

Euclid's Elements

Book II

It is a remarkable fact in the history of geometry, that the Elements of Euclid, written two thousand years ago, are still regarded by many as the best introduction to the mathematical sciences.

Florian Cajori,
A History of Mathematics (1893)

Definitions:

Any rectangular parallelogram is said to be contained by the two straight lines containing the right angle.

And in any parallelogrammic area let any one whatever of the parallelograms about its diameter with the two complements be called a gnomon.

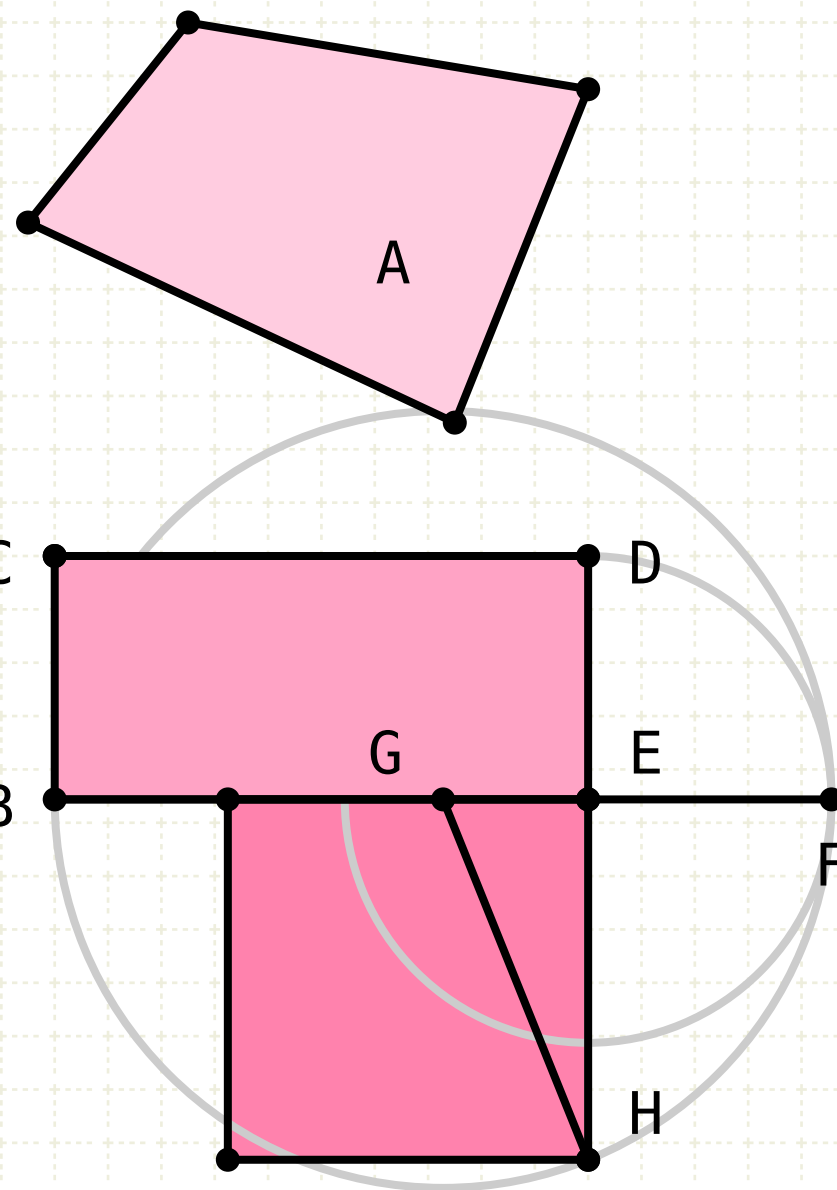
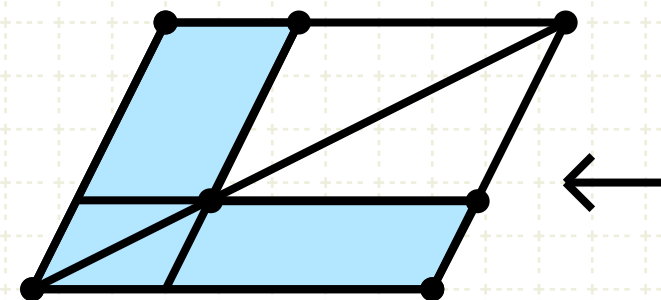


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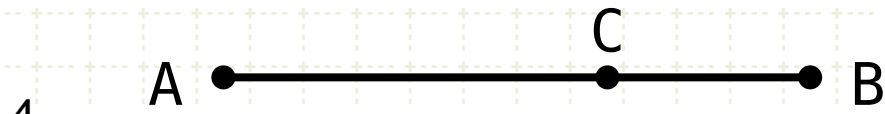
$A \cdot BC = A \cdot BD + A \cdot DE + A \cdot EC$



$AB^2 = AB \cdot AC + AB \cdot BC$



$AB \cdot CB = AC \cdot CB + CB^2$



$AB^2 = AC^2 + CB^2 + 2 \cdot AC \cdot CB$



$AD \cdot DB + CD^2 = CB^2$



$AD \cdot DB + CB^2 = CD^2$



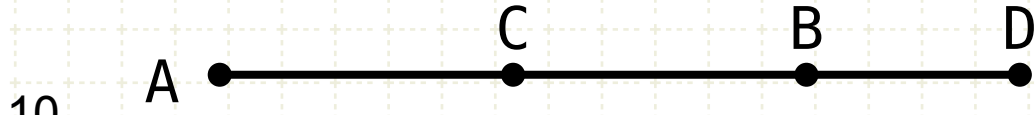
$AB^2 + BC^2 = AC^2 + 2 \cdot AB \cdot BC$



$4 \cdot AB \cdot BC + AC^2 = (AB + BC)^2$



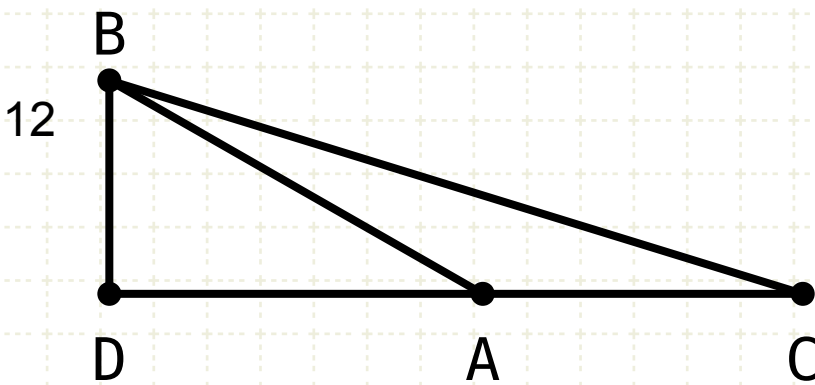
$AD^2 + DB^2 = 2 \cdot (AC^2 + CD^2)$



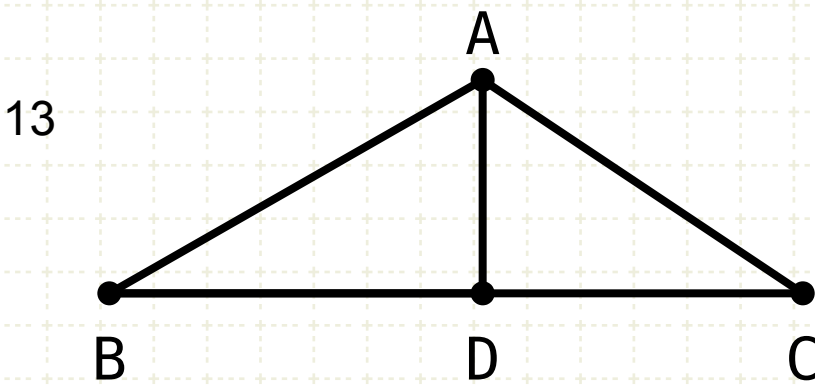
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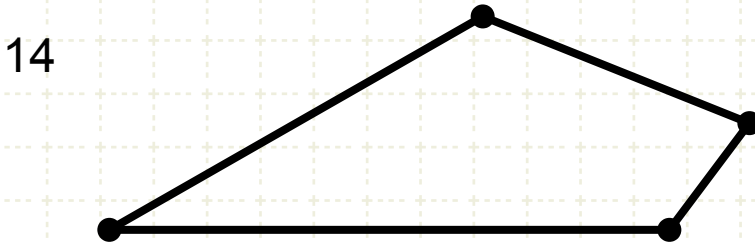
Find H. $AB \cdot BH = AH^2$



Cosine Law. $BC^2 = AB^2 + AC^2 + 2 \cdot AD \cdot AC$



Cosine Law. $AC^2 = AB^2 + BC^2 - 2 \cdot BD \cdot BC$



Find square of polygon



Proposition 6 of Book II

If a straight line be bisected and a straight line be added to it in a straight line, the rectangle contained by the whole with the added straight line and the added straight line together with the square on the half is equal to the square on the straight line made up of the half and the added straight line.



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In other words

Let AB be a straight line, bisected at point C, and extend the line AB to an arbitrary point D

$$AD=AB+BD, \quad CB=\frac{1}{2}AB, \quad CD=CB+BD$$



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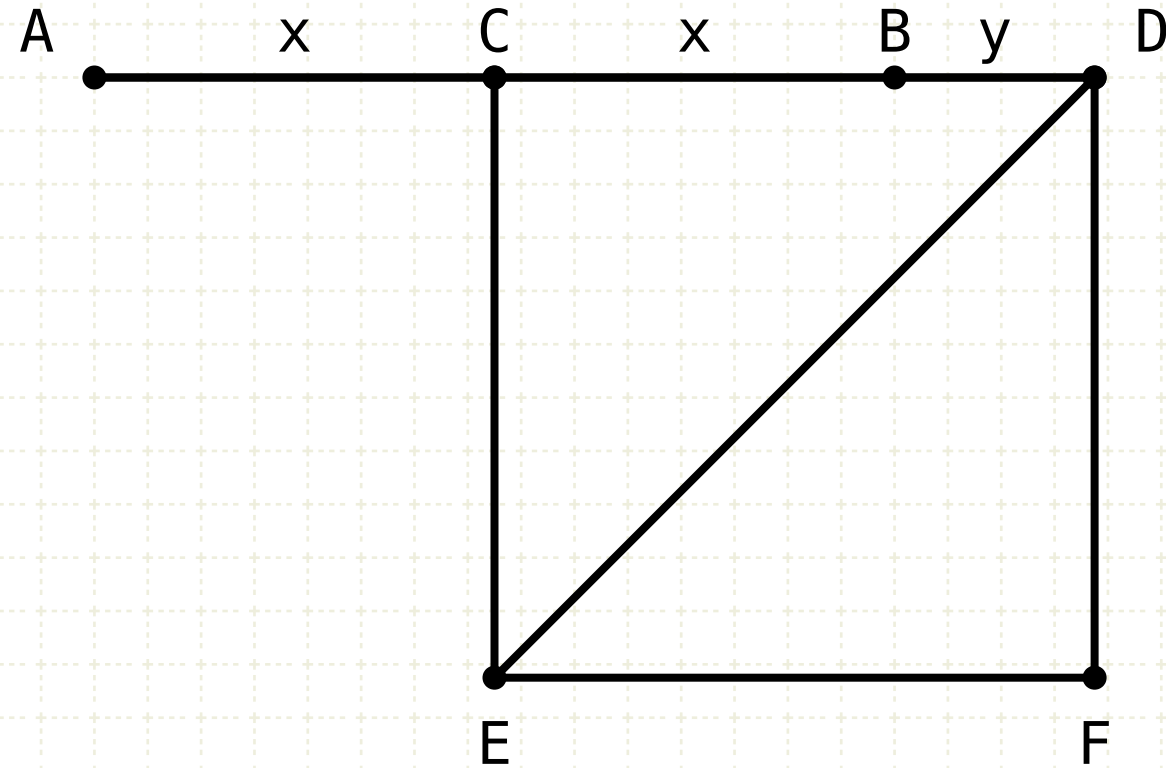
$$AD \cdot DB + CB \cdot CB = CD \cdot CD$$

$$(2x+y)y + x^2 = (x+y)^2$$



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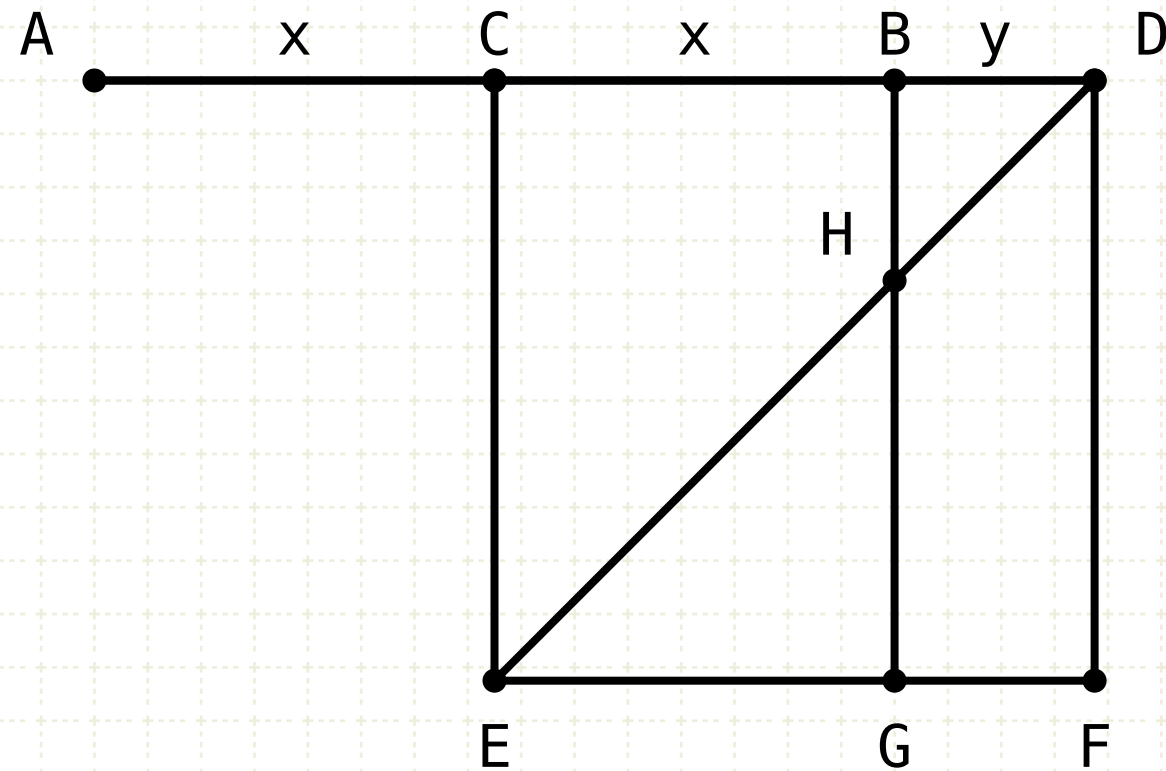
The rectangle formed by the extended line AD, and the extension BD plus the square on CB is equal to the square on CD

Construction:

Draw a square CEFB on the line CD (I-46) and draw the diagonal DE

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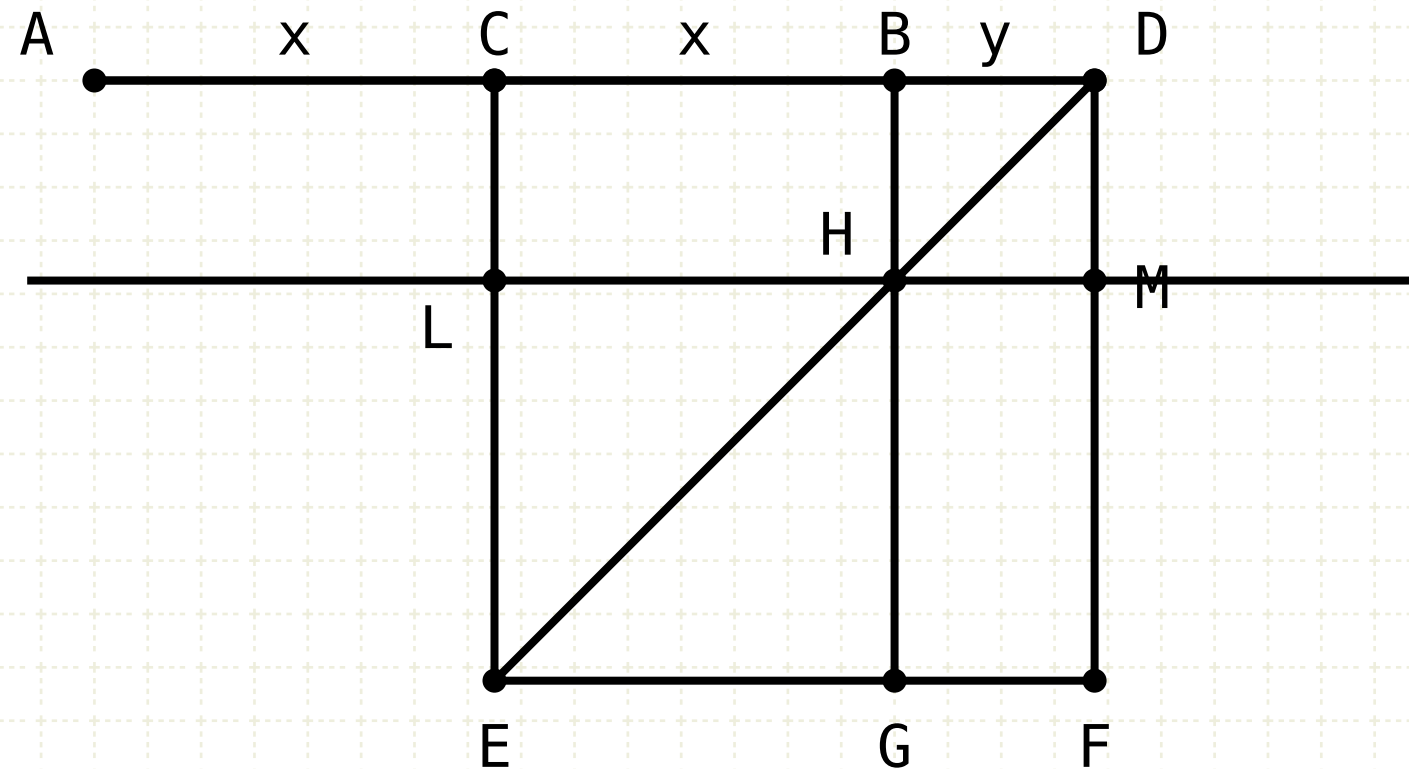
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Draw a square CEFG on the line CD (I·46) and draw the diagonal DE

From point B, draw a line parallel to either CE or DF (I·31)

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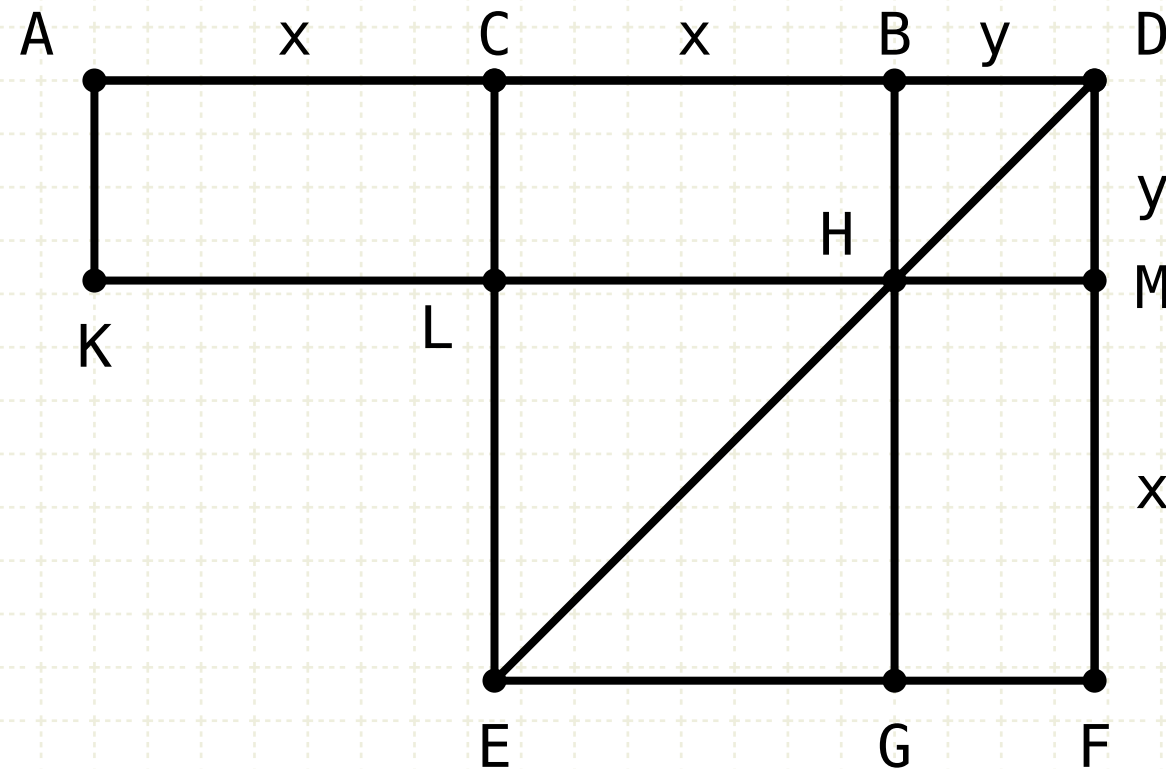
Draw a square CEFB on the line CD (I·46) and draw the diagonal DE

From point B, draw a line parallel to either CE or DF (I·31)

From point H, draw a line parallel to either AB or EF (I·31)

Proposition 6 of Book II

If a straight line be bisected and a straight line be added to it in a straight line, the rectangle contained by the whole with the added straight line and the added straight line together with the square on the half is equal to the square on the straight line made up of the half and the added straight line.



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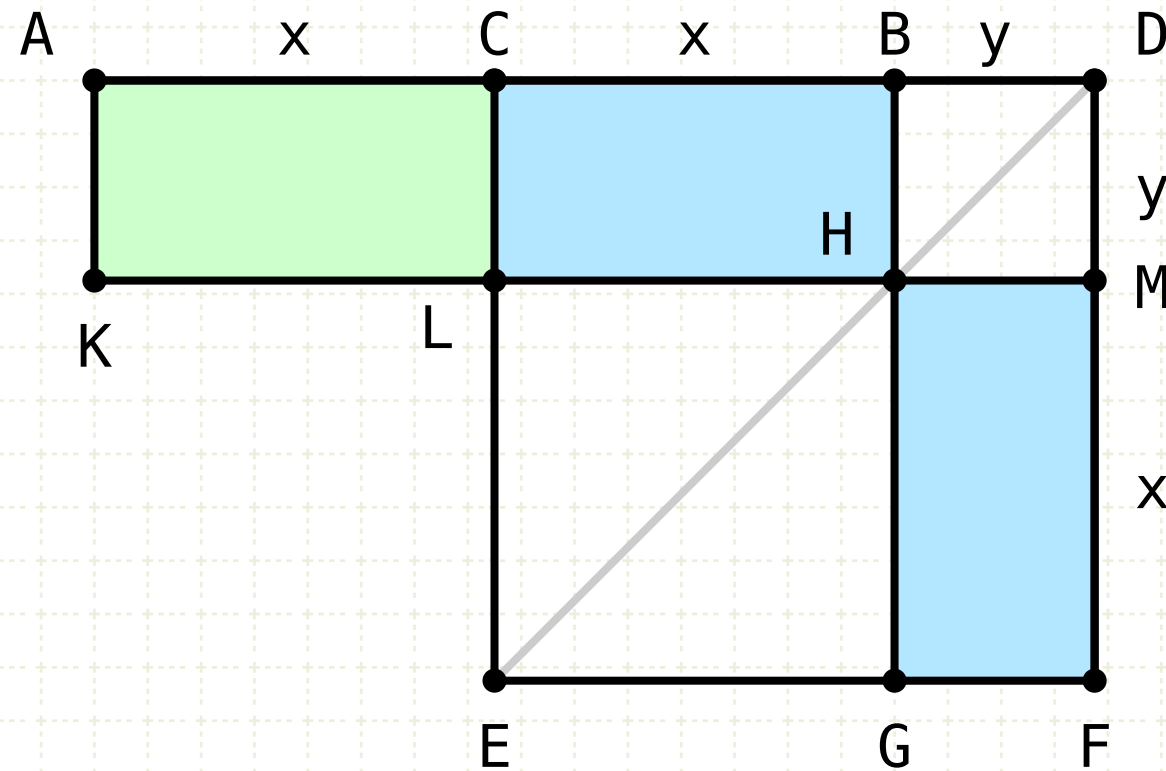
From point B, draw a line parallel to either CE or DF (I·31)

From point H, draw a line parallel to either AB or EF (I·31)

From point A, draw a line parallel to either CL or BH (I·31)

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Proof:

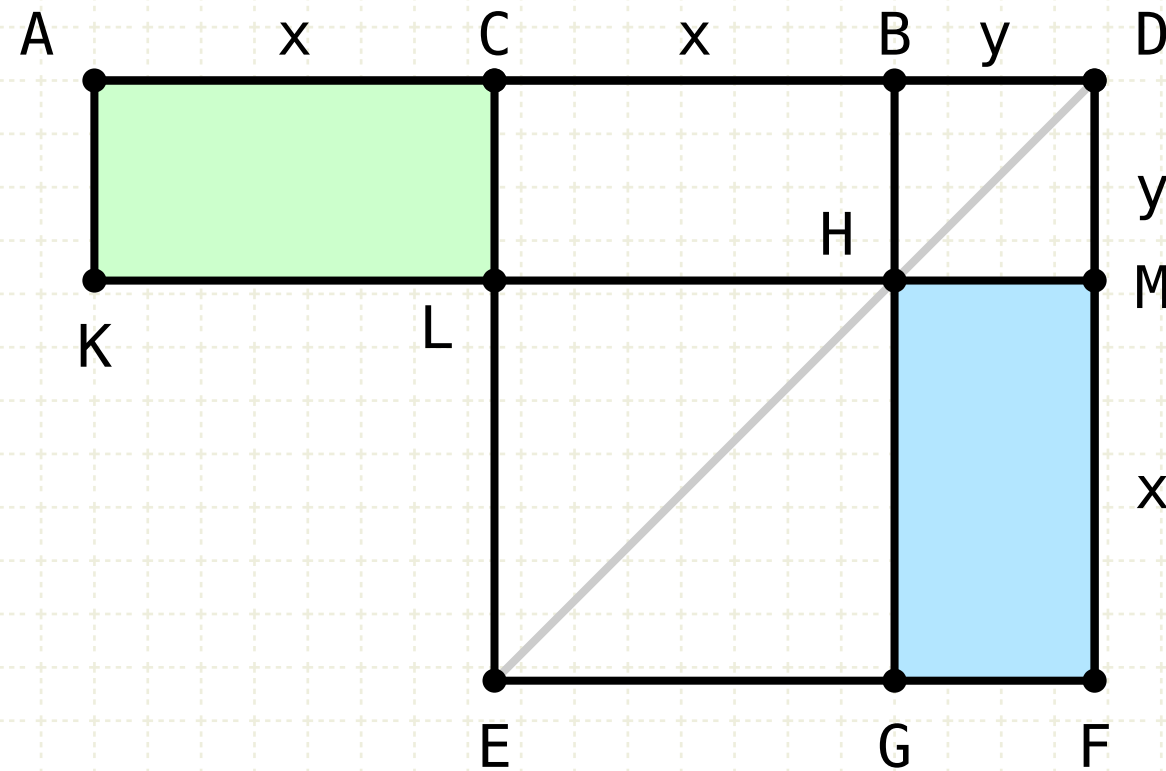
Since AC equals CB, then AL equals CH (I·36), and CH equals HF (I·43), then $AL = HF$

$$AD=AB+BD, \quad CB=\frac{1}{2}AB, \quad CD=CB+BD$$

$$AC=CB \quad \therefore \square AL=\square CH=\square HF \quad \therefore \square AL=\square HF$$

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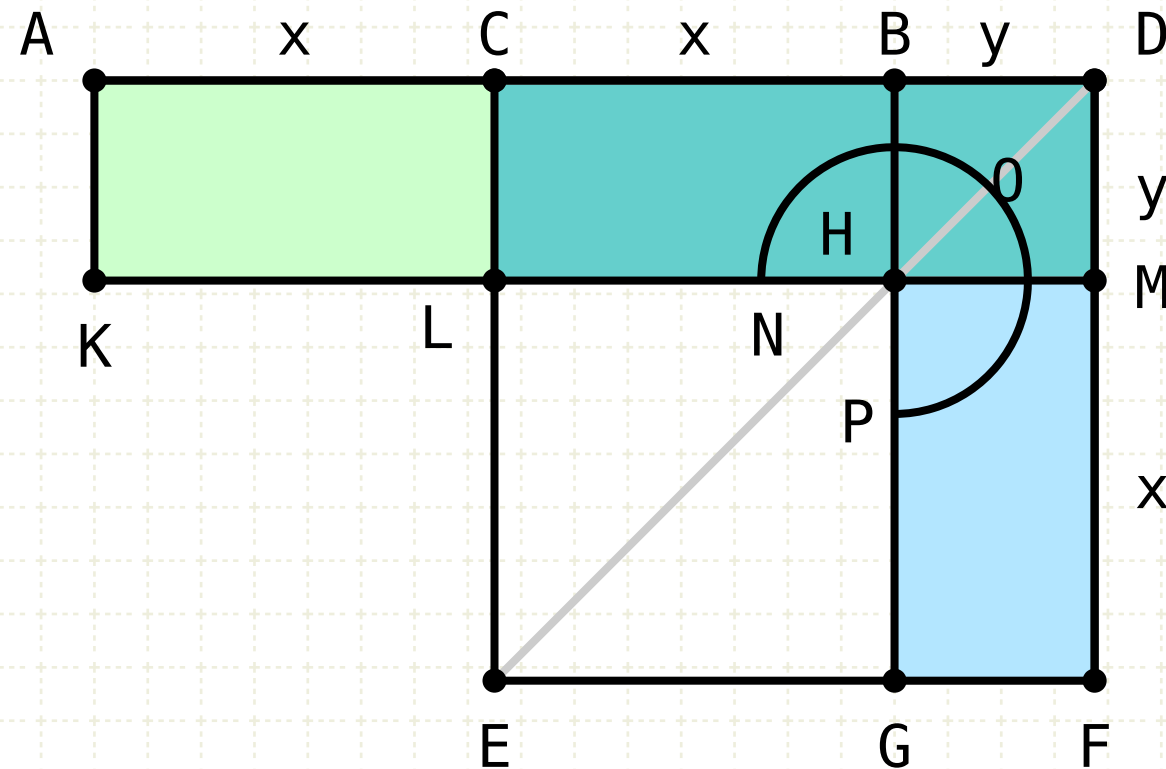
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Proof:

Since AC equals CB, then AL equals CH (I·36), and CH equals HF (I·43), then $AL = HF$

Add CM to both

$$AD = AB + BD, \quad CB = \frac{1}{2}AB, \quad CD = CB + BD$$

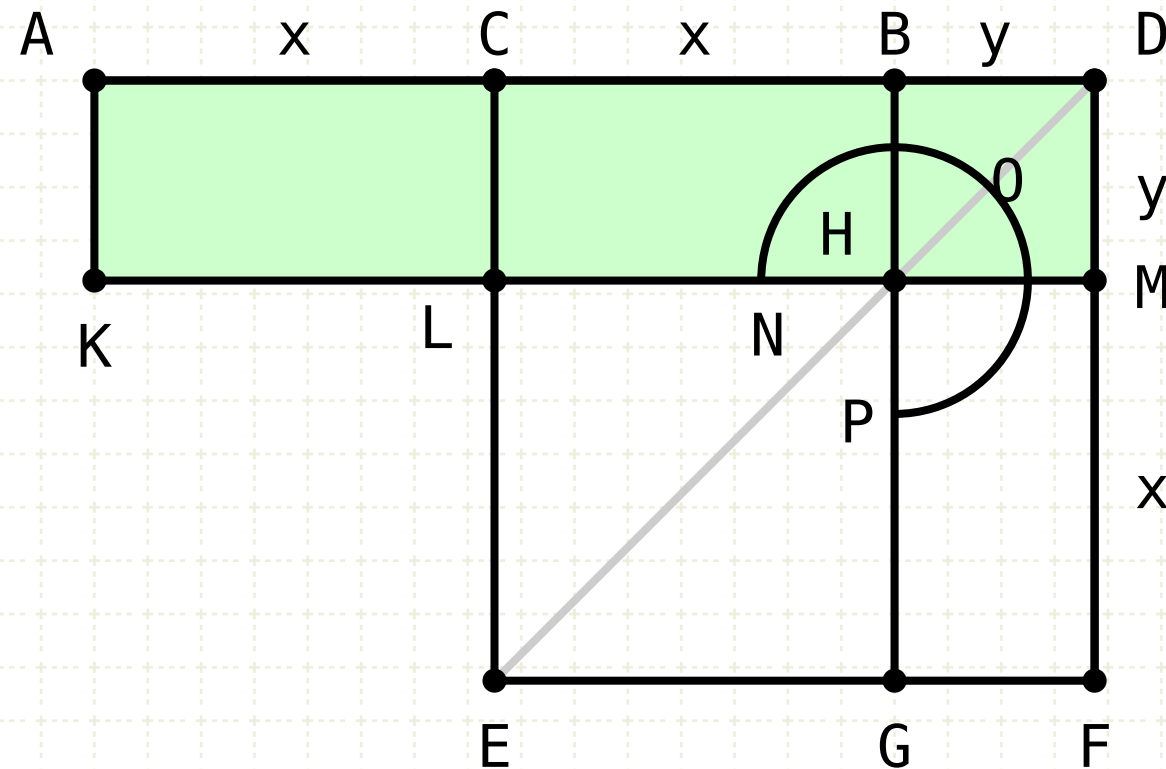
$$AC = CB \therefore \square AL = \square CH = \square HF \therefore \square AL = \square HF$$

$$\square AL + \square CM = \square AM = \square HF + \square CM$$

$$\square AM = \square NOP$$

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Proof:

Since AC equals CB, then AL equals CH (I·36), and CH equals HF (I·43), then AL = HF

Add CM to both

Since DB equals DM, AM is the rectangle formed from AD and DB

$$AD=AB+BD, \quad CB=\frac{1}{2}AB, \quad CD=CB+BD$$

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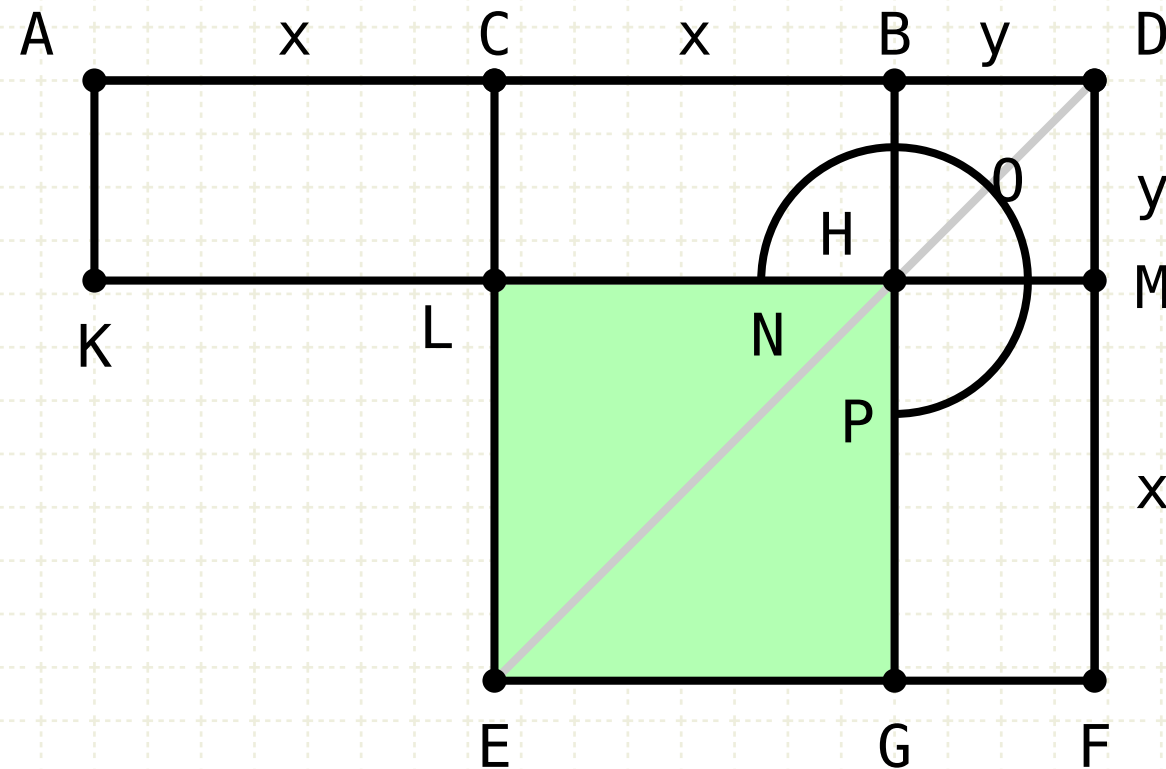
□ AM = NOP

$$\square AM = AD \cdot DB$$



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$$\square AM = AD \cdot DB$$

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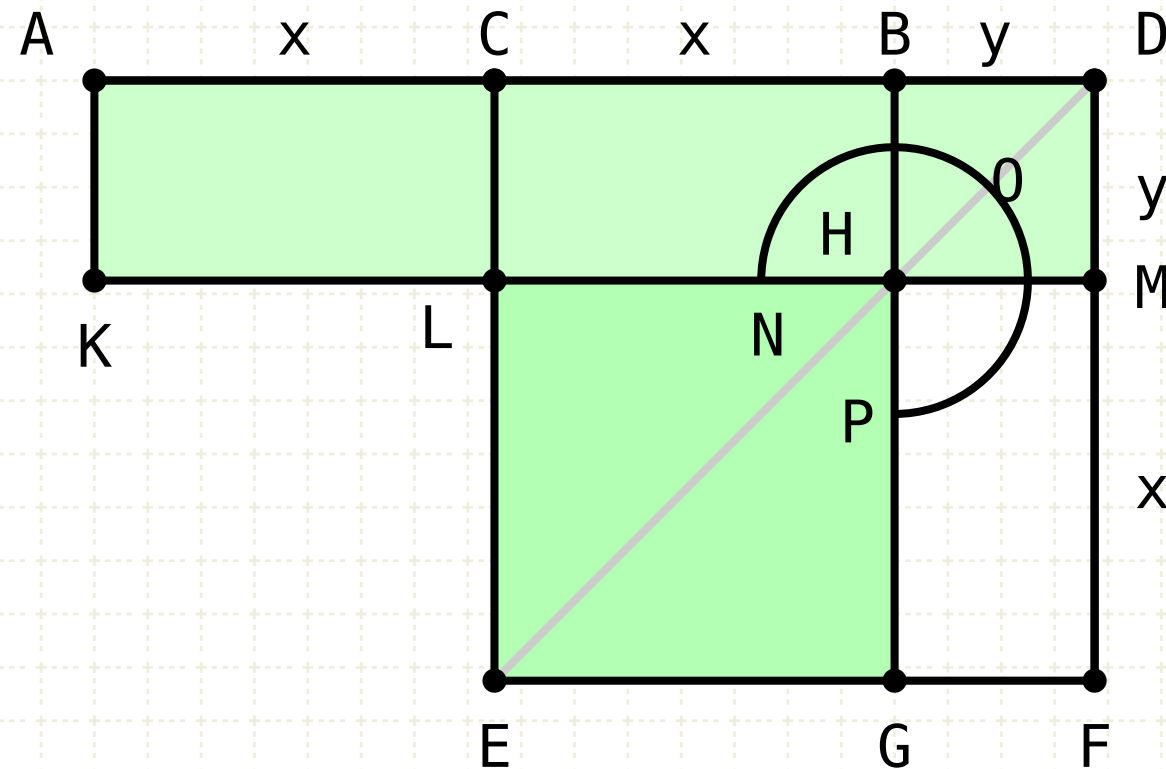
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Since DB equals DM, AM is the rectangle formed from AD and DB

LG is the square of CB

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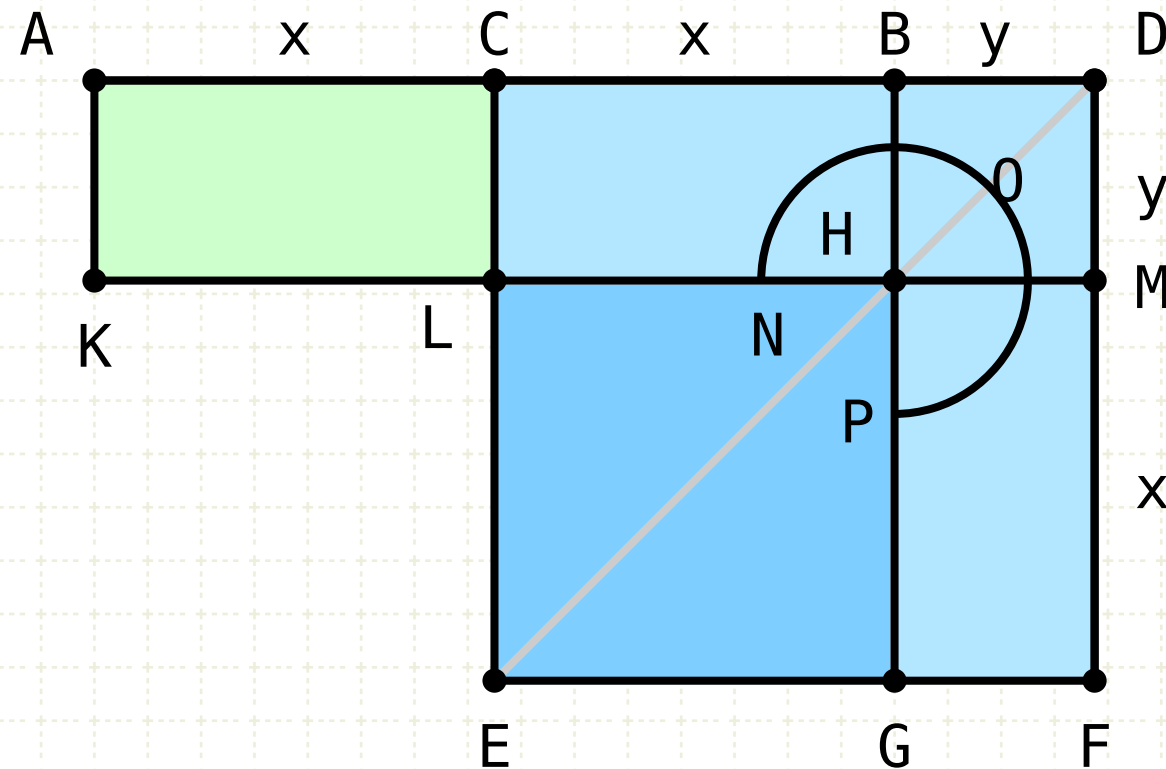
Since DB equals DM, AM is the rectangle formed from AD and DB

LG is the square of CB

Add LG to AM and the gnomon NOP

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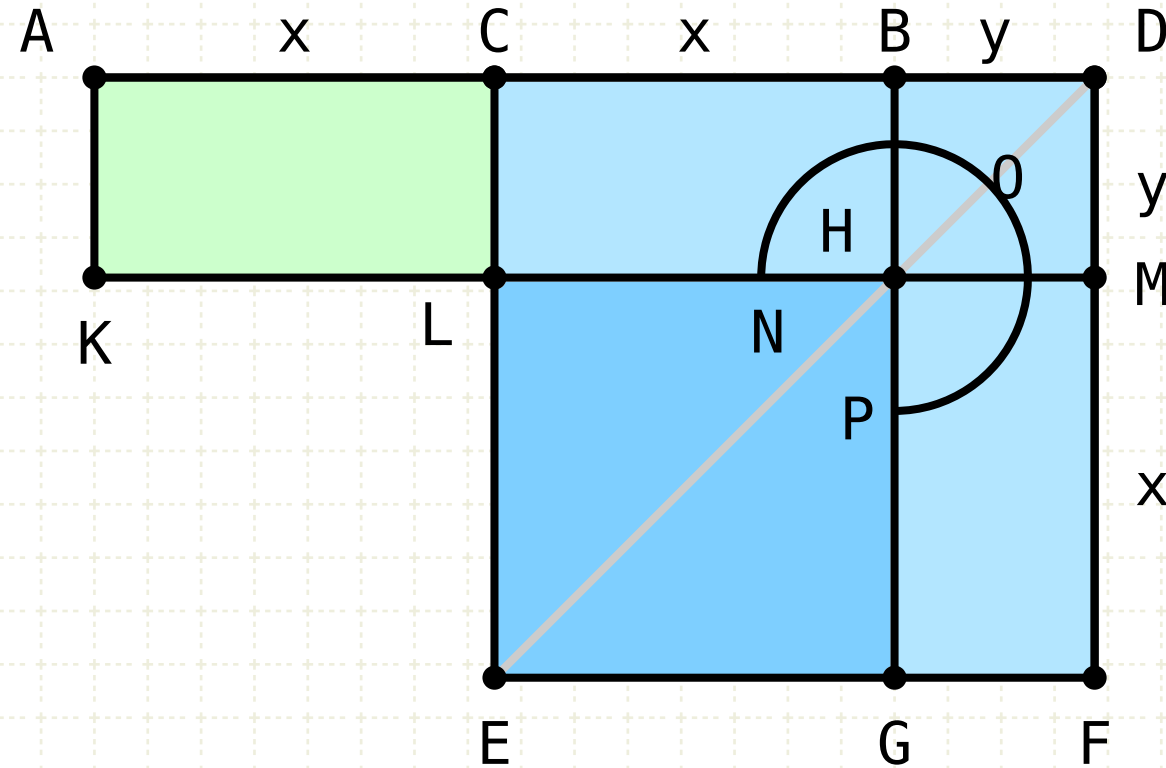
Add LG to AM and the gnomon NOP

But LG added to the gnomon NOP is also equal to the square on CD, therefore AM plus LG is equal to CF



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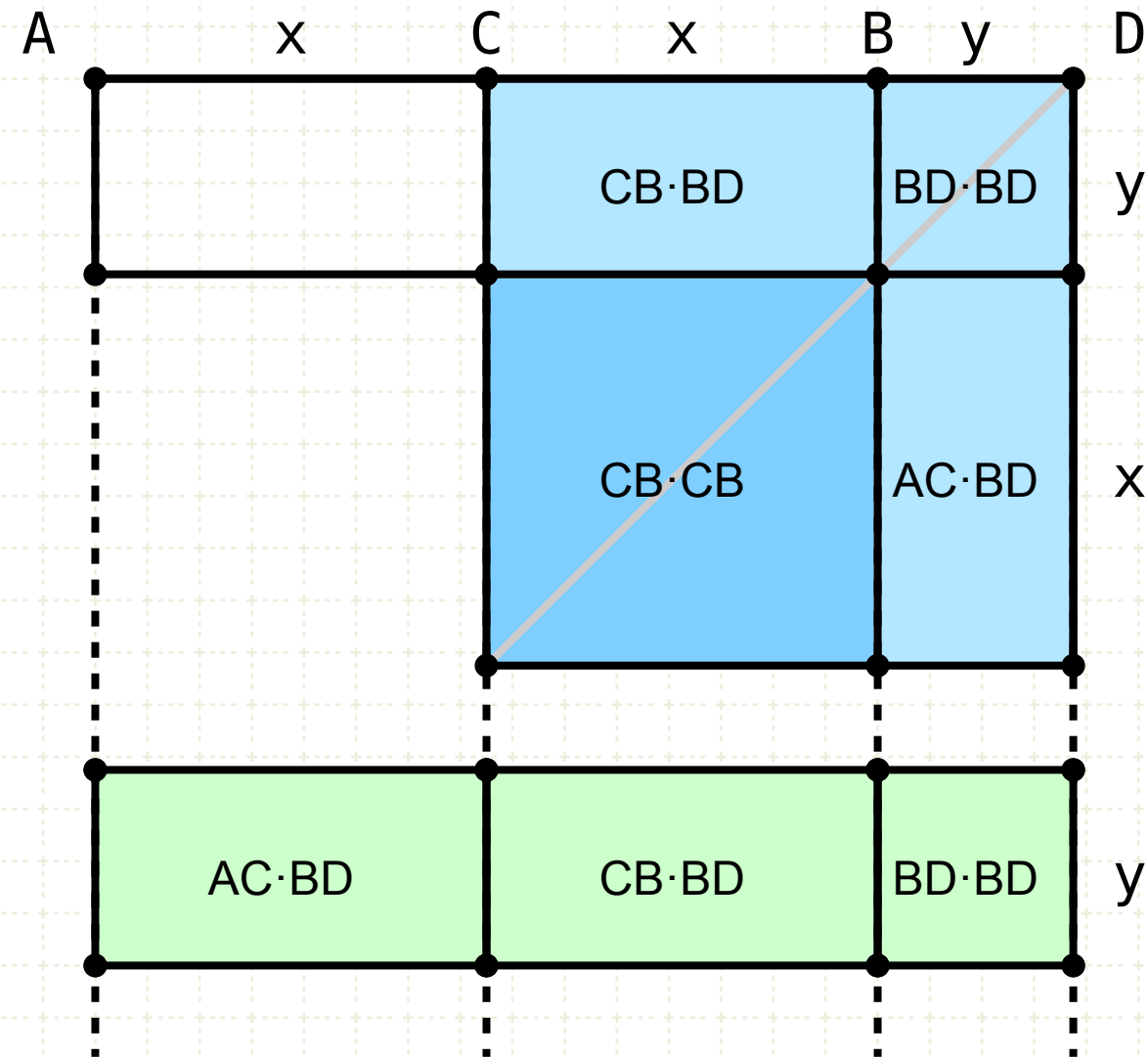
$$AD \cdot DB + CB \cdot CB = NOP + CB \cdot CB$$

$$AD \cdot DB + CB \cdot CB = CD \cdot CD$$



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