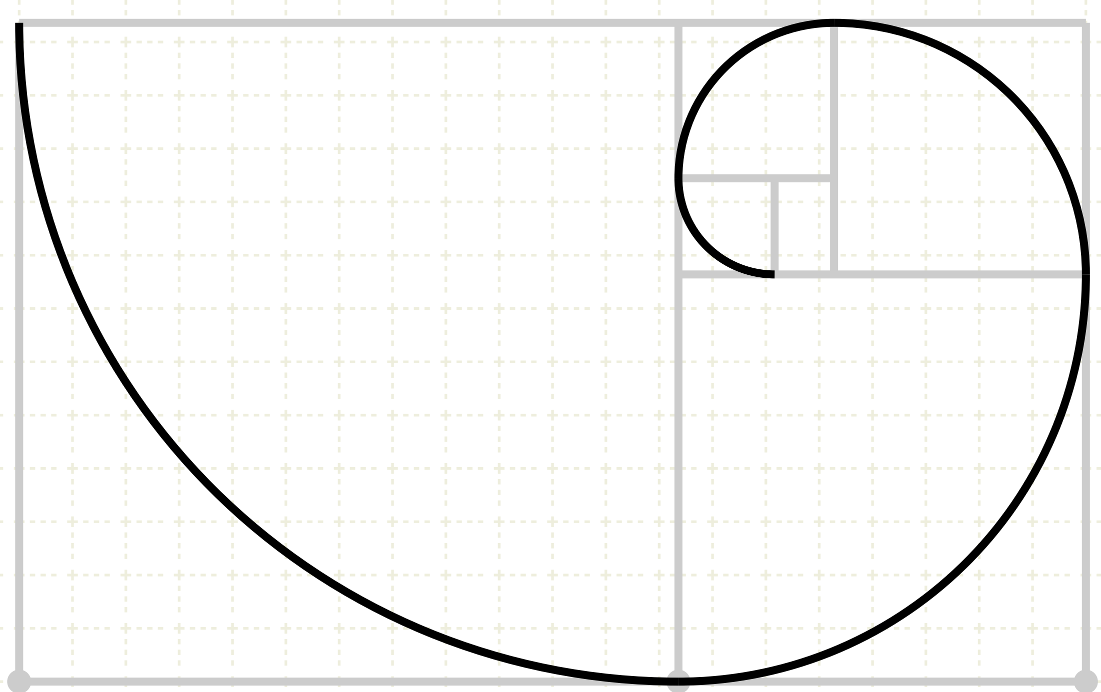
Euclid's Elements

Book VI



One can state, without exaggeration, that the observation of and the search for similarities and differences are the basis of all human knowledge.

Alfred Nobel

Table of Contents, Chapter 6

Table of Contents, Chapter 3

Proposition 15 of Book VI

In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and those triangles which have one angle equal to one angle, and in which the sides about the equal angles are reciprocally proportional, are equal.

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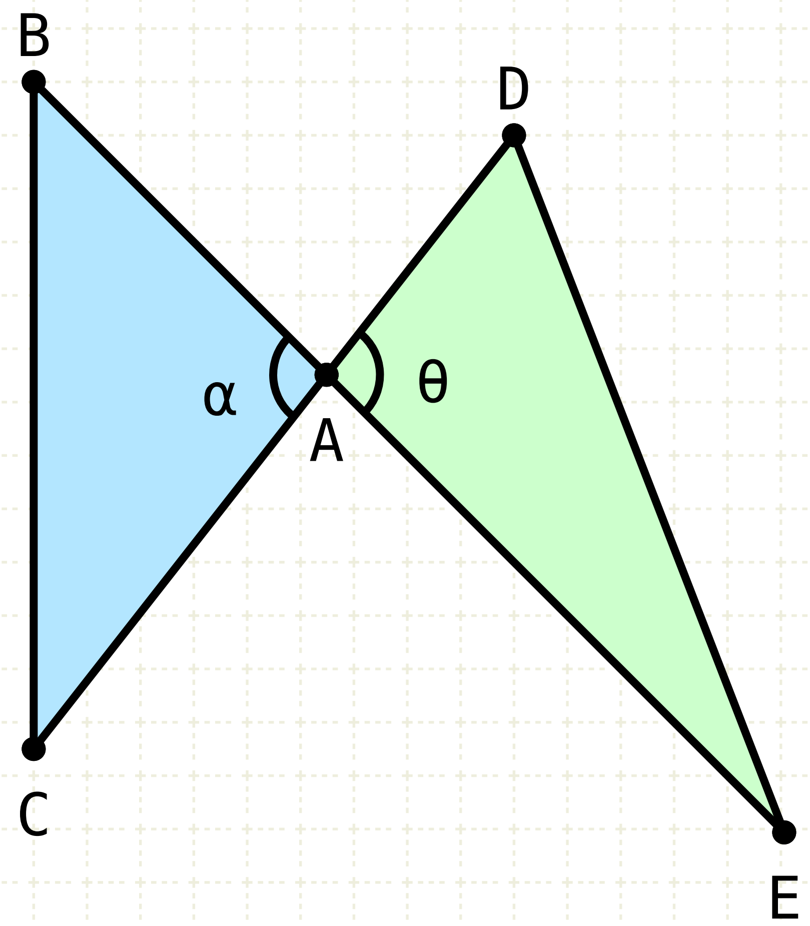

$\alpha = \theta$
$\Delta ABC = \Delta DAE$
$AC:AD = AE:AB$

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$\Delta ABC = \Delta DAE$

In other words

Given two triangles with one equal angle, and the areas of the triangle are equal, then the ratios of the sides around the equal angle is reciprocally proportional

… or … the multiplication of the two sides of two equal area triangles on either side of two equal angles remains constant

And the inverse

Note:

Assume two objects 'x' and 'y', both with properties '1' and '2'

Proportional:

$$x_1:y_1 = x_2:y_2$$

Reciprocally Proportional:

$$x_1:y_1 = y_2:x_2, \quad x_1 \cdot x_2 = y_1 \cdot y_2$$

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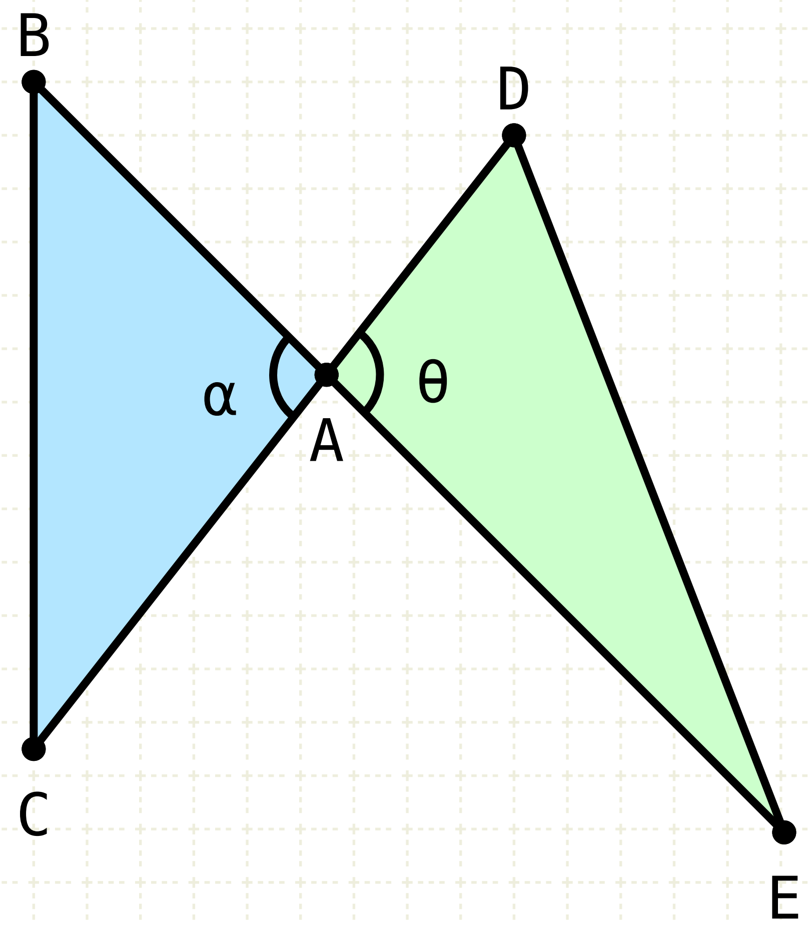

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Proof (Part 1)

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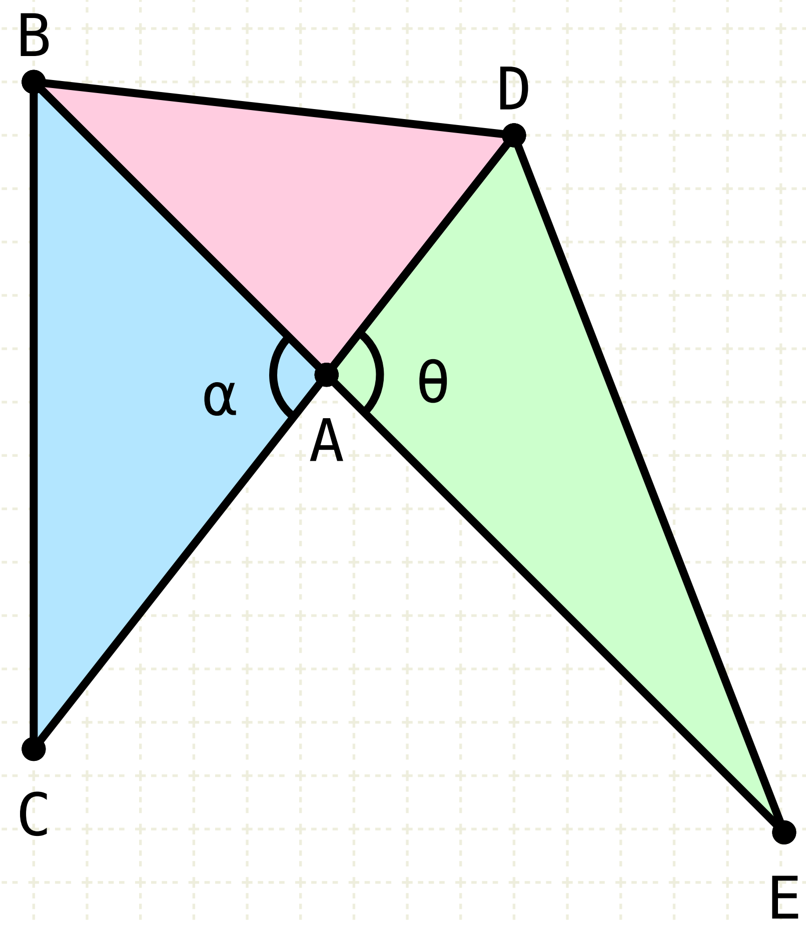

$\alpha = \theta$

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Proof (Part 1)

Let CA, AD be place in in a straight line, therefore EA, AB are also in a straight line (I·14)

Create the triangle BAD

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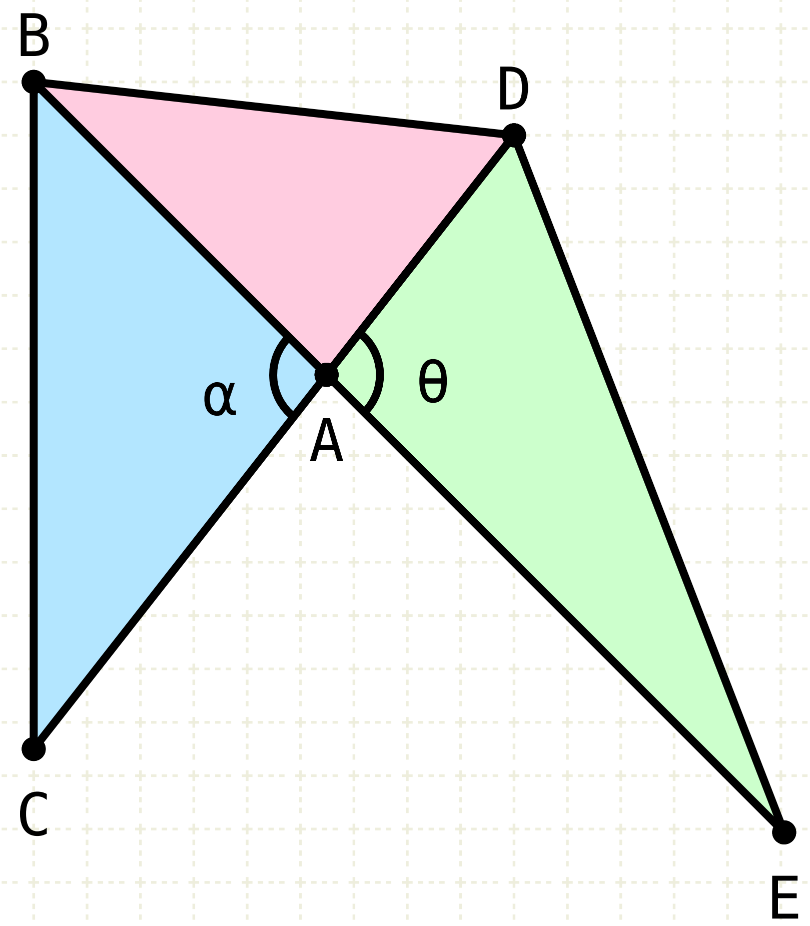

$\alpha = \theta$

$\Delta ABC = \Delta DAE$

$\Delta ABC : \Delta BAD = \Delta DAE : \Delta BAD$

Proof (Part 1)

Let CA, AD be place in in a straight line, therefore EA, AB are also in a straight line (I·14)

Create the triangle BAD

Since triangles ABC and DAE are equal, then the ratios of these to the triangle BAD will also be equal (V·7)

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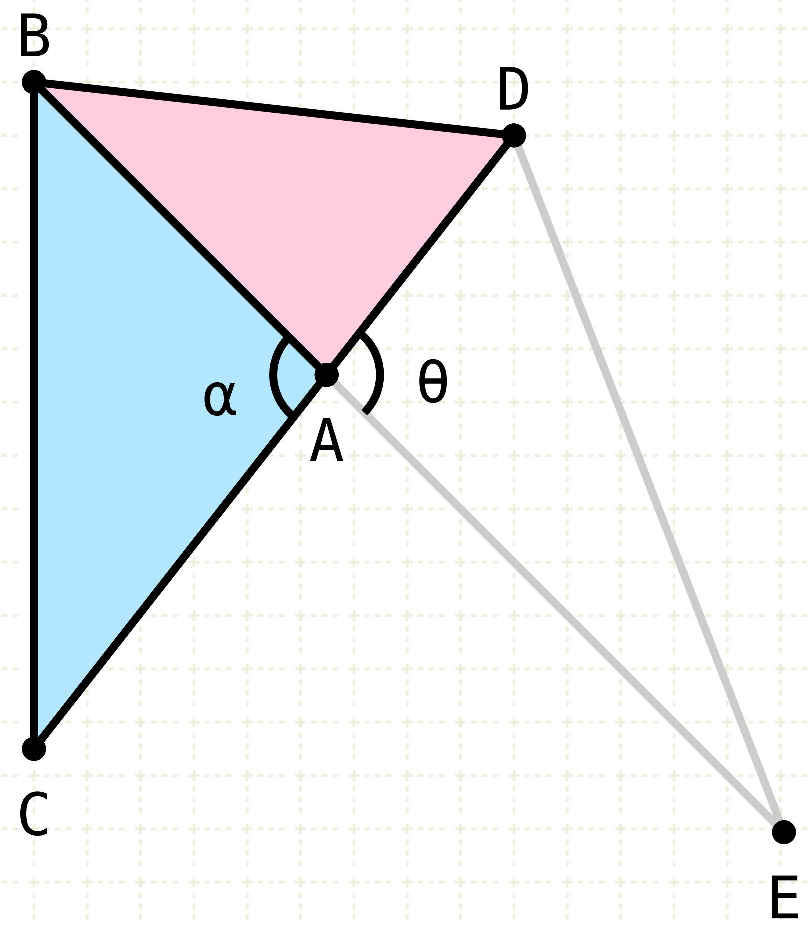
In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and those triangles which have one angle equal to one angle, and in which the sides about the equal angles are reciprocally proportional, are equal.



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$\Delta ABC : \Delta BAD = \Delta DAE : \Delta BAD$

$\Delta ABC : \Delta BAD = AC : AD$

Proof (Part 1)

Let CA, AD be place in in a straight line, therefore EA, AB are also in a straight line (I·14)

Create the triangle BAD

Since triangles ABC and DAE are equal, then the ratios of these to the triangle BAD will also be equal (V·7)

But, as the area ABC is to the area BAD, so is the base AC to the base AD (VI·1)

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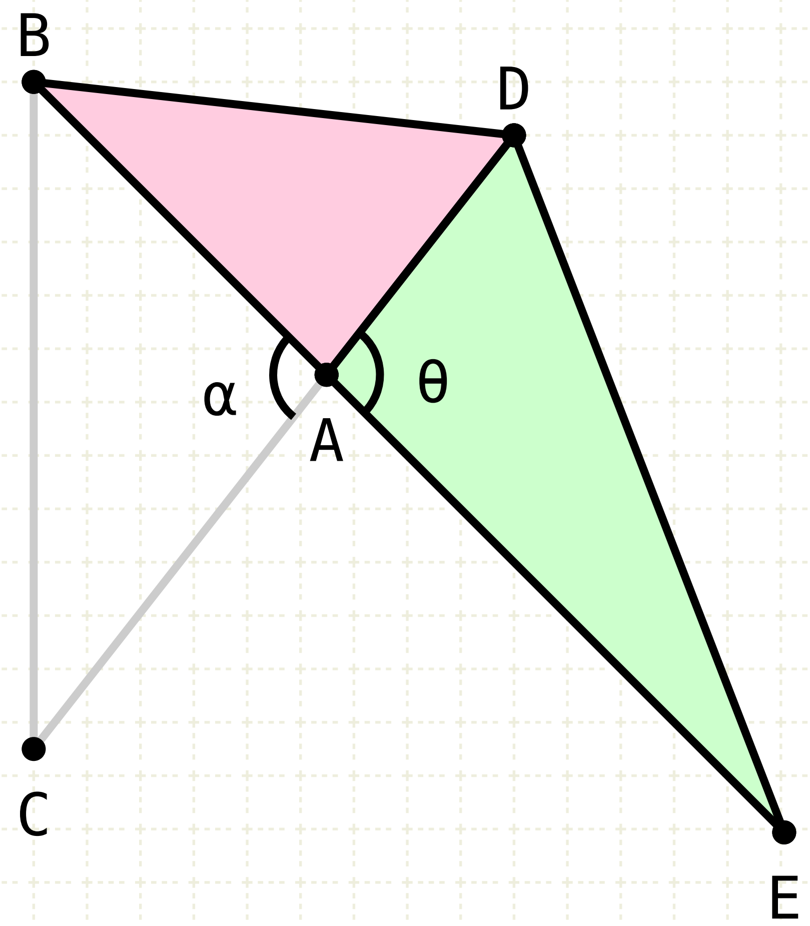

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Proof (Part 1)

Let CA, AD be place in in a straight line, therefore EA, AB are also in a straight line (I·14)

Create the triangle BAD

Since triangles ABC and DAE are equal, then the ratios of these to the triangle BAD will also be equal (V·7)

But, as the area ABC is to the area BAD, so is the base AC to the base AD (VI·1)

and as DAE is to BAD, so is AE to AB (VI·1)

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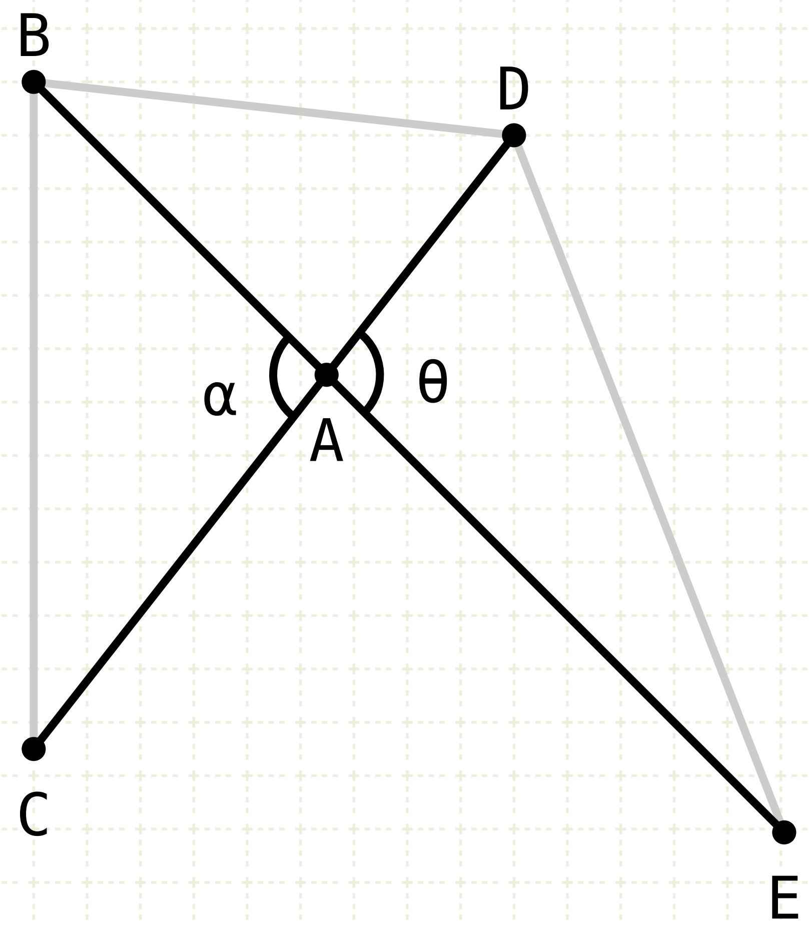

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$\Delta DAE : \Delta BAD = AE : AB$

$AC : AD = AE : AB$

Proof (Part 1)

Let CA, AD be place in in a straight line, therefore EA, AB are also in a straight line (I·14)

Create the triangle BAD

Since triangles ABC and DAE are equal, then the ratios of these to the triangle BAD will also be equal (V·7)

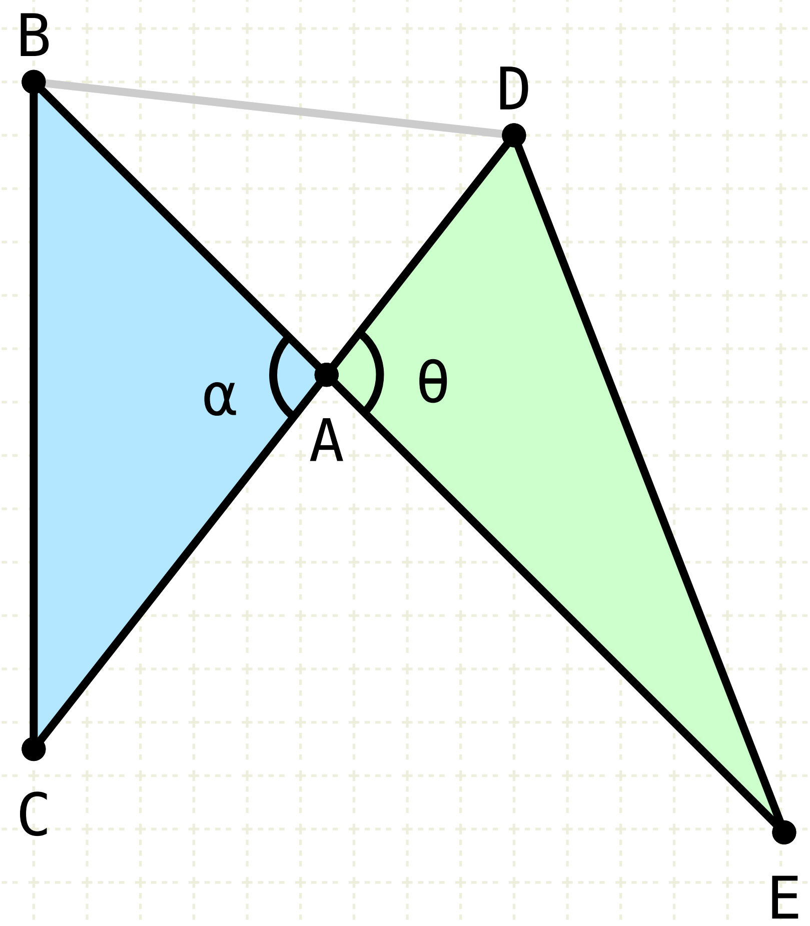
But, as the area ABC is to the area BAD, so is the base AC to the base AD (VI·1)

and as DAE is to BAD, so is AE to AB (VI·1)

Therefore, as AC is to AD, so is AE to AB (V·11)

Proposition 15 of Book VI

In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and those triangles which have one angle equal to one angle, and in which the sides about the equal angles are reciprocally proportional, are equal.



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Create the triangle BAD

Since triangles ABC and DAE are equal, then the ratios of these to the triangle BAD will also be equal (V·7)

But, as the area ABC is to the area BAD, so is the base AC to the base AD (VI·1)

and as DAE is to BAD, so is AE to AB (VI·1)

Therefore, as AC is to AD, so is AE to AB (V·11)

Thus, in the two triangles, the sides about the equal angles are reciprocally proportional

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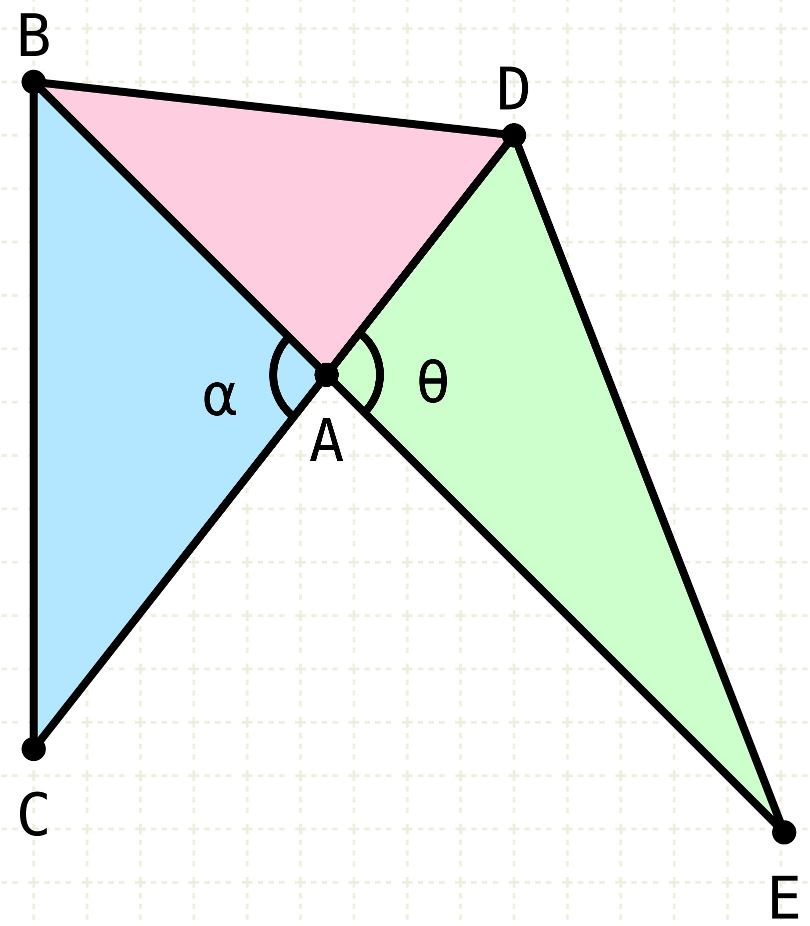

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$AE:AB = AC:AD$

Proof (Part 2)

Let CA, AD be place in in a straight line, therefore EA, AB are also in a straight line (I·14)

Create the triangle BAD

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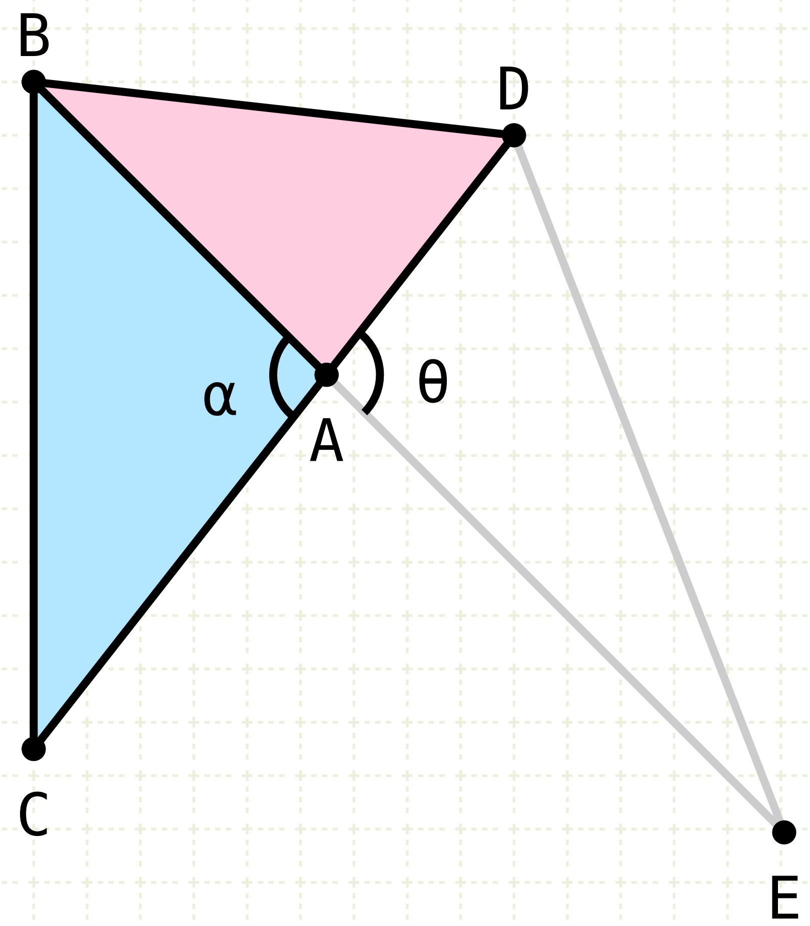

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$AE:AB = AC:AD$

$AC:AD = \Delta ABC:\Delta BAD$

Proof (Part 2)

Let CA, AD be place in in a straight line, therefore EA, AB are also in a straight line (I·14)

Create the triangle BAD

The ratio of AC to AD is equal to the ratio of the triangles ABC to BAD (VI·1)

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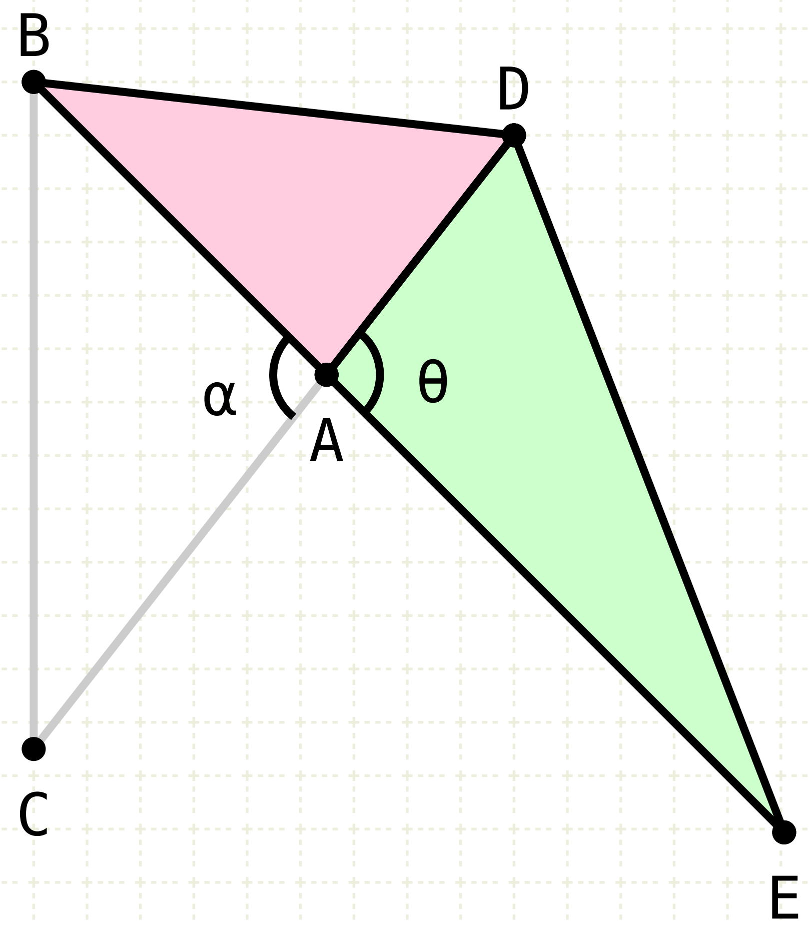

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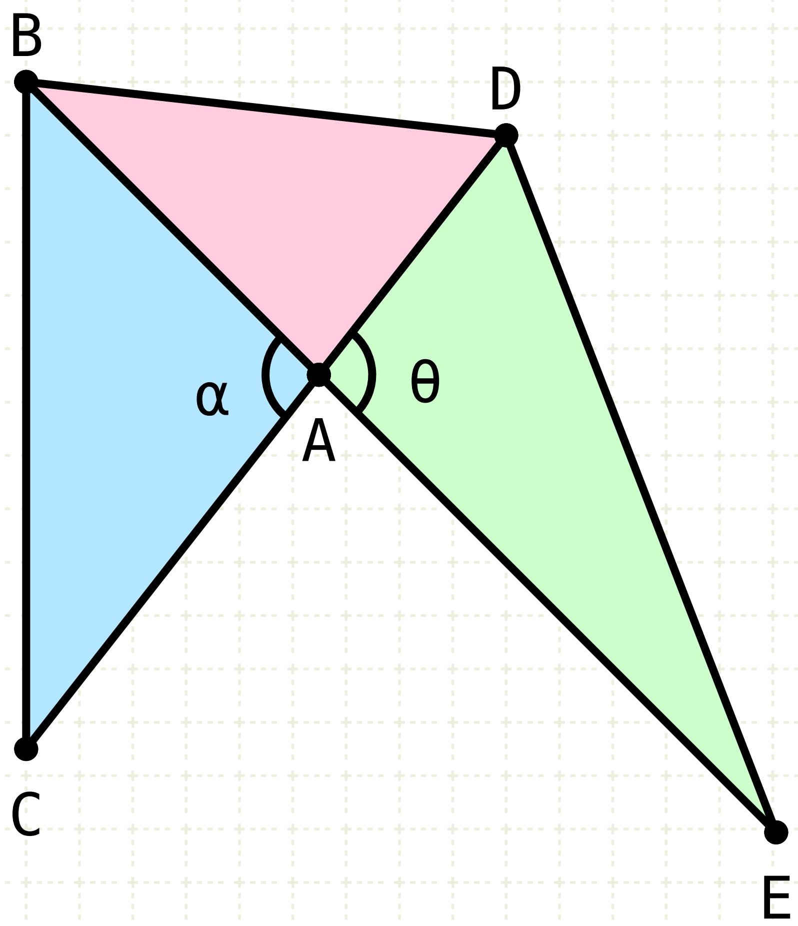

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The ratio of AC to AD is equal to the ratio of the triangles ABC to BAD (VI·1)

The ratio of AE to AB is equal to the ratio of the triangles DAE to BAD (VI·1)

Therefore the ratio of the triangles ABC to BAD is equal to DAE to BAD (V·11)

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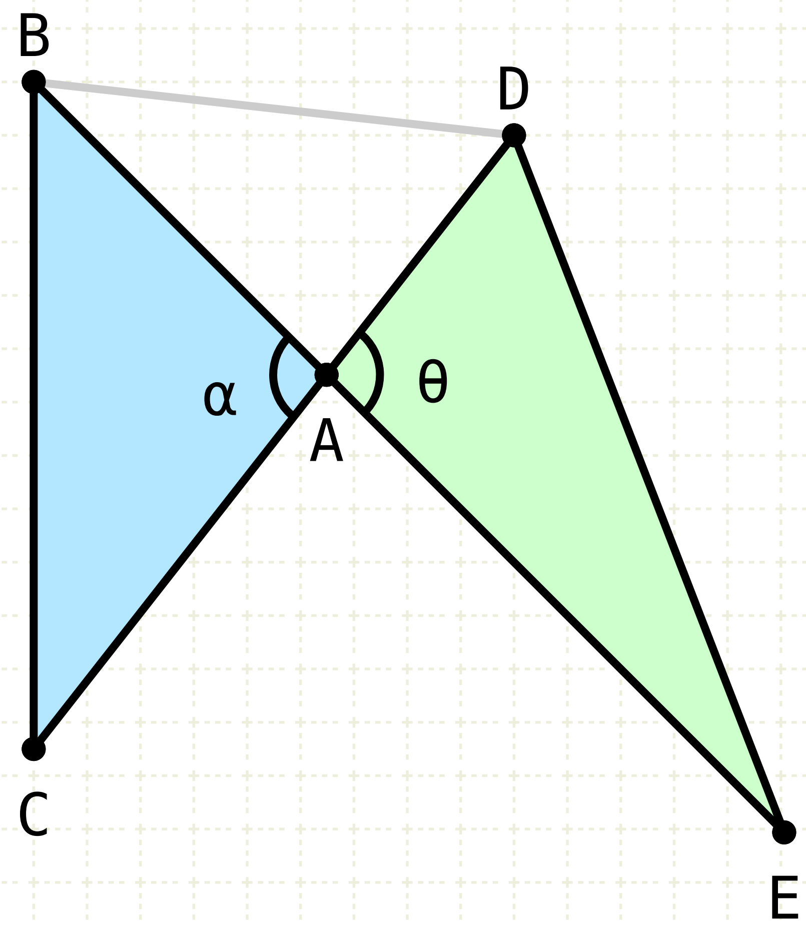

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Let CA, AD be place in in a straight line, therefore EA, AB are also in a straight line (I·14)

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The ratio of AC to AD is equal to the ratio of the triangles ABC to BAD (VI·1)

The ratio of AE to AB is equal to the ratio of the triangles DAE to BAD (VI·1)

Therefore the ratio of the triangles ABC to BAD is equal to DAE to BAD (V·11)

And thus the triangles ABC and DAE are equal (V·9)

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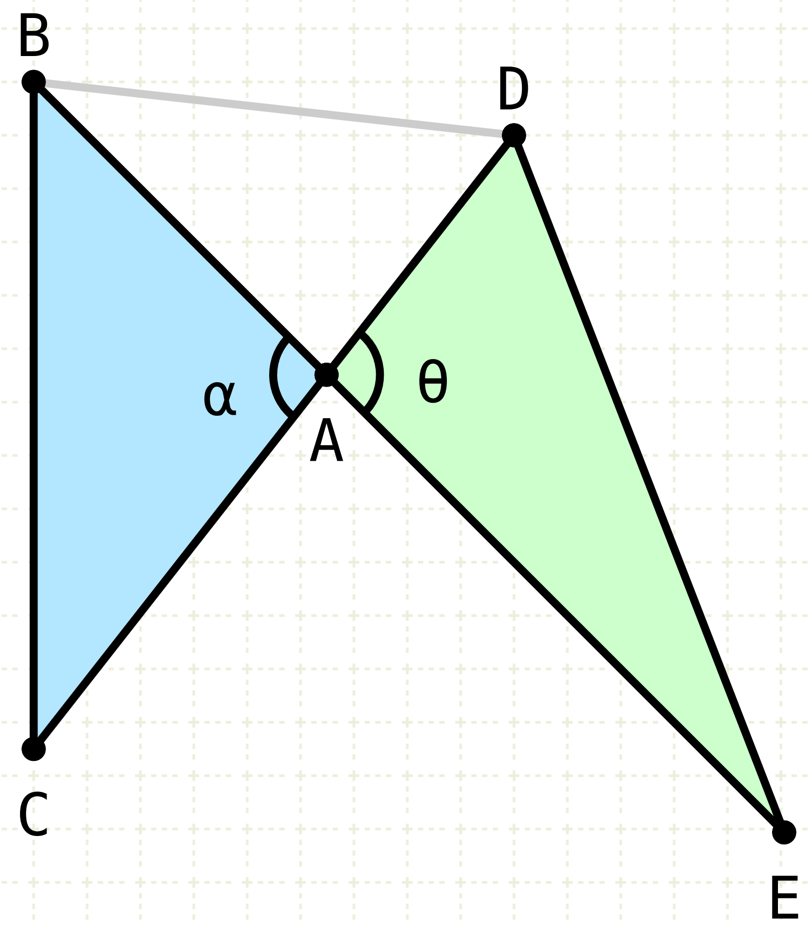

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