

Euclid's Elements

Book VI

One can state, without exaggeration, that the observation of and the search for similarities and differences are the basis of all human knowledge.

Alfred Nobel



Table of Contents, Chapter 6

1	If the height of two triangles are equal, then the ratio of the areas is equal to the ratio of the bases	7	If two triangles have one angle equal to one angle, and the sides about other angles are proportional, and the remaining angles either both less or both not less than a right angle, then triangles will be equiangular	14	In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and vice versa
2	If a line cuts a triangle, parallel to its base, it will cut the sides of the triangle proportionally			15	In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and vice versa
3	If an angle of a triangle is bisected and the straight line cutting the angle also cuts the base, the segments of the base will have the same ratio as the remaining sides of the triangle	8	If in a right-angled triangle a perpendicular be drawn from the right angle to the base, the triangles adjoining the perpendicular are similar both to the whole and to one another	16	If four straight lines are proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means, and vice versa
4	If two triangles have equal angles, then the sides opposite the equal angles are proportional, as well, the sides of the triangles on either side of the equal angles are also proportional	9	From a given straight line to cut off a given fraction	17	If three straight lines are proportional, the rectangle contained by the extremes is equal to the square on the mean; and vice versa
5	If two triangles have proportional sides, the triangles will be equiangular	10	To cut a given uncut straight line similarly to a given cut straight line	18	On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure
6	If two triangles have one angle equal to one angle and the sides about the equal angles are proportional, then the triangles will be equiangular	11	To two given straight lines to find a third proportional	19	Similar triangles are to one another in the duplicate ratio of the corresponding sides
		12	To three given straight lines to find a fourth proportional		
		13	To two given straight lines to find a mean proportional		



Table of Contents, Chapter 3

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|----|--|----|---|----|---|
| 20 | Similar polygons are divided into the same number of similar triangles, which have the same ratio as the wholes, and the polygons have duplicate ratios to their corresponding sides | 26 | If from a parallelogram a similar parallelogram with a common angle is subtracted, it is about the same diameter as the original | 31 | In right-angled triangles the figure on the side subtending the right angle is equal to the similar and similarly described figures on the sides containing the right angle |
| 21 | Figures which are similar to the same rectilineal figure are also similar to one another | 27 | Of all the parallelograms applied to the same straight line and deficient by parallelogrammic figures similar to a parallelogram drawn on half the said line, the largest will be one that is drawn on half of the straight line and is similar to the defect | | |
| 22 | If four straight lines are proportional, similar rectilineal figures will also be proportional; and vice versa | 28 | To a given straight line, apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one | | |
| 23 | Equiangular parallelograms have to one another the ratio compounded of the ratios of their sides | 29 | To a given straight line, apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one | | |
| 24 | In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another | 30 | To cut a finite straight line in extreme ratio | | |
| 25 | To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure | | | | |



Proposition 17 of Book VI

If three straight lines be proportional, the rectangle contained by the extremes is equal to the square on the mean; and, if the rectangle contained by the extremes be equal to the square on the mean, the three straight lines will be proportional



Proposition 17 of Book VI

If three straight lines be proportional, the rectangle contained by the extremes is equal to the square on the mean; and, if the rectangle contained by the extremes be equal to the square on the mean, the three straight lines will be proportional

A ●—————

B ●—————

C ●—————

$$A:B = B:C$$

$$\rightarrow A \times C = B^2$$

$$A \times C = B^2$$

$$\rightarrow A:B = B:C$$

In other words

Given three lines A,B,C that are proportional, A is to B as B is to C

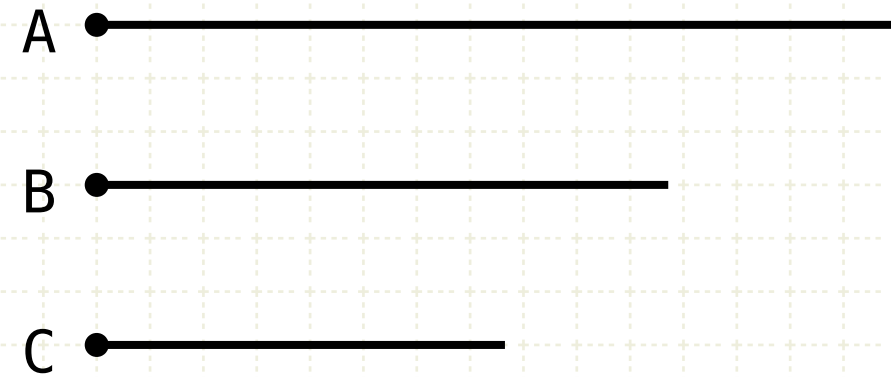
Then the product of A,C is equal to the square of B

And the inverse



Proposition 17 of Book VI

If three straight lines be proportional, the rectangle contained by the extremes is equal to the square on the mean; and, if the rectangle contained by the extremes be equal to the square on the mean, the three straight lines will be proportional



Proof (Part 1)

Assume A is to B as B is to C

$$A:B = B:C$$



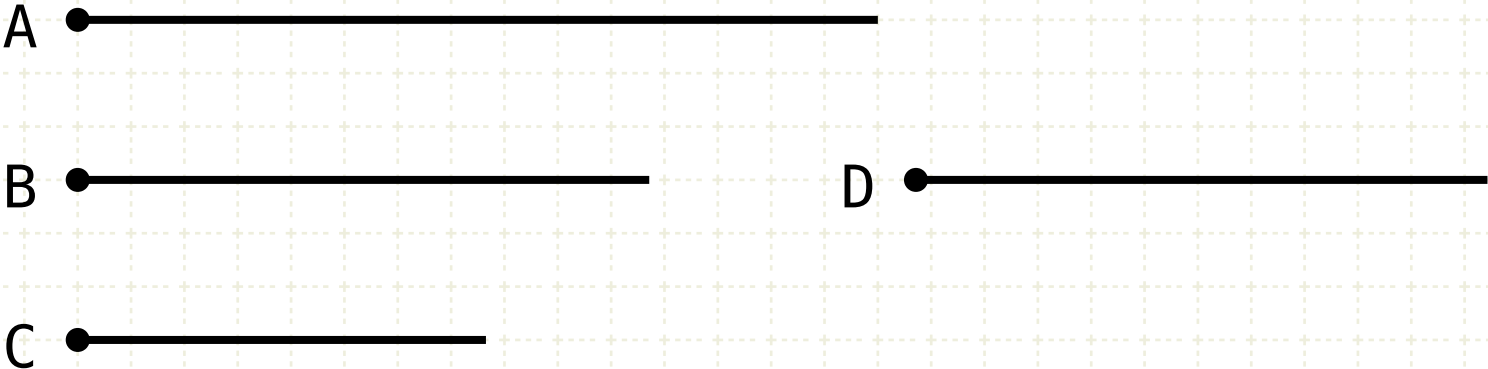
Proposition 17 of Book VI

If three straight lines be proportional, the rectangle contained by the extremes is equal to the square on the mean; and, if the rectangle contained by the extremes be equal to the square on the mean, the three straight lines will be proportional

Proof (Part 1)

Assume A is to B as B is to C

Let line D be equal to B



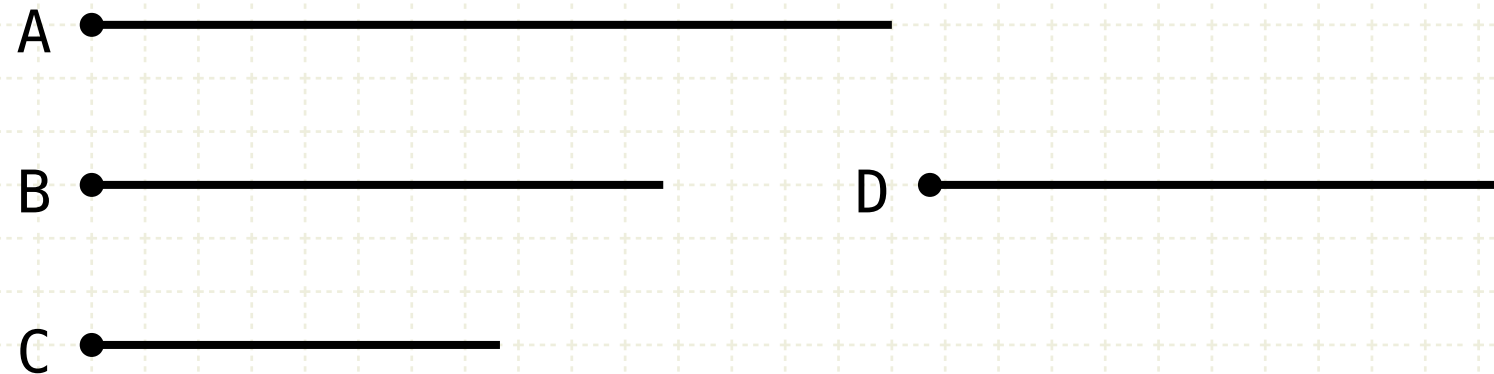
$$A:B = B:C$$

$$D = B$$



Proposition 17 of Book VI

If three straight lines be proportional, the rectangle contained by the extremes is equal to the square on the mean; and, if the rectangle contained by the extremes be equal to the square on the mean, the three straight lines will be proportional



$$A:B = B:C$$

$$D = B$$

$$A:B = D:C$$

Proof (Part 1)

Assume A is to B as B is to C

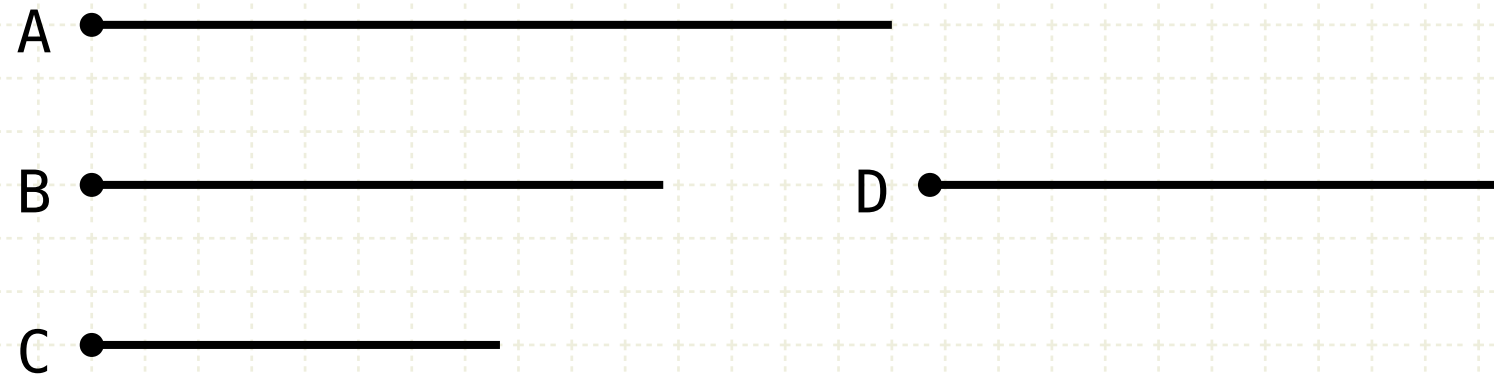
Let line D be equal to B

And since A is to B as B is to C, then A is to B as D is to C



Proposition 17 of Book VI

If three straight lines be proportional, the rectangle contained by the extremes is equal to the square on the mean; and, if the rectangle contained by the extremes be equal to the square on the mean, the three straight lines will be proportional



$$A:B = B:C$$

$$D = B$$

$$A:B = D:C$$

$$A \times C = B \times D$$

Proof (Part 1)

Assume A is to B as B is to C

Let line D be equal to B

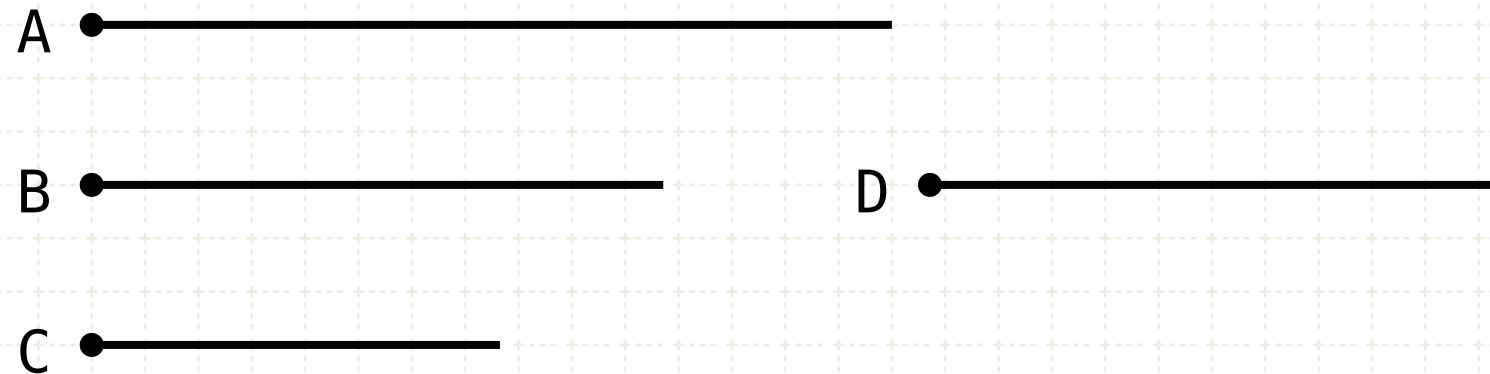
And since A is to B as B is to C, then A is to B as D is to C

If the four lines are proportional, then the rectangles contained by A,C will be equal to the rectangle contained by B,D (VI·16)



Proposition 17 of Book VI

If three straight lines be proportional, the rectangle contained by the extremes is equal to the square on the mean; and, if the rectangle contained by the extremes be equal to the square on the mean, the three straight lines will be proportional



$$A:B = B:C$$

$$D = B$$

$$A:B = D:C$$

$$A \times C = B \times D$$

$$A \times C = B \times B = B^2$$

Proof (Part 1)

Assume A is to B as B is to C

Let line D be equal to B

And since A is to B as B is to C, then A is to B as D is to C

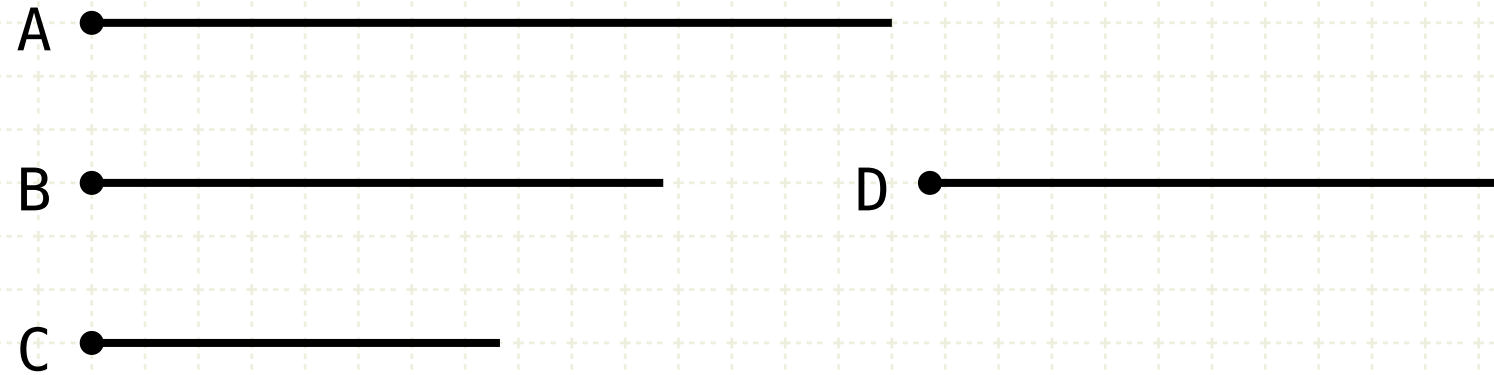
If the four lines are proportional, then the rectangles contained by A,C will be equal to the rectangle contained by B,D (VI-16)

Since B equals D, the rectangle formed by B,D is equivalent to the square of B



Proposition 17 of Book VI

If three straight lines be proportional, the rectangle contained by the extremes is equal to the square on the mean; and, if the rectangle contained by the extremes be equal to the square on the mean, the three straight lines will be proportional



$$A \times C = B^2$$

$$D = B$$

Proof (Part 2)

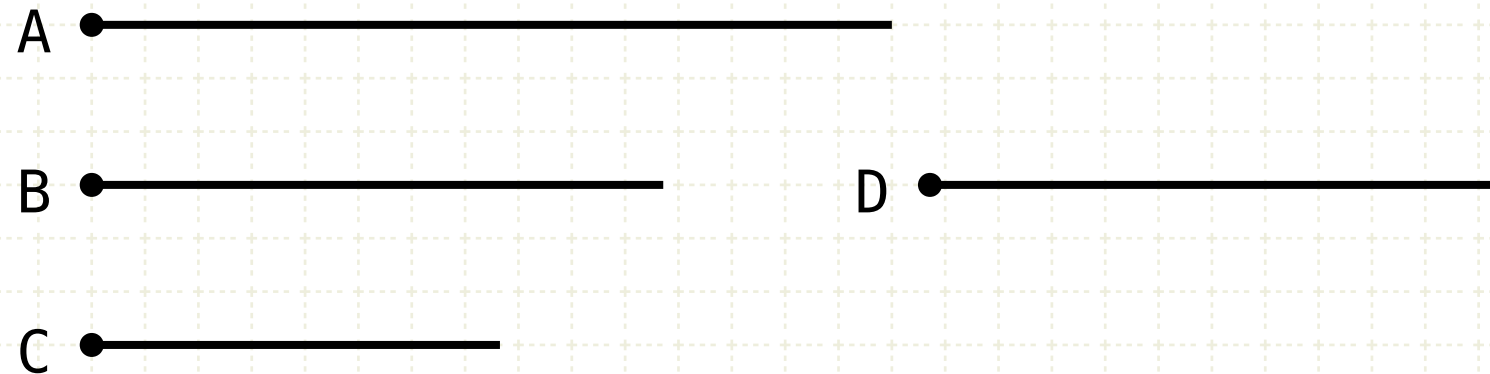
Assume the rectangle formed by A,C is equal to the square of B

Let line D be equal to B



Proposition 17 of Book VI

If three straight lines be proportional, the rectangle contained by the extremes is equal to the square on the mean; and, if the rectangle contained by the extremes be equal to the square on the mean, the three straight lines will be proportional



$$A \times C = B^2$$

$$D = B$$

$$B^2 = B \times B = B \times D$$

Proof (Part 2)

Assume the rectangle formed by A,C is equal to the square of B

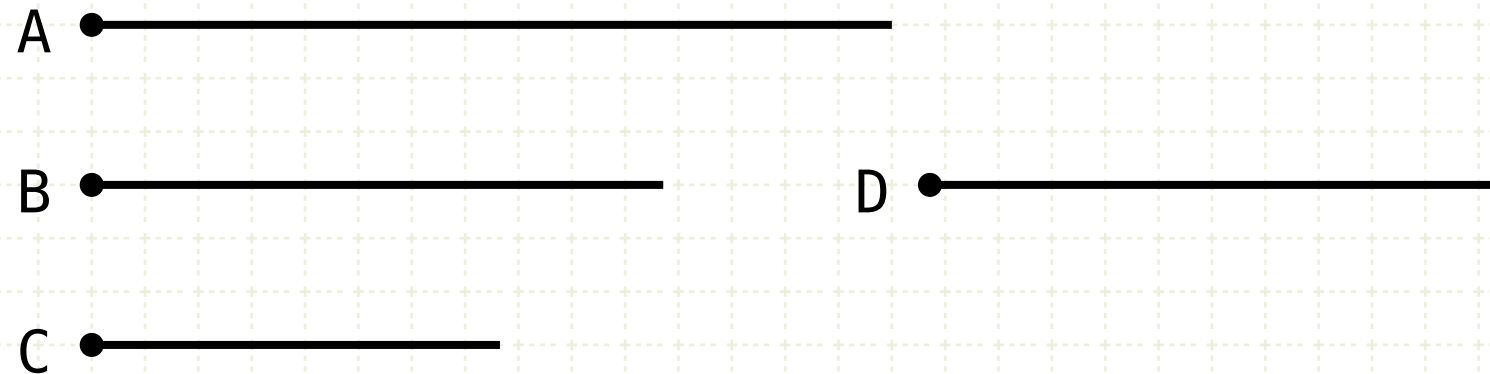
Let line D be equal to B

Since B equals D, square of B is equivalent to the the rectangle formed by B,D



Proposition 17 of Book VI

If three straight lines be proportional, the rectangle contained by the extremes is equal to the square on the mean; and, if the rectangle contained by the extremes be equal to the square on the mean, the three straight lines will be proportional



$$A \times C = B^2$$

$$D = B$$

$$B^2 = B \times B = B \times D$$

$$A \times C = B \times D$$

Proof (Part 2)

Assume the rectangle formed by A,C is equal to the square of B

Let line D be equal to B

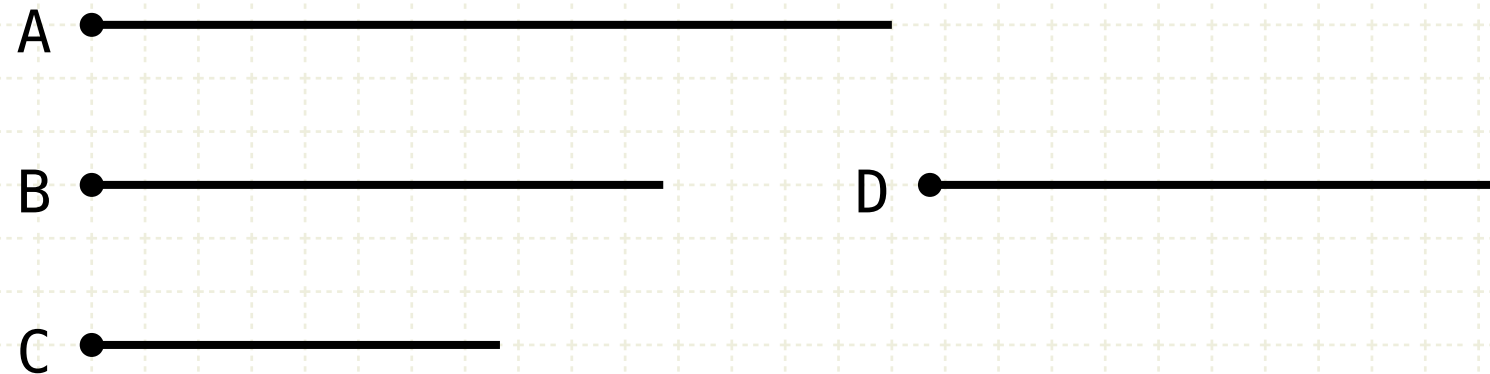
Since B equals D, square of B is equivalent to the the rectangle formed by B,D

Therefore the rectangle contained by A,C is equal to the rectangle contained by D,C



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If three straight lines be proportional, the rectangle contained by the extremes is equal to the square on the mean; and, if the rectangle contained by the extremes be equal to the square on the mean, the three straight lines will be proportional



$$A \times C = B^2$$

$$D = B$$

$$B^2 = B \times B = B \times D$$

$$A \times C = B \times D$$

$$A : B = D : C$$

Proof (Part 2)

Assume the rectangle formed by A,C is equal to the square of B

Let line D be equal to B

Since B equals D, square of B is equivalent to the the rectangle formed by B,D

Therefore the rectangle contained by A,C is equal to the rectangle contained by D,C

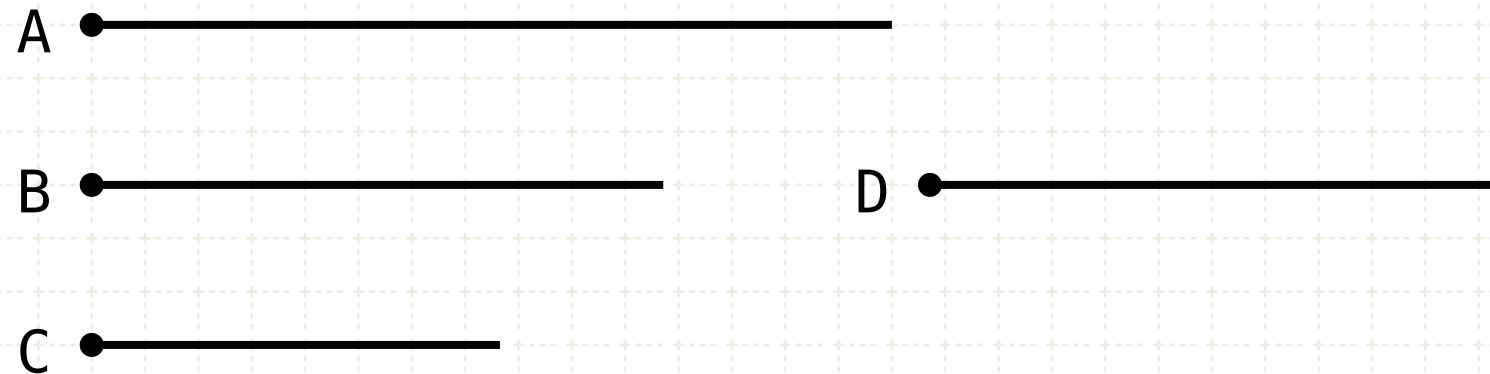
If the rectangle contained by the extremes (A,C) is equal to the rectangle contained by the means (B,D)...

... then the four lines are proportional (A is to B as D is to C)
(VI·16)



Proposition 17 of Book VI

If three straight lines be proportional, the rectangle contained by the extremes is equal to the square on the mean; and, if the rectangle contained by the extremes be equal to the square on the mean, the three straight lines will be proportional



$$A \times C = B^2$$

$$D = B$$

$$B^2 = B \times B = B \times D$$

$$A \times C = B \times D$$

$$A : B = D : C$$

$$A : B = B : C$$

Proof (Part 2)

Assume the rectangle formed by A,C is equal to the square of B

Let line D be equal to B

Since B equals D, square of B is equivalent to the the rectangle formed by B,D

Therefore the rectangle contained by A,C is equal to the rectangle contained by D,C

If the rectangle contained by the extremes (A,C) is equal to the rectangle contained by the means (B,D)...

... then the four lines are proportional (A is to B as D is to C)
(VI·16)

But B equals D, therefore A is to B as B is to C



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