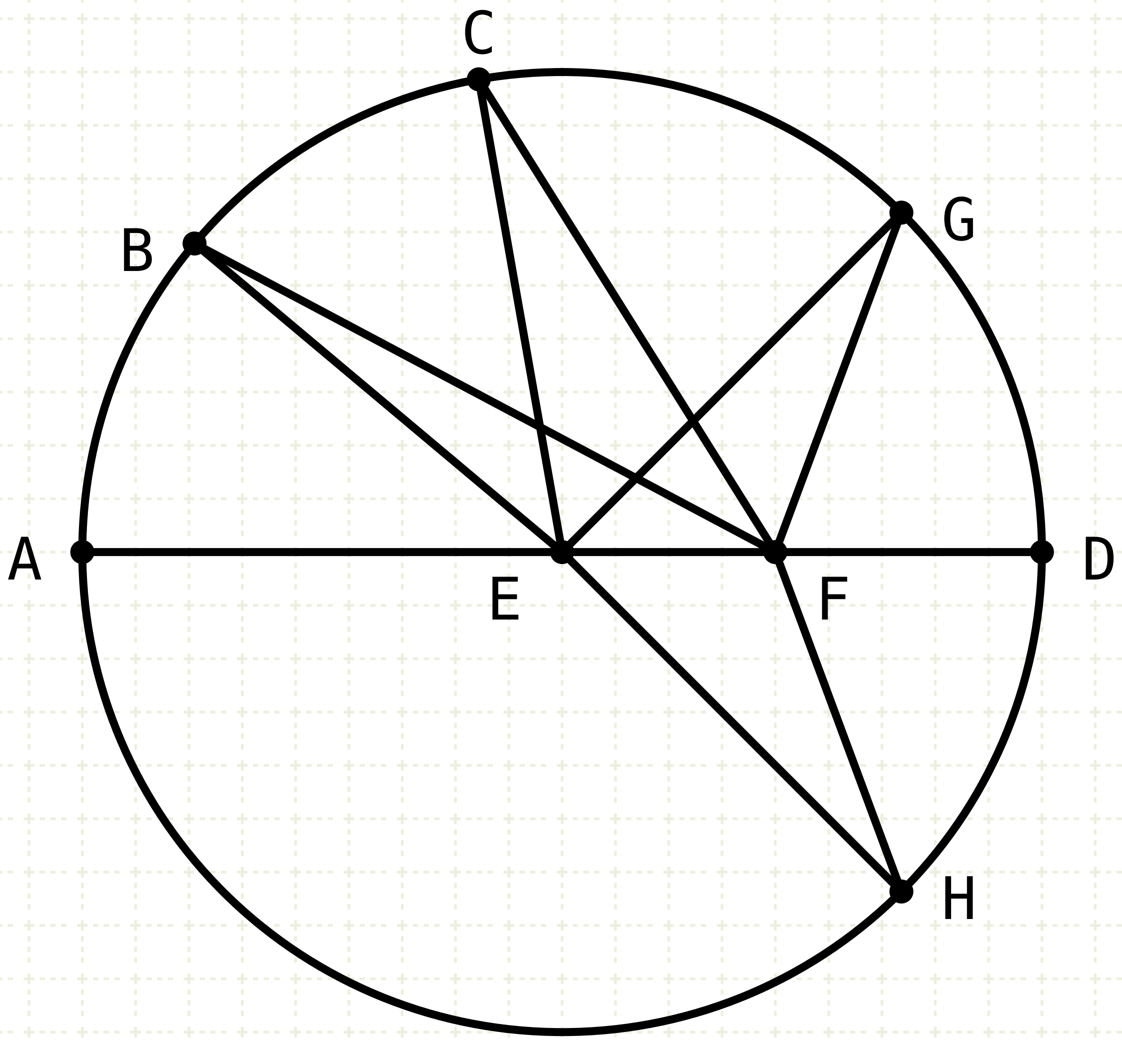

Euclid's Elements

Book III

A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)

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Proposition 10 of Book III

A circle does not cut a circle at more points than two.

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Proof by Contradiction

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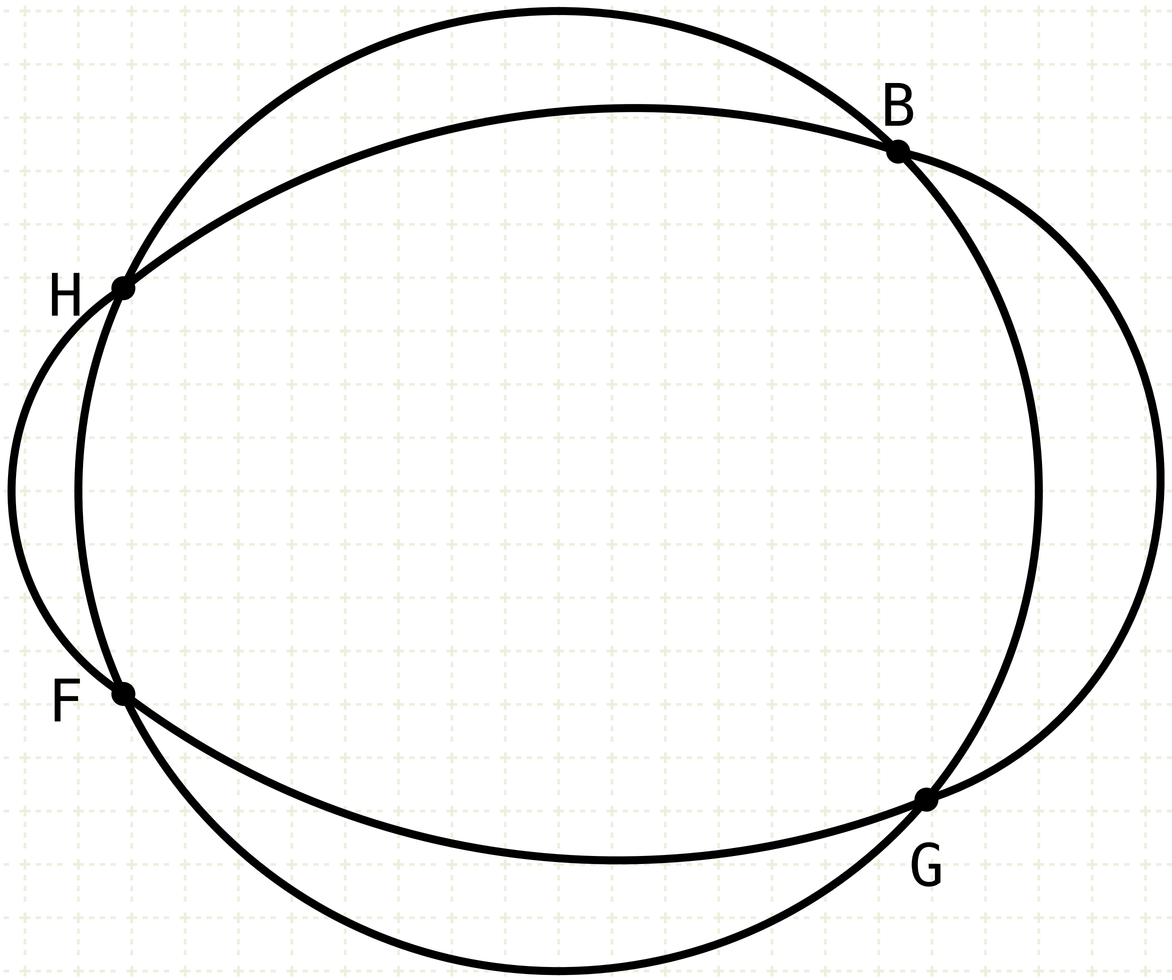

Proof by Contradiction

Assume we have two circles which intersect at 4 points

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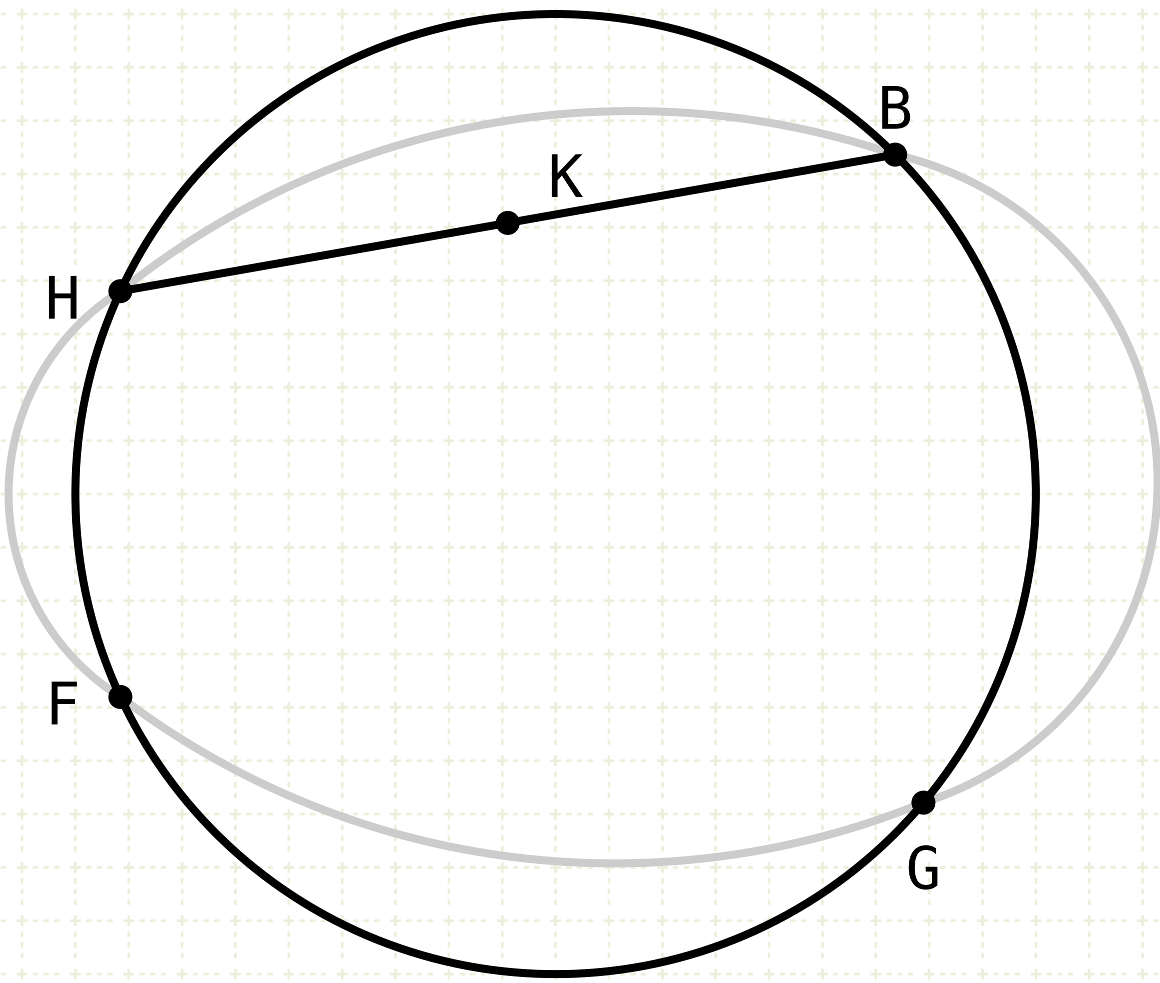

HK = KB

Proof by Contradiction

Assume we have two circles which intersect at 4 points

Join BH and bisect at point K

Proposition 10 of Book III

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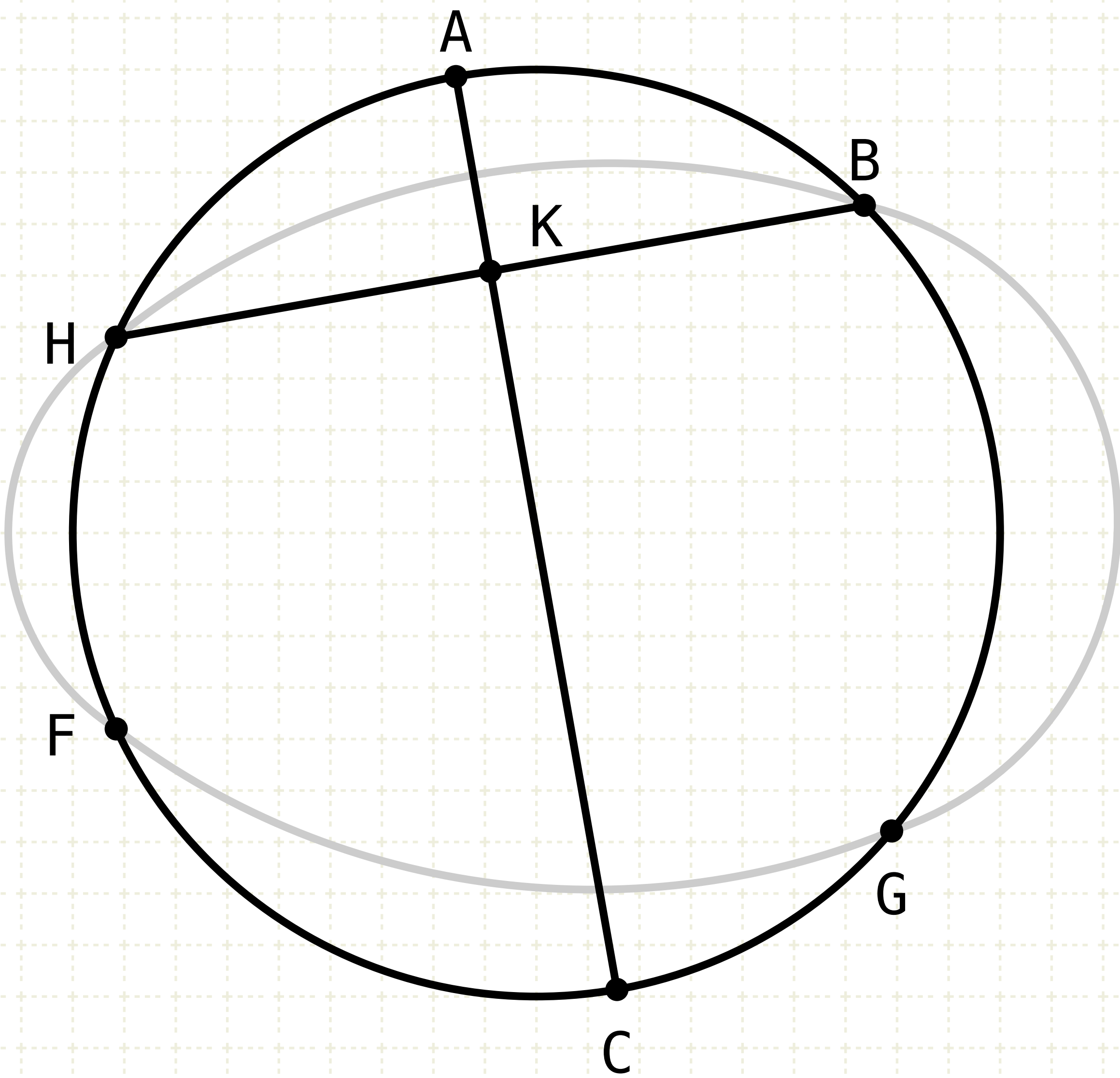

HK = KB

AC ⊥ HB

Proof by Contradiction

Assume we have two circles which intersect at 4 points

Join BH and bisect at point K

Construct a line perpendicular to HB, at point K

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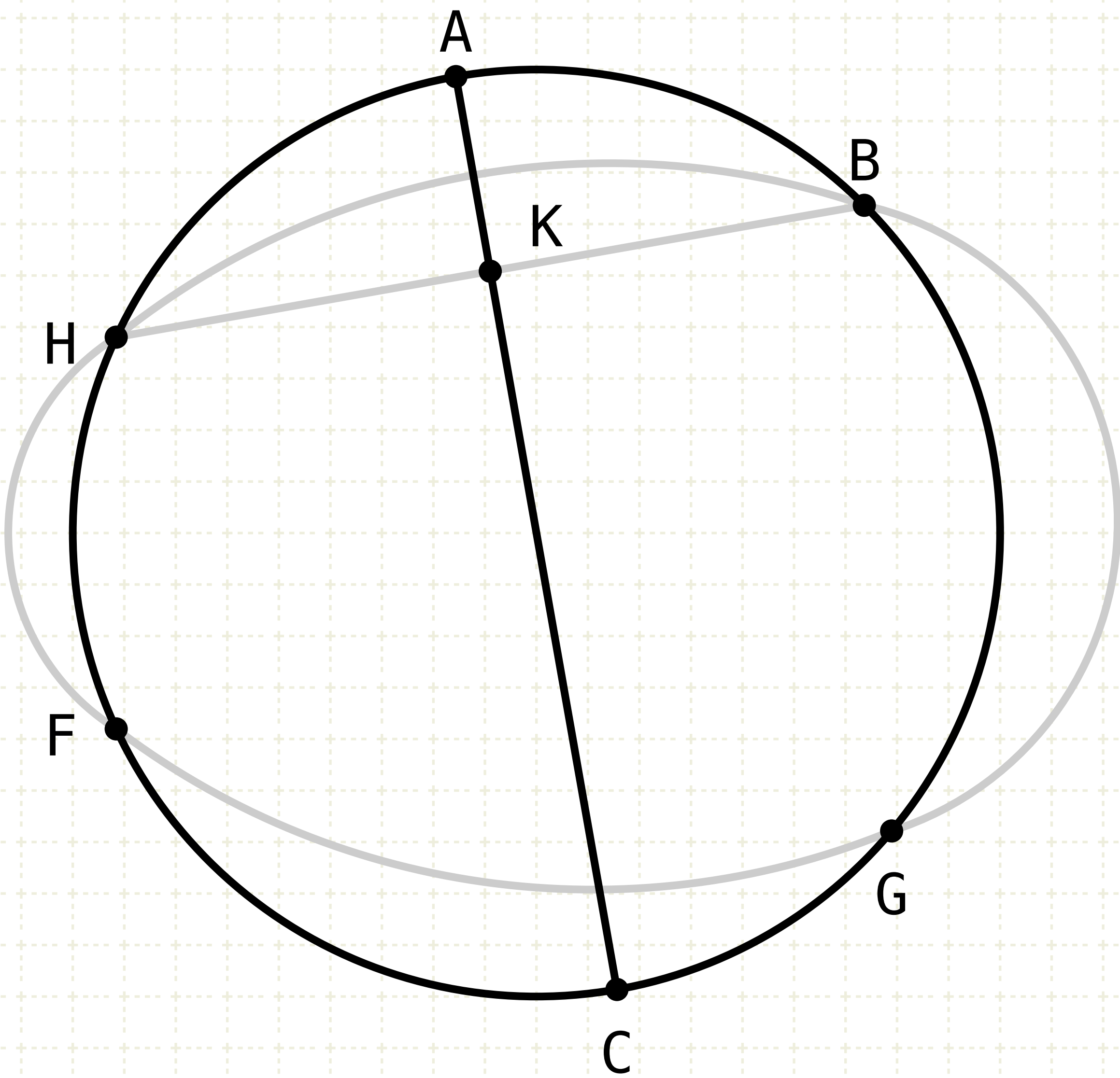

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Assume we have two circles which intersect at 4 points

Join BH and bisect at point K

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From proposition III·1, we know that the centre of the circle lies on the line AC

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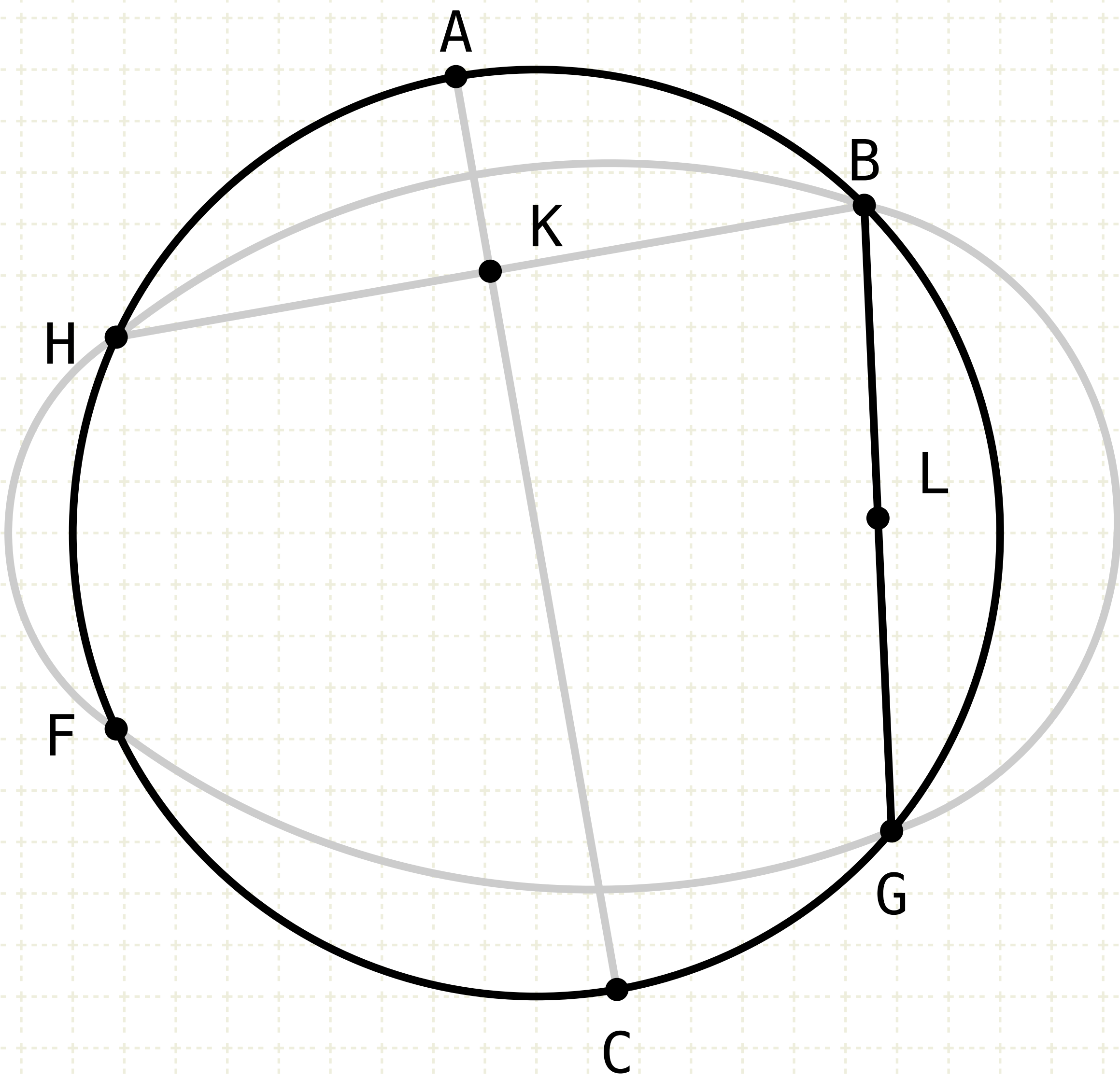

$HK = KB$

$AC \perp HB$

$BL = LG$

Proof by Contradiction

Assume we have two circles which intersect at 4 points

Join BH and bisect at point K

Construct a line perpendicular to HB, at point K

From proposition III·1, we know that the centre of the circle lies on the line AC

Join BG and bisect at point L

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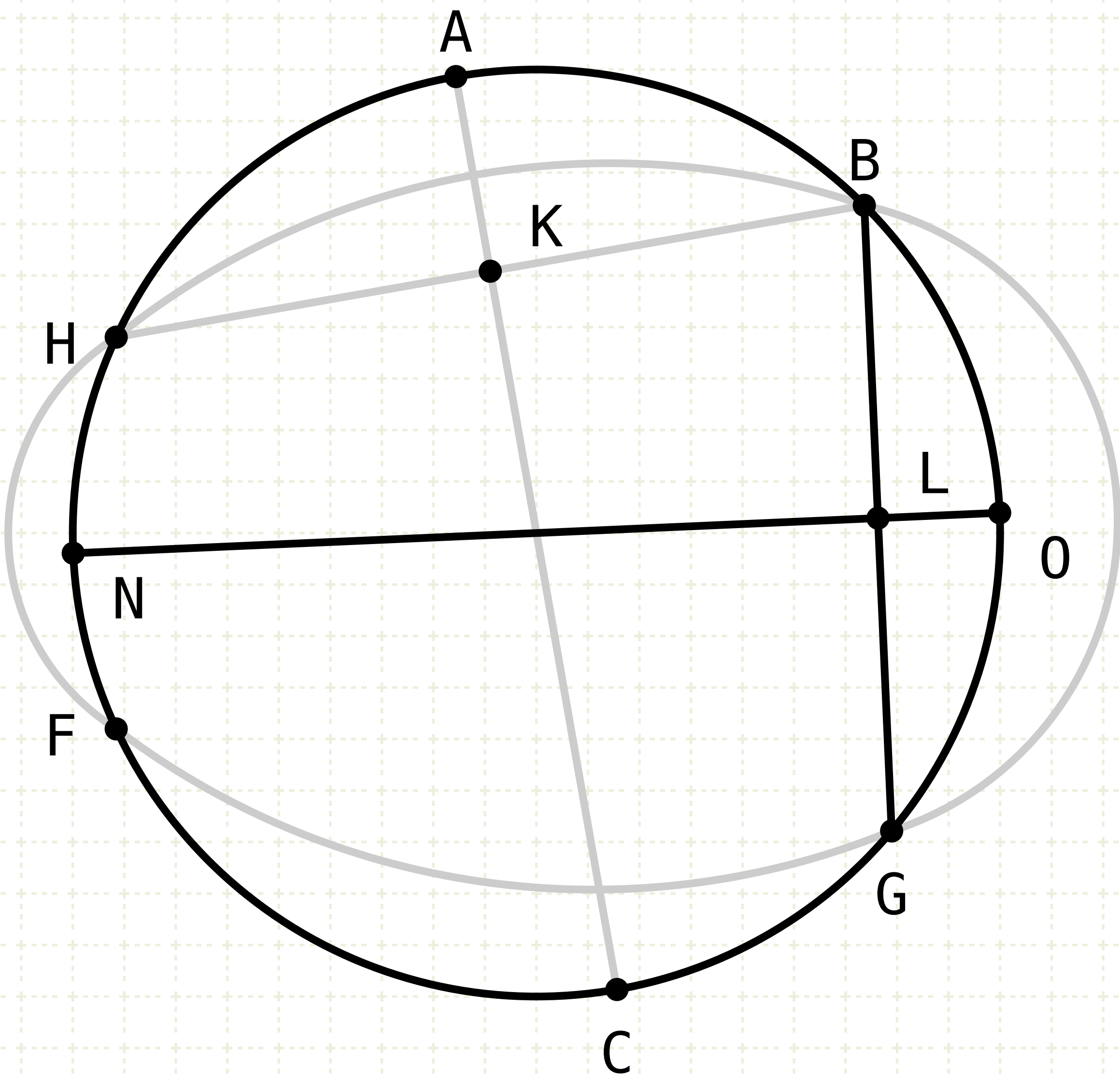

HK = KB
AC ⊥ HB
BL = LG
NO ⊥ BG

Proof by Contradiction

Assume we have two circles which intersect at 4 points

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Construct a line perpendicular to HB, at point K

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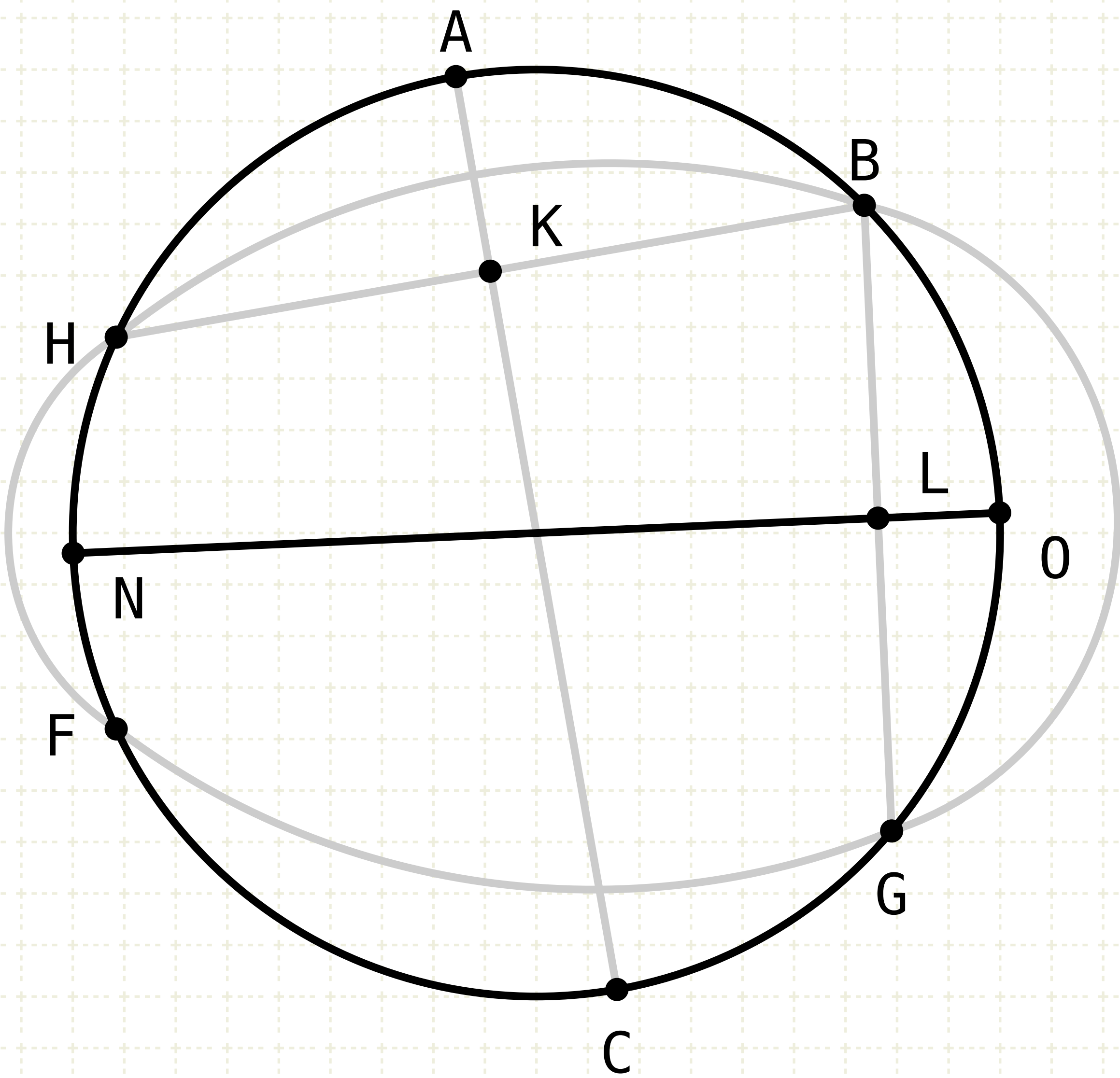

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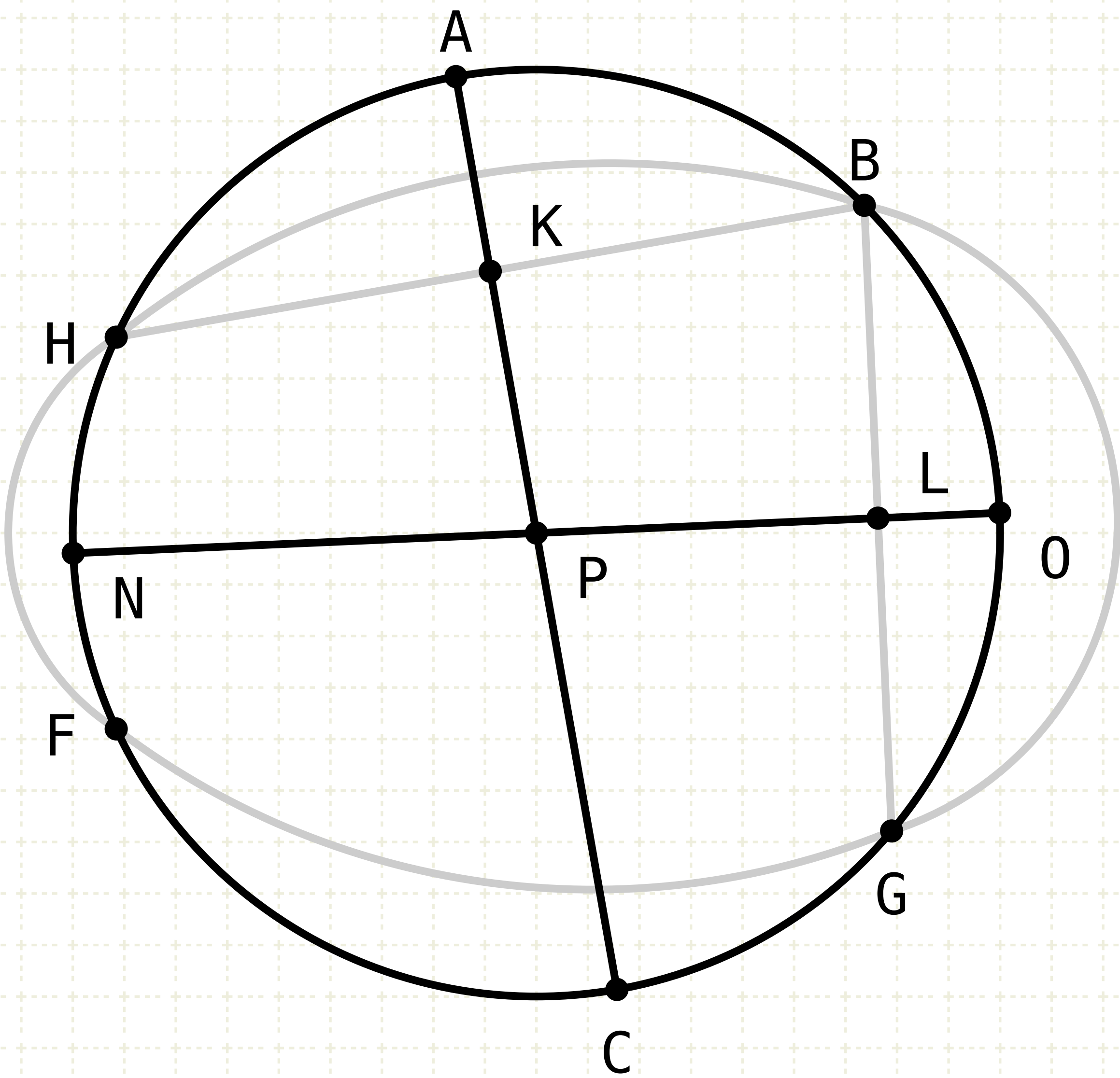

HK = KB

AC ⊥ HB

BL = LG

NO ⊥ BG

P is the centre of ABC

Proof by Contradiction

Assume we have two circles which intersect at 4 points

Join BH and bisect at point K

Construct a line perpendicular to HB, at point K

From proposition III·1, we know that the centre of the circle lies on the line AC

Join BG and bisect at point L

Construct a line perpendicular to BG, at point L

From proposition III·1, we know that the centre of the circle ABC lies on the line NO

If the centre of the circle is on AC and NO, then the centre of the circle ABC must be point P

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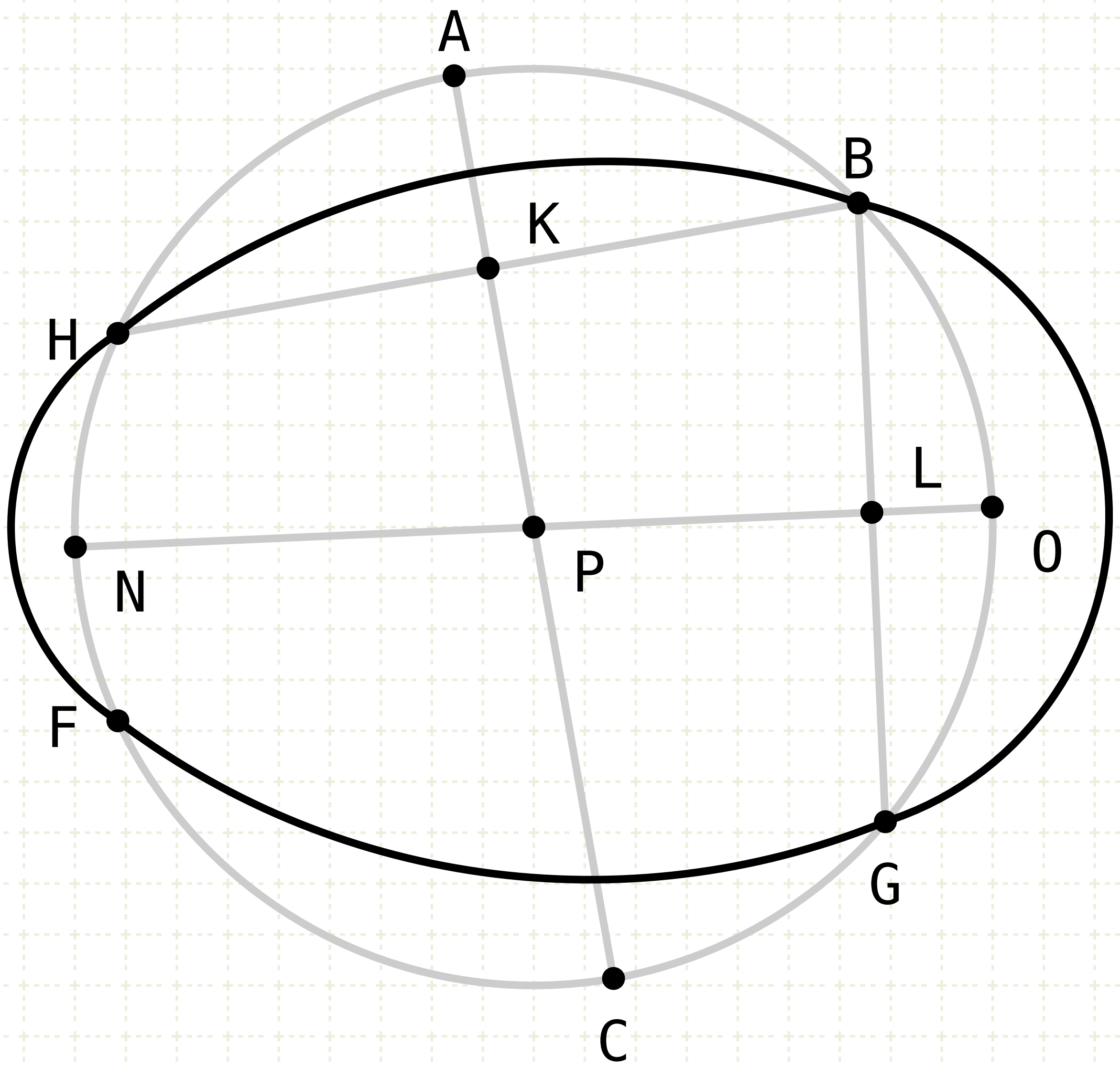

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Join BG and bisect at point L

Construct a line perpendicular to BG, at point L

From proposition III·1, we know that the centre of the circle ABC lies on the line NO

If the centre of the circle is on AC and NO, then the centre of the circle ABC must be point P

Using the same logic, it can be demonstrated that P is also the centre of DEF

Proposition 10 of Book III

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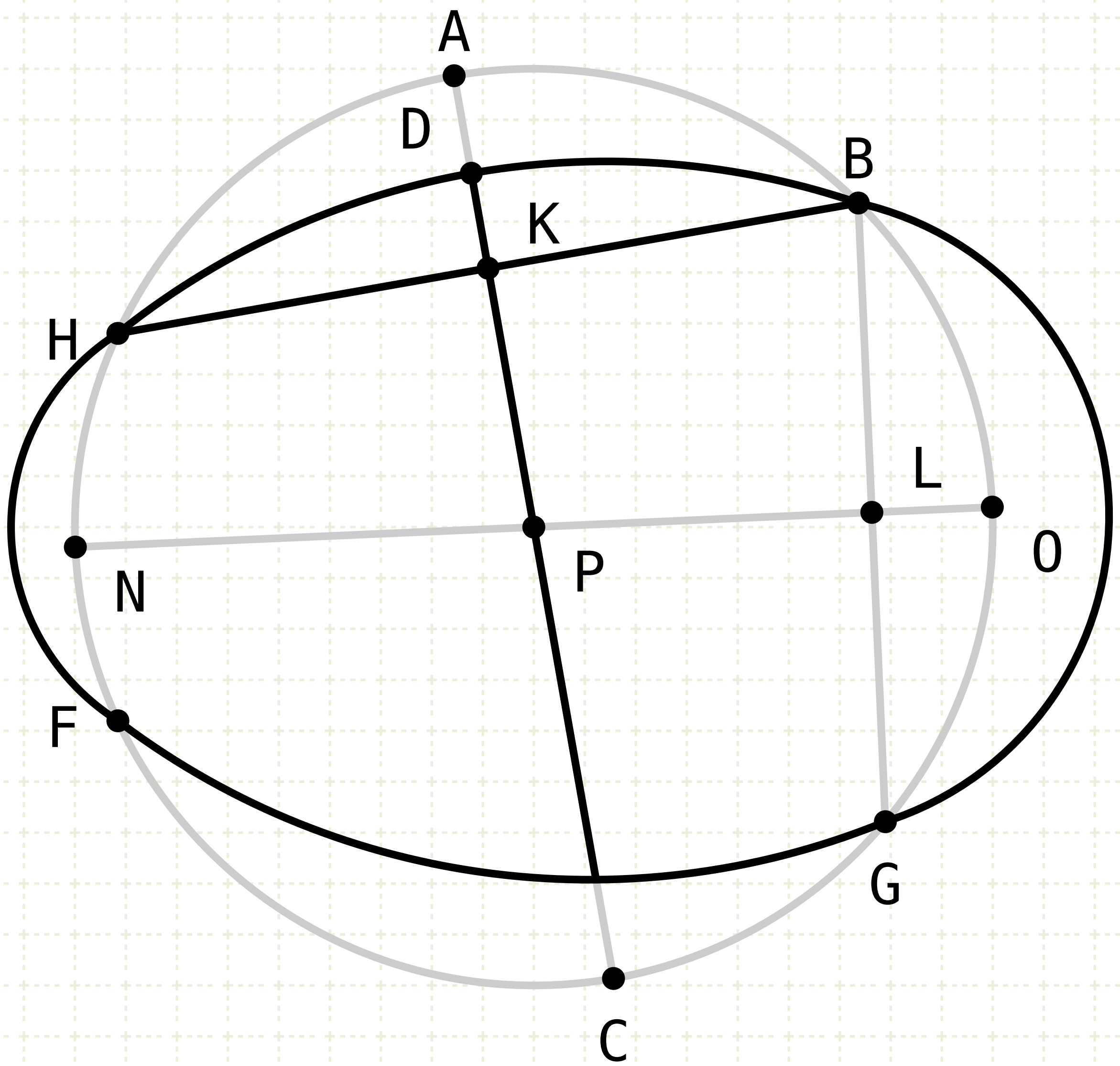

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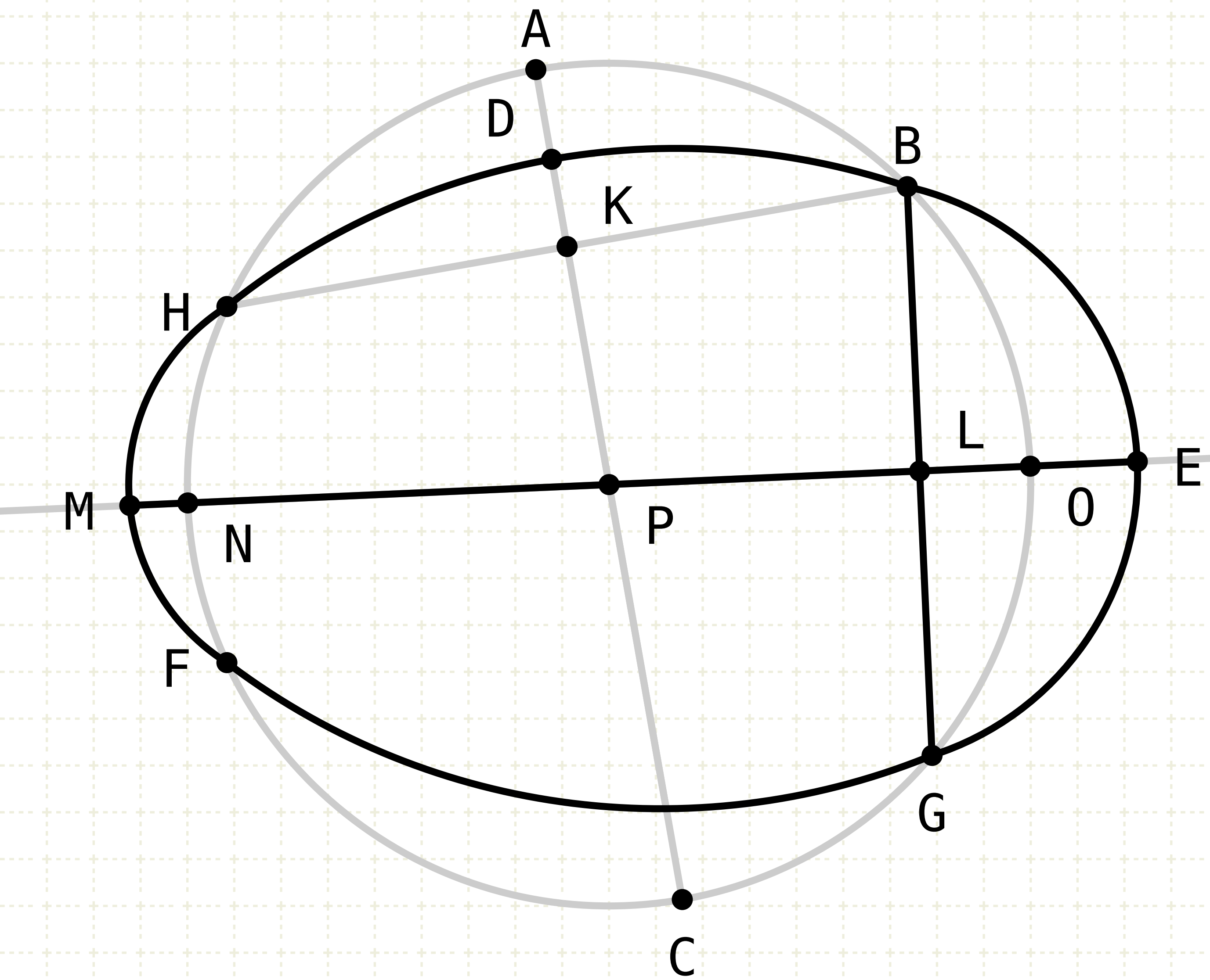

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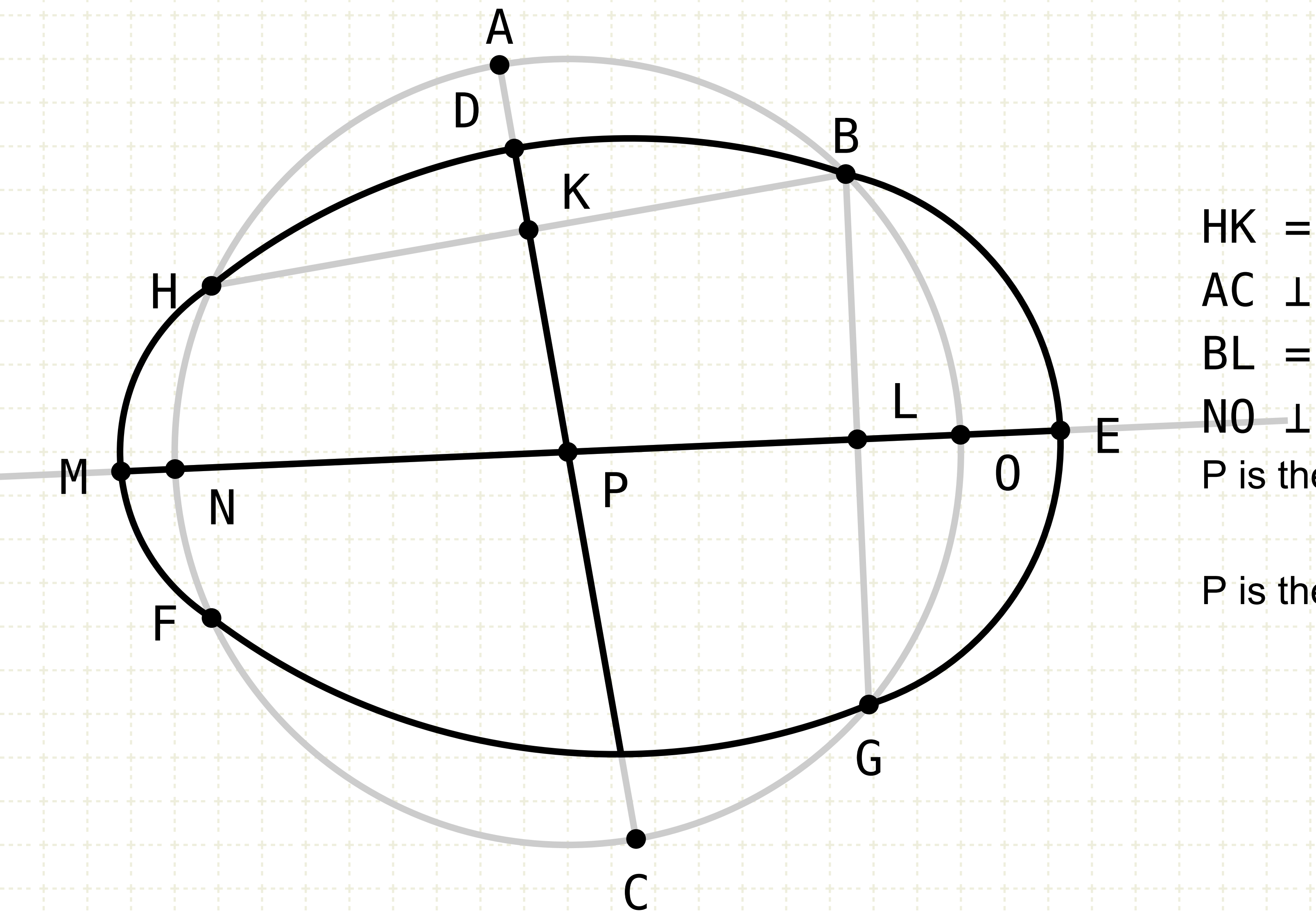

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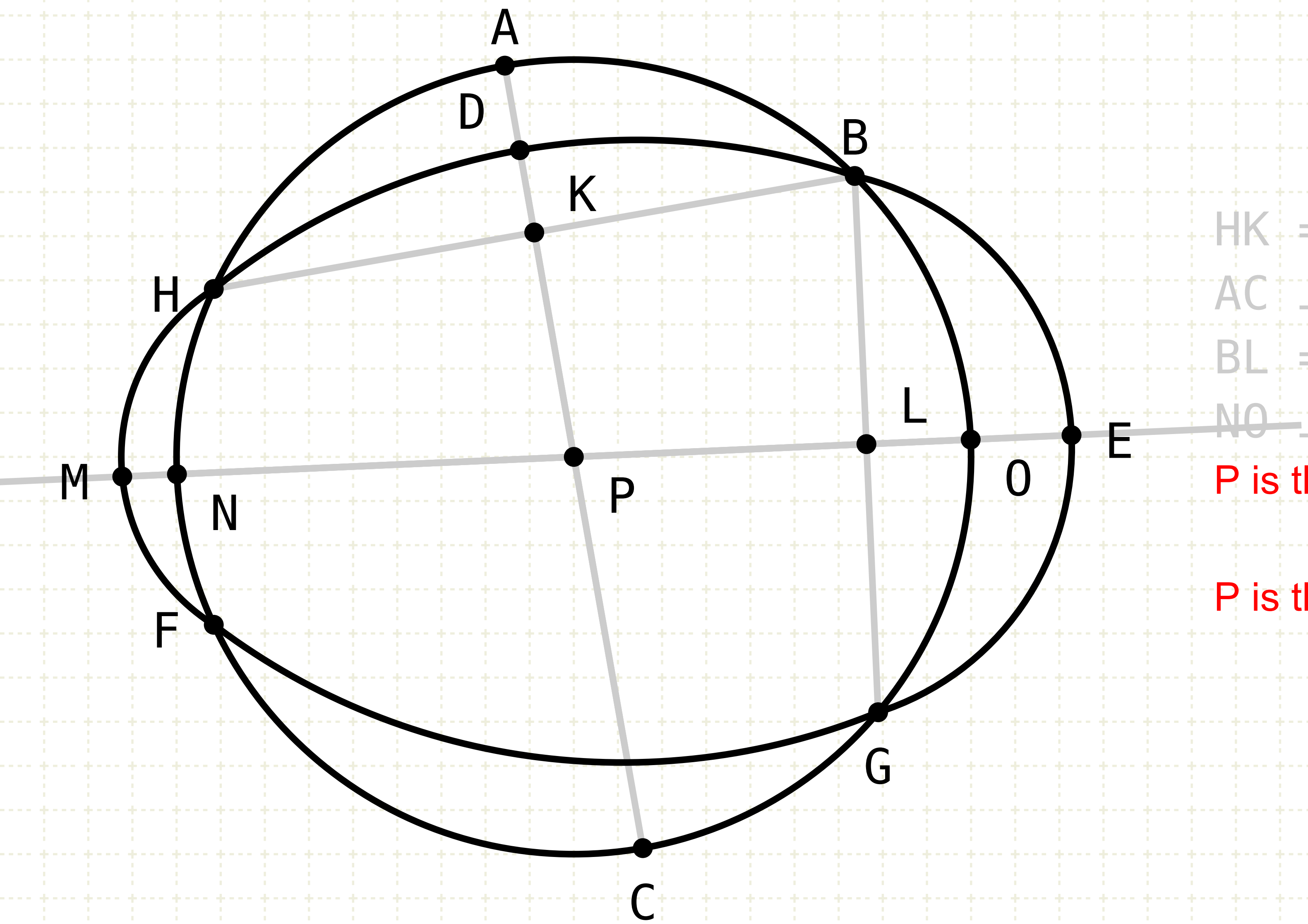

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But two intersecting circles cannot have the same centre (III·5), hence a contradiction

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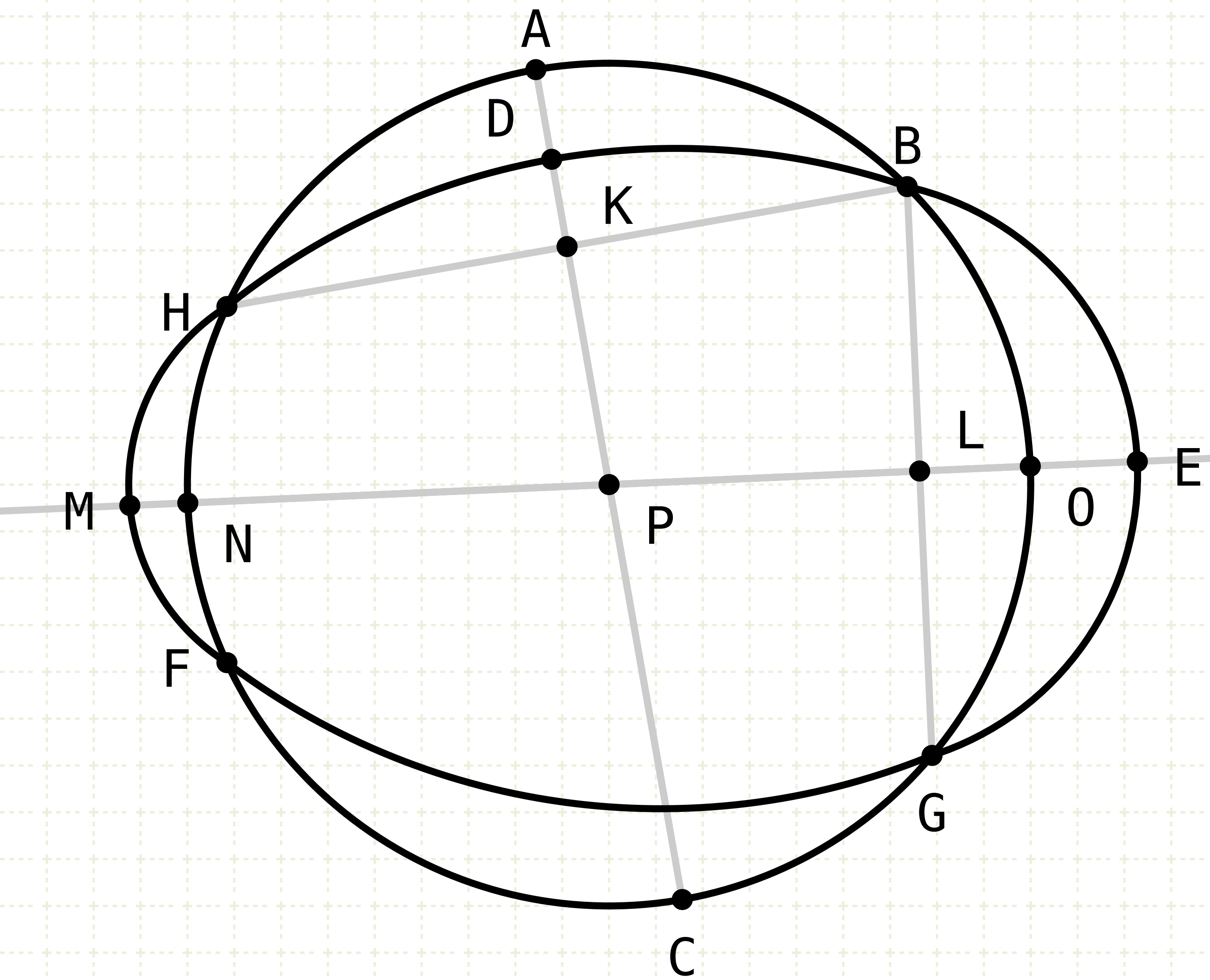

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Using the same logic, it can be demonstrated that P is also the centre of DEF

But two intersecting circles cannot have the same centre (III·5), hence a contradiction

Thus two circles cannot intersect at more than two points

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