

Euclid's Elements

Book III



A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)



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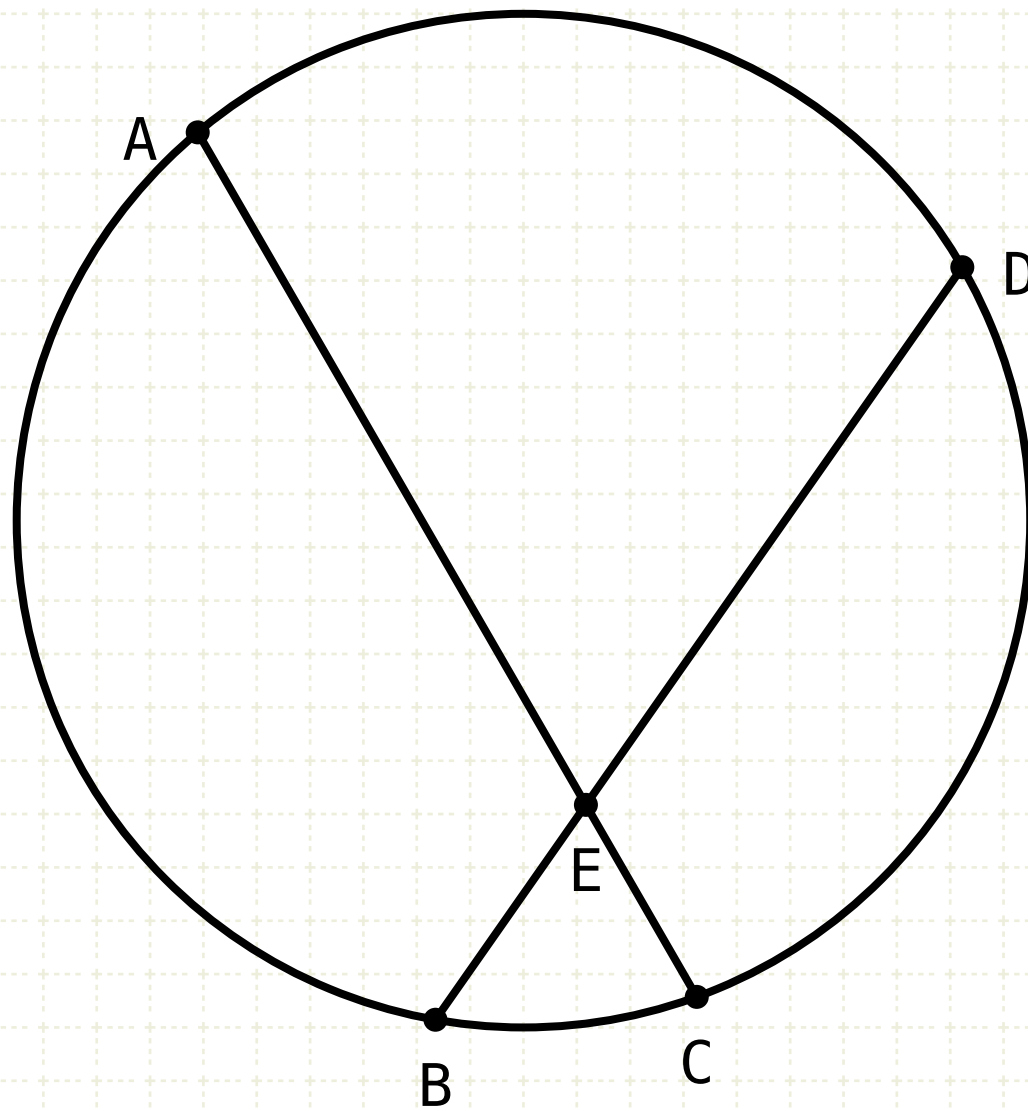
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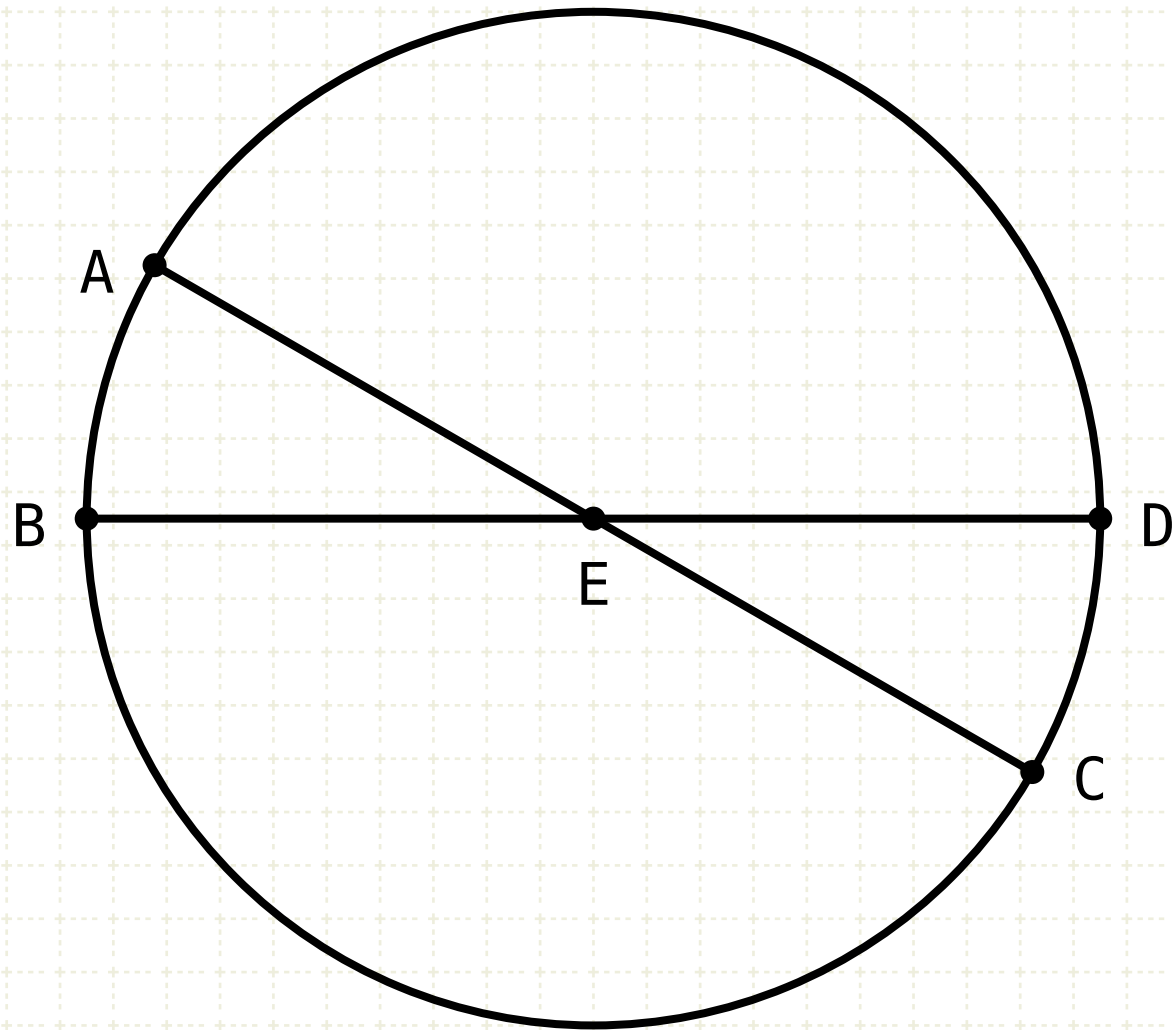
$$AE \cdot EC = BE \cdot ED$$

In other words

If lines AC and BD cross each other in a circle at point E, then the product of AE, EC is equal to the product BE, ED

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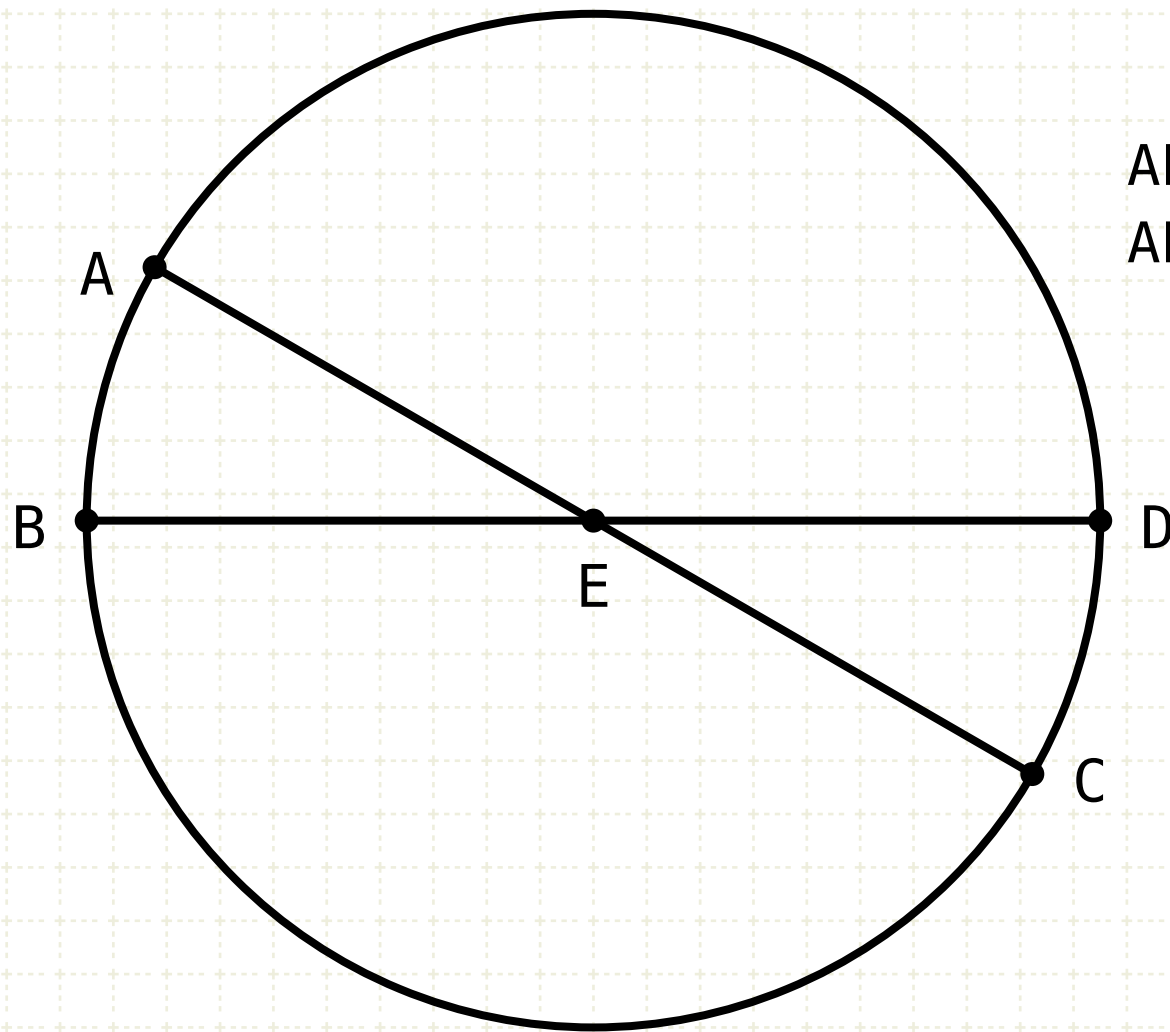
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Proof 1

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$$AE = BE = ED = EC$$
$$AE \cdot EC = BE \cdot ED$$

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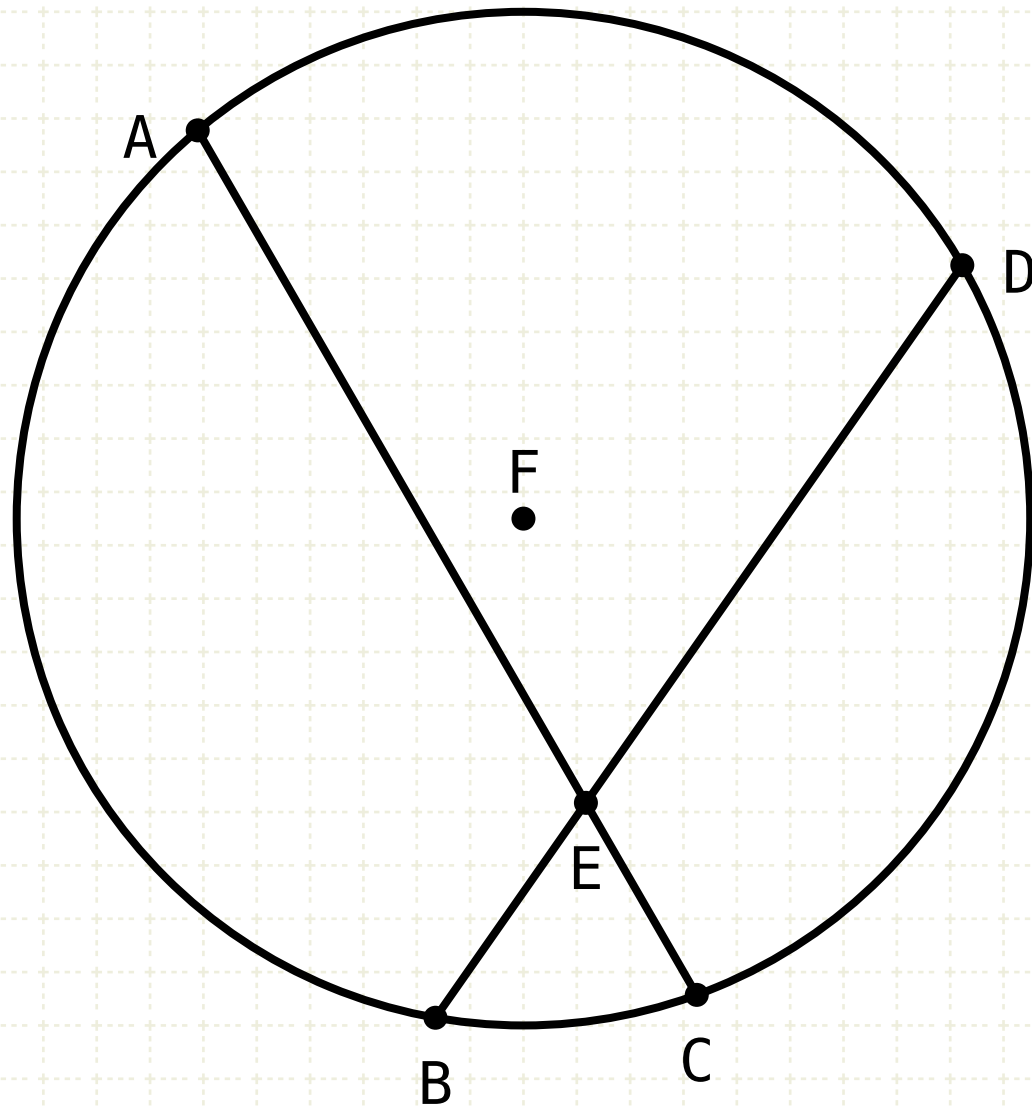
If lines AC and BD cross each other in a circle at point E, then the product of AE,EC is equal to the product BE,ED

Proof 1

If E is the centre of the circle, then AE,BE,DE,CE are all equal (radii of the same circle), so it is obvious that AE,CE equals BE, ED

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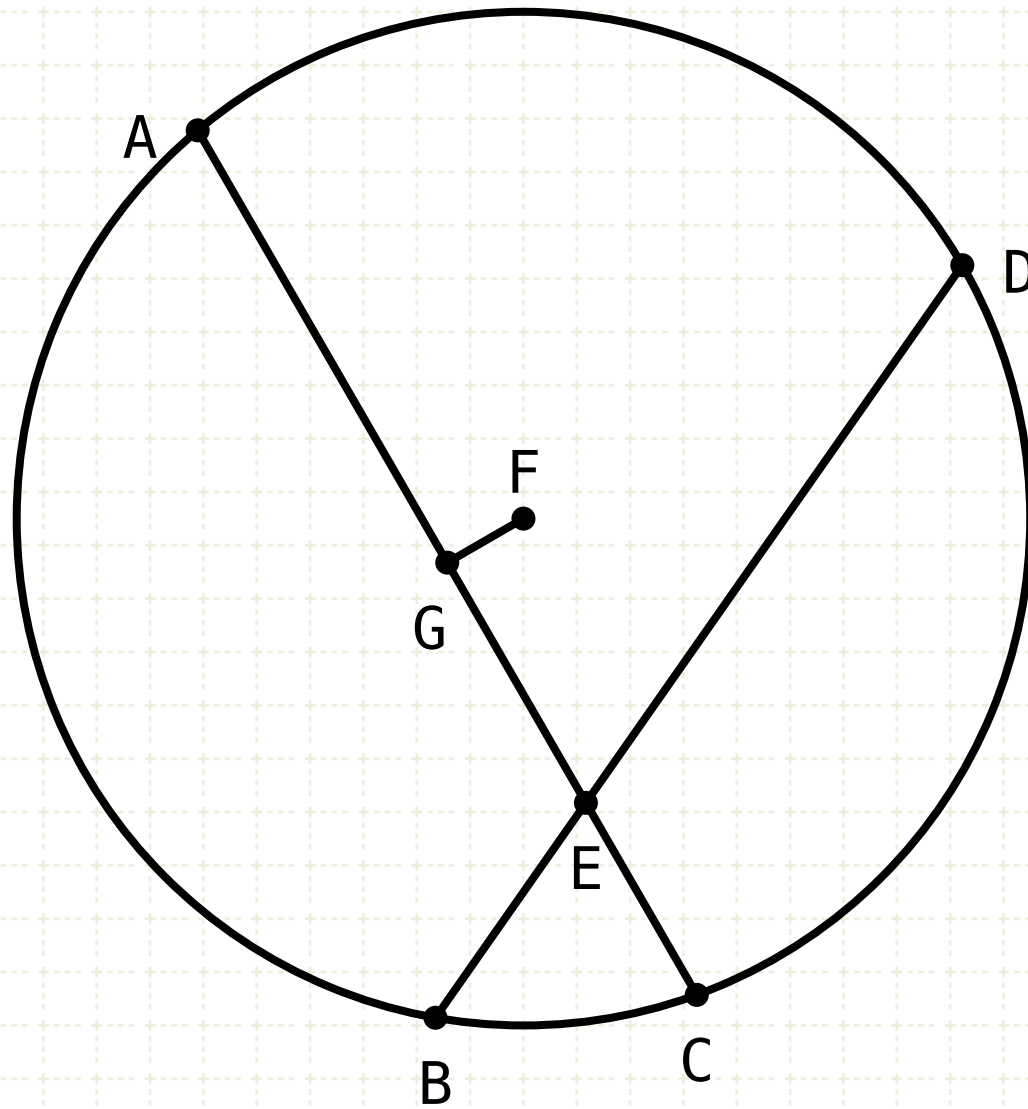


Proof 2

F is the centre of the circle, not E

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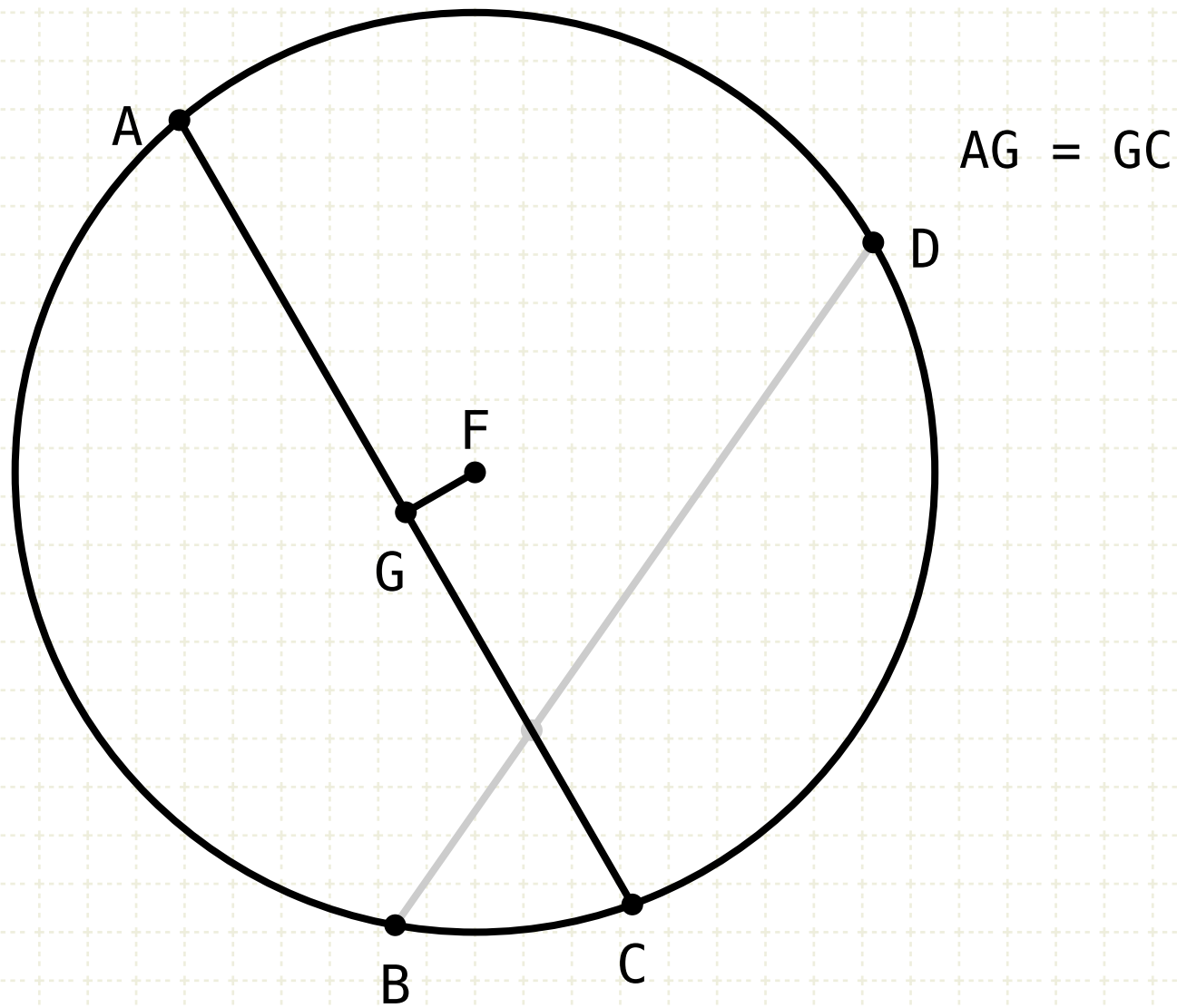
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From F, draw FG perpendicular to AC

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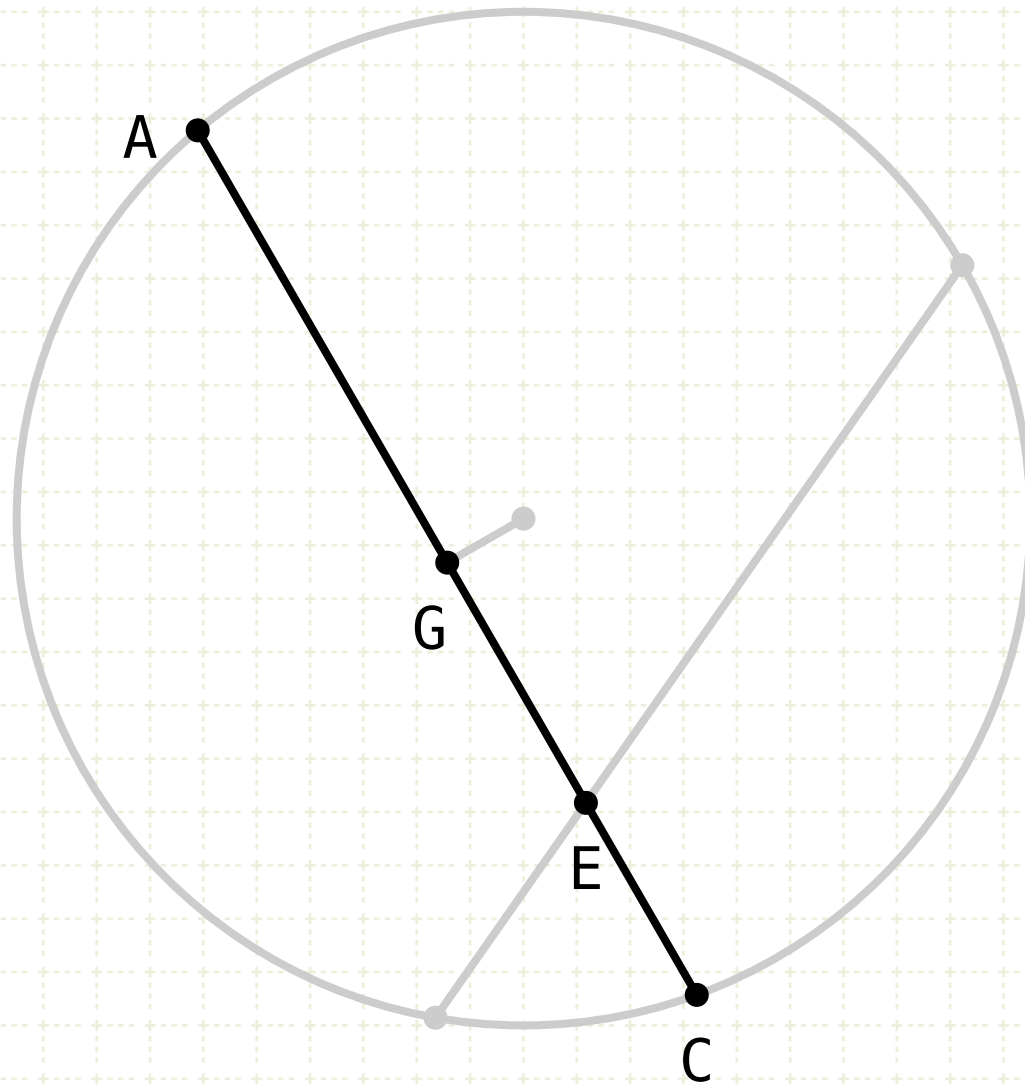
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$$\begin{aligned} AG &= GC \\ AE \cdot EC + GE^2 &= GC^2 \end{aligned}$$

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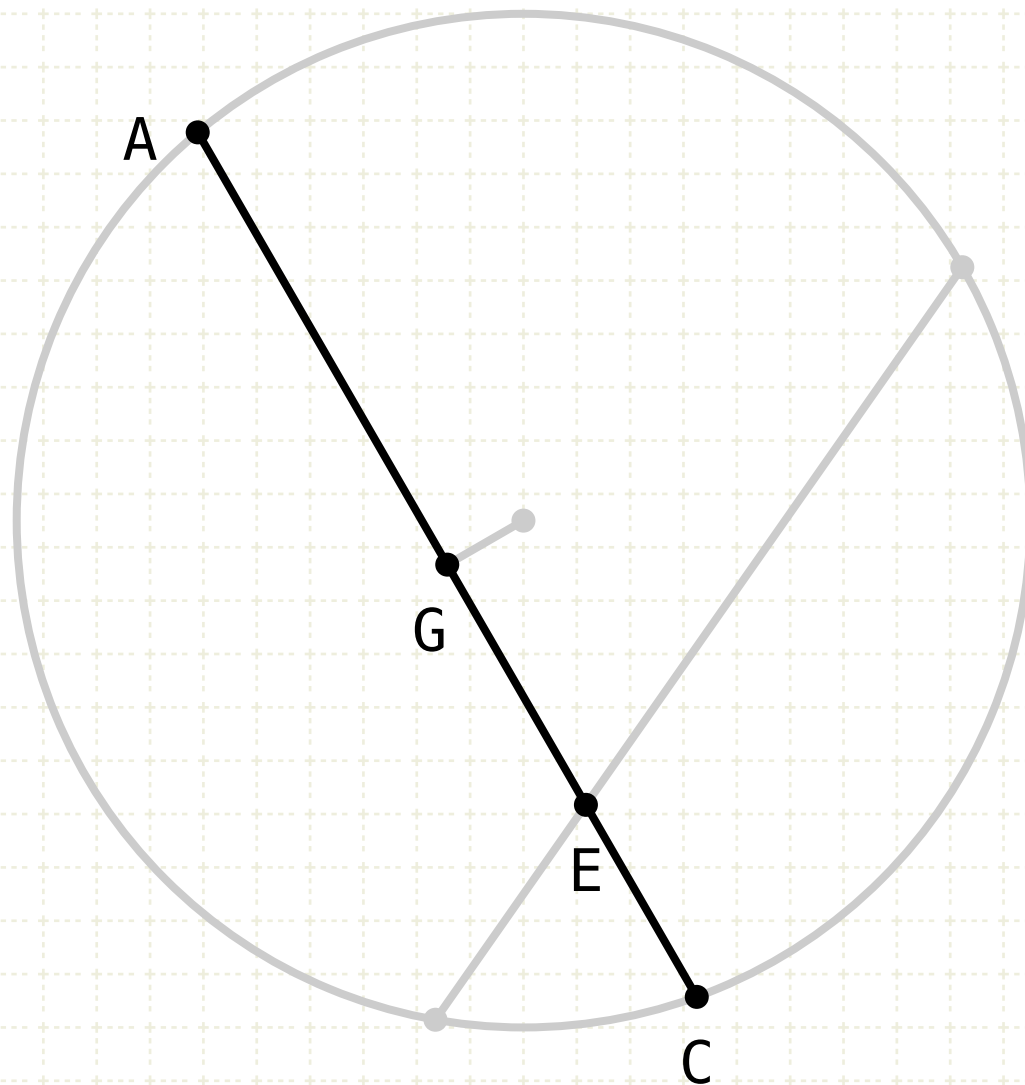
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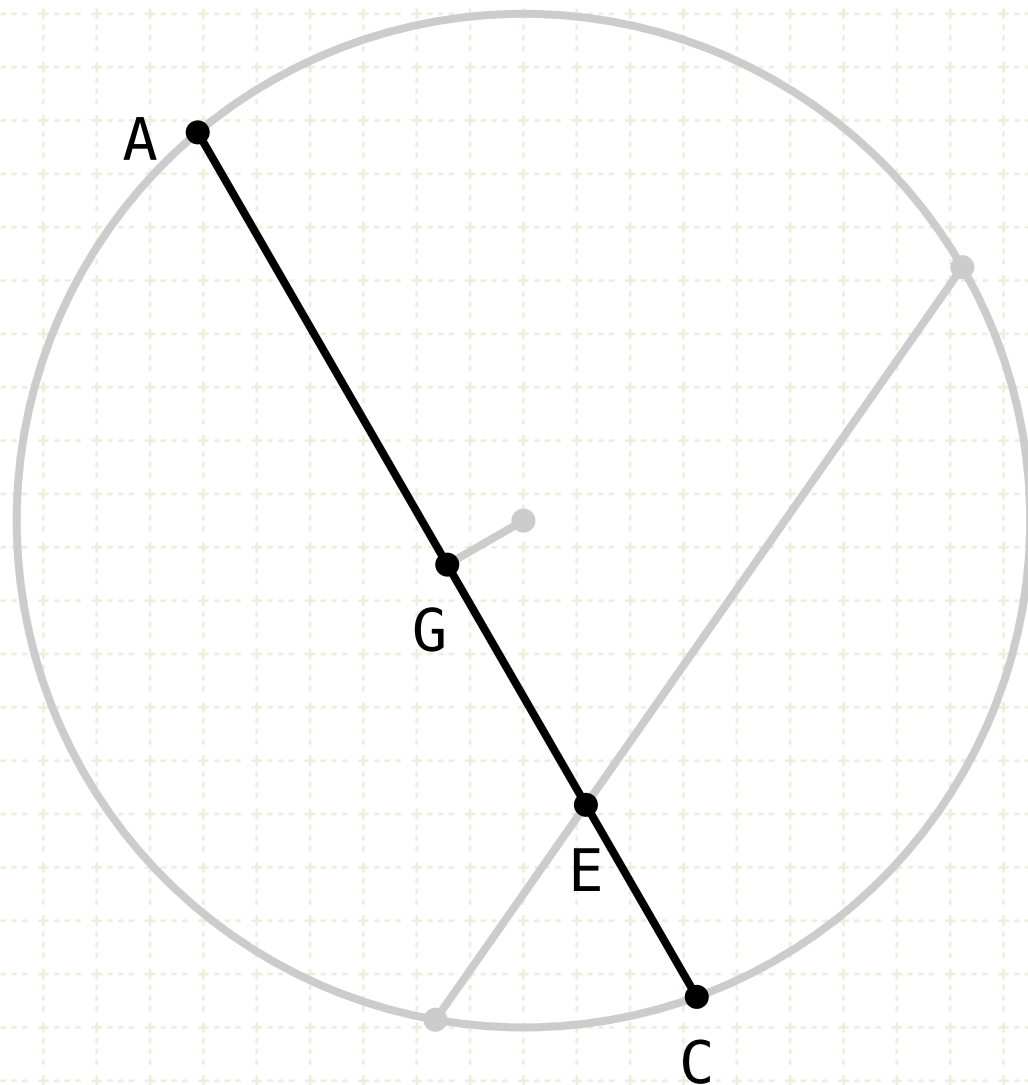
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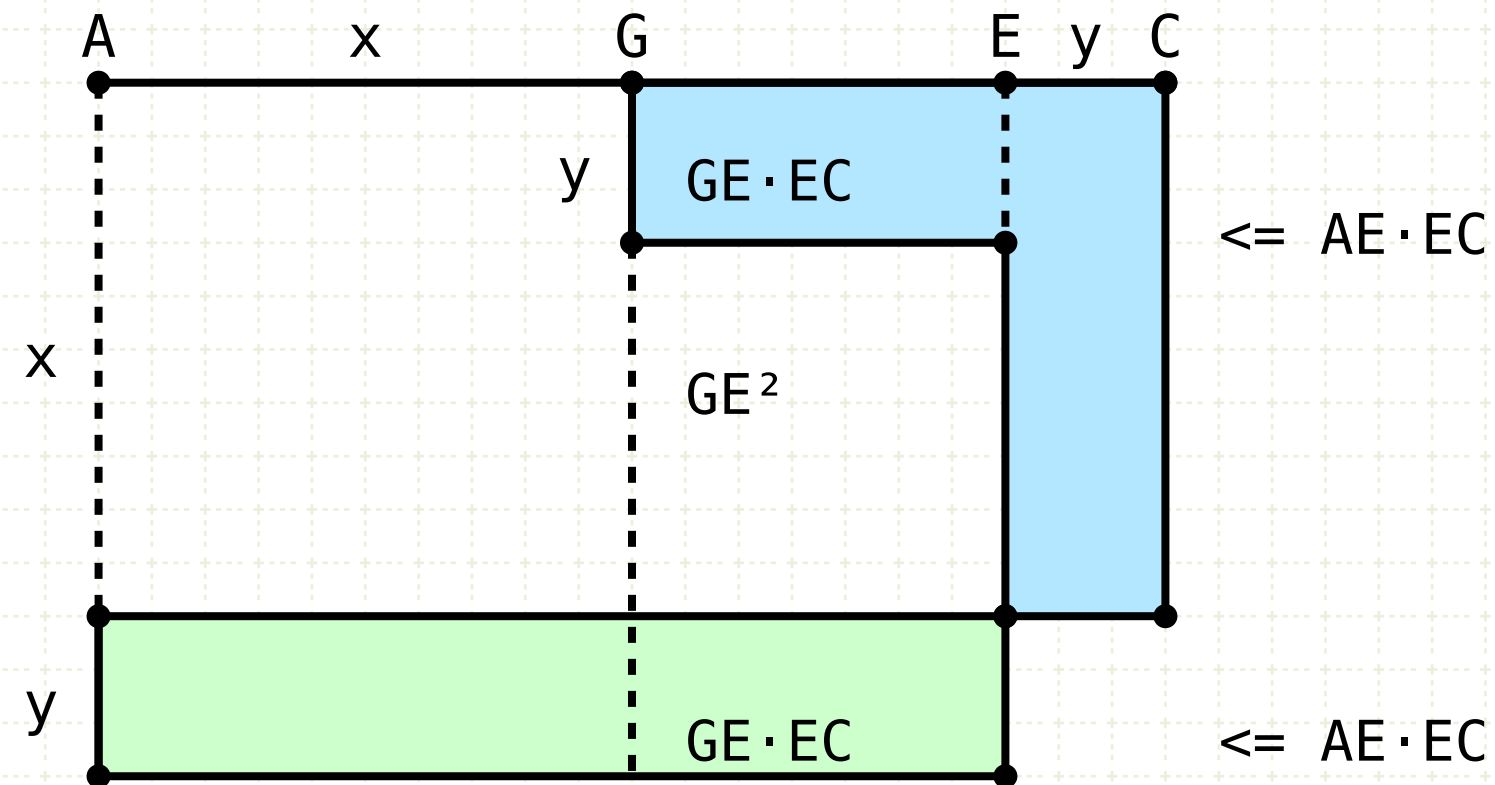
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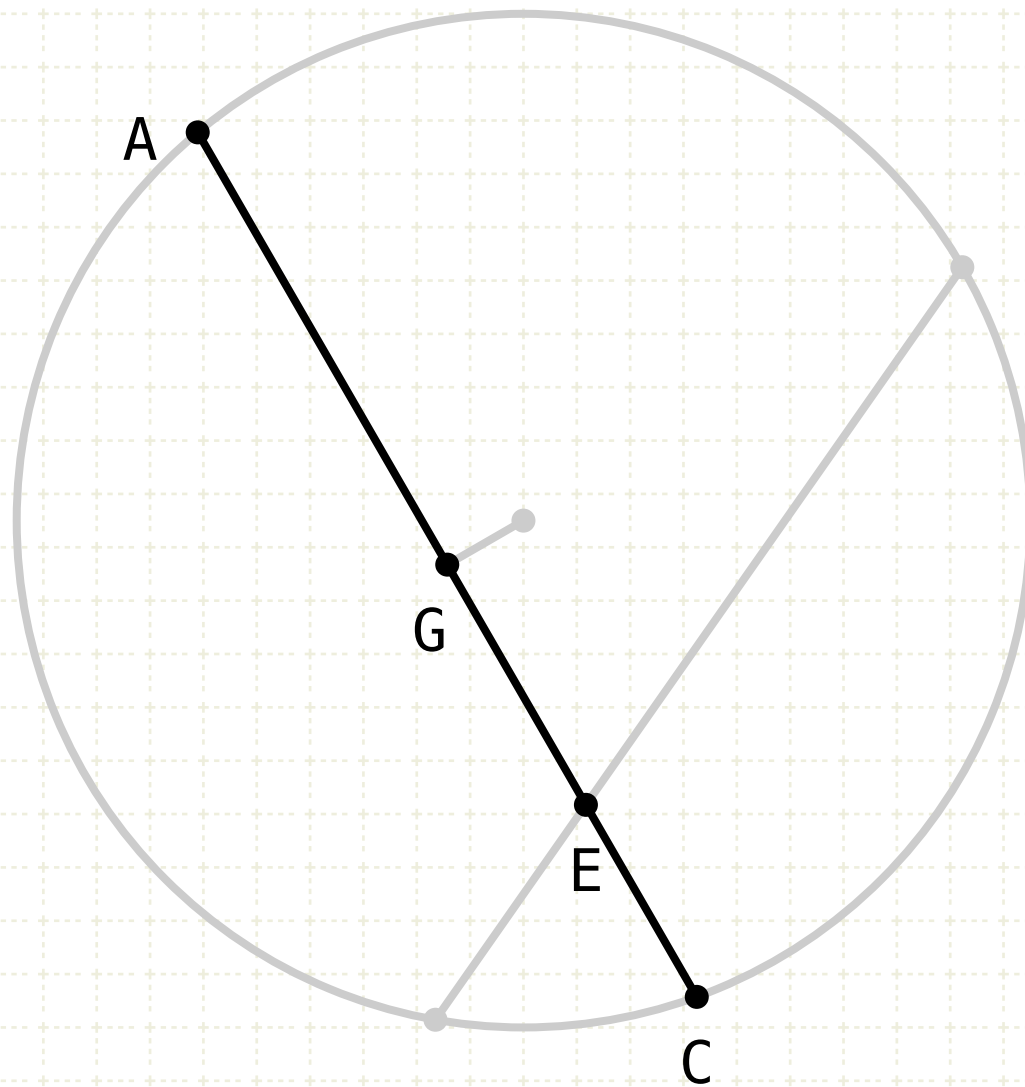
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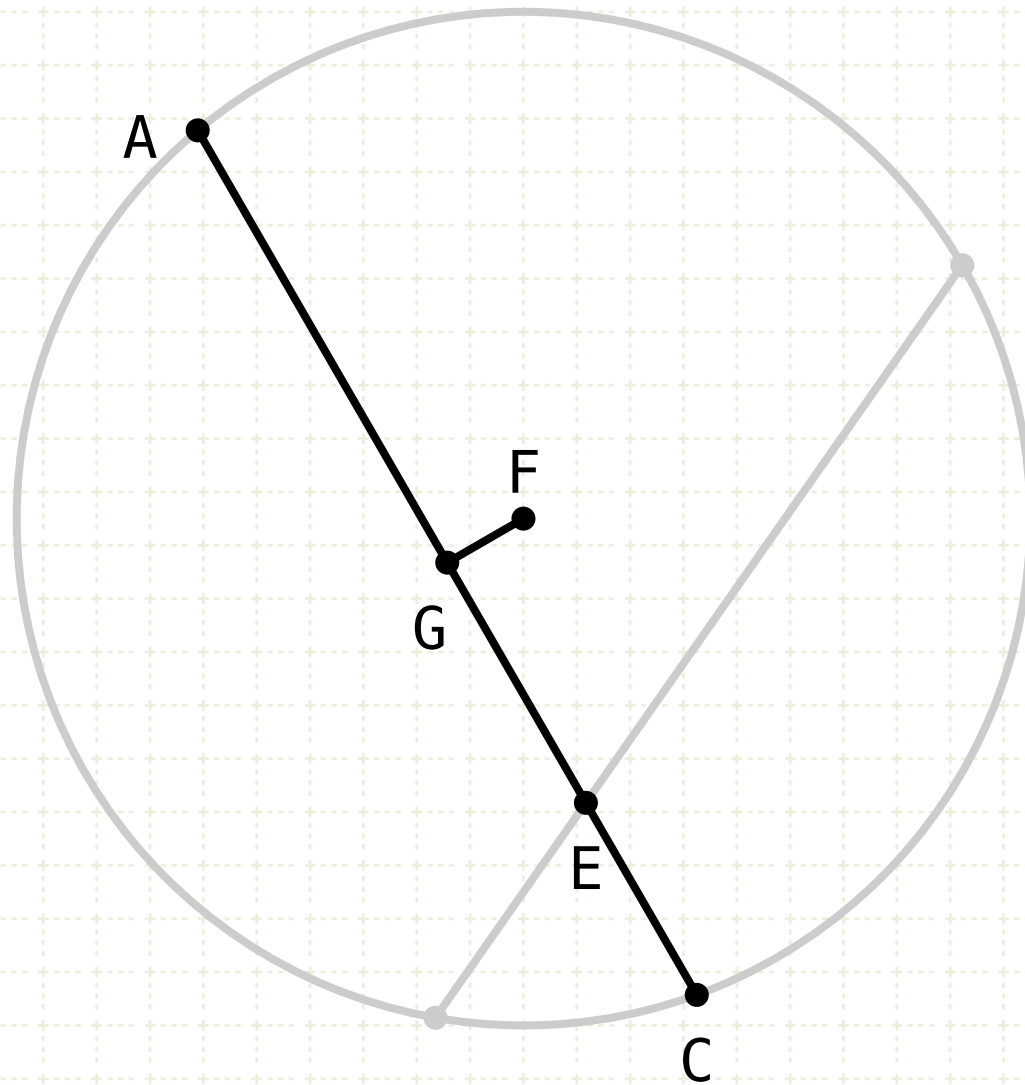
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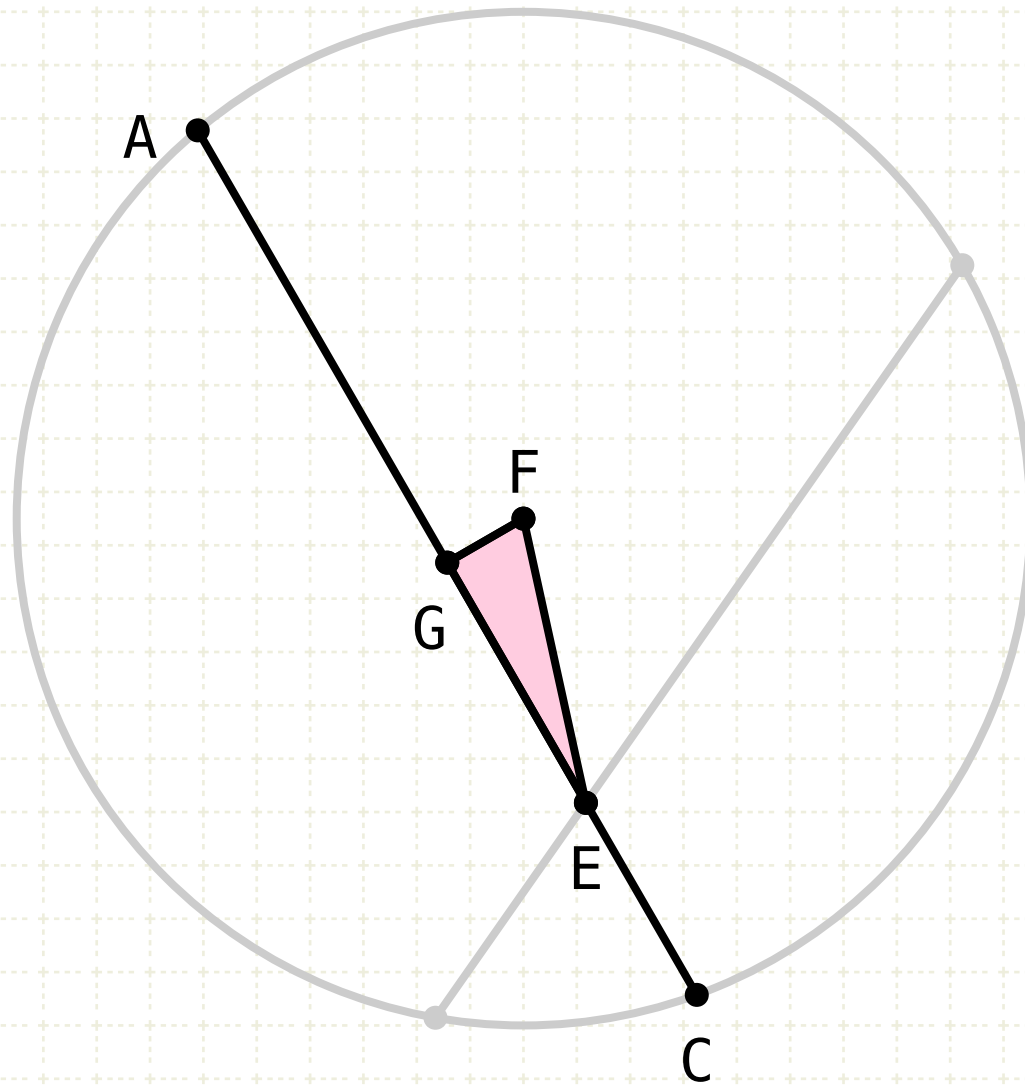
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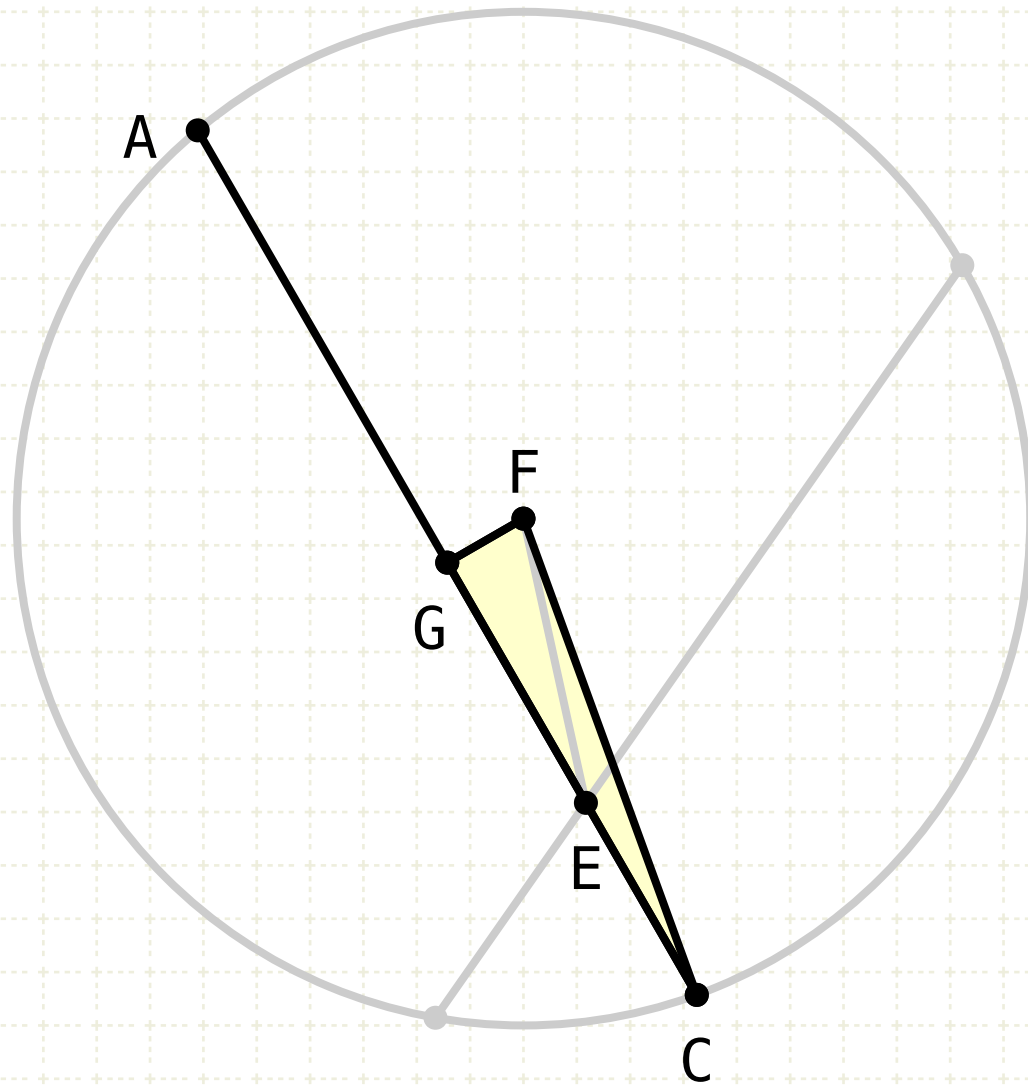
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$$GC^2 + FG^2 = FC^2 = r^2$$

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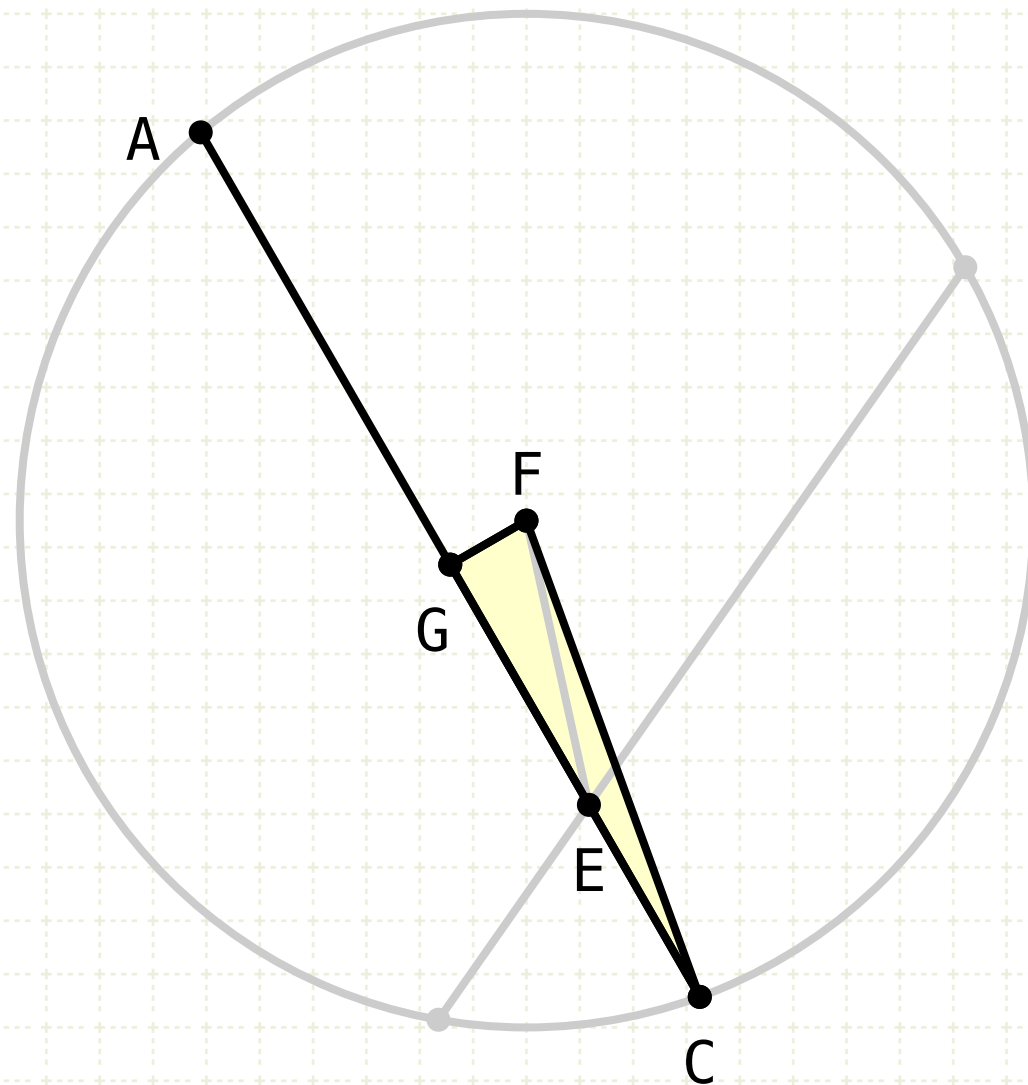
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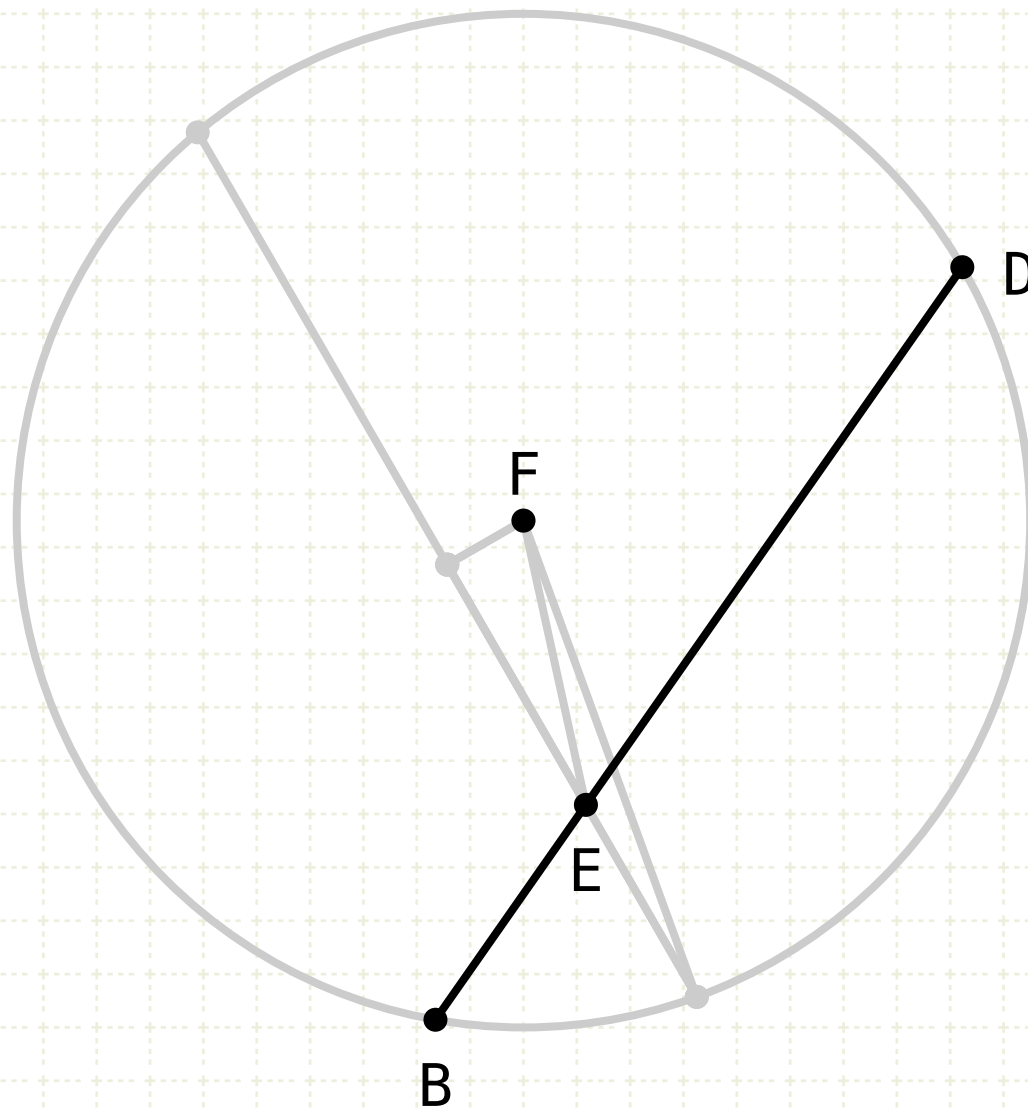
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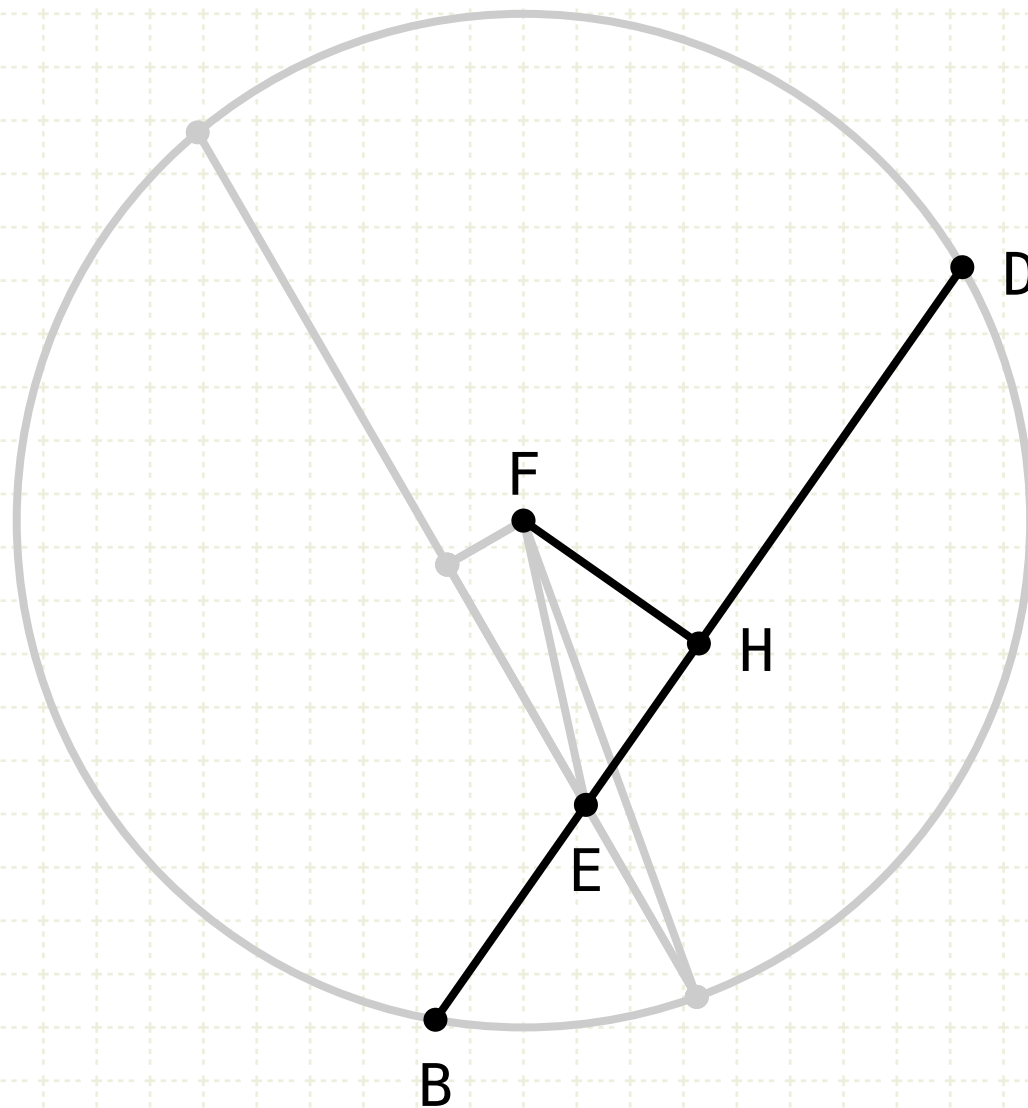
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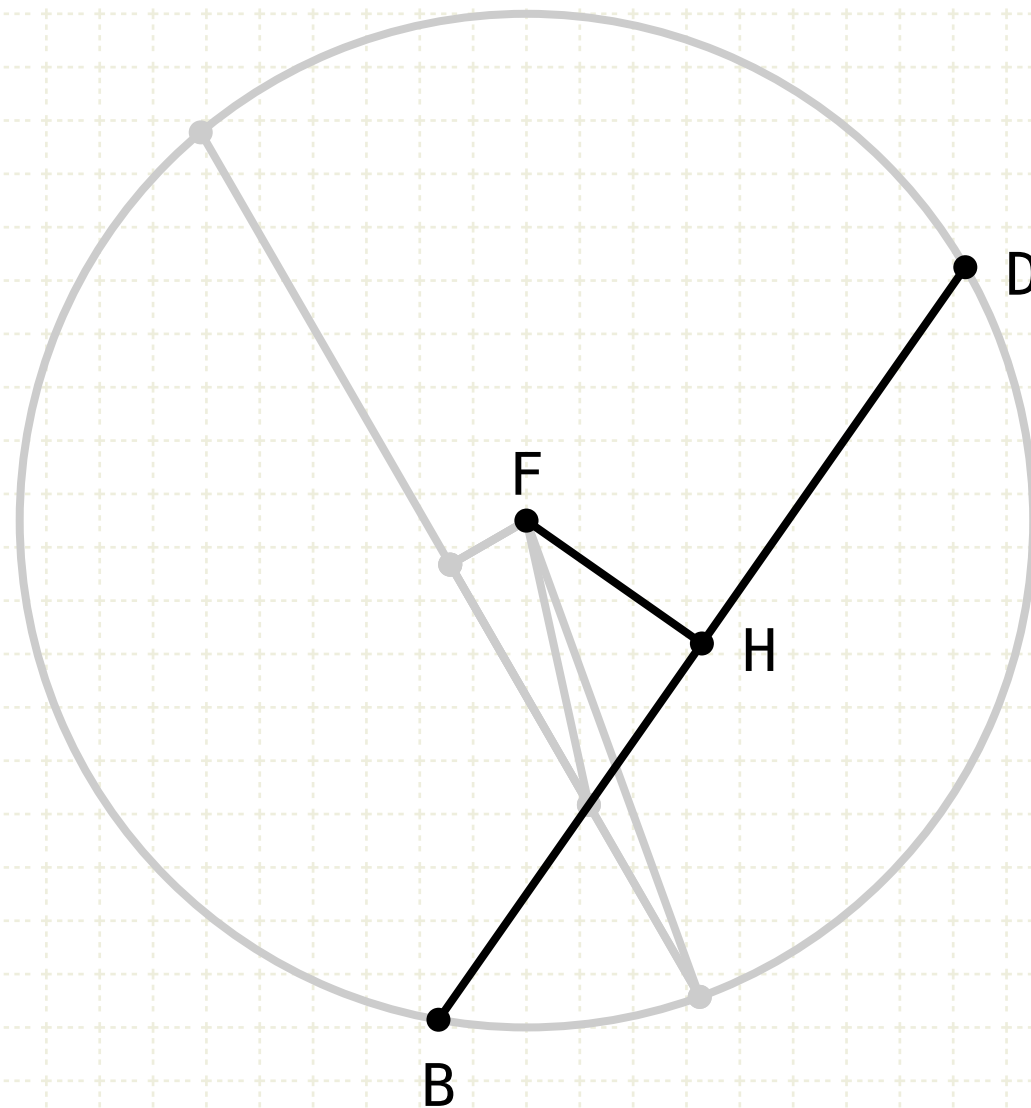
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$$BH = DH$$

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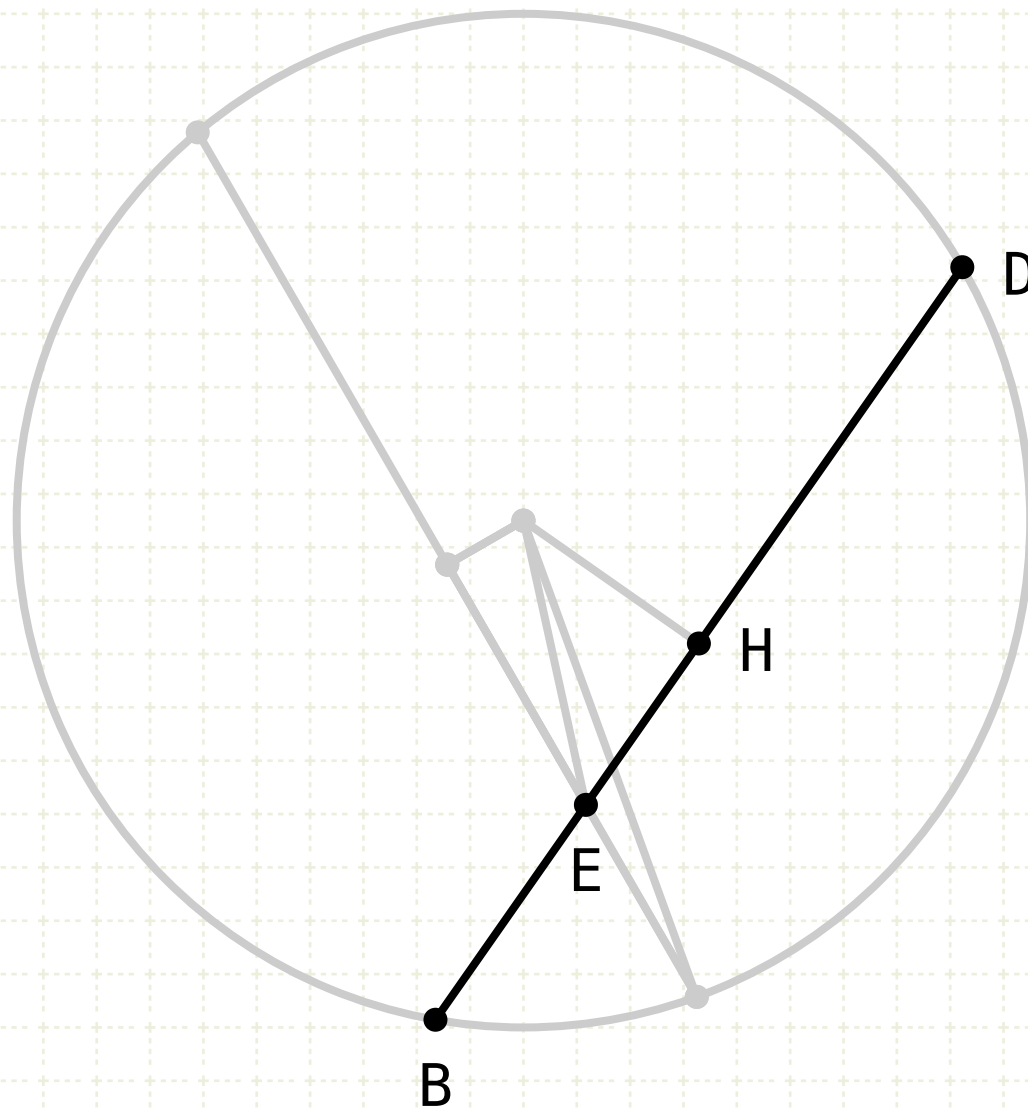
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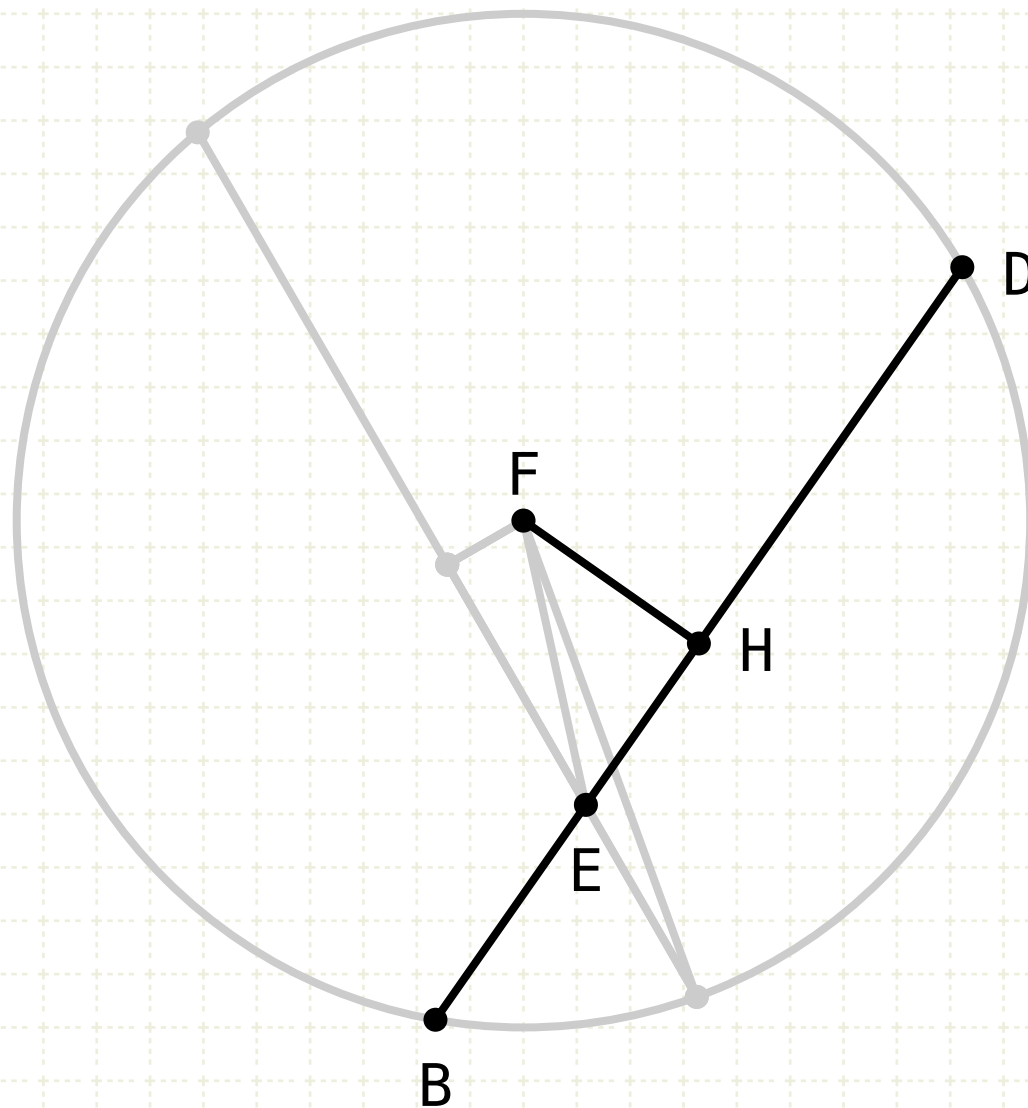
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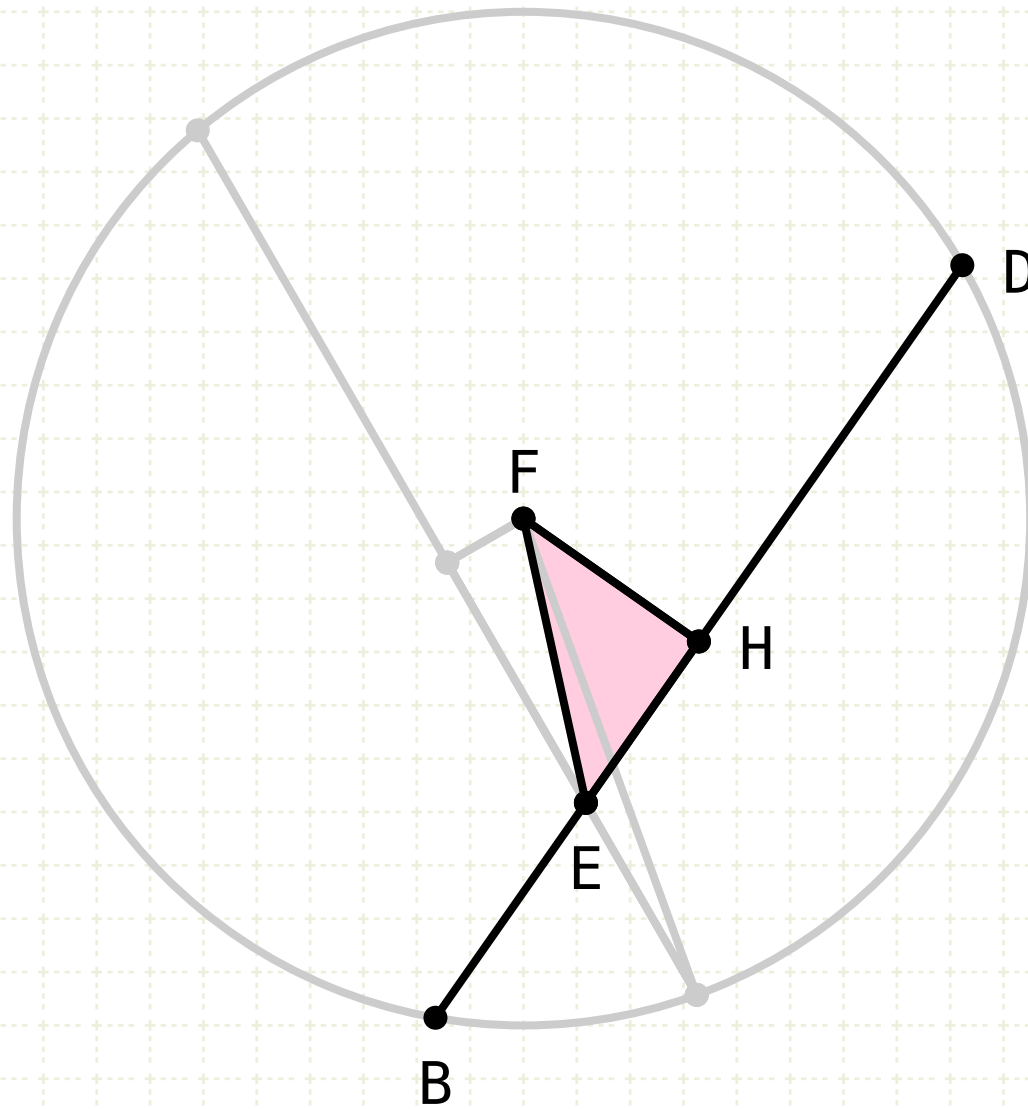
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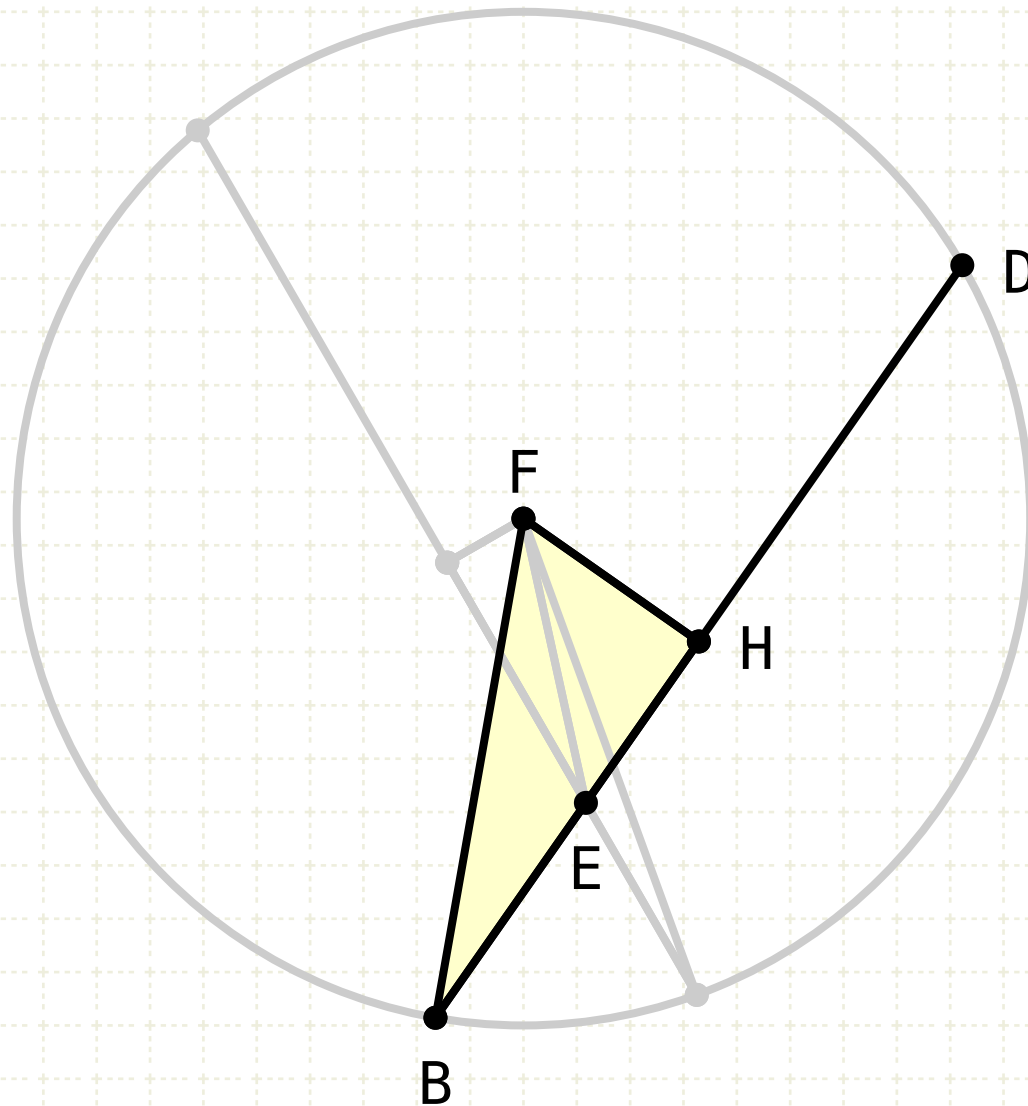
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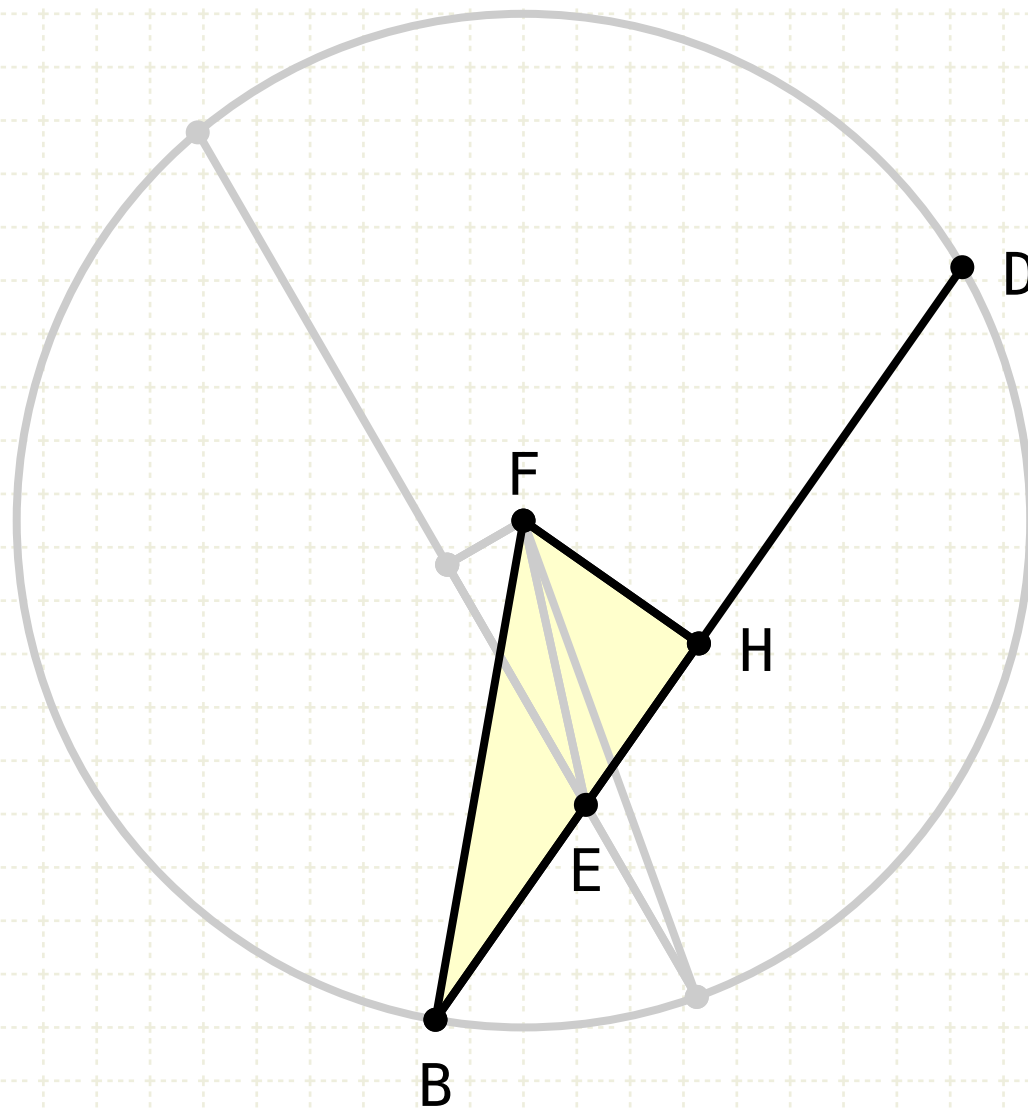
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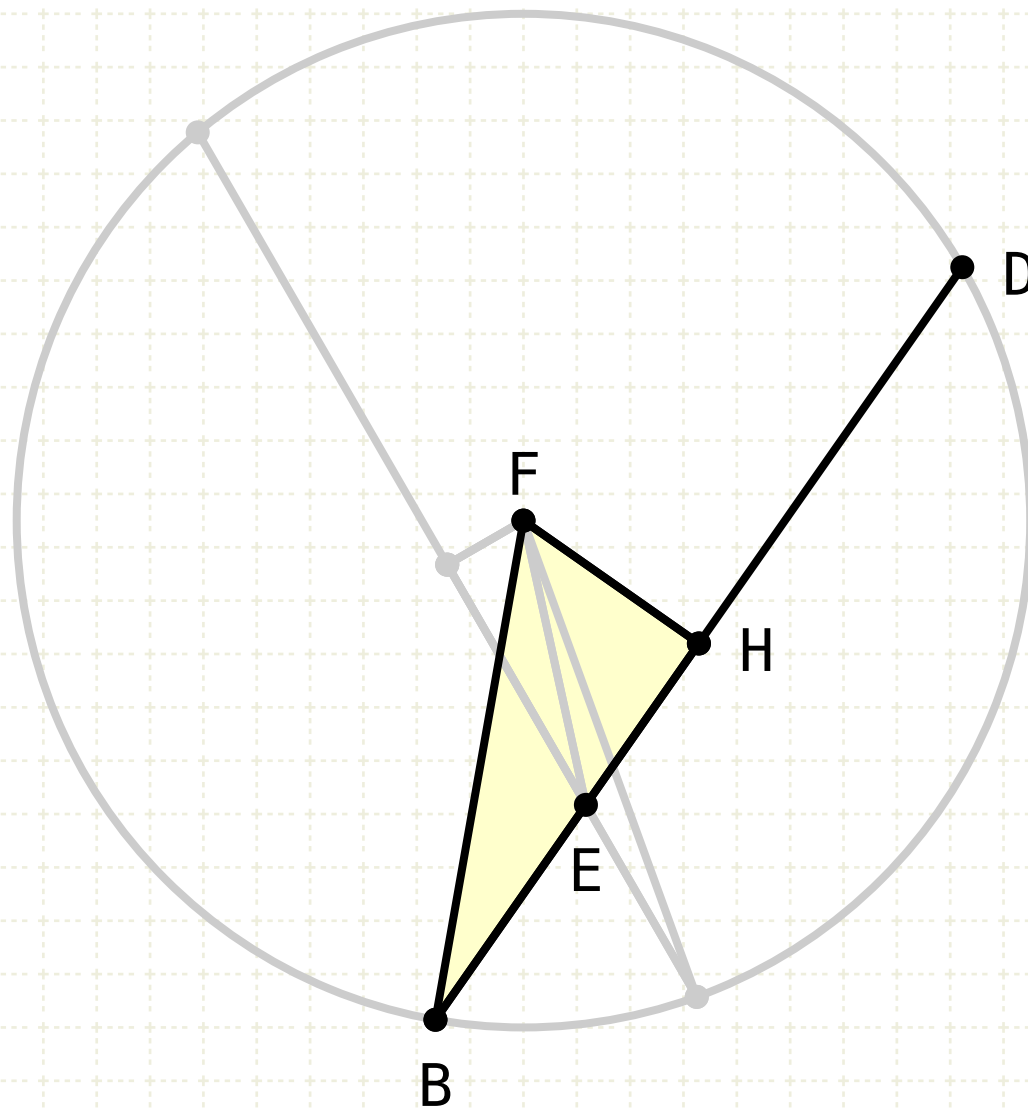
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Follow the same steps for line BD

Proposition 35 of Book III

If in a circle two straight lines cut one another, the rectangle contained by the segments of the one is equal to the rectangle contained by the segments of the other.



$$\begin{aligned} AG &= GC \\ AE \cdot EC + GE^2 &= GC^2 \\ AE \cdot EC + GE^2 + FG^2 &= GC^2 + FG^2 \\ GE^2 + FG^2 &= FE^2 \\ GC^2 + FG^2 &= FC^2 = r^2 \\ AE \cdot EC + FE^2 &= r^2 \end{aligned}$$

$$\begin{aligned} BH &= DH \\ DE \cdot EB + HE^2 &= HB^2 \end{aligned}$$

$$\begin{aligned} DE \cdot EB + HE^2 + HF^2 &= HB^2 + HF^2 \\ HE^2 + HF^2 &= FE^2 \\ HB^2 + HF^2 &= FB^2 = r^2 \\ DE \cdot EB + FE^2 &= r^2 \end{aligned}$$

$$AE \cdot EC = DE \cdot EB$$

Proof 2

F is the centre of the circle, not E

From F, draw FG perpendicular to AC

Since FG passes through the centre of the circle, and is at right angles to AC, it also bisects AC (III·3)

When a line (AC) is broken into equal parts (AG,GC) and unequal parts (AE,EC), then the product AE,EC plus the square of GE is equal to the square of GC (II·5)

Add the square of FG (the radius) to both sides of the equality

By pythagoras' theorem (I·47), the sum of the squares GE, GF is equal to the square of FE

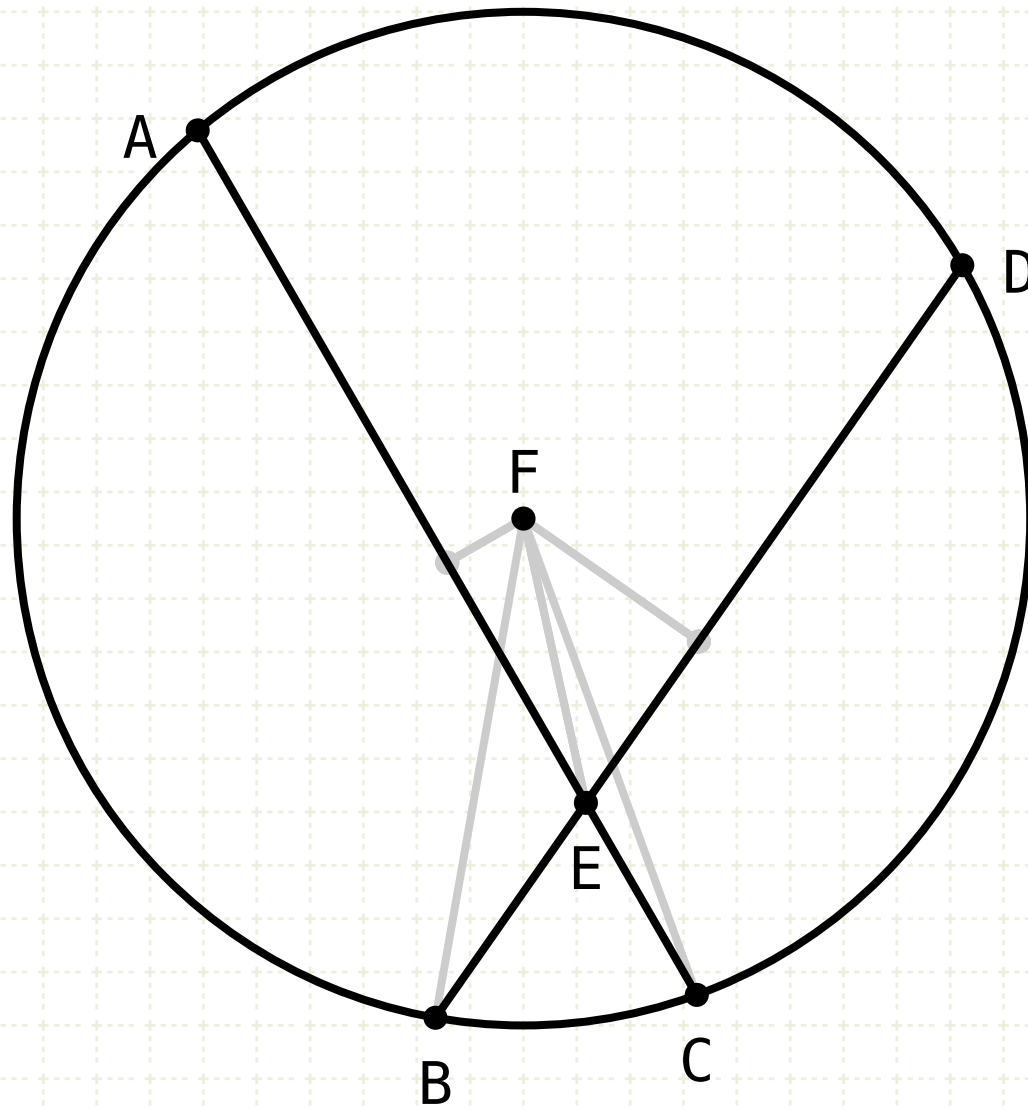
Similarly, the sum of the squares GC, GF is equal to the square of FC

Follow the same steps for line BD

Since FB and FC are both equal to the circle's radius, it can be easily seen that the rectangle formed by AE,EC is equal to the rectangle formed by DE,EB

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Proposition 35 of Book III

If in a circle two straight lines cut one another, the rectangle contained by the segments of the one is equal to the rectangle contained by the segments of the other.

Compare to II-14 - squaring a rectangle



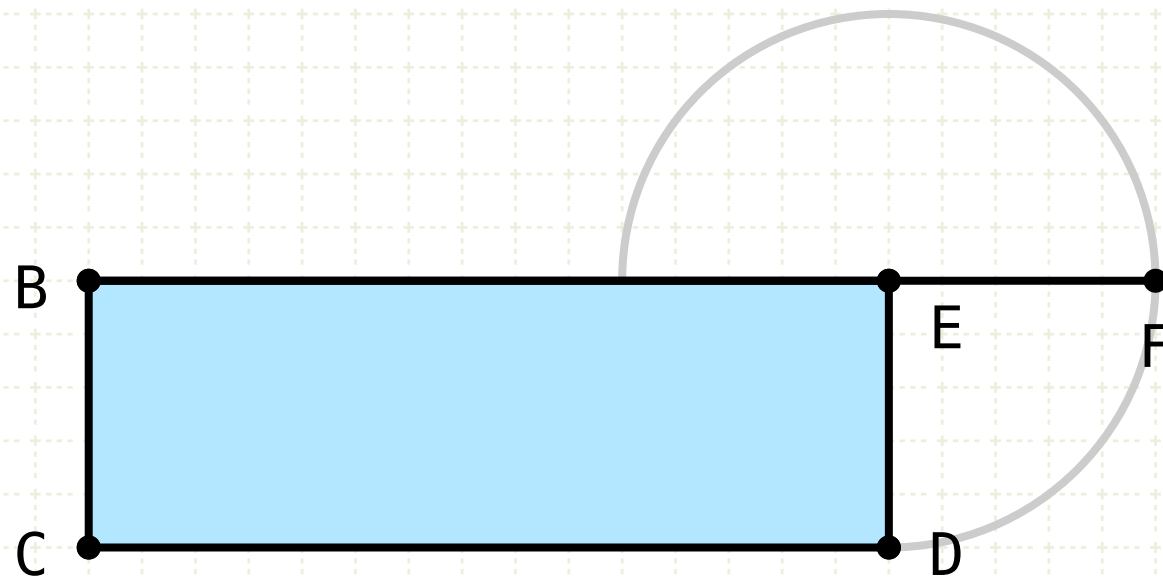
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If in a circle two straight lines cut one another, the rectangle contained by the segments of the one is equal to the rectangle contained by the segments of the other.

$$EF = ED$$

Compare to II-14 - squaring a rectangle

Extend BE to F, where EF equals ED



Proposition 35 of Book III

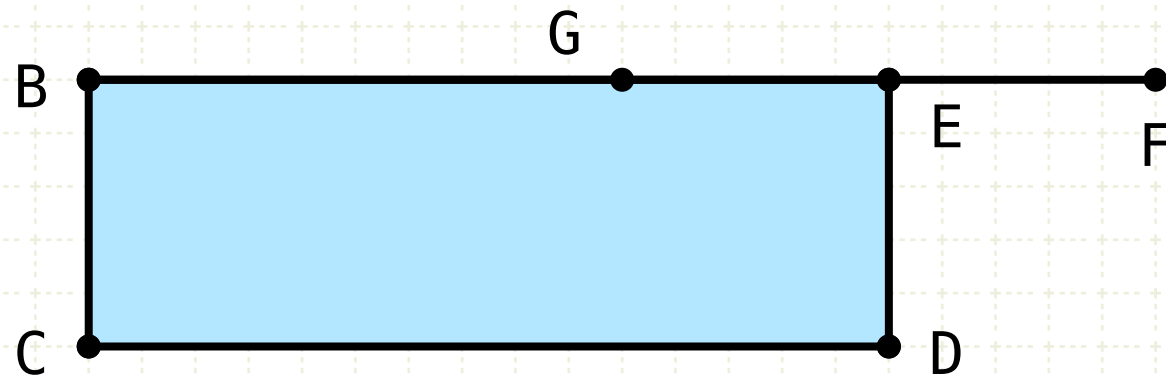
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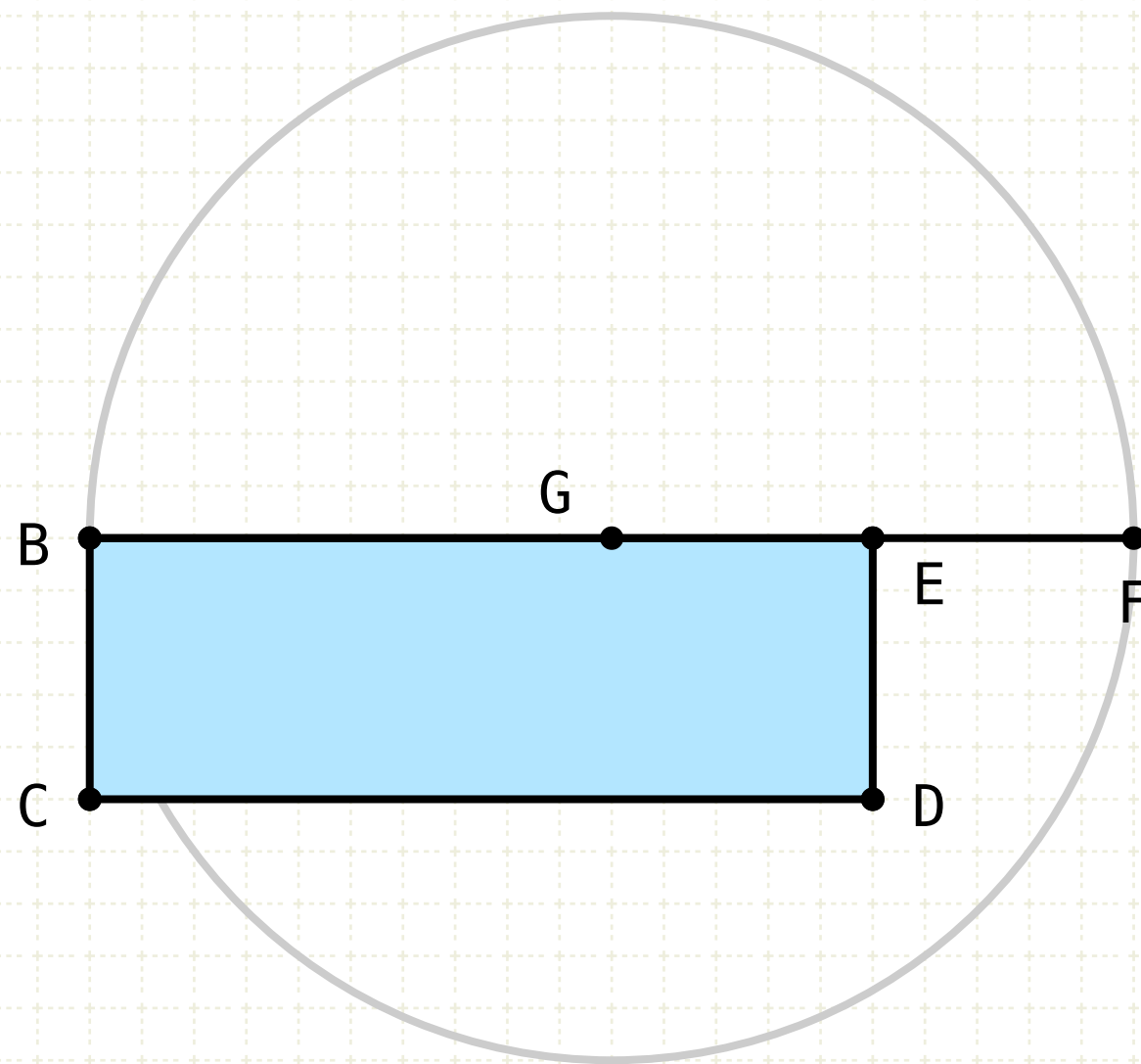
Bisect BF (and label it point G)



Proposition 35 of Book III

If in a circle two straight lines cut one another, the rectangle contained by the segments of the one is equal to the rectangle contained by the segments of the other.

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Compare to II-14 - squaring a rectangle

Extend BE to F, where EF equals ED

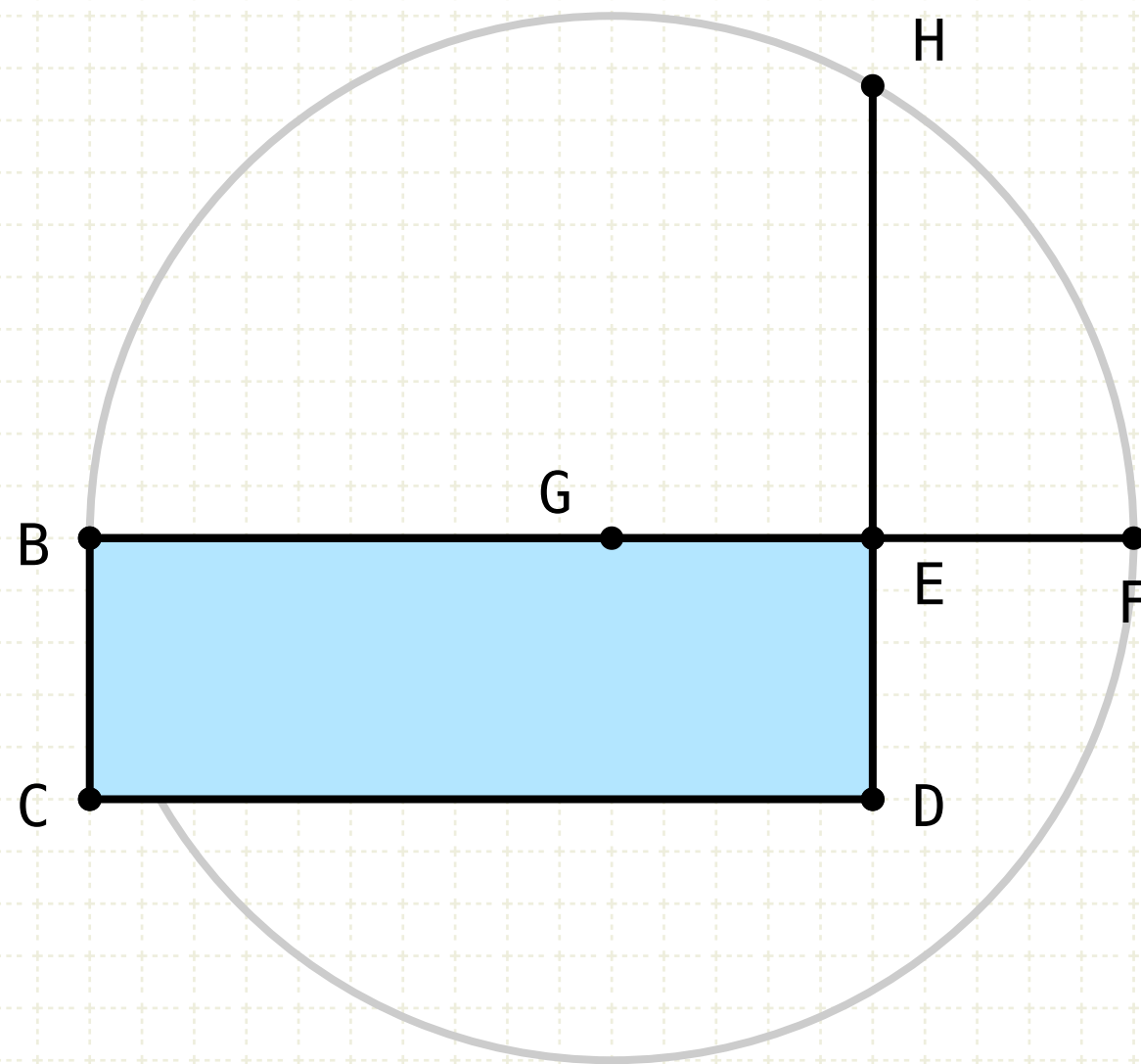
Bisect BF (and label it point G)

Draw a circle with G as the center and GF as the radius

Proposition 35 of Book III

If in a circle two straight lines cut one another, the rectangle contained by the segments of the one is equal to the rectangle contained by the segments of the other.

$$EF = ED$$



Compare to II-14 - squaring a rectangle

Extend BE to F, where EF equals ED

Bisect BF (and label it point G)

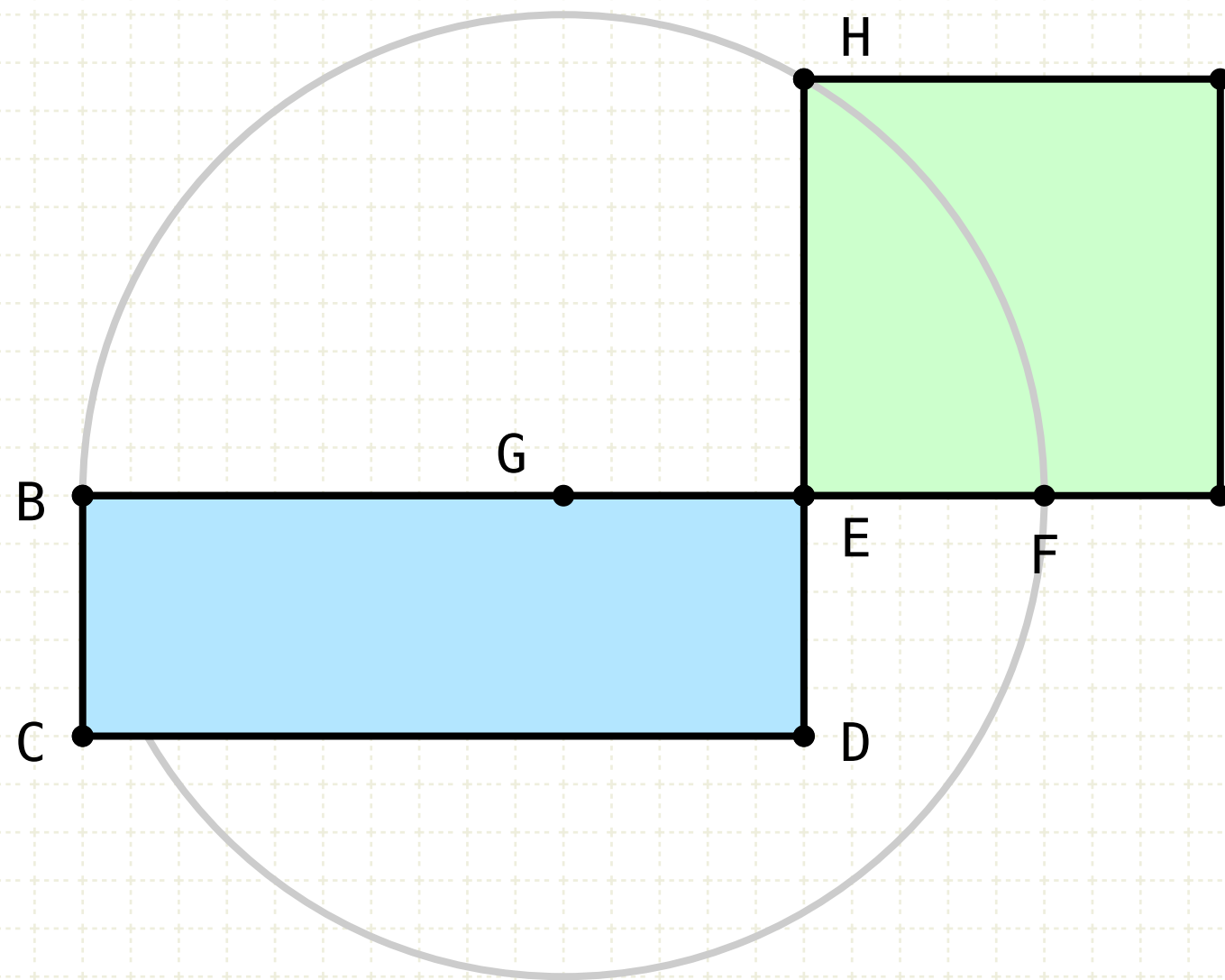
Draw a circle with G as the center and GF as the radius

Extend DE to intersect with the circle at point H

Proposition 35 of Book III

If in a circle two straight lines cut one another, the rectangle contained by the segments of the one is equal to the rectangle contained by the segments of the other.

$$EF = ED$$
$$BE \cdot ED = EH^2$$



Compare to II-14 - squaring a rectangle

Extend BE to F, where EF equals ED

Bisect BF (and label it point G)

Draw a circle with G as the center and GF as the radius

Extend DE to intersect with the circle at point H

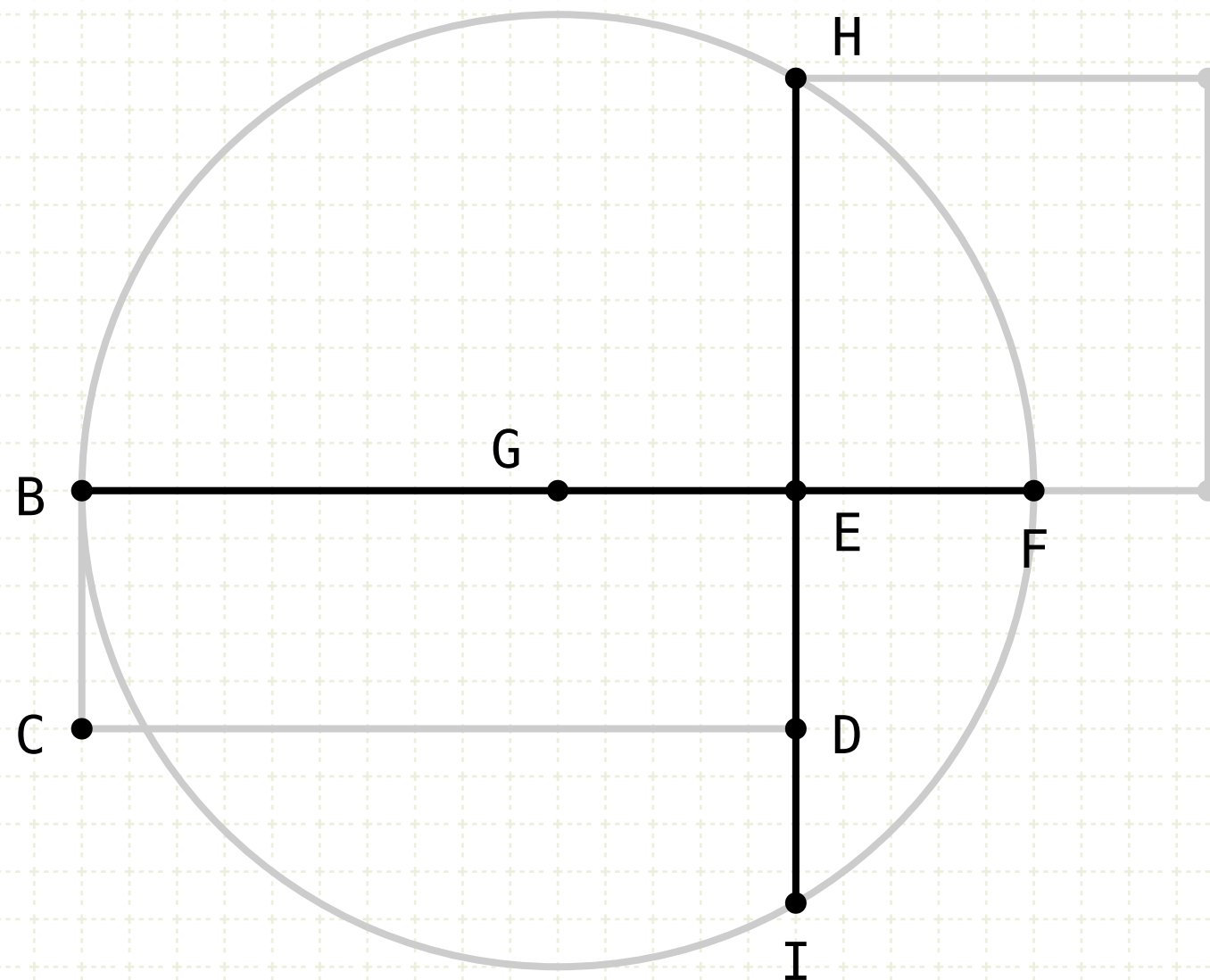
According to II-14, the square on HE is equal in area of the rectangle

Proposition 35 of Book III

If in a circle two straight lines cut one another, the rectangle contained by the segments of the one is equal to the rectangle contained by the segments of the other.

$$EF = BD$$

$$HE = EI$$



Compare to II-14 - squaring a rectangle

Extend BE to F, where EF equals ED

Bisect BF (and label it point G)

Draw a circle with G as the center and GF as the radius

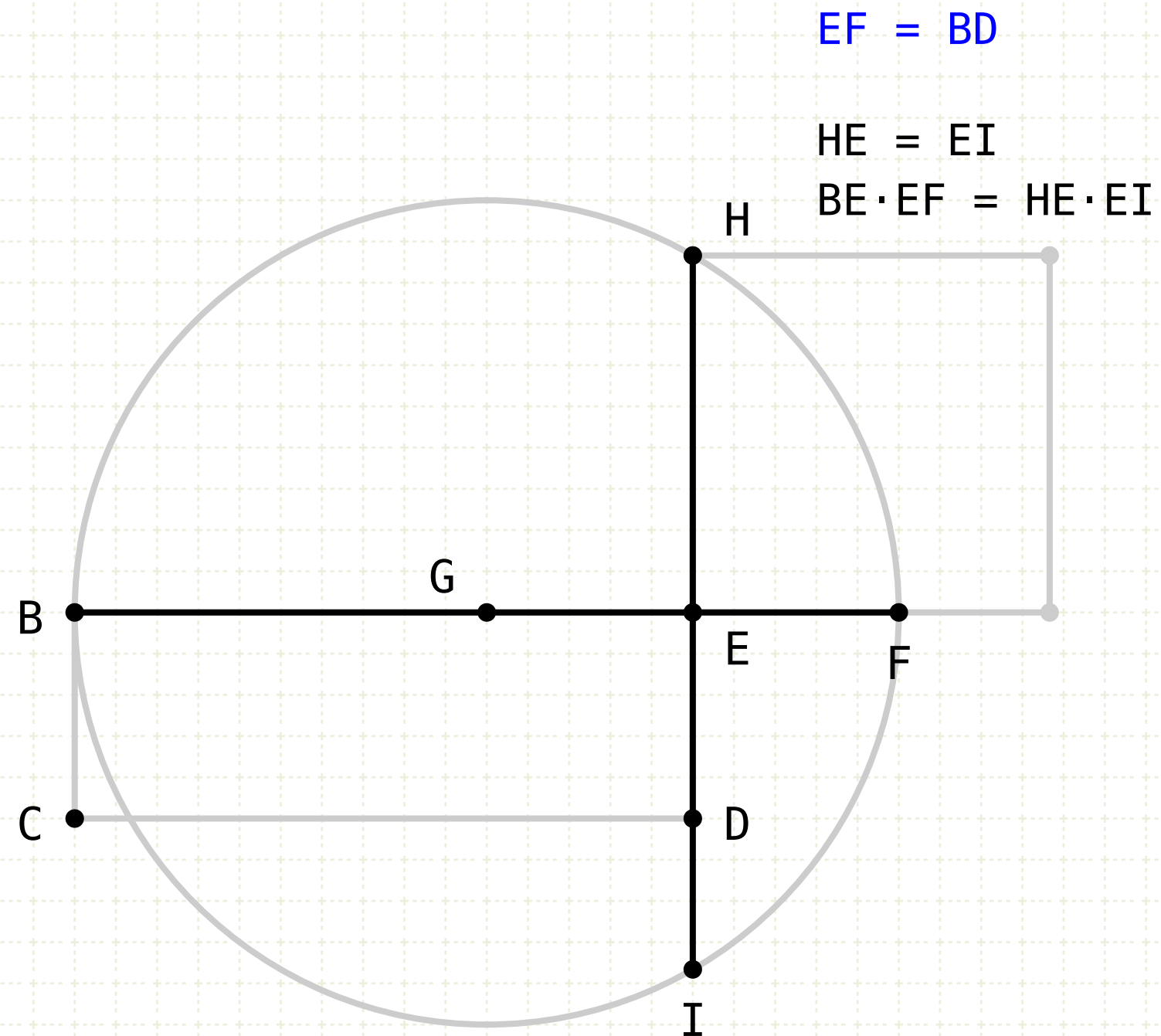
Extend DE to intersect with the circle at point H

According to II-14, the square on HE is equal in area of the rectangle

Since BF is perpendicular to HE (BCDE is a rectangle), and BF passes through the centre of the circle, HE is equal to EI (III-3)

Proposition 35 of Book III

If in a circle two straight lines cut one another, the rectangle contained by the segments of the one is equal to the rectangle contained by the segments of the other.



Compare to II-14 - squaring a rectangle

Extend BE to F, where EF equals ED

Bisect BF (and label it point G)

Draw a circle with G as the center and GF as the radius

Extend DE to intersect with the circle at point H

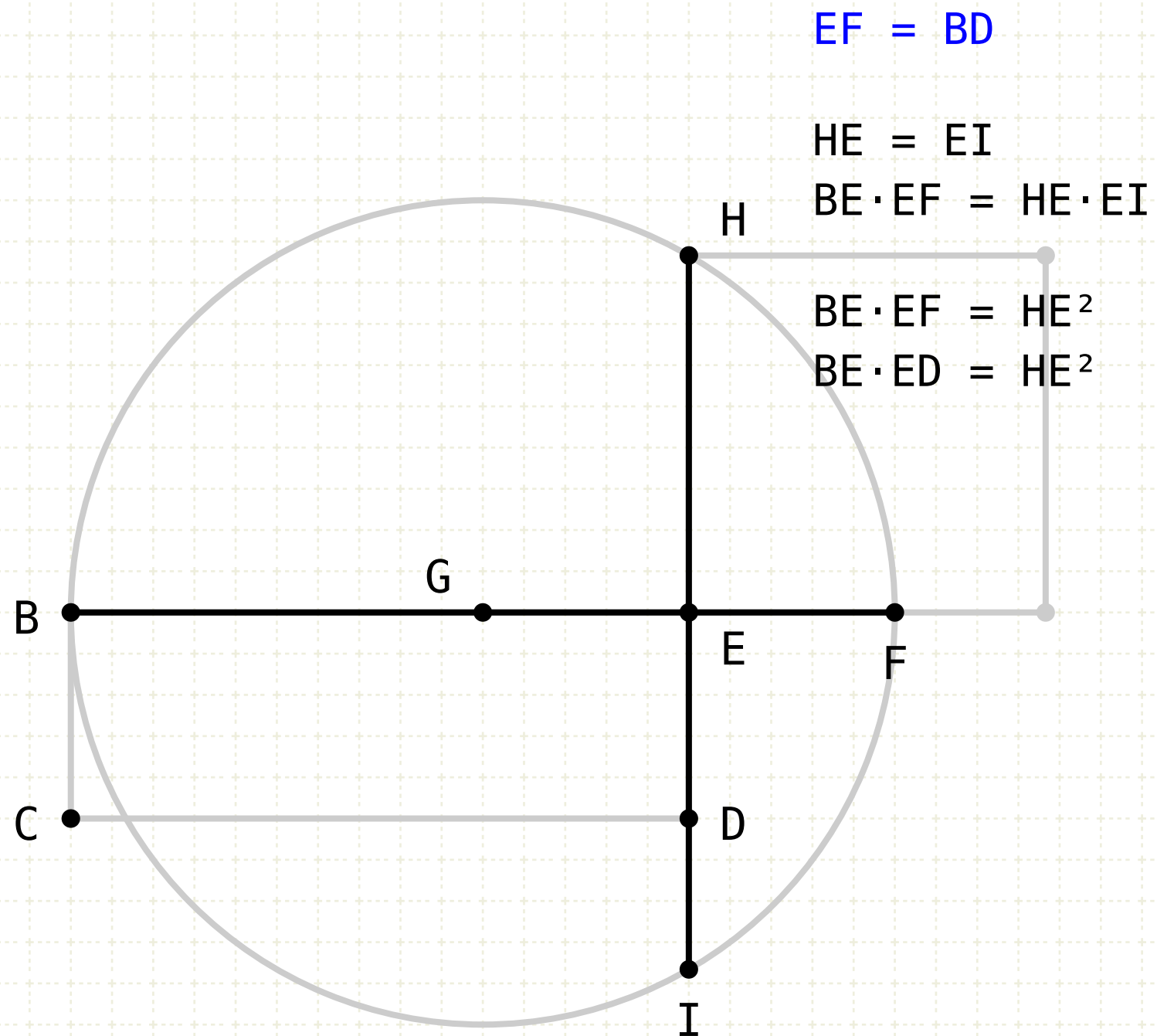
According to II-14, the square on HE is equal in area of the rectangle

Since BF is perpendicular to HE (BCDE is a rectangle), and BF passes through the centre of the circle, HE is equal to EI (III-3)

And, according to this proposition, BE,EF is equal to HE,EI

Proposition 35 of Book III

If in a circle two straight lines cut one another, the rectangle contained by the segments of the one is equal to the rectangle contained by the segments of the other.



Compare to II-14 - squaring a rectangle

Extend BE to F, where EF equals ED

Bisect BF (and label it point G)

Draw a circle with G as the center and GF as the radius

Extend DE to intersect with the circle at point H

According to II-14, the square on HE is equal in area of the rectangle

Since BF is perpendicular to HE (BCDE is a rectangle), and BF passes through the centre of the circle, HE is equal to EI (III-3)

And, according to this proposition, BE,EF is equal to HE,EI

With the appropriate substitutions, we get the same result as II-14

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