

Euclid's Elements

Book VII

Definitions:

- 1 A unit is that by virtue of which each of the things that exist is called one
- 2 A number is a multitude composed of units. (not one)
- 3 A number is part of a number, the less of the greater, when it measures the greater
- 11 A prime number is that which is measured by a unit alone.
- 12 Numbers prime to one another are those which are measured by a unit alone as a common measure
- 15 A number is said to multiply a number when that which is multiplied is added to itself as many times as there are units in the other, and thus some number is produced.
- 20 Numbers are proportional when the first is the same multiple, or the same part, or the same parts, of the second that the third is of the fourth.

As long as algebra and geometry have been separated, their progress have been slow and their uses limited; but when these two sciences have been united, they have lent each mutual forces, and have marched together towards perfection.

Joseph-Louis Lagrange
(1736 to 1813)



Table of Contents, Chapter 7

1	Determine if two numbers are relatively prime	10	If $B = (p/q) \cdot A$ and $D = (p/q) \cdot C$, and If $B = (r/s) \cdot D$, then $A = (r/s) \cdot C$	21	If A,B are relatively prime, then A,B are the smallest whole numbers that can be used to describe the ratio A:B
2	Find the greatest common divisor for two numbers	11	If $A:B = C:D$, then $(A-C):(B-D) = A:B$	22	If A,B are the smallest whole numbers that can be used to describe the ratio A:B, then A,B are relatively prime
3	Find the largest common divisor for three numbers	12	If $A:B = C:D$, then $(A+C):(B+C) = A:B$	23	If A,B are relatively prime and if $A = n \cdot C$, then B,C are relatively prime
4	Given two natural numbers, A and B, either B is part of A, or there exists a natural number (a part) that can measure both A and B	13	If $A:B = C:D$, then $A:C = B:D$	24	If A,C are relatively prime and B,C are relatively prime then the $A \times B$ is relatively prime to C
5	If $B = (1/q) \cdot A$ and $D = (1/q) \cdot C$, then $(B+D) = (1/q) \cdot (A+C)$	14	If $A:B = D:E$ and $B:C = E:F$, then $A:C = D:F$	25	If A,B are relatively prime then A^2, B are relatively prime
6	If $B = (p/q) \cdot A$ and $D = (p/q) \cdot C$, then $(B+D) = (p/q) \cdot (A+C)$	15	If $B = i \cdot 1$ and $E = i \cdot D$, and if $D = j \cdot 1$ then $E = j \cdot B$	26	If A is relatively prime to C and D, and if B is also relatively prime to C and D, then $A \times B$ is relatively prime to $C \times D$
7	If $B = A/q$ and $D = C/q$, $B > D$, then $(B-D) = (A-C)/q$	16	$A \times B = B \times A$	27	If A,B are relatively prime, then A^2, B^2 are relatively prime, and A^3, B^3 are relatively prime, and so on
8	If $B = (p/q) \cdot A$ and $D = (p/q) \cdot C$, $B > D$, then $(B-D) = (p/q) \cdot (A-C)$	17	If $D = A \times B$ and $E = A \times C$ then $D:E = B:C$		
9	If $B = (1/q) \cdot A$ and $D = (1/q) \cdot C$, and If $B = (r/s) \cdot D$, then $A = (r/s) \cdot C$	18	If $D = B \times A$ and $E = C \times A$ then $D:E = B:C$		
		19	If $A:B = C:D$ then $A \times D = B \times C$ If $A \times D = B \times C$ then $A:B = C:D$		
		20	Given the ratio A:B and C,D are the smallest numbers such that $A:B = C:D$ then $A = n \cdot C$ and $B = n \cdot D$		



Table of Contents, Chapter 7

- | | | | |
|----|--|----|---|
| 28 | If A,B are relatively prime, then A,(A+B) are relatively prime | 37 | If $A = p \cdot B$, then $A = q \cdot C$ where $C = p \cdot 1$ |
| 29 | If A is prime, and $B \neq n \cdot A$, then A,B are relatively prime | 38 | If $A = (1/c) \cdot B$ and $C = c \cdot 1$ then $A = n \cdot C$ |
| 30 | If $C = A \times B$ and $C = i \cdot D$ where D is prime, then either $A = j \cdot D$ or $B = j \cdot D$ | 39 | Find the smallest number that has the fractions $1/a, 1/b, 1/c$ |
| 31 | If $A = B \times C$, then $A = j \cdot D$ where D is prime | | |
| 32 | If A is a number then it is either prime, or $A = j \cdot D$ where D is prime | | |
| 33 | Find the smallest numbers X,Y,Z where the ratio X:Y:Z is equal to the given ratio A:B:C | | |
| 34 | Find the lowest common denominator of 2 numbers | | |
| 35 | If E is the lowest common denominator of A,B, and if $C = n \cdot A = m \cdot B$, then $C = i \cdot E$ | | |
| 36 | Find the least common multiple of 3 numbers | | |



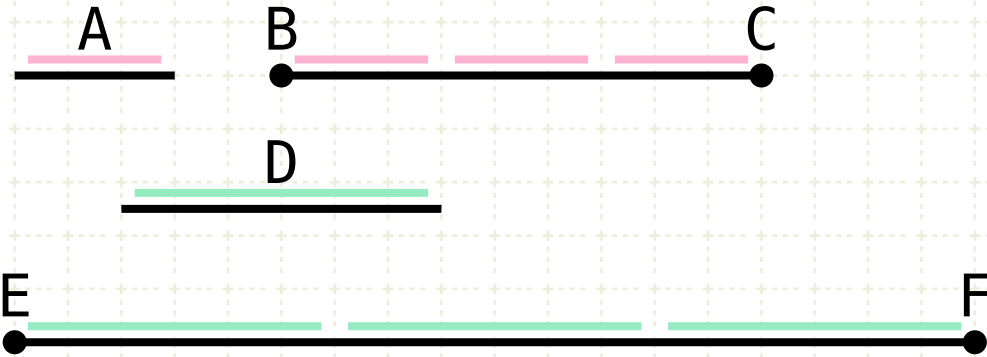
Proposition 15 of Book VII

If an unit measure any number, and another number measure any other number the same number of times, alternately also, the unit will measure the third number the same number of times that the second measures the fourth.



Proposition 15 of Book VII

If an unit measure any number, and another number measure any other number the same number of times, alternately also, the unit will measure the third number the same number of times that the second measures the fourth.



$$A = 1$$

$$BC = bc \cdot A$$

$$EF = bc \cdot D$$

In other words

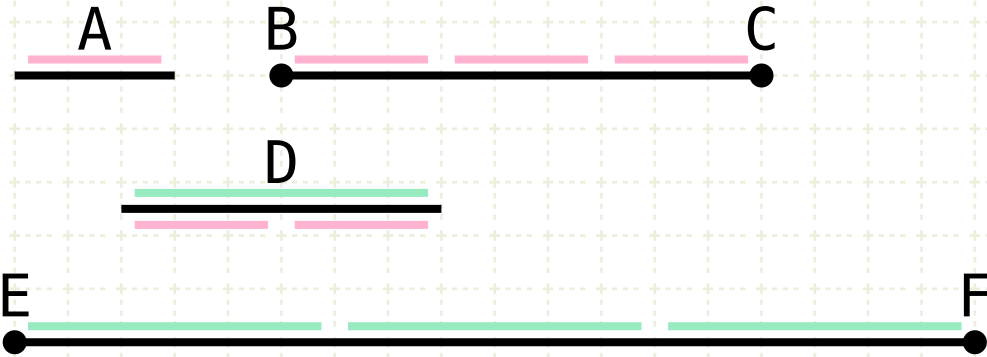
Let A (a unit measure) measure BC

Let the number D measure EF by the same amount



Proposition 15 of Book VII

If an unit measure any number, and another number measure any other number the same number of times, alternately also, the unit will measure the third number the same number of times that the second measures the fourth.



$$A = 1$$

$$BC = bc \cdot A$$

$$EF = bc \cdot D$$

$$D = d \cdot A$$

In other words

Let A (a unit measure) measure BC

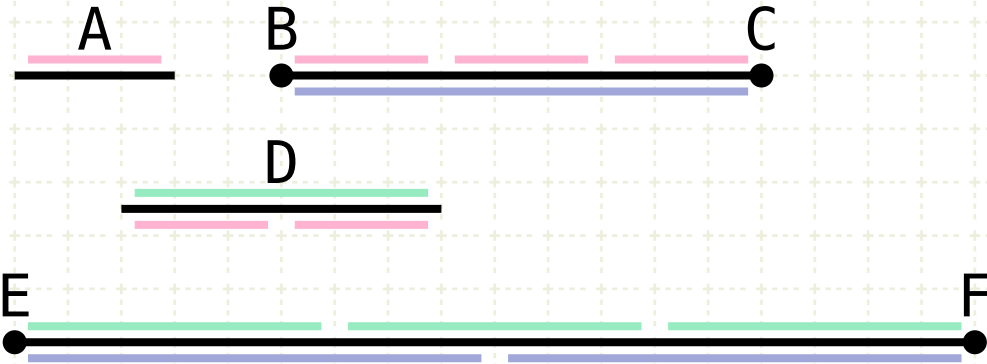
Let the number D measure EF by the same amount

Then, as A measures D, ...



Proposition 15 of Book VII

If an unit measure any number, and another number measure any other number the same number of times, alternately also, the unit will measure the third number the same number of times that the second measures the fourth.



$$A = 1$$

$$BC = bc \cdot A$$

$$EF = bc \cdot D$$

$$D = d \cdot A$$

$$EF = d \cdot BC$$

In other words

Let A (a unit measure) measure BC

Let the number D measure EF by the same amount

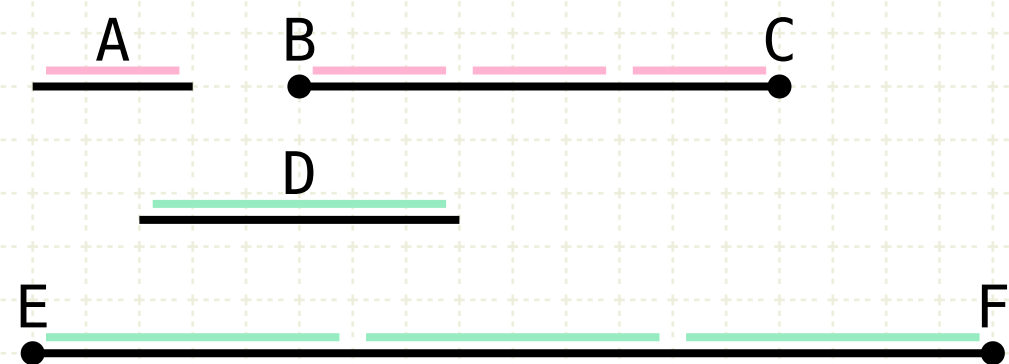
Then, as A measures D, ...

... the same number of times that BC measures EF



Proposition 15 of Book VII

If an unit measure any number, and another number measure any other number the same number of times, alternately also, the unit will measure the third number the same number of times that the second measures the fourth.



$$\begin{aligned} A &= 1 \\ BC &= bc \cdot A \\ EF &= bc \cdot D \end{aligned}$$

In other words

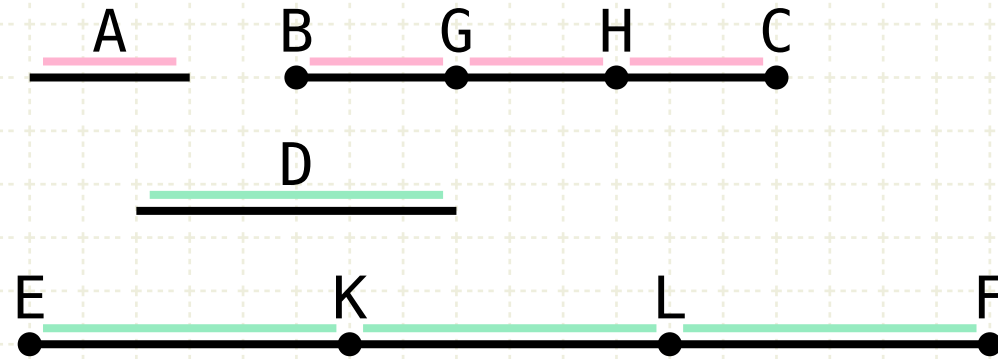
Let A (a unit measure) measure BC
Let the number D measure EF by the same amount
Then, as A measures D, ...
... the same number of times that BC measures EF

Proof



Proposition 15 of Book VII

If an unit measure any number, and another number measure any other number the same number of times, alternately also, the unit will measure the third number the same number of times that the second measures the fourth.



$$A = 1$$

$$BC = bc \cdot A$$

$$EF = bc \cdot D$$

$$BG = GH = HC = A$$

$$EK = KL = LF = D$$

In other words

Let A (a unit measure) measure BC

Let the number D measure EF by the same amount

Then, as A measures D, ...

... the same number of times that BC measures EF

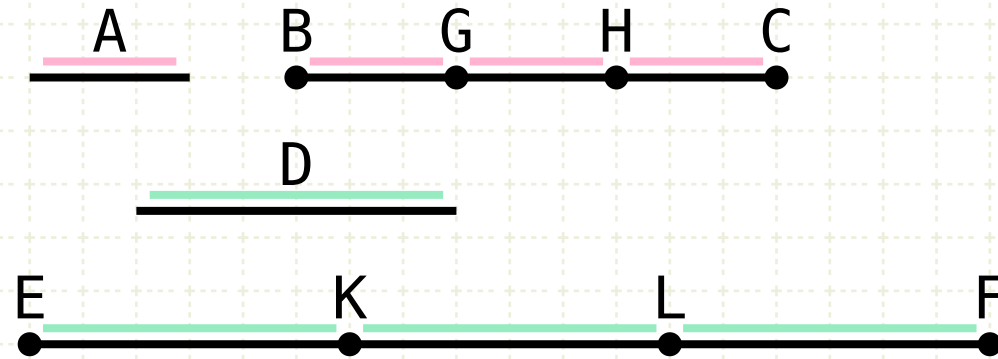
Proof

There are as many units in BC as there are parts equal to D in EF, so divide BC and EF into that many units/parts



Proposition 15 of Book VII

If an unit measure any number, and another number measure any other number the same number of times, alternately also, the unit will measure the third number the same number of times that the second measures the fourth.



$$A = 1$$

$$BC = bc \cdot A$$

$$EF = bc \cdot D$$

$$BG = GH = HC = A$$

$$EK = KL = LF = D$$

$$BG : EK = GH : KL = HC : LF$$

In other words

Let A (a unit measure) measure BC

Let the number D measure EF by the same amount

Then, as A measures D, ...

... the same number of times that BC measures EF

Proof

There are as many units in BC as there are parts equal to D in EF, so divide BC and EF into that many units/parts

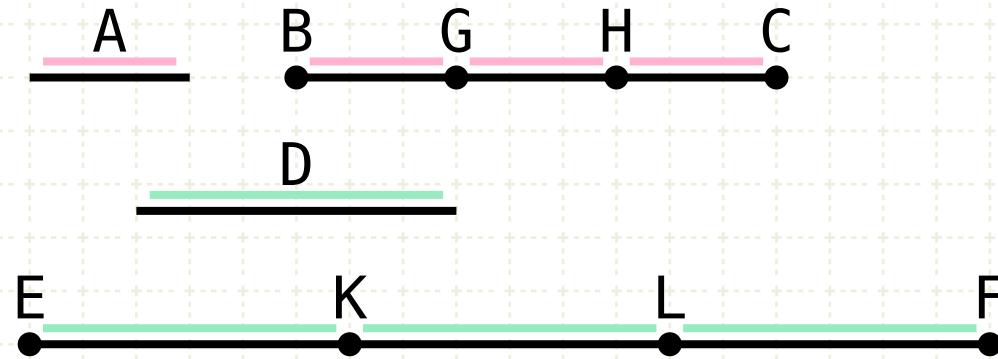
BG, GH, HC are all equal to each other, EK, KL, LF are all equal to each other, and there are as many numbers BG, GH, HC as there are numbers EK, KL, LF ...

... the ratio of BG to EK is equal to the ratio GH to KL is equal to the ratio HC to LF



Proposition 15 of Book VII

If an unit measure any number, and another number measure any other number the same number of times, alternately also, the unit will measure the third number the same number of times that the second measures the fourth.



$$A = 1$$

$$BC = bc \cdot A$$

$$EF = bc \cdot D$$

$$BG = GH = HC = A$$

$$EK = KL = LF = D$$

$$BG : EK = GH : KL = HC : LF$$

$$(BG + GH + HC) : (EK + KL + LF) = BC : EF = BG : EK$$

In other words

Let A (a unit measure) measure BC

Let the number D measure EF by the same amount

Then, as A measures D, ...

... the same number of times that BC measures EF

Proof

There are as many units in BC as there are parts equal to D in EF, so divide BC and EF into that many units/parts

BG, GH, HC are all equal to each other, EK, KL, LF are all equal to each other, and there are as many numbers BG, GH, HC as there are numbers EK, KL, LF ...

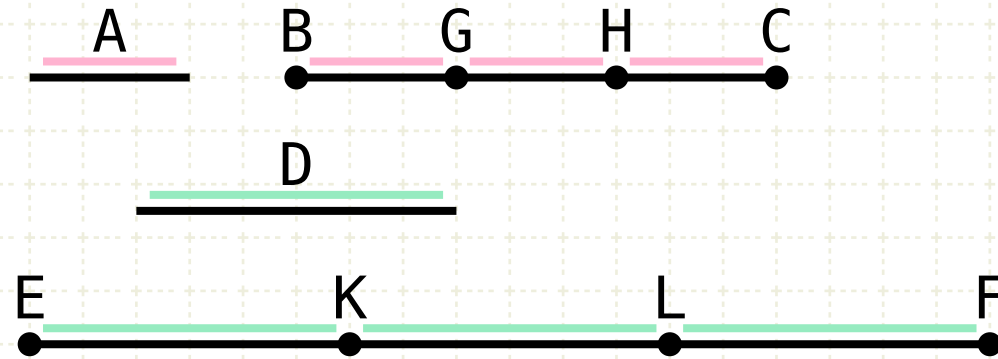
... the ratio of BG to EK is equal to the ratio GH to KL is equal to the ratio HC to LF

However, the ratio of the sum of the antecedents to the sum of the consequents will also be equal to BG to EK (VII.12)



Proposition 15 of Book VII

If an unit measure any number, and another number measure any other number the same number of times, alternately also, the unit will measure the third number the same number of times that the second measures the fourth.



$$A = 1$$

$$BC = bc \cdot A$$

$$EF = bc \cdot D$$

$$BG = GH = HC = A$$

$$EK = KL = LF = D$$

$$BG : EK = GH : KL = HC : LF$$

$$(BG + GH + HC) : (EK + KL + LF) = BC : EF = BG : EK$$

$$BC : EF = A : D$$

In other words

Let A (a unit measure) measure BC

Let the number D measure EF by the same amount

Then, as A measures D, ...

... the same number of times that BC measures EF

Proof

There are as many units in BC as there are parts equal to D in EF, so divide BC and EF into that many units/parts

BG, GH, HC are all equal to each other, EK, KL, LF are all equal to each other, and there are as many numbers BG, GH, HC as there are numbers EK, KL, LF ...

... the ratio of BG to EK is equal to the ratio GH to KL is equal to the ratio HC to LF

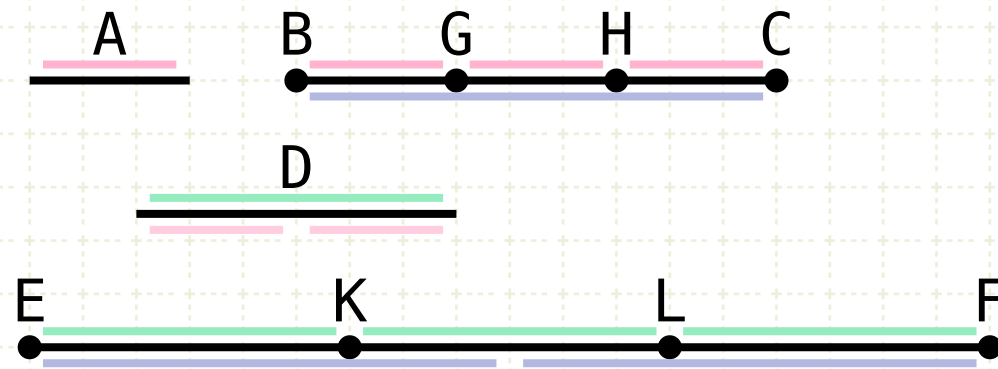
However, the ratio of the sum of the antecedents to the sum of the consequents will also be equal to BG to EK (VII-12)

But BG is equal to A, and EK equals D, so BC to EF equals A to D



Proposition 15 of Book VII

If an unit measure any number, and another number measure any other number the same number of times, alternately also, the unit will measure the third number the same number of times that the second measures the fourth.



$$A = 1$$

$$BC = bc \cdot A$$

$$EF = bc \cdot D$$

$$BG = GH = HC = A$$

$$EK = KL = LF = D$$

$$BG : EK = GH : KL = HC : LF$$

$$(BG + GH + HC) : (EK + KL + LF) = BC : EF = BG : EK$$

$$BC : EF = A : D$$

$$D = d \cdot A$$

$$EF = d \cdot BC$$

In other words

Let A (a unit measure) measure BC

Let the number D measure EF by the same amount

Then, as A measures D, ...

... the same number of times that BC measures EF

Proof

There are as many units in BC as there are parts equal to D in EF, so divide BC and EF into that many units/parts

BG, GH, HC are all equal to each other, EK, KL, LF are all equal to each other, and there are as many numbers BG, GH, HC as there are numbers EK, KL, LF ...

... the ratio of BG to EK is equal to the ratio GH to KL is equal to the ratio HC to LF

However, the ratio of the sum of the antecedents to the sum of the consequents will also be equal to BG to EK (VII.12)

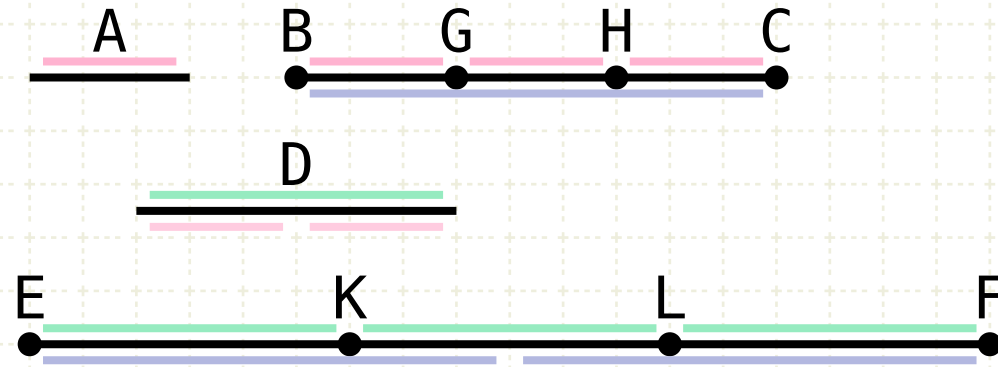
But BG is equal to A, and EK equals D, so BC to EF equals A to D

By the definition of proportional, the unit A measures the number D the same number of times that BC measures EF



Proposition 15 of Book VII

If an unit measure any number, and another number measure any other number the same number of times, alternately also, the unit will measure the third number the same number of times that the second measures the fourth.



$$A = 1$$

$$BC = bc \cdot A$$

$$EF = bc \cdot D$$

$$BG = GH = HC = A$$

$$EK = KL = LF = D$$

$$BG : EK = GH : KL = HC : LF$$

$$(BG + GH + HC) : (EK + KL + LF) = BC : EF = BG : EK$$

$$BC : EF = A : D$$

$$D = d \cdot A$$

$$EF = d \cdot BC$$

In other words

Let A (a unit measure) measure BC

Let the number D measure EF by the same amount

Then, as A measures D, ...

... the same number of times that BC measures EF

Proof

There are as many units in BC as there are parts equal to D in EF, so divide BC and EF into that many units/parts

BG, GH, HC are all equal to each other, EK, KL, LF are all equal to each other, and there are as many numbers BG, GH, HC as there are numbers EK, KL, LF ...

... the ratio of BG to EK is equal to the ratio GH to KL is equal to the ratio HC to LF

However, the ratio of the sum of the antecedents to the sum of the consequents will also be equal to BG to EK (VII.12)

But BG is equal to A, and EK equals D, so BC to EF equals A to D

By the definition of proportional, the unit A measures the number D the same number of times that BC measures EF



Youtube Videos

<https://www.youtube.com/c/SandyBultena>

Copyright © 2019 by Sandy Bultena.



Except where otherwise noted, this work is licensed under
<http://creativecommons.org/licenses/by-nc/3.0>