

# Euclid's Elements

## Book I

*If Euclid did not kindle your youthful enthusiasm, you  
were not born to be a scientific thinker.*

Albert Einstein



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# Proposition 48 of Book I

If in a triangle the square on one of the sides equals the sum of the squares on the remaining two sides of the triangle, then the angle contained by the remaining two sides of the triangle is right.

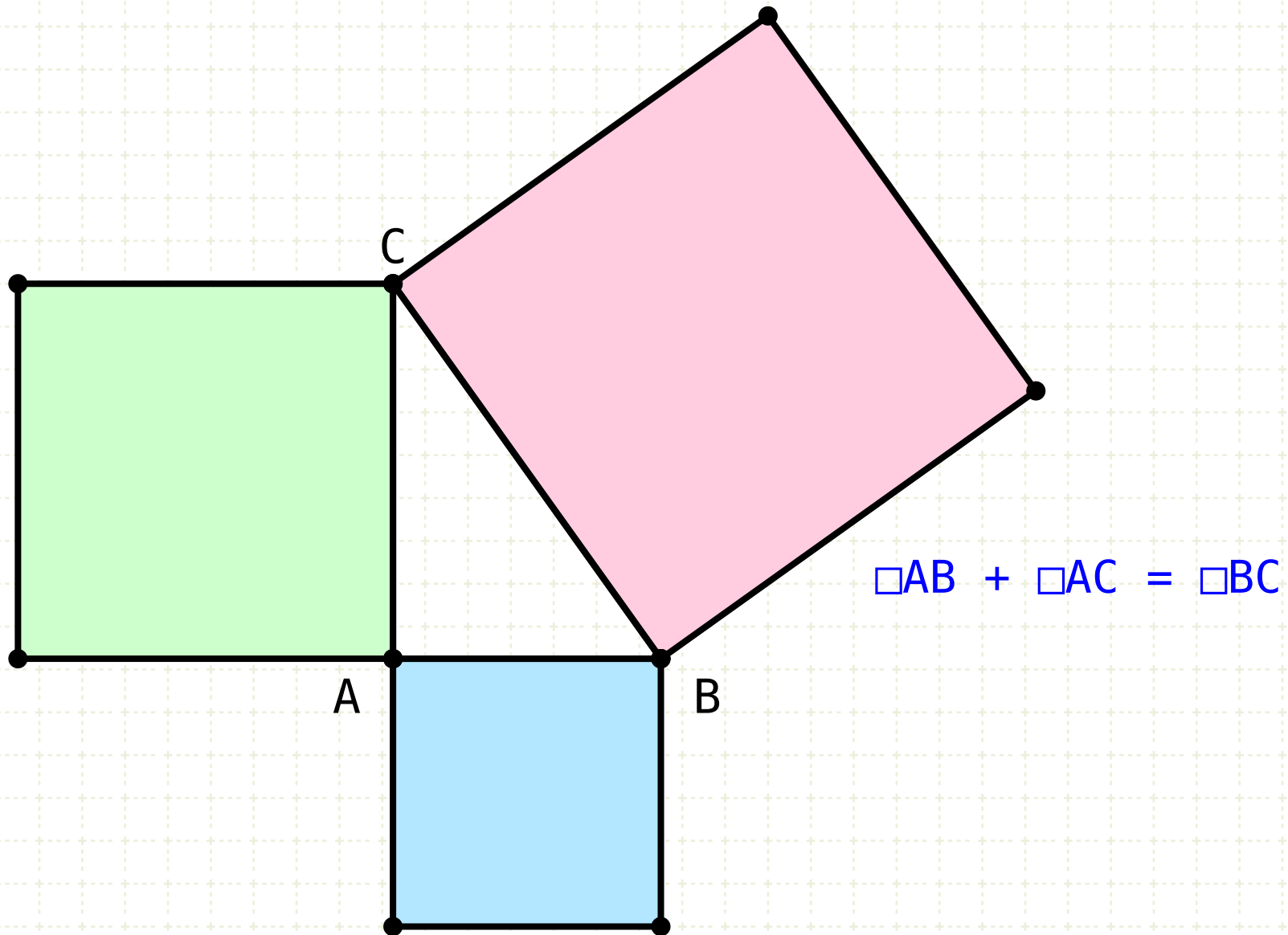


# Proposition 48 of Book I

If in a triangle the square on one of the sides equals the sum of the squares on the remaining two sides of the triangle, then the angle contained by the remaining two sides of the triangle is right.

## In other words

Given a triangle ABC, where the square of AB and AC equals the square of BC



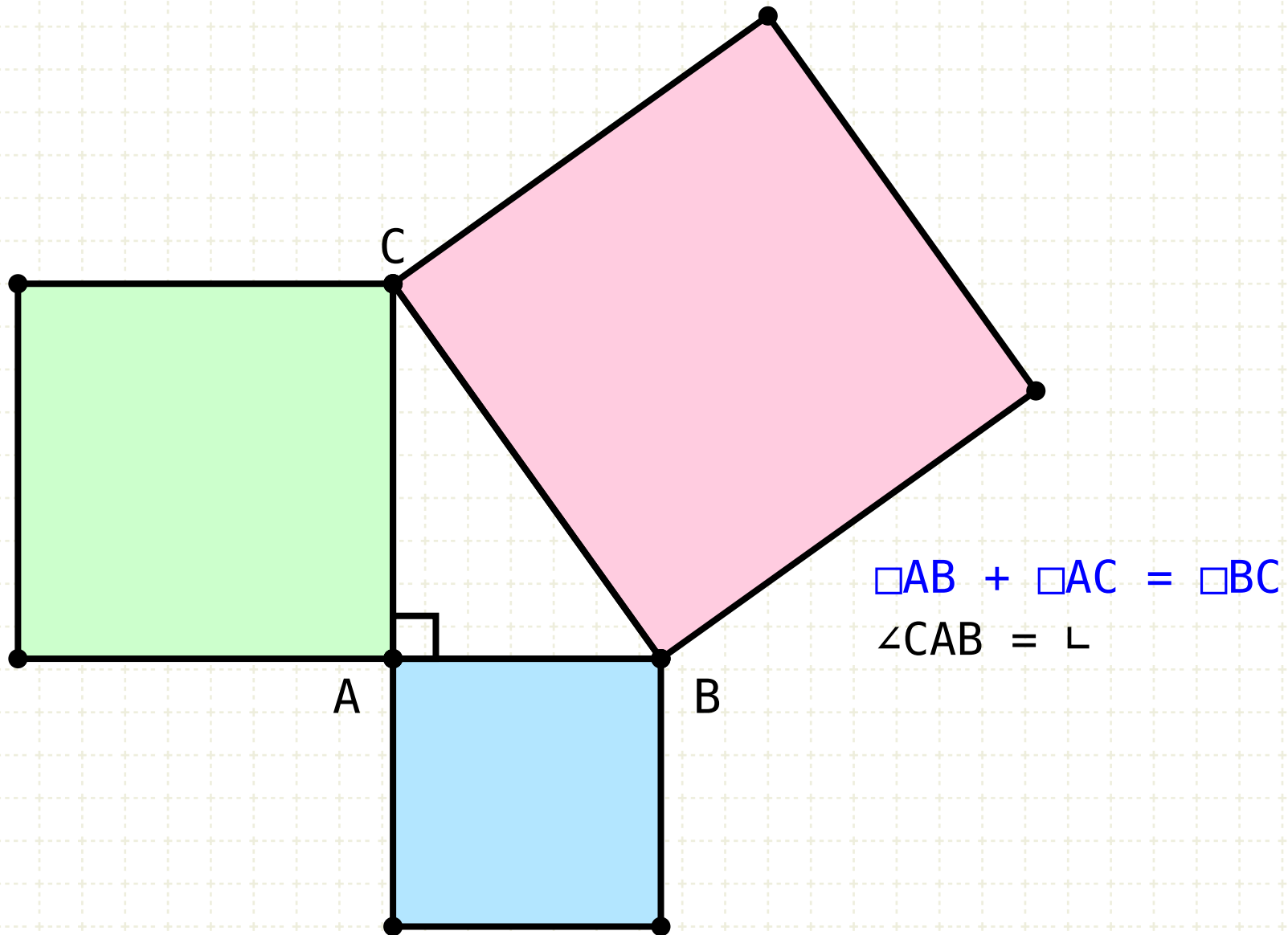
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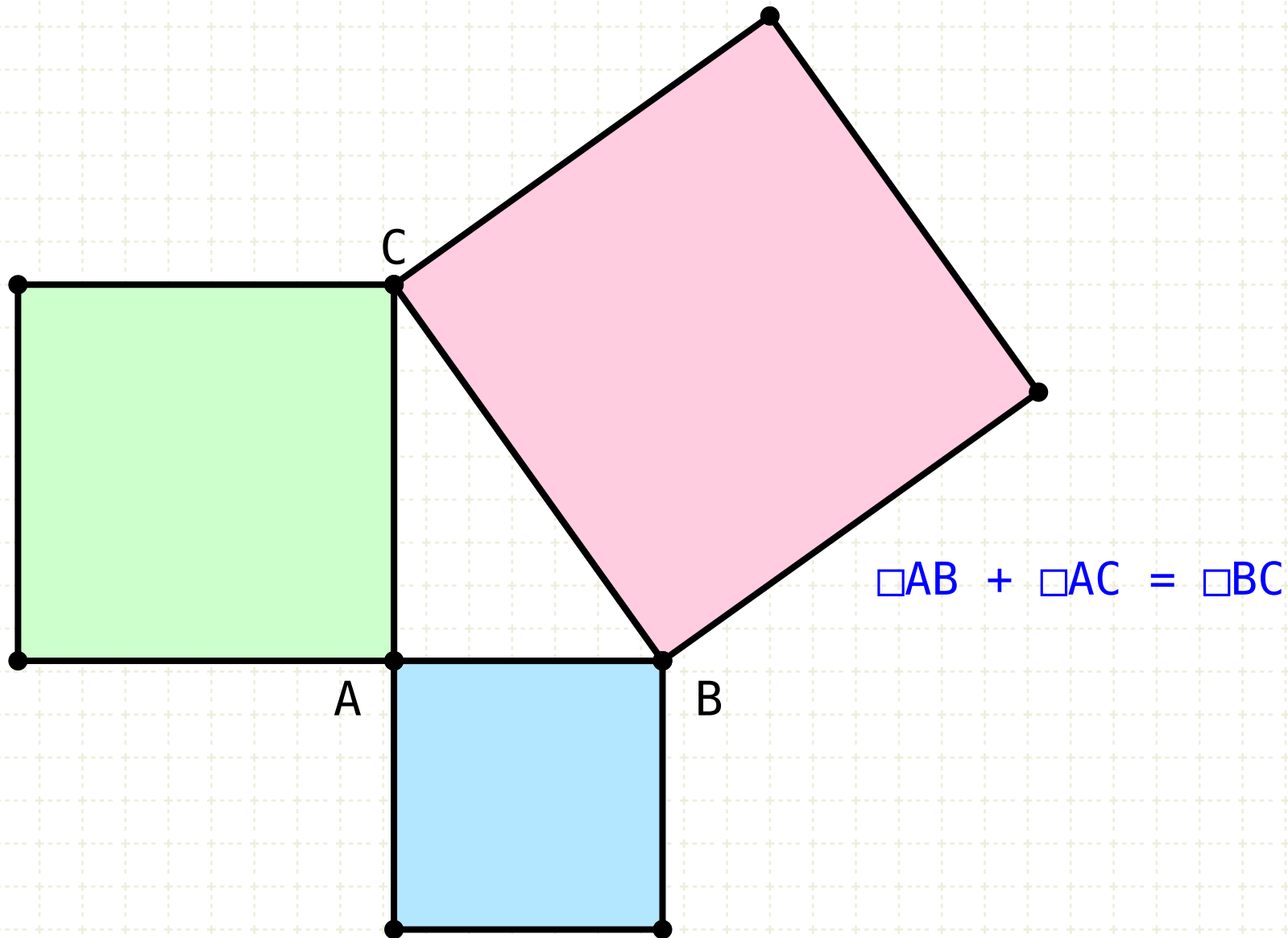
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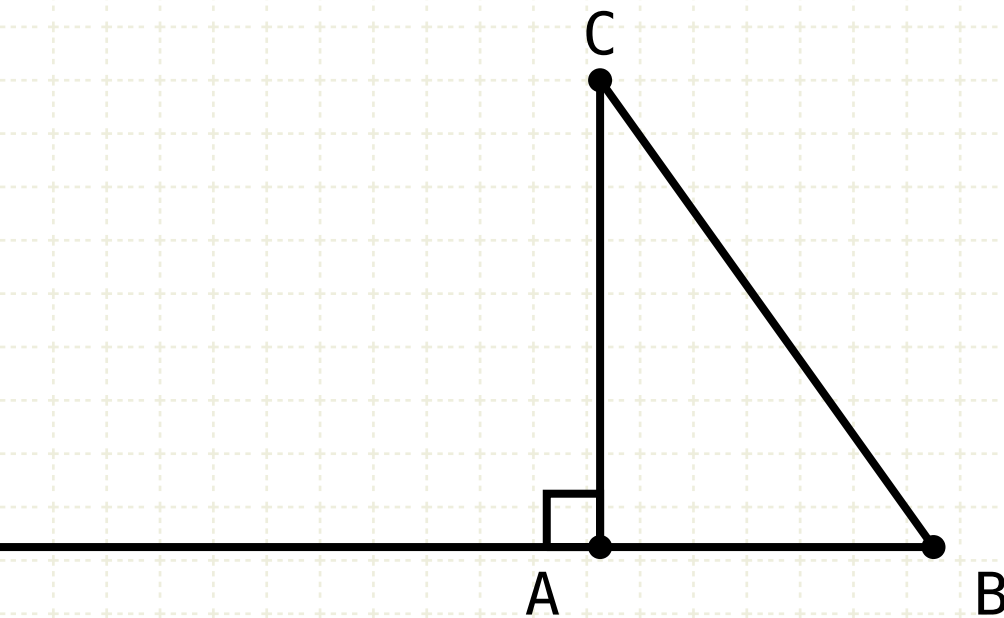
## In other words

Given a triangle ABC, where the square of AB and AC equals the square of BC

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## Proof:

Draw a line perpendicular to AC, from point A



$$AB^2 + AC^2 = BC^2$$

$$AC \perp AB$$



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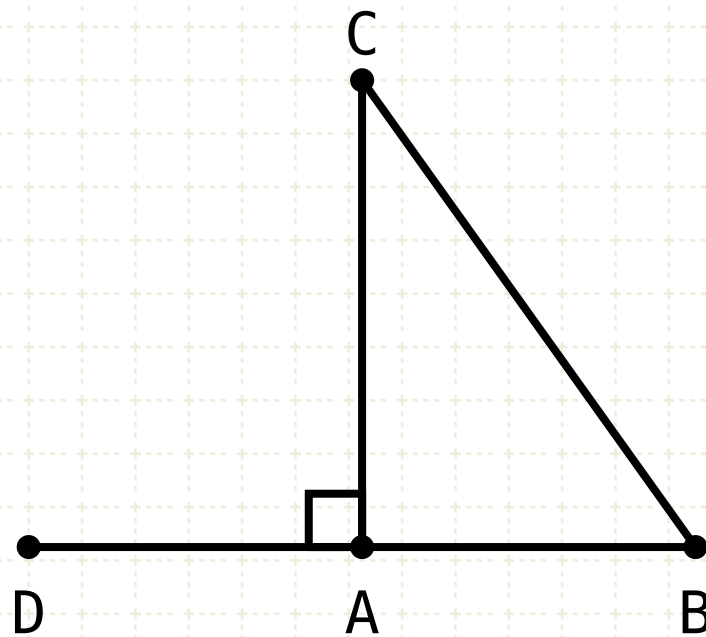
Given a triangle ABC, where the square of AB and AC equals the square of BC

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## Proof:

Draw a line perpendicular to AC, from point A

Define a point D such that AD equals AB



$$\square AB + \square AC = \square BC$$

$$AC \perp AD$$

$$AD = AB$$

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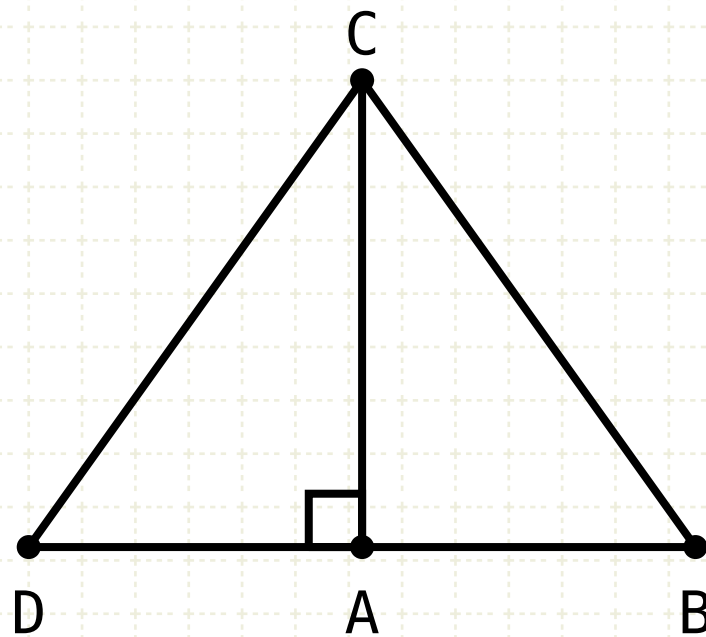
Then the angle CAB is a right angle

## Proof:

Draw a line perpendicular to AC, from point A

Define a point D such that AD equals AB

Draw line CD



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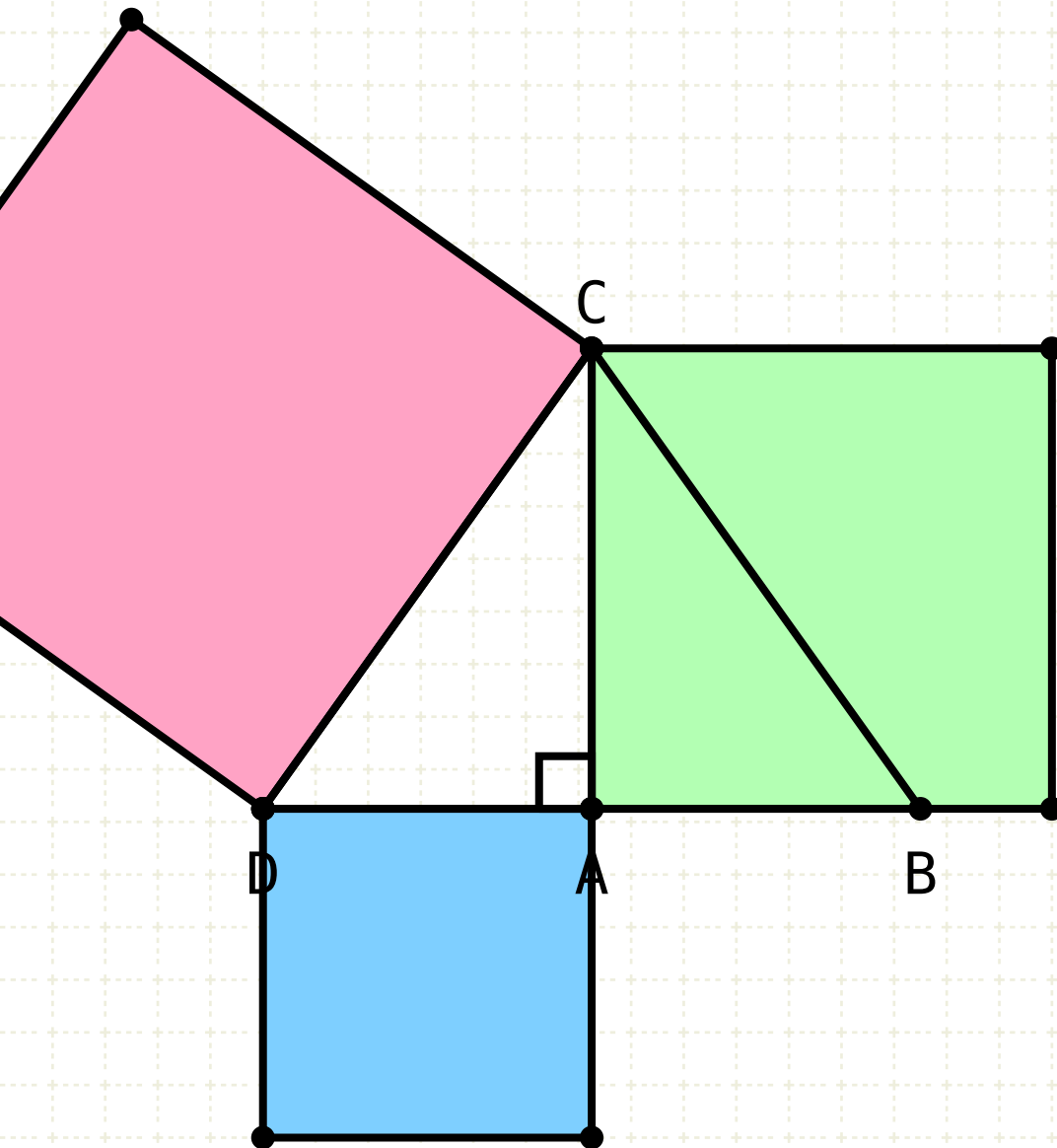
## Proof:

Draw a line perpendicular to AC, from point A

Define a point D such that AD equals AB

Draw line CD

Since the triangle CDA is a right angle triangle, the square of line CD equals the squares of AD and AC (I-47)



$$AB^2 + AC^2 = BC^2$$

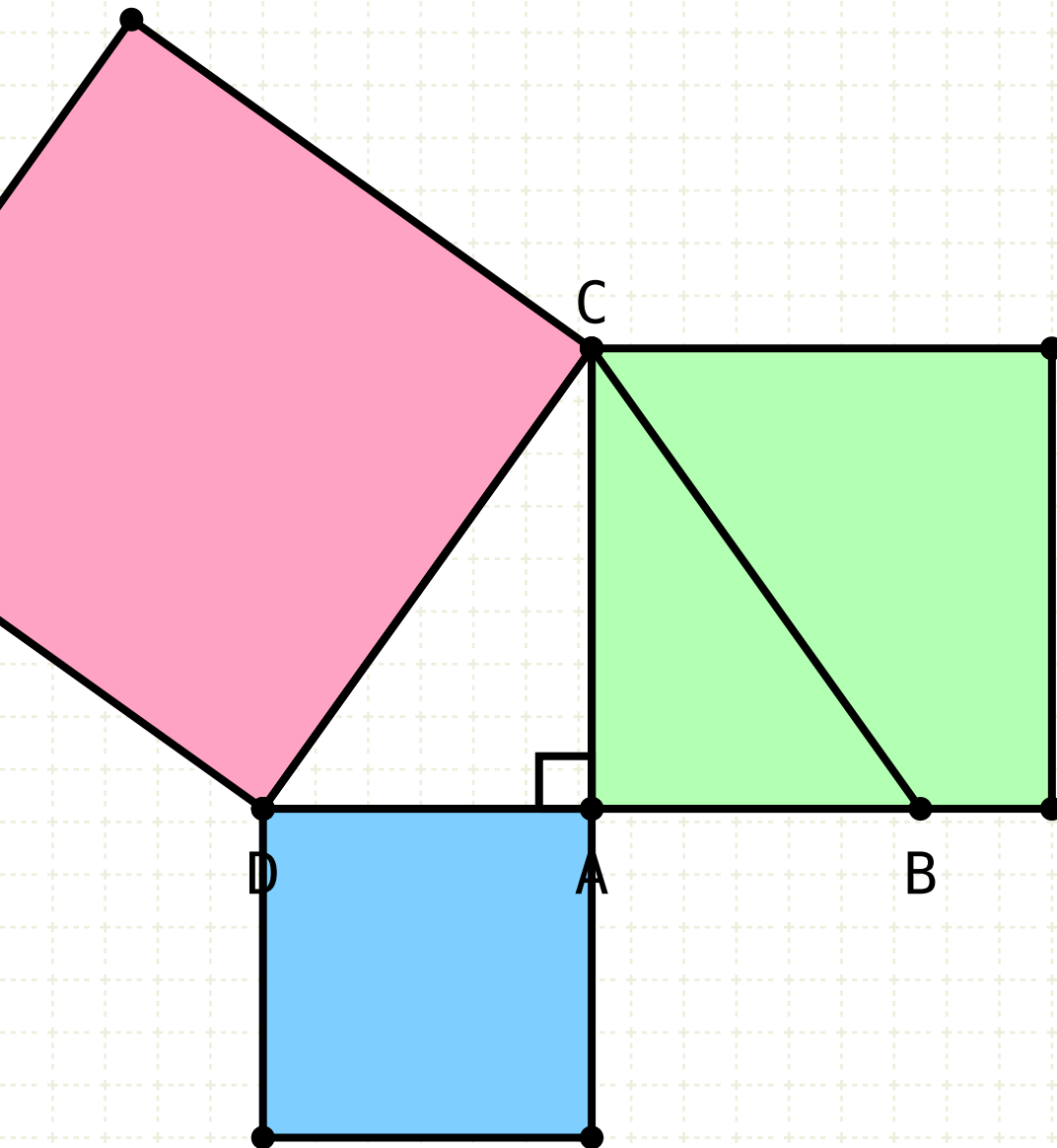
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$$AD^2 + AC^2 = CD^2$$

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If in a triangle the square on one of the sides equals the sum of the squares on the remaining two sides of the triangle, then the angle contained by the remaining two sides of the triangle is right.



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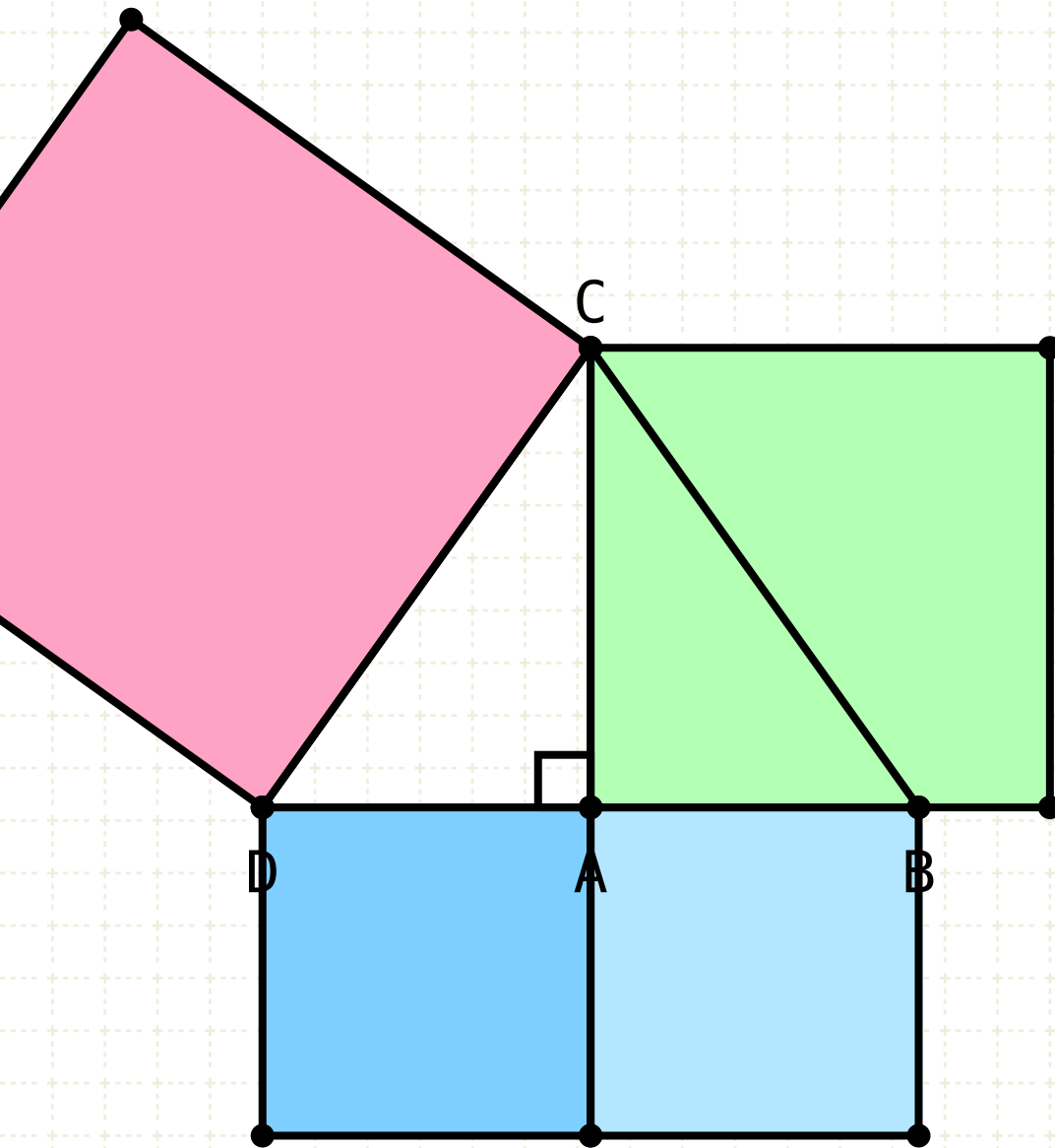
Draw line CD

Since the triangle CDA is a right angle triangle, the square of line CD equals the squares of AD and AC (I-47)

But since AD equals AB,

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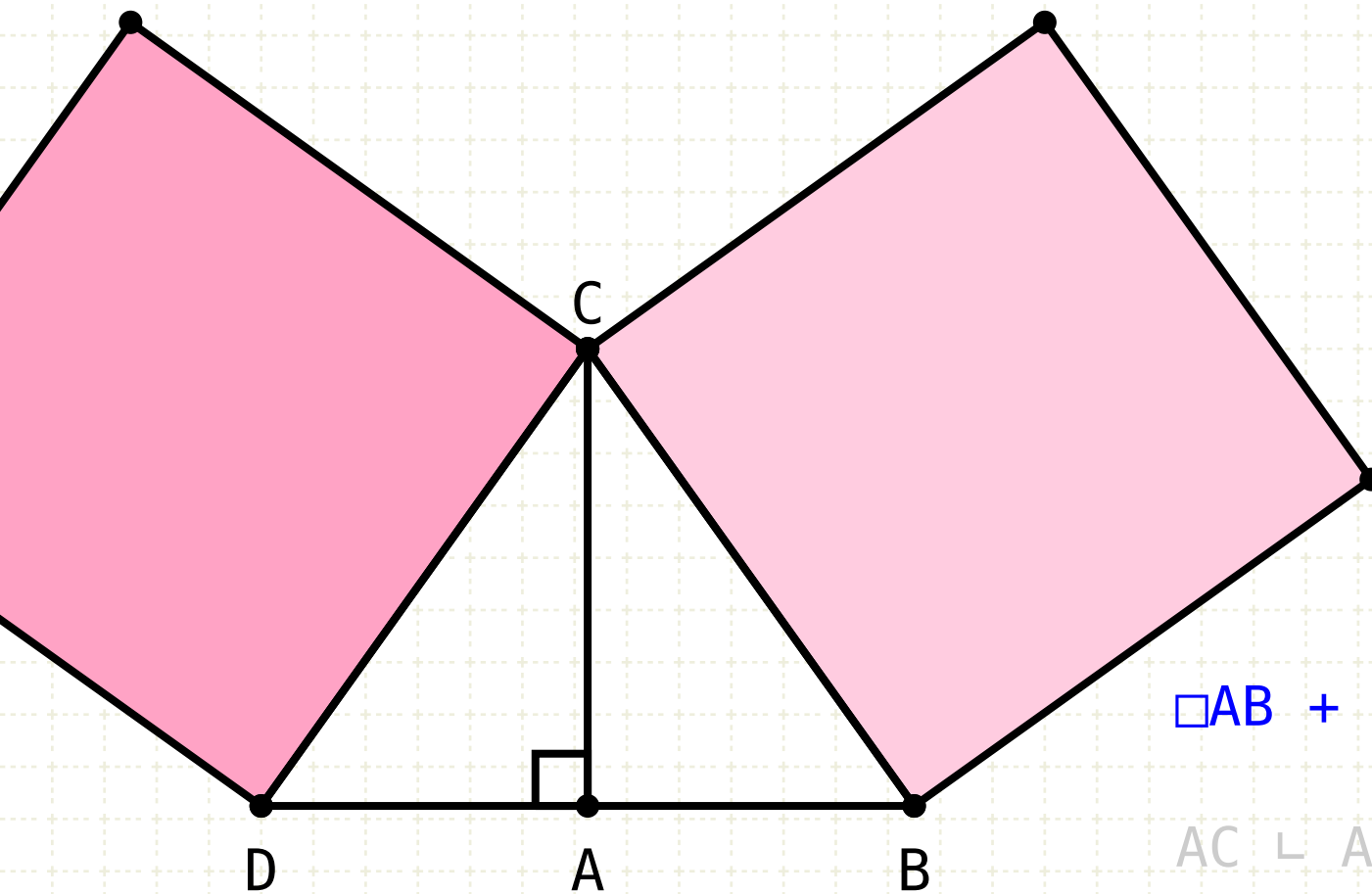
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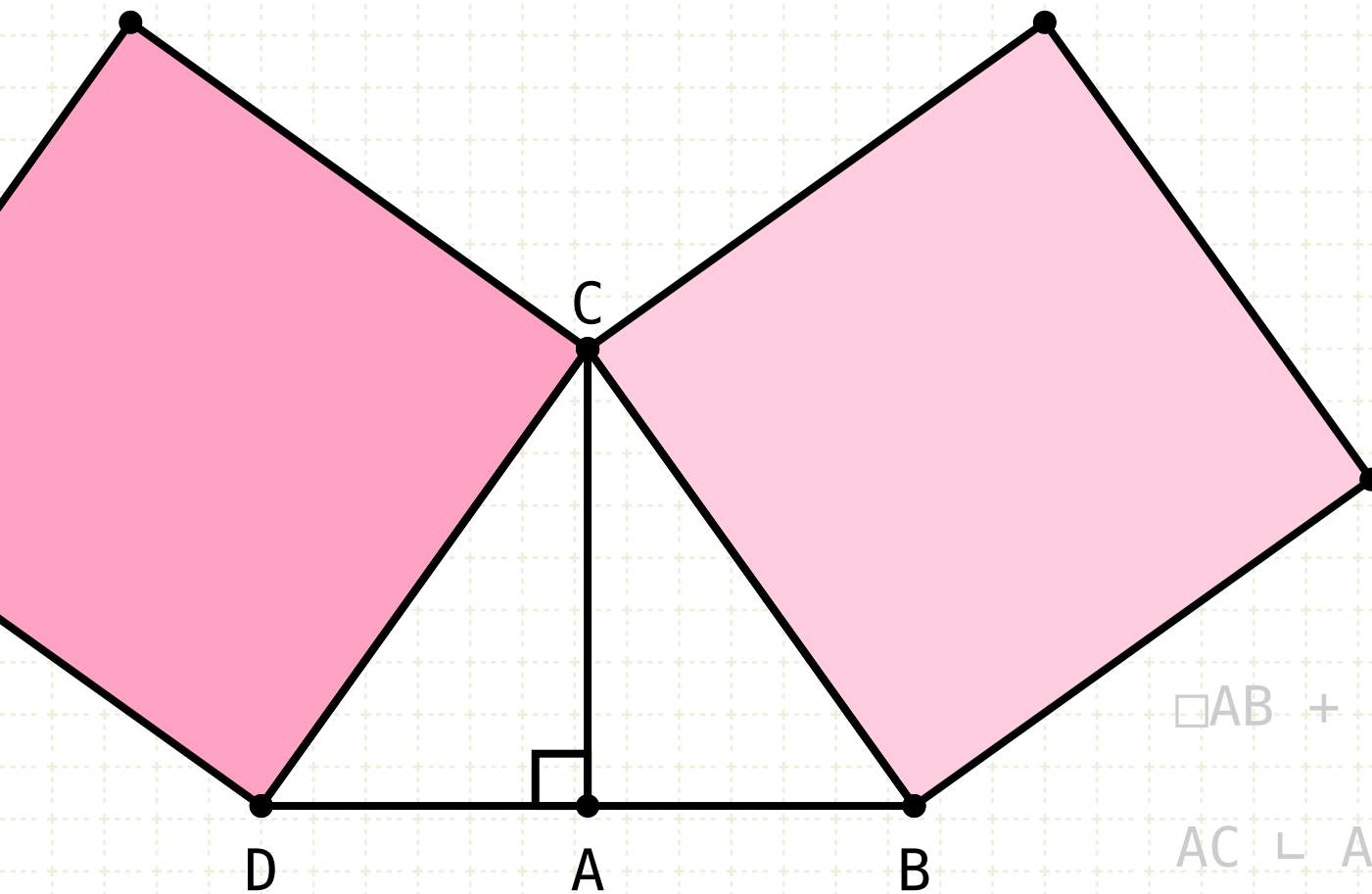
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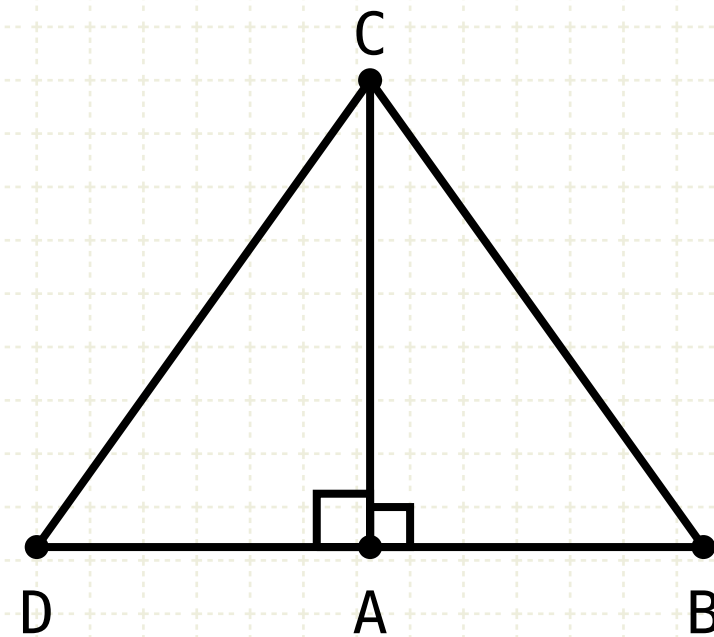
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Thus the square of CD equals the square of CB

If the squares are equal, so are the lines

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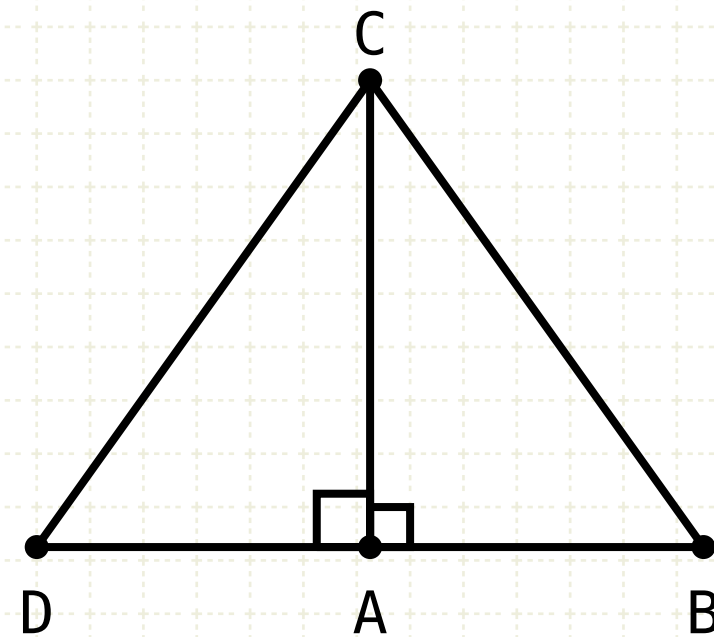
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Triangle ABC and ADC have three equal sides

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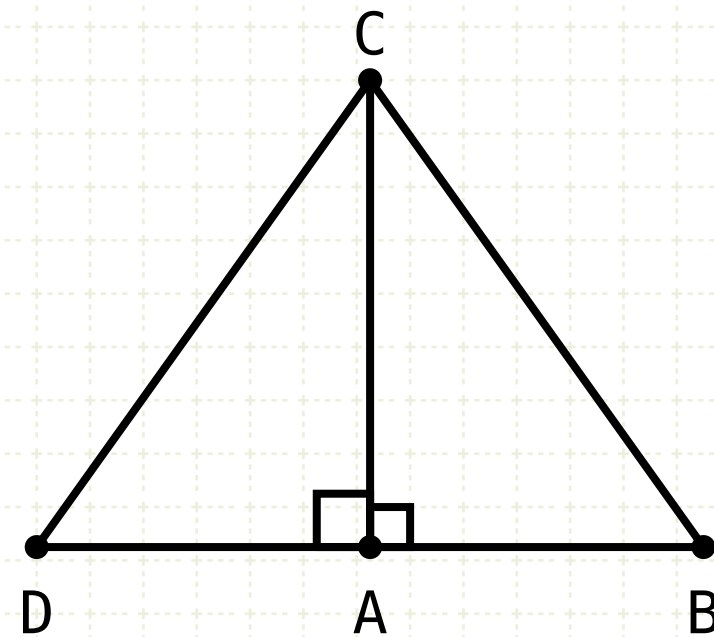
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