

Euclid's Elements

Book III



A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)



Table of Contents, Chapter 3

1	To find the centre of a circle	9	If three lines, starting at a point 'A' and touching the circle, are all equal, then 'A' is the centre of the circle	17	From a given point to draw a straight line touching a given circle
2	A chord of a circle always lies inside the circle	10	A circle does not cut a circle at more points than two	18	If line touches a circle, then it is perpendicular to the diameter that touches that point
3	A line through the centre of a circle bisects a chord, and vice versa	11	Point of contact between two internal circles, and their centres, are collinear	19	If line touches a circle, then the centre of the circle lies on a line perpendicular to the original
4	A line not through the centre of a circle does not bisect a chord	12	Point of contact between two external circles, and their centres, are collinear	20	The angle at the centre of a circle is twice that from an angle from the circumference
5	If two circles cut one another, they will not have the same center	13	A circle does not touch a circle at more points than one, whether it touch it internally or externally.	21	In a circle the angles in the same segment are equal to one another
6	If two circles touch one another, they will not have the same center	14	In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another.	22	The opposite angles of quadrilaterals in circles are equal to two right angles
7	Consider two lines from a point inside a circle to the edge, the longer one will be the one closest to the longest part of the diameter passing through the original point	15	The longest line in a circle is its diameter, shorter the farther away from the diameter	23	On the same straight line there cannot be constructed two similar and unequal segments of circles on the same side
8	Consider two lines from a point outside a circle to the edge, the line closest to the centre will be longer on the concave side and shorter on the convex side	16	A line on the circle, perpendicular to the diameter, lies outside the circle	24	Similar segments of circles on equal straight lines are equal to one another



Table of Contents, Chapter 3

- | | | | |
|----|---|----|--|
| 25 | Given a segment of a circle, to describe the complete circle of which it is a segment. | 34 | Construct a circle segment on a given circle, such that the angle within the segment is equal to a given angle |
| 26 | In equal circles equal angles stand on equal circumferences | 35 | If two circle chords intersect, the segments on one multiplied together equals the segments of the other multiplied together |
| 27 | In equal circles angles standing on equal circumferences are equal to one another | 36 | Secant-tangent law |
| 28 | In equal circles equal straight lines cut off equal circumferences | 37 | Converse of the secant-tangent law |
| 29 | In equal circles equal circumferences are subtended by equal straight lines | | |
| 30 | To bisect a given circumference | | |
| 31 | In a circle the angle in the semicircle is right ... | | |
| 32 | The angle between a tangent and a straight line cutting a circle is equal to the angle in the alternate segment | | |
| 33 | Construct a circle segment on a given line, such that the angle within the segment is equal to a given angle | | |



Proposition 25 of Book III

Given a segment of a circle, to describe the complete circle of which it is a segment.

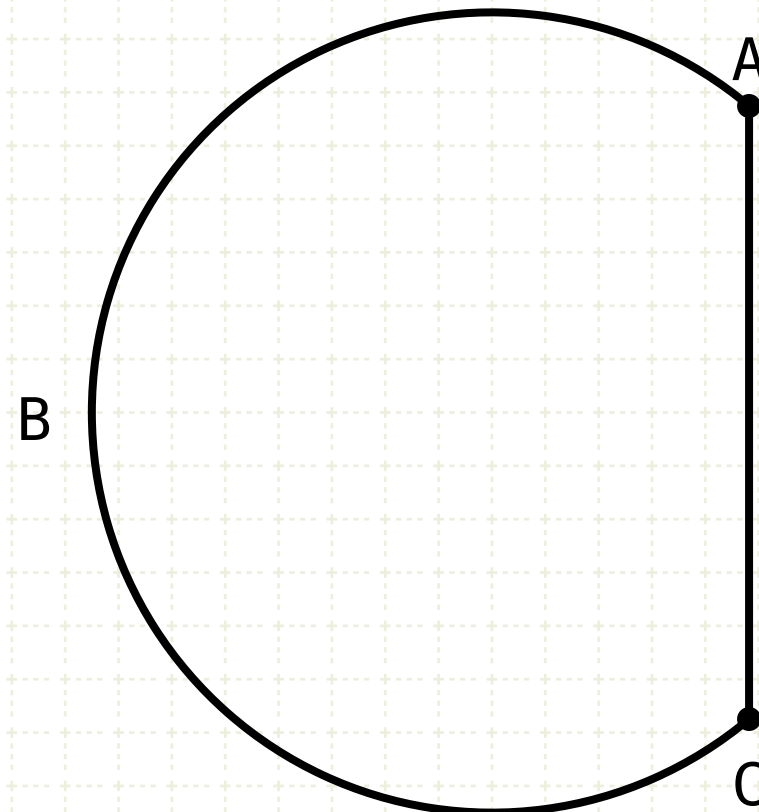
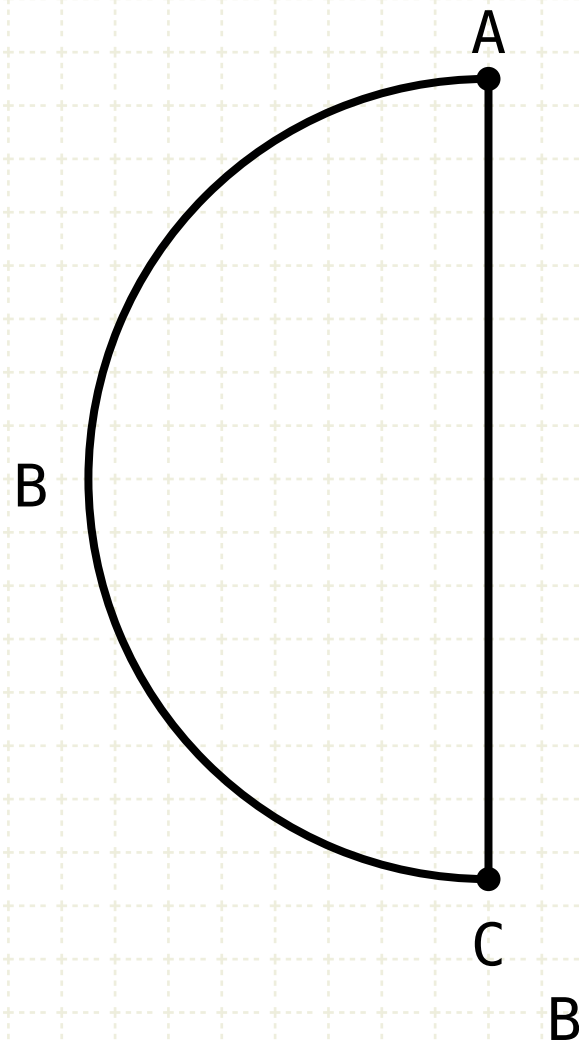
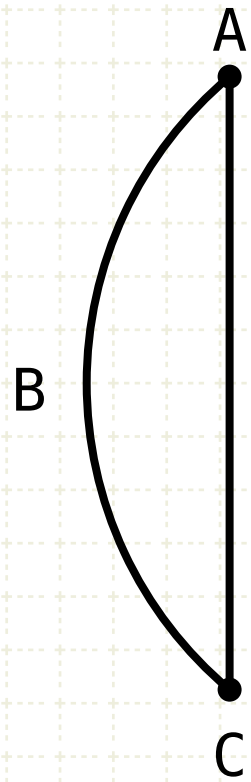


Proposition 25 of Book III

Given a segment of a circle, to describe the complete circle of which it is a segment.

In other words

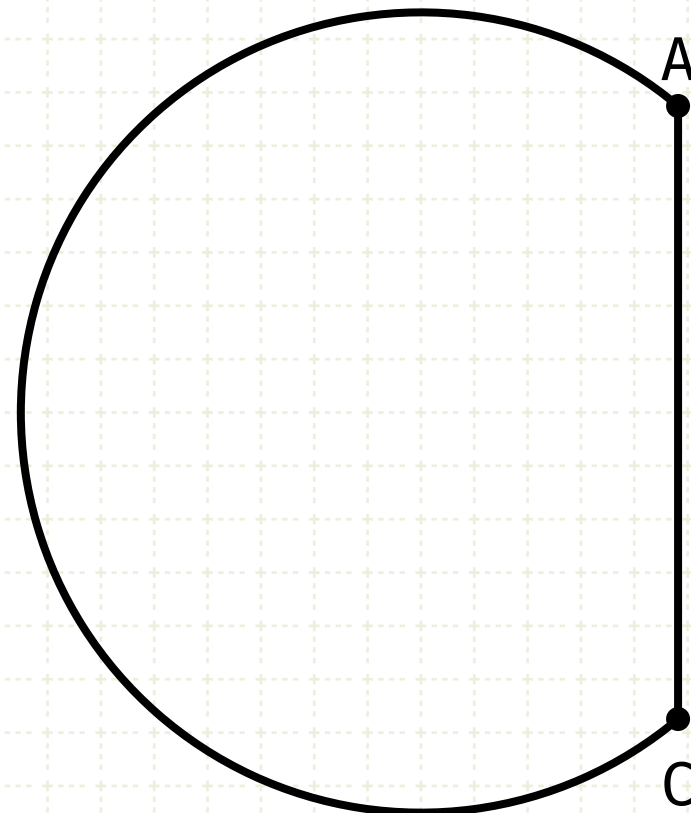
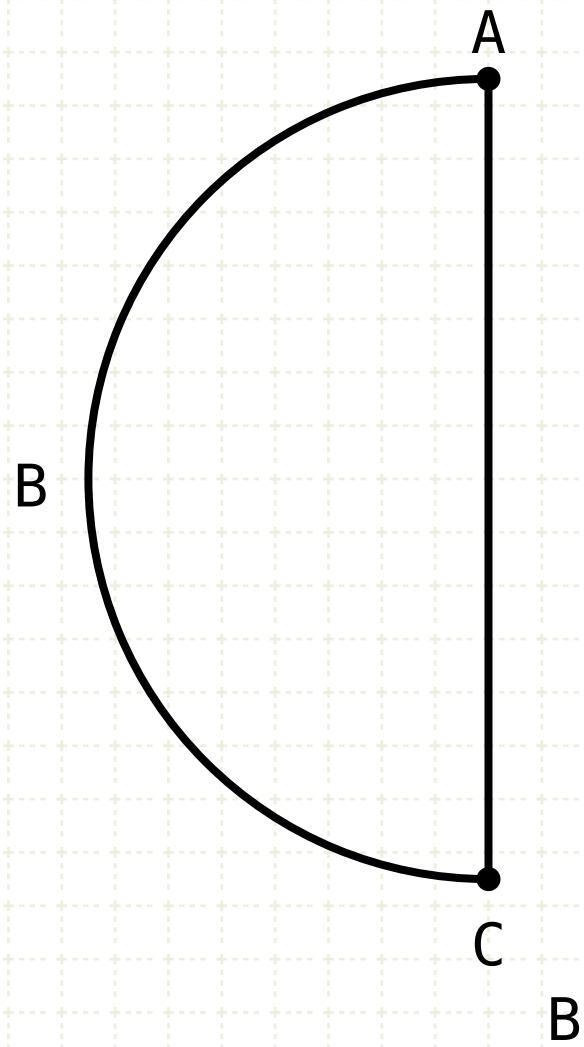
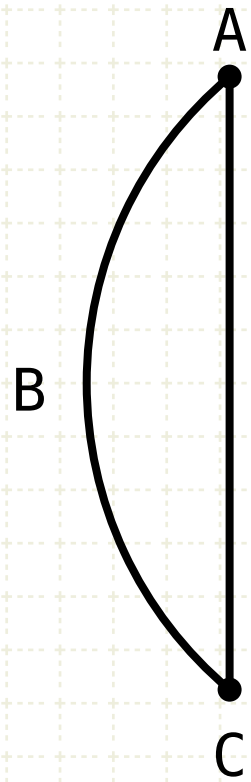
From a given segment ABC, find the radius and centre of the circle



Proposition 25 of Book III

Given a segment of a circle, to describe the complete circle of which it is a segment.

Construction



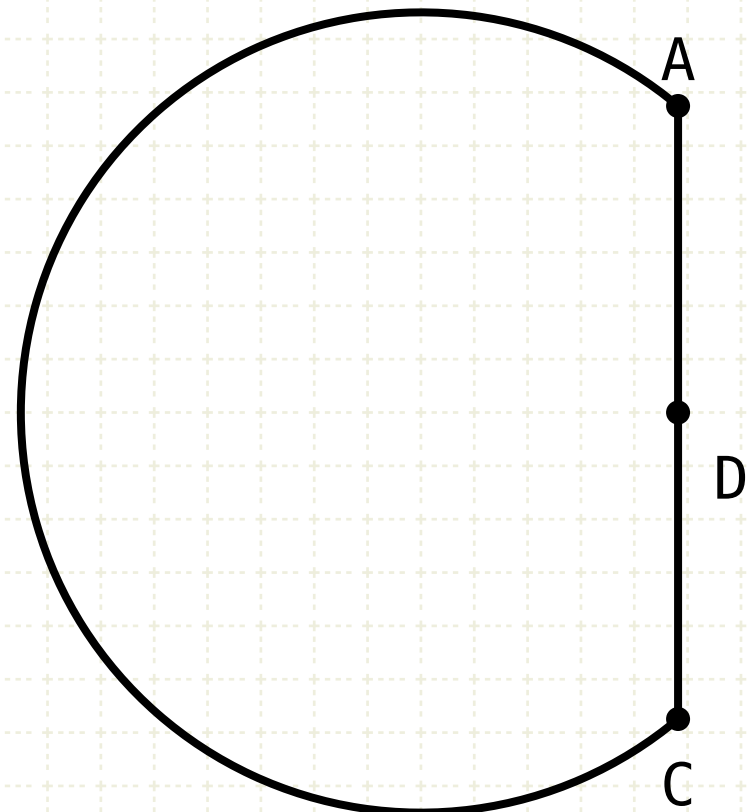
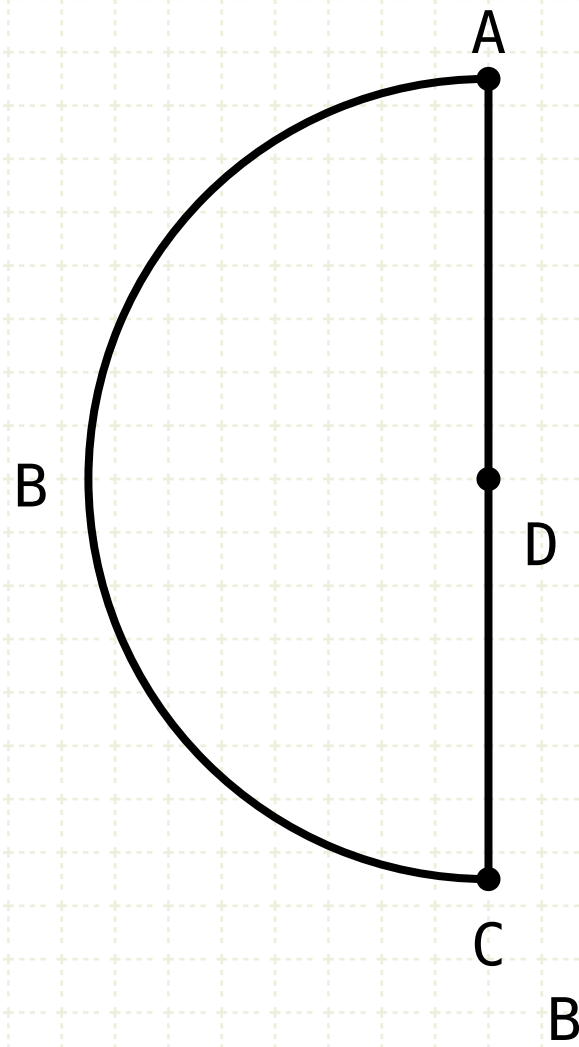
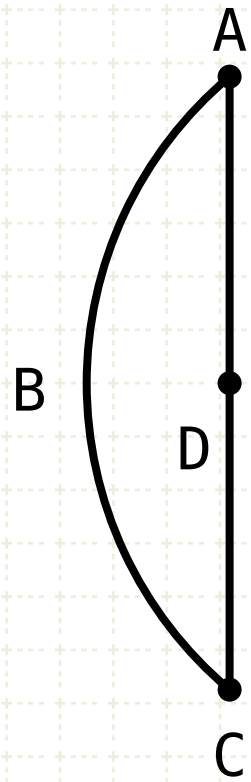
Proposition 25 of Book III

Given a segment of a circle, to describe the complete circle of which it is a segment.

Construction

Bisect line AC at point D

$$AD = DC$$



Proposition 25 of Book III

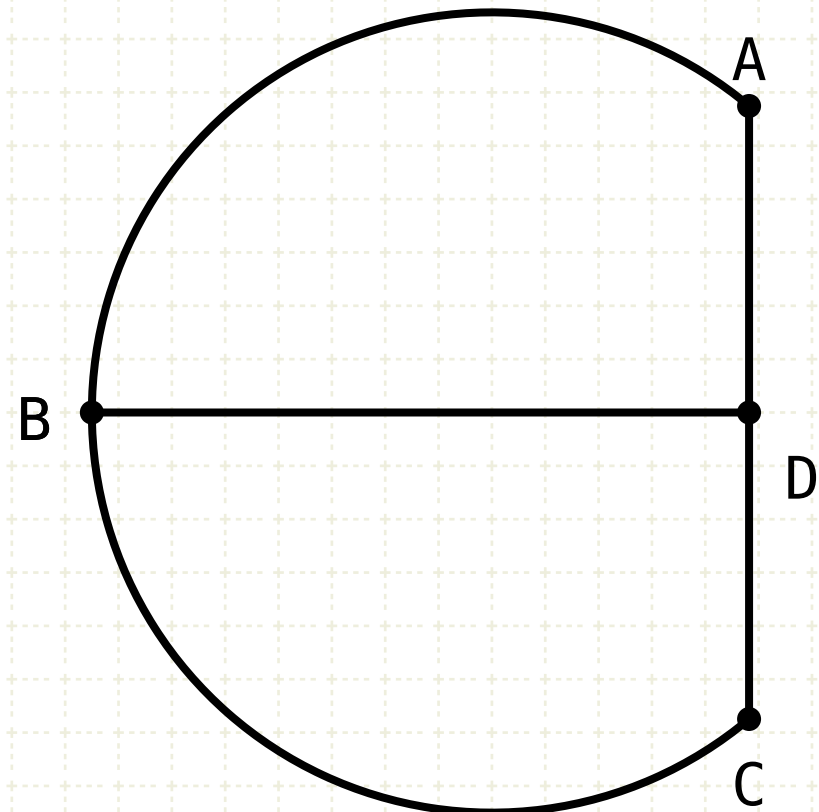
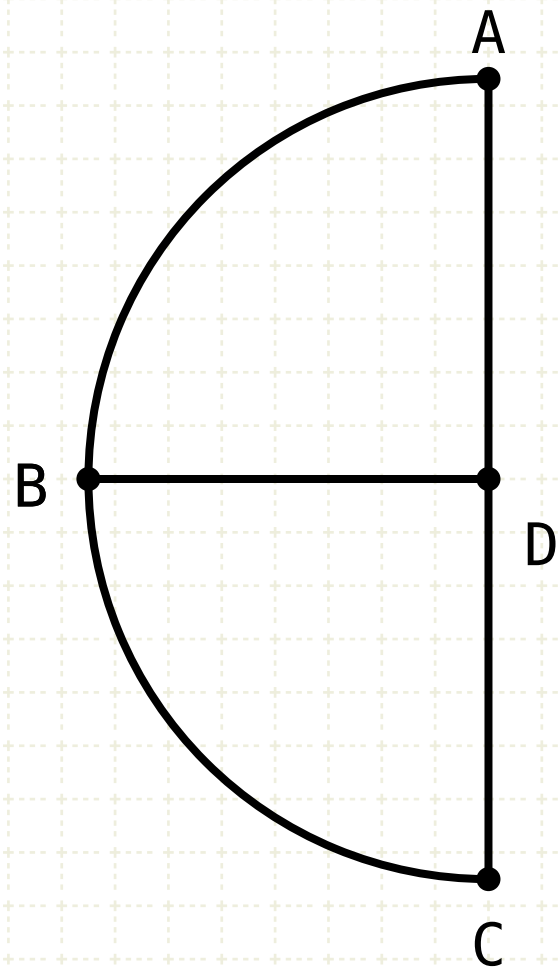
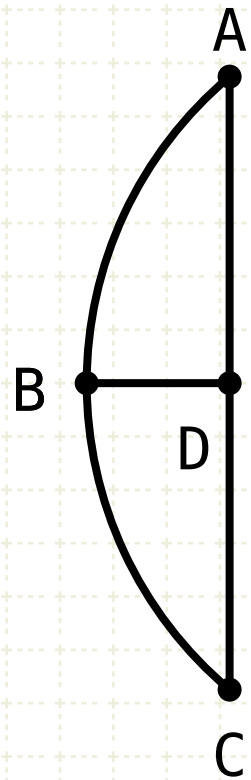
Given a segment of a circle, to describe the complete circle of which it is a segment.

Construction

Bisect line AC at point D

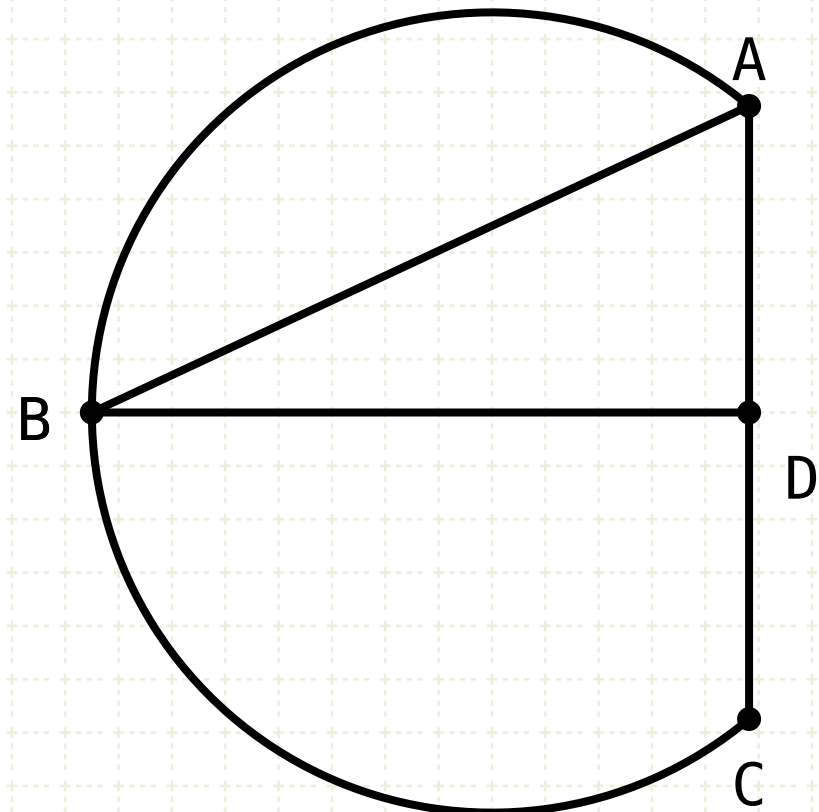
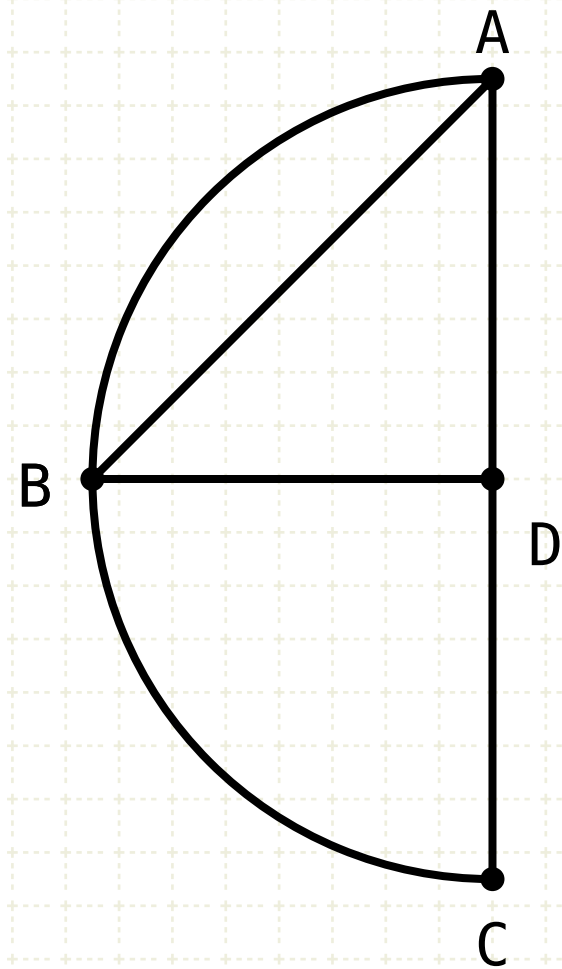
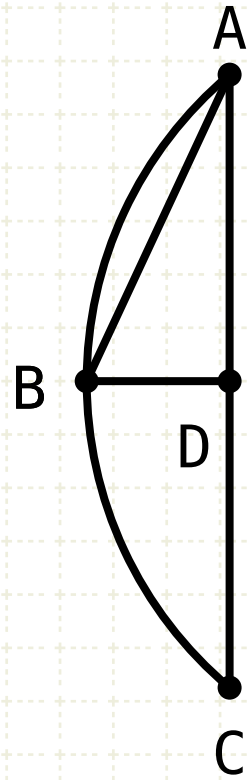
Draw line perpendicular to AC from D, intersecting the circumference at point B

$$AD = DC$$



Proposition 25 of Book III

Given a segment of a circle, to describe the complete circle of which it is a segment.



$$AD = DC$$

Construction

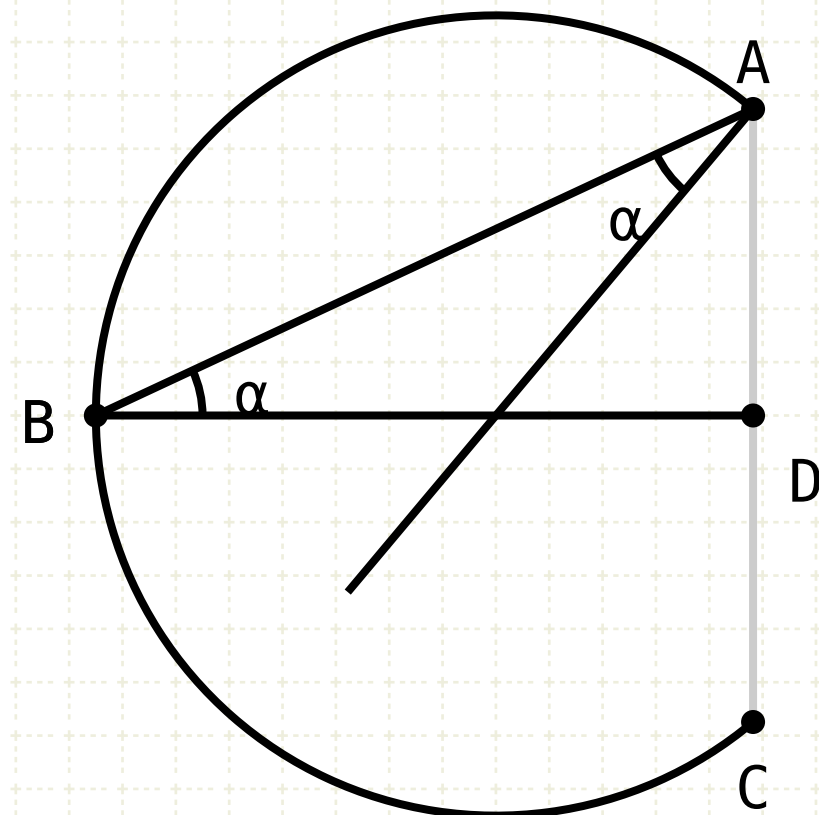
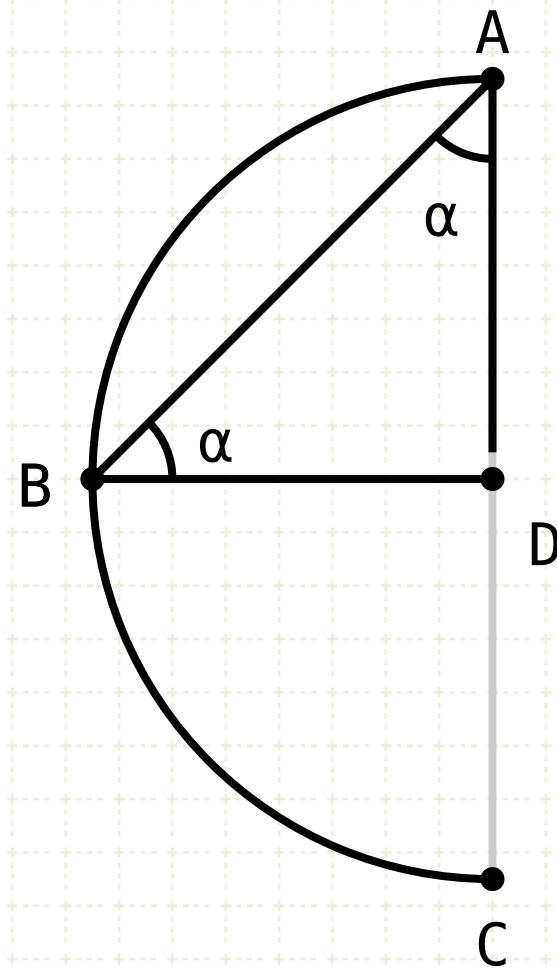
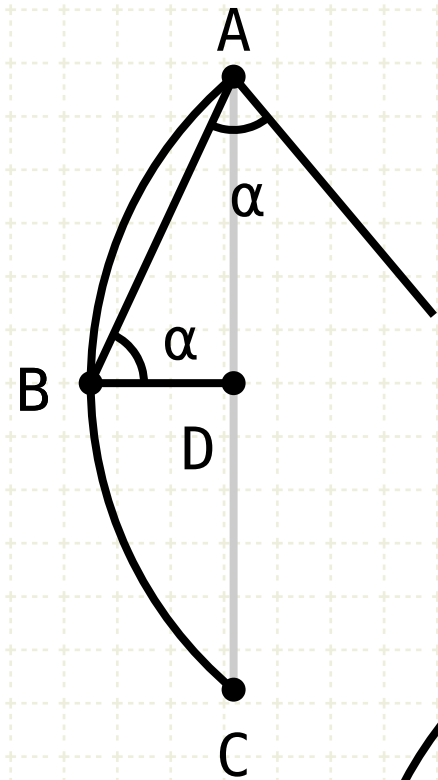
Bisect line AC at point D

Draw line perpendicular to AC from D, intersecting the circumference at point B

Join points A and B

Proposition 25 of Book III

Given a segment of a circle, to describe the complete circle of which it is a segment.



$$AD = DC$$

$$\angle BAE = \angle ABD$$

Construction

Bisect line AC at point D

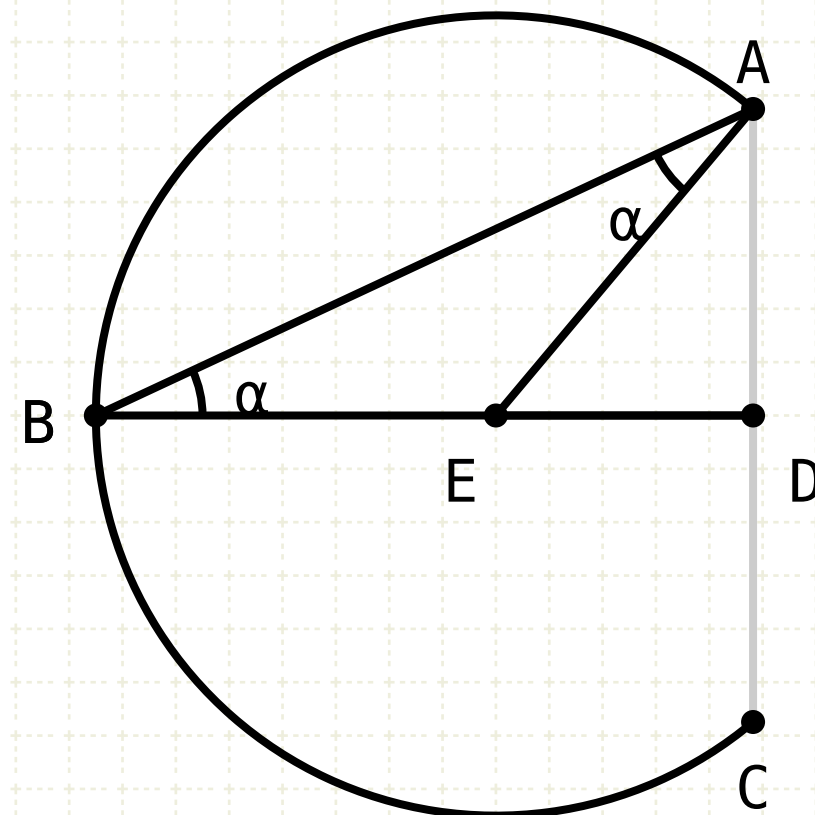
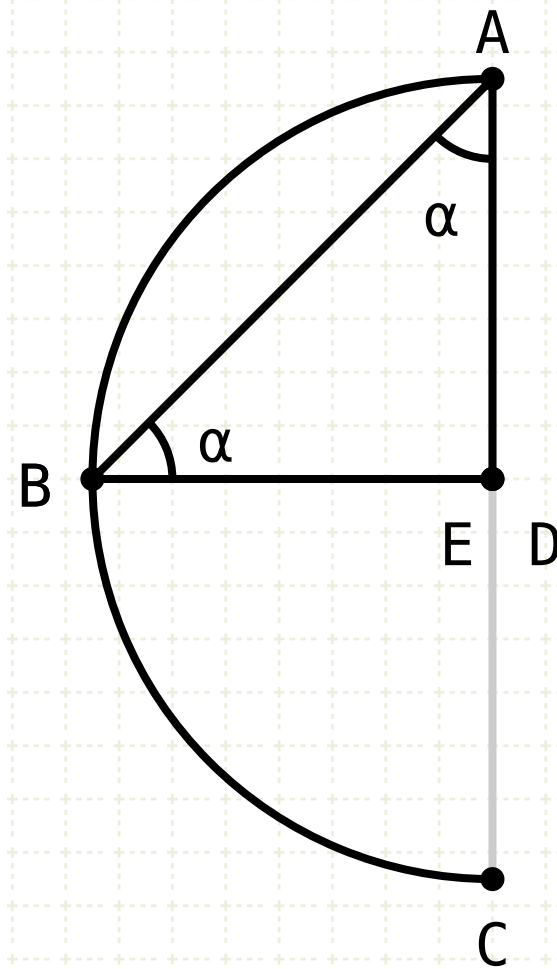
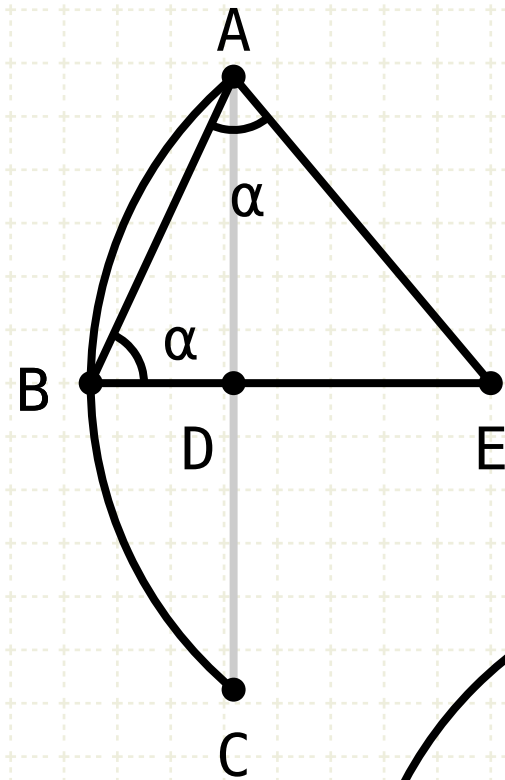
Draw line perpendicular to AC from D, intersecting the circumference at point B

Join points A and B

Construct an angle BAE equal to the angle ABD

Proposition 25 of Book III

Given a segment of a circle, to describe the complete circle of which it is a segment.



$$AD = DC$$
$$\angle BAE = \angle ABD$$

Construction

Bisect line AC at point D

Draw line perpendicular to AC from D, intersecting the circumference at point B

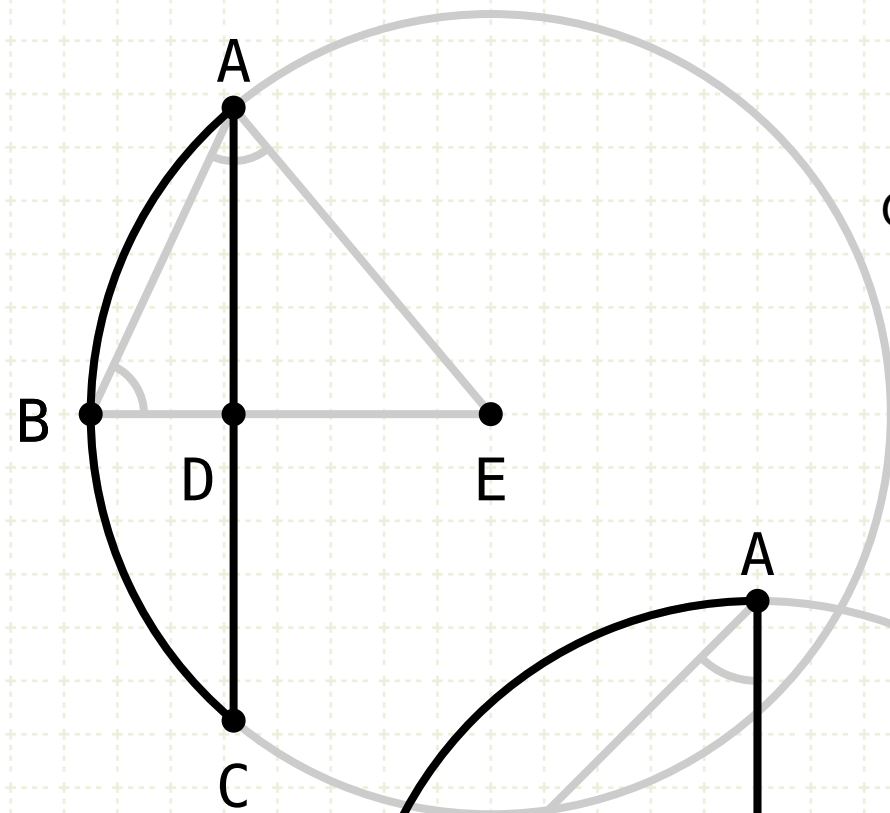
Join points A and B

Construct an angle BAE equal to the angle ABD

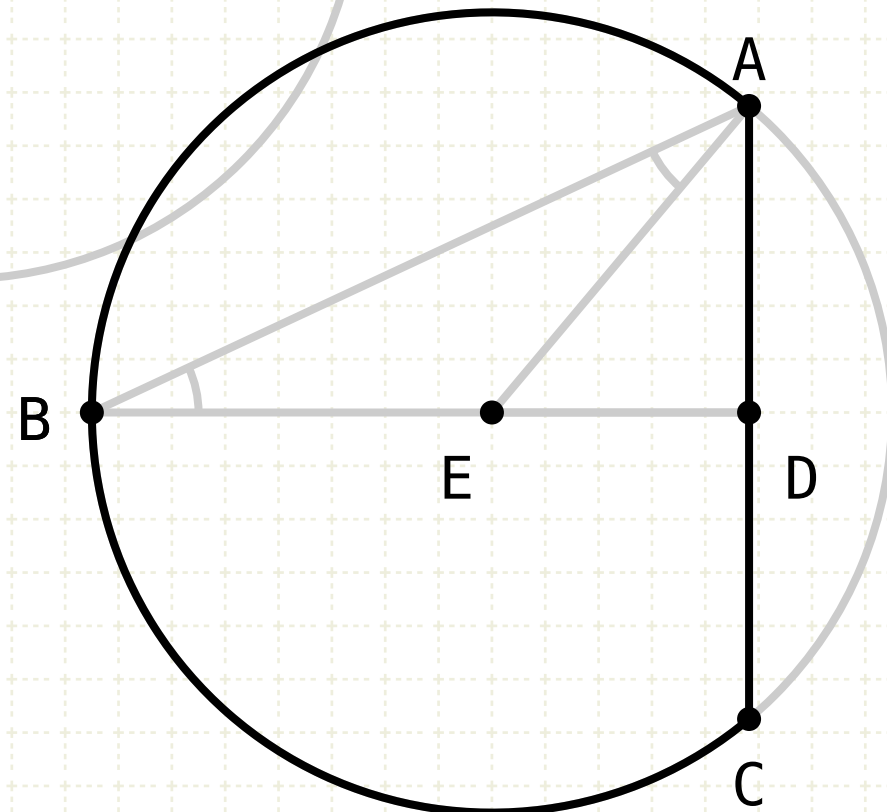
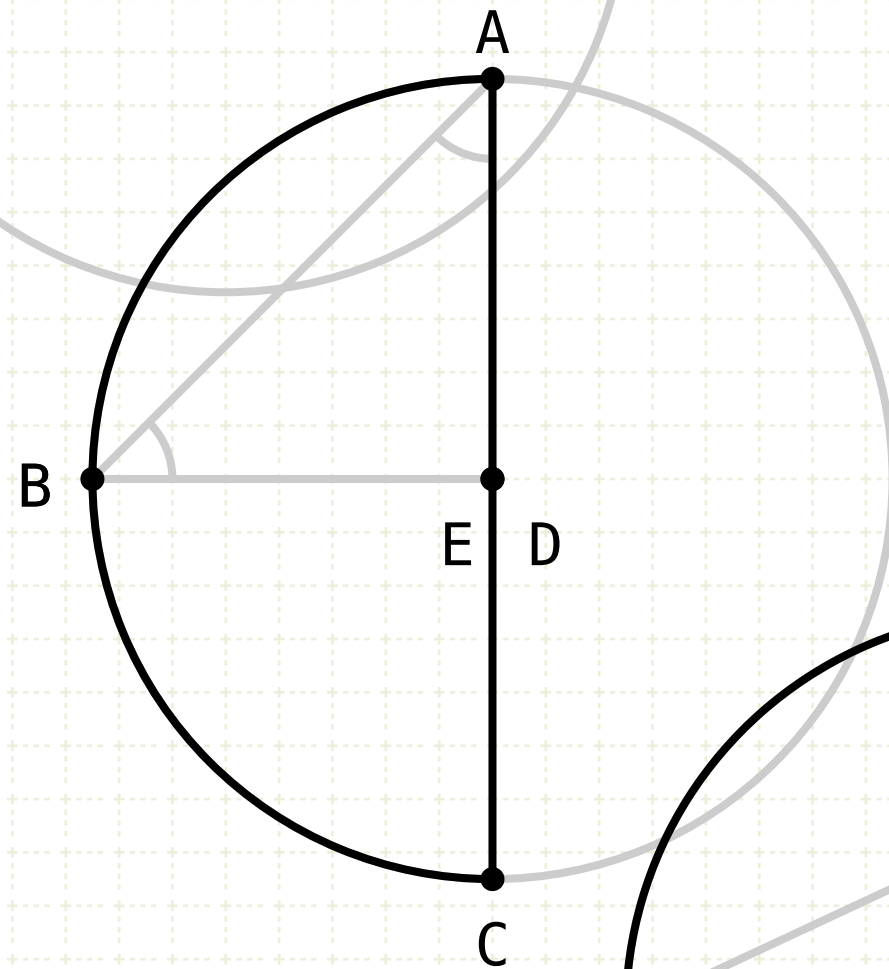
Extend the angled line and the line BD until they meet at the point E

Proposition 25 of Book III

Given a segment of a circle, to describe the complete circle of which it is a segment.



$$AD = DC$$
$$\angle BAE = \angle ABD$$



Construction

Bisect line AC at point D

Draw line perpendicular to AC from D, intersecting the circumference at point B

Join points A and B

Construct an angle BAE equal to the angle ABD

Extend the angled line and the line BD until they meet at the point E

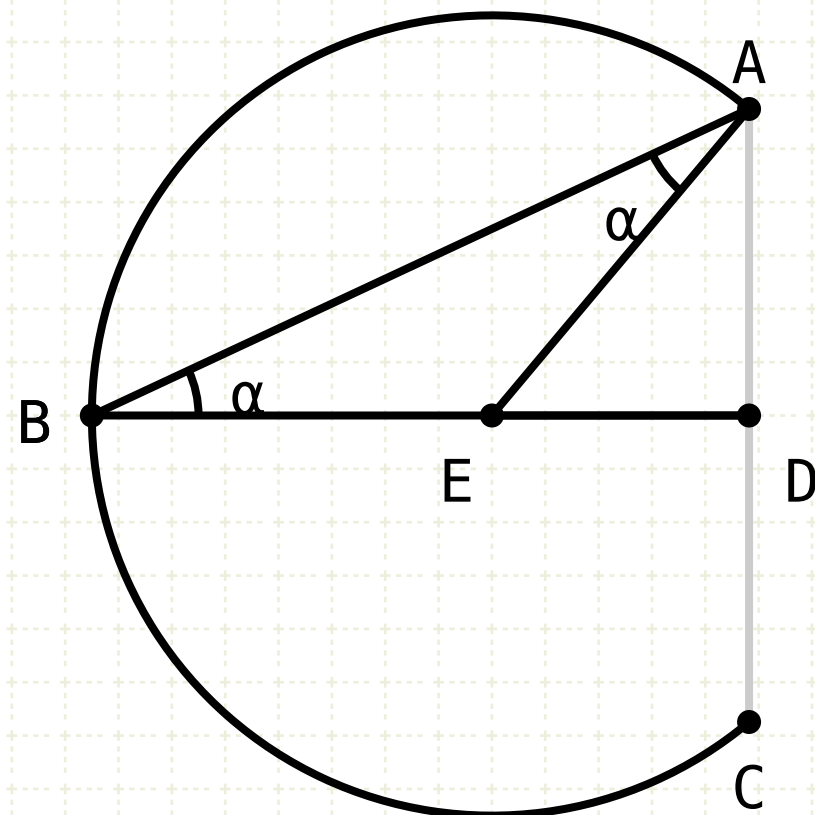
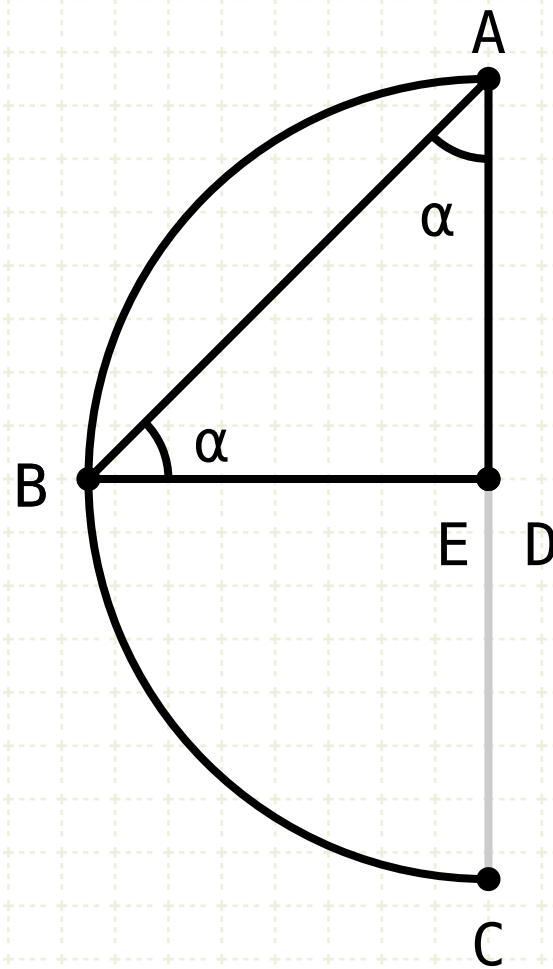
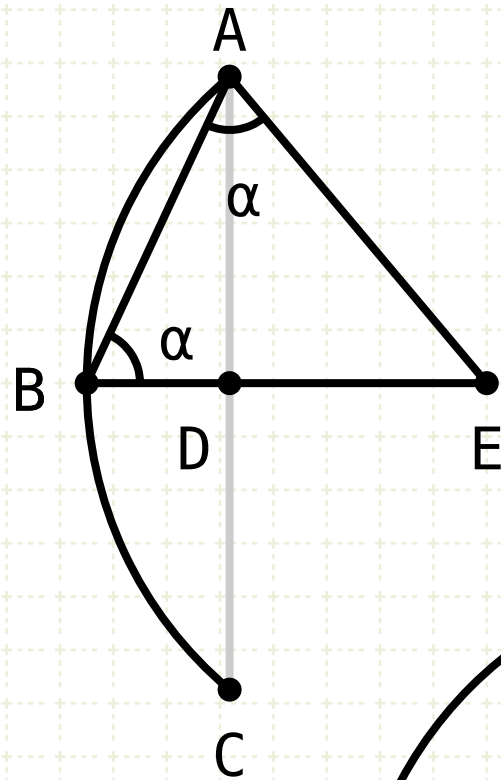
With E as the centre of a circle, and one of the three lines (AE, BE, CE) as radius, the circle will be complete

Proposition 25 of Book III

Given a segment of a circle, to describe the complete circle of which it is a segment.

Proof

$$\begin{aligned}AD &= DC \\ \angle BAE &= \angle ABD\end{aligned}$$



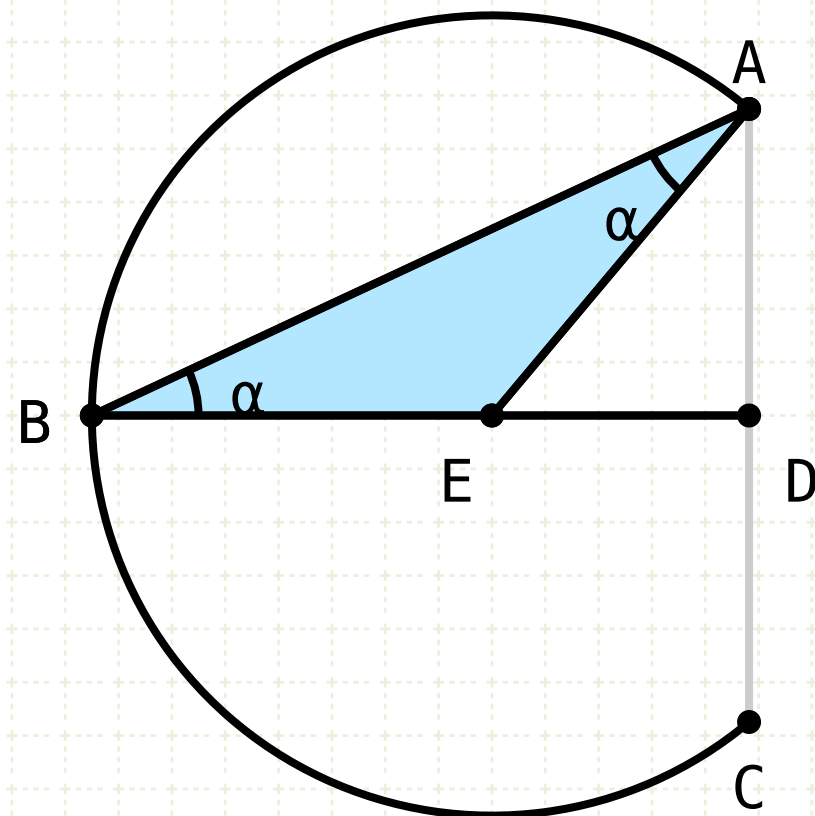
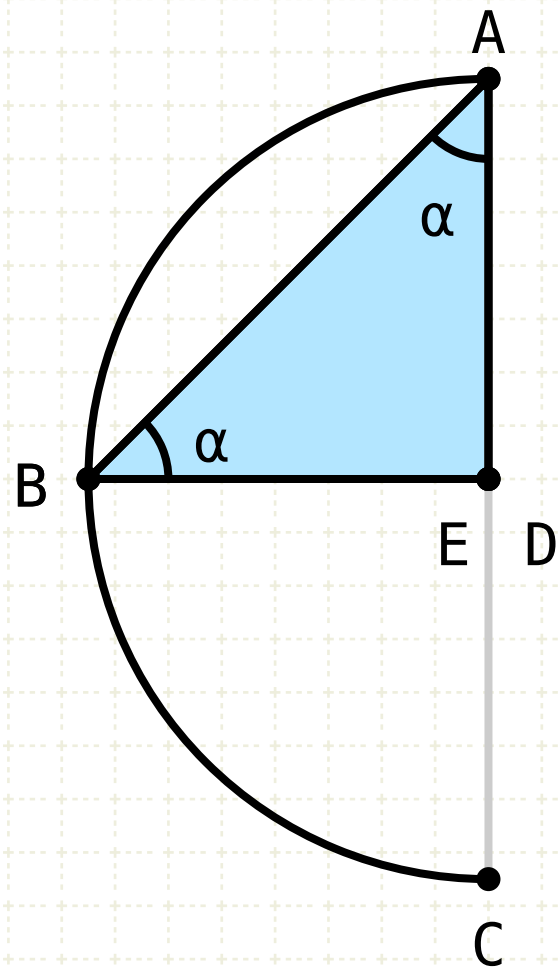
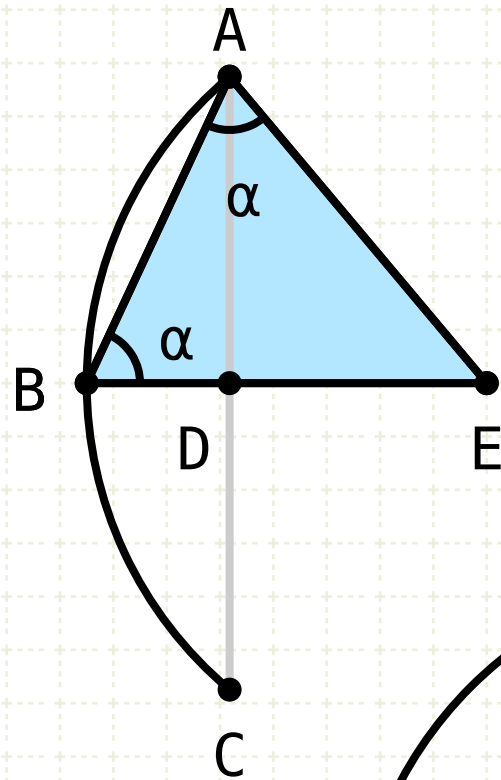
Proposition 25 of Book III

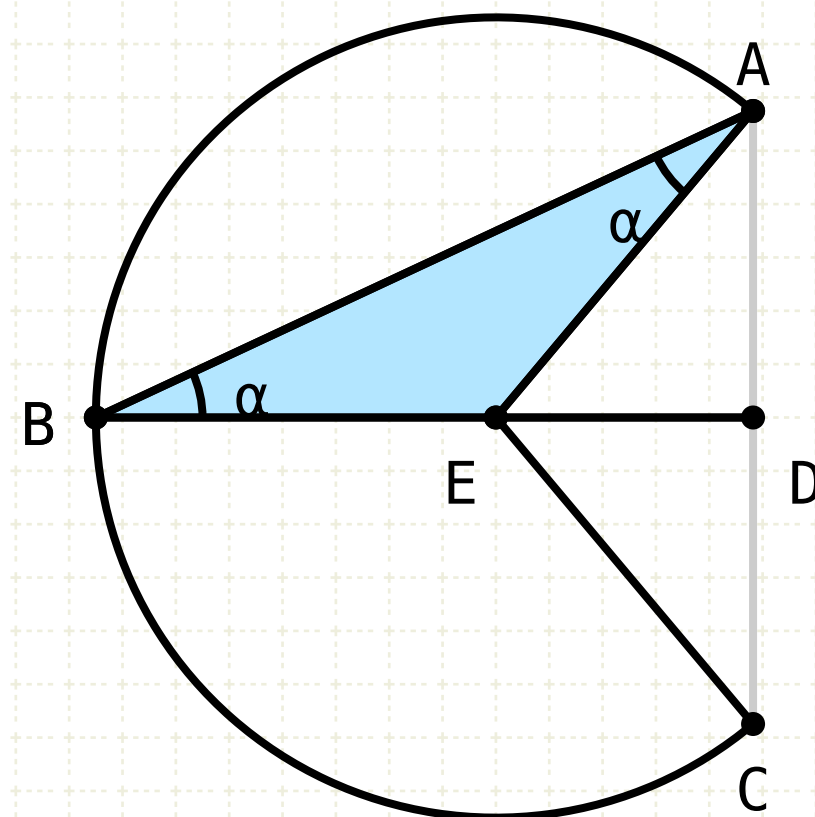
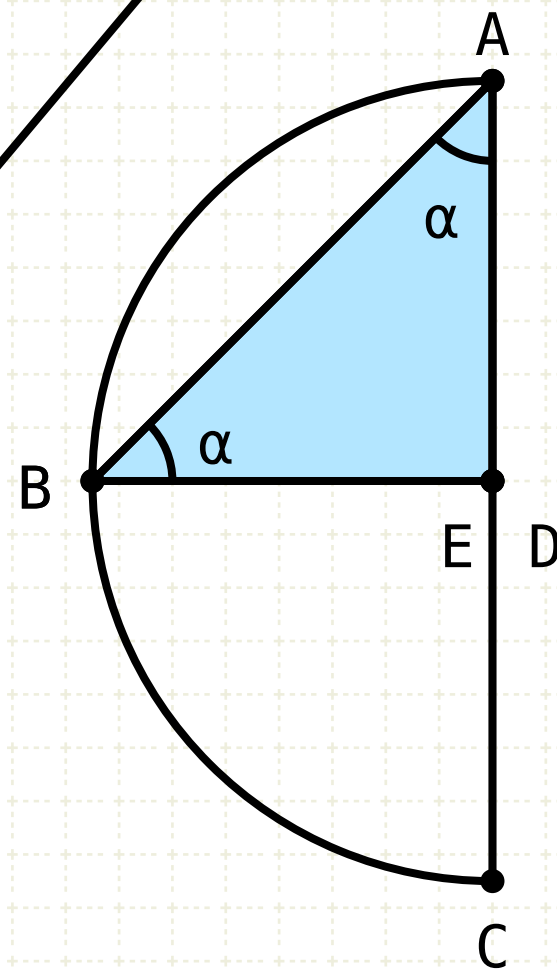
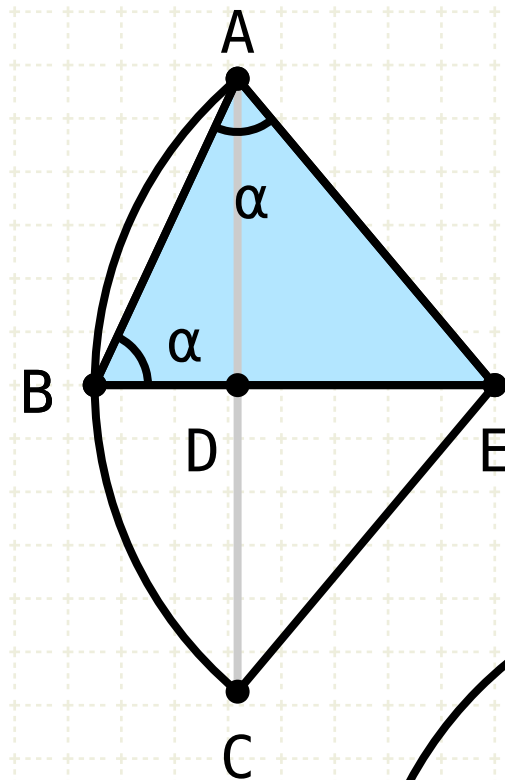
Given a segment of a circle, to describe the complete circle of which it is a segment.

Proof

Since angles EBA and EAB are equal, the triangle is an isosceles, and the lines AE and BE are equal (I-6)

$$\begin{aligned} AD &= DC \\ \angle BAE &= \angle ABD \\ AE &= BE \end{aligned}$$





Proposition 25 of Book III

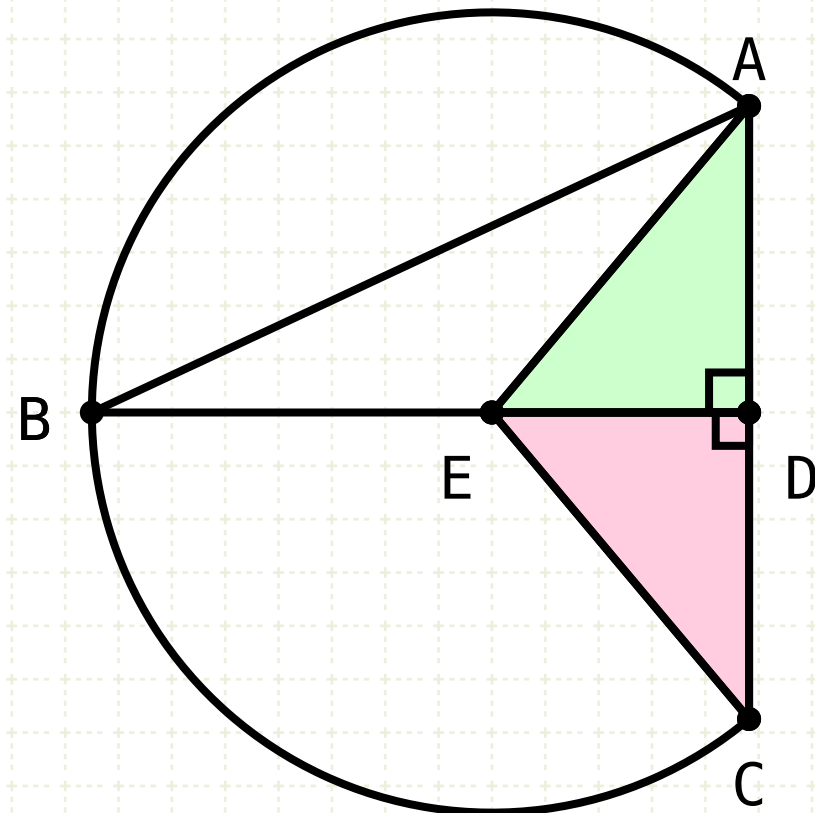
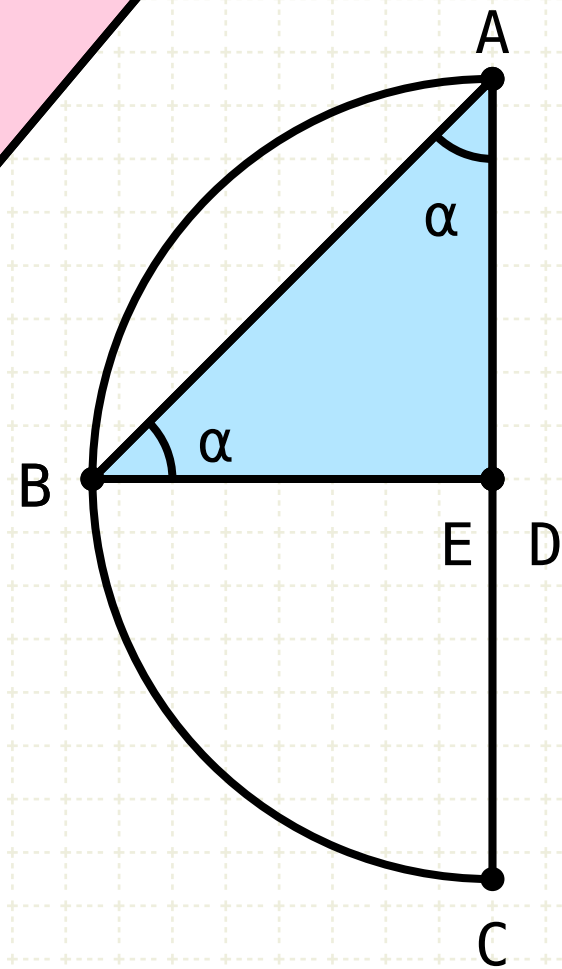
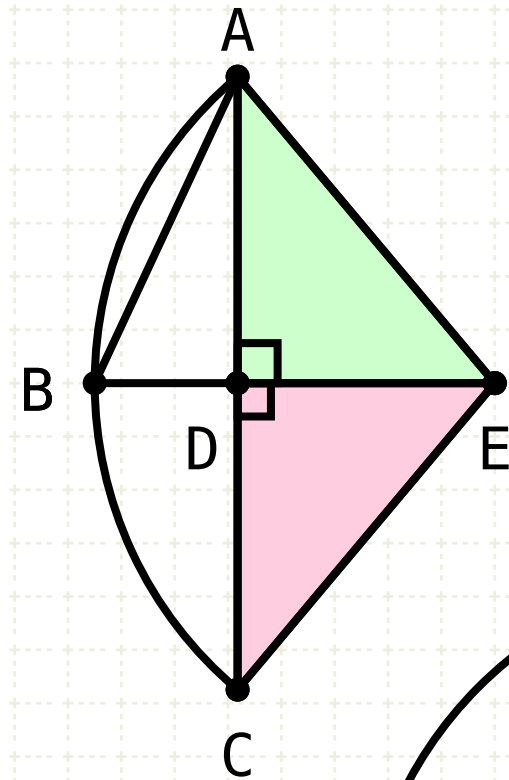
Given a segment of a circle, to describe the complete circle of which it is a segment.

$$\begin{aligned} AD &= DC \\ \angle BAE &= \angle ABD \\ AE &= BE \end{aligned}$$

Proof

Since angles EBA and EAB are equal, the triangle is an isosceles, and the lines AE and BE are equal (I·6)

Draw line CE



Proposition 25 of Book III

Given a segment of a circle, to describe the complete circle of which it is a segment.

$$\begin{aligned} AD &= DC \\ \angle BAE &= \angle ABD \\ AE &= BE \\ AD &= DC \end{aligned}$$

Proof

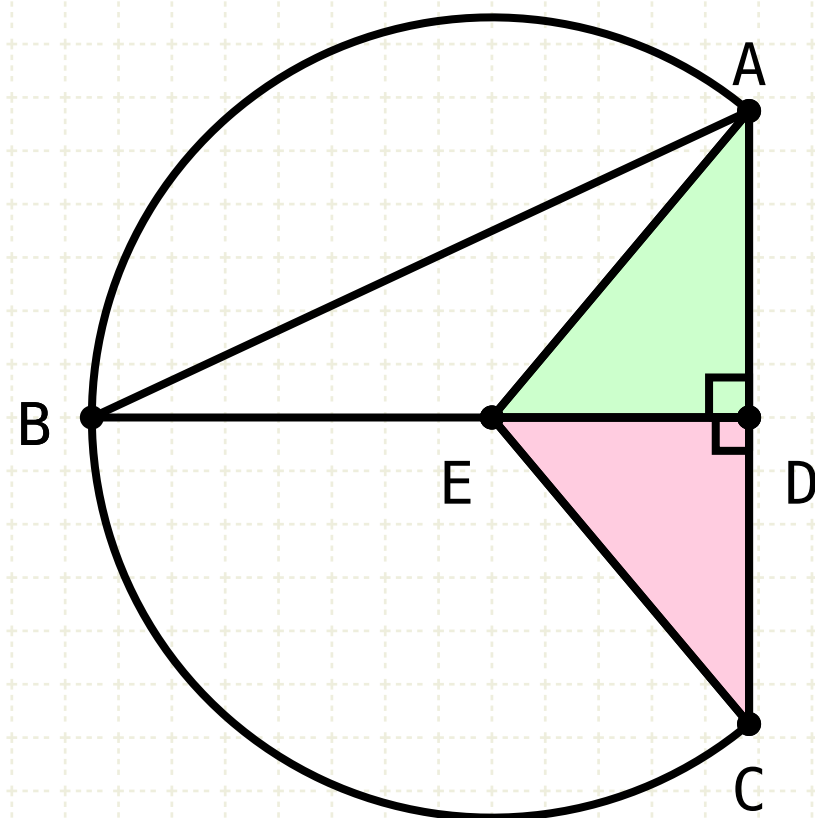
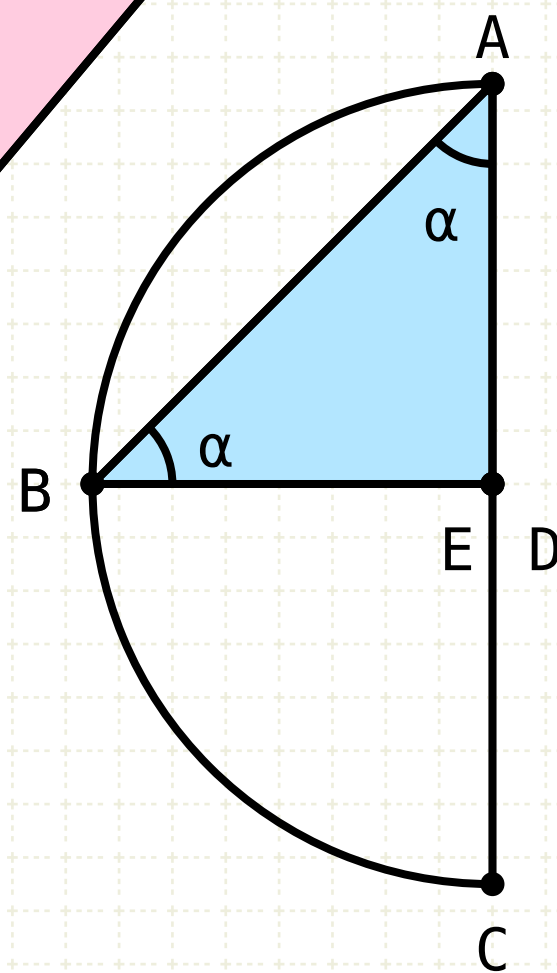
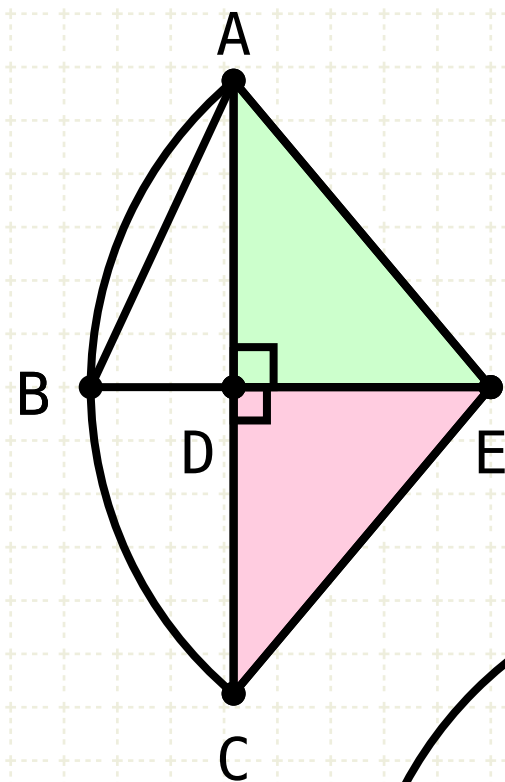
Since angles EBA and EAB are equal, the triangle is an isosceles, and the lines AE and BE are equal (I-6)

Draw line CE

If D and E are not the same point,

Consider triangles ADE and ACE

Two sides are equal, (D bisects AC, and DE is common), and the angles ADE and EDC are equal (BD is perpendicular to AC)



Proposition 25 of Book III

Given a segment of a circle, to describe the complete circle of which it is a segment.

$$\begin{aligned} AD &= DC \\ \angle BAE &= \angle ABD \\ AE &= BE \\ AD &= DC \\ \triangle ADE &\equiv \triangle DEC \\ AE &= EC \end{aligned}$$

Proof

Since angles EBA and EAB are equal, the triangle is an isosceles, and the lines AE and BE are equal (I·6)

Draw line CE

If D and E are not the same point,

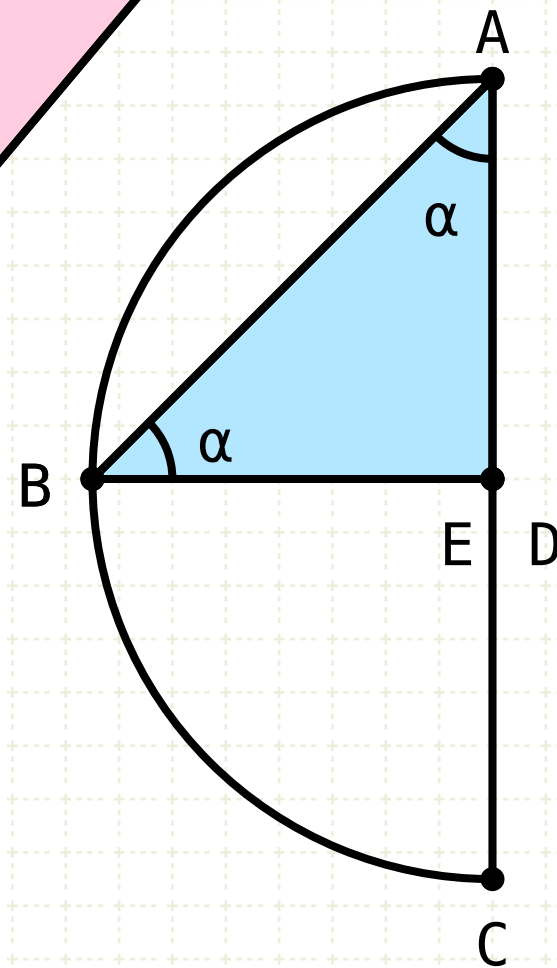
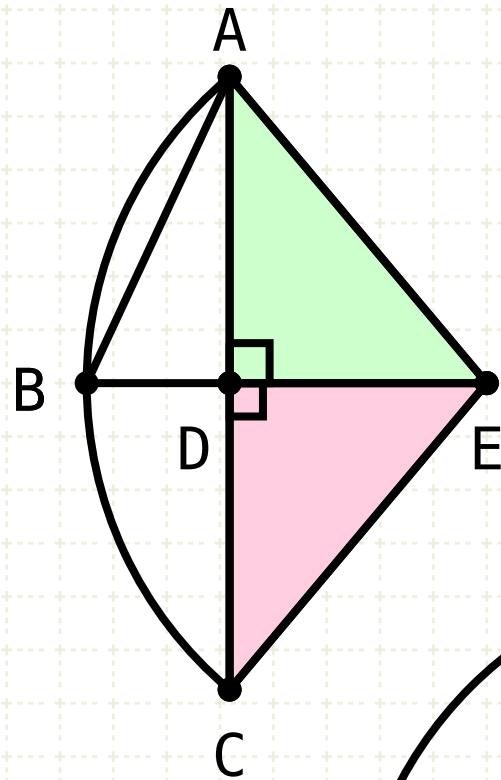
Consider triangles ADE and ACE

Two sides are equal, (D bisects AC, and DE is common), and the angles ADE and EDC are equal (BD is perpendicular to AC)

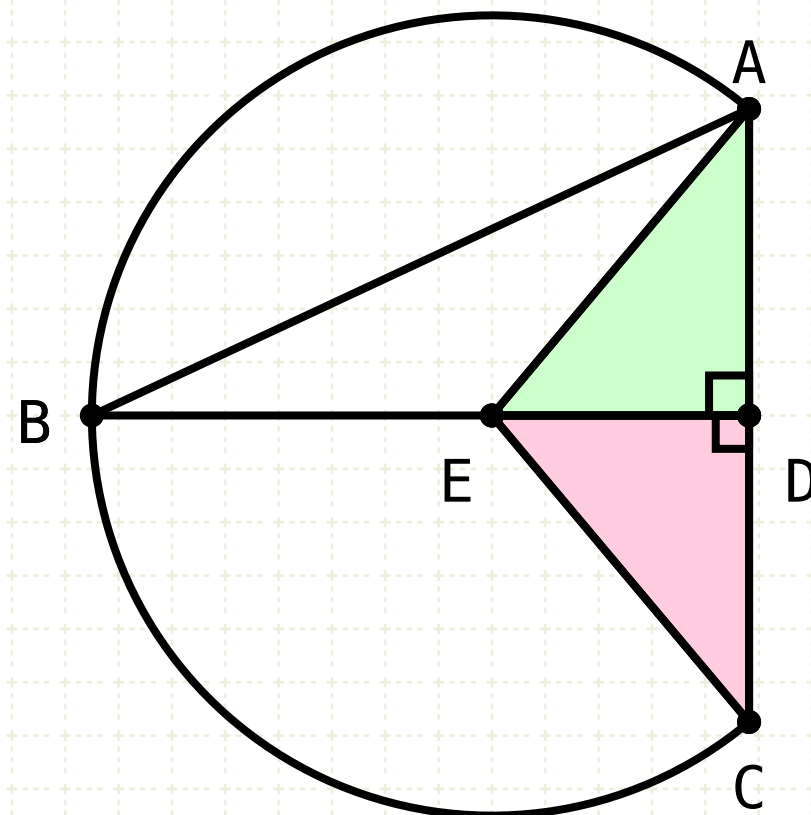
Therefore the triangles ADE and DEC are equal, and AE equals EC (I·4)

Proposition 25 of Book III

Given a segment of a circle, to describe the complete circle of which it is a segment.



$$\begin{aligned} AD &= DC \\ \angle BAE &= \angle ABD \\ AE &= BE \\ AD &= DC \\ \triangle ADE &\equiv \triangle DEC \\ AE &= EC \end{aligned}$$



Proof

Since angles EBA and EAB are equal, the triangle is an isosceles, and the lines AE and BE are equal (I·6)

Draw line CE

If D and E are not the same point,

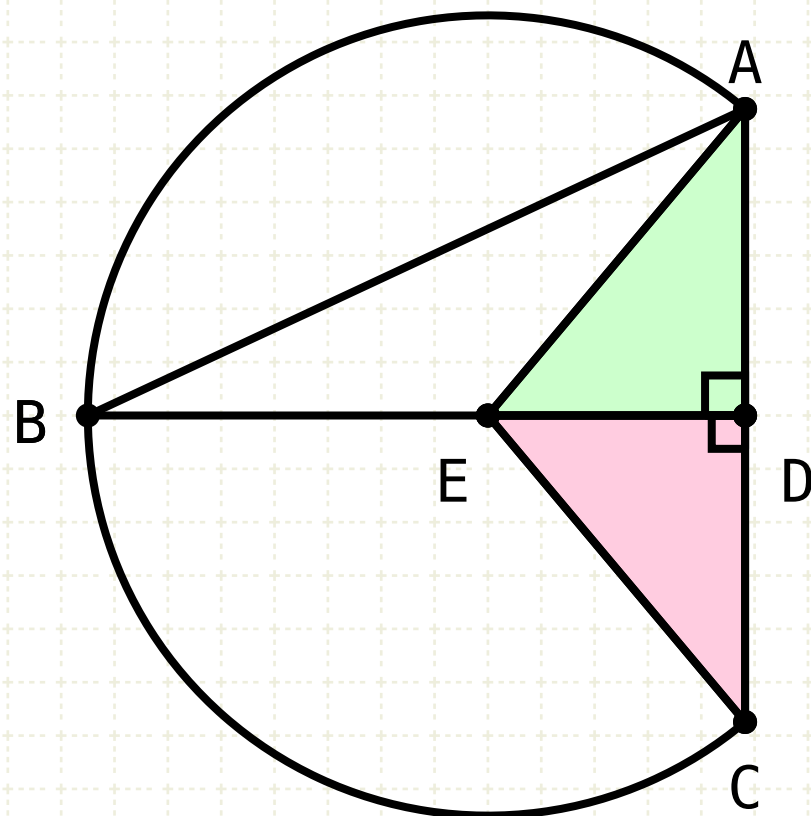
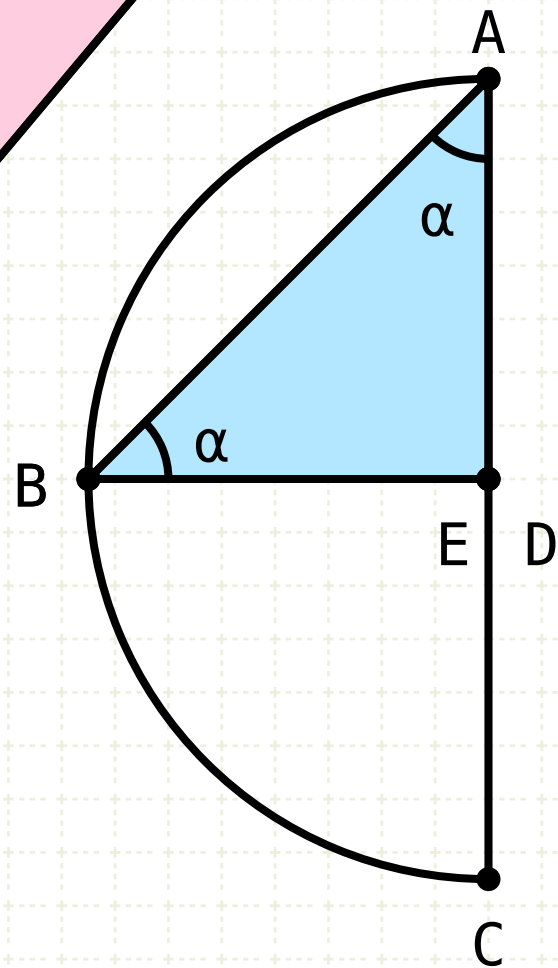
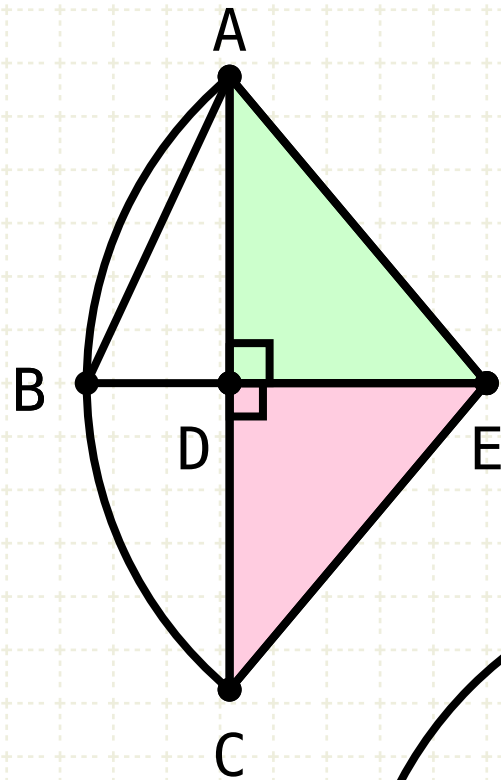
Consider triangles ADE and ACE

Two sides are equal, (D bisects AC, and DE is common), and the angles ADE and EDC are equal (BD is perpendicular to AC)

Therefore the triangles ADE and DEC are equal, and AE equals EC (I·4)

If D and E are the same point,

AE equals AD, and DC equals EC, so AE equals EC



Proposition 25 of Book III

Given a segment of a circle, to describe the complete circle of which it is a segment.

$$\begin{aligned} AD &= DC \\ \angle BAE &= \angle ABD \\ AE &= BE \\ AD &= DC \\ \triangle ADE &\equiv \triangle DEC \\ AE &= EC \end{aligned}$$

Proof

Since angles EBA and EAB are equal, the triangle is an isosceles, and the lines AE and BE are equal (I·6)

Draw line CE

If D and E are not the same point,

Consider triangles ADE and ACE

Two sides are equal, (D bisects AC, and DE is common), and the angles ADE and EDC are equal (BD is perpendicular to AC)

Therefore the triangles ADE and DEC are equal, and AE equals EC (I·4)

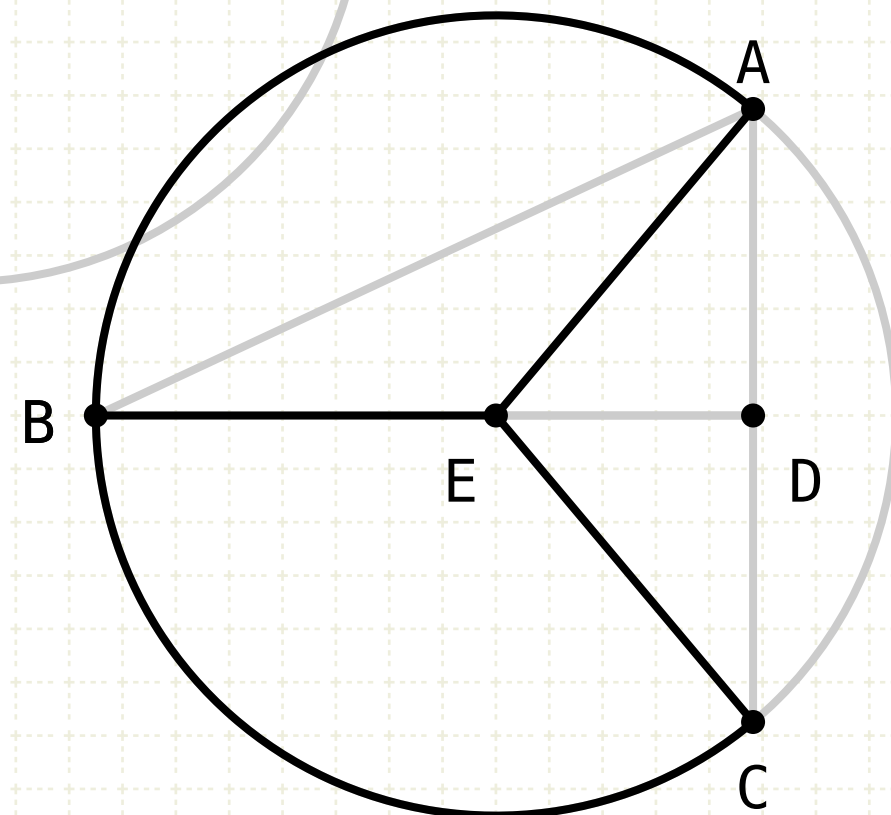
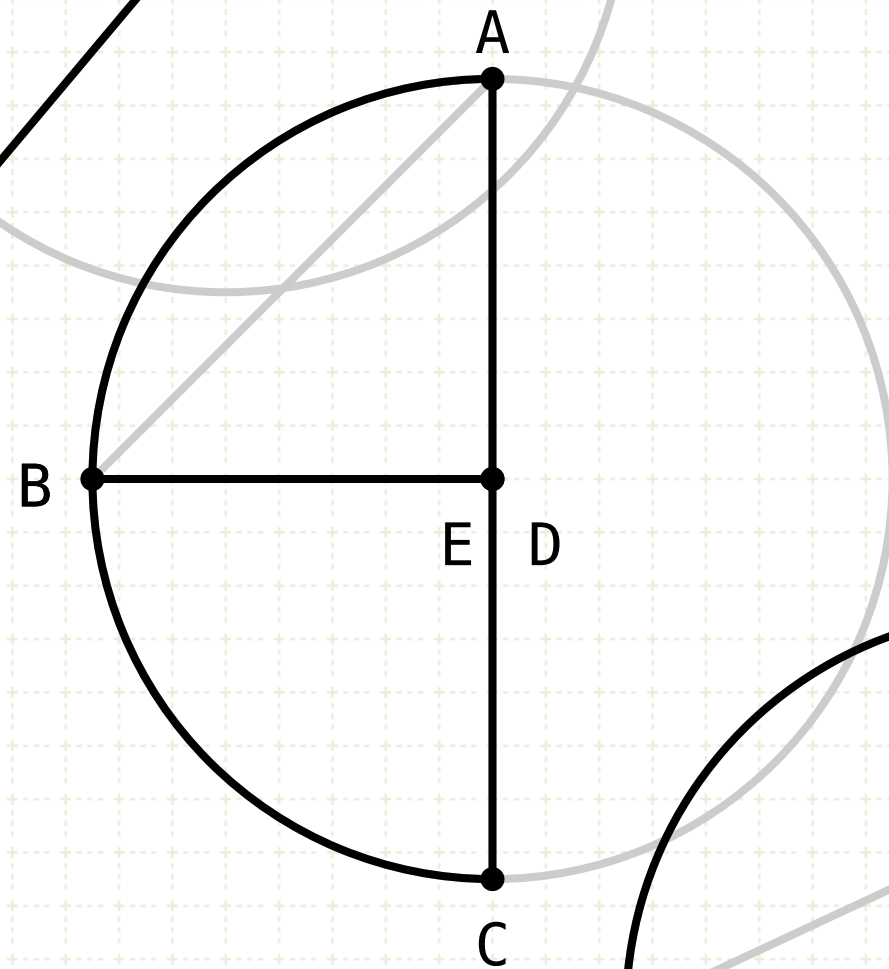
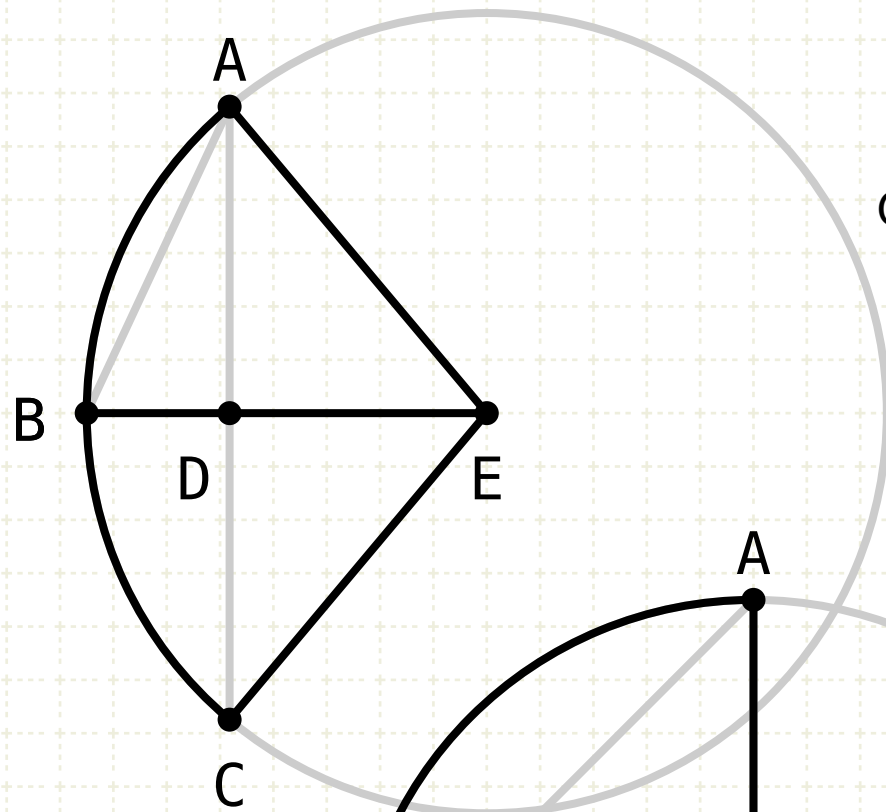
If D and E are the same point,

AE equals AD, and DC equals EC, so AE equals EC

The three lines AE, BE, and CE are equal

Proposition 25 of Book III

Given a segment of a circle, to describe the complete circle of which it is a segment.



$$\begin{aligned} AD &= DC \\ \angle BAE &= \angle ABD \\ AE &= BE \\ AD &= DC \\ \triangle ADE &\equiv \triangle DEC \\ AE &= EC \end{aligned}$$

Proof

Since angles EBA and EAB are equal, the triangle is an isosceles, and the lines AE and BE are equal (I·6)

Draw line CE

If D and E are not the same point,

Consider triangles ADE and ACE

Two sides are equal, (D bisects AC, and DE is common), and the angles ADE and EDC are equal (BD is perpendicular to AC)

Therefore the triangles ADE and DEC are equal, and AE equals EC (I·4)

If D and E are the same point,

AE equals AD, and DC equals EC, so AE equals EC

The three lines AE, BE, and CE are equal

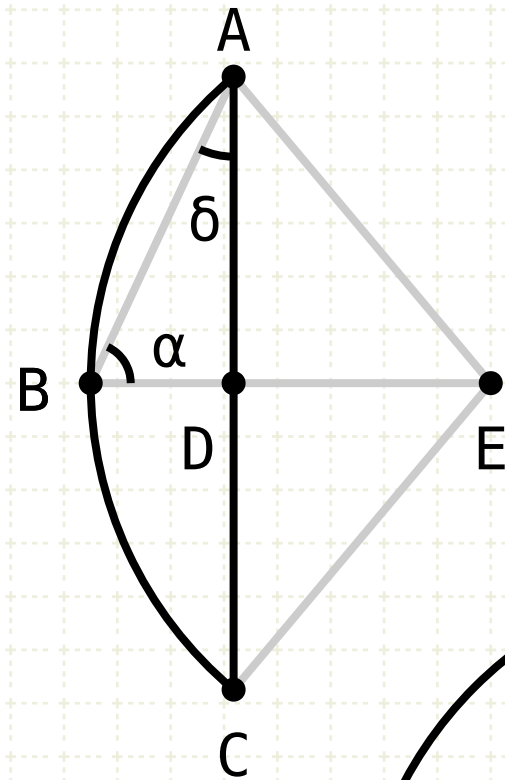
If more than two EQUAL lines fall from a point within a circle to the circumference of a circle, then that point is the centre (III·9)

Therefore E is the centre of the circle, and the radius is AE

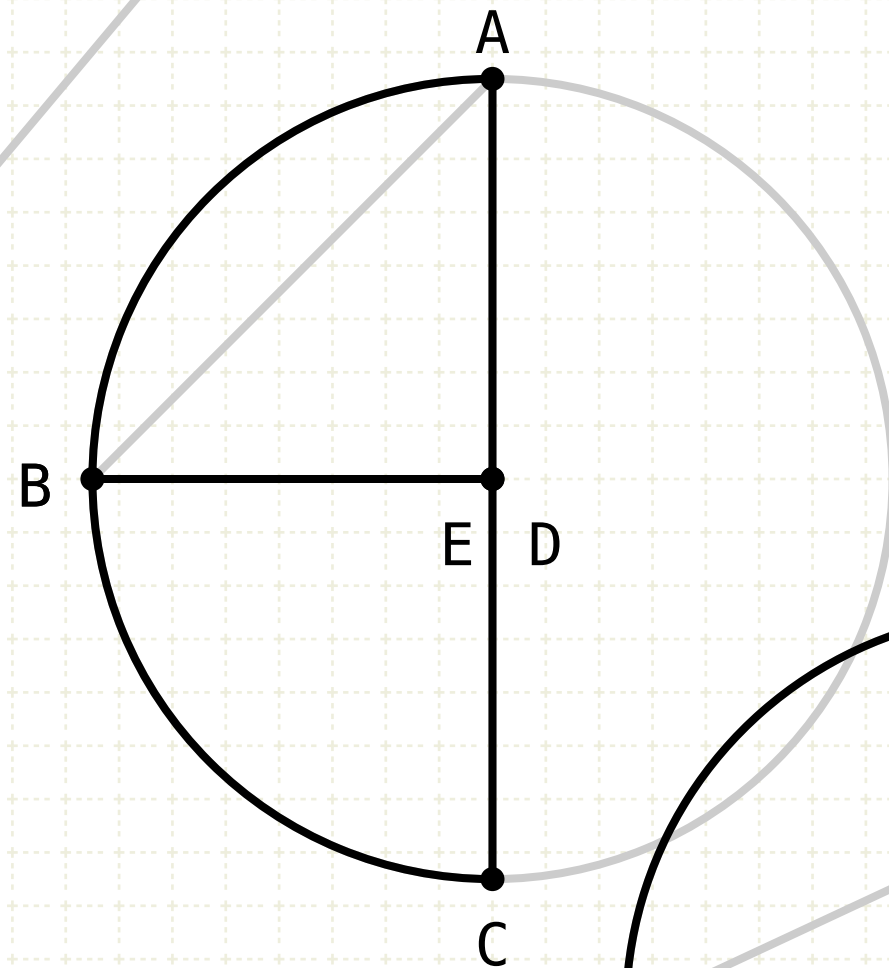


Proposition 25 of Book III

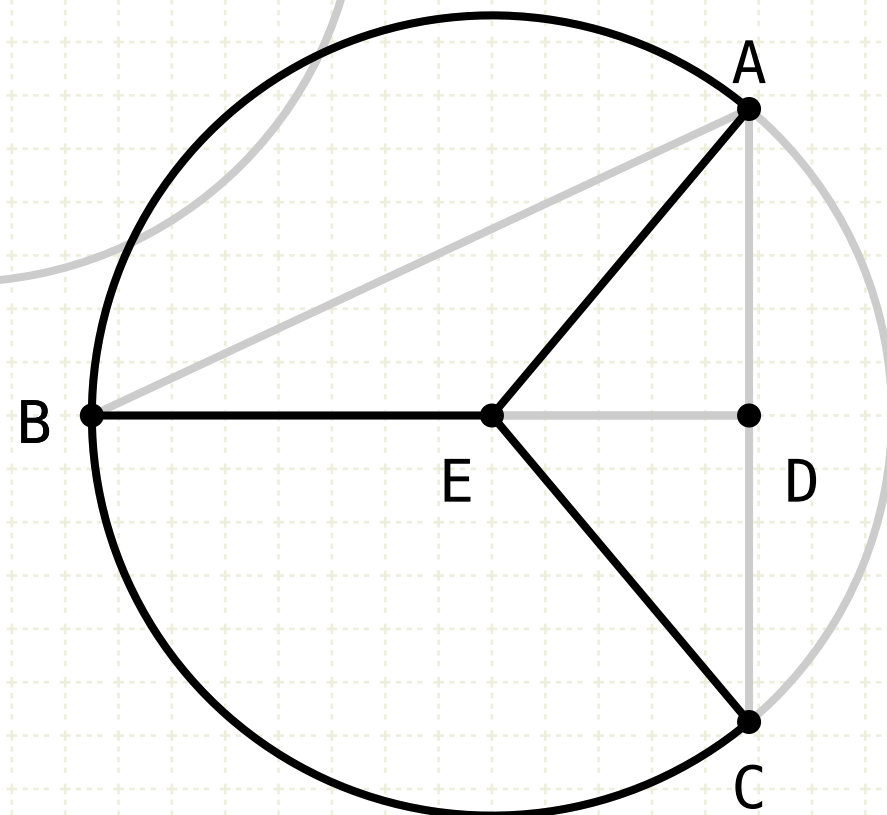
Given a segment of a circle, to describe the complete circle of which it is a segment.



If $\alpha > \delta$
then the segment is less than a semicircle



$$\begin{aligned} AD &= DC \\ \angle BAE &= \angle ABD \\ AE &= BE \\ AD &= DC \\ \triangle ADE &\equiv \triangle DEC \\ AE &= EC \end{aligned}$$



Proof

Since angles EBA and EAB are equal, the triangle is an isosceles, and the lines AE and BE are equal (I·6)

Draw line CE

If D and E are not the same point,

Consider triangles ADE and ACE

Two sides are equal, (D bisects AC, and DE is common), and the angles ADE and EDC are equal (BD is perpendicular to AC)

Therefore the triangles ADE and DEC are equal, and AE equals EC (I·4)

If D and E are the same point,

AE equals AD, and DC equals EC, so AE equals EC

The three lines AE, BE, and CE are equal

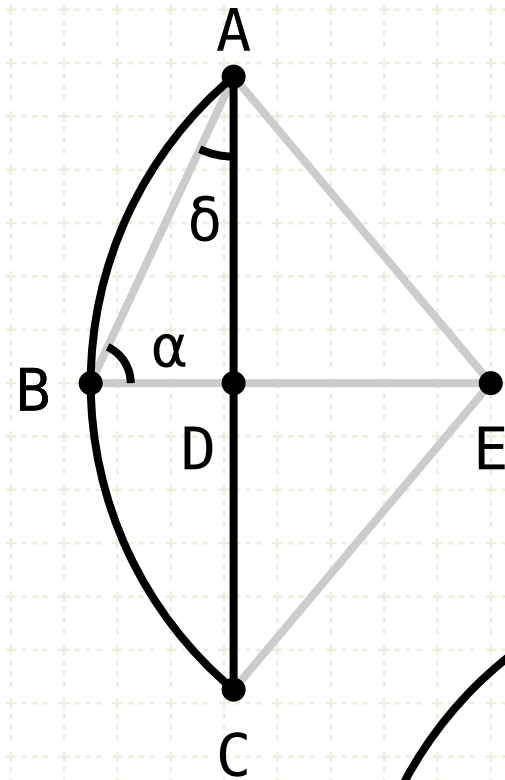
If more than two EQUAL lines fall from a point within a circle to the circumference of a circle, then that point is the centre (III·9)

Therefore E is the centre of the circle, and the radius is AE

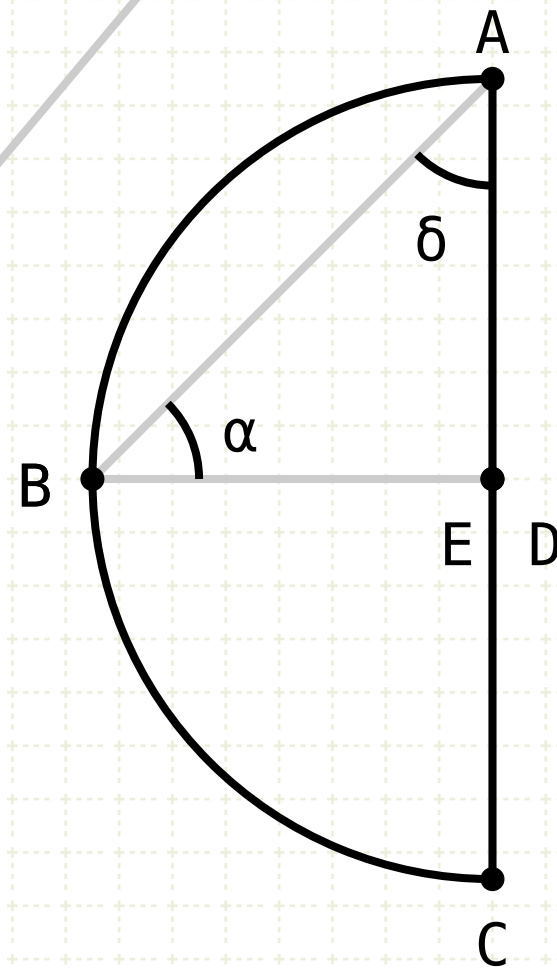


Proposition 25 of Book III

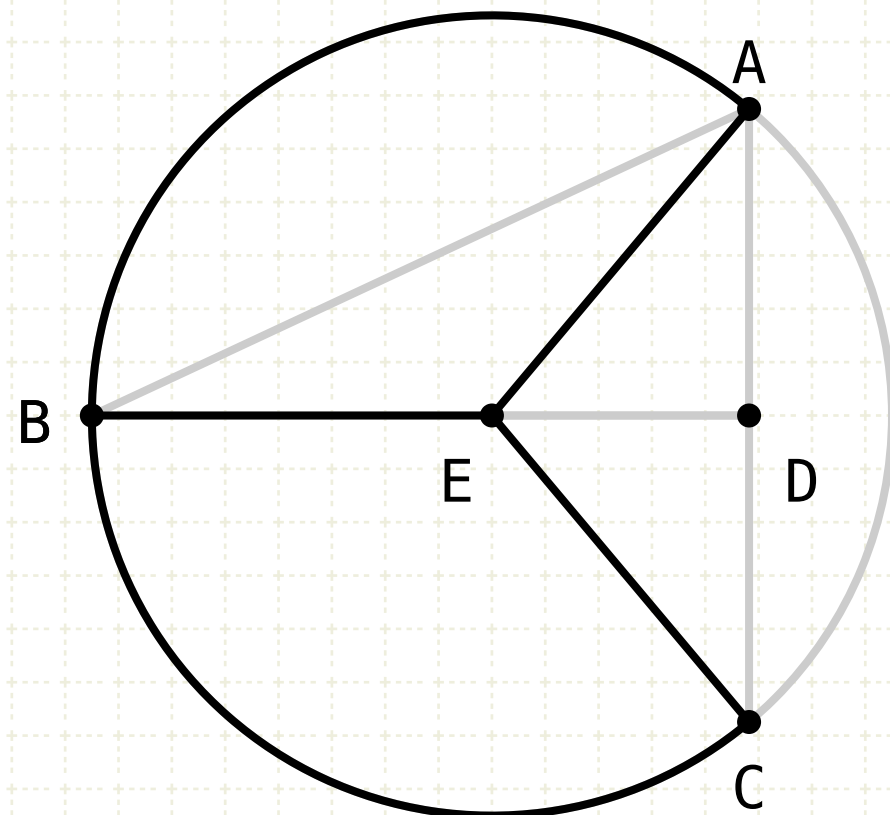
Given a segment of a circle, to describe the complete circle of which it is a segment.



If $\alpha > \delta$
then the segment is less than a semicircle



If $\alpha = \delta$
then the segment is a semicircle



$$\begin{aligned} AD &= DC \\ \angle BAE &= \angle ABD \\ AE &= BE \\ AD &= DC \\ \triangle ADE &\equiv \triangle DEC \\ AE &= EC \end{aligned}$$

Proof

Since angles EBA and EAB are equal, the triangle is an isosceles, and the lines AE and BE are equal (I·6)

Draw line CE

If D and E are not the same point,

Consider triangles ADE and ACE

Two sides are equal, (D bisects AC, and DE is common), and the angles ADE and EDC are equal (BD is perpendicular to AC)

Therefore the triangles ADE and DEC are equal, and AE equals EC (I·4)

If D and E are the same point,

AE equals AD, and DC equals EC, so AE equals EC

The three lines AE, BE, and CE are equal

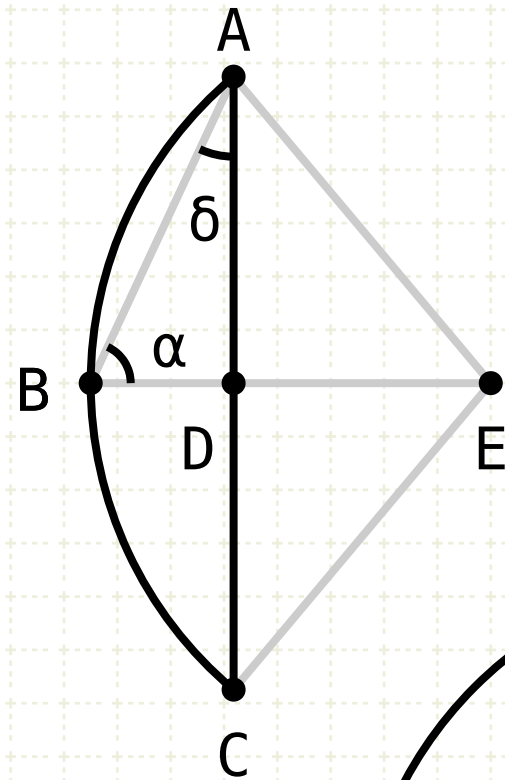
If more than two EQUAL lines fall from a point within a circle to the circumference of a circle, then that point is the centre (III·9)

Therefore E is the centre of the circle, and the radius is AE

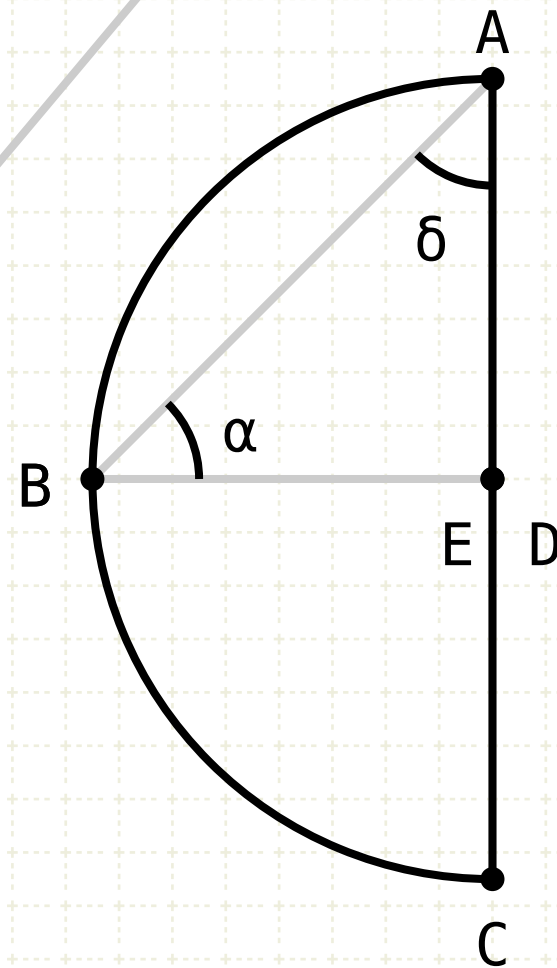


Proposition 25 of Book III

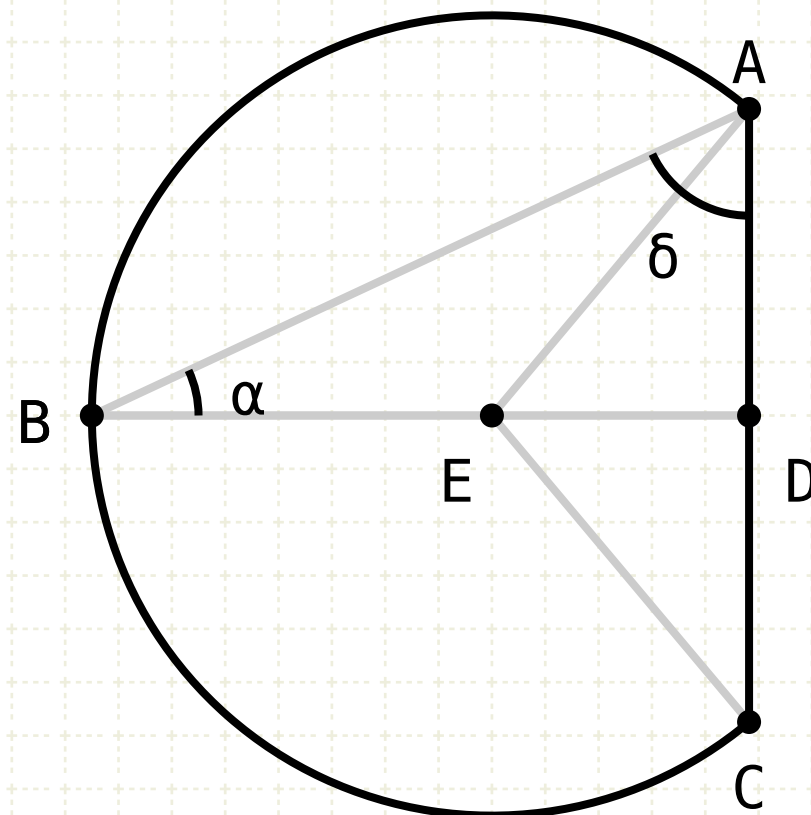
Given a segment of a circle, to describe the complete circle of which it is a segment.



If $\alpha > \delta$
then the segment is less than a
semicircle



If $\alpha = \delta$
then the segment is a semicircle



If $\alpha < \delta$
then the segment is larger than a semicircle

$$\begin{aligned} AD &= DC \\ \angle BAE &= \angle ABD \\ AE &= BE \\ AD &= DC \\ \triangle ADE &\equiv \triangle DEC \\ AE &= EC \end{aligned}$$

Proof

Since angles EBA and EAB are equal, the triangle is an isosceles, and the lines AE and BE are equal (I·6)

Draw line CE

If D and E are not the same point,

Consider triangles ADE and ACE

Two sides are equal, (D bisects AC, and DE is common), and the angles ADE and EDC are equal (BD is perpendicular to AC)

Therefore the triangles ADE and DEC are equal, and AE equals EC (I·4)

If D and E are the same point,

AE equals AD, and DC equals EC, so AE equals EC

The three lines AE, BE, and CE are equal

If more than two EQUAL lines fall from a point within a circle to the circumference of a circle, then that point is the centre (III·9)

Therefore E is the centre of the circle, and the radius is AE



Youtube Videos

<https://www.youtube.com/c/SandyBultena>

Copyright © 2019 by Sandy Bultena.



Except where otherwise noted, this work is licensed under
<http://creativecommons.org/licenses/by-nc/3.0>