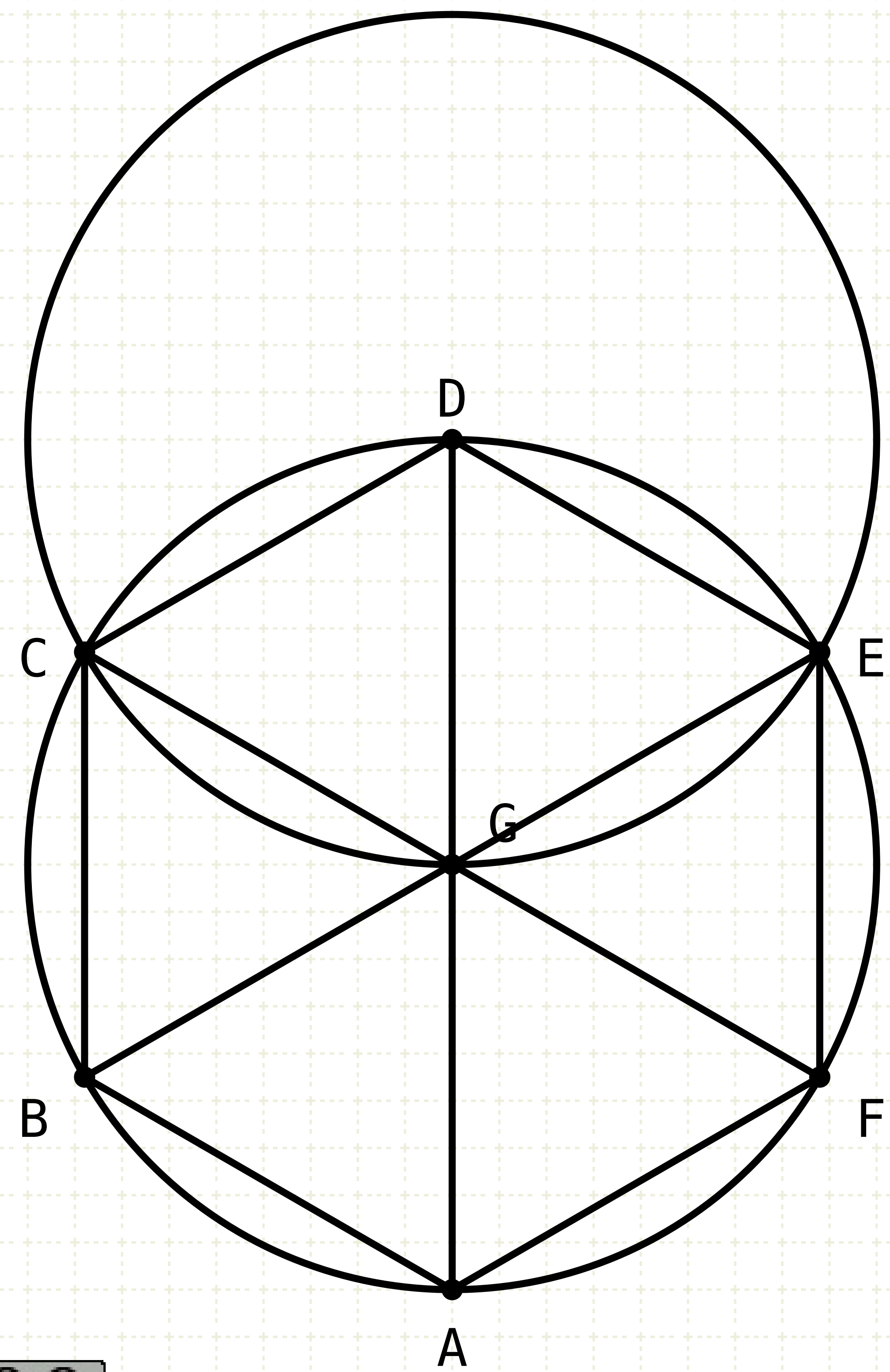
Euclid's Elements

Book IV



Philosophy (nature) is written in that great book which ever is before our eyes -- I mean the universe -- but we cannot understand it if we do not first learn the language and grasp the symbols in which it is written. The book is written in mathematical language, and the symbols are triangles, circles and other geometrical figures, without whose help it is impossible to comprehend a single word of it - without which one wanders in vain through a dark labyrinth.

Galileo Galilei

Proposition 10 of Book IV

To construct an isosceles triangle having each of the angles at the base double of the remaining one.

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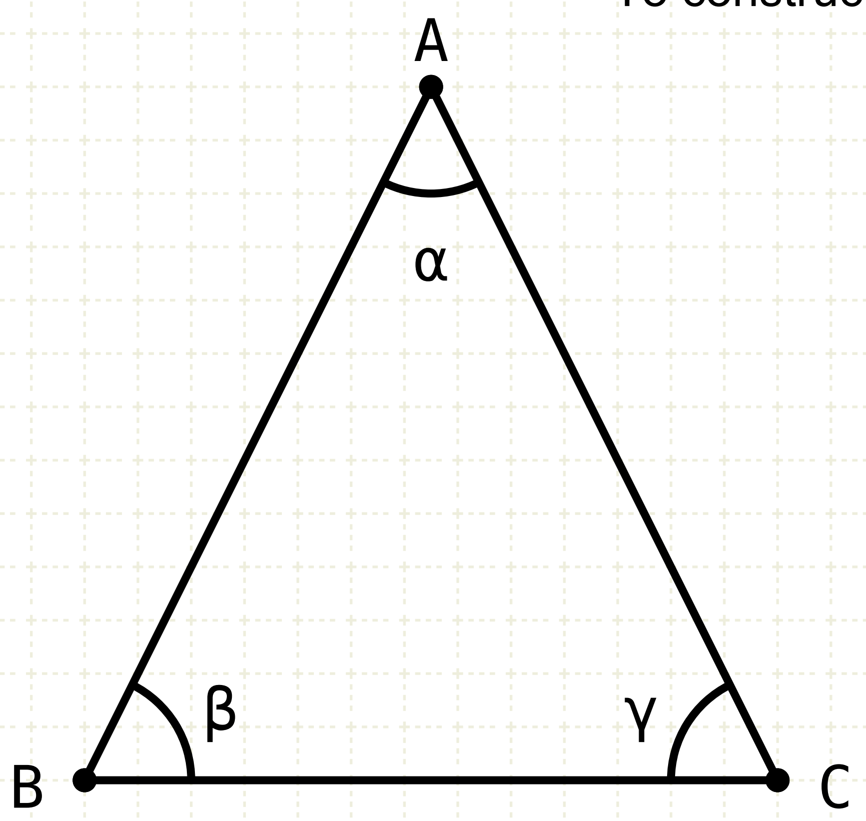


$\beta = \gamma = 2 \cdot \alpha$

In other words

Construct an isosceles triangle ABC such that the angle at A is half the angle at B and also half the angle at C

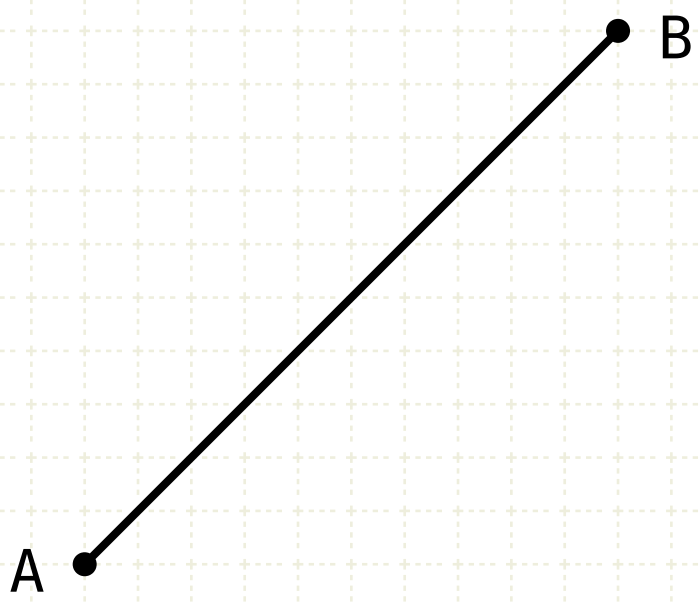
Proposition 10 of Book IV

To construct an isosceles triangle having each of the angles at the base double of the remaining one.

Construction

Proposition 10 of Book IV

To construct an isosceles triangle having each of the angles at the base double of the remaining one.

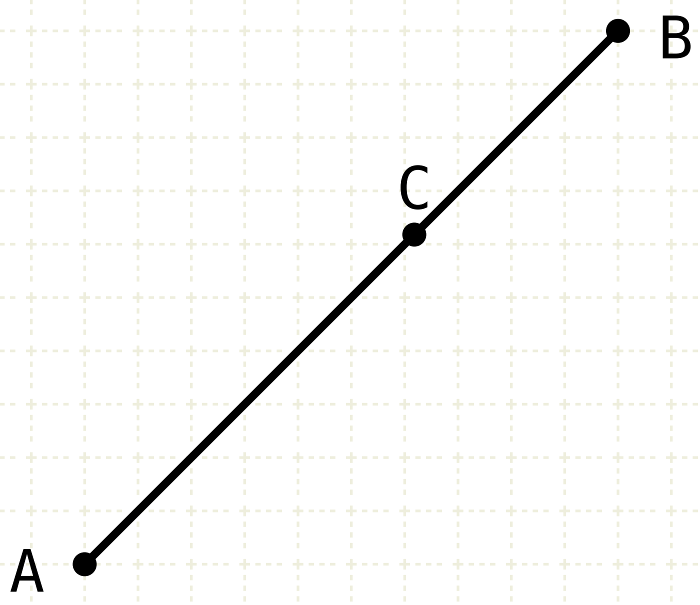


Construction

Start with any line AB

Proposition 10 of Book IV

To construct an isosceles triangle having each of the angles at the base double of the remaining one.



$AB \cdot BC = AC^2$

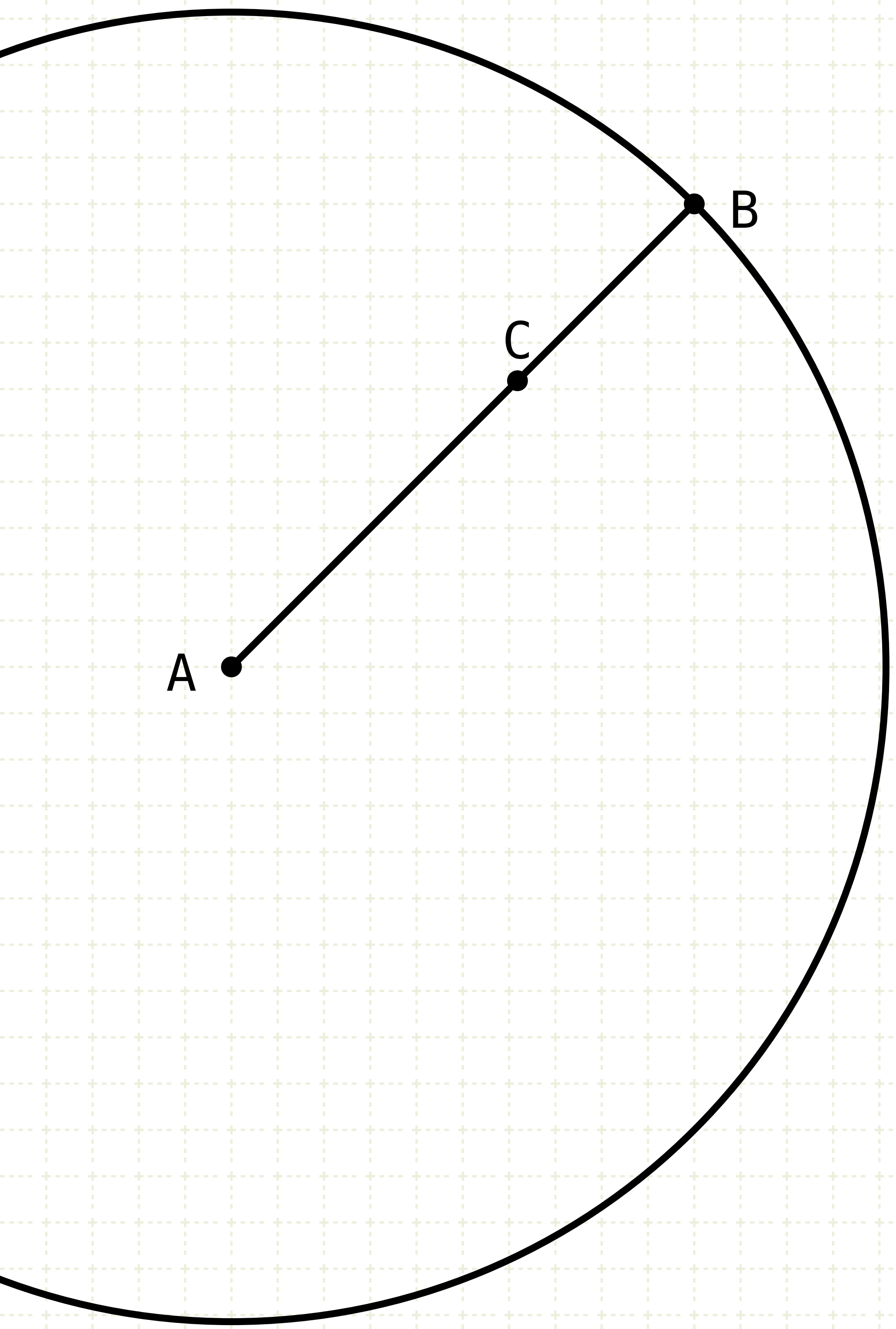
Construction

Start with any line AB

Let it be cut at point C such that the rectangle AB,BC equals the square on CA (II·11)

Proposition 10 of Book IV

To construct an isosceles triangle having each of the angles at the base double of the remaining one.



$AB \cdot BC = AC^2$

Construction

Start with any line AB

Let it be cut at point C such that the rectangle AB,BC equals the square on CA (II·11)

Draw a circle, with A as the center, and AB as the radius

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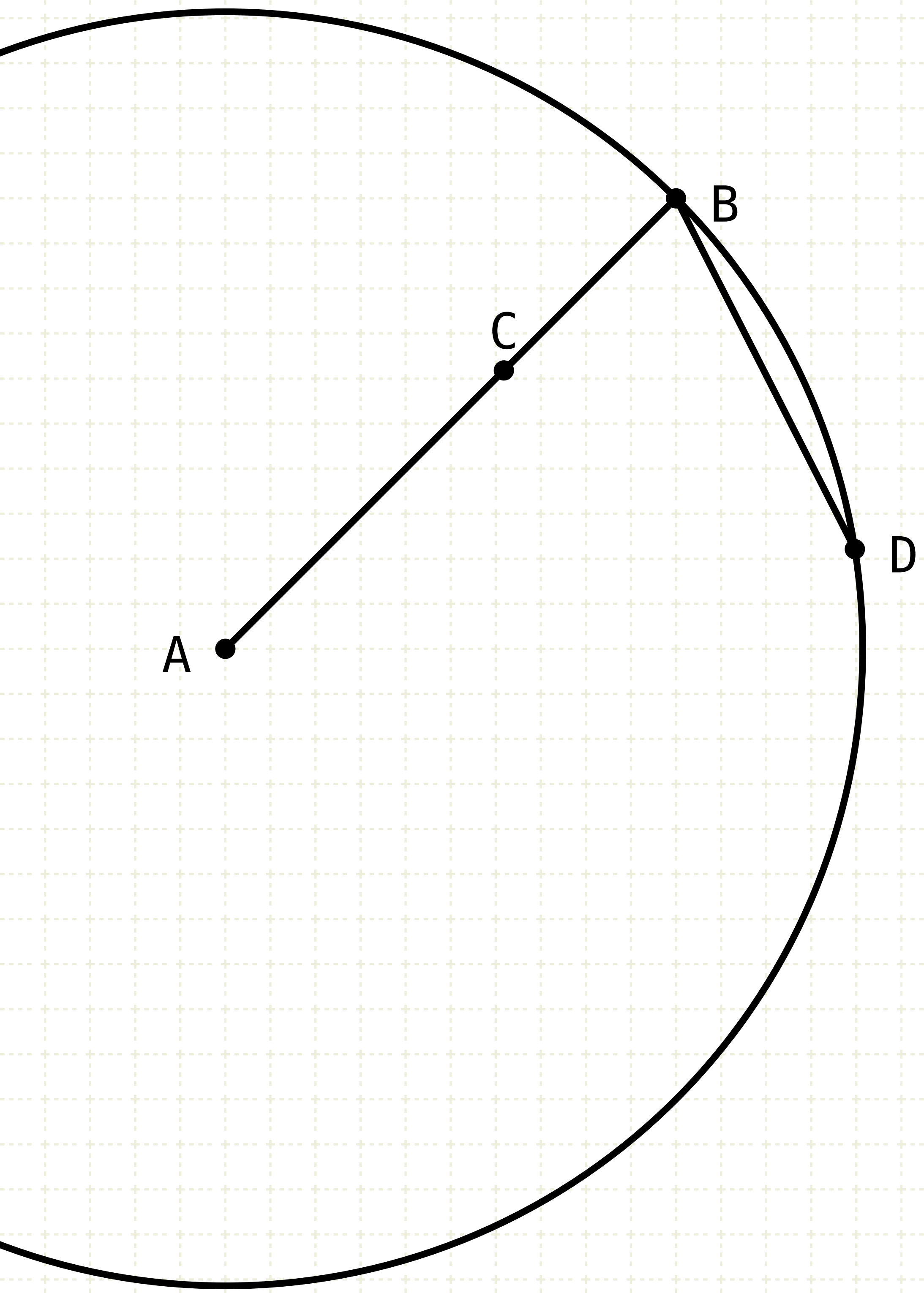


$$AB \cdot BC = AC^2$$
$$AC = BD$$

Construction

Start with any line AB

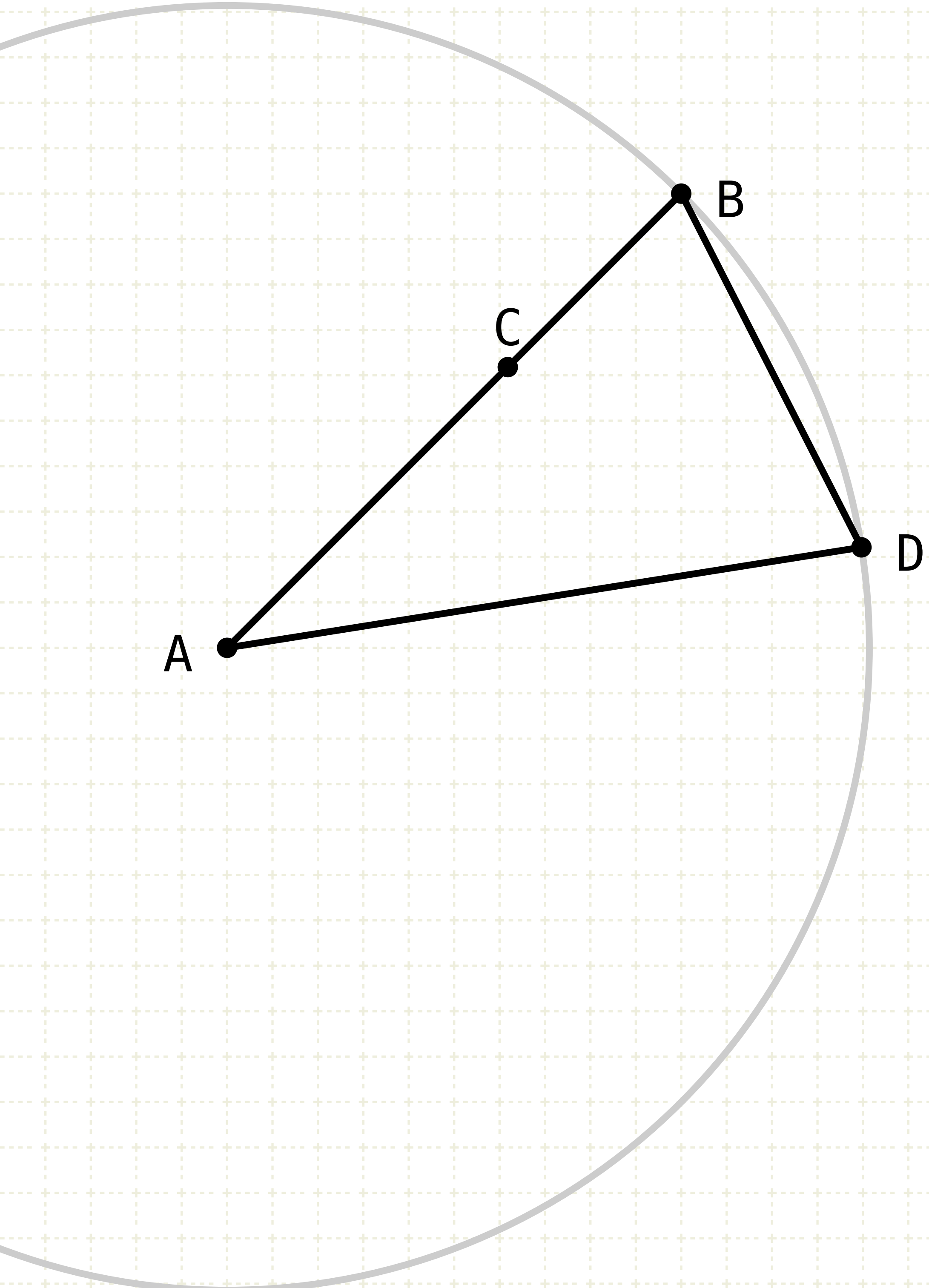
Let it be cut at point C such that the rectangle AB,BC equals the square on CA (II·11)

Draw a circle, with A as the center, and AB as the radius

Draw a line BD within the circle, equal to length AC (IV·1)

Proposition 10 of Book IV

To construct an isosceles triangle having each of the angles at the base double of the remaining one.



$AB \cdot BC = AC^2$

$AC = BD$

$\angle B = \angle D = 2 \cdot \angle A$

Construction

Start with any line AB

Let it be cut at point C such that the rectangle AB,BC equals the square on CA (II·11)

Draw a circle, with A as the center, and AB as the radius

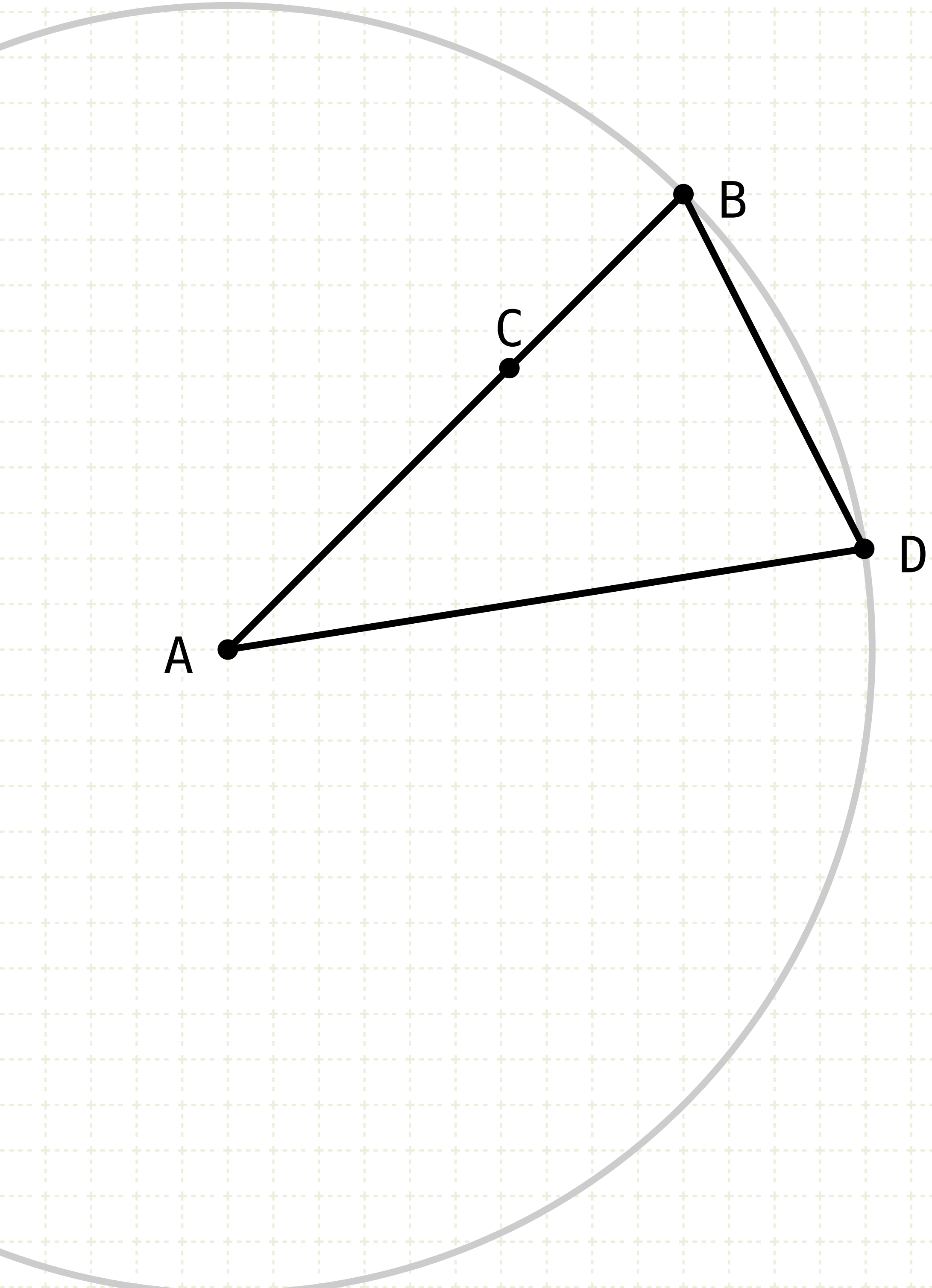
Draw a line BD within the circle, equal to length AC (IV·1)

Let AD be joined

The triangle ABC is an isosceles triangle, and the angle at B and D are equal, and are half the angle at A

Proposition 10 of Book IV

To construct an isosceles triangle having each of the angles at the base double of the remaining one.



Proof

$AB \cdot BC = AC^2$

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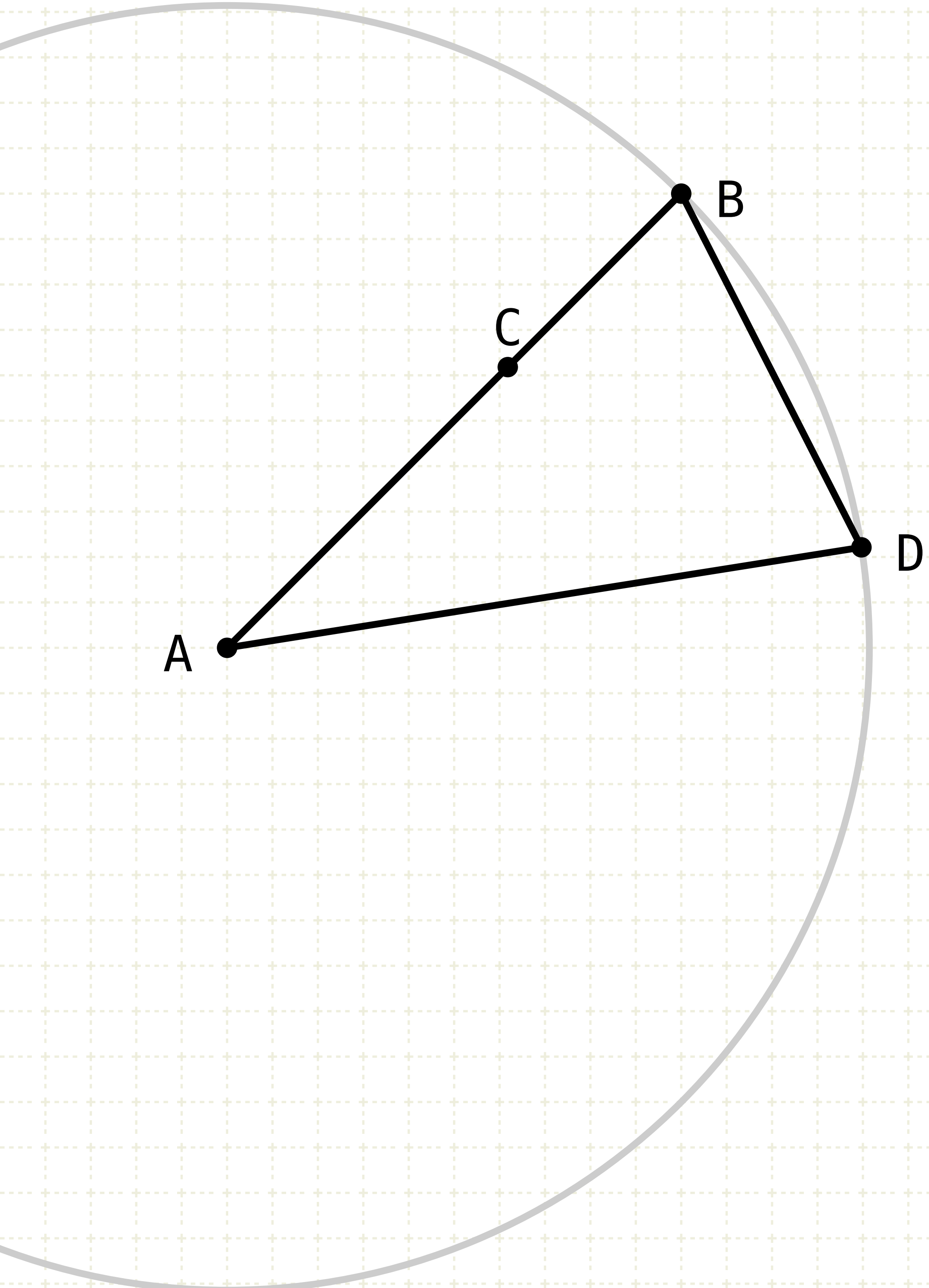

$AB \cdot BC = AC^2$
$AC = BD$

$AB \cdot BC = BD^2$

Proof

Since AC equals BD (by construction), and the rectangle AB,BC equals the square on AC (again by construction), then the rectangle AB,BC equals the square on BD

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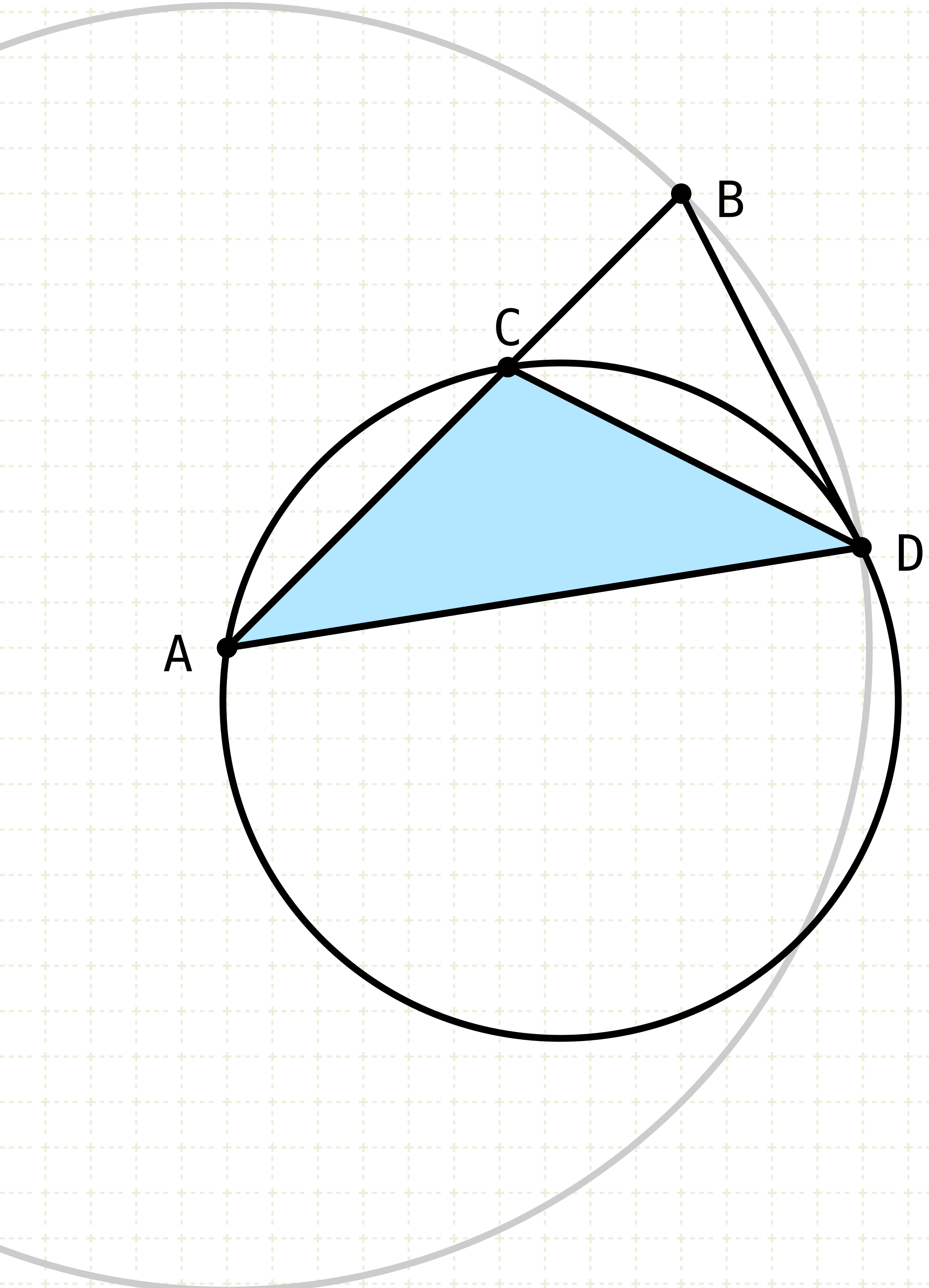

$AB \cdot BC = AC^2$
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Proof

Since AC equals BD (by construction), and the rectangle AB,BC equals the square on AC (again by construction), then the rectangle AB,BC equals the square on BD

Draw the line CD, and let a circle be drawn circumscribing the triangle ACD (IV·5)

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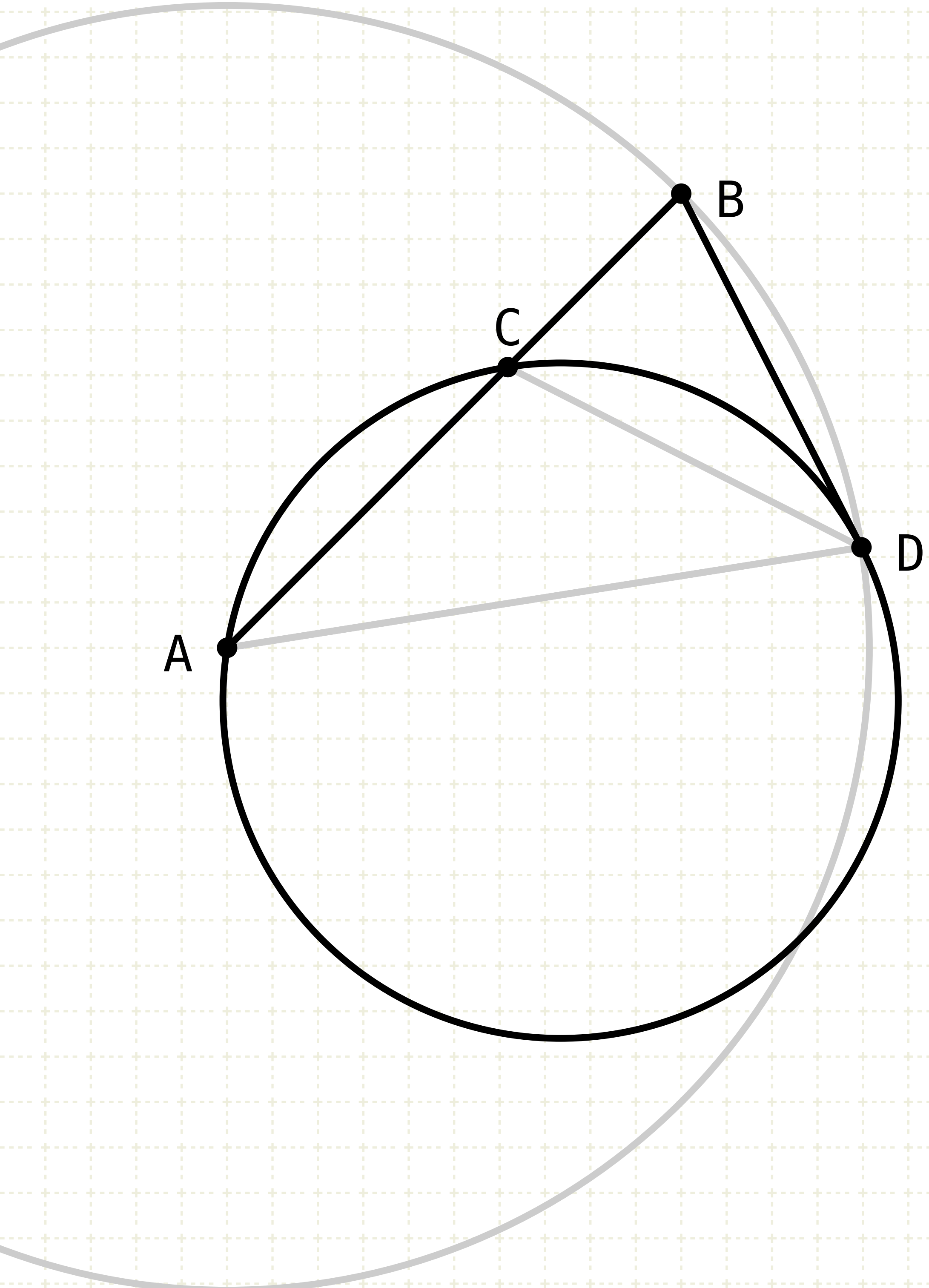

$AB \cdot BC = AC^2$

$AC = BD$

$AB \cdot BC = BD^2$

BD touches the circle

Proof

Since AC equals BD (by construction), and the rectangle AB,BC equals the square on AC (again by construction), then the rectangle AB,BC equals the square on BD

Draw the line CD, and let a circle be drawn circumscribing the triangle ACD (IV·5)

We have point B on the outside of the circle, and from B, two straight lines fall on the circle, AB cutting the circle, and AB falling on the circle

And since the rectangle AB,BC equals the square of BD, BD touches the circle (III·37)

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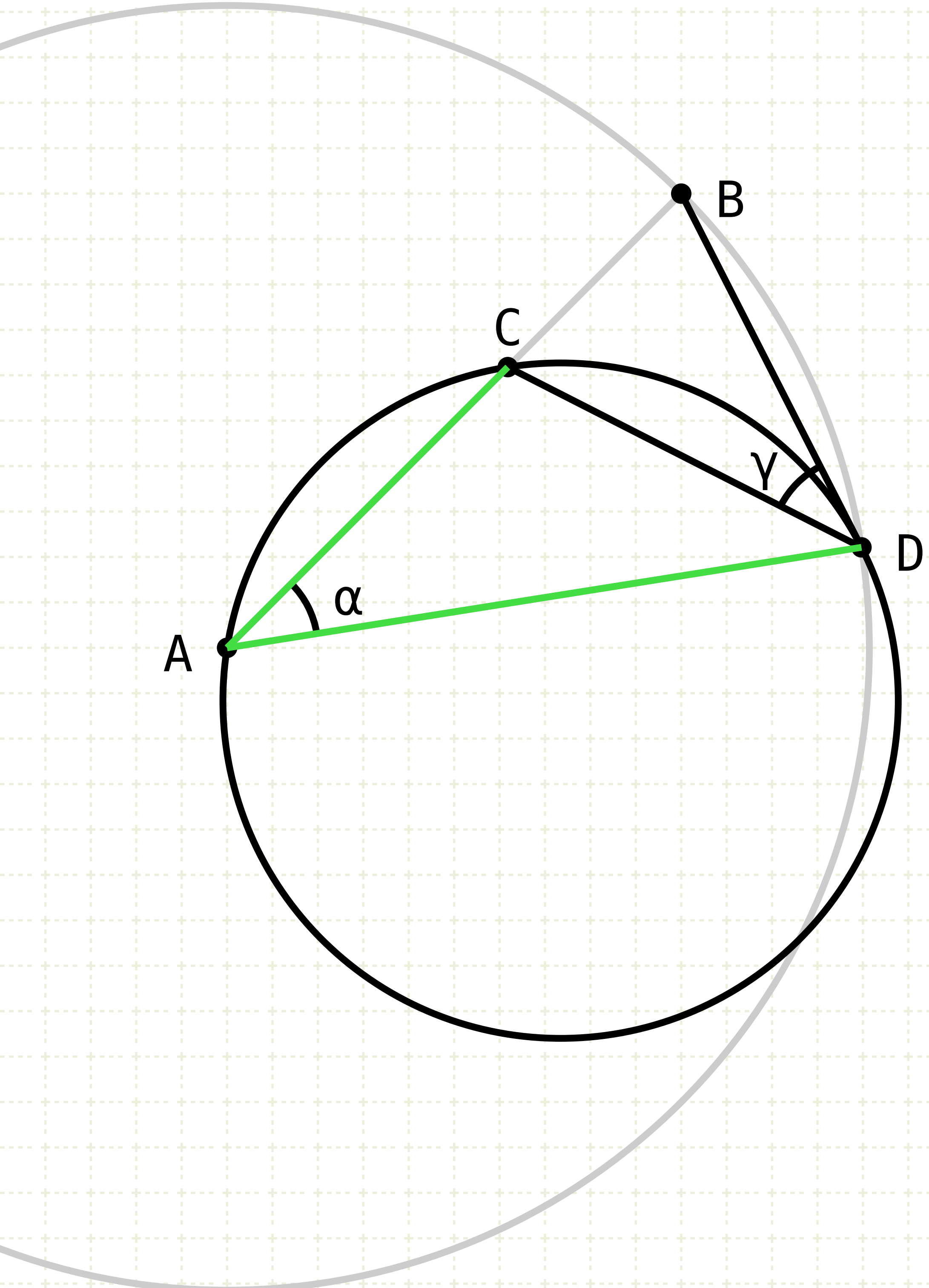

$AB \cdot BC = AC^2$

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BD touches the circle

$\alpha = \gamma$

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Now, since BD touches the circle, and DC cuts the circle, the angle BDC (γ) is equal to the angle in the alternating section of the circle DAC (α) (III·32)

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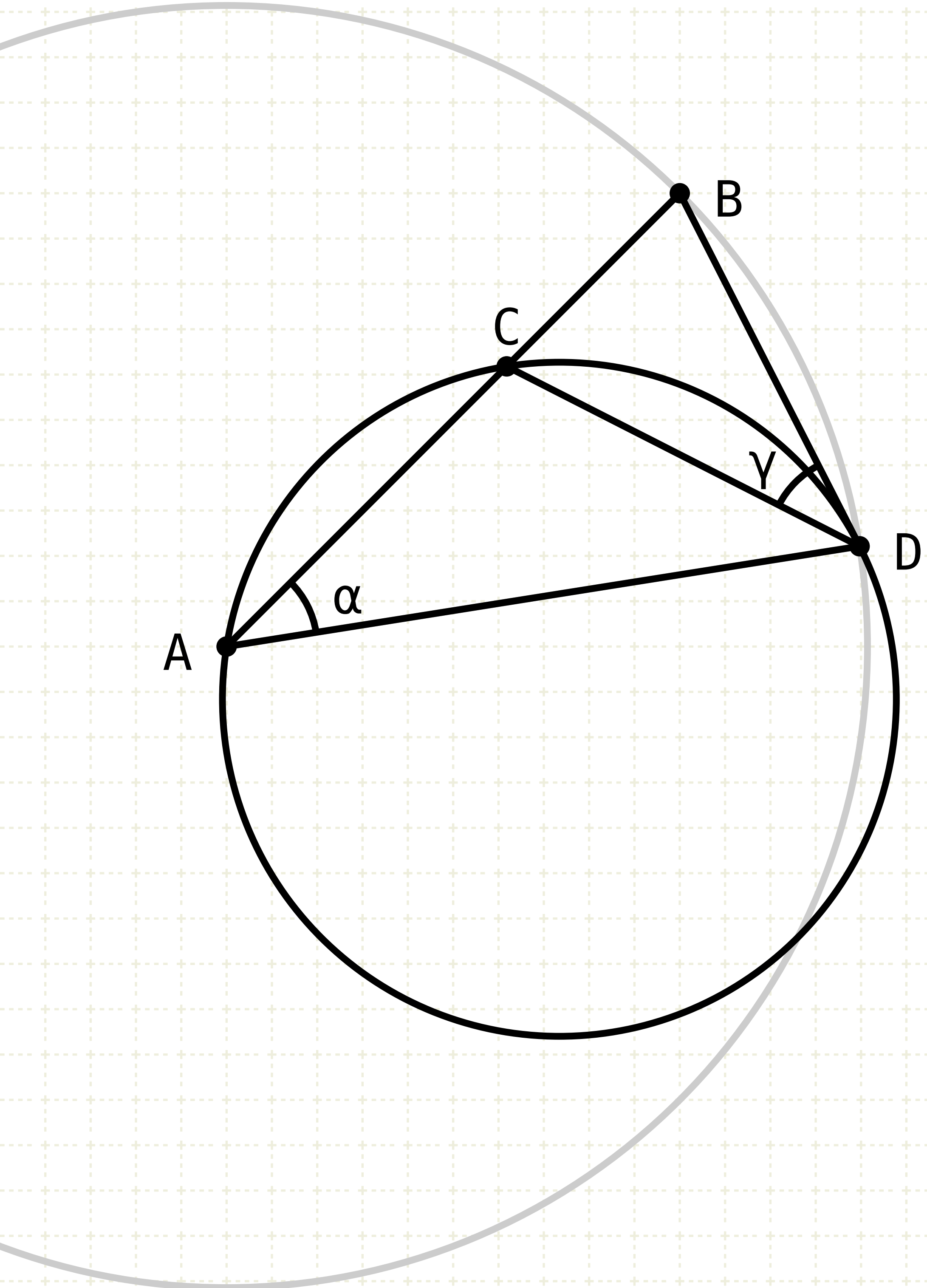

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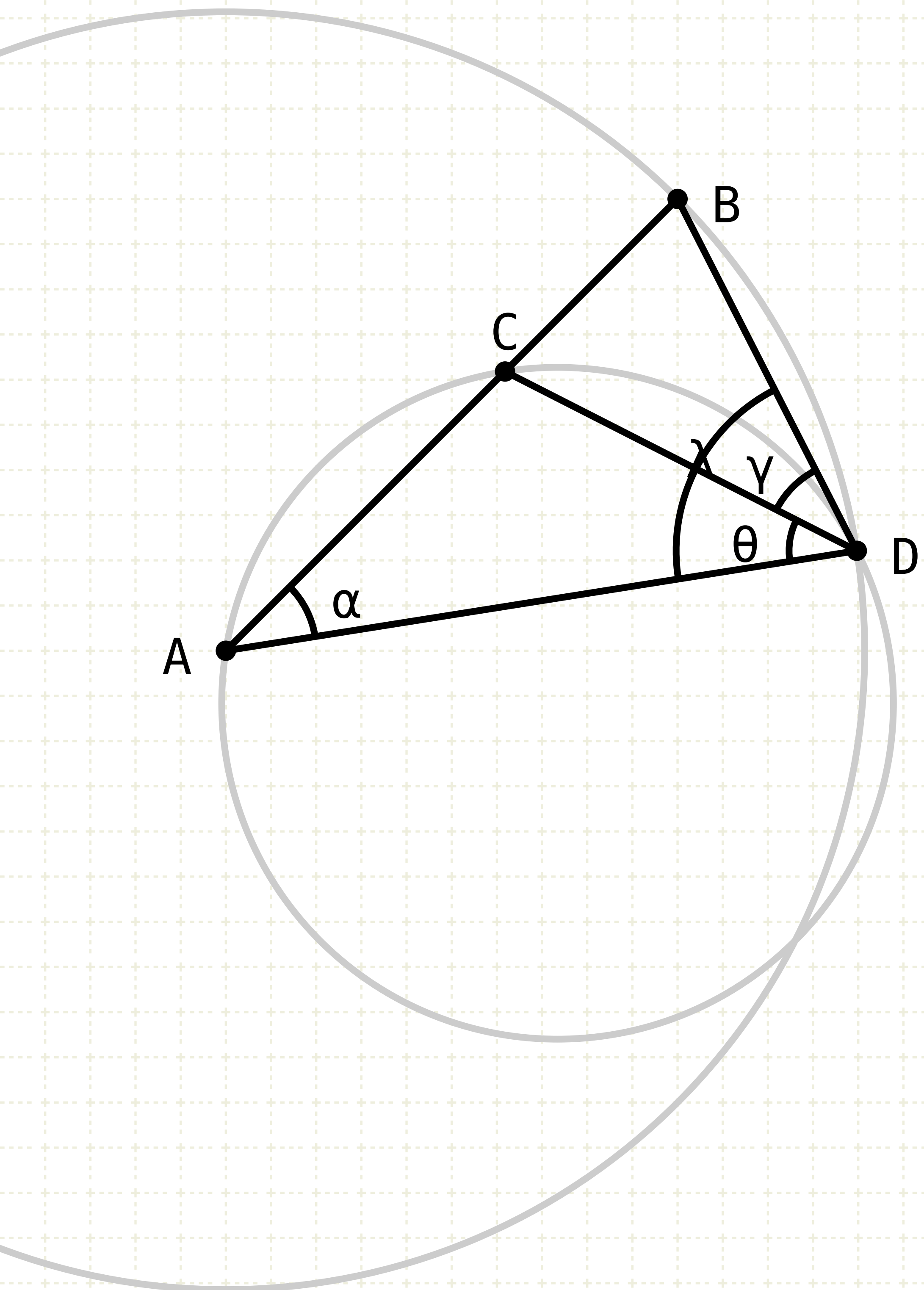

$AB \cdot BC = AC^2$

$AC = BD$

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BD touches the circle

$\alpha = \gamma$

$\alpha + \theta = \gamma + \theta = \lambda$

$\lambda = \alpha + \theta$

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Now, since BD touches the circle, and DC cuts the circle, the angle BDC (γ) is equal to the angle in the alternating section of the circle DAC (α) (III·32)

Add the angle CDA (θ) to both sides of the equality

Thus BDA (λ) is equal to the two angles CDA,DAC (θ,α)

Proposition 10 of Book IV

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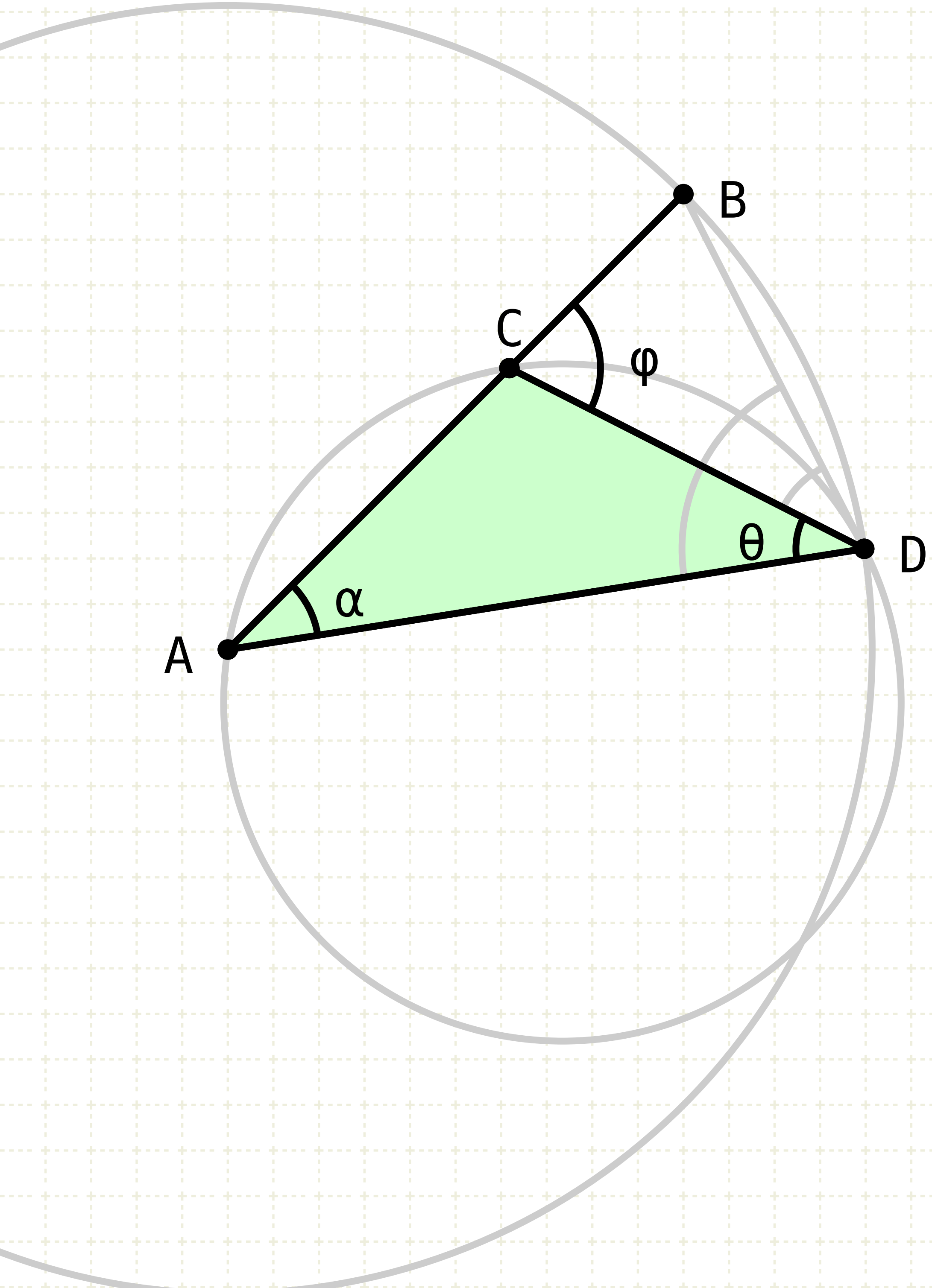

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Add the angle CDA (θ) to both sides of the equality

Thus BDA (λ) is equal to the two angles CDA,DAC (θ,α)

The exterior angle BCD (φ) is also equal to the sum of the interior and opposite angles, CDA,DAC (θ,α) (I·32)

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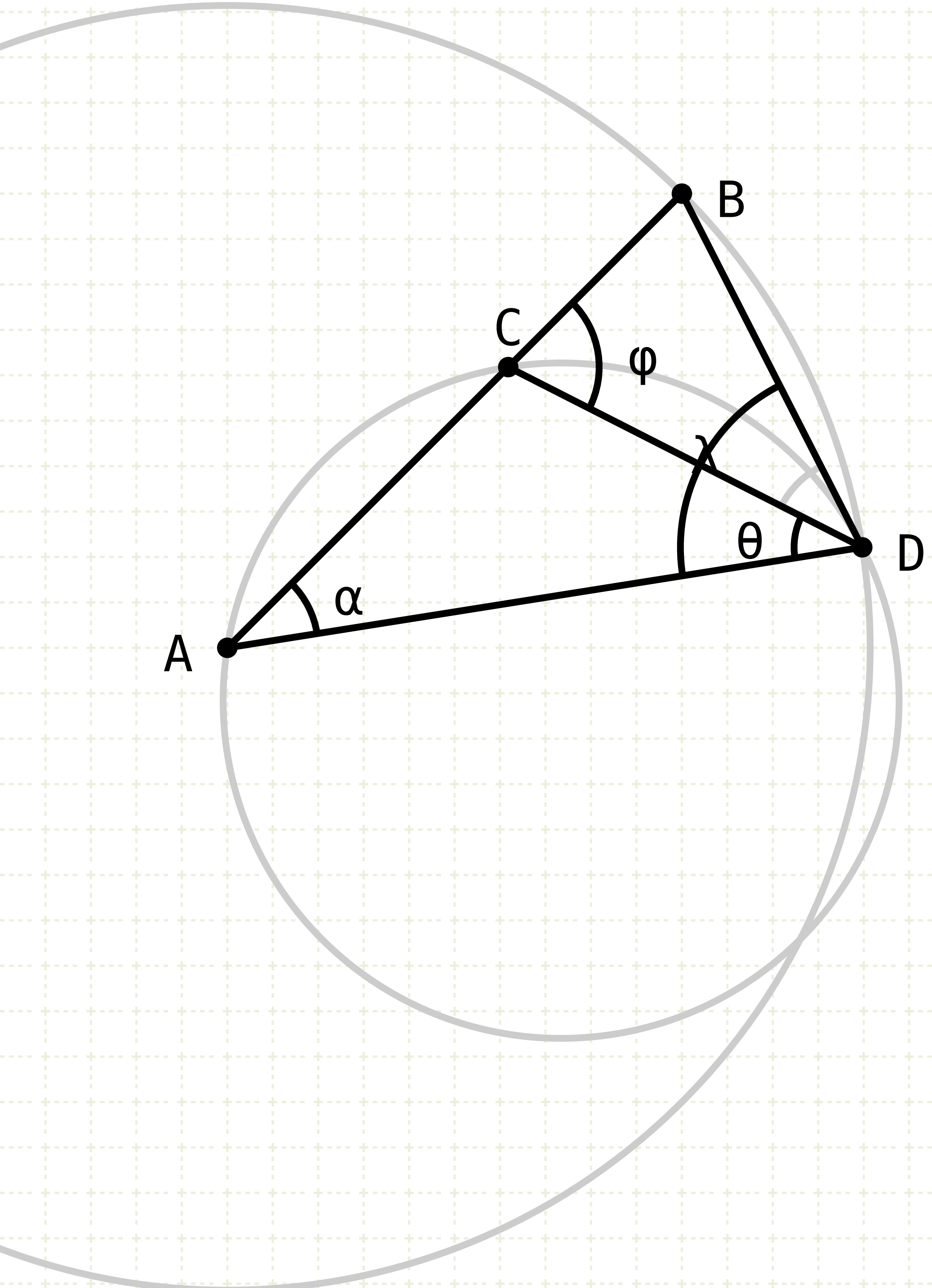
To construct an isosceles triangle having each of the angles at the base double of the remaining one.



$AB \cdot BC = AC^2$

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BD touches the circle

$\alpha = \gamma$

$\alpha + \theta = \gamma + \theta = \lambda$

$\lambda = \alpha + \theta$

$\varphi = \alpha + \theta$

$\therefore \lambda = \varphi$

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Thus BDA (λ) equals BCD (φ)

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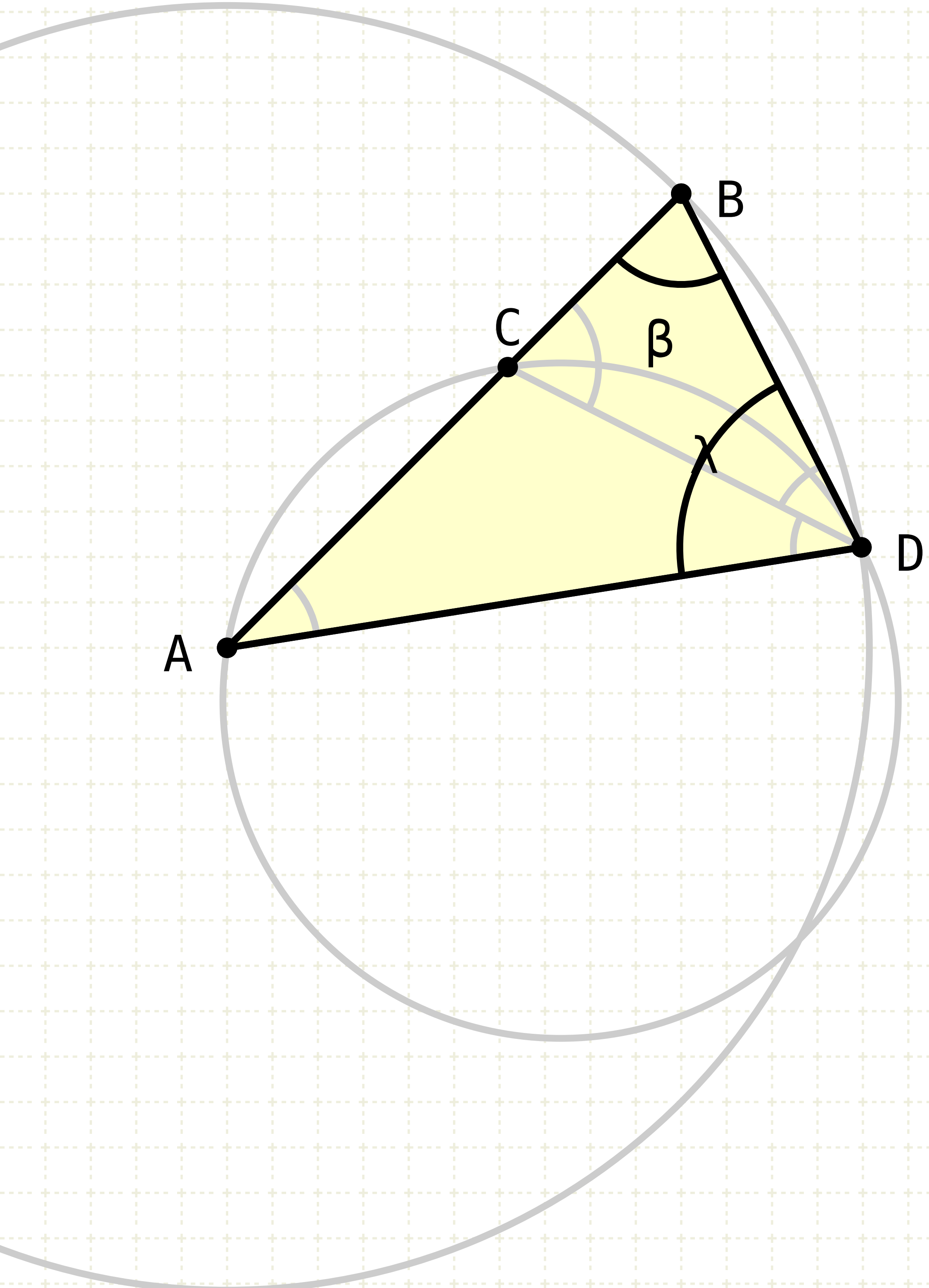

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$\varphi = \alpha + \theta$

$\therefore \lambda = \varphi$

$\beta = \lambda = \varphi$

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Thus BDA (λ) is equal to the two angles CDA,DAC (θ,α)

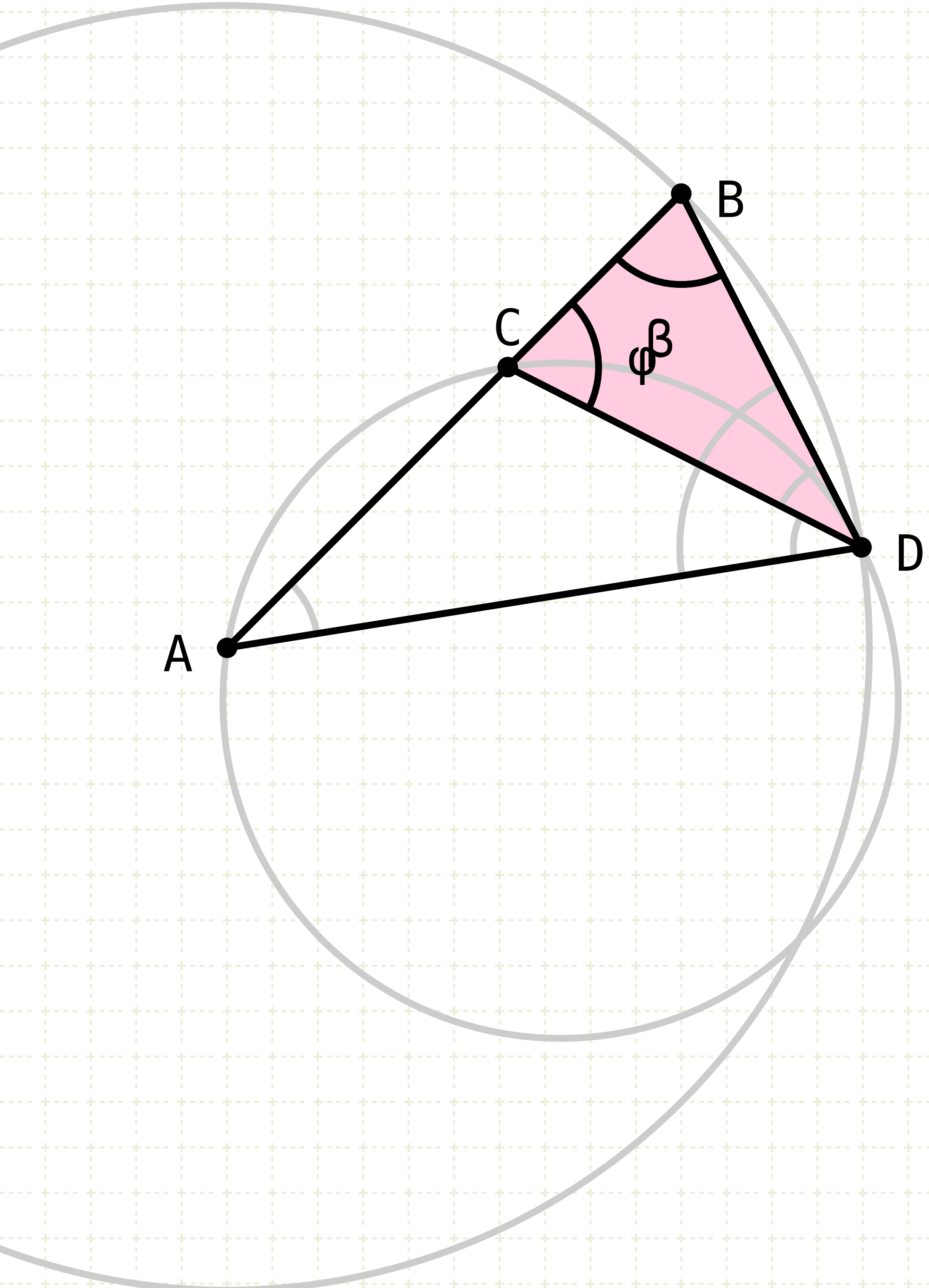
The exterior angle BCD (φ) is also equal to the sum of the interior and opposite angles, CDA,DAC (θ,α) (I·32)

Thus BDA (λ) equals BCD (φ)

ABD is an isosceles triangle since AB equals AD, and therefore angles BDA (λ) and DBA (β) are equal

Proposition 10 of Book IV

To construct an isosceles triangle having each of the angles at the base double of the remaining one.



$AB \cdot BC = AC^2$

$AC = BD$

$AB \cdot BC = BD^2$

BD touches the circle

$\alpha = \gamma$

$\alpha + \theta = \gamma + \theta = \lambda$

$\lambda = \alpha + \theta$

$\varphi = \alpha + \theta$

$\therefore \lambda = \varphi$

$\beta = \lambda = \varphi$

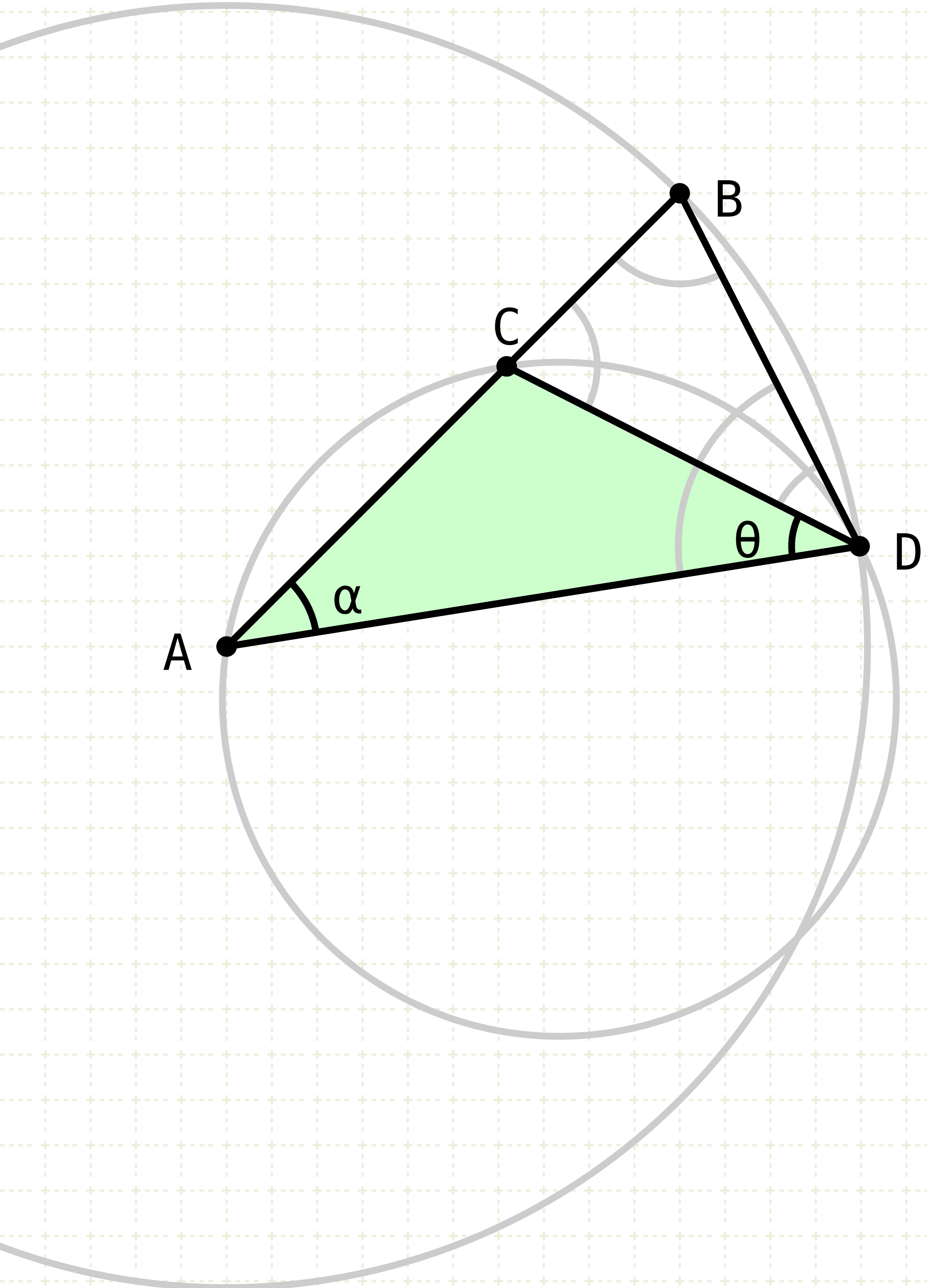
$CD = BD$

Proof (cont)

Since the angles DBA (β) and BCD (φ) are equal, BD is equal to CD (I·6)

Proposition 10 of Book IV

To construct an isosceles triangle having each of the angles at the base double of the remaining one.



$AB \cdot BC = AC^2$

$AC = BD$

$AB \cdot BC = BD^2$

BD touches the circle

$\alpha = \gamma$

$\alpha + \theta = \gamma + \theta = \lambda$

$\lambda = \alpha + \theta$

$\varphi = \alpha + \theta$

$\therefore \lambda = \varphi$

$\beta = \lambda = \varphi$

$CD = BD$

$\therefore AC = CD$

$\therefore \theta = \alpha$

Proof (cont)

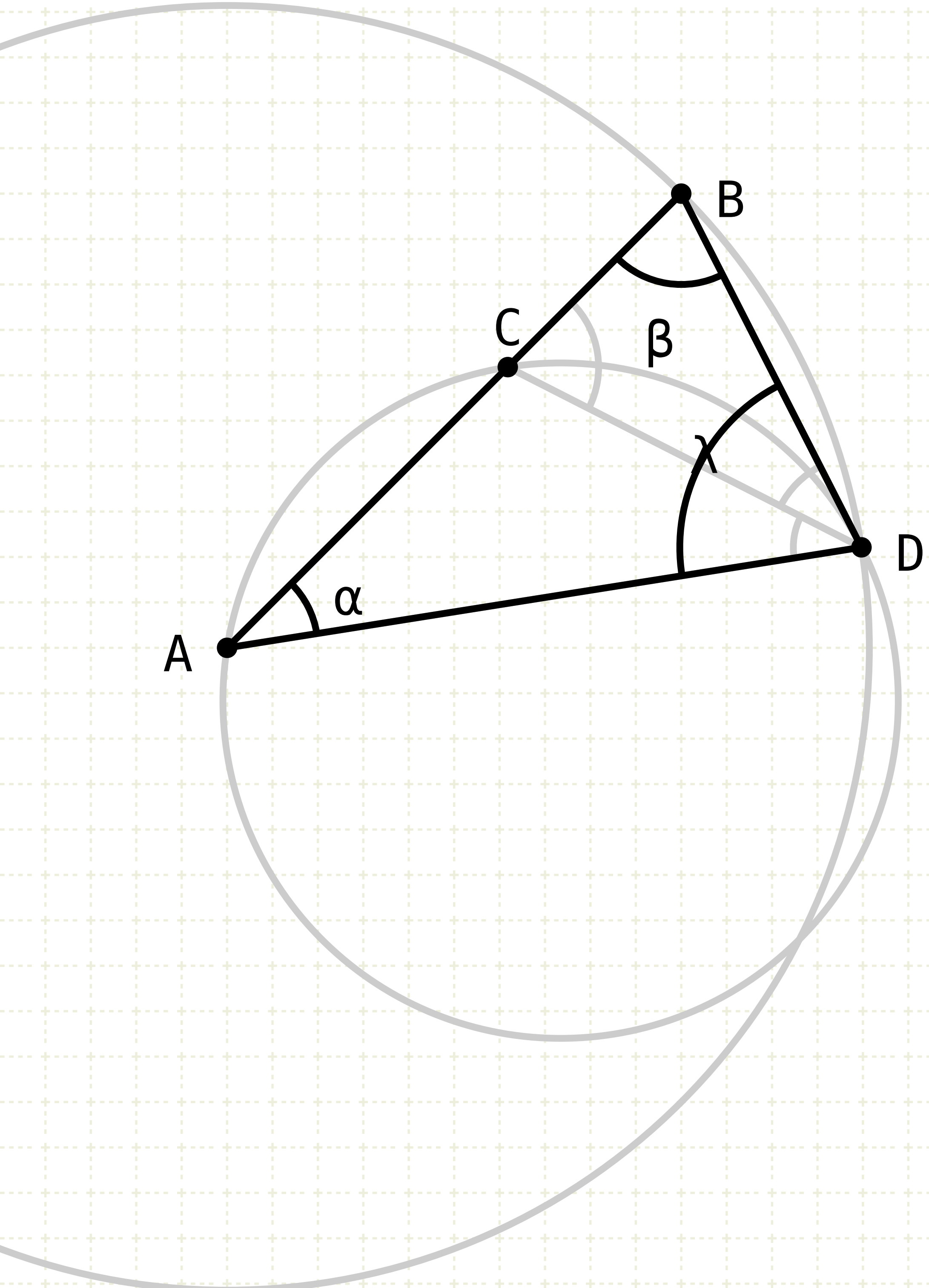
Since the angles DBA (β) and BCD (φ) are equal, BD is equal to CD (I·6)

But BD is equal to AC (by construction) so triangle ACD is an isosceles triangle

Thus angle CAD is equal to CDA (I·6)

Proposition 10 of Book IV

To construct an isosceles triangle having each of the angles at the base double of the remaining one.



$AB \cdot BC = AC^2$

$AC = BD$

$AB \cdot BC = BD^2$

BD touches the circle

$\alpha = \gamma$

$\alpha + \theta = \gamma + \theta = \lambda$

$\lambda = \alpha + \theta$

$\varphi = \alpha + \theta$

$\therefore \lambda = \varphi$

$\beta = \lambda = \varphi$

$CD = BD$

$\therefore AC = CD$

$\therefore \theta = \alpha$

$\lambda = \beta = \alpha + \theta = 2\alpha$

Proof (cont)

Since the angles DBA (β) and BCD (φ) are equal, BD is equal to CD (I·6)

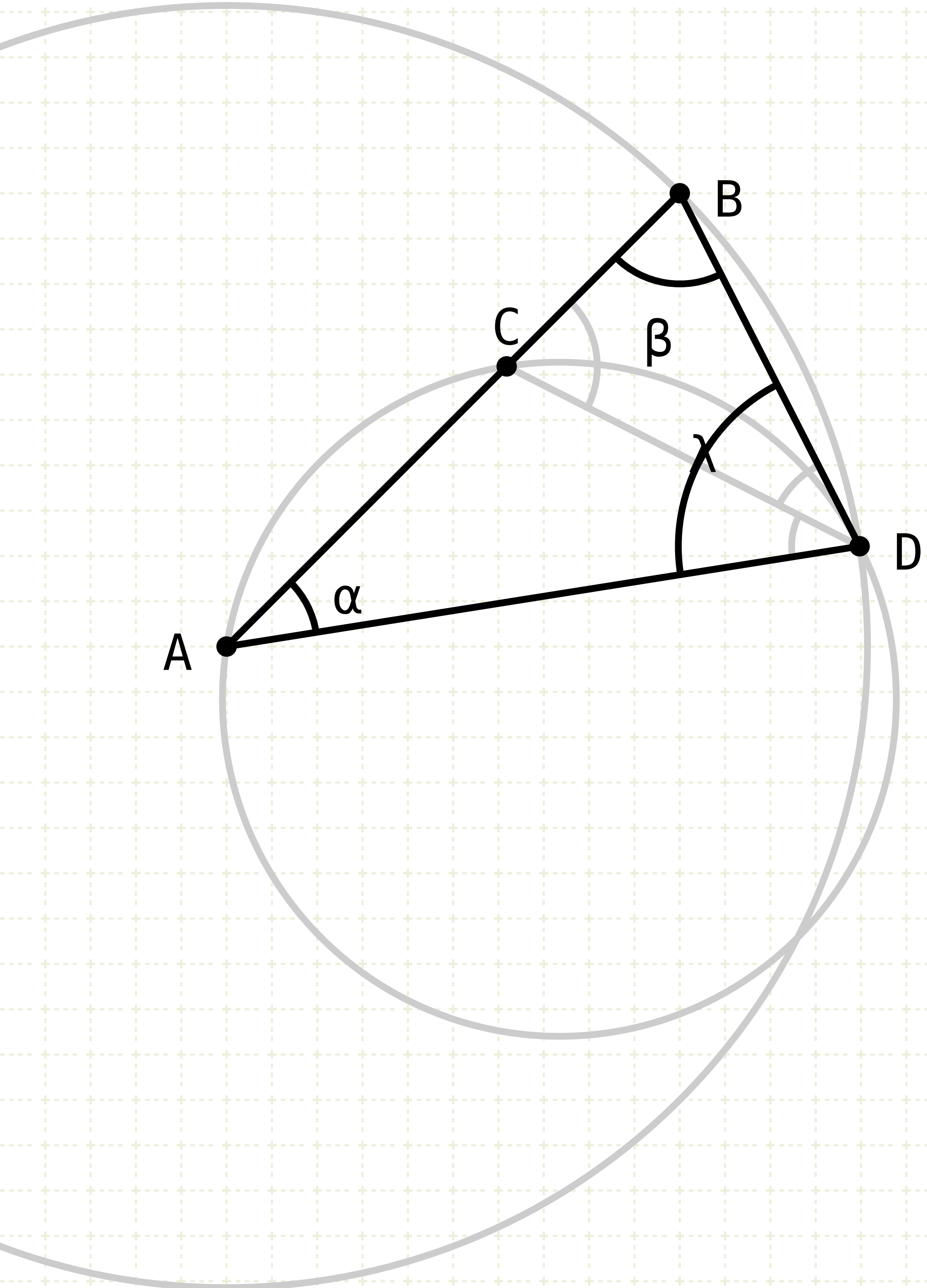
But BD is equal to AC (by construction) so triangle ACD is an isosceles triangle

Thus angle CAD is equal to CDA (I·6)

Angle ABD (β) equals BDA (λ), which in turn equals the sum of the equal angles BAD (α) and CDA (θ), thus ABD (β) and BDA (λ) equal twice BAD (α)

Proposition 10 of Book IV

To construct an isosceles triangle having each of the angles at the base double of the remaining one.



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BD touches the circle

$\alpha = \gamma$

$\alpha + \theta = \gamma + \theta = \lambda$

$\lambda = \alpha + \theta$

$\varphi = \alpha + \theta$

$\therefore \lambda = \varphi$

$\beta = \lambda = \varphi$

$CD = BD$

$\therefore AC = CD$

$\therefore \theta = \alpha$

$\lambda = \beta = \alpha + \theta = 2\alpha$

Proof (cont)

Since the angles DBA (β) and BCD (φ) are equal, BD is equal to CD (I·6)

But BD is equal to AC (by construction) so triangle ACD is an isosceles triangle

Thus angle CAD is equal to CDA (I·6)

Angle ABD (β) equals BDA (λ), which in turn equals the sum of the equal angles BAD (α) and CDA (θ), thus ABD (β) and BDA (λ) equal twice BAD (α)

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