Euclid's Elements

Book VII

Definitions:

1 A unit is that by virtue of which each of the things that exist is called one

2 A number is a multitude composed of units. (not one)

3 A number is part of a number, the less of the greater, when it measures the greater

11 A prime number is that which is measured by a unit alone.

12 Numbers prime to one another are those which are measured by a unit alone as a common measure

15 A number is said to multiply a number when that which is multiplied is added to itself as many times as there are units in the other, and thus some number is produced.

20 Numbers are proportional when the first is the same multiple, or the same part, or the same parts, of the second that the third is of the fourth.

As long as algebra and geometry have been separated, their progress have been slow and their uses limited; but when these two sciences have been united, they have lent each mutual forces, and have marched together towards perfection.

Joseph-Louis Lagrange
(1736 to 1813)

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1 Determine if two numbers are relatively prime

2 Find the greatest common divisor for two numbers

3 Find the largest common divisor for three numbers

4 Given two natural numbers, A and B, either B is part of A, or there exists a natural number (a part) that can measure both A and B

5 If B = (1/q)·A and D = (1/q)·C, then (B+D) = (1/q)·(A+C)

6 If B = (p/q)·A and D = (p/q)·C, then (B+D) = (p/q)·(A+C)

7 If B = A/q and D = C/q, B>D, then (B-D) = (A-C)/q

8 If B = (p/q)·A and D = (p/q)·C, B>D, then (B-D) = (p/q)·(A-C)

9 If B = (1/q)·A and D = (1/q)·C, and If B = (r/s)·D, then A = (r/s)·C

10 If B = (p/q)·A and D = (p/q)·C, and If B = (r/s)·D, then A = (r/s)·C

11 If A:B = C:D, then (A-C):(B-D) = A:B

12 If A:B = C:D, then (A+C):(B+C) = A:B

13 If A:B = C:D, then A:C = B:D

14 If A:B = D:E and B:C = E:F, then A:C = D:F

15 If B = i·1 and E = i·D, and if D = j·1 then E = j·B

16 A × B = B × A

17 If D = A × B and E = A × C then D:E = B:C

18 If D = B × A and E = C × A then D:E = B:C

19 If A:B = C:D then A × D = B × C
If A × D = B × C then A:B = C:D

20 Given the ratio A:B and C,D are the smallest numbers such that A:B = C:D then A = n·C and B = n·D

21 If A,B are relatively prime, then A,B are the smallest whole numbers that can be used to describe the ratio A:B

22 If A,B are the smallest whole numbers that can be used to describe the ratio A:B, then A,B are relatively prime

23 If A,B are relatively prime and if A = n·C, then B,C are relatively prime

24 If A,C are relatively prime and B,C are relatively prime then the A × B is relatively prime to C

25 If A,B are relatively prime then A^2,B are relatively prime

26 If A is relatively prime to C and D, and if B is also relatively prime to C and D, then A × B is relatively prime to C × D

27 If A,B are relatively prime, then A^2,B^2 are relatively prime, and A^3,B^3 are relatively prime, and so on

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28 If A,B are relatively prime, then A,(A+B) are relatively prime

29 If A is prime, and B ≠ n·A, then A,B are relatively prime

30 If C = A×B and C = i·D where D is prime, then either A = j·D or B = j·D

31 If A = B×C, then A = j·D where D is prime

32 If A is a number then it is either prime, or A = j·D where D is prime

33 Find the smallest numbers X,Y,Z where the ratio X:Y:Z is equal to the given ratio A:B:C

34 Find the lowest common denominator of 2 numbers

35 If E is the lowest common denominator of A,B, and if C = n ·A = m·B, then C = i·E

36 Find the least common multiple of 3 numbers

37 If A = p·B, then A = q·C where C = p·1

38 If A = (1/c)·B and C = c·1 then A = n·C

39 Find the smallest number that has the fractions 1/a, 1/b, 1/c

Proposition 6 of Book VII

If a number be parts of a number, and another be the same parts of another, the sum will also be the same parts of the sum that the one is of the one.

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If a number be parts of a number, and another be the same parts of another, the sum will also be the same parts of the sum that the one is of the one.

$$b = (p/q)\cdot a$$
$$d = (p/q)\cdot c$$
$$\rightarrow (b+d) = (p/q)\cdot(a+c)$$

In other words

If b is the same fraction of a as d is to c, then the sum b,d will also be the same fraction of the sum a,c

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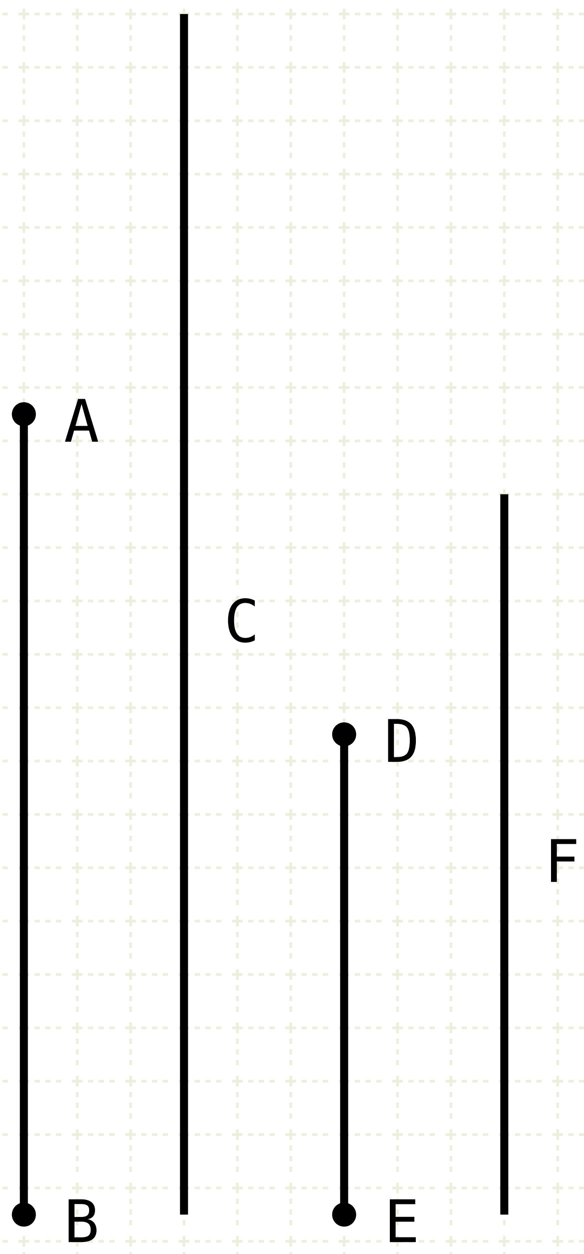

$$AB = p \cdot C/q = p \cdot C/q$$
$$DE = p \cdot F/q = p \cdot F/q$$

In other words

If b is the same fraction of a as d is to c, then the sum b,d will also be the same fraction of the sum a,c

Proof

Let the number AB be parts of C, and let another number DE be the same parts of F

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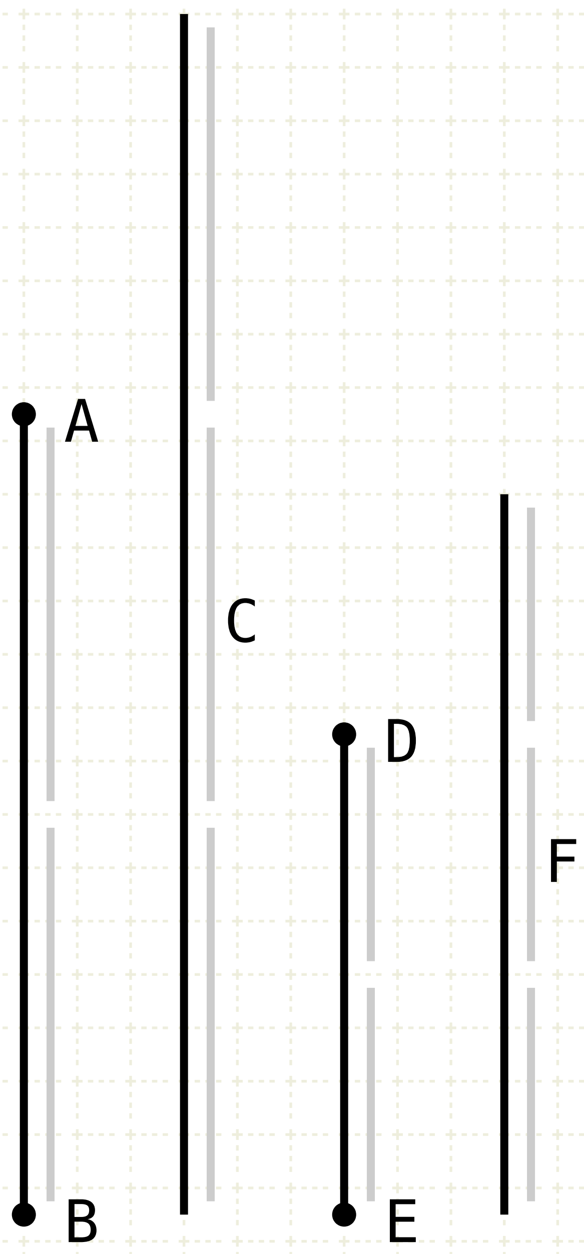

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If b is the same fraction of a as d is to c, then the sum b,d will also be the same fraction of the sum a,c

Proof

Let the number AB be parts of C, and let another number DE be the same parts of F

Since there as parts of DE in F as there are parts of AB in C, therefore there are as many parts of F in DE as there are parts of C in AB

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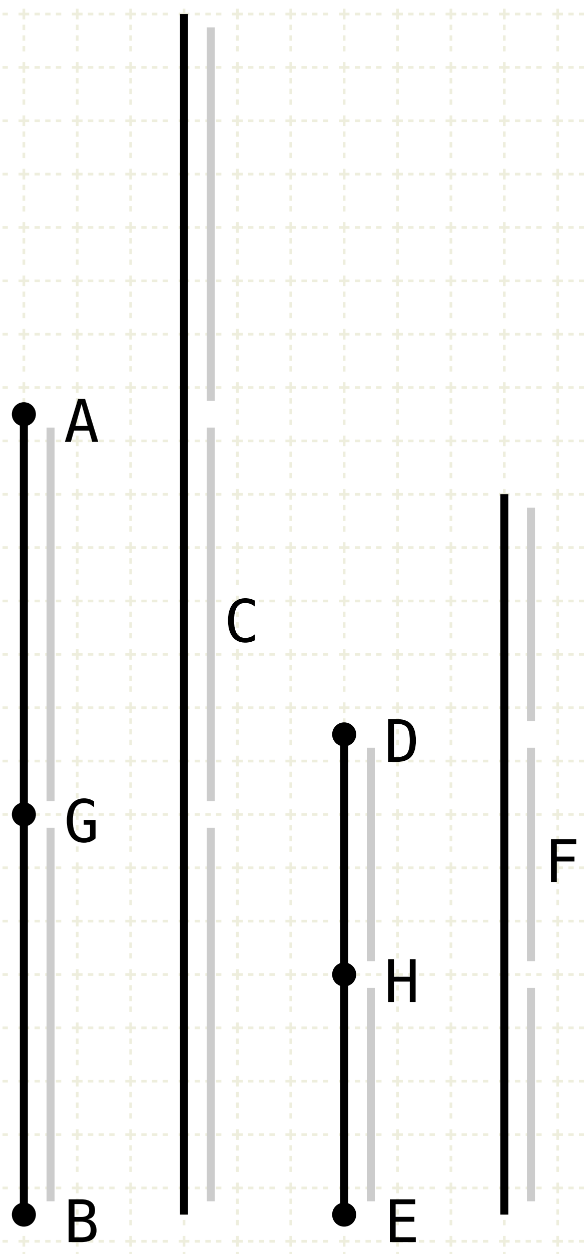

$AB = p \cdot C/q = p \cdot C/q$

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$AG = GB = C/q$

$DH = HE = F/q$

In other words

If b is the same fraction of a as d is to c, then the sum b,d will also be the same fraction of the sum a,c

Proof

Let the number AB be parts of C, and let another number DE be the same parts of F

Since there as parts of DE in F as there are parts of AB in C, therefore there are as many parts of F in DE as there are parts of C in AB

Let AB be divided into the parts of C

Let DE be divided into the parts of F

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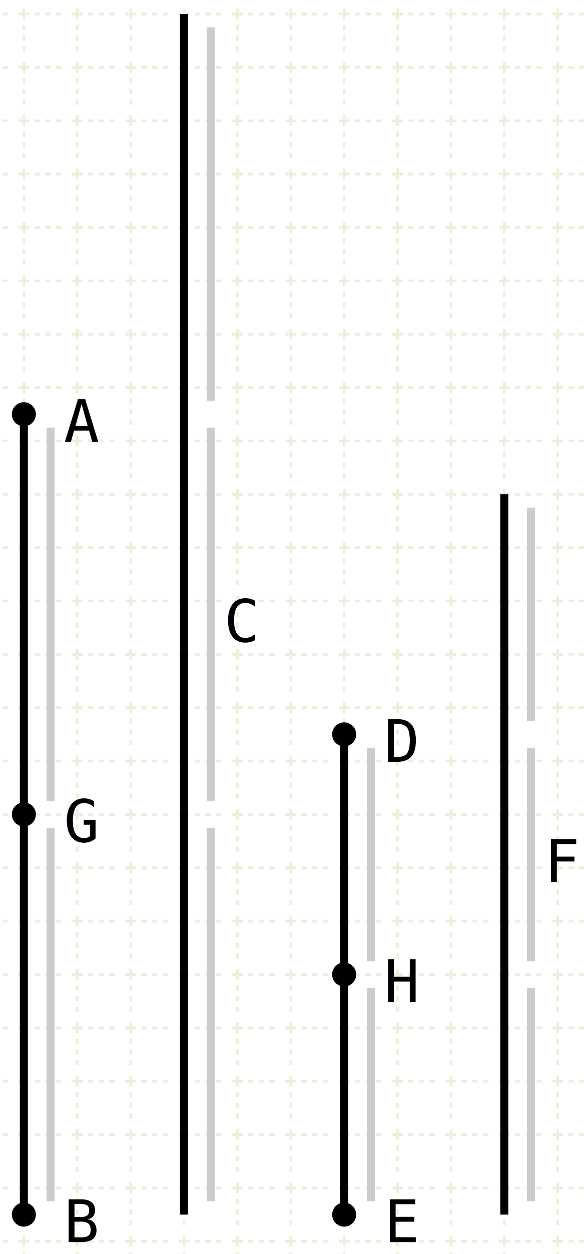

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$$AG + DH = (C+F)/q$$

In other words

If b is the same fraction of a as d is to c, then the sum b,d will also be the same fraction of the sum a,c

Proof

Let the number AB be parts of C, and let another number DE be the same parts of F

Since there as parts of DE in F as there are parts of AB in C, therefore there are as many parts of F in DE as there are parts of C in AB

Let AB be divided into the parts of C

Let DE be divided into the parts of F

Since DH is the same part of F that AG is of C, the sum of DH,AG will be the same part of the sum C,F as DH is to F (VII·5)

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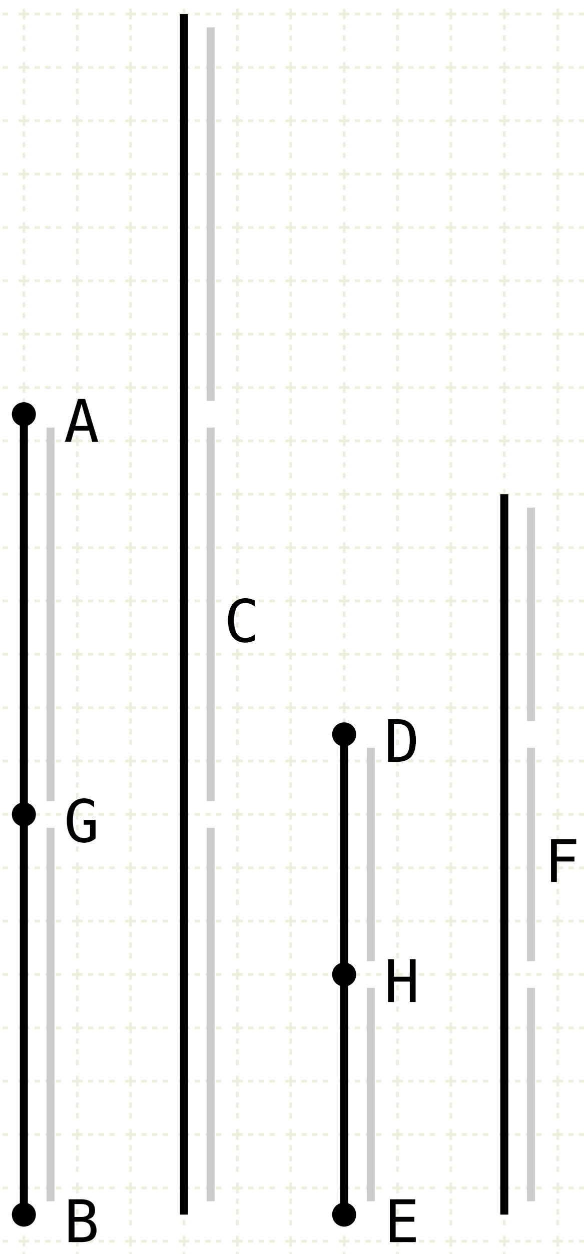

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If b is the same fraction of a as d is to c, then the sum b,d will also be the same fraction of the sum a,c

Proof

Let the number AB be parts of C, and let another number DE be the same parts of F

Since there as parts of DE in F as there are parts of AB in C, therefore there are as many parts of F in DE as there are parts of C in AB

Let AB be divided into the parts of C

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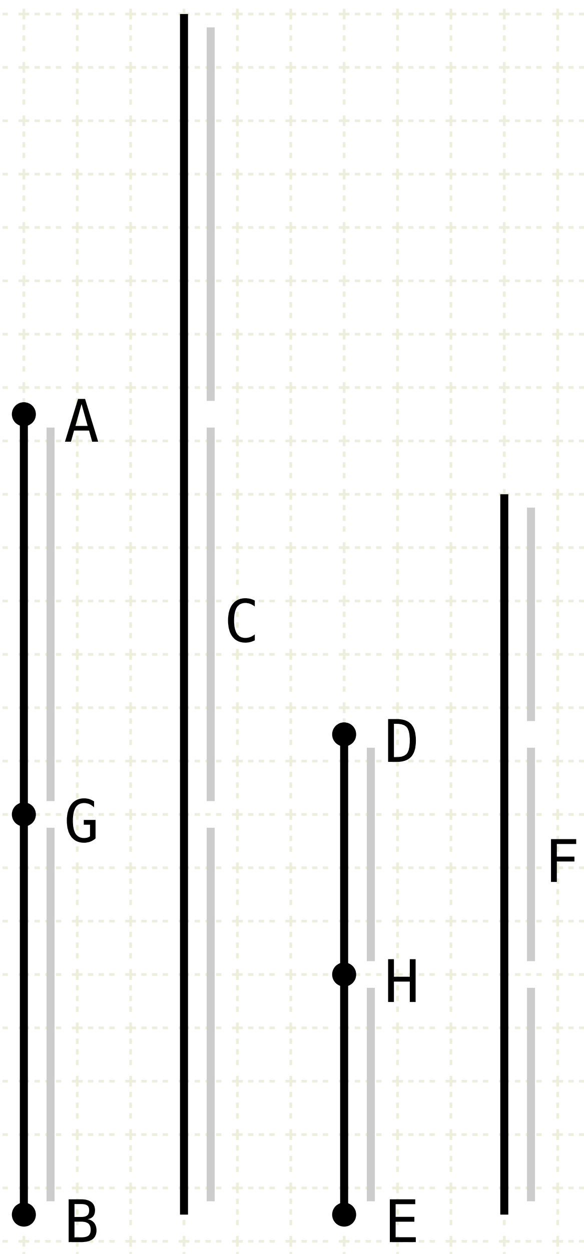

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$$GB + HE = (C+F)/q$$

$$AB + DE = p \cdot (C+F)/q$$

In other words

If b is the same fraction of a as d is to c, then the sum b,d will also be the same fraction of the sum a,c

Proof

Let the number AB be parts of C, and let another number DE be the same parts of F

Since there as parts of DE in F as there are parts of AB in C, therefore there are as many parts of F in DE as there are parts of C in AB

Let AB be divided into the parts of C

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Since DH is the same part of F that AG is of C, the sum of DH,AG will be the same part of the sum C,F as DH is to F (VII·5)

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Given that AB and DE have the same number of parts, the previous process can be repeated for every part in AB and DE

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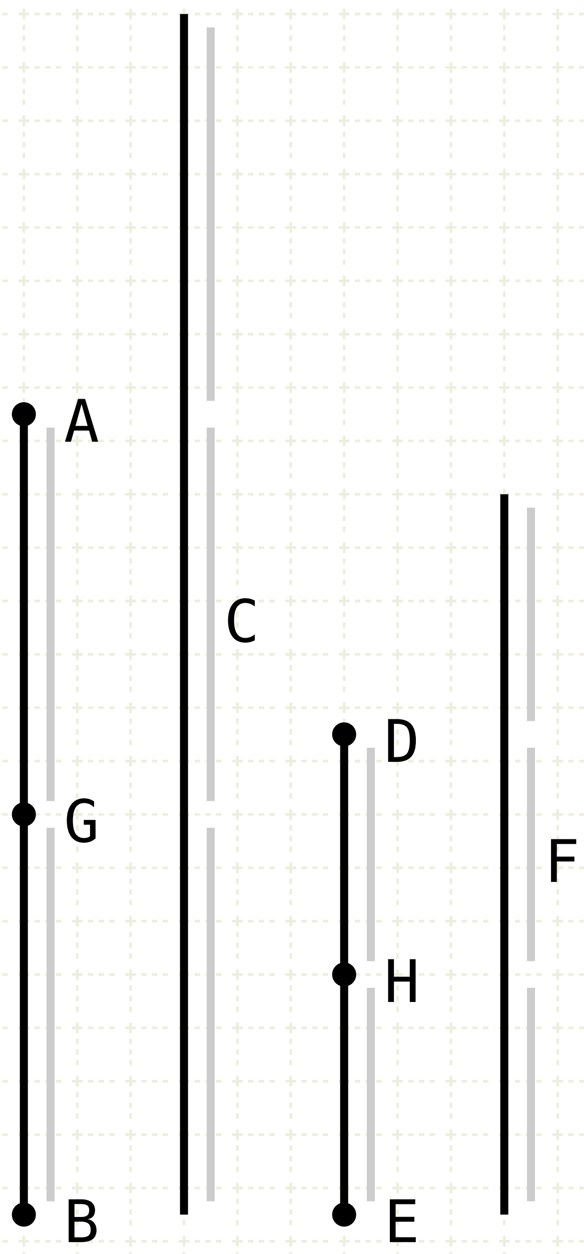

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$$AB + DE = p \cdot (C+F)/q$$

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If b is the same fraction of a as d is to c, then the sum b,d will also be the same fraction of the sum a,c

Proof

Let the number AB be parts of C, and let another number DE be the same parts of F

Since there as parts of DE in F as there are parts of AB in C, therefore there are as many parts of F in DE as there are parts of C in AB

Let AB be divided into the parts of C

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Since DH is the same part of F that AG is of C, the sum of DH,AG will be the same part of the sum C,F as DH is to F (VII·5)

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Given that AB and DE have the same number of parts, the previous process can be repeated for every part in AB and DE

Thus the sum of AB,DE will be the same parts of C,F as AB is to C

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