

Euclid's Elements

Book I

If Euclid did not kindle your youthful enthusiasm, you were not born to be a scientific thinker.

Albert Einstein



Table of Contents, Chapter 1

1	Construct an equilateral triangle	15	Vertical angles equal one another	29	Lines parallel, alternate angles are equal
2	Copy a line	16	Exterior angle larger than interior angle	30	Lines parallel to same line are parallel to themselves
3	Subtract one line from another	17	Sum of two interior angles less than 180	31	Construct one line parallel to another
4	Equal triangles if equal side-angle-side	18	Greater side opposite of greater angle	32	Sum of interior angles of a triangle = 180
5	Isosceles triangle gives equal base angles	19	Greater angle opposite of greater side	33	Lines joining ends of equal parallels are parallel
6	Equal base angles gives isosceles triangle	20	Sum of two angles greater than third	34	Opposite sides-angles equal in parallelogram
7	Two sides of triangle meet at unique point	21	Triangle within triangle has smaller sides	35	Parallelograms, same base-height have equal area
8	Equal triangles if equal side-side-side	22	Construct triangle from given lines	36	Parallelograms, equal base-height have equal area
9	How to bisect an angle	23	Copy an angle	37	Triangles, same base-height have equal area
10	Bisect a line	24	Larger angle gives larger base	38	Triangles, equal base-height have equal area
11	Construct right angle, point on line	25	Larger base gives larger angle		
12	Construct perpendicular, point to line	26	Equal triangles if equal angle-side-angle		
13	Sum of angles on straight line = 180	27	Alternate angles equal then lines parallel		
14	Two lines form a single line if angle = 180	28	Sum of interior angles = 180 , lines parallel		



Table of Contents, Chapter 1

39	Equal triangles on same base, have equal height
40	Equal triangles on equal base, have equal height
41	Triangle is half parallelogram with same base and height
42	Construct parallelogram with equal area as triangle
43	Parallelogram complements are equal
44	Construct parallelogram on line, equal to triangle
45	Construct parallelogram equal to polygon
46	Construct a square
47	Pythagoras' theorem
48	Inverse Pythagoras' theorem



Proposition 46 of Book I

To describe a square on a given straight line.



Proposition 46 of Book I

To describe a square on a given straight line.

Construction
Start with a given line AB



Proposition 46 of Book I

To describe a square on a given straight line.

Construction

Start with a given line AB

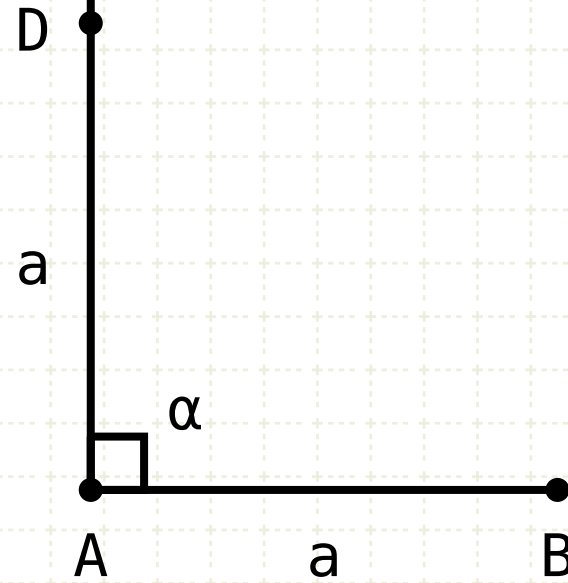
Draw a line AC perpendicular to AB, from point A (I·11)



$$\alpha = \angle$$

Proposition 46 of Book I

To describe a square on a given straight line.



$$\alpha = \angle$$
$$AD = AB$$

Construction

Start with a given line AB

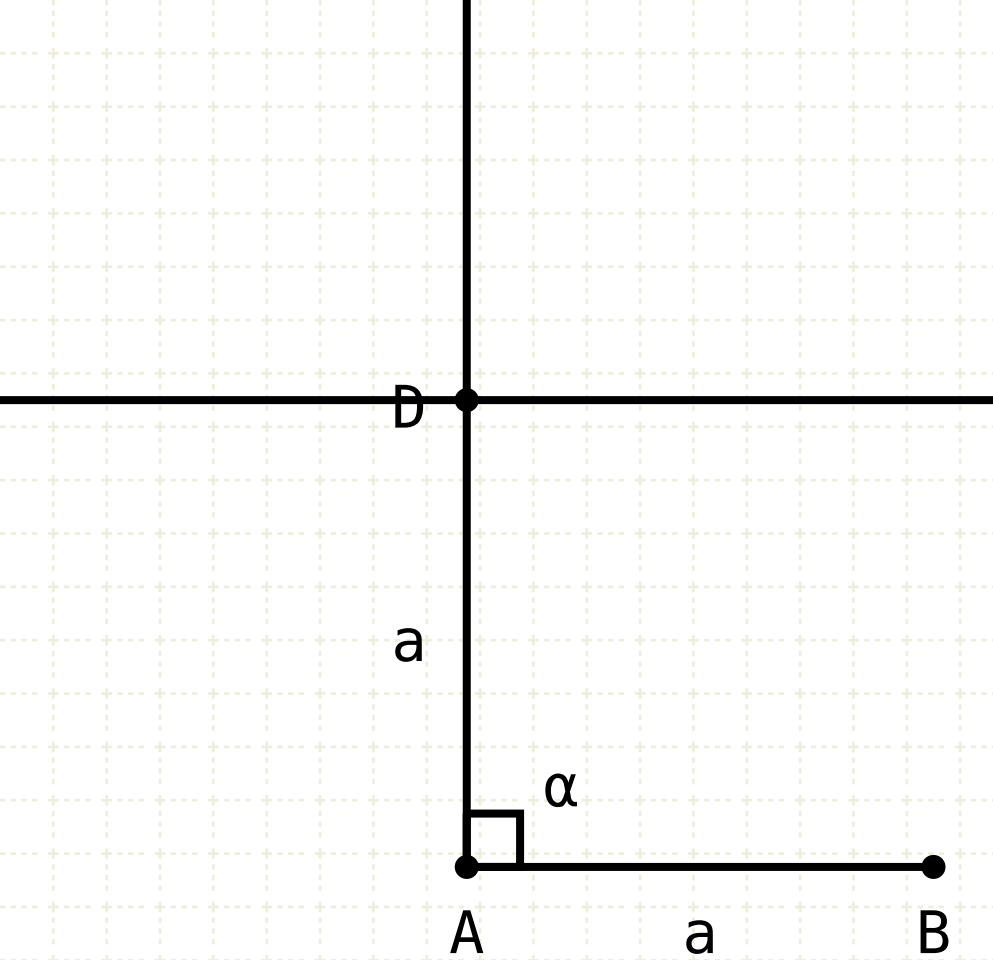
Draw a line AC perpendicular to AB, from point A (I·11)

Define point D such that AD equals AB (I·3)



Proposition 46 of Book I

To describe a square on a given straight line.



$$\alpha = \angle$$
$$AD = AB$$
$$AB \parallel DE$$

Construction

Start with a given line AB

Draw a line AC perpendicular to AB, from point A (I·11)

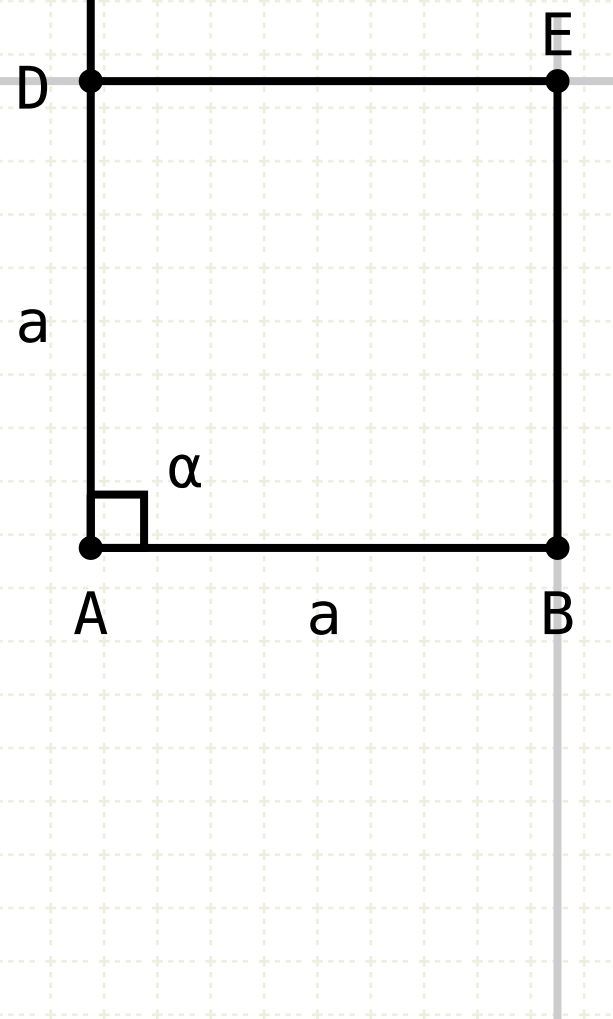
Define point D such that AD equals AB (I·3)

Draw a line DE through D parallel to line AB (I·31)



Proposition 46 of Book I

To describe a square on a given straight line.



$$\begin{aligned}\alpha &= \angle \\ AD &= AB \\ AB &\parallel DE \\ AD &\parallel BE\end{aligned}$$

Construction

Start with a given line AB

Draw a line AC perpendicular to AB, from point A (I·11)

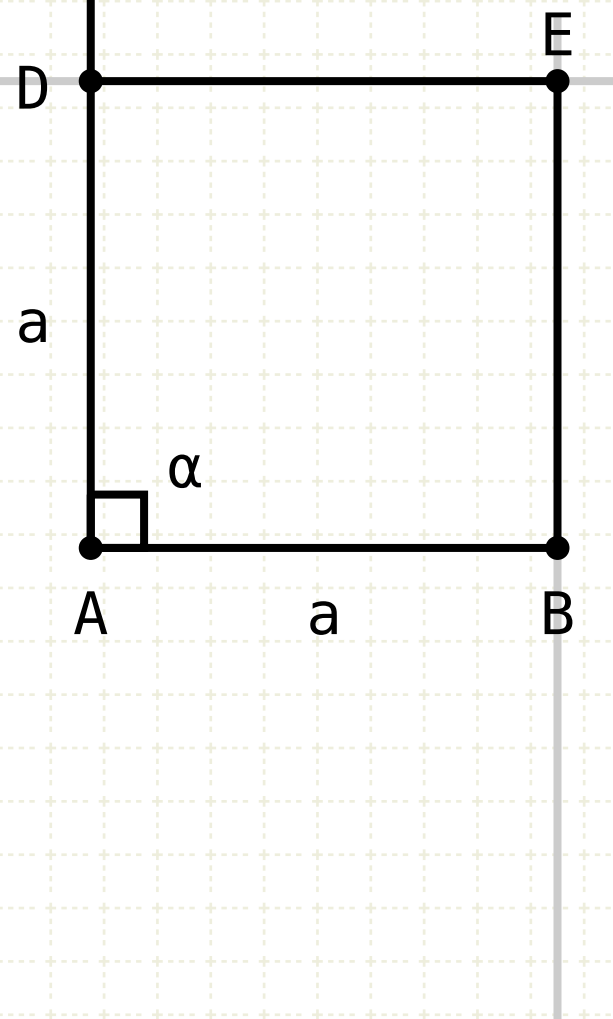
Define point D such that AD equals AB (I·3)

Draw a line DE through D parallel to line AB (I·31)

Draw a line through B parallel to line AD (I·31) intersecting the previous line at point E

Proposition 46 of Book I

To describe a square on a given straight line.



$$\begin{aligned}\alpha &= \angle \\ AD &= AB \\ AB &\parallel DE \\ AD &\parallel BE\end{aligned}$$

Construction

Start with a given line AB

Draw a line AC perpendicular to AB, from point A (I·11)

Define point D such that AD equals AB (I·3)

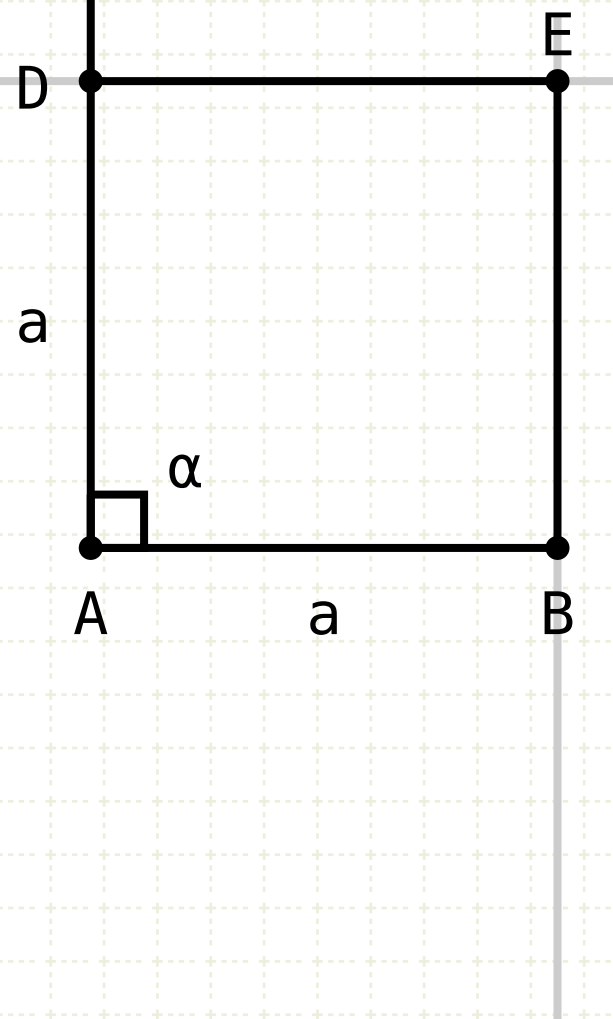
Draw a line DE through D parallel to line AB (I·31)

Draw a line through B parallel to line AD (I·31) intersecting the previous line at point E

ABED is a square

Proposition 46 of Book I

To describe a square on a given straight line.



$$\begin{aligned}\alpha &= \angle \\ AD &= AB \\ AB &\parallel DE \\ AD &\parallel BE\end{aligned}$$

Construction

Start with a given line AB

Draw a line AC perpendicular to AB, from point A (I·11)

Define point D such that AD equals AB (I·3)

Draw a line DE through D parallel to line AB (I·31)

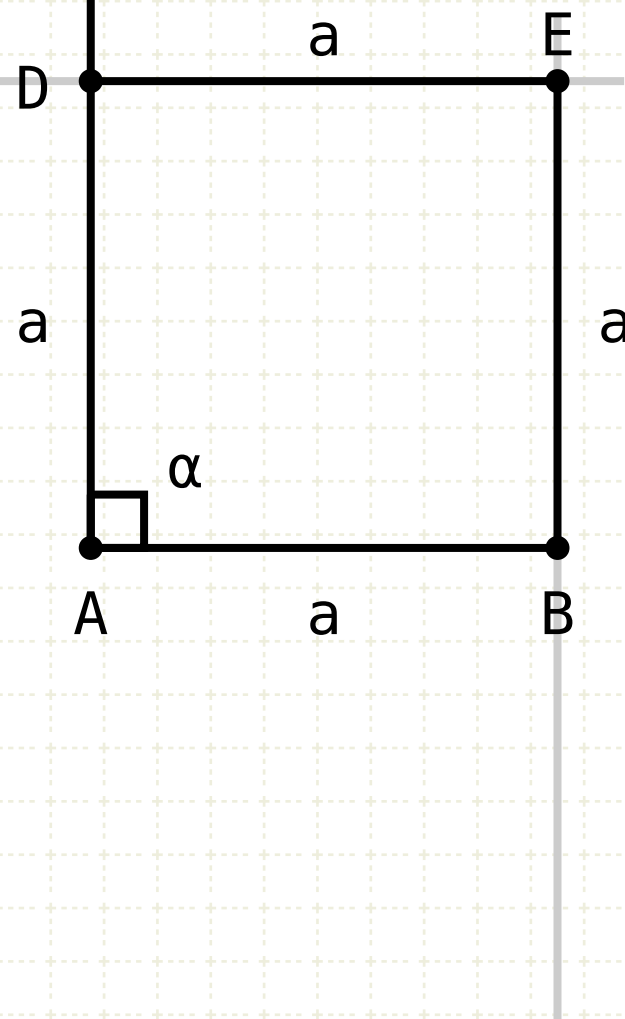
Draw a line through B parallel to line AD (I·31) intersecting the previous line at point E

ABED is a square

Proof:

Proposition 46 of Book I

To describe a square on a given straight line.



$$\begin{aligned}\alpha &= L \\ AD &= AB \\ AB &\parallel DE \\ AD &\parallel BE\end{aligned}$$

$$\begin{aligned}AB &= DE \\ AD &= BE\end{aligned}$$

Construction

Start with a given line AB

Draw a line AC perpendicular to AB, from point A (I·11)

Define point D such that AD equals AB (I·3)

Draw a line DE through D parallel to line AB (I·31)

Draw a line through B parallel to line AD (I·31) intersecting the previous line at point E

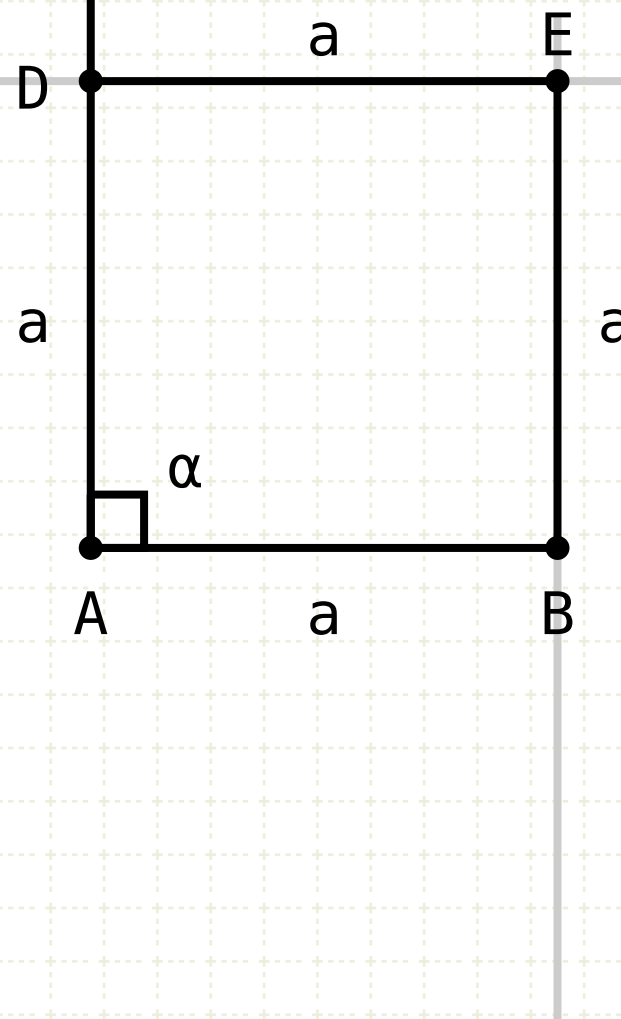
ABED is a square

Proof:

Since AB is parallel to DE, and AD is parallel to BE, ABED is a parallelogram, and AB equals DE and AD equals BE (I·34)

Proposition 46 of Book I

To describe a square on a given straight line.



$$\alpha = \angle$$
$$AD = AB$$
$$AB \parallel DE$$
$$AD \parallel BE$$

$$AB = DE$$
$$AD = BE$$

Construction

Start with a given line AB

Draw a line AC perpendicular to AB, from point A (I·11)

Define point D such that AD equals AB (I·3)

Draw a line DE through D parallel to line AB (I·31)

Draw a line through B parallel to line AD (I·31) intersecting the previous line at point E

ABED is a square

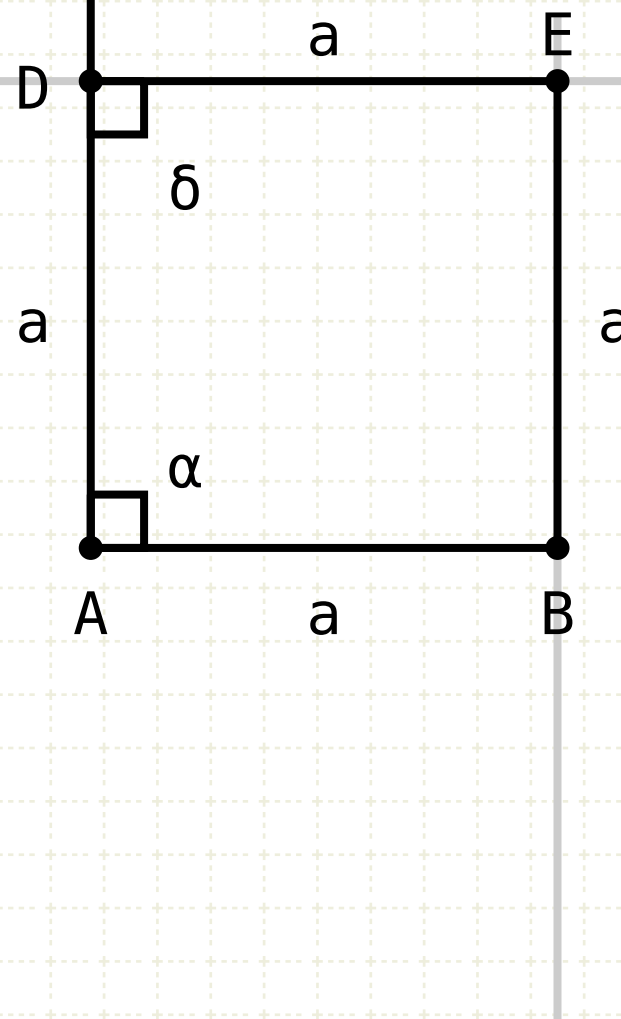
Proof:

Since AB is parallel to DE, and AD is parallel to BE, ABED is a parallelogram, and AB equals DE and AD equals BE (I·34)

But, AD equals AB, thus all the sides of the parallelogram are equal, so ABED is an equilateral

Proposition 46 of Book I

To describe a square on a given straight line.



$$\begin{aligned}\alpha &= L \\ AD &= AB \\ AB &\parallel DE \\ AD &\parallel BE\end{aligned}$$

$$\begin{aligned}AB &= DE \\ AD &= BE \\ \alpha + \delta &= 2L \\ \delta &= L\end{aligned}$$

Construction

Start with a given line AB

Draw a line AC perpendicular to AB, from point A (I·11)

Define point D such that AD equals AB (I·3)

Draw a line DE through D parallel to line AB (I·31)

Draw a line through B parallel to line AD (I·31) intersecting the previous line at point E

ABED is a square

Proof:

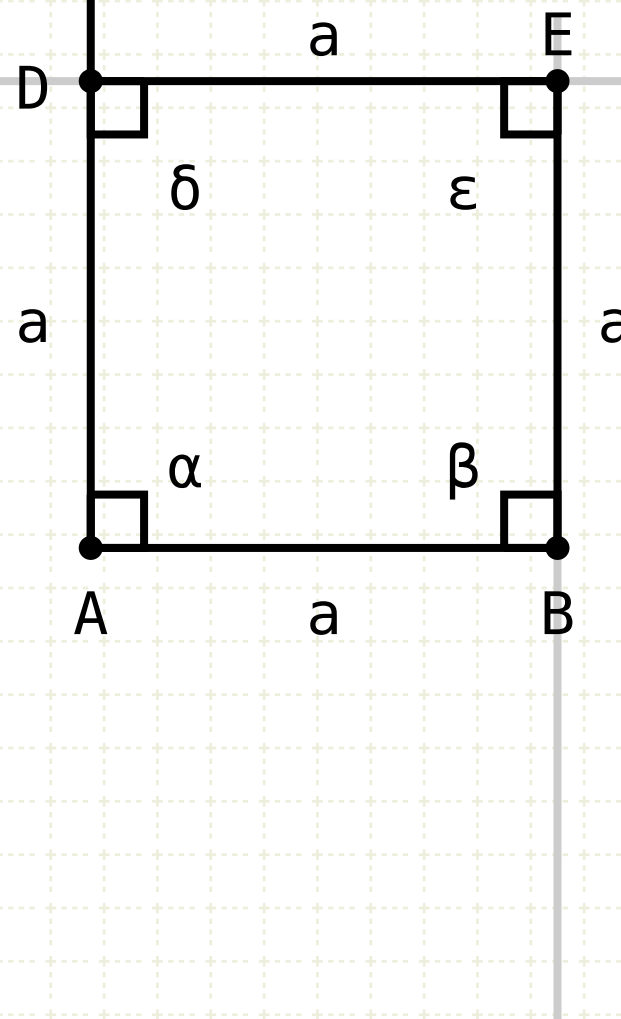
Since AB is parallel to DE, and AD is parallel to BE, ABED is a parallelogram, and AB equals DE and AD equals BE (I·34)

But, AD equals AB, thus all the sides of the parallelogram are equal, so ABED is an equilateral

Line DA intersects two parallel lines, thus the angles DAB and ADE equal two right angles (I·29)

Proposition 46 of Book I

To describe a square on a given straight line.



$$\alpha = \angle$$

$$AD = AB$$

$$AB \parallel DE$$

$$AD \parallel BE$$

$$AB = DE$$

$$AD = BE$$

$$\alpha + \delta = 2\angle$$

$$\delta = \angle$$

$$\epsilon = \angle$$

$$\beta = \angle$$

Construction

Start with a given line AB

Draw a line AC perpendicular to AB, from point A (I·11)

Define point D such that AD equals AB (I·3)

Draw a line DE through D parallel to line AB (I·31)

Draw a line through B parallel to line AD (I·31) intersecting the previous line at point E

ABED is a square

Proof:

Since AB is parallel to DE, and AD is parallel to BE, ABED is a parallelogram, and AB equals DE and AD equals BE (I·34)

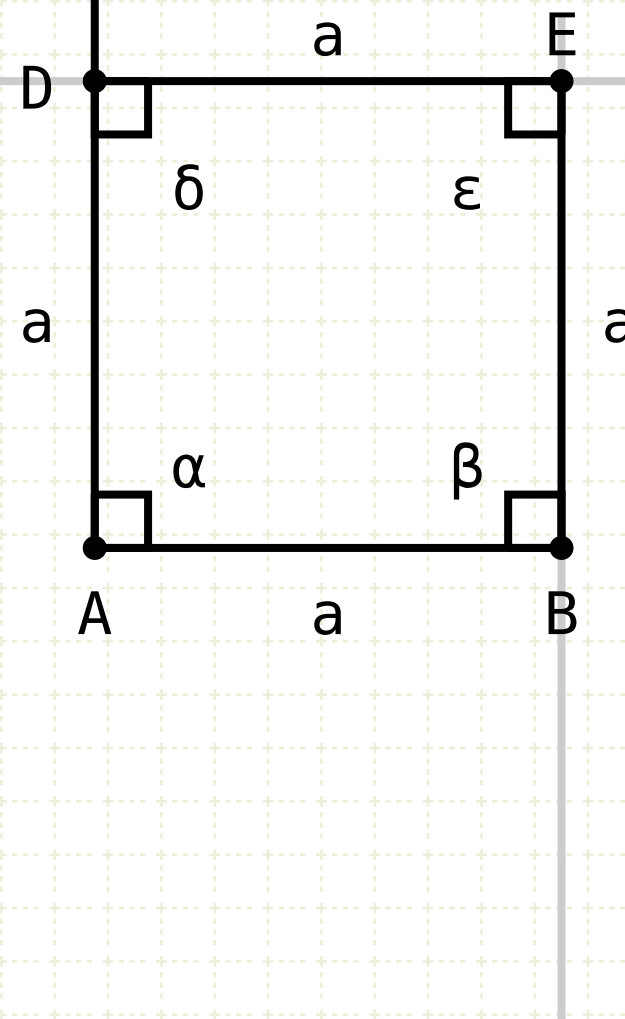
But, AD equals AB, thus all the sides of the parallelogram are equal, so ABED is an equilateral

Line DA intersects two parallel lines, thus the angles DAB and ADE equal two right angles (I·29)

In a parallelogram, the opposite angles are equal to one another, so angles DEB and EBA are also right angles (I·34)

Proposition 46 of Book I

To describe a square on a given straight line.



$$\begin{aligned}\alpha &= \angle \\ AD &= AB \\ AB &\parallel DE \\ AD &\parallel BE\end{aligned}$$

$$\begin{aligned}AB &= DE \\ AD &= BE \\ \alpha + \delta &= 2\angle \\ \delta &= \angle \\ \epsilon &= \angle \\ \beta &= \angle\end{aligned}$$

Construction

Start with a given line AB

Draw a line AC perpendicular to AB, from point A (I·11)

Define point D such that AD equals AB (I·3)

Draw a line DE through D parallel to line AB (I·31)

Draw a line through B parallel to line AD (I·31) intersecting the previous line at point E

ABED is a square

Proof:

Since AB is parallel to DE, and AD is parallel to BE, ABED is a parallelogram, and AB equals DE and AD equals BE (I·34)

But, AD equals AB, thus all the sides of the parallelogram are equal, so ABED is an equilateral

Line DA intersects two parallel lines, thus the angles DAB and ADE equal two right angles (I·29)

In a parallelogram, the opposite angles are equal to one another, so angles DEB and EBA are also right angles (I·34)

An equilateral parallelogram with all right angles is a square



Youtube Videos

<https://www.youtube.com/c/SandyBultena>

Copyright © 2019 by Sandy Bultena.



Except where otherwise noted, this work is licensed under
<http://creativecommons.org/licenses/by-nc/3.0>