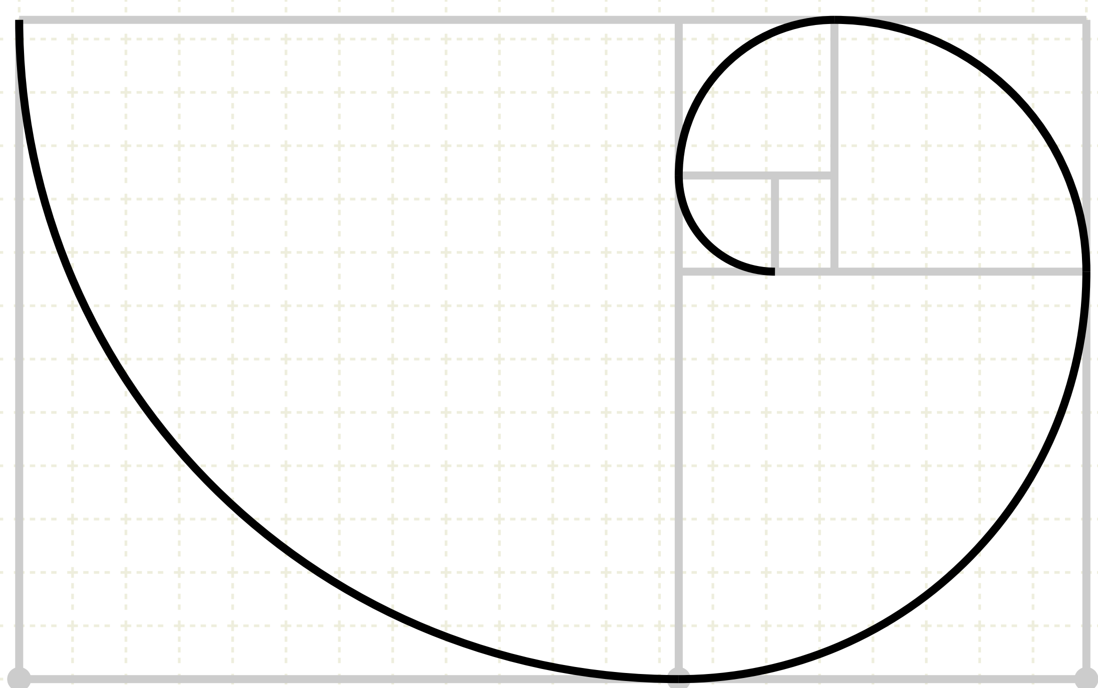
Euclid's Elements

Book VI



One can state, without exaggeration, that the observation of and the search for similarities and differences are the basis of all human knowledge.

Alfred Nobel

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Proposition 20 of Book VI

Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.

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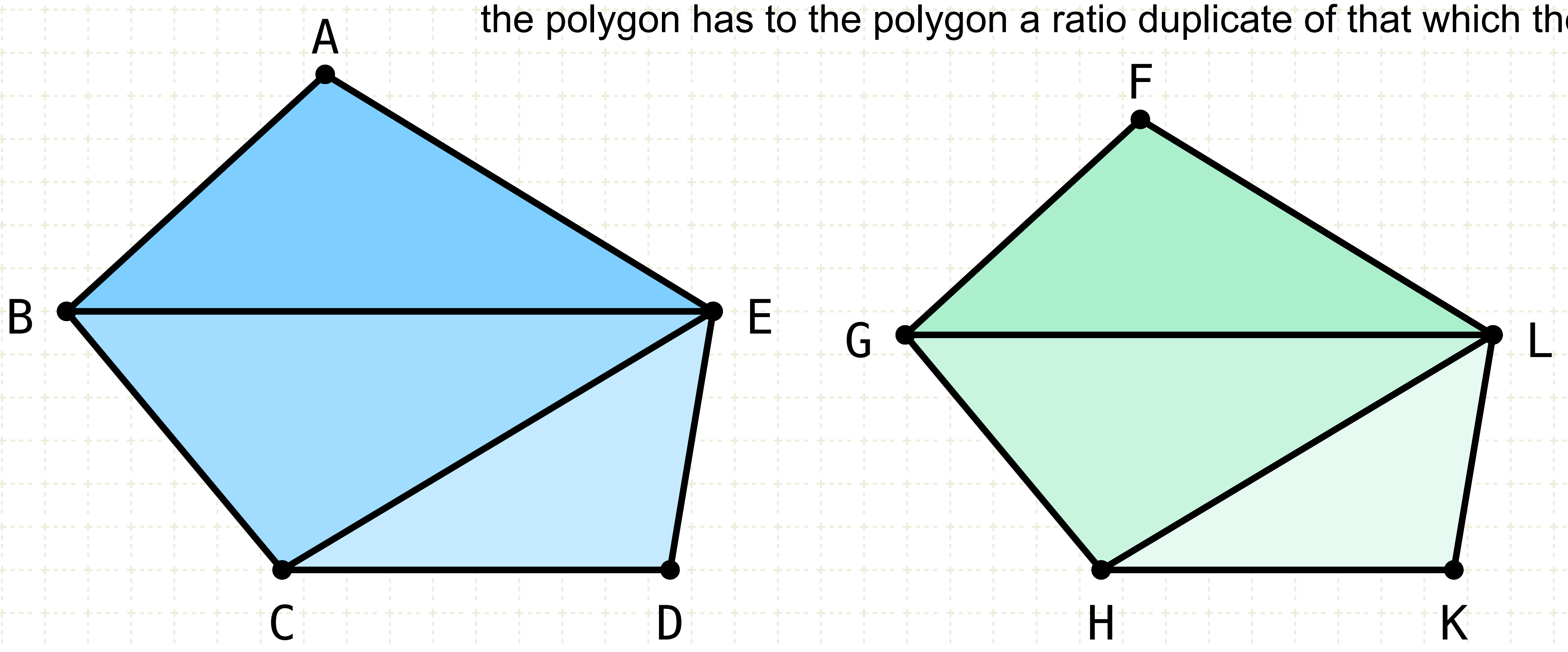


In other words

If there are two polygons which are similar (equal angles, sides proportional),

then the polygons can be divided into an equal number of similar triangles

And the ratio of all the triangles are equal, which is also equal to the ratio of the original polygons

And finally, the ratio of the areas of the polygons is in duplicate ratio to the ratio of the sides

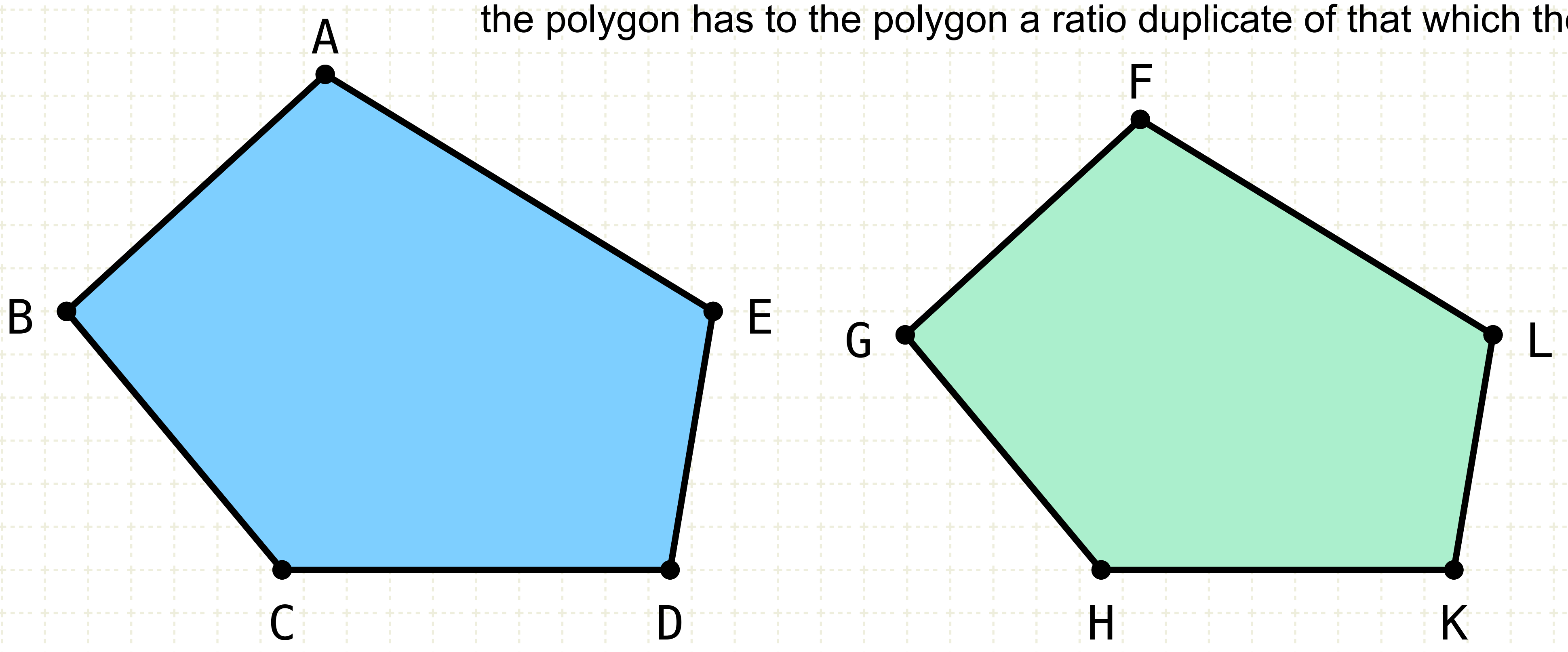
ABCDE ~ FGHKL
ΔABE ~ ΔFGL
ΔBEC ~ ΔGLH
ΔECD ~ ΔLHK

ΔABE:ΔFGL = ΔBEC:ΔLHK
= ΔECD:ΔGHK = ABCDE:FGHKL

ABCDE:FGHKL = $(AB:FG)^2$

Proposition 20 of Book VI

Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.



Proof - Similar Triangles

ABCDE ~ FGHKL

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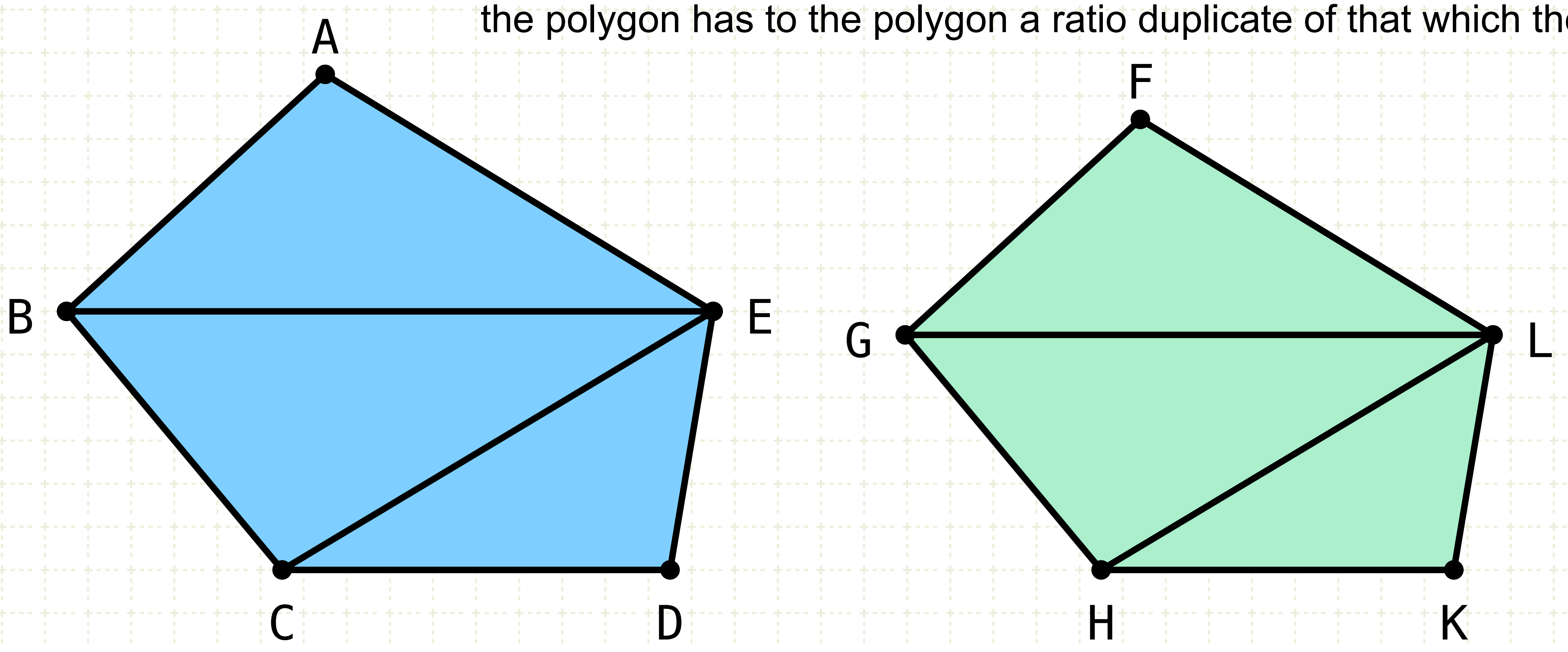
Proof - Similar Triangles

Draw lines BE, EC, GL and LH



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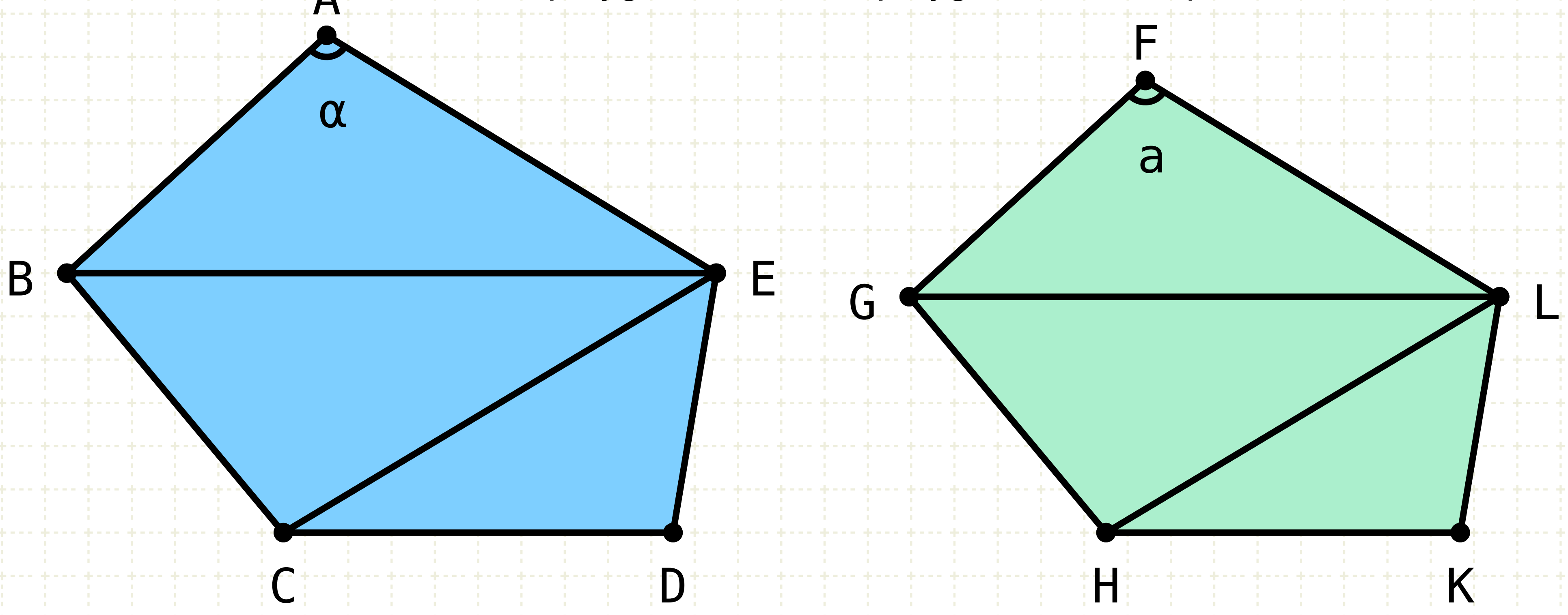

Proof - Similar Triangles

Draw lines BE, EC, GL and LH

The two polygons are similar, therefore angle BAE is equal to angle GFL,

and the ratio of BA to AE is equal to the ratio GF to FL (VI·Def·1)

ABCDE ~ FGHKL

a = α

AB:AE = FG:FL

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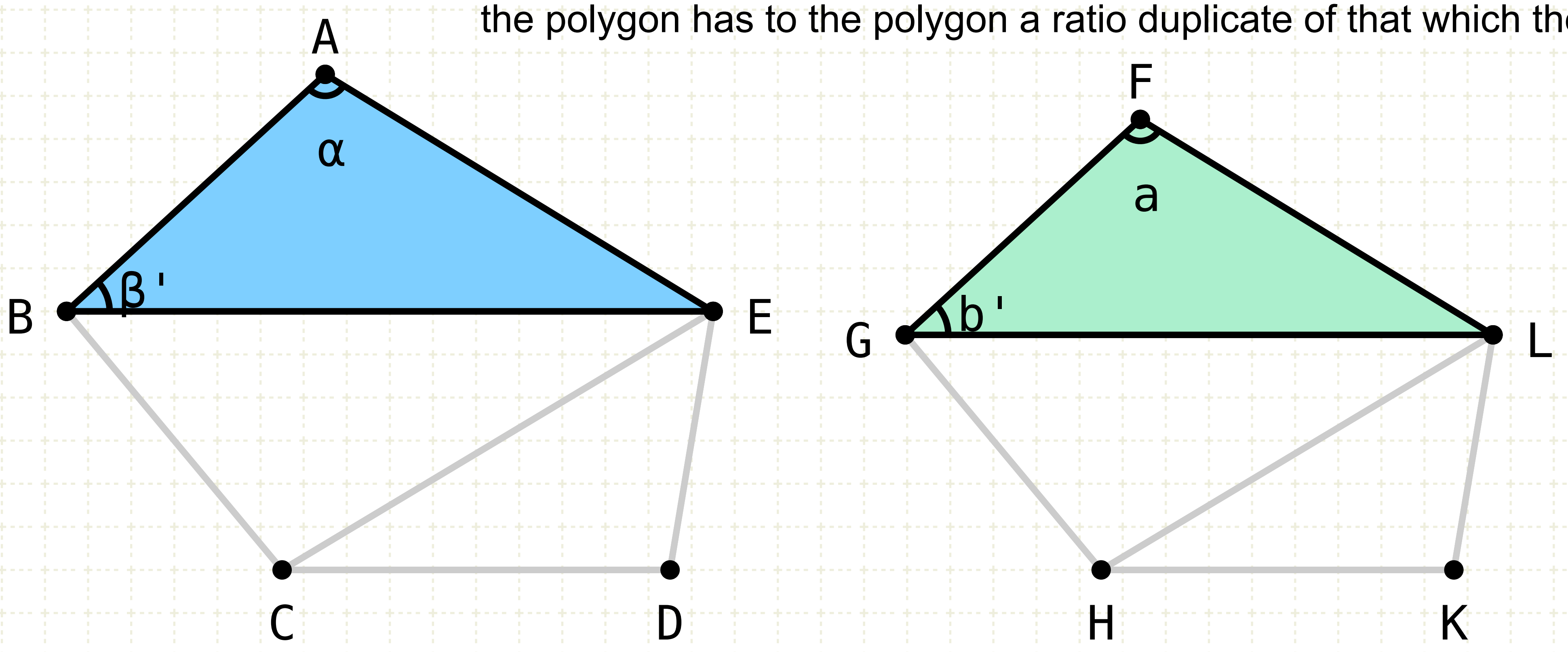

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So angle ABE is equal to angle FGL

ABCDE ~ FGHKL

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ΔABE ~ ΔFGL, b' = β'

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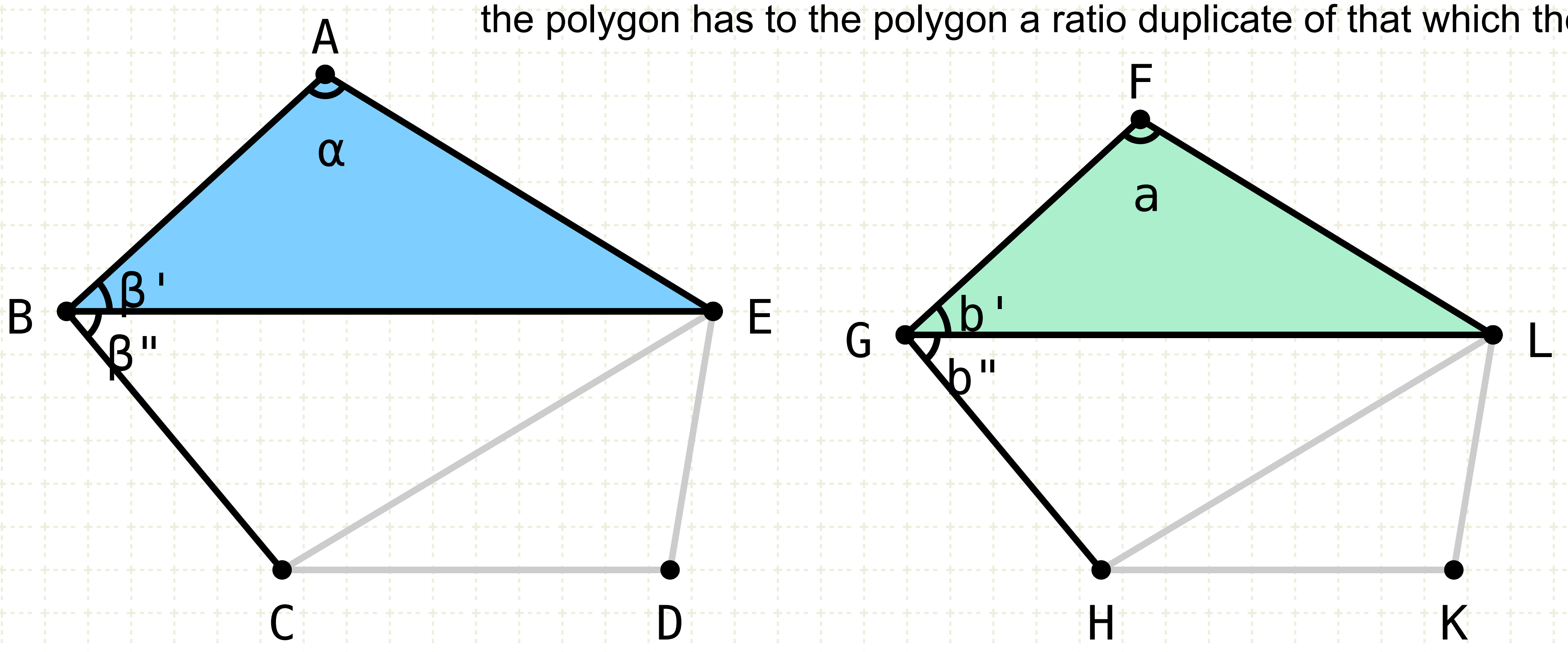

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So angle ABE is equal to angle FGL

But angle ABC is equal to angle FGH because the polygons are similar, therefore the angles EBC and LGH are also equal

ABCDE ~ FGHKL

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ΔABE ~ ΔFGL, b' = β'

b = b' + b" = β = β' + β"

b" = β"

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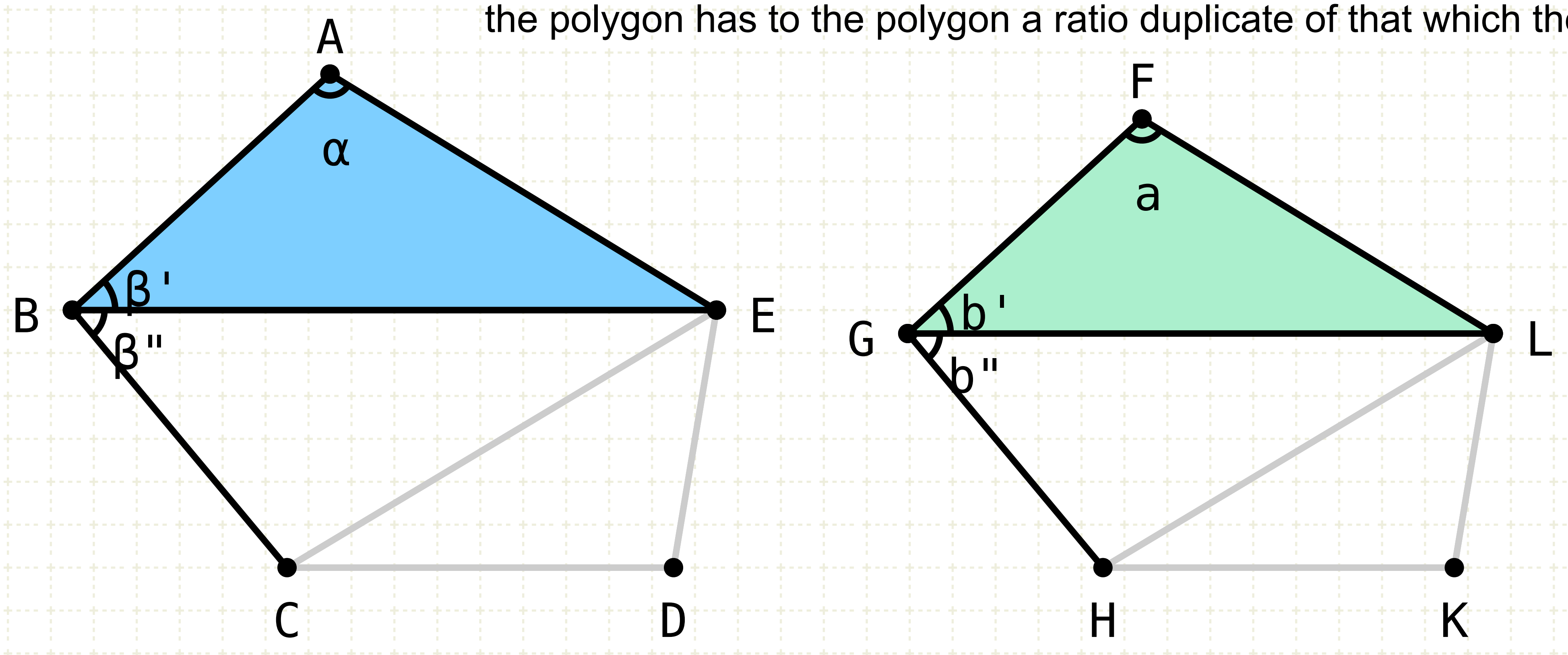

$ABCDE \sim FGHKL$

$a = \alpha$

$AB:AE = FG:FL$

$\Delta ABE \sim \Delta FGL,\ b' = \beta'$

$b = b' + b'' = \beta = \beta' + \beta''$

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$BE:AB = GL:FG$

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Because the triangle ABE is similar to the triangle FGL, BE is to AB as GL is to GF

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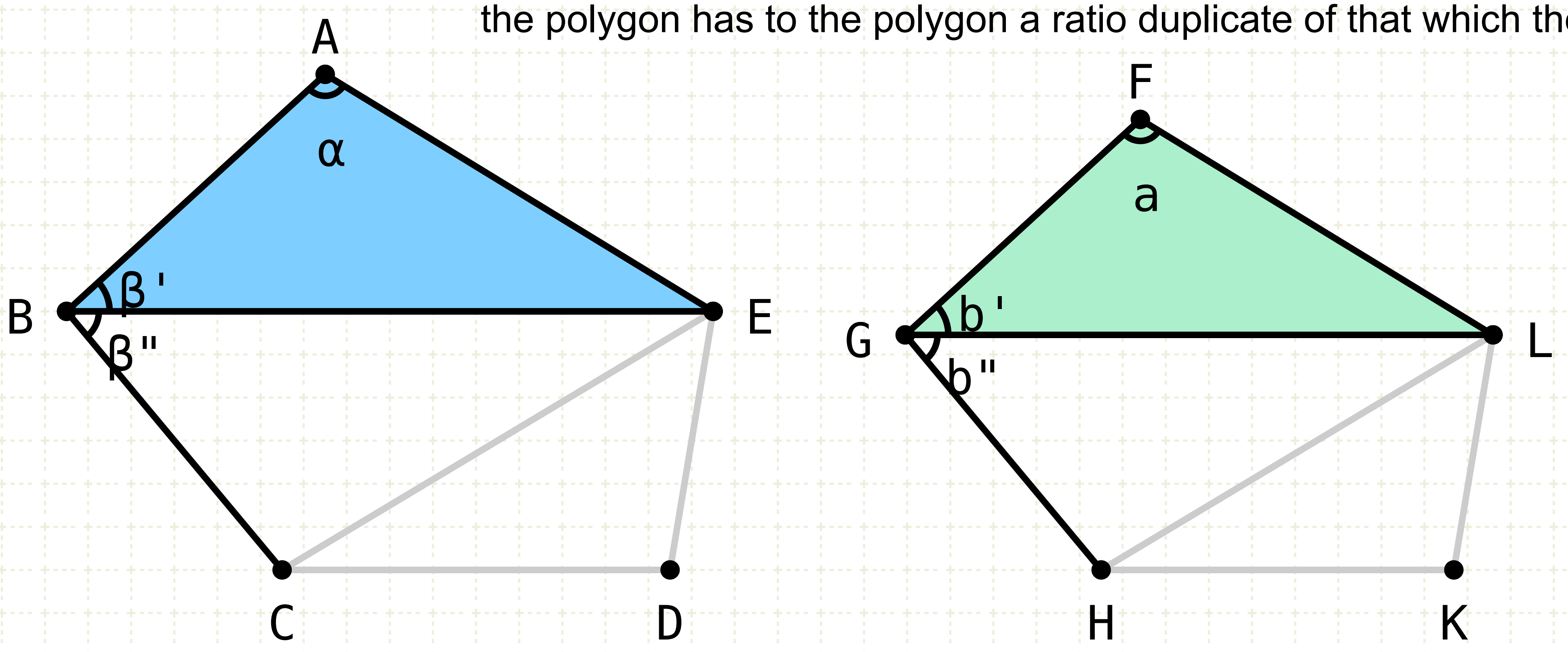

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Again, because the polygons are similar, AB is to BC as FG is to GH

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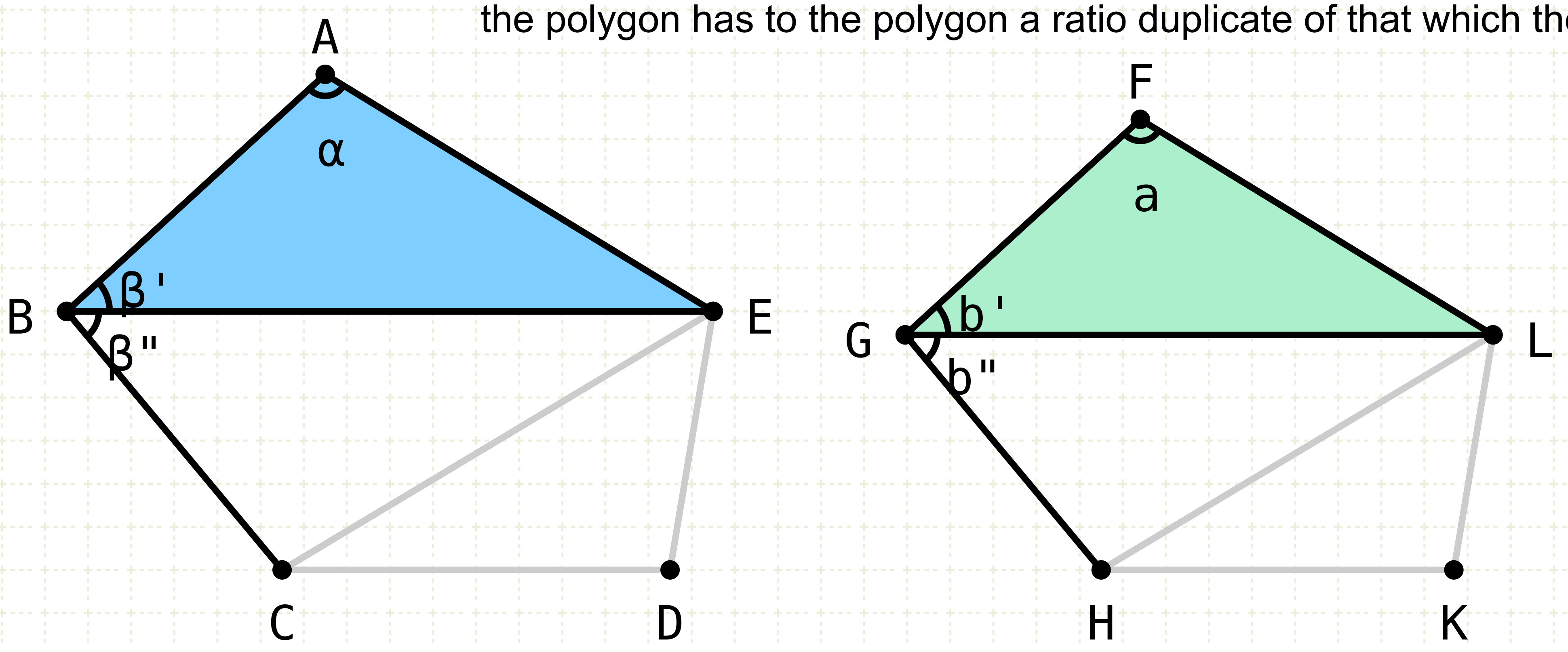

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So angle ABE is equal to angle FGL

But angle ABC is equal to angle FGH because the polygons are similar, therefore the angles EBC and LGH are also equal

Because the triangle ABE is similar to the triangle FGL, BE is to AB as GL is to GF

Again, because the polygons are similar, AB is to BC as FG is to GH

therefore BE is to BC as GL is to GH (V·22)

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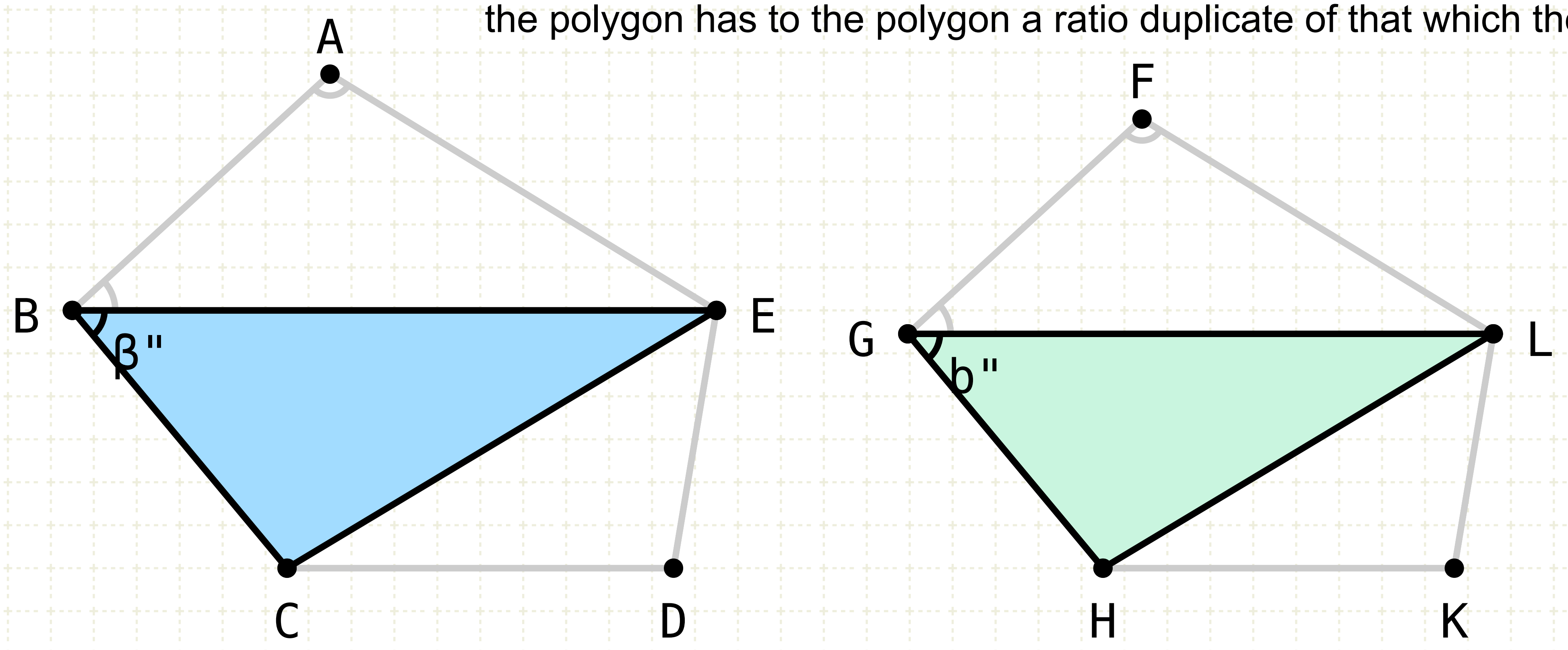

ABCDE ~ FGHKL

$a = \alpha$

AB:AE = FG:FL

$\Delta ABE \sim \Delta FGL$, $b' = \beta'$

$b = b' + b'' = \beta = \beta' + \beta''$

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BE:AB = GL:FG

AB:BC = FG:GH

BE:BC = GL:GH

$\Delta BEC \sim \Delta LGK$

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Because the triangle ABE is similar to the triangle FGL, BE is to AB as GL is to GF

Again, because the polygons are similar, AB is to BC as FG is to GH

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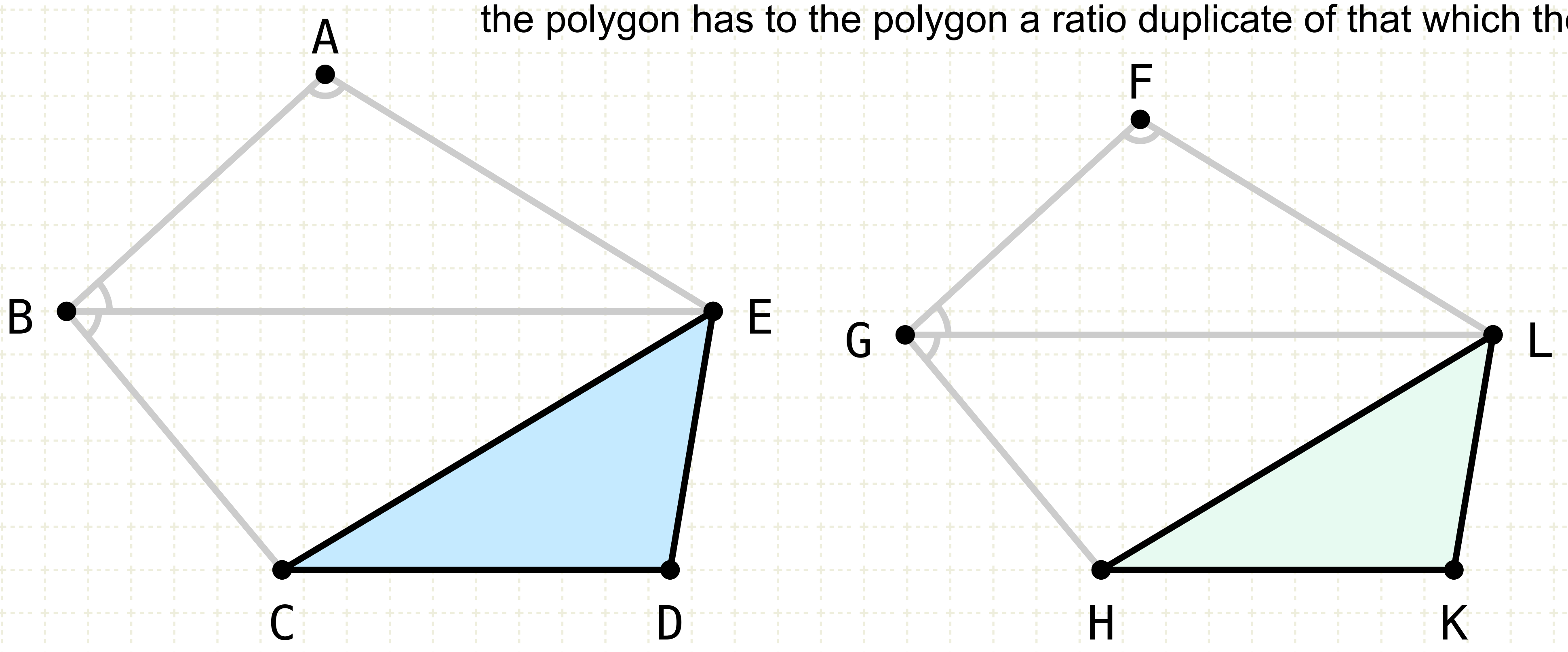

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$\Delta BEC \sim \Delta LGK$

$\Delta ECD \sim \Delta LHK$

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For the same reasons, the triangle ECD is similar to the triangle LHK

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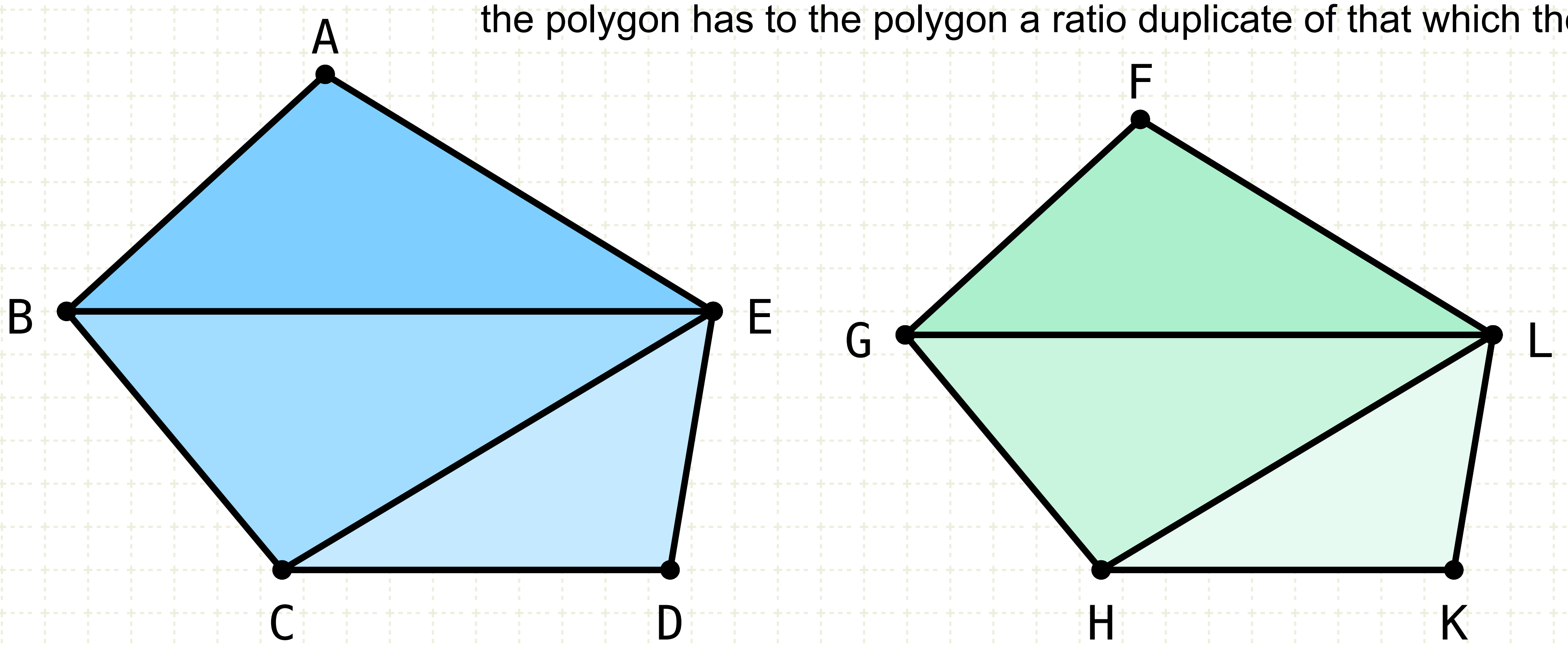

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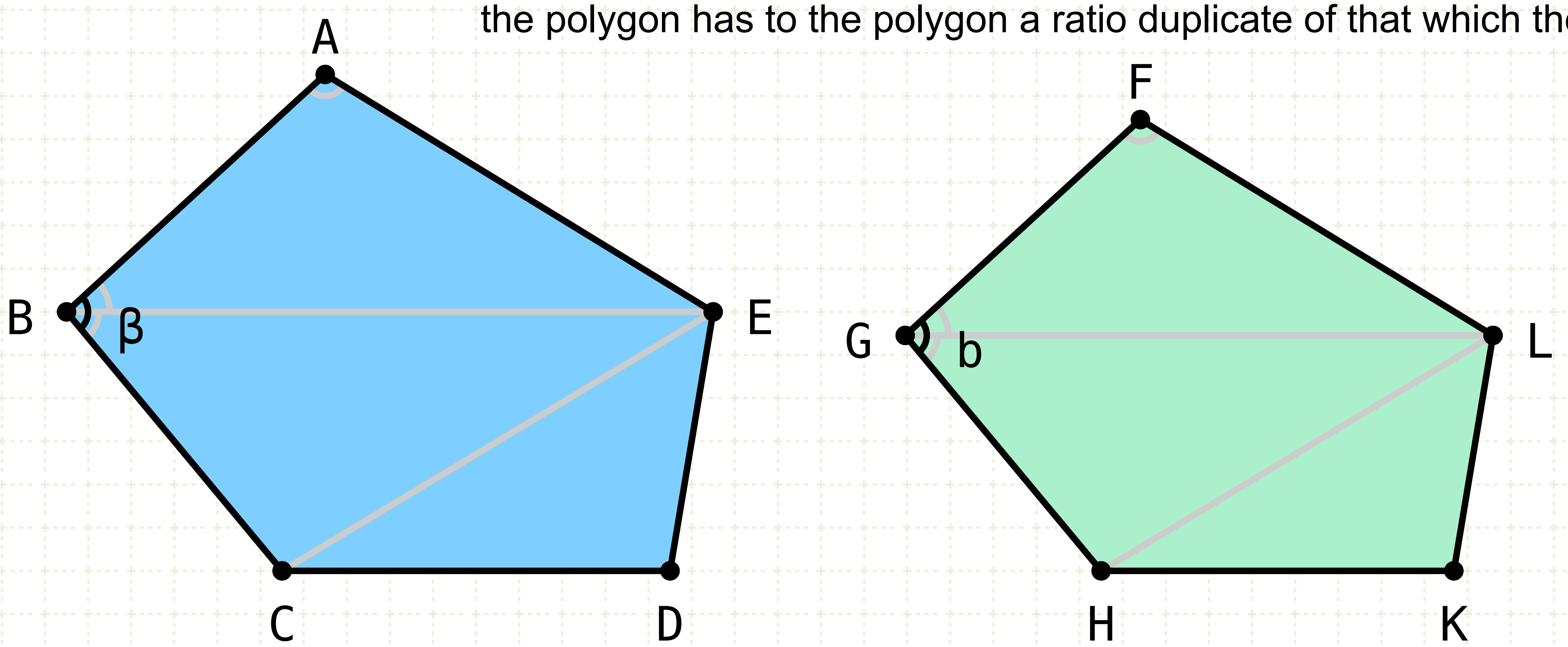
For the same reasons, the triangle ECD is similar to the triangle LHK

So the two similar polygon has been divided into similar triangles, equal in multitude

Proposition 20 of Book VI

Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.

Proof - Duplicate Ratio to Sides



ABCDE ~ FGHKL

b = β

b' = β'

AB:BC = FG:GH

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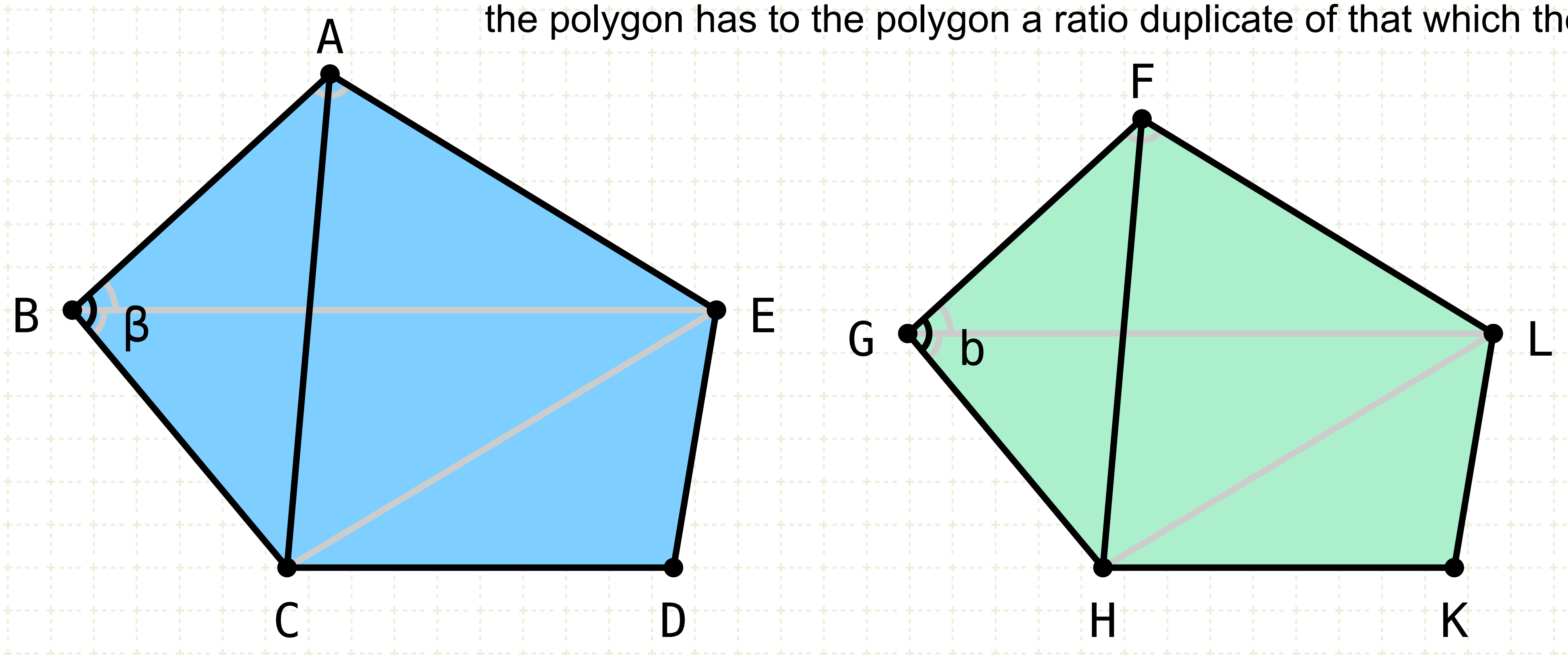

Proof - Duplicate Ratio to Sides

Draw AC, FH

ABCDE ~ FGHKL

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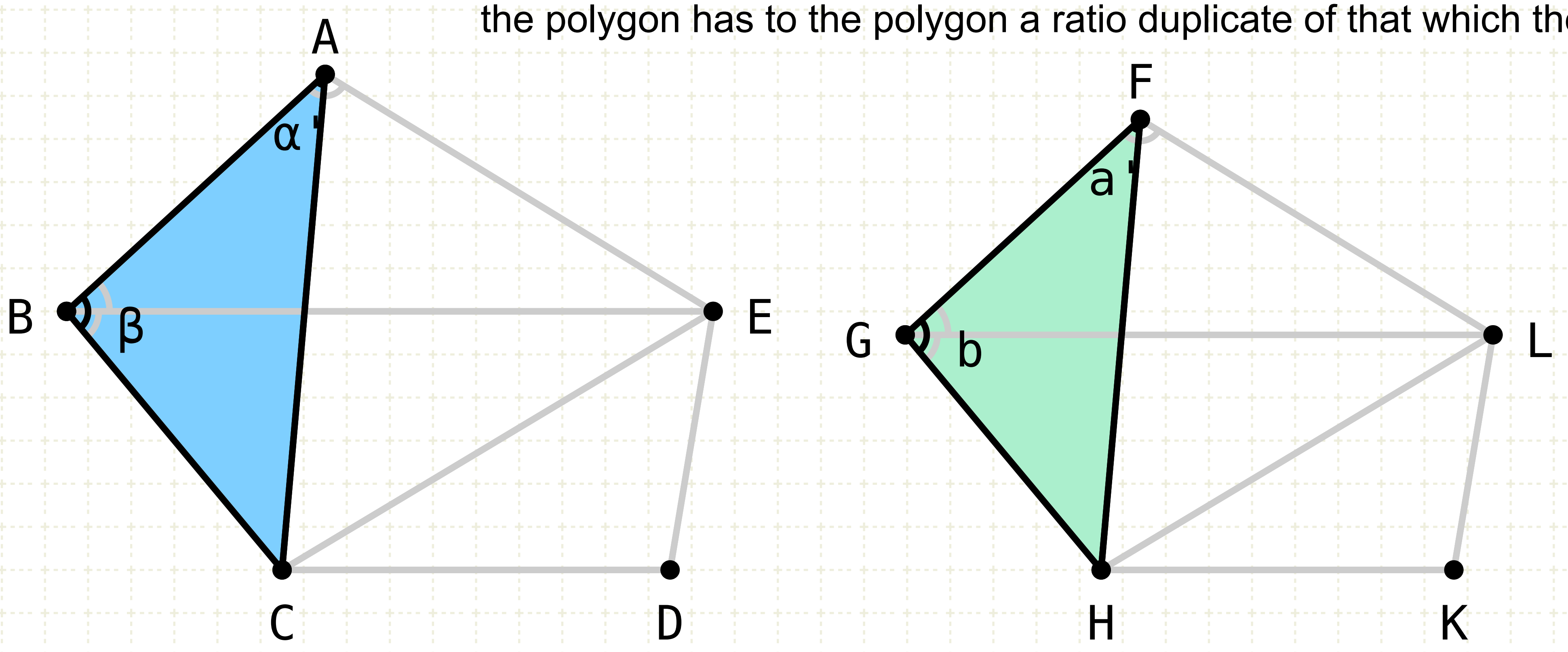

Proof - Duplicate Ratio to Sides

Draw AC, FH

Triangle ABC is similar to FGH (VI·6)

ABCDE ~ FGHKL

b = β

b' = β'

AB:BC = FG:GH

a' = α'

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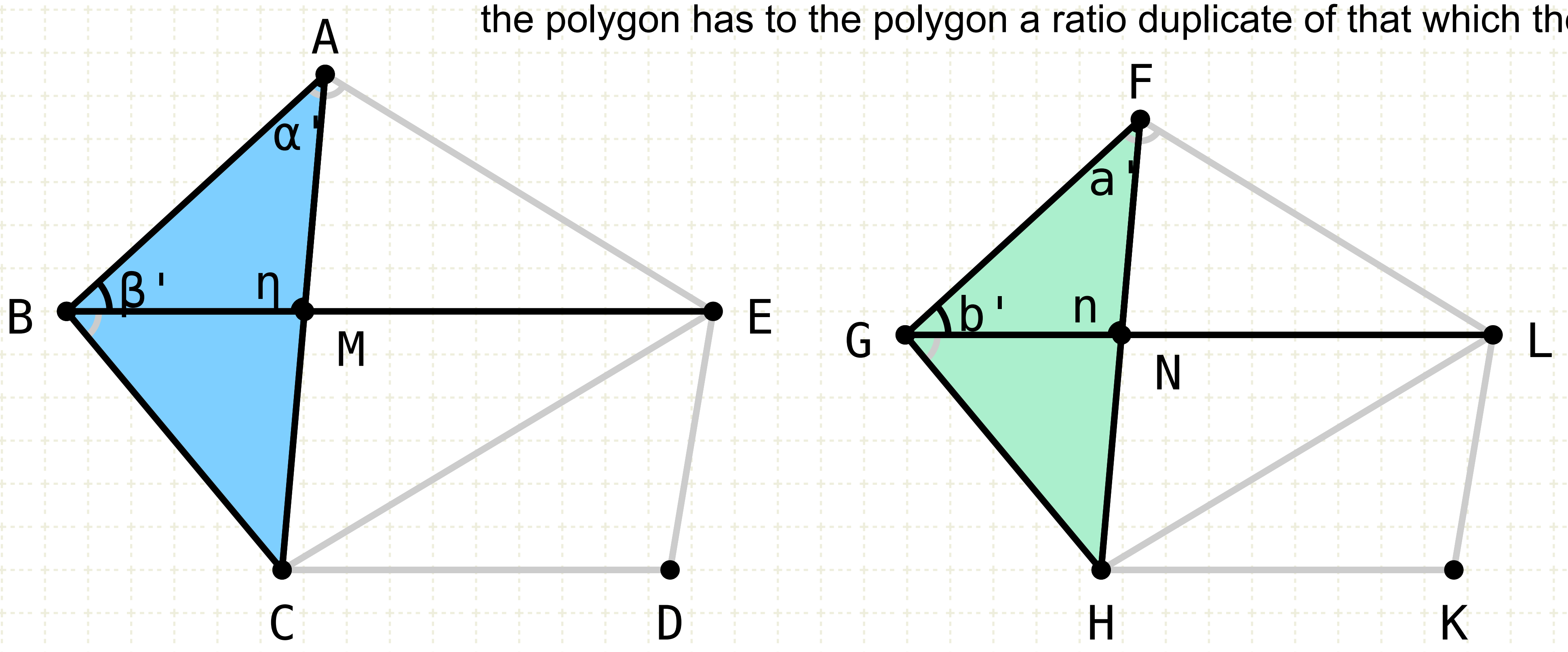

Proof - Duplicate Ratio to Sides

Draw AC, FH

Triangle ABC is similar to FGH (VI·6)

Since the angles ABM and FGN are equal, and the angles BAM and GFN are equal, the angles AMB and FNG are also equal (I·32)

ABCDE ~ FGHKL

b = β

b' = β'

AB:BC = FG:GH

a' = α'

n = η

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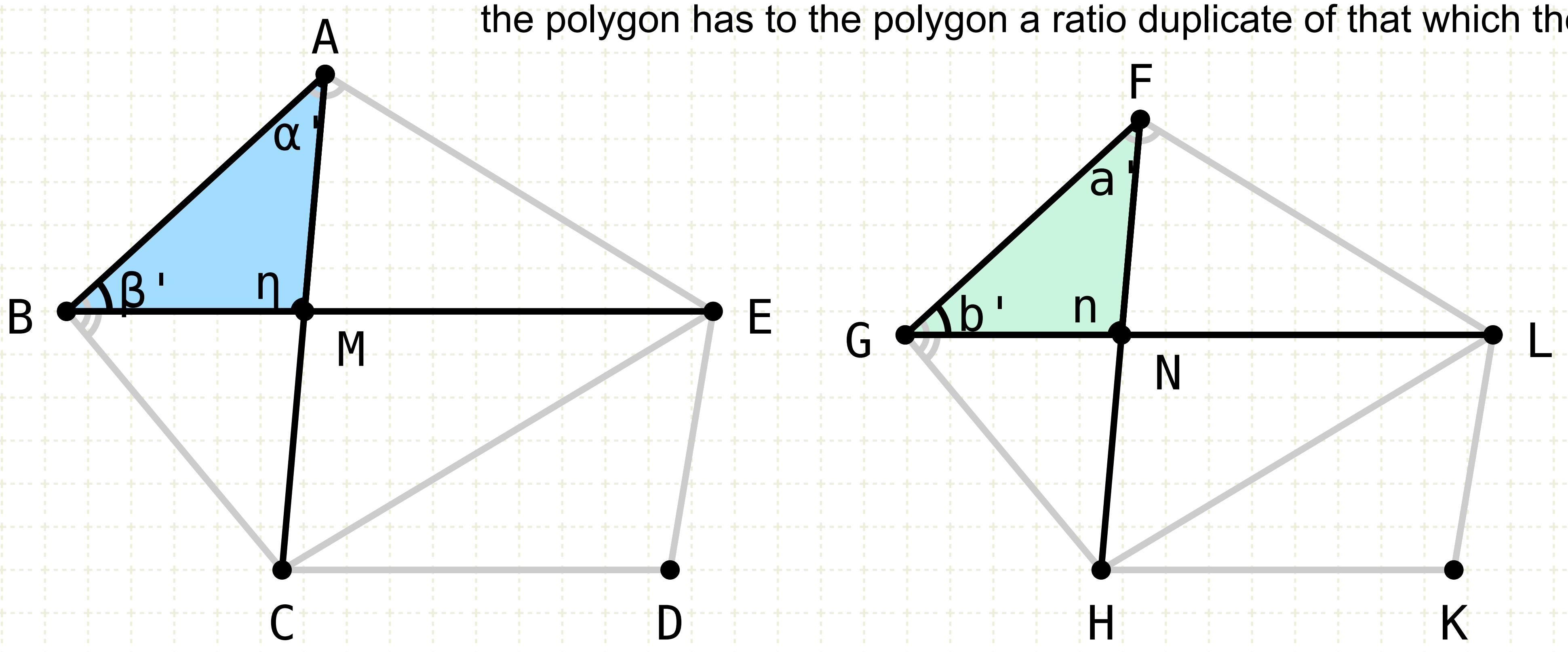

Proof - Duplicate Ratio to Sides

Draw AC, FH

Triangle ABC is similar to FGH (VI·6)

Since the angles ABM and FGN are equal, and the angles BAM and GFN are equal, the angles AMB and FNG are also equal (I·32)

Therefore triangle ABM is equiangular with triangle FGN and similar, so the ratio of AM to MB is equal to FN to NG

ABCDE ~ FGHKL

b = β

b' = β'

AB:BC = FG:GH

a' = α'

n = η

AM:BM = FN:NG

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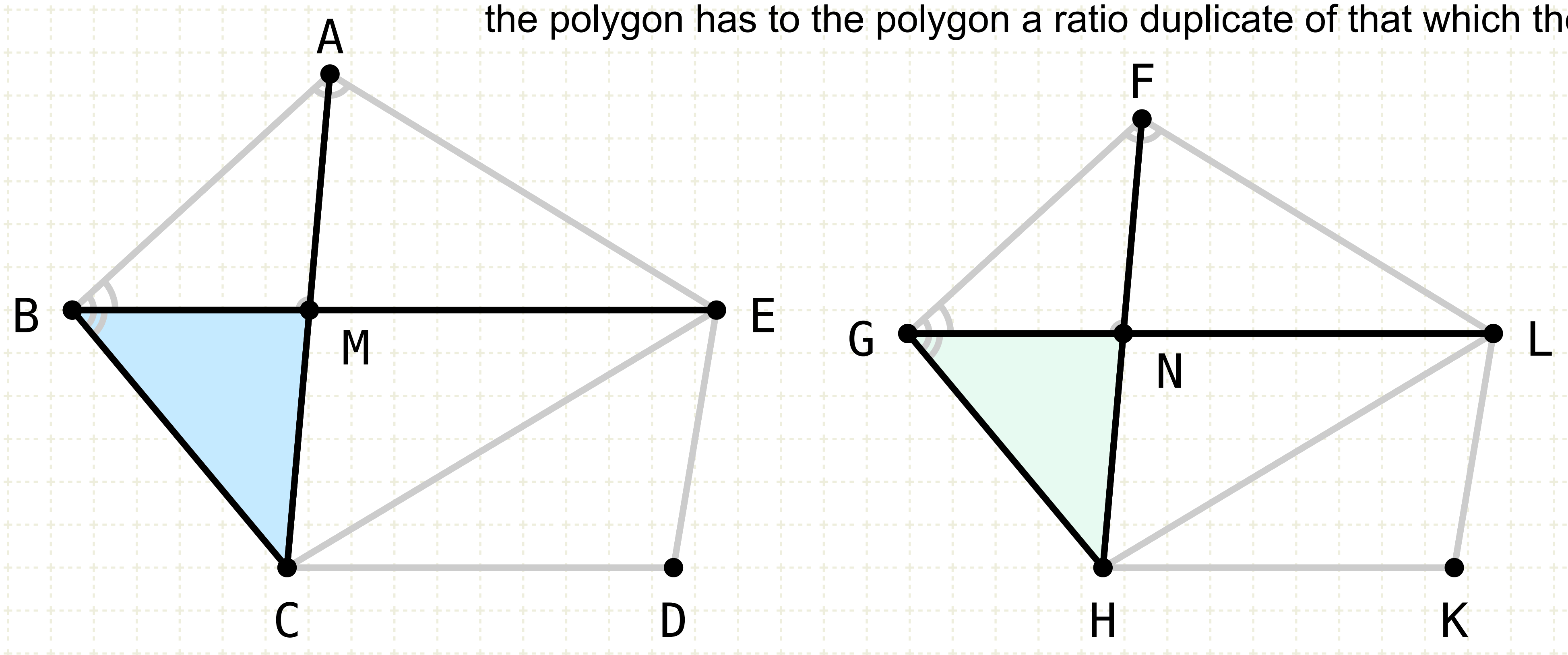

Proof - Duplicate Ratio to Sides

Draw AC, FH

Triangle ABC is similar to FGH (VI·6)

Since the angles ABM and FGN are equal, and the angles BAM and GFN are equal, the angles AMB and FNG are also equal (I·32)

Therefore triangle ABM is equiangular with triangle FGN and similar, so the ratio of AM to MB is equal to FN to NG

Similarly we can prove that triangle BMC is also equiangular with triangle GNH, thus BM is to MC as NG is to NH

ABCDE ~ FGHKL

b = β

b' = β'

AB:BC = FG:GH

a' = α'

n = η

AM:BM = FN:NG

BM:MC = NG:NH

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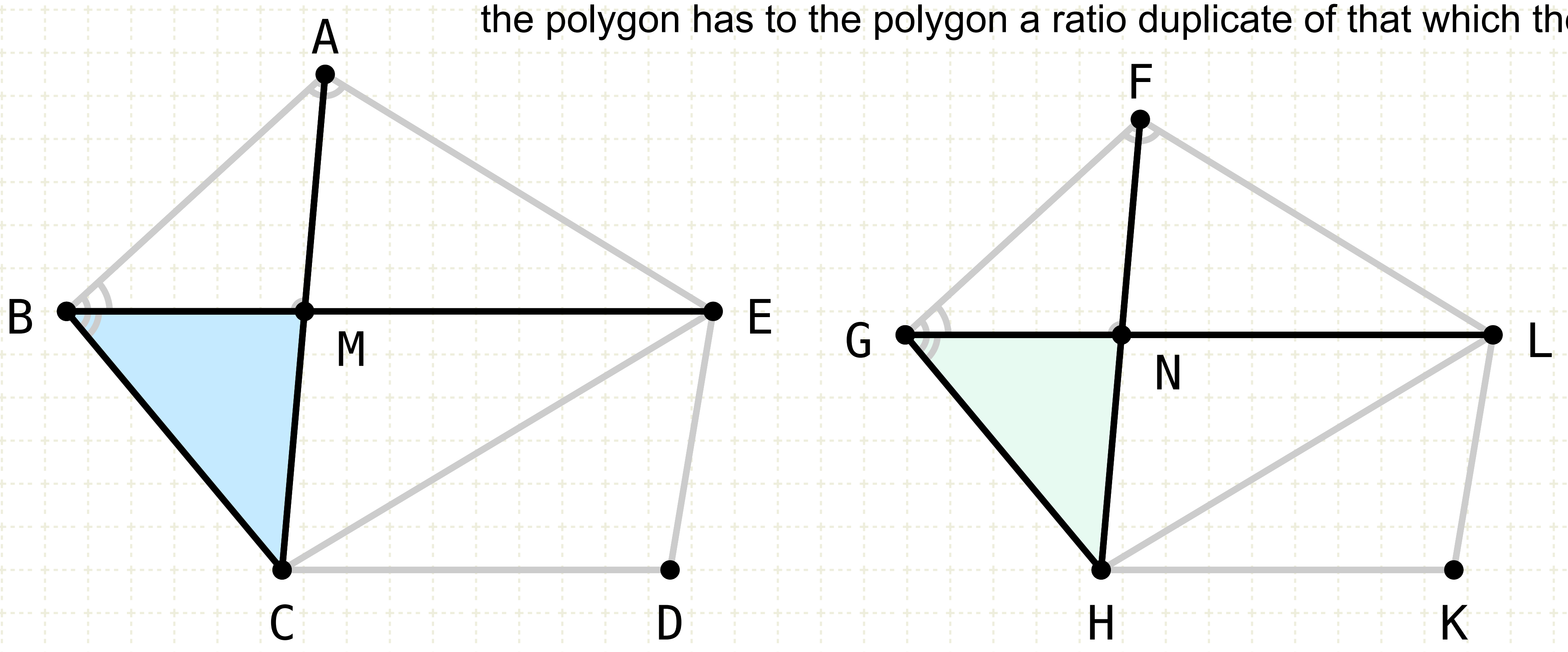

Proof - Duplicate Ratio to Sides

Draw AC, FH

Triangle ABC is similar to FGH (VI·6)

Since the angles ABM and FGN are equal, and the angles BAM and GFN are equal, the angles AMB and FNG are also equal (I·32)

Therefore triangle ABM is equiangular with triangle FGN and similar, so the ratio of AM to MB is equal to FN to NG

Similarly we can prove that triangle BMC is also equiangular with triangle GNH, thus BM is to MC as NG is to NH

In addition, ex aequali, AM is to MC as is FN to NH

ABCDE ~ FGHKL

b = β

b' = β'

AB:BC = FG:GH

a' = α'

n = η

AM:BM = FN:NG

BM:MC = NG:NH

AM:MC = FN:NH

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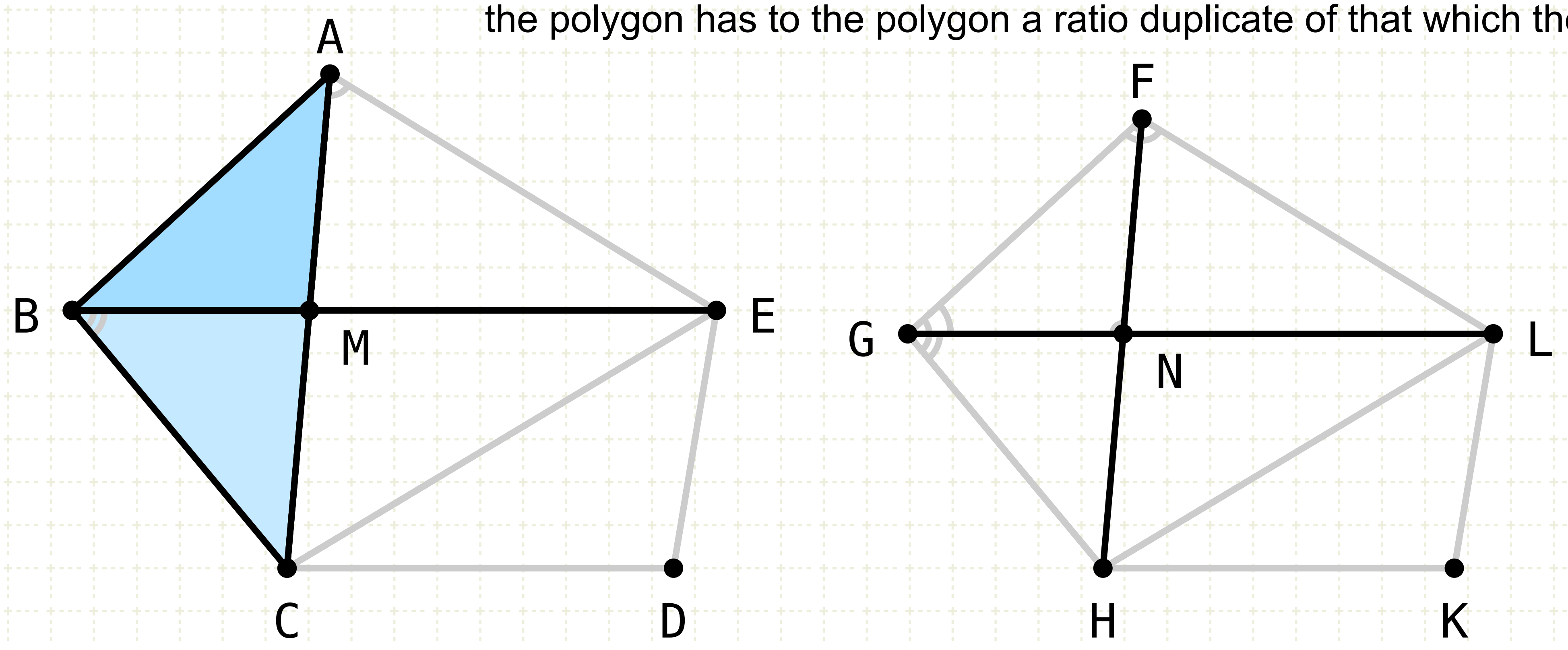

ABCDE ~ FGHKL

b = β

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a' = α'

n = η

AM:BM = FN:NG

BM:MC = NG:NH

AM:MC = FN:NH

AM:MC

= ΔABM:ΔMBC

Proof - Duplicate Ratio to Sides

Draw AC, FH

Triangle ABC is similar to FGH (VI·6)

Since the angles ABM and FGN are equal, and the angles BAM and GFN are equal, the angles AMB and FNG are also equal (I·32)

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Similarly we can prove that triangle BMC is also equiangular with triangle GNH, thus BM is to MC as NG is to NH

In addition, ex aequali, AM is to MC as is FN to NH

But, the ratio of AM to MC is equal to the ratio of the triangles ABM to MBC (VI·1)

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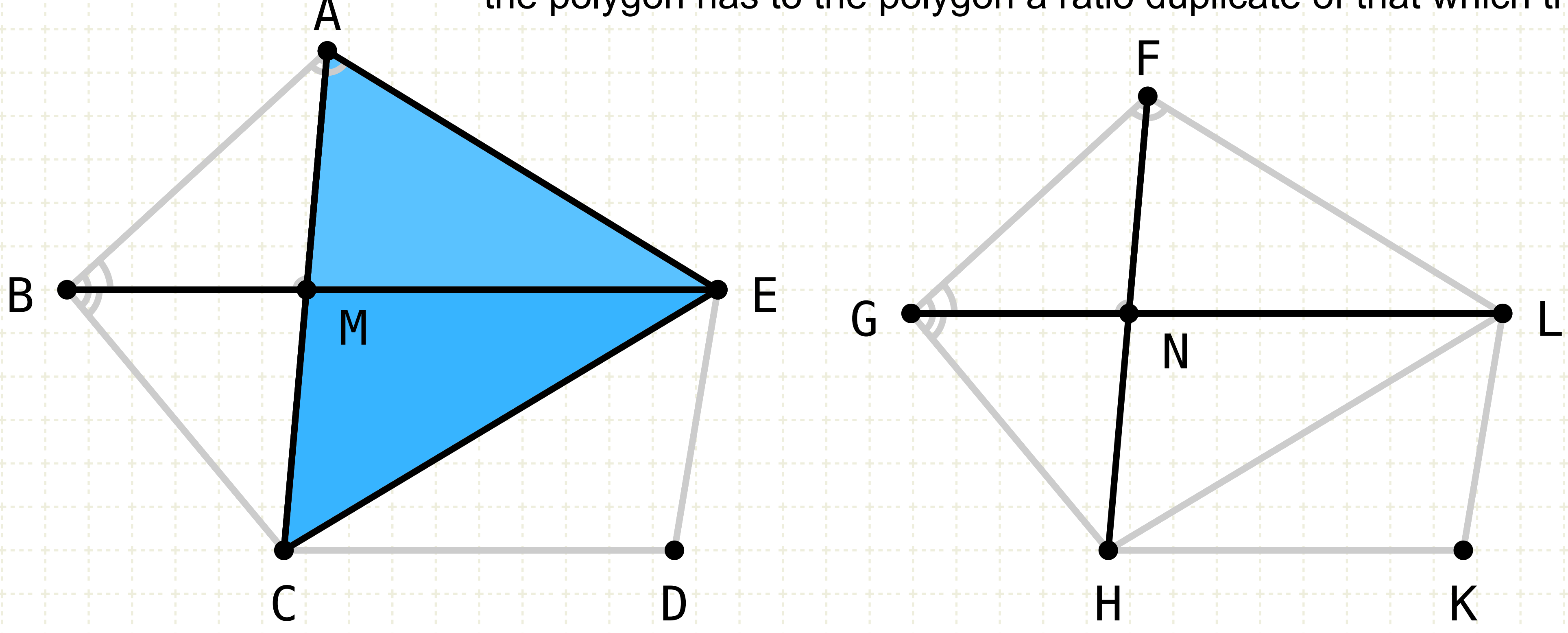

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AM:BM = FN:NG

BM:MC = NG:NH

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AM:MC

= ΔABM:ΔMBC = ΔAME:ΔEMC

Proof - Duplicate Ratio to Sides

Draw AC, FH

Triangle ABC is similar to FGH (VI·6)

Since the angles ABM and FGN are equal, and the angles BAM and GFN are equal, the angles AMB and FNG are also equal (I·32)

Therefore triangle ABM is equiangular with triangle FGN and similar, so the ratio of AM to MB is equal to FN to NG

Similarly we can prove that triangle BMC is also equiangular with triangle GNH, thus BM is to MC as NG is to NH

In addition, ex aequali, AM is to MC as is FN to NH

But, the ratio of AM to MC is equal to the ratio of the triangles ABM to MBC (VI·1)

Likewise, the ratio of AM to MC is equal to the ratio of the triangles AME to EMC (VI·1)

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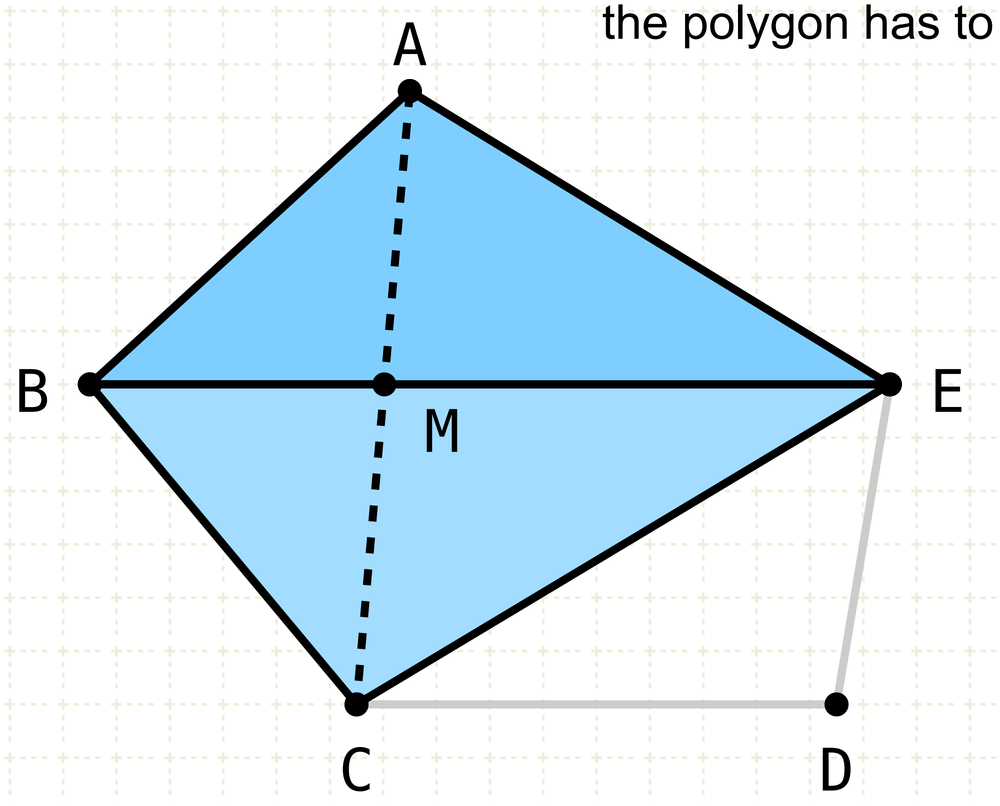

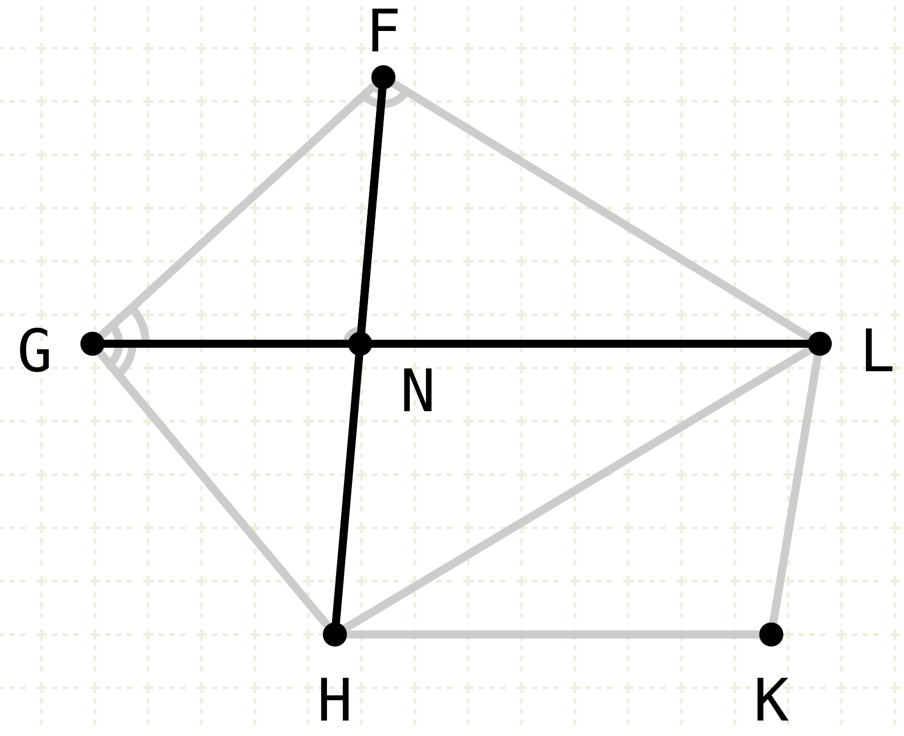

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= ΔABM:ΔMBC = ΔAME:ΔEMC

= (ΔABM+ΔAME):(ΔMBC+ΔEMC)

= ΔABE:ΔBCE

Proof - Duplicate Ratio to Sides

Draw AC, FH

Triangle ABC is similar to FGH (VI·6)

Since the angles ABM and FGN are equal, and the angles BAM and GFN are equal, the angles AMB and FNG are also equal (I·32)

Therefore triangle ABM is equiangular with triangle FGN and similar, so the ratio of AM to MB is equal to FN to NG

Similarly we can prove that triangle BMC is also equiangular with triangle GNH, thus BM is to MC as NG is to NH

In addition, ex aequali, AM is to MC as is FN to NH

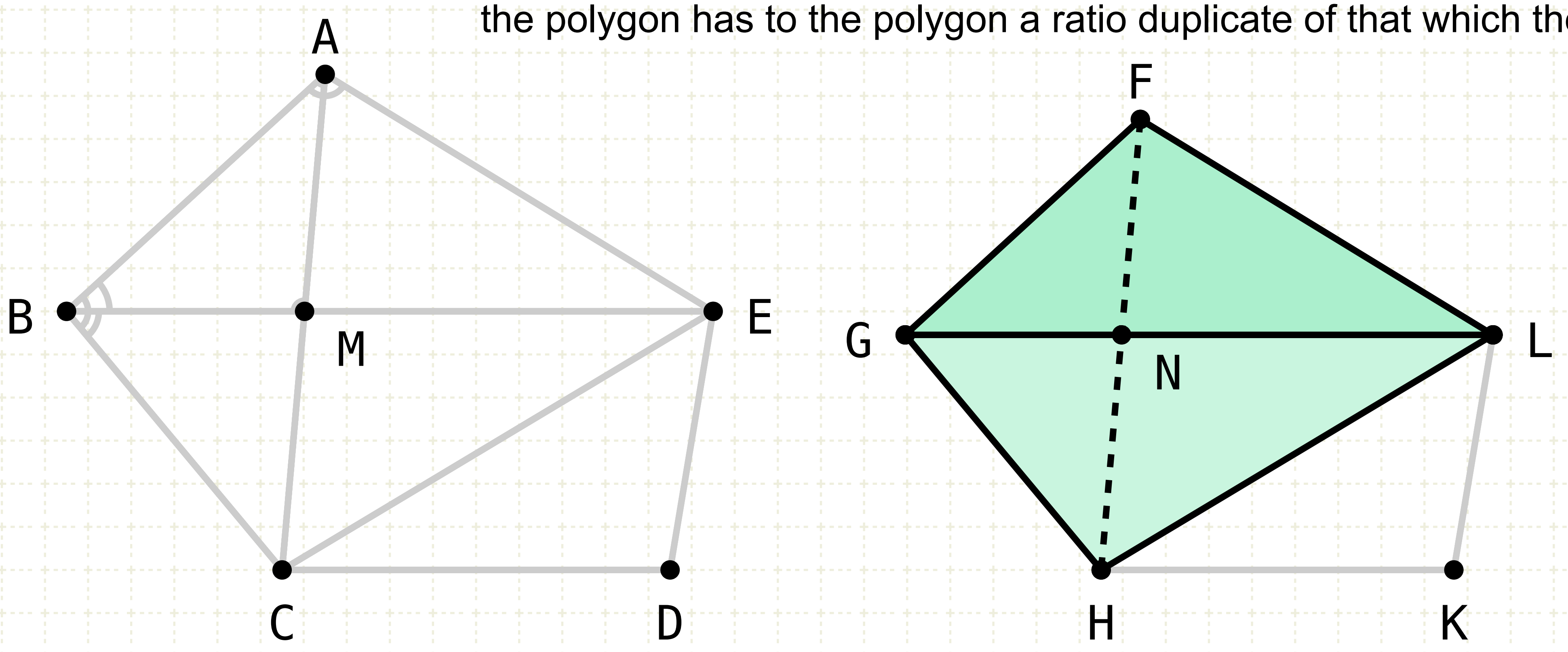
But, the ratio of AM to MC is equal to the ratio of the triangles ABM to MBC (VI·1)

Likewise, the ratio of AM to MC is equal to the ratio of the triangles AME to EMC (VI·1)

If two ratios are equal, so is this the ratio of the sum of antecedents to the sum of consequents (V·12), so AME to EMC is equal to ABE to BCE

Proposition 20 of Book VI

Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.



$$ABCDE \sim FGHKL$$
$$b = \beta$$
$$b' = \beta'$$
$$AB:BC = FG:GH$$
$$a' = \alpha'$$
$$n = \eta$$
$$AM:BM = FN:NG$$
$$BM:MC = NG:NH$$
$$AM:MC = FN:NH$$

$$\begin{aligned} AM:MC & \\ &= \Delta ABM:\Delta MBC = \Delta AME:\Delta EMC \\ &= (\Delta ABM+\Delta AME):(\Delta MBC+\Delta EMC) \\ &= \Delta ABE:\Delta BCE \end{aligned}$$
$$FN:NH = \Delta FGL:\Delta GHL$$

Proof - Duplicate Ratio to Sides

Draw AC, FH

Triangle ABC is similar to FGH (VI·6)

Since the angles ABM and FGN are equal, and the angles BAM and GFN are equal, the angles AMB and FNG are also equal (I·32)

Therefore triangle ABM is equiangular with triangle FGN and similar, so the ratio of AM to MB is equal to FN to NG

Similarly we can prove that triangle BMC is also equiangular with triangle GNH, thus BM is to MC as NG is to NH

In addition, ex aequali, AM is to MC as is FN to NH

But, the ratio of AM to MC is equal to the ratio of the triangles ABM to MBC (VI·1)

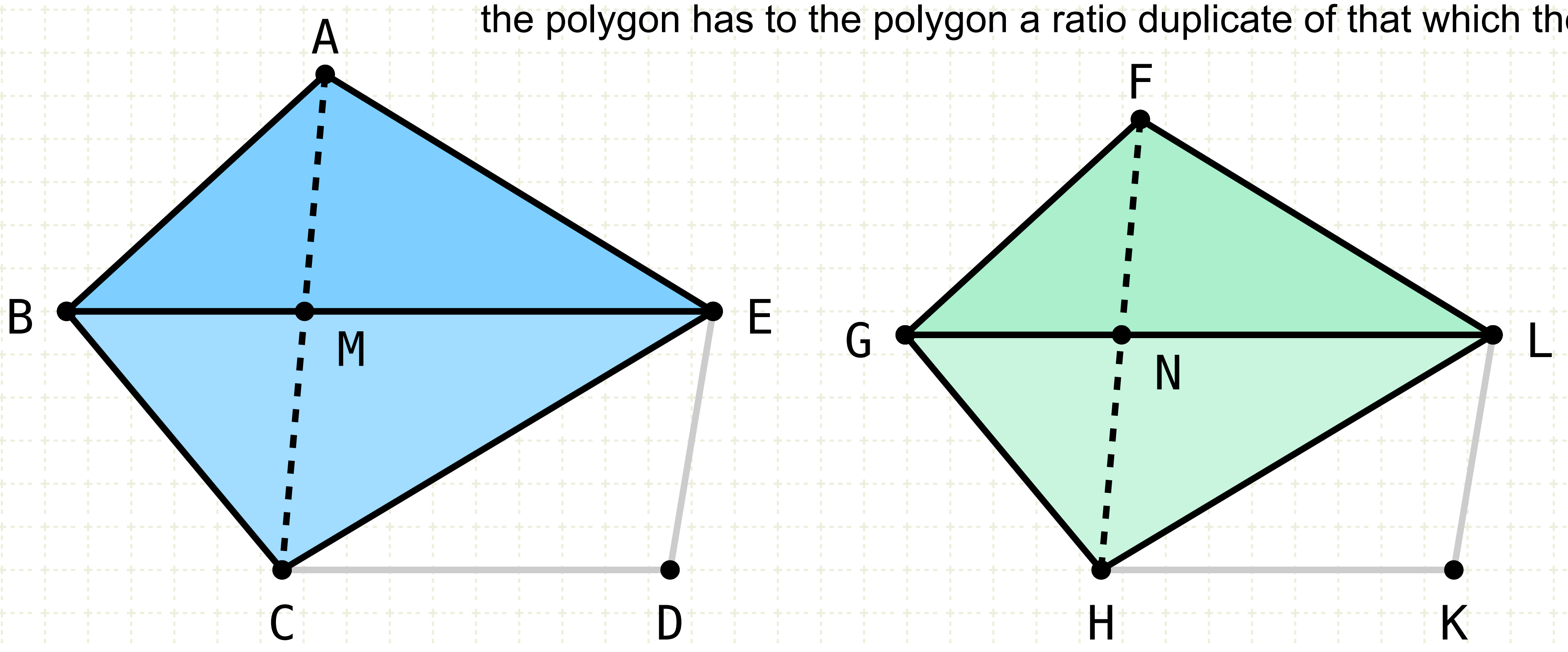
Likewise, the ratio of AM to MC is equal to the ratio of the triangles AME to EMC (VI·1)

If two ratios are equal, so is this the ratio of the sum of antecedents to the sum of consequents (V·12), so AME to EMC is equal to ABE to BCE

Similarly, FN to NH is equal to the triangles FGL to GHL

Proposition 20 of Book VI

Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.



Proof - Duplicate Ratio to Sides

ABCDE ~ FGHKL

AB:BC = FG:GH

AM:MC = FN:NH

AM:MC = ΔABE:ΔBCE

FN:NH = ΔFGL:ΔGHL

Proposition 20 of Book VI

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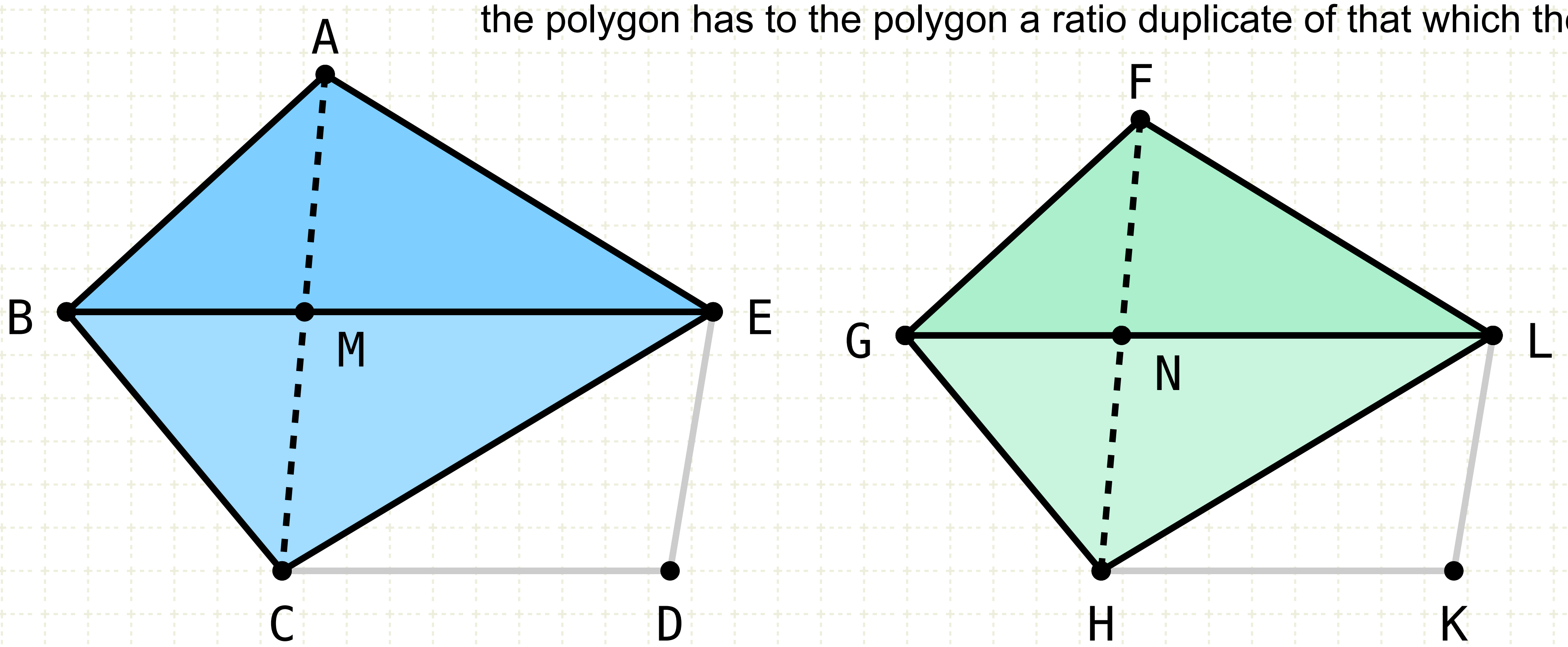

Proof - Duplicate Ratio to Sides

Substitute the triangle ratios for the ratios AM to MC and FN to NH with the result that the ratios of the triangles ABC to BCE equals the ratio of the triangles FGL to GHL

ABCDE ~ FGHKL

AB:BC = FG:GH

AM:MC = FN:NH

AM:MC = ΔABE:ΔBCE

FN:NH = ΔFGL:ΔGHL

ΔABE:ΔBCE = ΔFGL:ΔGHL

Proposition 20 of Book VI

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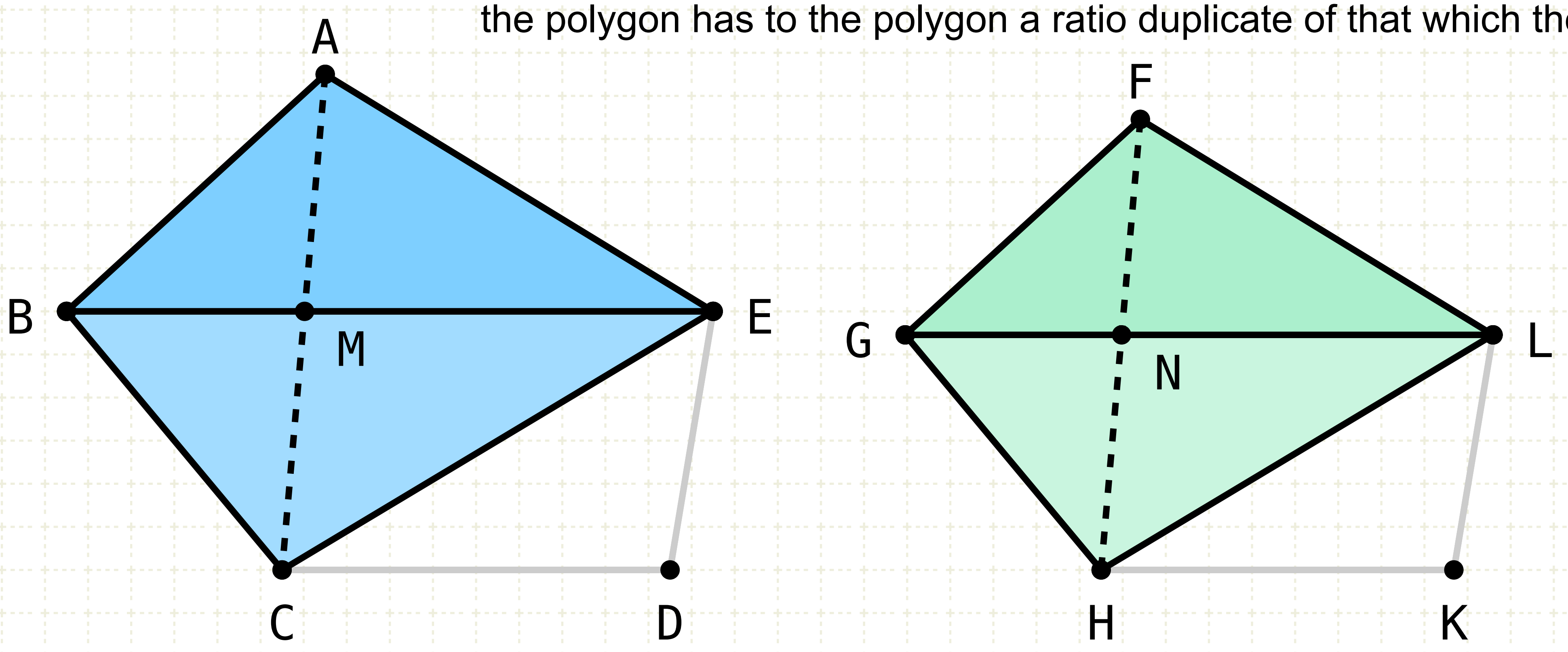

Proof - Duplicate Ratio to Sides

Substitute the triangle ratios for the ratios AM to MC and FN to NH with the result that the ratios of the triangles ABC to BCE equals the ratio of the triangles FGL to GHL

And alternately, the ratio of triangles ABE to FGL is equal to the ratio BCE to GHL

ABCDE ~ FGHKL
AB:BC = FG:GH
AM:MC = FN:NH
AM:MC = ΔABE:ΔBCE
FN:NH = ΔFGL:ΔGHL
ΔABE:ΔBCE = ΔFGL:ΔGHL
ΔABE:ΔFGL = ΔBCE:ΔGHL

Proposition 20 of Book VI

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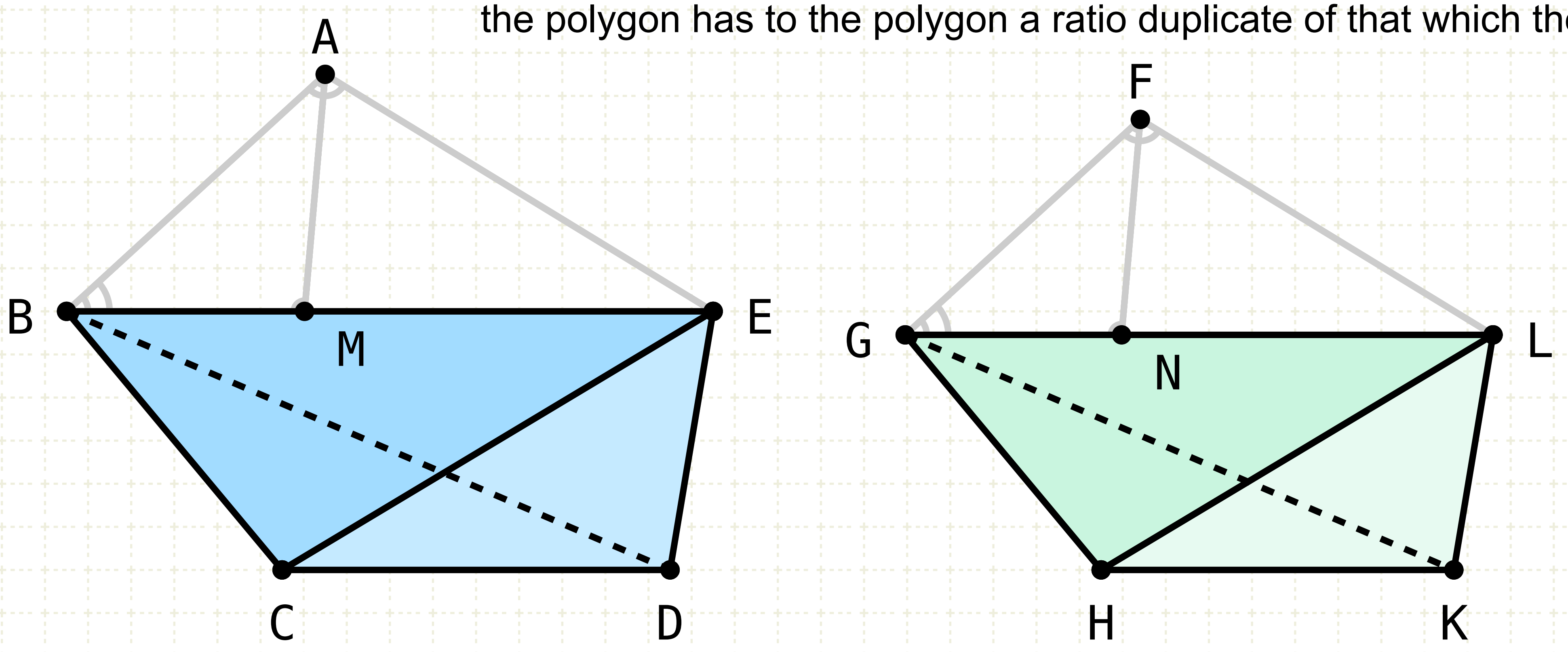

Proof - Duplicate Ratio to Sides

Substitute the triangle ratios for the ratios AM to MC and FN to NH with the result that the ratios of the triangles ABC to BCE equals the ratio of the triangles FGL to GHL

And alternately, the ratio of triangles ABE to FGL is equal to the ratio BCE to GHL

Similarly, we can show that if the lines BD and GK are drawn, the ratio of the triangles BCE to GHL is equal to the ratio of the triangles ECD to LHK.

$ABCDE \sim FGHKL$

$AB:BC = FG:GH$

$AM:MC = FN:NH$

$AM:MC = \Delta ABE:\Delta BCE$

$FN:NH = \Delta FGL:\Delta GHL$

$\Delta ABE:\Delta BCE = \Delta FGL:\Delta GHL$

$\Delta ABE:\Delta FGL = \Delta BCE:\Delta GHL$

$\Delta BCE:\Delta GHL = \Delta ECD:\Delta LHK$

Proposition 20 of Book VI

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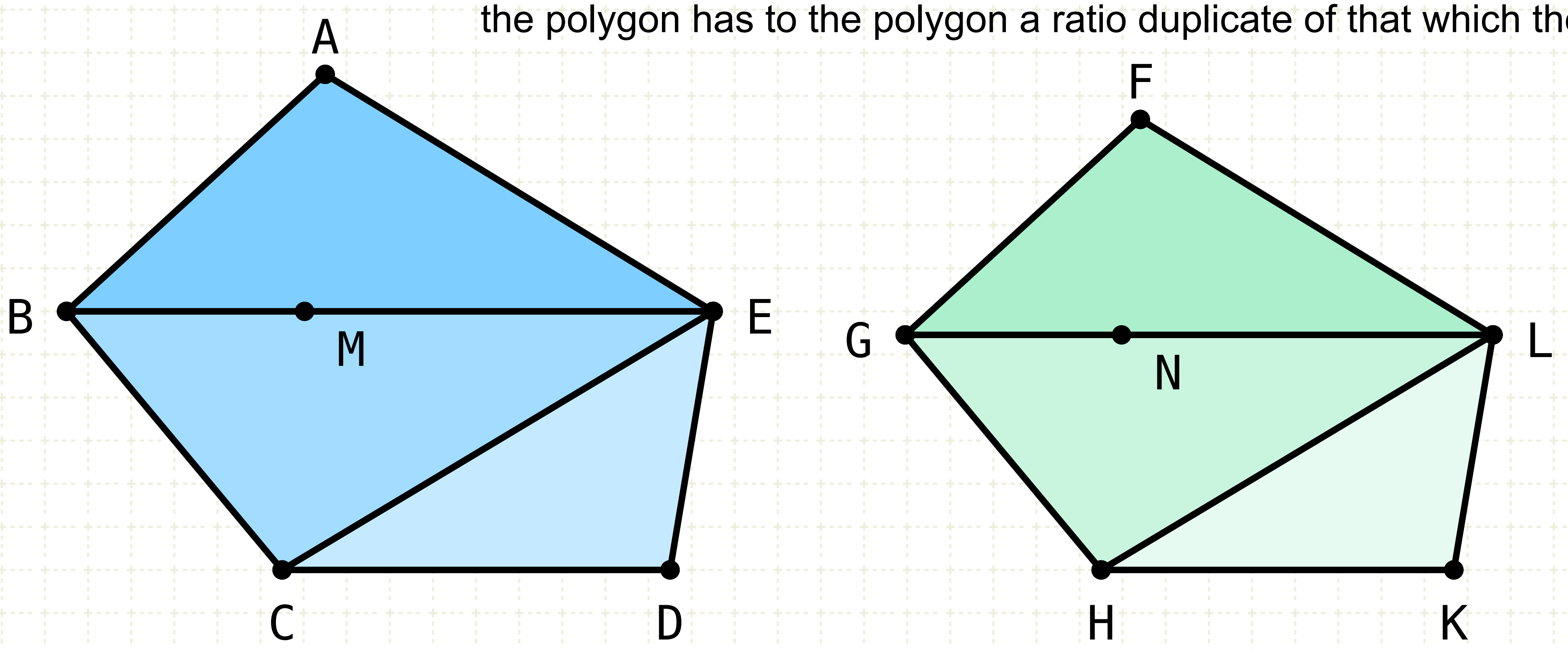


Proof - Duplicate Ratio to Sides

Substitute the triangle ratios for the ratios AM to MC and FN to NH with the result that the ratios of the triangles ABC to BCE equals the ratio of the triangles FGL to GHL

And alternately, the ratio of triangles ABE to FGL is equal to the ratio BCE to GHL

Similarly, we can show that if the lines BD and GK are drawn, the ratio of the triangles BCE to GHL is equal to the ratio of the triangles ECD to LHK.

ABCDE ~ FGHKL

AB:BC = FG:GH

AM:MC = FN:NH

AM:MC = ΔABE:ΔBCE

FN:NH = ΔFGL:ΔGHL

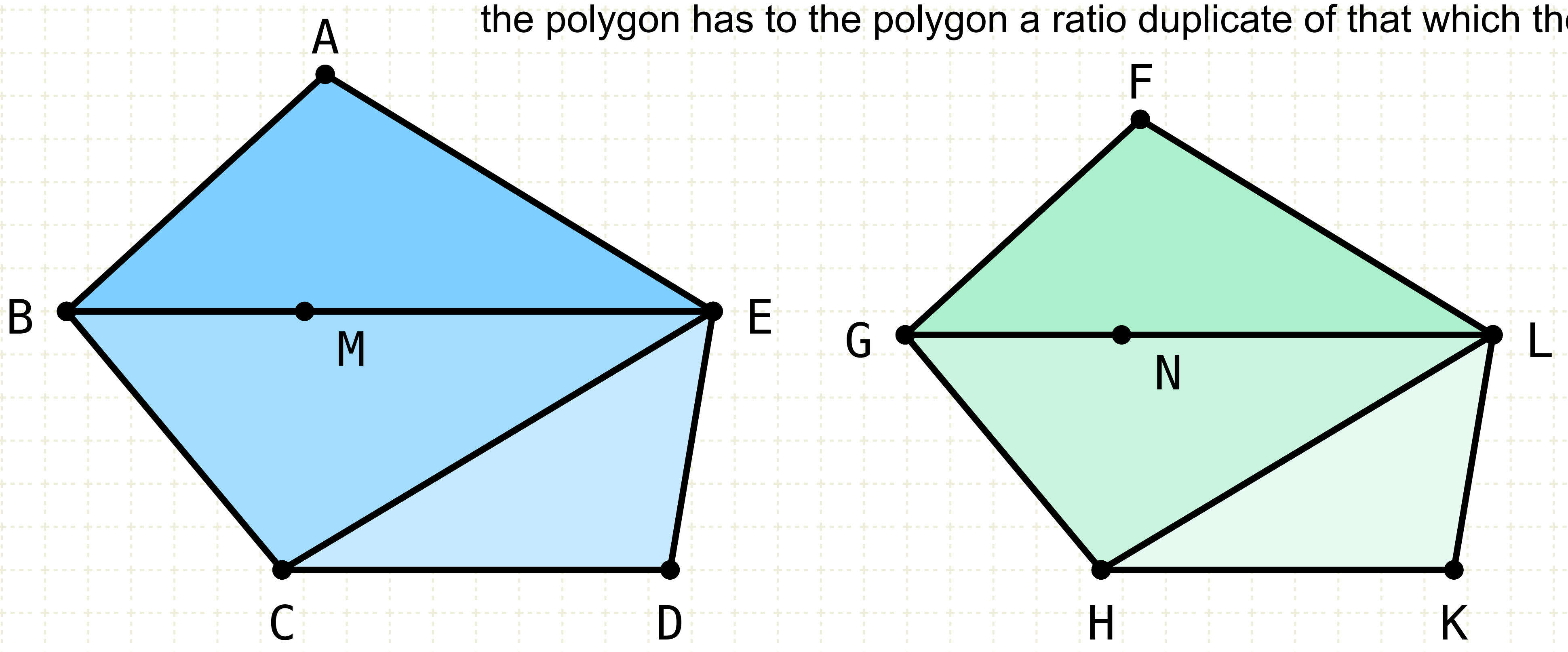
ΔABE:ΔBCE = ΔFGL:ΔGHL

ΔABE:ΔFGL = ΔBCE:ΔGHL

ΔBCE:ΔGHL = ΔECD:ΔLHK

Proposition 20 of Book VI

Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.



Proof - Duplicate Ratio to Sides

Substitute the triangle ratios for the ratios AM to MC and FN to NH with the result that the ratios of the triangles ABC to BCE equals the ratio of the triangles FGL to GHL

And alternately, the ratio of triangles ABE to FGL is equal to the ratio BCE to GHL

Similarly, we can show that if the lines BD and GK are drawn, the ratio of the triangles BCE to GHL is equal to the ratio of the triangles ECD to LHK.

If a number of ratios are equal, then they are also equal to the ratio of the sums of the antecedents to the sums of the consequents (V·12)

$ABCDE \sim FGHKL$

$AB:BC = FG:GH$

$AM:MC = FN:NH$

$AM:MC = \Delta ABE:\Delta BCE$

$FN:NH = \Delta FGL:\Delta GHL$

$\Delta ABE:\Delta BCE = \Delta FGL:\Delta GHL$

$\Delta ABE:\Delta FGL = \Delta BCE:\Delta GHL$

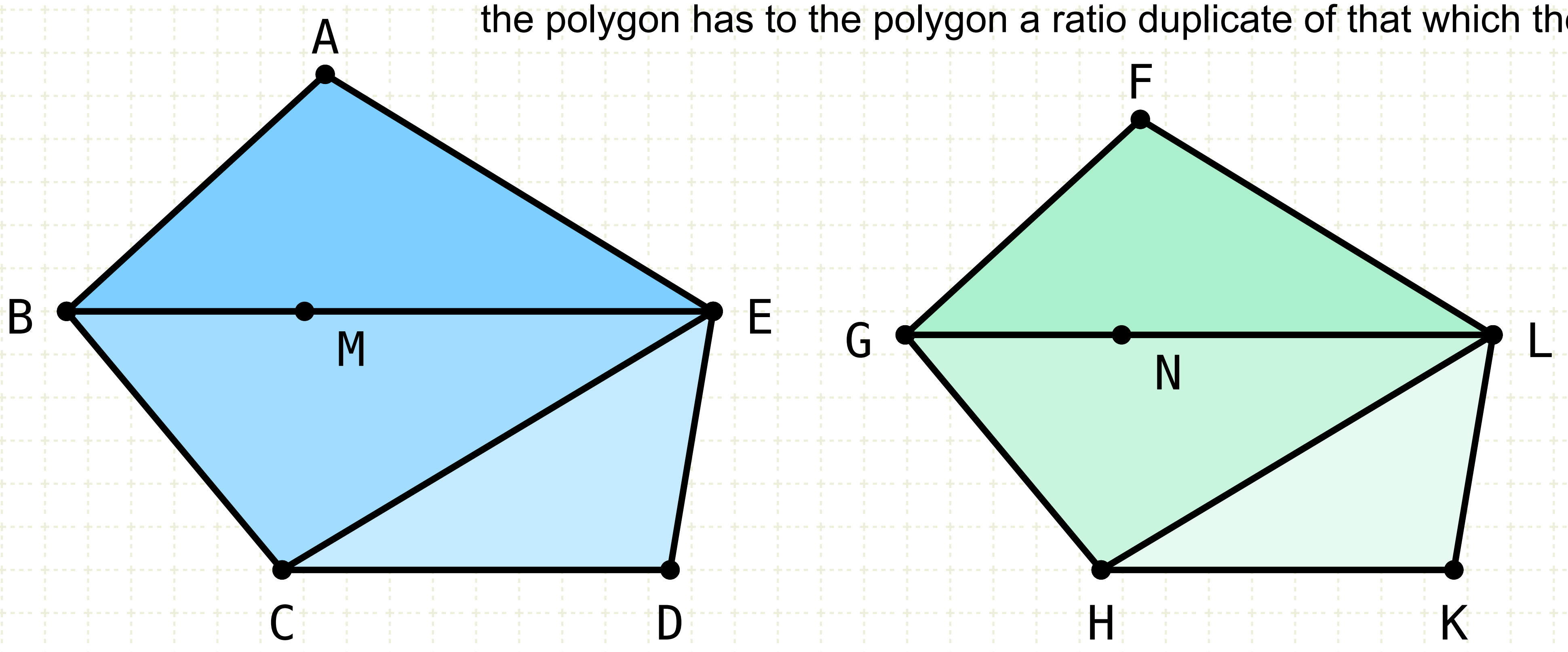
$\Delta BCE:\Delta GHL = \Delta ECD:\Delta LHK$

$= (\Delta ABE+\Delta BCE+\Delta ECD):(\Delta FGL+\Delta GHL+\Delta LHK)$

$\Delta ABE:\Delta FGL = ABCDE:FGHKL$

Proposition 20 of Book VI

Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.



ABCDE ~ FGHKL

AB:BC = FG:GH

AM:MC = FN:NH

AM:MC = ΔABE:ΔBCE

FN:NH = ΔFGL:ΔGHL

ΔABE:ΔBCE = ΔFGL:ΔGHL

ΔABE:ΔFGL = ΔBCE:ΔGHL

ΔBCE:ΔGHL = ΔECD:ΔLHK

= (ΔABE+ΔBCE+ΔECD):(ΔFGL+ΔGHL+ΔLHK)

ΔABE:ΔFGL = ABCDE:FGHKL

ΔABE:ΔFGL = $(AB:FG)^2$

Proof - Duplicate Ratio to Sides

Substitute the triangle ratios for the ratios AM to MC and FN to NH with the result that the ratios of the triangles ABC to BCE equals the ratio of the triangles FGL to GHL

And alternately, the ratio of triangles ABE to FGL is equal to the ratio BCE to GHL

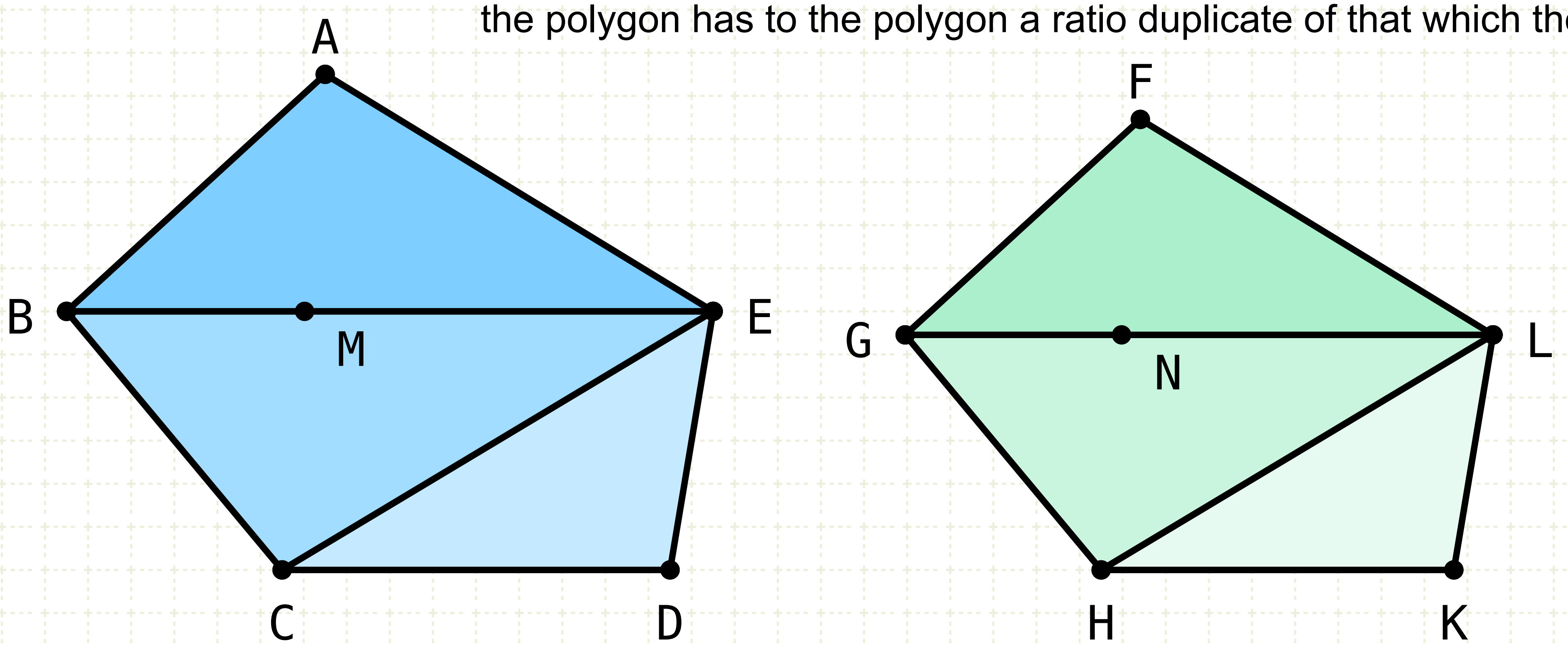
Similarly, we can show that if the lines BD and GK are drawn, the ratio of the triangles BCE to GHL is equal to the ratio of the triangles ECD to LHK.

If a number of ratios are equal, then they are also equal to the ratio of the sums of the antecedents to the sums of the consequents (V·12)

But the ratio of the triangles ABE to FGL is a duplicate ratio of the sides AB to FG (VI·19)

Proposition 20 of Book VI

Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.



Proof - Duplicate Ratio to Sides

Substitute the triangle ratios for the ratios AM to MC and FN to NH with the result that the ratios of the triangles ABC to BCE equals the ratio of the triangles FGL to GHL

And alternately, the ratio of triangles ABE to FGL is equal to the ratio BCE to GHL

Similarly, we can show that if the lines BD and GK are drawn, the ratio of the triangles BCE to GHL is equal to the ratio of the triangles ECD to LHK.

If a number of ratios are equal, then they are also equal to the ratio of the sums of the antecedents to the sums of the consequents (V·12)

But the ratio of the triangles ABE to FGL is a duplicate ratio of the sides AB to FG (VI·19)

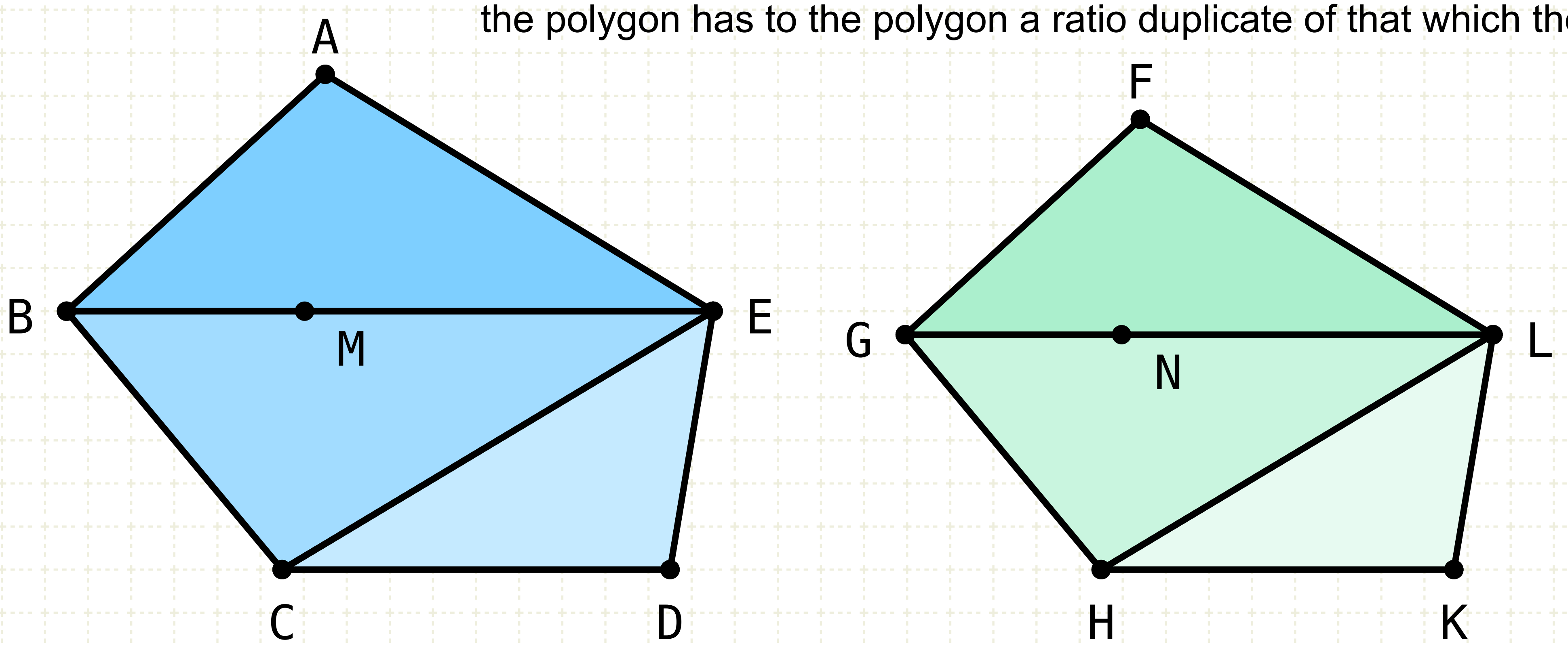
Therefore, the ratio of the two polygons ABCDE and FGHKL is a duplicate ratio of the sides AB to FG

$ABCDE \sim FGHKL$
$AB:BC = FG:GH$
$AM:MC = FN:NH$
$AM:MC = \Delta ABE:\Delta BCE$
$FN:NH = \Delta FGL:\Delta GHL$
$\Delta ABE:\Delta BCE = \Delta FGL:\Delta GHL$
$\Delta ABE:\Delta FGL = \Delta BCE:\Delta GHL$
$\Delta BCE:\Delta GHL = \Delta ECD:\Delta LHK$
$= (\Delta ABE+\Delta BCE+\Delta ECD):(\Delta FGL+\Delta GHL+\Delta LHK)$

$\Delta ABE:\Delta FGL = ABCDE:FGHKL$
$\Delta ABE:\Delta FGL = (AB:FG)^2$
$ABCDE:FGHKL = (AB:FG)^2$

Proposition 20 of Book VI

Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.



$ABCDE \sim FGHKL$

$AB:BC = FG:GH$

$AM:MC = FN:NH$

$AM:MC = \Delta ABE:\Delta BCE$

$FN:NH = \Delta FGL:\Delta GHL$

$\Delta ABE:\Delta BCE = \Delta FGL:\Delta GHL$

$\Delta ABE:\Delta FGL = \Delta BCE:\Delta GHL$

$\Delta BCE:\Delta GHL = \Delta ECD:\Delta LHK$

$= (\Delta ABE+\Delta BCE+\Delta ECD):(\Delta FGL+\Delta GHL+\Delta LHK)$

$\Delta ABE:\Delta FGL = ABCDE:FGHKL$

$\Delta ABE:\Delta FGL = (AB:FG)^2$

$ABCDE:FGHKL = (AB:FG)^2$

Proof - Duplicate Ratio to Sides

Substitute the triangle ratios for the ratios AM to MC and FN to NH with the result that the ratios of the triangles ABC to BCE equals the ratio of the triangles FGL to GHL

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But the ratio of the triangles ABE to FGL is a duplicate ratio of the sides AB to FG (VI·19)

Therefore, the ratio of the two polygons ABCDE and FGHKL is a duplicate ratio of the sides AB to FG

Thus, it has been shown that the ratio of the two polygons is a duplicate ratio of their respective sides

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