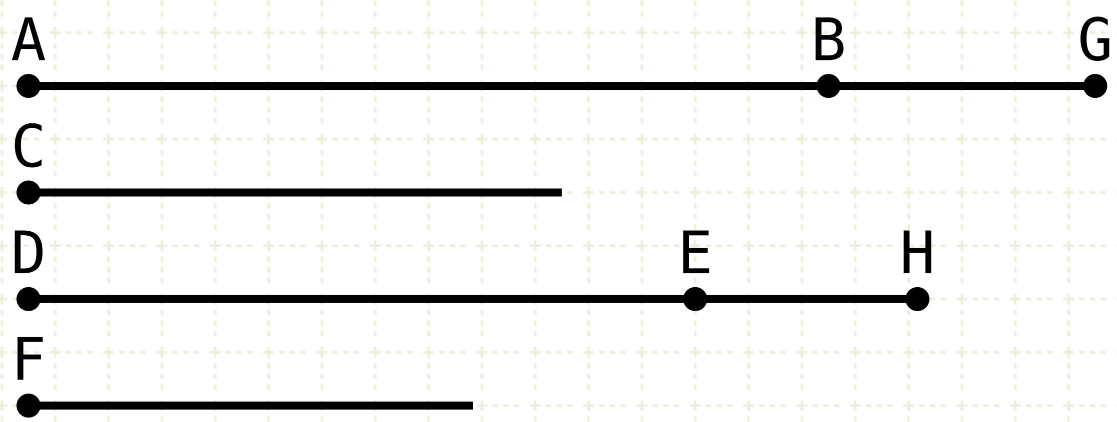
Euclid's Elements

Book V



AB:C = DE:F
BG:C = EH:F

AG:C = DH:F

Proportions are what makes the old Greek temples classic in their beauty. They are like huge blocks, from which the air has been literally hewn out between the columns.

Arne Jacobsen

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Proposition 3 of Book V

If a first magnitude be the same multiple of a second that a third is of a fourth, and if equimultiples be taken of the first and third, then also, ex aequali, the magnitudes taken will be equimultiples respectively, the one of the second, and the other of the fourth.

Proposition 3 of Book V

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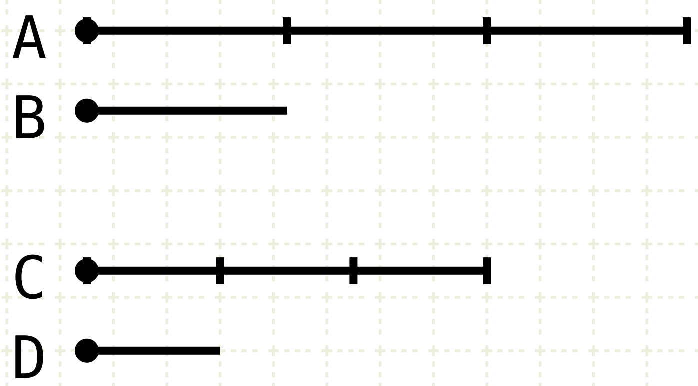


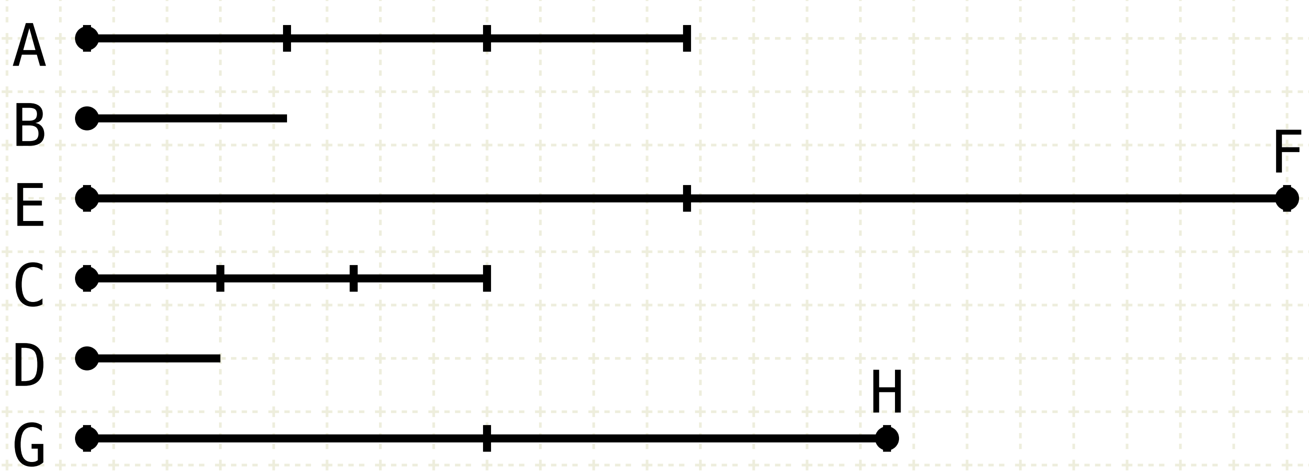
In other words

If we have two lines (A and C) that are equal multiples of two other lines (B and D respectively) and …

If $A = n \cdot B$, $C = n \cdot D$

Proposition 3 of Book V

If a first magnitude be the same multiple of a second that a third is of a fourth, and if equimultiples be taken of the first and third, then also, ex aequali, the magnitudes taken will be equimultiples respectively, the one of the second, and the other of the fourth.



If $A=n \cdot B$, $C=n \cdot D$
and $EF=m \cdot A$, $GH=m \cdot C$

In other words

If we have two lines (A and C) that are equal multiples of two other lines (B and D respectively) and …

we draw two new lines (E and G), equimultiple to A and C respectively …

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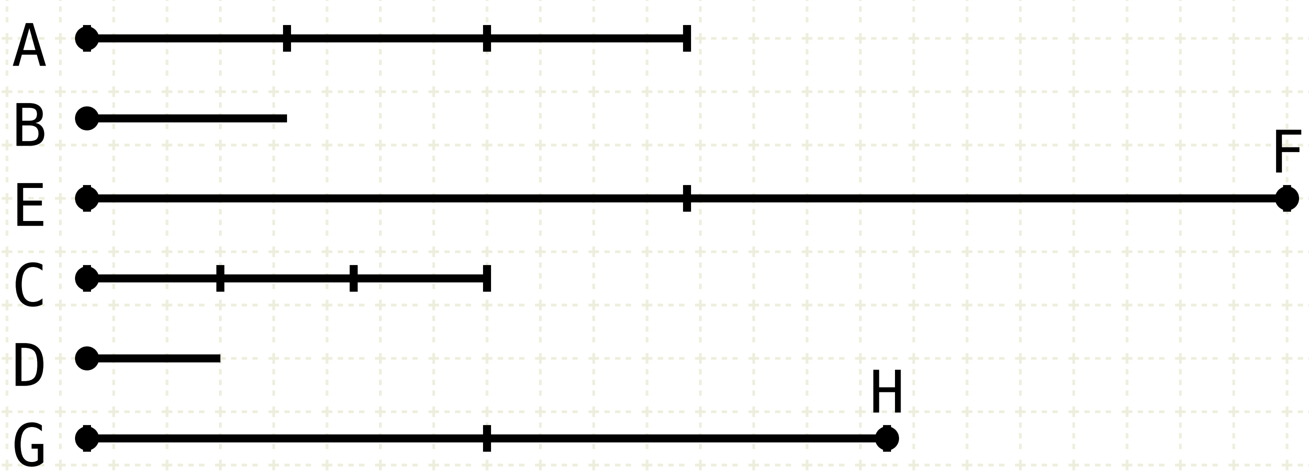


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If   A=n·B, C=n·D
and  EF=m·A, GH=m·C
then EF=k·B and GH=k·D
where n,m,k are integers
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In other words

If we have two lines (A and C) that are equal multiples of two other lines (B and D respectively) and …

we draw two new lines (E and G), equimultiple to A and C respectively …

then the lines EF and GH will be equimultiples of B and D respectively

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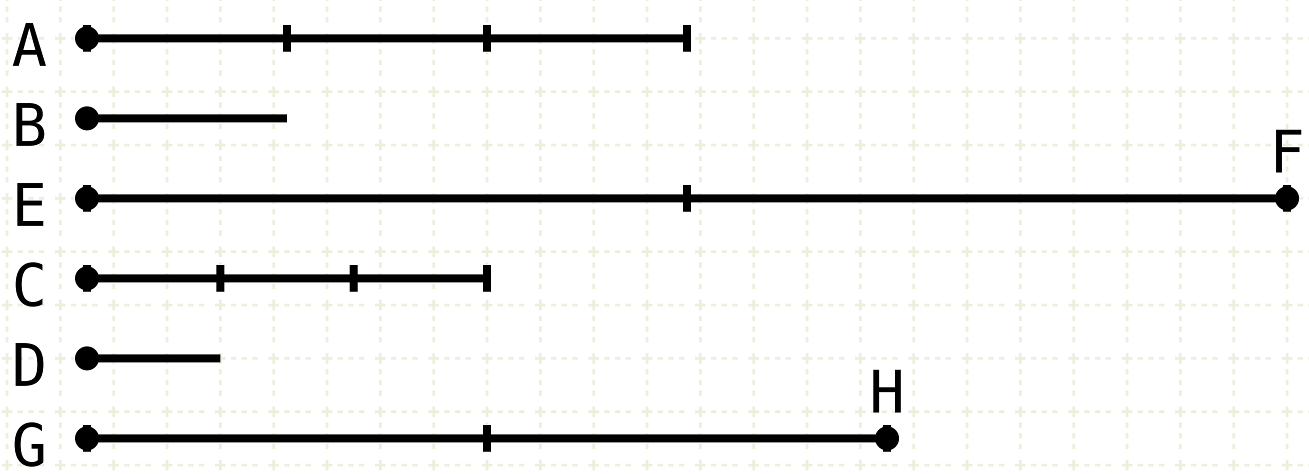

$A=n\cdot B,\quad C=n\cdot D$

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Proof

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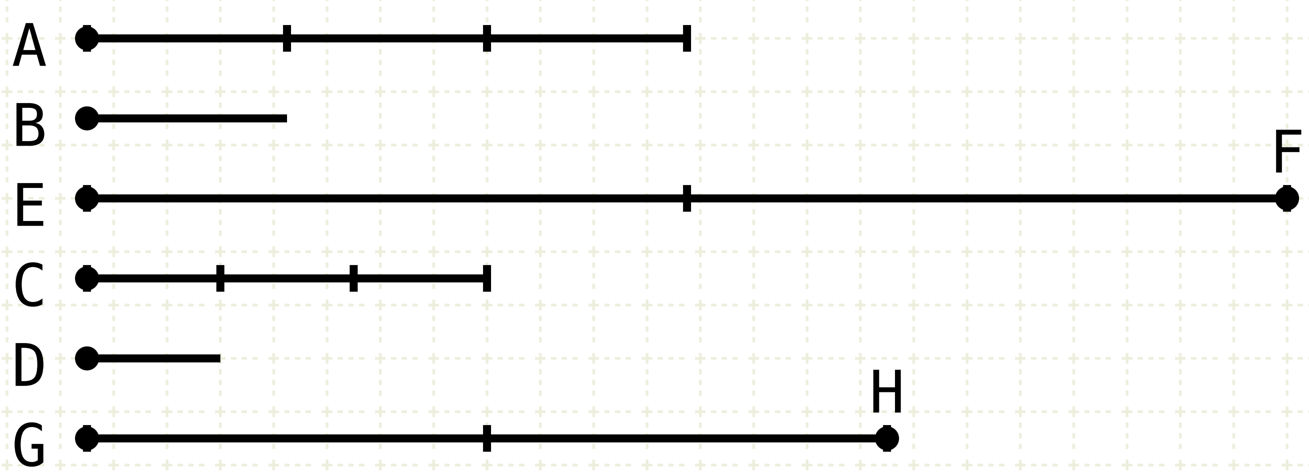

$A = n \cdot B, \quad C = n \cdot D$

$EF = m \cdot A, \quad GH = m \cdot C$

$EF = m \cdot A, \quad GH = m \cdot C, \; m = 2$

In other words

If we have two lines (A and C) that are equal multiples of two other lines (B and D respectively) and …

we draw two new lines (E and G), equimultiple to A and C respectively …

then the lines EF and GH will be equimultiples of B and D respectively

Proof

Since EF and GH are the same multiples of A and C respectively, then there are the an equal number of magnitudes in EF and GH

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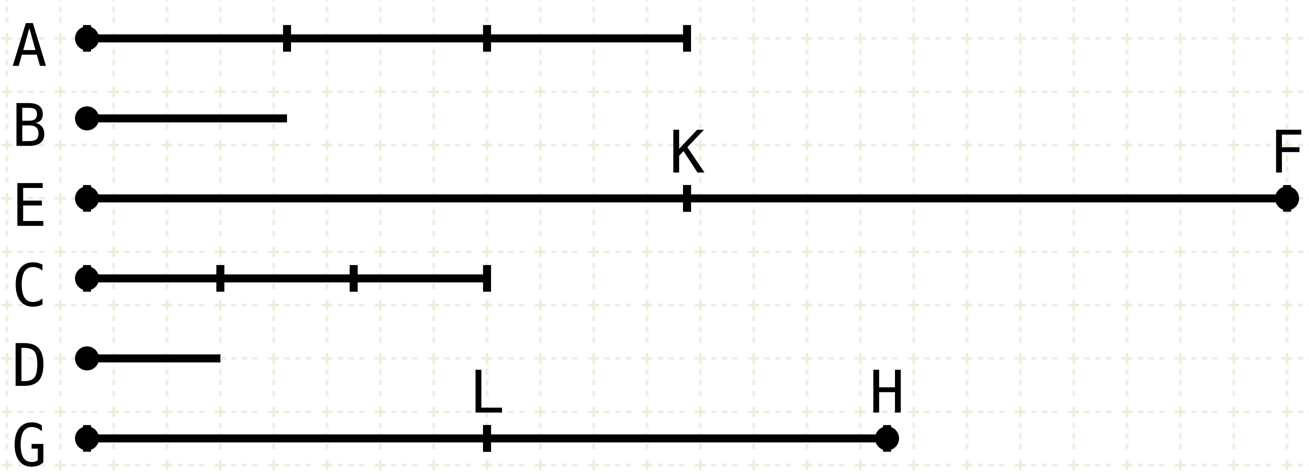

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$EK = KF = A$

$GL = LH = C$

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Since EF and GH are the same multiples of A and C respectively, then there are the an equal number of magnitudes in EF and GH

Divide EF into equal segments of length A (EK,KF) and divide GH into equal segments of length C (GL,LH)

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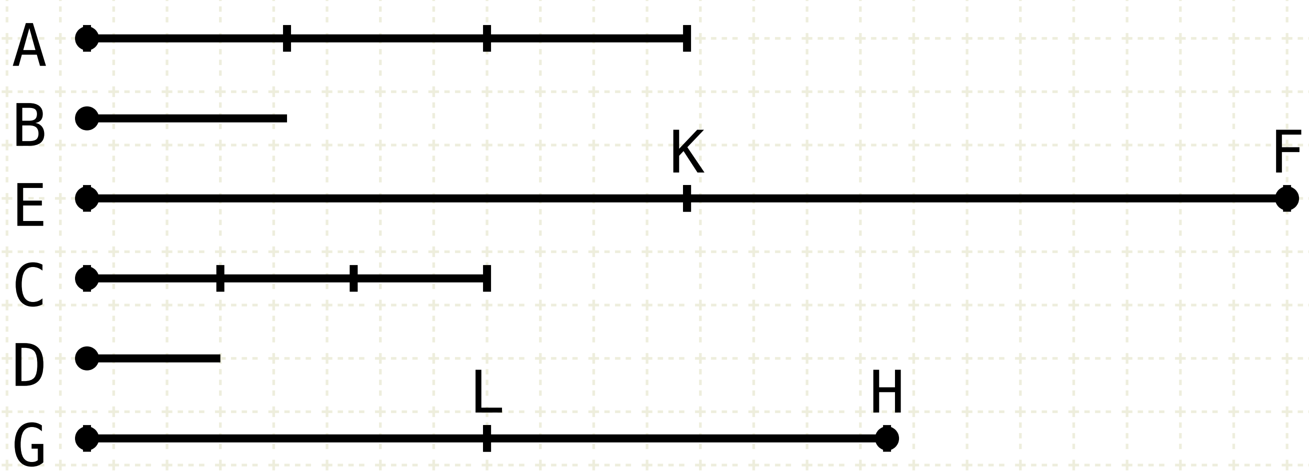

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$EK = KF = A$

$GL = LH = C$

$EK = A = n\cdot B$

$GL = C = n\cdot D$

EK = first line

B = second line

GL = third line

D = fourth line

In other words

If we have two lines (A and C) that are equal multiples of two other lines (B and D respectively) and …

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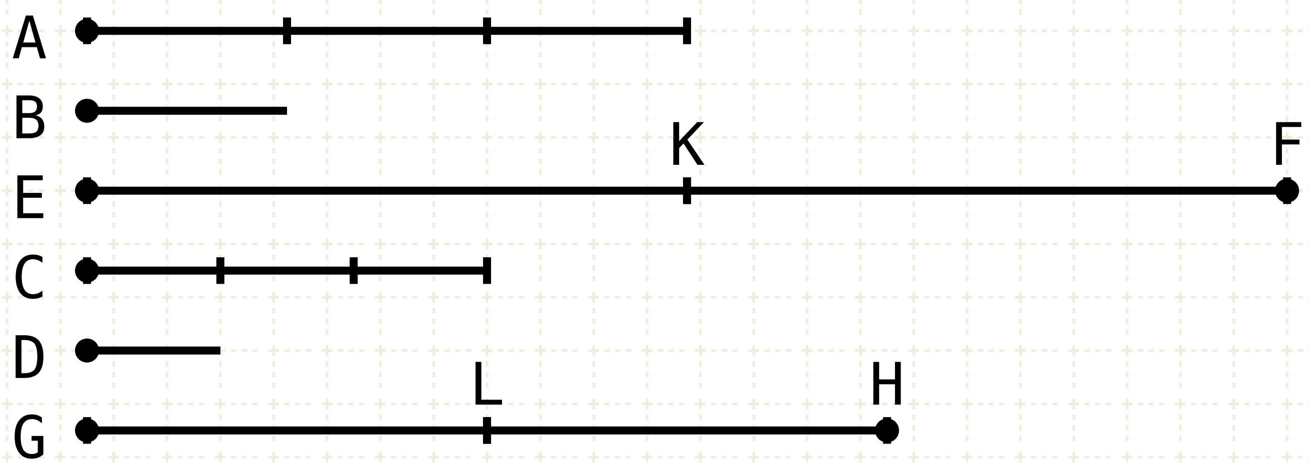

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$EK = A = n\cdot B$

$GL = C = n\cdot D$

$KF = A = n\cdot B$

$LH = C = n\cdot D$

EK = first line
B = second line
GL = third line
D = fourth line
KF = fifth line
LH = sixth line

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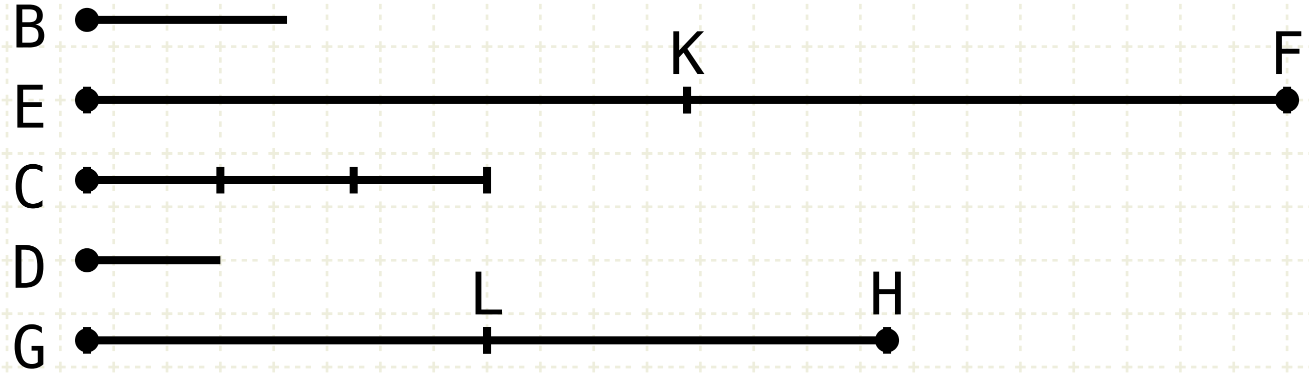

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$EK = A = n \cdot B$
$GL = C = n \cdot D$
$KF = A = n \cdot B$
$LH = C = n \cdot D$
$EF = EK + KF = n \cdot B + n \cdot B = m \cdot n \cdot B = k \cdot B$
$GH = GL + LH = n \cdot D + n \cdot D = m \cdot n \cdot D = k \cdot D$

EK = first line
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Thus EF is the same multiple of B as GH is of D (V·2)

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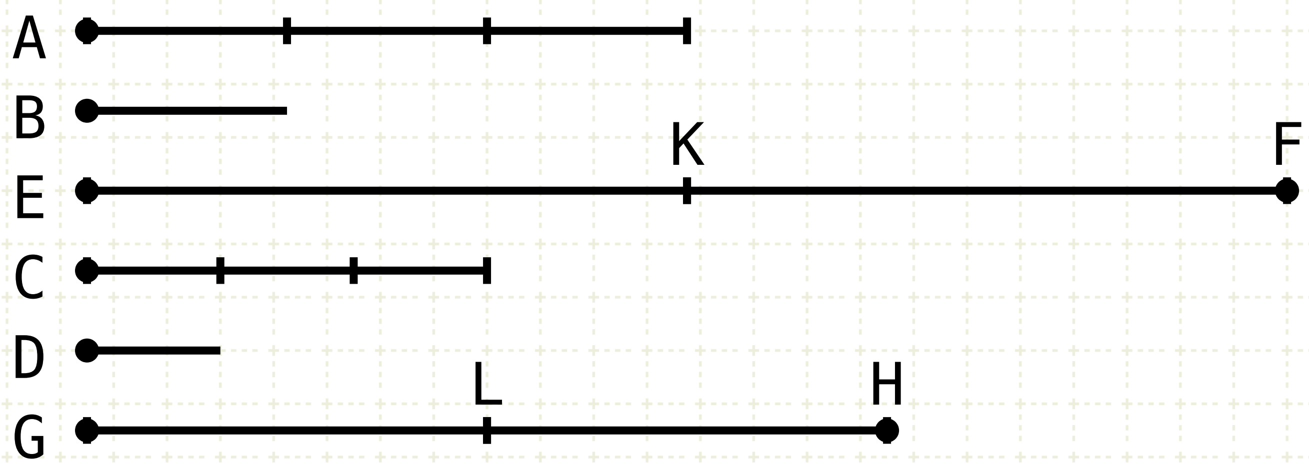

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