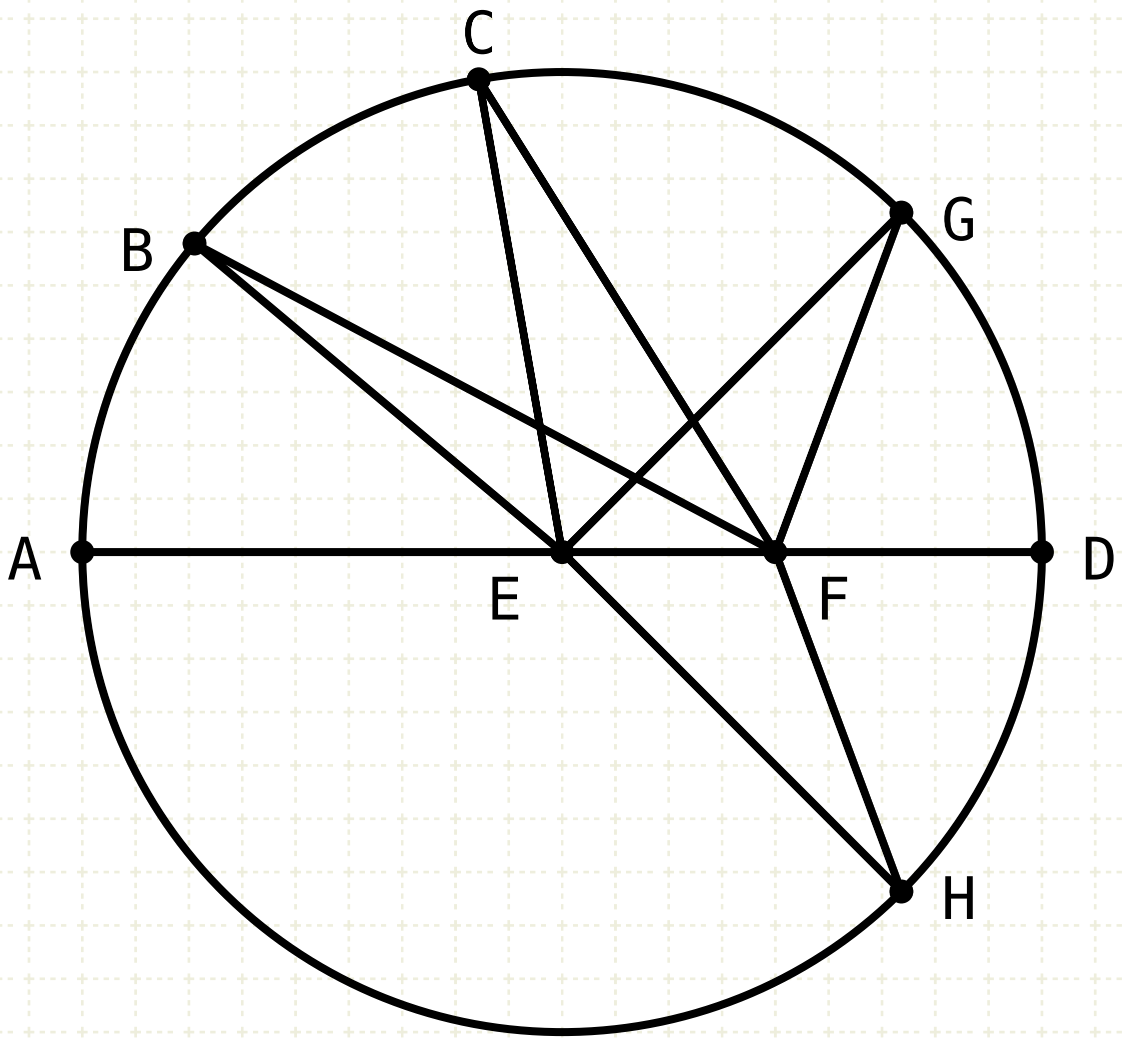
Euclid's Elements

Book III



A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)

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Proposition 14 of Book III

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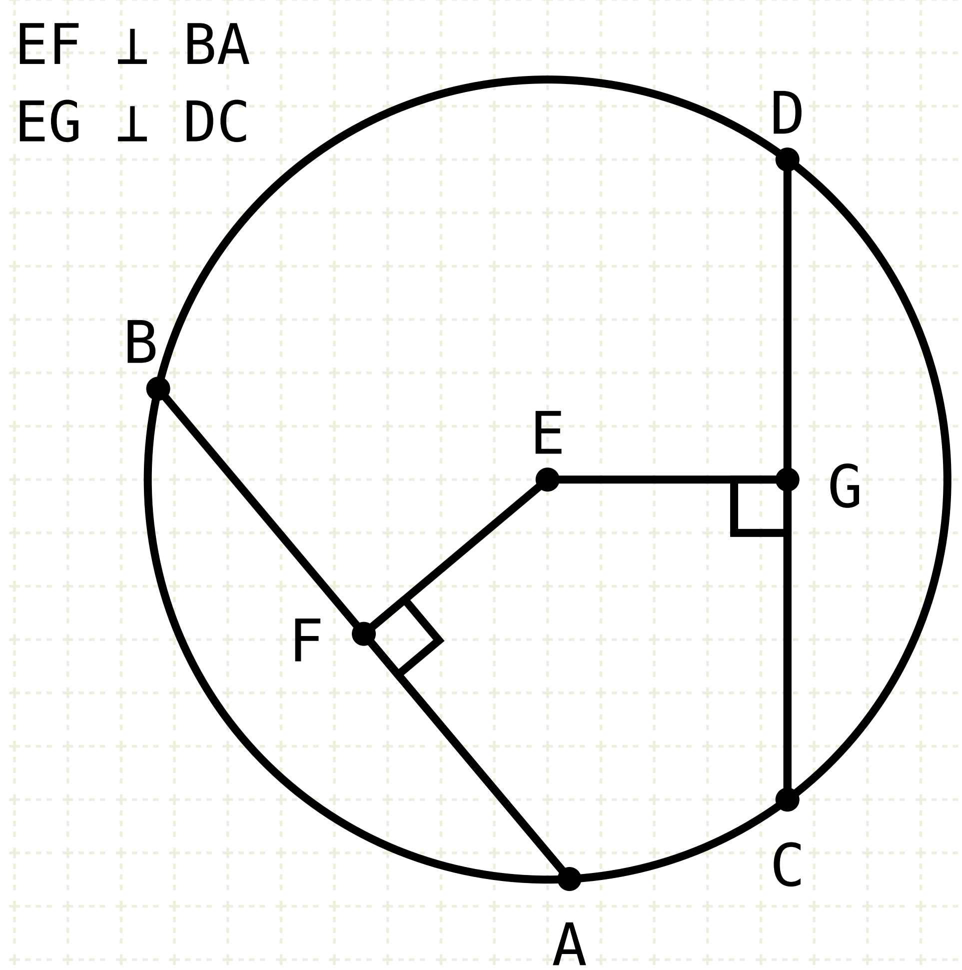

In other words

If E is the centre of the circle, EF perpendicular to BA, and EG perpendicular to DC, then

(1) If AB equals CD, then EG equals EF

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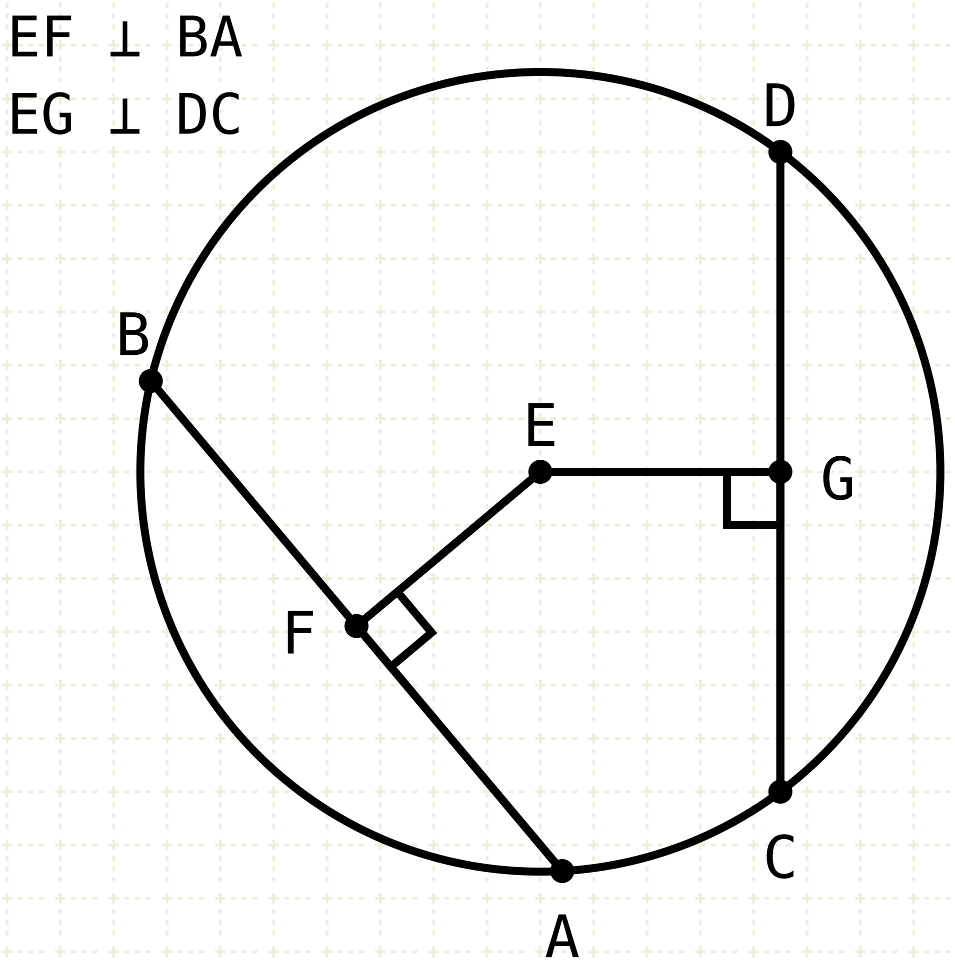

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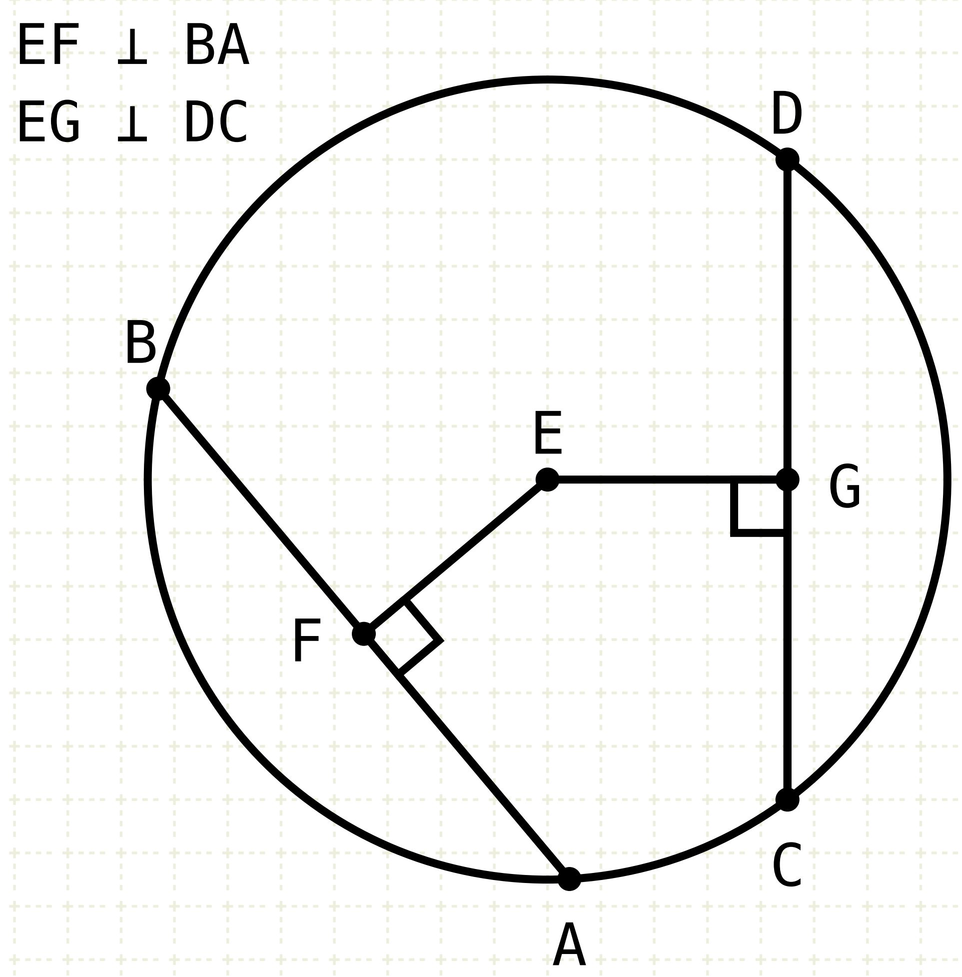

$AF = FB, \quad AB = 2 \cdot AF$

$DG = GC, \quad DC = 2 \cdot GC$

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The straight line EF bisects AB, and EG bisects DC (III·3), therefore AB is twice AF, and DC is twice GC

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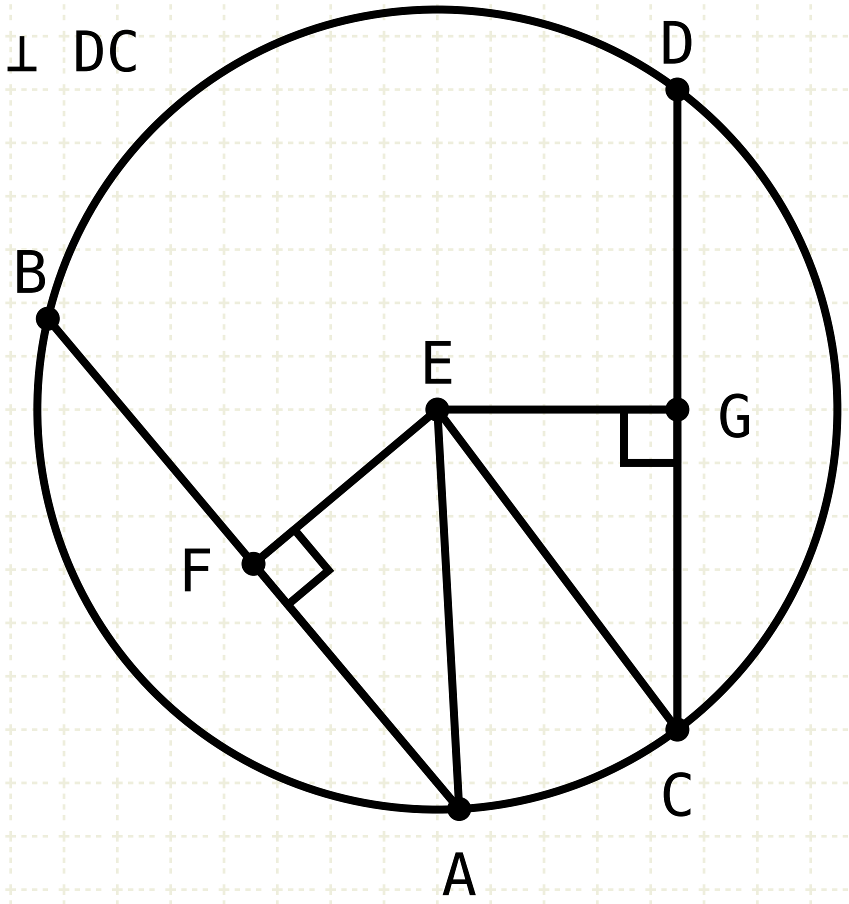

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Join AE and CE

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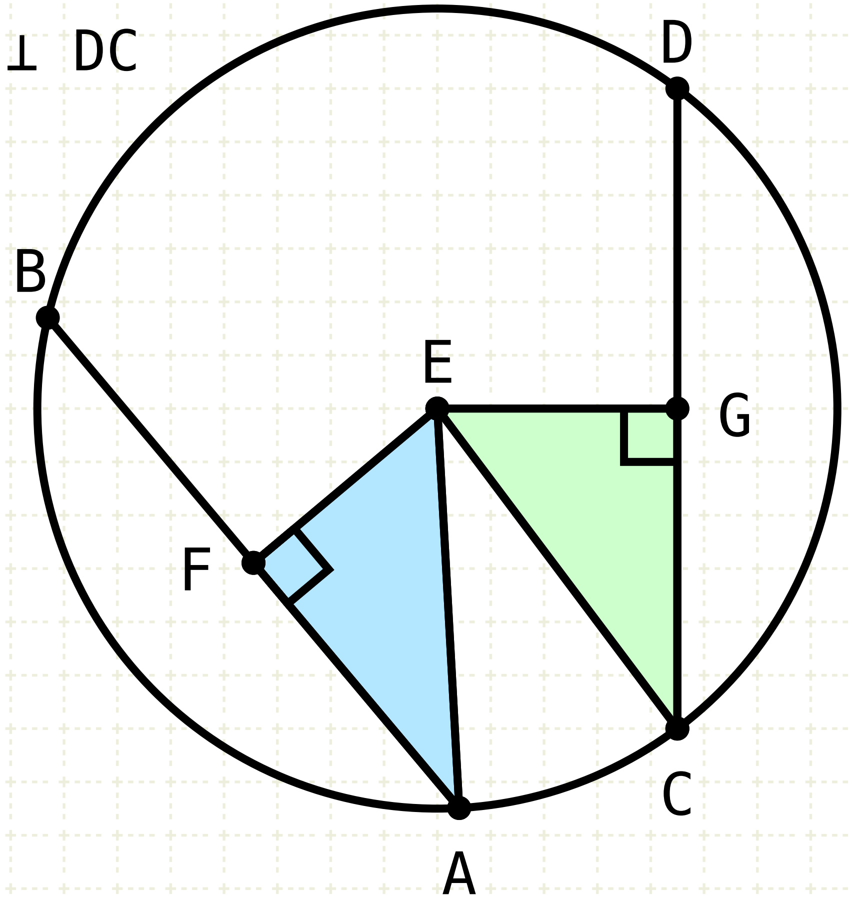

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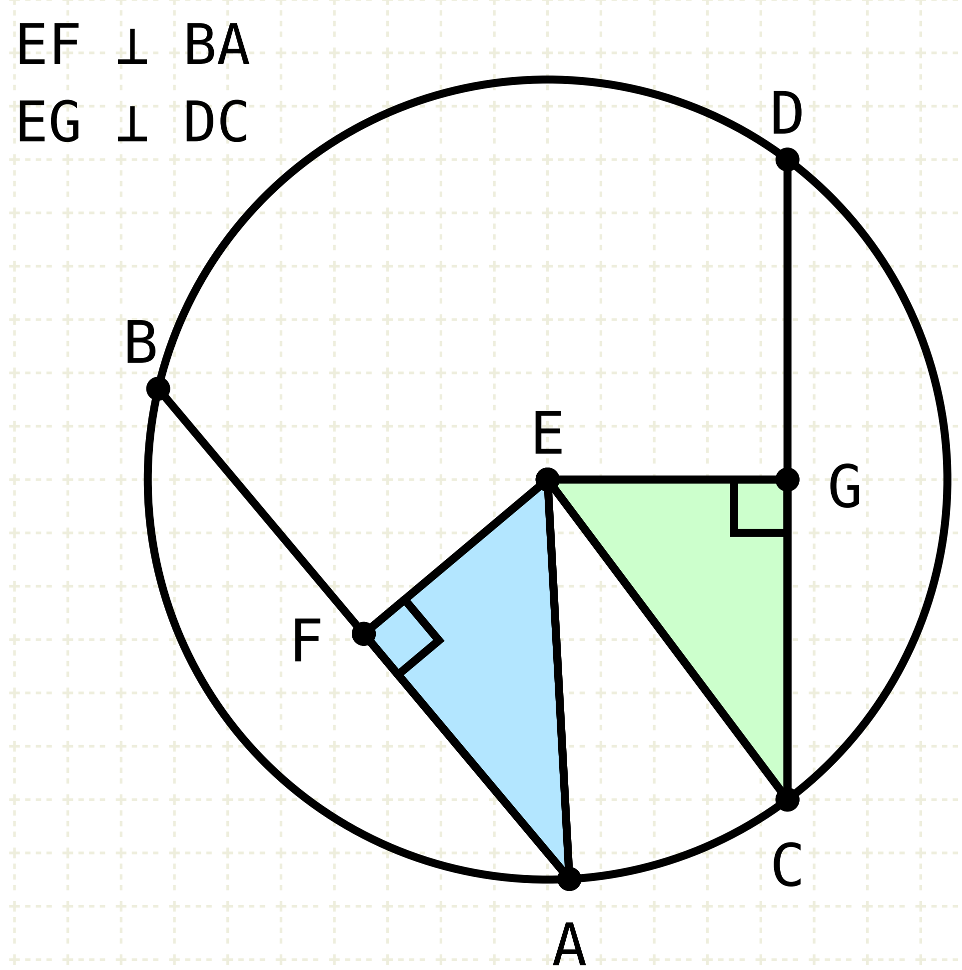

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AE and CE are equal since they are the radii of the circle

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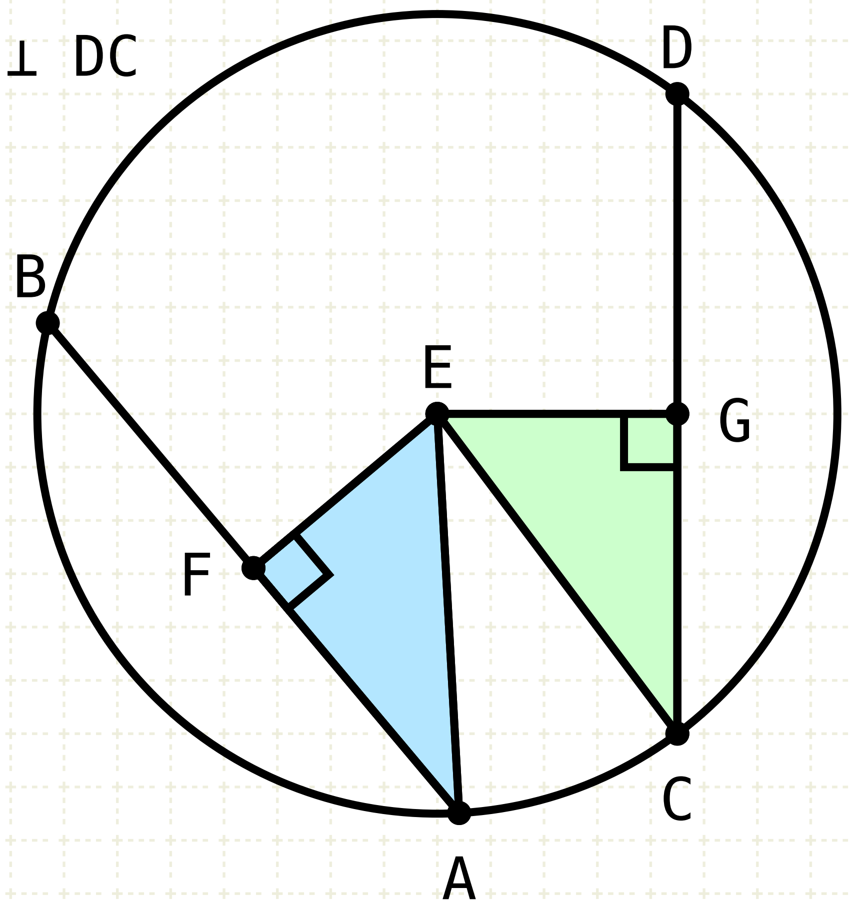

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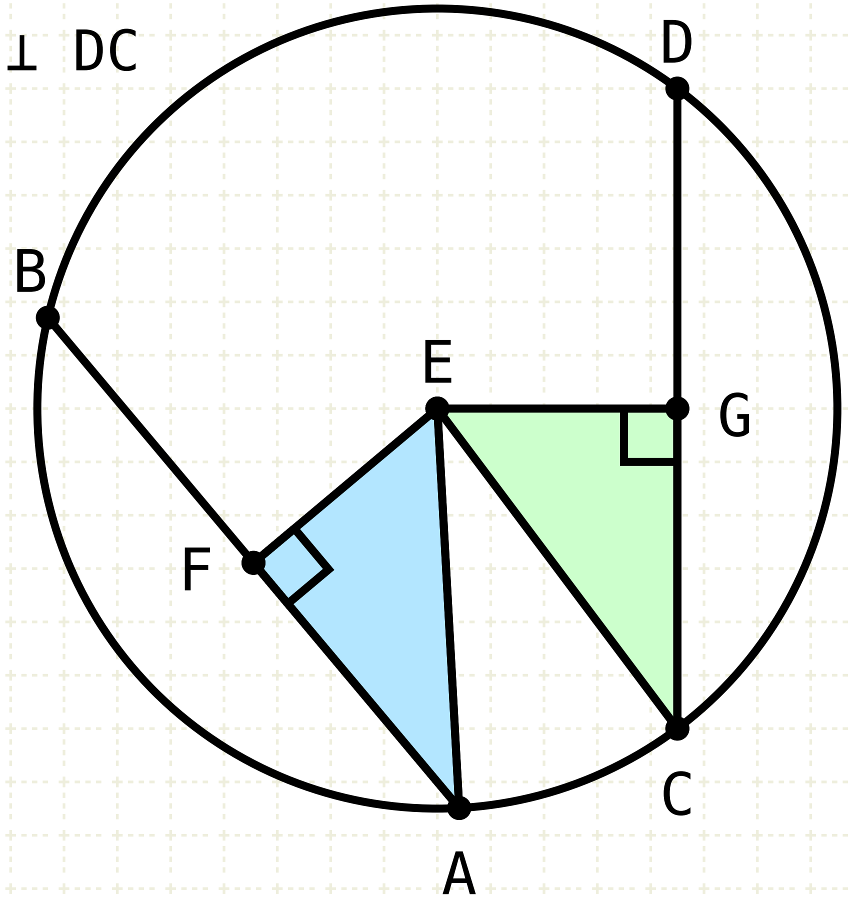

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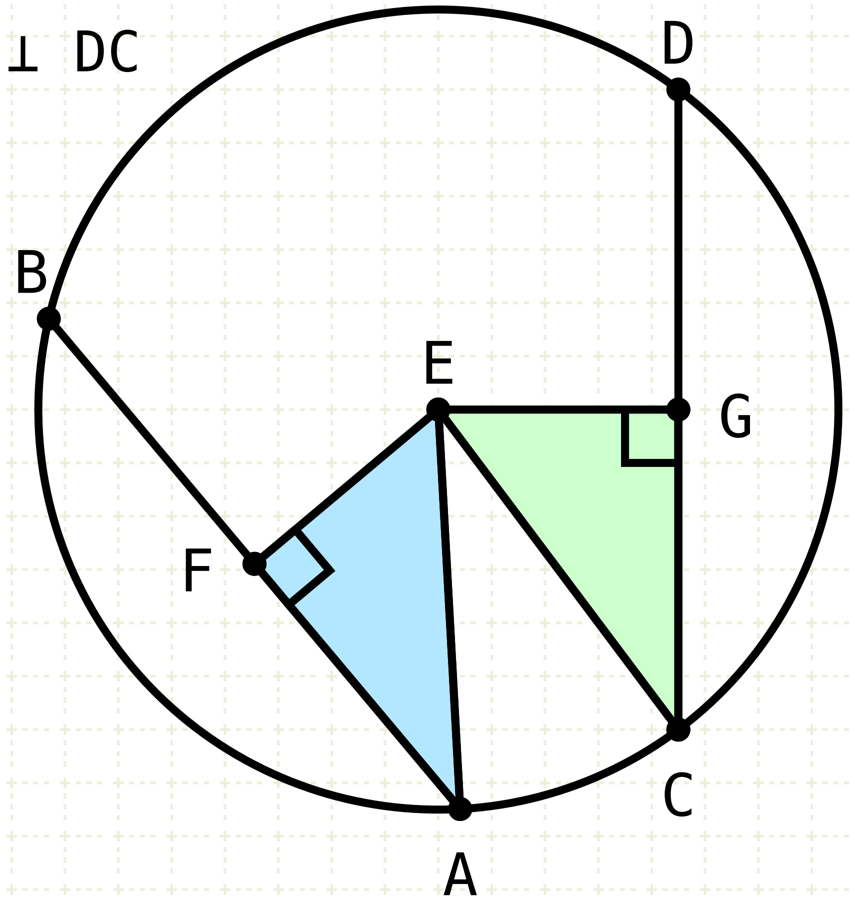

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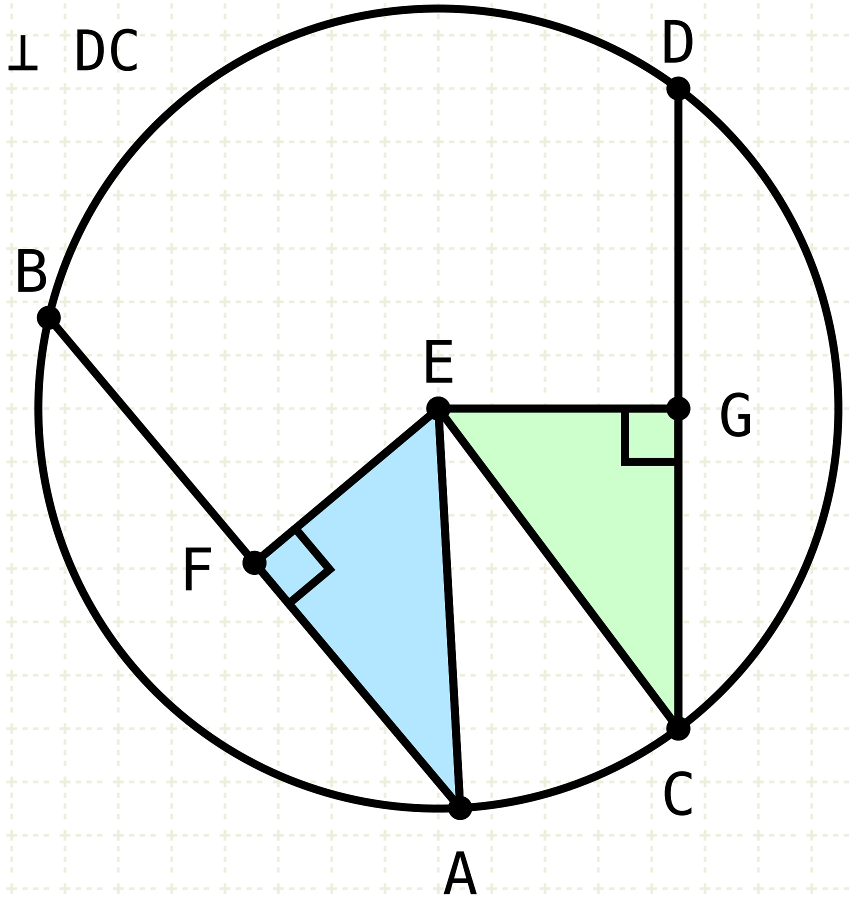

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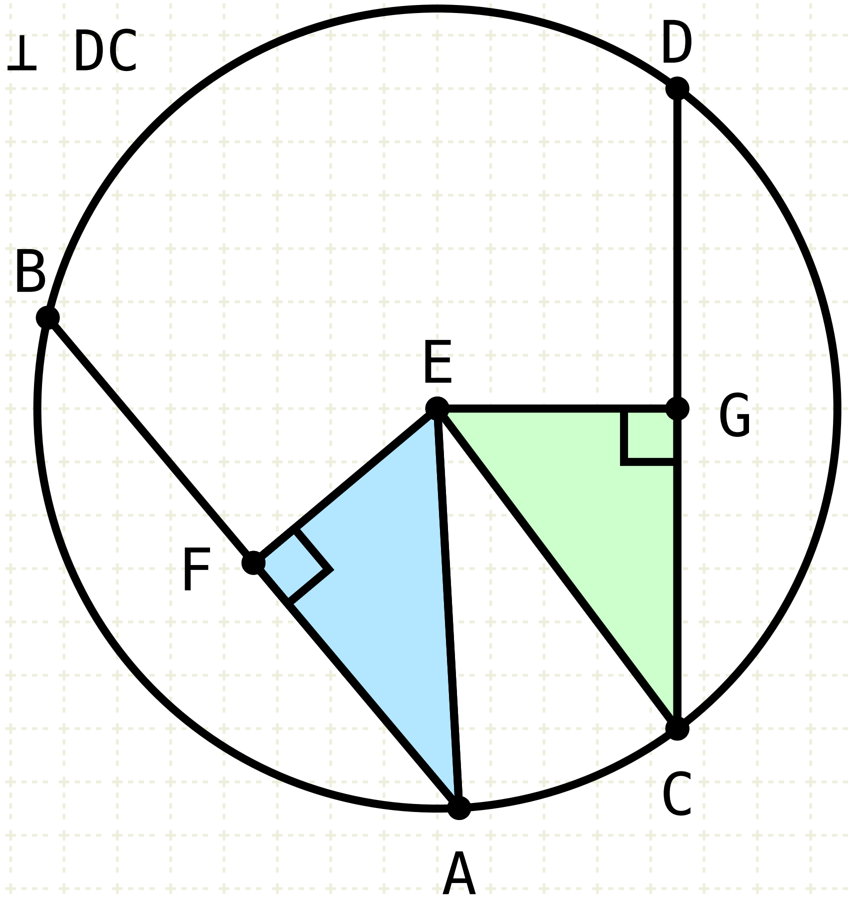

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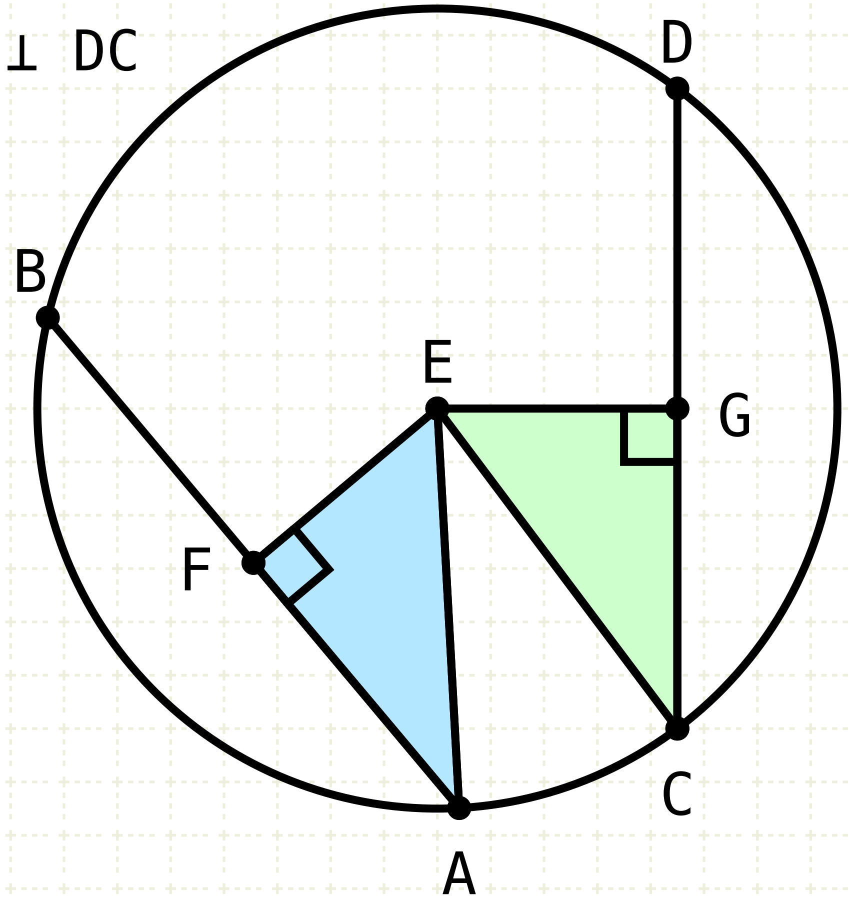

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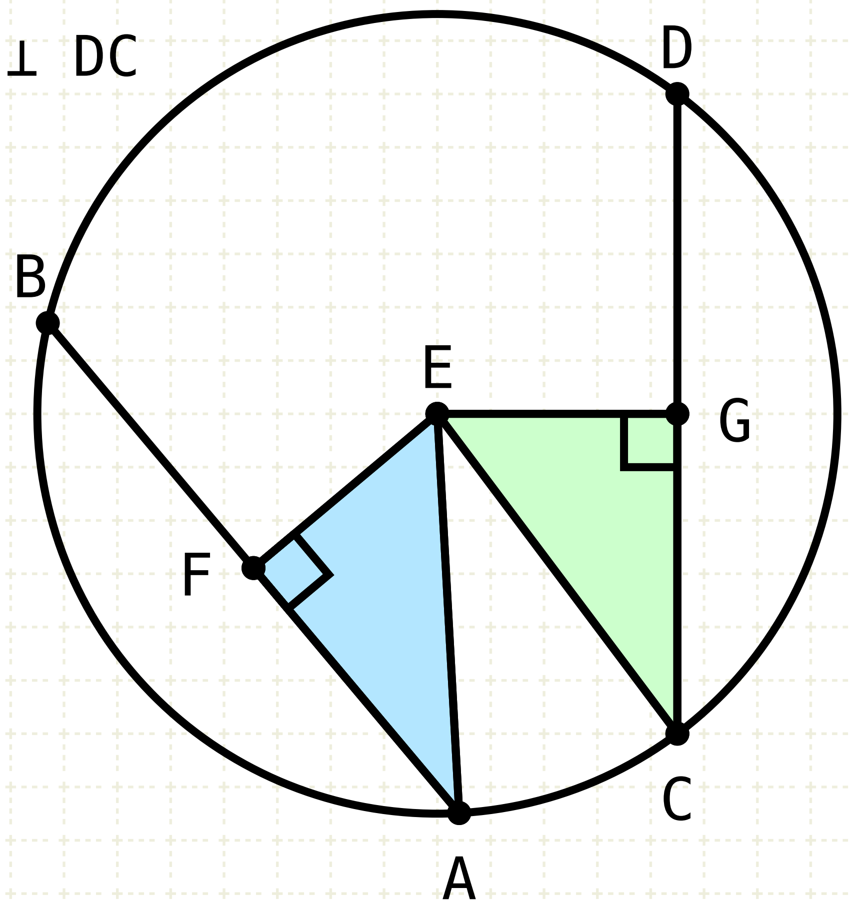

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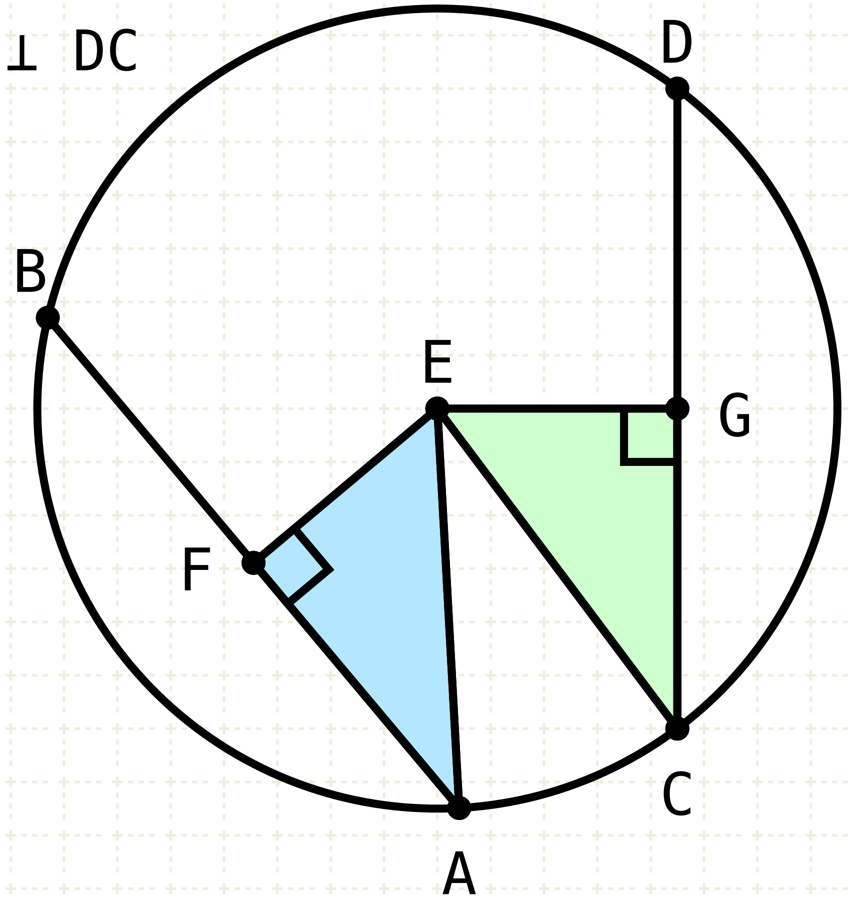

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