

# Euclid's Elements

## Book VI

*One can state, without exaggeration, that the observation of and the search for similarities and differences are the basis of all human knowledge.*

**Alfred Nobel**



# Table of Contents, Chapter 6

1	If the height of two triangles are equal, then the ratio of the areas is equal to the ratio of the bases	7	If two triangles have one angle equal to one angle, and the sides about other angles are proportional, and the remaining angles either both less or both not less than a right angle, then triangles will be equiangular	14	In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and vice versa
2	If a line cuts a triangle, parallel to its base, it will cut the sides of the triangle proportionally			15	In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and vice versa
3	If an angle of a triangle is bisected and the straight line cutting the angle also cuts the base, the segments of the base will have the same ratio as the remaining sides of the triangle	8	If in a right-angled triangle a perpendicular be drawn from the right angle to the base, the triangles adjoining the perpendicular are similar both to the whole and to one another	16	If four straight lines are proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means, and vice versa
4	If two triangles have equal angles, then the sides opposite the equal angles are proportional, as well, the sides of the triangles on either side of the equal angles are also proportional	9	From a given straight line to cut off a given fraction	17	If three straight lines are proportional, the rectangle contained by the extremes is equal to the square on the mean; and vice versa
5	It two triangles have proportional sides, the triangles will be equiangular	10	To cut a given uncut straight line similarly to a given cut straight line	18	On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure
6	<b>If two triangles have one angle equal to one angle and the sides about the equal angles are proportional, then the triangles will be equiangular</b>	11	To two given straight lines to find a third proportional	19	Similar triangles are to one another in the duplicate ratio of the corresponding sides
		12	To three given straight lines to find a fourth proportional		
		13	To two given straight lines to find a mean proportional		



## Table of Contents, Chapter 3

- |    |  |    |   |    |   |
|----|--|----|---|----|---|
| 20 | Similar polygons are divided into the same number of similar triangles, which have the same ratio as the wholes, and the polygons have duplicate ratios to their corresponding sides | 26 | If from a parallelogram a similar parallelogram with a common angle is subtracted, it is about the same diameter as the original  | 31 | In right-angled triangles the figure on the side subtending the right angle is equal to the similar and similarly described figures on the sides containing the right angle |
| 21 | Figures which are similar to the same rectilineal figure are also similar to one another   | 27 | Of all the parallelograms applied to the same straight line and deficient by parallelogrammic figures similar to a parallelogram drawn on half the said line, the largest will be one that is drawn on half of the straight line and is similar to the defect |    |   |
| 22 | If four straight lines are proportional, similar rectilineal figures will also be proportional; and vice versa   | 28 | To a given straight line, apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one   |    |   |
| 23 | Equiangular parallelograms have to one another the ratio compounded of the ratios of their sides   | 29 | To a given straight line, apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one   |    |   |
| 24 | In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another  | 30 | To cut a finite straight line in extreme ratio  |    |   |
| 25 | To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure   |    |   |    |   |



## Proposition 6 of Book VI

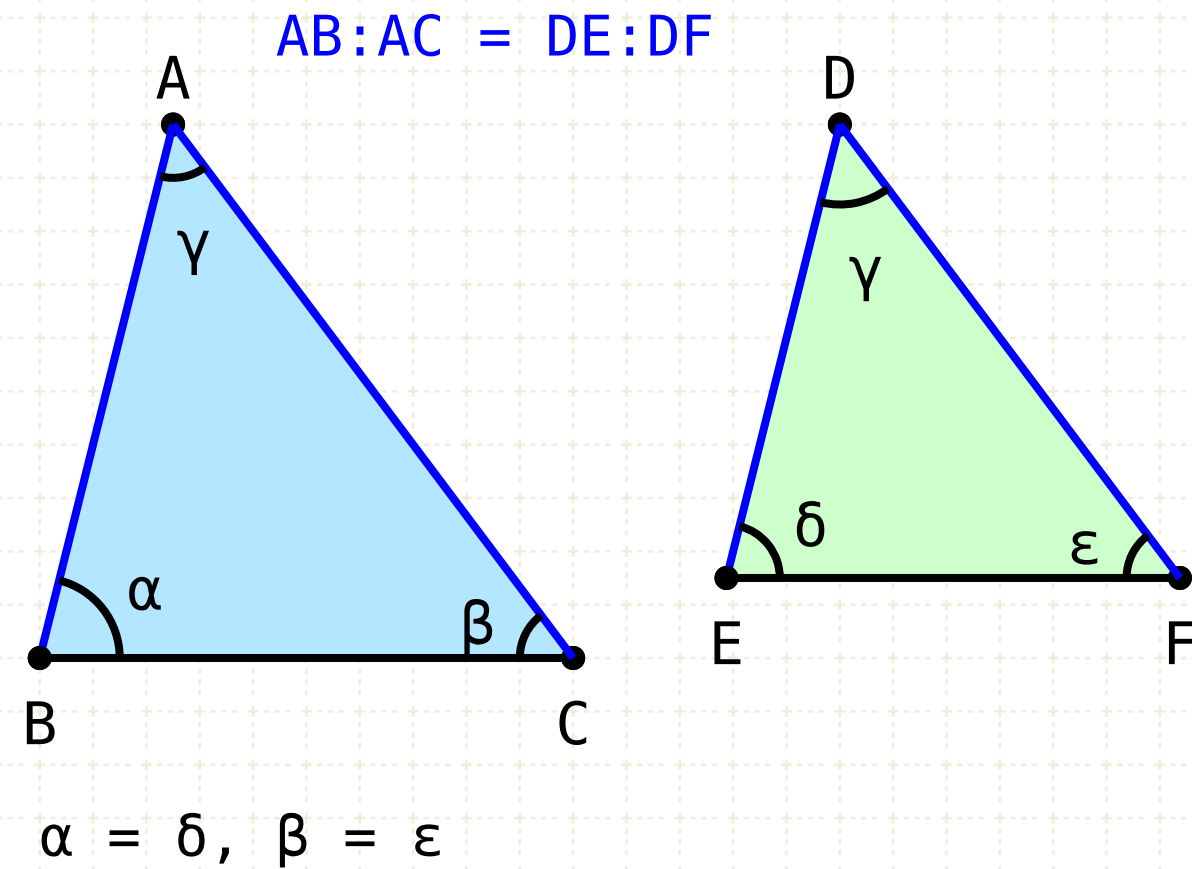
If two triangles have one angle equal to one angle and the sides about the equal angles proportional, the triangles will be equiangular and will have those angles equal which the corresponding sides subtend.





## Proposition 6 of Book VI

If two triangles have one angle equal to one angle and the sides about the equal angles proportional, the triangles will be equiangular and will have those angles equal which the corresponding sides subtend.

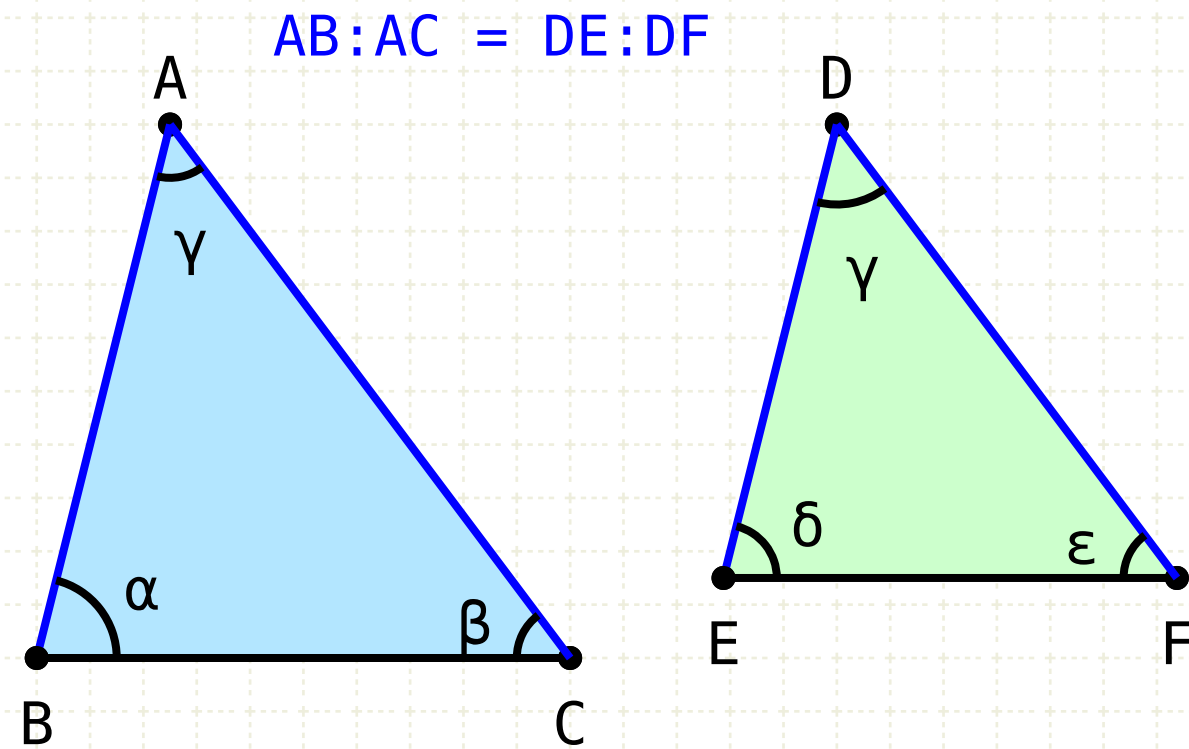


### In other words

If two triangles have one angle that is equal between them,  
AND the ratio of the sides of around that angle are also equal,  
then the two triangles are equiangular

## Proposition 6 of Book VI

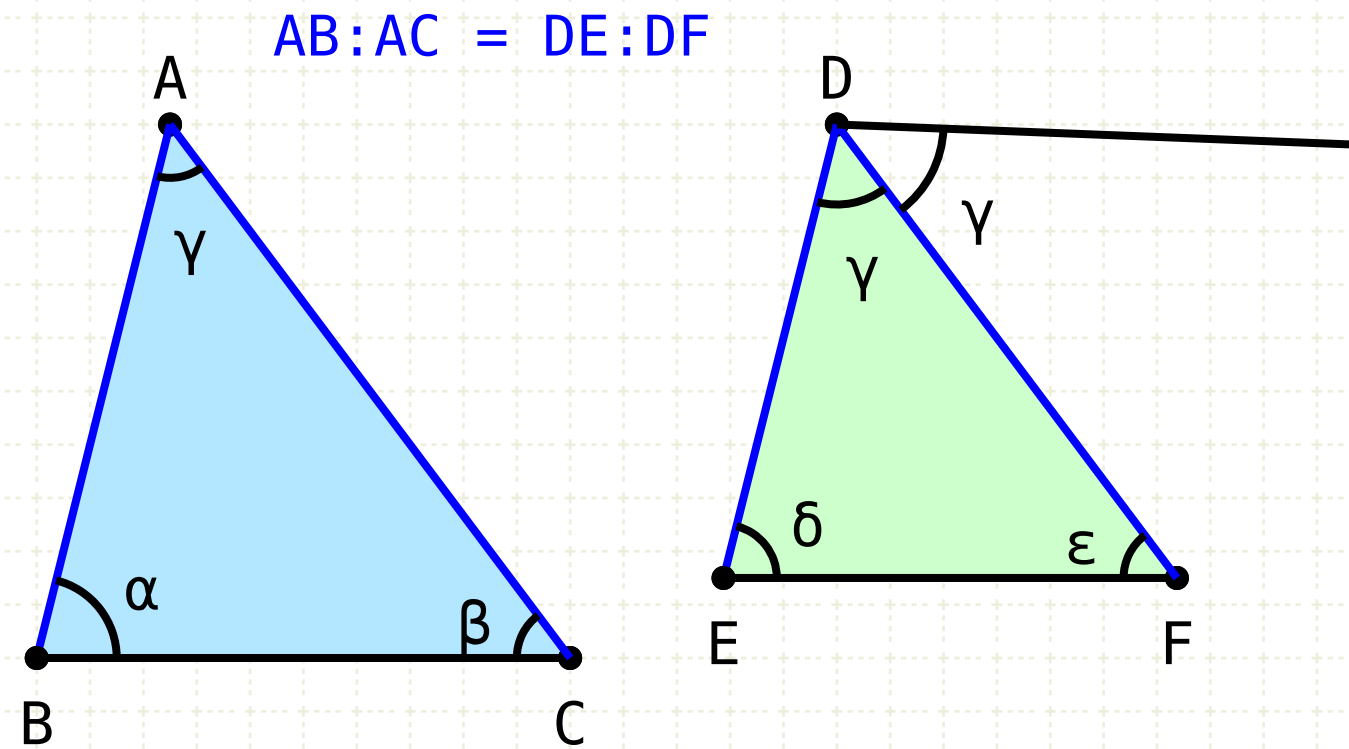
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### Proof

## Proposition 6 of Book VI

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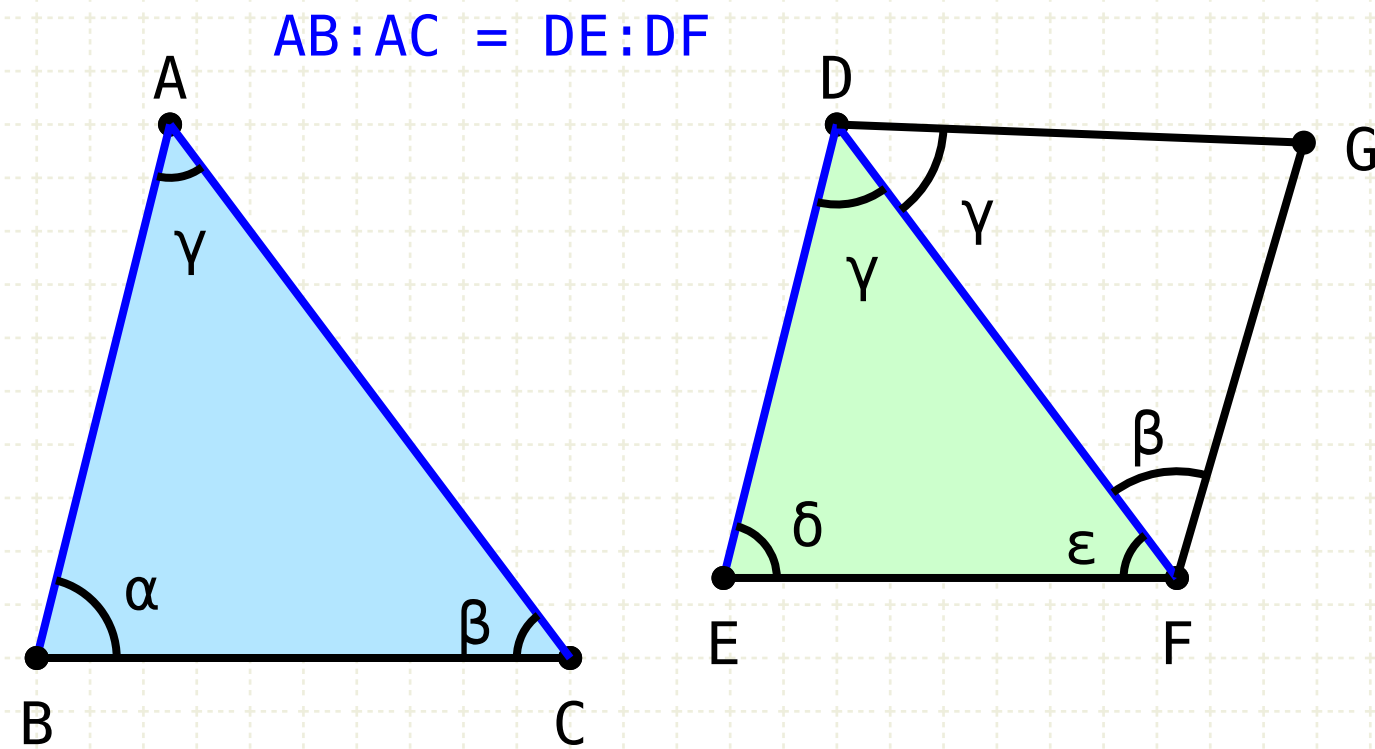


### Proof

On the point D, construct an angle FDG on the line DF equal to the angle  $\gamma$  (I.23)

## Proposition 6 of Book VI

If two triangles have one angle equal to one angle and the sides about the equal angles proportional, the triangles will be equiangular and will have those angles equal which the corresponding sides subtend.



### Proof

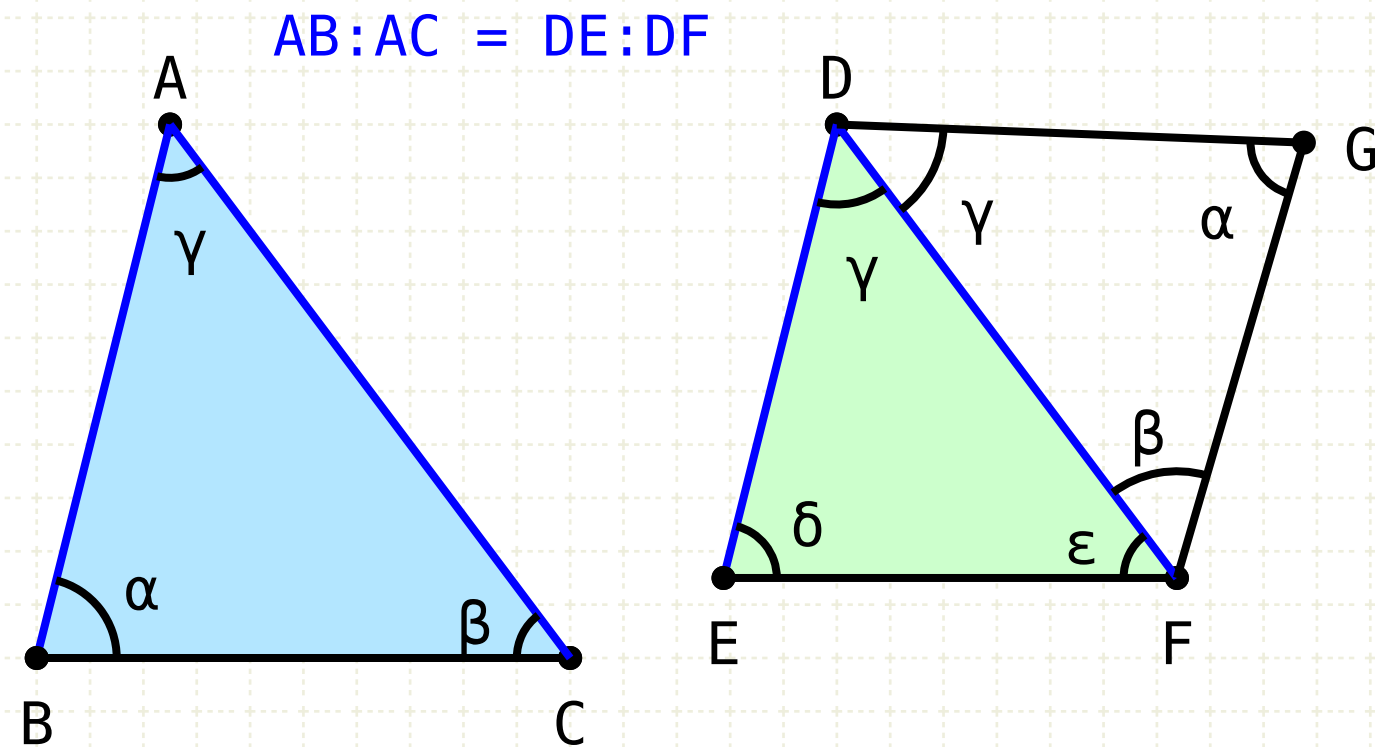
On the point D, construct an angle FDG on the line DF equal to the angle  $\gamma$  (I·23)

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## Proposition 6 of Book VI

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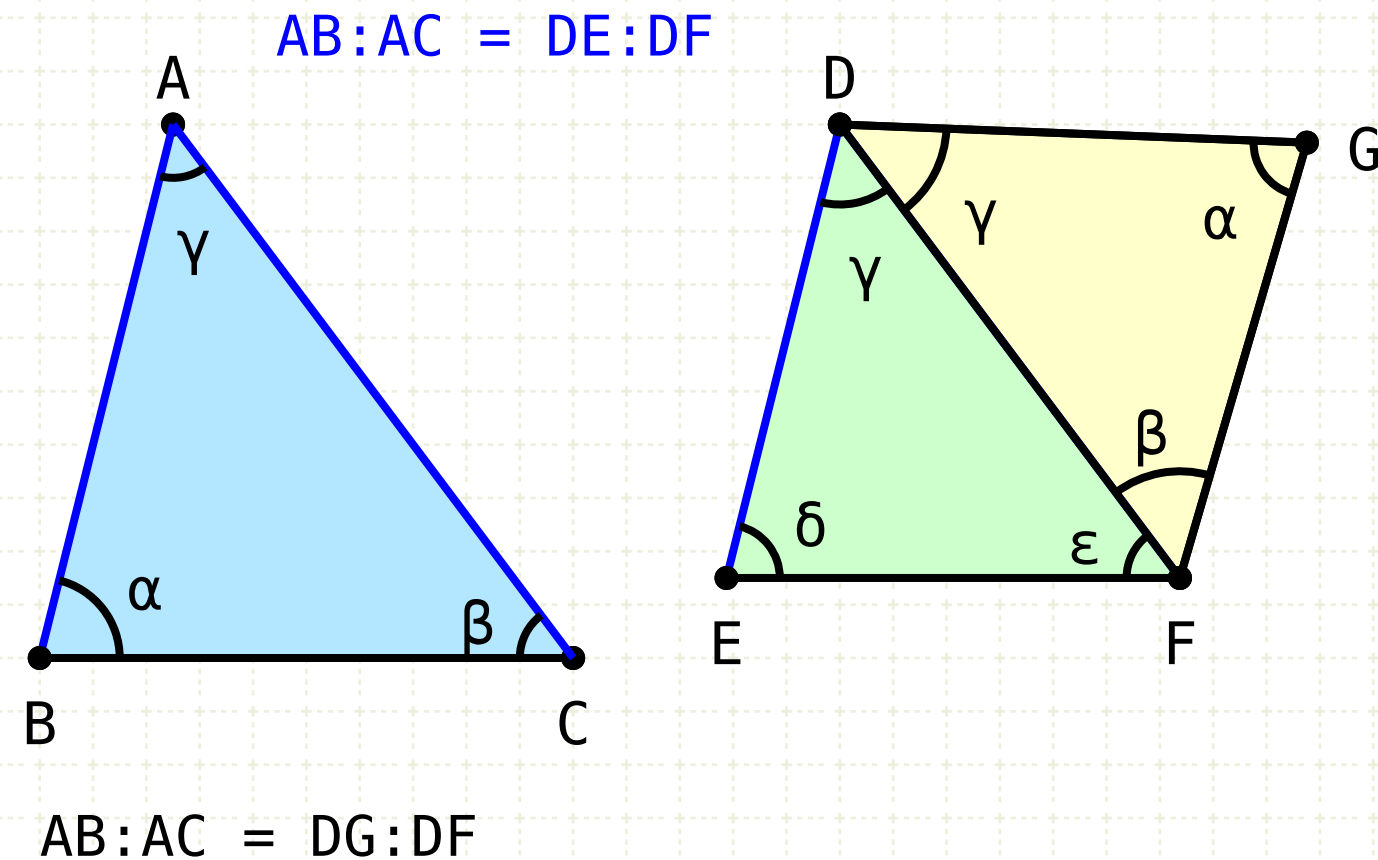
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On the point F, construct an angle DFG on the line DF equal to the angle  $\beta$  (I·23)

And thus, the angle at G will also be the angle at B (I·32)

## Proposition 6 of Book VI

If two triangles have one angle equal to one angle and the sides about the equal angles proportional, the triangles will be equiangular and will have those angles equal which the corresponding sides subtend.



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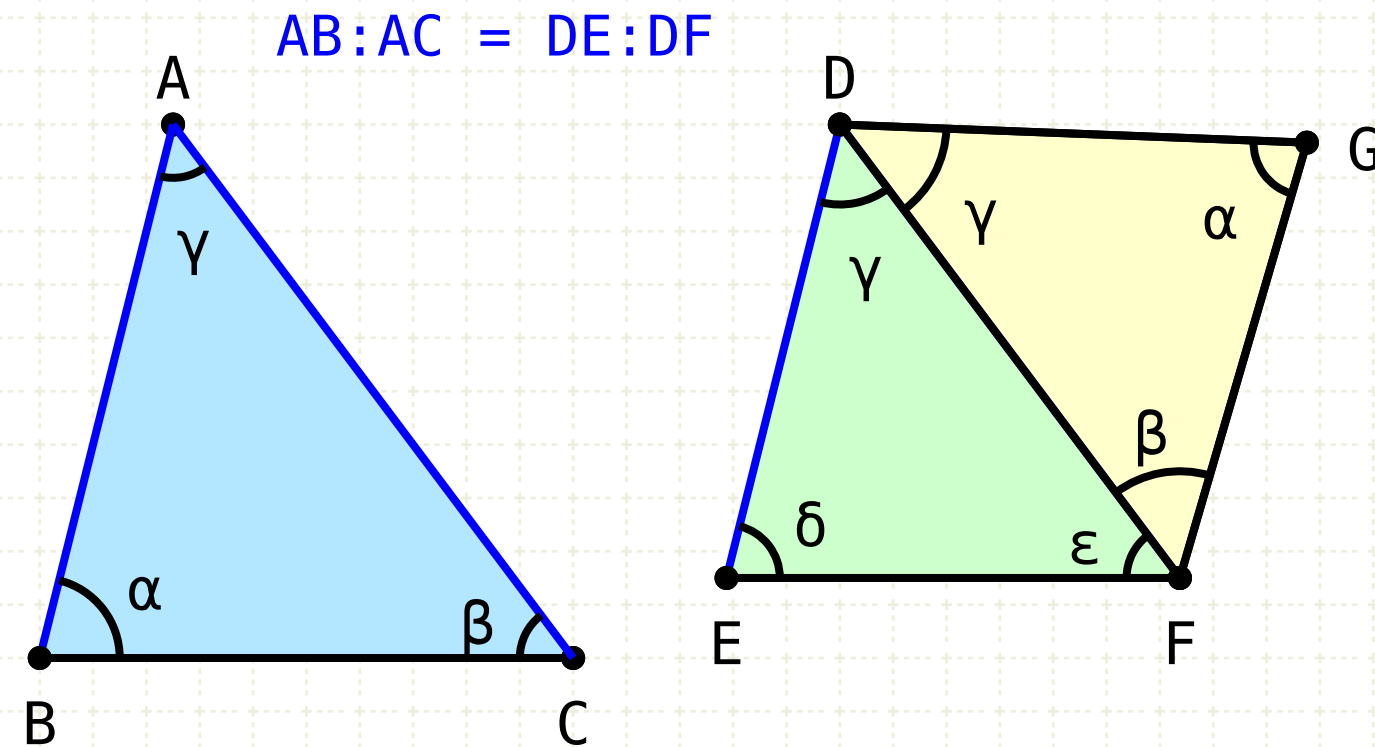
On the point F, construct an angle DFG on the line DF equal to the angle  $\beta$  (I·23)

And thus, the angle at G will also be the angle at B (I·32)

Therefore the triangle ABC is equiangular to DFG, and as such, the edges surrounding the equal angles will be in proportion, i.e. AB is to AC as DG to DF (VI·4)

## Proposition 6 of Book VI

If two triangles have one angle equal to one angle and the sides about the equal angles proportional, the triangles will be equiangular and will have those angles equal which the corresponding sides subtend.



$$AB:AC = DG:DF$$
$$DE:DF = DG:DF$$

### Proof

On the point D, construct an angle FDG on the line DF equal to the angle  $\gamma$  (I·23)

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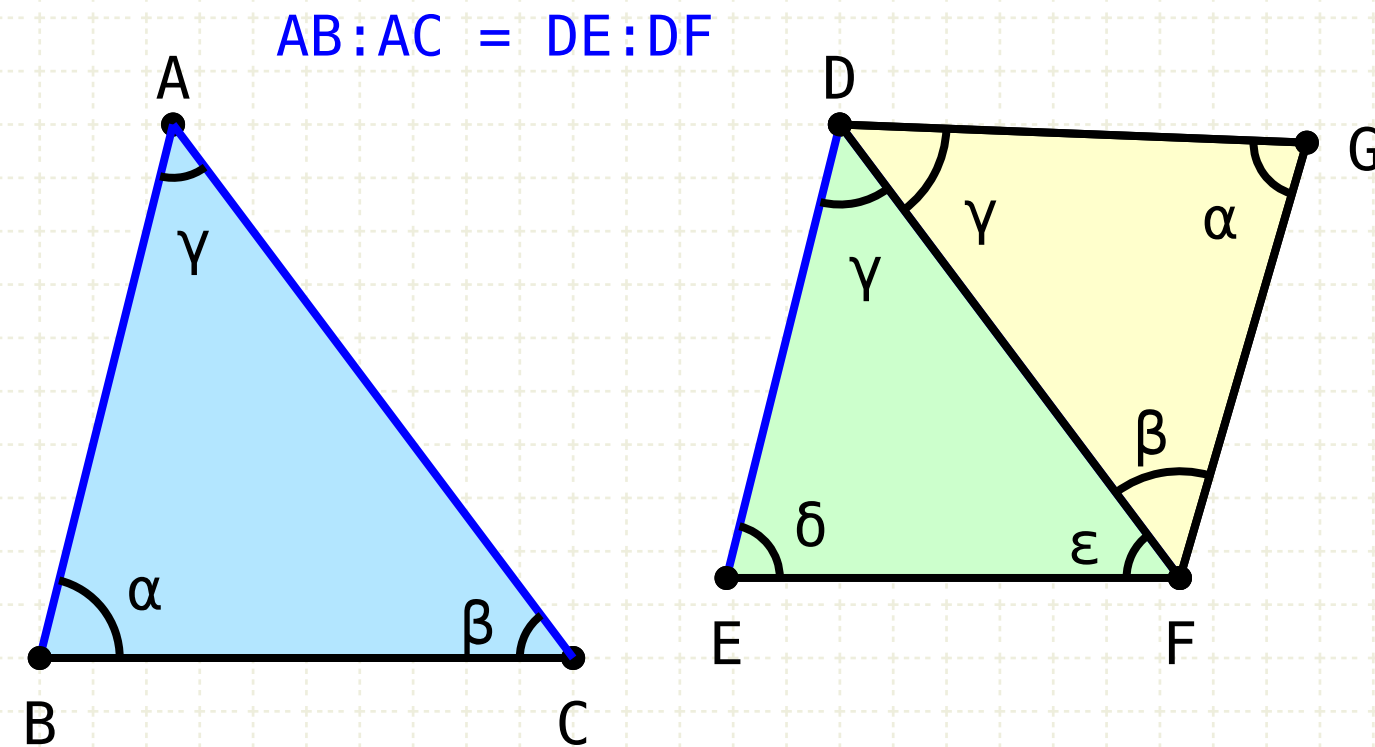
And thus, the angle at G will also be the angle at B (I·32)

Therefore the triangle ABC is equiangular to DFG, and as such, the edges surrounding the equal angles will be in proportion, i.e. AB is to AC as DG to DF (VI·4)

But the ratio AB to AC is equal to DE to DF, therefore the ratio DE to DF equals DG to DF (V·11)

## Proposition 6 of Book VI

If two triangles have one angle equal to one angle and the sides about the equal angles proportional, the triangles will be equiangular and will have those angles equal which the corresponding sides subtend.



$$\begin{aligned} AB:AC &= DG:DF \\ DE:DF &= DG:DF \\ DE &= DG \end{aligned}$$

### Proof

On the point D, construct an angle FDG on the line DF equal to the angle  $\gamma$  (I·23)

On the point F, construct an angle DFG on the line DF equal to the angle  $\beta$  (I·23)

And thus, the angle at G will also be the angle at B (I·32)

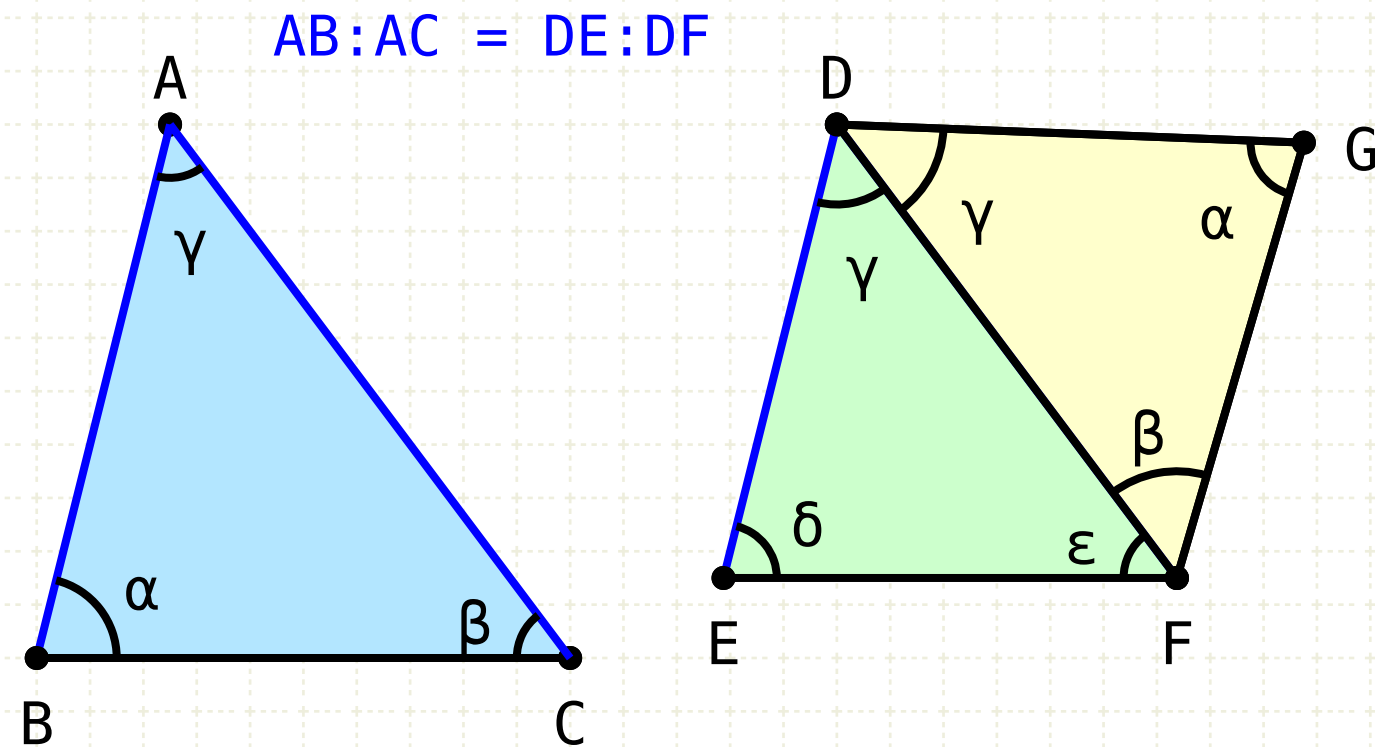
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Since DE and DG have the same ratio to DF, DE and DG are equal (V·9)

# Proposition 6 of Book VI

If two triangles have one angle equal to one angle and the sides about the equal angles proportional, the triangles will be equiangular and will have those angles equal which the corresponding sides subtend.



$$AB:AC = DG:DF$$

$$DE:DF = DG:DF$$

$$DE = DG$$

$$EF = FG$$

$$\alpha = a$$

$$\beta = b$$

## Proof

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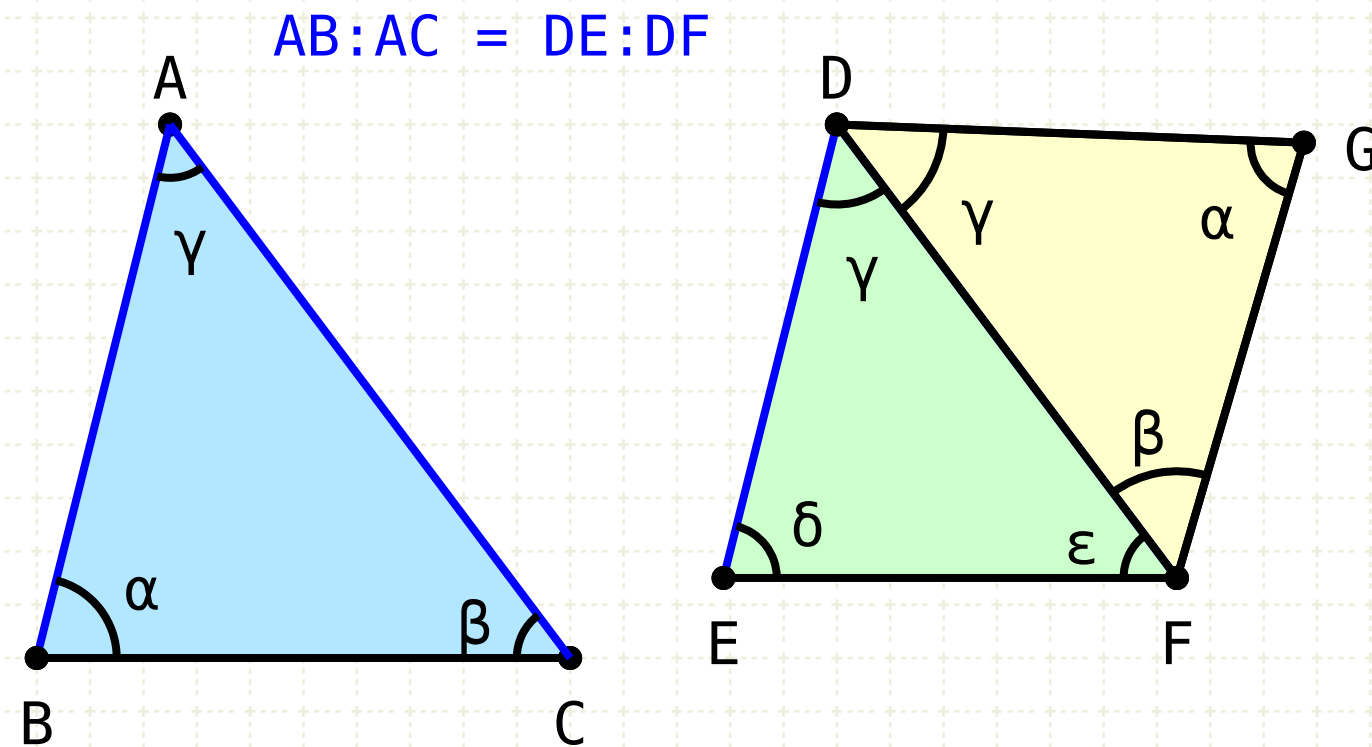
Since DE and DG have the same ratio to DF, DE and DG are equal (V·9)

Since ED equals DG, and DF is common, and the angles EDF and FDG are equal, triangle DEF is equal to the triangle DGF in all respects (I·4)



## Proposition 6 of Book VI

If two triangles have one angle equal to one angle and the sides about the equal angles proportional, the triangles will be equiangular and will have those angles equal which the corresponding sides subtend.



$$AB:AC = DG:DF$$

$$DE:DF = DG:DF$$

$$DE = DG$$

$$EF = FG$$

$$\alpha = a$$

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### Proof

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And thus, the angle at G will also be the angle at B (I·32)

Therefore the triangle ABC is equiangular to DFG, and as such, the edges surrounding the equal angles will be in proportion, i.e. AB is to AC as DG to DF (VI·4)

But the ratio AB to AC is equal to DE to DF, therefore the ratio DE to DF equals DG to DF (V·11)

Since DE and DG have the same ratio to DF, DE and DG are equal (V·9)

Since ED equals DG, and DF is common, and the angles EDF and FDG are equal, triangle DEF is equal to the triangle DGF in all respects (I·4)

So finally, the triangle DEF is equiangular to triangle ABC

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