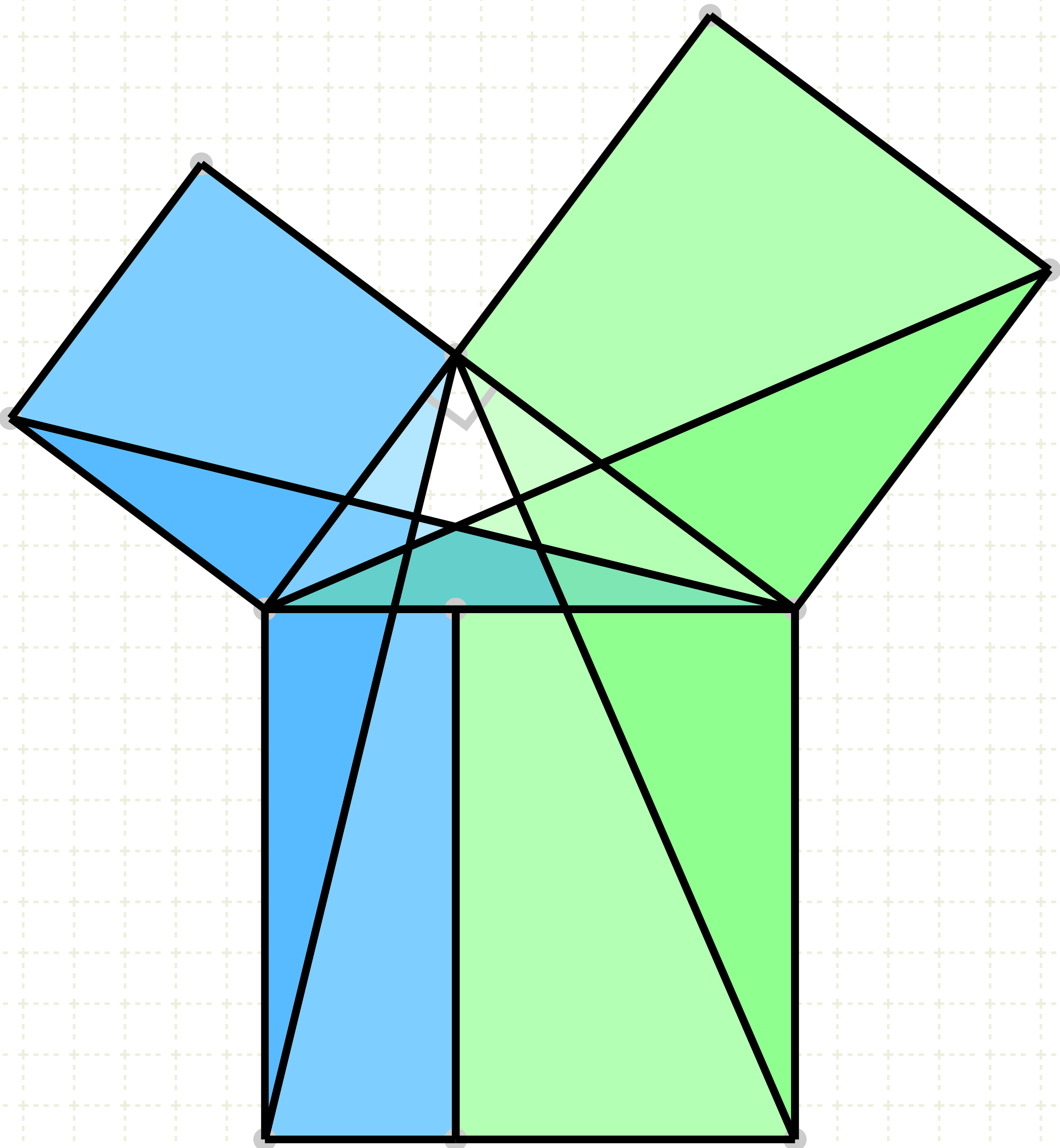
Euclid's Elements

Book I



If Euclid did not kindle your youthful enthusiasm, you were not born to be a scientific thinker.

Albert Einstein

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Proposition 34 of Book I

In parallelogrammic areas the opposite sides and angles equal one another, and the diameter bisects the areas.

Proposition 34 of Book I

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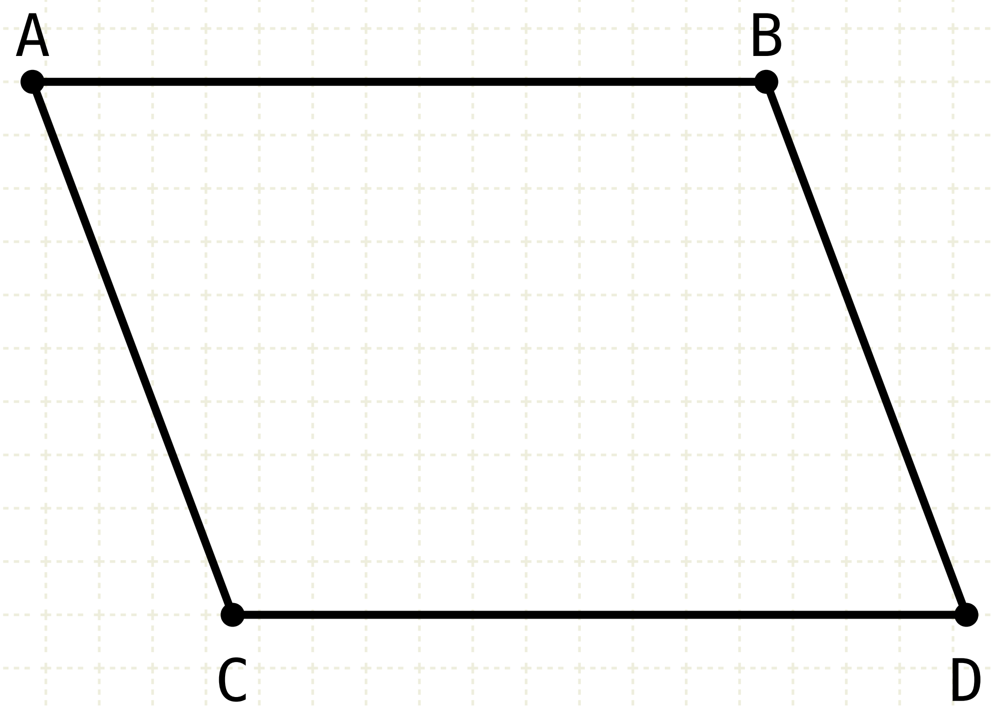


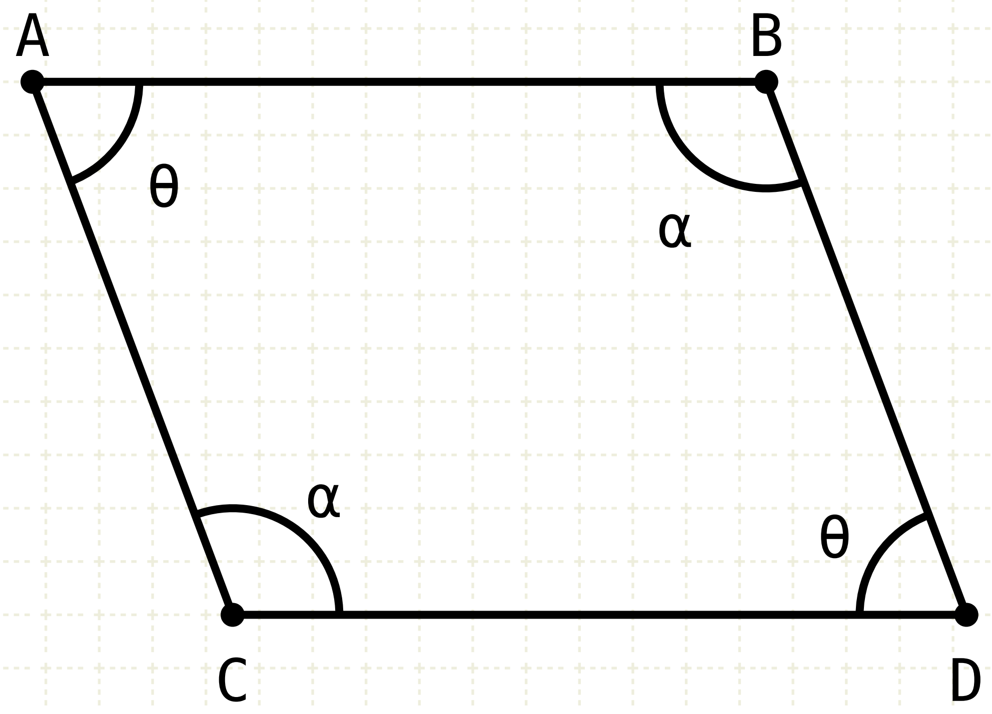
AB || CD
AC || BD

In other words

Let ABCD define a parallelogram

Proposition 34 of Book I

In parallelogrammic areas the opposite sides and angles equal one another, and the diameter bisects the areas.



AB || CD
AC || BD

∠BAC = ∠CDB = θ
∠DBA = ∠ACD = α

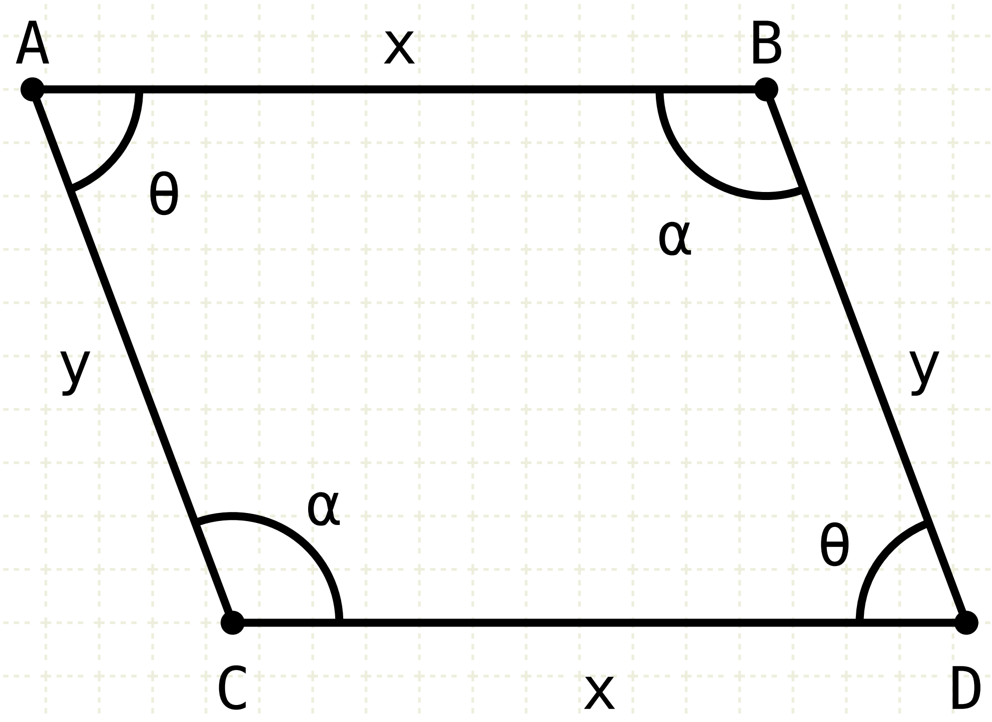
In other words

Let ABCD define a parallelogram

The opposite angles are equal

Proposition 34 of Book I

In parallelogrammic areas the opposite sides and angles equal one another, and the diameter bisects the areas.



AB || CD
AC || BD

∠BAC = ∠CDB = θ
∠DBA = ∠ACD = α
AB = CD = x
AC = BD = y

In other words

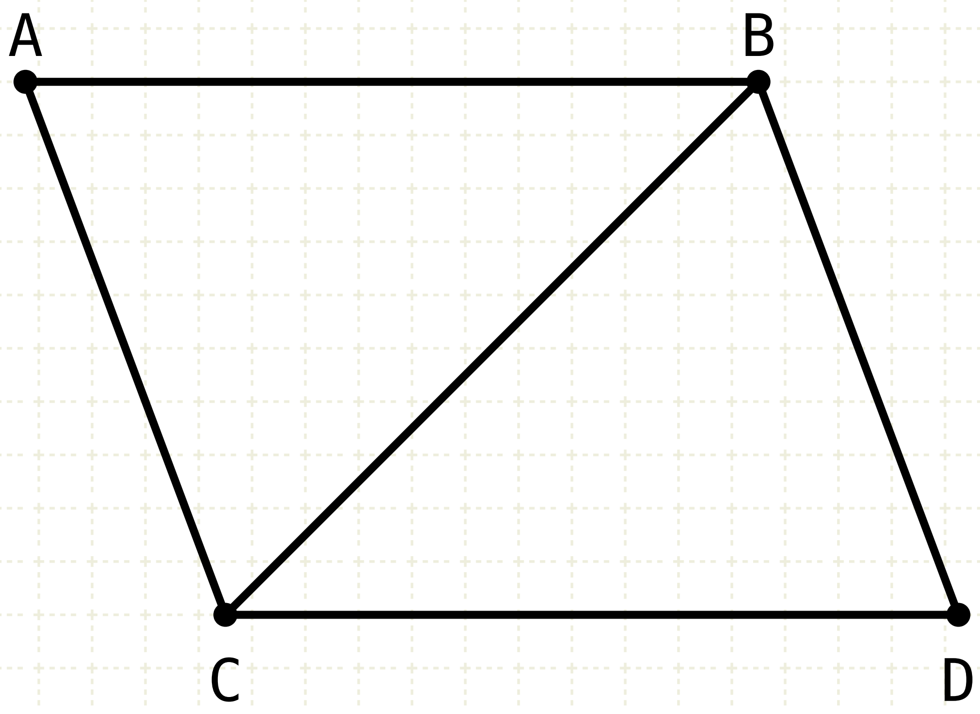
Let ABCD define a parallelogram

The opposite angles are equal

The opposite sides are equal

Proposition 34 of Book I

In parallelogrammic areas the opposite sides and angles equal one another, and the diameter bisects the areas.



$AB \parallel CD$
$AC \parallel BD$

$\angle BAC = \angle CDB = \theta$
$\angle DBA = \angle ACD = \alpha$
$AB = CD = x$
$AC = BD = y$

In other words

Let ABCD define a parallelogram

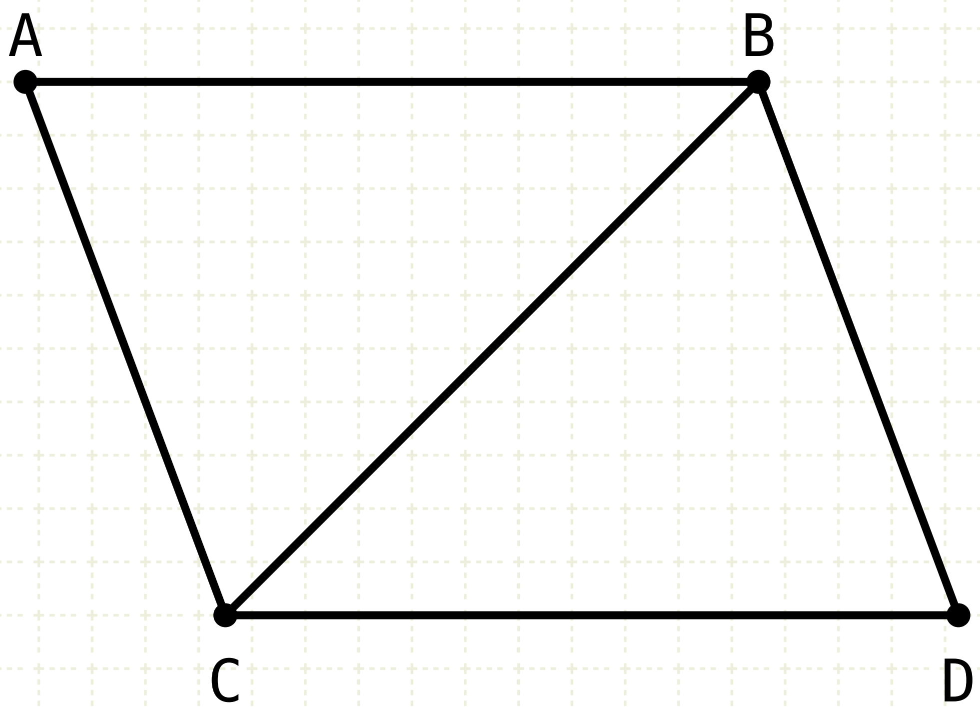
The opposite angles are equal

The opposite sides are equal

Let BC be the diameter (diagonal) of the parallelogram

Proposition 34 of Book I

In parallelogrammic areas the opposite sides and angles equal one another, and the diameter bisects the areas.



$AB \parallel CD$
$AC \parallel BD$

$\angle BAC = \angle CDB = \theta$
$\angle DBA = \angle ACD = \alpha$
$AB = CD = x$
$AC = BD = y$
$\Delta ABC = \Delta BCD$

In other words

Let ABCD define a parallelogram

The opposite angles are equal

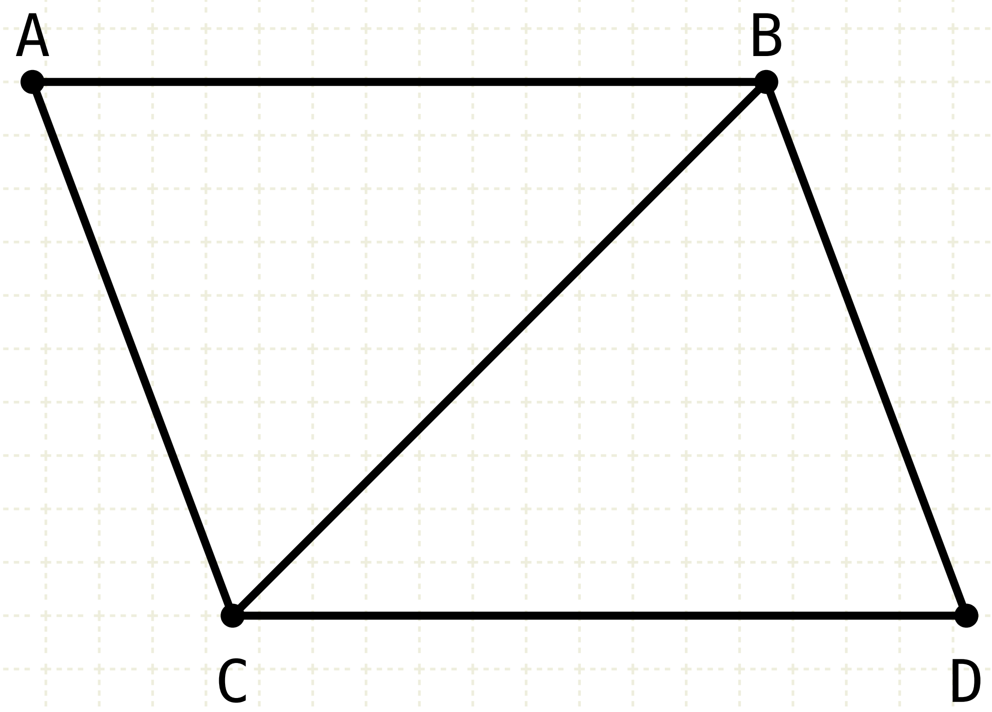
The opposite sides are equal

Let BC be the diameter (diagonal) of the parallelogram

The diameter BC bisects the parallelogram

Proposition 34 of Book I

In parallelogrammic areas the opposite sides and angles equal one another, and the diameter bisects the areas.



AB || CD
AC || BD

In other words

Let ABCD define a parallelogram

The opposite angles are equal

The opposite sides are equal

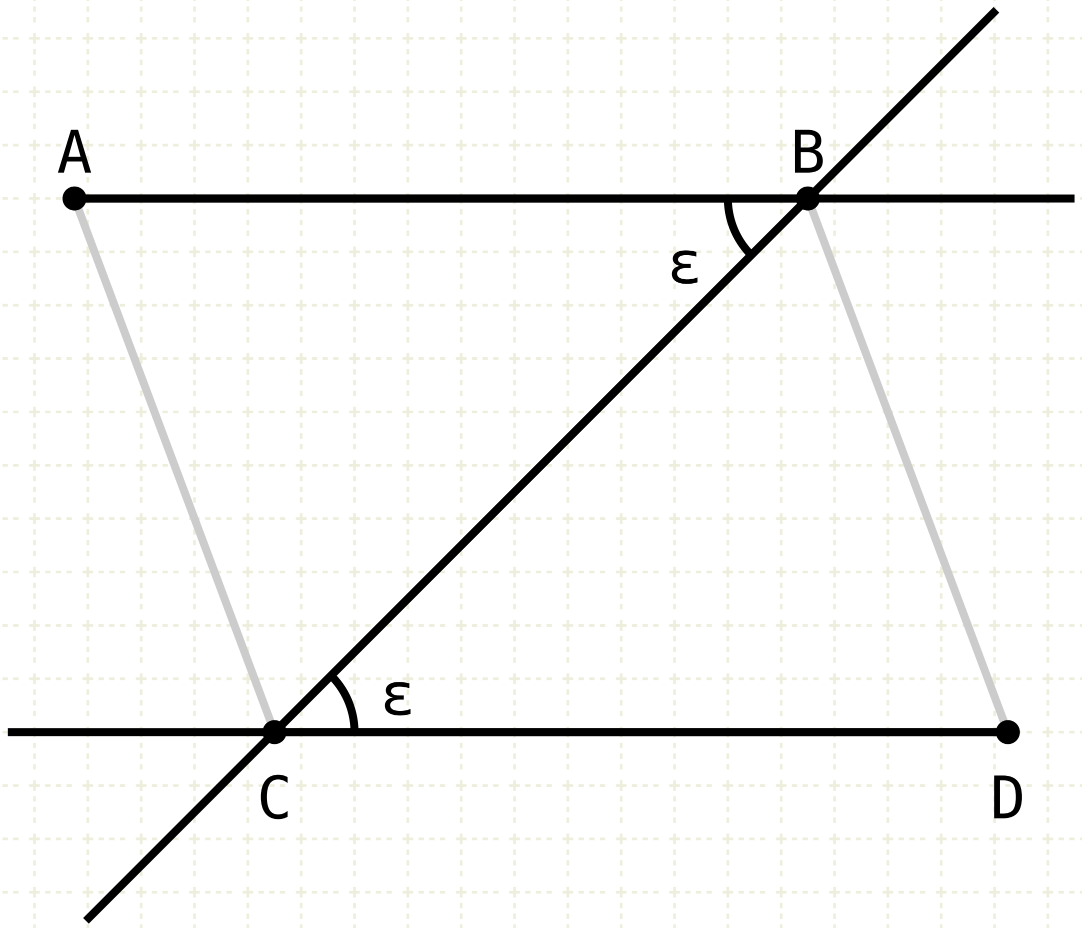
Let BC be the diameter (diagonal) of the parallelogram

The diameter BC bisects the parallelogram

Proof

Proposition 34 of Book I

In parallelogrammic areas the opposite sides and angles equal one another, and the diameter bisects the areas.



AB || CD
AC || BD

$\angle ABC = \angle BCD = \varepsilon$

In other words

Let ABCD define a parallelogram

The opposite angles are equal

The opposite sides are equal

Let BC be the diameter (diagonal) of the parallelogram

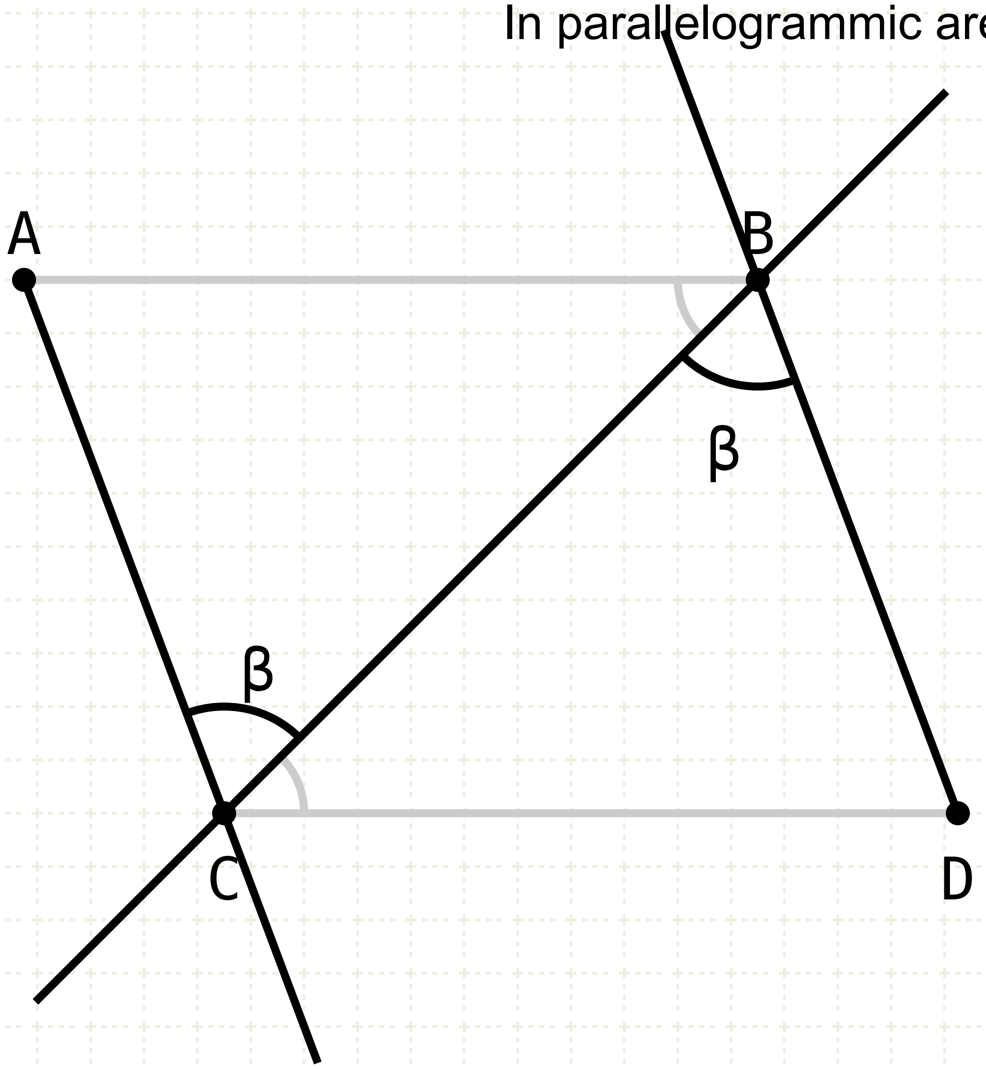
The diameter BC bisects the parallelogram

Proof

Since line BC intersects two parallel lines (AB and CD), angles ABC and BCD are equal (I·29)

Proposition 34 of Book I

In parallelogrammic areas the opposite sides and angles equal one another, and the diameter bisects the areas.



AB || CD

AC || BD

∠ABC = ∠BCD = ε

∠ACB = ∠CBD = β

In other words

Let ABCD define a parallelogram

The opposite angles are equal

The opposite sides are equal

Let BC be the diameter (diagonal) of the parallelogram

The diameter BC bisects the parallelogram

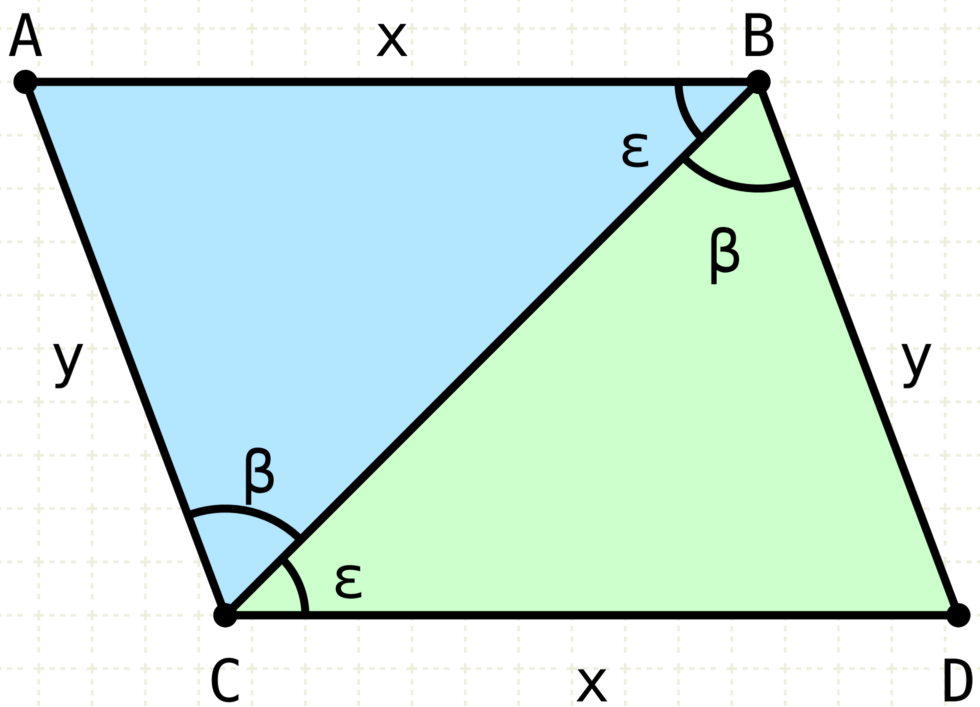
Proof

Since line BC intersects two parallel lines (AB and CD), angles ABC and BCD are equal (I·29)

Since line BC intersects two parallel lines (AC and BD), angles ACB and CBD are equal (I·29)

Proposition 34 of Book I

In parallelogrammic areas the opposite sides and angles equal one another, and the diameter bisects the areas.



AB || CD
AC || BD

$\angle ABC = \angle BCD = \varepsilon$
$\angle ACB = \angle CBD = \beta$
$\Delta ABC \equiv \Delta BCD$

In other words

Let ABCD define a parallelogram

The opposite angles are equal

The opposite sides are equal

Let BC be the diameter (diagonal) of the parallelogram

The diameter BC bisects the parallelogram

Proof

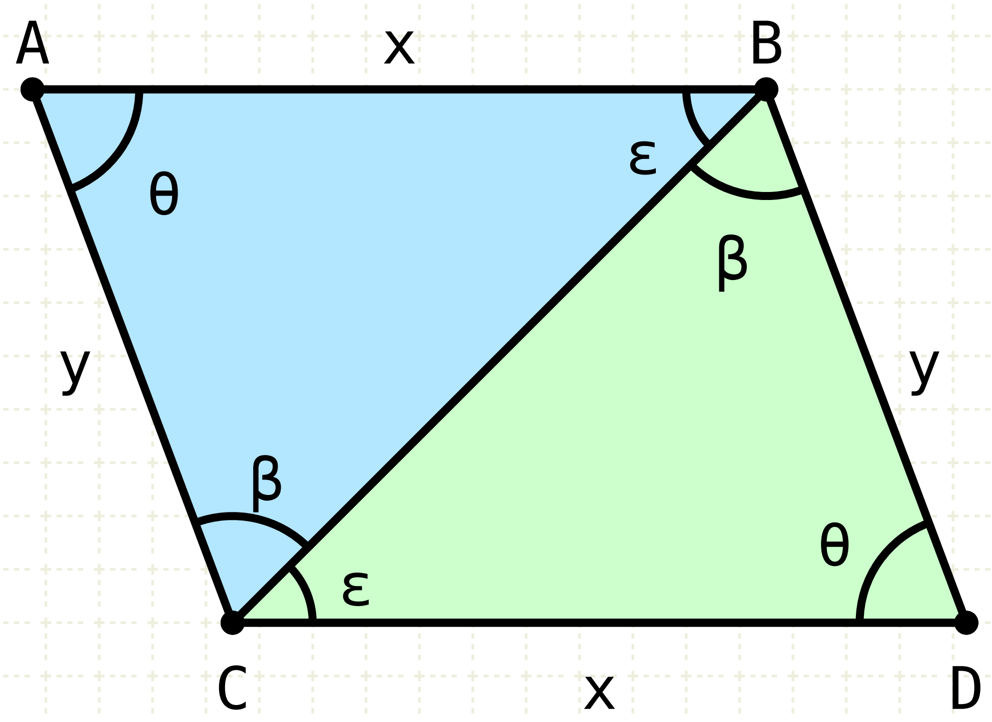
Since line BC intersects two parallel lines (AB and CD), angles ABC and BCD are equal (I·29)

Since line BC intersects two parallel lines (AC and BD), angles ACB and CBD are equal (I·29)

Triangles ABC and BDC have two equal angles, and one equal side (CB), hence they are equivalent (I·26),

Proposition 34 of Book I

In parallelogrammic areas the opposite sides and angles equal one another, and the diameter bisects the areas.



AB || CD
AC || BD

∠ABC = ∠BCD = ε
∠ACB = ∠CBD = β
ΔABC ≡ ΔBCD

AB = CD = x
AC = BD = y
∠BAC = ∠CBD = θ
ΔABC = ΔBCD

In other words

Let ABCD define a parallelogram

The opposite angles are equal

The opposite sides are equal

Let BC be the diameter (diagonal) of the parallelogram

The diameter BC bisects the parallelogram

Proof

Since line BC intersects two parallel lines (AB and CD), angles ABC and BCD are equal (I·29)

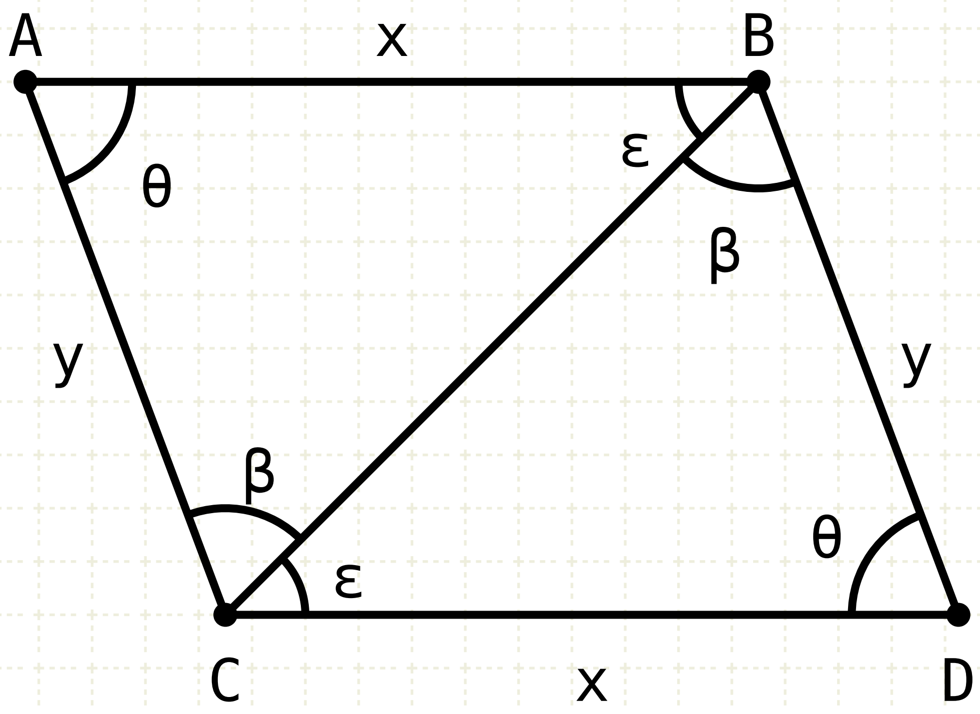
Since line BC intersects two parallel lines (AC and BD), angles ACB and CBD are equal (I·29)

Triangles ABC and BDC have two equal angles, and one equal side (CB), hence they are equivalent (I·26),

Which means all the sides, angles and areas are equal

Proposition 34 of Book I

In parallelogrammic areas the opposite sides and angles equal one another, and the diameter bisects the areas.



$AB \parallel CD$

$AC \parallel BD$

$\angle ABC = \angle BCD = \varepsilon$

$\angle ACB = \angle CBD = \beta$

$\Delta ABC \equiv \Delta BCD$

$AB = CD = x$

$AC = BD = y$

$\angle BAC = \angle CBD = \theta$

$\Delta ABC = \Delta BCD$

$\angle ABD = \varepsilon + \beta$

$\angle ACD = \beta + \varepsilon$

In other words

Let ABCD define a parallelogram

The opposite angles are equal

The opposite sides are equal

Let BC be the diameter (diagonal) of the parallelogram

The diameter BC bisects the parallelogram

Proof

Since line BC intersects two parallel lines (AB and CD), angles ABC and BCD are equal (I·29)

Since line BC intersects two parallel lines (AC and BD), angles ACB and CBD are equal (I·29)

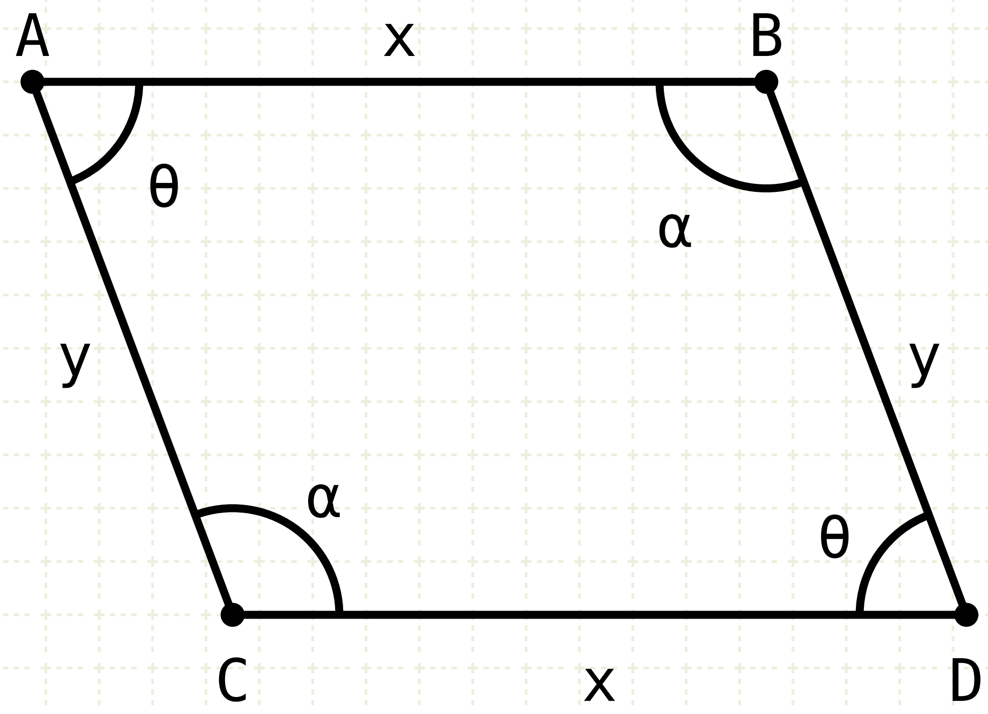
Triangles ABC and BDC have two equal angles, and one equal side (CB), hence they are equivalent (I·26),

Which means all the sides, angles and areas are equal

Angle ABD is equal to the sum of ABC and CBD and angle ACD is equal to the sum of ACB and BCD

Proposition 34 of Book I

In parallelogrammic areas the opposite sides and angles equal one another, and the diameter bisects the areas.



$AB \parallel CD$

$AC \parallel BD$

$\angle ABC = \angle BCD = \varepsilon$

$\angle ACB = \angle CBD = \beta$

$\Delta ABC \equiv \Delta BCD$

$AB = CD = x$

$AC = BD = y$

$\angle BAC = \angle CBD = \theta$

$\Delta ABC = \Delta BCD$

$\angle ABD = \varepsilon + \beta$

$\angle ACD = \beta + \varepsilon$

$\angle ABD = \angle ACD = \alpha$

In other words

Let ABCD define a parallelogram

The opposite angles are equal

The opposite sides are equal

Let BC be the diameter (diagonal) of the parallelogram

The diameter BC bisects the parallelogram

Proof

Since line BC intersects two parallel lines (AB and CD), angles ABC and BCD are equal (I·29)

Since line BC intersects two parallel lines (AC and BD), angles ACB and CBD are equal (I·29)

Triangles ABC and BDC have two equal angles, and one equal side (CB), hence they are equivalent (I·26),

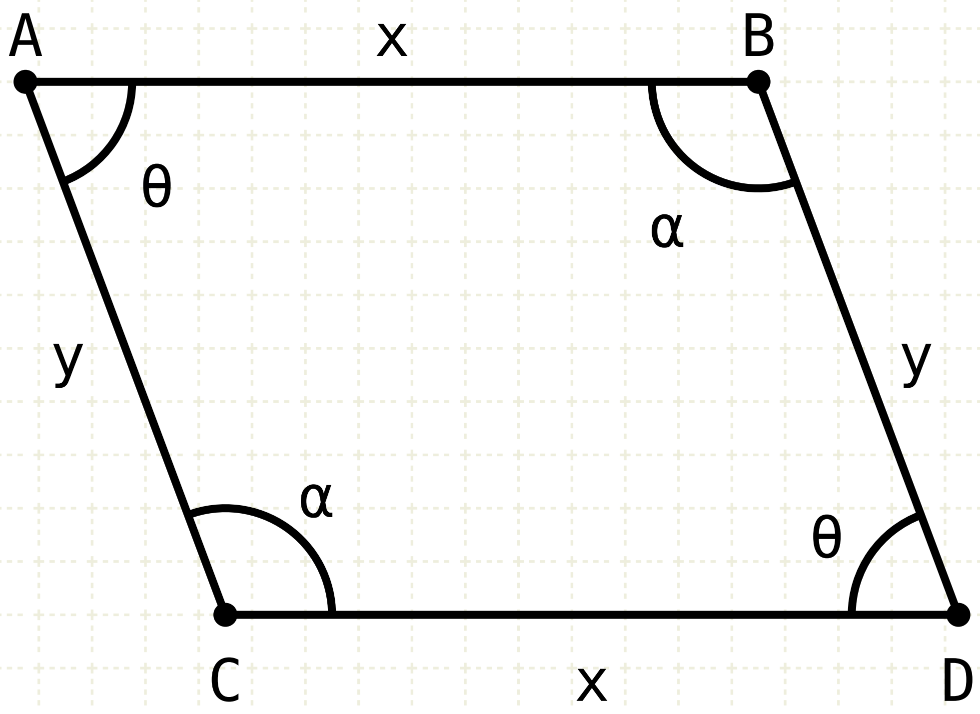
Which means all the sides, angles and areas are equal

Angle ABD is equal to the sum of ABC and CBD and angle ACD is equal to the sum of ACB and BCD

Therefore, angles ABD and ACD are equal

Proposition 34 of Book I

In parallelogrammic areas the opposite sides and angles equal one another, and the diameter bisects the areas.



$AB \parallel CD$
$AC \parallel BD$

$\angle ABC = \angle BCD = \varepsilon$
$\angle ACB = \angle CBD = \beta$
$\Delta ABC \equiv \Delta BCD$

$AB = CD = x$
$AC = BD = y$
$\angle BAC = \angle CBD = \theta$
$\Delta ABC = \Delta BCD$
$\angle ABD = \varepsilon + \beta$
$\angle ACD = \beta + \varepsilon$
$\angle ABD = \angle ACD = \alpha$

In other words

Let ABCD define a parallelogram

The opposite angles are equal

The opposite sides are equal

Let BC be the diameter (diagonal) of the parallelogram

The diameter BC bisects the parallelogram

Proof

Since line BC intersects two parallel lines (AB and CD), angles ABC and BCD are equal (I·29)

Since line BC intersects two parallel lines (AC and BD), angles ACB and CBD are equal (I·29)

Triangles ABC and BDC have two equal angles, and one equal side (CB), hence they are equivalent (I·26),

Which means all the sides, angles and areas are equal

Angle ABD is equal to the sum of ABC and CBD and angle ACD is equal to the sum of ACB and BCD

Therefore, angles ABD and ACD are equal

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