

Euclid's Elements

Book VI

One can state, without exaggeration, that the observation of and the search for similarities and differences are the basis of all human knowledge.

Alfred Nobel



Table of Contents, Chapter 6

1	If the height of two triangles are equal, then the ratio of the areas is equal to the ratio of the bases	7	If two triangles have one angle equal to one angle, and the sides about other angles are proportional, and the remaining angles either both less or both not less than a right angle, then triangles will be equiangular	14	In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and vice versa
2	If a line cuts a triangle, parallel to its base, it will cut the sides of the triangle proportionally			15	In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and vice versa
3	If an angle of a triangle is bisected and the straight line cutting the angle also cuts the base, the segments of the base will have the same ratio as the remaining sides of the triangle	8	If in a right-angled triangle a perpendicular be drawn from the right angle to the base, the triangles adjoining the perpendicular are similar both to the whole and to one another	16	If four straight lines are proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means, and vice versa
4	If two triangles have equal angles, then the sides opposite the equal angles are proportional, as well, the sides of the triangles on either side of the equal angles are also proportional	9	From a given straight line to cut off a given fraction	17	If three straight lines are proportional, the rectangle contained by the extremes is equal to the square on the mean; and vice versa
5	If two triangles have proportional sides, the triangles will be equiangular	10	To cut a given uncut straight line similarly to a given cut straight line	18	On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure
6	If two triangles have one angle equal to one angle and the sides about the equal angles are proportional, then the triangles will be equiangular	11	To two given straight lines to find a third proportional	19	Similar triangles are to one another in the duplicate ratio of the corresponding sides
		12	To three given straight lines to find a fourth proportional		
		13	To two given straight lines to find a mean proportional		



Table of Contents, Chapter 3

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|----|--|----|--|----|---|
| 20 | Similar polygons are divided into the same number of similar triangles, which have the same ratio as the wholes, and the polygons have duplicate ratios to their corresponding sides | 26 | If from a parallelogram a similar parallelogram with a common angle is subtracted, it is about the same diameter as the original | 31 | In right-angled triangles the figure on the side subtending the right angle is equal to the similar and similarly described figures on the sides containing the right angle |
| 21 | Figures which are similar to the same rectilineal figure are also similar to one another | 27 | Of all the parallelograms applied to the same straight line and deficient by parallelogrammic figures similar to a parallelogram drawn on half the said line, the largest will be one that is drawn on half of the straight line and is similar to the defect | | |
| 22 | If four straight lines are proportional, similar rectilineal figures will also be proportional; and vice versa | 28 | To a given straight line, apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one | | |
| 23 | Equiangular parallelograms have to one another the ratio compounded of the ratios of their sides | 29 | To a given straight line, apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one | | |
| 24 | In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another | 30 | To cut a finite straight line in extreme ratio | | |
| 25 | To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure | | | | |



Proposition 27 of Book VI

Of all the parallelograms applied to the same straight line and deficient by parallelogrammic figures similar and similarly situated to that described on the half of the straight line, that parallelogram is greatest which is applied to the half of the straight line and is similar to the defect



Proposition 27 of Book VI

Of all the parallelograms applied to the same straight line and deficient by parallelogrammic figures similar and similarly situated to that described on the half of the straight line, that parallelogram is greatest which is applied to the half of the straight line and is similar to the defect

In other words

Given a straight line AB

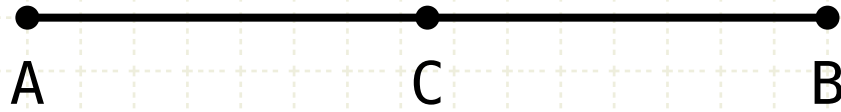


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Of all the parallelograms applied to the same straight line and deficient by parallelogrammic figures similar and similarly situated to that described on the half of the straight line, that parallelogram is greatest which is applied to the half of the straight line and is similar to the defect

In other words

Given a straight line AB
and a midpoint C

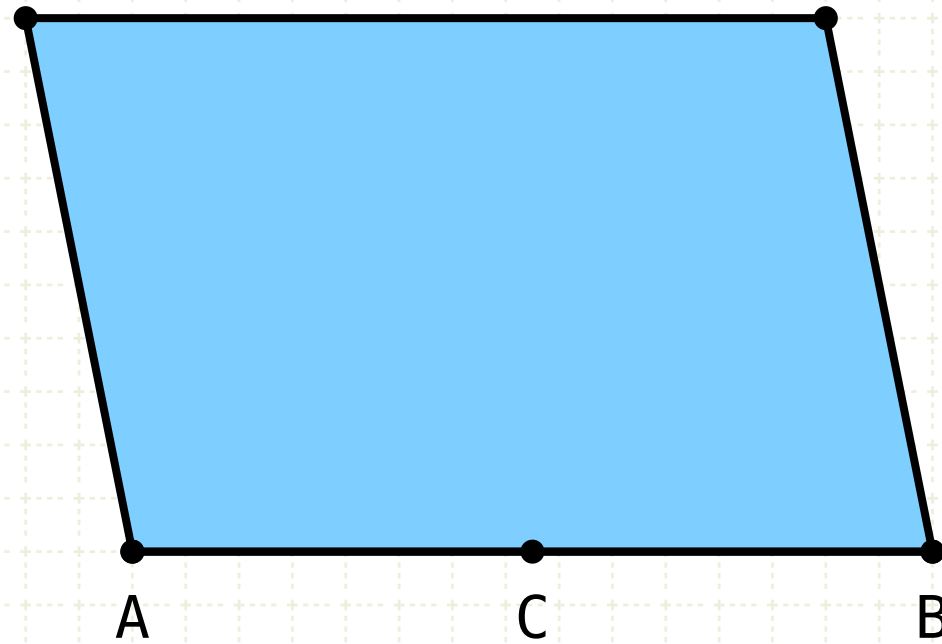


$$AC = \frac{1}{2} AB$$



Proposition 27 of Book VI

Of all the parallelograms applied to the same straight line and deficient by parallelogrammic figures similar and similarly situated to that described on the half of the straight line, that parallelogram is greatest which is applied to the half of the straight line and is similar to the defect



$$AC = \frac{1}{2} AB$$

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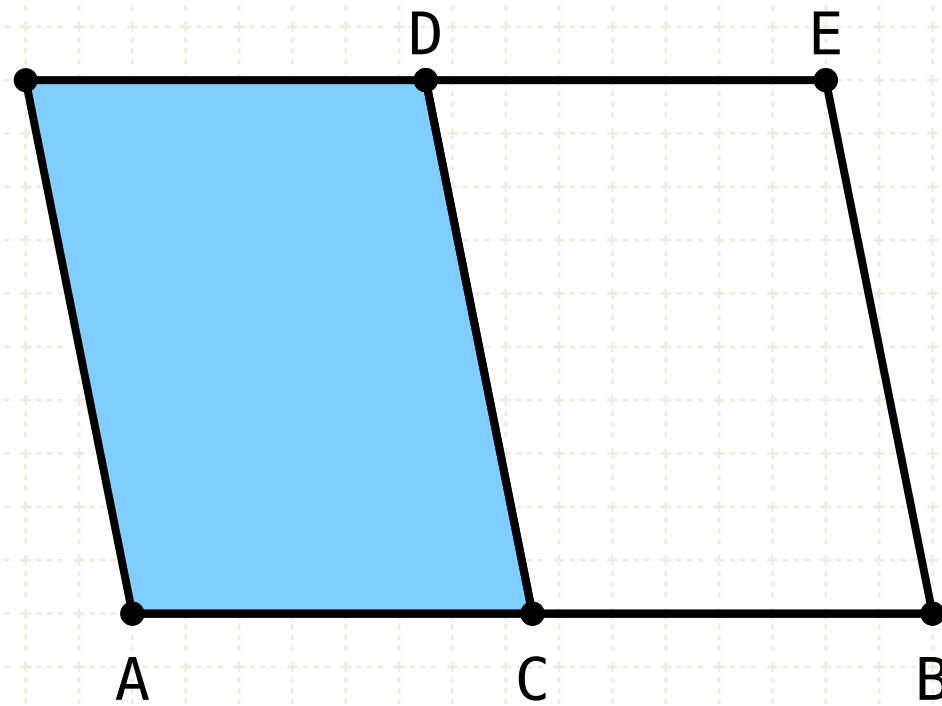
Given a straight line AB

and a midpoint C

Draw a parallelogram on AB

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Of all the parallelograms applied to the same straight line and deficient by parallelogrammic figures similar and similarly situated to that described on the half of the straight line, that parallelogram is greatest which is applied to the half of the straight line and is similar to the defect



$$AC = \frac{1}{2} AB$$
$$\square AD = \square DB$$

In other words

Given a straight line AB

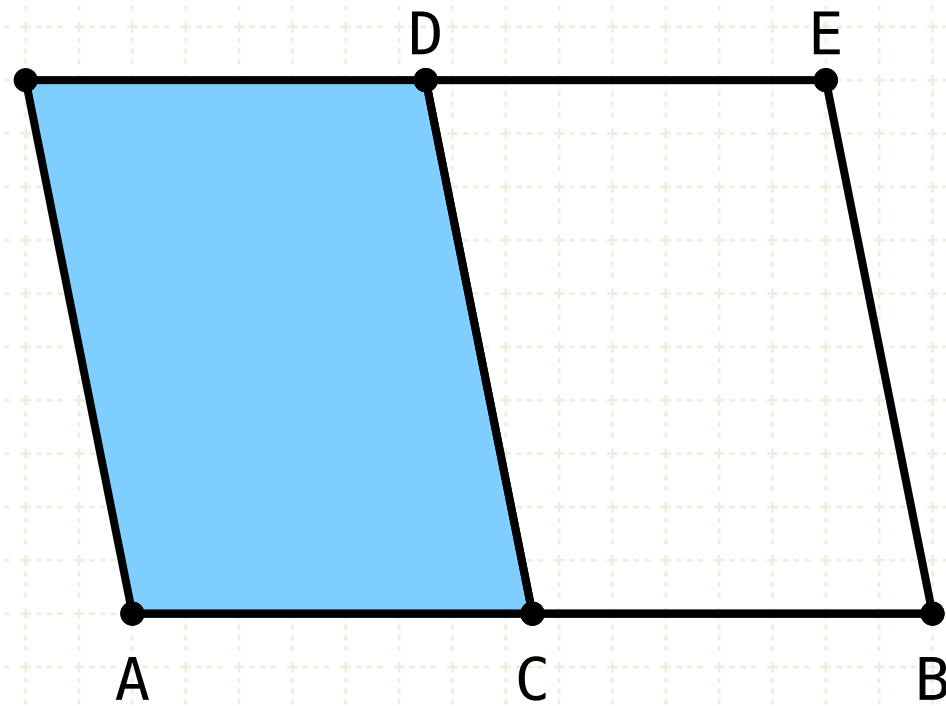
and a midpoint C

Draw a parallelogram on AB

Remove the section described by the parallelogram CE

Proposition 27 of Book VI

Of all the parallelograms applied to the same straight line and deficient by parallelogrammic figures similar and similarly situated to that described on the half of the straight line, that parallelogram is greatest which is applied to the half of the straight line and is similar to the defect



$$AC = \frac{1}{2} AB$$
$$\square AD = \square DB$$

In other words

Given a straight line AB

and a midpoint C

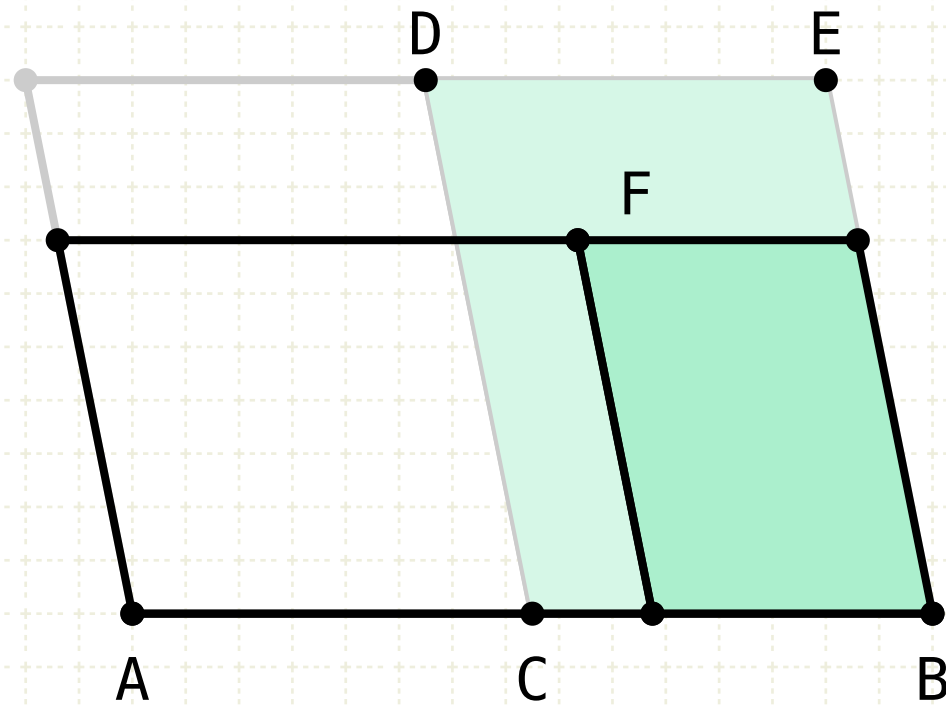
Draw a parallelogram on AB

Remove the section described by the parallelogram CE

Then the parallelogram AD is the largest of all parallelograms drawn on AB, where another parallelogram similar to CE (and similarly situated) is removed

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Of all the parallelograms applied to the same straight line and deficient by parallelogrammic figures similar and similarly situated to that described on the half of the straight line, that parallelogram is greatest which is applied to the half of the straight line and is similar to the defect



$$AC = \frac{1}{2} AB$$

$$\square AD = \square DB$$

$$\square DB \sim \square FB$$

In other words

Given a straight line AB

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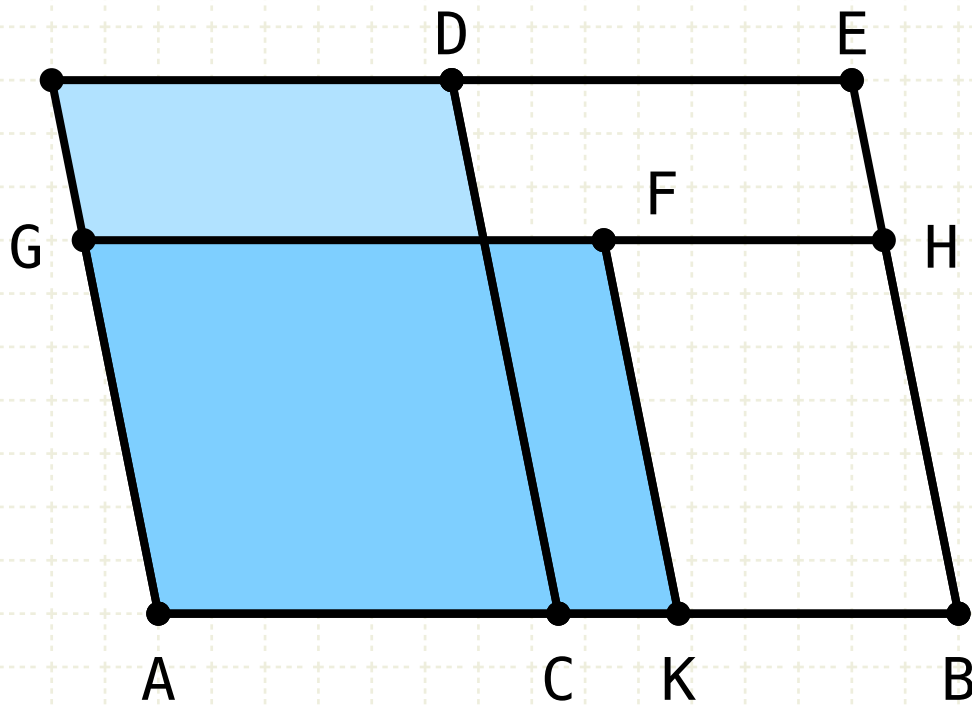
Draw a parallelogram on AB

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$$AC = \frac{1}{2} AB$$

$$\square AD = \square DB$$

$$\square DB \sim \square FB$$

$$\square AD > \square AF$$

In other words

Given a straight line AB

and a midpoint C

Draw a parallelogram on AB

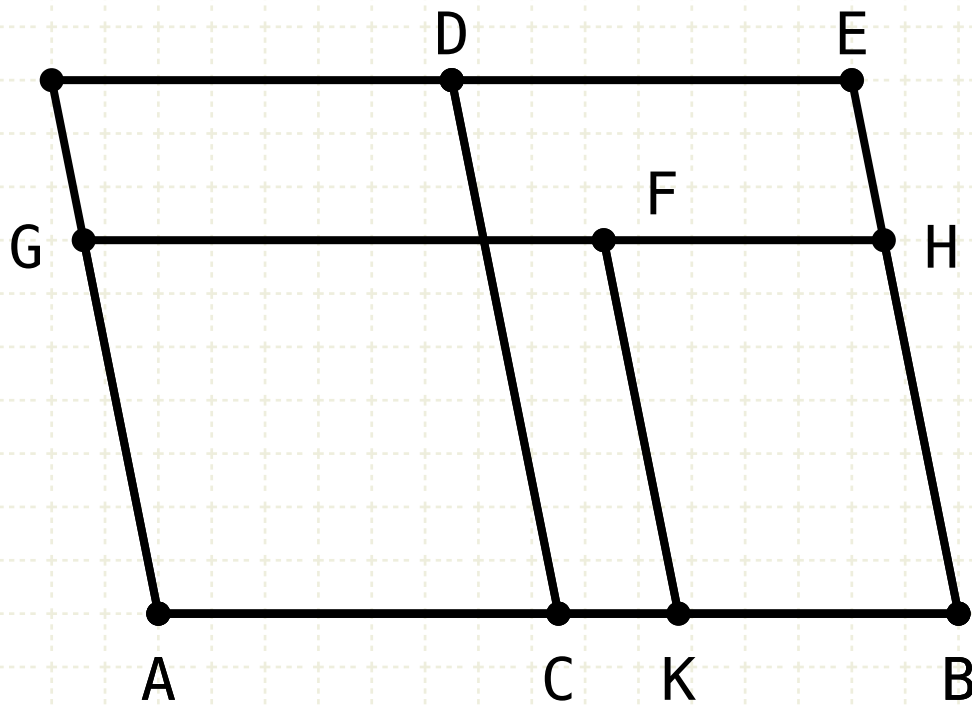
Remove the section described by the parallelogram CE

Then the parallelogram AD is the largest of all parallelograms drawn on AB, where another parallelogram similar to CE (and similarly situated) is removed



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Of all the parallelograms applied to the same straight line and deficient by parallelogrammic figures similar and similarly situated to that described on the half of the straight line, that parallelogram is greatest which is applied to the half of the straight line and is similar to the defect



Proof

$$AC = \frac{1}{2} AB$$

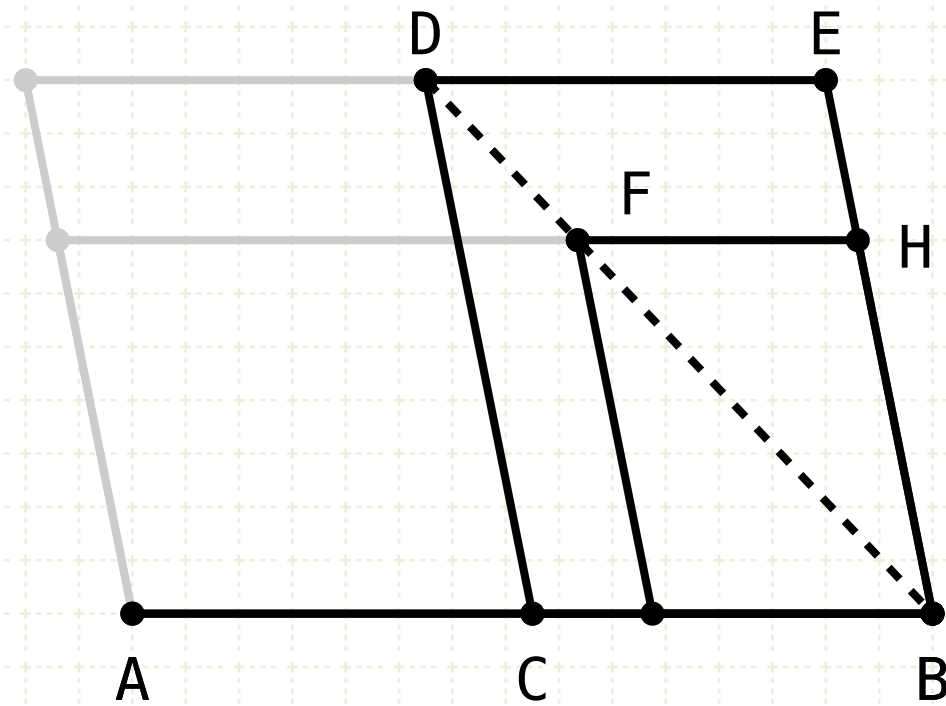
$$\square AD = \square DB$$

$$\square DB \sim \square FB$$



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Of all the parallelograms applied to the same straight line and deficient by parallelogrammic figures similar and similarly situated to that described on the half of the straight line, that parallelogram is greatest which is applied to the half of the straight line and is similar to the defect



$$AC = \frac{1}{2} AB$$

$$\square AD = \square DB$$

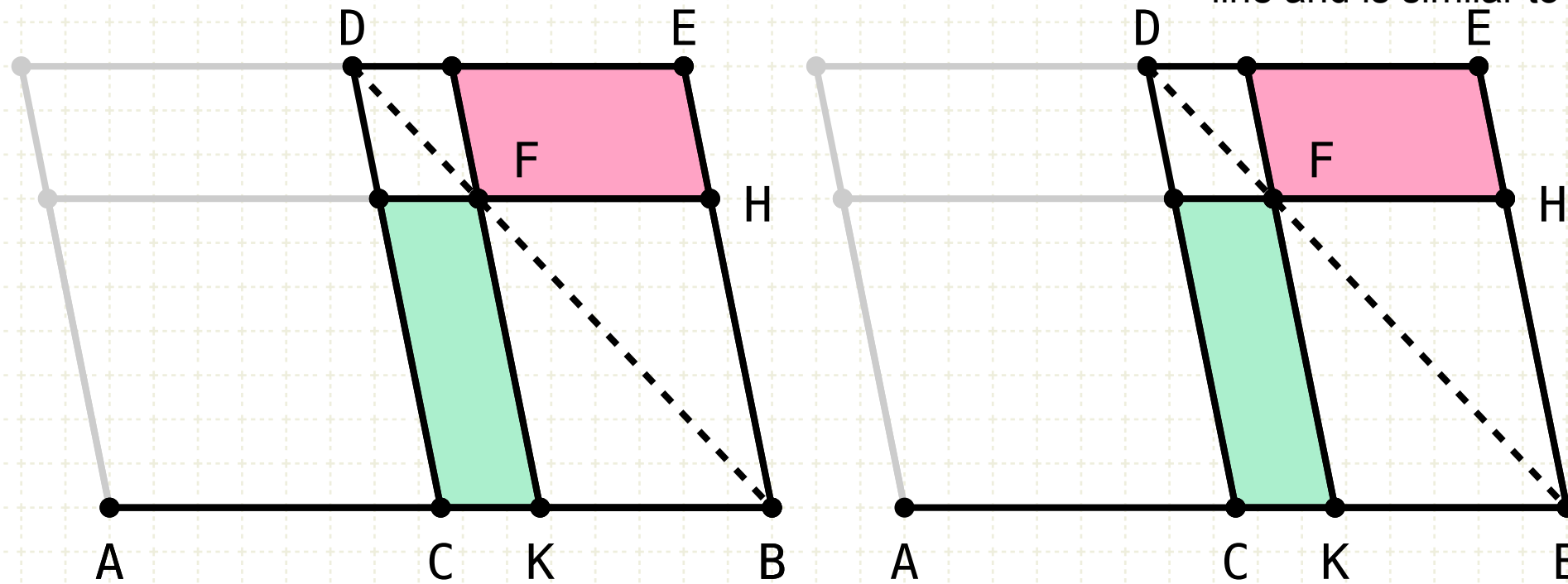
$$\square DB \sim \square FB$$

Proof

Since the parallelograms DB and FB are similar, they are both on the same diameter (VI·26)

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Of all the parallelograms applied to the same straight line and deficient by parallelogrammic figures similar and similarly situated to that described on the half of the straight line, that parallelogram is greatest which is applied to the half of the straight line and is similar to the defect



$$\begin{aligned} AC &= \frac{1}{2} AB \\ \square AD &= \square DB \\ \square DB &\sim \square FB \\ \square CF &= \square FE \end{aligned}$$

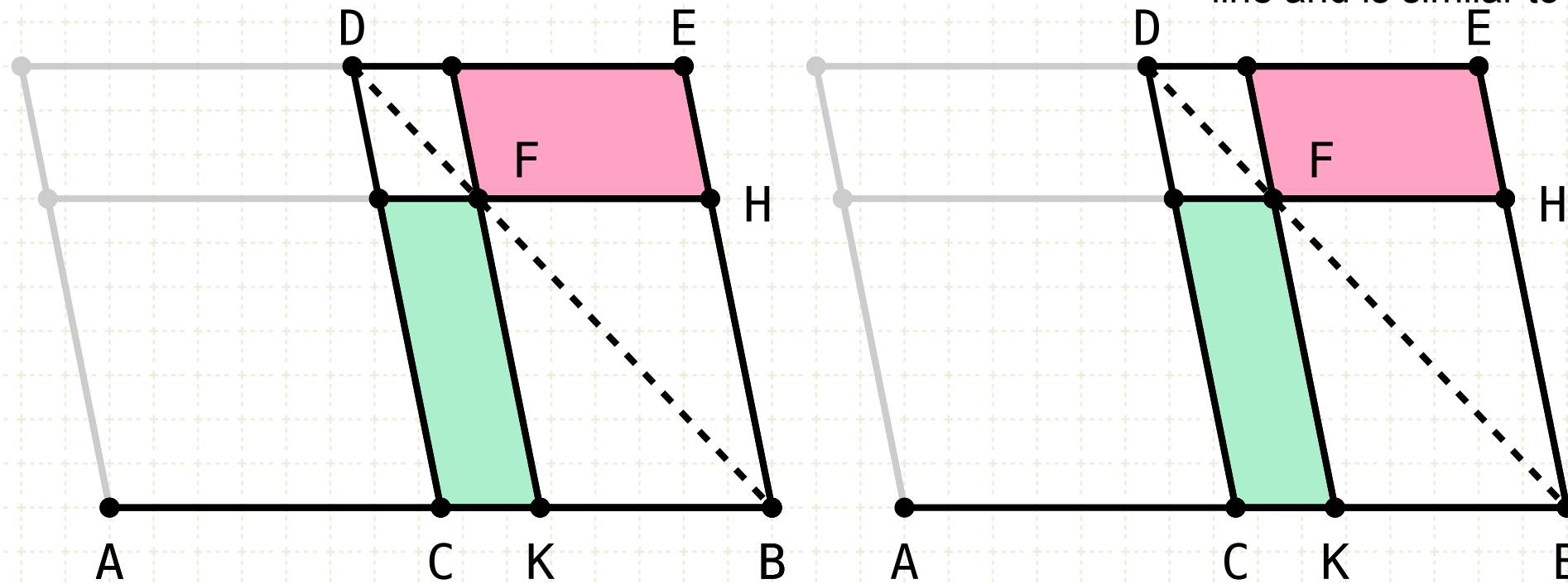
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Since the parallelograms DB and FB are similar, they are both on the same diameter (VI-26)

Parallelograms CF and FE are equal (I-43)

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Proof

Since the parallelograms DB and FB are similar, they are both on the same diameter (VI-26)

Parallelograms CF and FE are equal (I-43)

Since the parallelogram FB is common, the whole of CH is equal to the whole KE

$$AC = \frac{1}{2} AB$$

$$\square AD = \square DB$$

$$\square DB \sim \square FB$$

$$\square CF = \square FE$$

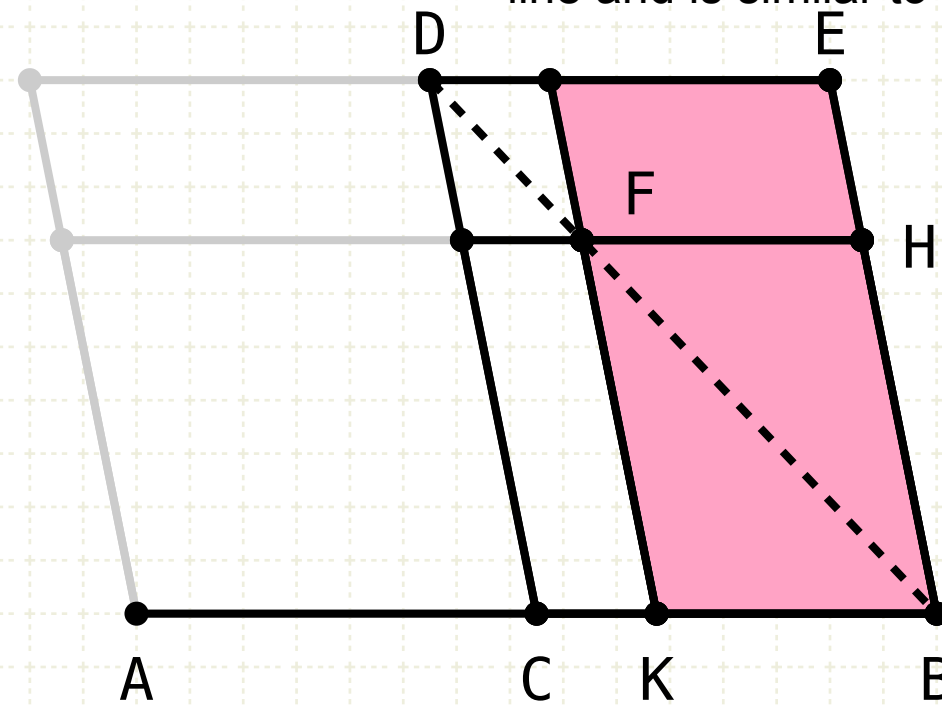
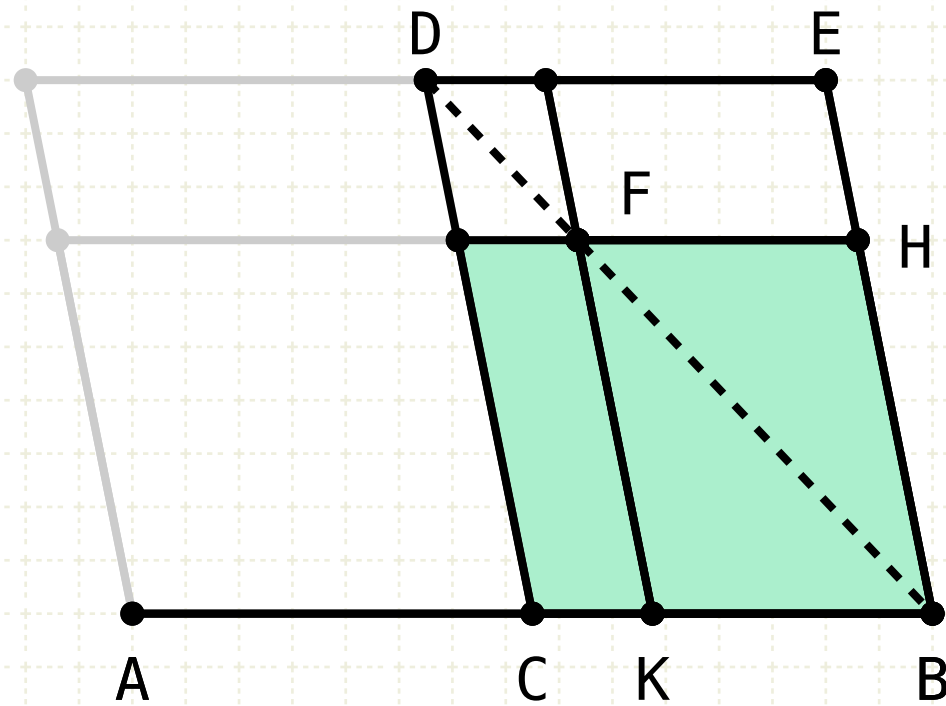
$$\square CF + \square FB = \square FE + \square FB$$

$$\square CH = \square KE$$



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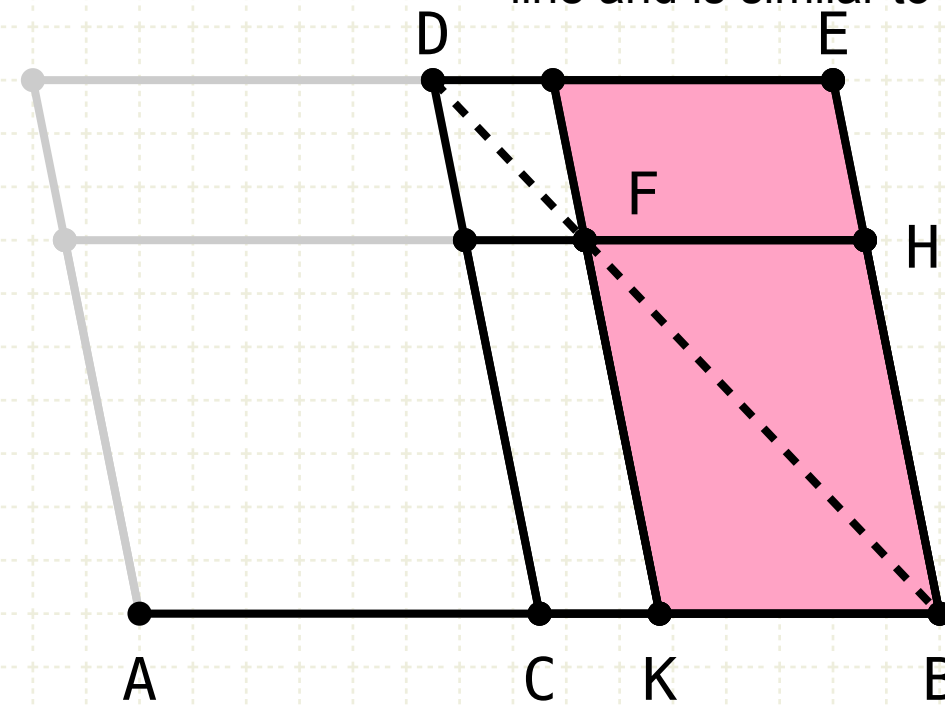
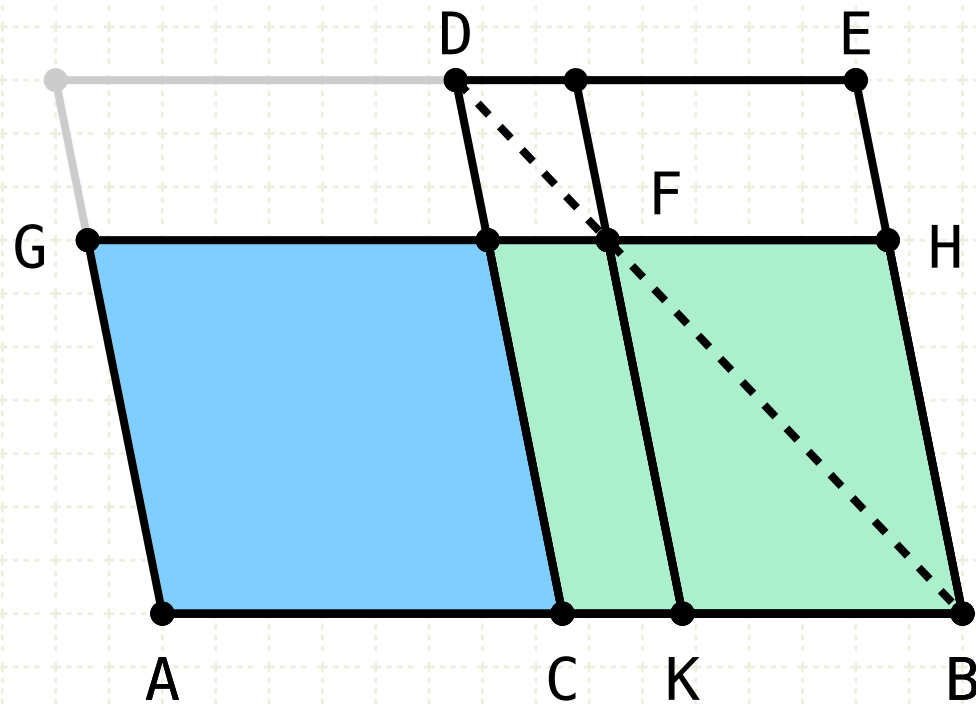
$$\square CF + \square FB = \square FE + \square FB$$

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Of all the parallelograms applied to the same straight line and deficient by parallelogrammic figures similar and similarly situated to that described on the half of the straight line, that parallelogram is greatest which is applied to the half of the straight line and is similar to the defect



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But CH is equal to CG, since AC is also equal to CB (I-36)

$$AC = \frac{1}{2} AB$$

$$\square AD = \square DB$$

$$\square DB \sim \square FB$$

$$\square CF = \square FE$$

$$\square CF + \square FB = \square FE + \square FB$$

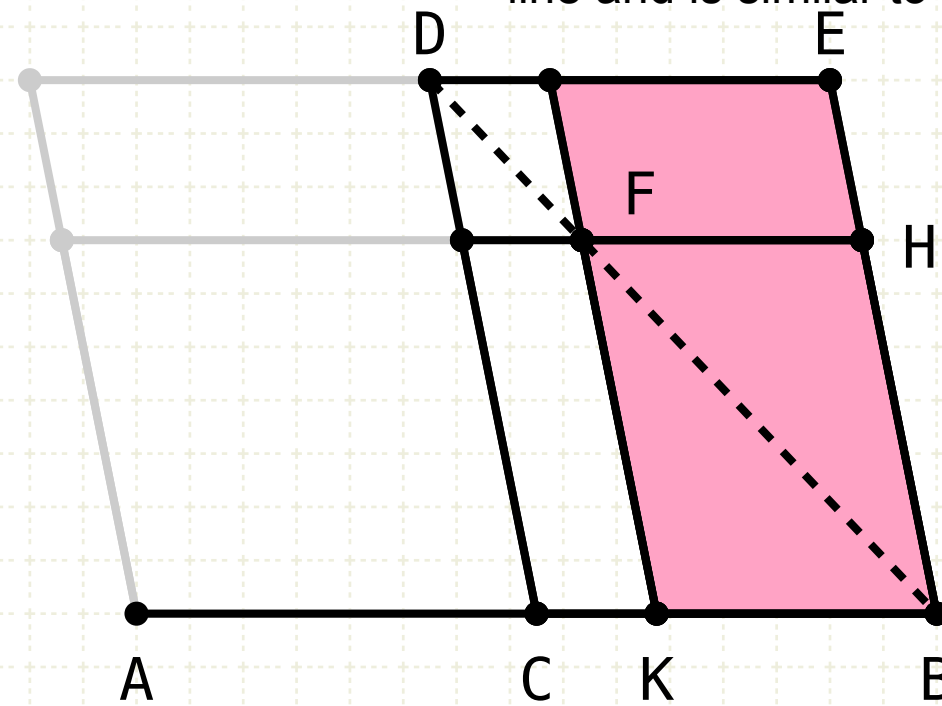
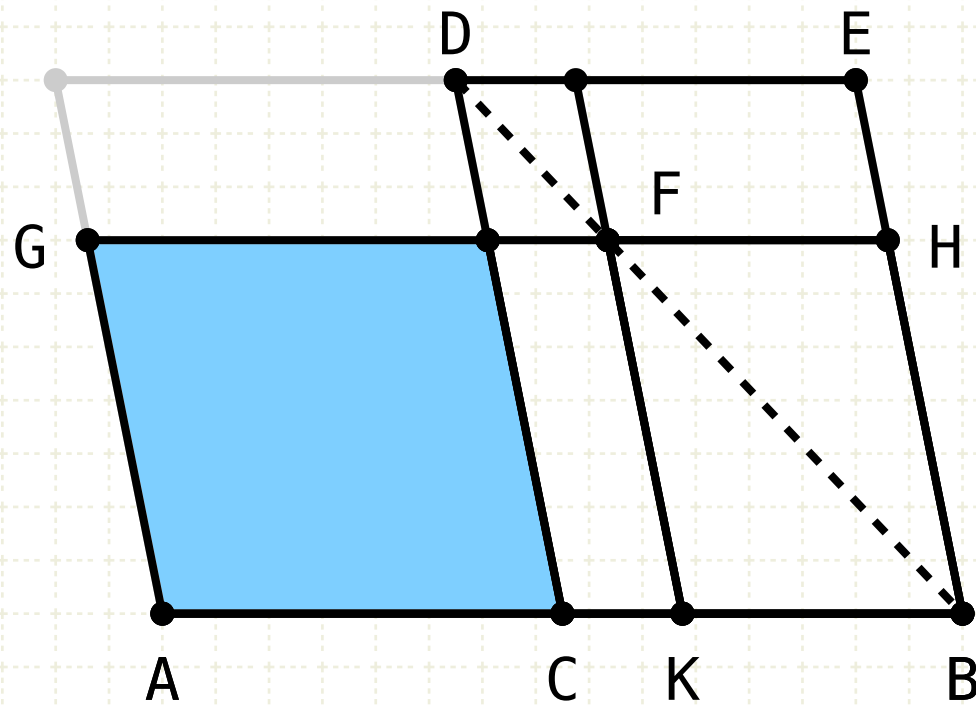
$$\square CH = \square KE$$

$$\square CH = \square CG$$



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$$\square AD = \square DB$$

$$\square DB \sim \square FB$$

$$\square CF = \square FE$$

$$\square CF + \square FB = \square FE + \square FB$$

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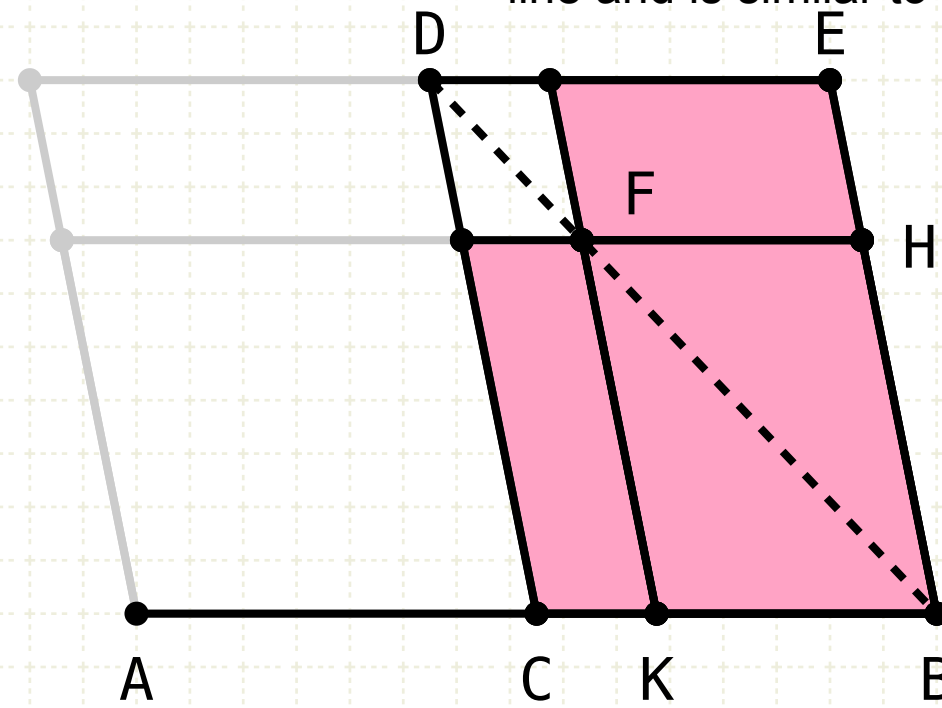
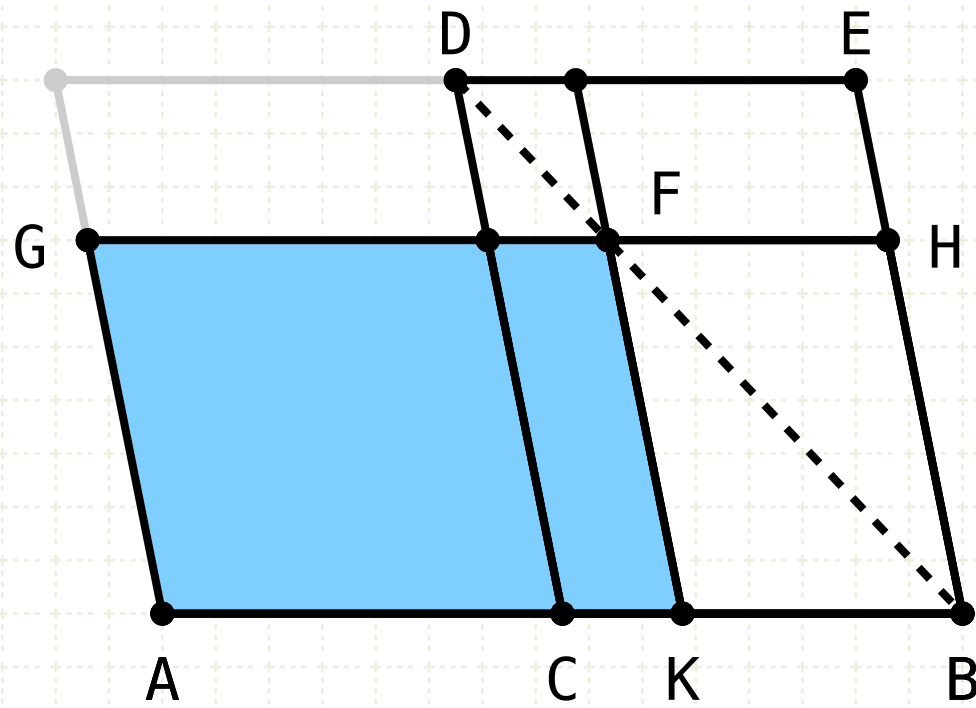
$$\square CH = \square CG$$

$$\square CG = \square KE$$



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Therefore CG is also equal to KE

Add the parallelogram CF to each, therefore the gnomon CBEF is equal to the parallelogram AF

$$AC = \frac{1}{2} AB$$

$$\square AD = \square DB$$

$$\square DB \sim \square FB$$

$$\square CF = \square FE$$

$$\square CF + \square FB = \square FE + \square FB$$

$$\square CH = \square KE$$

$$\square CH = \square CG$$

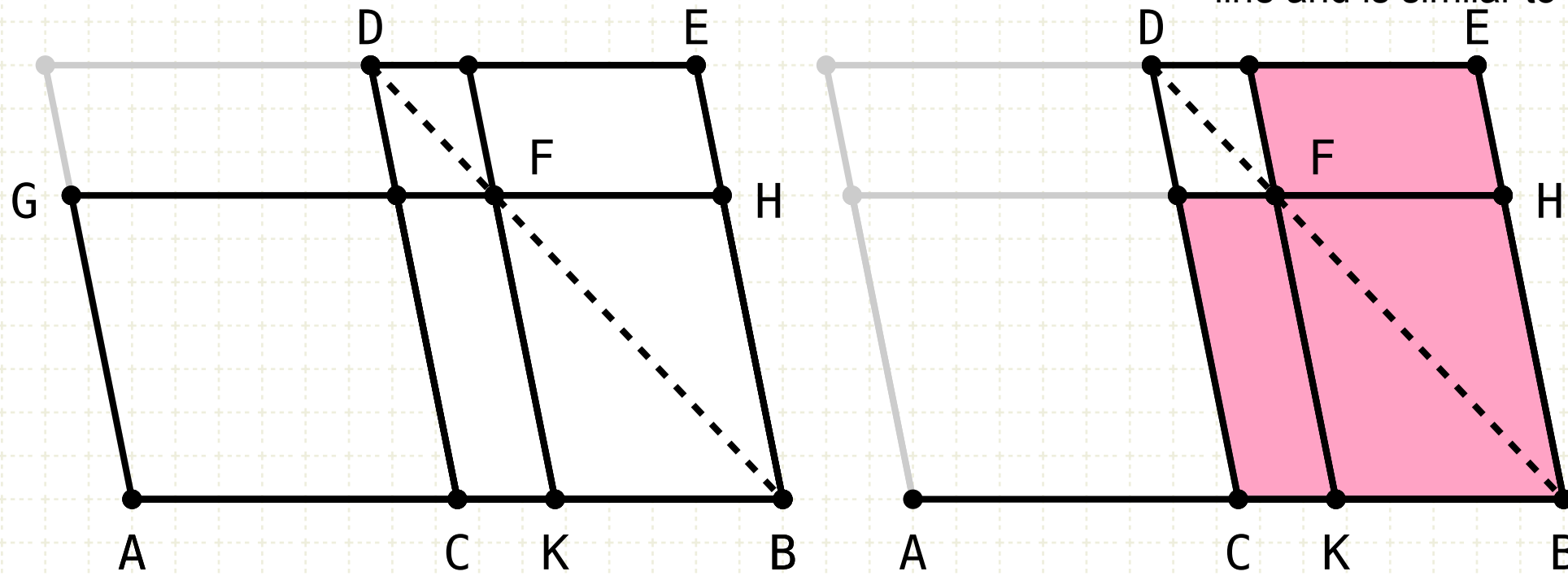
$$\square CG = \square KE$$

$$\square AF = \square CBEF$$



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Add the parallelogram CF to each, therefore the gnomon CBEF is equal to the parallelogram AF

The gnomon CBEF is less than the parallelogram DB

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$$\square AD = \square DB$$

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$$\square CF + \square FB = \square FE + \square FB$$

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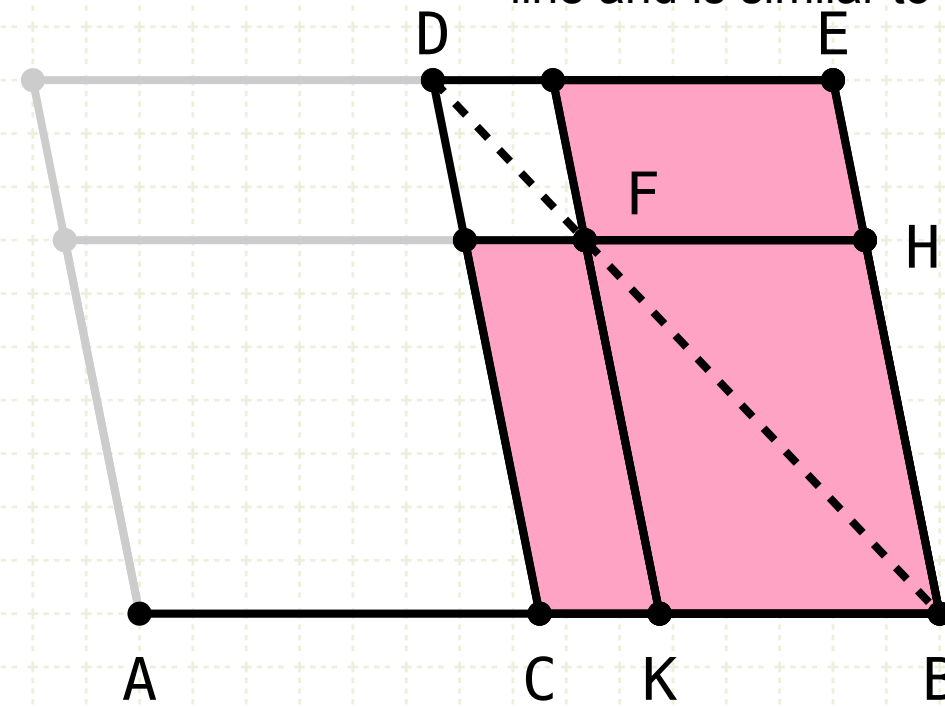
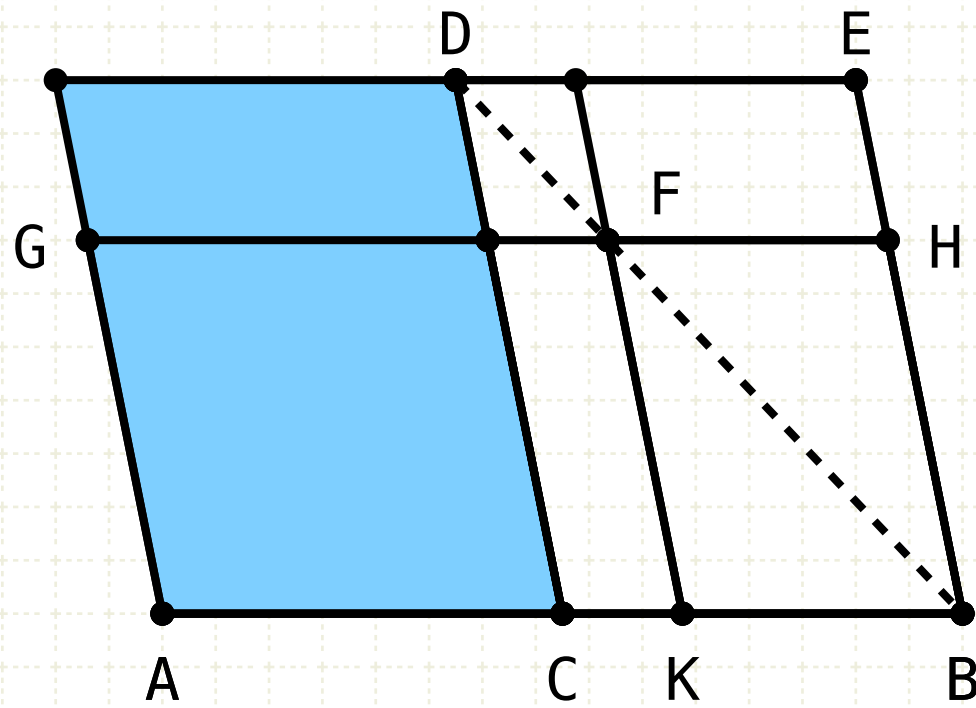
$$\square AF = \square CBEF$$

$$\square CBEF < \square DB$$



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Since the parallelograms DB and FB are similar, they are both on the same diameter (VI-26)

Parallelograms CF and FE are equal (I-43)

Since the parallelogram FB is common, the whole of CH is equal to the whole KE

But CH is equal to CG, since AC is also equal to CB (I-36)

Therefore CG is also equal to KE

Add the parallelogram CF to each, therefore the gnomon CBEF is equal to the parallelogram AF

The gnomon CBEF is less than the parallelogram DB

And thus, the gnomon is also less than AD

$$AC = \frac{1}{2} AB$$

$$\square AD = \square DB$$

$$\square DB \sim \square FB$$

$$\square CF = \square FE$$

$$\square CF + \square FB = \square FE + \square FB$$

$$\square CH = \square KE$$

$$\square CH = \square CG$$

$$\square CG = \square KE$$

$$\square AF = \square CBEF$$

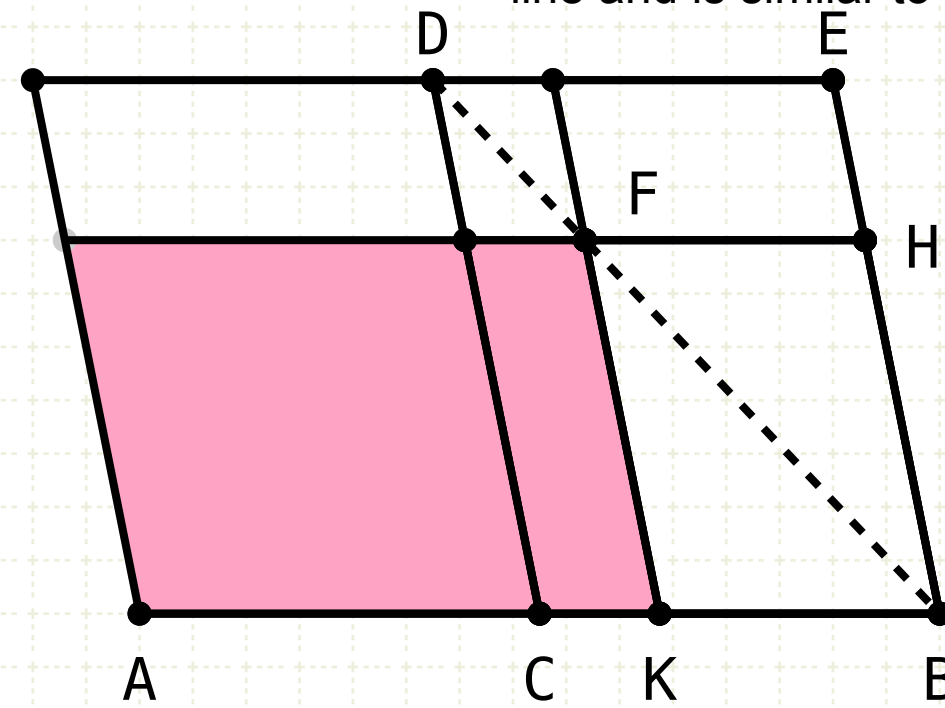
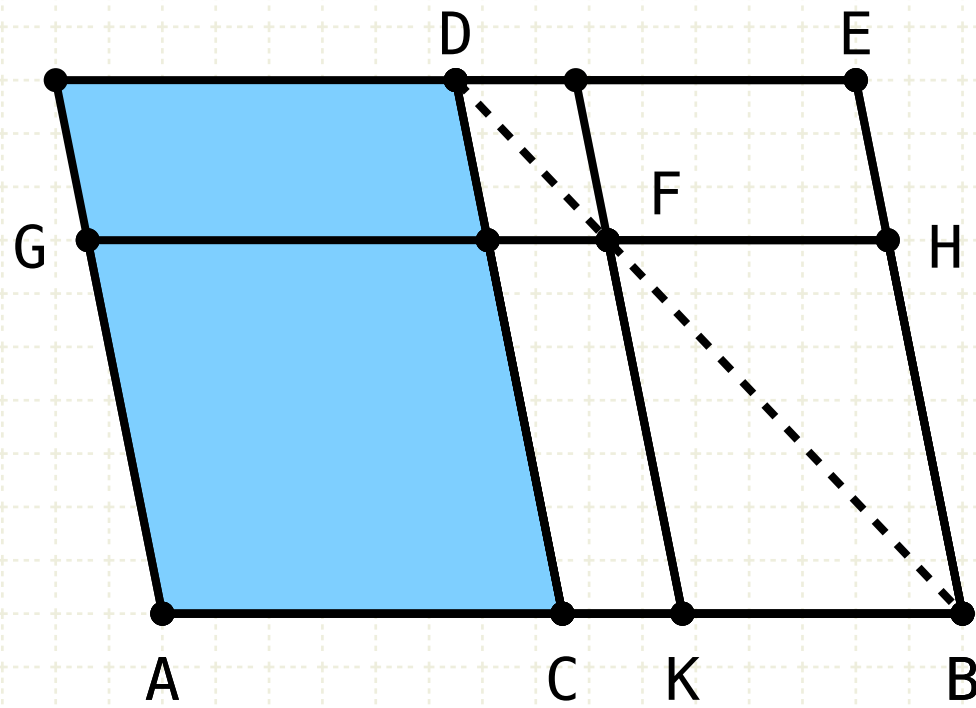
$$\square CBEF < \square DB$$

$$\square CBEF < \square AD$$



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Of all the parallelograms applied to the same straight line and deficient by parallelogrammic figures similar and similarly situated to that described on the half of the straight line, that parallelogram is greatest which is applied to the half of the straight line and is similar to the defect



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Parallelograms CF and FE are equal (I-43)

Since the parallelogram FB is common, the whole of CH is equal to the whole KE

But CH is equal to CG, since AC is also equal to CB (I-36)

Therefore CG is also equal to KE

Add the parallelogram CF to each, therefore the gnomon CBEF is equal to the parallelogram AF

The gnomon CBEF is less than the parallelogram DB

And thus, the gnomon is also less than AD

Finally, since CBEF is equal to AF, AF is less than the AD

$$AC = \frac{1}{2} AB$$

$$\square AD = \square DB$$

$$\square DB \sim \square FB$$

$$\square CF = \square FE$$

$$\square CF + \square FB = \square FE + \square FB$$

$$\square CH = \square KE$$

$$\square CH = \square CG$$

$$\square CG = \square KE$$

$$\square AF = CBEF$$

$$CBEF < \square DB$$

$$CBEF < \square AD$$

$$\square AF < \square AD$$



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