

Euclid's Elements

Book III



A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)



Table of Contents, Chapter 3

1	To find the centre of a circle	9	If three lines, starting at a point 'A' and touching the circle, are all equal, then 'A' is the centre of the circle	17	From a given point to draw a straight line touching a given circle
2	A chord of a circle always lies inside the circle	10	A circle does not cut a circle at more points than two	18	If line touches a circle, then it is perpendicular to the diameter that touches that point
3	A line through the centre of a circle bisects a chord, and vice versa	11	Point of contact between two internal circles, and their centres, are collinear	19	If line touches a circle, then the centre of the circle lies on a line perpendicular to the original
4	A line not through the centre of a circle does not bisect a chord	12	Point of contact between two external circles, and their centres, are collinear	20	The angle at the centre of a circle is twice that from an angle from the circumference
5	If two circles cut one another, they will not have the same center	13	A circle does not touch a circle at more points than one, whether it touch it internally or externally.	21	In a circle the angles in the same segment are equal to one another
6	If two circles touch one another, they will not have the same center	14	In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another.	22	The opposite angles of quadrilaterals in circles are equal to two right angles
7	Consider two lines from a point inside a circle to the edge, the longer one will be the one closest to the longest part of the diameter passing through the original point	15	The longest line in a circle is its diameter, shorter the farther away from the diameter	23	On the same straight line there cannot be constructed two similar and unequal segments of circles on the same side
8	Consider two lines from a point outside a circle to the edge, the line closest to the centre will be longer on the concave side and shorter on the convex side	16	A line on the circle, perpendicular to the diameter, lies outside the circle	24	Similar segments of circles on equal straight lines are equal to one another



Table of Contents, Chapter 3

- | | | | |
|----|---|----|--|
| 25 | Given a segment of a circle, to describe the complete circle of which it is a segment. | 34 | Construct a circle segment on a given circle, such that the angle within the segment is equal to a given angle |
| 26 | In equal circles equal angles stand on equal circumferences | 35 | If two circle chords intersect, the segments on one multiplied together equals the segments of the other multiplied together |
| 27 | In equal circles angles standing on equal circumferences are equal to one another | 36 | Secant-tangent law |
| 28 | In equal circles equal straight lines cut off equal circumferences | 37 | Converse of the secant-tangent law |
| 29 | In equal circles equal circumferences are subtended by equal straight lines | | |
| 30 | To bisect a given circumference | | |
| 31 | In a circle the angle in the semicircle is right ... | | |
| 32 | The angle between a tangent and a straight line cutting a circle is equal to the angle in the alternate segment | | |
| 33 | Construct a circle segment on a given line, such that the angle within the segment is equal to a given angle | | |



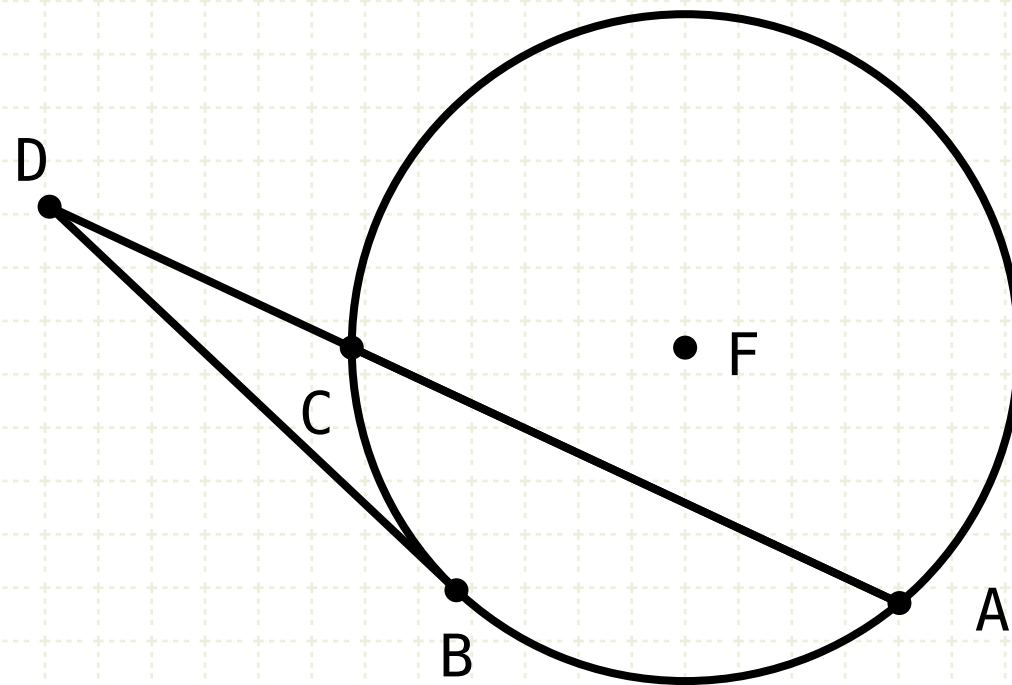
Proposition 37 of Book III

If a point be taken outside a circle and from the point there fall on the circle two straight lines, if one of them cut the circle, and the other fall on it, and if further the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference be equal to the square on the straight line which falls on the circle, and the straight line which falls on it will touch the circle.



Proposition 37 of Book III

If a point be taken outside a circle and from the point there fall on the circle two straight lines, if one of them cut the circle, and the other fall on it, and if further the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference be equal to the square on the straight line which falls on the circle, and the straight line which falls on it will touch the circle.



$$AD \cdot CD = BD^2$$

In other words

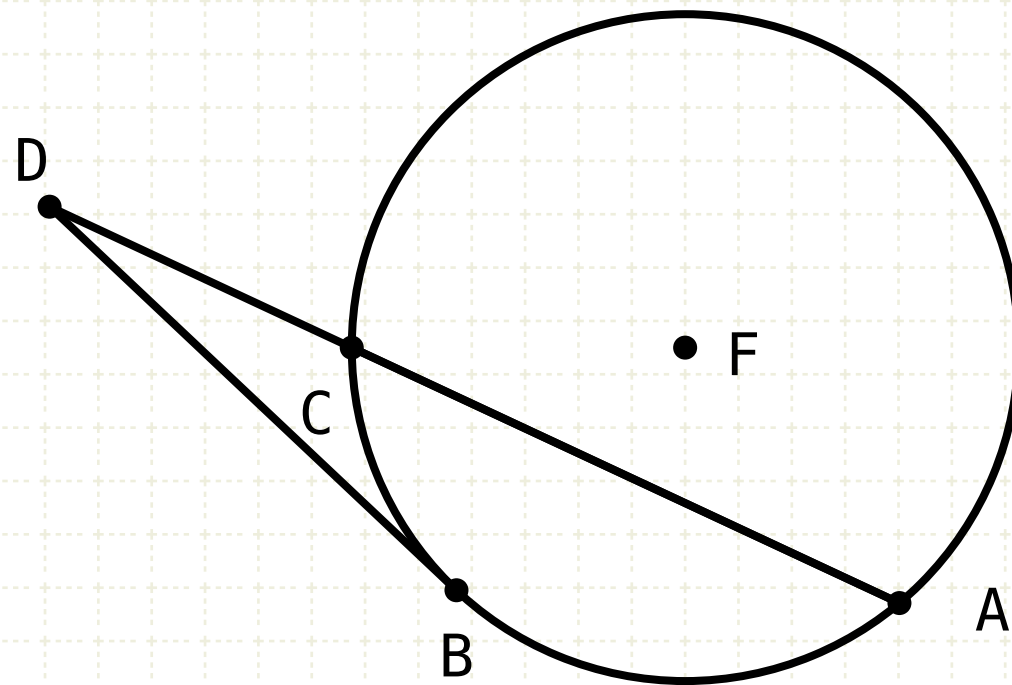
Let point D be outside of the circle

Let a line DA cut the circle at C and A, and let line DB fall on the circle

If the product AD,CD equals BD squared, then DB touches the circle

Proposition 37 of Book III

If a point be taken outside a circle and from the point there fall on the circle two straight lines, if one of them cut the circle, and the other fall on it, and if further the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference be equal to the square on the straight line which falls on the circle, and the straight line which falls on it will touch the circle.



$$AD \cdot CD = BD^2$$

In other words

Let point D be outside of the circle

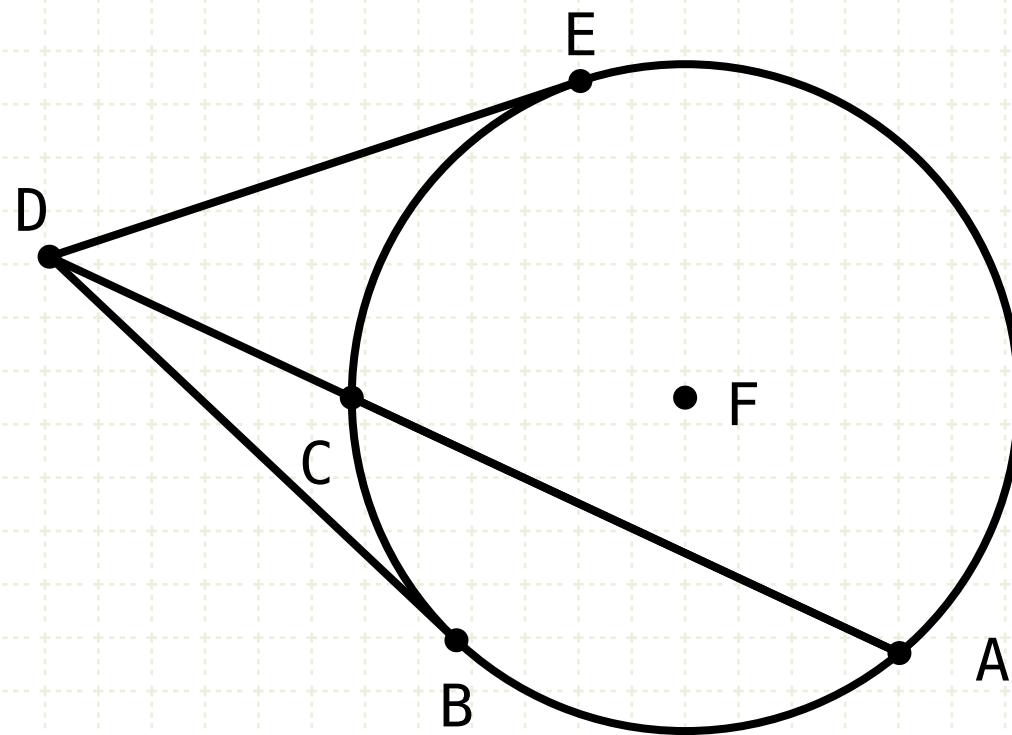
Let a line DA cut the circle at C and A, and let line DB fall on the circle

If the product AD,CD equals BD squared, then DB touches the circle

Proof

Proposition 37 of Book III

If a point be taken outside a circle and from the point there fall on the circle two straight lines, if one of them cut the circle, and the other fall on it, and if further the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference be equal to the square on the straight line which falls on the circle, and the straight line which falls on it will touch the circle.



$$AD \cdot CD = BD^2$$

In other words

Let point D be outside of the circle

Let a line DA cut the circle at C and A, and let line DB fall on the circle

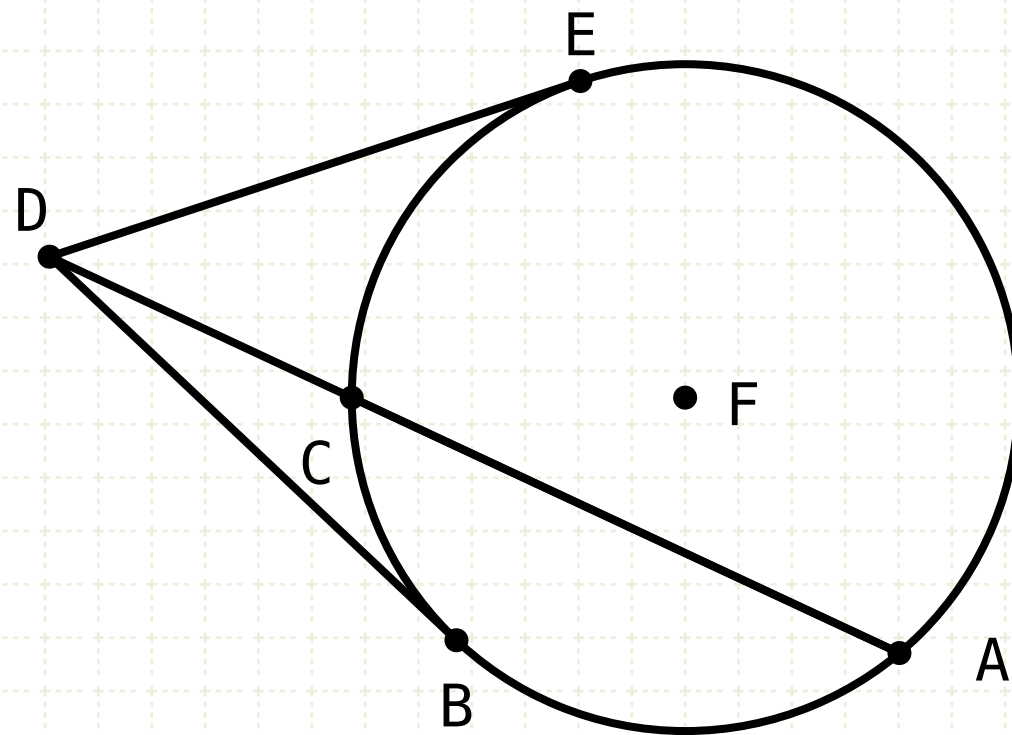
If the product AD,CD equals BD squared, then DB touches the circle

Proof

Draw DE such that it touches the circle (III·17)

Proposition 37 of Book III

If a point be taken outside a circle and from the point there fall on the circle two straight lines, if one of them cut the circle, and the other fall on it, and if further the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference be equal to the square on the straight line which falls on the circle, and the straight line which falls on it will touch the circle.



$$AD \cdot CD = BD^2$$

$$AD \cdot CD = DE^2$$

$$BD = DE$$

In other words

Let point D be outside of the circle

Let a line DA cut the circle at C and A, and let line DB fall on the circle

If the product AD,CD equals BD squared, then DB touches the circle

Proof

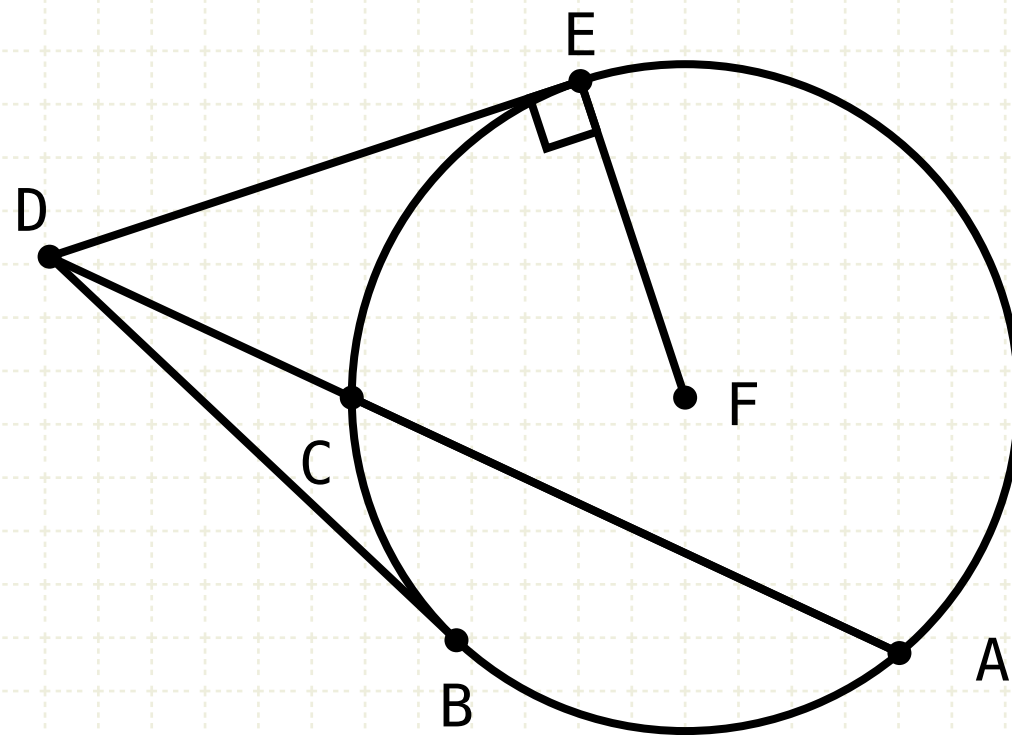
Draw DE such that it touches the circle (III·17)

Since DE touches the circle, the product AD,CD equals DE squared (III·36)

Therefore DE equals BD

Proposition 37 of Book III

If a point be taken outside a circle and from the point there fall on the circle two straight lines, if one of them cut the circle, and the other fall on it, and if further the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference be equal to the square on the straight line which falls on the circle, and the straight line which falls on it will touch the circle.



$$AD \cdot CD = BD^2$$

$$AD \cdot CD = DE^2$$

$$BD = DE$$

In other words

Let point D be outside of the circle

Let a line DA cut the circle at C and A, and let line DB fall on the circle

If the product AD,CD equals BD squared, then DB touches the circle

Proof

Draw DE such that it touches the circle (III·17)

Since DE touches the circle, the product AD,CD equals DE squared (III·36)

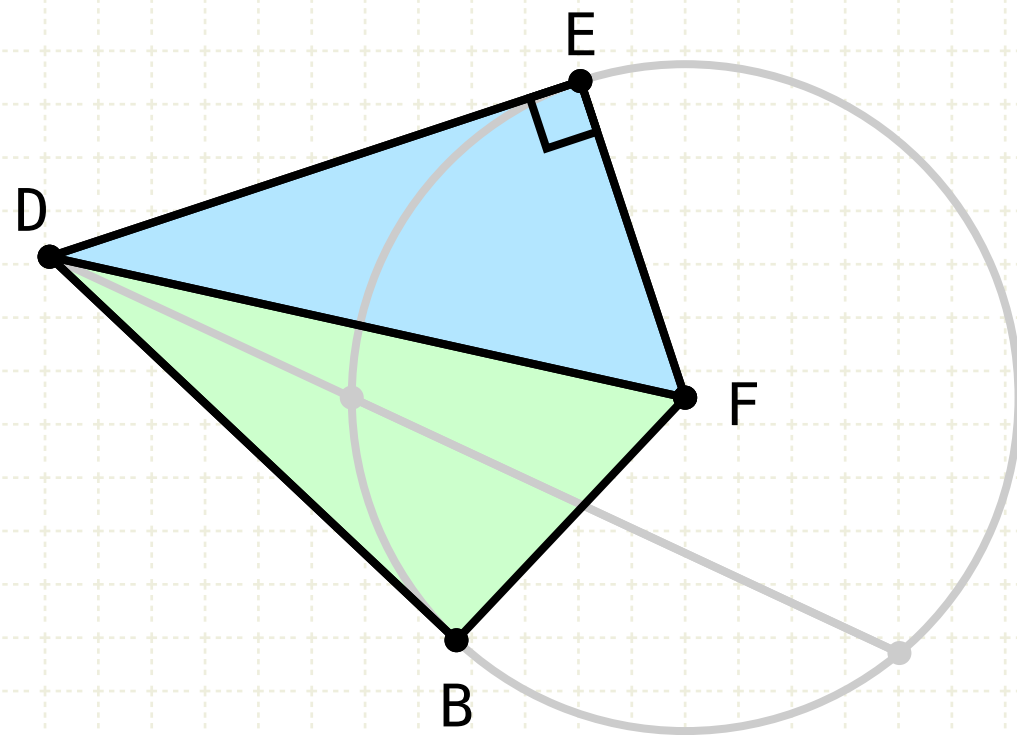
Therefore DE equals BD

Draw EF, where F is the centre of the circle

Angle FED is right (III·18)

Proposition 37 of Book III

If a point be taken outside a circle and from the point there fall on the circle two straight lines, if one of them cut the circle, and the other fall on it, and if further the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference be equal to the square on the straight line which falls on the circle, and the straight line which falls on it will touch the circle.



$$AD \cdot CD = BD^2$$

$$AD \cdot CD = DE^2$$

$$BD = DE$$

$$EF = BF$$

In other words

Let point D be outside of the circle

Let a line DA cut the circle at C and A, and let line DB fall on the circle

If the product AD,CD equals BD squared, then DB touches the circle

Proof

Draw DE such that it touches the circle (III·17)

Since DE touches the circle, the product AD,CD equals DE squared (III·36)

Therefore DE equals BD

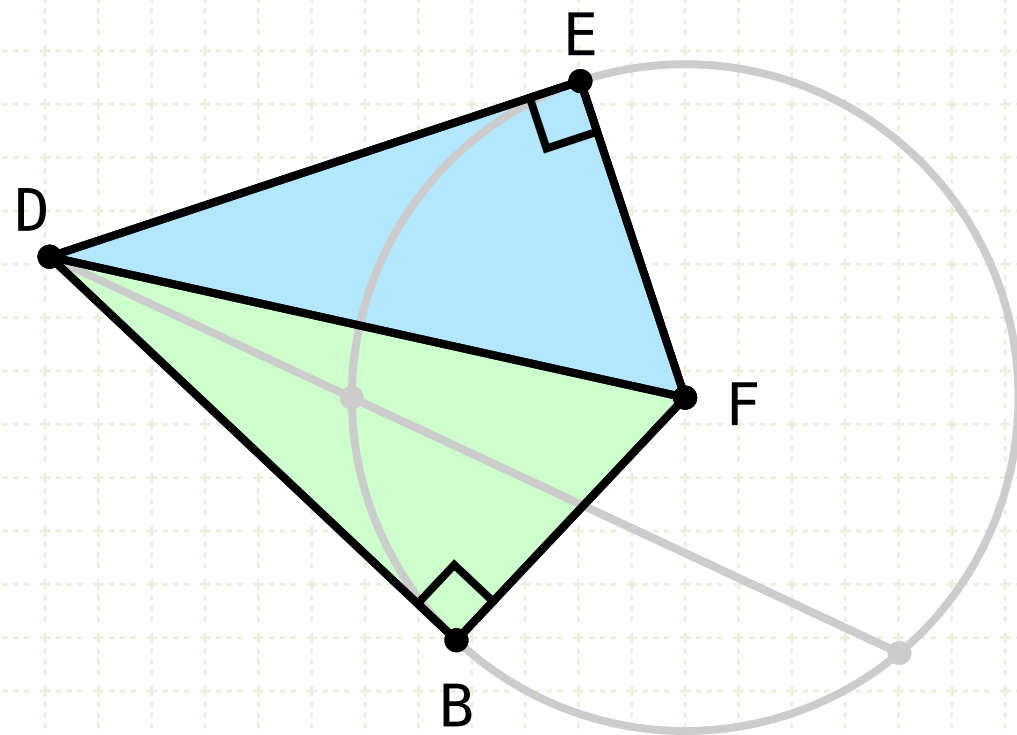
Draw EF, where F is the centre of the circle

Angle FED is right (III·18)

Compare the two triangles DEF and DBF

Proposition 37 of Book III

If a point be taken outside a circle and from the point there fall on the circle two straight lines, if one of them cut the circle, and the other fall on it, and if further the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference be equal to the square on the straight line which falls on the circle, and the straight line which falls on it will touch the circle.



$$AD \cdot CD = BD^2$$

$$AD \cdot CD = DE^2$$

$$BD = DE$$

$$EF = BF$$

In other words

Let point D be outside of the circle

Let a line DA cut the circle at C and A, and let line DB fall on the circle

If the product AD,CD equals BD squared, then DB touches the circle

Proof

Draw DE such that it touches the circle (III·17)

Since DE touches the circle, the product AD,CD equals DE squared (III·36)

Therefore DE equals BD

Draw EF, where F is the centre of the circle

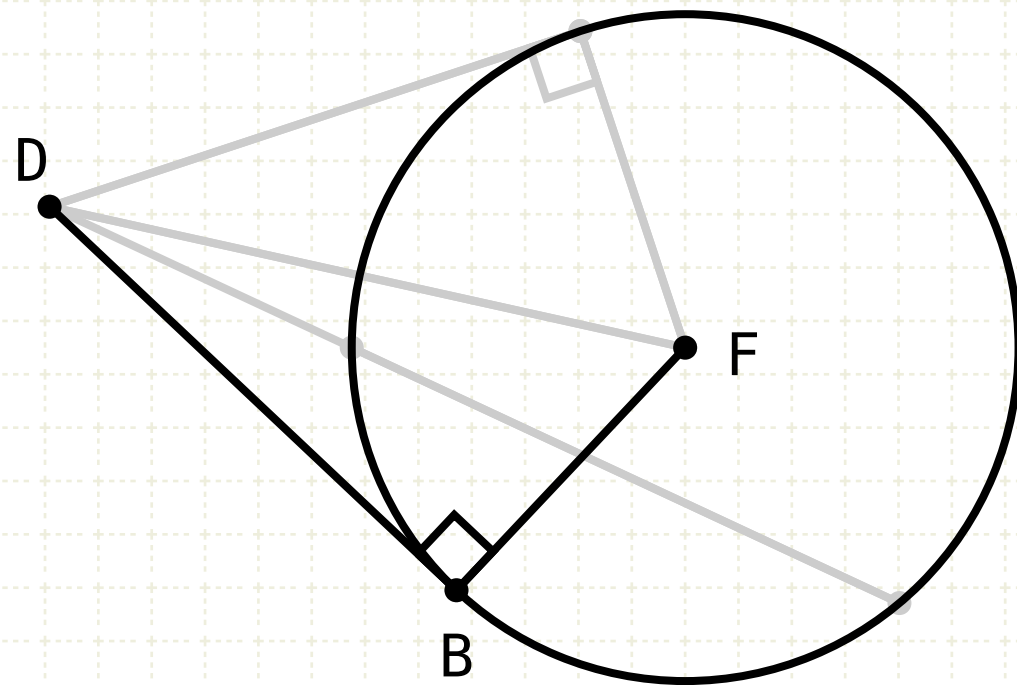
Angle FED is right (III·18)

Compare the two triangles DEF and DBF

Since all three sides of the triangle are equal (I·8), then angle FBD is also right

Proposition 37 of Book III

If a point be taken outside a circle and from the point there fall on the circle two straight lines, if one of them cut the circle, and the other fall on it, and if further the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference be equal to the square on the straight line which falls on the circle, and the straight line which falls on it will touch the circle.



$$AD \cdot CD = BD^2$$

$$AD \cdot CD = DE^2$$

$$BD = DE$$

$$EF = BF$$

In other words

Let point D be outside of the circle

Let a line DA cut the circle at C and A, and let line DB fall on the circle

If the product AD,CD equals BD squared, then DB touches the circle

Proof

Draw DE such that it touches the circle (III·17)

Since DE touches the circle, the product AD,CD equals DE squared (III·36)

Therefore DE equals BD

Draw EF, where F is the centre of the circle

Angle FED is right (III·18)

Compare the two triangles DEF and DBF

Since all three sides of the triangle are equal (I·8), then angle FBD is also right

If the angle FBD is right (and since B is at the extremity of the diameter), then BD touches the circle (III·16)

Youtube Videos

<https://www.youtube.com/c/SandyBultena>

Copyright © 2019 by Sandy Bultena.



Except where otherwise noted, this work is licensed under
<http://creativecommons.org/licenses/by-nc/3.0>