# Euclid's Elements

# Book I

If Euclid did not kindle your youthful enthusiasm, you were not born to be a scientific thinker.

Albert Einstein

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Proposition 11 of Book I

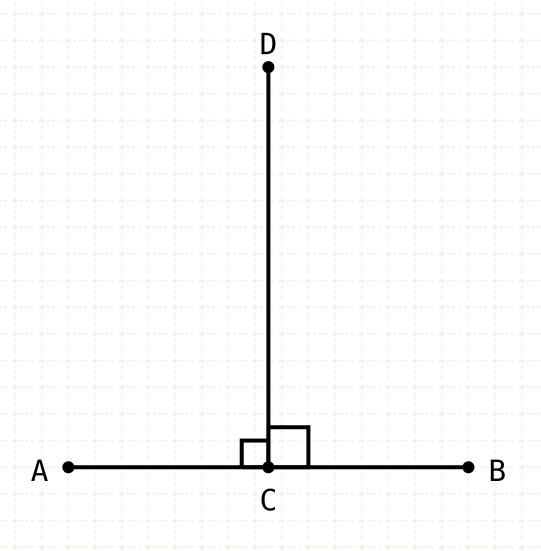
To draw a straight line at right angles to a given straight line from a given point on it.



To draw a straight line at right angles to a given straight line from a given point on it.

# **Definition - Right Angle**

When a straight line standing on a straight line makes the adjacent angles equal to one another, each of the equal angles is right, and the straight line standing on the other is called a perpendicular to that on which it stands.



$$\angle ACD = \angle BCD = \bot \text{ (right angle)}$$

Proposition 11 of Book I

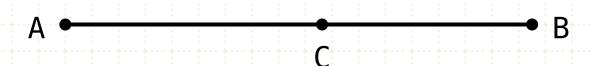
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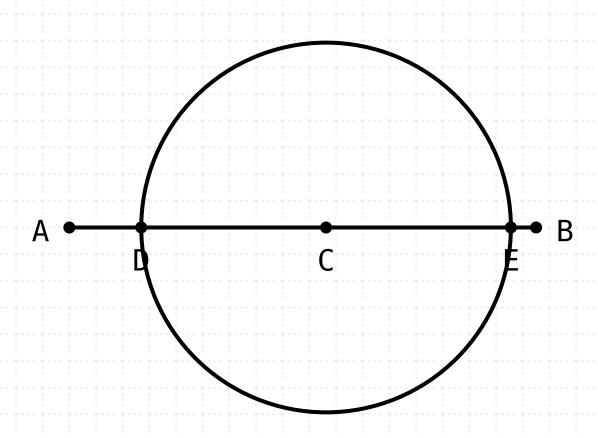
# Construction:

Start with a line segment AB, and an arbitrary point C on this line





To draw a straight line at right angles to a given straight line from a given point on it.



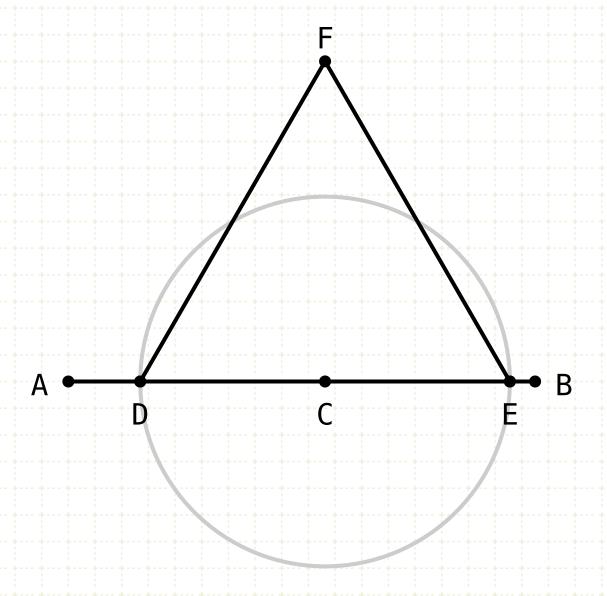
### Construction:

Start with a line segment AB, and an arbitrary point C on this line

Define another point D on line AB

Define point E such that EC equals CD

To draw a straight line at right angles to a given straight line from a given point on it.



#### **Construction:**

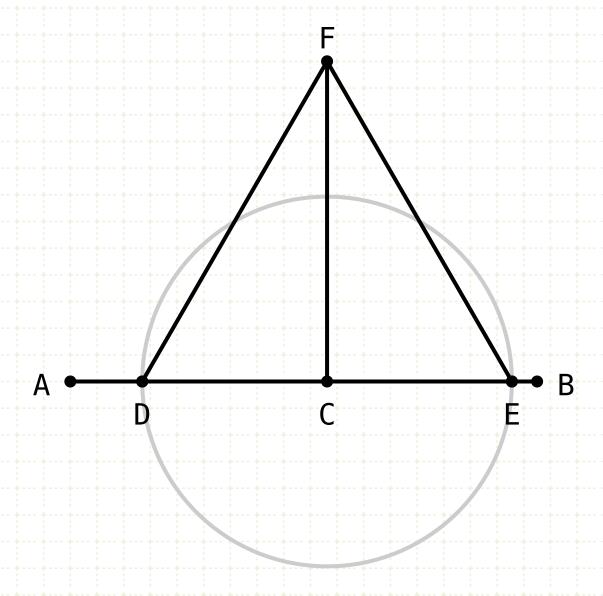
Start with a line segment AB, and an arbitrary point C on this line

Define another point D on line AB

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Construct an equilateral triangle on DE and label the vertex F (I·1)

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#### **Construction:**

Start with a line segment AB, and an arbitrary point C on this line

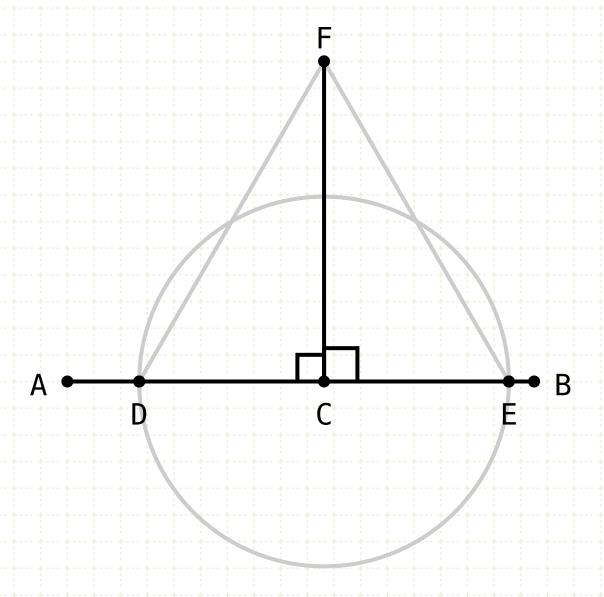
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Construct line segment FC

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#### **Construction:**

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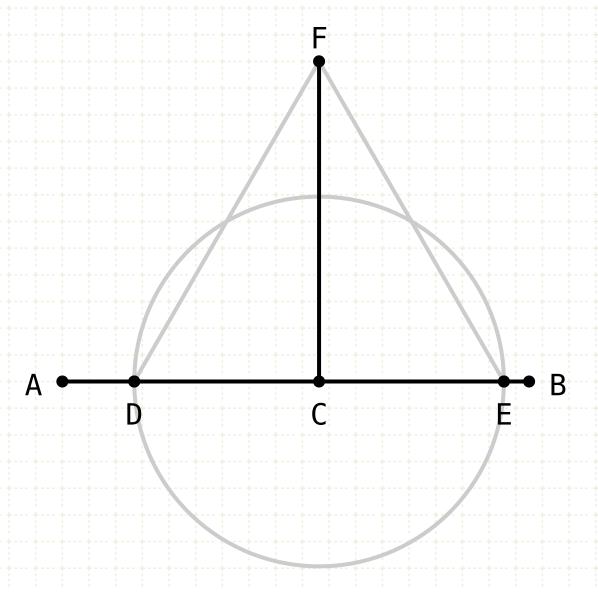
Define point E such that EC equals CD

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Angle ACF and angle BCF are right angles

To draw a straight line at right angles to a given straight line from a given point on it.



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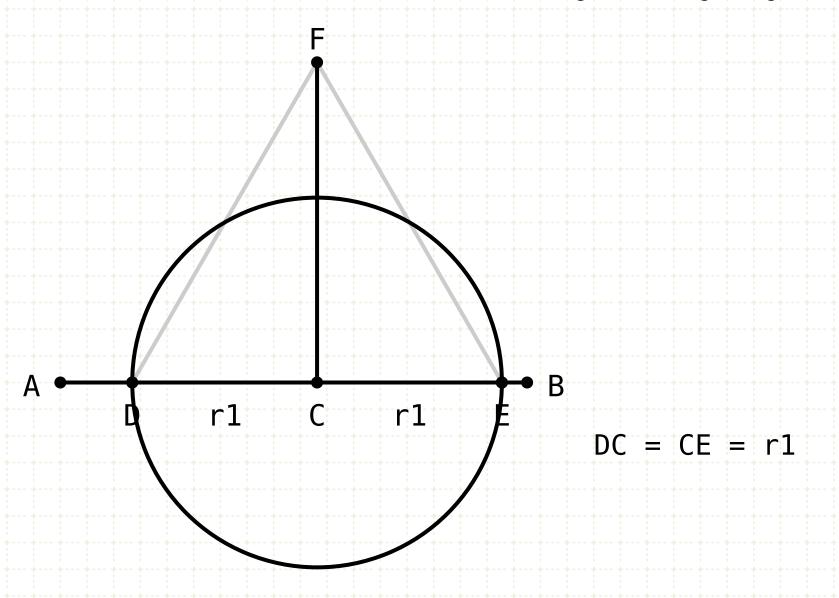
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#### **Proof**



To draw a straight line at right angles to a given straight line from a given point on it.



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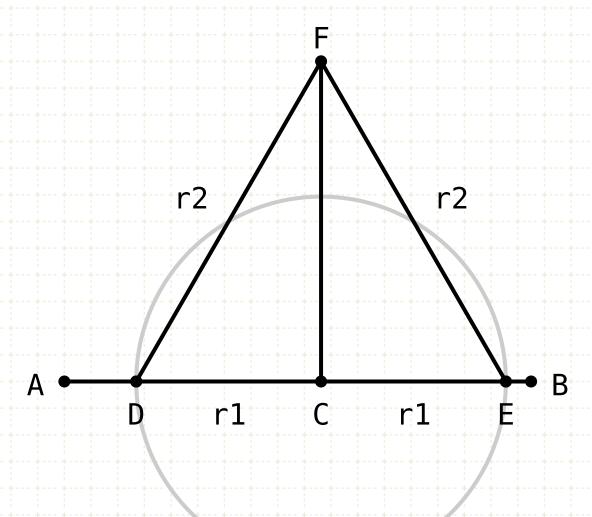
Construct line segment FC

Angle ACF and angle BCF are right angles

#### **Proof**

Line DC equals line CE since they are radii of the same circle

To draw a straight line at right angles to a given straight line from a given point on it.



$$FD = FE = r2$$

### **Construction:**

Start with a line segment AB, and an arbitrary point C on this line

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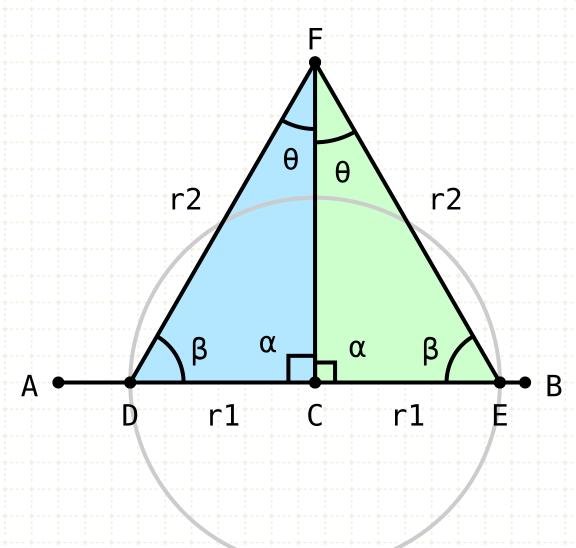
Angle ACF and angle BCF are right angles

#### **Proof**

Line DC equals line CE since they are radii of the same circle

FD and FE are equal since they are two sides of an equilateral triangle

To draw a straight line at right angles to a given straight line from a given point on it.



DC = CE = r1

FD = FE = r2

$$\angle$$
CDF =  $\angle$ CEF =  $\beta$ 
 $\angle$ DFC =  $\angle$ EFC =  $\theta$ 
 $\angle$ FCD =  $\angle$ FCE =  $\alpha$ 

#### Construction:

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Define another point D on line AB

Define point E such that EC equals CD

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Angle ACF and angle BCF are right angles

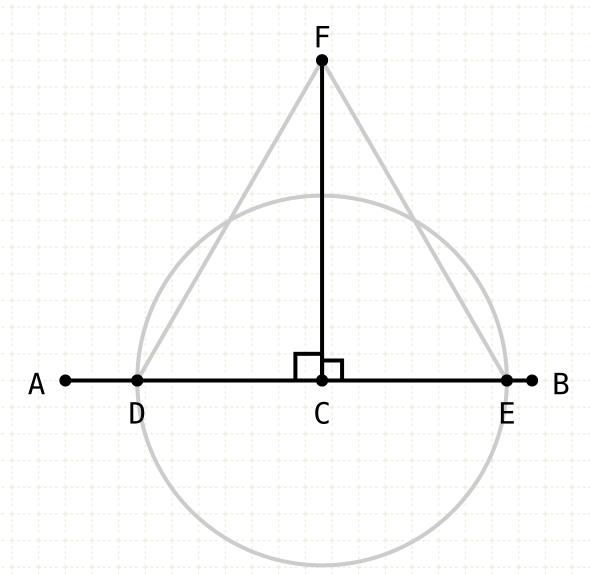
#### **Proof**

Line DC equals line CE since they are radii of the same circle

FD and FE are equal since they are two sides of an equilateral triangle

Triangle DCF and triangle FCE have all three sides equal to each other, thus all the angles are equal to each other (I·8)

To draw a straight line at right angles to a given straight line from a given point on it.



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Construct an equilateral triangle on DE and label the vertex F (I·1)

Construct line segment FC

Angle ACF and angle BCF are right angles

#### **Proof**

Line DC equals line CE since they are radii of the same circle

FD and FE are equal since they are two sides of an equilateral triangle

Triangle DCF and triangle FCE have all three sides equal to each other, thus all the angles are equal to each other (I·8)

Angles FCD and FCE are equal, and therefore are 'right angles'

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