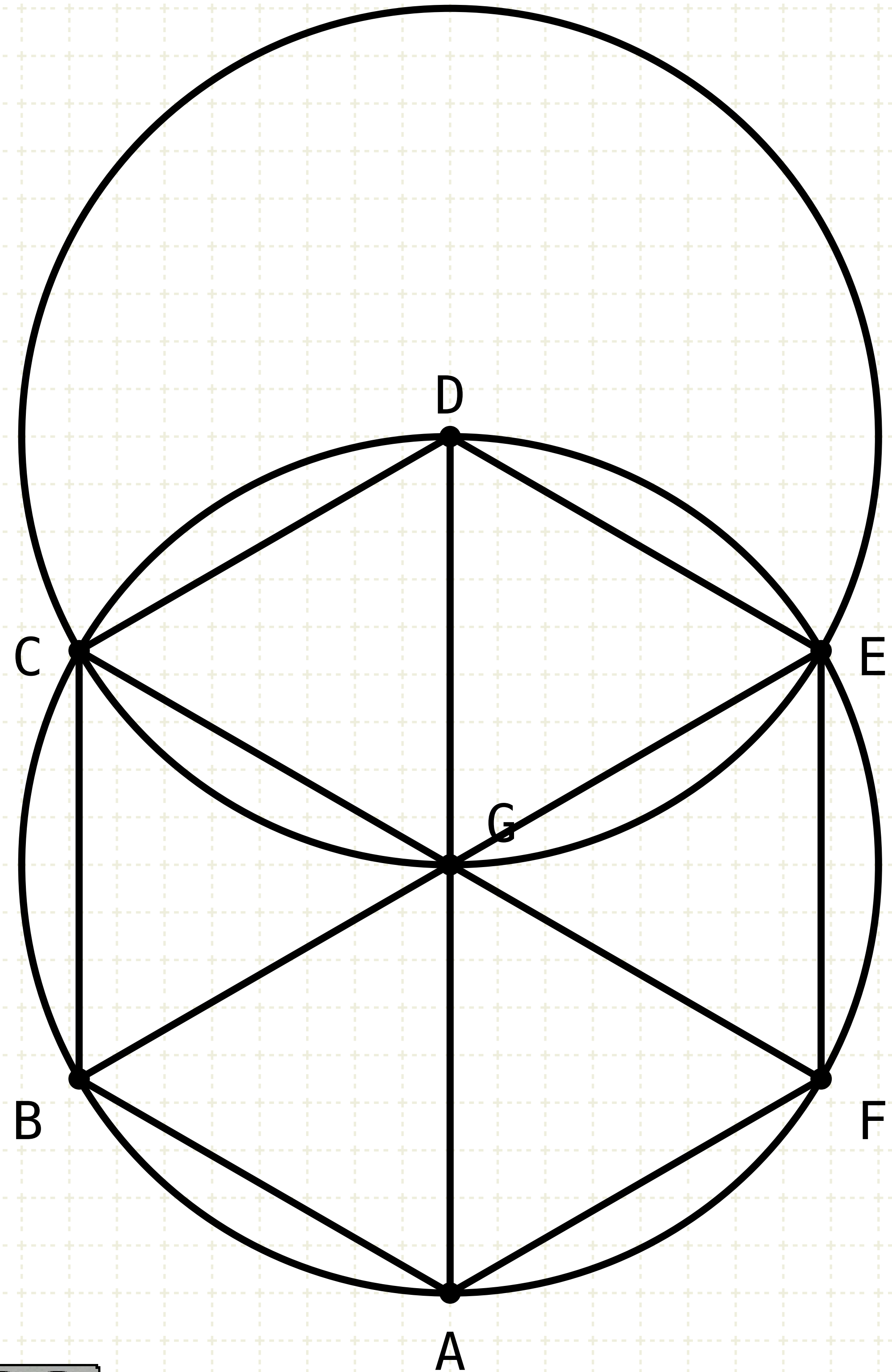
Euclid's Elements

Book IV



Philosophy (nature) is written in that great book which ever is before our eyes -- I mean the universe -- but we cannot understand it if we do not first learn the language and grasp the symbols in which it is written. The book is written in mathematical language, and the symbols are triangles, circles and other geometrical figures, without whose help it is impossible to comprehend a single word of it - without which one wanders in vain through a dark labyrinth.

Galileo Galilei

Proposition 15 of Book IV

In a given circle to inscribe an equilateral and equiangular hexagon.

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Proposition 15 of Book IV

In a given circle to inscribe an equilateral and equiangular hexagon.

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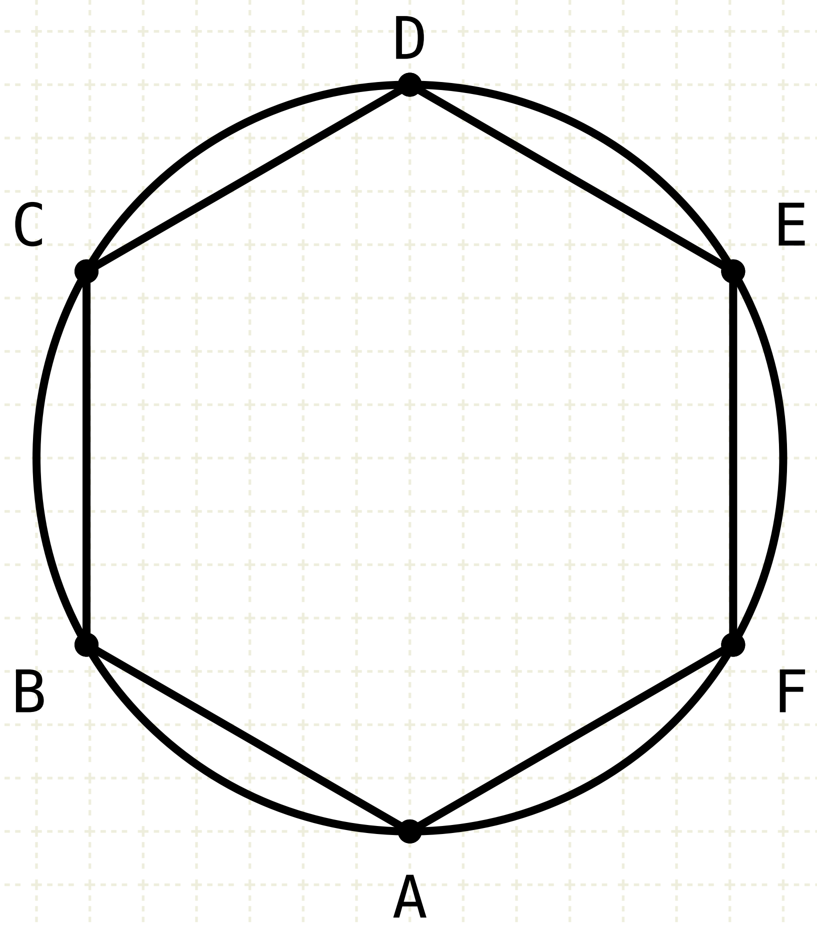
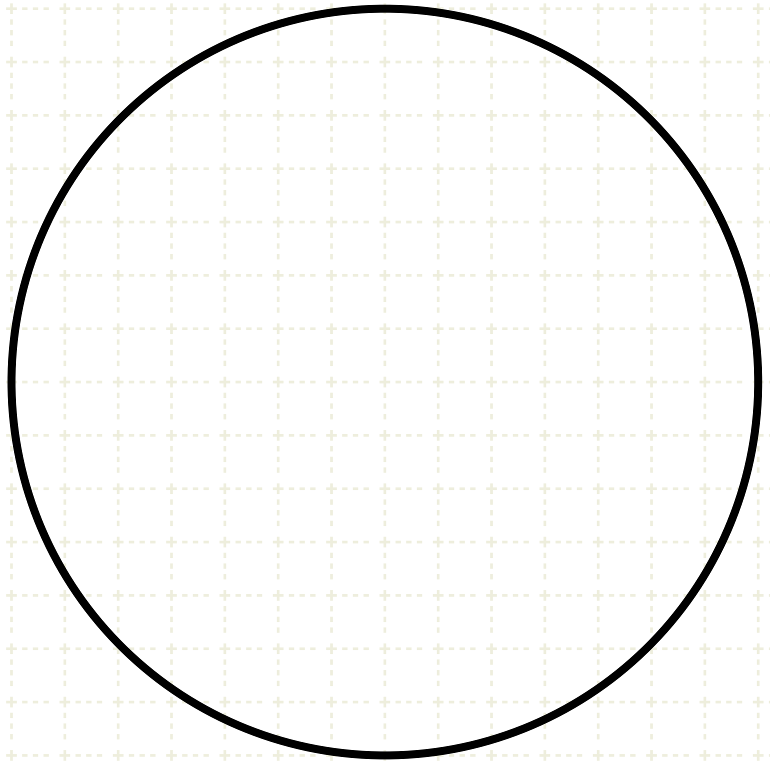


In other words

Given a circle, draw a six sided polygon with equal sides and equal angles on the inside of the circle.

Proposition 15 of Book IV

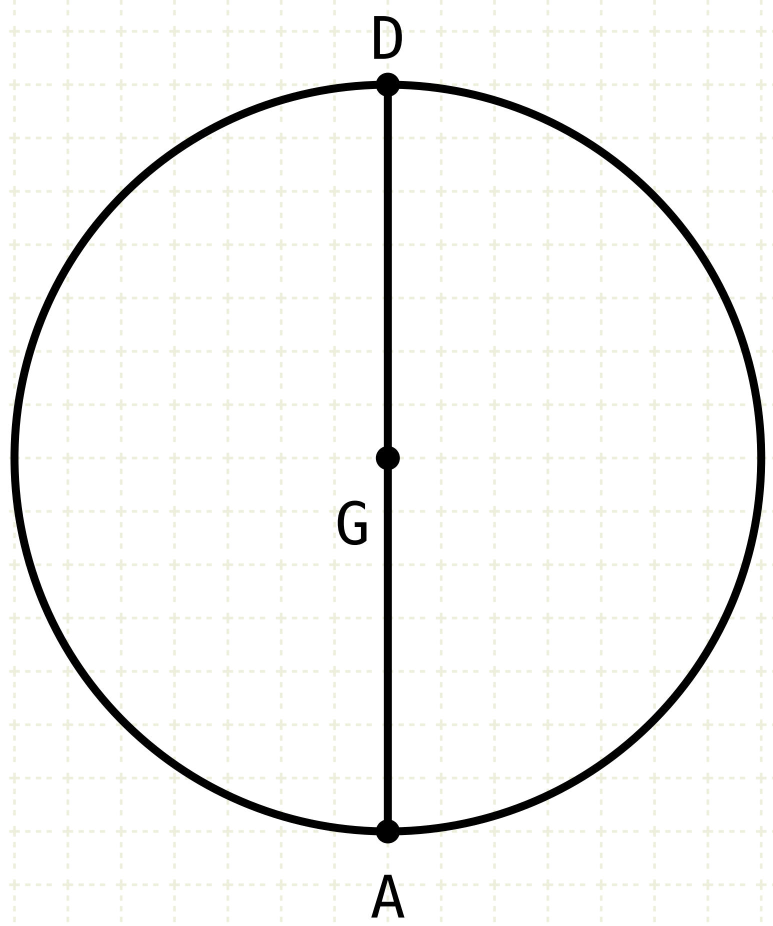
In a given circle to inscribe an equilateral and equiangular hexagon.



Construction

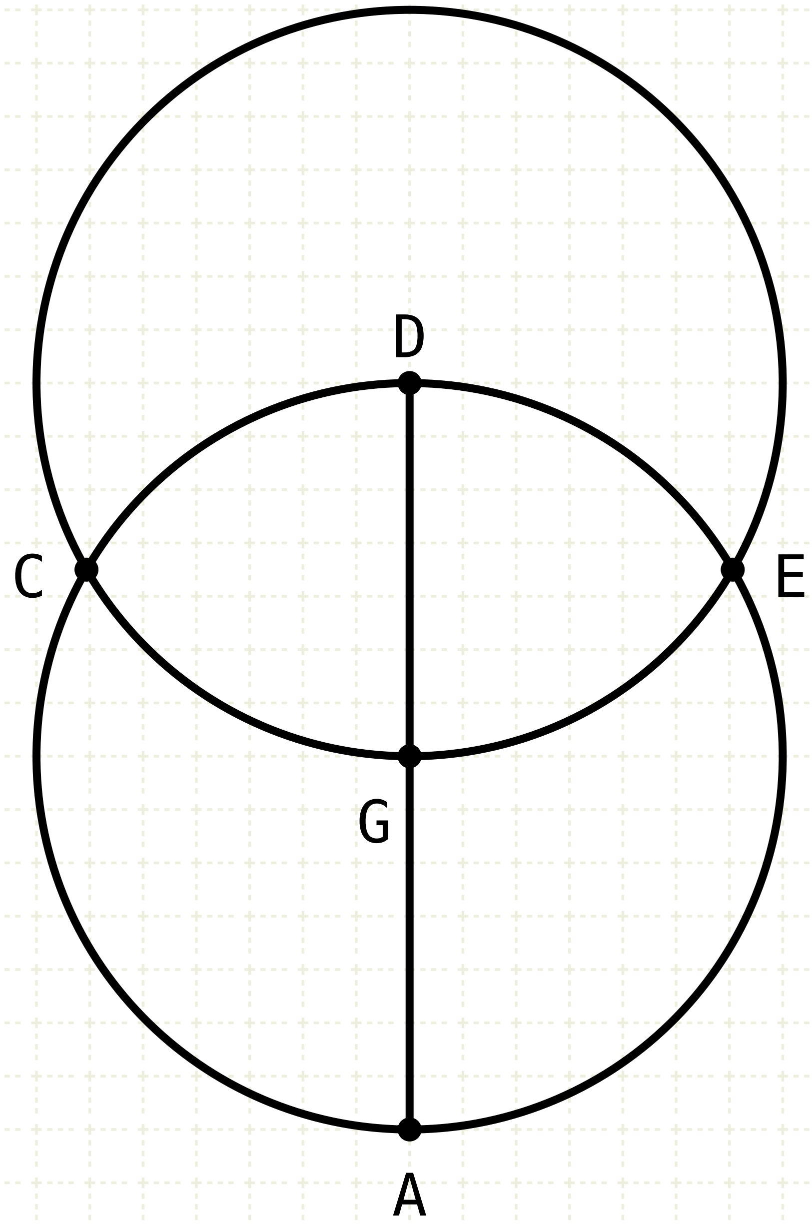
Proposition 15 of Book IV

In a given circle to inscribe an equilateral and equiangular hexagon.



Construction

Draw the diameter AD through the centre of the circle G



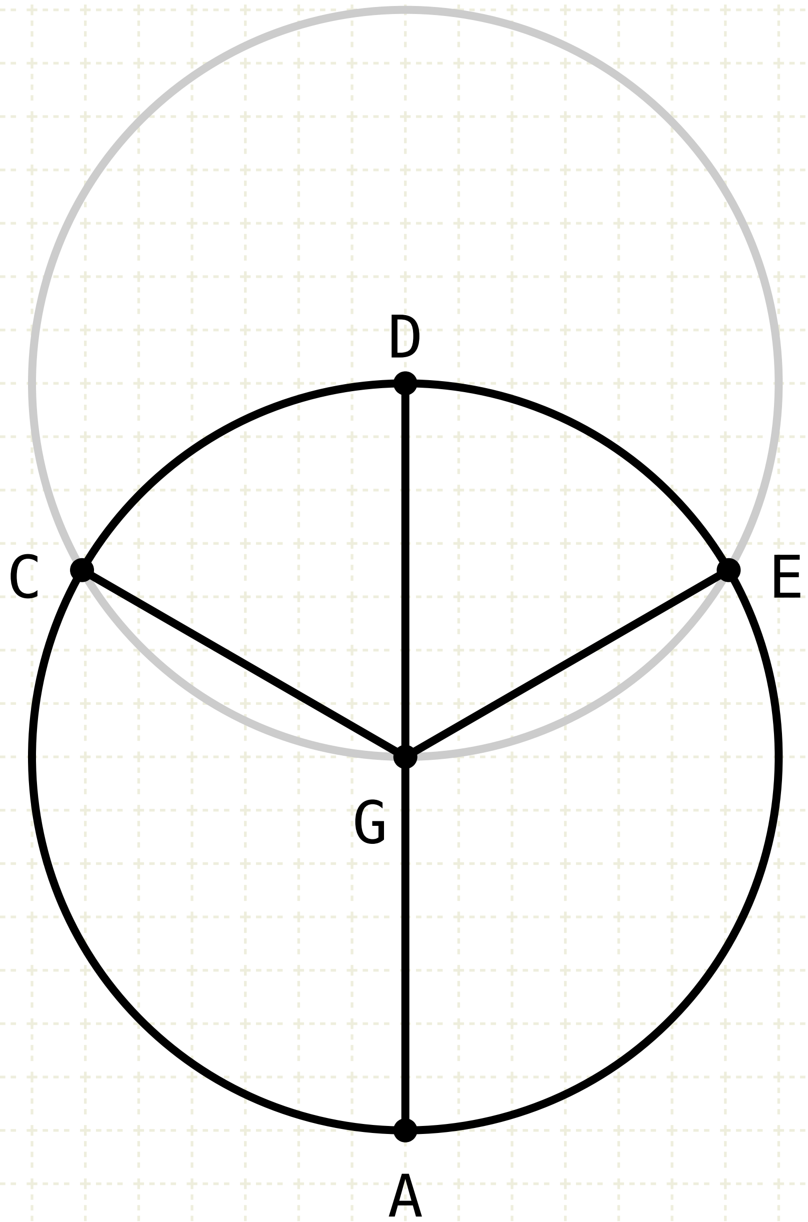
Proposition 15 of Book IV

In a given circle to inscribe an equilateral and equiangular hexagon.

Construction

Draw the diameter AD through the centre of the circle G

Draw a circle with D as the centre, and DG as the radius



Proposition 15 of Book IV

In a given circle to inscribe an equilateral and equiangular hexagon.

Construction

Draw the diameter AD through the centre of the circle G

Draw a circle with D as the centre, and DG as the radius

Draw the lines CG and EG

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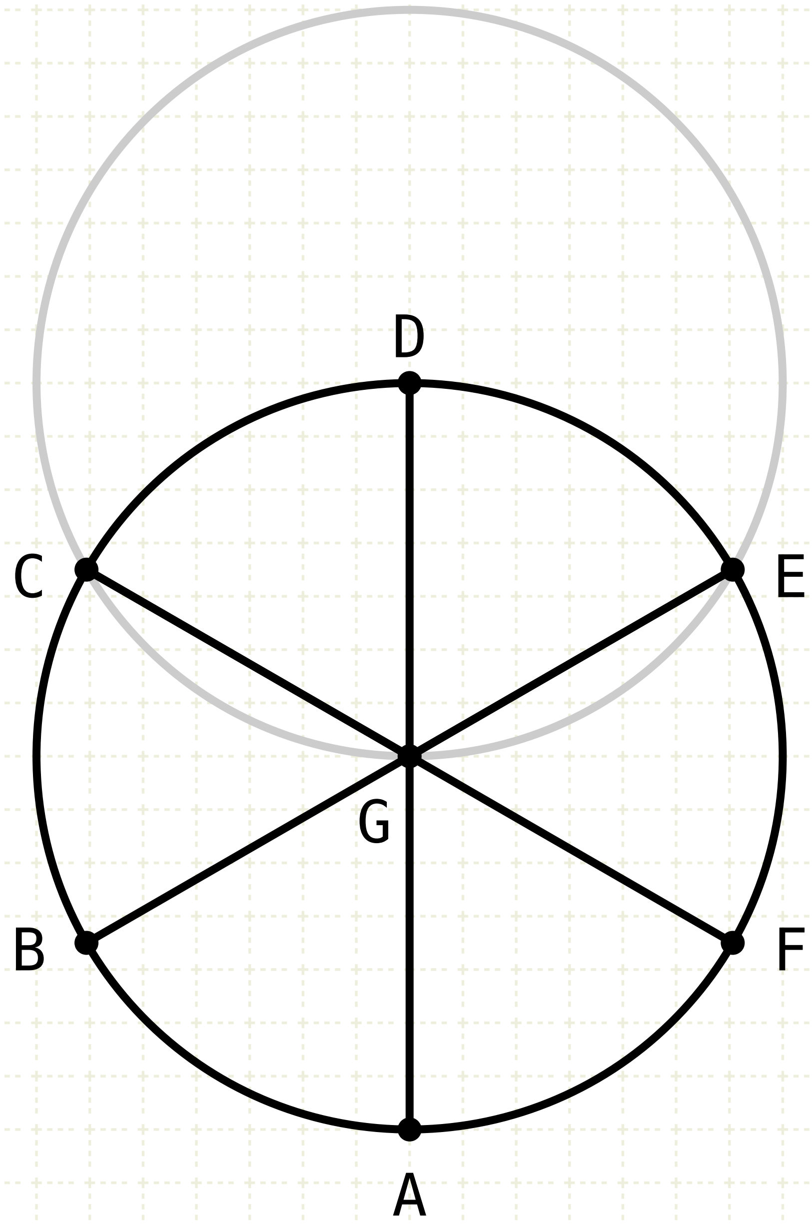


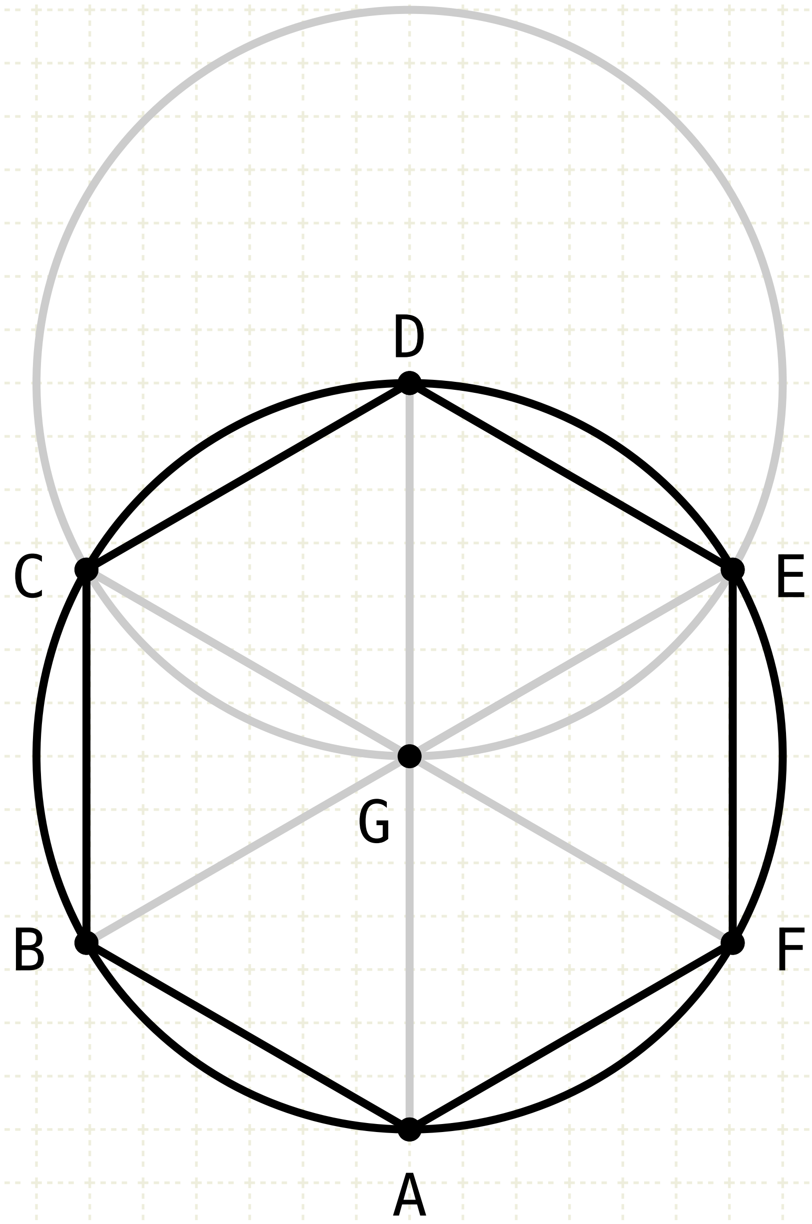
Construction

Draw the diameter AD through the centre of the circle G

Draw a circle with D as the centre, and DG as the radius

Draw the lines CG and EG

Extend the lines CG and EG to the other side of the circle at points F and B respectively



Proposition 15 of Book IV

In a given circle to inscribe an equilateral and equiangular hexagon.

Construction

Draw the diameter AD through the centre of the circle G

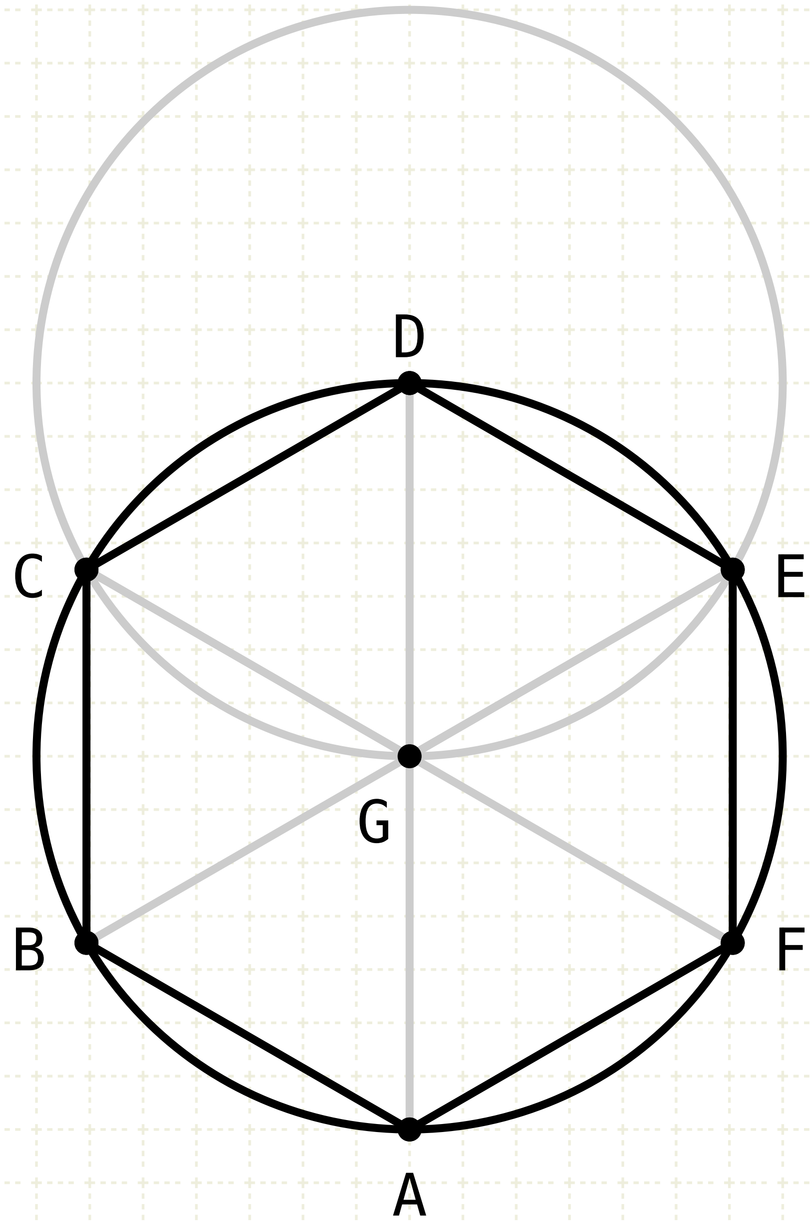
Draw a circle with D as the centre, and DG as the radius

Draw the lines CG and EG

Extend the lines CG and EG to the other side of the circle at points F and B respectively

Draw lines AB, BC, CD, DE, EA

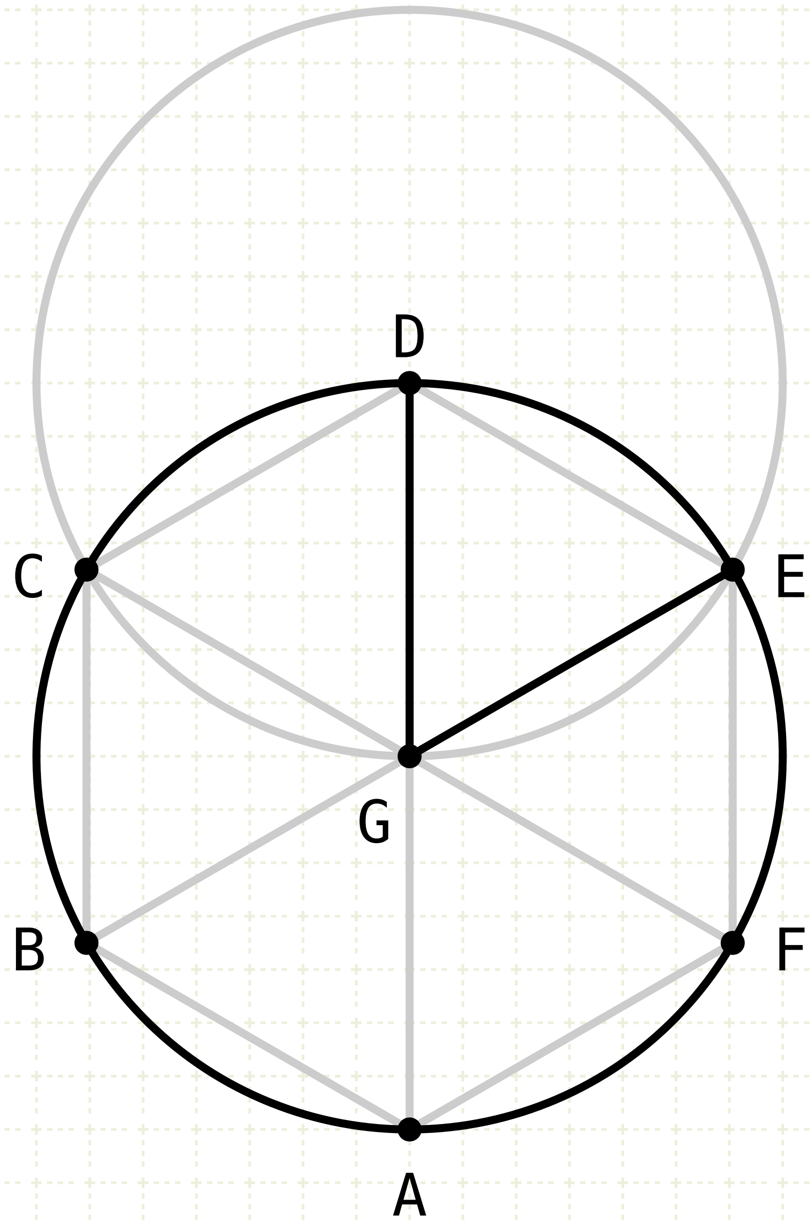
The resulting hexagon is equilateral and equiangular



Proposition 15 of Book IV

In a given circle to inscribe an equilateral and equiangular hexagon.

Proof



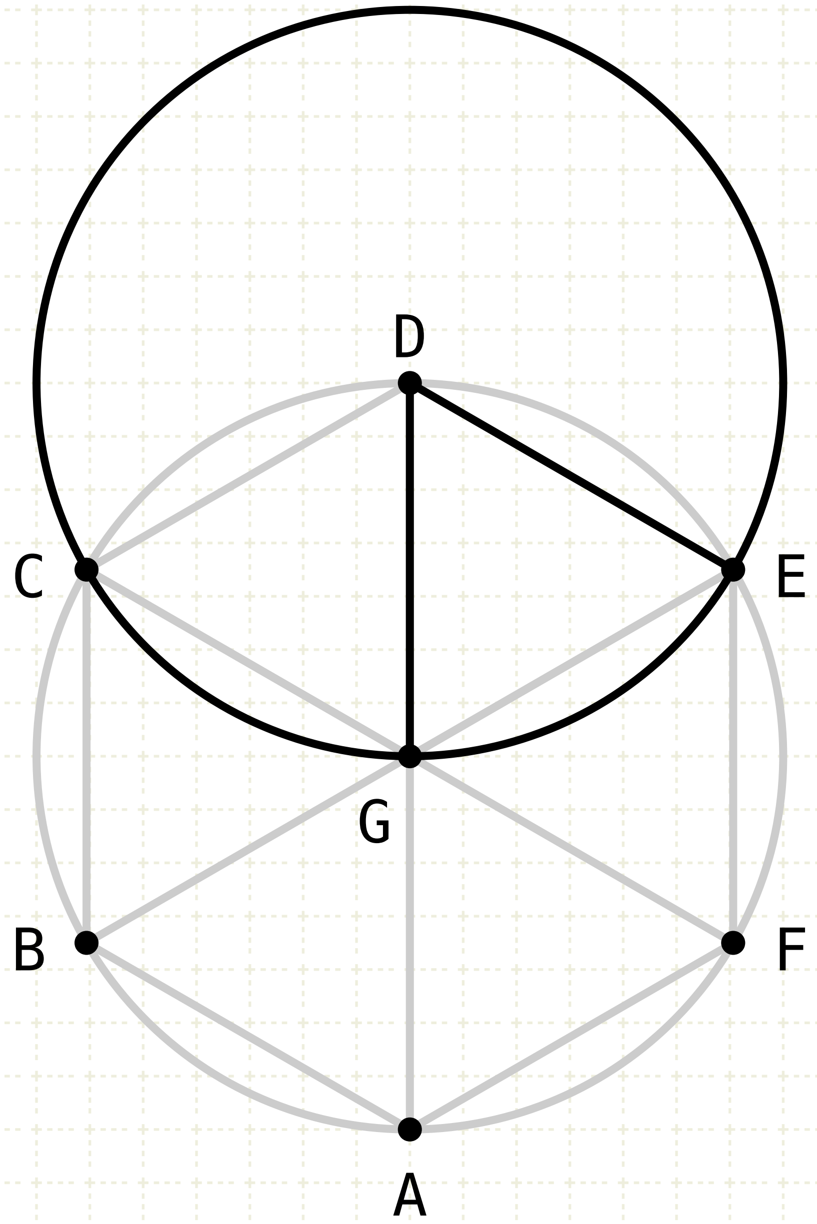
Proposition 15 of Book IV

In a given circle to inscribe an equilateral and equiangular hexagon.

$DG = EG$

Proof

The lines DG and EG are radii of the same circle, and thus are equal



Proposition 15 of Book IV

In a given circle to inscribe an equilateral and equiangular hexagon.

DG = EG
DG = DE

Proof

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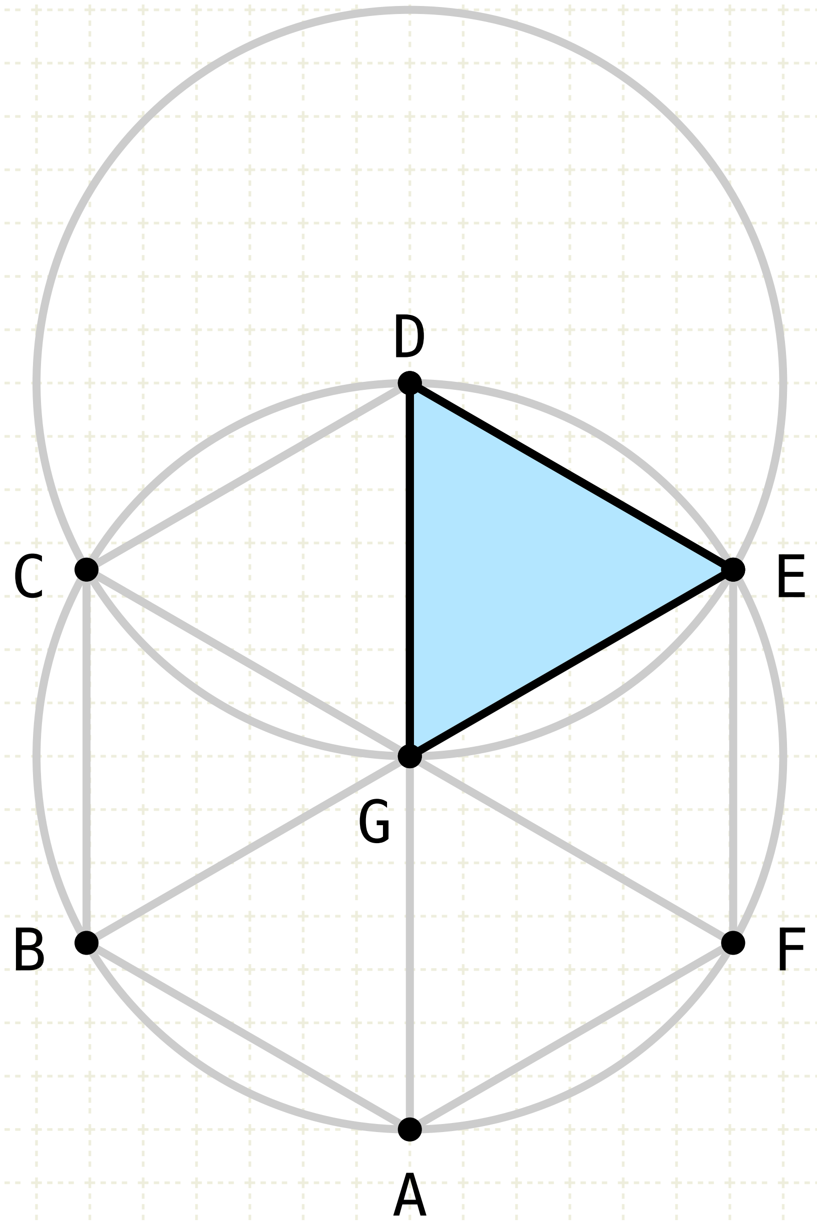

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In a given circle to inscribe an equilateral and equiangular hexagon.

DG = EG
DG = DE

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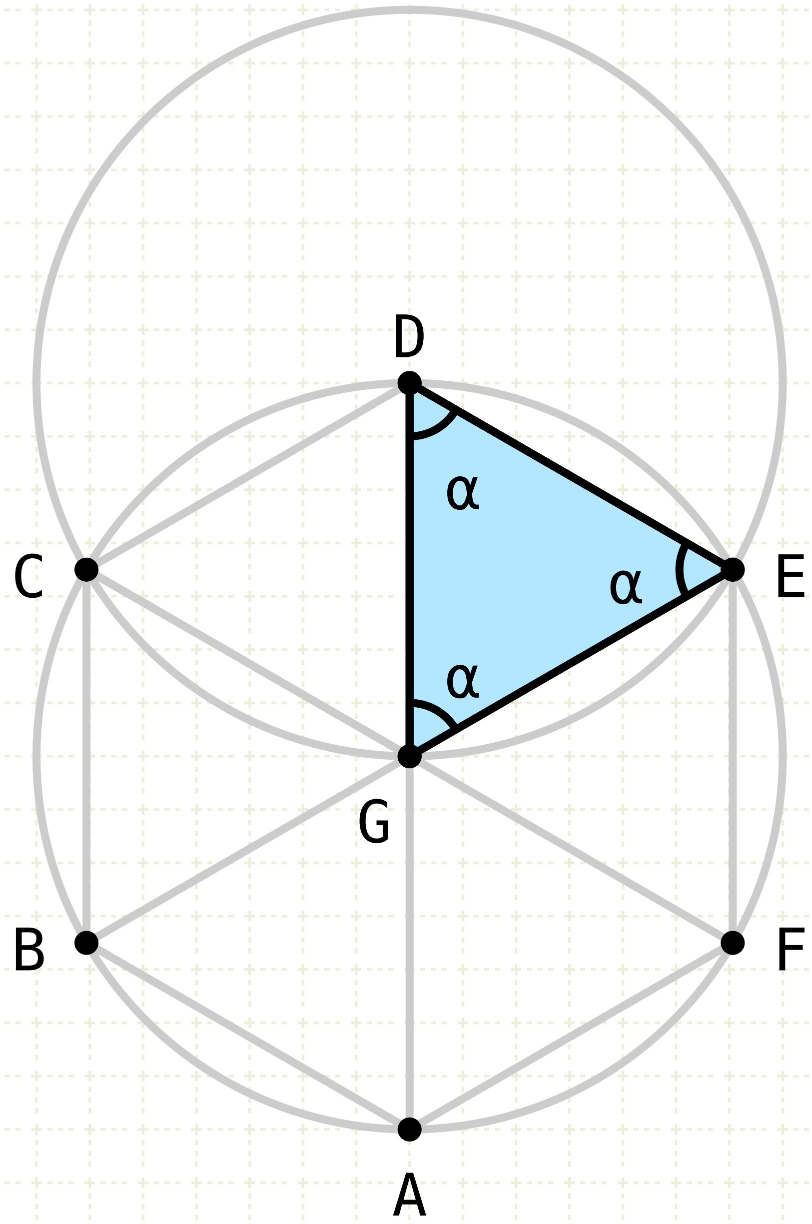
The lines DG and EG are radii of the same circle, and thus are equal

The lines DG and DE are radii of the same circle, and thus are equal

Therefore the triangle DGE is an equilateral triangle



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In a given circle to inscribe an equilateral and equiangular hexagon.

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DG = DE

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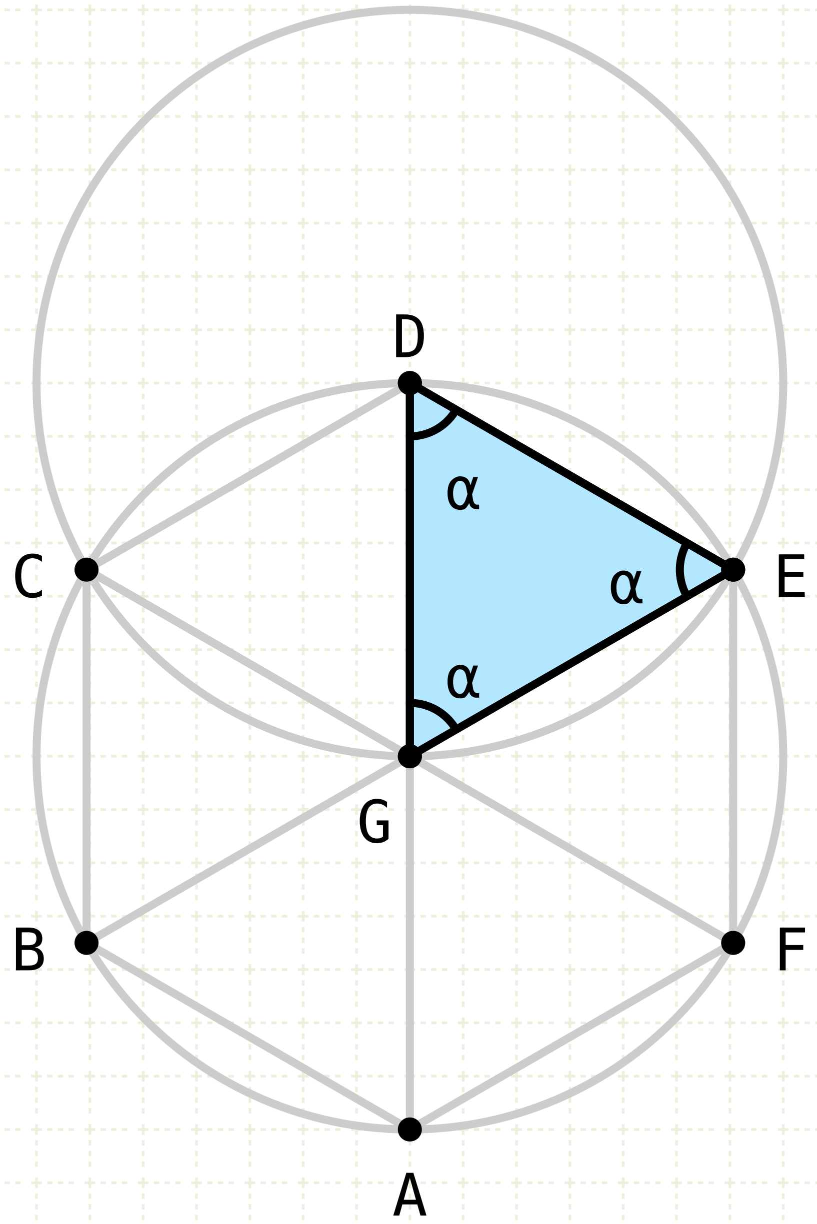
The lines DG and DE are radii of the same circle, and thus are equal

Therefore the triangle DGE is an equilateral triangle

An equilateral triangle is also an isosceles triangle, regardless of which side is chosen as its base, therefore, all the angles within the triangle are equal (I·5)



Proposition 15 of Book IV

In a given circle to inscribe an equilateral and equiangular hexagon.

$DG = EG$

$DG = DE$

$\angle DGE = \alpha = (1/3)\cdot 2\llcorner$

Proof

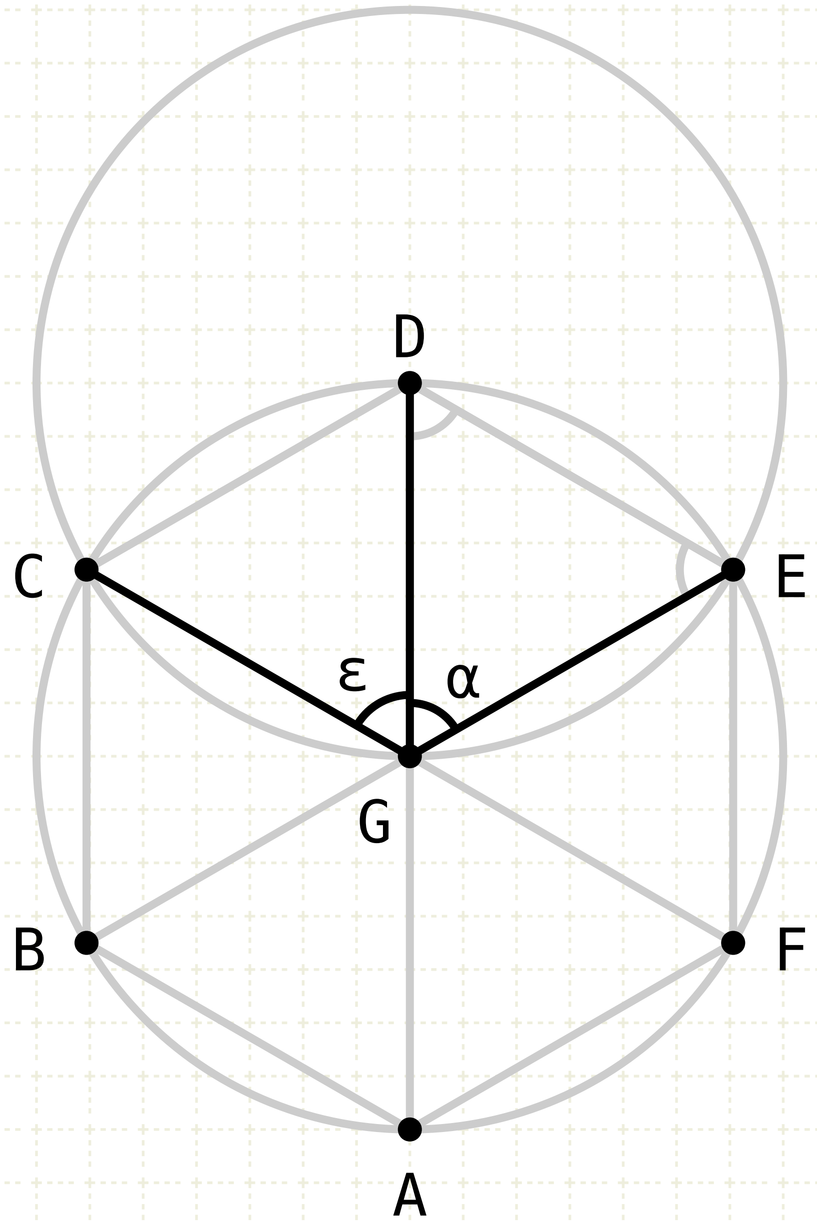
The lines DG and EG are radii of the same circle, and thus are equal

The lines DG and DE are radii of the same circle, and thus are equal

Therefore the triangle DGE is an equilateral triangle

An equilateral triangle is also an isosceles triangle, regardless of which side is chosen as its base, therefore, all the angles within the triangle are equal (I·5)

The sum of all the angles within a triangle is two right angles (I·32), therefore the angle EGD is one-third of two right angles



Proposition 15 of Book IV

In a given circle to inscribe an equilateral and equiangular hexagon.

$DG = EG$

$DG = DE$

$\angle DGE = \alpha = (1/3) \cdot 2∟$

$\angle CGD = \varepsilon = (1/3) \cdot 2∟$

Proof

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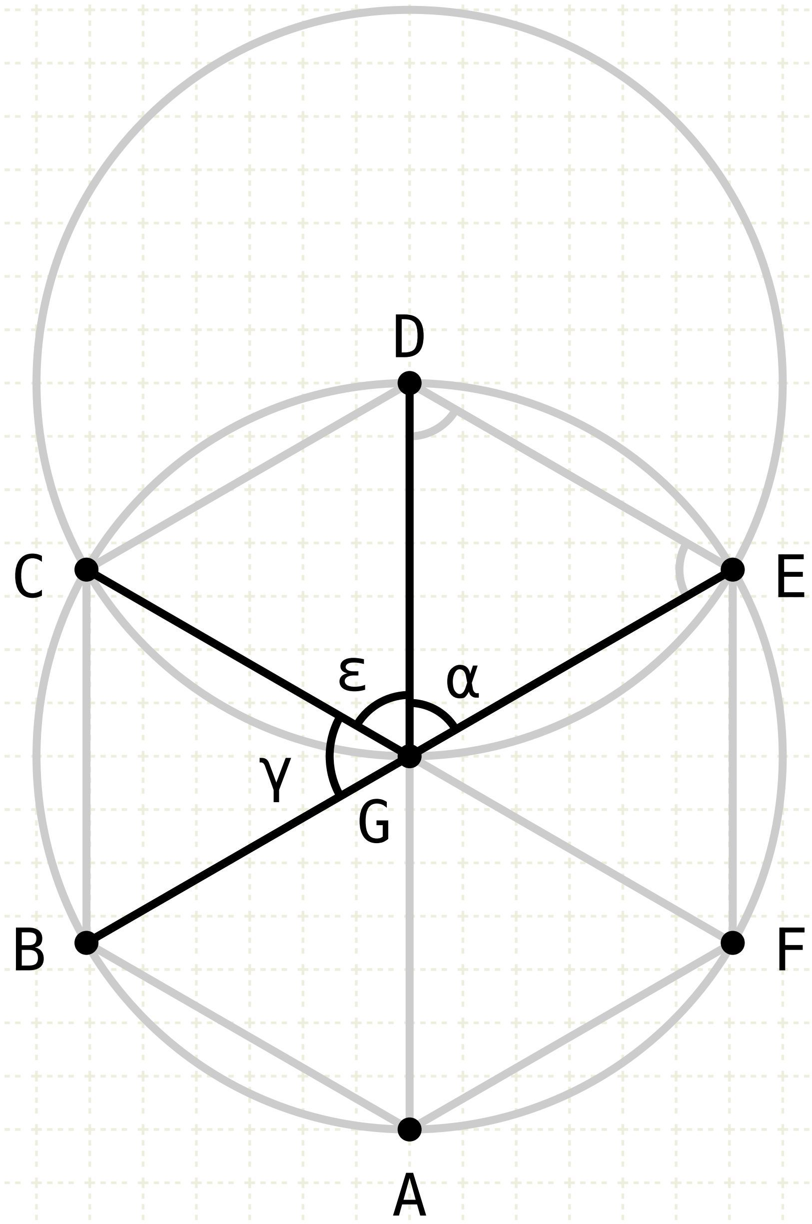
Similarly, it can be shown that CGD is also one-third of two right angles



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$(\varepsilon + \alpha) + \gamma = 2\llcorner$

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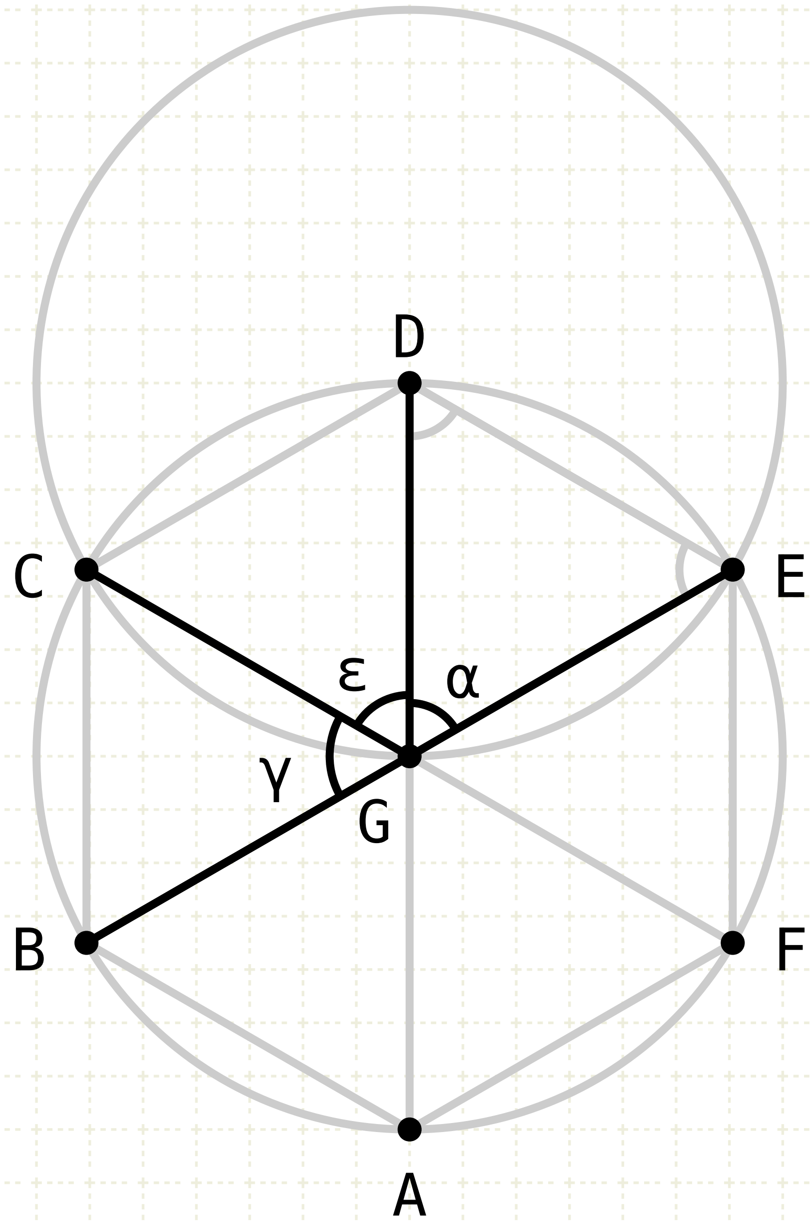
Similarly, it can be shown that CGD is also one-third of two right angles

The line CG cuts the straight line BE, therefore the angles CGE and CGB equal two right angles



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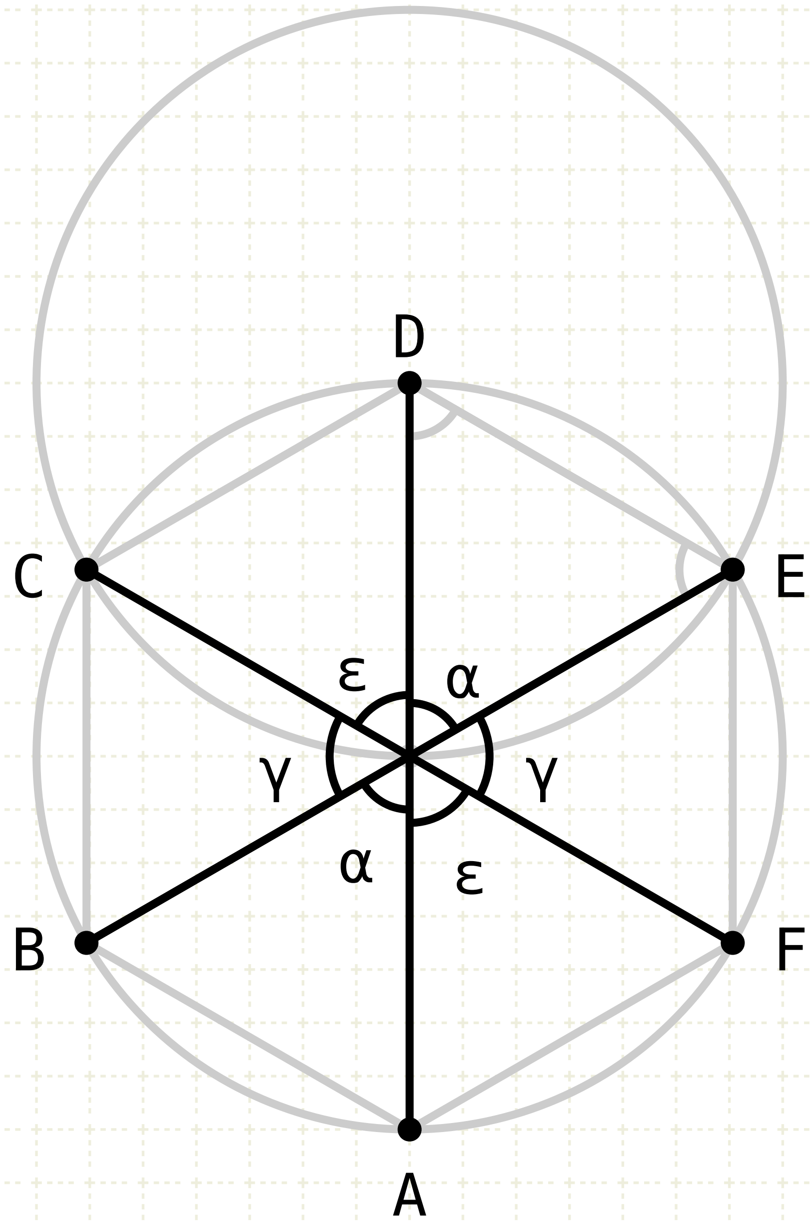
Similarly, it can be shown that CGD is also one-third of two right angles

The line CG cuts the straight line BE, therefore the angles CGE and CGB equal two right angles

Therefore, angle BGC is also one third of two right angles and all of the angles are equal



Proposition 15 of Book IV

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$$DG = EG$$
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The line CG cuts the straight line BE, therefore the angles CGE and CGB equal two right angles

Therefore, angle BGC is also one third of two right angles and all of the angles are equal

The angles vertical to BGC, CGD, DGE are also equal (I·15)

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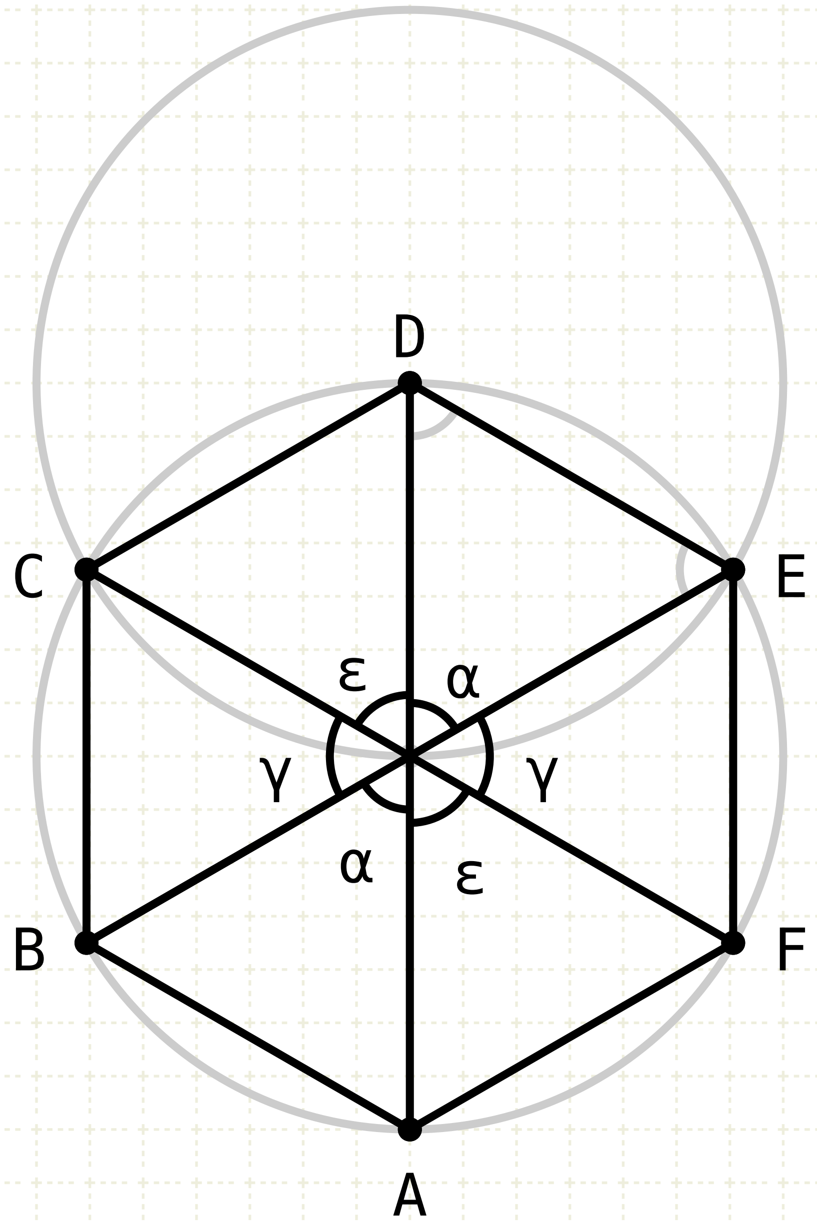

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$AB = BC = CD = DE = EF$

Proof

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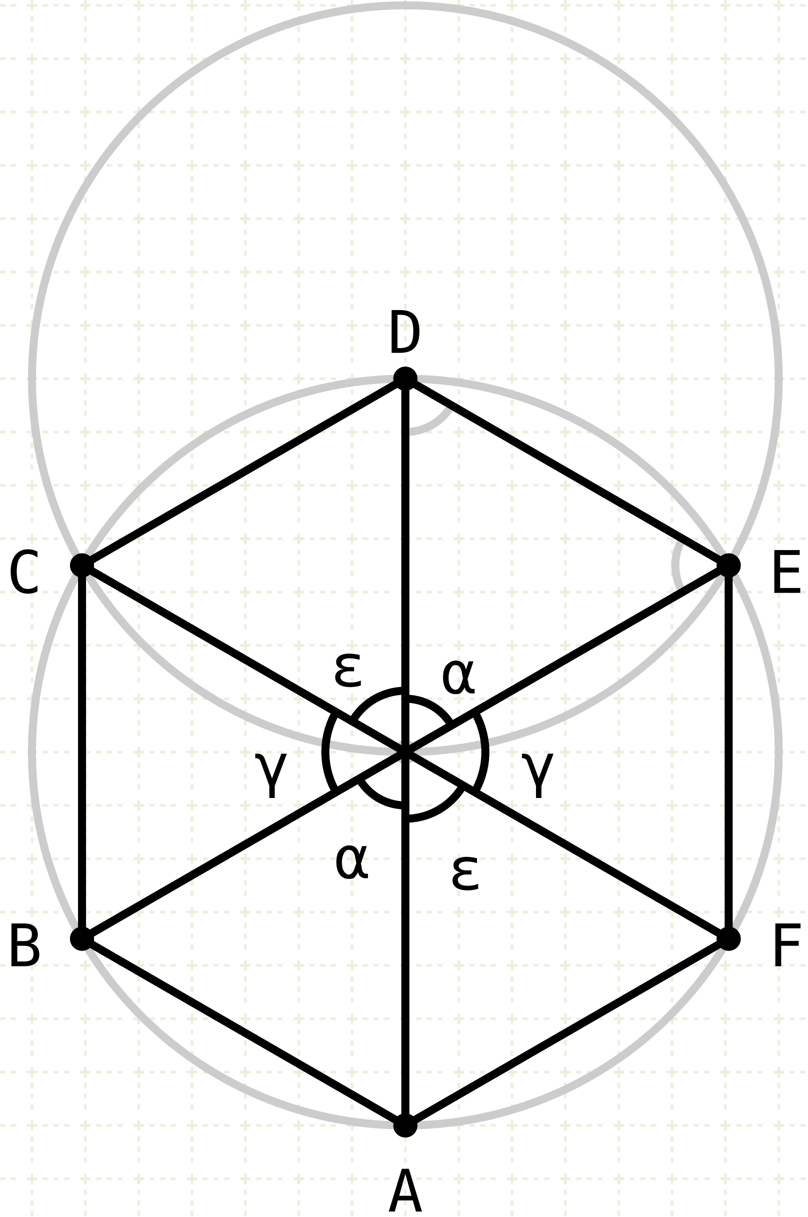
Therefore, angle BGC is also one third of two right angles and all of the angles are equal

The angles vertical to BGC, CGD, DGE are also equal (I·15)

Equal angles subtend equal circumferences (III·26) and equal circumferences are subtended by equal straight lines (III·29)

Therefore the six lines are equal



Proposition 15 of Book IV

In a given circle to inscribe an equilateral and equiangular hexagon.

The hexagon was proven to be equilateral

It is also equiangular

Proof (cont)

$DG = EG$

$DG = DE$

$\angle DGE = \alpha = (1/3)\cdot 2\llcorner$

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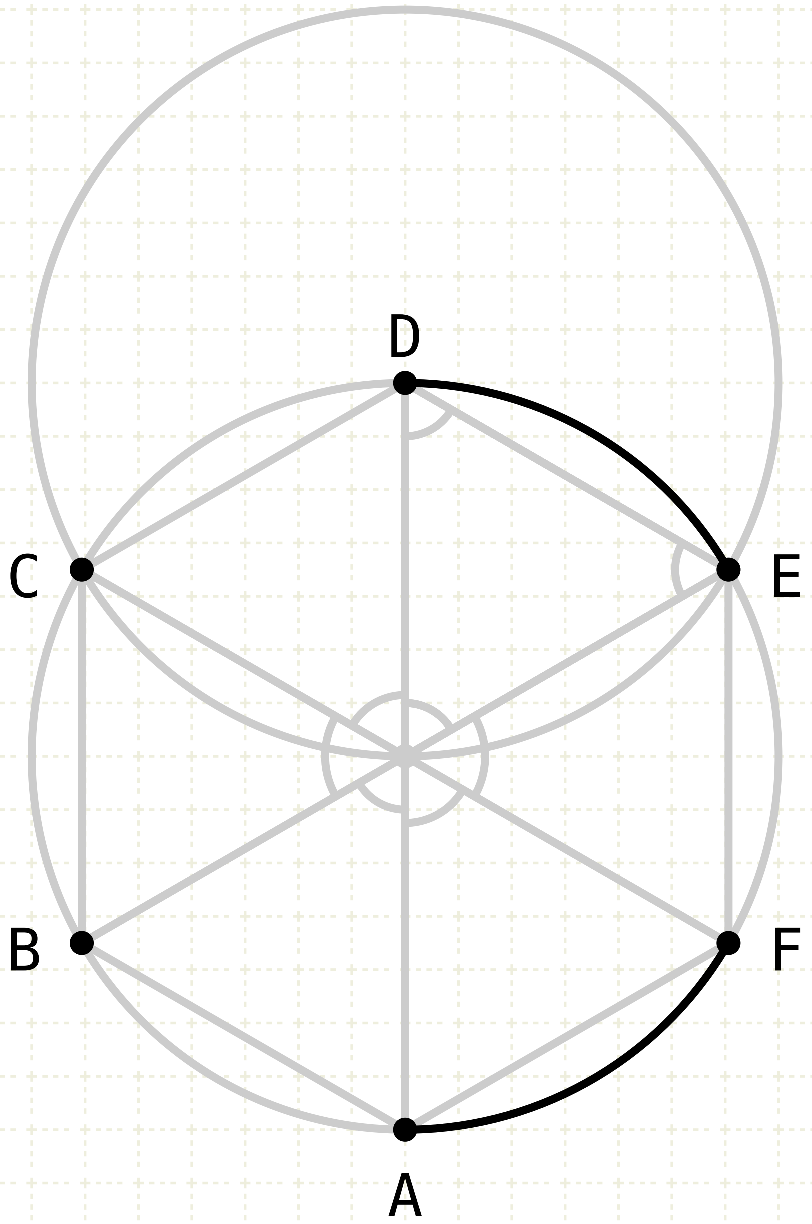

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$(\varepsilon+\alpha) + \gamma = 2\llcorner$

$\gamma = \varepsilon = \alpha = (1/3)\cdot 2\llcorner$

$AB = BC = CD = DE = EF$

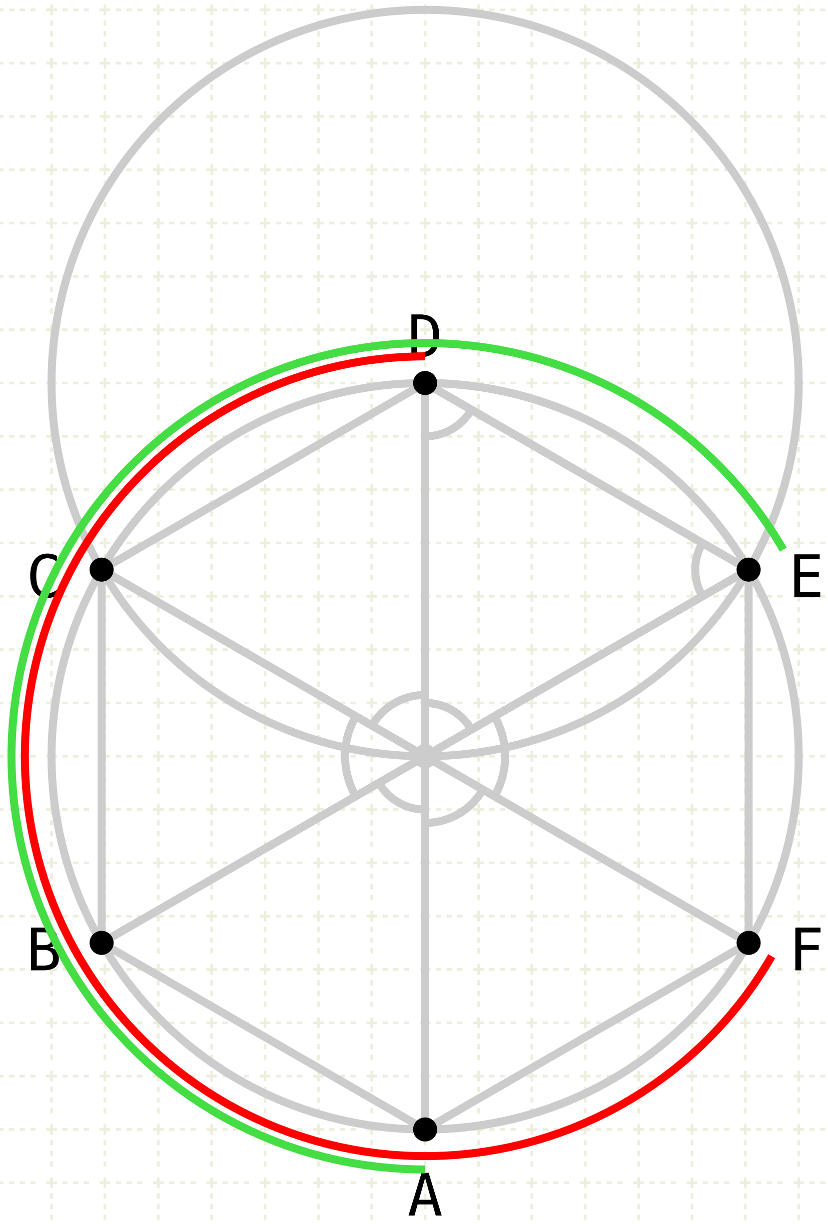
$\circlearrowleft DE = \circlearrowleft FA$

The hexagon was proven to be equilateral

It is also equiangular

Proof (cont)

The circumference FA is equal to the circumference ED



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$AB = BC = CD = DE = EF$

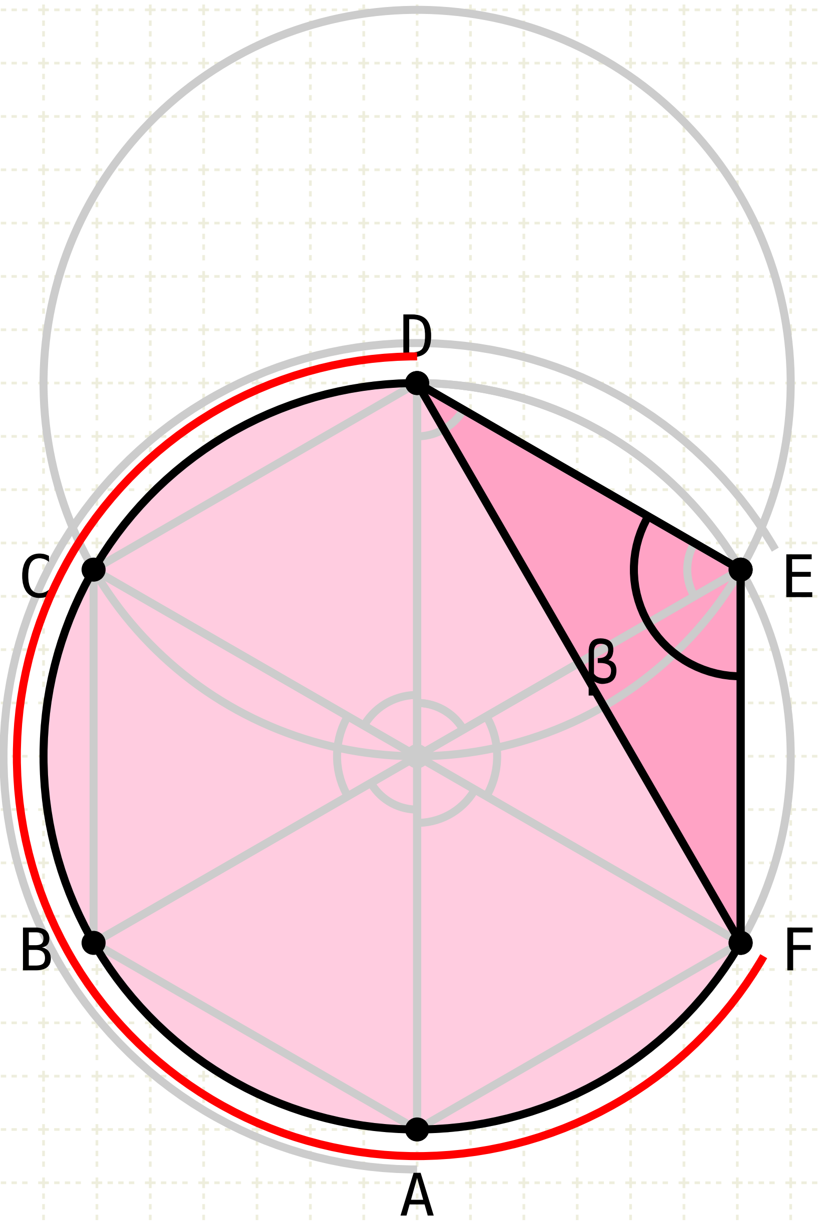
Proof (cont)

The circumference FA is equal to the circumference ED

Let the circumference ABCD be added to each of FA and ED maintaining the equality

↺DE = ↺FA
↺ABCD + ↺DE = ↺FA + ↺ABCD

↺ABCDE = ↺FABCD



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$AB = BC = CD = DE = EF$

$\circlearrowleft DE = \circlearrowleft FA$
$\circlearrowleft ABCD + \circlearrowleft DE = \circlearrowleft FA + \circlearrowleft ABCD$

$\circlearrowleft ABCDE = \circlearrowleft FABCD$
$\angle DEF =$

The hexagon was proven to be equilateral

It is also equiangular

Proof (cont)

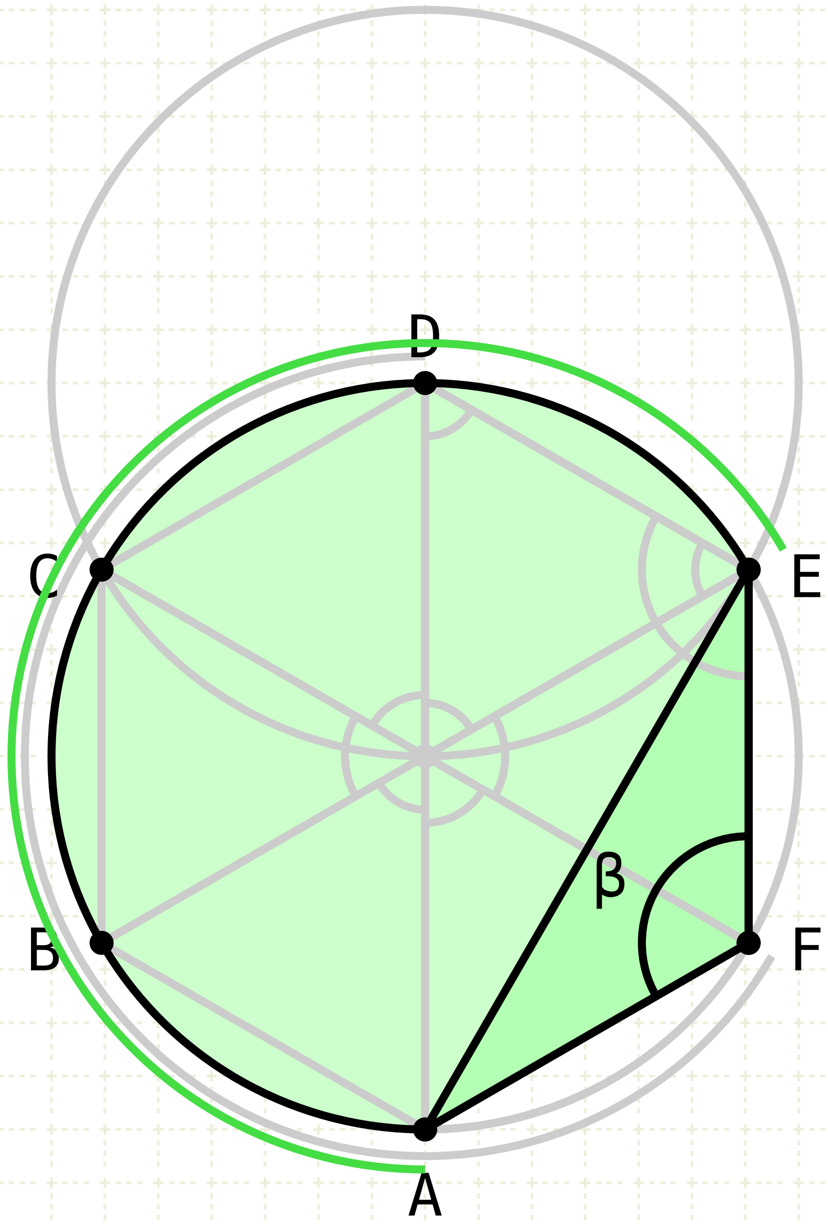
The circumference FA is equal to the circumference ED

Let the circumference ABCD be added to each of FA and ED maintaining the equality

Equal circumferences have equal angles, therefore the angles are also equal (III·27)

Proposition 15 of Book IV

In a given circle to inscribe an equilateral and equiangular hexagon.



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$\circlearrowleft ABCDE = \circlearrowleft FABCD$

$\angle DEF = \angle EFA$

The hexagon was proven to be equilateral

It is also equiangular

Proof (cont)

The circumference FA is equal to the circumference ED

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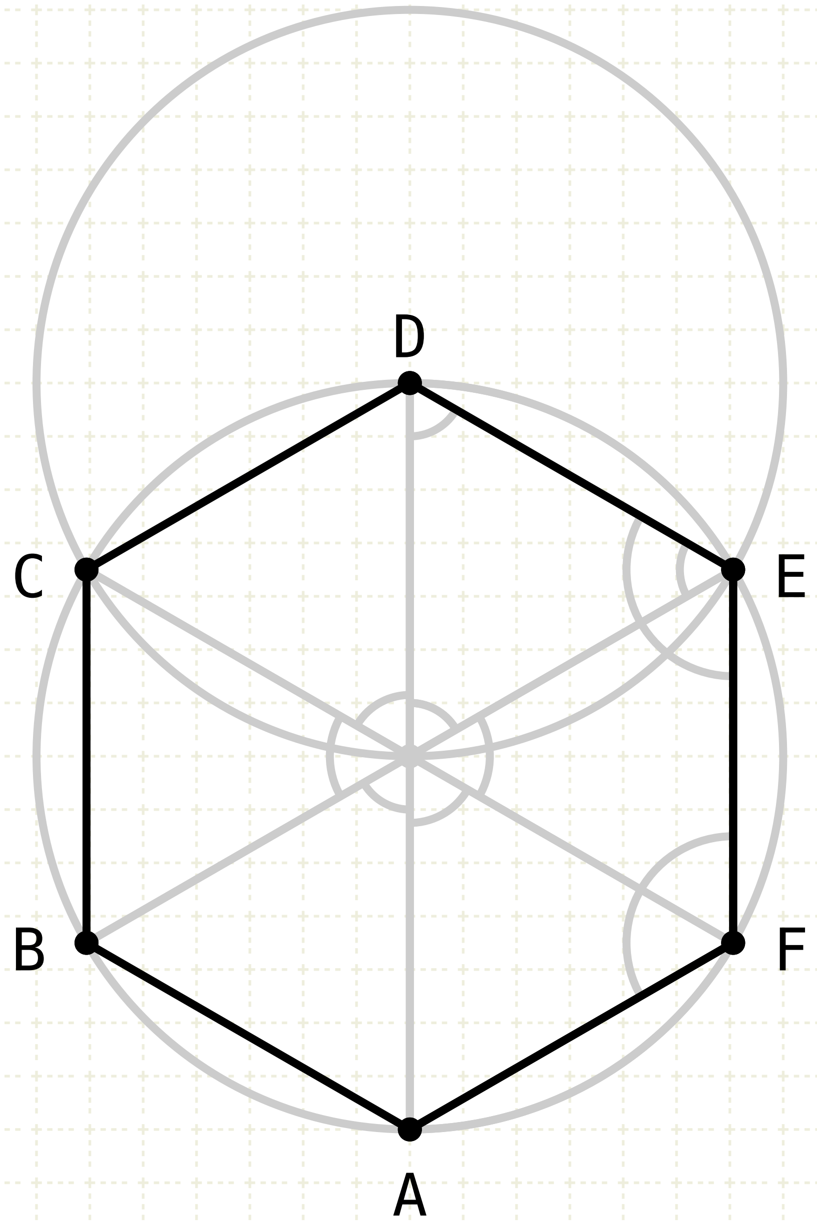

Proposition 15 of Book IV

In a given circle to inscribe an equilateral and equiangular hexagon.

DG = EG
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∠CGD = ε = (1/3)·2∟
(ε+α) + γ = 2∟
γ = ε = α = (1/3)·2∟
AB = BC = CD = DE = EF

↺DE = ↺FA
↺ABCD + ↺DE = ↺FA + ↺ABCD

↺ABCDE = ↺FABCD
∠DEF = ∠EFA
∠DCB = ∠CBA = ∠BAF
= ∠AFE = ∠FED = ∠EDC

The hexagon was proven to be equilateral

It is also equiangular

Proof (cont)

The circumference FA is equal to the circumference ED

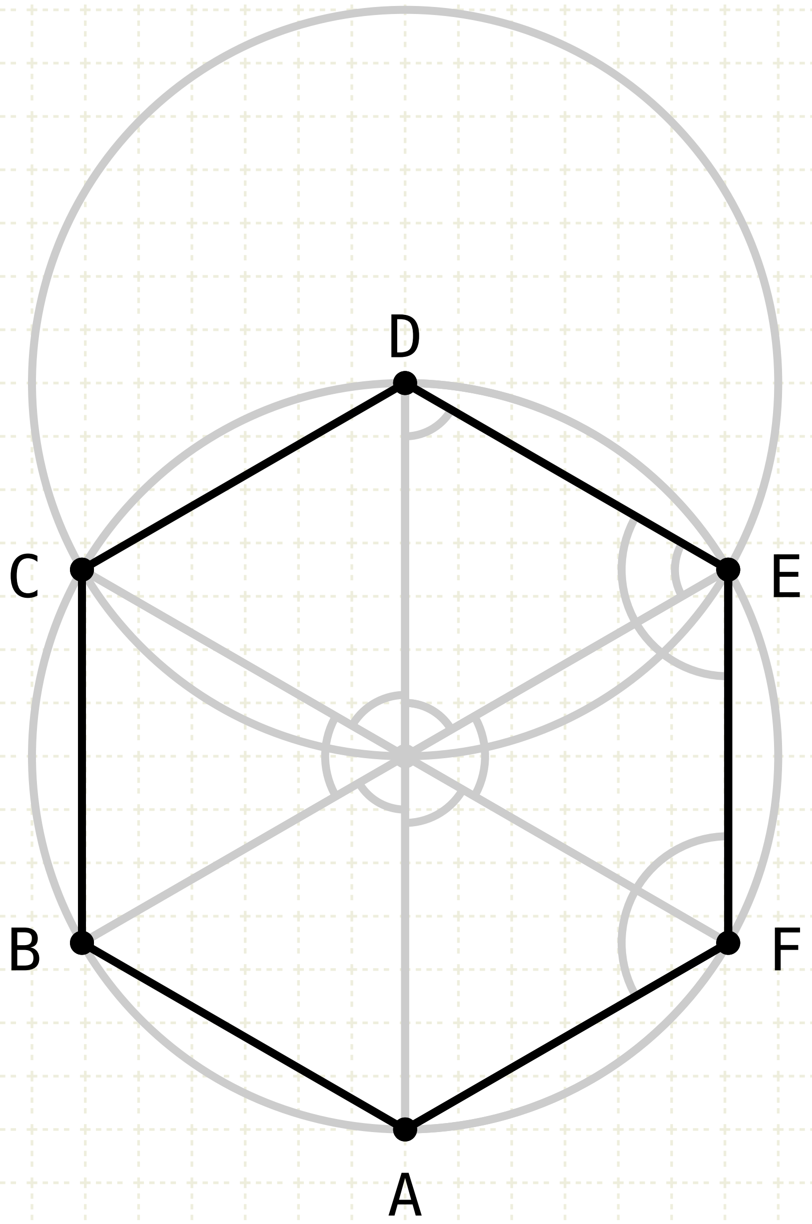
Let the circumference ABCD be added to each of FA and ED maintaining the equality

Equal circumferences have equal angles, therefore the angles are also equal (III·27)

Similarly, we can show that all the angles are equal

Proposition 15 of Book IV

In a given circle to inscribe an equilateral and equiangular hexagon.



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$\angle DEF = \angle EFA$

$\angle DCB = \angle CBA = \angle BAF$

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The hexagon was proven to be equilateral

It is also equiangular

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The circumference FA is equal to the circumference ED

Let the circumference ABCD be added to each of FA and ED maintaining the equality

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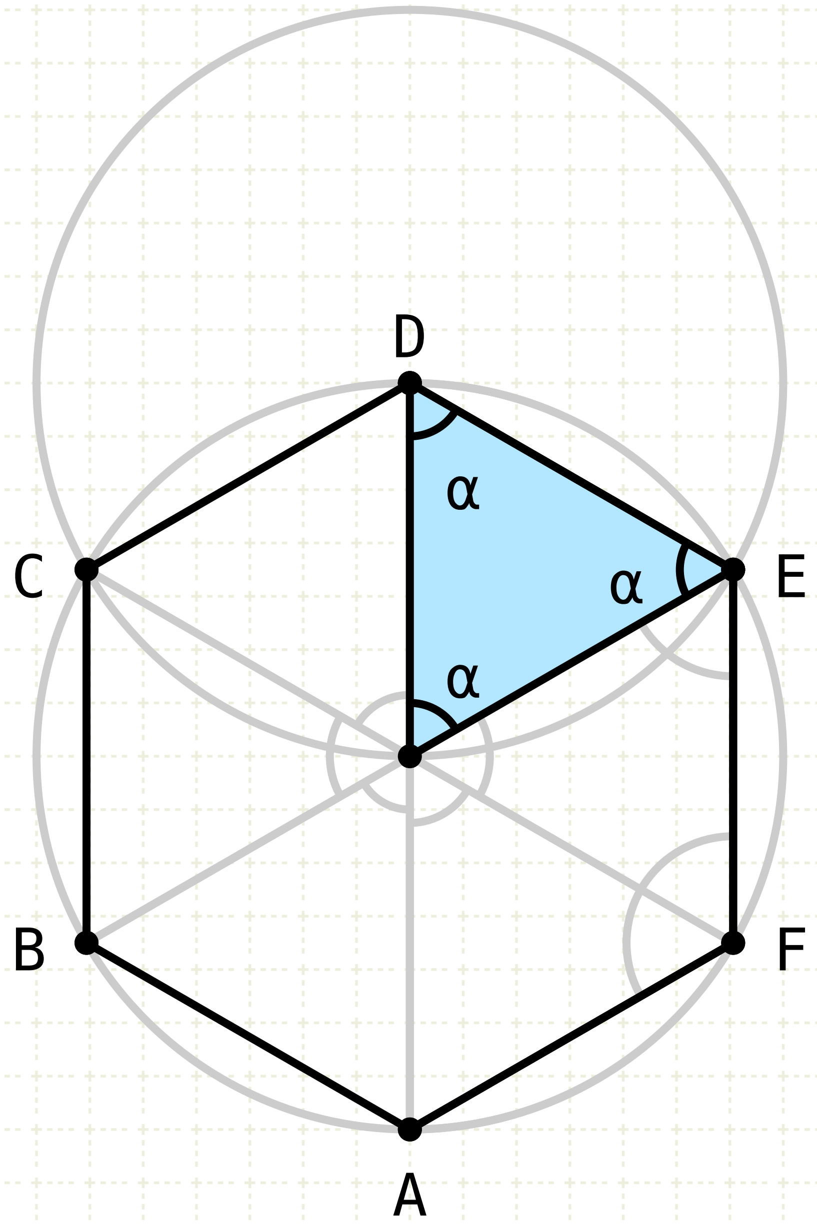
Similarly, we can show that all the angles are equal

The hexagon is both equilateral and equiangular

Proposition 15 of Book IV

In a given circle to inscribe an equilateral and equiangular hexagon.



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The hexagon was proven to be equilateral

It is also equiangular

Proof (cont)

The circumference FA is equal to the circumference ED

Let the circumference ABCD be added to each of FA and ED maintaining the equality

Equal circumferences have equal angles, therefore the angles are also equal (III·27)

Similarly, we can show that all the angles are equal

The hexagon is both equilateral and equiangular

Note:

From the first part of the proof, it can be noted that the side of a hexagon is equal to the radius of the circle.

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