

Euclid's Elements

Book VII

Definitions:

- 1 A unit is that by virtue of which each of the things that exist is called one
- 2 A number is a multitude composed of units. (not one)
- 3 A number is part of a number, the less of the greater, when it measures the greater
- 11 A prime number is that which is measured by a unit alone.
- 12 Numbers prime to one another are those which are measured by a unit alone as a common measure
- 15 A number is said to multiply a number when that which is multiplied is added to itself as many times as there are units in the other, and thus some number is produced.
- 20 Numbers are proportional when the first is the same multiple, or the same part, or the same parts, of the second that the third is of the fourth.

As long as algebra and geometry have been separated, their progress have been slow and their uses limited; but when these two sciences have been united, they have lent each mutual forces, and have marched together towards perfection.

Joseph-Louis Lagrange
(1736 to 1813)



Table of Contents, Chapter 7

1	Determine if two numbers are relatively prime	10	If $B = (p/q) \cdot A$ and $D = (p/q) \cdot C$, and If $B = (r/s) \cdot D$, then $A = (r/s) \cdot C$	21	If A,B are relatively prime, then A,B are the smallest whole numbers that can be used to describe the ratio A:B
2	Find the greatest common divisor for two numbers	11	If $A:B = C:D$, then $(A-C):(B-D) = A:B$	22	If A,B are the smallest whole numbers that can be used to describe the ratio A:B, then A,B are relatively prime
3	Find the largest common divisor for three numbers	12	If $A:B = C:D$, then $(A+C):(B+C) = A:B$	23	If A,B are relatively prime and if $A = n \cdot C$, then B,C are relatively prime
4	Given two natural numbers, A and B, either B is part of A, or there exists a natural number (a part) that can measure both A and B	13	If $A:B = C:D$, then $A:C = B:D$	24	If A,C are relatively prime and B,C are relatively prime then the $A \times B$ is relatively prime to C
5	If $B = (1/q) \cdot A$ and $D = (1/q) \cdot C$, then $(B+D) = (1/q) \cdot (A+C)$	14	If $A:B = D:E$ and $B:C = E:F$, then $A:C = D:F$	25	If A,B are relatively prime then A^2, B are relatively prime
6	If $B = (p/q) \cdot A$ and $D = (p/q) \cdot C$, then $(B+D) = (p/q) \cdot (A+C)$	15	If $B = i \cdot 1$ and $E = i \cdot D$, and if $D = j \cdot 1$ then $E = j \cdot B$	26	If A is relatively prime to C and D, and if B is also relatively prime to C and D, then $A \times B$ is relatively prime to $C \times D$
7	If $B = A/q$ and $D = C/q$, $B > D$, then $(B-D) = (A-C)/q$	16	$A \times B = B \times A$	27	If A,B are relatively prime, then A^2, B^2 are relatively prime, and A^3, B^3 are relatively prime, and so on
8	If $B = (p/q) \cdot A$ and $D = (p/q) \cdot C$, $B > D$, then $(B-D) = (p/q) \cdot (A-C)$	17	If $D = A \times B$ and $E = A \times C$ then $D:E = B:C$		
9	If $B = (1/q) \cdot A$ and $D = (1/q) \cdot C$, and If $B = (r/s) \cdot D$, then $A = (r/s) \cdot C$	18	If $D = B \times A$ and $E = C \times A$ then $D:E = B:C$		
		19	If $A:B = C:D$ then $A \times D = B \times C$ If $A \times D = B \times C$ then $A:B = C:D$		
		20	Given the ratio A:B and C,D are the smallest numbers such that $A:B = C:D$ then $A = n \cdot C$ and $B = n \cdot D$		



Table of Contents, Chapter 7

- | | | | |
|----|--|----|---|
| 28 | If A,B are relatively prime, then A,(A+B) are relatively prime | 37 | If $A = p \cdot B$, then $A = q \cdot C$ where $C = p \cdot 1$ |
| 29 | If A is prime, and $B \neq n \cdot A$, then A,B are relatively prime | 38 | If $A = (1/c) \cdot B$ and $C = c \cdot 1$ then $A = n \cdot C$ |
| 30 | If $C = A \times B$ and $C = i \cdot D$ where D is prime, then either $A = j \cdot D$ or $B = j \cdot D$ | 39 | Find the smallest number that has the fractions $1/a, 1/b, 1/c$ |
| 31 | If $A = B \times C$, then $A = j \cdot D$ where D is prime | | |
| 32 | If A is a number then it is either prime, or $A = j \cdot D$ where D is prime | | |
| 33 | Find the smallest numbers X,Y,Z where the ratio X:Y:Z is equal to the given ratio A:B:C | | |
| 34 | Find the lowest common denominator of 2 numbers | | |
| 35 | If E is the lowest common denominator of A,B, and if $C = n \cdot A = m \cdot B$, then $C = i \cdot E$ | | |
| 36 | Find the least common multiple of 3 numbers | | |



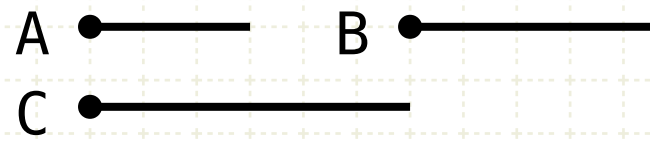
Proposition 36 of Book VII

Given three numbers, to find the least number which they measure.



Proposition 36 of Book VII

Given three numbers, to find the least number which they measure.



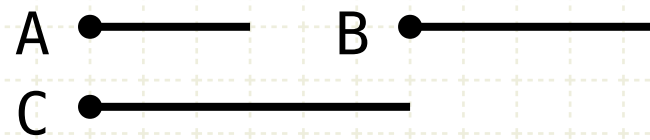
In other words

Let A,B,C be any three numbers



Proposition 36 of Book VII

Given three numbers, to find the least number which they measure.



In other words

Let A,B,C be any three numbers

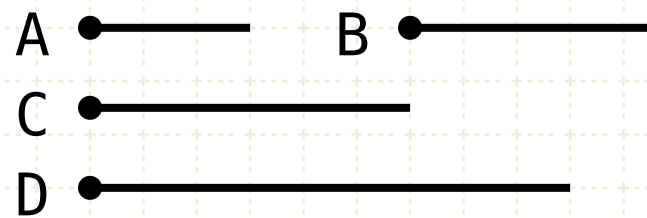
Find the least common multiple

$$\text{lcm}(A,B,C) = ?$$



Proposition 36 of Book VII

Given three numbers, to find the least number which they measure.



$$\text{lcm}(A, B) = D$$

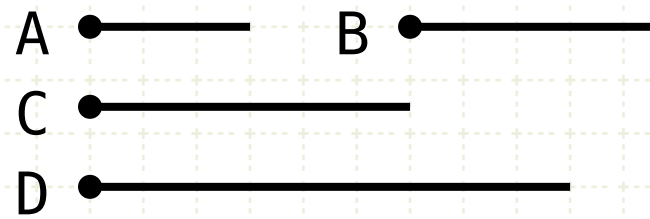
Method

Let D be the least common multiple of A and B (VII·34)



Proposition 36 of Book VII

Given three numbers, to find the least number which they measure.



$$\text{lcm}(A, B) = D$$

$$D = p \cdot C$$

Method

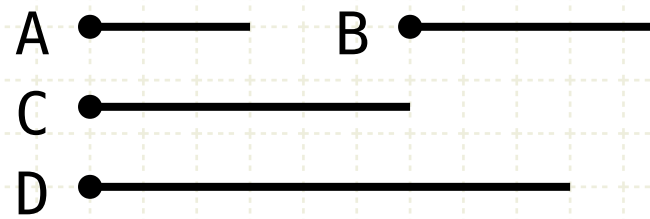
Let D be the least common multiple of A and B (VII·34)

If C either measures D , or it does not. If it does...



Proposition 36 of Book VII

Given three numbers, to find the least number which they measure.



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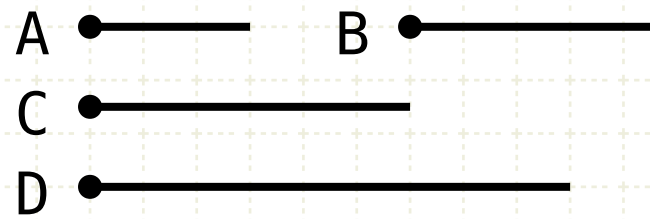
If C either measures D, or it does not. If it does...

... then D is the least common multiple of A,B and C



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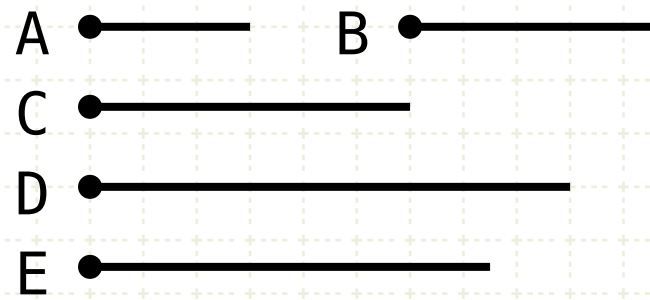
... then D is the least common multiple of A,B and C

Proof by Contradiction



Proposition 36 of Book VII

Given three numbers, to find the least number which they measure.



$$\text{lcm}(A, B) = D$$

$$D = p \cdot C$$

$$E < D$$

$$E = m \cdot A$$

$$E = n \cdot B$$

$$E = r \cdot C$$

Method

Let D be the least common multiple of A and B (VII·34)

If C either measures D, or it does not. If it does...

... then D is the least common multiple of A,B and C

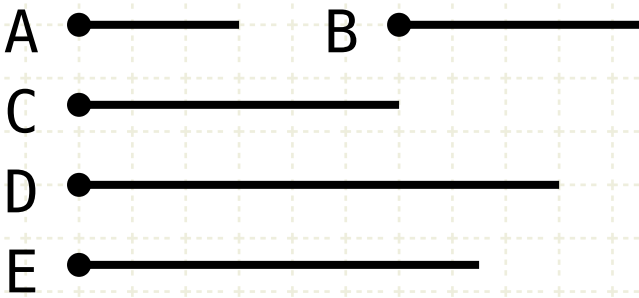
Proof by Contradiction

Assume that E, a number less than D, is measured by A, B and C



Proposition 36 of Book VII

Given three numbers, to find the least number which they measure.



$$\text{lcm}(A, B) = D$$

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Proof by Contradiction

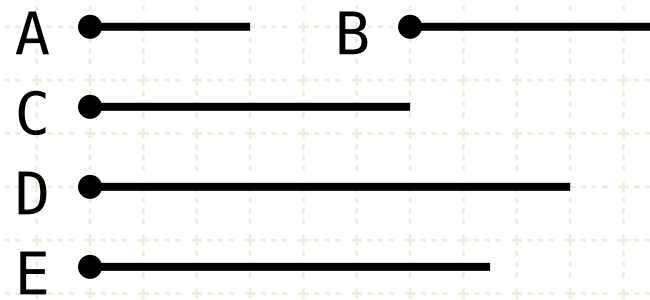
Assume that E, a number less than D, is measured by A, B and C

If E is measured by A,B and C, it is also measured by A and B



Proposition 36 of Book VII

Given three numbers, to find the least number which they measure.



$$\text{lcm}(A, B) = D$$

$$D = p \cdot C$$

$$E < D$$

$$E = m \cdot A$$

$$E = n \cdot B$$

$$E = r \cdot C$$

$$E = q \cdot D$$

Method

Let D be the least common multiple of A and B (VII·34)

If C either measures D, or it does not. If it does...

... then D is the least common multiple of A,B and C

Proof by Contradiction

Assume that E, a number less than D, is measured by A, B and C

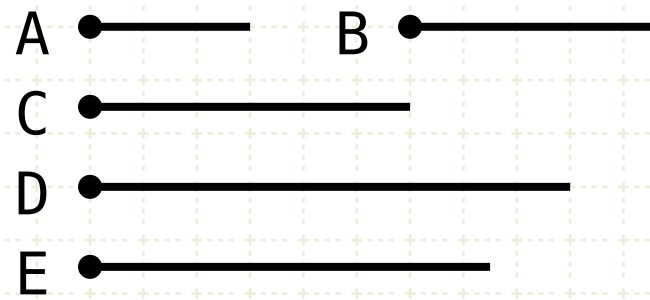
If E is measured by A,B and C, it is also measured by A and B

But if A and B both measure E, then the least common multiple (D) will also measure E (VII·35)



Proposition 36 of Book VII

Given three numbers, to find the least number which they measure.



$$\text{lcm}(A, B) = D$$

$$D = p \cdot C$$

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Let D be the least common multiple of A and B (VII·34)

If C either measures D, or it does not. If it does...

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Proof by Contradiction

Assume that E, a number less than D, is measured by A, B and C

If E is measured by A,B and C, it is also measured by A and B

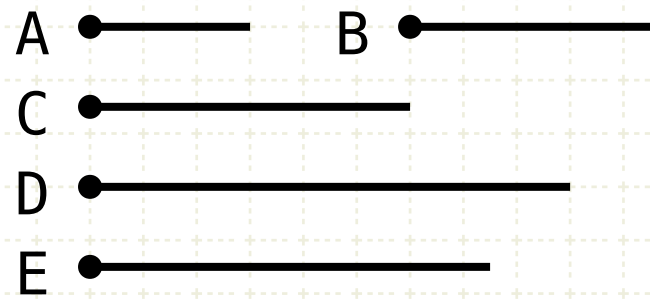
But if A and B both measure E, then the least common multiple (D) will also measure E (VII·35)

But D cannot measure E, and also be less than E



Proposition 36 of Book VII

Given three numbers, to find the least number which they measure.



$$\text{lcm}(A, B) = D$$

$$D = p \cdot C$$

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$$E = r \cdot C$$

$$E = q \cdot D$$

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Let D be the least common multiple of A and B (VII·34)

If C either measures D, or it does not. If it does...

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Assume that E, a number less than D, is measured by A, B and C

If E is measured by A,B and C, it is also measured by A and B

But if A and B both measure E, then the least common multiple (D) will also measure E (VII·35)

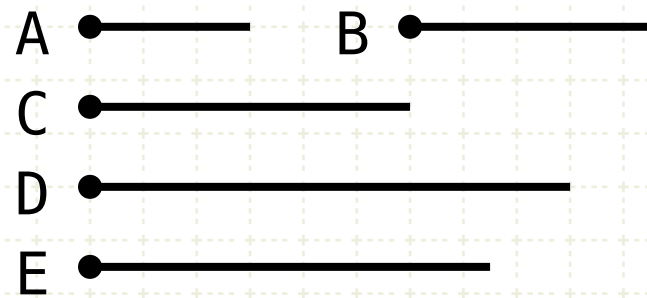
But D cannot measure E, and also be less than E

Thus, there is no number E, less than D, which is a multiple of A, B and C



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Given three numbers, to find the least number which they measure.



$$\text{lcm}(A, B) = D$$

$$D = p \cdot C$$

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$$E = m \cdot A$$

$$E = n \cdot B$$

$$E = r \cdot C$$

$$E = q \cdot D$$

$$\text{lcm}(A, B, C) = D$$

Method

Let D be the least common multiple of A and B (VII·34)

If C either measures D, or it does not. If it does...

... then D is the least common multiple of A, B and C

Proof by Contradiction

Assume that E, a number less than D, is measured by A, B and C

If E is measured by A, B and C, it is also measured by A and B

But if A and B both measure E, then the least common multiple (D) will also measure E (VII·35)

But D cannot measure E, and also be less than E

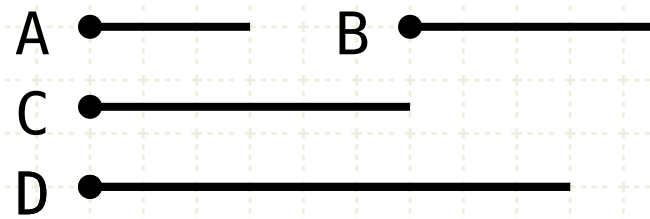
Thus, there is no number E, less than D, which is a multiple of A, B and C

So D is the least common multiple of A, B and C



Proposition 36 of Book VII

Given three numbers, to find the least number which they measure.



$$\text{lcm}(A, B) = D$$

$$D \neq p \cdot C$$

Method

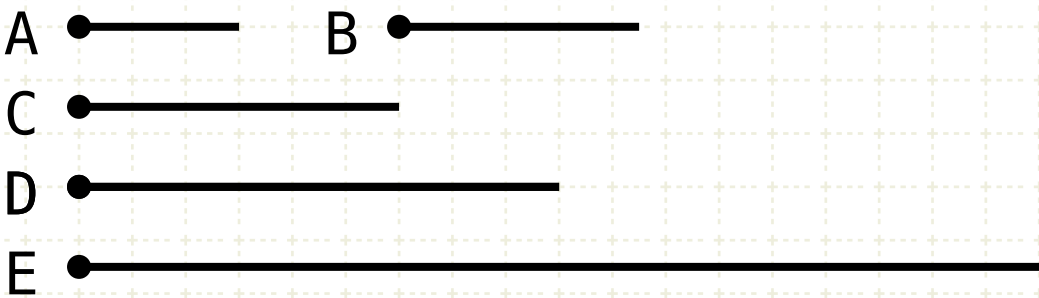
Let D be the least common multiple of A and B (VII·34)

If C either measures D, or it does not. If it does not...



Proposition 36 of Book VII

Given three numbers, to find the least number which they measure.



$$\text{lcm}(A, B) = D$$

$$D \neq p \cdot C$$

$$\text{lcm}(C, D) = E$$

Method

Let D be the least common multiple of A and B (VII·34)

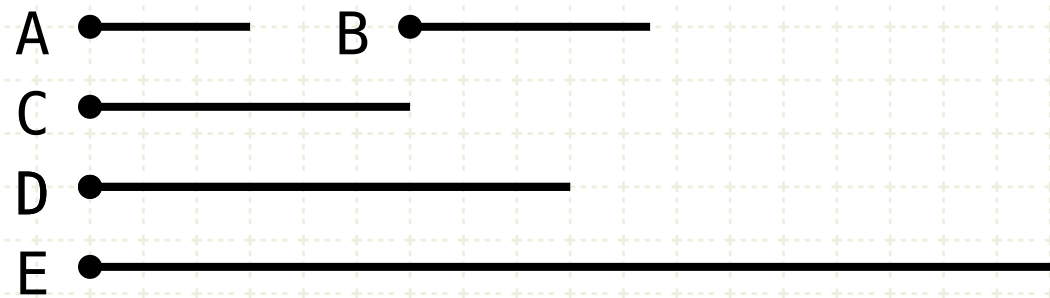
If C either measures D, or it does not. If it does not...

Let E be the least common measure of C and D (VII·34)



Proposition 36 of Book VII

Given three numbers, to find the least number which they measure.



$$\text{lcm}(A, B) = D$$

$$D \neq p \cdot C$$

$$\text{lcm}(C, D) = E$$

$$D = m \cdot A = n \cdot B$$

$$E = p \cdot C = q \cdot D$$

$$\therefore E = q \cdot (m \cdot A) = q \cdot (n \cdot B)$$

Method

Let D be the least common multiple of A and B (VII·34)

If C either measures D, or it does not. If it does not...

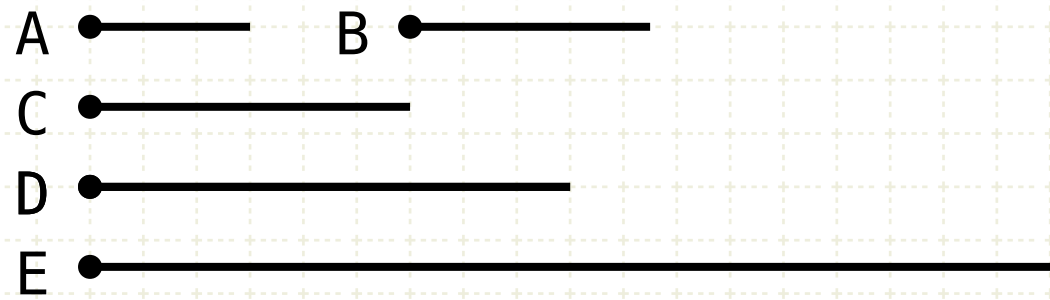
Let E be the least common measure of C and D (VII·34)

Since A and B measure D, and D measures E, then A and B measure E



Proposition 36 of Book VII

Given three numbers, to find the least number which they measure.



$$\text{lcm}(A, B) = D$$

$$D \neq p \cdot C$$

$$\text{lcm}(C, D) = E$$

$$D = m \cdot A = n \cdot B$$

$$E = p \cdot C = q \cdot D$$

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$$\text{lcm}(A, B, C) = E$$

Method

Let D be the least common multiple of A and B (VII·34)

If C either measures D, or it does not. If it does not...

Let E be the least common measure of C and D (VII·34)

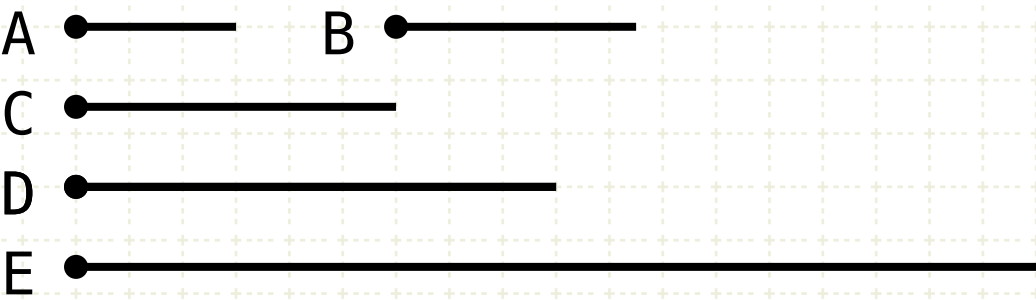
Since A and B measure D, and D measures E, then A and B measure E

Thus A, B and C measures E, and E is the least common multiple



Proposition 36 of Book VII

Given three numbers, to find the least number which they measure.



$$\text{lcm}(A, B) = D$$

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Let D be the least common multiple of A and B (VII·34)

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Let E be the least common measure of C and D (VII·34)

Since A and B measure D, and D measures E, then A and B measure E

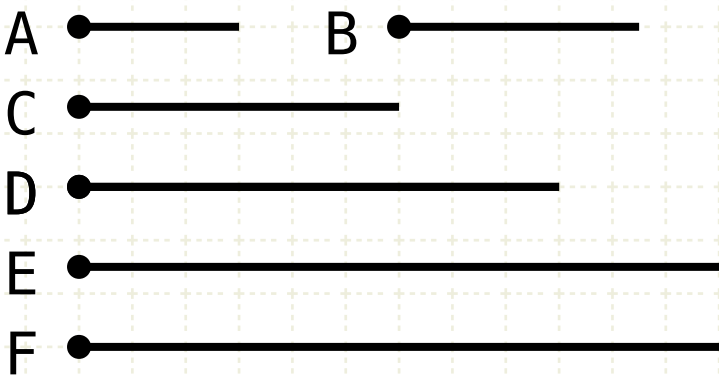
Thus A, B and C measures E, and E is the least common multiple

Proof by Contradiction



Proposition 36 of Book VII

Given three numbers, to find the least number which they measure.



$$\text{lcm}(A, B) = D$$

$$D \neq p \cdot C$$

$$\text{lcm}(C, D) = E$$

$$F = r \cdot A = s \cdot B = t \cdot C$$

$$F < E$$

Method

Let D be the least common multiple of A and B (VII·34)

If C either measures D, or it does not. If it does not...

Let E be the least common measure of C and D (VII·34)

Since A and B measure D, and D measures E, then A and B measure E

Thus A, B and C measures E, and E is the least common multiple

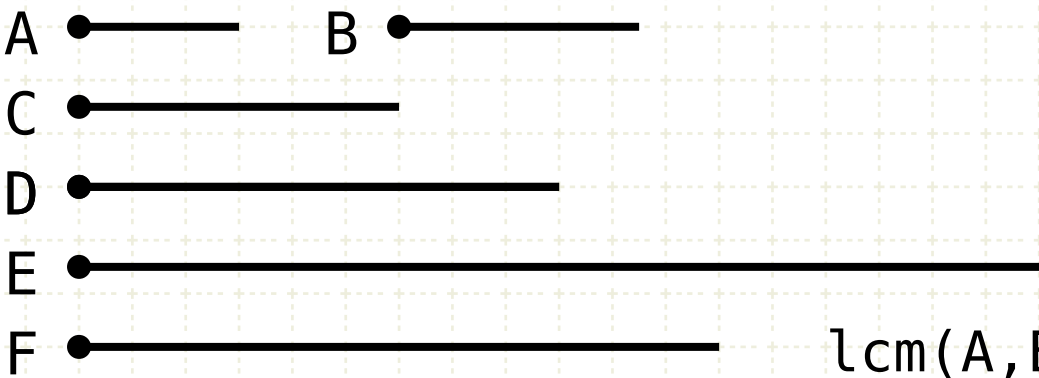
Proof by Contradiction

Let A, B and C measure F, which is smaller than E



Proposition 36 of Book VII

Given three numbers, to find the least number which they measure.



$$\text{lcm}(A,B) = D$$

$$D \neq p \cdot C$$

$$\text{lcm}(C,D) = E$$

$$F = r \cdot A = s \cdot B = t \cdot C$$

$$F < E$$

$$F = i \cdot D$$

Method

Let D be the least common multiple of A and B (VII·34)

If C either measures D, or it does not. If it does not...

Let E be the least common measure of C and D (VII·34)

Since A and B measure D, and D measures E, then A and B measure E

Thus A, B and C measures E, and E is the least common multiple

Proof by Contradiction

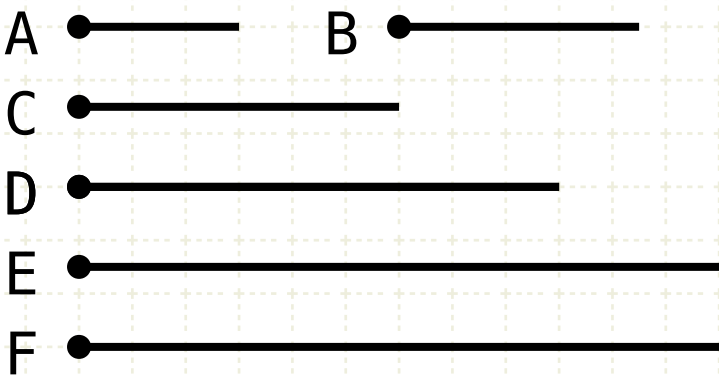
Let A, B and C measure F, which is smaller than E

If A and B measure F, then the least common multiple of A and B will also measure F (VII·35)



Proposition 36 of Book VII

Given three numbers, to find the least number which they measure.



$$\text{lcm}(A,B) = D$$

$$D \neq p \cdot C$$

$$\text{lcm}(C,D) = E$$

$$F = r \cdot A = s \cdot B = t \cdot C$$

$$F < E$$

$$F = i \cdot D$$

$$F = j \cdot E$$

Method

Let D be the least common multiple of A and B (VII·34)

If C either measures D, or it does not. If it does not...

Let E be the least common measure of C and D (VII·34)

Since A and B measure D, and D measures E, then A and B measure E

Thus A, B and C measures E, and E is the least common multiple

Proof by Contradiction

Let A, B and C measure F, which is smaller than E

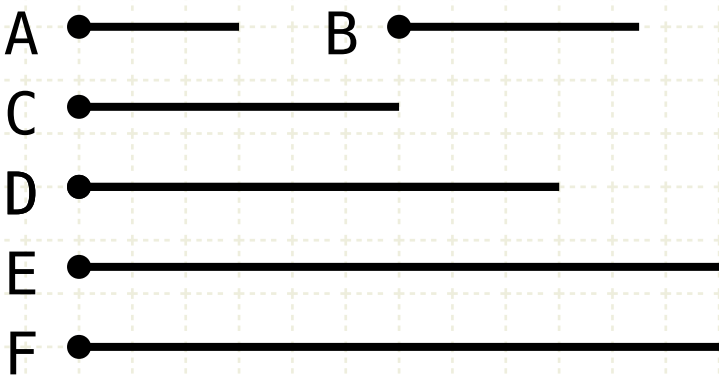
If A and B measure F, then the least common multiple of A and B will also measure F (VII·35)

But C and D also measures F, so the lowest common multiple of C and D, which is E, will also measure F (VII·35).



Proposition 36 of Book VII

Given three numbers, to find the least number which they measure.



$$\text{lcm}(A, B) = D$$

$$D \neq p \cdot C$$

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Let D be the least common multiple of A and B (VII·34)

If C either measures D, or it does not. If it does not...

Let E be the least common measure of C and D (VII·34)

Since A and B measure D, and D measures E, then A and B measure E

Thus A, B and C measures E, and E is the least common multiple

Proof by Contradiction

Let A, B and C measure F, which is smaller than E

If A and B measure F, then the least common multiple of A and B will also measure F (VII·35)

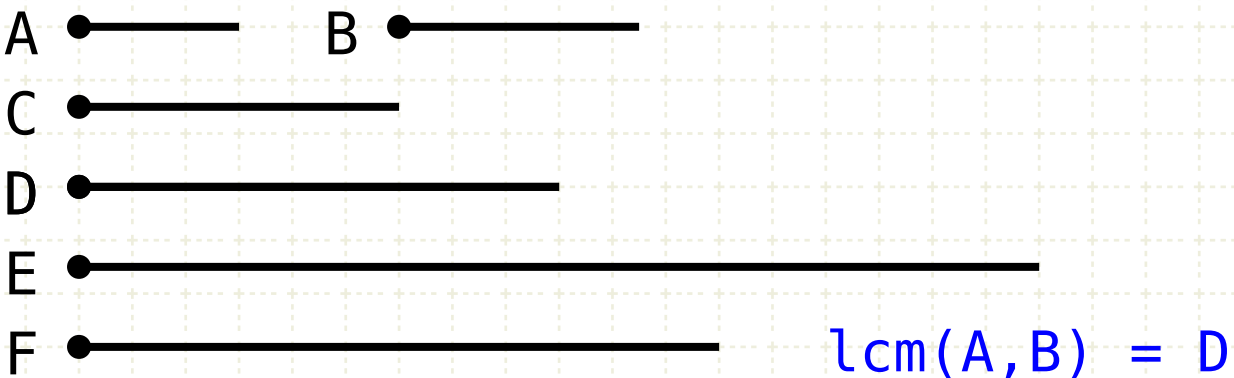
But C and D also measures F, so the lowest common multiple of C and D, which is E, will also measure F (VII·35).

But F cannot be measured by E while also be less than E



Proposition 36 of Book VII

Given three numbers, to find the least number which they measure.



$\text{lcm}(A,B) = D$

$D \neq p \cdot C$

$\text{lcm}(C,D) = E$

$F = r \cdot A = s \cdot B = t \cdot C$

$F < E$

$F = i \cdot D$

$F = j \cdot E$

$\text{lcm}(A,B,C) = E$

Method

Let D be the least common multiple of A and B (VII·34)

If C either measures D, or it does not. If it does not...

Let E be the least common measure of C and D (VII·34)

Since A and B measure D, and D measures E, then A and B measure E

Thus A, B and C measures E, and E is the least common multiple

Proof by Contradiction

Let A, B and C measure F, which is smaller than E

If A and B measure F, then the least common multiple of A and B will also measure F (VII·35)

But C and D also measures F, so the lowest common multiple of C and D, which is E, will also measure F (VII·35).

But F cannot be measured by E while also be less than E

Hence, no number less than E can measure A, B and C, and thus E is the lowest common multiple



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