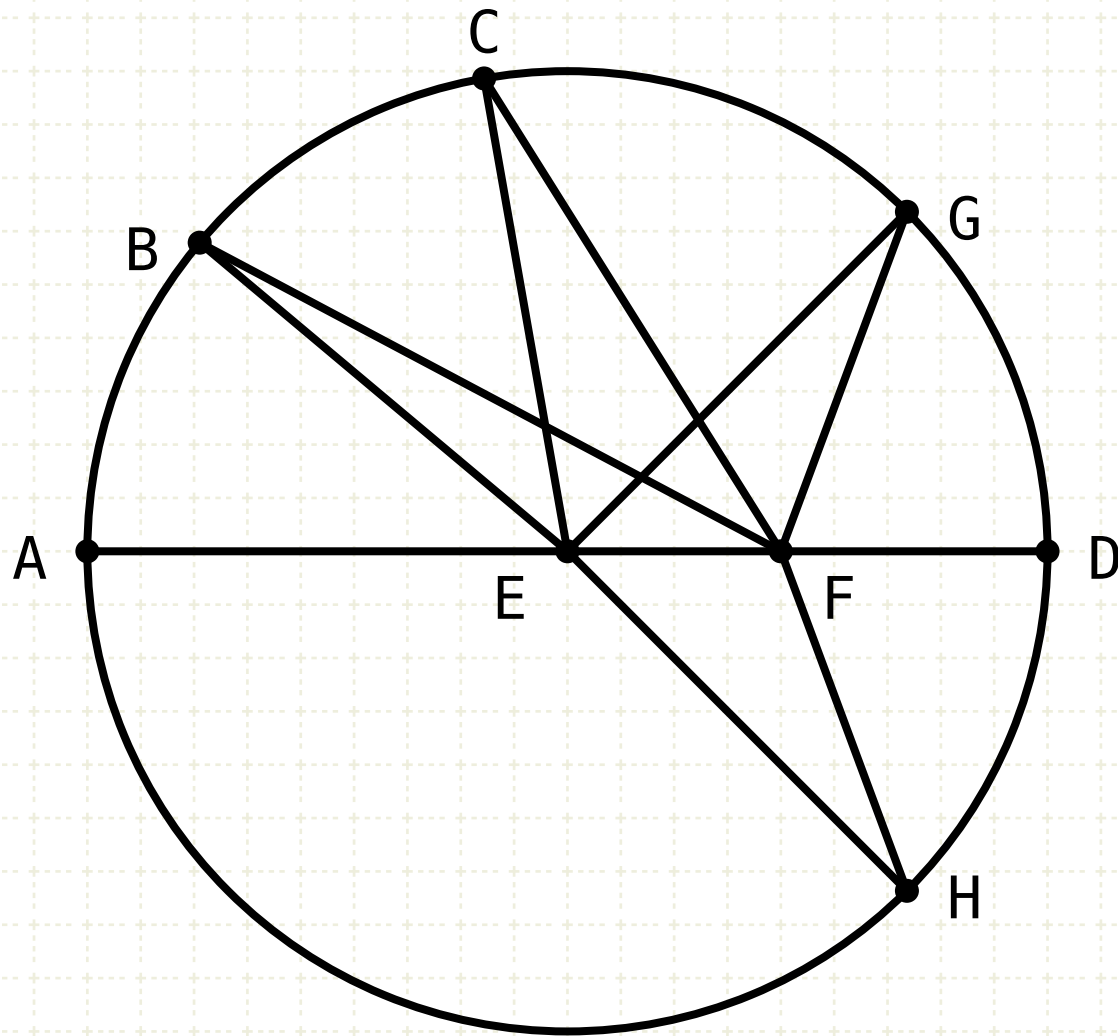


Euclid's Elements

Book III



A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)



Table of Contents, Chapter 3

1	To find the centre of a circle	9	If three lines, starting at a point 'A' and touching the circle, are all equal, then 'A' is the centre of the circle	17	From a given point to draw a straight line touching a given circle
2	A chord of a circle always lies inside the circle	10	A circle does not cut a circle at more points than two	18	If line touches a circle, then it is perpendicular to the diameter that touches that point
3	A line through the centre of a circle bisects a chord, and vice versa	11	Point of contact between two internal circles, and their centres, are collinear	19	If line touches a circle, then the centre of the circle lies on a line perpendicular to the original
4	A line not through the centre of a circle does not bisect a chord	12	Point of contact between two external circles, and their centres, are collinear	20	The angle at the centre of a circle is twice that from an angle from the circumference
5	If two circles cut one another, they will not have the same center	13	A circle does not touch a circle at more points than one, whether it touch it internally or externally.	21	In a circle the angles in the same segment are equal to one another
6	If two circles touch one another, they will not have the same center	14	In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another.	22	The opposite angles of quadrilaterals in circles are equal to two right angles
7	Consider two lines from a point inside a circle to the edge, the longer one will be the one closest to the longest part of the diameter passing through the original point	15	The longest line in a circle is its diameter, shorter the farther away from the diameter	23	On the same straight line there cannot be constructed two similar and unequal segments of circles on the same side
8	Consider two lines from a point outside a circle to the edge, the line closest to the centre will be longer on the concave side and shorter on the convex side	16	A line on the circle, perpendicular to the diameter, lies outside the circle	24	Similar segments of circles on equal straight lines are equal to one another



Table of Contents, Chapter 3

- | | | | |
|----|---|----|--|
| 25 | Given a segment of a circle, to describe the complete circle of which it is a segment. | 34 | Construct a circle segment on a given circle, such that the angle within the segment is equal to a given angle |
| 26 | In equal circles equal angles stand on equal circumferences | 35 | If two circle chords intersect, the segments on one multiplied together equals the segments of the other multiplied together |
| 27 | In equal circles angles standing on equal circumferences are equal to one another | | |
| 28 | In equal circles equal straight lines cut off equal circumferences | 36 | Secant-tangent law |
| 29 | In equal circles equal circumferences are subtended by equal straight lines | 37 | Converse of the secant-tangent law |
| 30 | To bisect a given circumference | | |
| 31 | In a circle the angle in the semicircle is right ... | | |
| 32 | The angle between a tangent and a straight line cutting a circle is equal to the angle in the alternate segment | | |
| 33 | Construct a circle segment on a given line, such that the angle within the segment is equal to a given angle | | |



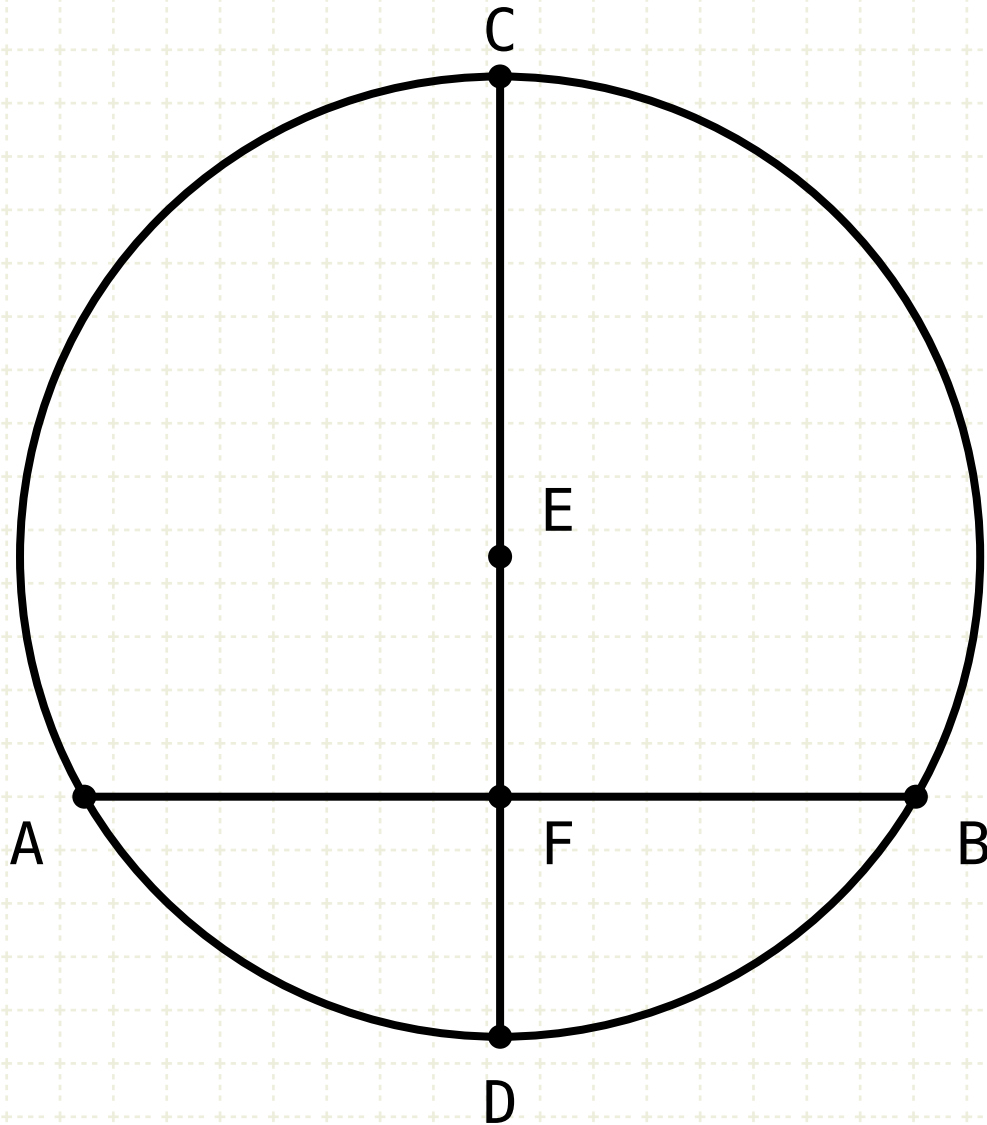
Proposition 3 of Book III

If in a circle a straight line through the center bisect a straight line not through the center, it also cuts it a right angles; and if it cut it at right angles it also bisects it.



Proposition 3 of Book III

If in a circle a straight line through the center bisect a straight line not through the center, it also cuts it a right angles; and if it cut it at right angles it also bisects it.



$$\begin{aligned} AF = FB &\rightarrow \angle AFE = \angle BFE \\ \angle AFE = \angle BFE &\rightarrow AF = FB \end{aligned}$$

In other words

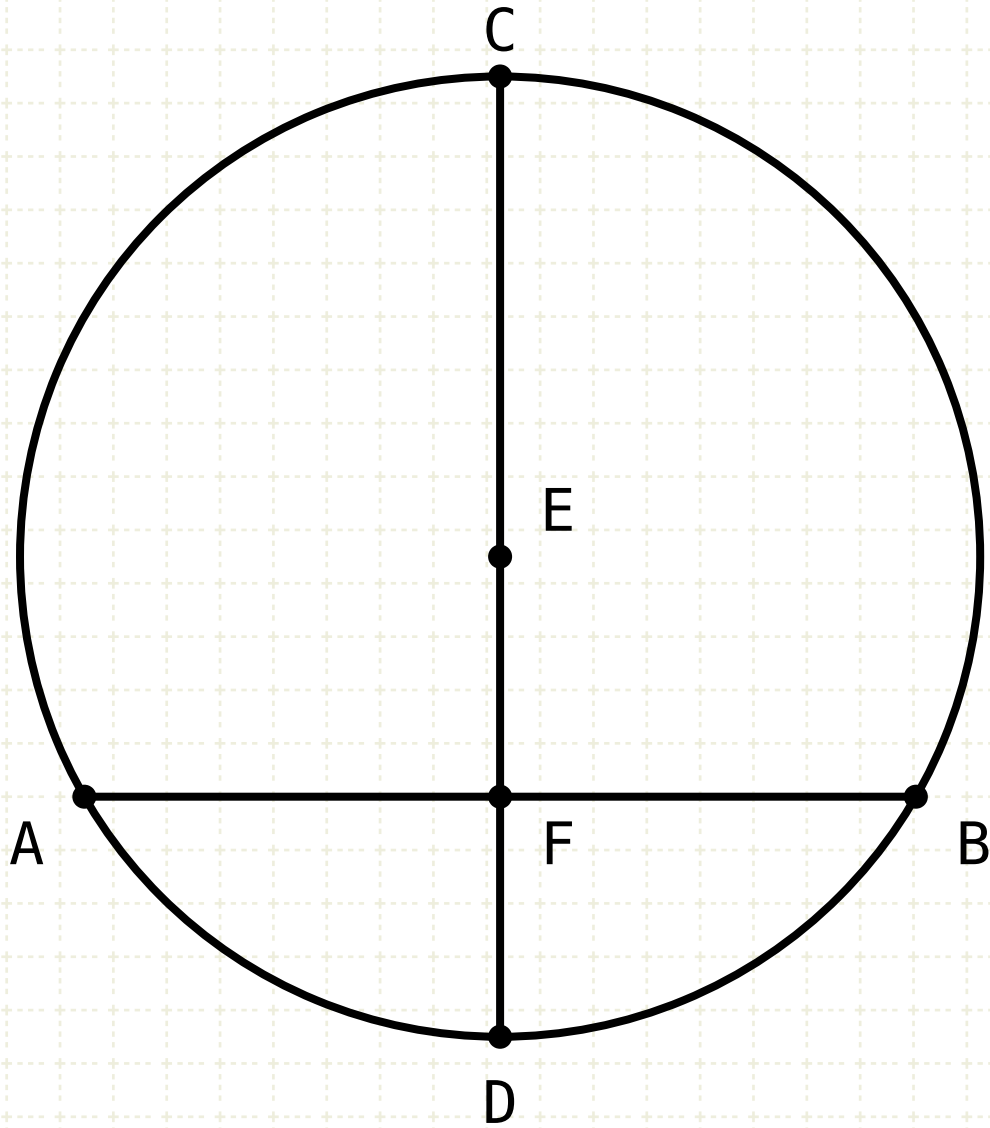
Let a line CD pass through the center of the circle (E), and cut line (AB) not through the center intersecting at point F.

- If F bisects AB, then CD cuts AB at right angles
- If CD cuts AB at right angles, then F bisects AB

Proposition 3 of Book III

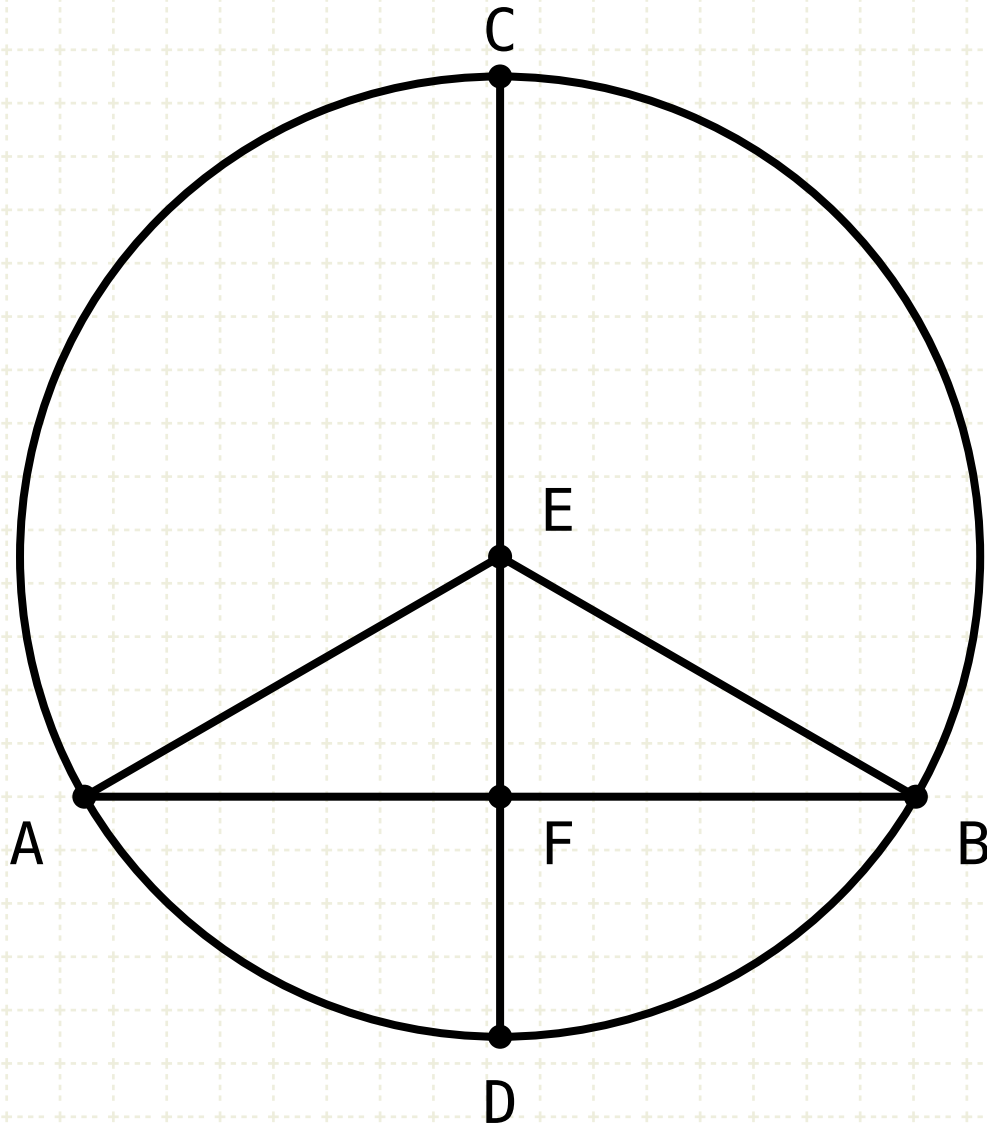
If in a circle a straight line through the center bisect a straight line not through the center, it also cuts it a right angles; and if it cut it at right angles it also bisects it.

Proof



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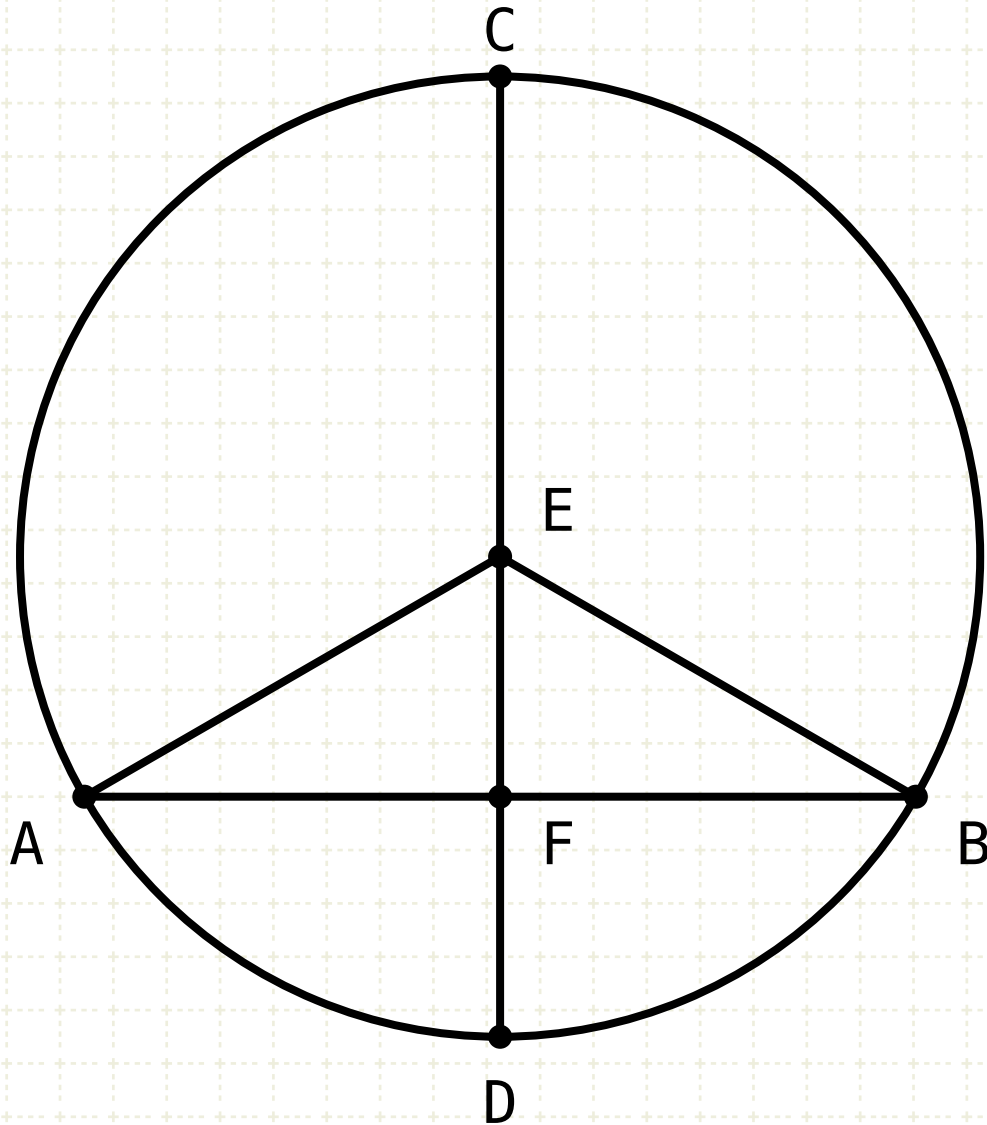
$$EA = EB$$

Proof

Join the points EA and EB. Because EA and EB are radii of the same circle they are equal.

Proposition 3 of Book III

If in a circle a straight line through the center bisect a straight line not through the center, it also cuts it a right angles; and if it cut it at right angles it also bisects it.



$$EA = EB$$

If F bisects AB

$$AF = FB$$

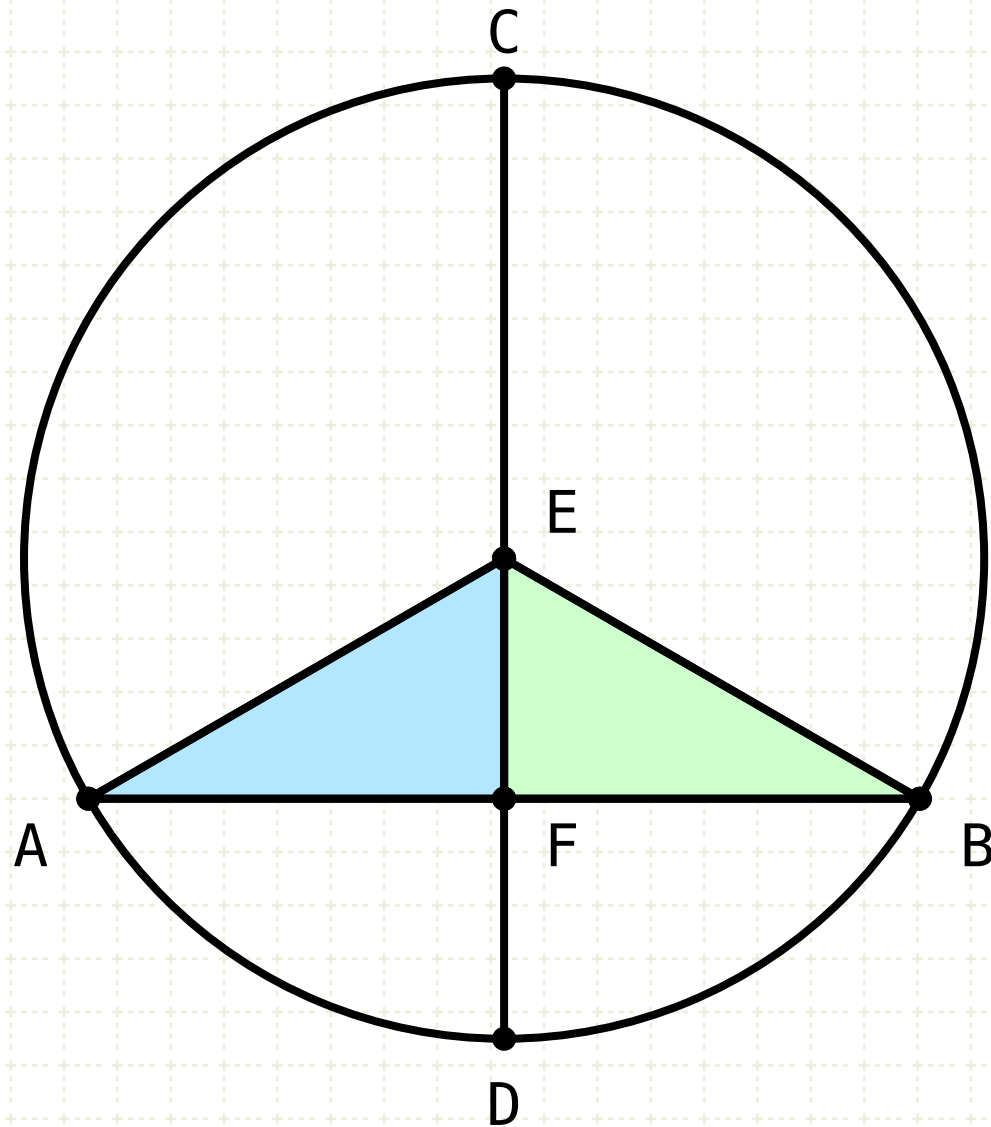
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Join the points EA and EB. Because EA and EB are radii of the same circle they are equal.

If F bisects AB, AF and FB are equal

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$$EA = EB$$

If F bisects AB

$$AF = FB$$

$$\triangle EAF \cong \triangle EFB$$

Proof

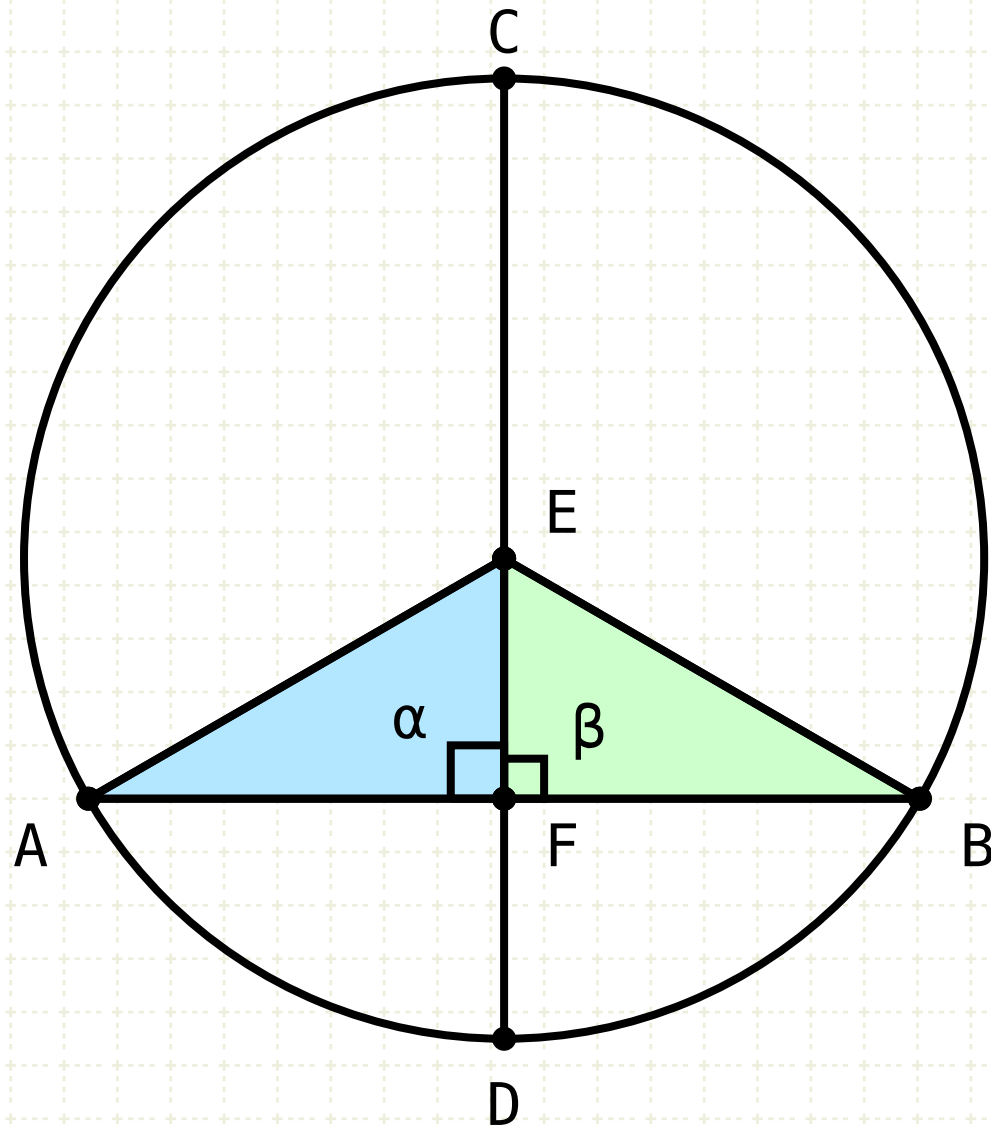
Join the points EA and EB. Because EA and EB are radii of the same circle they are equal.

If F bisects AB, AF and FB are equal

Since AF, FB are equal, EA, EB are equal, and EF is common, we have a two triangles with three equal sides, thus the two triangles are equal (I.8)

Proposition 3 of Book III

If in a circle a straight line through the center bisect a straight line not through the center, it also cuts it a right angles; and if it cut it at right angles it also bisects it.



$$EA = EB$$

If F bisects AB

$$AF = FB$$

$$\triangle EAF \equiv \triangle EFB$$

$$\alpha = \beta$$

Proof

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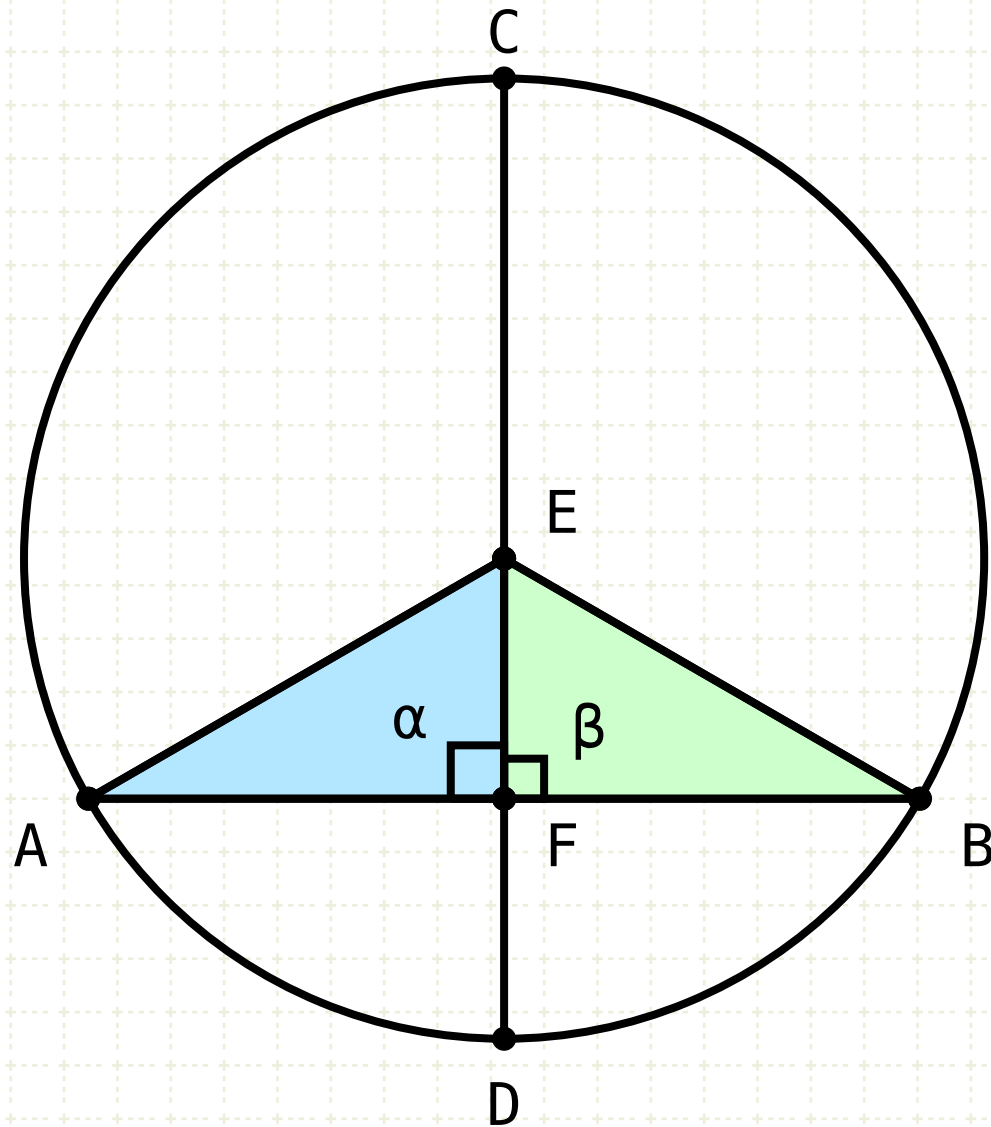
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EFA (α) equals angle EFB (β)

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$$EA = EB$$

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$$\alpha = \beta$$

$$\alpha = \beta = 90^\circ$$

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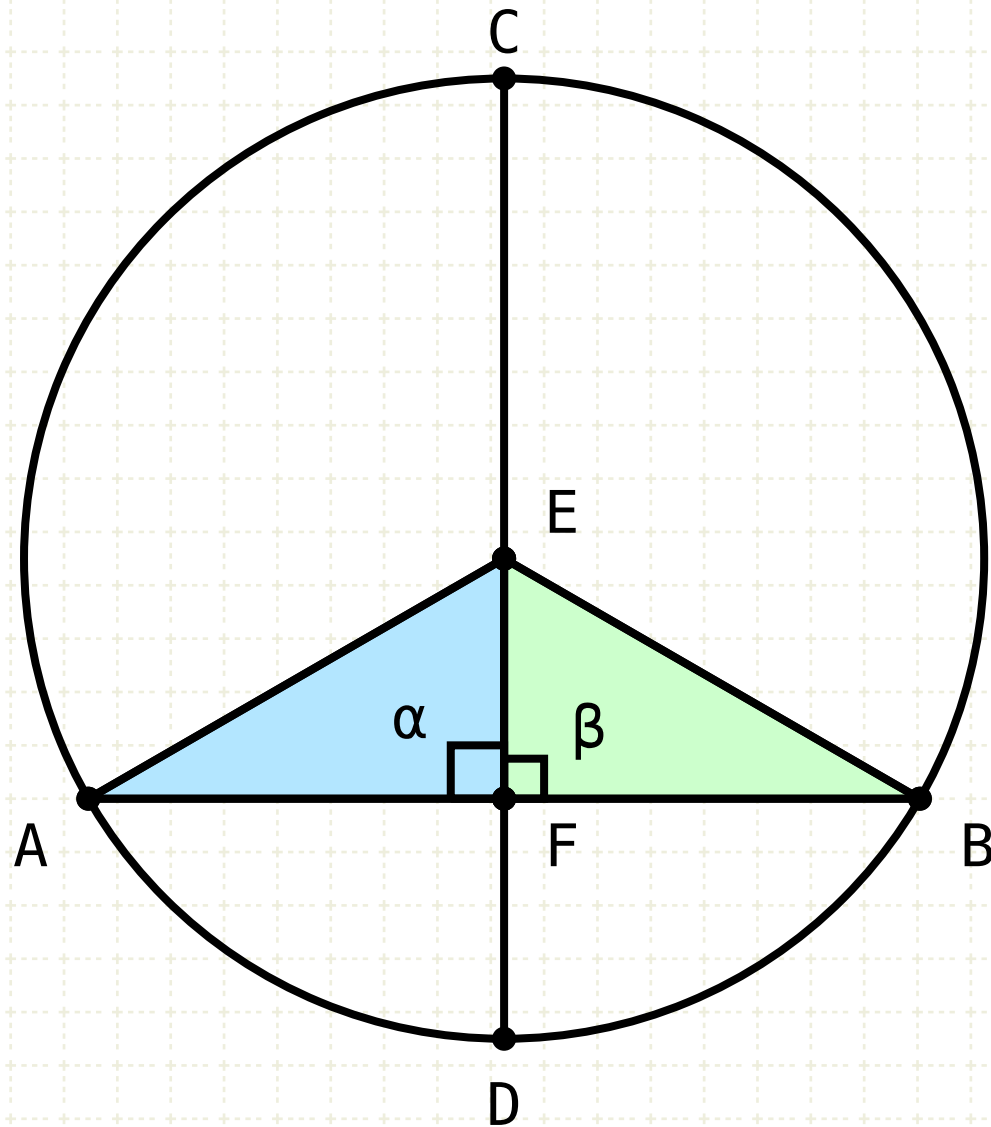
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If α and β are equal, then they are right angles by definition (I.Def.10)

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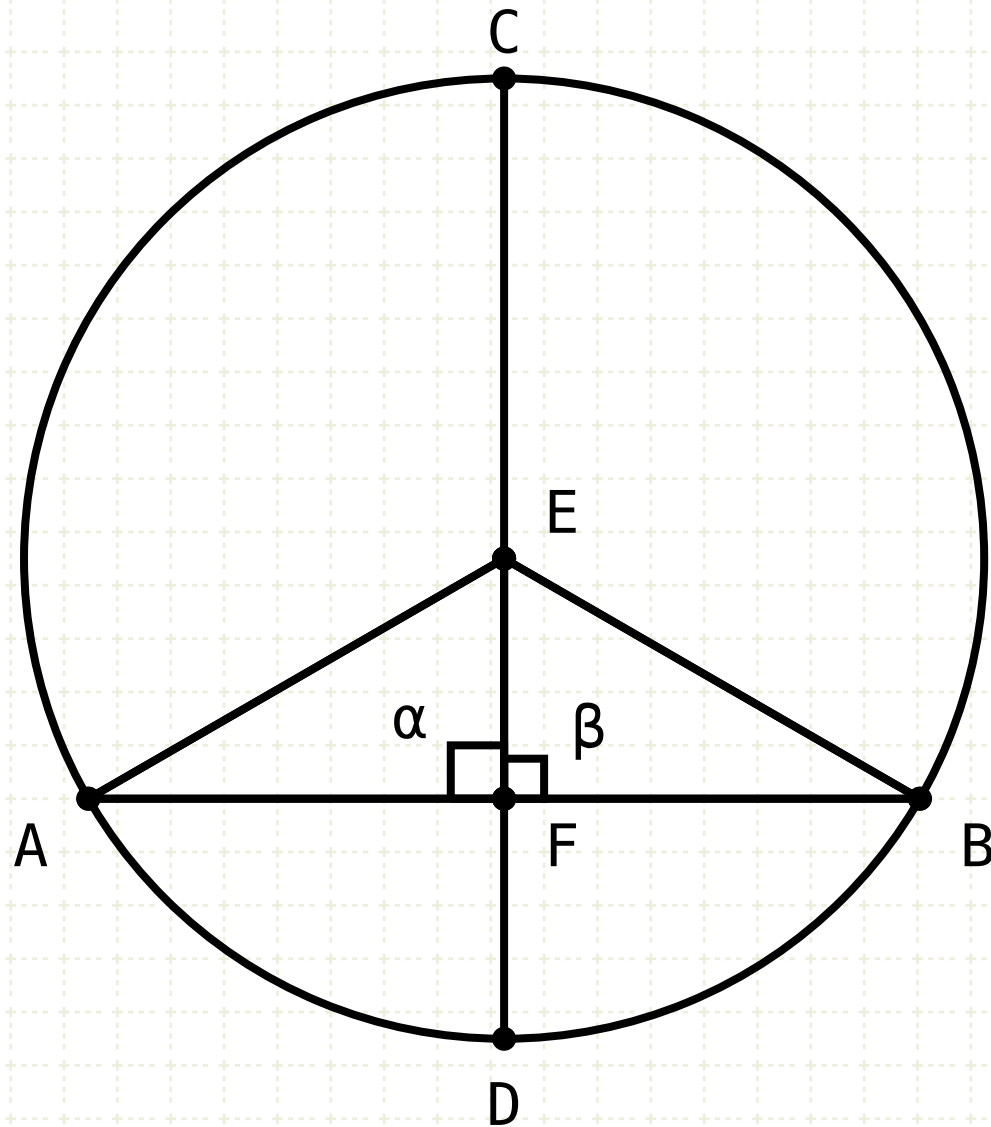
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If CD cuts AB at right angles

$$\alpha = \beta = \angle$$

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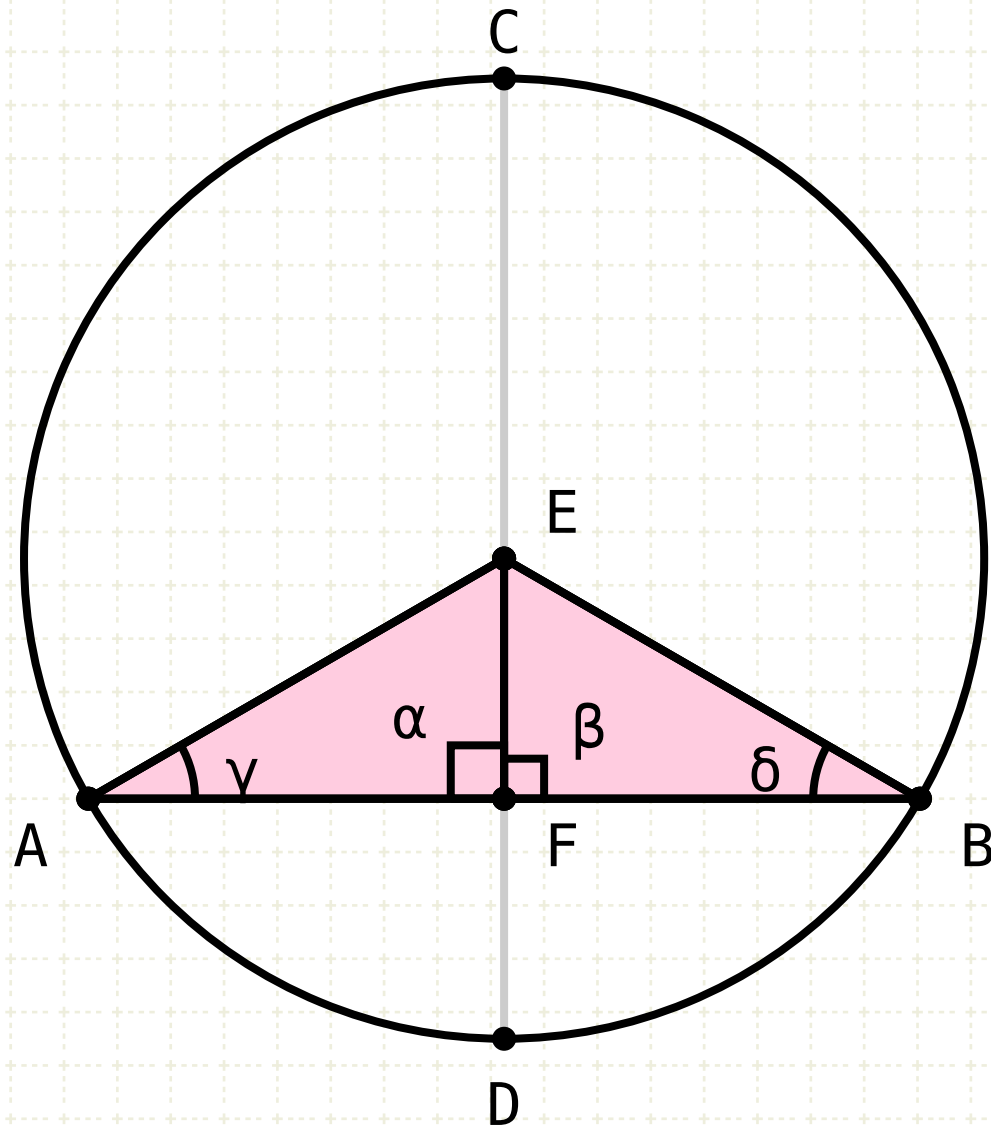
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$$\alpha = \beta = \text{L}$$

If CD cuts AB at right angles

$$\alpha = \beta = \text{L}$$

$$\gamma = \delta$$

Proof

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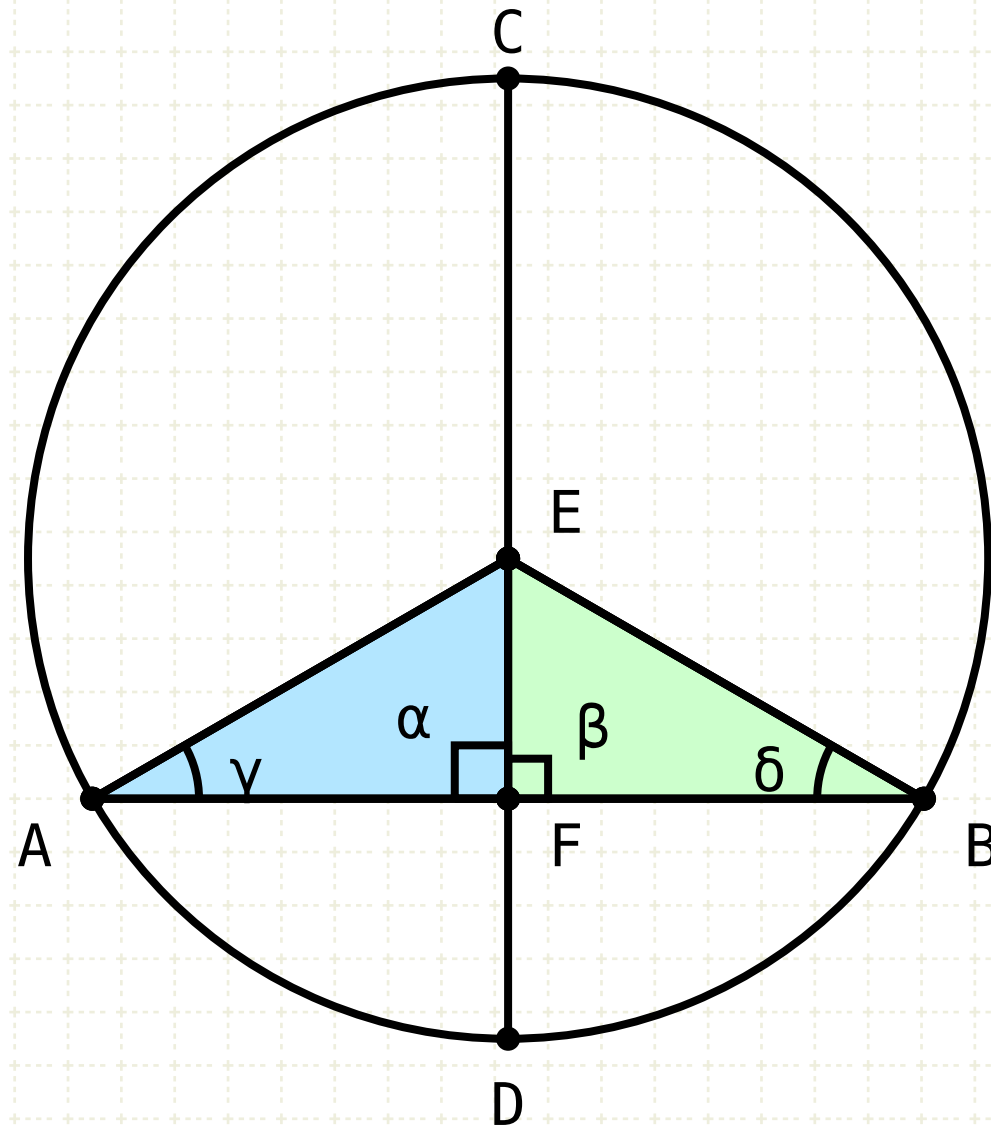
If α and β are equal, then they are right angles by definition (I.Def.10)

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Triangle EAB is an isosceles triangle, thus angles EAF (γ) and ABF (δ) are equal (I.5)

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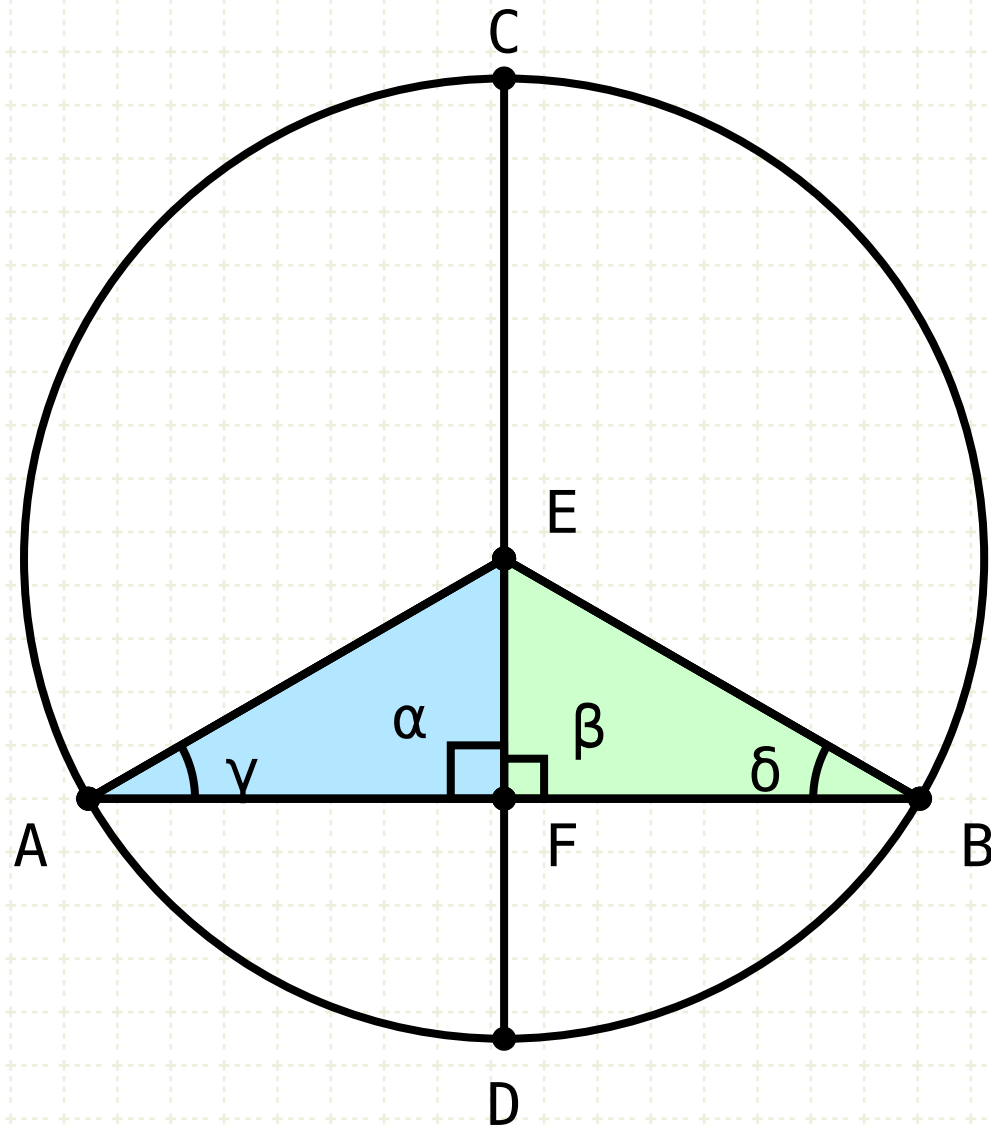
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Triangle EAB is an isosceles triangle, thus angles EAF (γ) and ABF (δ) are equal (I.5)

Since EF is common, $\gamma = \delta$, and $\alpha = \beta$ we have a two triangles with one side and two angles equal (SAA), thus the two triangles are equal (I.26)

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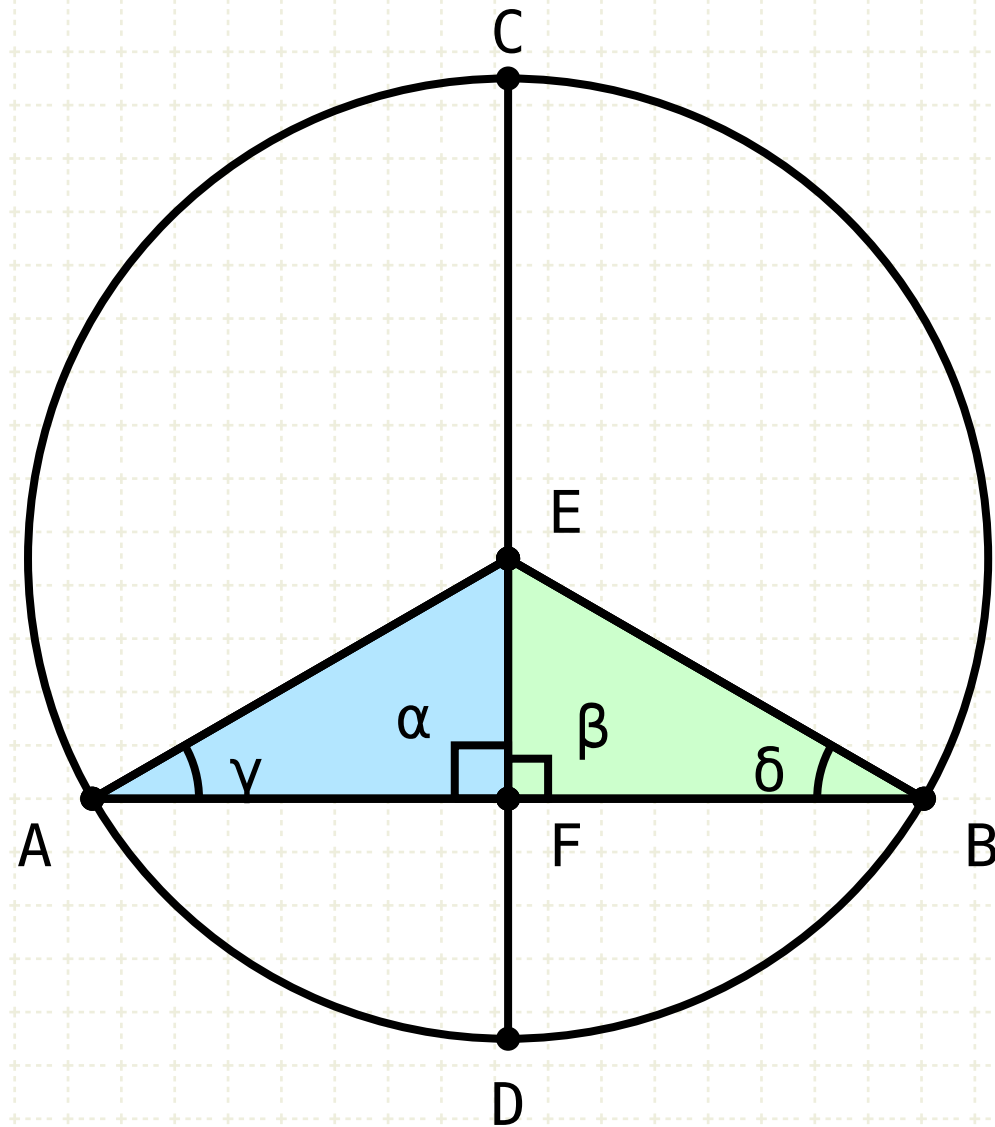
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