

# Euclid's Elements

## Book I

*If Euclid did not kindle your youthful enthusiasm, you were not born to be a scientific thinker.*

Albert Einstein



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# Proposition 7 of Book I

Given two straight lines constructed from the ends of a straight line and meeting in a point, there cannot be constructed from the ends of the same straight line, and on the same side of it, two other straight lines meeting in another point and equal to the former two respectively, namely each equal to that from the same end.



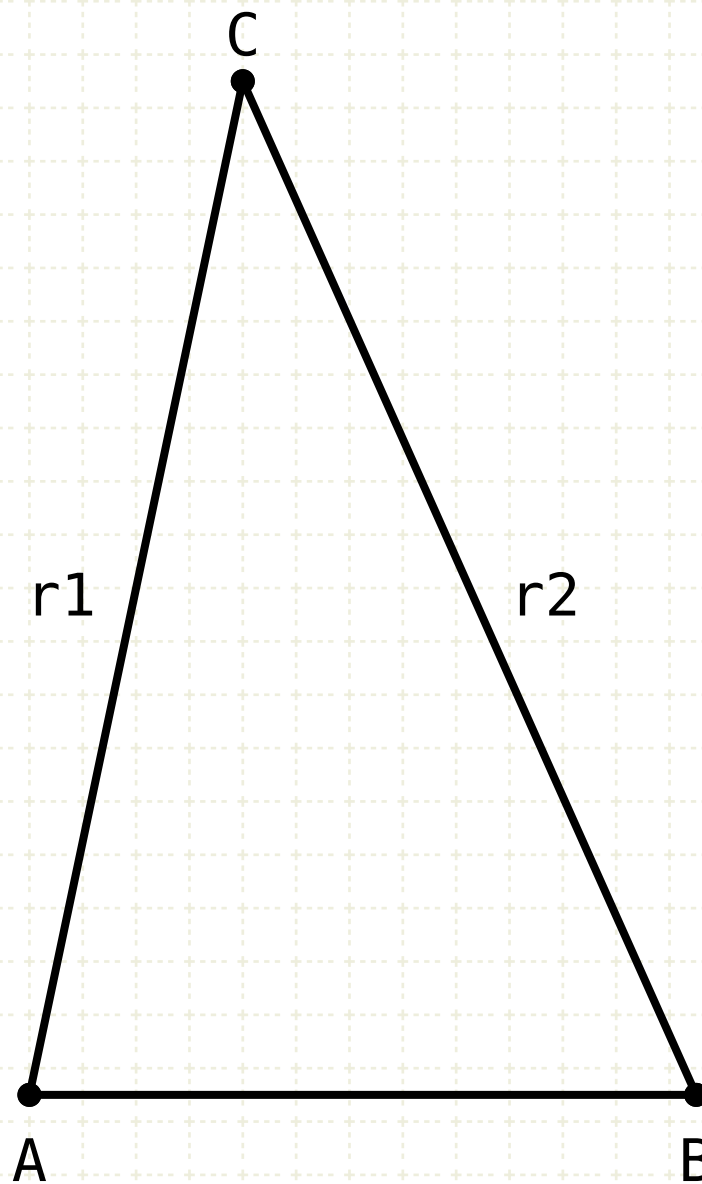
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## In other words

Given a triangle ABC

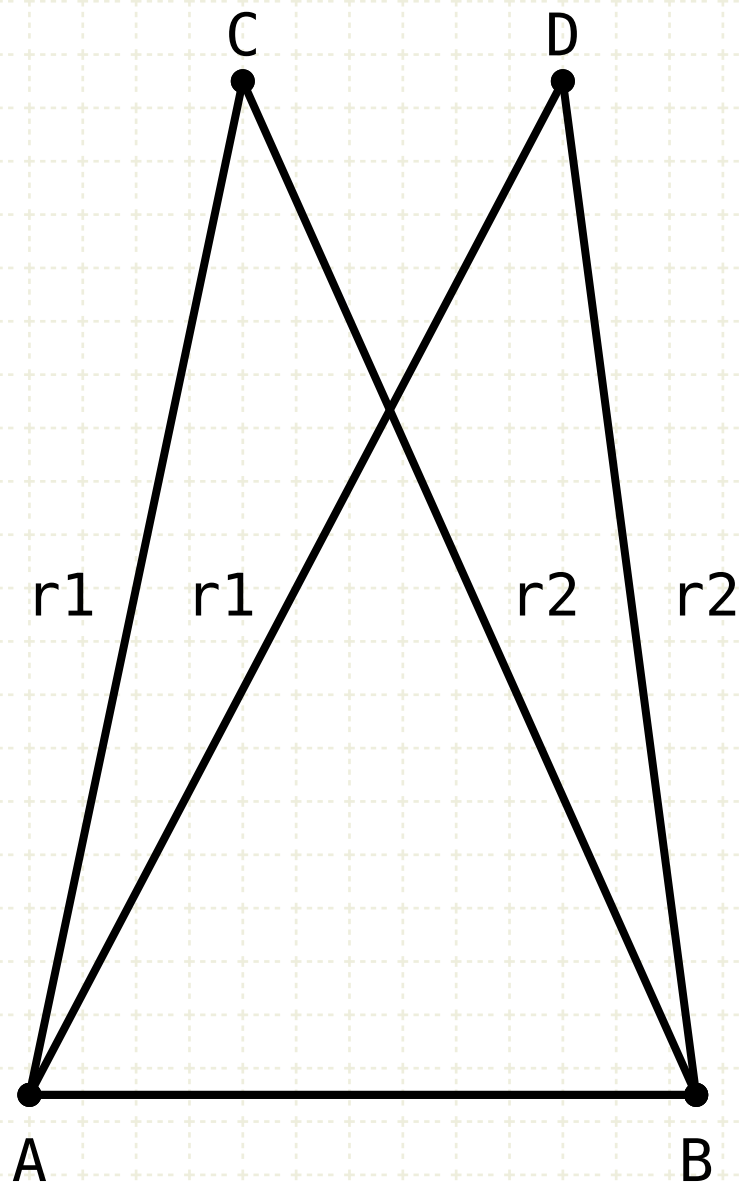
There is a unique point C where the sides of the triangle, AC and BC, meet





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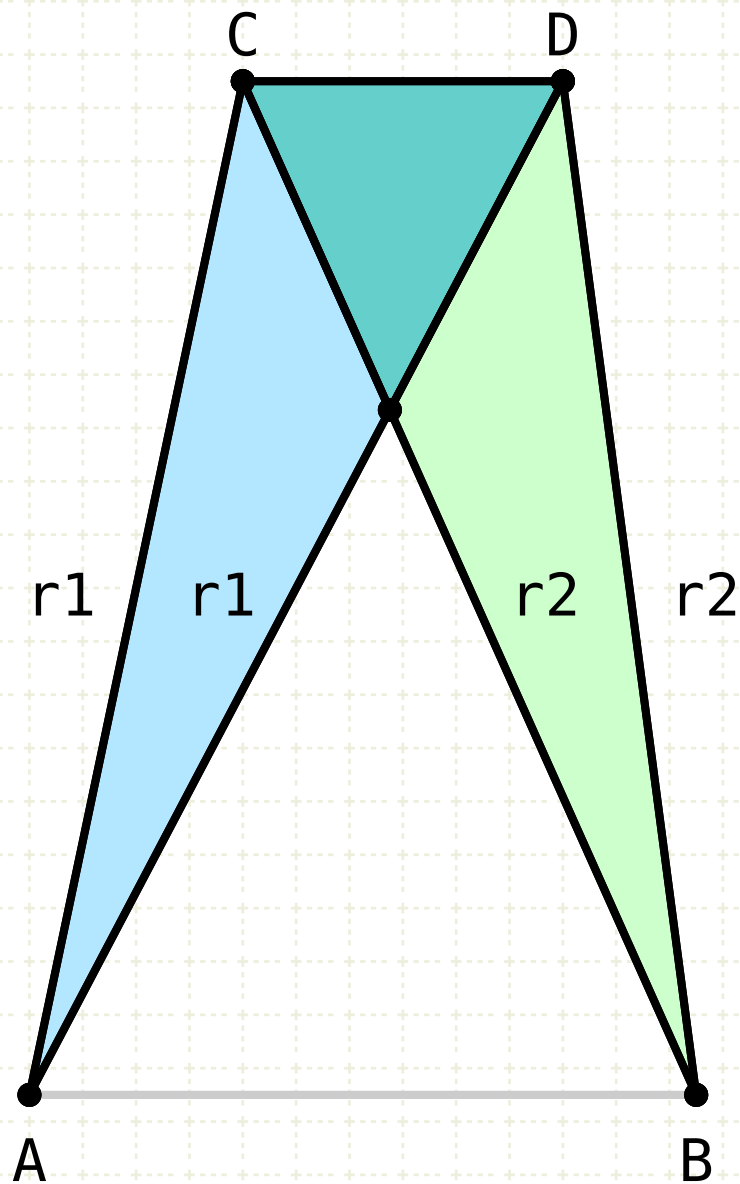
## Proof by Contradiction

Assume there is a point D where AD is equal in length to AC and BD is equal in length to BC

Create triangle ABD

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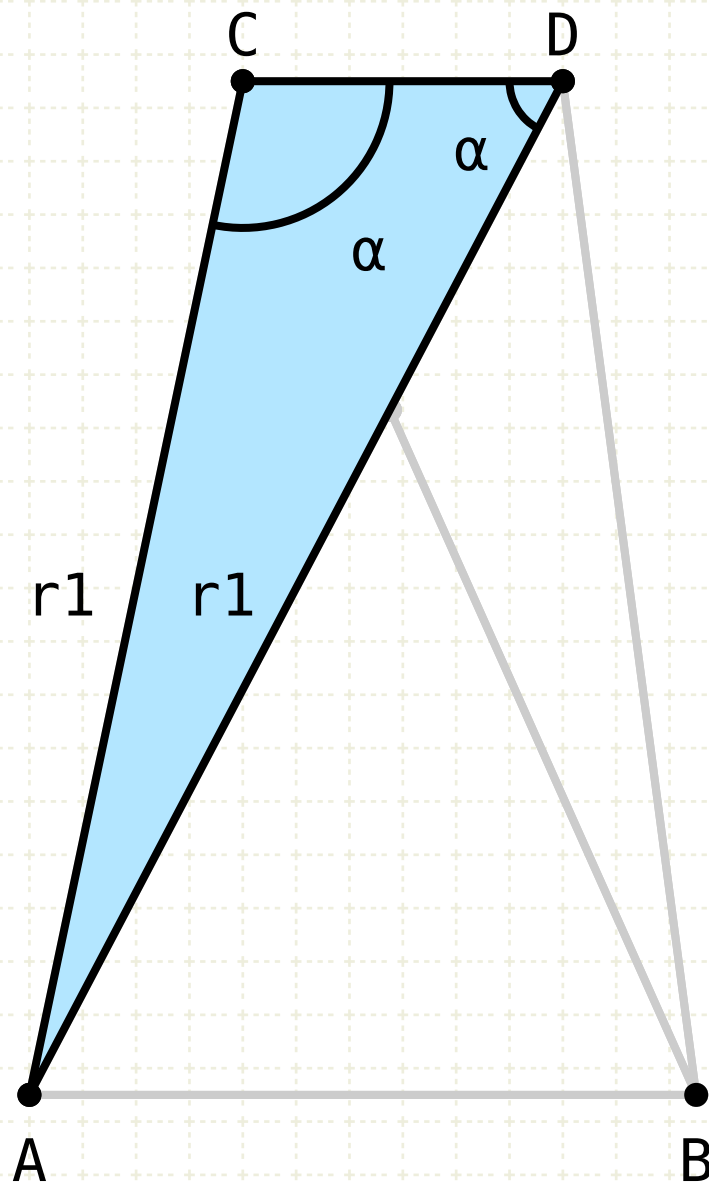
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Construct line CD, thus creating triangles ACD and BCD

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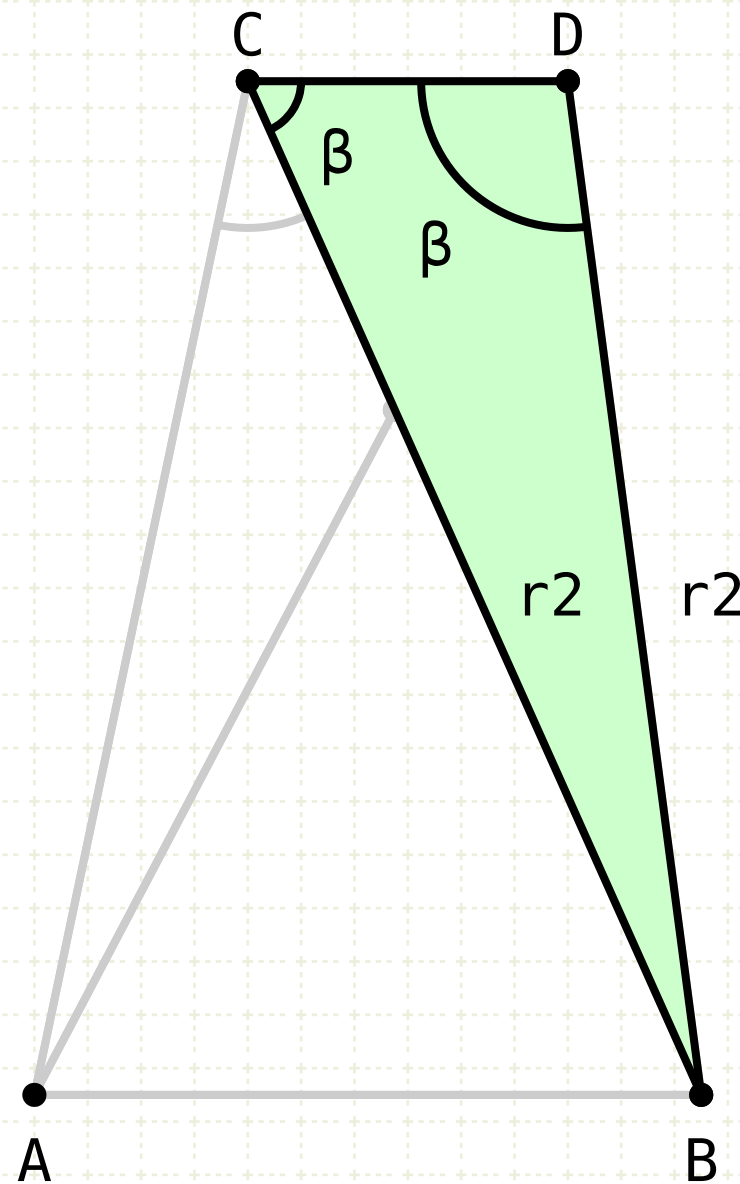
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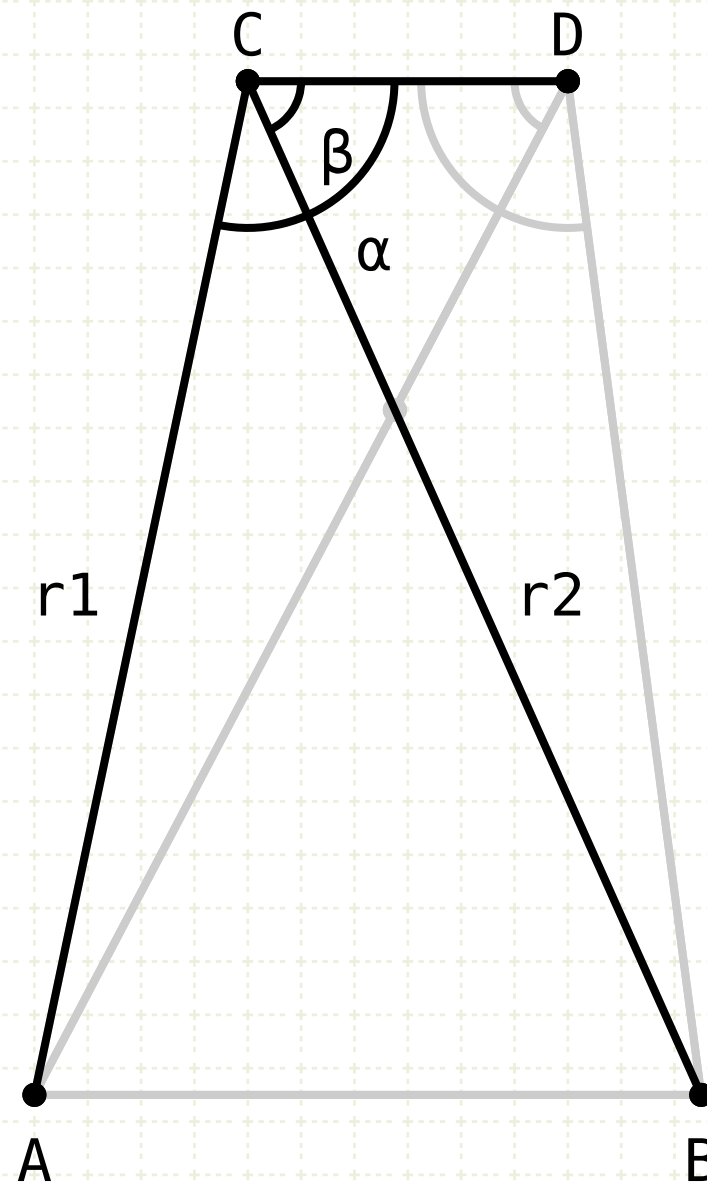
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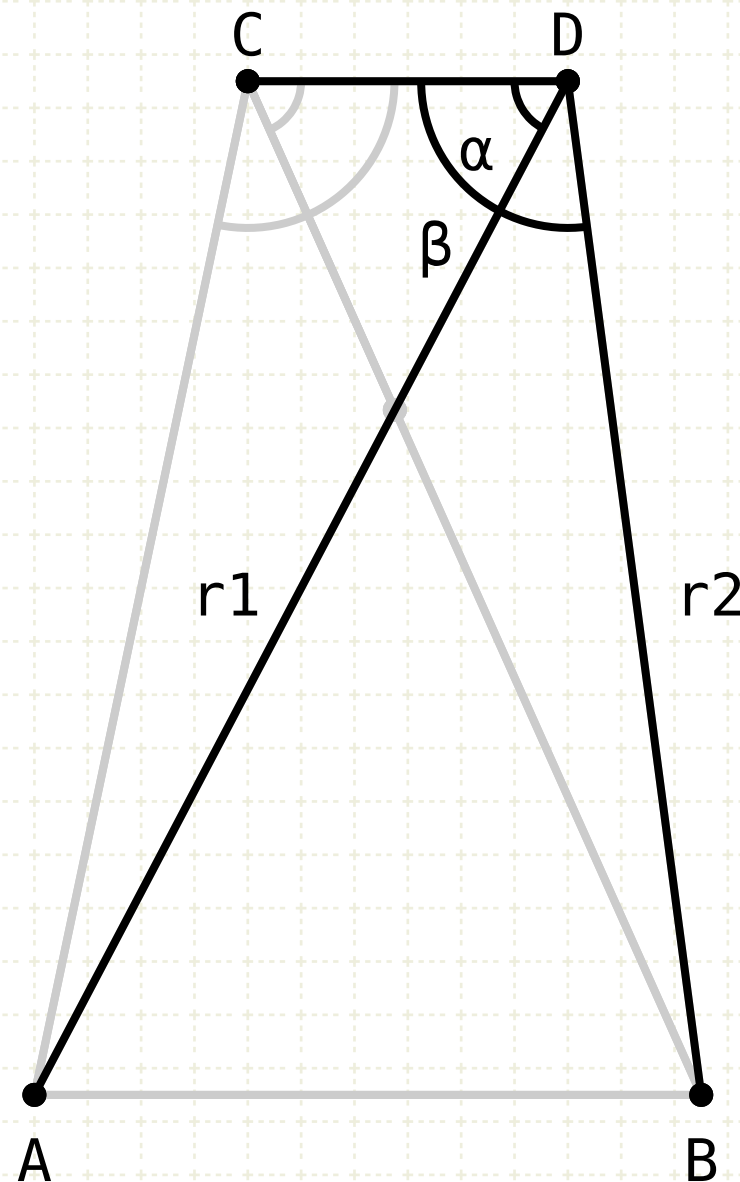
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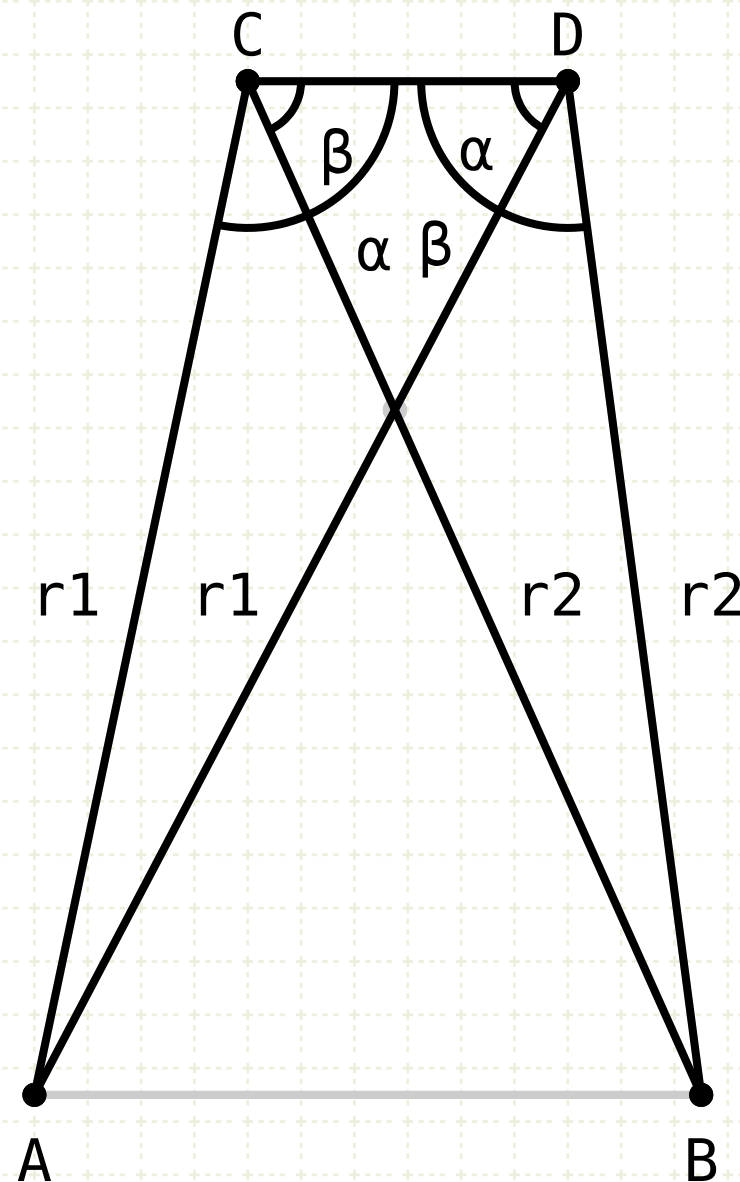
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But, since angle CDA is equal to DCA, and angle DCB is equal to angle CDB we have CDA simultaneously bigger and less than DBC

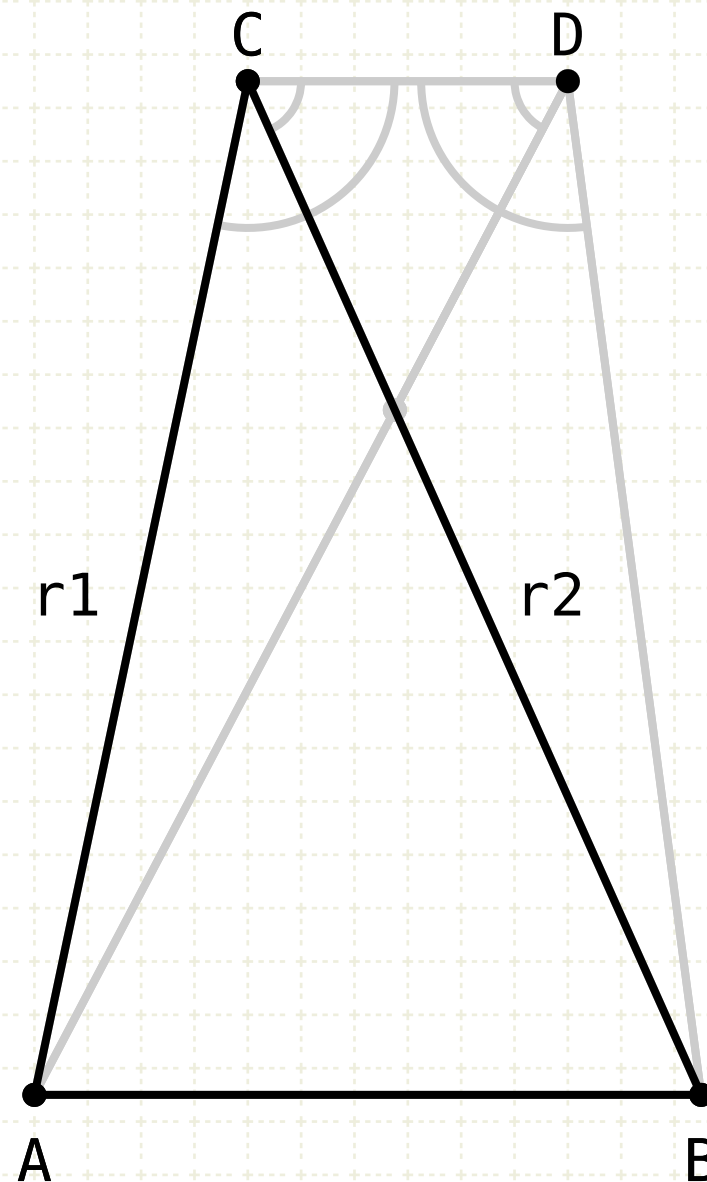
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But, since angle CDA is equal to DCA, and angle DCB is equal to angle CDB we have CDA simultaneously bigger and less than DBC

This is impossible

Thus, we have demonstrated that point D cannot exist, and the point C is unique.





# Youtube Videos

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