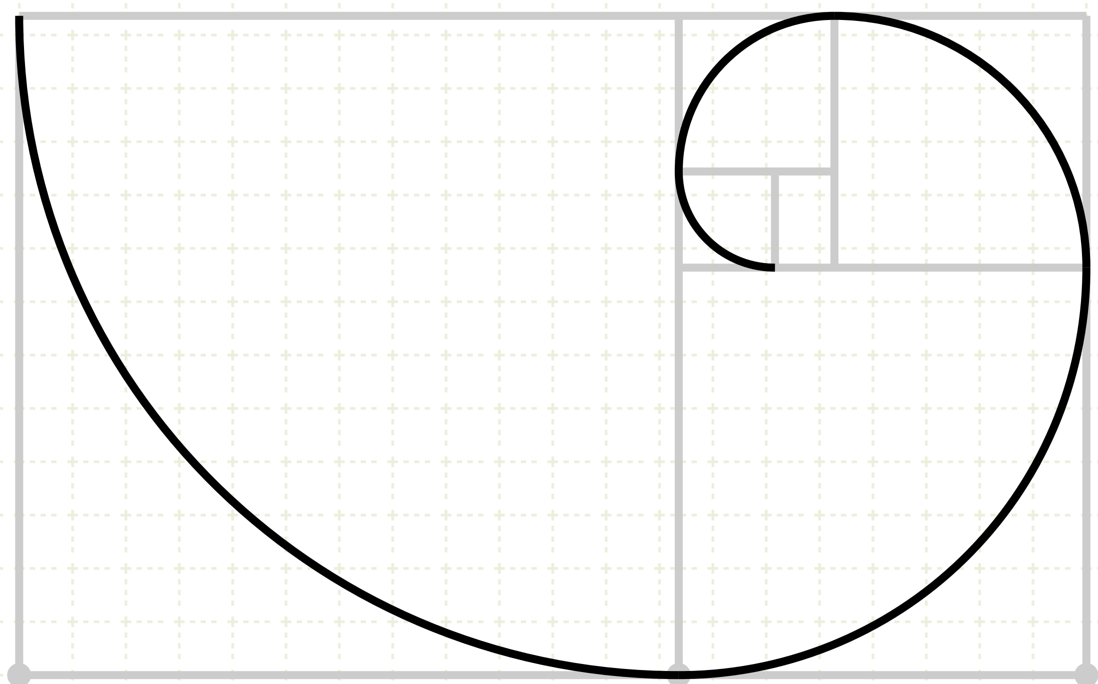
Euclid's Elements

Book VI



One can state, without exaggeration, that the observation of and the search for similarities and differences are the basis of all human knowledge.

Alfred Nobel

Table of Contents, Chapter 6

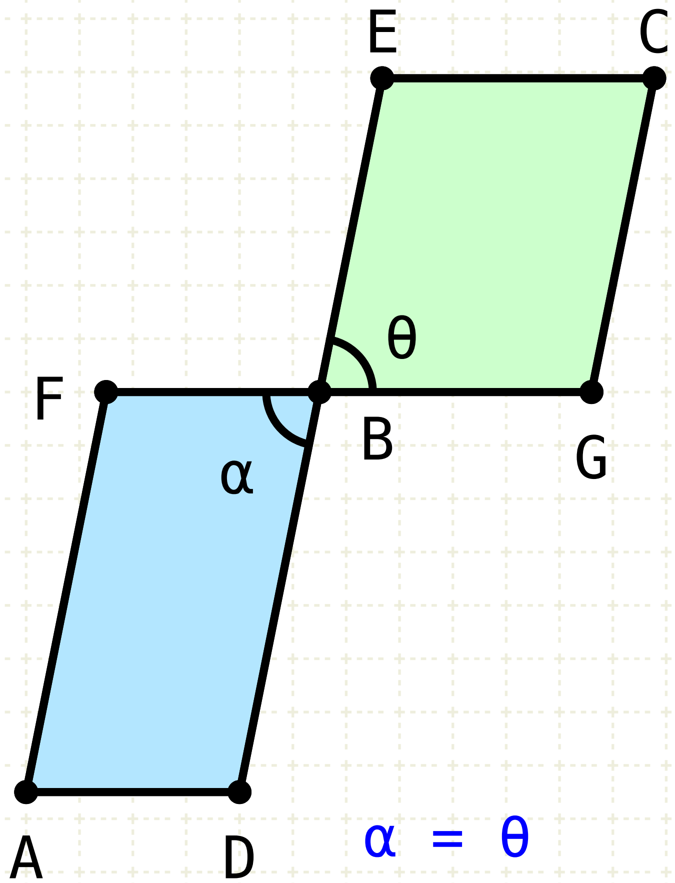
Table of Contents, Chapter 3

Proposition 14 of Book VI

In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal.

Proposition 14 of Book VI

In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal.



$\alpha = \theta$

$\square AB = \square BC$

$DB:BE = GB:BF \quad \rightarrow DB \times BF = GB \times BE$

$\alpha = \theta$

$DB:BE = GB:BF$

$\square AB = \square BC$

In other words

Given two equiangular parallelograms, where the areas are equal, then the ratios of the sides around the equal angle FB,BD and EB,BG are reciprocally proportional

… or … the multiplication of the two sides of the parallelogram remains constant as long as the area and the angles remain the same

And the inverse

Note:

Assume two objects 'x' and 'y', both with properties '1' and '2'

Proportional:

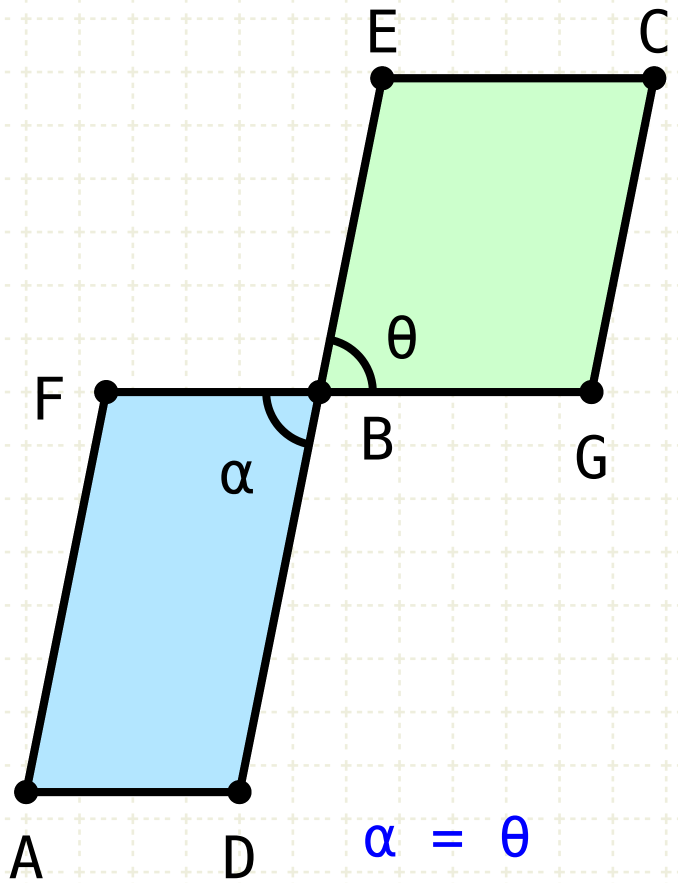
$$x_1:y_1 = x_2:y_2$$

Reciprocally Proportional:

$$x_1:y_1 = y_2:x_2, \quad x_1 \cdot x_2 = y_1 \cdot y_2$$

Proposition 14 of Book VI

In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal.



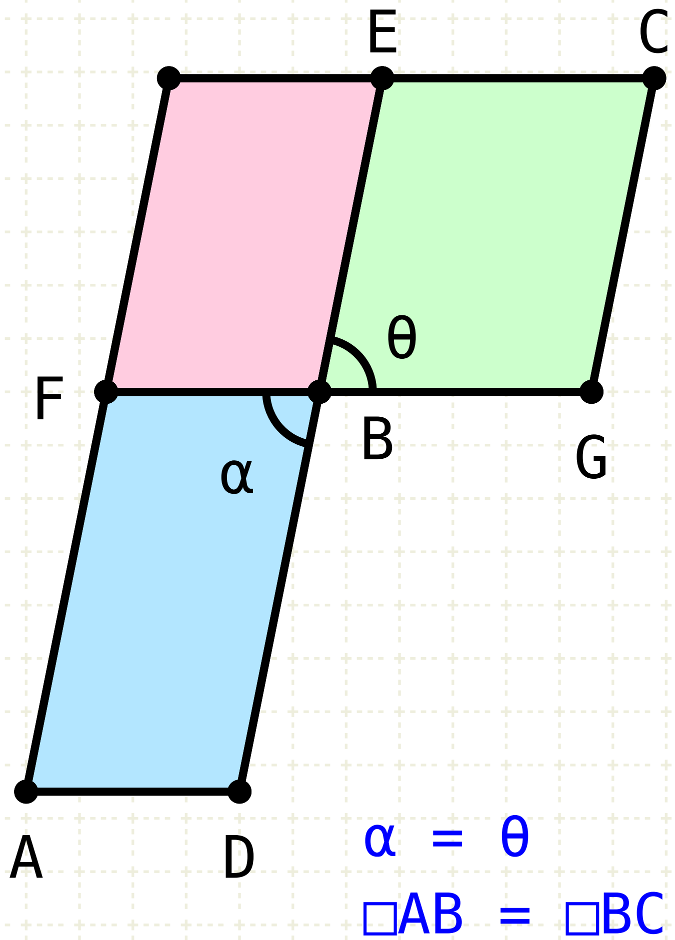
$\alpha = \theta$

□AB = □BC

Proof (Part 1)

Proposition 14 of Book VI

In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal.



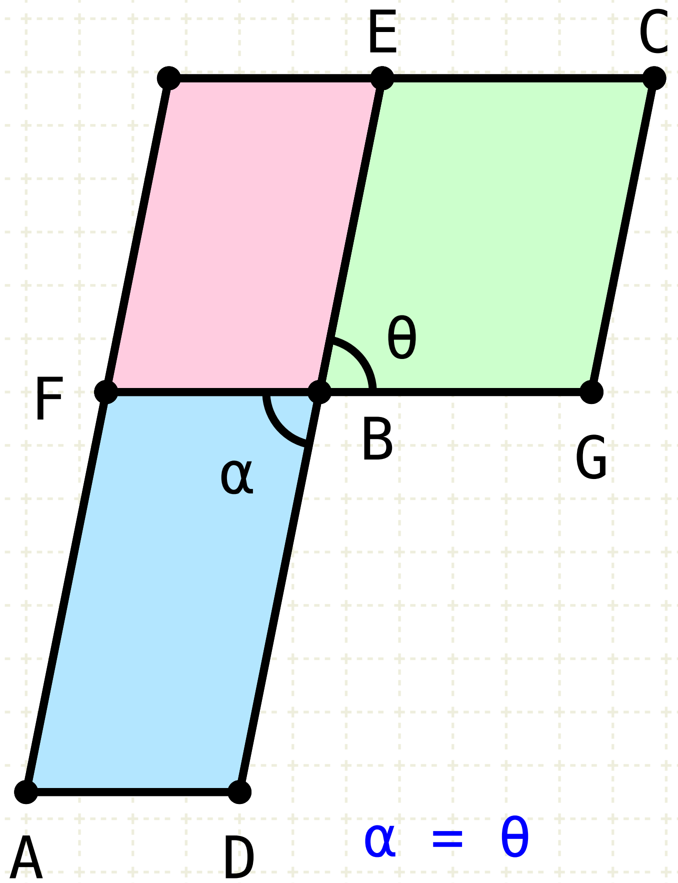
Proof (Part 1)

Let DB, BE be place in in a straight line, therefore FB, GG are also in a straight line (I·14)

Create the parallelogram FE

Proposition 14 of Book VI

In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal.



$\alpha = \theta$

$\square AB = \square BC$

$\square AB : \square FE = \square BC : \square EF$

Proof (Part 1)

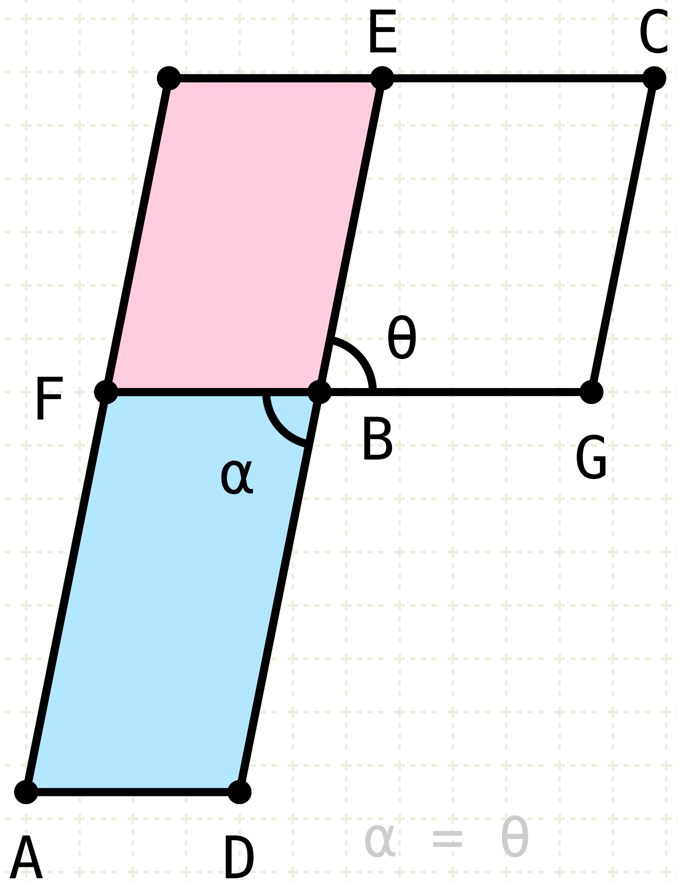
Let DB, BE be place in in a straight line, therefore FB, GG are also in a straight line (I·14)

Create the parallelogram FE

Since parallelograms AB and BC are equal, then the ratios of these to the parallelogram FE will also be equal (V·7)

Proposition 14 of Book VI

In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal.



α = θ

□AB = □BC

□AB:□FE = □BC:□EF

□AB:□FE = DB:BE

Proof (Part 1)

Let DB, BE be place in in a straight line, therefore FB, GG are also in a straight line (I·14)

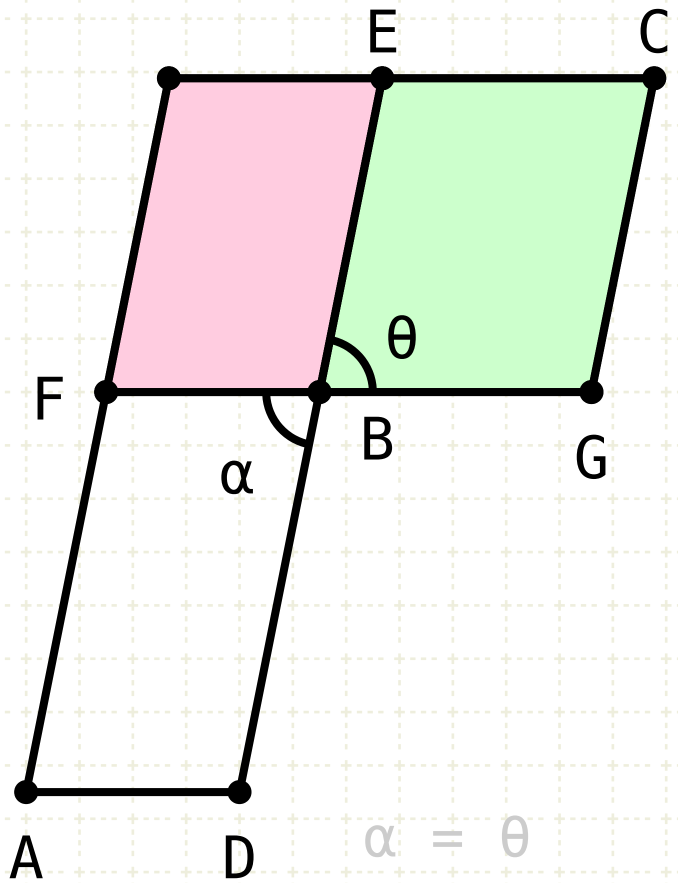
Create the parallelogram FE

Since parallelograms AB and BC are equal, then the ratios of these to the parallelogram FE will also be equal (V·7)

But, as AB is to FE, so is DB to BE (VI·1)

Proposition 14 of Book VI

In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal.



$\alpha = \theta$

$\square AB = \square BC$

$\square AB : \square FE = \square BC : \square EF$

$\square AB : \square FE = DB : BE$

$\square BC : \square FE = BG : BF$

Proof (Part 1)

Let DB, BE be place in in a straight line, therefore FB, GG are also in a straight line (I·14)

Create the parallelogram FE

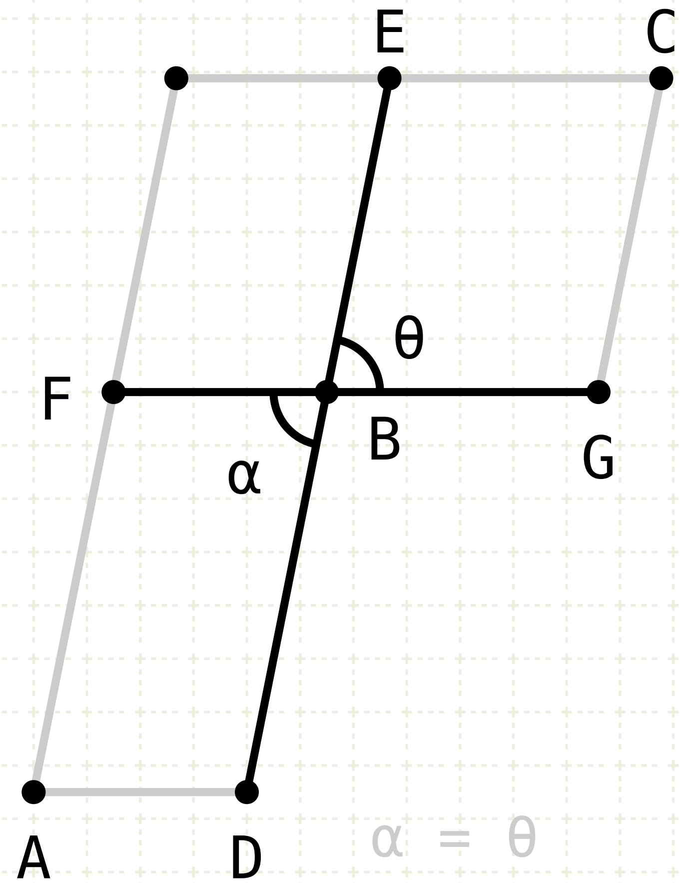
Since parallelograms AB and BC are equal, then the ratios of these to the parallelogram FE will also be equal (V·7)

But, as AB is to FE, so is DB to BE (VI·1)

and as BC is to FE, so is BG to BF (VI·1)

Proposition 14 of Book VI

In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal.



$\alpha = \theta$

$\square AB = \square BC$

$\square AB : \square FE = \square BC : \square EF$

$\square AB : \square FE = DB : BE$

$\square BC : \square FE = BG : BF$

$DB : BE = BG : BF$

Proof (Part 1)

Let DB, BE be place in in a straight line, therefore FB, GG are also in a straight line (I·14)

Create the parallelogram FE

Since parallelograms AB and BC are equal, then the ratios of these to the parallelogram FE will also be equal (V·7)

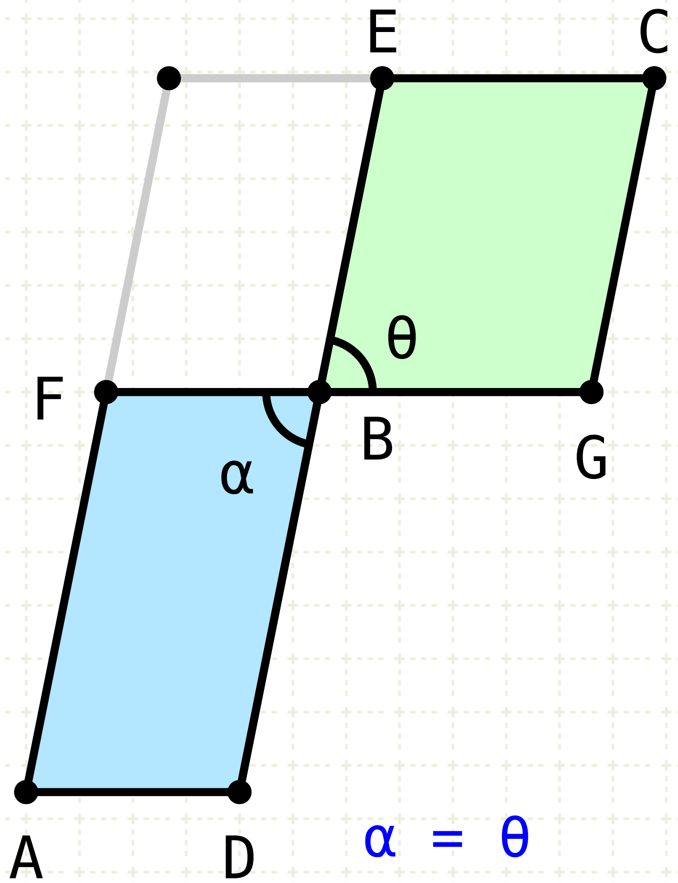
But, as AB is to FE, so is DB to BE (VI·1)

and as BC is to FE, so is BG to BF (VI·1)

Therefore, as DB is to BE, so is BG to BF (V·11)

Proposition 14 of Book VI

In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal.



$\alpha = \theta$

$\square AB = \square BC$

$\square AB : \square FE = \square BC : \square EF$

$\square AB : \square FE = DB : BE$

$\square BC : \square FE = BG : BF$

$DB : BE = BG : BF$

$\rightarrow DB \times BF = GB \times BE$

Proof (Part 1)

Let DB, BE be place in in a straight line, therefore FB, GG are also in a straight line (I·14)

Create the parallelogram FE

Since parallelograms AB and BC are equal, then the ratios of these to the parallelogram FE will also be equal (V·7)

But, as AB is to FE, so is DB to BE (VI·1)

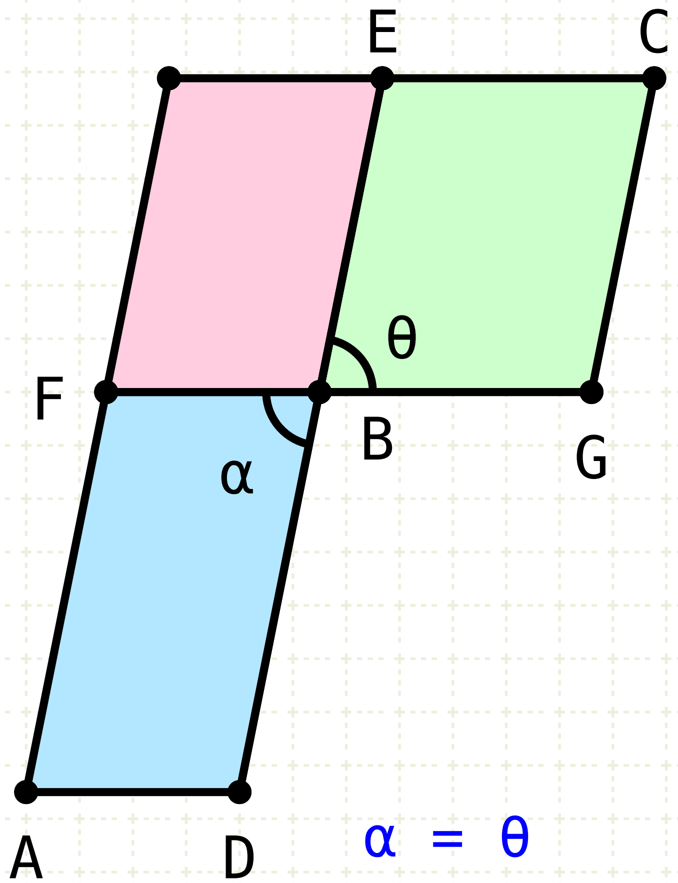
and as BC is to FE, so is BG to BF (VI·1)

Therefore, as DB is to BE, so is BG to BF (V·11)

Thus, in two equal parallelograms, the sides about the equal angles are reciprocally proportional

Proposition 14 of Book VI

In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal.



$\alpha = \theta$

DB:BE = GB:BF

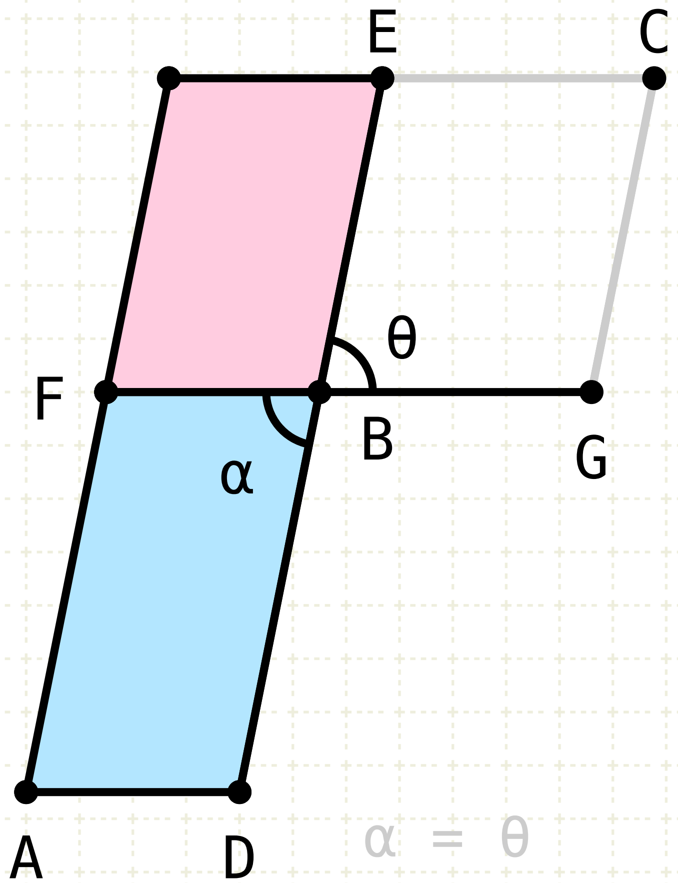
Proof (Part 2)

Let DB, BE be place in in a straight line, therefore FB, GG are also in a straight line (I·14)

Create the parallelogram FE

Proposition 14 of Book VI

In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal.



$\alpha = \theta$

DB:BE = GB:BF

DB:BE = □AB:□EF

Proof (Part 2)

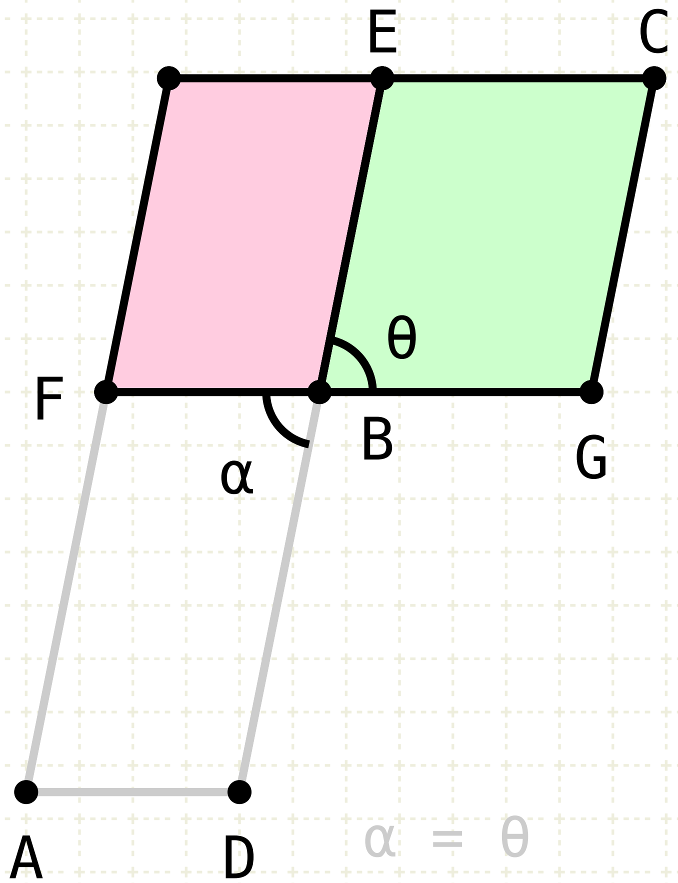
Let DB, BE be place in in a straight line, therefore FB, GG are also in a straight line (I·14)

Create the parallelogram FE

The ratio of DB to BE is equal to the ratio of the parallelograms AB to EF (VI·1)

Proposition 14 of Book VI

In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal.



α = θ

DB:BE = GB:BF

DB:BE = □AB:□EF

BG:BF = □BC:□EF

Proof (Part 2)

Let DB, BE be place in in a straight line, therefore FB, GG are also in a straight line (I·14)

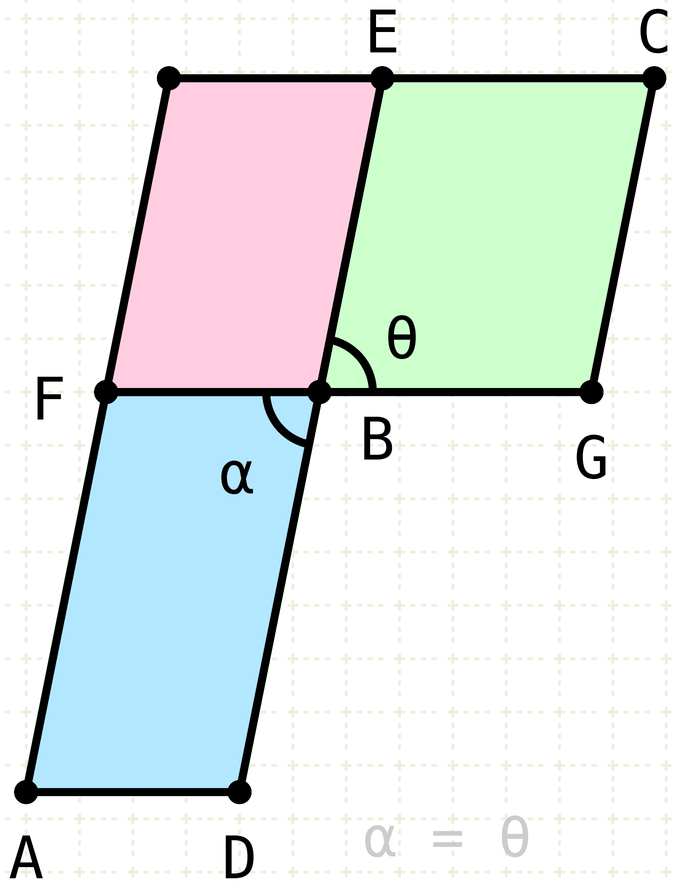
Create the parallelogram FE

The ratio of DB to BE is equal to the ratio of the parallelograms AB to EF (VI·1)

The ratio of BG to BF is equal to the ratio of the parallelograms BC to EF (VI·1)

Proposition 14 of Book VI

In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal.



α = θ

DB:BE = GB:BF

DB:BE = □AB:□EF

BG:BF = □BC:□EF

□AB:□EF = □BC:□EF

Proof (Part 2)

Let DB, BE be place in in a straight line, therefore FB, GG are also in a straight line (I·14)

Create the parallelogram FE

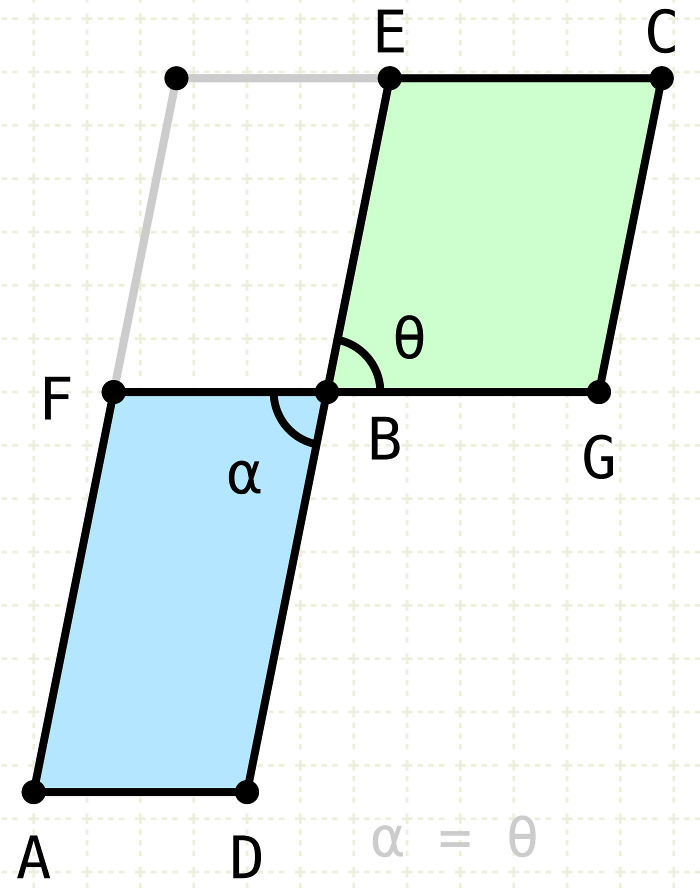
The ratio of DB to BE is equal to the ratio of the parallelograms AB to EF (VI·1)

The ratio of BG to BF is equal to the ratio of the parallelograms BC to EF (VI·1)

Therefore the ratio of the parallelograms AB to FE is equal to BC to FE (V·11)

Proposition 14 of Book VI

In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal.



$\alpha = \theta$

$DB:BE = GB:BF$

$DB:BE = \square AB:\square EF$

$BG:BF = \square BC:\square EF$

$\square AB:\square EF = \square BC:\square EF$

$\square AB = \square BC$

Proof (Part 2)

Let DB, BE be place in in a straight line, therefore FB, GG are also in a straight line (I·14)

Create the parallelogram FE

The ratio of DB to BE is equal to the ratio of the parallelograms AB to EF (VI·1)

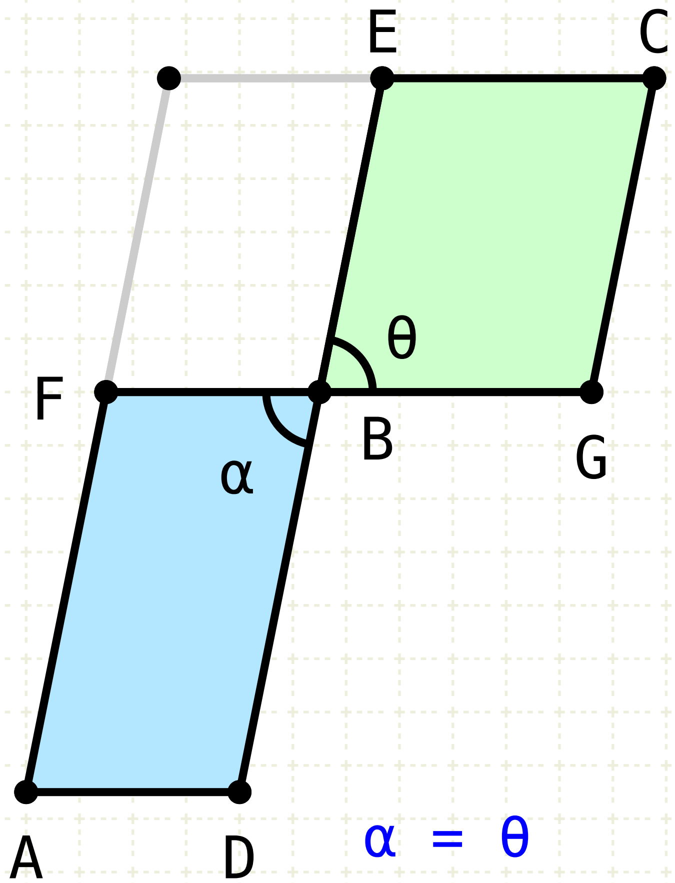
The ratio of BG to BF is equal to the ratio of the parallelograms BC to EF (VI·1)

Therefore the ratio of the parallelograms AB to FE is equal to BC to FE (V·11)

And thus the parallelograms AB and EF are equal(V·9)

Proposition 14 of Book VI

In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal.



$\alpha = \theta$

DB:BE = GB:BF

DB:BE = □AB:□EF

BG:BF = □BC:□EF

□AB:□EF = □BC:□EF

□AB = □BC

Proof (Part 2)

Let DB, BE be place in in a straight line, therefore FB, GG are also in a straight line (I·14)

Create the parallelogram FE

The ratio of DB to BE is equal to the ratio of the parallelograms AB to EF (VI·1)

The ratio of BG to BF is equal to the ratio of the parallelograms BC to EF (VI·1)

Therefore the ratio of the parallelograms AB to FE is equal to BC to FE (V·11)

And thus the parallelograms AB and EF are equal(V·9)

Youtube Videos

https://www.youtube.com/c/SandyBultena