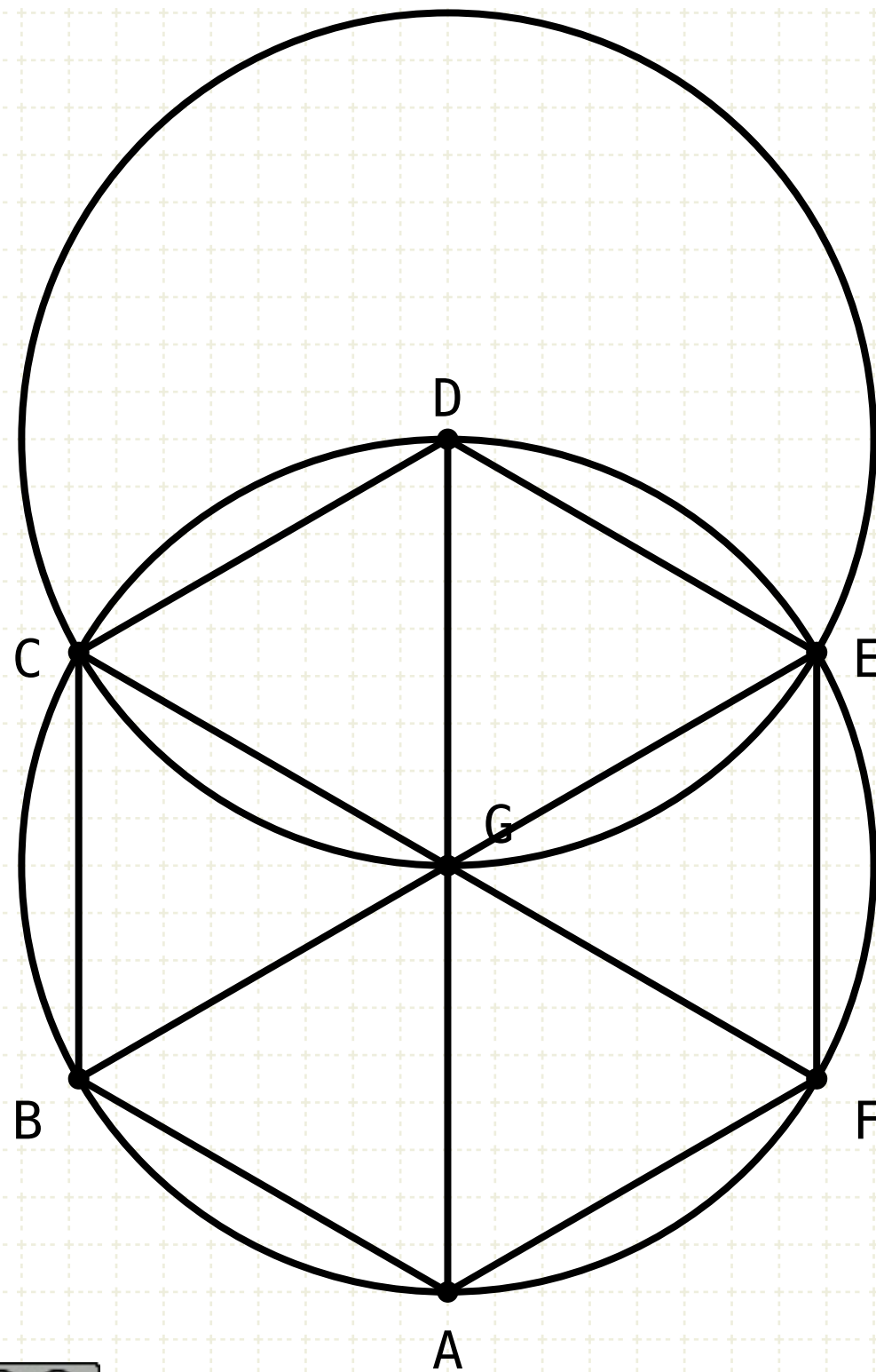


Euclid's Elements

Book IV



Philosophy (nature) is written in that great book which ever is before our eyes -- I mean the universe -- but we cannot understand it if we do not first learn the language and grasp the symbols in which it is written. The book is written in mathematical language, and the symbols are triangles, circles and other geometrical figures, without whose help it is impossible to comprehend a single word of it - without which one wanders in vain through a dark labyrinth.

Galileo Galilei



Proposition 4 of Book IV

In a given triangle, to inscribe a circle.



Table of Contents, Chapter 4

1	Fit a given straight line into a given circle, if the line is less than the diameter	11	In a given circle to inscribe an equilateral and equiangular pentagon
2	In a given circle to inscribe a triangle equiangular with a given triangle	12	About a given circle to circumscribe an equilateral and equiangular pentagon
3	About a given circle to circumscribe a triangle equiangular with a given triangle	13	In a given pentagon, which is equilateral and equiangular, to inscribe a circle
4	In a given triangle, to inscribe a circle	14	About a given pentagon, which is equilateral and equiangular, to circumscribe a circle
5	About a given triangle to circumscribe a circle	15	In a given circle to inscribe an equilateral and equiangular hexagon
6	In a given circle to inscribe a square	16	In a given circle to inscribe a fifteen angled figure which shall be both equilateral and equiangular
7	About a given circle to circumscribe a square		
8	In a given square, to inscribe a circle		
9	About a given square, to circumscribe a circle		
10	To construct an isosceles triangle having each of the angles at the base double of the remaining one		



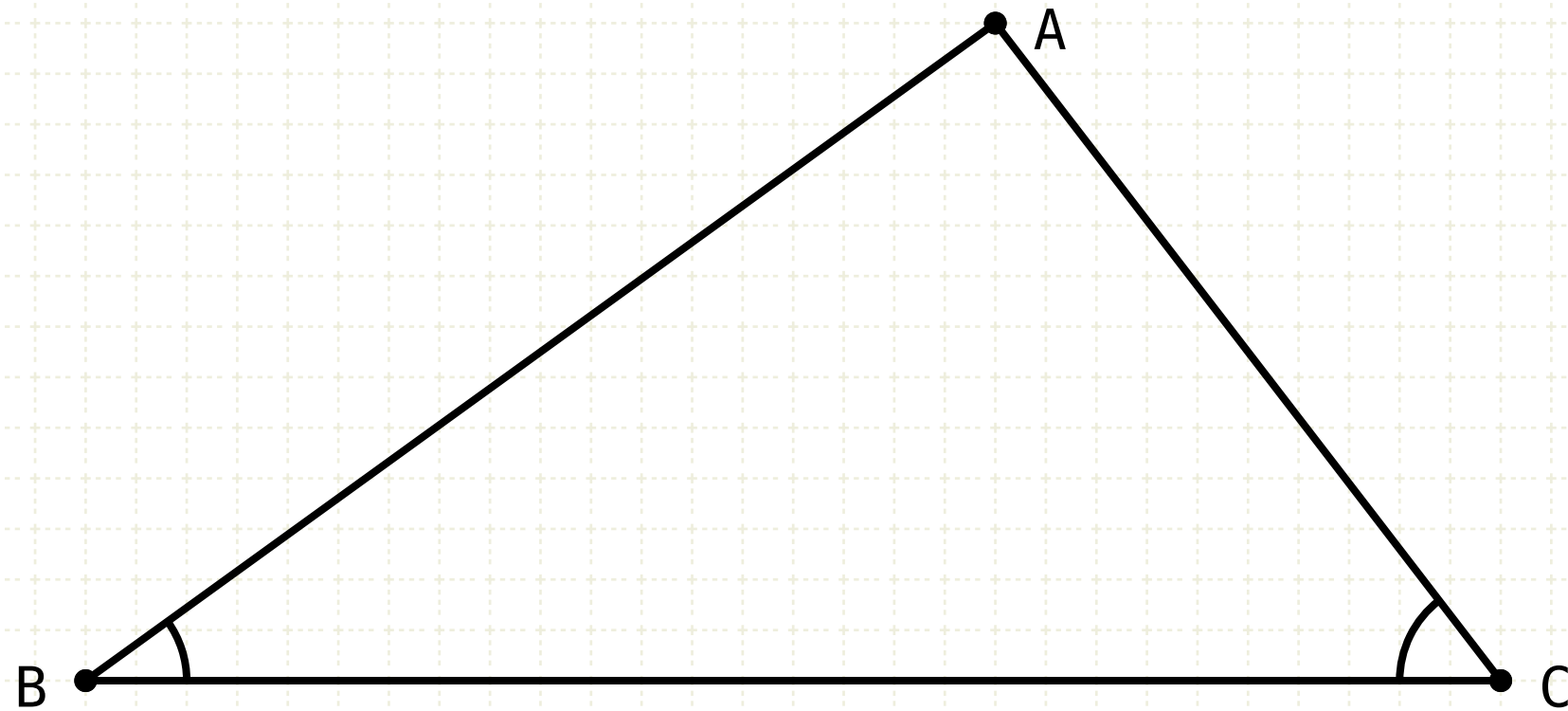
Proposition 4 of Book IV

In a given triangle, to inscribe a circle.



Proposition 4 of Book IV

In a given triangle, to inscribe a circle.



In other words

Given a triangle ABC

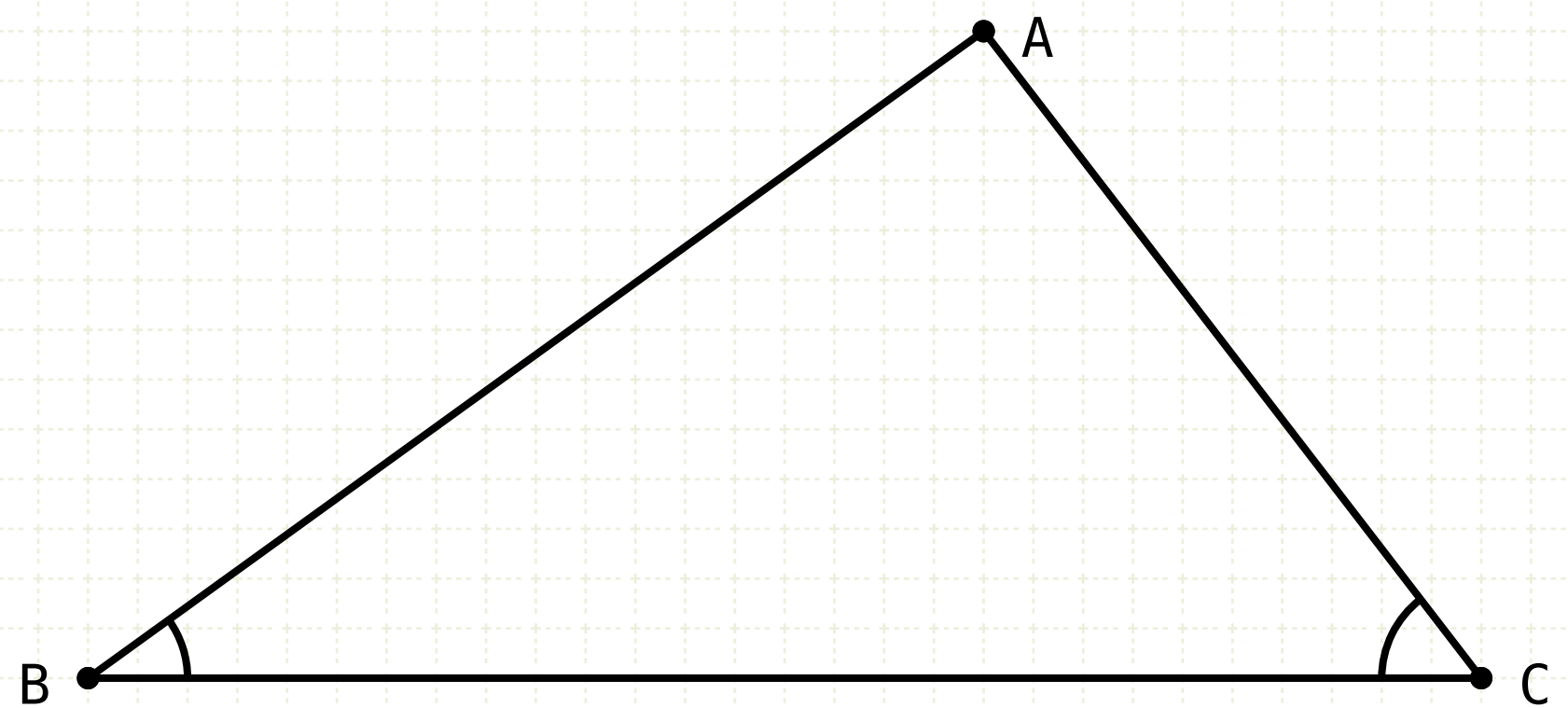
Draw a circle so that it touches all three sides of the triangle



Proposition 4 of Book IV

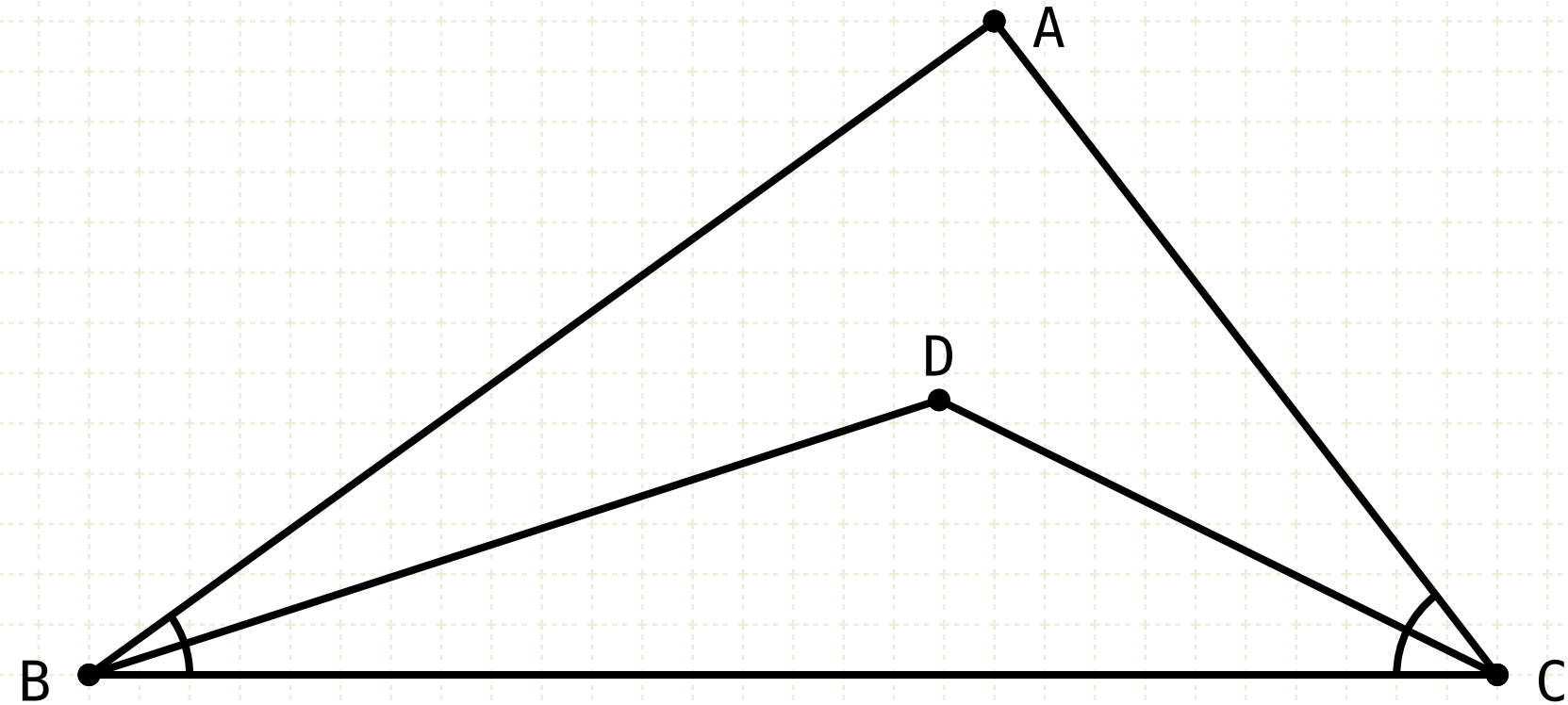
In a given triangle, to inscribe a circle.

Construction



Proposition 4 of Book IV

In a given triangle, to inscribe a circle.



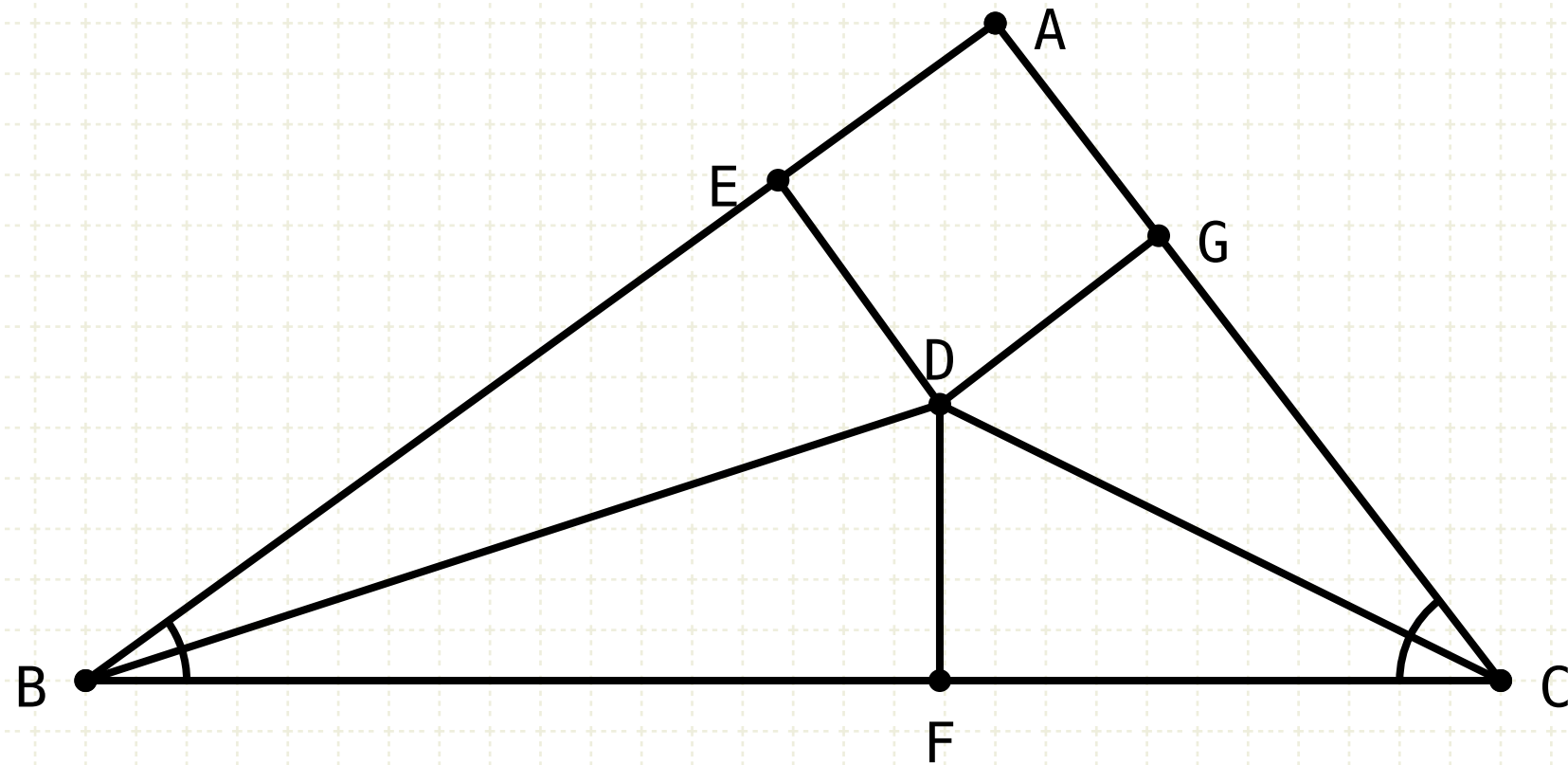
Construction

Bisect the angles at points B and C with lines BD and CD, intersecting at point D



Proposition 4 of Book IV

In a given triangle, to inscribe a circle.



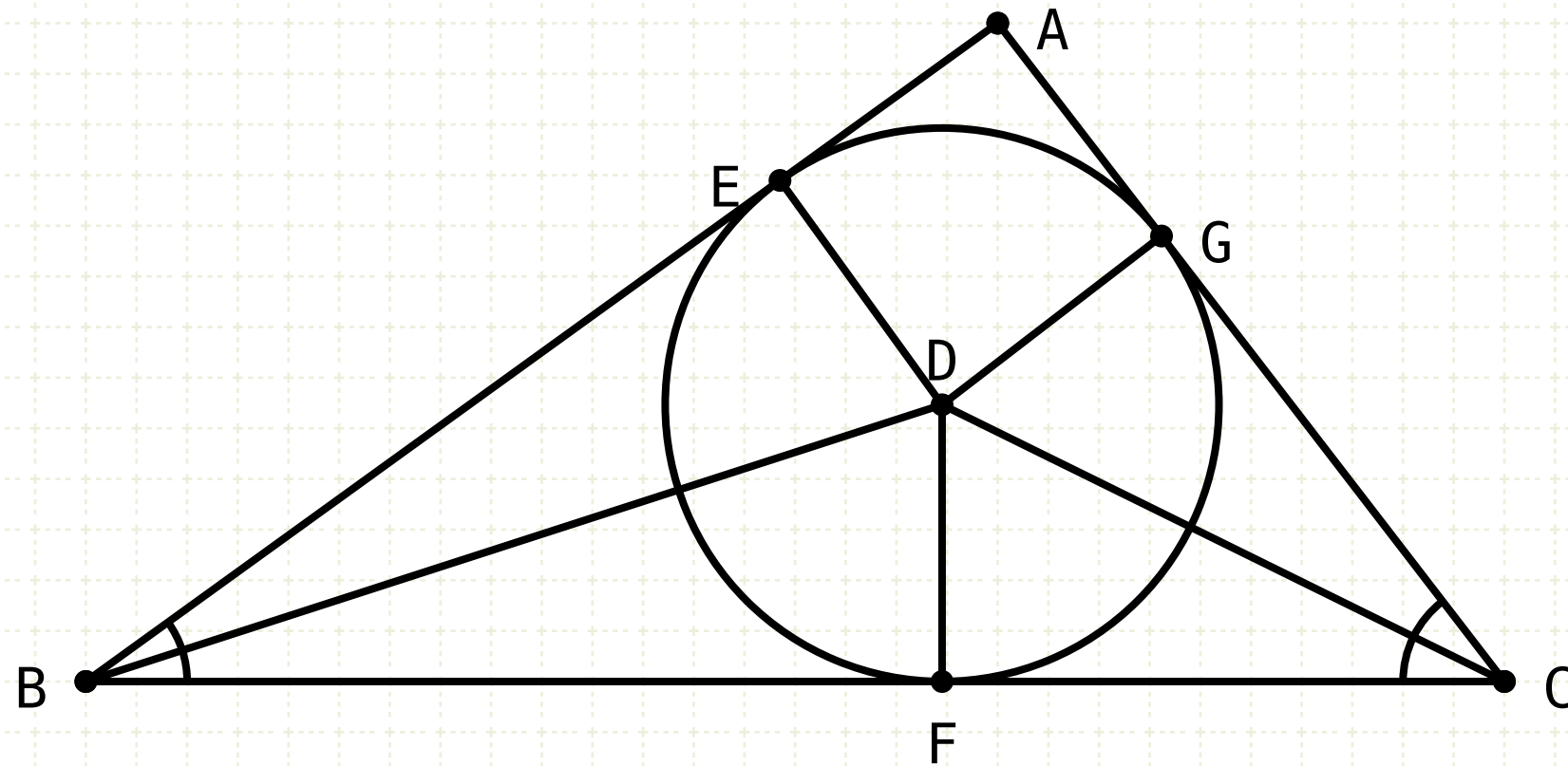
Construction

Bisect the angles at points B and C with lines BD and CD, intersecting at point D

Draw perpendicular lines from the point D to all three sides of the triangle

Proposition 4 of Book IV

In a given triangle, to inscribe a circle.



Construction

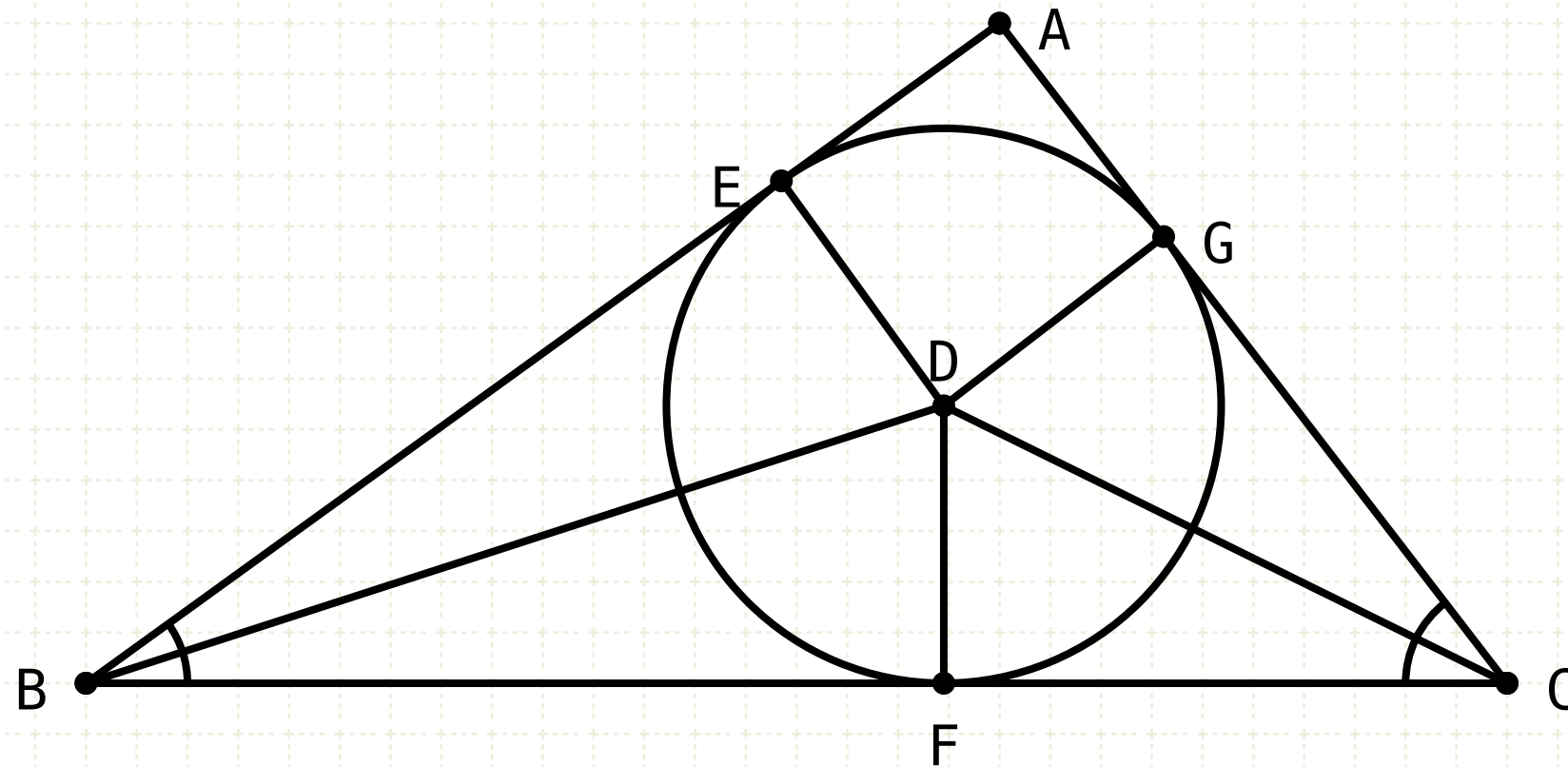
Bisect the angles at points B and C with lines BD and CD , intersecting at point D

Draw perpendicular lines from the point D to all three sides of the triangle

With D as the centre, and DF as the radius, it is possible to draw a circle that touches all three sides of the triangle

Proposition 4 of Book IV

In a given triangle, to inscribe a circle.



Construction

Bisect the angles at points B and C with lines BD and CD , intersecting at point D

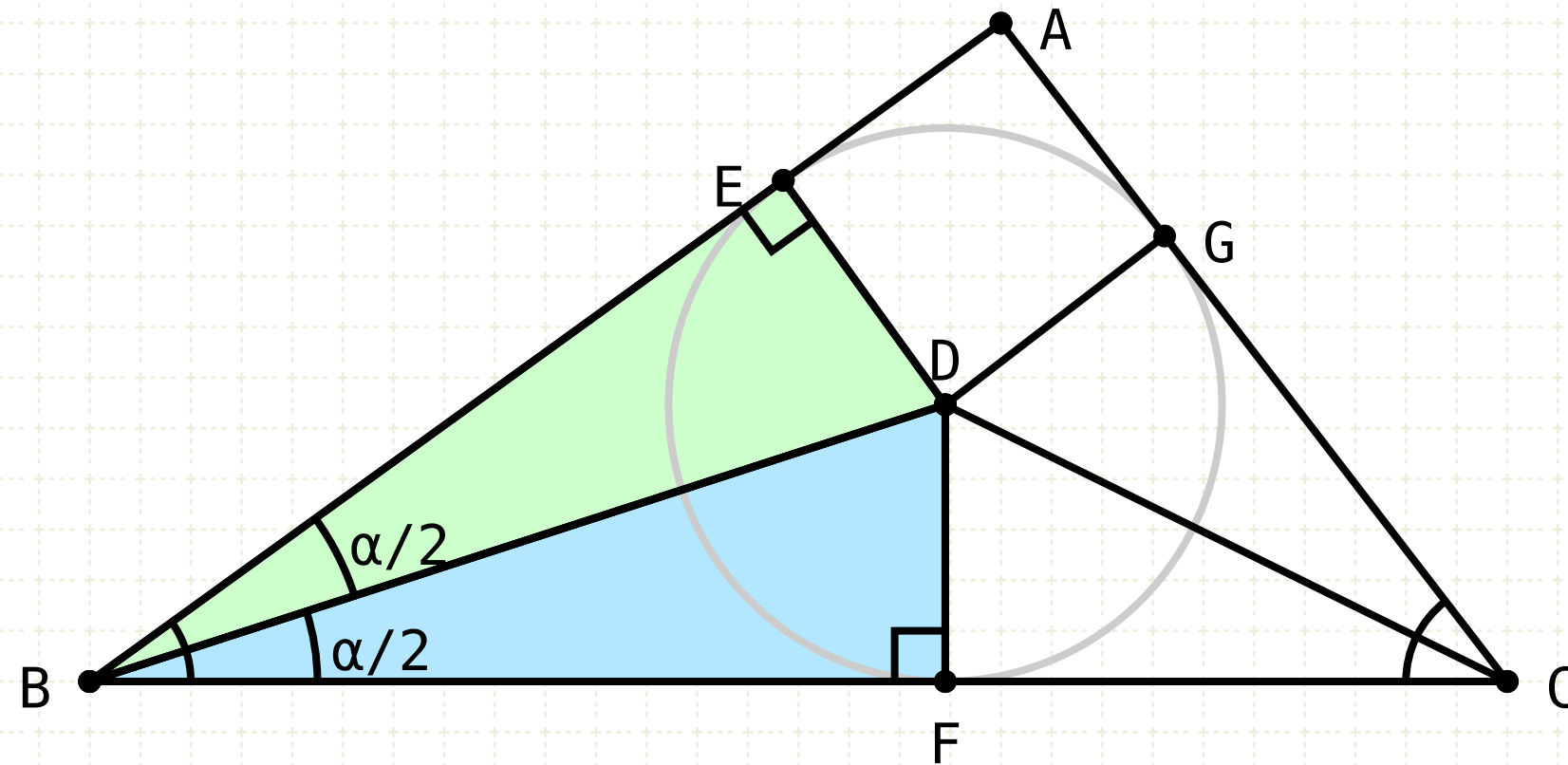
Draw perpendicular lines from the point D to all three sides of the triangle

With D as the centre, and DF as the radius, it is possible to draw a circle that touches all three sides of the triangle

Proof

Proposition 4 of Book IV

In a given triangle, to inscribe a circle.



$$\angle EBD = \angle DBF = \alpha/2$$

$$\angle DFE = \angle DEB = L$$

BD is common

Construction

Bisect the angles at points B and C with lines BD and CD , intersecting at point D

Draw perpendicular lines from the point D to all three sides of the triangle

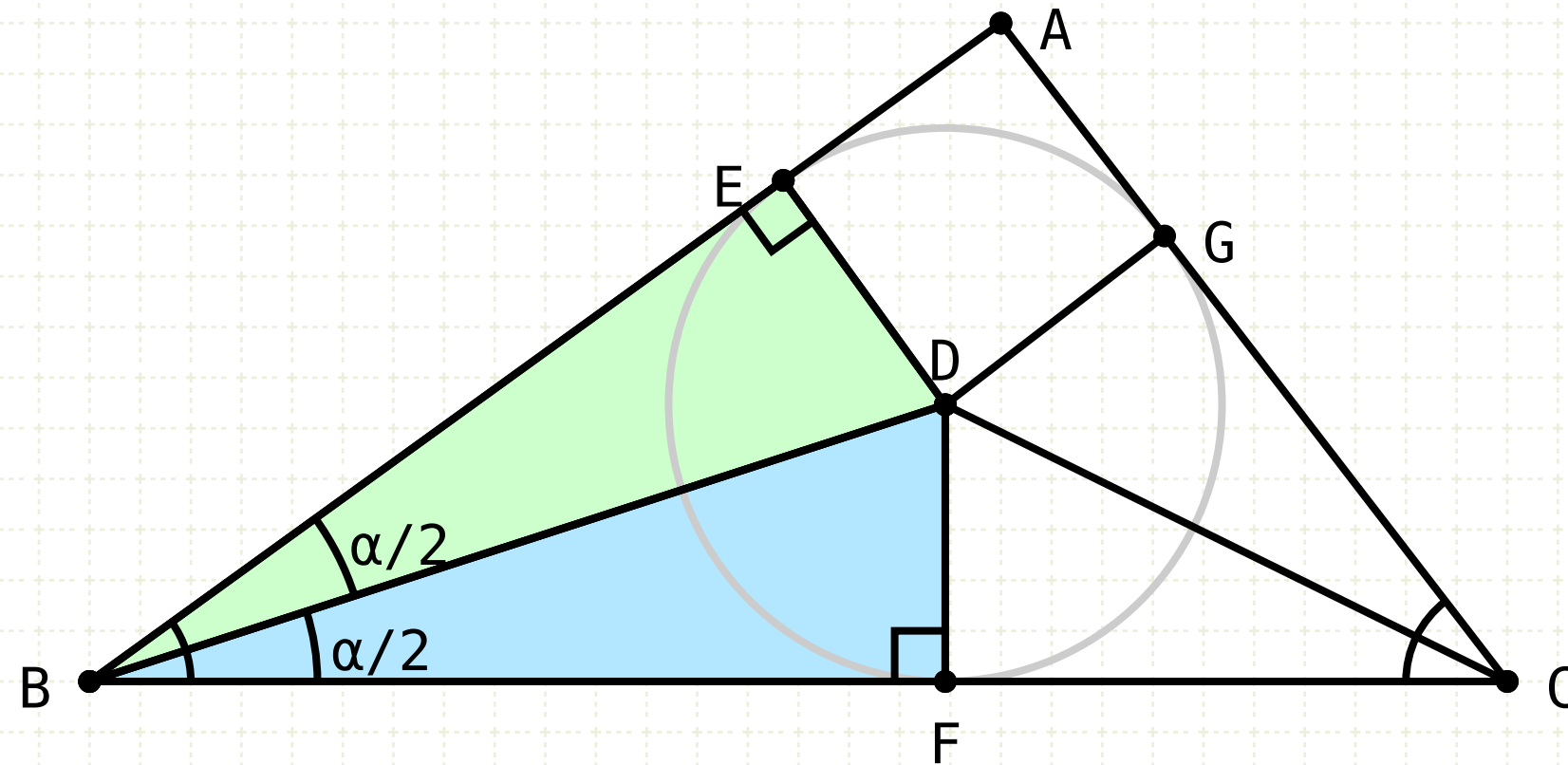
With D as the centre, and DF as the radius, it is possible to draw a circle that touches all three sides of the triangle

Proof

The two triangles DEB and DBF are equal in all respects, since they have an angle ($\frac{1}{2}\alpha$), side (BD) and angle (DFB and DEB) equal (I.26)

Proposition 4 of Book IV

In a given triangle, to inscribe a circle.



$$\angle EBD = \angle DBF = \alpha/2$$

$$\angle DFE = \angle DEB = L$$

BD is common

$$\therefore DF = DE$$

Construction

Bisect the angles at points B and C with lines BD and CD, intersecting at point D

Draw perpendicular lines from the point D to all three sides of the triangle

With D as the centre, and DF as the radius, it is possible to draw a circle that touches all three sides of the triangle

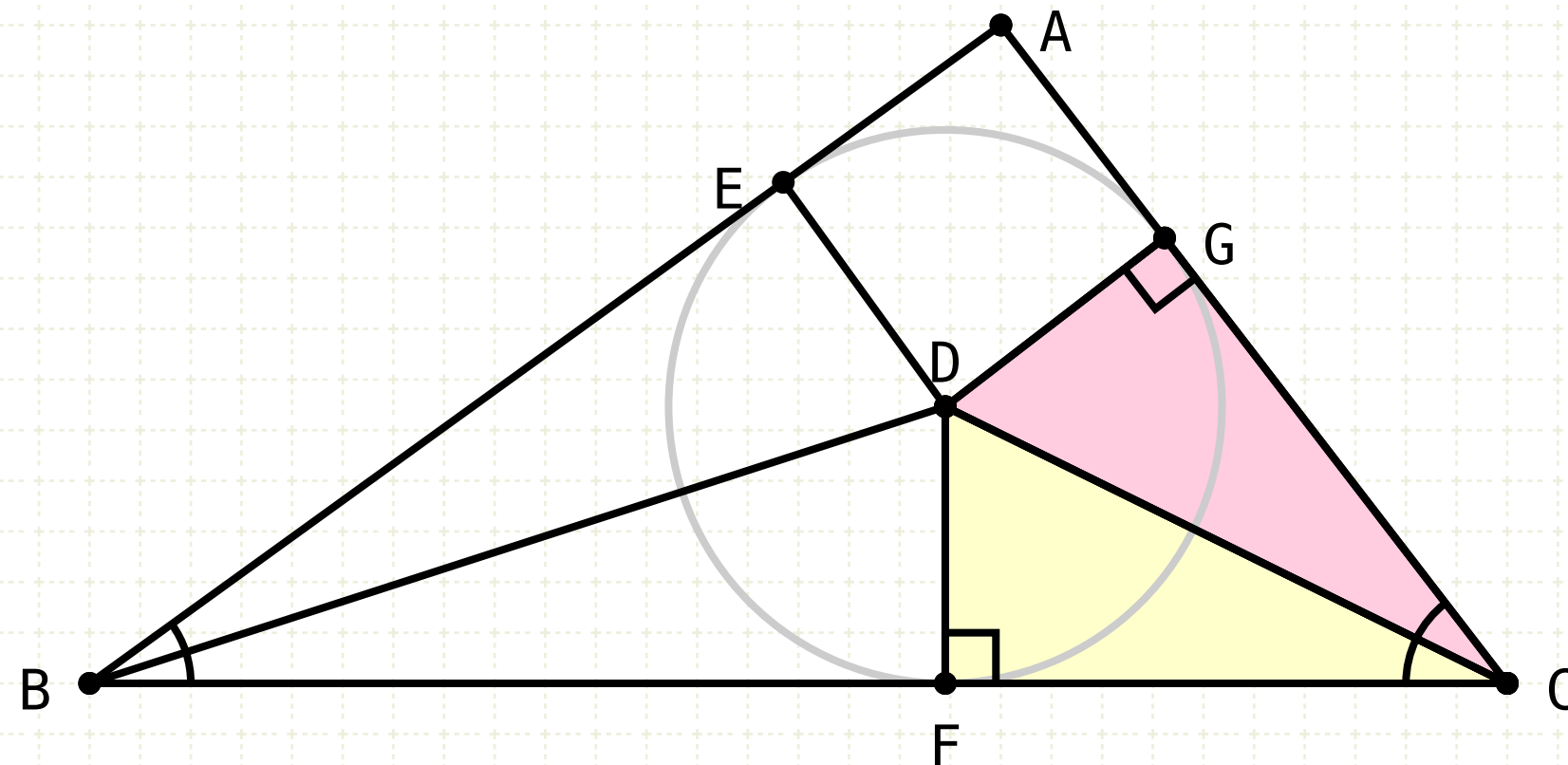
Proof

The two triangles DEB and DBF are equal in all respects, since they have an angle ($\frac{1}{2}\alpha$), side (BD) and angle (DFB and DEB) equal (I.26)

Hence DF equals DE

Proposition 4 of Book IV

In a given triangle, to inscribe a circle.



$$\angle EBD = \angle DBF = \alpha/2$$

$$\angle DFE = \angle DEB = L$$

BD is common

$$\therefore DF = DE$$

$$DF = DG$$

Construction

Bisect the angles at points B and C with lines BD and CD, intersecting at point D

Draw perpendicular lines from the point D to all three sides of the triangle

With D as the centre, and DF as the radius, it is possible to draw a circle that touches all three sides of the triangle

Proof

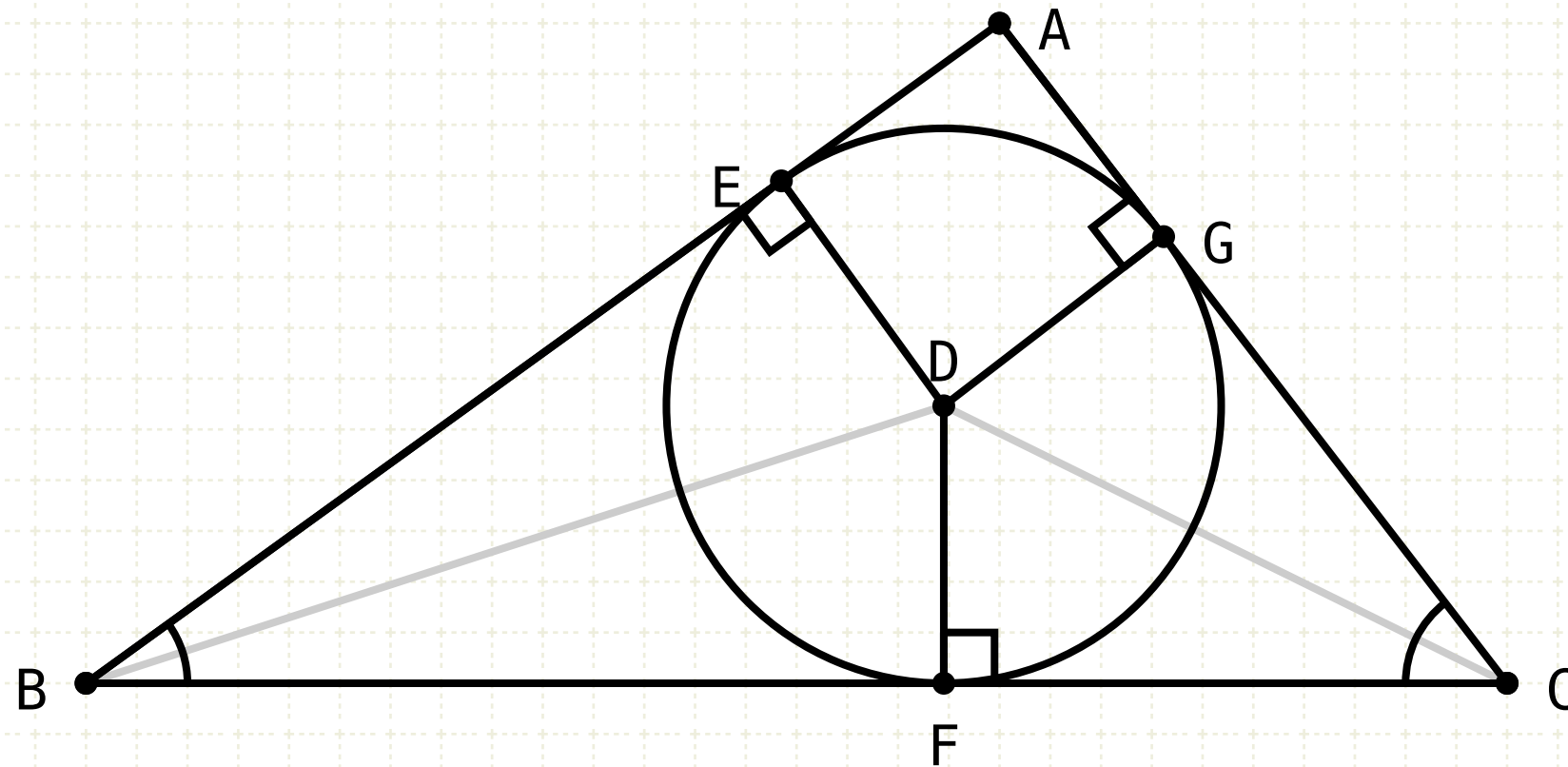
The two triangles DEB and DBF are equal in all respects, since they have an angle ($\frac{1}{2}\alpha$), side (BD) and angle (DFB and DEB) equal (I.26)

Hence DF equals DE

Similarly, it can be shown that DF equals DG

Proposition 4 of Book IV

In a given triangle, to inscribe a circle.



$$\angle EBD = \angle DBF = \alpha/2$$

$$\angle DFE = \angle DEB = L$$

BD is common

$$\therefore DF = DE$$

$$DF = DG$$

Construction

Bisect the angles at points B and C with lines BD and CD, intersecting at point D

Draw perpendicular lines from the point D to all three sides of the triangle

With D as the centre, and DF as the radius, it is possible to draw a circle that touches all three sides of the triangle

Proof

The two triangles DEB and DBF are equal in all respects, since they have an angle ($\frac{1}{2}\alpha$), side (BD) and angle (DFB and DEB) equal (I·26)

Hence DF equals DE

Similarly, it can be shown that DF equals DG

Since the sides of the triangles are at right angles to the radii of the circle, and are at the extremities of the radii, then they touch the circle (III·16)

Youtube Videos

<https://www.youtube.com/c/SandyBultena>

Copyright © 2019 by Sandy Bultena.



Except where otherwise noted, this work is licensed under
<http://creativecommons.org/licenses/by-nc/3.0>