# Euclid's Elements

# Book III



*A circle is a round straight line with a hole in the middle.*

**Mark Twain**

quoting a schoolchild in "-English as She Is Taught-"

*If people stand in a circle long enough, they'll eventually begin to dance.*

**George Carlin, Napalm and Silly Putty (2001)**

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# Proposition 19 of Book III

If a straight line touch a circle, and from the point of contact a straight line be drawn at right angles to the tangent, the centre of the circle will be on the straight line so drawn.

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## In other words

If line ED touches a circle, and a line AC is drawn perpendicular to ED at the point where it touches the circle, then AC will pass through the centre of the circle F

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Assume that AC does not pass through the centre of the circle F

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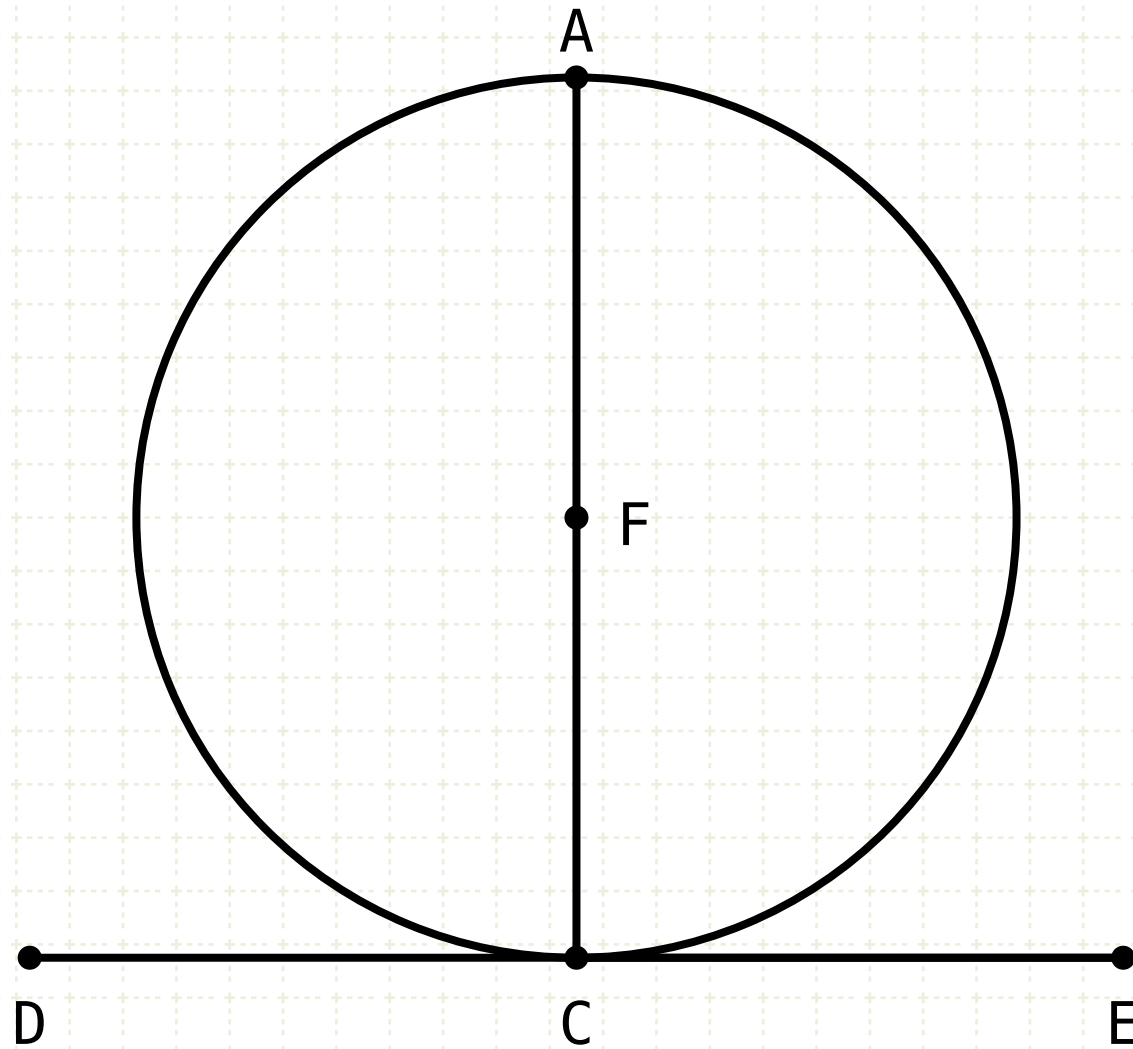
Draw the line CF

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A

$\alpha = L$
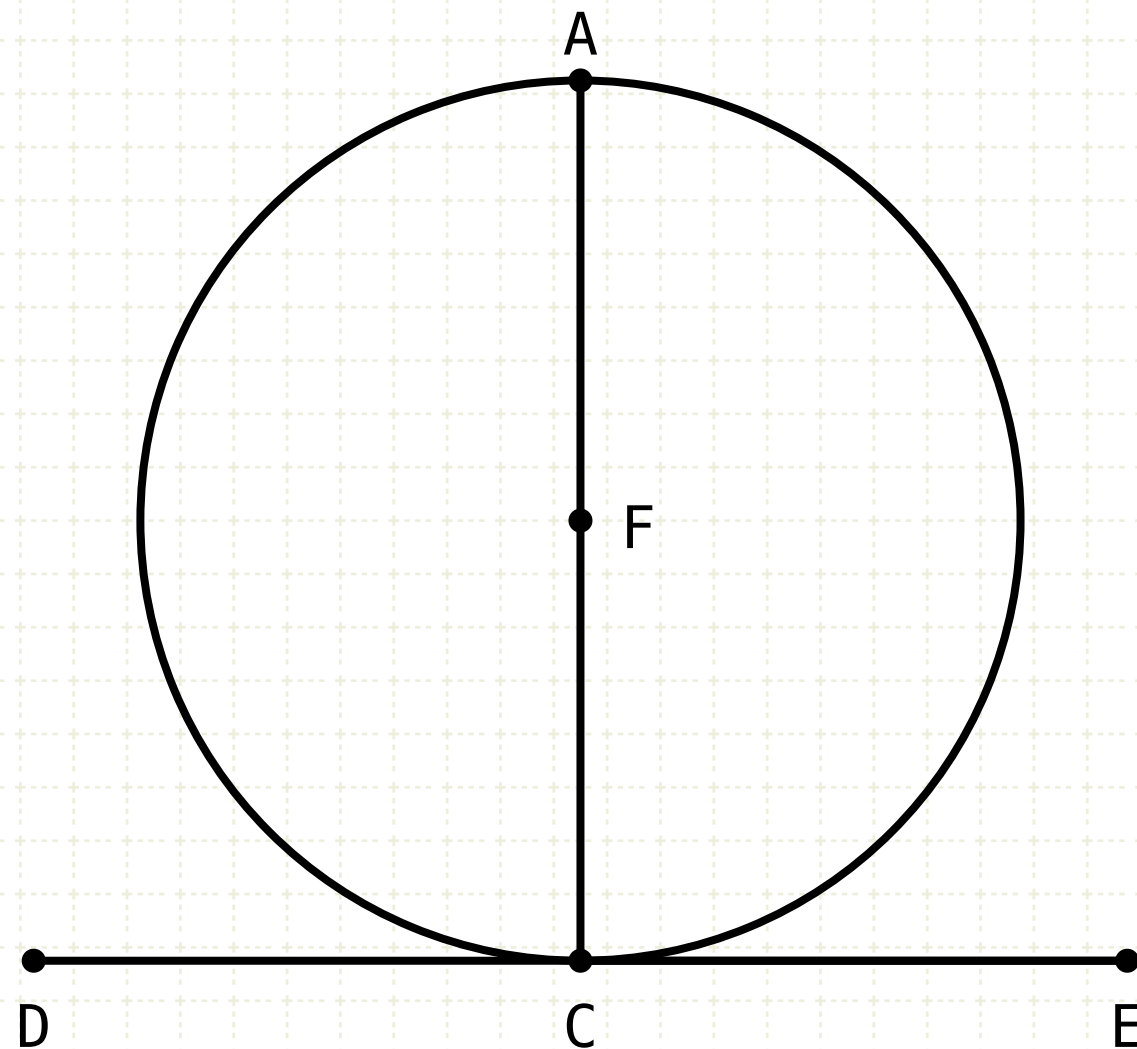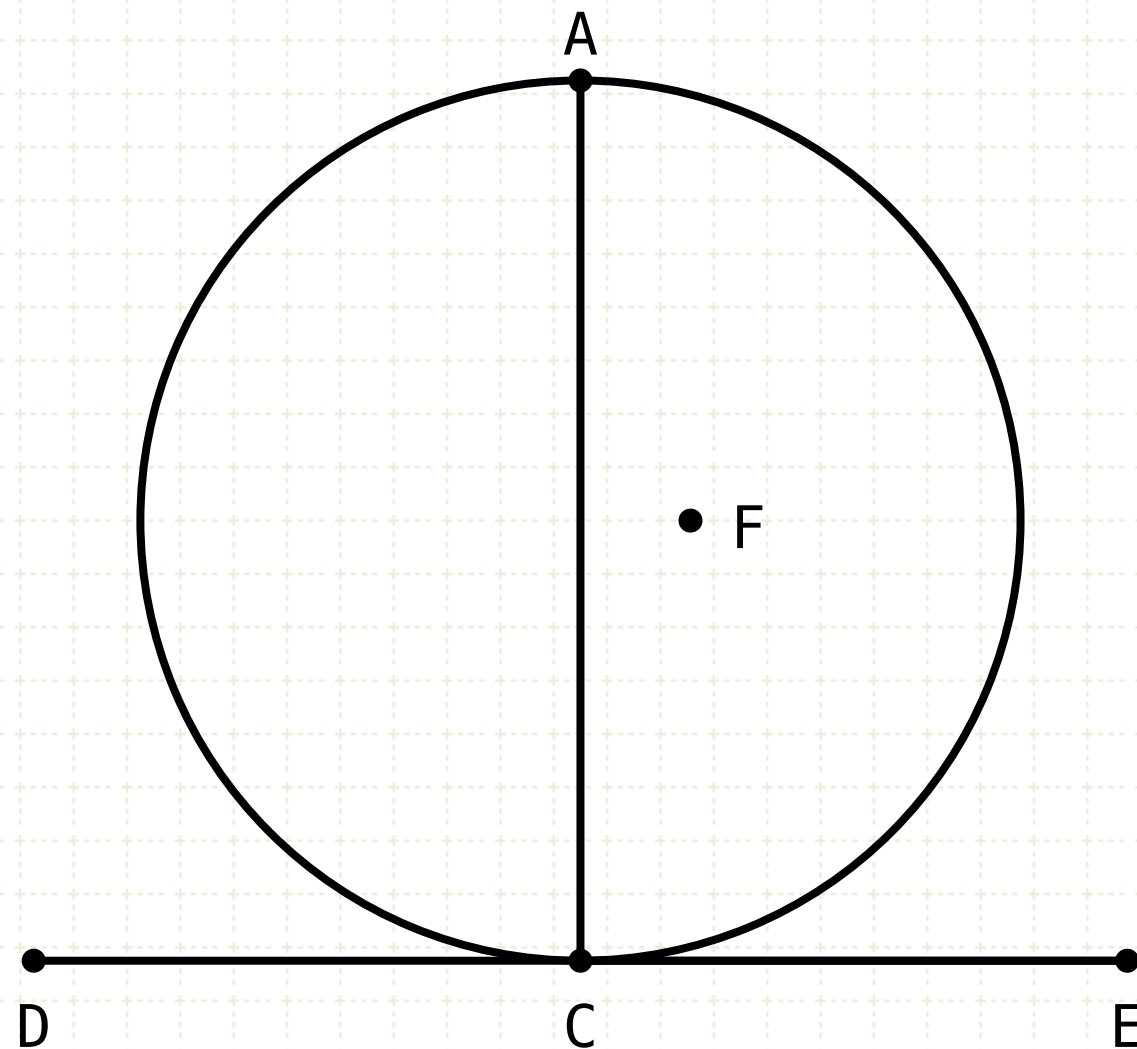
F

$\alpha$

D          C          E

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Assume that AC does not pass through the centre of the circle F

Draw the line CF

Since line DE touches the circle, a line drawn from the centre of the circle to the line DE forms a right angle (III·18), so FCE (α) is a right angle

# Proposition 19 of Book III

If a straight line touch a circle, and from the point of contact a straight line be drawn at right angles to the tangent, the centre of the circle will be on the straight line so drawn.

A

$\alpha = \llcorner$

$\varepsilon = \llcorner$

F

$\varepsilon$

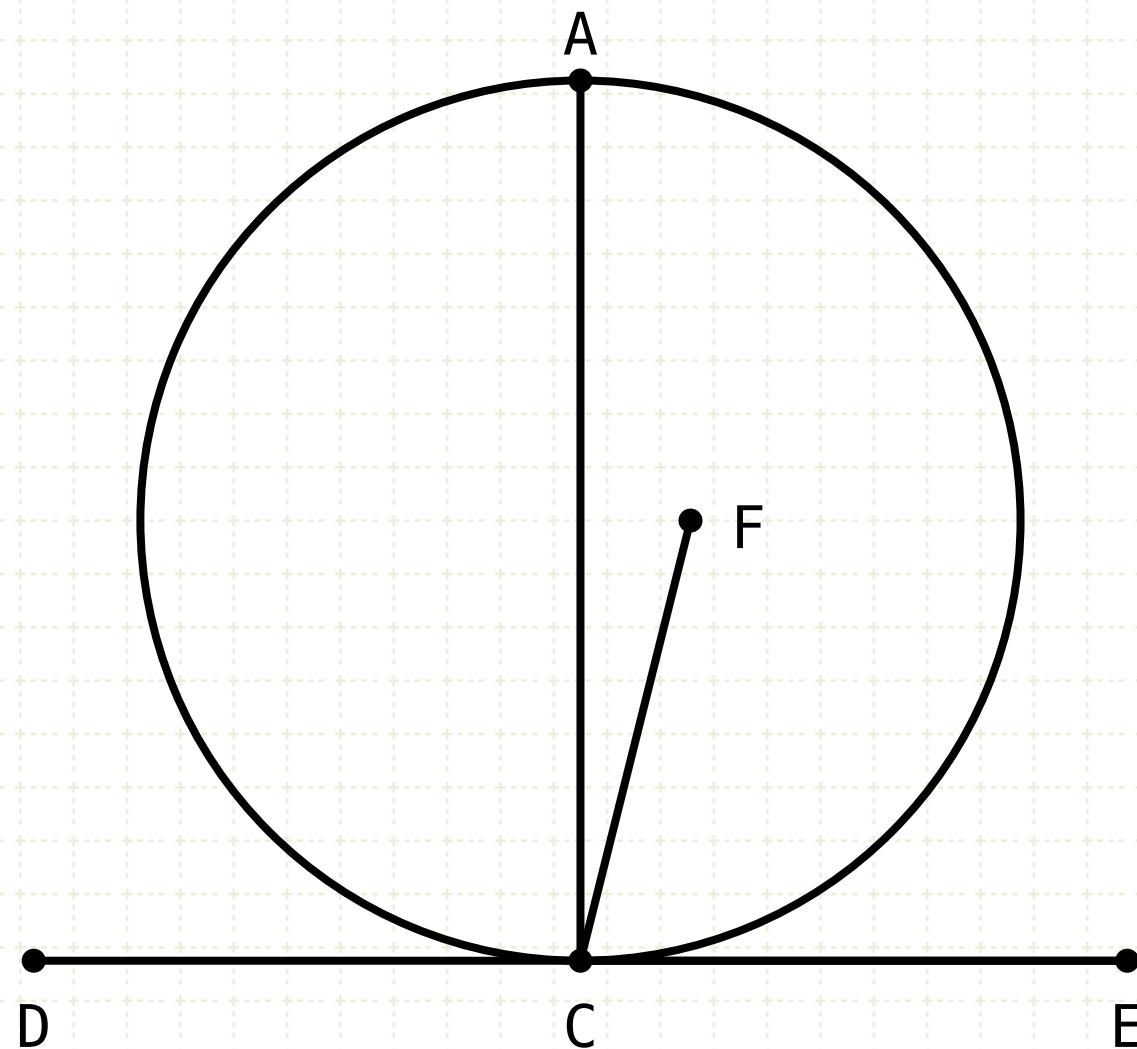D          C          E

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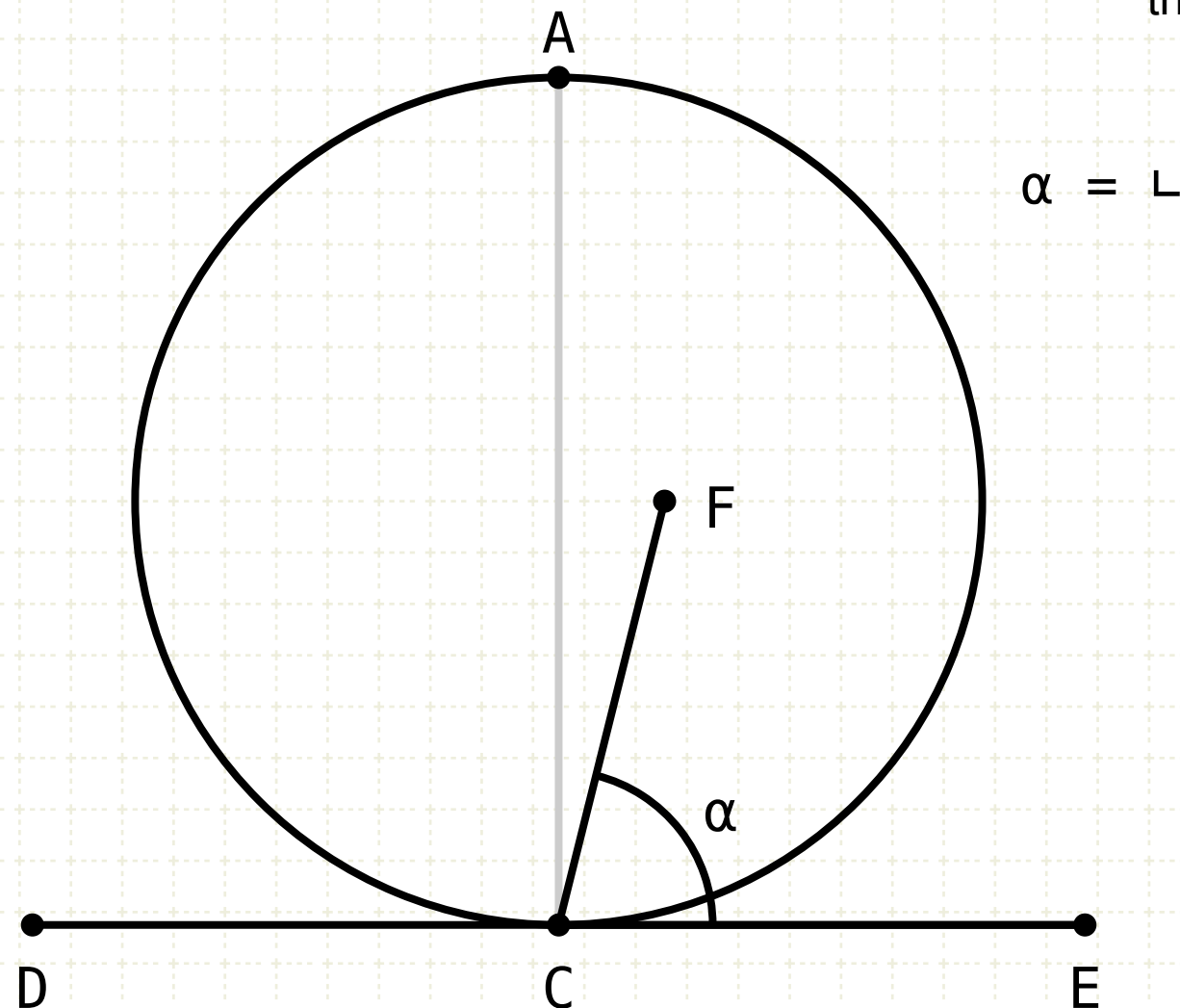
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Assume that AC does not pass through the centre of the circle F

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Since line DE touches the circle, a line drawn from the centre of the circle to the line DE forms a right angle (III·18), so FCE ($\alpha$) is a right angle

ACE ($\varepsilon$) is also a right angle (by construction)

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If a straight line touch a circle, and from the point of contact a straight line be drawn at right angles to the tangent, the centre of the circle will be on the straight line so drawn.

A

$\alpha = L$

$\epsilon = L$

F

$\therefore \; \epsilon = \alpha$

$\epsilon$

D          C          E

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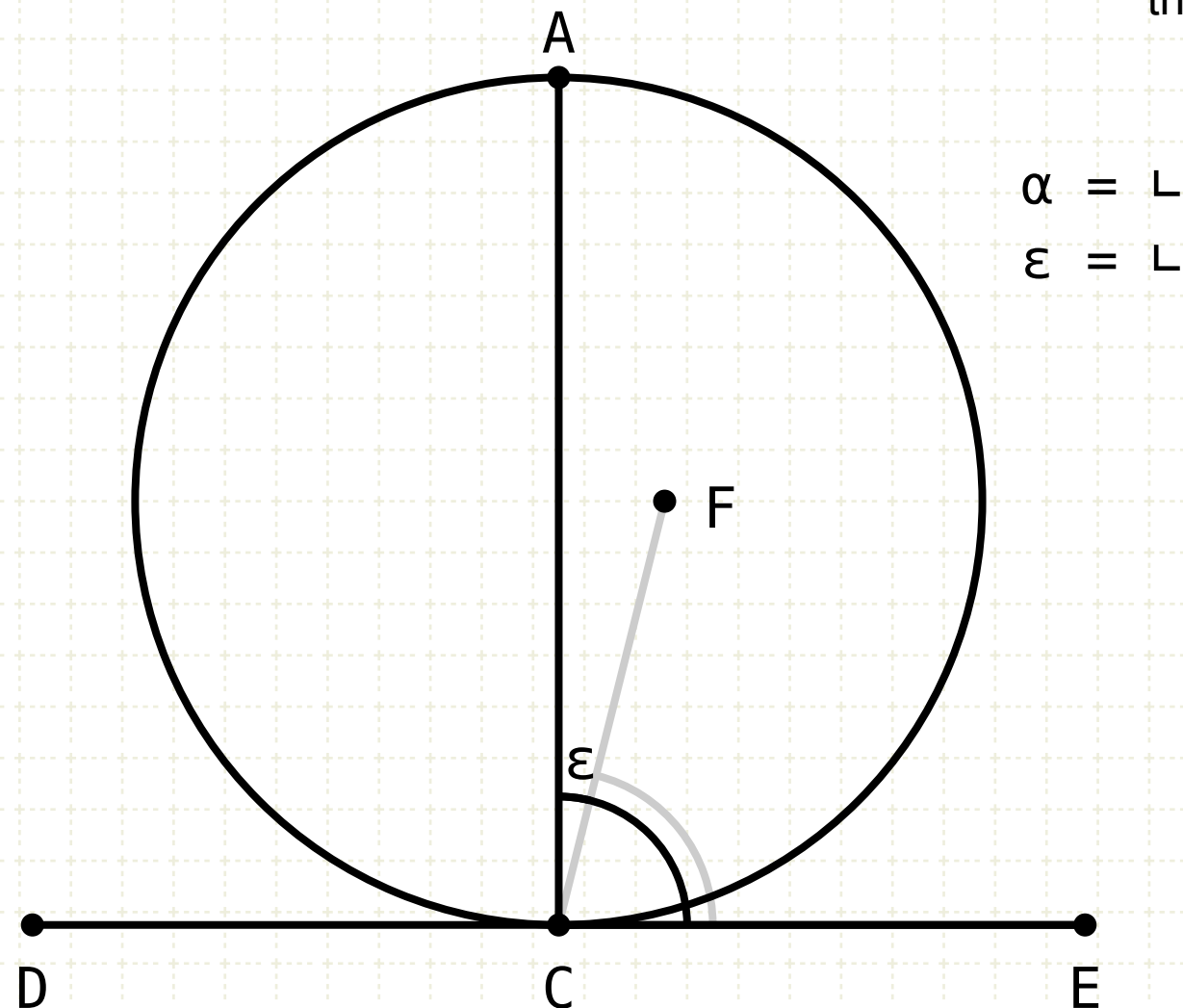
## Proof by Contradiction

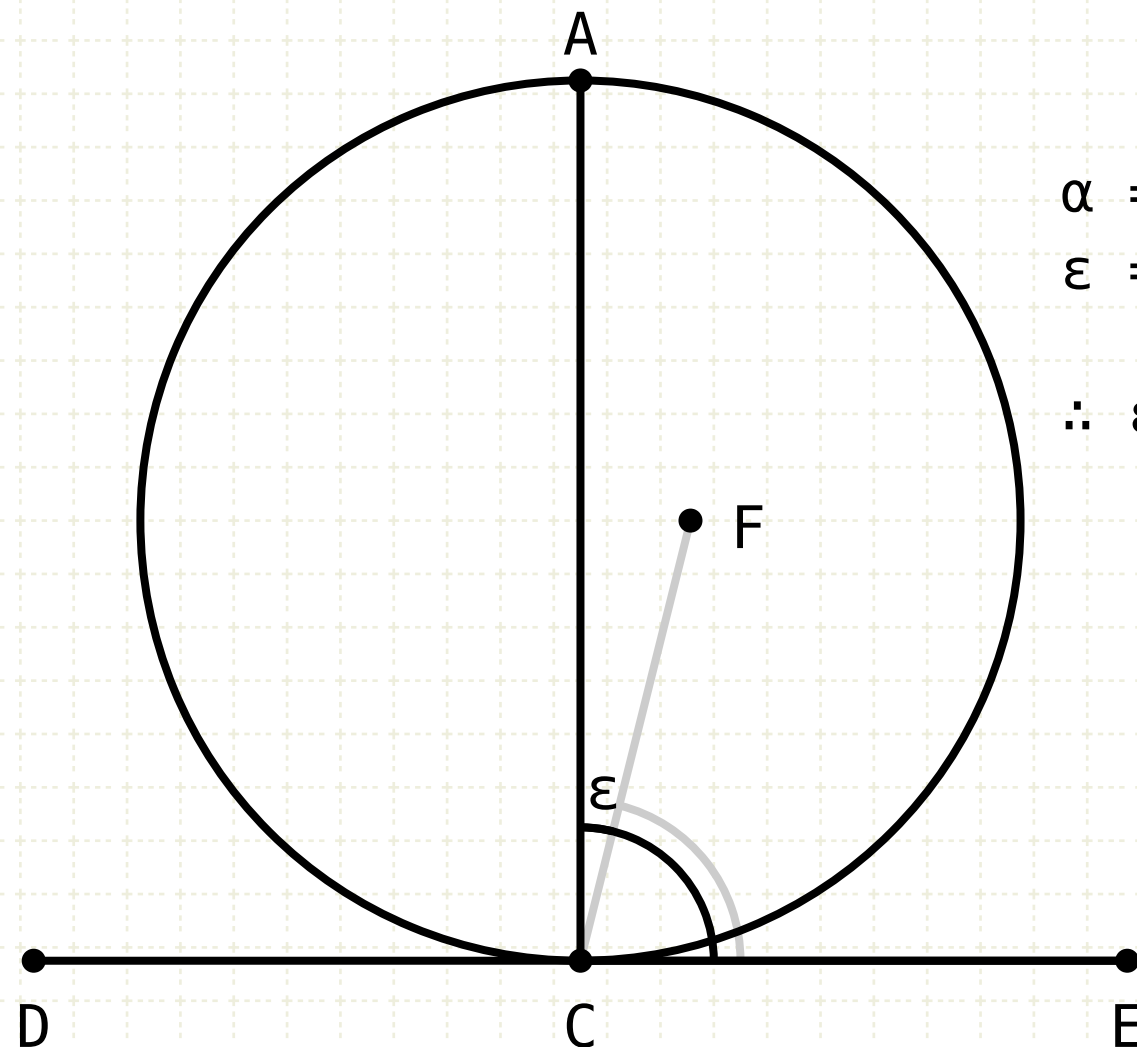Assume that AC does not pass through the centre of the circle F

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A

α = ∟

ε = ∟

∴ ε = α

F

ε

α

D          C          E

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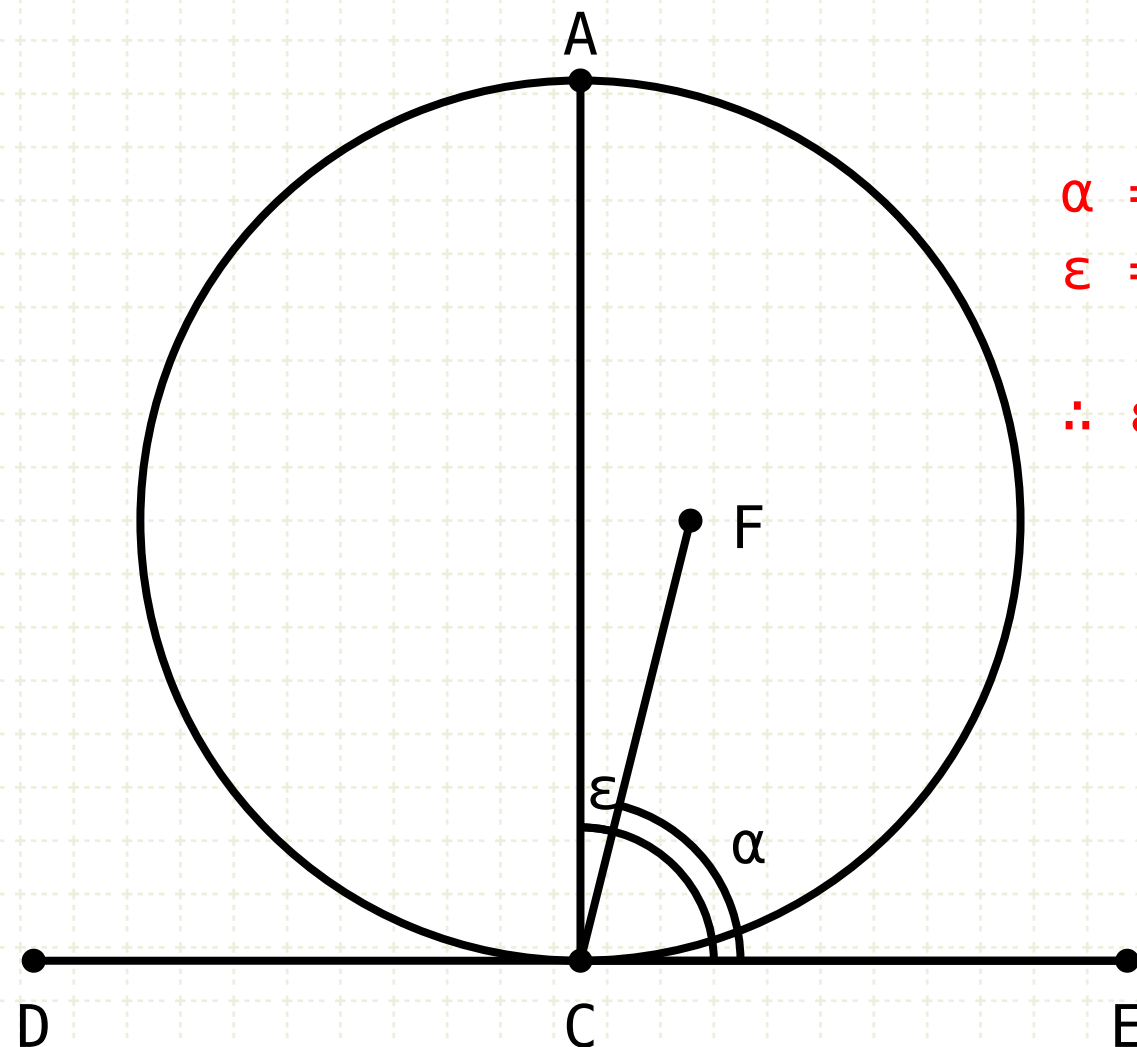
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So FCE equals ACE

This is impossible…

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$\alpha = \llcorner$

$\varepsilon = \llcorner$

$\therefore \ \varepsilon = \alpha$

F

D    C    E

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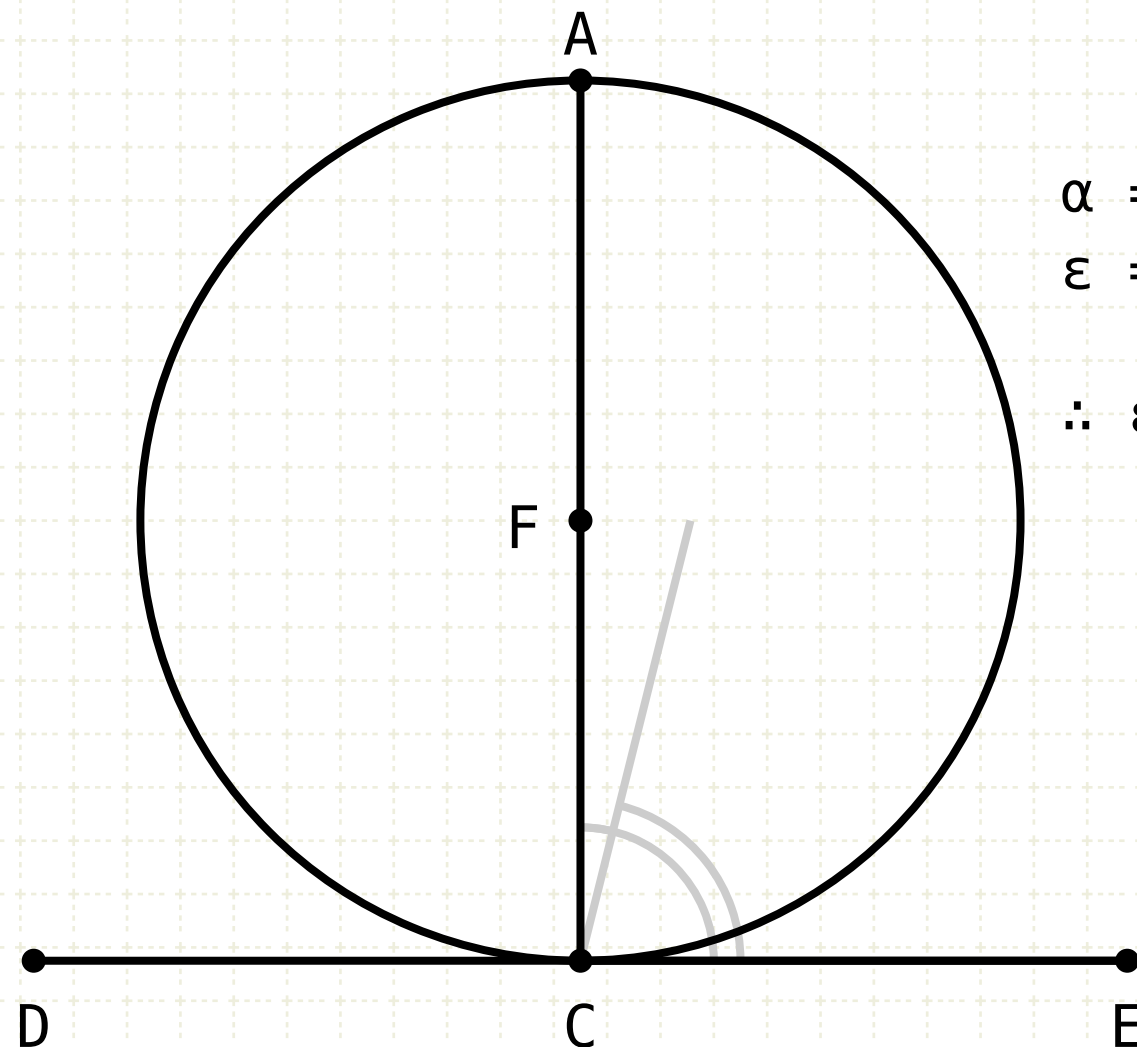
ACE (ε) is also a right angle (by construction)

So FCE equals ACE

This is impossible…

… unless the line AC passes through the centre F

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