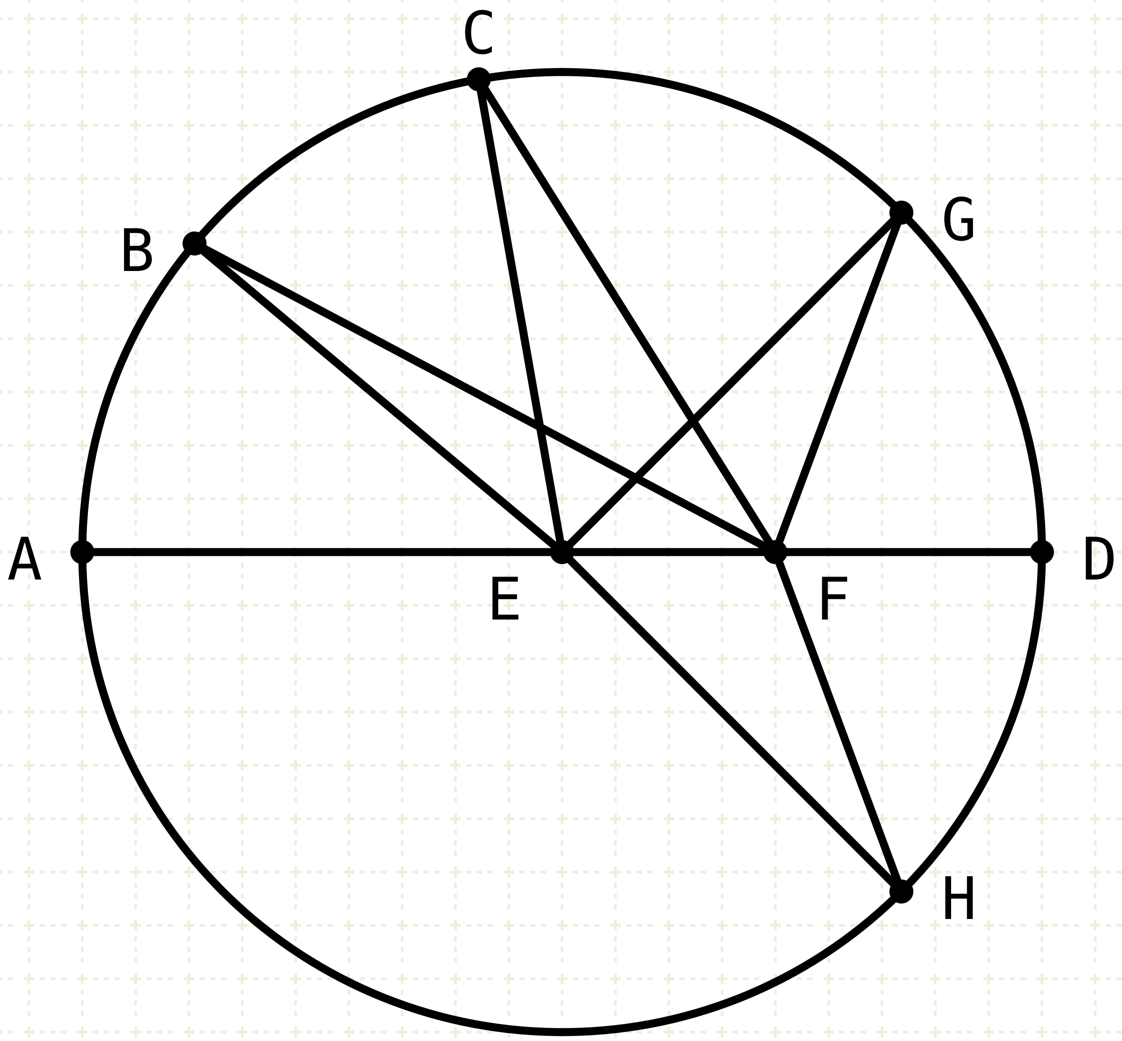
Euclid's Elements

Book III



A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)

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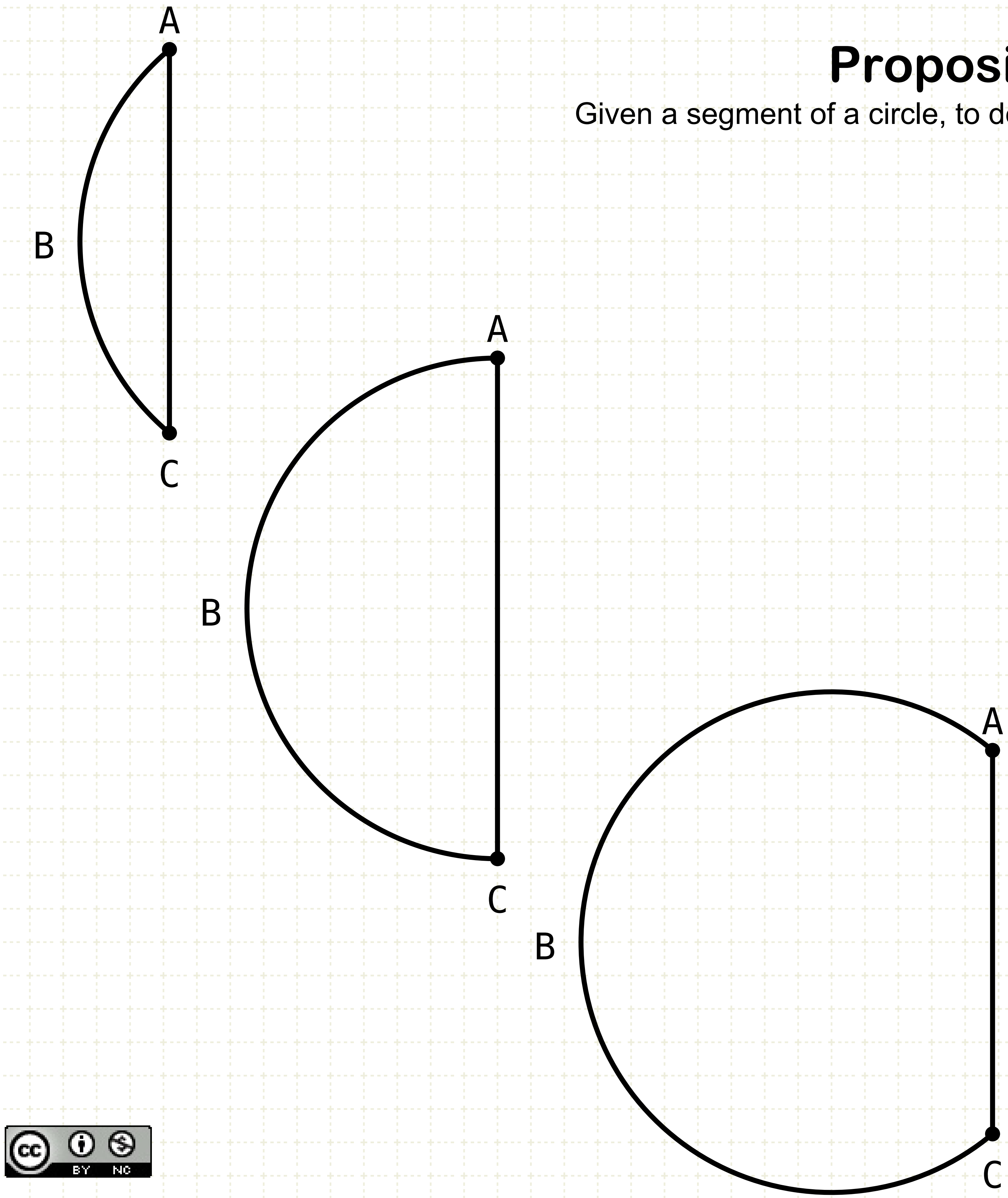
Proposition 25 of Book III

Given a segment of a circle, to describe the complete circle of which it is a segment.



Proposition 25 of Book III

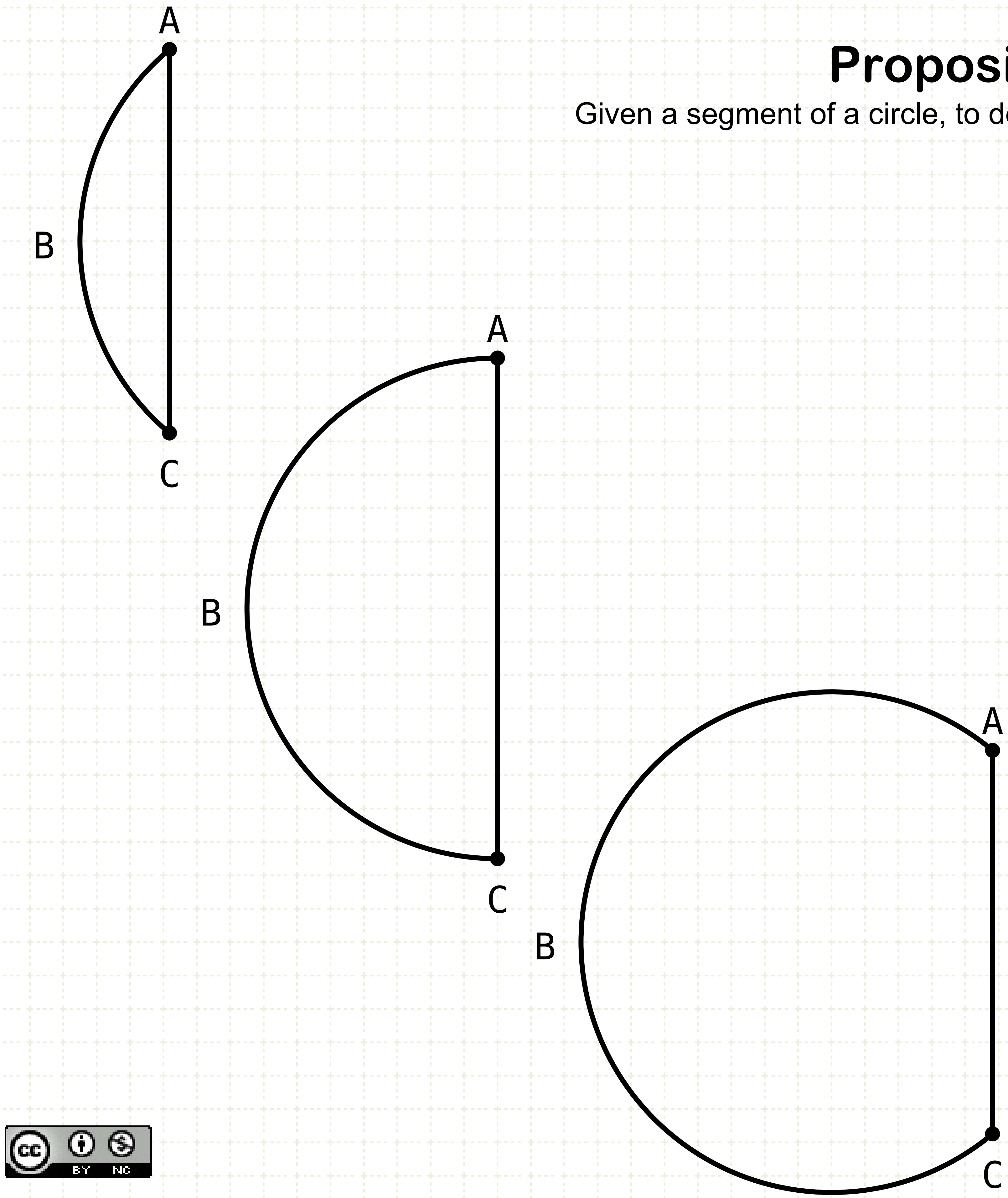
Given a segment of a circle, to describe the complete circle of which it is a segment.

In other words

From a given segment ABC, find the radius and centre of the circle

Proposition 25 of Book III

Given a segment of a circle, to describe the complete circle of which it is a segment.



Construction

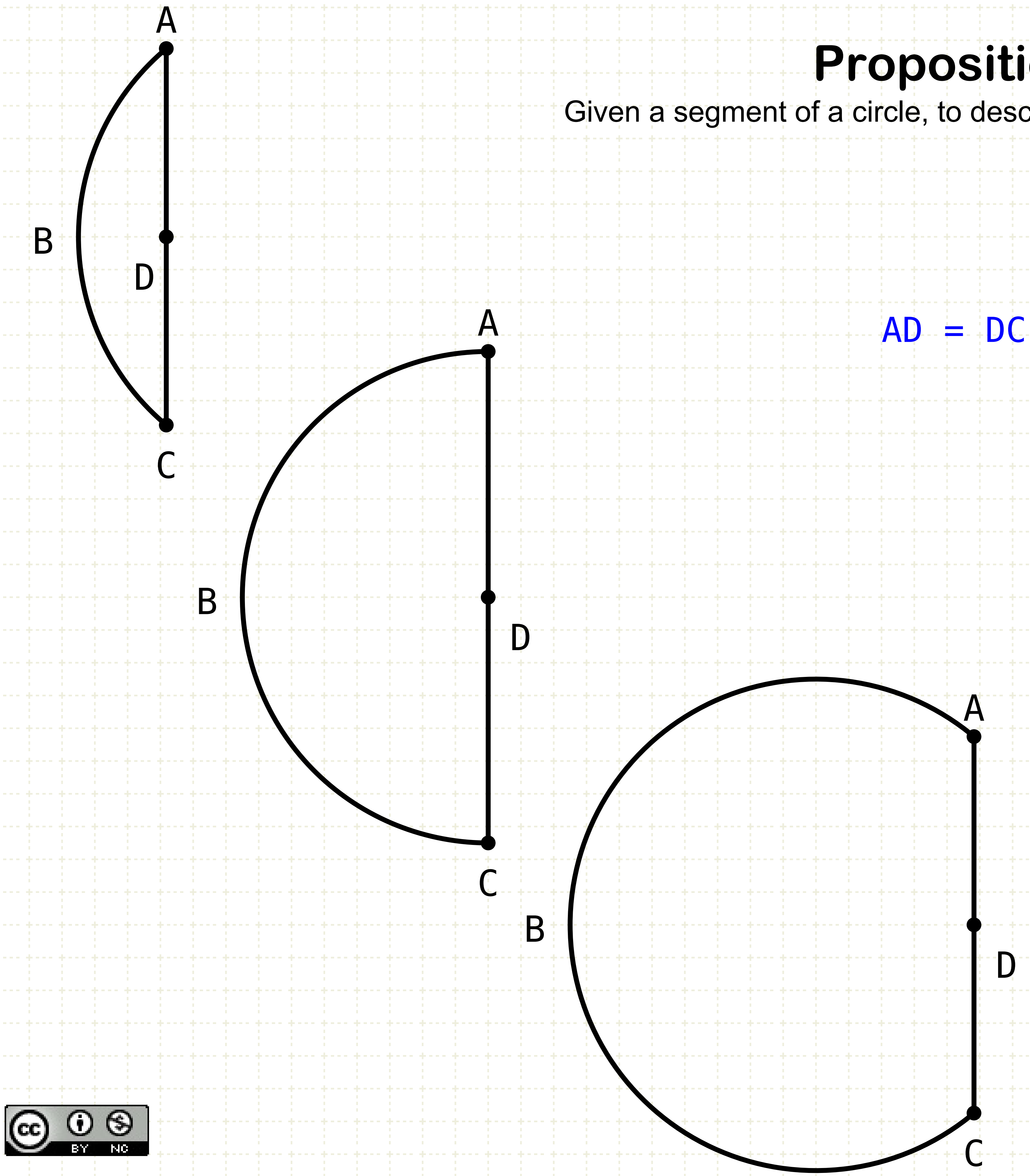
Proposition 25 of Book III

Given a segment of a circle, to describe the complete circle of which it is a segment.

Construction

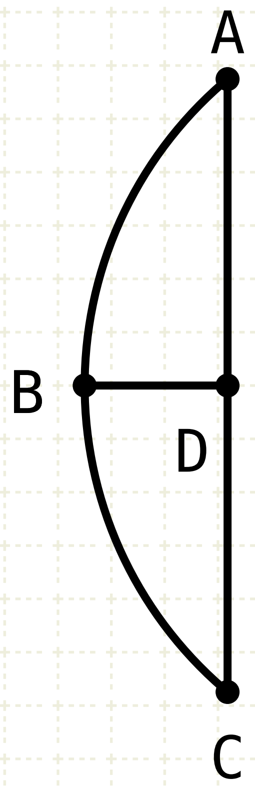
Bisect line AC at point D

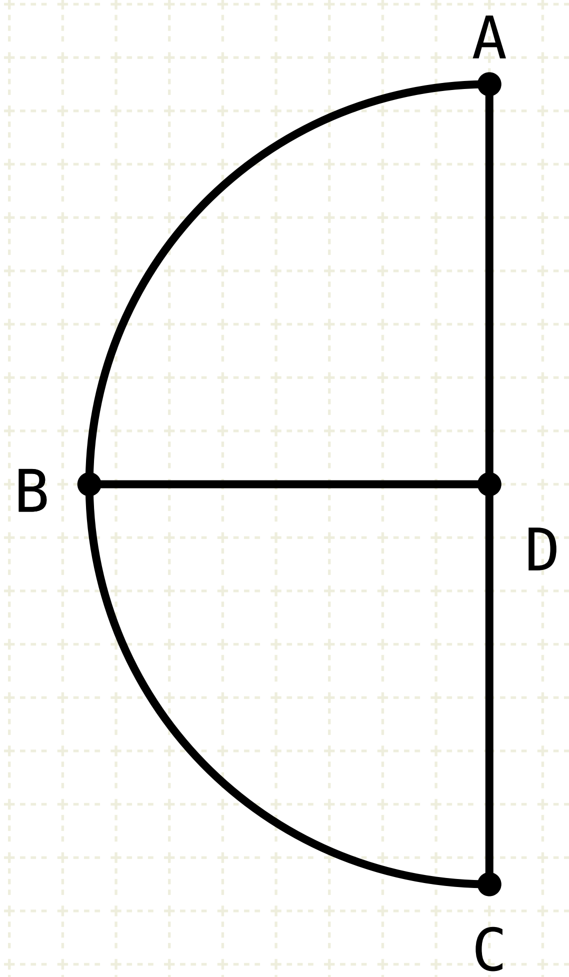
$AD = DC$



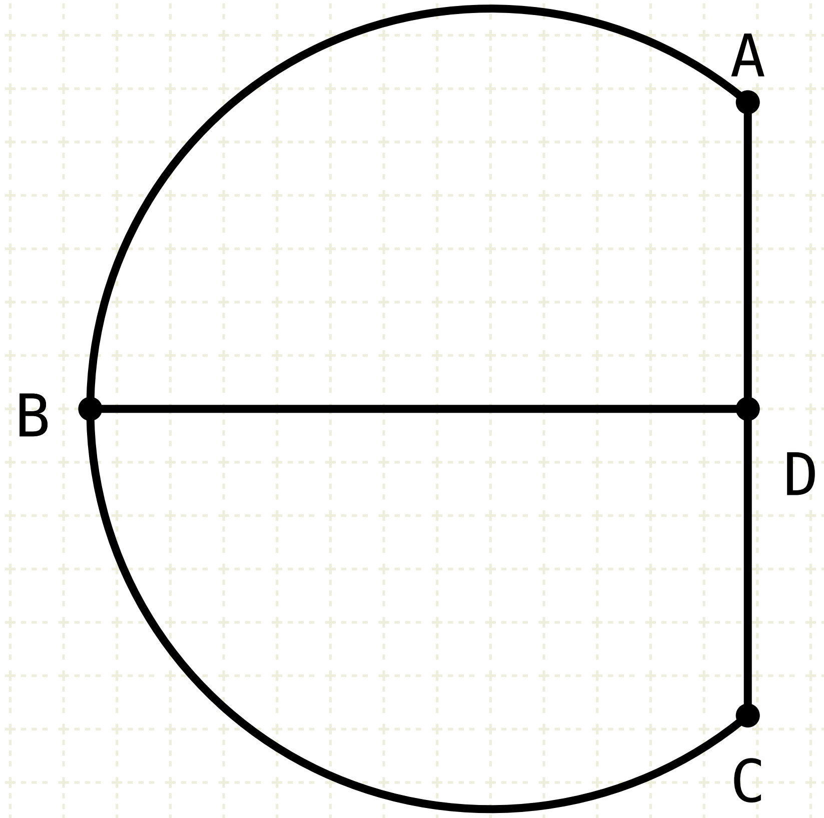
Proposition 25 of Book III

Given a segment of a circle, to describe the complete circle of which it is a segment.





AD = DC



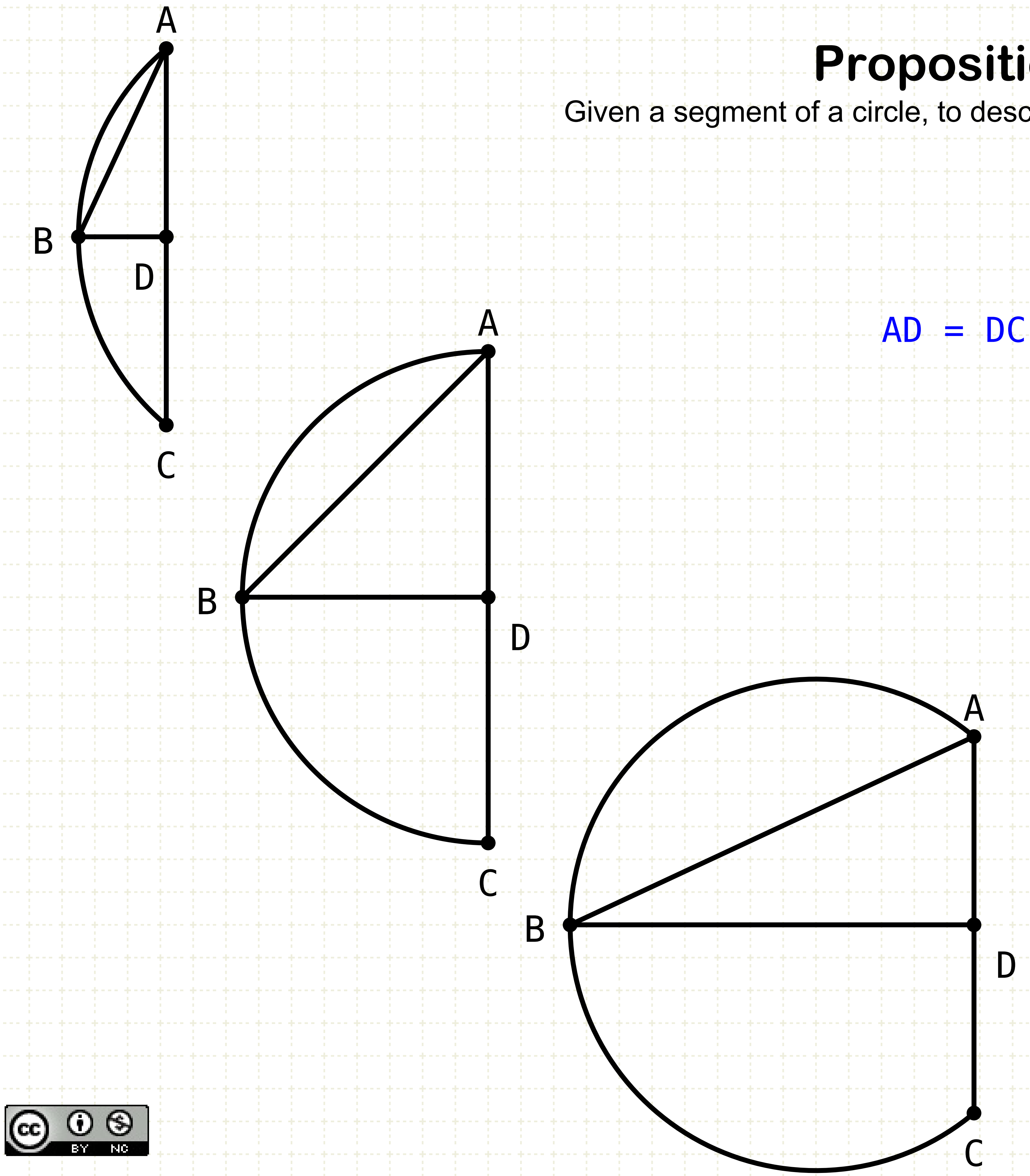
Construction

Bisect line AC at point D

Draw line perpendicular to AC from D, intersecting the circumference at point B

Proposition 25 of Book III

Given a segment of a circle, to describe the complete circle of which it is a segment.



AD = DC

Construction

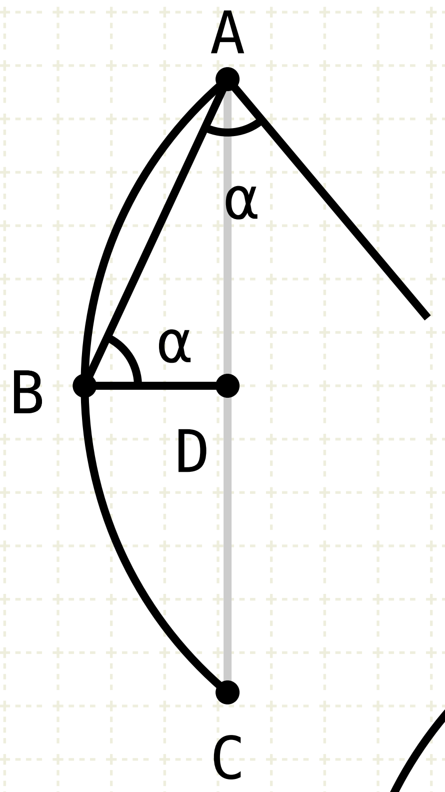
Bisect line AC at point D

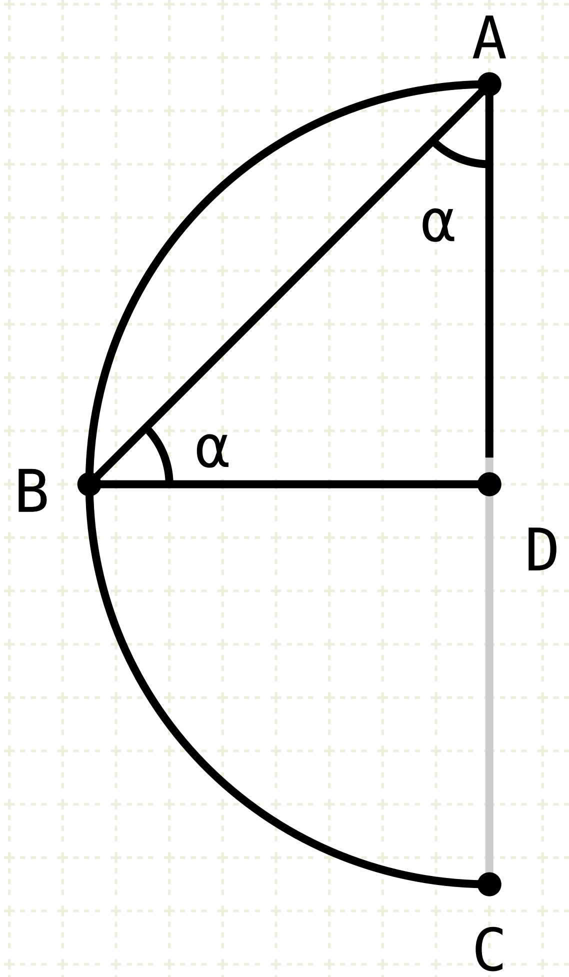
Draw line perpendicular to AC from D, intersecting the circumference at point B

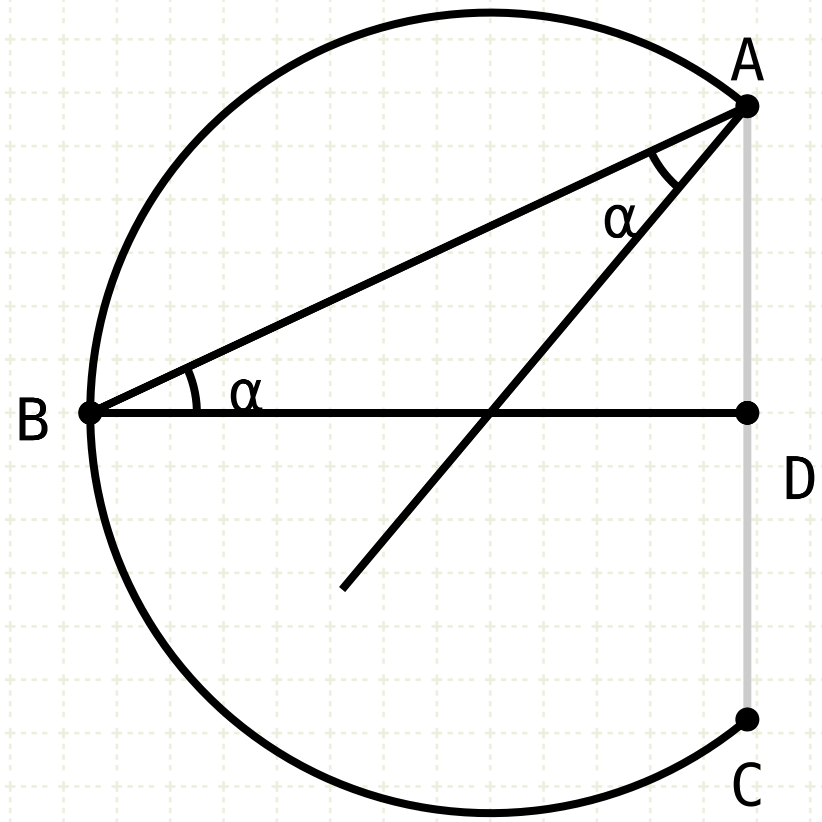
Join points A and B

Proposition 25 of Book III

Given a segment of a circle, to describe the complete circle of which it is a segment.







$AD = DC$

$\angle BAE = \angle ABD$

Construction

Bisect line AC at point D

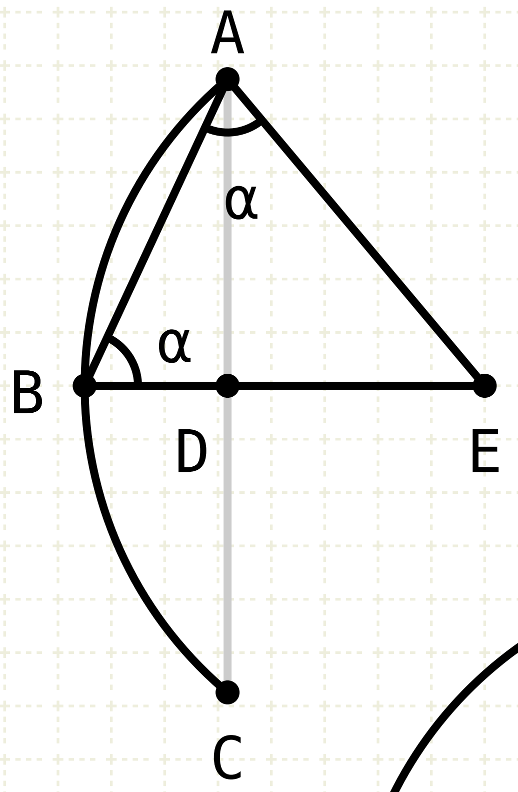
Draw line perpendicular to AC from D, intersecting the circumference at point B

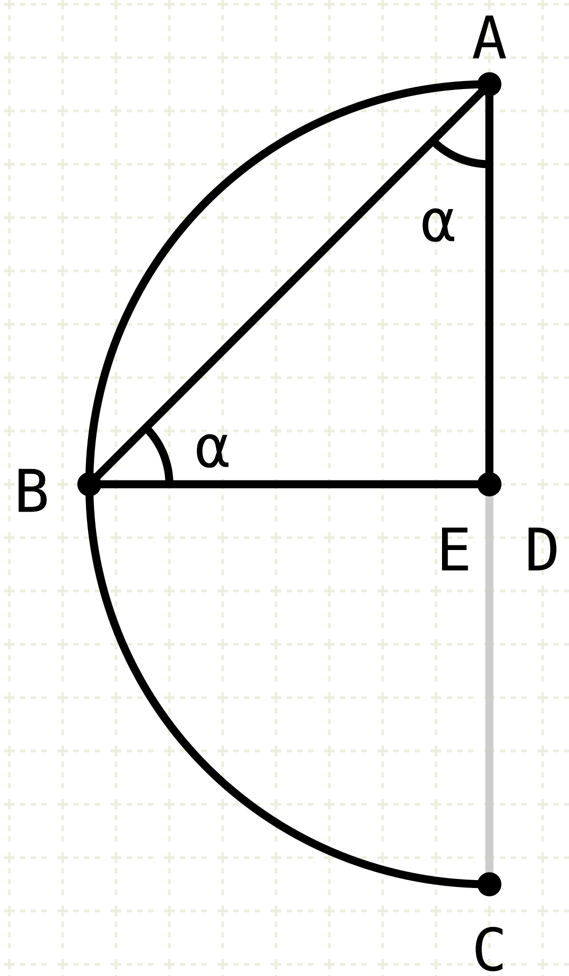
Join points A and B

Construct an angle BAE equal to the angle ABD

Proposition 25 of Book III

Given a segment of a circle, to describe the complete circle of which it is a segment.





$AD = DC$

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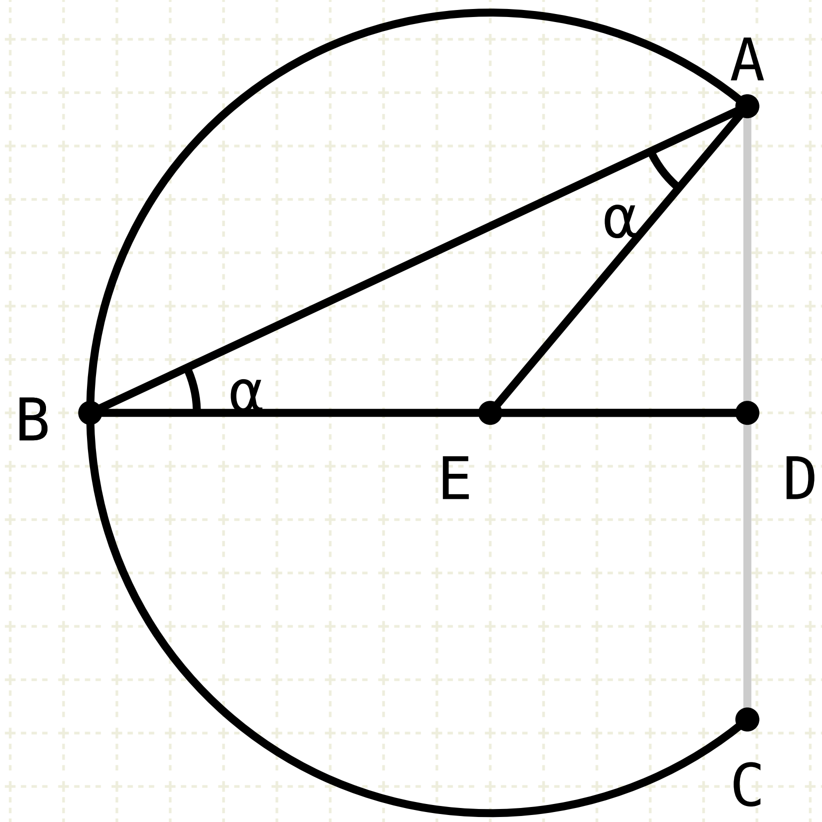


Construction

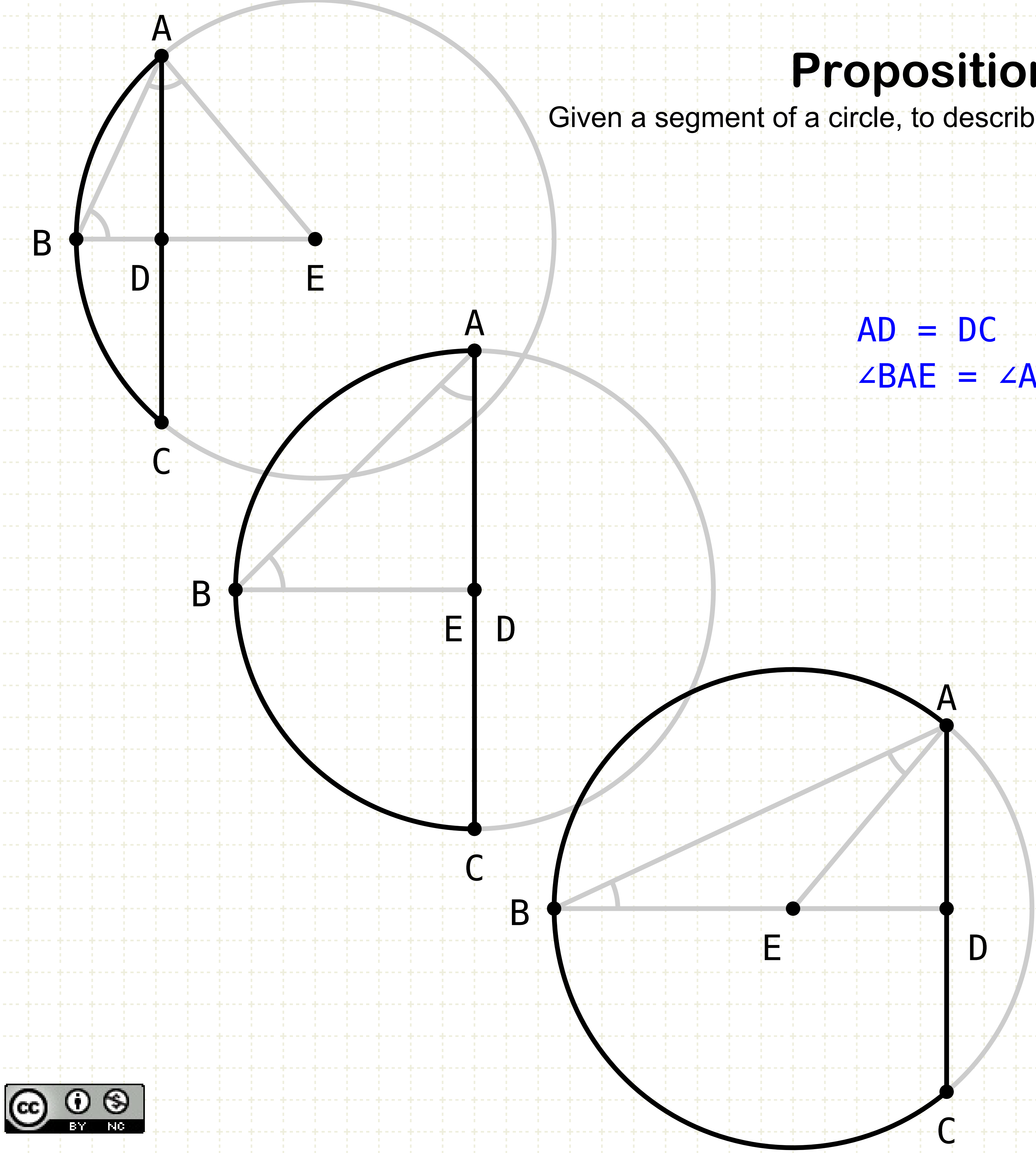
Bisect line AC at point D

Draw line perpendicular to AC from D, intersecting the circumference at point B

Join points A and B

Construct an angle BAE equal to the angle ABD

Extend the angled line and the line BD until they meet at the point E



Proposition 25 of Book III

Given a segment of a circle, to describe the complete circle of which it is a segment.

AD = DC

∠BAE = ∠ABD

Construction

Bisect line AC at point D

Draw line perpendicular to AC from D, intersecting the circumference at point B

Join points A and B

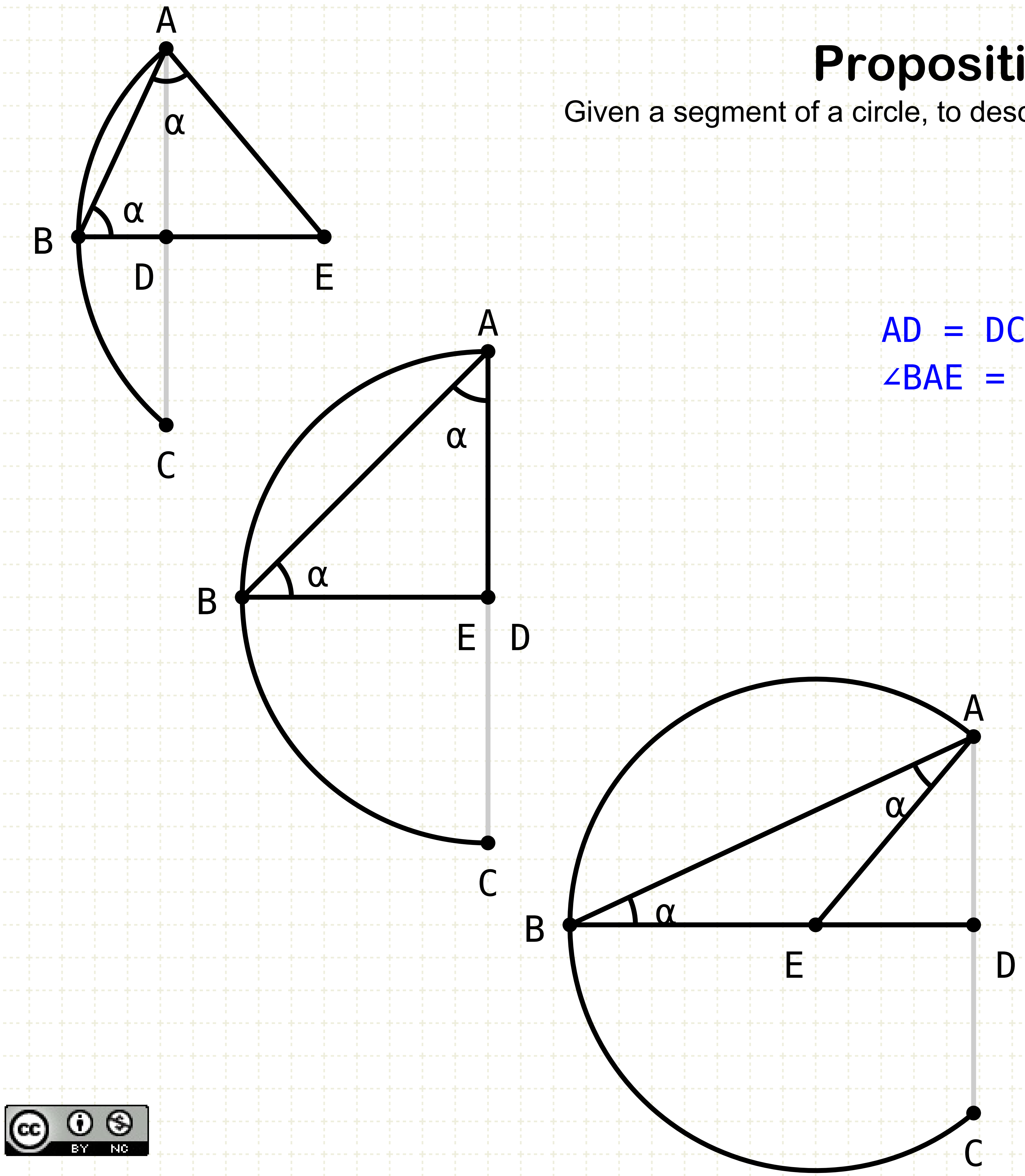
Construct an angle BAE equal to the angle ABD

Extend the angled line and the line BD until they meet at the point E

With E as the centre of a circle, and one of the three lines (AE, BE, CE) as radius, the circle will be complete

Proposition 25 of Book III

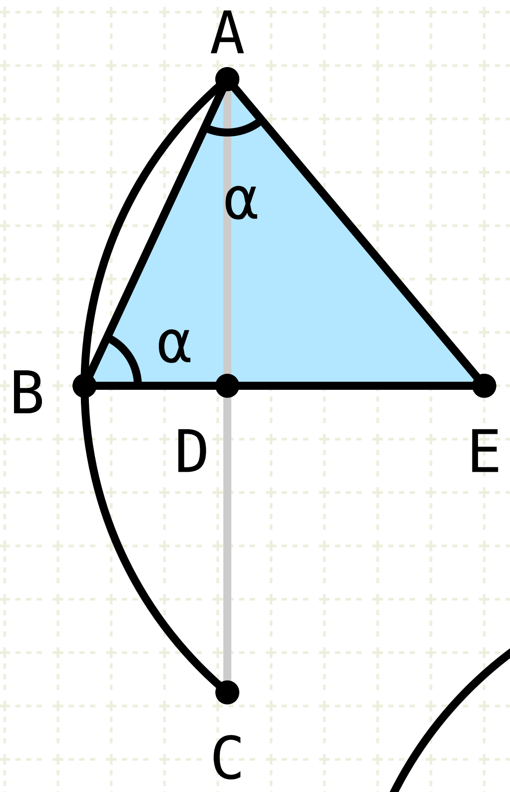
Given a segment of a circle, to describe the complete circle of which it is a segment.



Proof

AD = DC

∠BAE = ∠ABD



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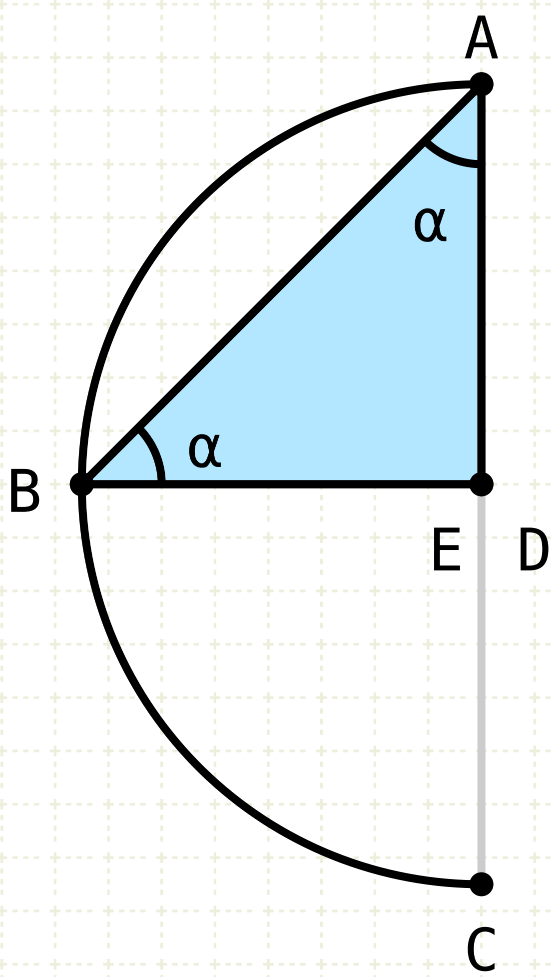
Given a segment of a circle, to describe the complete circle of which it is a segment.



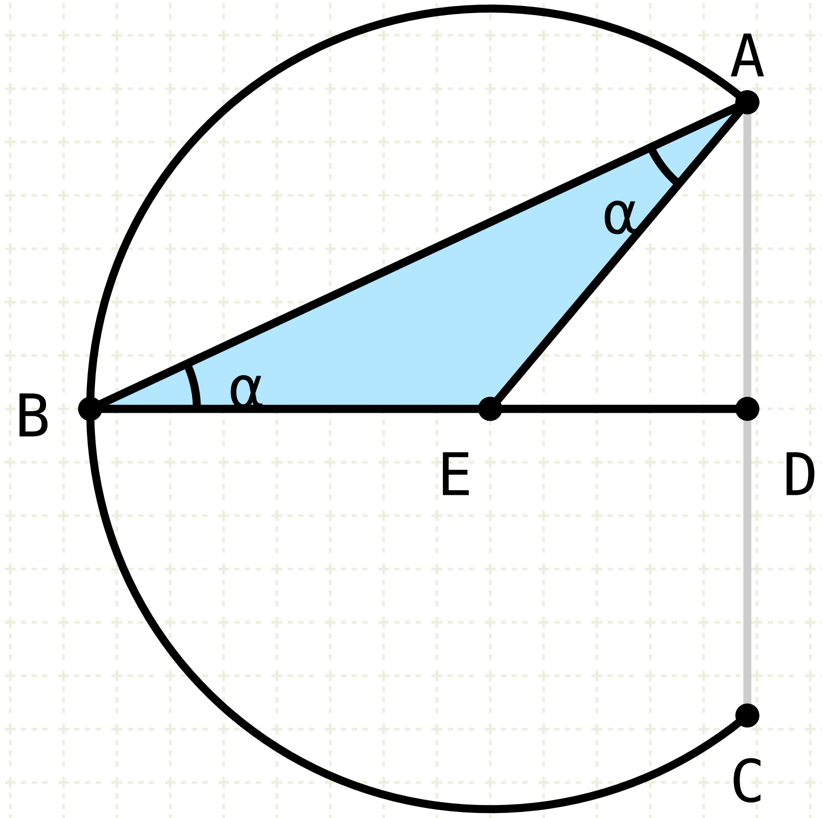
$AD = DC$

$\angle BAE = \angle ABD$

$AE = BE$

Proof

Since angles EBA and EAB are equal, the triangle is an isosceles, and the lines AE and BE are equal (I·6)



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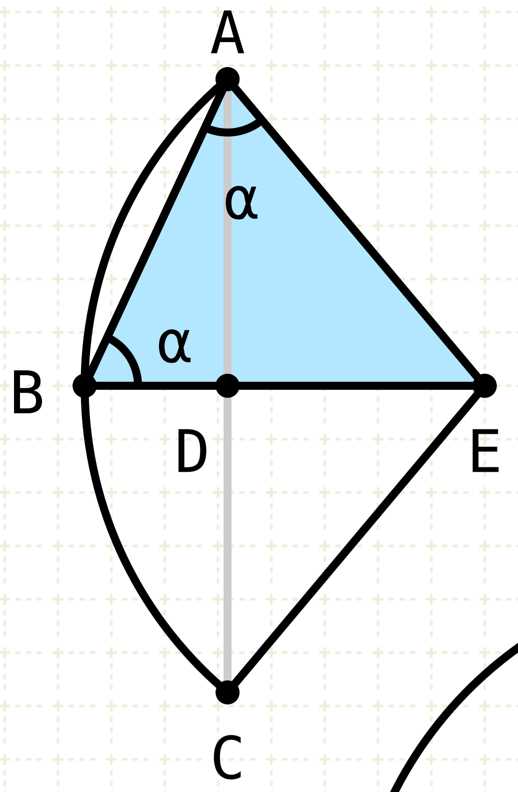
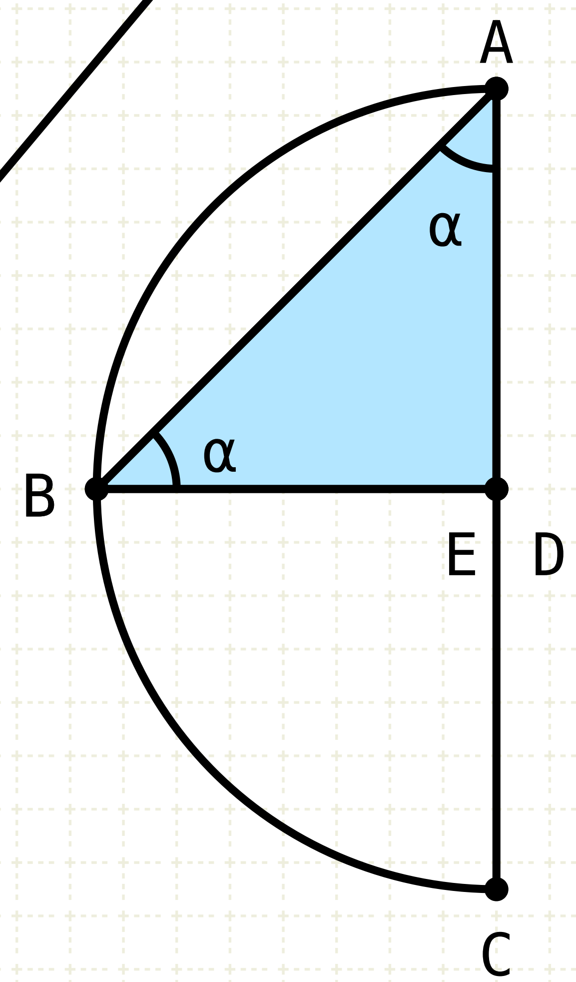
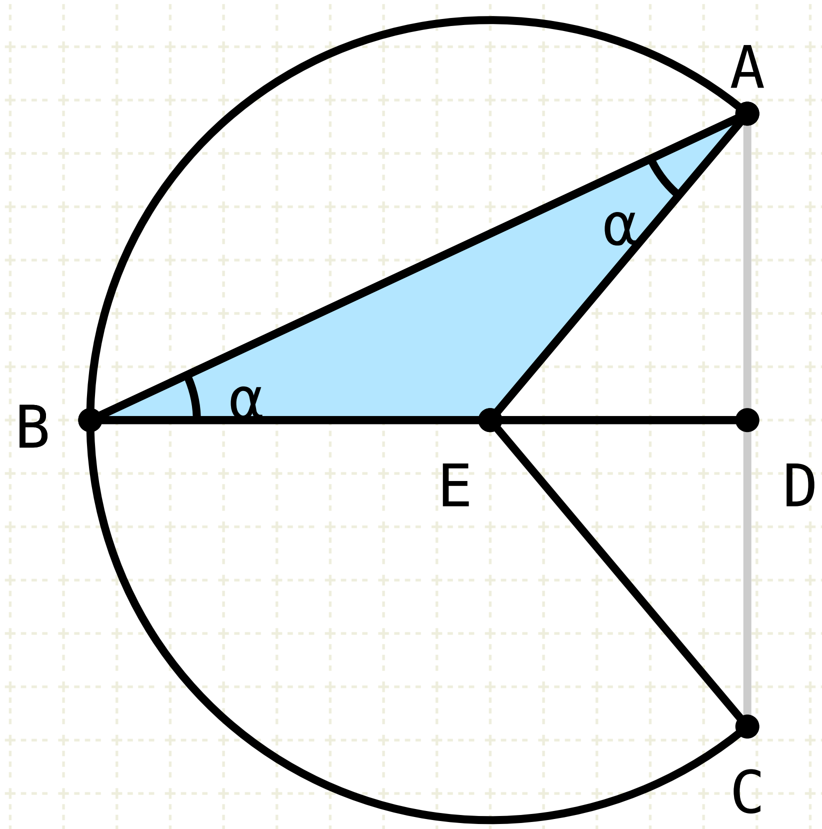
Given a segment of a circle, to describe the complete circle of which it is a segment.







$AD = DC$

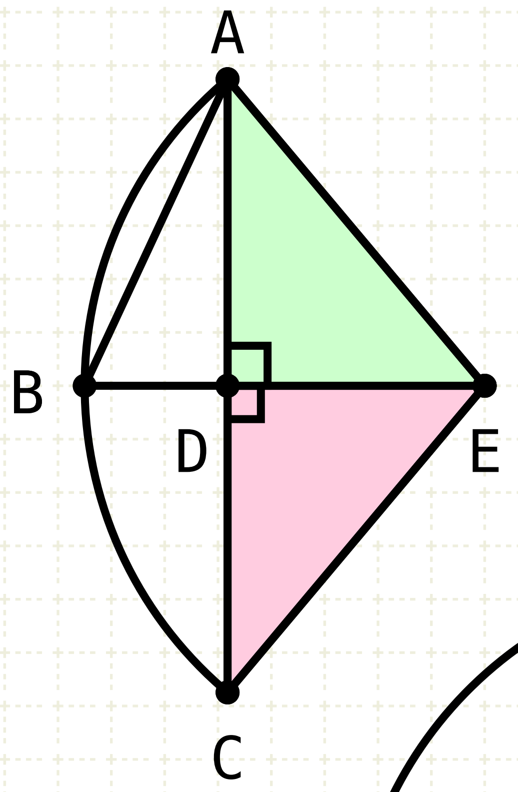
$\angle BAE = \angle ABD$

$AE = BE$

Proof

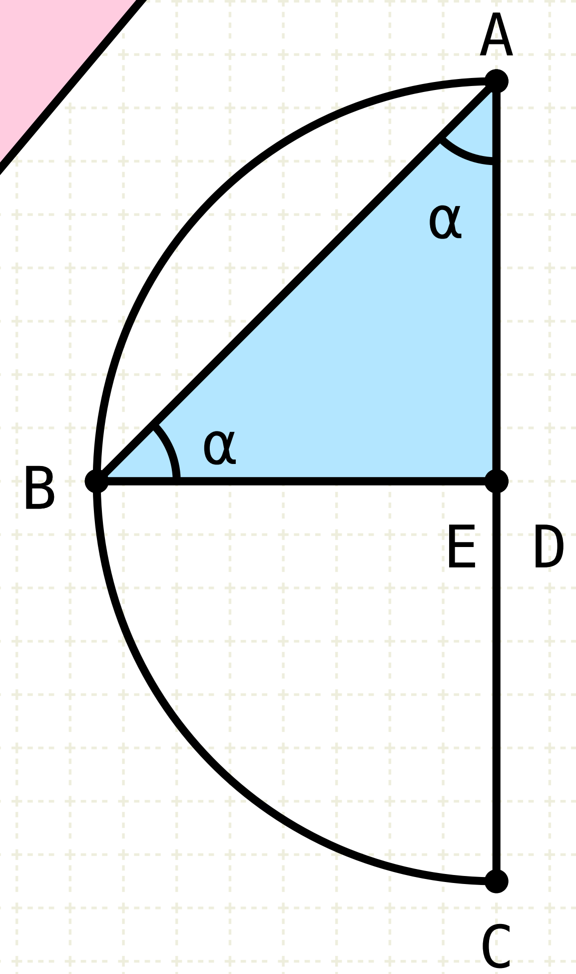
Since angles EBA and EAB are equal, the triangle is an isosceles, and the lines AE and BE are equal (I·6)

Draw line CE



Proposition 25 of Book III

Given a segment of a circle, to describe the complete circle of which it is a segment.



AD = DC
∠BAE = ∠ABD
AE = BE
AD = DC

Proof

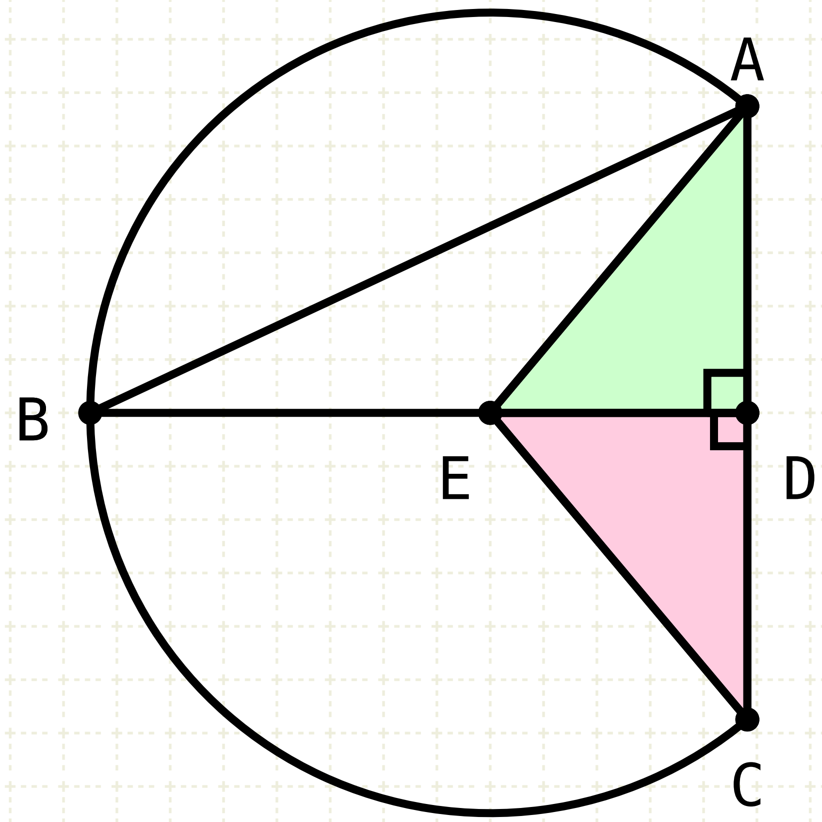
Since angles EBA and EAB are equal, the triangle is an isosceles, and the lines AE and BE are equal (I·6)

Draw line CE

If D and E are not the same point,

Consider triangles ADE and ACE

Two sides are equal, (D bisects AC, and DE is common), and the angles ADE and EDC are equal (BD is perpendicular to AC)



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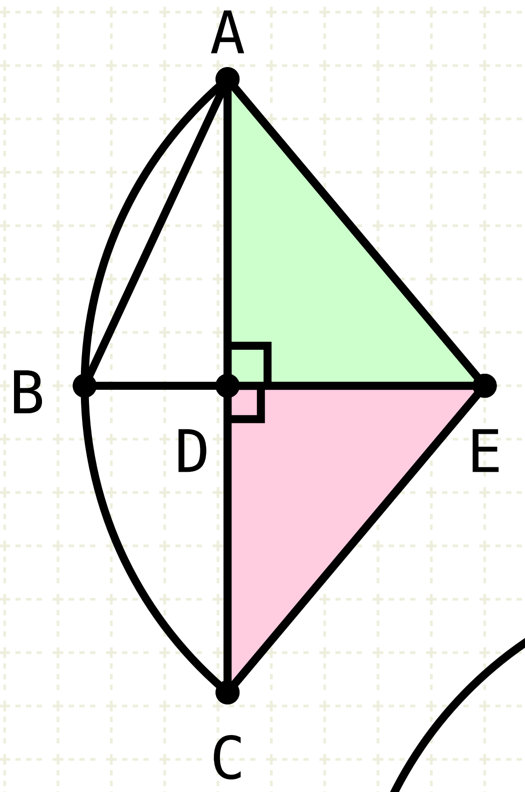
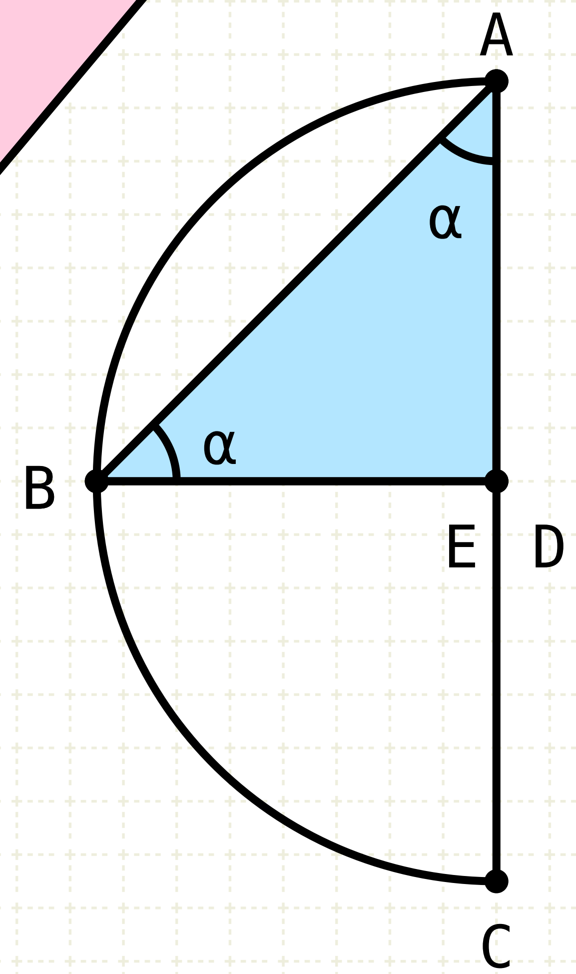
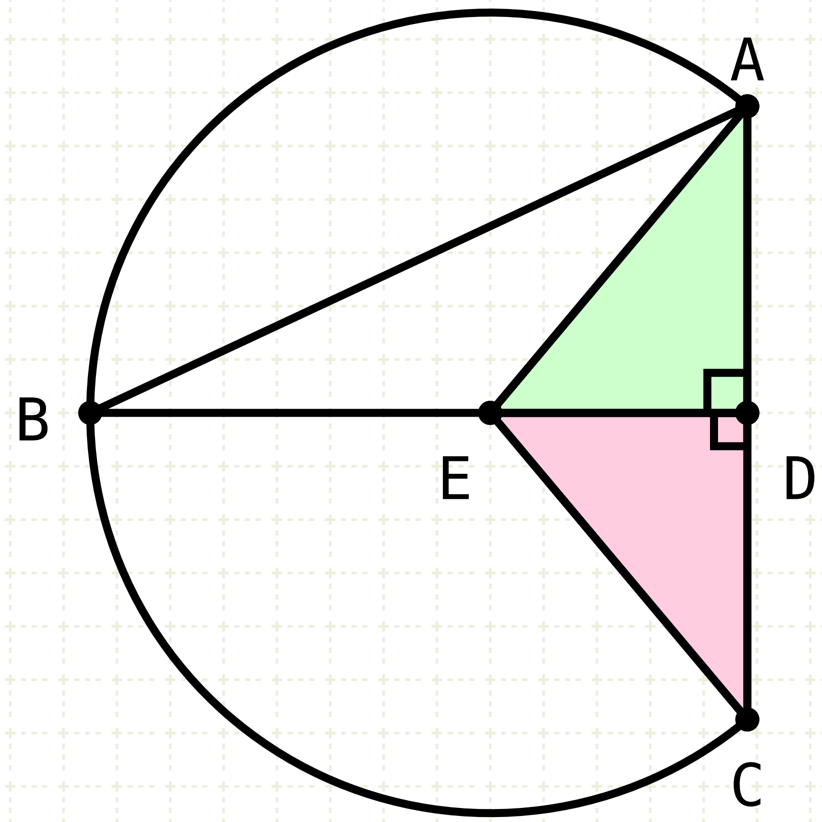






AD = DC
∠BAE = ∠ABD
AE = BE
AD = DC
ΔADE ≡ ΔDEC
AE = EC

Proof

Since angles EBA and EAB are equal, the triangle is an isosceles, and the lines AE and BE are equal (I·6)

Draw line CE

If D and E are not the same point,

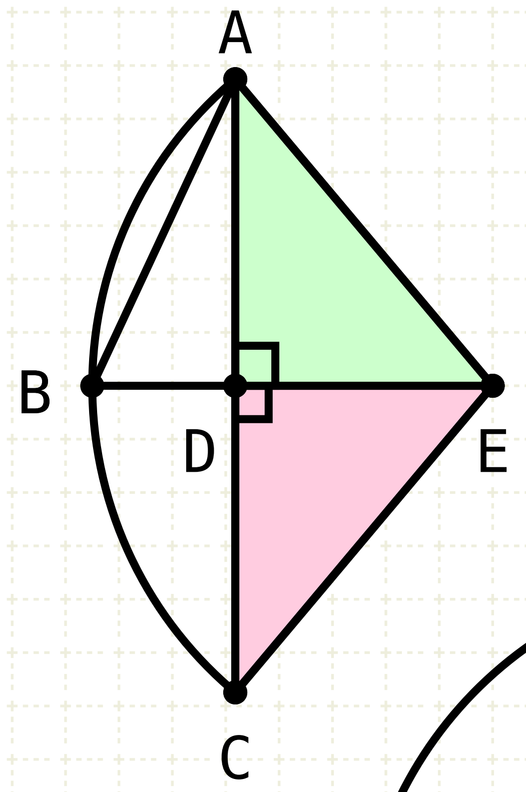
Consider triangles ADE and ACE

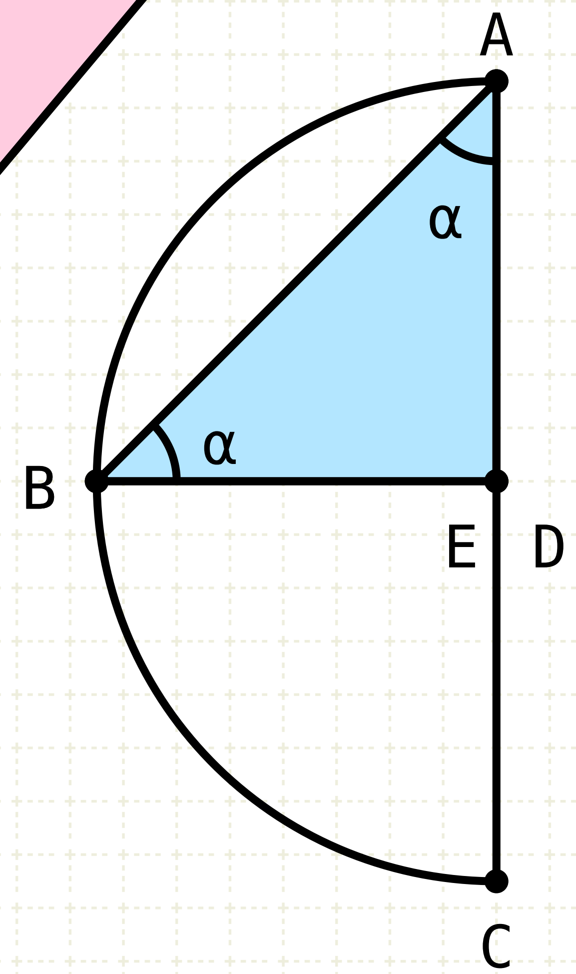
Two sides are equal, (D bisects AC, and DE is common), and the angles ADE and EDC are equal (BD is perpendicular to AC)

Therefore the triangles ADE and DEC are equal, and AE equals EC (I·4)

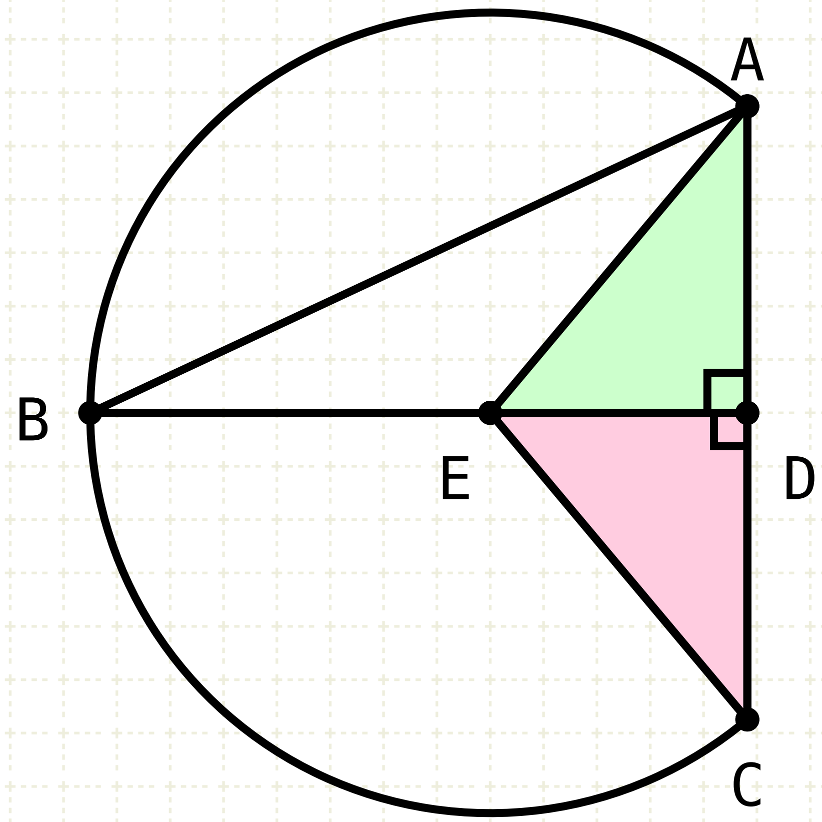
Proposition 25 of Book III

Given a segment of a circle, to describe the complete circle of which it is a segment.





$AD = DC$
$\angle BAE = \angle ABD$
$AE = BE$
$AD = DC$
$\Delta ADE \equiv \Delta DEC$
$AE = EC$



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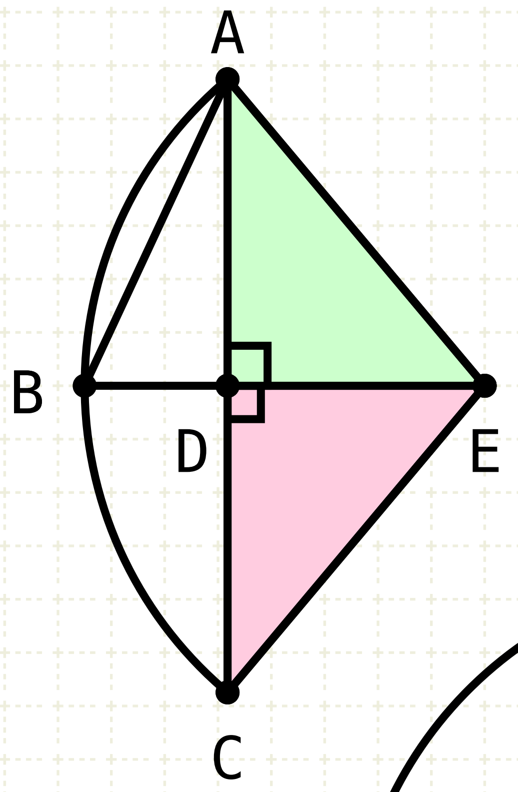
Therefore the triangles ADE and DEC are equal, and AE equals EC (I·4)

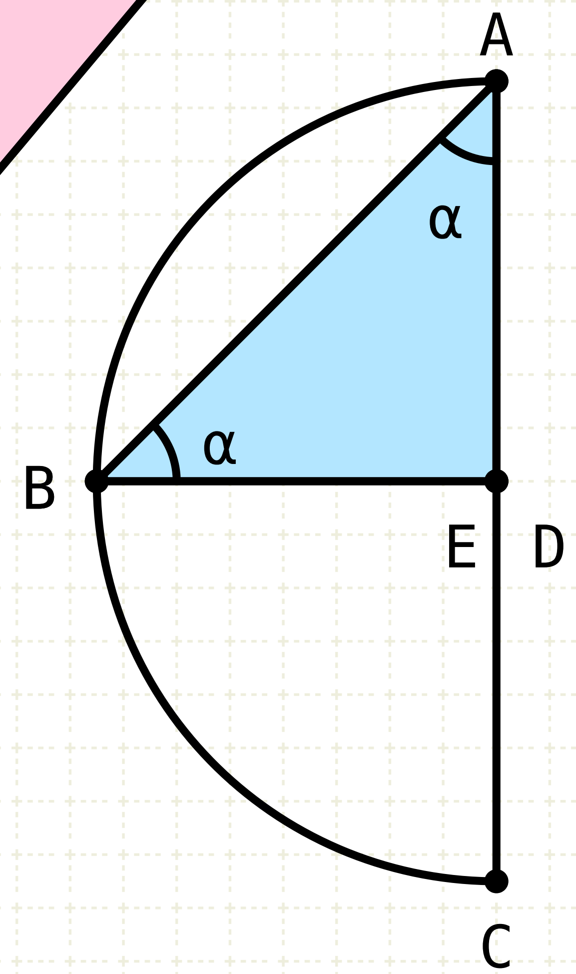
If D and E are the same point,

AE equals AD, and DC equals EC, so AE equals EC

Proposition 25 of Book III

Given a segment of a circle, to describe the complete circle of which it is a segment.





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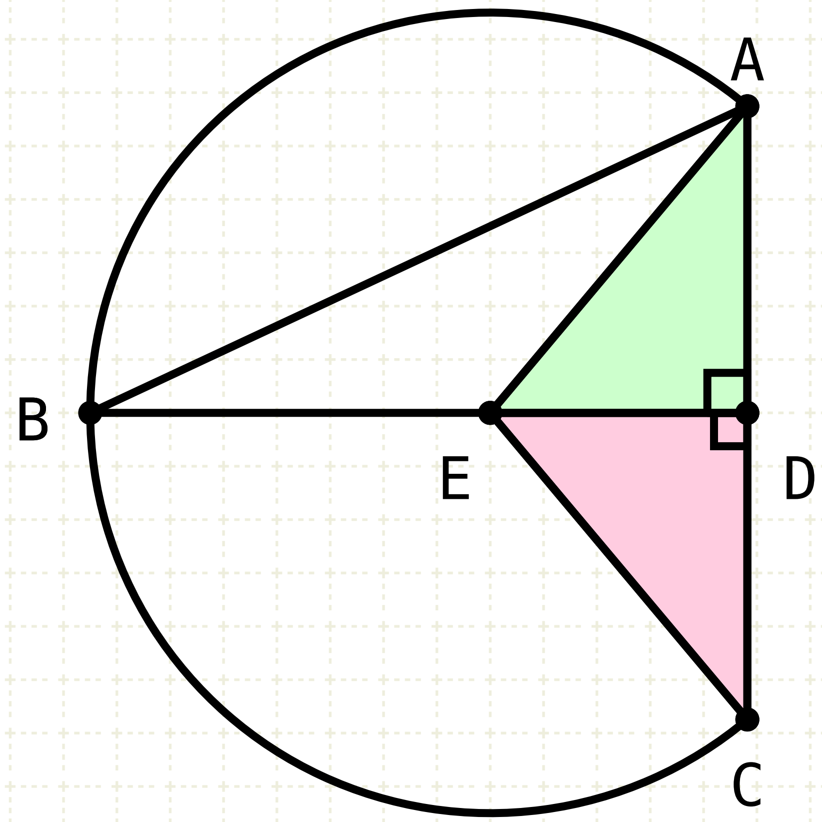

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Since angles EBA and EAB are equal, the triangle is an isosceles, and the lines AE and BE are equal (I·6)

Draw line CE

If D and E are not the same point,

Consider triangles ADE and ACE

Two sides are equal, (D bisects AC, and DE is common), and the angles ADE and EDC are equal (BD is perpendicular to AC)

Therefore the triangles ADE and DEC are equal, and AE equals EC (I·4)

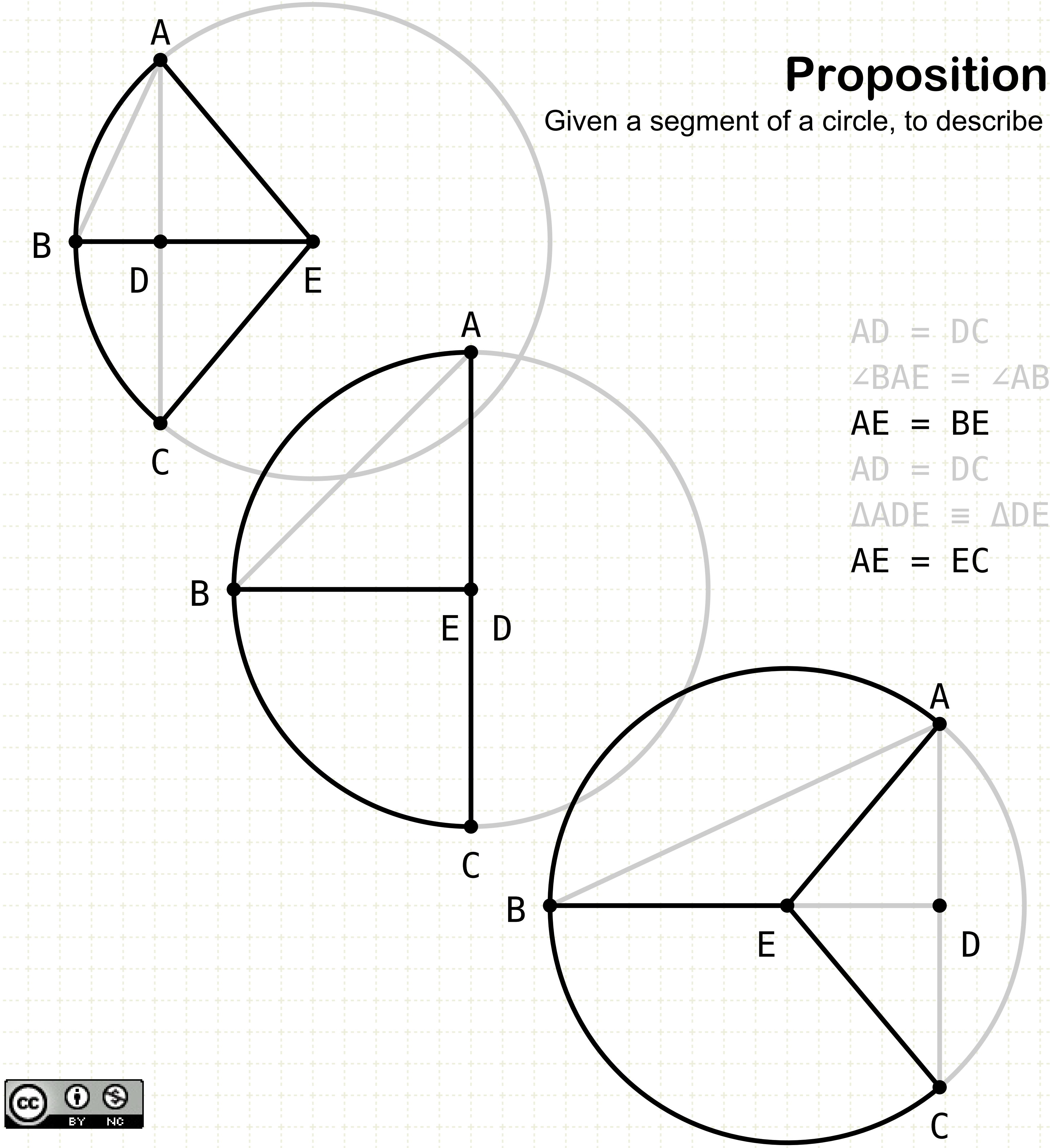
If D and E are the same point,

AE equals AD, and DC equals EC, so AE equals EC

The three lines AE, BE, and CE are equal

Proposition 25 of Book III

Given a segment of a circle, to describe the complete circle of which it is a segment.



$AD = DC$

$\angle BAE = \angle ABD$

$AE = BE$

$AD = DC$

$\Delta ADE \equiv \Delta DEC$

$AE = EC$

Proof

Since angles EBA and EAB are equal, the triangle is an isosceles, and the lines AE and BE are equal (I·6)

Draw line CE

If D and E are not the same point,

Consider triangles ADE and ACE

Two sides are equal, (D bisects AC, and DE is common), and the angles ADE and EDC are equal (BD is perpendicular to AC)

Therefore the triangles ADE and DEC are equal, and AE equals EC (I·4)

If D and E are the same point,

AE equals AD, and DC equals EC, so AE equals EC

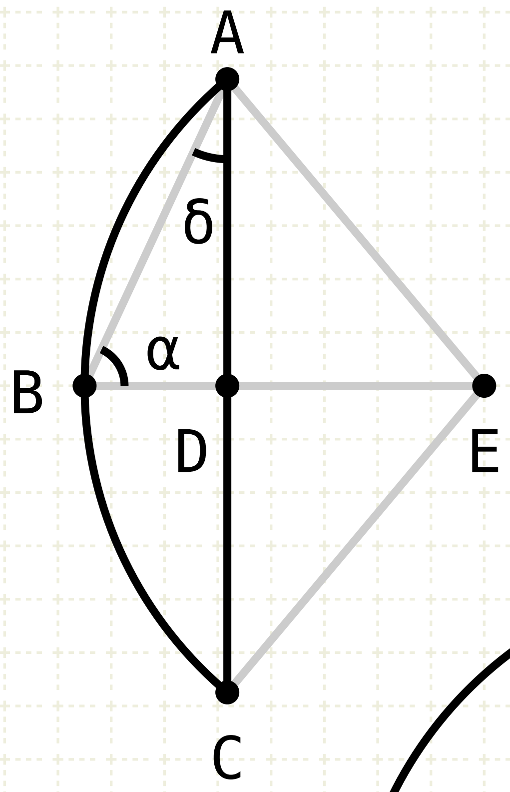
The three lines AE, BE, and CE are equal

If more than two EQUAL lines fall from a point within a circle to the circumference of a circle, then that point is the centre (III·9)

Therefore E is the centre of the circle, and the radius is AE

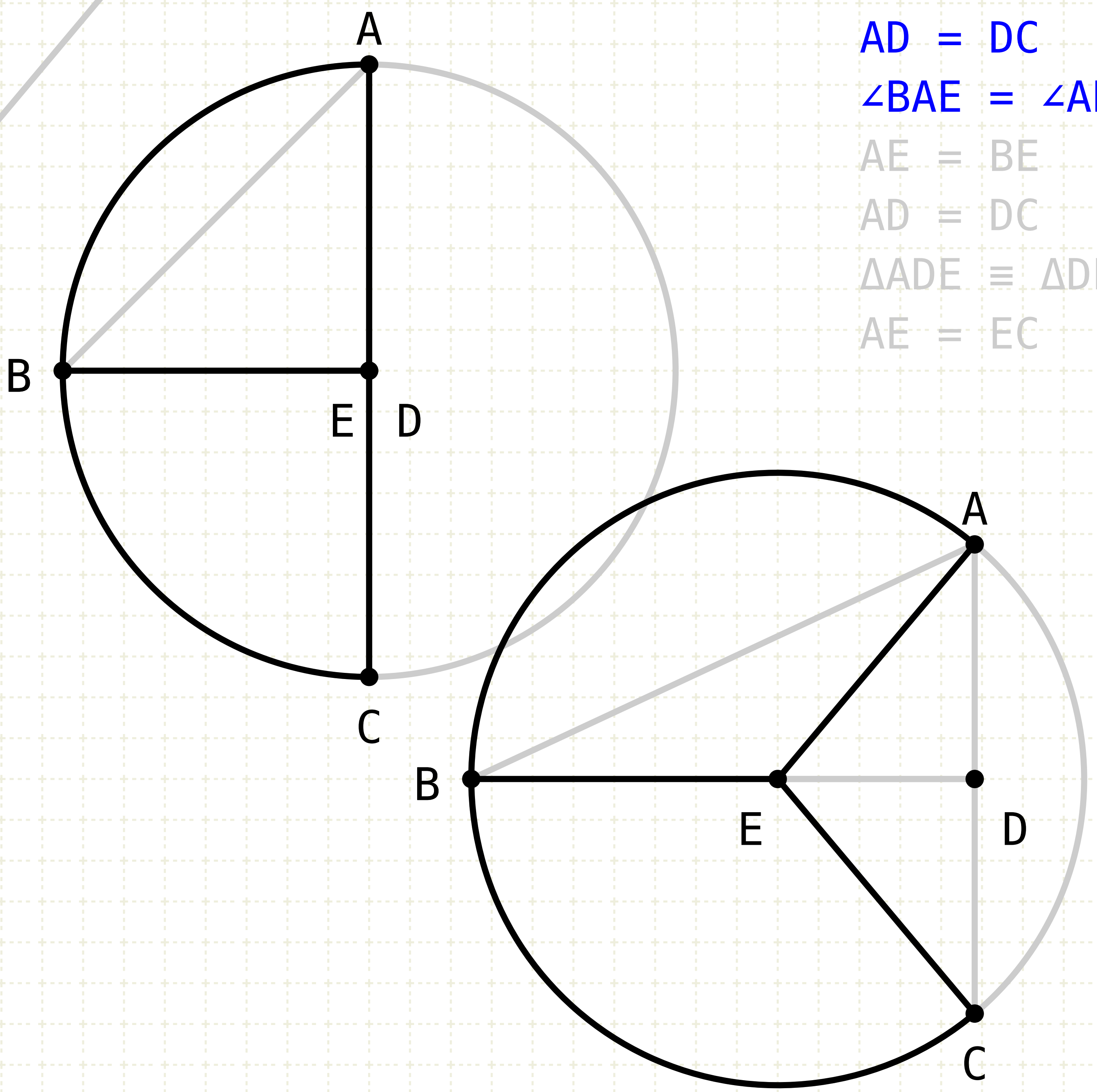
Proposition 25 of Book III

Given a segment of a circle, to describe the complete circle of which it is a segment.



If $\alpha > \delta$

then the segment is less than a semicircle



$AD = DC$

$\angle BAE = \angle ABD$

$AE = BE$

$AD = DC$

$\Delta ADE \equiv \Delta DEC$

$AE = EC$

Proof

Since angles EBA and EAB are equal, the triangle is an isosceles, and the lines AE and BE are equal (I·6)

Draw line CE

If D and E are not the same point,

Consider triangles ADE and ACE

Two sides are equal, (D bisects AC, and DE is common), and the angles ADE and EDC are equal (BD is perpendicular to AC)

Therefore the triangles ADE and DEC are equal, and AE equals EC (I·4)

If D and E are the same point,

AE equals AD, and DC equals EC, so AE equals EC

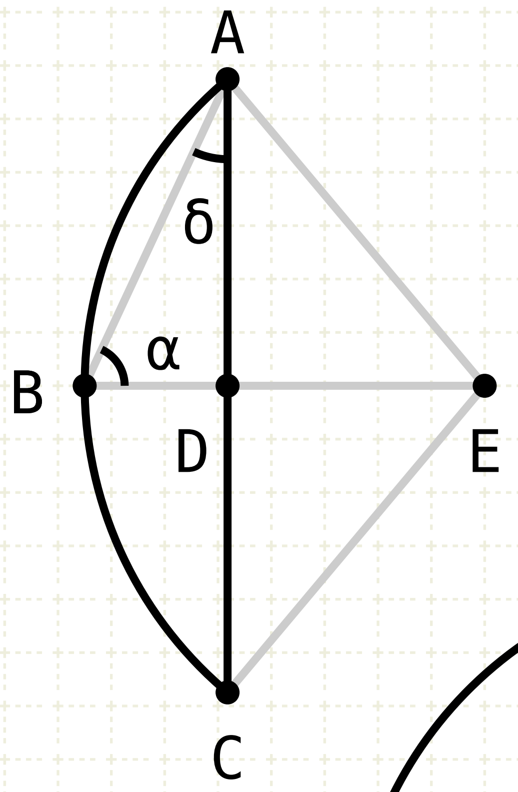
The three lines AE, BE, and CE are equal

If more than two EQUAL lines fall from a point within a circle to the circumference of a circle, then that point is the centre (III·9)

Therefore E is the centre of the circle, and the radius is AE

Proposition 25 of Book III

Given a segment of a circle, to describe the complete circle of which it is a segment.



If $\alpha > \delta$

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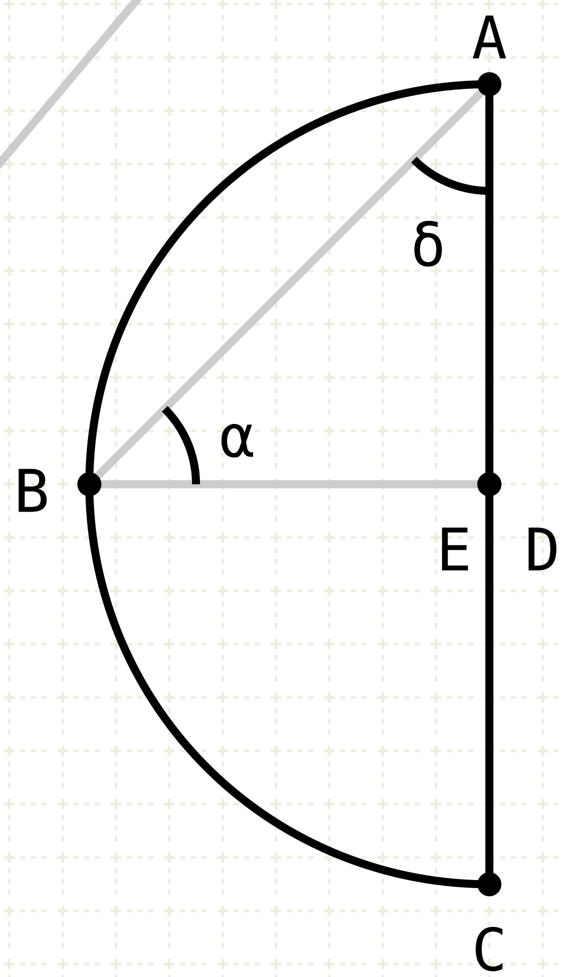
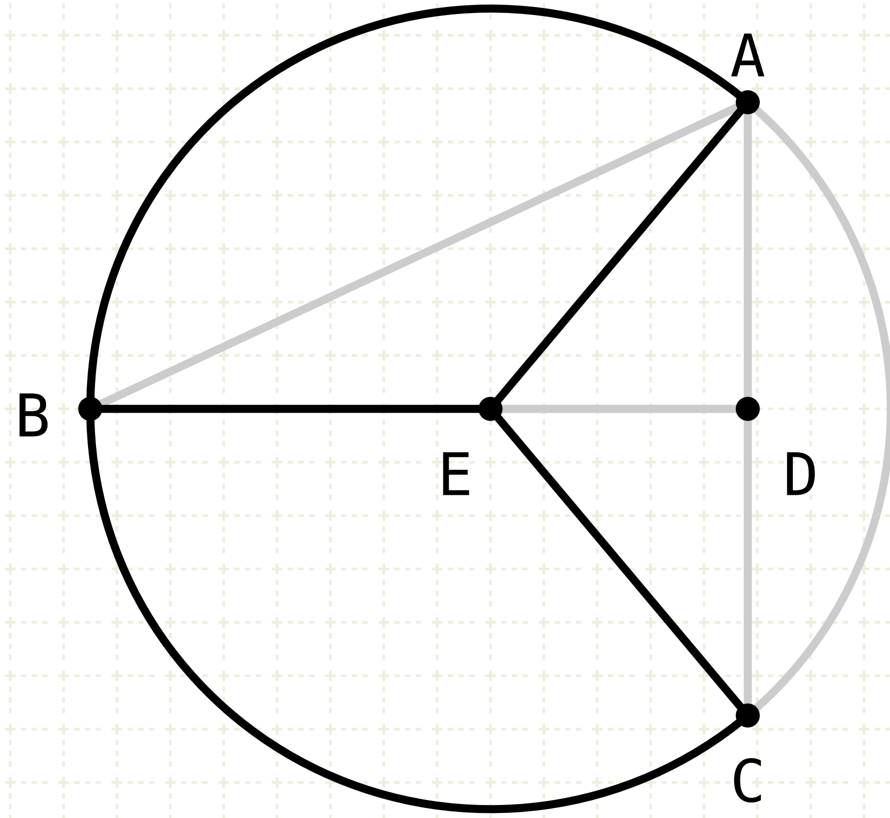


If $\alpha = \delta$

then the segment is a semicircle



$AD = DC$

$\angle BAE = \angle ABD$

$AE = BE$

$AD = DC$

$\Delta ADE \equiv \Delta DEC$

$AE = EC$

Proof

Since angles EBA and EAB are equal, the triangle is an isosceles, and the lines AE and BE are equal (I·6)

Draw line CE

If D and E are not the same point,

Consider triangles ADE and ACE

Two sides are equal, (D bisects AC, and DE is common), and the angles ADE and EDC are equal (BD is perpendicular to AC)

Therefore the triangles ADE and DEC are equal, and AE equals EC (I·4)

If D and E are the same point,

AE equals AD, and DC equals EC, so AE equals EC

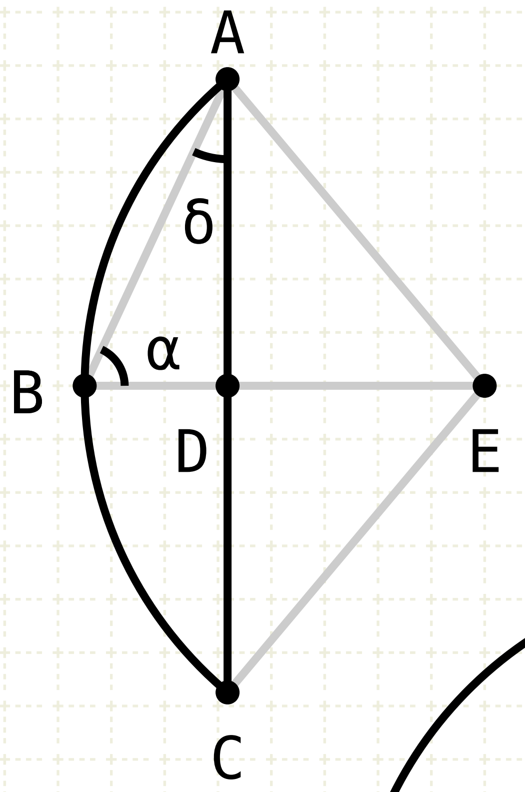
The three lines AE, BE, and CE are equal

If more than two EQUAL lines fall from a point within a circle to the circumference of a circle, then that point is the centre (III·9)

Therefore E is the centre of the circle, and the radius is AE

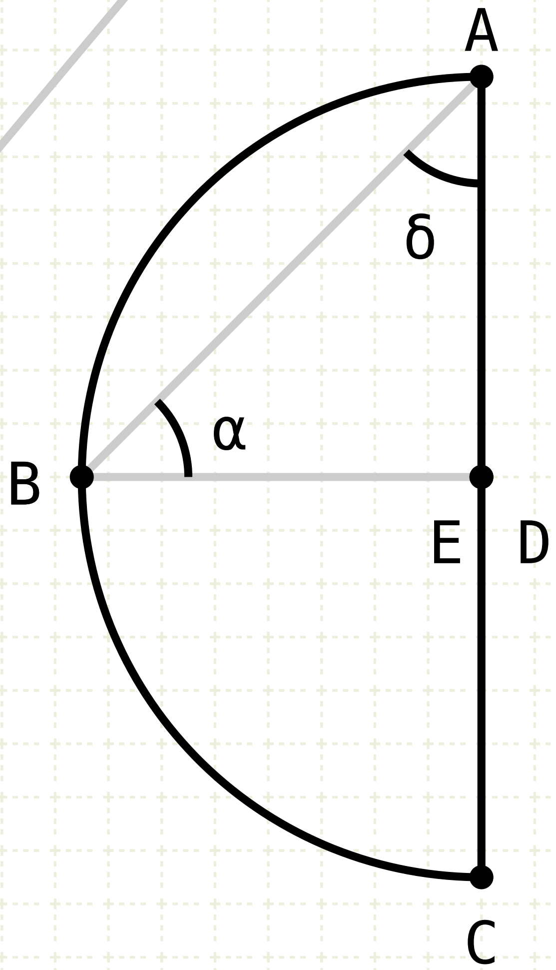
Proposition 25 of Book III

Given a segment of a circle, to describe the complete circle of which it is a segment.



If α > δ

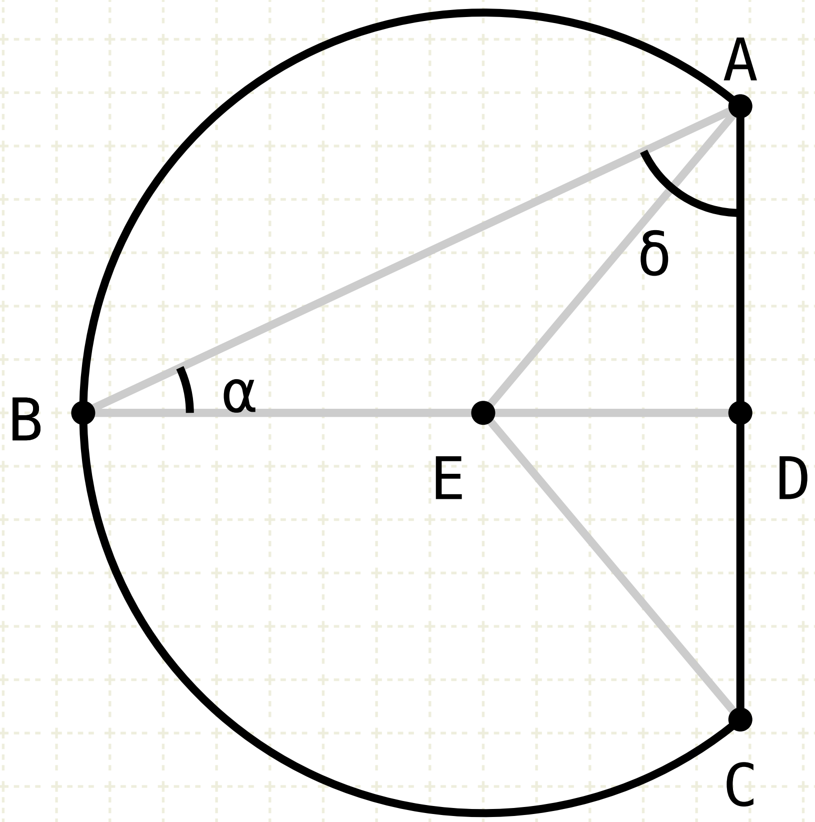
then the segment is less than a semicircle



If α = δ

then the segment is a semicircle

$AD = DC$
$\angle BAE = \angle ABD$
$AE = BE$
$AD = DC$
$\Delta ADE \equiv \Delta DEC$
$AE = EC$



If α < δ

then the segment is larger than a semicircle

Proof

Since angles EBA and EAB are equal, the triangle is an isosceles, and the lines AE and BE are equal (I·6)

Draw line CE

If D and E are not the same point,

Consider triangles ADE and ACE

Two sides are equal, (D bisects AC, and DE is common), and the angles ADE and EDC are equal (BD is perpendicular to AC)

Therefore the triangles ADE and DEC are equal, and AE equals EC (I·4)

If D and E are the same point,

AE equals AD, and DC equals EC, so AE equals EC

The three lines AE, BE, and CE are equal

If more than two EQUAL lines fall from a point within a circle to the circumference of a circle, then that point is the centre (III·9)

Therefore E is the centre of the circle, and the radius is AE

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