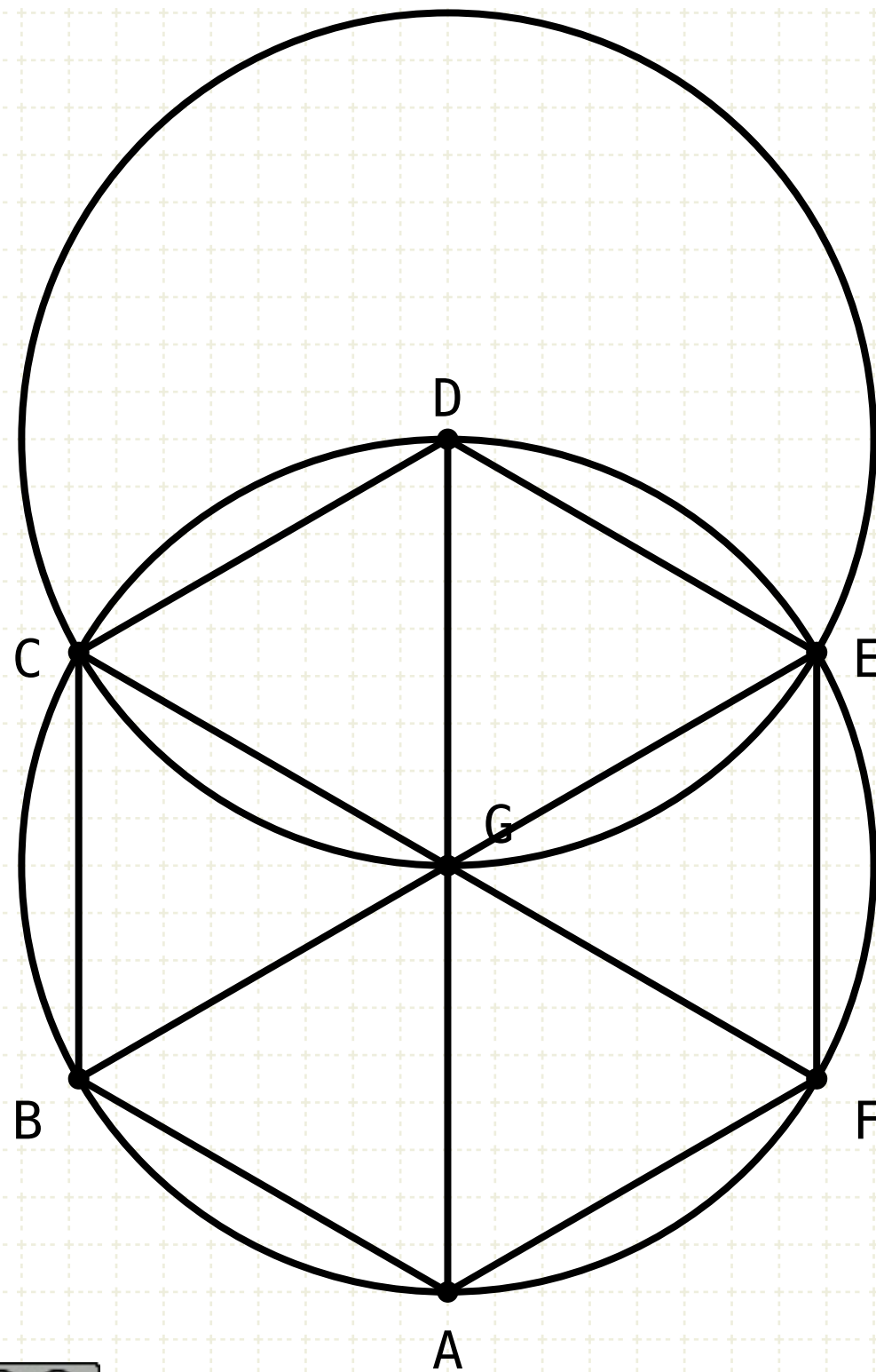


Euclid's Elements

Book IV



Philosophy (nature) is written in that great book which ever is before our eyes -- I mean the universe -- but we cannot understand it if we do not first learn the language and grasp the symbols in which it is written. The book is written in mathematical language, and the symbols are triangles, circles and other geometrical figures, without whose help it is impossible to comprehend a single word of it - without which one wanders in vain through a dark labyrinth.

Galileo Galilei



Proposition 8 of Book IV

In a given square, to inscribe a circle.



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3	About a given circle to circumscribe a triangle equiangular with a given triangle	13	In a given pentagon, which is equilateral and equiangular, to inscribe a circle
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5	About a given triangle to circumscribe a circle	15	In a given circle to inscribe an equilateral and equiangular hexagon
6	In a given circle to inscribe a square	16	In a given circle to inscribe a fifteen angled figure which shall be both equilateral and equiangular
7	About a given circle to circumscribe a square		
8	In a given square, to inscribe a circle		
9	About a given square, to circumscribe a circle		
10	To construct an isosceles triangle having each of the angles at the base double of the remaining one		



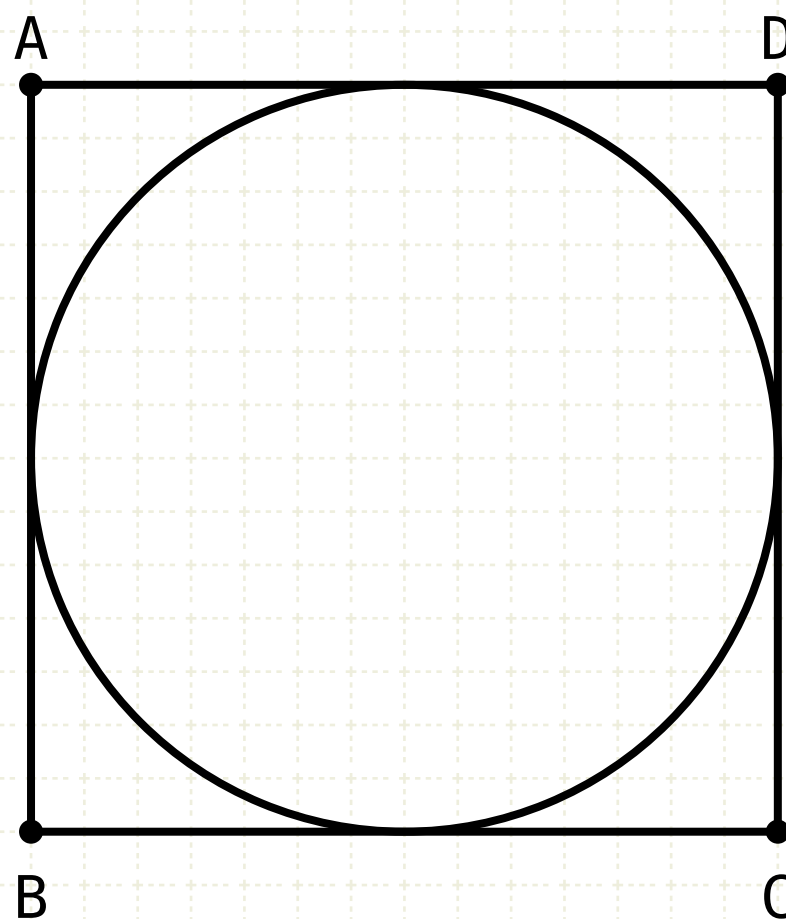
Proposition 8 of Book IV

In a given square, to inscribe a circle.



Proposition 8 of Book IV

In a given square, to inscribe a circle.



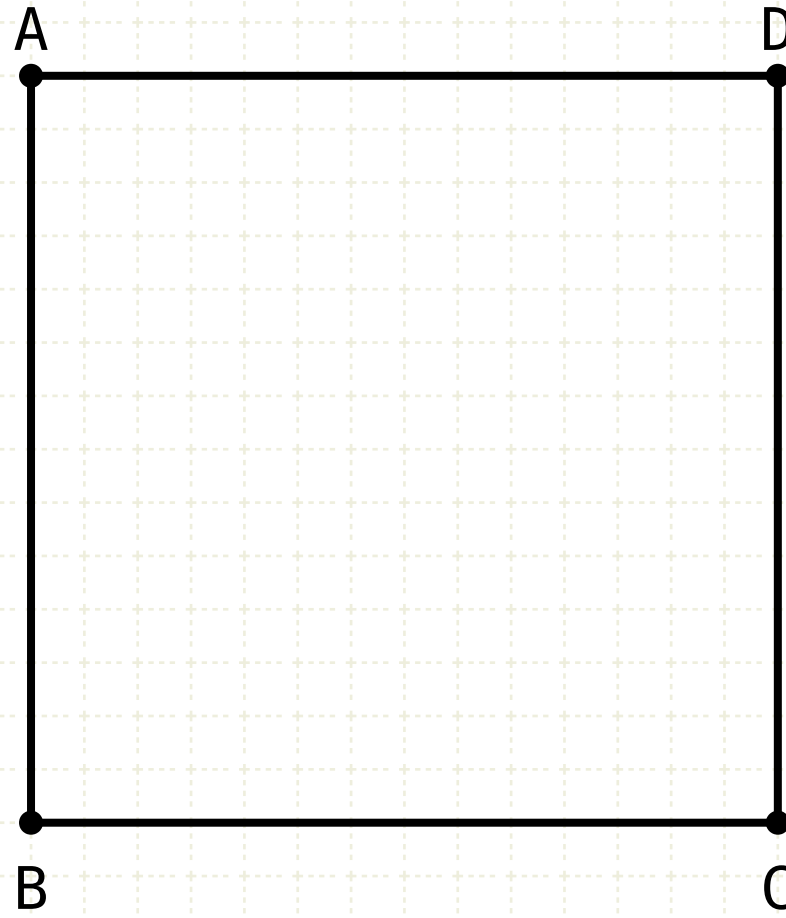
In other words

Given a square ABCD, draw a circle inside the square, touching each side of the square



Proposition 8 of Book IV

In a given square, to inscribe a circle.

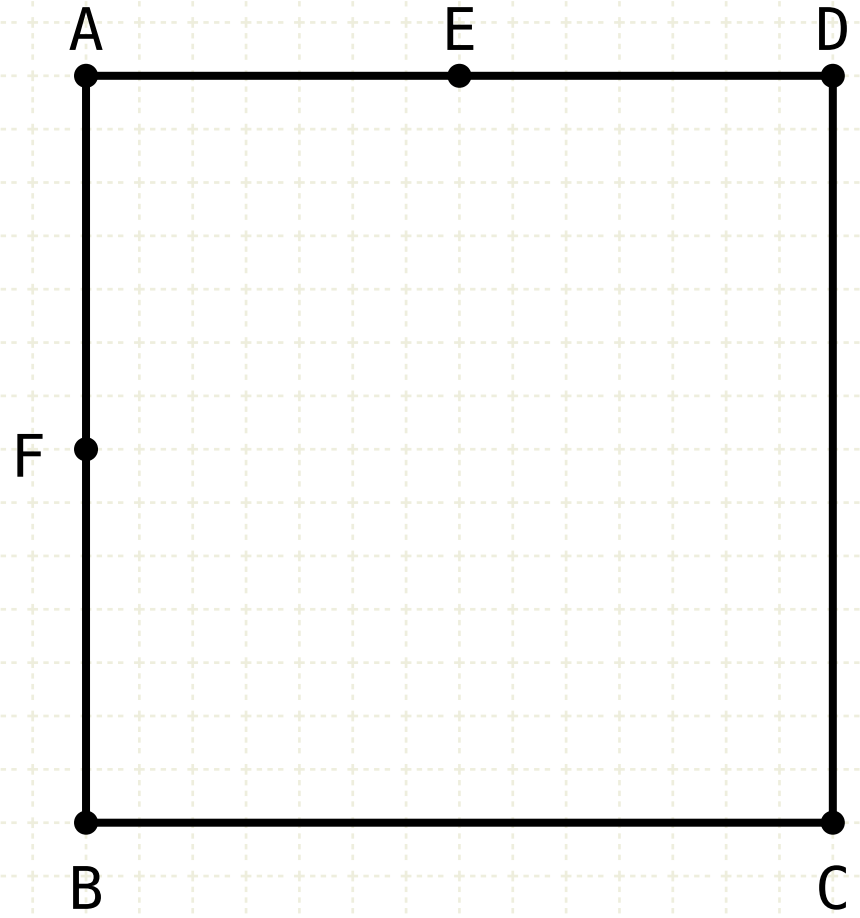


Construction



Proposition 8 of Book IV

In a given square, to inscribe a circle.



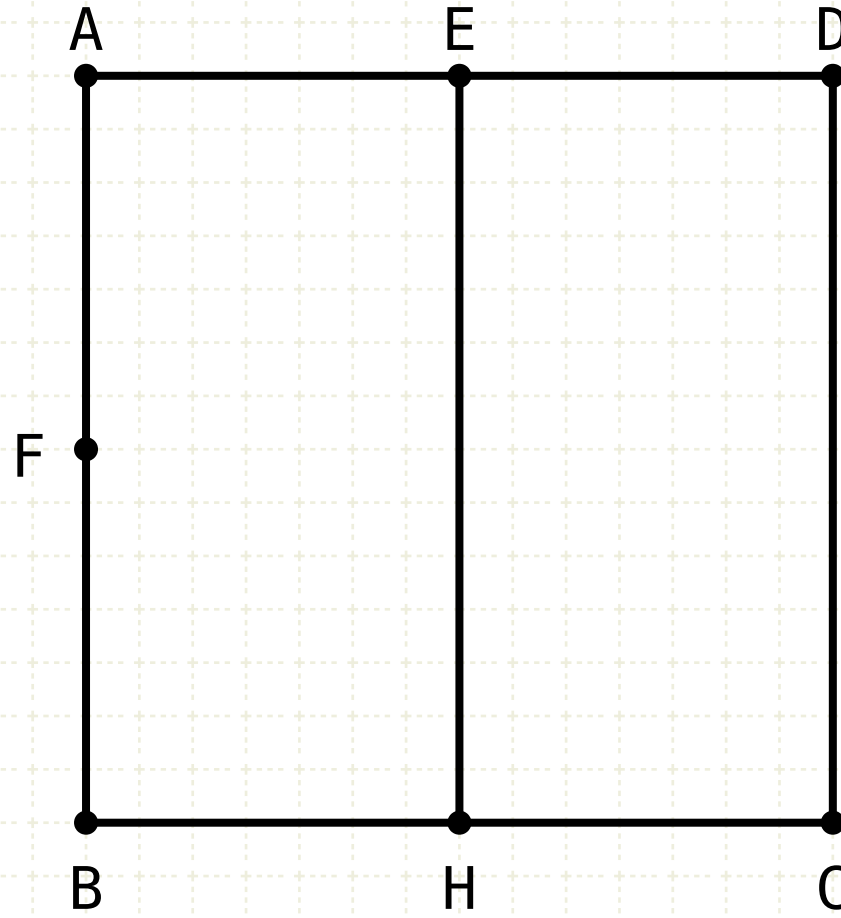
Construction

Bisect the lines AB and AD at points E and F (I·10)



Proposition 8 of Book IV

In a given square, to inscribe a circle.



Construction

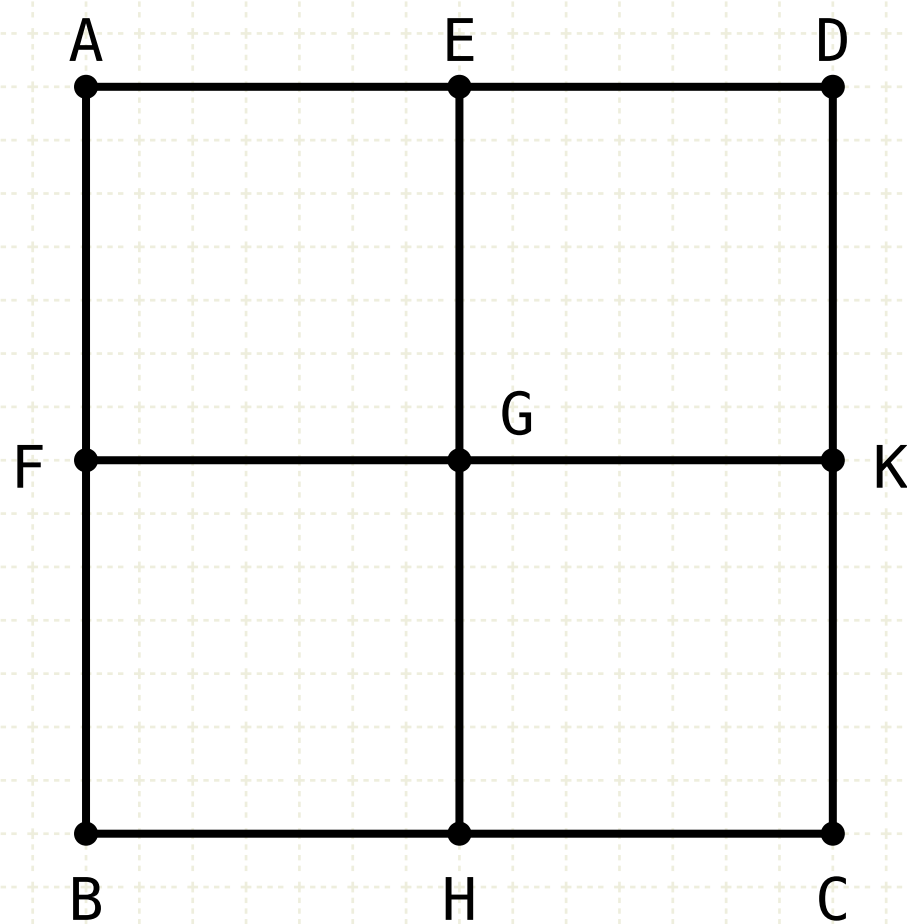
Bisect the lines AB and AD at points E and F (I·10)

Draw EH parallel to AB (I·31)



Proposition 8 of Book IV

In a given square, to inscribe a circle.



Construction

Bisect the lines AB and AD at points E and F (I·10)

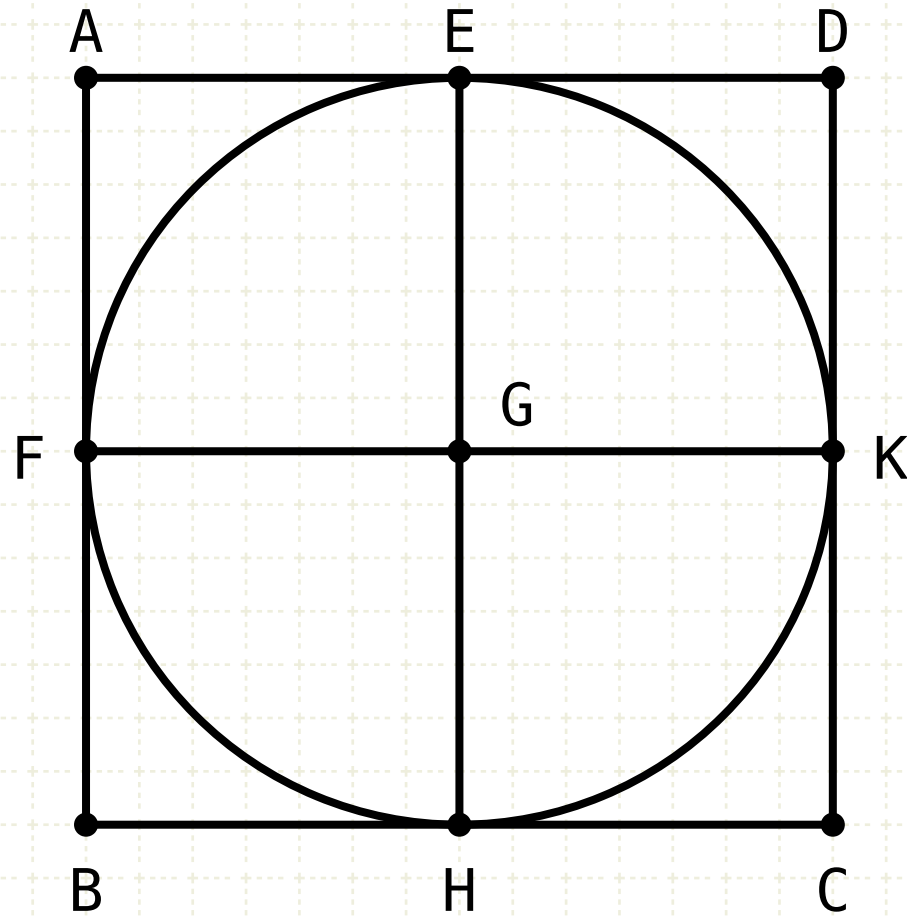
Draw EH parallel to AB (I·31)

Draw FK parallel to AD (I·31)



Proposition 8 of Book IV

In a given square, to inscribe a circle.



Construction

Bisect the lines AB and AD at points E and F (I·10)

Draw EH parallel to AB (I·31)

Draw FK parallel to AD (I·31)

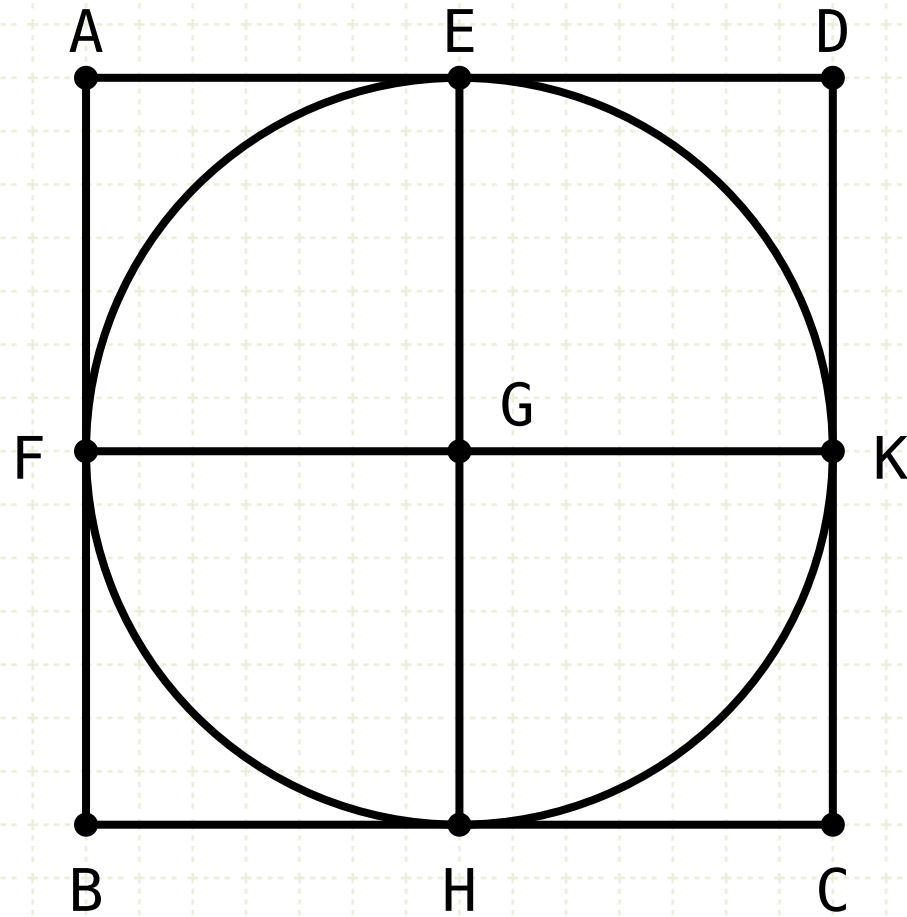
Draw a circle with centre G, and radius GE

This circle is inscribed in the square



Proposition 8 of Book IV

In a given square, to inscribe a circle.



Construction

Bisect the lines AB and AD at points E and F (I·10)

Draw EH parallel to AB (I·31)

Draw FK parallel to AD (I·31)

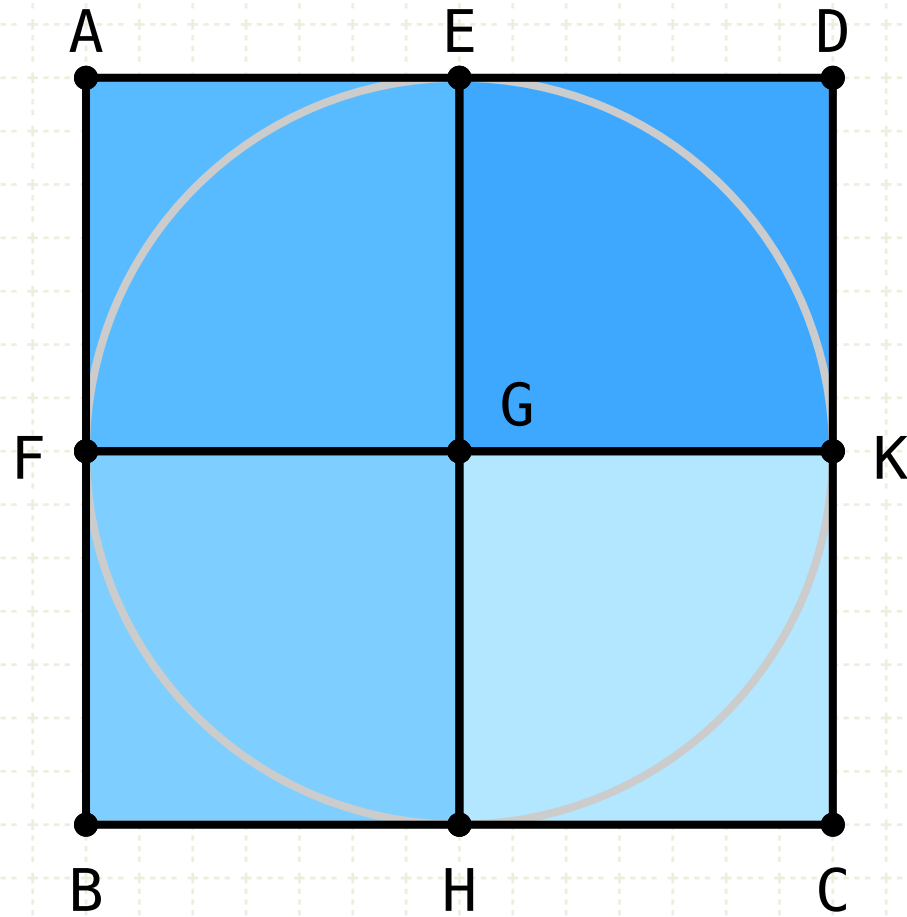
Draw a circle with centre G, and radius GE

This circle is inscribed in the square

Proof

Proposition 8 of Book IV

In a given square, to inscribe a circle.



$$AE = FG = BH$$

$$ED = GK = HC$$

$$AF = EG = DK$$

$$BF = HG = CK$$

Construction

Bisect the lines AB and AD at points E and F (I·10)

Draw EH parallel to AB (I·31)

Draw FK parallel to AD (I·31)

Draw a circle with centre G, and radius GE

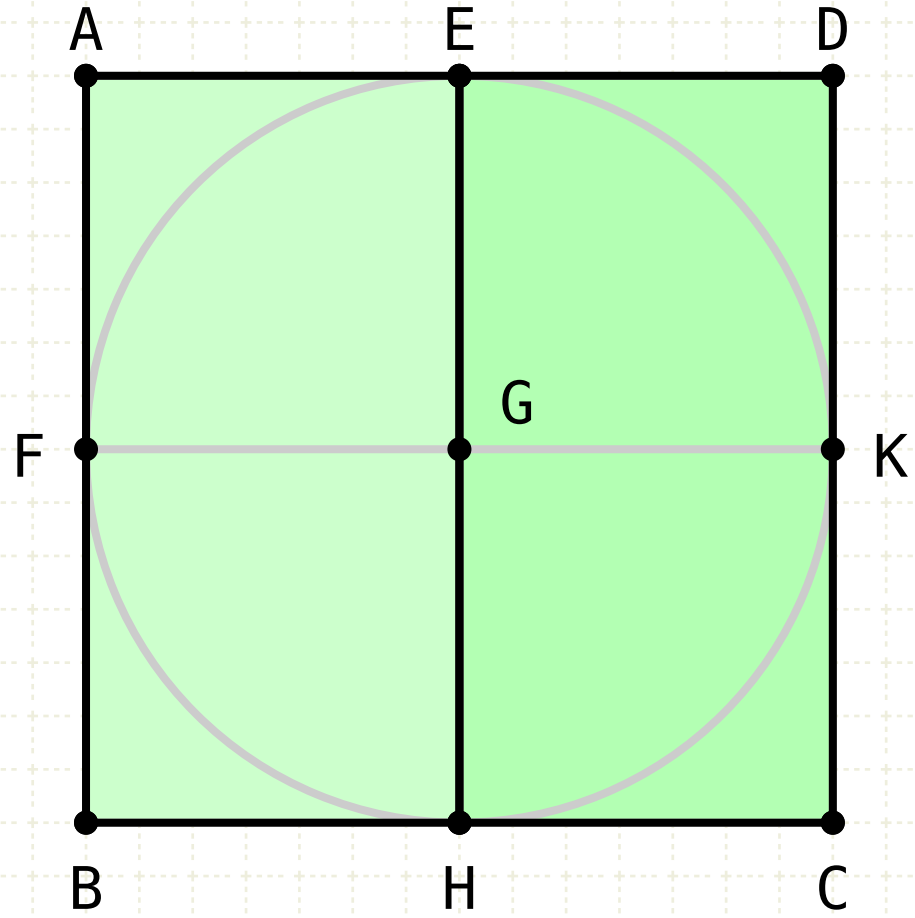
This circle is inscribed in the square

Proof

Because all of the lines are parallel to each other, each sub-figure (AK,KB,AH,HD,AG,GC,BG,GD) is also a parallelogram and their opposite sides are equal (I·34)

Proposition 8 of Book IV

In a given square, to inscribe a circle.



$$\begin{aligned} AE &= FG = BH \\ ED &= GK = HC \\ AF &= EG = DK \\ BF &= HG = CK \\ AB &= EH = DC \end{aligned}$$

Construction

Bisect the lines AB and AD at points E and F (I·10)

Draw EH parallel to AB (I·31)

Draw FK parallel to AD (I·31)

Draw a circle with centre G, and radius GE

This circle is inscribed in the square

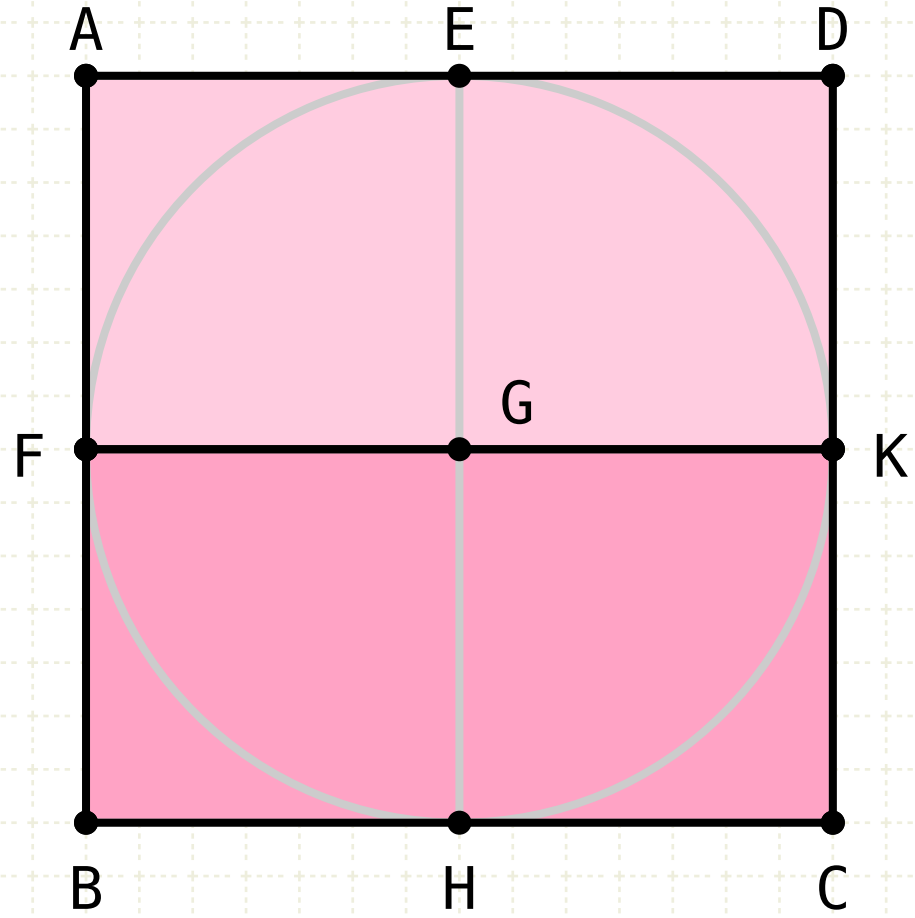
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Proposition 8 of Book IV

In a given square, to inscribe a circle.



$$\begin{aligned} AE &= FG = BH \\ ED &= GK = HC \\ AF &= EG = DK \\ BF &= HG = CK \\ AB &= EH = DC \\ AD &= FK = BC \end{aligned}$$

Construction

- Bisect the lines AB and AD at points E and F (I·10)
- Draw EH parallel to AB (I·31)
- Draw FK parallel to AD (I·31)
- Draw a circle with centre G, and radius GE
- This circle is inscribed in the square

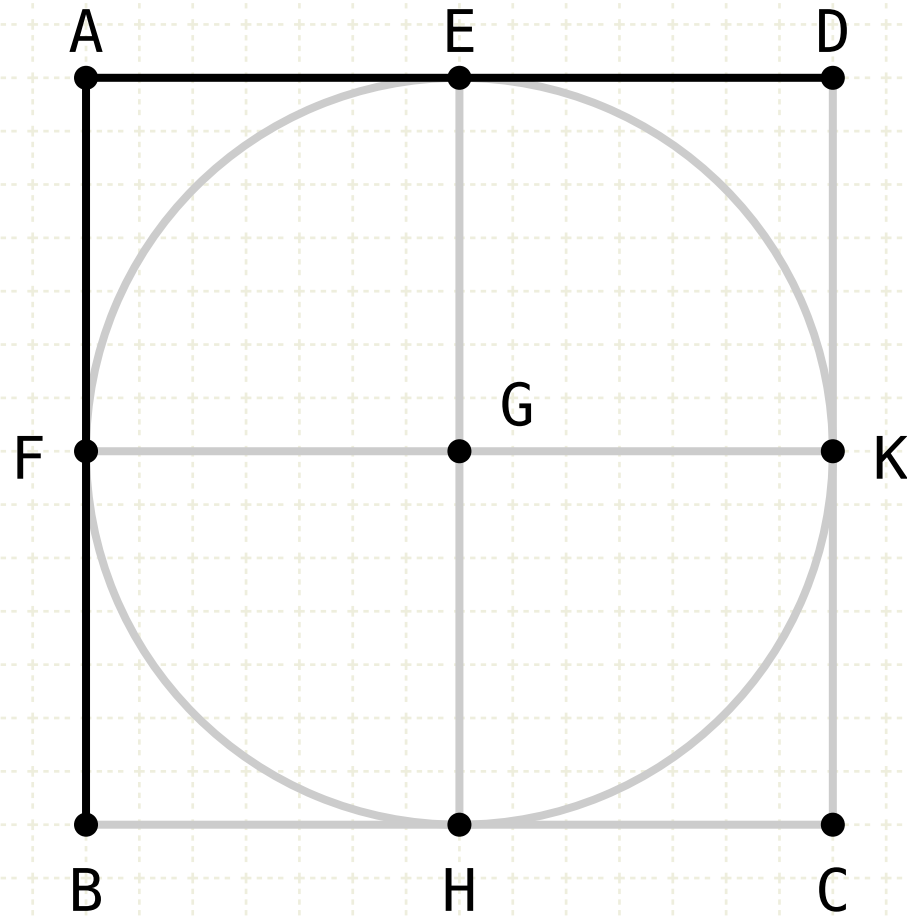
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Proposition 8 of Book IV

In a given square, to inscribe a circle.



$$AE = FG = BH$$

$$ED = GK = HC$$

$$AF = EG = DK$$

$$BF = HG = CK$$

$$AB = EH = DC$$

$$AD = FK = BC$$

$$AD = AB$$

$$AE = \frac{1}{2} AD, \quad AF = \frac{1}{2} AB$$

$$\therefore AE = AF$$

Construction

Bisect the lines AB and AD at points E and F (I·10)

Draw EH parallel to AB (I·31)

Draw FK parallel to AD (I·31)

Draw a circle with centre G, and radius GE

This circle is inscribed in the square

Proof

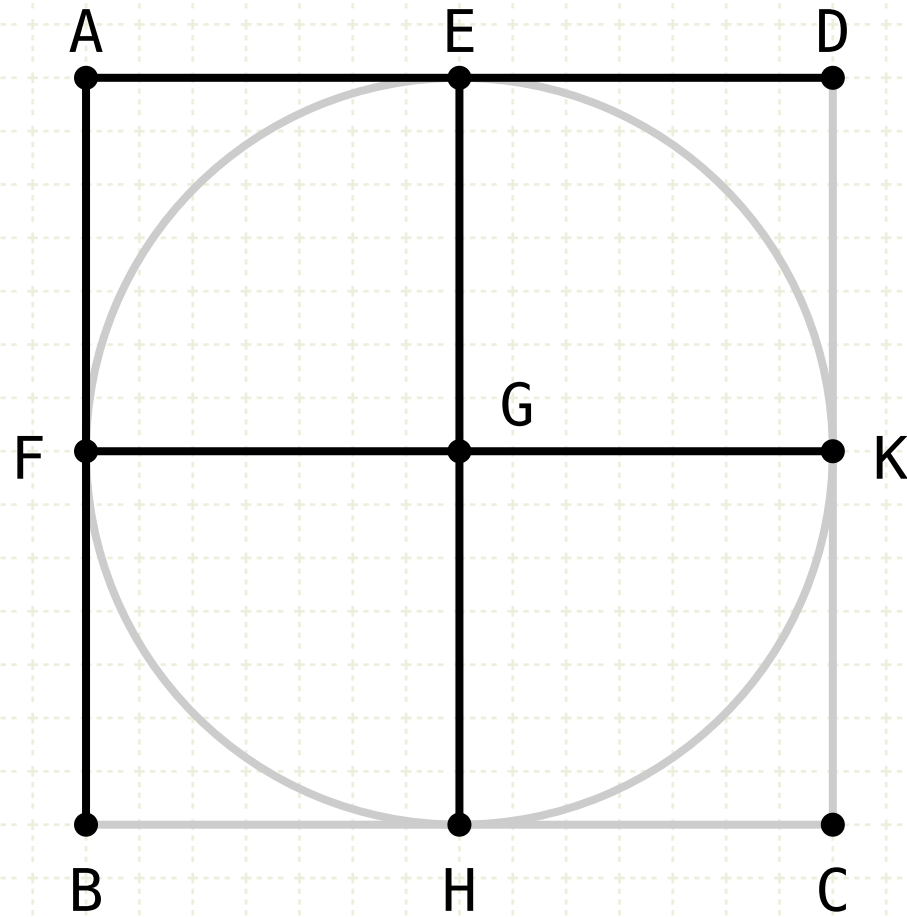
Because all of the lines are parallel to each other, each sub-figure (AK,KB,AH,HD,AG,GC,BG,GD) is also a parallelogram and their opposite sides are equal (I·34)

Since AD equals AB, and AE is half of AD and AF is half of AB,
 $AE = AF$



Proposition 8 of Book IV

In a given square, to inscribe a circle.



$$AE = FG = BH$$

$$ED = GK = HC$$

$$AF = EG = DK$$

$$BF = HG = CK$$

$$AB = EH = DC$$

$$AD = FK = BC$$

$$AD = AB$$

$$AE = \frac{1}{2} AD, \quad AF = \frac{1}{2} AB$$

$$\therefore AE = AF$$

$$\Rightarrow FG = GE$$

Construction

Bisect the lines AB and AD at points E and F (I·10)

Draw EH parallel to AB (I·31)

Draw FK parallel to AD (I·31)

Draw a circle with centre G, and radius GE

This circle is inscribed in the square

Proof

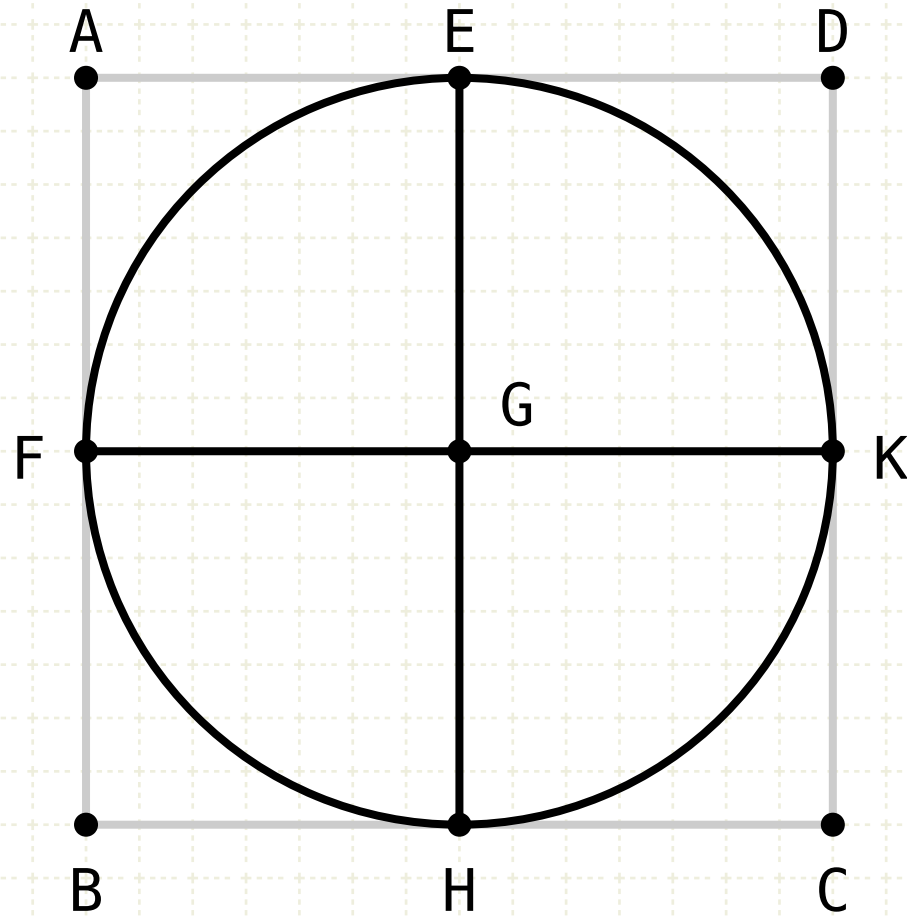
Because all of the lines are parallel to each other, each sub-figure (AK,KB,AH,HD,AG,GC,BG,GD) is also a parallelogram and their opposite sides are equal (I·34)

Since AD equals AB, and AE is half of AD and AF is half of AB, $AE = AF$

From our prior equalities, we know that AE equals FG and AF equals EG, so FG equals GE

Proposition 8 of Book IV

In a given square, to inscribe a circle.



$$AE = FG = BH$$

$$ED = GK = HC$$

$$AF = EG = DK$$

$$BF = HG = CK$$

$$AB = EH = DC$$

$$AD = FK = BC$$

$$AD = AB$$

$$AE = \frac{1}{2} AD, AF = \frac{1}{2} AB$$

$$\therefore AE = AF$$

$$\Rightarrow FG = GE$$

$$GE = GF = GH = GK$$

Construction

Bisect the lines AB and AD at points E and F (I·10)

Draw EH parallel to AB (I·31)

Draw FK parallel to AD (I·31)

Draw a circle with centre G, and radius GE

This circle is inscribed in the square

Proof

Because all of the lines are parallel to each other, each sub-figure (AK,KB,AH,HD,AG,GC,BG,GD) is also a parallelogram and their opposite sides are equal (I·34)

Since AD equals AB, and AE is half of AD and AF is half of AB, $AE = AF$

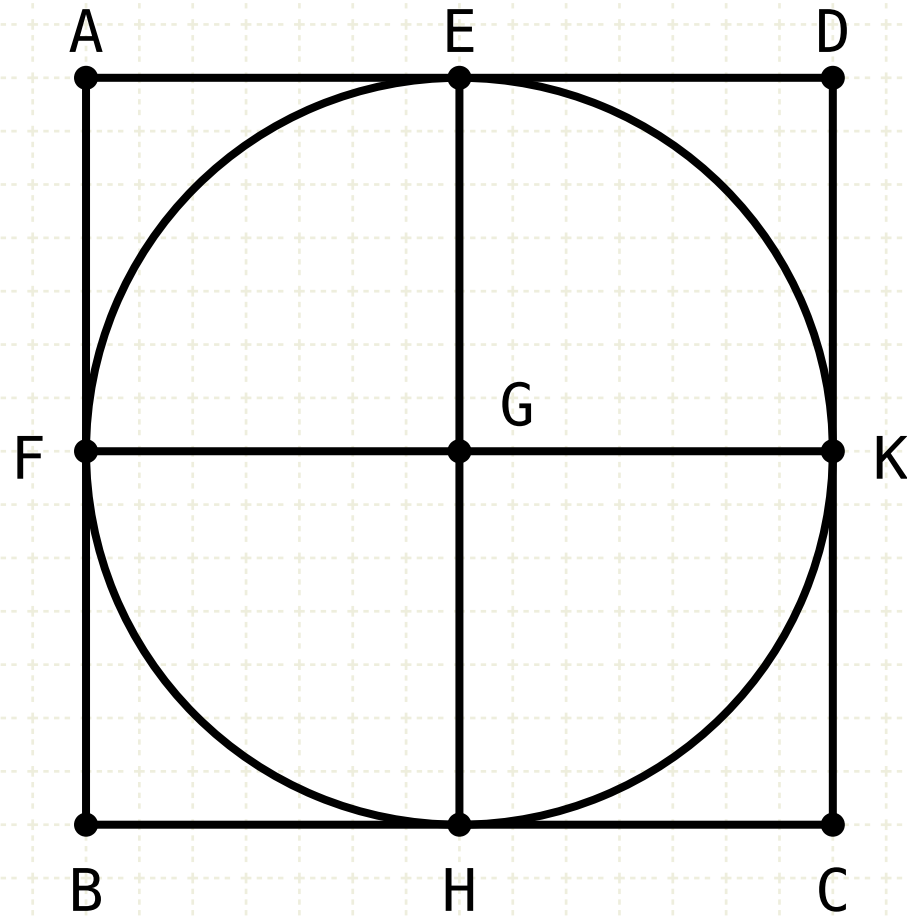
From our prior equalities, we know that AE equals FG and AF equals EG, so FG equals GE

Using similar logic, it can be shown that GK and GH are equal, thus a circle drawn with the centre at G, with radius EG will pass through the points E,F,H,K



Proposition 8 of Book IV

In a given square, to inscribe a circle.



$$AE = FG = BH$$

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$$AB = EH = DC$$

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$$AD = AB$$

$$AE = \frac{1}{2} AD, AF = \frac{1}{2} AB$$

$$\therefore AE = AF$$

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$$GE = GF = GH = GK$$

Construction

Bisect the lines AB and AD at points E and F (I·10)

Draw EH parallel to AB (I·31)

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Proof

Because all of the lines are parallel to each other, each sub-figure (AK,KB,AH,HD,AG,GC,BG,GD) is also a parallelogram and their opposite sides are equal (I·34)

Since AD equals AB, and AE is half of AD and AF is half of AB, $AE = AF$

From our prior equalities, we know that AE equals FG and AF equals EG, so FG equals GE

Using similar logic, it can be shown that GK and GH are equal, thus a circle drawn with the centre at G, with radius EG will pass through the points E,F,H,K

And finally, since the angles at E,F,H,K are right, the lines comprising the square touch the circle (III·16)



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