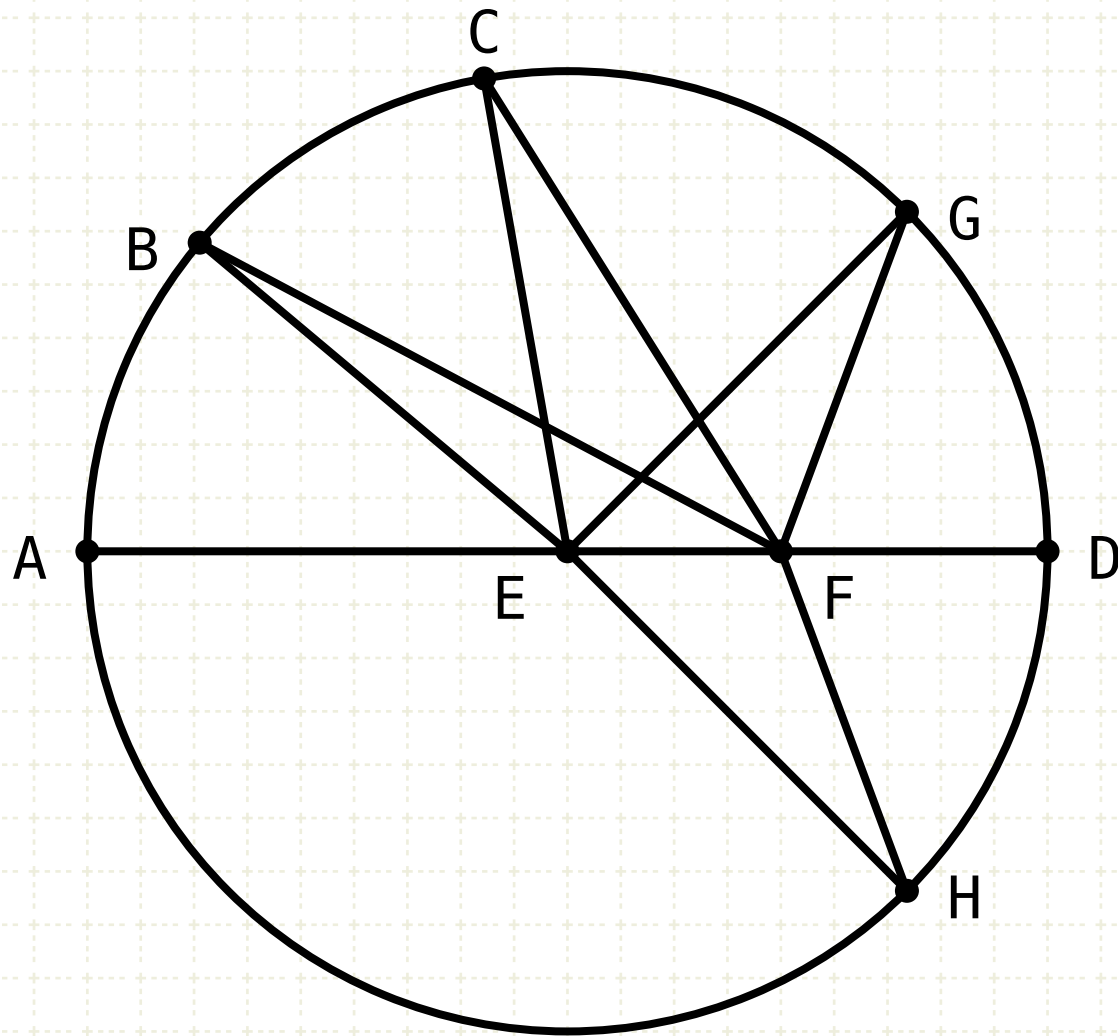


Euclid's Elements

Book III



A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)



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2	A chord of a circle always lies inside the circle	10	A circle does not cut a circle at more points than two	18	If line touches a circle, then it is perpendicular to the diameter that touches that point
3	A line through the centre of a circle bisects a chord, and vice versa	11	Point of contact between two internal circles, and their centres, are collinear	19	If line touches a circle, then the centre of the circle lies on a line perpendicular to the original
4	A line not through the centre of a circle does not bisect a chord	12	Point of contact between two external circles, and their centres, are collinear	20	The angle at the centre of a circle is twice that from an angle from the circumference
5	If two circles cut one another, they will not have the same center	13	A circle does not touch a circle at more points than one, whether it touch it internally or externally.	21	In a circle the angles in the same segment are equal to one another
6	If two circles touch one another, they will not have the same center	14	In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another.	22	The opposite angles of quadrilaterals in circles are equal to two right angles
7	Consider two lines from a point inside a circle to the edge, the longer one will be the one closest to the longest part of the diameter passing through the original point	15	The longest line in a circle is its diameter, shorter the farther away from the diameter	23	On the same straight line there cannot be constructed two similar and unequal segments of circles on the same side
8	Consider two lines from a point outside a circle to the edge, the line closest to the centre will be longer on the concave side and shorter on the convex side	16	A line on the circle, perpendicular to the diameter, lies outside the circle	24	Similar segments of circles on equal straight lines are equal to one another



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| 26 | In equal circles equal angles stand on equal circumferences | 35 | If two circle chords intersect, the segments on one multiplied together equals the segments of the other multiplied together |
| 27 | In equal circles angles standing on equal circumferences are equal to one another | 36 | Secant-tangent law |
| 28 | In equal circles equal straight lines cut off equal circumferences | 37 | Converse of the secant-tangent law |
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| 33 | Construct a circle segment on a given line, such that the angle within the segment is equal to a given angle | | |



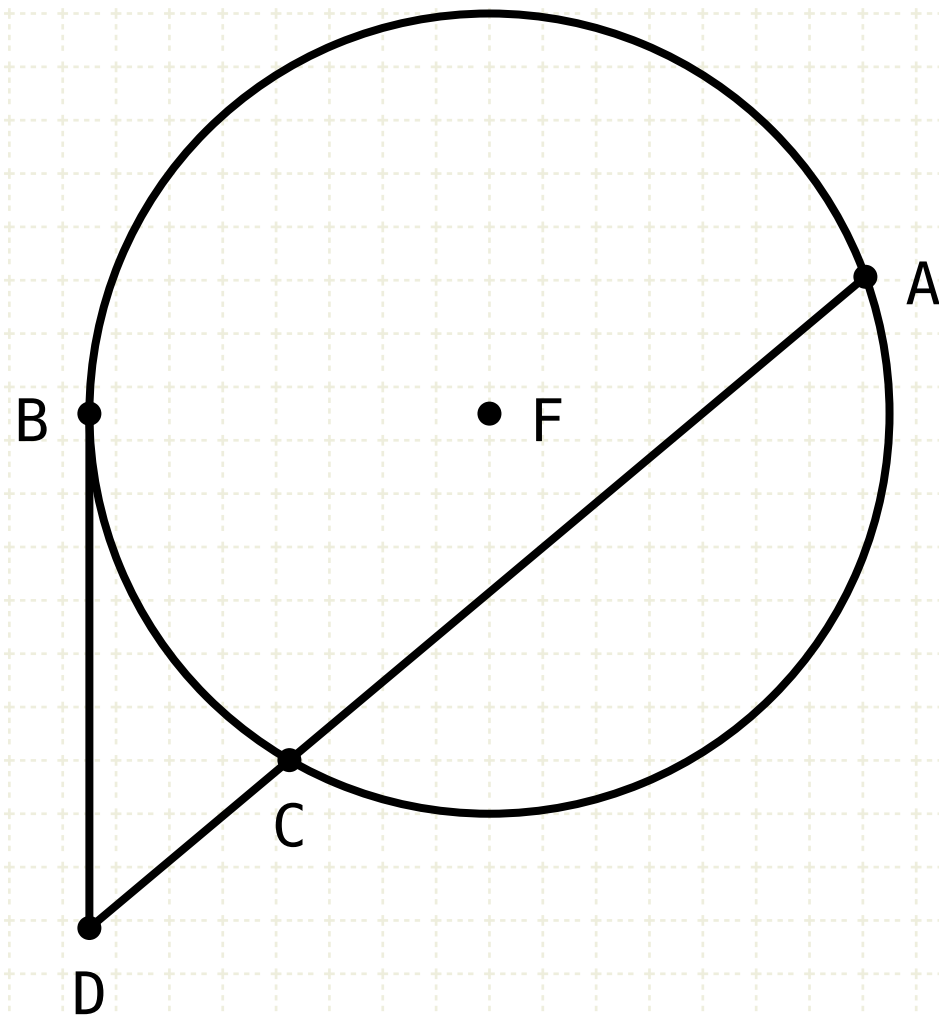
Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.



Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.



$$AD \cdot CD = BD^2$$

In other words

Let line AD cut a circle in two places, where A,C are the intersection points on the circumference of the circle

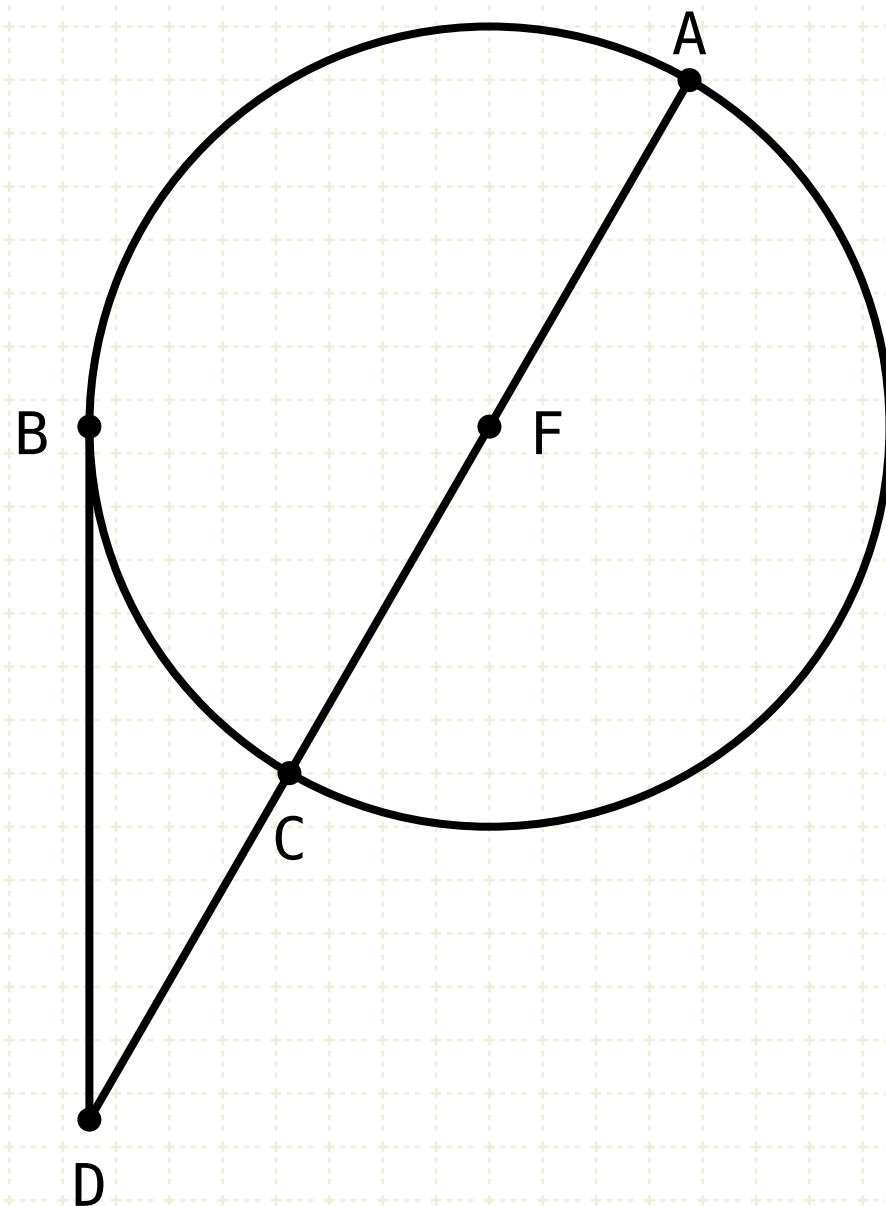
Definition of secant: a line that cuts a circle in two points

Let a line drawn from D touch the circle at point B

Then the product AD,CD equals BD squared

Proposition 36 of Book III

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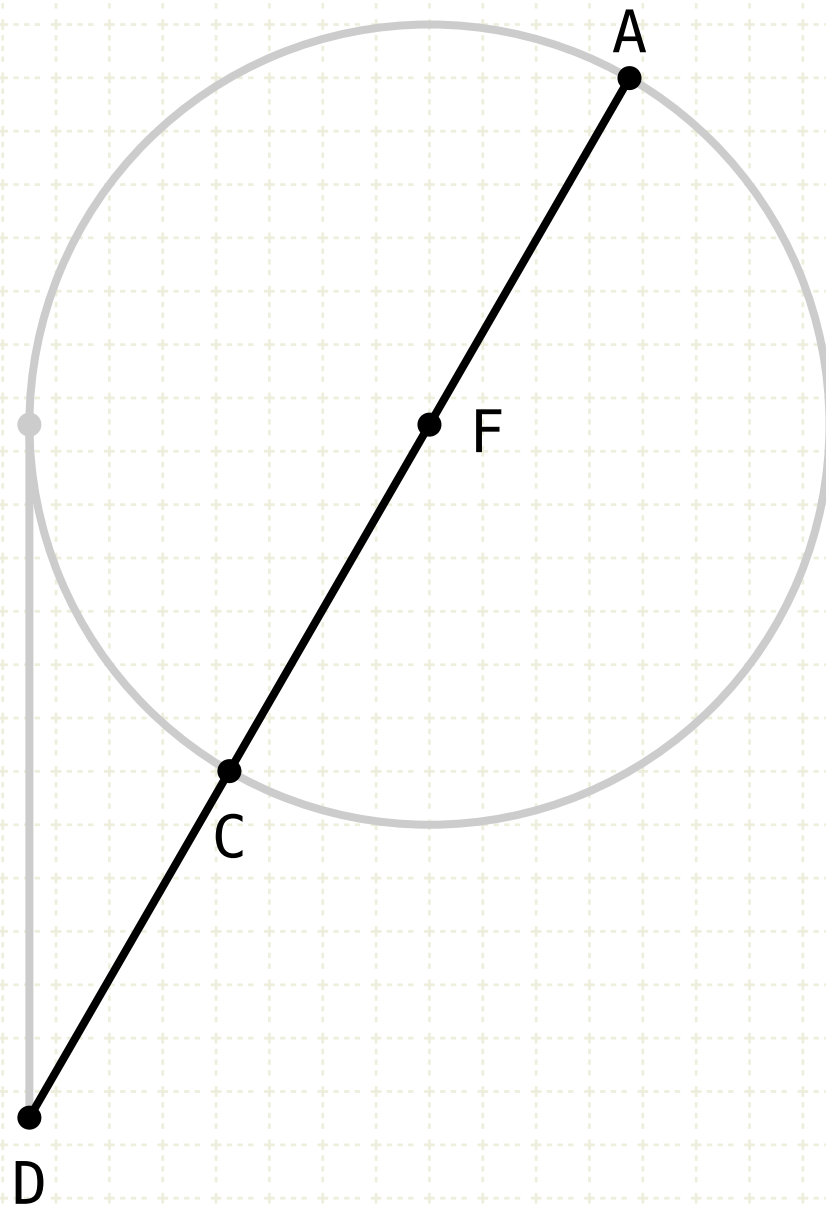


$$AF = FC$$

Proof - AD passes through centre of circle

Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.



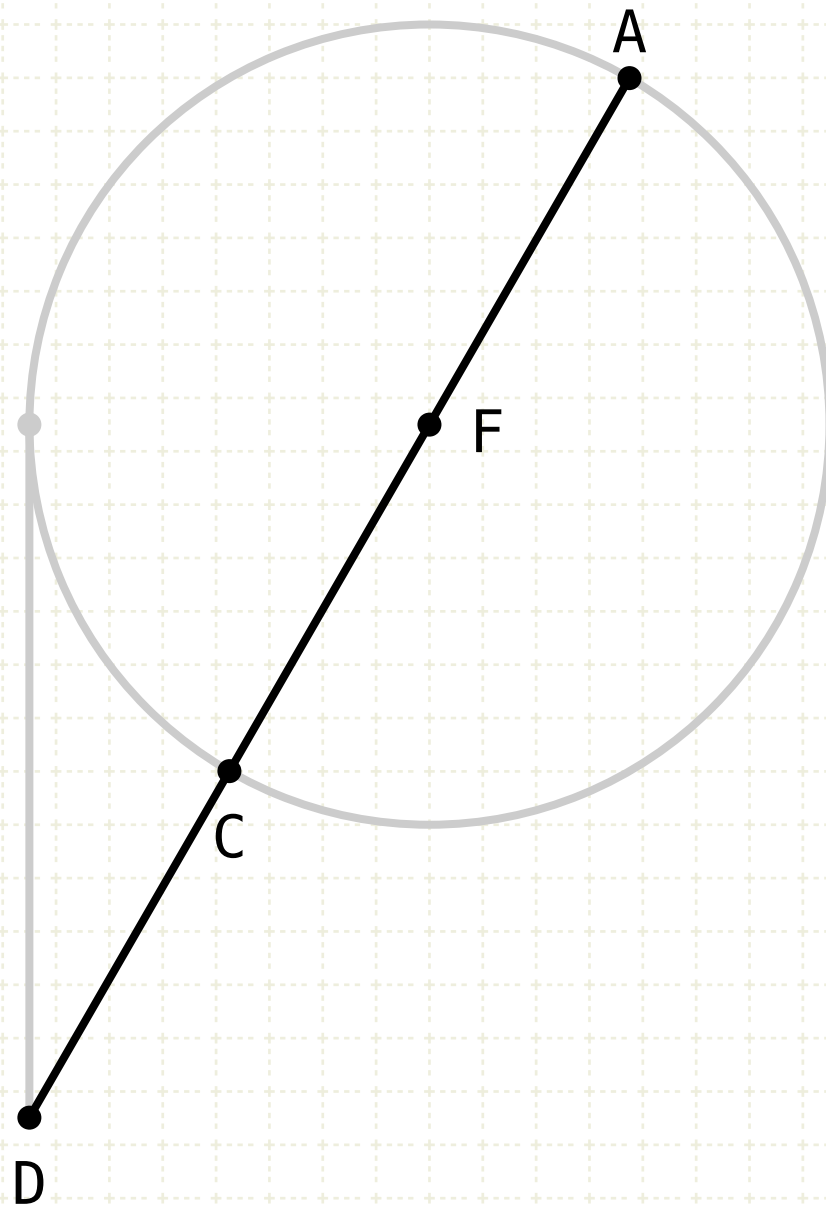
$$\begin{aligned} AF &= FC \\ AD \cdot CD + FC^2 &= FD^2 \end{aligned}$$

Proof - AD passes through centre of circle

If a line AC is bisected at point F, and extended from C to point D, the product AD,CD plus the square of FC equals the square of FD (II·6)

Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.



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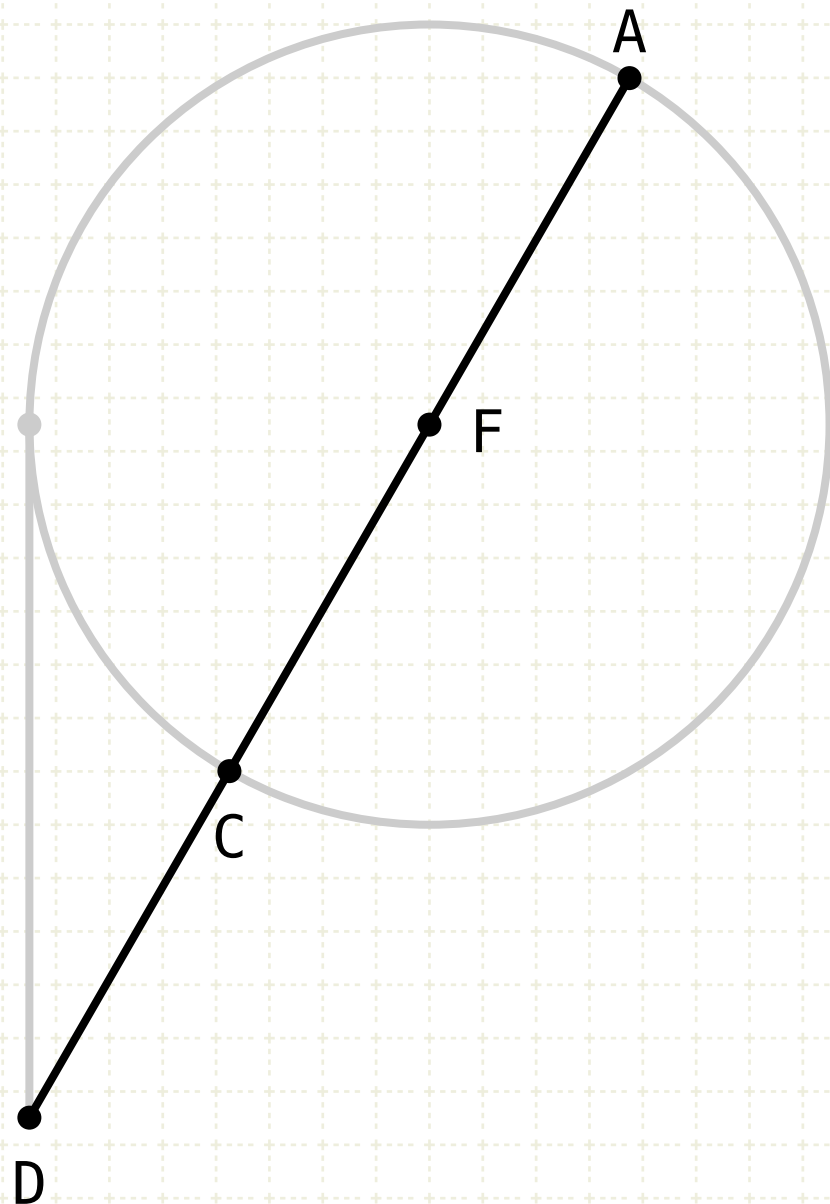
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If a line AC is bisected at point F, and extended from C to point D, the product AD,CD plus the square of FC equals the square of FD (II-6)



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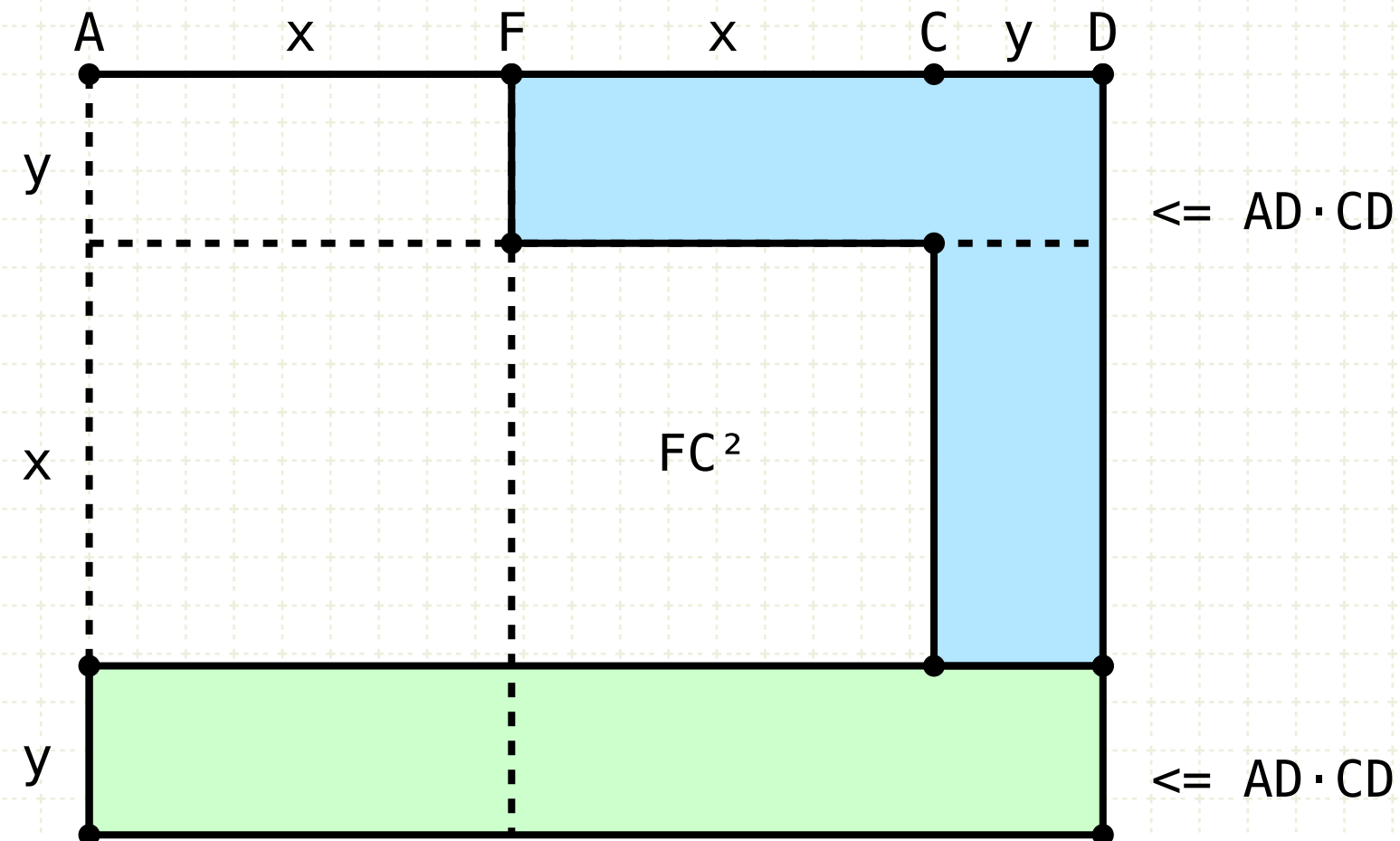


$$AF = FC$$

$$AD \cdot CD + FC^2 = FD^2$$

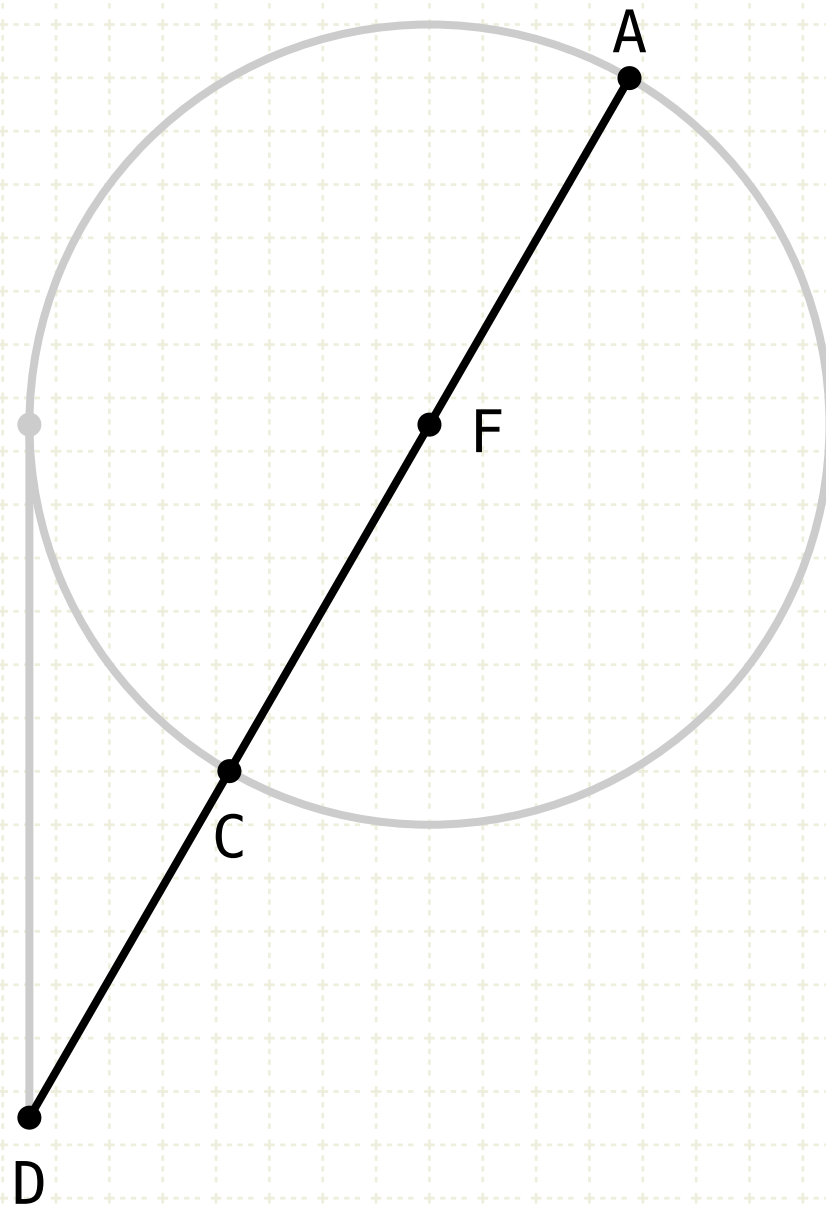
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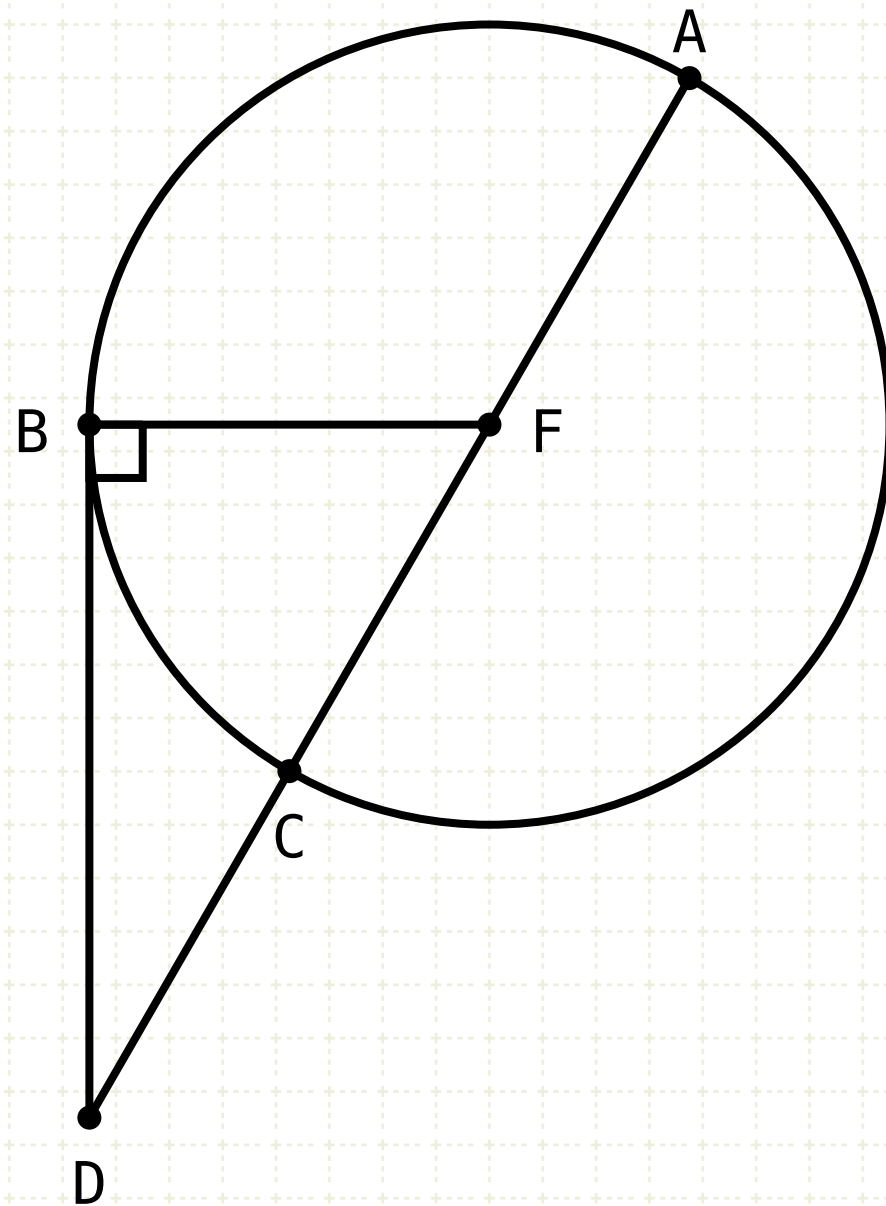
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$$AF = FC$$

$$AD \cdot CD + FC^2 = FD^2$$

$$BF = FC$$

Proof - AD passes through centre of circle

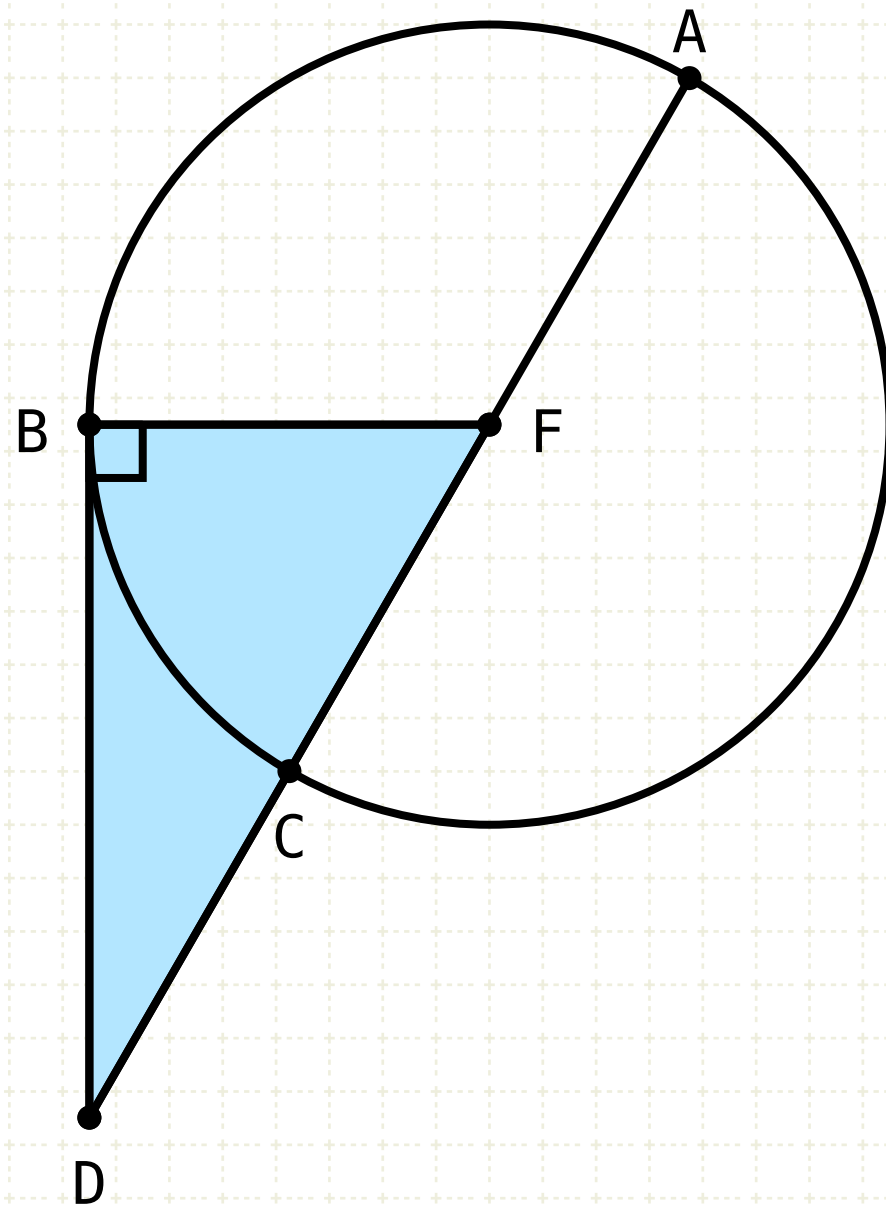
If a line AC is bisected at point F, and extended from C to point D, the product AD,CD plus the square of FC equals the square of FD (II·6)

Draw line FB

Angle FBD is right (III·18)

Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.



$$\begin{aligned}AF &= FC \\AD \cdot CD + FC^2 &= FD^2 \\BF &= FC \\BD^2 + BF^2 &= FD^2\end{aligned}$$

Proof - AD passes through centre of circle

If a line AC is bisected at point F, and extended from C to point D, the product AD,CD plus the square of FC equals the square of FD (II·6)

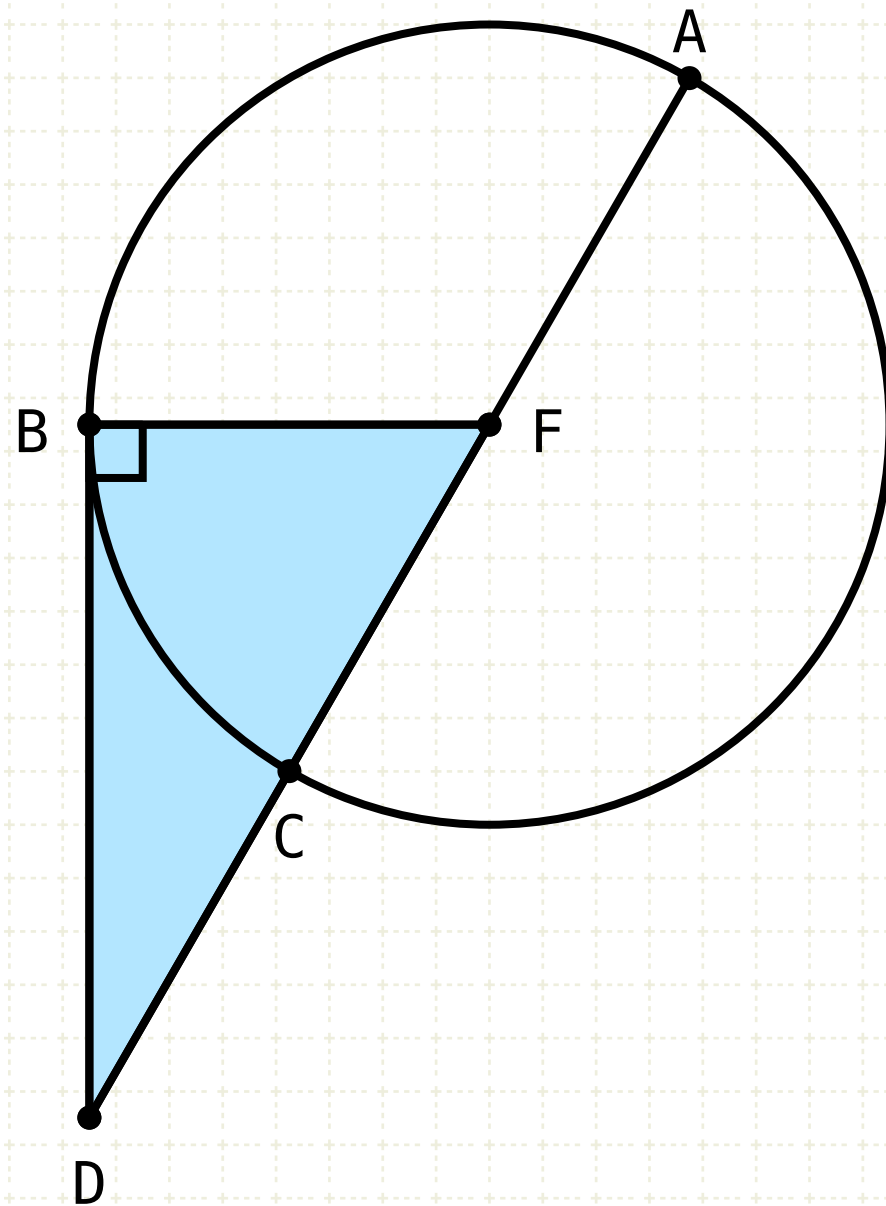
Draw line FB

Angle FBD is right (III·18)

The triangle FBD is a right triangle, and thus follows Pythagoras' rule (I·47), thus the sum of the squares BF,BD equals the square of FD

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$$AF = FC$$

$$AD \cdot CD + FC^2 = FD^2$$

$$BF = FC$$

$$BD^2 + BF^2 = FD^2$$

$$BD^2 + FC^2 = FD^2$$

Proof - AD passes through centre of circle

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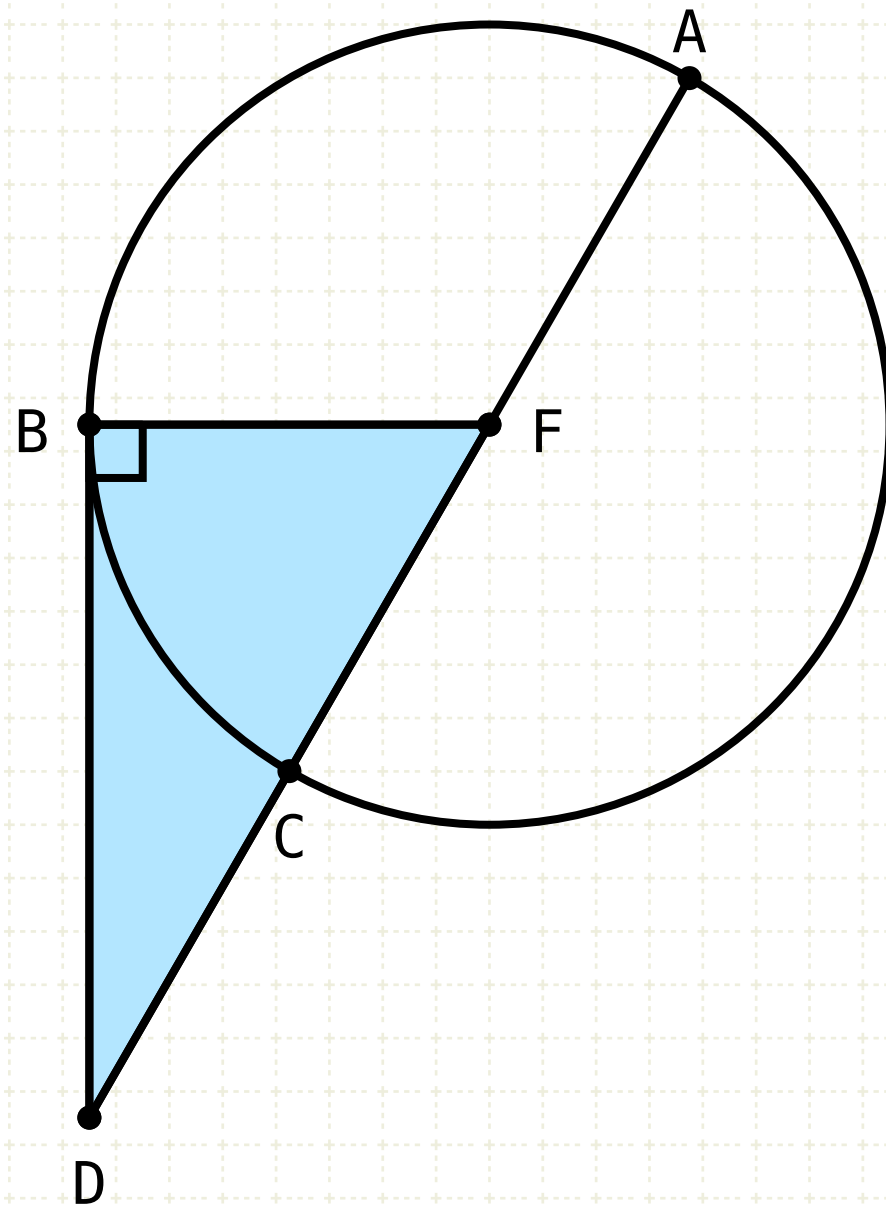
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Since BF equals FC, then the square of BF equals the square of FC

Proposition 36 of Book III

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$$AF = FC$$

$$AD \cdot CD + FC^2 = FD^2$$

$$BF = FC$$

$$BD^2 + BF^2 = FD^2$$

$$BD^2 + FC^2 = FD^2$$

$$BD^2 = FD^2 - FC^2$$

Proof - AD passes through centre of circle

If a line AC is bisected at point F, and extended from C to point D, the product AD,CD plus the square of FC equals the square of FD (II·6)

Draw line FB

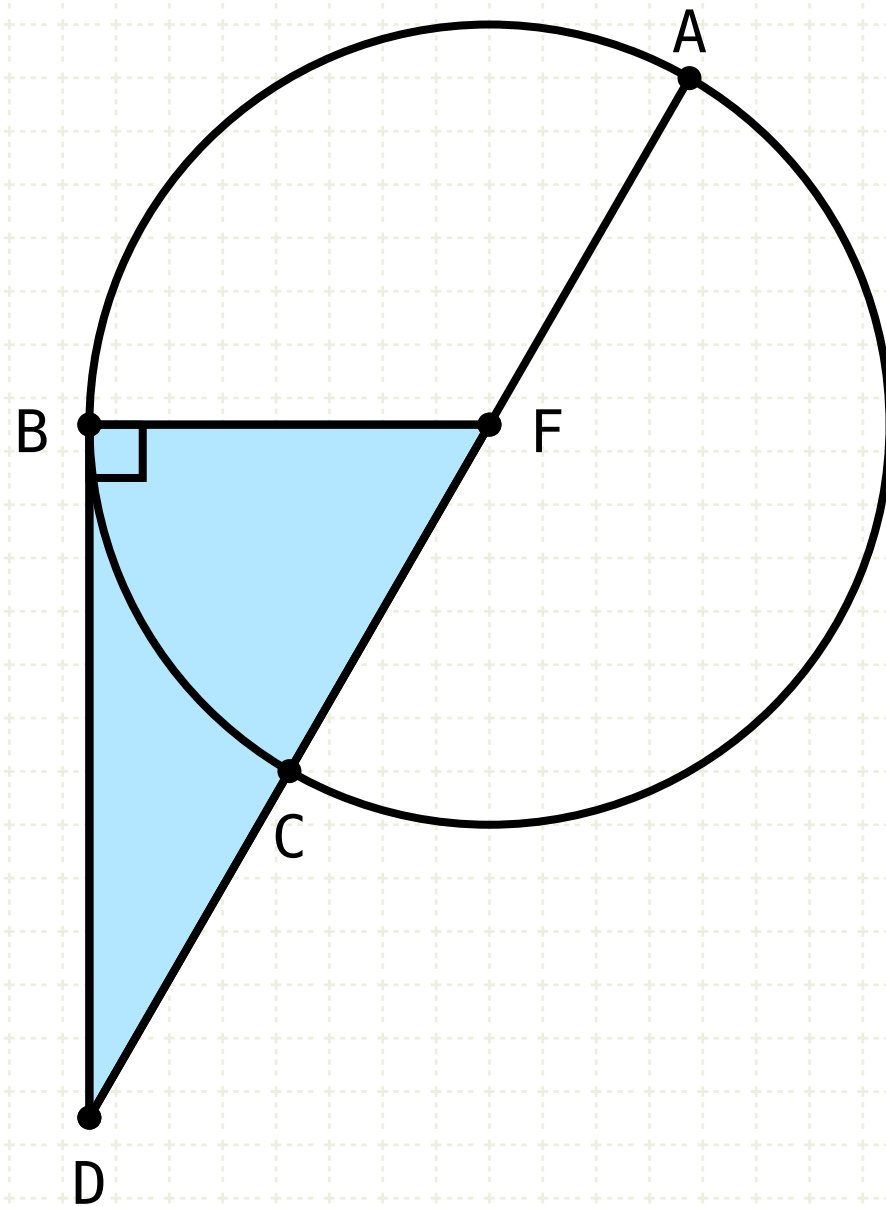
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Since BF equals FC, then the square of BF equals the square of FC

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$$\begin{aligned}AF &= FC \\AD \cdot CD + FC^2 &= FD^2 \\BF &= FC \\BD^2 + BF^2 &= FD^2 \\BD^2 + FC^2 &= FD^2\end{aligned}$$

$$\begin{aligned}BD^2 &= FD^2 - FC^2 \\AD \cdot CD &= FD^2 - FC^2\end{aligned}$$

Proof - AD passes through centre of circle

If a line AC is bisected at point F, and extended from C to point D, the product AD,CD plus the square of FC equals the square of FD (II·6)

Draw line FB

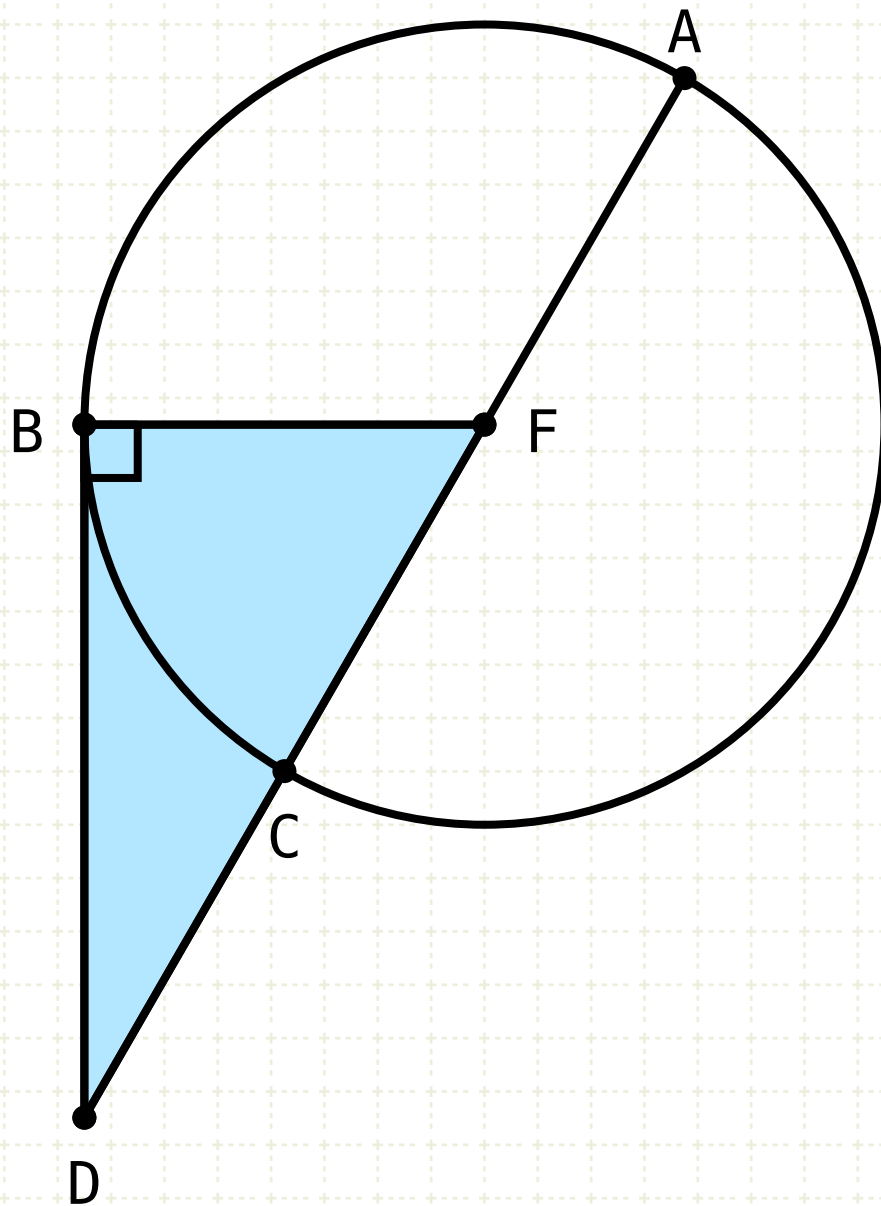
Angle FBD is right (III·18)

The triangle FBD is a right triangle, and thus follows Pythagoras' rule (I·47), thus the sum of the squares BF,BD equals the square of FD

Since BF equals FC, then the square of BF equals the square of FC

Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.



$$\begin{aligned}AF &= FC \\AD \cdot CD + FC^2 &= FD^2 \\BF &= FC \\BD^2 + BF^2 &= FD^2 \\BD^2 + FC^2 &= FD^2\end{aligned}$$

$$\begin{aligned}BD^2 &= FD^2 - FC^2 \\AD \cdot CD &= FD^2 - FC^2 \\AD \cdot CD &= BD^2\end{aligned}$$

Proof - AD passes through centre of circle

If a line AC is bisected at point F, and extended from C to point D, the product AD,CD plus the square of FC equals the square of FD (II·6)

Draw line FB

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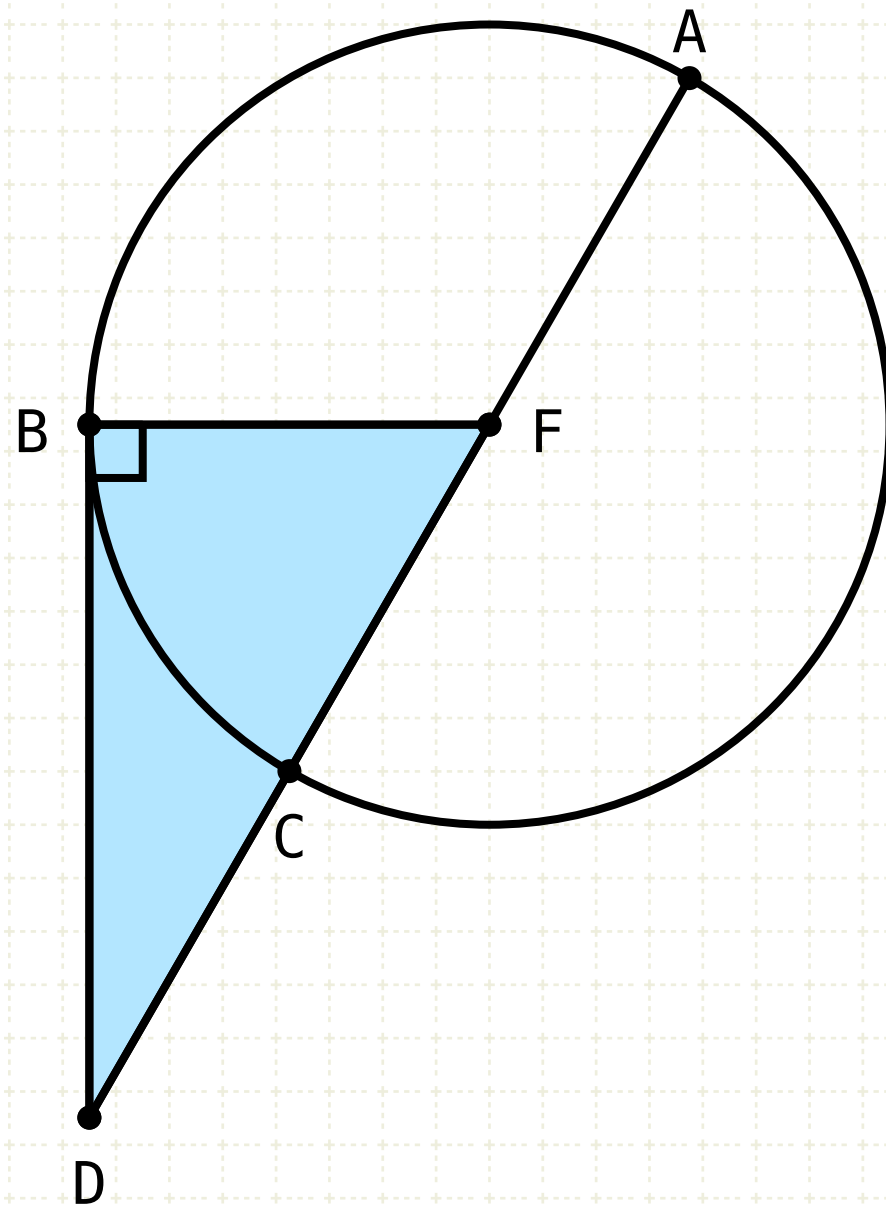
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Since BF equals FC, then the square of BF equals the square of FC

Thus, the product of AD,CD equals the square of BD

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$$AF = FC$$

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$$BF = FC$$

$$BD^2 + BF^2 = FD^2$$

$$BD^2 + FC^2 = FD^2$$

$$BD^2 = FD^2 - FC^2$$

$$AD \cdot CD = FD^2 - FC^2$$

$$AD \cdot CD = BD^2$$

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If a line AC is bisected at point F, and extended from C to point D, the product AD,CD plus the square of FC equals the square of FD (II·6)

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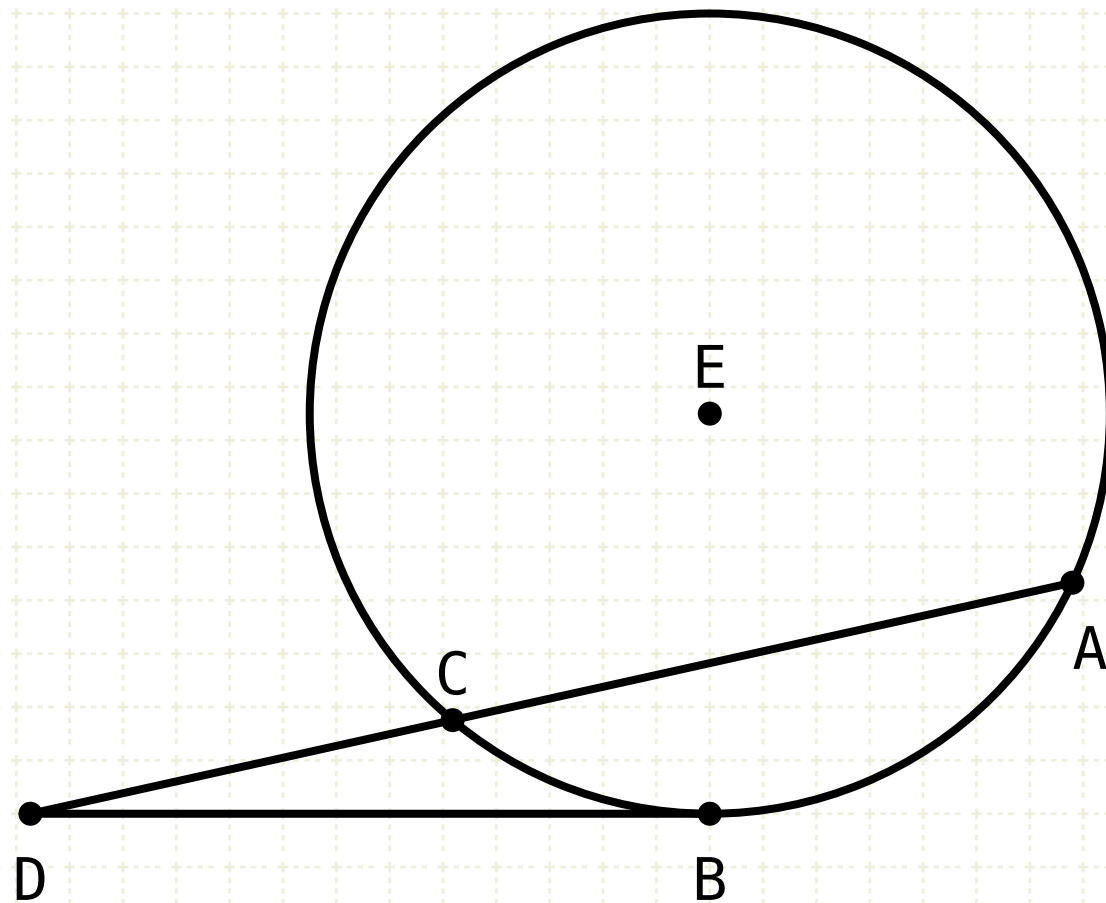
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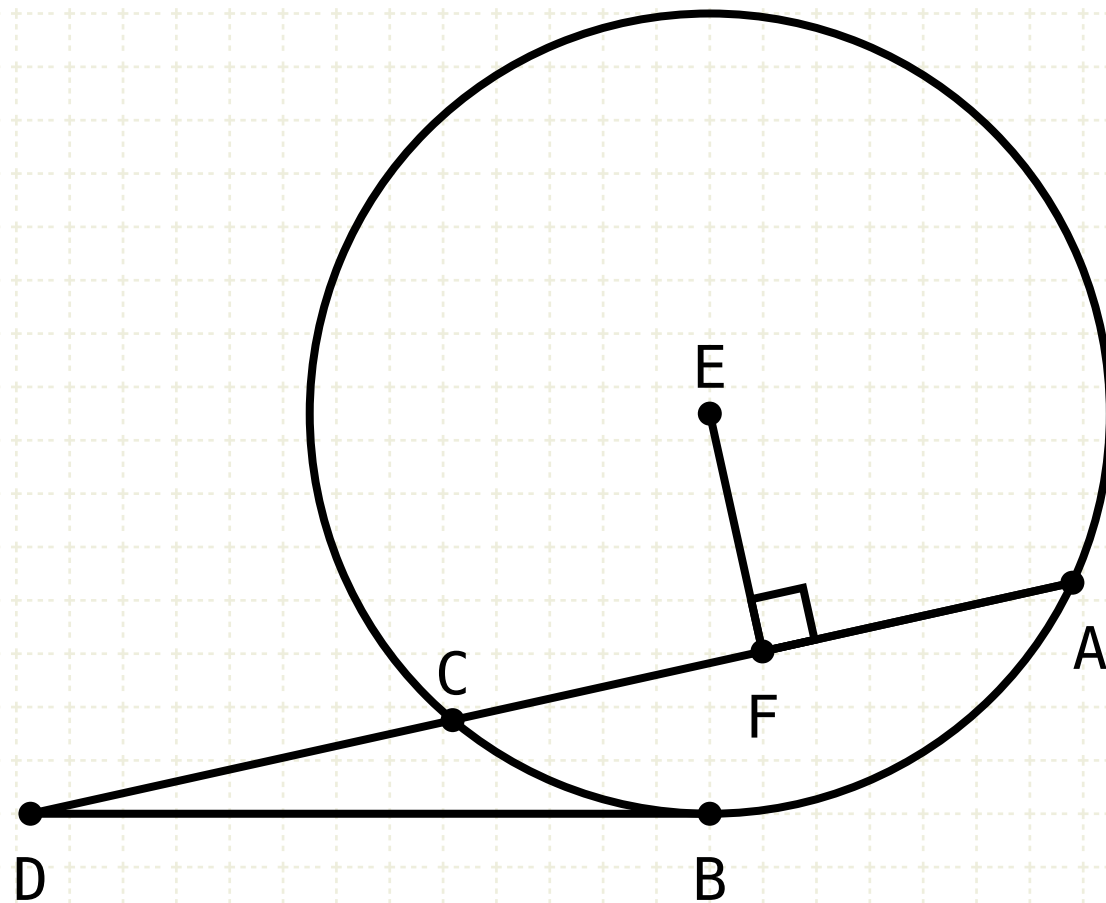
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**Proof - AD does not pass through
centre of circle**

Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.

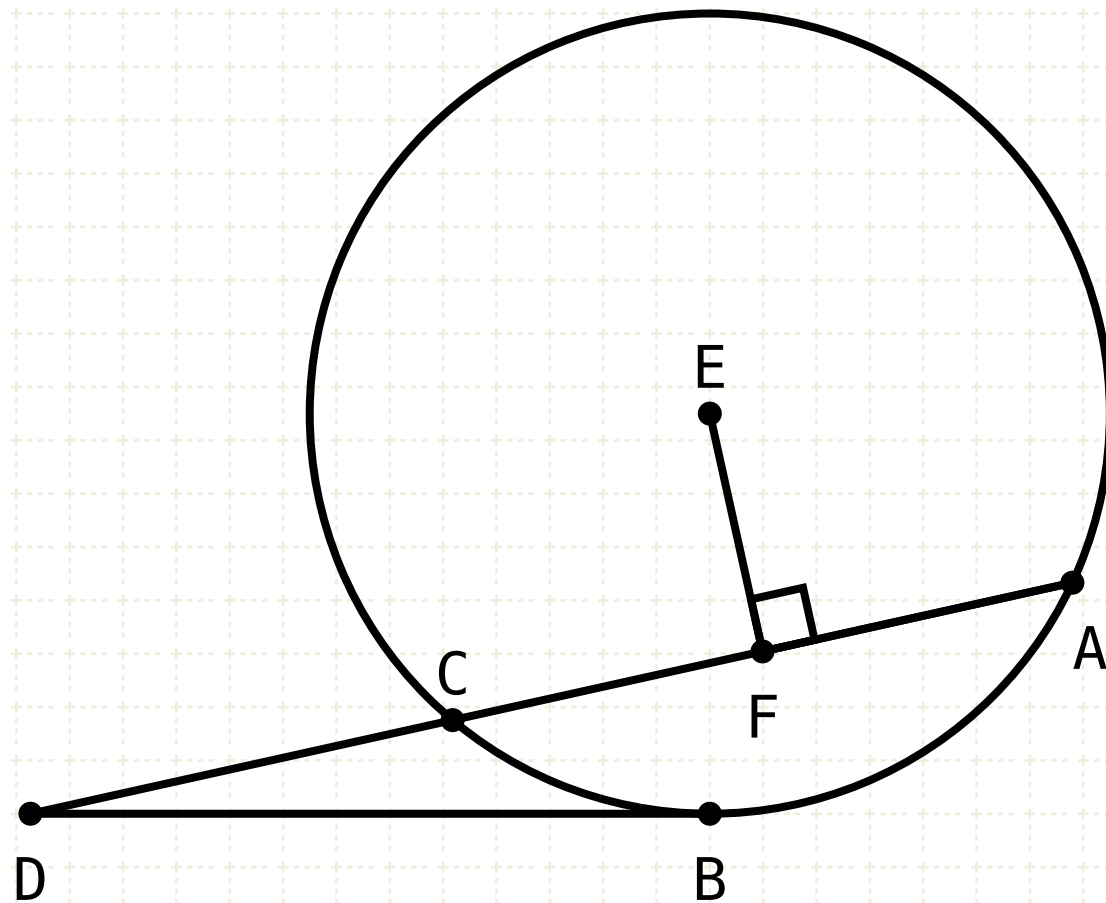


Proof - AD does not pass through centre of circle

Draw a line EF from the centre of the circle E , perpendicular to the line DA

Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.



$$CF = FA$$

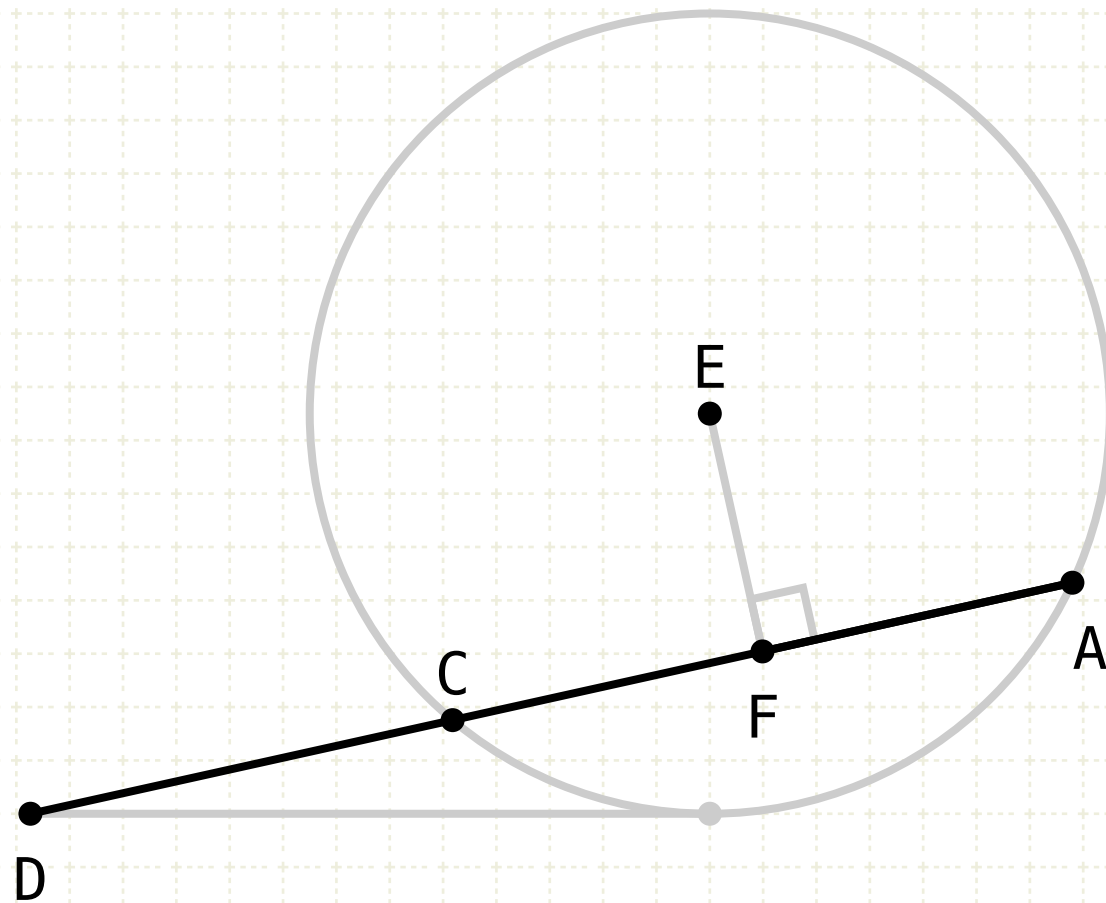
Proof - AD does not pass through centre of circle

Draw a line EF from the centre of the circle E, perpendicular to the line DA

Lines CF and FA are equal (III·18)

Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.



$$CF = FA$$
$$AD \cdot CD + FC^2 = FD^2$$

Proof - AD does not pass through centre of circle

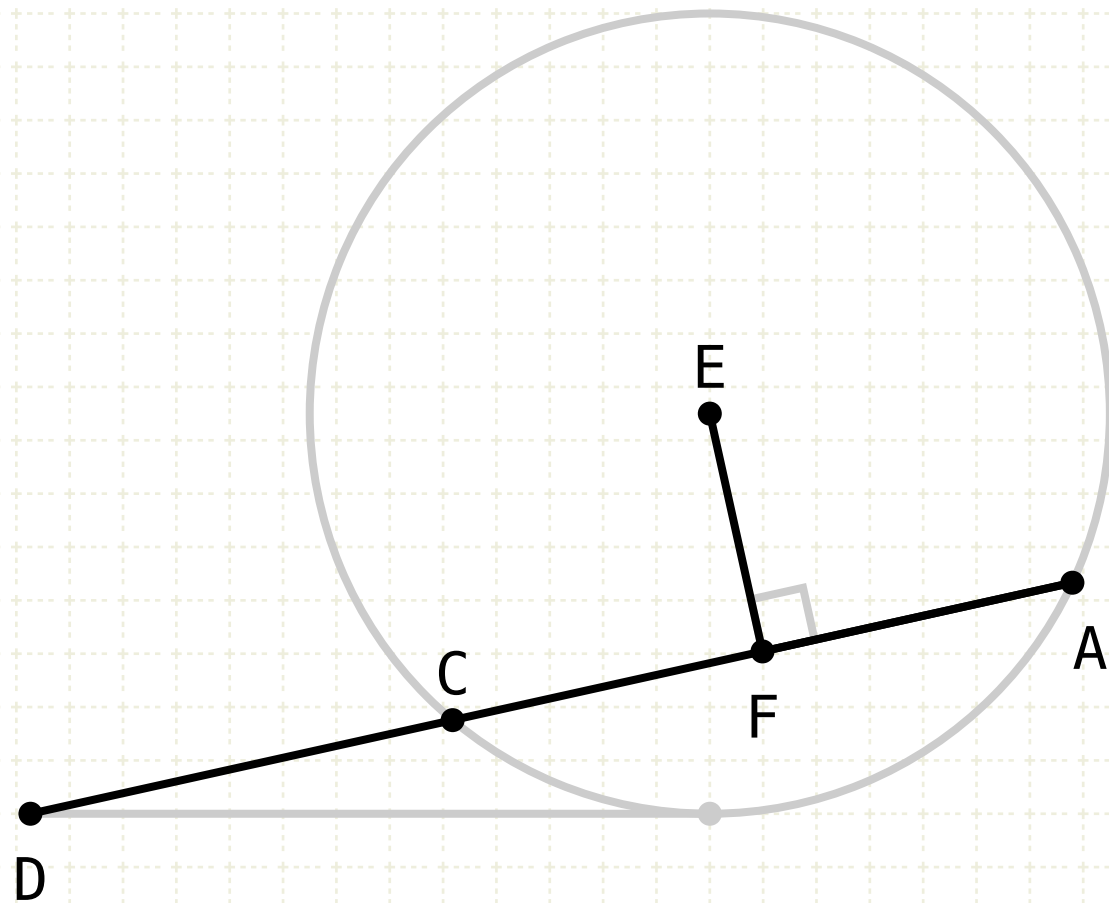
Draw a line EF from the centre of the circle E, perpendicular to the line DA

Lines CF and FA are equal (III·18)

If a line AC is bisected at point F, and extended from C to point D, the product AD,CD plus the square of FC equals the square of FD (II·6)

Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.



$$CF = FA$$

$$AD \cdot CD + FC^2 = FD^2$$

$$AD \cdot CD + FC^2 + EF^2 = FD^2 + EF^2$$

Proof - AD does not pass through centre of circle

Draw a line EF from the centre of the circle E, perpendicular to the line DA

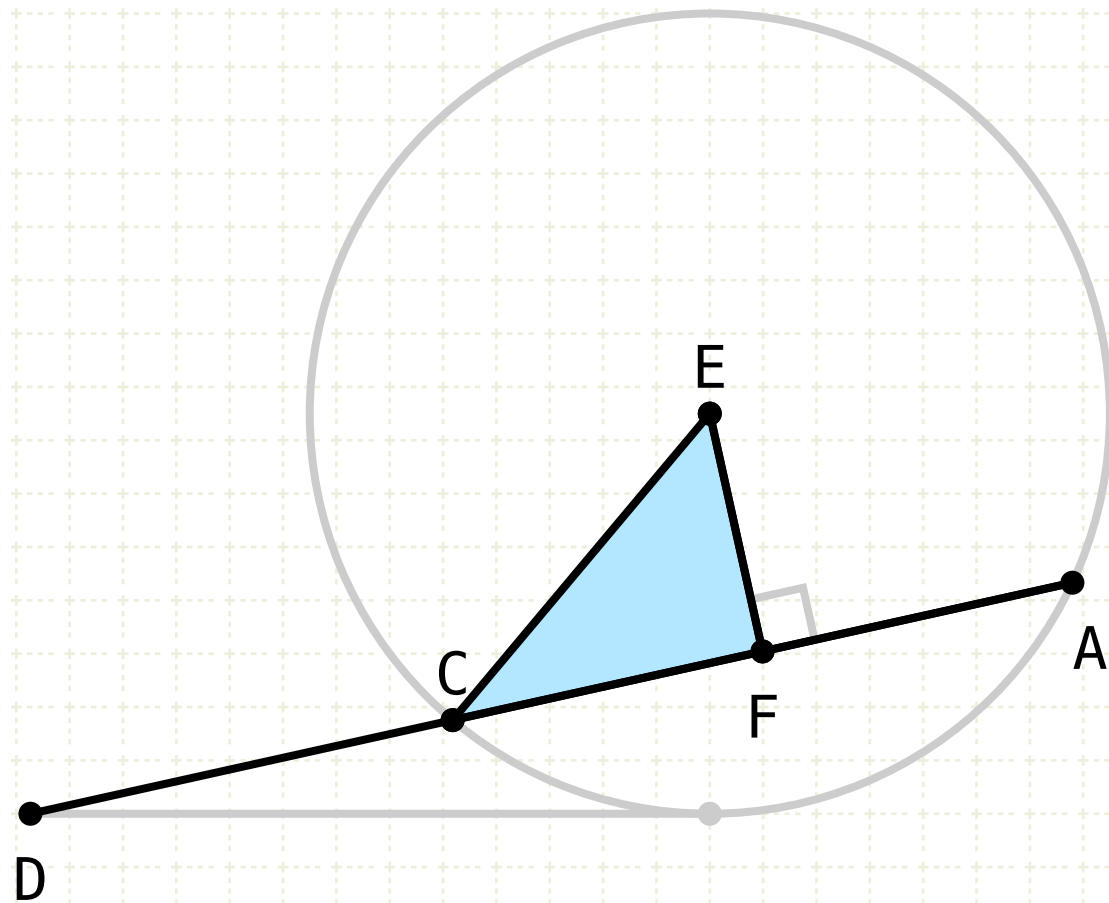
Lines CF and FA are equal (III·18)

If a line AC is bisected at point F, and extended from C to point D, the product AD,CD plus the square of FC equals the square of FD (II·6)

Add the square of EF to both sides of the equality

Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.



$$\begin{aligned}CF &= FA \\AD \cdot CD + FC^2 &= FD^2 \\AD \cdot CD + FC^2 + EF^2 &= FD^2 + EF^2 \\FC^2 + EF^2 &= EC^2\end{aligned}$$

Proof - AD does not pass through centre of circle

Draw a line EF from the centre of the circle E, perpendicular to the line DA

Lines CF and FA are equal (III·18)

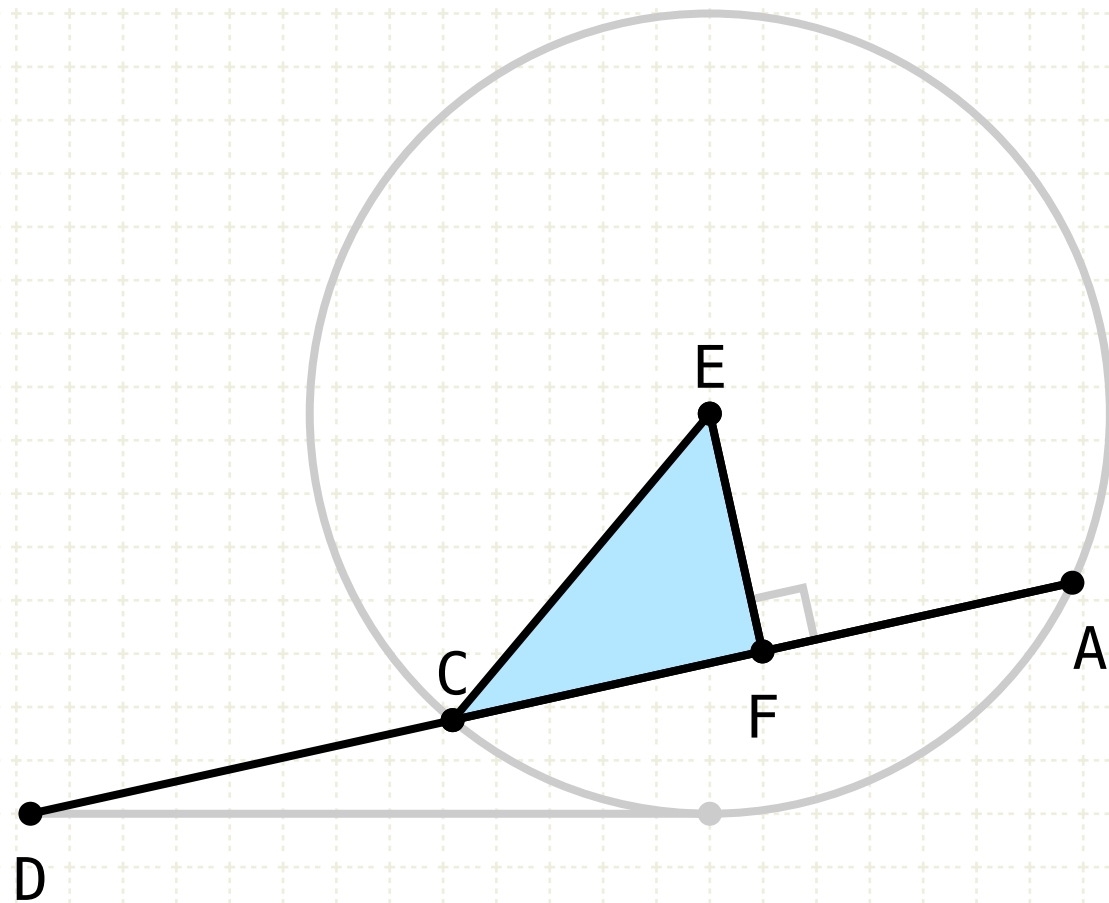
If a line AC is bisected at point F, and extended from C to point D, the product AD,CD plus the square of FC equals the square of FD (II·6)

Add the square of EF to both sides of the equality

In the triangle ECF, the sum of the squares EF,FC equal the square of EC (I·47)

Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.



$$\begin{aligned}CF &= FA \\AD \cdot CD + FC^2 &= FD^2 \\AD \cdot CD + FC^2 + EF^2 &= FD^2 + EF^2 \\FC^2 + EF^2 &= EC^2 \\AD \cdot CD + EC^2 &= FD^2 + EF^2\end{aligned}$$

Proof - AD does not pass through centre of circle

Draw a line EF from the centre of the circle E, perpendicular to the line DA

Lines CF and FA are equal (III·18)

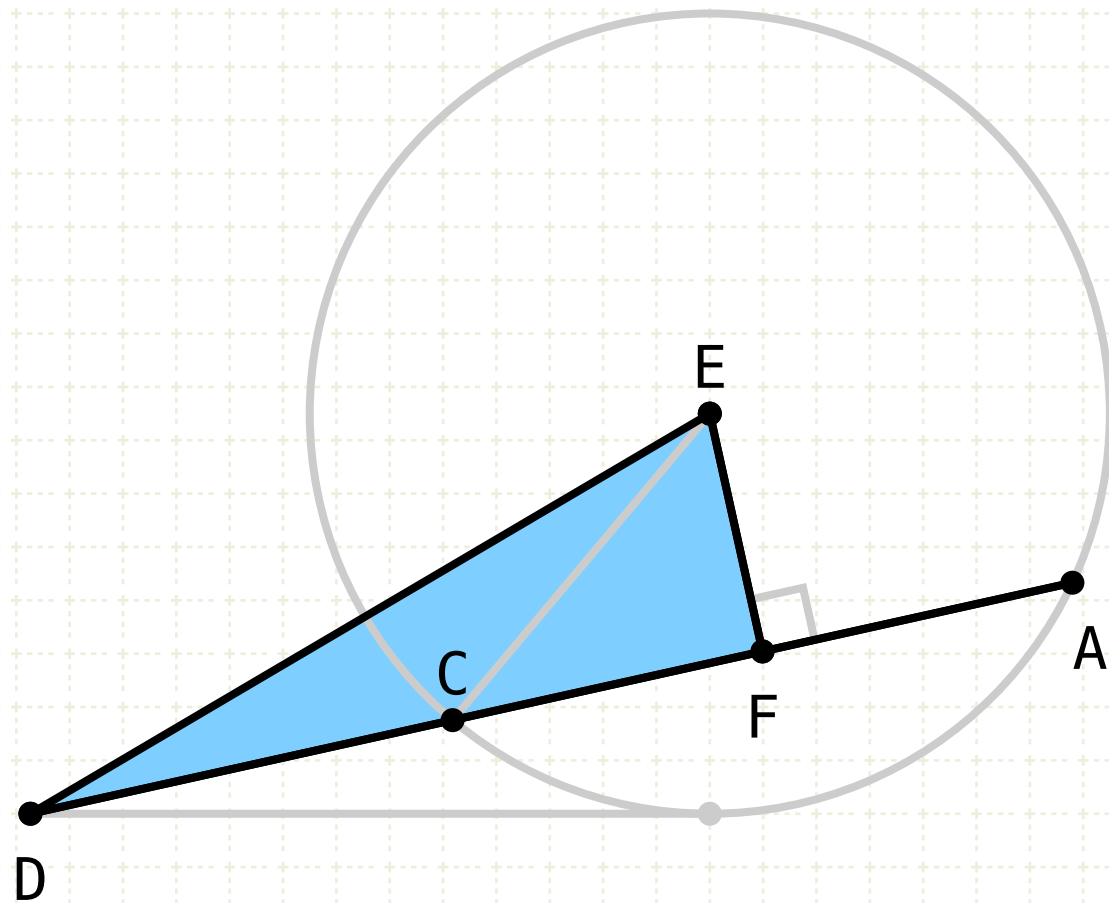
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Add the square of EF to both sides of the equality

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$$\begin{aligned}CF &= FA \\AD \cdot CD + FC^2 &= FD^2 \\AD \cdot CD + FC^2 + EF^2 &= FD^2 + EF^2 \\FC^2 + EF^2 &= EC^2 \\AD \cdot CD + EC^2 &= FD^2 + EF^2 \\FD^2 + EF^2 &= ED^2\end{aligned}$$

Proof - AD does not pass through centre of circle

Draw a line EF from the centre of the circle E, perpendicular to the line DA

Lines CF and FA are equal (III·18)

If a line AC is bisected at point F, and extended from C to point D, the product AD,CD plus the square of FC equals the square of FD (II·6)

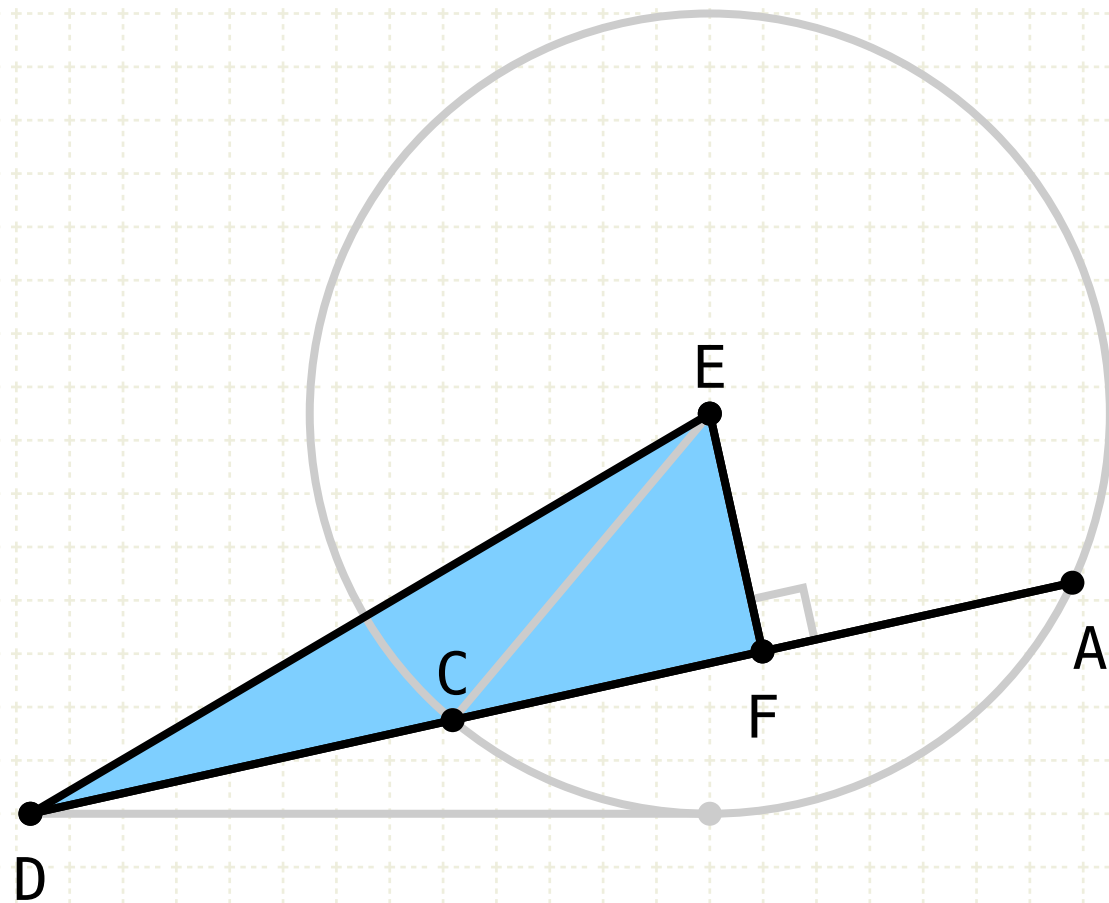
Add the square of EF to both sides of the equality

In the triangle ECF, the sum of the squares EF,FC equal the square of EC (I·47)

Similarly, the sum of the squares FD,EF equal the square of ED (I·47)

Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.



$$CF = FA$$

$$AD \cdot CD + FC^2 = FD^2$$

$$AD \cdot CD + FC^2 + EF^2 = FD^2 + EF^2$$

$$FC^2 + EF^2 = EC^2$$

$$AD \cdot CD + EC^2 = FD^2 + EF^2$$

$$FD^2 + EF^2 = ED^2$$

$$AD \cdot CD + EC^2 = ED^2$$

Proof - AD does not pass through centre of circle

Draw a line EF from the centre of the circle E, perpendicular to the line DA

Lines CF and FA are equal (III·18)

If a line AC is bisected at point F, and extended from C to point D, the product AD,CD plus the square of FC equals the square of FD (II·6)

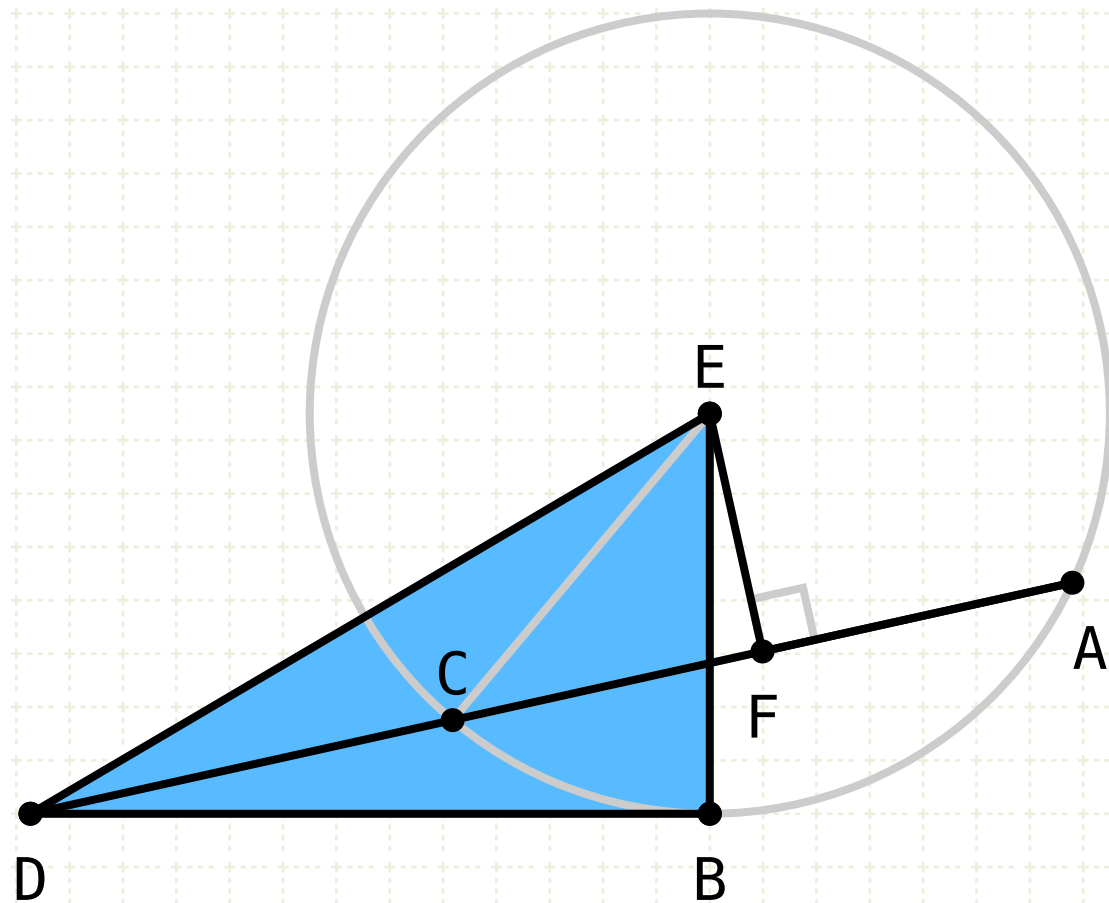
Add the square of EF to both sides of the equality

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Similarly, the sum of the squares FD,EF equal the square of ED (I·47)

Proposition 36 of Book III

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$$CF = FA$$

$$AD \cdot CD + FC^2 = FD^2$$

$$AD \cdot CD + FC^2 + EF^2$$

$$= FD^2 + EF^2$$

$$FC^2 + EF^2 = EC^2$$

$$AD \cdot CD + EC^2$$

$$= FD^2 + EF^2$$

$$FD^2 + EF^2 = ED^2$$

$$AD \cdot CD + EC^2 = ED^2$$

$$DB^2 + EB^2 = ED^2$$

Proof - AD does not pass through centre of circle

Draw a line EF from the centre of the circle E, perpendicular to the line DA

Lines CF and FA are equal (III·18)

If a line AC is bisected at point F, and extended from C to point D, the product AD,CD plus the square of FC equals the square of FD (II·6)

Add the square of EF to both sides of the equality

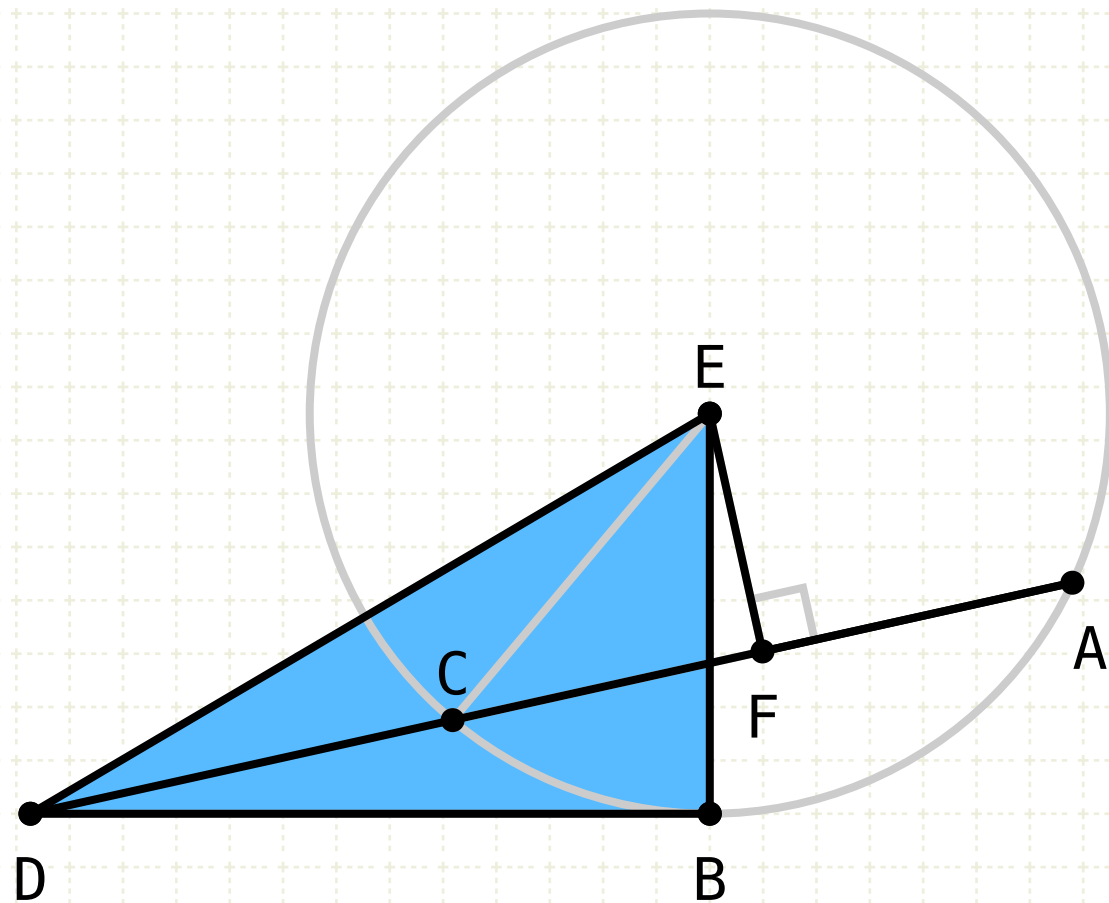
In the triangle ECF, the sum of the squares EF,FC equal the square of EC (I·47)

Similarly, the sum of the squares FD,EF equal the square of ED (I·47)

But the square of ED is just the sum of the squares of DB,EB

Proposition 36 of Book III

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal to the square on the tangent.



$$CF = FA$$

$$AD \cdot CD + FC^2 = FD^2$$

$$AD \cdot CD + FC^2 + EF^2$$

$$= FD^2 + EF^2$$

$$FC^2 + EF^2 = EC^2$$

$$AD \cdot CD + EC^2$$

$$= FD^2 + EF^2$$

$$FD^2 + EF^2 = ED^2$$

$$AD \cdot CD + EC^2 = ED^2$$

$$DB^2 + EB^2 = ED^2$$

$$EB = EC$$

$$DB^2 + EC^2 = ED^2$$

Proof - AD does not pass through centre of circle

Draw a line EF from the centre of the circle E, perpendicular to the line DA

Lines CF and FA are equal (III·18)

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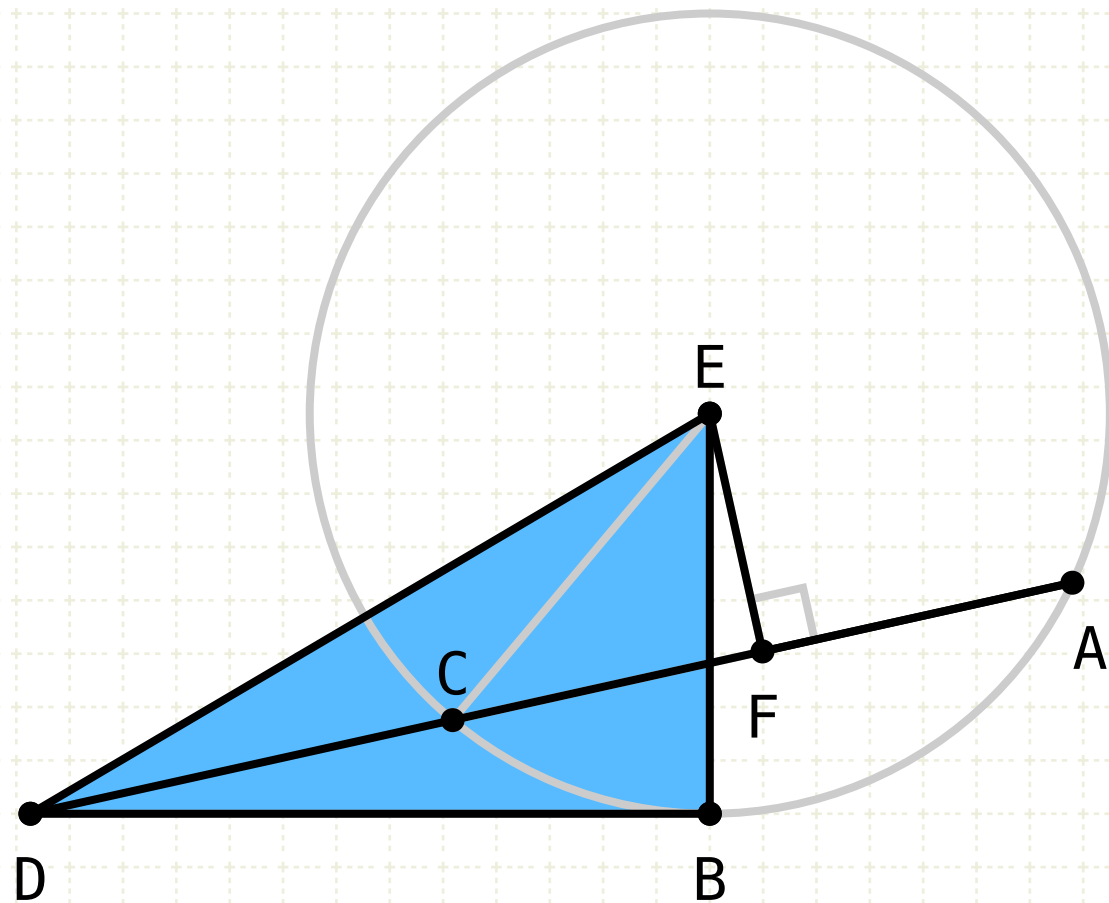
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$$AD \cdot CD = ED^2 - EC^2$$

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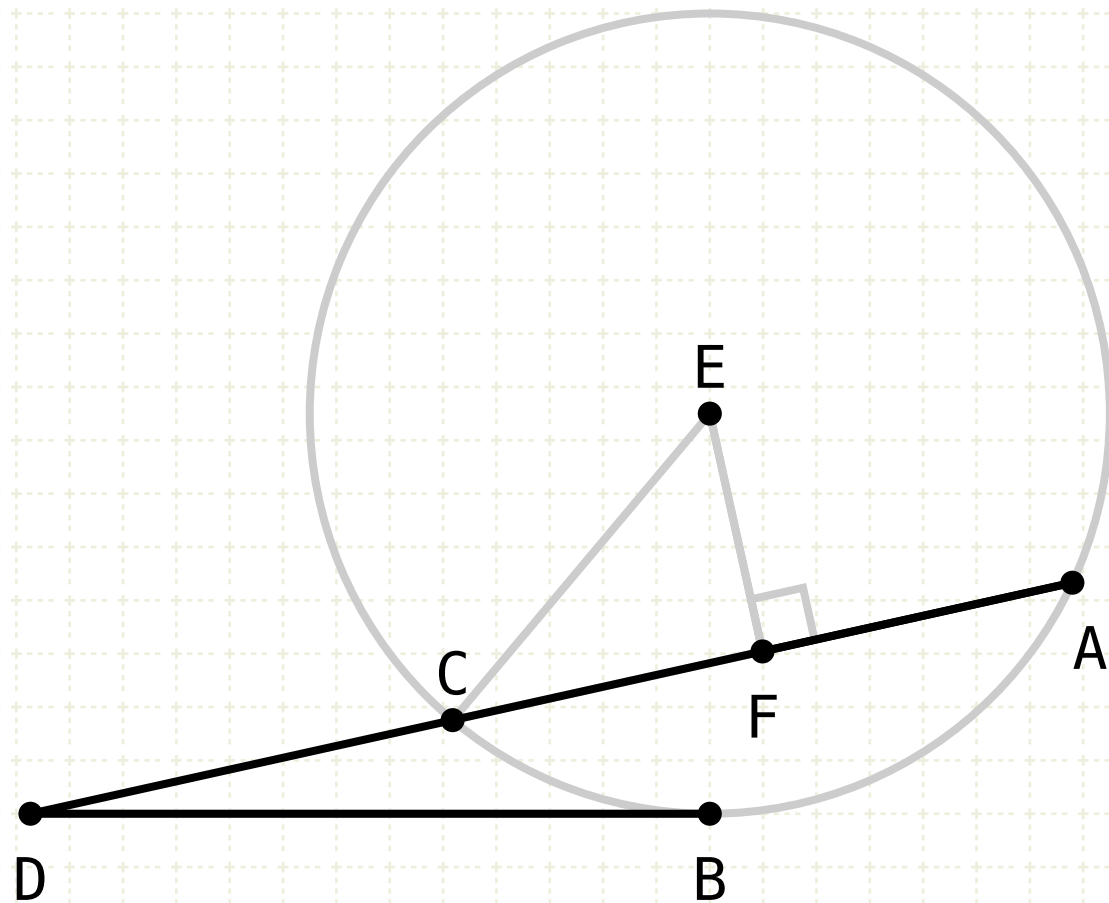
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Subtract EC from both sides of both equations

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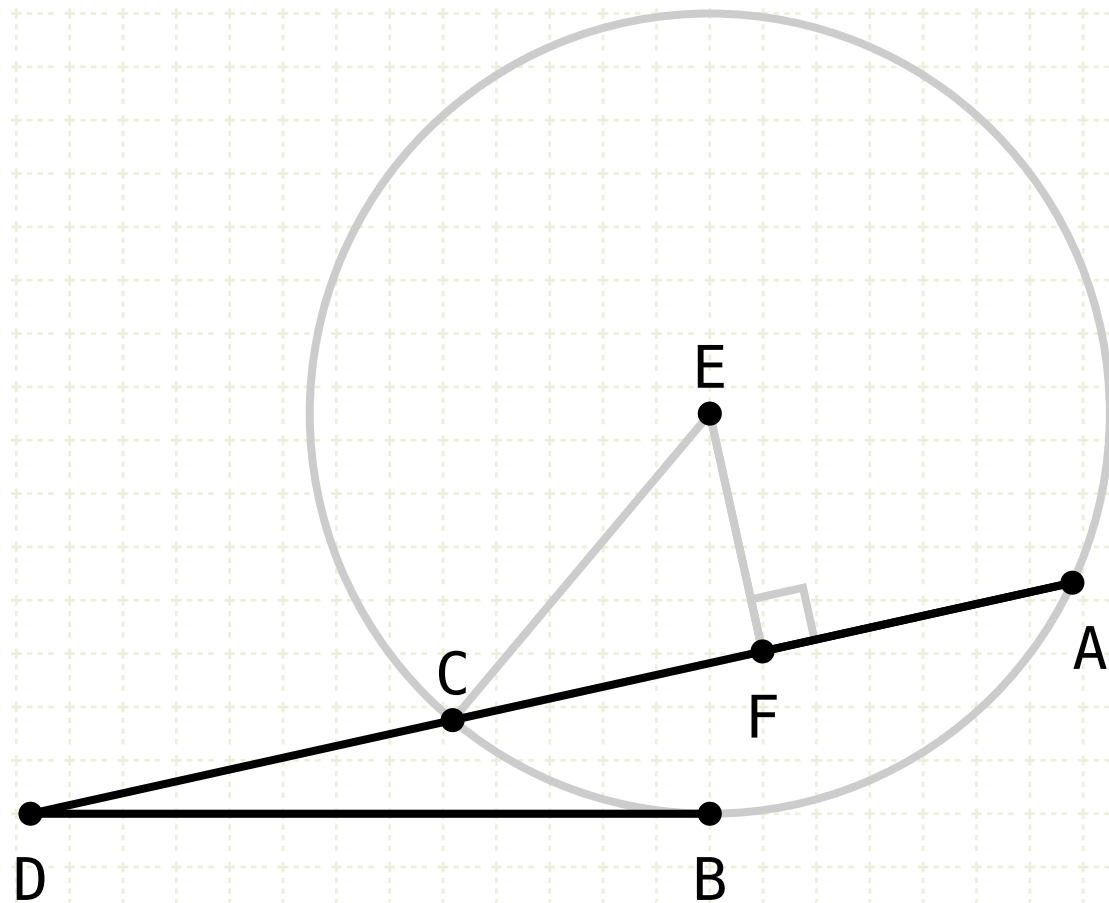
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Since equals are equal to equals, we have proven this proposition



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