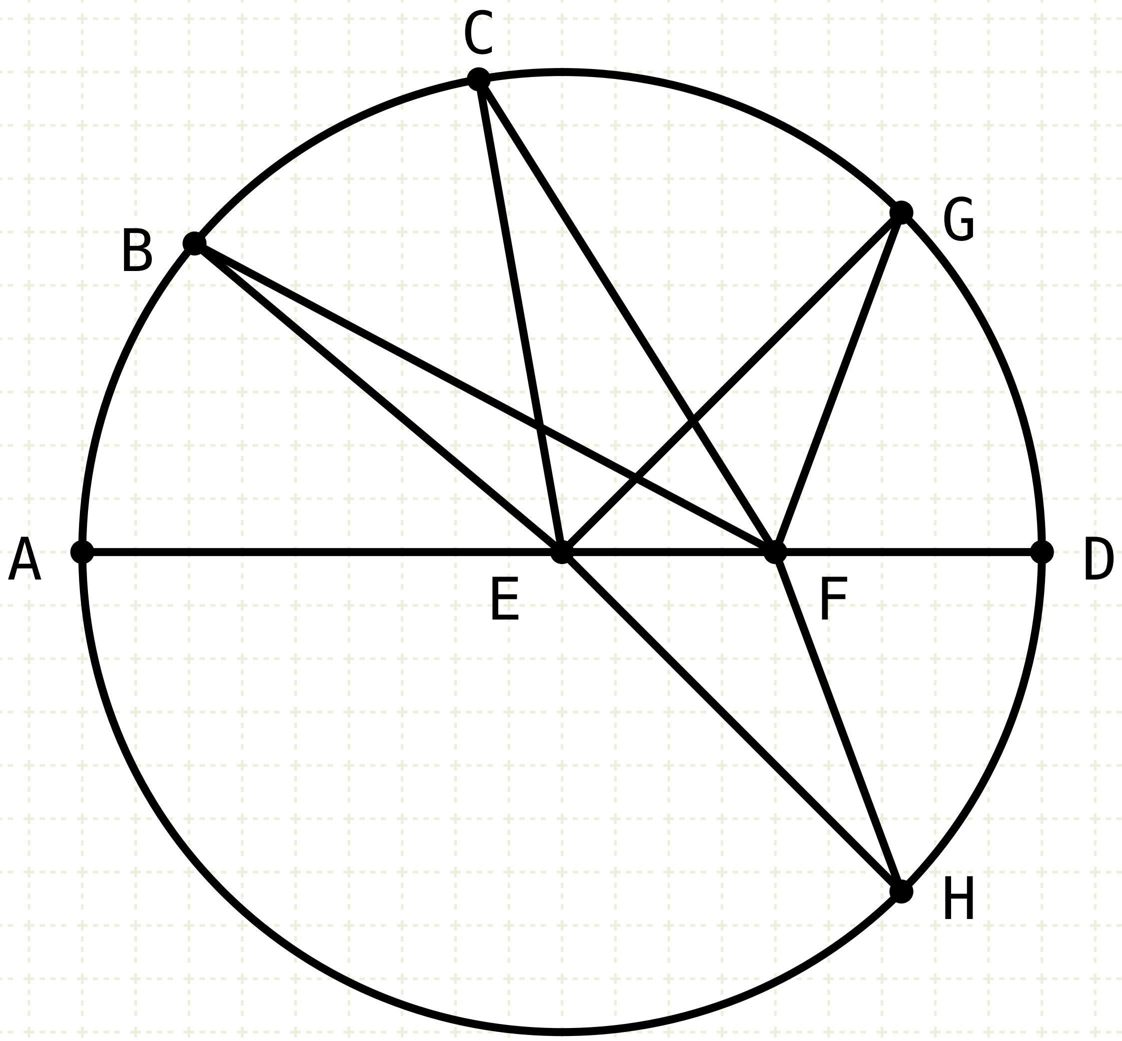
Euclid's Elements

Book III



A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)

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Proposition 1 of Book III

To find the center of a given circle.

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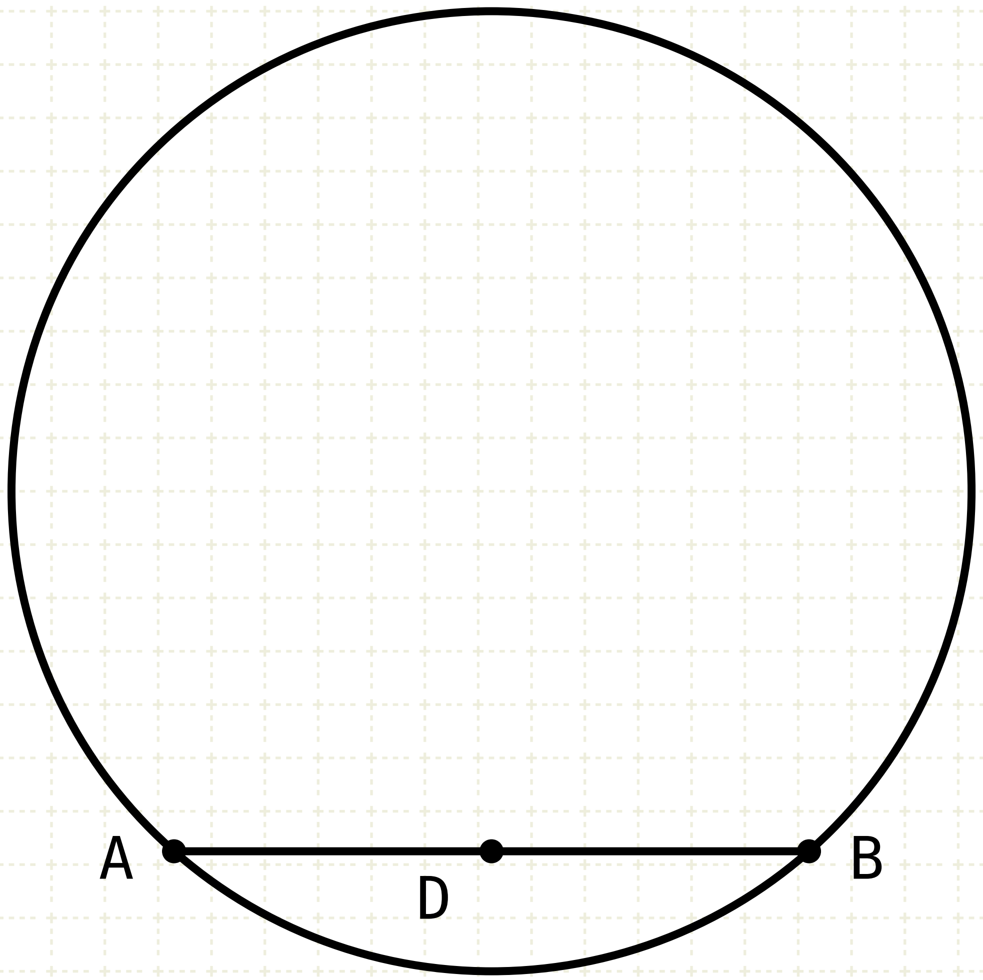
AD = DB

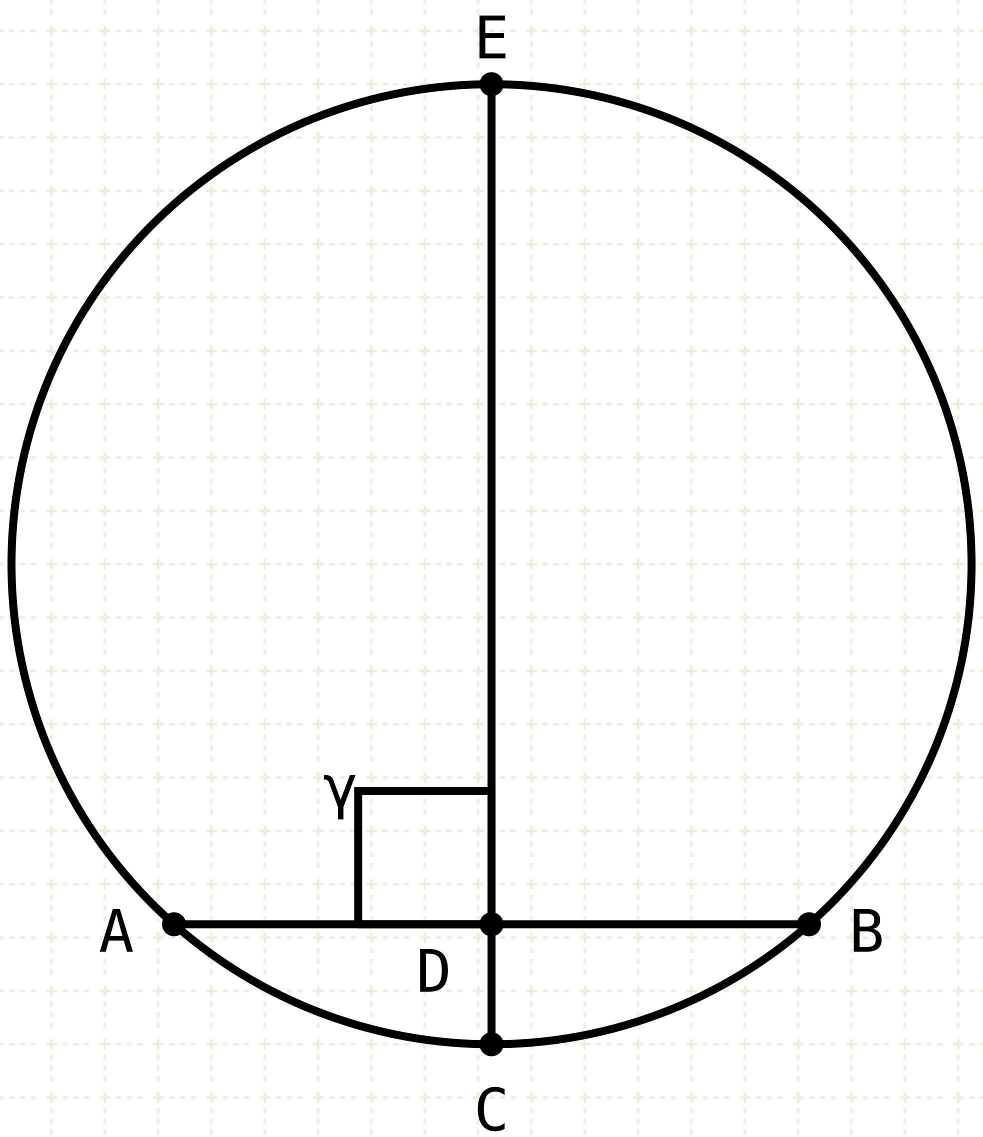


Construction

Let a straight line (AB) be drawn through the circle at random, and be bisected at point D

Proposition 1 of Book III

To find the center of a given circle.



$AD = DB$

$\gamma = \llcorner$

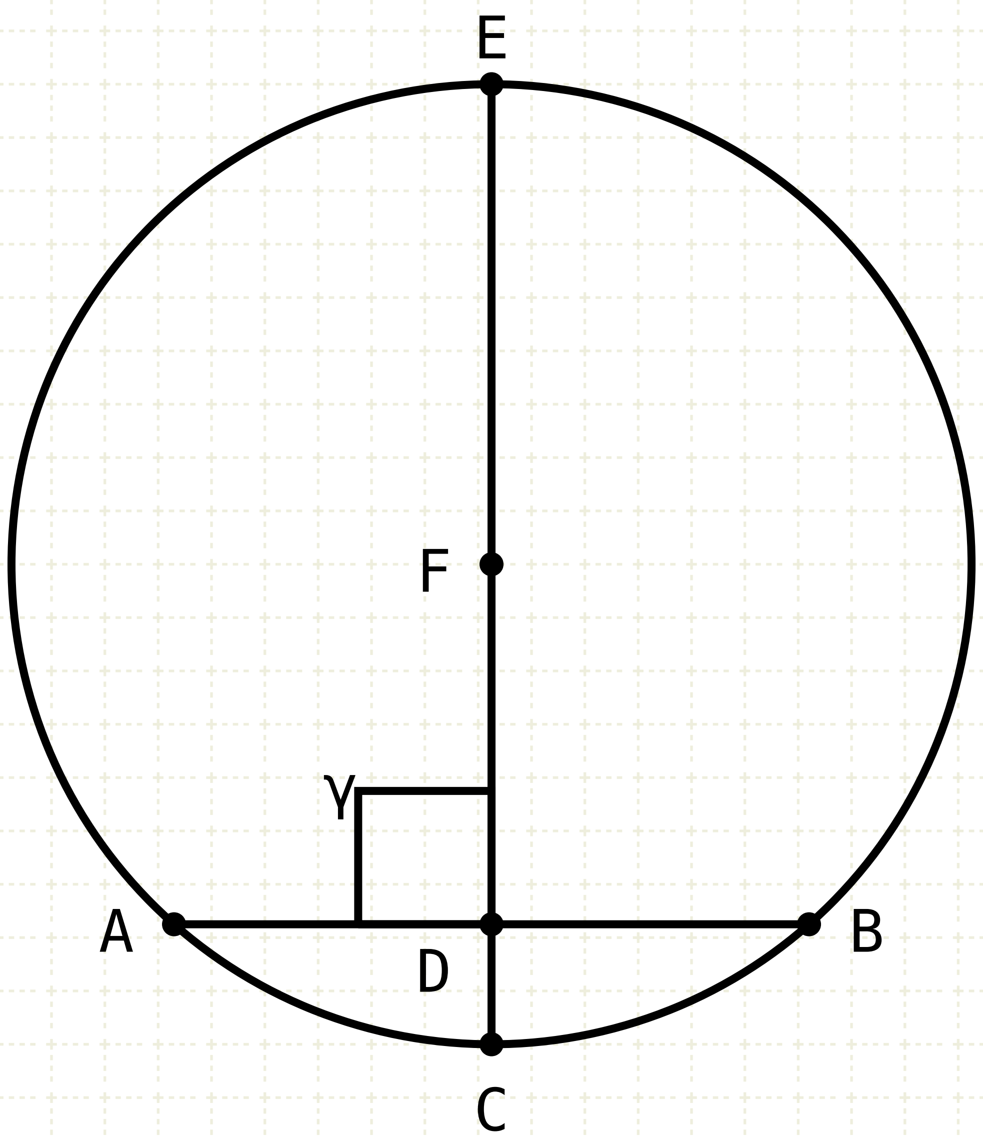
Construction

Let a straight line (AB) be drawn through the circle at random, and be bisected at point D

Draw a line perpendicular to AB through the point D, intersecting the circle at CE

Proposition 1 of Book III

To find the center of a given circle.



AD = DB

γ = ∟

CF = FE

Construction

Let a straight line (AB) be drawn through the circle at random, and be bisected at point D

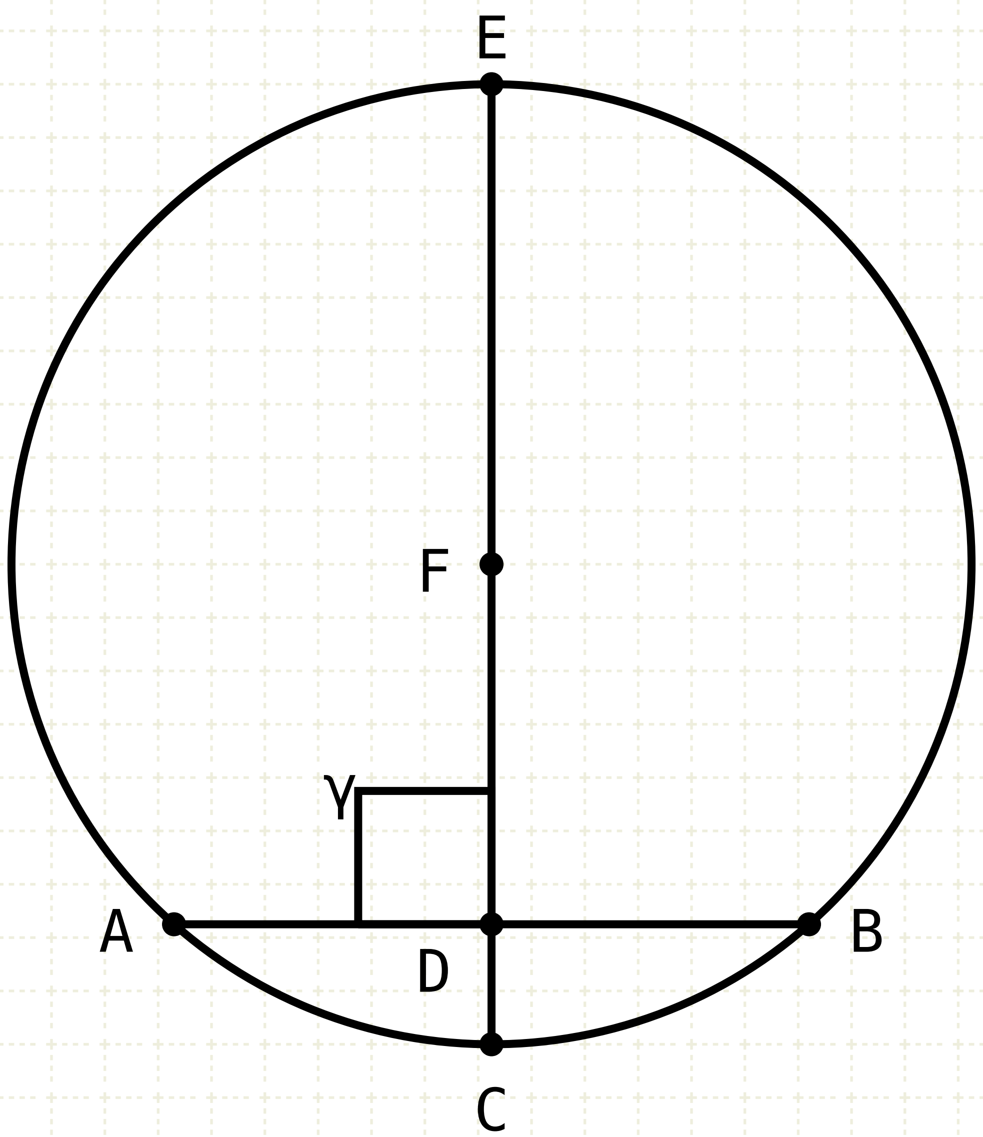
Draw a line perpendicular to AB through the point D, intersecting the circle at CE

Bisect CE at point F. Point F is the center of the circle

Proposition 1 of Book III

To find the center of a given circle.

Proof by Contradiction



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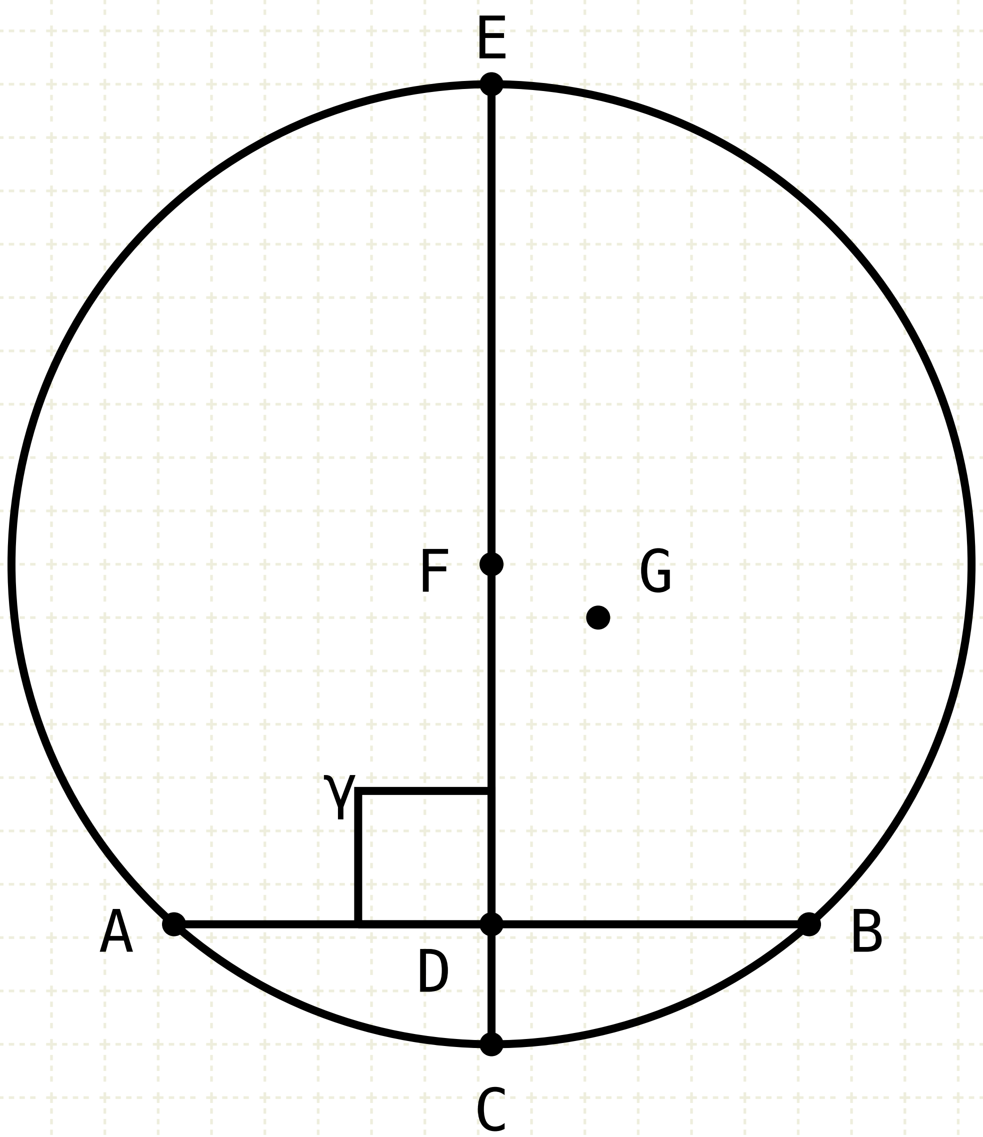

AD = DB

γ = ∟

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If G is the centre of the circle

Proof by Contradiction

Assume that G is the center of the circle, and that G does not lie on the line CE

Proposition 1 of Book III

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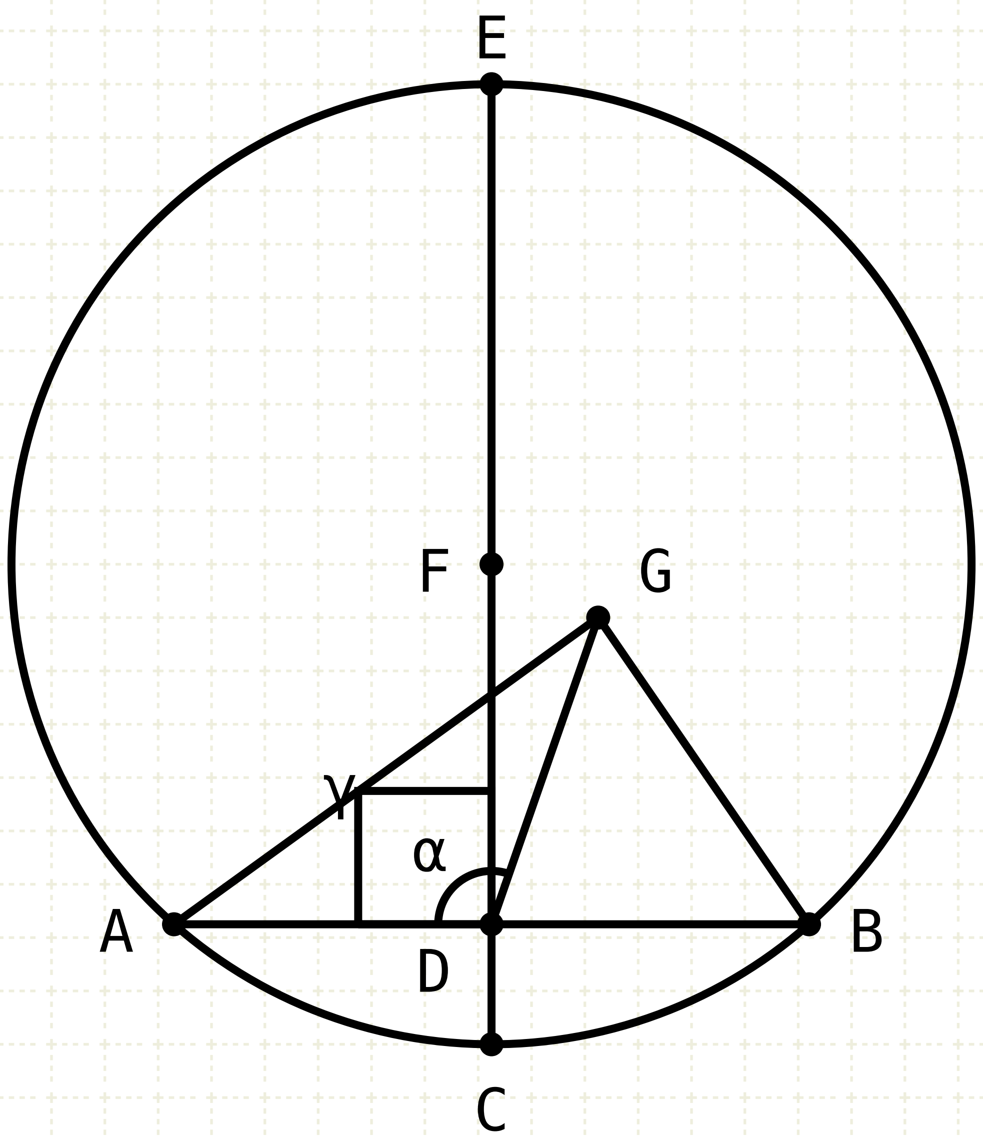

$AD = DB$

$\gamma = \llcorner$

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If G is the centre of the circle

$\alpha > \gamma$

Proof by Contradiction

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Join GA, GD and GB

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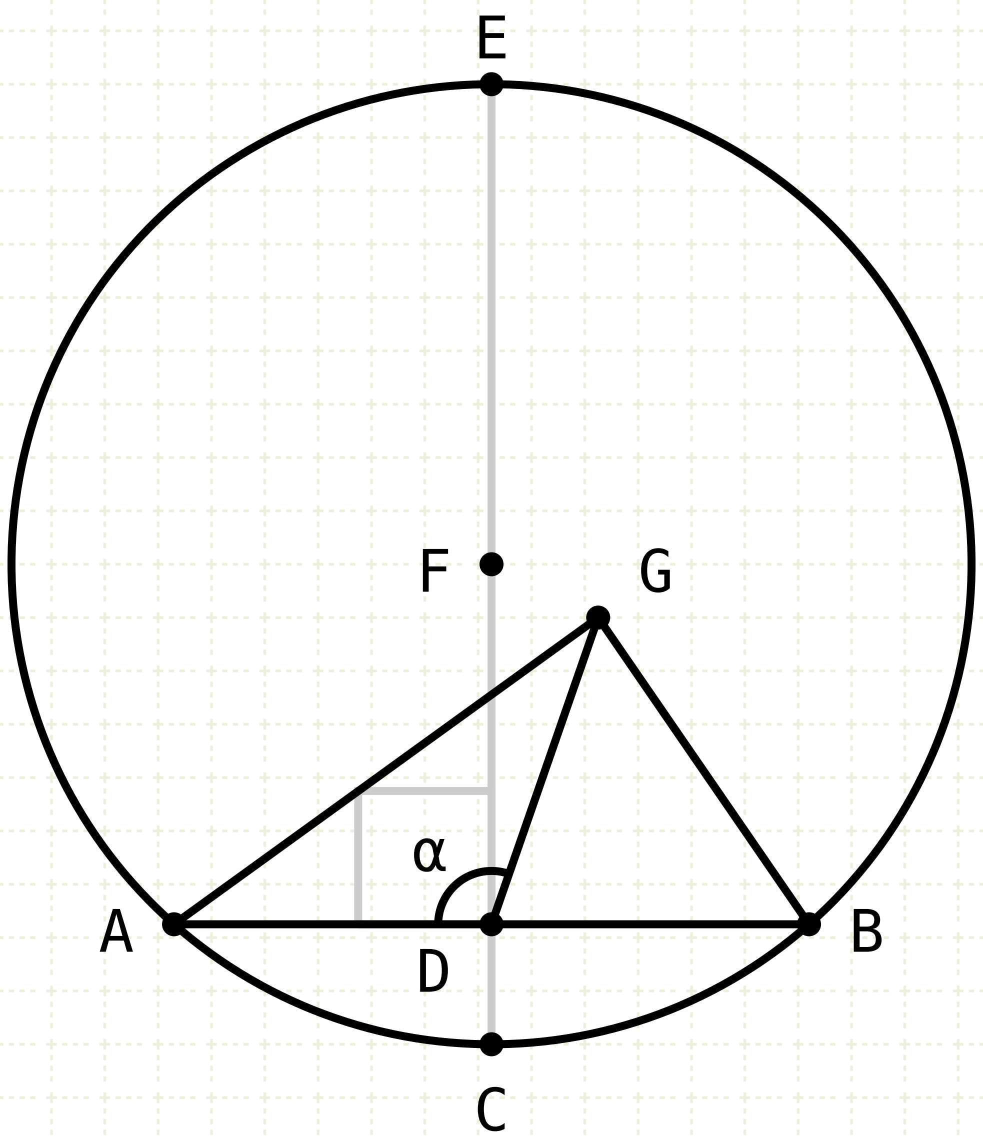

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AG and GB are radii, and thus are equal

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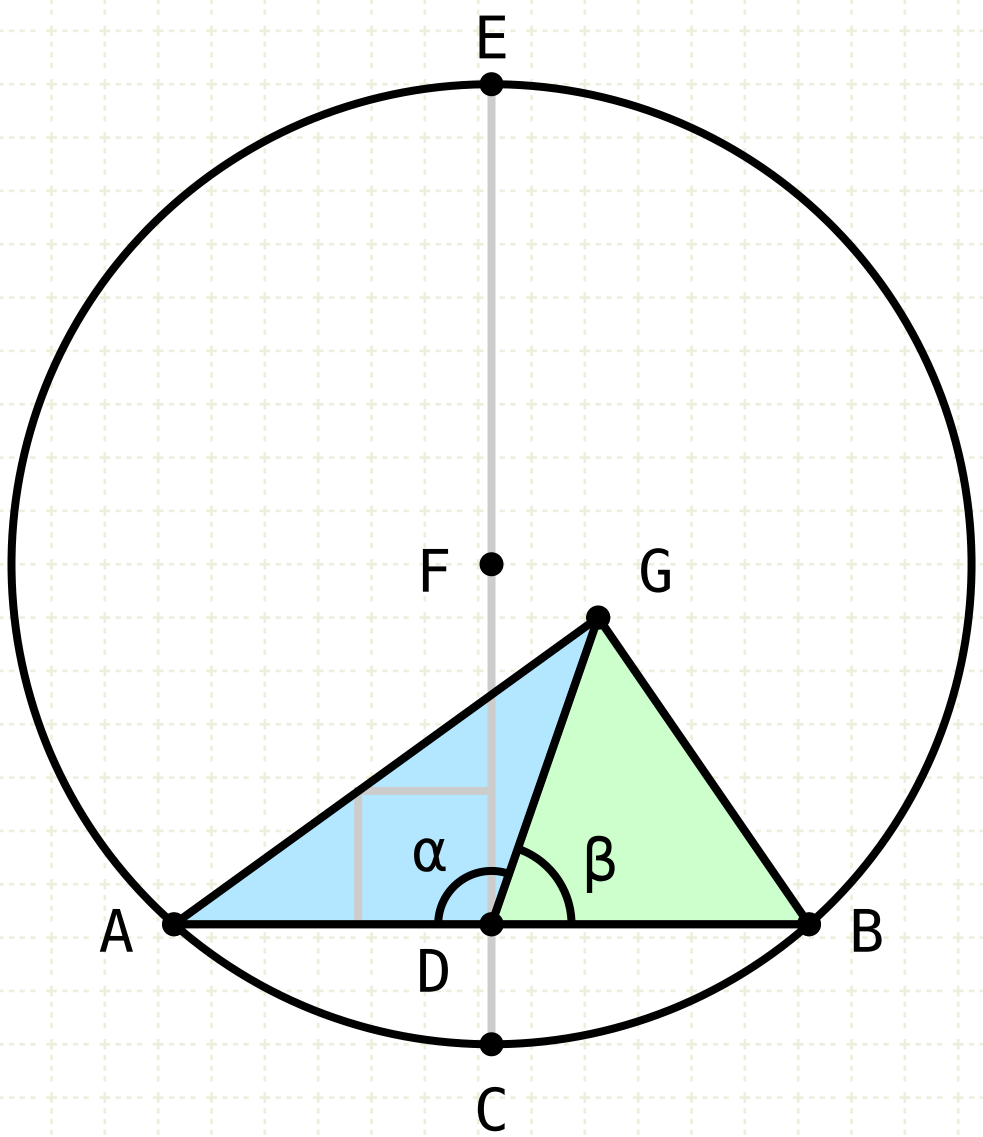

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Thus, since the side DG is shared between both triangles ADG and GDB, the triangles have three equal sides, therefore angle ADG (α) equals angle GDB (β) (I·8)

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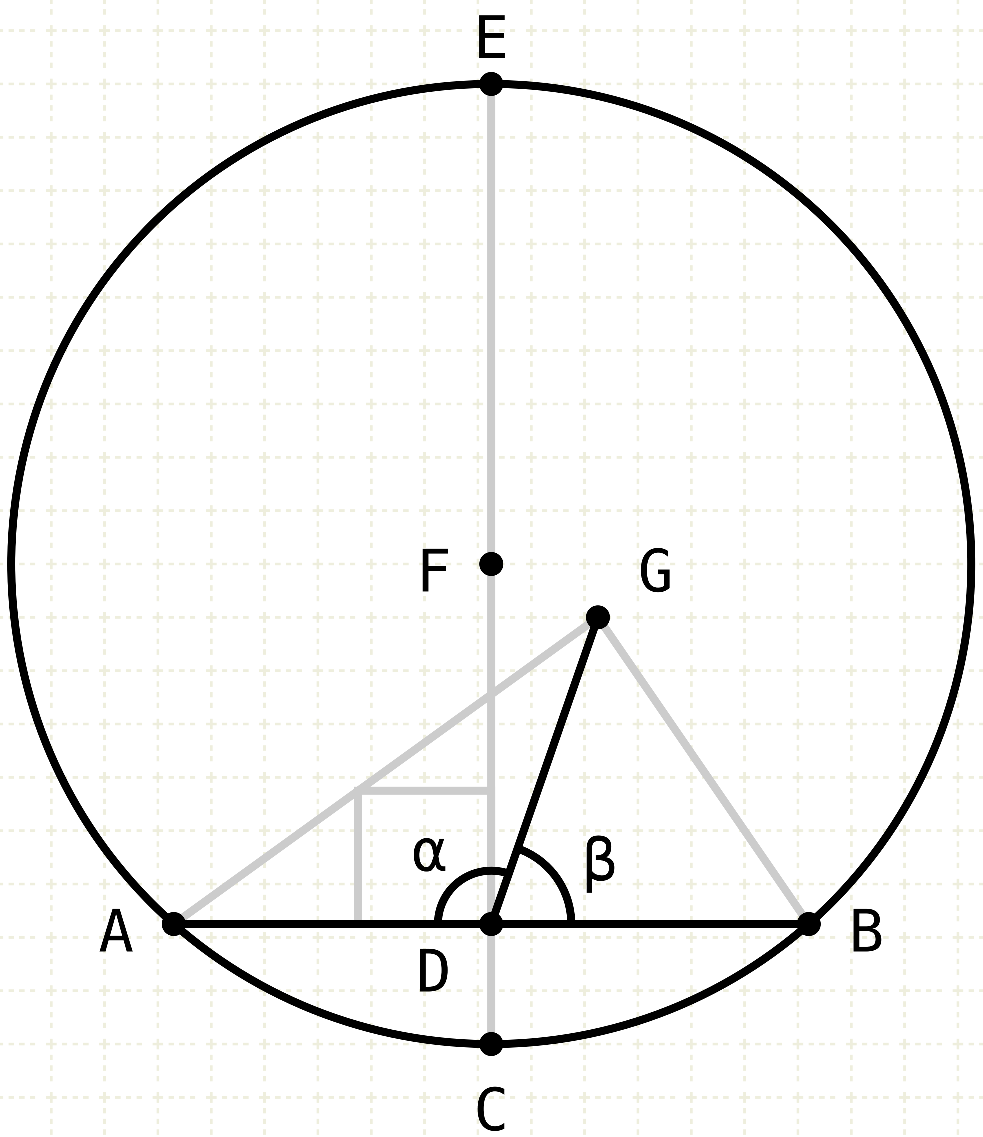

AD = DB
γ = ∟
CF = FE

If G is the centre of the circle

α > γ
AG = GB
α = β
α = β = ∟

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By definition (I.Def.10), two angles on a straight lines are right angles if they are equal to each other, thus α and β are right

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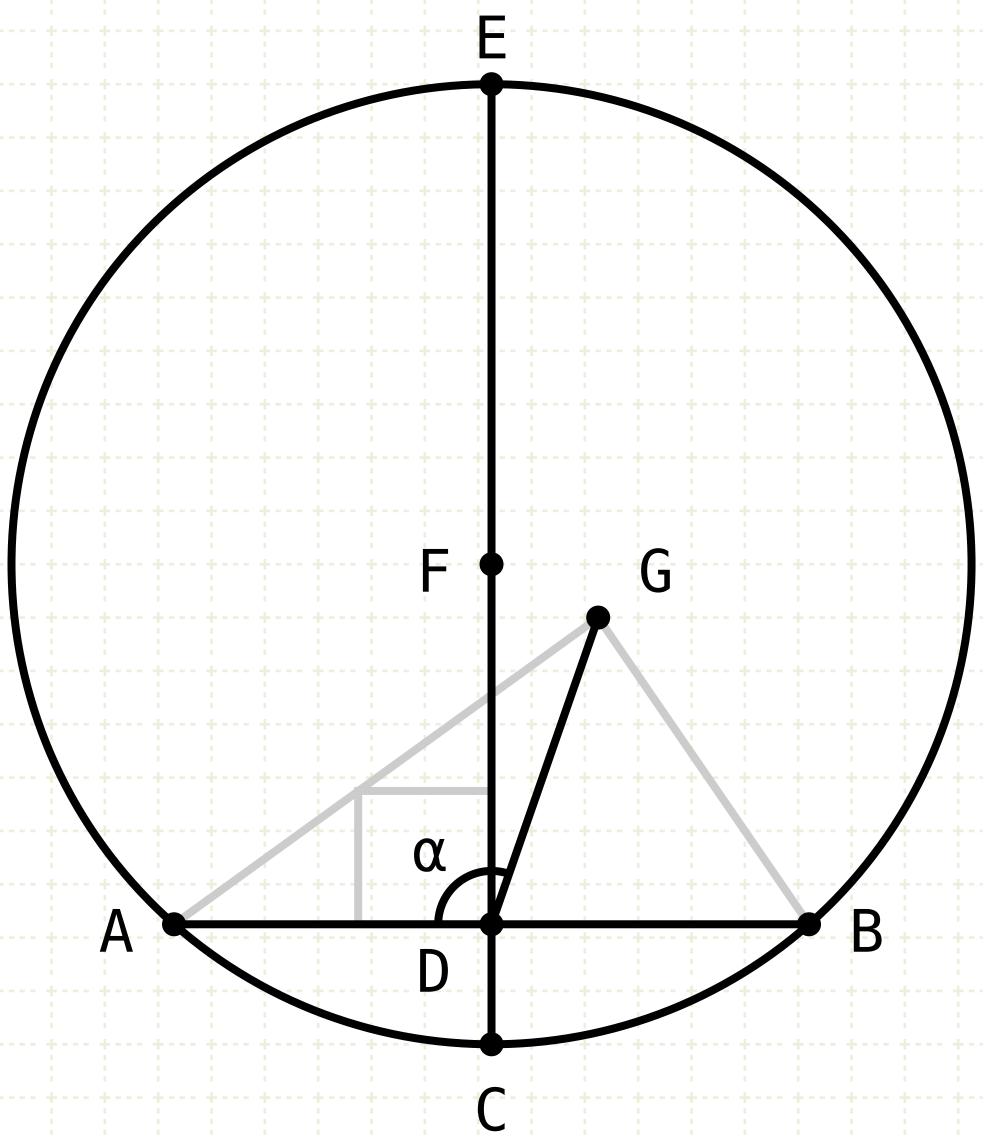

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But GDB (α) is greater than FDA (γ), which is also a right angle (by construction)

The angle α cannot be both greater than and equal to γ

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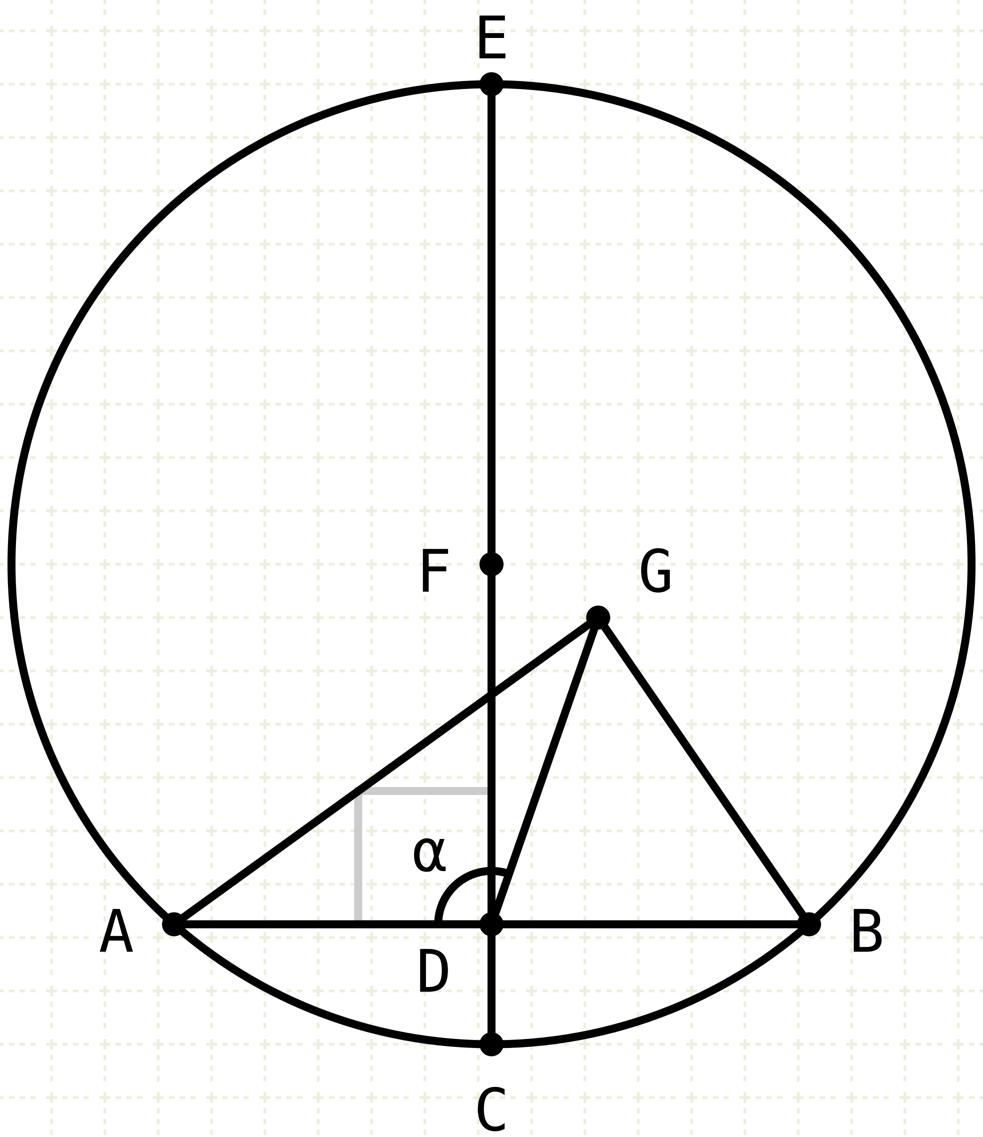

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So G is not the center of the circle

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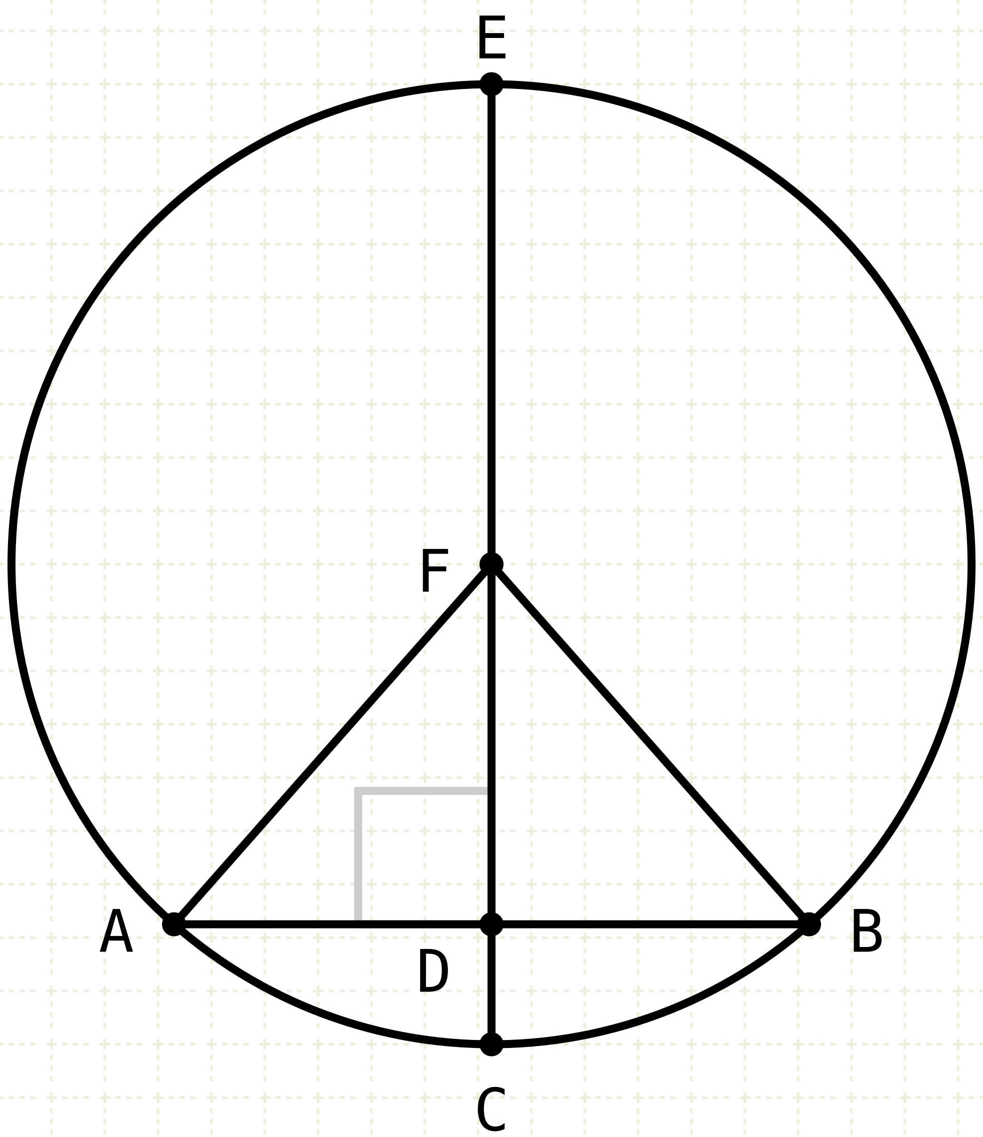

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Centre of circle lies on EC

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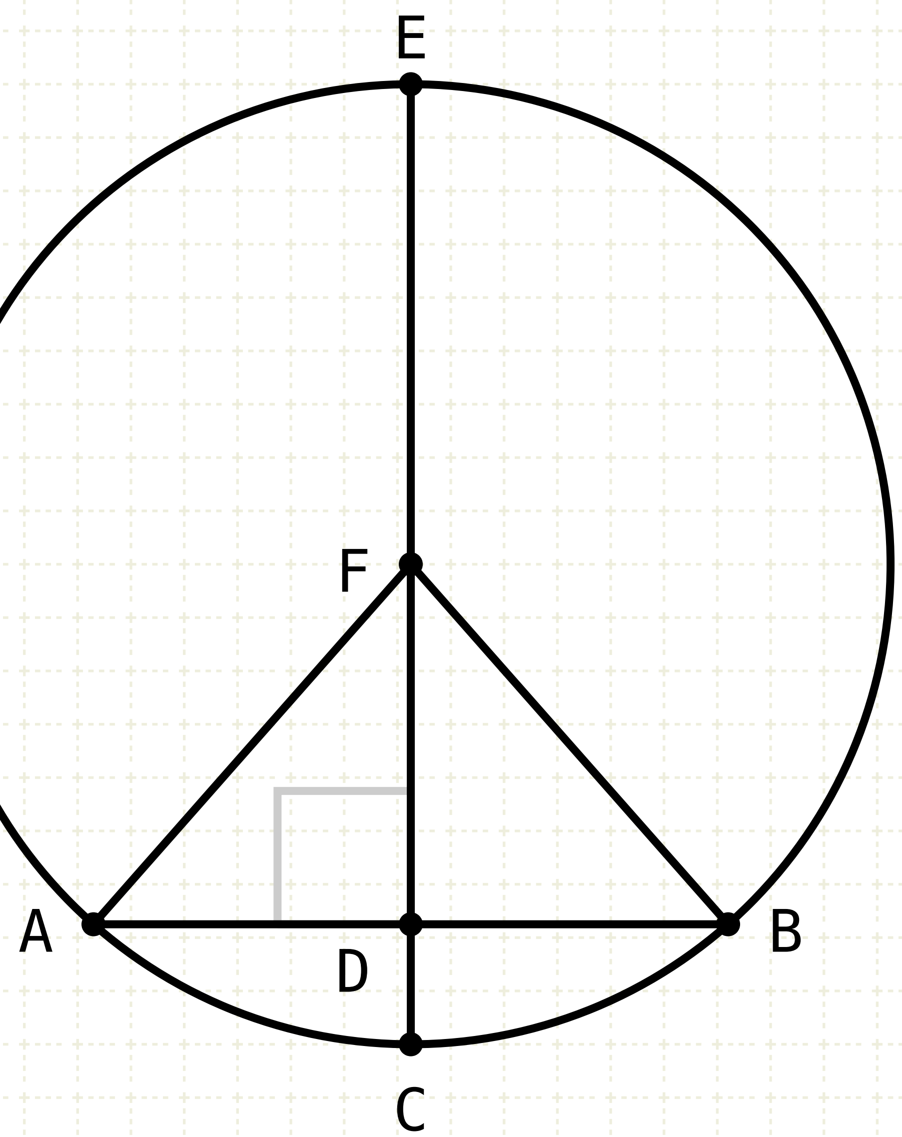

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F is the centre

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Thus, the centre of the circle must lie on the line CE

Note that if G lies on the line CE, and it is the center of the circle, it must coincide with the point F, since the points E and C must be equidistant from the center.

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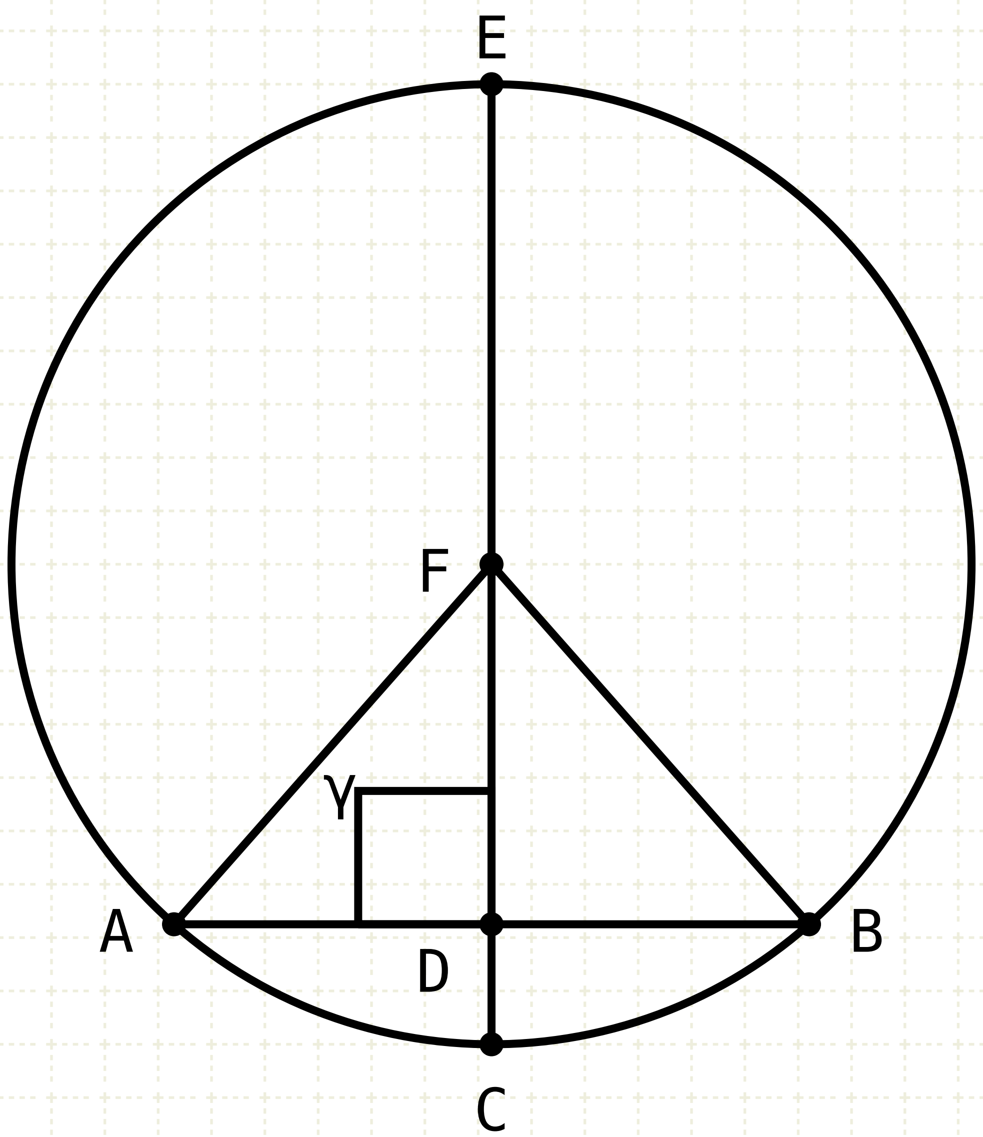

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