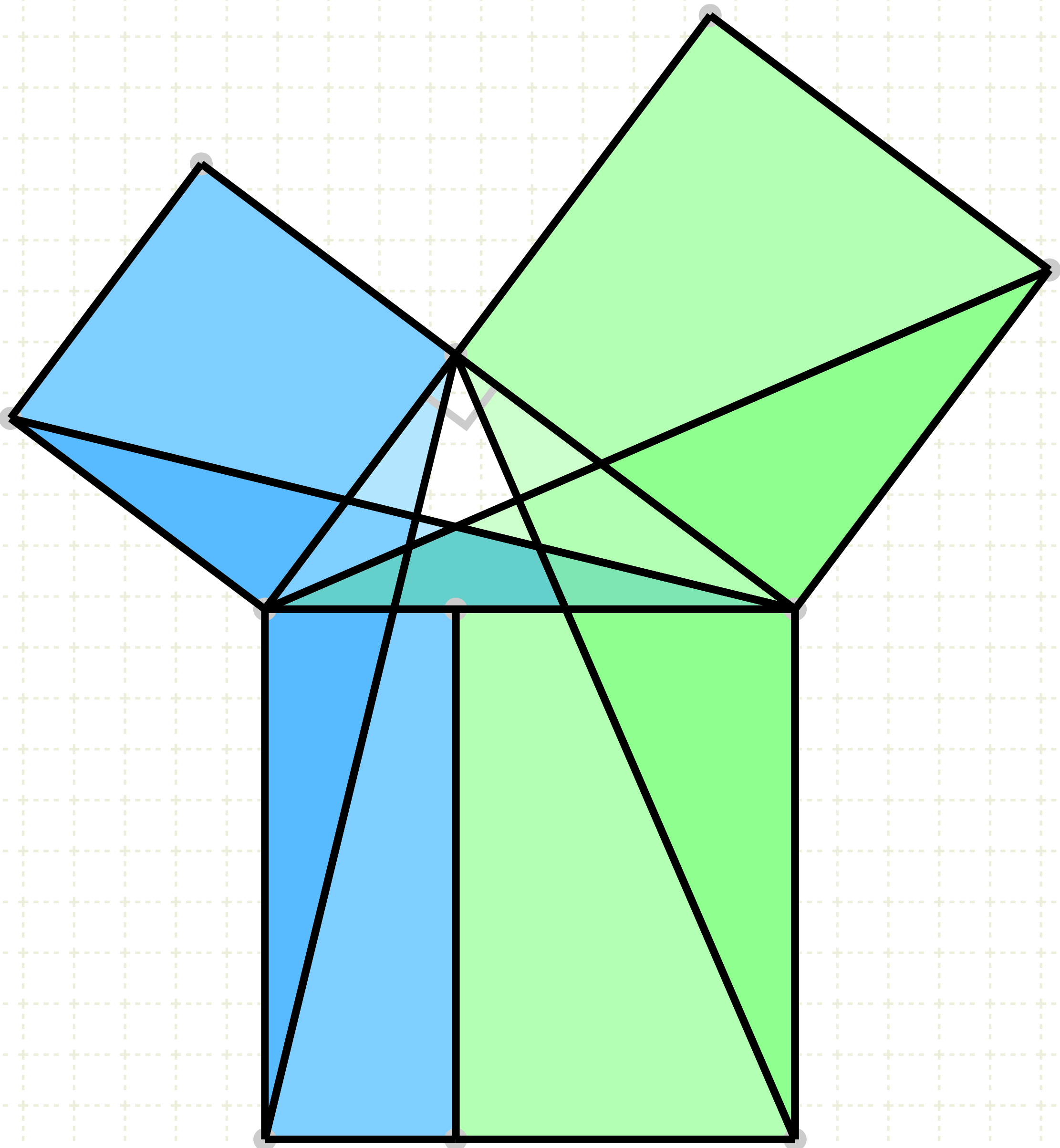
Euclid's Elements

Book I



If Euclid did not kindle your youthful enthusiasm, you were not born to be a scientific thinker.

Albert Einstein

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Proposition 21 of Book I

If from the ends of one of the sides of a triangle two straight lines are constructed meeting within the triangle, then the sum of the straight lines so constructed is less than the sum of the remaining two sides of the triangle, but the constructed straight lines contain a greater angle than the angle contained by the remaining two sides.

Proposition 21 of Book I

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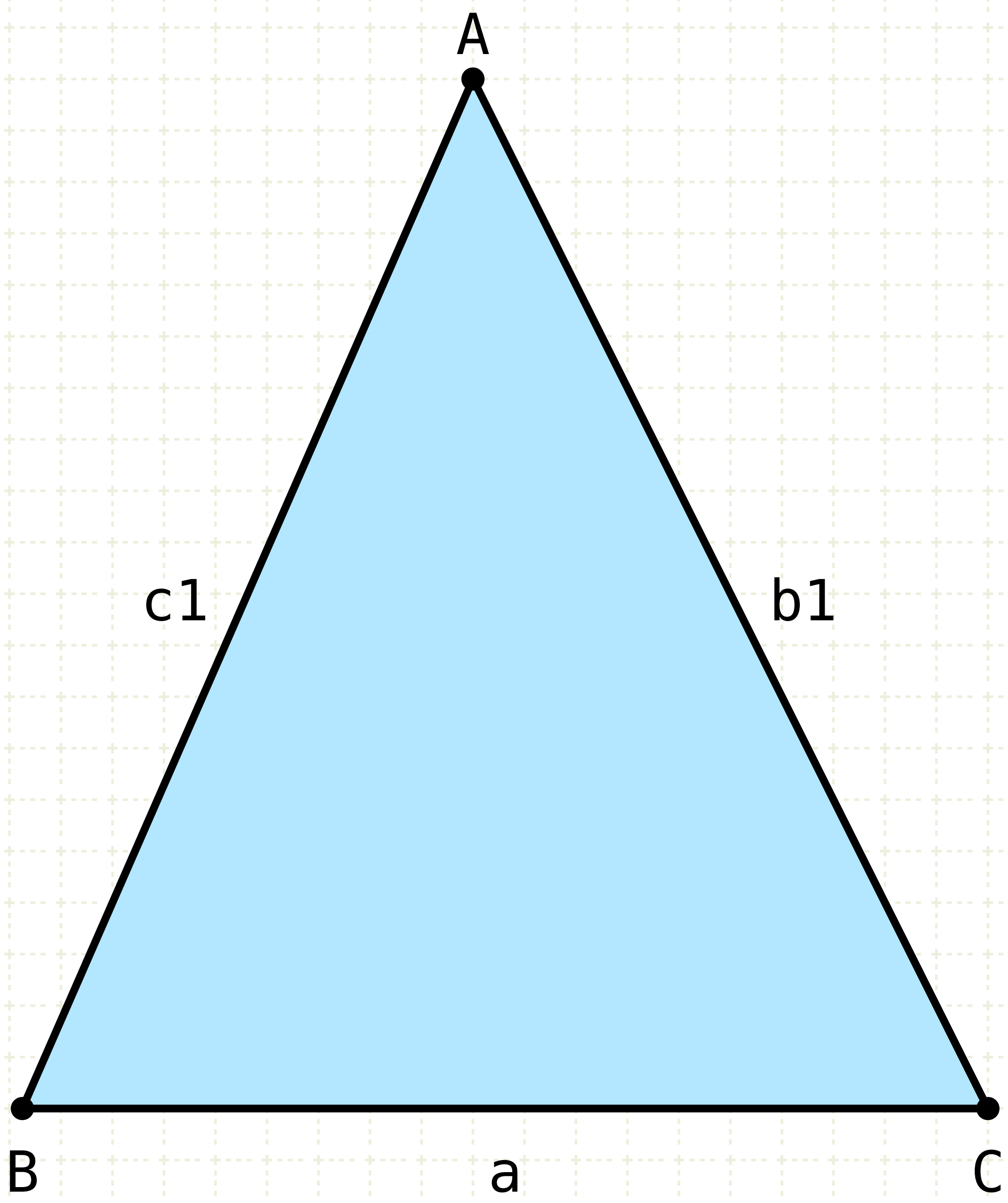

In other words

Given a triangle ABC

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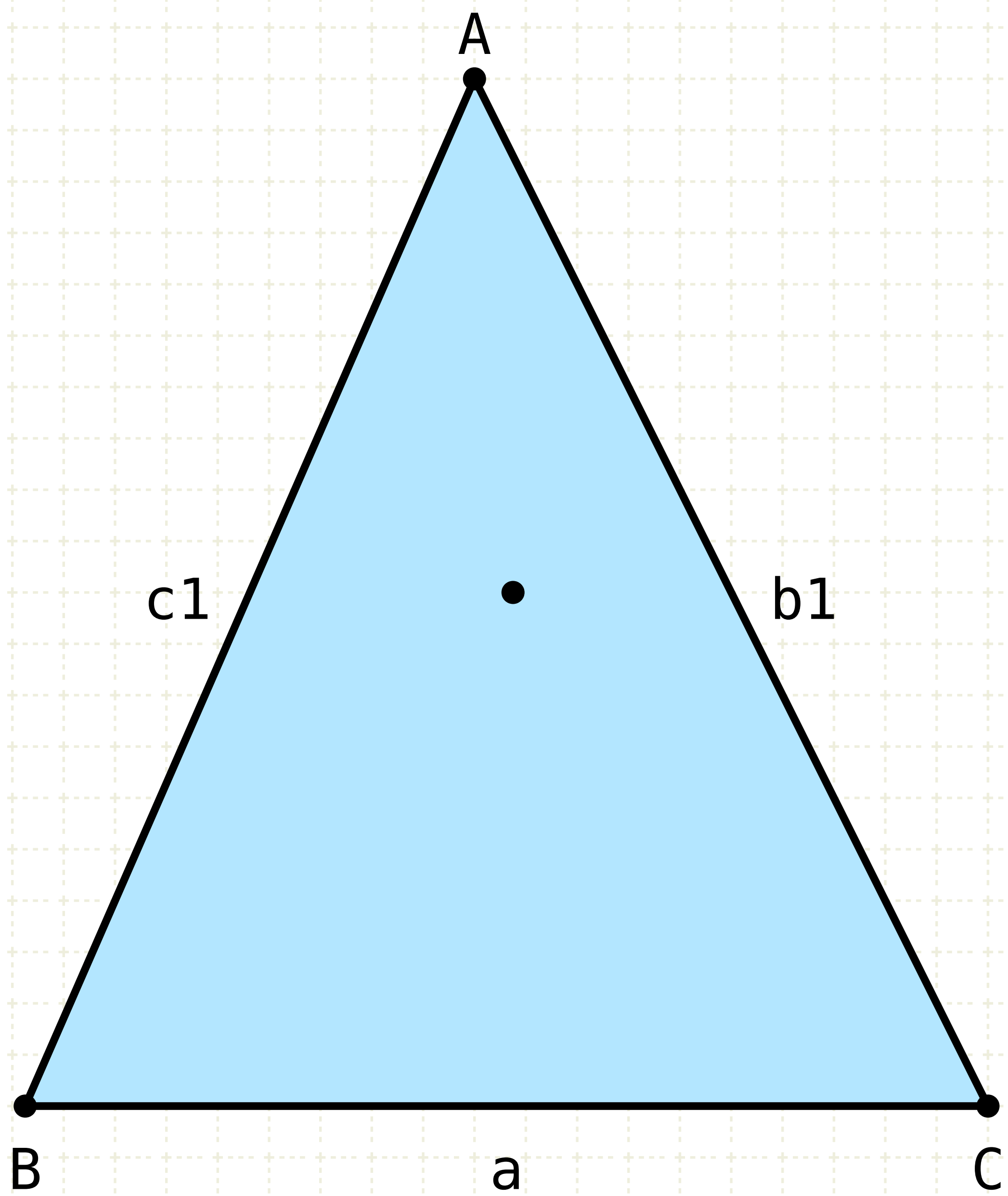
If from the ends of one of the sides of a triangle two straight lines are constructed meeting within the triangle, then the sum of the straight lines so constructed is less than the sum of the remaining two sides of the triangle, but the constructed straight lines contain a greater angle than the angle contained by the remaining two sides.



In other words

Given a triangle ABC

From a point within the triangle ABC…

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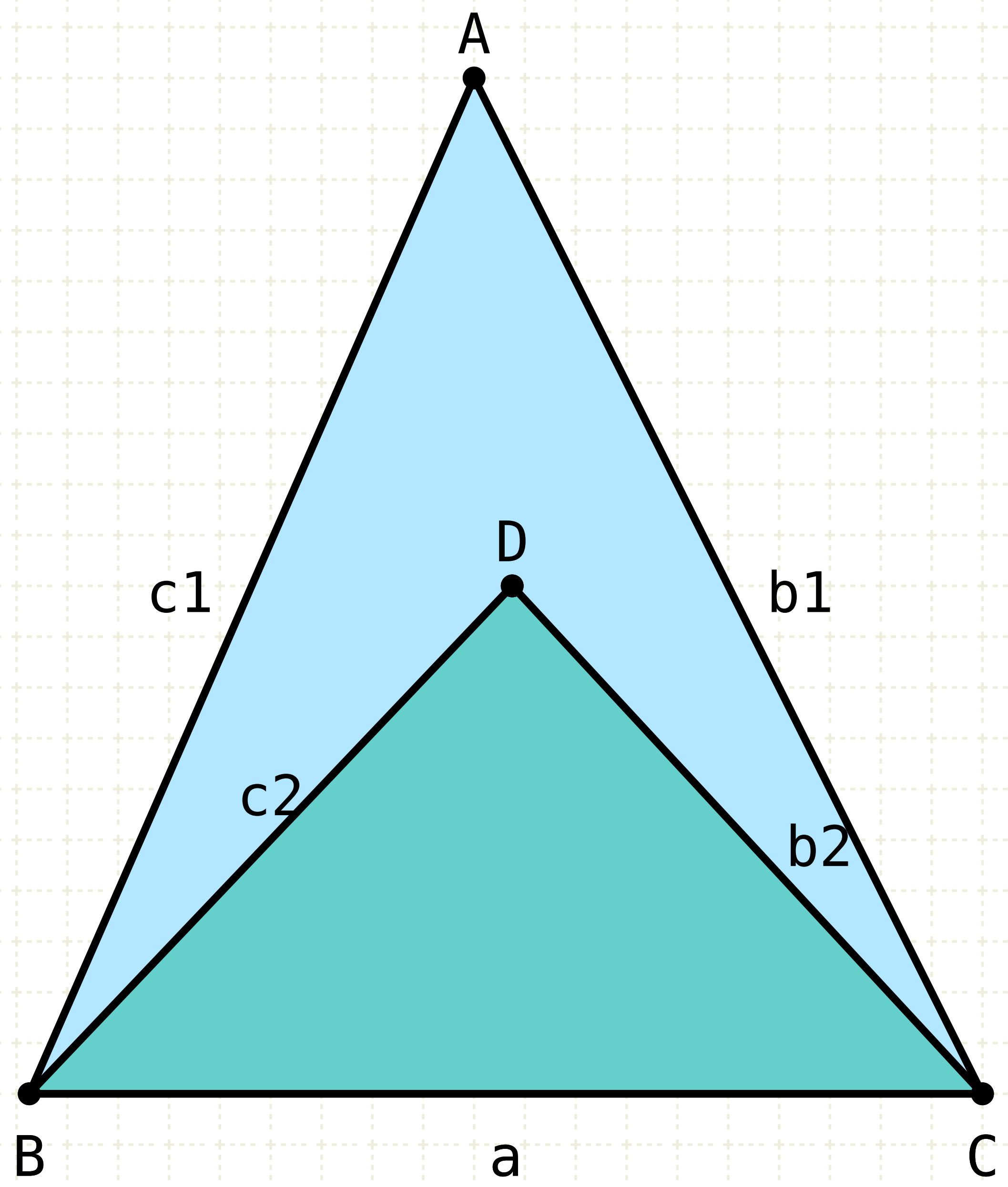

In other words

Given a triangle ABC

From a point within the triangle ABC…

… construct a second triangle DBC

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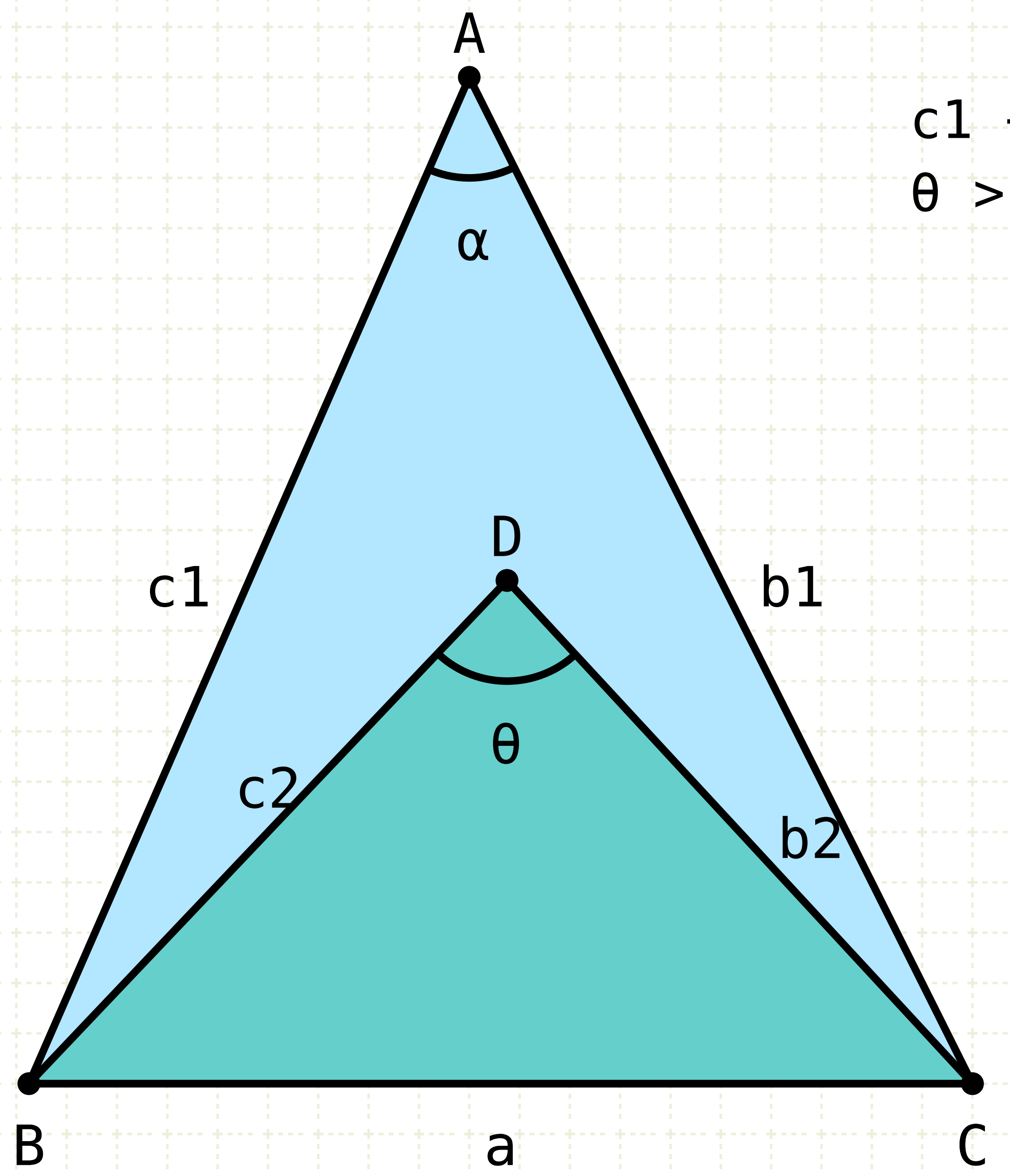

$c1 + b1 > c2 + b2$

$\theta > \alpha$

In other words

Given a triangle ABC

From a point within the triangle ABC…

… construct a second triangle DBC

The sum of the lines BD and DC is less than the sum of the lines BA and AC, and the angle BDC is greater than angle BAC

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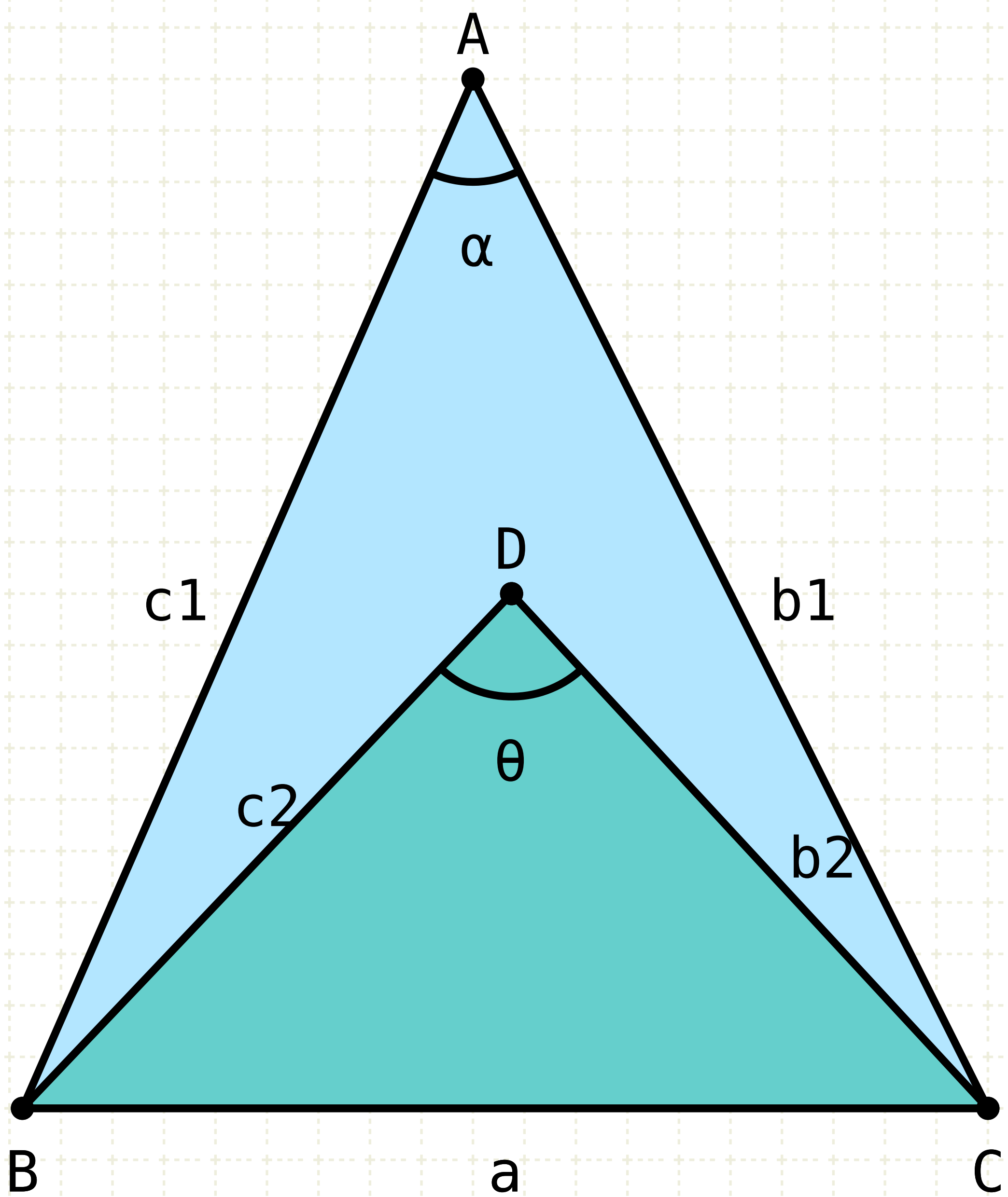

Proof

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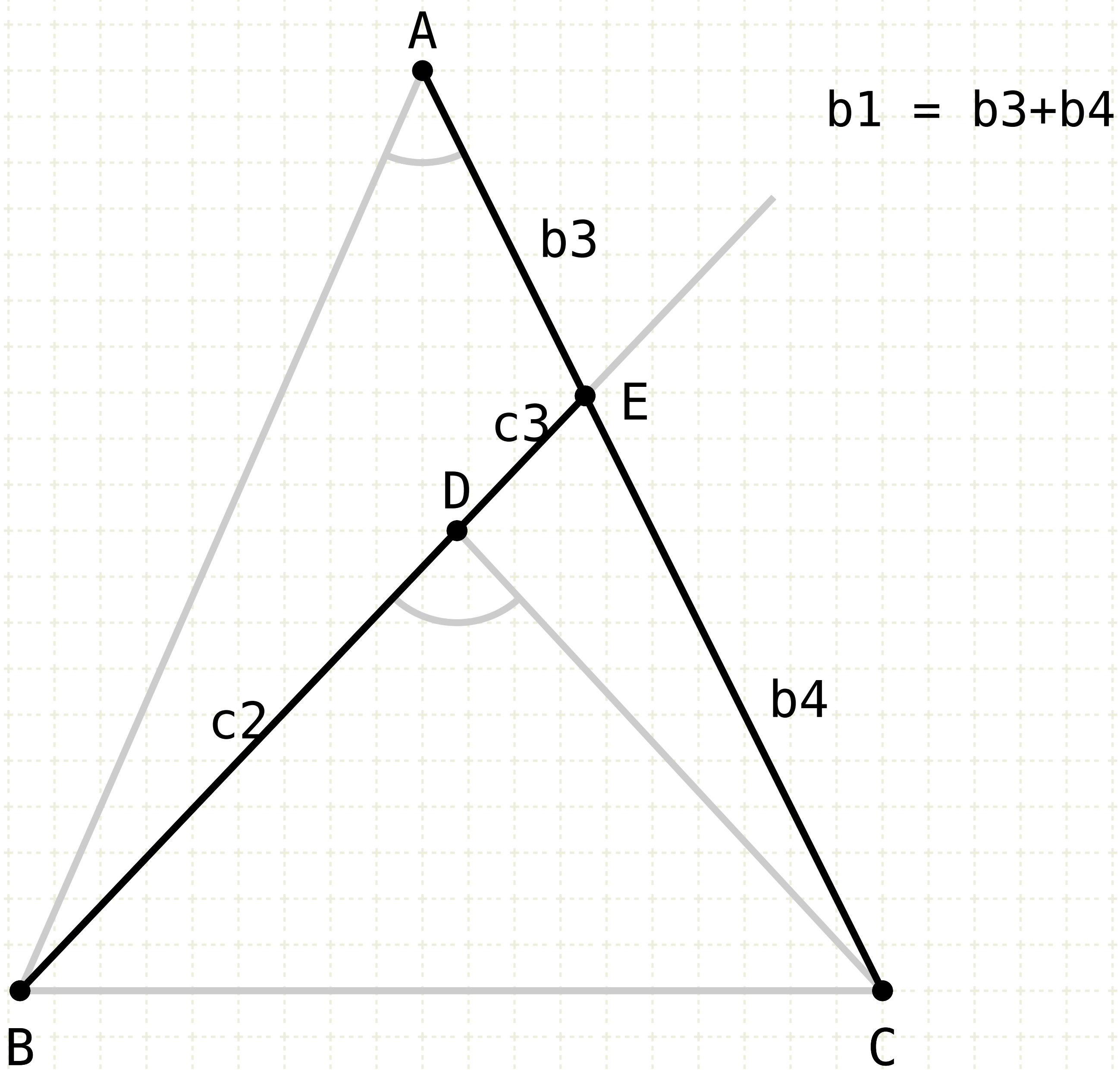

Proof

Extend BD such that it intersects AC at point E

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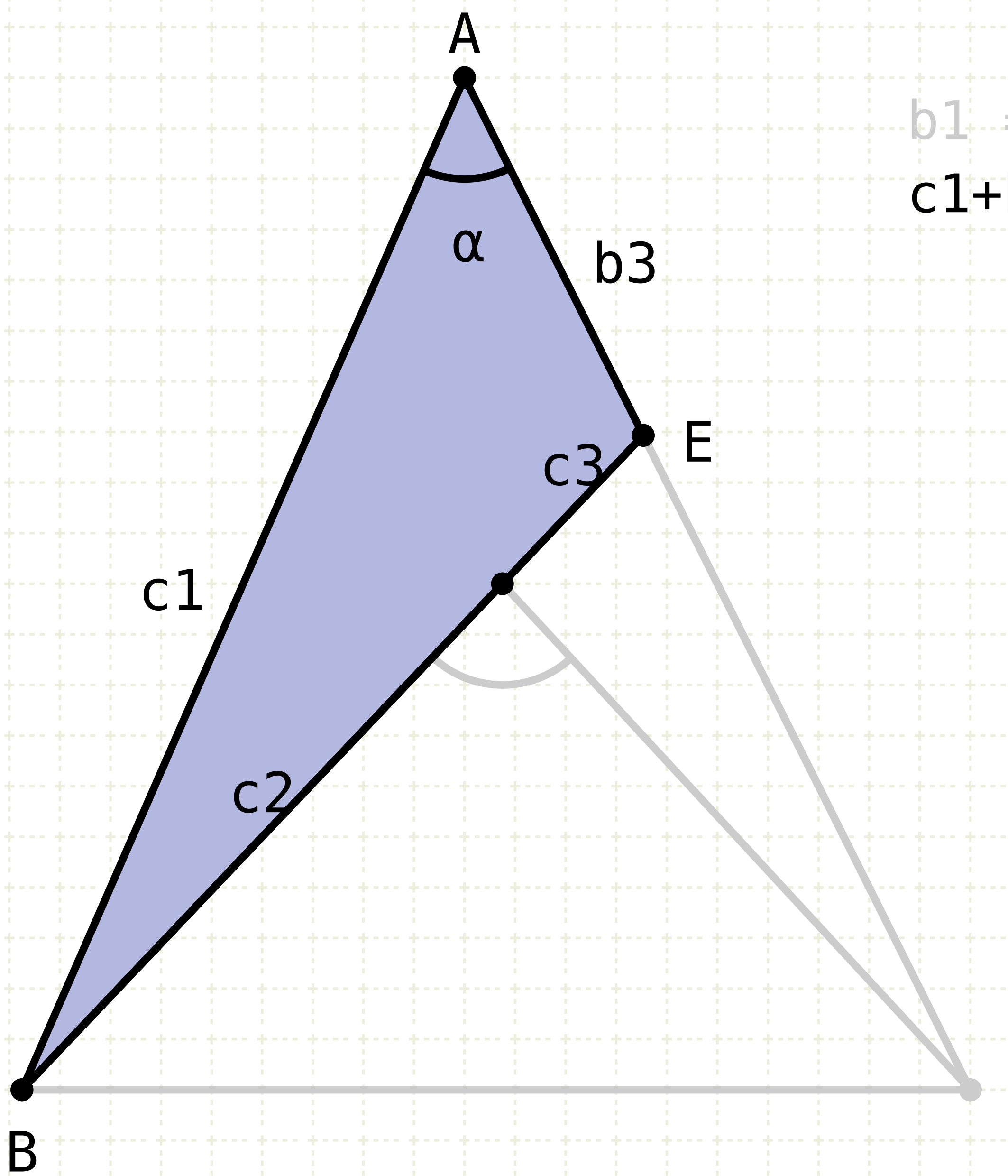

b1 = b3+b4

c1+b3 > c2+c3

Proof

Extend BD such that it intersects AC at point E

Consider triangle ABE

The sum of lines AB and AE is greater than BE (I·18)

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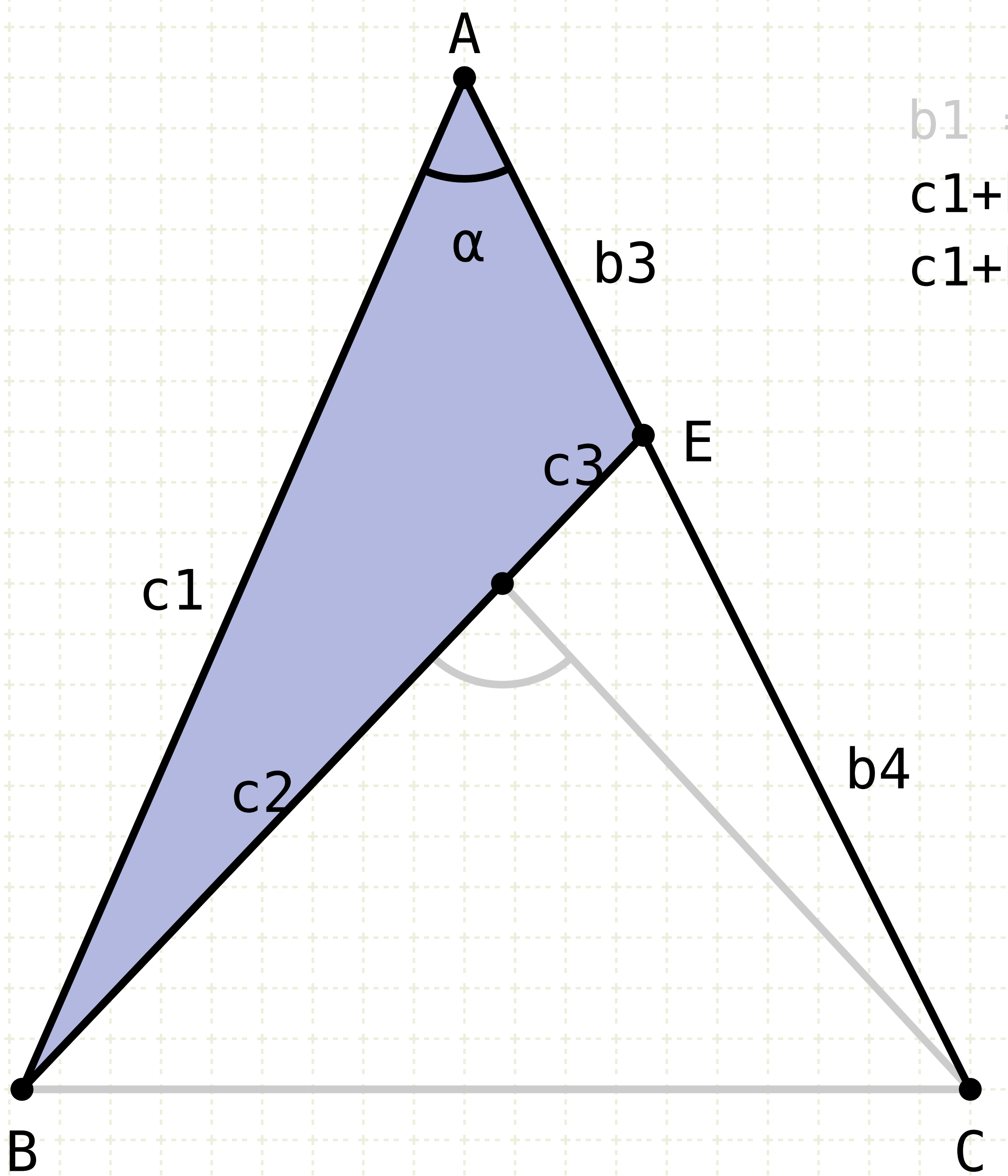


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b1 = b3+b4
c1+b3     > c2+c3
c1+b3+b4 > c2+c3+b4
```

Proof

Extend BD such that it intersects AC at point E

Consider triangle ABE

The sum of lines AB and AE is greater than BE (I·18)

Add length EC to both each part of the inequality

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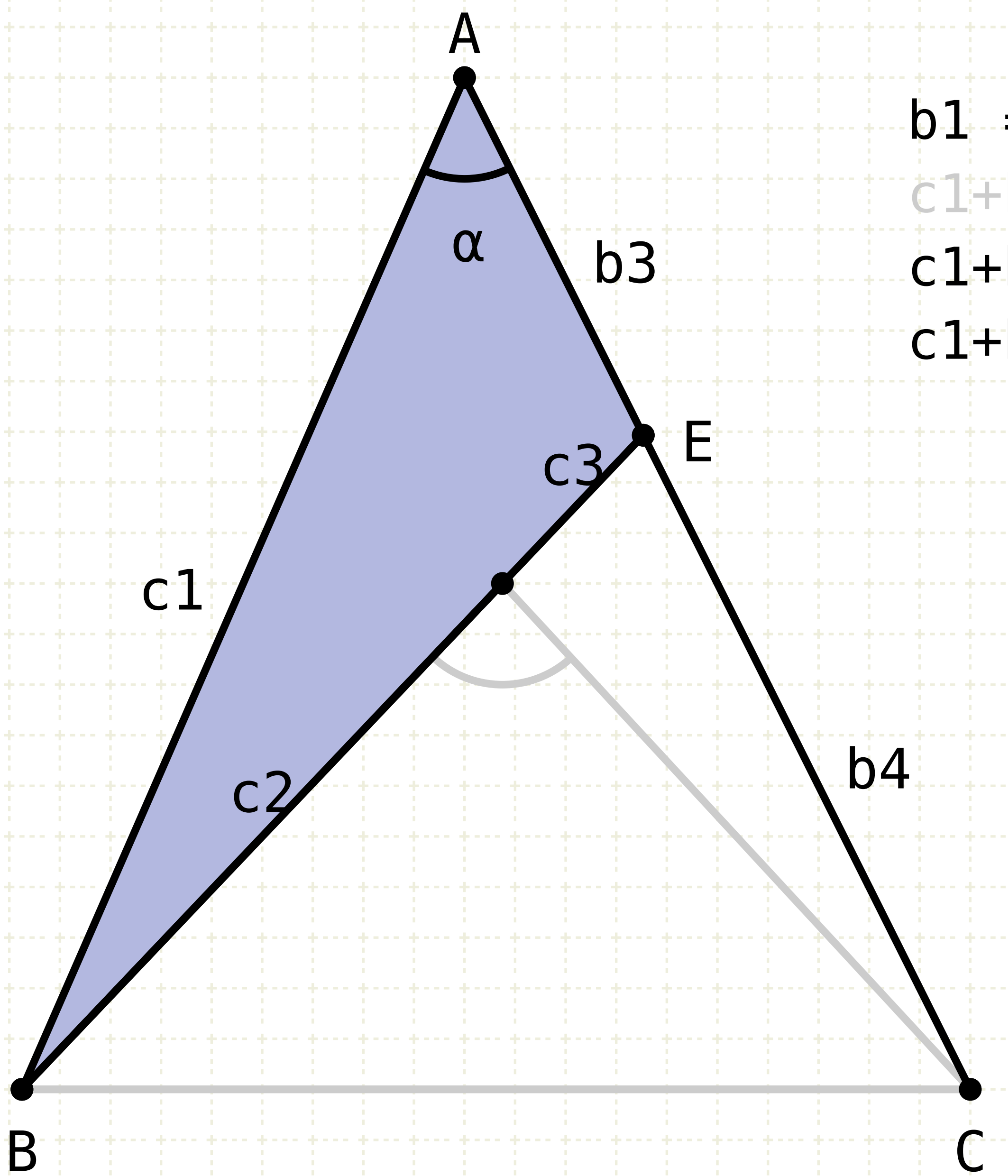

$$b1 = b3+b4$$
$$c1+b3 > c2+c3$$
$$c1+b3+b4 > c2+c3+b4$$
$$c1+b1 > c2+c3+b4$$

Proof

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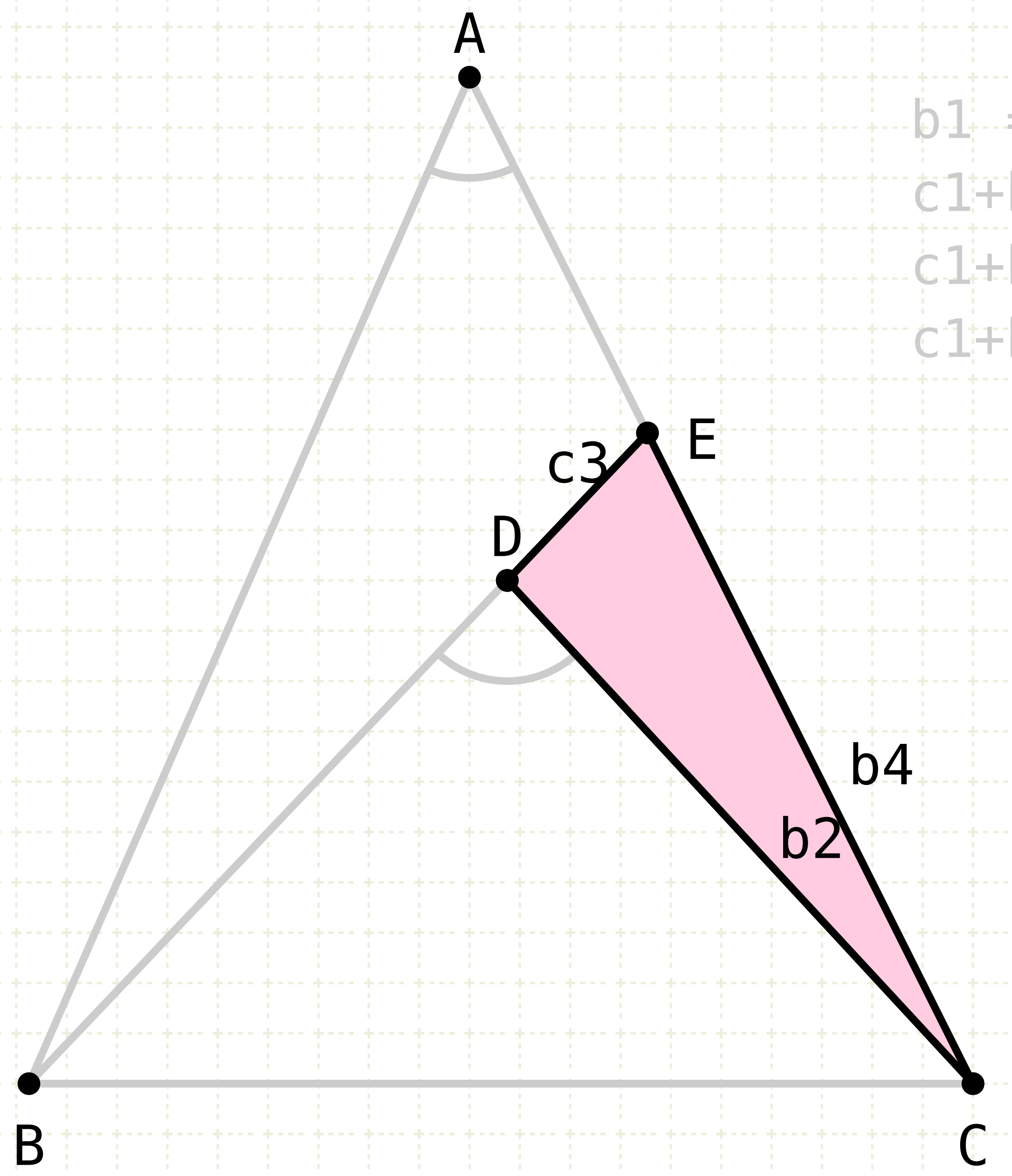


```
b1 = b3+b4
c1+b3     > c2+c3
c1+b3+b4  > c2+c3+b4
c1+b1     > c2+c3+b4
           c3+b4 >    b2
```

Proof

Extend BD such that it intersects AC at point E

Consider triangle ABE

The sum of lines AB and AE is greater than BE (I·18)

Add length EC to both each part of the inequality

Consider triangle DEC

The sum of lines DE and EC is greater than CD (I·18)

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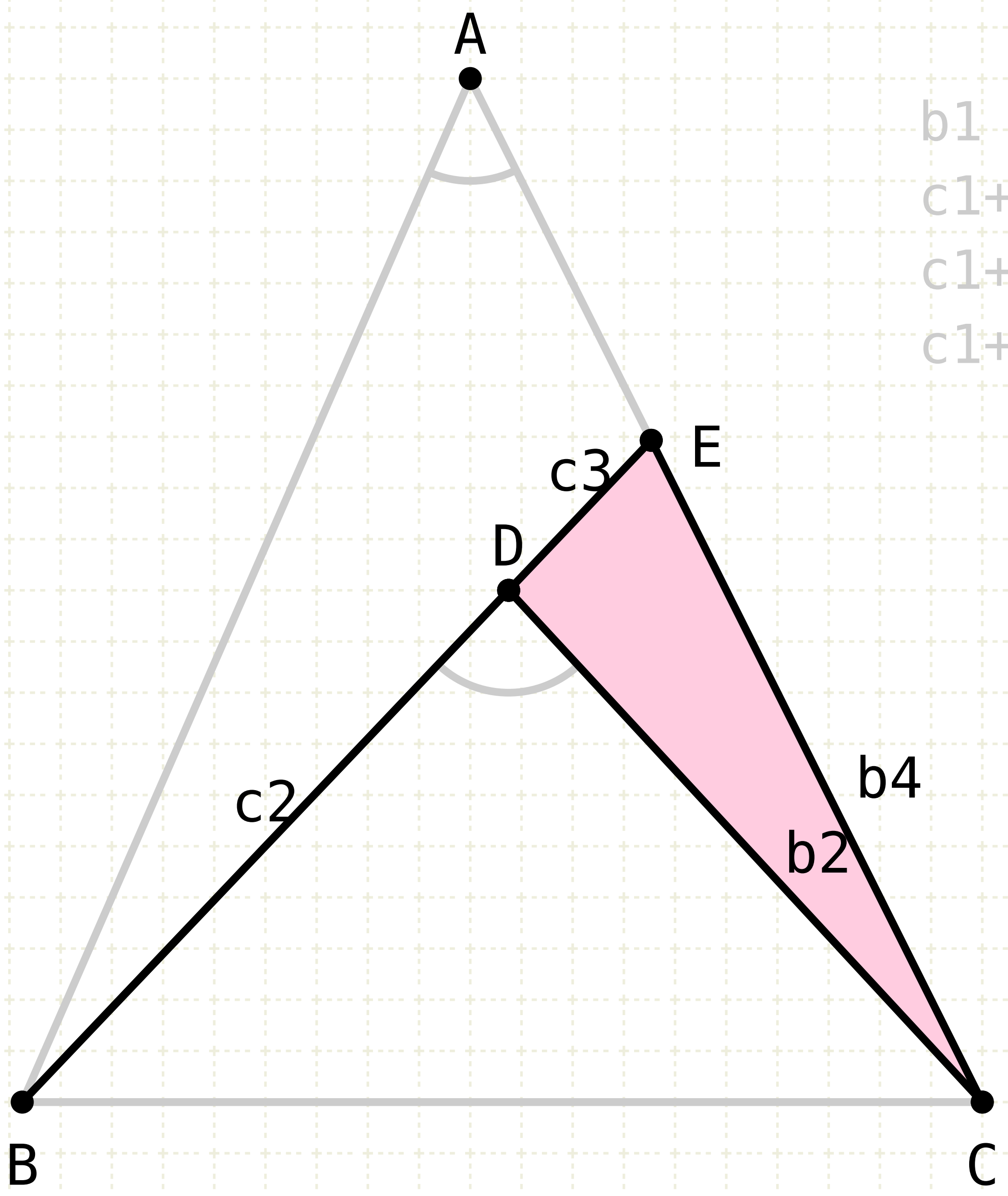

$$b1 = b3+b4$$
$$c1+b3 > c2+c3$$
$$c1+b3+b4 > c2+c3+b4$$
$$c1+b1 > c2+c3+b4$$
$$c3+b4 > b2$$
$$c2+c3+b4 > c2+b2$$

Proof

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Consider triangle ABE

The sum of lines AB and AE is greater than BE (I·18)

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Consider triangle DEC

The sum of lines DE and EC is greater than CD (I·18)

Add BD to both sides of the inequality

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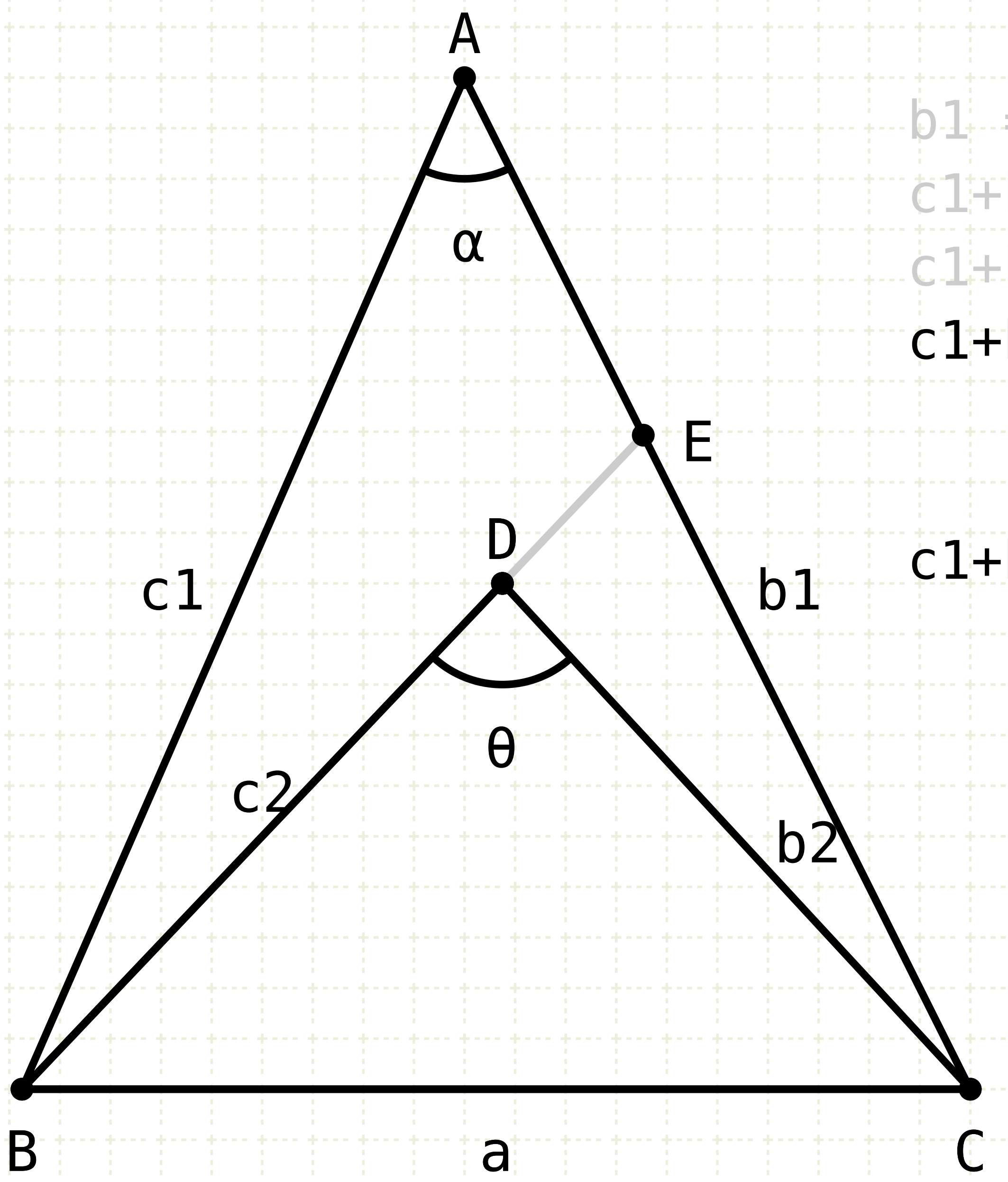

$$
\begin{aligned}
b1 &= b3+b4 \\
c1+b3 &> c2+c3 \\
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c1+b1 &> c2+c3+b4 \\
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Proof

Extend BD such that it intersects AC at point E

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The sum of lines AB and AE is greater than BE (I·18)

Add length EC to both each part of the inequality

Consider triangle DEC

The sum of lines DE and EC is greater than CD (I·18)

Add BD to both sides of the inequality

Thus, the sum of AB and AC is greater than the sum of DB and DC

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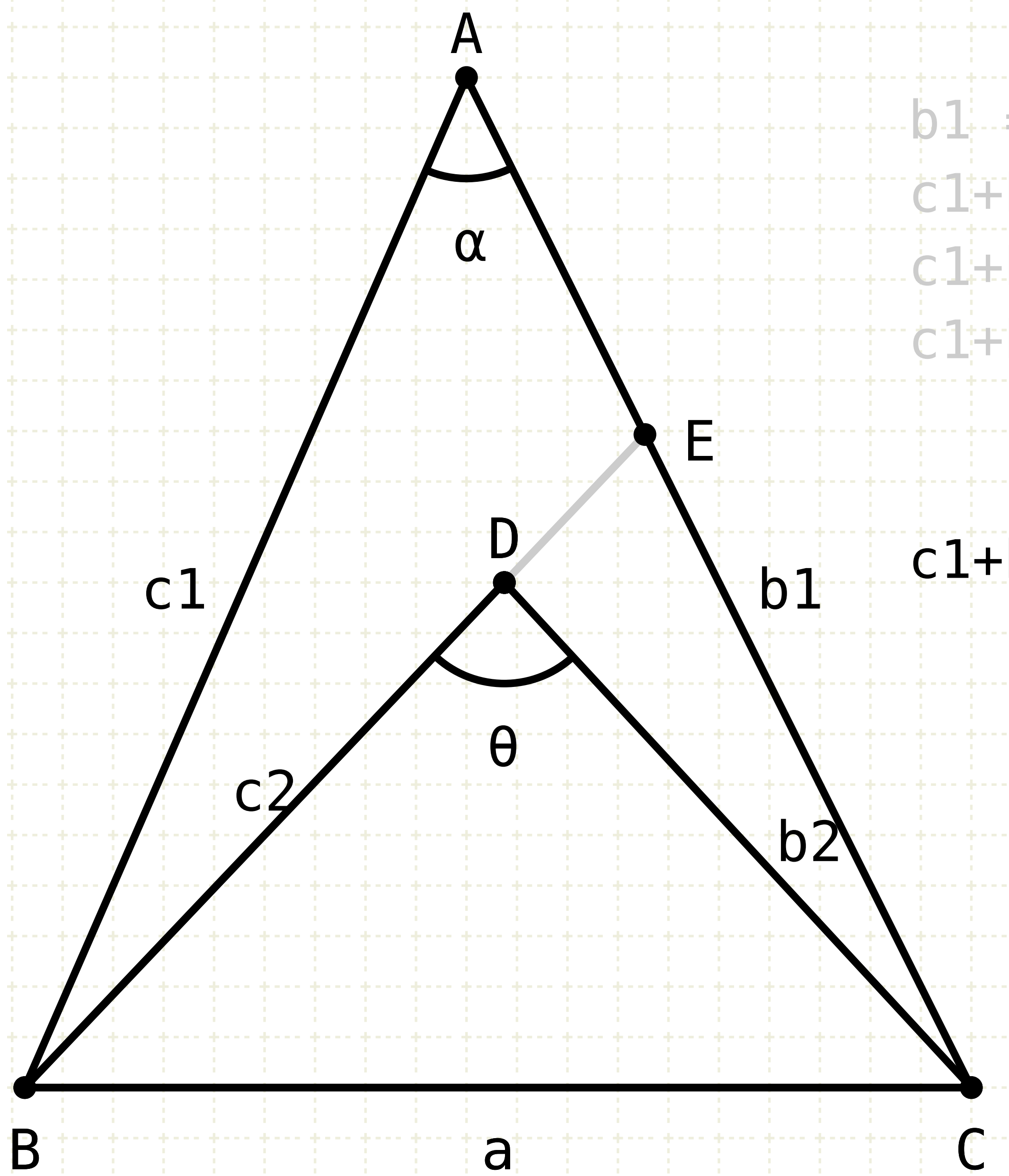


```
b1 = b3+b4
c1+b3     > c2+c3
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c1+b1     > c2+c3+b4
                c3+b4 >     b2
             c2+c3+b4 > c2+b2

c1+b1     >               c2+b2
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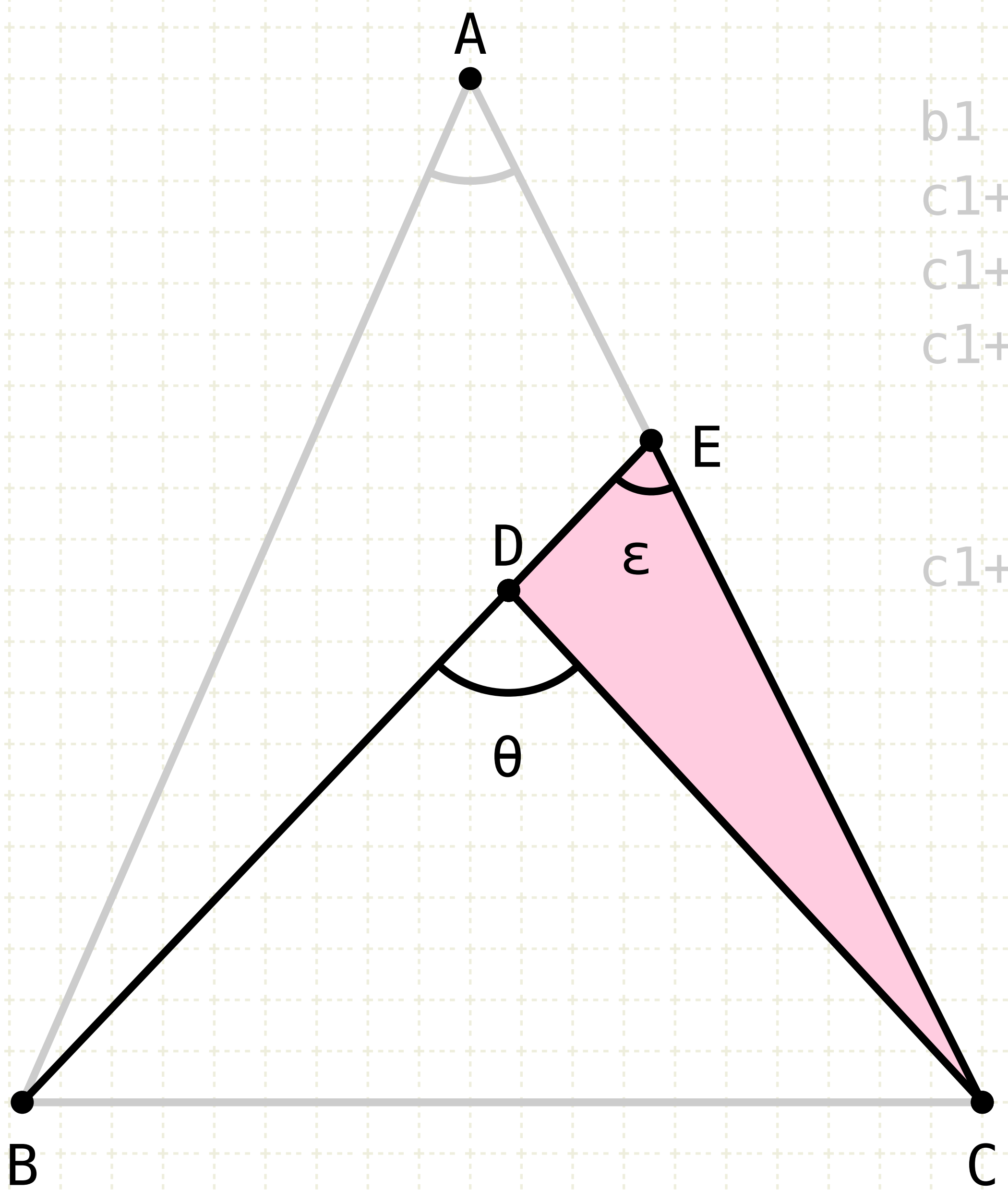

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c3+b4 &> b2 \\
c2+c3+b4 &> c2+b2 \\
c1+b1 &> c2+b2
\end{aligned}
$$

$\theta > \varepsilon$

Proof

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Consider triangle ABE

The sum of lines AB and AE is greater than BE (I·18)

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Consider triangle DEC

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Angle BDC is an exterior angle to triangle DCE, hence it is larger than the angle DEC (I·16)

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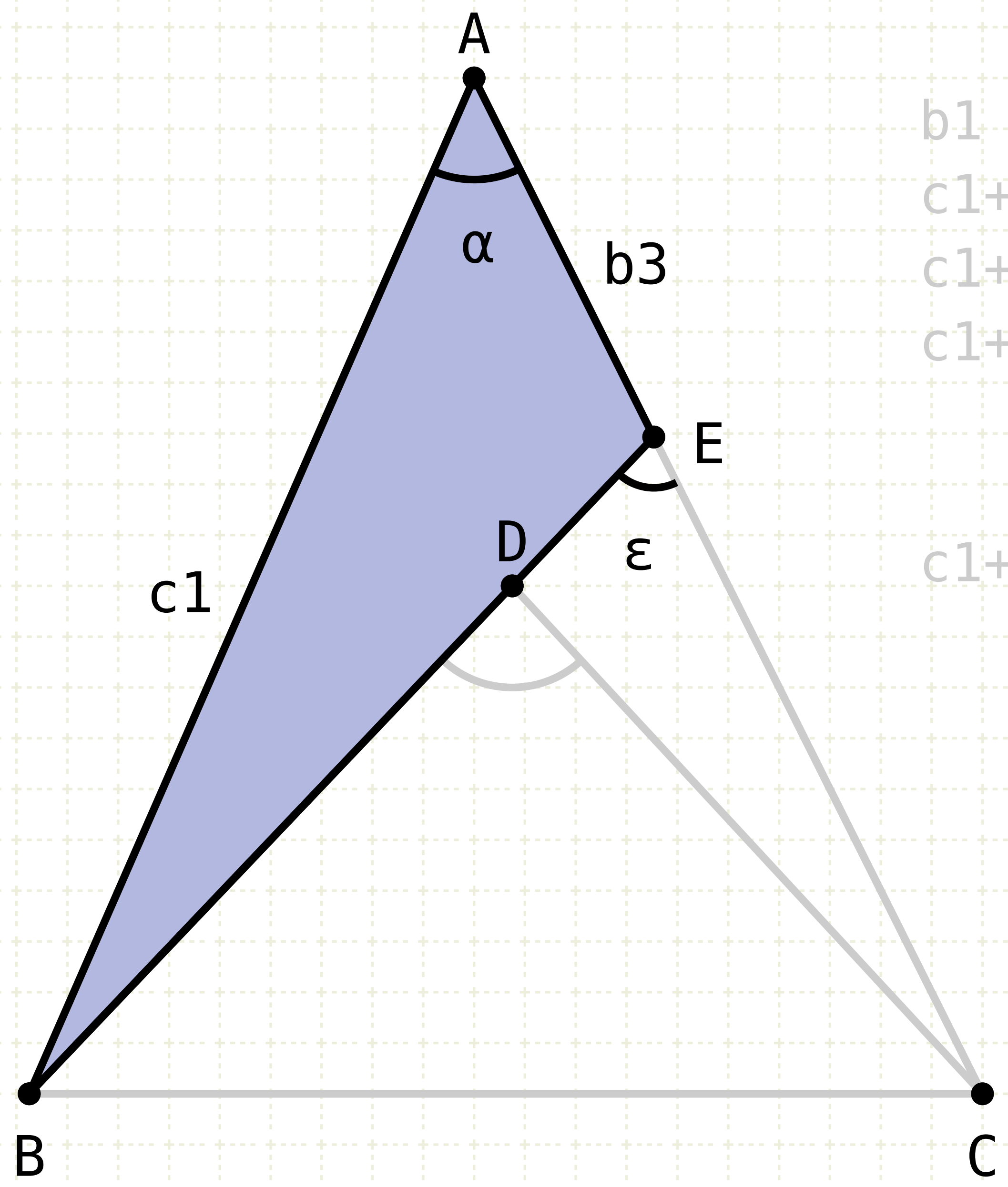

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c1+b1 &> c2+b2
\end{aligned}
$$

$$
\begin{aligned}
\theta &> \varepsilon \\
\varepsilon &> \alpha
\end{aligned}
$$

Proof

Extend BD such that it intersects AC at point E

Consider triangle ABE

The sum of lines AB and AE is greater than BE (I·18)

Add length EC to both each part of the inequality

Consider triangle DEC

The sum of lines DE and EC is greater than CD (I·18)

Add BD to both sides of the inequality

Thus, the sum of AB and AC is greater than the sum of DB and DC

Angle BDC is an exterior angle to triangle DCE, hence it is larger than the angle DEC (I·16)

Angle DEC is an exterior angle to triangle EAB, hence it is larger than the angle EAB (I·16)

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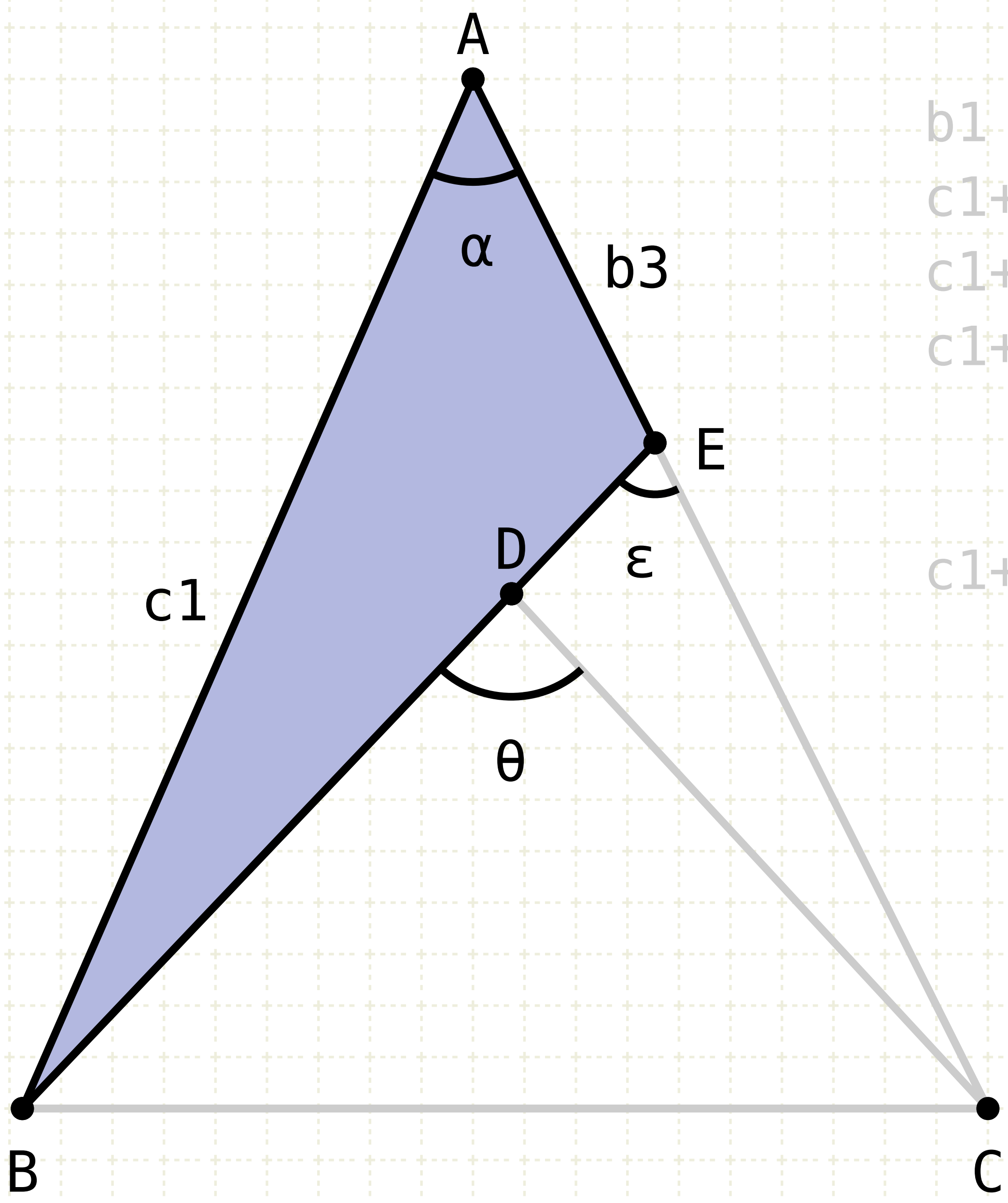

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\end{aligned}
$$

$$
\begin{aligned}
&\theta > \varepsilon \\
&\varepsilon > \alpha \\
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$$

Proof

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Consider triangle DEC

The sum of lines DE and EC is greater than CD (I·18)

Add BD to both sides of the inequality

Thus, the sum of AB and AC is greater than the sum of DB and DC

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Angle DEC is an exterior angle to triangle EAB, hence it is larger than the angle EAB (I·16)

Thus, angle BDC is greater than angle ABC

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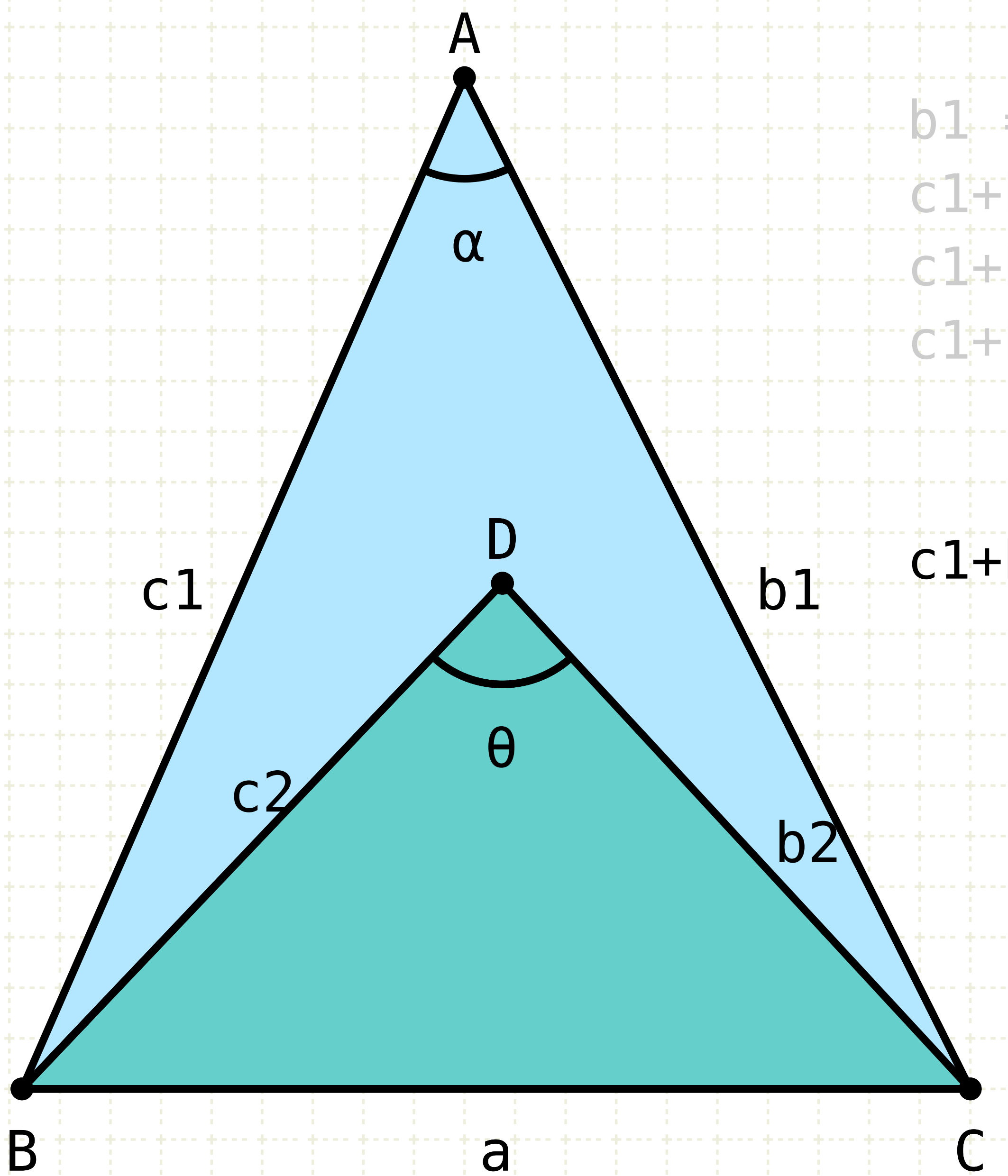

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\theta &> \varepsilon \\
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\theta &> \varepsilon > \alpha \\
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Extend BD such that it intersects AC at point E

Consider triangle ABE

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The sum of lines DE and EC is greater than CD (I·18)

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Thus, the sum of AB and AC is greater than the sum of DB and DC

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Angle DEC is an exterior angle to triangle EAB, hence it is larger than the angle EAB (I·16)

Thus, angle BDC is greater than angle ABC

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