

Euclid's Elements

Book V



Proportions are what makes the old Greek temples classic in their beauty. They are like huge blocks, from which the air has been literally hewn out between the columns.

$$AB:C = DE:F$$

$$BG:C = EH:F$$

$$AG:C = DH:F$$

Arne Jacobsen



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10	if $A:C > B:C$, or $A:C < B:C$ then $A > B$, or $A < C$, respectively				



Proposition 7 of Book V

Equal magnitudes have the same ratio, as also has the same to equal magnitudes



Proposition 7 of Book V

Equal magnitudes have the same ratio, as also has the same to equal magnitudes

Definitions

5. Magnitudes are said to BE IN THE SAME RATIO, the first to the second and the third to the fourth, when, if any equimultiples whatever be taken of the first and third, and any equimultiples whatever of the second and fourth, the former equimultiples alike exceed, are alike equal to, or alike fall short of, the latter equimultiples respectively taken in corresponding order

The ratio $A:B = C:D$ if, for any number 'p' and 'q'

if $pA > qB$ then $pC > qD$

if $pA < qB$ then $pC < qD$

if $pA = qB$ then $pC = qD$

if $A:B = C:D$

then $pA \geq qB \rightarrow pC \geq qD$



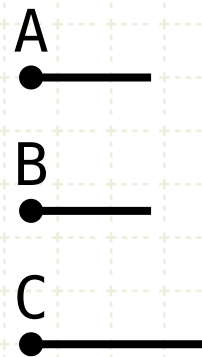
Proposition 7 of Book V

Equal magnitudes have the same ratio, as also has the same to equal magnitudes



Proposition 7 of Book V

Equal magnitudes have the same ratio, as also has the same to equal magnitudes



$$A = B \neq C$$

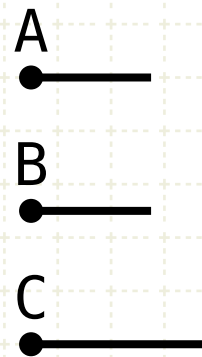
In other words

Let A,B be equal, and C not equal



Proposition 7 of Book V

Equal magnitudes have the same ratio, as also has the same to equal magnitudes



$$A = B \neq C$$

$$A:C = B:C$$

In other words

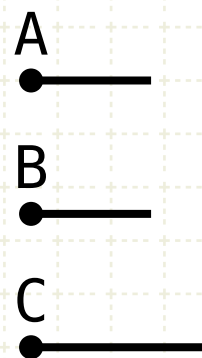
Let A,B be equal, and C not equal

Then the ratio of A to C is the same as the ratio of B to C



Proposition 7 of Book V

Equal magnitudes have the same ratio, as also has the same to equal magnitudes



$$A = B \neq C$$

$$A : C = B : C$$

$$C : A = C : B$$

In other words

Let A,B be equal, and C not equal

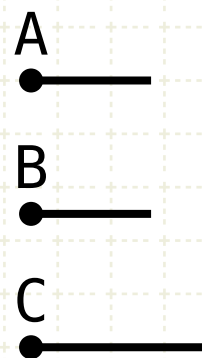
Then the ratio of A to C is the same as the ratio of B to C

And the ratio of C to A is the same as the ratio of C to B



Proposition 7 of Book V

Equal magnitudes have the same ratio, as also has the same to equal magnitudes



$$A = B \neq C$$

In other words

Let A,B be equal, and C not equal

Then the ratio of A to C is the same as the ratio of B to C

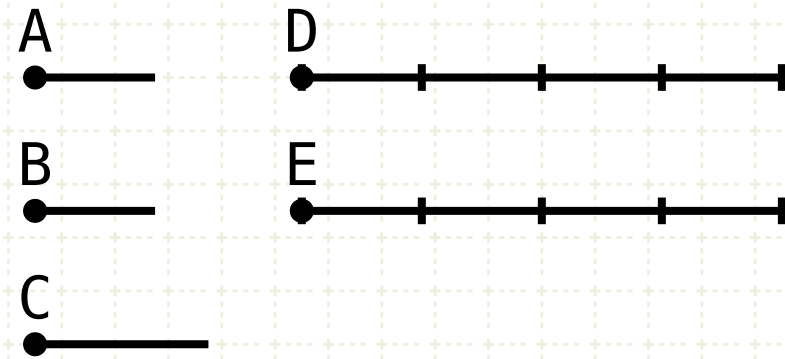
And the ratio of C to A is the same as the ratio of C to B

Proof



Proposition 7 of Book V

Equal magnitudes have the same ratio, as also has the same to equal magnitudes



$$A = B \neq C$$

$$D = nA$$

$$E = nB$$

In other words

Let A,B be equal, and C not equal

Then the ratio of A to C is the same as the ratio of B to C

And the ratio of C to A is the same as the ratio of C to B

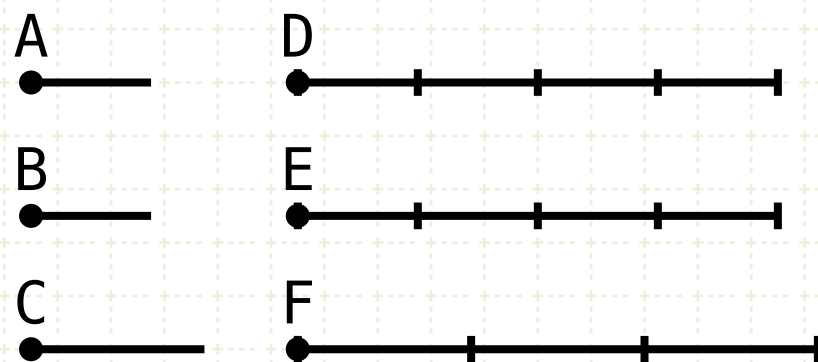
Proof

Let equimultiples D,E, be taken of magnitudes A,B



Proposition 7 of Book V

Equal magnitudes have the same ratio, as also has the same to equal magnitudes



$$A = B \neq C$$

$$D = nA$$

$$E = nB$$

$$F = mC$$

In other words

Let A,B be equal, and C not equal

Then the ratio of A to C is the same as the ratio of B to C

And the ratio of C to A is the same as the ratio of C to B

Proof

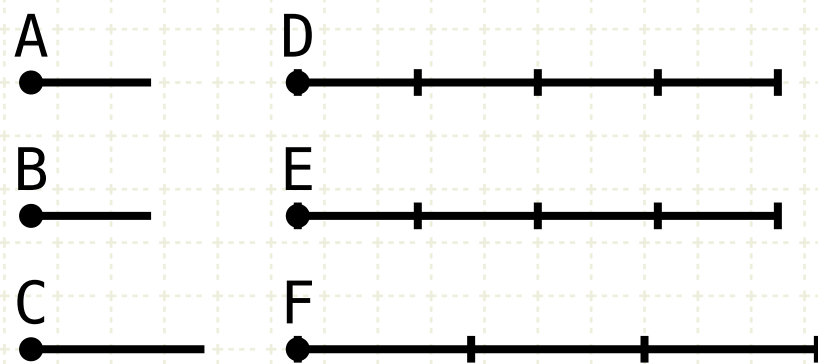
Let equimultiples D,E, be taken of magnitudes A,B

And let another multiple F be taken of C



Proposition 7 of Book V

Equal magnitudes have the same ratio, as also has the same to equal magnitudes



$$A = B \neq C$$

$$D = nA$$

$$E = nB$$

$$F = mC$$

$$D = E$$

In other words

Let A,B be equal, and C not equal

Then the ratio of A to C is the same as the ratio of B to C

And the ratio of C to A is the same as the ratio of C to B

Proof

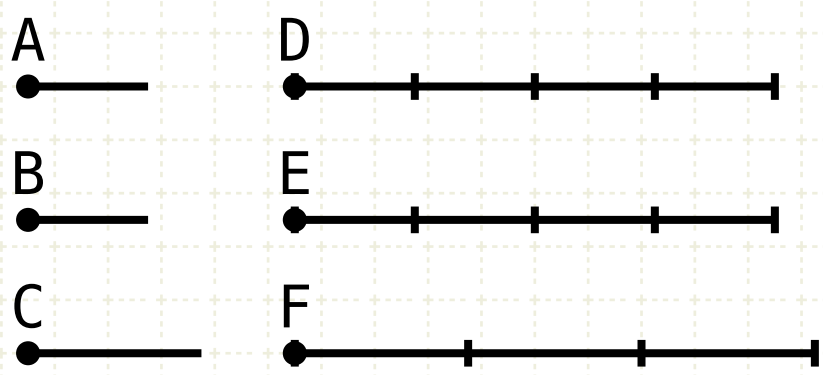
Let equimultiples D,E, be taken of magnitudes A,B

And let another multiple F be taken of C

Since D is the same multiple of A that E is of B, and A is equal to B, then D is equal to E

Proposition 7 of Book V

Equal magnitudes have the same ratio, as also has the same to equal magnitudes



$A = B \neq C$

$D = nA$
 $E = nB$
 $F = mC$
 $D = E$

$D \geq F \rightarrow E \geq F$
 $nA \geq mC \rightarrow nB \geq mC$

In other words

Let A,B be equal, and C not equal
Then the ratio of A to C is the same as the ratio of B to C
And the ratio of C to A is the same as the ratio of C to B

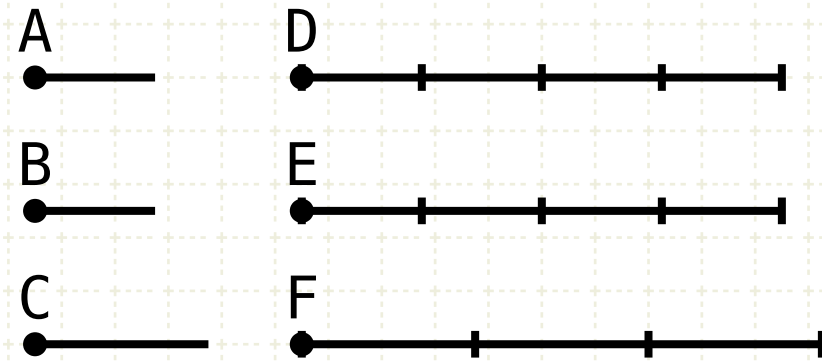
Proof

Let equimultiples D,E, be taken of magnitudes A,B
And let another multiple F be taken of C
Since D is the same multiple of A that E is of B, and A is equal to B, then D is equal to E
F is another arbitrary magnitude, so if D is greater than F, then so E is also greater than F, and so on



Proposition 7 of Book V

Equal magnitudes have the same ratio, as also has the same to equal magnitudes



$$A = B \neq C$$

$$D = nA$$

$$E = nB$$

$$F = mC$$

$$D = E$$

$$D \geqslant < F \rightarrow E \geqslant < F$$

$$nA \geqslant < mC \rightarrow nB \geqslant < mC$$

$$A:C = B:C$$

In other words

Let A,B be equal, and C not equal

Then the ratio of A to C is the same as the ratio of B to C

And the ratio of C to A is the same as the ratio of C to B

Proof

Let equimultiples D,E, be taken of magnitudes A,B

And let another multiple F be taken of C

Since D is the same multiple of A that E is of B, and A is equal to B, then D is equal to E

F is another arbitrary magnitude, so if D is greater than F, then so E is also greater than F, and so on

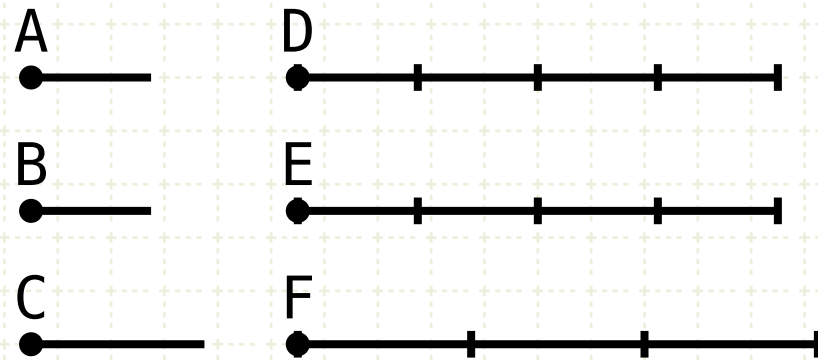
This is the requirement for equal ratios, so...

... A is to C as B is to C (V Def 5)



Proposition 7 of Book V

Equal magnitudes have the same ratio, as also has the same to equal magnitudes



$$A = B \neq C$$

$$D = nA$$

$$E = nB$$

$$F = mC$$

$$D = E$$

$$D \geq < F \rightarrow E \geq < F$$

$$nA \geq < mC \rightarrow nB \geq < mC$$

$$A:C = B:C$$

$$F \geq < D \rightarrow F \geq < E$$

$$mC \geq < nA \rightarrow mC \geq < nB$$

In other words

Let A,B be equal, and C not equal

Then the ratio of A to C is the same as the ratio of B to C

And the ratio of C to A is the same as the ratio of C to B

Proof

Let equimultiples D,E, be taken of magnitudes A,B

And let another multiple F be taken of C

Since D is the same multiple of A that E is of B, and A is equal to B, then D is equal to E

F is another arbitrary magnitude, so if D is greater than F, then so E is also greater than F, and so on

This is the requirement for equal ratios, so...

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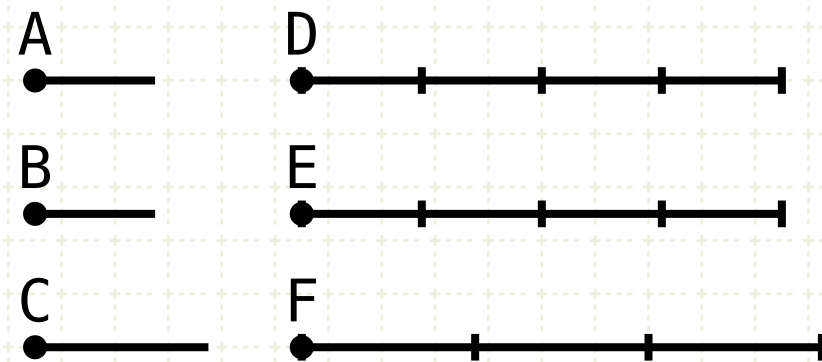
Going back to D is equal to E, and F being an arbitrary magnitude...

If F is greater than D, then so F is also greater than E, and so on



Proposition 7 of Book V

Equal magnitudes have the same ratio, as also has the same to equal magnitudes



$$A = B \neq C$$

$$D = nA$$

$$E = nB$$

$$F = mC$$

$$D = E$$

$$D \geq < F \rightarrow E \geq < F$$

$$nA \geq < mC \rightarrow nB \geq < mC$$

$$A:C = B:C$$

$$F \geq < D \rightarrow F \geq < E$$

$$mC \geq < nA \rightarrow mC \geq < nB$$

$$C:A = C:B$$

In other words

Let A,B be equal, and C not equal

Then the ratio of A to C is the same as the ratio of B to C

And the ratio of C to A is the same as the ratio of C to B

Proof

Let equimultiples D,E, be taken of magnitudes A,B

And let another multiple F be taken of C

Since D is the same multiple of A that E is of B, and A is equal to B, then D is equal to E

F is another arbitrary magnitude, so if D is greater than F, then so E is also greater than F, and so on

This is the requirement for equal ratios, so...

... A is to C as B is to C (V Def 5)

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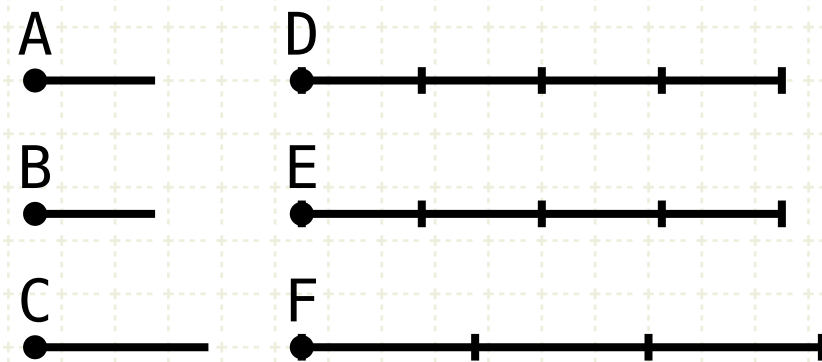
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$$A = B \neq C$$

$$D = nA$$

$$E = nB$$

$$F = mC$$

$$D = E$$

$$D \geqslant F \rightarrow E \geqslant F$$

$$nA \geqslant mC \rightarrow nB \geqslant mC$$

$$A:C = B:C$$

$$F \geqslant D \rightarrow F \geqslant E$$

$$mC \geqslant nA \rightarrow mC \geqslant nB$$

$$C:A = C:B$$

In other words

Let A,B be equal, and C not equal

Then the ratio of A to C is the same as the ratio of B to C

And the ratio of C to A is the same as the ratio of C to B

Proof

Let equimultiples D,E, be taken of magnitudes A,B

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