

Euclid's Elements

Book V



Proportions are what makes the old Greek temples classic in their beauty. They are like huge blocks, from which the air has been literally hewn out between the columns.

$$AB:C = DE:F$$

$$BG:C = EH:F$$

$$AG:C = DH:F$$

Arne Jacobsen



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Proposition 10 of Book V

Of magnitudes which have a ratio to the same, that which has a greater ratio is greater; and that to which the same has a greater ratio is less.



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In other words

Let the ratio of A to C be greater than the ratio B to C

Then A is greater than B

$$A:C > B:C \rightarrow A > B$$

$$C:B > C:A \rightarrow B < A$$



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$$A = B \rightarrow A:C = B:C$$

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Proof

If A is not greater than B, then A either equals B or is less than B

If A equals B then the ratios A to C and B to C would be equal (V·7)

If A is less than B then the ratio A to C would be less than B to C (V·8)



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$$\text{But } A:C > B:C \therefore A > B$$

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Then A is greater than B

Proof

If A is not greater than B, then A either equals B or is less than B

If A equals B then the ratios A to C and B to C would be equal (V.7)

If A is less than B then the ratio A to C would be less than B to C (V.8)

But A to C IS greater than B to C, therefore A is greater than B

If B is not less than A, then B either equals A or is greater than A

If B equals A then the ratios C to B and C to A would be equal (V.7)

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$$\text{But } C:B > C:A \therefore B < A$$

In other words

Let the ratio of A to C be greater than the ratio B to C

Then A is greater than B

Proof

If A is not greater than B, then A either equals B or is less than B

If A equals B then the ratios A to C and B to C would be equal (V.7)

If A is less than B then the ratio A to C would be less than B to C (V.8)

But A to C IS greater than B to C, therefore A is greater than B

If B is not less than A, then B either equals A or is greater than A

If B equals A then the ratios C to B and C to A would be equal (V.7)

If B is greater than A then the ratio C to B would be less than C to A (V.8)

But C to B IS greater than C to A, therefore B is less than A



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