

Euclid's Elements

Book V



Proportions are what makes the old Greek temples classic in their beauty. They are like huge blocks, from which the air has been literally hewn out between the columns.

$$AB:C = DE:F$$

$$BG:C = EH:F$$

$$AG:C = DH:F$$

Arne Jacobsen



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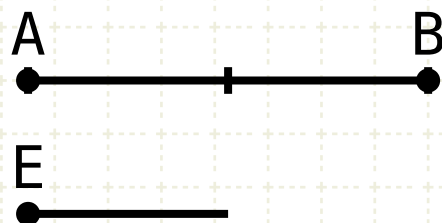
Proposition 1 of Book V

If there be any number of magnitudes whatever which are, respectively, equimultiples of any magnitudes equal in multitude, then, whatever multiple one the magnitudes is of one, that multiple also will all be of all



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Definitions

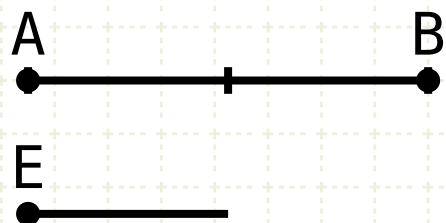
1. A magnitude is a PART of a magnitude, the less of the greater, when it measures the greater

(1) E is a part of AB, it measures AB



Proposition 1 of Book V

If there be any number of magnitudes whatever which are, respectively, equimultiples of any magnitudes equal in multitude, then, whatever multiple one the magnitudes is of one, that multiple also will all be of all



Definitions

1. A magnitude is a PART of a magnitude, the less of the greater, when it measures the greater
2. The greater is a MULTIPLE of the less when it is measured by the less

- (1) E is a part of AB, it measures AB
- (2) AB is a multiple of E



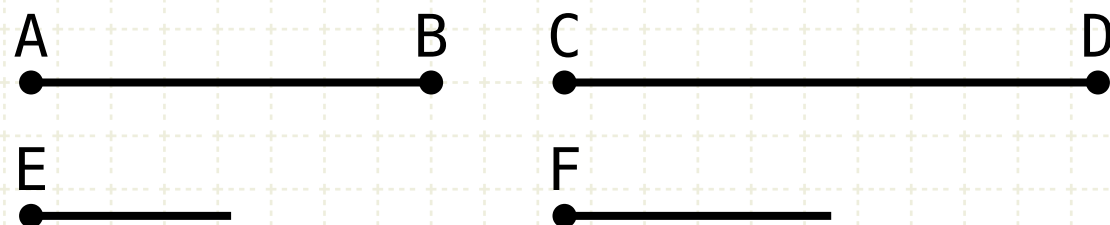
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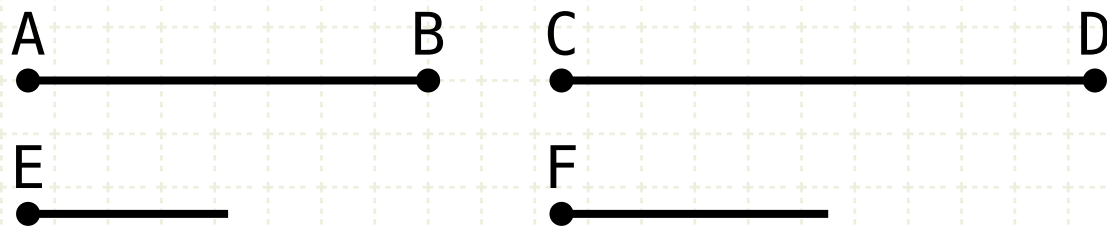
In other words

If we have two lines (AB and CD) that are equal multiples of two other lines (E and F respectively) then ...

If $AB = n \cdot E$, $CD = n \cdot F$

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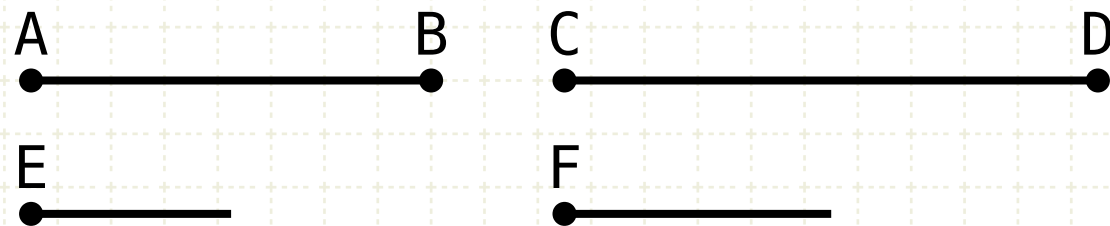
If we have two lines (AB and CD) that are equal multiples of two other lines (E and F respectively) then ...

The sum of AB and CD will also be an equal multiple of the sum of E and F

If $AB = n \cdot E$, $CD = n \cdot F$
then $AB + CD = n \cdot (E + F)$

Proposition 1 of Book V

If there be any number of magnitudes whatever which are, respectively, equimultiples of any magnitudes equal in multitude, then, whatever multiple one the magnitudes is of one, that multiple also will all be of all



$$AB = 2E, CD = 2F$$

In other words

If we have two lines (AB and CD) that are equal multiples of two other lines (E and F respectively) then ...

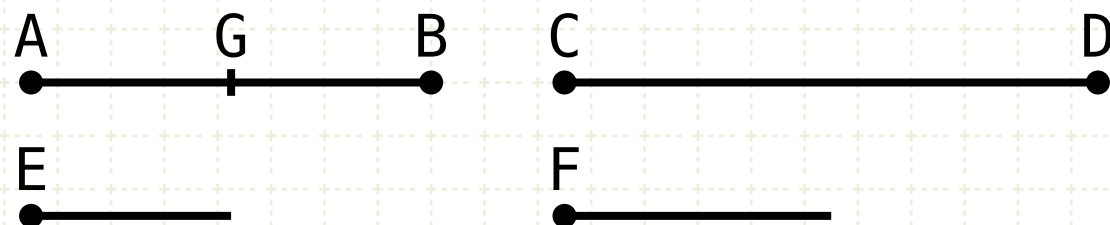
The sum of AB and CD will also be an equal multiple of the sum of E and F

Proof



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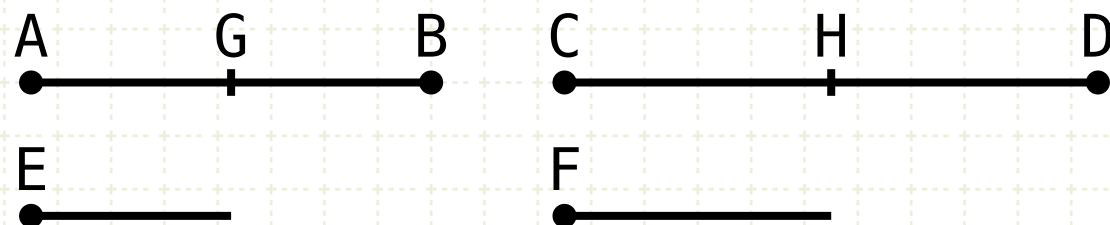
The sum of AB and CD will also be an equal multiple of the sum of E and F

Proof

Let AB be divided into segments (magnitudes) of equal lengths, where each magnitude is equal to E

Proposition 1 of Book V

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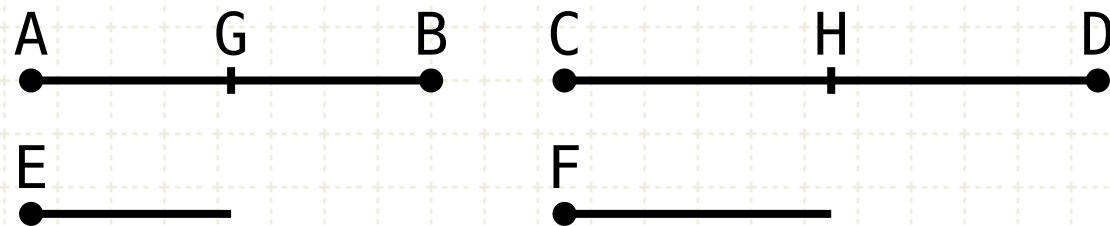
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$$AB = 2E, \quad CD = 2F$$

$$AG = GB = E$$

$$CH = HD = F$$

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If we have two lines (AB and CD) that are equal multiples of two other lines (E and F respectively) then ...

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Proof

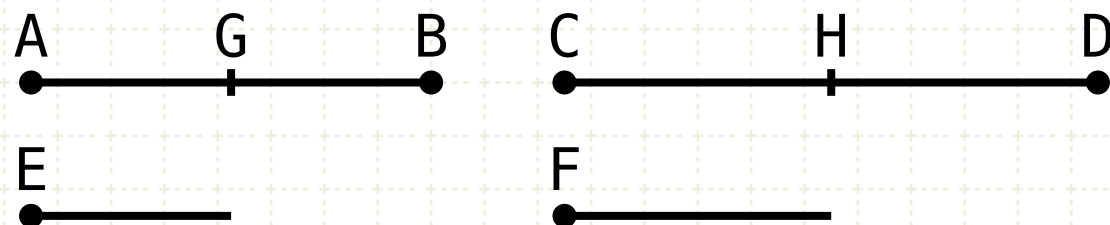
Let AB be divided into segments (magnitudes) of equal lengths, where each magnitude is equal to E

Let CD be divided into equal lengths, where each length is equal to F

Since AB and CD are equal multitudes of E and F respectively, they will be divided into the same number of magnitudes

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$$AB = 2E, \quad CD = 2F$$

$$AG = GB = E$$

$$CH = HD = F$$

$$AG + CH = E + F$$

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If we have two lines (AB and CD) that are equal multiples of two other lines (E and F respectively) then ...

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Let CD be divided into equal lengths, where each length is equal to F

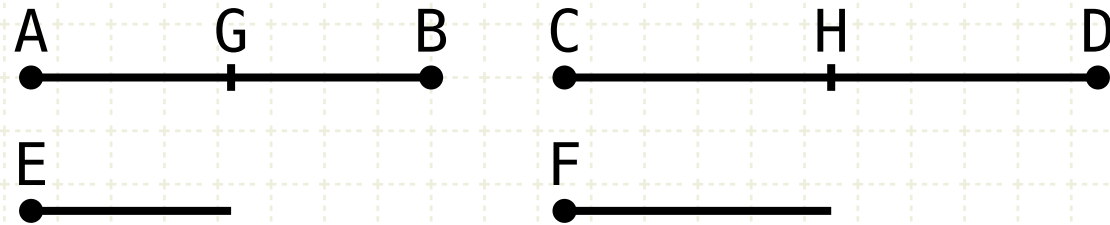
Since AB and CD are equal multitudes of E and F respectively, they will be divided into the same number of magnitudes

Now, since AG equals E, and CH equals F, then AG and CH together is equal to E and F



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If we have two lines (AB and CD) that are equal multiples of two other lines (E and F respectively) then ...

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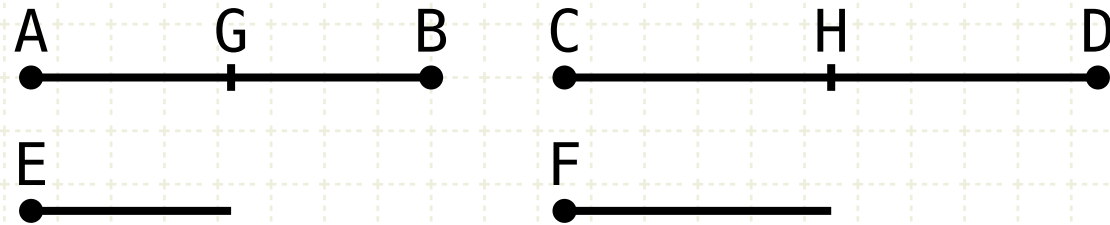
Now, since AG equals E, and CH equals F, then AG and CH together is equal to E and F

Similarly, since GB equals E, and HD equals F, then GB and HD together is equal to E and F



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If there be any number of magnitudes whatever which are, respectively, equimultiples of any magnitudes equal in multitude, then, whatever multiple one the magnitudes is of one, that multiple also will all be of all



$$AB = 2E, \quad CD = 2F$$

$$AG = GB = E$$

$$CH = HD = F$$

$$AG + CH = E + F$$

$$GB + HD = E + F$$

$$AG + CH + GB + HD = (E + F) + (E + F) = 2(E + F)$$

In other words

If we have two lines (AB and CD) that are equal multiples of two other lines (E and F respectively) then ...

The sum of AB and CD will also be an equal multiple of the sum of E and F

Proof

Let AB be divided into segments (magnitudes) of equal lengths, where each magnitude is equal to E

Let CD be divided into equal lengths, where each length is equal to F

Since AB and CD are equal multitudes of E and F respectively, they will be divided into the same number of magnitudes

Now, since AG equals E, and CH equals F, then AG and CH together is equal to E and F

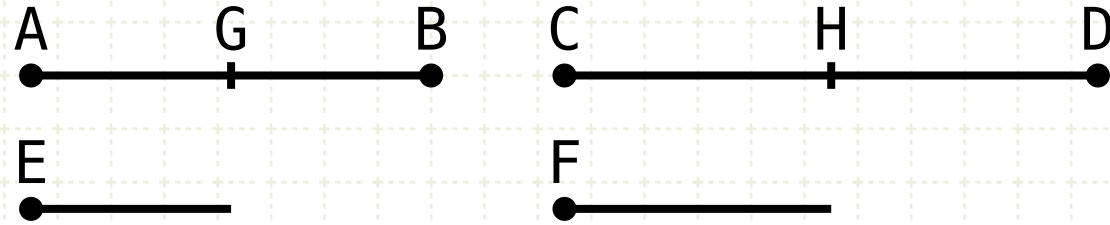
Similarly, since GB equals E, and HD equals F, then GB and HD together is equal to E and F

Therefore for every length E within the length AB there is a length E+F in the sum of AB and CD



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$$AB = 2E, \quad CD = 2F$$

$$AG = GB = E$$

$$CH = HD = F$$

$$AG + CH = E + F$$

$$GB + HD = E + F$$

$$AG + CH + GB + HD = (E + F) + (E + F) = 2(E + F)$$

$$\text{If } AB = n \cdot E \text{ and } CD = n \cdot F \text{ then } AB + CD = n \cdot (E + F)$$

In other words

If we have two lines (AB and CD) that are equal multiples of two other lines (E and F respectively) then ...

The sum of AB and CD will also be an equal multiple of the sum of E and F

Proof

Let AB be divided into segments (magnitudes) of equal lengths, where each magnitude is equal to E

Let CD be divided into equal lengths, where each length is equal to F

Since AB and CD are equal multitudes of E and F respectively, they will be divided into the same number of magnitudes

Now, since AG equals E, and CH equals F, then AG and CH together is equal to E and F

Similarly, since GB equals E, and HD equals F, then GB and HD together is equal to E and F

Therefore for every length E within the length AB there is a length E+F in the sum of AB and CD

In more general terms, for however many magnitudes in AB equal to E, there are that many magnitudes in CD that are equal to F.



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