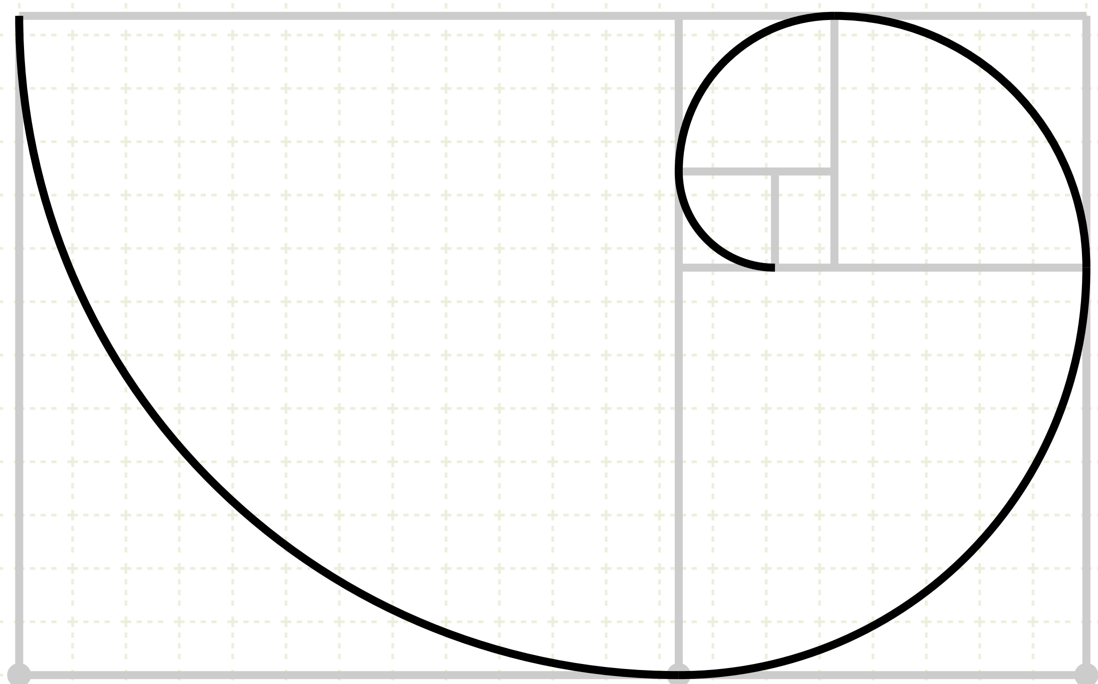
Euclid's Elements

Book VI



One can state, without exaggeration, that the observation of and the search for similarities and differences are the basis of all human knowledge.

Alfred Nobel

Table of Contents, Chapter 6

Table of Contents, Chapter 3

Proposition 1 of Book VI

Triangles and parallelograms which are under the same height are to one another as their bases

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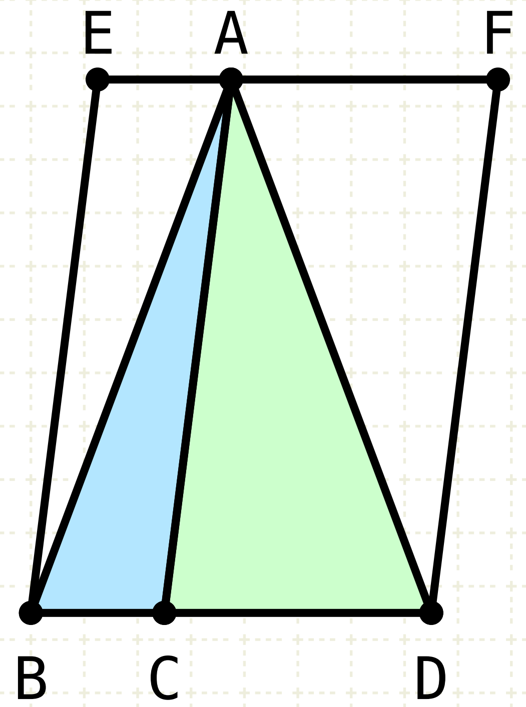
Triangles and parallelograms which are under the same height are to one another as their bases



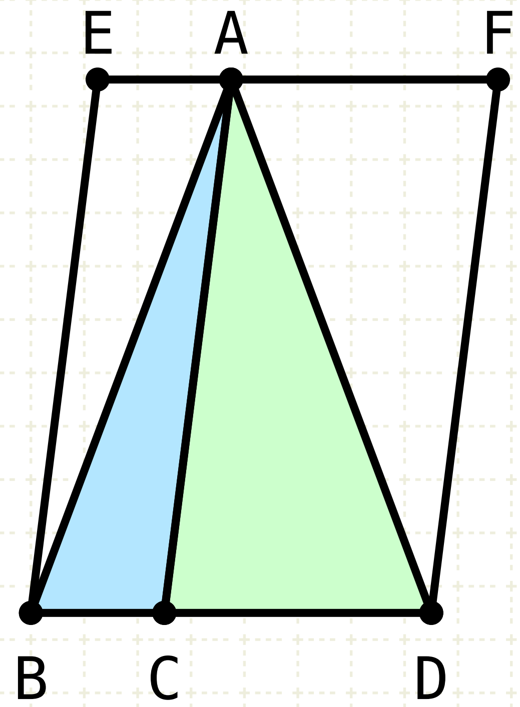
In other words

If we have two triangles ABC and ACD, or two parallelograms EC and CF, with the same height, then the ratio of BC to CD is the same as the ratios of the triangles, and parallelograms respectively

$$BC:CD = \Delta ABC:\Delta ACD = \square EC:\square CF$$

Proposition 1 of Book VI

Triangles and parallelograms which are under the same height are to one another as their bases



Proof - Triangles

Proposition 1 of Book VI

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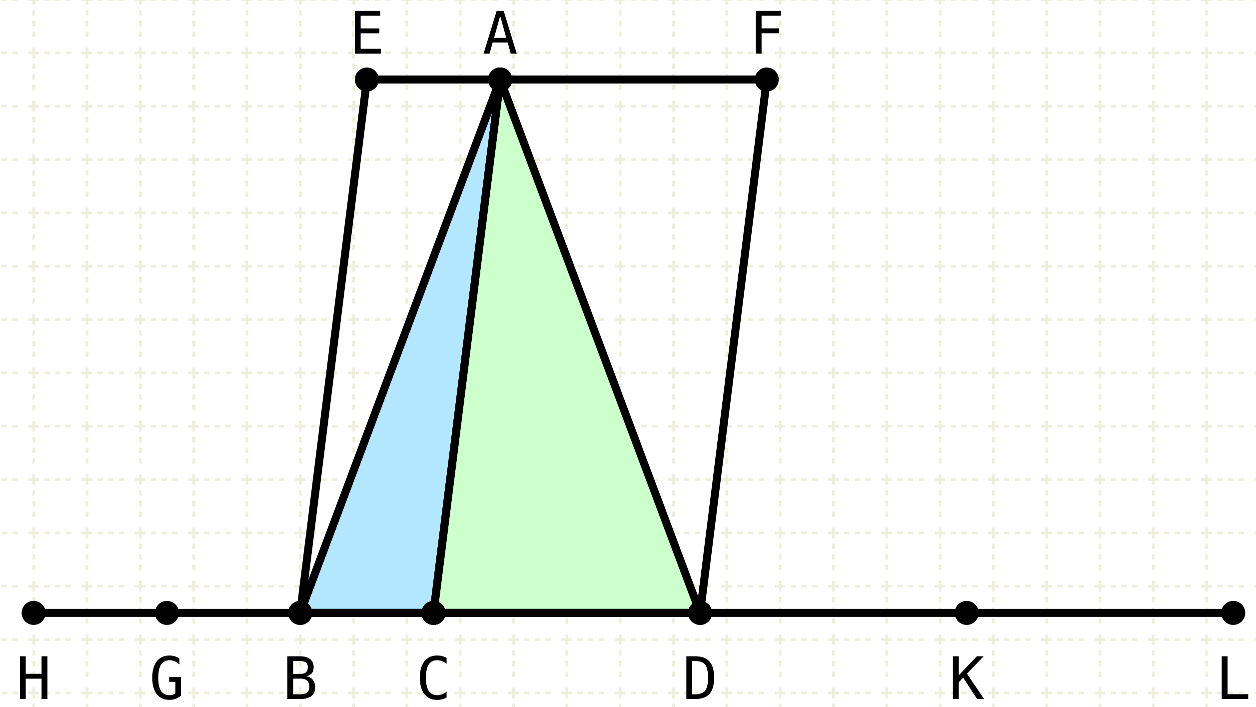

HG = GB = BC
CD = DK = KL

Proof - Triangles

Extend the line BD to HL, such that HB is composed of any number of lines equal to BC, and that DL is composed of any number of lines equal to CD

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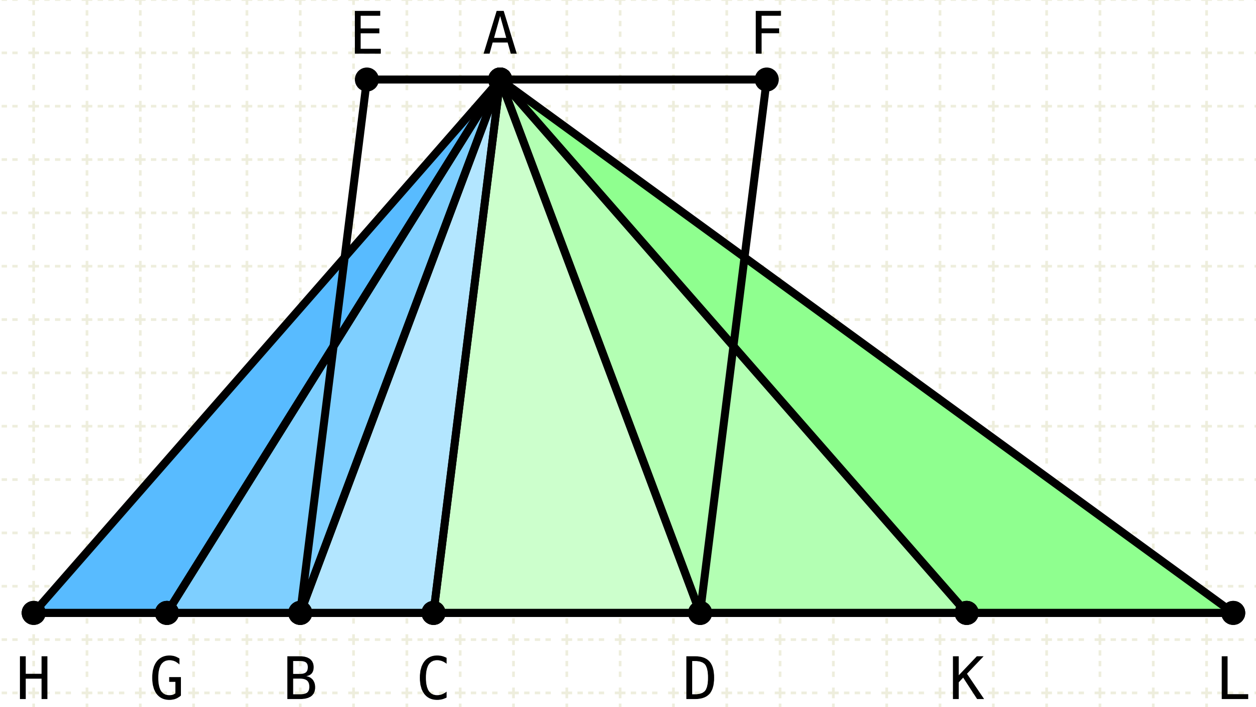

HG = GB = BC
CD = DK = KL

Proof - Triangles

Extend the line BD to HL, such that HB is composed of any number of lines equal to BC, and that DL is composed of any number of lines equal to CD

Draw the triangles AHG, AGB, and ADK and AKL

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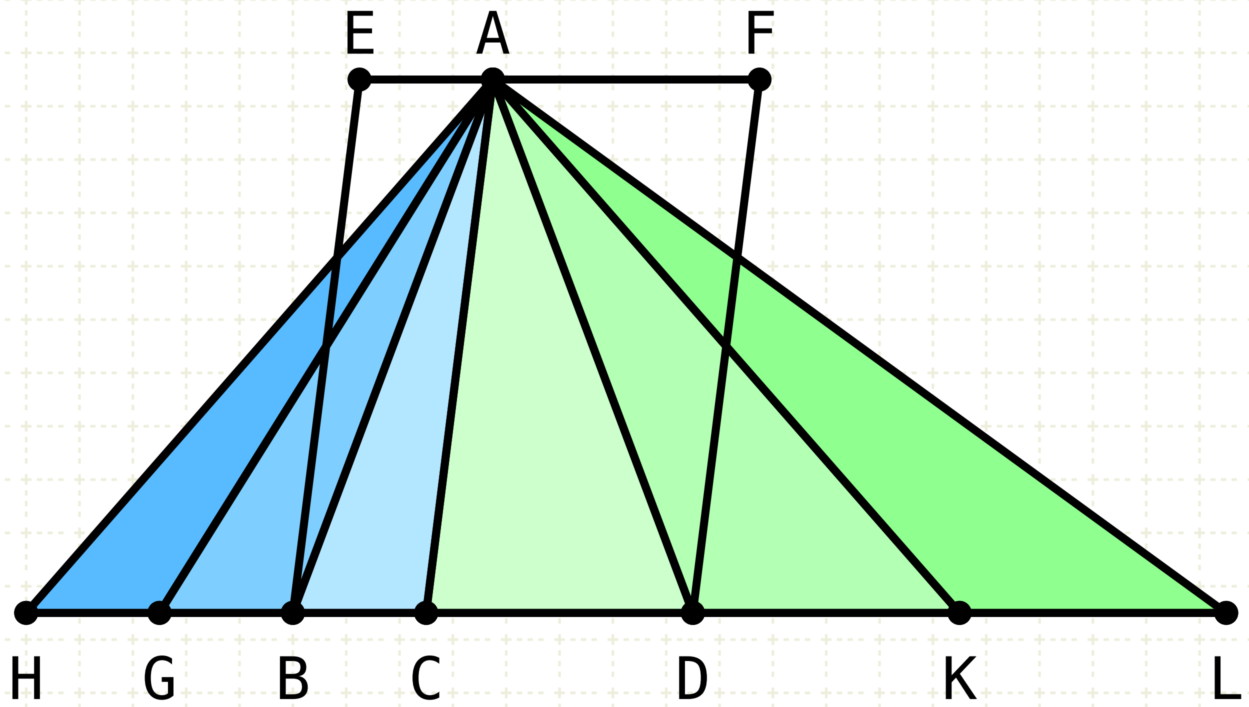

Proof - Triangles

Extend the line BD to HL, such that HB is composed of any number of lines equal to BC, and that DL is composed of any number of lines equal to CD

Draw the triangles AHG, AGB, and ADK and AKL

Since the bases are equal, triangles ABC, AGB and AHG are equal, and the triangles ACD, ADK and AKL are equal (I·38)

HG = GB = BC
CD = DK = KL
ΔABC = ΔAGB = ΔAHG
ΔACD = ΔADK = ΔAKL

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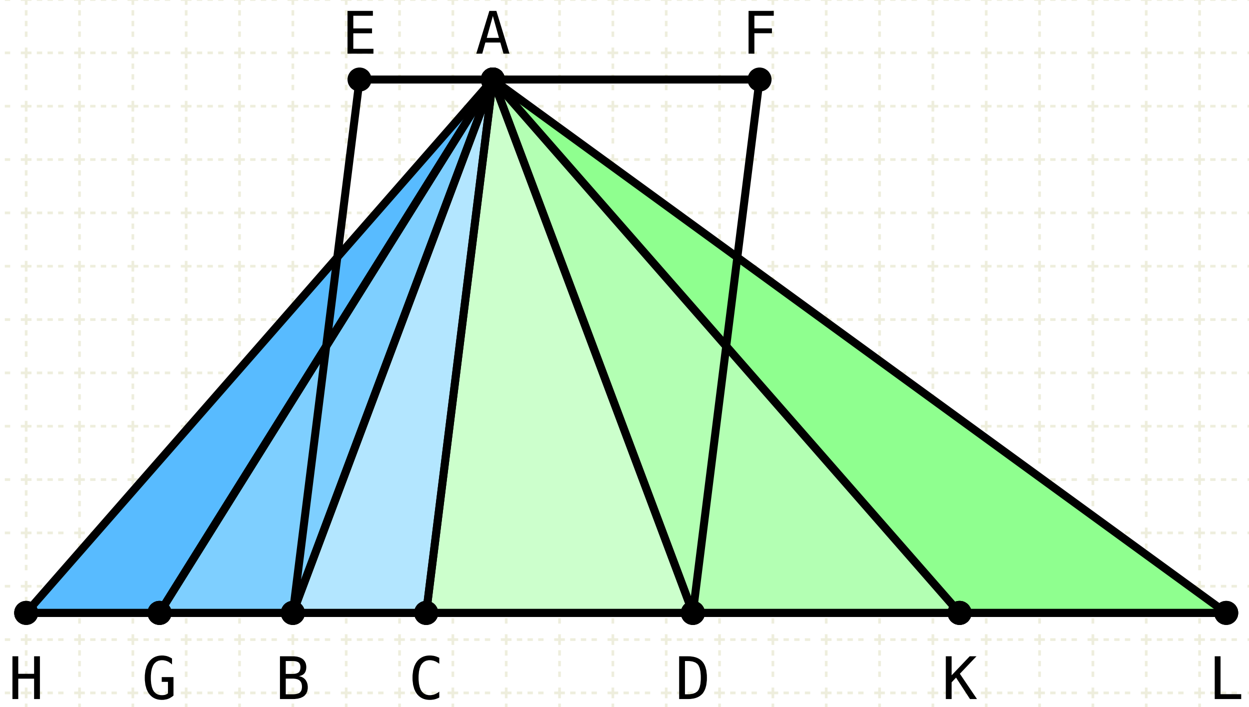

$$HG = GB = BC$$

$$CD = DK = KL$$

$$\Delta ABC = \Delta AGB = \Delta AHG$$

$$\Delta ACD = \Delta ADK = \Delta AKL$$

$$\Delta AHC = \Delta ABC + \Delta AGB + \Delta AHG = n \cdot \Delta ABC$$

$$HC = BC + GB + HG = n \cdot BC$$

Proof - Triangles

Extend the line BD to HL, such that HB is composed of any number of lines equal to BC, and that DL is composed of any number of lines equal to CD

Draw the triangles AHG, AGB, and ADK and AKL

Since the bases are equal, triangles ABC, AGB and AHG are equal, and the triangles ACD, ADK and AKL are equal (I·38)

Therefore, the triangle AHC is the same multiple of ABC as the line HC is to BC

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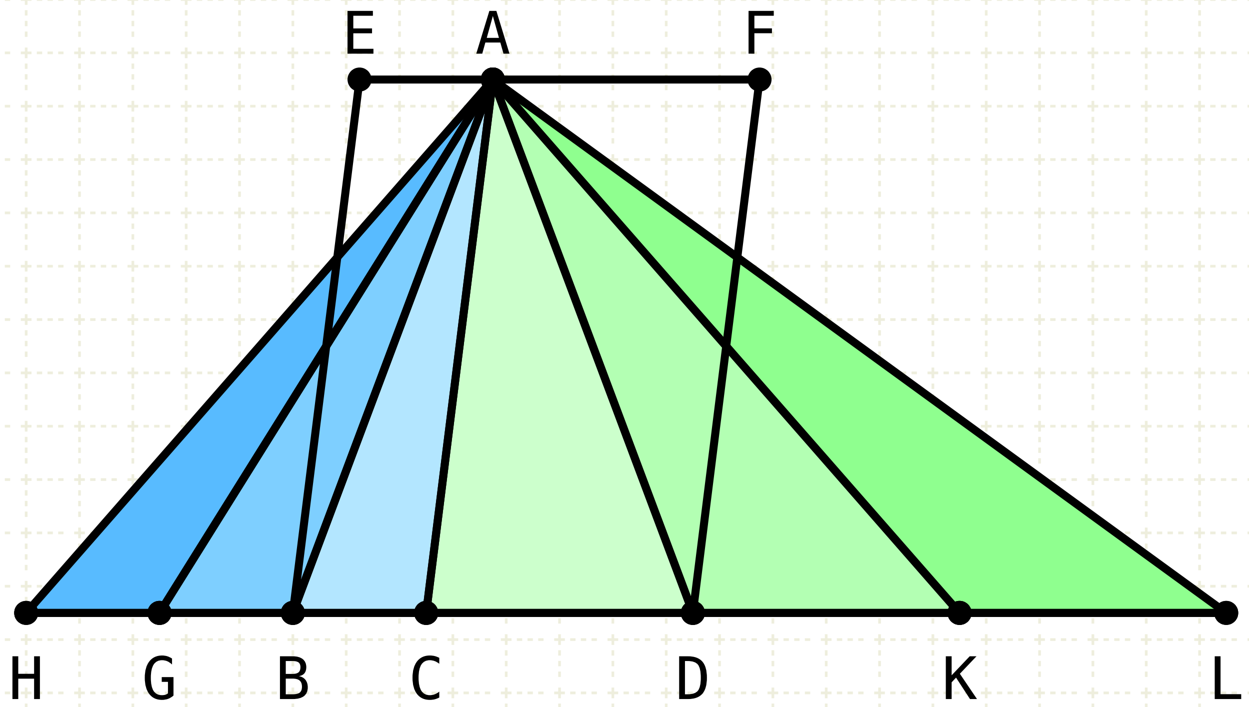

$$HG = GB = BC$$

$$CD = DK = KL$$

$$\Delta ABC = \Delta AGB = \Delta AHG$$

$$\Delta ACD = \Delta ADK = \Delta AKL$$

$$\Delta AHC = \Delta ABC + \Delta AGB + \Delta AHG = n \cdot \Delta ABC$$

$$HC = BC + GB + HG = n \cdot BC$$

$$\Delta ACL = \Delta ACD + \Delta ADK + \Delta AKL = m \cdot \Delta ACD$$

$$CL = CD + DK + KL = m \cdot CD$$

Proof - Triangles

Extend the line BD to HL, such that HB is composed of any number of lines equal to BC, and that DL is composed of any number of lines equal to CD

Draw the triangles AHG, AGB, and ADK and AKL

Since the bases are equal, triangles ABC, AGB and AHG are equal, and the triangles ACD, ADK and AKL are equal (I·38)

Therefore, the triangle AHC is the same multiple of ABC as the line HC is to BC

Similarly, the triangle ACL is the same multiple of ACD as the line CD is to CL

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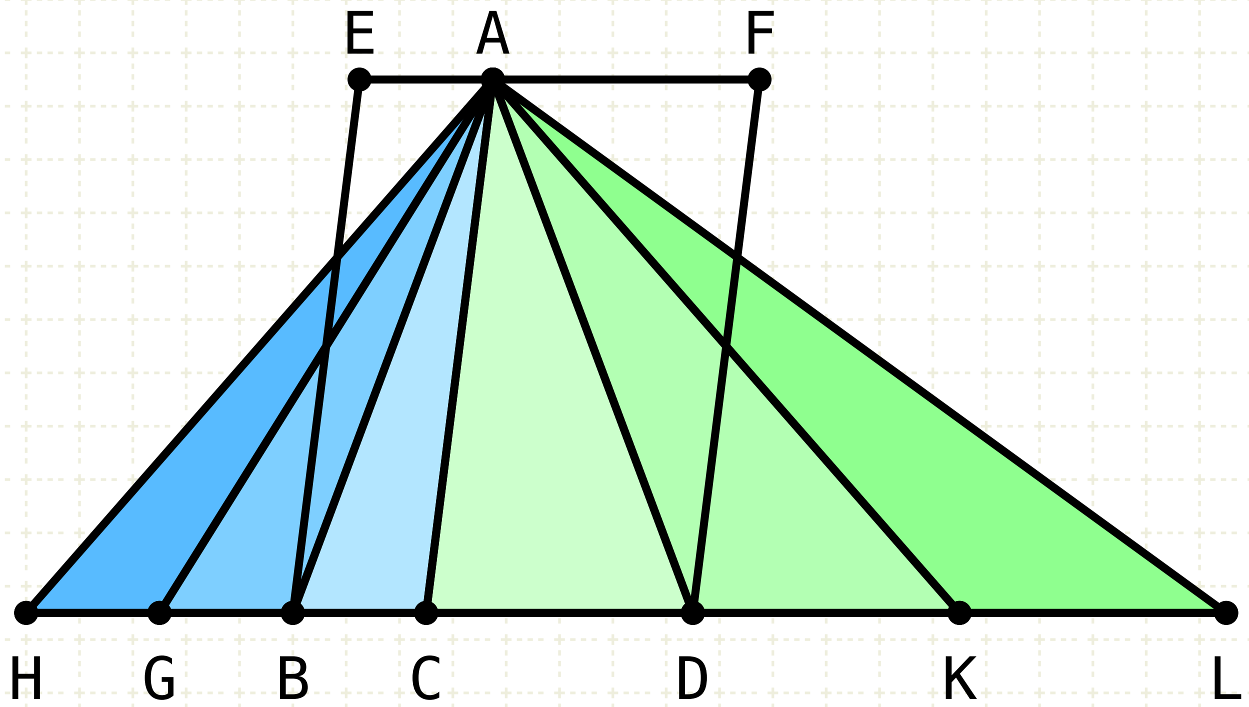

$HG = GB = BC,\ CD = DK = KL$

$\Delta AHC = n \cdot \Delta ABC,\quad HC = n \cdot BC$

$\Delta ACL = m \cdot \Delta ACD,\quad CL = m \cdot CD$

Proof - Triangles

Extend the line BD to HL, such that HB is composed of any number of lines equal to BC, and that DL is composed of any number of lines equal to CD

Draw the triangles AHG, AGB, and ADK and AKL

Since the bases are equal, triangles ABC, AGB and AHG are equal, and the triangles ACD, ADK and AKL are equal (I·38)

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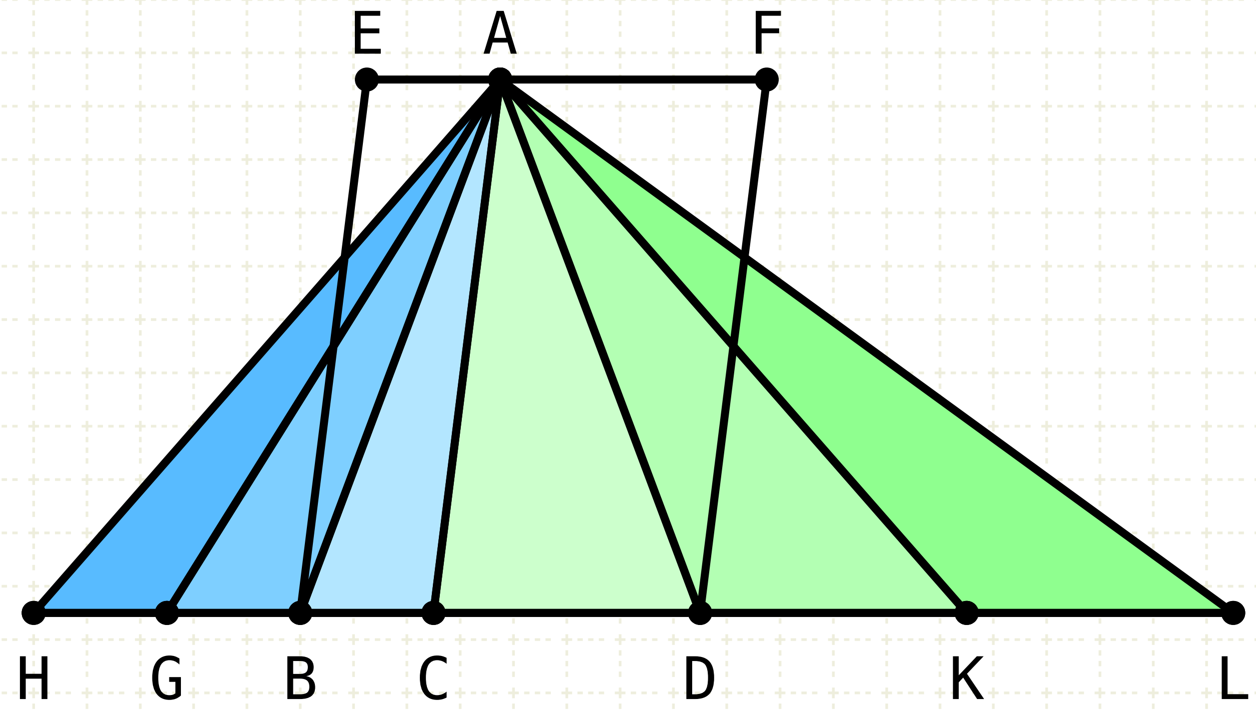

$HG = GB = BC,\ CD = DK = KL$

$\Delta AHC = n \cdot \Delta ABC,\quad HC = n \cdot BC$

$\Delta ACL = m \cdot \Delta ACD,\quad CL = m \cdot CD$

if $HC = CL \rightarrow \Delta AHC = \Delta ACL$

Proof - Triangles

Extend the line BD to HL, such that HB is composed of any number of lines equal to BC, and that DL is composed of any number of lines equal to CD

Draw the triangles AHG, AGB, and ADK and AKL

Since the bases are equal, triangles ABC, AGB and AHG are equal, and the triangles ACD, ADK and AKL are equal (I·38)

Therefore, the triangle AHC is the same multiple of ABC as the line HC is to BC

Similarly, the triangle ACL is the same multiple of ACD as the line CD is to CL

If HC is equal to CL, then the triangles AHC and ACL are equal (I·38)

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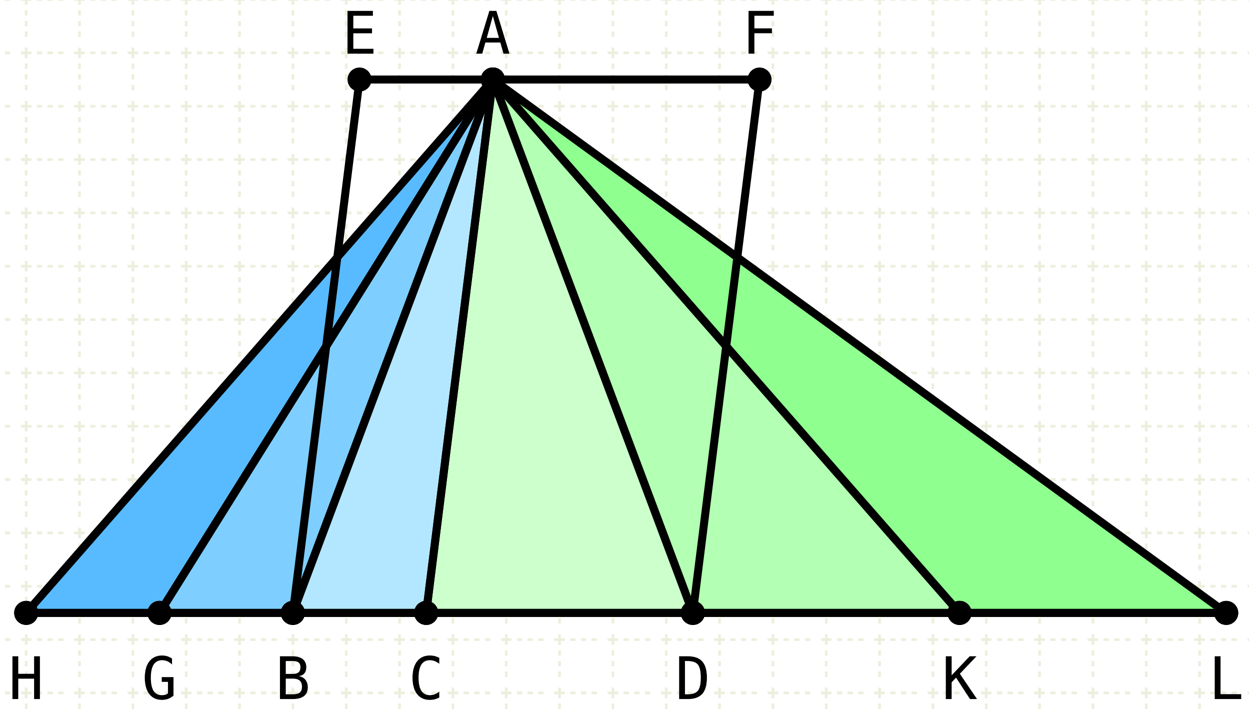

$HG = GB = BC,\ CD = DK = KL$

$\Delta AHC = n \cdot \Delta ABC,\quad HC = n \cdot BC$

$\Delta ACL = m \cdot \Delta ACD,\quad CL = m \cdot CD$

if $\quad HC = CL \rightarrow \Delta AHC = \Delta ACL$

$\quad HC <=> CL \rightarrow \Delta AHC <=> \Delta ACL$

Proof - Triangles

Extend the line BD to HL, such that HB is composed of any number of lines equal to BC, and that DL is composed of any number of lines equal to CD

Draw the triangles AHG, AGB, and ADK and AKL

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Similarly, the triangle ACL is the same multiple of ACD as the line CD is to CL

If HC is equal to CL, then the triangles AHC and ACL are equal (I·38)

IF HC is greater (or less) than CL, the the triangle AHC will be greater (or less) than ACL (I·38)

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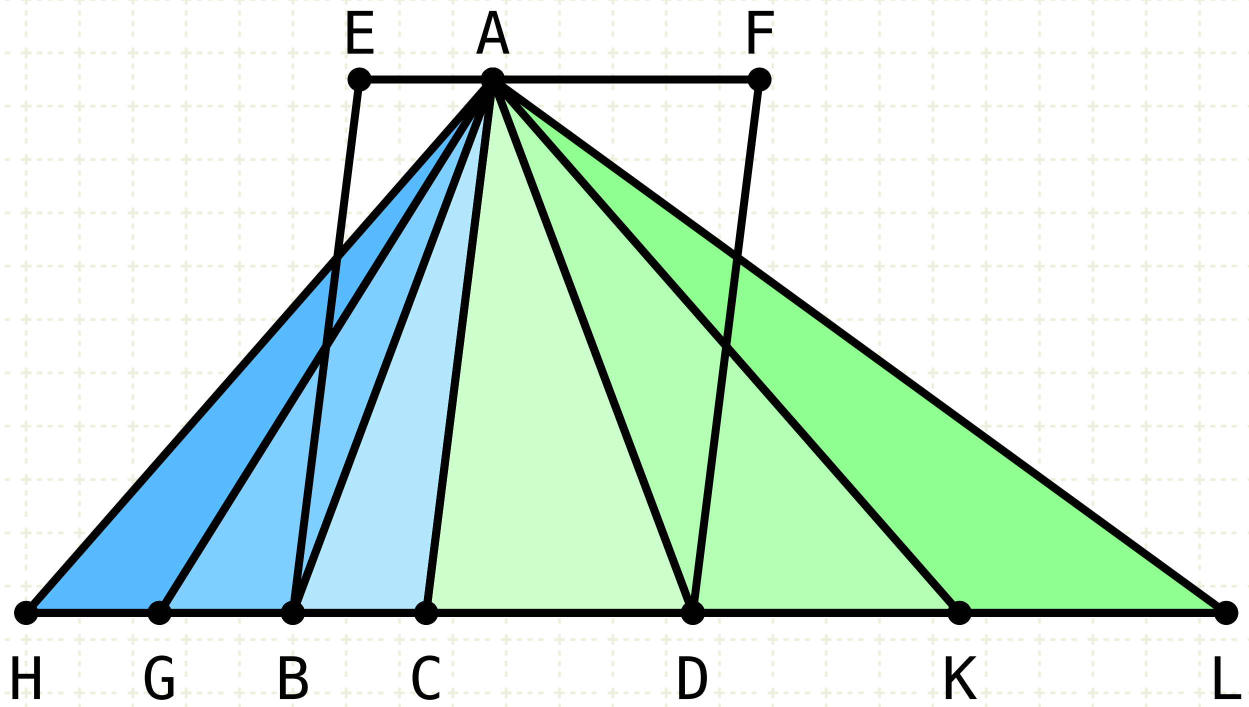

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Therefore, the triangle AHC is the same multiple of ABC as the line HC is to BC

Similarly, the triangle ACL is the same multiple of ACD as the line CD is to CL

If HC is equal to CL, then the triangles AHC and ACL are equal (I·38)

IF HC is greater (or less) than CL, the the triangle AHC will be greater (or less) than ACL (I·38)

By definition (V.Def.5), the ratio of BC to CD is equal to the ratio of the triangles ABC and ACD

$HG = GB = BC, \ CD = DK = KL$

$\Delta AHC = n \cdot \Delta ABC, \quad HC = n \cdot BC$

$\Delta ACL = m \cdot \Delta ACD, \quad CL = m \cdot CD$

if $\quad HC=CL \rightarrow \Delta AHC = \Delta ACL$

$\quad HC \Leftarrow\Rightarrow CL \rightarrow \Delta AHC \Leftarrow\Rightarrow \Delta ACL$

or $\quad n \cdot BC \Leftarrow\Rightarrow m \cdot CD \rightarrow n \cdot \Delta ABC \Leftarrow\Rightarrow m \cdot \Delta ACD$

$\therefore BC:CD = \Delta ABC:\Delta ACD$

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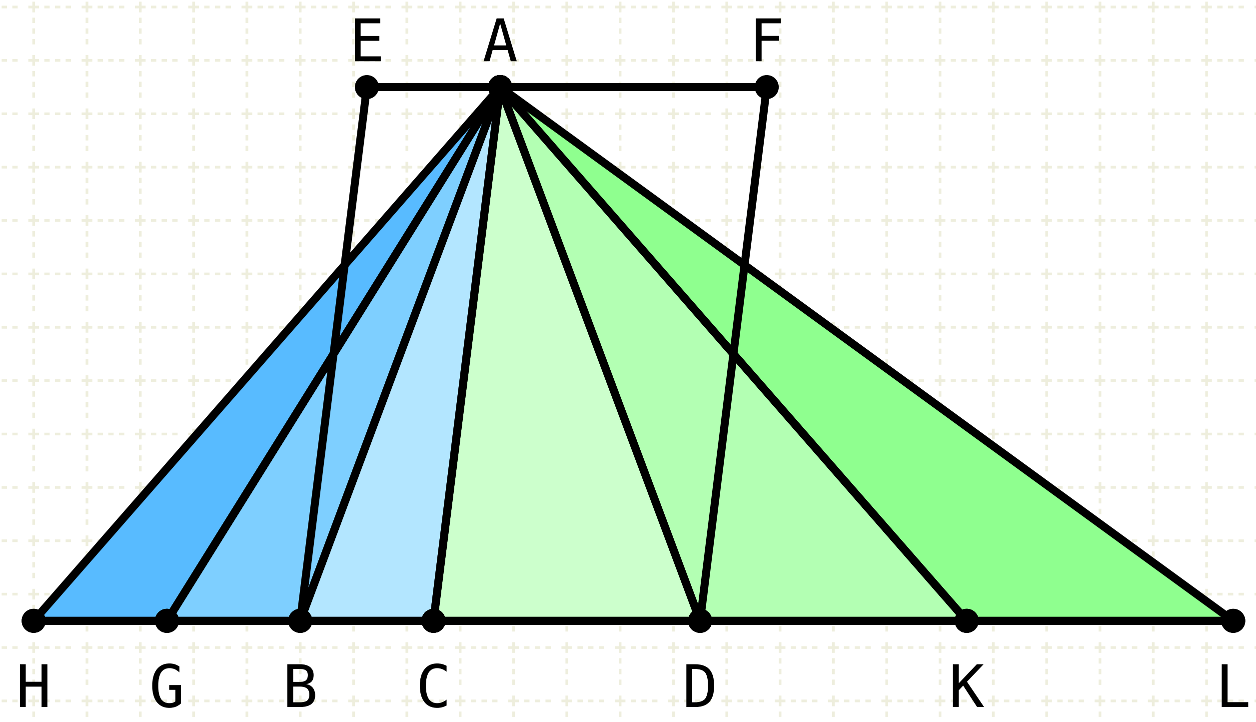

$HG = GB = BC, \quad CD = DK = KL$

$\Delta AHC = n \cdot \Delta ABC, \quad HC = n \cdot BC$

$\Delta ACL = m \cdot \Delta ACD, \quad CL = m \cdot CD$

if $\quad HC = CL \rightarrow \Delta AHC = \Delta ACL$

$\quad HC <=> CL \rightarrow \Delta AHC <=> \Delta ACL$

or $\quad n \cdot BC <=> m \cdot CD \rightarrow n \cdot \Delta ABC <=> m \cdot \Delta ACD$

$\therefore BC : CD = \Delta ABC : \Delta ACD$

Proof - Triangles

Extend the line BD to HL, such that HB is composed of any number of lines equal to BC, and that DL is composed of any number of lines equal to CD

Draw the triangles AHG, AGB, and ADK and AKL

Since the bases are equal, triangles ABC, AGB and AHG are equal, and the triangles ACD, ADK and AKL are equal (I·38)

Therefore, the triangle AHC is the same multiple of ABC as the line HC is to BC

Similarly, the triangle ACL is the same multiple of ACD as the line CD is to CL

If HC is equal to CL, then the triangles AHC and ACL are equal (I·38)

IF HC is greater (or less) than CL, the the triangle AHC will be greater (or less) than ACL (I·38)

By definition (V.Def.5), the ratio of BC to CD is equal to the ratio of the triangles ABC and ACD

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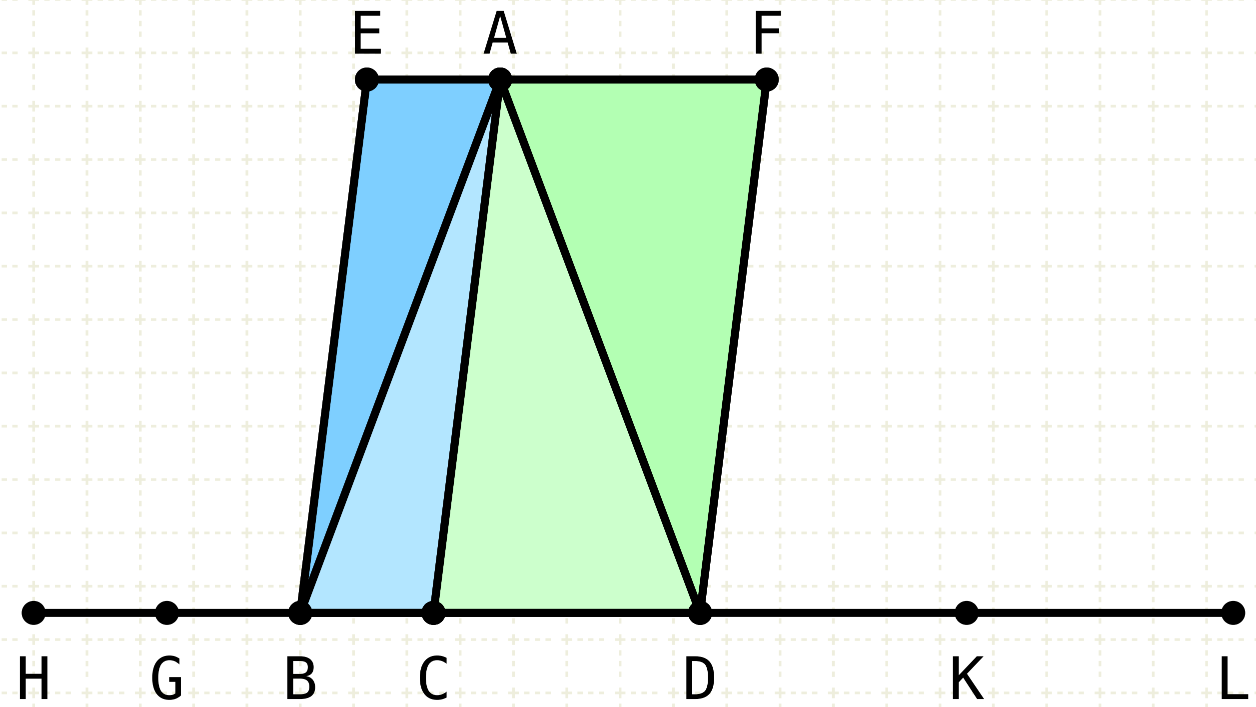

Proof - Parallelograms

BC:CD = ΔABC:ΔACD

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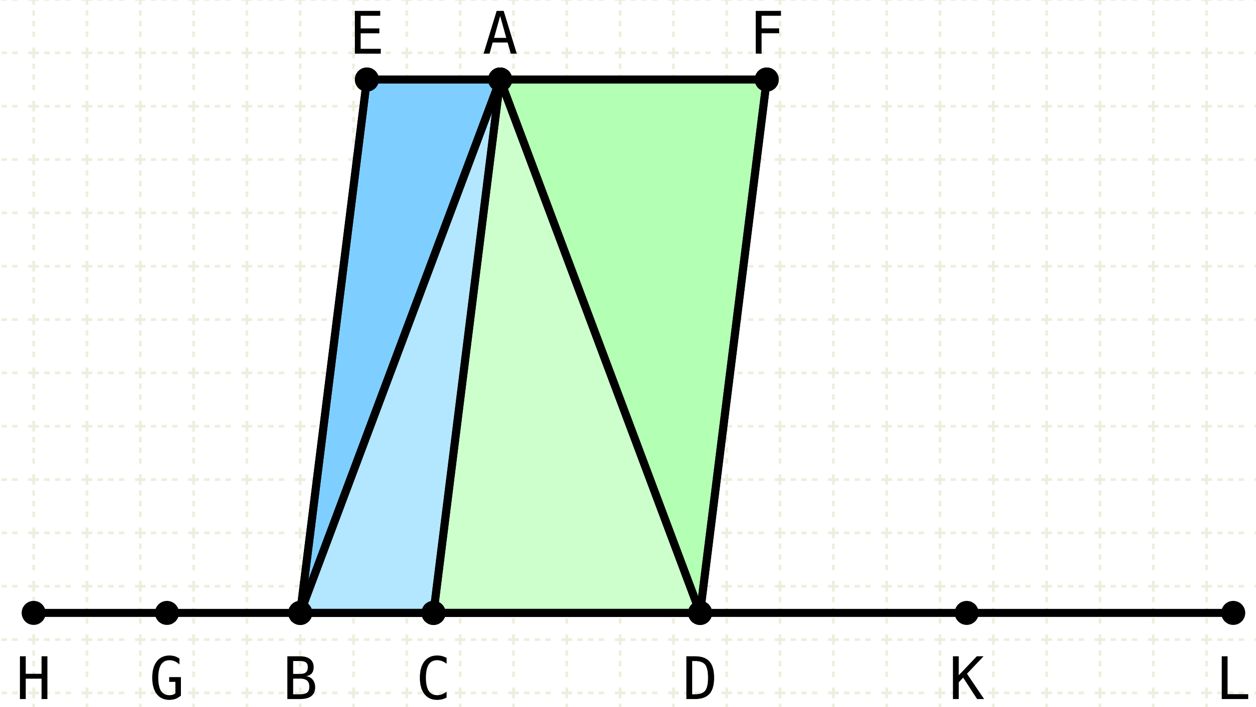

Proof - Parallelograms

The parallelograms EC and CF are twice the triangles ABC and ACD respectively (I·41)

BC:CD = ΔABC:ΔACD
□EC = 2ΔABC
□CF = 2ΔACD

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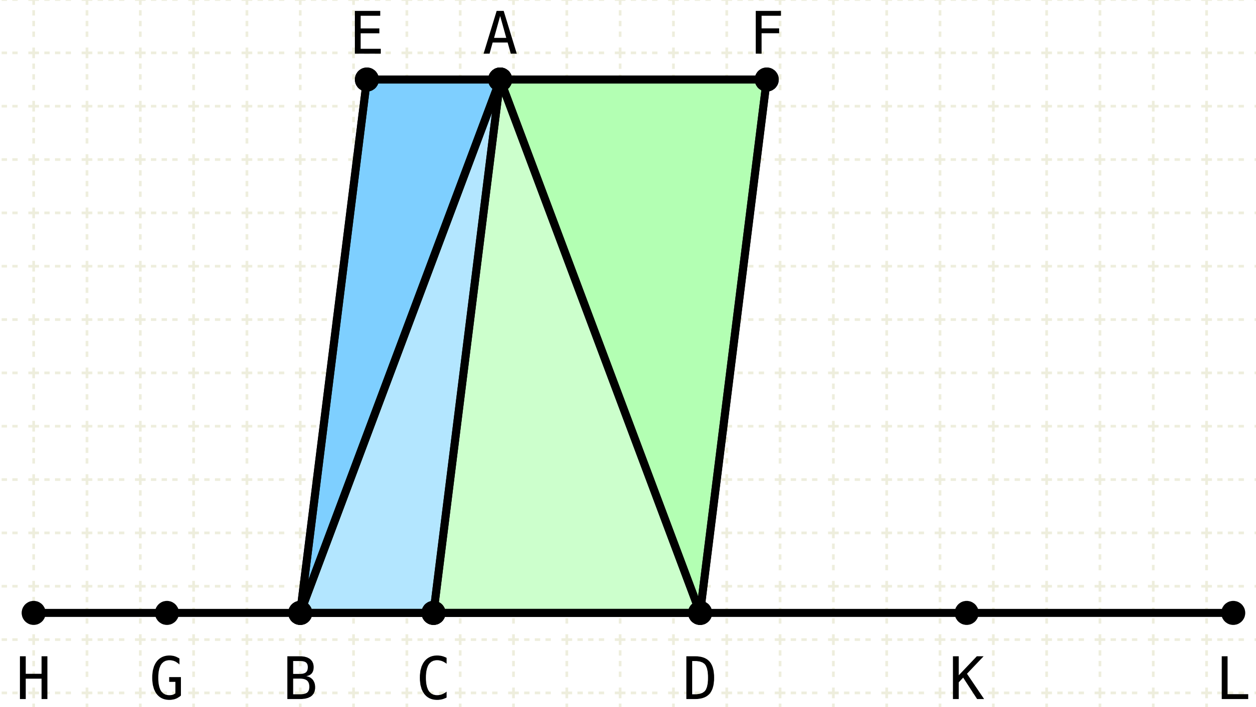

Proof - Parallelograms

The parallelograms EC and CF are twice the triangles ABC and ACD respectively (I·41)

If two magnitudes are multiplied by the same number, then the ratio will not be affected (V·15)

Therefore the ratio of the triangles to each other is the same as the ratio of the parallelograms

$BC:CD = \Delta ABC:\Delta ACD$

$\square EC = 2\Delta ABC$

$\square CF = 2\Delta ACD$

$\Delta ABC:\Delta ACD = \square EC:\square CF$

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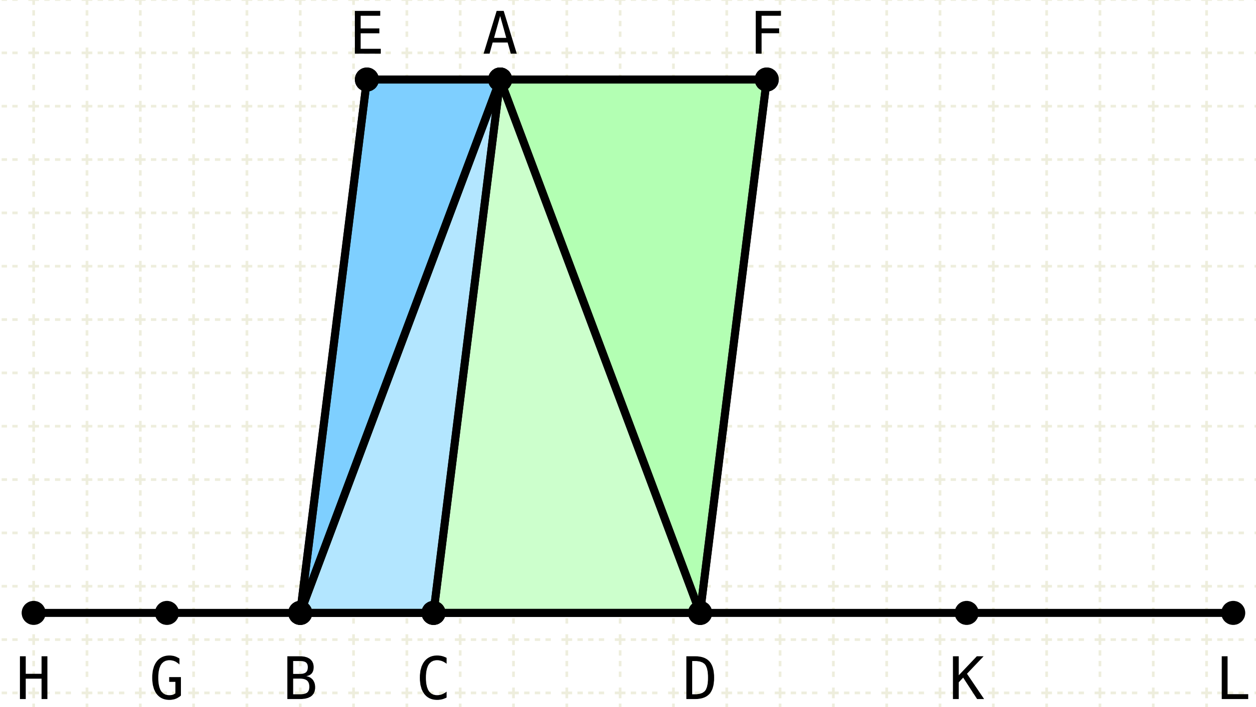

Proof - Parallelograms

The parallelograms EC and CF are twice the triangles ABC and ACD respectively (I·41)

If two magnitudes are multiplied by the same number, then the ratio will not be affected (V·15)

Therefore the ratio of the triangles to each other is the same as the ratio of the parallelograms

And since ratios are transitive (a=b b=c → a=c) (V·11), then the ratio of the parallelograms is equal to the ratio of the bases

$BC:CD = \Delta ABC:\Delta ACD$

$\square EC = 2\Delta ABC$

$\square CF = 2\Delta ACD$

$\Delta ABC:\Delta ACD = \square EC:\square CF$

$BC:CD = \Delta ABC:\Delta ACD = \square EC:\square CF$

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