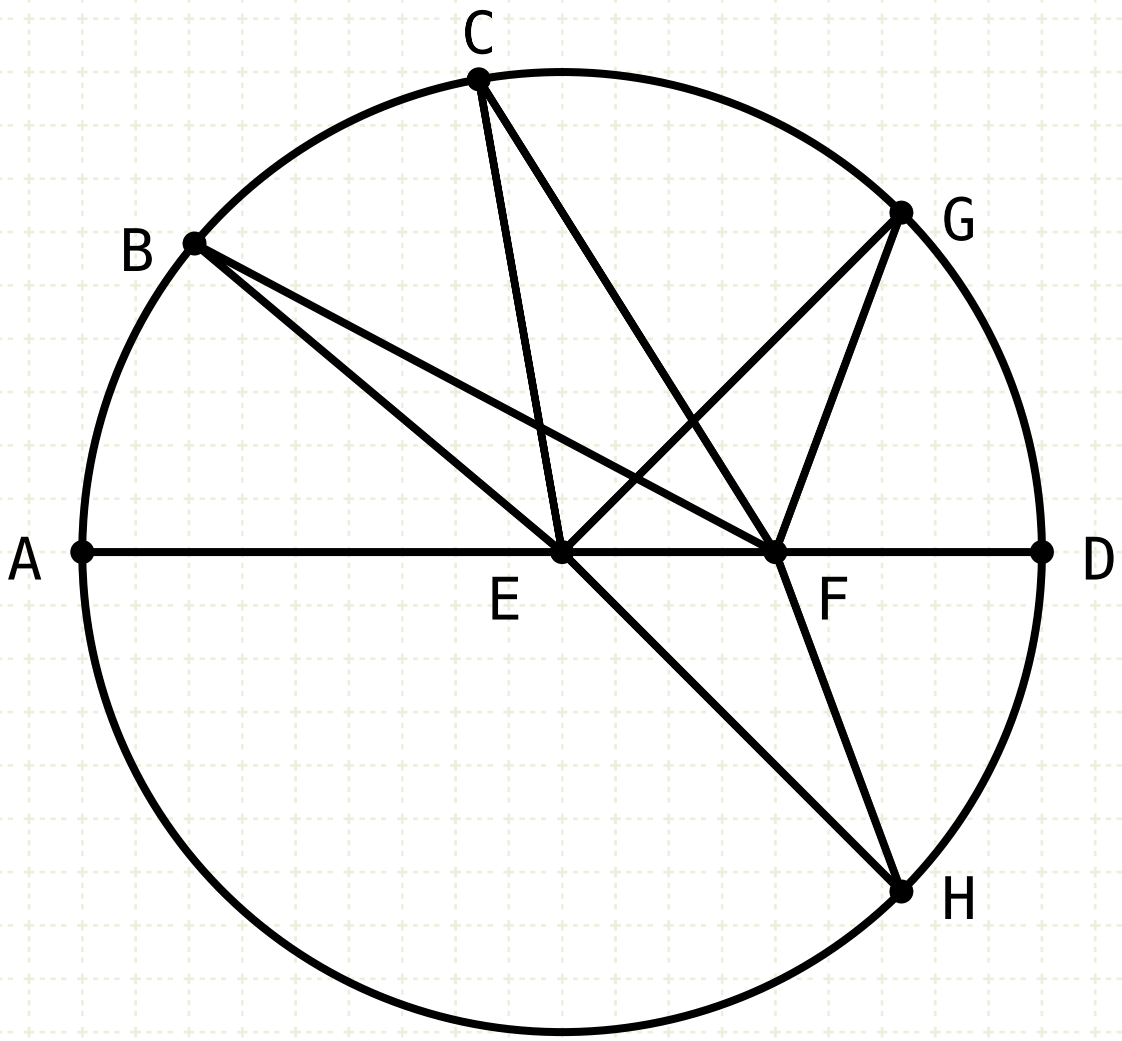
Euclid's Elements

Book III



A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)

Table of Contents, Chapter 3

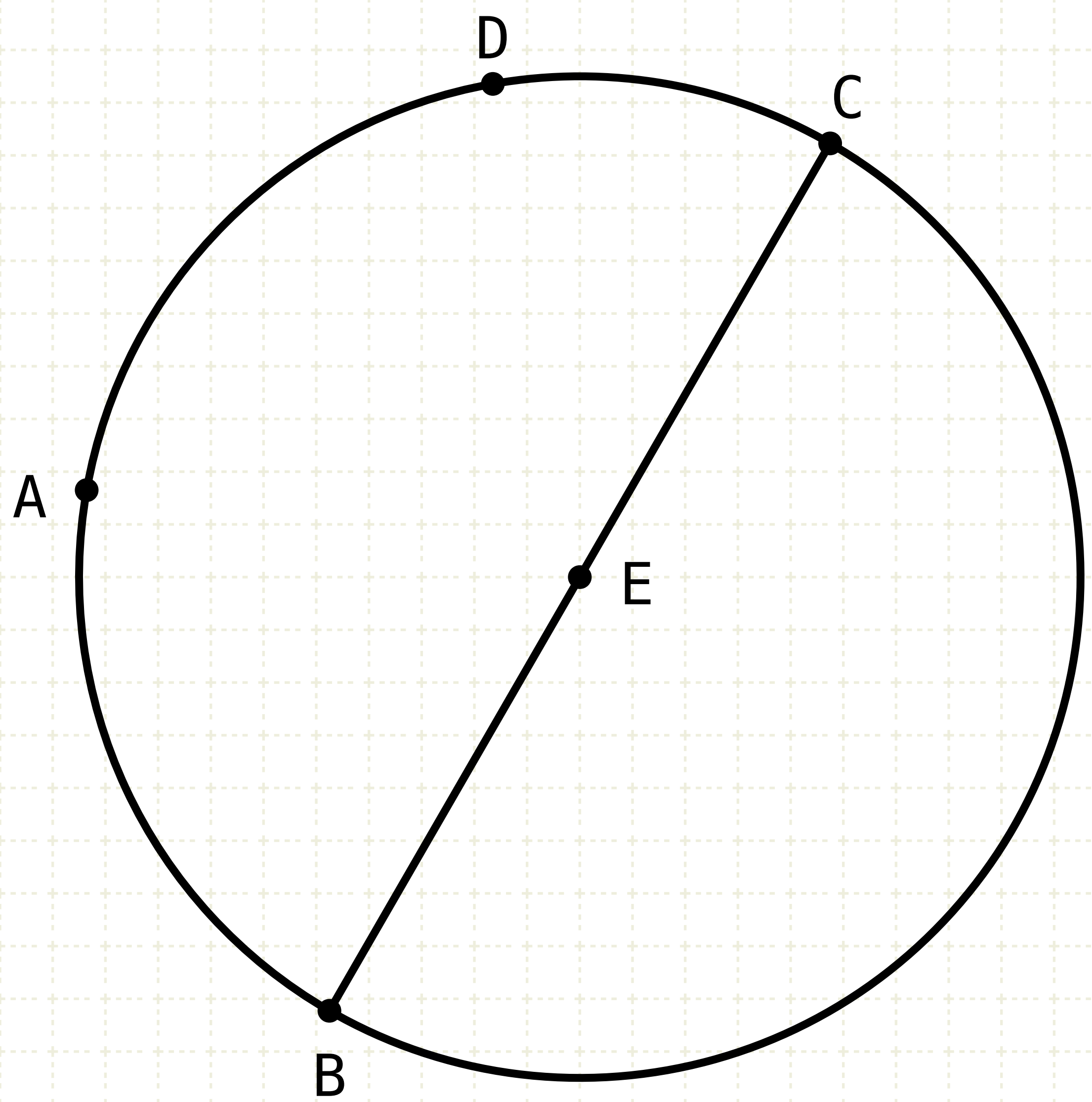
Table of Contents, Chapter 3

Proposition 31 of Book III

In a circle the angle in the semicircle is right, that in a greater segment less than a right angle, and that in a less segment greater than a right angle, and further the angle of the greater segment is greater than a right angle, and the angle of the less segment less than a right angle.

Proposition 31 of Book III

In a circle the angle in the semicircle is right, that in a greater segment less than a right angle, and that in a less segment greater than a right angle, and further the angle of the greater segment is greater than a right angle, and the angle of the less segment less than a right angle.

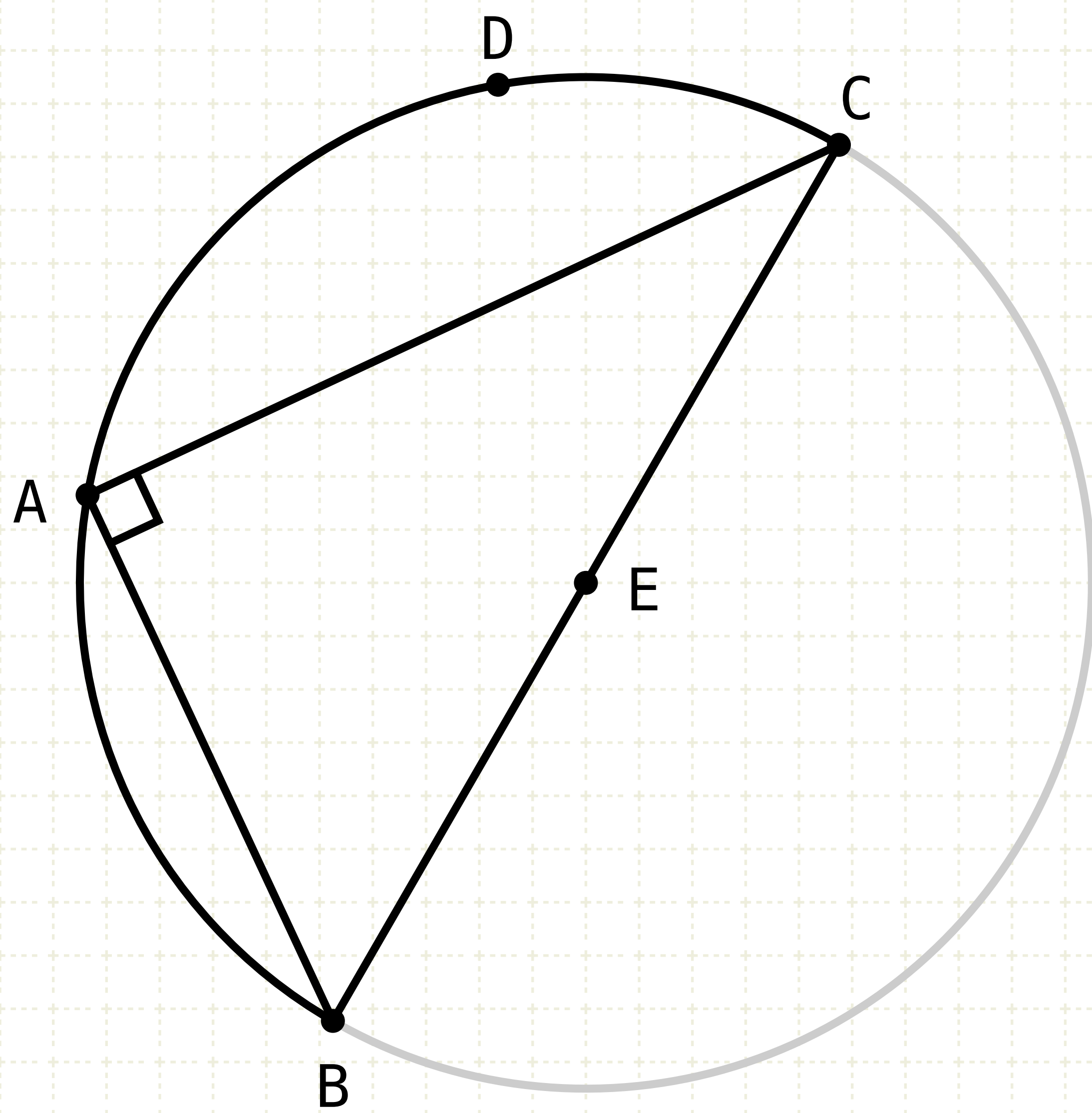


In other words

Let E be the centre of the circle and BC the diameter

Proposition 31 of Book III

In a circle the angle in the semicircle is right, that in a greater segment less than a right angle, and that in a less segment greater than a right angle, and further the angle of the greater segment is greater than a right angle, and the angle of the less segment less than a right angle.



∠BAC = ∟

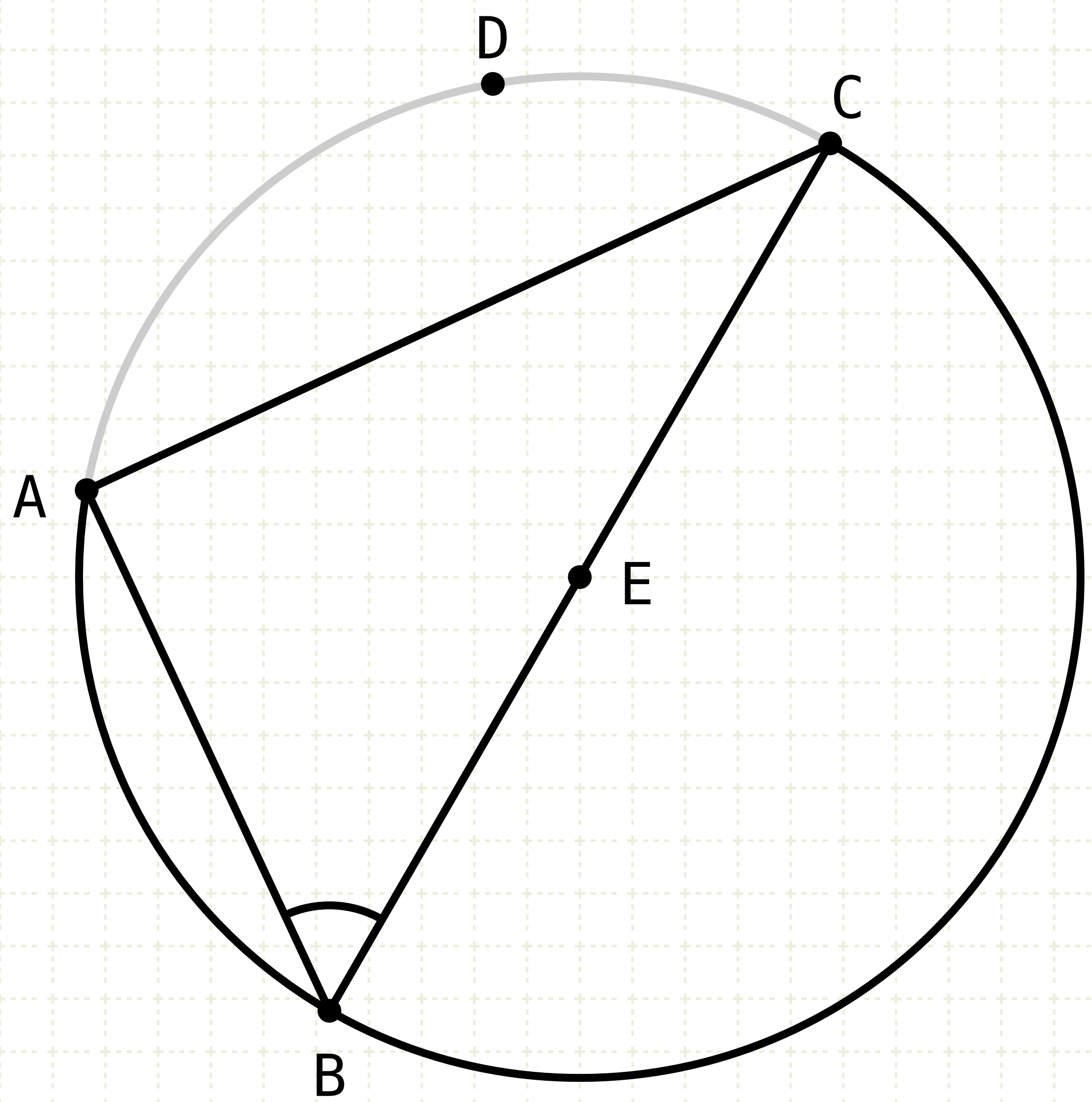
In other words

Let E be the centre of the circle and BC the diameter

The angle BAC in the semicircle segment BAC is right

Proposition 31 of Book III

In a circle the angle in the semicircle is right, that in a greater segment less than a right angle, and that in a less segment greater than a right angle, and further the angle of the greater segment is greater than a right angle, and the angle of the less segment less than a right angle.



∠BAC = ∟

∠ABC < ∟

In other words

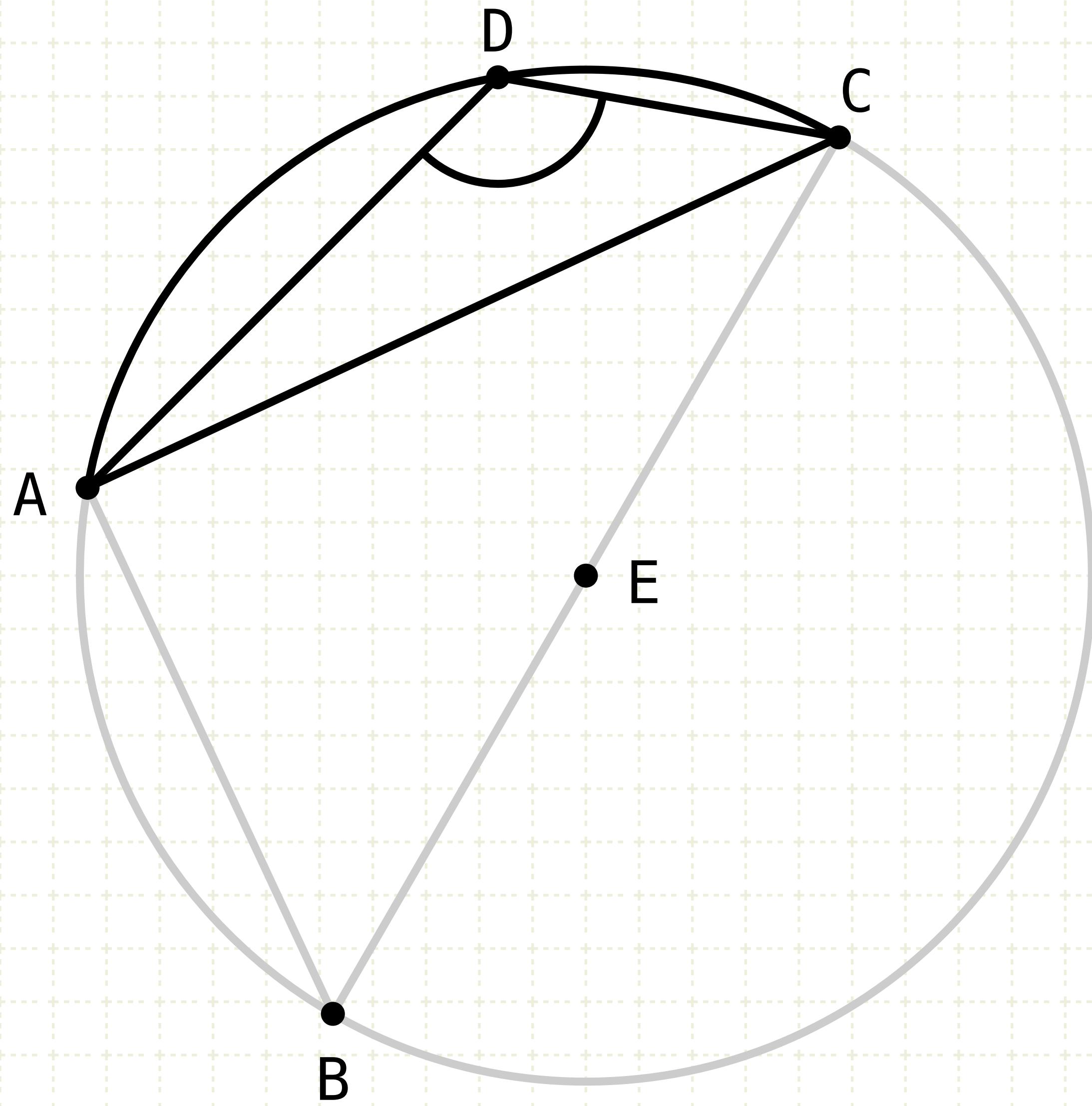
Let E be the centre of the circle and BC the diameter

The angle BAC in the semicircle segment BAC is right

The angle ABC in the 'greater than a semicircle' segment ABC is less than a right angle

Proposition 31 of Book III

In a circle the angle in the semicircle is right, that in a greater segment less than a right angle, and that in a less segment greater than a right angle, and further the angle of the greater segment is greater than a right angle, and the angle of the less segment less than a right angle.



∠BAC = ∟
∠ABC < ∟
∠ADC > ∟

In other words

Let E be the centre of the circle and BC the diameter

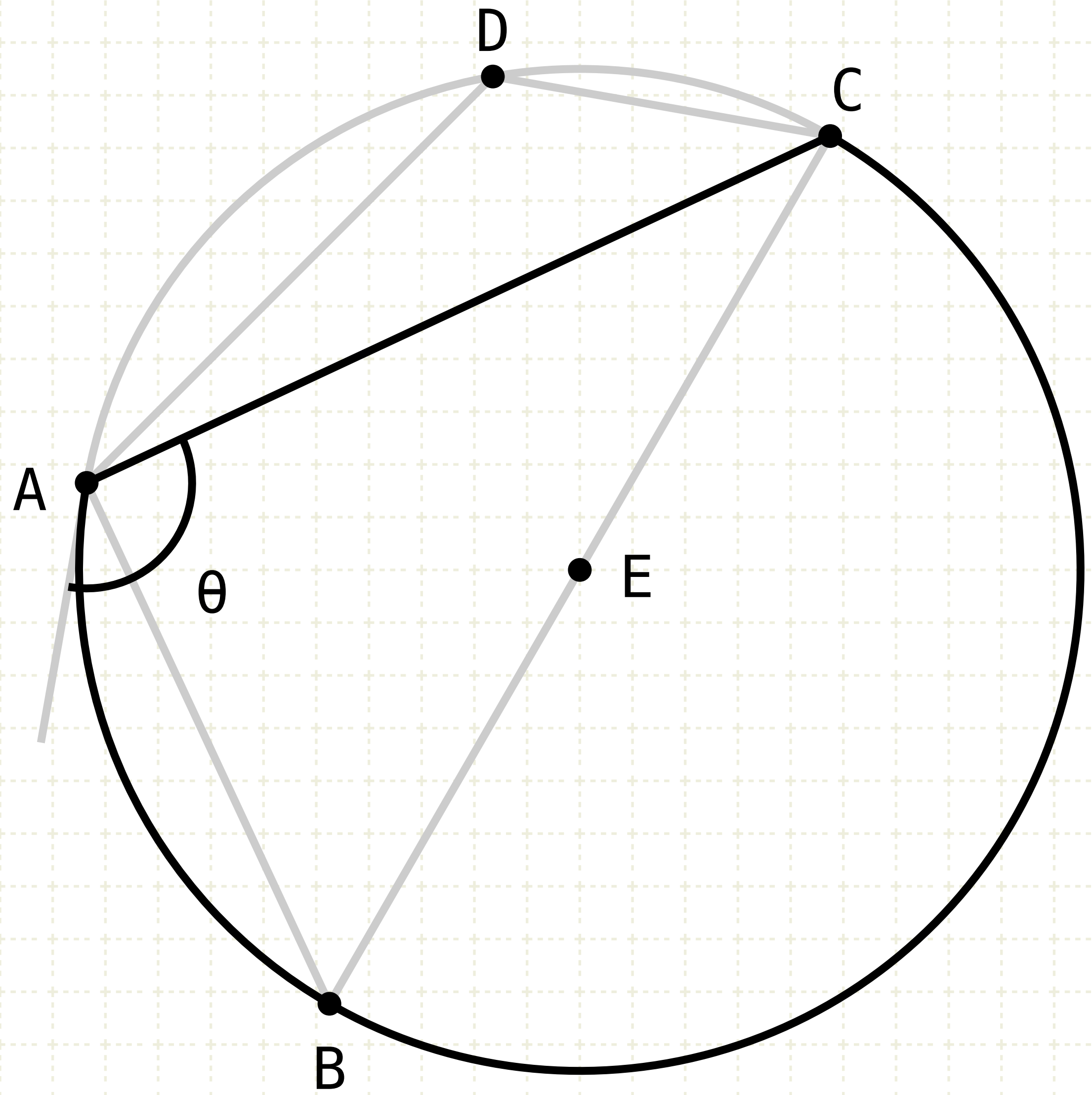
The angle BAC in the semicircle segment BAC is right

The angle ABC in the 'greater than a semicircle' segment ABC is less than a right angle

The angle ADC in the 'less than a semicircle' segment ADC is greater than a right angle

Proposition 31 of Book III

In a circle the angle in the semicircle is right, that in a greater segment less than a right angle, and that in a less segment greater than a right angle, and further the angle of the greater segment is greater than a right angle, and the angle of the less segment less than a right angle.



$\angle BAC = ∟$

$\angle ABC < ∟$

$\angle ADC > ∟$

$\theta > ∟$

In other words

Let E be the centre of the circle and BC the diameter

The angle BAC in the semicircle segment BAC is right

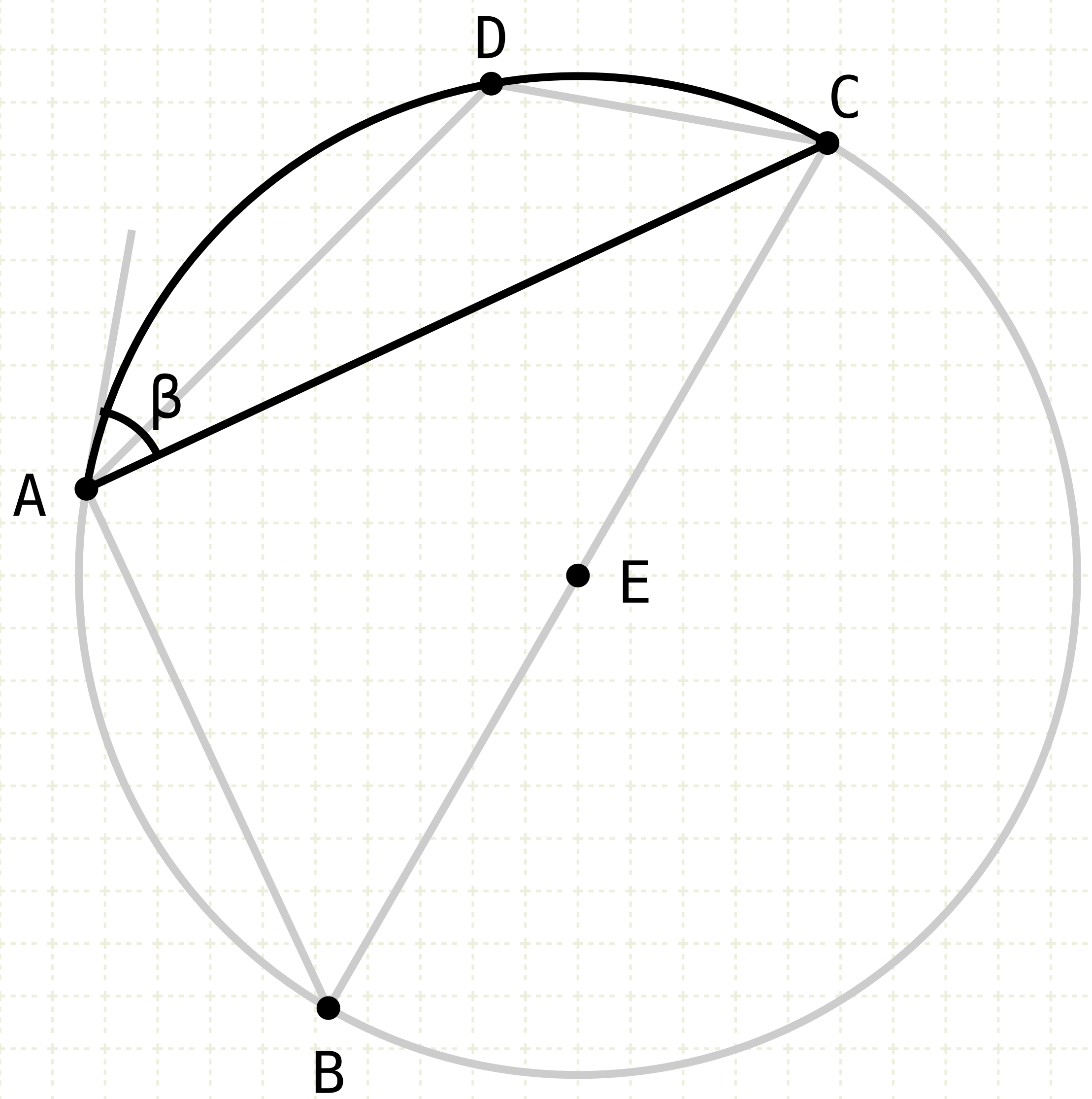
The angle ABC in the 'greater than a semicircle' segment ABC is less than a right angle

The angle ADC in the 'less than a semicircle' segment ADC is greater than a right angle

The angle between the line AC and the segment ABC is larger than a right angle

Proposition 31 of Book III

In a circle the angle in the semicircle is right, that in a greater segment less than a right angle, and that in a less segment greater than a right angle, and further the angle of the greater segment is greater than a right angle, and the angle of the less segment less than a right angle.



$\angle BAC = \llcorner$

$\angle ABC < \llcorner$

$\angle ADC > \llcorner$

$\theta > \llcorner$

$\beta < \llcorner$

In other words

Let E be the centre of the circle and BC the diameter

The angle BAC in the semicircle segment BAC is right

The angle ABC in the 'greater than a semicircle' segment ABC is less than a right angle

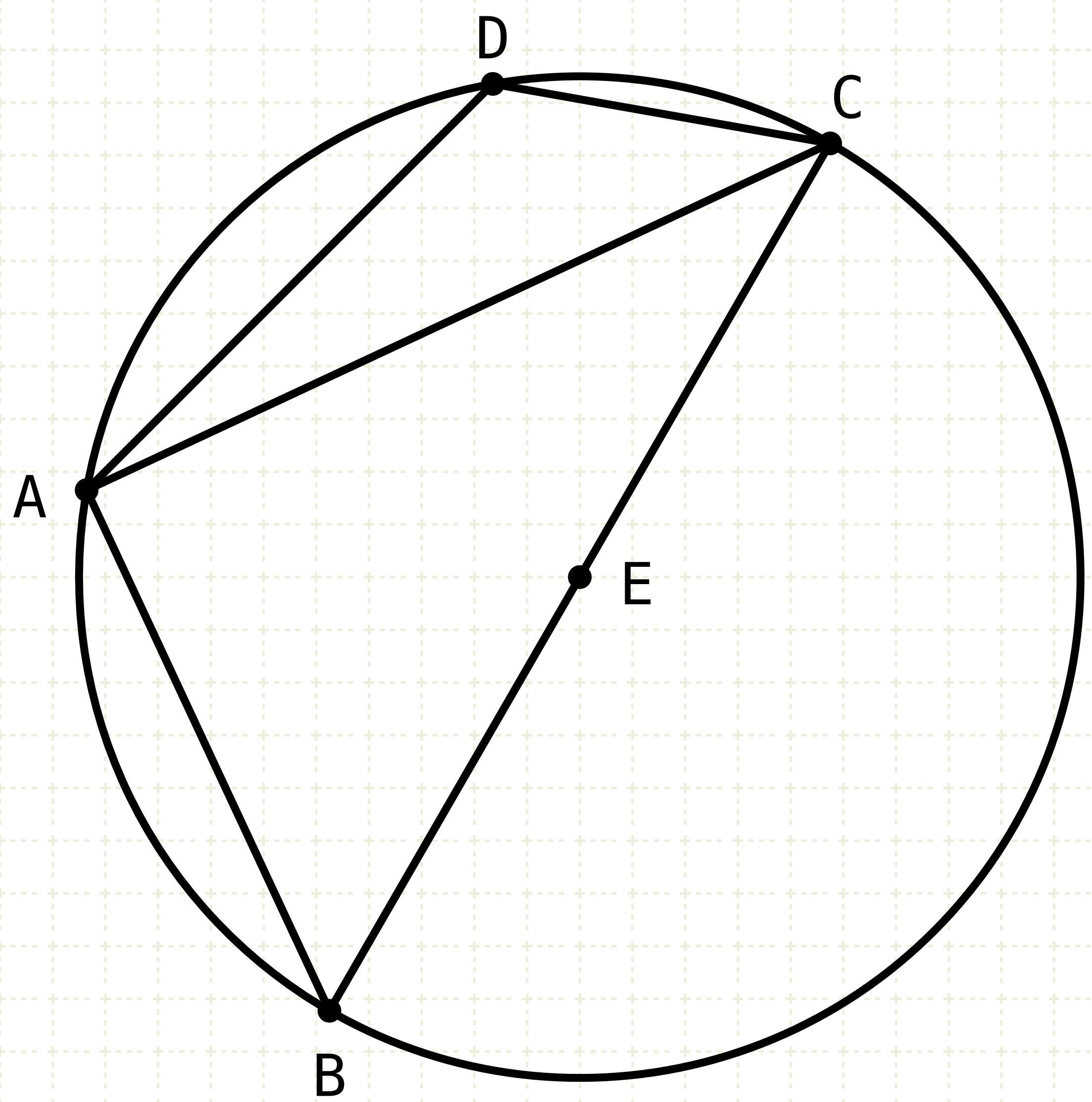
The angle ADC in the 'less than a semicircle' segment ADC is greater than a right angle

The angle between the line AC and the segment ABC is larger than a right angle

The angle between the line AC and the segment ADC is less than a right angle

Proposition 31 of Book III

In a circle the angle in the semicircle is right, that in a greater segment less than a right angle, and that in a less segment greater than a right angle, and further the angle of the greater segment is greater than a right angle, and the angle of the less segment less than a right angle.



BC = diameter
BE = EC

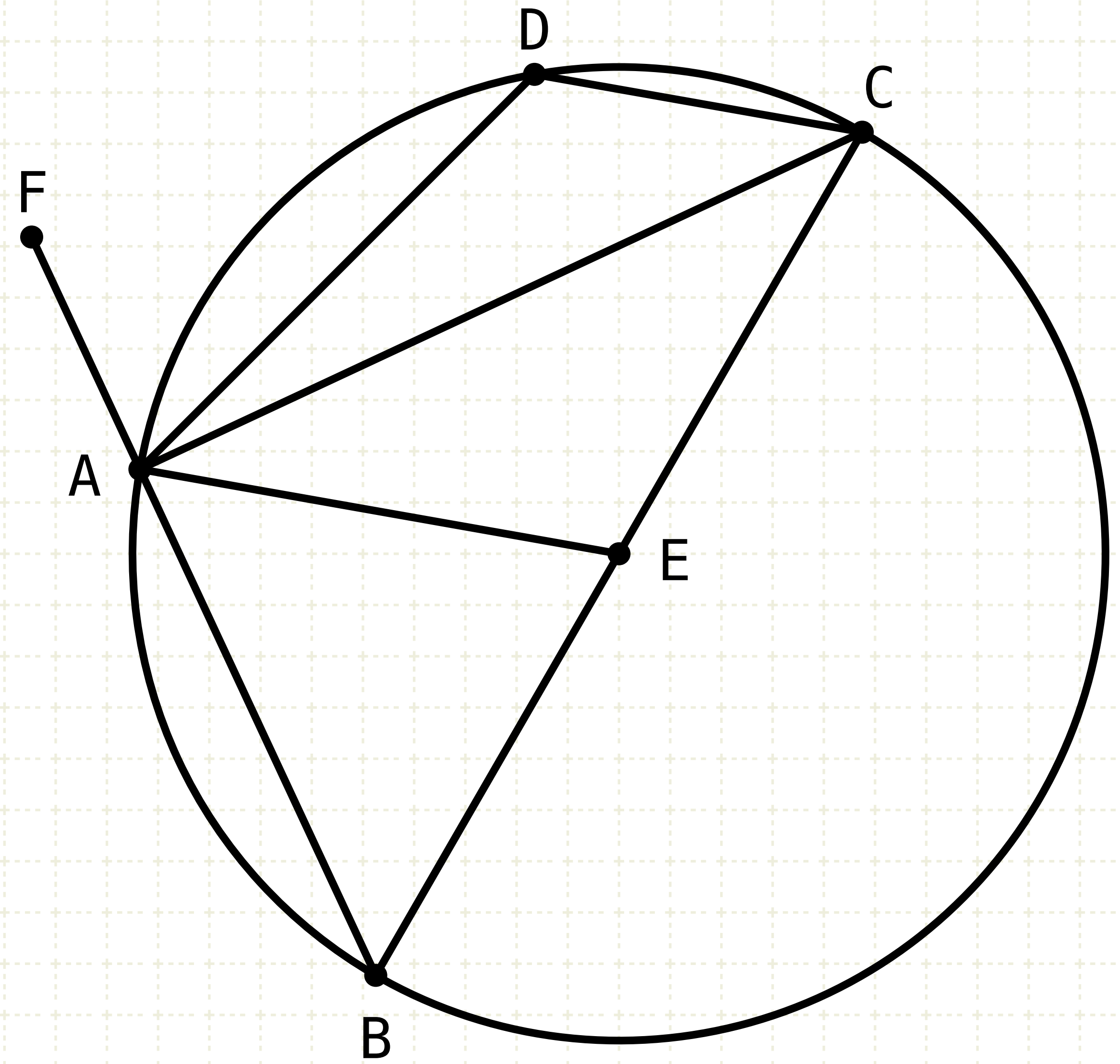
In other words

∠BAC = ∟, ∠ABC < ∟, ∠ADC > ∟

Proof

Proposition 31 of Book III

In a circle the angle in the semicircle is right, that in a greater segment less than a right angle, and that in a less segment greater than a right angle, and further the angle of the greater segment is greater than a right angle, and the angle of the less segment less than a right angle.



BC = diameter
BE = EC

In other words

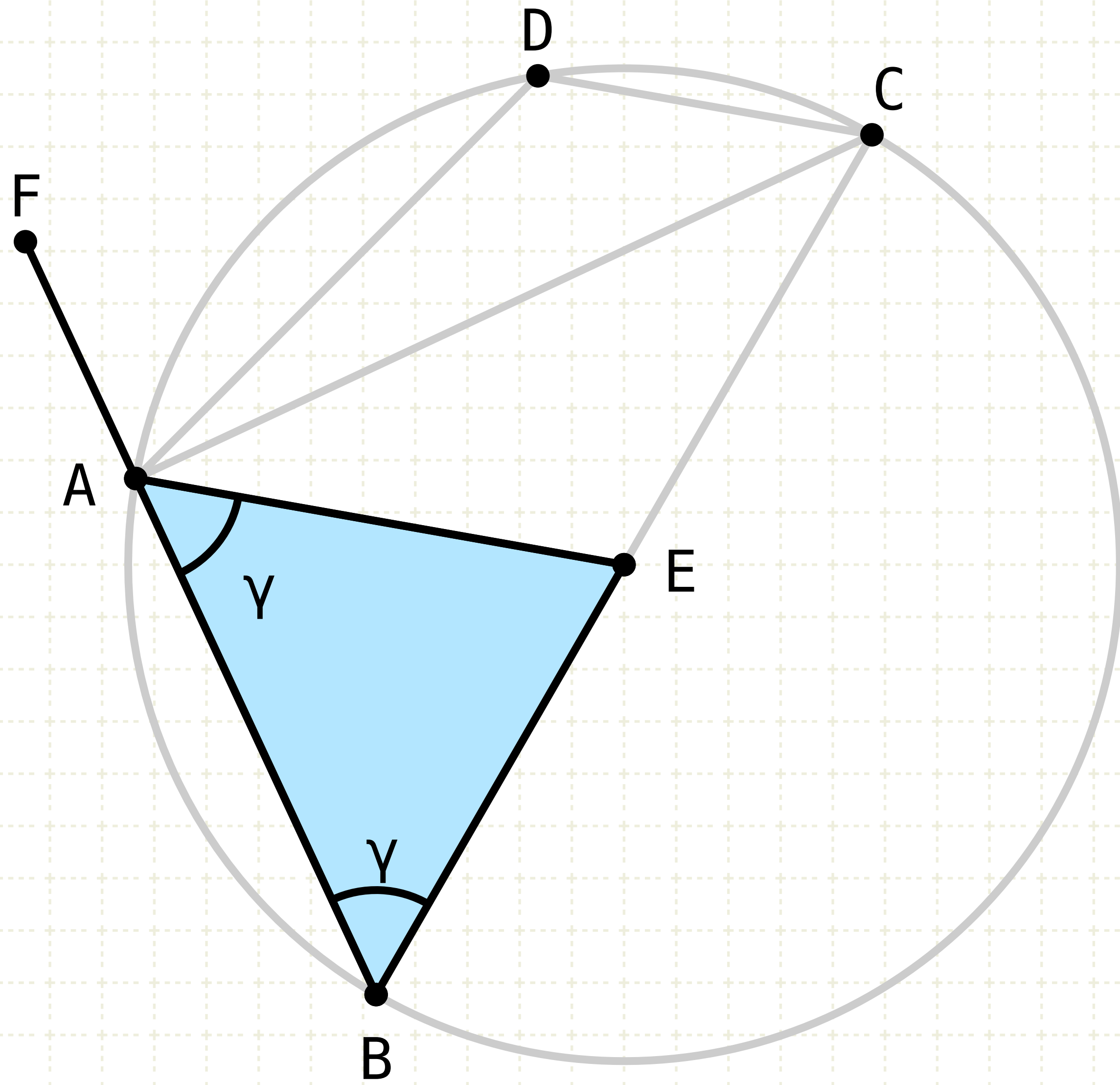
∠BAC = ∟, ∠ABC < ∟, ∠ADC > ∟

Proof

Draw line AE, and extend line BA to the point F

Proposition 31 of Book III

In a circle the angle in the semicircle is right, that in a greater segment less than a right angle, and that in a less segment greater than a right angle, and further the angle of the greater segment is greater than a right angle, and the angle of the less segment less than a right angle.



BC = diameter
BE = EC
∠ABE = ∠BAE = γ

In other words

∠BAC = ∟, ∠ABC < ∟, ∠ADC > ∟

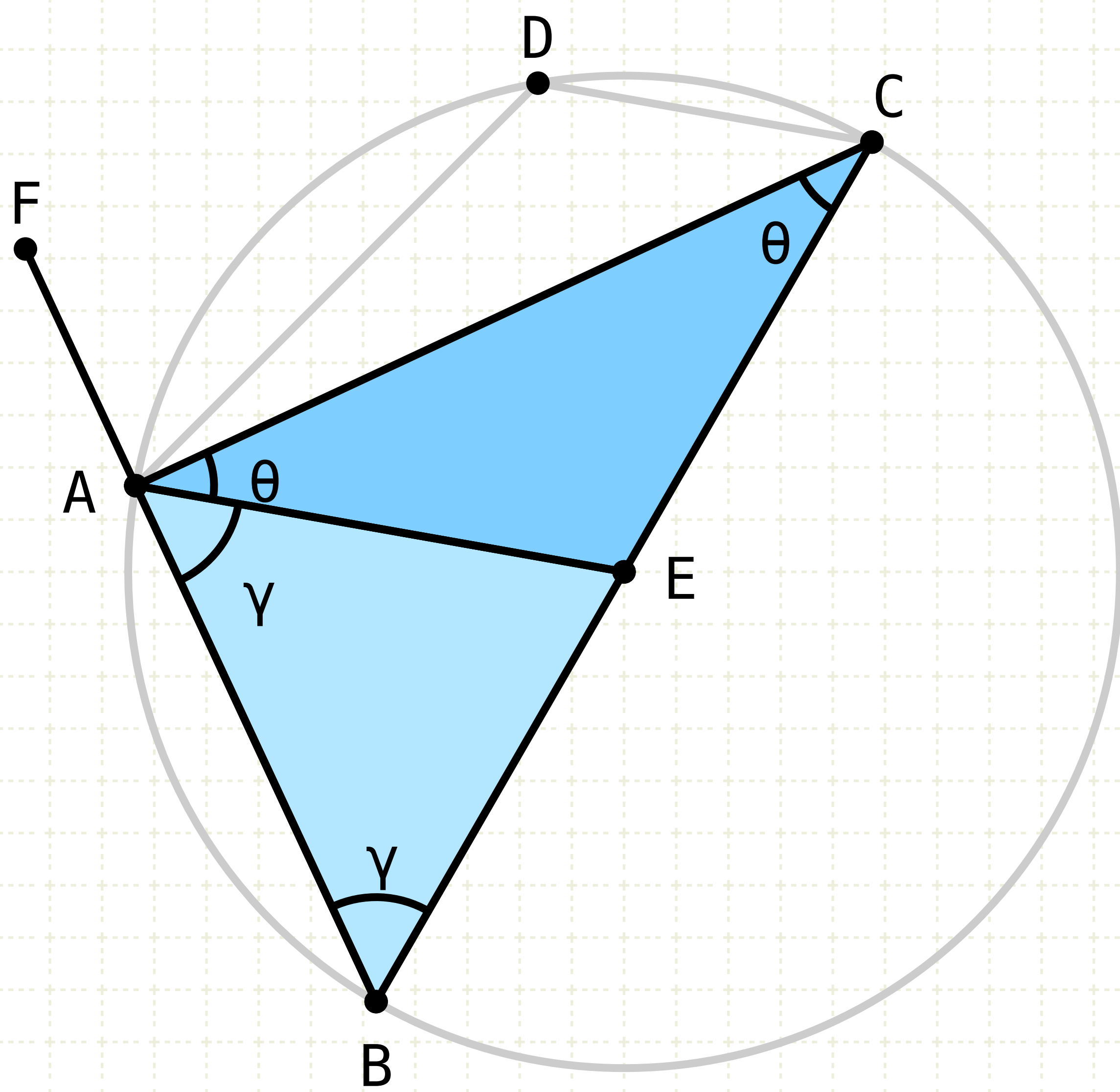
Proof

Draw line AE, and extend line BA to the point F

Since AE and BE are equal, ABE is an isosceles triangle, and the angles BAE and ABE are equal (I·5)

Proposition 31 of Book III

In a circle the angle in the semicircle is right, that in a greater segment less than a right angle, and that in a less segment greater than a right angle, and further the angle of the greater segment is greater than a right angle, and the angle of the less segment less than a right angle.



BC = diameter
BE = EC
∠ABE = ∠BAE = γ
∠CAE = ∠ACE = θ

In other words

∠BAC = ∟, ∠ABC < ∟, ∠ADC > ∟

Proof

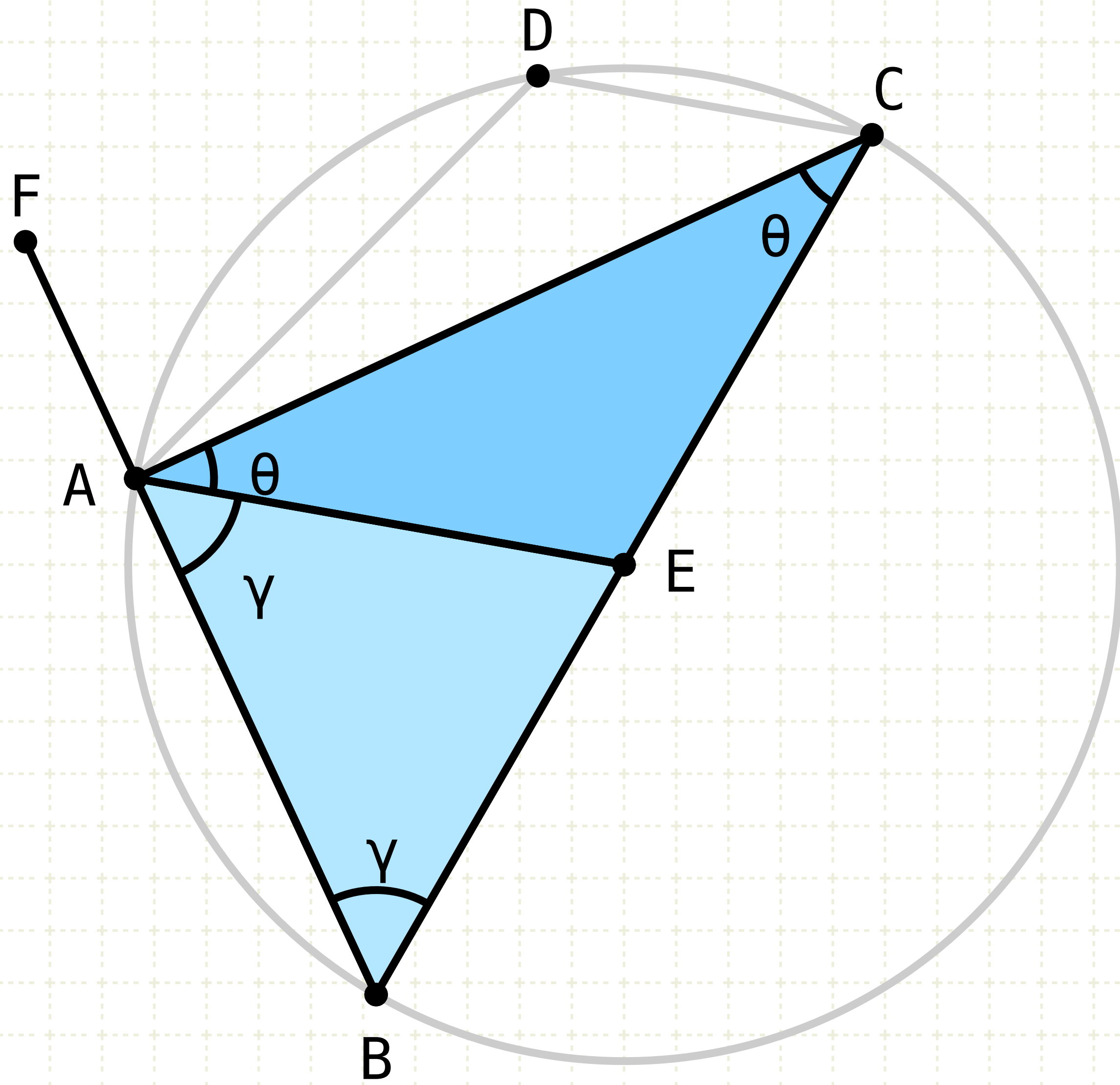
Draw line AE, and extend line BA to the point F

Since AE and BE are equal, ABE is an isosceles triangle, and the angles BAE and ABE are equal (I·5)

Similarly AE and CE are equal, ACE is an isosceles triangle, and the angles CAE and ACE are equal (I·5)

Proposition 31 of Book III

In a circle the angle in the semicircle is right, that in a greater segment less than a right angle, and that in a less segment greater than a right angle, and further the angle of the greater segment is greater than a right angle, and the angle of the less segment less than a right angle.



BC = diameter
BE = EC
∠ABE = ∠BAE = γ
∠CAE = ∠ACE = θ
∠BAC = γ + θ

In other words

∠BAC = ∟, ∠ABC < ∟, ∠ADC > ∟

Proof

Draw line AE, and extend line BA to the point F

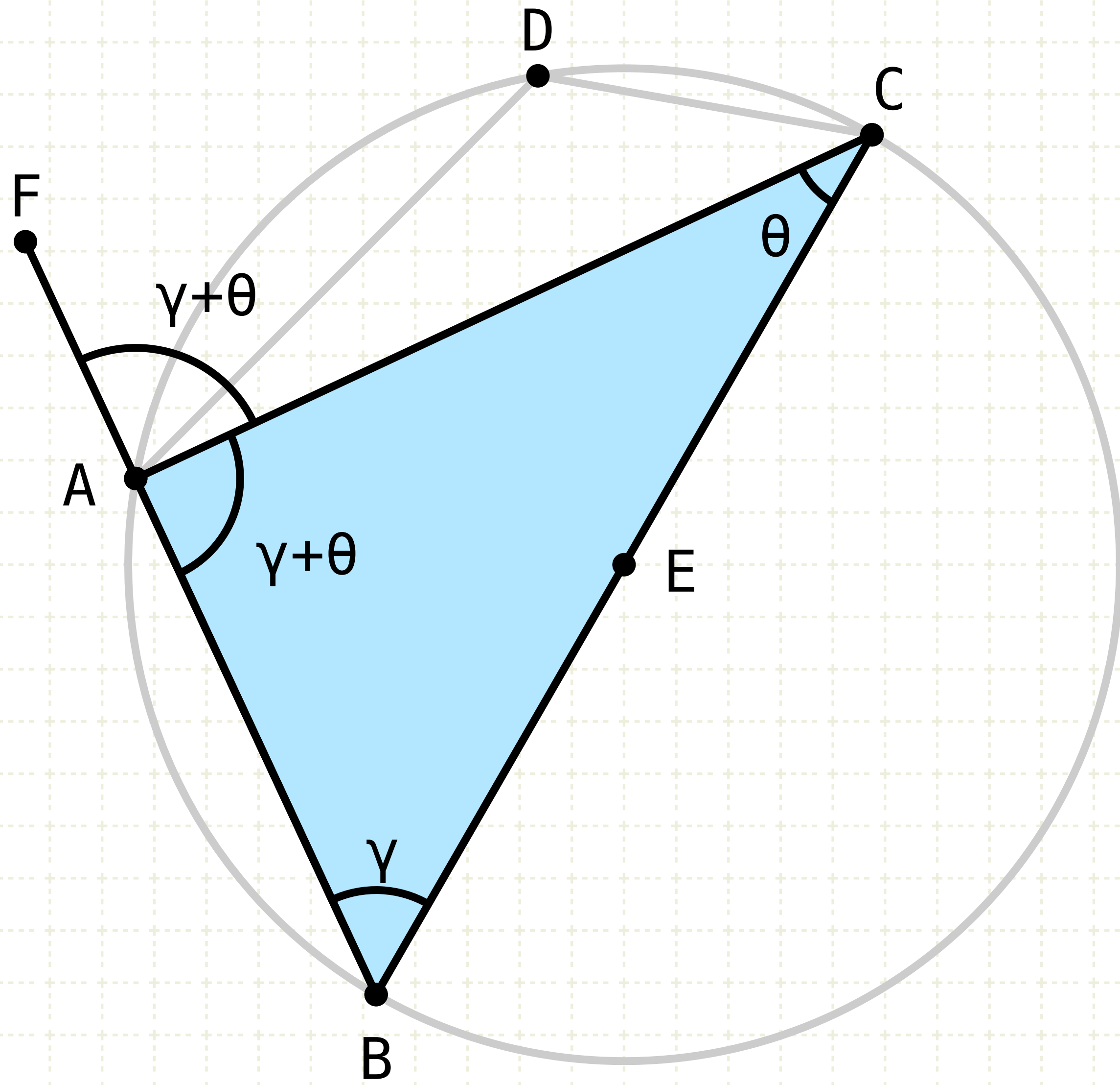
Since AE and BE are equal, ABE is an isosceles triangle, and the angles BAE and ABE are equal (I·5)

Similarly AE and CE are equal, ACE is an isosceles triangle, and the angles CAE and ACE are equal (I·5)

Thus, the angle BAC is the sum of CAE,BAE, or ACE,ABE

Proposition 31 of Book III

In a circle the angle in the semicircle is right, that in a greater segment less than a right angle, and that in a less segment greater than a right angle, and further the angle of the greater segment is greater than a right angle, and the angle of the less segment less than a right angle.



$BC = \text{diameter}$

$BE = EC$

$\angle ABE = \angle BAE = \gamma$

$\angle CAE = \angle ACE = \theta$

$\angle BAC = \gamma + \theta$

$\angle FAC = \gamma + \theta$

In other words

$\angle BAC = \llcorner, \ \angle ABC < \llcorner, \ \angle ADC > \llcorner$

Proof

Draw line AE, and extend line BA to the point F

Since AE and BE are equal, ABE is an isosceles triangle, and the angles BAE and ABE are equal (I·5)

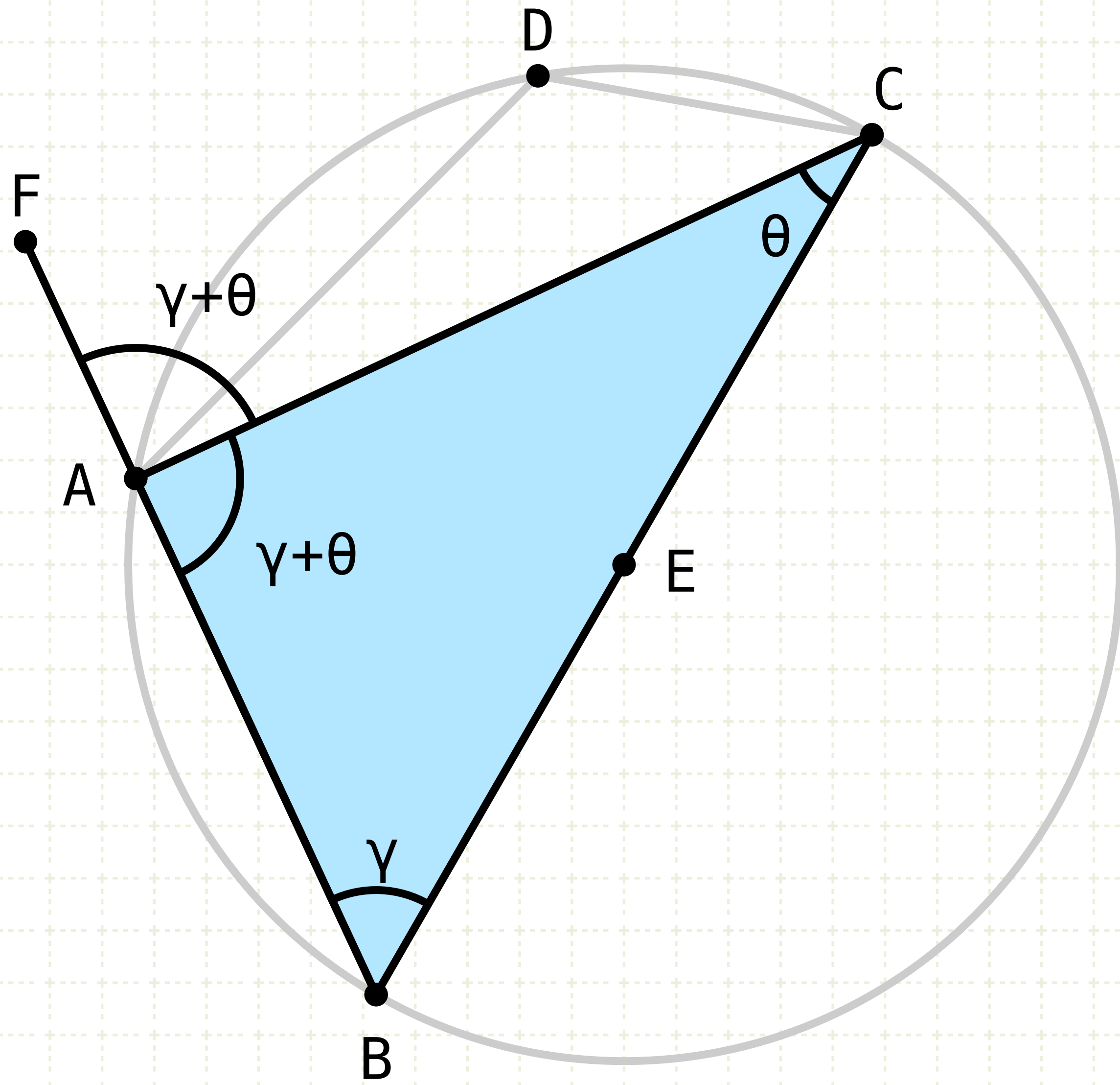
Similarly AE and CE are equal, ACE is an isosceles triangle, and the angles CAE and ACE are equal (I·5)

Thus, the angle BAC is the sum of CAE,BAE, or ACE,ABE

The exterior angle FAC is equal to the sum of the opposite interior angles of the triangle BAC (I·32)

Proposition 31 of Book III

In a circle the angle in the semicircle is right, that in a greater segment less than a right angle, and that in a less segment greater than a right angle, and further the angle of the greater segment is greater than a right angle, and the angle of the less segment less than a right angle.



BC = diameter
BE = EC
∠ABE = ∠BAE = γ
∠CAE = ∠ACE = θ
∠BAC = γ + θ
∠FAC = γ + θ
∠FAC = ∠BAC = α = ∟

In other words

∠BAC = ∟, ∠ABC < ∟, ∠ADC > ∟

Proof

Draw line AE, and extend line BA to the point F

Since AE and BE are equal, ABE is an isosceles triangle, and the angles BAE and ABE are equal (I·5)

Similarly AE and CE are equal, ACE is an isosceles triangle, and the angles CAE and ACE are equal (I·5)

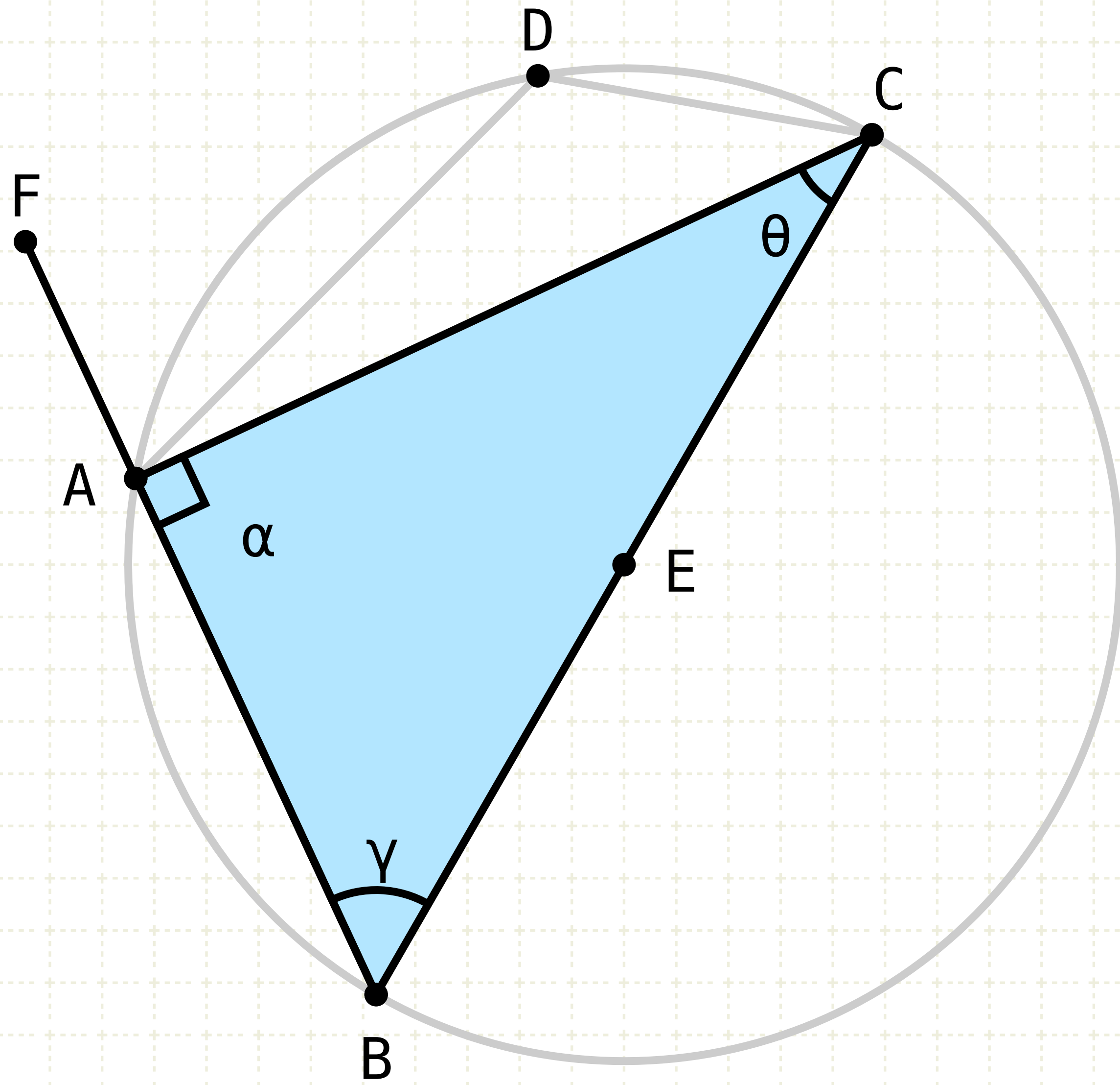
Thus, the angle BAC is the sum of CAE,BAE, or ACE,ABE

The exterior angle FAC is equal to the sum of the opposite interior angles of the triangle BAC (I·32)

By definition, if angle FAC equals angle BAC, then they are both right angles

Proposition 31 of Book III

In a circle the angle in the semicircle is right, that in a greater segment less than a right angle, and that in a less segment greater than a right angle, and further the angle of the greater segment is greater than a right angle, and the angle of the less segment less than a right angle.



BC = diameter
BE = EC
∠ABE = ∠BAE = γ
∠CAE = ∠ACE = θ
∠BAC = γ + θ
∠FAC = γ + θ
∠FAC = ∠BAC = α = ∟

In other words

∠BAC = ∟, ∠ABC < ∟, ∠ADC > ∟

Proof

Draw line AE, and extend line BA to the point F

Since AE and BE are equal, ABE is an isosceles triangle, and the angles BAE and ABE are equal (I·5)

Similarly AE and CE are equal, ACE is an isosceles triangle, and the angles CAE and ACE are equal (I·5)

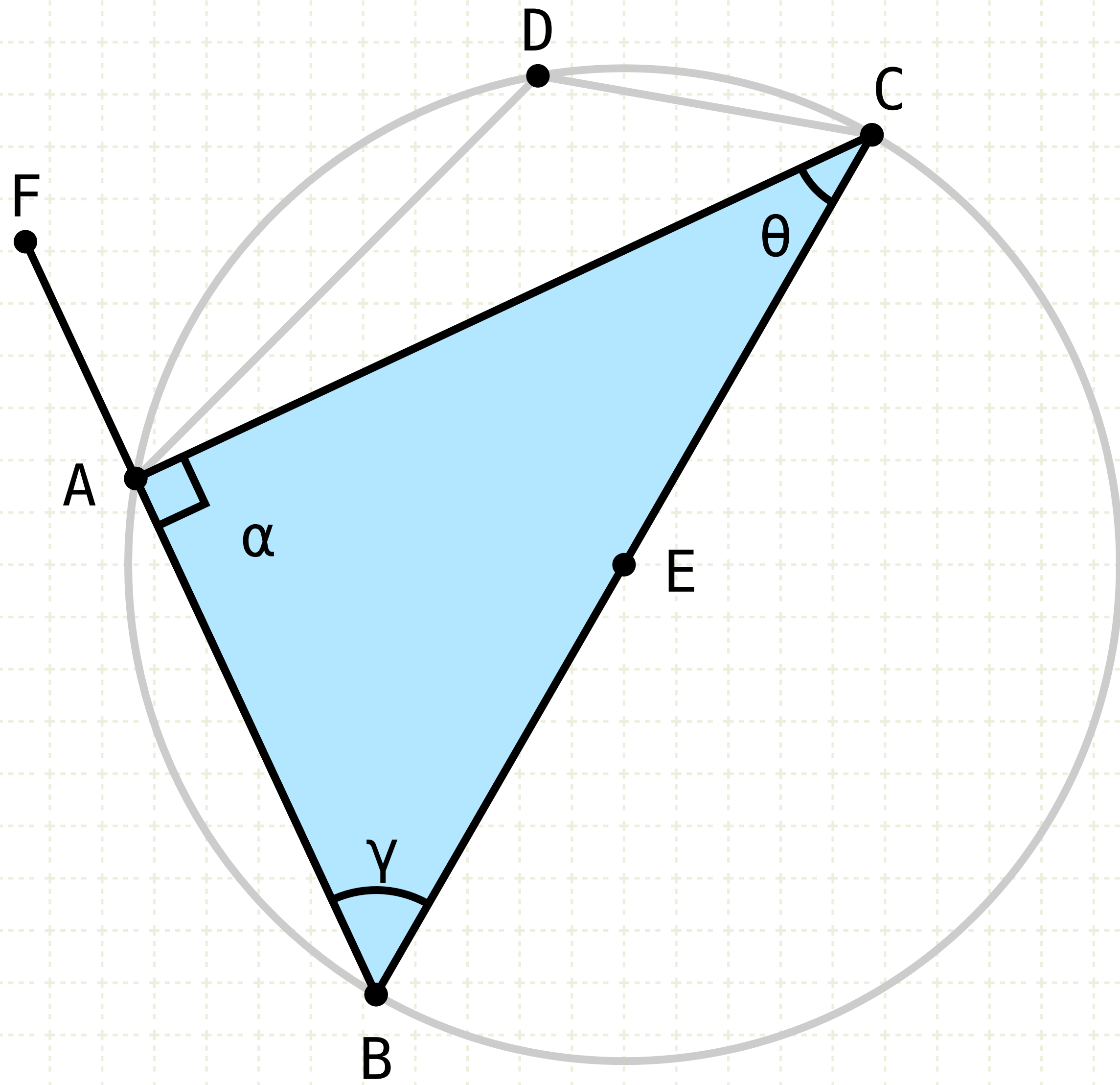
Thus, the angle BAC is the sum of CAE,BAE, or ACE,ABE

The exterior angle FAC is equal to the sum of the opposite interior angles of the triangle BAC (I·32)

By definition, if angle FAC equals angle BAC, then they are both right angles

Proposition 31 of Book III

In a circle the angle in the semicircle is right, that in a greater segment less than a right angle, and that in a less segment greater than a right angle, and further the angle of the greater segment is greater than a right angle, and the angle of the less segment less than a right angle.



BC = diameter
BE = EC
∠ABE = ∠BAE = γ
∠CAE = ∠ACE = θ
∠BAC = γ + θ
∠FAC = γ + θ
∠FAC = ∠BAC = α = ∟

α + γ < 2∟

In other words

∠BAC = ∟, ∠ABC < ∟, ∠ADC > ∟

Proof

Draw line AE, and extend line BA to the point F

Since AE and BE are equal, ABE is an isosceles triangle, and the angles BAE and ABE are equal (I·5)

Similarly AE and CE are equal, ACE is an isosceles triangle, and the angles CAE and ACE are equal (I·5)

Thus, the angle BAC is the sum of CAE,BAE, or ACE,ABE

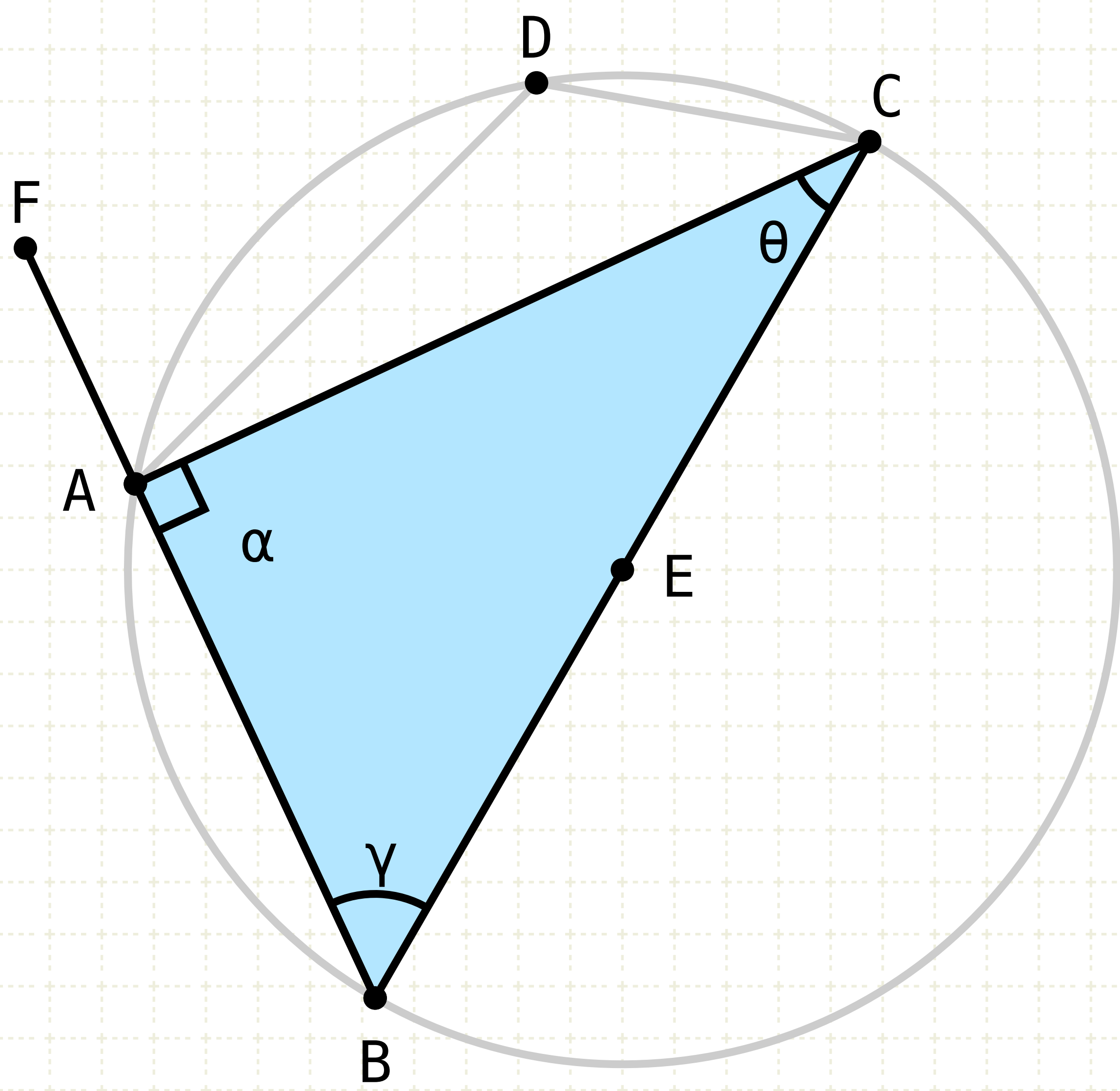
The exterior angle FAC is equal to the sum of the opposite interior angles of the triangle BAC (I·32)

By definition, if angle FAC equals angle BAC, then they are both right angles

In the triangle ABC, the angles ABC,BAC are less than two right angles (I·17)

Proposition 31 of Book III

In a circle the angle in the semicircle is right, that in a greater segment less than a right angle, and that in a less segment greater than a right angle, and further the angle of the greater segment is greater than a right angle, and the angle of the less segment less than a right angle.



$BC = \text{diameter}$

$BE = EC$

$\angle ABE = \angle BAE = \gamma$

$\angle CAE = \angle ACE = \theta$

$\angle BAC = \gamma + \theta$

$\angle FAC = \gamma + \theta$

$\angle FAC = \angle BAC = \alpha = \llcorner$

$\alpha + \gamma < 2\llcorner$

$\gamma < \llcorner$

In other words

$\angle BAC = \llcorner,\ \angle ABC < \llcorner,\ \angle ADC > \llcorner$

Proof

Draw line AE, and extend line BA to the point F

Since AE and BE are equal, ABE is an isosceles triangle, and the angles BAE and ABE are equal (I·5)

Similarly AE and CE are equal, ACE is an isosceles triangle, and the angles CAE and ACE are equal (I·5)

Thus, the angle BAC is the sum of CAE,BAE, or ACE,ABE

The exterior angle FAC is equal to the sum of the opposite interior angles of the triangle BAC (I·32)

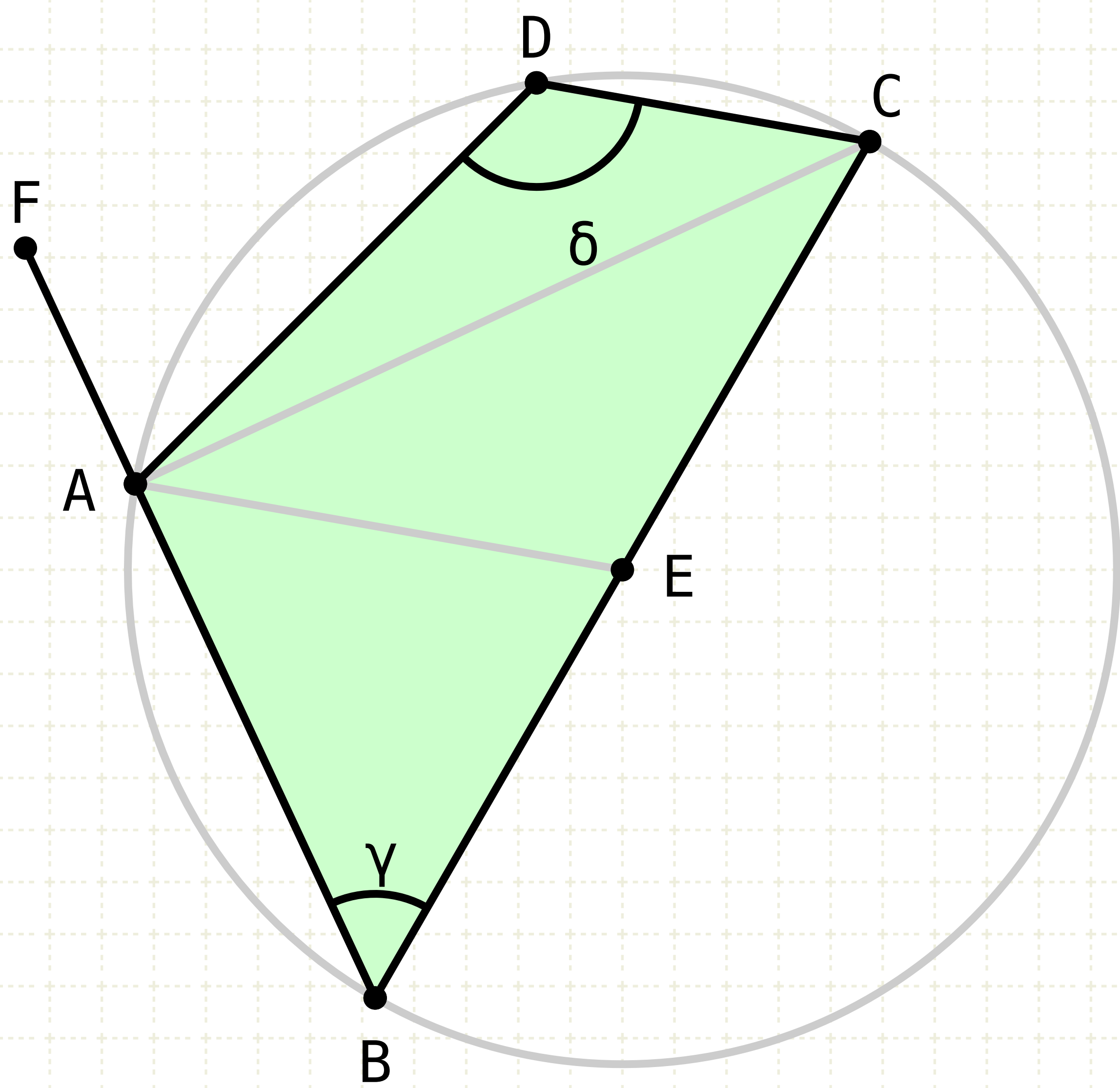
By definition, if angle FAC equals angle BAC, then they are both right angles

In the triangle ABC, the angles ABC,BAC are less than two right angles (I·17)

… and since BAC is a right angle, angle ABC is less than a right angle

Proposition 31 of Book III

In a circle the angle in the semicircle is right, that in a greater segment less than a right angle, and that in a less segment greater than a right angle, and further the angle of the greater segment is greater than a right angle, and the angle of the less segment less than a right angle.



$BC = \text{diameter}$
$BE = EC$
$\angle ABE = \angle BAE = \gamma$
$\angle CAE = \angle ACE = \theta$
$\angle BAC = \gamma + \theta$
$\angle FAC = \gamma + \theta$
$\angle FAC = \angle BAC = \alpha = \llcorner$

$\alpha + \gamma < 2\llcorner$
$\gamma < \llcorner$

$\gamma + \delta = 2\llcorner$

In other words

$\angle BAC = \llcorner,\ \angle ABC < \llcorner,\ \angle ADC > \llcorner$

Proof

Draw line AE, and extend line BA to the point F

Since AE and BE are equal, ABE is an isosceles triangle, and the angles BAE and ABE are equal (I·5)

Similarly AE and CE are equal, ACE is an isosceles triangle, and the angles CAE and ACE are equal (I·5)

Thus, the angle BAC is the sum of CAE,BAE, or ACE,ABE

The exterior angle FAC is equal to the sum of the opposite interior angles of the triangle BAC (I·32)

By definition, if angle FAC equals angle BAC, then they are both right angles

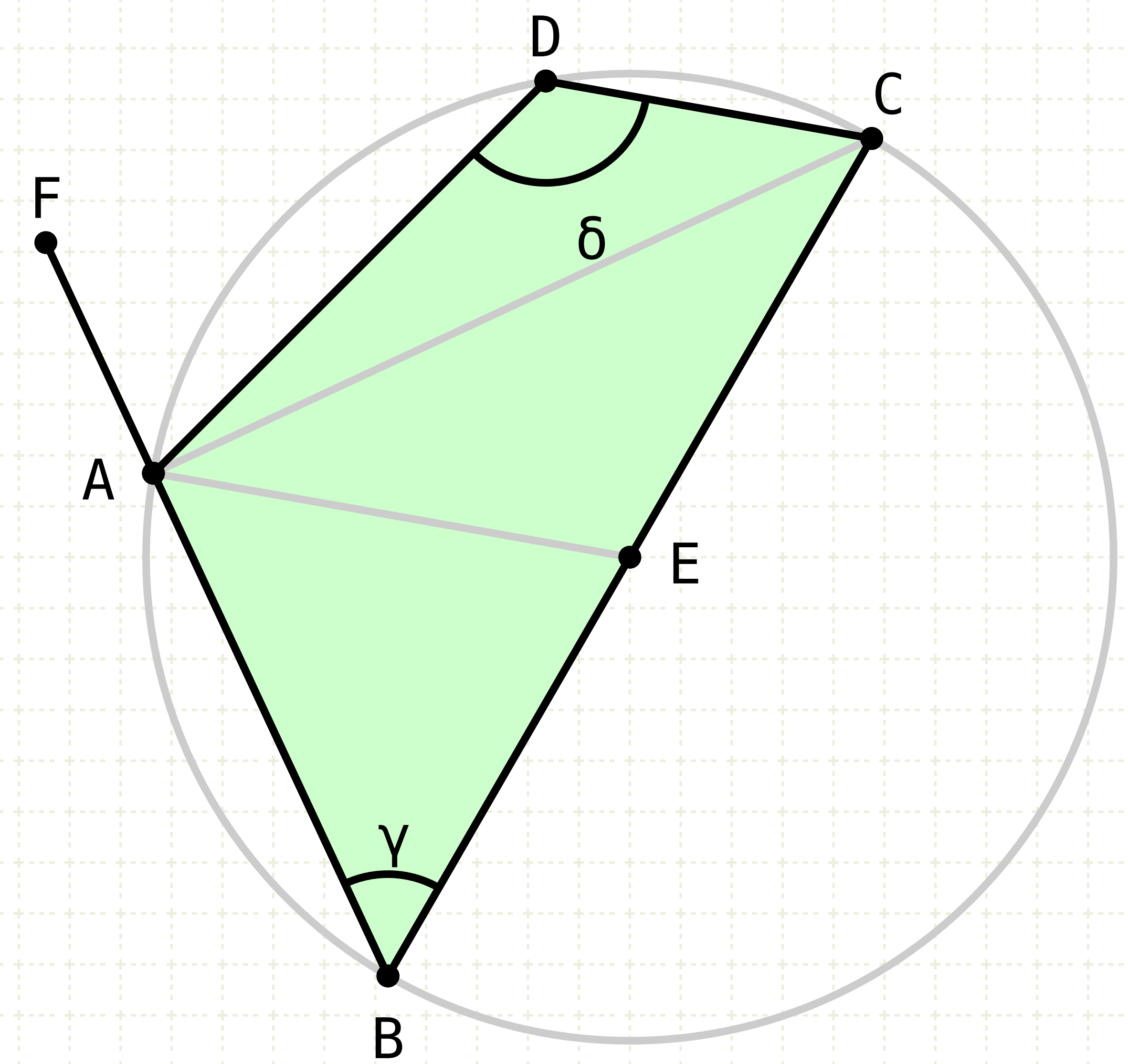
In the triangle ABC, the angles ABC,BAC are less than two right angles (I·17)

… and since BAC is a right angle, angle ABC is less than a right angle

A quadilateral inscribed in a circle has the sum of the opposite angles equal to two right angles (III·22), so ADC,ABC equals two right angles

Proposition 31 of Book III

In a circle the angle in the semicircle is right, that in a greater segment less than a right angle, and that in a less segment greater than a right angle, and further the angle of the greater segment is greater than a right angle, and the angle of the less segment less than a right angle.



$BC = \text{diameter}$

$BE = EC$

$\angle ABE = \angle BAE = \gamma$

$\angle CAE = \angle ACE = \theta$

$\angle BAC = \gamma + \theta$

$\angle FAC = \gamma + \theta$

$\angle FAC = \angle BAC = \alpha = \llcorner$

$\alpha + \gamma < 2\llcorner$

$\gamma < \llcorner$

$\gamma + \delta = 2\llcorner$

$\delta > \llcorner$

In other words

$\angle BAC = \llcorner,\ \angle ABC < \llcorner,\ \angle ADC > \llcorner$

Proof

Draw line AE, and extend line BA to the point F

Since AE and BE are equal, ABE is an isosceles triangle, and the angles BAE and ABE are equal (I·5)

Similarly AE and CE are equal, ACE is an isosceles triangle, and the angles CAE and ACE are equal (I·5)

Thus, the angle BAC is the sum of CAE,BAE, or ACE,ABE

The exterior angle FAC is equal to the sum of the opposite interior angles of the triangle BAC (I·32)

By definition, if angle FAC equals angle BAC, then they are both right angles

In the triangle ABC, the angles ABC,BAC are less than two right angles (I·17)

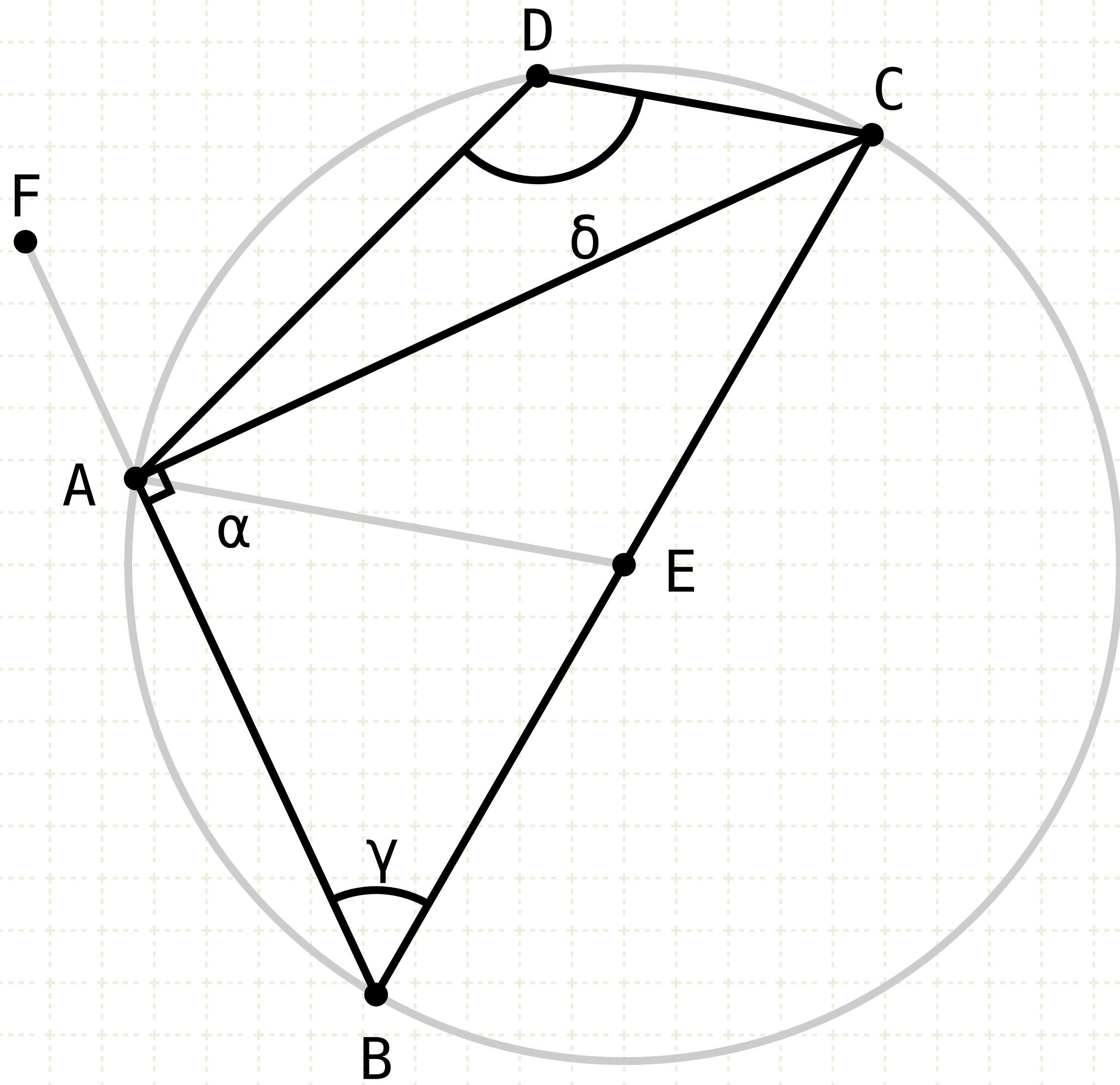
… and since BAC is a right angle, angle ABC is less than a right angle

A quadilateral inscribed in a circle has the sum of the opposite angles equal to two right angles (III·22), so ADC,ABC equals two right angles

Since ABC is less than one right angle, then ADC must be larger than a right angle

Proposition 31 of Book III

In a circle the angle in the semicircle is right, that in a greater segment less than a right angle, and that in a less segment greater than a right angle, and further the angle of the greater segment is greater than a right angle, and the angle of the less segment less than a right angle.



$BC = \text{diameter}$

$BE = EC$

$\angle ABE = \angle BAE = \gamma$

$\angle CAE = \angle ACE = \theta$

$\angle BAC = \gamma + \theta$

$\angle FAC = \gamma + \theta$

$\angle FAC = \angle BAC = \alpha = \llcorner$

$\alpha + \gamma < 2\llcorner$

$\gamma < \llcorner$

$\gamma + \delta = 2\llcorner$

$\delta > \llcorner$

In other words

$\angle BAC = \llcorner,\ \angle ABC < \llcorner,\ \angle ADC > \llcorner$

Proof

Draw line AE, and extend line BA to the point F

Since AE and BE are equal, ABE is an isosceles triangle, and the angles BAE and ABE are equal (I·5)

Similarly AE and CE are equal, ACE is an isosceles triangle, and the angles CAE and ACE are equal (I·5)

Thus, the angle BAC is the sum of CAE,BAE, or ACE,ABE

The exterior angle FAC is equal to the sum of the opposite interior angles of the triangle BAC (I·32)

By definition, if angle FAC equals angle BAC, then they are both right angles

In the triangle ABC, the angles ABC,BAC are less than two right angles (I·17)

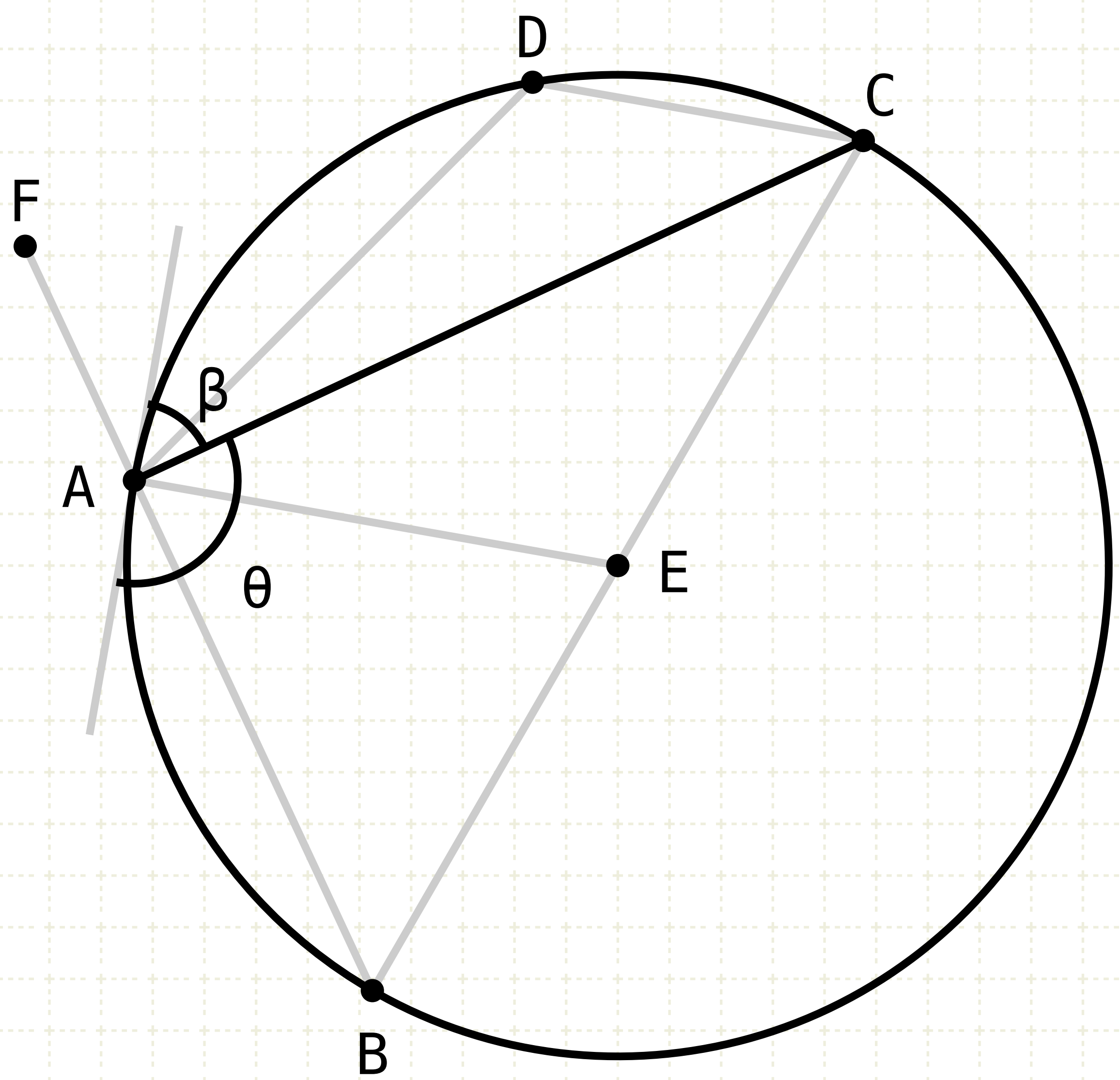
… and since BAC is a right angle, angle ABC is less than a right angle

A quadilateral inscribed in a circle has the sum of the opposite angles equal to two right angles (III·22), so ADC,ABC equals two right angles

Since ABC is less than one right angle, then ADC must be larger than a right angle

Proposition 31 of Book III

In a circle the angle in the semicircle is right, that in a greater segment less than a right angle, and that in a less segment greater than a right angle, and further the angle of the greater segment is greater than a right angle, and the angle of the less segment less than a right angle.



BC = diameter
BE = EC

In other words

∠BAC = ∟, ∠ABC < ∟, ∠ADC > ∟

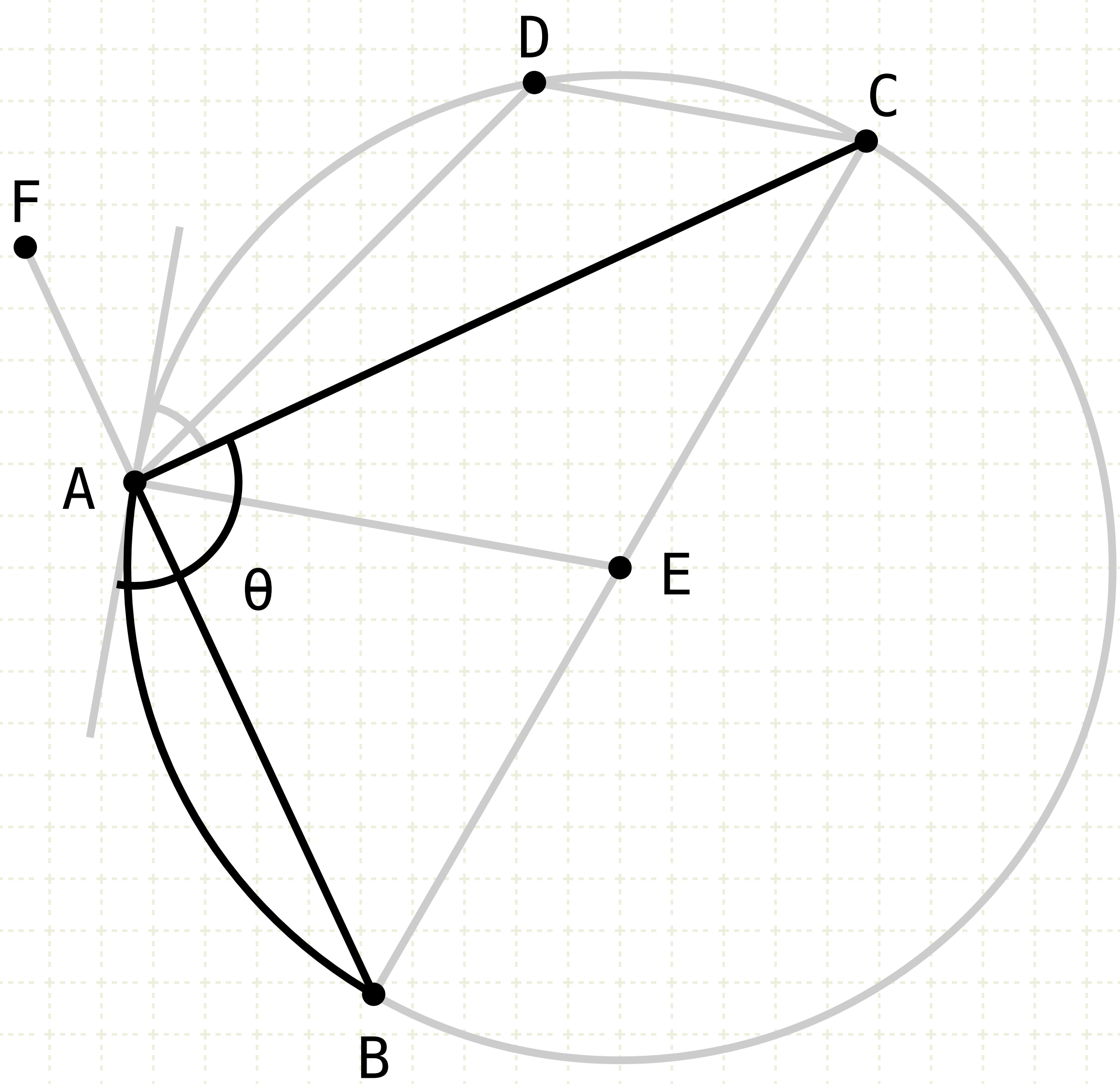
The angle between the line AC and the segment ADC is less than a right angle

The angle between the line AC and the segment ABC is greater than a right angle

Proof

Proposition 31 of Book III

In a circle the angle in the semicircle is right, that in a greater segment less than a right angle, and that in a less segment greater than a right angle, and further the angle of the greater segment is greater than a right angle, and the angle of the less segment less than a right angle.



BC = diameter
BE = EC
θ > ∟

In other words

∠BAC = ∟, ∠ABC < ∟, ∠ADC > ∟

The angle between the line AC and the segment ADC is less than a right angle

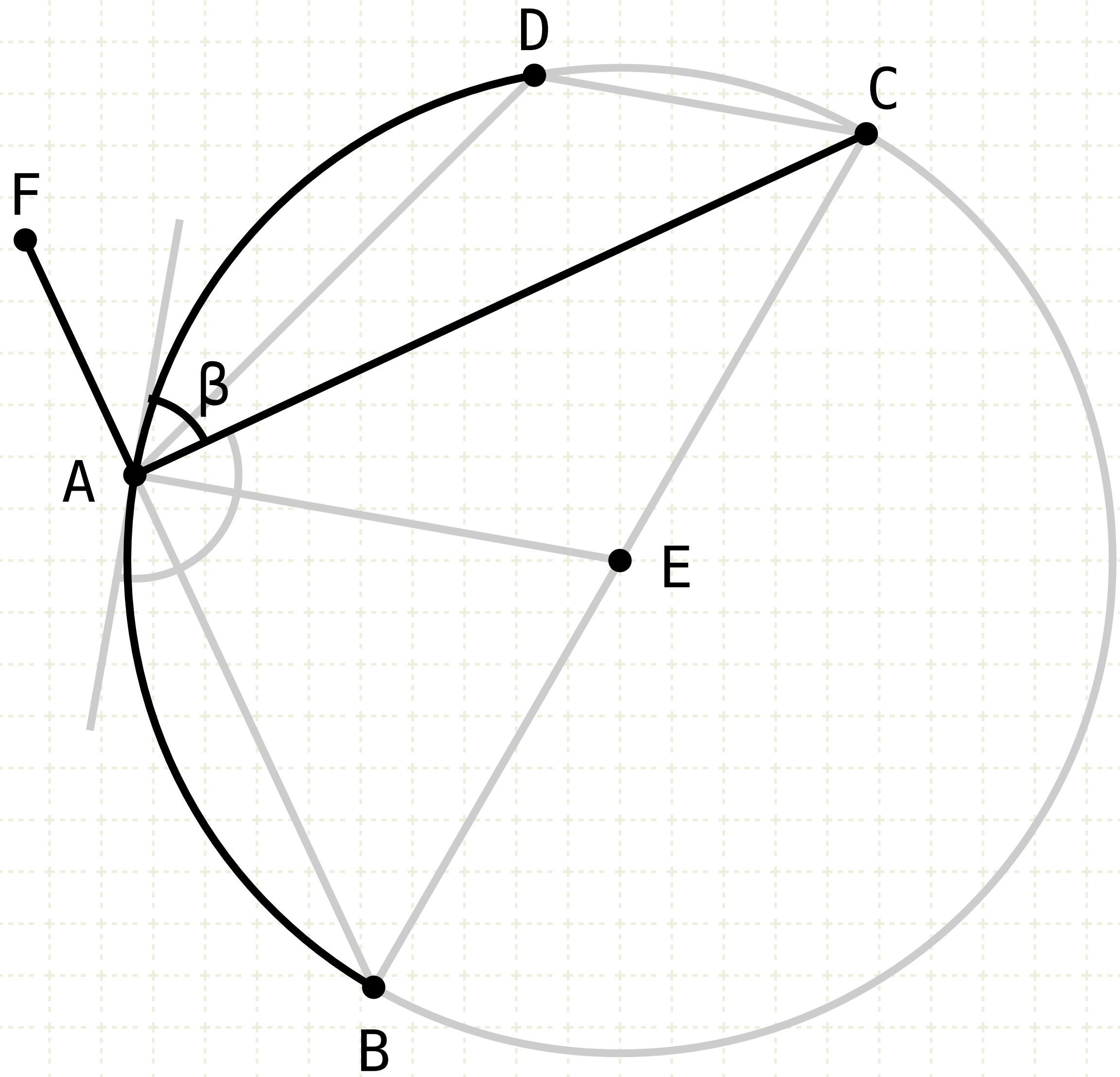
The angle between the line AC and the segment ABC is greater than a right angle

Proof

Angle BAC is right, and it is obvious that θ is greater than BAC, thus θ is greater than a right angle

Proposition 31 of Book III

In a circle the angle in the semicircle is right, that in a greater segment less than a right angle, and that in a less segment greater than a right angle, and further the angle of the greater segment is greater than a right angle, and the angle of the less segment less than a right angle.



BC = diameter
BE = EC
θ > ∟
β < ∟

In other words

∠BAC = ∟, ∠ABC < ∟, ∠ADC > ∟

The angle between the line AC and the segment ADC is less than a right angle

The angle between the line AC and the segment ABC is greater than a right angle

Proof

Angle BAC is right, and it is obvious that θ is greater than BAC, thus θ is greater than a right angle

Angle FAB is right, and it is obvious that β is less than FAB, thus β is less than a right angle

Youtube Videos

https://www.youtube.com/c/SandyBultena