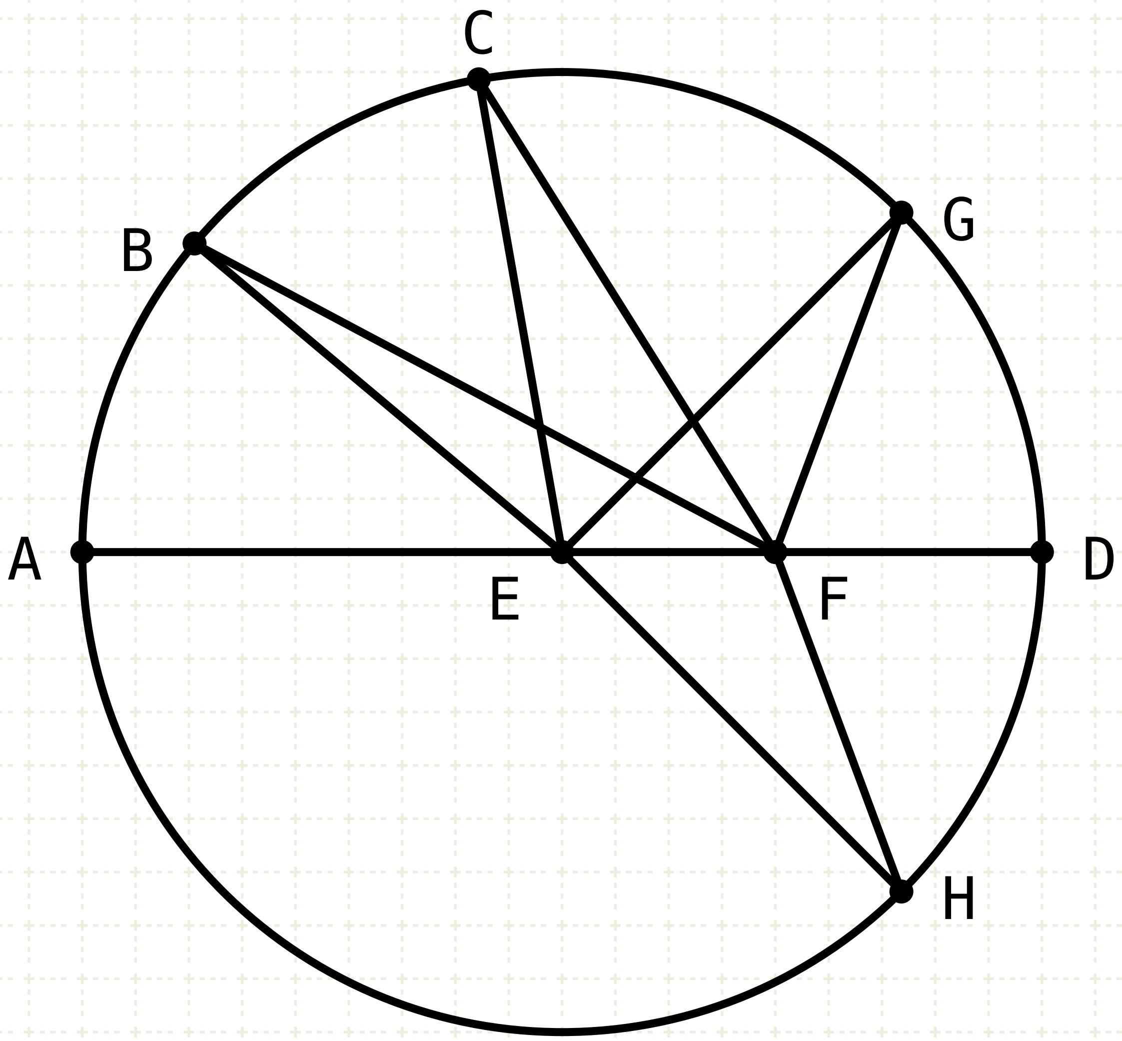
Euclid's Elements

Book III



A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)

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Proposition 22 of Book III

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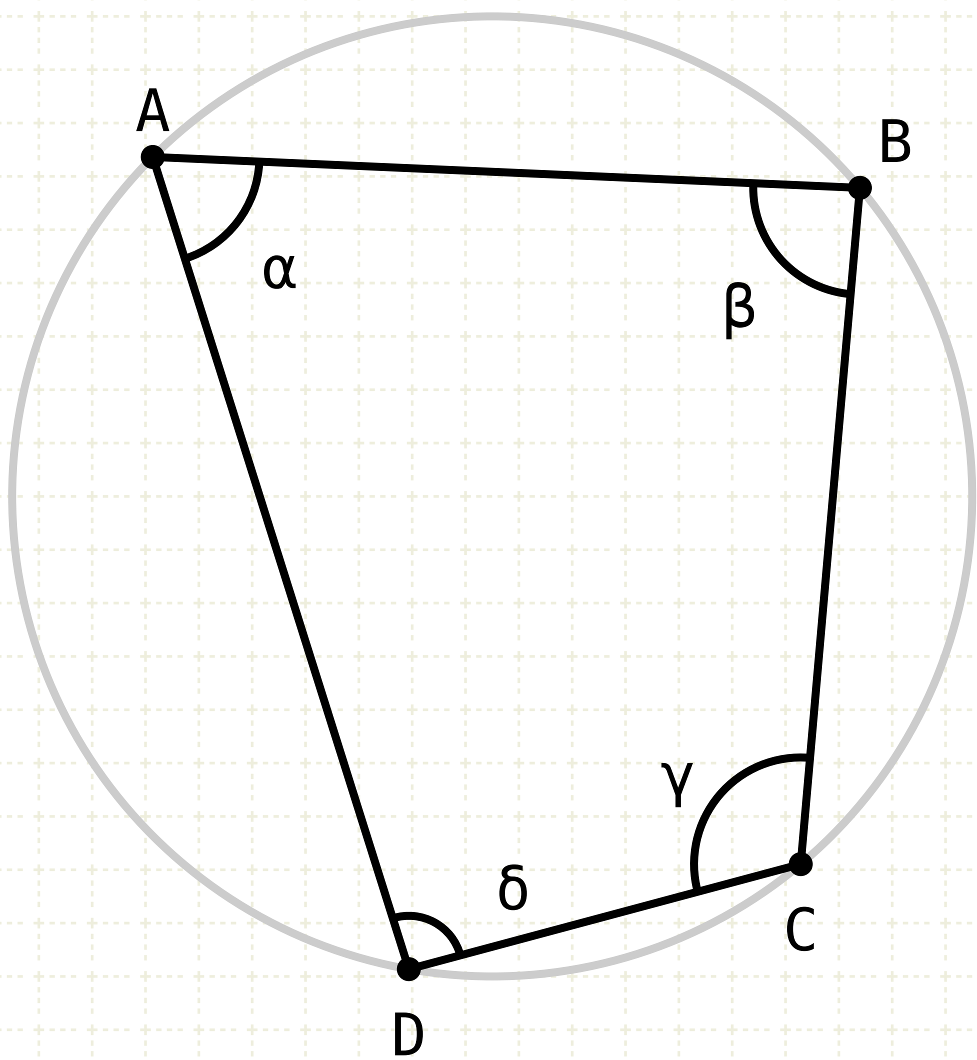


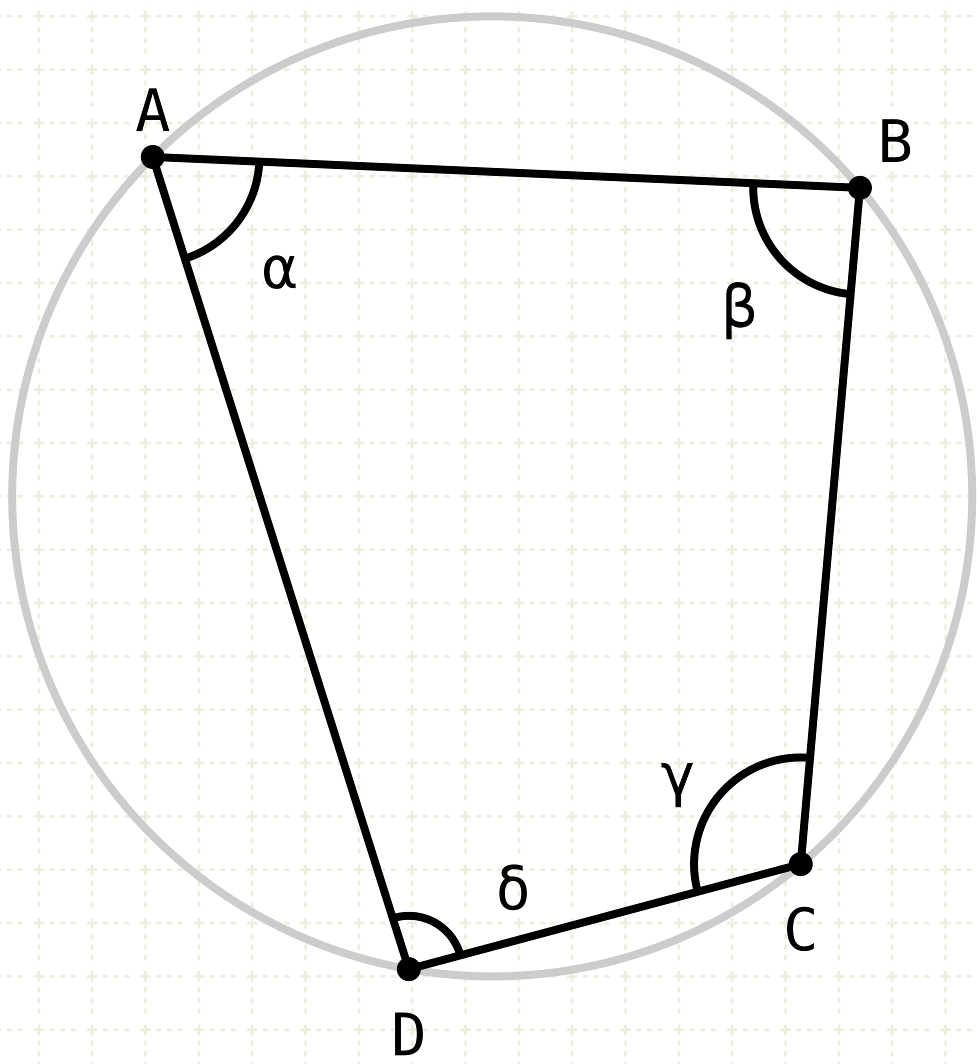
$$\alpha + \gamma = 2\llcorner$$
$$\delta + \beta = 2\llcorner$$

In other words

If a quadilateral ABCD is drawn within a circle, then the sum of the angles at A and C (α and γ) equals two right angles, similarly, the angles at B and D (β and δ) sum to two right angles

Proposition 22 of Book III

The opposite angles of quadrilaterals in circles are equal to two right angles.



$\angle A = \alpha$, $\angle B = \beta$,
$\angle C = \gamma$, $\angle D = \delta$

In other words

If a quadilateral ABCD is drawn within a circle, then the sum of the angles at A and C (α and γ) equals two right angles, similarly, the angles at B and D (β and δ) sum to two right angles

Proof

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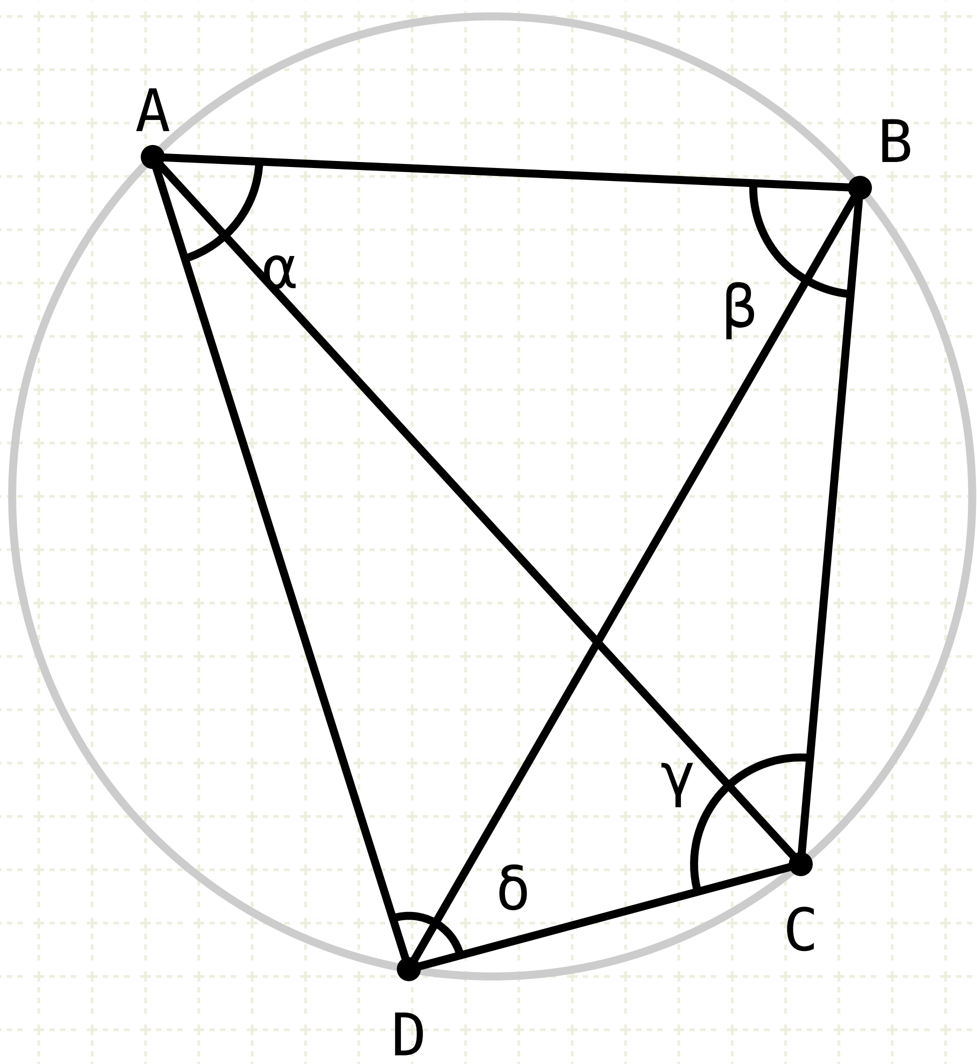

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Draw the lines AC and BD

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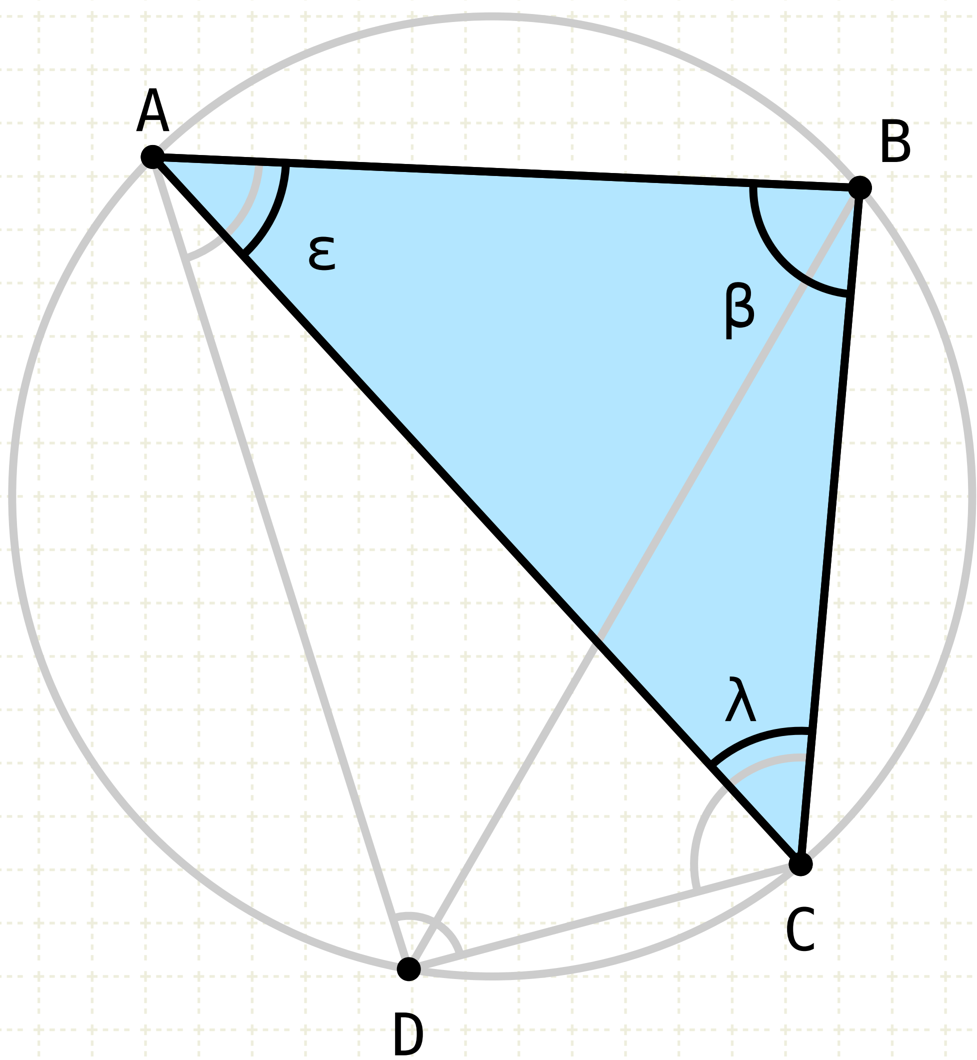

$\angle A = \alpha$, $\angle B = \beta$,
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$\varepsilon + \lambda + \beta = 2\llcorner$

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Draw the lines AC and BD

The sum of the angles inside the triangle ABC equals two right angles (I·32)

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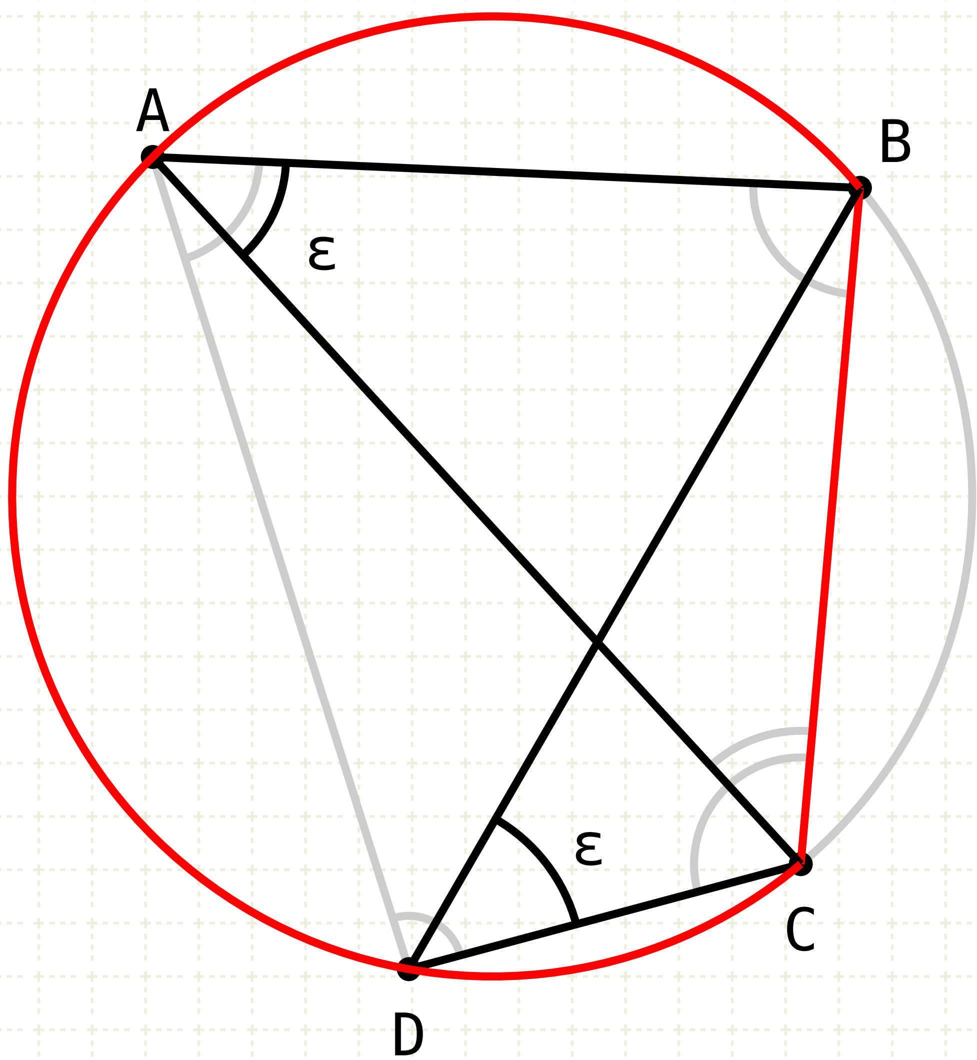

$\angle A = \alpha,\ \angle B = \beta,$
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$\varepsilon + \lambda + \beta = 2\llcorner$
$\angle CAB = \angle BDC = \varepsilon$

In other words

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Angle CAB is equal to angle BDC because they are in the same circular segment (III·21)

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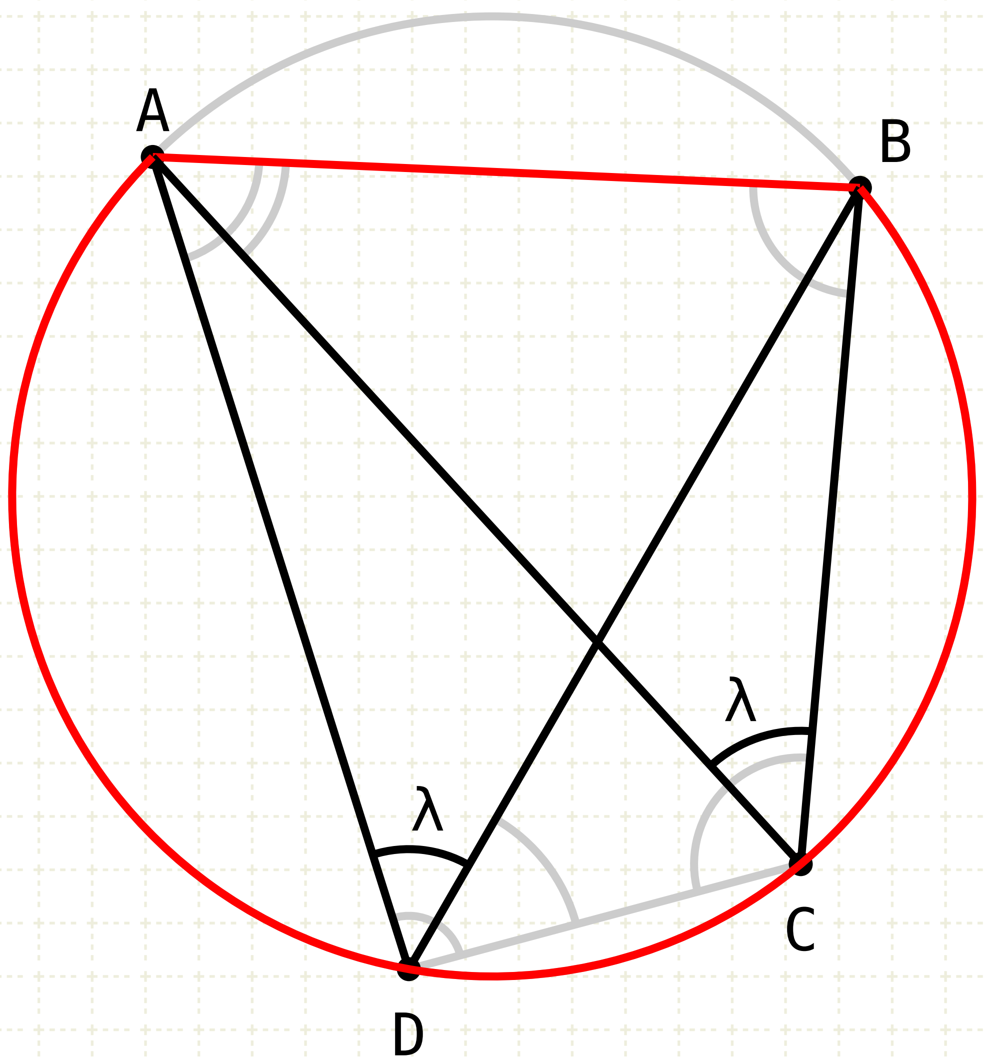

$\angle A = \alpha,\ \angle B = \beta,$
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$\varepsilon + \lambda + \beta = 2∟$
$\angle CAB = \angle BDC = \varepsilon$
$\angle ACB = \angle ADB = \lambda$

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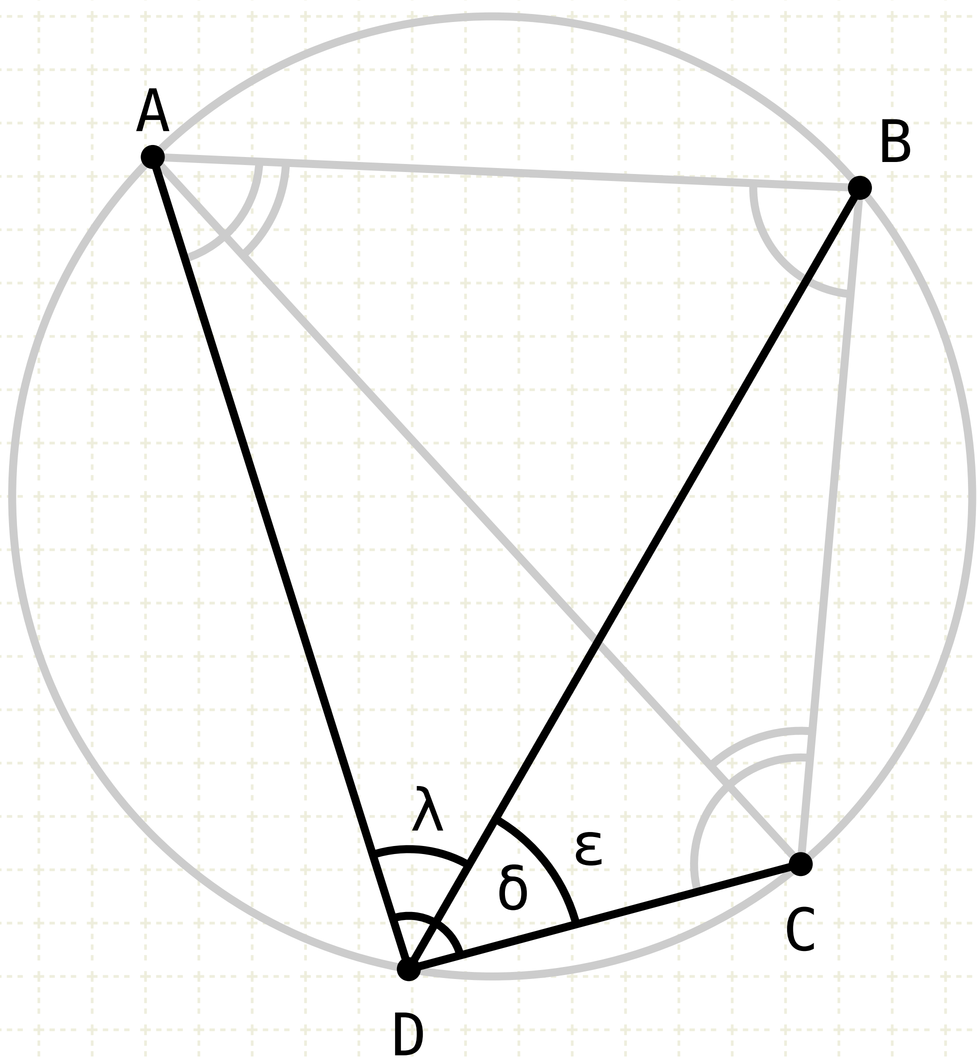

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$\delta = \varepsilon + \lambda$

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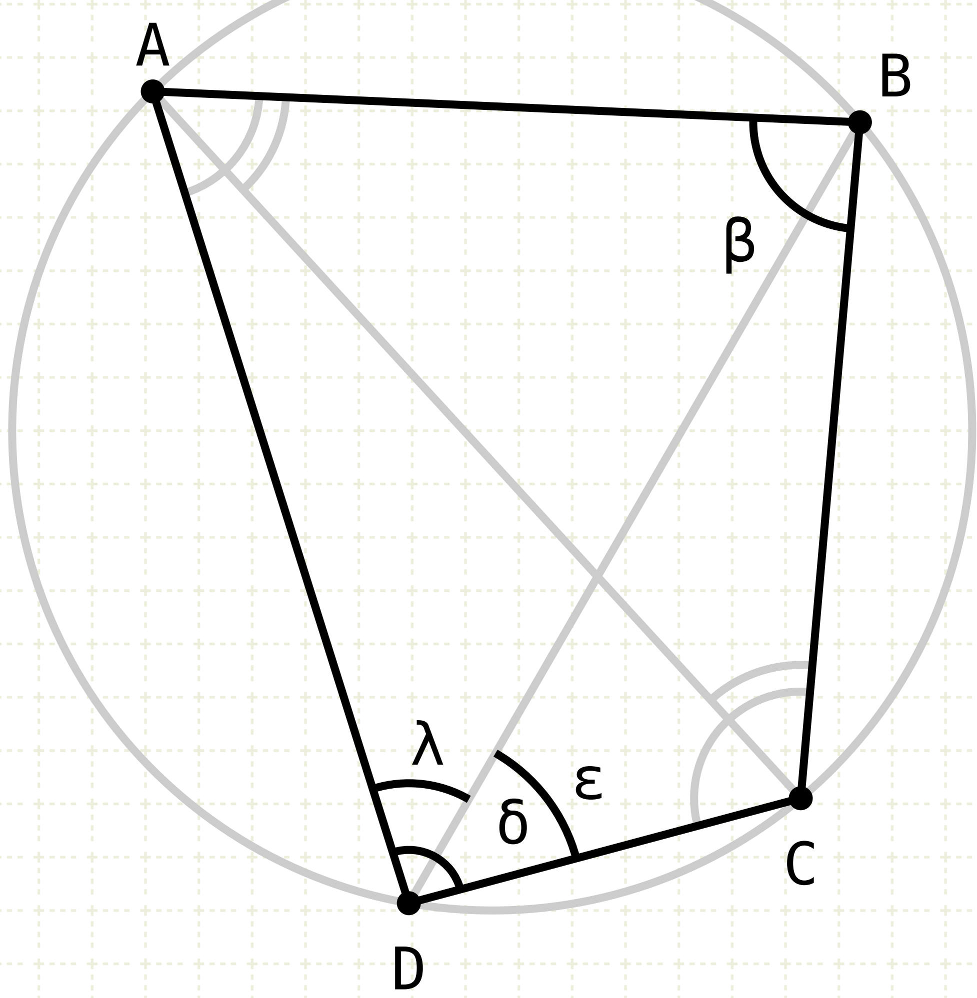

$\angle A = \alpha,\ \angle B = \beta,$
$\angle C = \gamma,\ \angle D = \delta$

$\varepsilon + \lambda + \beta = 2\llcorner$
$\angle CAB = \angle BDC = \varepsilon$
$\angle ACB = \angle ADB = \lambda$
$\delta = \varepsilon + \lambda$

$\angle B + \angle D$
$= \beta + \delta$
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So the sum of the angles as B and D is equal to the sum of the angles at B (β), ADB (λ), and BDC (ε)

Proposition 22 of Book III

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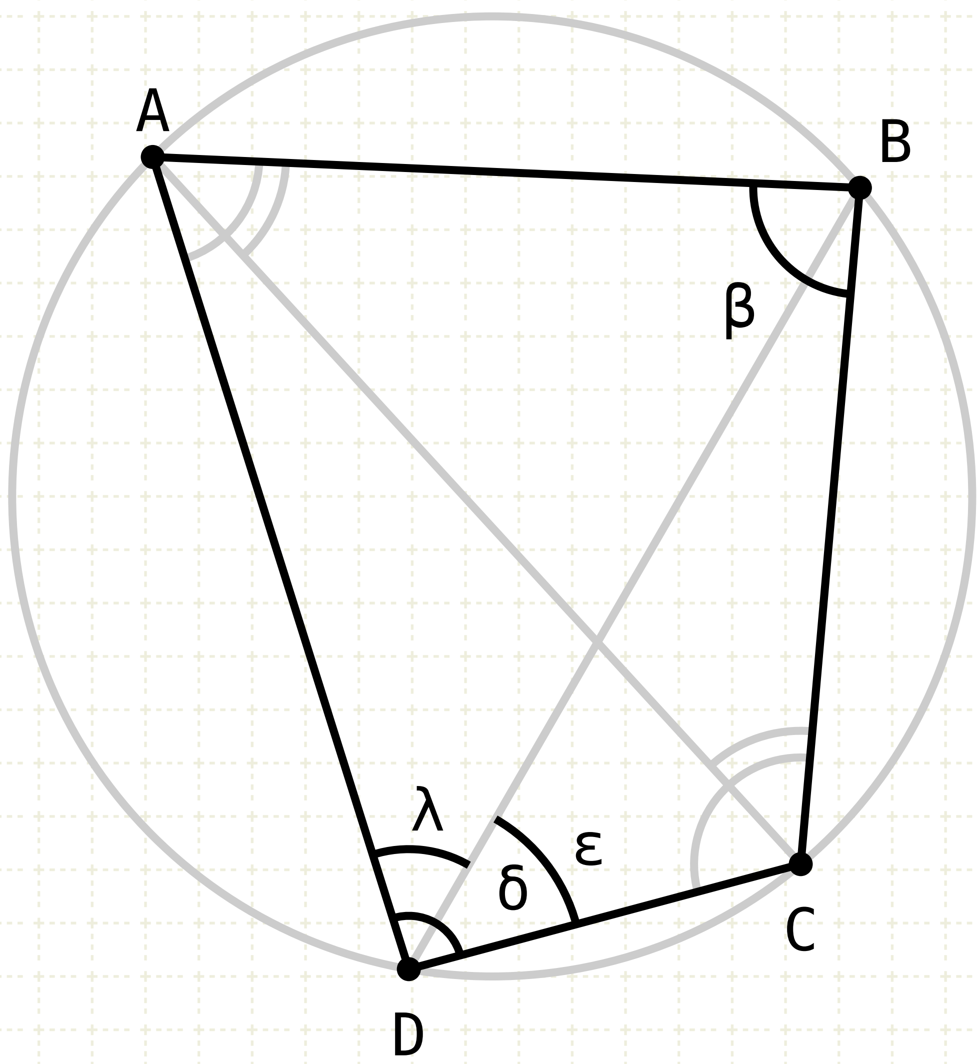

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Which is equal to two right angles

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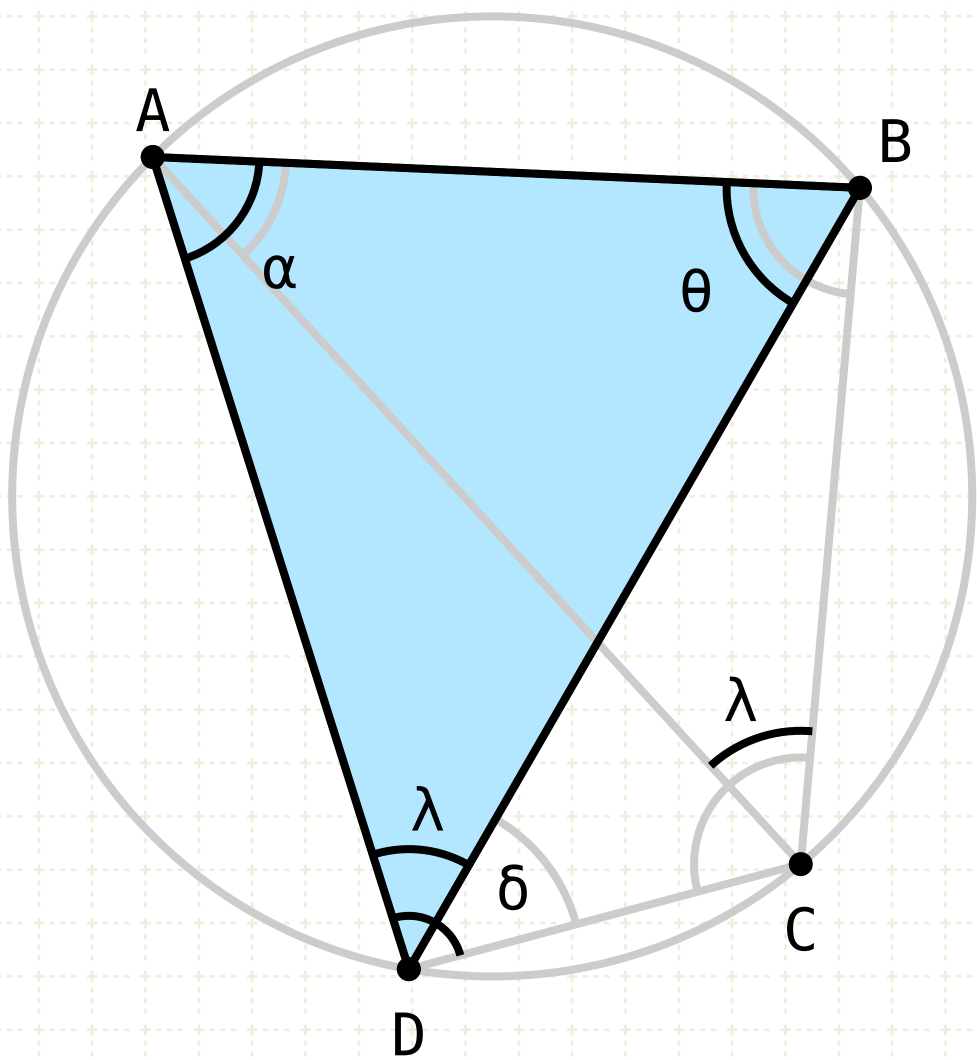

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$\alpha + \theta + \lambda = 2\llcorner$

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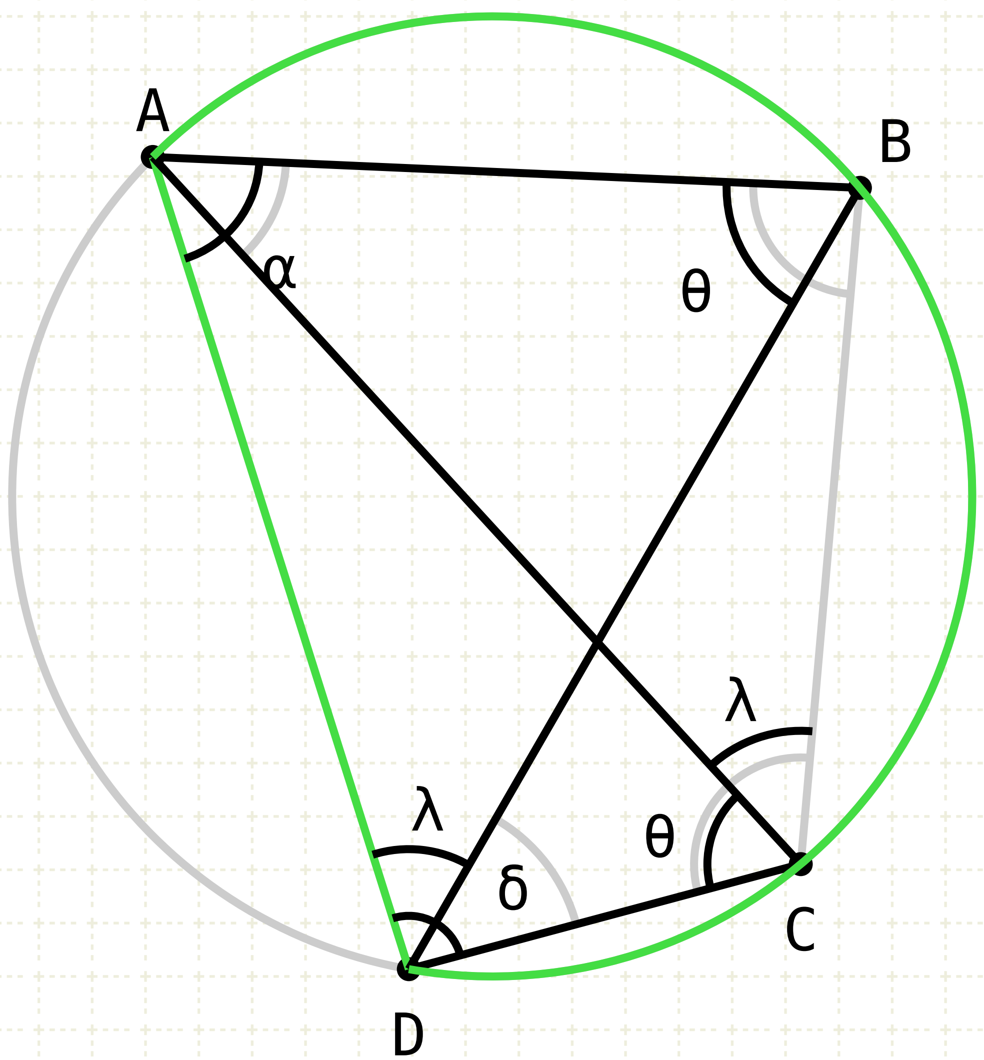

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$\alpha + \theta + \lambda = 2\llcorner$
$\angle DBA = \angle ACD = \theta$

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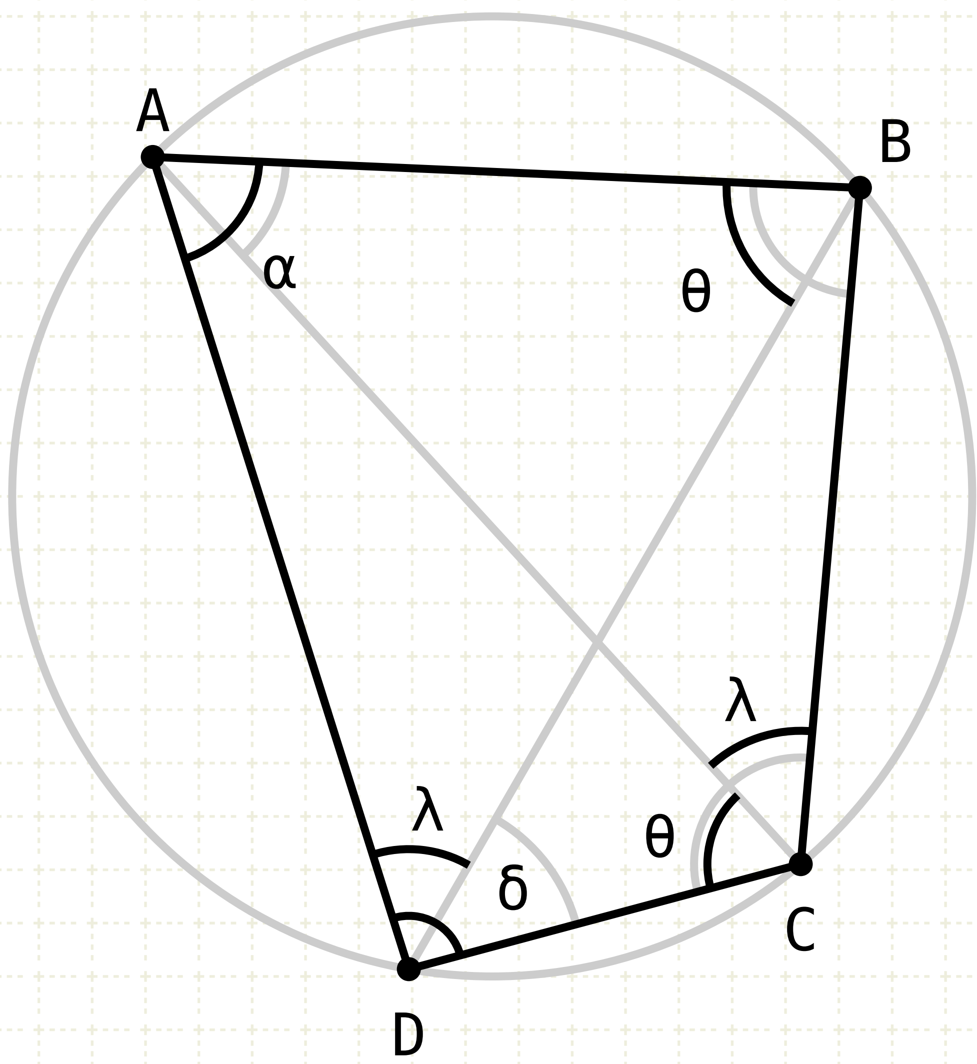

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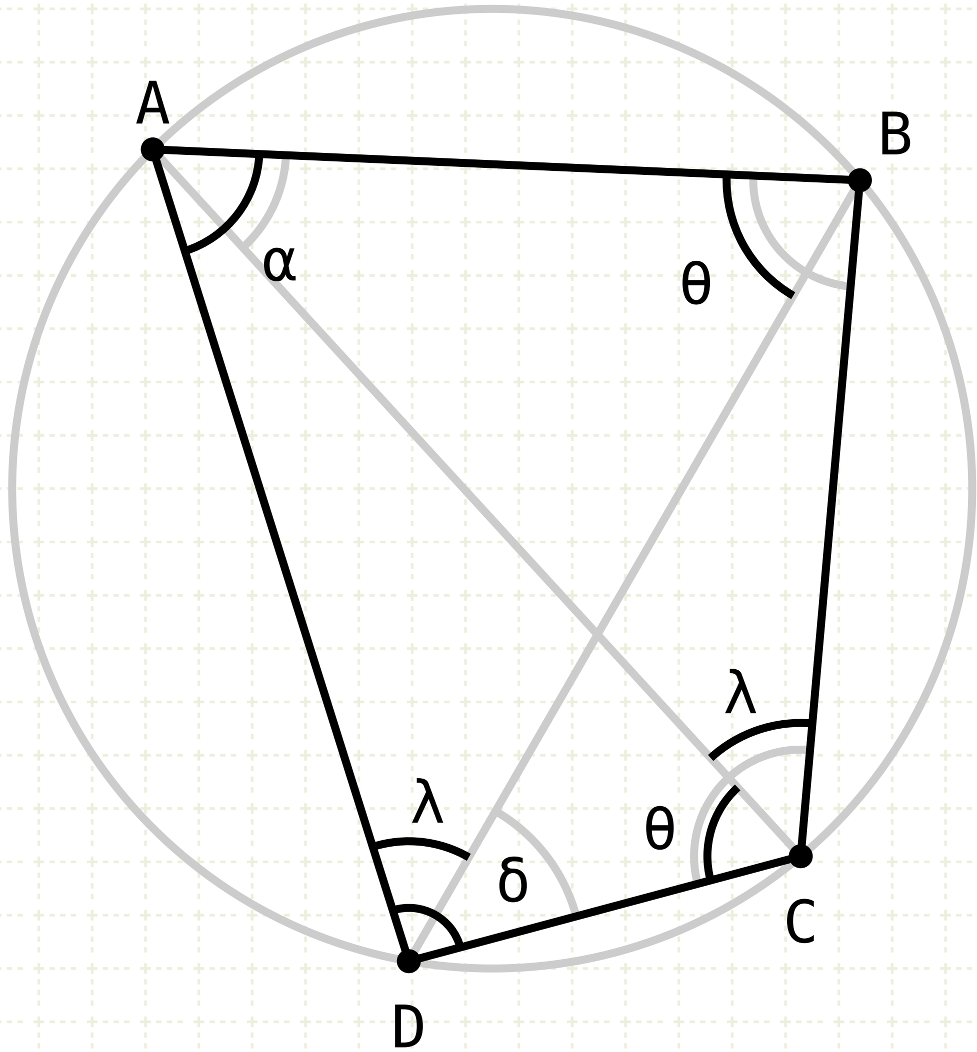

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