Euclid's Elements

Book VII

Definitions:

1 A unit is that by virtue of which each of the things that exist is called one

2 A number is a multitude composed of units. (not one)

3 A number is part of a number, the less of the greater, when it measures the greater

11 A prime number is that which is measured by a unit alone.

12 Numbers prime to one another are those which are measured by a unit alone as a common measure

15 A number is said to multiply a number when that which is multiplied is added to itself as many times as there are units in the other, and thus some number is produced.

20 Numbers are proportional when the first is the same multiple, or the same part, or the same parts, of the second that the third is of the fourth.

As long as algebra and geometry have been separated, their progress have been slow and their uses limited; but when these two sciences have been united, they have lent each mutual forces, and have marched together towards perfection.

**Joseph-Louis Lagrange
(1736 to 1813)**

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Proposition 10 of Book VII

If a number be a parts of a number, and another be the same parts of another, alternately also, whatever part or parts the first of the third, the same part, or the same parts, will the second also be of the fourth

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In other words

If b is the same fraction of a as d is to c, and b another fraction of d, then the fraction of b to d is the same fraction of a to c

$$b = (p/q) \cdot a$$

$$d = (p/q) \cdot c$$

$$\text{if } b = (m/n) \cdot d \rightarrow a = (m/n) \cdot c$$

$$\frac{b}{a} = \frac{d}{c} \rightarrow \frac{b}{d} = \frac{a}{c}$$

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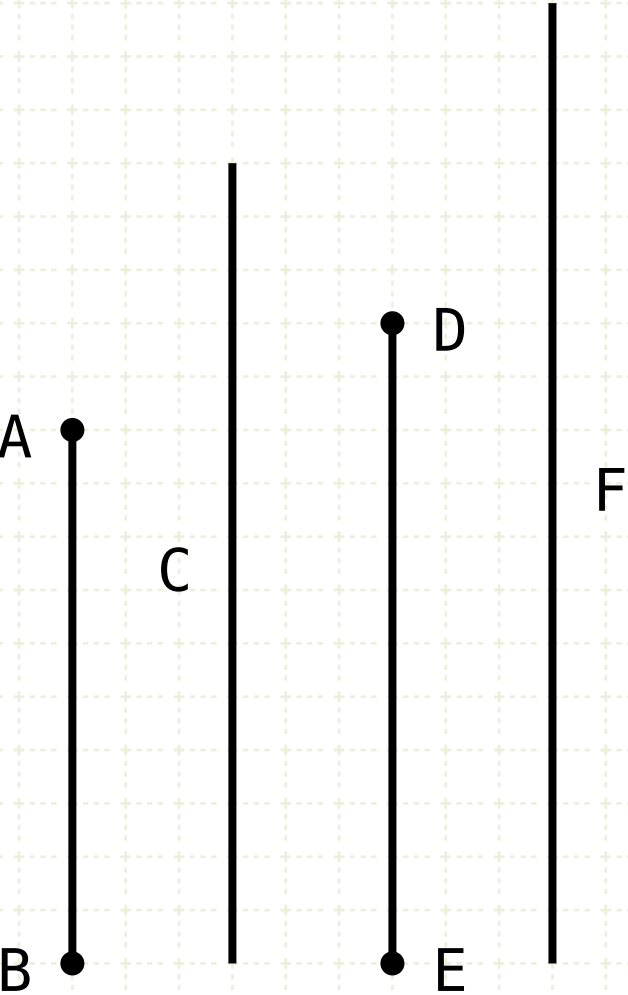
$$AB = (p/q)C$$
$$DE = (p/q)F$$

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Proof

Let AB be parts of the number C, and let DE be the same parts of F

A

C

D

F

B

E

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Since AB and DE are the same part of C,F respectively, then there is an equal number of parts of AB in C as there is of DE in F

Divide AB into the number of parts it has of C, and DE into the number of parts it has of F

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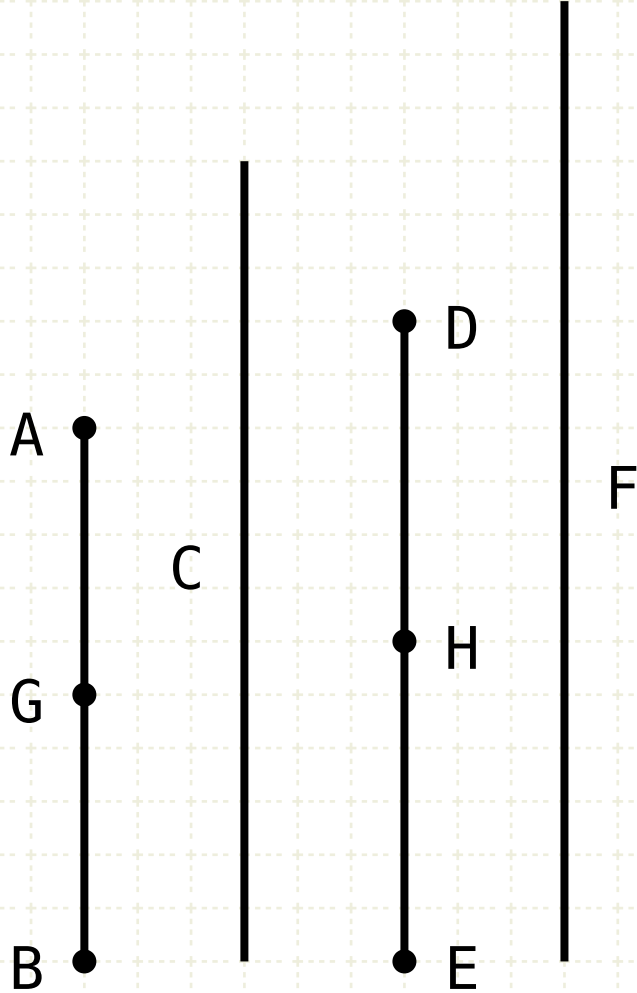
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AG = (r/s)DH
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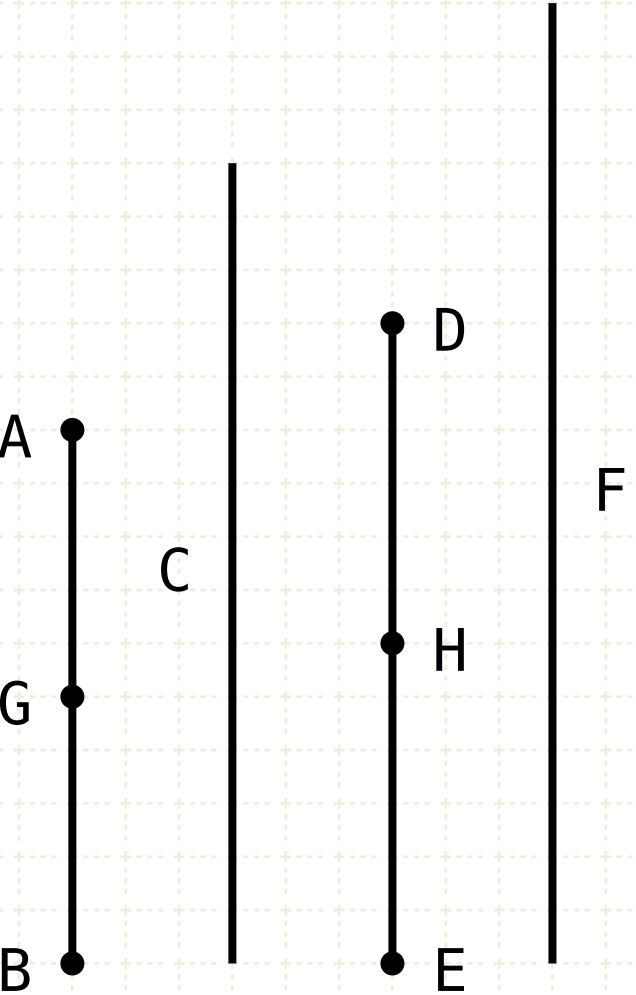Let AB be parts of the number C, and let DE be the same parts of F

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Whatever parts AG is of DH, So is C of F (VII·9)

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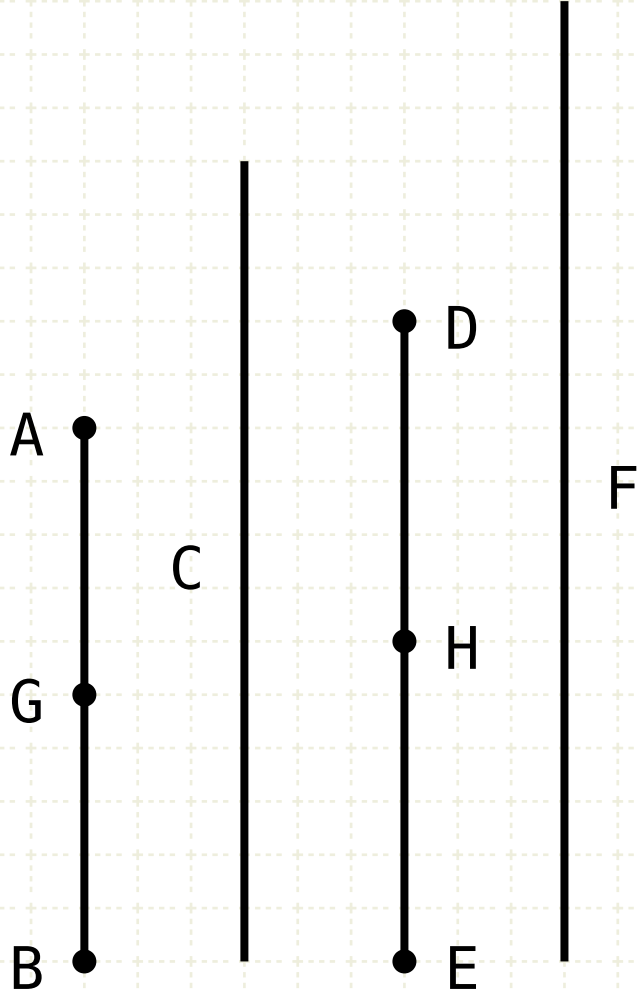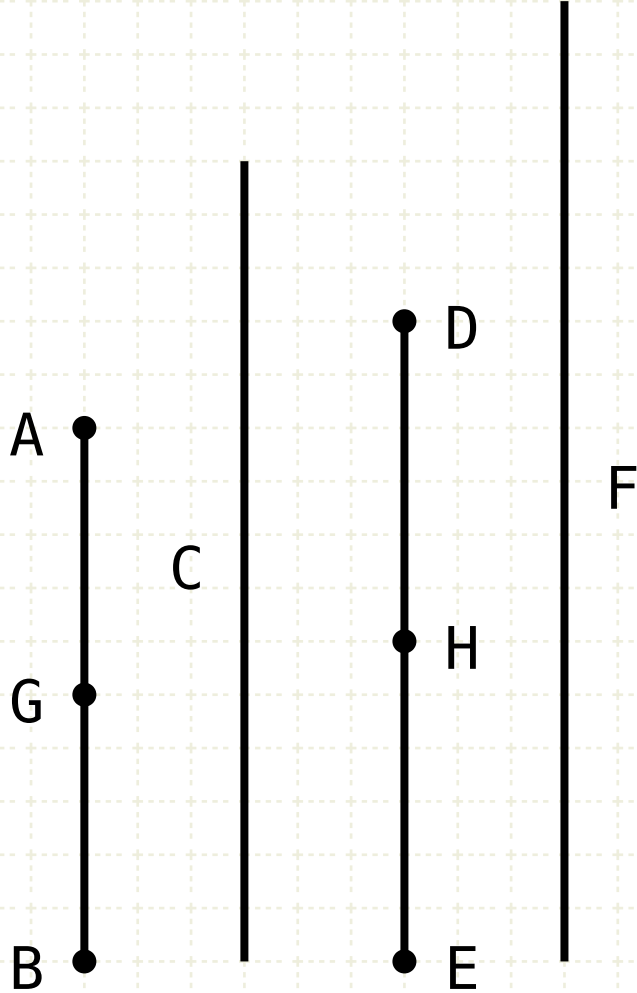Divide AB into the number of parts it has of C, and DE into the number of parts it has of F
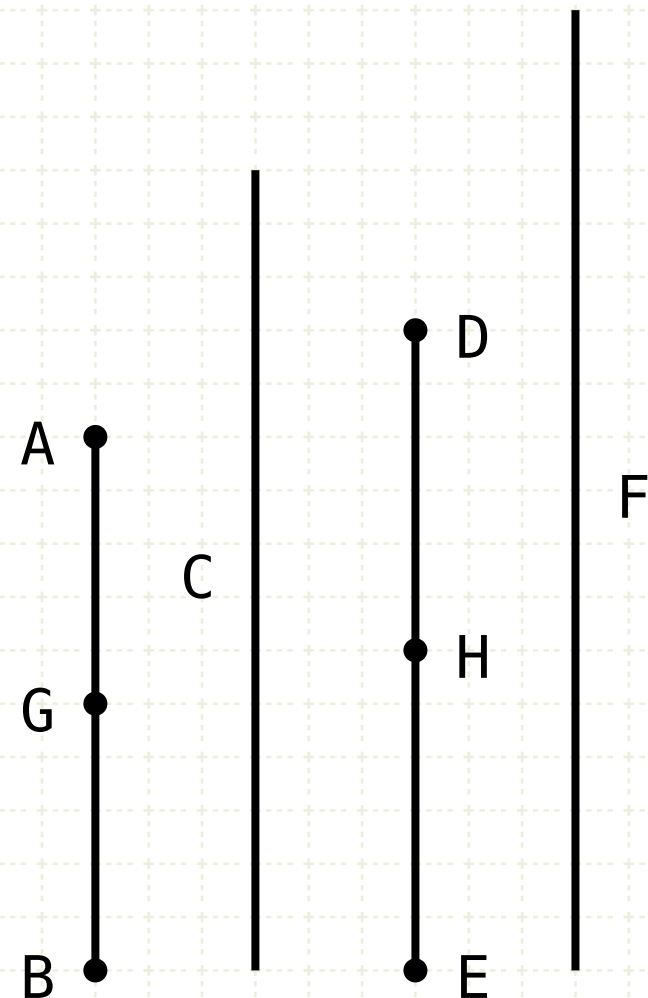
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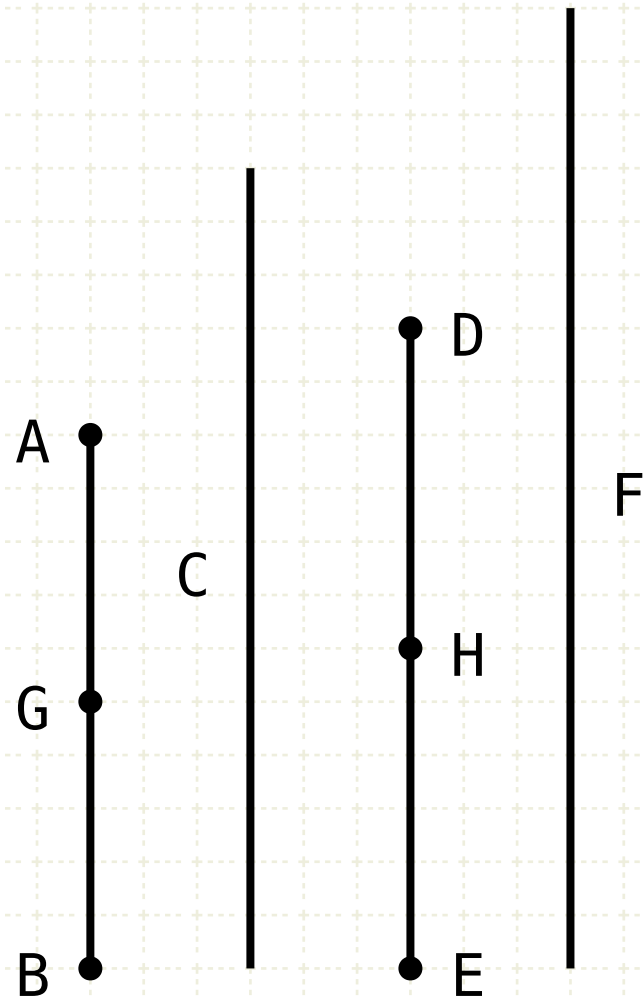Whatever parts AG is of DH, So is C of F (VII·9)

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Thus, the sum of AG,GB will have the same parts of the sum EH,HD as GB has of HE (VII·5), (VII·6)

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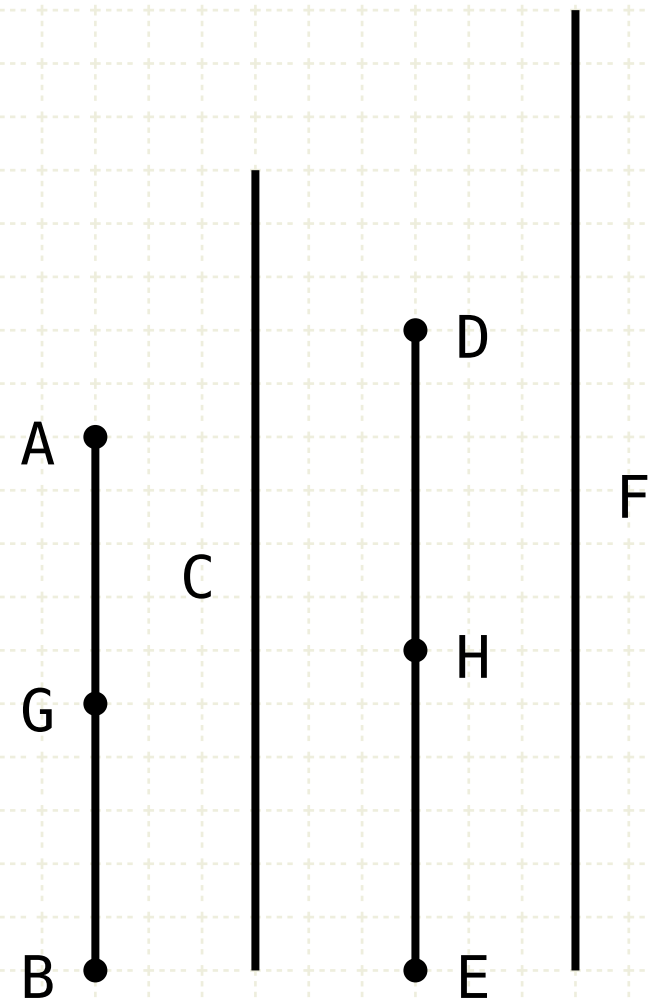Whatever parts AG is of DH, So is C of F (VII·9)

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