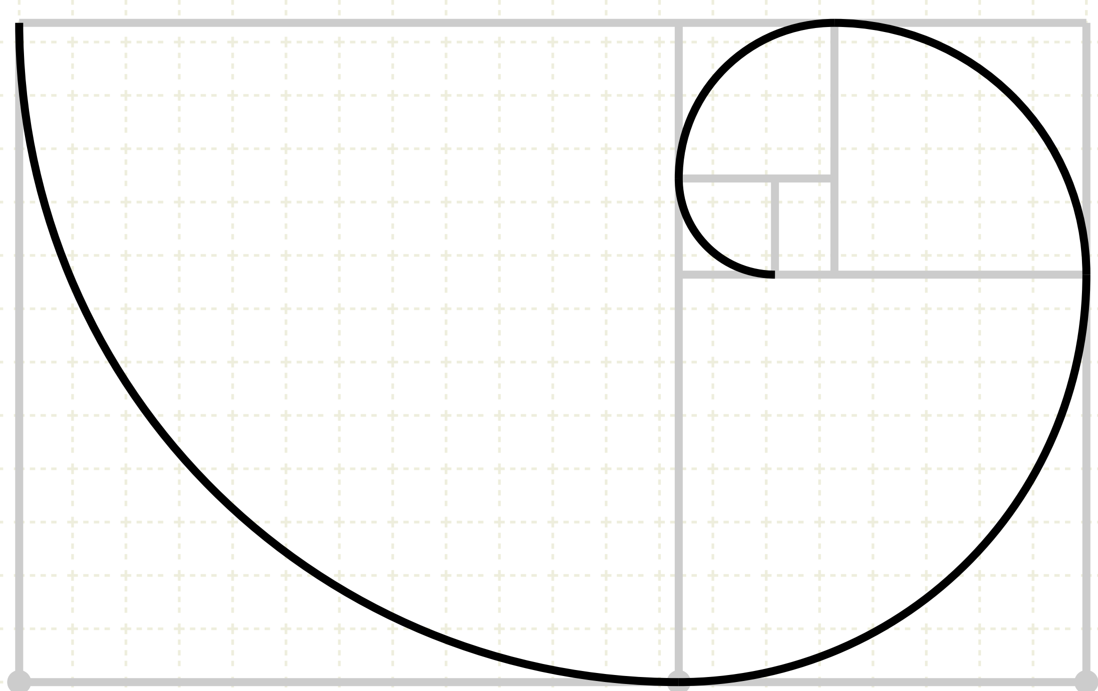
Euclid's Elements

Book VI



One can state, without exaggeration, that the observation of and the search for similarities and differences are the basis of all human knowledge.

Alfred Nobel

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Proposition 8 of Book VI

If in a right-angled triangle a perpendicular be drawn from the right angle to the base, the triangles adjoining the perpendicular are similar both to the whole and to one another.

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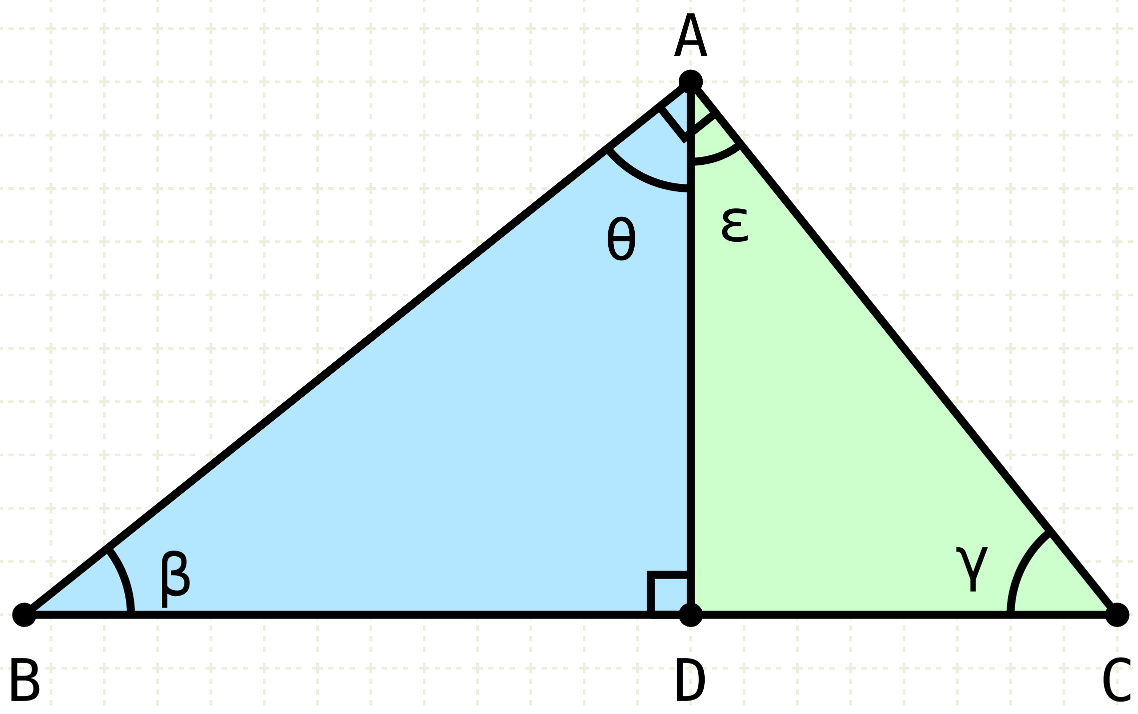

$\gamma = \theta$, $\beta = \varepsilon$

AD:AB = AC:BC = CD:AC

AB:BD = BC:AB = AC:AD

AD:BD = AC:AB = CD:AD

In other words

If we take a right angle triangle (ABC), where the angle BAC is a right angle, and we drop a perpendicular AD to the side BC, then…

The triangles BAD and ADC will be similar to the original triangle ABC, AND they will be similar to each other

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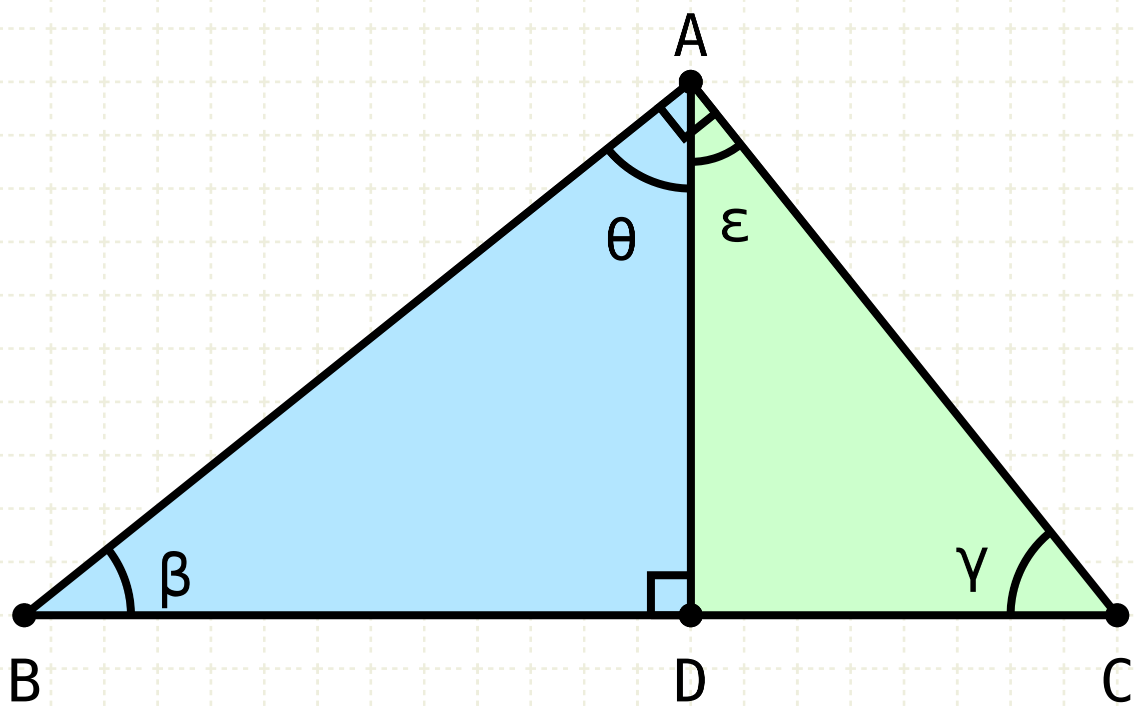

Proof

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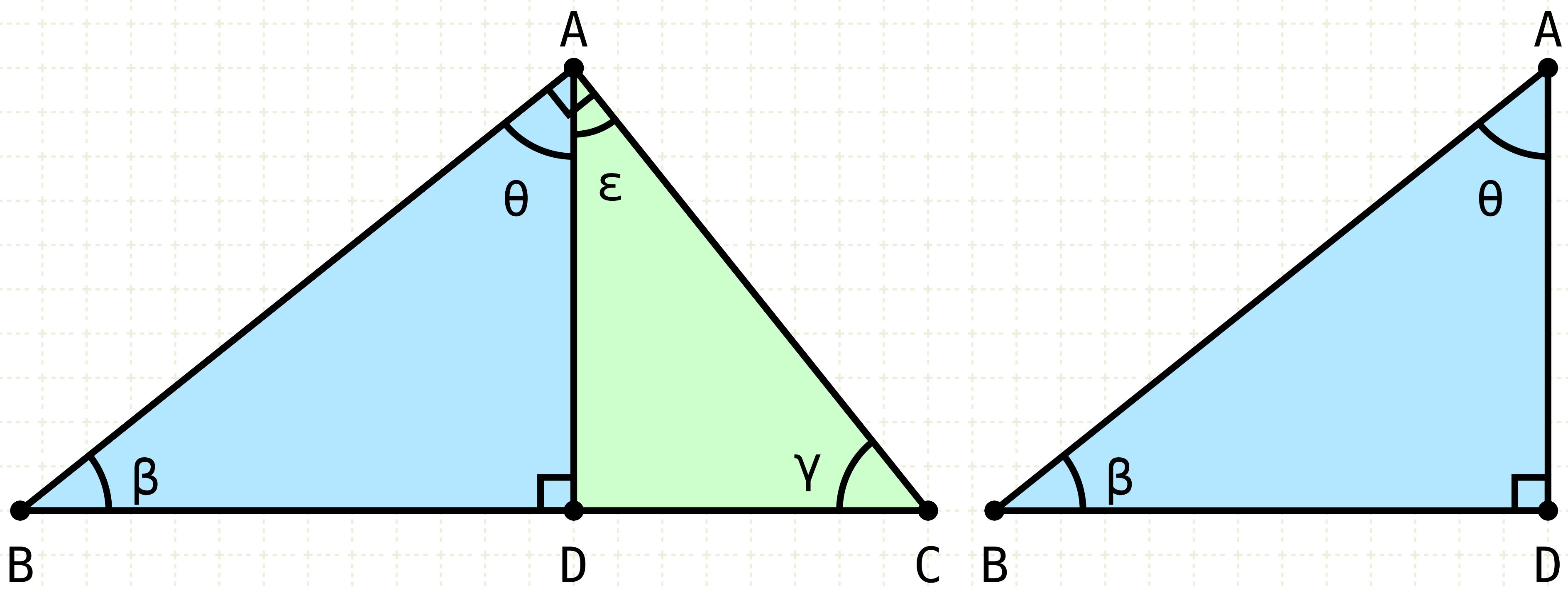

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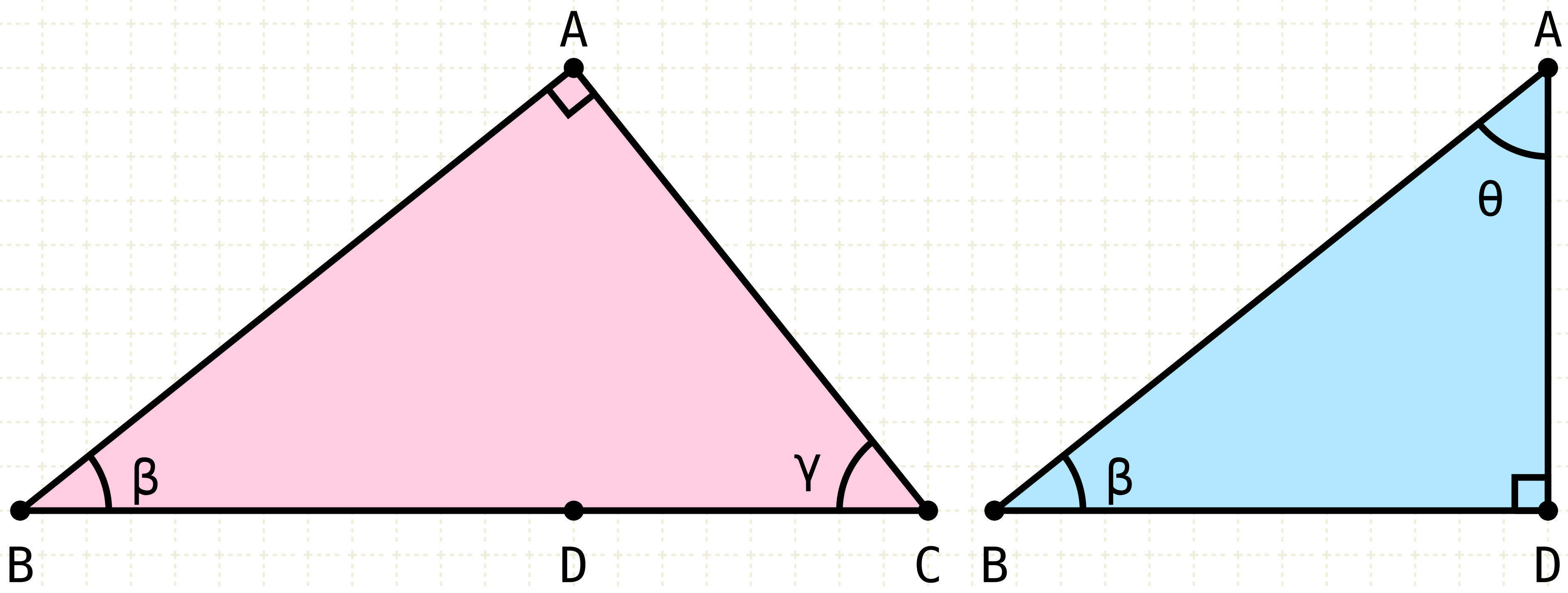

Proof

Angle BAC equals angle ADB (both are right angles), and the angle β is common to the two triangles ABD and ABC

Thus the remaining angles (BCA and BAD) must be equal (I·32)

$\angle BAC = \angle BDA = ∟$

$\theta = \gamma$

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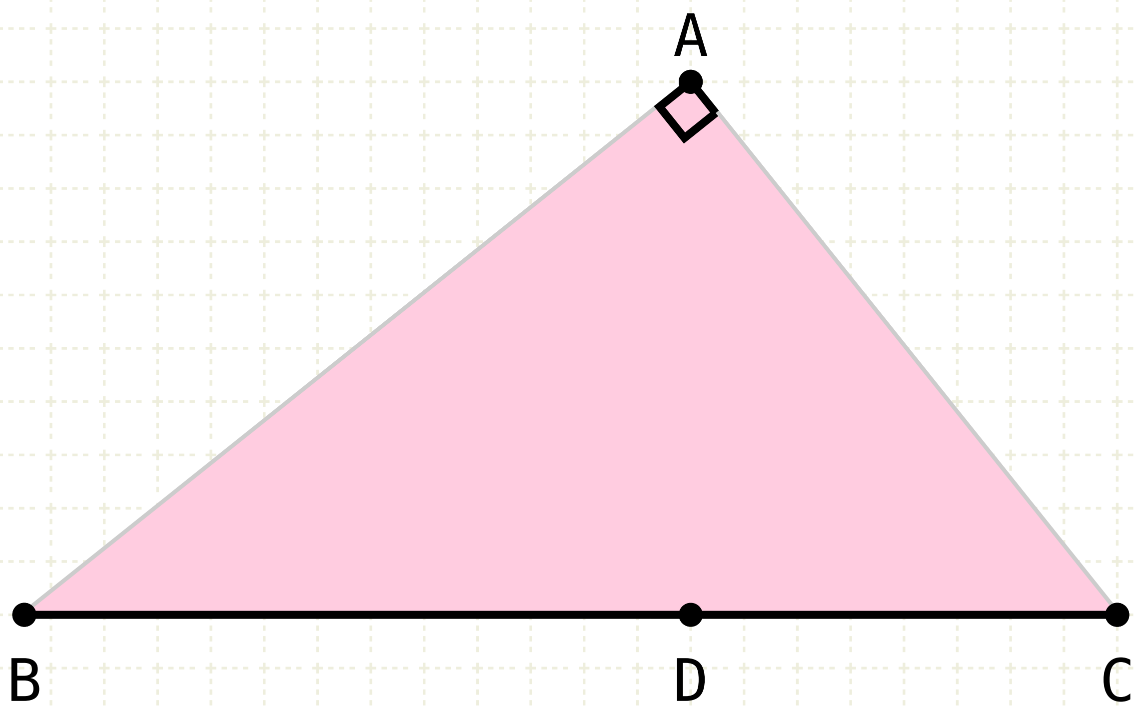

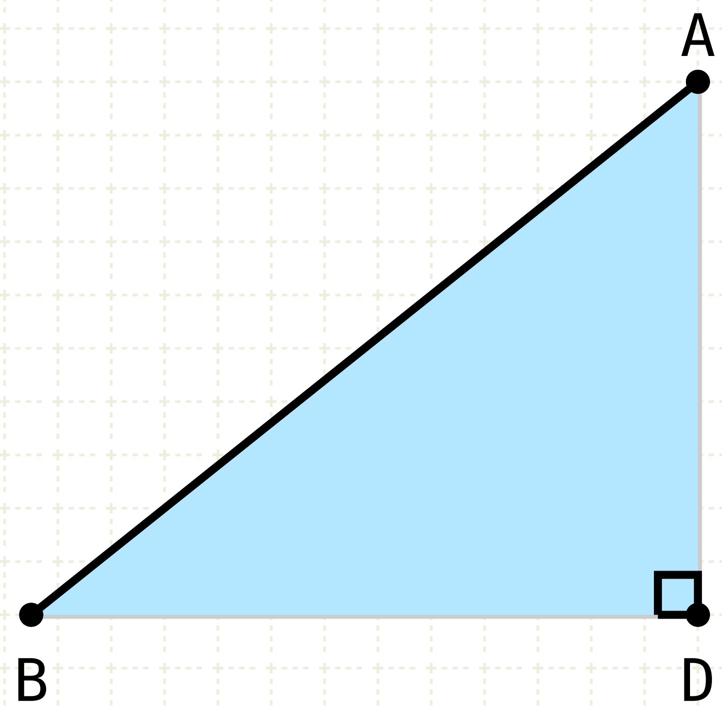

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Therefore triangle ABC is equiangular with triangle ABD

Therefore, as BC (subtends the right angle in ABC) is to BA (subtends the right angle in the triangle ABD),

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BC:BA

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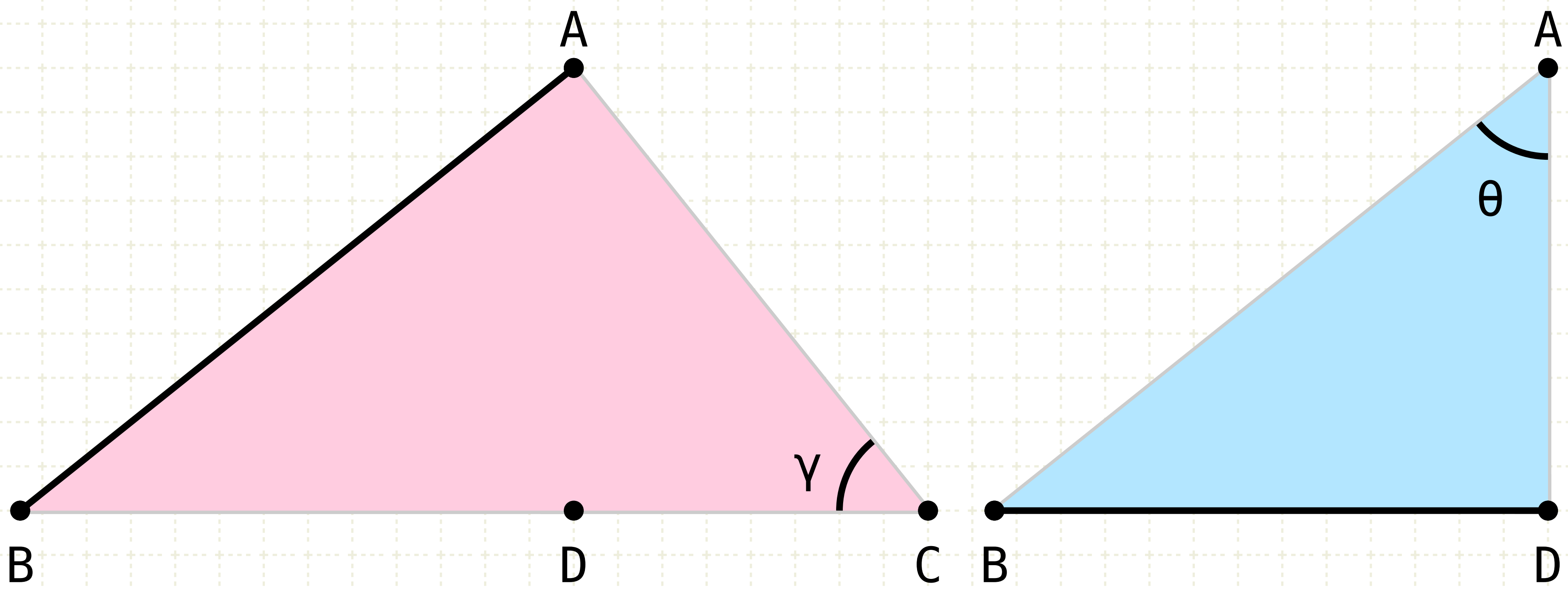

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$BC:BA = AB:BD$

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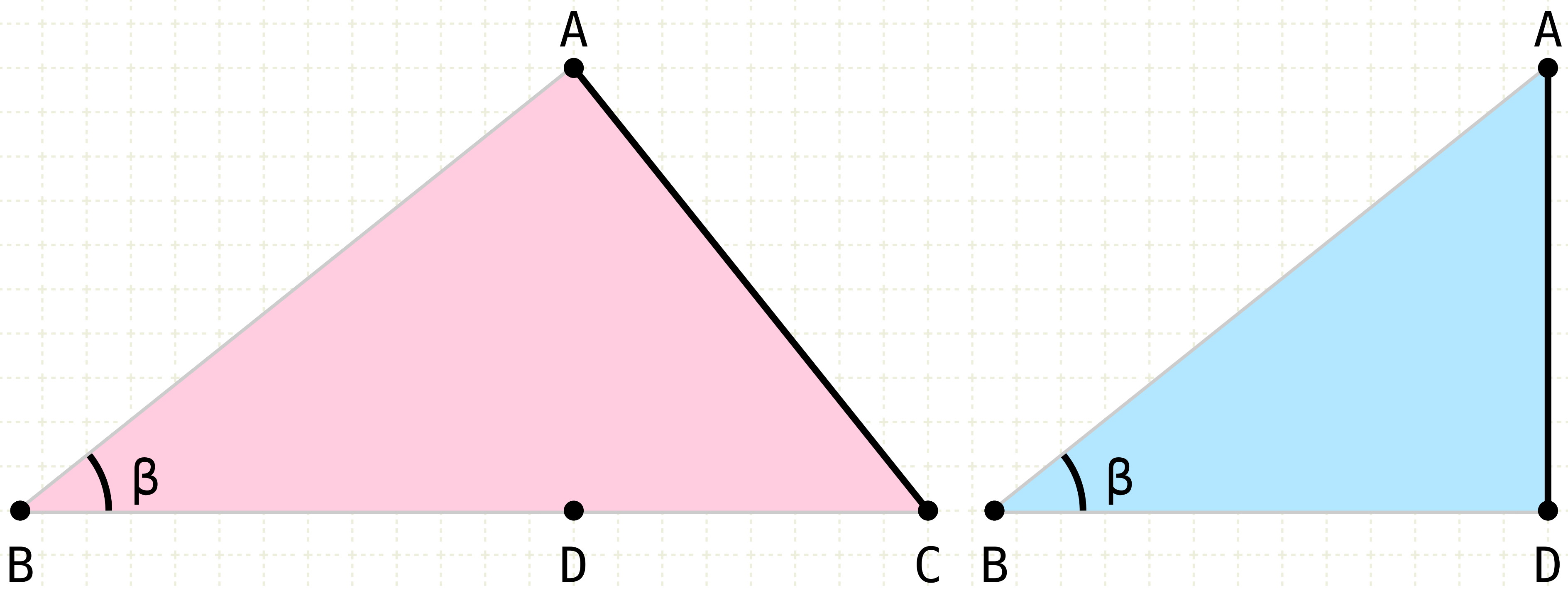

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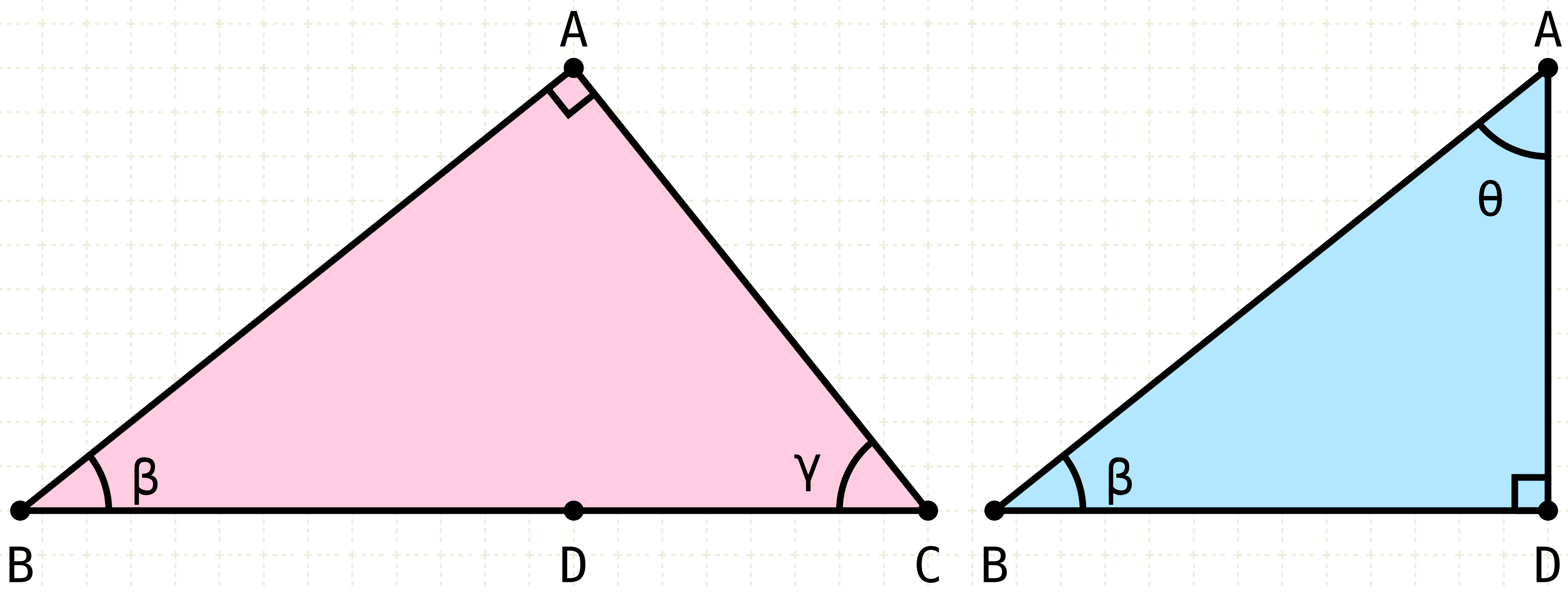

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and so is AC (subtends angle β in ABC) to AD (subtends the angle β in ABD) (VI·4)

So, ABC and ABD are equiangular, and their sides are proportional, which, by definition (VI.Def.1), means they are similar triangles

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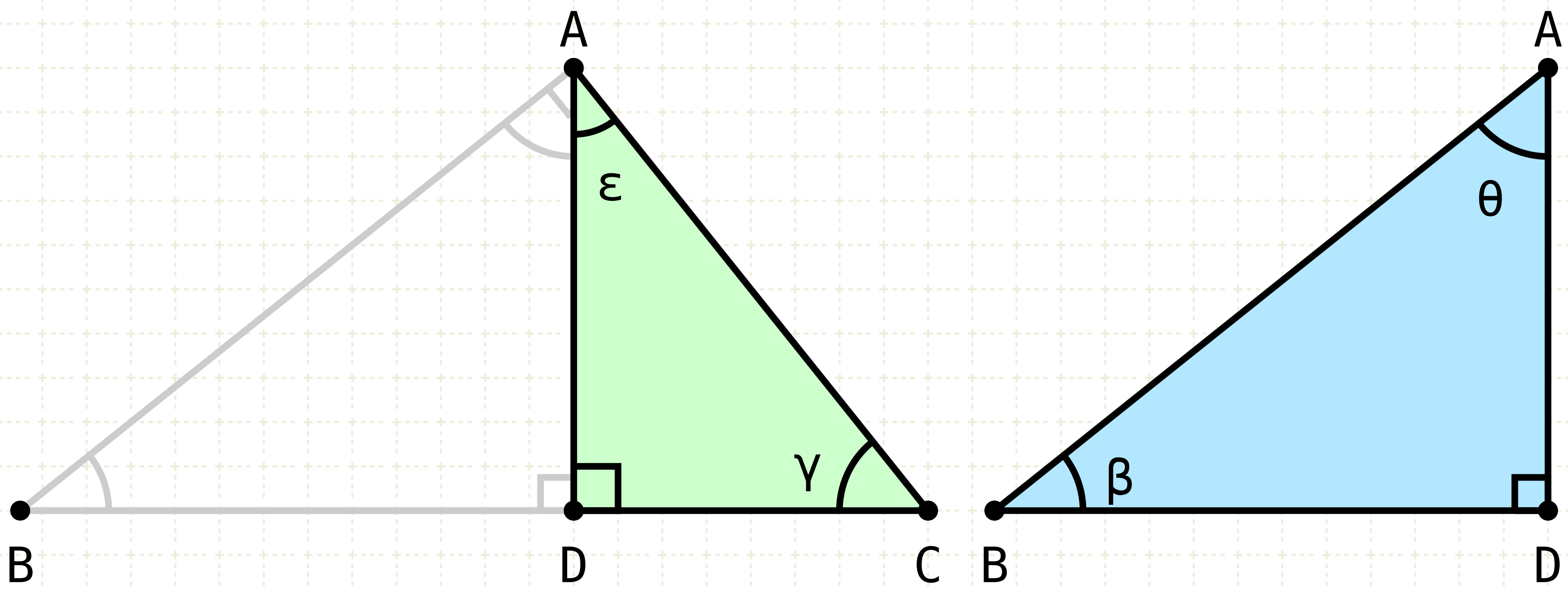

Proof (cont)

∠BAC = ∠BDA = ∟

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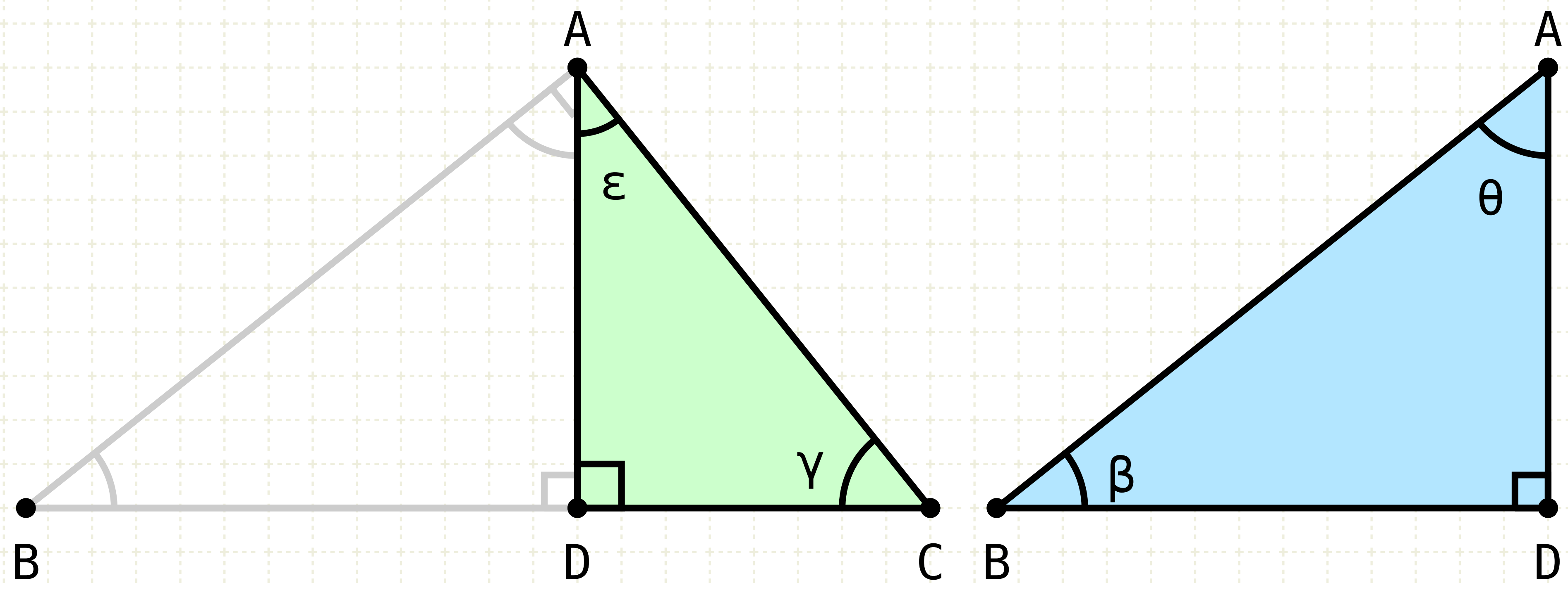

Proof (cont)

Angle BDA equals angle ADC (both are right angles), and the angle θ is already been proven equal to γ

Thus the remaining angles (β and ε) must be equal (I·32)

∠BAC = ∠BDA = ∟

θ = γ

BC:BA = AB:BD = AC:AD

∠BDC = ∠BDA = ∟

ε = β

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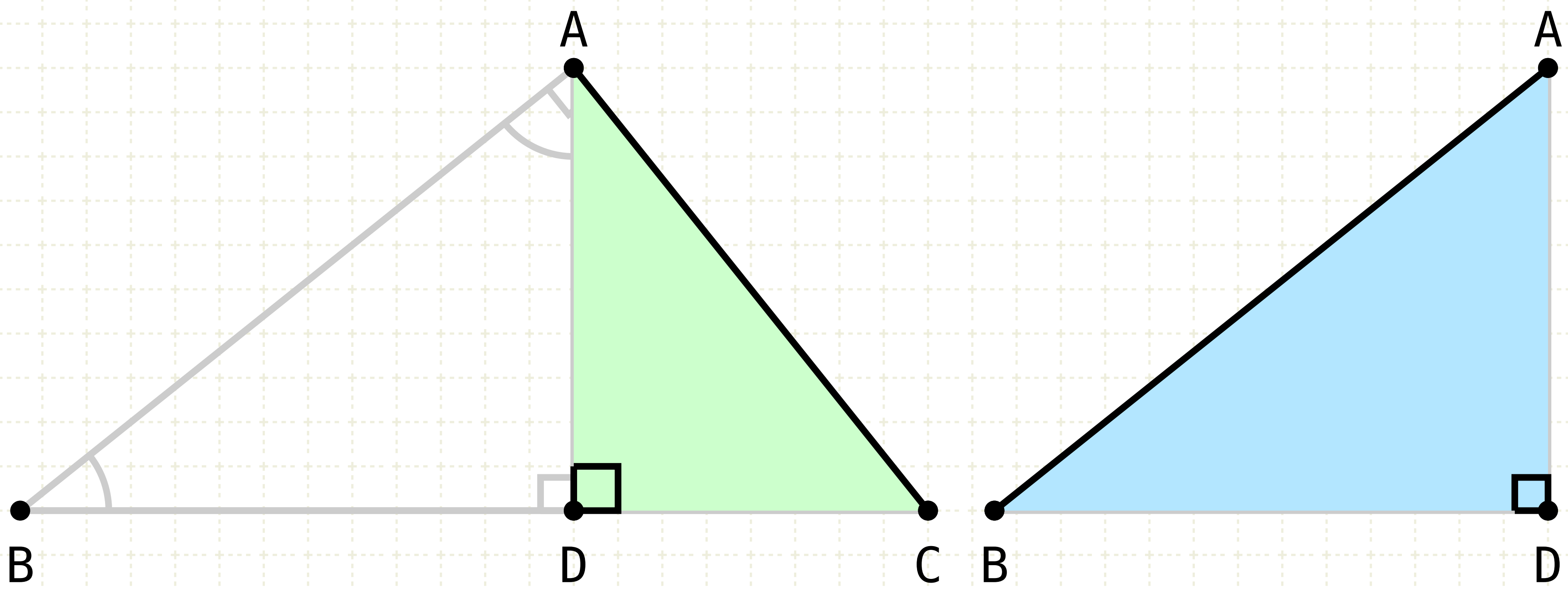

Proof (cont)

Angle BDA equals angle ADC (both are right angles), and the angle θ is already been proven equal to γ

Thus the remaining angles (β and ε) must be equal (I·32)

Therefore triangle ABD is equiangular with triangle ADC

Therefore, as AB (subtends the right angle in ABD) is to AC (subtends the right angle in the triangle ADC),

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BC:BA = AB:BD = AC:AD

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AB:AC

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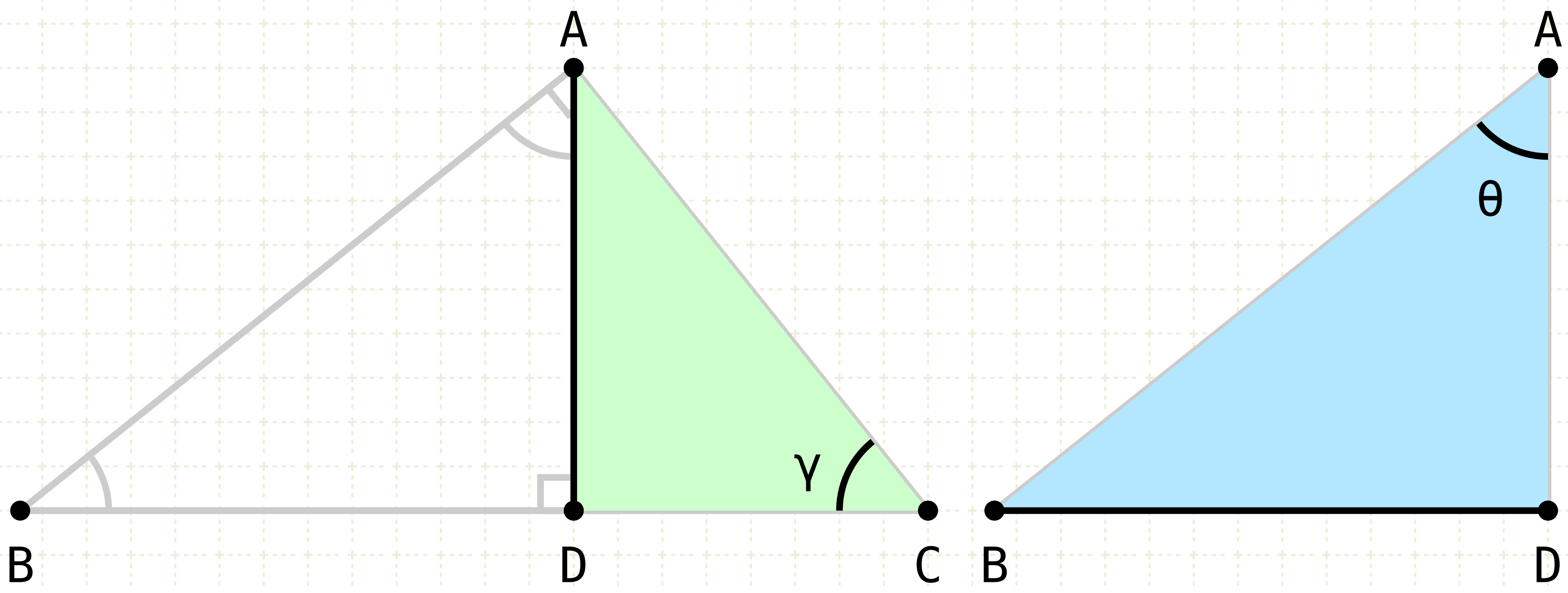

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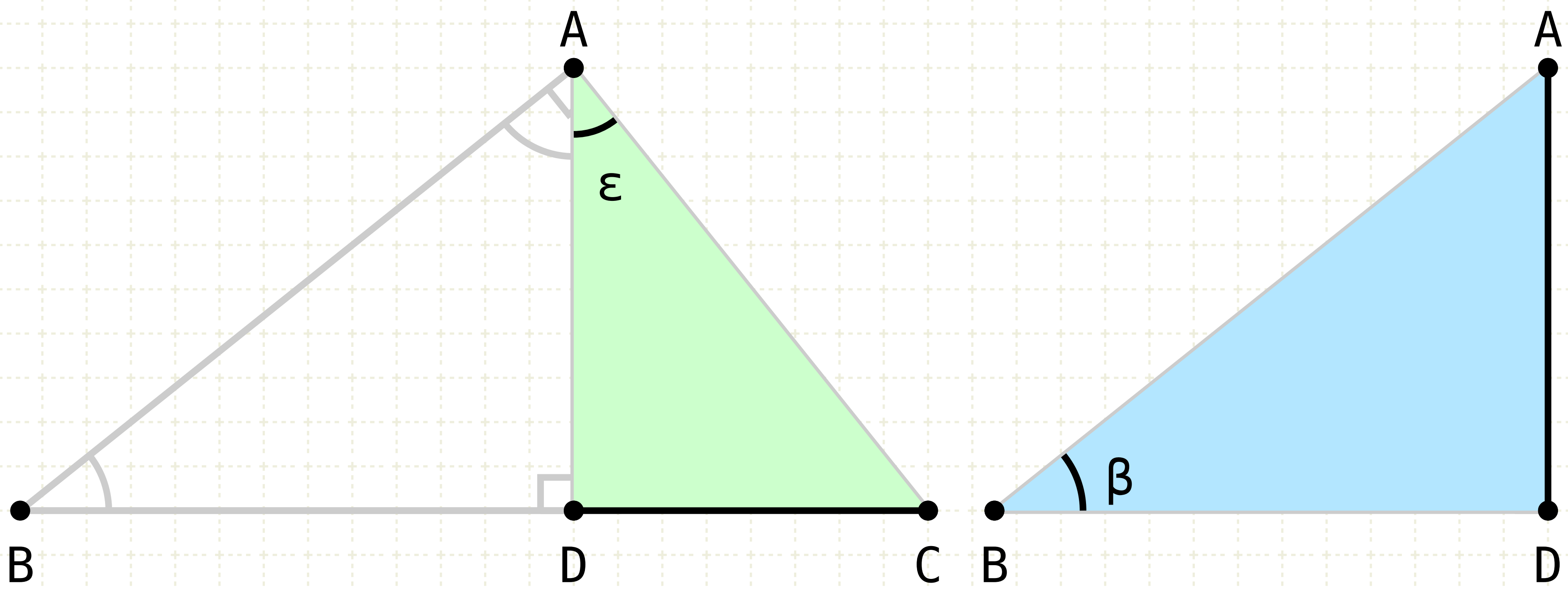

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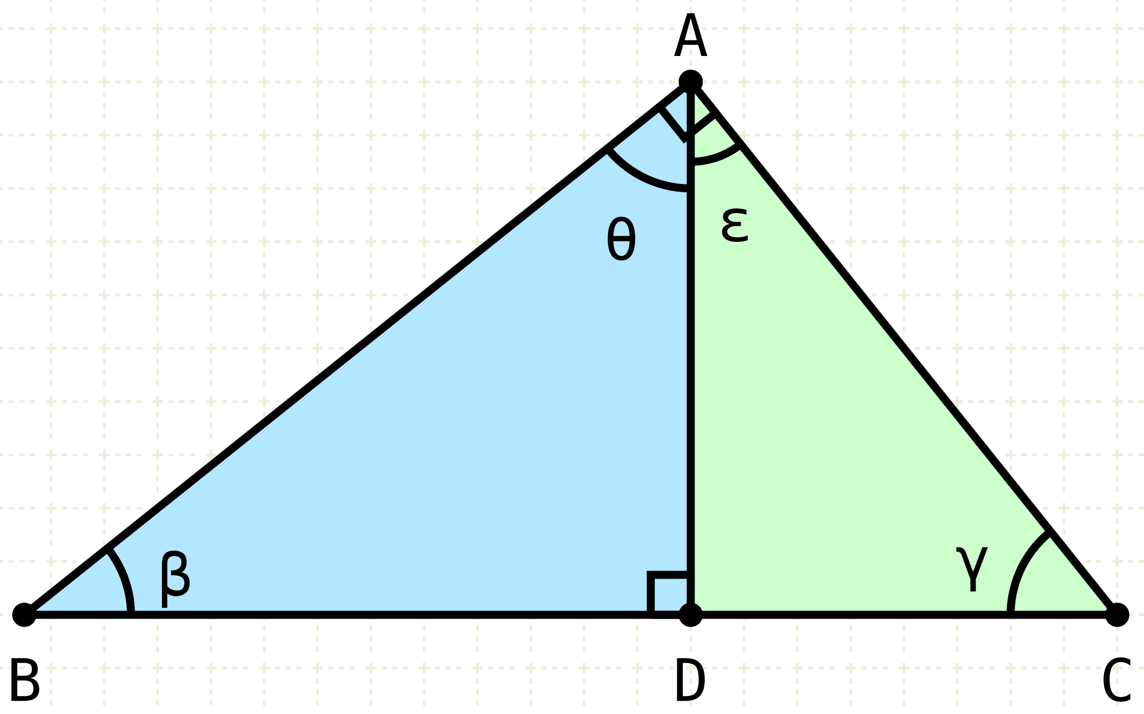

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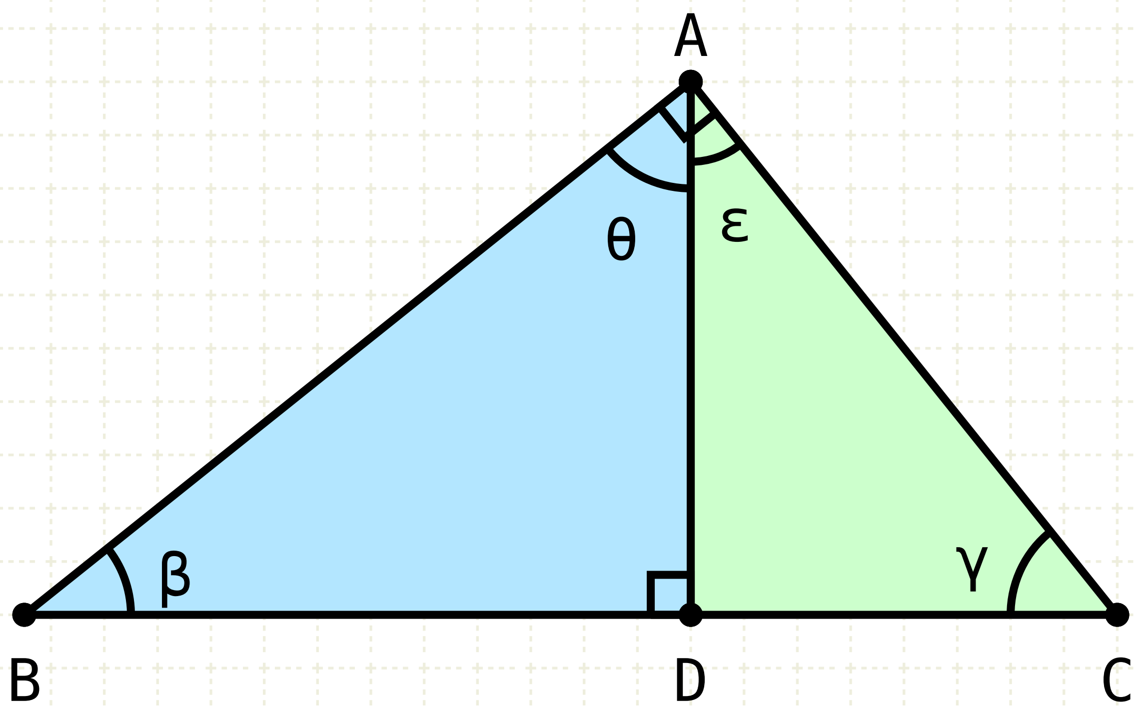

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Porism

In a right angle triangle, if a line be drawn from the right angle, perpendicular to the base, then this line will be the mean proportional between the segments of the base.

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