

Euclid's Elements

Book I

*If Euclid did not kindle your youthful enthusiasm, you
were not born to be a scientific thinker.*

Albert Einstein

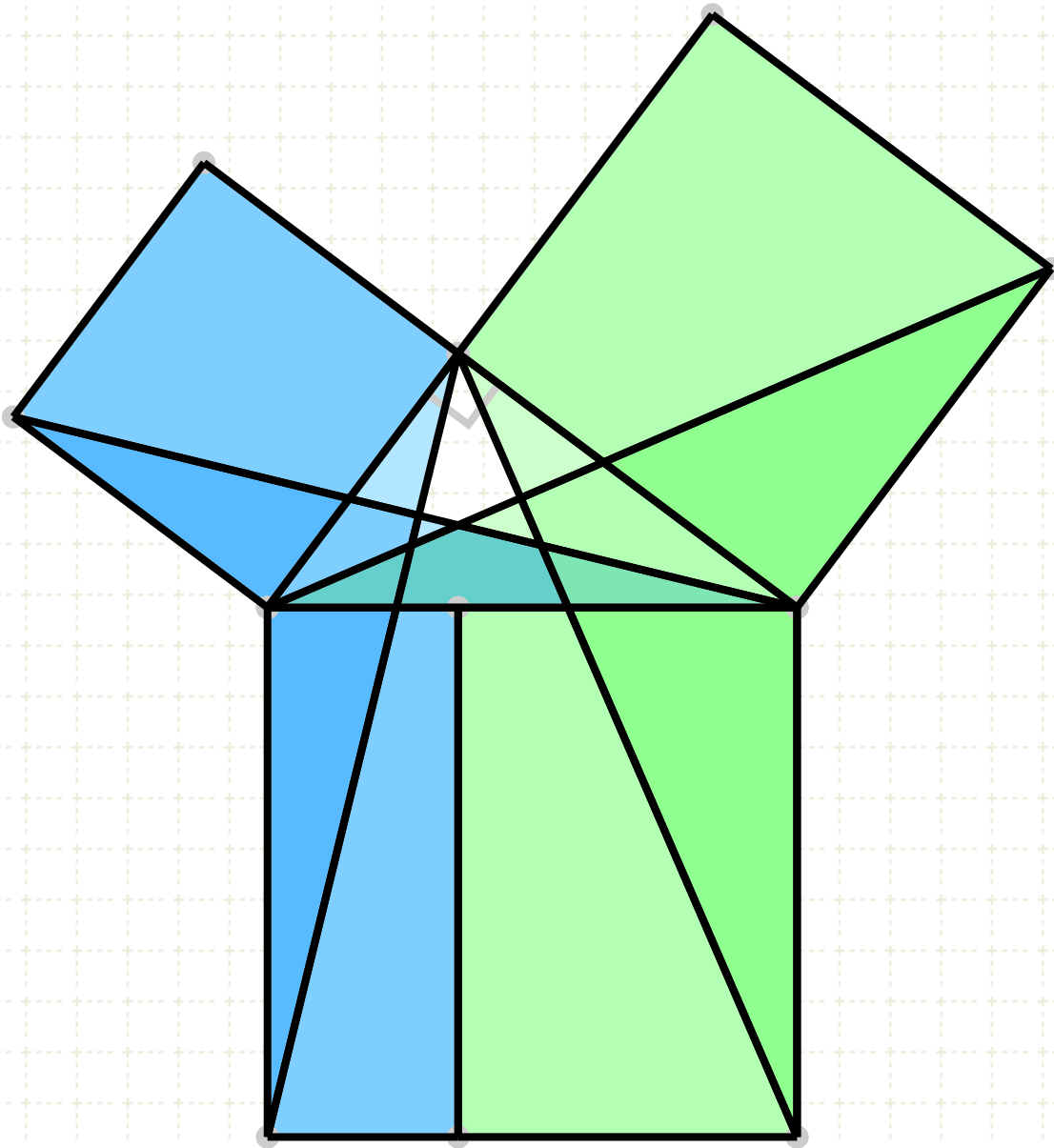


Table of Contents, Chapter 1

1	Construct an equilateral triangle	15	Vertical angles equal one another	29	Lines parallel, alternate angles are equal
2	Copy a line	16	Exterior angle larger than interior angle	30	Lines parallel to same line are parallel to themselves
3	Subtract one line from another	17	Sum of two interior angles less than 180	31	Construct one line parallel to another
4	Equal triangles if equal side-angle-side	18	Greater side opposite of greater angle	32	Sum of interior angles of a triangle = 180
5	Isosceles triangle gives equal base angles	19	Greater angle opposite of greater side	33	Lines joining ends of equal parallels are parallel
6	Equal base angles gives isosceles triangle	20	Sum of two angles greater than third	34	Opposite sides-angles equal in parallelogram
7	Two sides of triangle meet at unique point	21	Triangle within triangle has smaller sides	35	Parallelograms, same base-height have equal area
8	Equal triangles if equal side-side-side	22	Construct triangle from given lines	36	Parallelograms, equal base-height have equal area
9	How to bisect an angle	23	Copy an angle	37	Triangles, same base-height have equal area
10	Bisect a line	24	Larger angle gives larger base	38	Triangles, equal base-height have equal area
11	Construct right angle, point on line	25	Larger base gives larger angle		
12	Construct perpendicular, point to line	26	Equal triangles if equal angle-side-angle		
13	Sum of angles on straight line = 180	27	Alternate angles equal then lines parallel		
14	Two lines form a single line if angle = 180	28	Sum of interior angles = 180 , lines parallel		



Table of Contents, Chapter 1

39	Equal triangles on same base, have equal height
40	Equal triangles on equal base, have equal height
41	Triangle is half parallelogram with same base and height
42	Construct parallelogram with equal area as triangle
43	Parallelogram complements are equal
44	Construct parallelogram on line, equal to triangle
45	Construct parallelogram equal to polygon
46	Construct a square
47	Pythagoras' theorem
48	Inverse Pythagoras' theorem



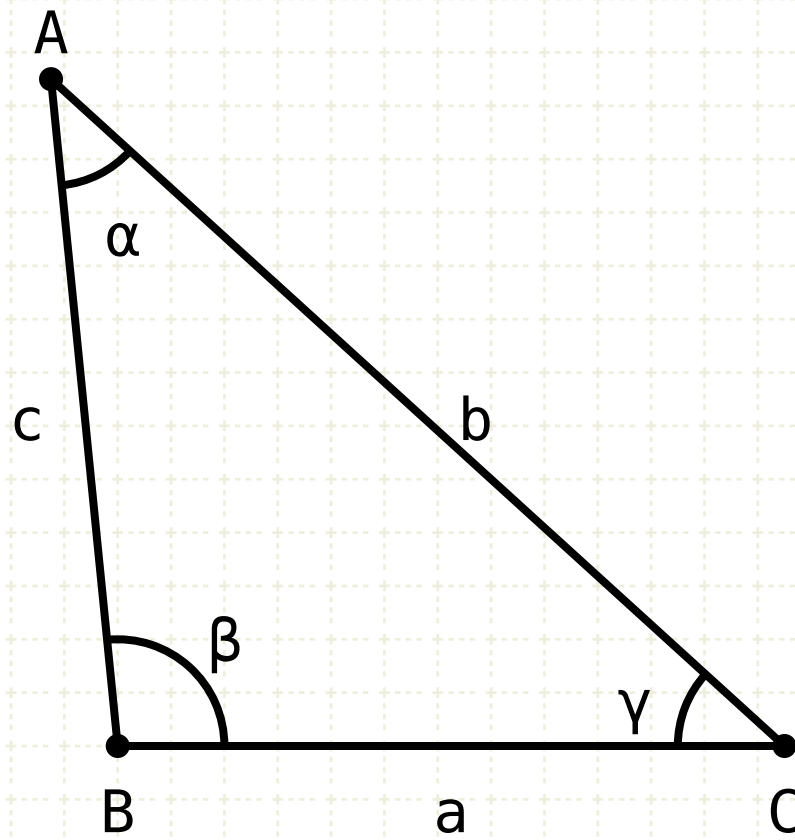
Proposition 19 of Book I

A greater angle of a triangle is opposite a greater side.



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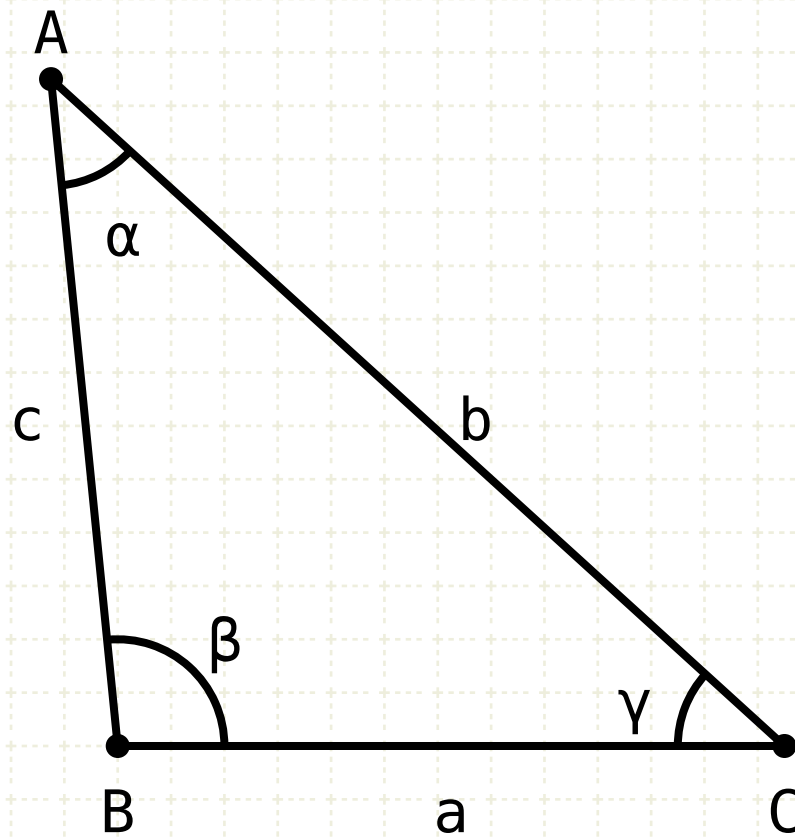
In other words

Given a triangle ABC

$$\beta > \alpha \quad \beta > \gamma$$

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A greater angle of a triangle is opposite a greater side.



$$\begin{array}{cc} \beta > \alpha & \beta > \gamma \\ b > a & b > c \end{array}$$

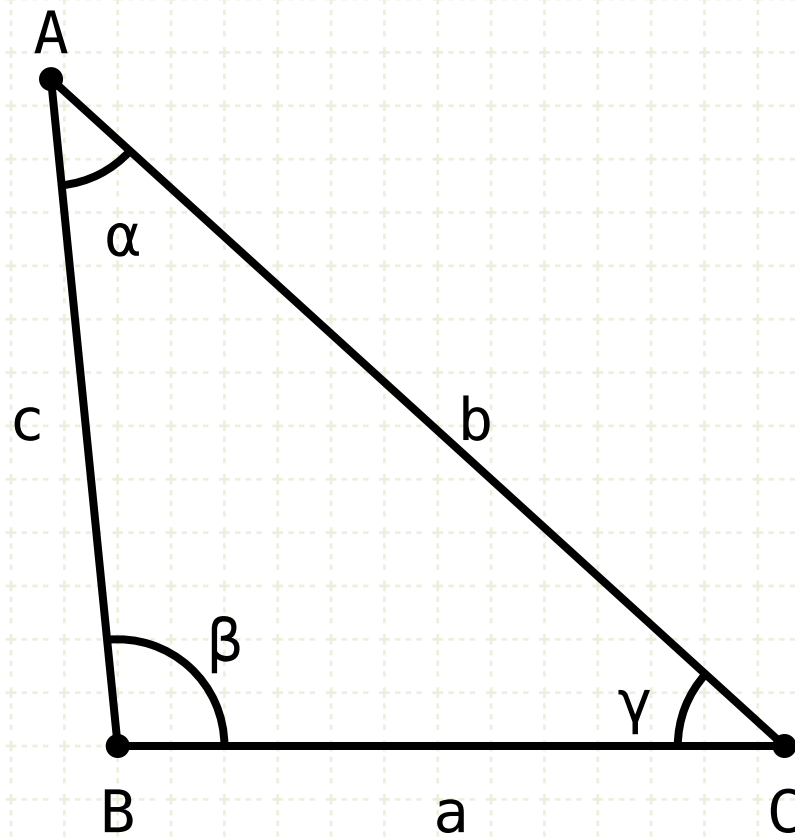
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Given a triangle ABC

If angle ABC is greater than angle BCA and CAB, then the side AC is greater than the other two sides of the triangle

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$$AC \leq AB$$

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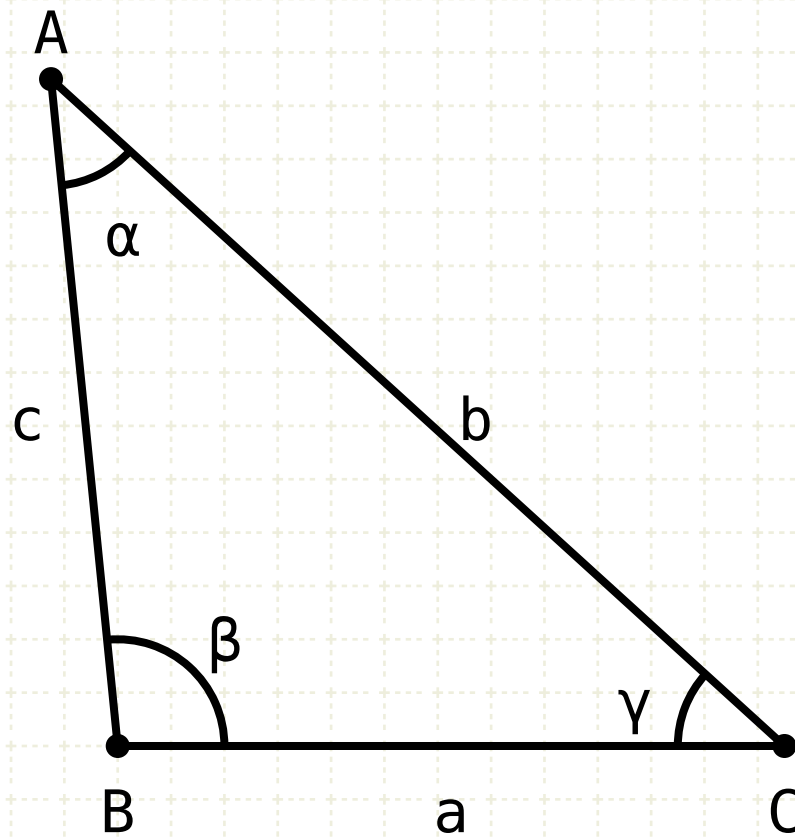
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If AC is not greater than AB, then it must be less than or equal to AB

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If line AC equals AB, then the triangle would be an isosceles triangle, where angle ABC equals angle ACB (I·5)

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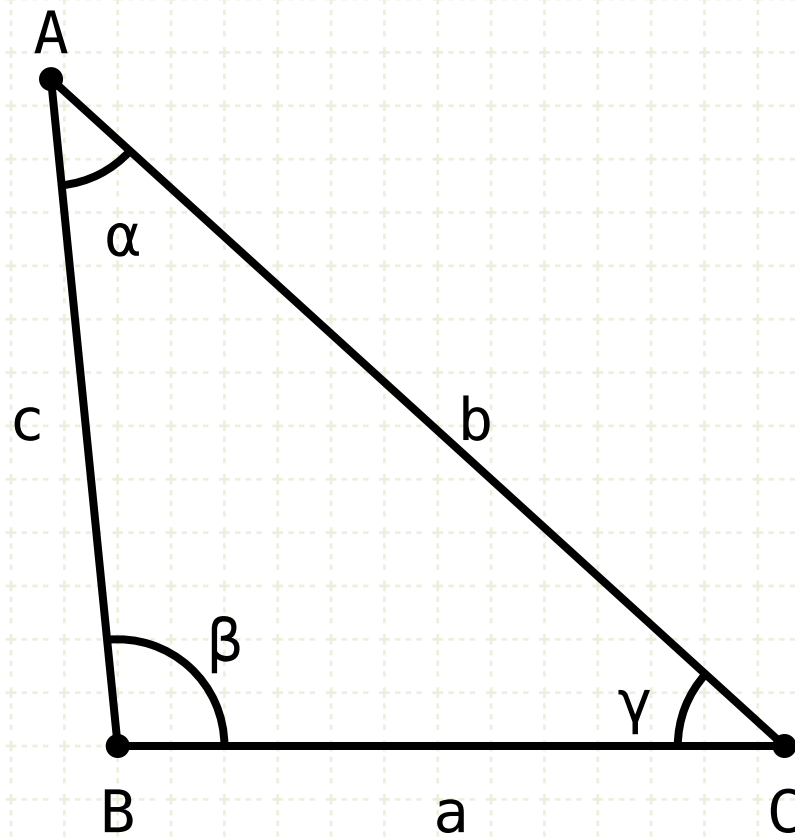
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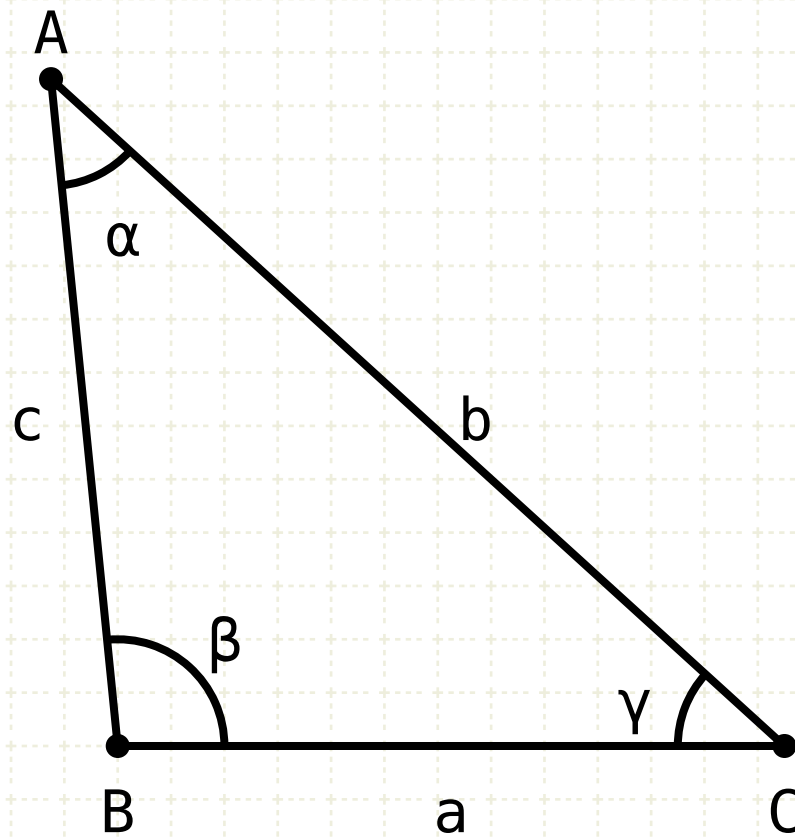
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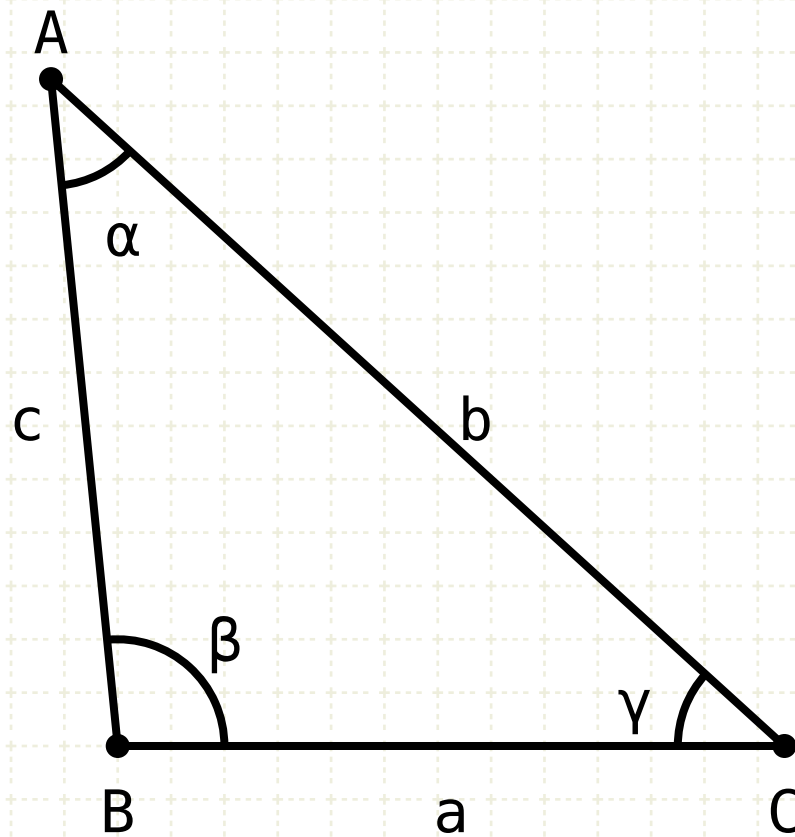
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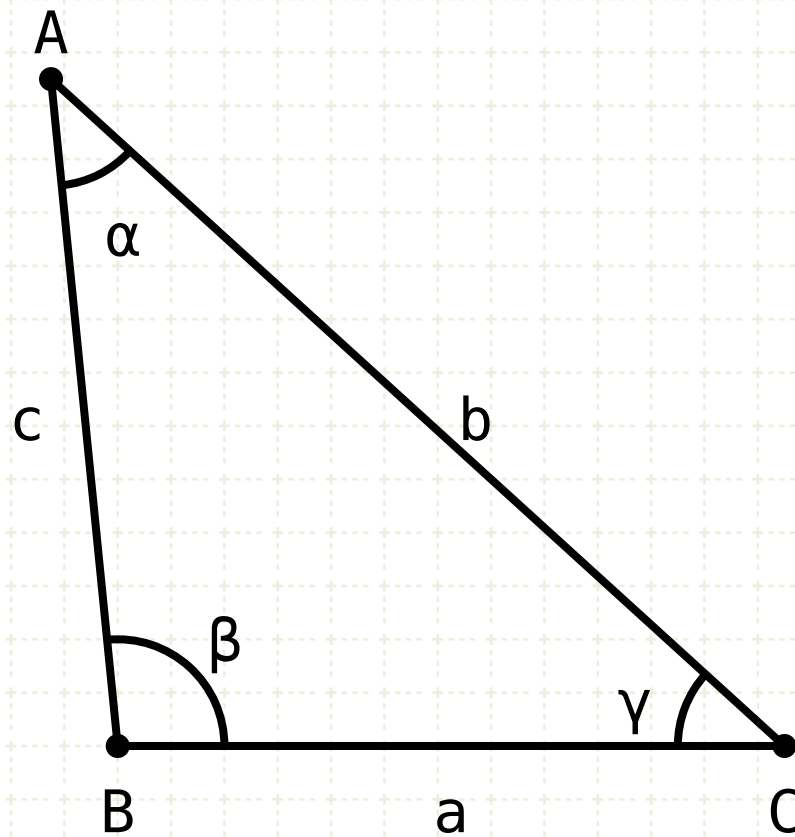
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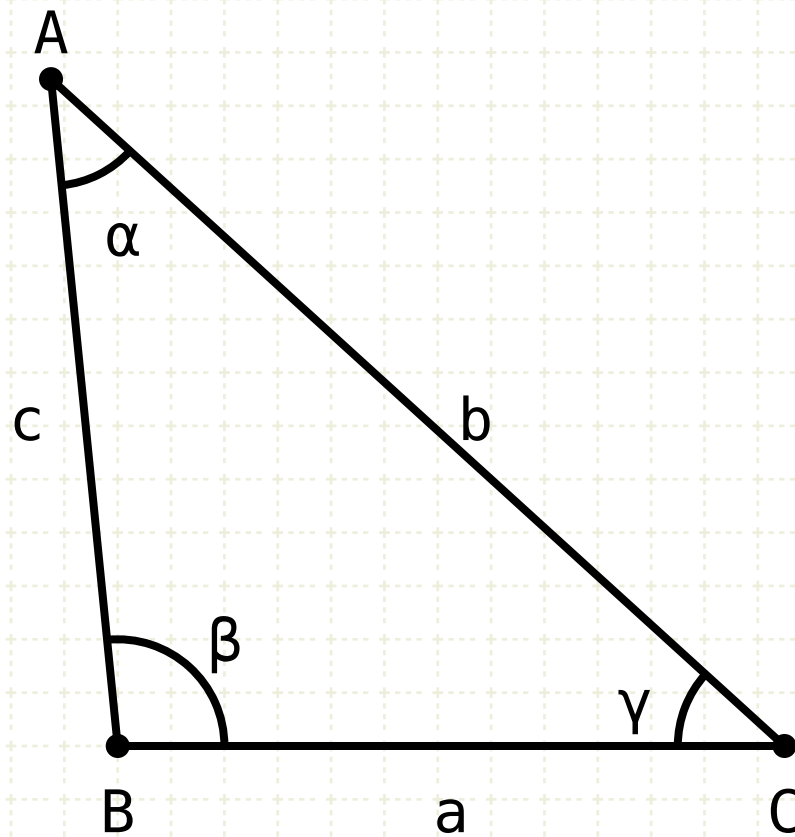
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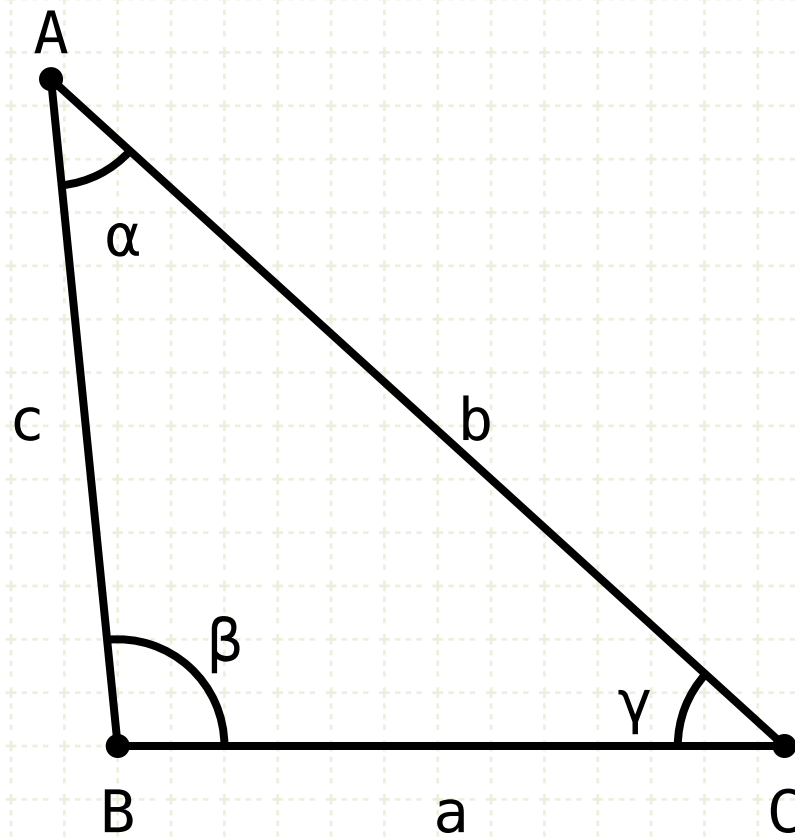
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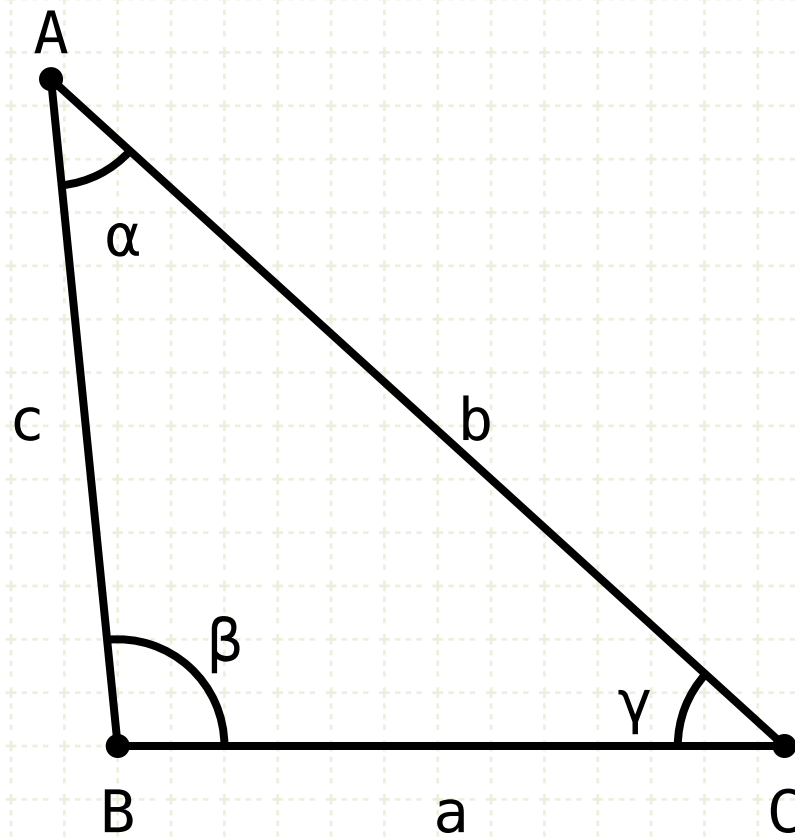
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