Euclid's Elements

Book VII

Definitions:

1 A unit is that by virtue of which each of the things that exist is called one

2 A number is a multitude composed of units. (not one)

3 A number is part of a number, the less of the greater, when it measures the greater

11 A prime number is that which is measured by a unit alone.

12 Numbers prime to one another are those which are measured by a unit alone as a common measure

15 A number is said to multiply a number when that which is multiplied is added to itself as many times as there are units in the other, and thus some number is produced.

20 Numbers are proportional when the first is the same multiple, or the same part, or the same parts, of the second that the third is of the fourth.

As long as algebra and geometry have been separated, their progress have been slow and their uses limited; but when these two sciences have been united, they have lent each mutual forces, and have marched together towards perfection.

Joseph-Louis Lagrange
(1736 to 1813)

Table of Contents, Chapter 7

1 Determine if two numbers are relatively prime

2 Find the greatest common divisor for two numbers

3 Find the largest common divisor for three numbers

4 Given two natural numbers, A and B, either B is part of A, or there exists a natural number (a part) that can measure both A and B

5 If $B = (1/q)\cdot A$ and $D = (1/q)\cdot C$, then $(B+D) = (1/q)\cdot(A+C)$

6 If $B = (p/q)\cdot A$ and $D = (p/q)\cdot C$, then $(B+D) = (p/q)\cdot(A+C)$

7 If $B = A/q$ and $D = C/q$, $B>D$, then $(B-D) = (A-C)/q$

8 If $B = (p/q)\cdot A$ and $D = (p/q)\cdot C$, $B>D$, then $(B-D) = (p/q)\cdot(A-C)$

9 If $B = (1/q)\cdot A$ and $D = (1/q)\cdot C$, and If $B = (r/s)\cdot D$, then $A = (r/s)\cdot C$

10 If $B = (p/q)\cdot A$ and $D = (p/q)\cdot C$, and If $B = (r/s)\cdot D$, then $A = (r/s)\cdot C$

11 If $A:B = C:D$, then $(A-C):(B-D) = A:B$

12 If $A:B = C:D$, then $(A+C):(B+C) = A:B$

13 If $A:B = C:D$, then $A:C = B:D$

14 If $A:B = D:E$ and $B:C = E:F$, then $A:C = D:F$

15 If $B = i\cdot 1$ and $E = i\cdot D$, and if $D = j\cdot 1$ then $E = j\cdot B$

16 $A \times B = B \times A$

17 If $D = A \times B$ and $E = A \times C$ then $D:E = B:C$

18 If $D = B \times A$ and $E = C \times A$ then $D:E = B:C$

19 If $A:B = C:D$ then $A \times D = B \times C$
If $A \times D = B \times C$ then $A:B = C:D$

20 Given the ratio A:B and C,D are the smallest numbers such that $A:B = C:D$ then $A = n\cdot C$ and $B = n\cdot D$

21 If A,B are relatively prime, then A,B are the smallest whole numbers that can be used to describe the ratio A:B

22 If A,B are the smallest whole numbers that can be used to describe the ratio A:B, then A,B are relatively prime

23 If A,B are relatively prime and if $A = n\cdot C$, then B,C are relatively prime

24 If A,C are relatively prime and B,C are relatively prime then the $A \times B$ is relatively prime to C

25 If A,B are relatively prime then A^2,B are relatively prime

26 If A is relatively prime to C and D, and if B is also relatively prime to C and D, then $A \times B$ is relatively prime to $C \times D$

27 If A,B are relatively prime, then A^2,B^2 are relatively prime, and A^3,B^3 are relatively prime, and so on

Table of Contents, Chapter 7

28 If A,B are relatively prime, then A,(A+B) are relatively prime

29 If A is prime, and $B \neq n \cdot A$, then A,B are relatively prime

30 If $C = A \times B$ and $C = i \cdot D$ where D is prime, then either $A = j \cdot D$ or $B = j \cdot D$

31 If $A = B \times C$, then $A = j \cdot D$ where D is prime

32 If A is a number then it is either prime, or $A = j \cdot D$ where D is prime

33 Find the smallest numbers X,Y,Z where the ratio X:Y:Z is equal to the given ratio A:B:C

34 Find the lowest common denominator of 2 numbers

35 If E is the lowest common denominator of A,B, and if $C = n \cdot A = m \cdot B$, then $C = i \cdot E$

36 Find the least common multiple of 3 numbers

37 If $A = p \cdot B$, then $A = q \cdot C$ where $C = p \cdot 1$

38 If $A = (1/c) \cdot B$ and $C = c \cdot 1$ then $A = n \cdot C$

39 Find the smallest number that has the fractions 1/a, 1/b, 1/c

Proposition 2 of Book VII

Given two numbers not prime to one another, to find their greatest common measure.

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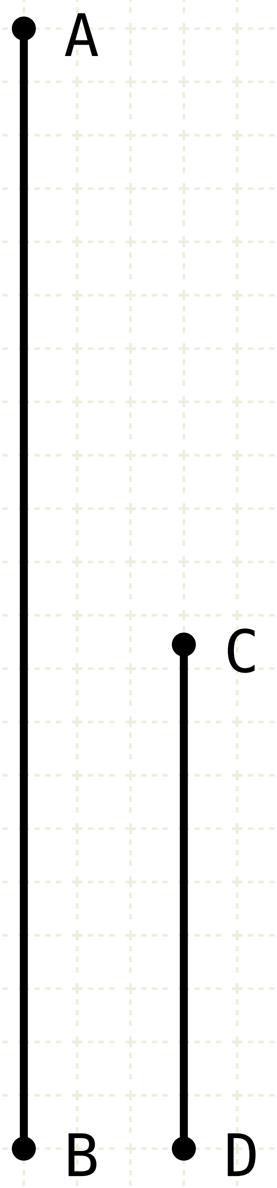
Given two numbers not prime to one another, to find their greatest common measure.

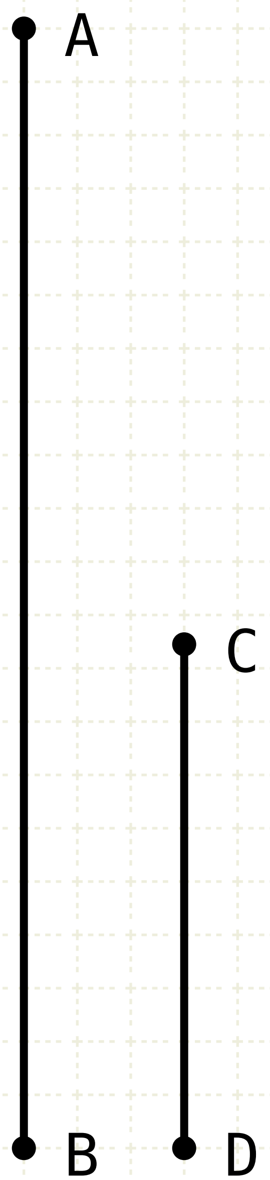


In other words

Find the greatest common divisor for two numbers

Proposition 2 of Book VII

Given two numbers not prime to one another, to find their greatest common measure.



Finding gcd()

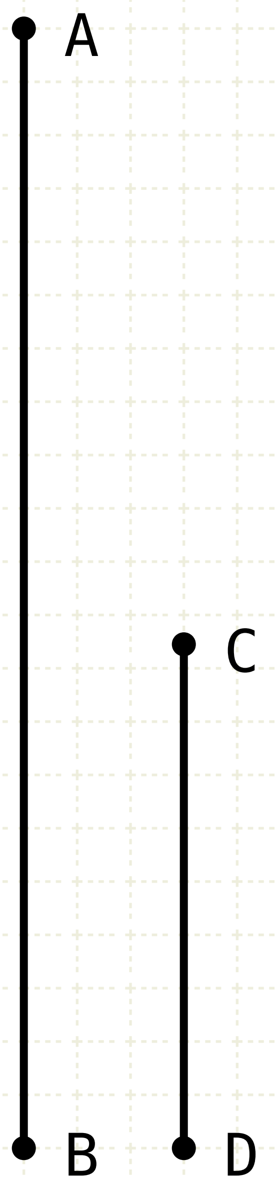
Continuously subtract the smaller number from the larger, until one number measures the other

This number will not be 1, as AB,CD are not relatively prime (VII·1)

This number is the largest common divisor

Proposition 2 of Book VII

Given two numbers not prime to one another, to find their greatest common measure.



AB = 140, CD = 63

Finding gcd()

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Example

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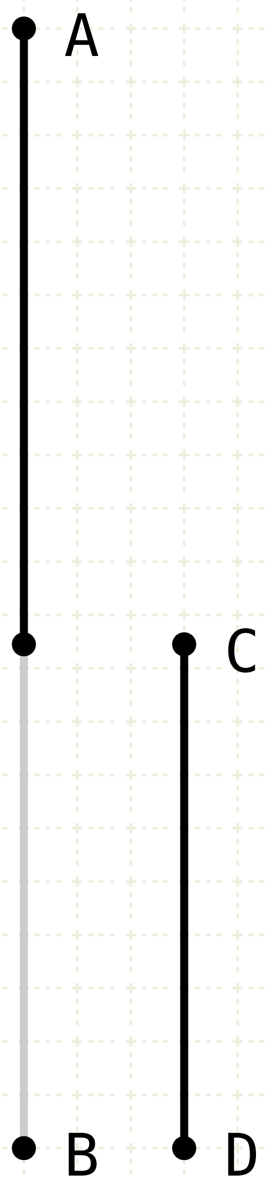

AB = 140, CD = 63

140- 63 = 77

Finding gcd()

Continuously subtract the smaller number from the larger, until one number measures the other

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This number is the largest common divisor

Example

Let CD measure BE with the remainder AE less than CD,

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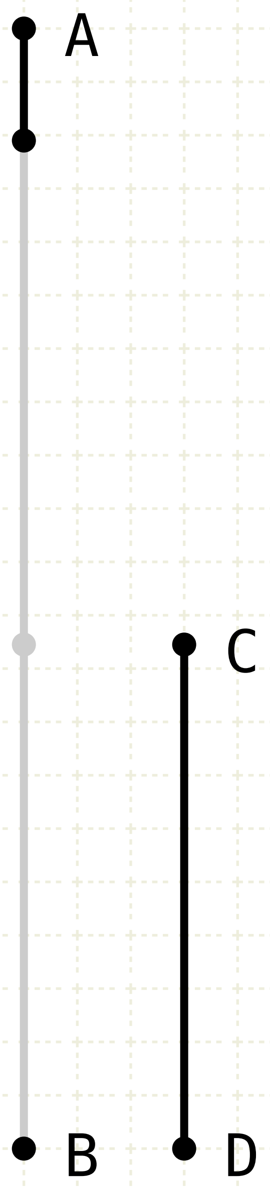

AB = 140, CD = 63

140 - 63 = 77

77 - 63 = 14

Finding gcd()

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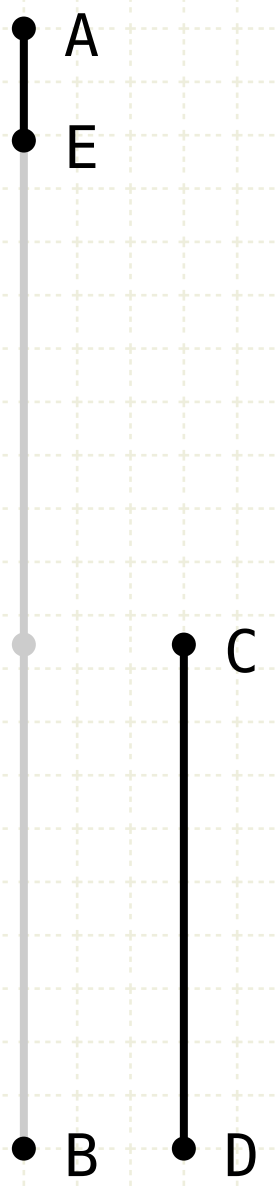


```
AB = 140, CD = 63
        140- 63 = 77
         77- 63 = 14
AE=14
```

Finding gcd()

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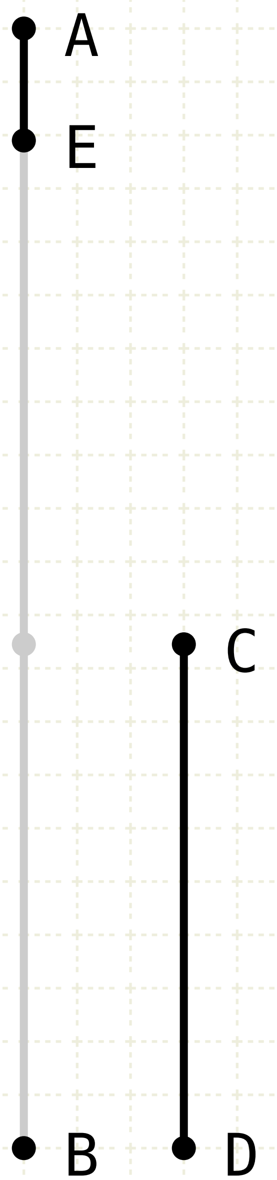

AB = 140, CD = 63

140- 63 = 77

77- 63 = 14

AE=14

Finding gcd()

Continuously subtract the smaller number from the larger, until one number measures the other

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Example

Let CD measure BE with the remainder AE less than CD,

And AE measure DF, with CF less than AE

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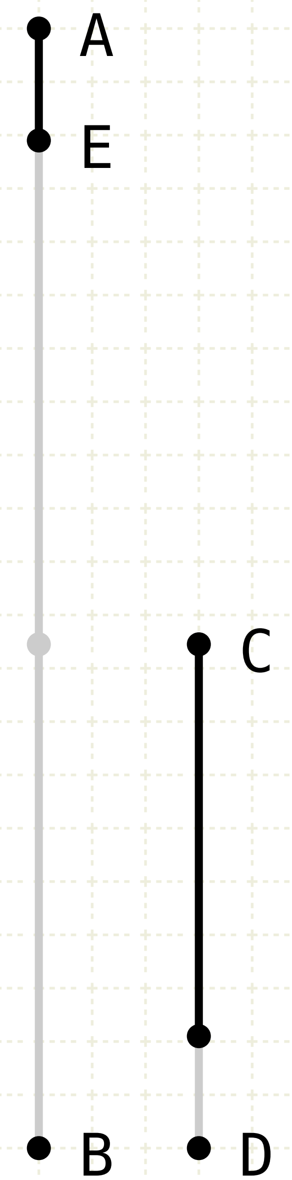

AB = 140, CD = 63

140- 63 = 77

77- 63 = 14

AE=14

63- 14 = 49

Finding gcd()

Continuously subtract the smaller number from the larger, until one number measures the other

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Let CD measure BE with the remainder AE less than CD,

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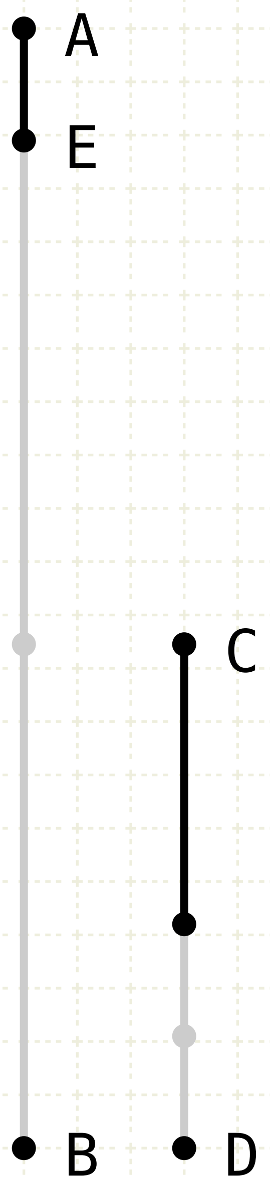

AB = 140, CD = 63

140- 63 = 77

77- 63 = 14

AE=14

63- 14 = 49

49- 14 = 35

Finding gcd()

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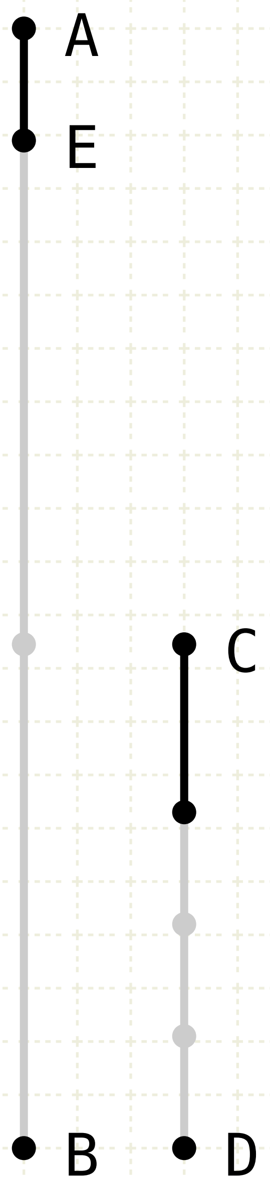


```
AB = 140, CD = 63
        140- 63 = 77
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AE=14
         63- 14 = 49
         49- 14 = 35
         35- 14 = 21
```

Finding gcd()

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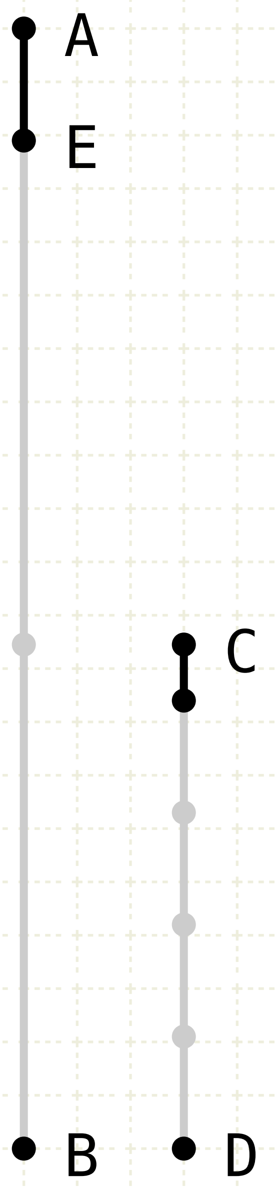


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AB = 140, CD = 63
        140- 63 = 77
         77- 63 = 14
AE=14
         63- 14 = 49
         49- 14 = 35
         35- 14 = 21
         21- 14 =  7
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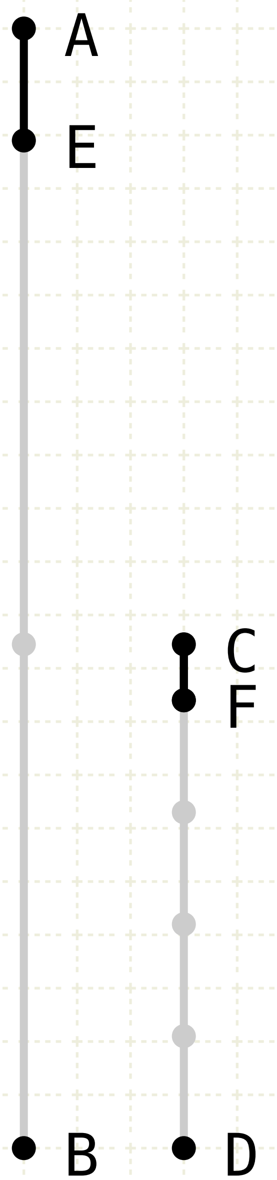


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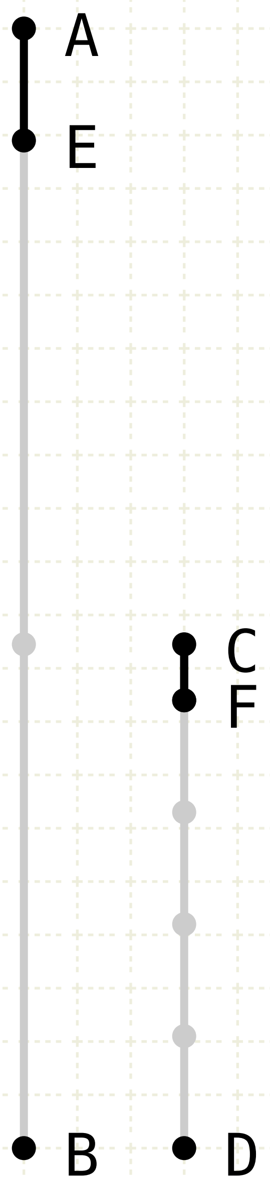


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And let CF measure AE…

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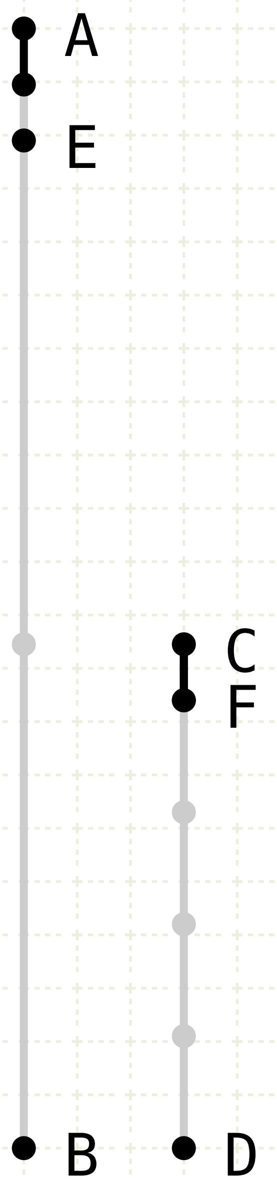


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         77- 63 = 14
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         63- 14 = 49
         49- 14 = 35
         35- 14 = 21
         21- 14 =  7
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         14-  7 =  7
```

Finding gcd()

Continuously subtract the smaller number from the larger, until one number measures the other

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Let CD measure BE with the remainder AE less than CD,

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And let CF measure AE…

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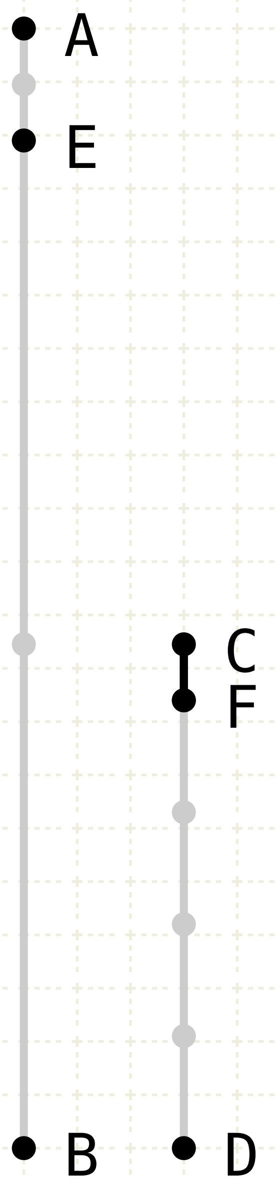


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         63- 14 = 49
         49- 14 = 35
         35- 14 = 21
         21- 14 =  7
CF=7
         14-  7 =  7
          7-  7 =  0
```

Finding gcd()

Continuously subtract the smaller number from the larger, until one number measures the other

This number will not be 1, as AB,CD are not relatively prime (VII·1)

This number is the largest common divisor

Example

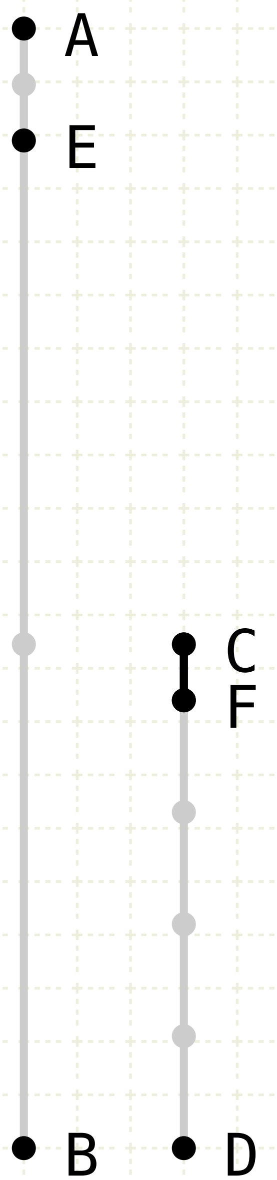
Let CD measure BE with the remainder AE less than CD,

And AE measure DF, with CF less than AE

And let CF measure AE…

Proposition 2 of Book VII

Given two numbers not prime to one another, to find their greatest common measure.

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        140- 63 = 77
         77- 63 = 14
AE=14
         63- 14 = 49
         49- 14 = 35
         35- 14 = 21
         21- 14 =  7
CF=7
         14-  7 =  7
          7-  7 =  0
AE = 2×CF
```

Finding gcd()

Continuously subtract the smaller number from the larger, until one number measures the other

This number will not be 1, as AB,CD are not relatively prime (VII·1)

This number is the largest common divisor

Example

Let CD measure BE with the remainder AE less than CD,

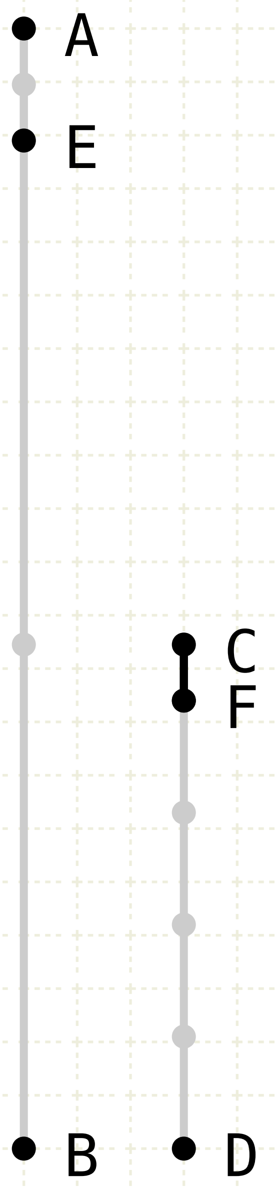
And AE measure DF, with CF less than AE

And let CF measure AE…

… leaving NO remainder

Proposition 2 of Book VII

Given two numbers not prime to one another, to find their greatest common measure.

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AB = 140, CD = 63
        140- 63 = 77
         77- 63 = 14
AE=14
         63- 14 = 49
         49- 14 = 35
         35- 14 = 21
         21- 14 =  7
CF=7
         14-  7 =  7
          7-  7 =  0
AE = 2×CF
→ gcd(AB,CD) = CF = 7
```

Finding gcd()

Continuously subtract the smaller number from the larger, until one number measures the other

This number will not be 1, as AB,CD are not relatively prime (VII·1)

This number is the largest common divisor

Example

Let CD measure BE with the remainder AE less than CD,

And AE measure DF, with CF less than AE

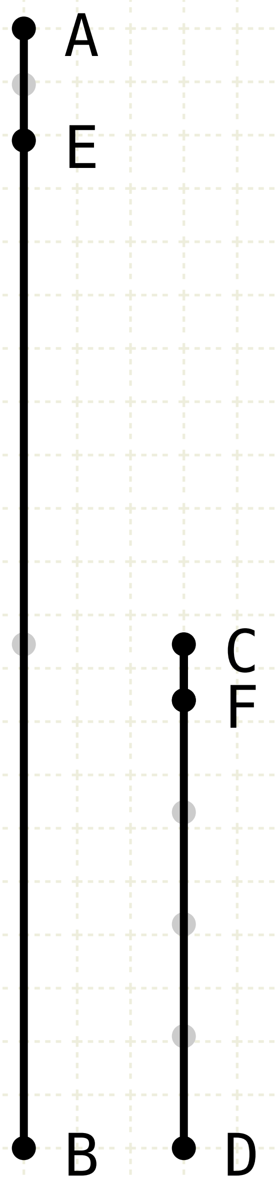
And let CF measure AE…

… leaving NO remainder

Since the smaller number (7) measures the larger number (14) it is the greatest common divisor

Proposition 2 of Book VII

Given two numbers not prime to one another, to find their greatest common measure.



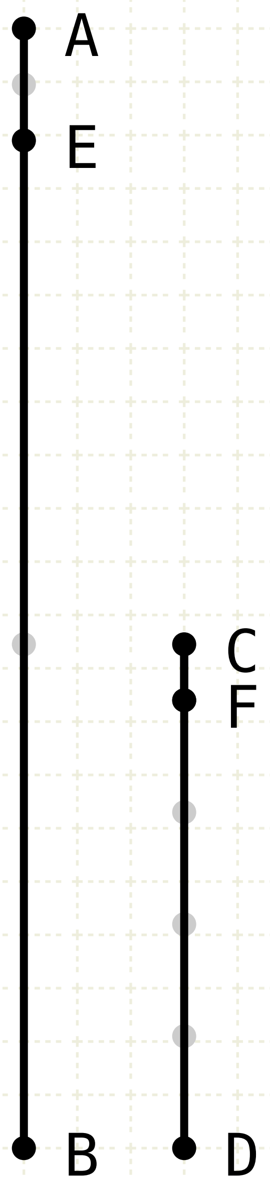

```
if AB = n·CD
gcd(AB,CD) = CD
```

Proof

If CD measures AB, then CD is the largest common divisor since it measures AB and itself, and no larger number can measure CD

Proposition 2 of Book VII

Given two numbers not prime to one another, to find their greatest common measure.



$BE = a \cdot CD,\ AE < CD$

$DF = b \cdot AE,\ FC < AE$

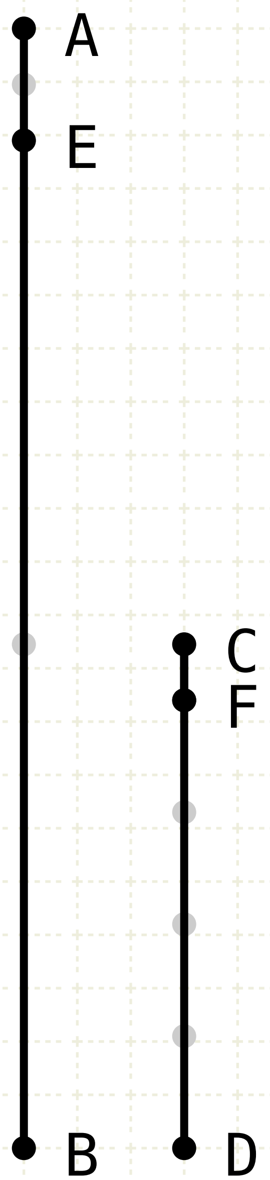
$AE = c \cdot CF$

Proof

Let CD measure BE, leaving AE less than CD, let AE measure DF, leaving FC less than DF, and let CF measure AE

Proposition 2 of Book VII

Given two numbers not prime to one another, to find their greatest common measure.



$BE = a \cdot CD,\ AE < CD$

$DF = b \cdot AE,\ FC < AE$

$AE = c \cdot CF$

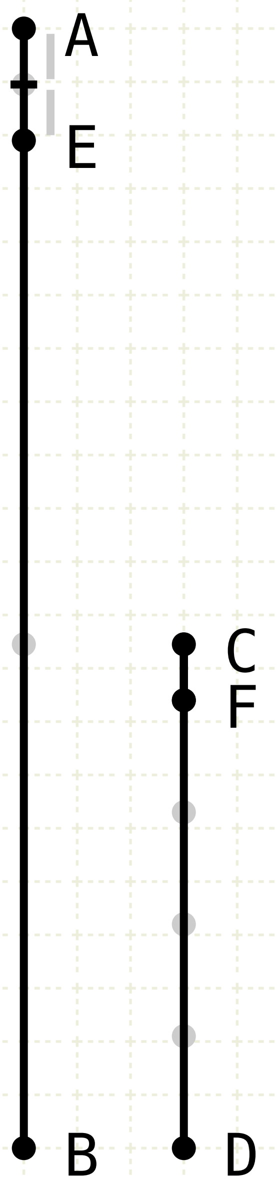
Proof

Let CD measure BE, leaving AE less than CD, let AE measure DF, leaving FC less than DF, and let CF measure AE

Proof that CF is a common measure

Proposition 2 of Book VII

Given two numbers not prime to one another, to find their greatest common measure.



$BE = a \cdot CD,\ AE < CD$

$DF = b \cdot AE,\ FC < AE$

$AE = c \cdot CF$

Proof

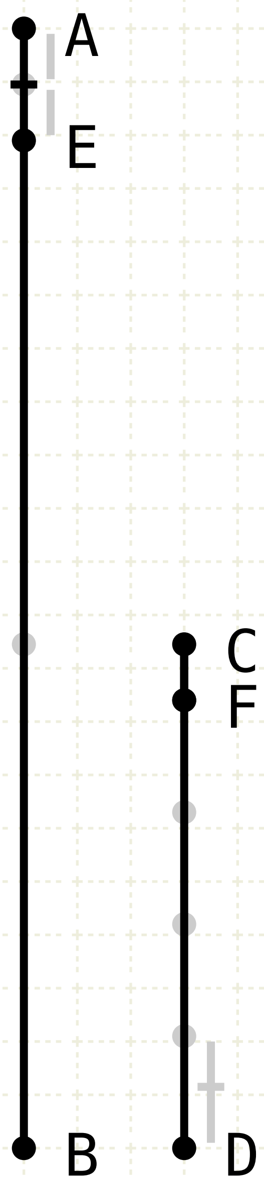
Let CD measure BE, leaving AE less than CD, let AE measure DF, leaving FC less than DF, and let CF measure AE

Proof that CF is a common measure

Since CF measures AE, and AE measures DF, then CF will also measure DF

Proposition 2 of Book VII

Given two numbers not prime to one another, to find their greatest common measure.



$BE = a \cdot CD,\ AE < CD$

$DF = b \cdot AE,\ FC < AE$

$AE = c \cdot CF$

$DF = c \cdot CF + \ldots$

Proof

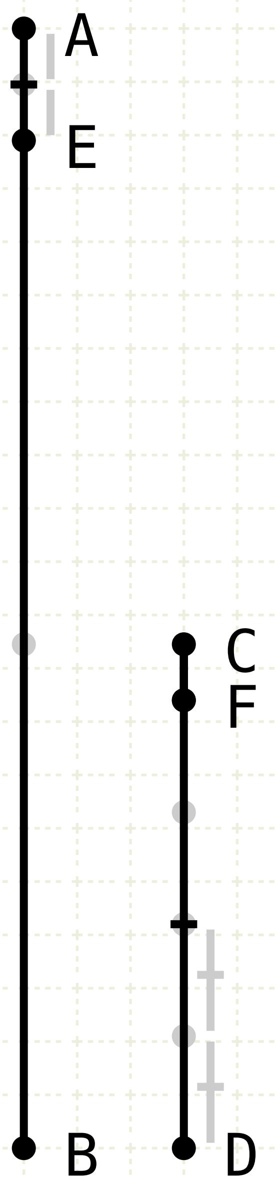
Let CD measure BE, leaving AE less than CD, let AE measure DF, leaving FC less than DF, and let CF measure AE

Proof that CF is a common measure

Since CF measures AE, and AE measures DF, then CF will also measure DF

Proposition 2 of Book VII

Given two numbers not prime to one another, to find their greatest common measure.



$BE = a \cdot CD,\ AE < CD$

$DF = b \cdot AE,\ FC < AE$

$AE = c \cdot CF$

$DF = c \cdot CF + c \cdot CF + \ldots$

Proof

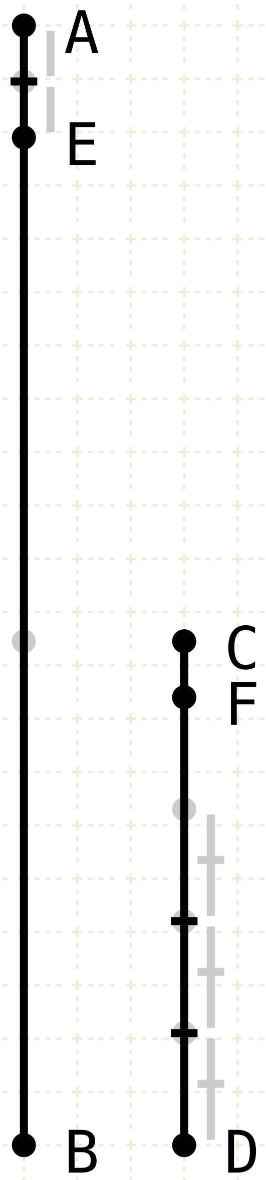
Let CD measure BE, leaving AE less than CD, let AE measure DF, leaving FC less than DF, and let CF measure AE

Proof that CF is a common measure

Since CF measures AE, and AE measures DF, then CF will also measure DF

Proposition 2 of Book VII

Given two numbers not prime to one another, to find their greatest common measure.



$BE = a \cdot CD,\ AE < CD$

$DF = b \cdot AE,\ FC < AE$

$AE = c \cdot CF$

$DF = c \cdot CF + c \cdot CF + c \cdot CF + \ldots$

Proof

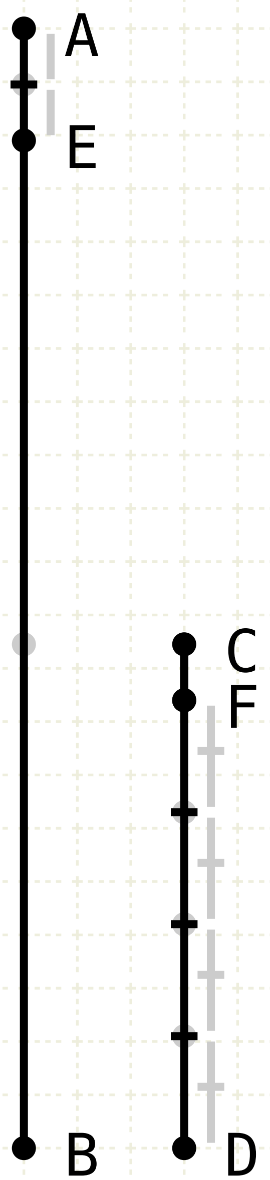
Let CD measure BE, leaving AE less than CD, let AE measure DF, leaving FC less than DF, and let CF measure AE

Proof that CF is a common measure

Since CF measures AE, and AE measures DF, then CF will also measure DF

Proposition 2 of Book VII

Given two numbers not prime to one another, to find their greatest common measure.



$BE = a \cdot CD,\ AE < CD$

$DF = b \cdot AE,\ FC < AE$

$AE = c \cdot CF$

$DF = c \cdot CF + c \cdot CF + \ldots = p \cdot CF$

Proof

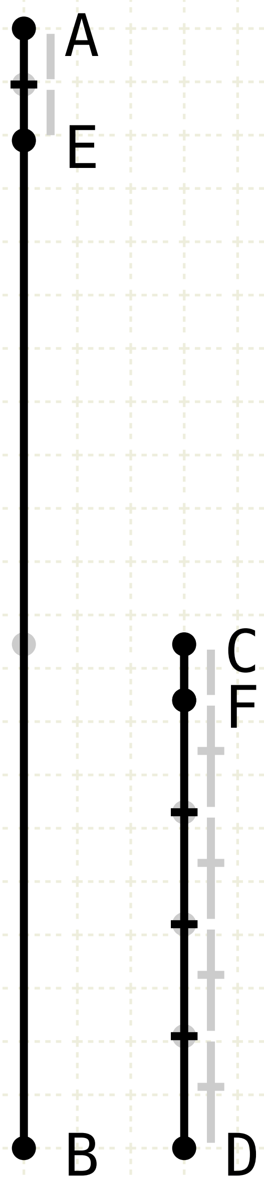
Let CD measure BE, leaving AE less than CD, let AE measure DF, leaving FC less than DF, and let CF measure AE

Proof that CF is a common measure

Since CF measures AE, and AE measures DF, then CF will also measure DF

Proposition 2 of Book VII

Given two numbers not prime to one another, to find their greatest common measure.



$$BE = a \cdot CD,\ AE < CD$$

$$DF = b \cdot AE,\ FC < AE$$

$$AE = c \cdot CF$$

$$DF = c \cdot CF + c \cdot CF + \dots = p \cdot CF$$

$$CD = CF + DF = CF + p \cdot CF = n \cdot CF$$

Proof

Let CD measure BE, leaving AE less than CD, let AE measure DF, leaving FC less than DF, and let CF measure AE

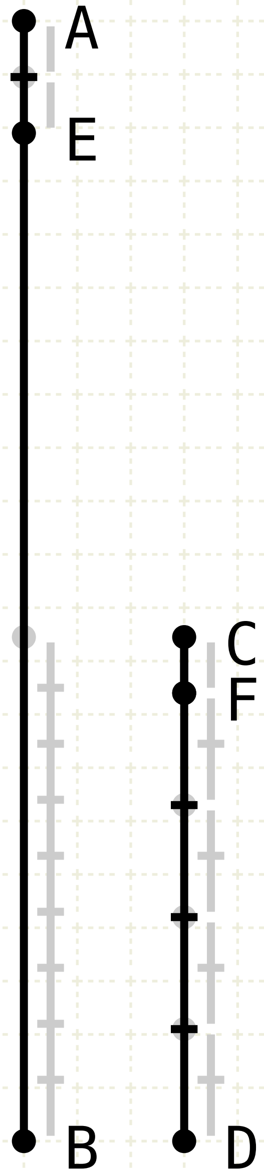
Proof that CF is a common measure

Since CF measures AE, and AE measures DF, then CF will also measure DF

CF also measures itself, therefore it measures all of CD

Proposition 2 of Book VII

Given two numbers not prime to one another, to find their greatest common measure.



$BE = a \cdot CD,\ AE < CD$

$DF = b \cdot AE,\ FC < AE$

$AE = c \cdot CF$

$DF = c \cdot CF + c \cdot CF + \ldots = p \cdot CF$

$CD = CF + DF = CF + p \cdot CF = n \cdot CF$

$BE = n \cdot CF + n \cdot CF + \ldots = q \cdot CF$

Proof

Let CD measure BE, leaving AE less than CD, let AE measure DF, leaving FC less than DF, and let CF measure AE

Proof that CF is a common measure

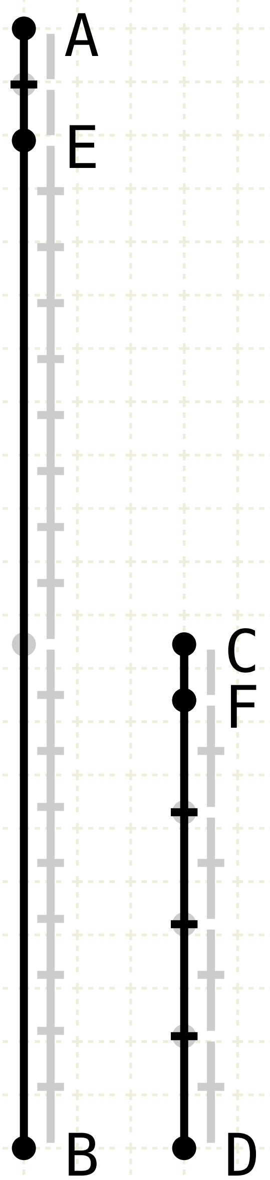
Since CF measures AE, and AE measures DF, then CF will also measure DF

CF also measures itself, therefore it measures all of CD

But CD measures BE, therefore CF will also measure BE

Proposition 2 of Book VII

Given two numbers not prime to one another, to find their greatest common measure.



$BE = a \cdot CD,\ AE < CD$

$DF = b \cdot AE,\ FC < AE$

$AE = c \cdot CF$

$DF = c \cdot CF + c \cdot CF + \ldots = p \cdot CF$

$CD = CF + DF = CF + p \cdot CF = n \cdot CF$

$BE = n \cdot CF + n \cdot CF + \ldots = q \cdot CF$

Proof

Let CD measure BE, leaving AE less than CD, let AE measure DF, leaving FC less than DF, and let CF measure AE

Proof that CF is a common measure

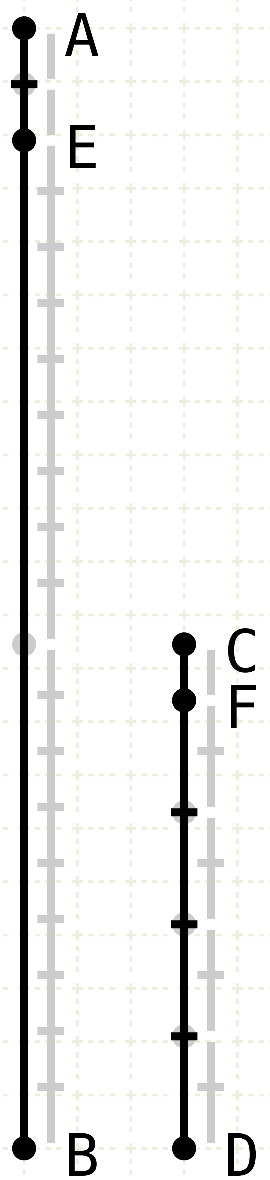
Since CF measures AE, and AE measures DF, then CF will also measure DF

CF also measures itself, therefore it measures all of CD

But CD measures BE, therefore CF will also measure BE

Proposition 2 of Book VII

Given two numbers not prime to one another, to find their greatest common measure.



$BE = a \cdot CD,\ AE < CD$

$DF = b \cdot AE,\ FC < AE$

$AE = c \cdot CF$

$DF = c \cdot CF + c \cdot CF + \dots = p \cdot CF$

$CD = CF + DF = CF + p \cdot CF = n \cdot CF$

$BE = n \cdot CF + n \cdot CF + \dots = q \cdot CF$

$AB = AE + BE = c \cdot CF + q \cdot CF = m \cdot CF$

Proof

Let CD measure BE, leaving AE less than CD, let AE measure DF, leaving FC less than DF, and let CF measure AE

Proof that CF is a common measure

Since CF measures AE, and AE measures DF, then CF will also measure DF

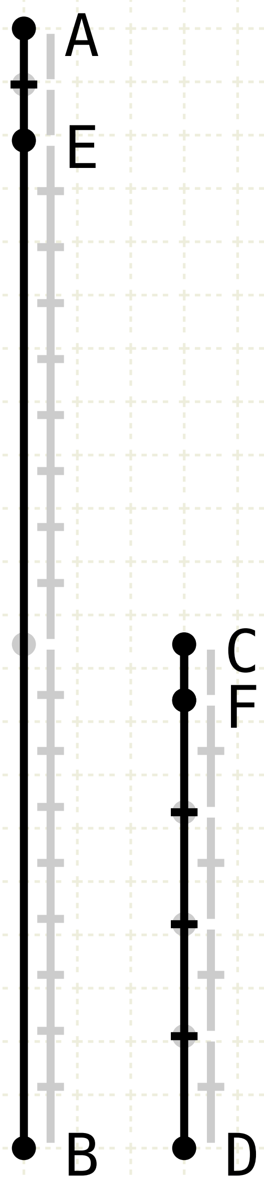
CF also measures itself, therefore it measures all of CD

But CD measures BE, therefore CF will also measure BE

CF measures AE, therefore it measures all of AB

Proposition 2 of Book VII

Given two numbers not prime to one another, to find their greatest common measure.



$BE = a \cdot CD, \; AE < CD$

$DF = b \cdot AE, \; FC < AE$

$AE = c \cdot CF$

$DF = c \cdot CF + c \cdot CF + \ldots = p \cdot CF$

$CD = CF + DF = CF + p \cdot CF = n \cdot CF$

$BE = n \cdot CF + n \cdot CF + \ldots = q \cdot CF$

$AB = AE + BE = c \cdot CF + q \cdot CF = m \cdot CF$

Proof

Let CD measure BE, leaving AE less than CD, let AE measure DF, leaving FC less than DF, and let CF measure AE

Proof that CF is a common measure

Since CF measures AE, and AE measures DF, then CF will also measure DF

CF also measures itself, therefore it measures all of CD

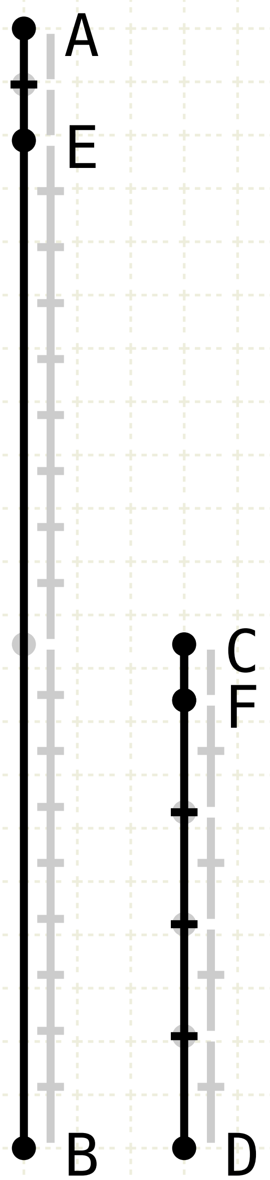
But CD measures BE, therefore CF will also measure BE

CF measures AE, therefore it measures all of AB

CF measures both AB and CD

Proposition 2 of Book VII

Given two numbers not prime to one another, to find their greatest common measure.



$$BE = a \cdot CD,\ AE < CD$$

$$DF = b \cdot AE,\ FC < AE$$

$$AE = c \cdot CF$$

$$CD = n \cdot CF$$

$$AB = m \cdot CF$$

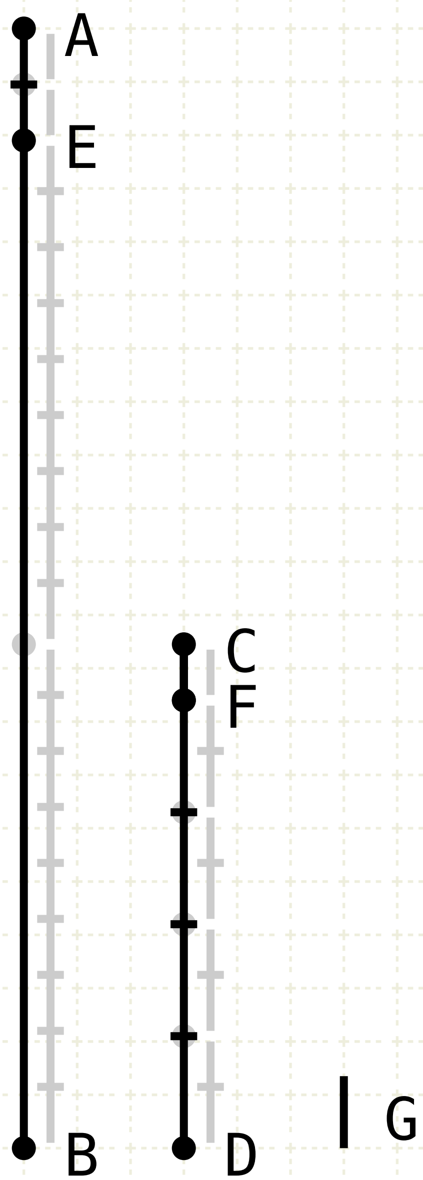
Proof

Let CD measure BE, leaving AE less than CD, let AE measure DF, leaving FC less than DF, and let CF measure AE

Proof that CD is the greatest common divisor

Proposition 2 of Book VII

Given two numbers not prime to one another, to find their greatest common measure.



$BE = a \cdot CD,\ AE < CD$

$DF = b \cdot AE,\ FC < AE$

$AE = c \cdot CF$

$CD = n \cdot CF$

$AB = m \cdot CF$

Assume

$G > CF$

$AB = p \cdot G$

$CD = q \cdot G$

Proof

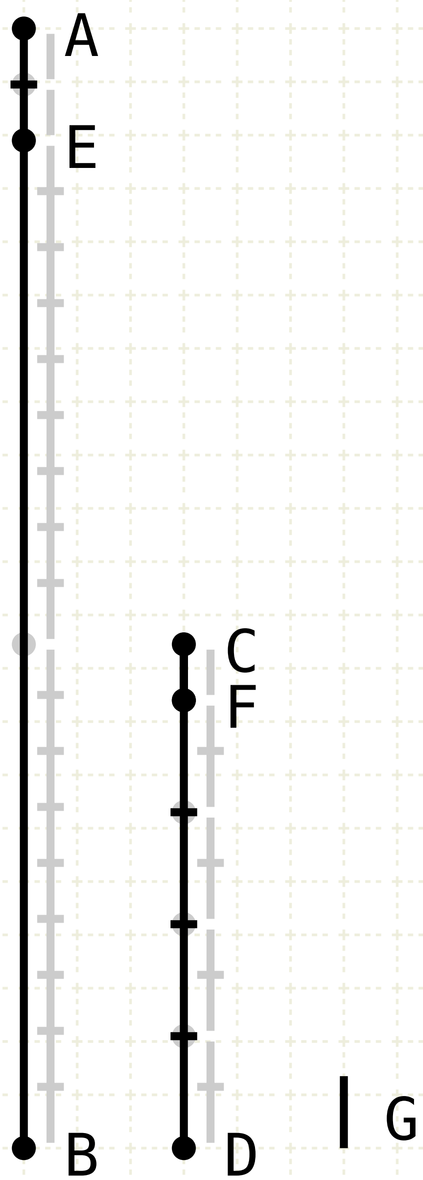
Let CD measure BE, leaving AE less than CD, let AE measure DF, leaving FC less than DF, and let CF measure AE

Proof that CD is the greatest common divisor

Assume that G, larger than CF, is also a common divisor

Proposition 2 of Book VII

Given two numbers not prime to one another, to find their greatest common measure.



$BE = a \cdot CD,\ AE < CD$

$DF = b \cdot AE,\ FC < AE$

$AE = c \cdot CF$

$CD = n \cdot CF$

$AB = m \cdot CF$

Assume

$G > CF$

$AB = p \cdot G$

$CD = q \cdot G$

$BE = q \cdot G + q \cdot G + \ldots = r \cdot G$

Proof

Let CD measure BE, leaving AE less than CD, let AE measure DF, leaving FC less than DF, and let CF measure AE

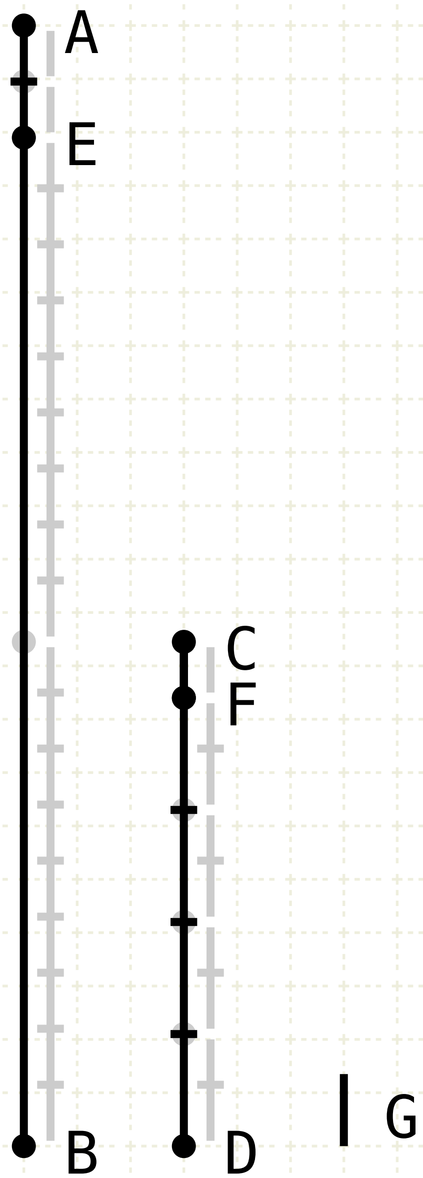
Proof that CD is the greatest common divisor

Assume that G, larger than CF, is also a common divisor

Since G measures CD, and CD measures BE, G also measures BE

Proposition 2 of Book VII

Given two numbers not prime to one another, to find their greatest common measure.



$BE = a \cdot CD,\ AE < CD$

$DF = b \cdot AE,\ FC < AE$

$AE = c \cdot CF$

$CD = n \cdot CF$

$AB = m \cdot CF$

Assume

$G > CF$

$AB = p \cdot G$

$CD = q \cdot G$

$BE = q \cdot G + q \cdot G + \ldots = r \cdot G$

$AE = AB - BE = p \cdot G - r \cdot G = s \cdot G$

Proof

Let CD measure BE, leaving AE less than CD, let AE measure DF, leaving FC less than DF, and let CF measure AE

Proof that CD is the greatest common divisor

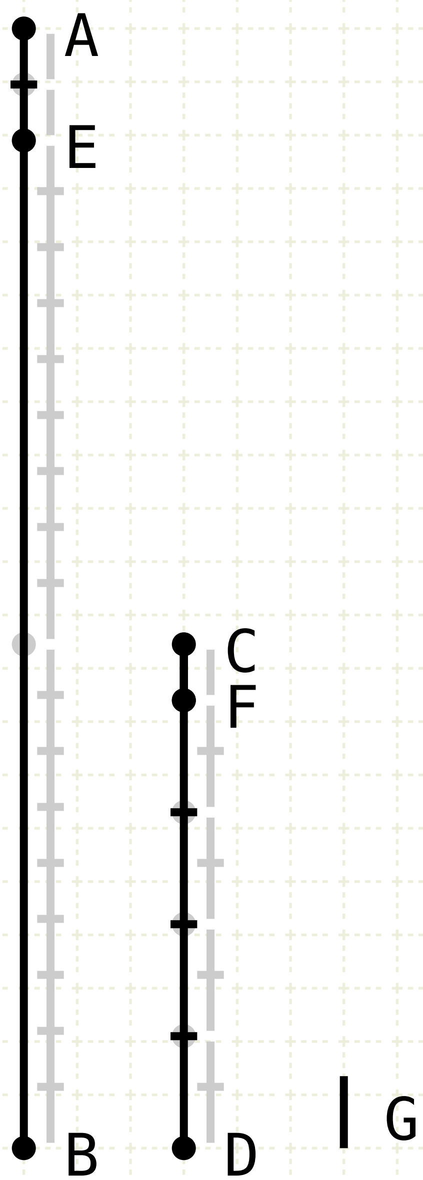
Assume that G, larger than CF, is also a common divisor

Since G measures CD, and CD measures BE, G also measures BE

Since G also measures AB, it must measure AE

Proposition 2 of Book VII

Given two numbers not prime to one another, to find their greatest common measure.



$BE = a \cdot CD,\ AE < CD$

$DF = b \cdot AE,\ FC < AE$

$AE = c \cdot CF$

$CD = n \cdot CF$

$AB = m \cdot CF$

Assume

$G > CF$

$AB = p \cdot G$

$CD = q \cdot G$

$BE = q \cdot G + q \cdot G + \ldots = r \cdot G$

$AE = AB - BE = p \cdot G - r \cdot G = s \cdot G$

$DF = s \cdot G + s \cdot G + \ldots = t \cdot G$

Proof

Let CD measure BE, leaving AE less than CD, let AE measure DF, leaving FC less than DF, and let CF measure AE

Proof that CD is the greatest common divisor

Assume that G, larger than CF, is also a common divisor

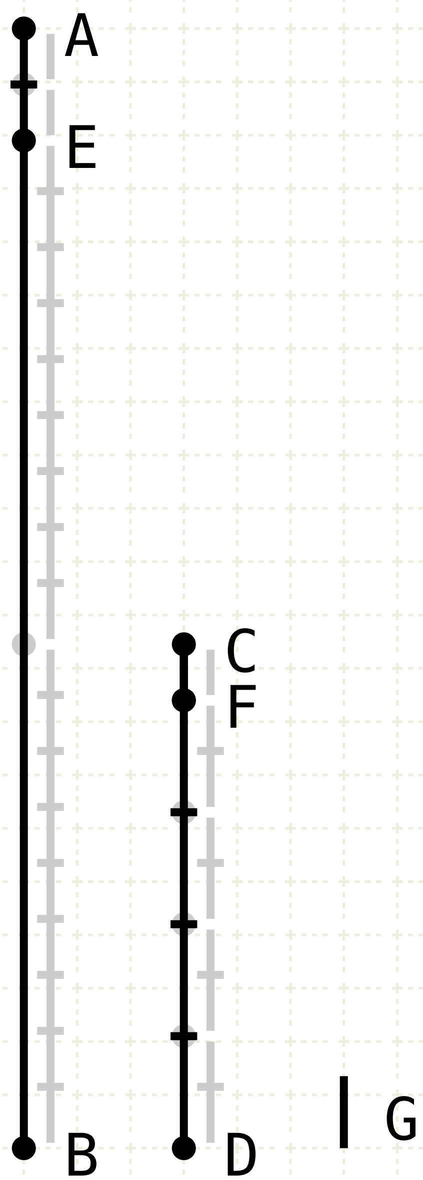
Since G measures CD, and CD measures BE, G also measures BE

Since G also measures AB, it must measure AE

But AE measures DF, therefore G will also measure DF

Proposition 2 of Book VII

Given two numbers not prime to one another, to find their greatest common measure.



$BE = a \cdot CD,\ AE < CD$

$DF = b \cdot AE,\ FC < AE$

$AE = c \cdot CF$

$CD = n \cdot CF$

$AB = m \cdot CF$

Assume

$G > CF$

$AB = p \cdot G$

$CD = q \cdot G$

$BE = q \cdot G + q \cdot G + \ldots = r \cdot G$

$AE = AB - BE = p \cdot G - r \cdot G = s \cdot G$

$DF = s \cdot G + s \cdot G + \ldots = t \cdot G$

$CF = CD - DF = q \cdot G - t \cdot G = u \cdot G$

Proof

Let CD measure BE, leaving AE less than CD, let AE measure DF, leaving FC less than DF, and let CF measure AE

Proof that CD is the greatest common divisor

Assume that G, larger than CF, is also a common divisor

Since G measures CD, and CD measures BE, G also measures BE

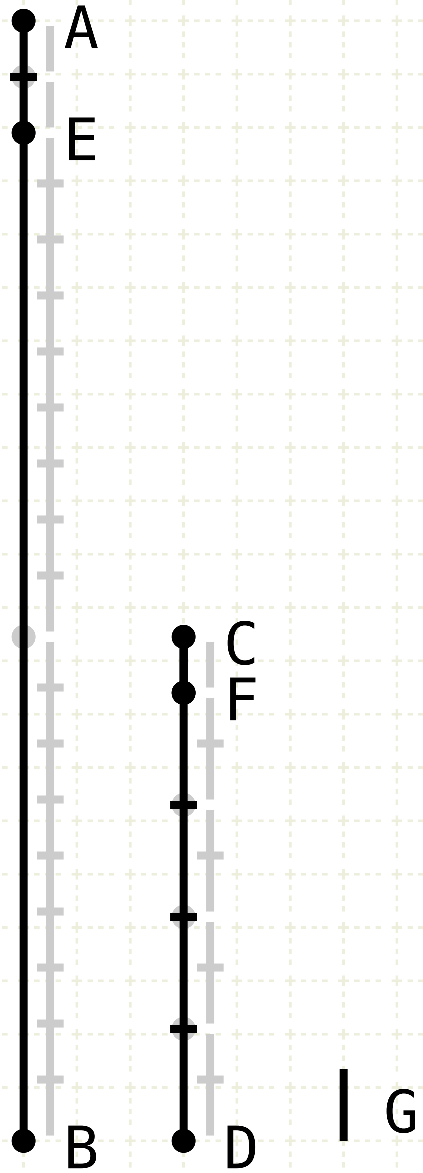
Since G also measures AB, it must measure AE

But AE measures DF, therefore G will also measure DF

Since G also measures DC, it must measure CF

Proposition 2 of Book VII

Given two numbers not prime to one another, to find their greatest common measure.



$BE = a \cdot CD,\ AE < CD$

$DF = b \cdot AE,\ FC < AE$

$AE = c \cdot CF$

$CD = n \cdot CF$

$AB = m \cdot CF$

Assume

$G > CF$

$AB = p \cdot G$

$CD = q \cdot G$

$BE = q \cdot G + q \cdot G + \ldots = r \cdot G$

$AE = AB - BE = p \cdot G - r \cdot G = s \cdot G$

$DF = s \cdot G + s \cdot G + \ldots = t \cdot G$

$CF = CD - DF = q \cdot G - t \cdot G = u \cdot G$

$CF \neq u \cdot G$

Proof

Let CD measure BE, leaving AE less than CD, let AE measure DF, leaving FC less than DF, and let CF measure AE

Proof that CD is the greatest common divisor

Assume that G, larger than CF, is also a common divisor

Since G measures CD, and CD measures BE, G also measures BE

Since G also measures AB, it must measure AE

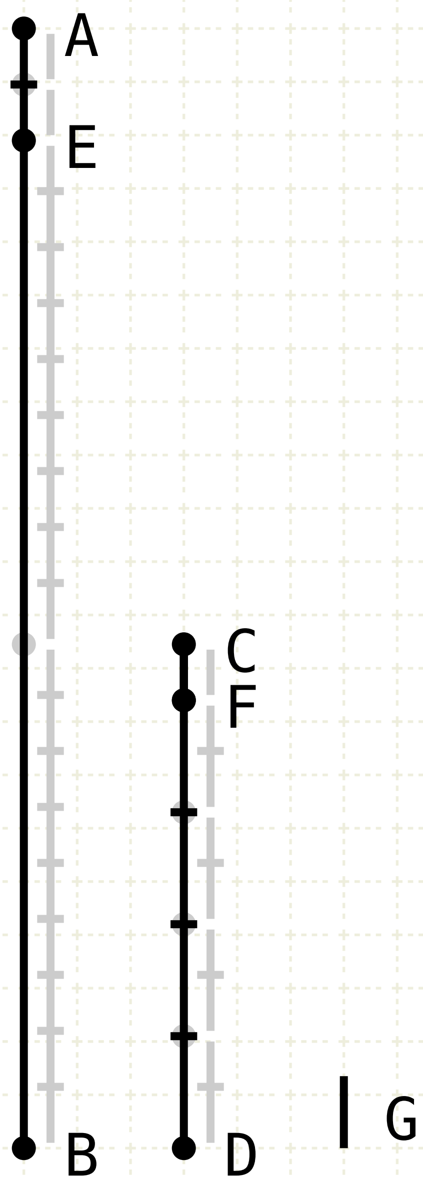
But AE measures DF, therefore G will also measure DF

Since G also measures DC, it must measure CF

But G cannot measure CF, because CF is less than G

Proposition 2 of Book VII

Given two numbers not prime to one another, to find their greatest common measure.



$BE = a \cdot CD,\ AE < CD$

$DF = b \cdot AE,\ FC < AE$

$AE = c \cdot CF$

$CD = n \cdot CF$

$AB = m \cdot CF$

Assume

$G > CF$

$AB = p \cdot G$

$CD = q \cdot G$

$BE = q \cdot G + q \cdot G + \ldots = r \cdot G$

$AE = AB - BE = p \cdot G - r \cdot G = s \cdot G$

$DF = s \cdot G + s \cdot G + \ldots = t \cdot G$

$CF = CD - DF = q \cdot G - t \cdot G = u \cdot G$

$CF \neq u \cdot G$

Proof

Let CD measure BE, leaving AE less than CD, let AE measure DF, leaving FC less than DF, and let CF measure AE

Proof that CD is the greatest common divisor

Assume that G, larger than CF, is also a common divisor

Since G measures CD, and CD measures BE, G also measures BE

Since G also measures AB, it must measure AE

But AE measures DF, therefore G will also measure DF

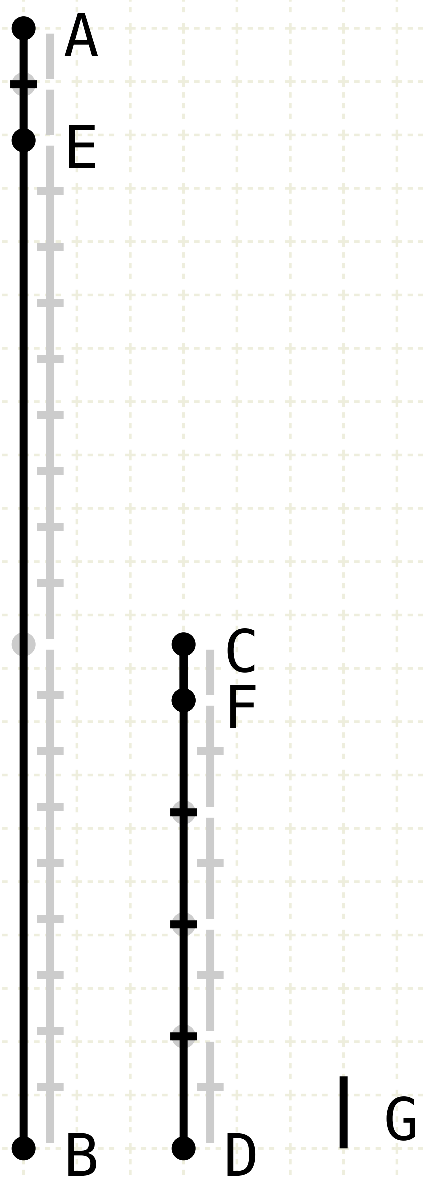
Since G also measures DC, it must measure CF

But G cannot measure CF, because CF is less than G

Therefore there is a contradiction, and there is no number G, larger than CF, that measures AB and CD

Proposition 2 of Book VII

Given two numbers not prime to one another, to find their greatest common measure.



$BE = a \cdot CD,\ AE < CD$

$DF = b \cdot AE,\ FC < AE$

$AE = c \cdot CF$

$CD = n \cdot CF$

$AB = m \cdot CF$

Assume

$G > CF$

$AB = p \cdot G$

$CD = q \cdot G$

$BE = q \cdot G + q \cdot G + \dots = r \cdot G$

$AE = AB - BE = p \cdot G - r \cdot G = s \cdot G$

$DF = s \cdot G + s \cdot G + \dots = t \cdot G$

$CF = CD - DF = q \cdot G - t \cdot G = u \cdot G$

$CF \neq u \cdot G$

CF is the greatest common divisor

Proof

Let CD measure BE, leaving AE less than CD, let AE measure DF, leaving FC less than DF, and let CF measure AE

Proof that CD is the greatest common divisor

Assume that G, larger than CF, is also a common divisor

Since G measures CD, and CD measures BE, G also measures BE

Since G also measures AB, it must measure AE

But AE measures DF, therefore G will also measure DF

Since G also measures DC, it must measure CF

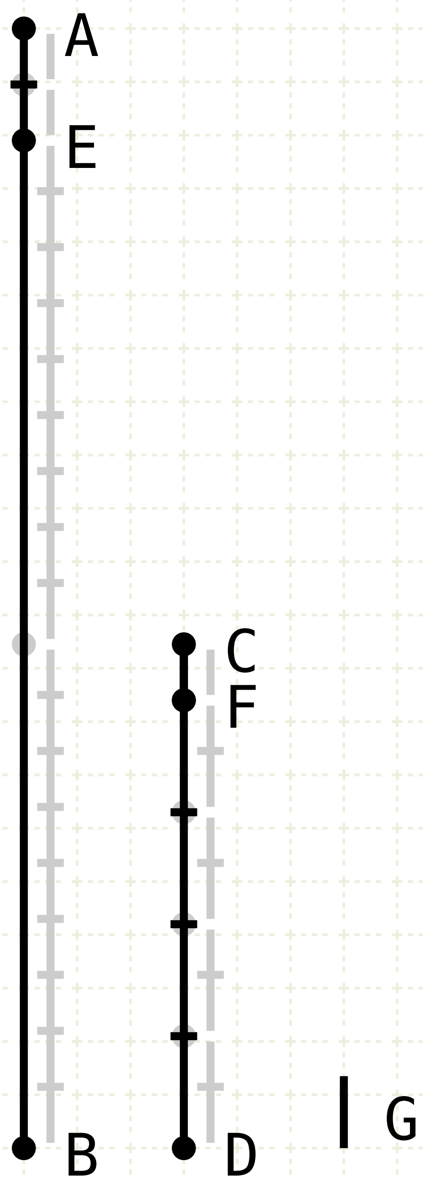
But G cannot measure CF, because CF is less than G

Therefore there is a contradiction, and there is no number G, larger than CF, that measures AB and CD

Therefore, there is no number G, greater than CF, that will measure AB and CD

Proposition 2 of Book VII

Given two numbers not prime to one another, to find their greatest common measure.



$BE = a \cdot CD,\ AE < CD$

$DF = b \cdot AE,\ FC < AE$

$AE = c \cdot CF$

$CD = n \cdot CF$

$AB = m \cdot CF$

Assume

$G > CF$

$AB = p \cdot G$

$CD = q \cdot G$

$BE = q \cdot G + q \cdot G + \ldots = r \cdot G$

$AE = AB - BE = p \cdot G - r \cdot G = s \cdot G$

$DF = s \cdot G + s \cdot G + \ldots = t \cdot G$

$CF = CD - DF = q \cdot G - t \cdot G = u \cdot G$

$CF \neq u \cdot G$

CF is the greatest common divisor

Proof

Let CD measure BE, leaving AE less than CD, let AE measure DF, leaving FC less than DF, and let CF measure AE

Proof that CD is the greatest common divisor

Assume that G, larger than CF, is also a common divisor

Since G measures CD, and CD measures BE, G also measures BE

Since G also measures AB, it must measure AE

But AE measures DF, therefore G will also measure DF

Since G also measures DC, it must measure CF

But G cannot measure CF, because CF is less than G

Therefore there is a contradiction, and there is no number G, larger than CF, that measures AB and CD

Therefore, there is no number G, greater than CF, that will measure AB and CD

Porism

If a number measures two numbers, it must also measure the greatest common divisor

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