

# Euclid's Elements

## Book III



*A circle is a round straight line with a hole in the middle.*

**Mark Twain**

quoting a schoolchild in "-English as She Is Taught-"

*If people stand in a circle long enough, they'll eventually begin to dance.*

**George Carlin, Napalm and Silly Putty (2001)**



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| 2 | A chord of a circle always lies inside the circle  | 10 | A circle does not cut a circle at more points than two  | <b>18</b> | <b>If line touches a circle, then it is perpendicular to the diameter that touches that point</b>                  |
| 3 | A line through the centre of a circle bisects a chord, and vice versa  | 11 | Point of contact between two internal circles, and their centres, are collinear   | 19        | If line touches a circle, then the centre of the circle lies on a line perpendicular to the original               |
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| 6 | If two circles touch one another, they will not have the same center   | 14 | In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another. | 22        | The opposite angles of quadrilaterals in circles are equal to two right angles                                     |
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| 8 | Consider two lines from a point outside a circle to the edge, the line closest to the centre will be longer on the concave side and shorter on the convex side             | 16 | A line on the circle, perpendicular to the diameter, lies outside the circle  | 24        | Similar segments of circles on equal straight lines are equal to one another                                       |



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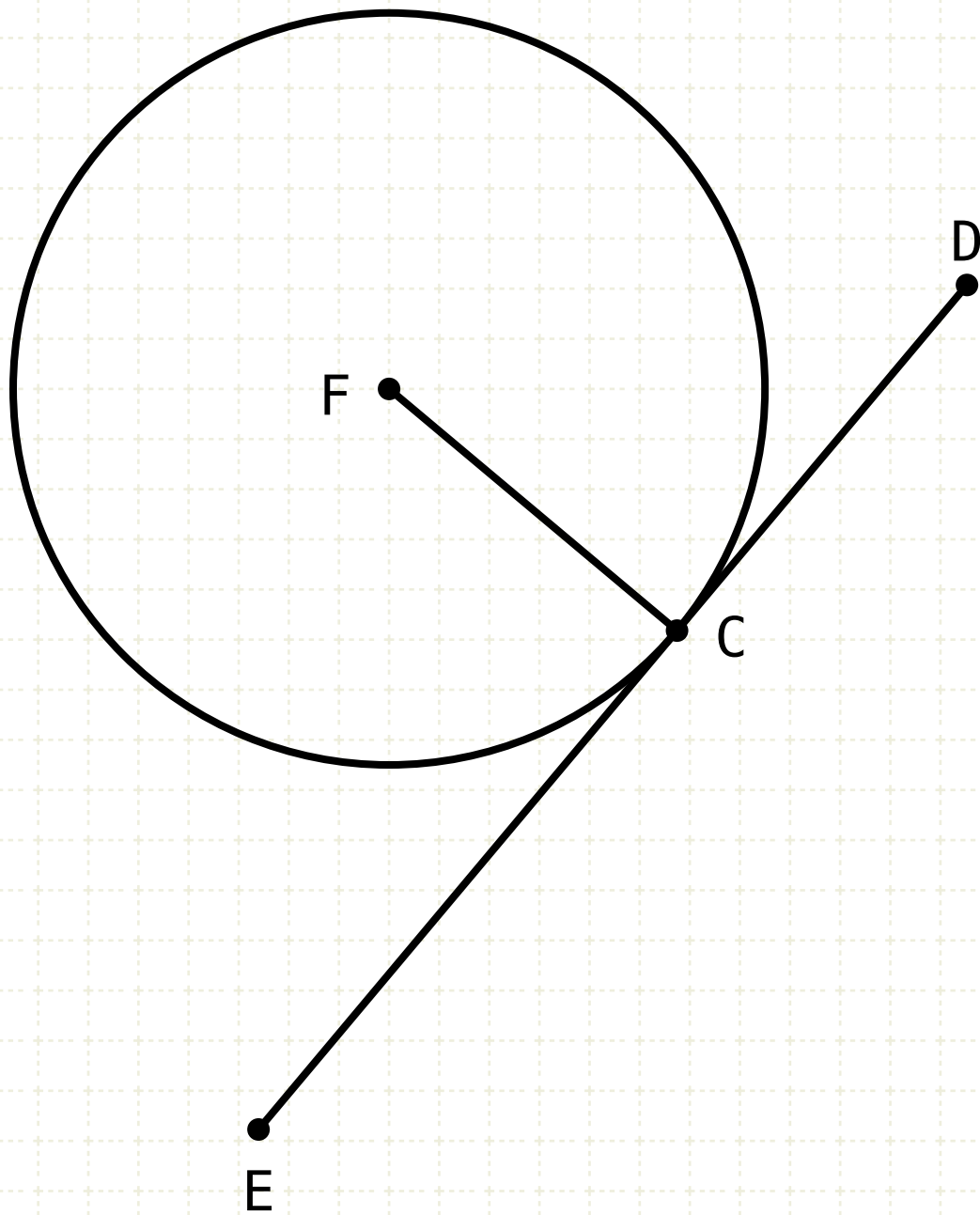
## Proposition 18 of Book III

If a straight line touch a circle, and a straight line be joined from the centre to the point of contact, the straight line so joined will be perpendicular to the tangent.



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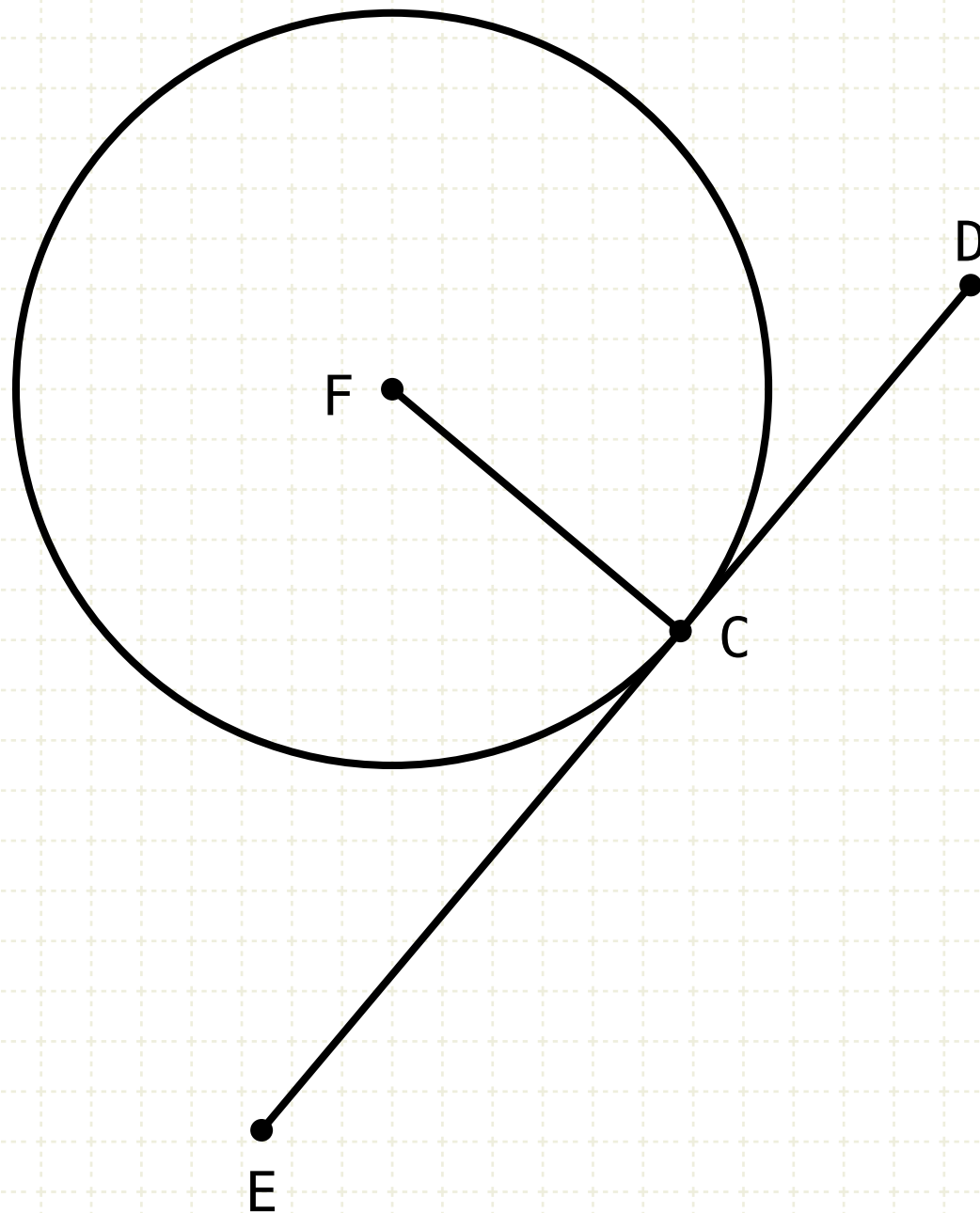
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If line ED touches a circle at point C, then it is perpendicular to the radius FC



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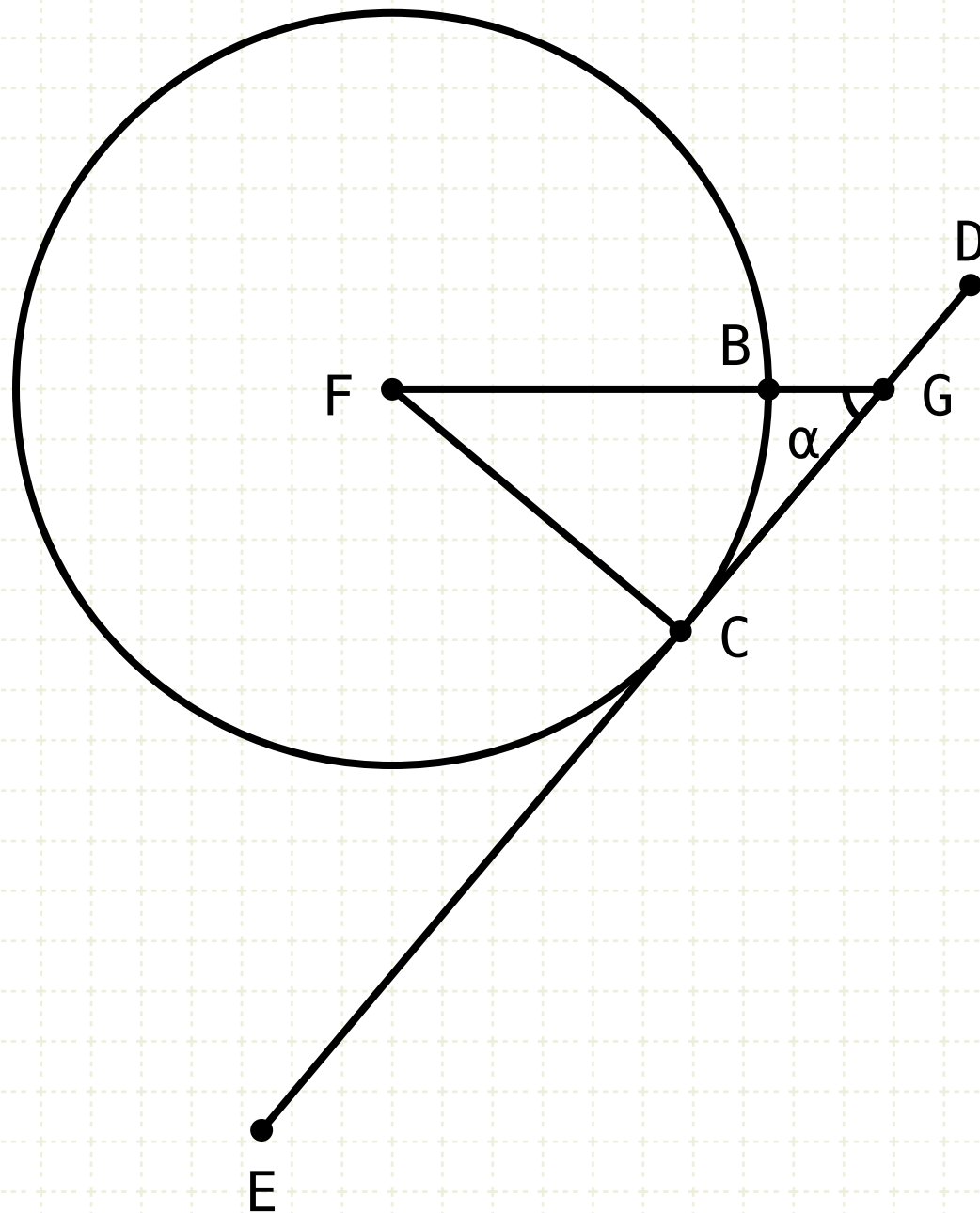
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$$\beta \neq \angle$$

$$\alpha = \angle$$

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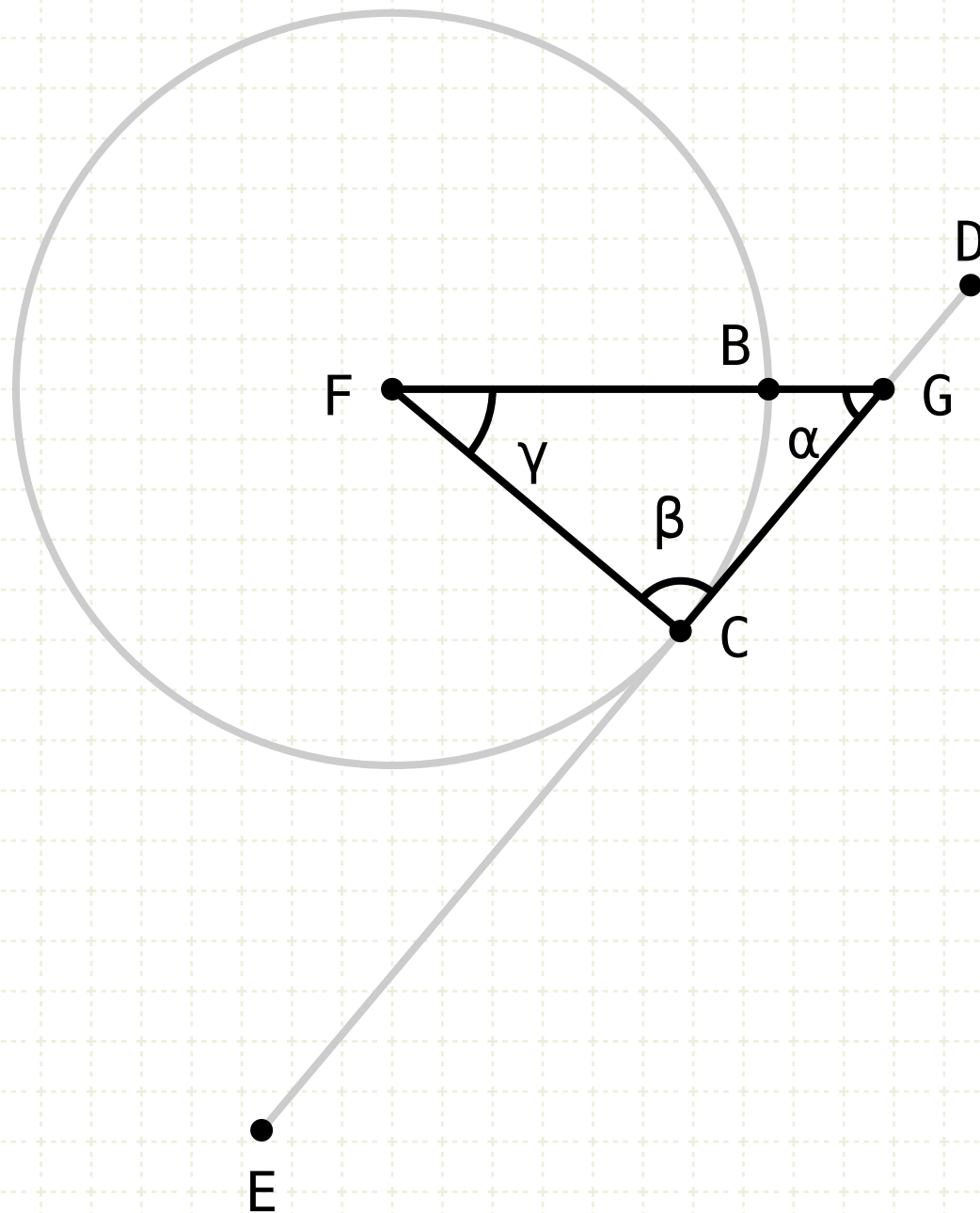
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$$\beta \neq L$$

$$\alpha = L$$

$$\alpha + \beta + \gamma = 2 \cdot L$$

$$L + \beta + \gamma = 2 \cdot L$$

$$\beta + \gamma = L$$

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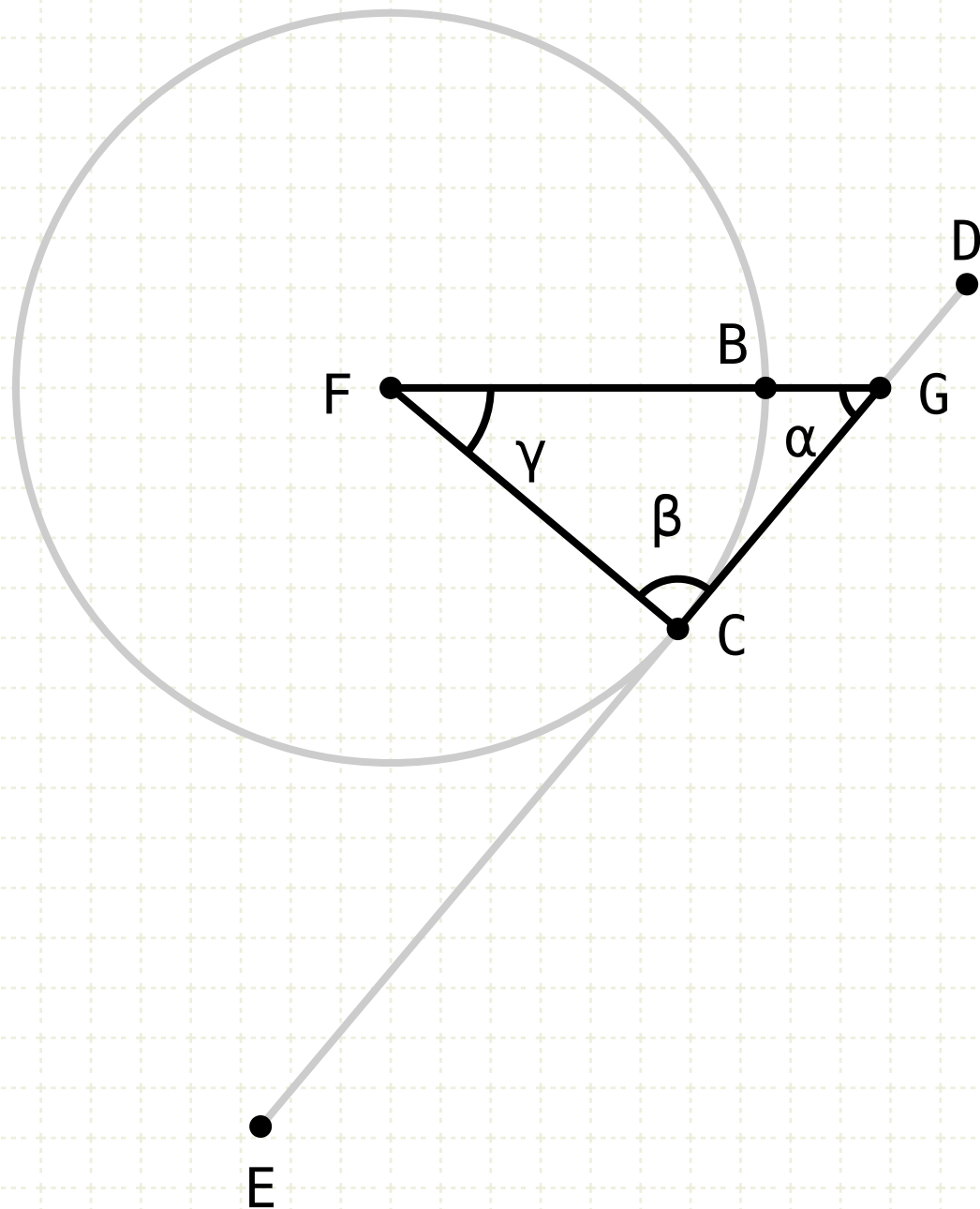
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The sum of the angles in a triangle is less than two right angles (I·17) so if CGF ( $\alpha$ ) is a right angle, then FCG ( $\beta$ ) must be less than a right angle



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$$FC > FG$$

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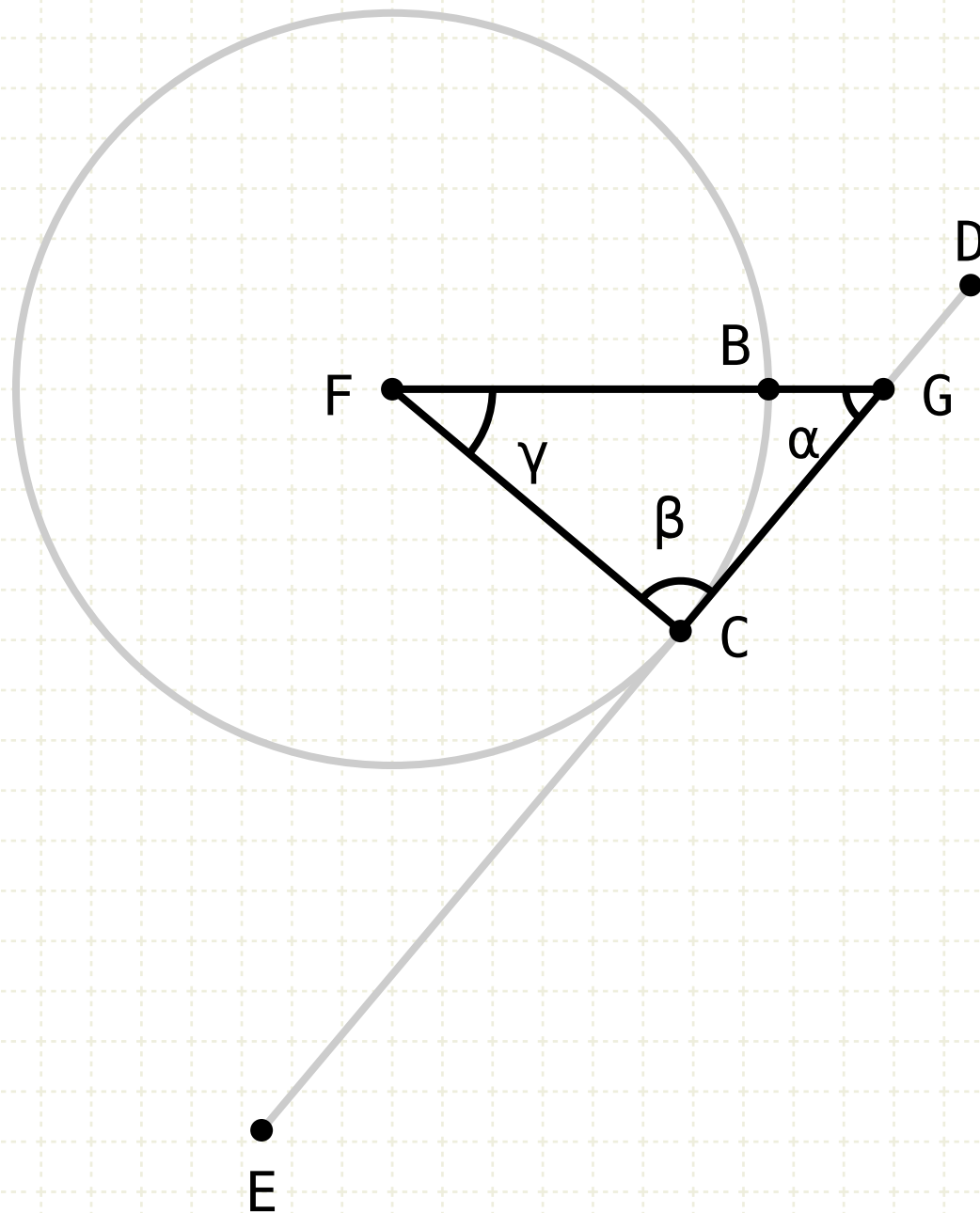
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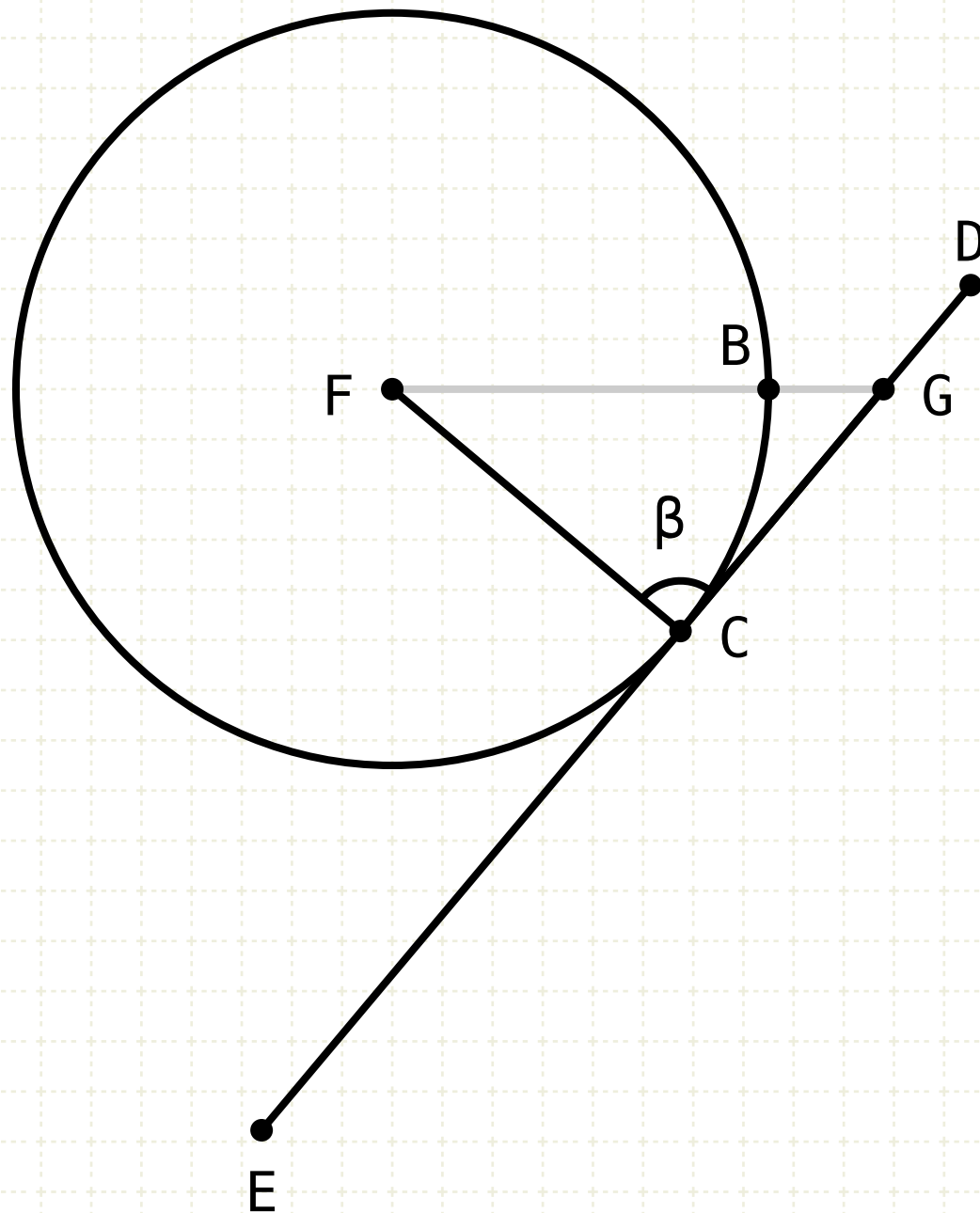
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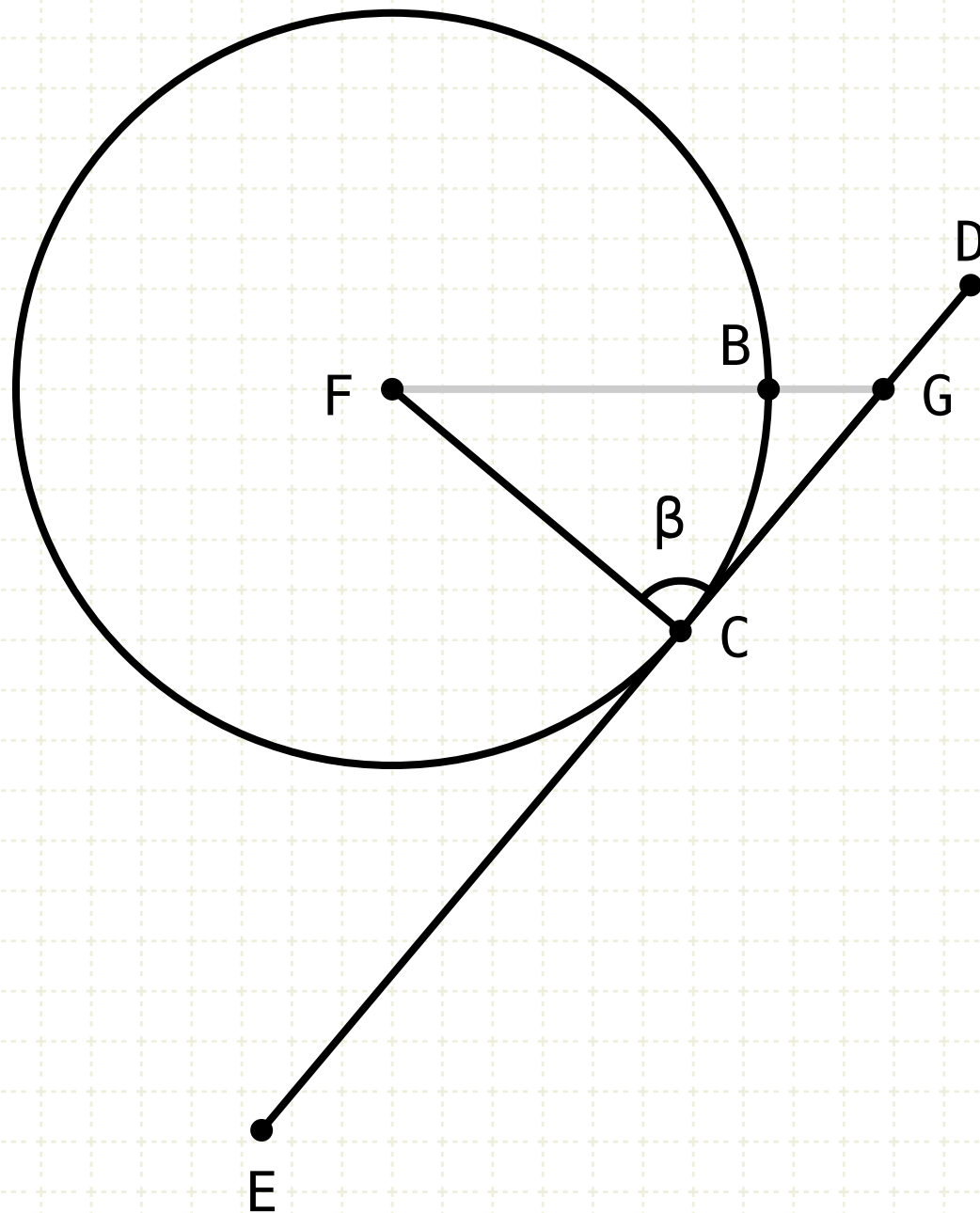
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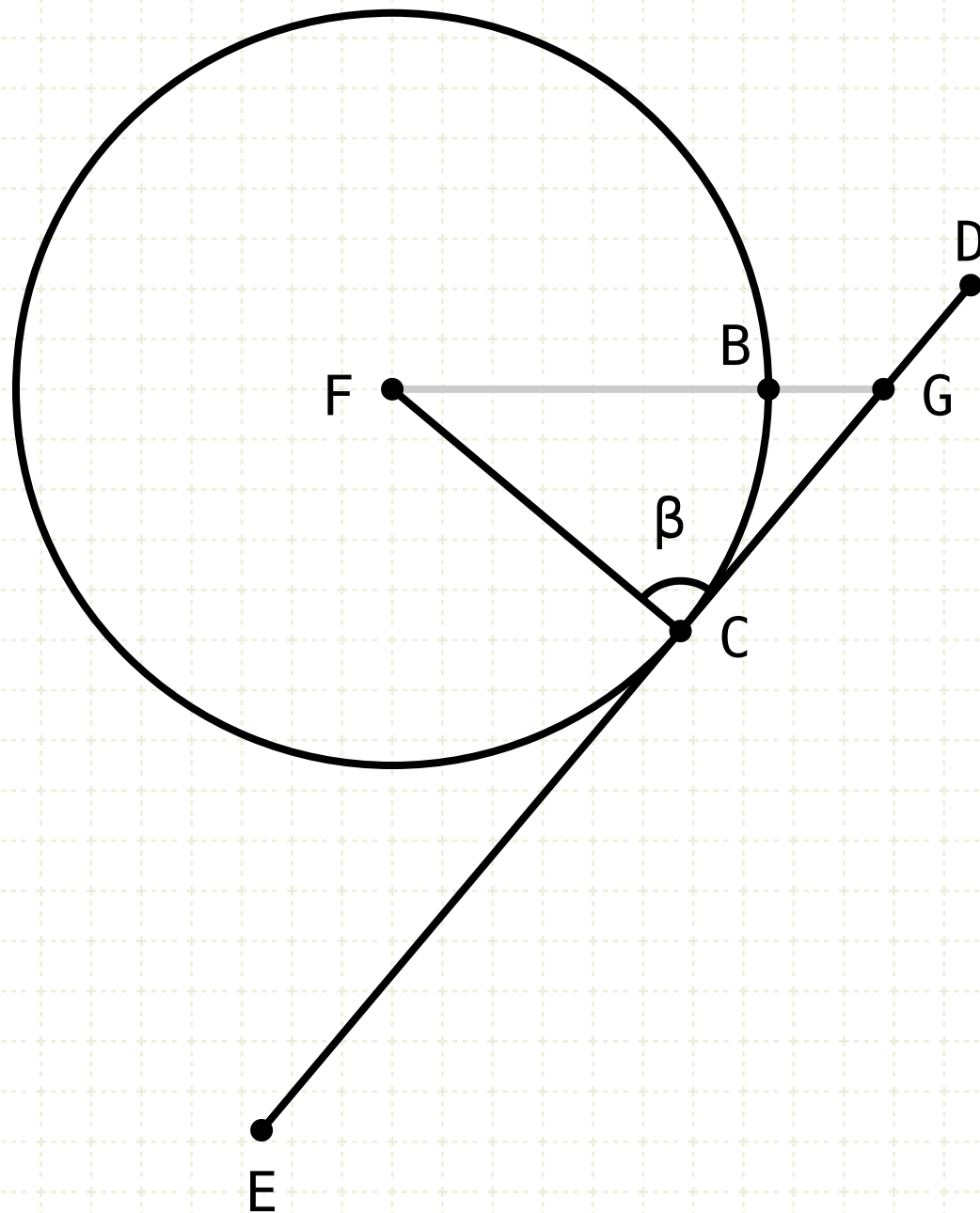
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$$\therefore \beta = L$$

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