

Euclid's Elements

Book VI

One can state, without exaggeration, that the observation of and the search for similarities and differences are the basis of all human knowledge.

Alfred Nobel



Table of Contents, Chapter 6

1	If the height of two triangles are equal, then the ratio of the areas is equal to the ratio of the bases	7	If two triangles have one angle equal to one angle, and the sides about other angles are proportional, and the remaining angles either both less or both not less than a right angle, then triangles will be equiangular	14	In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and vice versa
2	If a line cuts a triangle, parallel to its base, it will cut the sides of the triangle proportionally			15	In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and vice versa
3	If an angle of a triangle is bisected and the straight line cutting the angle also cuts the base, the segments of the base will have the same ratio as the remaining sides of the triangle	8	If in a right-angled triangle a perpendicular be drawn from the right angle to the base, the triangles adjoining the perpendicular are similar both to the whole and to one another	16	If four straight lines are proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means, and vice versa
4	If two triangles have equal angles, then the sides opposite the equal angles are proportional, as well, the sides of the triangles on either side of the equal angles are also proportional	9	From a given straight line to cut off a given fraction	17	If three straight lines are proportional, the rectangle contained by the extremes is equal to the square on the mean; and vice versa
5	If two triangles have proportional sides, the triangles will be equiangular	10	To cut a given uncut straight line similarly to a given cut straight line	18	On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure
6	If two triangles have one angle equal to one angle and the sides about the equal angles are proportional, then the triangles will be equiangular	11	To two given straight lines to find a third proportional	19	Similar triangles are to one another in the duplicate ratio of the corresponding sides
		12	To three given straight lines to find a fourth proportional		
		13	To two given straight lines to find a mean proportional		



Table of Contents, Chapter 3

20	Similar polygons are divided into the same number of similar triangles, which have the same ratio as the wholes, and the polygons have duplicate ratios to their corresponding sides	26	If from a parallelogram a similar parallelogram with a common angle is subtracted, it is about the same diameter as the original	31	In right-angled triangles the figure on the side subtending the right angle is equal to the similar and similarly described figures on the sides containing the right angle
21	Figures which are are similar to the same rectilineal figure are also similar to one another	27	Of all the parallelograms applied to the same straight line and deficient by parallelogrammic figures similar to a parallelogram drawn on half the said line, the largest will be one that is drawn on half of the straight line and is similar to the defect		
22	If four straight lines are proportional, similar rectilineal figures will also be proportional; and vice versa	28	To a given straight line, apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one		
23	Equiangular parallelograms have to one another the ratio compounded of the ratios of their sides	29	To a given straight line, apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one		
24	In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another	30	To cut a finite straight line in extreme ratio		
25	To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure				



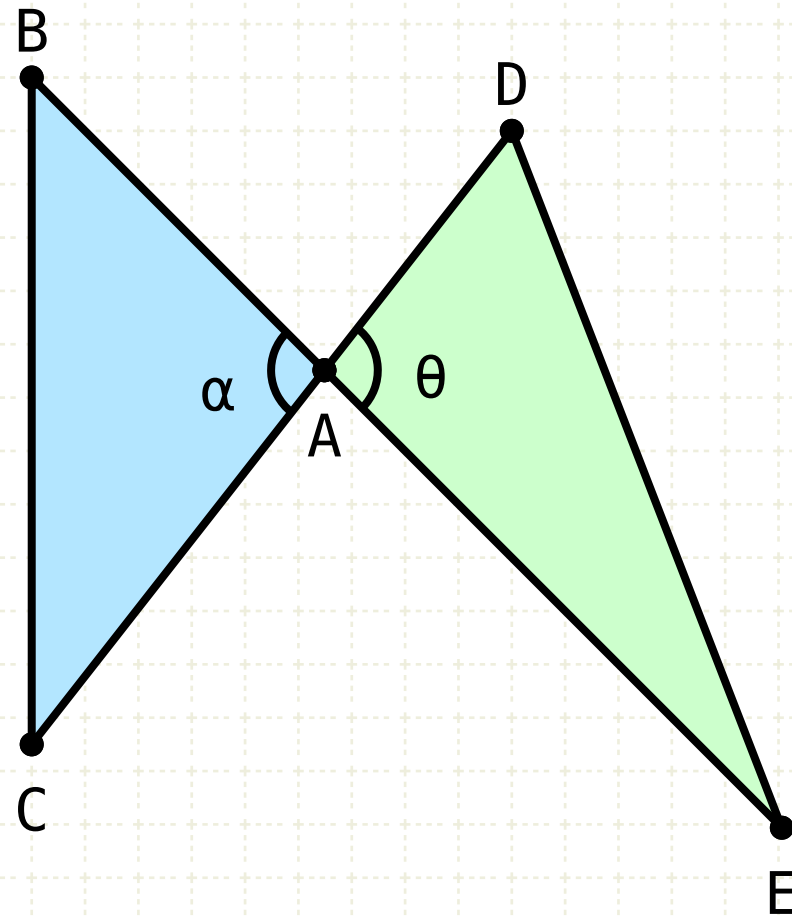
Proposition 15 of Book VI

In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and those triangles which have one angle equal to one angle, and in which the sides about the equal angles are reciprocally proportional, are equal.



Proposition 15 of Book VI

In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and those triangles which have one angle equal to one angle, and in which the sides about the equal angles are reciprocally proportional, are equal.



$$\alpha = \theta$$

$$\triangle ABC = \triangle DAE$$

$$AC:AD = AE:AB$$

$$\alpha = \theta$$

$$AC:AD = AE:AB$$

$$\triangle ABC = \triangle DAE$$

In other words

Given two triangles with one equal angle, and the areas of the triangle are equal, then the ratios of the sides around the equal angle is reciprocally proportional

... or ... the multiplication of the two sides of two equal area triangles on either side of two equal angles remains constant

And the inverse

Note:

Assume two objects 'x' and 'y', both with properties '1' and '2'

Proportional:

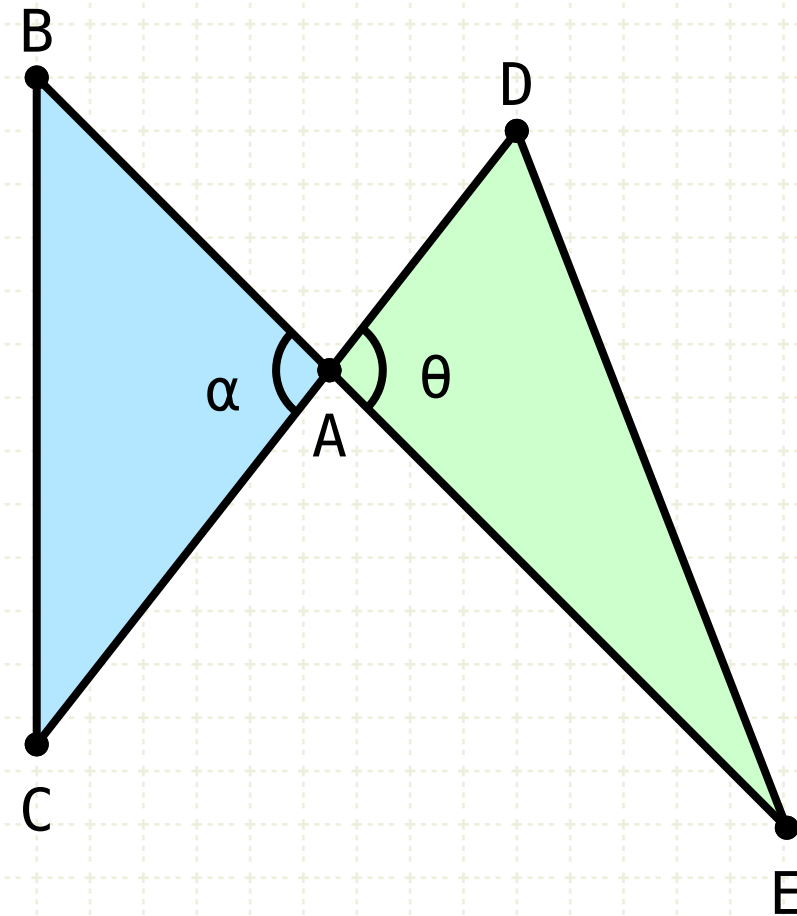
$$x_1:y_1 = x_2:y_2$$

Reciprocally Proportional:

$$x_1:y_1 = y_2:x_2, \quad x_1 \cdot x_2 = y_1 \cdot y_2$$

Proposition 15 of Book VI

In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and those triangles which have one angle equal to one angle, and in which the sides about the equal angles are reciprocally proportional, are equal.

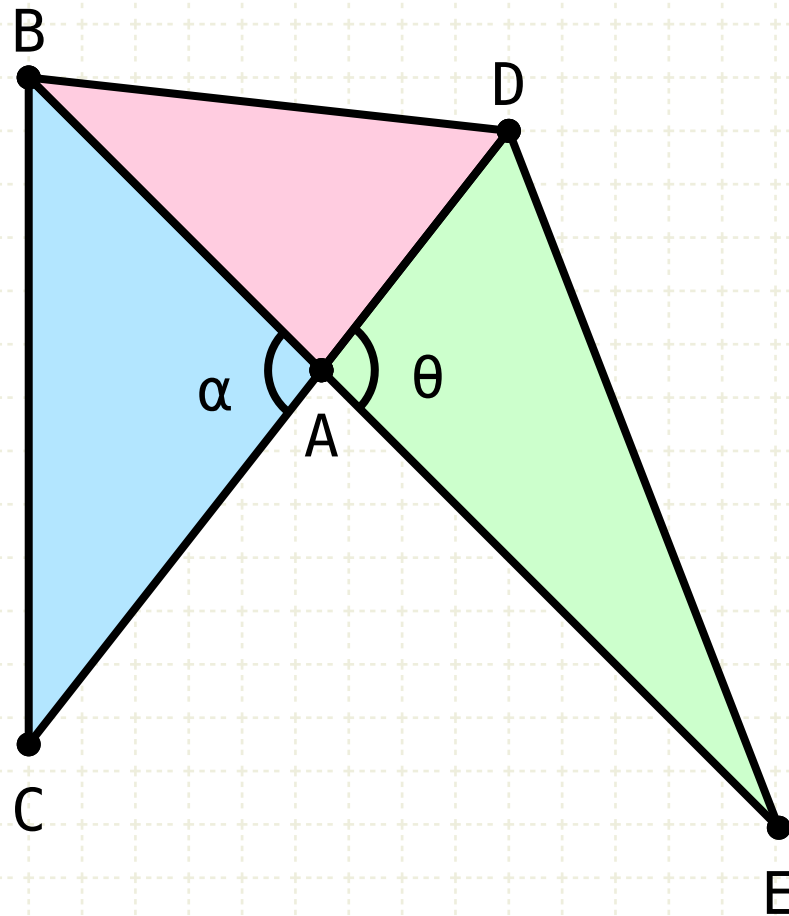


$$\alpha = \theta$$
$$\triangle ABC = \triangle DAE$$

Proof (Part 1)

Proposition 15 of Book VI

In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and those triangles which have one angle equal to one angle, and in which the sides about the equal angles are reciprocally proportional, are equal.



$$\alpha = \theta$$
$$\Delta ABC = \Delta DAE$$

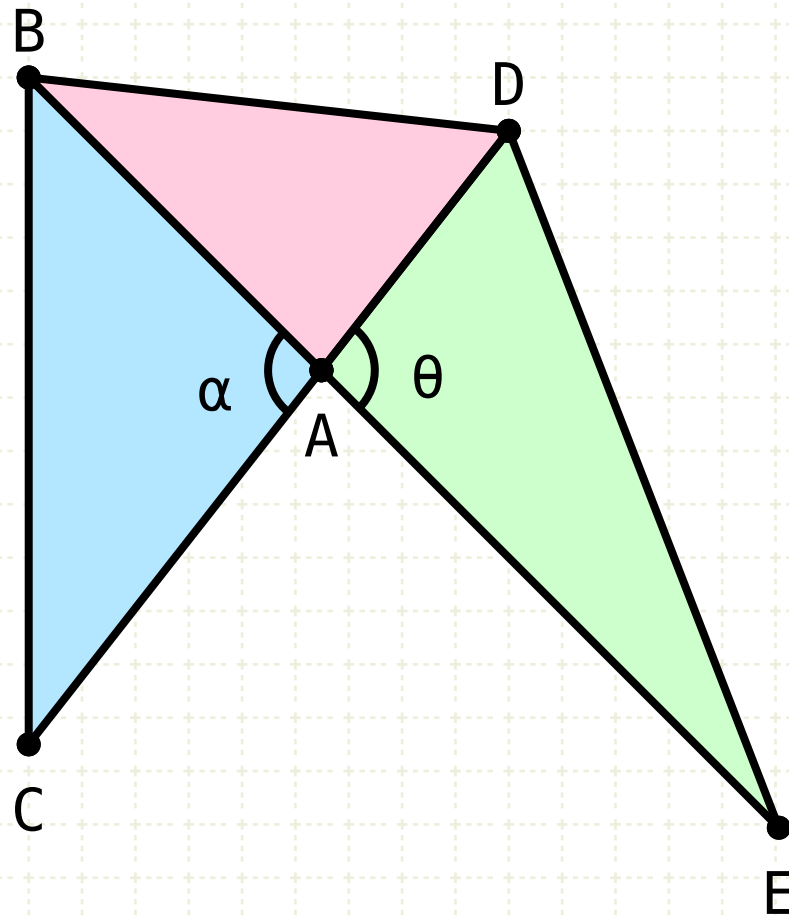
Proof (Part 1)

Let CA, AD be placed in a straight line, therefore EA, AB are also in a straight line (I-14)

Create the triangle BAD

Proposition 15 of Book VI

In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and those triangles which have one angle equal to one angle, and in which the sides about the equal angles are reciprocally proportional, are equal.



$$\alpha = \theta$$
$$\triangle ABC = \triangle DAE$$

$$\triangle ABC : \triangle BAD = \triangle DAE : \triangle BAD$$

Proof (Part 1)

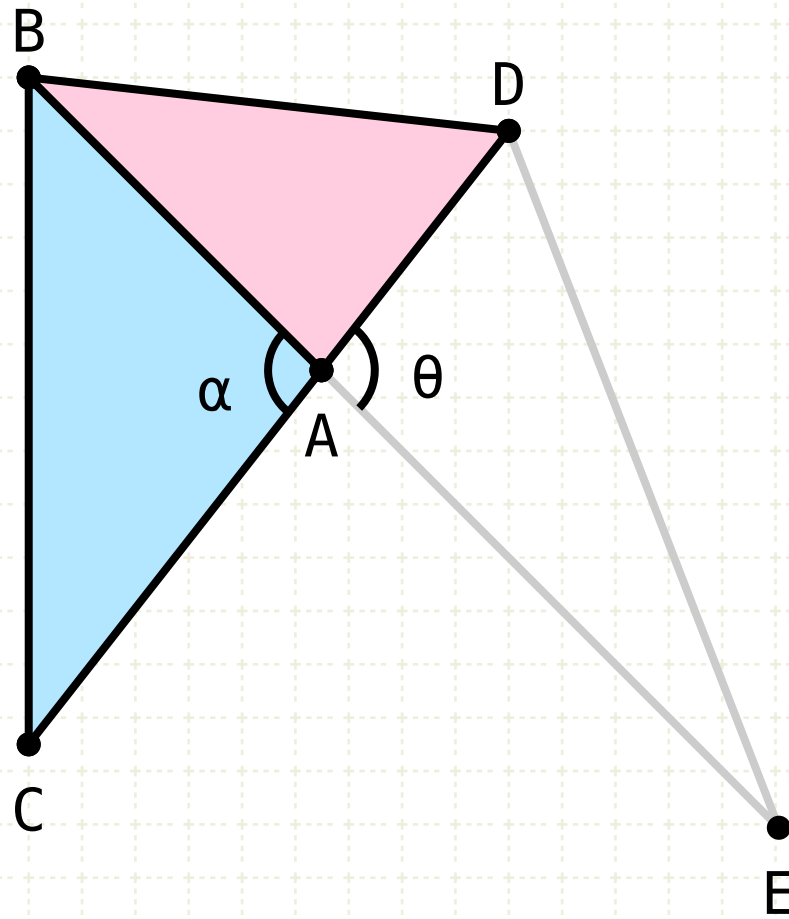
Let CA, AD be placed in a straight line, therefore EA, AB are also in a straight line (I·14)

Create the triangle BAD

Since triangles ABC and DAE are equal, then the ratios of these to the triangle BAD will also be equal (V·7)

Proposition 15 of Book VI

In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and those triangles which have one angle equal to one angle, and in which the sides about the equal angles are reciprocally proportional, are equal.



$$\alpha = \theta$$

$$\triangle ABC = \triangle DAE$$

$$\triangle ABC : \triangle BAD = \triangle DAE : \triangle BAD$$

$$\triangle ABC : \triangle BAD = AC : AD$$

Proof (Part 1)

Let CA, AD be placed in a straight line, therefore EA, AB are also in a straight line (I·14)

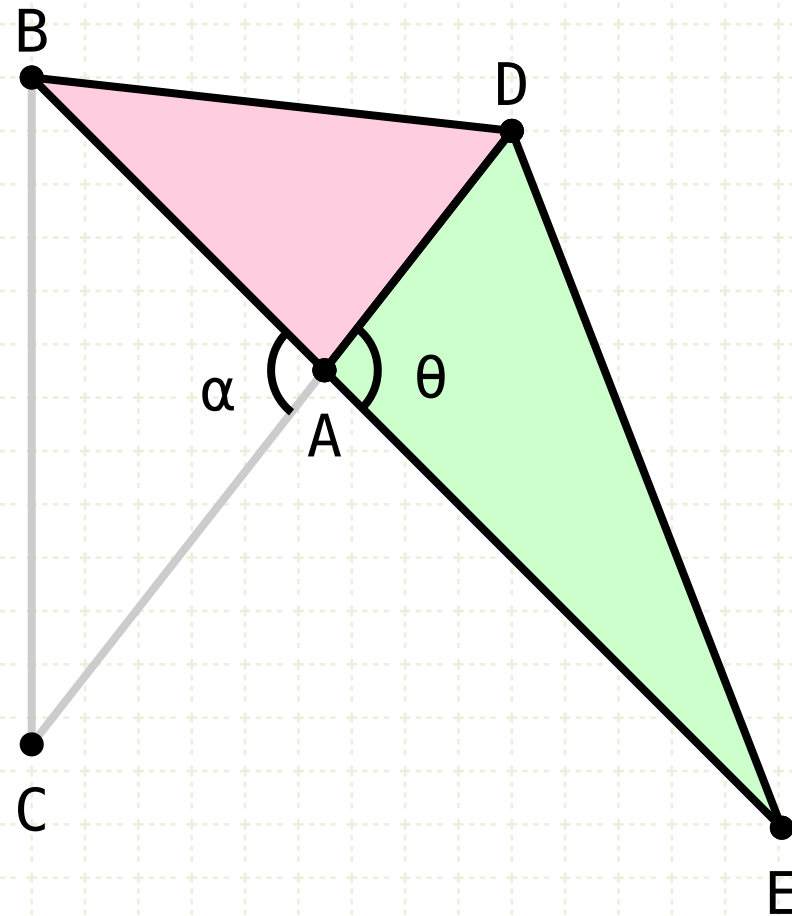
Create the triangle BAD

Since triangles ABC and DAE are equal, then the ratios of these to the triangle BAD will also be equal (V·7)

But, as the area ABC is to the area BAD, so is the base AC to the base AD (VI·1)

Proposition 15 of Book VI

In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and those triangles which have one angle equal to one angle, and in which the sides about the equal angles are reciprocally proportional, are equal.



$$\alpha = \theta$$

$$\triangle ABC = \triangle DAE$$

$$\triangle ABC : \triangle BAD = \triangle DAE : \triangle BAD$$

$$\triangle ABC : \triangle BAD = AC : AD$$

$$\triangle DAE : \triangle BAD = AE : AB$$

Proof (Part 1)

Let CA, AD be placed in a straight line, therefore EA, AB are also in a straight line (I·14)

Create the triangle BAD

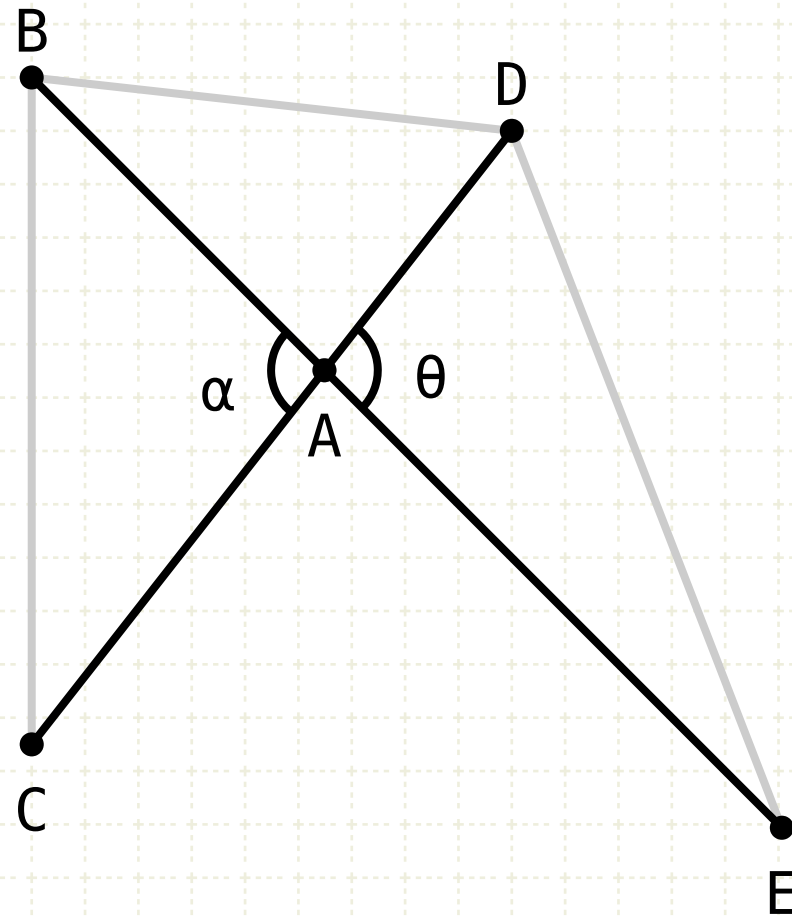
Since triangles ABC and DAE are equal, then the ratios of these to the triangle BAD will also be equal (V·7)

But, as the area ABC is to the area BAD, so is the base AC to the base AD (VI·1)

and as DAE is to BAD, so is AE to AB (VI·1)

Proposition 15 of Book VI

In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and those triangles which have one angle equal to one angle, and in which the sides about the equal angles are reciprocally proportional, are equal.



$$\alpha = \theta$$
$$\triangle ABC = \triangle DAE$$

$$\triangle ABC : \triangle BAD = \triangle DAE : \triangle BAD$$

$$\triangle ABC : \triangle BAD = AC : AD$$

$$\triangle DAE : \triangle BAD = AE : AB$$

$$AC : AD = AE : AB$$

Proof (Part 1)

Let CA, AD be placed in a straight line, therefore EA, AB are also in a straight line (I·14)

Create the triangle BAD

Since triangles ABC and DAE are equal, then the ratios of these to the triangle BAD will also be equal (V·7)

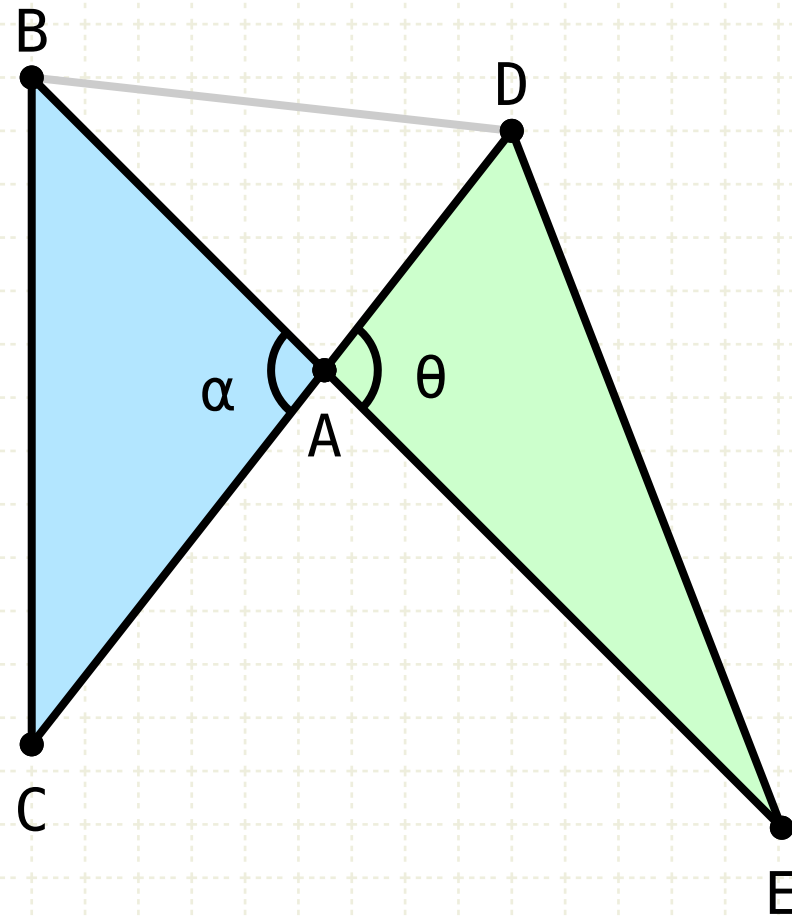
But, as the area ABC is to the area BAD, so is the base AC to the base AD (VI·1)

and as DAE is to BAD, so is AE to AB (VI·1)

Therefore, as AC is to AD, so is AE to AB (V·11)

Proposition 15 of Book VI

In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and those triangles which have one angle equal to one angle, and in which the sides about the equal angles are reciprocally proportional, are equal.



$$\alpha = \theta$$

$$\triangle ABC = \triangle DAE$$

$$\triangle ABC : \triangle BAD = \triangle DAE : \triangle BAD$$

$$\triangle ABC : \triangle BAD = AC : AD$$

$$\triangle DAE : \triangle BAD = AE : AB$$

$$AC : AD = AE : AB$$

Proof (Part 1)

Let CA, AD be placed in a straight line, therefore EA, AB are also in a straight line (I·14)

Create the triangle BAD

Since triangles ABC and DAE are equal, then the ratios of these to the triangle BAD will also be equal (V·7)

But, as the area ABC is to the area BAD, so is the base AC to the base AD (VI·1)

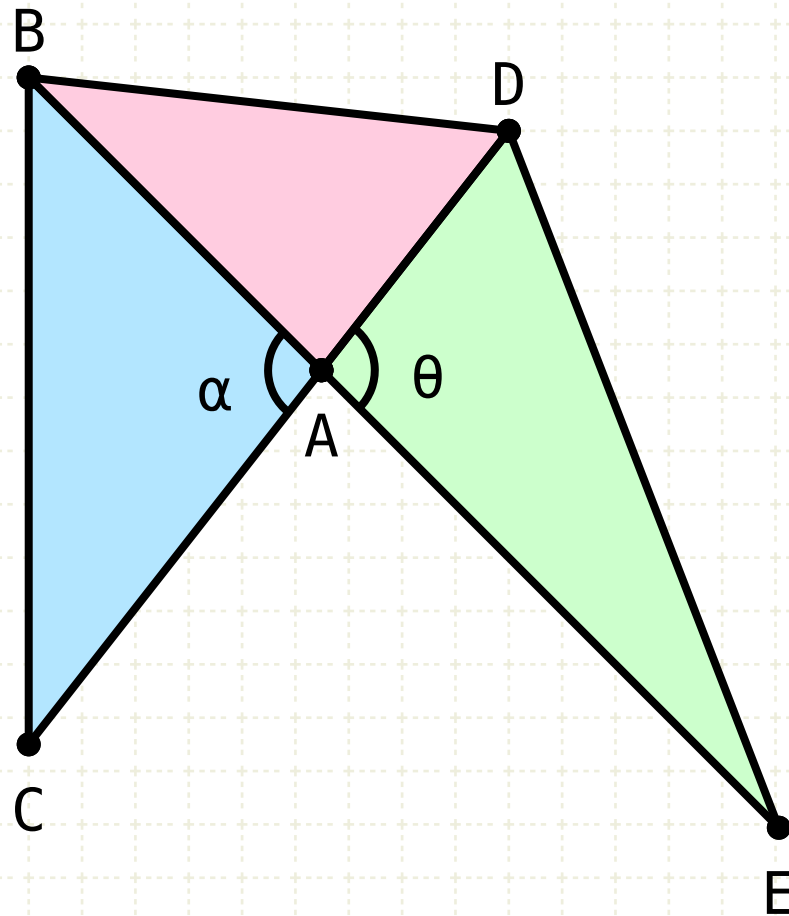
and as DAE is to BAD, so is AE to AB (VI·1)

Therefore, as AC is to AD, so is AE to AB (V·11)

Thus, in the two triangles, the sides about the equal angles are reciprocally proportional

Proposition 15 of Book VI

In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and those triangles which have one angle equal to one angle, and in which the sides about the equal angles are reciprocally proportional, are equal.



$$\alpha = \theta$$
$$AE:AB = AC:AD$$

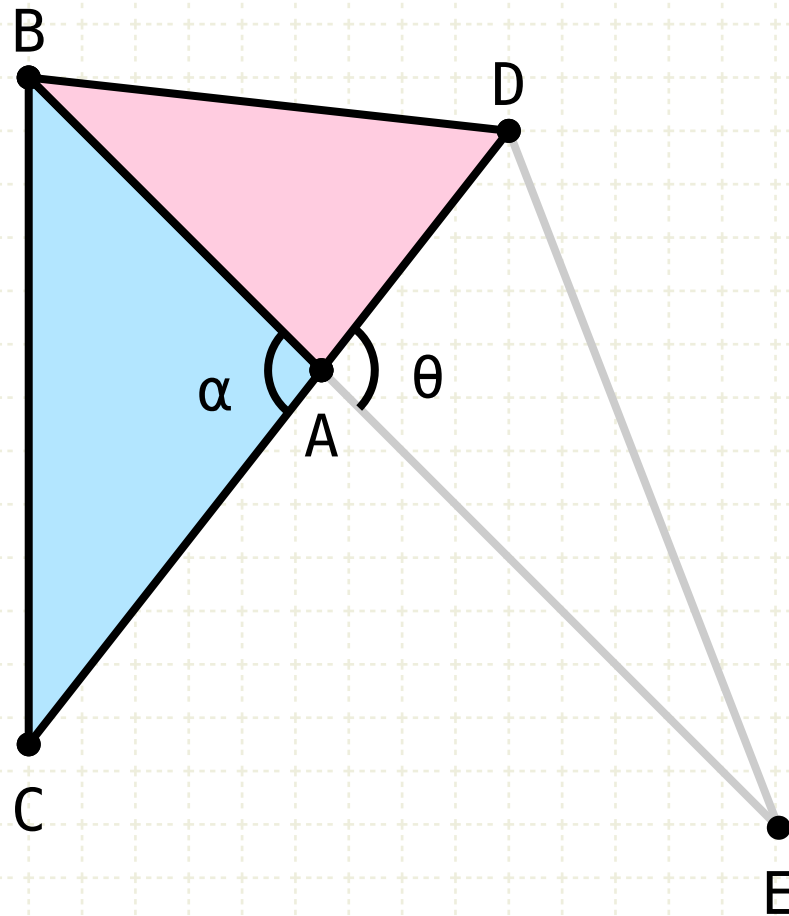
Proof (Part 2)

Let CA, AD be place in in a straight line, therefore EA, AB are also in a straight line (I·14)

Create the triangle BAD

Proposition 15 of Book VI

In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and those triangles which have one angle equal to one angle, and in which the sides about the equal angles are reciprocally proportional, are equal.



$$\alpha = \theta$$

$$AE : AB = AC : AD$$

$$AC : AD = \triangle ABC : \triangle BAD$$

Proof (Part 2)

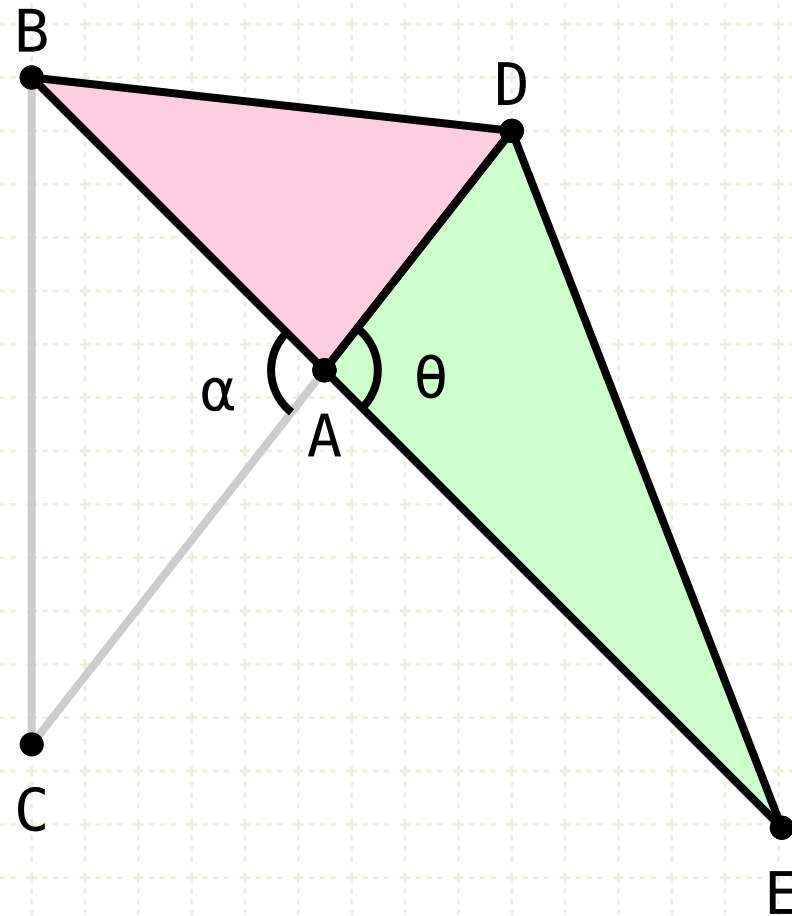
Let CA, AD be placed in a straight line, therefore EA, AB are also in a straight line (I·14)

Create the triangle BAD

The ratio of AC to AD is equal to the ratio of the triangles ABC to BAD (VI·1)

Proposition 15 of Book VI

In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and those triangles which have one angle equal to one angle, and in which the sides about the equal angles are reciprocally proportional, are equal.



$$\alpha = \theta$$

$$AE : AB = AC : AD$$

$$AC : AD = \triangle ABC : \triangle BAD$$

$$AE : AB = \triangle DAE : \triangle BAD$$

Proof (Part 2)

Let CA, AD be placed in a straight line, therefore EA, AB are also in a straight line (I·14)

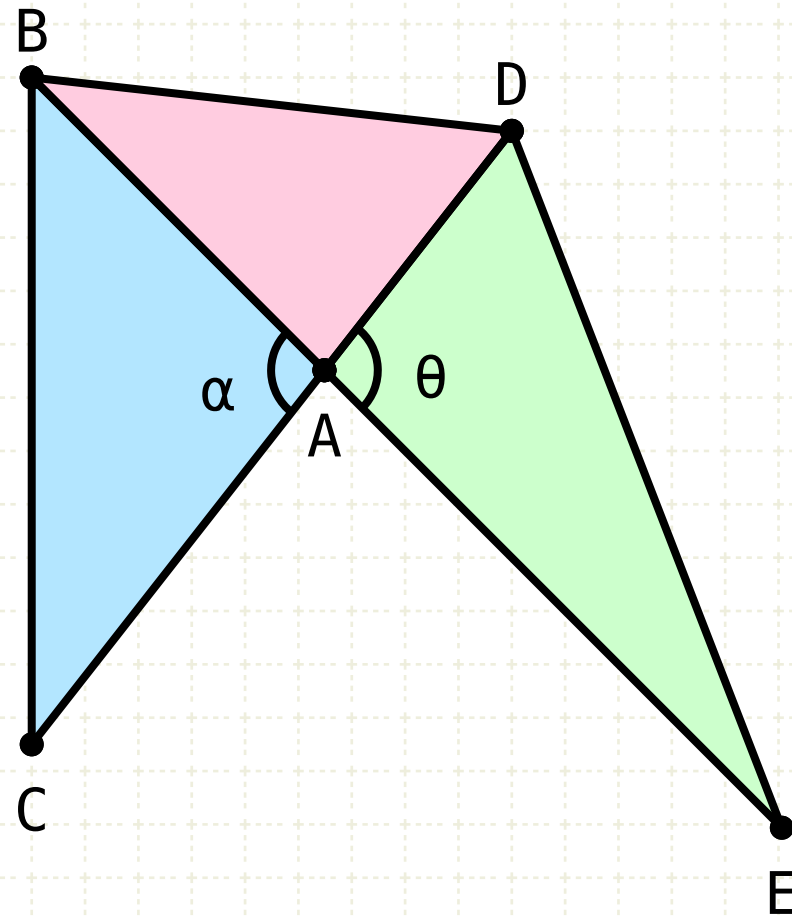
Create the triangle BAD

The ratio of AC to AD is equal to the ratio of the triangles ABC to BAD (VI·1)

The ratio of AE to AB is equal to the ratio of the triangles DAE to BAD (VI·1)

Proposition 15 of Book VI

In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and those triangles which have one angle equal to one angle, and in which the sides about the equal angles are reciprocally proportional, are equal.



$$\alpha = \theta$$

$$AE:AB = AC:AD$$

$$AC:AD = \triangle ABC:\triangle BAD$$

$$AE:AB = \triangle DAE:\triangle BAD$$

$$\triangle ABC:\triangle BAD = \triangle DAE:\triangle BAD$$

Proof (Part 2)

Let CA, AD be placed in a straight line, therefore EA, AB are also in a straight line (I·14)

Create the triangle BAD

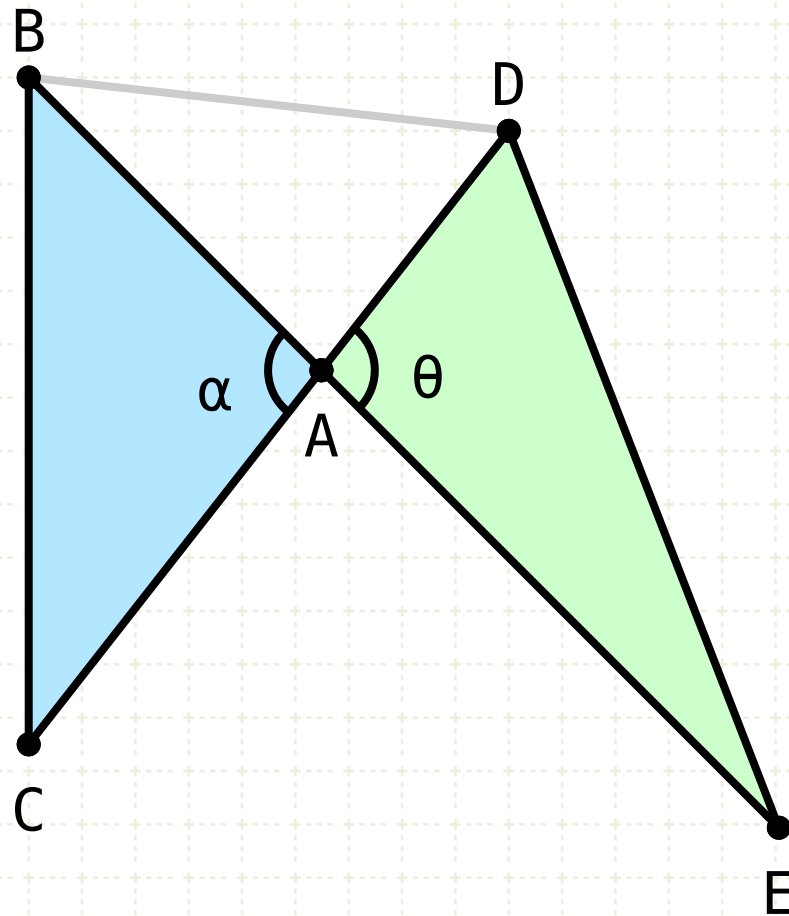
The ratio of AC to AD is equal to the ratio of the triangles ABC to BAD (VI·1)

The ratio of AE to AB is equal to the ratio of the triangles DAE to BAD (VI·1)

Therefore the ratio of the triangles ABC to BAD is equal to DAE to BAD (V·11)

Proposition 15 of Book VI

In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and those triangles which have one angle equal to one angle, and in which the sides about the equal angles are reciprocally proportional, are equal.



$$\alpha = \theta$$

$$AE:AB = AC:AD$$

$$AC:AD = \triangle ABC:\triangle BAD$$

$$AE:AB = \triangle DAE:\triangle BAD$$

$$\triangle ABC:\triangle BAD = \triangle DAE:\triangle BAD$$

$$\triangle ABC = \triangle DAE$$

Proof (Part 2)

Let CA, AD be placed in a straight line, therefore EA, AB are also in a straight line (I·14)

Create the triangle BAD

The ratio of AC to AD is equal to the ratio of the triangles ABC to BAD (VI·1)

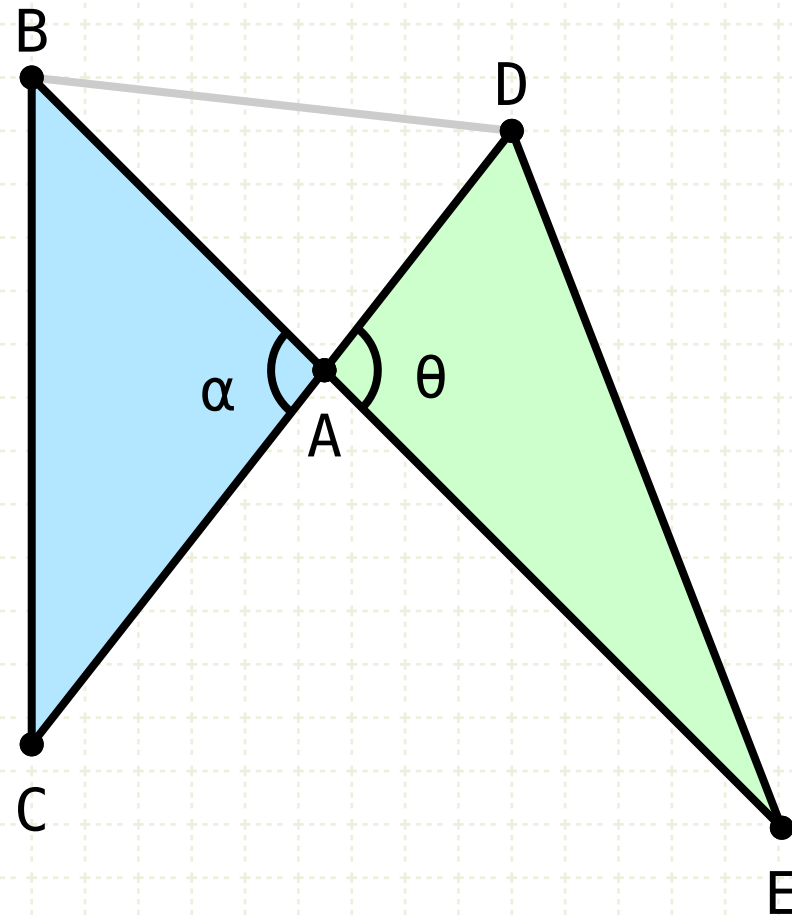
The ratio of AE to AB is equal to the ratio of the triangles DAE to BAD (VI·1)

Therefore the ratio of the triangles ABC to BAD is equal to DAE to BAD (V·11)

And thus the triangles ABC and DAE are equal (V·9)

Proposition 15 of Book VI

In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and those triangles which have one angle equal to one angle, and in which the sides about the equal angles are reciprocally proportional, are equal.



$$\alpha = \theta$$

$$AE:AB = AC:AD$$

$$AC:AD = \triangle ABC:\triangle BAD$$

$$AE:AB = \triangle DAE:\triangle BAD$$

$$\triangle ABC:\triangle BAD = \triangle DAE:\triangle BAD$$

$$\triangle ABC = \triangle DAE$$

Proof (Part 2)

Let CA, AD be placed in a straight line, therefore EA, AB are also in a straight line (I·14)

Create the triangle BAD

The ratio of AC to AD is equal to the ratio of the triangles ABC to BAD (VI·1)

The ratio of AE to AB is equal to the ratio of the triangles DAE to BAD (VI·1)

Therefore the ratio of the triangles ABC to BAD is equal to DAE to BAD (V·11)

And thus the triangles ABC and DAE are equal (V·9)

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