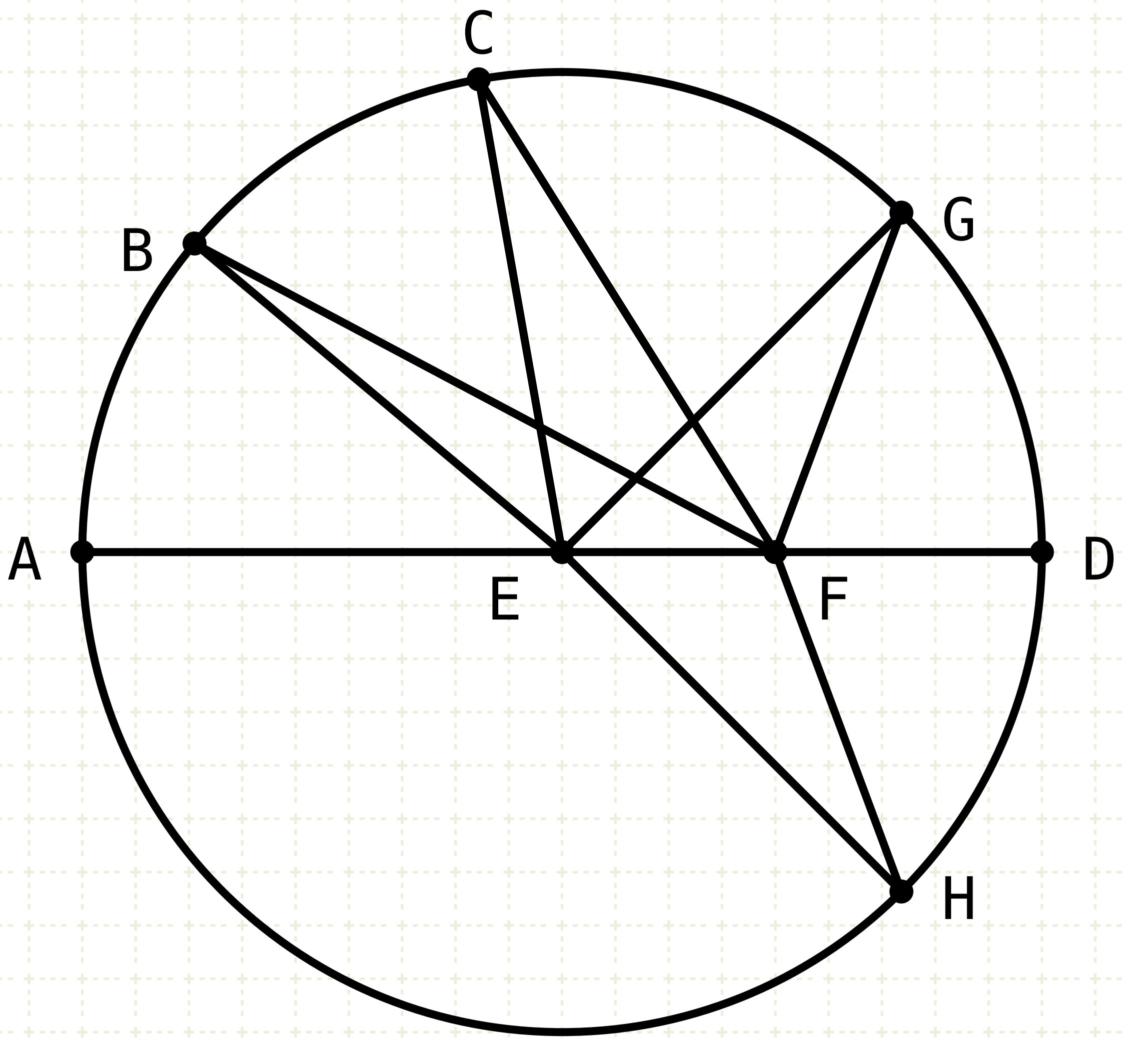
Euclid's Elements

Book III



A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)

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Proposition 37 of Book III

If a point be taken outside a circle and from the point there fall on the circle two straight lines, if one of them cut the circle, and the other fall on it, and if further the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference be equal to the square on the straight line which falls on the circle, and the straight line which falls on it will touch the circle.

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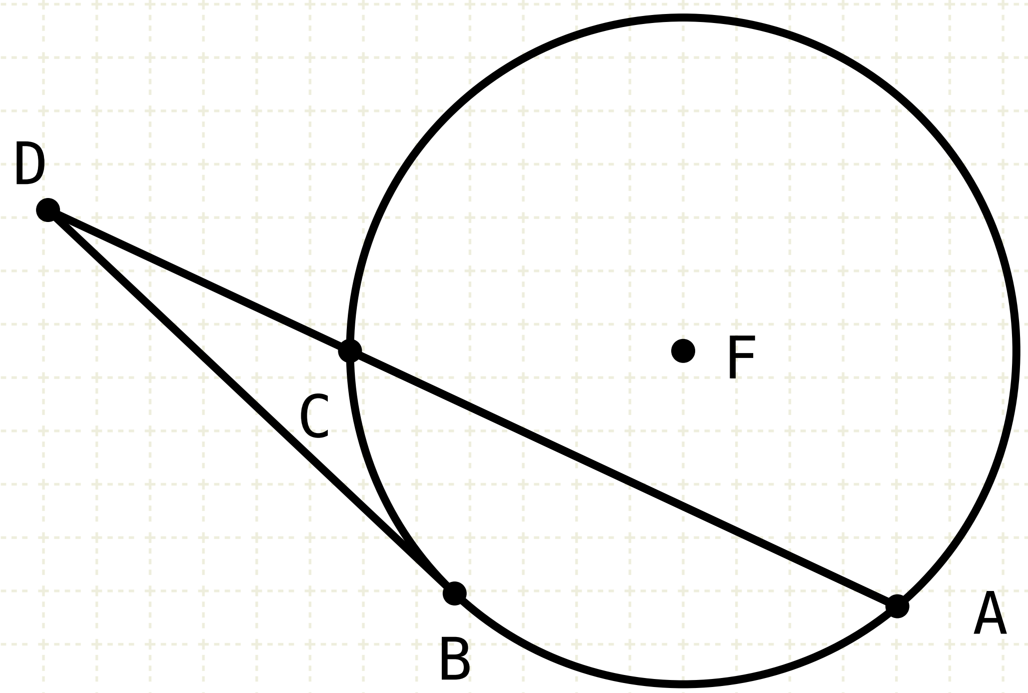

$AD \cdot CD = BD^2$

In other words

Let point D be outside of the circle

Let a line DA cut the circle at C and A, and let line DB fall on the circle

If the product AD,CD equals BD squared, then DB touches the circle

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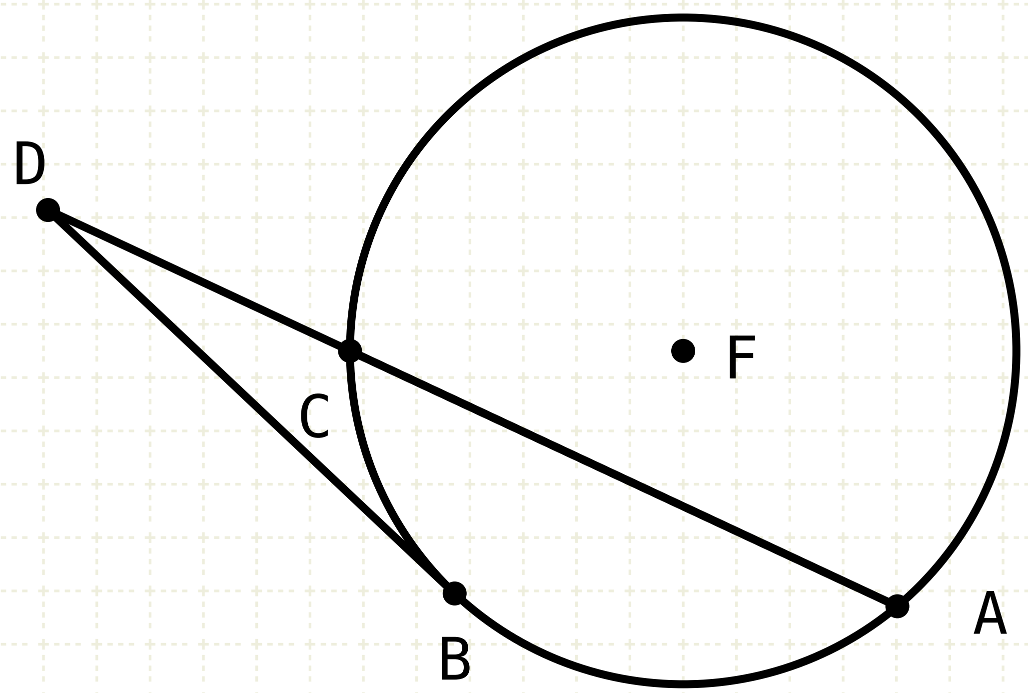

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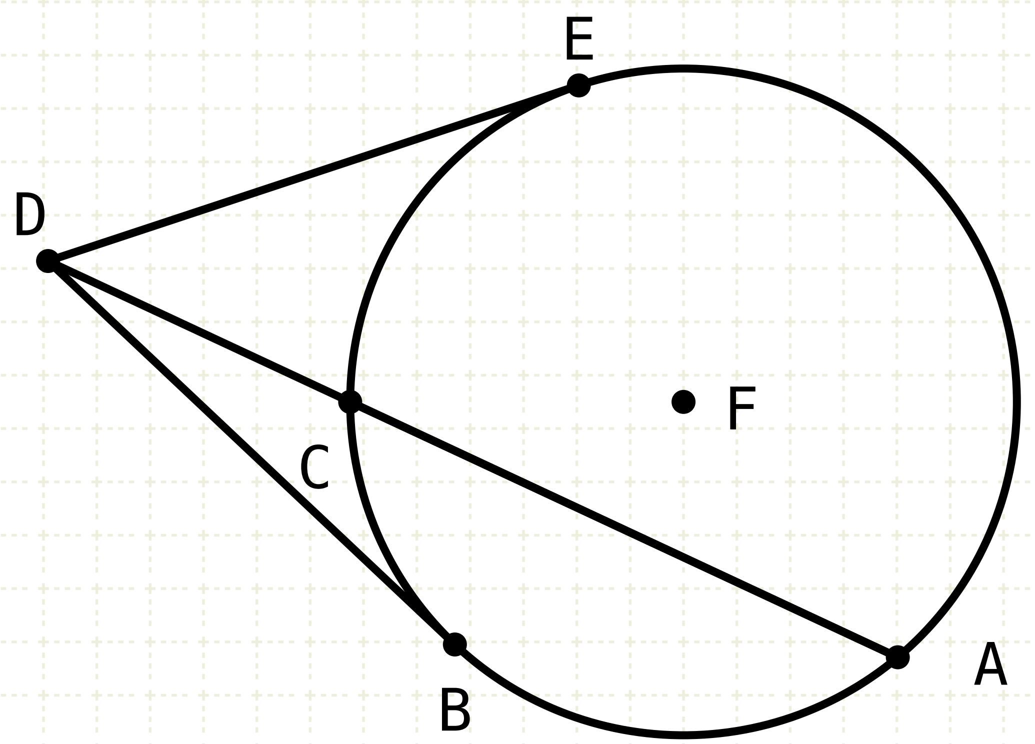

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Draw DE such that it touches the circle (III·17)

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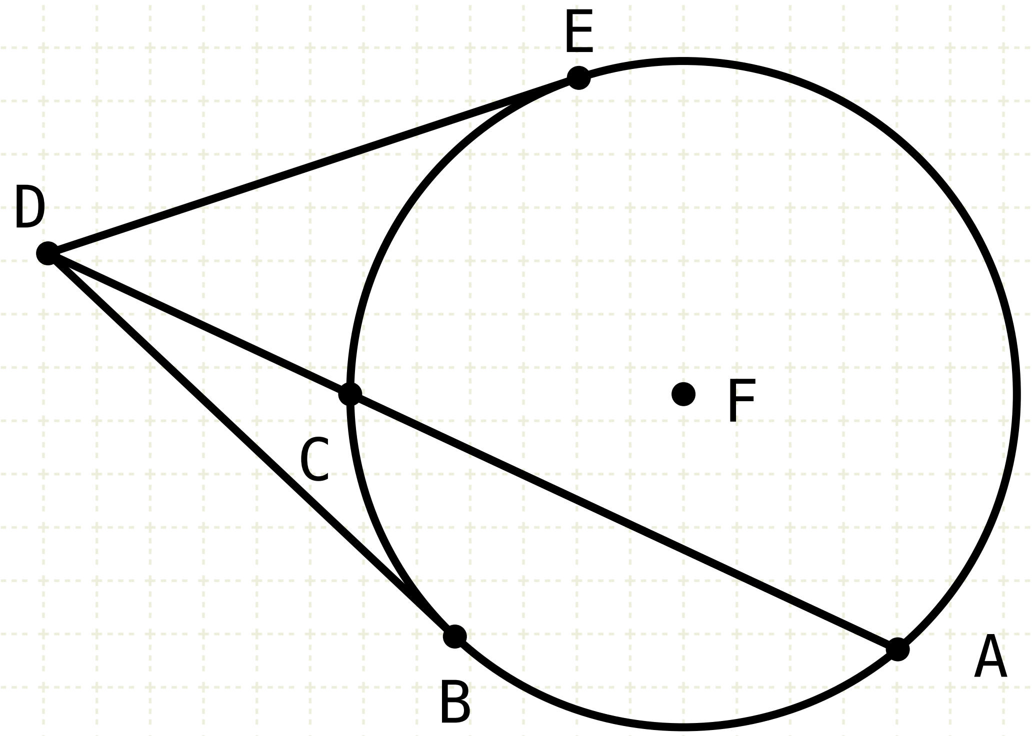

$AD \cdot CD = BD^2$

$AD \cdot CD = DE^2$

$BD = DE$

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Let point D be outside of the circle

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Draw DE such that it touches the circle (III·17)

Since DE touches the circle, the product AD,CD equals DE squared (III·36)

Therefore DE equals BD

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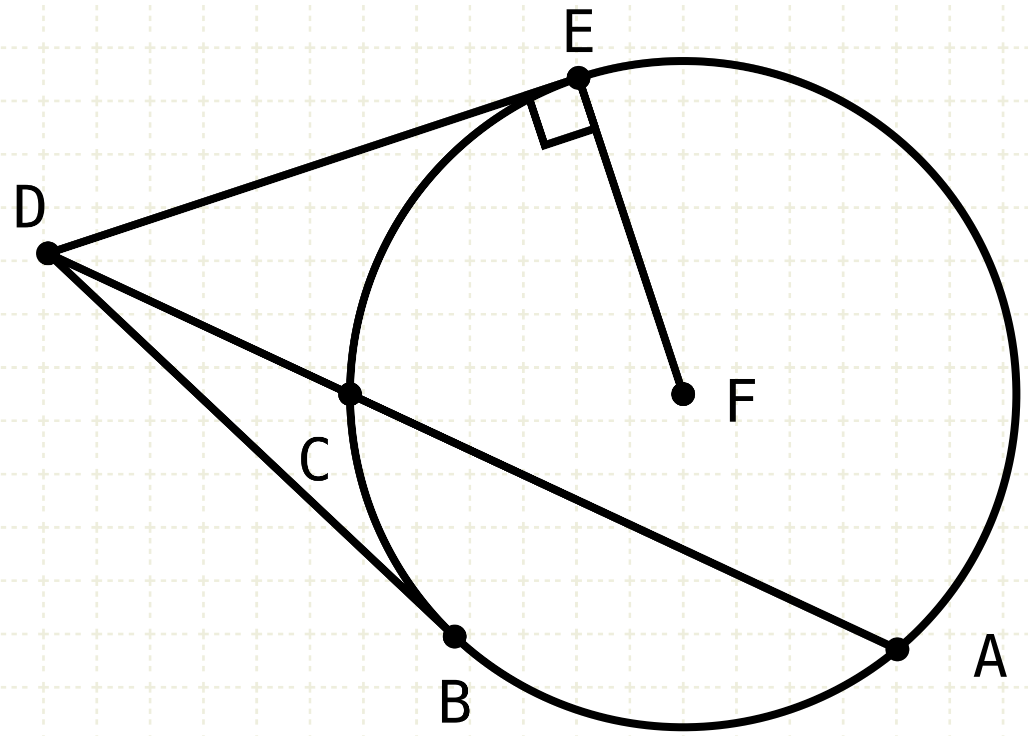

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Draw EF, where F is the centre of the circle

Angle FED is right (III·18)

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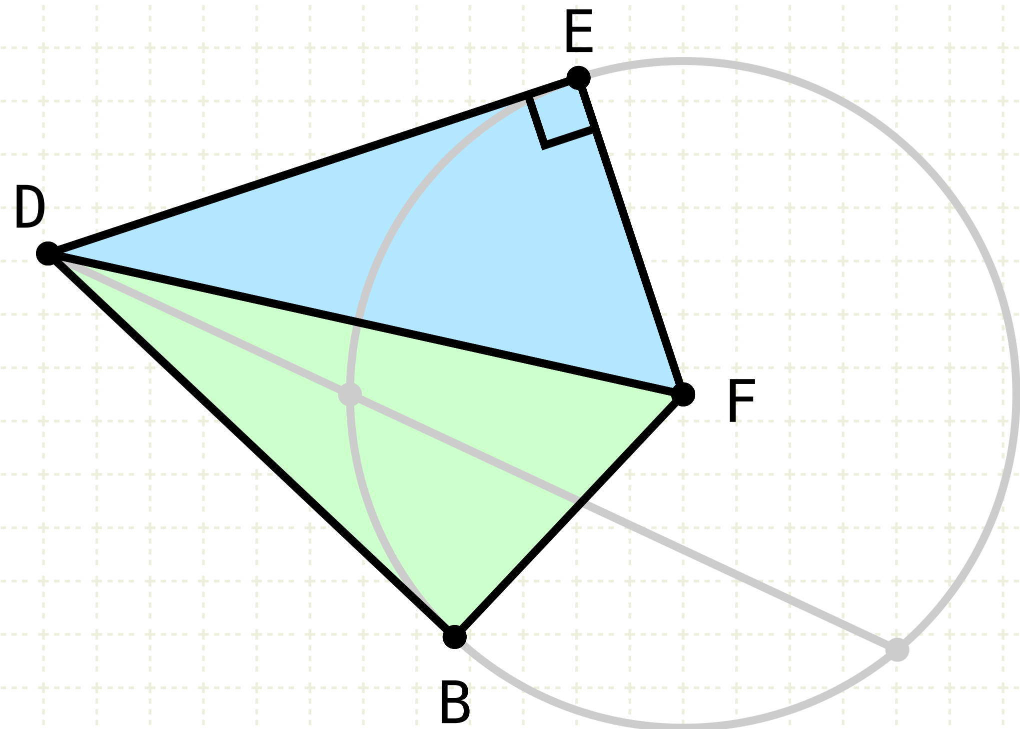

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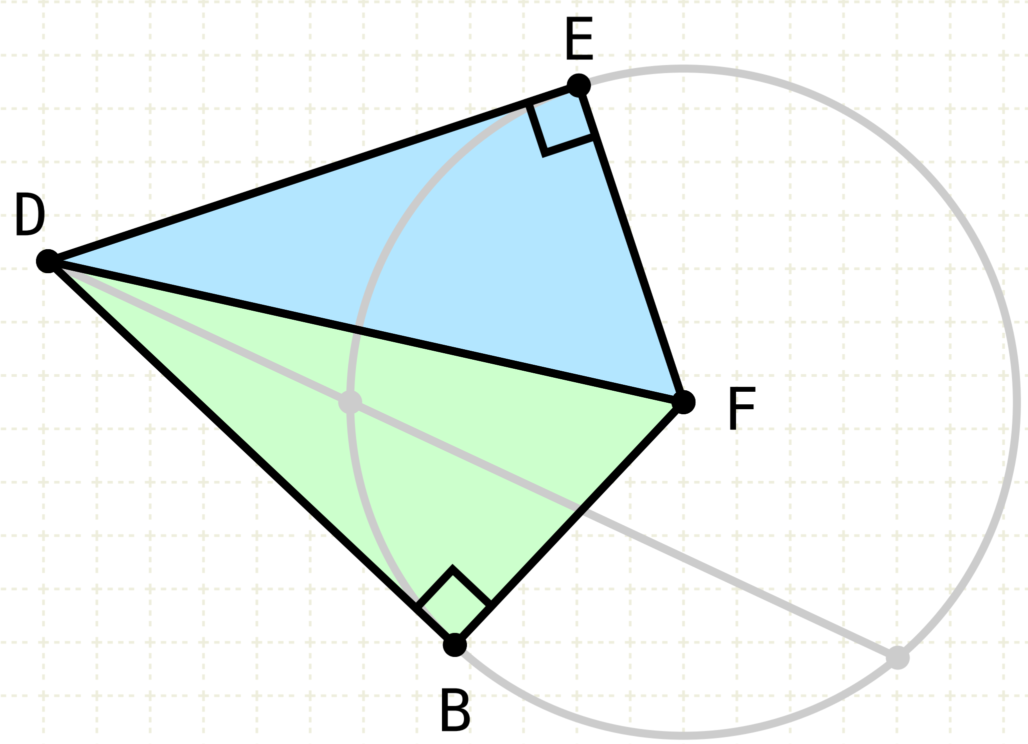

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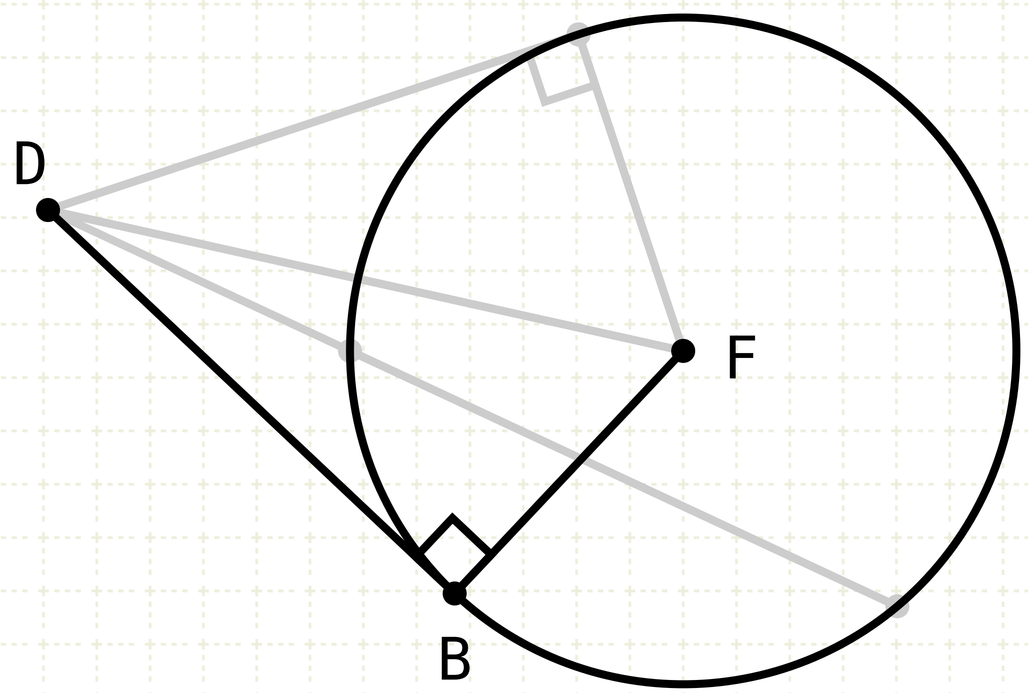

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If the angle FBD is right (and since B is at the extremity of the diameter), then BD touches the circle (III·16)

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