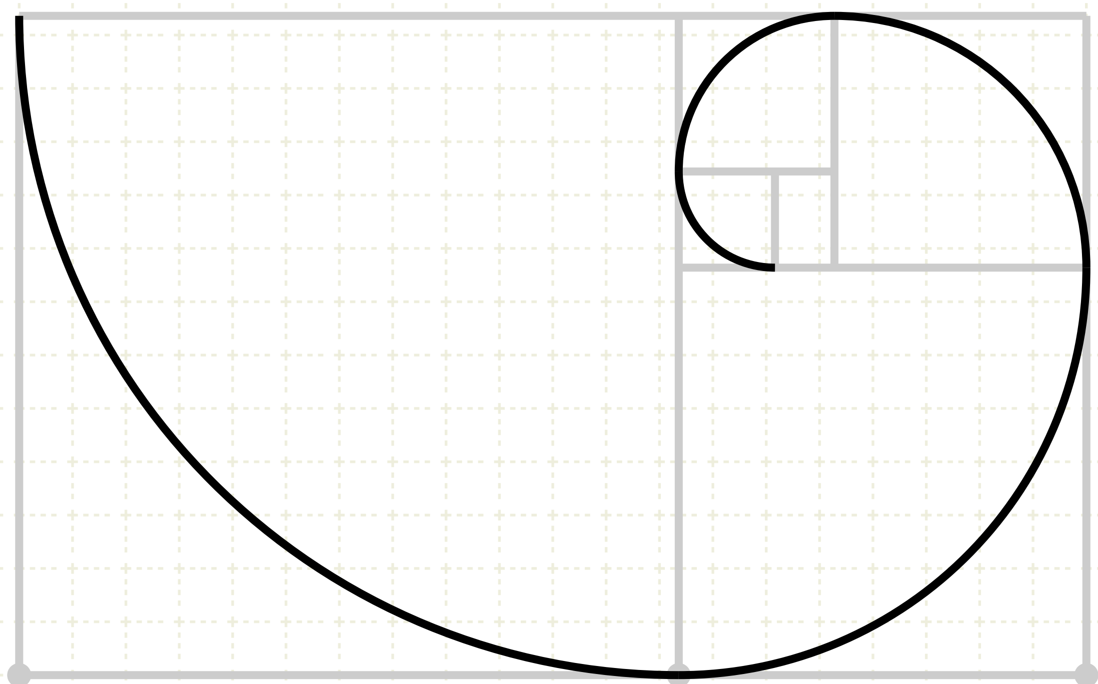
Euclid's Elements

Book VI



One can state, without exaggeration, that the observation of and the search for similarities and differences are the basis of all human knowledge.

Alfred Nobel

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Proposition 3 of Book VI

If an angle of a triangle be bisected and the straight line cutting the angle cut the base also, the segments of the base will have the same ratio as the remaining sides of the triangle; and, if the segments of the base have the same ratio as the remaining sides of the triangle, the straight line joined from the vertex to the point of section will bisect the angle of the triangle.

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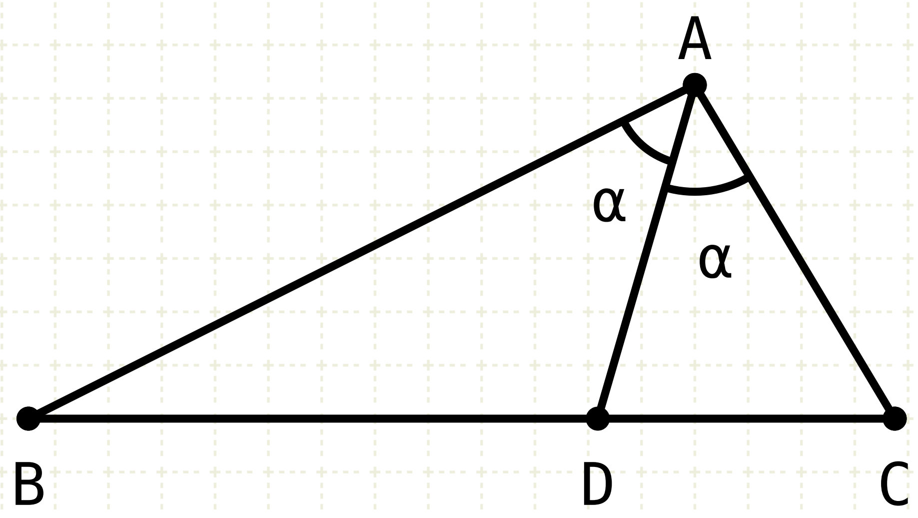


$\angle BAD = \angle DAC \rightarrow BD:DC = BA:AC$

$BD:DC = BA:AC \rightarrow \angle BAD = \angle DAC$

In other words

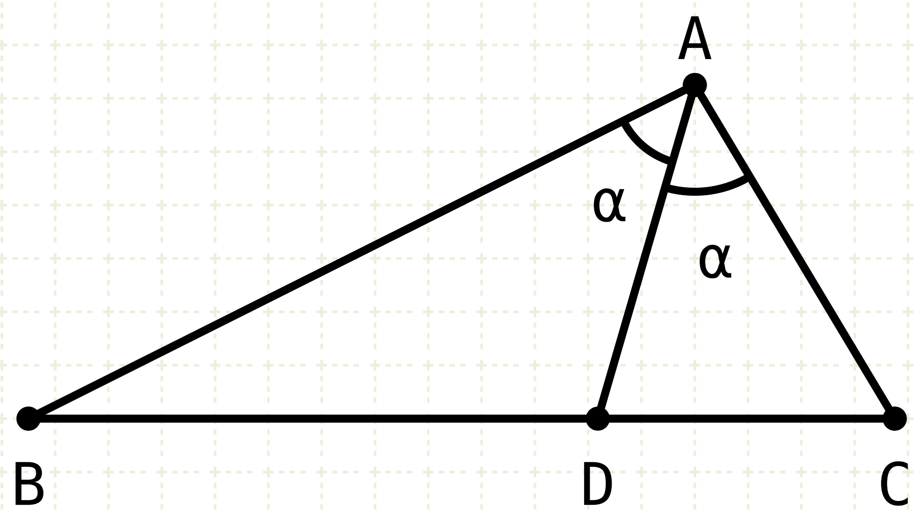
Given a triangle ABC, and let angle BAC be bisected by the straight line BD

Then the ratio of BD to CD is equal to the ratio BA to AC

Proposition 3 of Book VI

If an angle of a triangle be bisected and the straight line cutting the angle cut the base also, the segments of the base will have the same ratio as the remaining sides of the triangle; and, if the segments of the base have the same ratio as the remaining sides of the triangle, the straight line joined from the vertex to the point of section will bisect the angle of the triangle.

Proof (part 1)



$\angle BAD = \angle DAC$

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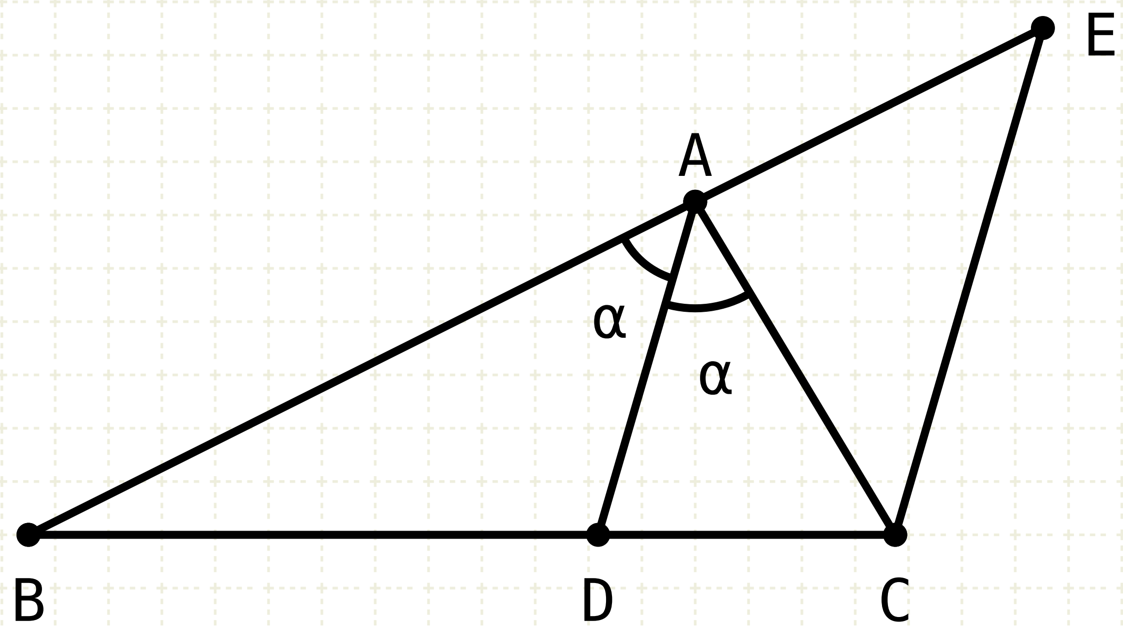

$\angle BAD = \angle DAC$

$CE \parallel DA$

Proof (part 1)

Draw a line parallel to DA from point C, and let the line BA extend to it at point E

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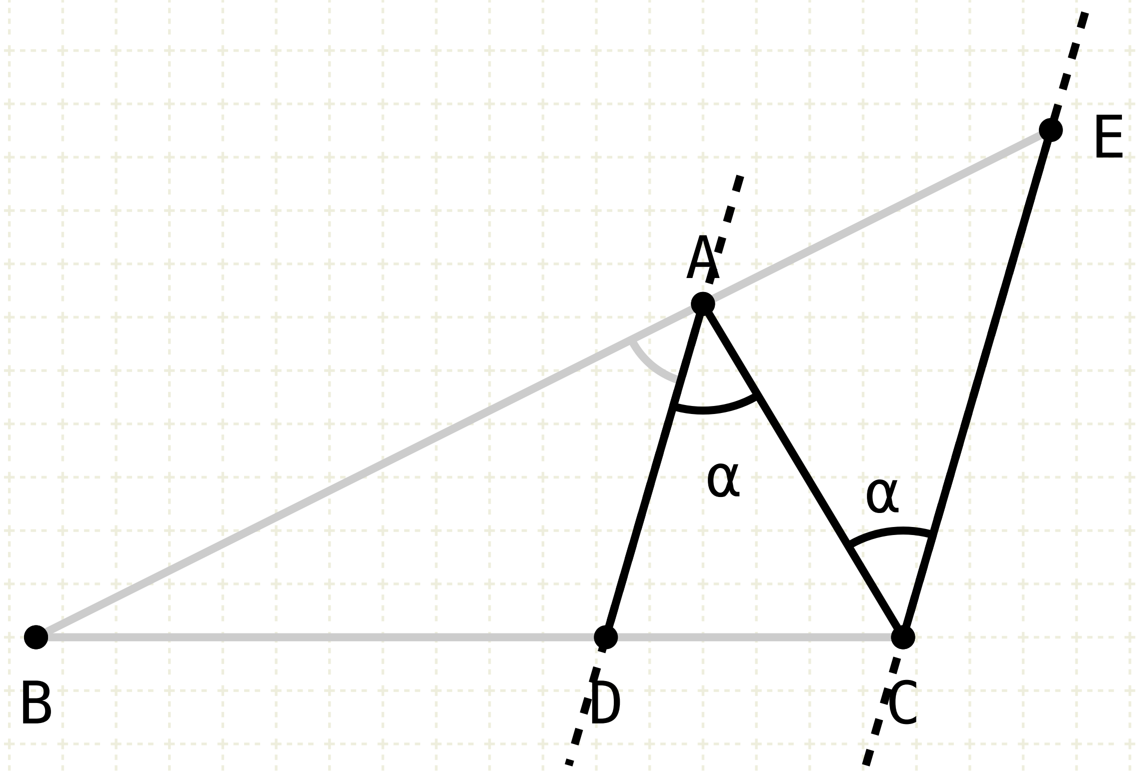
If an angle of a triangle be bisected and the straight line cutting the angle cut the base also, the segments of the base will have the same ratio as the remaining sides of the triangle; and, if the segments of the base have the same ratio as the remaining sides of the triangle, the straight line joined from the vertex to the point of section will bisect the angle of the triangle.



$\angle BAD = \angle DAC$

$CE \parallel DA$

$\angle DAC = \angle ACE$

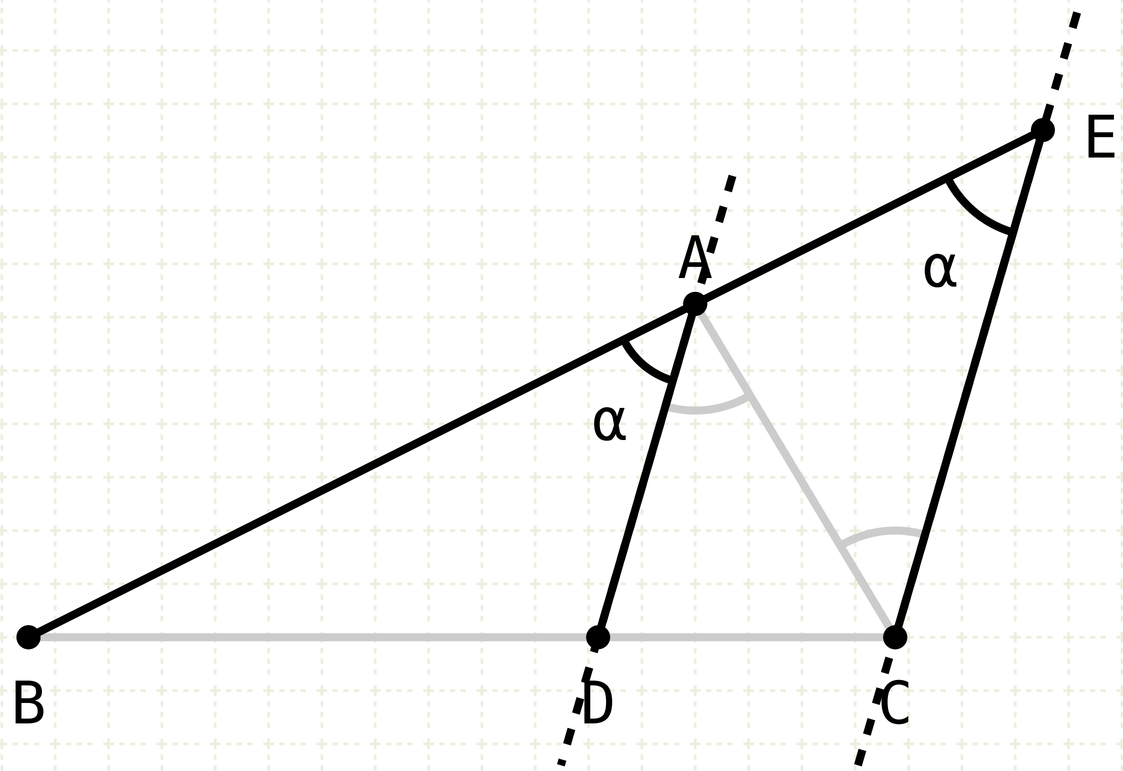
Proof (part 1)

Draw a line parallel to DA from point C, and let the line BA extend to it at point E

AC cuts two parallel lines AD and EC, therefore the opposite interior angle ACE is equal to DAC (I·29)

Proposition 3 of Book VI

If an angle of a triangle be bisected and the straight line cutting the angle cut the base also, the segments of the base will have the same ratio as the remaining sides of the triangle; and, if the segments of the base have the same ratio as the remaining sides of the triangle, the straight line joined from the vertex to the point of section will bisect the angle of the triangle.



Proof (part 1)

Draw a line parallel to DA from point C, and let the line BA extend to it at point E

AC cuts two parallel lines AD and EC, therefore the opposite interior angle ACE is equal to DAC (I·29)

BE cuts two parallel lines AD and EC, therefore the outside angle BAD is equal to interior angle AEC (I·29)

$\angle BAD = \angle DAC$

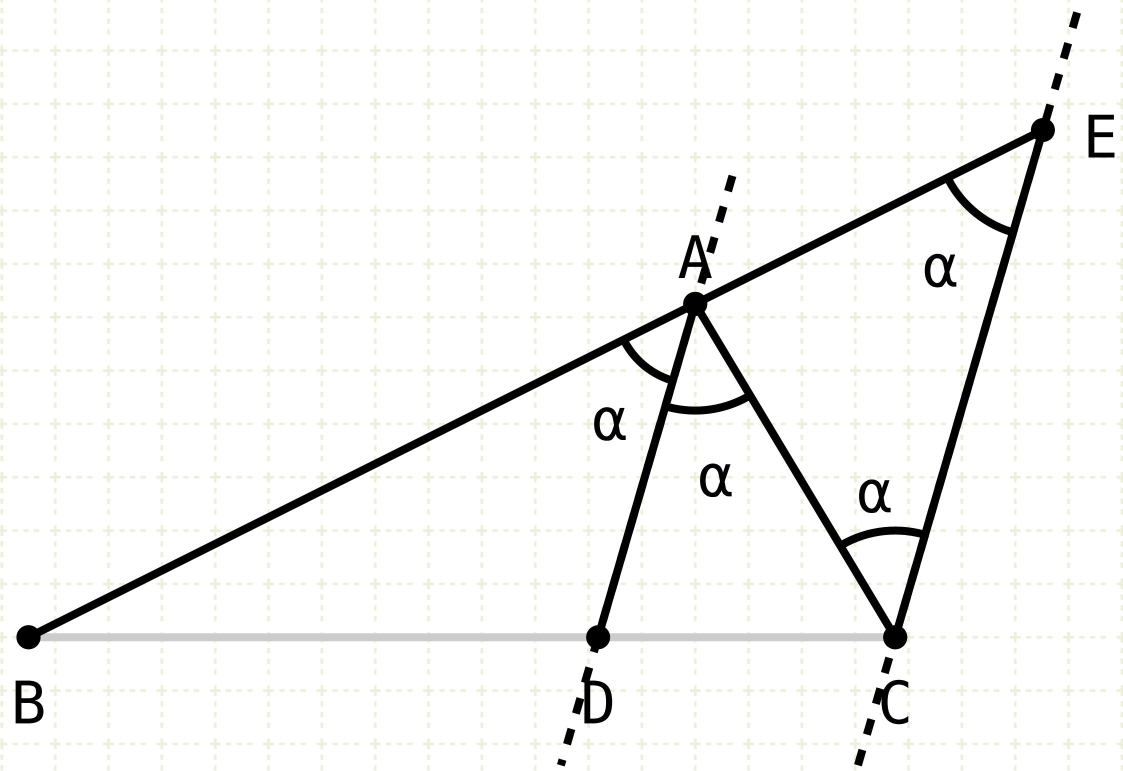
$CE \parallel DA$

$\angle DAC = \angle ACE$

$\angle BAD = \angle AEC$

Proposition 3 of Book VI

If an angle of a triangle be bisected and the straight line cutting the angle cut the base also, the segments of the base will have the same ratio as the remaining sides of the triangle; and, if the segments of the base have the same ratio as the remaining sides of the triangle, the straight line joined from the vertex to the point of section will bisect the angle of the triangle.



$\angle BAD = \angle DAC$

$CE \parallel DA$

$\angle DAC = \angle ACE$

$\angle BAD = \angle AEC$

Proof (part 1)

Draw a line parallel to DA from point C, and let the line BA extend to it at point E

AC cuts two parallel lines AD and EC, therefore the opposite interior angle ACE is equal to DAC (I·29)

BE cuts two parallel lines AD and EC, therefore the outside angle BAD is equal to interior angle AEC (I·29)

Since angles BAD and DAC are equal by construction, angles ACE and AEC are also equal

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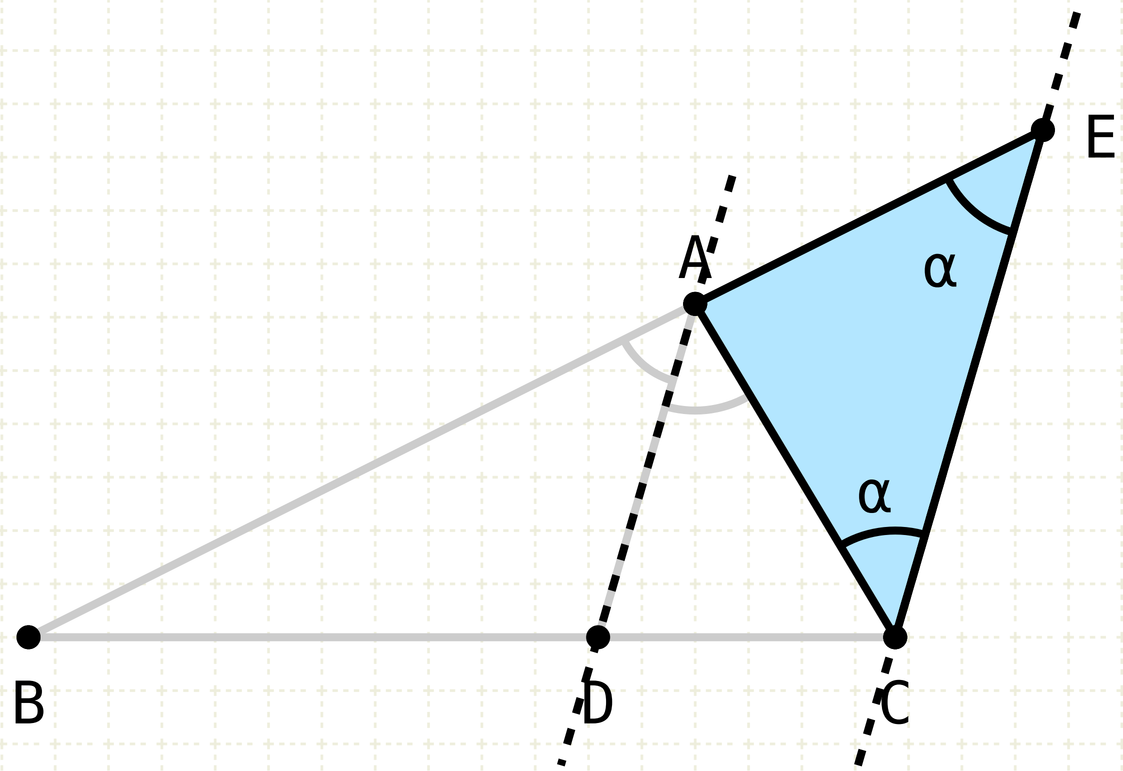
If an angle of a triangle be bisected and the straight line cutting the angle cut the base also, the segments of the base will have the same ratio as the remaining sides of the triangle; and, if the segments of the base have the same ratio as the remaining sides of the triangle, the straight line joined from the vertex to the point of section will bisect the angle of the triangle.



Proof (part 1)

Draw a line parallel to DA from point C, and let the line BA extend to it at point E

AC cuts two parallel lines AD and EC, therefore the opposite interior angle ACE is equal to DAC (I·29)

BE cuts two parallel lines AD and EC, therefore the outside angle BAD is equal to interior angle AEC (I·29)

Since angles BAD and DAC are equal by construction, angles ACE and AEC are also equal

Therefore, triangle AEC is an isosceles triangle, and the sides AE and AC are equal

$\angle BAD = \angle DAC$

$CE \parallel DA$

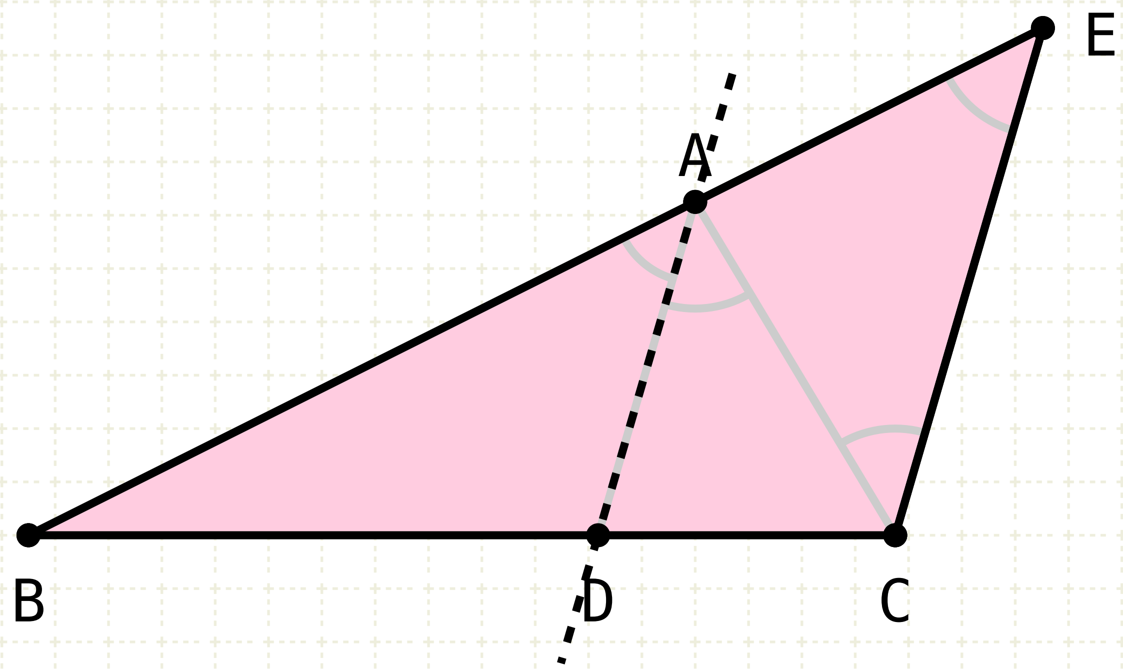
$\angle DAC = \angle ACE$

$\angle BAD = \angle AEC$

$AE = AC$

Proposition 3 of Book VI

If an angle of a triangle be bisected and the straight line cutting the angle cut the base also, the segments of the base will have the same ratio as the remaining sides of the triangle; and, if the segments of the base have the same ratio as the remaining sides of the triangle, the straight line joined from the vertex to the point of section will bisect the angle of the triangle.



$\angle BAD = \angle DAC$

$CE \parallel DA$

$\angle DAC = \angle ACE$

$\angle BAD = \angle AEC$

$AE = AC$

$BA:AE = BD:DC$

Proof (part 1)

Draw a line parallel to DA from point C, and let the line BA extend to it at point E

AC cuts two parallel lines AD and EC, therefore the opposite interior angle ACE is equal to DAC (I·29)

BE cuts two parallel lines AD and EC, therefore the outside angle BAD is equal to interior angle AEC (I·29)

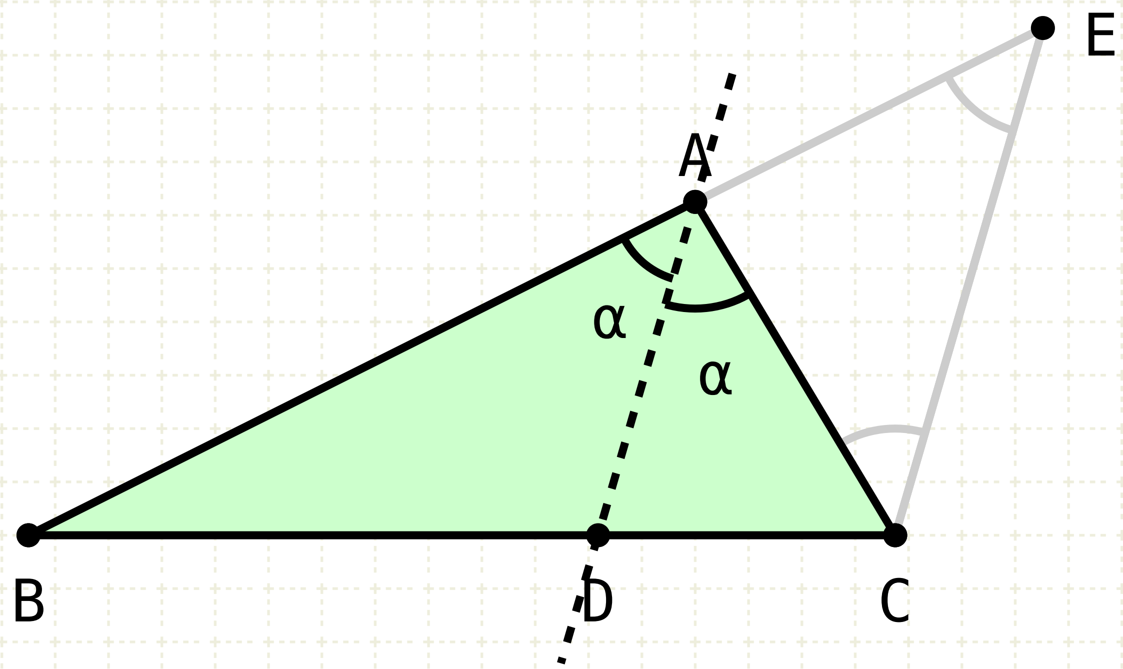
Since angles BAD and DAC are equal by construction, angles ACE and AEC are also equal

Therefore, triangle AEC is an isosceles triangle, and the sides AE and AC are equal

Consider triangle BCE, with a line AD drawn parallel to one of the triangle's sides. The ratio BA to AE is equal to the ratio BD to DC (VI·2)

Proposition 3 of Book VI

If an angle of a triangle be bisected and the straight line cutting the angle cut the base also, the segments of the base will have the same ratio as the remaining sides of the triangle; and, if the segments of the base have the same ratio as the remaining sides of the triangle, the straight line joined from the vertex to the point of section will bisect the angle of the triangle.



∠BAD = ∠DAC

CE || DA

∠DAC = ∠ACE

∠BAD = ∠AEC

AE = AC

BA:AE = BD:DC

BA:AC = BD:DC

Proof (part 1)

Draw a line parallel to DA from point C, and let the line BA extend to it at point E

AC cuts two parallel lines AD and EC, therefore the opposite interior angle ACE is equal to DAC (I·29)

BE cuts two parallel lines AD and EC, therefore the outside angle BAD is equal to interior angle AEC (I·29)

Since angles BAD and DAC are equal by construction, angles ACE and AEC are also equal

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Consider triangle BCE, with a line AD drawn parallel to one of the triangle's sides. The ratio BA to AE is equal to the ratio BD to DC (VI·2)

And AE is equal to AC, therefore ratio BA to AC is equal to the ratio BD to DC

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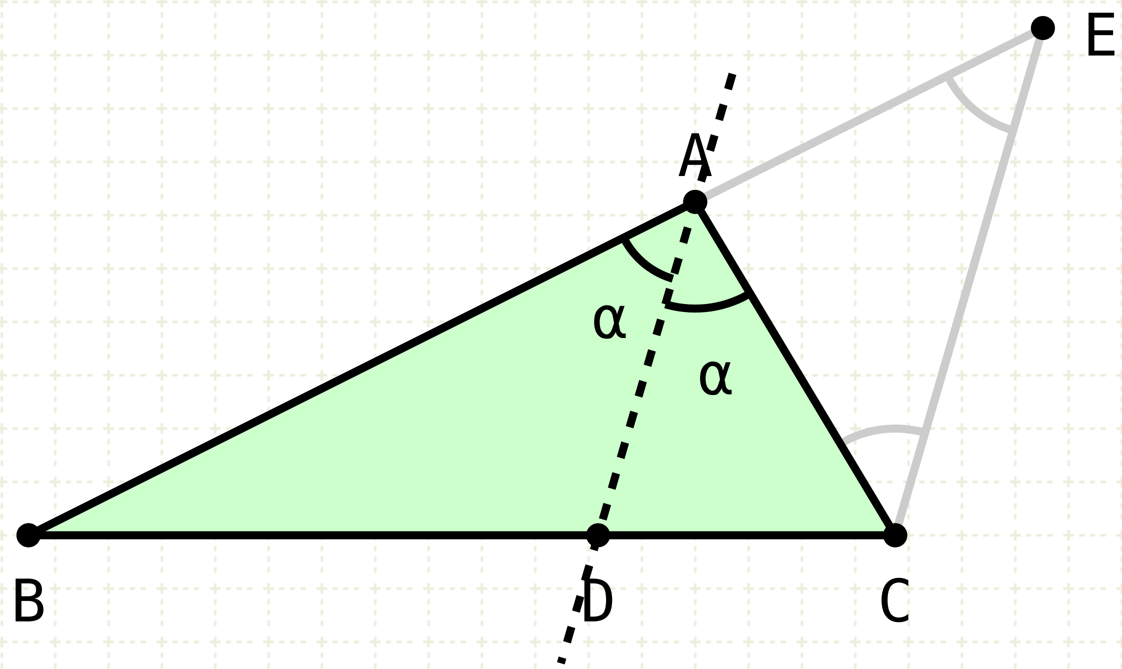
If an angle of a triangle be bisected and the straight line cutting the angle cut the base also, the segments of the base will have the same ratio as the remaining sides of the triangle; and, if the segments of the base have the same ratio as the remaining sides of the triangle, the straight line joined from the vertex to the point of section will bisect the angle of the triangle.



$\angle BAD = \angle DAC$

$CE \parallel DA$

$\angle DAC = \angle ACE$

$\angle BAD = \angle AEC$

$AE = AC$

$BA:AE = BD:DC$

$BA:AC = BD:DC$

Proof (part 1)

Draw a line parallel to DA from point C, and let the line BA extend to it at point E

AC cuts two parallel lines AD and EC, therefore the opposite interior angle ACE is equal to DAC (I·29)

BE cuts two parallel lines AD and EC, therefore the outside angle BAD is equal to interior angle AEC (I·29)

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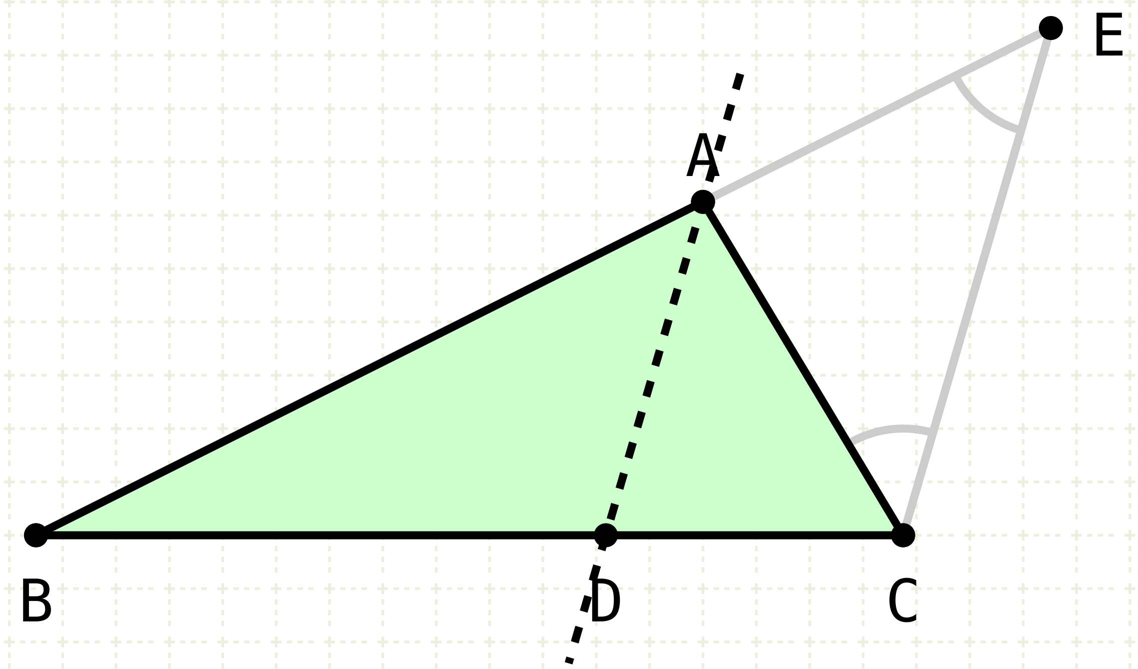

Proof (part 2)

BA:AC = BD:DC

DA || CE

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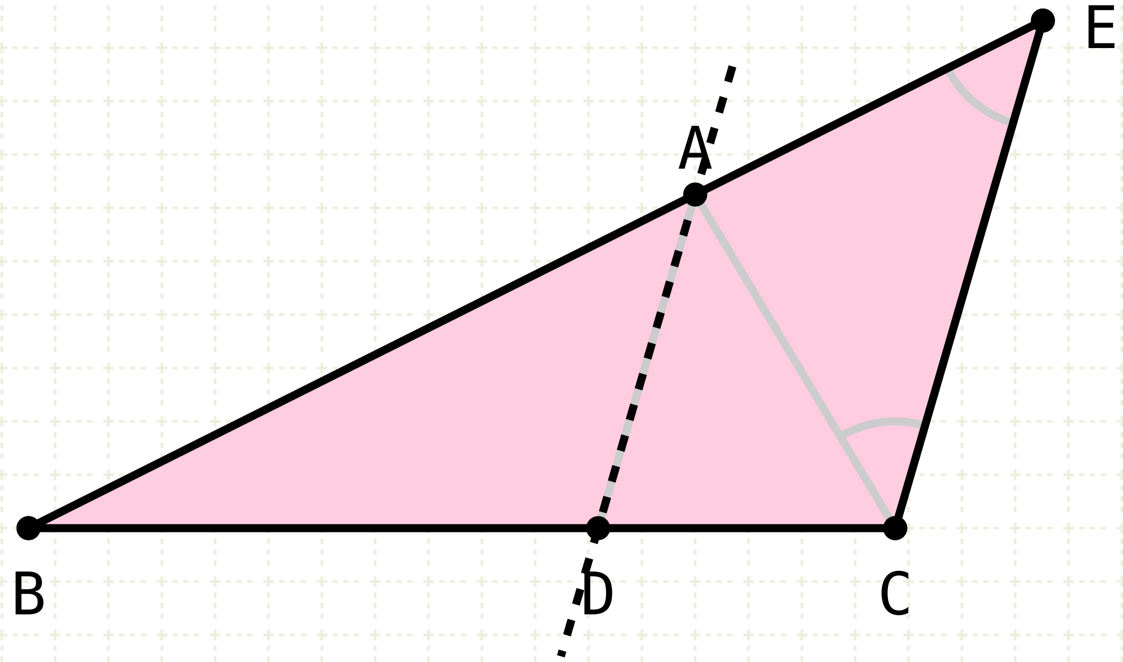

BA:AC = BD:DC

DA || CE

BA:AE = BD:DC

Proof (part 2)

Since DA is parallel to CE, the ratio AB to AE is equal to the ratio BD to DC (VI·2)

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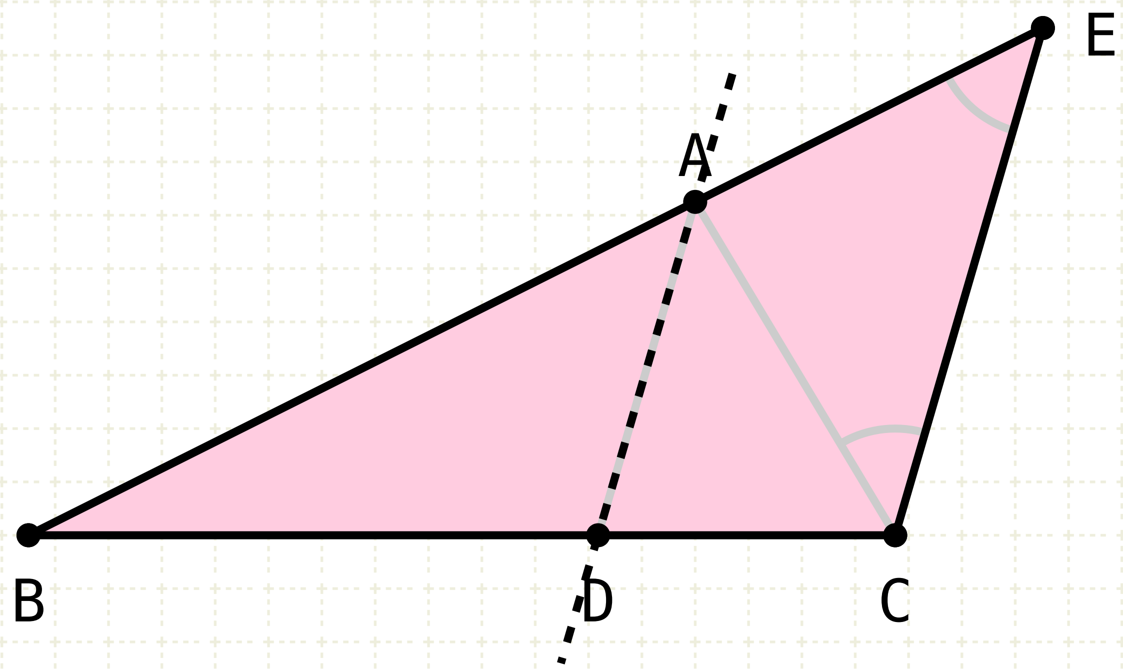

BA:AC = BD:DC

DA || CE

BA:AE = BD:DC

BA:AC = BA:AE

Proof (part 2)

Since DA is parallel to CE, the ratio AB to AE is equal to the ratio BD to DC (VI·2)

If two ratios are equal to the same, they are equal to each other (V·11), so the ratios BA to AC and BA to AE are also equal

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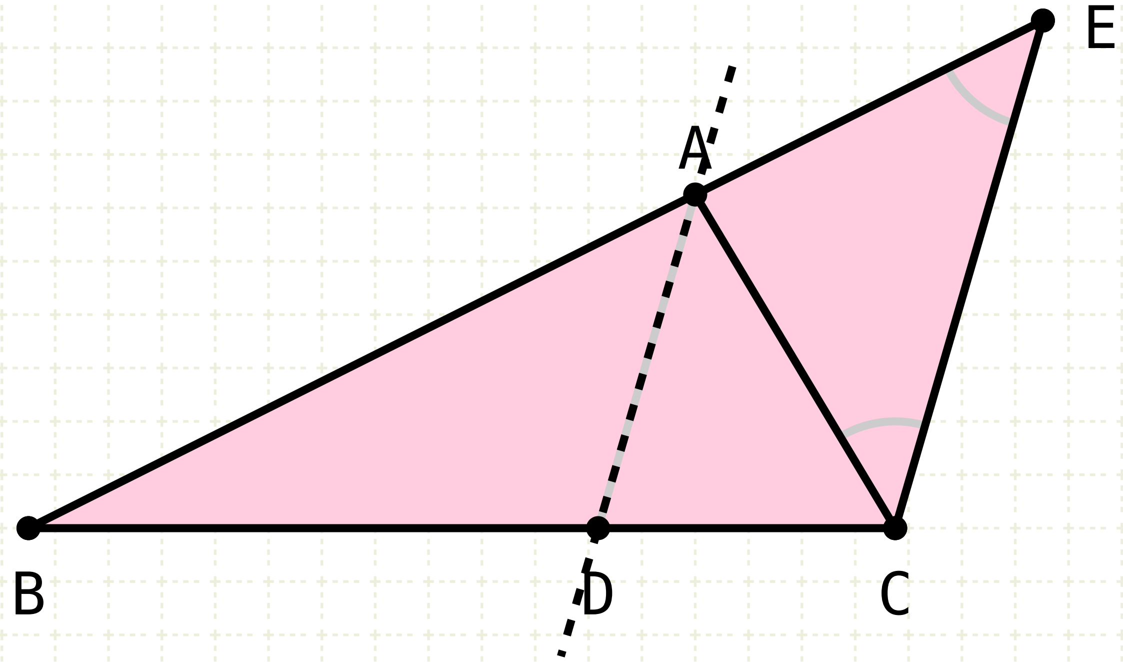
If an angle of a triangle be bisected and the straight line cutting the angle cut the base also, the segments of the base will have the same ratio as the remaining sides of the triangle; and, if the segments of the base have the same ratio as the remaining sides of the triangle, the straight line joined from the vertex to the point of section will bisect the angle of the triangle.



BA:AC = BD:DC

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BA:AE = BD:DC

BA:AC = BA:AE

AC = AE

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Since DA is parallel to CE, the ratio AB to AE is equal to the ratio BD to DC (VI·2)

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Therefore, AC equals AE (V·9)

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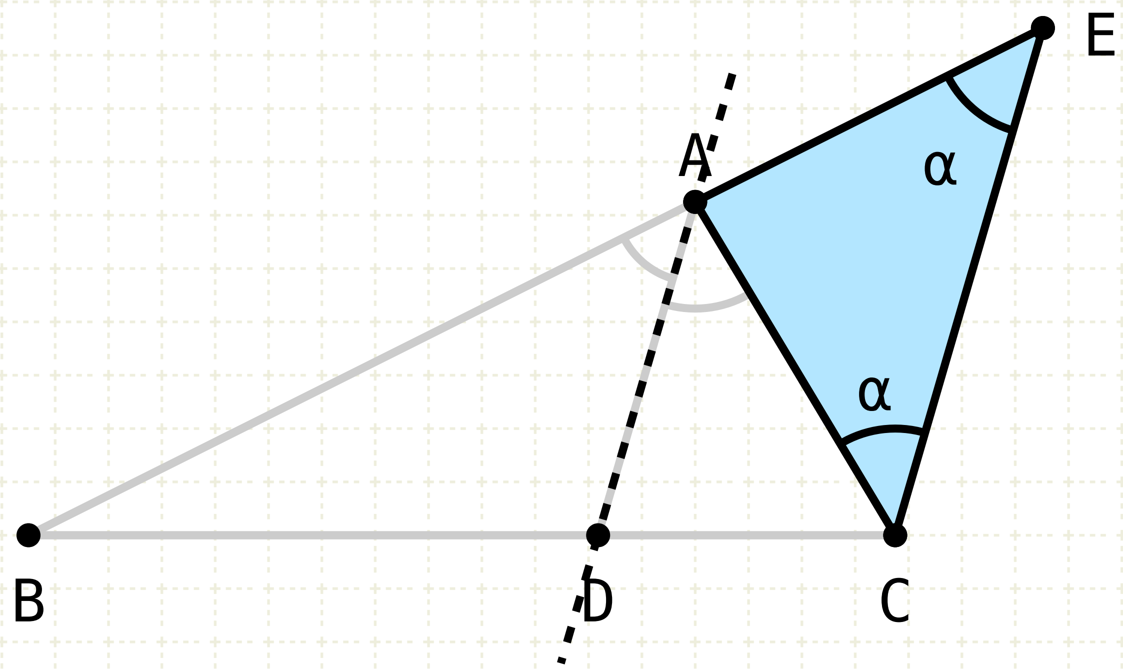

BA:AC = BD:DC
DA || CE
BA:AE = BD:DC
BA:AC = BA:AE
AC = AE
∠ACE = ∠AEC

Proof (part 2)

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If AC and AE are equal, then triangle ACE is an isosceles triangle, and the angles AEC and ACE are equal (I·5)

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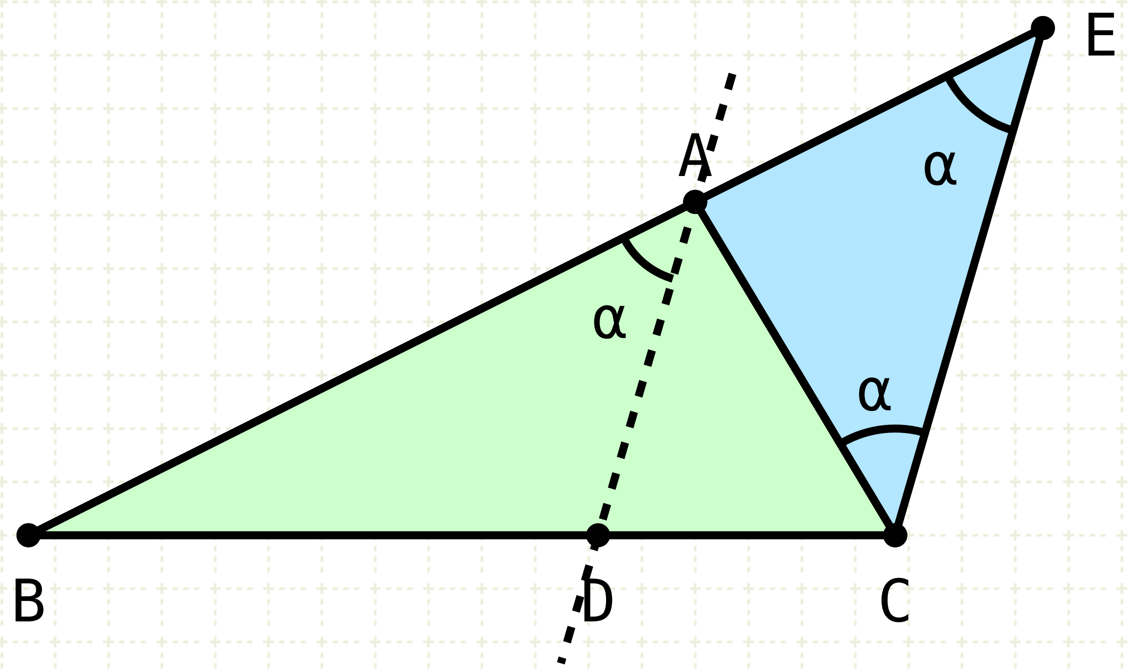

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If AC and AE are equal, then triangle ACE is an isosceles triangle, and the angles AEC and ACE are equal (I·5)

The angle AEC is equal to the exterior angle BAD of the parallel lines AD and EC (I·29)

BA:AC = BD:DC
DA || CE
BA:AE = BD:DC
BA:AC = BA:AE
AC = AE
∠ACE = ∠AEC
∠AEC = ∠BAD

Proposition 3 of Book VI

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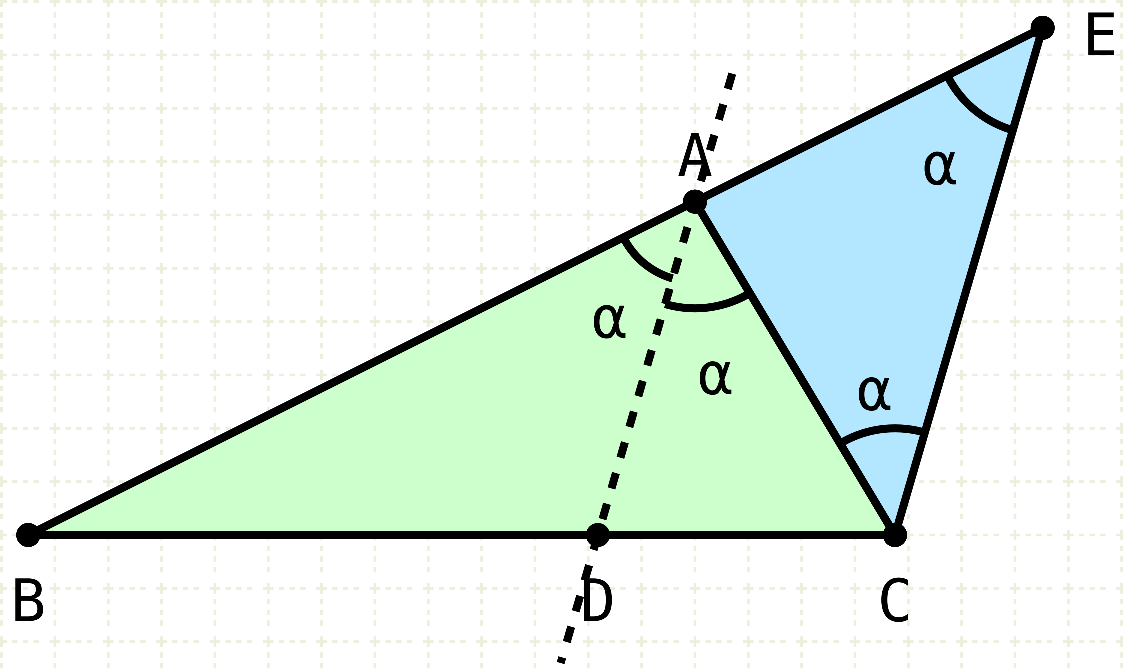

BA:AC = BD:DC
DA || CE
BA:AE = BD:DC
BA:AC = BA:AE
AC = AE
∠ACE = ∠AEC
∠AEC = ∠BAD
∠ACE = CAD

Proof (part 2)

Since DA is parallel to CE, the ratio AB to AE is equal to the ratio BD to DC (VI·2)

If two ratios are equal to the same, they are equal to each other (V·11), so the ratios BA to AC and BA to AE are also equal

Therefore, AC equals AE (V·9)

If AC and AE are equal, then triangle ACE is an isosceles triangle, and the angles AEC and ACE are equal (I·5)

The angle AEC is equal to the exterior angle BAD of the parallel lines AD and EC (I·29)

and the angle ACE is equal to the alternate angle CAD (I·29)

Proposition 3 of Book VI

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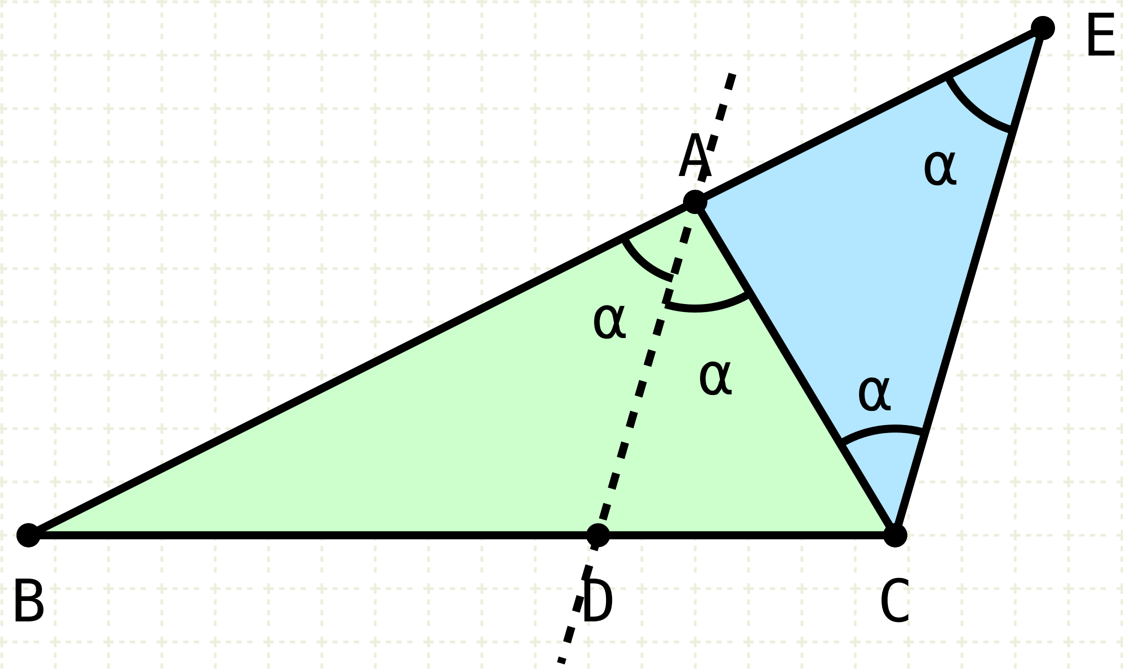

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BA:AC = BA:AE
AC = AE
∠ACE = ∠AEC
∠AEC = ∠BAD
∠ACE = CAD
∠BAD = ∠CAD

Proof (part 2)

Since DA is parallel to CE, the ratio AB to AE is equal to the ratio BD to DC (VI·2)

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If AC and AE are equal, then triangle ACE is an isosceles triangle, and the angles AEC and ACE are equal (I·5)

The angle AEC is equal to the exterior angle BAD of the parallel lines AD and EC (I·29)

and the angle ACE is equal to the alternate angle CAD (I·29)

Therefore, the line AD bisects the angle BAC

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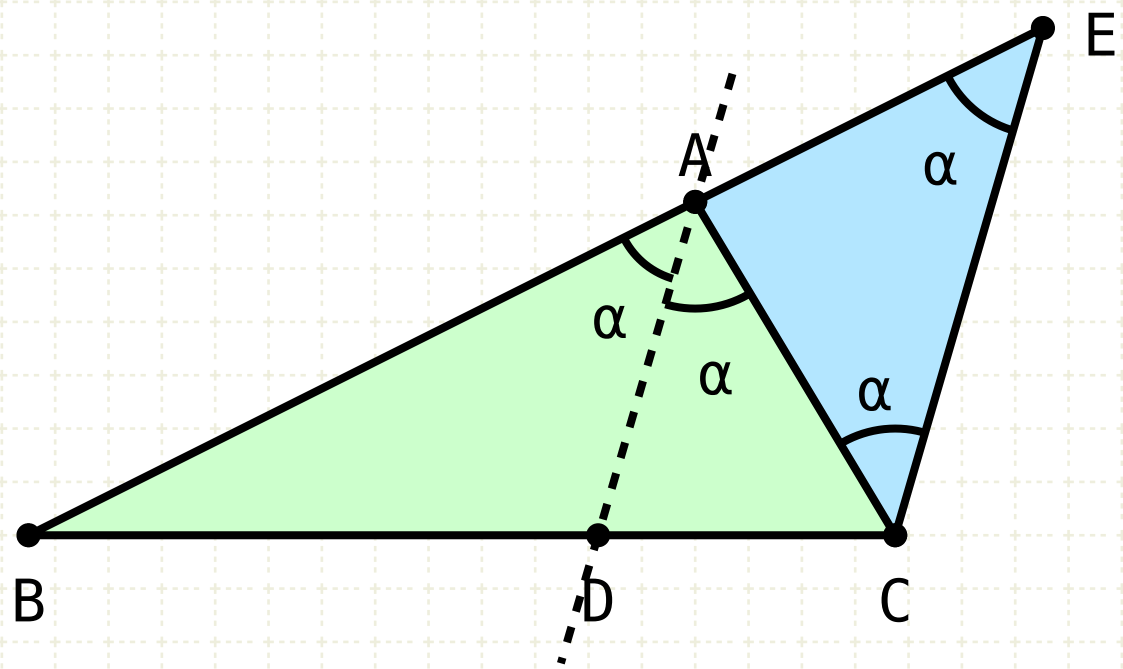

BA:AC = BD:DC

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∠AEC = ∠BAD

∠ACE = CAD

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Proof (part 2)

Since DA is parallel to CE, the ratio AB to AE is equal to the ratio BD to DC (VI·2)

If two ratios are equal to the same, they are equal to each other (V·11), so the ratios BA to AC and BA to AE are also equal

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The angle AEC is equal to the exterior angle BAD of the parallel lines AD and EC (I·29)

and the angle ACE is equal to the alternate angle CAD (I·29)

Therefore, the line AD bisects the angle BAC

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