Euclid's Elements

Book VII

Definitions:

1 A unit is that by virtue of which each of the things that exist is called one

2 A number is a multitude composed of units. (not one)

3 A number is part of a number, the less of the greater, when it measures the greater

11 A prime number is that which is measured by a unit alone.

12 Numbers prime to one another are those which are measured by a unit alone as a common measure

15 A number is said to multiply a number when that which is multiplied is added to itself as many times as there are units in the other, and thus some number is produced.

20 Numbers are proportional when the first is the same multiple, or the same part, or the same parts, of the second that the third is of the fourth.

As long as algebra and geometry have been separated, their progress have been slow and their uses limited; but when these two sciences have been united, they have lent each mutual forces, and have marched together towards perfection.

Joseph-Louis Lagrange
(1736 to 1813)

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Proposition 3 of Book VII

Given three numbers not prime to one another, to find their greatest common measure.

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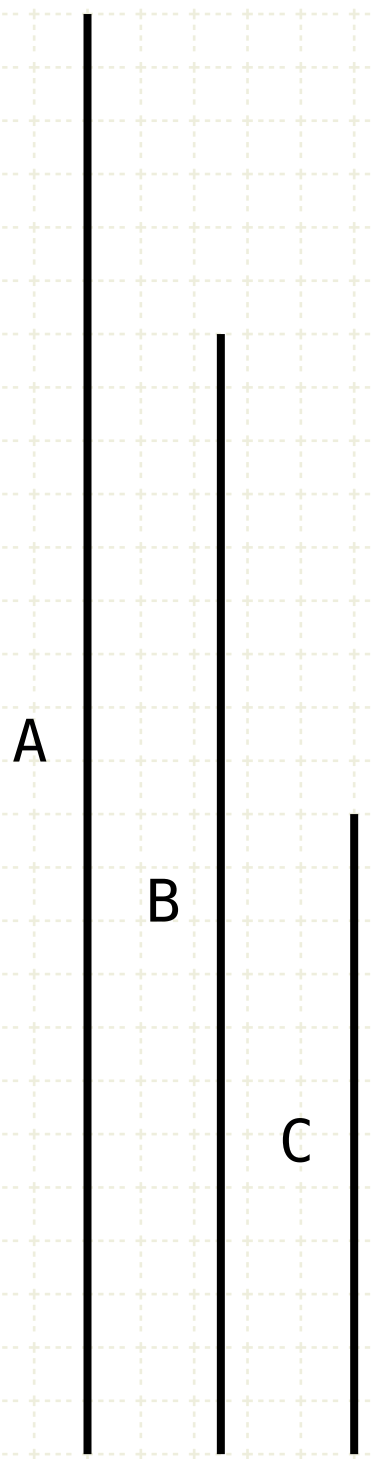
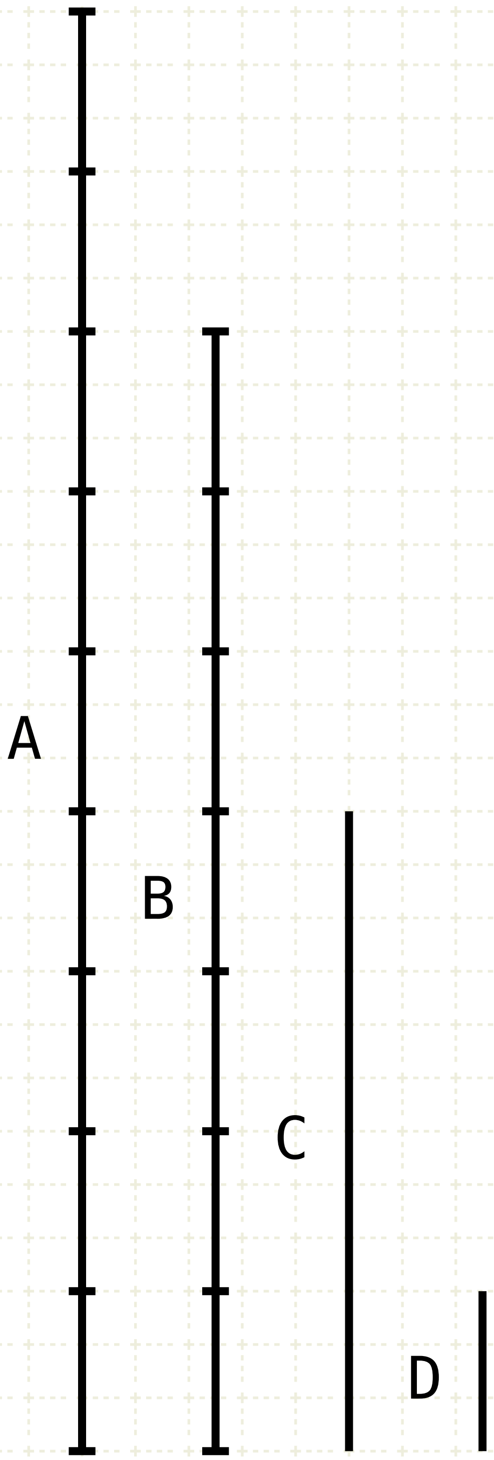


In other words

Find the largest common divisor for three numbers

Proposition 3 of Book VII

Given three numbers not prime to one another, to find their greatest common measure.



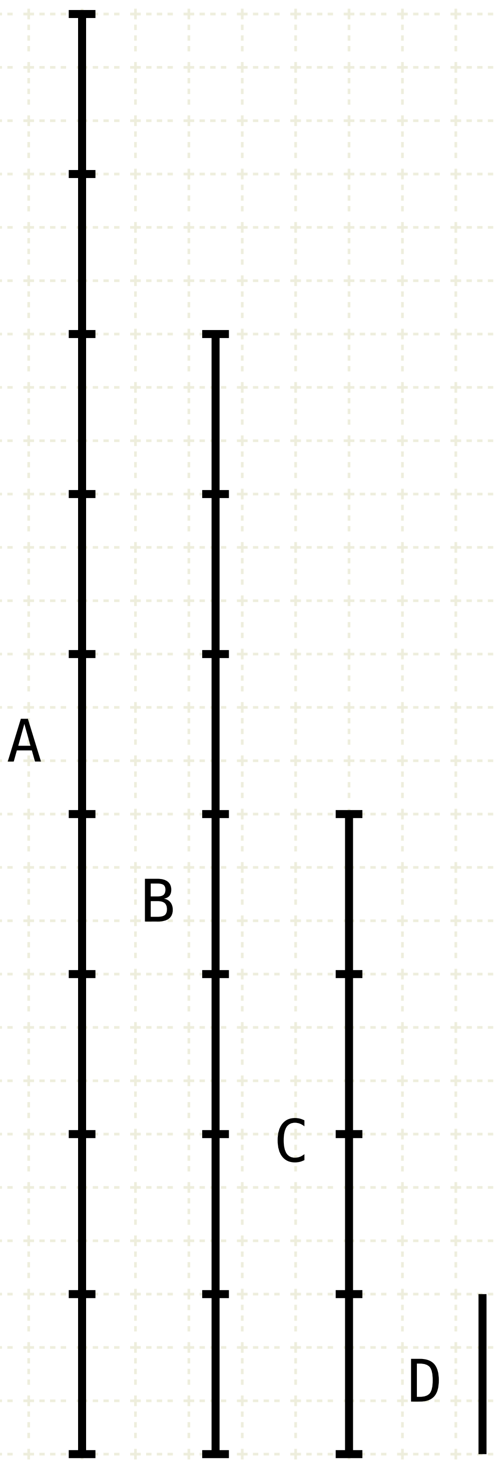
gcd(A,B) = D

Method

Find D, the greatest measure of A and B (VII·2)

Proposition 3 of Book VII

Given three numbers not prime to one another, to find their greatest common measure.



$\gcd(A,B) = D$

let $C = q \cdot D$

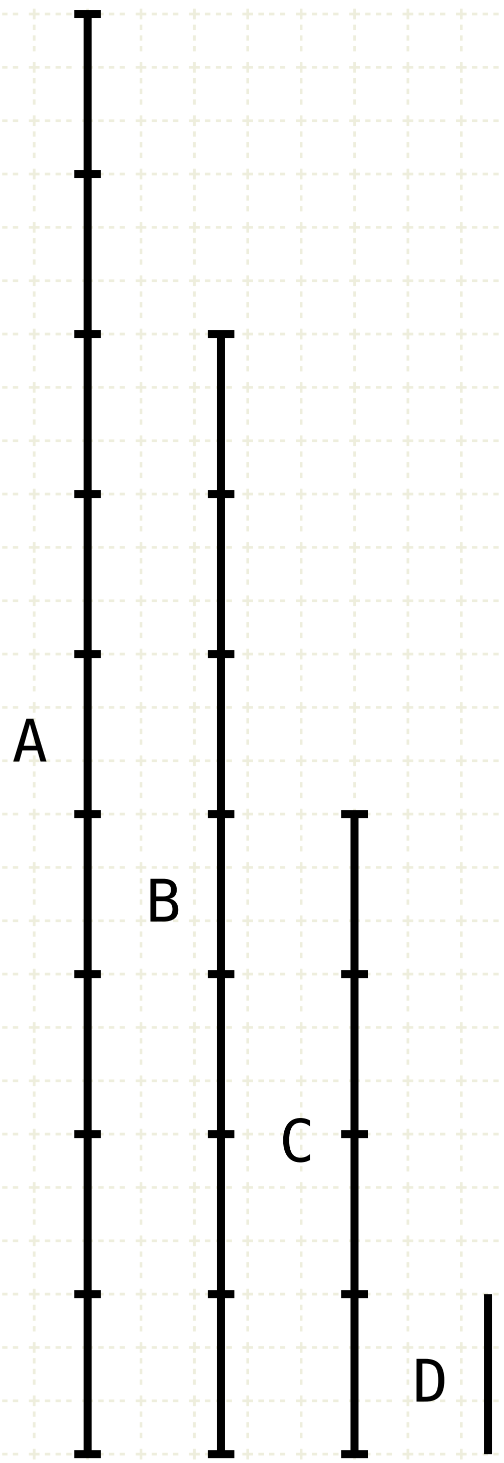
Method

Find D, the greatest measure of A and B (VII·2)

If D measure C, then D is a common divisor for A,B,C

Proposition 3 of Book VII

Given three numbers not prime to one another, to find their greatest common measure.



gcd(A,B) = D

let C = q·D

Method

Find D, the greatest measure of A and B (VII·2)

If D measure C, then D is a common divisor for A,B,C

D is the greatest common divisor

Proposition 3 of Book VII

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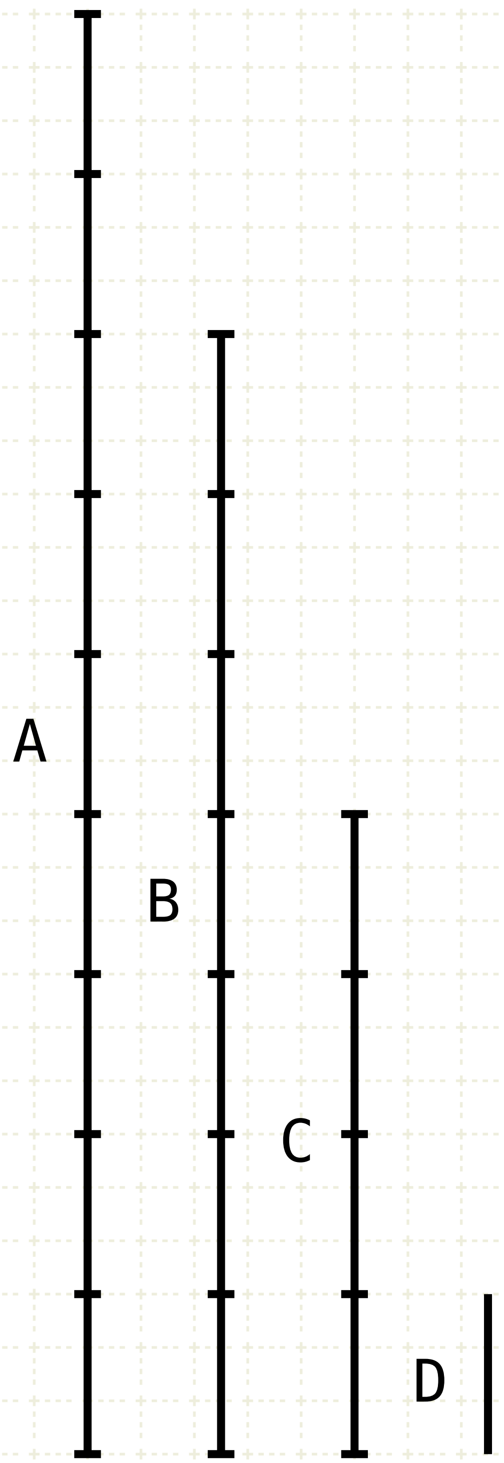

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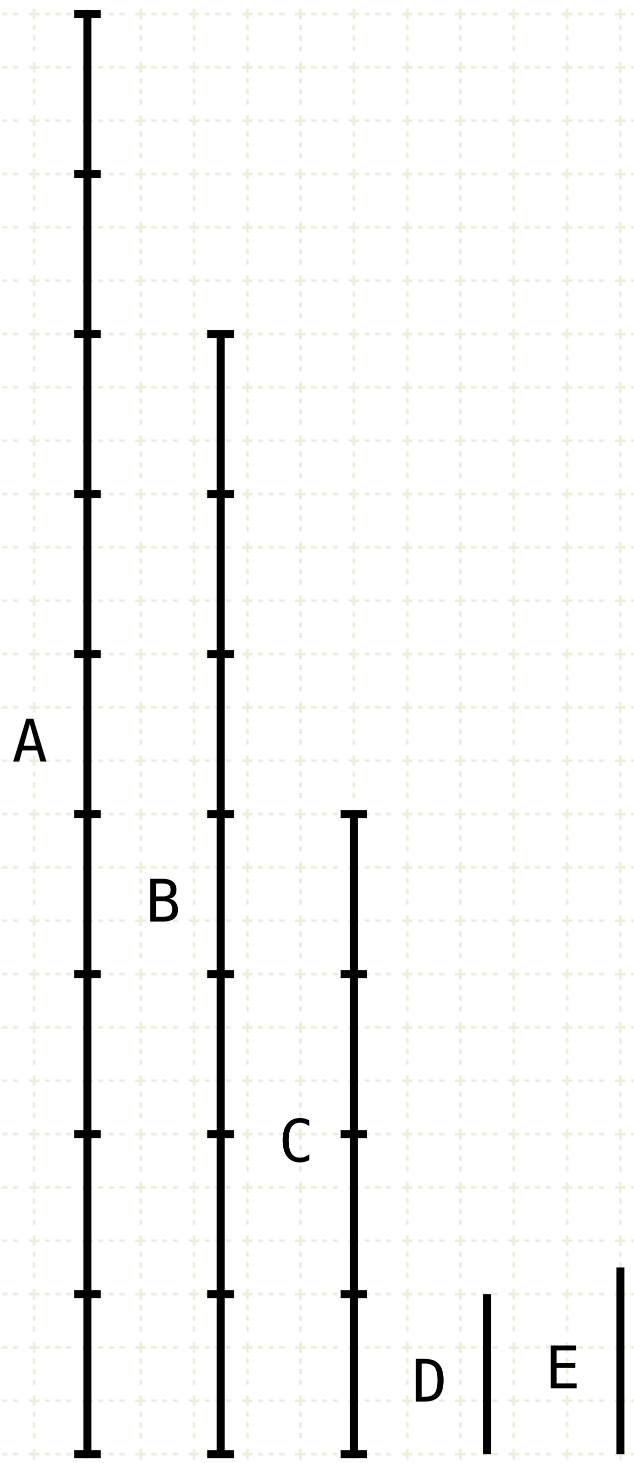
If D measure C, then D is a common divisor for A,B,C

D is the greatest common divisor

Proof by contradiction

Proposition 3 of Book VII

Given three numbers not prime to one another, to find their greatest common measure.



$\gcd(A,B) = D$

let $C = q \cdot D$

let $E = \gcd(A,B,C)$

$D < E$

$A = p \cdot E$

$B = q \cdot E$

$C = r \cdot E$

Method

Find D, the greatest measure of A and B (VII·2)

If D measure C, then D is a common divisor for A,B,C

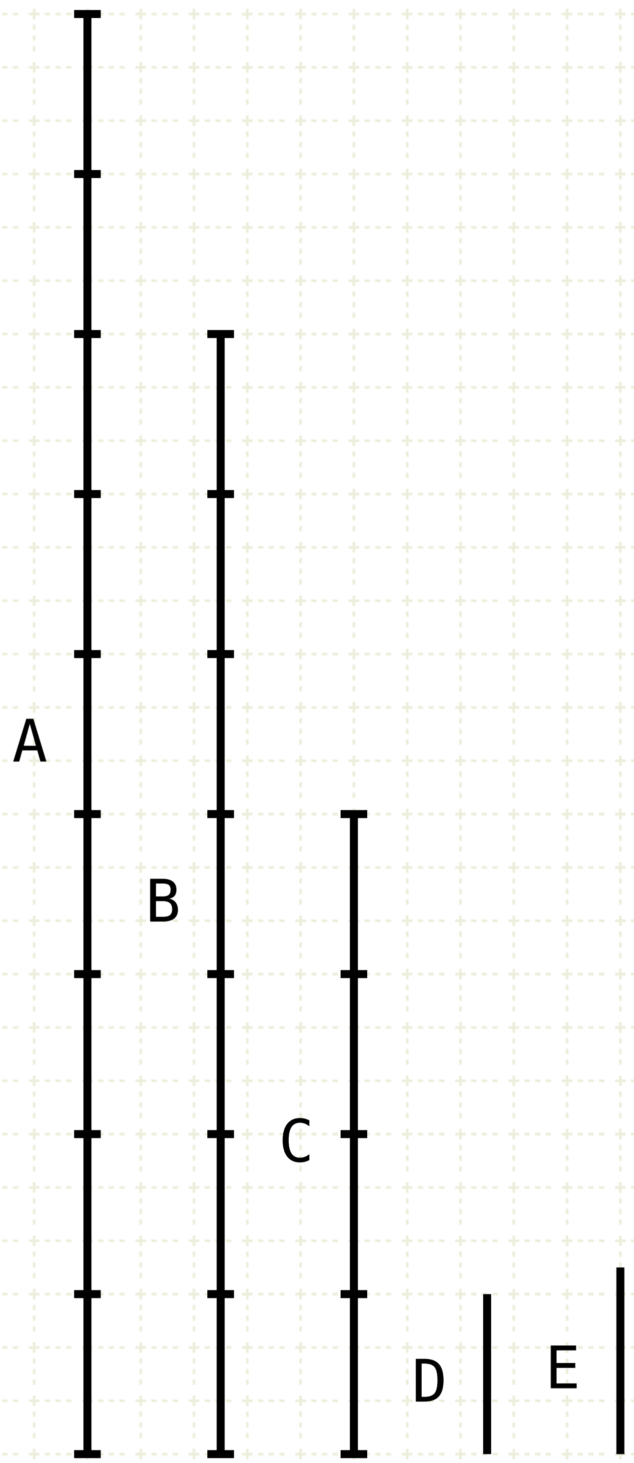
D is the greatest common divisor

Proof by contradiction

If D is not the greatest common divisor, let E be the greatest common divisor

Proposition 3 of Book VII

Given three numbers not prime to one another, to find their greatest common measure.



gcd(A,B) = D

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D < E

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B = q·E

C = r·E

D = s·E

Method

Find D, the greatest measure of A and B (VII·2)

If D measure C, then D is a common divisor for A,B,C

D is the greatest common divisor

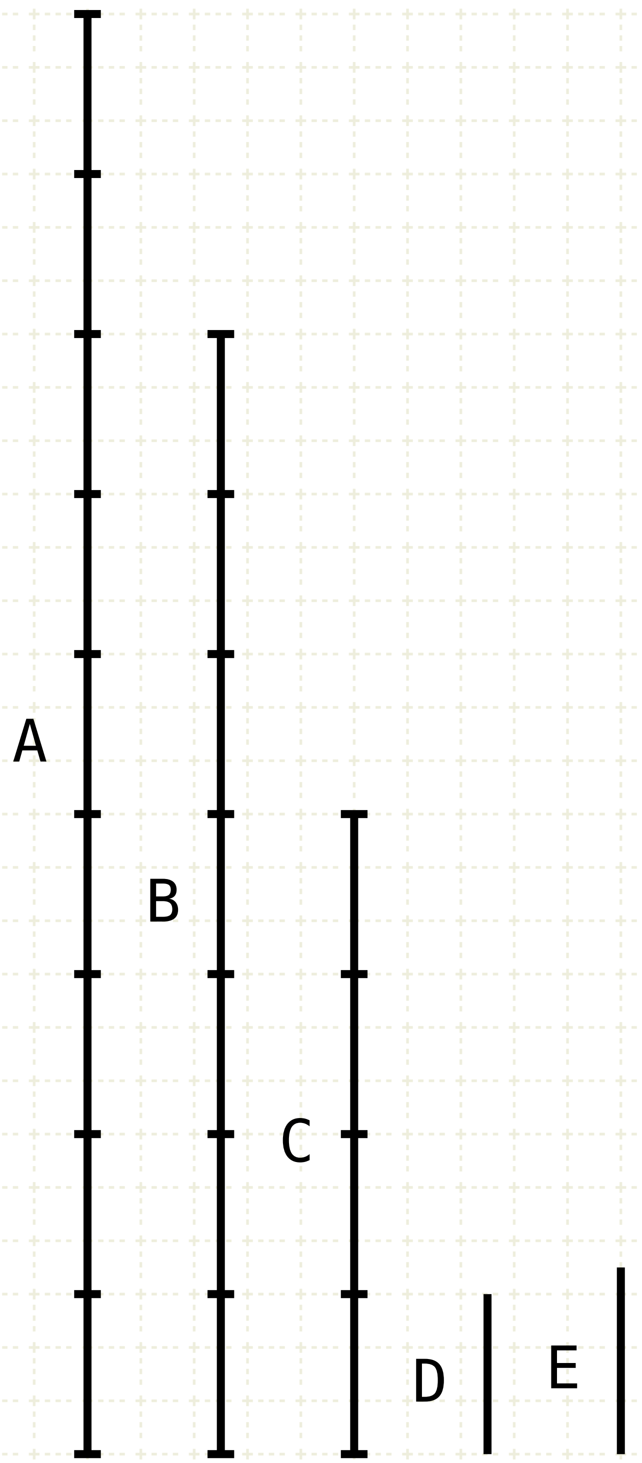
Proof by contradiction

If D is not the greatest common divisor, let E be the greatest common divisor

Since E measures A,B,C, it will also measure the greatest common divisor of A,B, which is D (VII·2 Por)

Proposition 3 of Book VII

Given three numbers not prime to one another, to find their greatest common measure.



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Method

Find D, the greatest measure of A and B (VII·2)

If D measure C, then D is a common divisor for A,B,C

D is the greatest common divisor

Proof by contradiction

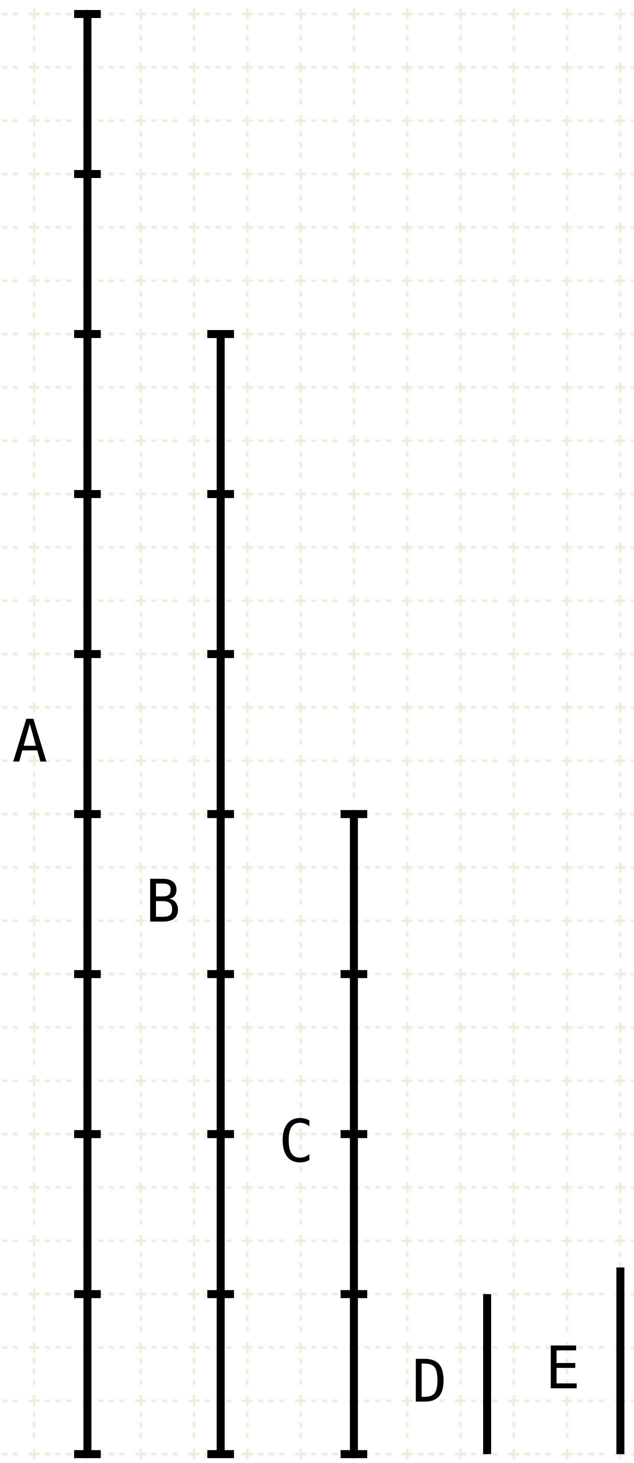
If D is not the greatest common divisor, let E be the greatest common divisor

Since E measures A,B,C, it will also measure the greatest common divisor of A,B, which is D (VII·2 Por)

But D is less than E, so it cannot be measured by E,

Proposition 3 of Book VII

Given three numbers not prime to one another, to find their greatest common measure.




```
gcd(A,B) = D

let C = q·D

let E = gcd(A,B,C)
D < E
A = p·E
B = q·E
C = r·E

D = s·E
∴ gcd(A,B,C) = D
```

Method

Find D, the greatest measure of A and B (VII·2)

If D measure C, then D is a common divisor for A,B,C

D is the greatest common divisor

Proof by contradiction

If D is not the greatest common divisor, let E be the greatest common divisor

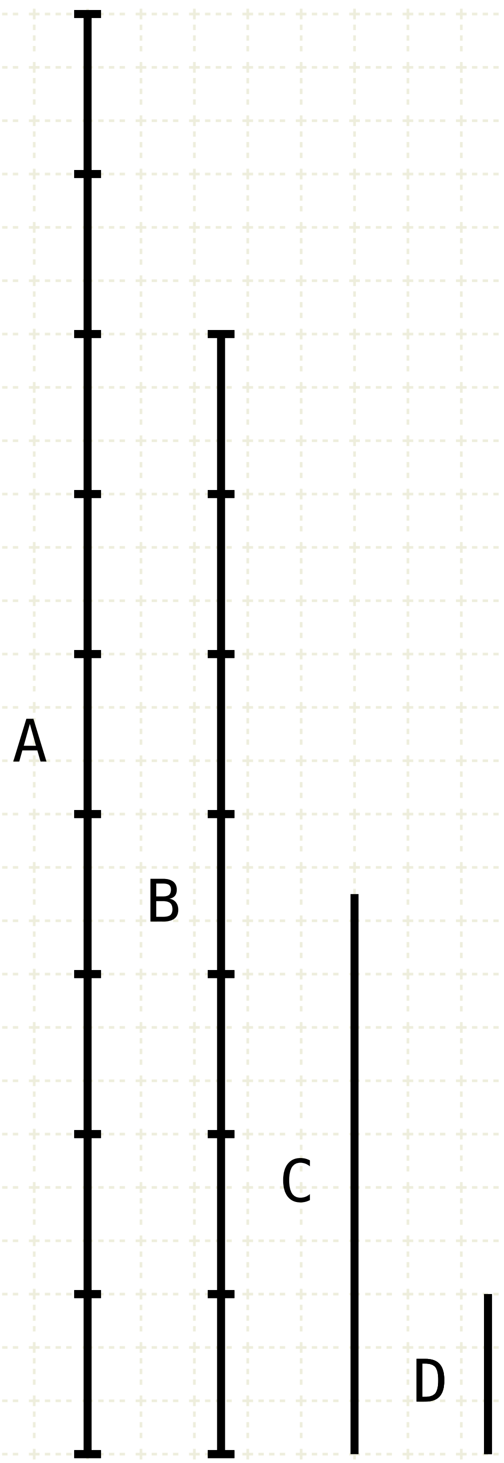
Since E measures A,B,C, it will also measure the greatest common divisor of A,B, which is D (VII·2 Por)

But D is less than E, so it cannot be measured by E,

therefore D is the greatest common divisor

Proposition 3 of Book VII

Given three numbers not prime to one another, to find their greatest common measure.



$\text{gcd}(A,B) = D$

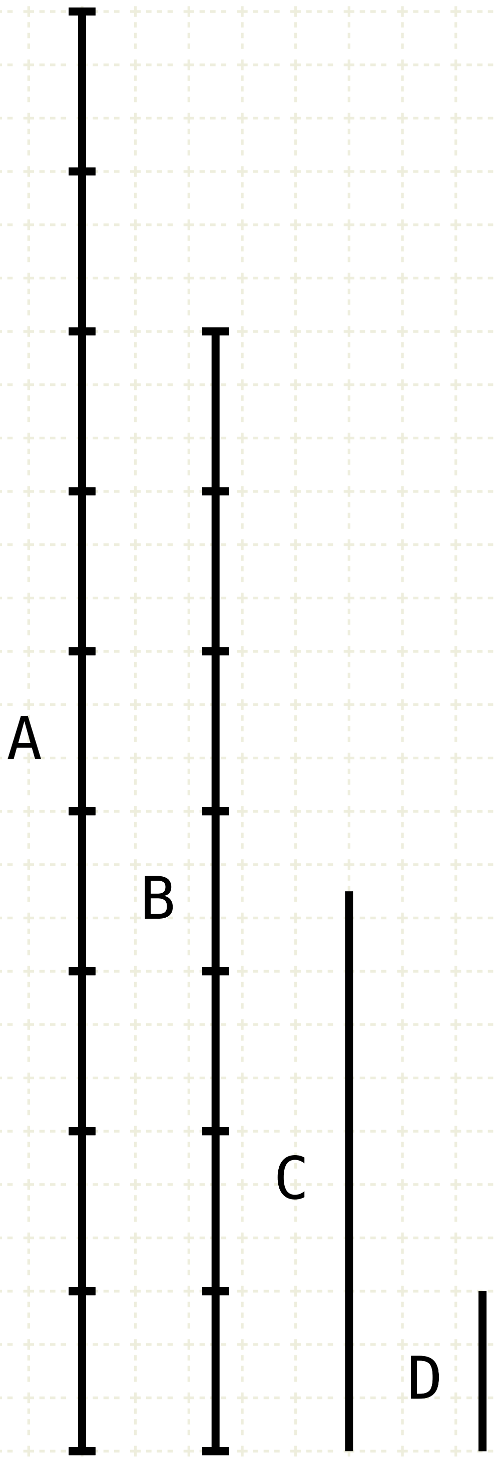
let $C \neq q \cdot D$

Method

Find D, the greatest measure of A and B (VII·2)

Proposition 3 of Book VII

Given three numbers not prime to one another, to find their greatest common measure.



gcd(A,B) = D

let C ≠ q·D

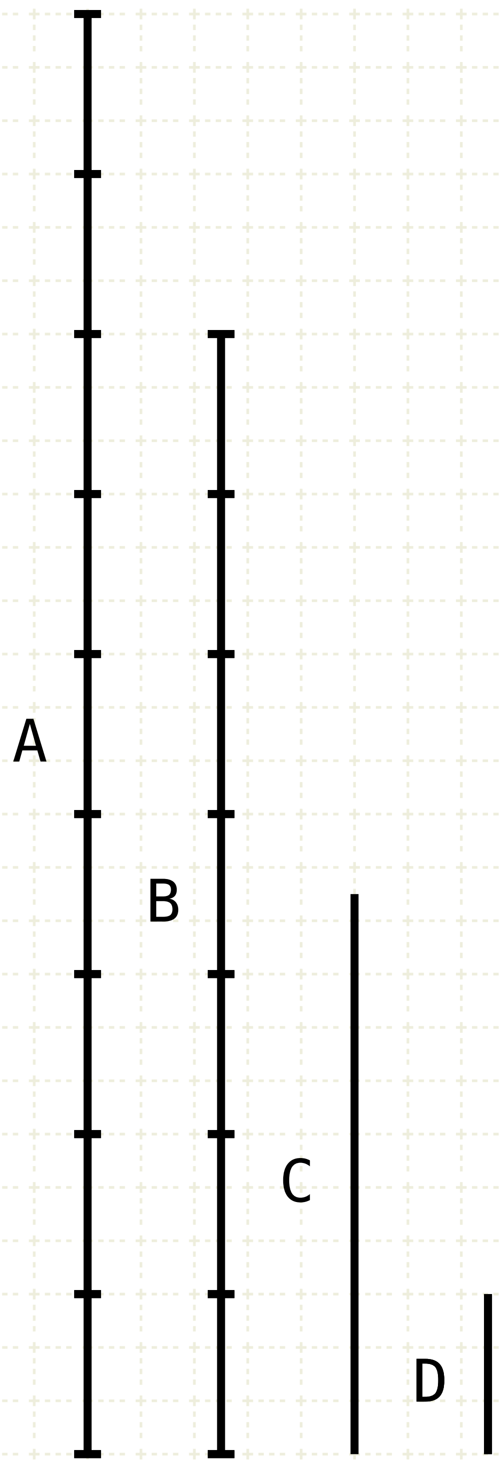
Method

Find D, the greatest measure of A and B (VII·2)

C and D are not prime to one another

Proposition 3 of Book VII

Given three numbers not prime to one another, to find their greatest common measure.



$\gcd(A,B) = D$

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Method

Find D, the greatest measure of A and B (VII·2)

C and D are not prime to one another

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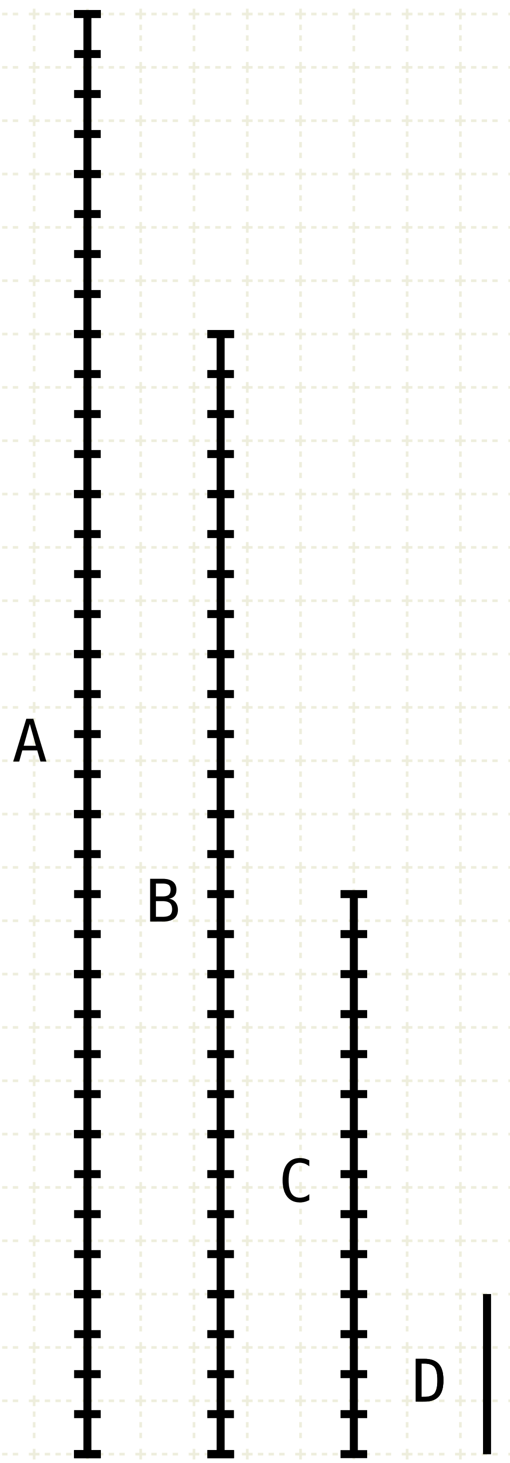
Proposition 3 of Book VII

Given three numbers not prime to one another, to find their greatest common measure.



$\gcd(A,B) = D$

let $C \neq q \cdot D$

$A = p \cdot x$

$B = q \cdot x$

$C = r \cdot x$

Method

Find D, the greatest measure of A and B (VII·2)

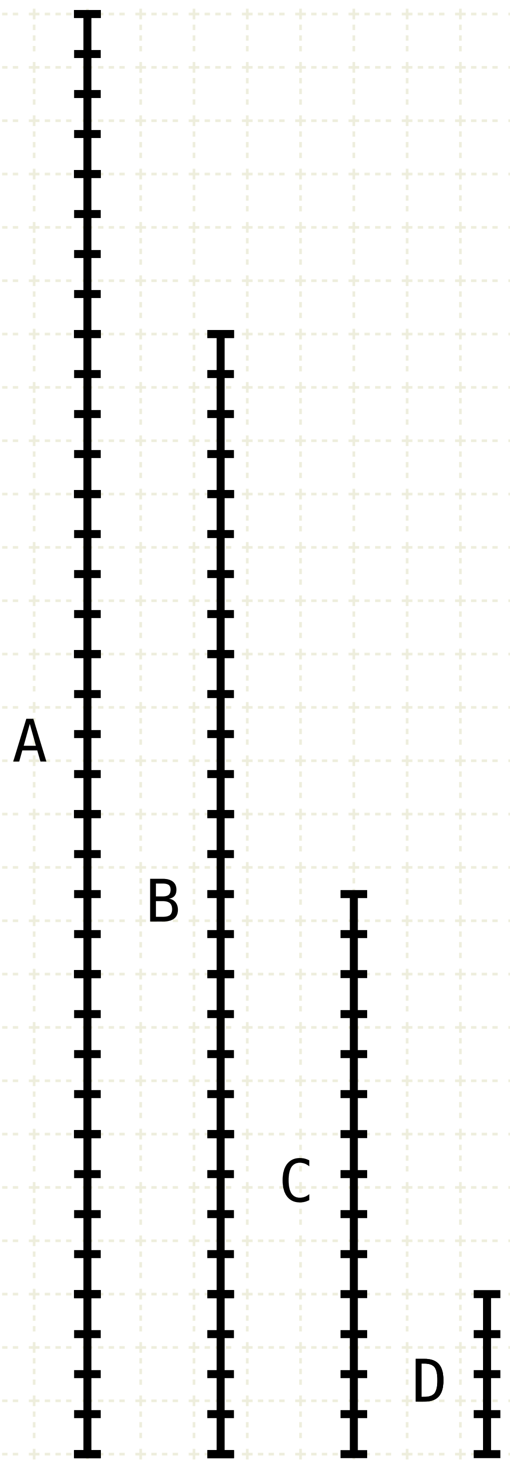
C and D are not prime to one another

Proof

Since A,B,C are not prime to one another, there is a number x that measures A,B,C

Proposition 3 of Book VII

Given three numbers not prime to one another, to find their greatest common measure.



gcd(A,B) = D

let C ≠ q·D

A = p·x

B = q·x

C = r·x

D = s·x

Method

Find D, the greatest measure of A and B (VII·2)

C and D are not prime to one another

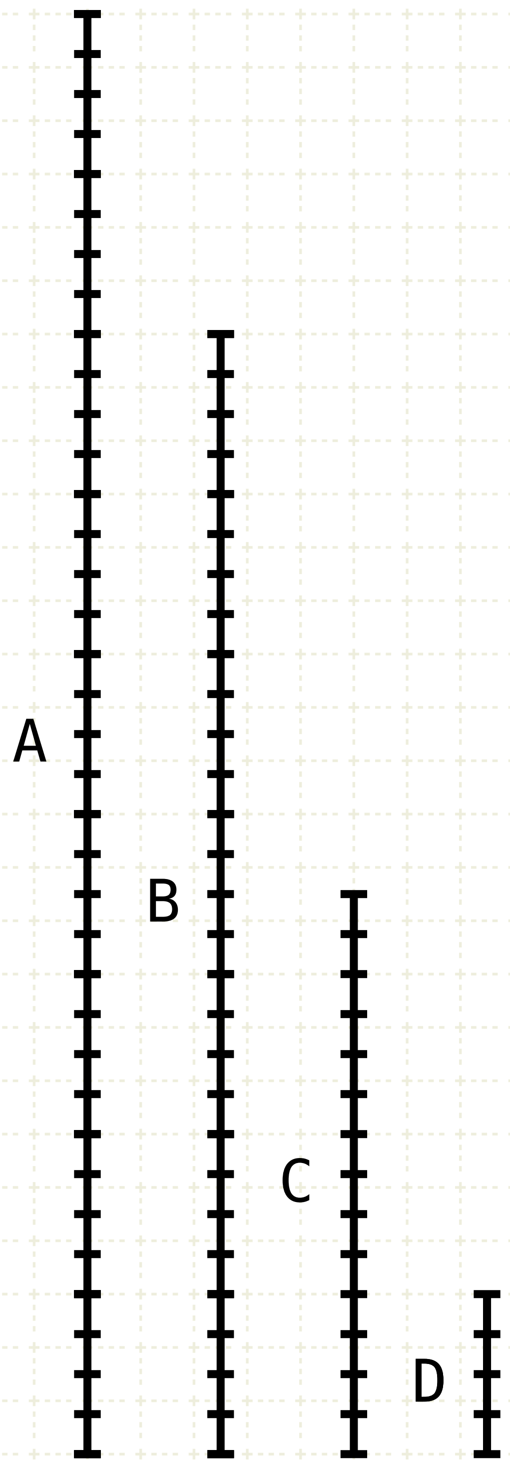
Proof

Since A,B,C are not prime to one another, there is a number x that measures A,B,C

This number x will also measure D, the greatest common divisor of A,B (VII·2 Por)

Proposition 3 of Book VII

Given three numbers not prime to one another, to find their greatest common measure.



$\gcd(A,B) = D$

let $C \neq q \cdot D$

$A = p \cdot x$

$B = q \cdot x$

$C = r \cdot x$

$D = s \cdot x$

Method

Find D, the greatest measure of A and B (VII·2)

C and D are not prime to one another

Proof

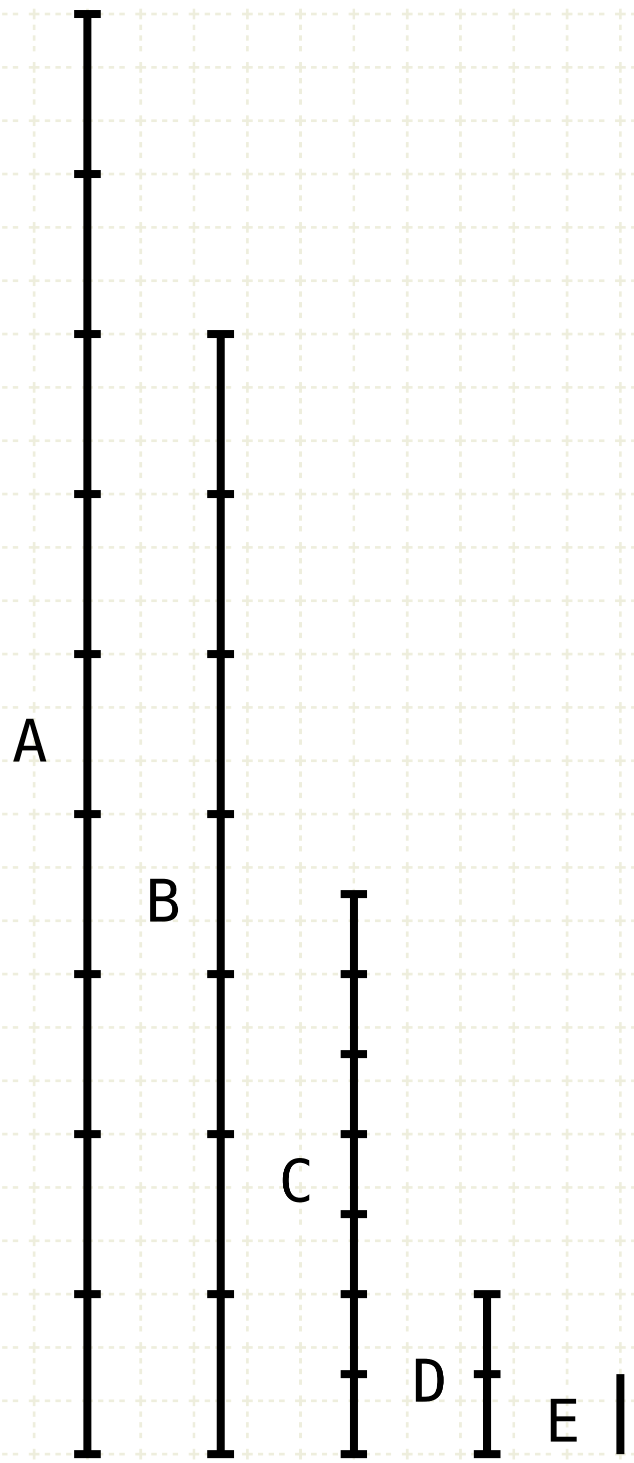
Since A,B,C are not prime to one another, there is a number x that measures A,B,C

This number x will also measure D, the greatest common divisor of A,B (VII·2 Por)

Since x measures D and C, D and C are not prime to each other

Proposition 3 of Book VII

Given three numbers not prime to one another, to find their greatest common measure.

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gcd(A,B) = D
let C ≠ q·D
gcd(C,D) = E
```

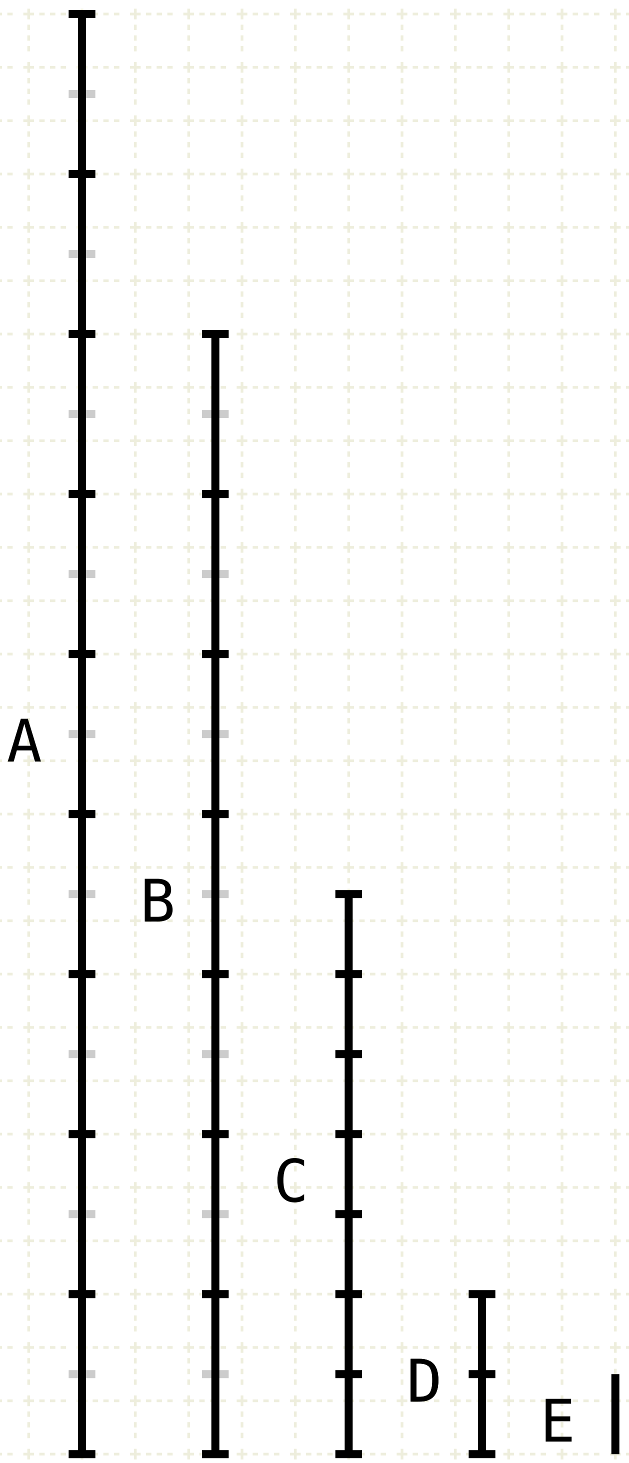
Method

Find D, the greatest measure of A and B (VII·2)

Find the greatest common divisor E for C and D (VII·2)

Proposition 3 of Book VII

Given three numbers not prime to one another, to find their greatest common measure.




```
gcd(A,B) = D
let C ≠ q·D
gcd(C,D) = E

A = a·E
B = b·E
```

Method

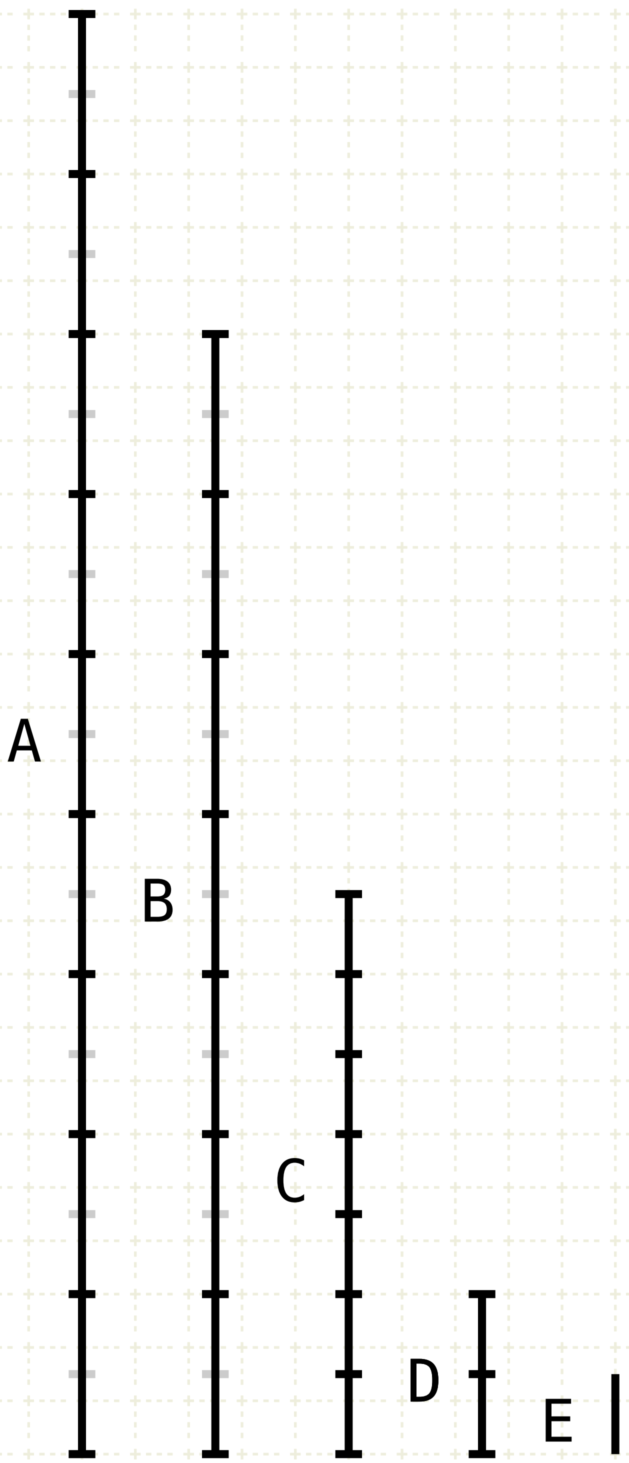
Find D, the greatest measure of A and B (VII·2)

Find the greatest common divisor E for C and D (VII·2)

Since E is a measure of D, and since D is a measure of A,B, E is also a measure of A,B

Proposition 3 of Book VII

Given three numbers not prime to one another, to find their greatest common measure.




```
gcd(A,B) = D
let C ≠ q·D
gcd(C,D) = E

A = a·E
B = b·E
```

Method

Find D, the greatest measure of A and B (VII·2)

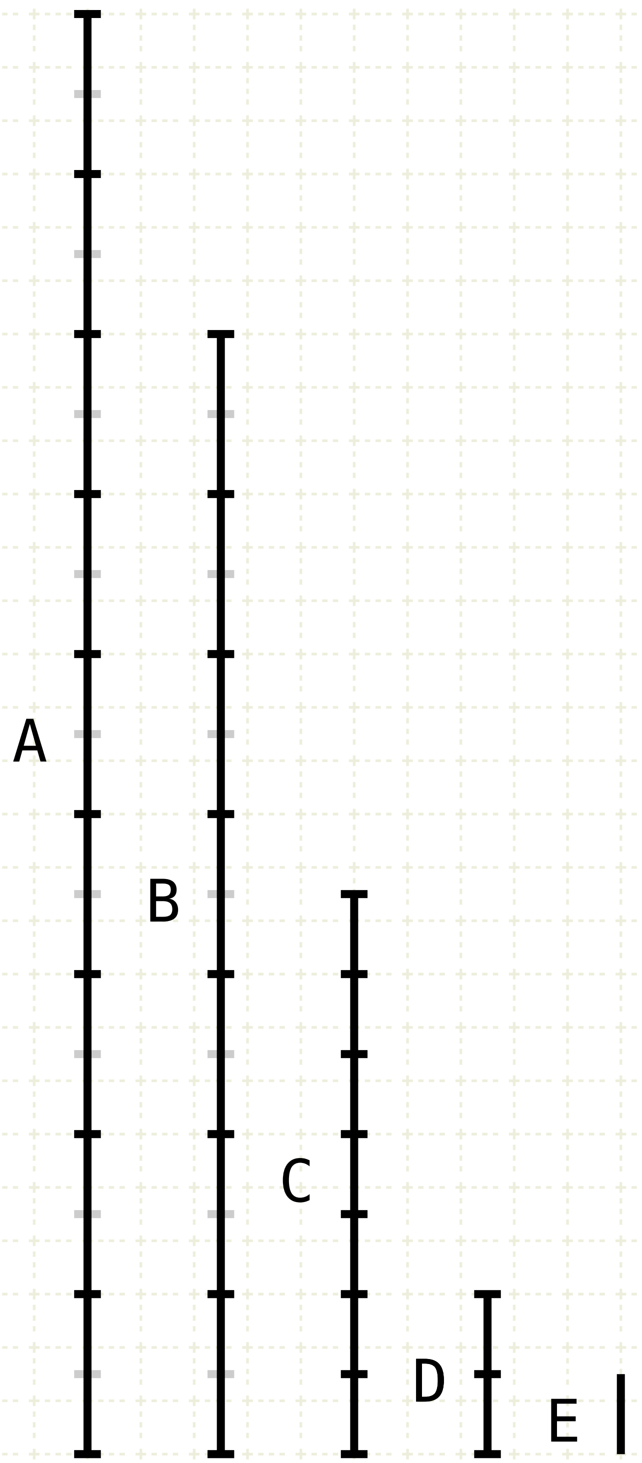
Find the greatest common divisor E for C and D (VII·2)

Since E is a measure of D, and since D is a measure of A,B, E is also a measure of A,B

And since E also measures C, E is a common measure of A,B,C

Proposition 3 of Book VII

Given three numbers not prime to one another, to find their greatest common measure.




```
gcd(A,B) = D
gcd(C,D) = E
```

Method

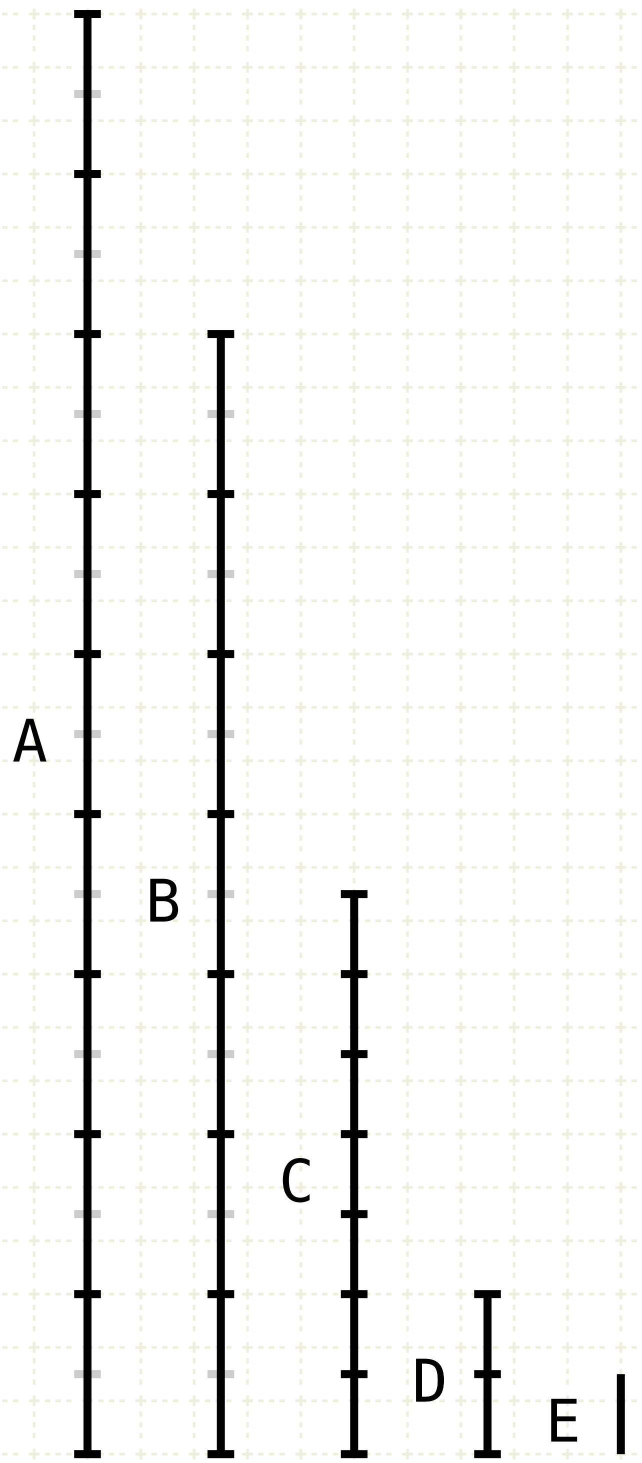
Find D, the greatest measure of A and B (VII·2)

Find the greatest common divisor E for C and D (VII·2)

E is the Greatest Common Divisor of A,B,C

Proposition 3 of Book VII

Given three numbers not prime to one another, to find their greatest common measure.

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gcd(A,B) = D
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```

Method

Find D, the greatest measure of A and B (VII·2)

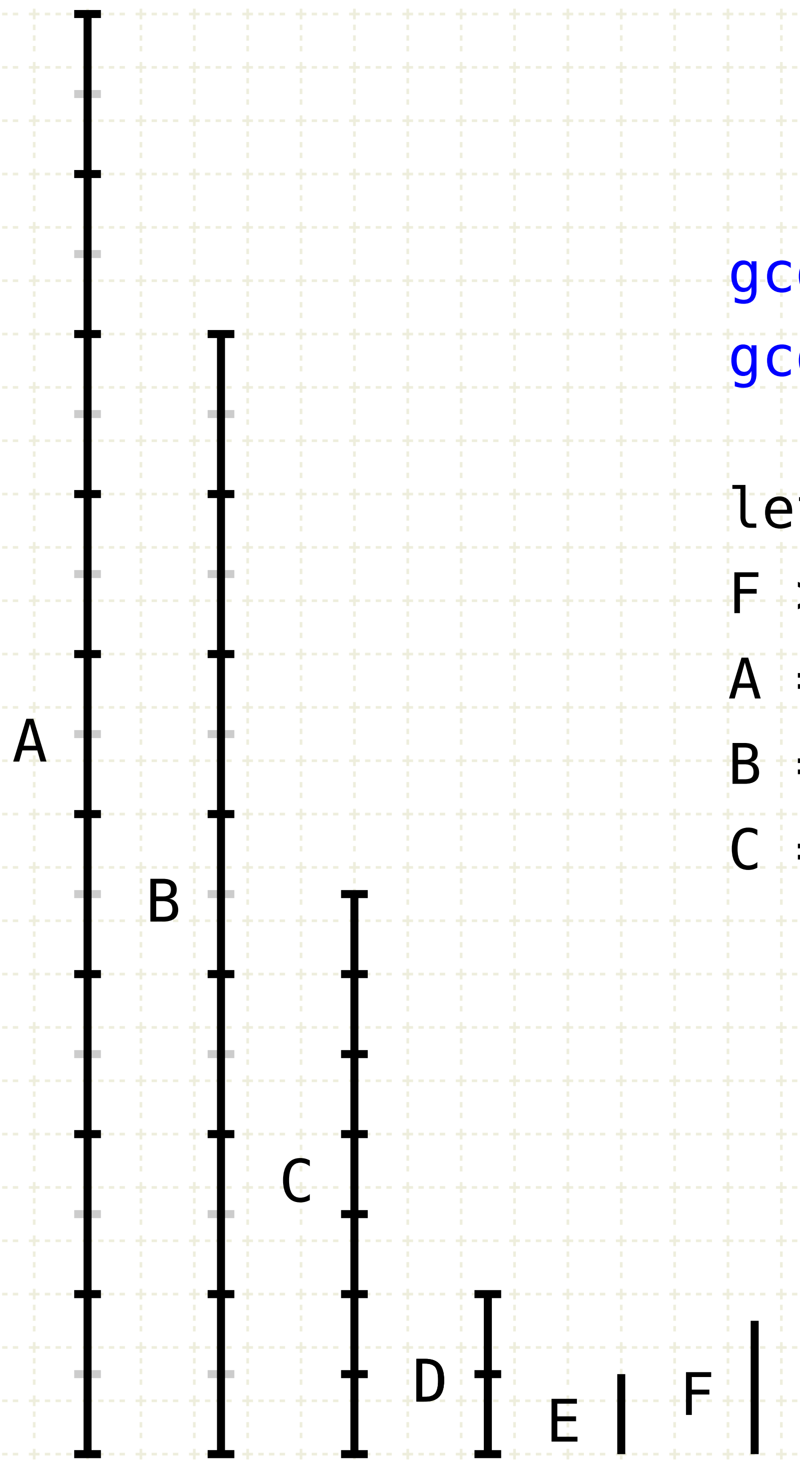
Find the greatest common divisor E for C and D (VII·2)

E is the Greatest Common Divisor of A,B,C

Proof by Contradiction

Proposition 3 of Book VII

Given three numbers not prime to one another, to find their greatest common measure.




```
gcd(A,B) = D
gcd(C,D) = E

let F = gcd(A,B,C)
F > E
A = p·F
B = q·F
C = r·F
```

Method

Find D, the greatest measure of A and B (VII·2)

Find the greatest common divisor E for C and D (VII·2)

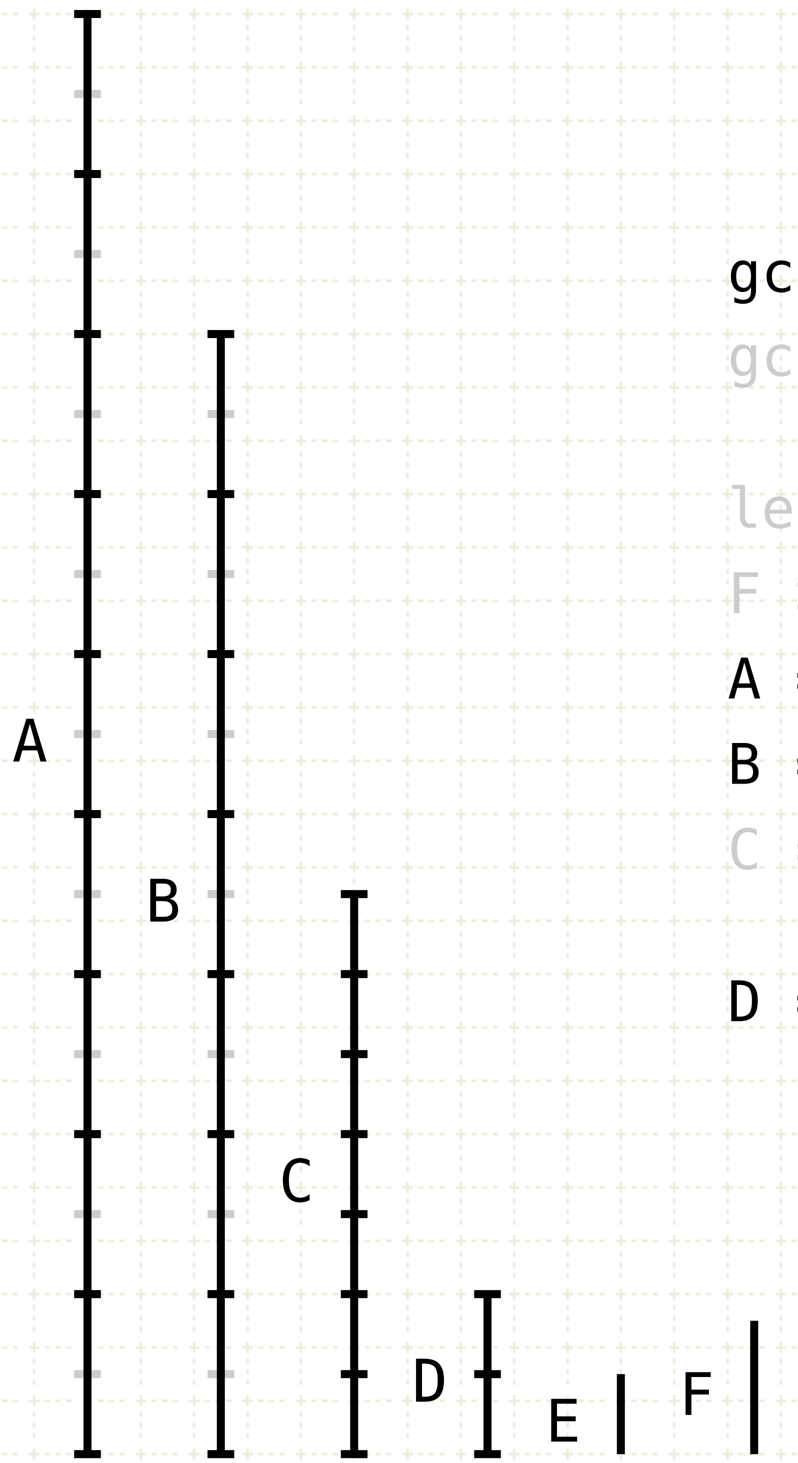
E is the Greatest Common Divisor of A,B,C

Proof by Contradiction

Assume that F, which is greater than E, is the largest common divisor

Proposition 3 of Book VII

Given three numbers not prime to one another, to find their greatest common measure.

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let F = gcd(A,B,C)
F > E
A = p·F
B = q·F
C = r·F

D = s·F
```

Method

Find D, the greatest measure of A and B (VII·2)

Find the greatest common divisor E for C and D (VII·2)

E is the Greatest Common Divisor of A,B,C

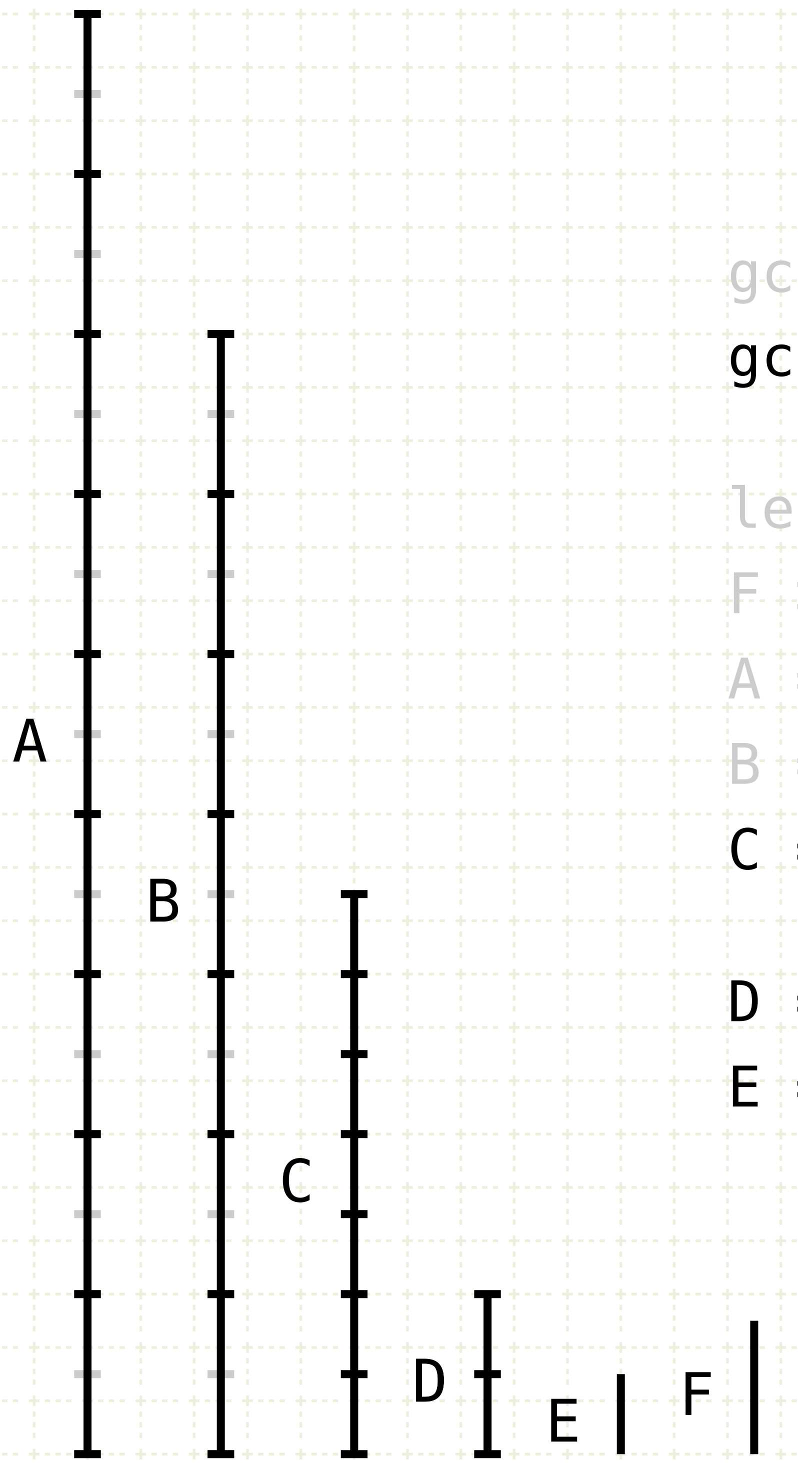
Proof by Contradiction

Assume that F, which is greater than E, is the largest common divisor

F measures A,B so it also measures D, the greatest common divisor of A,B (VII·2 Por)

Proposition 3 of Book VII

Given three numbers not prime to one another, to find their greatest common measure.

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gcd(A,B) = D
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let F = gcd(A,B,C)
F > E
A = p·F
B = q·F
C = r·F

D = s·F
E = n·F
```

Method

Find D, the greatest measure of A and B (VII·2)

Find the greatest common divisor E for C and D (VII·2)

E is the Greatest Common Divisor of A,B,C

Proof by Contradiction

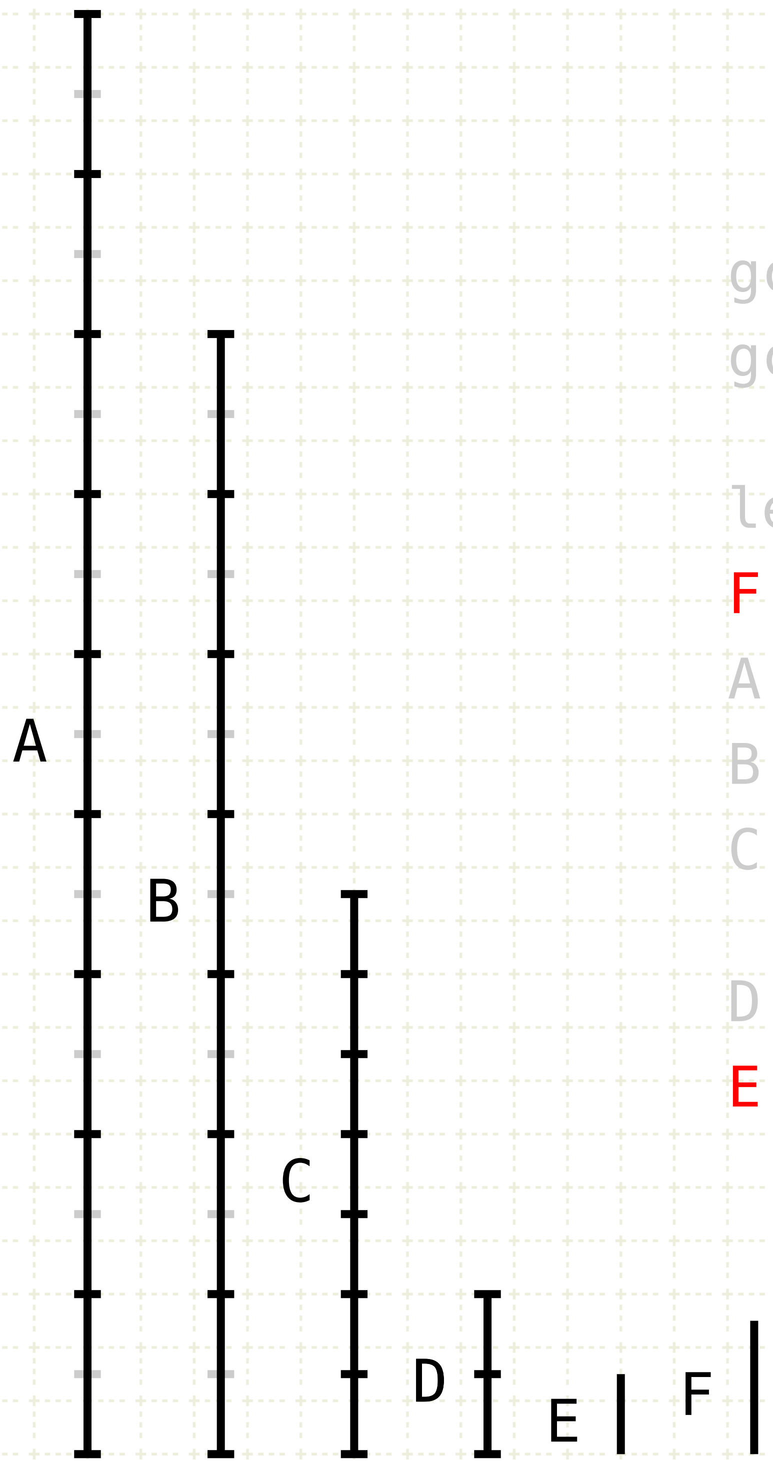
Assume that F, which is greater than E, is the largest common divisor

F measures A,B so it also measures D, the greatest common divisor of A,B (VII·2 Por)

F measures D,C so it also measures E, the greatest common divisor of D,C (VII·2 Por)

Proposition 3 of Book VII

Given three numbers not prime to one another, to find their greatest common measure.



$\gcd(A,B) = D$

$\gcd(C,D) = E$

let $F = \gcd(A,B,C)$

$F > E$

$A = p \cdot F$

$B = q \cdot F$

$C = r \cdot F$

$D = s \cdot F$

$E = n \cdot F$

Method

Find D, the greatest measure of A and B (VII·2)

Find the greatest common divisor E for C and D (VII·2)

E is the Greatest Common Divisor of A,B,C

Proof by Contradiction

Assume that F, which is greater than E, is the largest common divisor

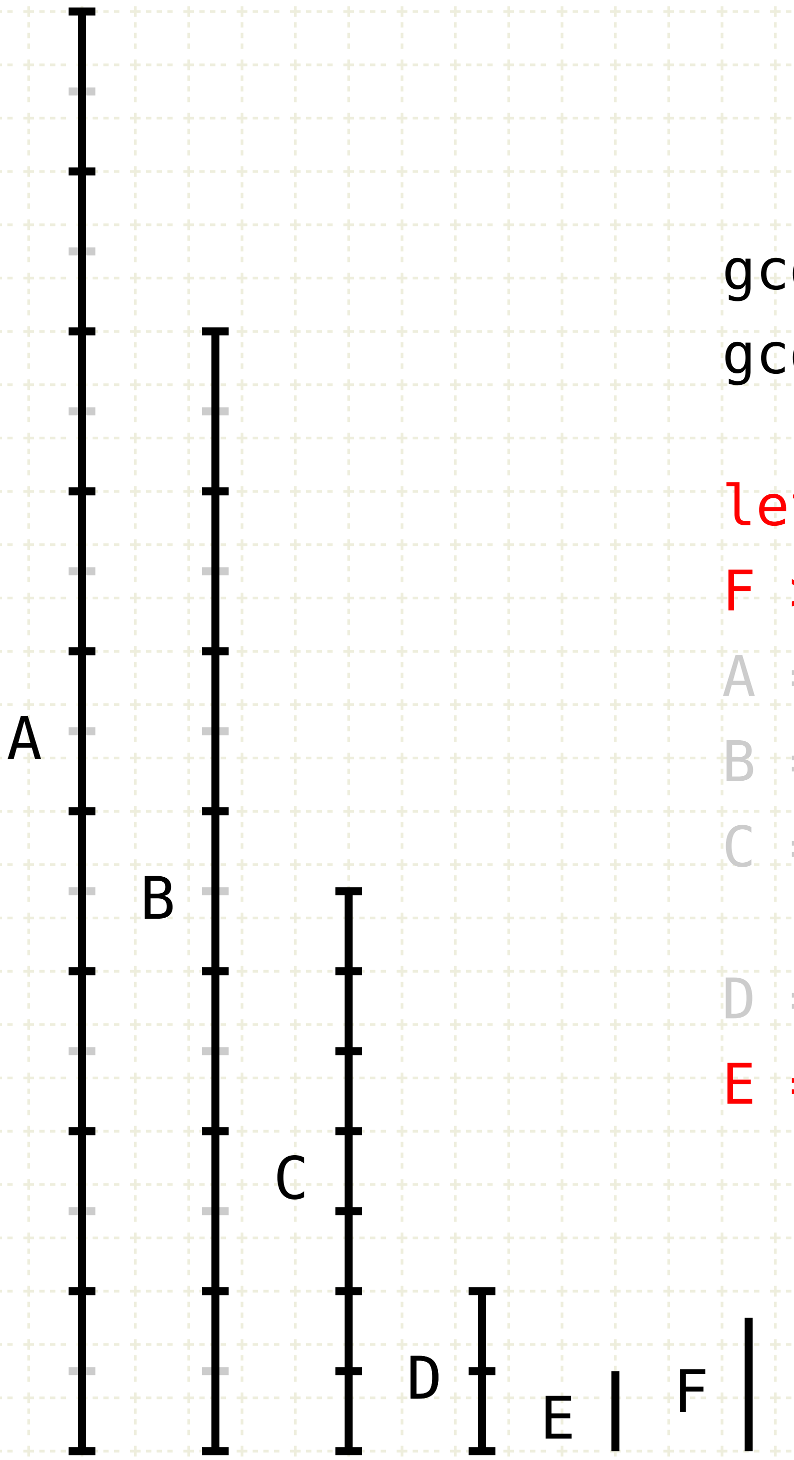
F measures A,B so it also measures D, the greatest common divisor of A,B (VII·2 Por)

F measures D,C so it also measures E, the greatest common divisor of D,C (VII·2 Por)

But F, being larger than E, cannot measure E, hence F cannot be larger than E

Proposition 3 of Book VII

Given three numbers not prime to one another, to find their greatest common measure.

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gcd(A,B) = D
gcd(C,D) = E

let F = gcd(A,B,C)
F > E
A = p·F
B = q·F
C = r·F

D = s·F
E = n·F
```

Method

Find D, the greatest measure of A and B (VII·2)

Find the greatest common divisor E for C and D (VII·2)

E is the Greatest Common Divisor of A,B,C

Proof by Contradiction

Assume that F, which is greater than E, is the largest common divisor

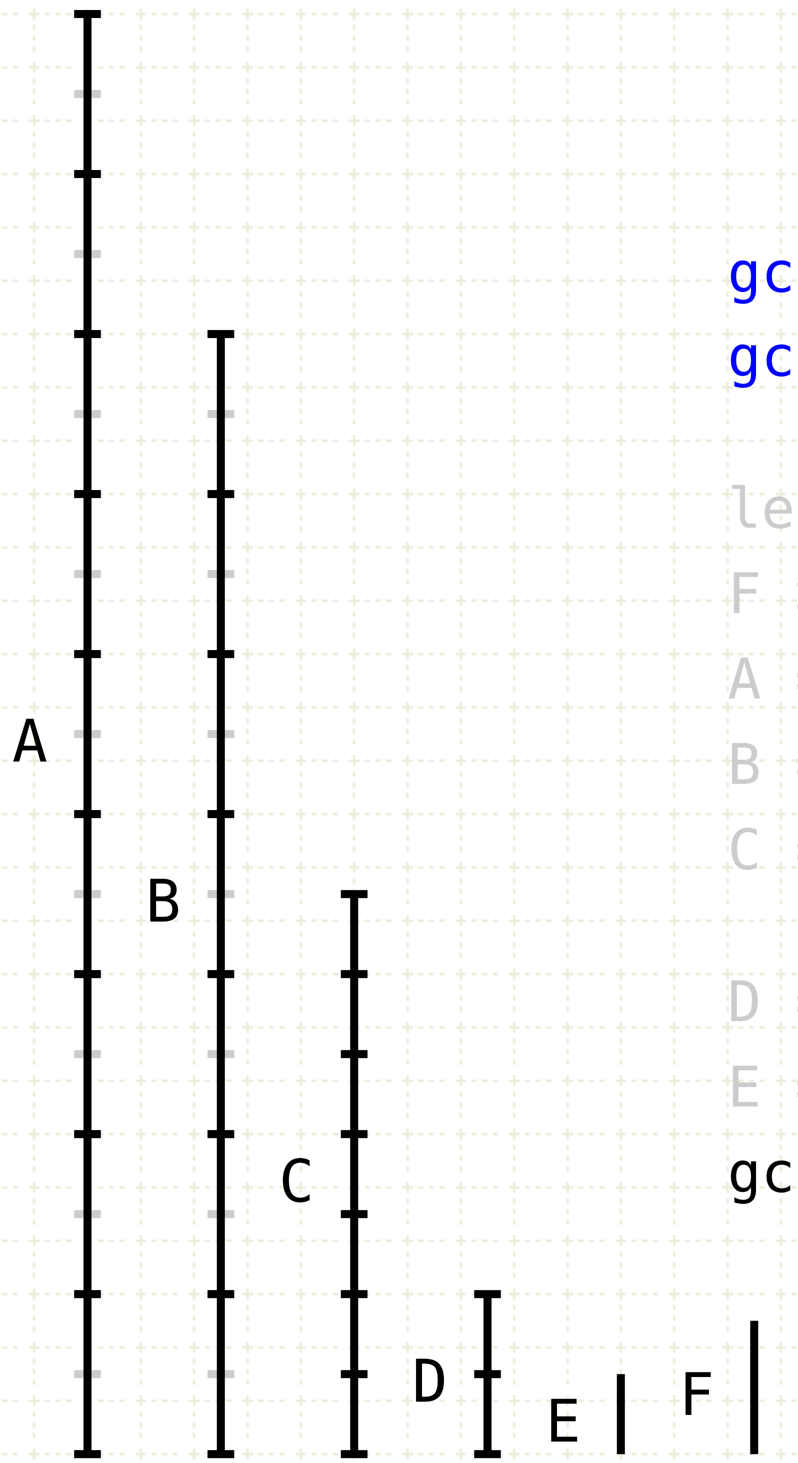
F measures A,B so it also measures D, the greatest common divisor of A,B (VII·2 Por)

F measures D,C so it also measures E, the greatest common divisor of D,C (VII·2 Por)

But F, being larger than E, cannot measure E, hence F cannot be larger than E

Proposition 3 of Book VII

Given three numbers not prime to one another, to find their greatest common measure.




```
gcd(A,B) = D
gcd(C,D) = E

let F = gcd(A,B,C)
F > E
A = p·F
B = q·F
C = r·F

D = s·F
E = n·F
gcd(A,B,C) = E
```

Method

Find D, the greatest measure of A and B (VII·2)

Find the greatest common divisor E for C and D (VII·2)

E is the Greatest Common Divisor of A,B,C

Proof by Contradiction

Assume that F, which is greater than E, is the largest common divisor

F measures A,B so it also measures D, the greatest common divisor of A,B (VII·2 Por)

F measures D,C so it also measures E, the greatest common divisor of D,C (VII·2 Por)

But F, being larger than E, cannot measure E, hence F cannot be larger than E

Therefore E is the greatest common divisor

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