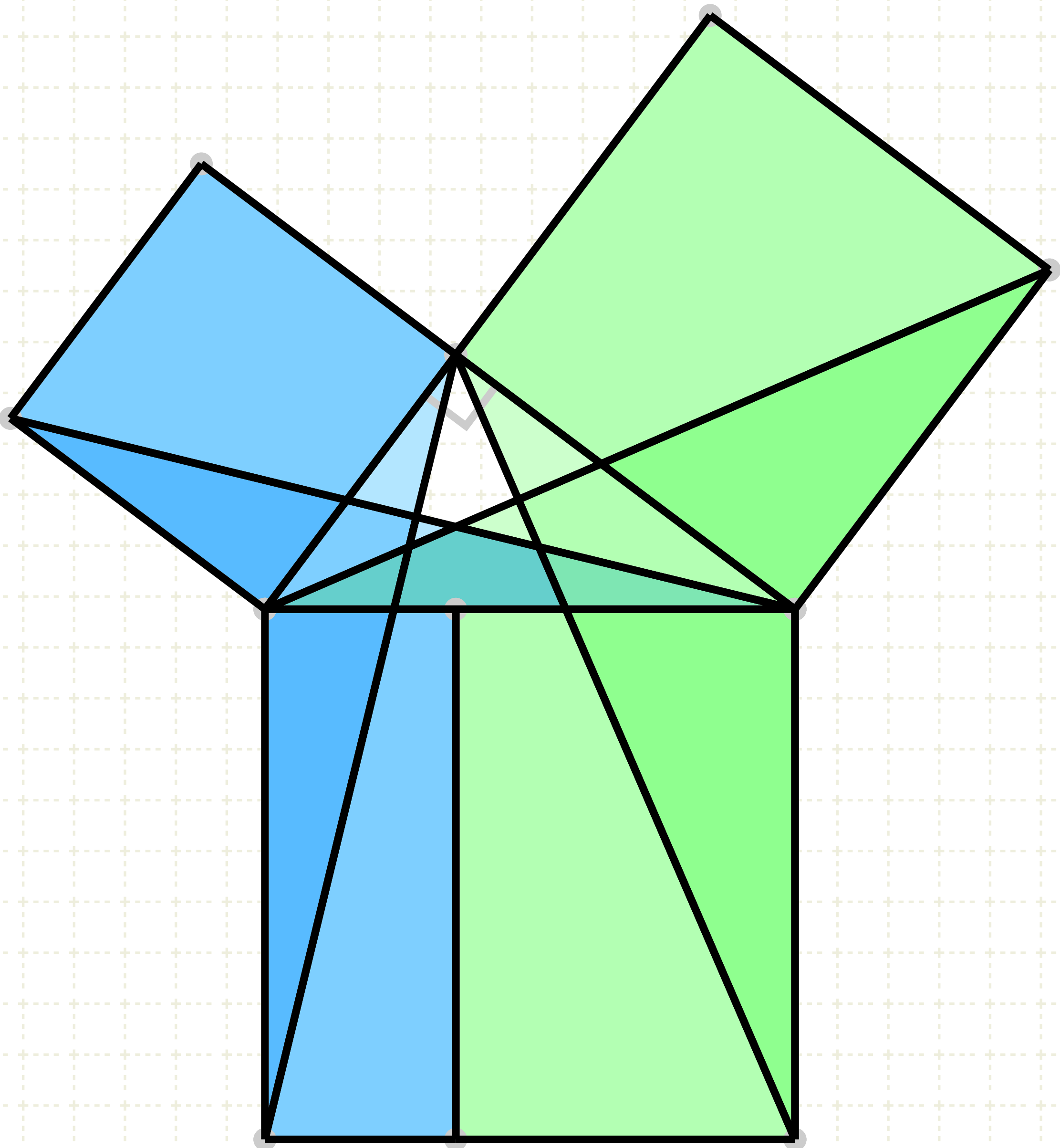
Euclid's Elements

Book I



If Euclid did not kindle your youthful enthusiasm, you were not born to be a scientific thinker.

Albert Einstein

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Proposition 5 of Book I

In isosceles triangles the angles at the base equal one another, and, if the equal straight lines are produced further, then the angles under the base equal one another.

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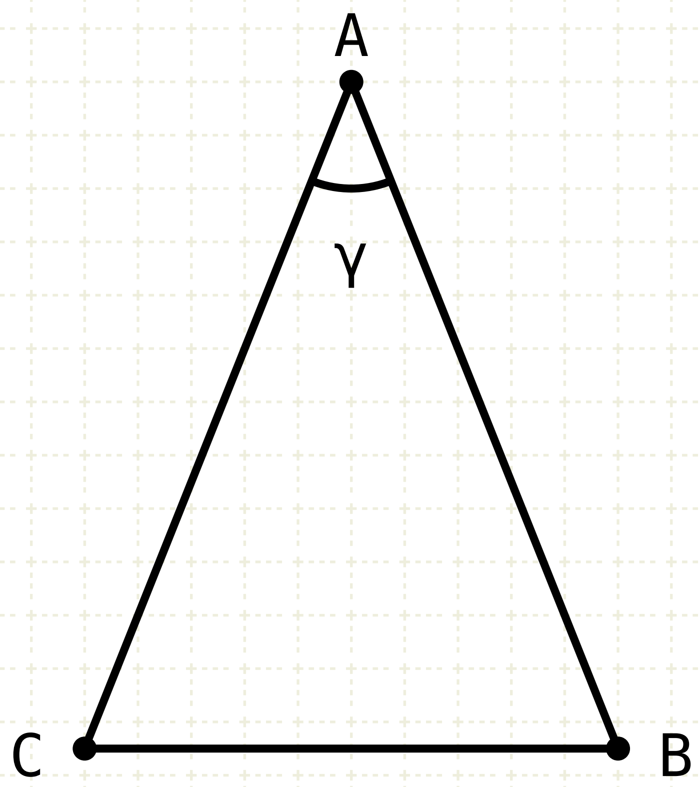

AB = AC

In other words

Given an isosceles triangle ABC

Proposition 5 of Book I

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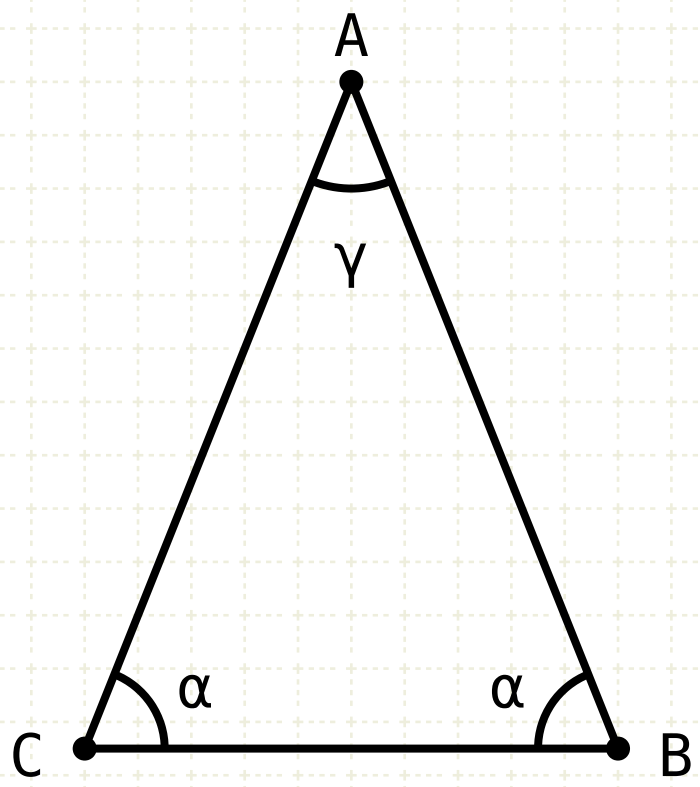

$AB = AC$

$\angle ACB = \angle ABC$

In other words

Given an isosceles triangle ABC

Then the angles at the base ACB and ABC are equal

Proposition 5 of Book I

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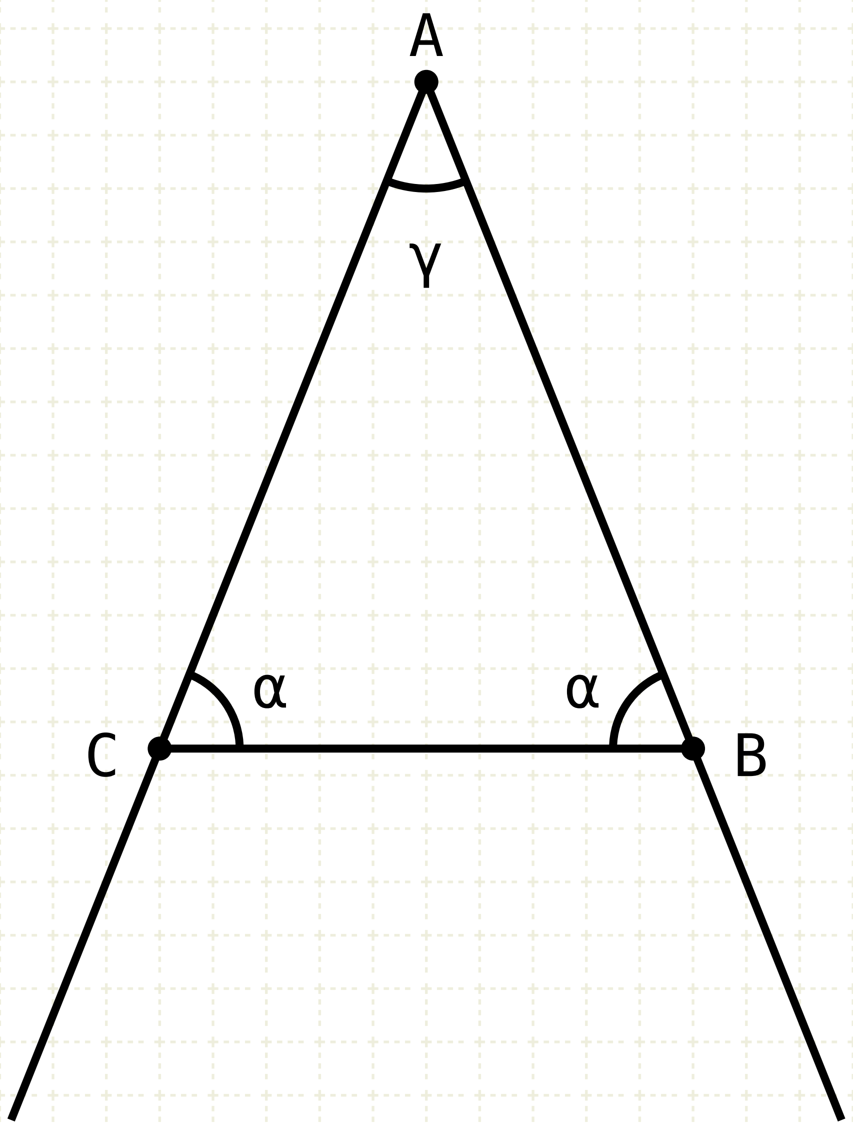

AB = AC

∠ACB = ∠ABC

In other words

Given an isosceles triangle ABC

Then the angles at the base ACB and ABC are equal

In addition, if we extend lines AB and AC

Proposition 5 of Book I

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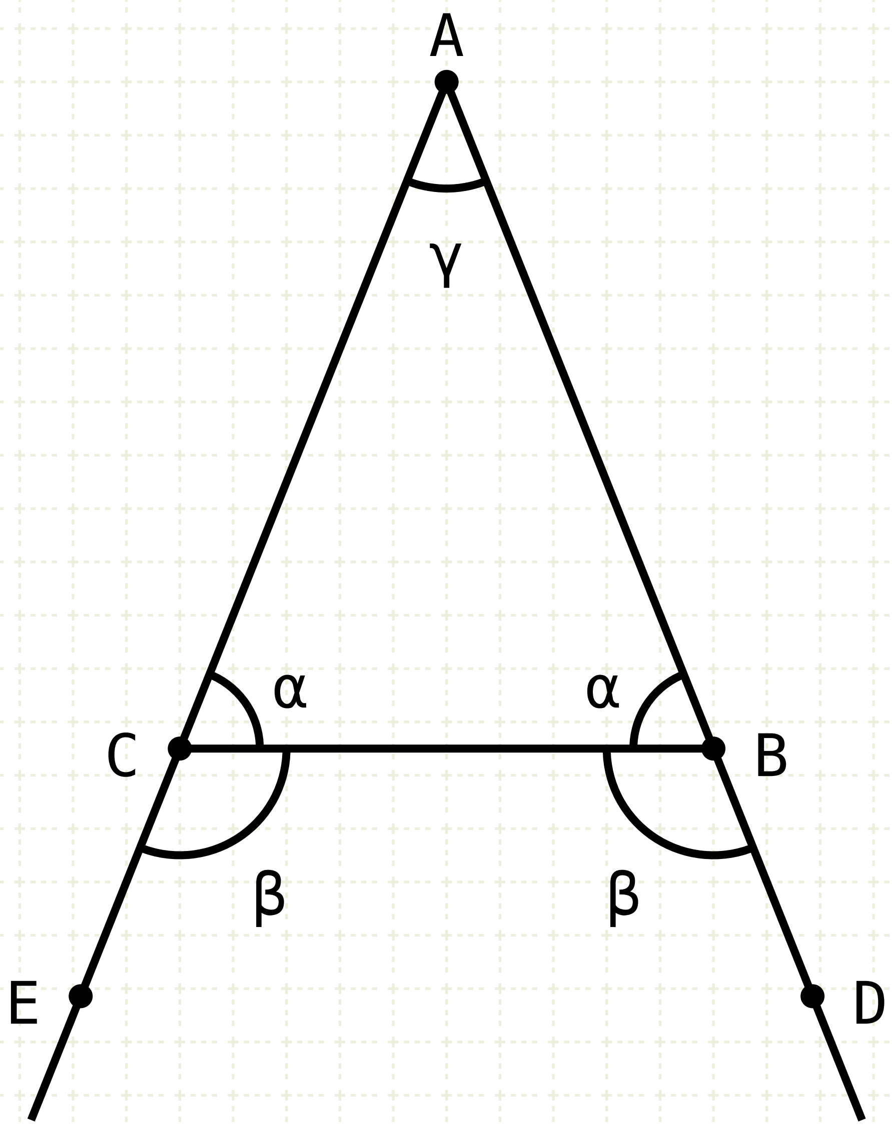


AB = AC

∠ACB = ∠ABC

∠BCE = ∠CBD

In other words

Given an isosceles triangle ABC

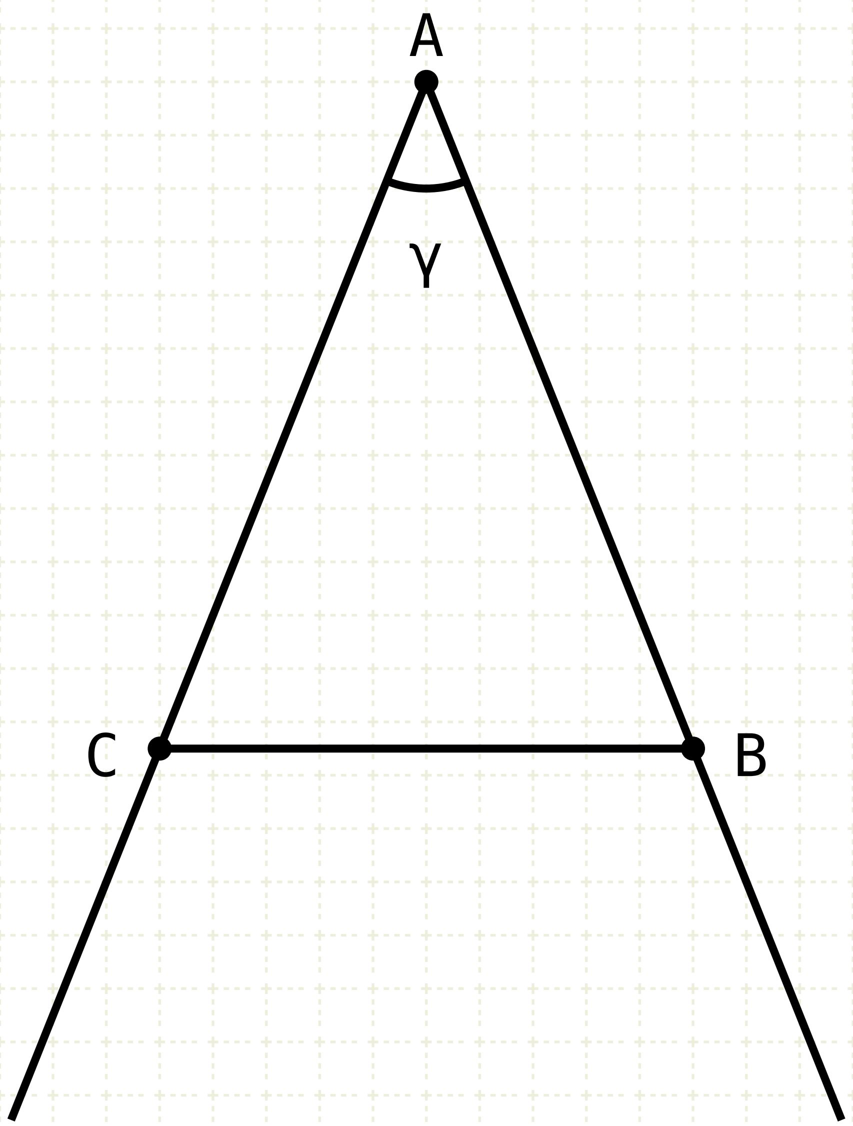
Then the angles at the base ACB and ABC are equal

In addition, if we extend lines AB and AC

Then the exterior angles are equal

Proposition 5 of Book I

In isosceles triangles the angles at the base equal one another, and, if the equal straight lines are produced further, then the angles under the base equal one another.

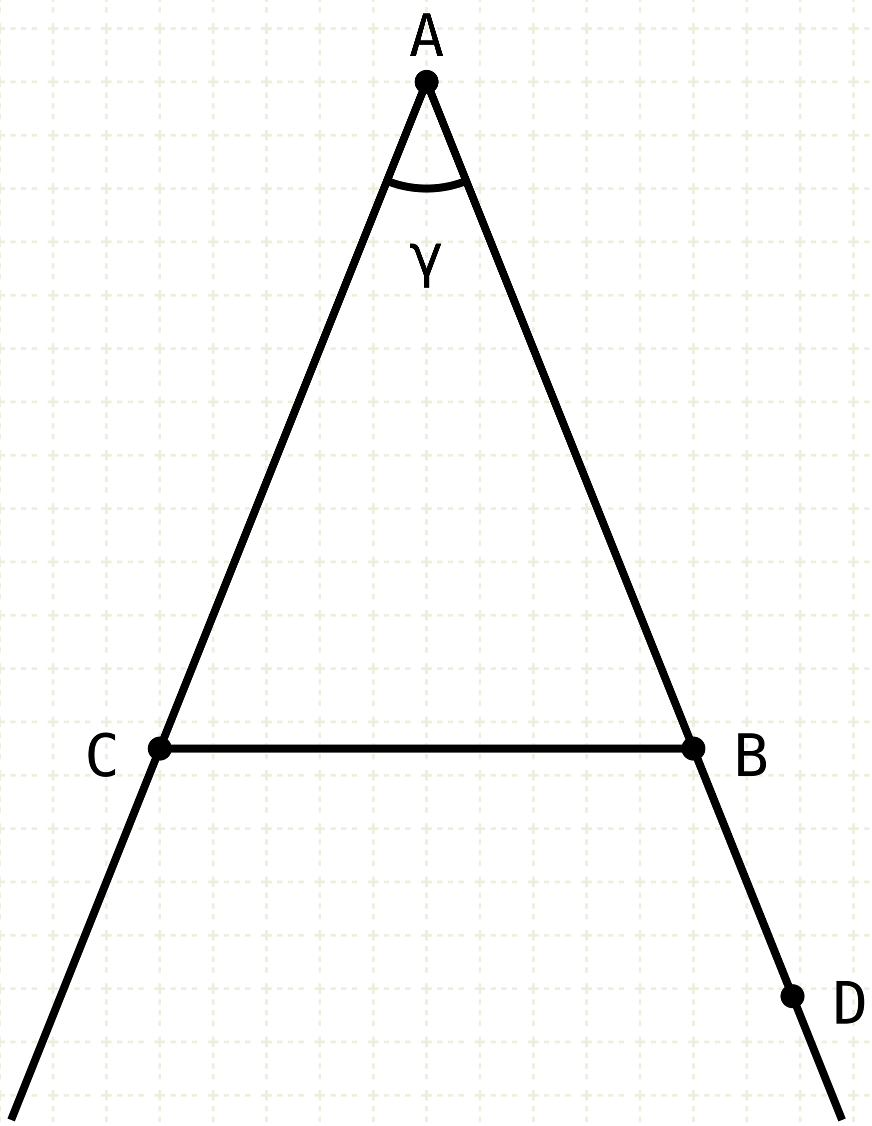


AB = AC

Proof

Proposition 5 of Book I

In isosceles triangles the angles at the base equal one another, and, if the equal straight lines are produced further, then the angles under the base equal one another.



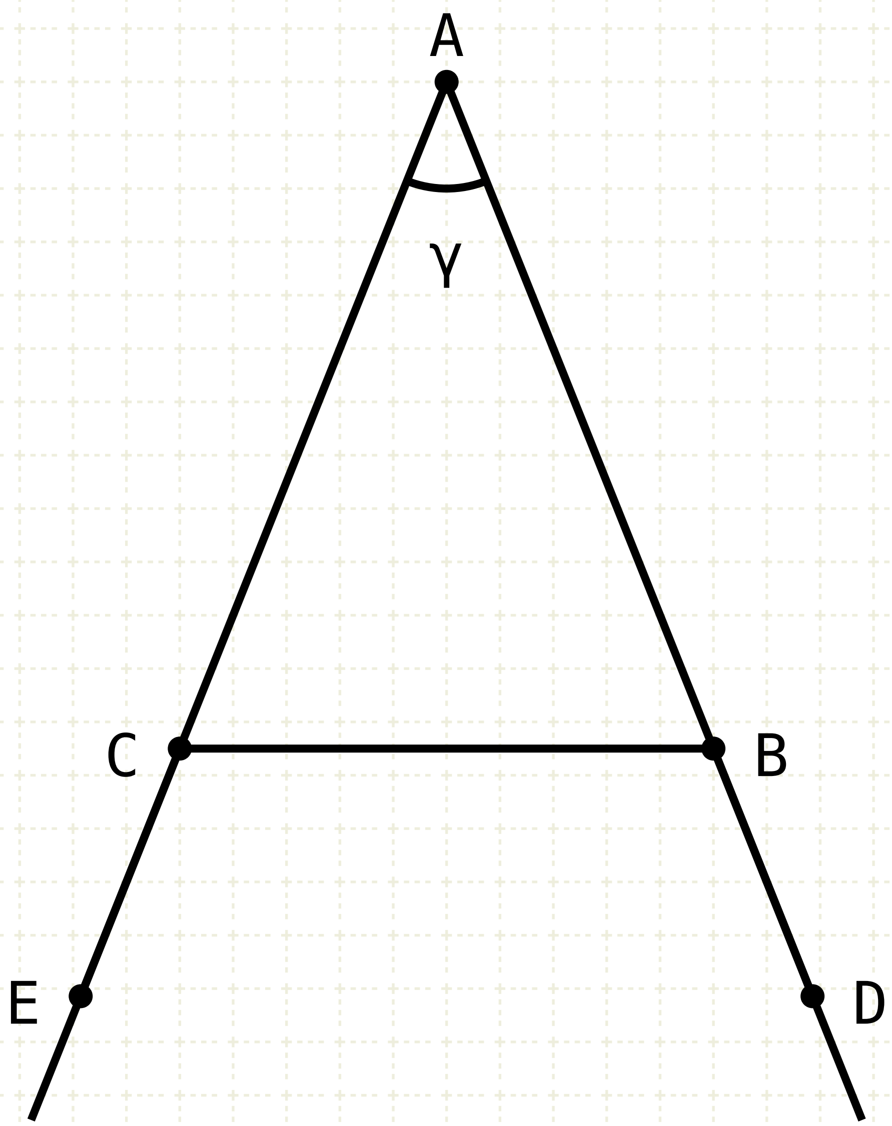
AB = AC

Proof

Define a point along the extension of AB

Proposition 5 of Book I

In isosceles triangles the angles at the base equal one another, and, if the equal straight lines are produced further, then the angles under the base equal one another.



AB = AC
BD = CE

Proof

Define a point along the extension of AB

Construct a line starting at C, with length BD, on the line segment of AC (I·2)

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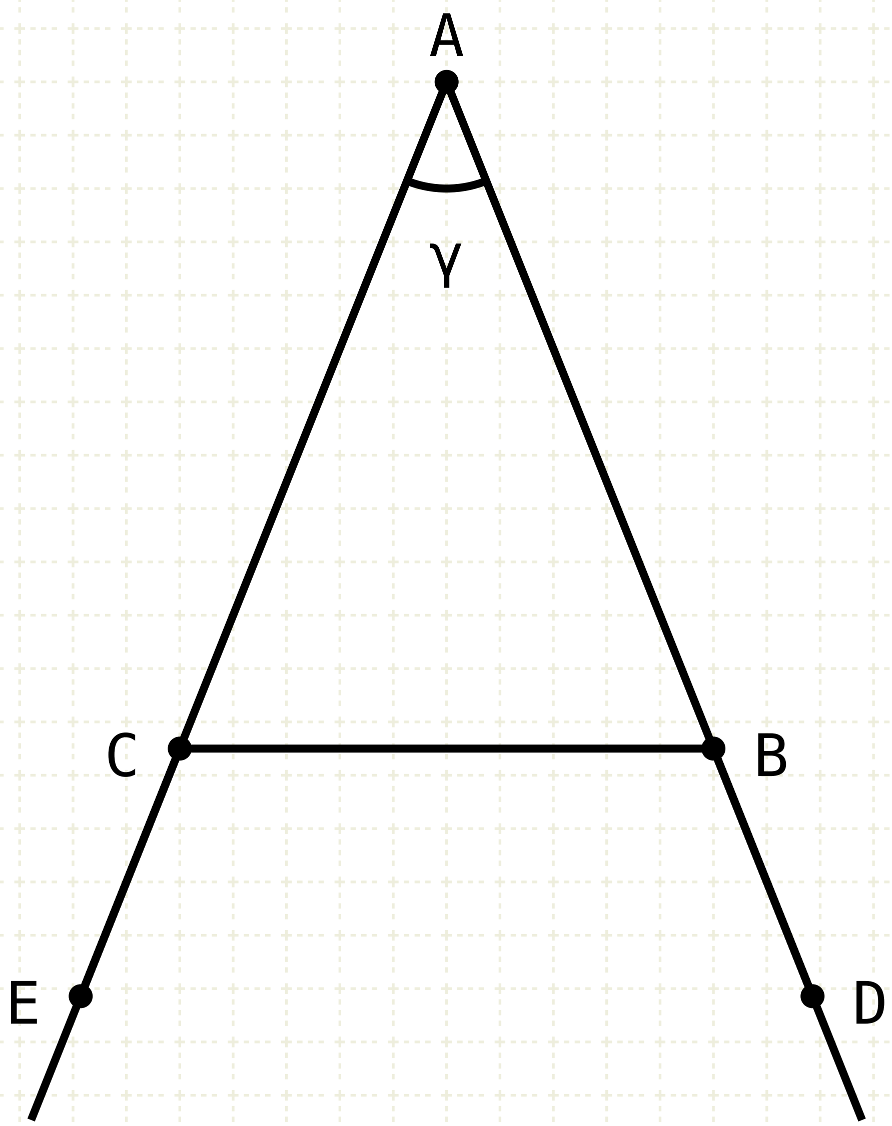
In isosceles triangles the angles at the base equal one another, and, if the equal straight lines are produced further, then the angles under the base equal one another.



$AB = AC$

$BD = CE$

$AE = AC + CE$

$AD = AB + BD$

$AE = AD$

Proof

Define a point along the extension of AB

Construct a line starting at C, with length BD, on the line segment of AC (I·2)

AC and AB are equal, as are BD and CE, thus AE and AD which are the sum of AC,CE and AB,BD respectively, are also equal

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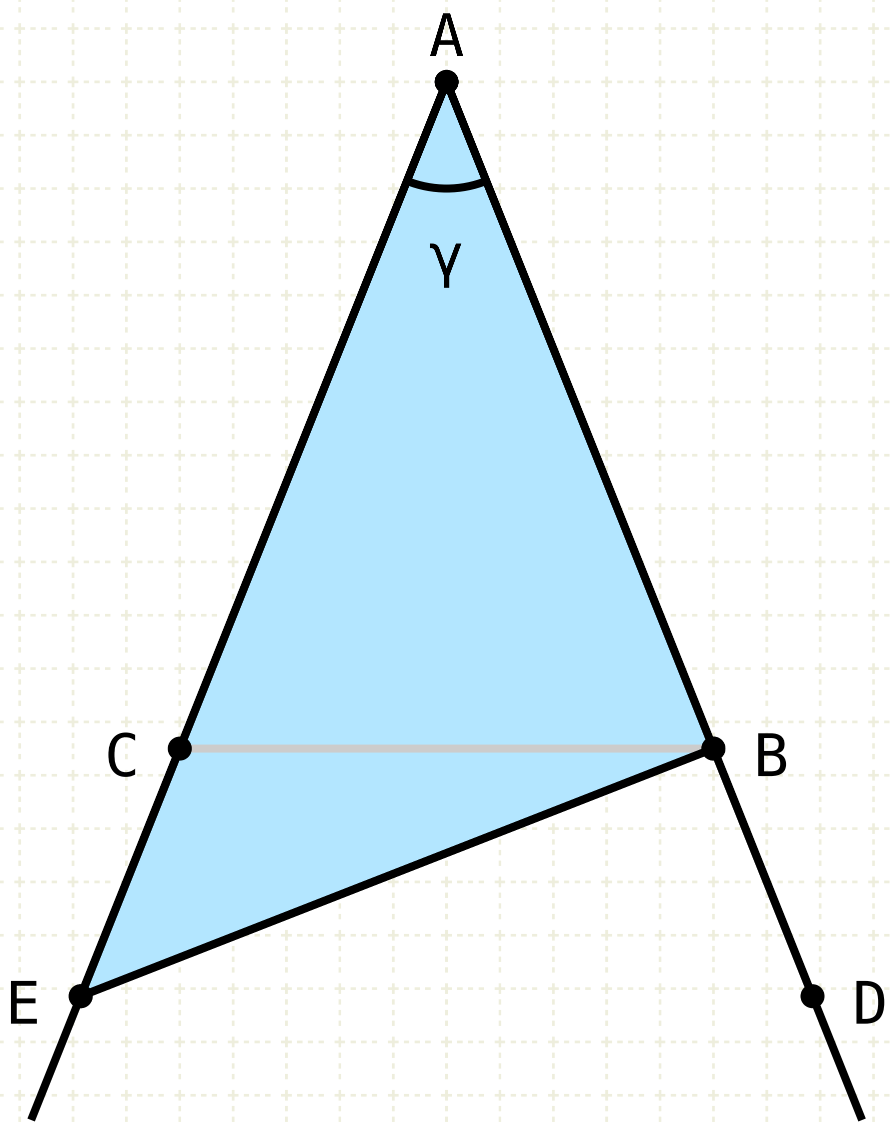

AB = AC
BD = CE
AE = AC + CE
AD = AB + BD
AE = AD

AE, ∠EAB=γ, AB

Proof

Define a point along the extension of AB

Construct a line starting at C, with length BD, on the line segment of AC (I·2)

AC and AB are equal, as are BD and CE, thus AE and AD which are the sum of AC,CE and AB,BD respectively, are also equal

Create triangle AEB

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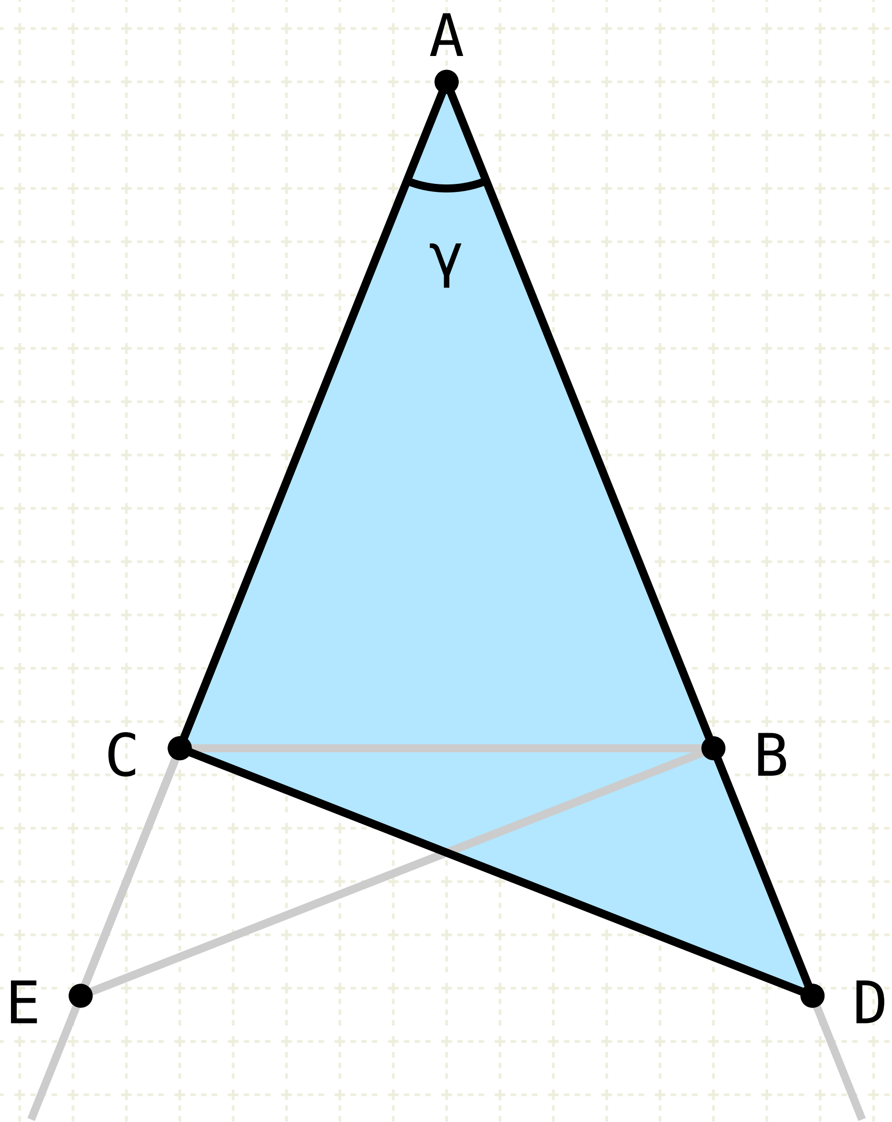
In isosceles triangles the angles at the base equal one another, and, if the equal straight lines are produced further, then the angles under the base equal one another.



AB = AC
BD = CE
AE = AC + CE
AD = AB + BD
AE = AD

AE, ∠EAB=γ, AB
AD, ∠DAC=γ, AC

Proof

Define a point along the extension of AB

Construct a line starting at C, with length BD, on the line segment of AC (I·2)

AC and AB are equal, as are BD and CE, thus AE and AD which are the sum of AC,CE and AB,BD respectively, are also equal

Create triangle AEB

Create triangle ADC

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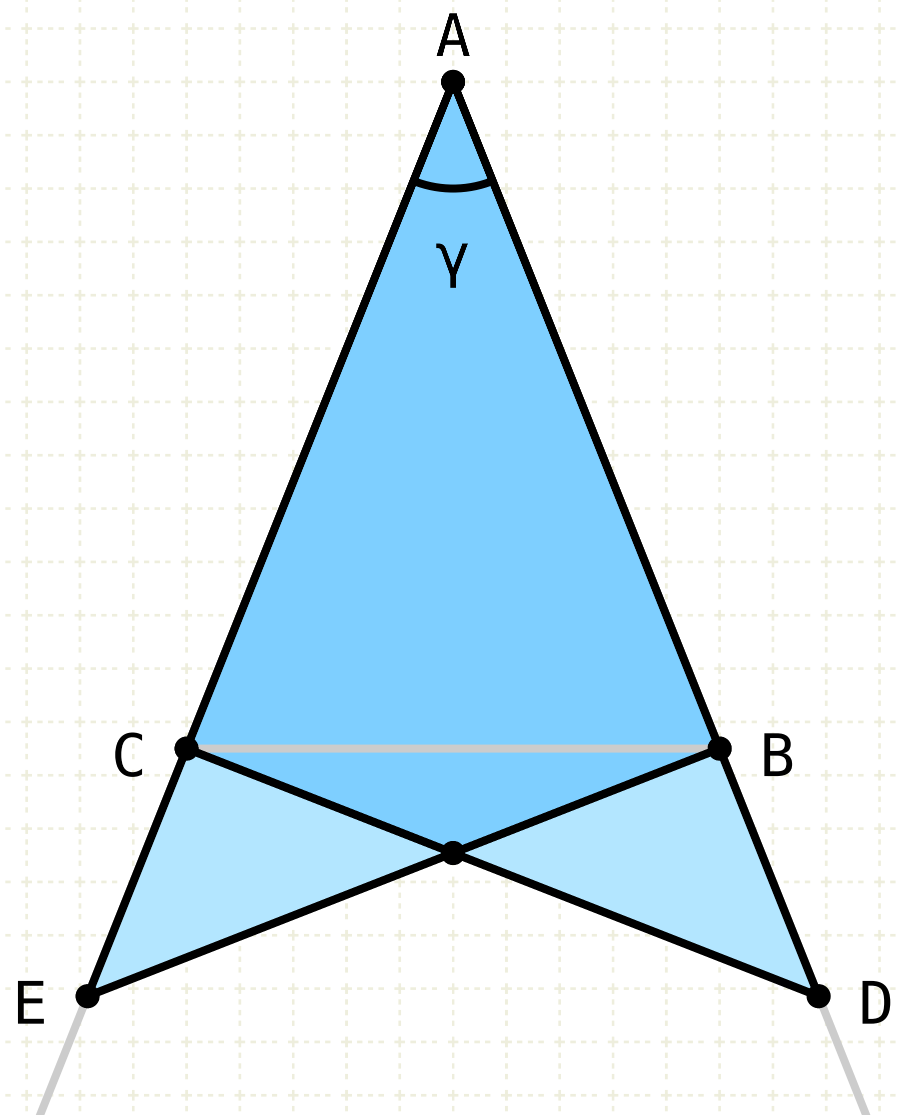

AB = AC

BD = CE

AE = AC + CE

AD = AB + BD

AE = AD

AE, ∠EAB=γ, AB

AD, ∠DAC=γ, AC

Proof

Define a point along the extension of AB

Construct a line starting at C, with length BD, on the line segment of AC (I·2)

AC and AB are equal, as are BD and CE, thus AE and AD which are the sum of AC,CE and AB,BD respectively, are also equal

Create triangle AEB

Create triangle ADC

Since two sides and the angle between are the same for both triangles,

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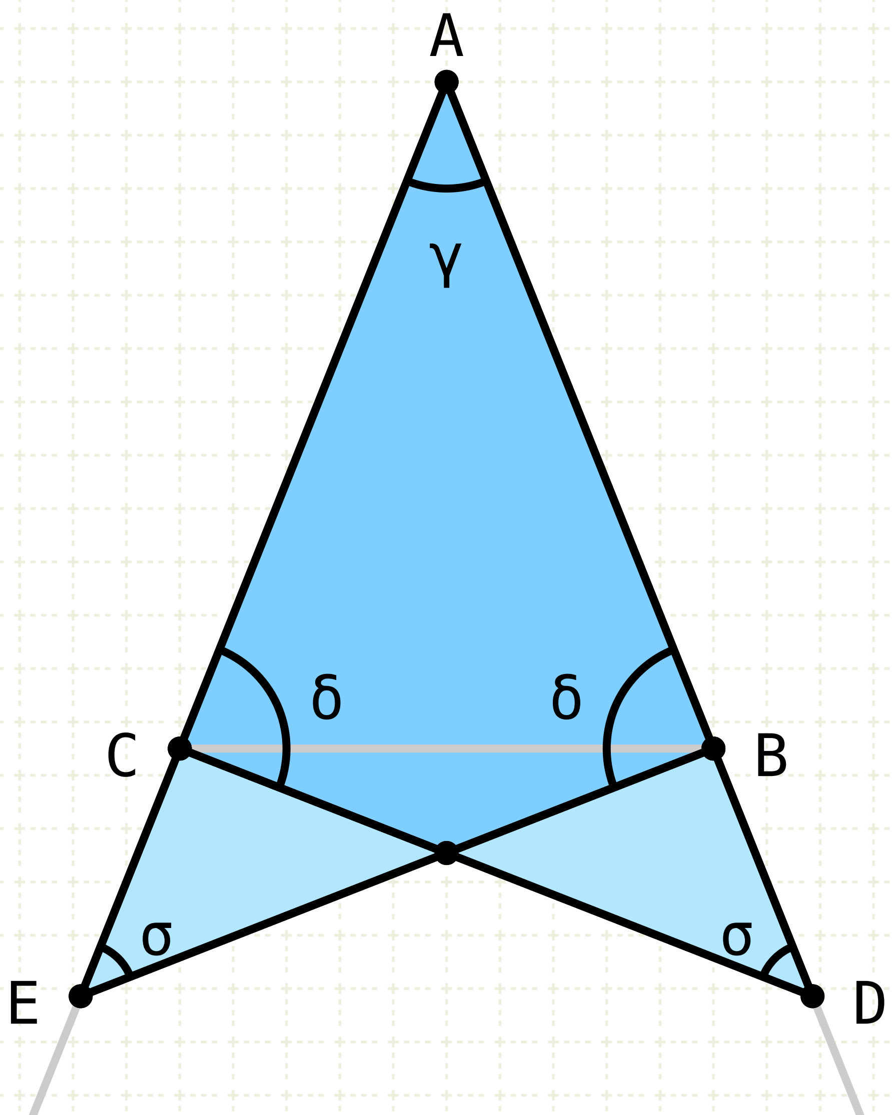

AB = AC
BD = CE
AE = AC + CE
AD = AB + BD
AE = AD

AE, ∠EAB=γ, AB
AD, ∠DAC=γ, AC

CD = BE
∠ACD = ∠ABE = δ
∠CDA = ∠BEA = σ

Proof

Define a point along the extension of AB

Construct a line starting at C, with length BD, on the line segment of AC (I·2)

AC and AB are equal, as are BD and CE, thus AE and AD which are the sum of AC,CE and AB,BD respectively, are also equal

Create triangle AEB

Create triangle ADC

Since two sides and the angle between are the same for both triangles,

then all the sides and angles are equal (I·4)

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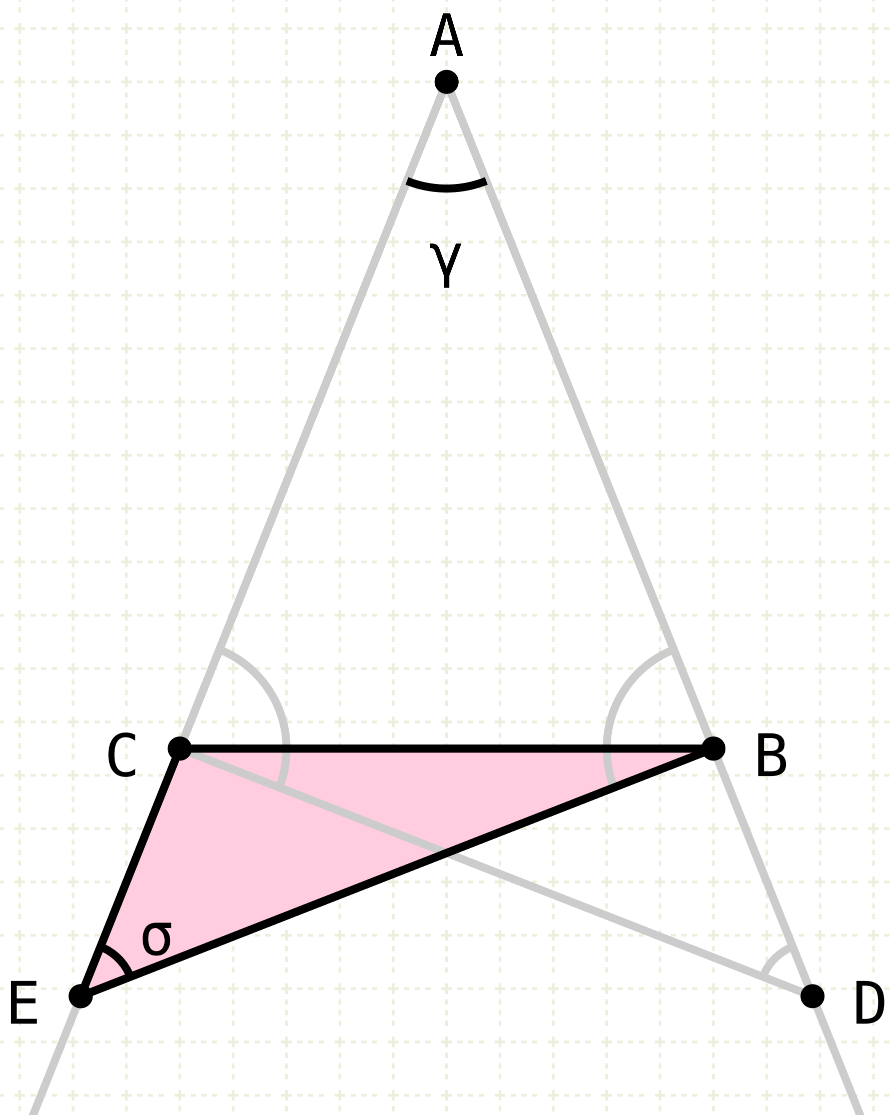

AB = AC
BD = CE
AE = AC + CE
AD = AB + BD
AE = AD

AE, ∠EAB=γ, AB
AD, ∠DAC=γ, AC

CD = BE
∠ACD = ∠ABE = δ
∠CDA = ∠BEA = σ

CE, ∠BEA=∠BEC=σ, BE

Proof

Define a point along the extension of AB

Construct a line starting at C, with length BD, on the line segment of AC (I·2)

AC and AB are equal, as are BD and CE, thus AE and AD which are the sum of AC,CE and AB,BD respectively, are also equal

Create triangle AEB

Create triangle ADC

Since two sides and the angle between are the same for both triangles,

then all the sides and angles are equal (I·4)

Lets look at triangle CEB

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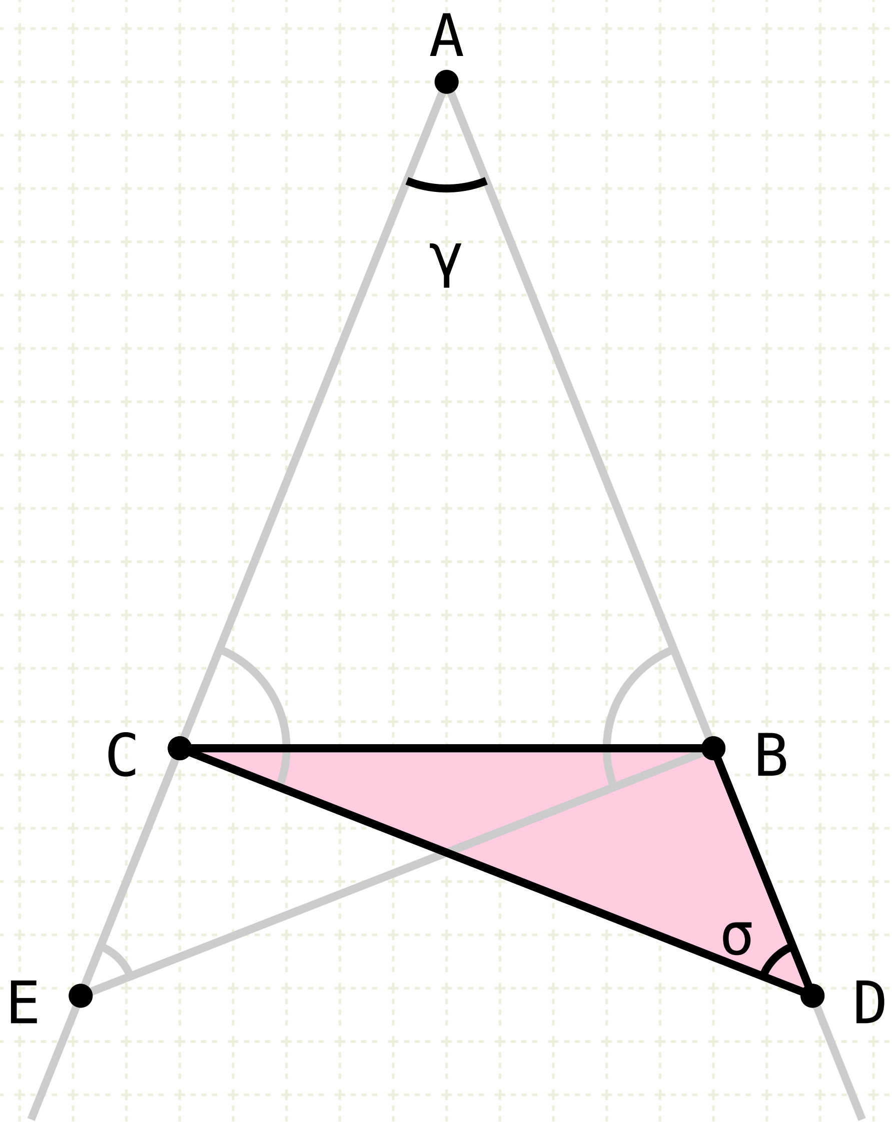
In isosceles triangles the angles at the base equal one another, and, if the equal straight lines are produced further, then the angles under the base equal one another.



$AB = AC$
$BD = CE$
$AE = AC + CE$
$AD = AB + BD$
$AE = AD$

$AE, \angle EAB = \gamma, AB$
$AD, \angle DAC = \gamma, AC$

$CD = BE$
$\angle ACD = \angle ABE = \delta$
$\angle CDA = \angle BEA = \sigma$

$CE, \angle BEA = \angle BEC = \sigma, BE$
$BD, \angle CDA = \angle CDB = \sigma, CD$

Proof

Define a point along the extension of AB

Construct a line starting at C, with length BD, on the line segment of AC (I·2)

AC and AB are equal, as are BD and CE, thus AE and AD which are the sum of AC,CE and AB,BD respectively, are also equal

Create triangle AEB

Create triangle ADC

Since two sides and the angle between are the same for both triangles,

then all the sides and angles are equal (I·4)

Lets look at triangle CEB

And at triangle CDB

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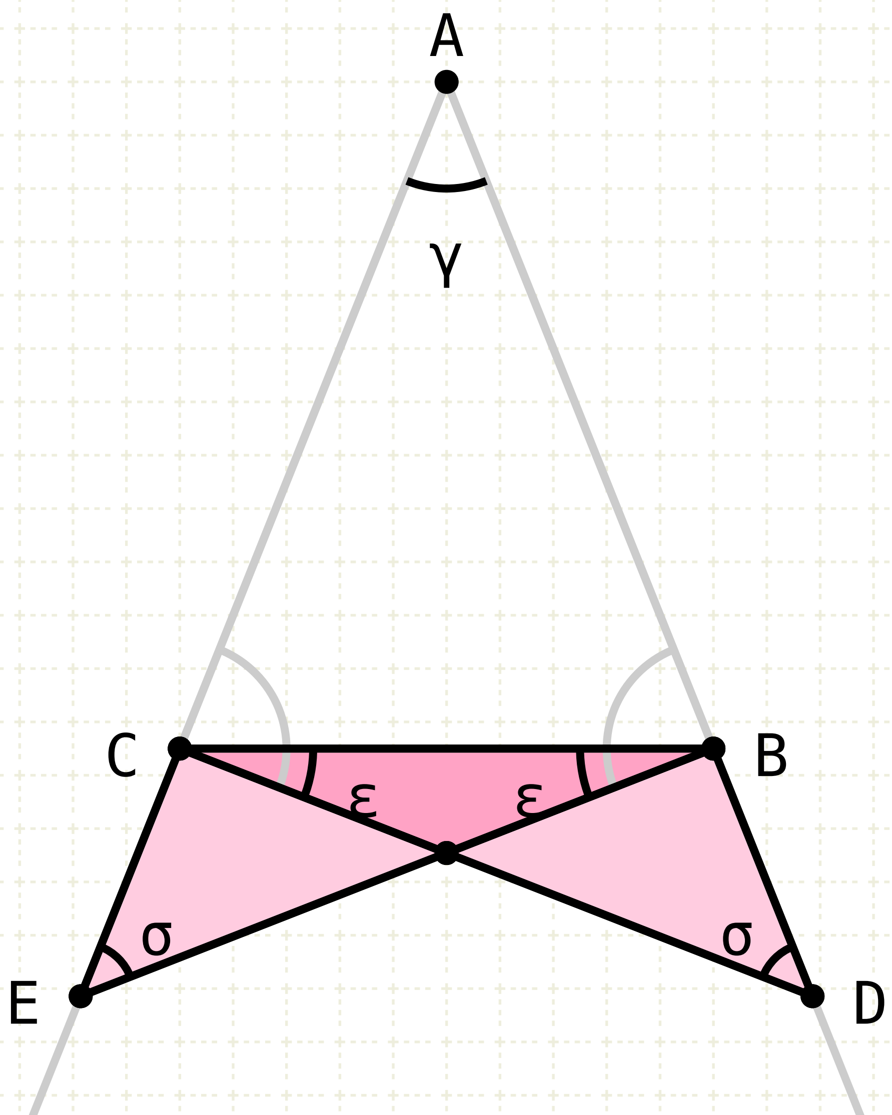

AB = AC
BD = CE
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AE = AD

AE, ∠EAB=γ, AB
AD, ∠DAC=γ, AC

CD = BE
∠ACD = ∠ABE = δ
∠CDA = ∠BEA = σ

CE, ∠BEA=∠BEC=σ, BE
BD, ∠CDA=∠CDB=σ, CD
∠CBE = ∠BCD = ε

Proof

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Construct a line starting at C, with length BD, on the line segment of AC (I·2)

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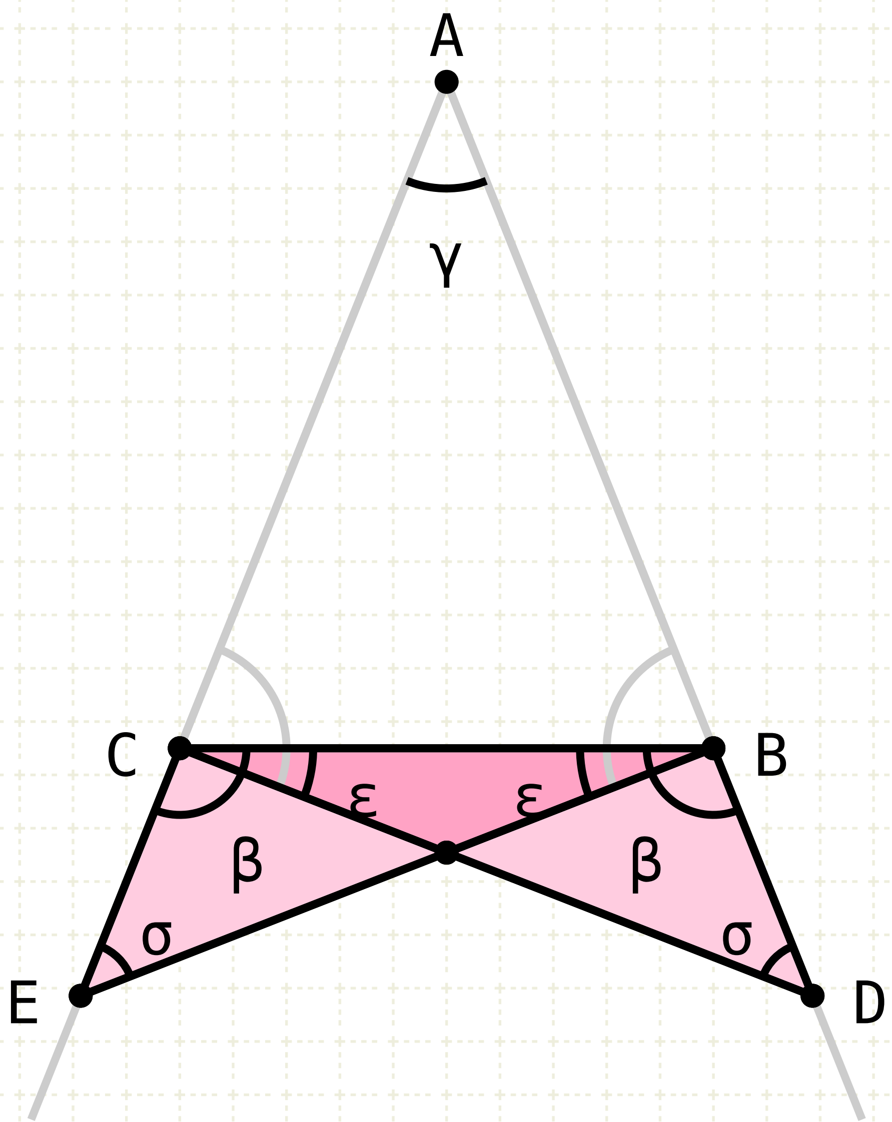

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$AE = AD$

AE, $\angle EAB = \gamma$, AB
AD, $\angle DAC = \gamma$, AC

$CD = BE$
$\angle ACD = \angle ABE = \delta$
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CE, $\angle BEA = \angle BEC = \sigma$, BE
BD, $\angle CDA = \angle CDB = \sigma$, CD
$\angle CBE = \angle BCD = \varepsilon$
$\angle BCE = \angle CBD = \beta$

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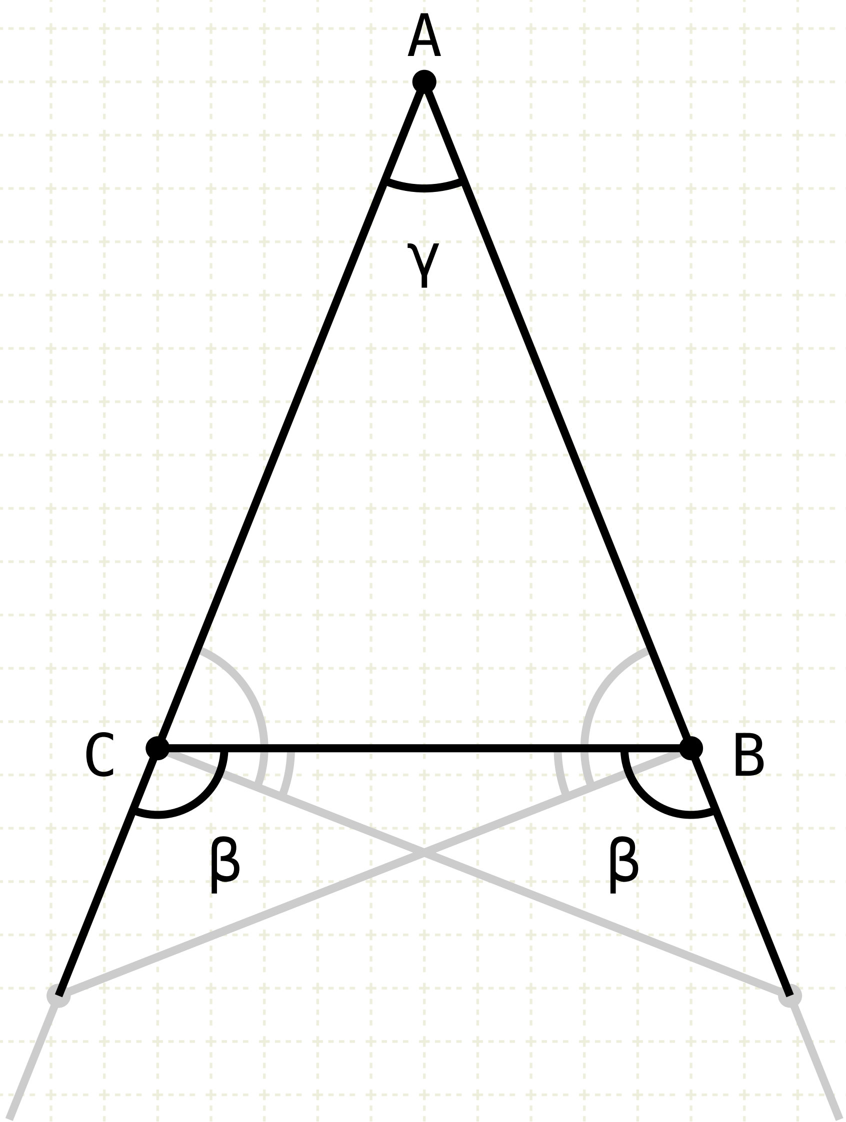

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Proof

Define a point along the extension of AB

Construct a line starting at C, with length BD, on the line segment of AC (I·2)

AC and AB are equal, as are BD and CE, thus AE and AD which are the sum of AC,CE and AB,BD respectively, are also equal

Create triangle AEB

Create triangle ADC

Since two sides and the angle between are the same for both triangles,

then all the sides and angles are equal (I·4)

Lets look at triangle CEB

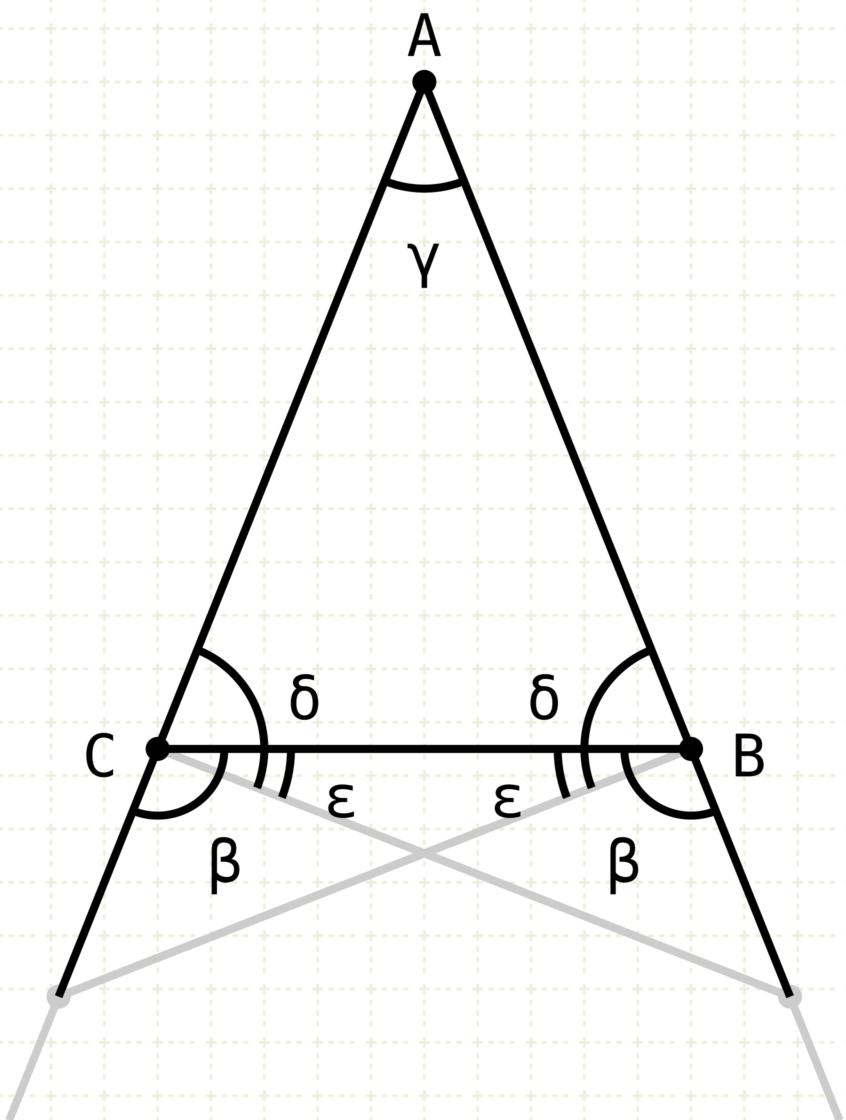
And at triangle CDB

Since two sides and the angle between are the same for both triangles, then all the sides and angles are equal (I·4)

And, we have just shown that the exterior angles are equal

Proposition 5 of Book I

In isosceles triangles the angles at the base equal one another, and, if the equal straight lines are produced further, then the angles under the base equal one another.



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$\angle BCE = \angle CBD = \beta$

$\angle BCE = \angle CBD = \beta$

$\angle ABC = \angle ACB = \delta - \varepsilon = \alpha$

Proof

Define a point along the extension of AB

Construct a line starting at C, with length BD, on the line segment of AC (I·2)

AC and AB are equal, as are BD and CE, thus AE and AD which are the sum of AC,CE and AB,BD respectively, are also equal

Create triangle AEB

Create triangle ADC

Since two sides and the angle between are the same for both triangles,

then all the sides and angles are equal (I·4)

Lets look at triangle CEB

And at triangle CDB

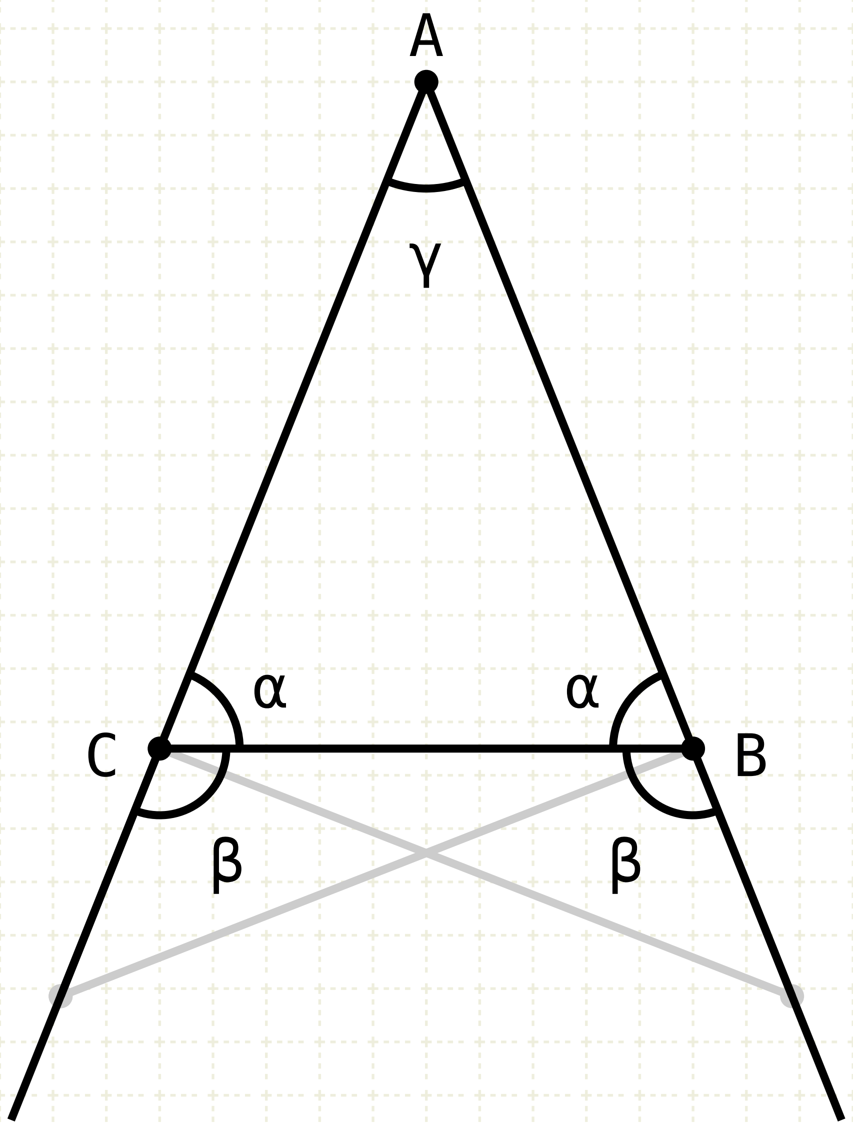
Since two sides and the angle between are the same for both triangles, then all the sides and angles are equal (I·4)

And, we have just shown that the exterior angles are equal

Let's look now at the interior angles. The differences between equals are equal so that means the interior angles are the same

Proposition 5 of Book I

In isosceles triangles the angles at the base equal one another, and, if the equal straight lines are produced further, then the angles under the base equal one another.



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Proof

Define a point along the extension of AB

Construct a line starting at C, with length BD, on the line segment of AC (I·2)

AC and AB are equal, as are BD and CE, thus AE and AD which are the sum of AC,CE and AB,BD respectively, are also equal

Create triangle AEB

Create triangle ADC

Since two sides and the angle between are the same for both triangles,

then all the sides and angles are equal (I·4)

Lets look at triangle CEB

And at triangle CDB

Since two sides and the angle between are the same for both triangles, then all the sides and angles are equal (I·4)

And, we have just shown that the exterior angles are equal

Let's look now at the interior angles. The differences between equals are equal so that means the interior angles are the same

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