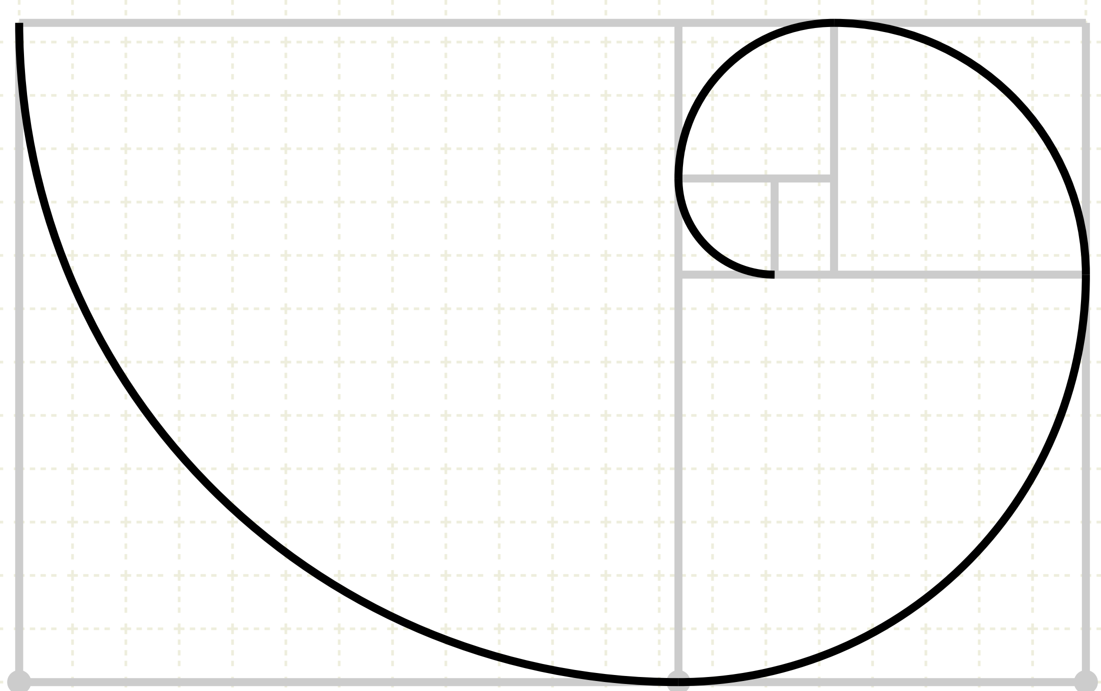
Euclid's Elements

Book VI



One can state, without exaggeration, that the observation of and the search for similarities and differences are the basis of all human knowledge.

Alfred Nobel

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Proposition 18 of Book VI

On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure.

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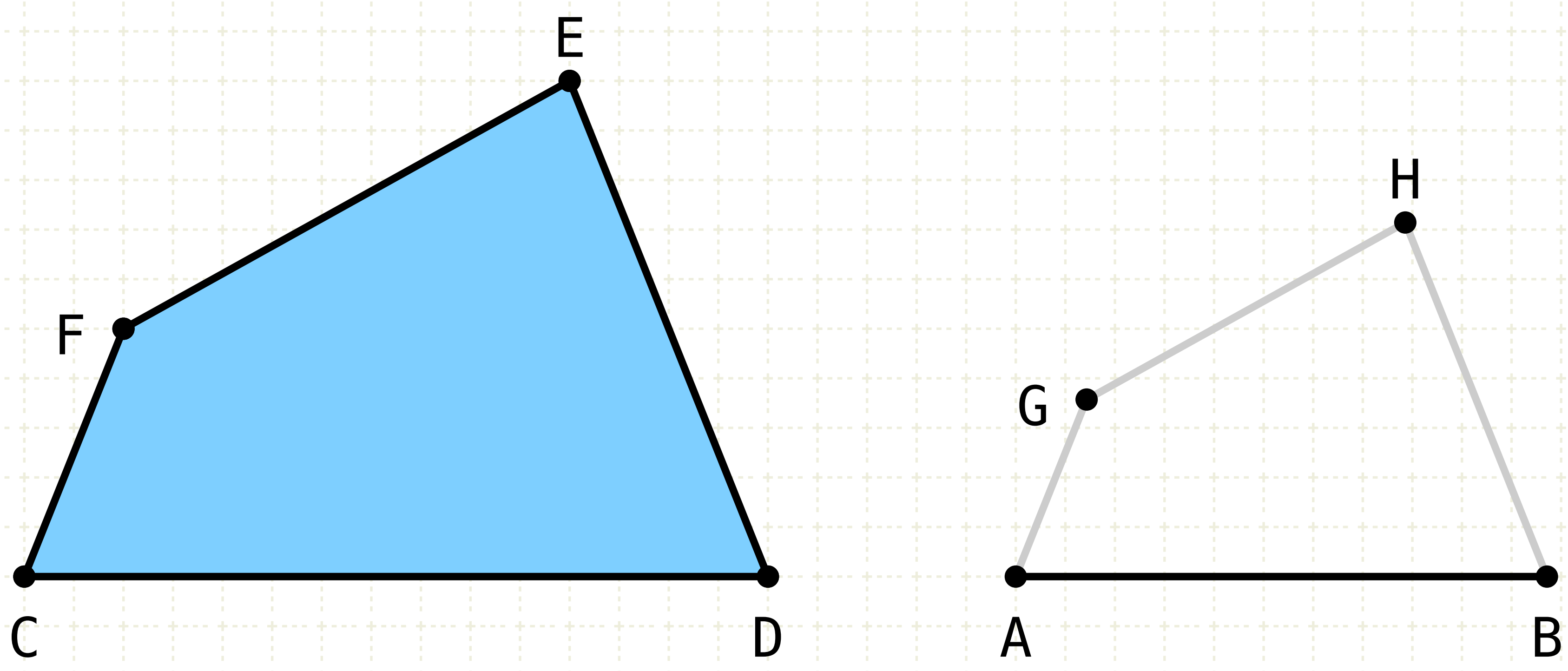


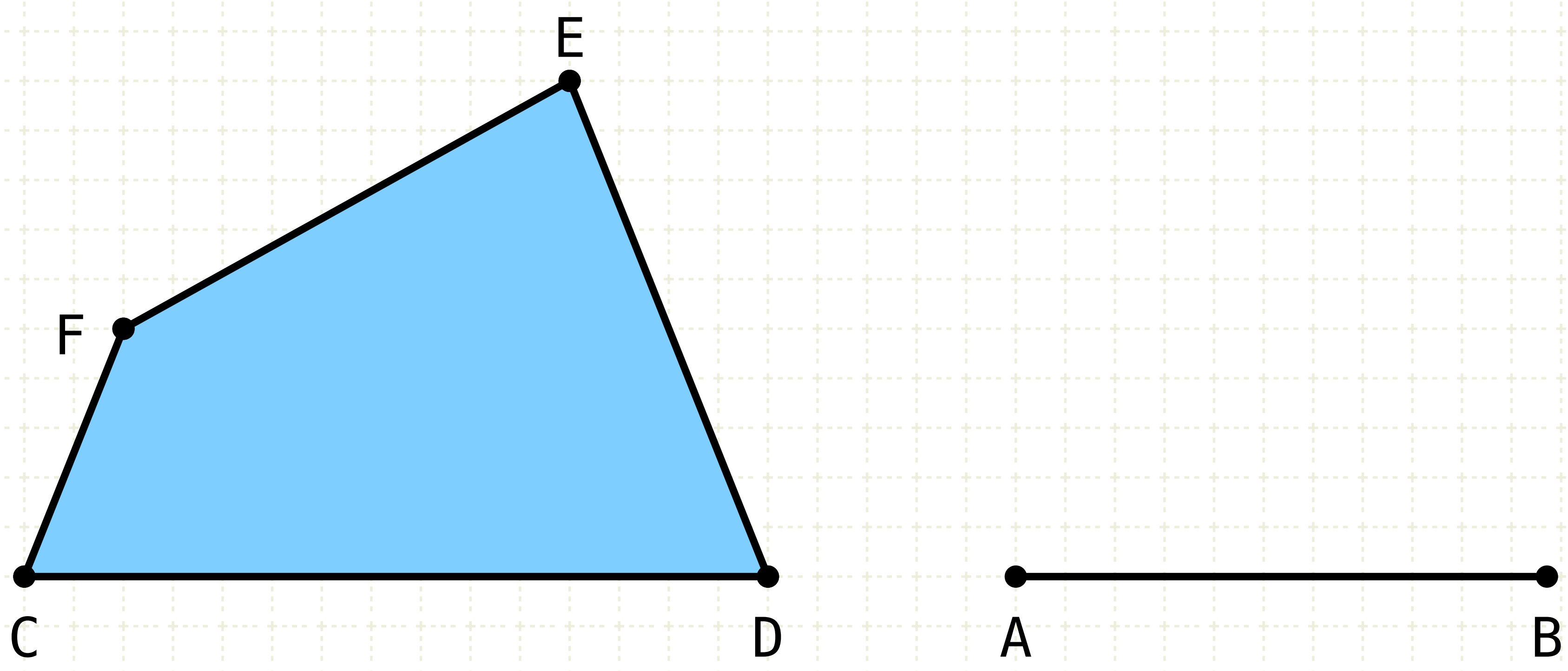
In other words

Copy one figure onto another straight line so that both figures are similar, i.e. equiangular and proportional sides

□CDEF ~ □ABHG

Proposition 18 of Book VI

On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure.



Construction

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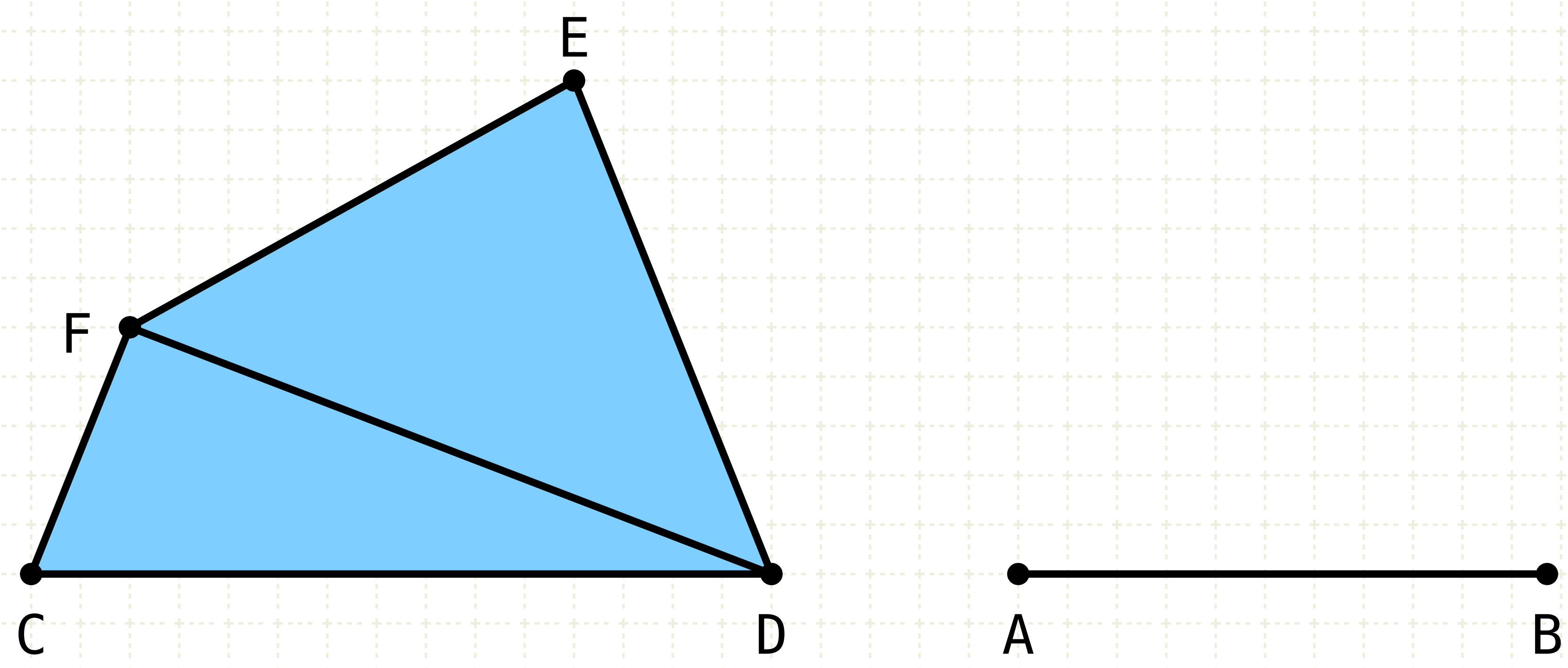
On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure.



Construction

Let line DF be drawn

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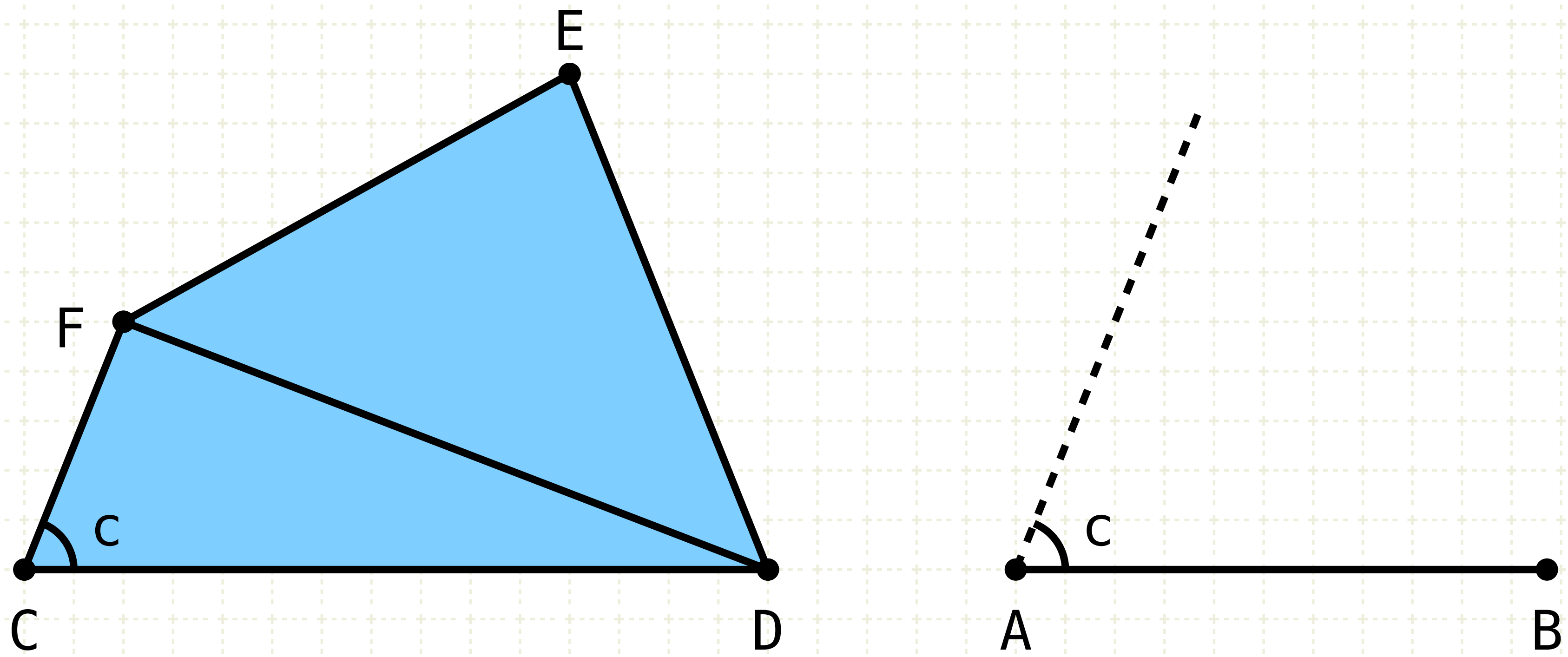


∠FCD = ∠GAB = c

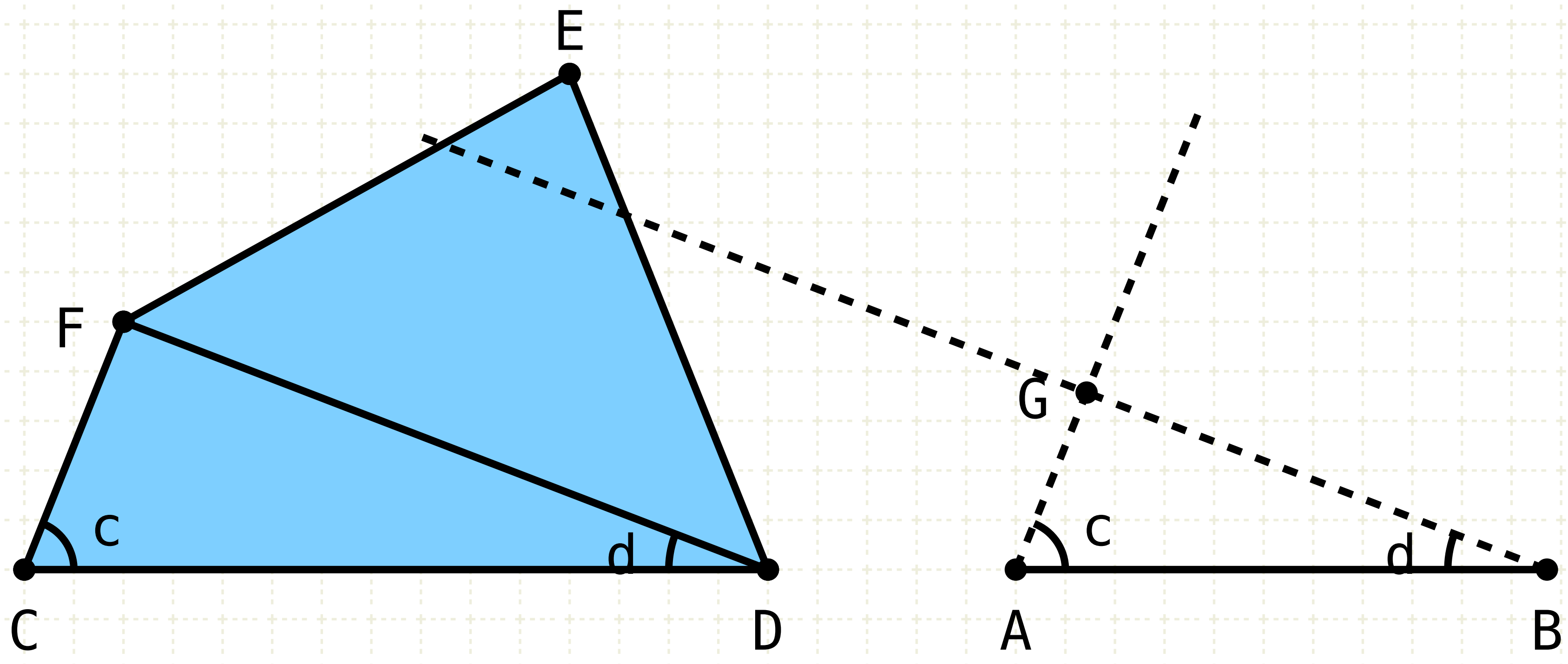
Construction

Let line DF be drawn

Copy the angle FCD to point A on line AB (I·23)

Proposition 18 of Book VI

On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure.



$\angle FCD = \angle GAB = c$

$\angle CDF = \angle ABG = d$

Construction

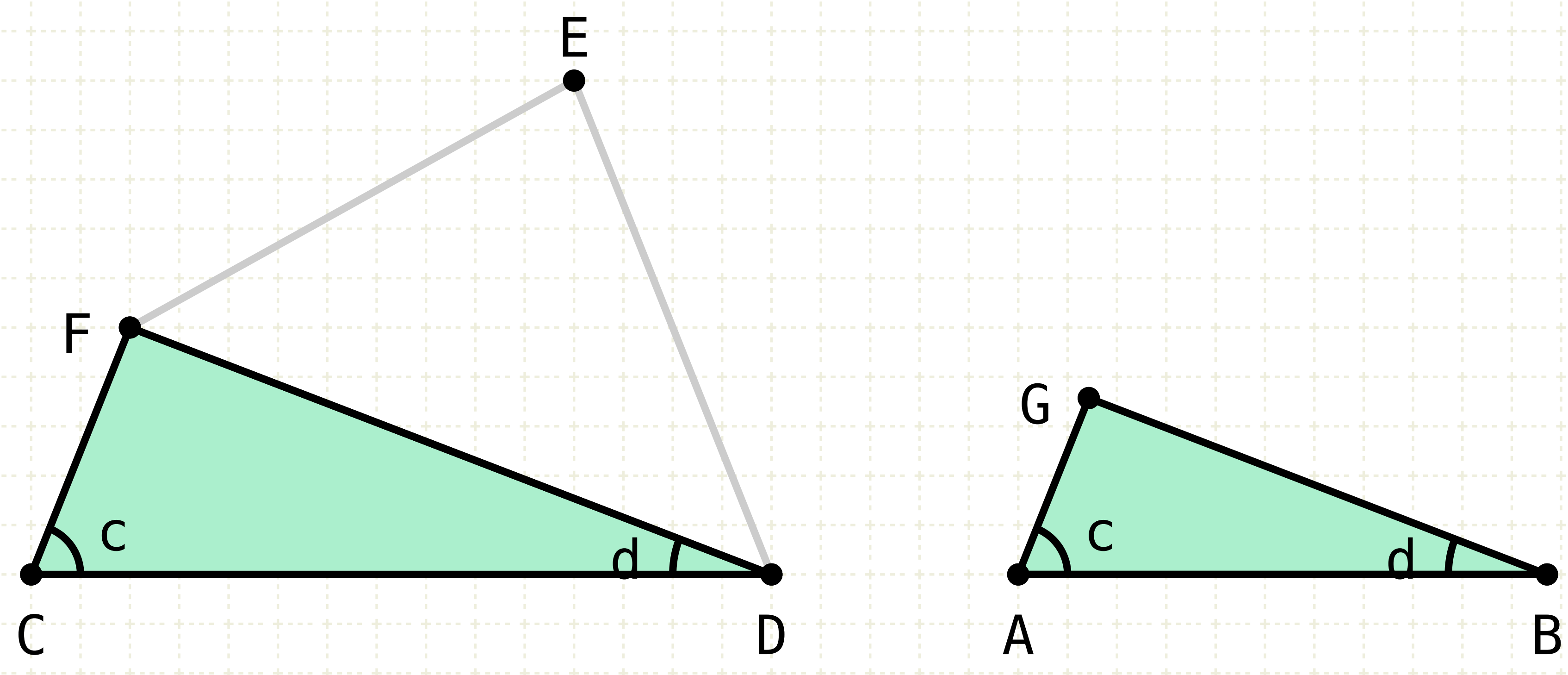
Let line DF be drawn

Copy the angle FCD to point A on line AB (I·23)

Copy the angle CDF to point B on line AB (I·23)

Proposition 18 of Book VI

On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure.



$\angle FCD = \angle GAB = c$

$\angle CDF = \angle ABG = d$

Construction

Let line DF be drawn

Copy the angle FCD to point A on line AB (I·23)

Copy the angle CDF to point B on line AB (I·23)

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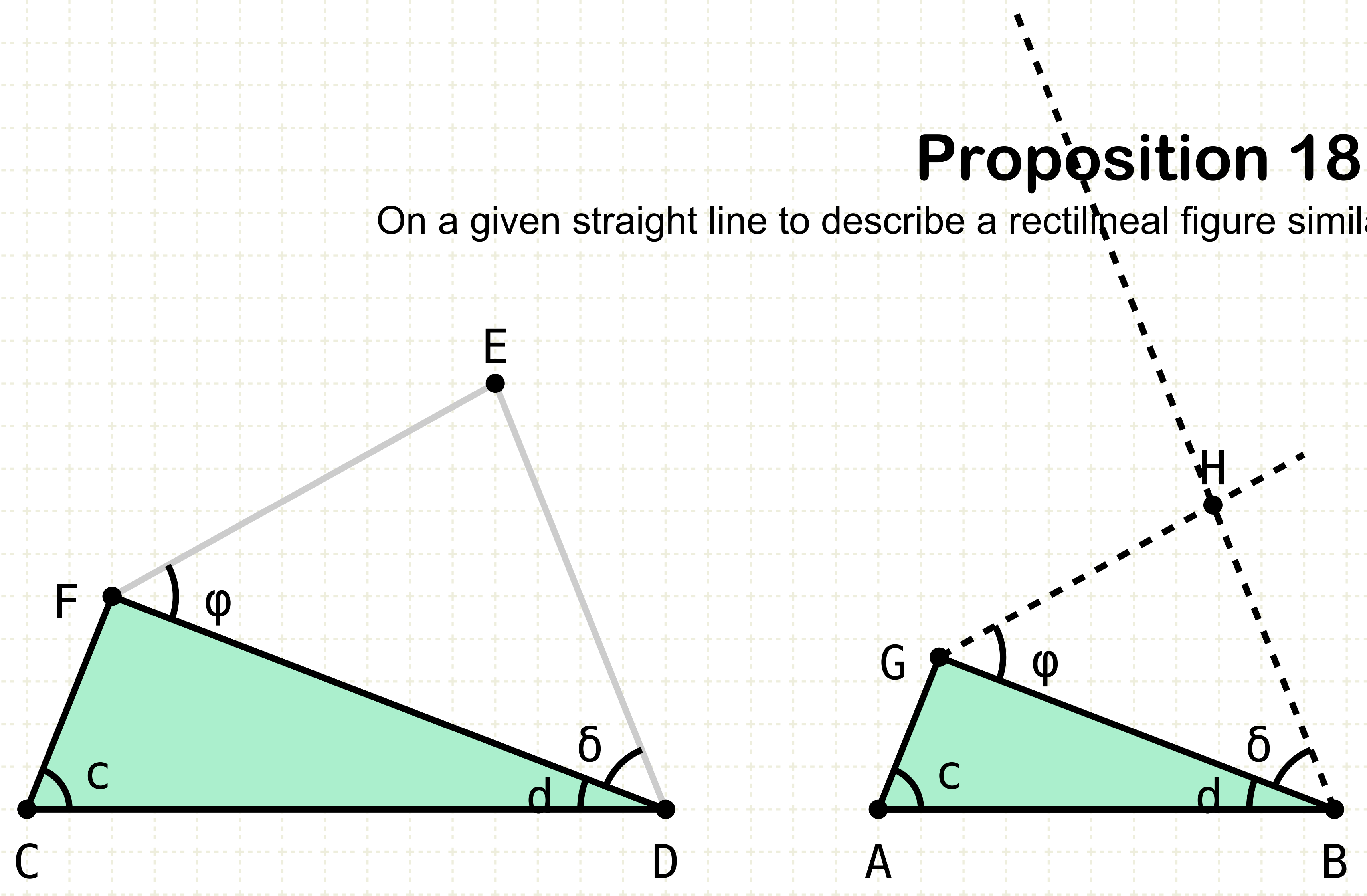

Construction

Let line DF be drawn

Copy the angle FCD to point A on line AB (I·23)

Copy the angle CDF to point B on line AB (I·23)

On the straight line BG, construct BGH equal to angle DFE and angle GBH equal to angle FDE (I·23)

$\angle FCD = \angle GAB = c$

$\angle CDF = \angle ABG = d$

$\angle DFE = \angle BGH = \varphi$

$\angle FDE = \angle GBH = \delta$

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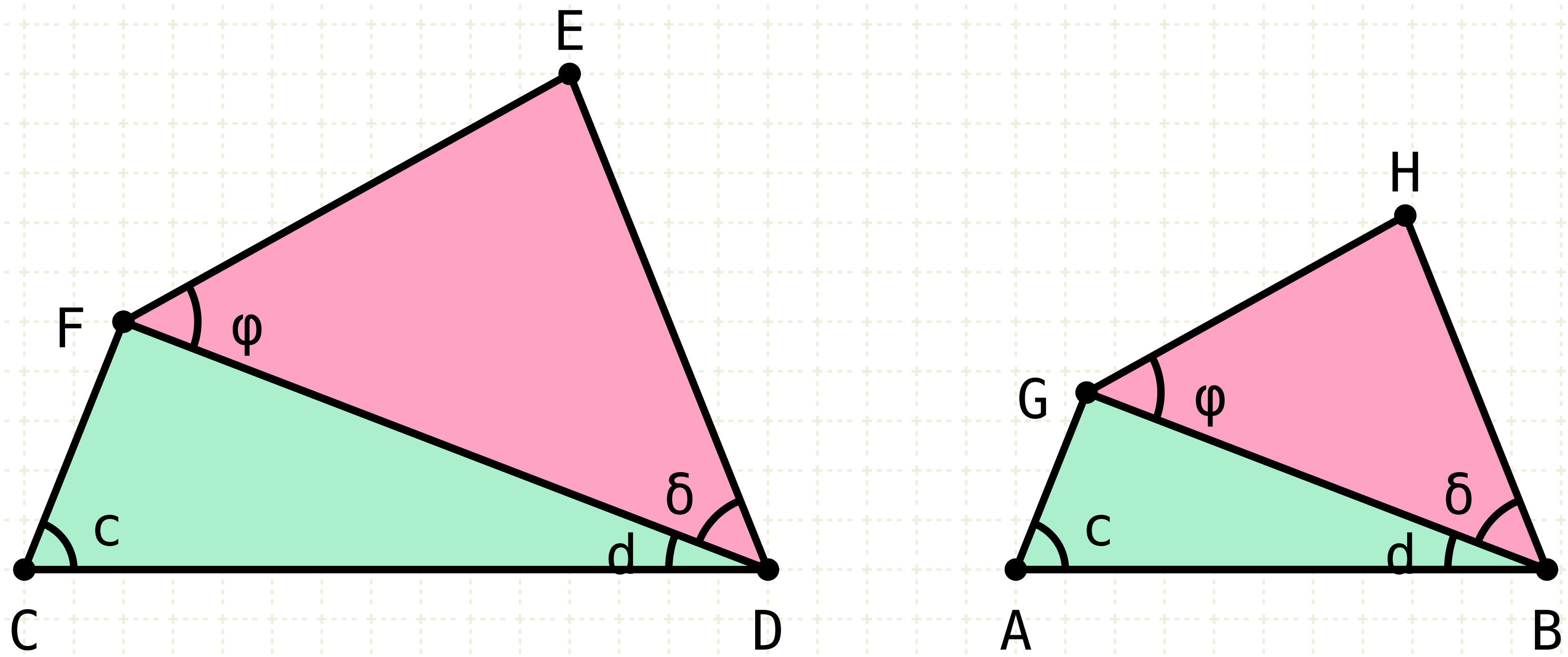

Construction

Let line DF be drawn

Copy the angle FCD to point A on line AB (I·23)

Copy the angle CDF to point B on line AB (I·23)

On the straight line BG, construct BGH equal to angle DFE and angle GBH equal to angle FDE (I·23)

$\angle FCD = \angle GAB = c$
$\angle CDF = \angle ABG = d$
$\angle DFE = \angle BGH = \varphi$
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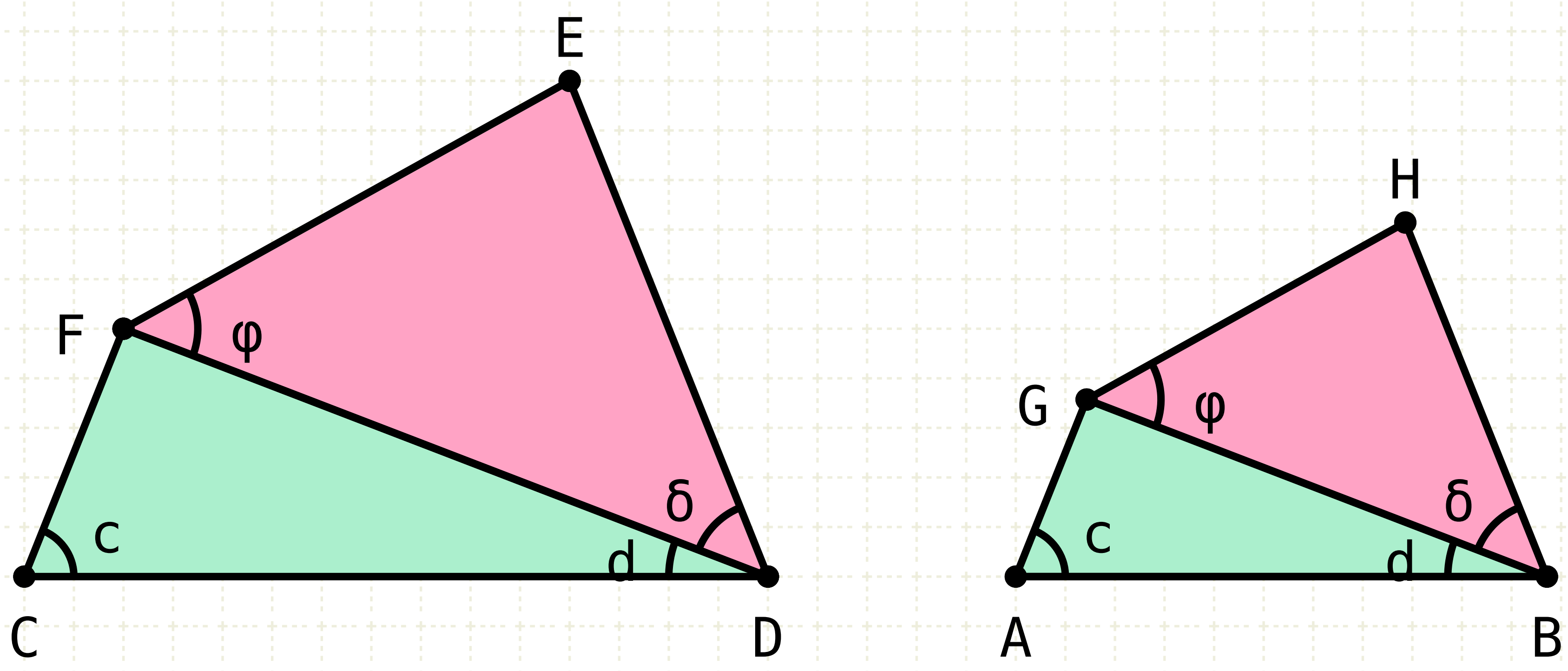

Construction

Let line DF be drawn

Copy the angle FCD to point A on line AB (I·23)

Copy the angle CDF to point B on line AB (I·23)

On the straight line BG, construct BGH equal to angle DFE and angle GBH equal to angle FDE (I·23)

The rectilineal figure ABHG is similar to CDEF

$\angle FCD = \angle GAB = c$
$\angle CDF = \angle ABG = d$
$\angle DFE = \angle BGH = \varphi$
$\angle FDE = \angle GBH = \delta$

$\square CDEF \sim \square ABHG$

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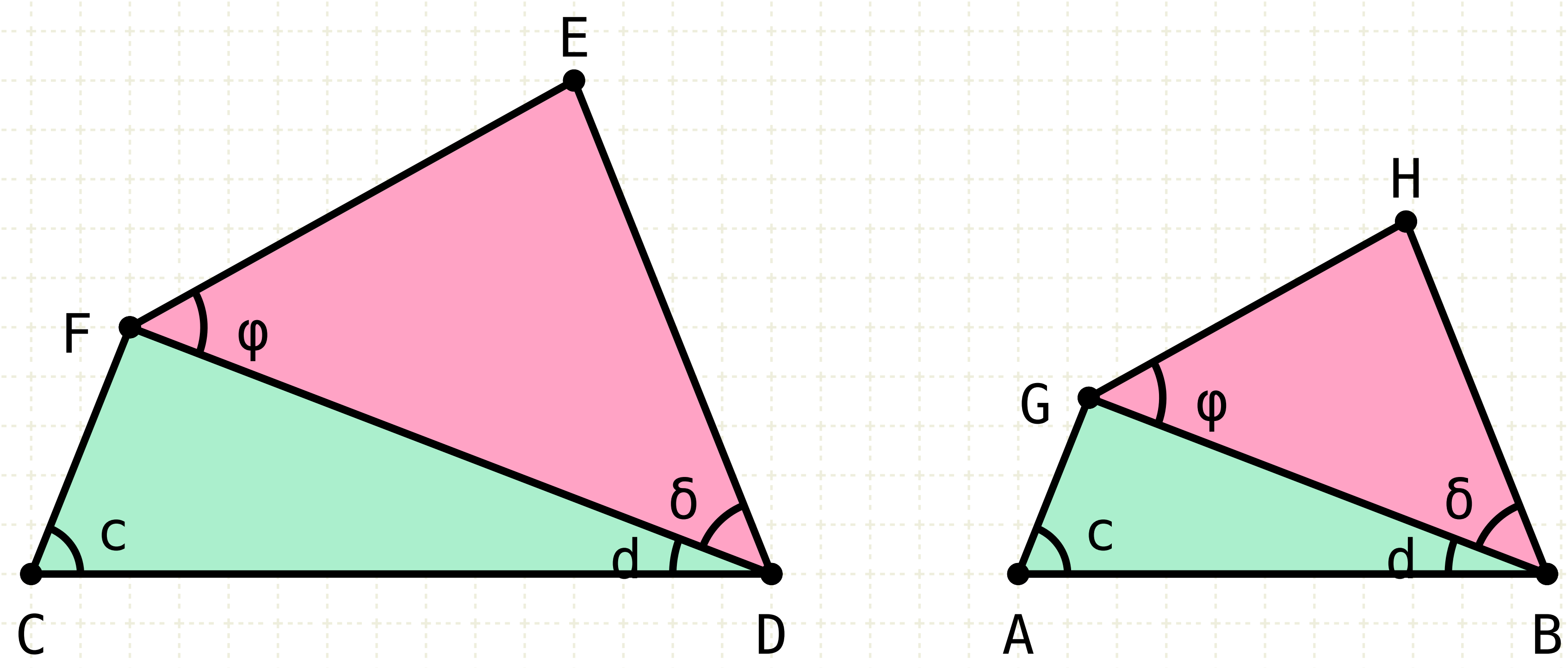

Proof

∠FCD = ∠GAB = c
∠CDF = ∠ABG = d
∠DFE = ∠BGH = φ
∠FDE = ∠GBH = δ

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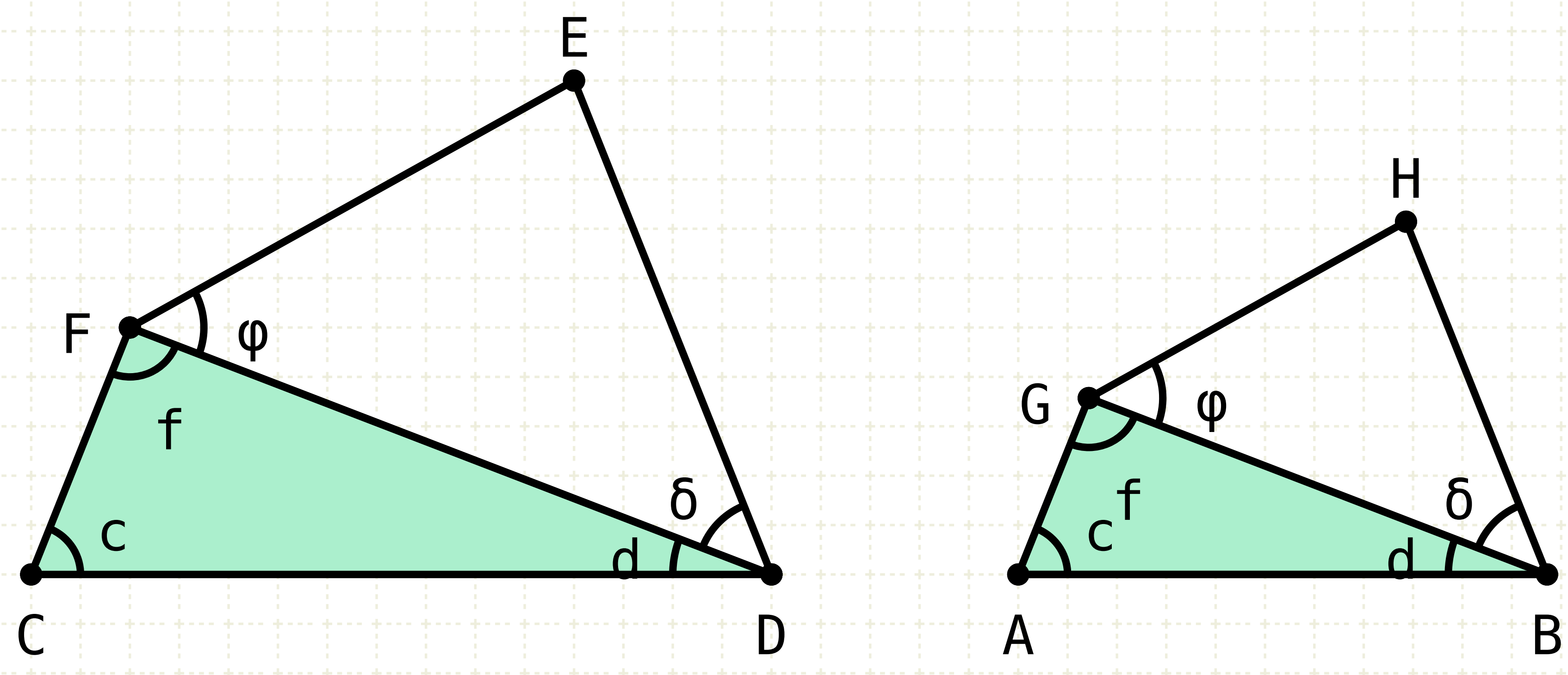

∠FCD = ∠GAB = c
∠CDF = ∠ABG = d
∠DFE = ∠BGH = φ
∠FDE = ∠GBH = δ

∠CFD = ∠AGB = f

Proof

Compare triangles FCD and GAB, the remaining angle AGB is equal to the angle CFD (I·32), thus the two triangles are equiangular

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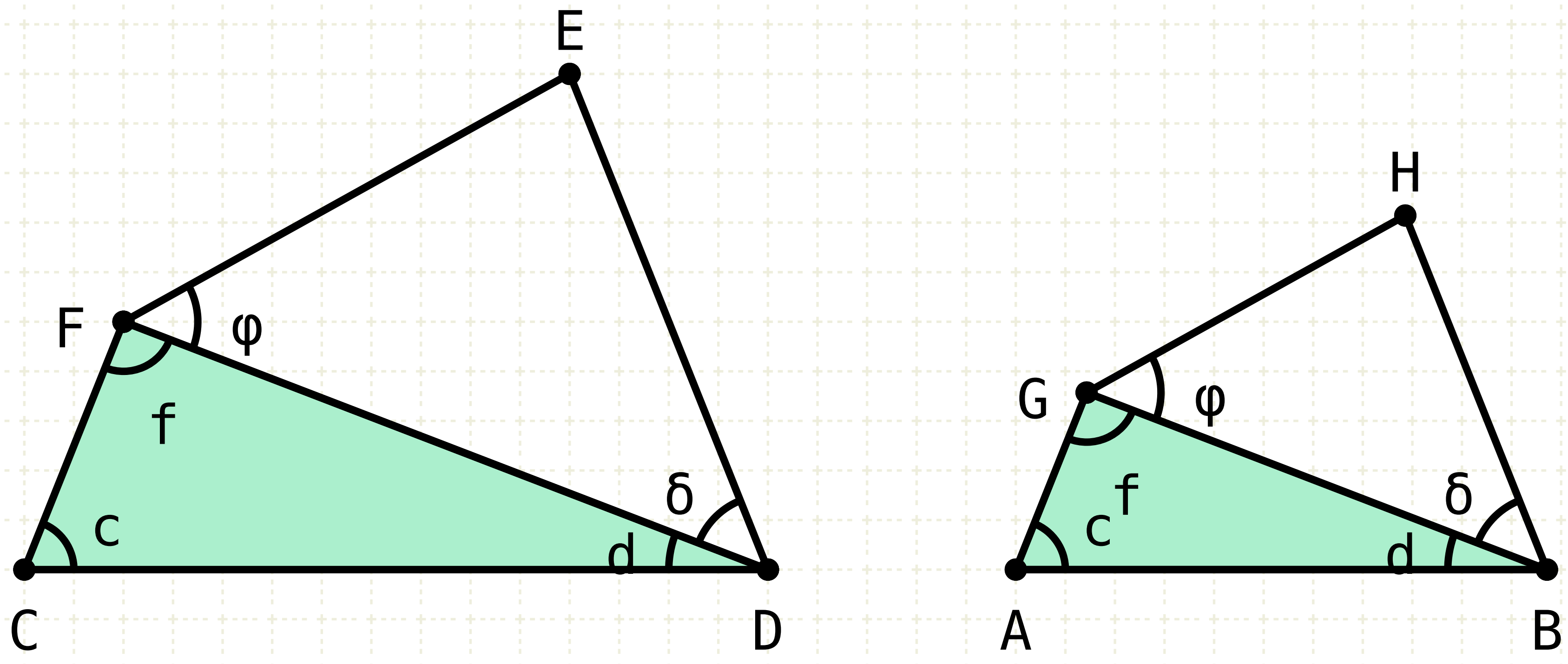


$\angle FCD = \angle GAB = c$
$\angle CDF = \angle ABG = d$
$\angle DFE = \angle BGH = \varphi$
$\angle FDE = \angle GBH = \delta$

$\angle CFD = \angle AGB = f$
$FD:GB = FC:GA = CD:AB$

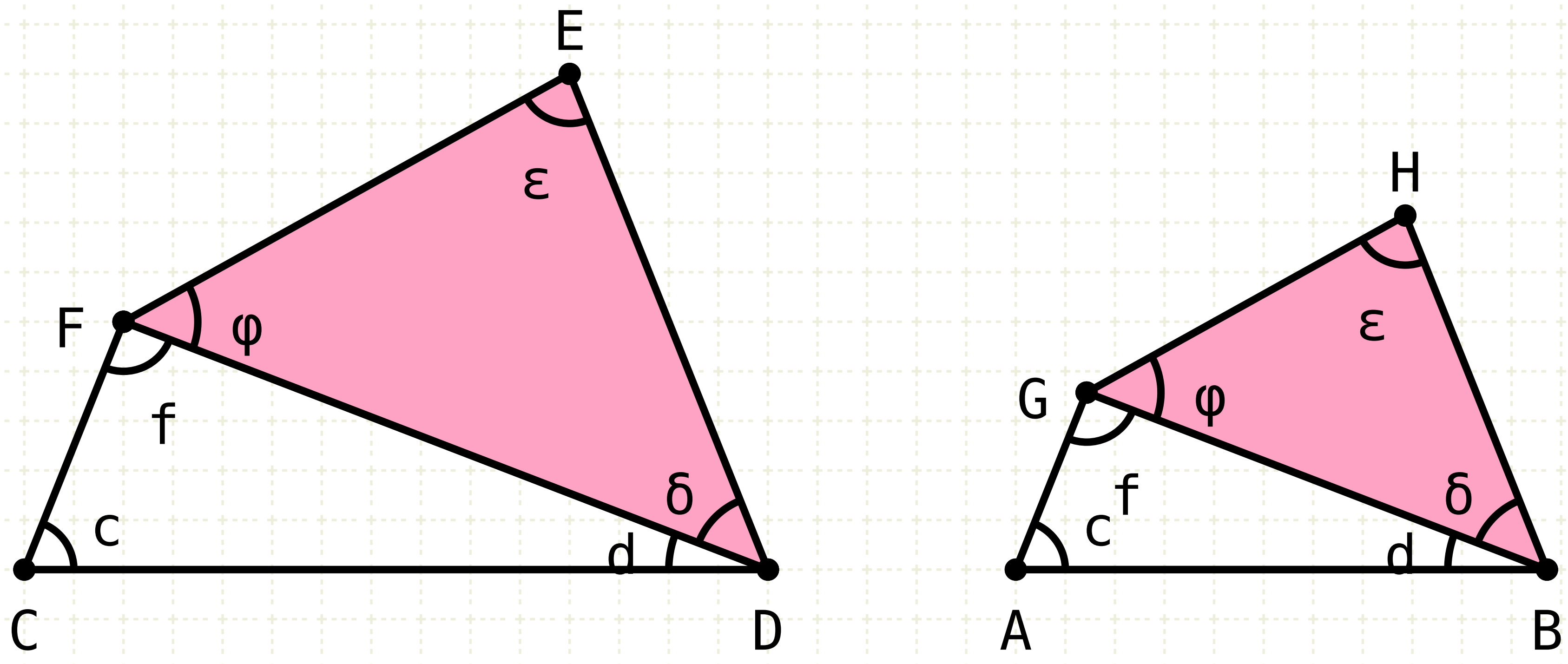
Proof

Compare triangles FCD and GAB, the remaining angle AGB is equal to the angle CFD (I·32), thus the two triangles are equiangular

Thus the sides of FCD and GAB are proportional (VI·4)

Proposition 18 of Book VI

On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure.



∠FCD = ∠GAB = c
∠CDF = ∠ABG = d
∠DFE = ∠BGH = φ
∠FDE = ∠GBH = δ

∠CFD = ∠AGB = f
FD:GB = FC:GA = CD:AB
∠GHB = ∠FED = ε

Proof

Compare triangles FCD and GAB, the remaining angle AGB is equal to the angle CFD (I·32), thus the two triangles are equiangular

Thus the sides of FCD and GAB are proportional (VI·4)

Compare the triangles FED and GHB, the remaining angle GHB is equal to the angle FED (I·32), thus the two triangles are equiangular

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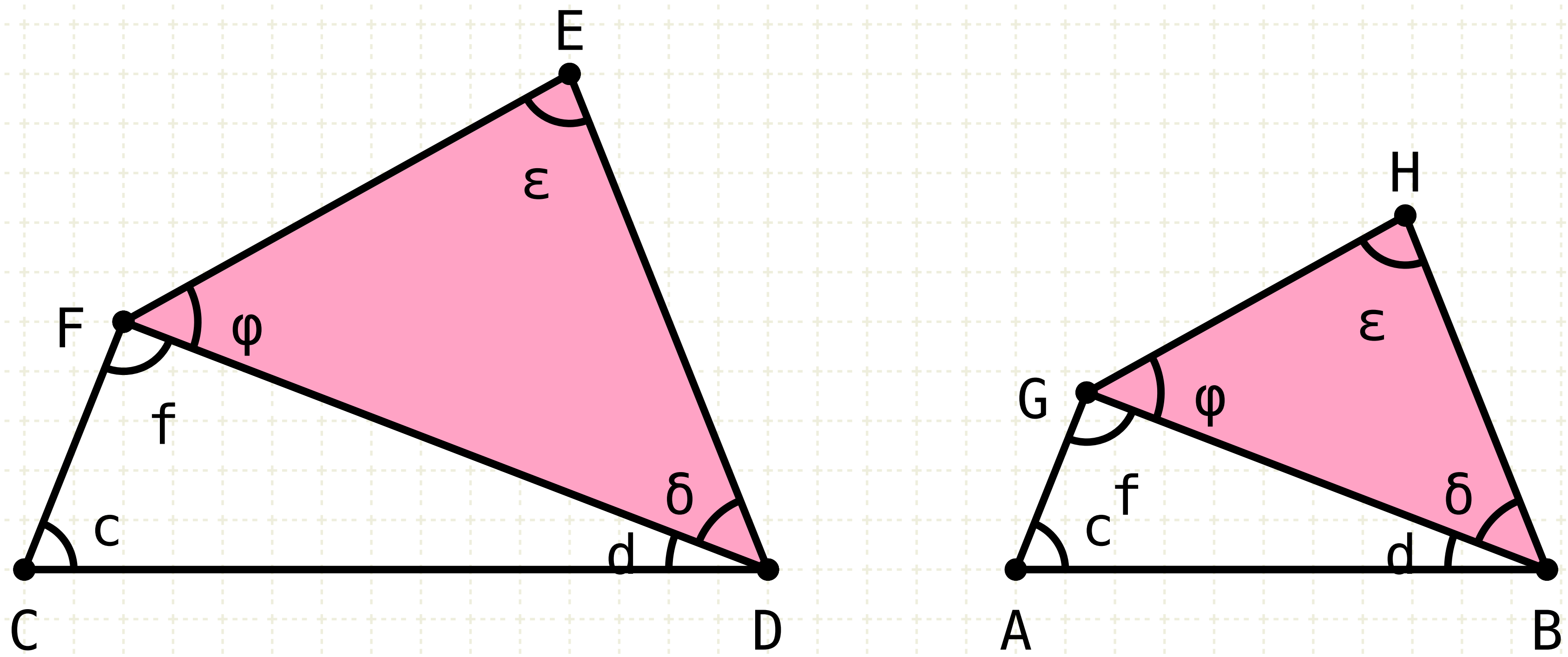

∠FCD = ∠GAB = c
∠CDF = ∠ABG = d
∠DFE = ∠BGH = φ
∠FDE = ∠GBH = δ

∠CFD = ∠AGB = f
FD:GB = FC:GA = CD:AB
∠GHB = ∠FED = ε
FD:GB = FE:GH = ED:BH

Proof

Compare triangles FCD and GAB, the remaining angle AGB is equal to the angle CFD (I·32), thus the two triangles are equiangular

Thus the sides of FCD and GAB are proportional (VI·4)

Compare the triangles FED and GHB, the remaining angle GHB is equal to the angle FED (I·32), thus the two triangles are equiangular

Thus the sides of FED and GHB are proportional (VI·4)

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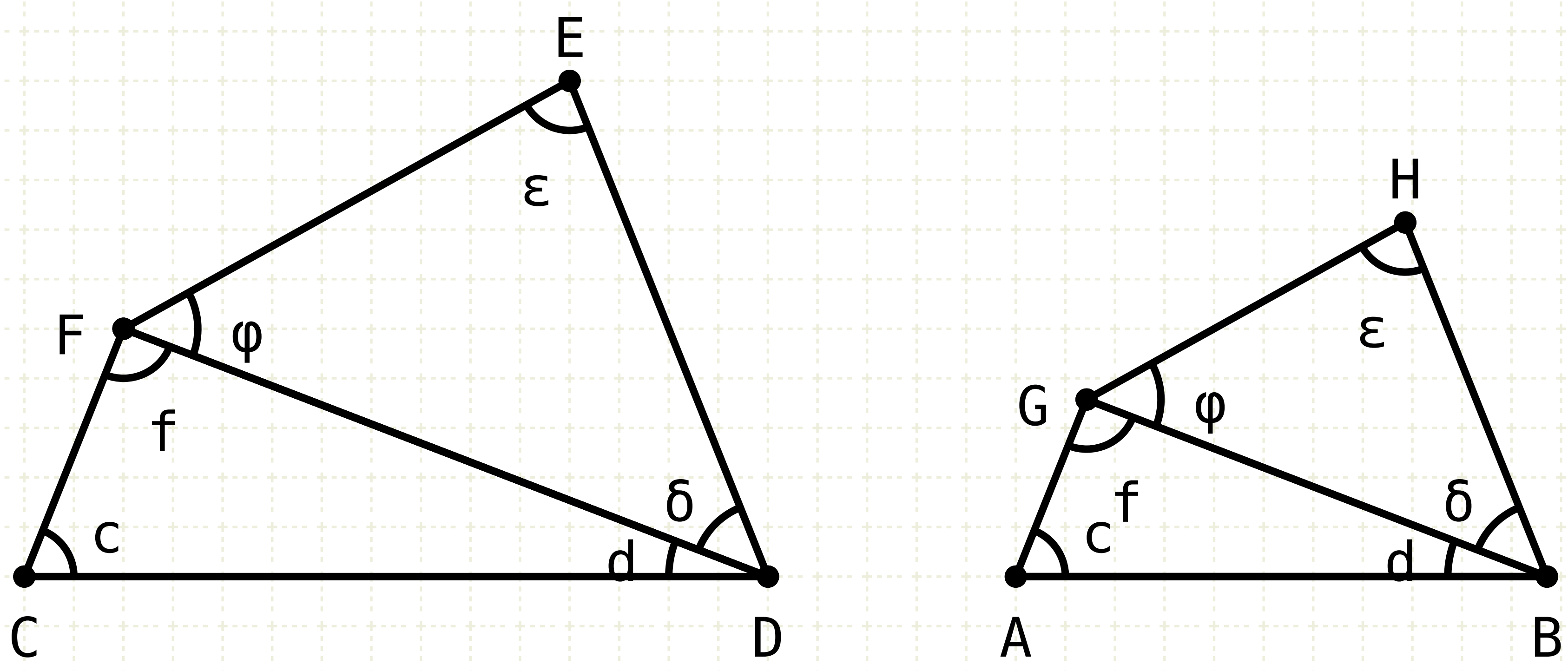
On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure.



∠FCD = ∠GAB = c
∠CDF = ∠ABG = d
∠DFE = ∠BGH = φ
∠FDE = ∠GBH = δ

∠CFD = ∠AGB = f
FD:GB = FC:GA = CD:AB
∠GHB = ∠FED = ε
FD:GB = FE:GH = ED:BH

FD:GB = FC:AG = CD:AB
= FE:GH = ED:HB

Proof

Compare triangles FCD and GAB, the remaining angle AGB is equal to the angle CFD (I·32), thus the two triangles are equiangular

Thus the sides of FCD and GAB are proportional (VI·4)

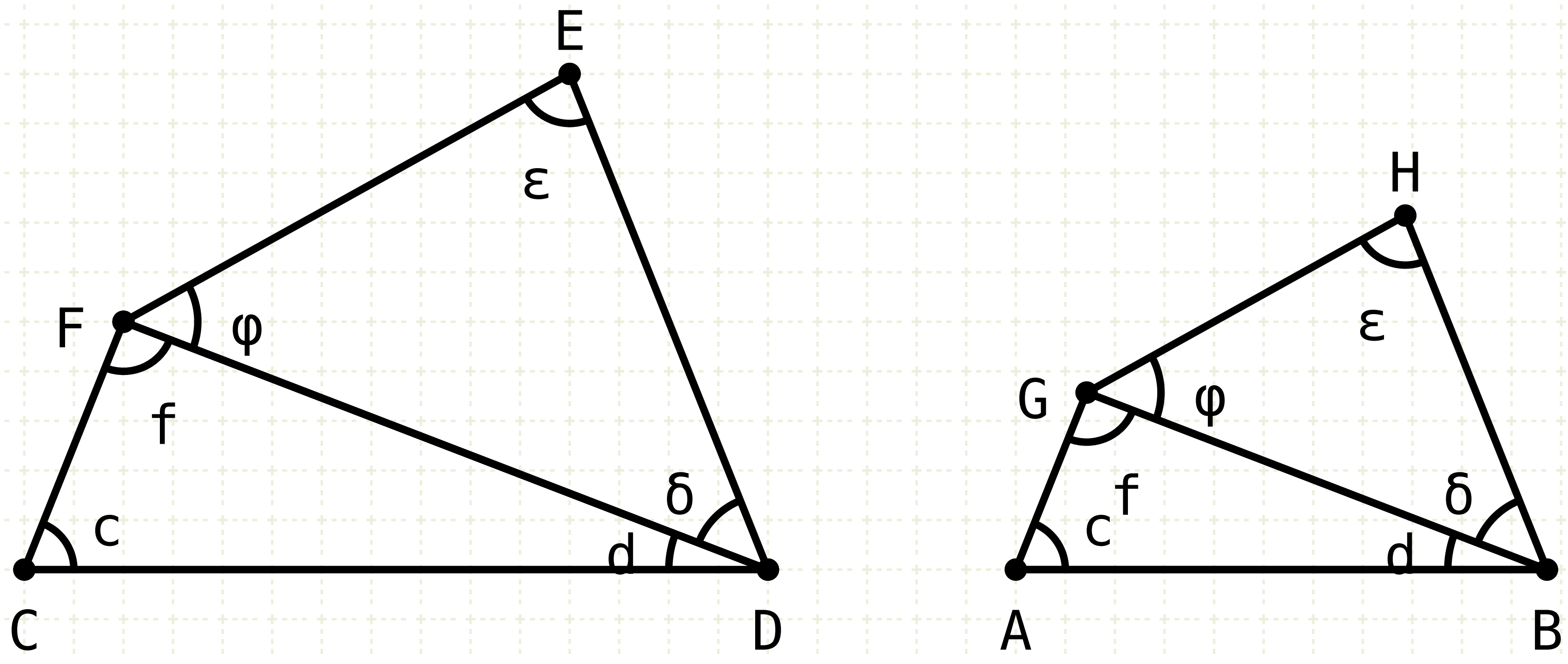
Compare the triangles FED and GHB, the remaining angle GHB is equal to the angle FED (I·32), thus the two triangles are equiangular

Thus the sides of FED and GHB are proportional (VI·4)

The ratio of FD to GB is simultaneously equal to the ratios of the sides FC,AG, CD,AB, FE,GH and ED,HB

Proposition 18 of Book VI

On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure.



$\angle FCD = \angle GAB = c$

$\angle CDF = \angle ABG = d$

$\angle DFE = \angle BGH = \varphi$

$\angle FDE = \angle GBH = \delta$

$\angle CFD = \angle AGB = f$

$FD:GB = FC:GA = CD:AB$

$\angle GHB = \angle FED = \varepsilon$

$FD:GB = FE:GH = ED:BH$

$FD:GB = FC:AG = CD:AB$
$\quad = FE:GH = ED:HB$

$\angle AGH = \angle CFE = f + \varphi$

Proof

Compare triangles FCD and GAB, the remaining angle AGB is equal to the angle CFD (I·32), thus the two triangles are equiangular

Thus the sides of FCD and GAB are proportional (VI·4)

Compare the triangles FED and GHB, the remaining angle GHB is equal to the angle FED (I·32), thus the two triangles are equiangular

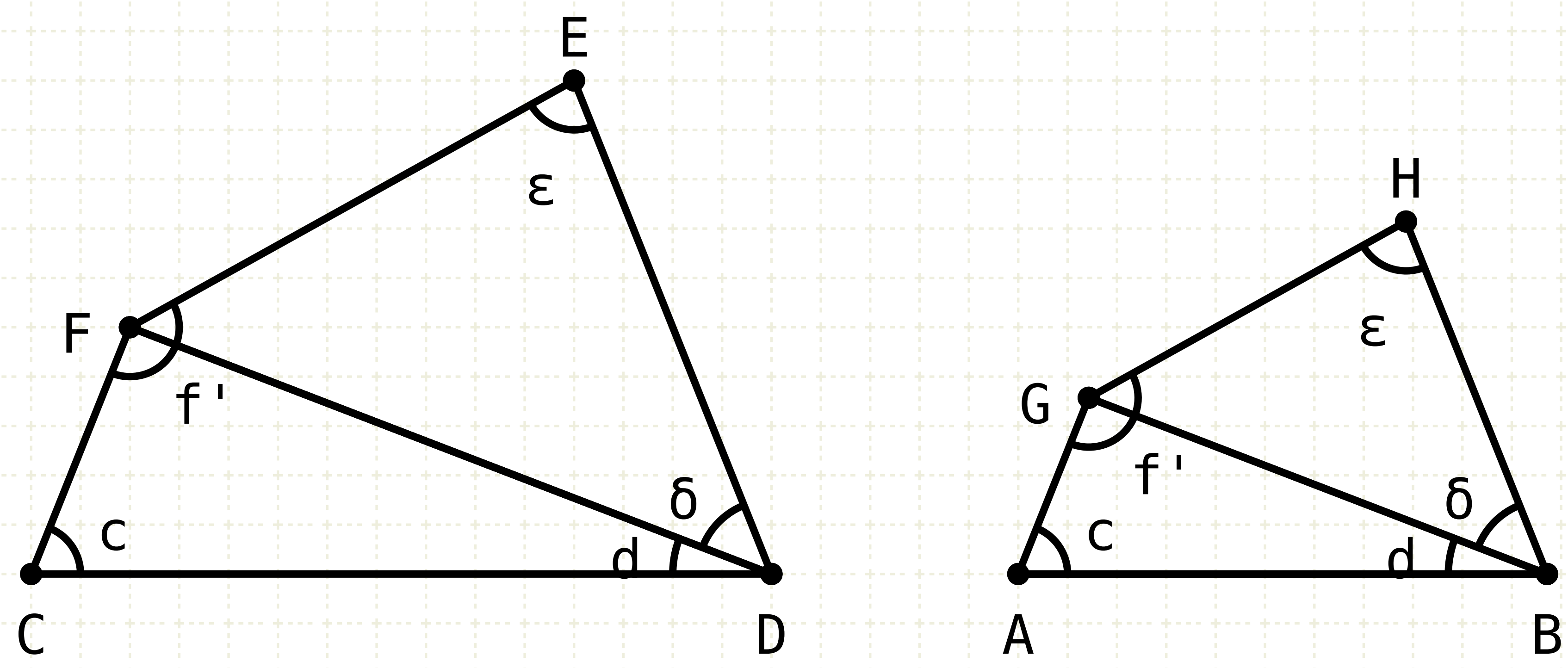
Thus the sides of FED and GHB are proportional (VI·4)

The ratio of FD to GB is simultaneously equal to the ratios of the sides FC,AG, CD,AB, FE,GH and ED,HB

The angles at AGB,BGH are equal to the angles at CFD,DFE, so their sums are also equal, therefore angle CFE equals AGH

Proposition 18 of Book VI

On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure.



∠FCD = ∠GAB = c

∠CDF = ∠ABG = d

∠DFE = ∠BGH = φ

∠FDE = ∠GBH = δ

∠CFD = ∠AGB = f

FD:GB = FC:GA = CD:AB

∠GHB = ∠FED = ε

FD:GB = FE:GH = ED:BH

FD:GB = FC:AG = CD:AB

= FE:GH = ED:HB

∠AGH = ∠CFE = f + φ = f'

Proof

Compare triangles FCD and GAB, the remaining angle AGB is equal to the angle CFD (I·32), thus the two triangles are equiangular

Thus the sides of FCD and GAB are proportional (VI·4)

Compare the triangles FED and GHB, the remaining angle GHB is equal to the angle FED (I·32), thus the two triangles are equiangular

Thus the sides of FED and GHB are proportional (VI·4)

The ratio of FD to GB is simultaneously equal to the ratios of the sides FC,AG, CD,AB, FE,GH and ED,HB

The angles at AGB,BGH are equal to the angles at CFD,DFE, so their sums are also equal, therefore angle CFE equals AGH

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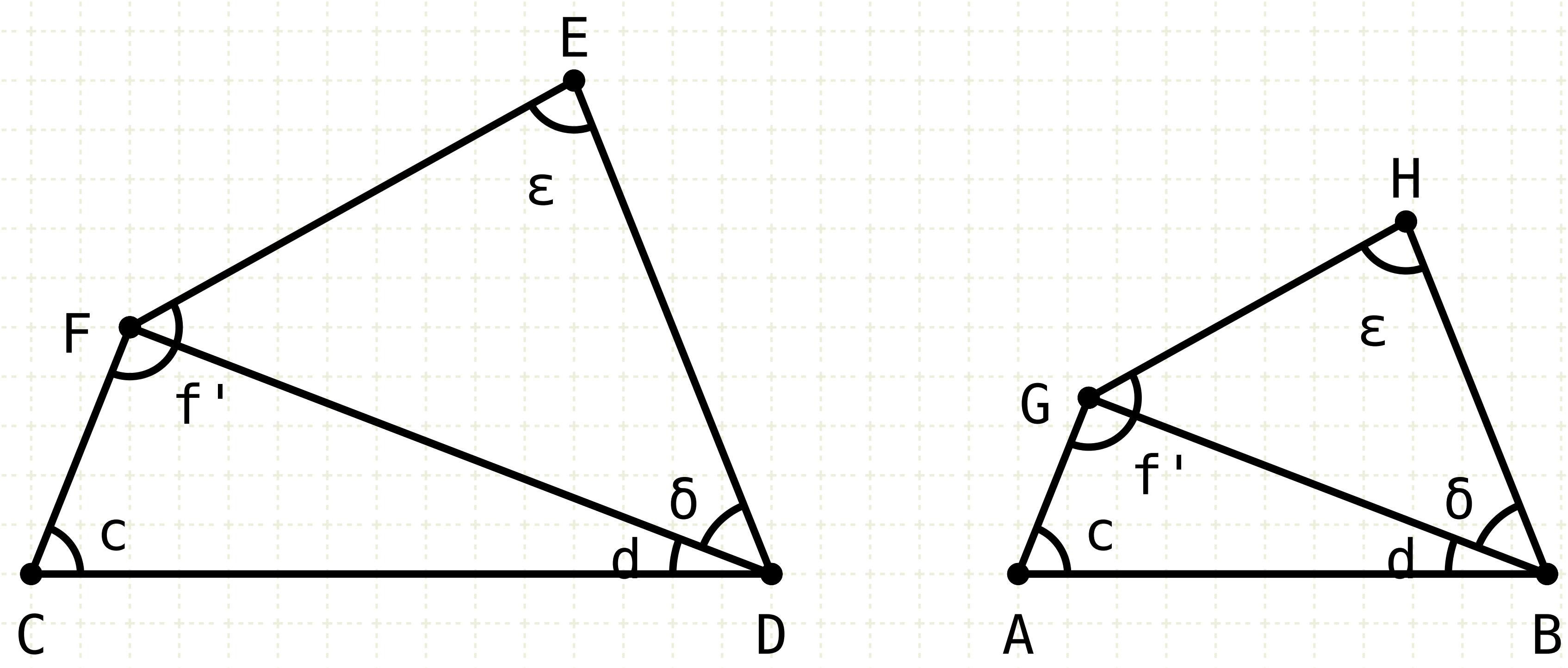

$\angle FCD = \angle GAB = c$

$\angle CDF = \angle ABG = d$

$\angle DFE = \angle BGH = \varphi$

$\angle FDE = \angle GBH = \delta$

$\angle CFD = \angle AGB = f$

$FD:GB = FC:GA = CD:AB$

$\angle GHB = \angle FED = \varepsilon$

$FD:GB = FE:GH = ED:BH$

$FD:GB = FC:AG = CD:AB$

$\quad = FE:GH = ED:HB$

$\angle AGH = \angle CFE = f + \varphi = f'$

$\angle ABH = \angle CDE = d + \delta$

Proof

Compare triangles FCD and GAB, the remaining angle AGB is equal to the angle CFD (I·32), thus the two triangles are equiangular

Thus the sides of FCD and GAB are proportional (VI·4)

Compare the triangles FED and GHB, the remaining angle GHB is equal to the angle FED (I·32), thus the two triangles are equiangular

Thus the sides of FED and GHB are proportional (VI·4)

The ratio of FD to GB is simultaneously equal to the ratios of the sides FC,AG, CD,AB, FE,GH and ED,HB

The angles at AGB,BGH are equal to the angles at CFD,DFE, so their sums are also equal, therefore angle CFE equals AGH

The angles at ABG,GBH are equal to the angles at CDF,FDE, so their sums are also equal, therefore angle ABH equals CDE

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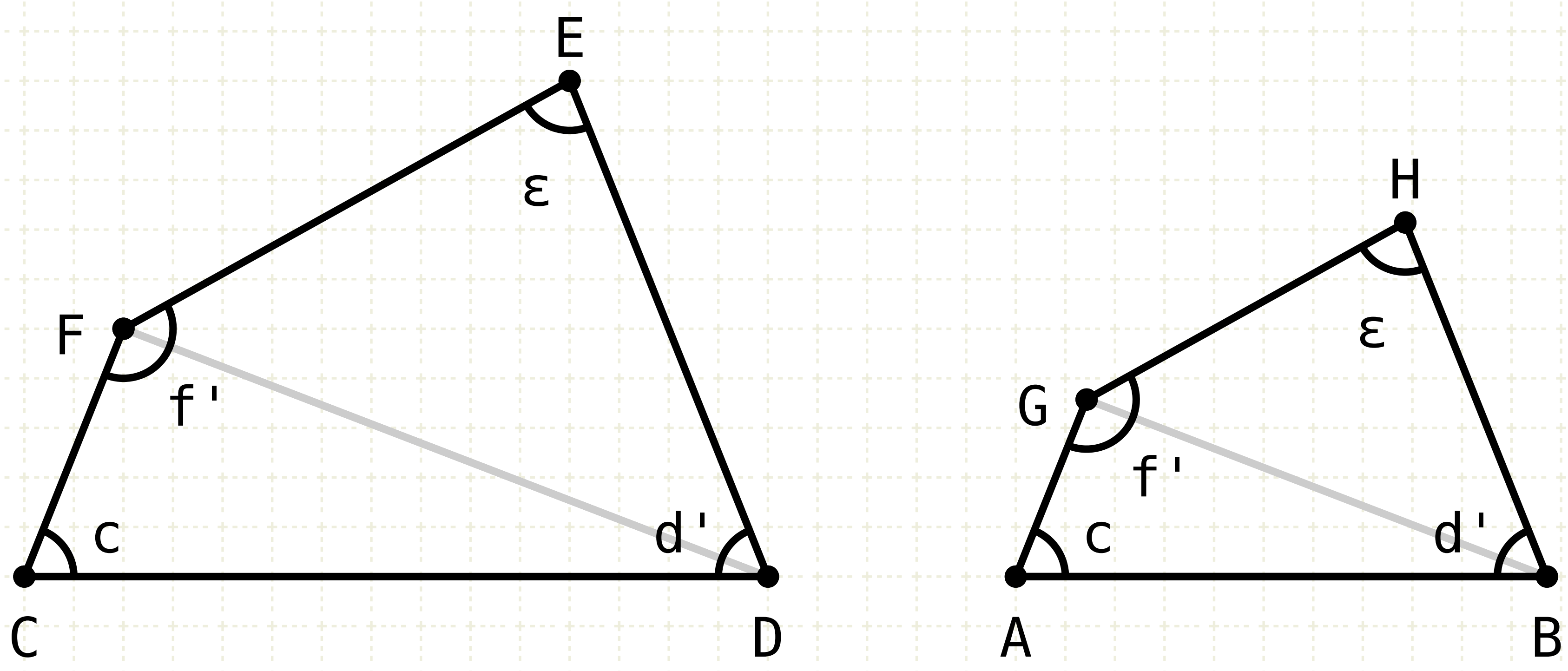

∠FCD = ∠GAB = c
∠CDF = ∠ABG = d
∠DFE = ∠BGH = φ
∠FDE = ∠GBH = δ

FD:GB = FC:AG = CD:AB
= FE:GH = ED:HB
∠AGH = ∠CFE = f + φ = f'
∠ABH = ∠CDE = d + δ = d'

∠CFD = ∠AGB = f
FD:GB = FC:GA = CD:AB
∠GHB = ∠FED = ε
FD:GB = FE:GH = ED:BH

Proof

Compare triangles FCD and GAB, the remaining angle AGB is equal to the angle CFD (I·32), thus the two triangles are equiangular

Thus the sides of FCD and GAB are proportional (VI·4)

Compare the triangles FED and GHB, the remaining angle GHB is equal to the angle FED (I·32), thus the two triangles are equiangular

Thus the sides of FED and GHB are proportional (VI·4)

The ratio of FD to GB is simultaneously equal to the ratios of the sides FC,AG, CD,AB, FE,GH and ED,HB

The angles at AGB,BGH are equal to the angles at CFD,DFE, so their sums are also equal, therefore angle CFE equals AGH

The angles at ABG,GBH are equal to the angles at CDF,FDE, so their sums are also equal, therefore angle ABH equals CDE

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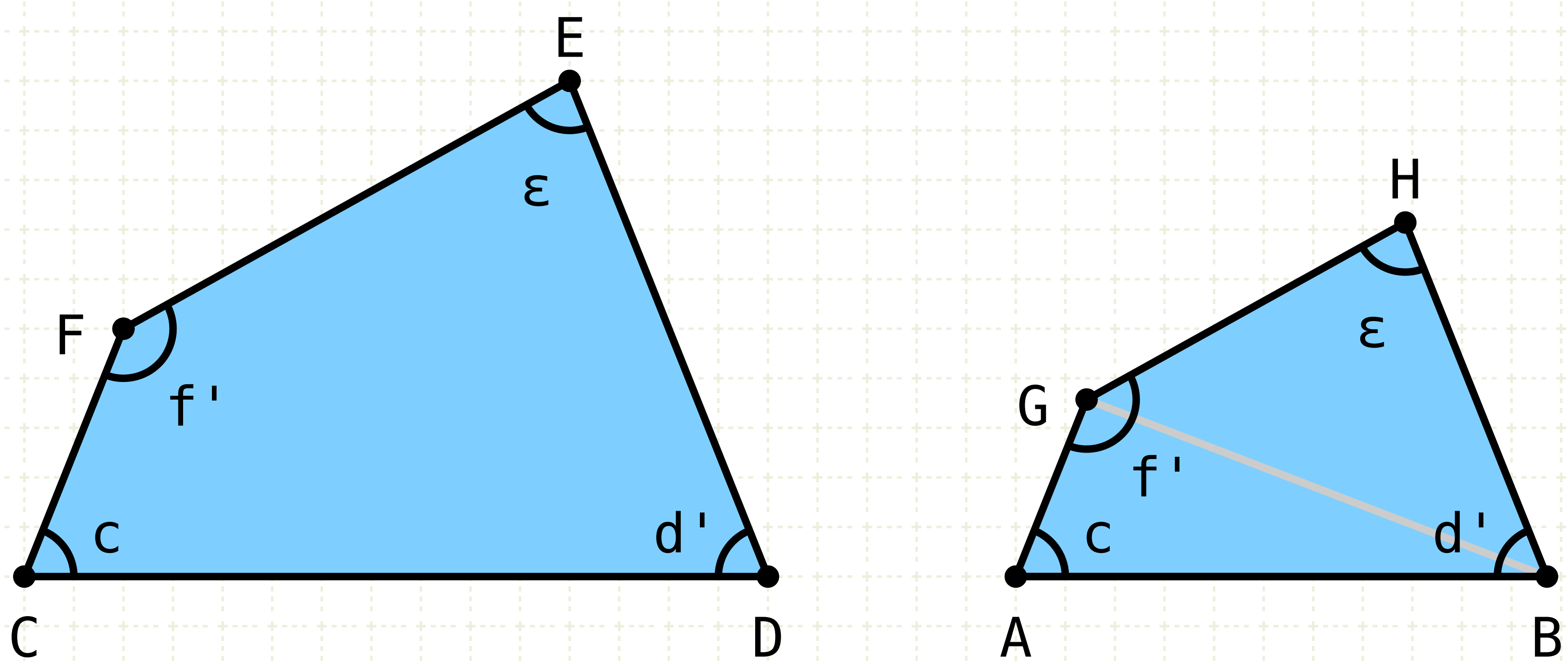
On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure.



∠FCD = ∠GAB = c
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∠DFE = ∠BGH = φ
∠FDE = ∠GBH = δ

∠CFD = ∠AGB = f
FD:GB = FC:GA = CD:AB
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FD:GB = FE:GH = ED:BH

FD:GB = FC:AG = CD:AB
= FE:GH = ED:HB
∠AGH = ∠CFE = f + φ = f'
∠ABH = ∠CDE = d + δ = d'

□CDEF ~ □ABHG

Proof

Compare triangles FCD and GAB, the remaining angle AGB is equal to the angle CFD (I·32), thus the two triangles are equiangular

Thus the sides of FCD and GAB are proportional (VI·4)

Compare the triangles FED and GHB, the remaining angle GHB is equal to the angle FED (I·32), thus the two triangles are equiangular

Thus the sides of FED and GHB are proportional (VI·4)

The ratio of FD to GB is simultaneously equal to the ratios of the sides FC,AG, CD,AB, FE,GH and ED,HB

The angles at AGB,BGH are equal to the angles at CFD,DFE, so their sums are also equal, therefore angle CFE equals AGH

The angles at ABG,GBH are equal to the angles at CDF,FDE, so their sums are also equal, therefore angle ABH equals CDE

If the angles are all equal, and the sides about the equal angles are proportional, then the two figures are similar (VI.Def.1)

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