

Euclid's Elements

Book VI

One can state, without exaggeration, that the observation of and the search for similarities and differences are the basis of all human knowledge.

Alfred Nobel



Table of Contents, Chapter 6

1	If the height of two triangles are equal, then the ratio of the areas is equal to the ratio of the bases	7	If two triangles have one angle equal to one angle, and the sides about other angles are proportional, and the remaining angles either both less or both not less than a right angle, then triangles will be equiangular	14	In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and vice versa
2	If a line cuts a triangle, parallel to its base, it will cut the sides of the triangle proportionally	8	If in a right-angled triangle a perpendicular be drawn from the right angle to the base, the triangles adjoining the perpendicular are similar both to the whole and to one another	15	In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and vice versa
3	If an angle of a triangle is bisected and the straight line cutting the angle also cuts the base, the segments of the base will have the same ratio as the remaining sides of the triangle	9	From a given straight line to cut off a given fraction	16	If four straight lines are proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means, and vice versa
4	If two triangles have equal angles, then the sides opposite the equal angles are proportional, as well, the sides of the triangles on either side of the equal angles are also proportional	10	To cut a given uncut straight line similarly to a given cut straight line	17	If three straight lines are proportional, the rectangle contained by the extremes is equal to the square on the mean; and vice versa
5	If two triangles have proportional sides, the triangles will be equiangular	11	To two given straight lines to find a third proportional	18	On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure
6	If two triangles have one angle equal to one angle and the sides about the equal angles are proportional, then the triangles will be equiangular	12	To three given straight lines to find a fourth proportional	19	Similar triangles are to one another in the duplicate ratio of the corresponding sides
		13	To two given straight lines to find a mean proportional		



Table of Contents, Chapter 3

20	Similar polygons are divided into the same number of similar triangles, which have the same ratio as the wholes, and the polygons have duplicate ratios to their corresponding sides	26	If from a parallelogram a similar parallelogram with a common angle is subtracted, it is about the same diameter as the original	31	In right-angled triangles the figure on the side subtending the right angle is equal to the similar and similarly described figures on the sides containing the right angle
21	Figures which are are similar to the same rectilineal figure are also similar to one another	27	Of all the parallelograms applied to the same straight line and deficient by parallelogrammic figures similar to a parallelogram drawn on half the said line, the largest will be one that is drawn on half of the straight line and is similar to the defect		
22	If four straight lines are proportional, similar rectilineal figures will also be proportional; and vice versa	28	To a given straight line, apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one		
23	Equiangular parallelograms have to one another the ratio compounded of the ratios of their sides	29	To a given straight line, apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one		
24	In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another	30	To cut a finite straight line in extreme ratio		
25	To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure				



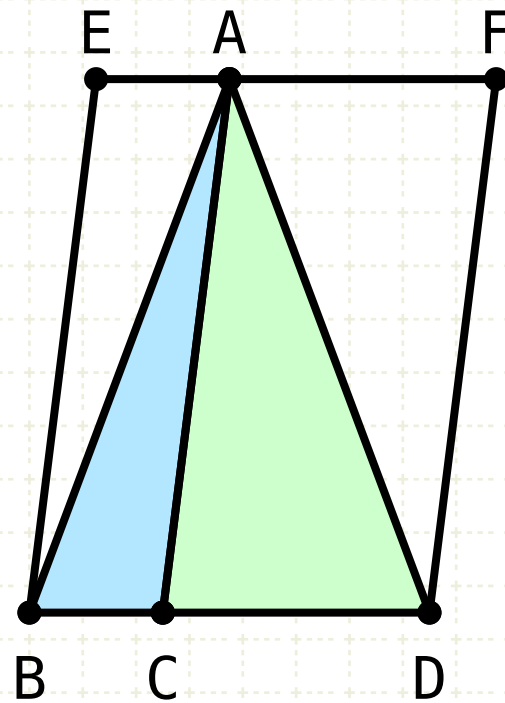
Proposition 1 of Book VI

Triangles and parallelograms which are under the same height are to one another as their bases



Proposition 1 of Book VI

Triangles and parallelograms which are under the same height are to one another as their bases



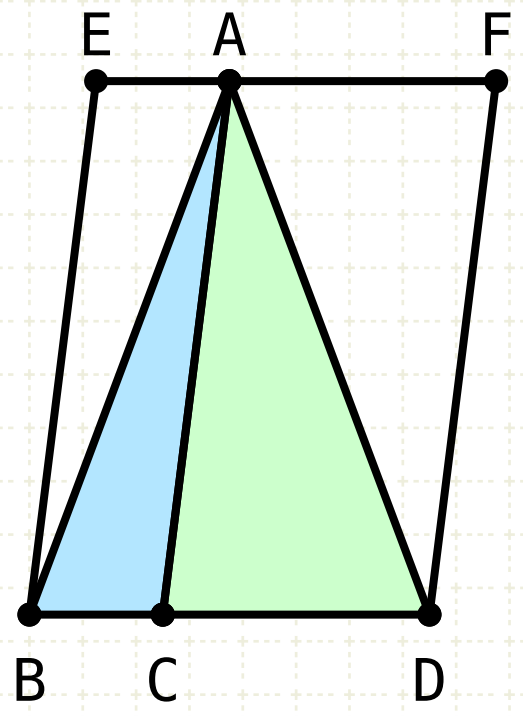
In other words

If we have two triangles ABC and ACD, or two parallelograms EC and CF, with the same height, then the ratio of BC to CD is the same as the ratios of the triangles, and parallelograms respectively

$$BC : CD = \triangle ABC : \triangle ACD = \square EC : \square CF$$

Proposition 1 of Book VI

Triangles and parallelograms which are under the same height are to one another as their bases

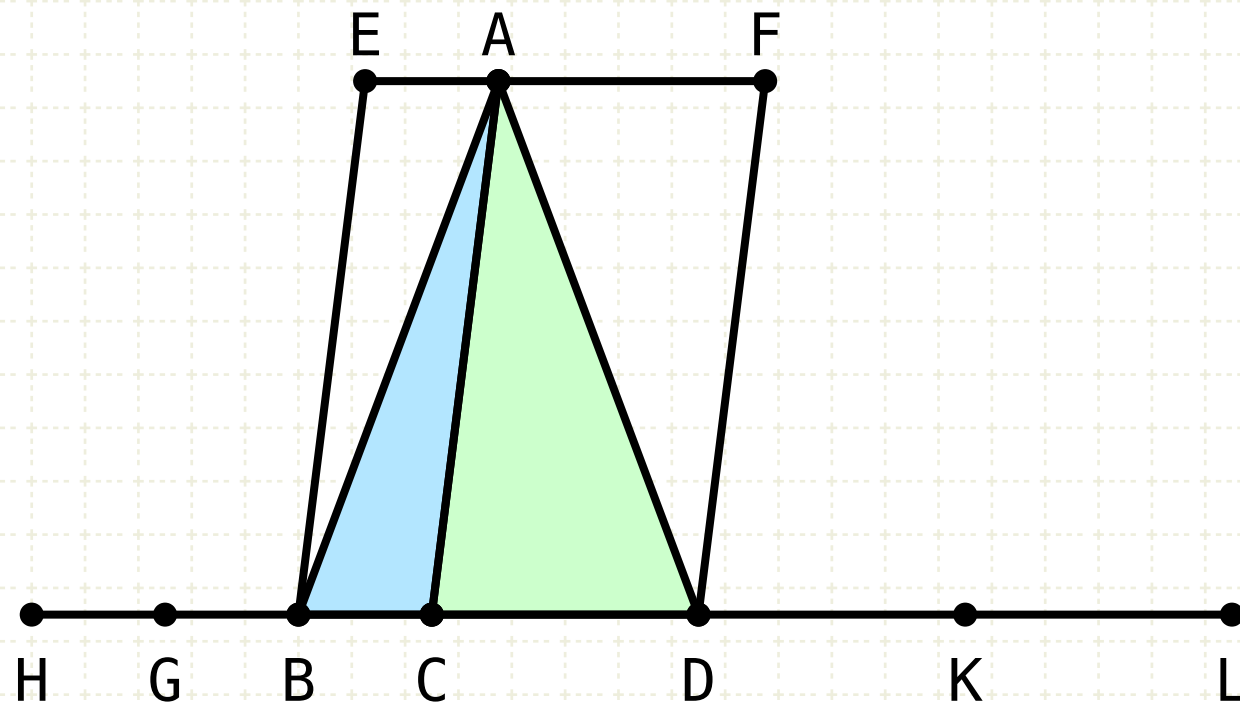


Proof - Triangles



Proposition 1 of Book VI

Triangles and parallelograms which are under the same height are to one another as their bases



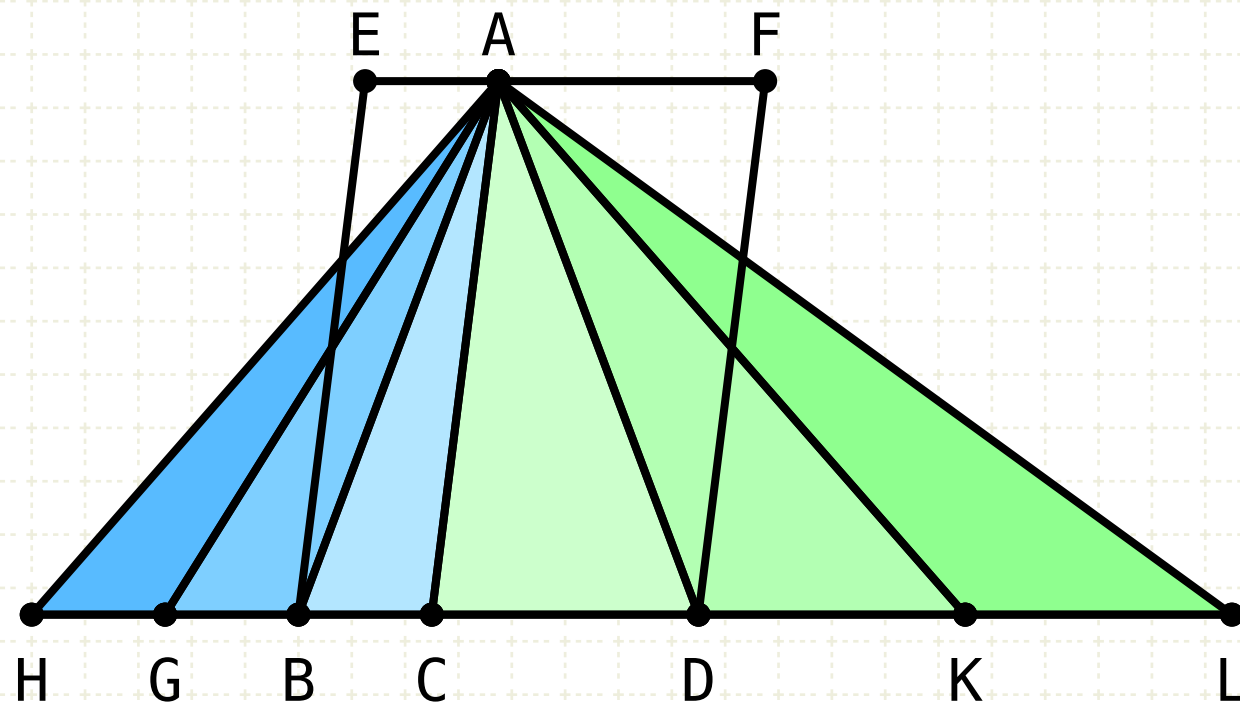
$$\begin{aligned} HG &= GB = BC \\ CD &= DK = KL \end{aligned}$$

Proof - Triangles

Extend the line BD to HL, such that HB is composed of any number of lines equal to BC, and that DL is composed of any number of lines equal to CD

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$$\begin{aligned} HG &= GB = BC \\ CD &= DK = KL \end{aligned}$$

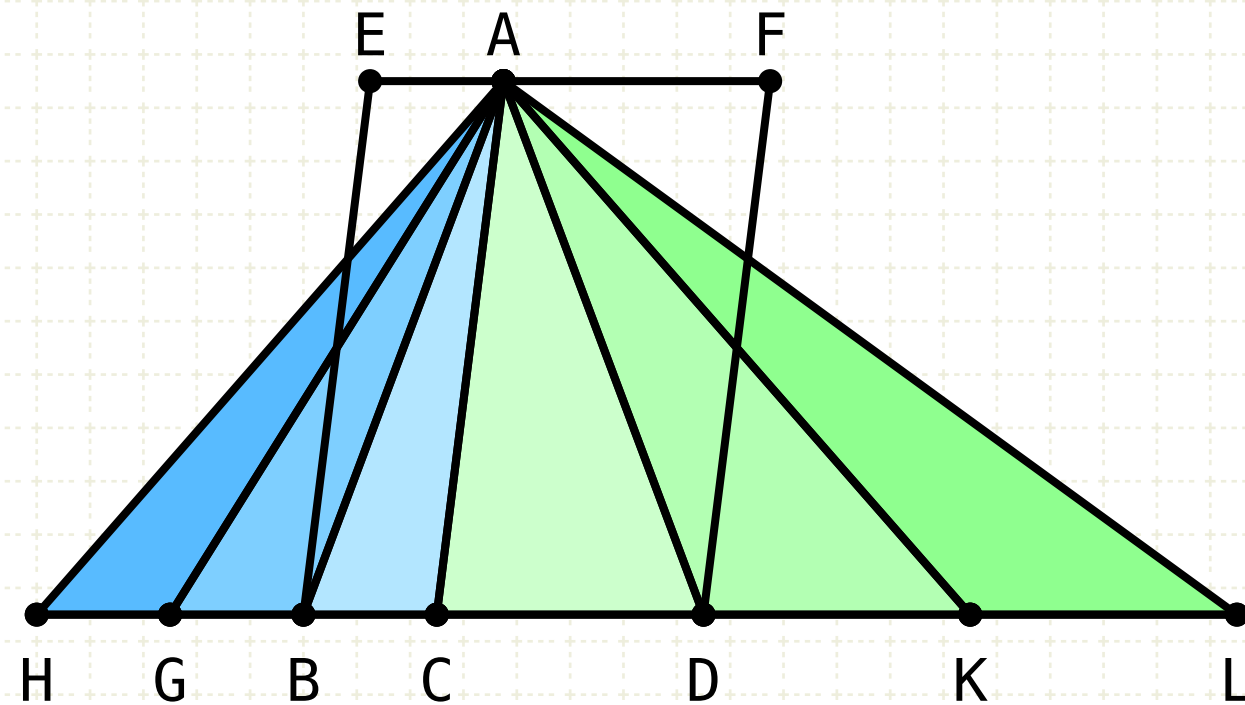
Proof - Triangles

Extend the line BD to HL, such that HB is composed of any number of lines equal to BC, and that DL is composed of any number of lines equal to CD

Draw the triangles AHG, AGB, and ADK and AKL

Proposition 1 of Book VI

Triangles and parallelograms which are under the same height are to one another as their bases



$$HG = GB = BC$$

$$CD = DK = KL$$

$$\triangle ABC = \triangle AGB = \triangle AHG$$

$$\triangle ACD = \triangle ADK = \triangle AKL$$

Proof - Triangles

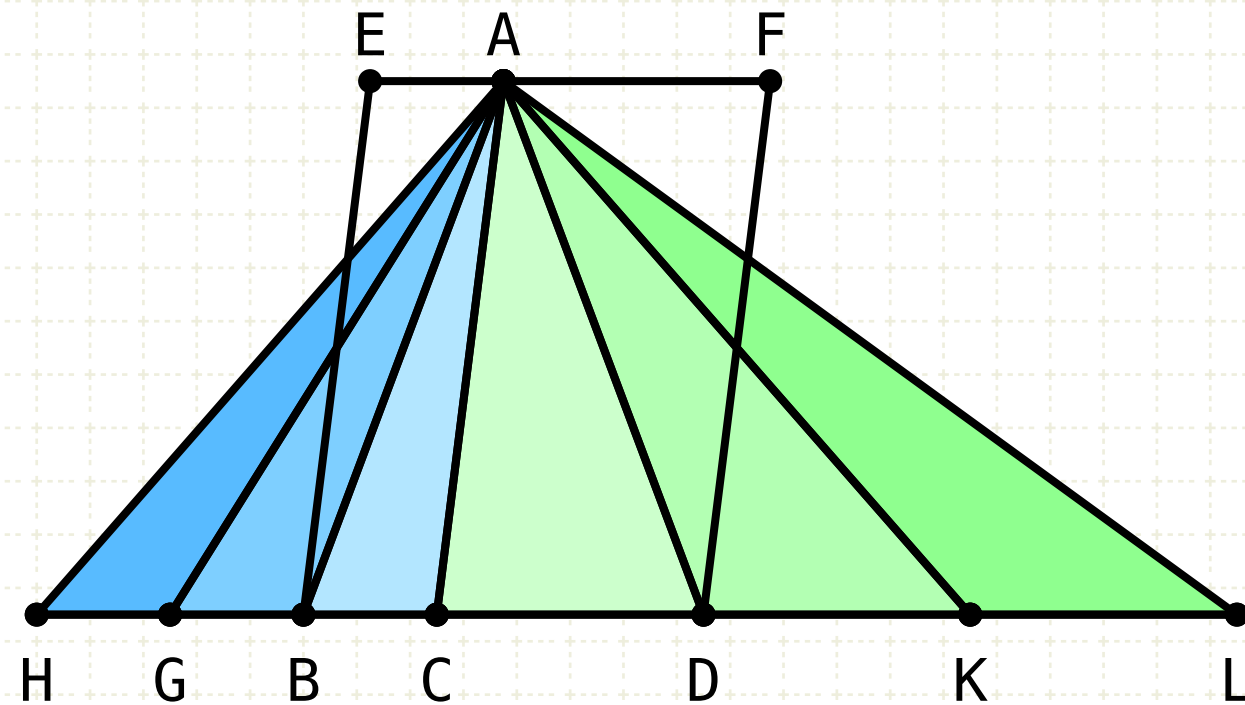
Extend the line BD to HL, such that HB is composed of any number of lines equal to BC, and that DL is composed of any number of lines equal to CD

Draw the triangles AHG, AGB, and ADK and AKL

Since the bases are equal, triangles ABC, AGB and AHG are equal, and the triangles ACD, ADK and AKL are equal (I-38)

Proposition 1 of Book VI

Triangles and parallelograms which are under the same height are to one another as their bases



Proof - Triangles

Extend the line BD to HL, such that HB is composed of any number of lines equal to BC, and that DL is composed of any number of lines equal to CD

Draw the triangles AHG, AGB, and ADK and AKL

Since the bases are equal, triangles ABC, AGB and AHG are equal, and the triangles ACD, ADK and AKL are equal (I-38)

Therefore, the triangle AHC is the same multiple of ABC as the line HC is to BC

$$HG = GB = BC$$

$$CD = DK = KL$$

$$\triangle ABC = \triangle AGB = \triangle AHG$$

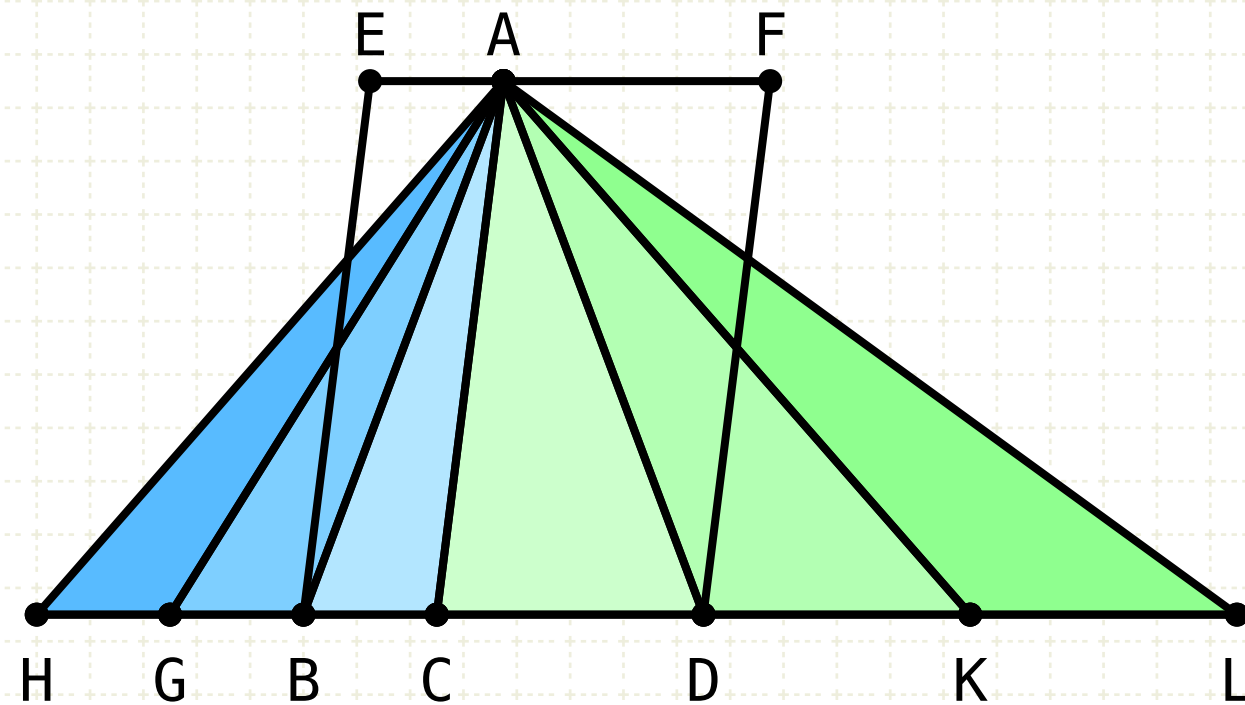
$$\triangle ACD = \triangle ADK = \triangle AKL$$

$$\triangle AHC = \triangle ABC + \triangle AGB + \triangle AHG = n \cdot \triangle ABC$$

$$HC = BC + GB + HG = n \cdot BC$$

Proposition 1 of Book VI

Triangles and parallelograms which are under the same height are to one another as their bases



Proof - Triangles

Extend the line BD to HL, such that HB is composed of any number of lines equal to BC, and that DL is composed of any number of lines equal to CD

Draw the triangles AHG, AGB, and ADK and AKL

Since the bases are equal, triangles ABC, AGB and AHG are equal, and the triangles ACD, ADK and AKL are equal (I-38)

Therefore, the triangle AHC is the same multiple of ABC as the line HC is to BC

Similarly, the triangle ACL is the same multiple of ACD as the line CD is to CL

$$HG = GB = BC$$

$$CD = DK = KL$$

$$\triangle ABC = \triangle AGB = \triangle AHG$$

$$\triangle ACD = \triangle ADK = \triangle AKL$$

$$\triangle AHC = \triangle ABC + \triangle AGB + \triangle AHG = n \cdot \triangle ABC$$

$$HC = BC + GB + HG = n \cdot BC$$

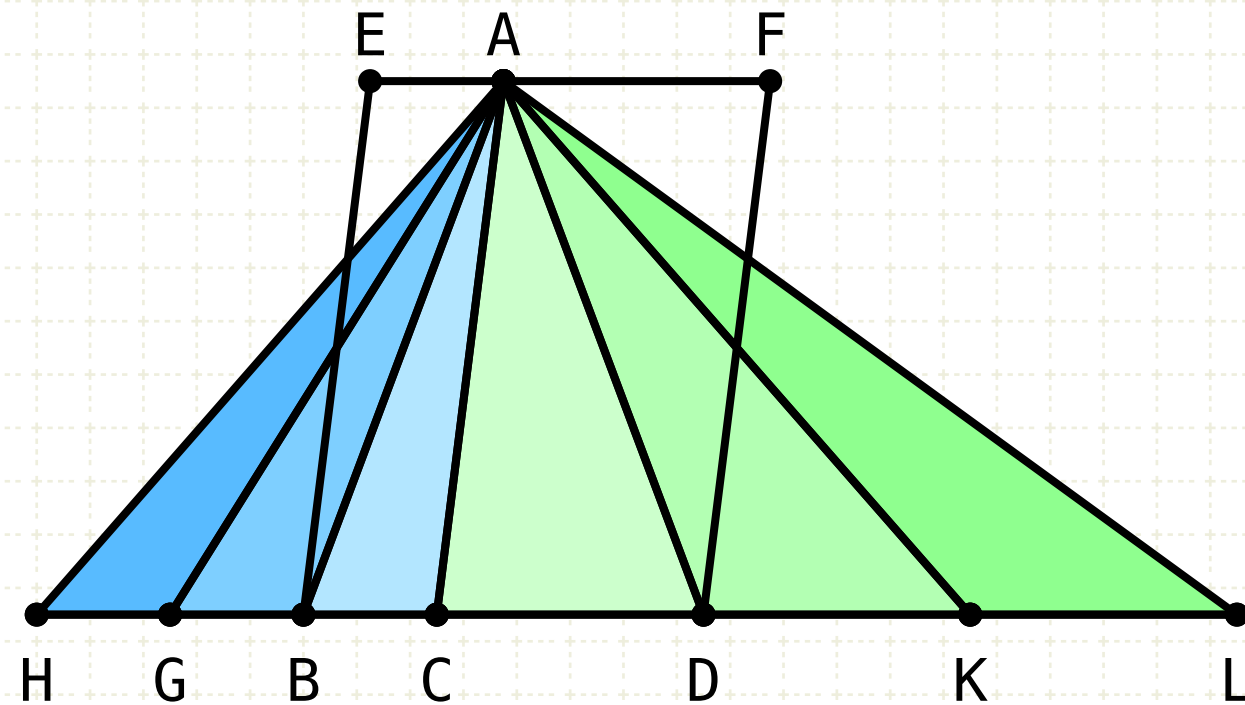
$$\triangle ACL = \triangle ACD + \triangle ADK + \triangle AKL = m \cdot \triangle ACD$$

$$CL = CD + DK + KL = m \cdot CD$$



Proposition 1 of Book VI

Triangles and parallelograms which are under the same height are to one another as their bases



$$\begin{aligned} HG &= GB = BC, & CD &= DK = KL \\ \Delta AHC &= n \cdot \Delta ABC, & HC &= n \cdot BC \\ \Delta ACL &= m \cdot \Delta ACD, & CL &= m \cdot CD \end{aligned}$$

Proof - Triangles

Extend the line BD to HL, such that HB is composed of any number of lines equal to BC, and that DL is composed of any number of lines equal to CD

Draw the triangles AHG, AGB, and ADK and AKL

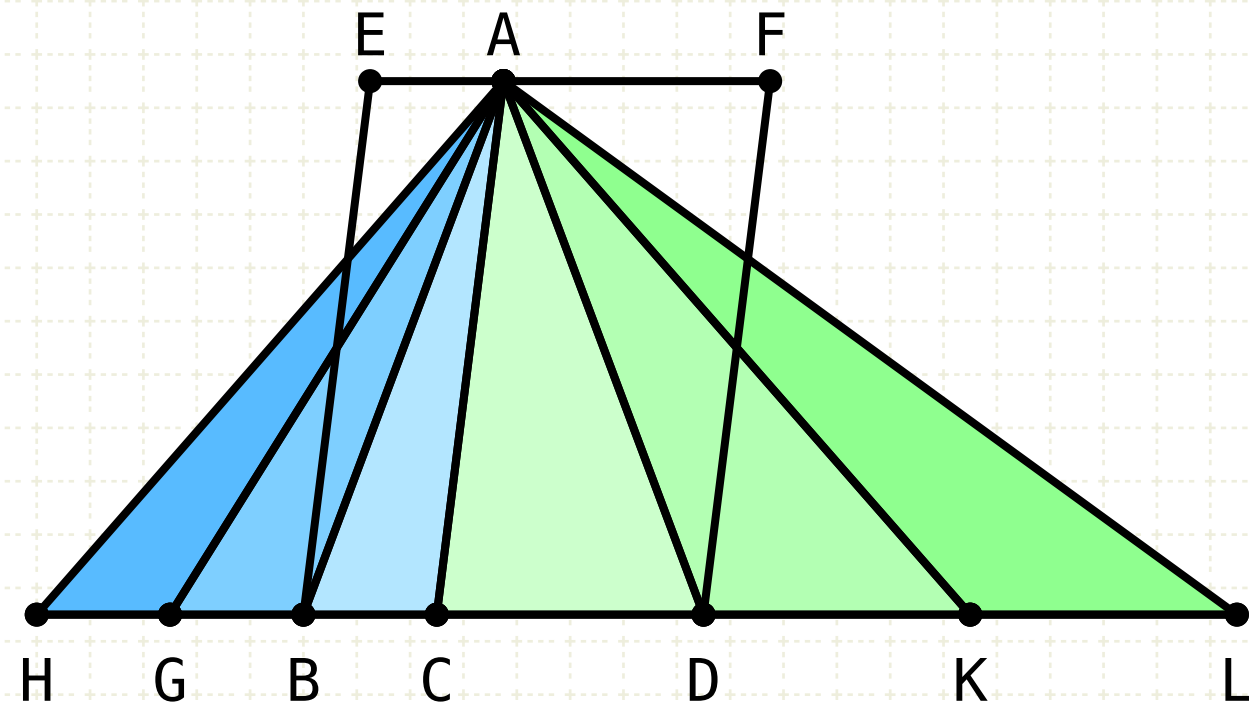
Since the bases are equal, triangles ABC, AGB and AHG are equal, and the triangles ACD, ADK and AKL are equal (I-38)

Therefore, the triangle AHC is the same multiple of ABC as the line HC is to BC

Similarly, the triangle ACL is the same multiple of ACD as the line CD is to CL

Proposition 1 of Book VI

Triangles and parallelograms which are under the same height are to one another as their bases



$$\begin{aligned} HG &= GB = BC, & CD &= DK = KL \\ \Delta AHC &= n \cdot \Delta ABC, & HC &= n \cdot BC \\ \Delta ACL &= m \cdot \Delta ACD, & CL &= m \cdot CD \\ \text{if } HC &= CL \rightarrow \Delta AHC &= \Delta ACL \end{aligned}$$

Proof - Triangles

Extend the line BD to HL, such that HB is composed of any number of lines equal to BC, and that DL is composed of any number of lines equal to CD

Draw the triangles AHG, AGB, and ADK and AKL

Since the bases are equal, triangles ABC, AGB and AHG are equal, and the triangles ACD, ADK and AKL are equal (I-38)

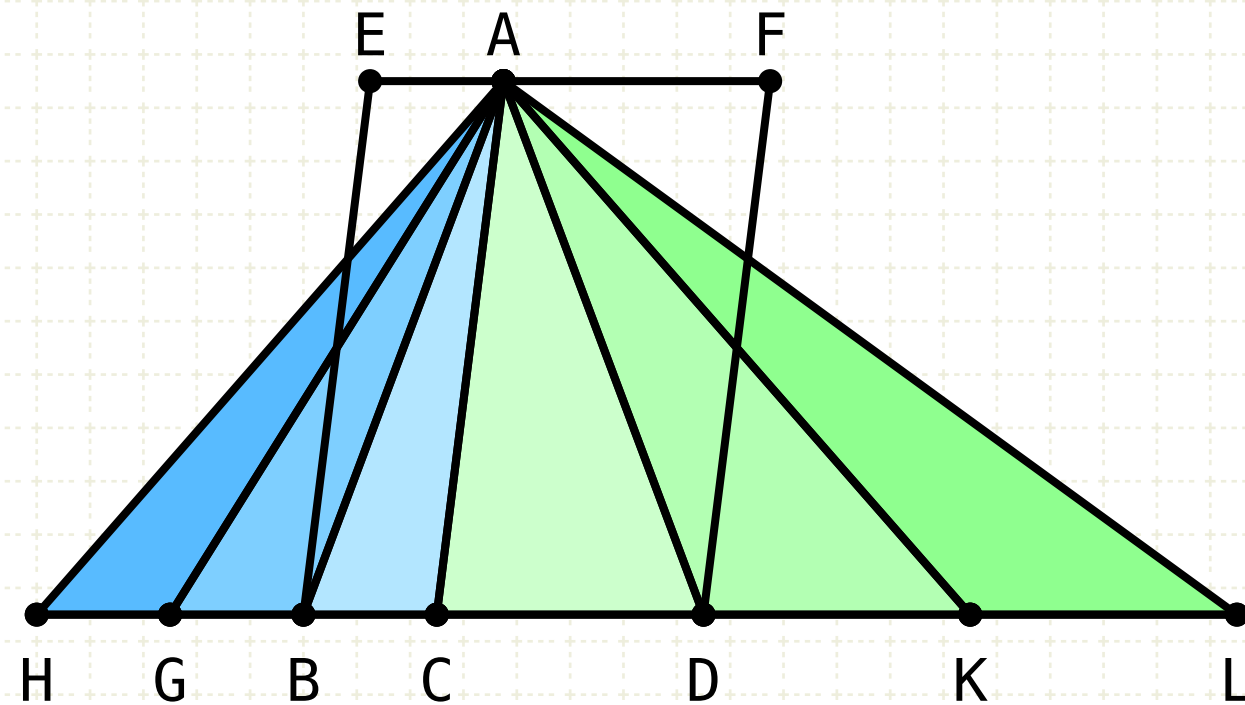
Therefore, the triangle AHC is the same multiple of ABC as the line HC is to BC

Similarly, the triangle ACL is the same multiple of ACD as the line CD is to CL

If HC is equal to CL, then the triangles AHC and ACL are equal (I-38)

Proposition 1 of Book VI

Triangles and parallelograms which are under the same height are to one another as their bases



$$HG = GB = BC, \quad CD = DK = KL$$

$$\Delta AHC = n \cdot \Delta ABC, \quad HC = n \cdot BC$$

$$\Delta ACL = m \cdot \Delta ACD, \quad CL = m \cdot CD$$

$$\text{if } HC = CL \rightarrow \Delta AHC = \Delta ACL$$

$$HC \Leftrightarrow CL \rightarrow \Delta AHC \Leftrightarrow \Delta ACL$$

Proof - Triangles

Extend the line BD to HL, such that HB is composed of any number of lines equal to BC, and that DL is composed of any number of lines equal to CD

Draw the triangles AHG, AGB, and ADK and AKL

Since the bases are equal, triangles ABC, AGB and AHG are equal, and the triangles ACD, ADK and AKL are equal (I-38)

Therefore, the triangle AHC is the same multiple of ABC as the line HC is to BC

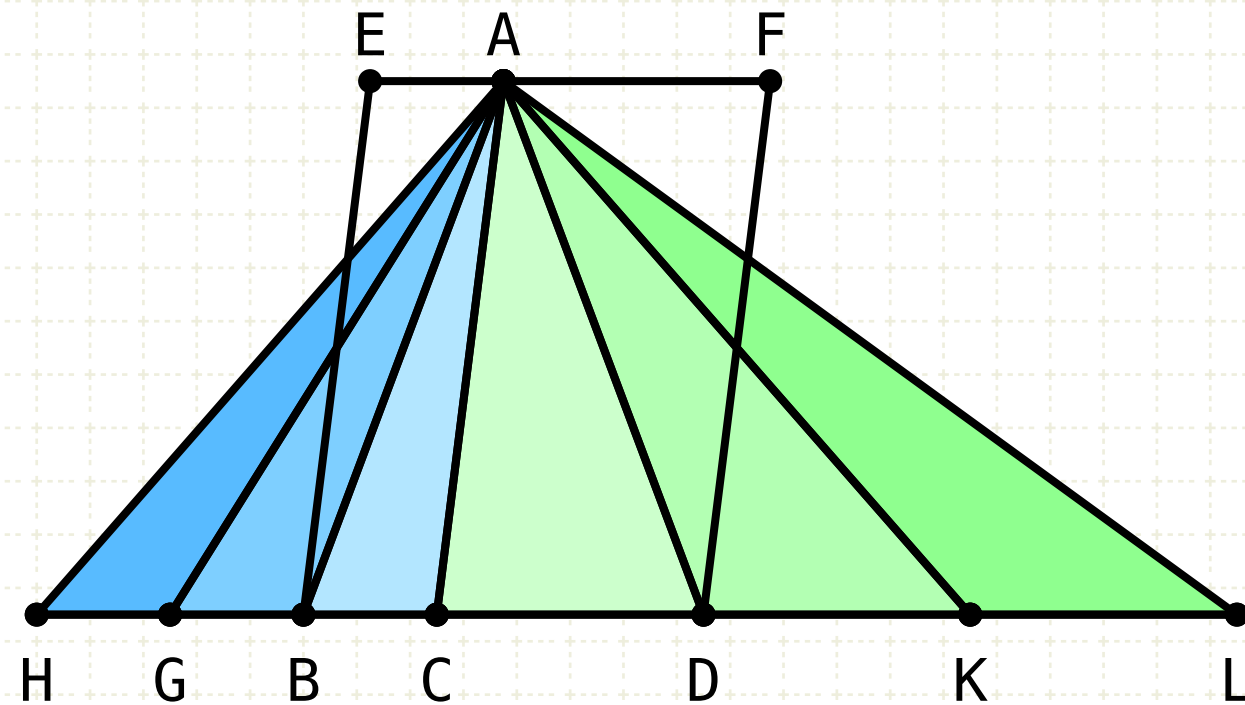
Similarly, the triangle ACL is the same multiple of ACD as the line CD is to CL

If HC is equal to CL, then the triangles AHC and ACL are equal (I-38)

IF HC is greater (or less) than CL, the the triangle AHC will be greater (or less) than ACL (I-38)

Proposition 1 of Book VI

Triangles and parallelograms which are under the same height are to one another as their bases



$$HG = GB = BC, \quad CD = DK = KL$$

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$$HC \Leftrightarrow CL \rightarrow \Delta AHC \Leftrightarrow \Delta ACL$$

$$\text{or } n \cdot BC \Leftrightarrow m \cdot CD \rightarrow n \cdot \Delta ABC \Leftrightarrow m \cdot \Delta ACD$$

$$\therefore BC : CD = \Delta ABC : \Delta ACD$$

Proof - Triangles

Extend the line BD to HL, such that HB is composed of any number of lines equal to BC, and that DL is composed of any number of lines equal to CD

Draw the triangles AHG, AGB, and ADK and AKL

Since the bases are equal, triangles ABC, AGB and AHG are equal, and the triangles ACD, ADK and AKL are equal (I-38)

Therefore, the triangle AHC is the same multiple of ABC as the line HC is to BC

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If HC is equal to CL, then the triangles AHC and ACL are equal (I-38)

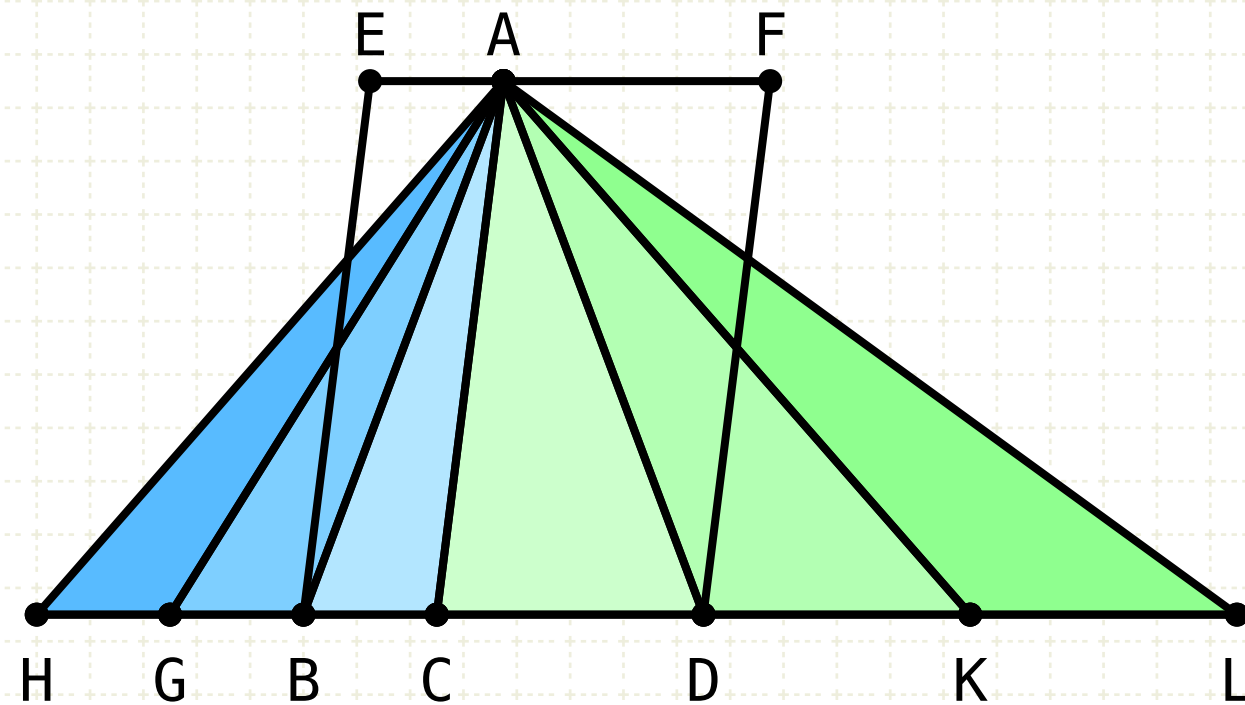
IF HC is greater (or less) than CL, the the triangle AHC will be greater (or less) than ACL (I-38)

By definition (V.Def.5), the ratio of BC to CD is equal to the ratio of the triangles ABC and ACD



Proposition 1 of Book VI

Triangles and parallelograms which are under the same height are to one another as their bases



$$HG = GB = BC, \quad CD = DK = KL$$

$$\Delta AHC = n \cdot \Delta ABC, \quad HC = n \cdot BC$$

$$\Delta ACL = m \cdot \Delta ACD, \quad CL = m \cdot CD$$

$$\text{if } HC = CL \rightarrow \Delta AHC = \Delta ACL$$

$$HC \Leftrightarrow CL \rightarrow \Delta AHC \Leftrightarrow \Delta ACL$$

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If HC is equal to CL, then the triangles AHC and ACL are equal (I-38)

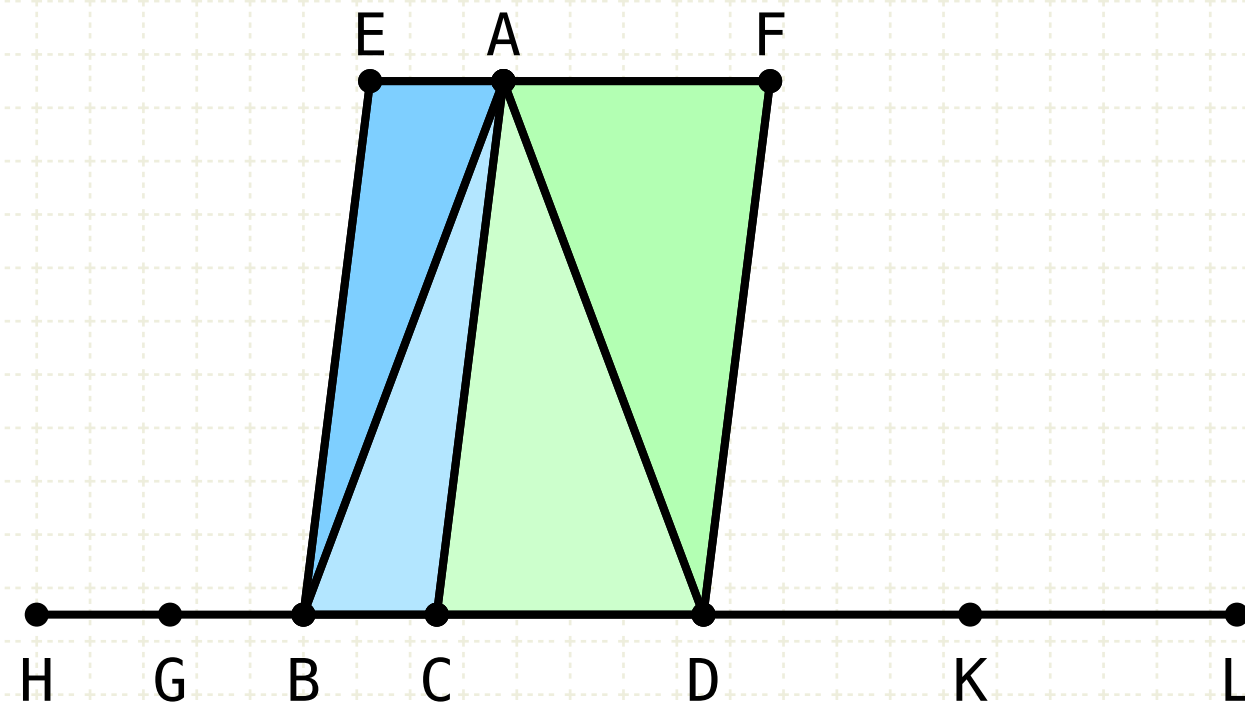
If HC is greater (or less) than CL, the the triangle AHC will be greater (or less) than ACL (I-38)

By definition (V.Def.5), the ratio of BC to CD is equal to the ratio of the triangles ABC and ACD



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Triangles and parallelograms which are under the same height are to one another as their bases

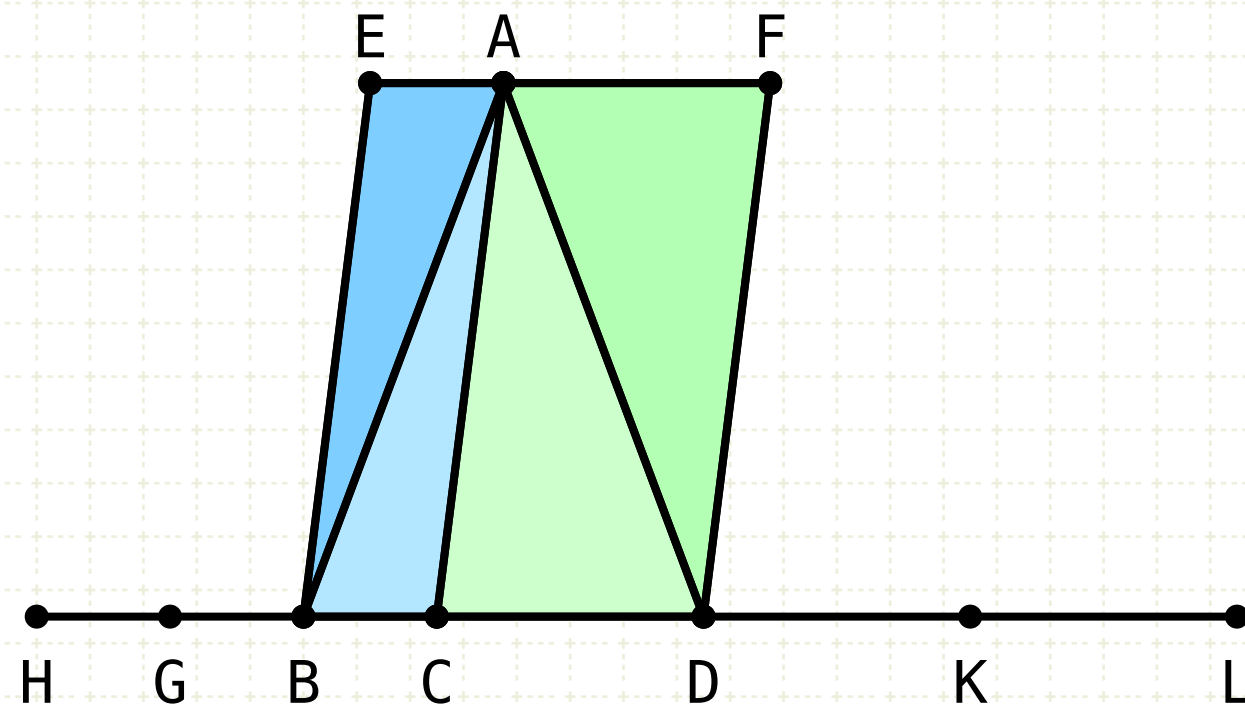


Proof - Parallelograms

$$BC : CD = \Delta ABC : \Delta ACD$$

Proposition 1 of Book VI

Triangles and parallelograms which are under the same height are to one another as their bases



Proof - Parallelograms

The parallelograms EC and CF are twice the triangles ABC and ACD respectively (I·41)

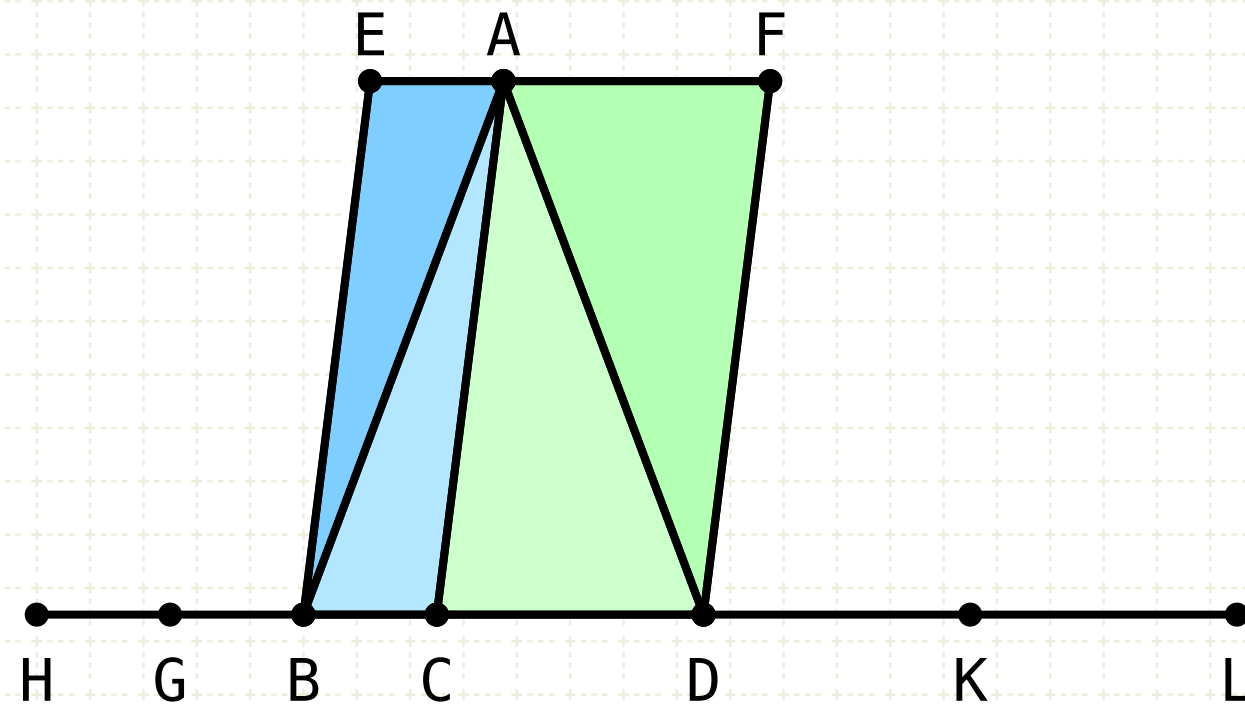
$$BC : CD = \triangle ABC : \triangle ACD$$

$$\square EC = 2\triangle ABC$$

$$\square CF = 2\triangle ACD$$

Proposition 1 of Book VI

Triangles and parallelograms which are under the same height are to one another as their bases



$$BC : CD = \triangle ABC : \triangle ACD$$

$$\square EC = 2\triangle ABC$$

$$\square CF = 2\triangle ACD$$

$$\triangle ABC : \triangle ACD = \square EC : \square CF$$

Proof - Parallelograms

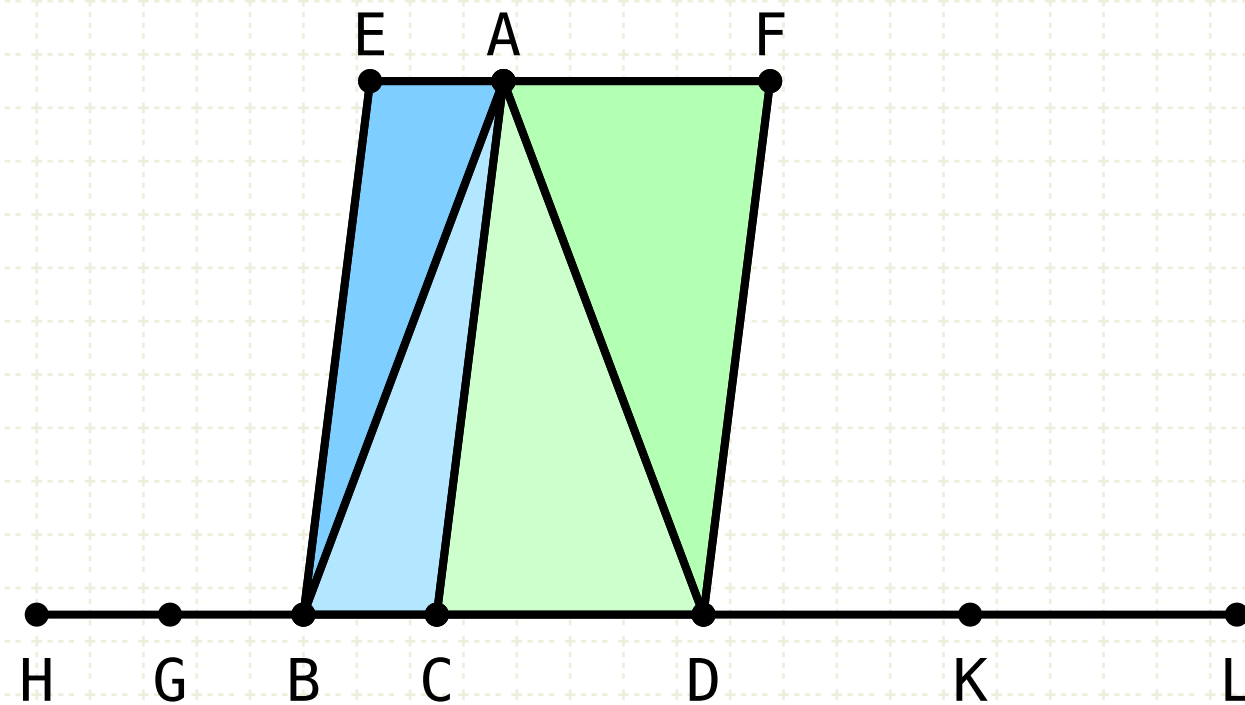
The parallelograms EC and CF are twice the triangles ABC and ACD respectively (I·41)

If two magnitudes are multiplied by the same number, then the ratio will not be affected (V·15)

Therefore the ratio of the triangles to each other is the same as the ratio of the parallelograms

Proposition 1 of Book VI

Triangles and parallelograms which are under the same height are to one another as their bases



Proof - Parallelograms

The parallelograms EC and CF are twice the triangles ABC and ACD respectively (I·41)

If two magnitudes are multiplied by the same number, then the ratio will not be affected (V·15)

Therefore the ratio of the triangles to each other is the same as the ratio of the parallelograms

And since ratios are transitive ($a=b$ $b=c \rightarrow a=c$) (V·11), then the ratio of the parallelograms is equal to the ratio of the bases

$$BC : CD = \triangle ABC : \triangle ACD$$

$$\square EC = 2\triangle ABC$$

$$\square CF = 2\triangle ACD$$

$$\triangle ABC : \triangle ACD = \square EC : \square CF$$

$$BC : CD = \triangle ABC : \triangle ACD = \square EC : \square CF$$

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