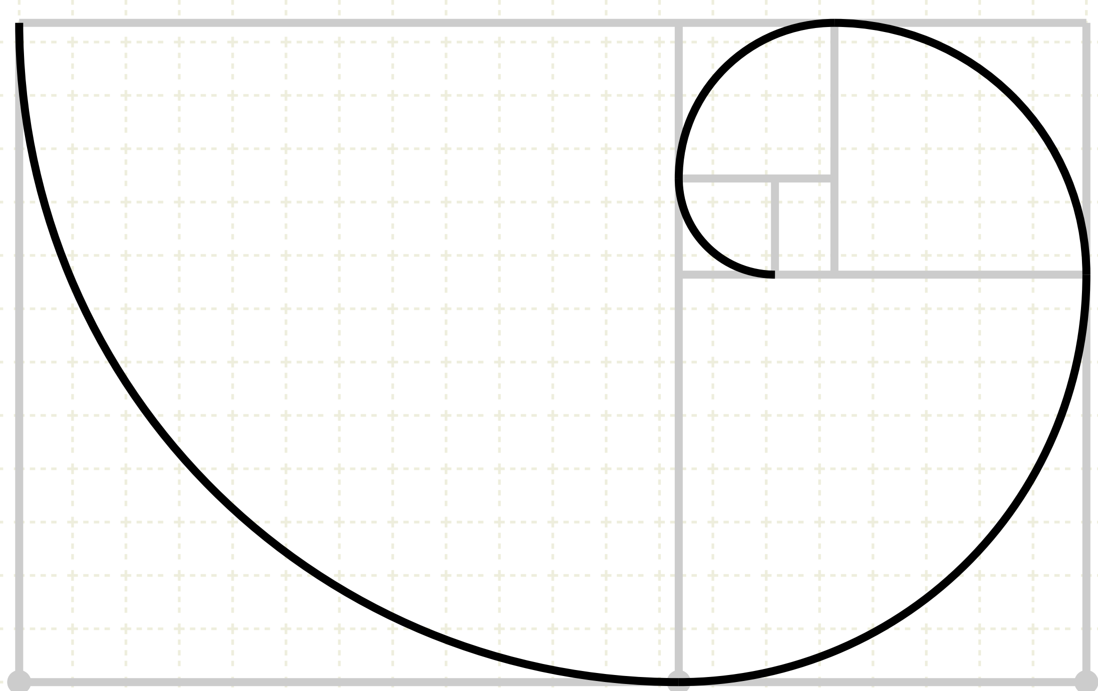
Euclid's Elements

Book VI



One can state, without exaggeration, that the observation of and the search for similarities and differences are the basis of all human knowledge.

Alfred Nobel

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Proposition 7 of Book VI

If two triangles have one angle equal to one angle, the sides about other angles proportional, and the remaining angles either both less or both not less than a right angle, the triangles will be equiangular and will have those angles equal, the sides about which are proportional.

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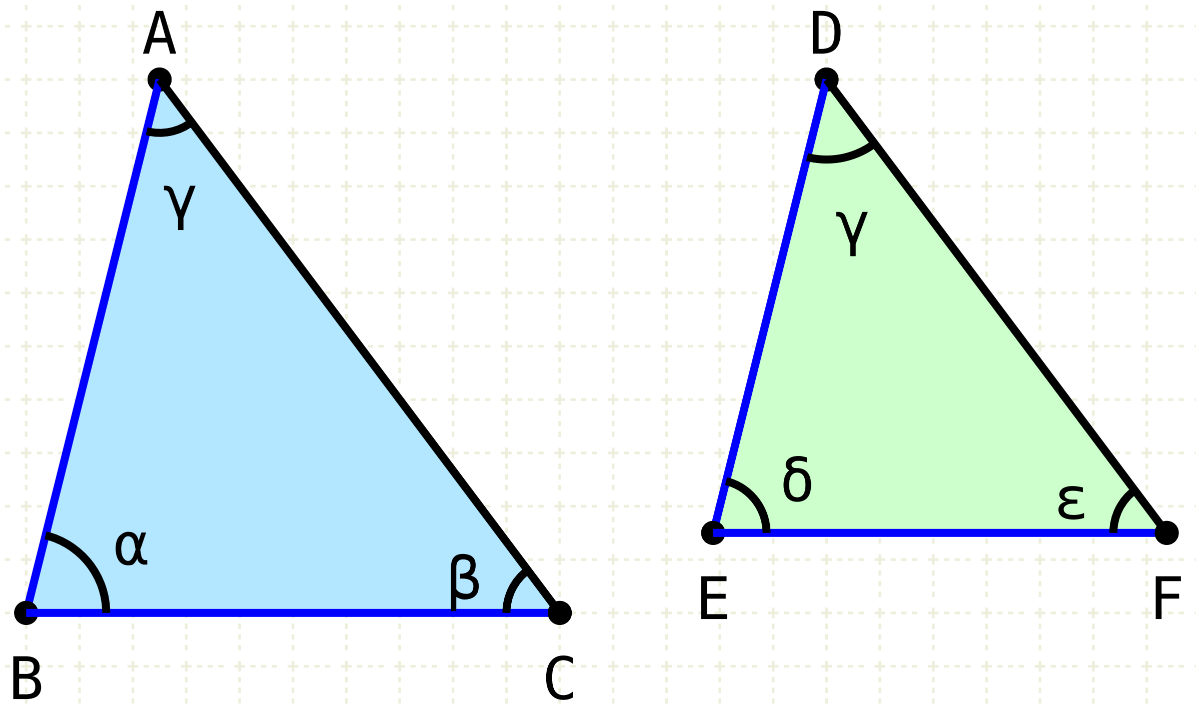

AB:BC = DE:EF

β < ∟, ε < ∟

α = δ, β = ε

In other words

If two triangles have one angle that is equal between them, AND the ratio of the sides of around a different angle are also equal, AND that the remaining angles are either both less than, or both greater than a right angle, then the two triangles will be equiangular

Let the angles at C and F be less than a right angle

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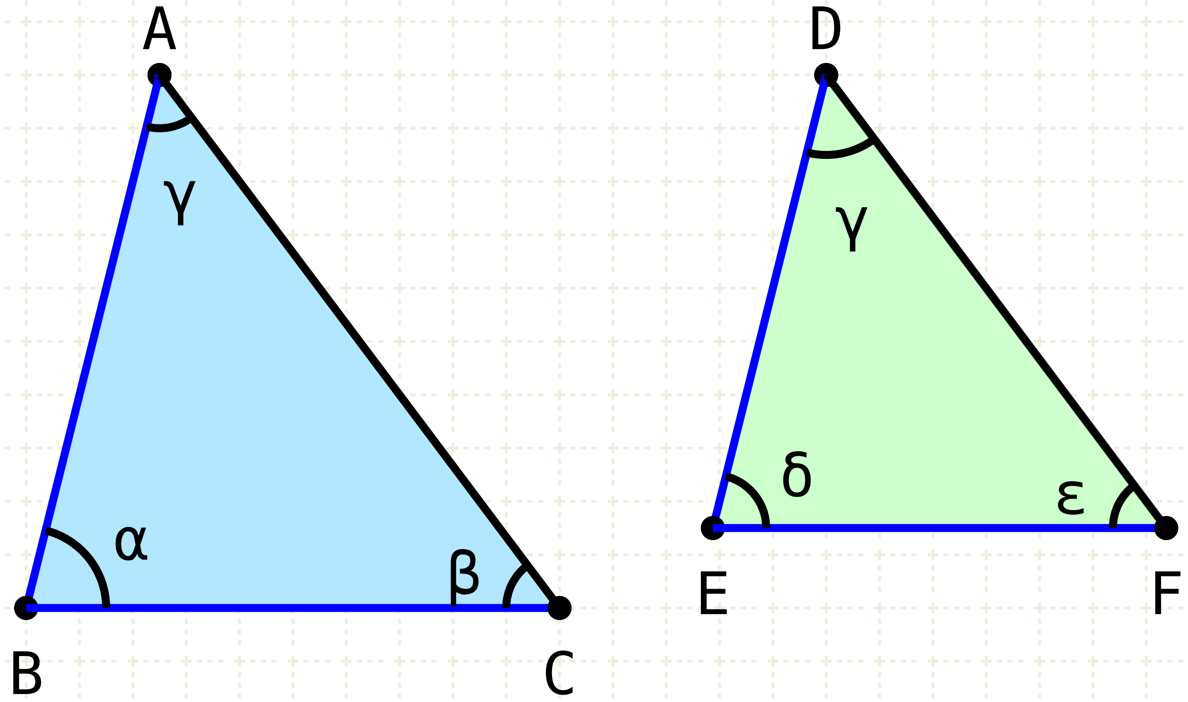

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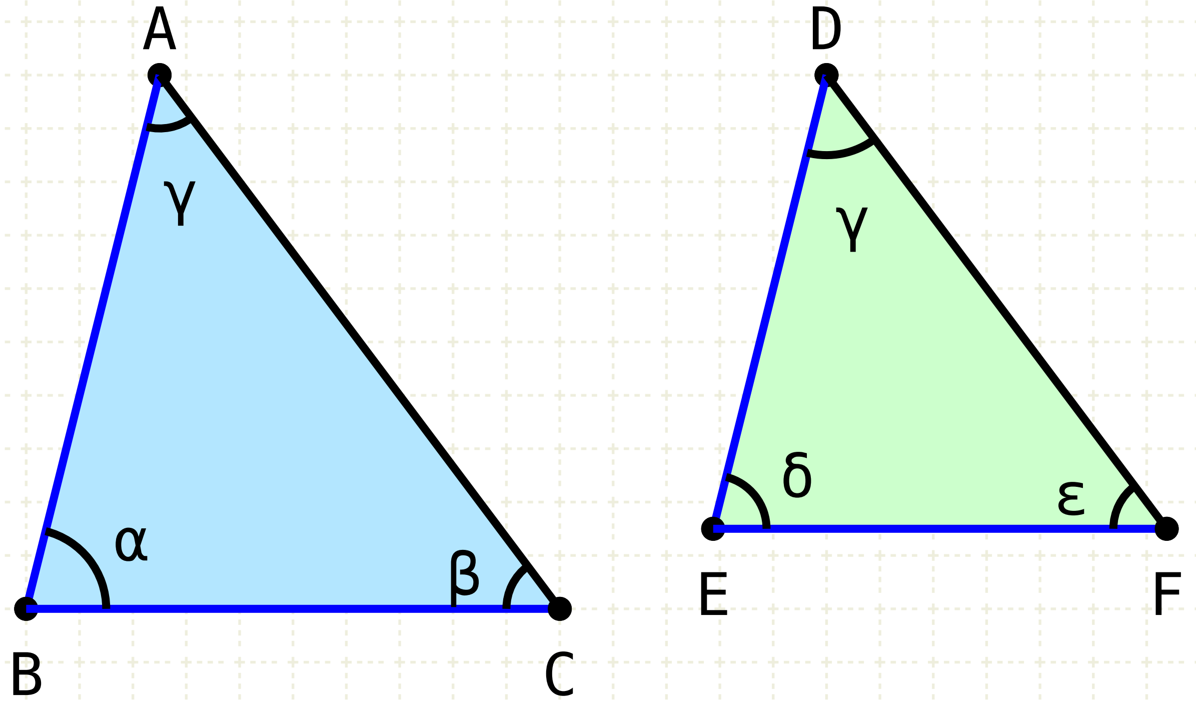

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β < ∟, ε < ∟

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Assume that the angle α is unequal to the angle δ, and that α is the greater

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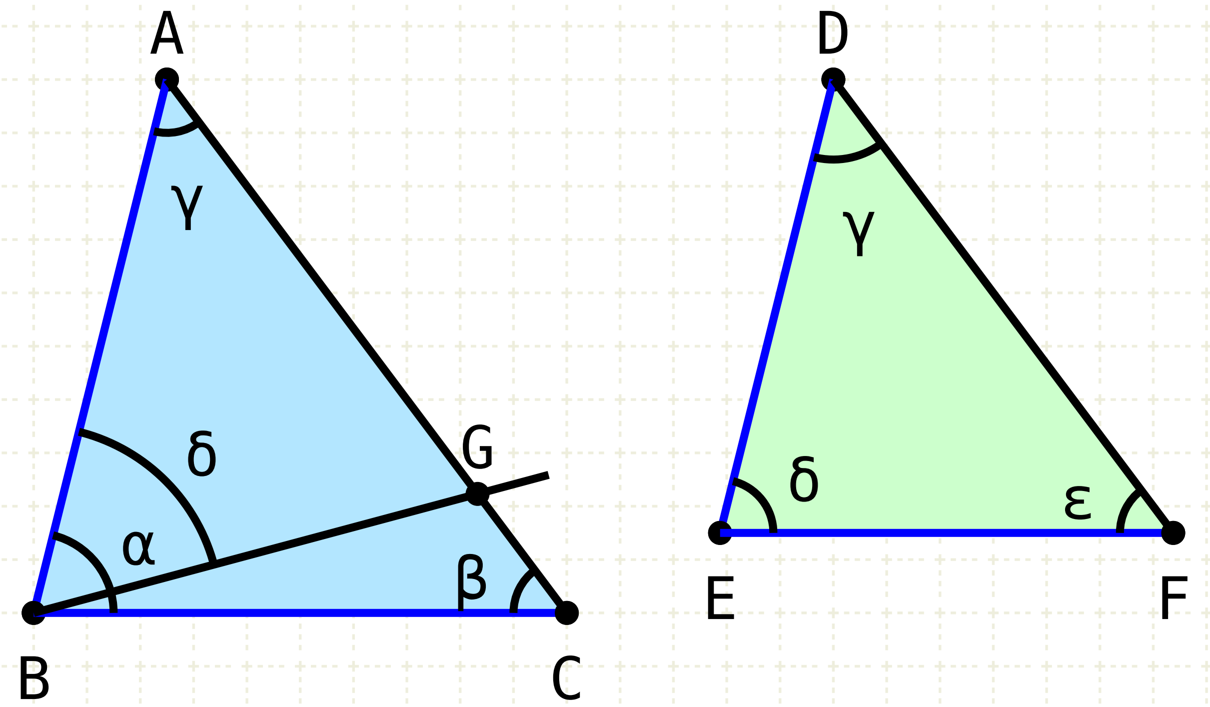

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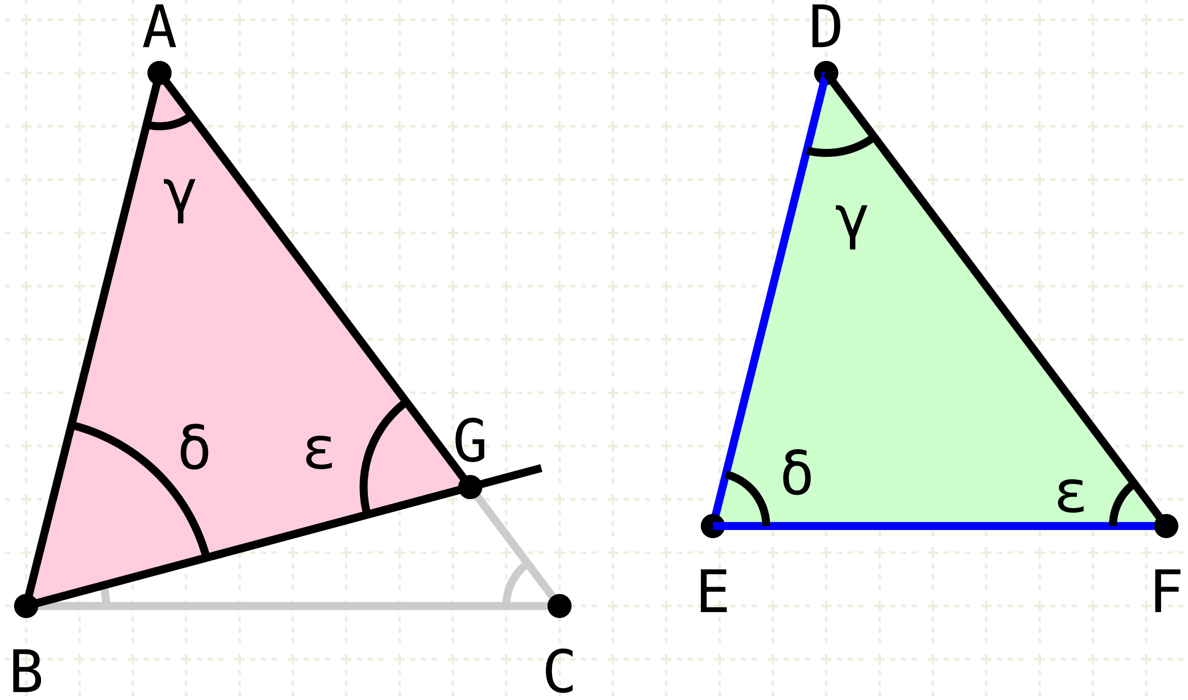

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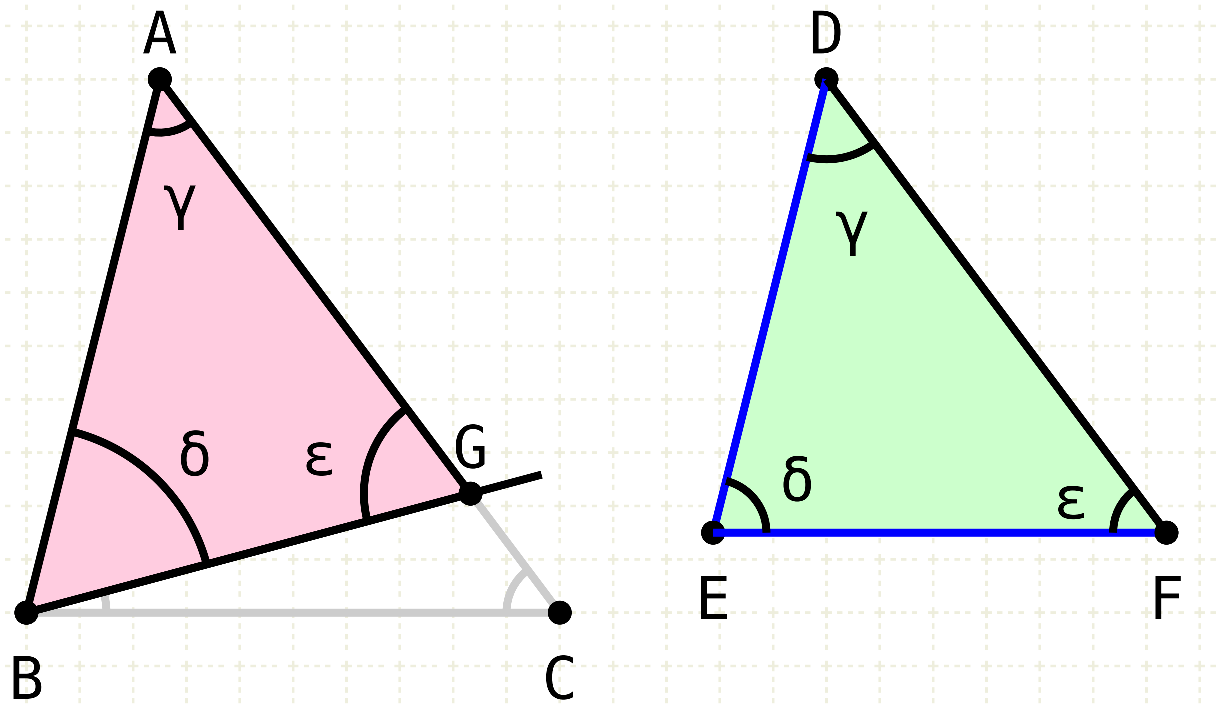

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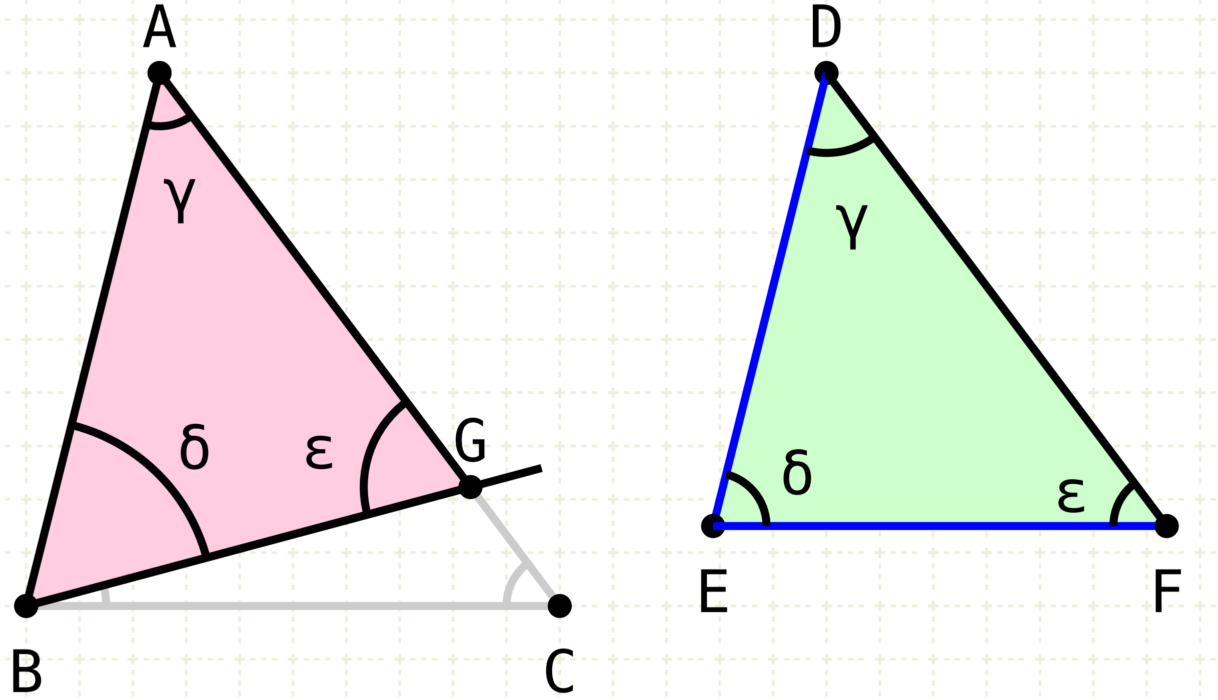

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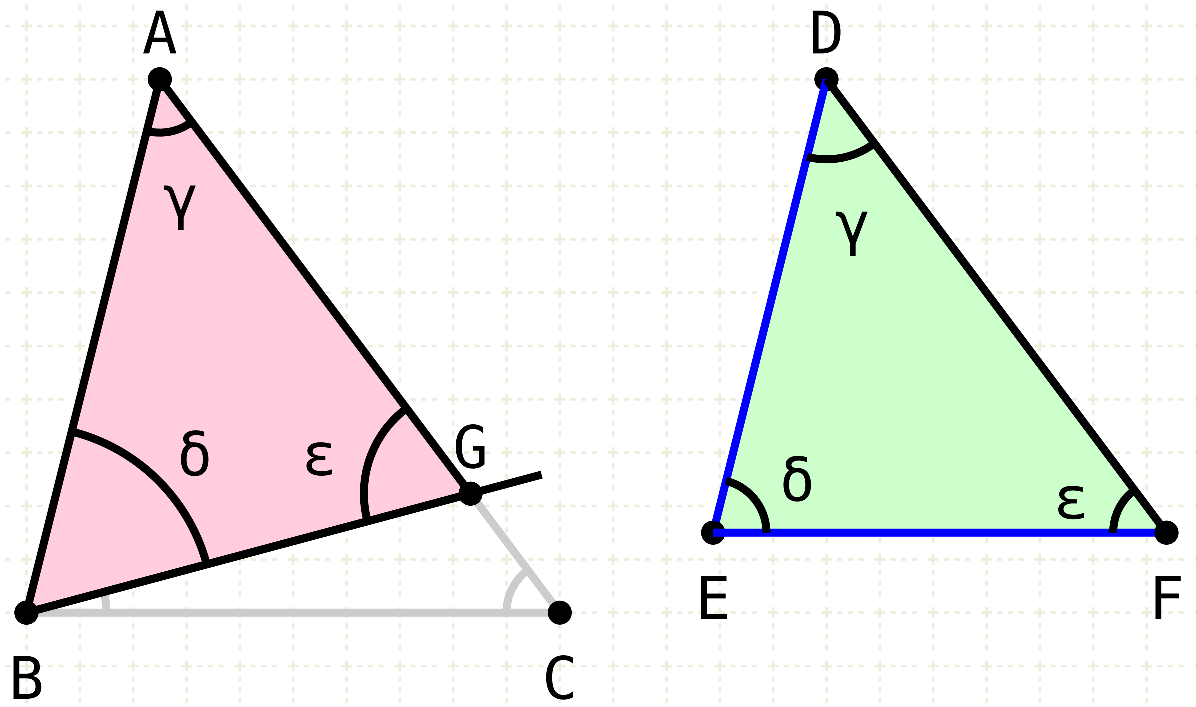

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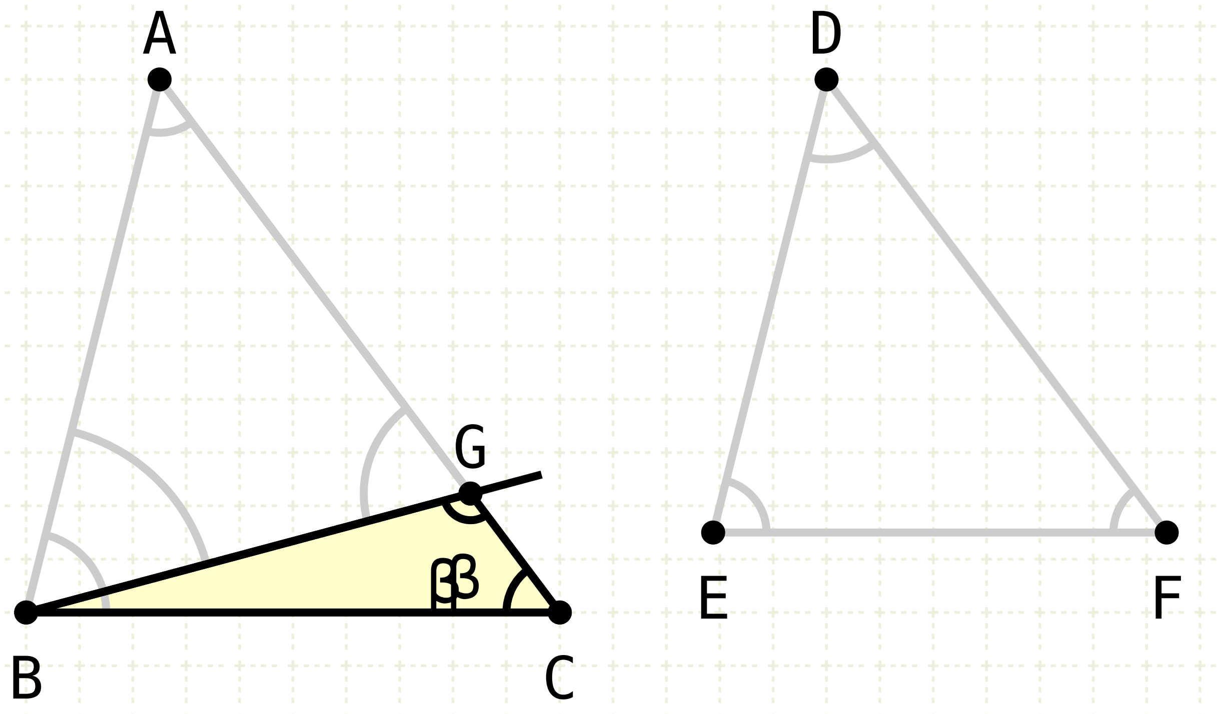

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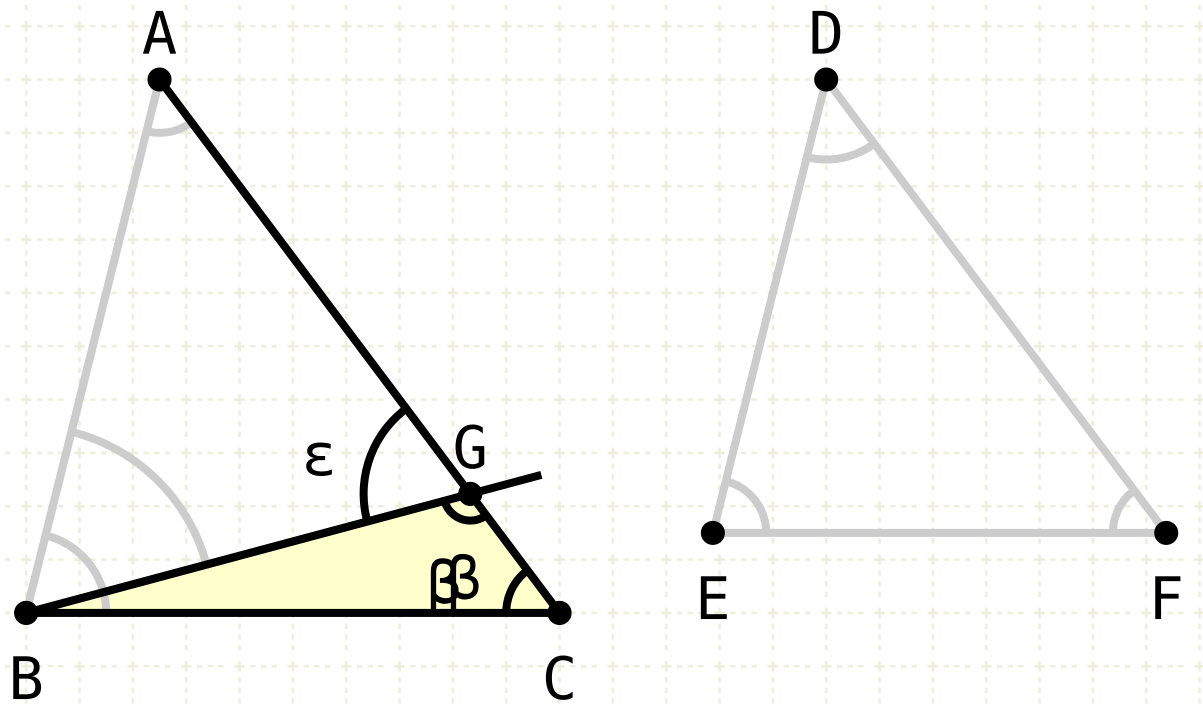

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$\varepsilon + \beta = 2\cdot ∟$

$\therefore\ \varepsilon > ∟$

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By definition, angle ACB is less than a right angle, so therefore so is angle BGC

Thus, the outside exterior angle must be greater than one right angle (the sum of the angles AGB and BGC equal two right angles) (I·13)

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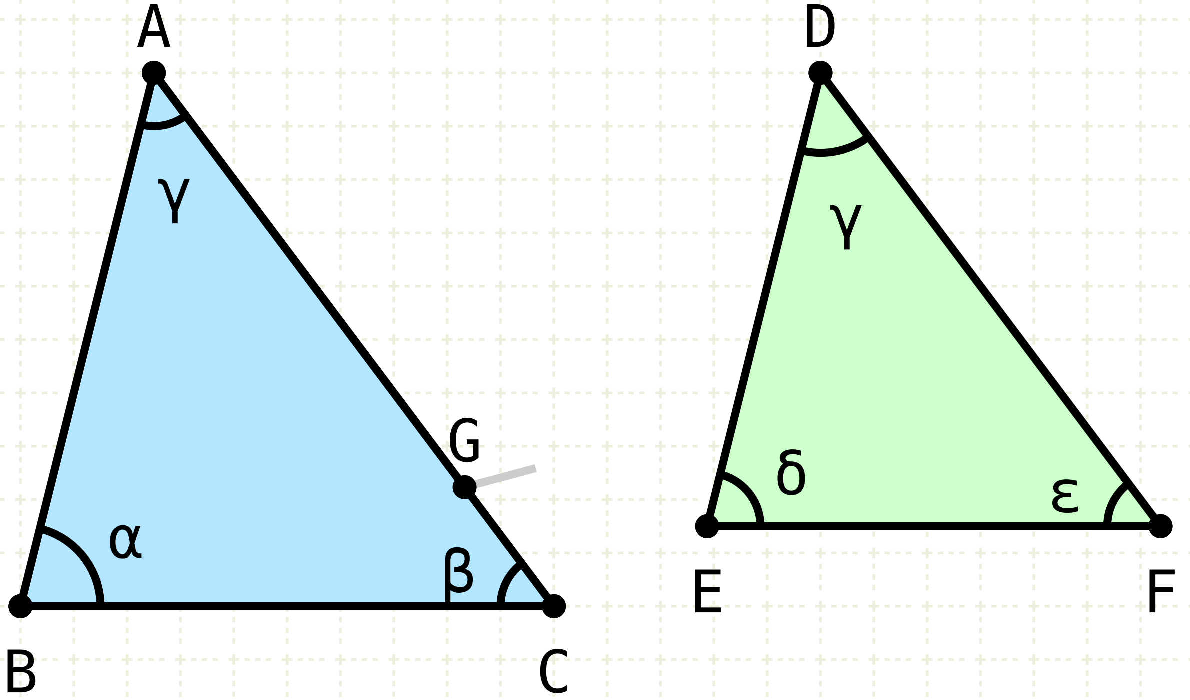

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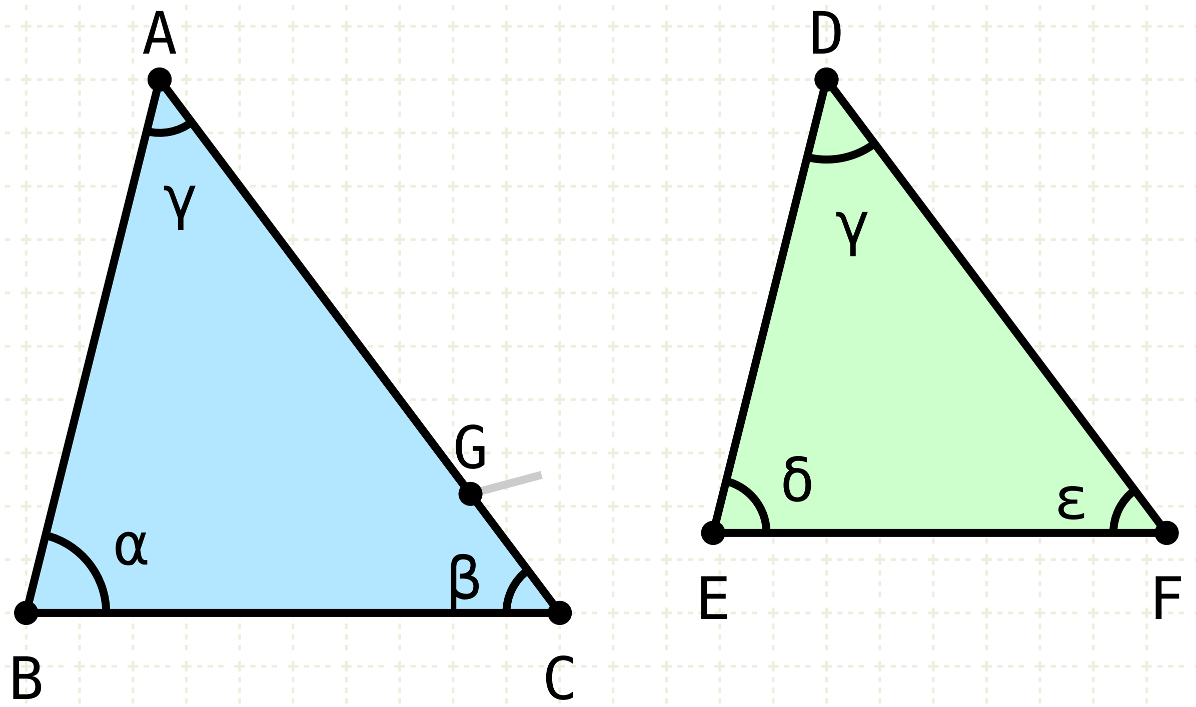

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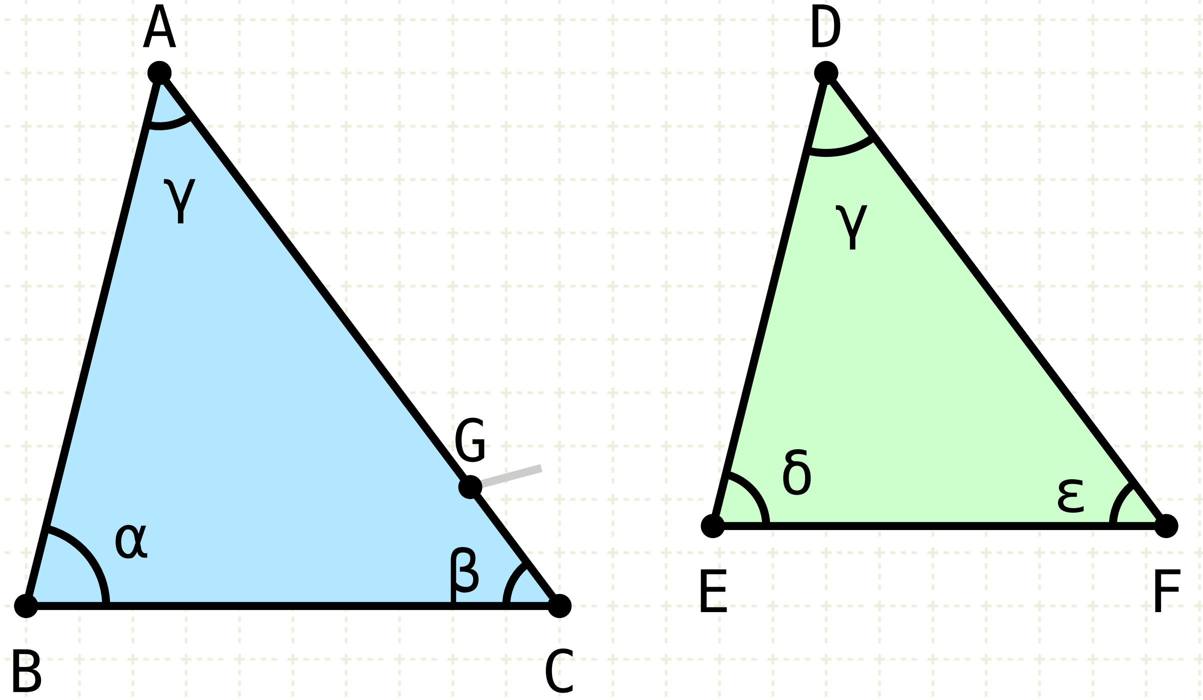

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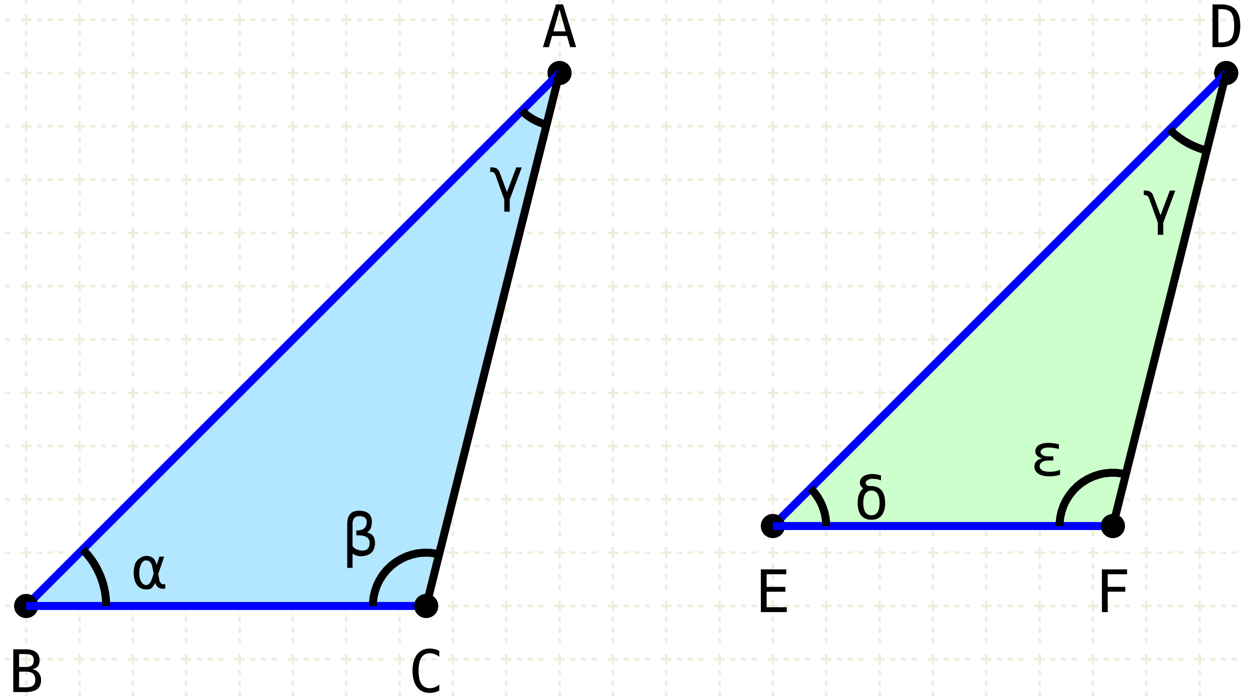

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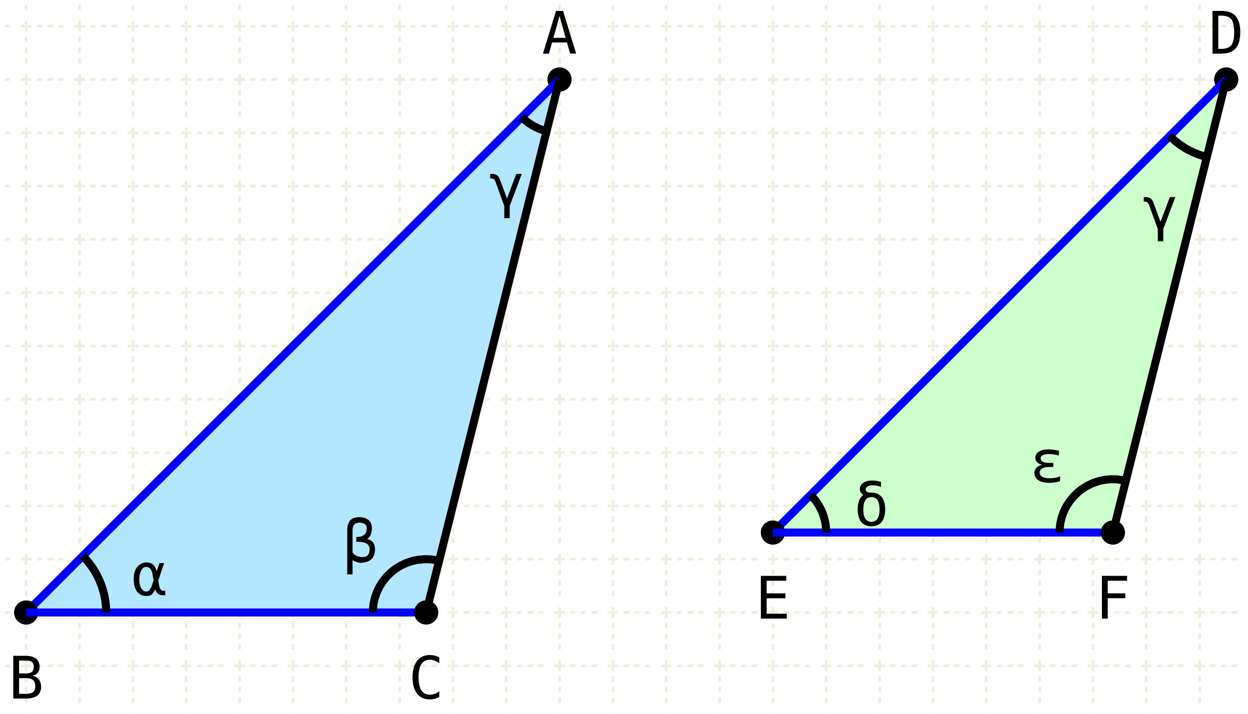

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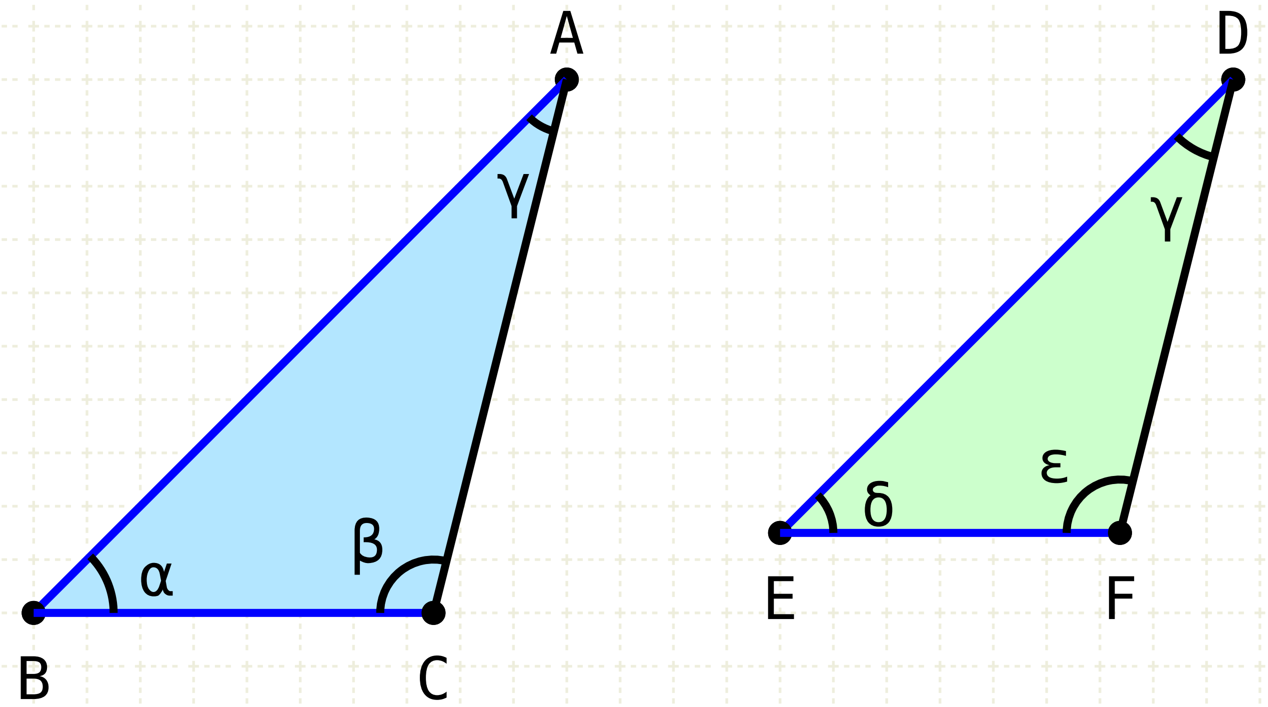

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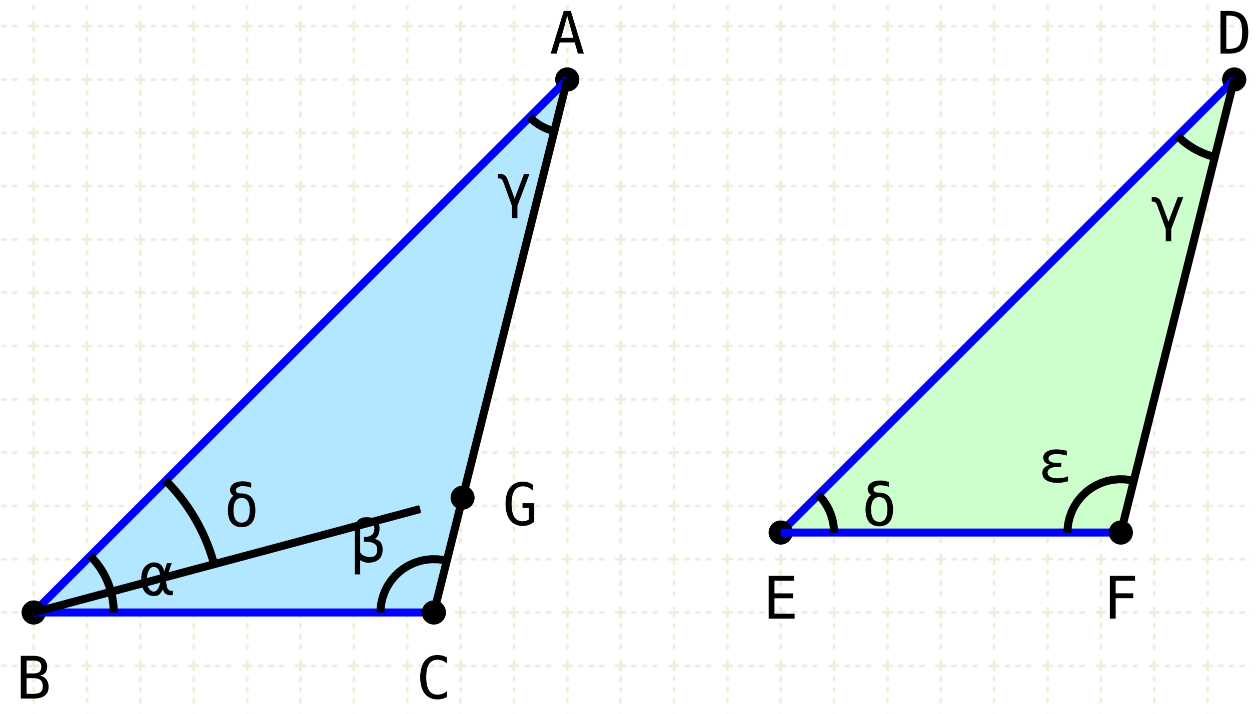

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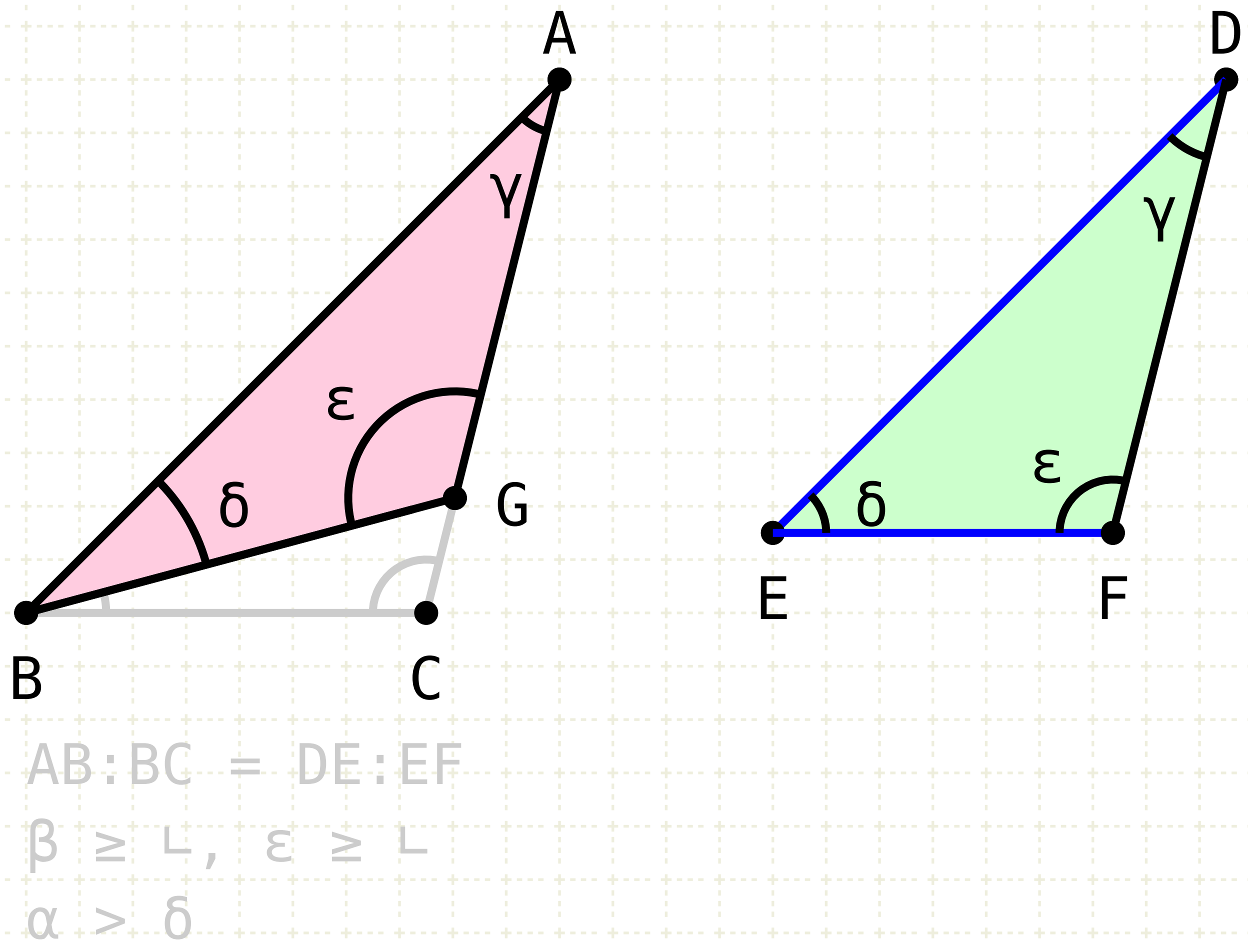

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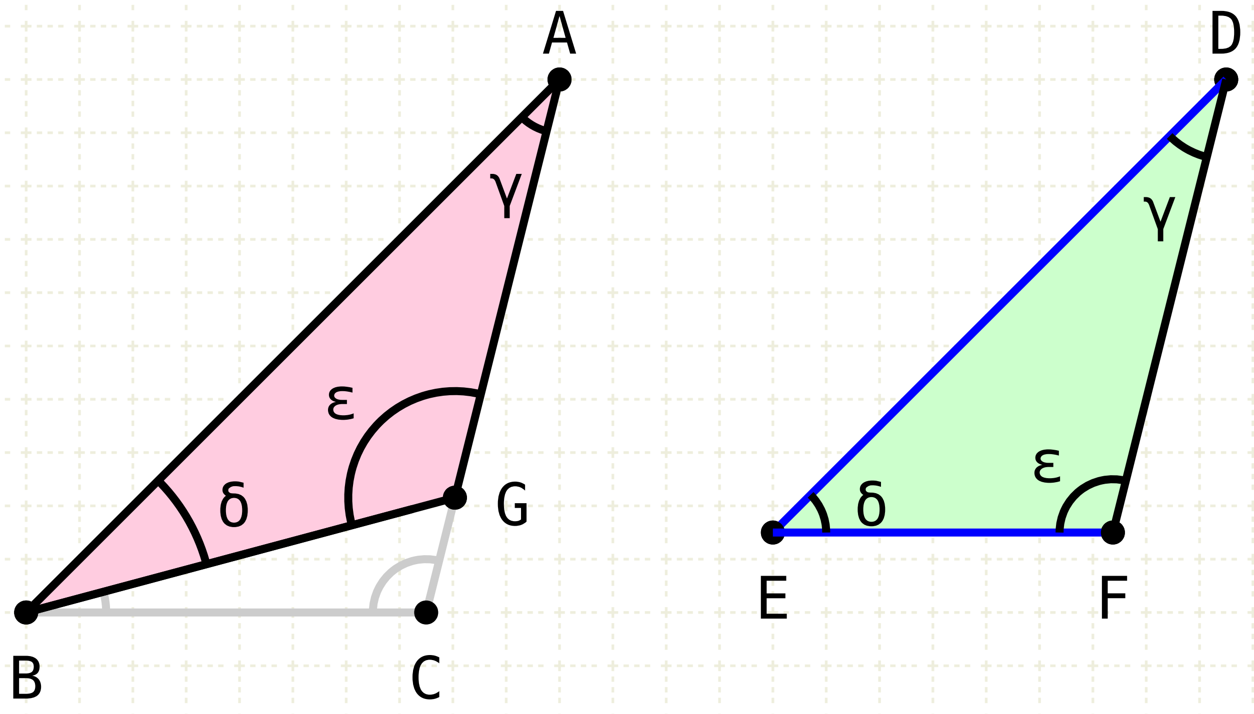

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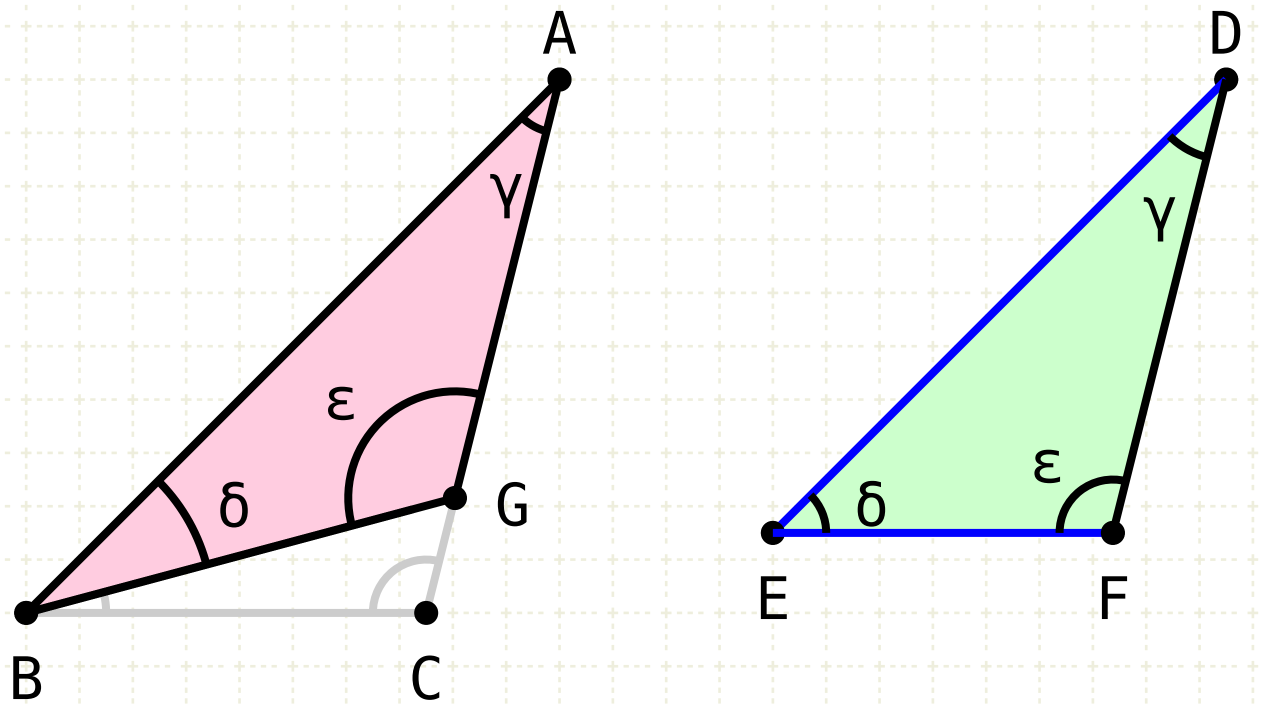

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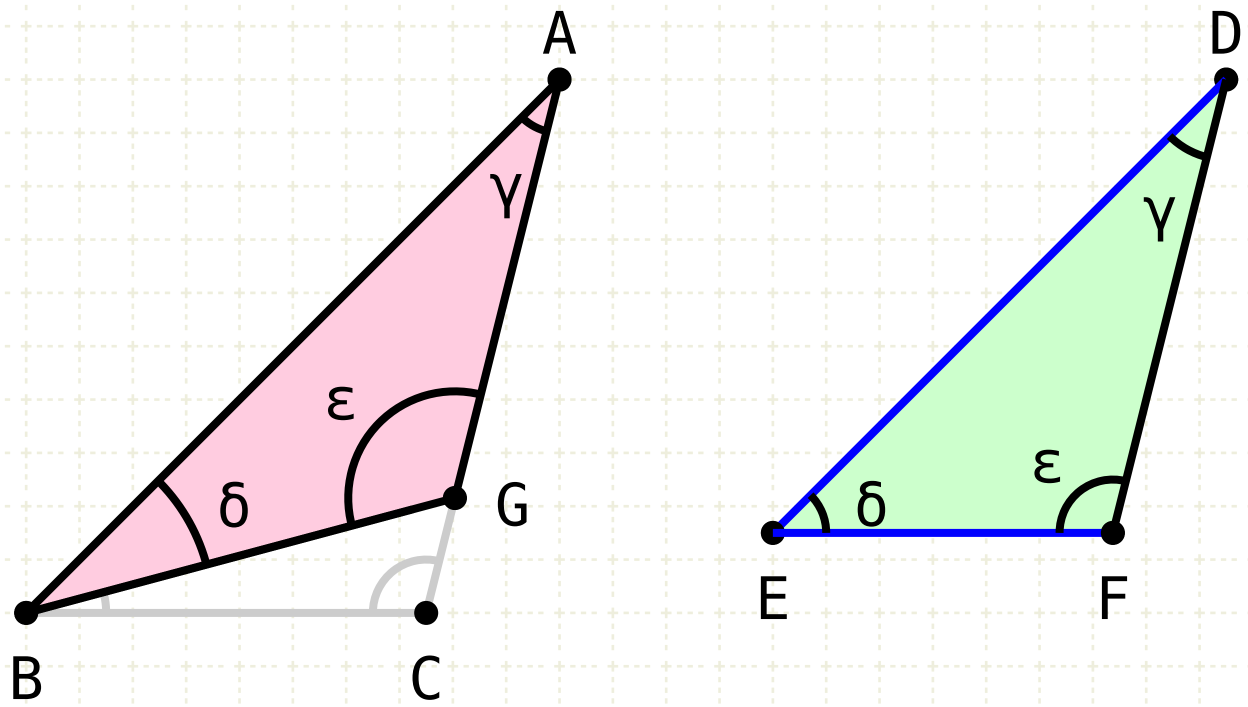

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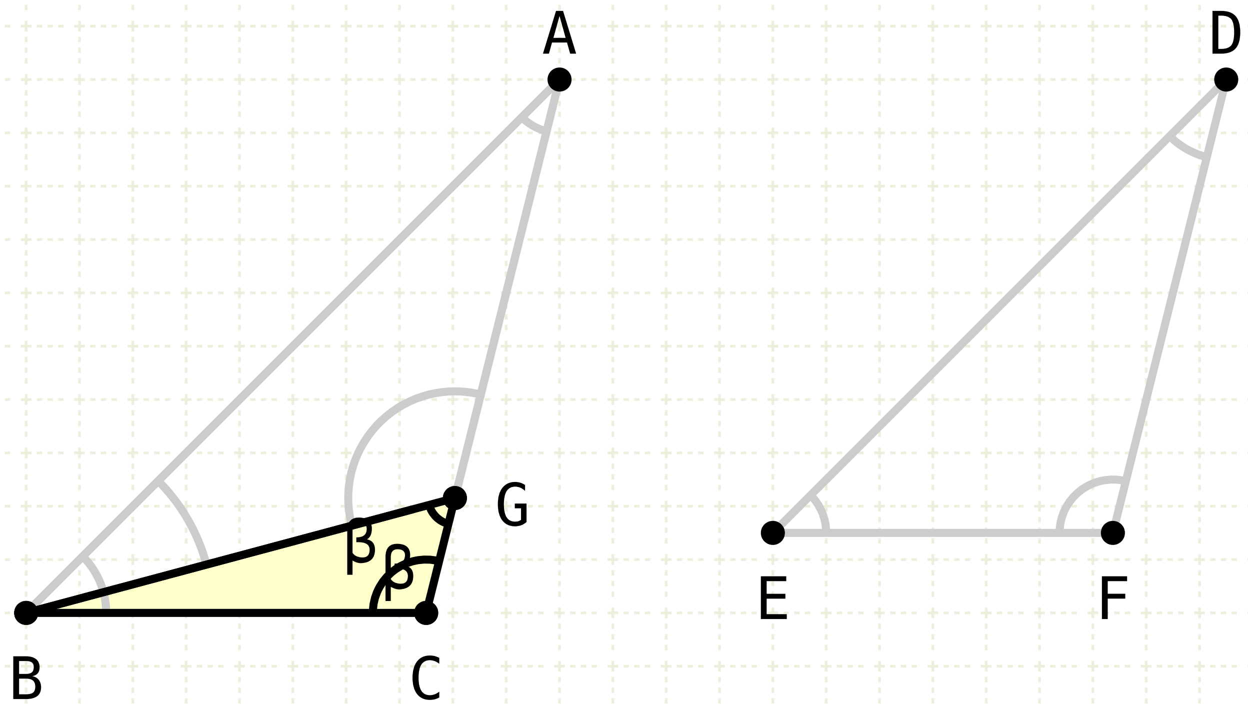

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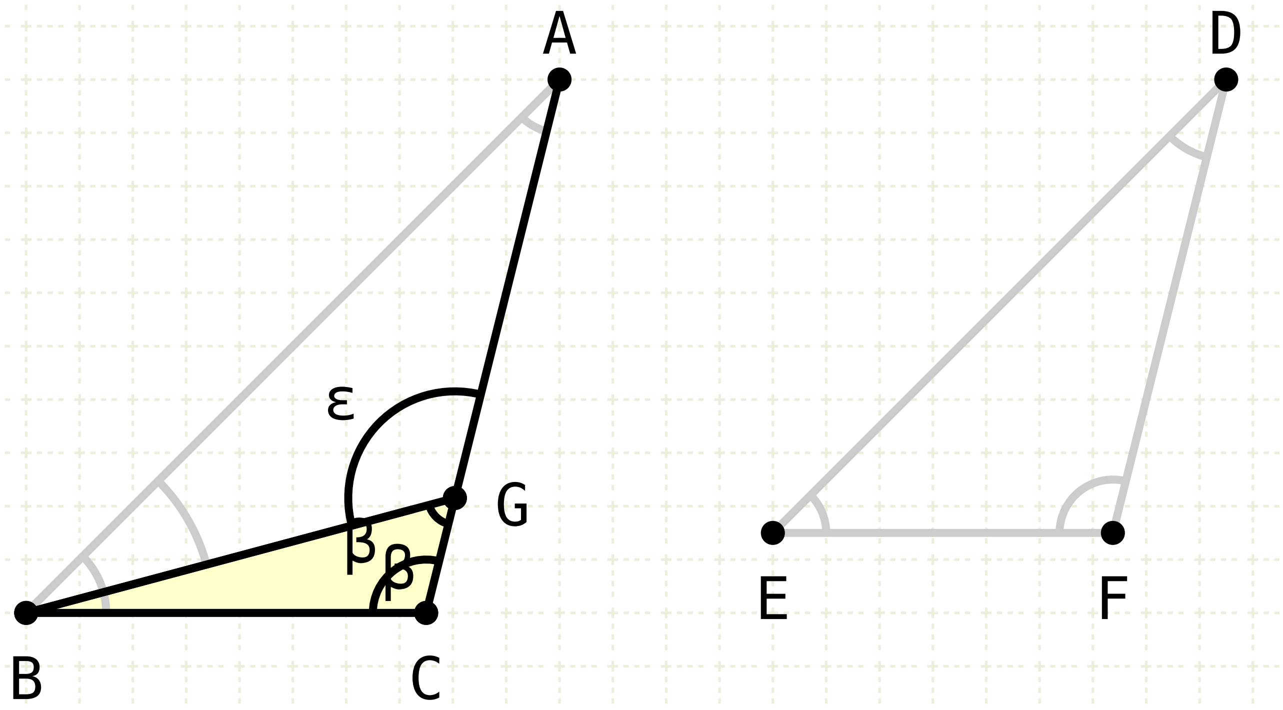

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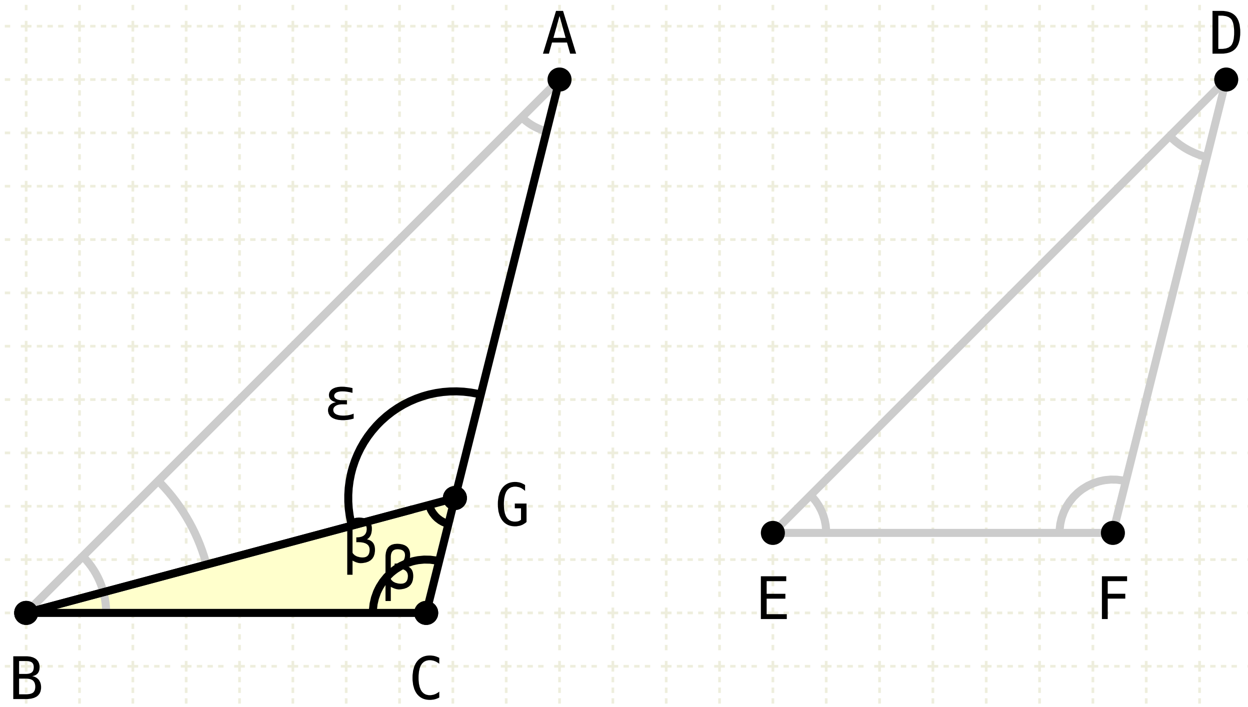
By definition, angle β is greater than or equal to a right angle, so therefore so is angle BGC

Proposition 7 of Book VI

If two triangles have one angle equal to one angle, the sides about other angles proportional, and the remaining angles either both less or both not less than a right angle, the triangles will be equiangular and will have those angles equal, the sides about which are proportional.



$AB:BC = DE:EF$

$\beta \geq \llcorner,\ \varepsilon \geq \llcorner$

$\alpha > \delta$

$AB:BG = DE:EF$

$AB:BG = AB:BC$

$BG = BC$

$\angle BGC = \angle GCB$

$\angle BGC = \angle GCB \geq \llcorner$

Proof by Contradiction (2)

Assume that the angle α is unequal to the angle δ, and that α is the greater

Construct the angle ABG such that is is equal to the angle δ (I·23)

Now, since triangles AGB and AFE have two angles equal, the third must also be equal (I·32)

ABG is equiangular to DEF, so AB is to BG, as DE is to EF (VI·4)

The ratios AB to BG and AB to BC are both equal to DE to EF, then they are equal to each other (V·11)

Therefore BG equals BC (V·9)

Thus the triangle BGC is an isosceles, and angle BGC is equal to angle BCG (I·5)

By definition, angle β is greater than or equal to a right angle, so therefore so is angle BGC

Thus we have two angles inside a triangle, each of which is greater than or equal to a right angle, which is impossible (I·17)

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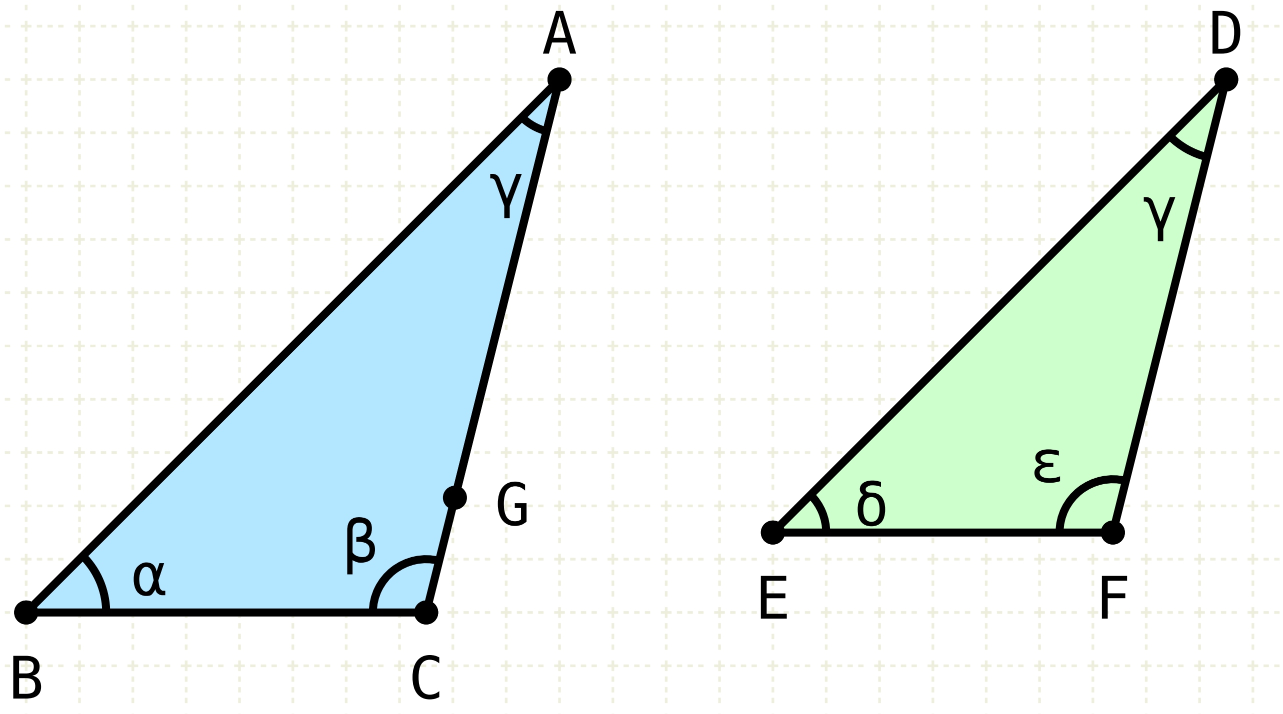

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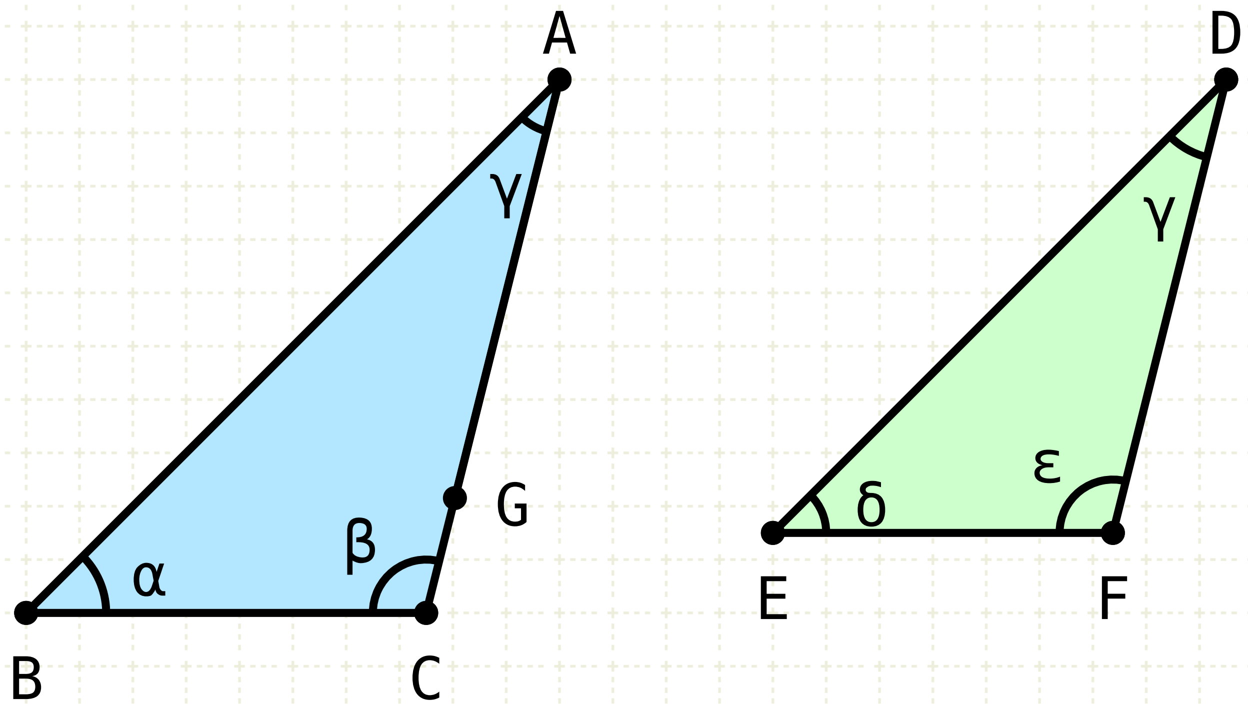

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The two triangles are equiangular

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