK equals BD), hence four AK is equal to four times AB, BD, which is also equal to the gnomon STU

Add the square of AC (which is also equal to OH) and we have four times the rectangle AB,BD plus the square of AC is equal to the gnomon plus OH, which is equal to the square of AD

Proposition 8 of Book II

If a straight line be cut at random, four times the rectangle contained by the whole and one of the segments together with the square on the remaining segment is equal to the square described on the whole and aforesaid segment as on one straight line.



$AB = AC+CB, \quad CB = BD$
$GK = KN$
$QR = RP$
$\square CK = \square BN$
$\square GR = \square KP$
$\square CK = \square KP$
$CG = GQ$
$\square AG = \square MQ$
$\square QL = \square RF$
$\square MQ = \square QL$

$\square CK = \square BN = \square GR = \square KP$
$\square MQ = \square QL = \square AG = \square RF$
$STU = 4 \cdot (\square AG + \square CK) = 4 \square AK$
$STU = 4 \square AK = 4 \cdot AB \cdot BD \quad \therefore STU = 4 \cdot AB \cdot BD$
$STU + \square OH = AD \cdot AD = 4 \cdot AB \cdot BD + AC \cdot AC$
$(AB + BC) \cdot (AB + BC) = 4 \cdot AB \cdot BC + AC \cdot AC$

In other words

Let AB be a straight line, arbitrarily cut at point C

Then four times the rectangle formed by lines AB and BC plus the square of AC is equal to the square of AB added to BC

Proof:

Therefore CK, BN, GR, KP are all equal, and the sum equals four CK

Therefore MQ,QL,AG,RF are all equal, and the sum of the areas is four AG

The gnomon STU is equal to the sum of all eight areas, which is also equal to four times AG plus CK

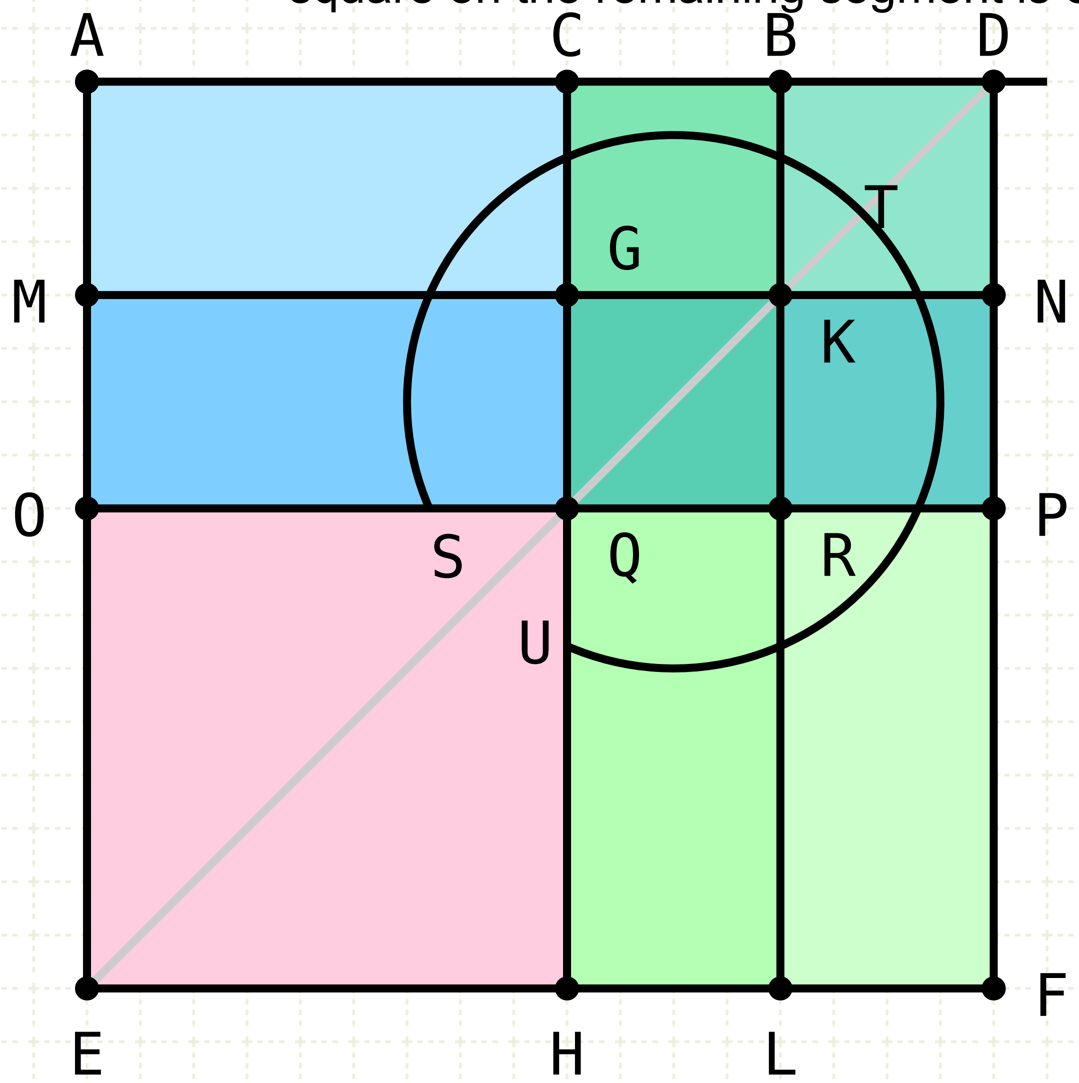
AK is the rectangle formed by AB, BD (since BK equals BD), hence four AK is equal to four times AB, BD, which is also equal to the gnomon STU

Add the square of AC (which is also equal to OH) and we have four times the rectangle AB,BD plus the square of AC is equal to the gnomon plus OH, which is equal to the square of AD

And finally, since BD is equal to CB, and AD is equal to AB with AC added in a straight line, the square of AC added with quadruple the rectangle of AB and AC, is equal to the square of AB added to BC

Proposition 8 of Book II

If a straight line be cut at random, four times the rectangle contained by the whole and one of the segments together with the square on the remaining segment is equal to the square described on the whole and aforesaid segment as on one straight line.



AB = AC+CB, CB = BD

GK = KN

QR = RP

□CK = □BN

□GR = □KP

□CK = □KP

CG = GQ

□AG = □MQ

□QL = □RF

□MQ = □QL

□CK = □BN = □GR = □KP

□MQ = □QL = □AG = □RF

STU = 4·(□AG + □CK) = 4□AK

STU = 4□AK = 4·AB·BD ∴ STU = 4·AB·BD

STU + □OH = AD·AD = 4·AB·BD + AC·AC

(AB + BC)·(AB + BC) = 4·AB·BC + AC·AC

In other words

Let AB be a straight line, arbitrarily cut at point C

Then four times the rectangle formed by lines AB and BC plus the square of AC is equal to the square of AB added to BC

Proof:

Therefore CK, BN, GR, KP are all equal, and the sum equals four CK

Therefore MQ,QL,AG,RF are all equal, and the sum of the areas is four AG

The gnomon STU is equal to the sum of all eight areas, which is also equal to four times AG plus CK

AK is the rectangle formed by AB, BD (since BK equals BD), hence four AK is equal to four times AB, BD, which is also equal to the gnomon STU

Add the square of AC (which is also equal to OH) and we have four times the rectangle AB,BD plus the square of AC is equal to the gnomon plus OH, which is equal to the square of AD

And finally, since BD is equal to CB, and AD is equal to AB with AC added in a straight line, the square of AC added with quadruple the rectangle of AB and AC, is equal to the square of AB added to BC

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