

Euclid's Elements

Book V



Proportions are what makes the old Greek temples classic in their beauty. They are like huge blocks, from which the air has been literally hewn out between the columns.

$$AB:C = DE:F$$

$$BG:C = EH:F$$

$$AG:C = DH:F$$

Arne Jacobsen



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10	if $A:C > B:C$, or $A:C < B:C$ then $A > B$, or $A < C$, respectively				



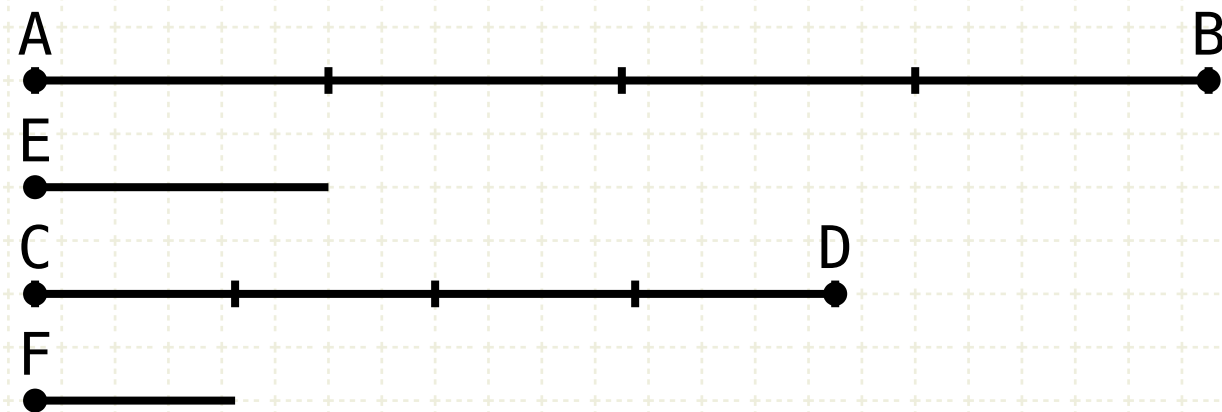
Proposition 6 of Book V

If two magnitudes be equimultiples of two magnitudes, and any magnitudes subtracted from them be equimultiples of the same, the remainders are also either equal to the same or equimultiples of them



Proposition 6 of Book V

If two magnitudes be equimultiples of two magnitudes, and any magnitudes subtracted from them be equimultiples of the same, the remainders are also either equal to the same or equimultiples of them



$$AB = nE$$

$$CD = nF$$

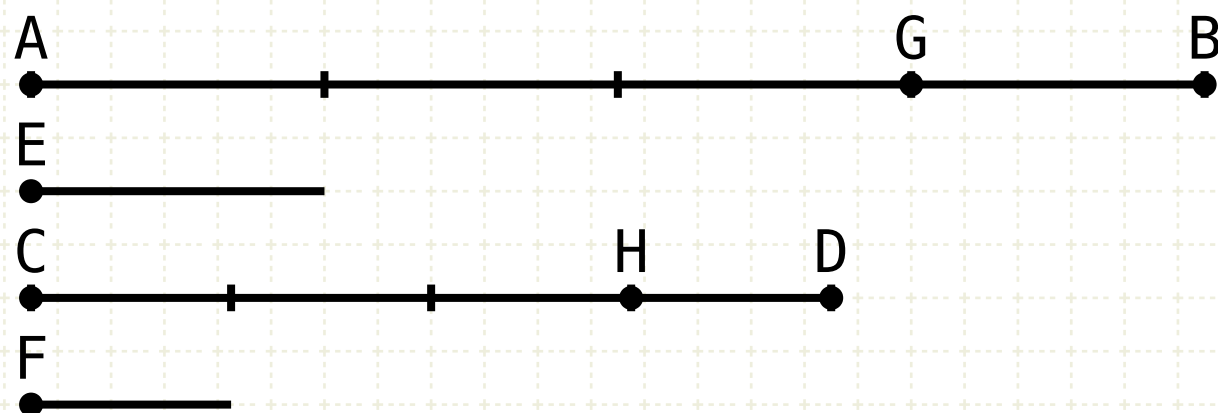
In other words

Let AB be the same multiple of E as CD is of F



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$$AB = nE$$

$$CD = nF$$

$$AG = mE$$

$$CH = mF$$

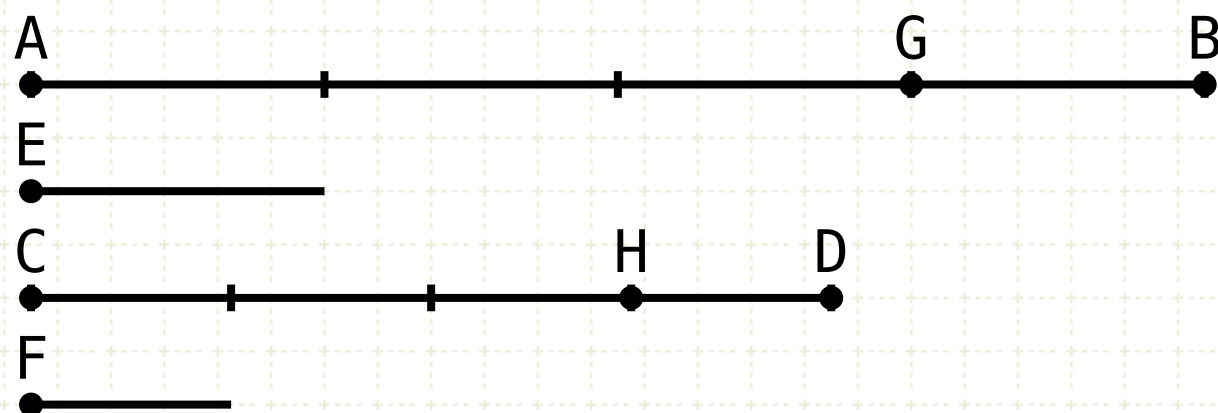
In other words

Let AB be the same multiple of E as CD is of F

Subtract AG and CH be from AB and CD, where AG,CH are equimultiples of E and F respectively

Proposition 6 of Book V

If two magnitudes be equimultiples of two magnitudes, and any magnitudes subtracted from them be equimultiples of the same, the remainders are also either equal to the same or equimultiples of them



$$AB = nE$$

$$CD = nF$$

$$AG = mE$$

$$CH = mF$$

$$GB = AB - AG = nE - mE = kE$$

$$HD = CD - CH = nF - mF = kF$$

In other words

Let AB be the same multiple of E as CD is of F

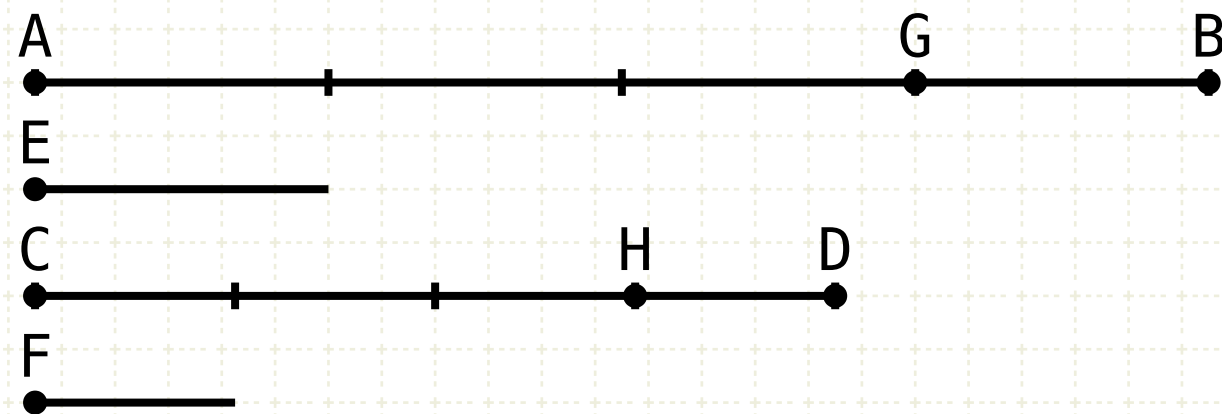
Subtract AG and CH from AB and CD, where AG, CH are equimultiples of E and F respectively

Then, either GB, HD are both equal to E, F, or they are equimultiples of them



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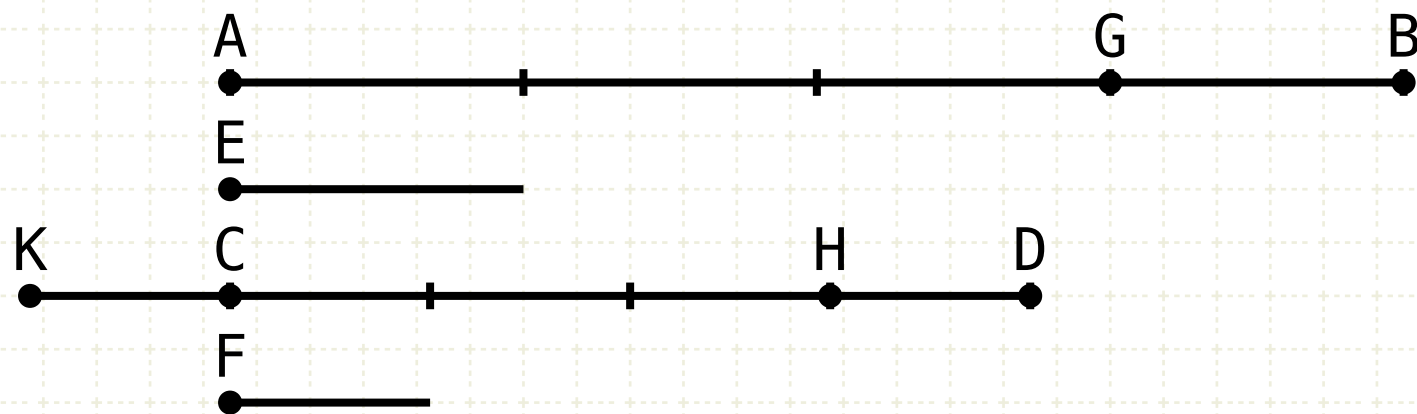
Then, either GB, HD are both equal to E, F, or they are equimultiples of them

Proof



Proposition 6 of Book V

If two magnitudes be equimultiples of two magnitudes, and any magnitudes subtracted from them be equimultiples of the same, the remainders are also either equal to the same or equimultiples of them



$$AB = nE$$

$$CD = nF$$

$$AG = mE$$

$$\text{CH} = \text{mF}$$

Let $GB = E$

Let $KC = F$

In other words

Let AB be the same multiple of E as CD is of F

Subtract AG and CH be from AB and CD, where AG,CH are equimultiples of E and F respectively

Then, either GB, HD are both equal to E, F , or they are equimultiples of them

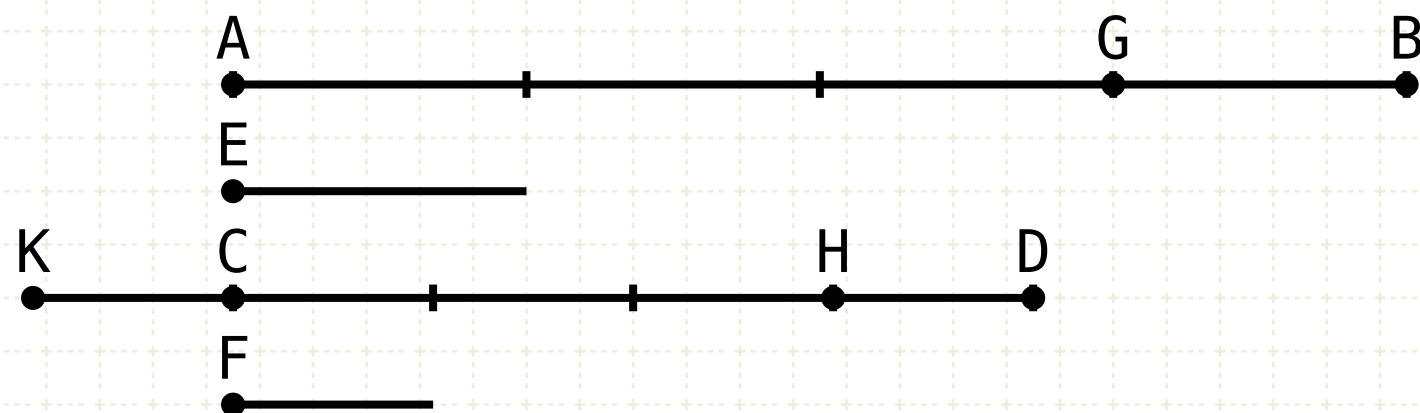
Proof

Let GB equal E, and create a line KC equal to F



Proposition 6 of Book V

If two magnitudes be equimultiples of two magnitudes, and any magnitudes subtracted from them be equimultiples of the same, the remainders are also either equal to the same or equimultiples of them



$$AB = nE$$

$$CD = nF$$

$$AG = mE$$

$$CH = mF$$

$$\text{Let } GB = E$$

$$\text{Let } KC = F$$

$$AG + GB = AB = nE$$

$$KC + CH = KH = nF$$

In other words

Let AB be the same multiple of E as CD is of F

Subtract AG and CH from AB and CD, where AG, CH are equimultiples of E and F respectively

Then, either GB, HD are both equal to E, F, or they are equimultiples of them

Proof

Let GB equal E, and create a line KC equal to F

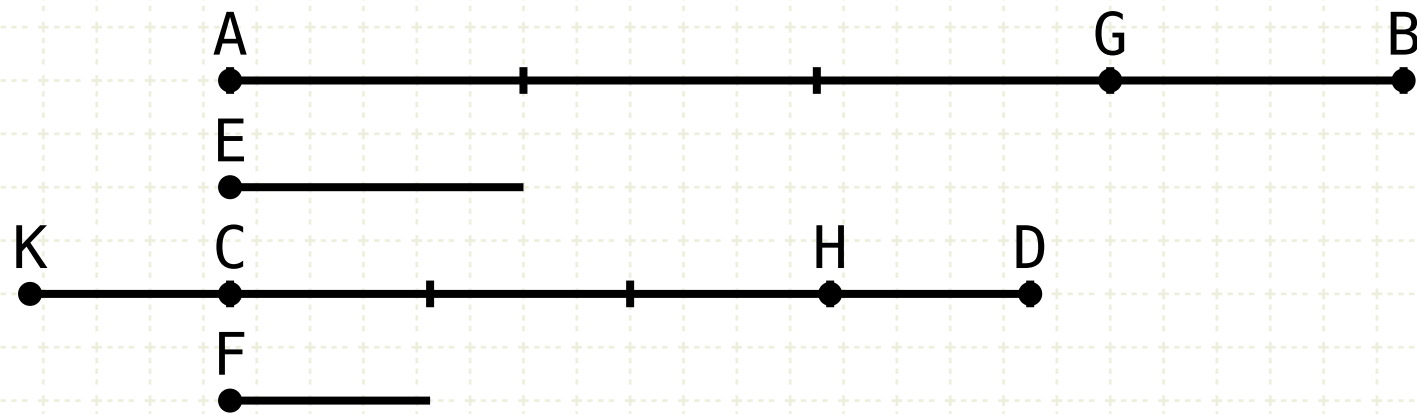
AG, CH are the same multiples of E, F, while GB equals E and KC equals F ...

... therefore, AB, KH are equimultiple to E, F (V.2)



Proposition 6 of Book V

If two magnitudes be equimultiples of two magnitudes, and any magnitudes subtracted from them be equimultiples of the same, the remainders are also either equal to the same or equimultiples of them



$$AB = nE$$

$$CD = nF$$

$$AG = mE$$

$$CH = mF$$

$$\text{Let } GB = E$$

$$\text{Let } KC = F$$

$$AG + GB = AB = nE$$

$$KC + CH = KH = nF$$

$$KH = CD$$

In other words

Let AB be the same multiple of E as CD is of F

Subtract AG and CH from AB and CD, where AG, CH are equimultiples of E and F respectively

Then, either GB, HD are both equal to E, F, or they are equimultiples of them

Proof

Let GB equal E, and create a line KC equal to F

AG, CH are the same multiples of E, F, while GB equals E and KC equals F ...

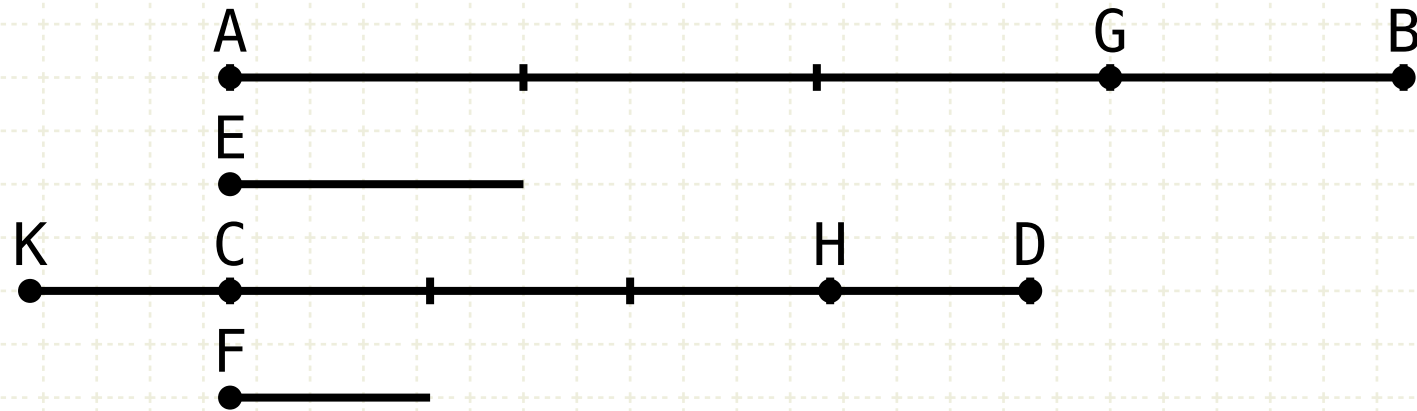
... therefore, AB, KH are equimultiple to E, F (V.2)

Since CD, KC are equimultiples of F, then KC must equal CD



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If two magnitudes be equimultiples of two magnitudes, and any magnitudes subtracted from them be equimultiples of the same, the remainders are also either equal to the same or equimultiples of them



$$AB = nE$$

$$CD = nF$$

$$AG = mE$$

$$CH = mF$$

$$KH = CD$$

$$KH - CH = CD - CH$$

$$KC = HD$$

$$\text{Let } GB = E$$

$$\text{Let } KC = F$$

$$AG + GB = AB = nE$$

$$KC + CH = KH = nF$$

$$KH = CD$$

In other words

Let AB be the same multiple of E as CD is of F

Subtract AG and CH from AB and CD, where AG, CH are equimultiples of E and F respectively

Then, either GB, HD are both equal to E, F, or they are equimultiples of them

Proof

Let GB equal E, and create a line KC equal to F

AG, CH are the same multiples of E, F, while GB equals E and KC equals F ...

... therefore, AB, KH are equimultiple to E, F (V.2)

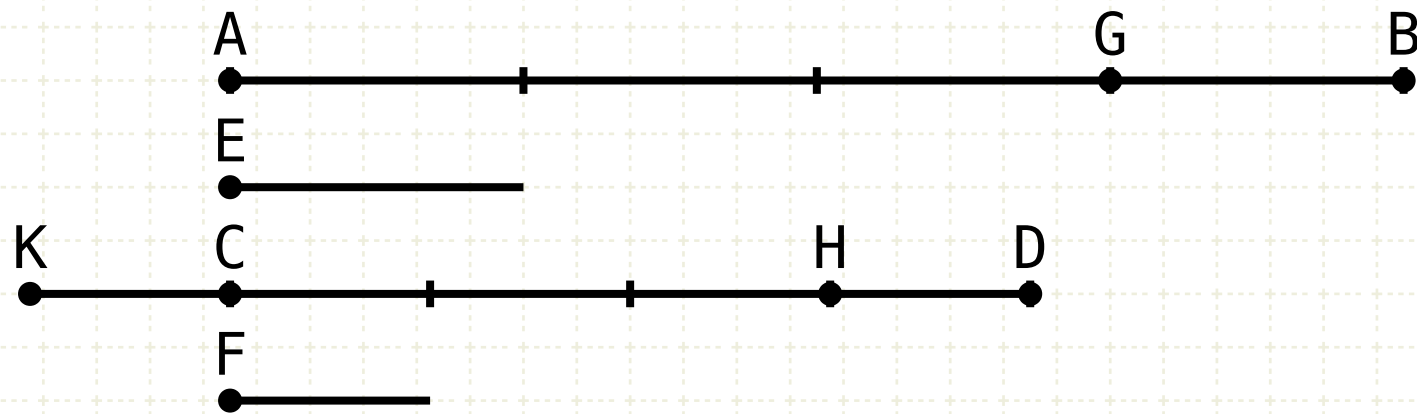
Since CD, KC are equimultiples of F, then KC must equal CD

Subtract CH from CH and CD, therefore the remainders KC, HD are equal



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If two magnitudes be equimultiples of two magnitudes, and any magnitudes subtracted from them be equimultiples of the same, the remainders are also either equal to the same or equimultiples of them



$$AB = nE$$

$$CD = nF$$

$$AG = mE$$

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$$\text{Let } GB = E$$

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$$AG + GB = AB = nE$$

$$KC + CH = KH = nF$$

$$KH = CD$$

$$KH = CD$$

$$KH - CH = CD - CH$$

$$KC = HD$$

$$HD = F$$

In other words

Let AB be the same multiple of E as CD is of F

Subtract AG and CH from AB and CD, where AG, CH are equimultiples of E and F respectively

Then, either GB, HD are both equal to E, F, or they are equimultiples of them

Proof

Let GB equal E, and create a line KC equal to F

AG, CH are the same multiples of E, F, while GB equals E and KC equals F ...

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Since CD, KC are equimultiples of F, then KC must equal CD

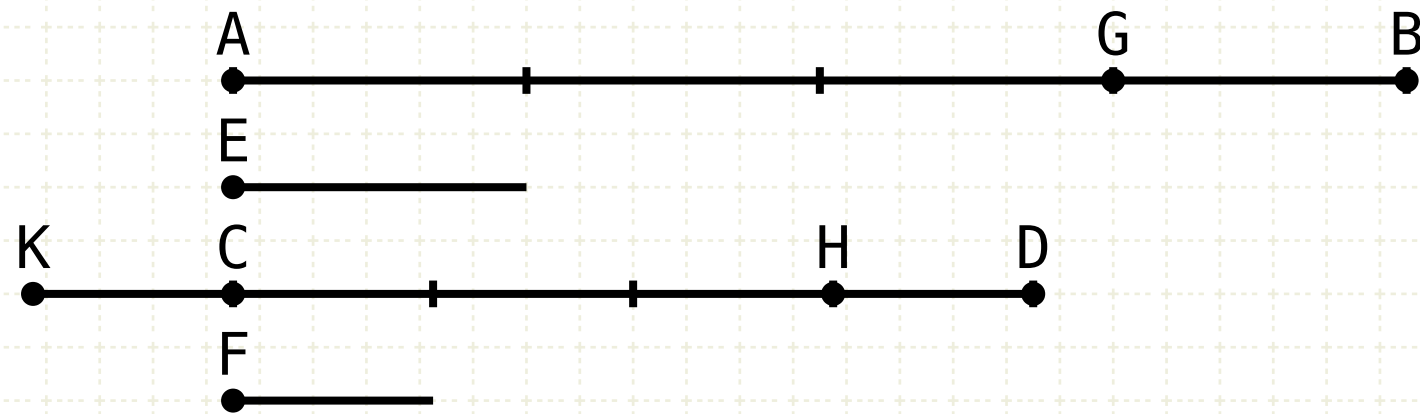
Subtract CH from CH and CD, therefore the remainders KC, HD are equal

But KC is equal to F, so HD must also be equal to F



Proposition 6 of Book V

If two magnitudes be equimultiples of two magnitudes, and any magnitudes subtracted from them be equimultiples of the same, the remainders are also either equal to the same or equimultiples of them



$$AB = nE$$

$$CD = nF$$

$$AG = mE$$

$$CH = mF$$

$$KH = CD$$

$$KH - CH = CD - CH$$

$$KC = HD$$

$$HD = F$$

$$\text{Let } GB = E$$

$$\text{Let } KC = F$$

$$AG + GB = AB = nE$$

$$KC + CH = KH = nF$$

$$KH = CD$$

In other words

Let AB be the same multiple of E as CD is of F

Subtract AG and CH from AB and CD, where AG, CH are equimultiples of E and F respectively

Then, either GB, HD are both equal to E, F, or they are equimultiples of them

Proof

Let GB equal E, and create a line KC equal to F

AG, CH are the same multiples of E, F, while GB equals E and KC equals F ...

... therefore, AB, KH are equimultiple to E, F (V.2)

Since CD, KC are equimultiples of F, then KC must equal CD

Subtract CH from CH and CD, therefore the remainders KC, HD are equal

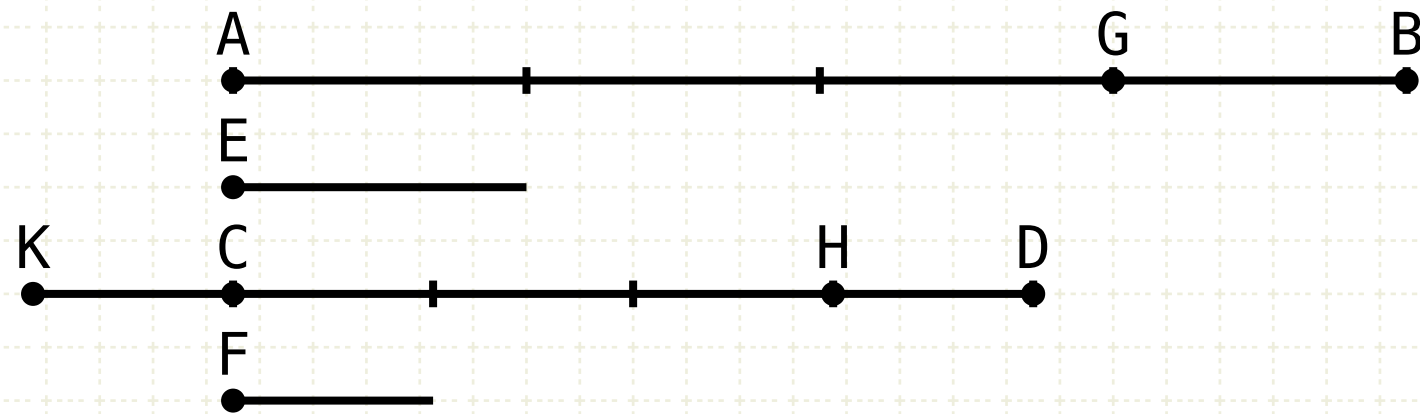
But KC is equal to F, so HD must also be equal to F

Hence, if GB is equal to E, HD must be equal to F



Proposition 6 of Book V

If two magnitudes be equimultiples of two magnitudes, and any magnitudes subtracted from them be equimultiples of the same, the remainders are also either equal to the same or equimultiples of them



$$AB = nE$$

$$CD = nF$$

$$AG = mE$$

$$CH = mF$$

$$KH = CD$$

$$KH - CH = CD - CH$$

$$KC = HD$$

$$HD = F$$

$$\text{Let } GB = E$$

$$\text{Let } KC = F$$

$$AG + GB = AB = nE$$

$$KC + CH = KH = nF$$

$$KH = CD$$

$$AB - AG = nE - mE = (n-m)E = kE = GB$$

$$CD - CH = nF - mF = (n-m)F = kF = HD$$

In other words

Let AB be the same multiple of E as CD is of F

Subtract AG and CH from AB and CD, where AG,CH are equimultiples of E and F respectively

Then, either GB,HD are both equal to E,F, or they are equimultiples of them

Proof

Let GB equal E, and create a line KC equal to F

AG,CH are the same multiples of E,F, while GB equals E and KC equals F ...

... therefore, AB,KH are equimultiple to E,F (V.2)

Since CD,KC are equimultiples of F, then KC must equal CD

Subtract CH from CH and CD, therefore the remainders KC,HD are equal

But KC is equal to F, so HD must also be equal to F

Hence, if GB is equal to E, HD must be equal to F

Using the same method, we can prove that if GB is a multiple of E, then HD will be the same multiple of F



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