Euclid's Elements

Book VII

Definitions:

1 A unit is that by virtue of which each of the things that exist is called one

2 A number is a multitude composed of units. (not one)

3 A number is part of a number, the less of the greater, when it measures the greater

11 A prime number is that which is measured by a unit alone.

12 Numbers prime to one another are those which are measured by a unit alone as a common measure

15 A number is said to multiply a number when that which is multiplied is added to itself as many times as there are units in the other, and thus some number is produced.

20 Numbers are proportional when the first is the same multiple, or the same part, or the same parts, of the second that the third is of the fourth.

As long as algebra and geometry have been separated, their progress have been slow and their uses limited; but when these two sciences have been united, they have lent each mutual forces, and have marched together towards perfection.

Joseph-Louis Lagrange
(1736 to 1813)

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Proposition 5 of Book VII

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Definitions

3. A number is a 'part' of a number, the less of the greater, when it measures the greater

$A = 10,\ B = 2,$

B is part of A

$A = B + B + B + B + B$

4. but 'parts' when it does not measure it

$A=10,\ B=6$

Let the part of A be 2

$p = 2,\ A = p + p + p + p + p$

B is a multiple of the part of A (B is parts of A)

$B = p + p + p$

A part of one number is the same as the part of another number if it is the same fraction

$A = 10,\ B = 4$

$p_A = (1/2)A = 5$

$p_B = (1/2)B = 2$

p_A same as p_B

Proposition 5 of Book VII

If a number be a part of a number, and another be the same part of another, the sum will also be the same part of the sum that the one is of the one.

$$b = (1/q)a$$

$$d = (1/q)c$$

$$\rightarrow (b+d) = (1/q)(a+c)$$

In other words

If b is the same fraction of a as d is to c, then the sum b,d will also be the same fraction of the sum a,c

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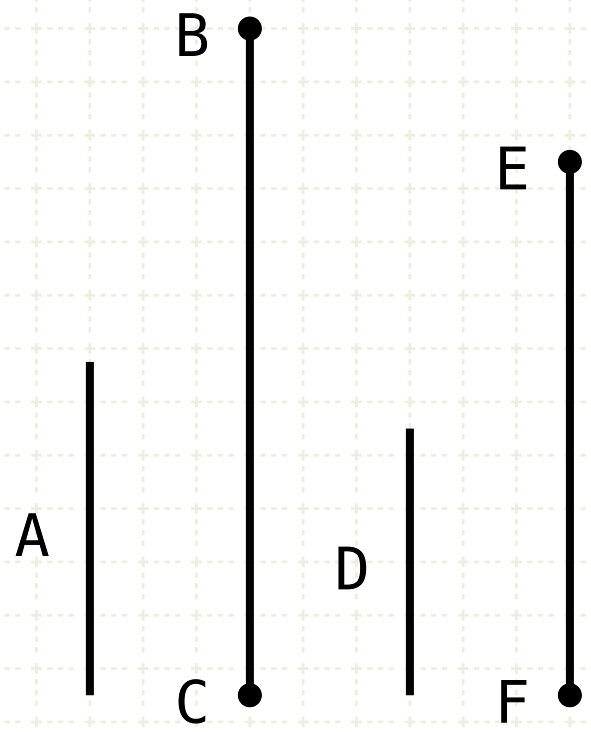

$BC = q \cdot A,\ A = BC/q$

$EF = q \cdot D,\ D = EF/q$

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Proof

Let the number A be part (fraction) of BC, and D be the same part (fraction) of EF

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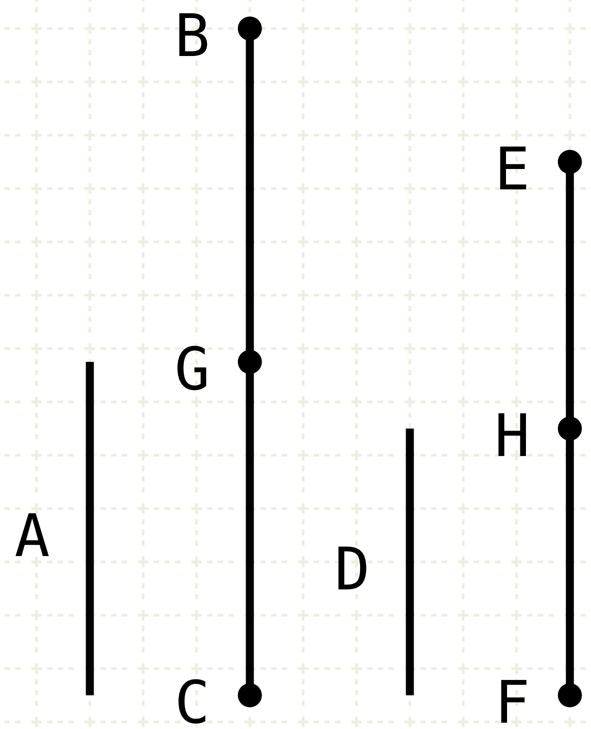

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$A = BG = GC$

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Let the number A be part (fraction) of BC, and D be the same part (fraction) of EF

Let BC be divided into the numbers equal to A, namely BG, GC

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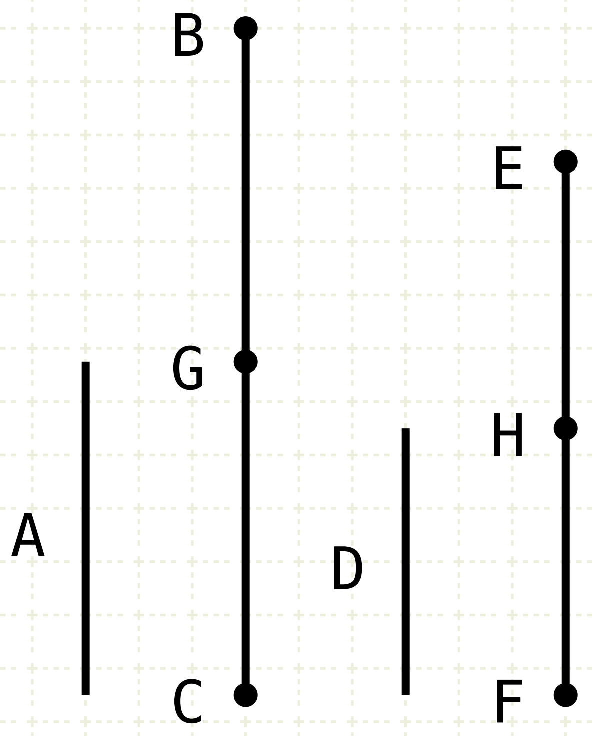

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The sum of BG,EH equals the sum of A,D, since A equals BG, and D equals EH

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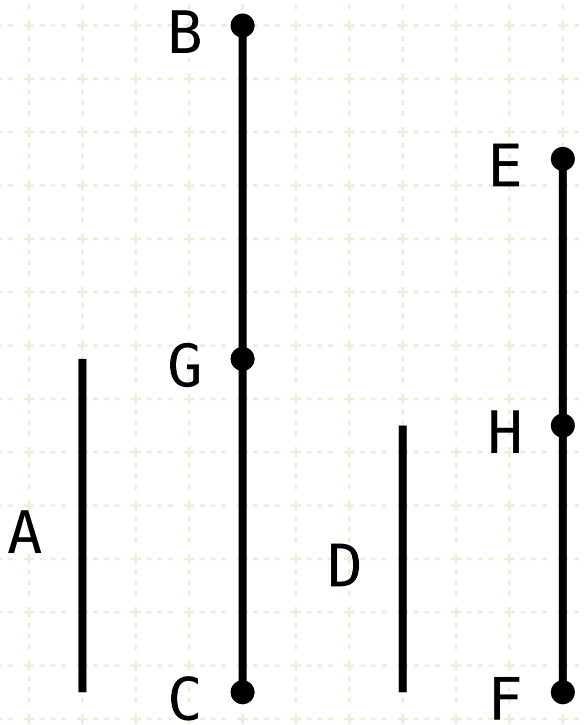

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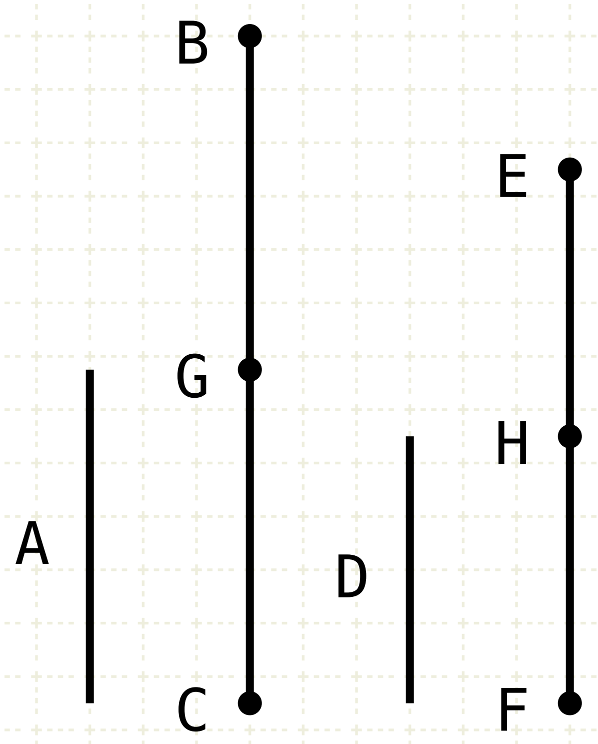

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Given that BC and EF have the same number of parts, the previous process can be repeated for every part in BC and EF, repeatedly adding A,D

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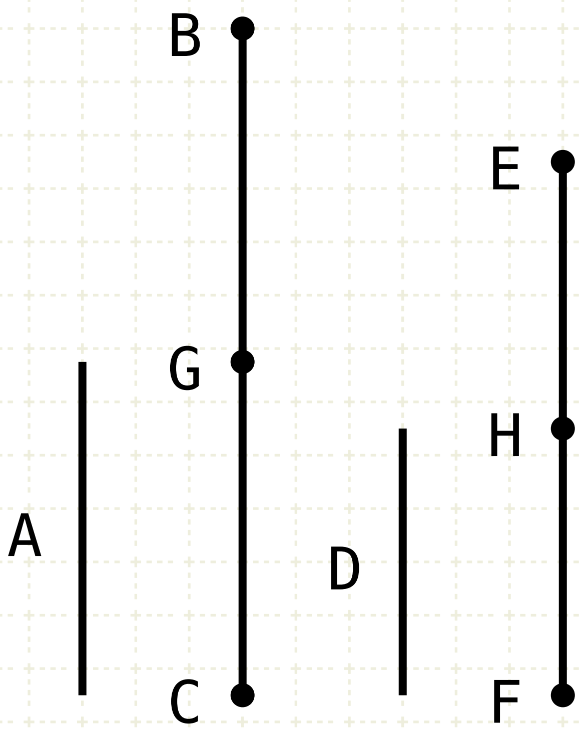

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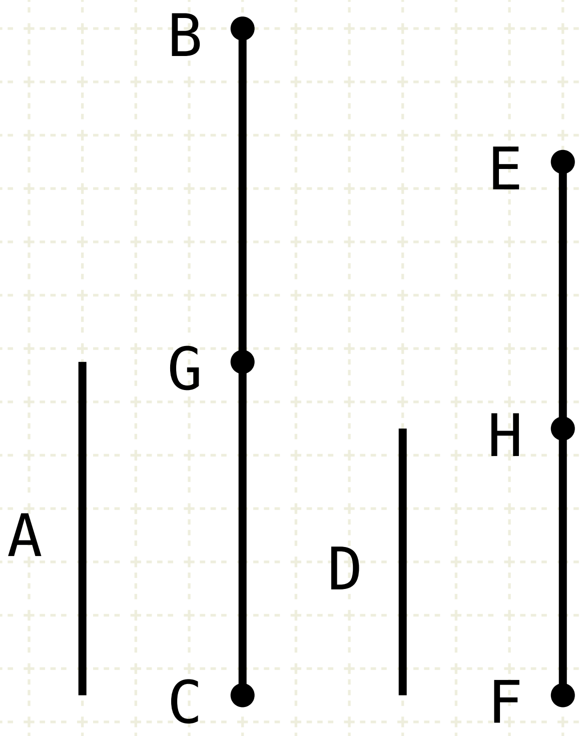

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$$BC + EF = q \cdot (A+D),$$

$$A + D = (BC + EF)/q$$

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Thus the sum of A,D will be the same part (fraction) of BC,EF as A is to BC D is to EF

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