

Euclid's Elements

Book VII

Definitions:

- 1 A unit is that by virtue of which each of the things that exist is called one
- 2 A number is a multitude composed of units. (not one)
- 3 A number is part of a number, the less of the greater, when it measures the greater
- 11 A prime number is that which is measured by a unit alone.
- 12 Numbers prime to one another are those which are measured by a unit alone as a common measure
- 15 A number is said to multiply a number when that which is multiplied is added to itself as many times as there are units in the other, and thus some number is produced.
- 20 Numbers are proportional when the first is the same multiple, or the same part, or the same parts, of the second that the third is of the fourth.

As long as algebra and geometry have been separated, their progress have been slow and their uses limited; but when these two sciences have been united, they have lent each mutual forces, and have marched together towards perfection.

Joseph-Louis Lagrange
(1736 to 1813)



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1	Determine if two numbers are relatively prime	10	If $B = (p/q) \cdot A$ and $D = (p/q) \cdot C$, and If $B = (r/s) \cdot D$, then $A = (r/s) \cdot C$	21	If A,B are relatively prime, then A,B are the smallest whole numbers that can be used to describe the ratio A:B
2	Find the greatest common divisor for two numbers	11	If $A:B = C:D$, then $(A-C):(B-D) = A:B$	22	If A,B are the smallest whole numbers that can be used to describe the ratio A:B, then A,B are relatively prime
3	Find the largest common divisor for three numbers	12	If $A:B = C:D$, then $(A+C):(B+C) = A:B$	23	If A,B are relatively prime and if $A = n \cdot C$, then B,C are relatively prime
4	Given two natural numbers, A and B, either B is part of A, or there exists a natural number (a part) that can measure both A and B	13	If $A:B = C:D$, then $A:C = B:D$	24	If A,C are relatively prime and B,C are relatively prime then the $A \times B$ is relatively prime to C
5	If $B = (1/q) \cdot A$ and $D = (1/q) \cdot C$, then $(B+D) = (1/q) \cdot (A+C)$	14	If $A:B = D:E$ and $B:C = E:F$, then $A:C = D:F$	25	If A,B are relatively prime then A^2, B are relatively prime
6	If $B = (p/q) \cdot A$ and $D = (p/q) \cdot C$, then $(B+D) = (p/q) \cdot (A+C)$	15	If $B = i \cdot 1$ and $E = i \cdot D$, and if $D = j \cdot 1$ then $E = j \cdot B$	26	If A is relatively prime to C and D, and if B is also relatively prime to C and D, then $A \times B$ is relatively prime to $C \times D$
7	If $B = A/q$ and $D = C/q$, $B > D$, then $(B-D) = (A-C)/q$	16	$A \times B = B \times A$	27	If A,B are relatively prime, then A^2, B^2 are relatively prime, and A^3, B^3 are relatively prime, and so on
8	If $B = (p/q) \cdot A$ and $D = (p/q) \cdot C$, $B > D$, then $(B-D) = (p/q) \cdot (A-C)$	17	If $D = A \times B$ and $E = A \times C$ then $D:E = B:C$		
9	If $B = (1/q) \cdot A$ and $D = (1/q) \cdot C$, and If $B = (r/s) \cdot D$, then $A = (r/s) \cdot C$	18	If $D = B \times A$ and $E = C \times A$ then $D:E = B:C$		
		19	If $A:B = C:D$ then $A \times D = B \times C$ If $A \times D = B \times C$ then $A:B = C:D$		
		20	Given the ratio A:B and C,D are the smallest numbers such that $A:B = C:D$ then $A = n \cdot C$ and $B = n \cdot D$		



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28 If A,B are relatively prime, then A,(A+B) are relatively prime

29 If A is prime, and $B \neq n \cdot A$, then A,B are relatively prime

30 If $C = A \times B$ and $C = i \cdot D$ where D is prime, then either $A = j \cdot D$ or $B = j \cdot D$

31 If $A = B \times C$, then $A = j \cdot D$ where D is prime

32 If A is a number then it is either prime, or $A = j \cdot D$ where D is prime

33 Find the smallest numbers X,Y,Z where the ratio X:Y:Z is equal to the given ratio A:B:C

34 Find the lowest common denominator of 2 numbers

35 If E is the lowest common denominator of A,B, and if $C = n \cdot A = m \cdot B$, then $C = i \cdot E$

36 Find the least common multiple of 3 numbers

37 If $A = p \cdot B$, then $A = q \cdot C$ where $C = p \cdot 1$

38 If $A = (1/c) \cdot B$ and $C = c \cdot 1$ then $A = n \cdot C$

39 Find the smallest number that has the fractions $1/a$, $1/b$, $1/c$



Proposition 38 of Book VII

If a number have any part whatever, it will be measured by a number called by the same name as the part



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In other words

If A is divisible by any number it can also be measured by that number



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$$B = (1/c) \cdot A$$
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If B is a part (fraction) of A, and C is a number equal to the inverse of the fraction, then ...



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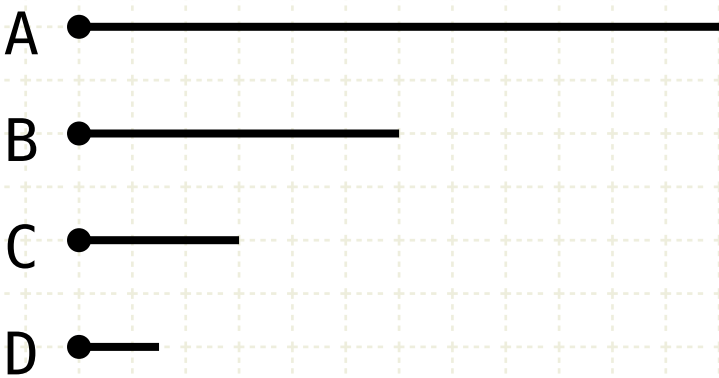
Proof

Let the number A have a part (fraction) B, and let that fraction be called C



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$$\begin{aligned} B &= (1/c) \cdot A \\ C &= c \\ D &= 1 \end{aligned}$$

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Proof

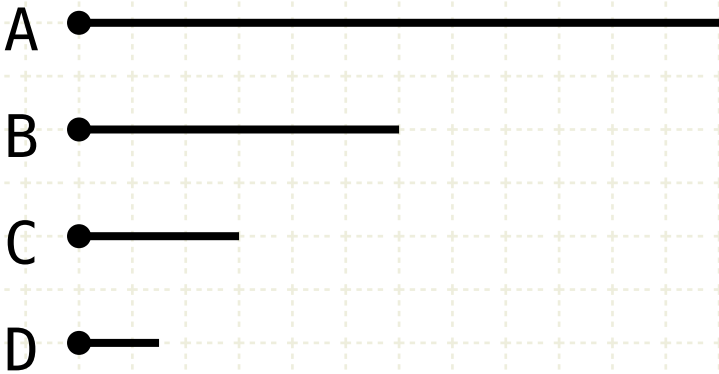
Let the number A have a part (fraction) B, and let that fraction be called C

Let the unit measure be D



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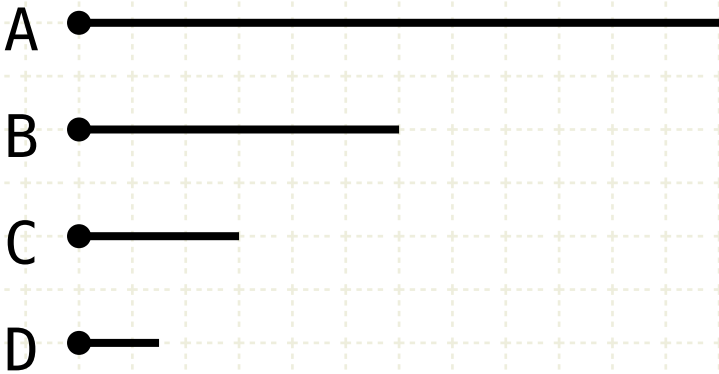
Let the unit measure be D

Since B is the same fraction of A as D is of C...



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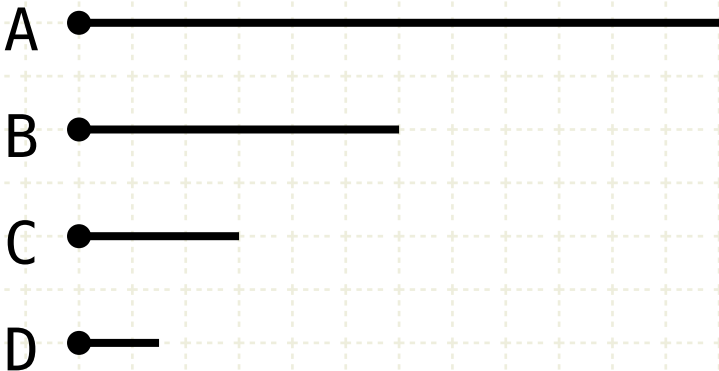
Since B is the same fraction of A as D is of C...

Then the unit D measures the number C the same number of times that B measures A



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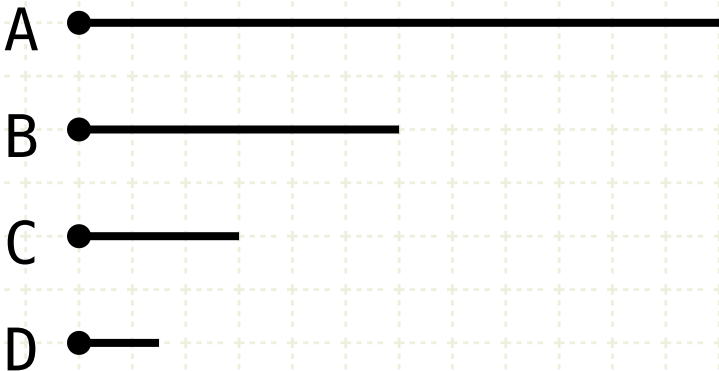
Then the unit D measures the number C the same number of times that B measures A

Alternatively, D measures the number B the same number of times that C measures A (VII·15)



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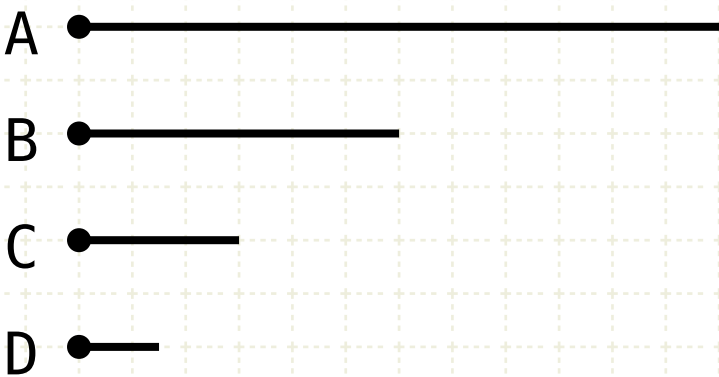
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Therefore C measures A



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Since B is the same fraction of A as D is of C...

Then the unit D measures the number C the same number of times that B measures A

Alternatively, D measures the number B the same number of times that C measures A (VII·15)

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