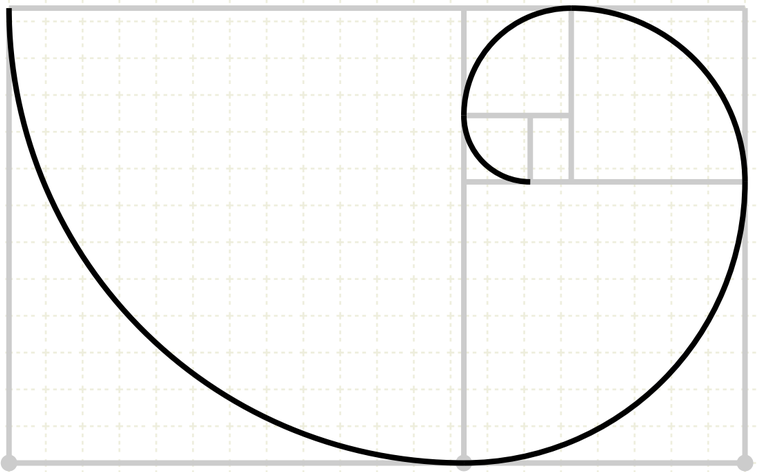
Euclid's Elements

Book VI



One can state, without exaggeration, that the observation of and the search for similarities and differences are the basis of all human knowledge.

Alfred Nobel

Table of Contents, Chapter 6

1 If the height of two triangles are equal, then the ratio of the areas is equal to the ratio of the bases

2 If a line cuts a triangle, parallel to its base, it will cut the sides of the triangle proportionally

3 If an angle of a triangle is bisected and the straight line cutting the angle also cuts the base, the segments of the base will have the same ratio as the remaining sides of the triangle

4 If two triangles have equal angles, then the sides opposite the equal angles are proportional, as well, the sides of the triangles on either side of the equal angles are also proportional

5 It two triangles have proportional sides, the triangles will be equiangular

6 If two triangles have one angle equal to one angle and the sides about the equal angles are proportional, then the triangles will be equiangular

7 If two triangles have one angle equal to one angle, and the sides about other angles are proportional, and the remaining angles either both less or both not less than a right angle, then triangles will be equiangular

8 If in a right-angled triangle a perpendicular be drawn from the right angle to the base, the triangles adjoining the perpendicular are similar both to the whole and to one another

9 From a given straight line to cut off a given fraction

10 To cut a given uncut straight line similarly to a given cut straight line

11 To two given straight lines to find a third proportional

12 To three given straight lines to find a fourth proportional

13 To two given straight lines to find a mean proportional

14 In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and vice versa

15 In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and vice versa

16 If four straight lines are proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means, and vice versa

17 If three straight lines are proportional, the rectangle contained by the extremes is equal to the square on the mean; and vice versa

18 On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure

19 Similar triangles are to one another in the duplicate ratio of the corresponding sides

Table of Contents, Chapter 3

20 Similar polygons are divided into the same number of similar triangles, which have the same ratio as the wholes, and the polygons have duplicate ratios to their corresponding sides

21 Figures which are are similar to the same rectilineal figure are also similar to one another

22 If four straight lines are proportional, similar rectilineal figures will also be proportional; and vice versa

23 Equiangular parallelograms have to one another the ratio compounded of the ratios of their sides

24 In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another

25 To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure

26 If from a parallelogram a similar parallelogram with a common angle is subtracted, it is about the same diameter as the original

27 Of all the parallelograms applied to the same straight line and deficient by parallelogrammic figures similar to a parallelogram drawn on half the said line, the largest will be one that is drawn on half of the straight line and is similar to the defect

28 To a given straight line, apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one

29 To a given straight line, apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one

30 To cut a finite straight line in extreme ratio

31 In right-angled triangles the figure on the side subtending the right angle is equal to the similar and similarly described figures on the sides containing the right angle

Proposition 30 of Book VI

To cut a finite straight line in extreme ratio

Proposition 30 of Book VI

To cut a finite straight line in extreme ratio

In other words

Given a straight line AB

Construct a point E such that

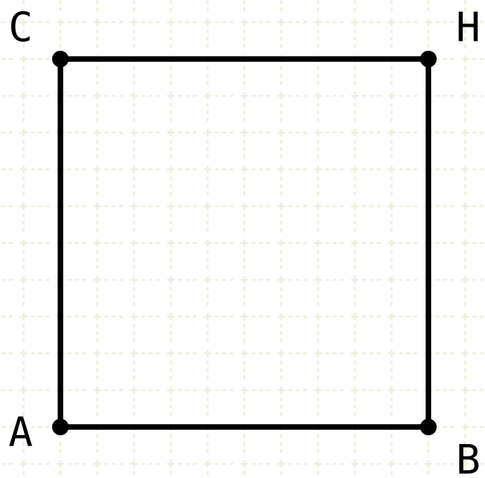
AB is to AE as AE is to EB and AE is greater than EB

```
AB:AE = AE:EB
AE > EB
```

Proposition 30 of Book VI

To cut a finite straight line in extreme ratio

Construction

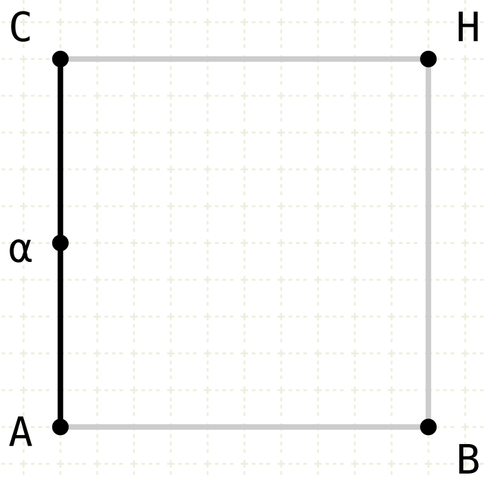


Proposition 30 of Book VI

To cut a finite straight line in extreme ratio

Construction

Draw a square on AB



Proposition 30 of Book VI

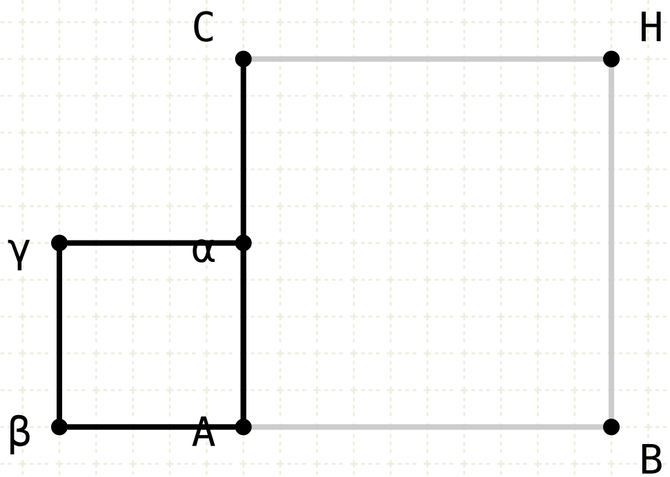
To cut a finite straight line in extreme ratio

Construction

Draw a square on AB

On the line AC, draw a parallelogram that is equal to the square BC, and whose excess (the part that is drawn past the line AC) is similar to the square BC (VI·29)

* Bisect the line AC at point α



Proposition 30 of Book VI

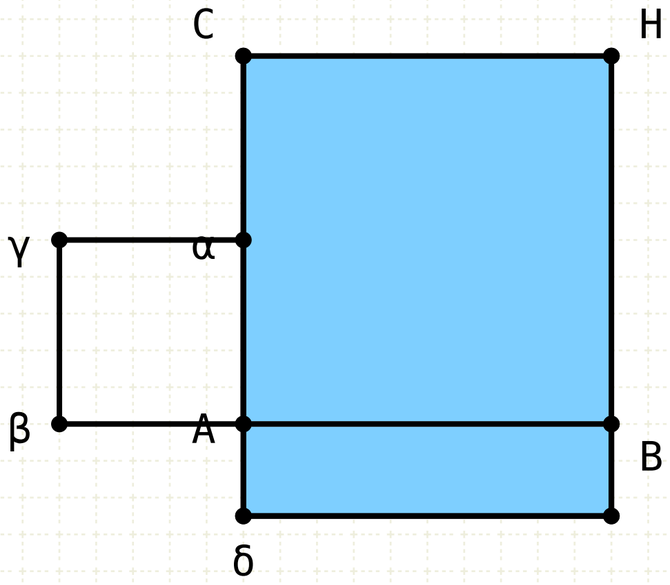
To cut a finite straight line in extreme ratio

Construction

Draw a square on AB

On the line AC, draw a parallelogram that is equal to the square BC, and whose excess (the part that is drawn past the line AC) is similar to the square BC (VI·29)

* Bisect the line AC at point α
* Create a square on the line Aα



Proposition 30 of Book VI

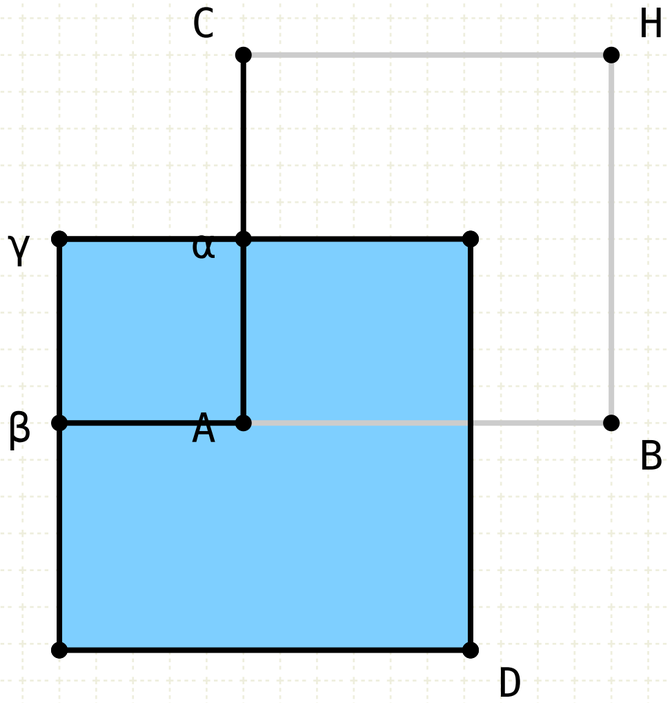
To cut a finite straight line in extreme ratio

Construction

Draw a square on AB

On the line AC, draw a parallelogram that is equal to the square BC, and whose excess (the part that is drawn past the line AC) is similar to the square BC (VI·29)

* Bisect the line AC at point α
* Create a square on the line Aα
* Let a parallelogram Hδ be constructed such that is equal to the area of Aγ plus the area of BC



Proposition 30 of Book VI

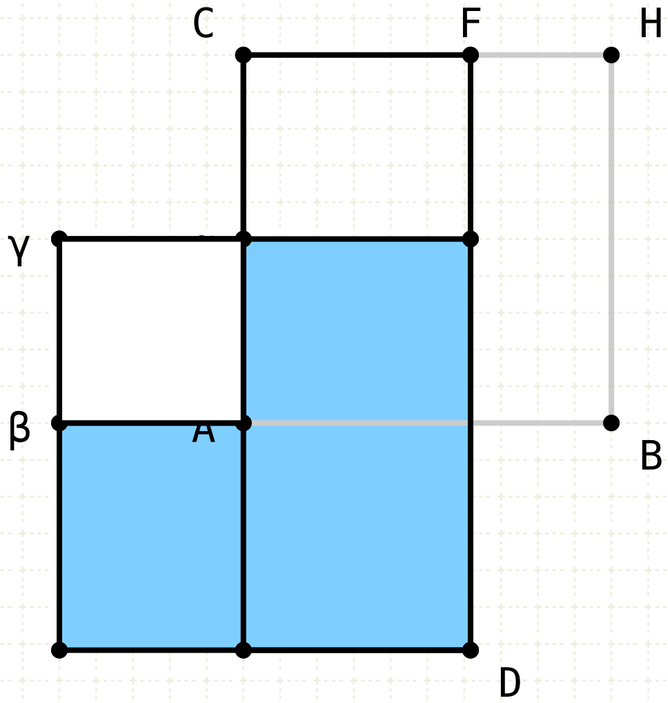
To cut a finite straight line in extreme ratio

Construction

Draw a square on AB

On the line AC, draw a parallelogram that is equal to the square BC, and whose excess (the part that is drawn past the line AC) is similar to the square BC (VI·29)

* Bisect the line AC at point α
* Create a square on the line Aα
* Let a parallelogram Hδ be constructed such that is equal to the area of Aγ plus the area of BC
* Copy this parallelogram to a square and move it such that the top left corner coincides with gamma



Proposition 30 of Book VI

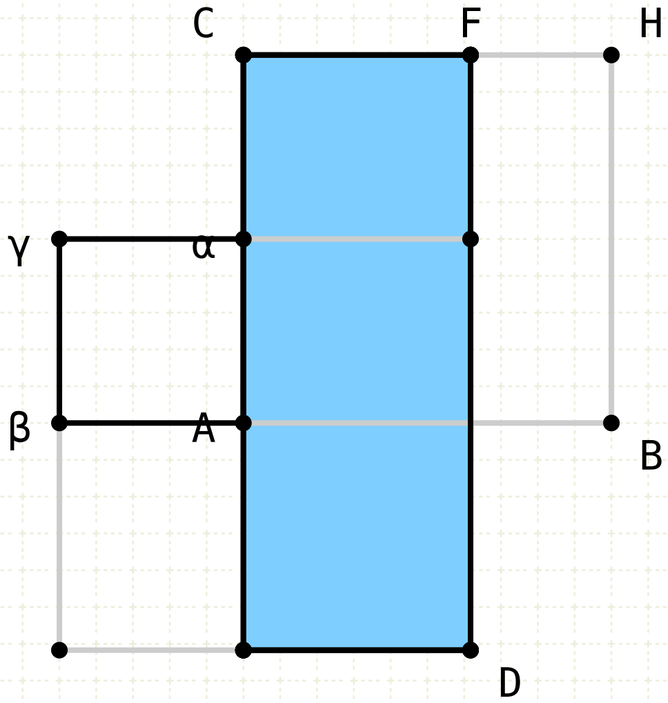
To cut a finite straight line in extreme ratio

Construction

Draw a square on AB

On the line AC, draw a parallelogram that is equal to the square BC, and whose excess (the part that is drawn past the line AC) is similar to the square BC (VI·29)

* Bisect the line AC at point α
* Create a square on the line Aα
* Let a parallelogram Hδ be constructed such that is equal to the area of Aγ plus the area of BC
* Copy this parallelogram to a square and move it such that the top left corner coincides with gamma
* The resulting gnomon is equal to the square BC



Proposition 30 of Book VI

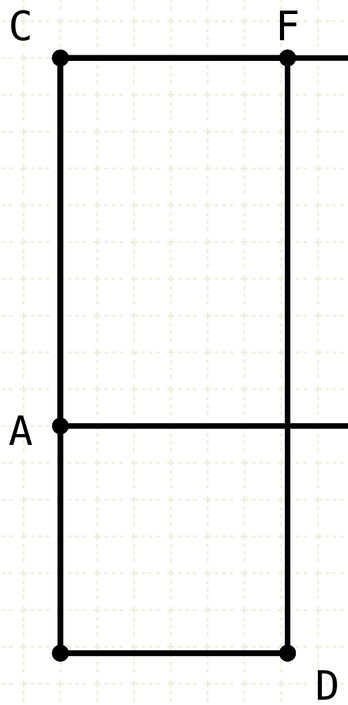
To cut a finite straight line in extreme ratio

Construction

Draw a square on AB

On the line AC, draw a parallelogram that is equal to the square BC, and whose excess (the part that is drawn past the line AC) is similar to the square BC (VI·29)

* Bisect the line AC at point α
* Create a square on the line Aα
* Let a parallelogram Hδ be constructed such that is equal to the area of Aγ plus the area of BC
* Copy this parallelogram to a square and move it such that the top left corner coincides with gamma
* The resulting gnomon is equal to the square BC
* And therefore the area CD is equal to the area BC



CB = CD

AD = □

Proposition 30 of Book VI

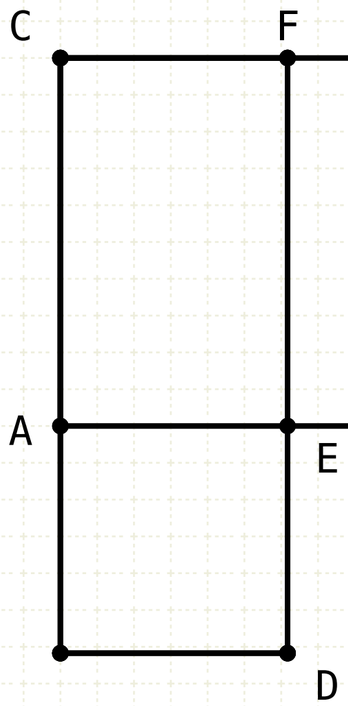
To cut a finite straight line in extreme ratio

Construction

Draw a square on AB

On the line AC, draw a parallelogram that is equal to the square BC, and whose excess (the part that is drawn past the line AC) is similar to the square BC (VI·29)

* Bisect the line AC at point α
* Create a square on the line Aα
* Let a parallelogram Hδ be constructed such that is equal to the area of Aγ plus the area of BC
* Copy this parallelogram to a square and move it such that the top left corner coincides with gamma
* The resulting gnomon is equal to the square BC
* And therefore the area CD is equal to the area BC



CB = CD

AD = □

AB:AE = AE:EB

Proposition 30 of Book VI

To cut a finite straight line in extreme ratio

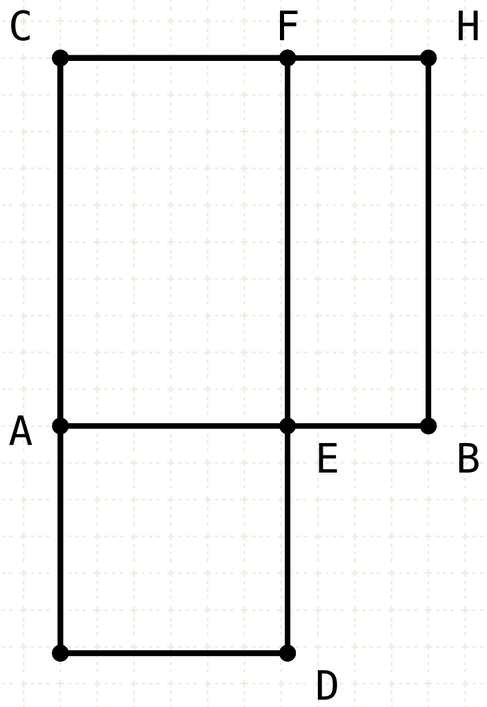
Construction

Draw a square on AB

On the line AC, draw a parallelogram that is equal to the square BC, and whose excess (the part that is drawn past the line AC) is similar to the square BC (VI·29)

* Bisect the line AC at point α
* Create a square on the line Aα
* Let a parallelogram Hδ be constructed such that is equal to the area of Aγ plus the area of BC
* Copy this parallelogram to a square and move it such that the top left corner coincides with gamma
* The resulting gnomon is equal to the square BC
* And therefore the area CD is equal to the area BC

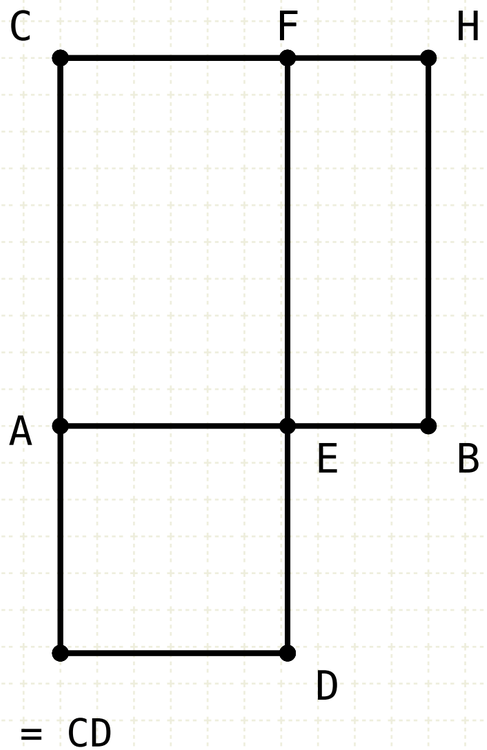
The intersection E of the polygon and line AB cuts the line into the extreme and mean ratio



Proposition 30 of Book VI

To cut a finite straight line in extreme ratio

Proof



Proposition 30 of Book VI

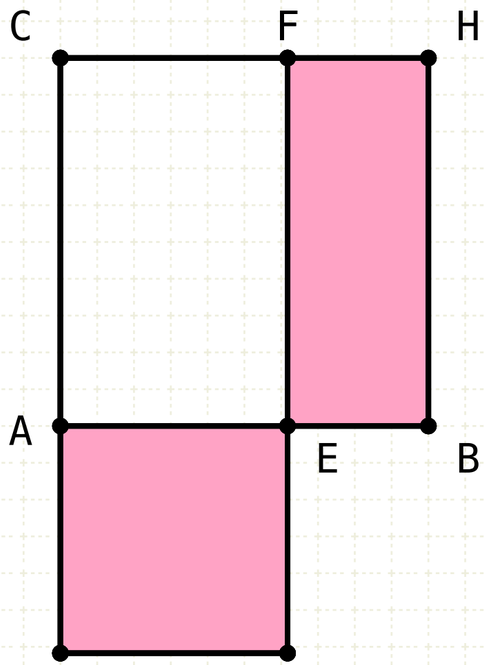
To cut a finite straight line in extreme ratio

Proof

BC is equal to CD (by construction)

CB = CD

AD = □



Proposition 30 of Book VI

To cut a finite straight line in extreme ratio

Proof

BC is equal to CD (by construction)

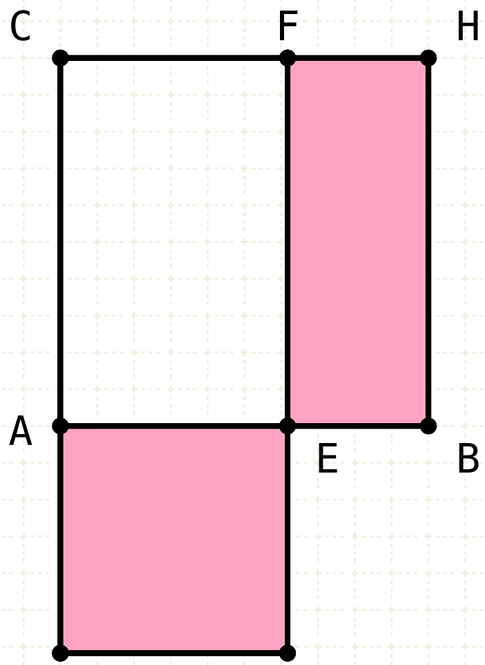
Subtract CE from BC and CD, and the remainders FB and AD are equal

CB = CD

AD = □

BC - CE = CD - CE

AD = FB



Proposition 30 of Book VI

To cut a finite straight line in extreme ratio

Proof

BC is equal to CD (by construction)

Subtract CE from BC and CD, and the remainders FB and AD are equal

AD and FB are equiangular, and equal, therefore the sides about the equal angles are reciprocally proportional (VI·14)

```
CB = CD
AD = □
BC - CE = CD - CE
AD = FB
FE:ED = AE:EB
```

Proposition 30 of Book VI

To cut a finite straight line in extreme ratio

Proof

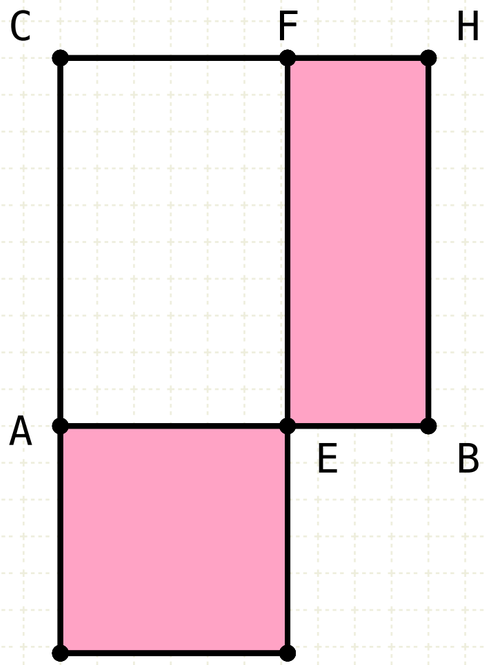
BC is equal to CD (by construction)

Subtract CE from BC and CD, and the remainders FB and AD are equal

AD and FB are equiangular, and equal, therefore the sides about the equal angles are reciprocally proportional (VI·14)

AB is a square, so therefore FE is equal to AB …

```
CB = CD
AD = □
BC - CE = CD - CE
AD = FB
FE:ED = AE:EB
AB:ED = AE:EB
```



Proposition 30 of Book VI

To cut a finite straight line in extreme ratio

Proof

BC is equal to CD (by construction)

Subtract CE from BC and CD, and the remainders FB and AD are equal

AD and FB are equiangular, and equal, therefore the sides about the equal angles are reciprocally proportional (VI·14)

AB is a square, so therefore FE is equal to AB …

… and AD is a square, so therefore ED is equal to AE

Therefore AB is to AE as AE is to EB

D

CB = CD

AD = □

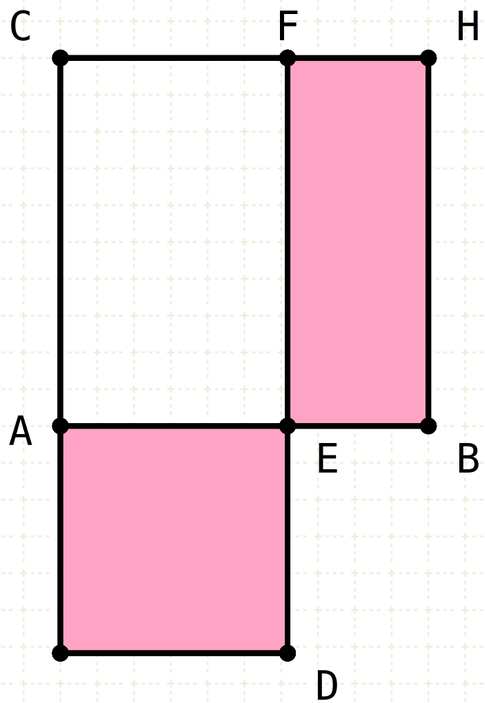
BC - CE = CD - CE

AD = FB

FE:ED = AE:EB

AB:ED = AE:EB

AB:AE = AE:EB



Proposition 30 of Book VI

To cut a finite straight line in extreme ratio

CB = CD

AD = □

BC - CE = CD - CE

AD = FB

FE:ED = AE:EB

AB:ED = AE:EB

AB:AE = AE:EB

AB > AE ∴ AE > EB

Proof

BC is equal to CD (by construction)

Subtract CE from BC and CD, and the remainders FB and AD are equal

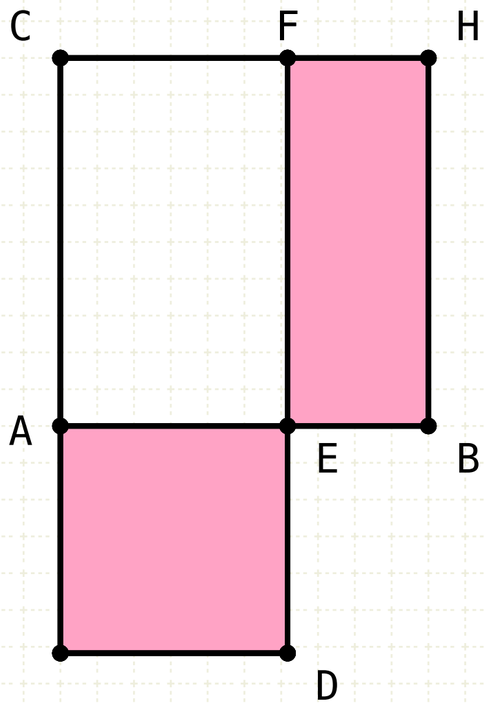
AD and FB are equiangular, and equal, therefore the sides about the equal angles are reciprocally proportional (VI·14)

AB is a square, so therefore FE is equal to AB …

… and AD is a square, so therefore ED is equal to AE

Therefore AB is to AE as AE is to EB

AB is greater than AE, therefore AE is greater than EB



Proposition 30 of Book VI

To cut a finite straight line in extreme ratio

CB = CD

AD = □

BC - CE = CD - CE

AD = FB

FE:ED = AE:EB

AB:ED = AE:EB

AB:AE = AE:EB

AB > AE ∴ AE > EB

Proof

BC is equal to CD (by construction)

Subtract CE from BC and CD, and the remainders FB and AD are equal

AD and FB are equiangular, and equal, therefore the sides about the equal angles are reciprocally proportional (VI·14)

AB is a square, so therefore FE is equal to AB …

… and AD is a square, so therefore ED is equal to AE

Therefore AB is to AE as AE is to EB

AB is greater than AE, therefore AE is greater than EB

Therefore AB has been cut in extreme and mean ratio at E, where AE is the larger segment

Youtube Videos

https://www.youtube.com/c/SandyBultena