

Euclid's Elements

Book VI

One can state, without exaggeration, that the observation of and the search for similarities and differences are the basis of all human knowledge.

Alfred Nobel



Table of Contents, Chapter 6

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| 1 | If the height of two triangles are equal, then the ratio of the areas is equal to the ratio of the bases | 7 | If two triangles have one angle equal to one angle, and the sides about other angles are proportional, and the remaining angles either both less or both not less than a right angle, then triangles will be equiangular | 14 | In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and vice versa |
| 2 | If a line cuts a triangle, parallel to its base, it will cut the sides of the triangle proportionally | | | 15 | In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and vice versa |
| 3 | If an angle of a triangle is bisected and the straight line cutting the angle also cuts the base, the segments of the base will have the same ratio as the remaining sides of the triangle | 8 | If in a right-angled triangle a perpendicular be drawn from the right angle to the base, the triangles adjoining the perpendicular are similar both to the whole and to one another | 16 | If four straight lines are proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means, and vice versa |
| 4 | If two triangles have equal angles, then the sides opposite the equal angles are proportional, as well, the sides of the triangles on either side of the equal angles are also proportional | 9 | From a given straight line to cut off a given fraction | 17 | If three straight lines are proportional, the rectangle contained by the extremes is equal to the square on the mean; and vice versa |
| 5 | If two triangles have proportional sides, the triangles will be equiangular | 10 | To cut a given uncut straight line similarly to a given cut straight line | 18 | On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure |
| 6 | If two triangles have one angle equal to one angle and the sides about the equal angles are proportional, then the triangles will be equiangular | 11 | To two given straight lines to find a third proportional | 19 | Similar triangles are to one another in the duplicate ratio of the corresponding sides |
| | | 12 | To three given straight lines to find a fourth proportional | | |
| | | 13 | To two given straight lines to find a mean proportional | | |



Table of Contents, Chapter 3

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|----|--|----|---|----|---|
| 20 | Similar polygons are divided into the same number of similar triangles, which have the same ratio as the wholes, and the polygons have duplicate ratios to their corresponding sides | 26 | If from a parallelogram a similar parallelogram with a common angle is subtracted, it is about the same diameter as the original | 31 | In right-angled triangles the figure on the side subtending the right angle is equal to the similar and similarly described figures on the sides containing the right angle |
| 21 | Figures which are similar to the same rectilineal figure are also similar to one another | 27 | Of all the parallelograms applied to the same straight line and deficient by parallelogrammic figures similar to a parallelogram drawn on half the said line, the largest will be one that is drawn on half of the straight line and is similar to the defect | | |
| 22 | If four straight lines are proportional, similar rectilineal figures will also be proportional; and vice versa | 28 | To a given straight line, apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one | | |
| 23 | Equiangular parallelograms have to one another the ratio compounded of the ratios of their sides | 29 | To a given straight line, apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one | | |
| 24 | In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another | 30 | To cut a finite straight line in extreme ratio | | |
| 25 | To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure | | | | |



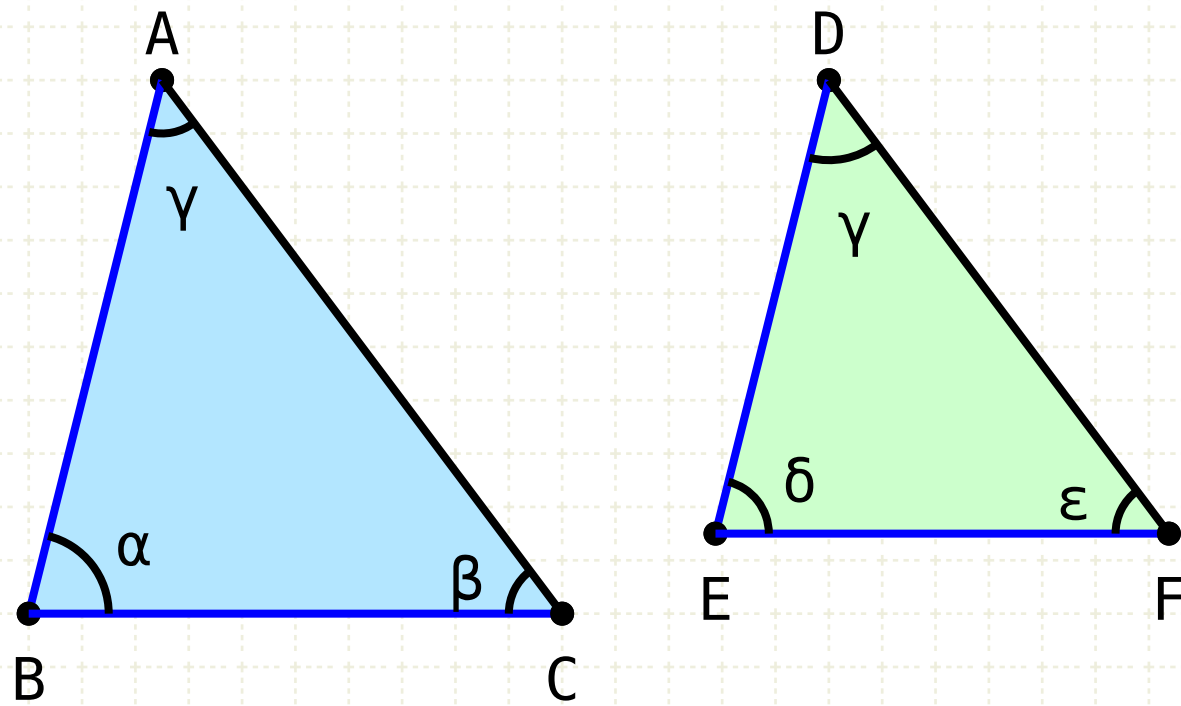
Proposition 7 of Book VI

If two triangles have one angle equal to one angle, the sides about other angles proportional, and the remaining angles either both less or both not less than a right angle, the triangles will be equiangular and will have those angles equal, the sides about which are proportional.



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$$AB:BC = DE:EF$$

$$\beta < \text{L}, \quad \varepsilon < \text{L}$$

$$\alpha = \delta, \quad \beta = \varepsilon$$

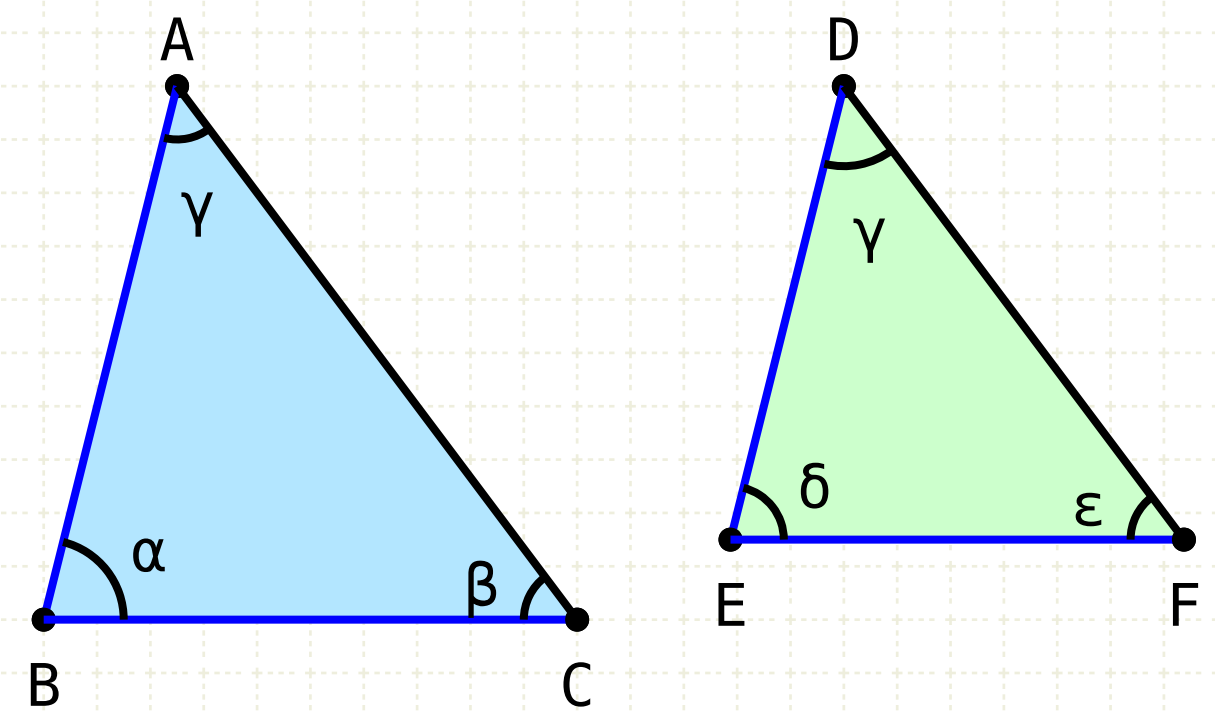
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If two triangles have one angle that is equal between them, AND the ratio of the sides of around a different angle are also equal, AND that the remaining angles are either both less than, or both greater than a right angle, then the two triangles will be equiangular

Let the angles at C and F be less than a right angle

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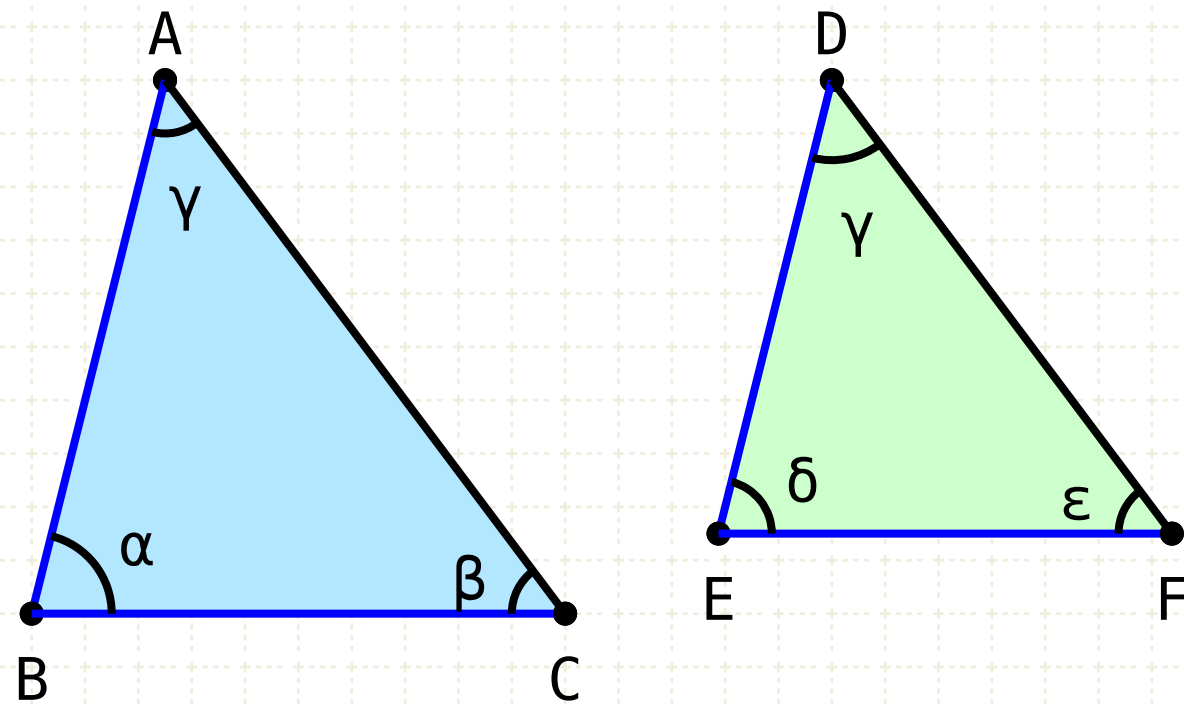
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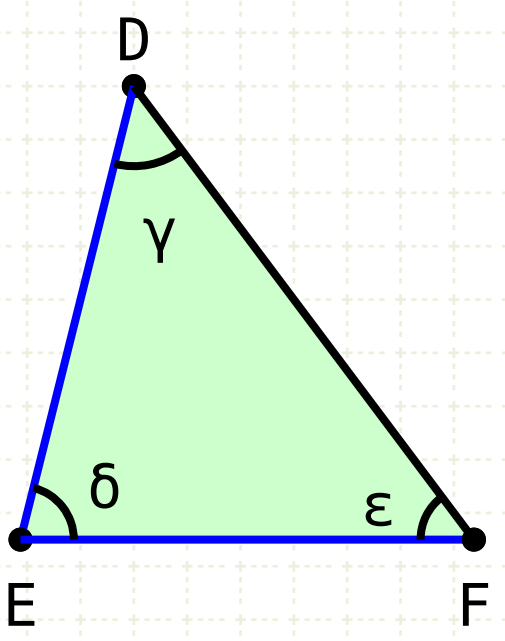
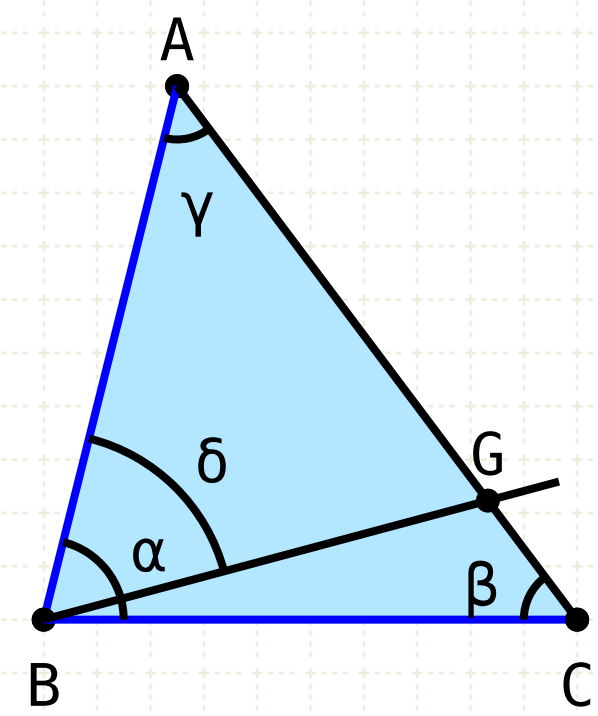
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Assume that the angle α is unequal to the angle δ , and that α is the greater

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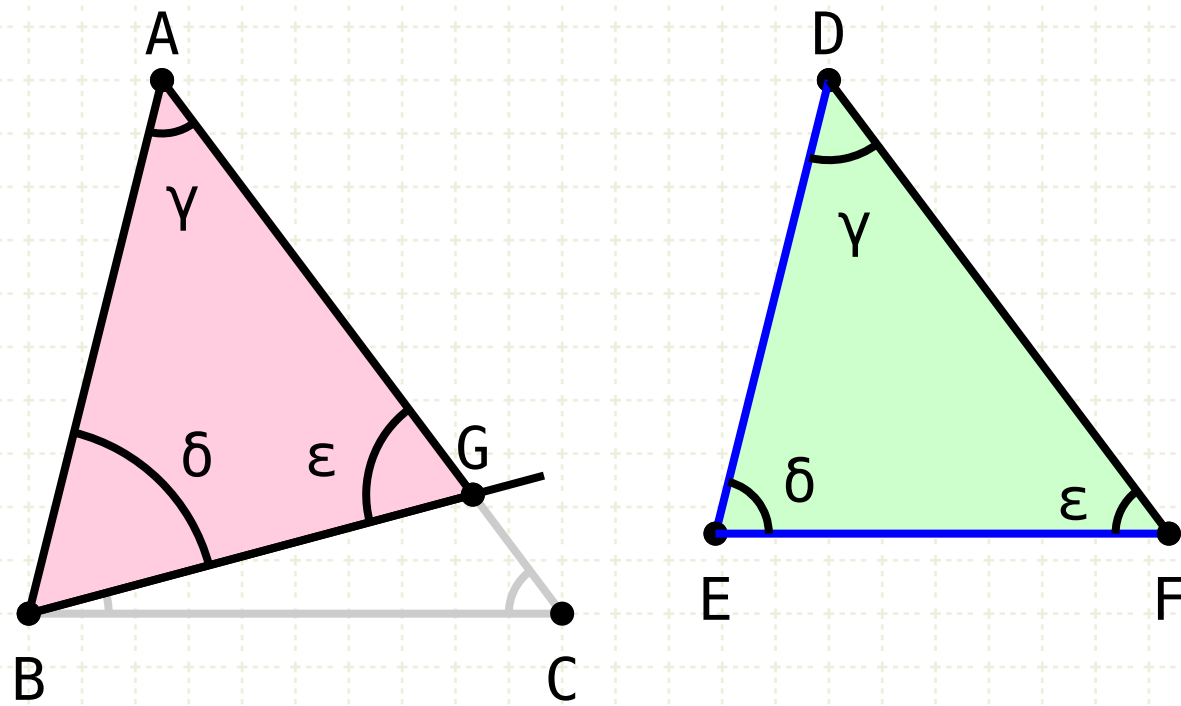
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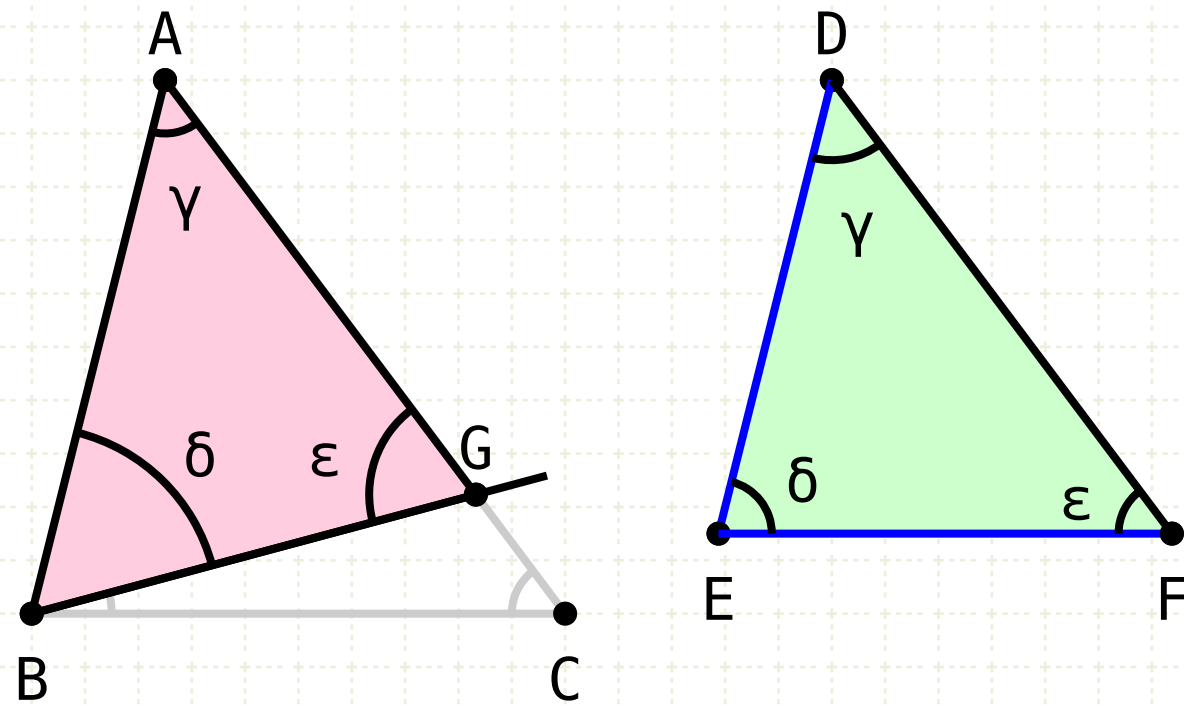
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Now, since triangles AGB and AFE have two angles equal, the third must also be equal (I-32)

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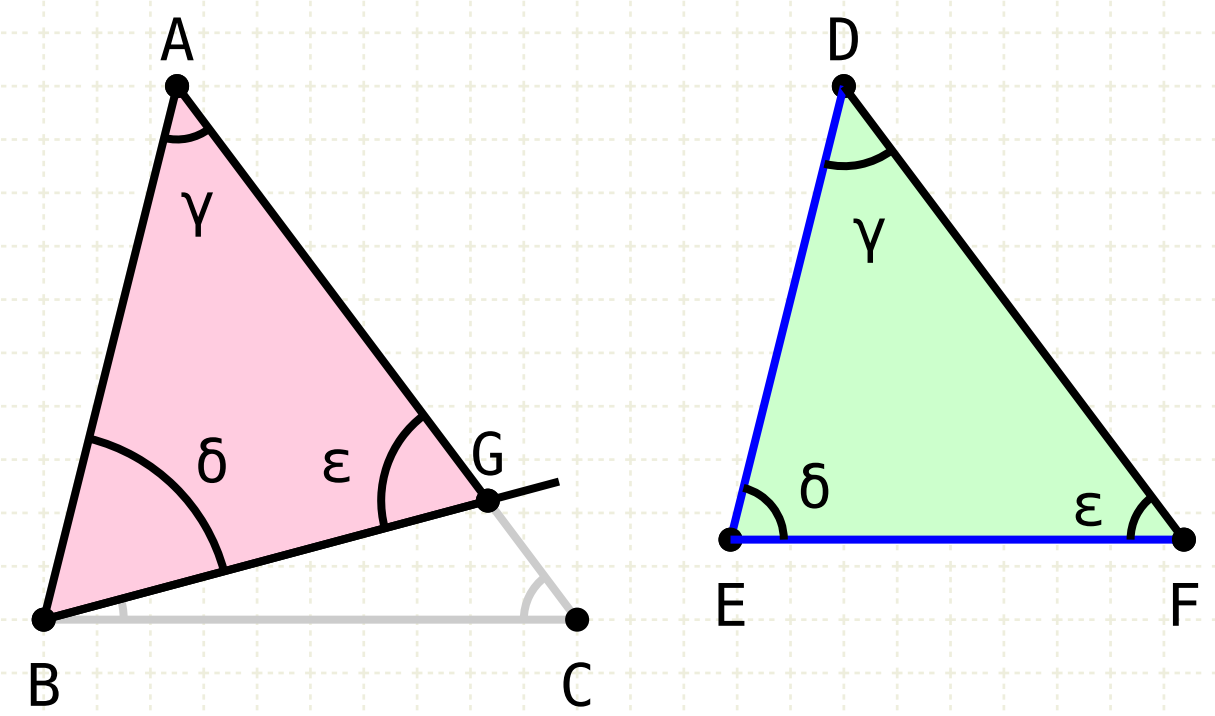
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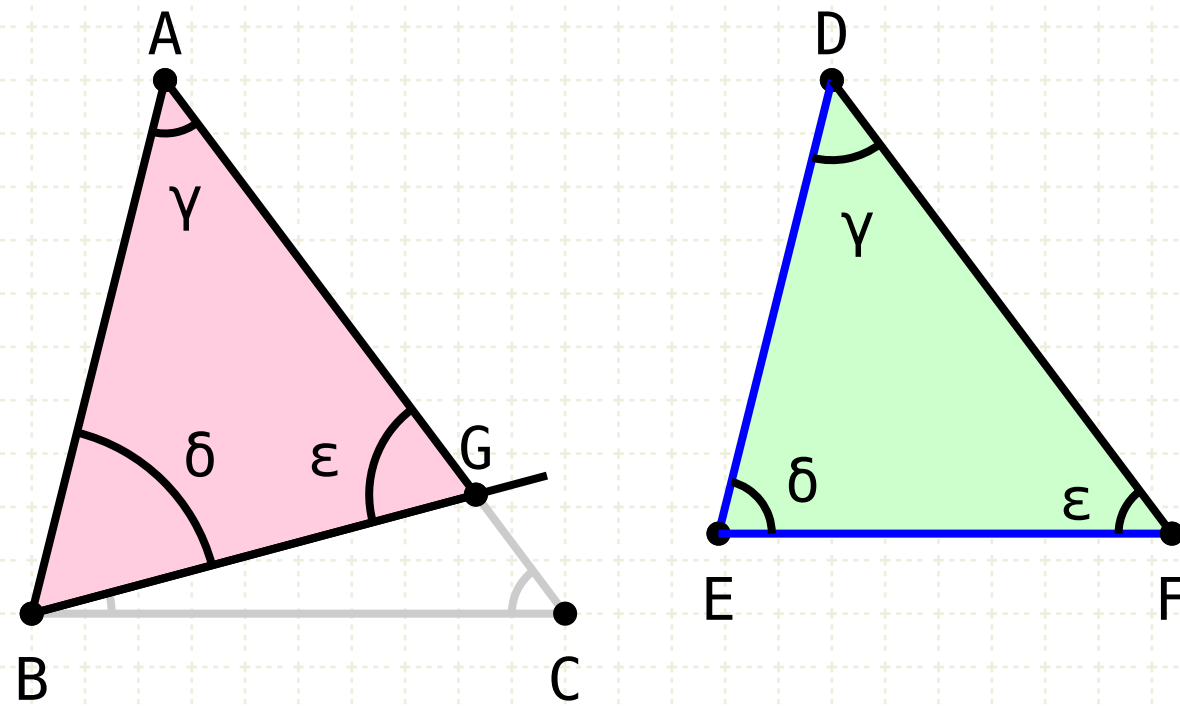
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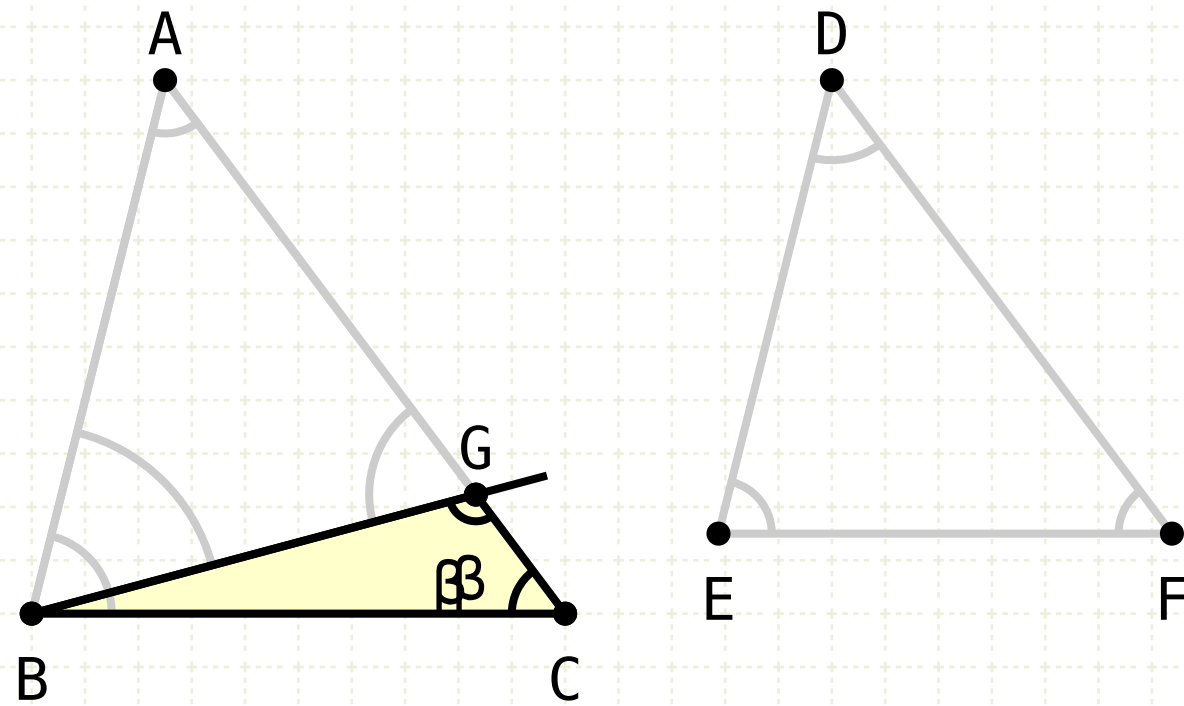
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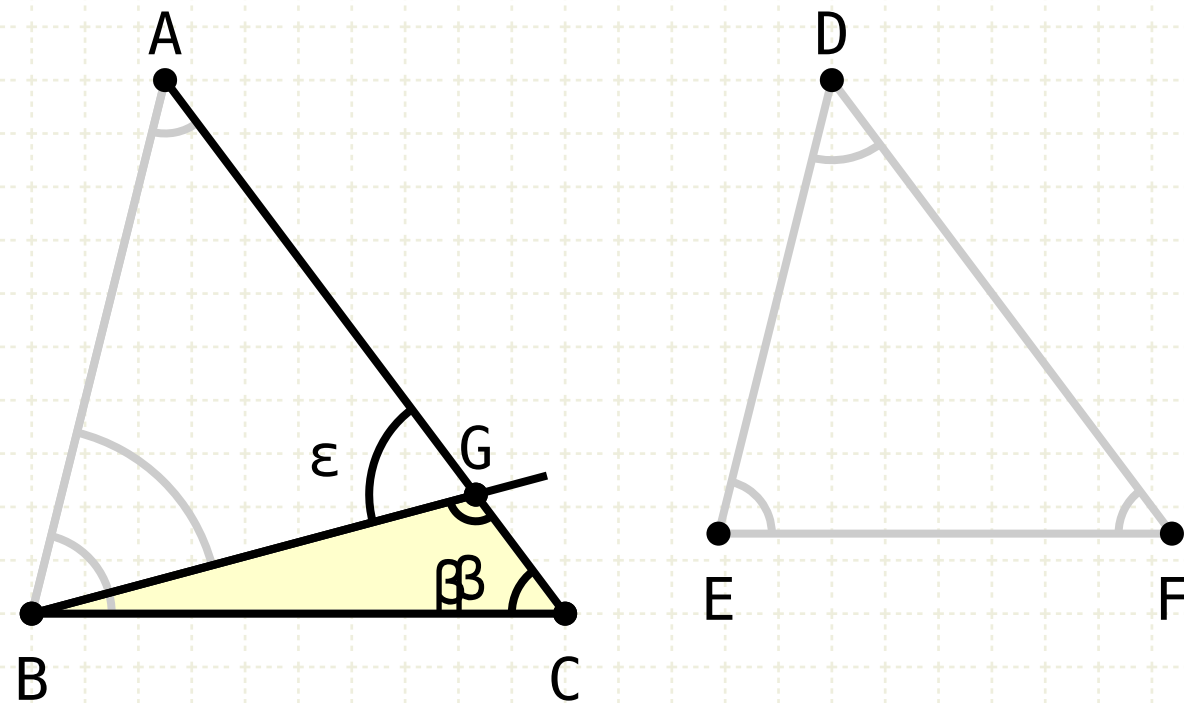
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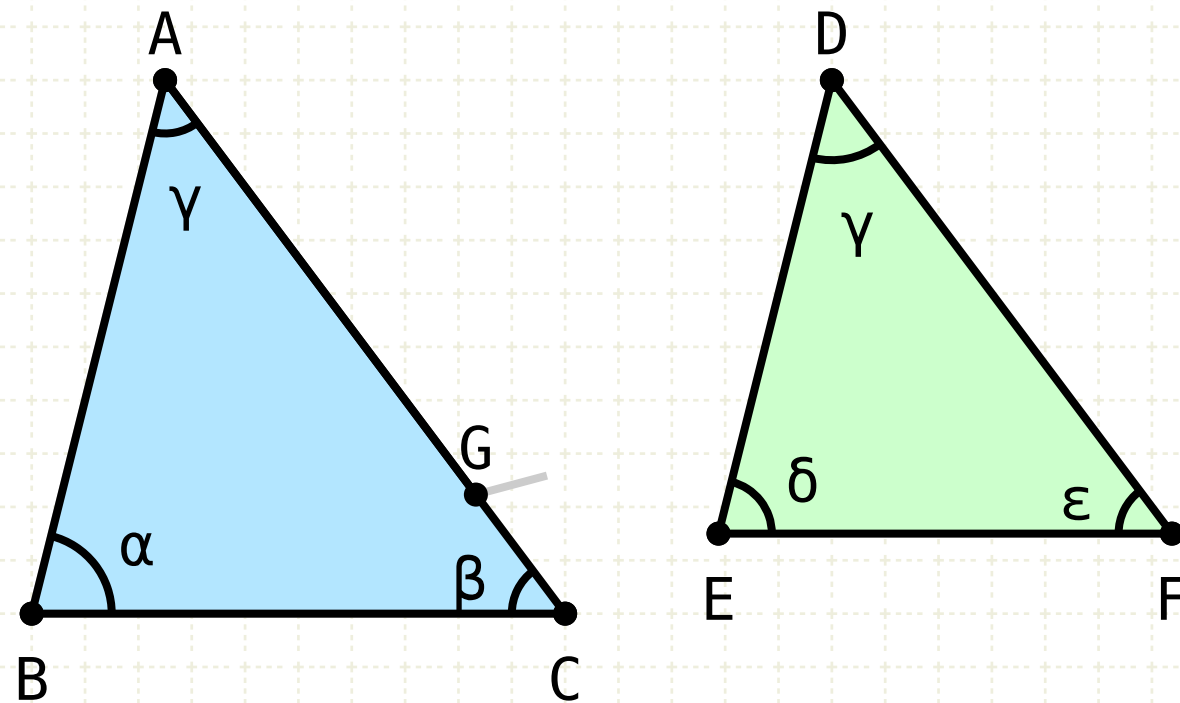
By definition, angle ACB is less than a right angle, so therefore so is angle BGC

Thus, the outside exterior angle must be greater than one right angle (the sum of the angles AGB and BGC equal two right angles) (I·13)



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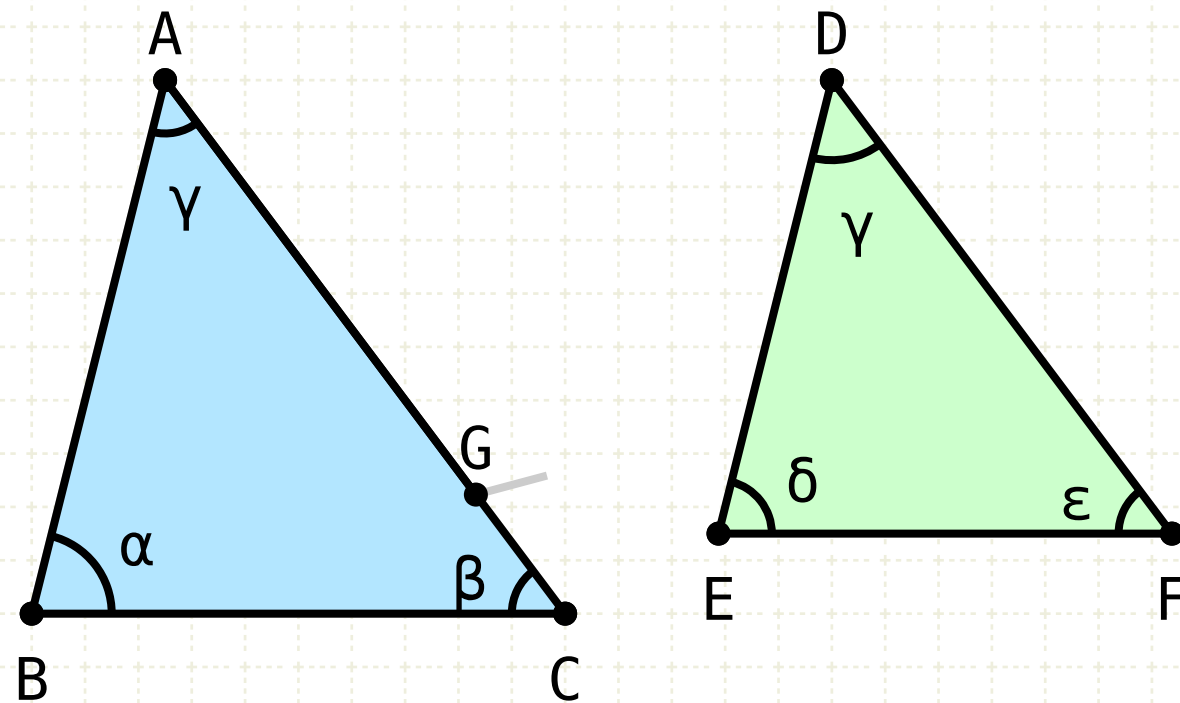
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Since there is a contradiction, the original assumption was wrong, so we know that angle α equals δ



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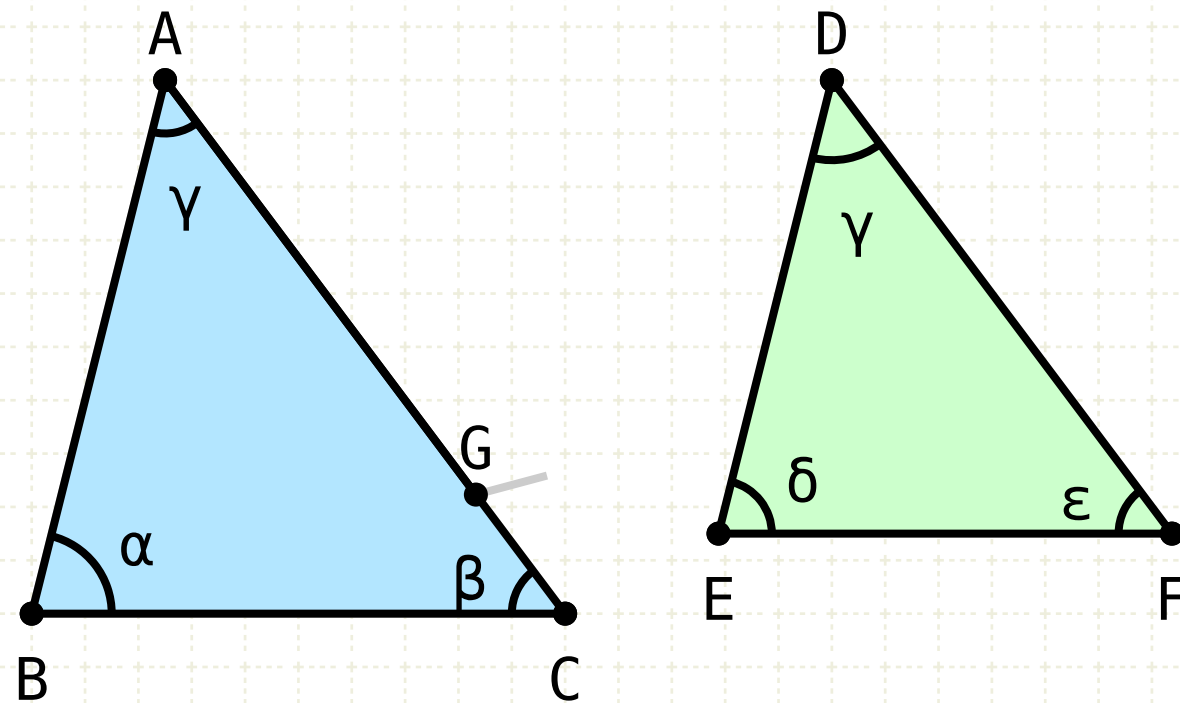
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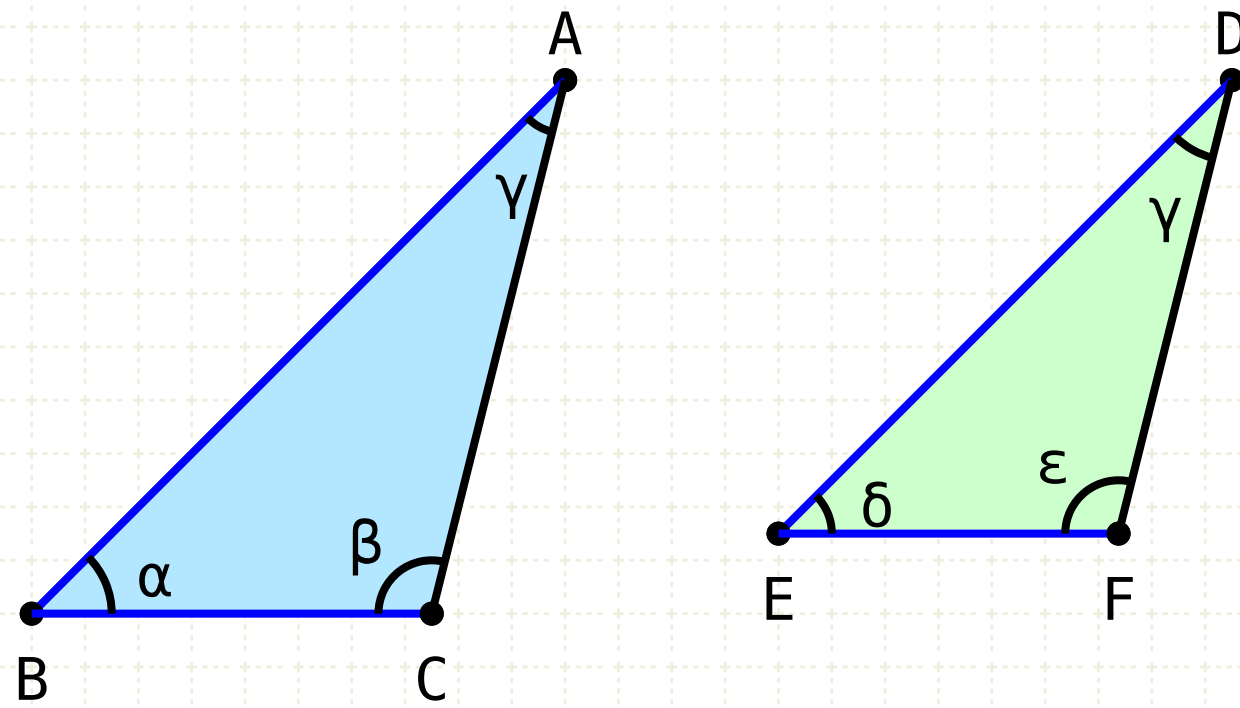
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The two triangles are equiangular



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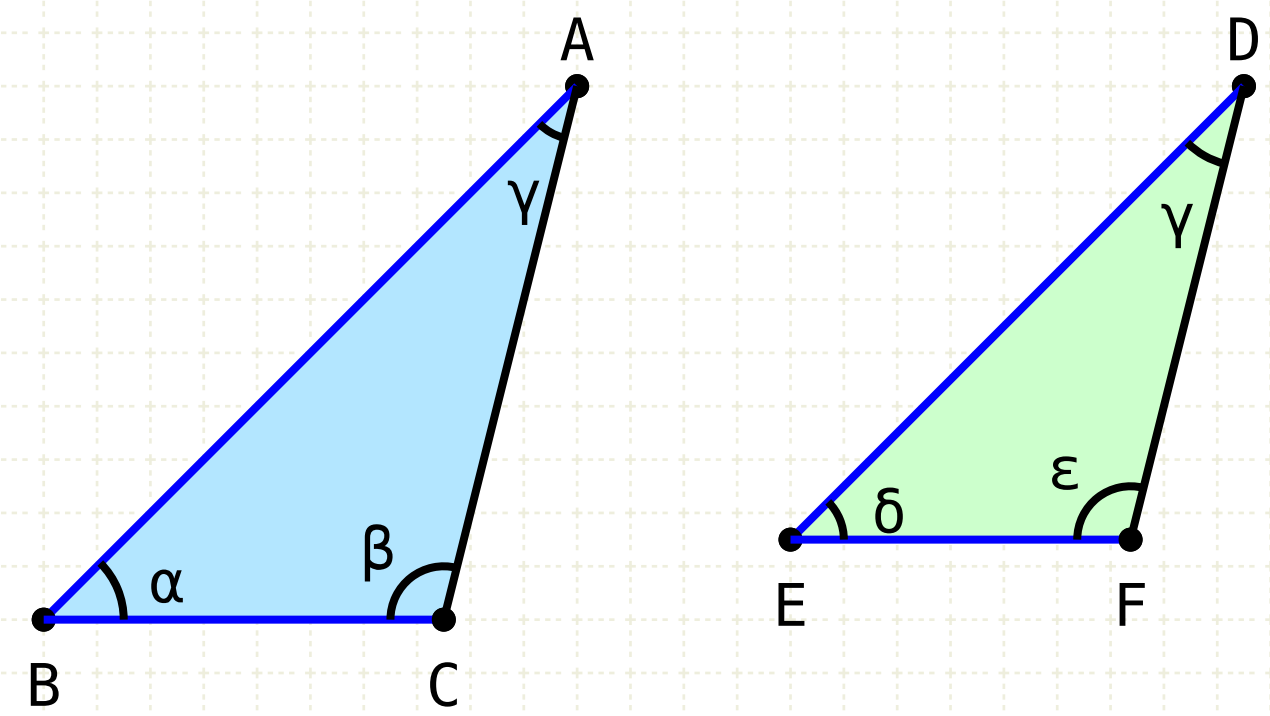
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If two triangles have one angle that is equal between them, AND the ratio of the sides of around a different angle are also equal, AND that the remaining angles are either both less than, or both greater than a right angle, then the two triangles will be equiangular

Let the angles at C and F greater than or less than a right angle

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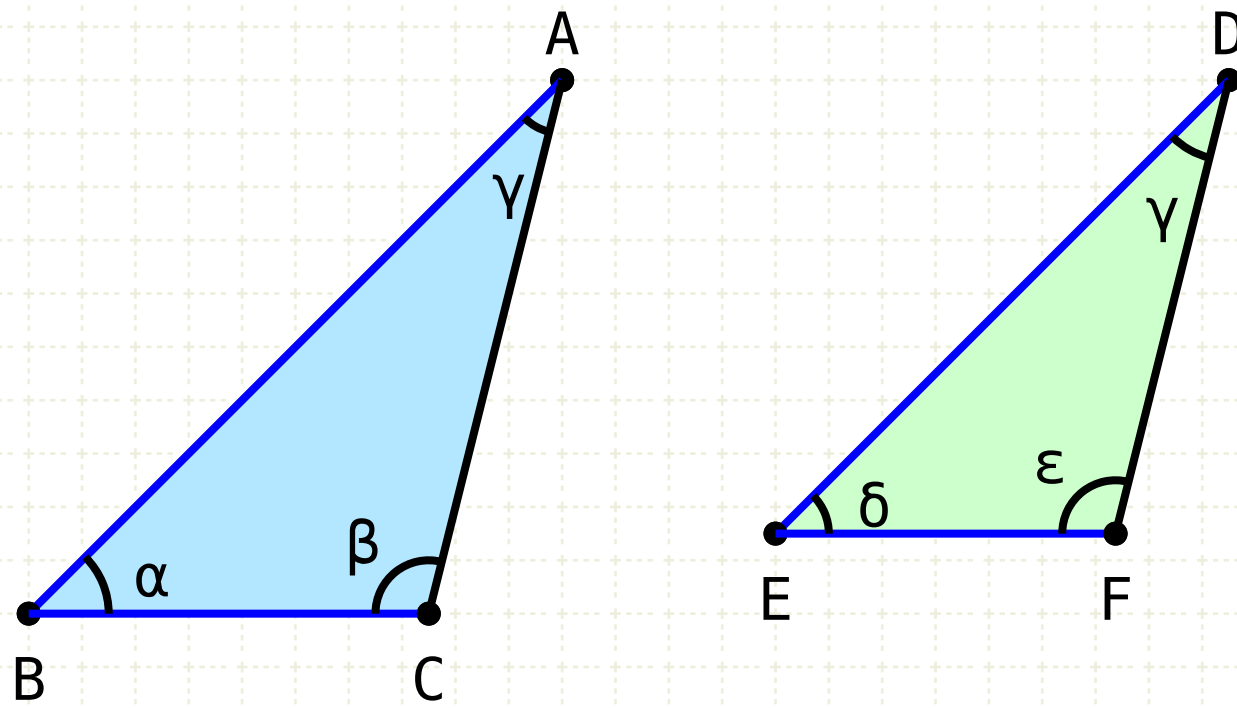
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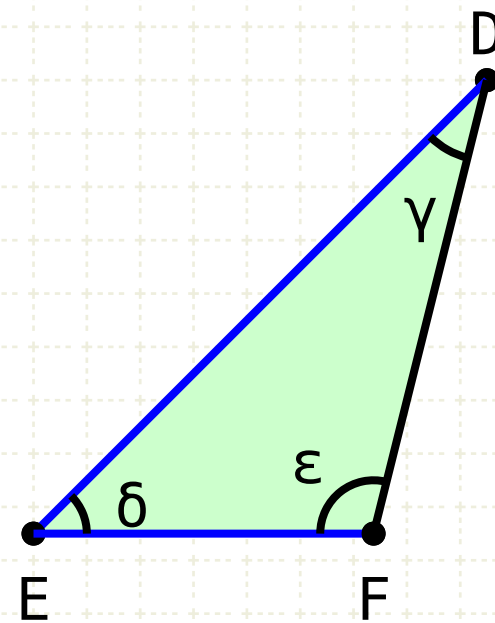
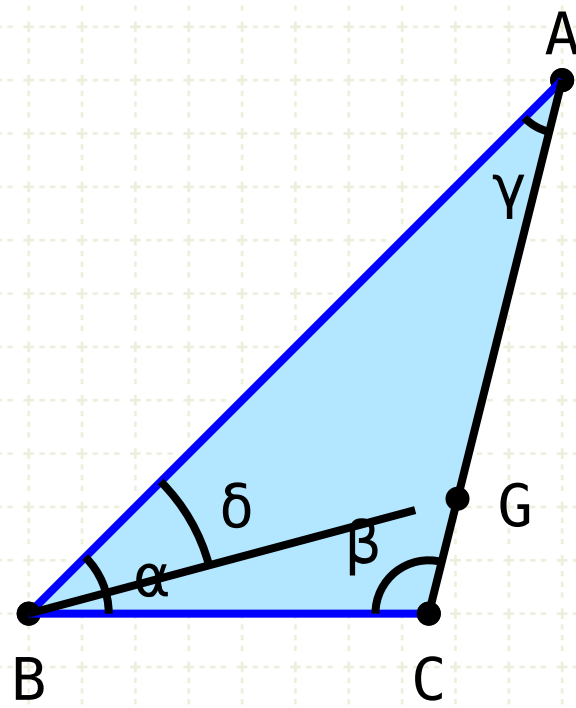
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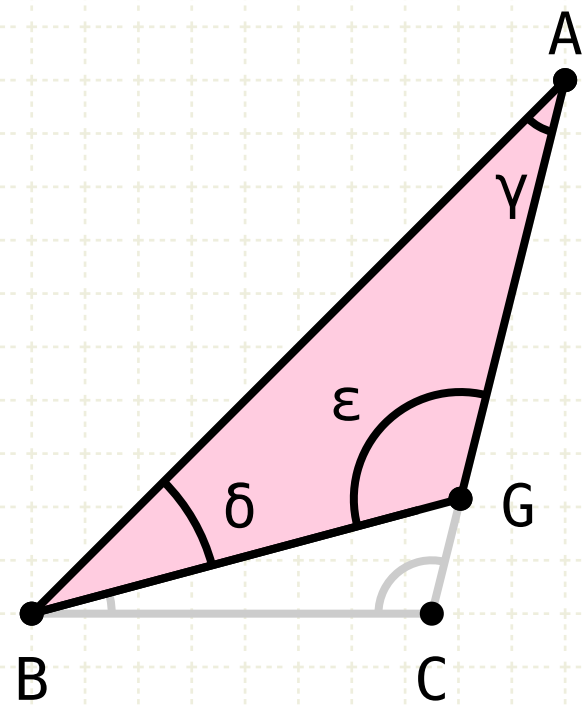
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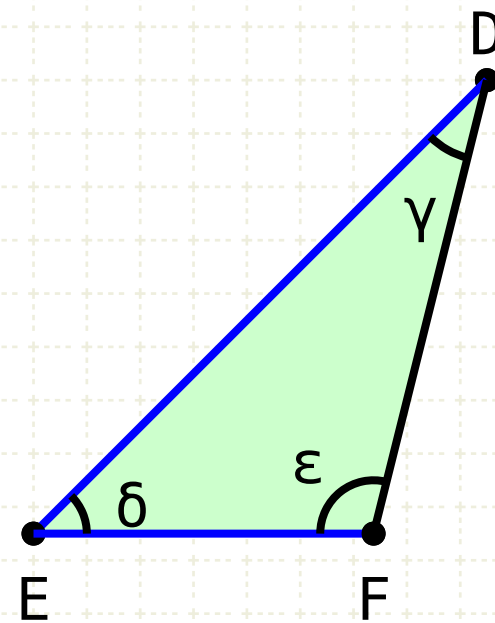
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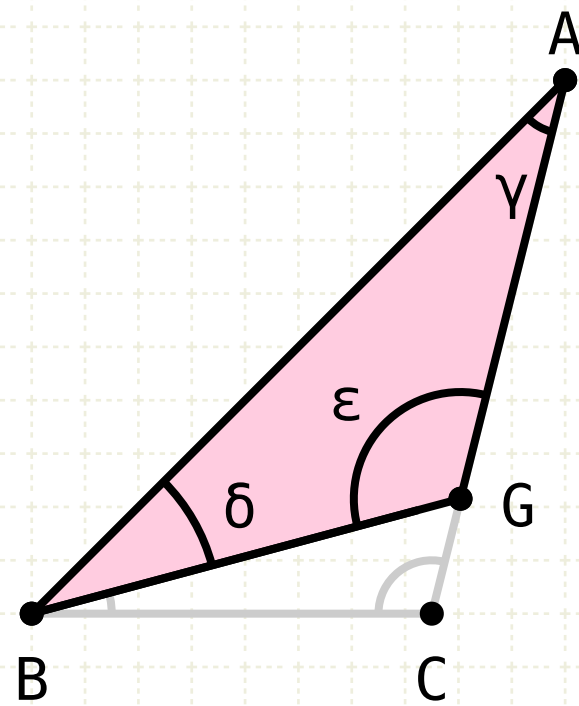
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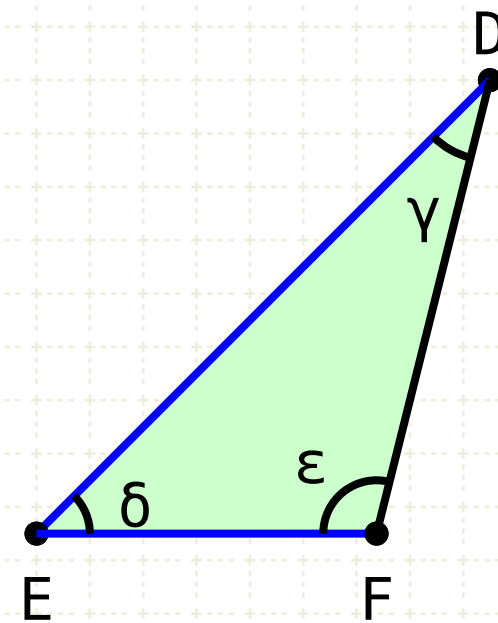


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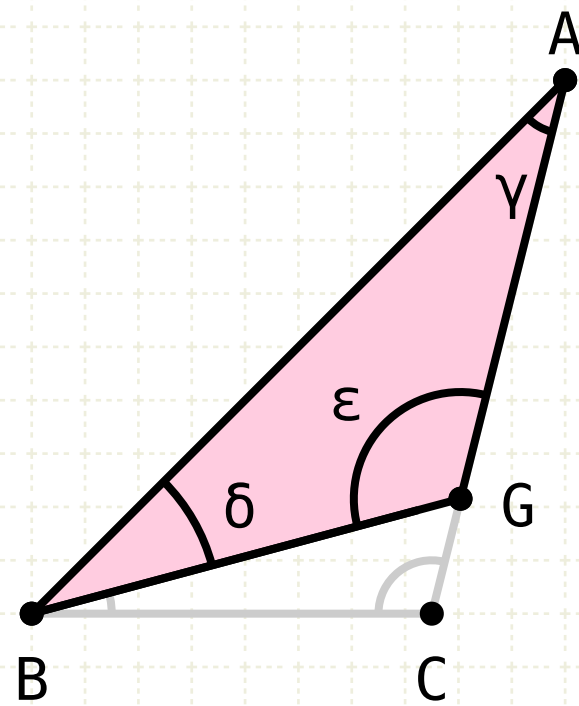
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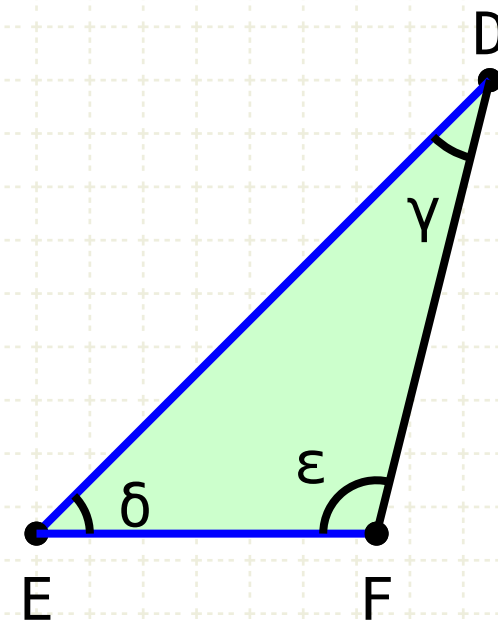
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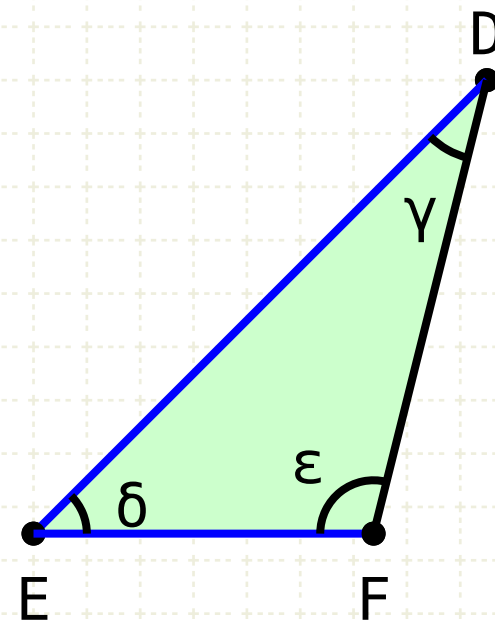
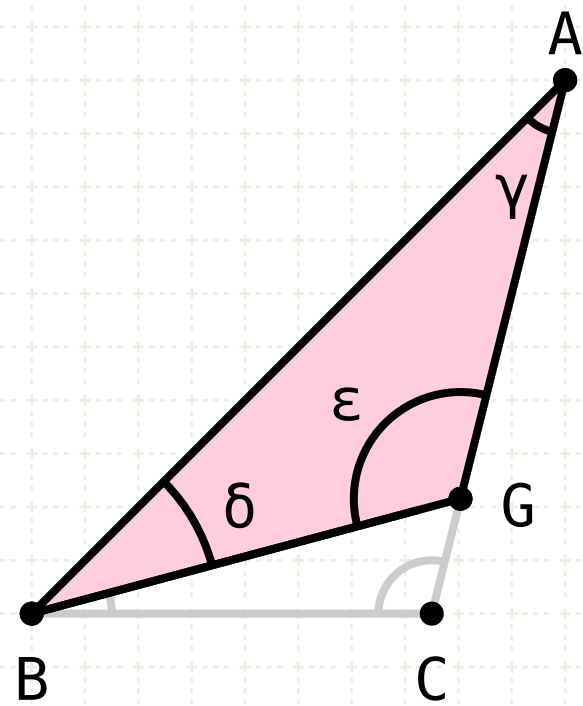
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$$BG = BC$$

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Assume that the angle α is unequal to the angle δ , and that α is the greater

Construct the angle ABG such that is is equal to the angle δ (I·23)

Now, since triangles AGB and AFE have two angles equal, the third must also be equal (I·32)

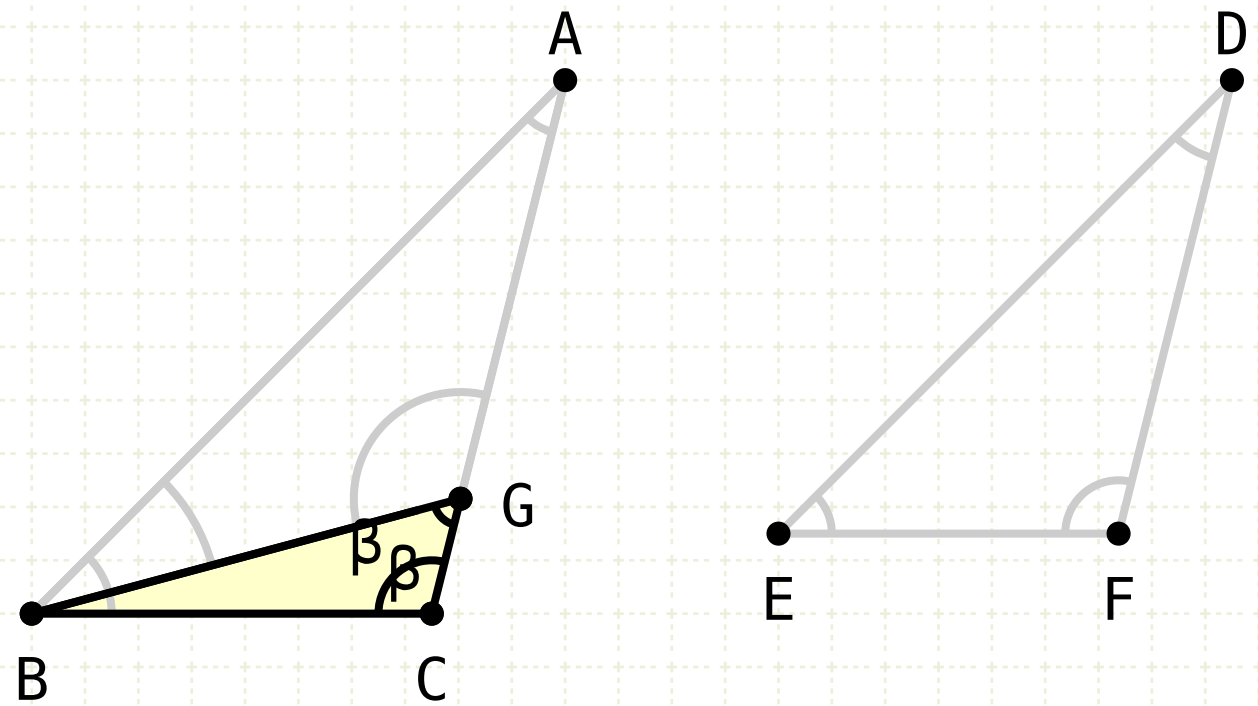
ABG is equiangular to DEF, so AB is to BG, as DE is to EF (VI·4)

The ratios AB to BG and AB to BC are both equal to DE to EF, then they are equal to each other (V·11)

Therefore BG equals BC (V·9)

Proposition 7 of Book VI

If two triangles have one angle equal to one angle, the sides about other angles proportional, and the remaining angles either both less or both not less than a right angle, the triangles will be equiangular and will have those angles equal, the sides about which are proportional.



$$AB:BC = DE:EF$$

$$\beta \geq \epsilon, \epsilon \geq \epsilon$$

$$\alpha > \delta$$

$$AB:BG = DE:EF$$

$$AB:BG = AB:BC$$

$$BG = BC$$

$$\angle BGC = \angle GCB$$

Proof by Contradiction (2)

Assume that the angle α is unequal to the angle δ , and that α is the greater

Construct the angle ABG such that is is equal to the angle δ (I·23)

Now, since triangles AGB and AFE have two angles equal, the third must also be equal (I·32)

ABG is equiangular to DEF , so AB is to BG , as DE is to EF (VI·4)

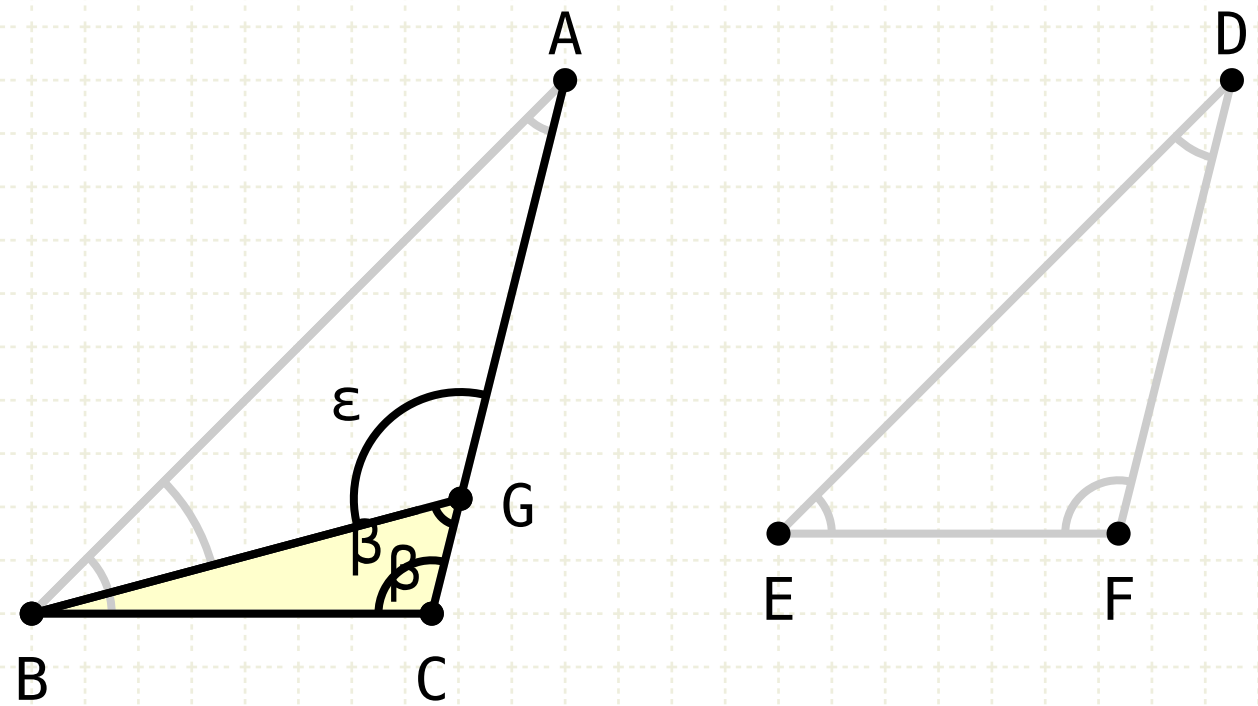
The ratios AB to BG and AB to BC are both equal to DE to EF , then they are equal to each other (V·11)

Therefore BG equals BC (V·9)

Thus the triangle BGC is an isosceles, and angle BGC is equal to angle BCG (I·5)

Proposition 7 of Book VI

If two triangles have one angle equal to one angle, the sides about other angles proportional, and the remaining angles either both less or both not less than a right angle, the triangles will be equiangular and will have those angles equal, the sides about which are proportional.



$$AB:BC = DE:EF$$

$$\beta \geq L, \epsilon \geq L$$

$$\alpha > \delta$$

$$AB:BG = DE:EF$$

$$AB:BG = AB:BC$$

$$BG = BC$$

$$\angle BGC = \angle GCB$$

$$\angle BGC = \angle GCB \geq L$$

Proof by Contradiction (2)

Assume that the angle α is unequal to the angle δ , and that α is the greater

Construct the angle ABG such that is is equal to the angle δ (I·23)

Now, since triangles AGB and AFE have two angles equal, the third must also be equal (I·32)

ABG is equiangular to DEF , so AB is to BG , as DE is to EF (VI·4)

The ratios AB to BG and AB to BC are both equal to DE to EF , then they are equal to each other (V·11)

Therefore BG equals BC (V·9)

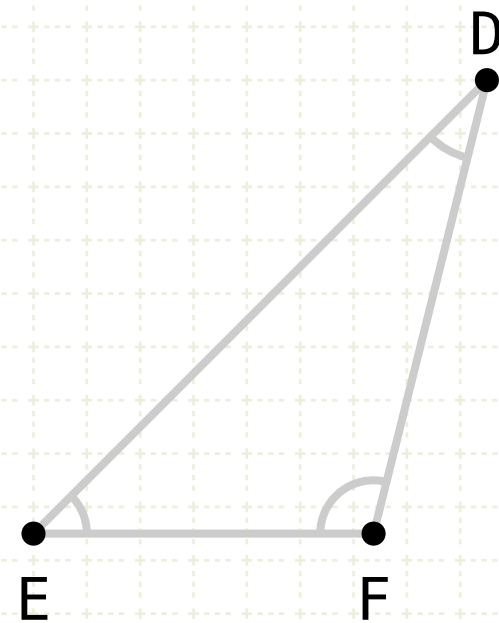
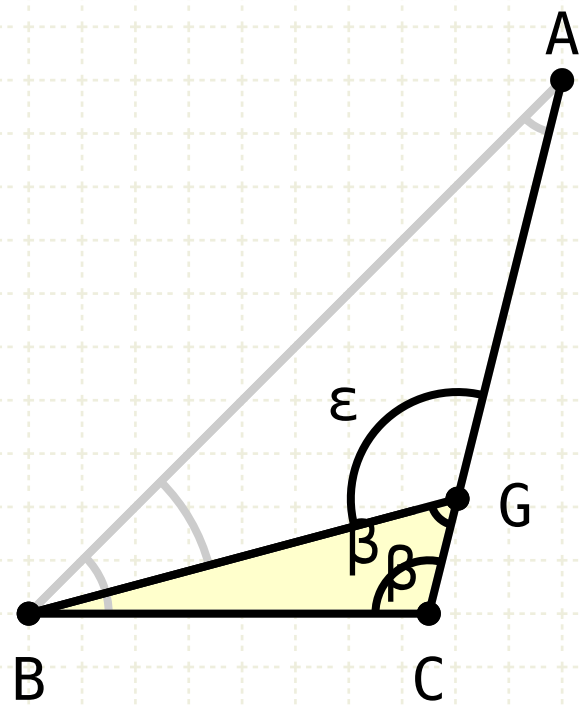
Thus the triangle BGC is an isosceles, and angle BGC is equal to angle BCG (I·5)

By definition, angle β is greater than or equal to a right angle, so therefore so is angle BGC



Proposition 7 of Book VI

If two triangles have one angle equal to one angle, the sides about other angles proportional, and the remaining angles either both less or both not less than a right angle, the triangles will be equiangular and will have those angles equal, the sides about which are proportional.



$$AB:BC = DE:EF$$

$$\beta \geq L, \epsilon \geq L$$

$$\alpha > \delta$$

$$AB:BG = DE:EF$$

$$AB:BG = AB:BC$$

$$BG = BC$$

$$\angle BGC = \angle GCB$$

$$\angle BGC = \angle GCB \geq L$$

Proof by Contradiction (2)

Assume that the angle α is unequal to the angle δ , and that α is the greater

Construct the angle ABG such that is is equal to the angle δ (I·23)

Now, since triangles AGB and AFE have two angles equal, the third must also be equal (I·32)

ABG is equiangular to DEF, so AB is to BG, as DE is to EF (VI·4)

The ratios AB to BG and AB to BC are both equal to DE to EF, then they are equal to each other (V·11)

Therefore BG equals BC (V·9)

Thus the triangle BGC is an isosceles, and angle BGC is equal to angle BCG (I·5)

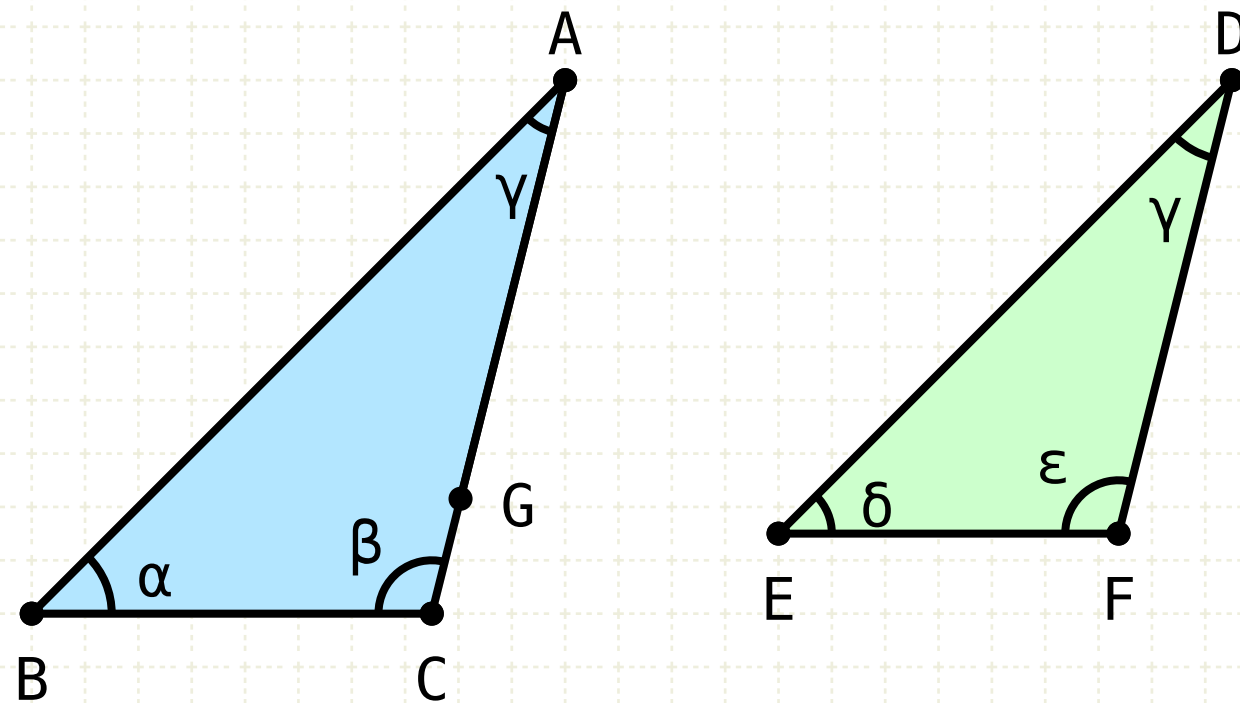
By definition, angle β is greater than or equal to a right angle, so therefore so is angle BGC

Thus we have two angles inside a triangle, each of which is greater than or equal to a right angle, which is impossible (I·17)



Proposition 7 of Book VI

If two triangles have one angle equal to one angle, the sides about other angles proportional, and the remaining angles either both less or both not less than a right angle, the triangles will be equiangular and will have those angles equal, the sides about which are proportional.



$$AB:BC = DE:EF$$

$$\beta \geq \text{L}, \epsilon \geq \text{L}$$

$$\alpha > \delta$$

$$AB:BG = DE:EF$$

$$AB:BG = AB:BC$$

$$BG = BC$$

$$\angle BGC = \angle GCB$$

$$\angle BGC = \angle GCB \geq \text{L}$$

$$\alpha = \delta$$

$$\beta = \epsilon$$

Proof by Contradiction (2)

Assume that the angle α is unequal to the angle δ , and that α is the greater

Construct the angle ABG such that is is equal to the angle δ (I·23)

Now, since triangles AGB and AFE have two angles equal, the third must also be equal (I·32)

ABG is equiangular to DEF, so AB is to BG, as DE is to EF (VI·4)

The ratios AB to BG and AB to BC are both equal to DE to EF, then they are equal to each other (V·11)

Therefore BG equals BC (V·9)

Thus the triangle BGC is an isosceles, and angle BGC is equal to angle BCG (I·5)

By definition, angle β is greater than or equal to a right angle, so therefore so is angle BGC

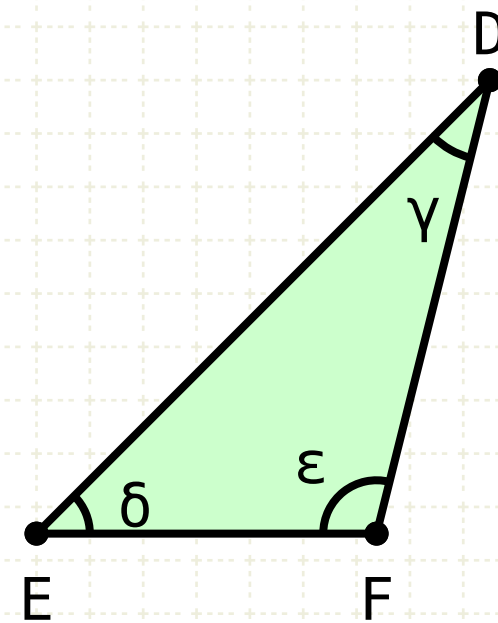
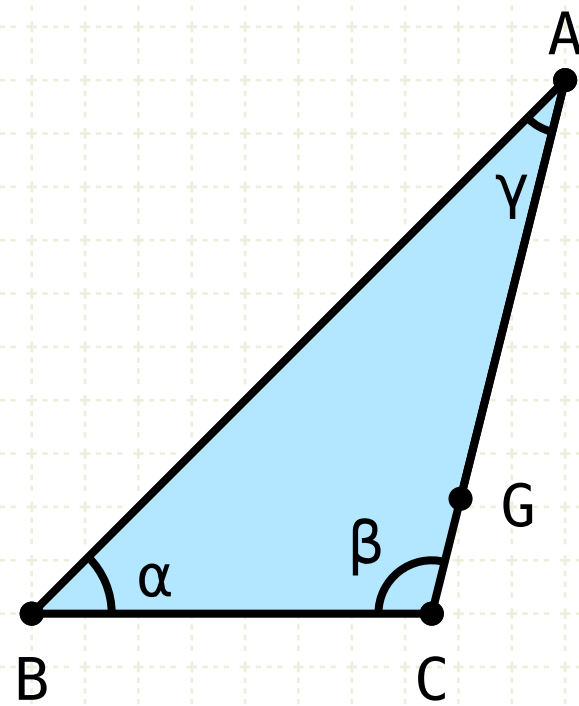
Thus we have two angles inside a triangle, each of which is greater than or equal to a right angle, which is impossible (I·17)

Since there is a contradiction, we know that angle α equals δ , and since BAC and EDF are equal, β equals ϵ (I·32)



Proposition 7 of Book VI

If two triangles have one angle equal to one angle, the sides about other angles proportional, and the remaining angles either both less or both not less than a right angle, the triangles will be equiangular and will have those angles equal, the sides about which are proportional.



$$AB:BC = DE:EF$$

$$\beta \geq \text{L}, \epsilon \geq \text{L}$$

$$\alpha > \delta$$

$$AB:BG = DE:EF$$

$$AB:BG = AB:BC$$

$$BG = BC$$

$$\angle BGC = \angle GCB$$

$$\angle BGC = \angle GCB \geq \text{L}$$

$$\alpha = \delta$$

$$\beta = \epsilon$$

Proof by Contradiction (2)

Assume that the angle α is unequal to the angle δ , and that α is the greater

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Now, since triangles AGB and AFE have two angles equal, the third must also be equal (I·32)

ABG is equiangular to DEF, so AB is to BG, as DE is to EF (VI·4)

The ratios AB to BG and AB to BC are both equal to DE to EF, then they are equal to each other (V·11)

Therefore BG equals BC (V·9)

Thus the triangle BGC is an isosceles, and angle BGC is equal to angle BCG (I·5)

By definition, angle β is greater than or equal to a right angle, so therefore so is angle BGC

Thus we have two angles inside a triangle, each of which is greater than or equal to a right angle, which is impossible (I·17)

Since there is a contradiction, we know that angle α equals δ , and since BAC and EDF are equal, β equals ϵ (I·32)

The two triangles are equiangular



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