

Euclid's Elements

Book VI

One can state, without exaggeration, that the observation of and the search for similarities and differences are the basis of all human knowledge.

Alfred Nobel



Table of Contents, Chapter 6

1	If the height of two triangles are equal, then the ratio of the areas is equal to the ratio of the bases	7	If two triangles have one angle equal to one angle, and the sides about other angles are proportional, and the remaining angles either both less or both not less than a right angle, then triangles will be equiangular	14	In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and vice versa
2	If a line cuts a triangle, parallel to its base, it will cut the sides of the triangle proportionally			15	In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and vice versa
3	If an angle of a triangle is bisected and the straight line cutting the angle also cuts the base, the segments of the base will have the same ratio as the remaining sides of the triangle	8	If in a right-angled triangle a perpendicular be drawn from the right angle to the base, the triangles adjoining the perpendicular are similar both to the whole and to one another	16	If four straight lines are proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means, and vice versa
4	If two triangles have equal angles, then the sides opposite the equal angles are proportional, as well, the sides of the triangles on either side of the equal angles are also proportional	9	From a given straight line to cut off a given fraction	17	If three straight lines are proportional, the rectangle contained by the extremes is equal to the square on the mean; and vice versa
5	If two triangles have proportional sides, the triangles will be equiangular	10	To cut a given uncut straight line similarly to a given cut straight line	18	On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure
6	If two triangles have one angle equal to one angle and the sides about the equal angles are proportional, then the triangles will be equiangular	11	To two given straight lines to find a third proportional	19	Similar triangles are to one another in the duplicate ratio of the corresponding sides
		12	To three given straight lines to find a fourth proportional		
		13	To two given straight lines to find a mean proportional		



Table of Contents, Chapter 3

20	Similar polygons are divided into the same number of similar triangles, which have the same ratio as the wholes, and the polygons have duplicate ratios to their corresponding sides	26	If from a parallelogram a similar parallelogram with a common angle is subtracted, it is about the same diameter as the original	31	In right-angled triangles the figure on the side subtending the right angle is equal to the similar and similarly described figures on the sides containing the right angle
21	Figures which are similar to the same rectilineal figure are also similar to one another	27	Of all the parallelograms applied to the same straight line and deficient by parallelogrammic figures similar to a parallelogram drawn on half the said line, the largest will be one that is drawn on half of the straight line and is similar to the defect		
22	If four straight lines are proportional, similar rectilineal figures will also be proportional; and vice versa	28	To a given straight line, apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one		
23	Equiangular parallelograms have to one another the ratio compounded of the ratios of their sides	29	To a given straight line, apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one		
24	In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another	30	To cut a finite straight line in extreme ratio		
25	To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure				



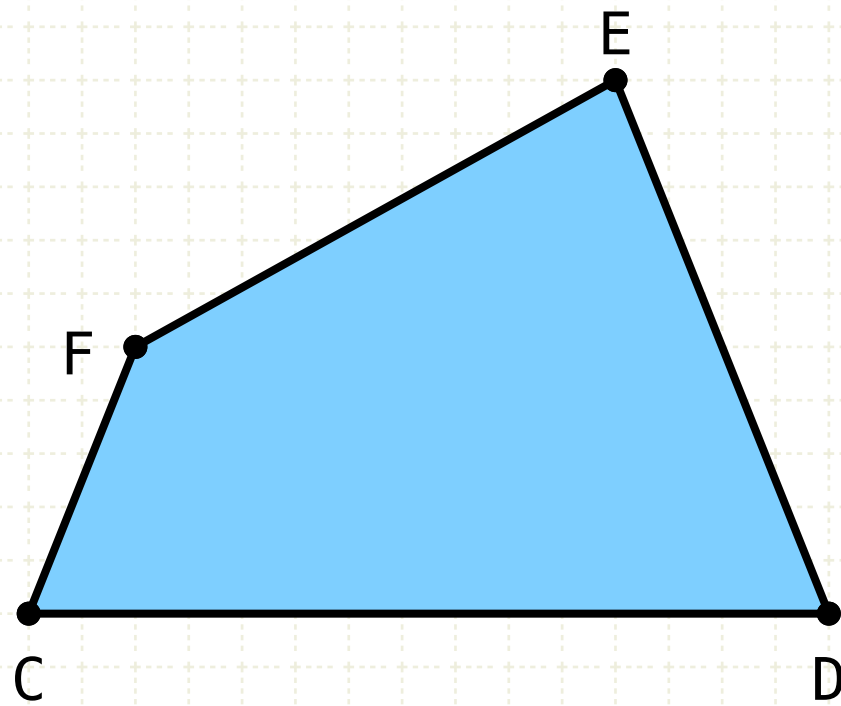
Proposition 18 of Book VI

On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure.

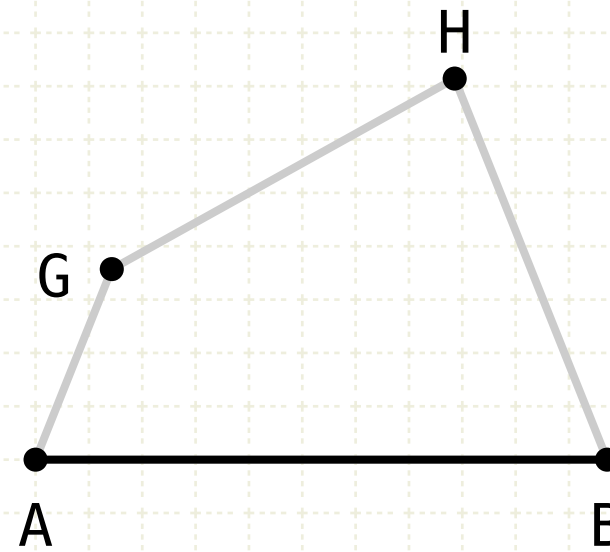


Proposition 18 of Book VI

On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure.



$\square CDEF \sim \square ABHG$

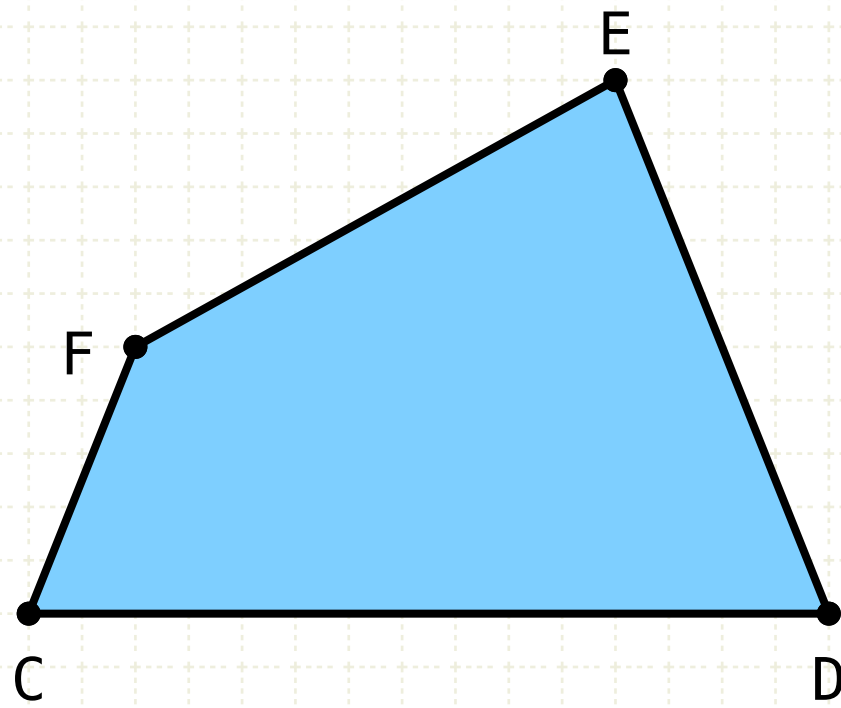


In other words

Copy one figure onto another straight line so that both figures are similar, i.e. equiangular and proportional sides

Proposition 18 of Book VI

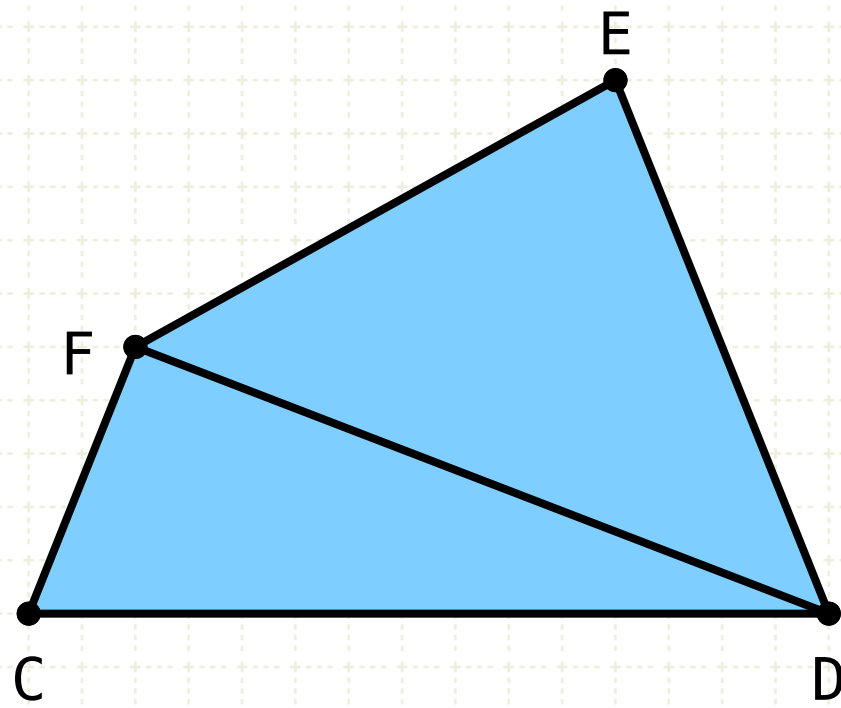
On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure.



Construction

Proposition 18 of Book VI

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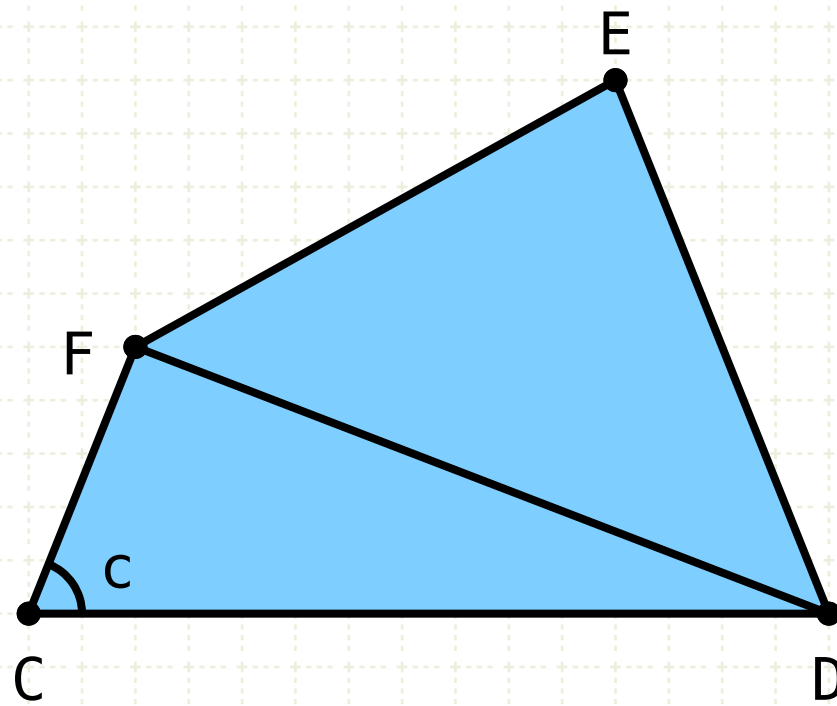


Construction

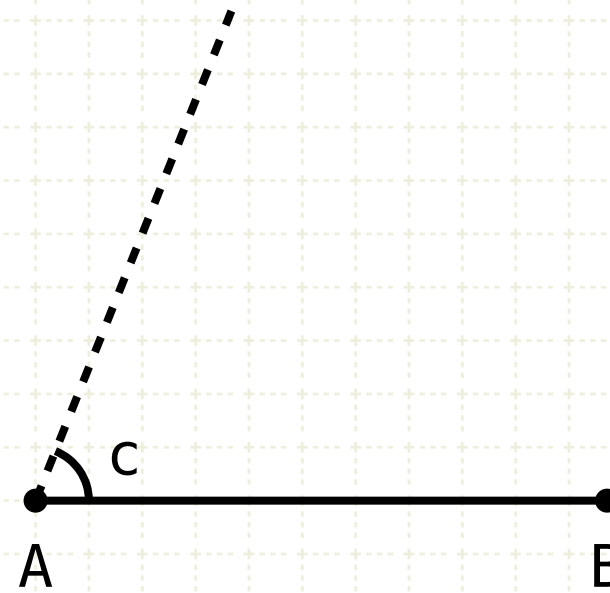
Let line DF be drawn

Proposition 18 of Book VI

On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure.



$$\angle FCD = \angle GAB = c$$



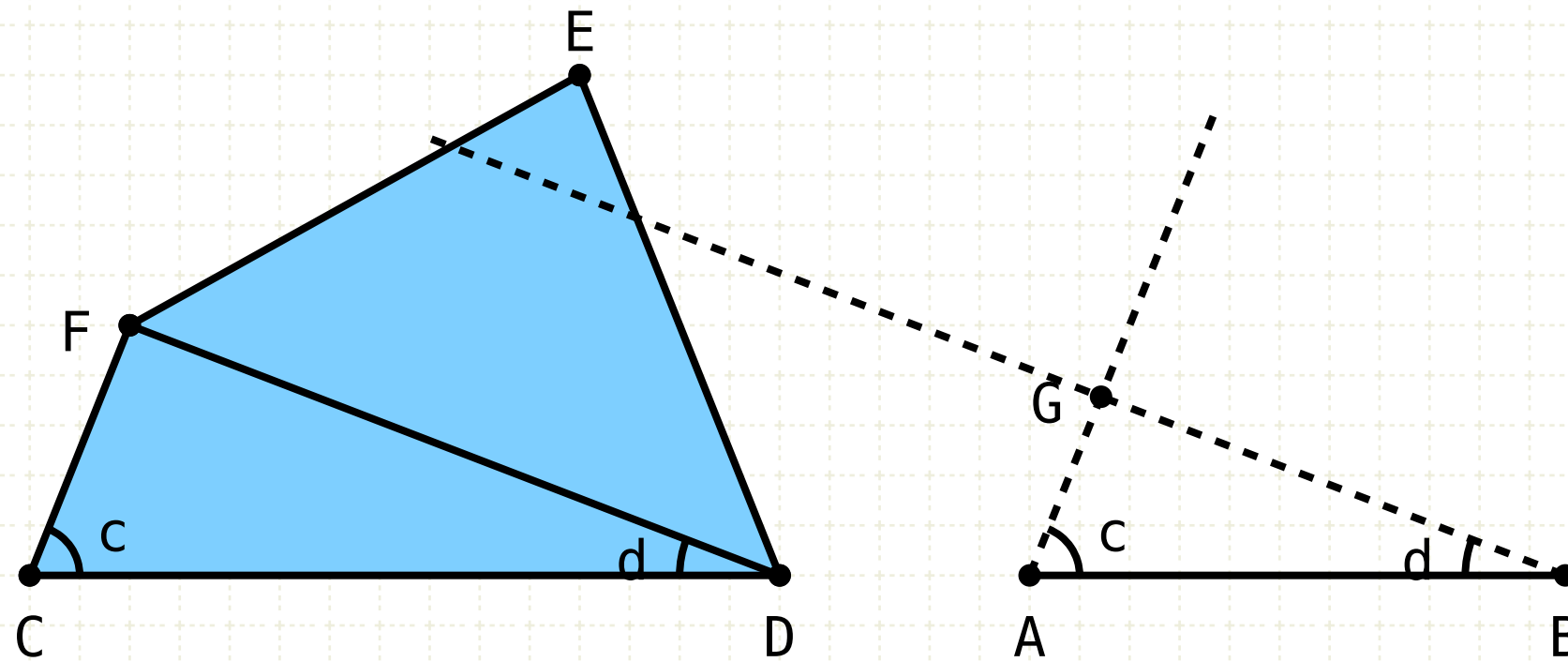
Construction

Let line DF be drawn

Copy the angle FCD to point A on line AB (I-23)

Proposition 18 of Book VI

On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure.



$$\begin{aligned}\angle FCD &= \angle GAB = c \\ \angle CDF &= \angle ABG = d\end{aligned}$$

Construction

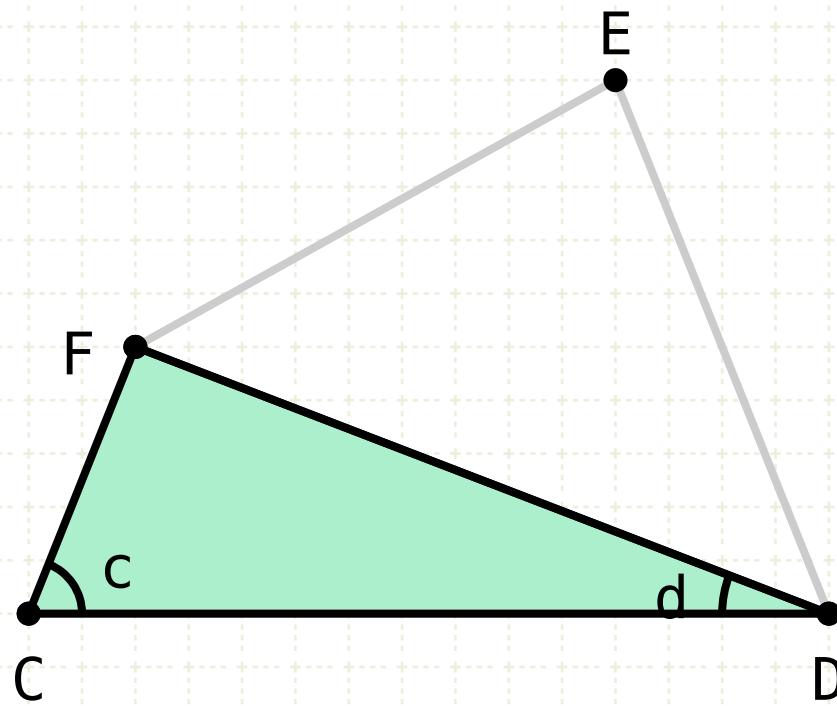
Let line DF be drawn

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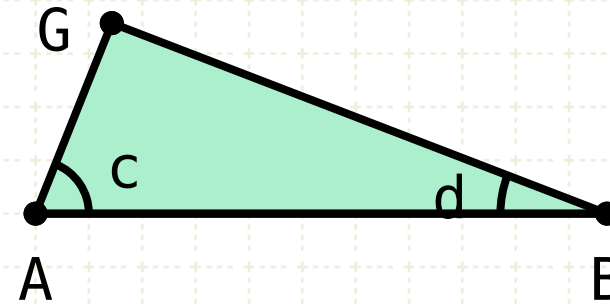
Copy the angle CDF to point B on line AB (I-23)

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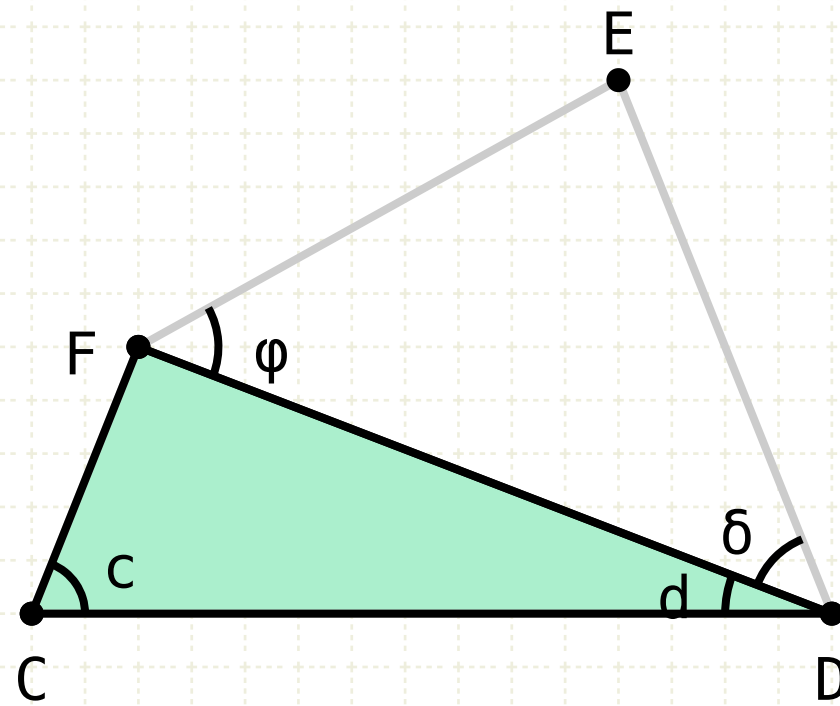
Let line DF be drawn

Copy the angle FCD to point A on line AB (I-23)

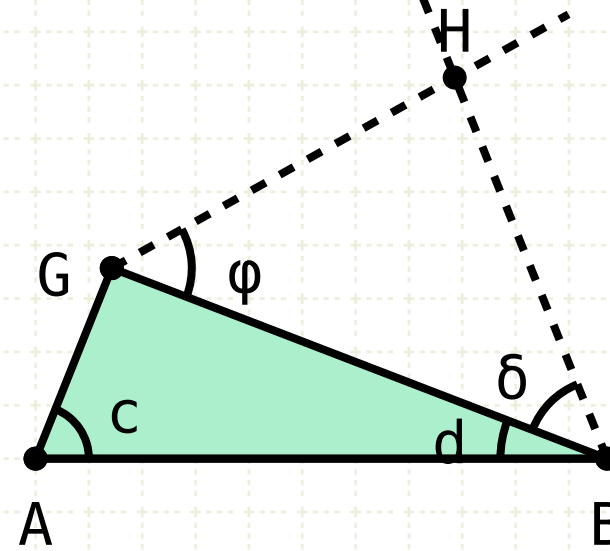
Copy the angle CDF to point B on line AB (I-23)

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$$\begin{aligned}\angle FCD &= \angle GAB = c \\ \angle CDF &= \angle ABG = d \\ \angle DFE &= \angle BGH = \varphi \\ \angle FDE &= \angle GBH = \delta\end{aligned}$$



Construction

Let line DF be drawn

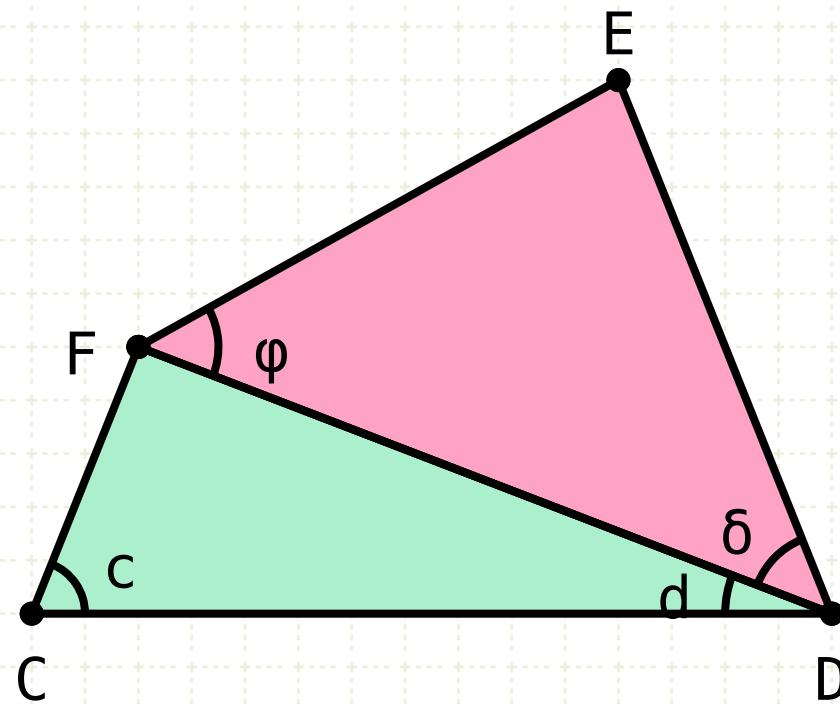
Copy the angle FCD to point A on line AB (I·23)

Copy the angle CDF to point B on line AB (I·23)

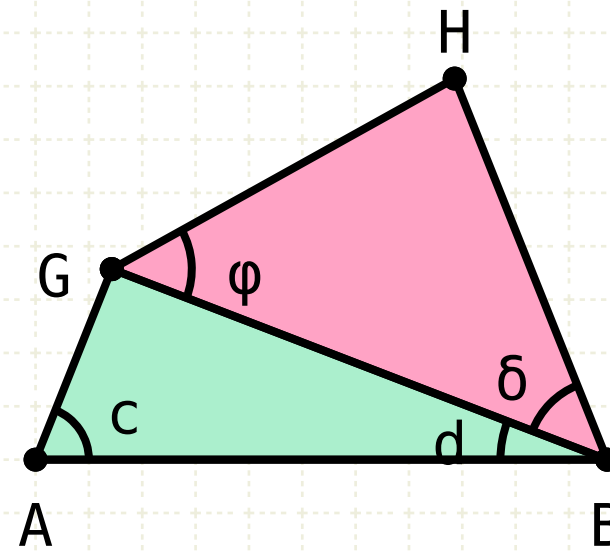
On the straight line BG, construct BGH equal to angle DFE and angle GBH equal to angle FDE (I·23)

Proposition 18 of Book VI

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Construction

Let line DF be drawn

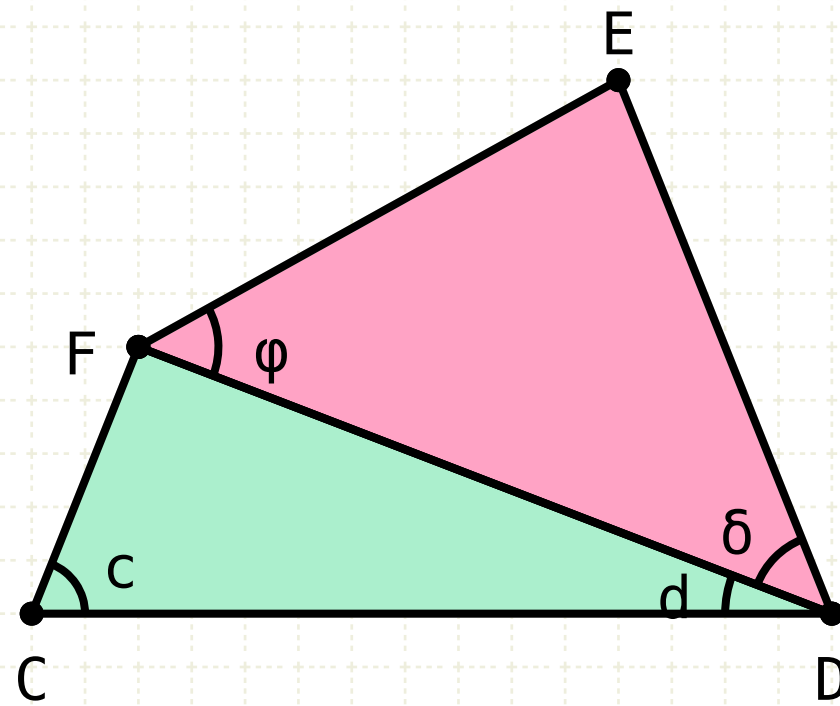
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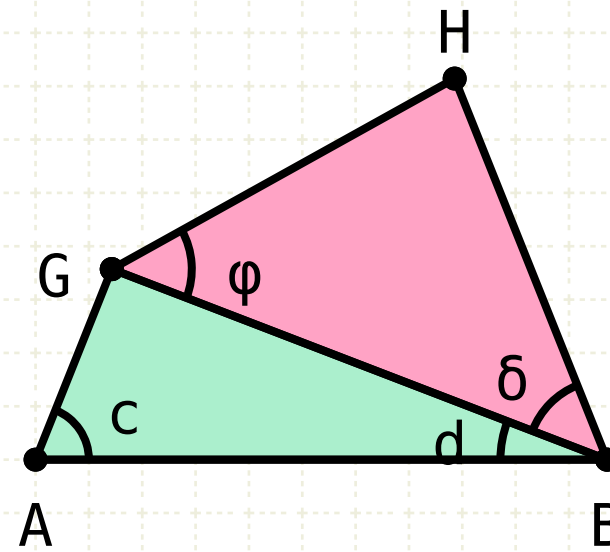
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On a given straight line to describe a rectilinear figure similar and similarly situated to a given rectilinear figure.



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$$\square CDEF \sim \square ABHG$$



Construction

Let line DF be drawn

Copy the angle FCD to point A on line AB (I·23)

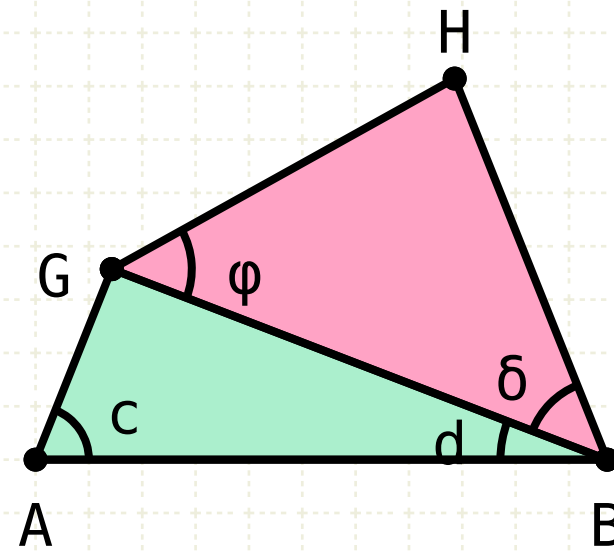
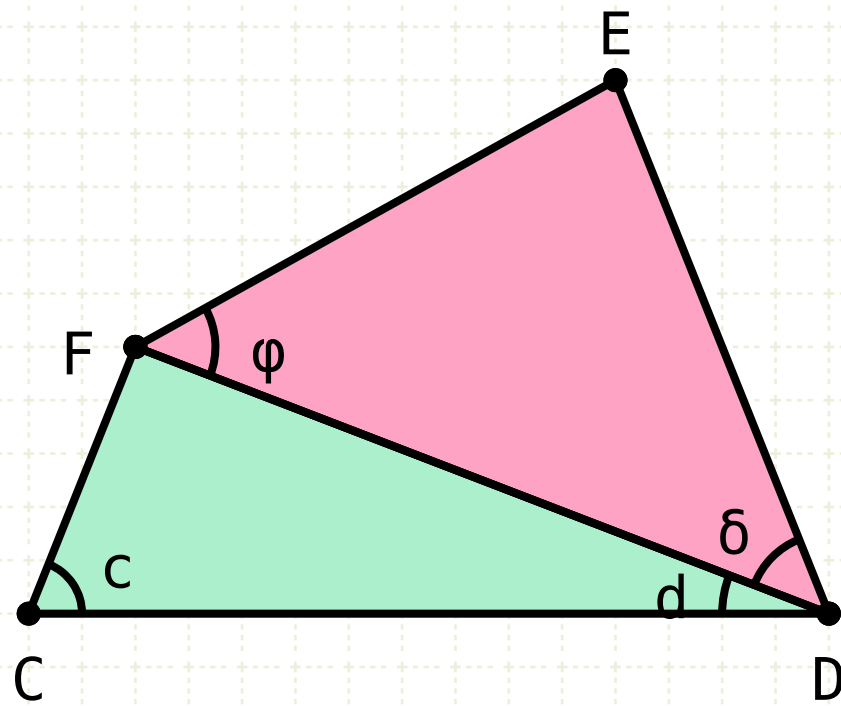
Copy the angle CDF to point B on line AB (I·23)

On the straight line BG, construct BGH equal to angle DFE and angle GBH equal to angle FDE (I·23)

The rectilinear figure ABHG is similar to CDEF

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On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure.

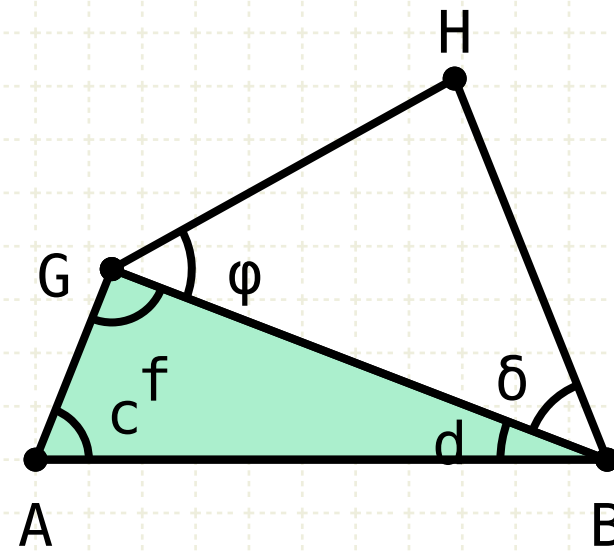
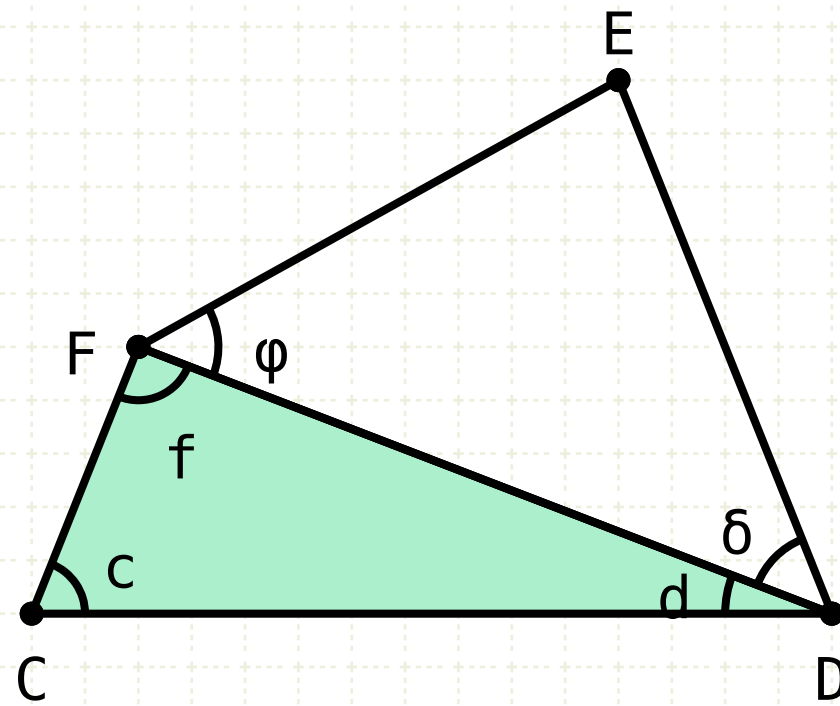


Proof

$$\begin{aligned}\angle FCD &= \angle GAB = c \\ \angle CDF &= \angle ABG = d \\ \angle DFE &= \angle BGH = \varphi \\ \angle FDE &= \angle GBH = \delta\end{aligned}$$

Proposition 18 of Book VI

On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure.



Proof

Compare triangles FCD and GAB, the remaining angle AGB is equal to the angle CFD (I-32), thus the two triangles are equiangular

$$\angle FCD = \angle GAB = c$$

$$\angle CDF = \angle ABG = d$$

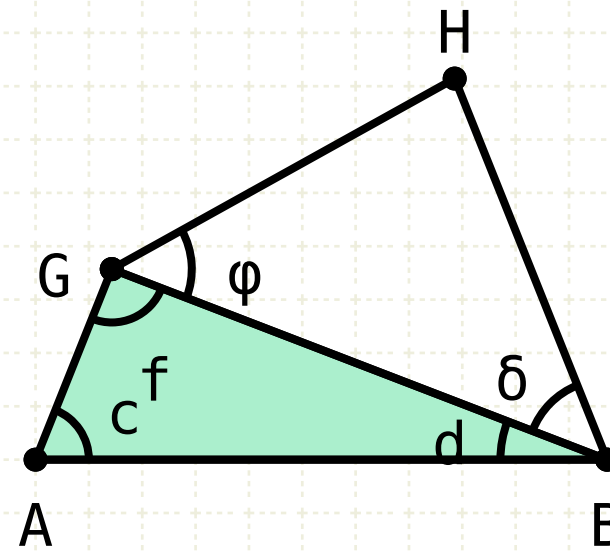
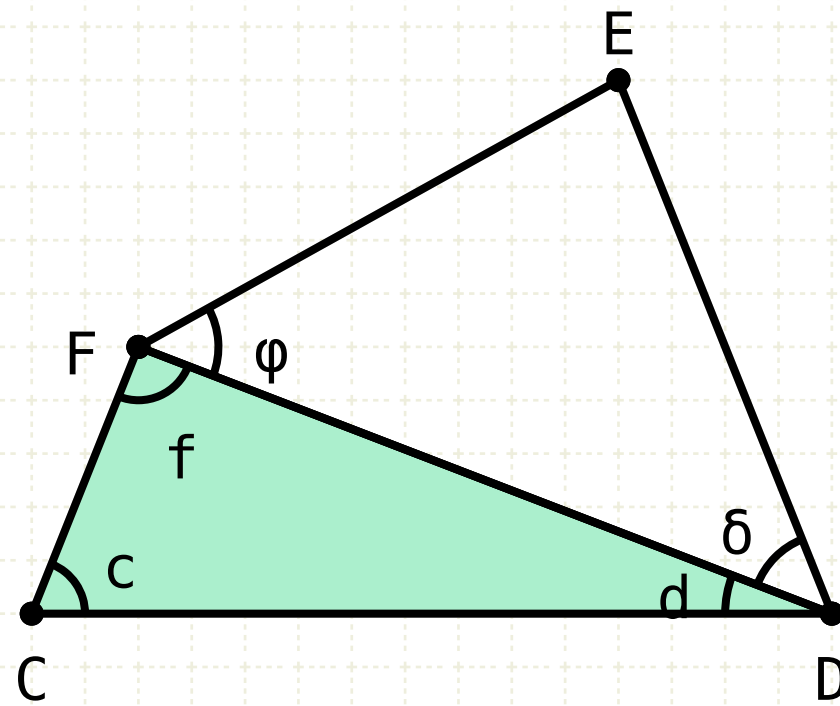
$$\angle DFE = \angle BGH = \varphi$$

$$\angle FDE = \angle GBH = \delta$$

$$\angle CFD = \angle AGB = f$$

Proposition 18 of Book VI

On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure.



Proof

Compare triangles FCD and GAB, the remaining angle AGB is equal to the angle CFD (I-32), thus the two triangles are equiangular

Thus the sides of FCD and GAB are proportional (VI-4)

$$\angle FCD = \angle GAB = c$$

$$\angle CDF = \angle ABG = d$$

$$\angle DFE = \angle BGH = \varphi$$

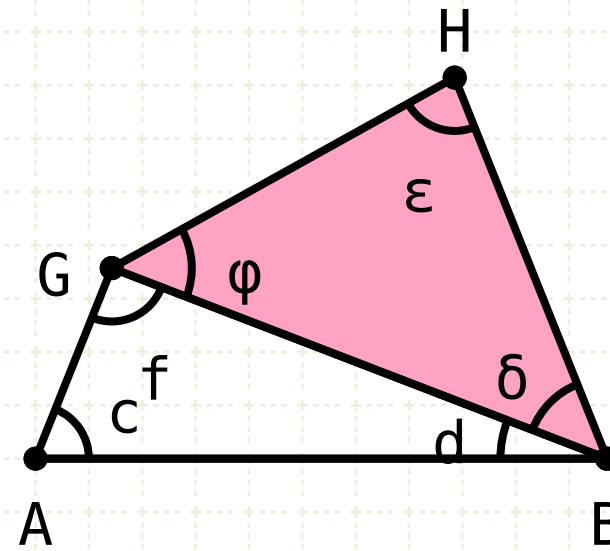
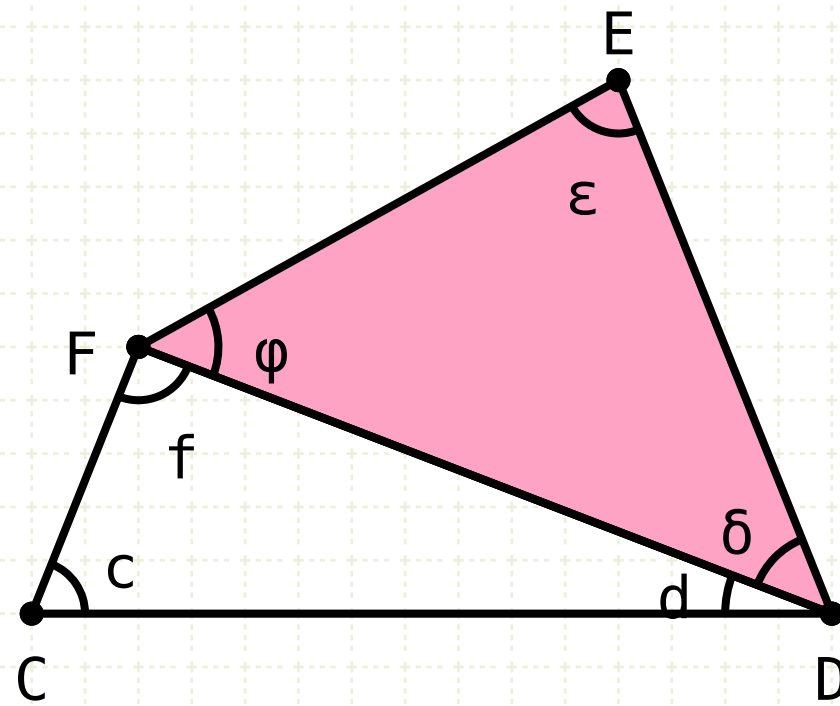
$$\angle FDE = \angle GBH = \delta$$

$$\angle CFD = \angle AGB = f$$

$$FD:GB = FC:GA = CD:AB$$

Proposition 18 of Book VI

On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure.



Proof

Compare triangles FCD and GAB, the remaining angle AGB is equal to the angle CFD (I·32), thus the two triangles are equiangular

Thus the sides of FCD and GAB are proportional (VI·4)

Compare the triangles FED and GHB, the remaining angle GHB is equal to the angle FED (I·32), thus the two triangles are equiangular

$$\angle FCD = \angle GAB = c$$

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$$\angle DFE = \angle BGH = \varphi$$

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$$\angle CFD = \angle AGB = f$$

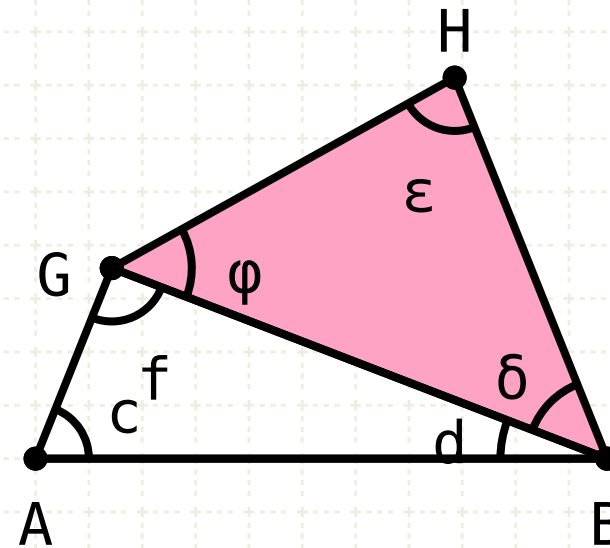
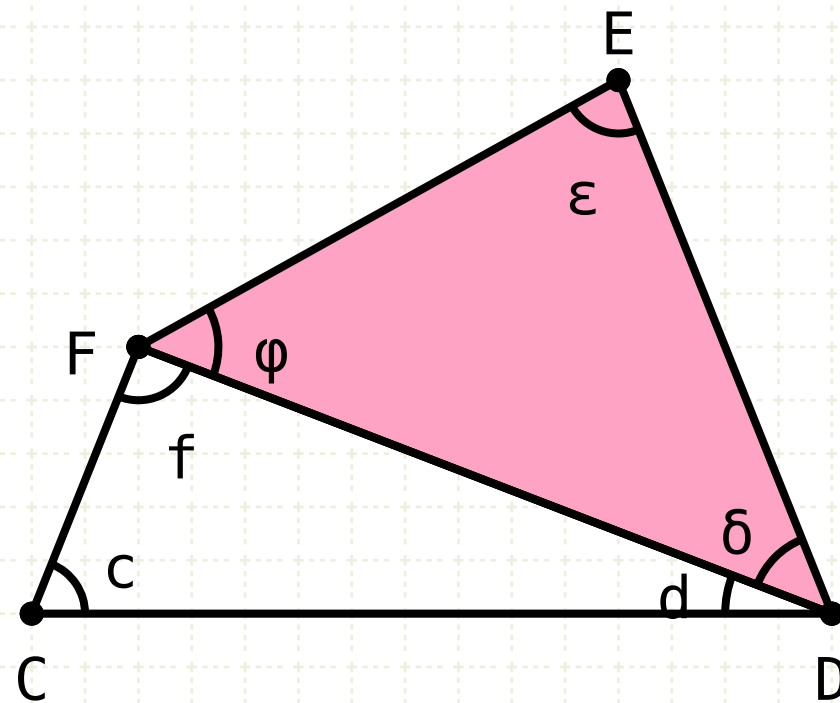
$$FD : GB = FC : GA = CD : AB$$

$$\angle GHB = \angle FED = \varepsilon$$



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$$\angle CFD = \angle AGB = f$$

$$FD:GB = FC:GA = CD:AB$$

$$\angle GHB = \angle FED = \epsilon$$

$$FD:GB = FE:GH = ED:BH$$

Proof

Compare triangles FCD and GAB, the remaining angle AGB is equal to the angle CFD (I-32), thus the two triangles are equiangular

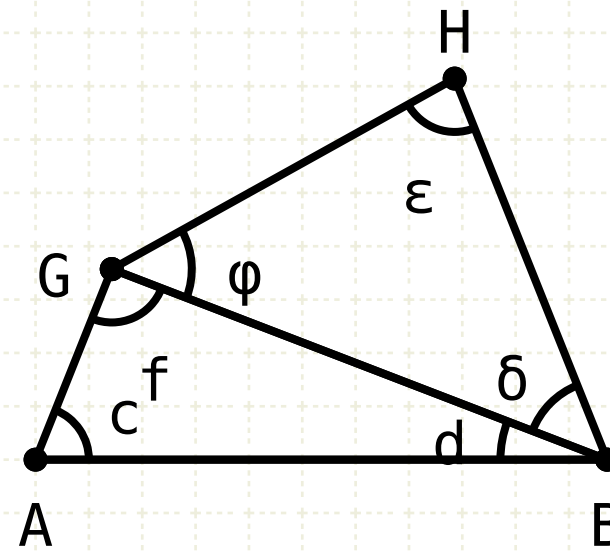
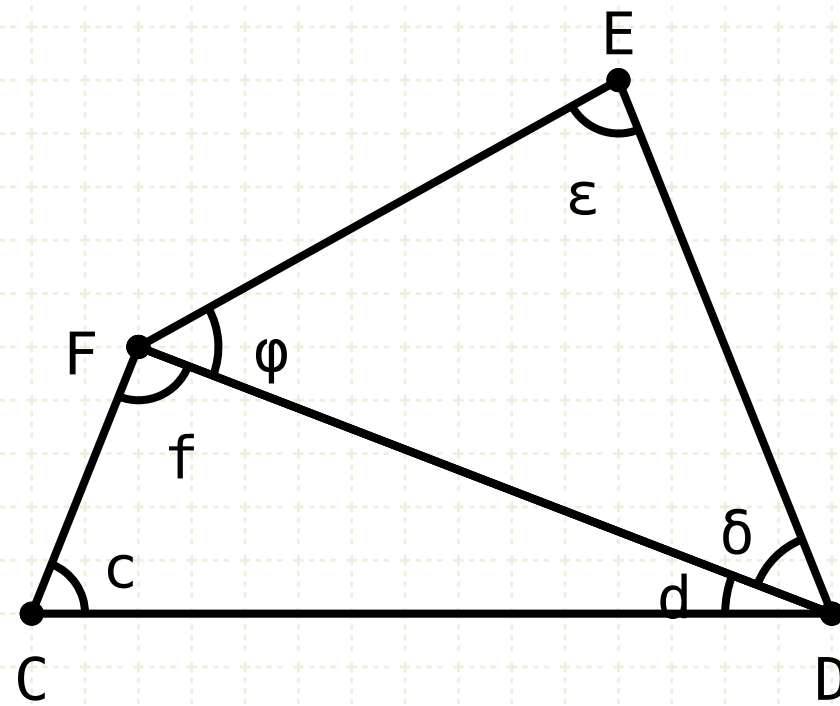
Thus the sides of FCD and GAB are proportional (VI-4)

Compare the triangles FED and GHB, the remaining angle GHB is equal to the angle FED (I-32), thus the two triangles are equiangular

Thus the sides of FED and GHB are proportional (VI-4)

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On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure.



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$$\begin{aligned}FD:GB &= FC:AG = CD:AB \\ &= FE:GH = ED:HB\end{aligned}$$

Proof

Compare triangles FCD and GAB, the remaining angle AGB is equal to the angle CFD (I-32), thus the two triangles are equiangular

Thus the sides of FCD and GAB are proportional (VI-4)

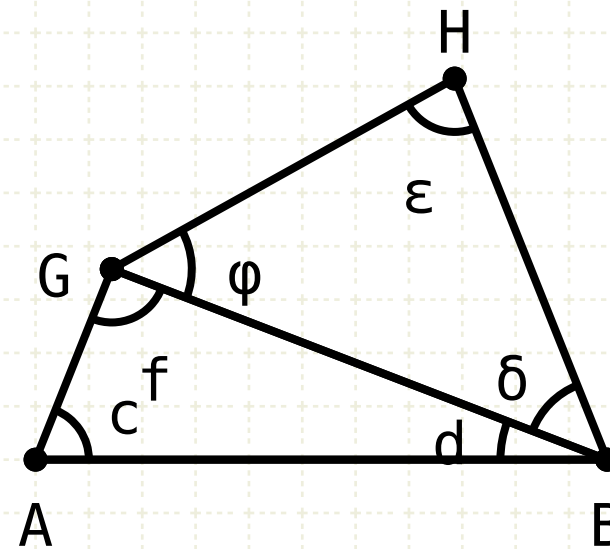
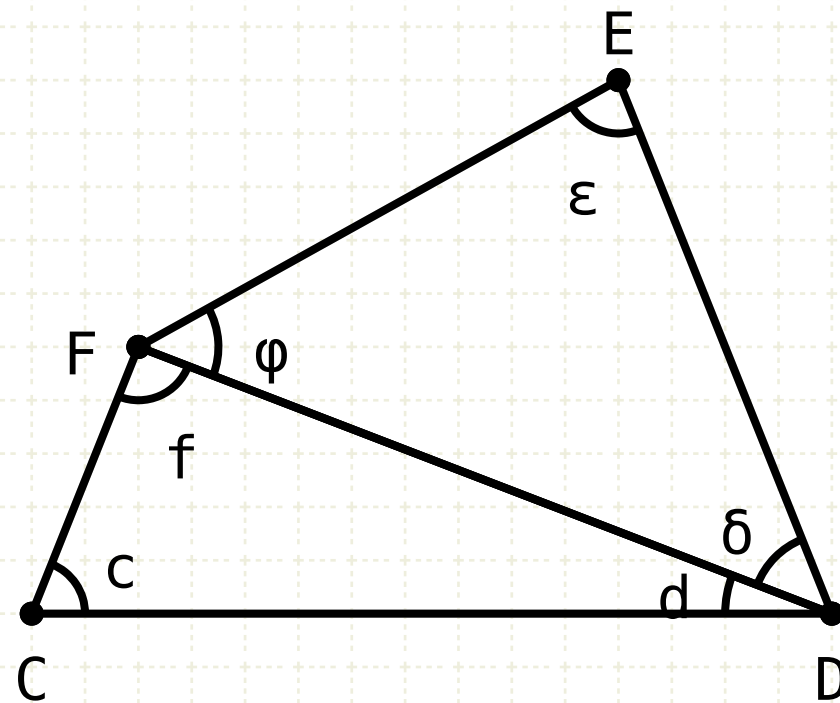
Compare the triangles FED and GHB, the remaining angle GHB is equal to the angle FED (I-32), thus the two triangles are equiangular

Thus the sides of FED and GHB are proportional (VI-4)

The ratio of FD to GB is simultaneously equal to the ratios of the sides FC,AG, CD,AB, FE,GH and ED,HB

Proposition 18 of Book VI

On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure.



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$$\begin{aligned}\angle CFD &= \angle AGB = f \\ FD:GB &= FC:GA = CD:AB \\ \angle GHB &= \angle FED = \varepsilon \\ FD:GB &= FE:GH = ED:BH\end{aligned}$$

$$\begin{aligned}FD:GB &= FC:AG = CD:AB \\ &= FE:GH = ED:HB \\ \angle AGH &= \angle CFE = f + \varphi\end{aligned}$$

Proof

Compare triangles FCD and GAB, the remaining angle AGB is equal to the angle CFD (I·32), thus the two triangles are equiangular

Thus the sides of FCD and GAB are proportional (VI·4)

Compare the triangles FED and GHB, the remaining angle GHB is equal to the angle FED (I·32), thus the two triangles are equiangular

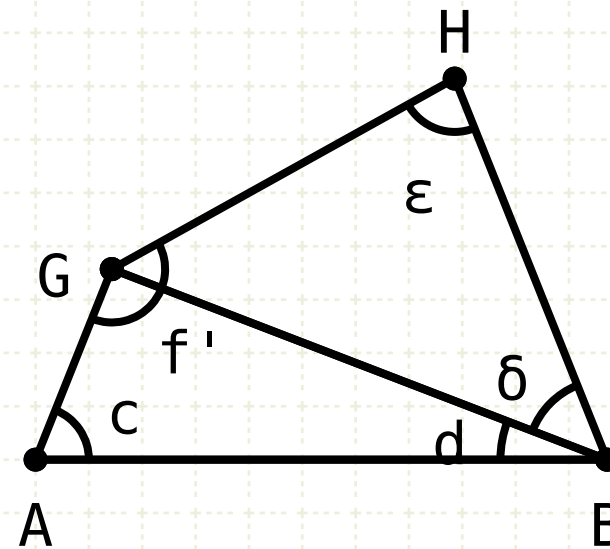
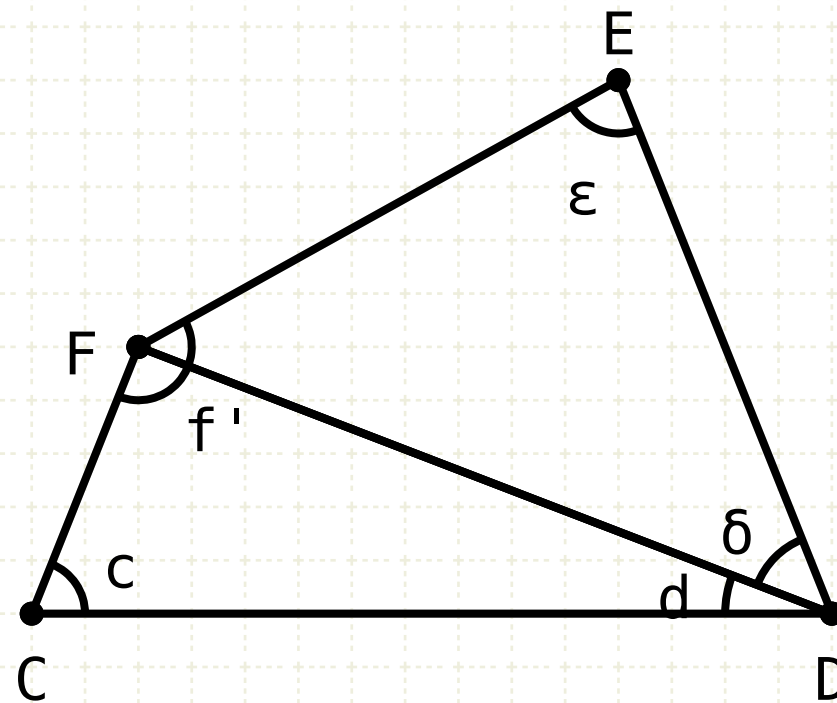
Thus the sides of FED and GHB are proportional (VI·4)

The ratio of FD to GB is simultaneously equal to the ratios of the sides FC,AG, CD,AB, FE,GH and ED,HB

The angles at AGB,BGH are equal to the angles at CFD,DFE, so their sums are also equal, therefore angle CFE equals AGH

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On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure.



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$$FD:GB = FC:GA = CD:AB$$

$$\angle GHB = \angle FED = \varepsilon$$

$$FD:GB = FE:GH = ED:HB$$

$$FD:GB = FC:AG = CD:AB$$

$$= FE:GH = ED:HB$$

$$\angle AGH = \angle CFE = f + \varphi = f'$$

Proof

Compare triangles FCD and GAB, the remaining angle AGB is equal to the angle CFD (I·32), thus the two triangles are equiangular

Thus the sides of FCD and GAB are proportional (VI·4)

Compare the triangles FED and GHB, the remaining angle GHB is equal to the angle FED (I·32), thus the two triangles are equiangular

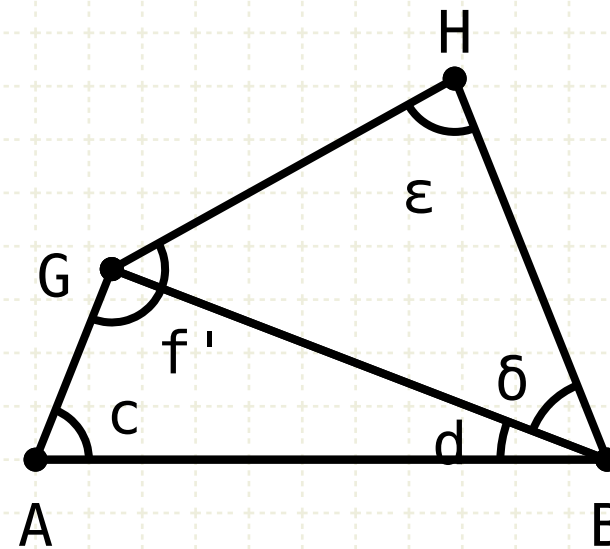
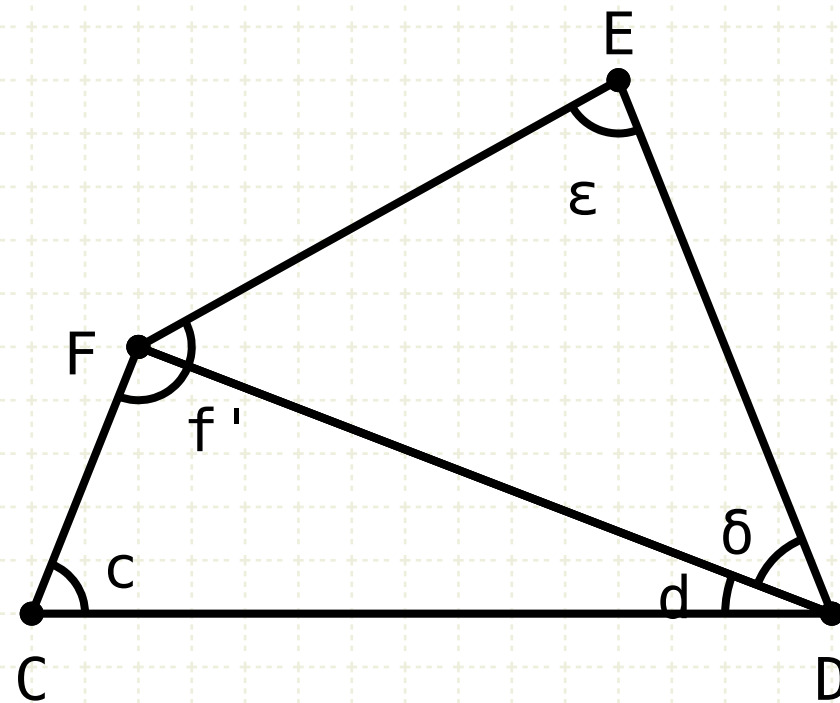
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Compare triangles FCD and GAB, the remaining angle AGB is equal to the angle CFD (I-32), thus the two triangles are equiangular

Thus the sides of FCD and GAB are proportional (VI-4)

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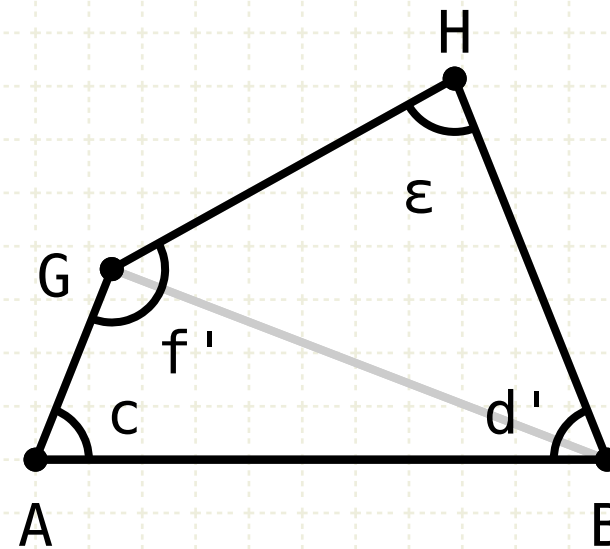
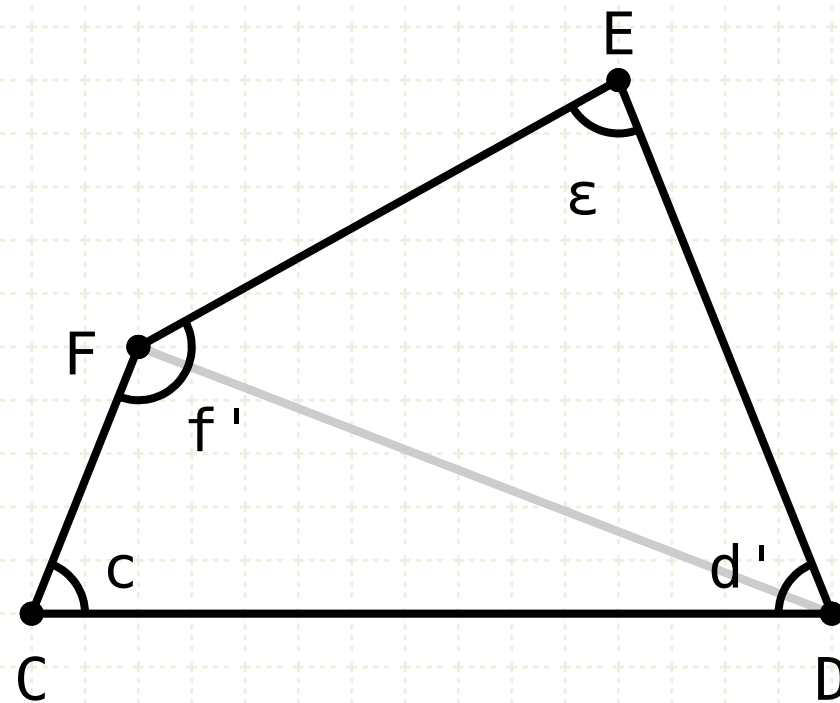
The ratio of FD to GB is simultaneously equal to the ratios of the sides FC,AG, CD,AB, FE,GH and ED,HB

The angles at AGB,BGH are equal to the angles at CFD,DFE, so their sums are also equal, therefore angle CFE equals AGH

The angles at ABG,GBH are equal to the angles at CDF,FDE, so their sums are also equal, therefore angle ABH equals CDE

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Proof

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Thus the sides of FCD and GAB are proportional (VI-4)

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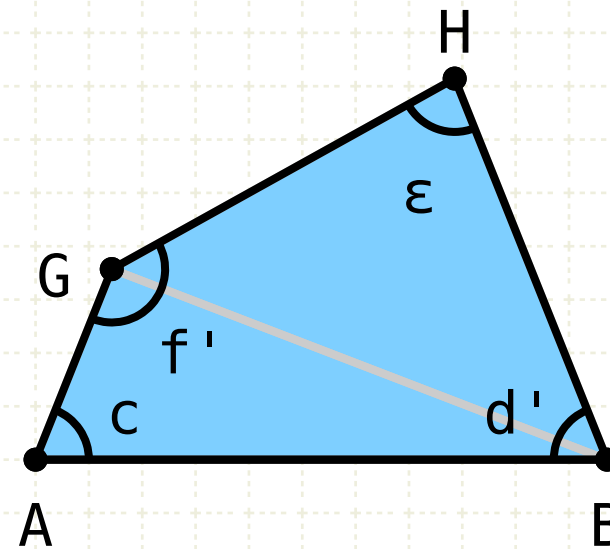
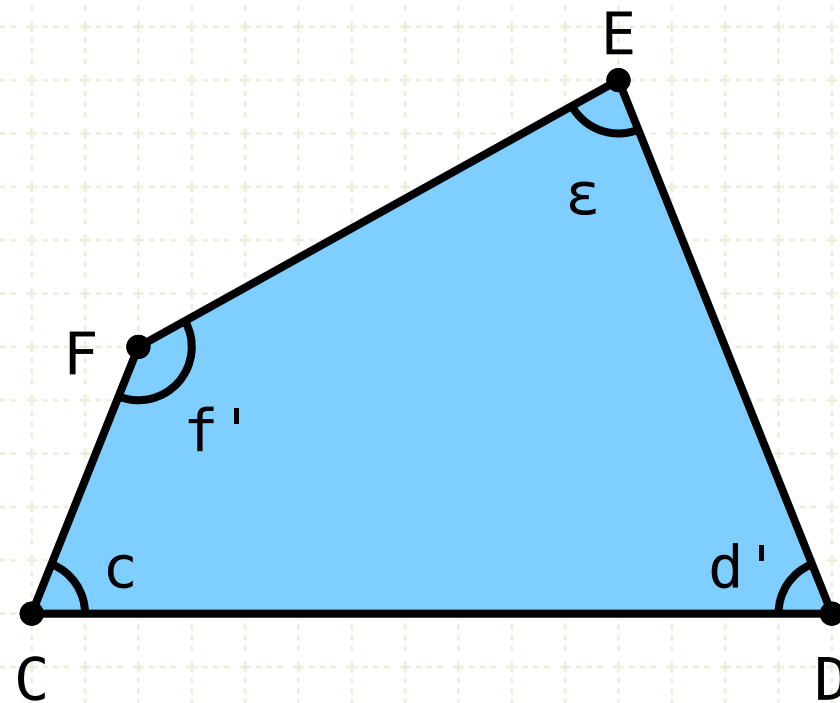
The angles at AGB,BGH are equal to the angles at CFD,DFE, so their sums are also equal, therefore angle CFE equals AGH

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Proof

Compare triangles FCD and GAB, the remaining angle AGB is equal to the angle CFD (I·32), thus the two triangles are equiangular

Thus the sides of FCD and GAB are proportional (VI·4)

Compare the triangles FED and GHB, the remaining angle GHB is equal to the angle FED (I·32), thus the two triangles are equiangular

Thus the sides of FED and GHB are proportional (VI·4)

The ratio of FD to GB is simultaneously equal to the ratios of the sides FC,AG, CD,AB, FE,GH and ED,HB

The angles at AGB,BGH are equal to the angles at CFD,DFE, so their sums are also equal, therefore angle CFE equals AGH

The angles at ABG,GBH are equal to the angles at CDF,FDE, so their sums are also equal, therefore angle ABH equals CDE

If the angles are all equal, and the sides about the equal angles are proportional, then the two figures are similar (VI.Def.1)

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