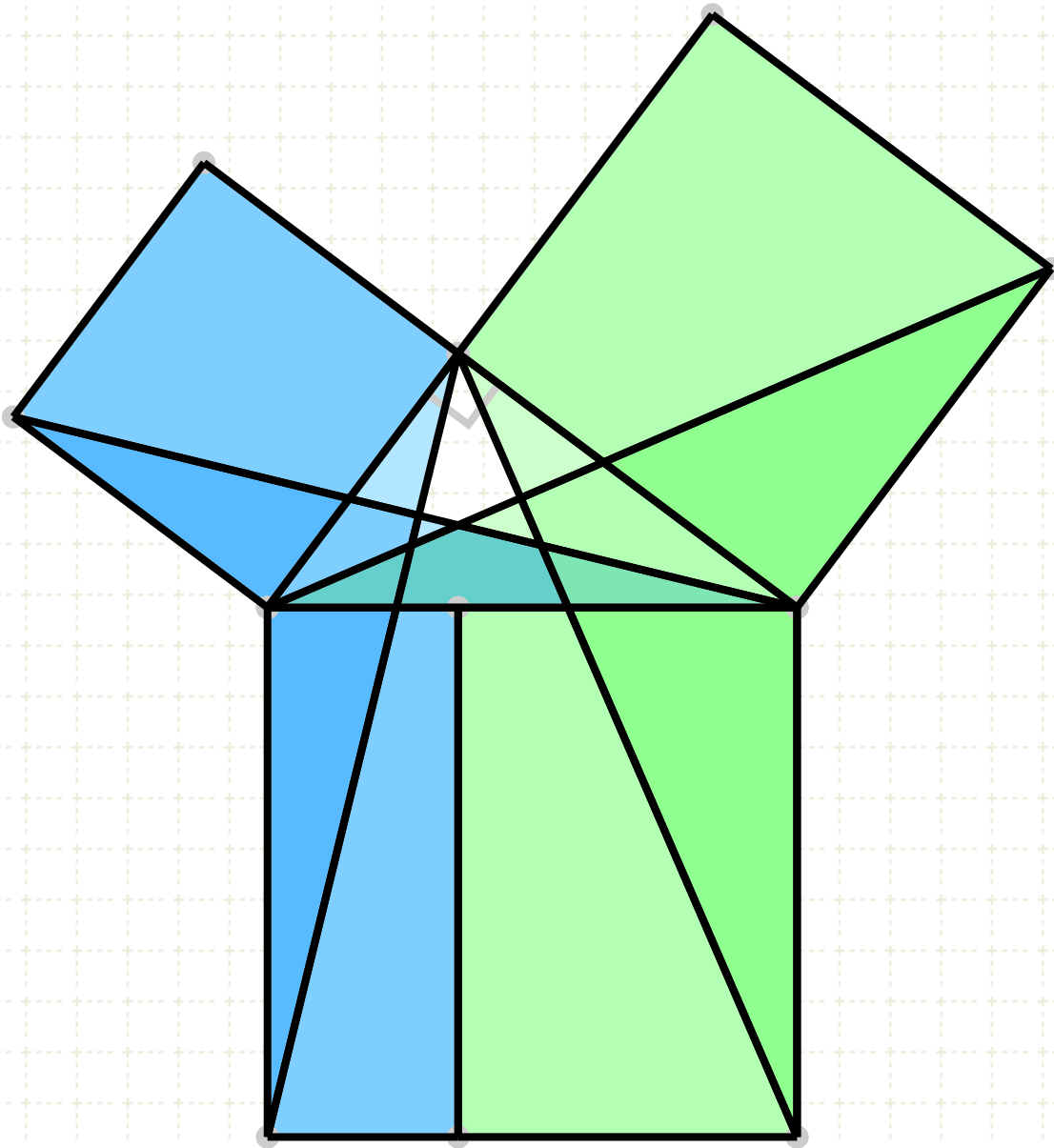


# Euclid's Elements

## Book I

*If Euclid did not kindle your youthful enthusiasm, you  
were not born to be a scientific thinker.*

Albert Einstein



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# Proposition 35 of Book I

Parallelograms which are on the same base and in the same parallels equal one another.



# Proposition 35 of Book I

Parallelograms which are on the same base and in the same parallels equal one another.

## In other words

Given two parallel lines

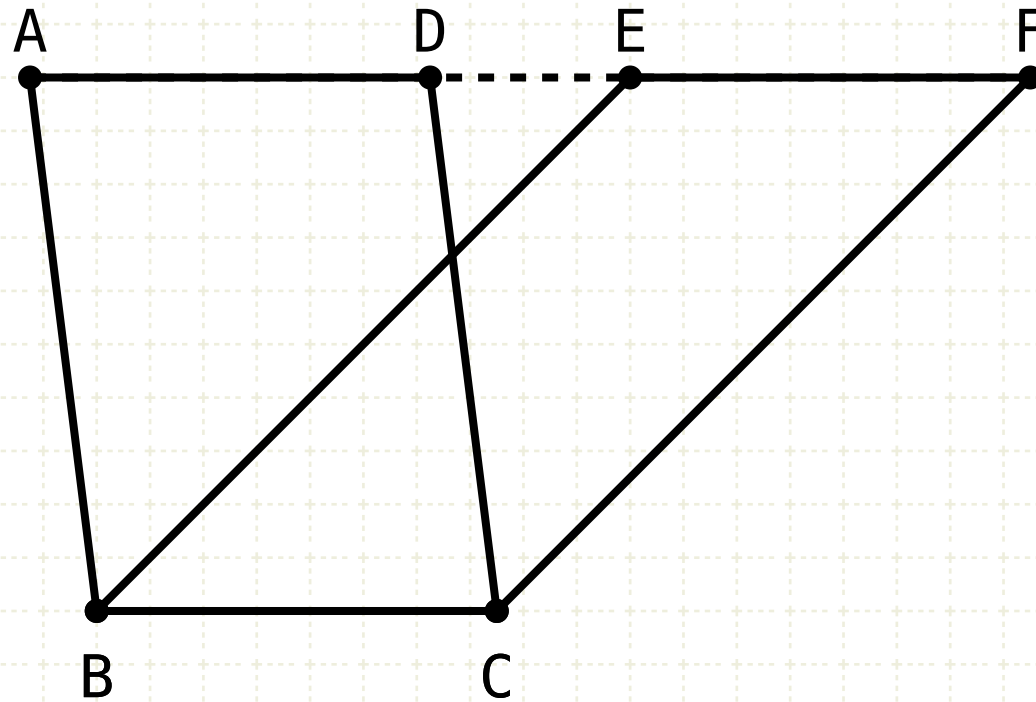
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# Proposition 35 of Book I

Parallelograms which are on the same base and in the same parallels equal one another.



$AD \parallel BC \parallel EF$   
 $AB \parallel DC$   
 $EB \parallel FC$

## In other words

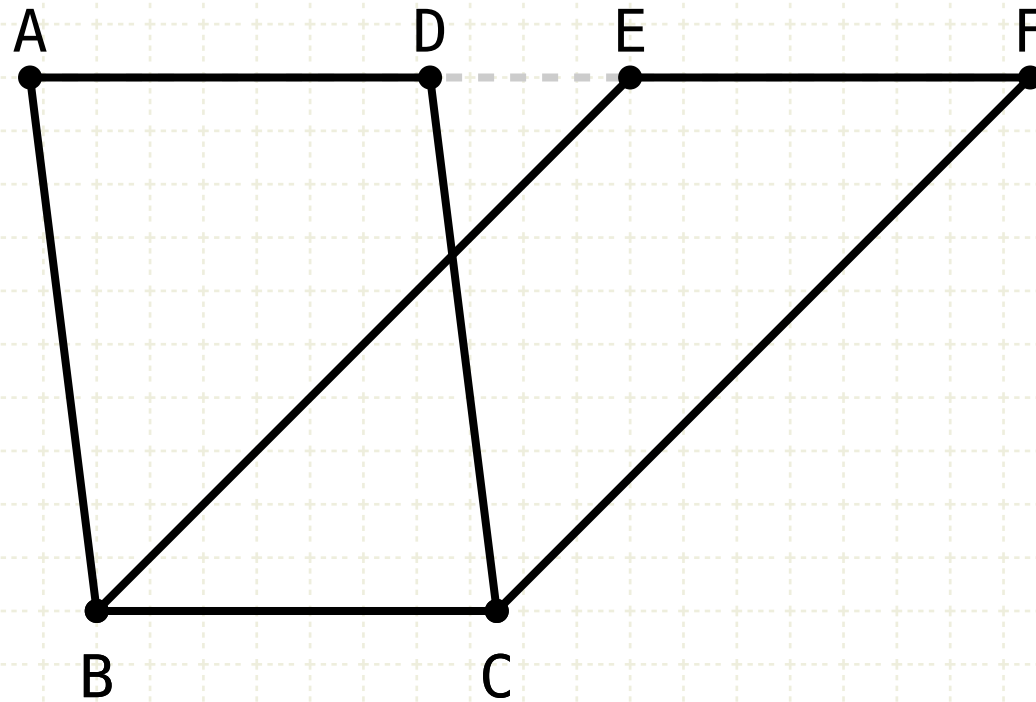
Given two parallel lines

Let ABCD and EBCF be parallelograms with the same base BC and the same height, (congruent with AF, a line parallel to the base)



# Proposition 35 of Book I

Parallelograms which are on the same base and in the same parallels equal one another.



$AD \parallel BC \parallel EF$

$AB \parallel DC$

$EB \parallel FC$

$\square ABCD = \square EBCF$

## In other words

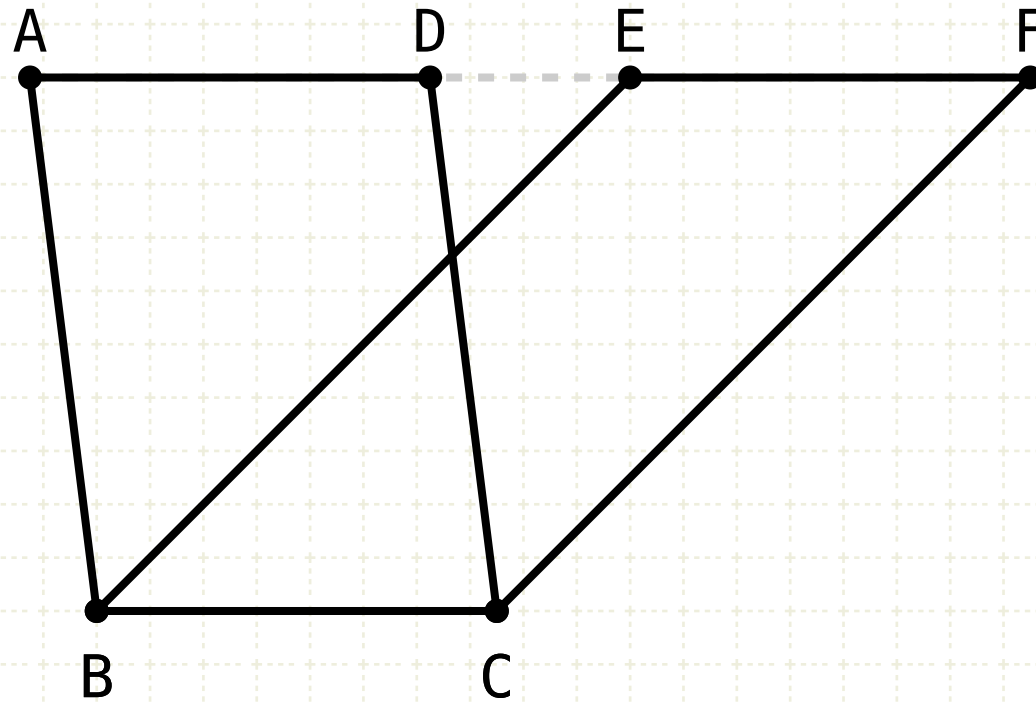
Given two parallel lines

Let ABCD and EBCF be parallelograms with the same base BC and the same height, (congruent with AF, a line parallel to the base)

The area ABCD is equal to EBCF

# Proposition 35 of Book I

Parallelograms which are on the same base and in the same parallels equal one another.



$AD \parallel BC \parallel EF$   
 $AB \parallel DC$   
 $EB \parallel FC$

## In other words

Given two parallel lines

Let ABCD and EBCF be parallelograms with the same base BC and the same height, (congruent with AF, a line parallel to the base)

The area ABCD is equal to EBCF

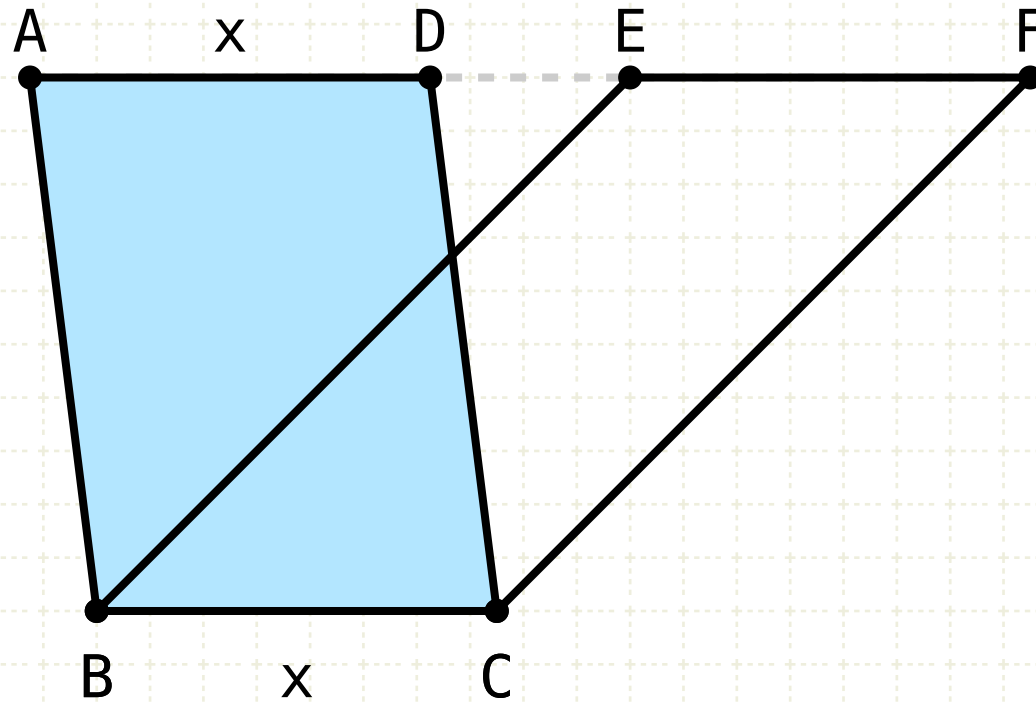
## Proof





# Proposition 35 of Book I

Parallelograms which are on the same base and in the same parallels equal one another.



$AD \parallel BC \parallel EF$   
 $AB \parallel DC$   
 $EB \parallel FC$

$$AD = BC = x$$

## In other words

Given two parallel lines

Let ABCD and EBCF be parallelograms with the same base BC and the same height, (congruent with AF, a line parallel to the base)

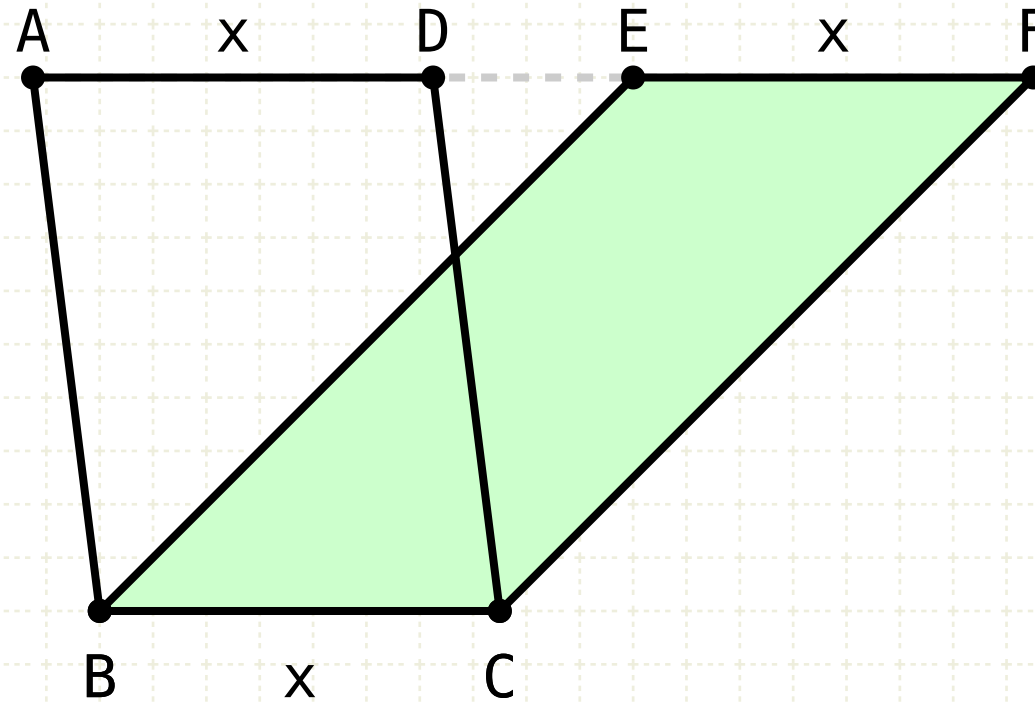
The area ABCD is equal to EBCF

## Proof

Since ABCD is a parallelogram, AD is equal to BC (I.34)

# Proposition 35 of Book I

Parallelograms which are on the same base and in the same parallels equal one another.



$AD \parallel BC \parallel EF$

$AB \parallel DC$

$EB \parallel FC$

$AD = BC = x$

$EF = BC = x$

## In other words

Given two parallel lines

Let ABCD and EBCF be parallelograms with the same base BC and the same height, (congruent with AF, a line parallel to the base)

The area ABCD is equal to EBCF

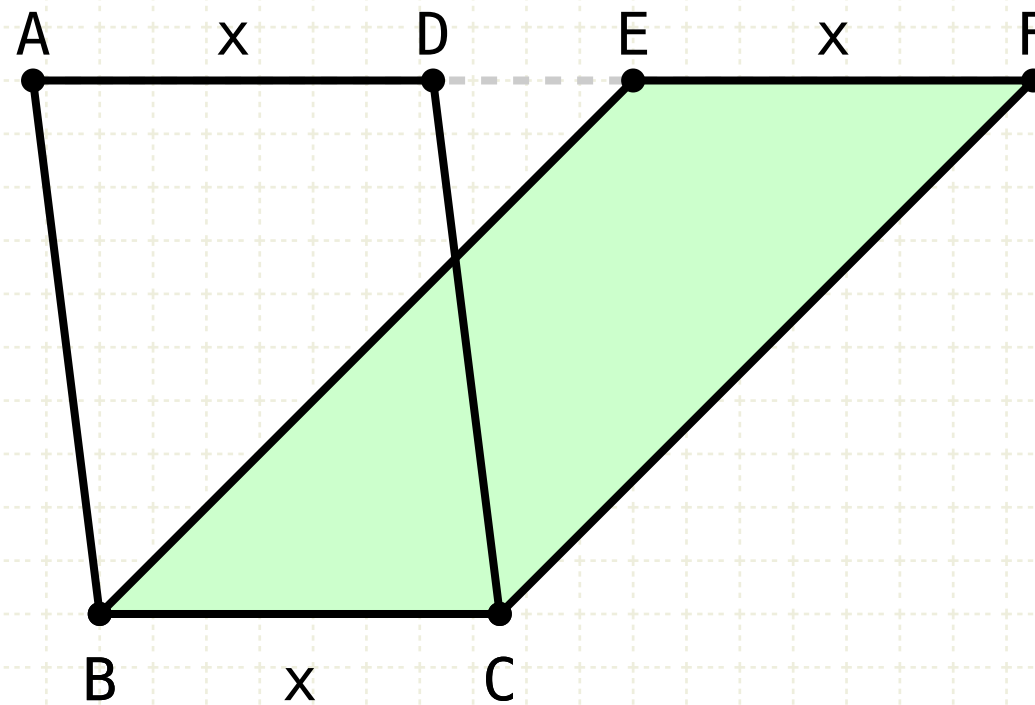
## Proof

Since ABCD is a parallelogram, AD is equal to BC (I·34)

Since EBCF is a parallelogram, EF is equal to BC (I·34)

# Proposition 35 of Book I

Parallelograms which are on the same base and in the same parallels equal one another.



$AD \parallel BC \parallel EF$   
 $AB \parallel DC$   
 $EB \parallel FC$

$AD = BC = x$   
 $EF = BC = x$   
 $AD = EF = x$

## In other words

Given two parallel lines

Let ABCD and EBCF be parallelograms with the same base BC and the same height, (congruent with AF, a line parallel to the base)

The area ABCD is equal to EBCF

## Proof

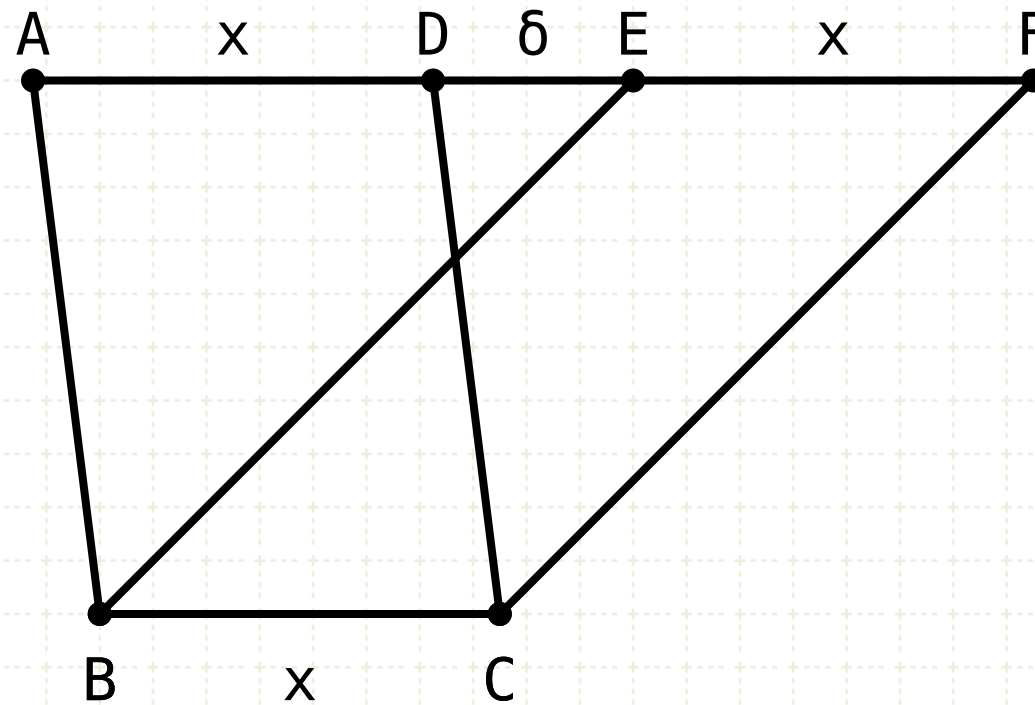
Since ABCD is a parallelogram, AD is equal to BC (I·34)

Since EBCF is a parallelogram, EF is equal to BC (I·34)

Hence AD is equal to EF

# Proposition 35 of Book I

Parallelograms which are on the same base and in the same parallels equal one another.



$$\begin{aligned} AD &\parallel BC \parallel EF \\ AB &\parallel DC \\ EB &\parallel FC \end{aligned}$$

$$\begin{aligned} AD &= BC = x \\ EF &= BC = x \\ AD &= EF = x \\ AE &= DF = x + \delta \end{aligned}$$

## In other words

Given two parallel lines

Let ABCD and EBCF be parallelograms with the same base BC and the same height, (congruent with AF, a line parallel to the base)

The area ABCD is equal to EBCF

## Proof

Since ABCD is a parallelogram, AD is equal to BC (I·34)

Since EBCF is a parallelogram, EF is equal to BC (I·34)

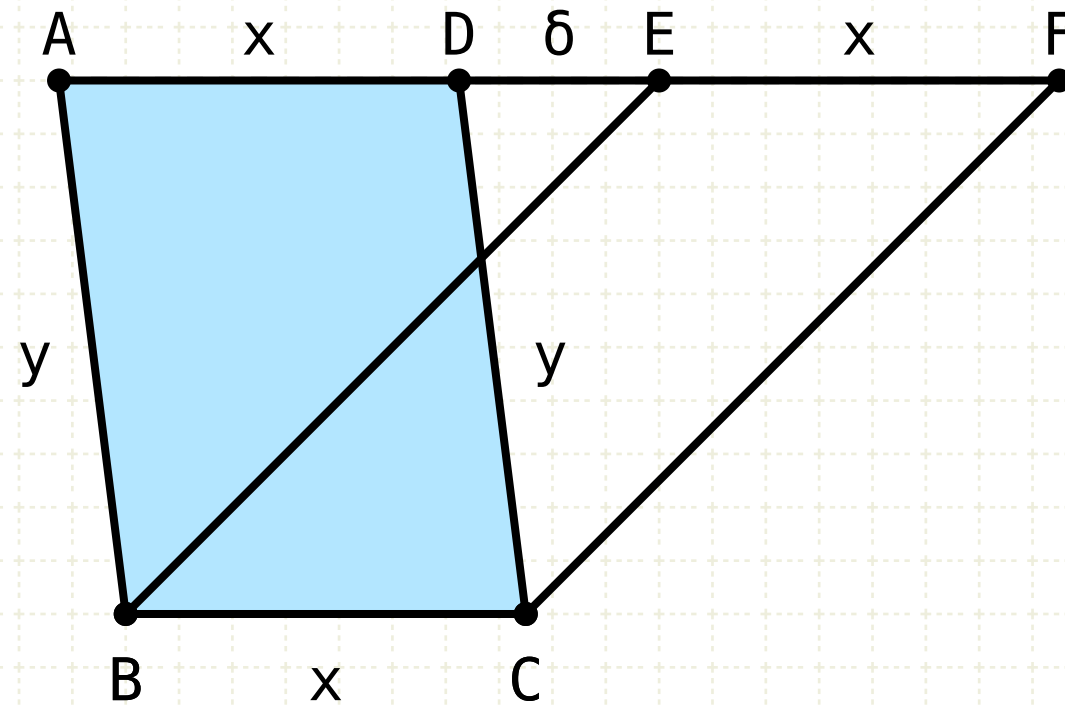
Hence AD is equal to EF

Add DE to both AD and EF, then AE is equal to DF



# Proposition 35 of Book I

Parallelograms which are on the same base and in the same parallels equal one another.



$$\begin{aligned} AD &\parallel BC \parallel EF \\ AB &\parallel DC \\ EB &\parallel FC \end{aligned}$$

$$\begin{aligned} AD &= BC = x \\ EF &= BC = x \\ AD &= EF = x \\ AE &= DF = x + \delta \\ AB &= DC = y \end{aligned}$$

## In other words

Given two parallel lines

Let ABCD and EBCF be parallelograms with the same base BC and the same height, (congruent with AF, a line parallel to the base)

The area ABCD is equal to EBCF

## Proof

Since ABCD is a parallelogram, AD is equal to BC (I-34)

Since EBCF is a parallelogram, EF is equal to BC (I-34)

Hence AD is equal to EF

Add DE to both AD and EF, then AE is equal to DF

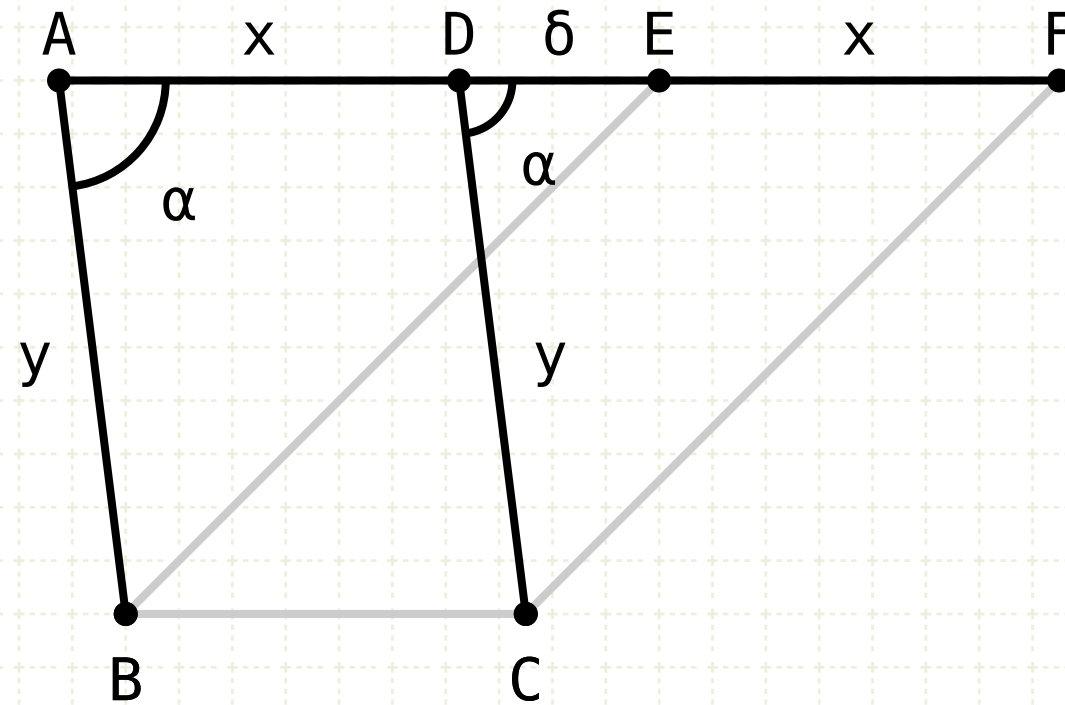
Since ABCD is a parallelogram, AB is equal to DC (I-34)





# Proposition 35 of Book I

Parallelograms which are on the same base and in the same parallels equal one another.



$AD \parallel BC \parallel EF$   
 $AB \parallel DC$   
 $EB \parallel FC$

$AD = BC = x$   
 $EF = BC = x$   
 $AD = EF = x$   
 $AE = DF = x + \delta$   
 $AB = DC = y$   
 $\angle EAB = \angle FDC$

## In other words

Given two parallel lines

Let ABCD and EBCF be parallelograms with the same base BC and the same height, (congruent with AF, a line parallel to the base)

The area ABCD is equal to EBCF

## Proof

Since ABCD is a parallelogram, AD is equal to BC (I·34)

Since EBCF is a parallelogram, EF is equal to BC (I·34)

Hence AD is equal to EF

Add DE to both AD and EF, then AE is equal to DF

Since ABCD is a parallelogram, AB is equal to DC (I·34)

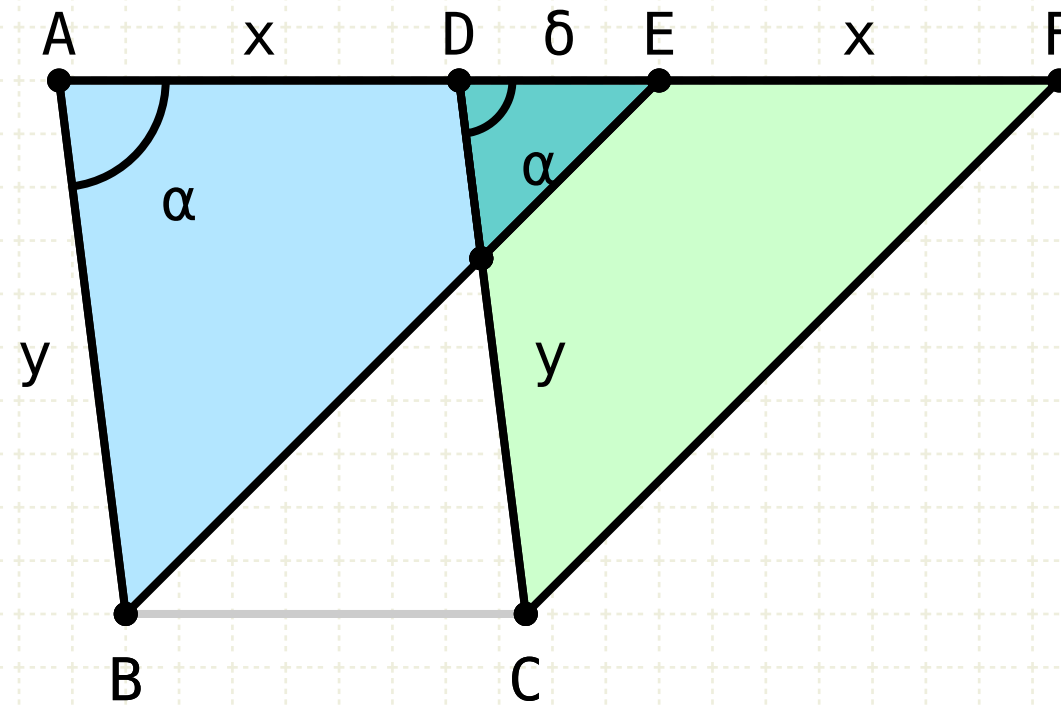
Angle DAB and FDC are equal (interior and exterior angles), since AF intersects two parallel lines AB and DC (I·29)





# Proposition 35 of Book I

Parallelograms which are on the same base and in the same parallels equal one another.



$$\begin{aligned} AD &\parallel BC \parallel EF \\ AB &\parallel DC \\ EB &\parallel FC \end{aligned}$$

$$\begin{aligned} AD &= BC = x \\ EF &= BC = x \\ AD &= EF = x \\ AE &= DF = x + \delta \\ AB &= DC = y \\ \angle EAB &= \angle FDC \\ \triangle ABE &= \triangle DCF \end{aligned}$$

## In other words

Given two parallel lines

Let ABCD and EBCF be parallelograms with the same base BC and the same height, (congruent with AF, a line parallel to the base)

The area ABCD is equal to EBCF

## Proof

Since ABCD is a parallelogram, AD is equal to BC (I·34)

Since EBCF is a parallelogram, EF is equal to BC (I·34)

Hence AD is equal to EF

Add DE to both AD and EF, then AE is equal to DF

Since ABCD is a parallelogram, AB is equal to DC (I·34)

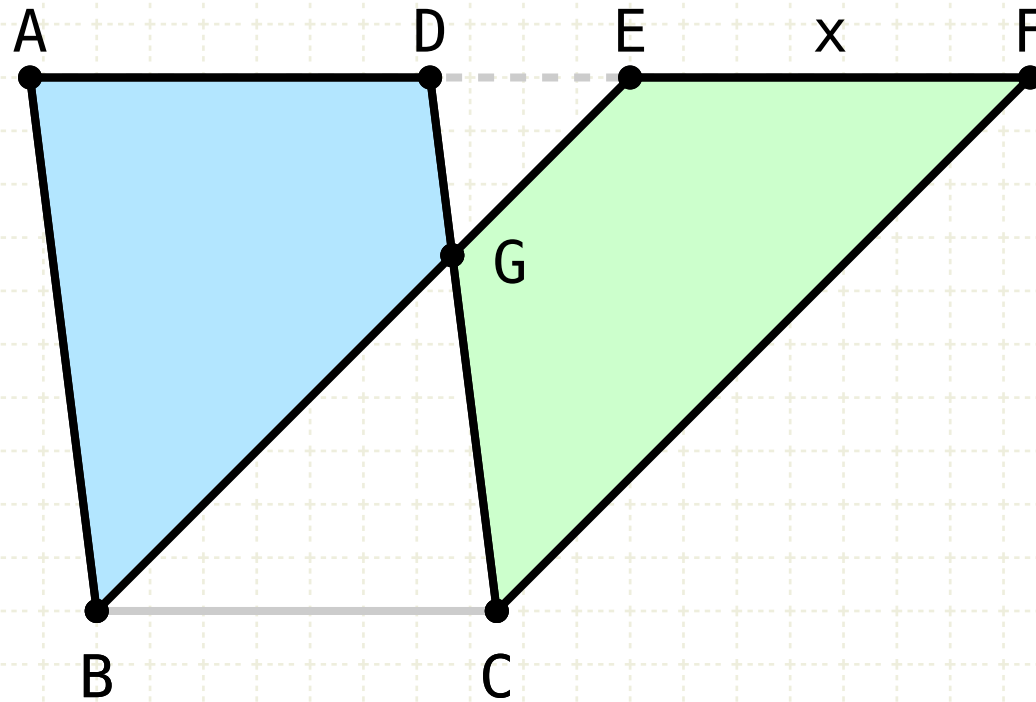
Angle DAB and FDC are equal (interior and exterior angles), since AF intersects two parallel lines AB and DC (I·29)

Triangles ABE and DFC are equivalent (I·4), thus equal in area



# Proposition 35 of Book I

Parallelograms which are on the same base and in the same parallels equal one another.



$$\begin{aligned} AD &\parallel BC \parallel EF \\ AB &\parallel DC \\ EB &\parallel FC \end{aligned}$$

$$\begin{aligned} AD &= BC = x \\ EF &= BC = x \\ AD &= EF = x \\ AE &= DF = x + \delta \\ AB &= DC = y \\ \angle EAB &= \angle FDC \\ \triangle ABE &= \triangle DCF \\ \triangle ABE - \triangle DGE &= \triangle DCF - \triangle DGE \\ \square ADGB &= \square EGCF \end{aligned}$$

## In other words

Given two parallel lines

Let ABCD and EBCF be parallelograms with the same base BC and the same height, (congruent with AF, a line parallel to the base)

The area ABCD is equal to EBCF

## Proof

Since ABCD is a parallelogram, AD is equal to BC (I·34)

Since EBCF is a parallelogram, EF is equal to BC (I·34)

Hence AD is equal to EF

Add DE to both AD and EF, then AE is equal to DF

Since ABCD is a parallelogram, AB is equal to DC (I·34)

Angle DAB and FDC are equal (interior and exterior angles), since AF intersects two parallel lines AB and DC (I·29)

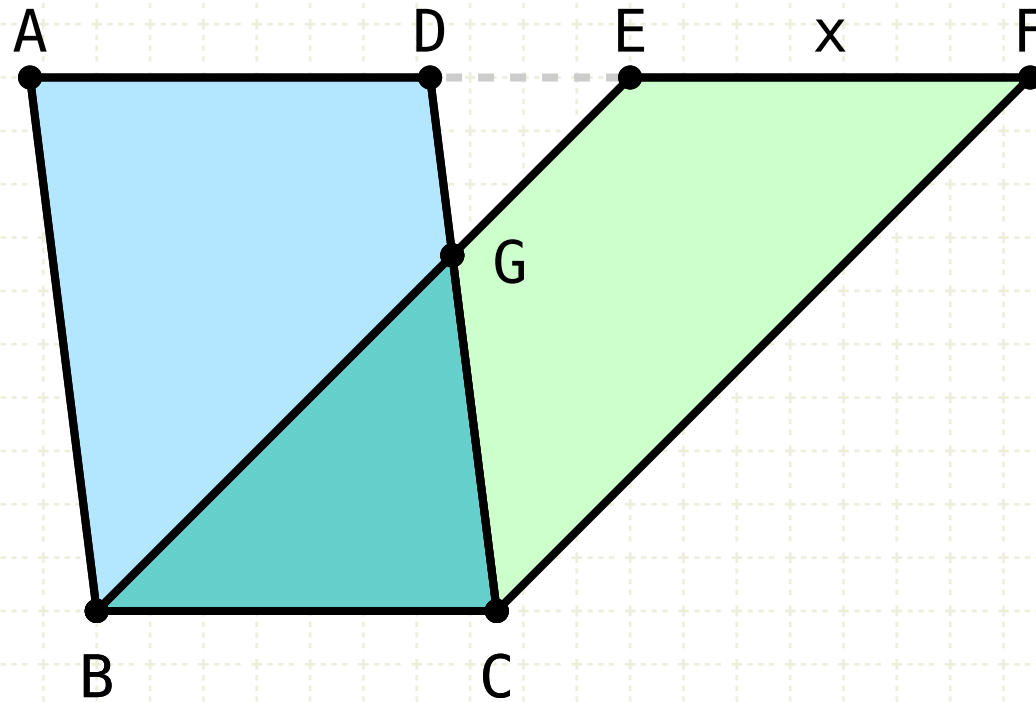
Triangles ABE and DFC are equivalent (I·4), thus equal in area

Remove EDG from ABE and DFC, the resulting trapezoids ADGB and EGCF are equal



# Proposition 35 of Book I

Parallelograms which are on the same base and in the same parallels equal one another.



$$\begin{aligned} AD &\parallel BC \parallel EF \\ AB &\parallel DC \\ EB &\parallel FC \end{aligned}$$

$$\begin{aligned} AD &= BC = x \\ EF &= BC = x \\ AD &= EF = x \\ AE &= DF = x + \delta \\ AB &= DC = y \end{aligned}$$

$$\angle EAB = \angle FDC$$

$$\triangle ABE = \triangle DCF$$

$$\triangle ABE - \triangle DGE = \triangle DCF - \triangle DGE$$

$$\square ADGB = \square EGCF$$

$$\square ADGB + \triangle BGC = \square EGCF + \triangle BGC$$

$$\square ABCD = \square EBCF$$

## In other words

Given two parallel lines

Let ABCD and EBCF be parallelograms with the same base BC and the same height, (congruent with AF, a line parallel to the base)

The area ABCD is equal to EBCF

## Proof

Since ABCD is a parallelogram, AD is equal to BC (I·34)

Since EBCF is a parallelogram, EF is equal to BC (I·34)

Hence AD is equal to EF

Add DE to both AD and EF, then AE is equal to DF

Since ABCD is a parallelogram, AB is equal to DC (I·34)

Angle DAB and FDC are equal (interior and exterior angles), since AF intersects two parallel lines AB and DC (I·29)

Triangles ABE and DFC are equivalent (I·4), thus equal in area

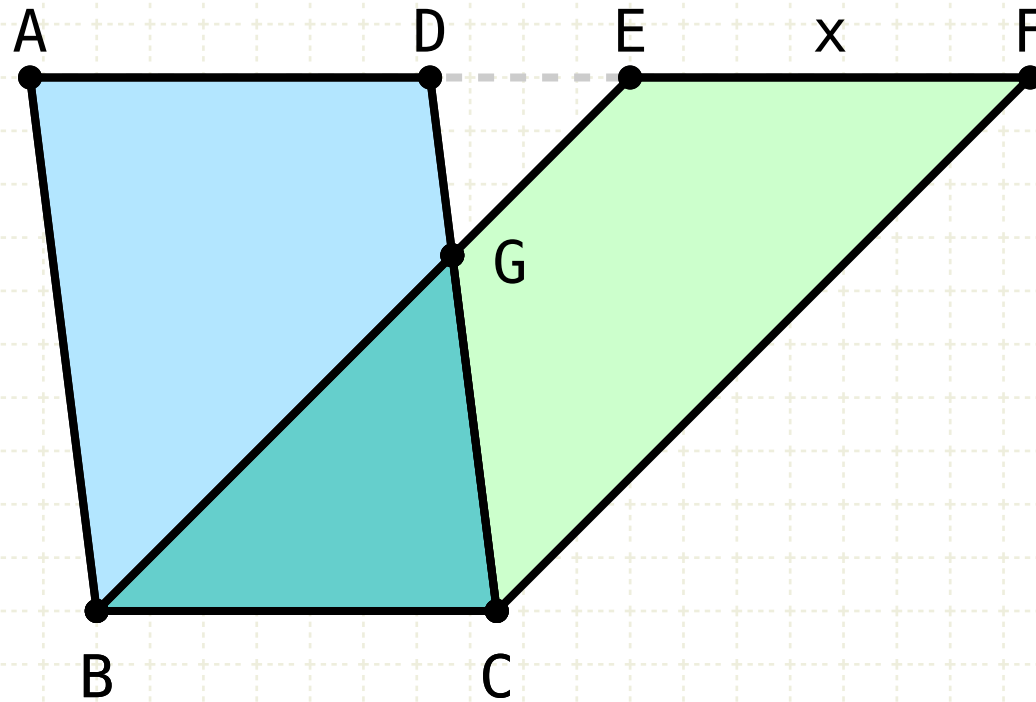
Remove EDG from ABE and DFC, the resulting trapezoids ADGB and EGCF are equal

Add BGC to both trapezoids, and the result is that the parallelograms ABCD and EBCF are equal



# Proposition 35 of Book I

Parallelograms which are on the same base and in the same parallels equal one another.



$$\begin{aligned} AD &\parallel BC \parallel EF \\ AB &\parallel DC \\ EB &\parallel FC \end{aligned}$$

$$\begin{aligned} AD &= BC = x \\ EF &= BC = x \\ AD &= EF = x \\ AE &= DF = x + \delta \\ AB &= DC = y \\ \angle EAB &= \angle FDC \\ \triangle ABE &= \triangle DCF \\ \triangle ABE - \triangle DGE &= \triangle DCF - \triangle DGE \\ \square ADGB &= \square EGCF \\ \square ADGB + \triangle BGC &= \square EGCF + \triangle BGC \end{aligned}$$

$$\square ABCD = \square EBCF$$

## In other words

Given two parallel lines

Let ABCD and EBCF be parallelograms with the same base BC and the same height, (congruent with AF, a line parallel to the base)

The area ABCD is equal to EBCF

## Proof

Since ABCD is a parallelogram, AD is equal to BC (I·34)

Since EBCF is a parallelogram, EF is equal to BC (I·34)

Hence AD is equal to EF

Add DE to both AD and EF, then AE is equal to DF

Since ABCD is a parallelogram, AB is equal to DC (I·34)

Angle DAB and FDC are equal (interior and exterior angles), since AF intersects two parallel lines AB and DC (I·29)

Triangles ABE and DFC are equivalent (I·4), thus equal in area

Remove EDG from ABE and DFC, the resulting trapezoids ADGB and EGCF are equal

Add BGC to both trapezoids, and the result is that the parallelograms ABCD and EBCF are equal





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