

Euclid's Elements

Book V



Proportions are what makes the old Greek temples classic in their beauty. They are like huge blocks, from which the air has been literally hewn out between the columns.

$$AB:C = DE:F$$

$$BG:C = EH:F$$

$$AG:C = DH:F$$

Arne Jacobsen



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10	if $A:C > B:C$, or $A:C < B:C$ then $A > B$, or $A < C$, respectively				



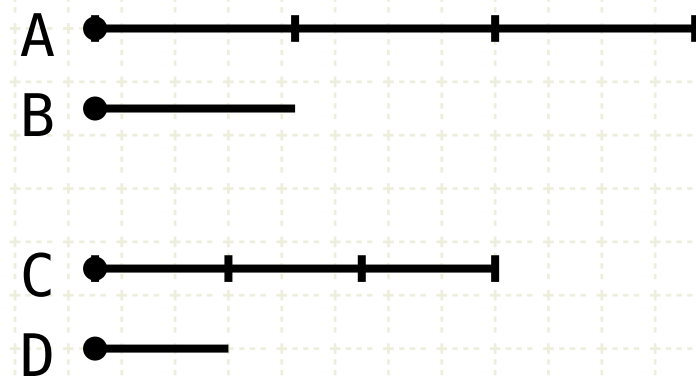
Proposition 3 of Book V

If a first magnitude be the same multiple of a second that a third is of a fourth, and if equimultiples be taken of the first and third, then also, ex aequali, the magnitudes taken will be equimultiples respectively, the one of the second, and the other of the fourth.



Proposition 3 of Book V

If a first magnitude be the same multiple of a second that a third is of a fourth, and if equimultiples be taken of the first and third, then also, ex aequali, the magnitudes taken will be equimultiples respectively, the one of the second, and the other of the fourth.



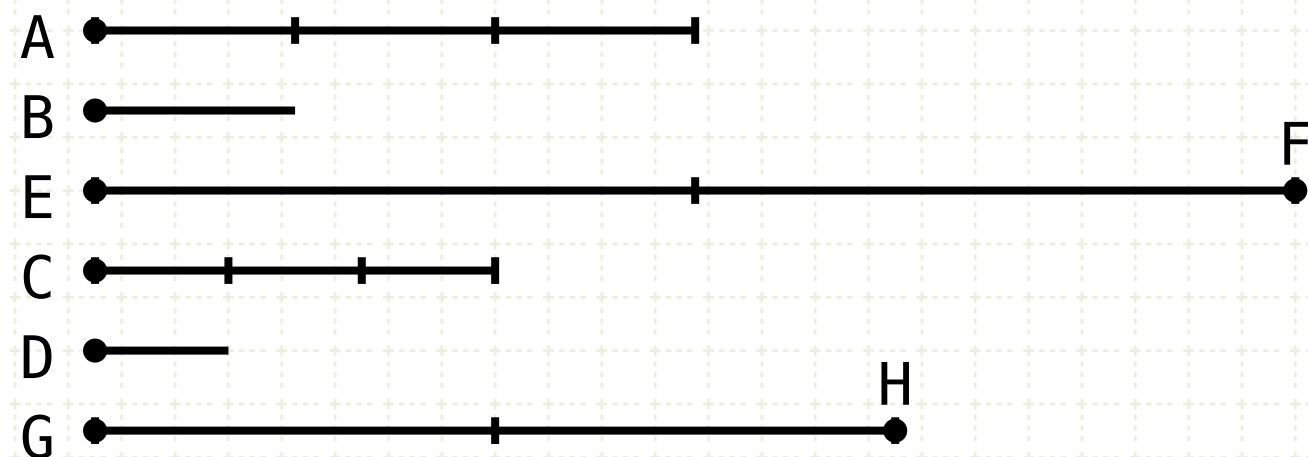
If $A = n \cdot B$, $C = n \cdot D$

In other words

If we have two lines (A and C) that are equal multiples of two other lines (B and D respectively) and ...

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If $A = n \cdot B$, $C = n \cdot D$
and $EF = m \cdot A$, $GH = m \cdot C$

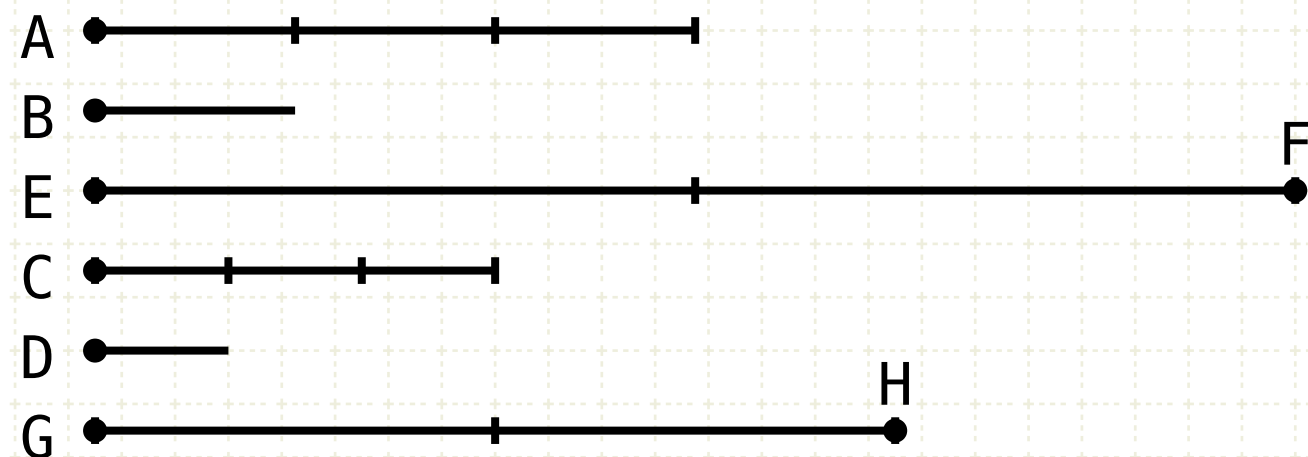
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If $A = n \cdot B$, $C = n \cdot D$
and $EF = m \cdot A$, $GH = m \cdot C$
then $EF = k \cdot B$ and $GH = k \cdot D$
where n, m, k are integers

In other words

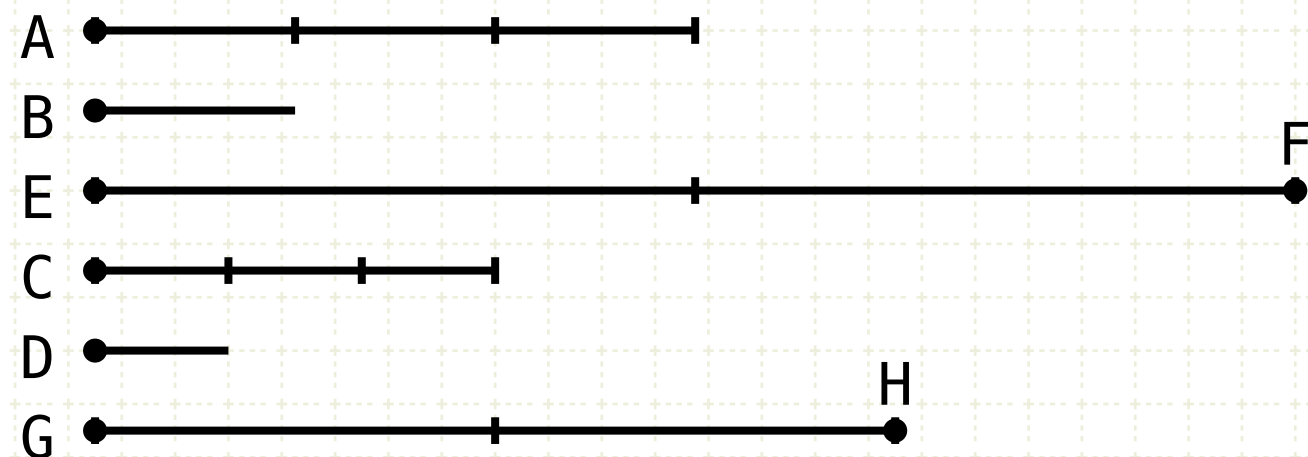
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$$A = n \cdot B, \quad C = n \cdot D$$
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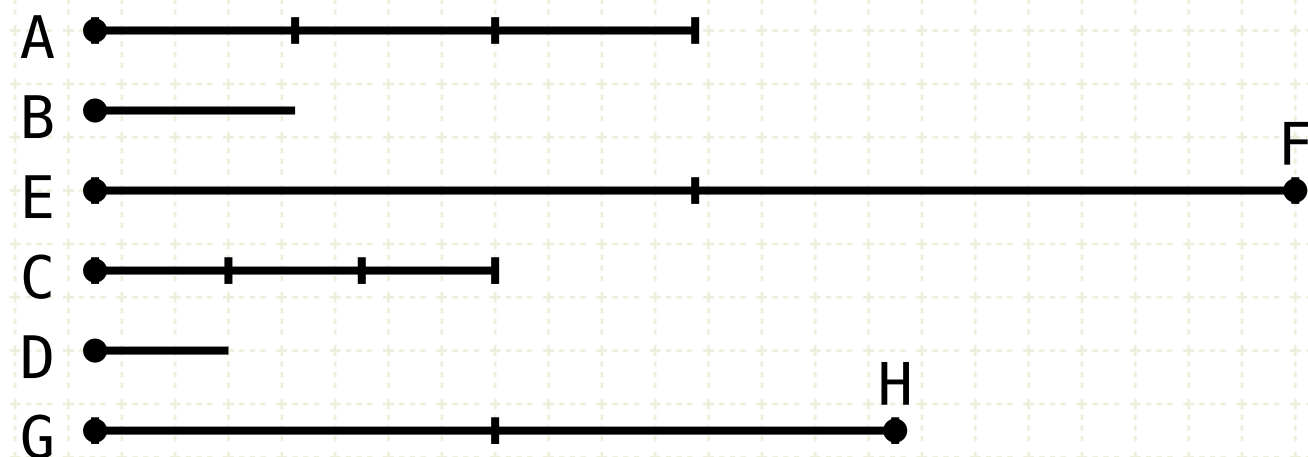
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$$A = n \cdot B, \quad C = n \cdot D$$
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$$EF = m \cdot A, \quad GH = m \cdot C, \quad m = 2$$

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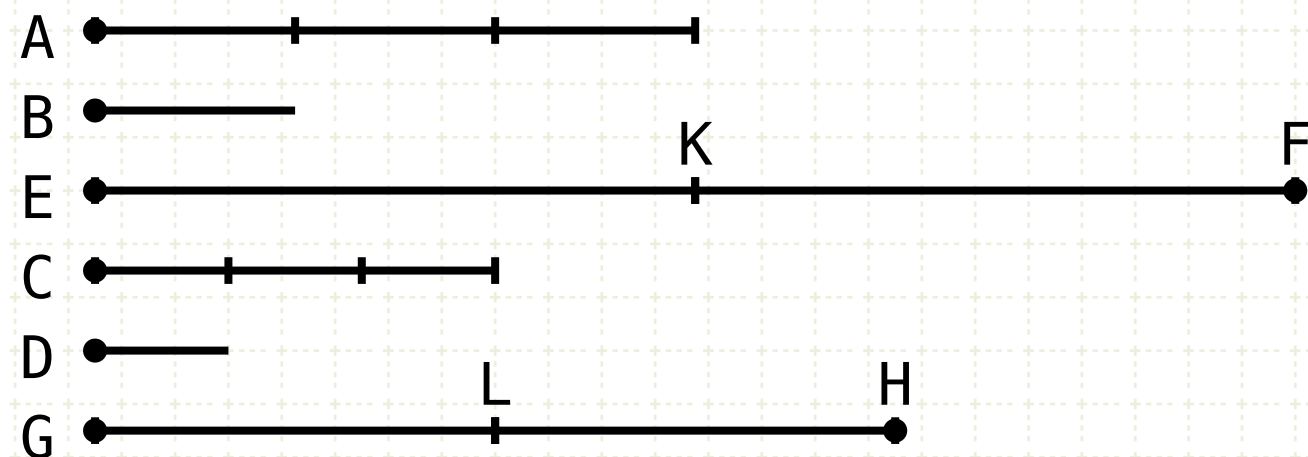
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Proof

Since EF and GH are the same multiples of A and C respectively, then there are the an equal number of magnitudes in EF and GH

Proposition 3 of Book V

If a first magnitude be the same multiple of a second that a third is of a fourth, and if equimultiples be taken of the first and third, then also, ex aequali, the magnitudes taken will be equimultiples respectively, the one of the second, and the other of the fourth.



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$$EF = m \cdot A, \quad GH = m \cdot C$$

$$EF = m \cdot A, \quad GH = m \cdot C, \quad m = 2$$
$$EK = KF = A$$
$$GL = LH = C$$

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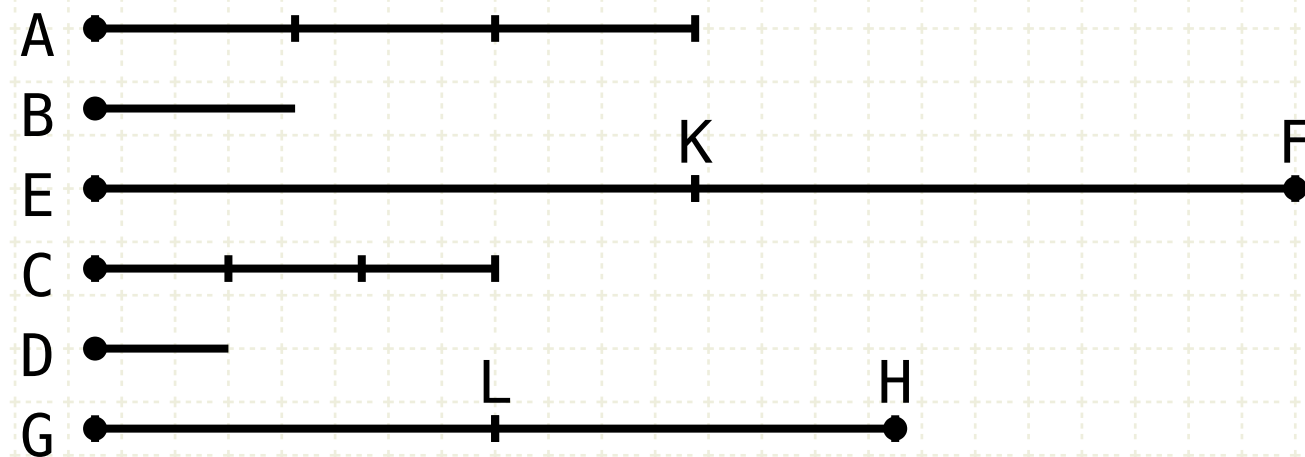
Proof

Since EF and GH are the same multiples of A and C respectively, then there are the an equal number of magnitudes in EF and GH

Divide EF into equal segments of length A (EK, KF) and divide GH into equal segments of length C (GL, LH)

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If a first magnitude be the same multiple of a second that a third is of a fourth, and if equimultiples be taken of the first and third, then also, ex aequali, the magnitudes taken will be equimultiples respectively, the one of the second, and the other of the fourth.



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$$EF = m \cdot A, \quad GH = m \cdot C, \quad m = 2$$

$$EK = KF = A$$

$$GL = LH = C$$

$$EK = A = n \cdot B$$

$$GL = C = n \cdot D$$

EK = first line

B = second line

GL = third line

D = fourth line

In other words

If we have two lines (A and C) that are equal multiples of two other lines (B and D respectively) and ...

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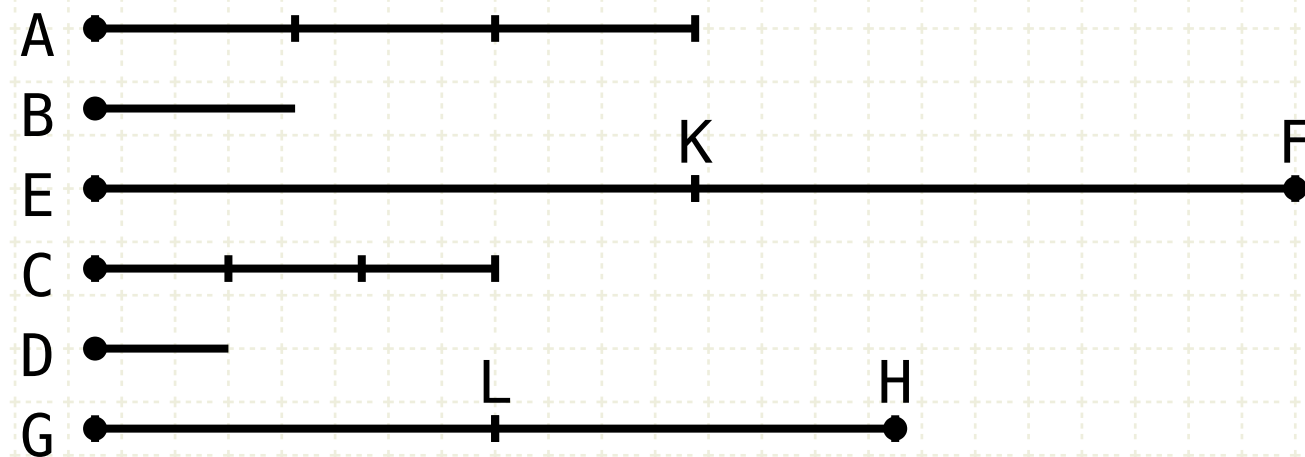
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So now, a first magnitude EK is the same multiple of a second B that a third GL is of a fourth D,



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$$EK = A = n \cdot B$$

$$GL = C = n \cdot D$$

$$KF = A = n \cdot B$$

$$LH = C = n \cdot D$$

EK = first line

B = second line

GL = third line

D = fourth line

KF = fifth line

LH = sixth line

In other words

If we have two lines (A and C) that are equal multiples of two other lines (B and D respectively) and ...

we draw two new lines (E and G), equimultiple to A and C respectively ...

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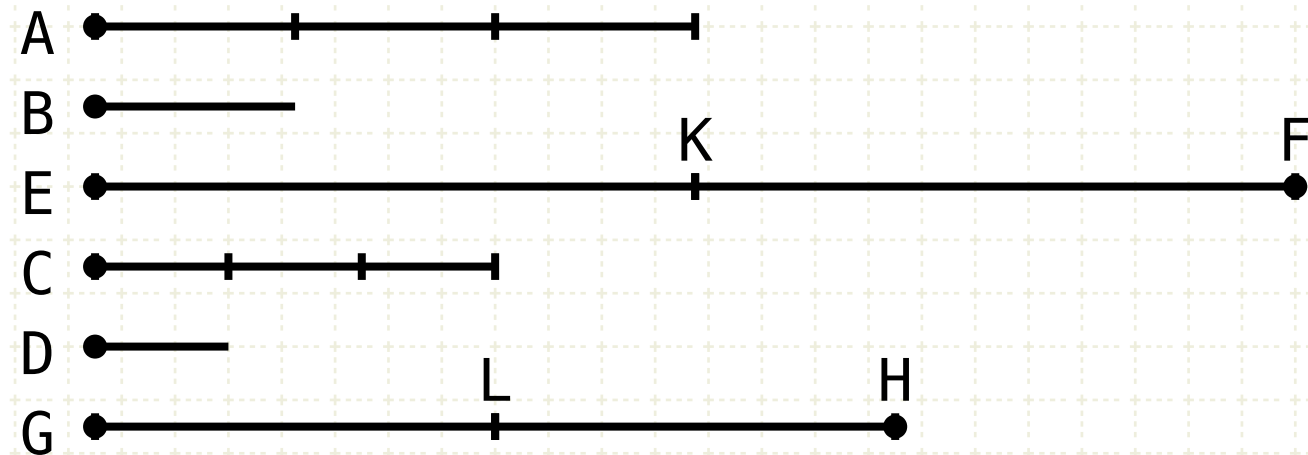
So now, a first magnitude EK is the same multiple of a second B that a third GL is of a fourth D,

And a fifth KF is also the same multiple of the second B that a sixth LH is of the fourth D,



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$$EK = A = n \cdot B$$

$$GL = C = n \cdot D$$

$$KF = A = n \cdot B$$

$$LH = C = n \cdot D$$

$$EF = EK + KF = n \cdot B + n \cdot B = m \cdot n \cdot B = k \cdot B$$

$$GH = GL + LH = n \cdot D + n \cdot D = m \cdot n \cdot D = k \cdot D$$

EK = first line

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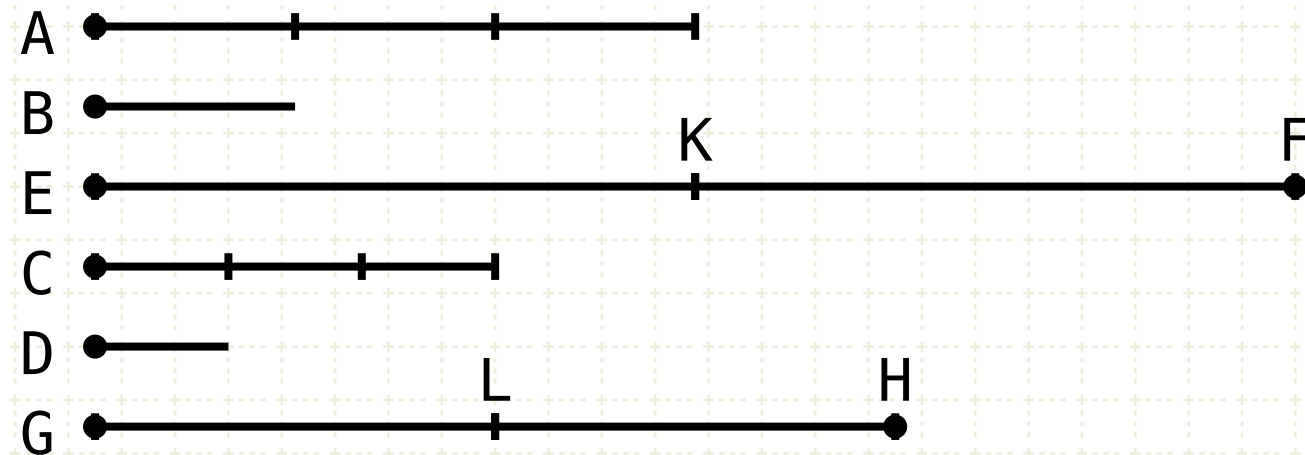
And a fifth KF is also the same multiple of the second B that a sixth LH is of the fourth D,

Thus EF is the same multiple of B as GH is of D (V.2)



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