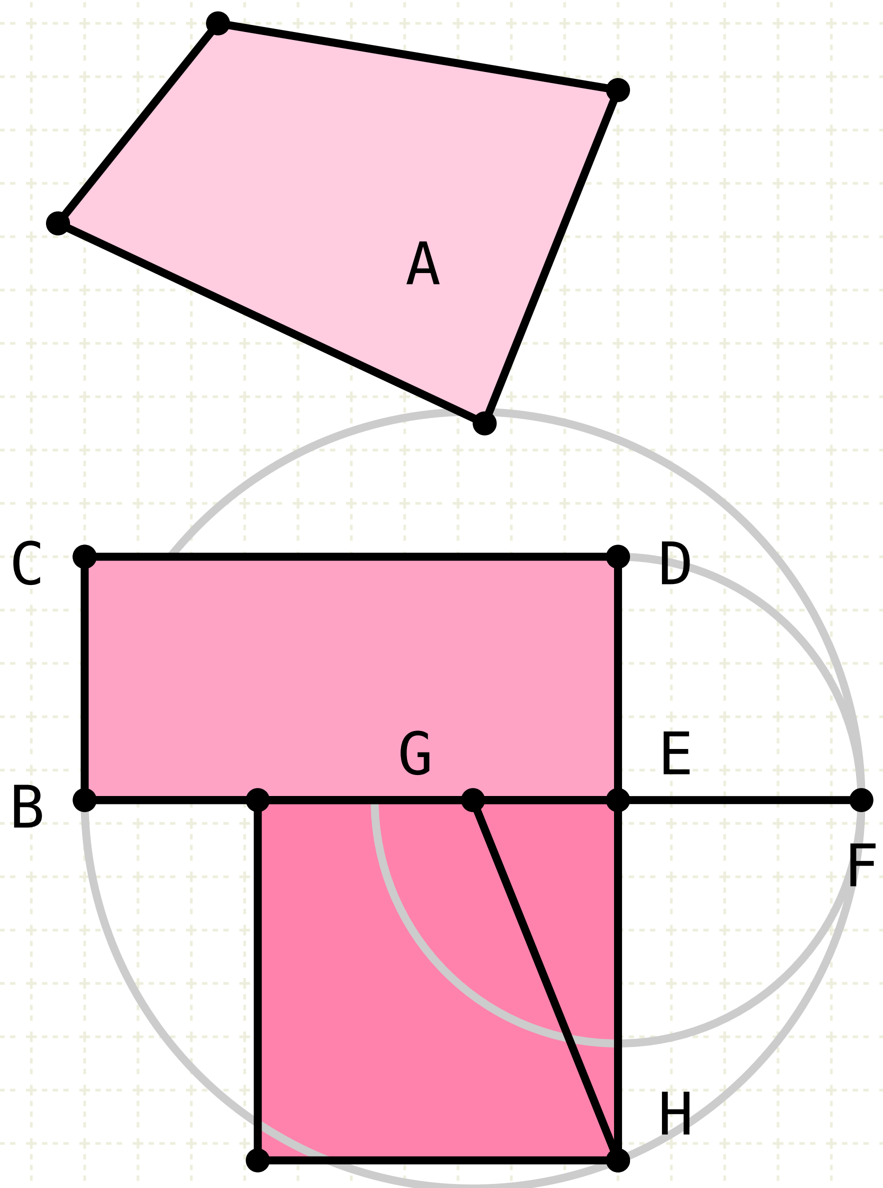
Euclid's Elements

Book II



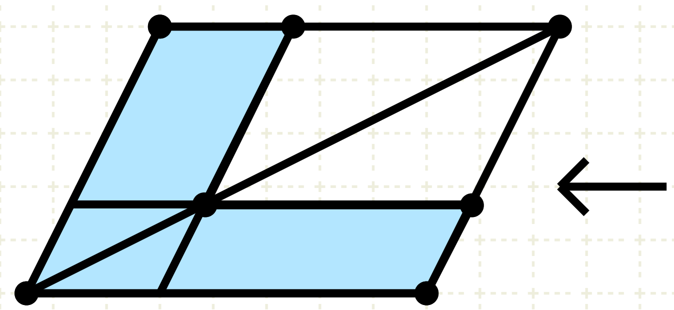
It is a remarkable fact in the history of geometry, that the Elements of Euclid, written two thousand years ago, are still regarded by many as the best introduction to the mathematical sciences.

Florian Cajori,

A History of Mathematics (1893)

Definitions:

Any rectangular parallelogram is said to be contained by the two straight lines containing the right angle.



And in any parallelogrammic area let any one whatever of the parallelograms about its diameter with the two complements be called a gnomon.

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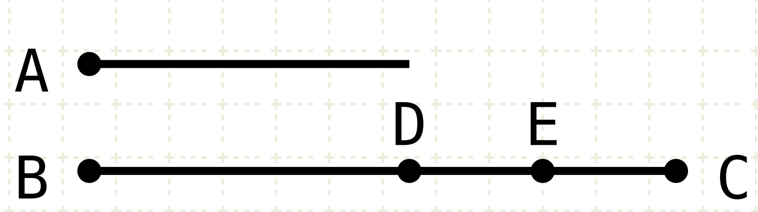
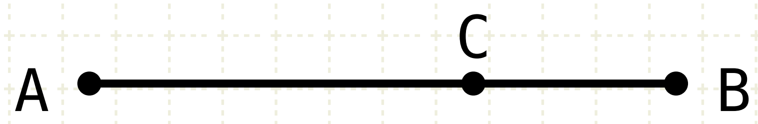


$A \cdot BC = A \cdot BD + A \cdot DE + A \cdot EC$

2

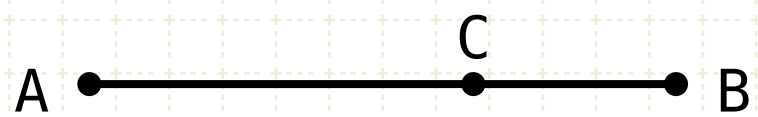


$AB^2 = AB \cdot AC + AB \cdot BC$

3

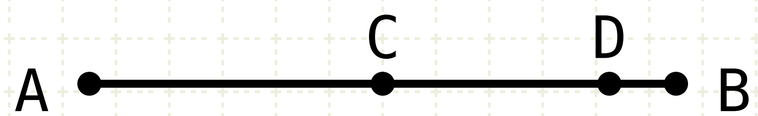
$AB \cdot CB = AC \cdot CB + CB^2$

4



$AB^2 = AC^2 + CB^2 + 2 \cdot AC \cdot CB$

5

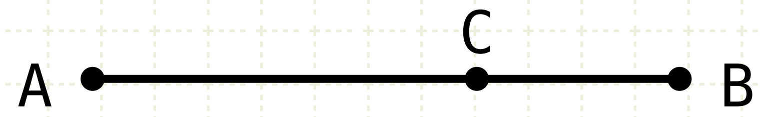


$AD \cdot DB + CD^2 = CB^2$

6

$AD \cdot DB + CB^2 = CD^2$

7

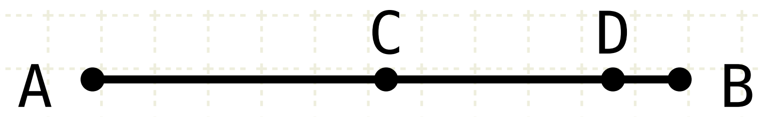


$\mathbf{AB^2 + BC^2 = AC^2 + 2 \cdot AB \cdot BC}$

8

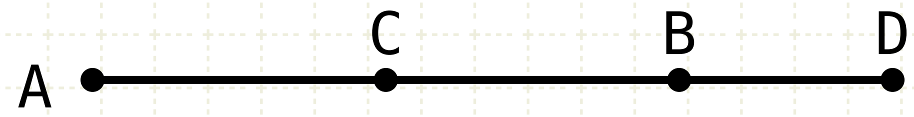
$4 \cdot AB \cdot BC + AC^2 = (AB+BC)^2$

9



$AD^2 + DB^2 = 2 \cdot (AC^2 + CD^2)$

10



$AD^2 + DB^2 = 2 \cdot (AC^2 + CD^2)$

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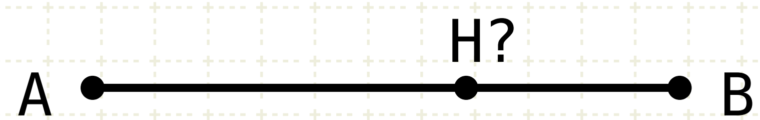
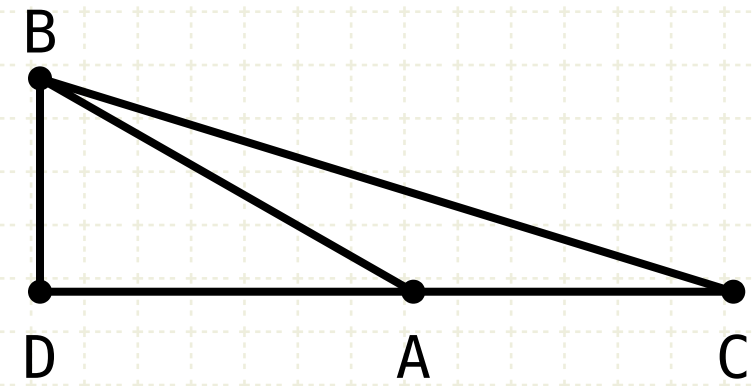


Find H. $AB \cdot BH = AH^2$

12



Cosine Law. $BC^2 = AB^2+AC^2+2 \cdot AD \cdot AC$

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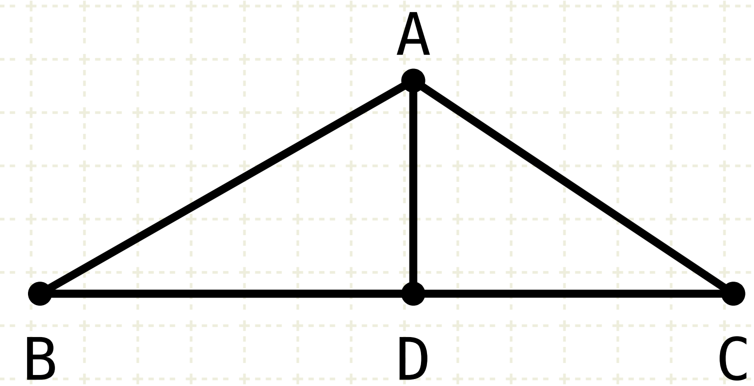
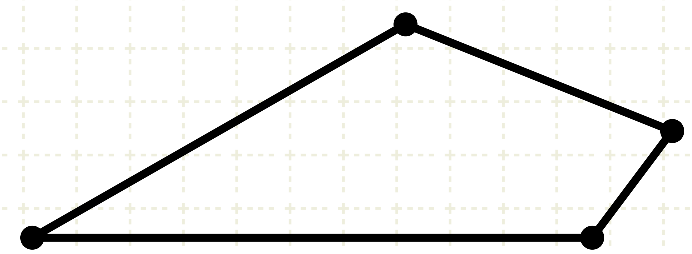


Cosine Law. $AC^2 = AB^2+BC^2-2 \cdot BD \cdot BC$

14



Find square of polygon

Proposition 7 of Book II

If a straight line be cut at random, the square on the whole and that of one of the segments both together are equal to twice the rectangle contained by the whole and the said segment and the square on the remaining segment.

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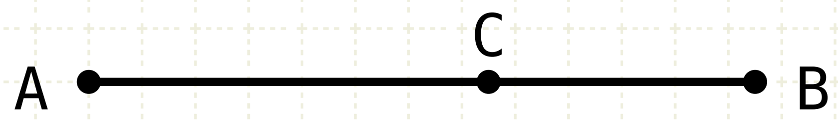

In other words

Let AB be a straight line, arbitrarily cut at point C

AB = AC + CB

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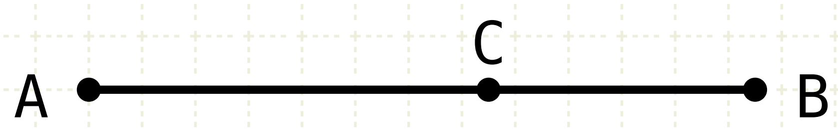

In other words

Let AB be a straight line, arbitrarily cut at point C

Then the squares formed by lines AB and BC are equal in area to the sum of the square formed by line AC, plus twice the area of the rectangle formed by lines AB and CB

$AB = AC + CB$

$AB \cdot AB + BC \cdot BC = AC \cdot AC + 2 \cdot AB \cdot BC$

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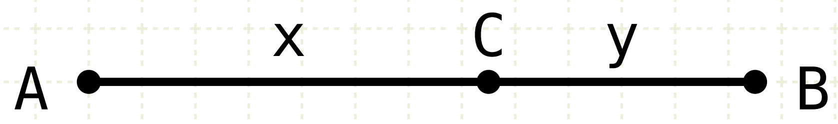

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$(x+y)^2 + y^2 = x^2 + 2(x+y)y$

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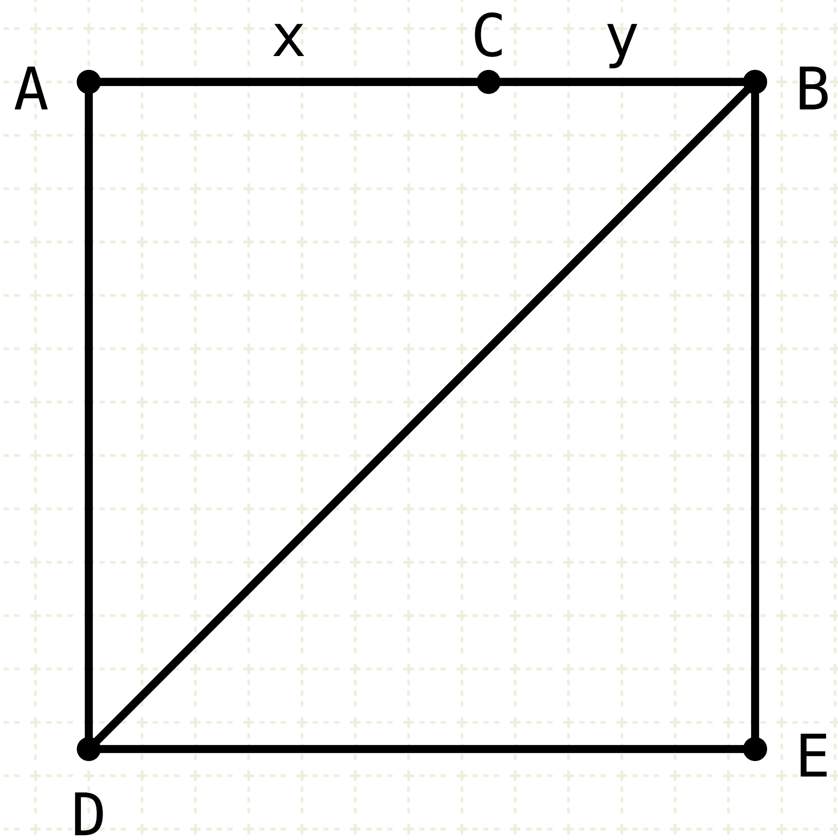

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Then the squares formed by lines AB and BC are equal in area to the sum of the square formed by line AC, plus twice the area of the rectangle formed by lines AB and CB

Construction:

Draw a square ADEB on the line AB (I·46), and draw the diagonal BD

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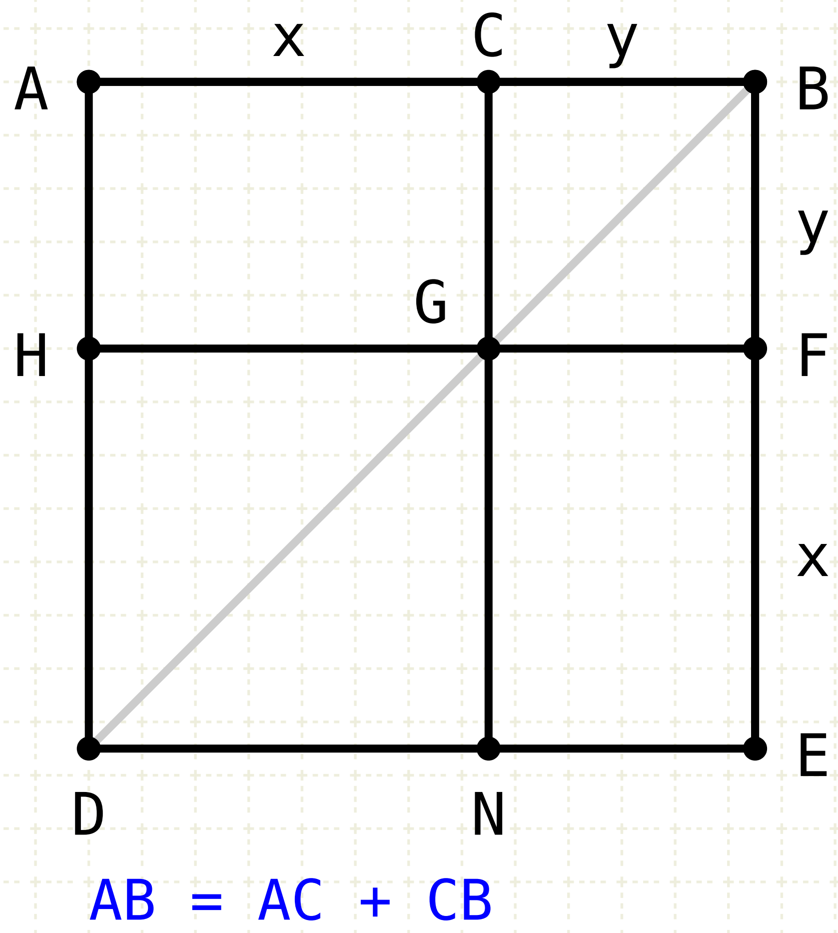

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Construction:

Draw a square ADEB on the line AB (I·46), and draw the diagonal BD

Draw a line CN parallel to AD (I·31), labelling the intersection with the diagonal as G

Draw a line parallel to AB through the point G (I·31).

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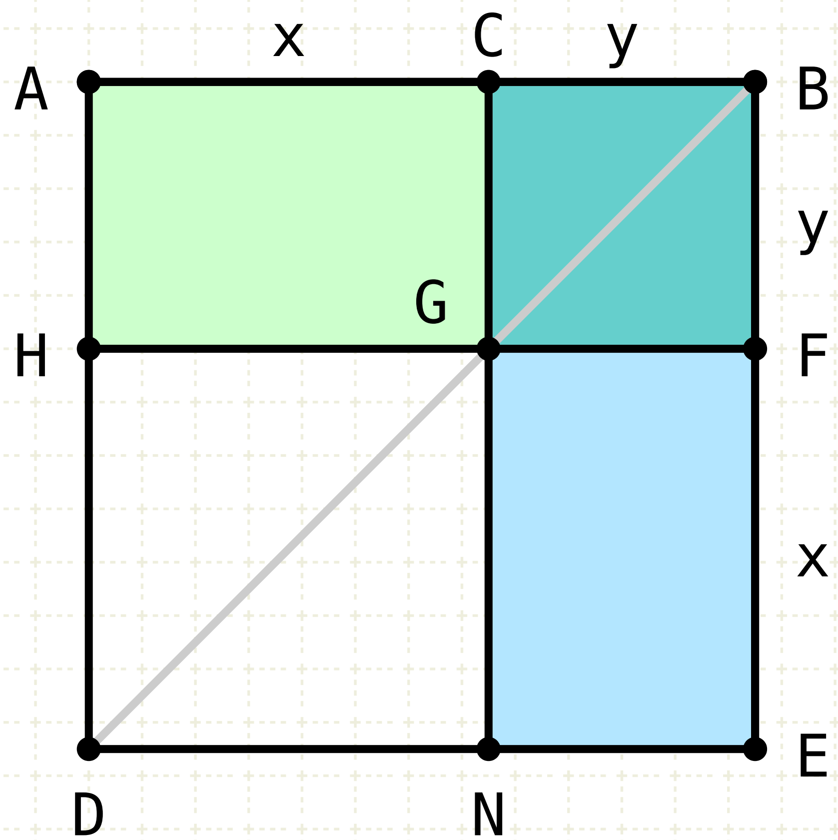

AB = AC + CB

□AF = □CE, □AF + □CE = 2□AF

In other words

Let AB be a straight line, arbitrarily cut at point C

Then the squares formed by lines AB and BC are equal in area to the sum of the square formed by line AC, plus twice the area of the rectangle formed by lines AB and CB

Proof:

AG equals GE (I·43), add CF to both, thus preserving the equality

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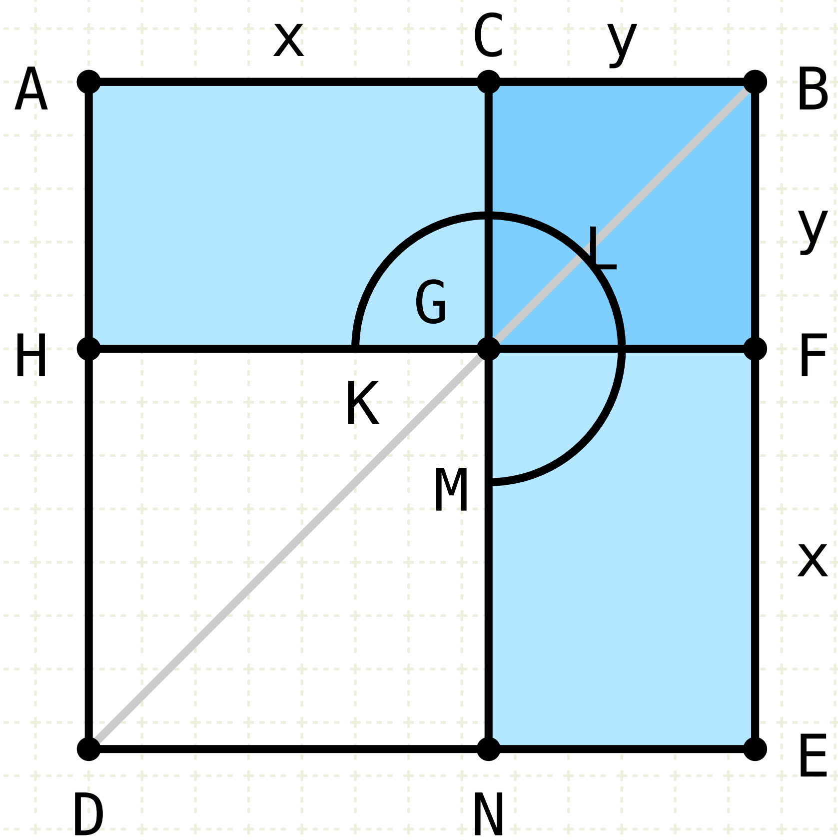

$AB = AC + CB$

$\square AF = \square CE, \quad \square AF + \square CE = 2\square AF$

$KLM + \square CF = \square AF + \square CE = 2\square AF$

In other words

Let AB be a straight line, arbitrarily cut at point C

Then the squares formed by lines AB and BC are equal in area to the sum of the square formed by line AC, plus twice the area of the rectangle formed by lines AB and CB

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But AF plus CE is equal to the gnomon KLM plus CE, therefore the gnomon KLM and CE is twice AF

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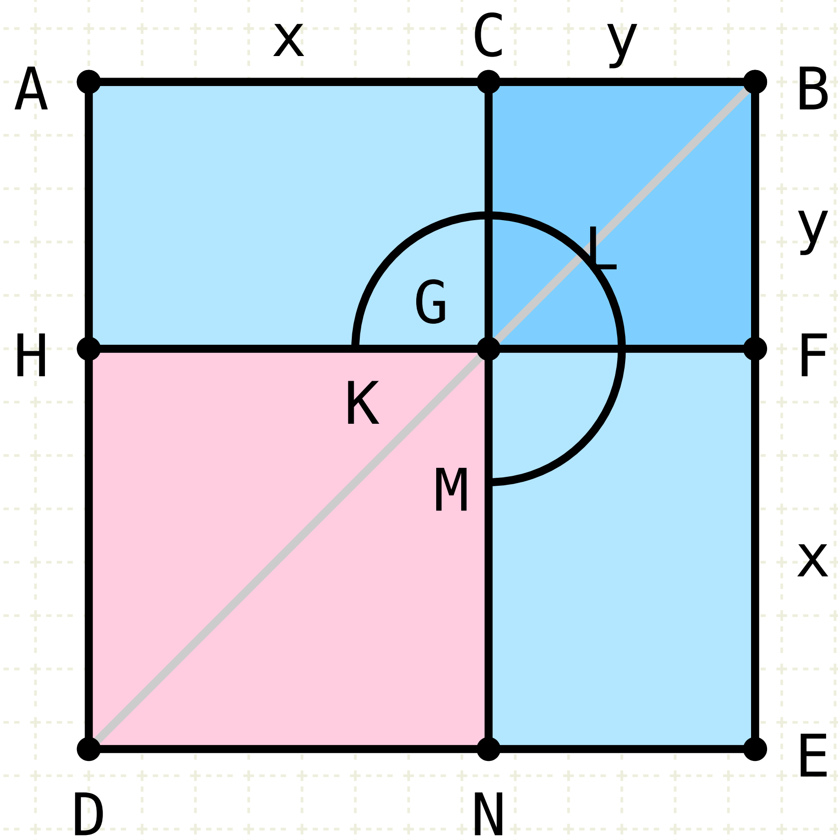

AB = AC + CB

□AF = □CE, □AF + □CE = 2□AF

KLM + □CF = □AF + □CE = 2□AF

KLM + □DG + □CF = 2□AF + □DG

In other words

Let AB be a straight line, arbitrarily cut at point C

Then the squares formed by lines AB and BC are equal in area to the sum of the square formed by line AC, plus twice the area of the rectangle formed by lines AB and CB

Proof:

AG equals GE (I·43), add CF to both, thus preserving the equality

But AF plus CE is equal to the gnomon KLM plus CE, therefore the gnomon KLM and CE is twice AF

Let DG, the square on AC, be added to each

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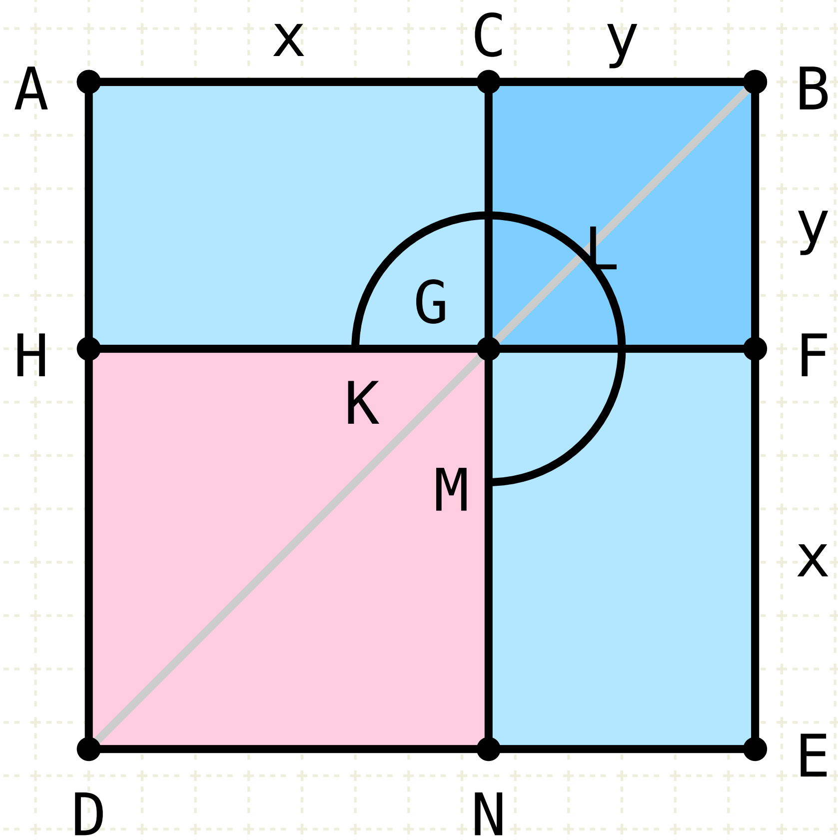

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KLM + □CF = □AF + □CE = 2□AF

KLM + □DG + □CF = 2□AF + □DG

□AE + □CF = 2□AF + □DG

In other words

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Proof:

AG equals GE (I·43), add CF to both, thus preserving the equality

But AF plus CE is equal to the gnomon KLM plus CE, therefore the gnomon KLM and CE is twice AF

Let DG, the square on AC, be added to each

Let DG, the square on AC, be added to each

But the gnomon KLM and DG equals the square AE

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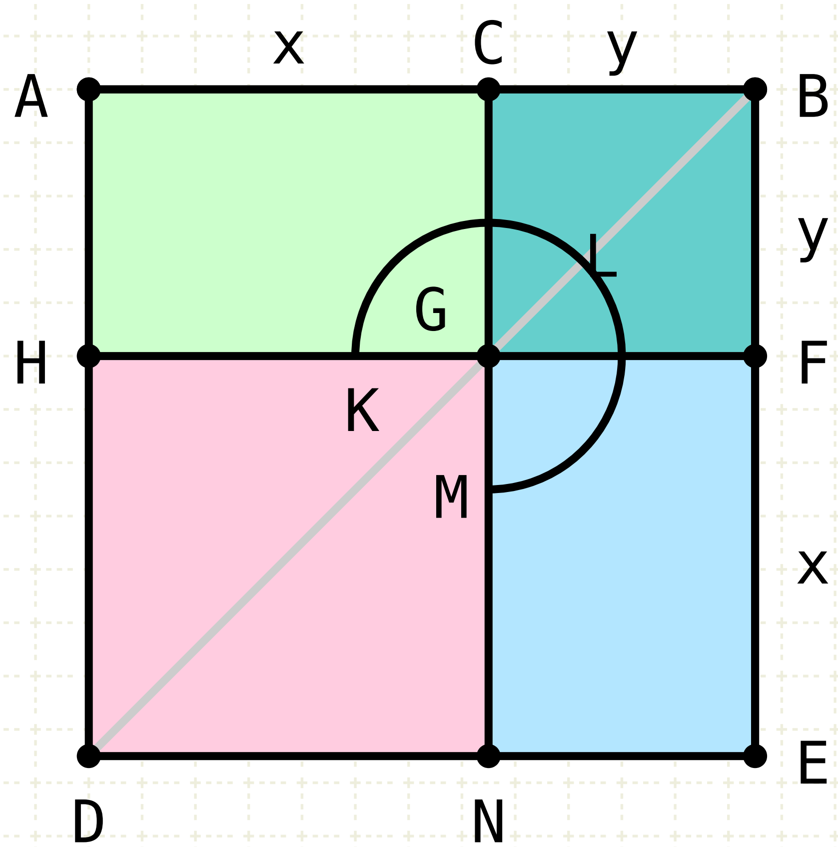

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KLM + □DG + □CF = 2□AF + □DG

□AE + □CF = 2□AF + □DG

AB·AB + CB·CB = 2AB·BC + AC·AC

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But the gnomon KLM and DG equals the square AE

AE equals the square on AB, CF equals the square on CB, AF is the rectangle formed by AB and BC, and finally DG is the square on AC

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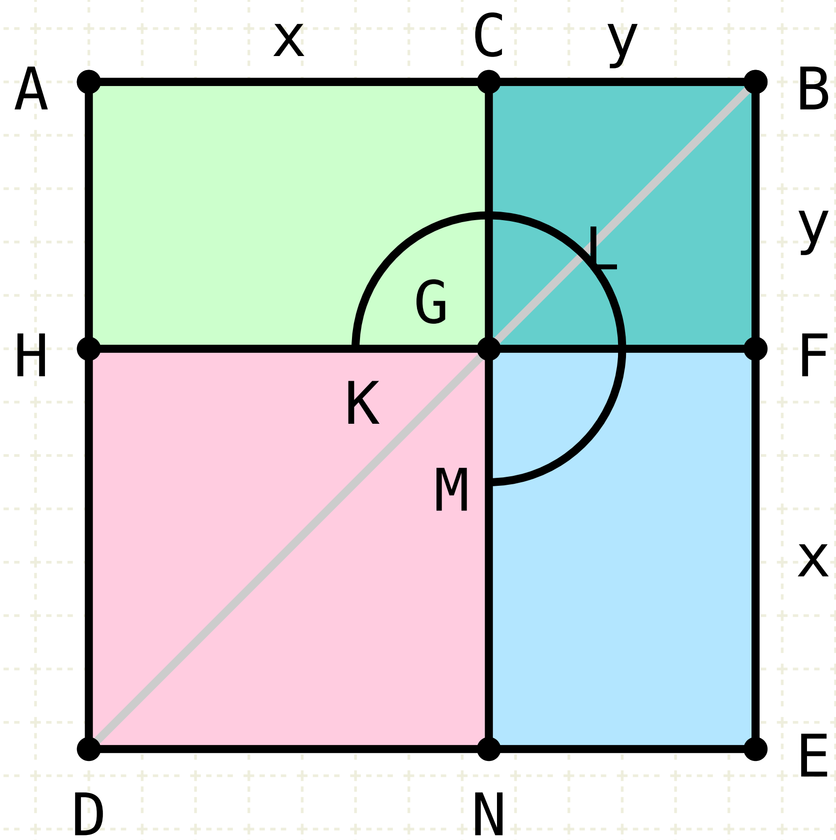

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$$KLM + \square DG + \square CF = 2\square AF + \square DG$$

$$\square AE + \square CF = 2\square AF + \square DG$$

$$AB\cdot AB + CB\cdot CB = 2AB\cdot BC + AC\cdot AC$$

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But the gnomon KLM and DG equals the square AE

AE equals the square on AB, CF equals the square on CB, AF is the rectangle formed by AB and BC, and finally DG is the square on AC

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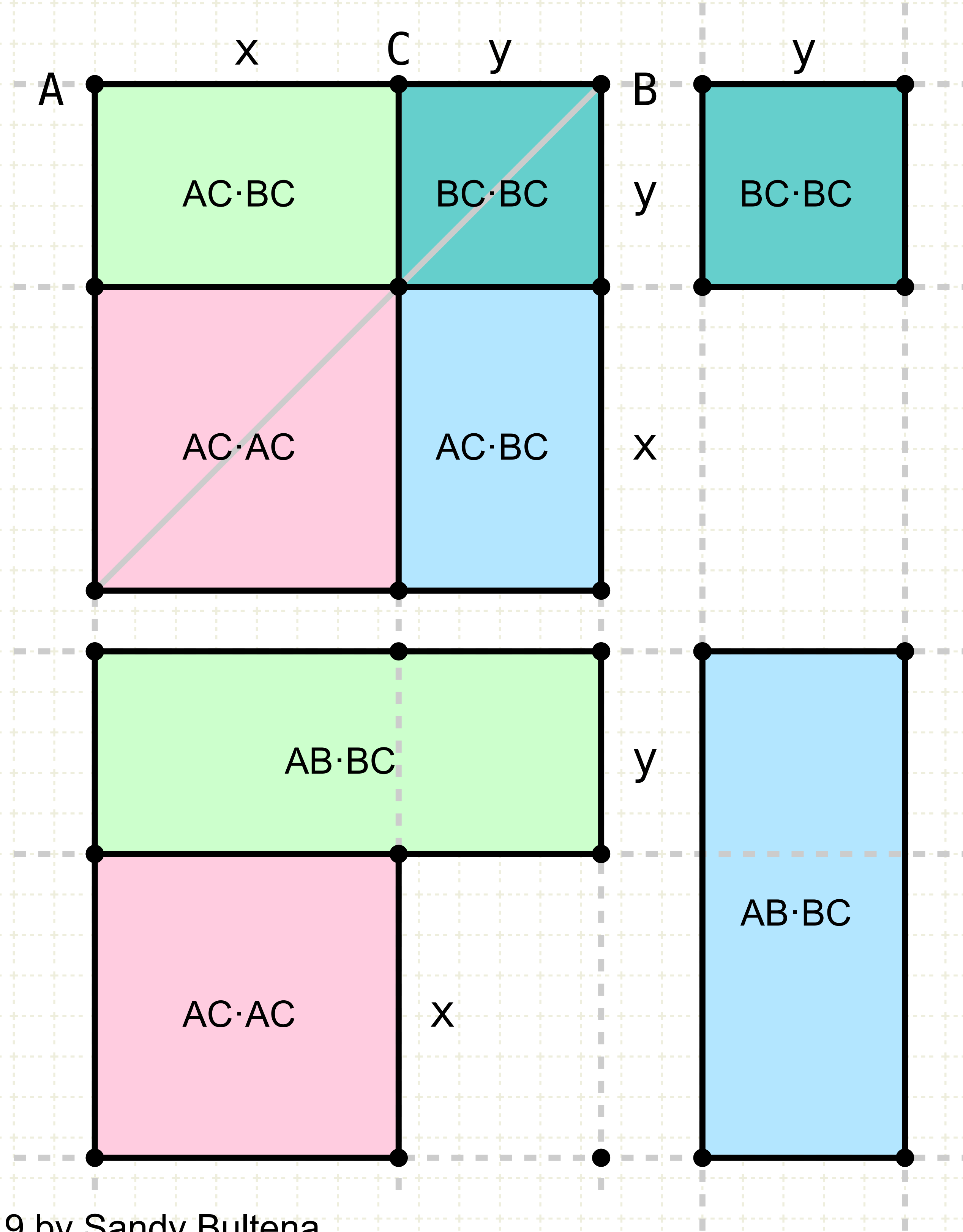

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But the gnomon KLM and DG equals the square AE

AE equals the square on AB, CF equals the square on CB, AF is the rectangle formed by AB and BC, and finally DG is the square on AC

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