

Euclid's Elements

Book VI

One can state, without exaggeration, that the observation of and the search for similarities and differences are the basis of all human knowledge.

Alfred Nobel



Table of Contents, Chapter 6

1	If the height of two triangles are equal, then the ratio of the areas is equal to the ratio of the bases	7	If two triangles have one angle equal to one angle, and the sides about other angles are proportional, and the remaining angles either both less or both not less than a right angle, then triangles will be equiangular	14	In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and vice versa
2	If a line cuts a triangle, parallel to its base, it will cut the sides of the triangle proportionally			15	In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and vice versa
3	If an angle of a triangle is bisected and the straight line cutting the angle also cuts the base, the segments of the base will have the same ratio as the remaining sides of the triangle	8	If in a right-angled triangle a perpendicular be drawn from the right angle to the base, the triangles adjoining the perpendicular are similar both to the whole and to one another	16	If four straight lines are proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means, and vice versa
4	If two triangles have equal angles, then the sides opposite the equal angles are proportional, as well, the sides of the triangles on either side of the equal angles are also proportional	9	From a given straight line to cut off a given fraction	17	If three straight lines are proportional, the rectangle contained by the extremes is equal to the square on the mean; and vice versa
5	If two triangles have proportional sides, the triangles will be equiangular	10	To cut a given uncut straight line similarly to a given cut straight line	18	On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure
6	If two triangles have one angle equal to one angle and the sides about the equal angles are proportional, then the triangles will be equiangular	11	To two given straight lines to find a third proportional	19	Similar triangles are to one another in the duplicate ratio of the corresponding sides
		12	To three given straight lines to find a fourth proportional		
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Table of Contents, Chapter 3

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| 20 | Similar polygons are divided into the same number of similar triangles, which have the same ratio as the wholes, and the polygons have duplicate ratios to their corresponding sides | 26 | If from a parallelogram a similar parallelogram with a common angle is subtracted, it is about the same diameter as the original | 31 | In right-angled triangles the figure on the side subtending the right angle is equal to the similar and similarly described figures on the sides containing the right angle |
| 21 | Figures which are similar to the same rectilineal figure are also similar to one another | 27 | Of all the parallelograms applied to the same straight line and deficient by parallelogrammic figures similar to a parallelogram drawn on half the said line, the largest will be one that is drawn on half of the straight line and is similar to the defect | | |
| 22 | If four straight lines are proportional, similar rectilineal figures will also be proportional; and vice versa | 28 | To a given straight line, apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one | | |
| 23 | Equiangular parallelograms have to one another the ratio compounded of the ratios of their sides | 29 | To a given straight line, apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one | | |
| 24 | In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another | 30 | To cut a finite straight line in extreme ratio | | |
| 25 | To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure | | | | |



Proposition 22 of Book VI

If four straight lines be proportional, the rectilineal figures similar and similarly described upon them will also be proportional;
and if the rectilineal figures similar and similarly described upon them be proportional,
the straight lines will themselves also be proportional



Proposition 22 of Book VI

If four straight lines be proportional, the rectilineal figures similar and similarly described upon them will also be proportional;
and if the rectilineal figures similar and similarly described upon them be proportional,
the straight lines will themselves also be proportional

In other words

If the line segments AB, CD, EF, GH are proportional, then if similar polygons are drawn on AB,CD and other similar polygons are drawn on EF,GH, these polygons will also be similar

And vice-versa

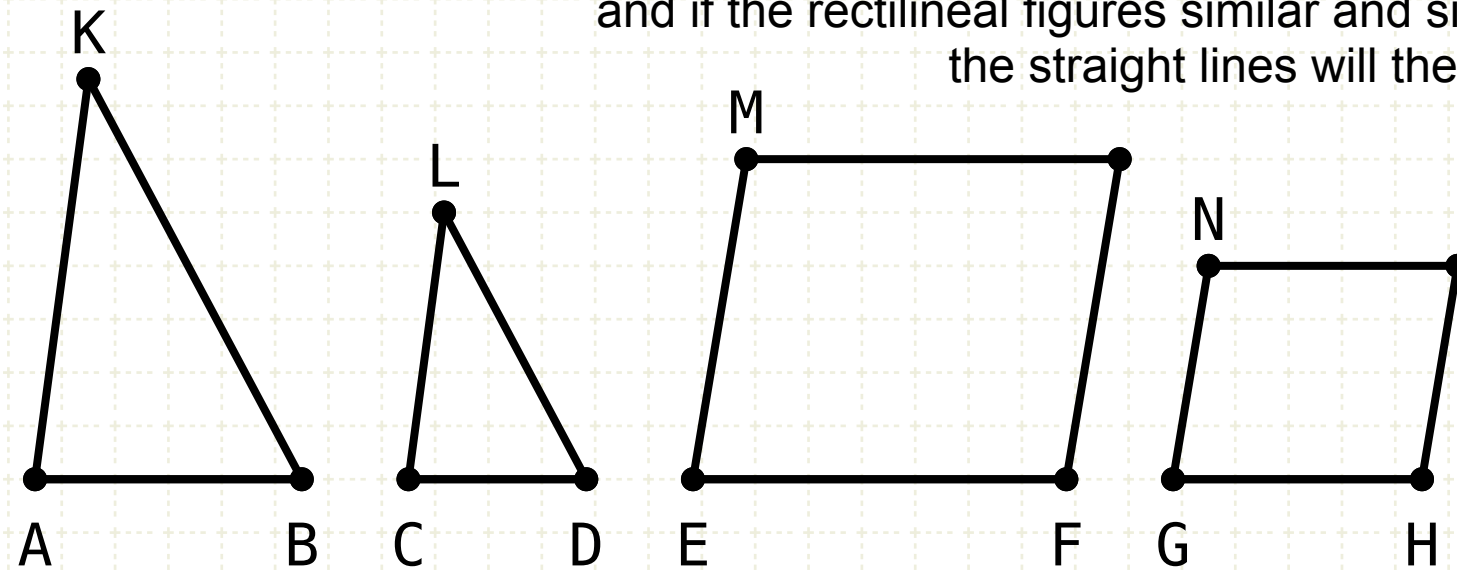


$$AB:CD = EF:GH$$



Proposition 22 of Book VI

If four straight lines be proportional, the rectilineal figures similar and similarly described upon them will also be proportional;
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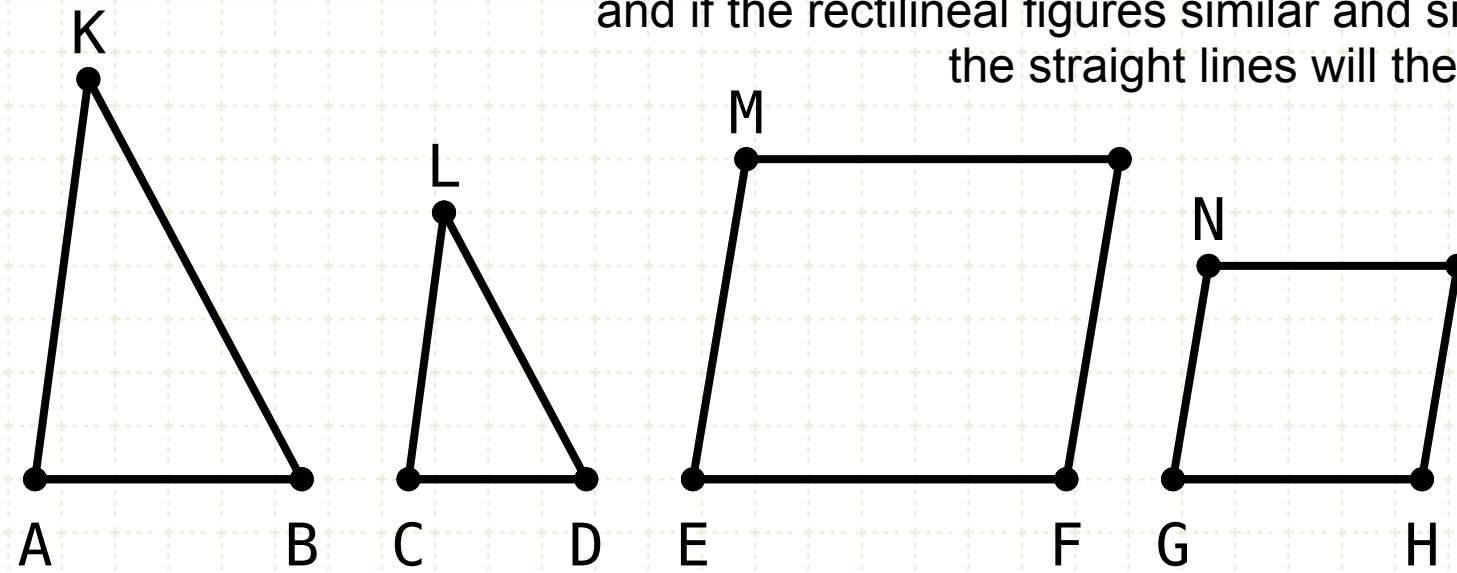
And vice-versa

$$AB:CD = EF:GH$$

$$\triangle KAB:\triangle LCD = \square MF:\square NH$$

Proposition 22 of Book VI

If four straight lines be proportional, the rectilineal figures similar and similarly described upon them will also be proportional;
and if the rectilineal figures similar and similarly described upon them be proportional,
the straight lines will themselves also be proportional

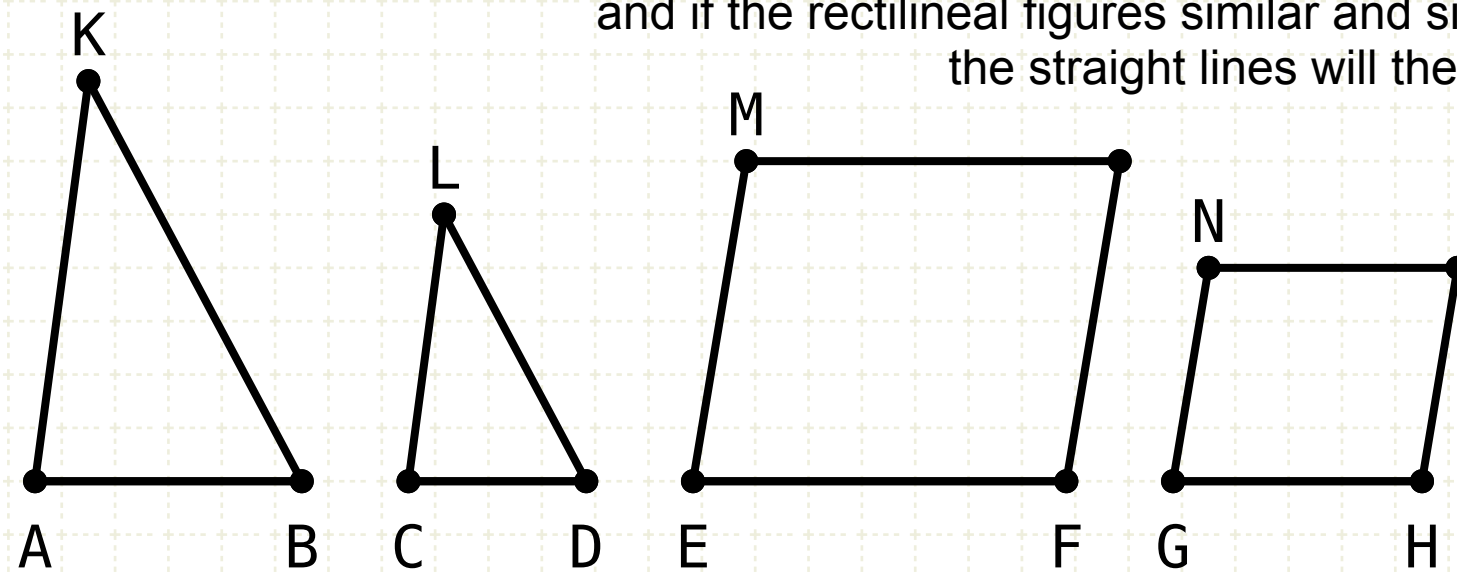


Proof - Part 1

$$AB:CD = EF:GH$$

Proposition 22 of Book VI

If four straight lines be proportional, the rectilineal figures similar and similarly described upon them will also be proportional;
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the straight lines will themselves also be proportional



O — P —

$$AB:CD = EF:GH$$

$$AB:CD = CD:O$$

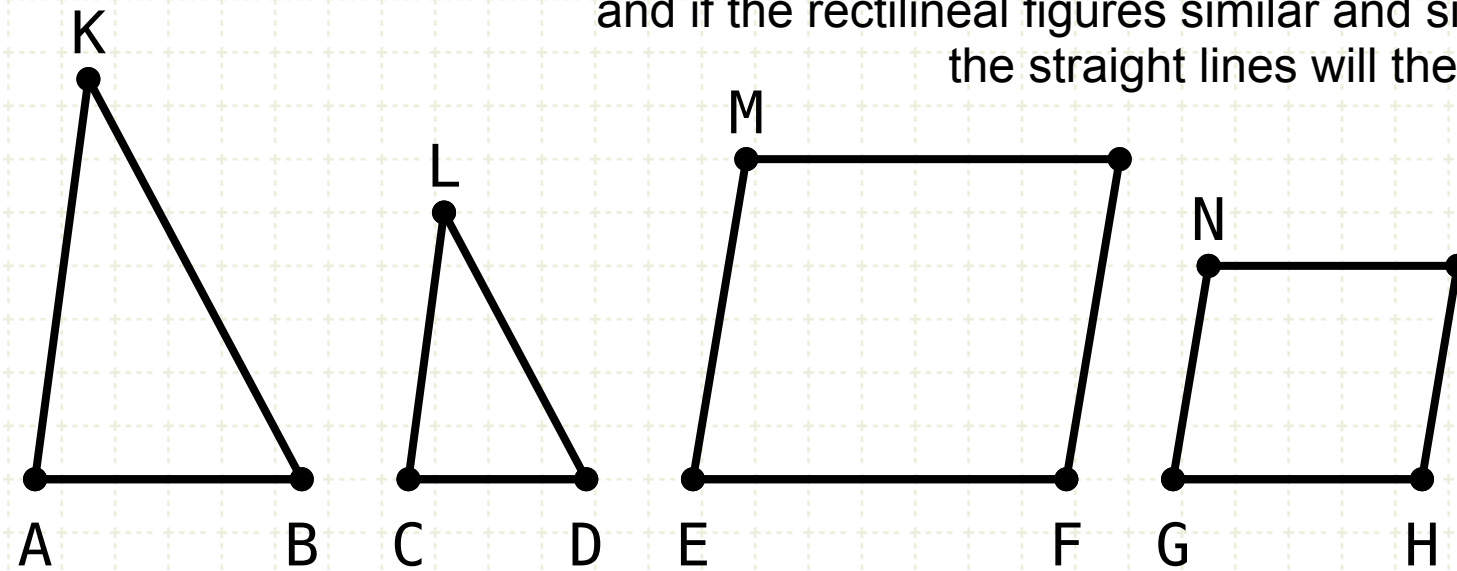
$$EF:GH = GH:P$$

Proof - Part 1

Create a third proportional (O) to AB, CD, and another third proportional (P) to EF, GH (VI·11)

Proposition 22 of Book VI

If four straight lines be proportional, the rectilineal figures similar and similarly described upon them will also be proportional;
and if the rectilineal figures similar and similarly described upon them be proportional,
the straight lines will themselves also be proportional



$$AB:CD = EF:GH$$

$$AB:CD = CD:O$$

$$EF:GH = GH:P$$

$$CD:O = GH:P$$

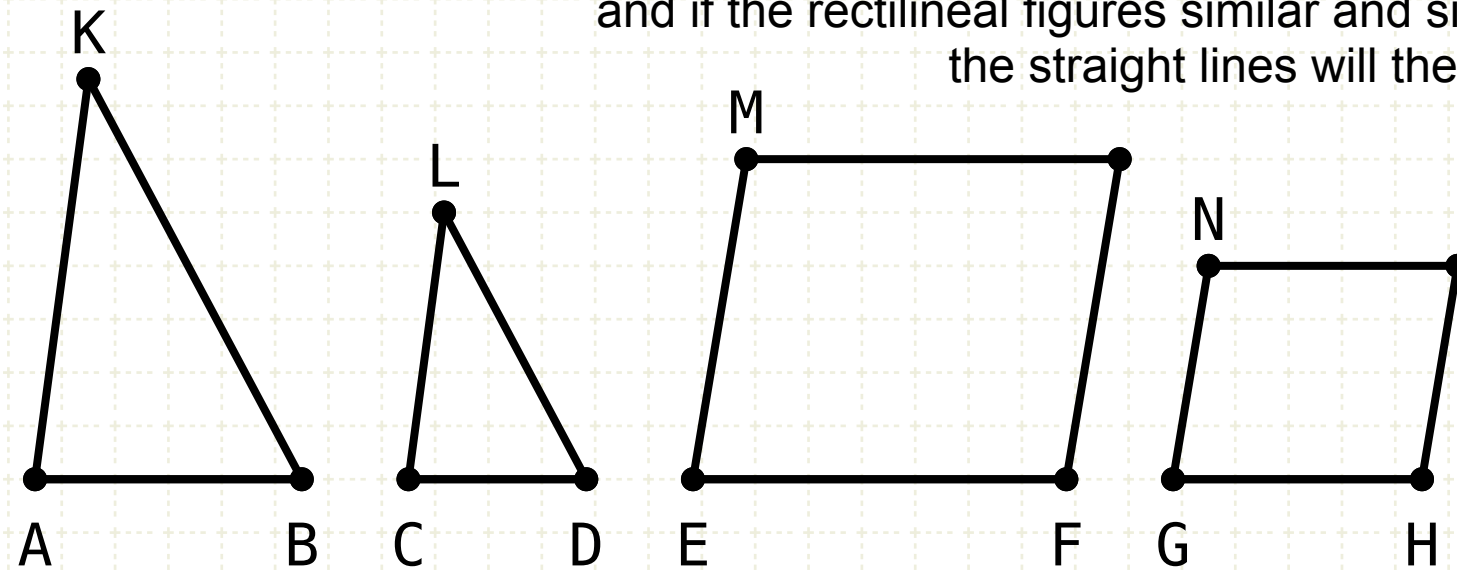
Proof - Part 1

Create a third proportional (O) to AB, CD, and another third proportional (P) to EF, GH (VI·11)

Thus the ratios CD to O and GH to P are also equal

Proposition 22 of Book VI

If four straight lines be proportional, the rectilineal figures similar and similarly described upon them will also be proportional;
and if the rectilineal figures similar and similarly described upon them be proportional,
the straight lines will themselves also be proportional



$$AB:CD = EF:GH$$

$$AB:CD = CD:O$$

$$EF:GH = GH:P$$

$$CD:O = GH:P$$

$$AB:O = EF:P$$

Proof - Part 1

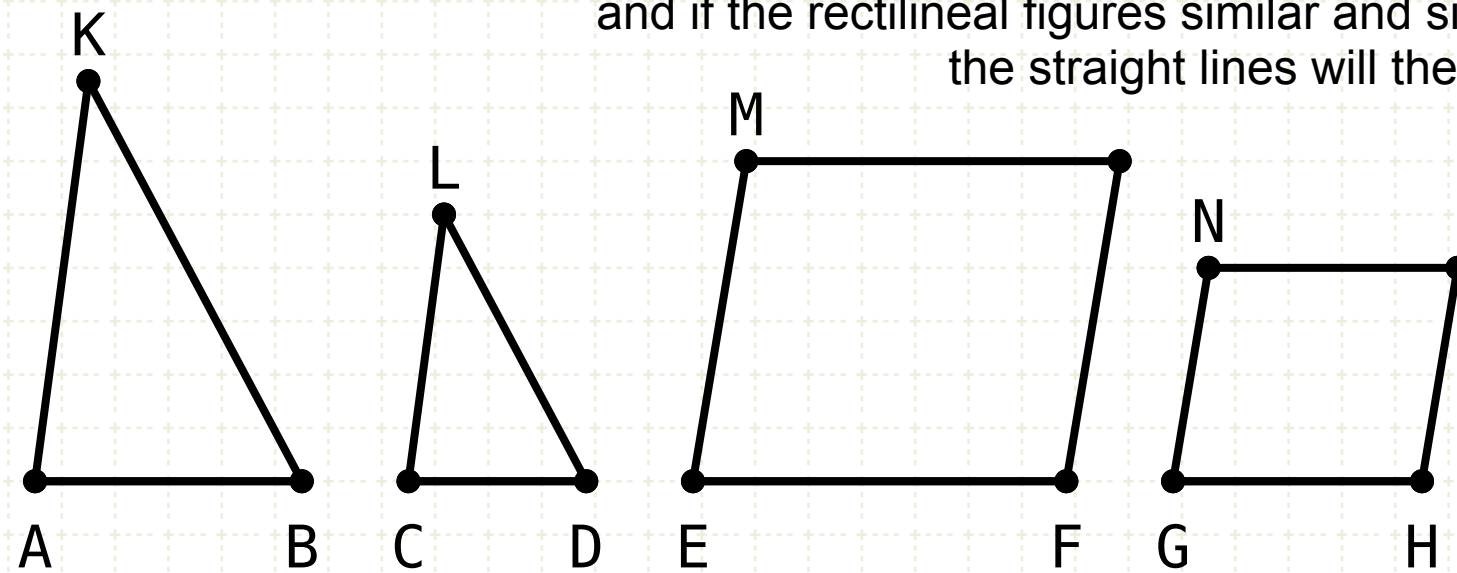
Create a third proportional (O) to AB, CD, and another third proportional (P) to EF, GH (VI·11)

Thus the ratios CD to O and GH to P are also equal

Therefore, ex aequali, AB is to O as EF is to P (V·22)

Proposition 22 of Book VI

If four straight lines be proportional, the rectilineal figures similar and similarly described upon them will also be proportional;
and if the rectilineal figures similar and similarly described upon them be proportional,
the straight lines will themselves also be proportional



$$AB:CD = EF:GH$$

$$AB:CD = CD:O$$

$$EF:GH = GH:P$$

$$CD:O = GH:P$$

$$AB:O = EF:P$$

$$\Delta KAB:\Delta LCD = AB:O$$

Proof - Part 1

Create a third proportional (O) to AB, CD, and another third proportional (P) to EF, GH (VI·11)

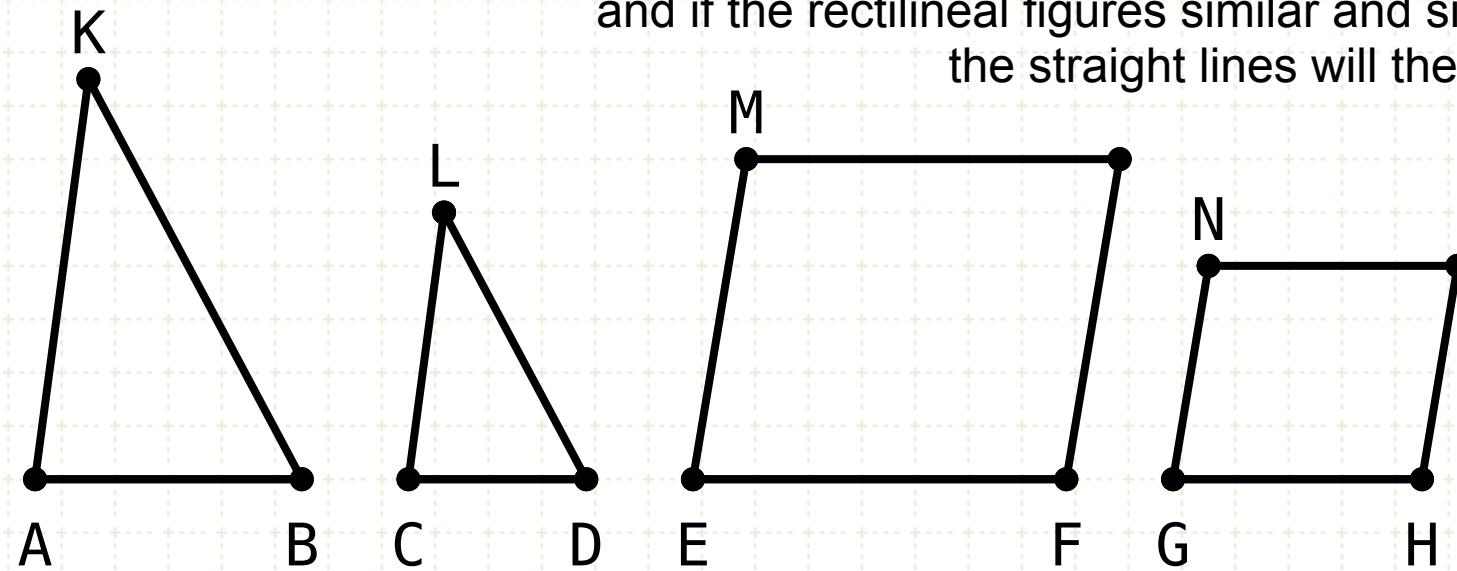
Thus the ratios CD to O and GH to P are also equal

Therefore, ex aequali, AB is to O as EF is to P (V·22)

The ratios of similar triangles is equal to the duplicate ratio of their sides about an equal angle (VI·19.Por), so KAB is to LCD as AB is to O

Proposition 22 of Book VI

If four straight lines be proportional, the rectilineal figures similar and similarly described upon them will also be proportional;
and if the rectilineal figures similar and similarly described upon them be proportional,
the straight lines will themselves also be proportional



O — P

$$AB:CD = EF:GH$$

$$AB:CD = CD:O$$

$$EF:GH = GH:P$$

$$CD:O = GH:P$$

$$AB:O = EF:P$$

$$\triangle KAB:\triangle LCD = AB:O$$

$$\square MF:\square NH = EF:P$$

Proof - Part 1

Create a third proportional (O) to AB, CD, and another third proportional (P) to EF, GH (VI·11)

Thus the ratios CD to O and GH to P are also equal

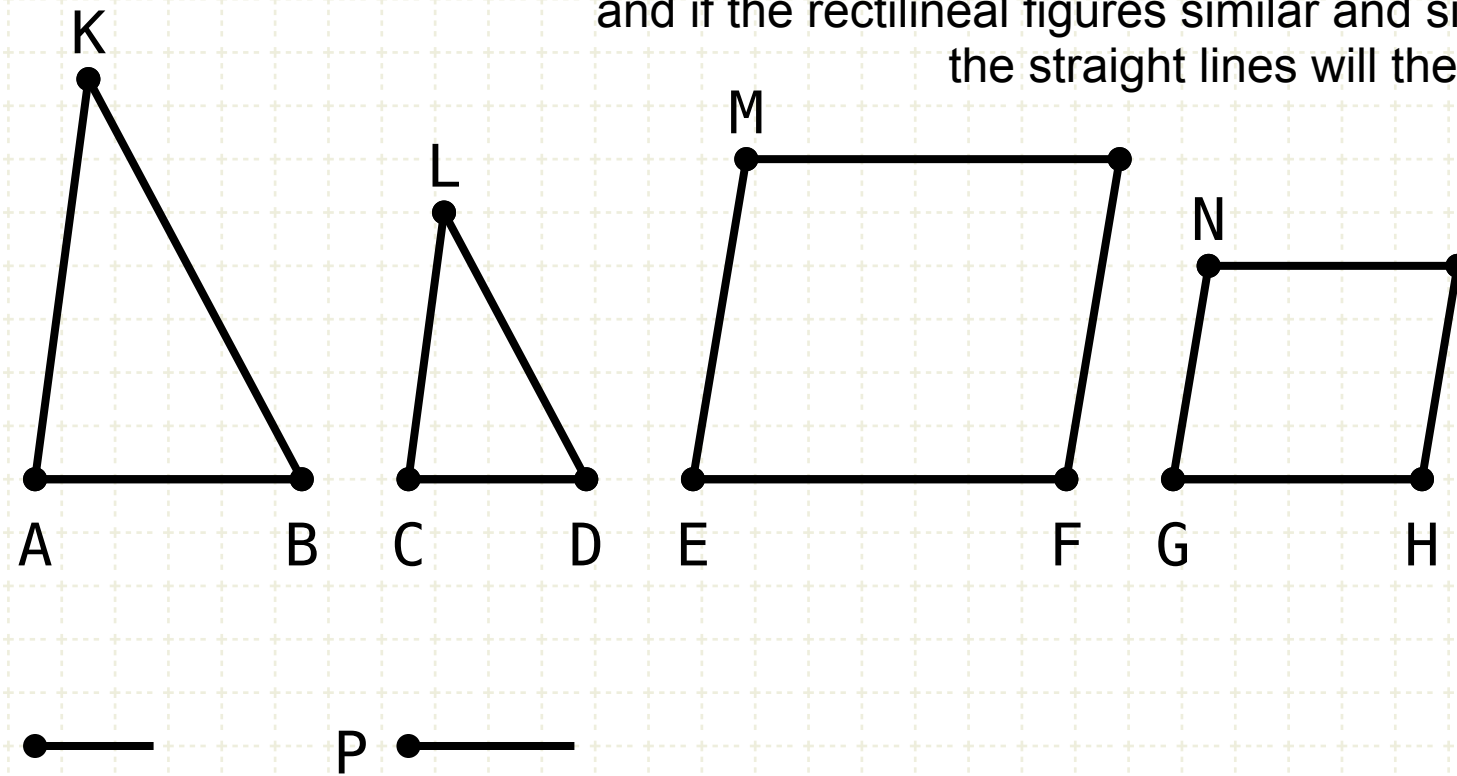
Therefore, ex aequali, AB is to O as EF is to P (V·22)

The ratios of similar triangles is equal to the duplicate ratio of their sides about an equal angle (VI·19.Por), so KAB is to LCD as AB is to O

Likewise, MF is to NH as EF is to P

Proposition 22 of Book VI

If four straight lines be proportional, the rectilineal figures similar and similarly described upon them will also be proportional;
and if the rectilineal figures similar and similarly described upon them be proportional,
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Proof - Part 1

Create a third proportional (O) to AB, CD, and another third proportional (P) to EF, GH (VI·11)

Thus the ratios CD to O and GH to P are also equal

Therefore, ex aequali, AB is to O as EF is to P (V·22)

The ratios of similar triangles is equal to the duplicate ratio of their sides about an equal angle (VI·19.Por), so KAB is to LCD as AB is to O

Likewise, MF is to NH as EF is to P

Therefore, KAB is to LCD as MF is to NH (V·11)

$$AB:CD = EF:GH$$

$$AB:CD = CD:O$$

$$EF:GH = GH:P$$

$$CD:O = GH:P$$

$$AB:O = EF:P$$

$$\Delta KAB:\Delta LCD = AB:O$$

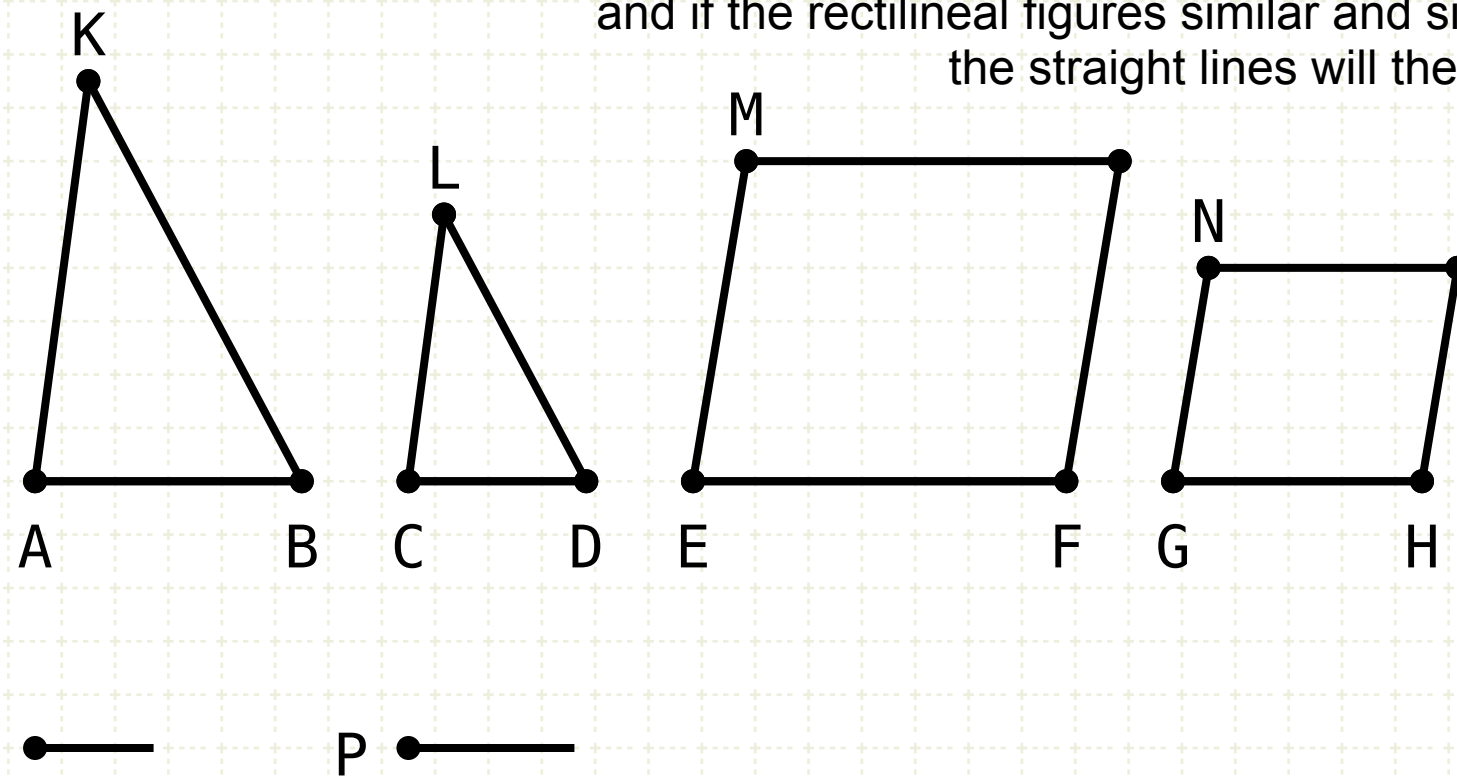
$$\square MF:\square NH = EF:P$$

$$\Delta KAB:\Delta LCD = \square MF:\square NH$$



Proposition 22 of Book VI

If four straight lines be proportional, the rectilineal figures similar and similarly described upon them will also be proportional;
and if the rectilineal figures similar and similarly described upon them be proportional,
the straight lines will themselves also be proportional



$$AB:CD = EF:GH$$

$$AB:CD = CD:O$$

$$EF:GH = GH:P$$

$$CD:O = GH:P$$

$$AB:O = EF:P$$

$$\triangle KAB:\triangle LCD = AB:O$$

$$\square MF:\square NH = EF:P$$

$$\triangle KAB:\triangle LCD = \square MF:\square NH$$

Proof - Part 1

Create a third proportional (O) to AB , CD , and another third proportional (P) to EF , GH (VI·11)

Thus the ratios CD to O and GH to P are also equal

Therefore, ex aequali, AB is to O as EF is to P (V·22)

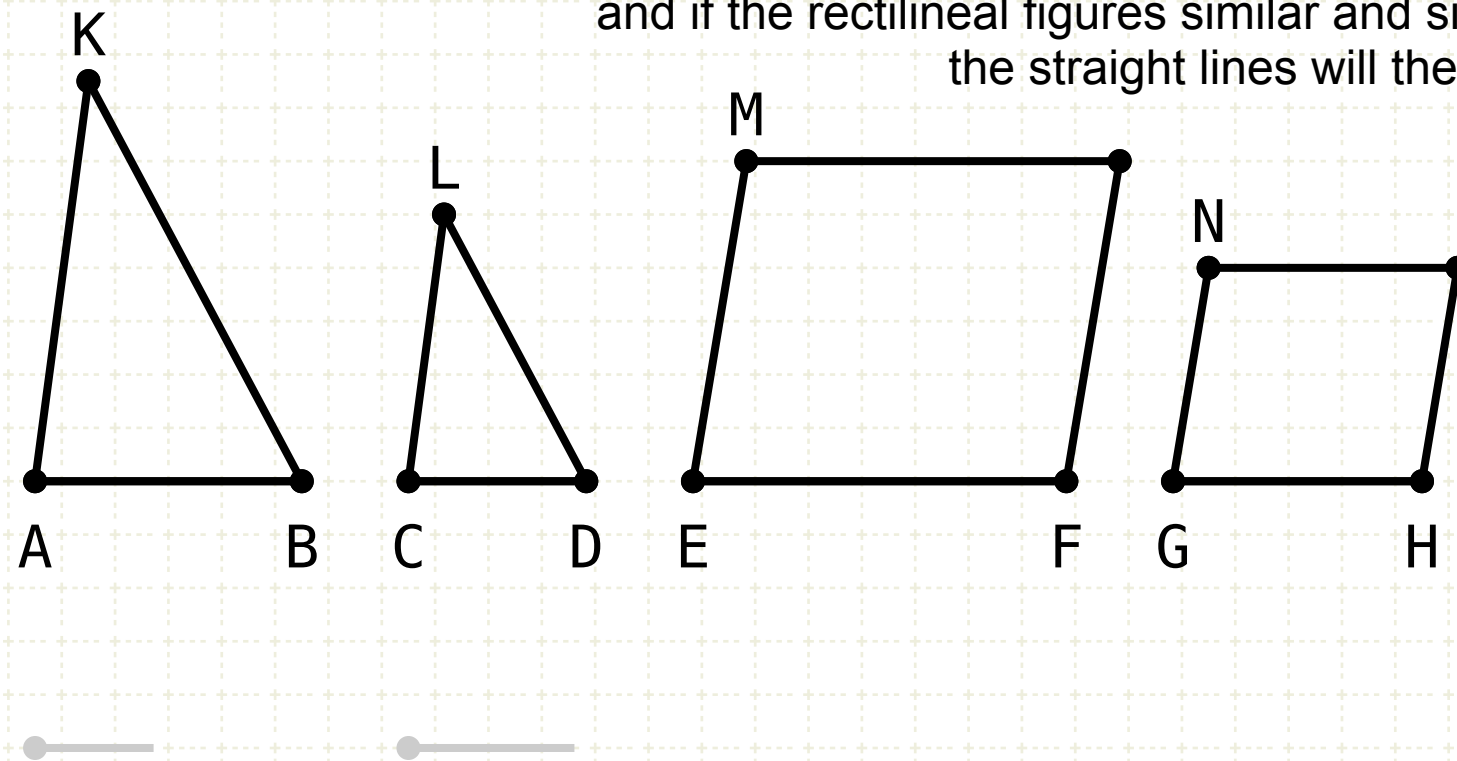
The ratios of similar triangles is equal to the duplicate ratio of their sides about an equal angle (VI·19.Por), so KAB is to LCD as AB is to O

Likewise, MF is to NH as EF is to P

Therefore, KAB is to LCD as MF is to NH (V·11)

Proposition 22 of Book VI

If four straight lines be proportional, the rectilineal figures similar and similarly described upon them will also be proportional;
and if the rectilineal figures similar and similarly described upon them be proportional,
the straight lines will themselves also be proportional



In other words - Part 2

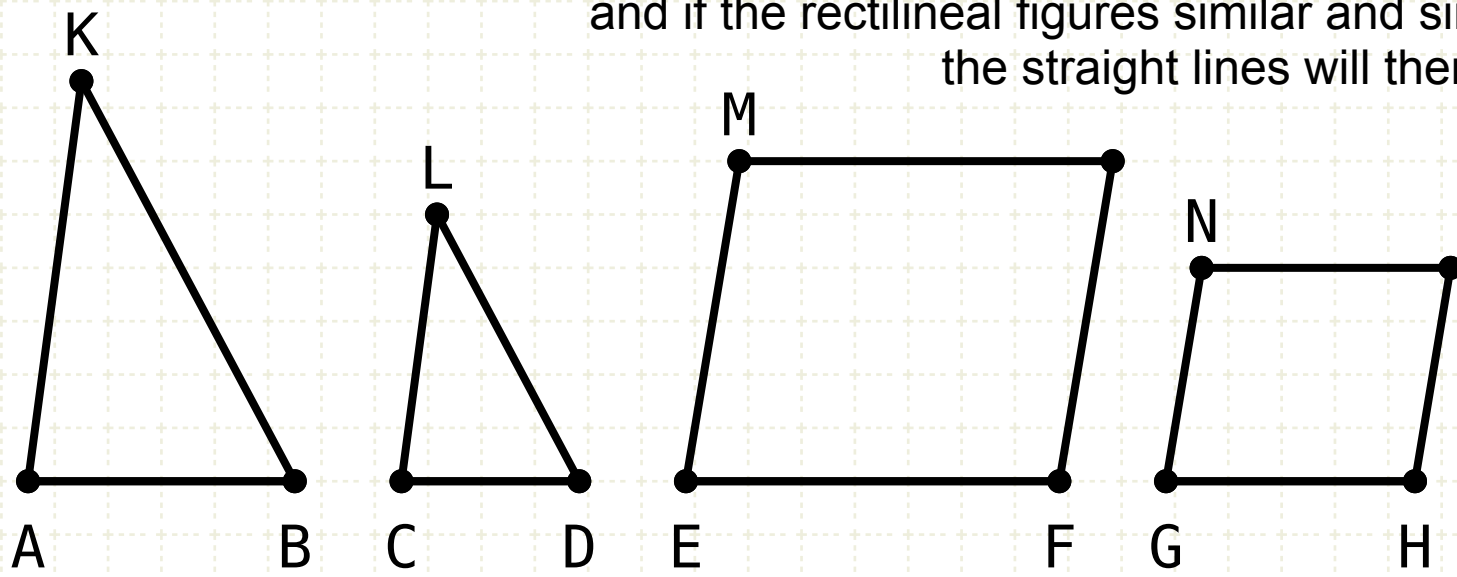
If KAB is to LCD as MF is to NH, then AB is to CD as EF is to GH

$$\triangle KAB : \triangle LCD = \square MF : \square NH$$

$$\rightarrow AB : CD = EF : GH$$

Proposition 22 of Book VI

If four straight lines be proportional, the rectilineal figures similar and similarly described upon them will also be proportional;
and if the rectilineal figures similar and similarly described upon them be proportional,
the straight lines will themselves also be proportional



$$\Delta KAB : \Delta LCD = \square MF : \square NH$$

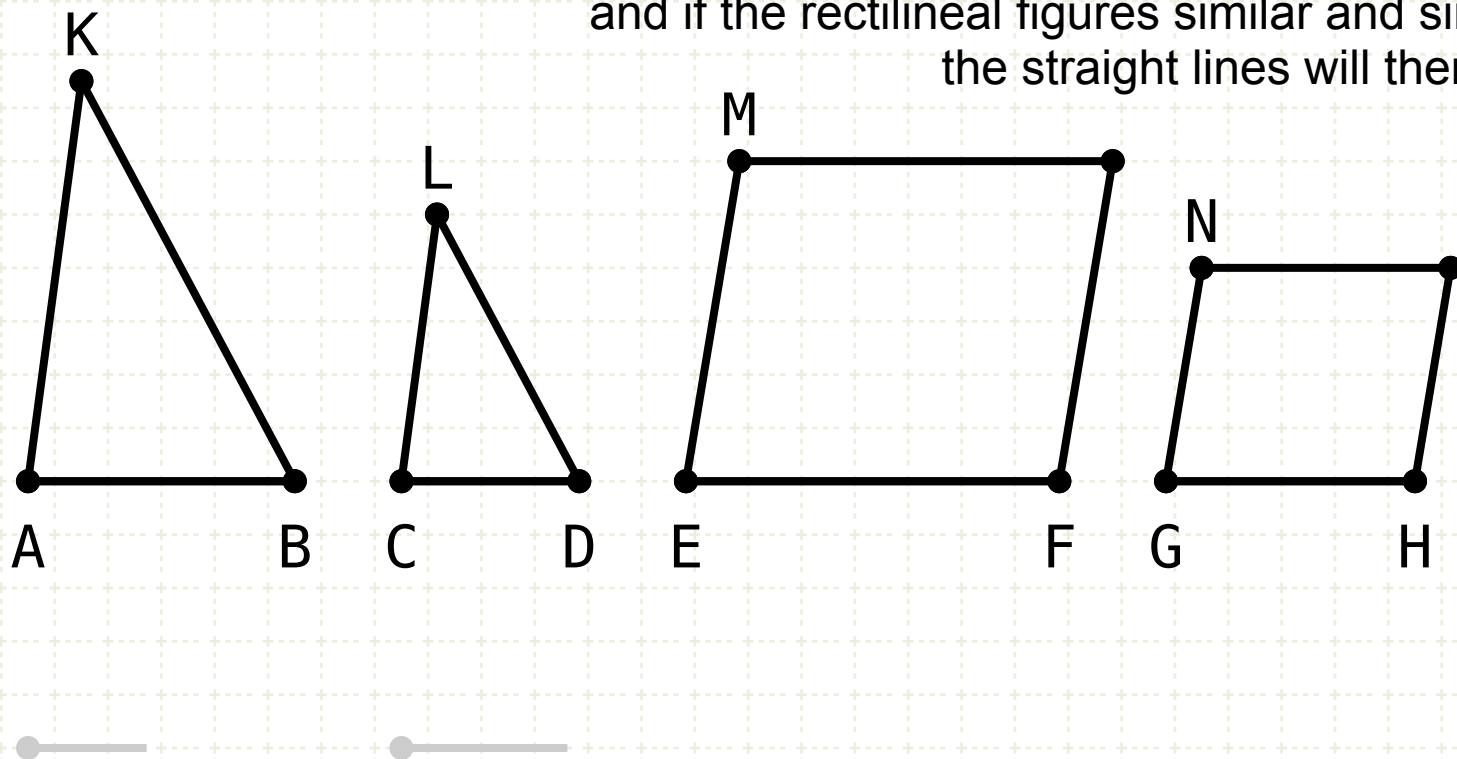
In other words - Part 2

If KAB is to LCD as MF is to NH, then AB is to CD as EF is to GH

Proof by Contradiction

Proposition 22 of Book VI

If four straight lines be proportional, the rectilineal figures similar and similarly described upon them will also be proportional;
 and if the rectilineal figures similar and similarly described upon them be proportional,
 the straight lines will themselves also be proportional



$$\Delta KAB : \Delta LCD = \square MF : \square NH$$

$$AB : CD \neq EF : GH$$

In other words - Part 2

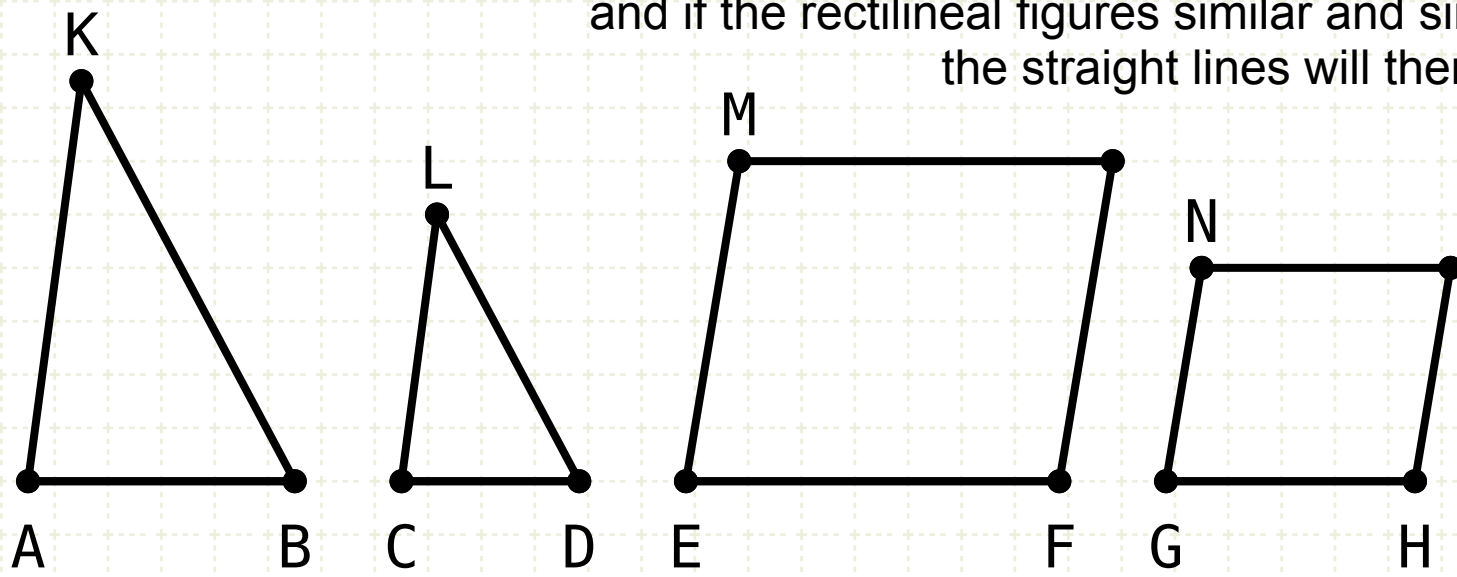
If KAB is to LCD as MF is to NH , then AB is to CD as EF is to GH

Proof by Contradiction

Assume that the ratio EF to GH is not equal to the ratio AB to CD

Proposition 22 of Book VI

If four straight lines be proportional, the rectilineal figures similar and similarly described upon them will also be proportional;
and if the rectilineal figures similar and similarly described upon them be proportional,
the straight lines will themselves also be proportional



$\Delta KAB : \Delta LCD = \square MF : \square NH$
 $AB : CD \neq EF : GH$
 $AB : CD = EF : QR$

In other words - Part 2

If KAB is to LCD as MF is to NH, then AB is to CD as EF is to GH

Proof by Contradiction

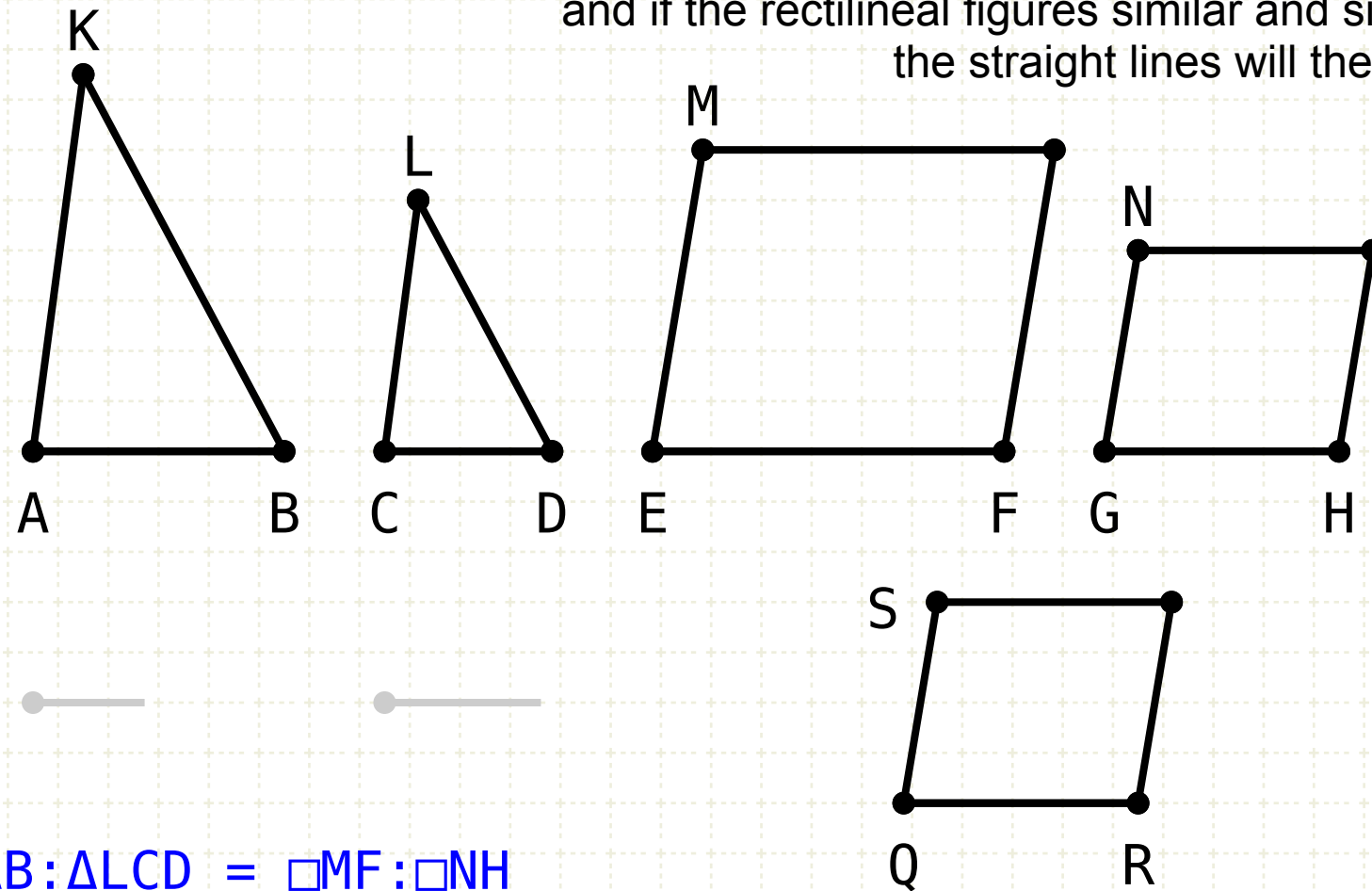
Assume that the ratio EF to GH is not equal to the ratio AB to CD

Define QR such that EF to QR is equal to AB to CD



Proposition 22 of Book VI

If four straight lines be proportional, the rectilineal figures similar and similarly described upon them will also be proportional;
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the straight lines will themselves also be proportional



$$\triangle KAB : \triangle LCD = \square MFEF : \square NHHH$$

$$AB : CD \neq EF : GH$$

$$AB : CD = EF : QR$$

$$\square SRQR \sim \square NHHH$$

In other words - Part 2

If KAB is to LCD as MF is to NH, then AB is to CD as EF is to GH

Proof by Contradiction

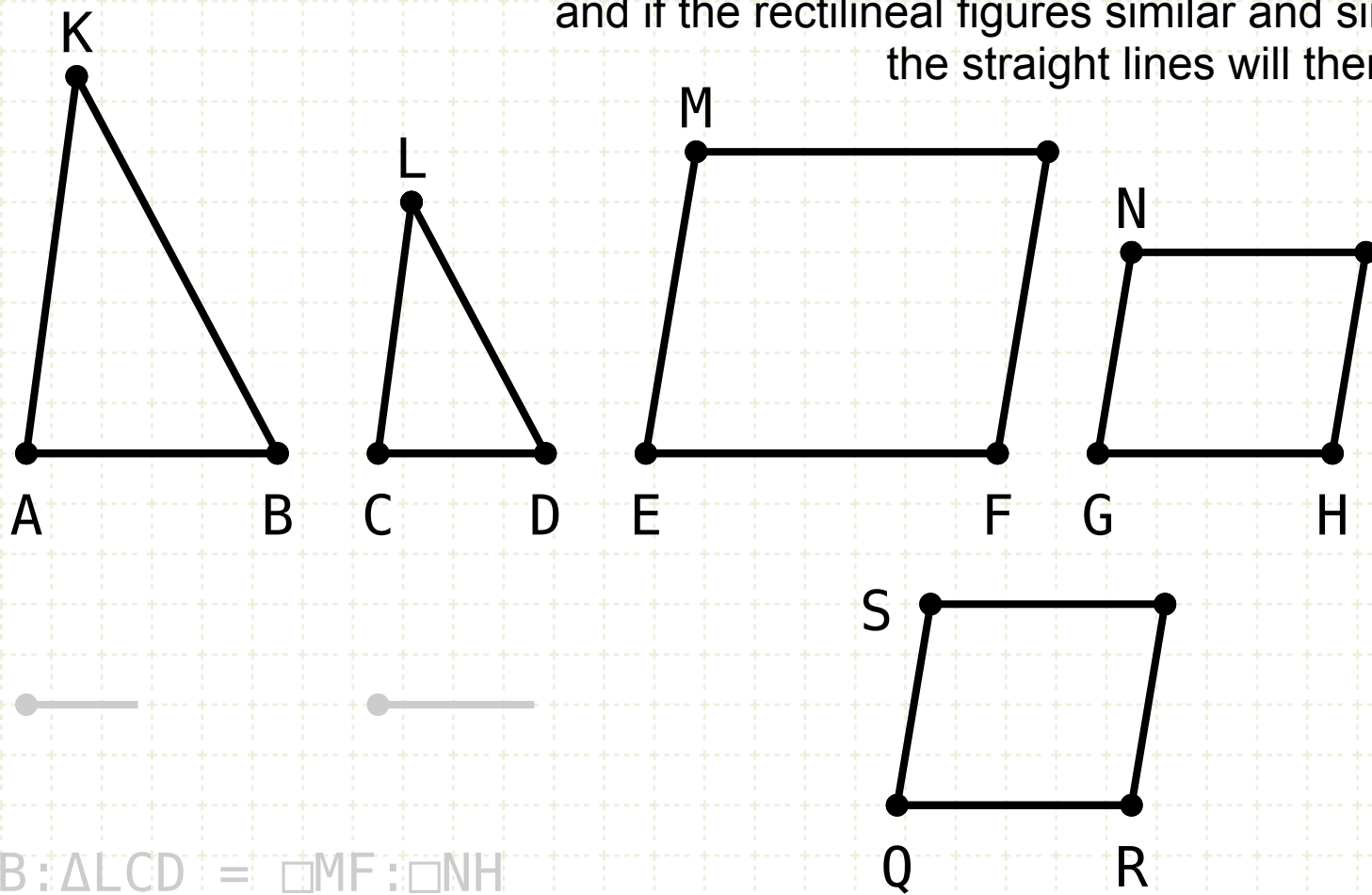
Assume that the ratio EF to GH is not equal to the ratio AB to CD

Define QR such that EF to QR is equal to AB to CD

Draw SR similar to either MF or NH (VI.18)

Proposition 22 of Book VI

If four straight lines be proportional, the rectilineal figures similar and similarly described upon them will also be proportional;
 and if the rectilineal figures similar and similarly described upon them be proportional,
 the straight lines will themselves also be proportional



$$\begin{aligned} \Delta KAB : \Delta LCD &= \square MF : \square NH \\ AB : CD &\neq EF : GH \\ AB : CD &= EF : QR \\ \Delta KAB : \Delta LCD &= \square MF : \square SR \end{aligned}$$

$$\square SR \sim \square NH$$

In other words - Part 2

If KAB is to LCD as MF is to NH , then AB is to CD as EF is to GH

Proof by Contradiction

Assume that the ratio EF to GH is not equal to the ratio AB to CD

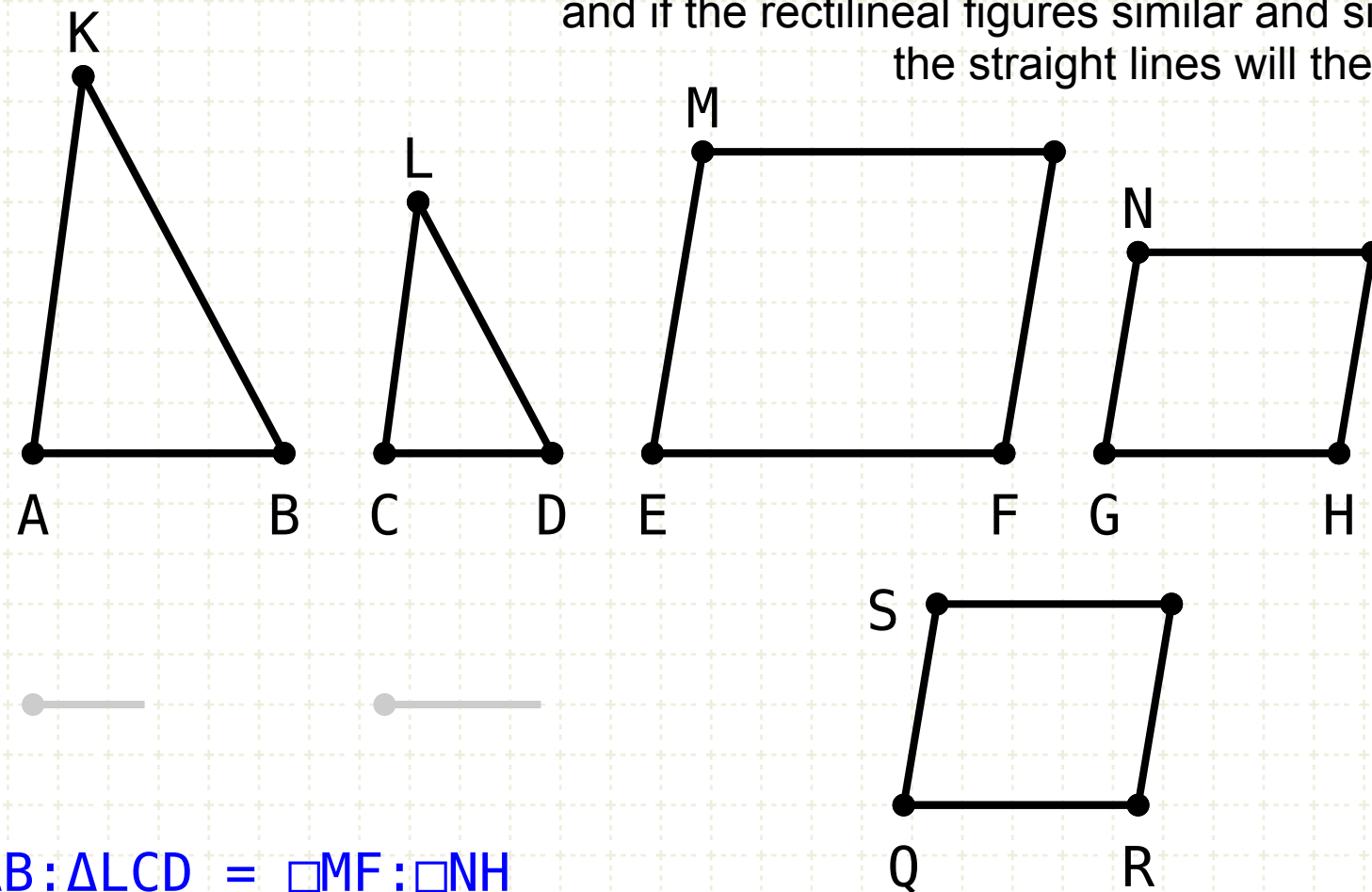
Define QR such that EF to QR is equal to AB to CD

Draw SR similar to either MF or NH (VI-18)

Since AB is to CD as EF is to QR , then KAB is to LCD , so is MF to SR

Proposition 22 of Book VI

If four straight lines be proportional, the rectilineal figures similar and similarly described upon them will also be proportional;
and if the rectilineal figures similar and similarly described upon them be proportional,
the straight lines will themselves also be proportional



$$\Delta KAB : \Delta LCD = \square MF : \square NH$$

$$AB : CD \neq EF : GH$$

$$AB : CD = EF : QR$$

$$\Delta KAB : \Delta LCD = \square MF : \square SR$$

$$\square MF : \square NH = \square MF : \square SR$$

$$\square SR \sim \square NH$$

In other words - Part 2

If KAB is to LCD as MF is to NH, then AB is to CD as EF is to GH

Proof by Contradiction

Assume that the ratio EF to GH is not equal to the ratio AB to CD

Define QR such that EF to QR is equal to AB to CD

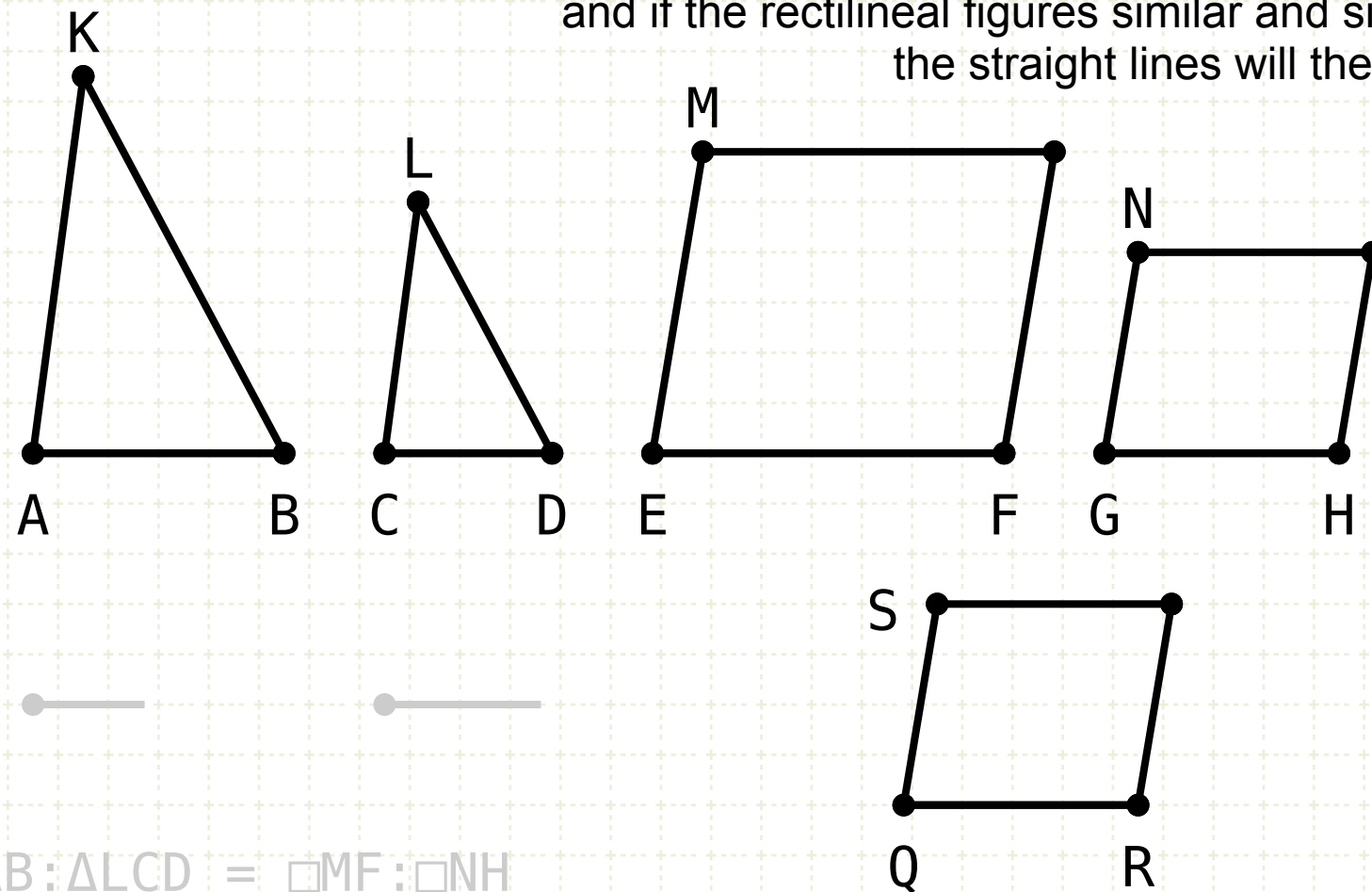
Draw SR similar to either MF or NH (VI·18)

Since AB is to CD as EF is to QR, then KAB is to LCD, so is MF to SR

But MF to NH is also equal to KAB to LCD, therefore MF is to SR so is MF to NH (V·11)

Proposition 22 of Book VI

If four straight lines be proportional, the rectilineal figures similar and similarly described upon them will also be proportional;
and if the rectilineal figures similar and similarly described upon them be proportional,
the straight lines will themselves also be proportional



$$\Delta KAB : \Delta LCD = \square MF : \square NH$$

$$AB : CD \neq EF : GH$$

$$AB : CD = EF : QR$$

$$\Delta KAB : \Delta LCD = \square MF : \square SR$$

$$\square MF : \square NH = \square MF : \square SR$$

$$\square NH = \square SR$$

$$\square SR \sim \square NH$$

In other words - Part 2

If KAB is to LCD as MF is to NH, then AB is to CD as EF is to GH

Proof by Contradiction

Assume that the ratio EF to GH is not equal to the ratio AB to CD

Define QR such that EF to QR is equal to AB to CD

Draw SR similar to either MF or NH (VI·18)

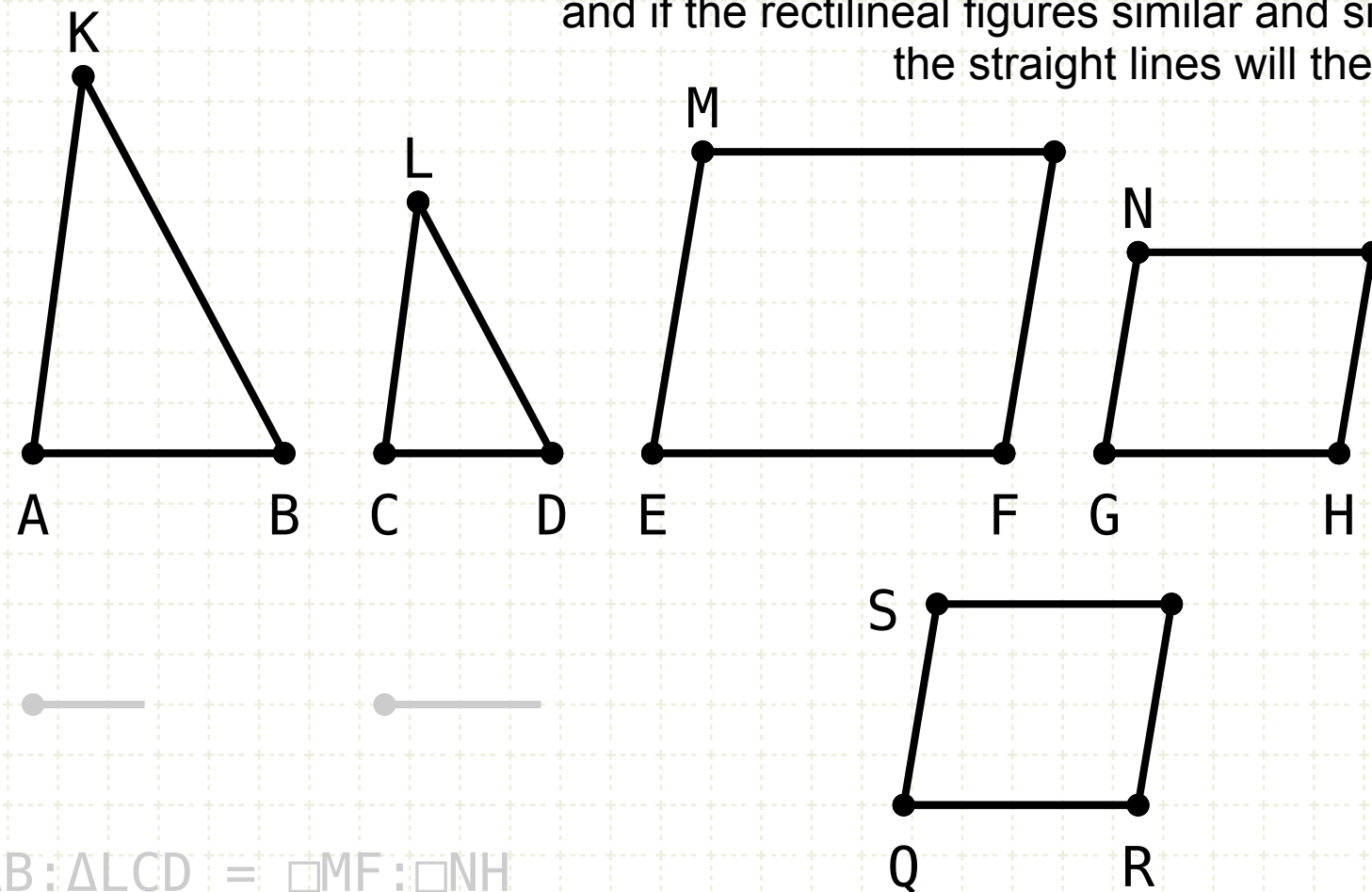
Since AB is to CD as EF is to QR, then KAB is to LCD, so is MF to SR

But MF to NH is also equal to KAB to LCD, therefore MF is to SR so is MF to NH (V·11)

MF has the same ratio to SR as it does to NH, therefore SR is equal to NH (V·9)

Proposition 22 of Book VI

If four straight lines be proportional, the rectilineal figures similar and similarly described upon them will also be proportional;
and if the rectilineal figures similar and similarly described upon them be proportional,
the straight lines will themselves also be proportional



$$\Delta KAB : \Delta LCD = \square MF : \square NH$$

$$AB : CD \neq EF : GH$$

$$AB : CD = EF : QR$$

$$\Delta KAB : \Delta LCD = \square MF : \square SR$$

$$\square MF : \square NH = \square MF : \square SR$$

$$\square NH = \square SR$$

$$QH = GR$$

$$\square SR \sim \square NH$$

In other words - Part 2

If KAB is to LCD as MF is to NH, then AB is to CD as EF is to GH

Proof by Contradiction

Assume that the ratio EF to GH is not equal to the ratio AB to CD

Define QR such that EF to QR is equal to AB to CD

Draw SR similar to either MF or NH (VI·18)

Since AB is to CD as EF is to QR, then KAB is to LCD, so is MF to SR

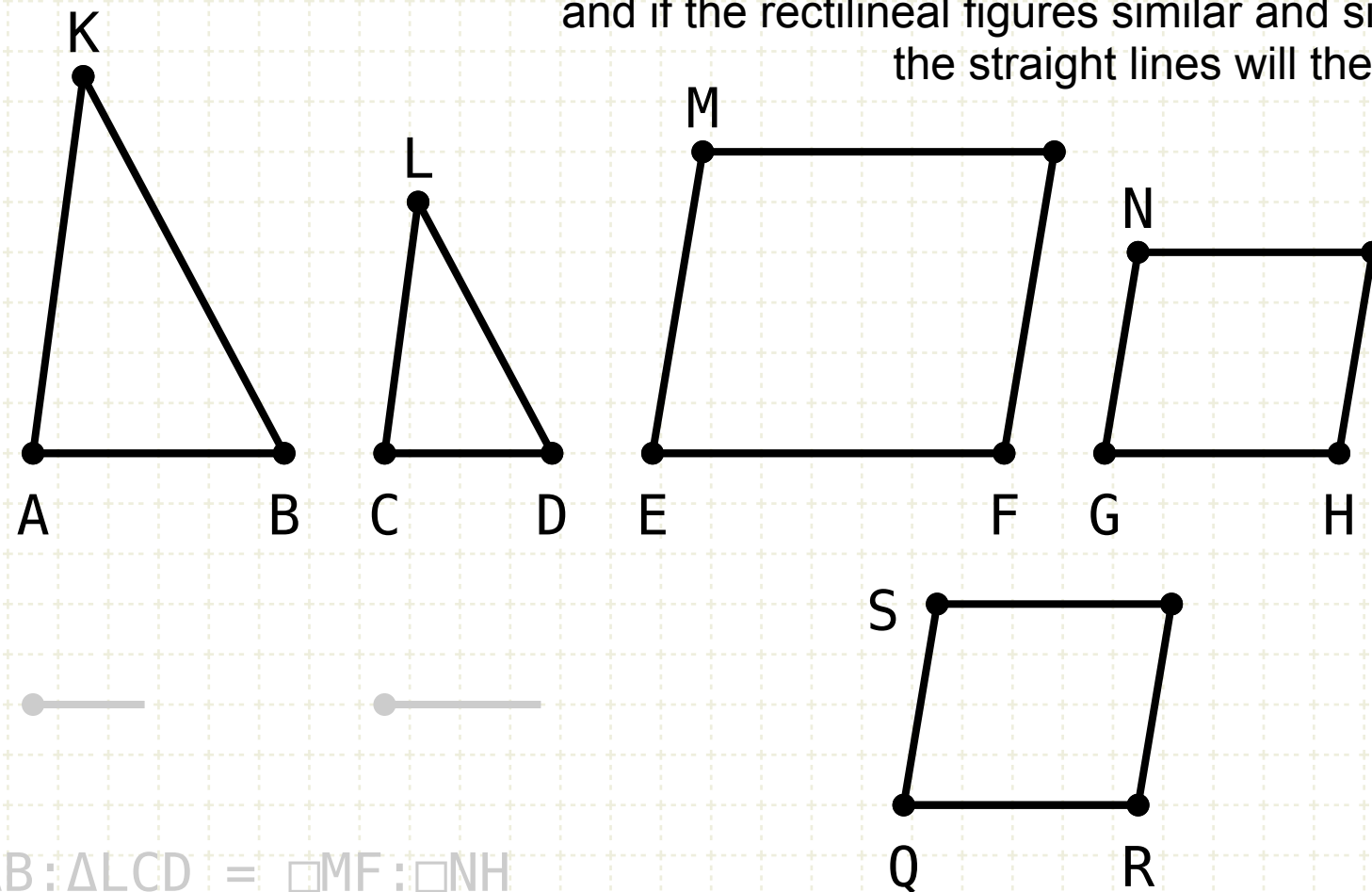
But MF to NH is also equal to KAB to LCD, therefore MF is to SR so is MF to NH (V·11)

MF has the same ratio to SR as it does to NH, therefore SR is equal to NH (V·9)

But NH and SR are similar, therefore GH = QR

Proposition 22 of Book VI

If four straight lines be proportional, the rectilineal figures similar and similarly described upon them will also be proportional;
and if the rectilineal figures similar and similarly described upon them be proportional,
the straight lines will themselves also be proportional



$$\Delta KAB : \Delta LCD = \square MF : \square NH$$

$$AB : CD \neq EF : GH$$

$$AB : CD = EF : QR$$

$$\Delta KAB : \Delta LCD = \square MF : \square SR$$

$$\square MF : \square NH = \square MF : \square SR$$

$$\square NH = \square SR$$

$$QH = GR$$

$$\square SR \sim \square NH$$

$$AB : CD = EF : GH$$

In other words - Part 2

If KAB is to LCD as MF is to NH, then AB is to CD as EF is to GH

Proof by Contradiction

Assume that the ratio EF to GH is not equal to the ratio AB to CD

Define QR such that EF to QR is equal to AB to CD

Draw SR similar to either MF or NH (VI·18)

Since AB is to CD as EF is to QR, then KAB is to LCD, so is MF to SR

But MF to NH is also equal to KAB to LCD, therefore MF is to SR so is MF to NH (V·11)

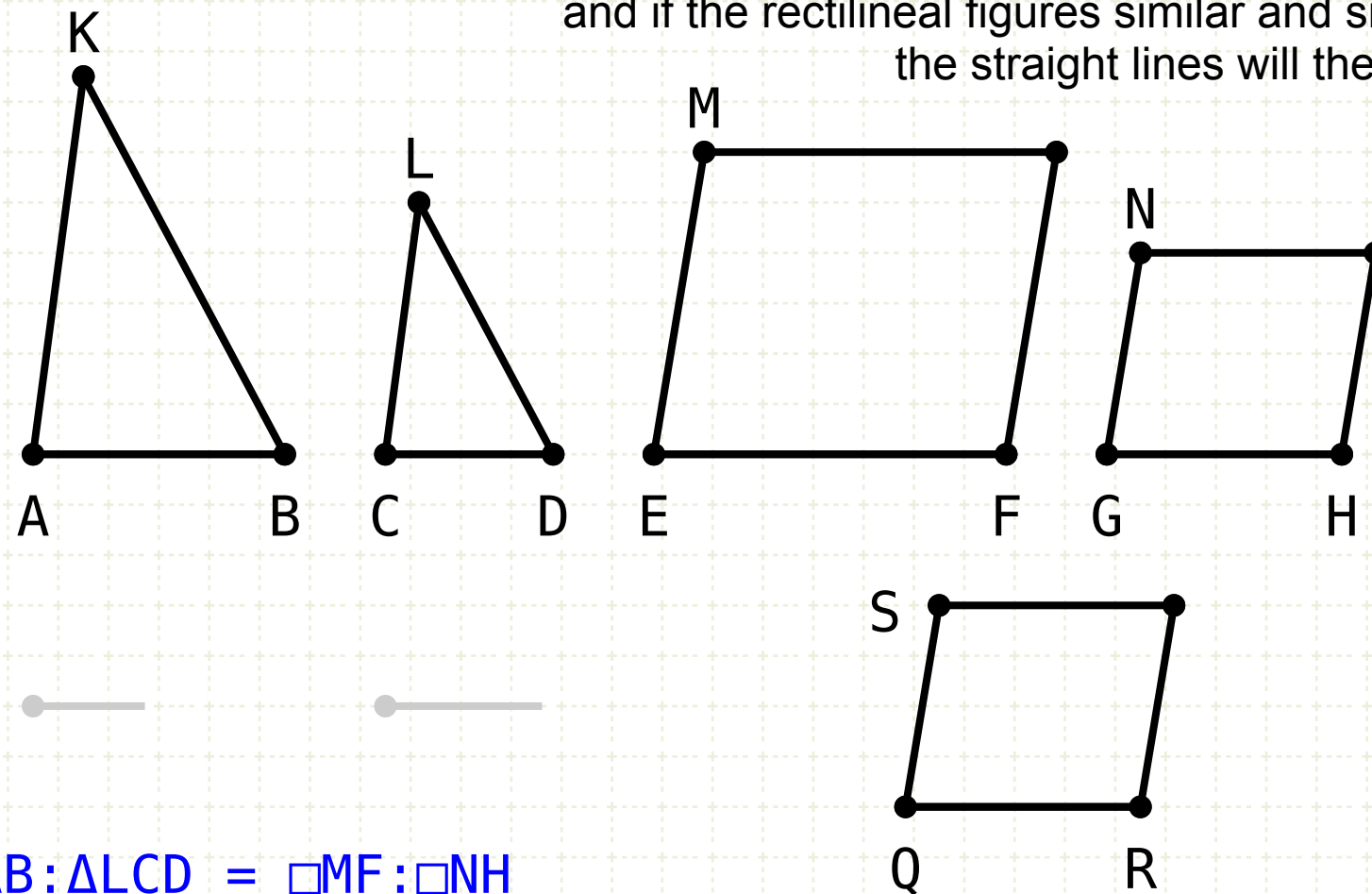
MF has the same ratio to SR as it does to NH, therefore SR is equal to NH (V·9)

But NH and SR are similar, therefore GH = QR

If AB is to CD as EF is to QR, and QR is equal to GH, then AB is to CE as EF is to GH

Proposition 22 of Book VI

If four straight lines be proportional, the rectilineal figures similar and similarly described upon them will also be proportional;
and if the rectilineal figures similar and similarly described upon them be proportional,
the straight lines will themselves also be proportional



$$\triangle KAB : \triangle LCD = \square MF : \square NH$$

$$AB : CD \neq EF : GH$$

$$AB : CD = EF : QR$$

$$\triangle KAB : \triangle LCD = \square MF : \square SR$$

$$\square MF : \square NH = \square MF : \square SR$$

$$\square NH = \square SR$$

$$QH = GR$$

$$AB : CD = EF : GH$$

In other words - Part 2

If KAB is to LCD as MF is to NH , then AB is to CD as EF is to GH

Proof by Contradiction

Assume that the ratio EF to GH is not equal to the ratio AB to CD

Define QR such that EF to QR is equal to AB to CD

Draw SR similar to either MF or NH (VI-18)

Since AB is to CD as EF is to QR , then KAB is to LCD , so is MF to SR

But MF to NH is also equal to KAB to LCD , therefore MF is to SR so is MF to NH (V-11)

MF has the same ratio to SR as it does to NH , therefore SR is equal to NH (V-9)

But NH and SR are similar, therefore $GH = QR$

If AB is to CD as EF is to QR , and QR is equal to GH , then AB is to CD as EF is to GH

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