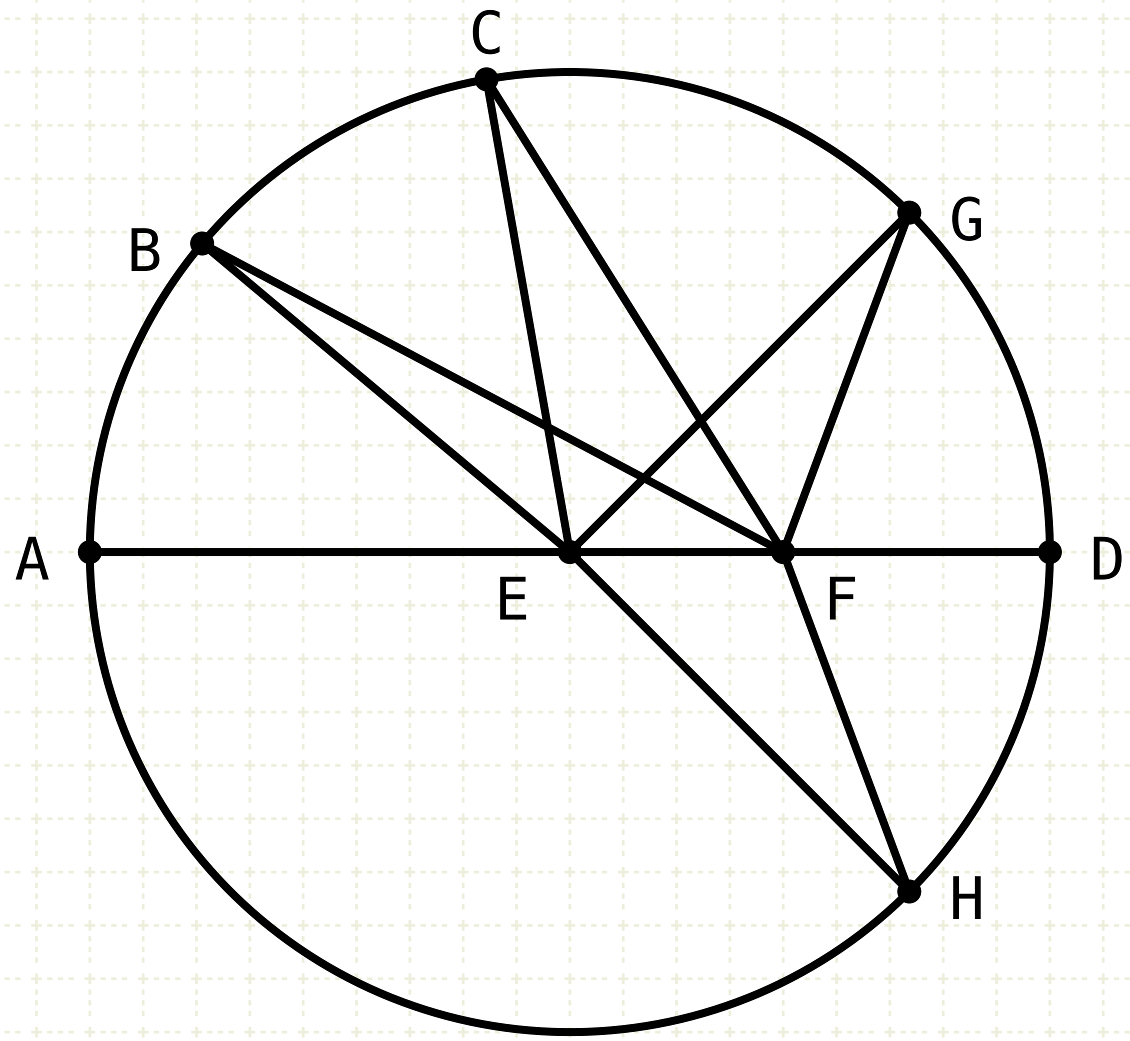
Euclid's Elements

Book III



A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)

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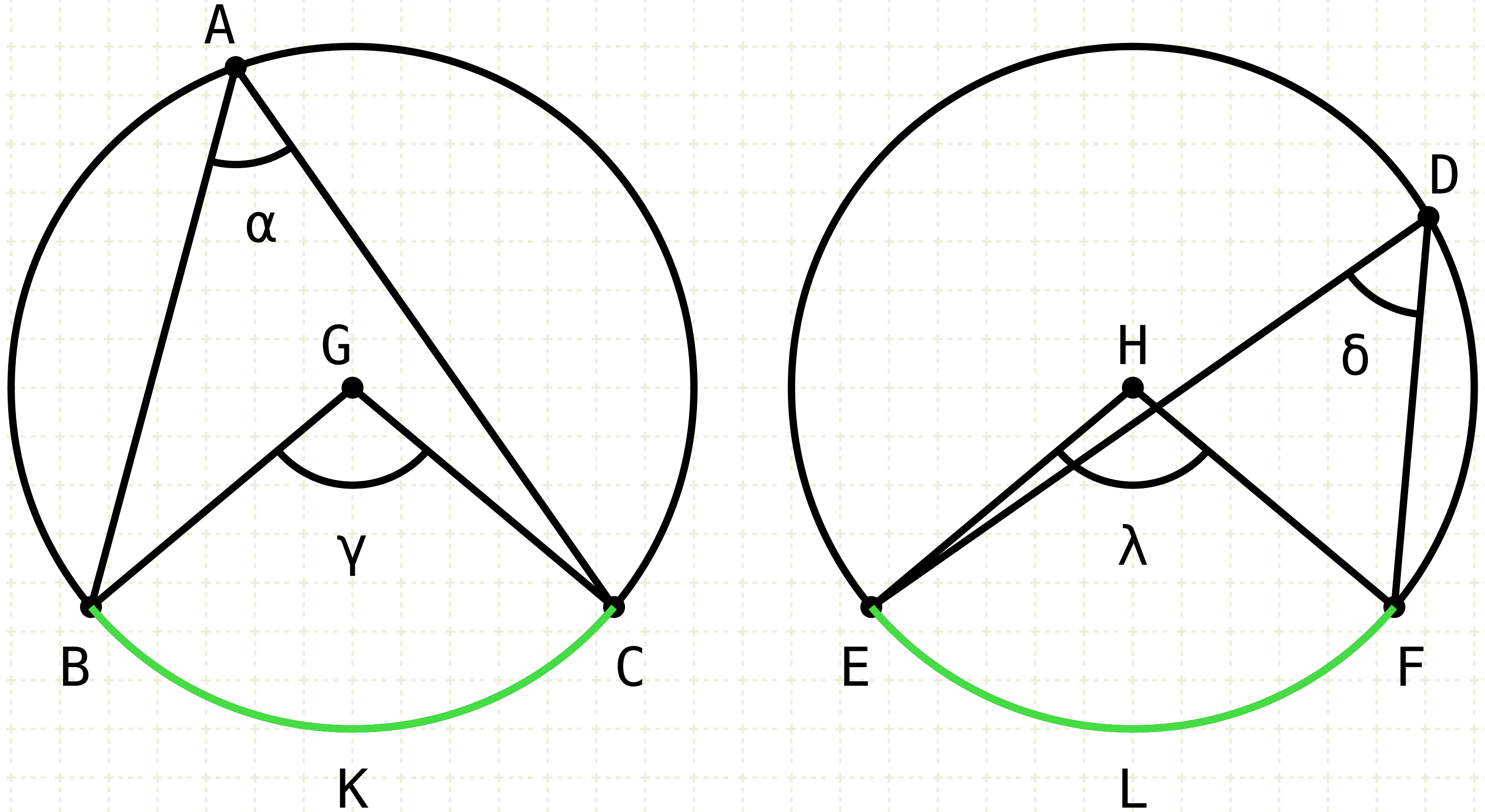

⊙ABC = ⊙EDF

α = δ

γ = λ

↺K = ↺L

In other words

Given two equal circles (as shown)

If α equals δ OR γ equals λ, then

the circumference K equals the circumference L

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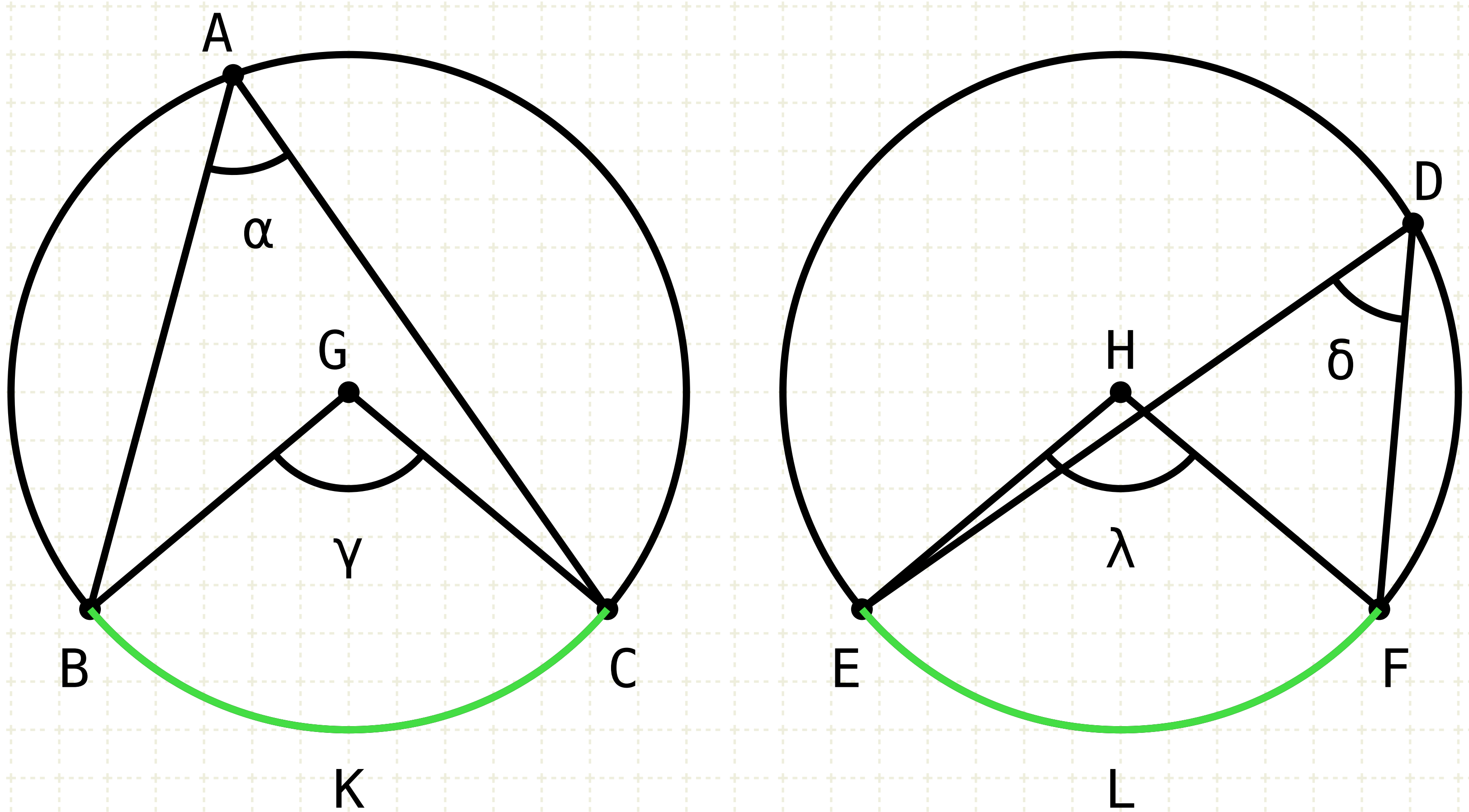

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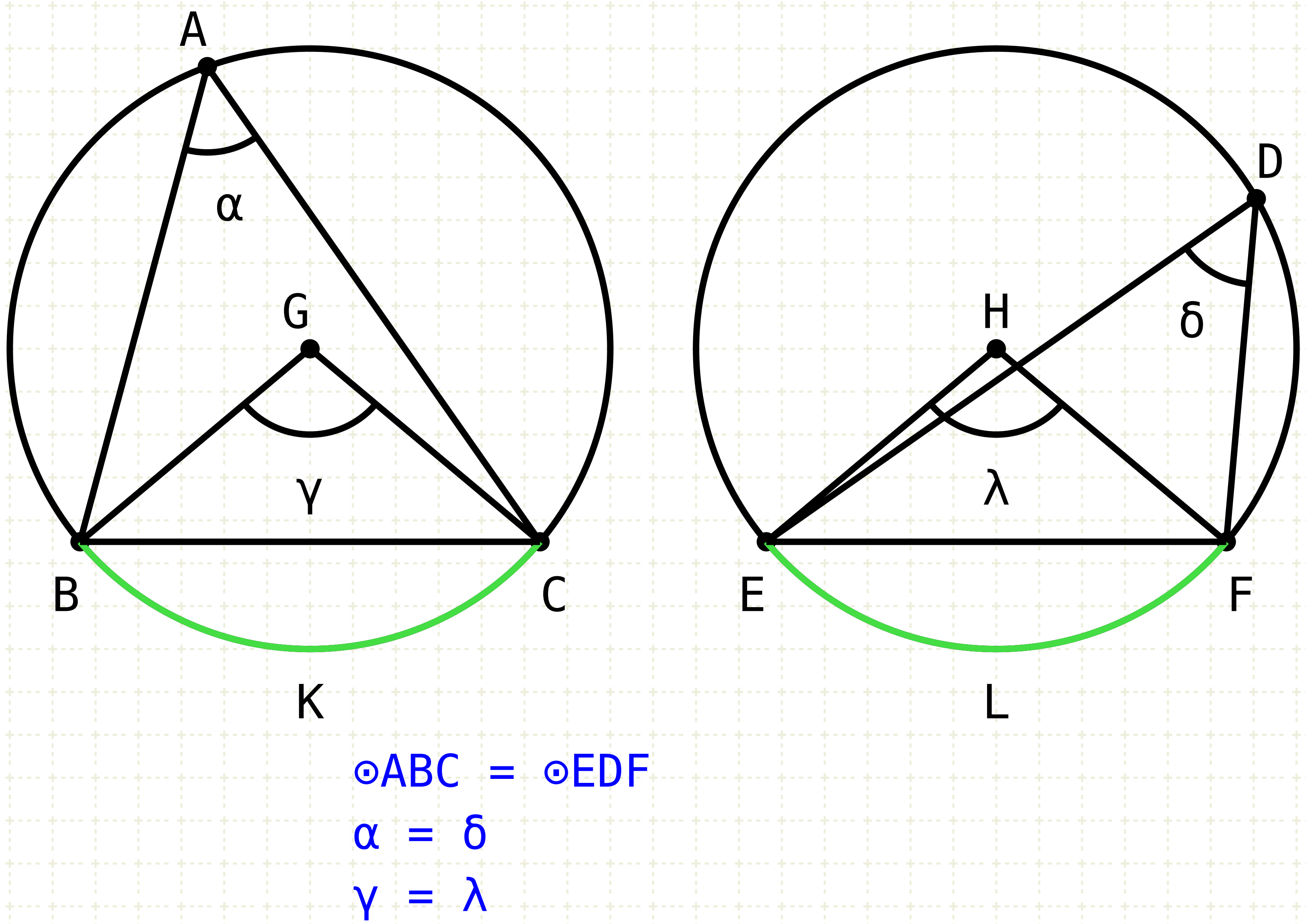

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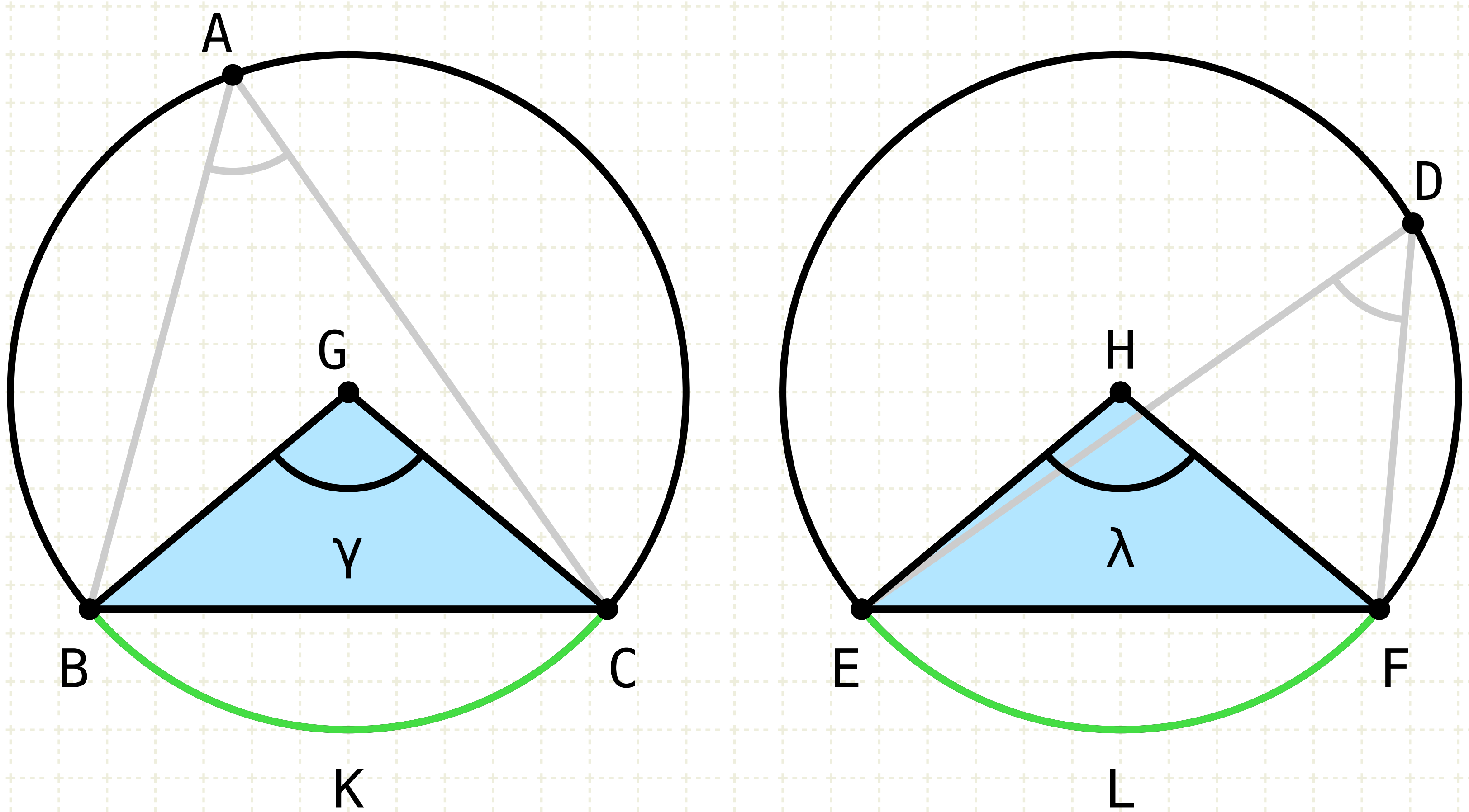

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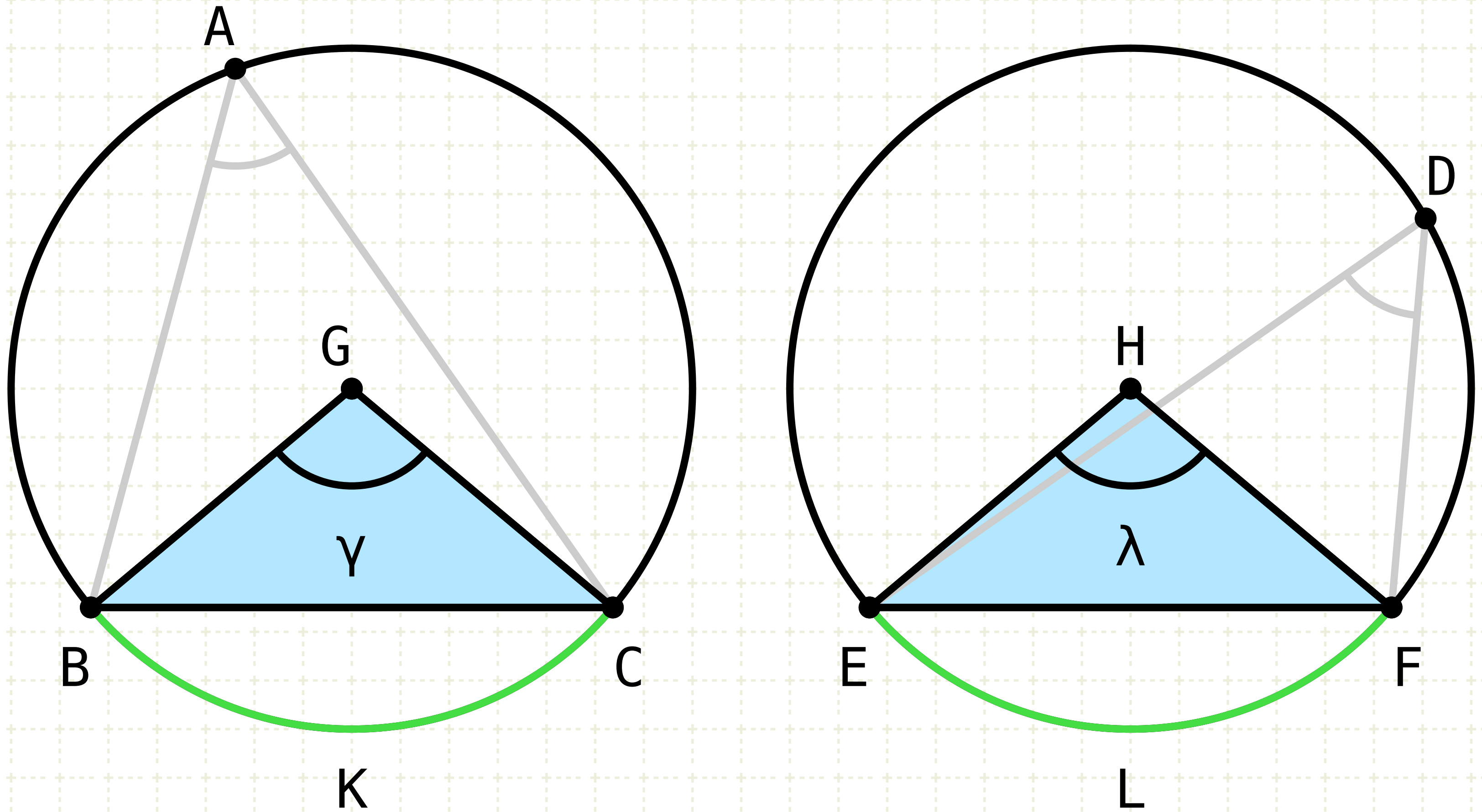

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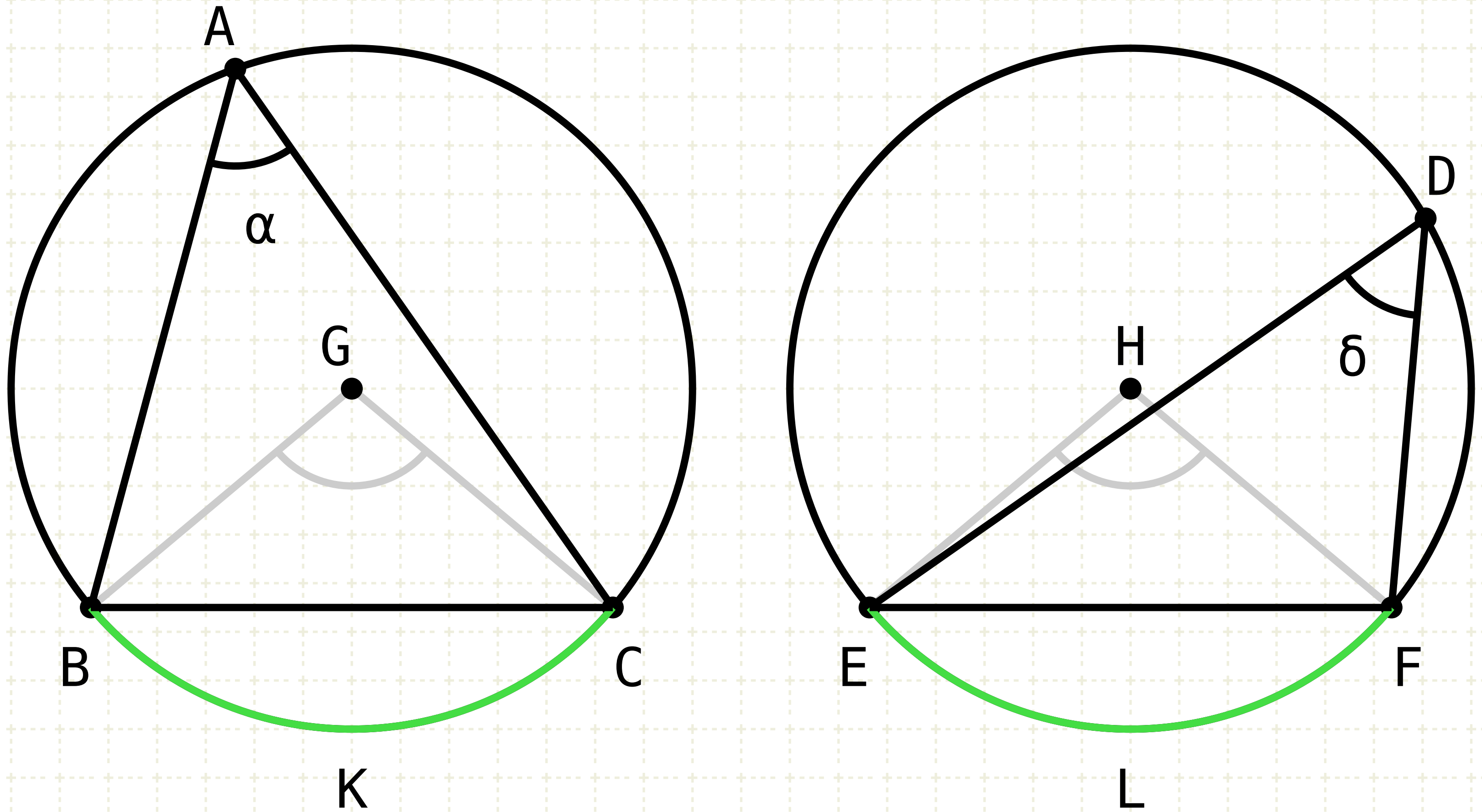

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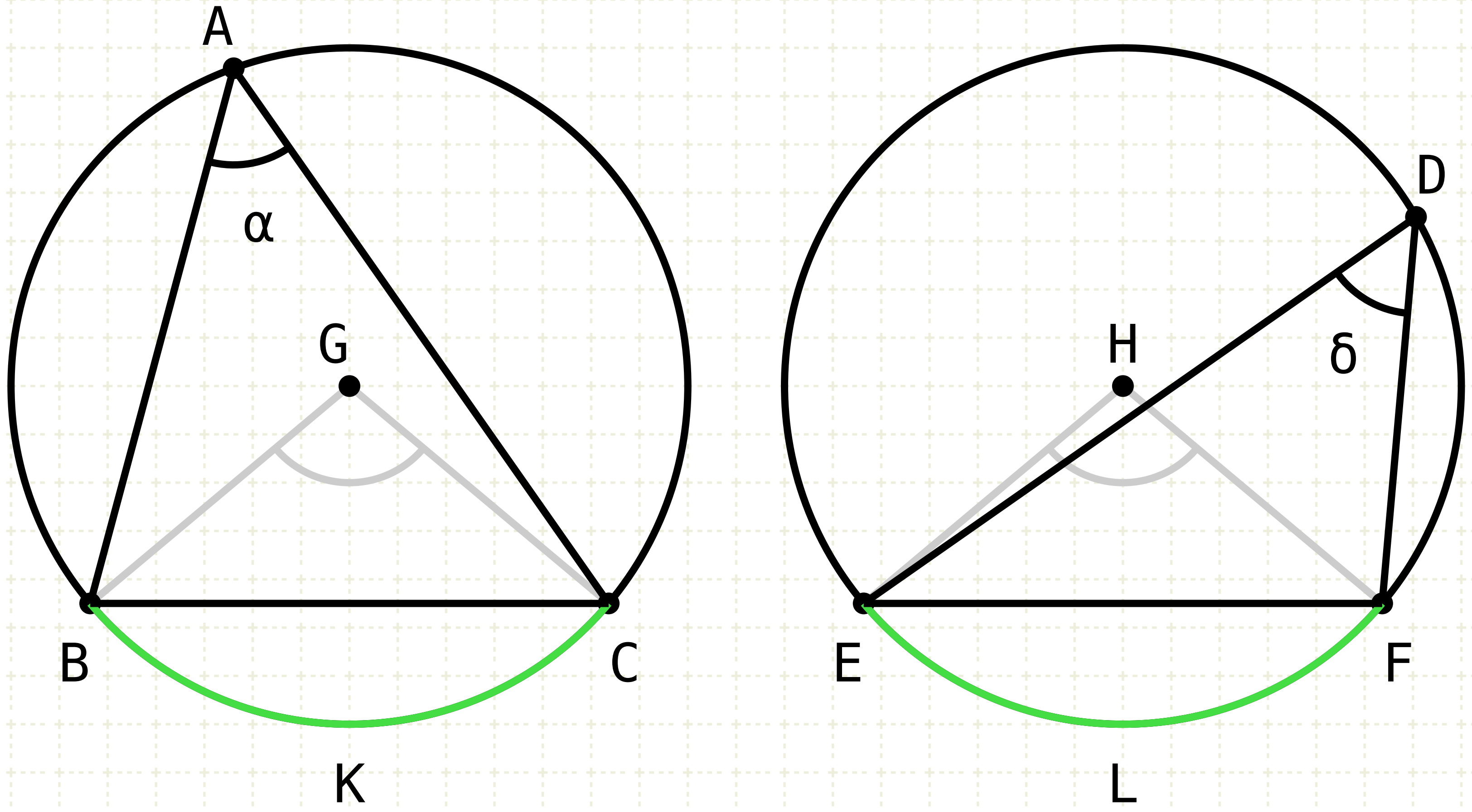

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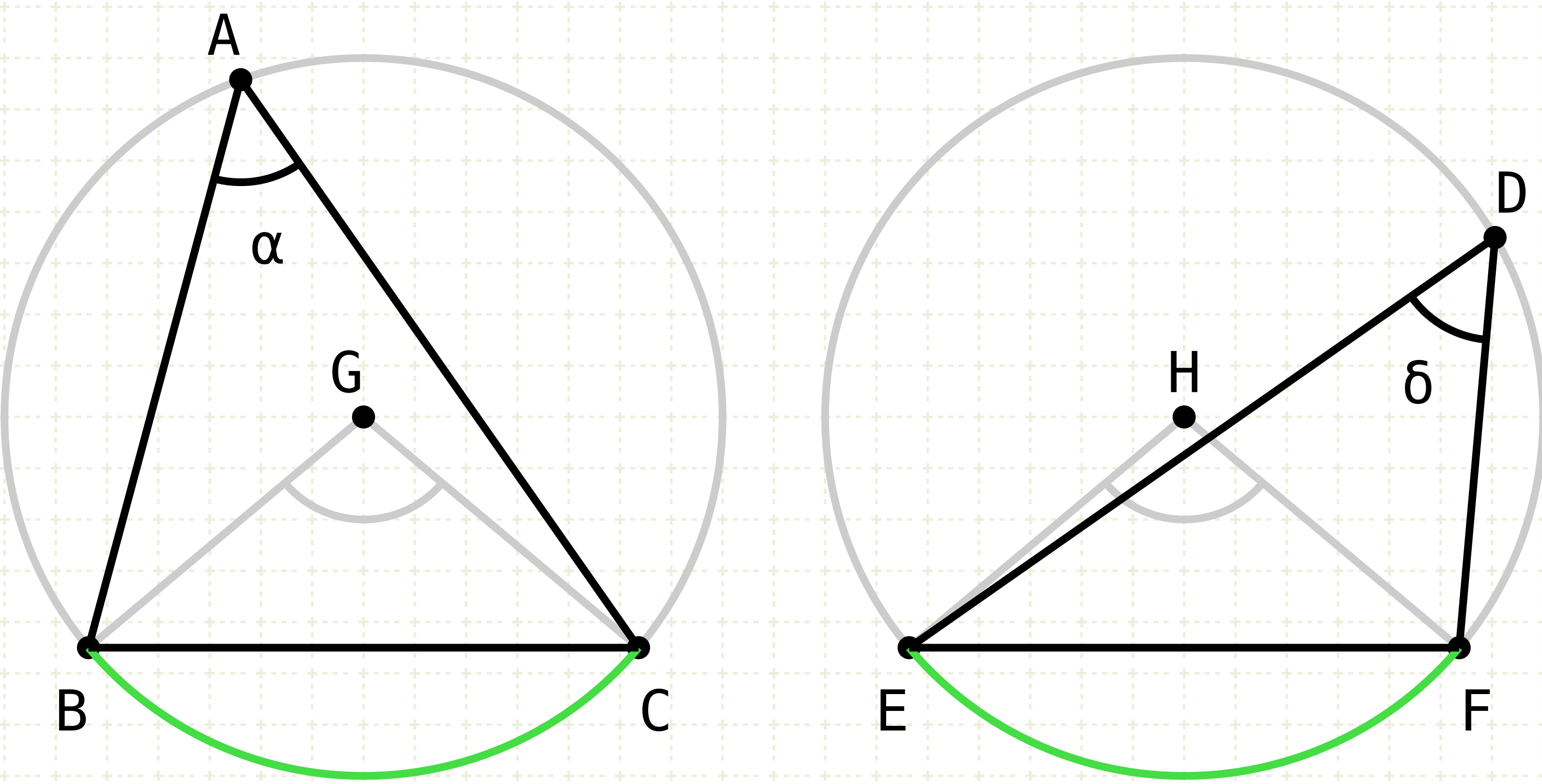

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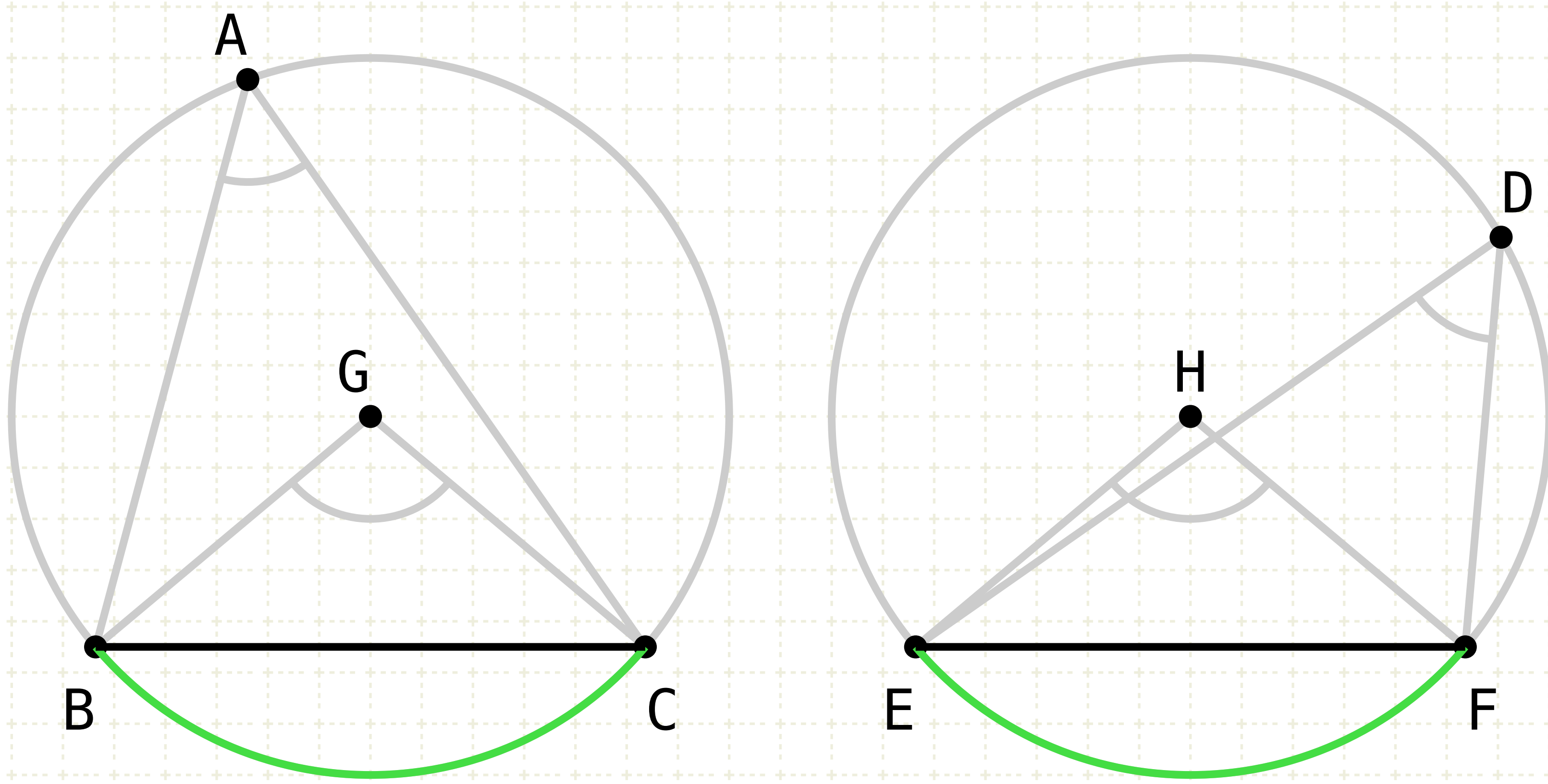

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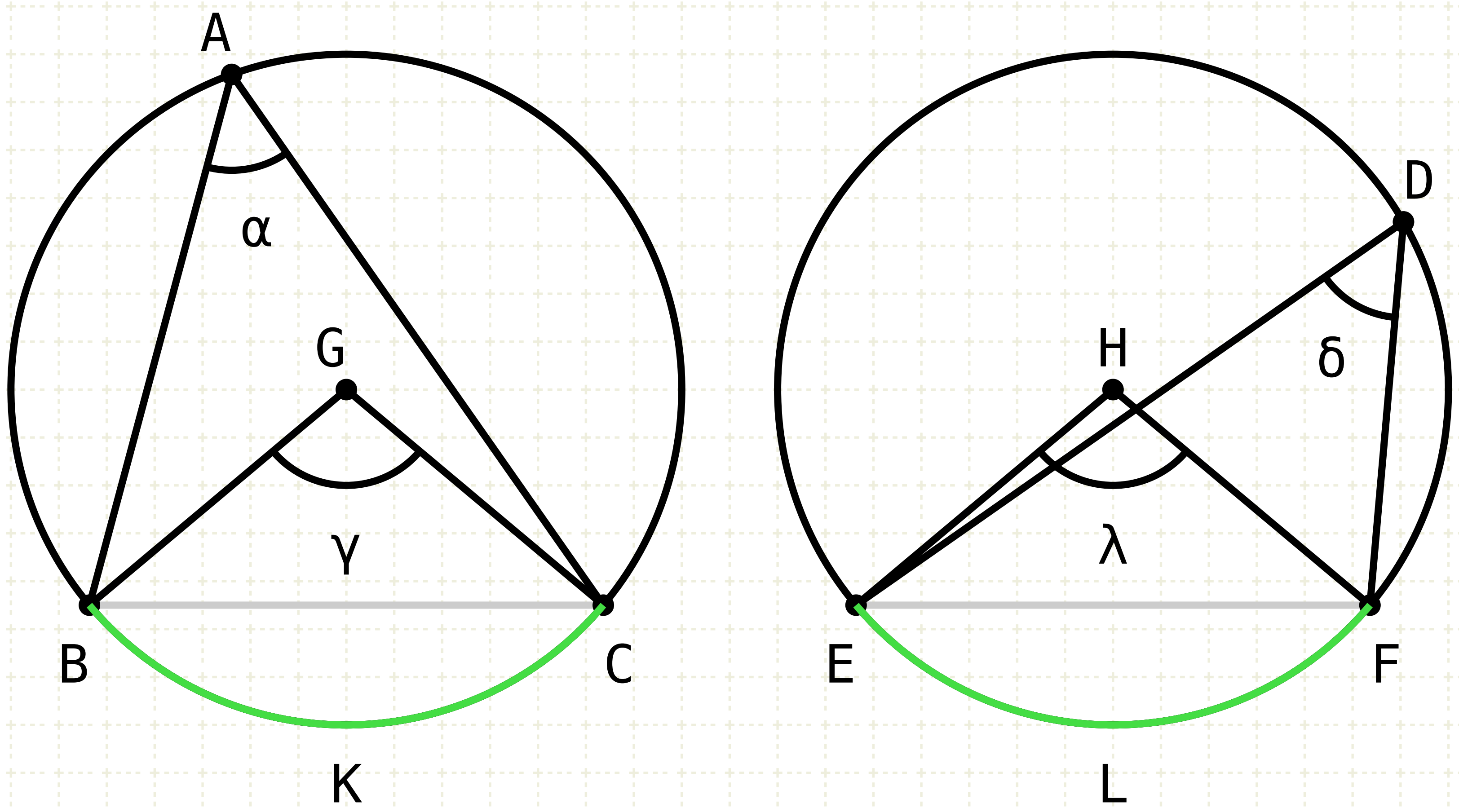

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