

Euclid's Elements

Book V



Proportions are what makes the old Greek temples classic in their beauty. They are like huge blocks, from which the air has been literally hewn out between the columns.

$$AB:C = DE:F$$

$$BG:C = EH:F$$

$$AG:C = DH:F$$

Arne Jacobsen



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10	if $A:C > B:C$, or $A:C < B:C$ then $A > B$, or $A < C$, respectively				



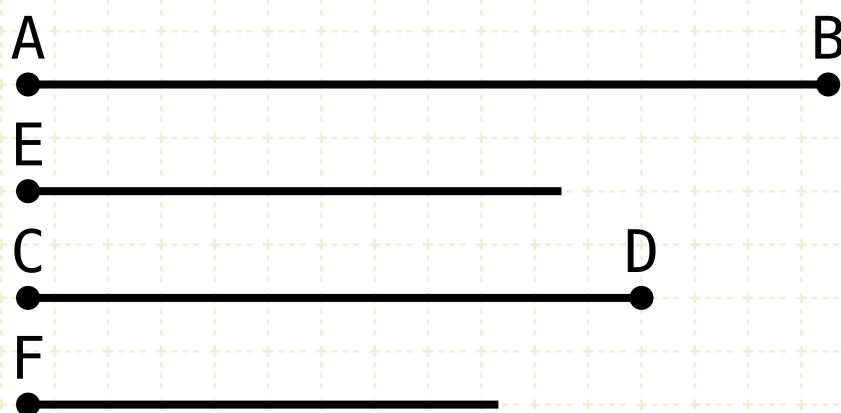
Proposition 25 of Book V

If four magnitudes be proportional, the greatest and the least are greater than the remaining two



Proposition 25 of Book V

If four magnitudes be proportional, the greatest and the least are greater than the remaining two



$$AB:CD = E:F$$

$$AB > CD, E, F \qquad F < CD, E, AB$$

$$\rightarrow AB + F > CD + E$$

In other words

Let AB, CD, E, F be proportional so that AB is to CD as is E to F, and let AB be the greatest of them, and F the least

Then the sum of AB,F is greater than the sum of CD,E



Proposition 25 of Book V

If four magnitudes be proportional, the greatest and the least are greater than the remaining two



$$AB:CD = E:F$$

$$AB > CD, E, F$$

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In other words

Let AB, CD, E, F be proportional so that AB is to CD as is E to F, and let AB be the greatest of them, and F the least

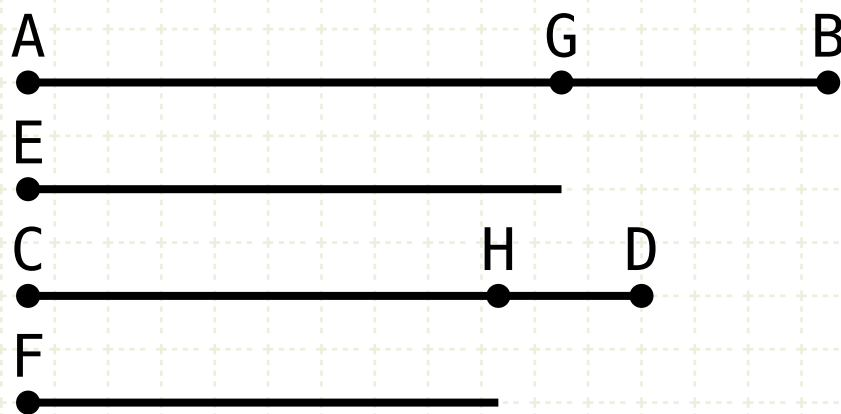
Then the sum of AB,F is greater than the sum of CD,E

Proof



Proposition 25 of Book V

If four magnitudes be proportional, the greatest and the least are greater than the remaining two



$AB:CD = E:F$
 $AB > CD, E, F$ $F < CD, E, AB$
 $AG = E$
 $CH = F$

In other words

Let AB, CD, E, F be proportional so that AB is to CD as is E to F, and let AB be the greatest of them, and F the least

Then the sum of AB,F is greater than the sum of CD,E

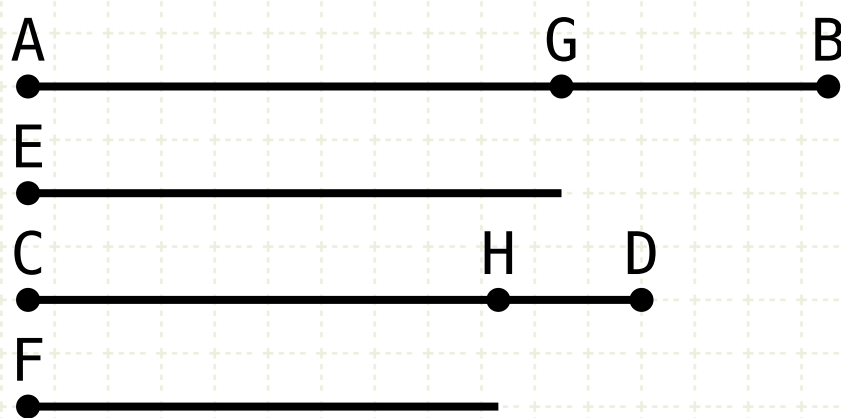
Proof

Let AG equal E, and CH equal F



Proposition 25 of Book V

If four magnitudes be proportional, the greatest and the least are greater than the remaining two



$AB:CD = E:F$

$AB > CD, E, F$ $F < CD, E, AB$

$AG = E$

$CH = F$

$AB:CD = AG:CH$

In other words

Let AB, CD, E, F be proportional so that AB is to CD as is E to F, and let AB be the greatest of them, and F the least

Then the sum of AB,F is greater than the sum of CD,E

Proof

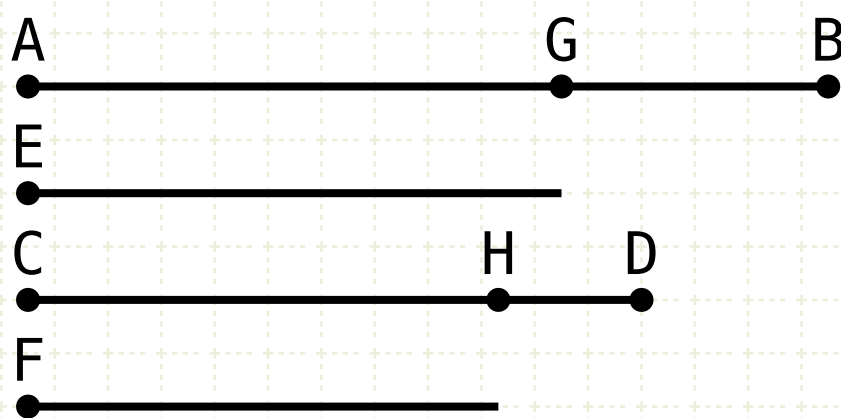
Let AG equal E, and CH equal F

Thus, AB is to CD as AG is to CH



Proposition 25 of Book V

If four magnitudes be proportional, the greatest and the least are greater than the remaining two



$AB:CD = E:F$
 $AB > CD, E, F$ $F < CD, E, AB$
 $AG = E$
 $CH = F$

$AB:CD = AG:CH$
 $AB:CD = GB:HD$

In other words

Let AB, CD, E, F be proportional so that AB is to CD as is E to F, and let AB be the greatest of them, and F the least

Then the sum of AB,F is greater than the sum of CD,E

Proof

Let AG equal E, and CH equal F

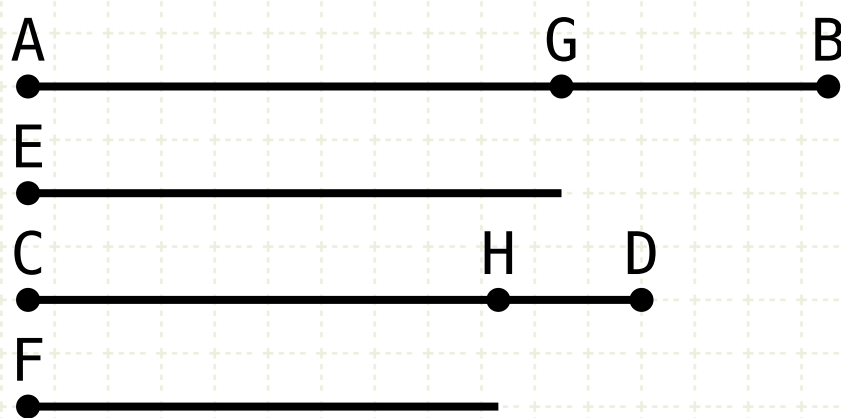
Thus, AB is to CD as AG is to CH

GB,HD are the remainder of AB,CD less AG,CH, therefore the remainder GB will be to the remainder HD as the whole AB is the whole CD (V.19)



Proposition 25 of Book V

If four magnitudes be proportional, the greatest and the least are greater than the remaining two



$$AB:CD = E:F$$

$$AB > CD, E, F \quad F < CD, E, AB$$

$$AG = E$$

$$CH = F$$

$$AB:CD = AG:CH$$

$$AB:CD = GB:HD$$

$$GB > HD$$

In other words

Let AB, CD, E, F be proportional so that AB is to CD as is E to F, and let AB be the greatest of them, and F the least

Then the sum of AB,F is greater than the sum of CD,E

Proof

Let AG equal E, and CH equal F

Thus, AB is to CD as AG is to CH

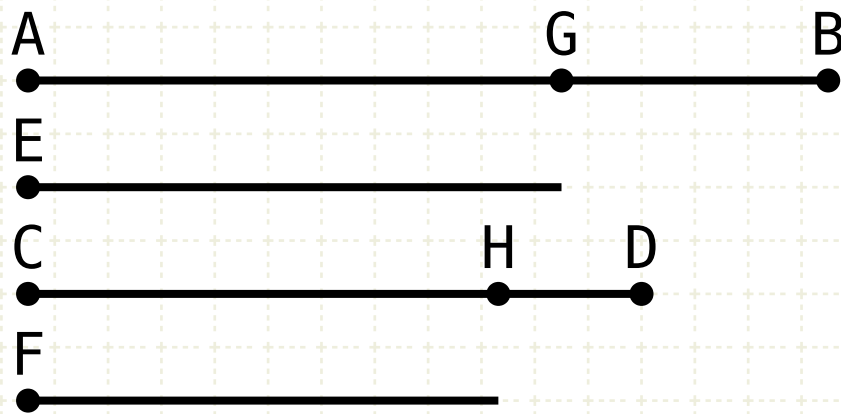
GB,HD are the remainder of AB,CD less AG,CH, therefore the remainder GB will be to the remainder HD as the whole AB is the whole CD (V·19)

But AB is greater than CD; therefore GB is greater than HD (V·def·5)



Proposition 25 of Book V

If four magnitudes be proportional, the greatest and the least are greater than the remaining two



$$AB:CD = E:F$$

$$AB > CD, E, F \quad F < CD, E, AB$$

$$AG = E$$

$$CH = F$$

$$AB:CD = AG:CH$$

$$AB:CD = GB:HD$$

$$GB > HD$$

$$AG + F = CH + E$$

In other words

Let AB, CD, E, F be proportional so that AB is to CD as is E to F, and let AB be the greatest of them, and F the least

Then the sum of AB,F is greater than the sum of CD,E

Proof

Let AG equal E, and CH equal F

Thus, AB is to CD as AG is to CH

GB,HD are the remainder of AB,CD less AG,CH, therefore the remainder GB will be to the remainder HD as the whole AB is the whole CD (V·19)

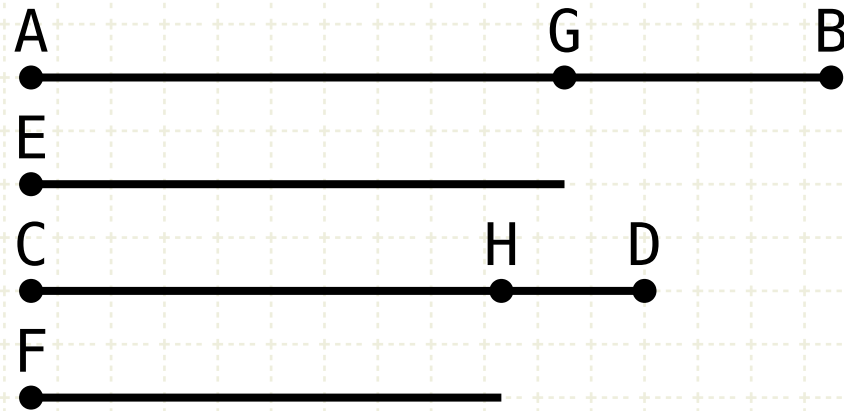
But AB is greater than CD; therefore GB is greater than HD (V·def·5)

Since AG is equal to E, and CH to F, The sum AG,F is equal to the sum CH,E



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If four magnitudes be proportional, the greatest and the least are greater than the remaining two



$$AB:CD = E:F$$

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$$AG = E$$

$$CH = F$$

$$AB:CD = AG:CH$$

$$AB:CD = GB:HD$$

$$GB > HD$$

$$AG + F = CH + E$$

$$GB + (AG+F) > HD + (CH+E)$$

In other words

Let AB, CD, E, F be proportional so that AB is to CD as is E to F, and let AB be the greatest of them, and F the least

Then the sum of AB,F is greater than the sum of CD,E

Proof

Let AG equal E, and CH equal F

Thus, AB is to CD as AG is to CH

GB,HD are the remainder of AB,CD less AG,CH, therefore the remainder GB will be to the remainder HD as the whole AB is the whole CD (V·19)

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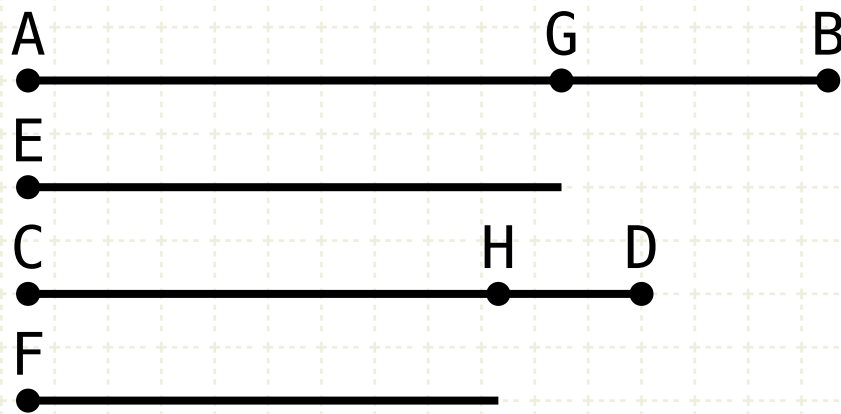
Since AG is equal to E, and CH to F, The sum AG,F is equal to the sum CH,E

GB is greater than HD, add AG,F to GB, and add CH,E to HD ...



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If four magnitudes be proportional, the greatest and the least are greater than the remaining two



$AB:CD = E:F$
 $AB > CD, E, F$ $F < CD, E, AB$
 $AG = E$
 $CH = F$

$AB:CD = AG:CH$
 $AB:CD = GB:HD$
 $GB > HD$
 $AG + F = CH + E$
 $GB + (AG+F) > HD + (CH+E)$
 $(GB+AG) + F > (HD+CH) + E$

In other words

Let AB, CD, E, F be proportional so that AB is to CD as is E to F, and let AB be the greatest of them, and F the least

Then the sum of AB,F is greater than the sum of CD,E

Proof

Let AG equal E, and CH equal F

Thus, AB is to CD as AG is to CH

GB,HD are the remainder of AB,CD less AG,CH, therefore the remainder GB will be to the remainder HD as the whole AB is the whole CD (V·19)

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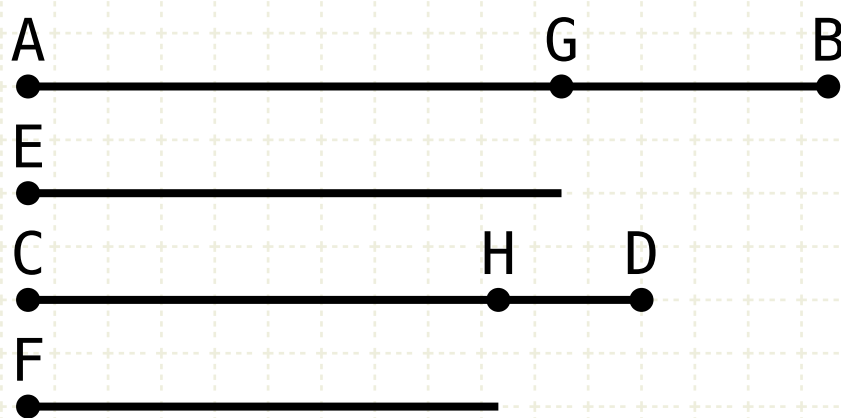
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$AB:CD = AG:CH$
 $AB:CD = GB:HD$
 $GB > HD$
 $AG + F = CH + E$
 $GB + (AG+F) > HD + (CH+E)$
 $(GB+AG) + F > (HD+CH) + E$
 $AB + F > CD + E$

In other words

Let AB, CD, E, F be proportional so that AB is to CD as is E to F, and let AB be the greatest of them, and F the least

Then the sum of AB,F is greater than the sum of CD,E

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Let AG equal E, and CH equal F

Thus, AB is to CD as AG is to CH

GB,HD are the remainder of AB,CD less AG,CH, therefore the remainder GB will be to the remainder HD as the whole AB is the whole CD (V·19)

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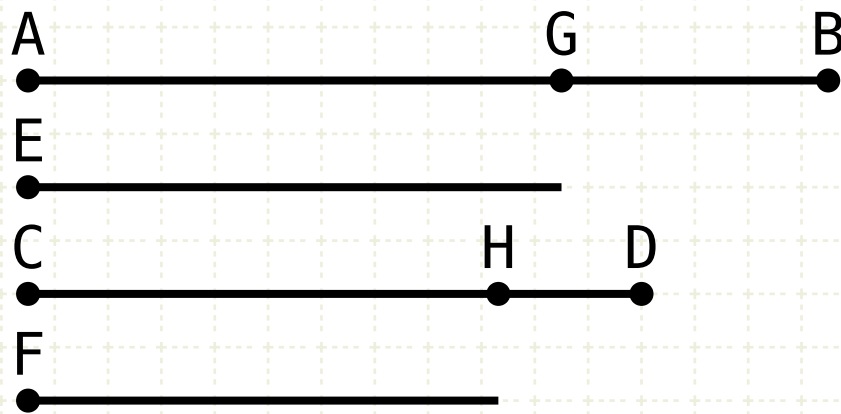
GB is greater than HD, add AG,F to GB, and add CH,E to HD ...

... if follows that the sum AB,F is greater than the sum CD,E



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 $GB > HD$
 $AG + F = CH + E$
 $GB + (AG+F) > HD + (CH+E)$
 $(GB+AG) + F > (HD+CH) + E$
 $AB + F > CD + E$

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GB is greater than HD, add AG,F to GB, and add CH,E to HD ...

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