

Euclid's Elements

Book I

*If Euclid did not kindle your youthful enthusiasm, you
were not born to be a scientific thinker.*

Albert Einstein

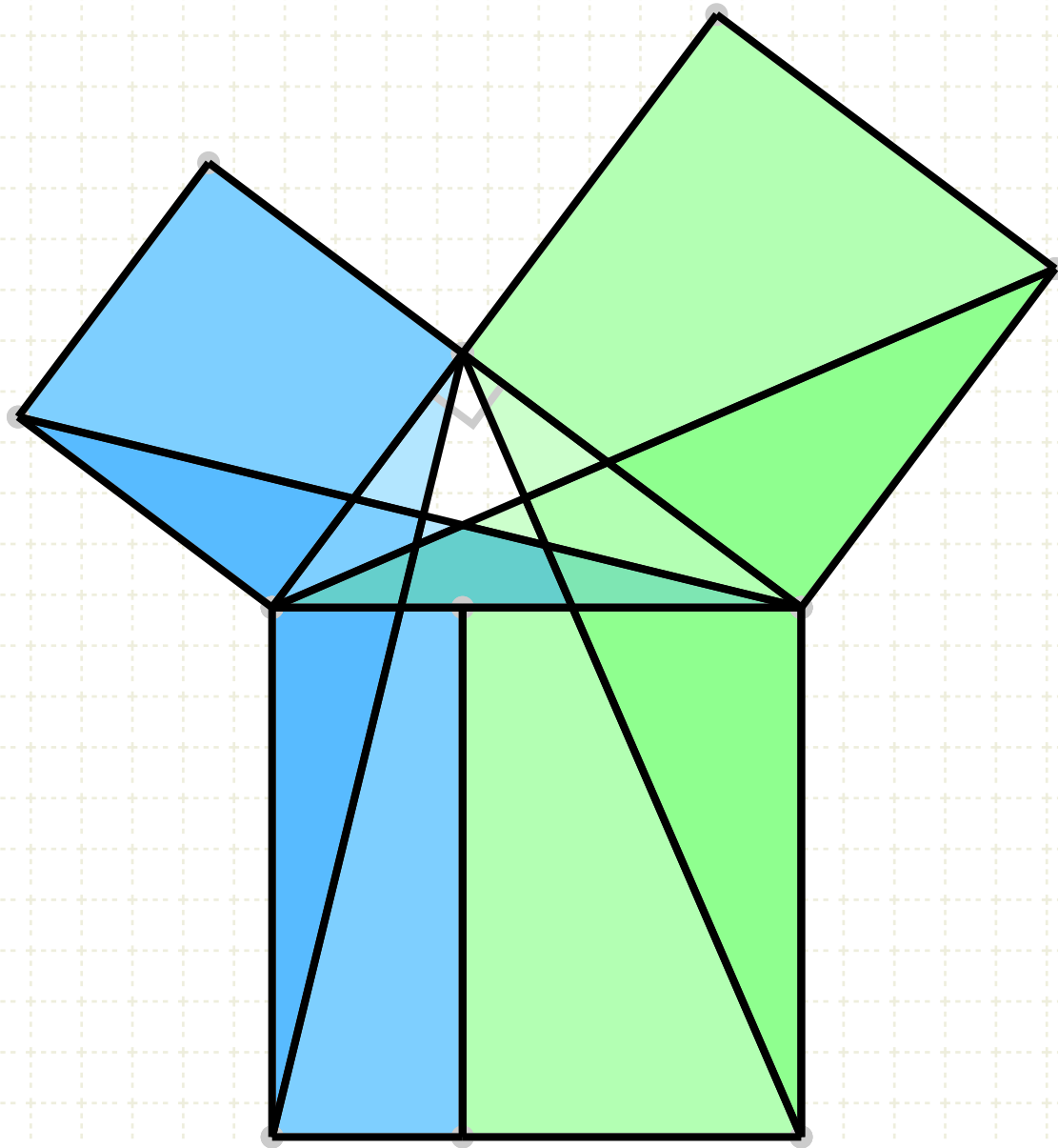


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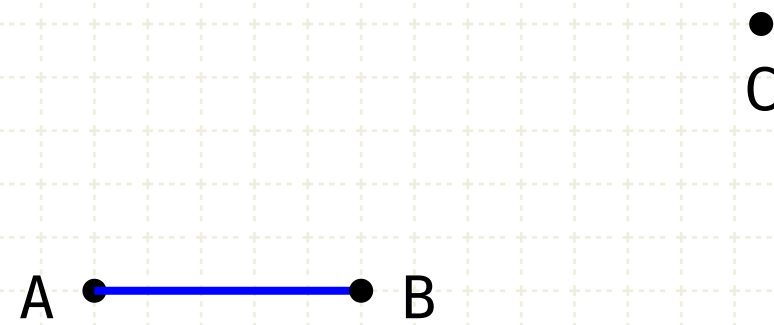
Proposition 2 of Book I

To place a straight line equal to a given straight line with one end at a given point.



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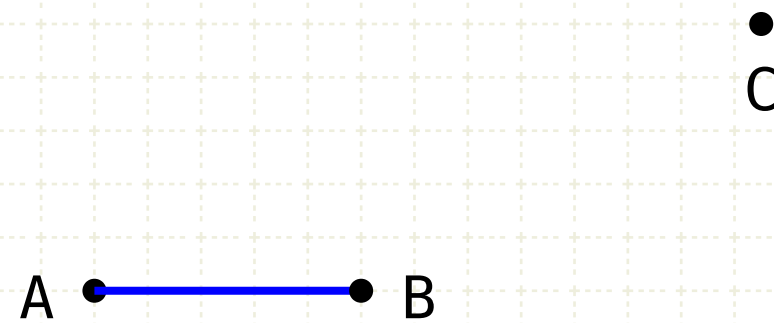


Proposition 2 of Book I

To place a straight line equal to a given straight line with one end at a given point.

Construction:

Start with line segment AB and point C



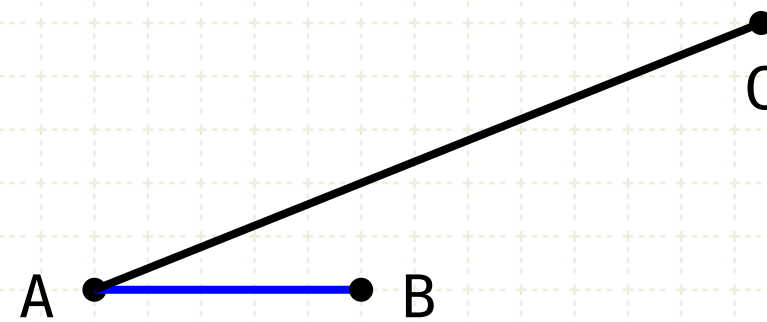
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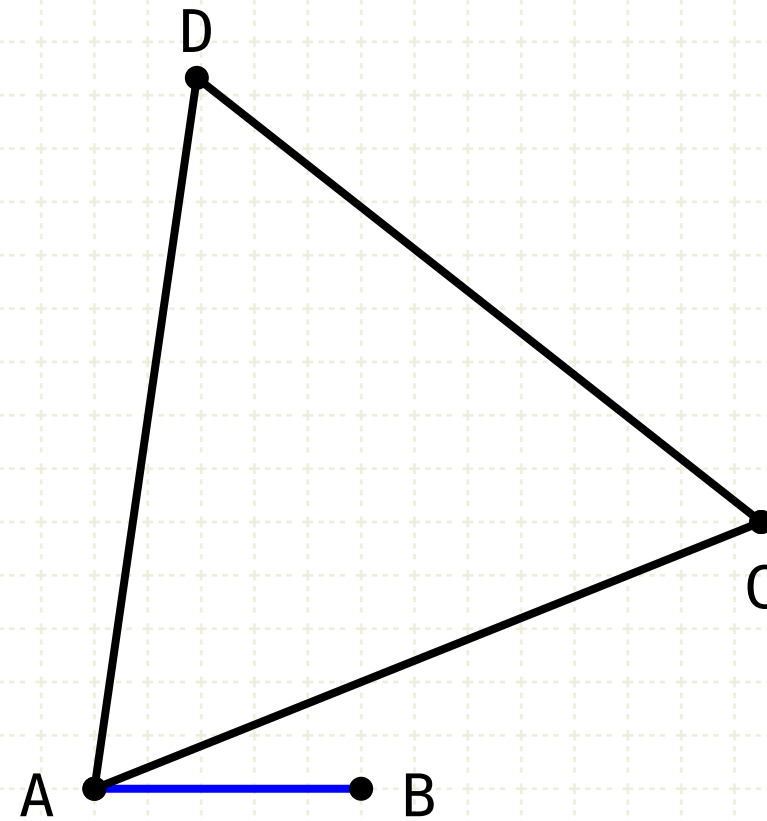
Start with line segment AB and point C

Construct line segment AC



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Construction:

Start with line segment AB and point C

Construct line segment AC

Construct an equilateral triangle on line AC (I·1)

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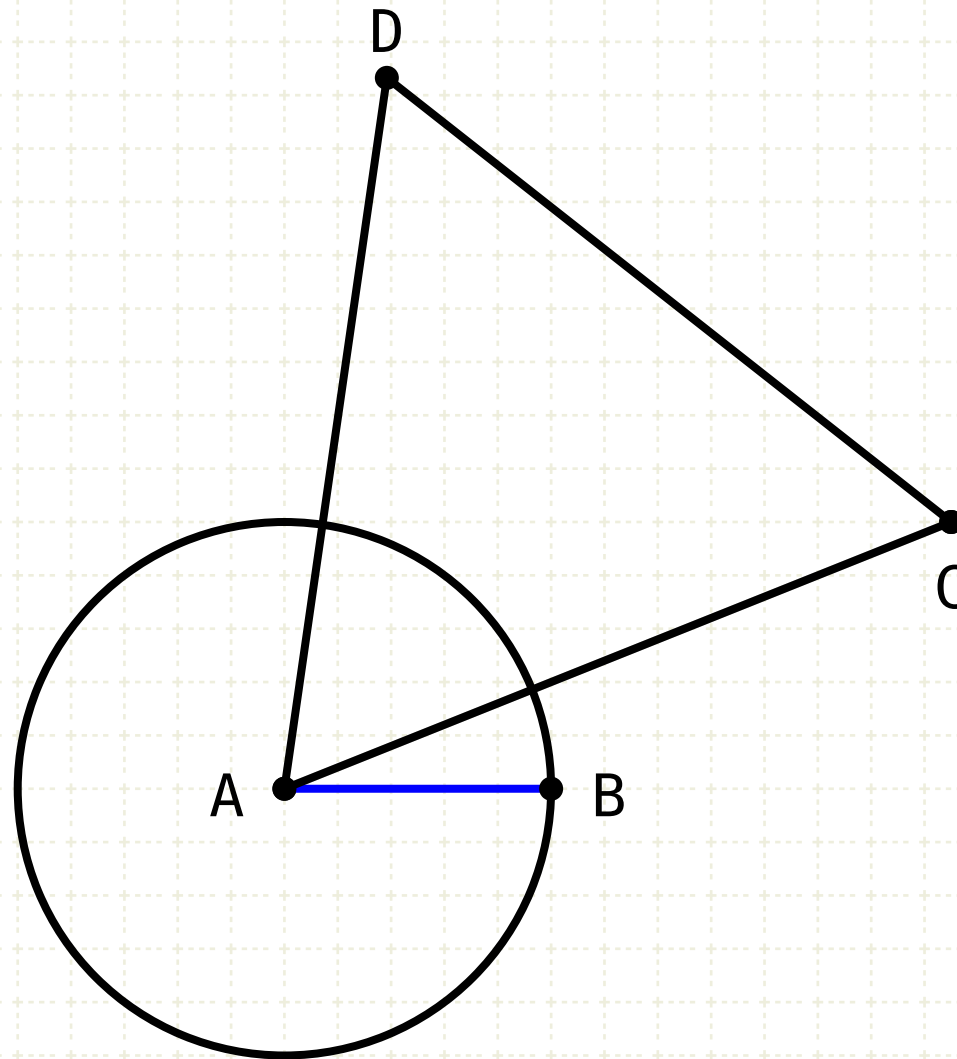
Construction:

Start with line segment AB and point C

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Construct an equilateral triangle on line AC (I·1)

Draw a circle with A as the center and AB as the radius



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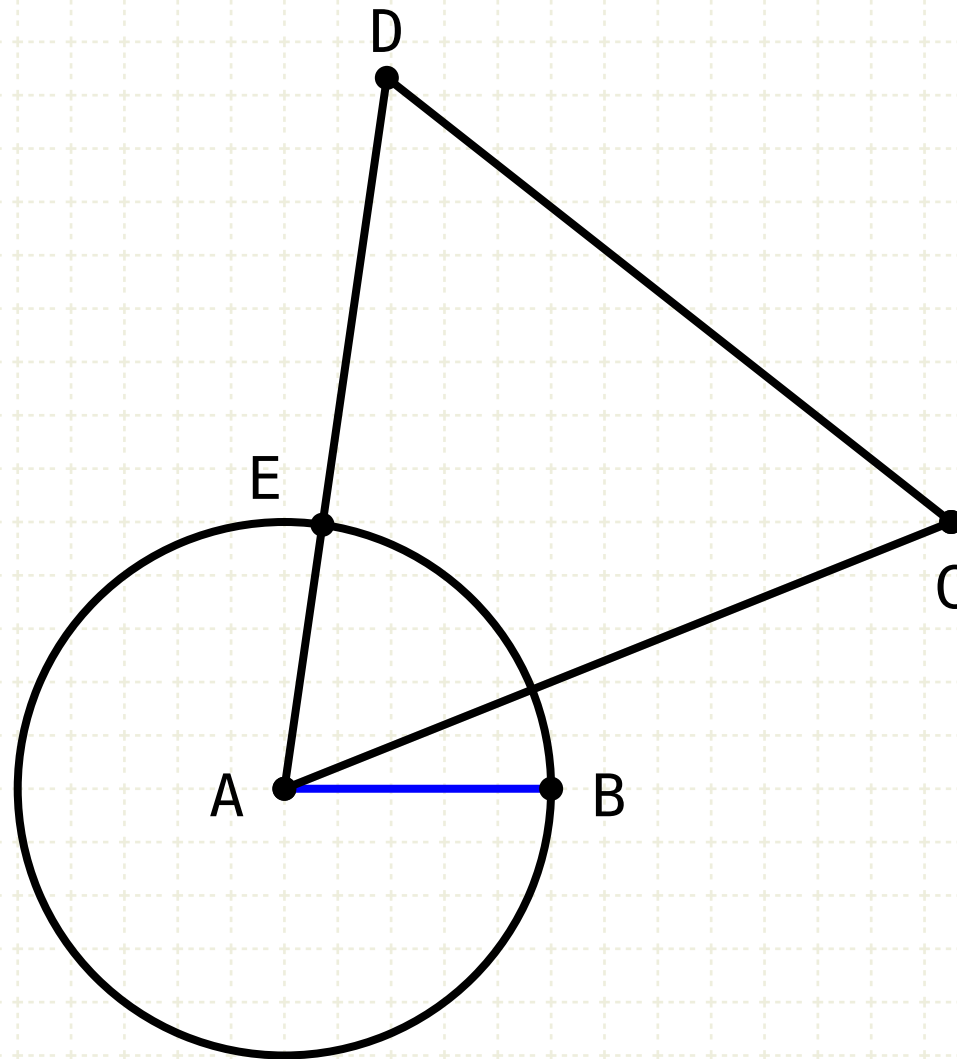
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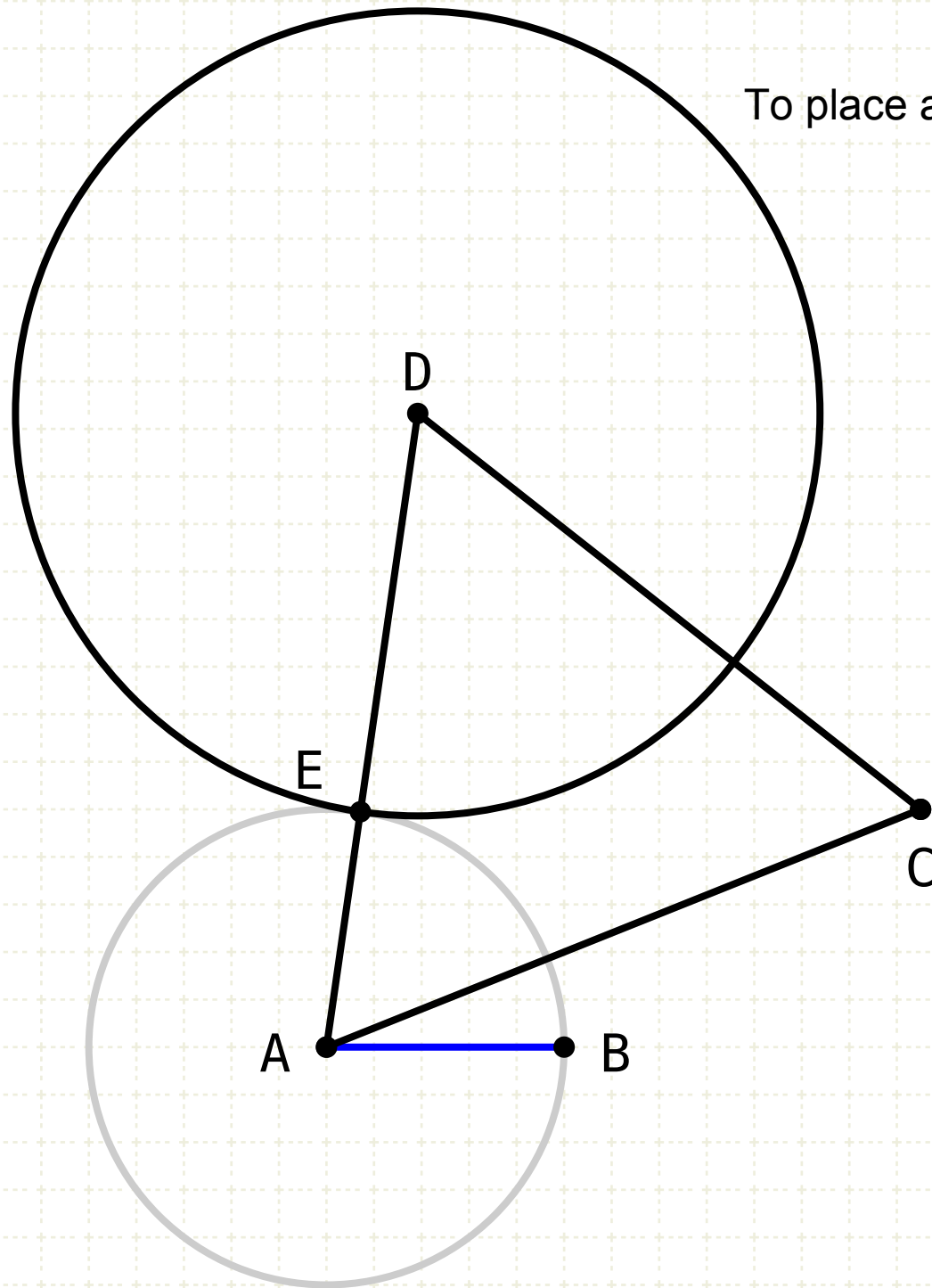
Draw a circle with A as the center and AB as the radius

Label the intersection of the circle and line AD as E



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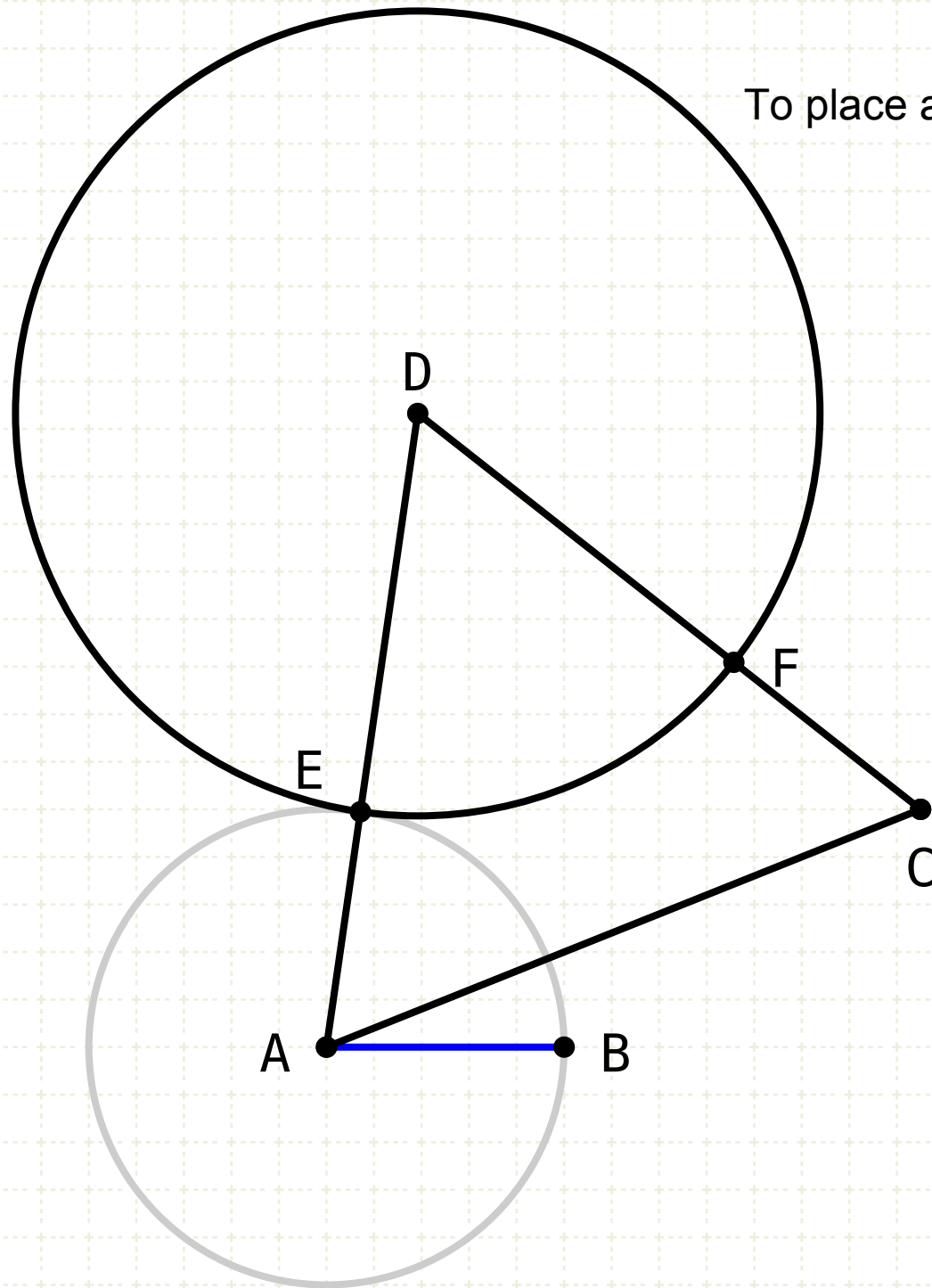
Draw a circle with A as the center and AB as the radius

Label the intersection of the circle and line AD as E

Draw a circle with D as the center and ED as the radius

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Start with line segment AB and point C

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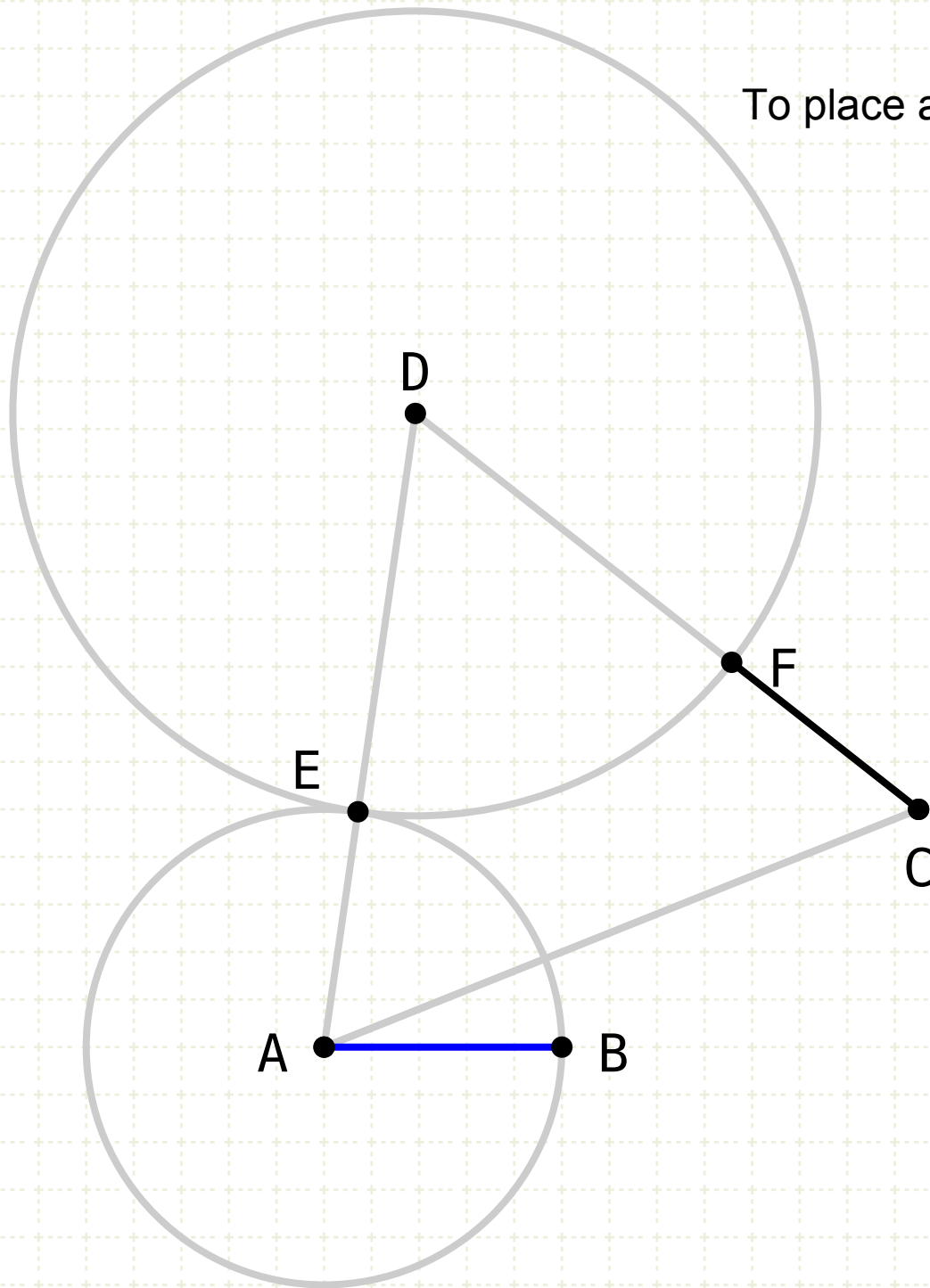
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Draw a circle with D as the center and ED as the radius

Label the intersection of the circle and line CD as F

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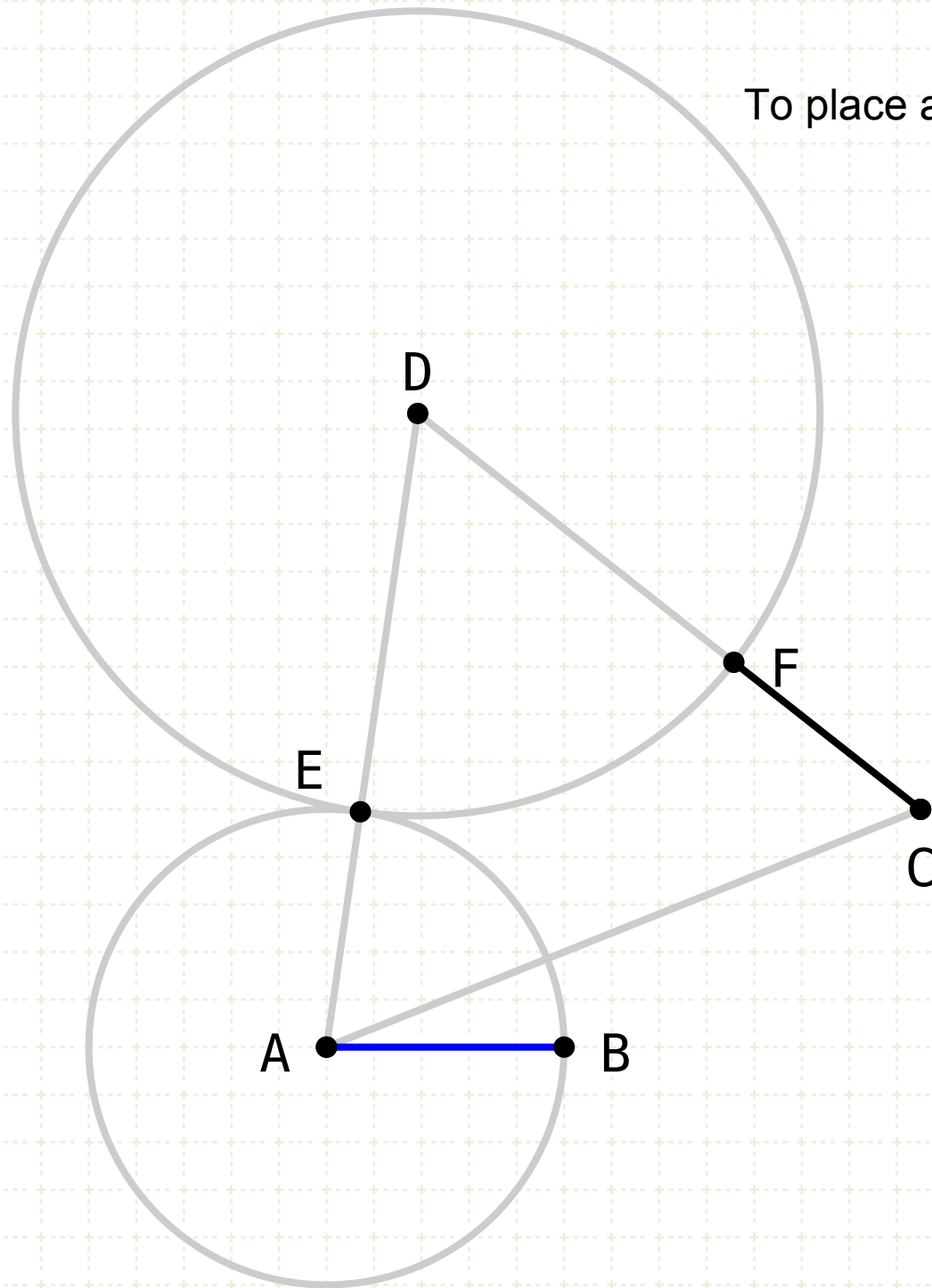
Draw a circle with D as the center and ED as the radius

Label the intersection of the circle and line CD as F

Line AB is equal to line CF

Proposition 2 of Book I

To place a straight line equal to a given straight line with one end at a given point.



Construction:

Start with line segment AB and point C

Construct line segment AC

Construct an equilateral triangle on line AC (I·1)

Draw a circle with A as the center and AB as the radius

Label the intersection of the circle and line AD as E

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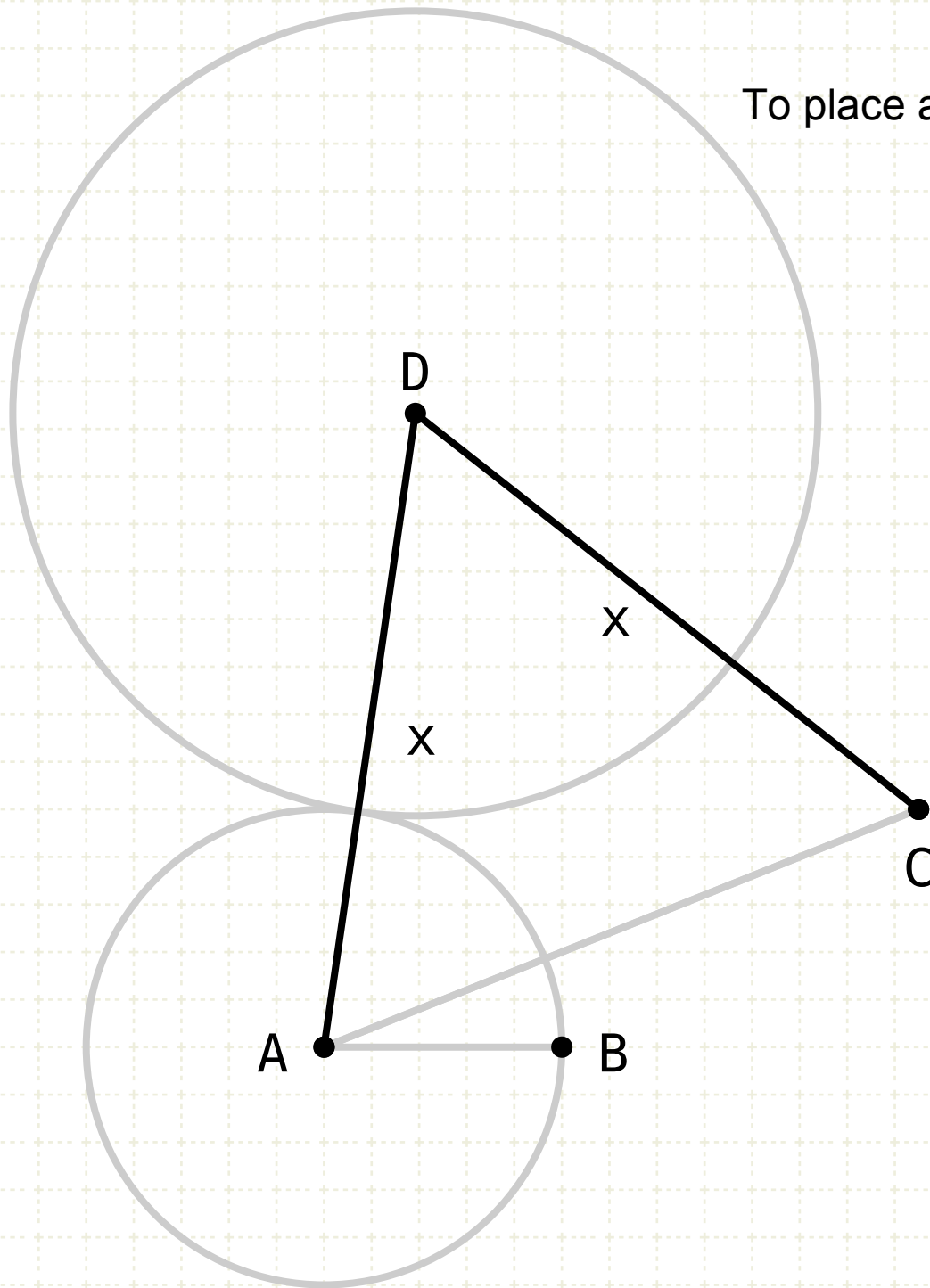
Label the intersection of the circle and line CD as F

Line AB is equal to line CF

Proof:

Proposition 2 of Book I

To place a straight line equal to a given straight line with one end at a given point.



$$AD = DC = x$$

Construction:

Start with line segment AB and point C

Construct line segment AC

Construct an equilateral triangle on line AC (I·1)

Draw a circle with A as the center and AB as the radius

Label the intersection of the circle and line AD as E

Draw a circle with D as the center and ED as the radius

Label the intersection of the circle and line CD as F

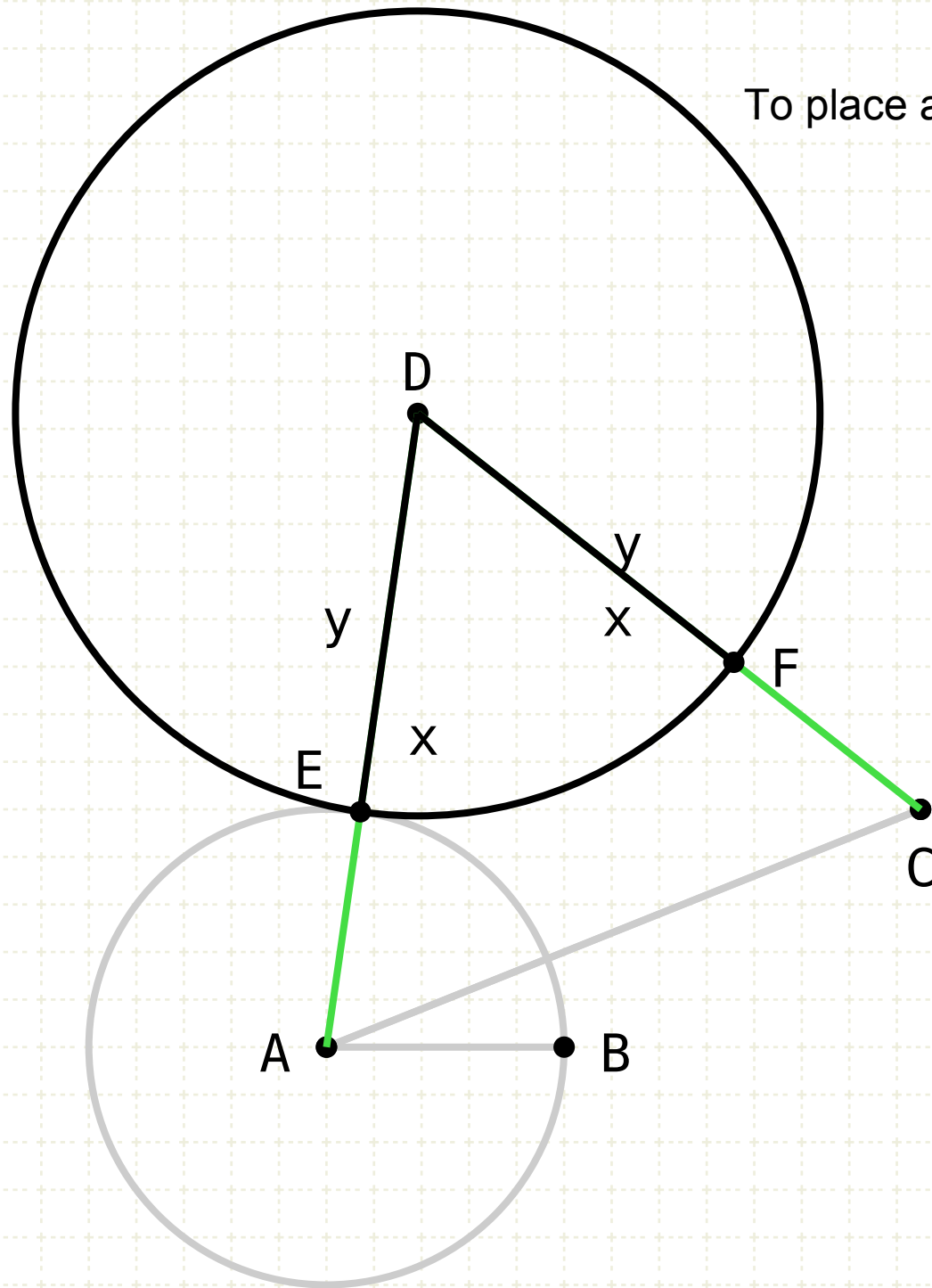
Line AB is equal to line CF

Proof:

Line AD is equal to line DC (equilateral triangle)

Proposition 2 of Book I

To place a straight line equal to a given straight line with one end at a given point.



$$\begin{aligned} AD &= DC = x \\ DE &= DF = y \end{aligned}$$

Construction:

Start with line segment AB and point C

Construct line segment AC

Construct an equilateral triangle on line AC (I.1)

Draw a circle with A as the center and AB as the radius

Label the intersection of the circle and line AD as E

Draw a circle with D as the center and ED as the radius

Label the intersection of the circle and line CD as F

Line AB is equal to line CF

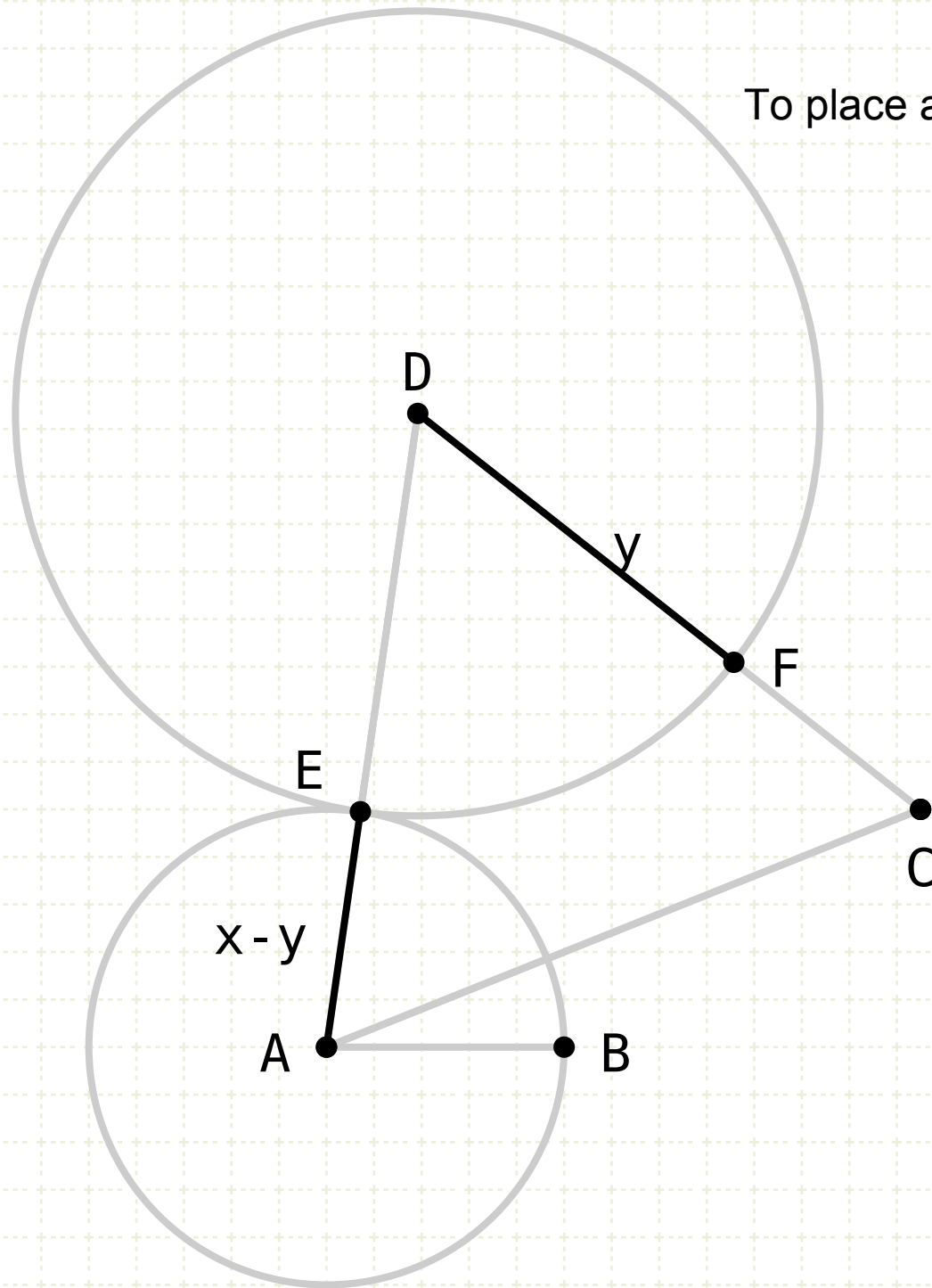
Proof:

Line AD is equal to line DC (equilateral triangle)

DE and DF are equal (radii of the same circle)

Proposition 2 of Book I

To place a straight line equal to a given straight line with one end at a given point.



$$\begin{aligned}AD &= DC = x \\DE &= DF = y \\AE &= DA - DE \\AE &= x - y\end{aligned}$$

Construction:

Start with line segment AB and point C

Construct line segment AC

Construct an equilateral triangle on line AC (I.1)

Draw a circle with A as the center and AB as the radius

Label the intersection of the circle and line AD as E

Draw a circle with D as the center and ED as the radius

Label the intersection of the circle and line CD as F

Line AB is equal to line CF

Proof:

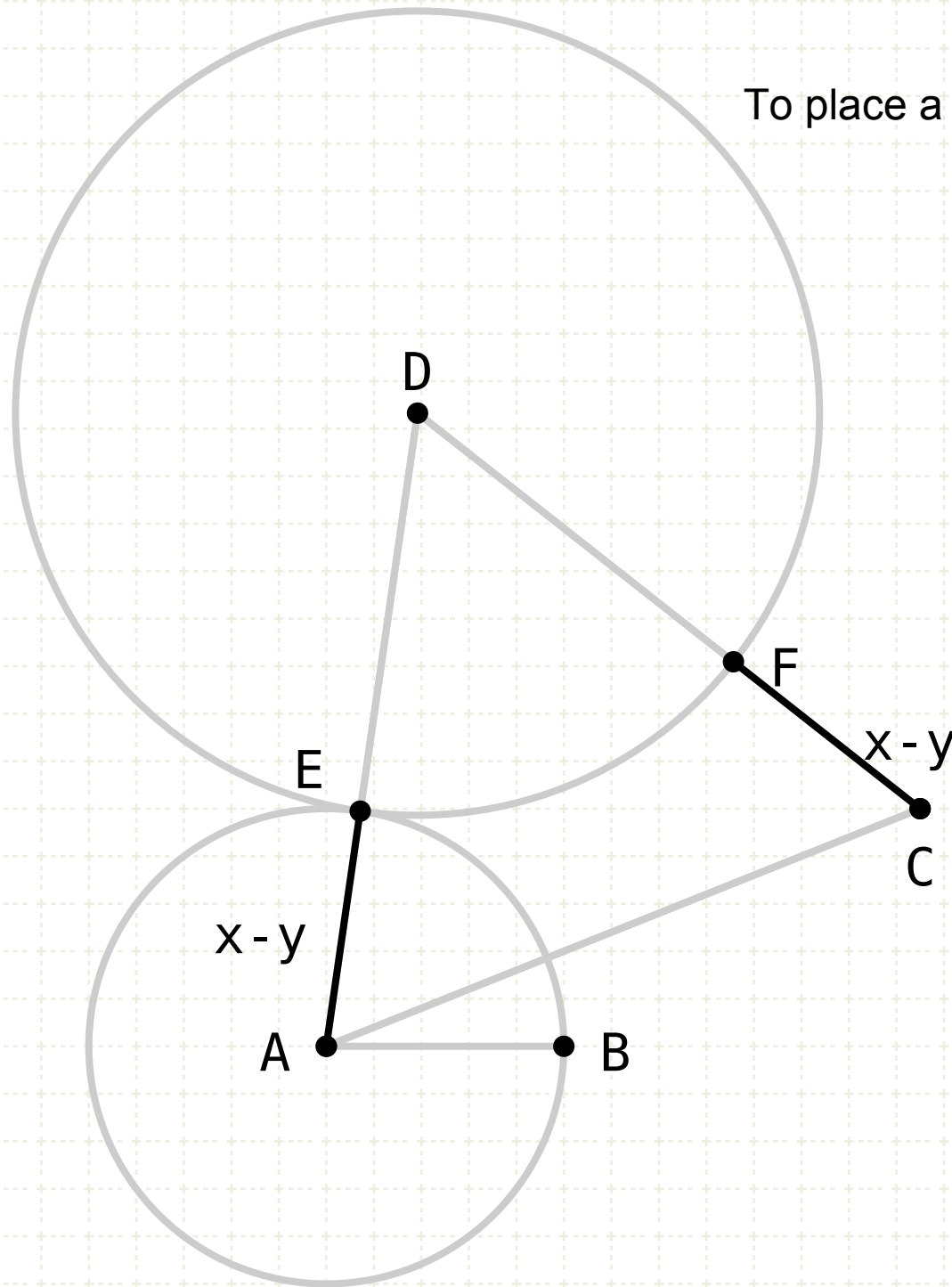
Line AD is equal to line DC (equilateral triangle)

DE and DF are equal (radii of the same circle)

AE is the difference between DA and DE

Proposition 2 of Book I

To place a straight line equal to a given straight line with one end at a given point.



Construction:

Start with line segment AB and point C

Construct line segment AC

Construct an equilateral triangle on line AC (I.1)

Draw a circle with A as the center and AB as the radius

Label the intersection of the circle and line AD as E

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Label the intersection of the circle and line CD as F

Line AB is equal to line CF

$$\begin{aligned} AD &= DC = x \\ DE &= DF = y \\ AE &= DA - DE \\ AE &= x - y \\ CF &= DC - DF \\ CF &= x - y \end{aligned}$$

Proof:

Line AD is equal to line DC (equilateral triangle)

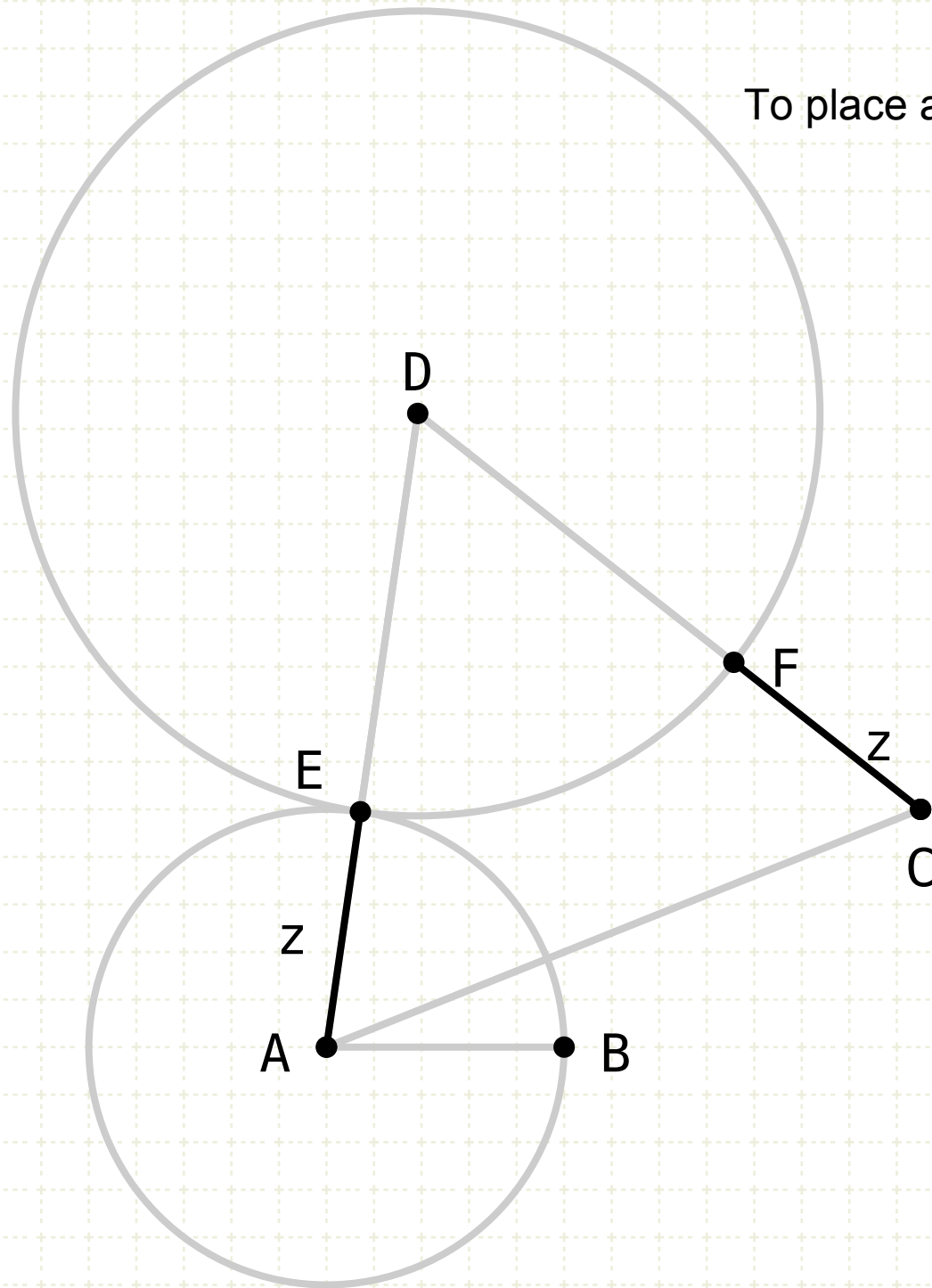
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Line AB is equal to line CF

$$\begin{aligned}AD &= DC = x \\DE &= DF = y \\AE &= DA - DE \\AE &= x - y \\CF &= DC - DF \\CF &= x - y \\AE &= CF = z\end{aligned}$$

Proof:

Line AD is equal to line DC (equilateral triangle)

DE and DF are equal (radii of the same circle)

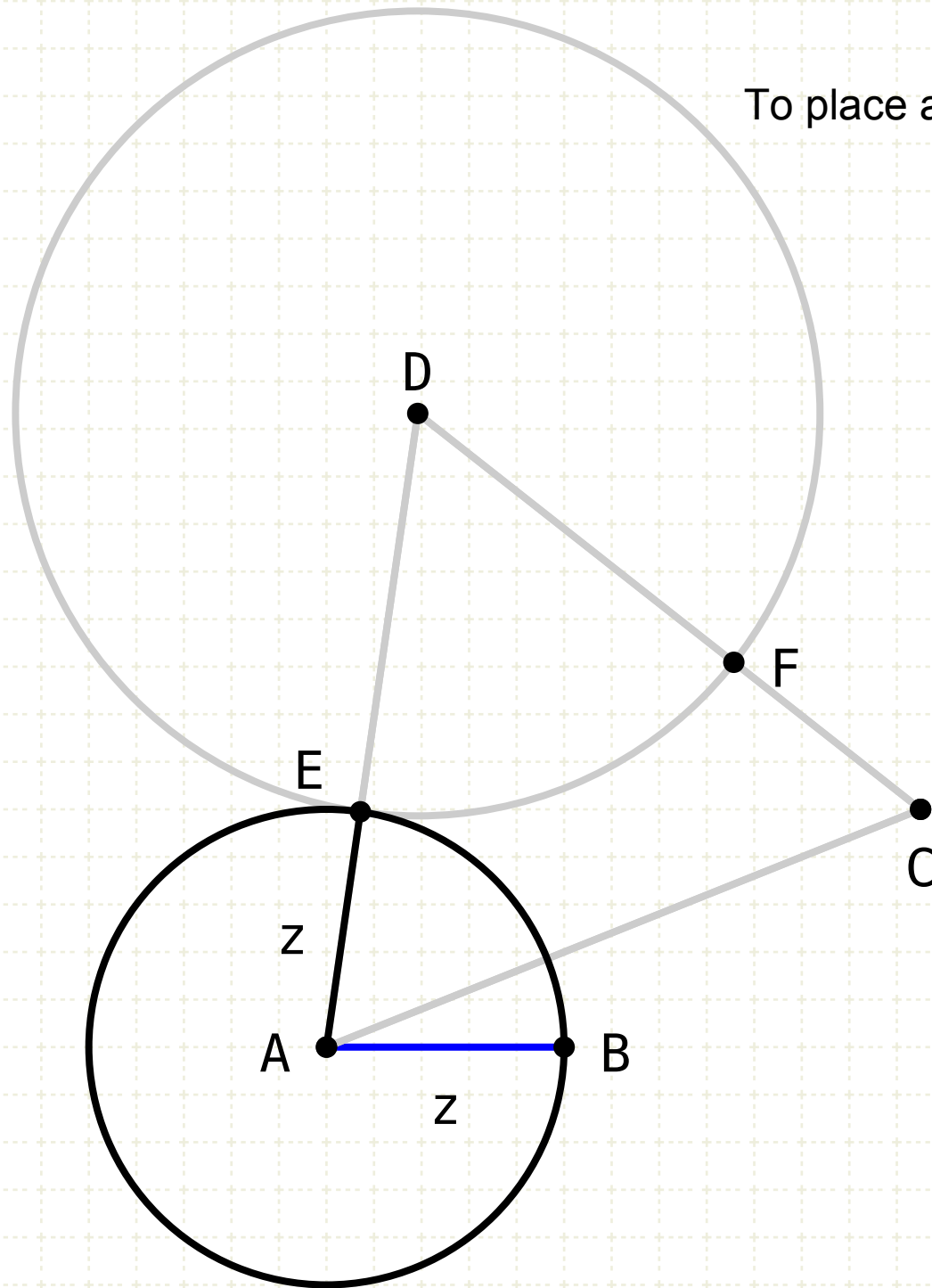
AE is the difference between DA and DE

CF is the difference between DC and DF

AE and FC are the differences of equals, so they are equal

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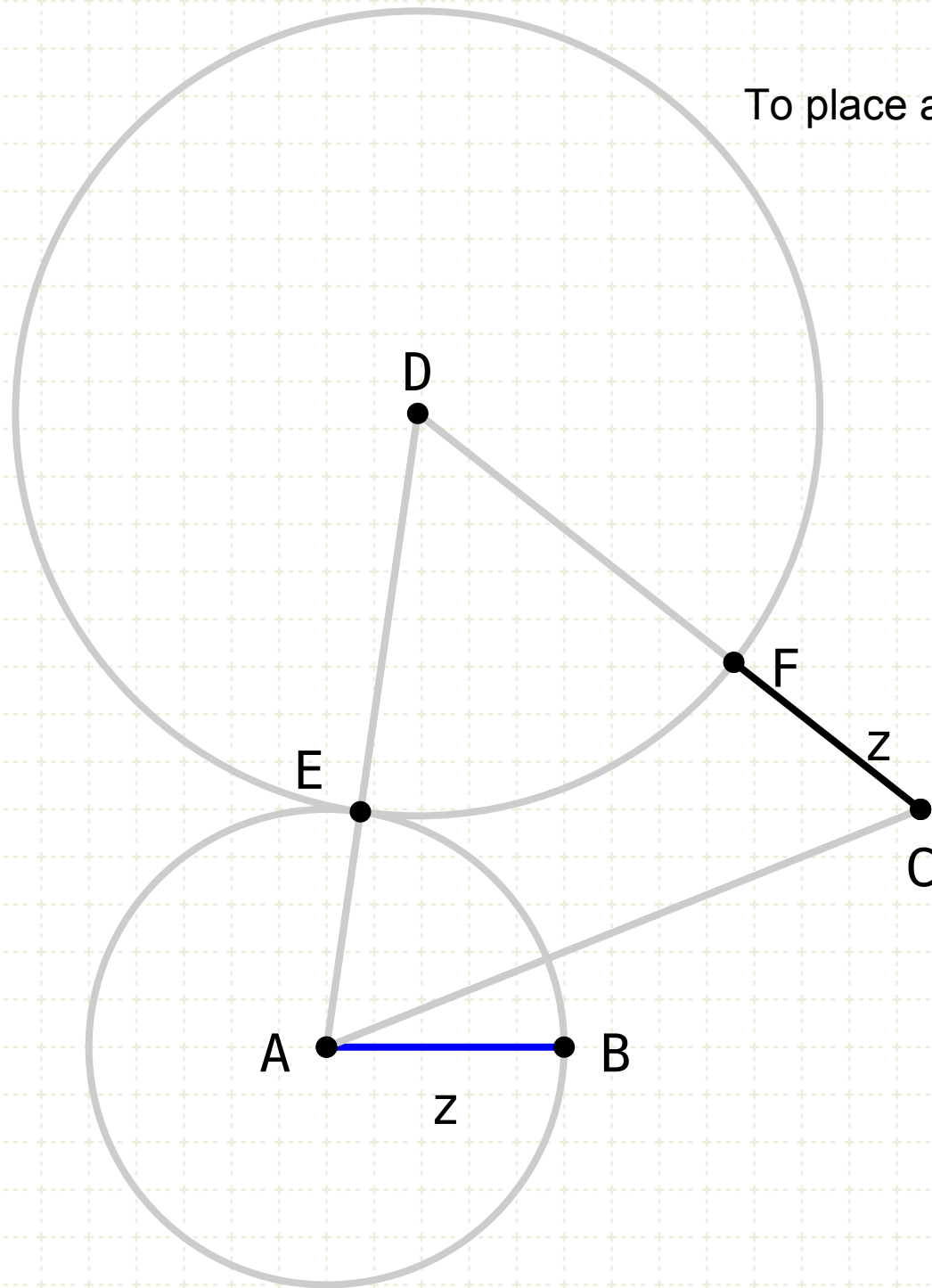
AE and FC are the differences of equals, so they are equal

AB and AE are radii of the same circle

$$\begin{aligned}AD &= DC = x \\DE &= DF = y \\AE &= DA - DE \\AE &= x - y \\CF &= DC - DF \\CF &= x - y \\AE &= CF = z \\AB &= AE = z\end{aligned}$$

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DE and DF are equal (radii of the same circle)

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AE and FC are the differences of equals, so they are equal

AB and AE are radii of the same circle

AB and CF are equal

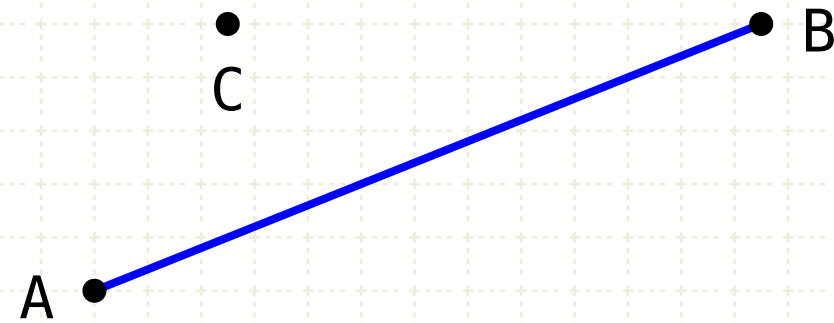
$$\begin{aligned}AD &= DC = x \\DE &= DF = y \\AE &= DA - DE \\AE &= x - y \\CF &= DC - DF \\CF &= x - y \\AE &= CF = z \\AB &= AE = z \\AB &= CF = z\end{aligned}$$

Proposition 2 of Book I

To place a straight line equal to a given straight line with one end at a given point.

But what if?

Start with line segment AB and point C



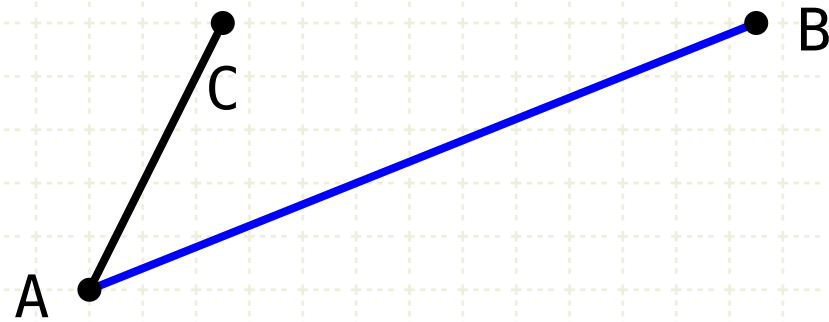
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Proposition 2 of Book I

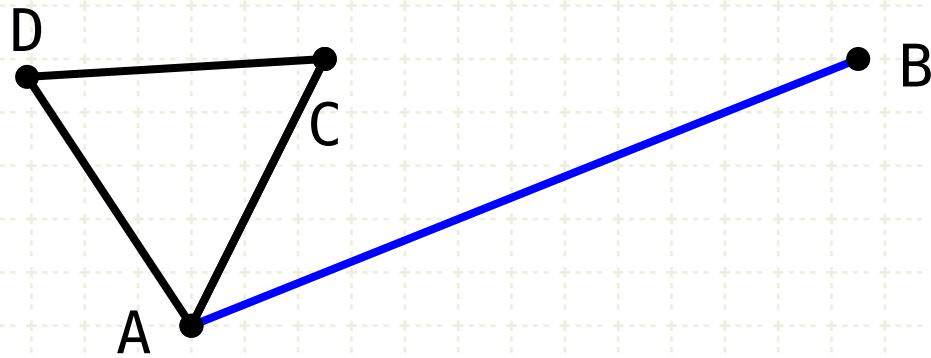
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Construct an equilateral triangle on line AC



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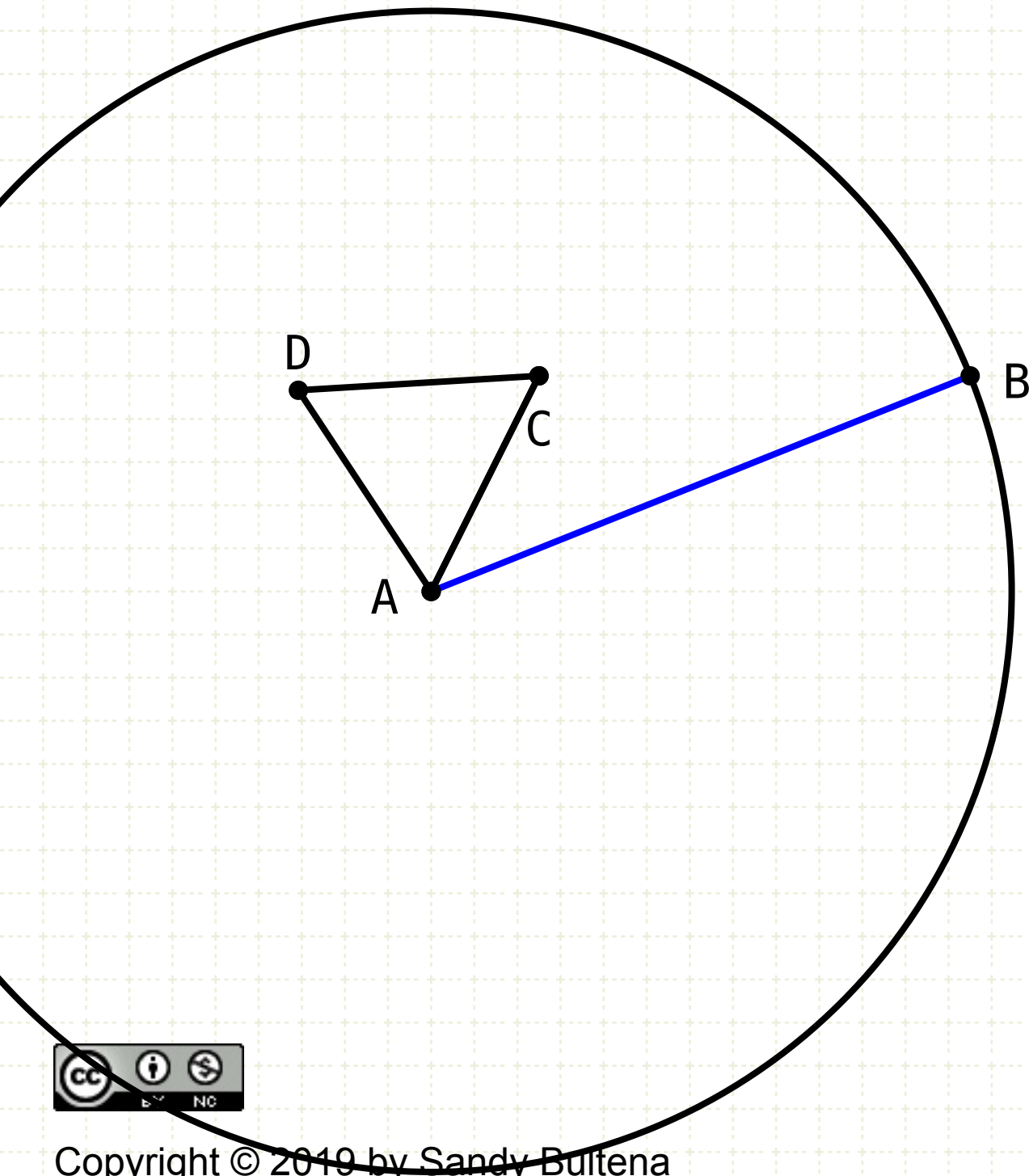
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Construct an equilateral triangle on line AC

Draw a circle with A as the center and AB as the radius



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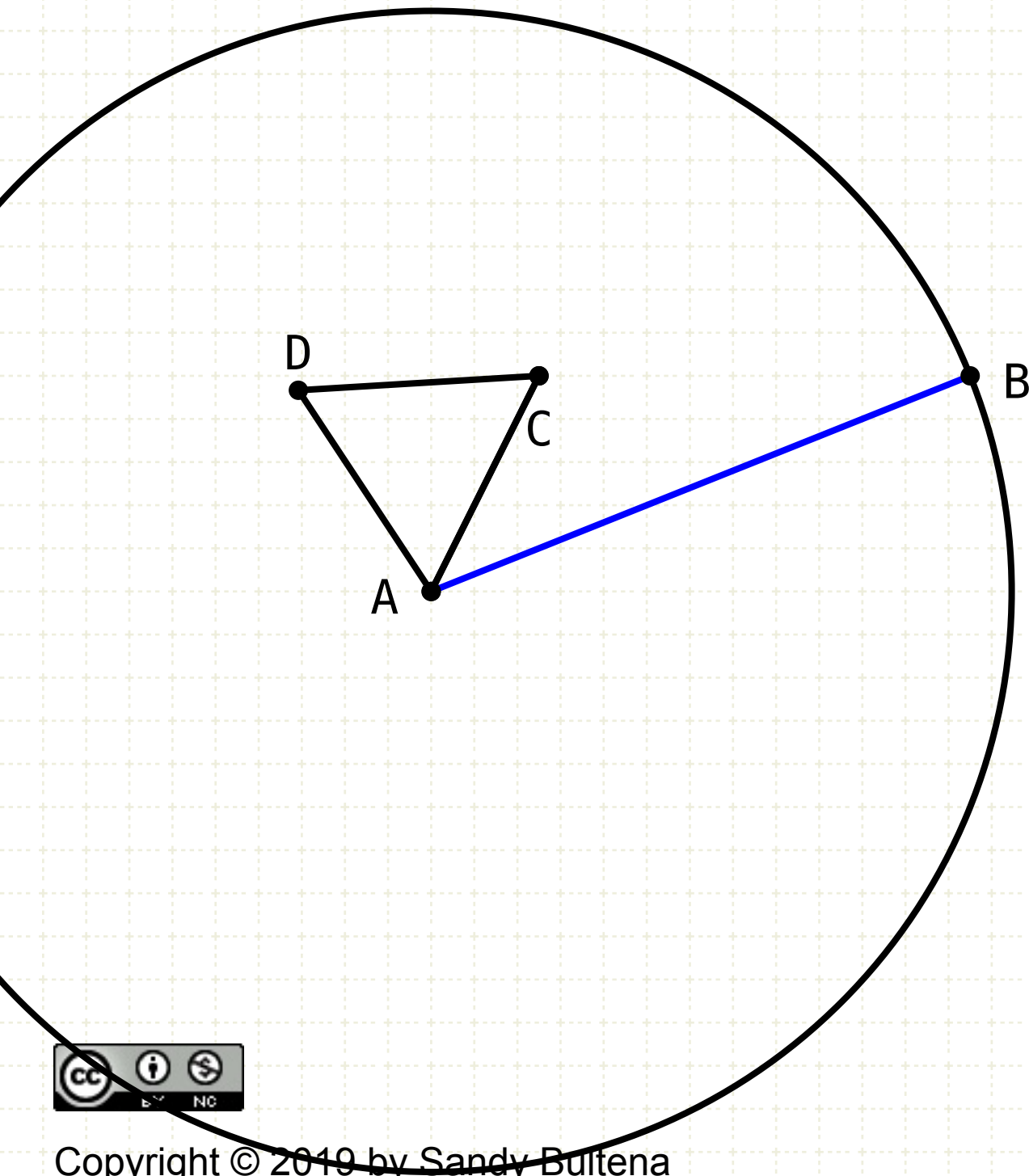
Construct line segment AC

Construct an equilateral triangle on line AC

Draw a circle with A as the center and AB as the radius

Label the intersection of the circle and line AD as E

...hang on... there isn't any intersection point, what now?



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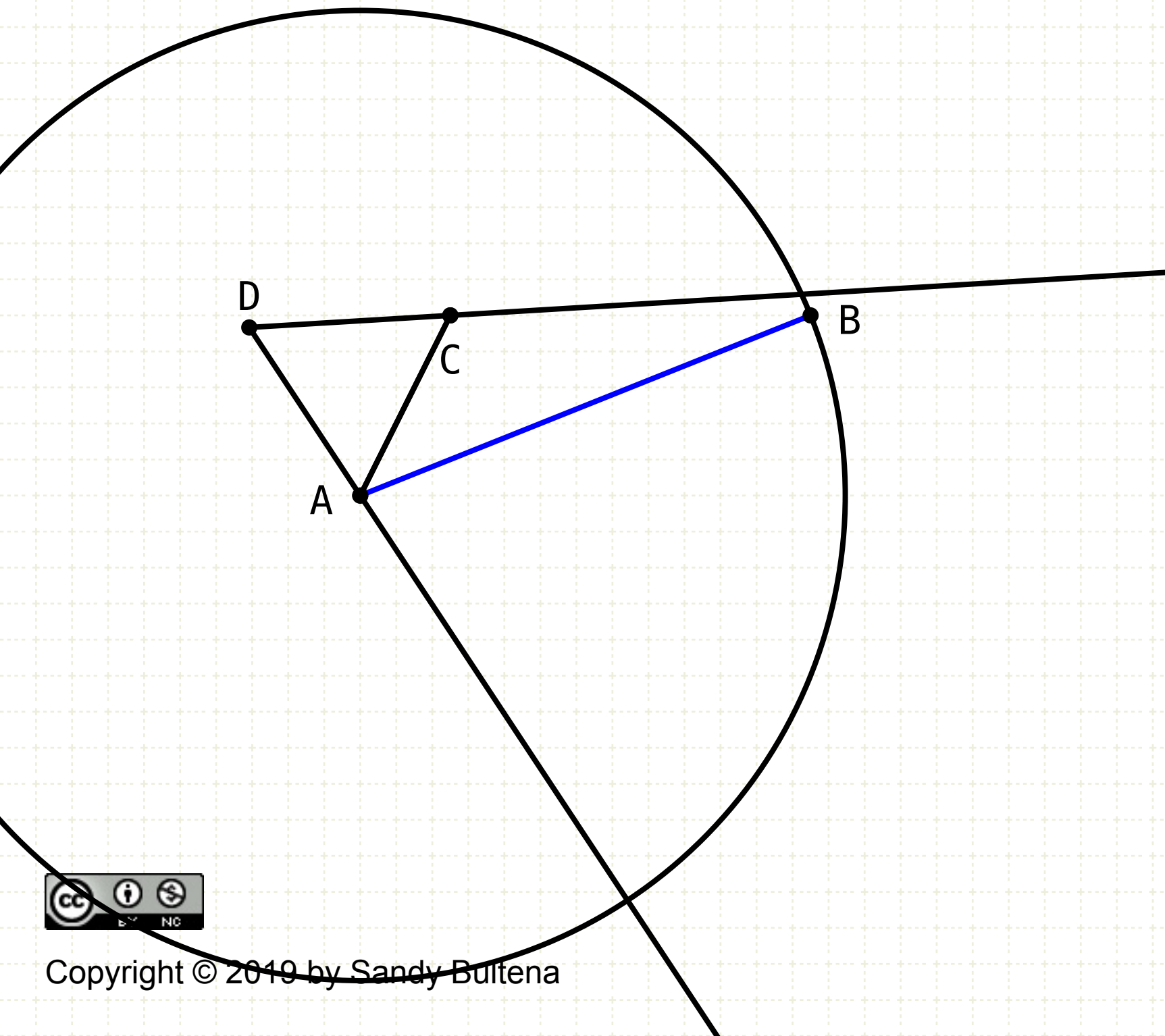
Construct an equilateral triangle on line AC

Draw a circle with A as the center and AB as the radius

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Extend DA and DC such that they intersect the circle



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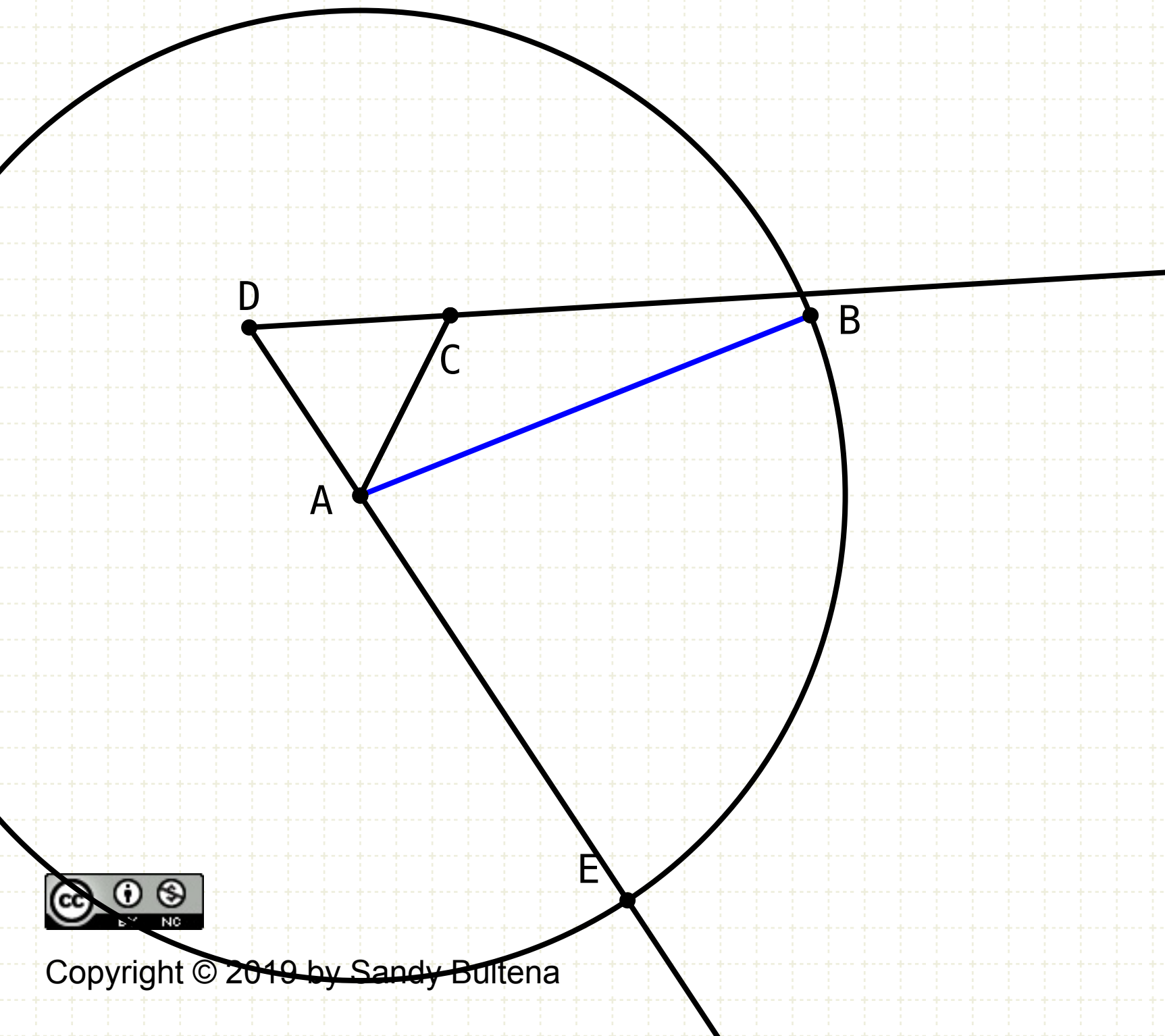
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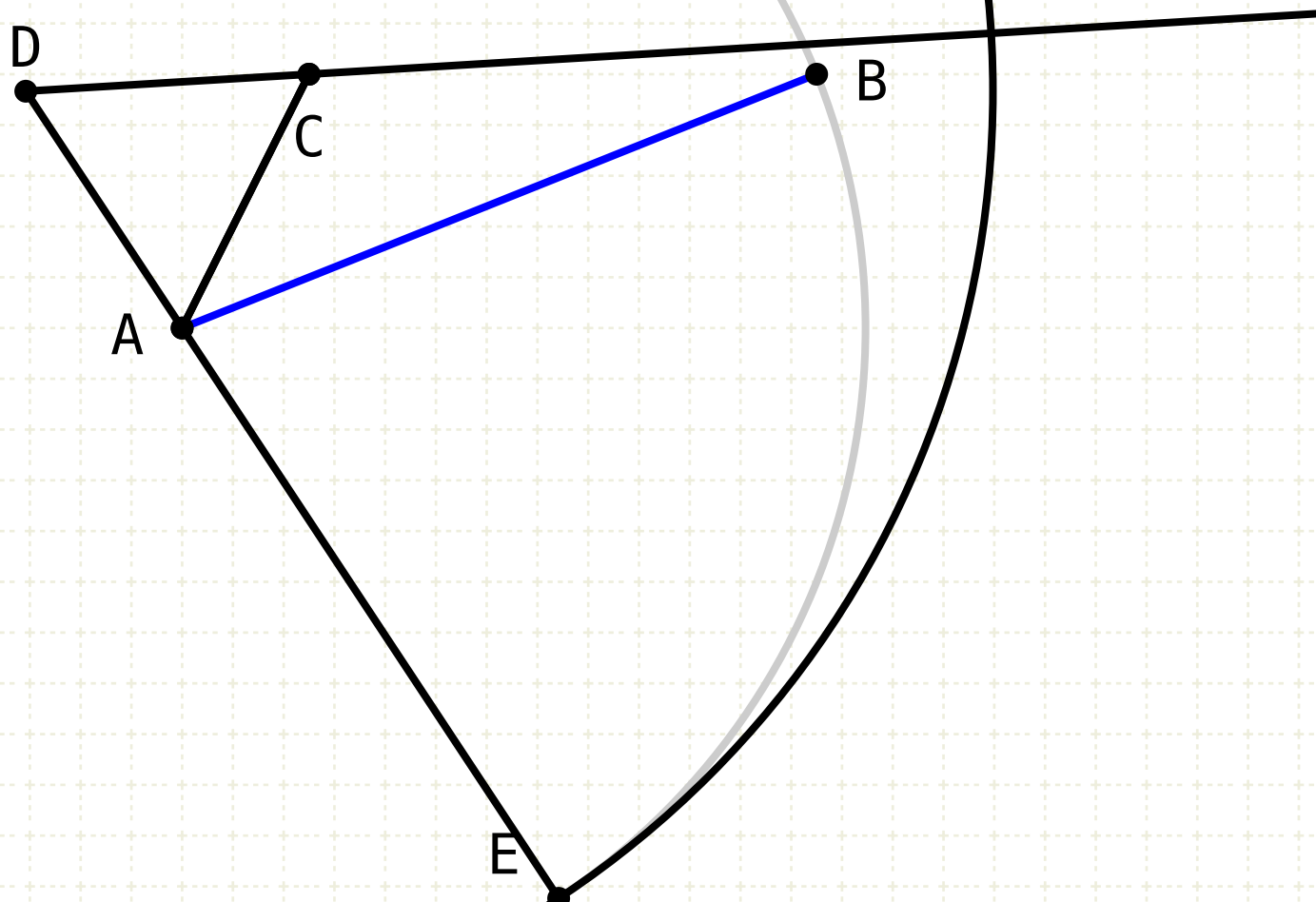
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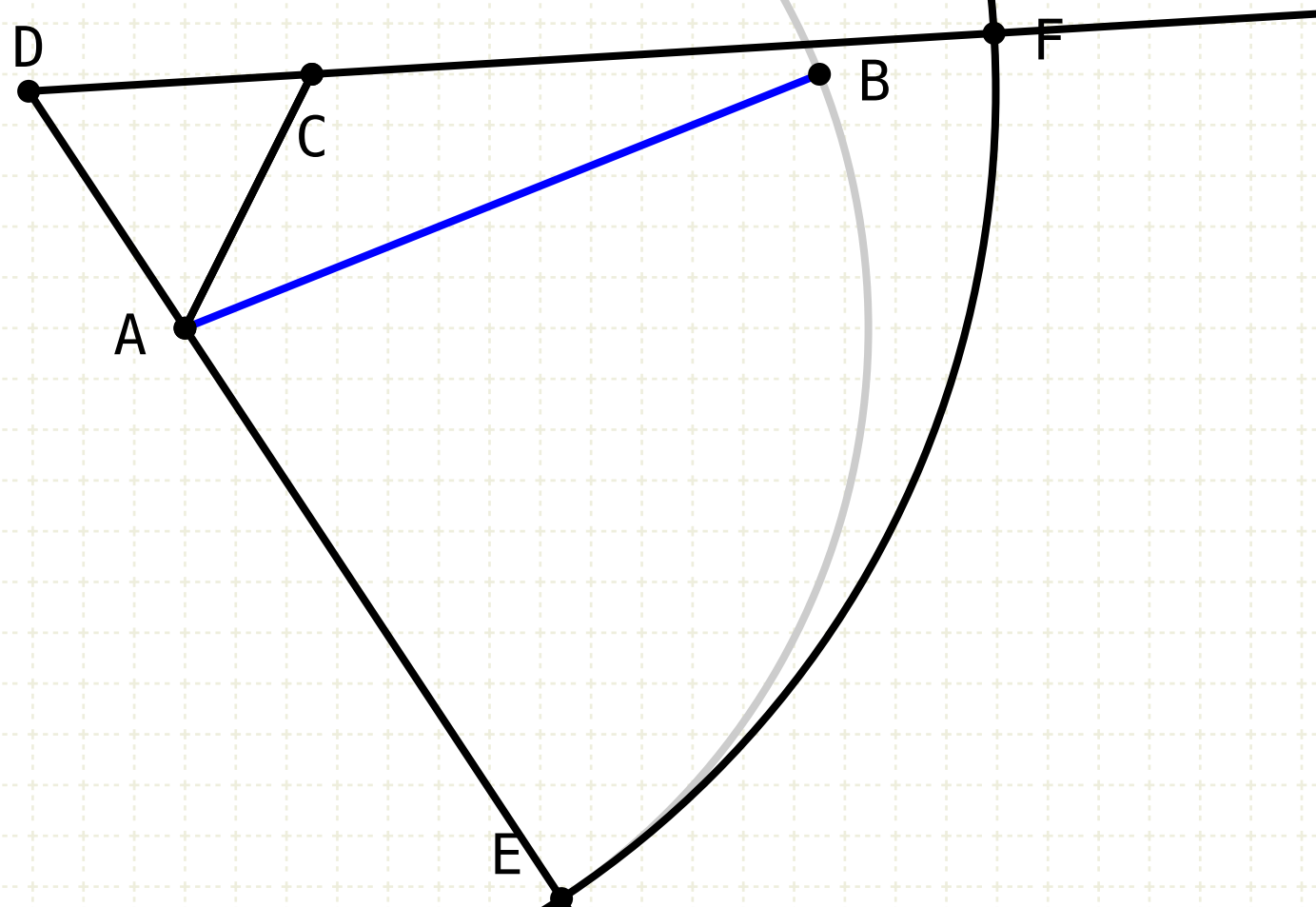
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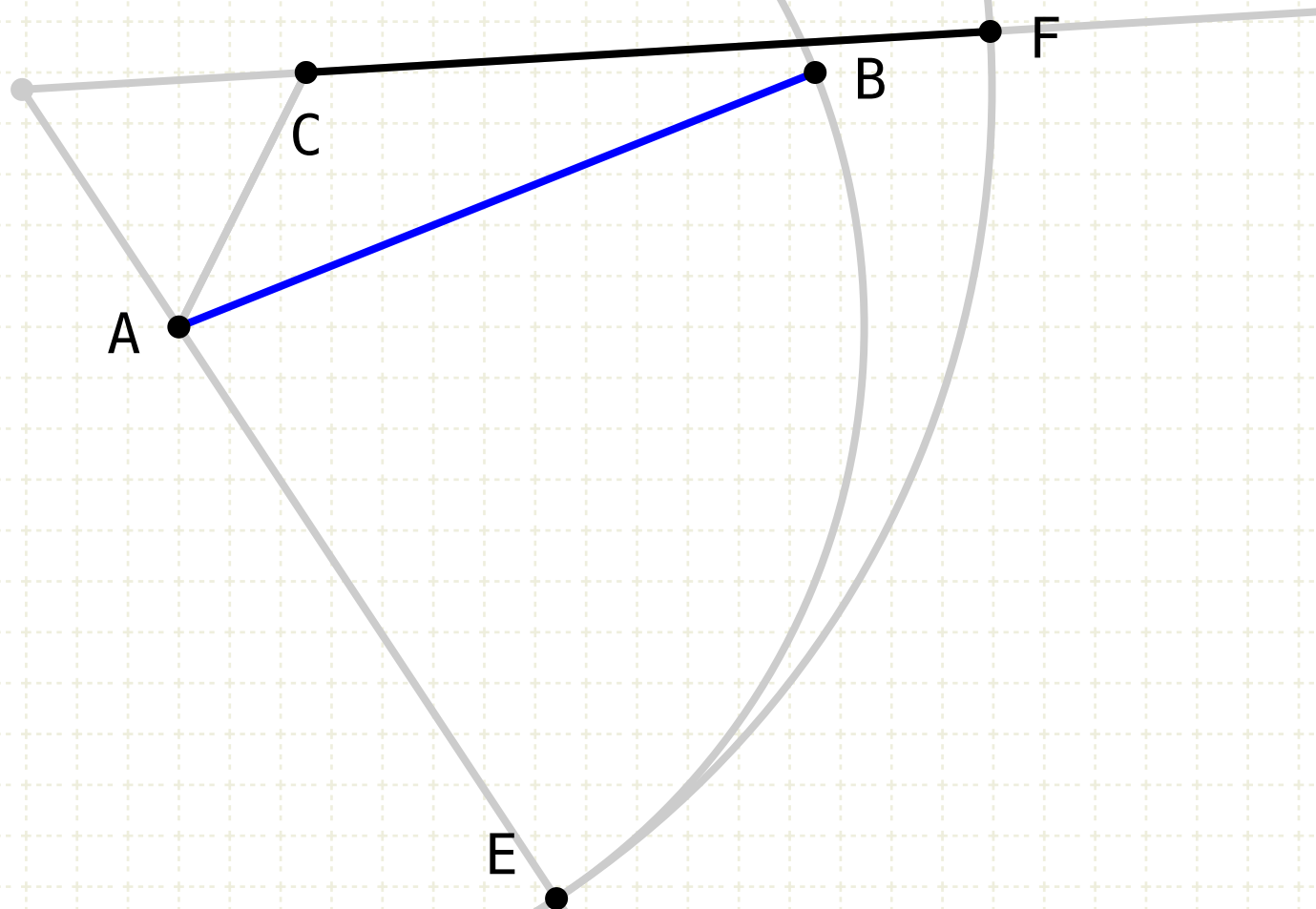
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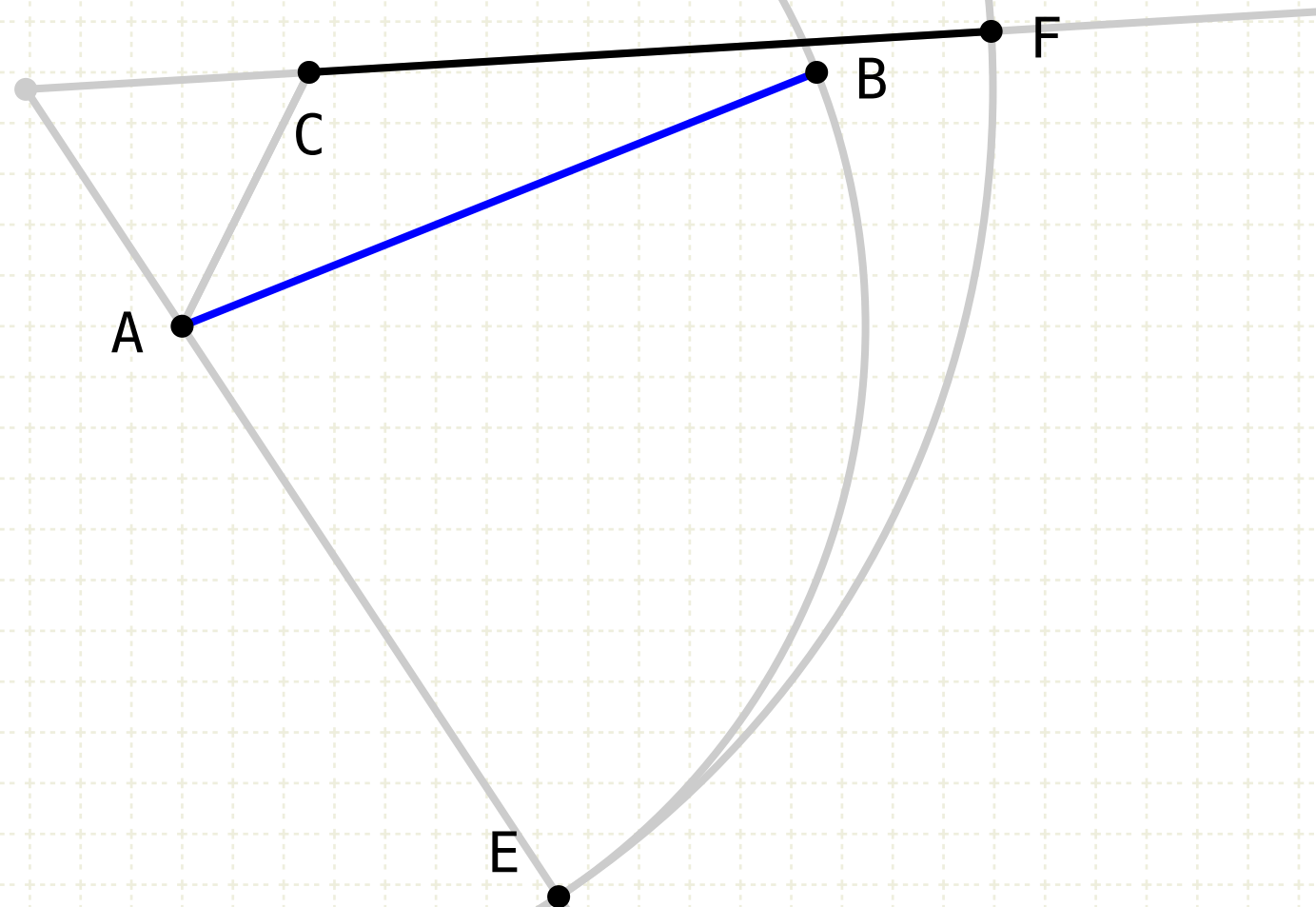
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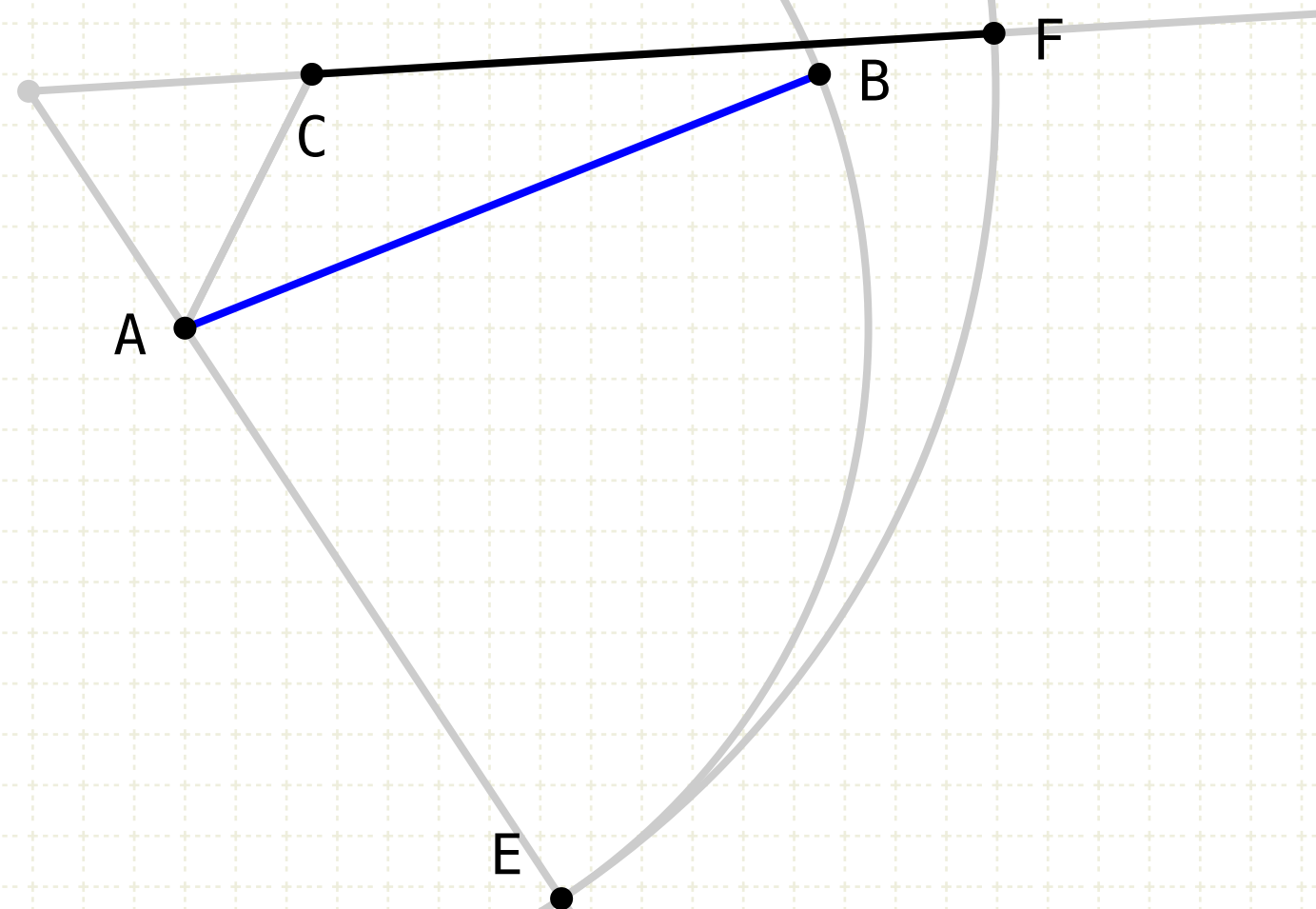
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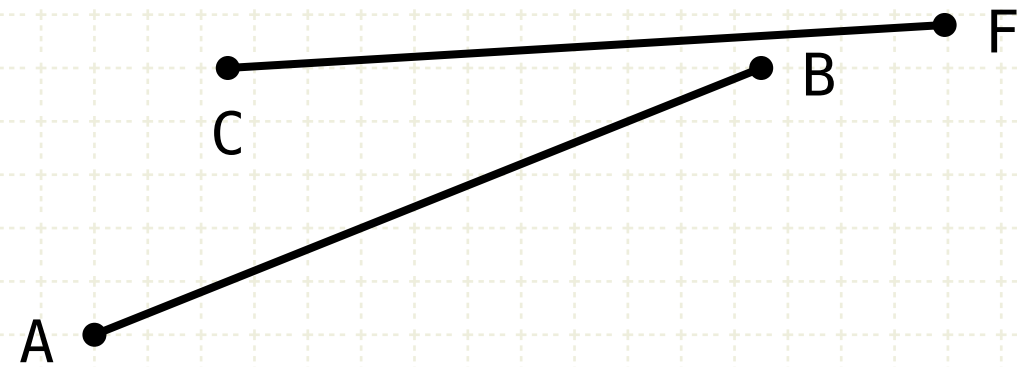
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