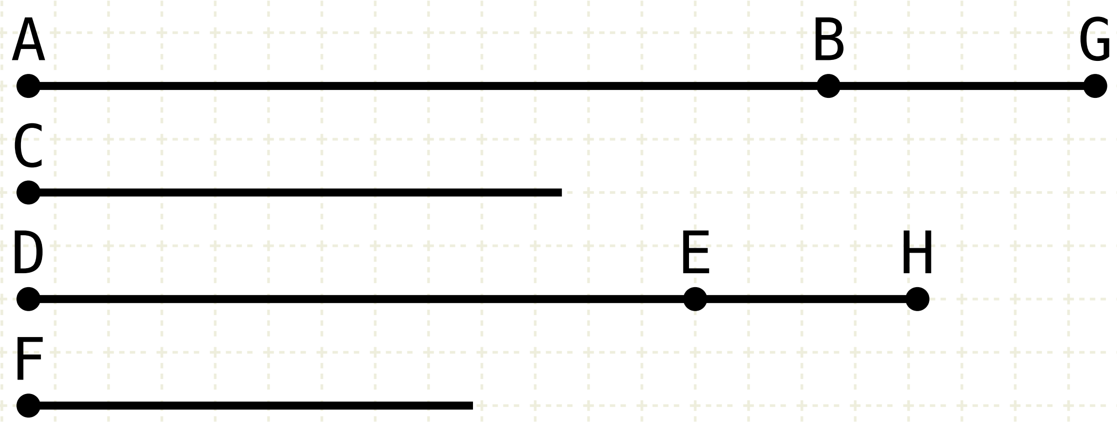
Euclid's Elements

Book V



$AB:C = DE:F$
$BG:C = EH:F$

$AG:C = DH:F$

Proportions are what makes the old Greek temples classic in their beauty. They are like huge blocks, from which the air has been literally hewn out between the columns.

Arne Jacobsen

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Proposition 18 of Book V

If magnitudes be proportional SEPARANDO, they will also be proportional COMPONENDO

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the componendo (composition) ratio of A:B is (A+B):B

the separando (separated) ratio of (A+B):B is A:B

the convertendo (in conversion) ratio of (A+B):B is (A+B):A

Definitions

14. COMPOSITION OF A RATIO means taking the antecedent together with the consequent as one in relation to the consequent by itself
15. SEPARATION OF A RATIO means taking the excess by which the antecedent exceeds the consequent in relation to the consequent by itself
16. CONVERSION OF A RATIO means taking the antecedent in relation to the excess by which the antecedent exceeds the consequent.

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If magnitudes be proportional SEPARANDO, they will also be proportional COMPONENDO

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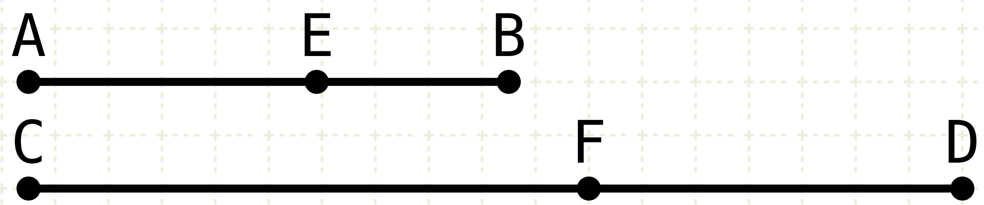

In other words

If AE, BE, CF, DF are proportional

… then COMPENDO (joint) ratios will also be equal

AE:EB = CF:FD

(AE+EB):EB = (CF+FD):FD

AB:EB = CD:FD

a:b = c:d → (a+b):b = (c+d):d

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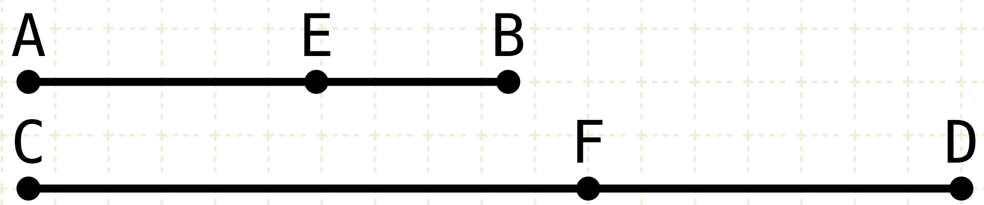
If magnitudes be proportional SEPARANDO, they will also be proportional COMPONENDO



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Proof by Contradiction

AE:EB = CF:FD

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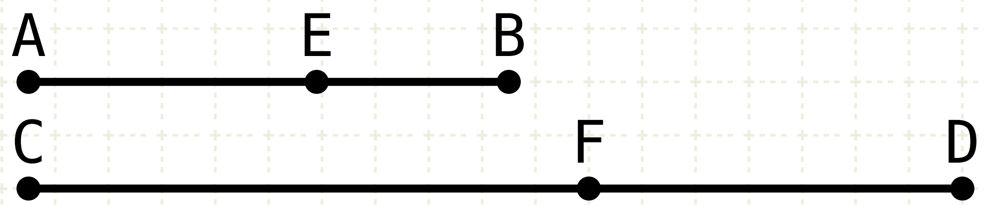

$AE:EB = CF:FD$

$CD:DF \neq AB:BE$

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If AE, BE, CF, DF are proportional

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Proof by Contradiction

Assume CD is not to DF as AB is to BE

Then the ratio of AB to BE will be equal to CD to some magnitude not equal to DF

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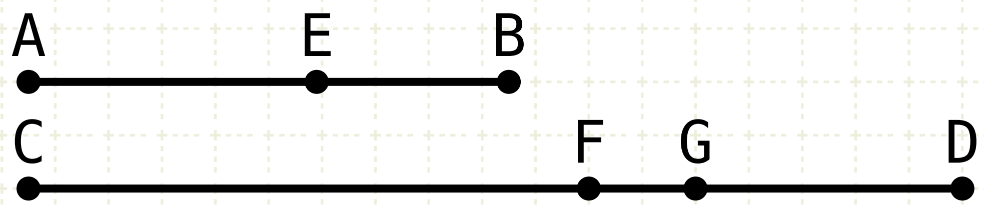

$AE:EB = CF:FD$

$CD:DF \neq AB:BE$

$GD < FD$

$AB:EB = CD:DG$

In other words

If AE, BE, CF, DF are proportional

… then COMPENDO (joint) ratios will also be equal

Proof by Contradiction

Assume CD is not to DF as AB is to BE

Then the ratio of AB to BE will be equal to CD to some magnitude not equal to DF

Assume that CD to GD is the correct ratio, where GD is less than FD

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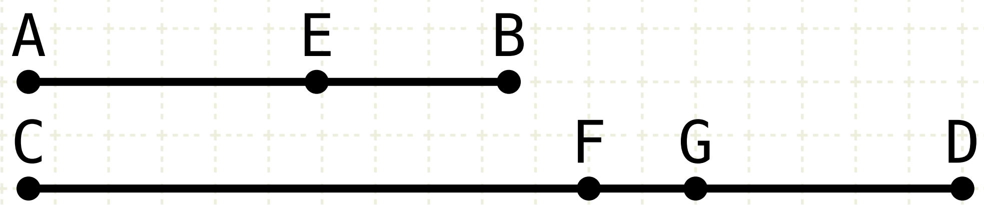

AE:EB = CF:FD

CD:DF ≠ AB:BE

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AB:EB = CD:DG

AE:EB = CG:GD

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If AE, BE, CF, DF are proportional

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Assume CD is not to DF as AB is to BE

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Assume that CD to GD is the correct ratio, where GD is less than FD

Since ratios that are proportional COMPRENDO will be proportional SEPARANDO, AE is to EB as CG is to GD (V·17)

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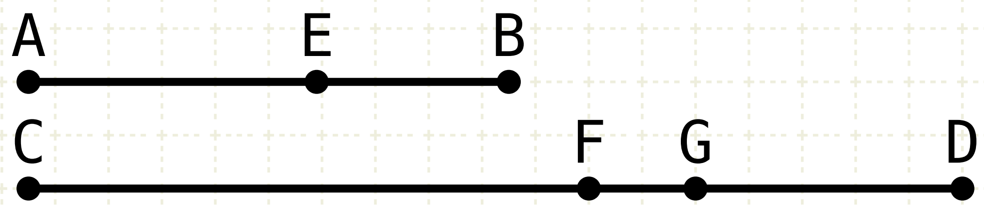

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Since ratios that are proportional COMPRENDO will be proportional SEPARANDO, AE is to EB as CG is to GD (V·17)

But AE is to EB is the same as CF to FD by definition so CF to FD is also the same as CG to GD (V·11)

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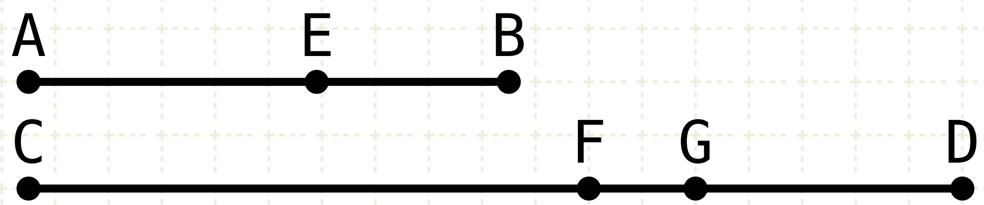

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AE:EB = CG:GD

CG:GD = CF:FD

CG > CF → GD > FD

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When two ratios are equal, if the first magnitude is larger than the third, then the second is larger than the fourth, which means…

… Since CG is greater than CF, GD is greater than FD (V·14)

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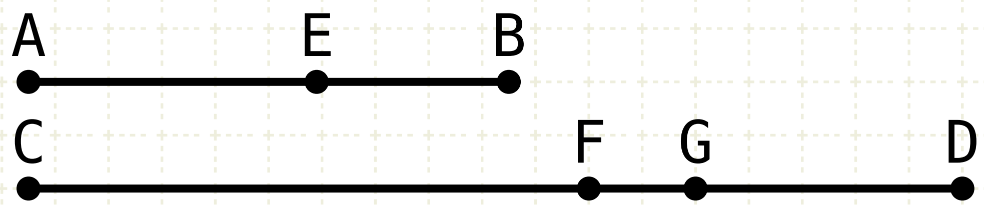

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When two ratios are equal, if the first magnitude is larger than the third, then the second is larger than the fourth, which means…

… Since CG is greater than CF, GD is greater than FD (V·14)

But that is impossible, so AB to EB cannot be the same as CG is to GD, where GD is less than FD

Similar arguments can show that the relationship cannot hold true even if GD is greater than FD

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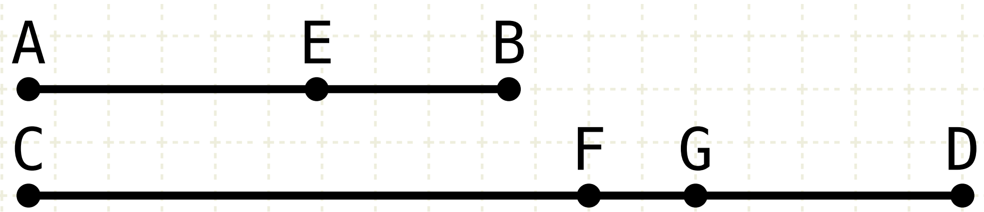

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$AE:EB = CG:GD$

$CG:GD = CF:FD$

$CG > CF \rightarrow GD > FD$

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