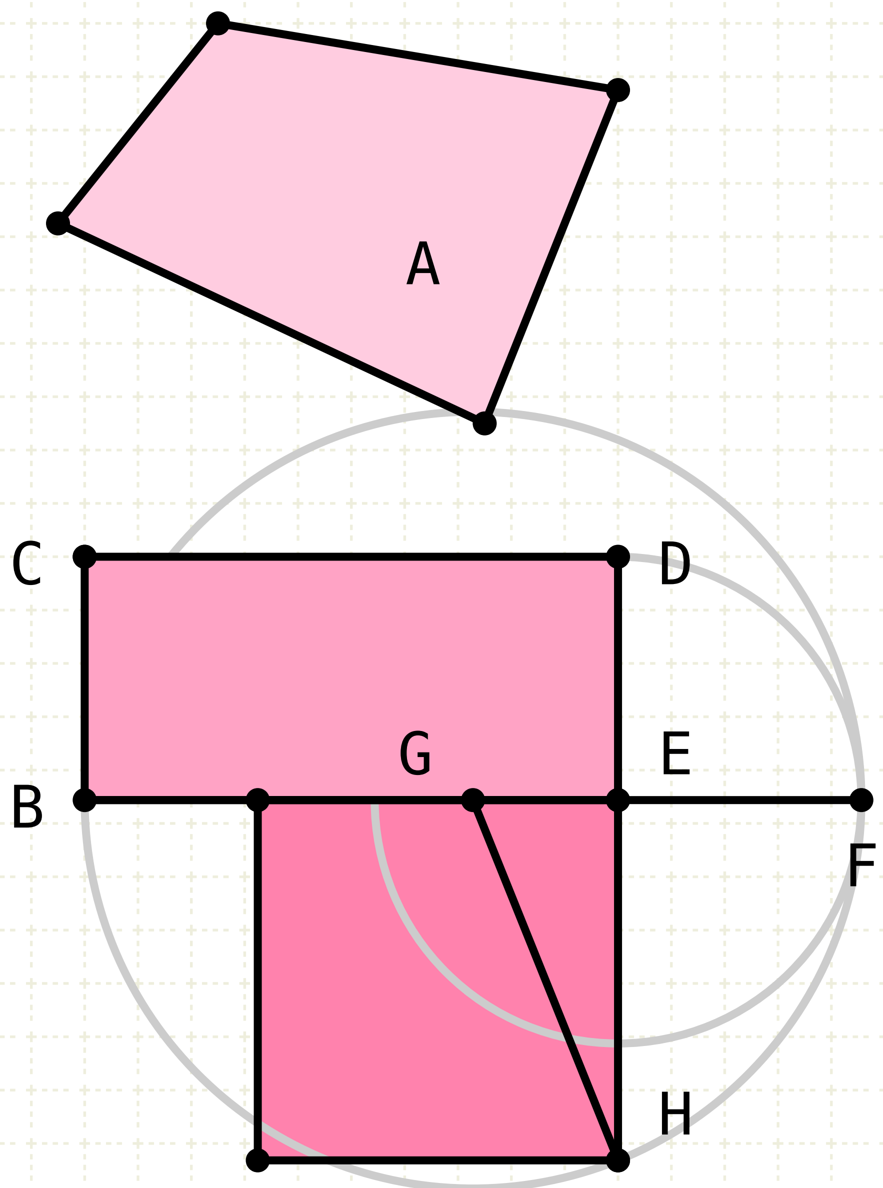
Euclid's Elements

Book II



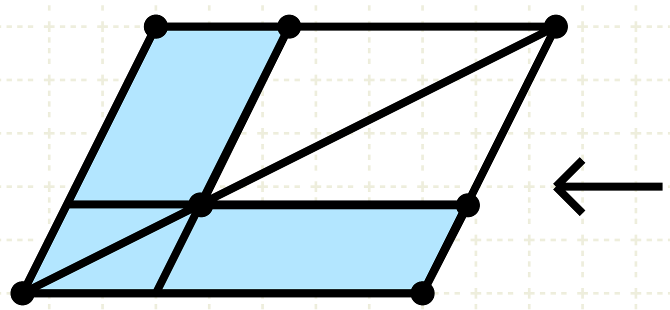
It is a remarkable fact in the history of geometry, that the Elements of Euclid, written two thousand years ago, are still regarded by many as the best introduction to the mathematical sciences.

Florian Cajori,

A History of Mathematics (1893)

Definitions:

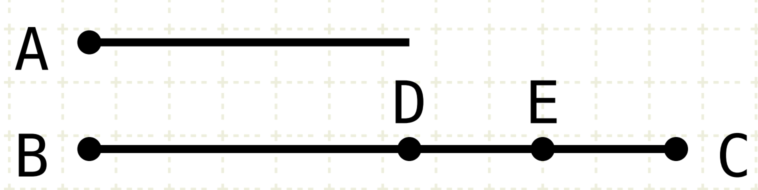
Any rectangular parallelogram is said to be contained by the two straight lines containing the right angle.



And in any parallelogrammic area let any one whatever of the parallelograms about its diameter with the two complements be called a gnomon.

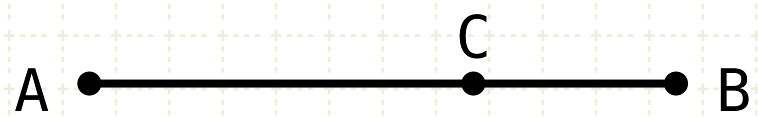
Table of Contents, Chapter 2

1



$A \cdot BC = A \cdot BD + A \cdot DE + A \cdot EC$

2

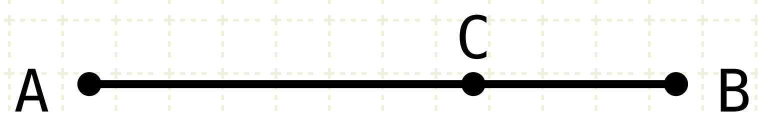


$AB^2 = AB \cdot AC + AB \cdot BC$

3

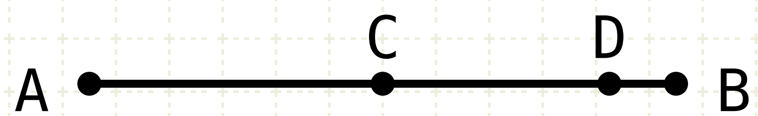
$AB \cdot CB = AC \cdot CB + CB^2$

4



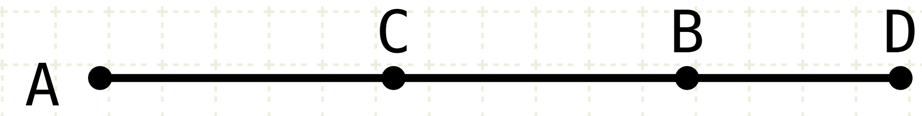
$AB^2 = AC^2 + CB^2 + 2 \cdot AC \cdot CB$

5



$AD \cdot DB + CD^2 = CB^2$

6



$AD \cdot DB + CB^2 = CD^2$

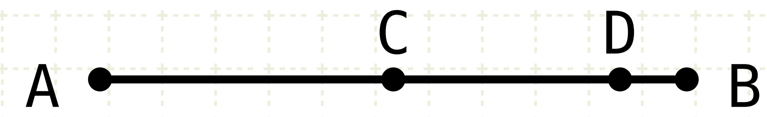
7

$AB^2 + BC^2 = AC^2 + 2 \cdot AB \cdot BC$

8

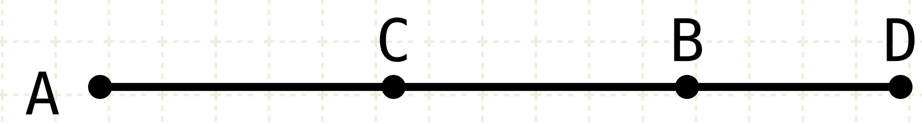
$4 \cdot AB \cdot BC + AC^2 = (AB+BC)^2$

9



$AD^2 + DB^2 = 2 \cdot (AC^2 + CD^2)$

10

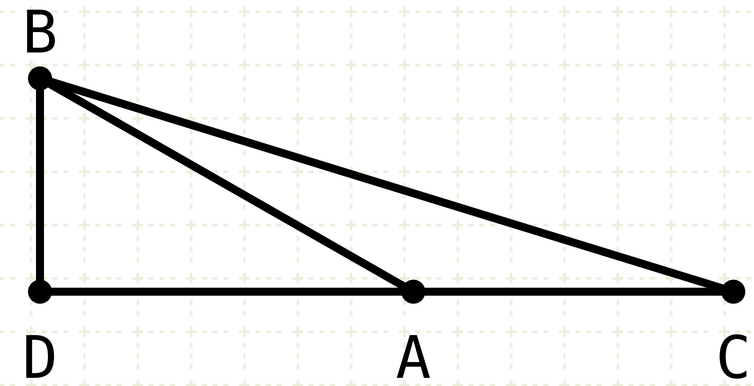


$AD^2 + DB^2 = 2 \cdot (AC^2 + CD^2)$

11

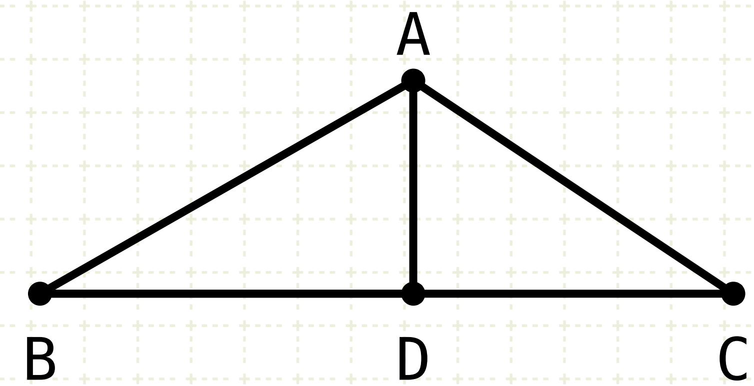
Find H. $AB \cdot BH = AH^2$

12



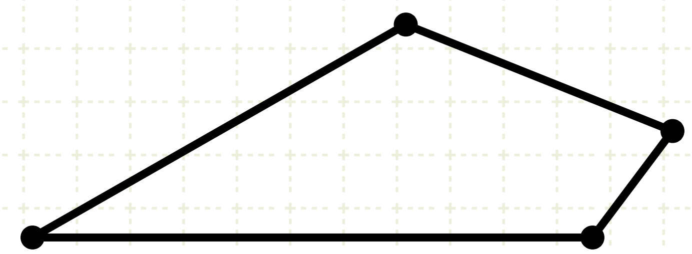
Cosine Law. $BC^2 = AB^2+AC^2+2 \cdot AD \cdot AC$

13



Cosine Law. $AC^2 = AB^2+BC^2-2 \cdot BD \cdot BC$

14



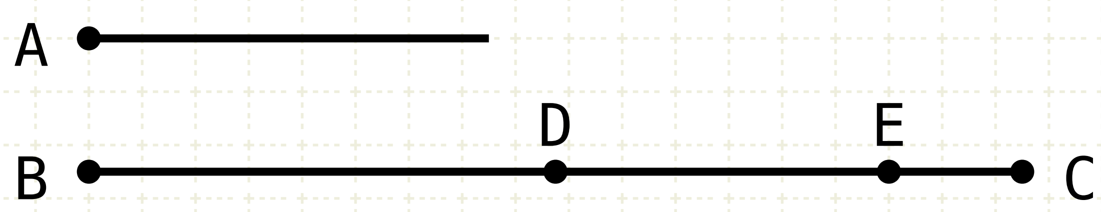
Find square of polygon

Proposition 1 of Book II

If there be two straight lines, and of of them be cut into any number of segments whatever, the rectangle contained by the two straight lines is equal to the rectangles contained by the uncut straight line and each of the segments.

Proposition 1 of Book II

If there be two straight lines, and of of them be cut into any number of segments whatever, the rectangle contained by the two straight lines is equal to the rectangles contained by the uncut straight line and each of the segments.



In other words

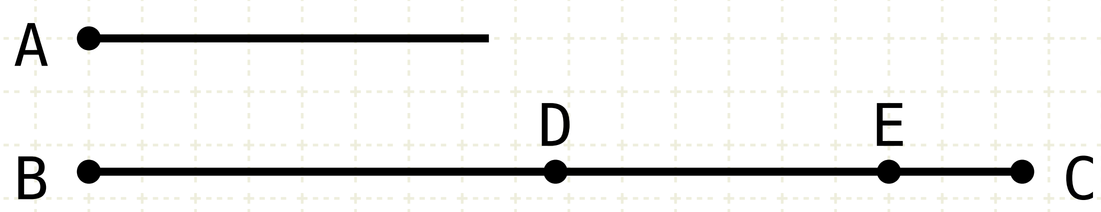
Let A and BC be two straight lines

Let BC be arbitrarily cut at points D and E

$BC = BD + DE + CE$

Proposition 1 of Book II

If there be two straight lines, and of of them be cut into any number of segments whatever, the rectangle contained by the two straight lines is equal to the rectangles contained by the uncut straight line and each of the segments.



In other words

Let A and BC be two straight lines

Let BC be arbitrarily cut at points D and E

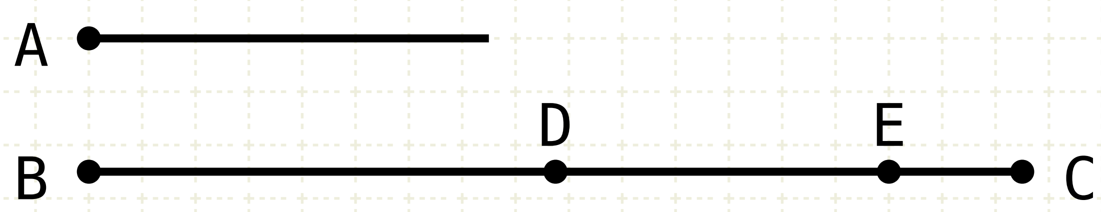
Then the area of the rectangle formed by line A and BC is equal in area to the sum of the rectangles formed by line A and BD, line A and DE, and line A and EC

$$BC = BD + DE + CE$$

$$A \cdot BC = A \cdot BD + A \cdot DE + A \cdot EC$$

Proposition 1 of Book II

If there be two straight lines, and of of them be cut into any number of segments whatever, the rectangle contained by the two straight lines is equal to the rectangles contained by the uncut straight line and each of the segments.



$BC = BD + DE + CE$

In other words

Let A and BC be two straight lines

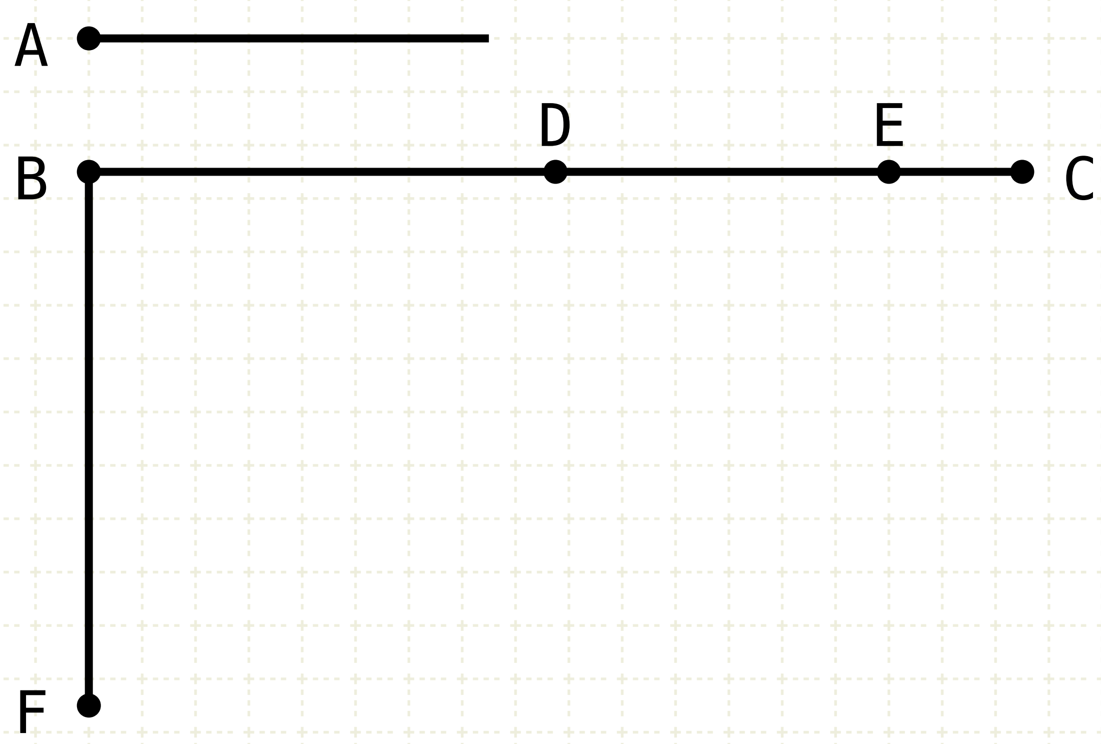
Let BC be arbitrarily cut at points D and E

Then the area of the rectangle formed by line A and BC is equal in area to the sum of the rectangles formed by line A and BD, line A and DE, and line A and EC

Proof:

Proposition 1 of Book II

If there be two straight lines, and of of them be cut into any number of segments whatever, the rectangle contained by the two straight lines is equal to the rectangles contained by the uncut straight line and each of the segments.



BC = BD + DE + CE

In other words

Let A and BC be two straight lines

Let BC be arbitrarily cut at points D and E

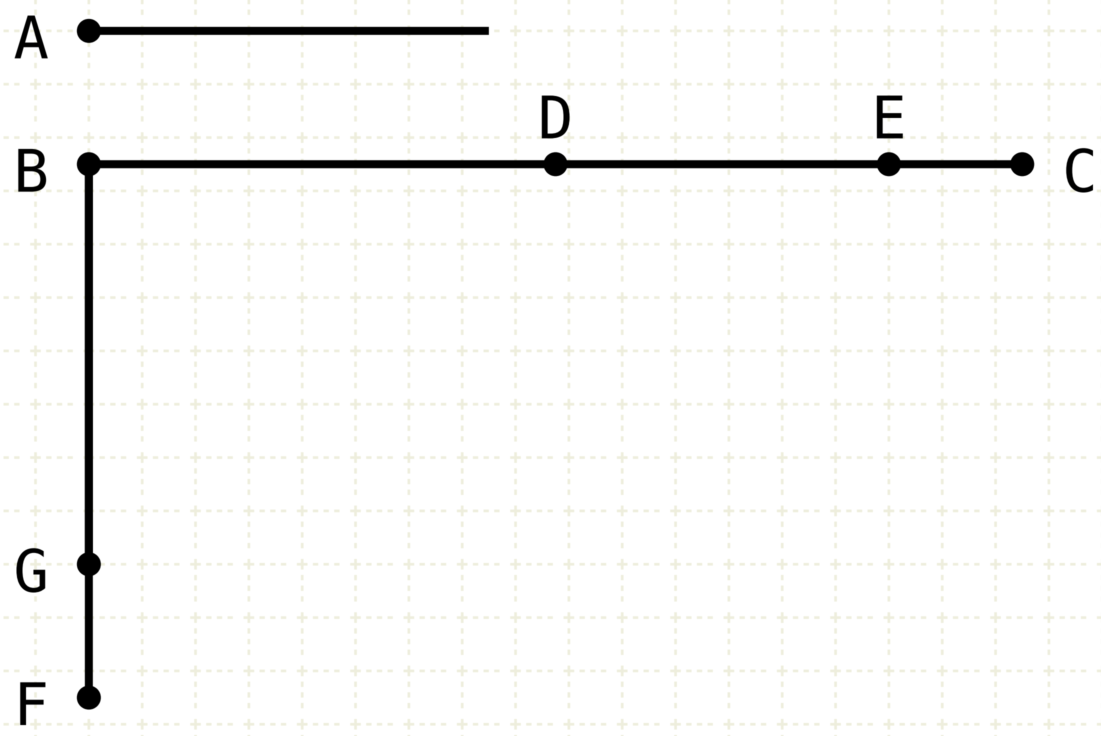
Then the area of the rectangle formed by line A and BC is equal in area to the sum of the rectangles formed by line A and BD, line A and DE, and line A and EC

Proof:

Draw a line BF perpendicular to BC (I·11)

Proposition 1 of Book II

If there be two straight lines, and of of them be cut into any number of segments whatever, the rectangle contained by the two straight lines is equal to the rectangles contained by the uncut straight line and each of the segments.



BC = BD + DE + CE

A = BG

In other words

Let A and BC be two straight lines

Let BC be arbitrarily cut at points D and E

Then the area of the rectangle formed by line A and BC is equal in area to the sum of the rectangles formed by line A and BD, line A and DE, and line A and EC

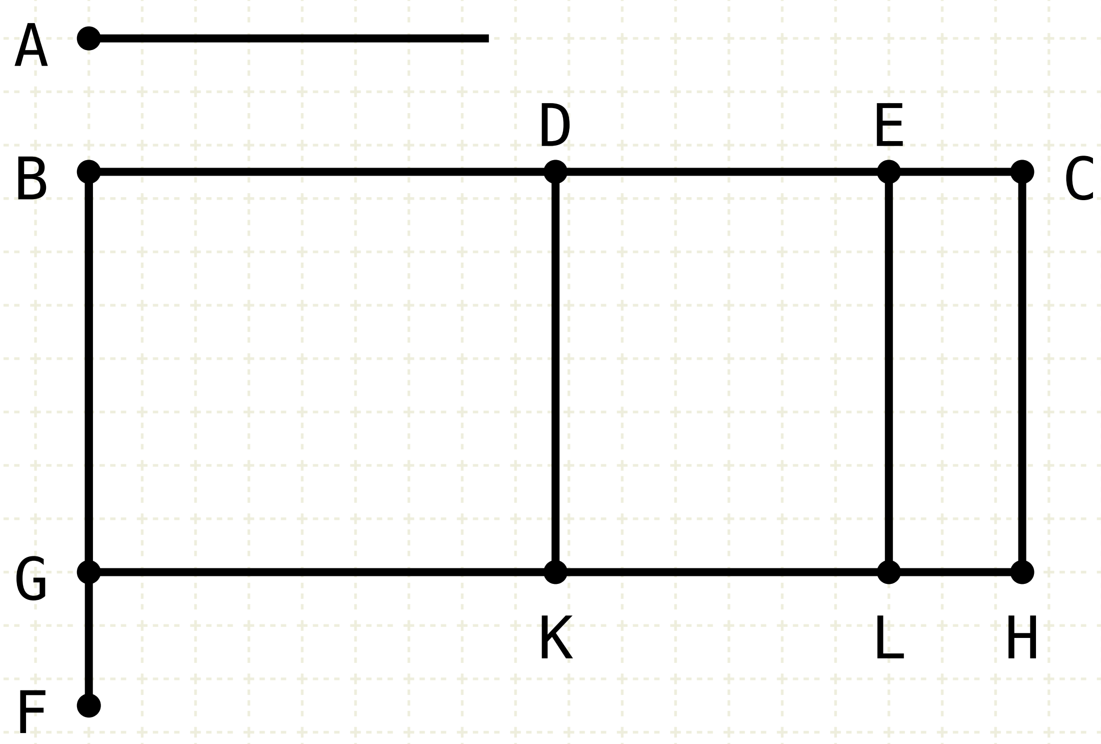
Proof:

Draw a line BF perpendicular to BC (I·11)

Define point G such that BG equals A (I·3)

Proposition 1 of Book II

If there be two straight lines, and of of them be cut into any number of segments whatever, the rectangle contained by the two straight lines is equal to the rectangles contained by the uncut straight line and each of the segments.



BC = BD + DE + CE

A = BG = DK = EL = CH

In other words

Let A and BC be two straight lines

Let BC be arbitrarily cut at points D and E

Then the area of the rectangle formed by line A and BC is equal in area to the sum of the rectangles formed by line A and BD, line A and DE, and line A and EC

Proof:

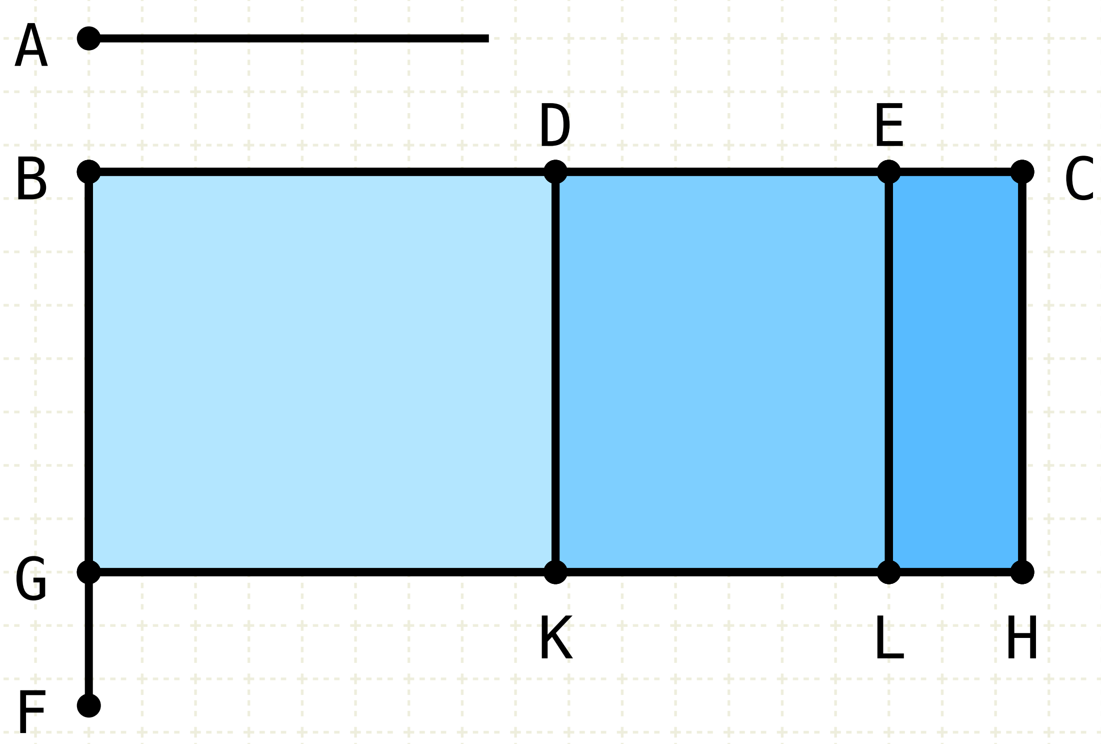
Draw a line BF perpendicular to BC (I·11)

Define point G such that BG equals A (I·3)

Draw GH parallel to BC, and DK, EL, and CH parallel to BG (I·31)

Proposition 1 of Book II

If there be two straight lines, and of of them be cut into any number of segments whatever, the rectangle contained by the two straight lines is equal to the rectangles contained by the uncut straight line and each of the segments.



$BC = BD + DE + CE$

$A = BG = DK = EL = CH$

$\square BH = \square BK + \square DL + \square EH$

In other words

Let A and BC be two straight lines

Let BC be arbitrarily cut at points D and E

Then the area of the rectangle formed by line A and BC is equal in area to the sum of the rectangles formed by line A and BD, line A and DE, and line A and EC

Proof:

Draw a line BF perpendicular to BC (I·11)

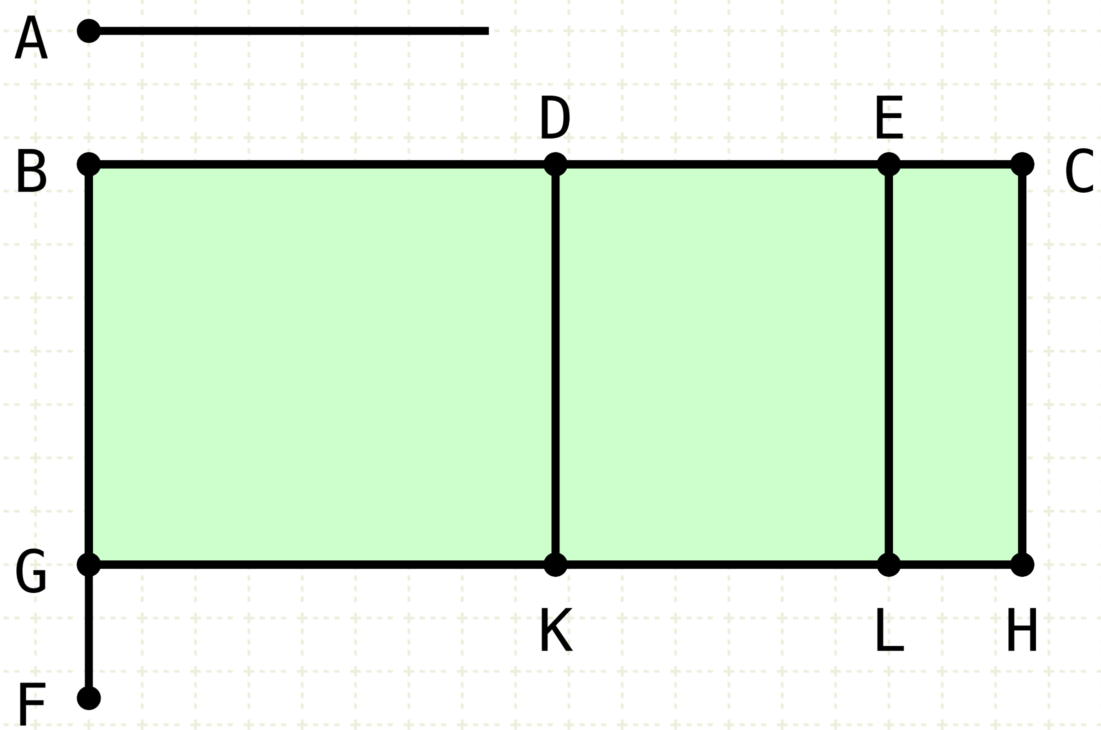
Define point G such that BG equals A (I·3)

Draw GH parallel to BC, and DK, EL, and CH parallel to BG (I·31)

The rectangle BH is the sum of the rectangles BK DL and EH

Proposition 1 of Book II

If there be two straight lines, and of of them be cut into any number of segments whatever, the rectangle contained by the two straight lines is equal to the rectangles contained by the uncut straight line and each of the segments.



$BC = BD + DE + CE$

$A = BG = DK = EL = CH$

$\square BH = \square BK + \square DL + \square EH$

$\square BH = BG \cdot BC, \quad \therefore \square BH = A \cdot BC$

In other words

Let A and BC be two straight lines

Let BC be arbitrarily cut at points D and E

Then the area of the rectangle formed by line A and BC is equal in area to the sum of the rectangles formed by line A and BD, line A and DE, and line A and EC

Proof:

Draw a line BF perpendicular to BC (I·11)

Define point G such that BG equals A (I·3)

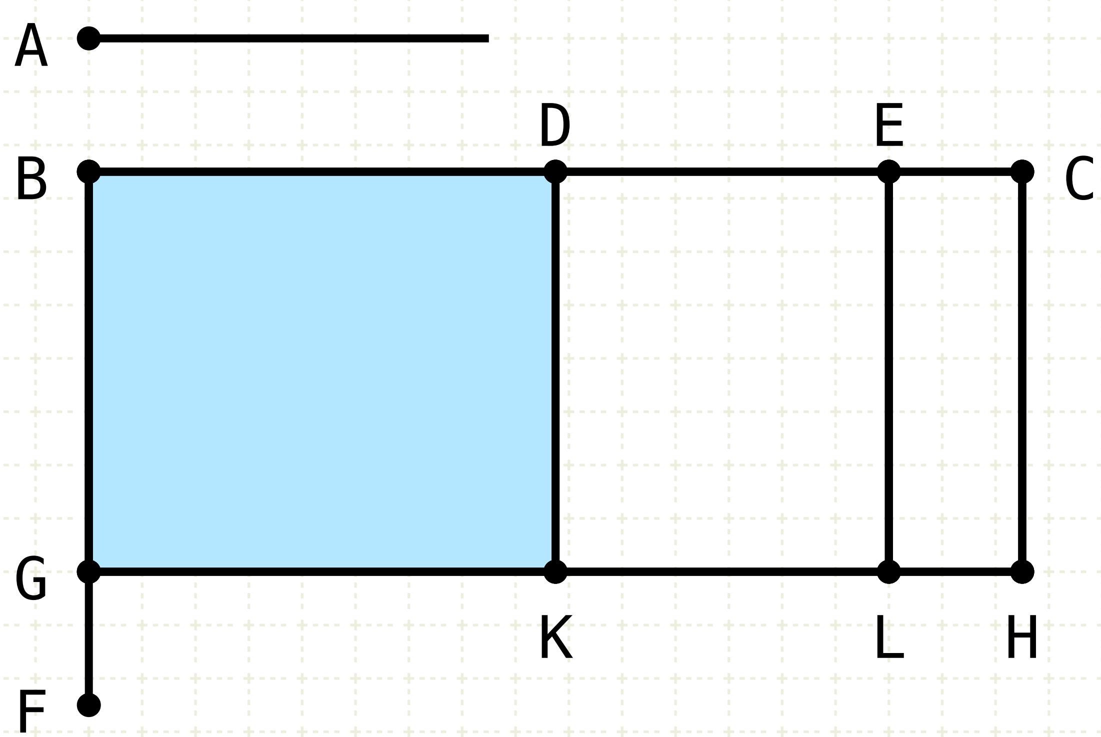
Draw GH parallel to BC, and DK, EL, and CH parallel to BG (I·31)

The rectangle BH is the sum of the rectangles BK DL and EH

Since BG is equal in length to A, the rectangle BH is equal to the rectangle contained by lines A and BC

Proposition 1 of Book II

If there be two straight lines, and of of them be cut into any number of segments whatever, the rectangle contained by the two straight lines is equal to the rectangles contained by the uncut straight line and each of the segments.



$BC = BD + DE + CE$

$A = BG = DK = EL = CH$

$\square BH = \square BK + \square DL + \square EH$

$\square BH = BG \cdot BC, \quad \therefore \square BH = A \cdot BC$

$\square BK = BG \cdot BD, \quad \therefore \square BK = A \cdot BD$

In other words

Let A and BC be two straight lines

Let BC be arbitrarily cut at points D and E

Then the area of the rectangle formed by line A and BC is equal in area to the sum of the rectangles formed by line A and BD, line A and DE, and line A and EC

Proof:

Draw a line BF perpendicular to BC (I·11)

Define point G such that BG equals A (I·3)

Draw GH parallel to BC, and DK, EL, and CH parallel to BG (I·31)

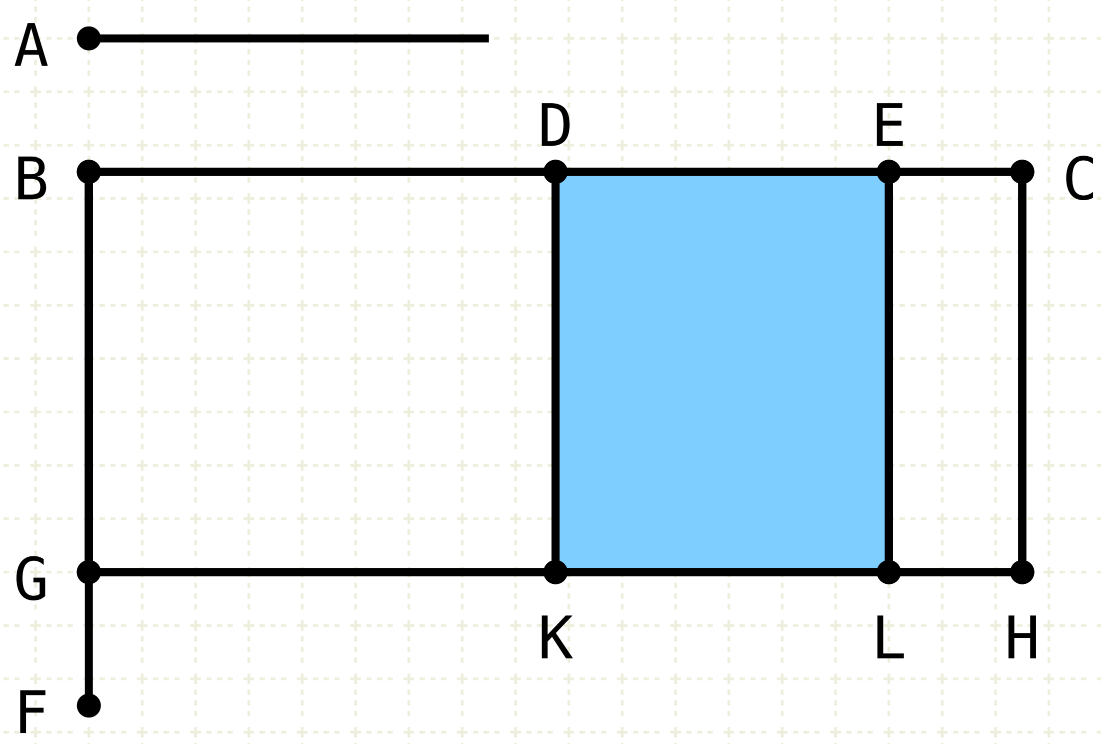
The rectangle BH is the sum of the rectangles BK DL and EH

Since BG is equal in length to A, the rectangle BH is equal to the rectangle contained by lines A and BC

Similarly, the rectangle BK is equal to the rectangle contained by lines A and BD

Proposition 1 of Book II

If there be two straight lines, and of of them be cut into any number of segments whatever, the rectangle contained by the two straight lines is equal to the rectangles contained by the uncut straight line and each of the segments.



$$BC = BD + DE + CE$$

$$A = BG = DK = EL = CH$$

$$\square BH = \square BK + \square DL + \square EH$$

$$\square BH = BG \cdot BC, \quad \therefore \square BH = A \cdot BC$$

$$\square BK = BG \cdot BD, \quad \therefore \square BK = A \cdot BD$$

$$\square DL = DK \cdot DE, \quad \therefore \square DL = A \cdot DE$$

In other words

Let A and BC be two straight lines

Let BC be arbitrarily cut at points D and E

Then the area of the rectangle formed by line A and BC is equal in area to the sum of the rectangles formed by line A and BD, line A and DE, and line A and EC

Proof:

Draw a line BF perpendicular to BC (I·11)

Define point G such that BG equals A (I·3)

Draw GH parallel to BC, and DK, EL, and CH parallel to BG (I·31)

The rectangle BH is the sum of the rectangles BK DL and EH

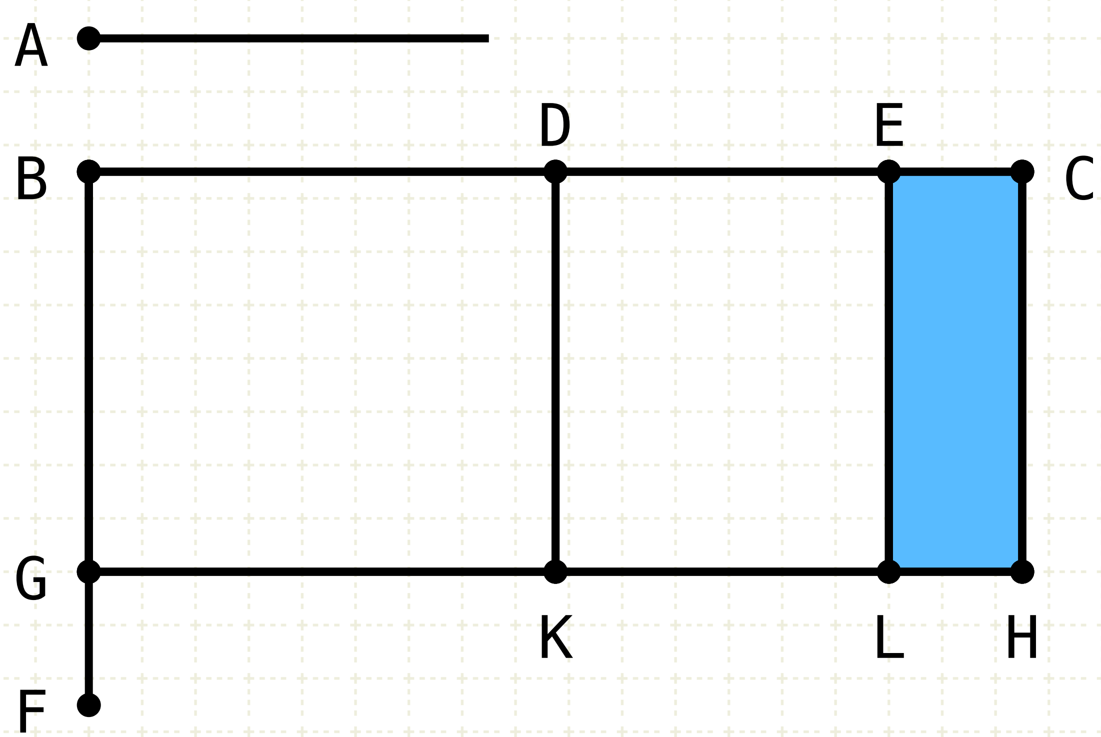
Since BG is equal in length to A, the rectangle BH is equal to the rectangle contained by lines A and BC

Similarly, the rectangle BK is equal to the rectangle contained by lines A and BD

Since BG equals DK (I·34), DL is equal to the rectangle contained by lines A and DE

Proposition 1 of Book II

If there be two straight lines, and of of them be cut into any number of segments whatever, the rectangle contained by the two straight lines is equal to the rectangles contained by the uncut straight line and each of the segments.



$BC = BD + DE + CE$

$A = BG = DK = EL = CH$

$\square BH = \square BK + \square DL + \square EH$

$\square BH = BG \cdot BC, \quad \therefore \square BH = A \cdot BC$

$\square BK = BG \cdot BD, \quad \therefore \square BK = A \cdot BD$

$\square DL = DK \cdot DE, \quad \therefore \square DL = A \cdot DE$

$\square EH = EL \cdot EC, \quad \therefore \square EH = A \cdot EC$

In other words

Let A and BC be two straight lines

Let BC be arbitrarily cut at points D and E

Then the area of the rectangle formed by line A and BC is equal in area to the sum of the rectangles formed by line A and BD, line A and DE, and line A and EC

Proof:

Draw a line BF perpendicular to BC (I·11)

Define point G such that BG equals A (I·3)

Draw GH parallel to BC, and DK, EL, and CH parallel to BG (I·31)

The rectangle BH is the sum of the rectangles BK DL and EH

Since BG is equal in length to A, the rectangle BH is equal to the rectangle contained by lines A and BC

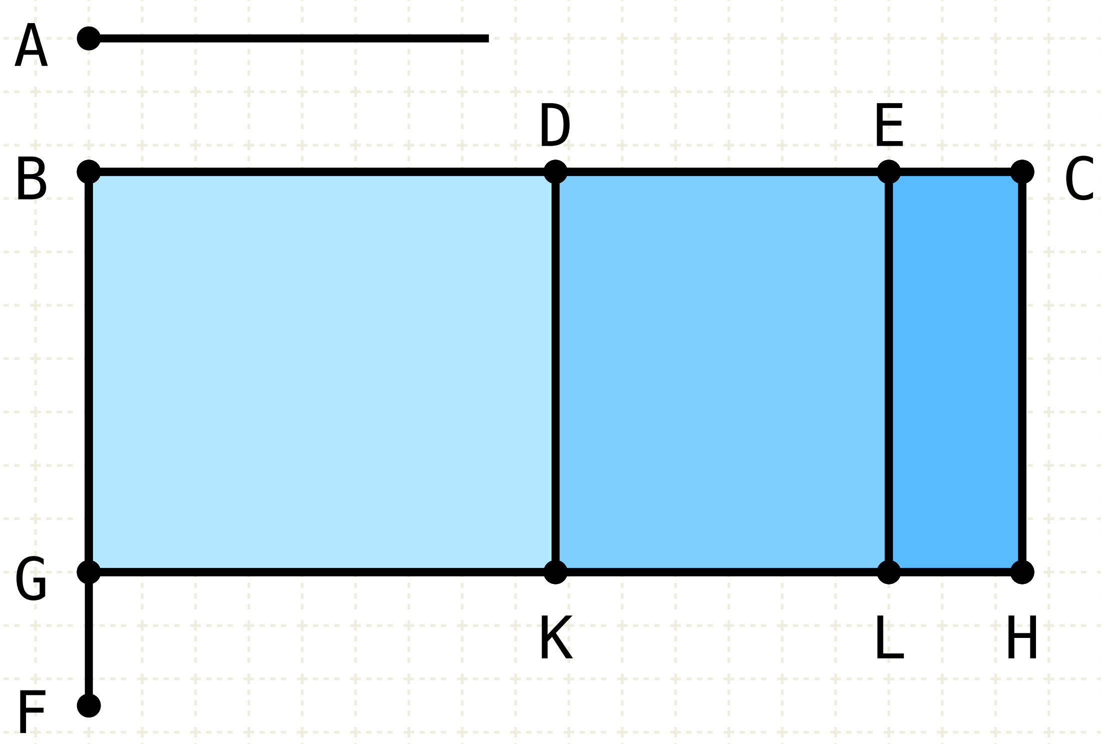
Similarly, the rectangle BK is equal to the rectangle contained by lines A and BD

Since BG equals DK (I·34), DL is equal to the rectangle contained by lines A and DE

And finally, EH is equal to the rectangle contained by lines A and EC

Proposition 1 of Book II

If there be two straight lines, and of of them be cut into any number of segments whatever, the rectangle contained by the two straight lines is equal to the rectangles contained by the uncut straight line and each of the segments.



$BC = BD + DE + CE$

$A = BG = DK = EL = CH$

$\square BH = \square BK + \square DL + \square EH$

$\square BH = BG \cdot BC, \quad \therefore \square BH = A \cdot BC$

$\square BK = BG \cdot BD, \quad \therefore \square BK = A \cdot BD$

$\square DL = DK \cdot DE, \quad \therefore \square DL = A \cdot DE$

$\square EH = EL \cdot EC, \quad \therefore \square EH = A \cdot EC$

$A \cdot BC = A \cdot BD + A \cdot DE + A \cdot EC$

In other words

Let A and BC be two straight lines

Let BC be arbitrarily cut at points D and E

Then the area of the rectangle formed by line A and BC is equal in area to the sum of the rectangles formed by line A and BD, line A and DE, and line A and EC

Proof:

Draw a line BF perpendicular to BC (I·11)

Define point G such that BG equals A (I·3)

Draw GH parallel to BC, and DK, EL, and CH parallel to BG (I·31)

The rectangle BH is the sum of the rectangles BK DL and EH

Since BG is equal in length to A, the rectangle BH is equal to the rectangle contained by lines A and BC

Similarly, the rectangle BK is equal to the rectangle contained by lines A and BD

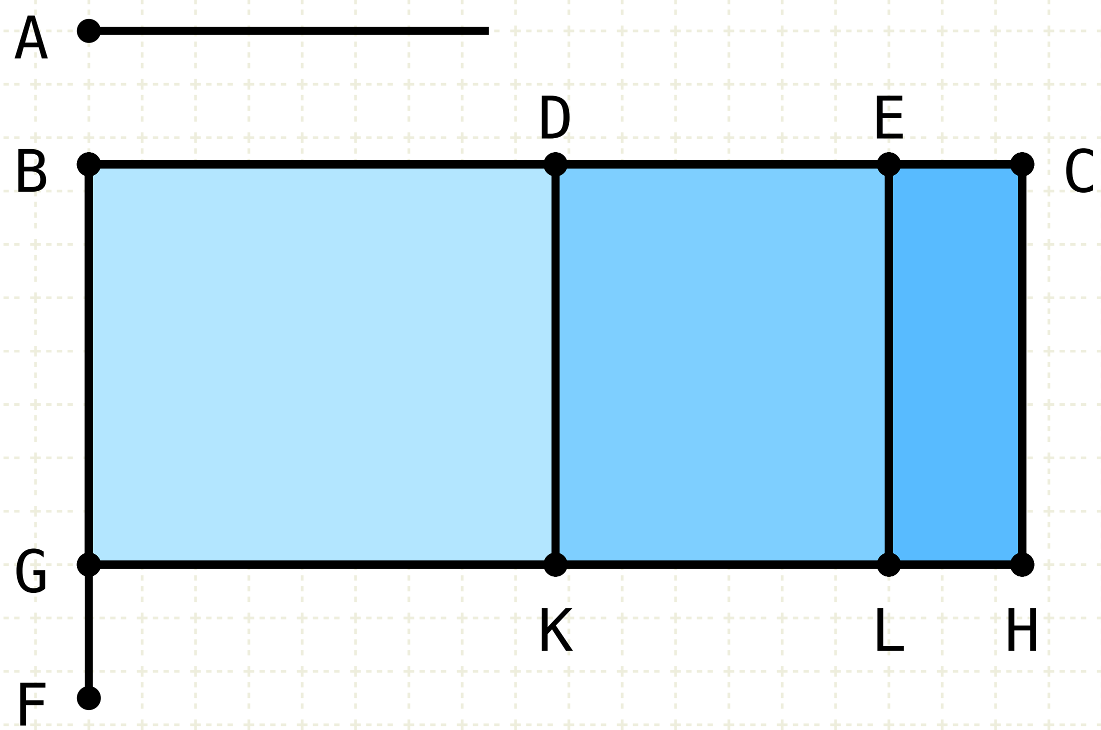
Since BG equals DK (I·34), DL is equal to the rectangle contained by lines A and DE

And finally, EH is equal to the rectangle contained by lines A and EC

Thus the rectangle formed by A,BC is equal to the sum of the rectangles formed by A,BD, A,DE and A,EC

Proposition 1 of Book II

If there be two straight lines, and of of them be cut into any number of segments whatever, the rectangle contained by the two straight lines is equal to the rectangles contained by the uncut straight line and each of the segments.



$BC = BD + DE + CE$

$A = BG = DK = EL = CH$

$\square BH = \square BK + \square DL + \square EH$

$\square BH = BG \cdot BC, \quad \therefore \square BH = A \cdot BC$

$\square BK = BG \cdot BD, \quad \therefore \square BK = A \cdot BD$

$\square DL = DK \cdot DE, \quad \therefore \square DL = A \cdot DE$

$\square EH = EL \cdot EC, \quad \therefore \square EH = A \cdot EC$

$A \cdot BC = A \cdot BD + A \cdot DE + A \cdot EC$

In other words

Let A and BC be two straight lines

Let BC be arbitrarily cut at points D and E

Then the area of the rectangle formed by line A and BC is equal in area to the sum of the rectangles formed by line A and BD, line A and DE, and line A and EC

Proof:

Draw a line BF perpendicular to BC (I·11)

Define point G such that BG equals A (I·3)

Draw GH parallel to BC, and DK, EL, and CH parallel to BG (I·31)

The rectangle BH is the sum of the rectangles BK DL and EH

Since BG is equal in length to A, the rectangle BH is equal to the rectangle contained by lines A and BC

Similarly, the rectangle BK is equal to the rectangle contained by lines A and BD

Since BG equals DK (I·34), DL is equal to the rectangle contained by lines A and DE

And finally, EH is equal to the rectangle contained by lines A and EC

Thus the rectangle formed by A,BC is equal to the sum of the rectangles formed by A,BD, A,DE and A,EC

Youtube Videos

https://www.youtube.com/c/SandyBultena