# Euclid's Elements

# Book I

If Euclid did not kindle your youthful enthusiasm, you were not born to be a scientific thinker.

Albert Einstein

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- 38 Triangles, equal base-height have equal area



# **Table of Contents, Chapter 1**

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Proposition 30 of Book I
Straight lines parallel to the same straight line are also parallel to one another.

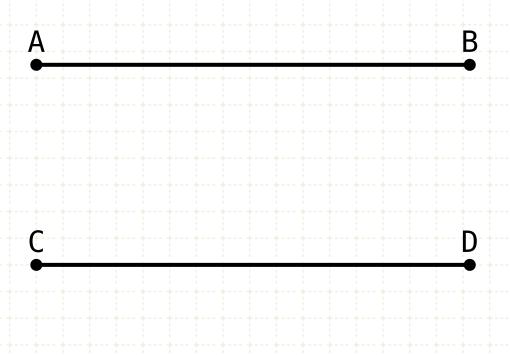


Straight lines parallel to the same straight line are also parallel to one another.

if AB || EF and CD || EF

## In other words

Given two lines AB and CD which are both parallel to EF

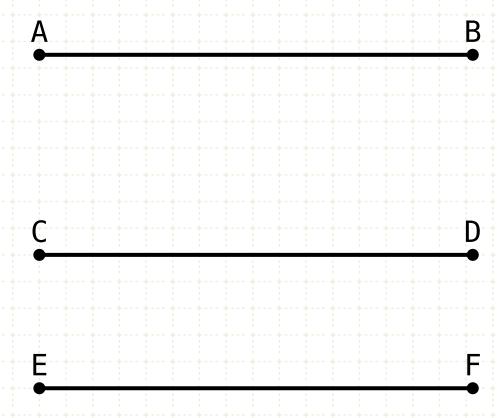


Straight lines parallel to the same straight line are also parallel to one another.

if AB || EF and CD || EF then AB || CD

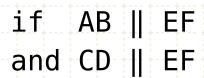
### In other words

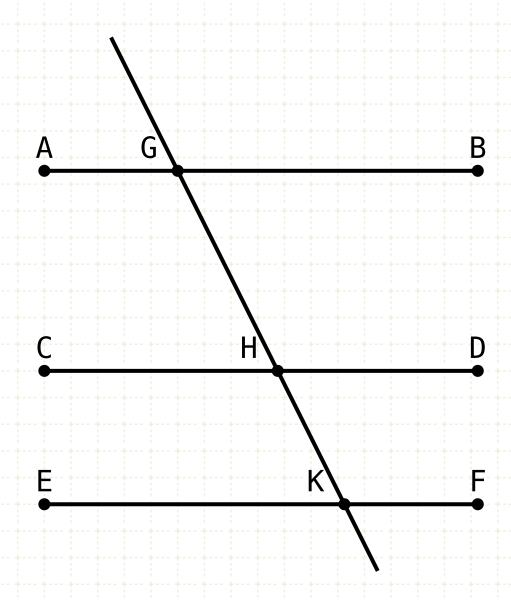
Given two lines AB and CD which are both parallel to EF Then lines AB and CD are parallel to each other





Straight lines parallel to the same straight line are also parallel to one another.





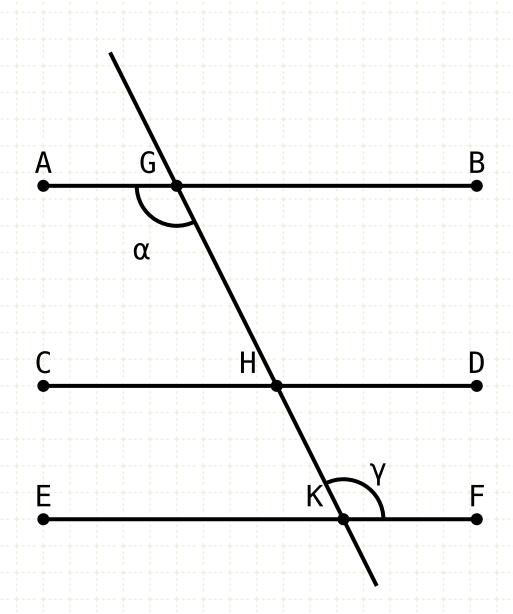
## In other words

Given two lines AB and CD which are both parallel to EF Then lines AB and CD are parallel to each other

#### **Proof**

Construct a fourth line GK such that it intersects lines AB, CD and EF

Straight lines parallel to the same straight line are also parallel to one another.



AB || EF 
$$\alpha = \gamma$$

#### In other words

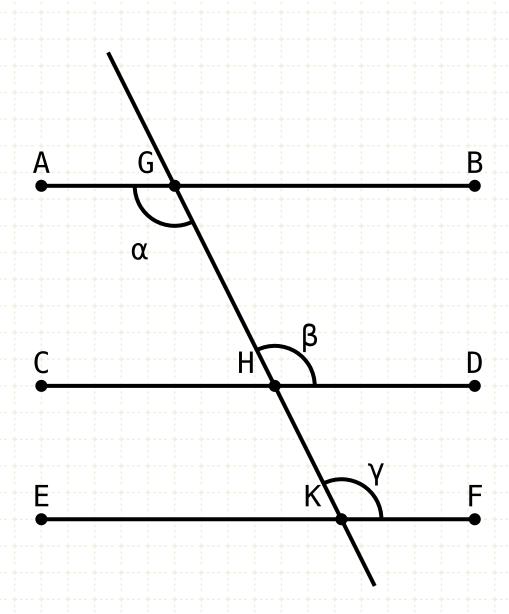
Given two lines AB and CD which are both parallel to EF Then lines AB and CD are parallel to each other

#### **Proof**

Construct a fourth line GK such that it intersects lines AB, CD and EF

Since lines AB and EF are parallel, angle AGH equals HKF (I-29)

Straight lines parallel to the same straight line are also parallel to one another.



AB || EF 
$$\alpha = \gamma$$

CD || EF 
$$\beta = \gamma$$

#### In other words

Given two lines AB and CD which are both parallel to EF
Then lines AB and CD are parallel to each other

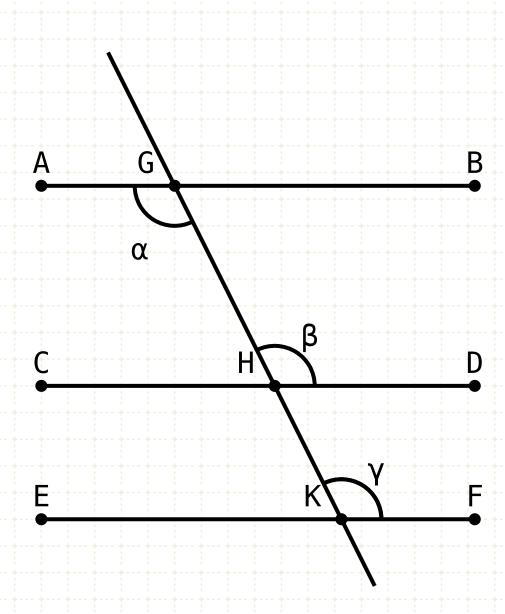
#### **Proof**

Construct a fourth line GK such that it intersects lines AB, CD and EF

Since lines AB and EF are parallel, angle AGH equals HKF (I·29)

Since lines CD and EF are parallel, angle GHD equals HKF (I·29)

Straight lines parallel to the same straight line are also parallel to one another.



AB || EF 
$$\alpha = \gamma$$

CD || EF 
$$\beta = \gamma$$

$$\alpha = \gamma = \beta$$
 $\alpha = \beta$ 

#### In other words

Given two lines AB and CD which are both parallel to EF Then lines AB and CD are parallel to each other

#### **Proof**

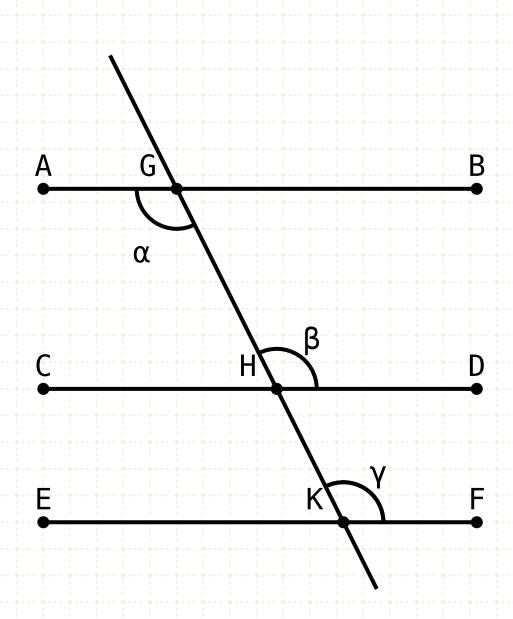
Construct a fourth line GK such that it intersects lines AB, CD and EF

Since lines AB and EF are parallel, angle AGH equals HKF (I·29)

Since lines CD and EF are parallel, angle GHD equals HKF (I·29)

Therefore angles AGH and GHD are equal

Straight lines parallel to the same straight line are also parallel to one another.



AB || EF 
$$\alpha = \gamma$$

CD 
$$\parallel$$
 EF  $\beta = \gamma$ 

$$\alpha = \gamma = \beta$$
 $\alpha = \beta$ 

#### In other words

Given two lines AB and CD which are both parallel to EF
Then lines AB and CD are parallel to each other

#### **Proof**

Construct a fourth line GK such that it intersects lines AB, CD and EF

Since lines AB and EF are parallel, angle AGH equals HKF (I·29)

Since lines CD and EF are parallel, angle GHD equals HKF (I·29)

Therefore angles AGH and GHD are equal

Since angles AGH and GHD are equal and they are opposite, then lines AB and CD are parallel (I·28)

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