

Euclid's Elements

Book III



A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)



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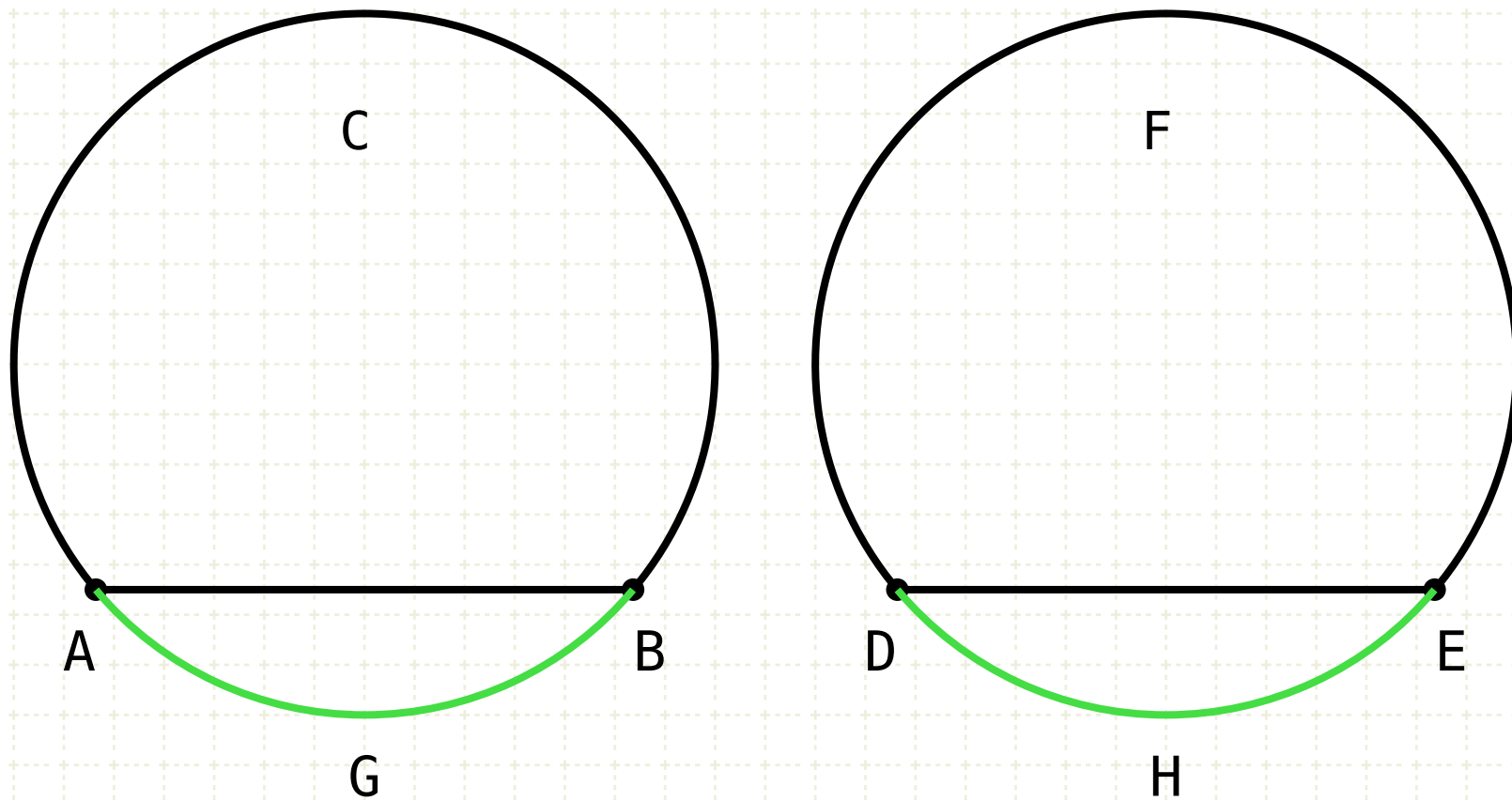
Proposition 28 of Book III

In equal circles equal straight lines cut off equal circumferences, the greater equal to the greater and the less to the less.



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$$\odot C = \odot F$$

$$AB = DE$$

$$\smile AGB = \smile DHE$$

$$\smile ACB = \smile DFE$$

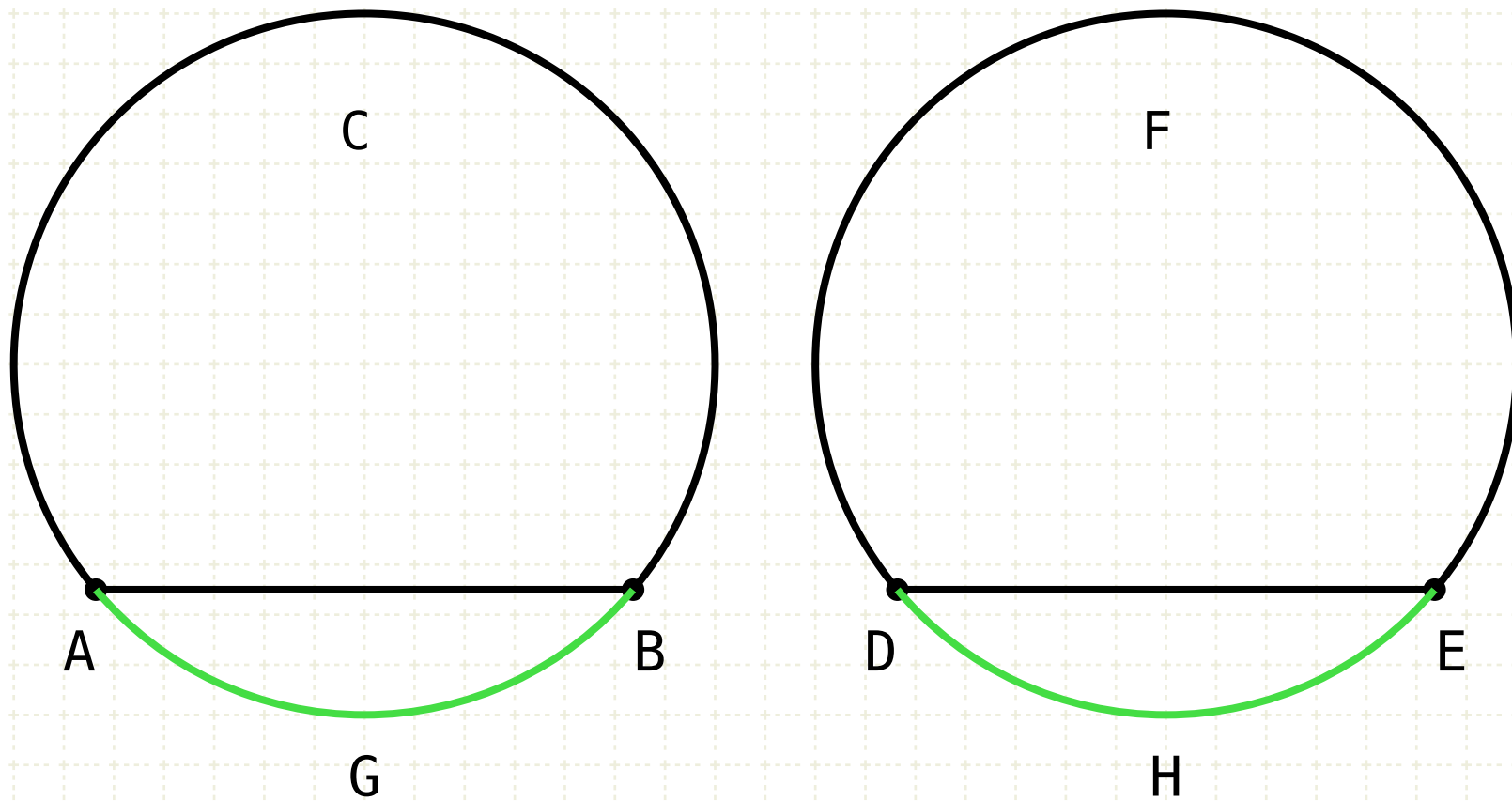
In other words

Given two equal circles (as shown)

If line AB equals line DE, then the circumference AGB equals DHE, and the circumference ACB equals DFE

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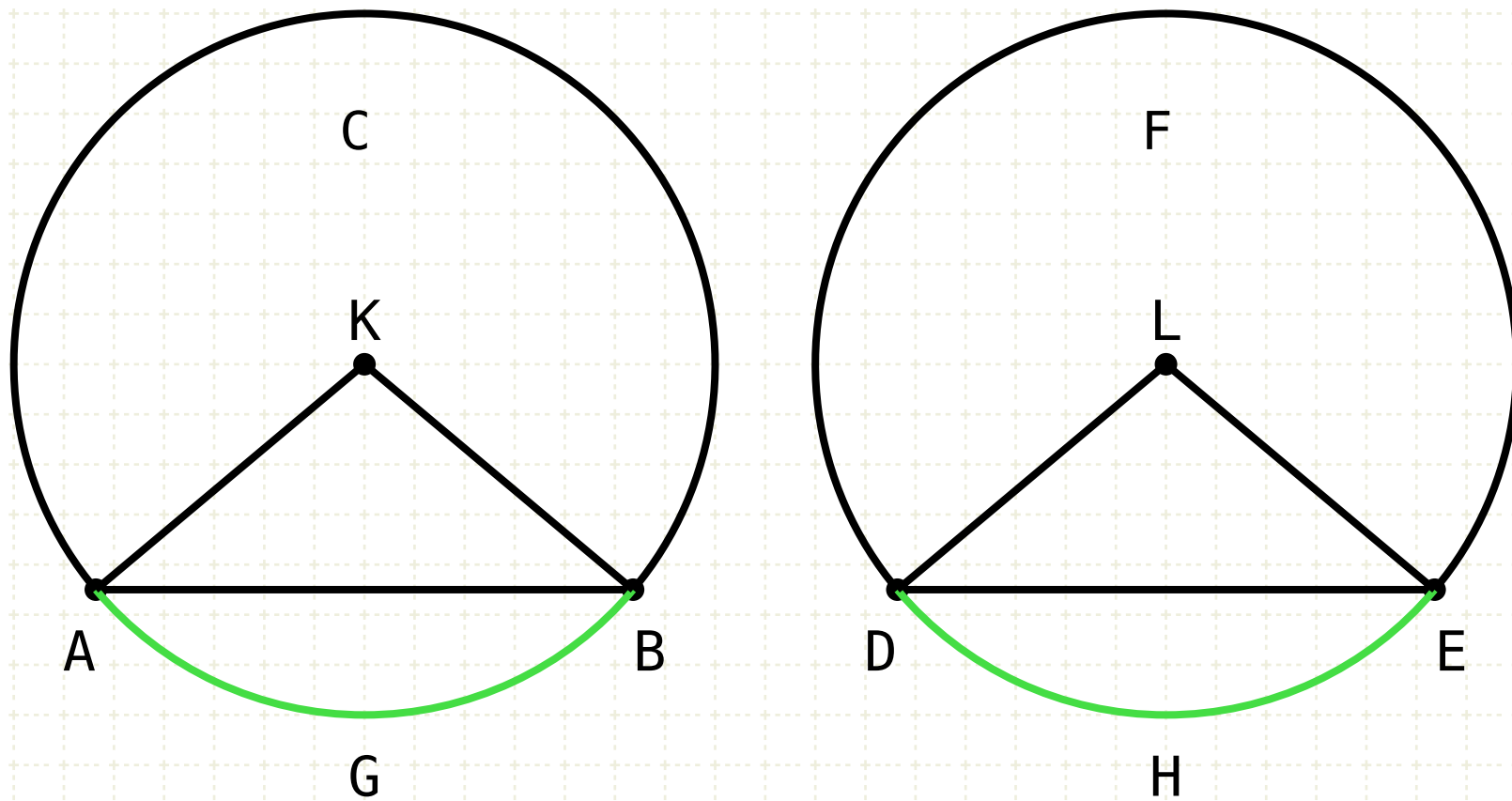
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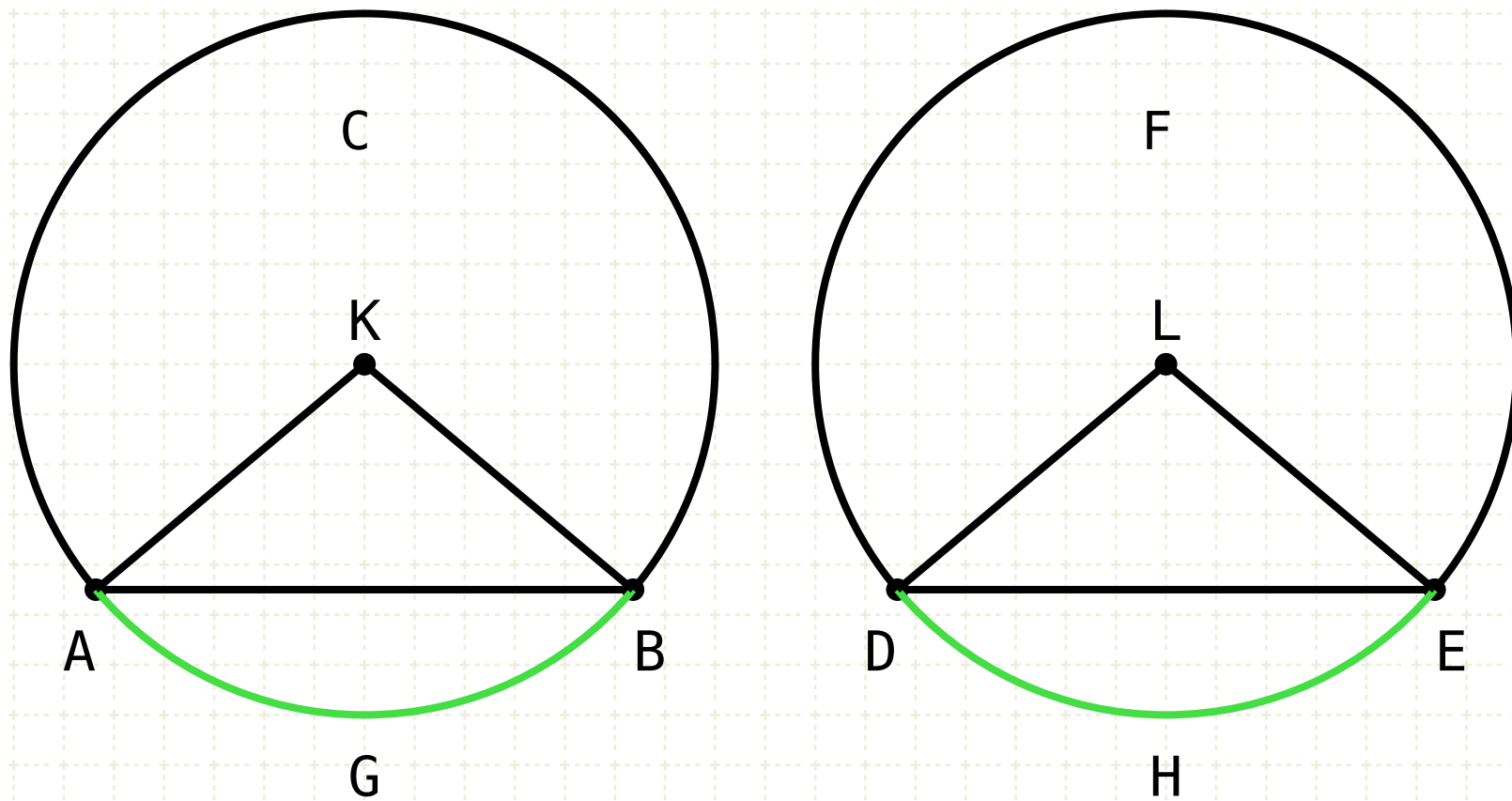
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Take the centre (K,L) of the circles C and F and draw the radii KA, KB, LD and LE

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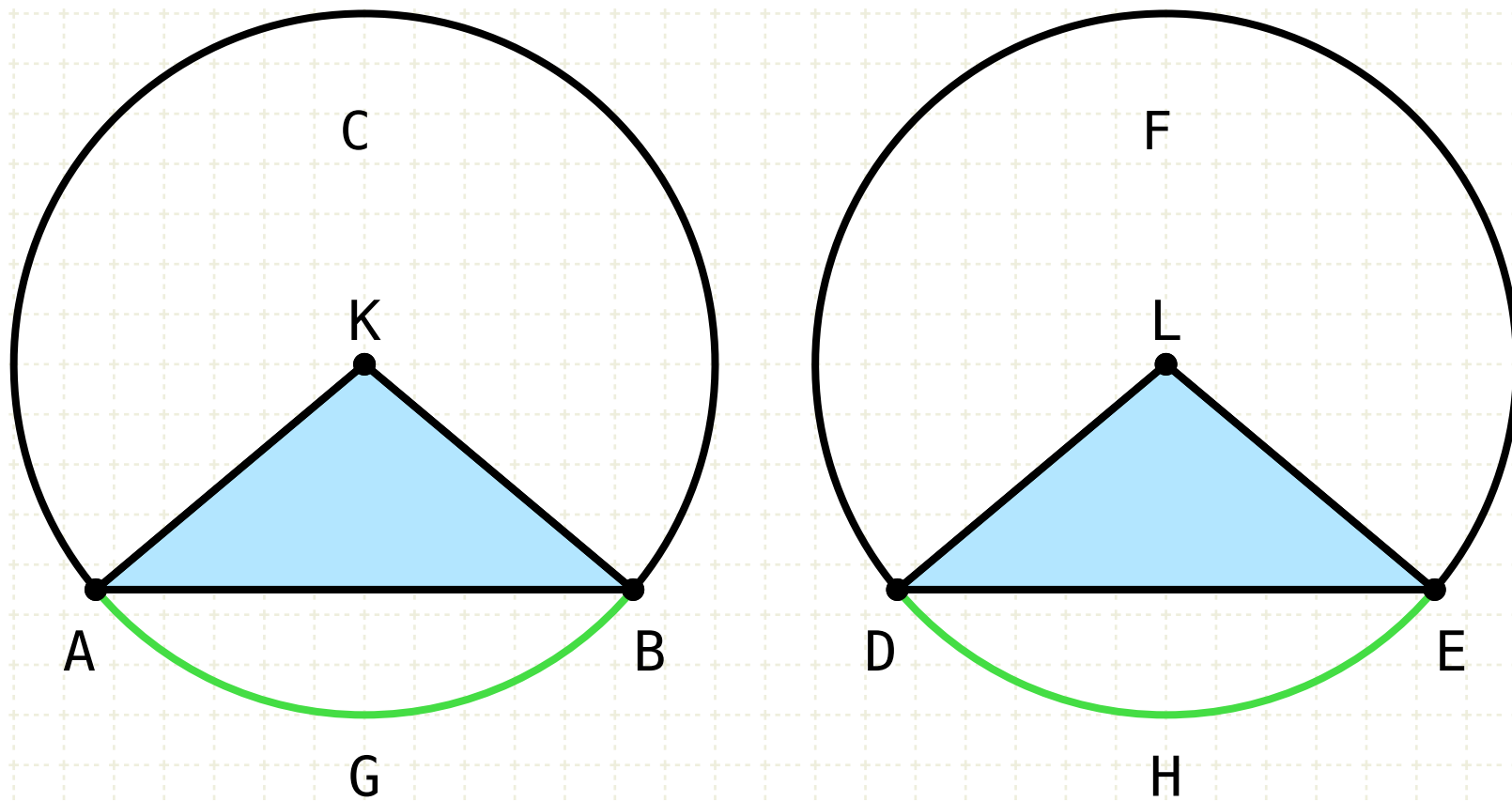
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Since they are all radii of equal circles, then they are all equal

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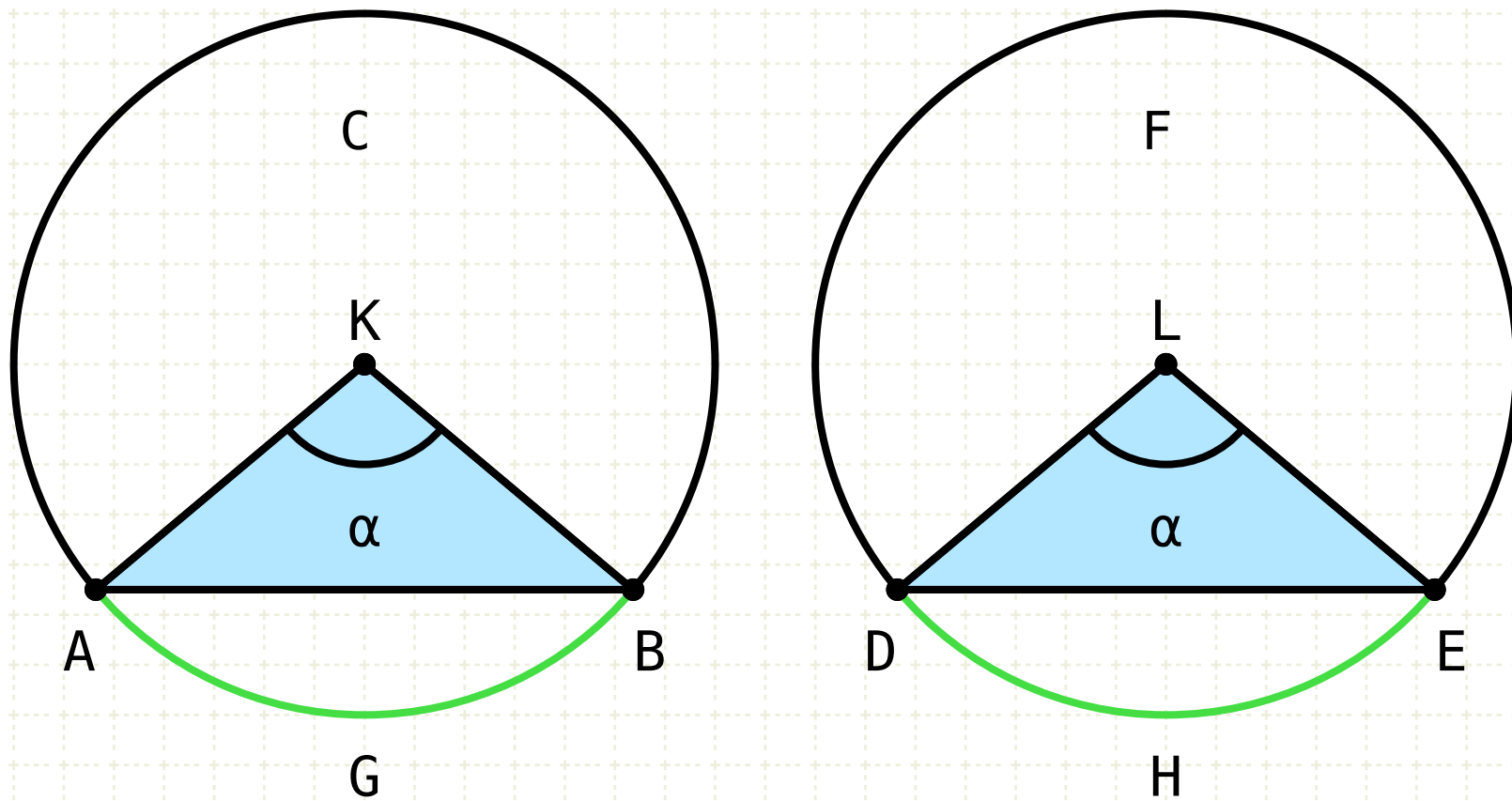
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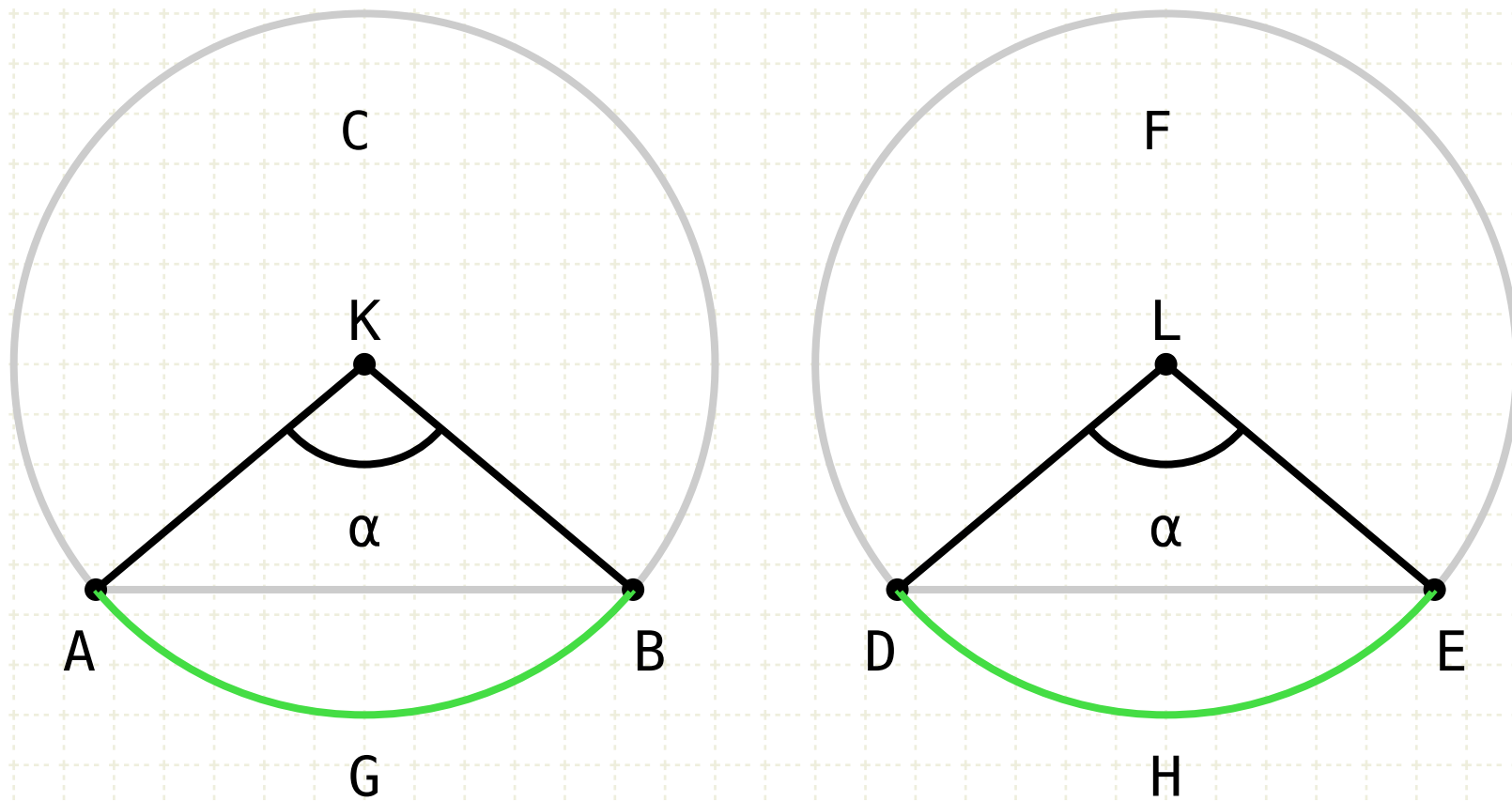
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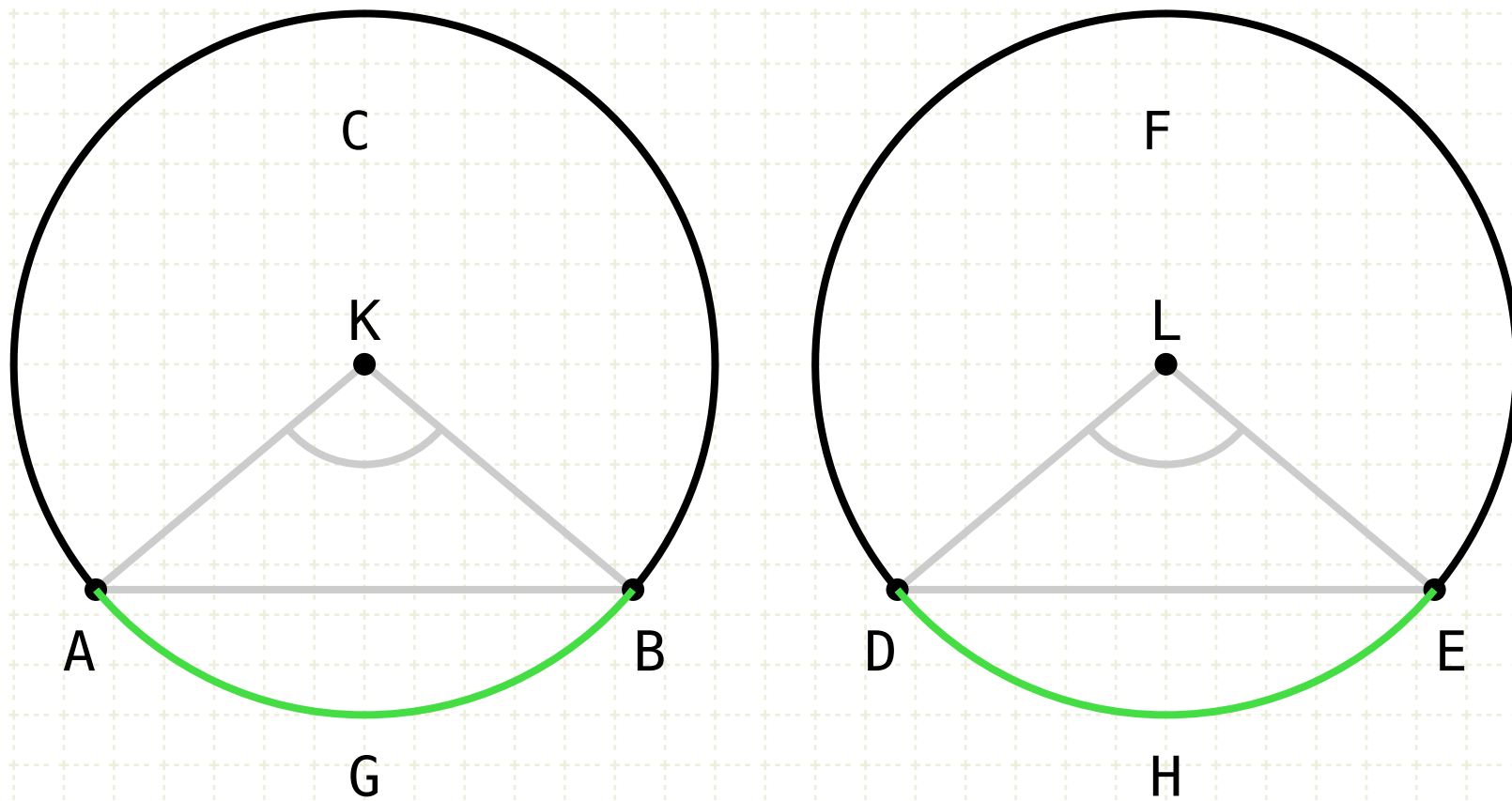
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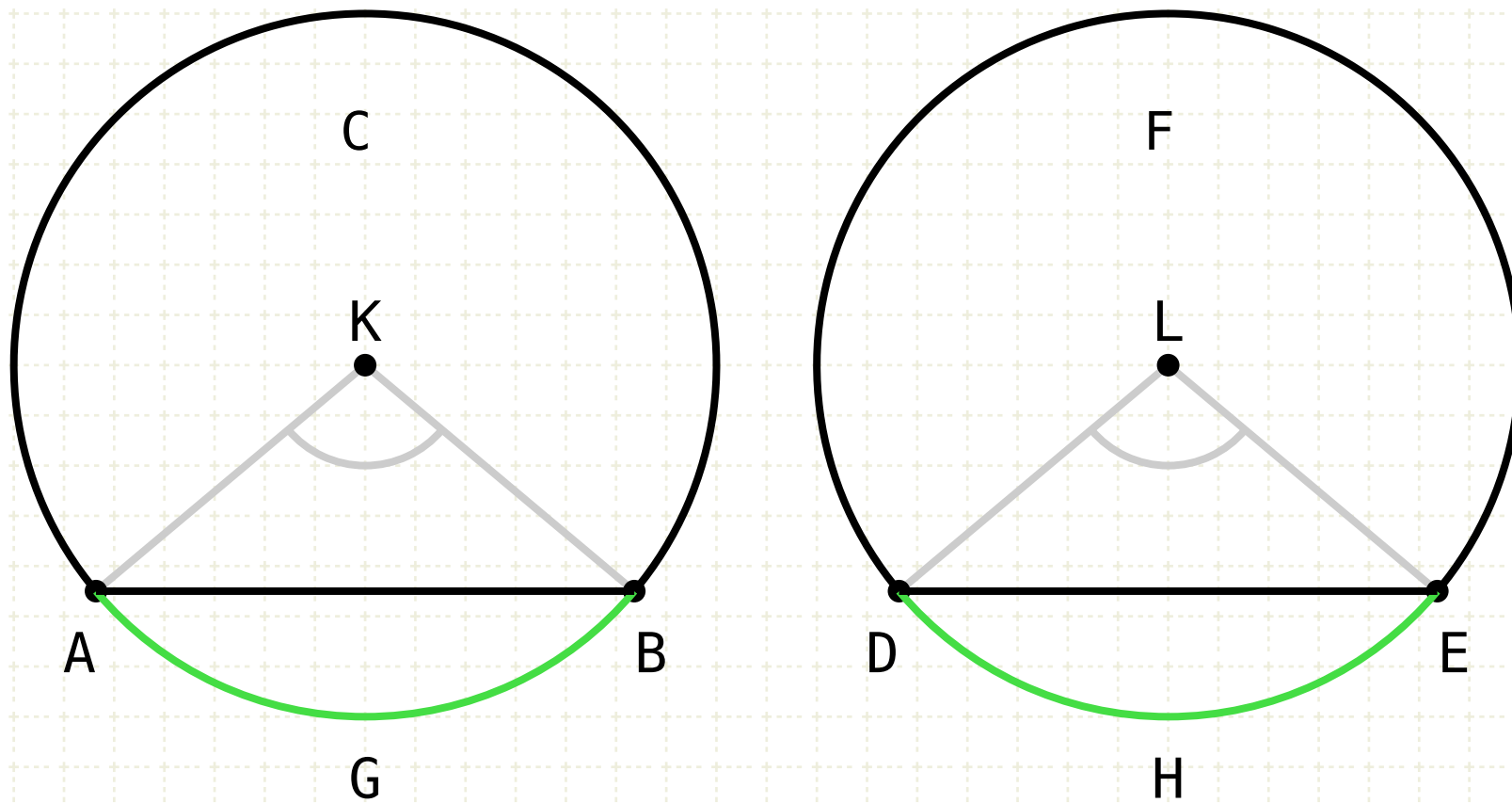
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Since the circles are equal, the remainder of the circumference after AGB and DHE are removed, will also be equal



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