Euclid's Elements

Book VII

Definitions:

1 A unit is that by virtue of which each of the things that exist is called one

2 A number is a multitude composed of units. (not one)

3 A number is part of a number, the less of the greater, when it measures the greater

11 A prime number is that which is measured by a unit alone.

12 Numbers prime to one another are those which are measured by a unit alone as a common measure

15 A number is said to multiply a number when that which is multiplied is added to itself as many times as there are units in the other, and thus some number is produced.

20 Numbers are proportional when the first is the same multiple, or the same part, or the same parts, of the second that the third is of the fourth.

As long as algebra and geometry have been separated, their progress have been slow and their uses limited; but when these two sciences have been united, they have lent each mutual forces, and have marched together towards perfection.

Joseph-Louis Lagrange
(1736 to 1813)

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Proposition 1 of Book VII

Two unequal numbers being set out, and the less being continuously subtracted in turn from the greater, if the number which is left never measures the one before it until a unit is left, the original numbers will be prime to one another

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Proposition 1 of Book VII

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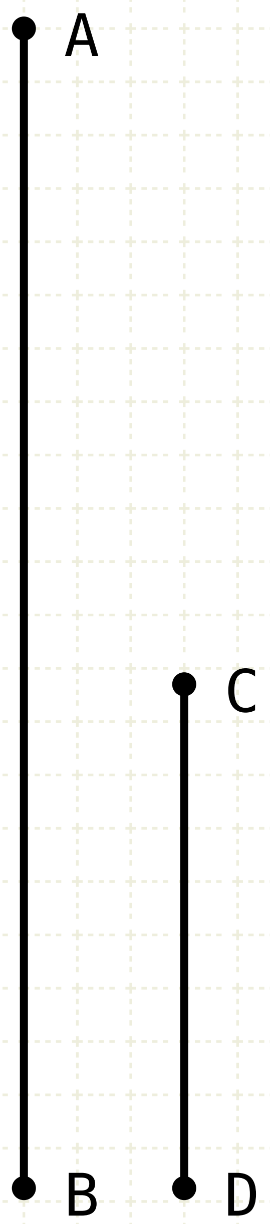

In other words

Start with two unequal natural numbers, continuously subtract the smaller from the larger as long as one number is not a multiple of the other

If the resulting number is the number one, then the two numbers are relatively prime

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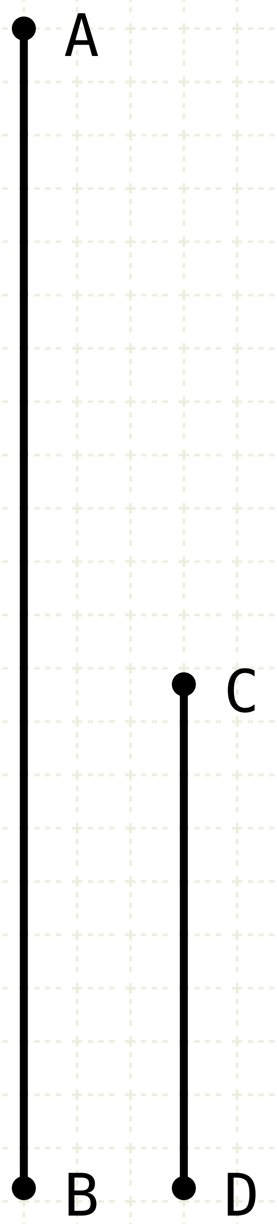

AB = 145, CD = 63

In other words

Start with two unequal natural numbers, continuously subtract the smaller from the larger as long as one number is not a multiple of the other

If the resulting number is the number one, then the two numbers are relatively prime

Example

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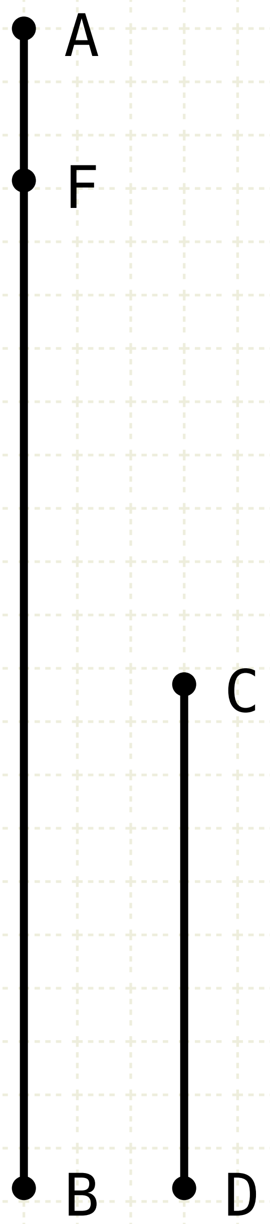

AB = 145, CD = 63

In other words

Start with two unequal natural numbers, continuously subtract the smaller from the larger as long as one number is not a multiple of the other

If the resulting number is the number one, then the two numbers are relatively prime

Example

Let CD measure BF with the remainder AF less than CD,

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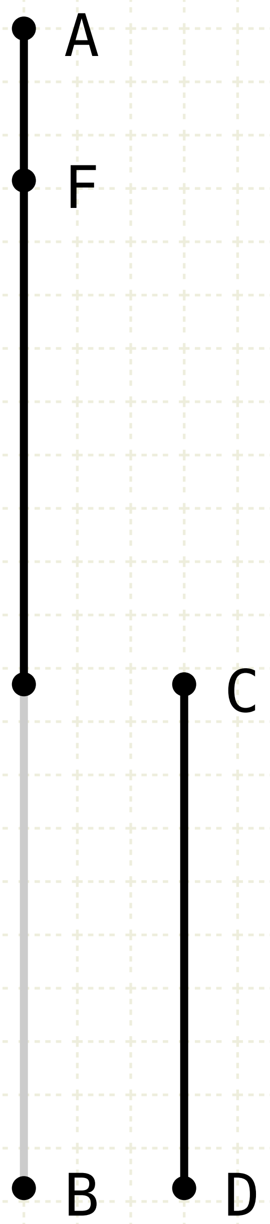

AB = 145, CD = 63

145- 63 = 82

In other words

Start with two unequal natural numbers, continuously subtract the smaller from the larger as long as one number is not a multiple of the other

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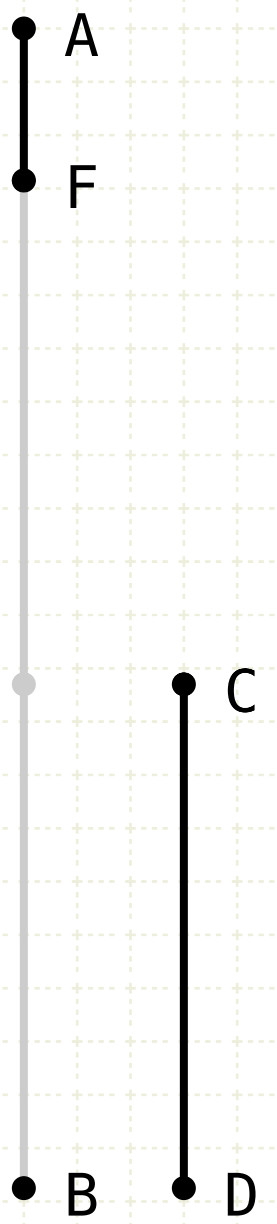

AB = 145, CD = 63

$145 - 63 = 82$

$82 - 63 = 19$

In other words

Start with two unequal natural numbers, continuously subtract the smaller from the larger as long as one number is not a multiple of the other

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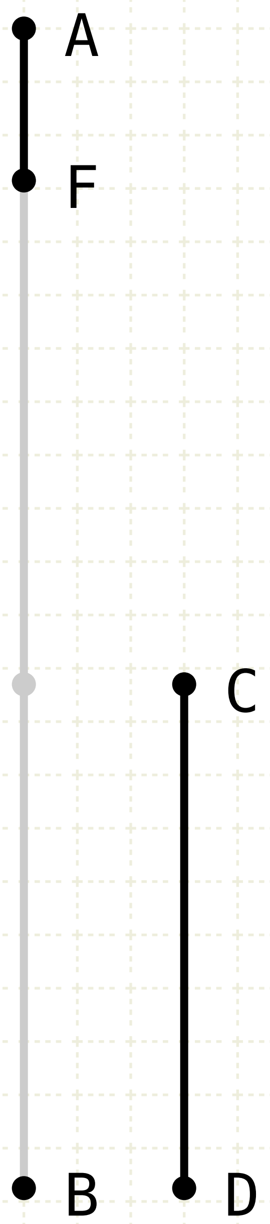

AB = 145, CD = 63

145- 63 = 82

82- 63 = 19

AF=19

In other words

Start with two unequal natural numbers, continuously subtract the smaller from the larger as long as one number is not a multiple of the other

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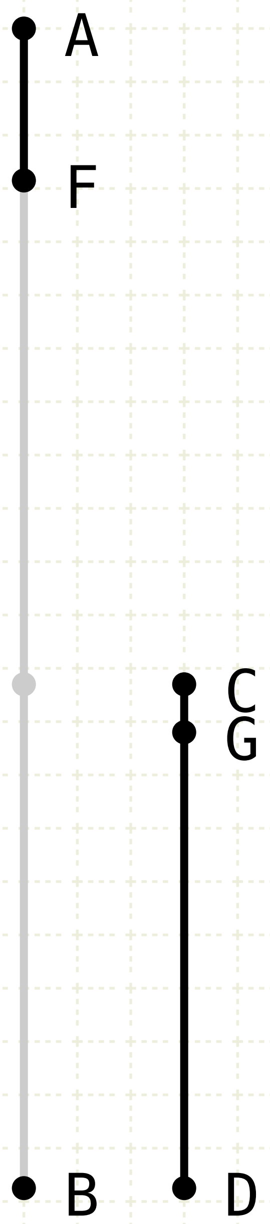

AB = 145, CD = 63

145 - 63 = 82

82 - 63 = 19

AF=19

In other words

Start with two unequal natural numbers, continuously subtract the smaller from the larger as long as one number is not a multiple of the other

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Example

Let CD measure BF with the remainder AF less than CD,

Let AF measure DG, with CG less than AF

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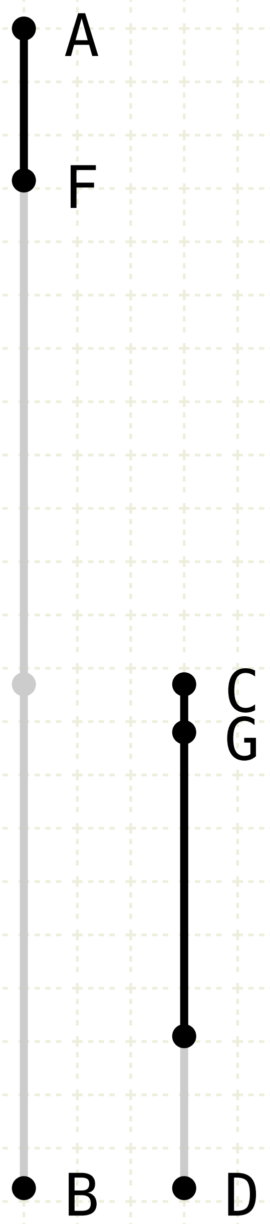

AB = 145, CD = 63

145 - 63 = 82

82 - 63 = 19

AF=19

63 - 19 = 44

In other words

Start with two unequal natural numbers, continuously subtract the smaller from the larger as long as one number is not a multiple of the other

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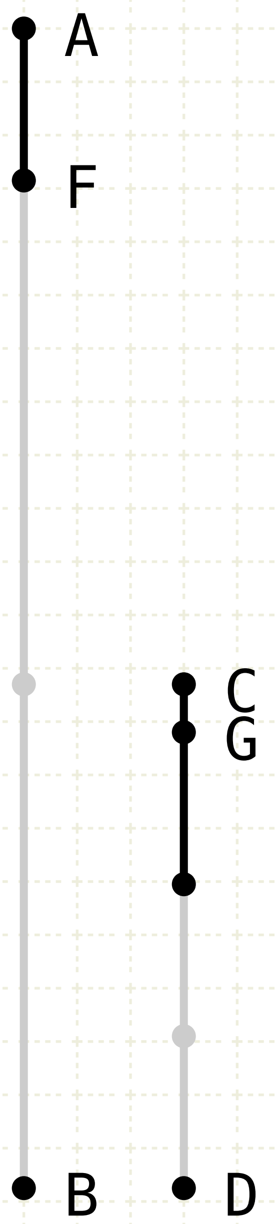


```
AB = 145, CD = 63
        145- 63 = 82
         82- 63 = 19
AF=19
         63- 19 = 44
         44- 19 = 25
```

In other words

Start with two unequal natural numbers, continuously subtract the smaller from the larger as long as one number is not a multiple of the other

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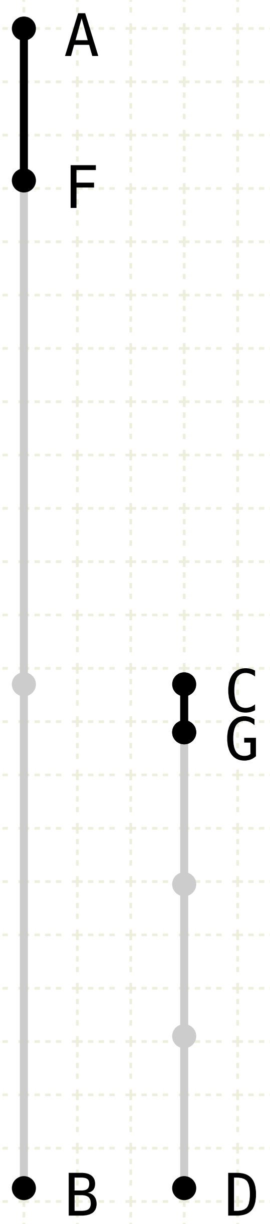


```
AB = 145, CD = 63
        145- 63 = 82
         82- 63 = 19
AF=19
         63- 19 = 44
         44- 19 = 25
         25- 19 =  6
```

In other words

Start with two unequal natural numbers, continuously subtract the smaller from the larger as long as one number is not a multiple of the other

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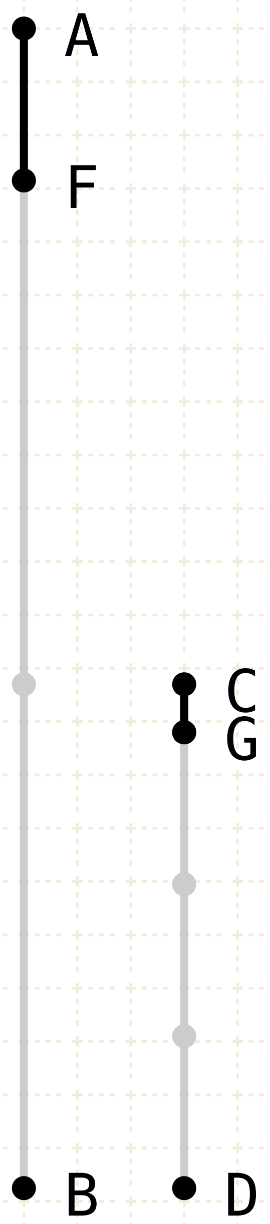

AB = 145, CD = 63

145 - 63 = 82

82 - 63 = 19

AF=19

63 - 19 = 44

44 - 19 = 25

25 - 19 = 6

CG=6

In other words

Start with two unequal natural numbers, continuously subtract the smaller from the larger as long as one number is not a multiple of the other

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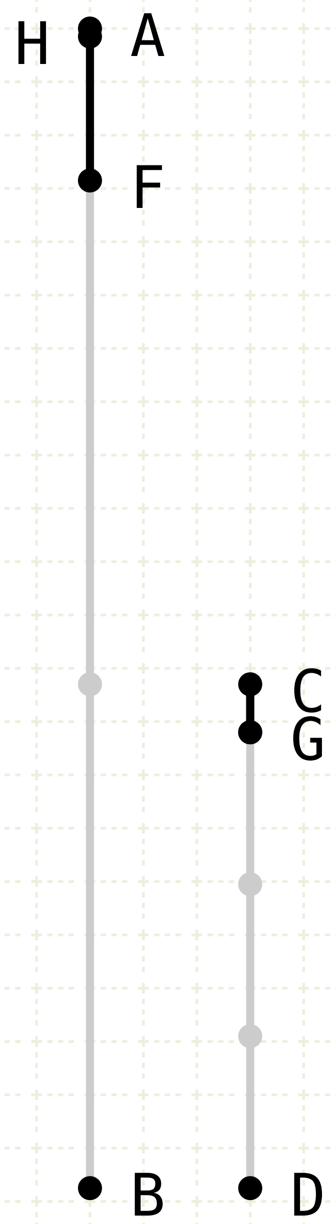

AB = 145, CD = 63

145 - 63 = 82

82 - 63 = 19

AF=19

63 - 19 = 44

44 - 19 = 25

25 - 19 = 6

CG=6

In other words

Start with two unequal natural numbers, continuously subtract the smaller from the larger as long as one number is not a multiple of the other

If the resulting number is the number one, then the two numbers are relatively prime

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Let CD measure BF with the remainder AF less than CD,

Let AF measure DG, with CG less than AF

And let CG measure FH …

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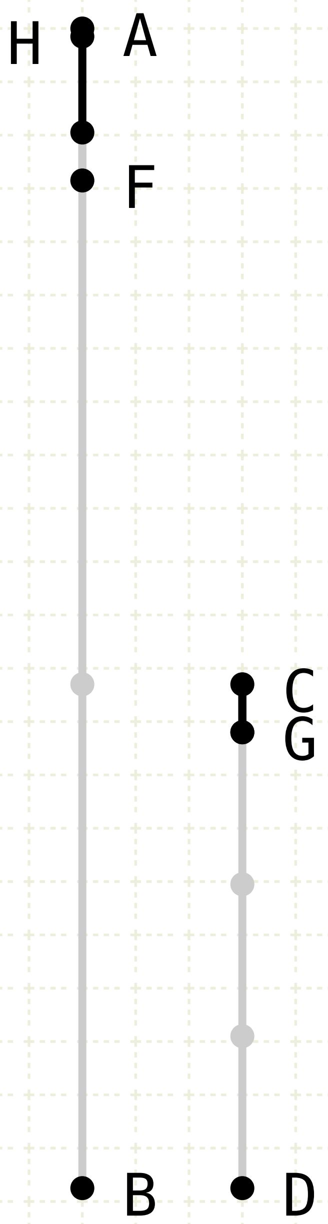


```
AB = 145, CD = 63
        145- 63 = 82
         82- 63 = 19
AF=19
         63- 19 = 44
         44- 19 = 25
         25- 19 =  6
CG=6
         19-  6 =  13
```

In other words

Start with two unequal natural numbers, continuously subtract the smaller from the larger as long as one number is not a multiple of the other

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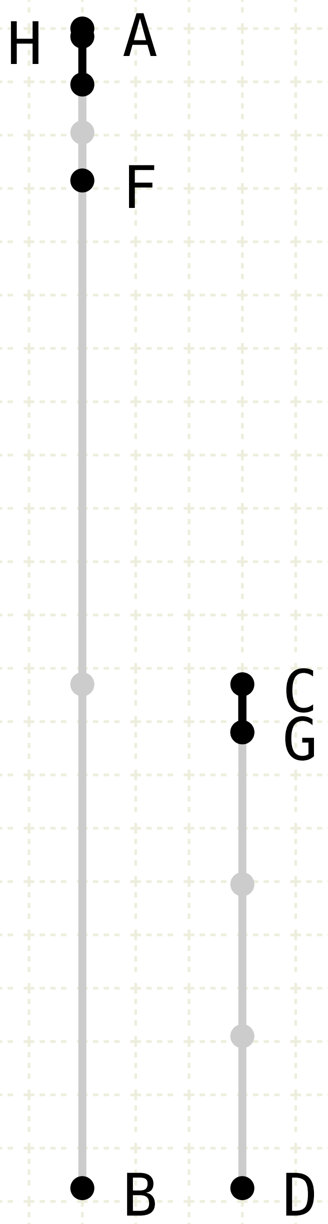

AB = 145, CD = 63

145 - 63 = 82
82 - 63 = 19

AF=19

63 - 19 = 44
44 - 19 = 25
25 - 19 = 6

CG=6

19 - 6 = 13
13 - 6 = 7

In other words

Start with two unequal natural numbers, continuously subtract the smaller from the larger as long as one number is not a multiple of the other

If the resulting number is the number one, then the two numbers are relatively prime

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Let CD measure BF with the remainder AF less than CD,

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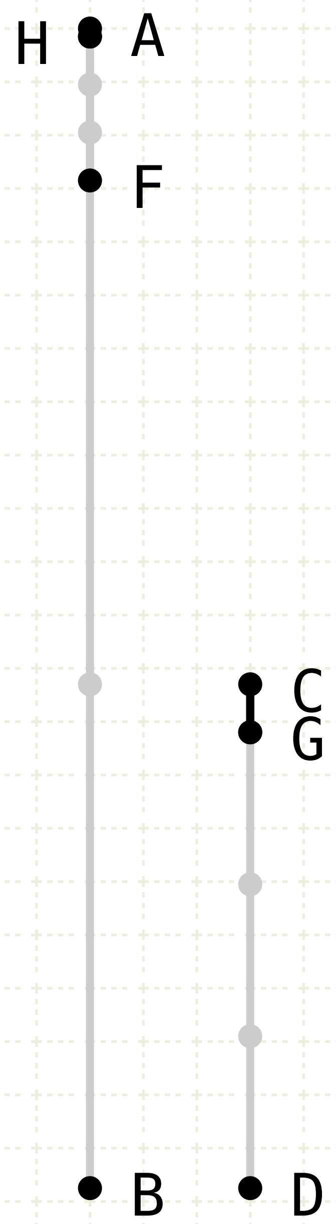


```
AB = 145, CD = 63
        145- 63 = 82
         82- 63 = 19
AF=19
         63- 19 = 44
         44- 19 = 25
         25- 19 =  6
CG=6
         19-  6 =  13
         13-  6 =   7
          7-  6 =   1
```

In other words

Start with two unequal natural numbers, continuously subtract the smaller from the larger as long as one number is not a multiple of the other

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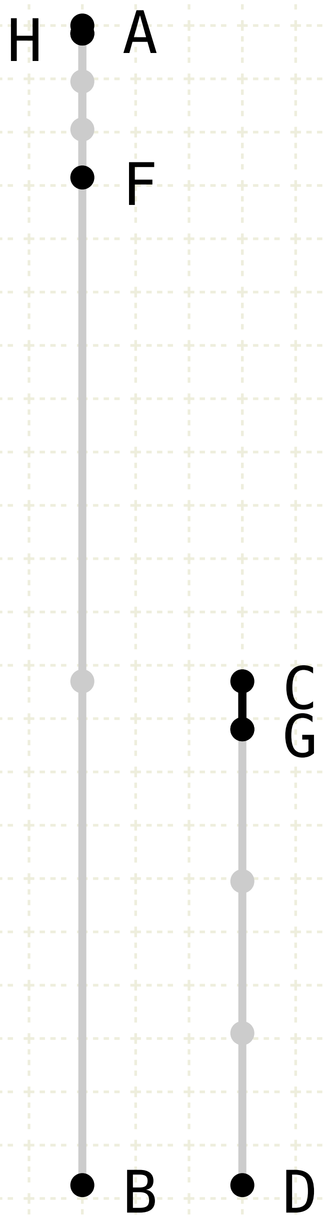

AB = 145, CD = 63

145 - 63 = 82
82 - 63 = 19

AF=19

63 - 19 = 44
44 - 19 = 25
25 - 19 = 6

CG=6

19 - 6 = 13
13 - 6 = 7
7 - 6 = 1

AH = 1

In other words

Start with two unequal natural numbers, continuously subtract the smaller from the larger as long as one number is not a multiple of the other

If the resulting number is the number one, then the two numbers are relatively prime

Example

Let CD measure BF with the remainder AF less than CD,

Let AF measure DG, with CG less than AF

And let CG measure FH …

… leaving a single unit as the remainder

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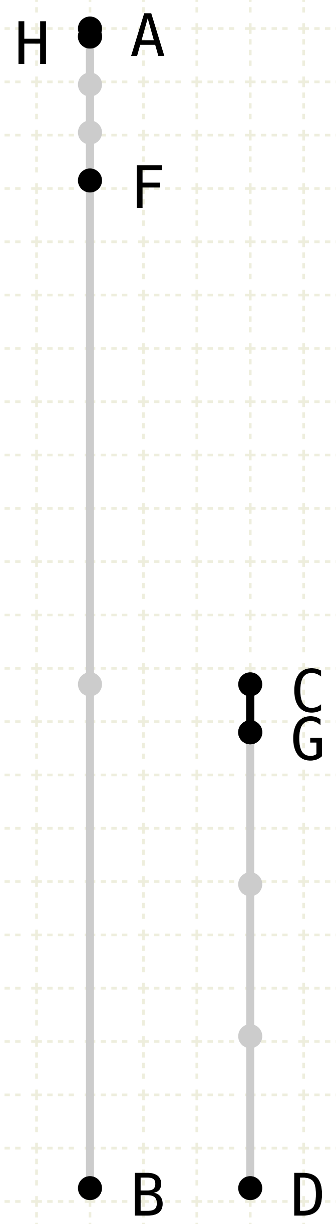


```
AB = 145, CD = 63
        145- 63 = 82
         82- 63 = 19
AF=19
         63- 19 = 44
         44- 19 = 25
         25- 19 =  6
CG=6
         19-  6 =  13
         13-  6 =   7
          7-  6 =   1
AH = 1

gcd(145,63) = 1
```

In other words

Start with two unequal natural numbers, continuously subtract the smaller from the larger as long as one number is not a multiple of the other

If the resulting number is the number one, then the two numbers are relatively prime

Example

Let CD measure BF with the remainder AF less than CD,

Let AF measure DG, with CG less than AF

And let CG measure FH …

… leaving a single unit as the remainder

145 and 63 are prime to one another

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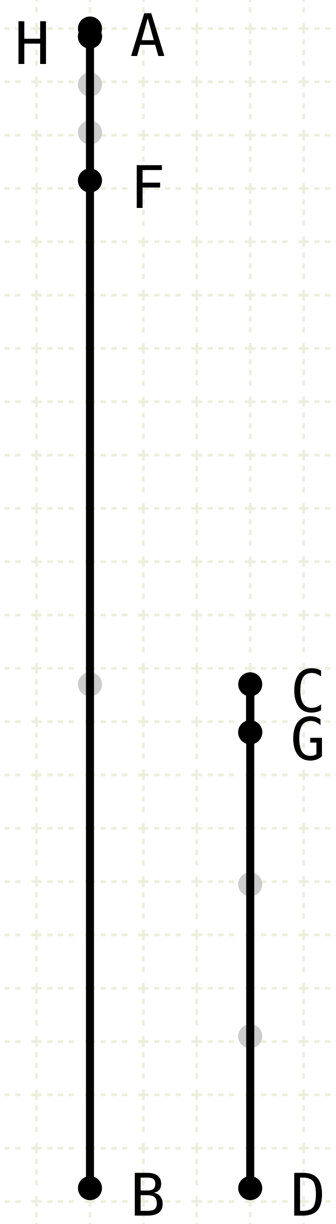

$BF = CD + CD + \ldots = a \cdot CD,\ AF < CD$

$DG = AF + AF + \ldots = b \cdot AF,\ CG < AF$

$FH = CG + CG + \ldots = c \cdot CG$

$AH = 1$

Proof by Contradiction

Let CD measure BF with the remainder AF less than CD,

And AF measure DG, with CG less than AF

And let CG measure FH, leaving AH equal to one

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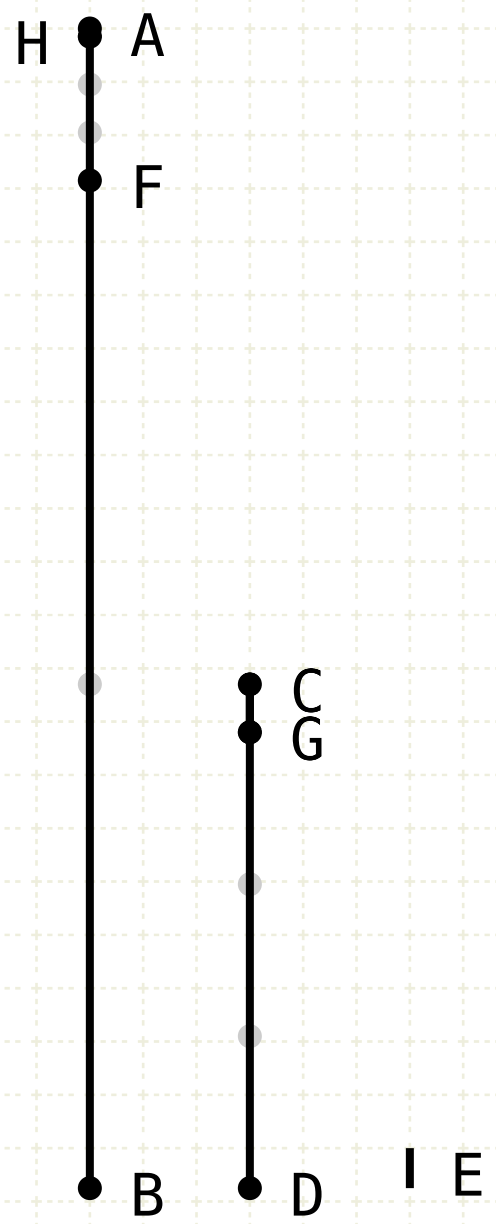

$BF = CD + CD + \dots = a \cdot CD, \; AF < CD$

$DG = AF + AF + \dots = b \cdot AF, \; CG < AF$

$FH = CG + CG + \dots = c \cdot CG$

$AH = 1$

Let

$AB = p \cdot E$

$CD = q \cdot E$

$E \neq 1$

Proof by Contradiction

Let CD measure BF with the remainder AF less than CD,

And AF measure DG, with CG less than AF

And let CG measure FH, leaving AH equal to one

Assume that AB,CD are not relatively prime

Therefore there is some natural number 'E' which measures both AB and CD

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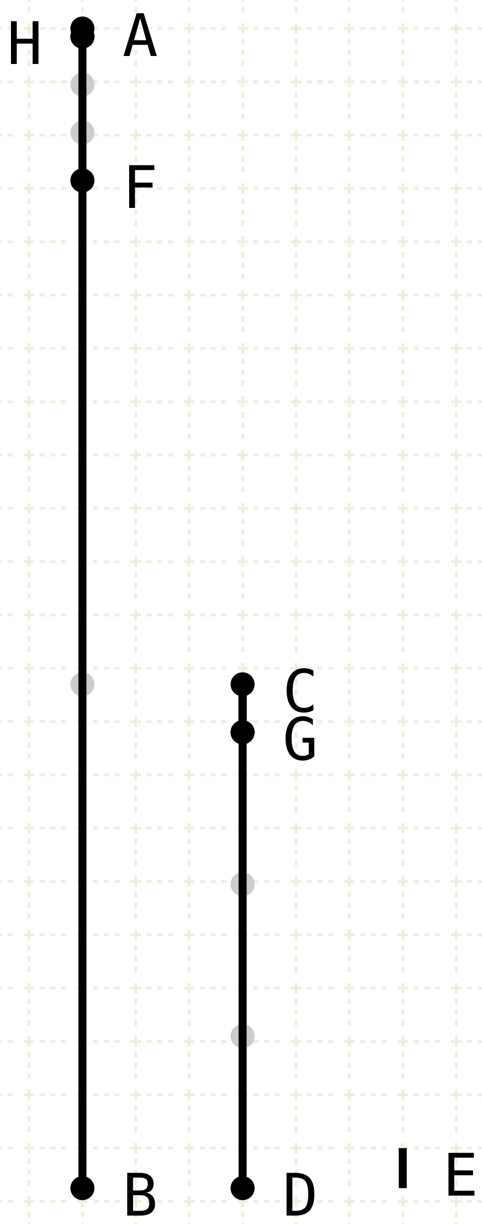

$BF = CD + CD + \ldots = a \cdot CD,\ AF < CD$

$DG = AF + AF + \ldots = b \cdot AF,\ CG < AF$

$FH = CG + CG + \ldots = c \cdot CG$

$AH = 1$

Let

$AB = p \cdot E$

$CD = q \cdot E$

$E \neq 1$

$BF = q \cdot E + q \cdot E + \ldots = d \cdot E$

Proof by Contradiction

Let CD measure BF with the remainder AF less than CD,

And AF measure DG, with CG less than AF

And let CG measure FH, leaving AH equal to one

Assume that AB,CD are not relatively prime

Therefore there is some natural number 'E' which measures both AB and CD

Since E measures CD, and CD measures BF, E also measures BF

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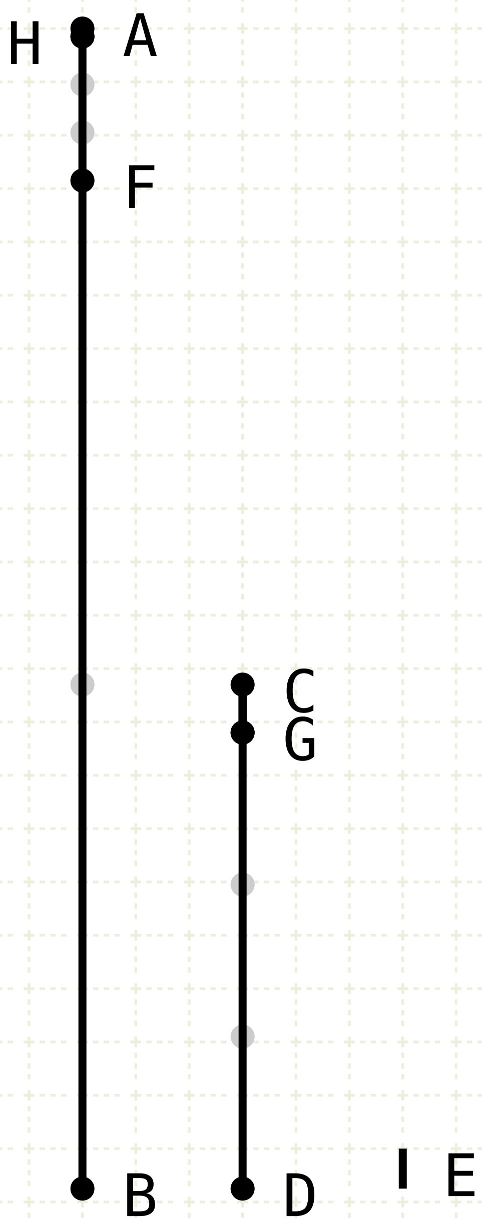

$BF = CD + CD + \ldots = a \cdot CD,\ AF < CD$

$DG = AF + AF + \ldots = b \cdot AF,\ CG < AF$

$FH = CG + CG + \ldots = c \cdot CG$

$AH = 1$

Let

$AB = p \cdot E$

$CD = q \cdot E$

$E \neq 1$

$BF = q \cdot E + q \cdot E + \ldots = d \cdot E$

$AF = AB - BF = p \cdot E - d \cdot E = e \cdot E$

Proof by Contradiction

Let CD measure BF with the remainder AF less than CD,

And AF measure DG, with CG less than AF

And let CG measure FH, leaving AH equal to one

Assume that AB,CD are not relatively prime

Therefore there is some natural number 'E' which measures both AB and CD

Since E measures CD, and CD measures BF, E also measures BF

But E also measures AB, therefore it also measures AF

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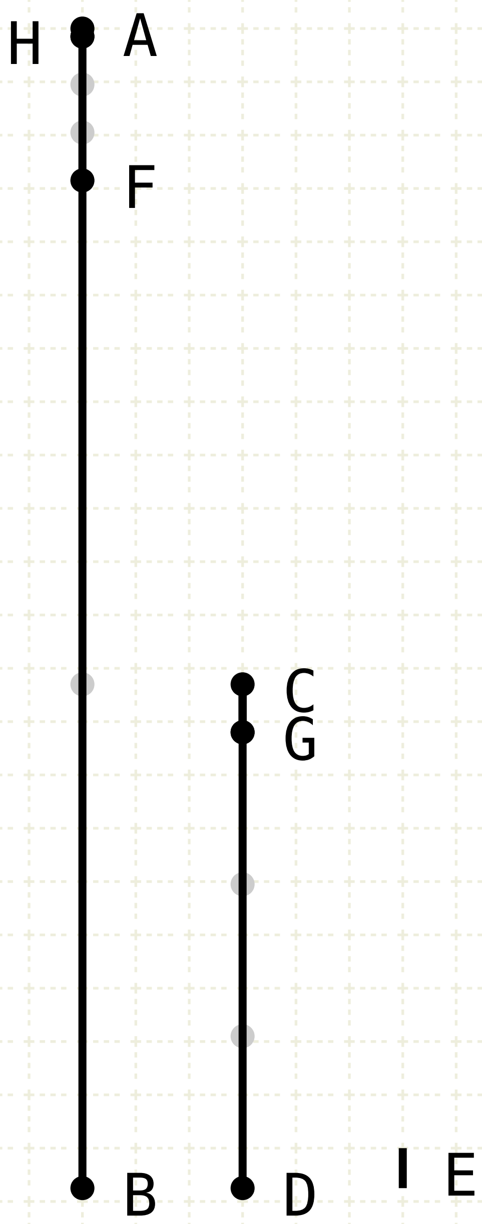

$BF = CD + CD + \ldots = a \cdot CD,\ AF < CD$

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$FH = CG + CG + \ldots = c \cdot CG$

$AH = 1$

Let

$AB = p \cdot E$

$CD = q \cdot E$

$E \neq 1$

$BF = q \cdot E + q \cdot E + \ldots = d \cdot E$

$AF = AB - BF = p \cdot E - d \cdot E = e \cdot E$

$DG = e \cdot E + e \cdot E + \ldots = f \cdot E$

Proof by Contradiction

Let CD measure BF with the remainder AF less than CD,

And AF measure DG, with CG less than AF

And let CG measure FH, leaving AH equal to one

Assume that AB,CD are not relatively prime

Therefore there is some natural number 'E' which measures both AB and CD

Since E measures CD, and CD measures BF, E also measures BF

But E also measures AB, therefore it also measures AF

But AF measures DG, therefore E also measures DG

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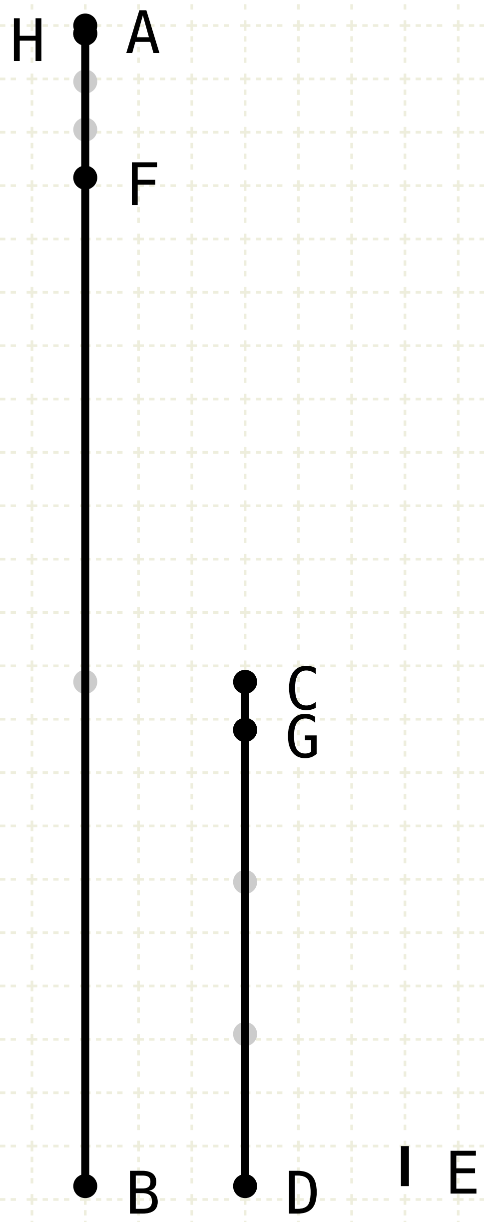

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$DG = AF + AF + \ldots = b \cdot AF, \; CG < AF$

$FH = CG + CG + \ldots = c \cdot CG$

$AH = 1$

Let

$AB = p \cdot E$

$CD = q \cdot E$

$E \neq 1$

$BF = q \cdot E + q \cdot E + \ldots = d \cdot E$

$AF = AB - BF = p \cdot E - d \cdot E = e \cdot E$

$DG = e \cdot E + e \cdot E + \ldots = f \cdot E$

$CG = CD - DG = q \cdot E - f \cdot E = g \cdot E$

Proof by Contradiction

Let CD measure BF with the remainder AF less than CD,

And AF measure DG, with CG less than AF

And let CG measure FH, leaving AH equal to one

Assume that AB,CD are not relatively prime

Therefore there is some natural number 'E' which measures both AB and CD

Since E measures CD, and CD measures BF, E also measures BF

But E also measures AB, therefore it also measures AF

But AF measures DG, therefore E also measures DG

But it also measures the whole CD, therefore it also measures CG

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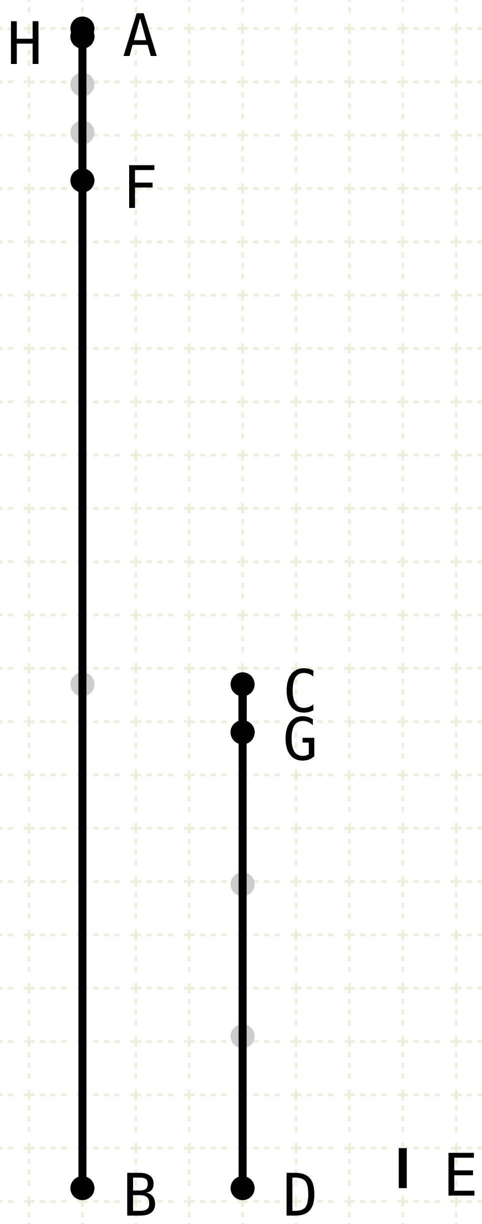

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$FH = CG + CG + \ldots = c \cdot CG$

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Let

$AB = p \cdot E$

$CD = q \cdot E$

$E \neq 1$

$BF = q \cdot E + q \cdot E + \ldots = d \cdot E$

$AF = AB - BF = p \cdot E - d \cdot E = e \cdot E$

$DG = e \cdot E + e \cdot E + \ldots = f \cdot E$

$CG = CD - DG = q \cdot E - f \cdot E = g \cdot E$

$FH = g \cdot E + g \cdot E + \ldots = h \cdot E$

Proof by Contradiction

Let CD measure BF with the remainder AF less than CD,

And AF measure DG, with CG less than AF

And let CG measure FH, leaving AH equal to one

Assume that AB,CD are not relatively prime

Therefore there is some natural number 'E' which measures both AB and CD

Since E measures CD, and CD measures BF, E also measures BF

But E also measures AB, therefore it also measures AF

But AF measures DG, therefore E also measures DG

But it also measures the whole CD, therefore it also measures CG

But CG measures FH, therefore E also measures FH

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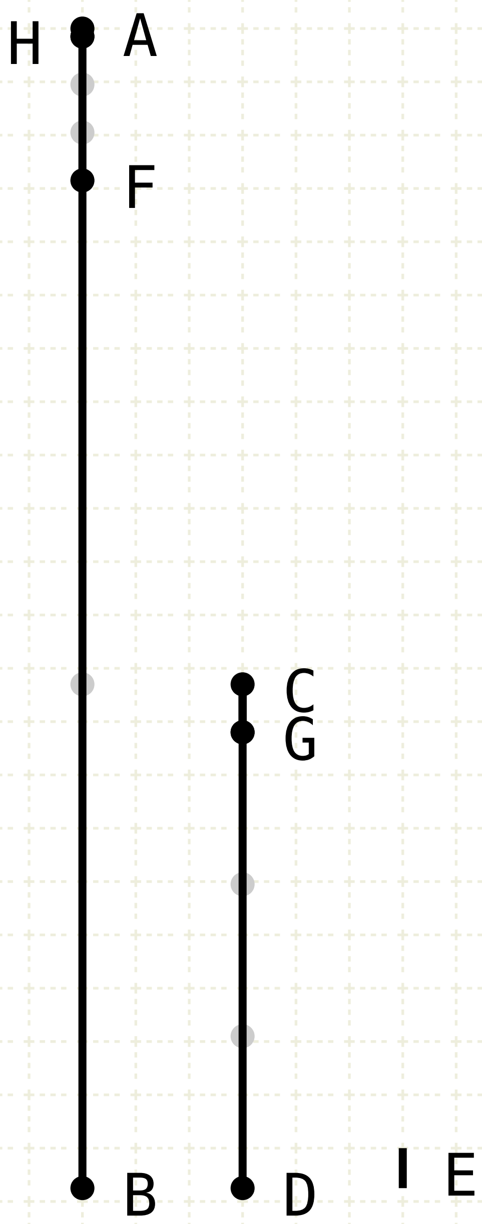
Proposition 1 of Book VII

Two unequal numbers being set out, and the less being continuously subtracted in turn from the greater, if the number which is left never measures the one before it until a unit is left, the original numbers will be prime to one another



$BF = CD + CD + \ldots = a \cdot CD,\ AF < CD$

$DG = AF + AF + \ldots = b \cdot AF,\ CG < AF$

$FH = CG + CG + \ldots = c \cdot CG$

$AH = 1$

Let

$AB = p \cdot E$

$CD = q \cdot E$

$E \neq 1$

$BF = q \cdot E + q \cdot E + \ldots = d \cdot E$

$AF = AB - BF = p \cdot E - d \cdot E = e \cdot E$

$DG = e \cdot E + e \cdot E + \ldots = f \cdot E$

$CG = CD - DG = q \cdot E - f \cdot E = g \cdot E$

$FH = g \cdot E + g \cdot E + \ldots = h \cdot E$

$AH = AF - FH = e \cdot E - h \cdot E = m \cdot E$

Proof by Contradiction

Let CD measure BF with the remainder AF less than CD,

And AF measure DG, with CG less than AF

And let CG measure FH, leaving AH equal to one

Assume that AB,CD are not relatively prime

Therefore there is some natural number 'E' which measures both AB and CD

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But AF measures DG, therefore E also measures DG

But it also measures the whole CD, therefore it also measures CG

But CG measures FH, therefore E also measures FH

But E also measures the whole of AF, therefore it will also measure the remainder AH

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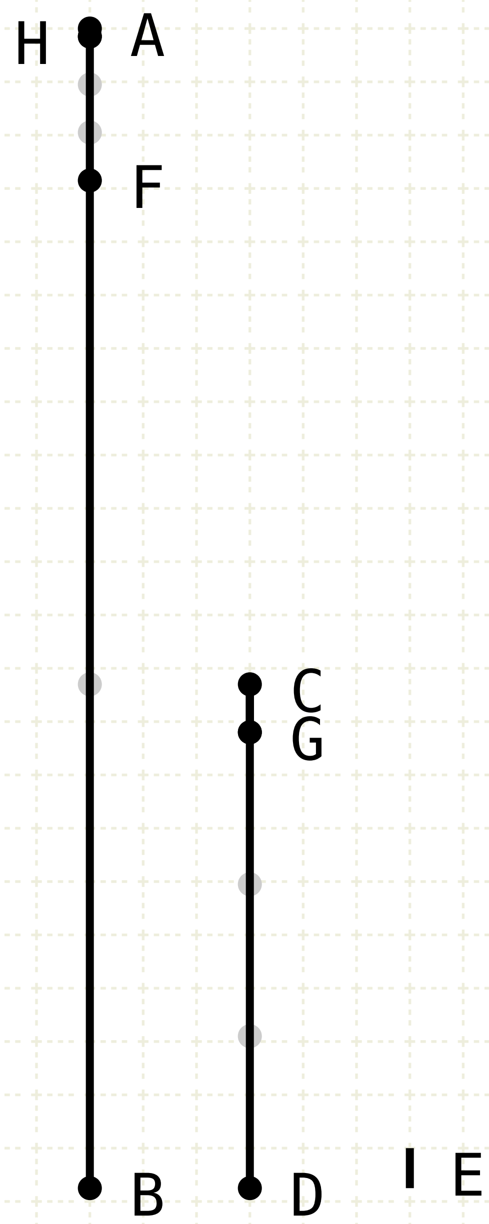

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But CG measures FH, therefore E also measures FH

But E also measures the whole of AF, therefore it will also measure the remainder AH

But… AH cannot be simultaneously be equal to one and a multiple of a number greater than one

Therefore we have a contradiction, and AB and CD must be relatively prime

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Proposition 1 of Book VII

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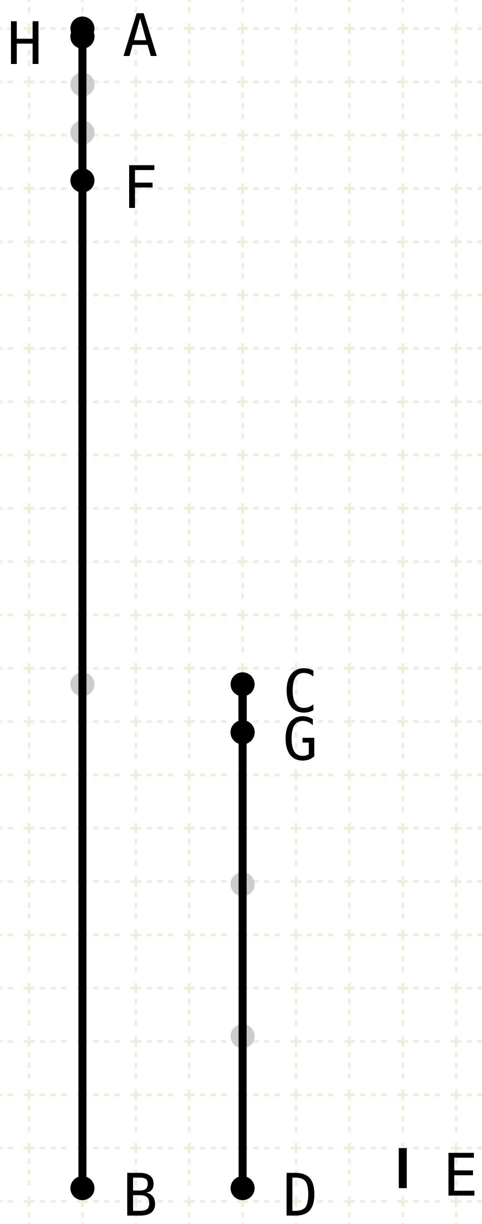

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$AH = m \cdot E = 1$ ✗

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