

Euclid's Elements

Book I

If Euclid did not kindle your youthful enthusiasm, you were not born to be a scientific thinker.

Albert Einstein



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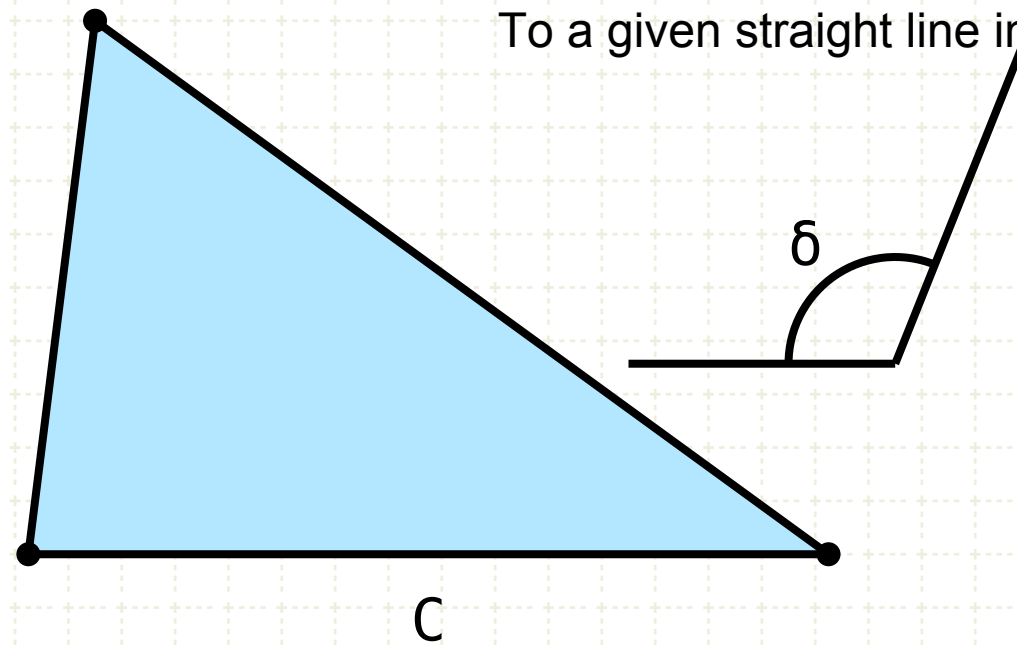
Proposition 44 of Book I

To a given straight line in a given rectilinear angle, to apply a parallelogram equal to a given triangle.



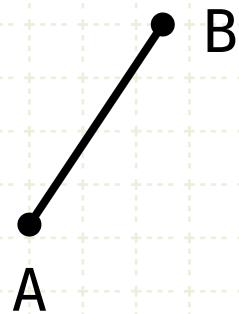
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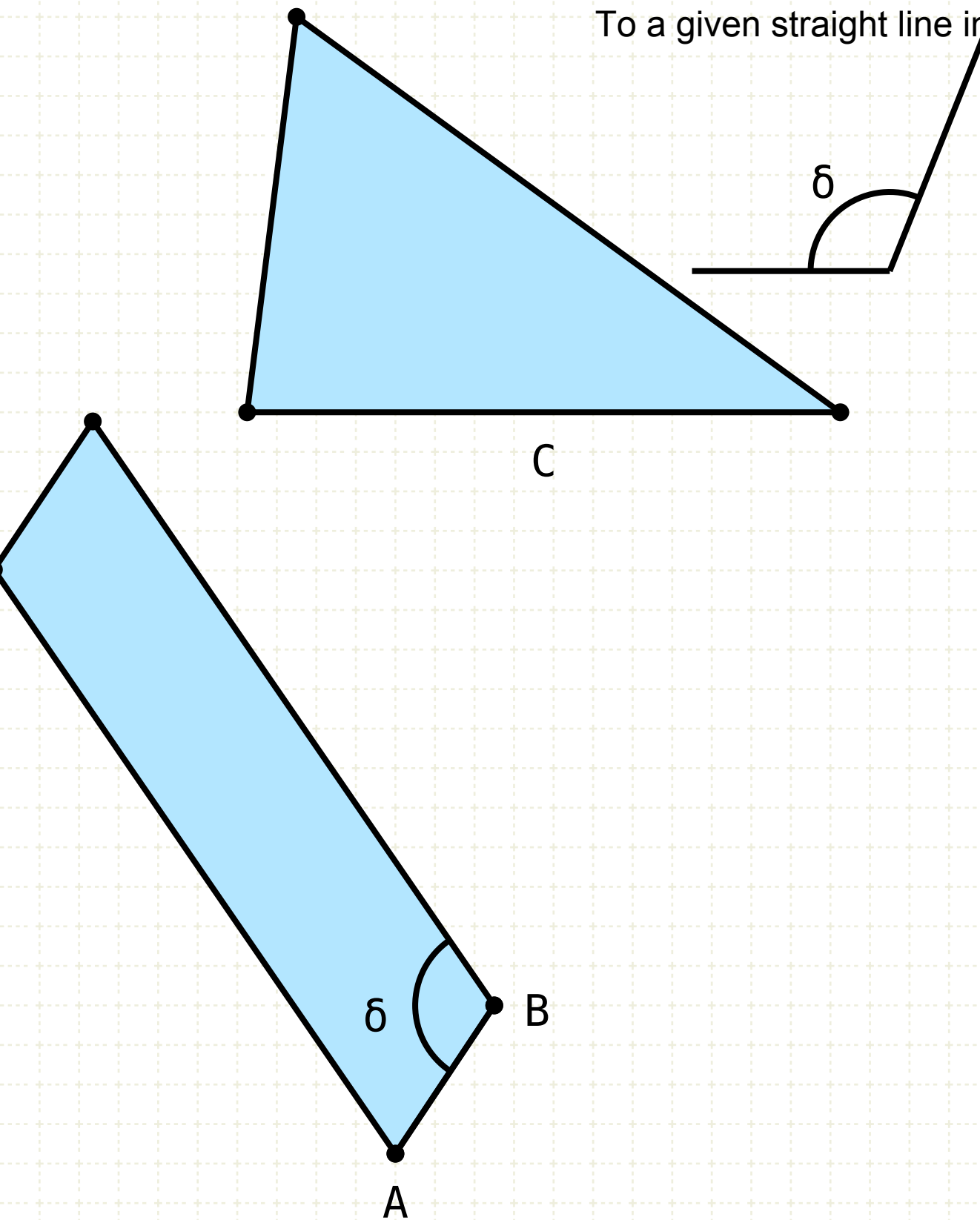
In other words

Start with a given triangle C, a straight line AB and an angle δ



Proposition 44 of Book I

To a given straight line in a given rectilinear angle, to apply a parallelogram equal to a given triangle.



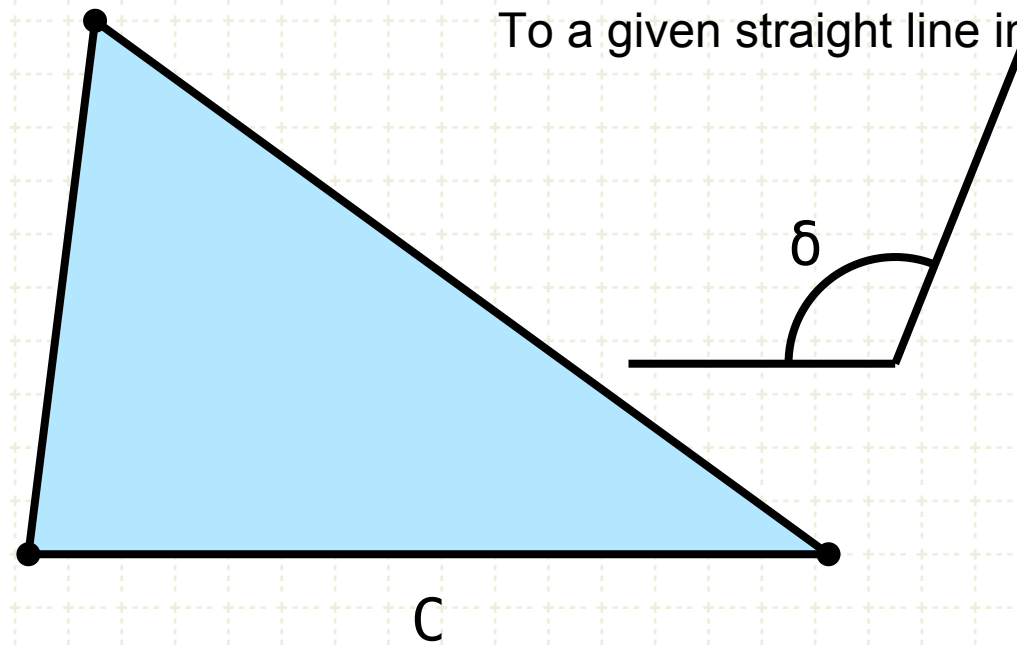
In other words

Start with a given triangle C, a straight line AB and an angle δ

Create a parallelogram, on the line AB, with an angle δ , such that it is equal in area to the triangle C

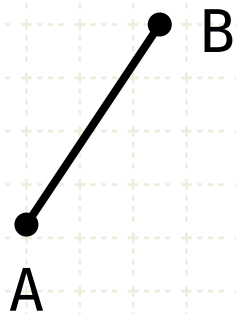
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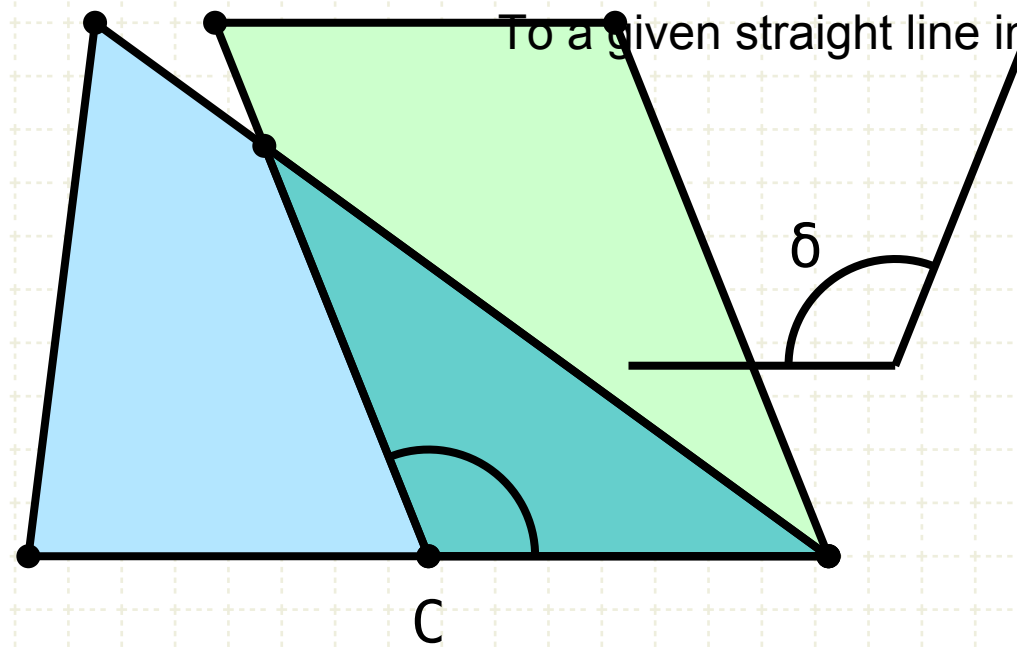
Construction

Start with a given triangle C, a straight line AB and an angle δ



Proposition 44 of Book I

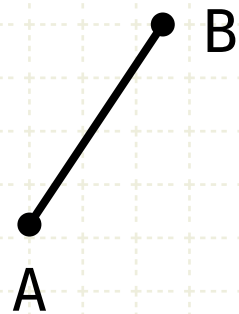
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Construction

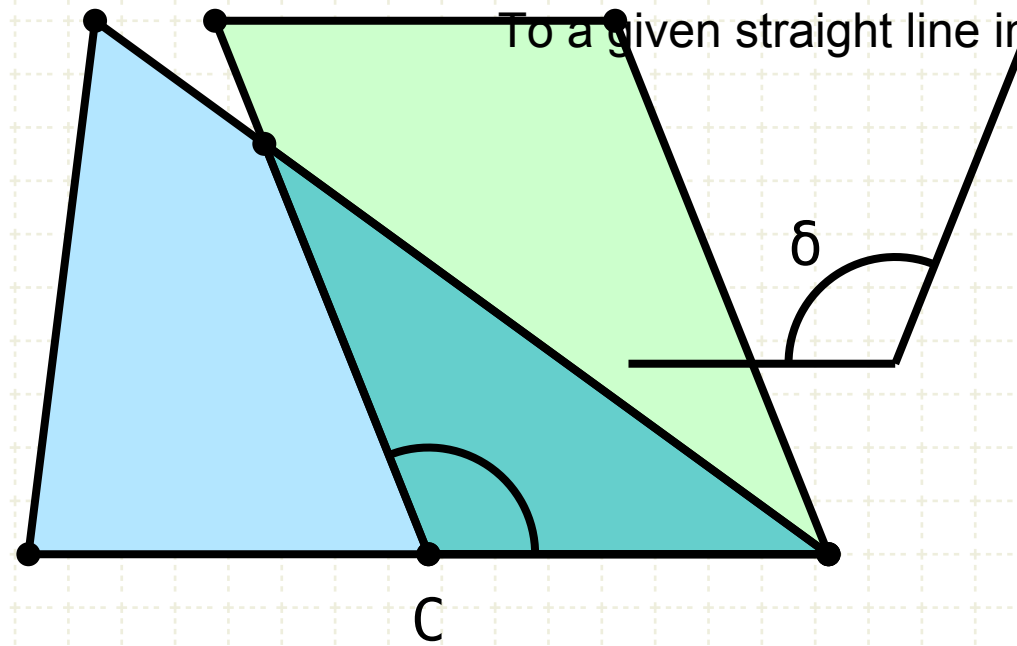
Start with a given triangle C, a straight line AB and an angle δ

Create a parallelogram equal to triangle C, with angle δ
(I.42)



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To a given straight line in a given rectilinear angle, to apply a parallelogram equal to a given triangle.

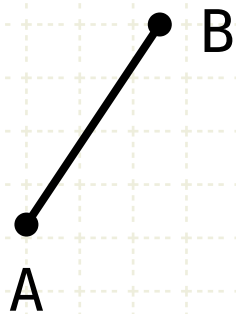


Construction

Start with a given triangle C, a straight line AB and an angle δ

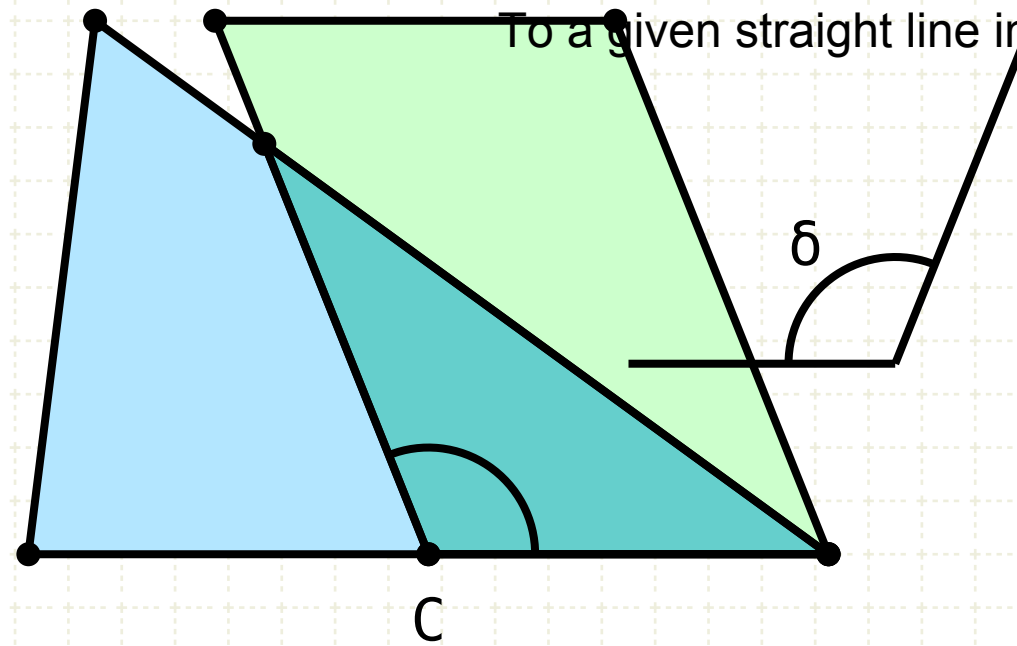
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Copy the parallelogram so that one side is in a straight line
with AB



Proposition 44 of Book I

To a given straight line in a given rectilinear angle, to apply a parallelogram equal to a given triangle.



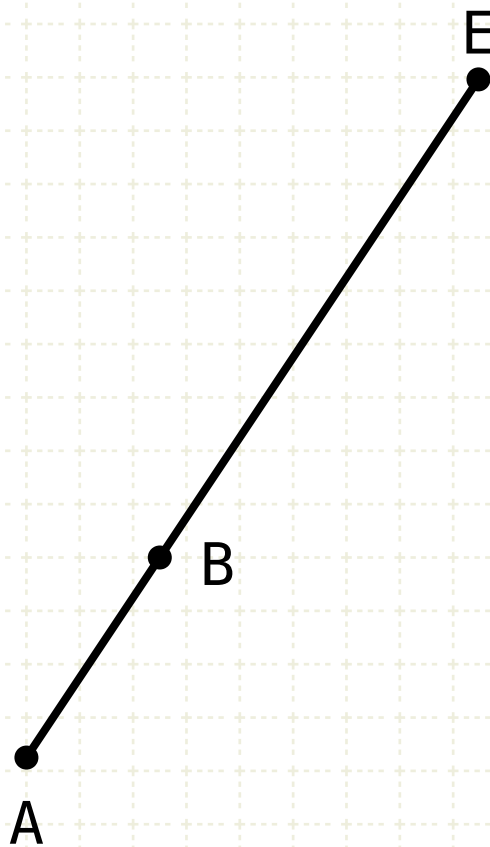
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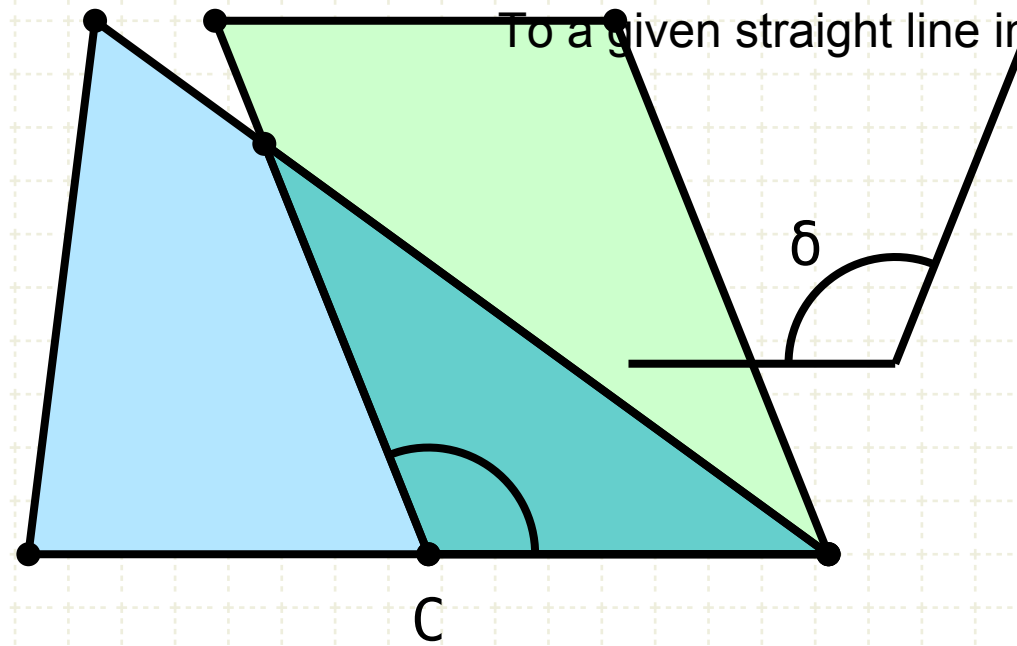
Copy the parallelogram so that one side is in a straight line
with AB

copy one side of parallelogram to AB



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To a given straight line in a given rectilinear angle, to apply a parallelogram equal to a given triangle.



Construction

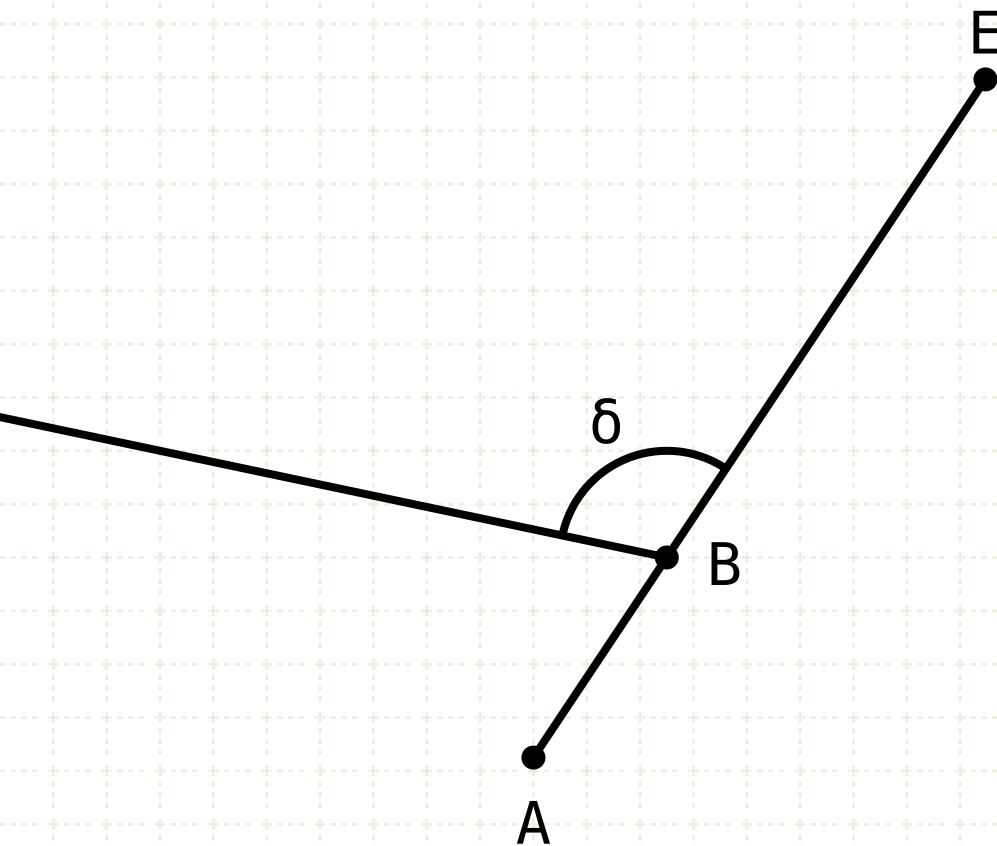
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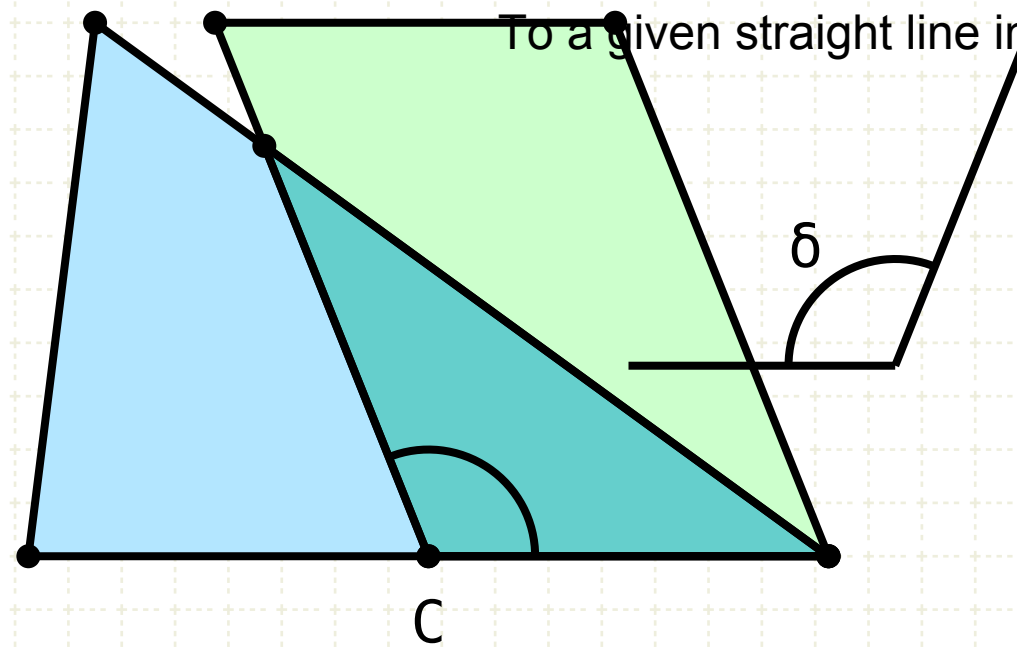
copy one side of parallelogram to AB

copy angle δ to BE



Proposition 44 of Book I

To a given straight line in a given rectilinear angle, to apply a parallelogram equal to a given triangle.



Construction

Start with a given triangle C, a straight line AB and an angle δ

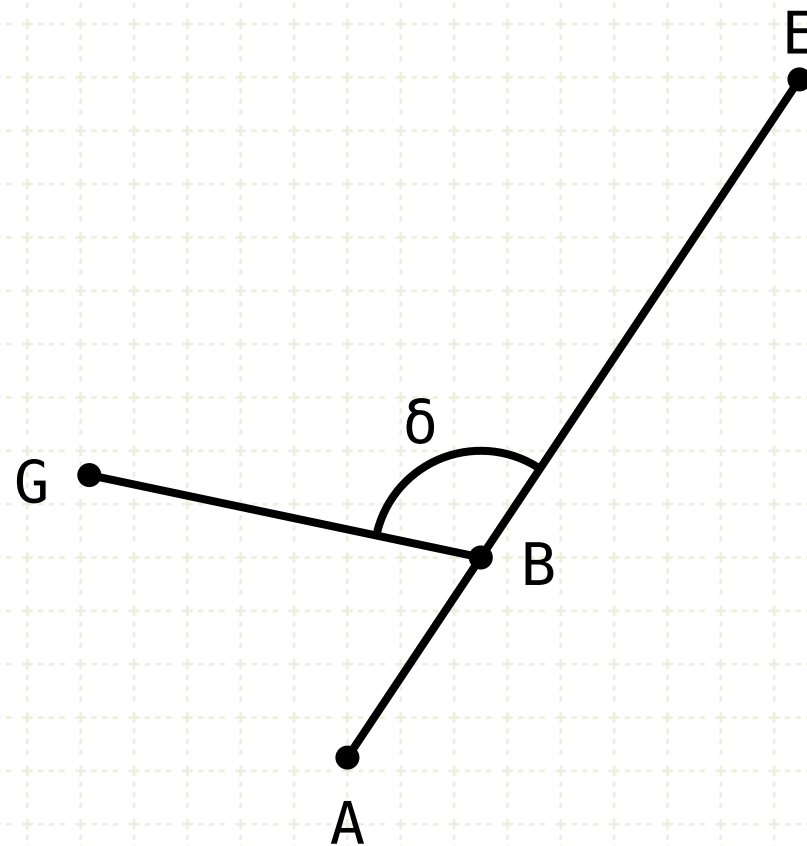
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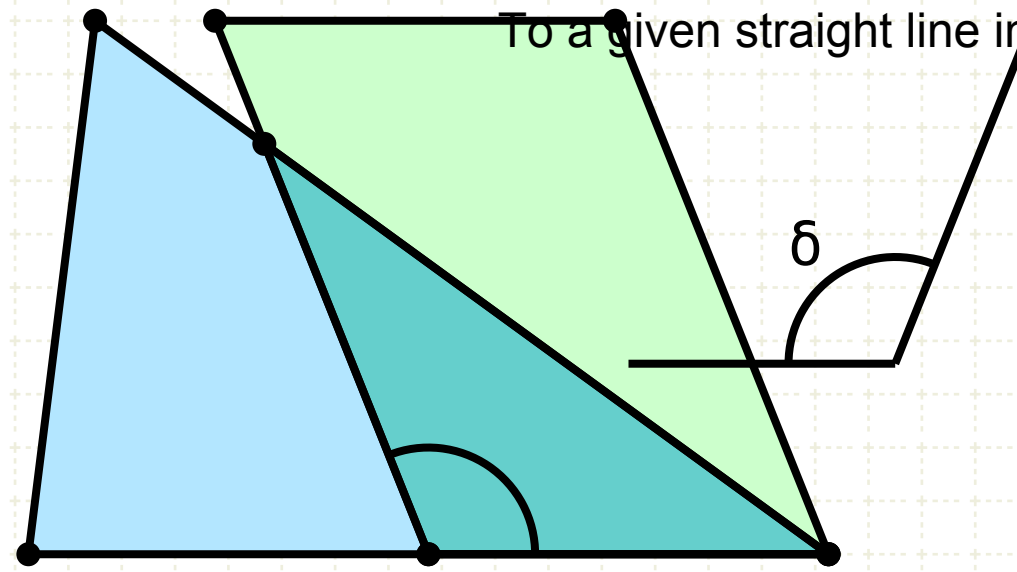
copy angle δ to BE

copy the other side of parallelogram to BG



Proposition 44 of Book I

To a given straight line in a given rectilinear angle, to apply a parallelogram equal to a given triangle.



Construction

Start with a given triangle C, a straight line AB and an angle δ

Create a parallelogram equal to triangle C, with angle δ
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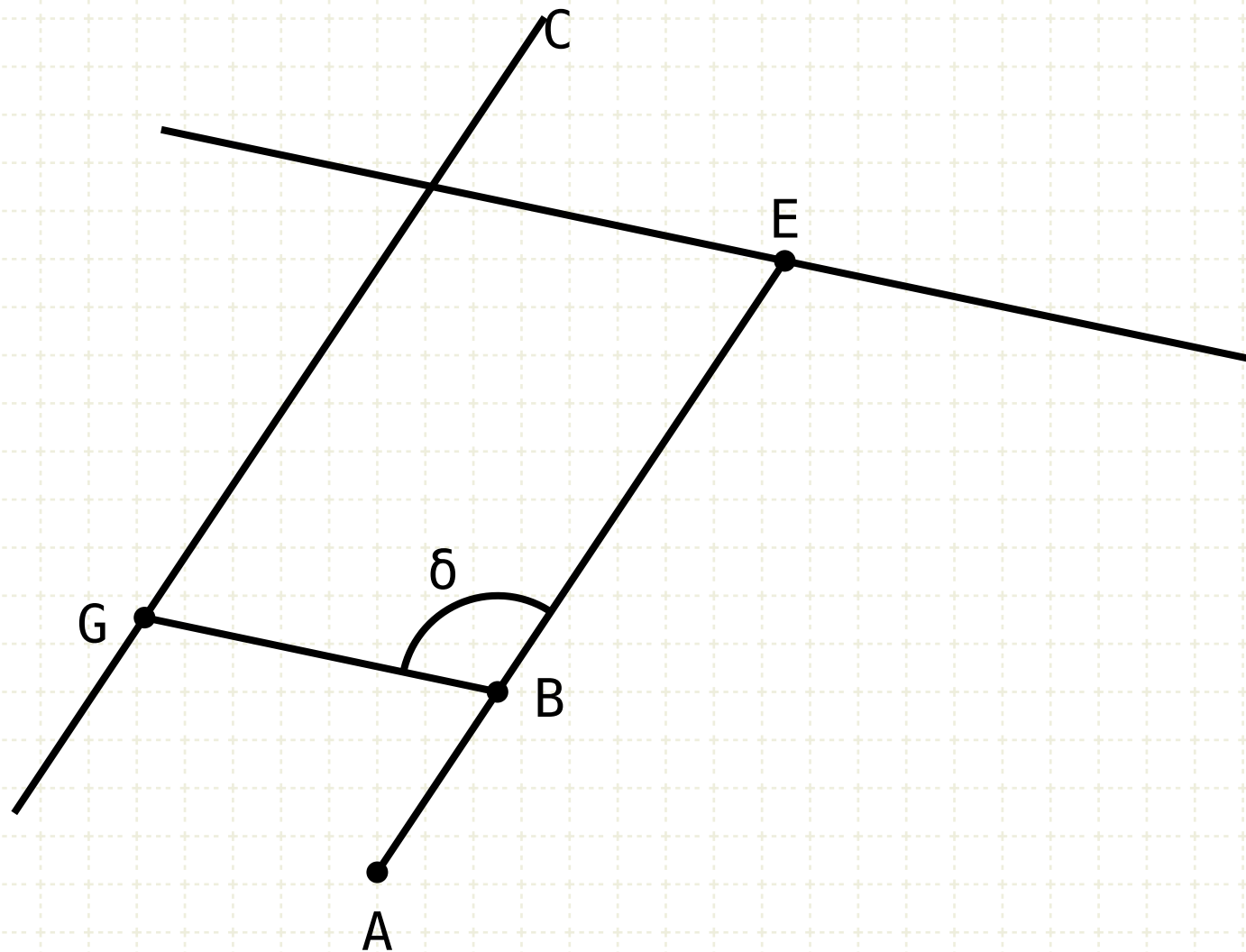
Copy the parallelogram so that one side is in a straight line with AB

copy one side of parallelogram to AB

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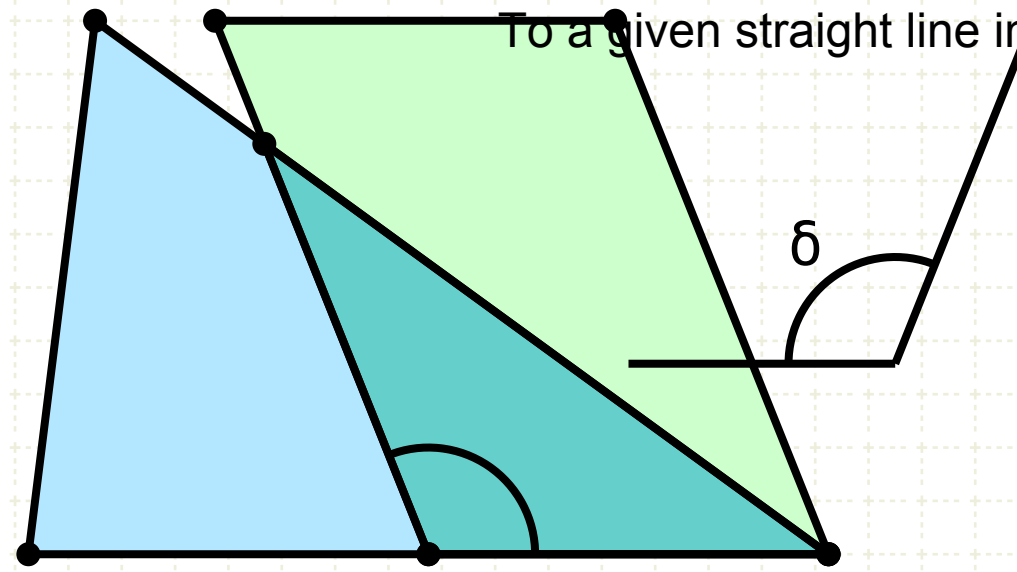
copy the other side of parallelogram to BG

create parallel lines to existing lines



Proposition 44 of Book I

To a given straight line in a given rectilinear angle, to apply a parallelogram equal to a given triangle.



Construction

Start with a given triangle C, a straight line AB and an angle δ

Create a parallelogram equal to triangle C, with angle δ
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Copy the parallelogram so that one side is in a straight line with AB

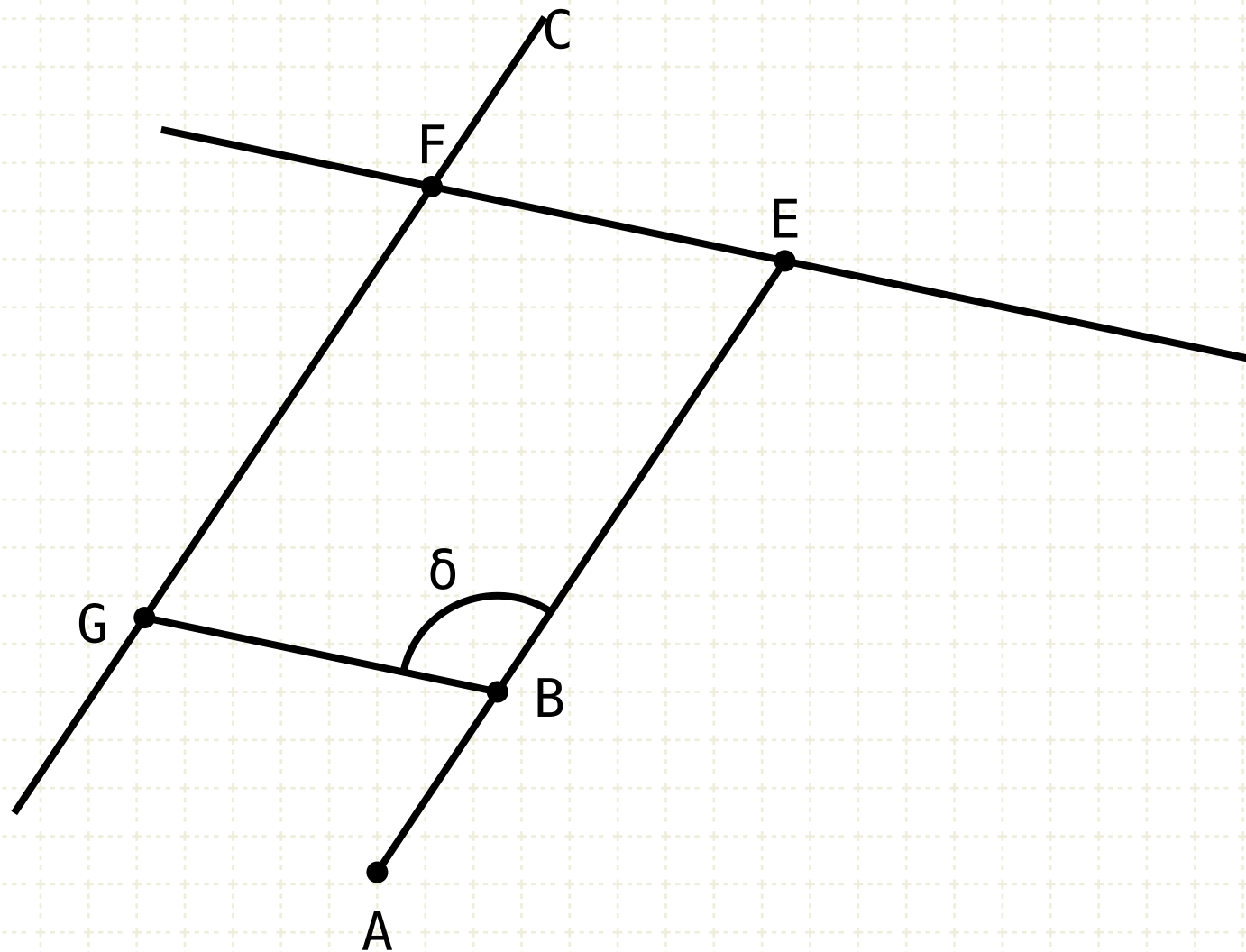
copy one side of parallelogram to AB

copy angle δ to BE

copy the other side of parallelogram to BG

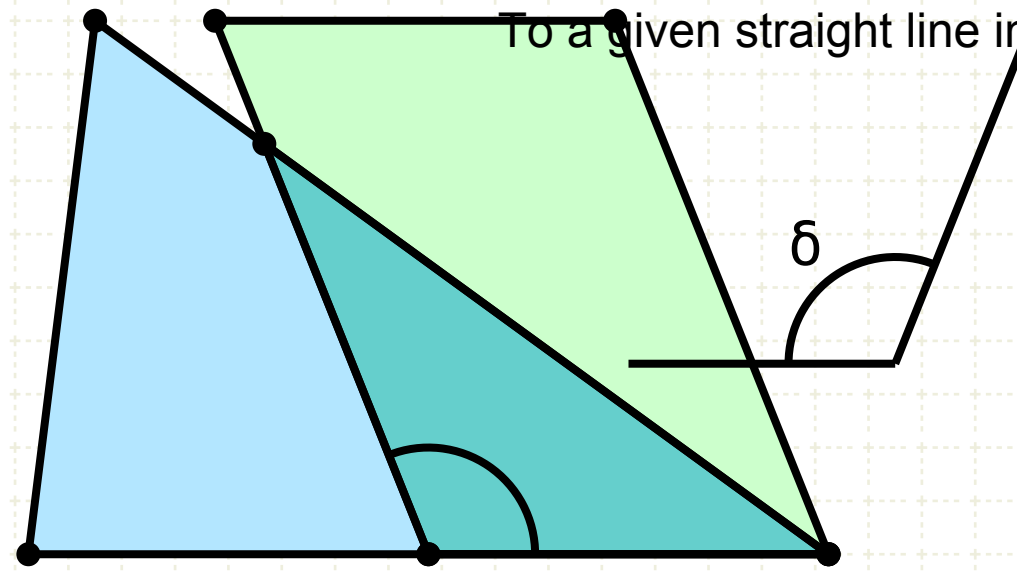
create parallel lines to existing lines

get last and final point



Proposition 44 of Book I

To a given straight line in a given rectilinear angle, to apply a parallelogram equal to a given triangle.



Construction

Start with a given triangle C, a straight line AB and an angle δ

Create a parallelogram equal to triangle C, with angle δ (I.42)

Copy the parallelogram so that one side is in a straight line with AB

copy one side of parallelogram to AB

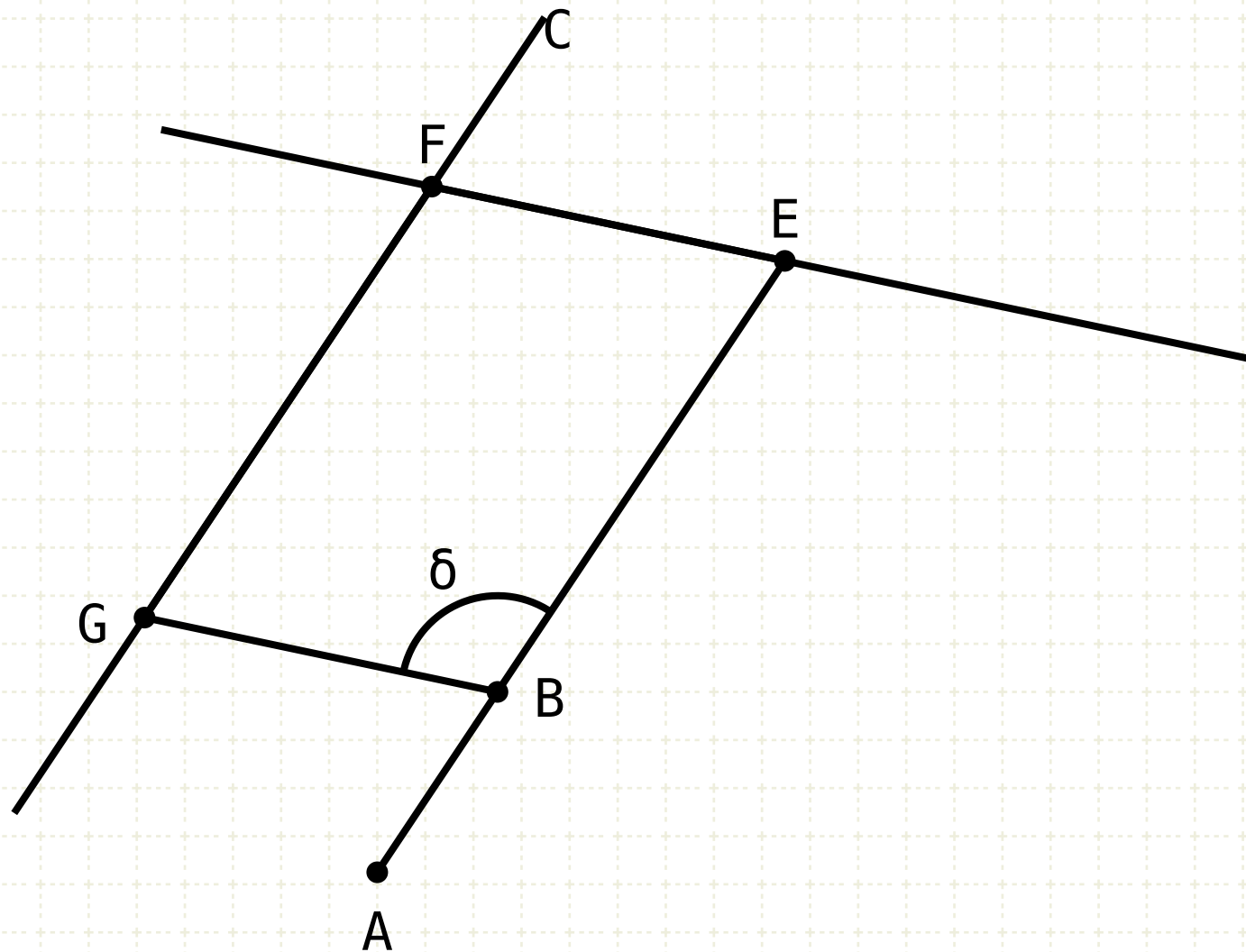
copy angle δ to BE

copy the other side of parallelogram to BG

create parallel lines to existing lines

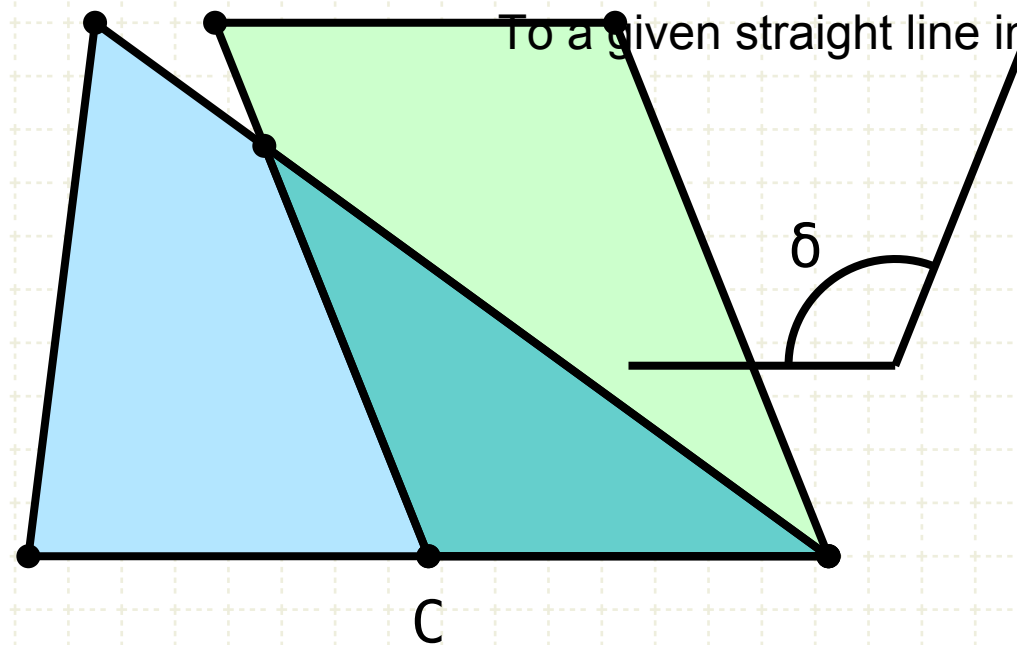
get last and final point

construct the last two lines of the polygon



Proposition 44 of Book I

To a given straight line in a given rectilinear angle, to apply a parallelogram equal to a given triangle.



$$EFGB = \Delta C$$

Construction

Start with a given triangle C, a straight line AB and an angle δ

Create a parallelogram equal to triangle C, with angle δ
(I.42)

Copy the parallelogram so that one side is in a straight line with AB

copy one side of parallelogram to AB

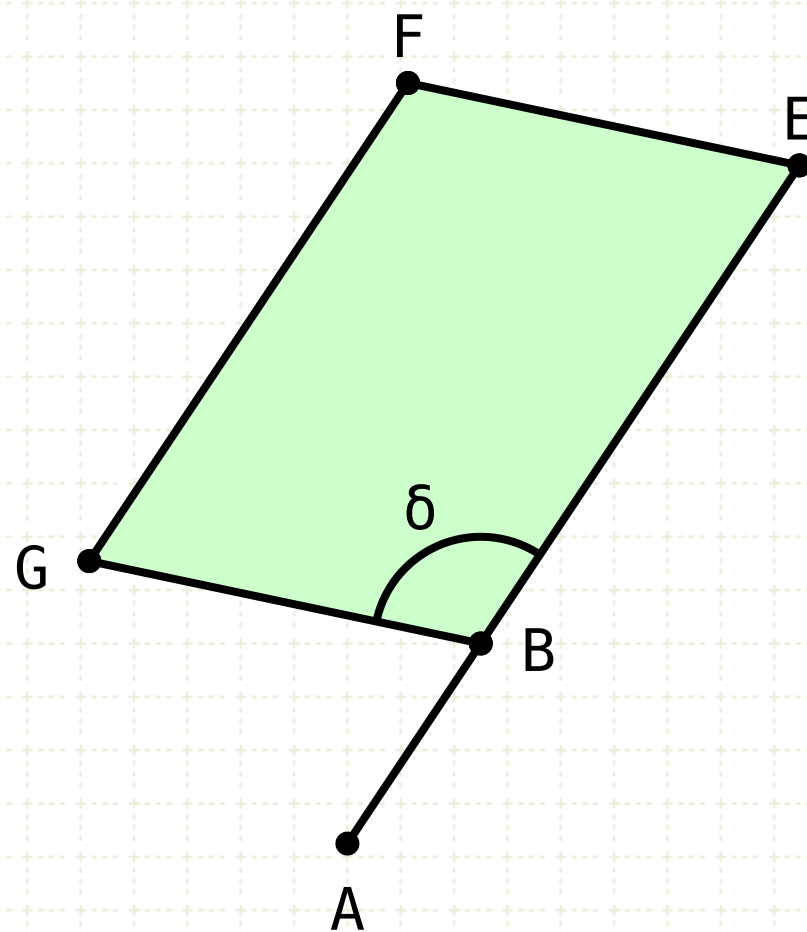
copy angle δ to BE

copy the other side of parallelogram to BG

create parallel lines to existing lines

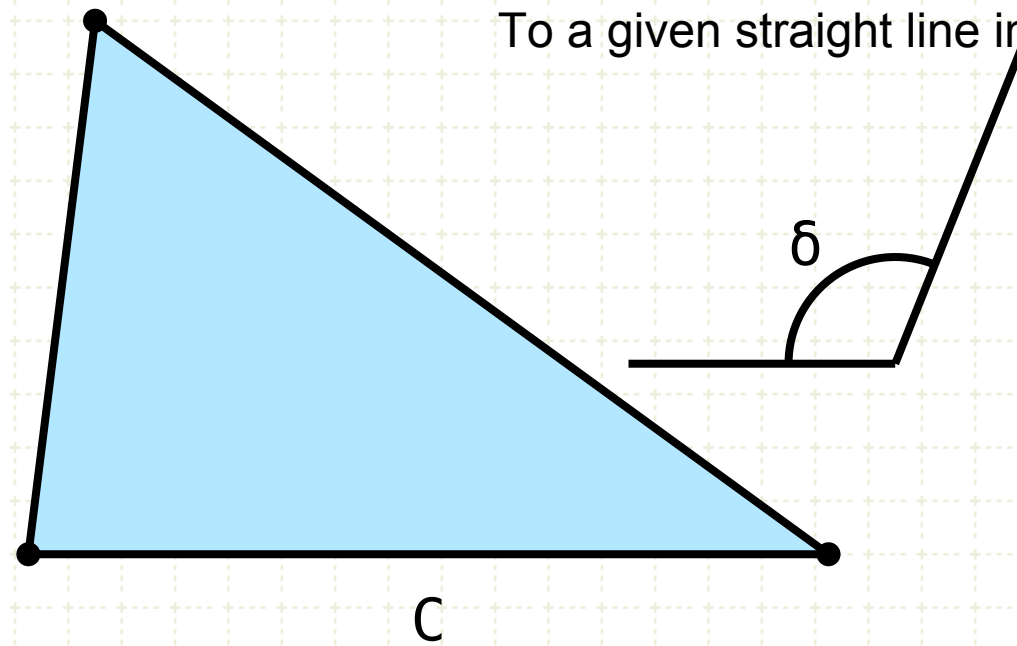
get last and final point

construct the last two lines of the polygon

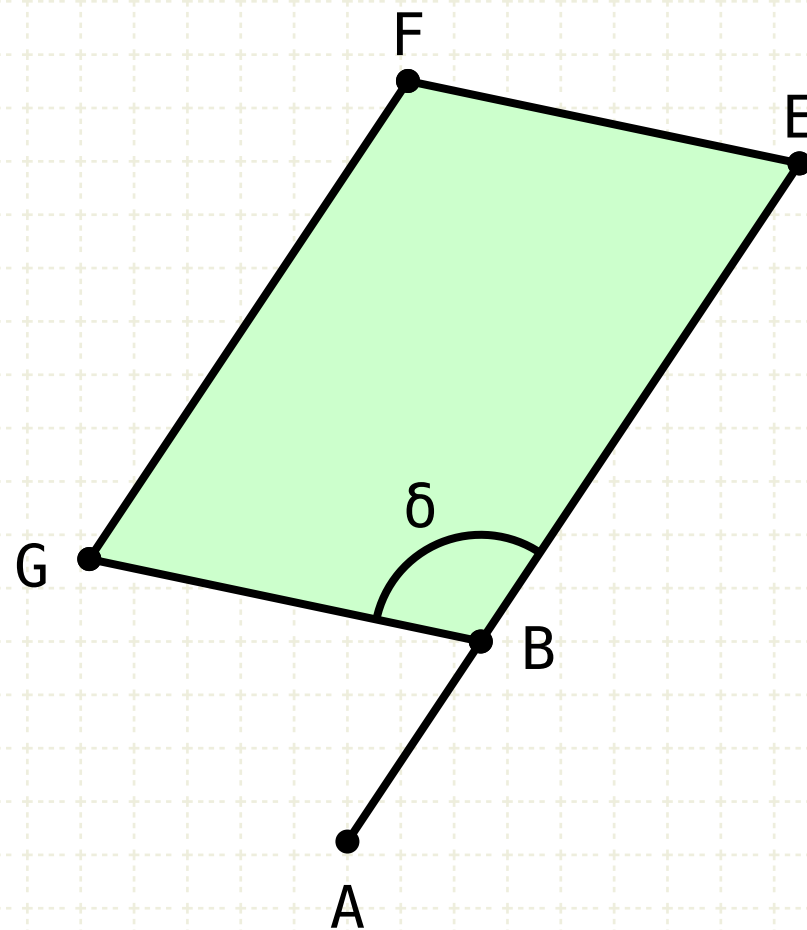


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To a given straight line in a given rectilinear angle, to apply a parallelogram equal to a given triangle.



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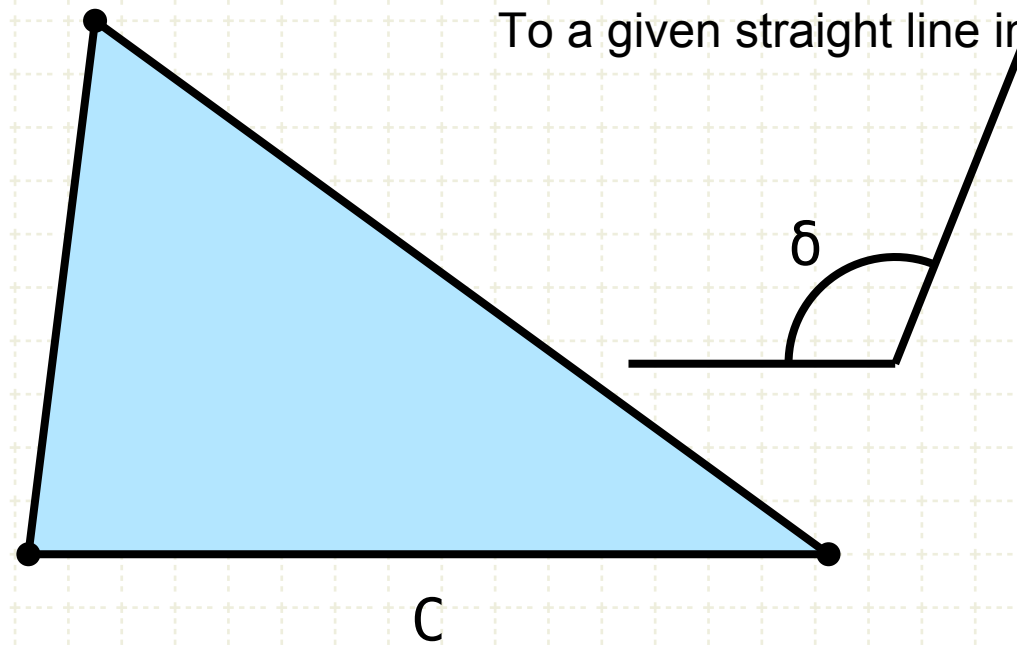
create parallel lines to existing lines

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$$EFGB = \Delta C$$

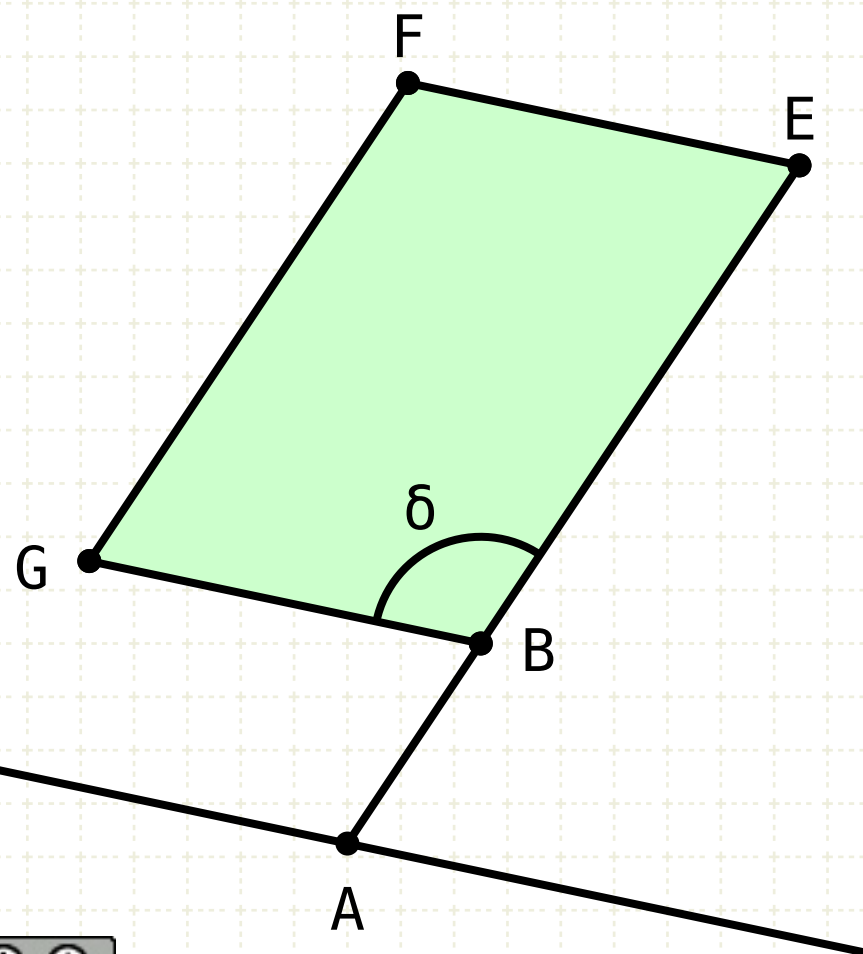
Construction

Start with a given triangle C , a straight line AB and an angle δ

Create a parallelogram equal to triangle C , with angle δ (I·42)

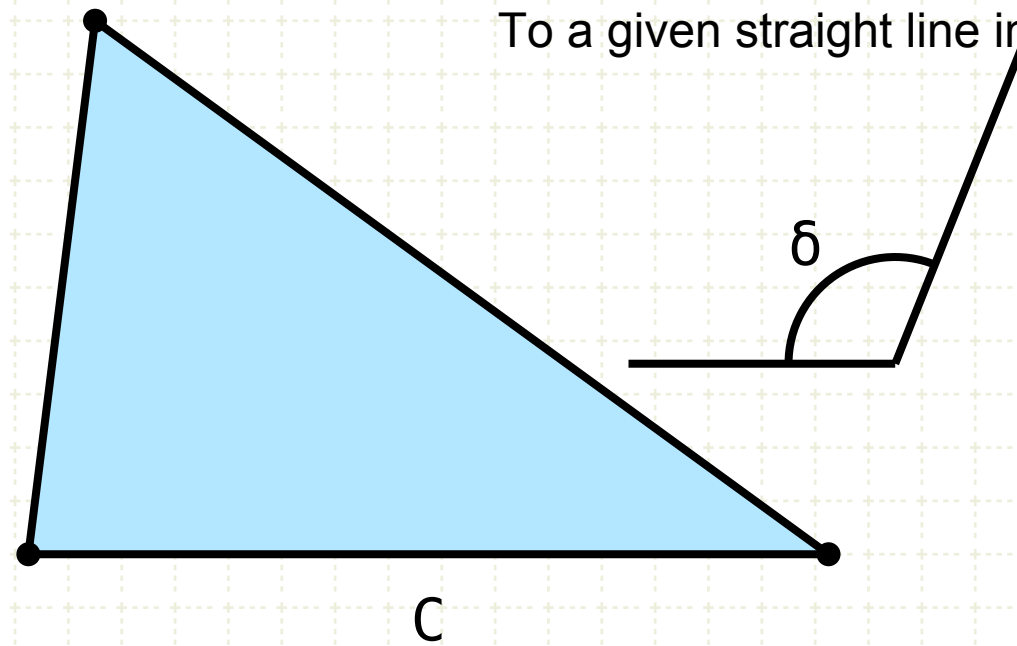
Copy the parallelogram so that one side is in a straight line with AB

Draw a line parallel to GB , through A (I·31)



Proposition 44 of Book I

To a given straight line in a given rectilinear angle, to apply a parallelogram equal to a given triangle.



$$EFGB = \Delta C$$

Construction

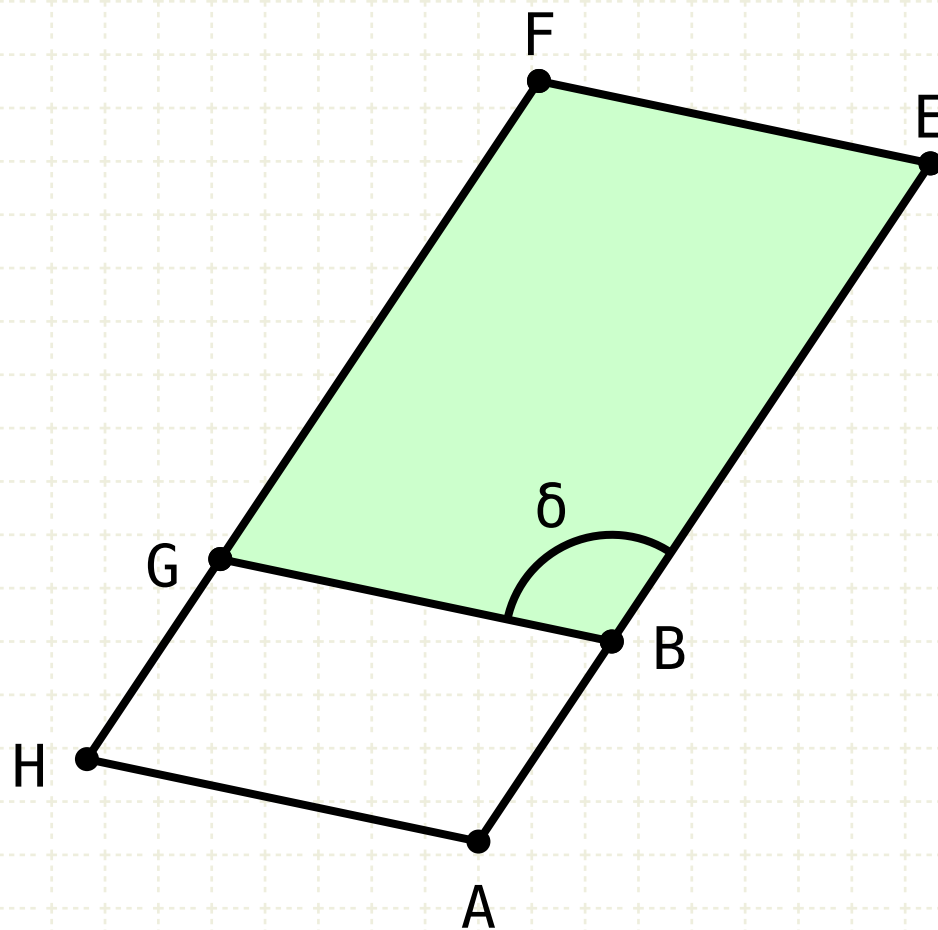
Start with a given triangle C , a straight line AB and an angle δ

Create a parallelogram equal to triangle C , with angle δ (I.42)

Copy the parallelogram so that one side is in a straight line with AB

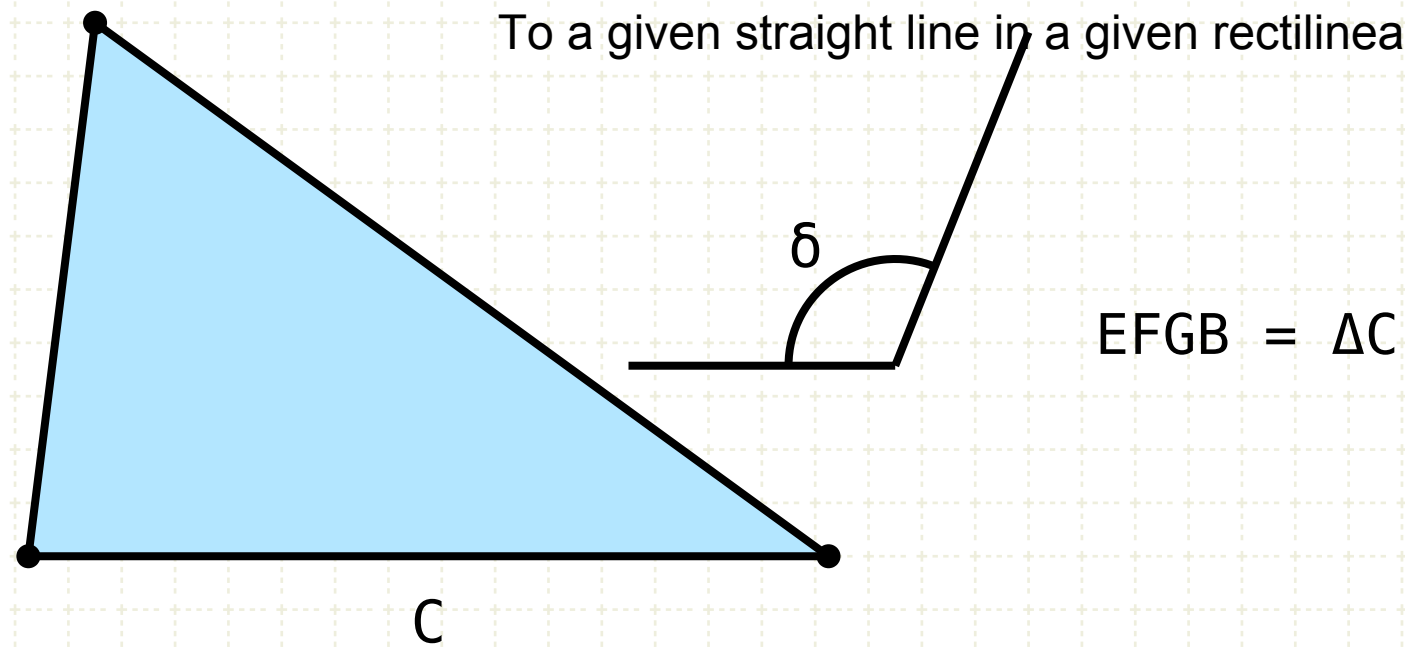
Draw a line parallel to GB , through A (I.31)

Extend line FG so that it intersects with the previously drawn line, at point H



Proposition 44 of Book I

To a given straight line in a given rectilinear angle, to apply a parallelogram equal to a given triangle.



Construction

Start with a given triangle C , a straight line AB and an angle δ

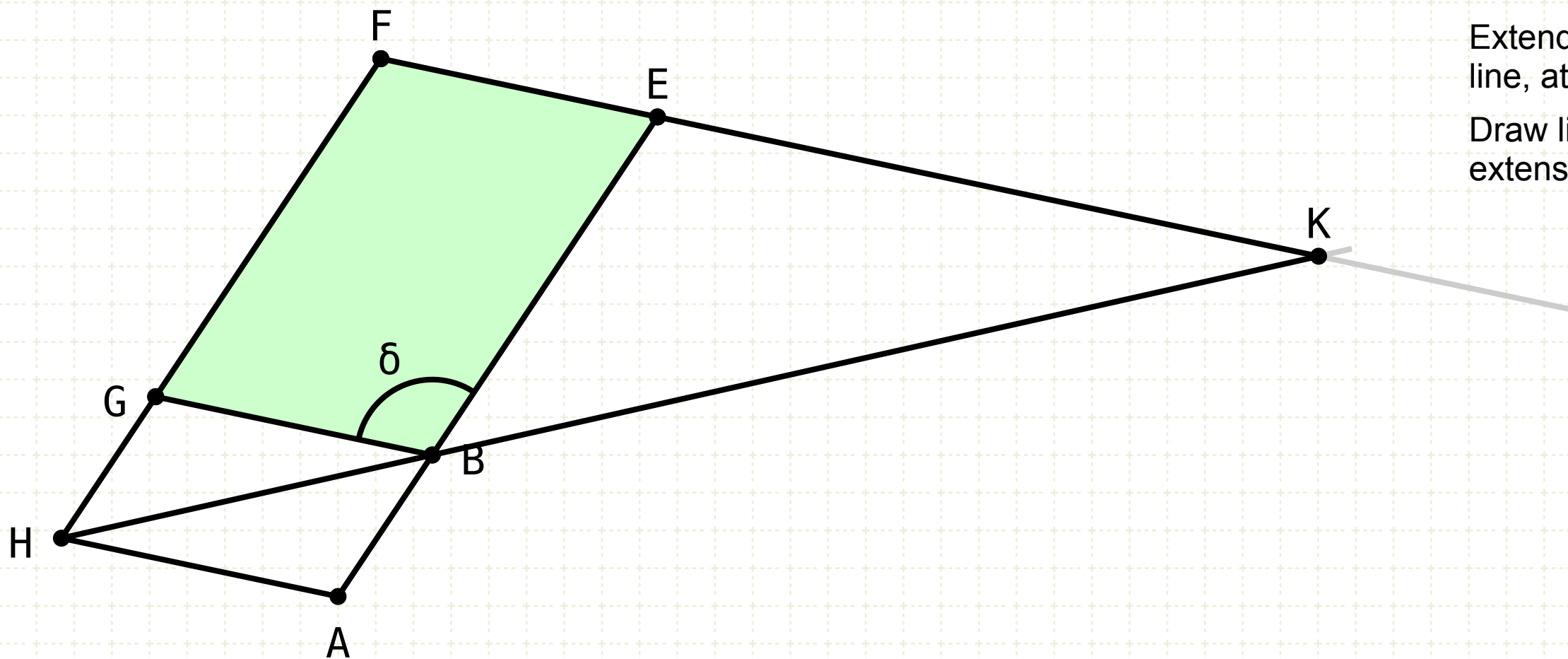
Create a parallelogram equal to triangle C , with angle δ (I·42)

Copy the parallelogram so that one side is in a straight line with AB

Draw a line parallel to GB , through A (I·31)

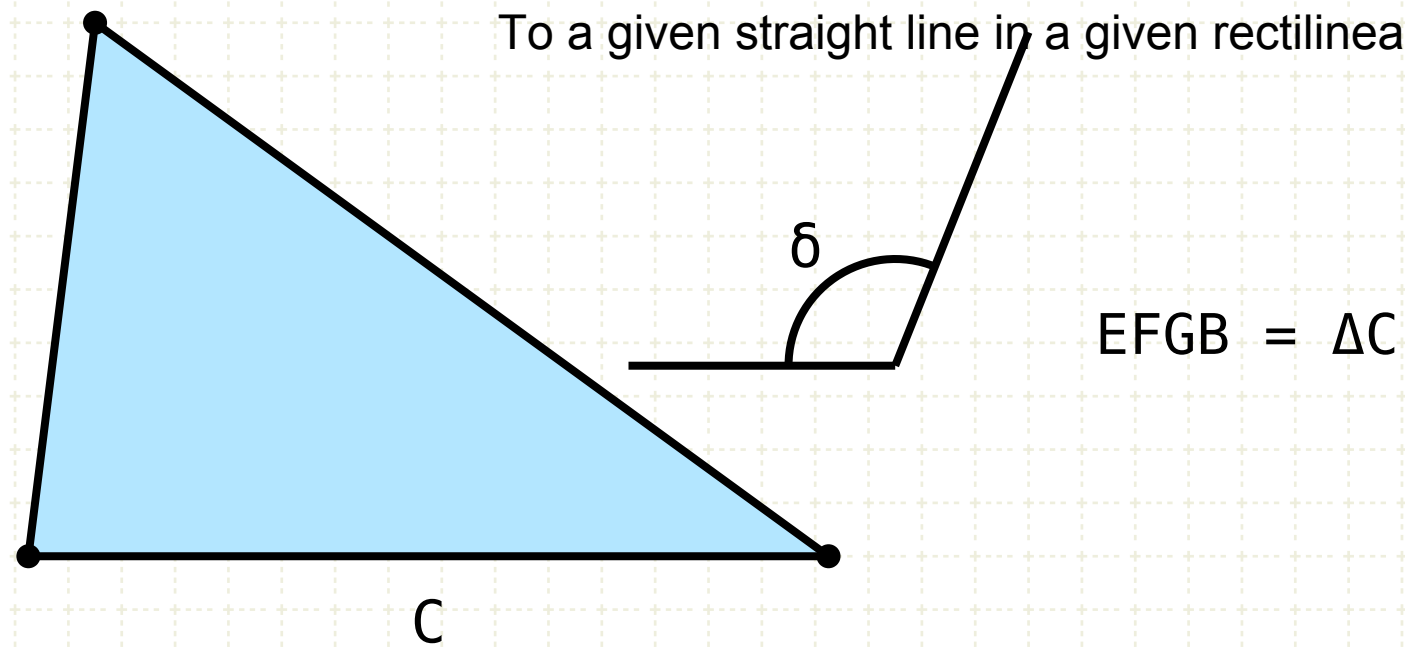
Extend line FG so that it intersects with the previously drawn line, at point H

Draw line HB , extending the line so that it intersects the extension of line FE at point K



Proposition 44 of Book I

To a given straight line in a given rectilinear angle, to apply a parallelogram equal to a given triangle.



Construction

Start with a given triangle C, a straight line AB and an angle δ

Create a parallelogram equal to triangle C, with angle δ
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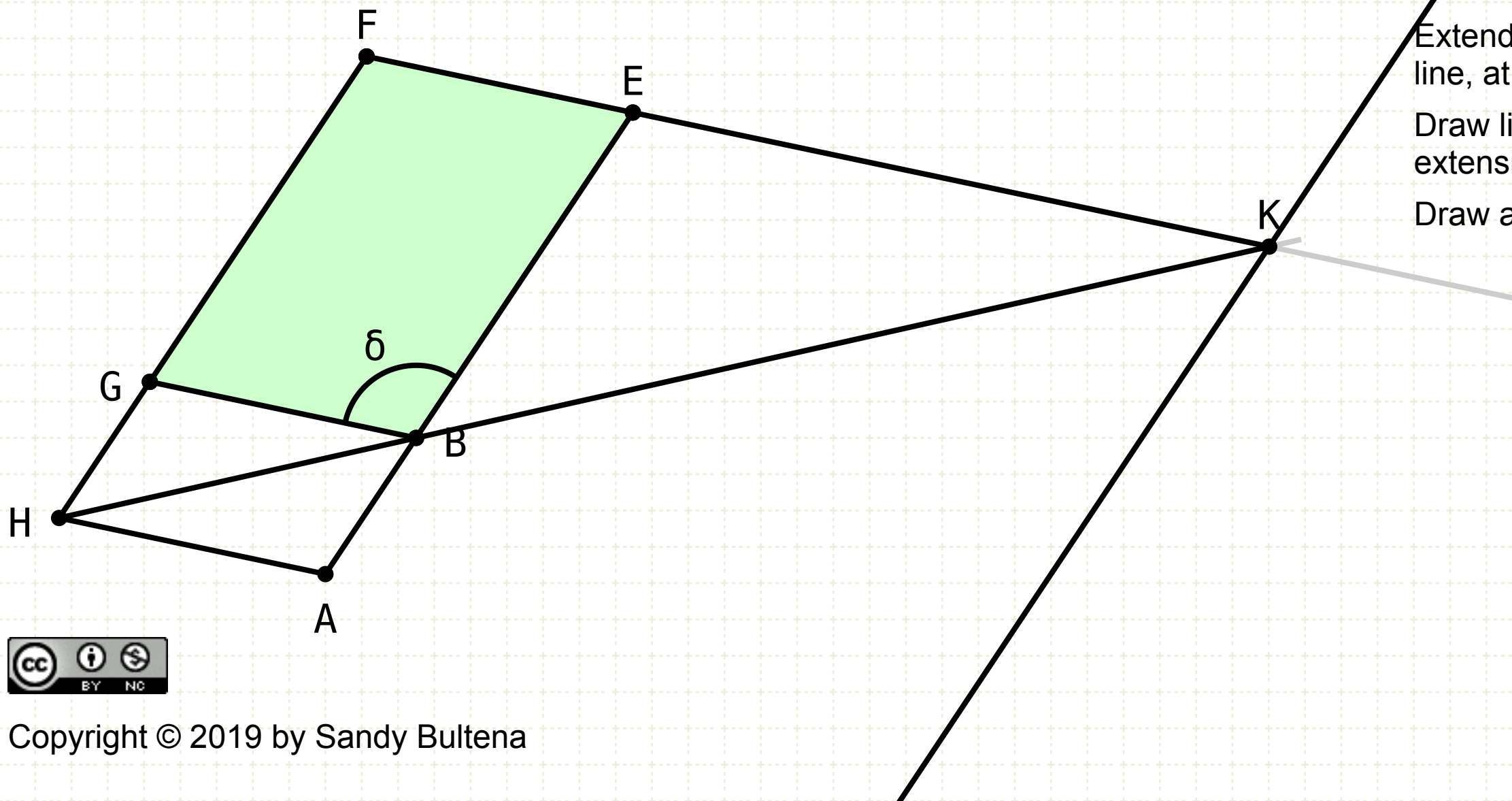
Copy the parallelogram so that one side is in a straight line with AB

~~Draw a line parallel to GB, through A (I-31)~~

Extend line FG so that it intersects with the previously drawn line, at point H

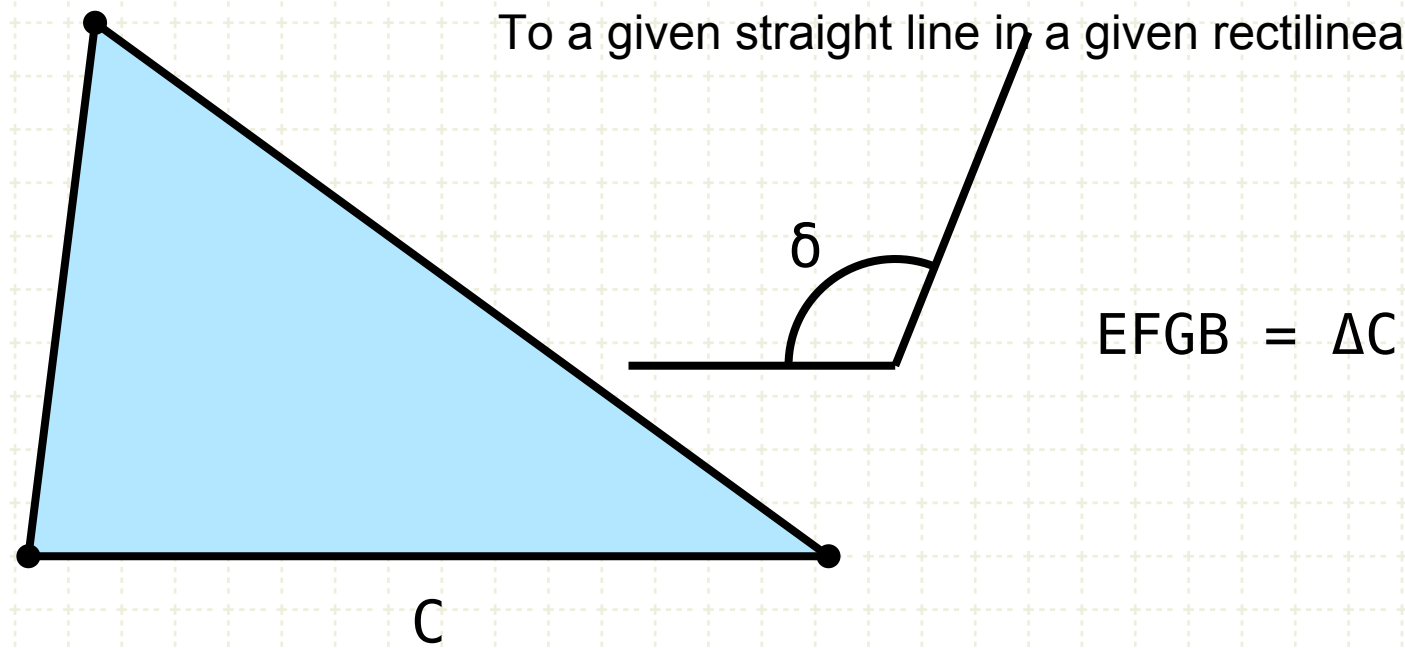
Draw line HB, extending the line so that it intersects the extension of line FE at point K

Draw a line parallel to BE, through K (I-31)



Proposition 44 of Book I

To a given straight line in a given rectilinear angle, to apply a parallelogram equal to a given triangle.



Construction

Start with a given triangle C , a straight line AB and an angle δ

Create a parallelogram equal to triangle C , with angle δ (I·42)

Copy the parallelogram so that one side is in a straight line with AB

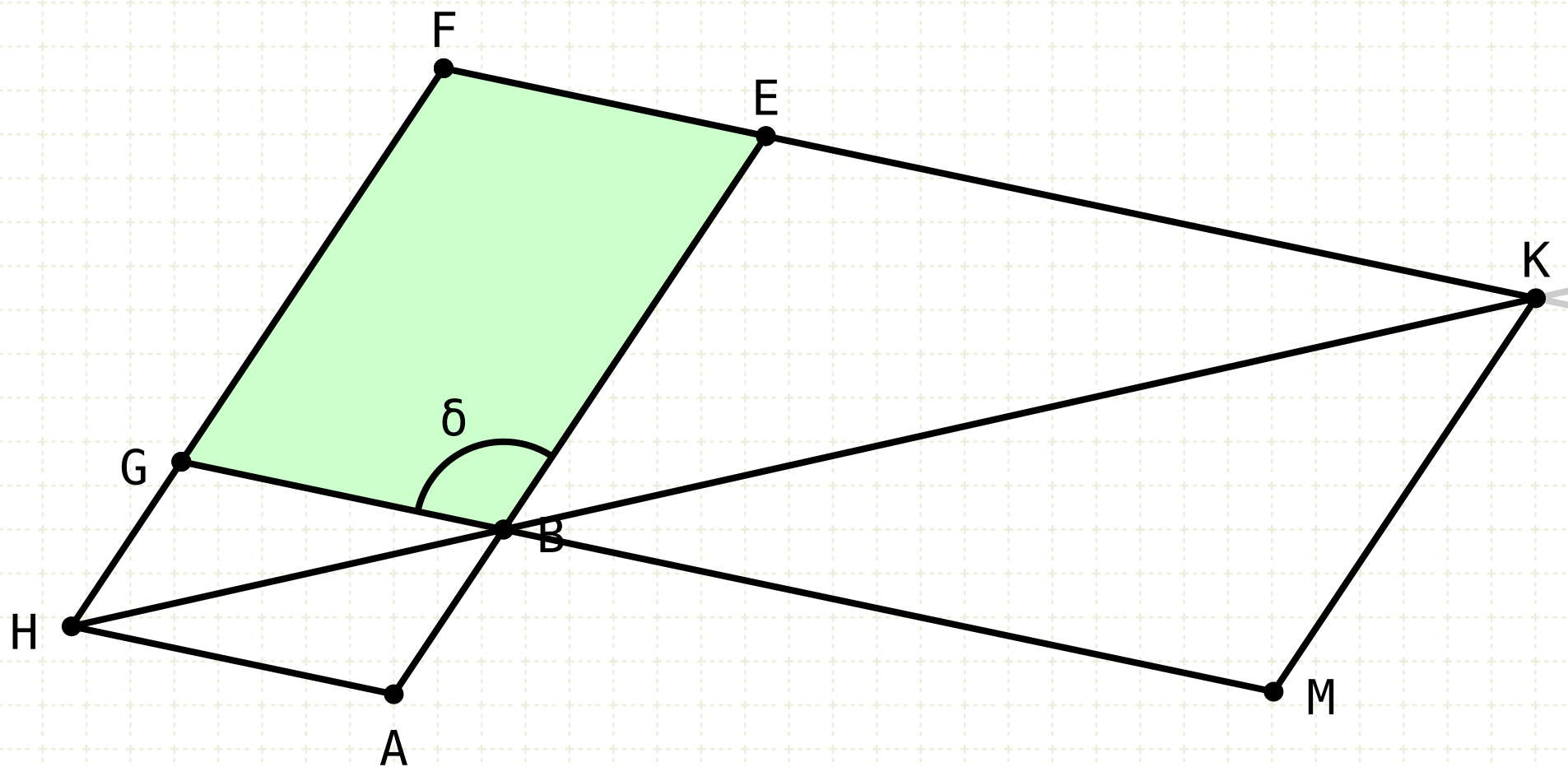
Draw a line parallel to GB , through A (I·31)

Extend line FG so that it intersects with the previously drawn line, at point H

Draw line HB , extending the line so that it intersects the extension of line FE at point K

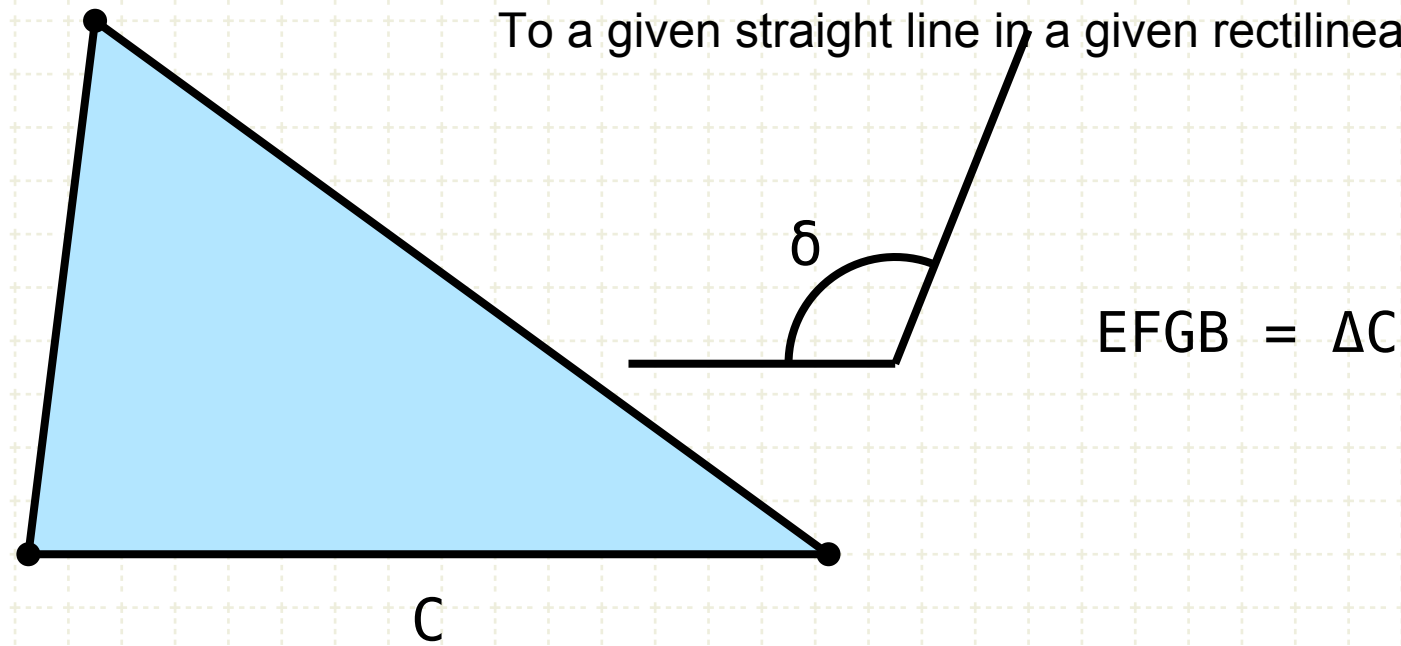
Draw a line parallel to BE , through K (I·31)

Extend GB so that it intersects with the previously drawn line, at point M



Proposition 44 of Book I

To a given straight line in a given rectilinear angle, to apply a parallelogram equal to a given triangle.



Construction

Start with a given triangle C , a straight line AB and an angle δ

Create a parallelogram equal to triangle C , with angle δ (I·42)

Copy the parallelogram so that one side is in a straight line with AB

Draw a line parallel to GB , through A (I·31)

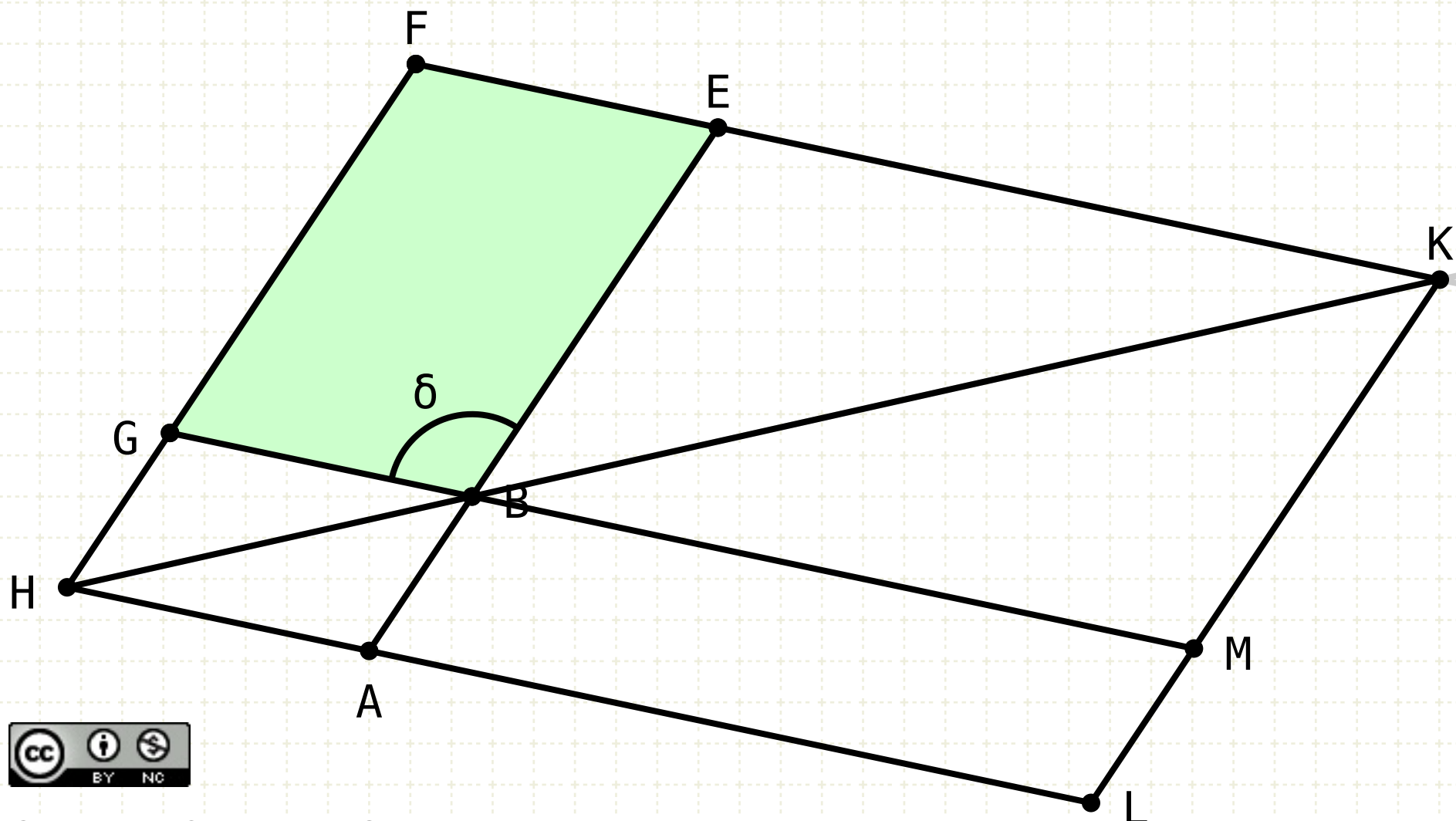
Extend line FG so that it intersects with the previously drawn line, at point H

Draw line HB , extending the line so that it intersects the extension of line FE at point K

Draw a line parallel to BE , through K (I·31)

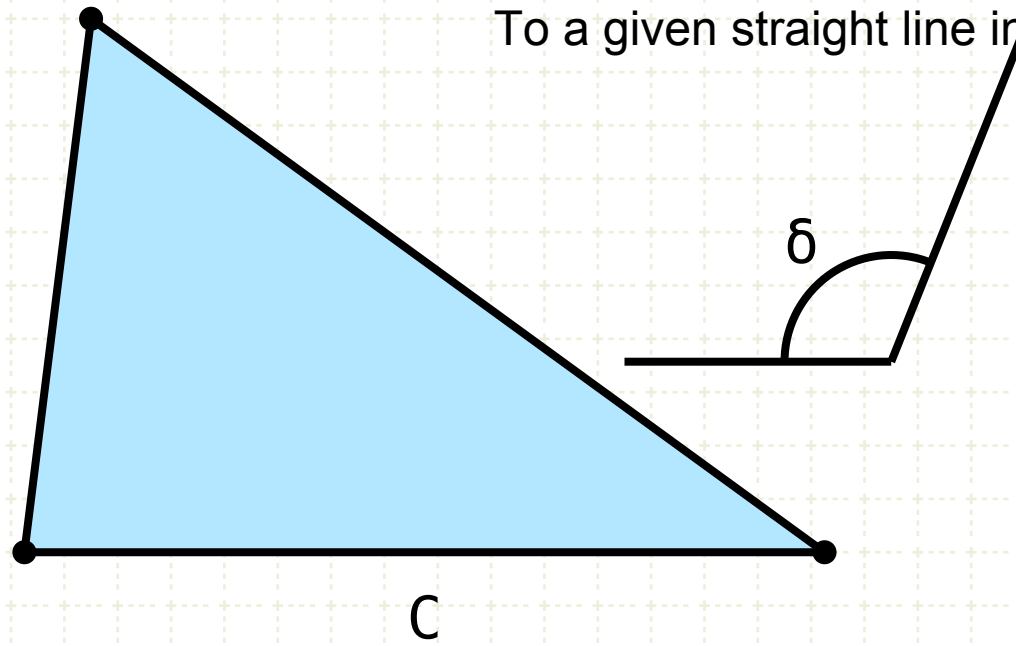
Extend GB so that it intersects with the previously drawn line, at point M

Extend HA and KM so that they intersect at point L



Proposition 44 of Book I

To a given straight line in a given rectilinear angle, to apply a parallelogram equal to a given triangle.



$$EFGB = \Delta C$$

$$\text{BALM} = \Delta C$$

Construction

Start with a given triangle C, a straight line AB and an angle δ

Create a parallelogram equal to triangle C, with angle δ
(I.42)

Copy the parallelogram so that one side is in a straight line with AB

Draw a line parallel to GB, through A (I-31)

Extend line FG so that it intersects with the previously drawn line, at point H

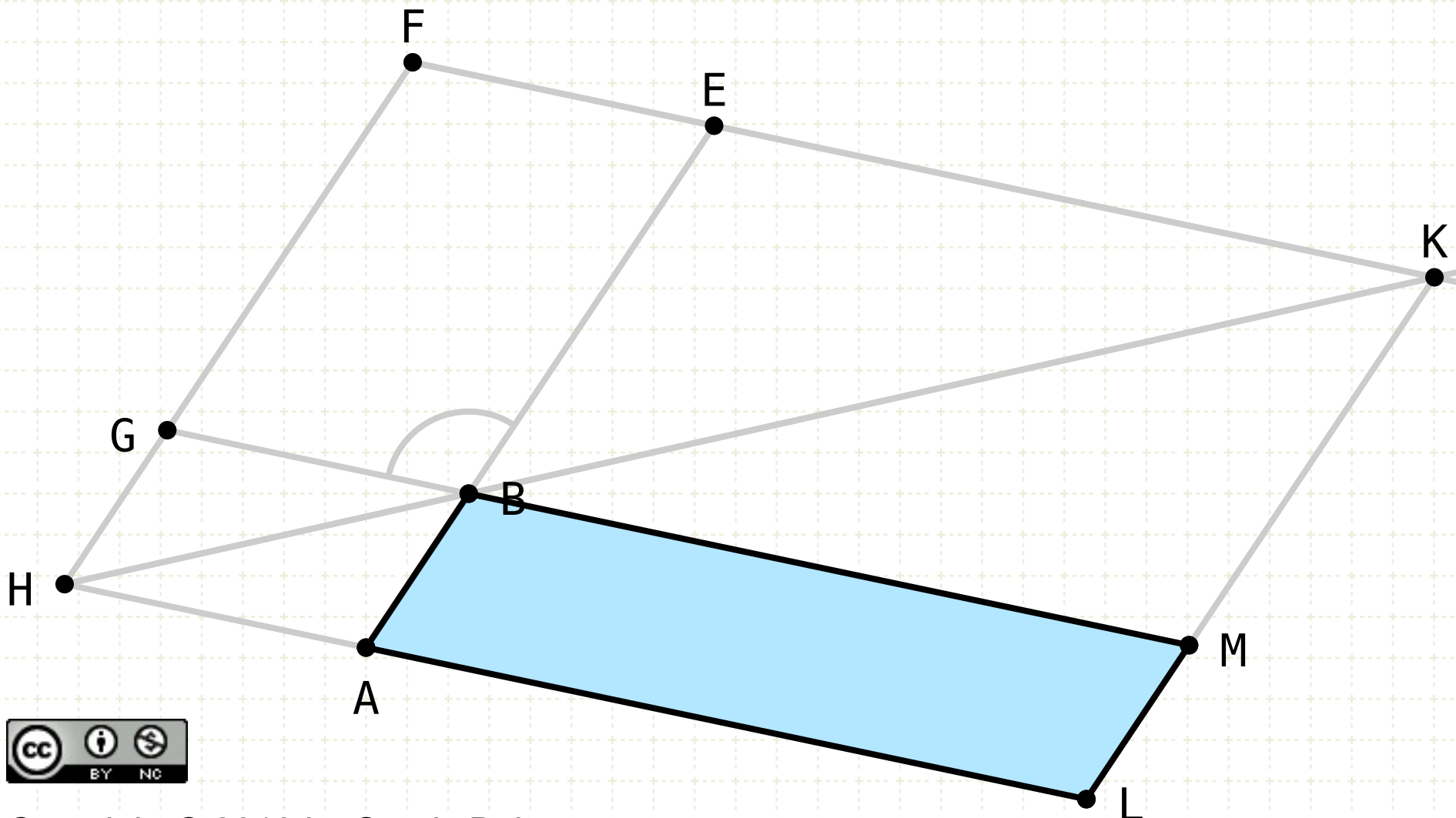
Draw line HB, extending the line so that it intersects the extension of line FE at point K

Draw a line parallel to BE, through K (I-31)

Extend GB so that it intersects with the previously drawn line, at point M

Extend HA and KM so that they intersect at point L

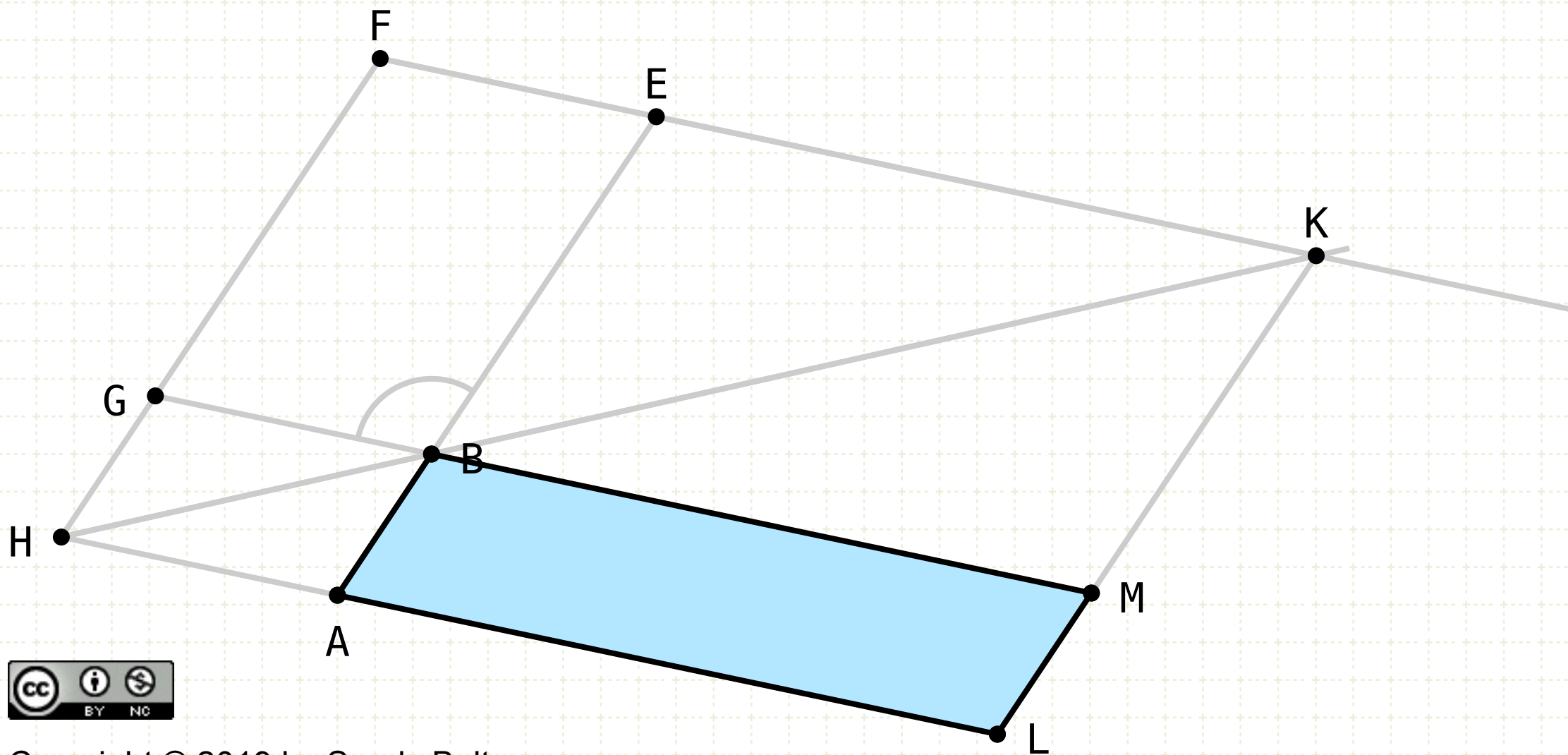
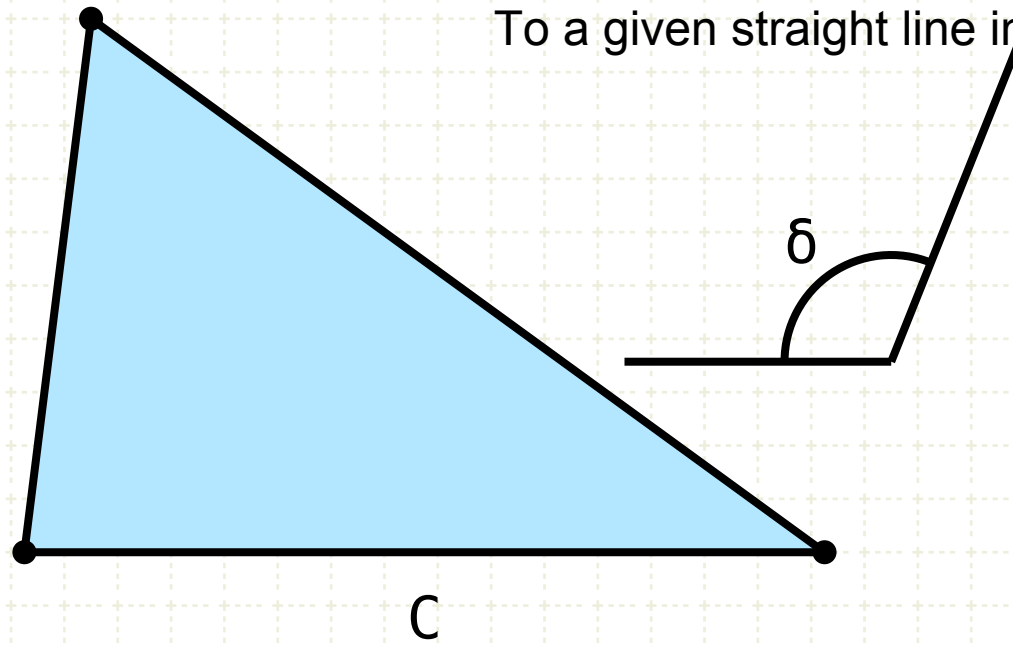
Parallelogram BALM is equal in area to triangle C, and it contains the angle δ and one of it's sides is the given line AB



Proposition 44 of Book I

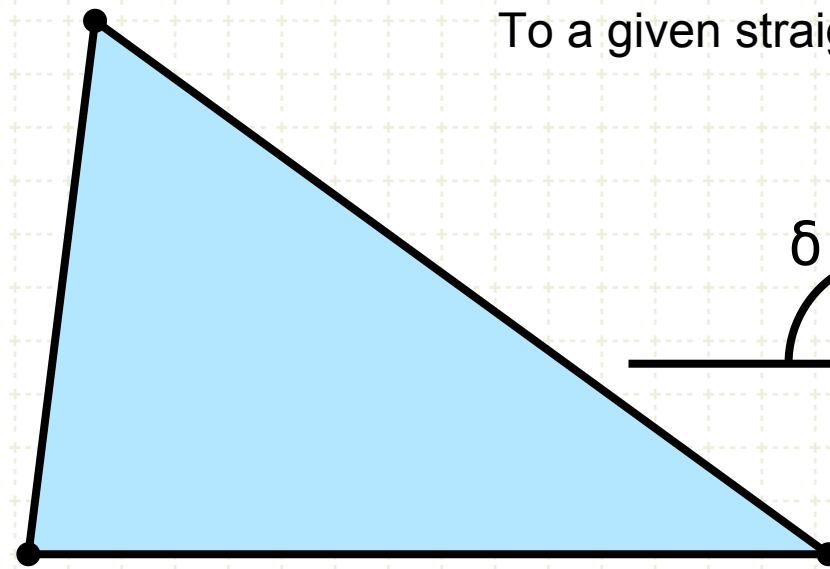
To a given straight line in a given rectilinear angle, to apply a parallelogram equal to a given triangle.

Proof:

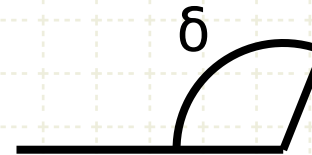


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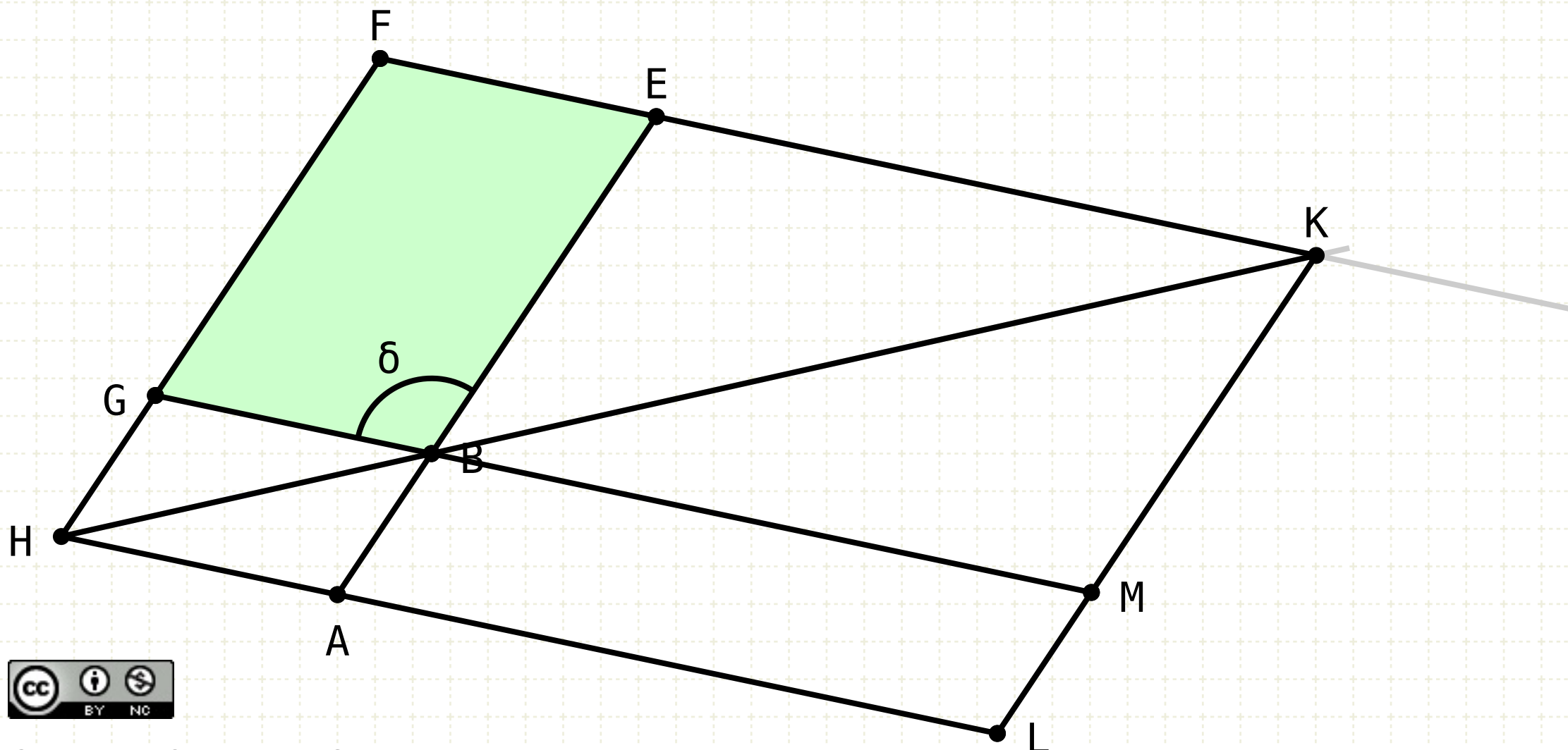
C



$$EFGB = \Delta C$$

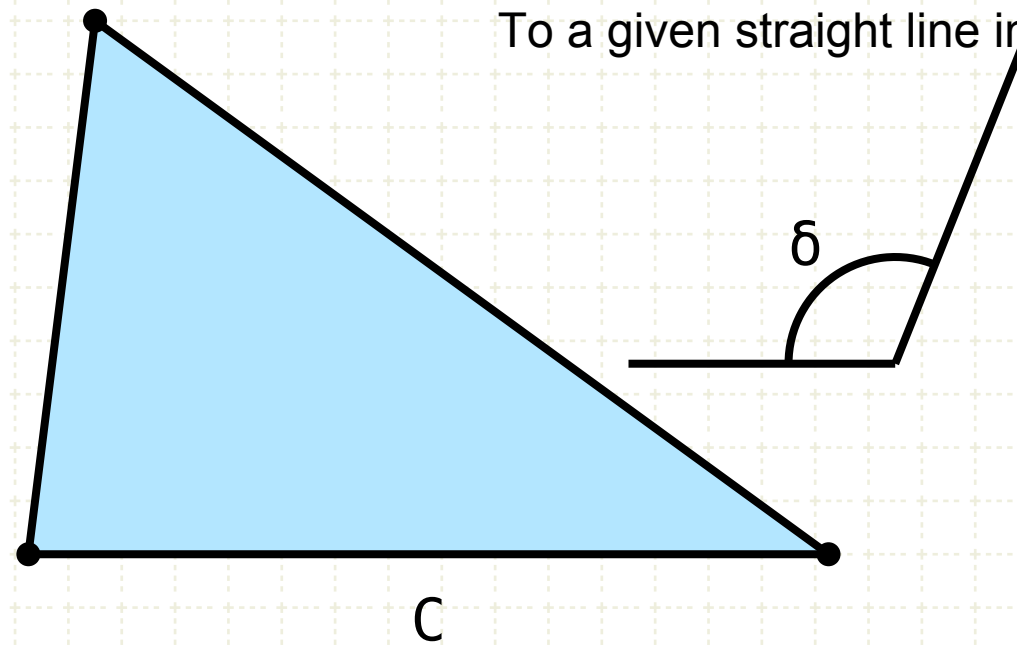
Proof:

By definition of the construction techniques EFGB is equal to the given triangle C (I-42)



Proposition 44 of Book I

To a given straight line in a given rectilinear angle, to apply a parallelogram equal to a given triangle.



$$EFGB = \triangle C$$

$$FE \parallel HA$$

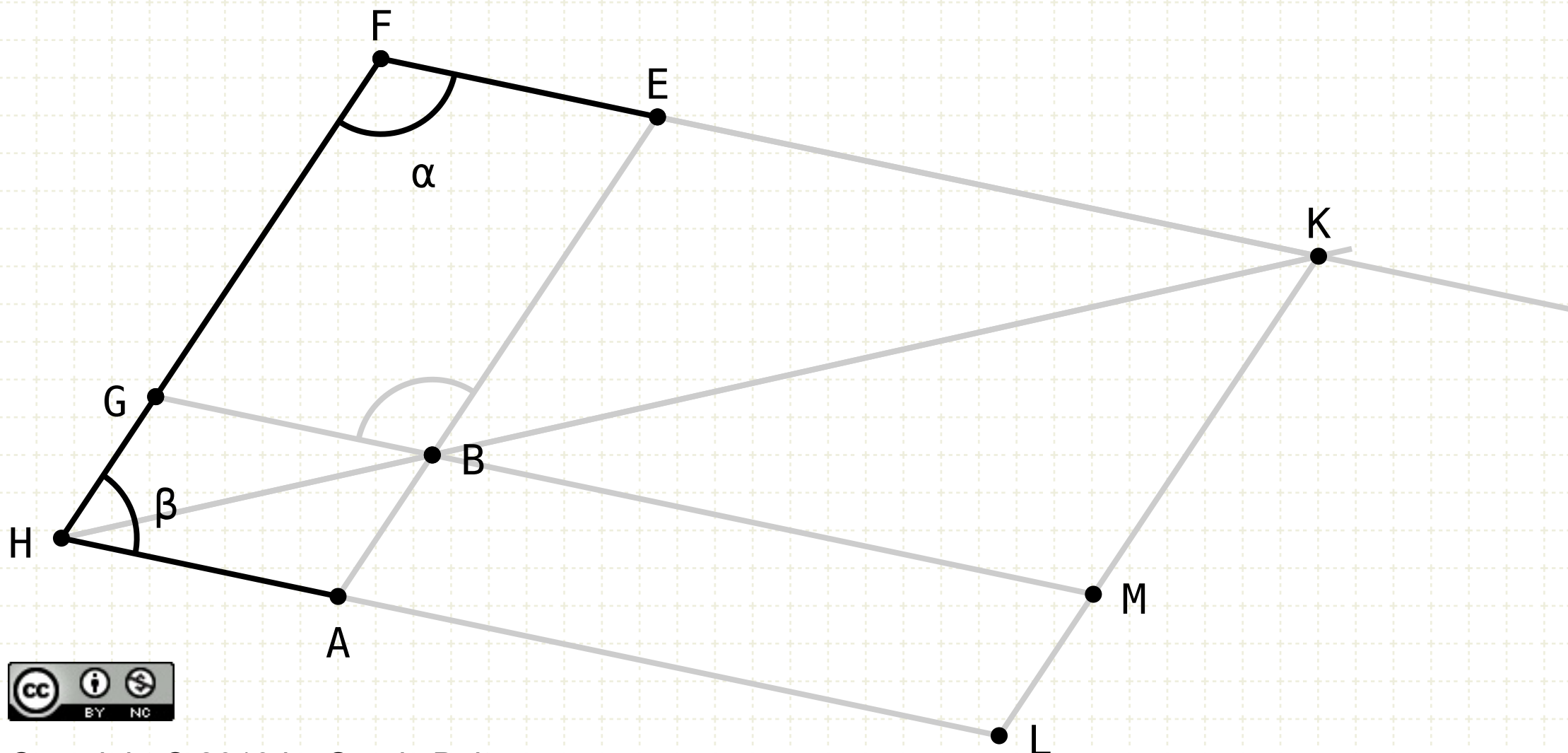
$$\alpha + \beta = L + L$$

Proof:

By definition of the construction techniques EFGB is equal to the given triangle C (I-42)

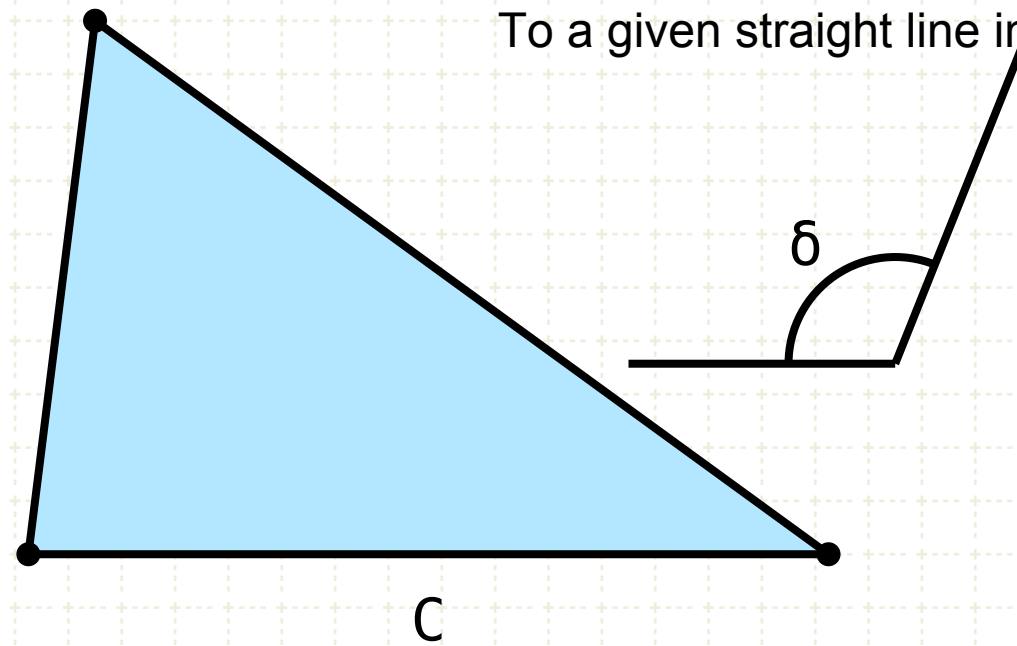
The extension of HB will intersect the extension of line FK because...

Lines FE and HA are parallel,
therefore the sum of the angles EFG and GHA is equal
the sum of two right angles (I-42)



Proposition 44 of Book I

To a given straight line in a given rectilinear angle, to apply a parallelogram equal to a given triangle.



$$EFGB = \triangle C$$

$$FE \parallel HA$$

$$\alpha + \beta = L + L$$

$$\varepsilon < \beta$$

$$\varepsilon + \alpha < L + L$$

Proof:

By definition of the construction techniques EFGB is equal to the given triangle C (I-42)

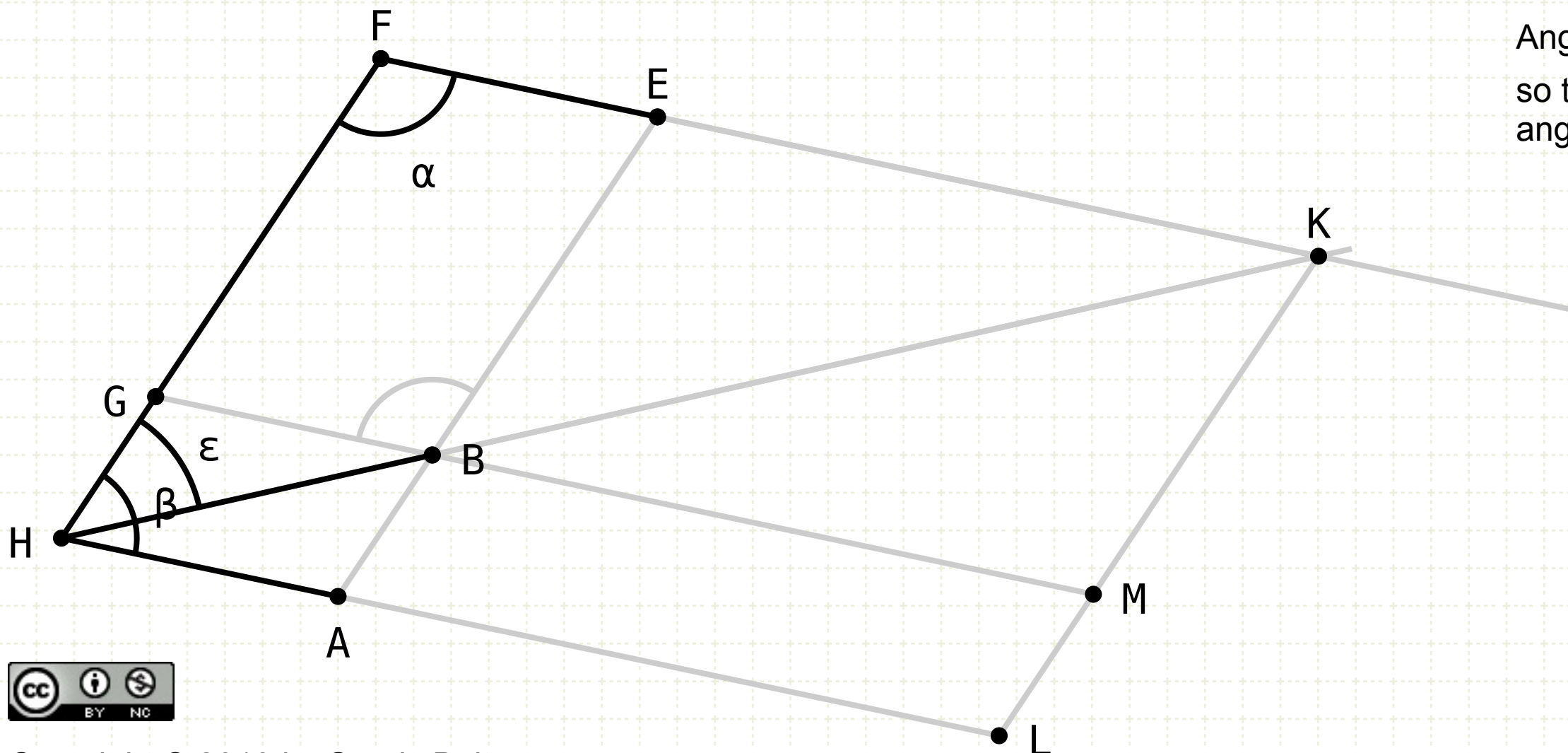
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Lines FE and HA are parallel,

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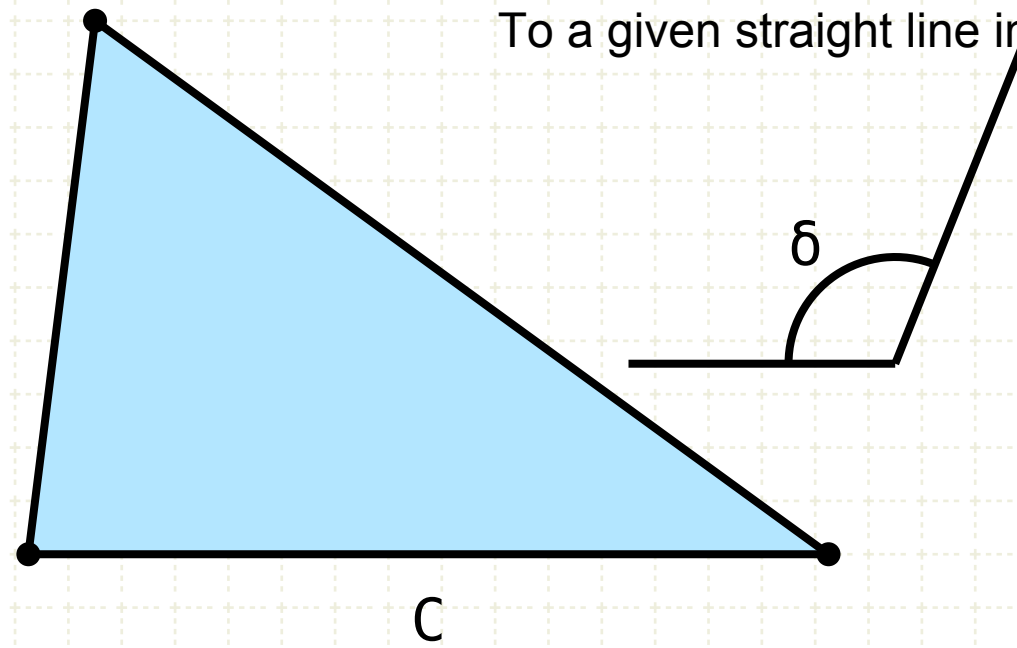
Angle BHG is less than GHA,

so the sum of angles EFG and GHA is less than two right angles



Proposition 44 of Book I

To a given straight line in a given rectilinear angle, to apply a parallelogram equal to a given triangle.



$$EFGB = \Delta C$$

$$\text{FE} \quad || \quad \text{HA}$$

$$\alpha + \beta = L + L$$

$$\varepsilon > \beta$$

$$\varepsilon + \alpha < L + L$$

Proof:

By definition of the construction techniques EFGB is equal to the given triangle C (I-42)

The extension of HB will intersect the extension of line FK because...

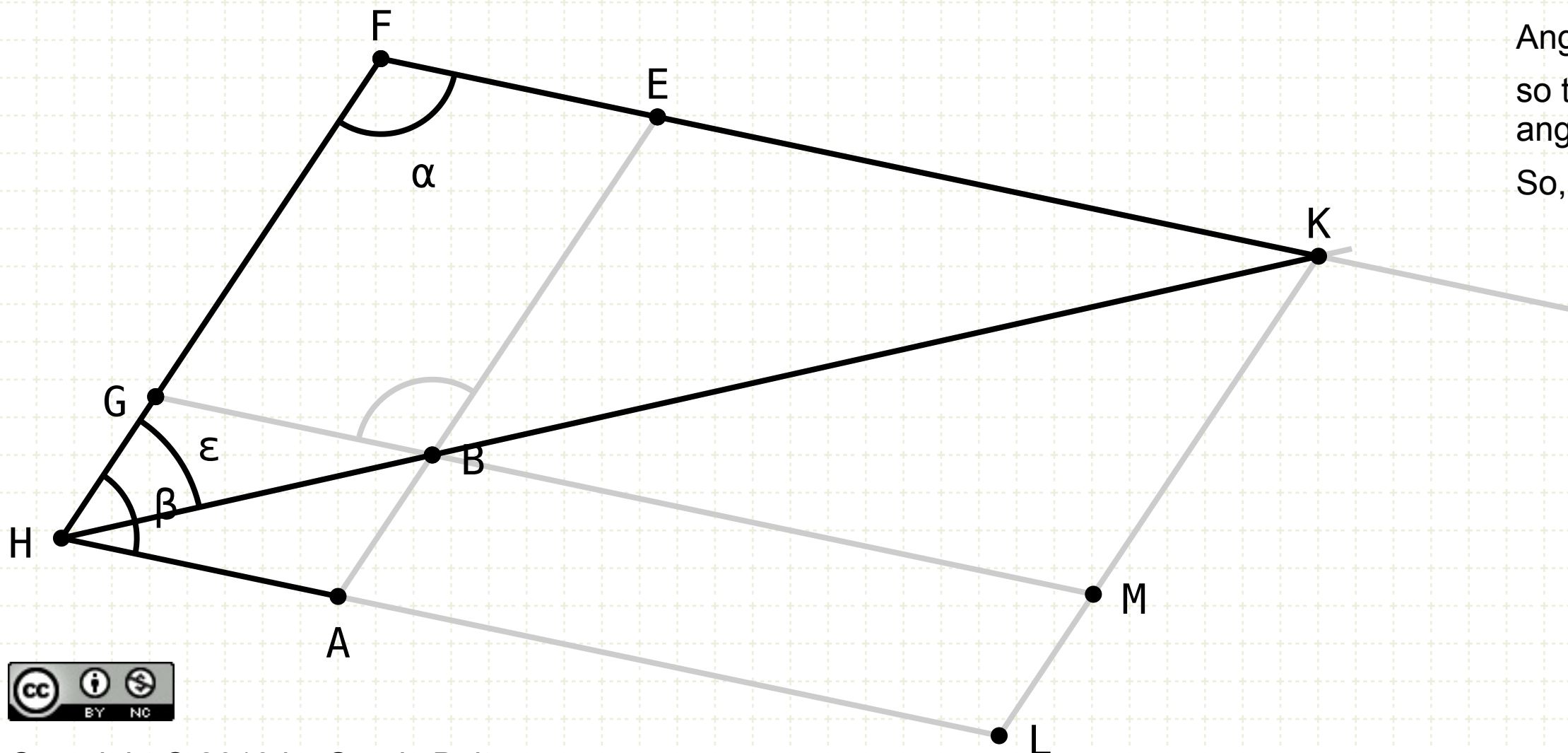
Lines FE and HA are parallel,

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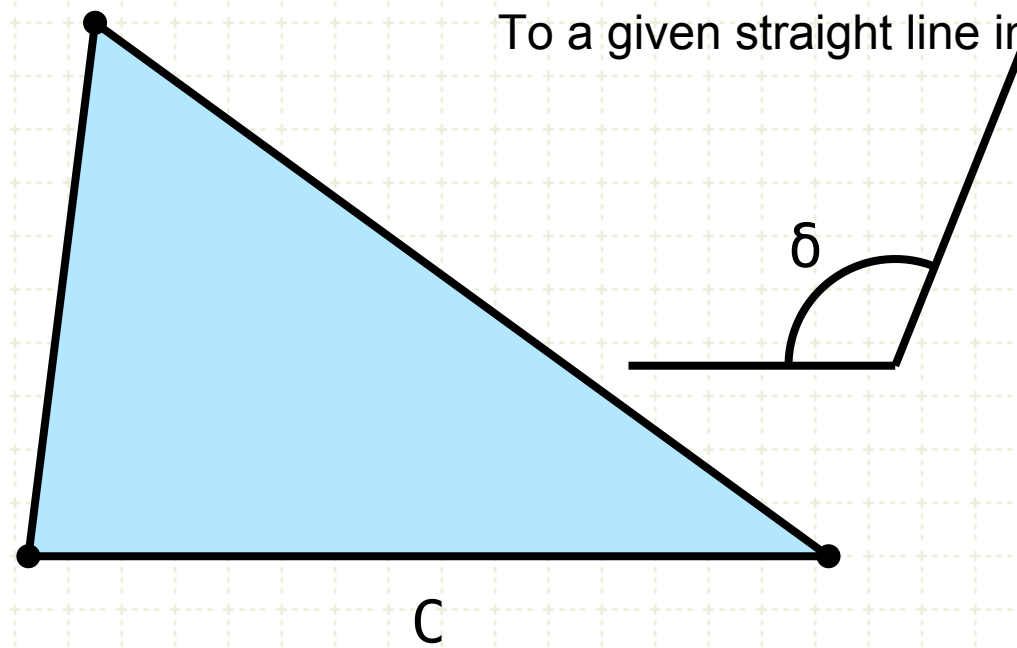
so the sum of angles EFG and GHA is less than two right angles

So, by Postulate 5, HB and FE will intersect



Proposition 44 of Book I

To a given straight line in a given rectilinear angle, to apply a parallelogram equal to a given triangle.



$$EFGB = \Delta C$$

$$\text{Fe}^{2+} \parallel \text{HA}^{\ominus}$$

$$\alpha + \beta = L + L$$

$\varepsilon < \beta$

$$\varepsilon + \alpha < L + L$$

BALM = EFGB

BALM = ΔC

Proof:

By definition of the construction techniques EFGB is equal to the given triangle C (I-42)

The extension of HB will intersect the extension of line FK because...

Lines FE and HA are parallel,

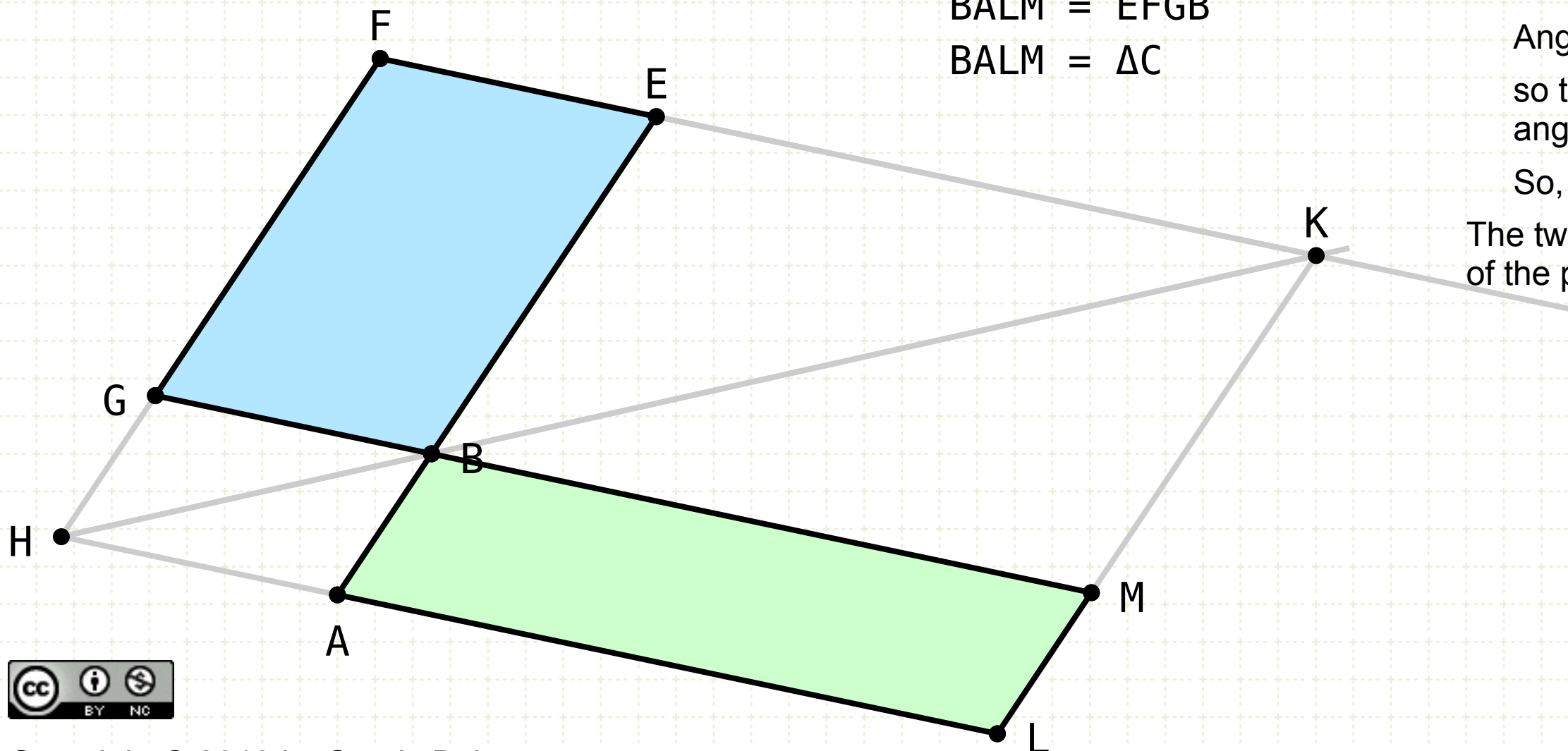
therefore the sum of the angles EFG and GHA is equal the sum of two right angles (I-42)

Angle BHG is less than GHA,

so the sum of angles EFG and GHA is less than two right angles

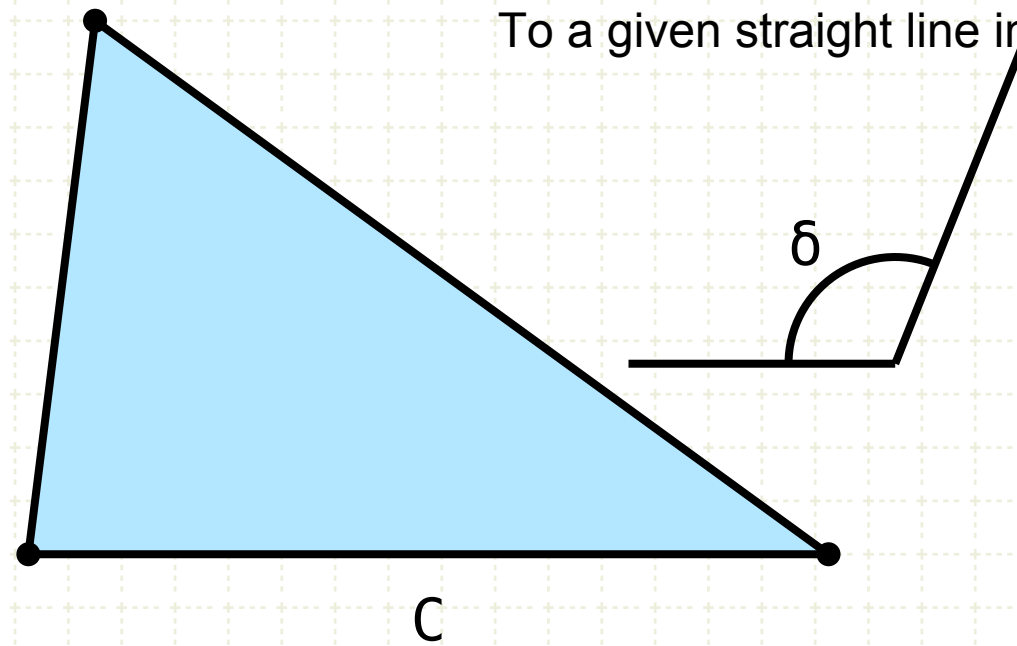
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The two parallelograms FGBE and BALM are complements of the parallelogram FHLK, thus they are equal in area (I-43)



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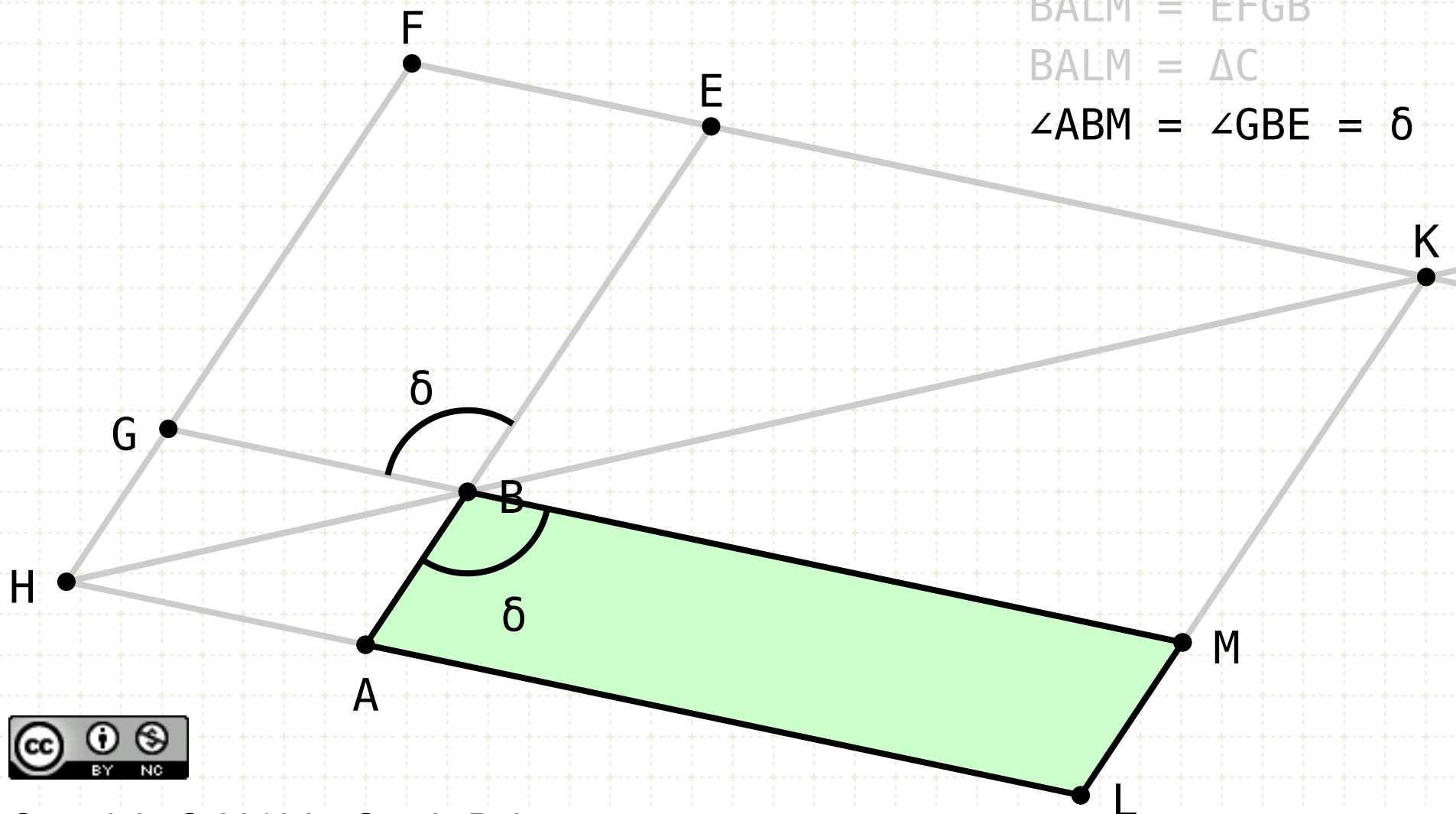
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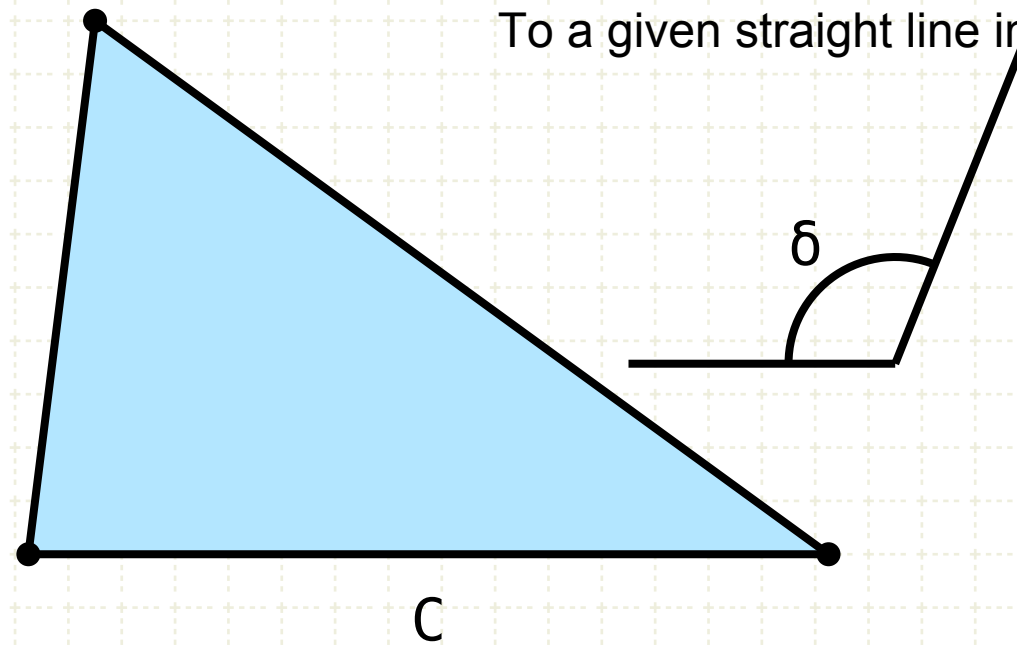
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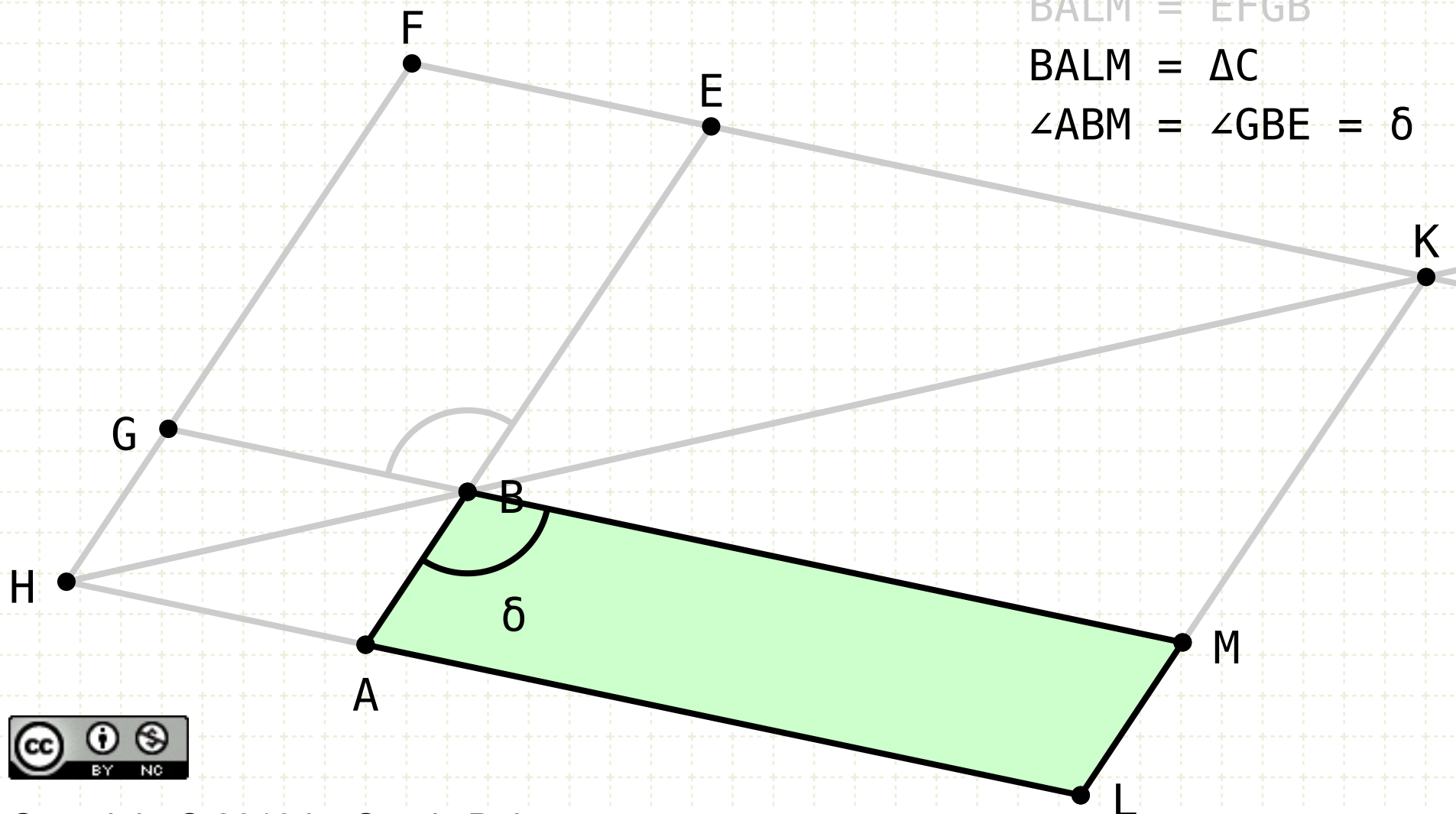
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