

Euclid's Elements

Book VII

Definitions:

- 1 A unit is that by virtue of which each of the things that exist is called one
- 2 A number is a multitude composed of units. (not one)
- 3 A number is part of a number, the less of the greater, when it measures the greater
- 11 A prime number is that which is measured by a unit alone.
- 12 Numbers prime to one another are those which are measured by a unit alone as a common measure
- 15 A number is said to multiply a number when that which is multiplied is added to itself as many times as there are units in the other, and thus some number is produced.
- 20 Numbers are proportional when the first is the same multiple, or the same part, or the same parts, of the second that the third is of the fourth.

As long as algebra and geometry have been separated, their progress have been slow and their uses limited; but when these two sciences have been united, they have lent each mutual forces, and have marched together towards perfection.

Joseph-Louis Lagrange
(1736 to 1813)



Table of Contents, Chapter 7

1	Determine if two numbers are relatively prime	10	If $B = (p/q) \cdot A$ and $D = (p/q) \cdot C$, and If $B = (r/s) \cdot D$, then $A = (r/s) \cdot C$	21	If A,B are relatively prime, then A,B are the smallest whole numbers that can be used to describe the ratio A:B
2	Find the greatest common divisor for two numbers	11	If $A:B = C:D$, then $(A-C):(B-D) = A:B$	22	If A,B are the smallest whole numbers that can be used to describe the ratio A:B, then A,B are relatively prime
3	Find the largest common divisor for three numbers	12	If $A:B = C:D$, then $(A+C):(B+C) = A:B$	23	If A,B are relatively prime and if $A = n \cdot C$, then B,C are relatively prime
4	Given two natural numbers, A and B, either B is part of A, or there exists a natural number (a part) that can measure both A and B	13	If $A:B = C:D$, then $A:C = B:D$	24	If A,C are relatively prime and B,C are relatively prime then the $A \times B$ is relatively prime to C
5	If $B = (1/q) \cdot A$ and $D = (1/q) \cdot C$, then $(B+D) = (1/q) \cdot (A+C)$	14	If $A:B = D:E$ and $B:C = E:F$, then $A:C = D:F$	25	If A,B are relatively prime then A^2, B are relatively prime
6	If $B = (p/q) \cdot A$ and $D = (p/q) \cdot C$, then $(B+D) = (p/q) \cdot (A+C)$	15	If $B = i \cdot 1$ and $E = i \cdot D$, and if $D = j \cdot 1$ then $E = j \cdot B$	26	If A is relatively prime to C and D, and if B is also relatively prime to C and D, then $A \times B$ is relatively prime to $C \times D$
7	If $B = A/q$ and $D = C/q$, $B > D$, then $(B-D) = (A-C)/q$	16	$A \times B = B \times A$	27	If A,B are relatively prime, then A^2, B^2 are relatively prime, and A^3, B^3 are relatively prime, and so on
8	If $B = (p/q) \cdot A$ and $D = (p/q) \cdot C$, $B > D$, then $(B-D) = (p/q) \cdot (A-C)$	17	If $D = A \times B$ and $E = A \times C$ then $D:E = B:C$		
9	If $B = (1/q) \cdot A$ and $D = (1/q) \cdot C$, and If $B = (r/s) \cdot D$, then $A = (r/s) \cdot C$	18	If $D = B \times A$ and $E = C \times A$ then $D:E = B:C$		
		19	If $A:B = C:D$ then $A \times D = B \times C$ If $A \times D = B \times C$ then $A:B = C:D$		
		20	Given the ratio A:B and C,D are the smallest numbers such that $A:B = C:D$ then $A = n \cdot C$ and $B = n \cdot D$		



Table of Contents, Chapter 7

- | | | | |
|----|---|----|---|
| 28 | If A,B are relatively prime, then A,(A+B) are relatively prime | 37 | If $A = p \cdot B$, then $A = q \cdot C$ where $C = p \cdot 1$ |
| 29 | If A is prime, and $B \neq n \cdot A$, then A,B are relatively prime | 38 | If $A = (1/c) \cdot B$ and $C = c \cdot 1$ then $A = n \cdot C$ |
| 30 | If $C = A \times B$ and $C = i \cdot D$ where D is prime, then either $A = j \cdot D$ or $B = j \cdot D$ | 39 | Find the smallest number that has the fractions $1/a$, $1/b$, $1/c$ |
| 31 | If $A = B \times C$, then $A = j \cdot D$ where D is prime | | |
| 32 | If A is a number then it is either prime, or $A = j \cdot D$ where D is prime | | |
| 33 | Find the smallest numbers X,Y,Z where the ratio X:Y:Z is equal to the given ratio A:B:C | | |
| 34 | Find the lowest common denominator of 2 numbers | | |
| 35 | If E is the lowest common denominator of A,B, and if $C = n \cdot A = m \cdot B$, then $C = i \cdot E$ | | |
| 36 | Find the least common multiple of 3 numbers | | |



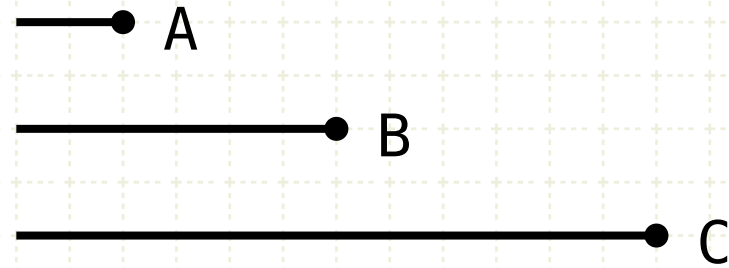
Proposition 30 of Book VII

If two numbers by multiplying one another make some number, and any prime number measure the product, it will also measure one of the original numbers.



Proposition 30 of Book VII

If two numbers by multiplying one another make some number, and any prime number measure the product, it will also measure one of the original numbers.



$C = A \times B$

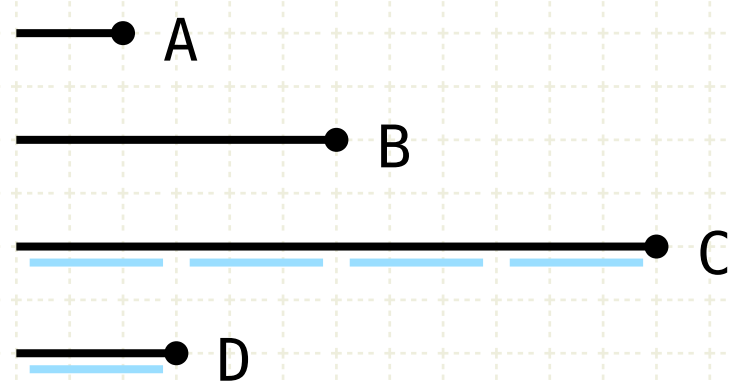
In other words

Let C equal A multiplied by B



Proposition 30 of Book VII

If two numbers by multiplying one another make some number, and any prime number measure the product, it will also measure one of the original numbers.



$$C = A \times B$$

D is prime

$$C = q \cdot D$$

In other words

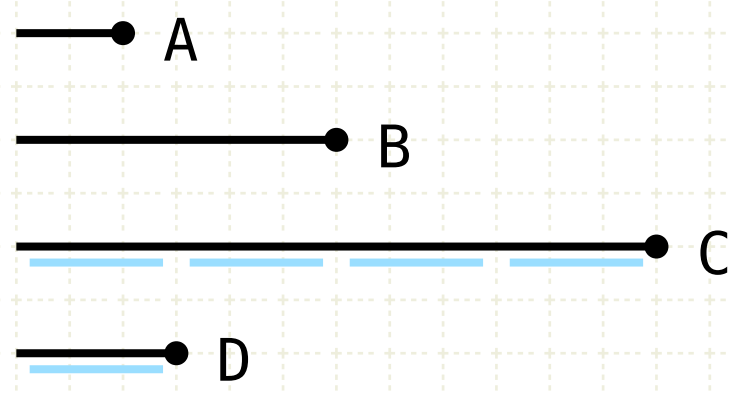
Let C equal A multiplied by B

Let D, a prime number, measure C



Proposition 30 of Book VII

If two numbers by multiplying one another make some number, and any prime number measure the product, it will also measure one of the original numbers.



$$C = A \times B$$

D is prime

$$C = q \cdot D$$

$$A = r \cdot D \text{ , or } B = r \cdot D$$

In other words

Let C equal A multiplied by B

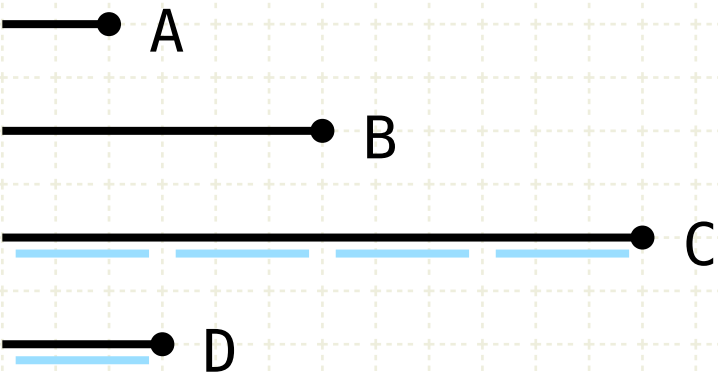
Let D, a prime number, measure C

then D measures either A or B



Proposition 30 of Book VII

If two numbers by multiplying one another make some number, and any prime number measure the product, it will also measure one of the original numbers.



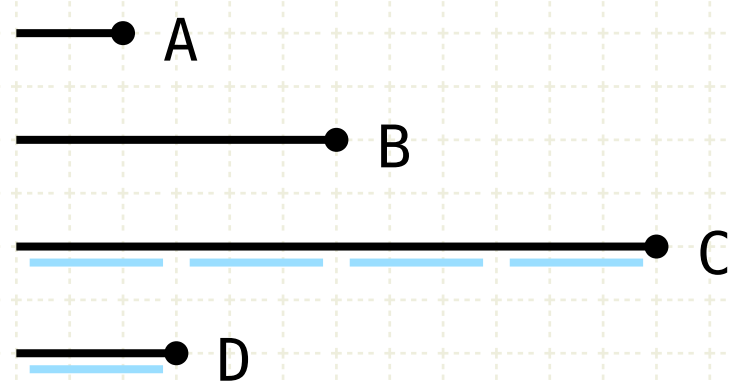
$C = A \times B$
D is prime
 $C = q \cdot D$
 $A = r \cdot D$, or $B = r \cdot D$

Proof



Proposition 30 of Book VII

If two numbers by multiplying one another make some number, and any prime number measure the product, it will also measure one of the original numbers.



$C = A \times B$
D is prime
 $C = q \cdot D$
 $A = r \cdot D$, or $B = r \cdot D$

 $A \neq r \cdot D$

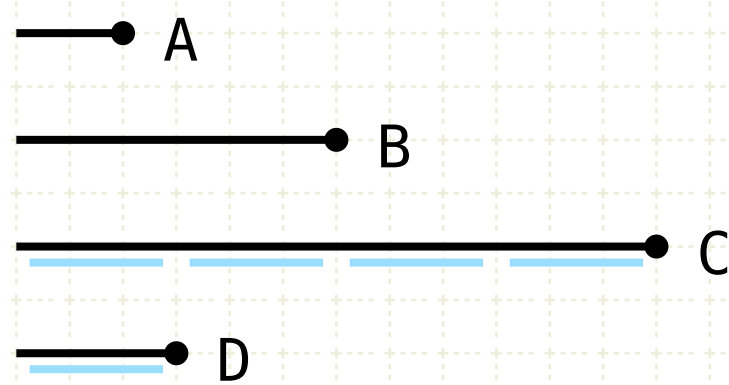
Proof

Assume that D does not measure A



Proposition 30 of Book VII

If two numbers by multiplying one another make some number, and any prime number measure the product, it will also measure one of the original numbers.



$$C = A \times B$$

D is prime

$$C = q \cdot D$$

$$A = r \cdot D, \text{ or } B = r \cdot D$$

$$A \neq r \cdot D$$

$$\gcd(D, A) = 1$$

Proof

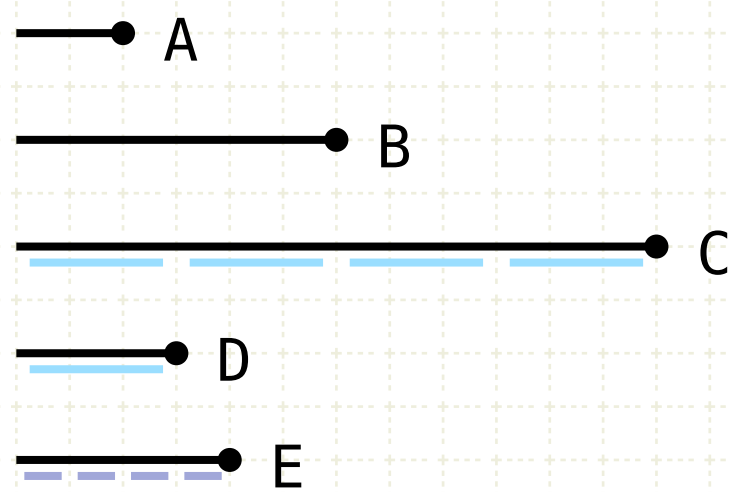
Assume that D does not measure A

D is prime, and does not measure A, so therefore A and D are prime to one another (VII-29)



Proposition 30 of Book VII

If two numbers by multiplying one another make some number, and any prime number measure the product, it will also measure one of the original numbers.



$$C = A \times B$$

D is prime

$$C = q \cdot D$$

$$A = r \cdot D, \text{ or } B = r \cdot D$$

$$A \neq r \cdot D$$

$$\gcd(D, A) = 1$$

$$E = q \cdot 1 = q$$

Proof

Assume that D does not measure A

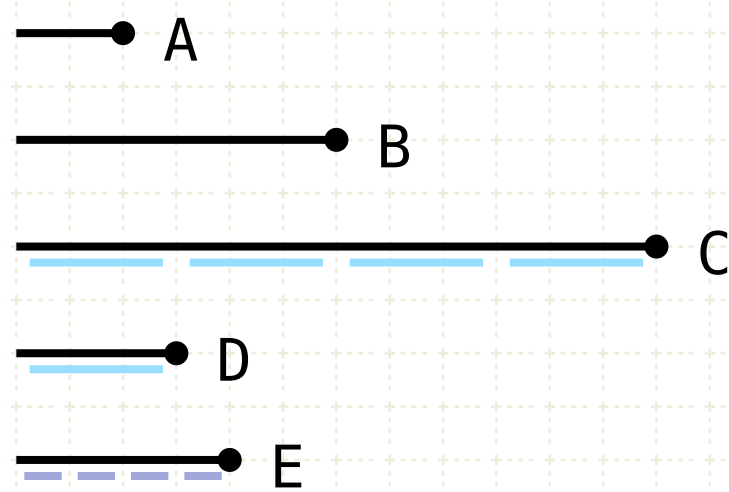
D is prime, and does not measure A, so therefore A and D are prime to one another (VII-29)

As many times as D measures C, let there be so many units (ones) in E



Proposition 30 of Book VII

If two numbers by multiplying one another make some number, and any prime number measure the product, it will also measure one of the original numbers.



$$C = A \times B$$

D is prime

$$C = q \cdot D$$

$$A = r \cdot D, \text{ or } B = r \cdot D$$

$$A \neq r \cdot D$$

$$\gcd(D, A) = 1$$

$$E = q \cdot 1 = q$$

$$C = D \times E$$

Proof

Assume that D does not measure A

D is prime, and does not measure A, so therefore A and D are prime to one another (VII-29)

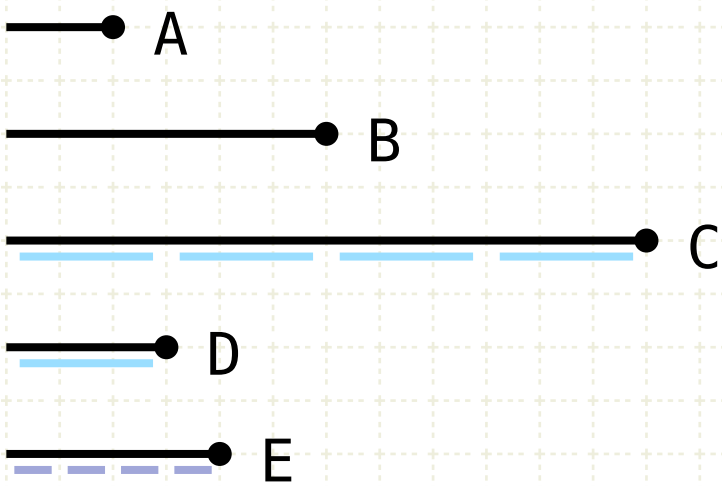
As many times as D measures C, let there be so many units (ones) in E

Since D measures C according to the units in E, then C is equal to D multiplied by E (VII.Def.15)



Proposition 30 of Book VII

If two numbers by multiplying one another make some number, and any prime number measure the product, it will also measure one of the original numbers.



$$C = A \times B$$

D is prime

$$C = q \cdot D$$

$$A = r \cdot D, \text{ or } B = r \cdot D$$

$$A \neq r \cdot D$$

$$\gcd(D, A) = 1$$

$$E = q \cdot 1 = q$$

$$C = D \times E$$

$$D \times E = B \times A$$

Proof

Assume that D does not measure A

D is prime, and does not measure A, so therefore A and D are prime to one another (VII-29)

As many times as D measures C, let there be so many units (ones) in E

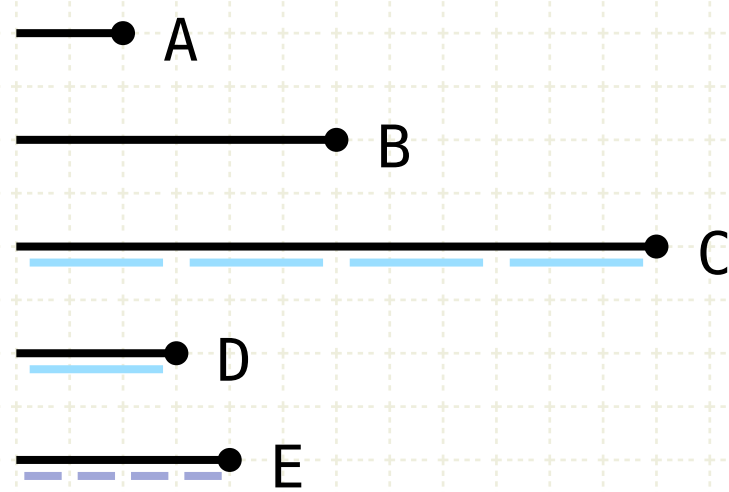
Since D measures C according to the units in E, then C is equal to D multiplied by E (VII.Def.15)

C is the product of A and B and also equal to the product of A and B



Proposition 30 of Book VII

If two numbers by multiplying one another make some number, and any prime number measure the product, it will also measure one of the original numbers.



$$C = A \times B$$

D is prime

$$C = q \cdot D$$

$$A = r \cdot D, \text{ or } B = r \cdot D$$

$$A \neq r \cdot D$$

$$\gcd(D, A) = 1$$

$$E = q \cdot 1 = q$$

$$C = D \times E$$

$$D \times E = B \times A$$

$$D : A = B : E$$

Proof

Assume that D does not measure A

D is prime, and does not measure A, so therefore A and D are prime to one another (VII-29)

As many times as D measures C, let there be so many units (ones) in E

Since D measures C according to the units in E, then C is equal to D multiplied by E (VII.Def.15)

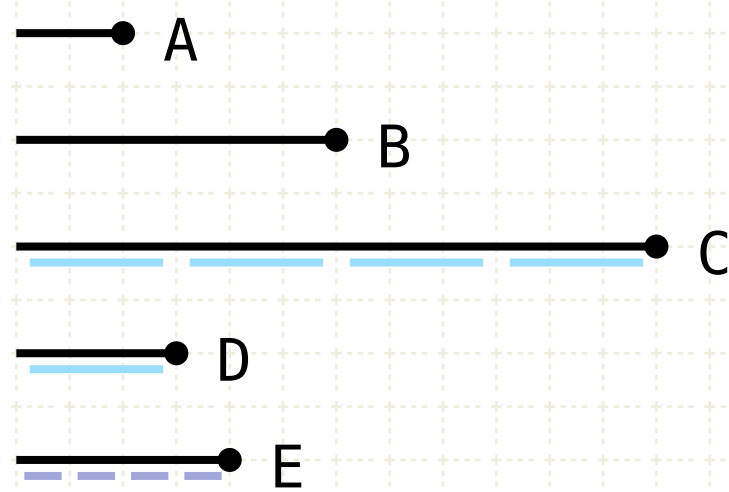
C is the product of A and B and also equal to the product of A and B

Thus D is to A as B is to E (VII-19)



Proposition 30 of Book VII

If two numbers by multiplying one another make some number, and any prime number measure the product, it will also measure one of the original numbers.



$$C = A \times B$$

D is prime

$$C = q \cdot D$$

$$A = r \cdot D, \text{ or } B = r \cdot D$$

$$A \neq r \cdot D$$

$$\gcd(D, A) = 1$$

$$E = q \cdot 1 = q$$

$$C = D \times E$$

$$D \times E = B \times A$$

$$D : A = B : E$$

$$D < B, A < E$$

Proof

Assume that D does not measure A

D is prime, and does not measure A, so therefore A and D are prime to one another (VII·29)

As many times as D measures C, let there be so many units (ones) in E

Since D measures C according to the units in E, then C is equal to D multiplied by E (VII.Def.15)

C is the product of A and B and also equal to the product of A and B

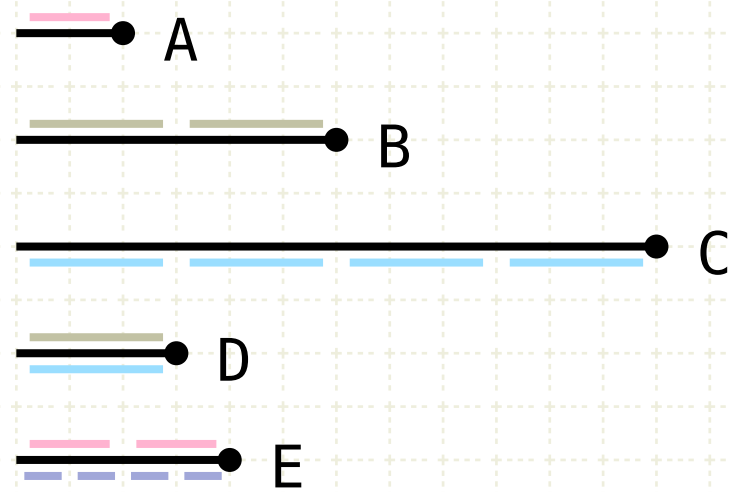
Thus D is to A as B is to E (VII·19)

But D and A are prime to one another, and as such they are the smallest possible numbers that can create the ratio of B to E (VII·21)



Proposition 30 of Book VII

If two numbers by multiplying one another make some number, and any prime number measure the product, it will also measure one of the original numbers.



$$C = A \times B$$

D is prime

$$C = q \cdot D$$

$$A = r \cdot D, \text{ or } B = r \cdot D$$

$$A \neq r \cdot D$$

$$\gcd(D, A) = 1$$

$$E = q \cdot 1 = q$$

$$C = D \times E$$

$$D \times E = B \times A$$

$$D : A = B : E$$

$$D < B, A < E$$

$$E = s \cdot A$$

$$B = s \cdot D$$

Proof

Assume that D does not measure A

D is prime, and does not measure A, so therefore A and D are prime to one another (VII·29)

As many times as D measures C, let there be so many units (ones) in E

Since D measures C according to the units in E, then C is equal to D multiplied by E (VII.Def.15)

C is the product of A and B and also equal to the product of A and B

Thus D is to A as B is to E (VII·19)

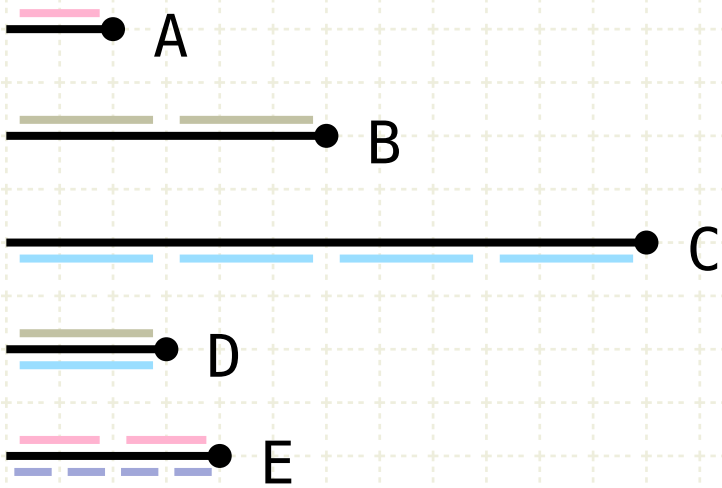
But D and A are prime to one another, and as such they are the smallest possible numbers that can create the ratio of B to E (VII·21)

Thus D measures B and A measures E equally (VII·20)



Proposition 30 of Book VII

If two numbers by multiplying one another make some number, and any prime number measure the product, it will also measure one of the original numbers.



$$C = A \times B$$

D is prime

$$C = q \cdot D$$

$$A = r \cdot D, \text{ or } B = r \cdot D$$

$$A \neq r \cdot D$$

$$\gcd(D, A) = 1$$

$$E = q \cdot 1 = q$$

$$C = D \times E$$

$$D \times E = B \times A$$

$$D : A = B : E$$

$$D < B, A < E$$

$$E = s \cdot A$$

$$B = s \cdot D$$

Proof

Assume that D does not measure A

D is prime, and does not measure A, so therefore A and D are prime to one another (VII·29)

As many times as D measures C, let there be so many units (ones) in E

Since D measures C according to the units in E, then C is equal to D multiplied by E (VII.Def.15)

C is the product of A and B and also equal to the product of A and B

Thus D is to A as B is to E (VII·19)

But D and A are prime to one another, and as such they are the smallest possible numbers that can create the ratio of B to E (VII·21)

Thus D measures B and A measures E equally (VII·20)

Therefore D measures B



Youtube Videos

<https://www.youtube.com/c/SandyBultena>

Copyright © 2019 by Sandy Bultena.



Except where otherwise noted, this work is licensed under
<http://creativecommons.org/licenses/by-nc/3.0>