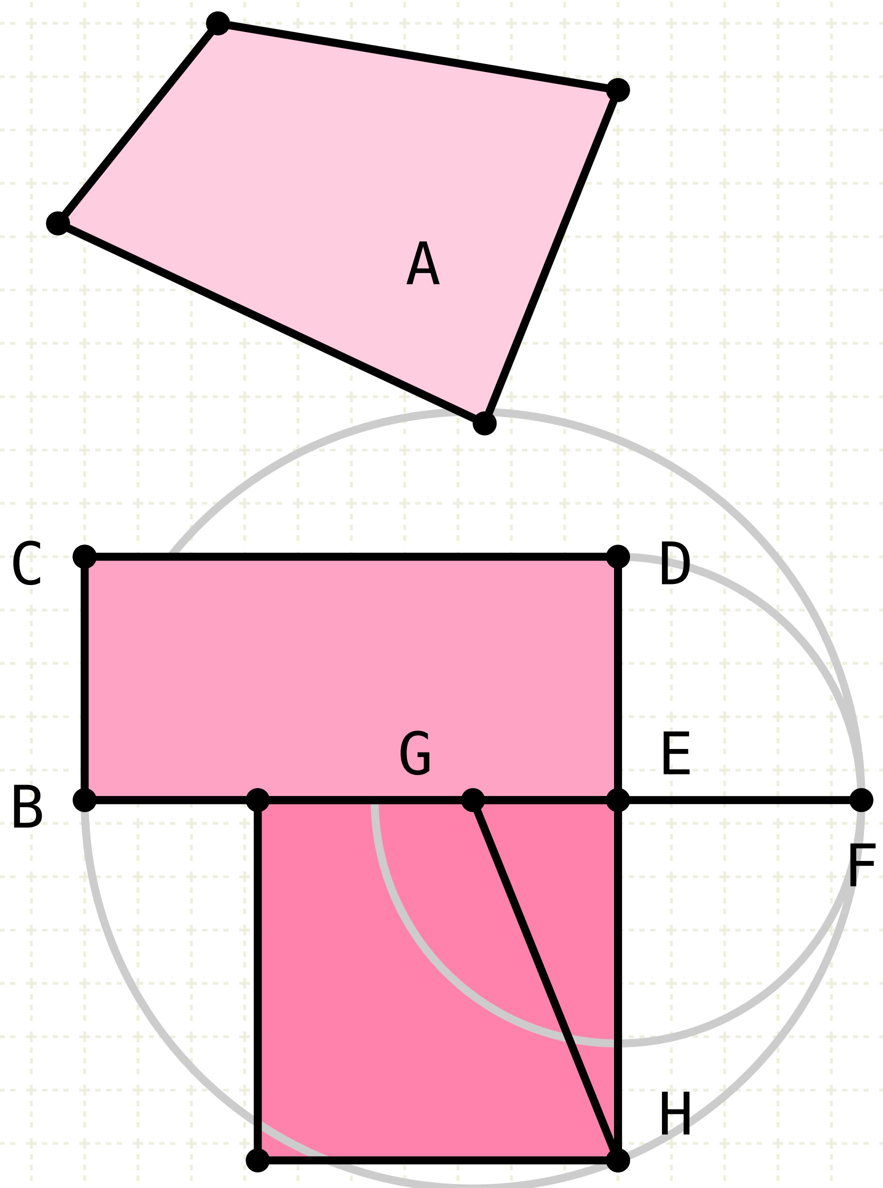
Euclid's Elements

Book II

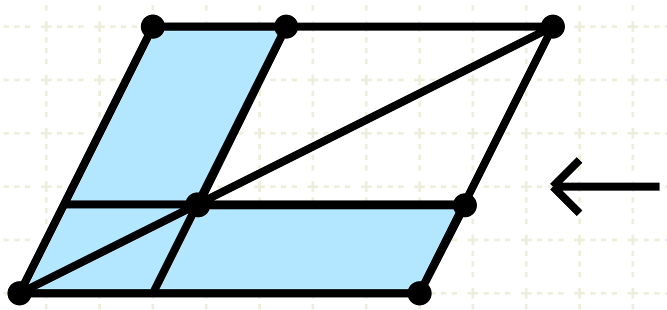


It is a remarkable fact in the history of geometry, that the Elements of Euclid, written two thousand years ago, are still regarded by many as the best introduction to the mathematical sciences.

Florian Cajori,
A History of Mathematics (1893)

Definitions:

Any rectangular parallelogram is said to be contained by the two straight lines containing the right angle.



And in any parallelogrammic area let any one whatever of the parallelograms about its diameter with the two complements be called a gnomon.

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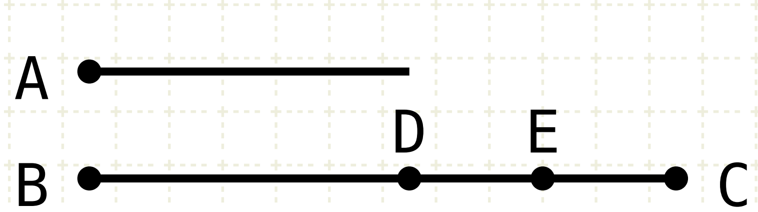
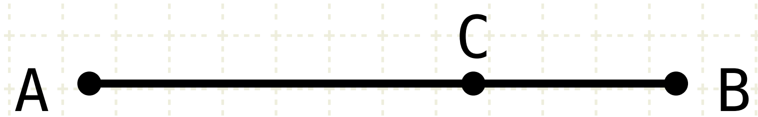
1 

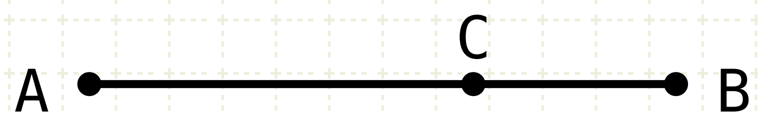
$A \cdot BC = A \cdot BD + A \cdot DE + A \cdot EC$

2 

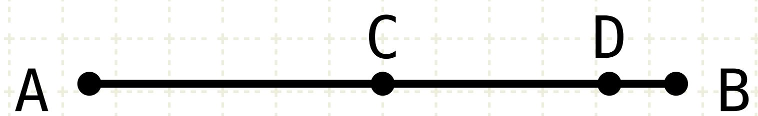
$AB^2 = AB \cdot AC + AB \cdot BC$

3 

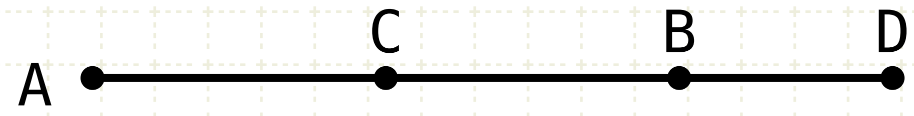
$\mathbf{AB \cdot CB = AC \cdot CB + CB^2}$

4 

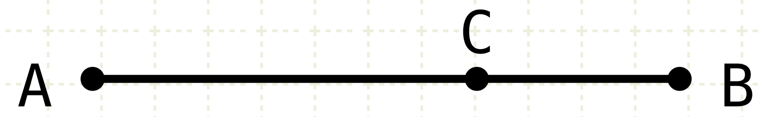
$AB^2 = AC^2 + CB^2 + 2 \cdot AC \cdot CB$

5 

$AD \cdot DB + CD^2 = CB^2$

6 

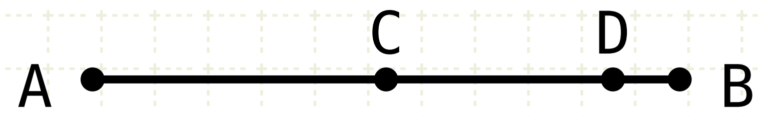
$AD \cdot DB + CB^2 = CD^2$

7 

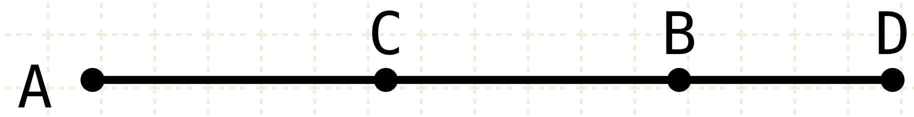
$AB^2 + BC^2 = AC^2 + 2 \cdot AB \cdot BC$

8 

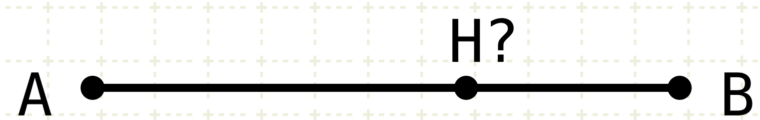
$4 \cdot AB \cdot BC + AC^2 = (AB+BC)^2$

9 

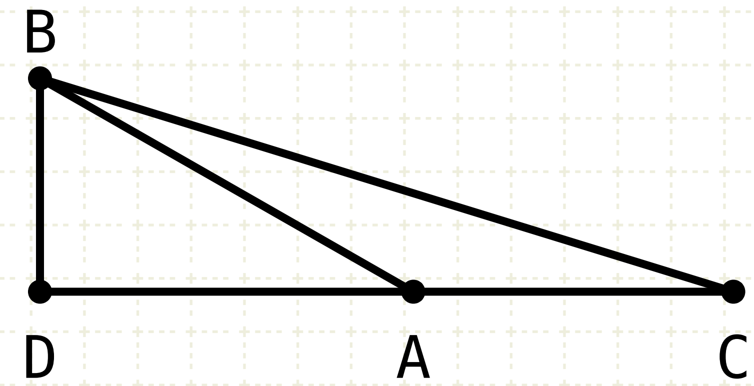
$AD^2 + DB^2 = 2 \cdot (AC^2 + CD^2)$

10 

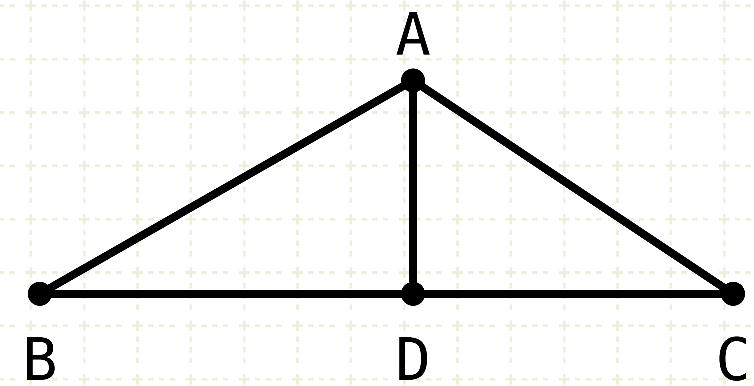
$AD^2 + DB^2 = 2 \cdot (AC^2 + CD^2)$

11 

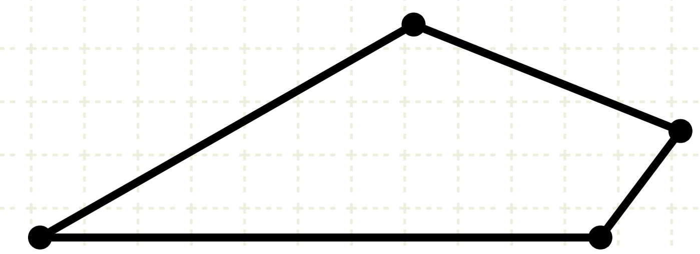
Find H. $AB \cdot BH = AH^2$

12 

Cosine Law. $BC^2 = AB^2+AC^2+2 \cdot AD \cdot AC$

13 

Cosine Law. $AC^2 = AB^2+BC^2-2 \cdot BD \cdot BC$

14 

Find square of polygon

Proposition 3 of Book II

If a straight line be cut at random, the rectangle contained by the whole and one of the segments is equal to the rectangle contained by the segments and the square on the aforesaid segment.

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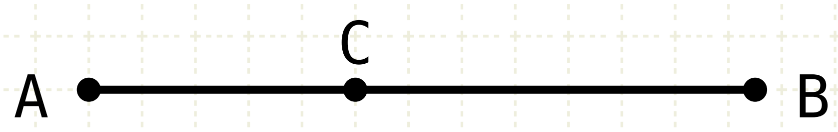

$$AB = AC + CB$$

In other words

Let AB be a straight line, arbitrarily cut at point C

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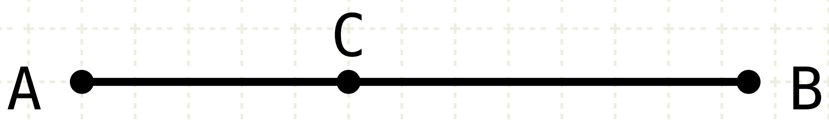

In other words

Let AB be a straight line, arbitrarily cut at point C

Then the area of the rectangle formed by line AB and CB is equal in area to the sum of the rectangles formed by line CB and AC, and line CB and CB

$$AB = AC + CB$$

$$CB \cdot AB = CB \cdot AC + CB \cdot CB$$

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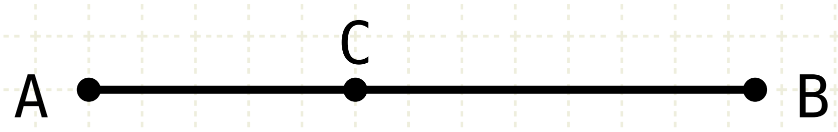

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Proof:

AB = AC + CB

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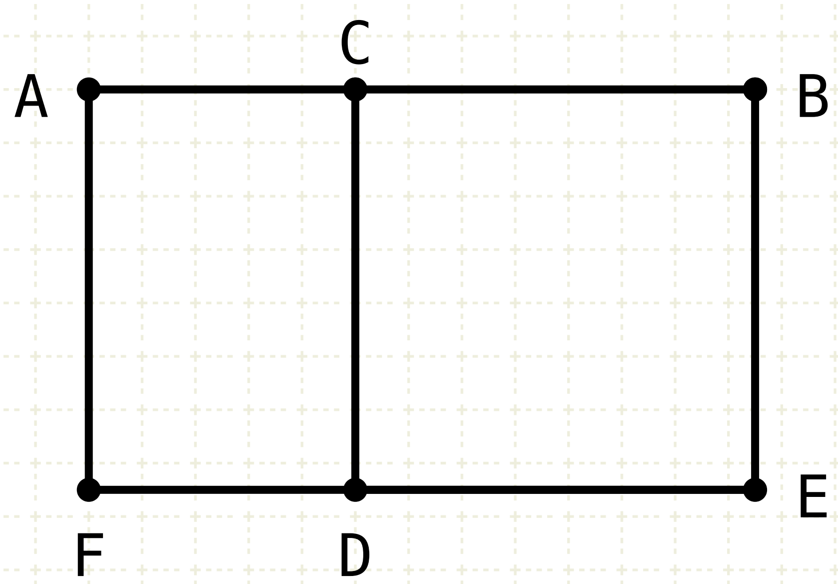

In other words

Let AB be a straight line, arbitrarily cut at point C

Then the area of the rectangle formed by line AB and CB is equal in area to the sum of the rectangles formed by line CB and AC, and line CB and CB

Proof:

Draw a square CDEB on the line CB (I·46) and draw a line AF parallel to either CD or BE (I·31). Extend DE to the point F

$AB = AC + CB$

$CB = BE = CD = AF$

Proposition 3 of Book II

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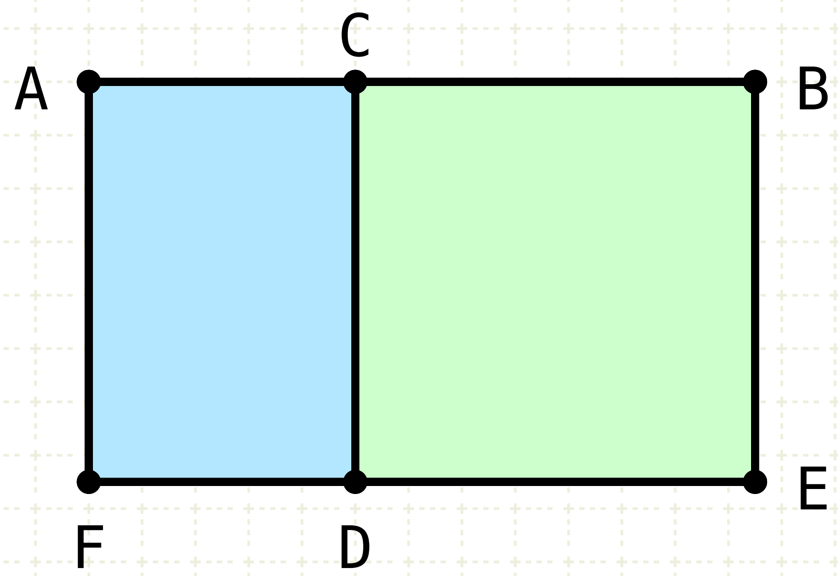

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Let AB be a straight line, arbitrarily cut at point C

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Proof:

Draw a square CDEB on the line CB (I·46) and draw a line AF parallel to either CD or BE (I·31). Extend DE to the point F

The rectangle AE is the sum of the rectangles AF and CE

AB = AC + CB

CB = BE = CD = AF

□AE = □AD + □CE

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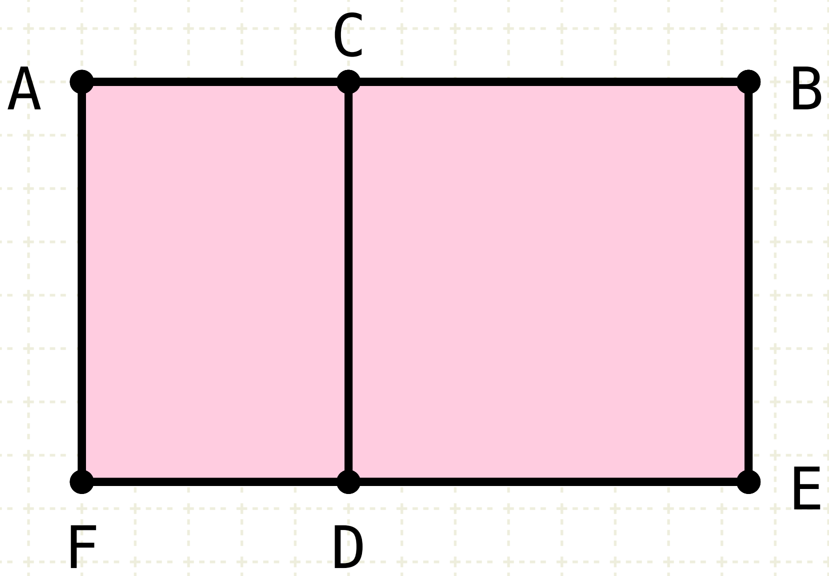

$AB = AC + CB$

$CB = BE = CD = AF$

$\square AE = \square AD + \square CE$

$\square AE = AF \cdot AB, \quad \therefore \square AE = CB \cdot AB$

In other words

Let AB be a straight line, arbitrarily cut at point C

Then the area of the rectangle formed by line AB and CB is equal in area to the sum of the rectangles formed by line CB and AC, and line CB and CB

Proof:

Draw a square CDEB on the line CB (I·46) and draw a line AF parallel to either CD or BE (I·31). Extend DE to the point F

The rectangle AE is the sum of the rectangles AF and CE

Since AF is equal in length to CB (I·34), the rectangle AE is equal to the rectangle contained by lines AB and CB

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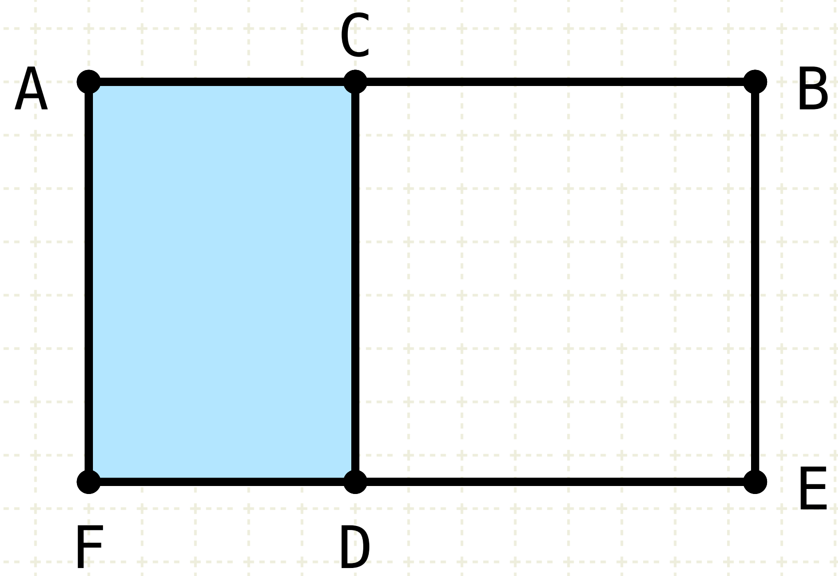

AB = AC + CB
CB = BE = CD = AF
□AE = □AD + □CE
□AE = AF·AB, ∴ □AE = CB·AB
□AD = AF·AC, ∴ □AD = CB·AC

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Let AB be a straight line, arbitrarily cut at point C

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Proof:

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The rectangle AE is the sum of the rectangles AF and CE

Since AF is equal in length to CB (I·34), the rectangle AE is equal to the rectangle contained by lines AB and CB

Similarly, the rectangle AD is equal to the rectangle contained by lines CB and AC

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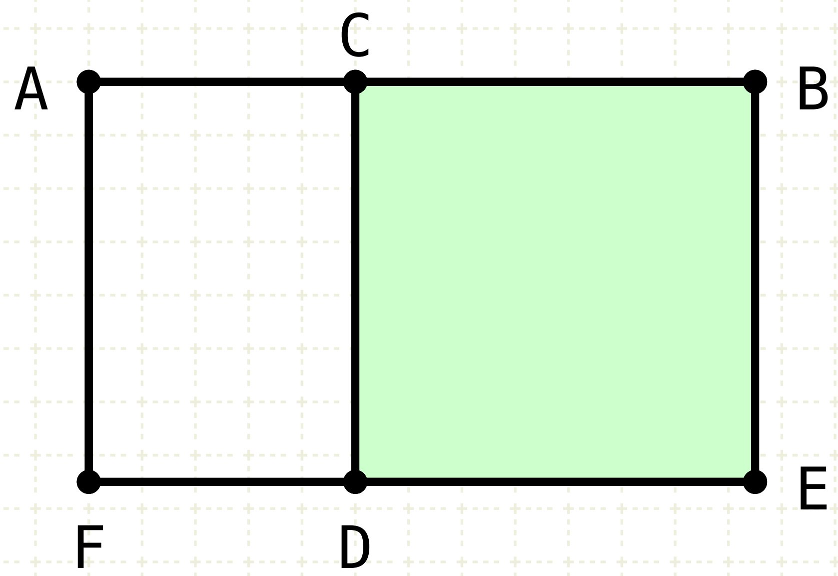

$AB = AC + CB$

$CB = BE = CD = AF$

$\square AE = \square AD + \square CE$

$\square AE = AF \cdot AB, \quad \therefore \square AE = CB \cdot AB$

$\square AD = AF \cdot AC, \quad \therefore \square AD = CB \cdot AC$

$\square CE = CD \cdot CB, \quad \therefore \square CE = CB \cdot CB$

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Let AB be a straight line, arbitrarily cut at point C

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Since CD equals AF (I·34), CE is equal to the rectangle contained by lines CB and CB

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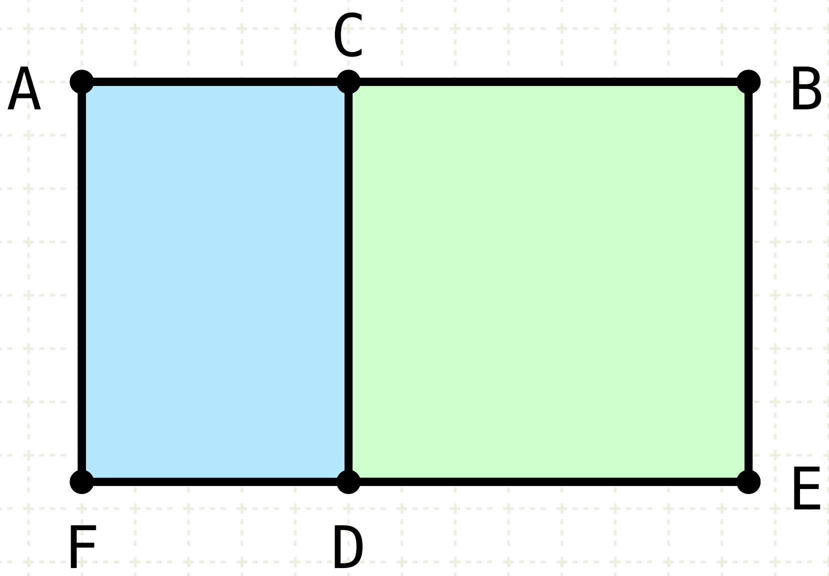

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$$\square AD = AF \cdot AC, \quad \therefore \square AD = CB \cdot AC$$

$$\square CE = CD \cdot CB, \quad \therefore \square CE = CB \cdot CB$$

$$CB \cdot AB = CB \cdot AC + CB \cdot CB$$

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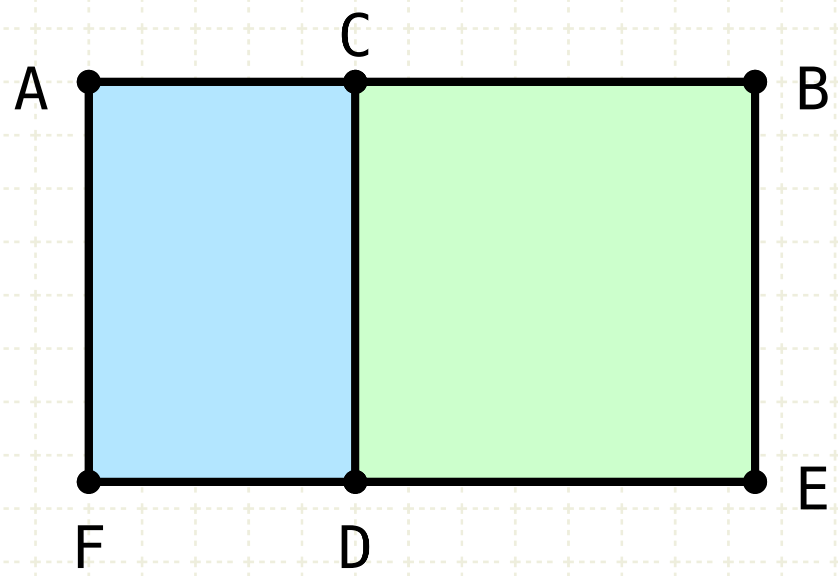
Similarly, the rectangle AD is equal to the rectangle contained by lines CB and AC

Since CD equals AF (I·34), CE is equal to the rectangle contained by lines CB and CB

Thus, the rectangle formed by CB,AB is equal to the sum of the rectangles formed by CB,AC and CB,CB

Proposition 3 of Book II

If a straight line be cut at random, the rectangle contained by the whole and one of the segments is equal to the rectangle contained by the segments and the square on the aforesaid segment.



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$$\square AD = AF \cdot AC, \quad \therefore \square AD = CB \cdot AC$$

$$\square CE = CD \cdot CB, \quad \therefore \square CE = CB \cdot CB$$

$$CB \cdot AB = CB \cdot AC + CB \cdot CB$$

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