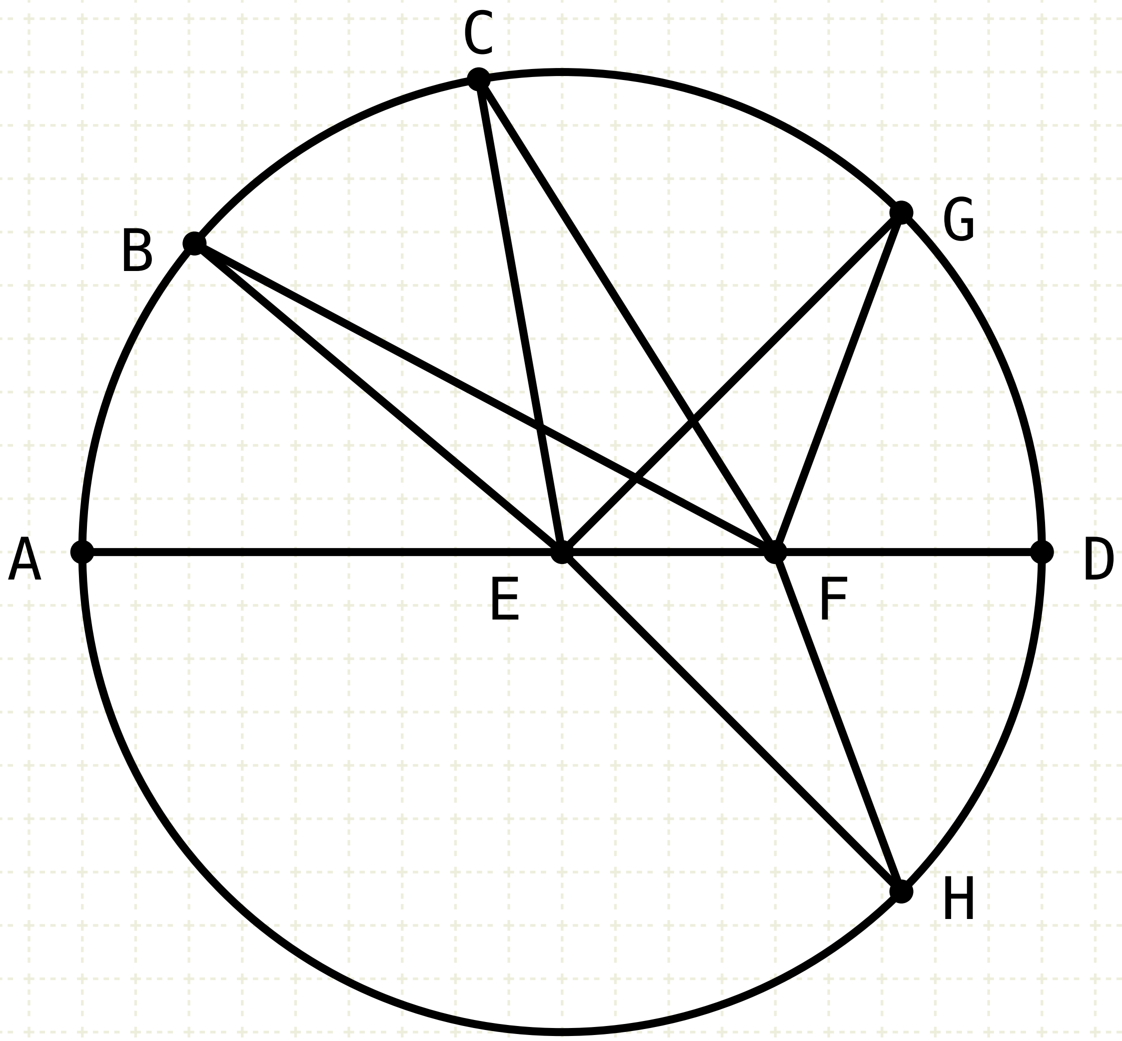
Euclid's Elements

Book III



A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)

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Proposition 8 of Book III

If a point be taken outside a circle and from the point straight lines be drawn through to the circle, one of which is through the centre and the others are drawn at random, then, of the straight lines which fall on the concave circumference, that through the centre is greatest, while of the rest the nearer to that through the centre is always greater than the more remote, but, of the straight lines falling on the convex circumference, that between the point and the diameter is least, while of the rest the nearer to the least is always less than the more remote, and only two equal straight lines will fall on the circle from the point, one on each side of the least.

Proposition 8 of Book III

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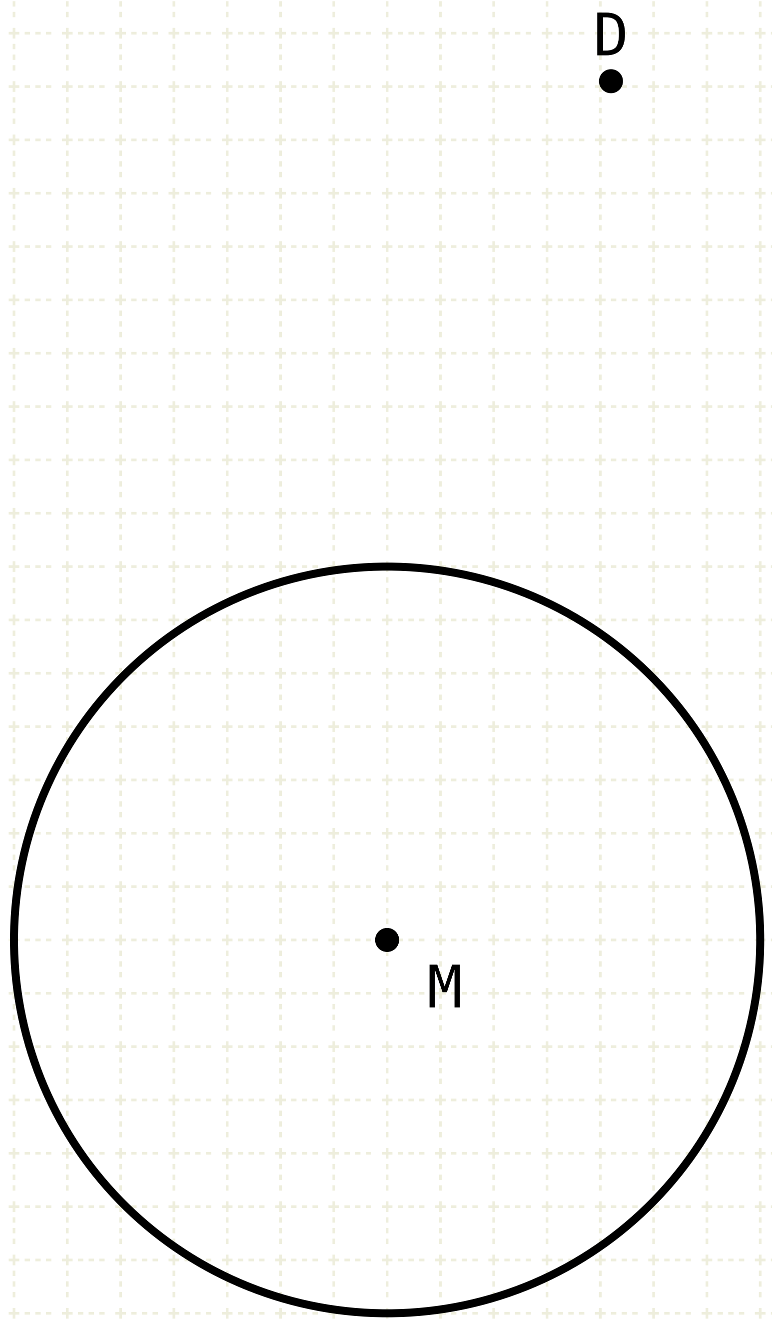
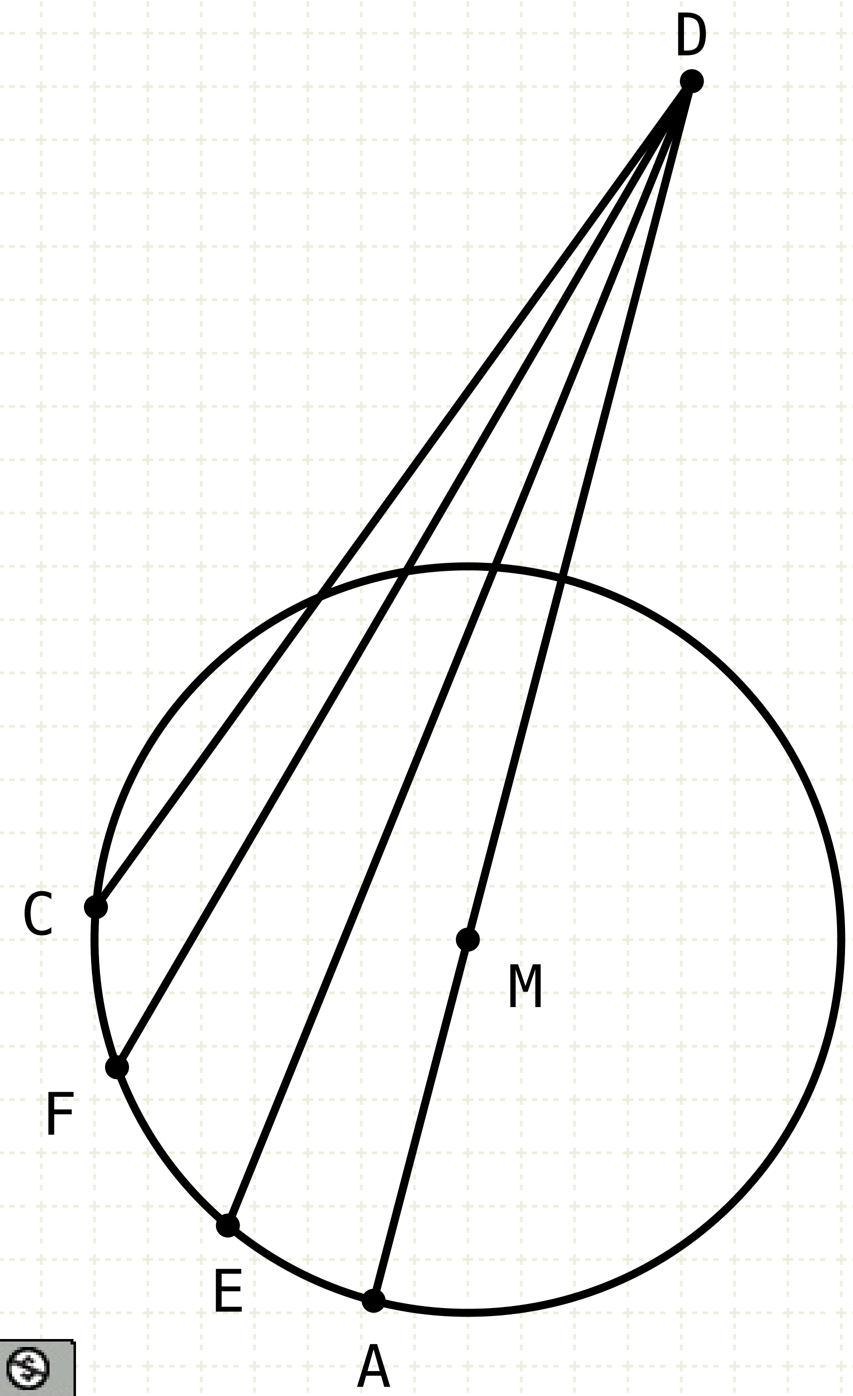


In other words

Let M be the center of a circle, and D be a point outside of the circle

Proposition 8 of Book III

If a point be taken outside a circle and from the point straight lines be drawn through to the circle, one of which is through the centre and the others are drawn at random, then, of the straight lines which fall on the concave circumference, that through the centre is greatest, while of the rest the nearer to that through the centre is always greater than the more remote, but, of the straight lines falling on the convex circumference, that between the point and the diameter is least, while of the rest the nearer to the least is always less than the more remote, and only two equal straight lines will fall on the circle from the point, one on each side of the least.



In other words

Let M be the center of a circle, and D be a point outside of the circle

Draw lines from D to points A, E, F, C on the far end of the circle, where DA passes through the center of the circle at point M

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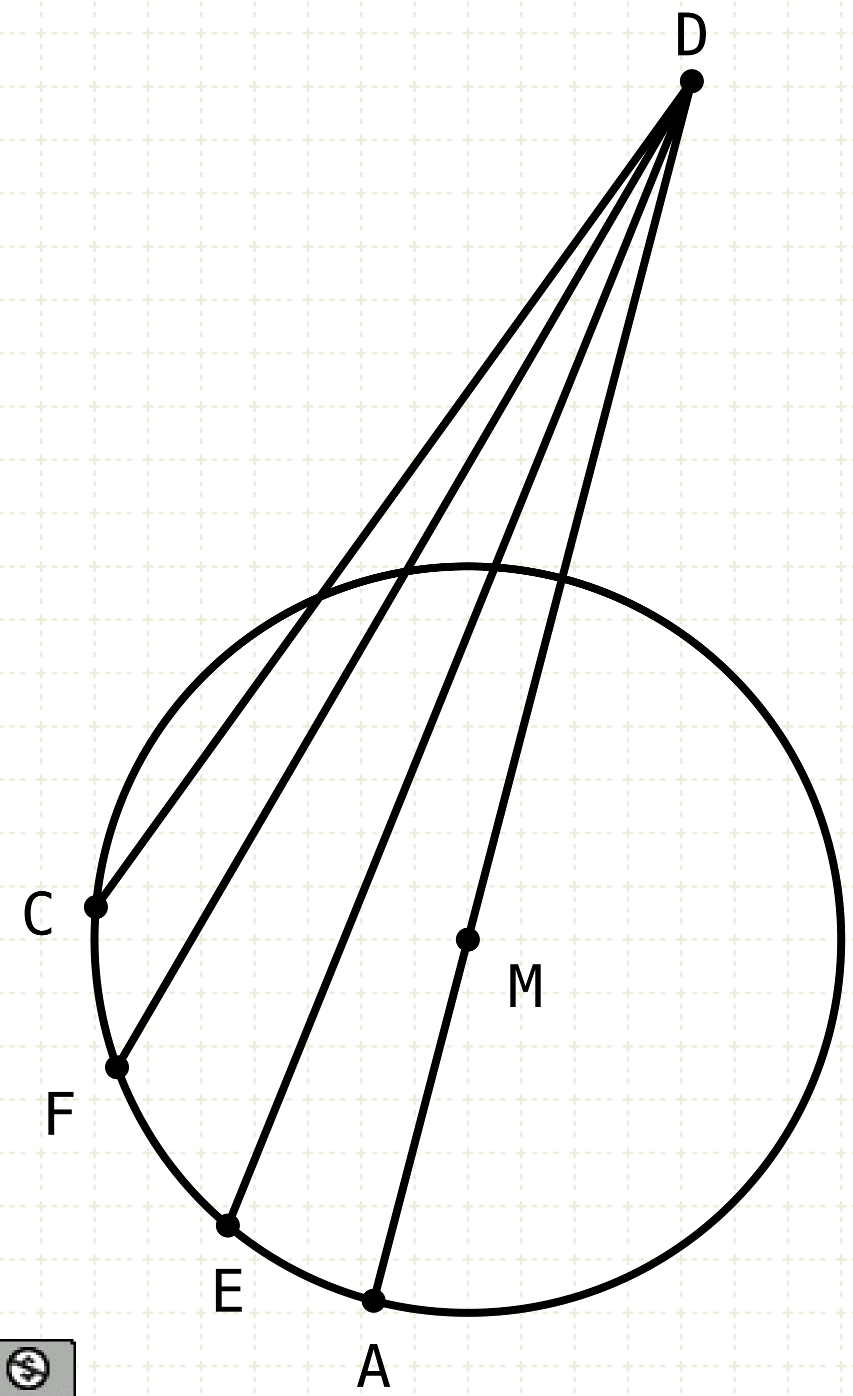
If a point be taken outside a circle and from the point straight lines be drawn through to the circle, one of which is through the centre and the others are drawn at random, then, of the straight lines which fall on the concave circumference, that through the centre is greatest, while of the rest the nearer to that through the centre is always greater than the more remote, but, of the straight lines falling on the convex circumference, that between the point and the diameter is least, while of the rest the nearer to the least is always less than the more remote, and only two equal straight lines will fall on the circle from the point, one on each side of the least.



$DA > DE > DF > DC$

In other words

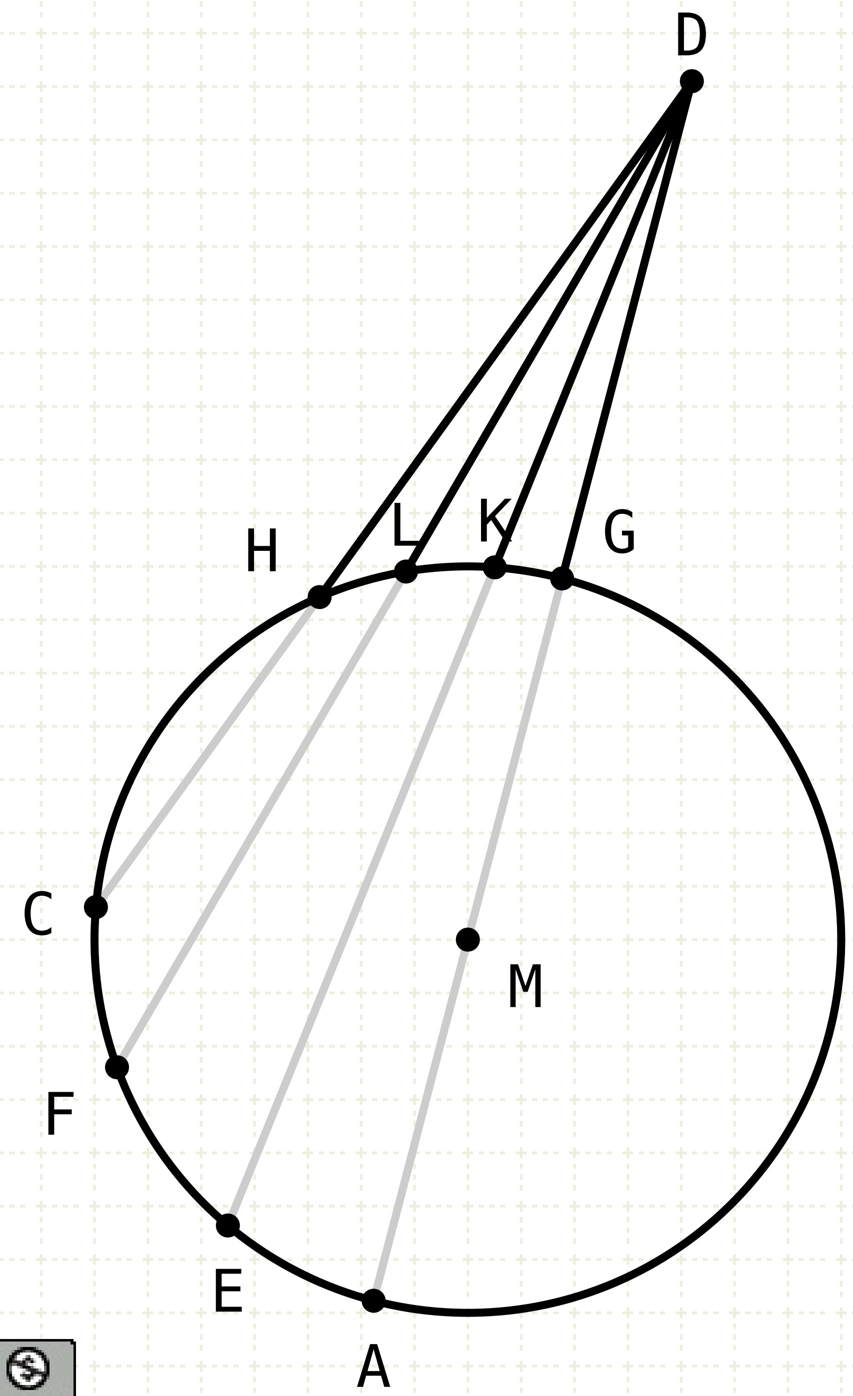
Let M be the center of a circle, and D be a point outside of the circle

Draw lines from D to points A, E, F, C on the far end of the circle, where DA passes through the center of the circle at point M

Of the lines falling on the concave part of the circle, DA is the largest, DE the next, and so on

Proposition 8 of Book III

If a point be taken outside a circle and from the point straight lines be drawn through to the circle, one of which is through the centre and the others are drawn at random, then, of the straight lines which fall on the concave circumference, that through the centre is greatest, while of the rest the nearer to that through the centre is always greater than the more remote, but, of the straight lines falling on the convex circumference, that between the point and the diameter is least, while of the rest the nearer to the least is always less than the more remote, and only two equal straight lines will fall on the circle from the point, one on each side of the least.



$DA > DE > DF > DC$

$DG < DK < DL < DH$

In other words

Let M be the center of a circle, and D be a point outside of the circle

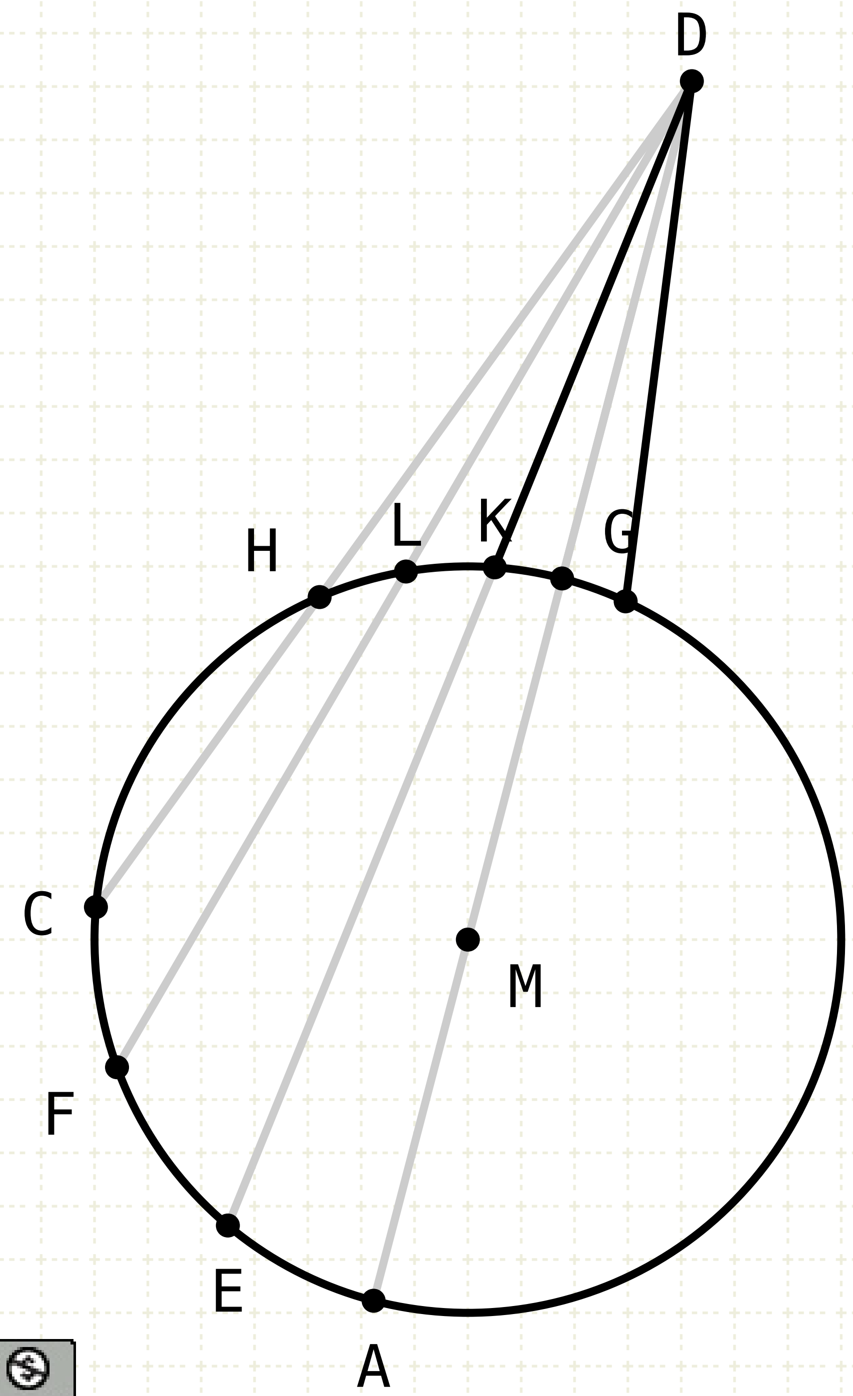
Draw lines from D to points A, E, F, C on the far end of the circle, where DA passes through the center of the circle at point M

Of the lines falling on the concave part of the circle, DA is the largest, DE the next, and so on

Of the lines falling on the convex part of the circle, DG is the least, DK the next, and so on

Proposition 8 of Book III

If a point be taken outside a circle and from the point straight lines be drawn through to the circle, one of which is through the centre and the others are drawn at random, then, of the straight lines which fall on the concave circumference, that through the centre is greatest, while of the rest the nearer to that through the centre is always greater than the more remote, but, of the straight lines falling on the convex circumference, that between the point and the diameter is least, while of the rest the nearer to the least is always less than the more remote, and only two equal straight lines will fall on the circle from the point, one on each side of the least.



$$DA > DE > DF > DC$$

$$DG < DK < DL < DH$$

In other words

Let M be the center of a circle, and D be a point outside of the circle

Draw lines from D to points A, E, F, C on the far end of the circle, where DA passes through the center of the circle at point M

Of the lines falling on the concave part of the circle, DA is the largest, DE the next, and so on

Of the lines falling on the convex part of the circle, DG is the least, DK the next, and so on

Finally, only two straight and equal lines from point D will fall on the circle, one on either side of DG

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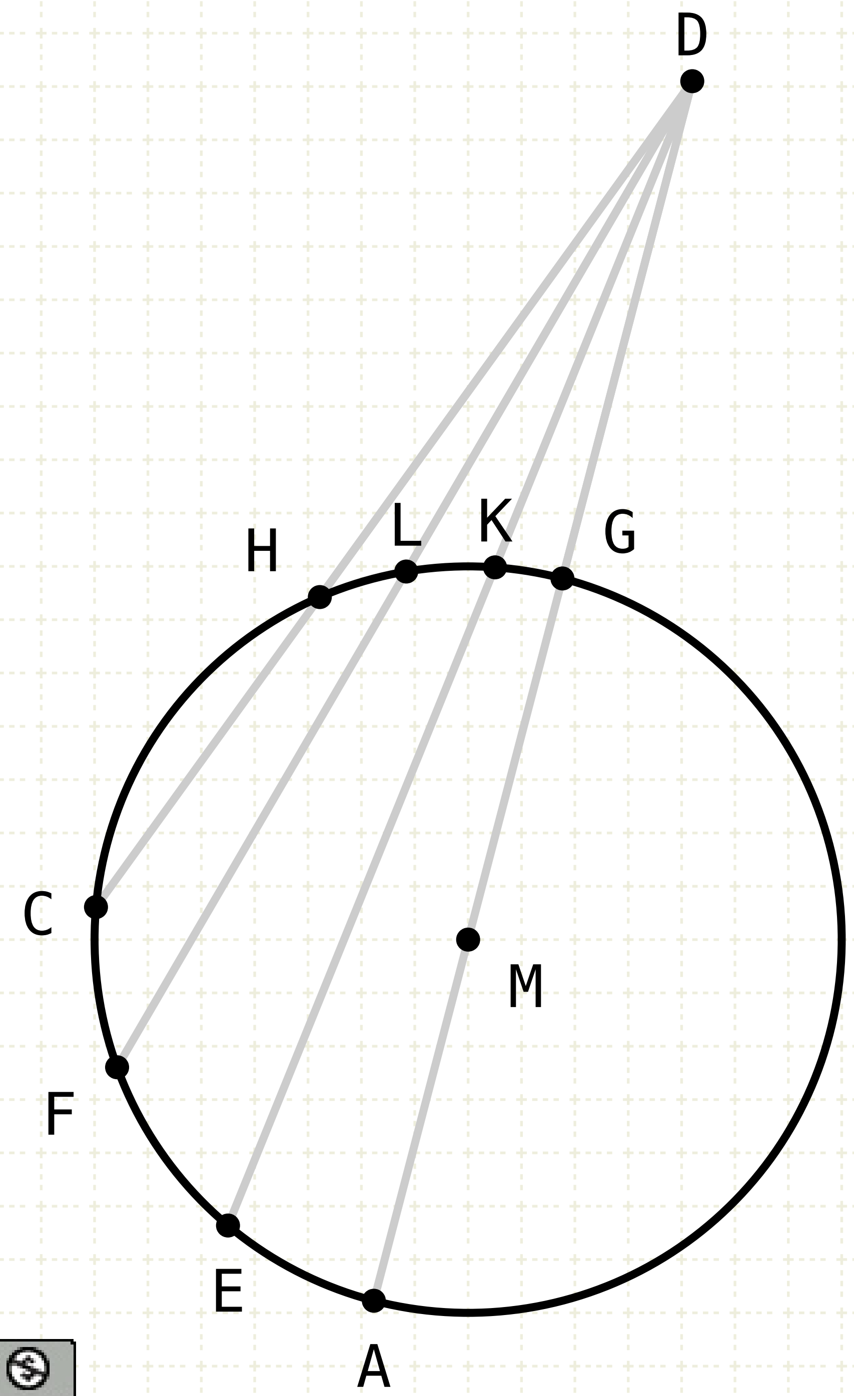
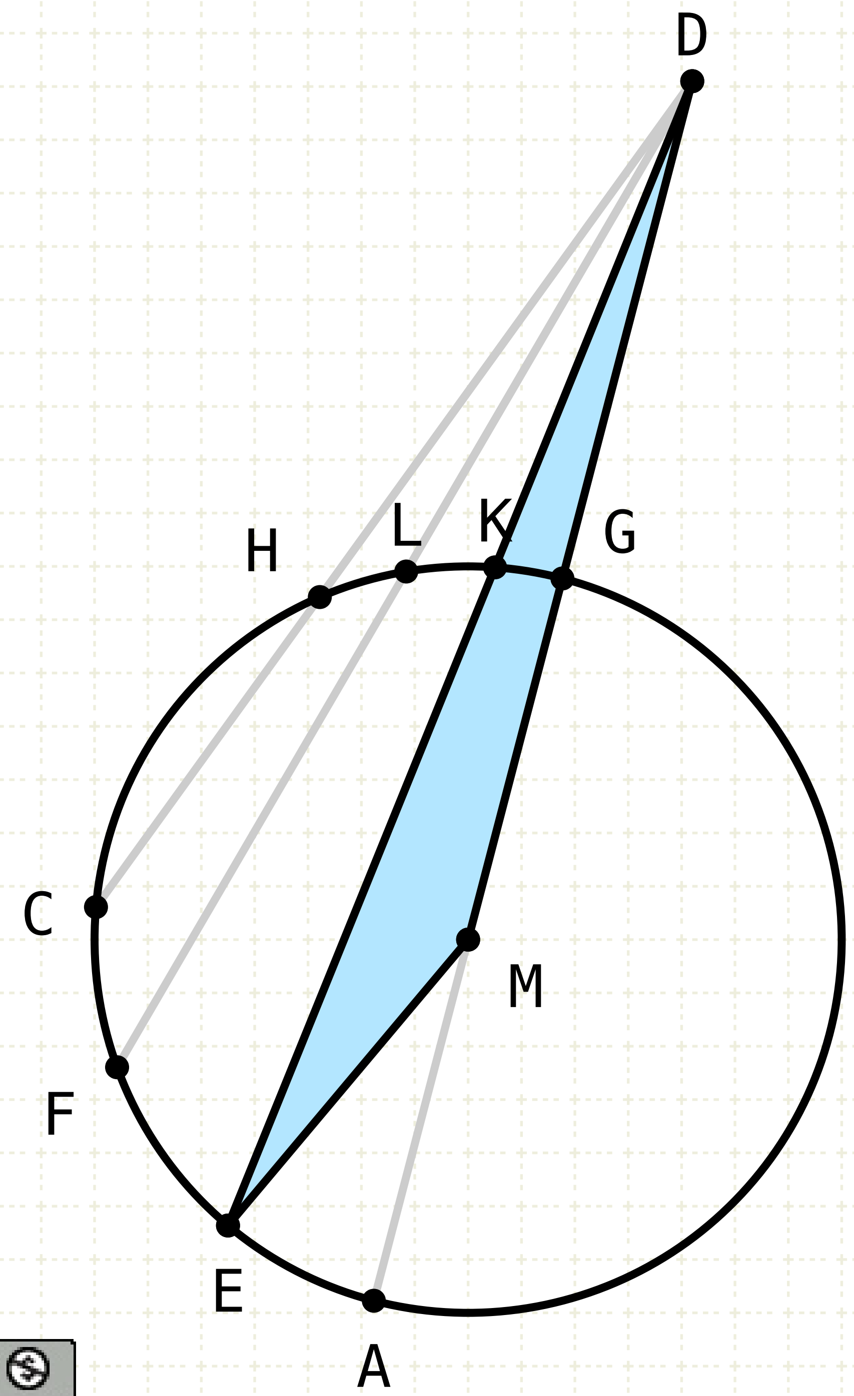


Proof (part 1)

Proposition 8 of Book III

If a point be taken outside a circle and from the point straight lines be drawn through to the circle, one of which is through the centre and the others are drawn at random, then, of the straight lines which fall on the concave circumference, that through the centre is greatest, while of the rest the nearer to that through the centre is always greater than the more remote, but, of the straight lines falling on the convex circumference, that between the point and the diameter is least, while of the rest the nearer to the least is always less than the more remote, and only two equal straight lines will fall on the circle from the point, one on each side of the least.



EM + DM > DE

Proof (part 1)

Consider the triangle DEM, the sum of two sides of any triangle is larger than the third (I·20)

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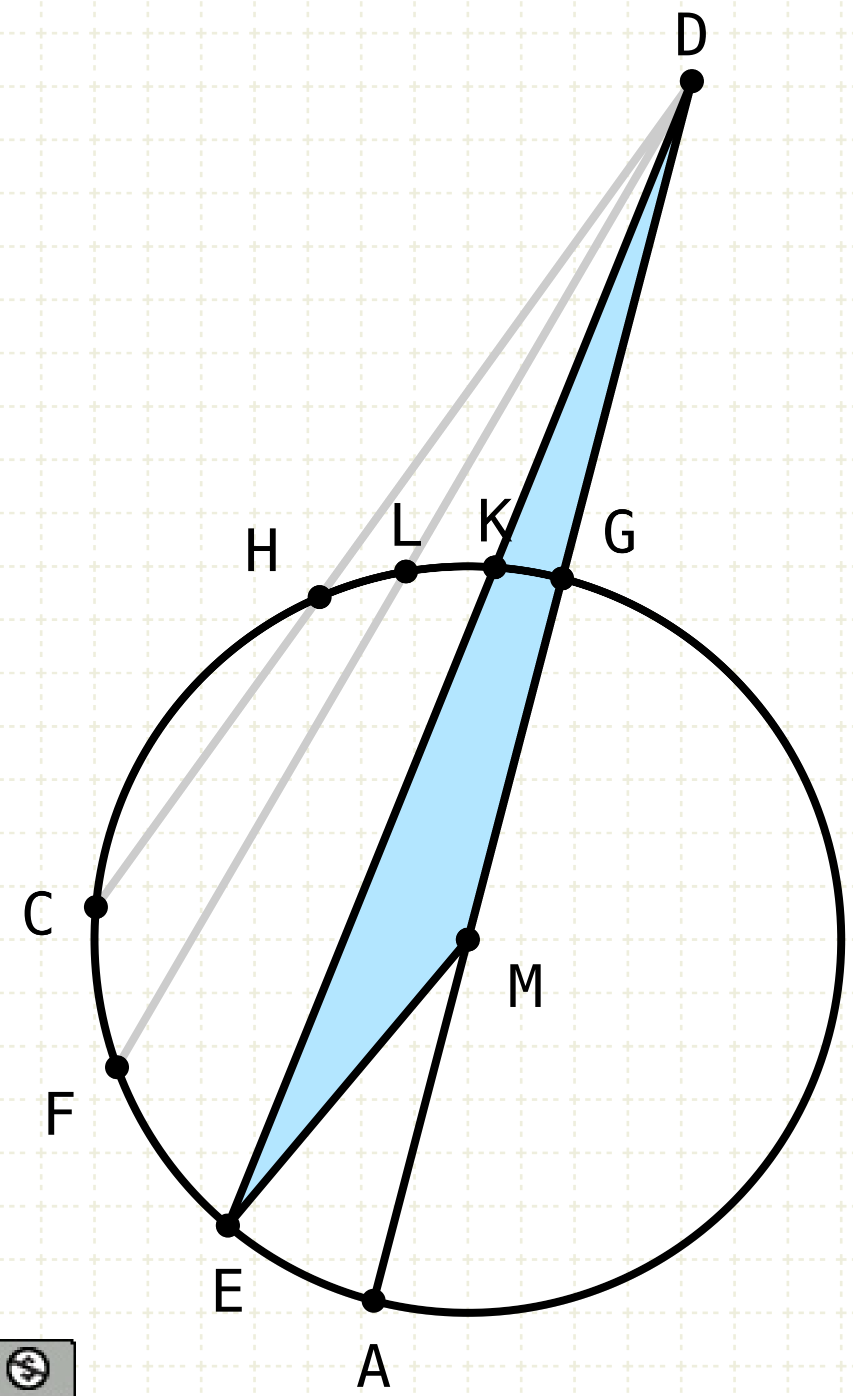
If a point be taken outside a circle and from the point straight lines be drawn through to the circle, one of which is through the centre and the others are drawn at random, then, of the straight lines which fall on the concave circumference, that through the centre is greatest, while of the rest the nearer to that through the centre is always greater than the more remote, but, of the straight lines falling on the convex circumference, that between the point and the diameter is least, while of the rest the nearer to the least is always less than the more remote, and only two equal straight lines will fall on the circle from the point, one on each side of the least.



$$EM + DM > DE$$
$$EM = AM$$

Proof (part 1)

Consider the triangle DEM, the sum of two sides of any triangle is larger than the third (I·20)

The lines EM and AM are radii of the same circle, and thus are equal

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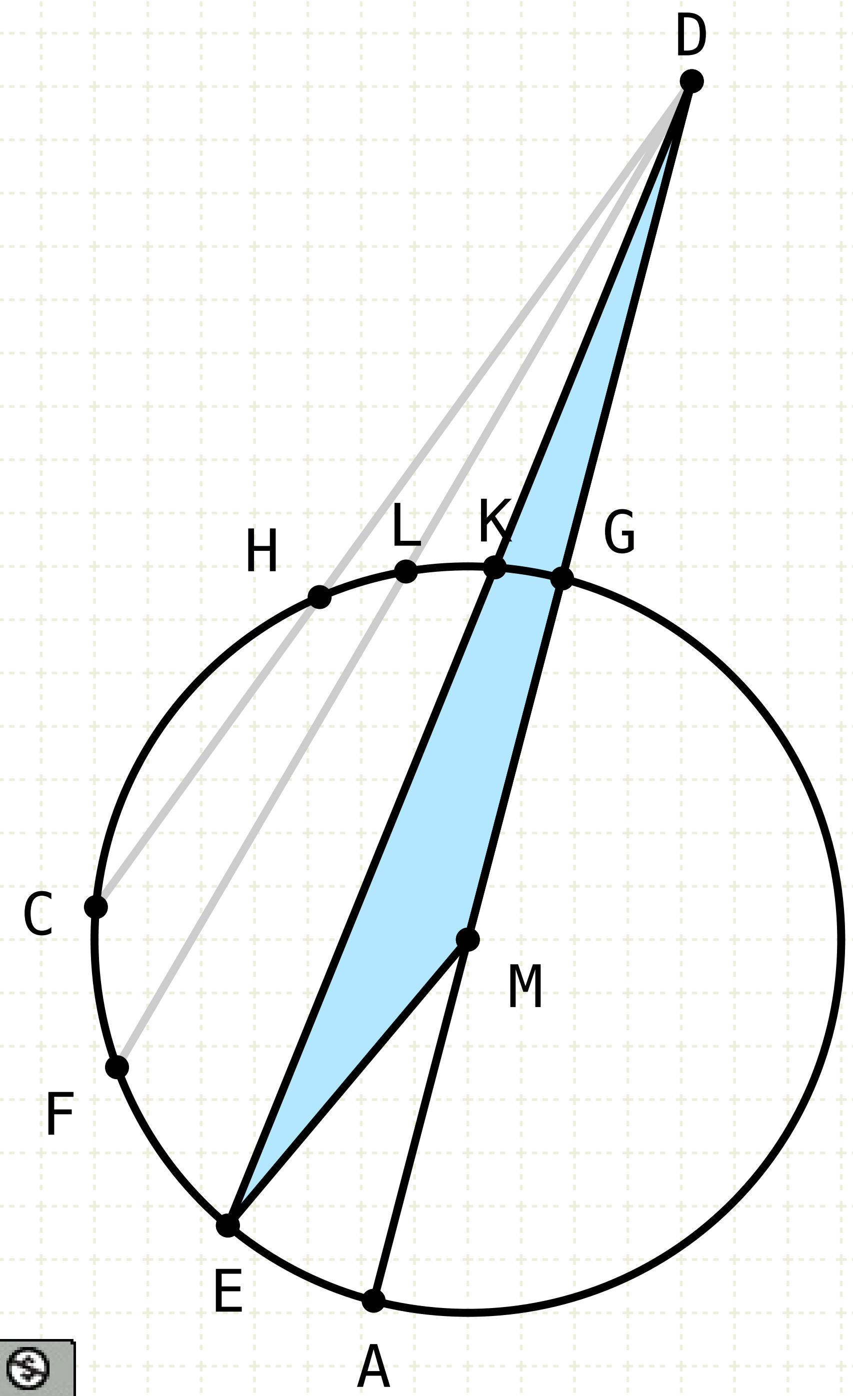

$$EM + DM > DE$$
$$EM = AM$$
$$AM + DM > DE$$
$$DA > DE$$

Proof (part 1)

Consider the triangle DEM, the sum of two sides of any triangle is larger than the third (I·20)

The lines EM and AM are radii of the same circle, and thus are equal

Thus, AM plus DM is greater than DE, and since DA equal AM,DM, DA is greater than DE

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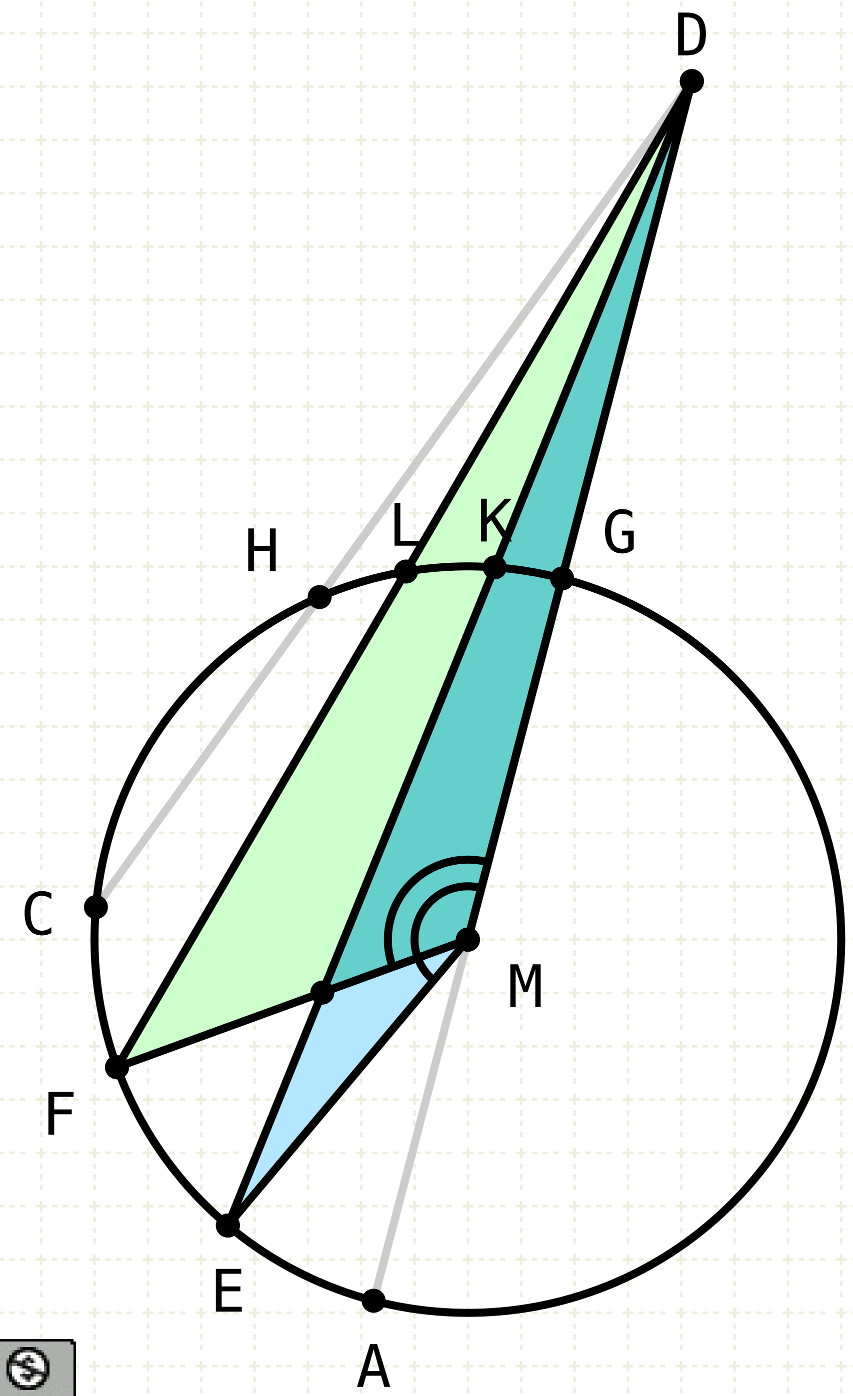

$EM + DM > DE$
$EM = AM$
$AM + DM > DE$
$DA > DE$

$FM = EM$

Proof (part 1)

Consider the triangle DEM, the sum of two sides of any triangle is larger than the third (I·20)

The lines EM and AM are radii of the same circle, and thus are equal

Thus, AM plus DM is greater than DE, and since DA equal AM,DM, DA is greater than DE

Compare the triangles DFM and DEM, FM and EM are equal, and DM is common to both, so we have two triangles with two equal sides,

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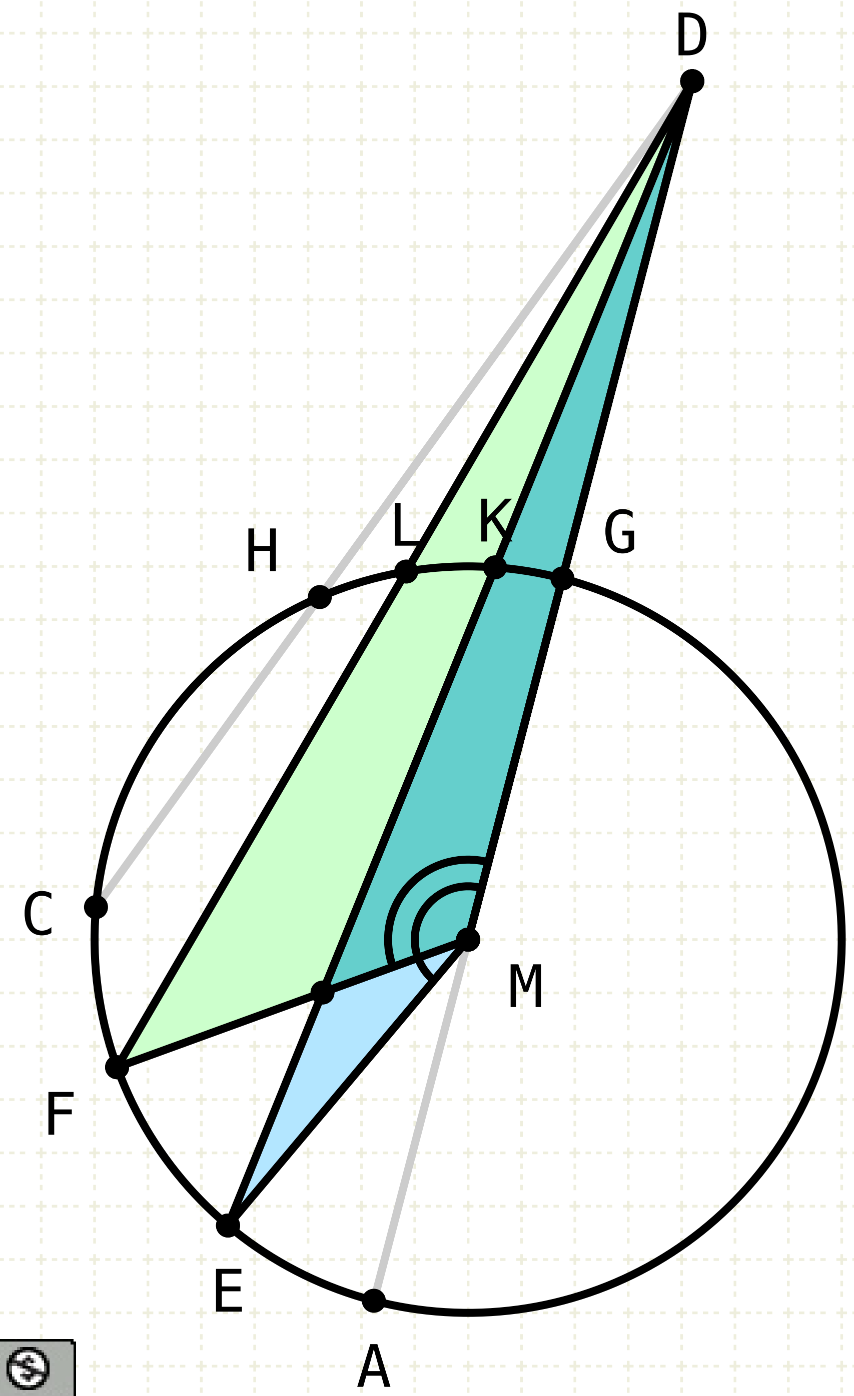

$EM + DM > DE$
$EM = AM$
$AM + DM > DE$
$DA > DE$

$FM = EM$
$DE > DF$

Proof (part 1)

Consider the triangle DEM, the sum of two sides of any triangle is larger than the third (I·20)

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Compare the triangles DFM and DEM, FM and EM are equal, and DM is common to both, so we have two triangles with two equal sides,

Since the angle DME is larger than the angle DMF, DE is larger than DF (I·24)

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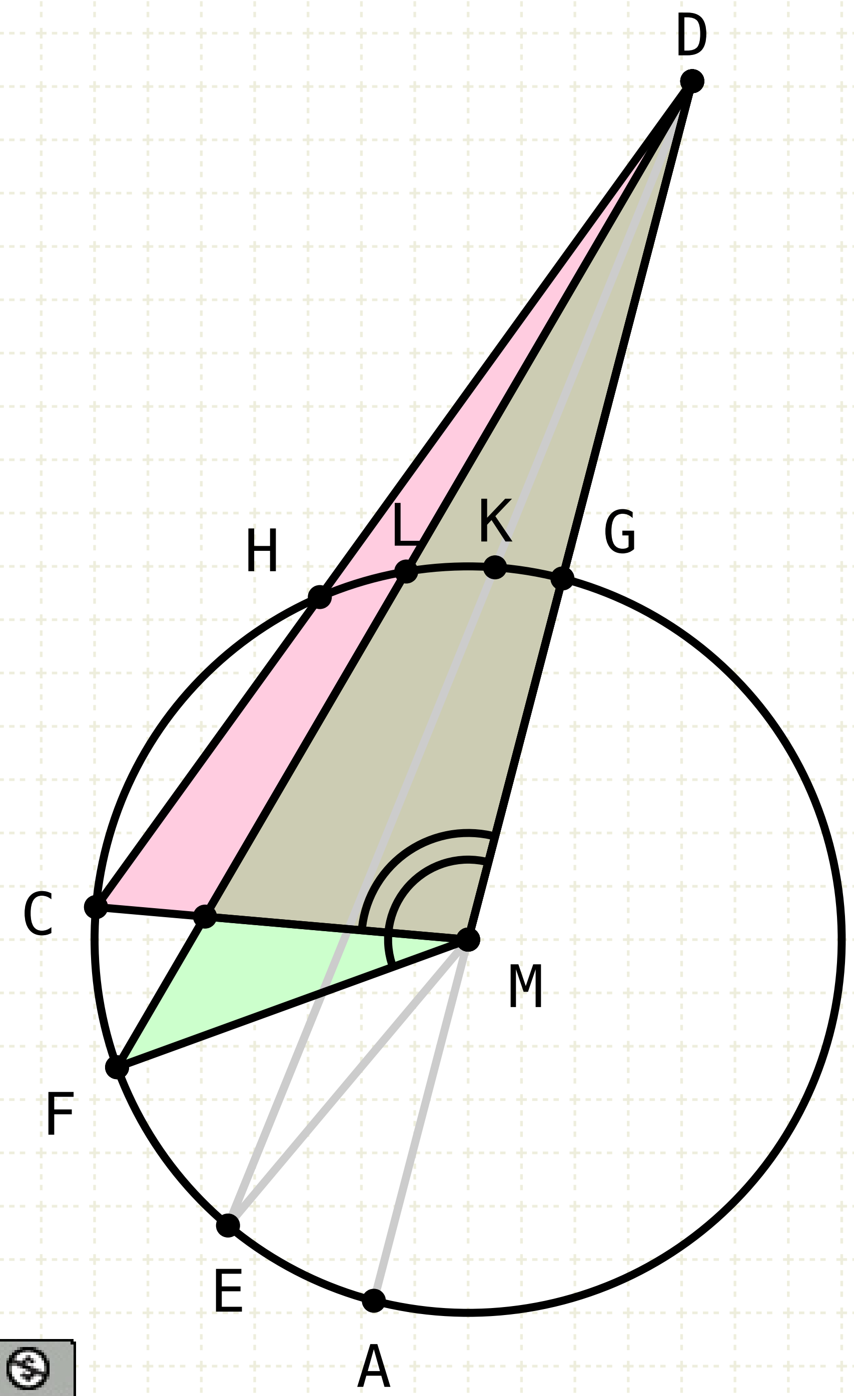

EM + DM > DE
EM = AM
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FM = EM
DE > DF

DF > DC

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Since the angle DME is larger than the angle DMF, DE is larger than DF (I·24)

Similarly, DF is larger than DC

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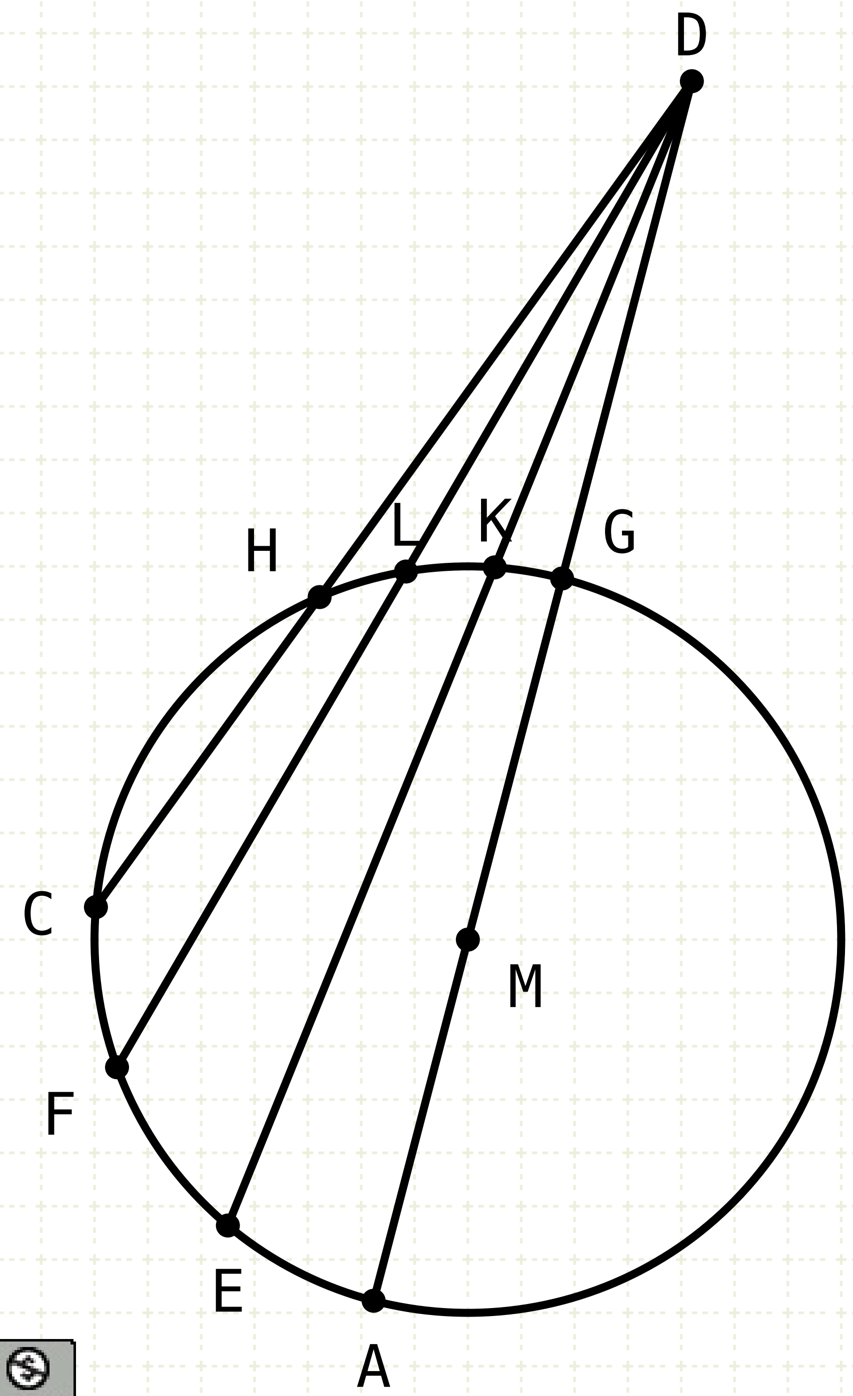

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DA>DE>DF>DC

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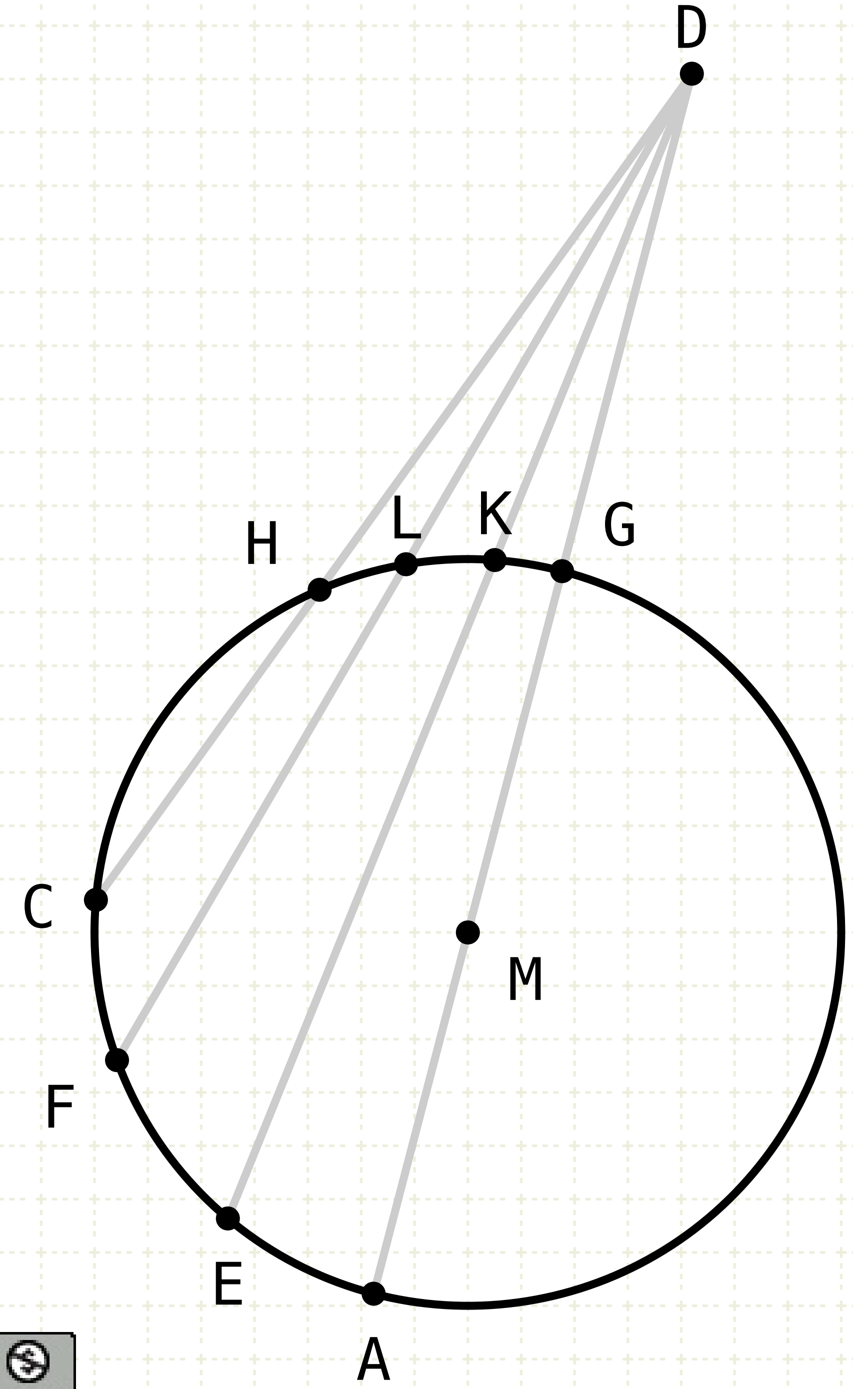

Proof (part 2)

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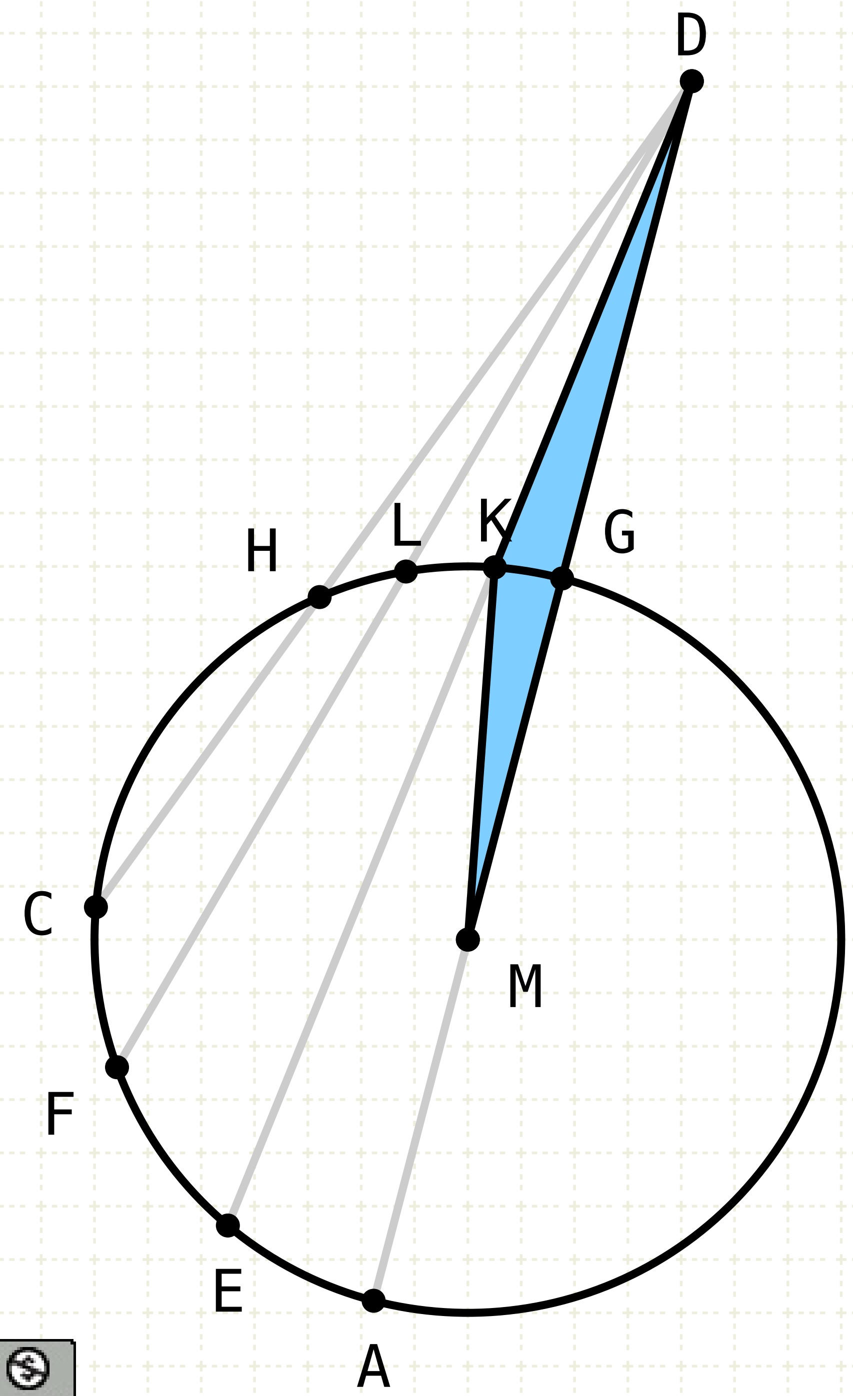

$$DK + KM > DM$$
$$DK + KM > DG + GM$$
$$DK > DG$$

Proof (part 2)

Consider triangle DKM, DK plus KM is greater than DM (I·20)

But KM is equal to GM

Subtract GM from both sides of the inequality gives DK is greater than DG

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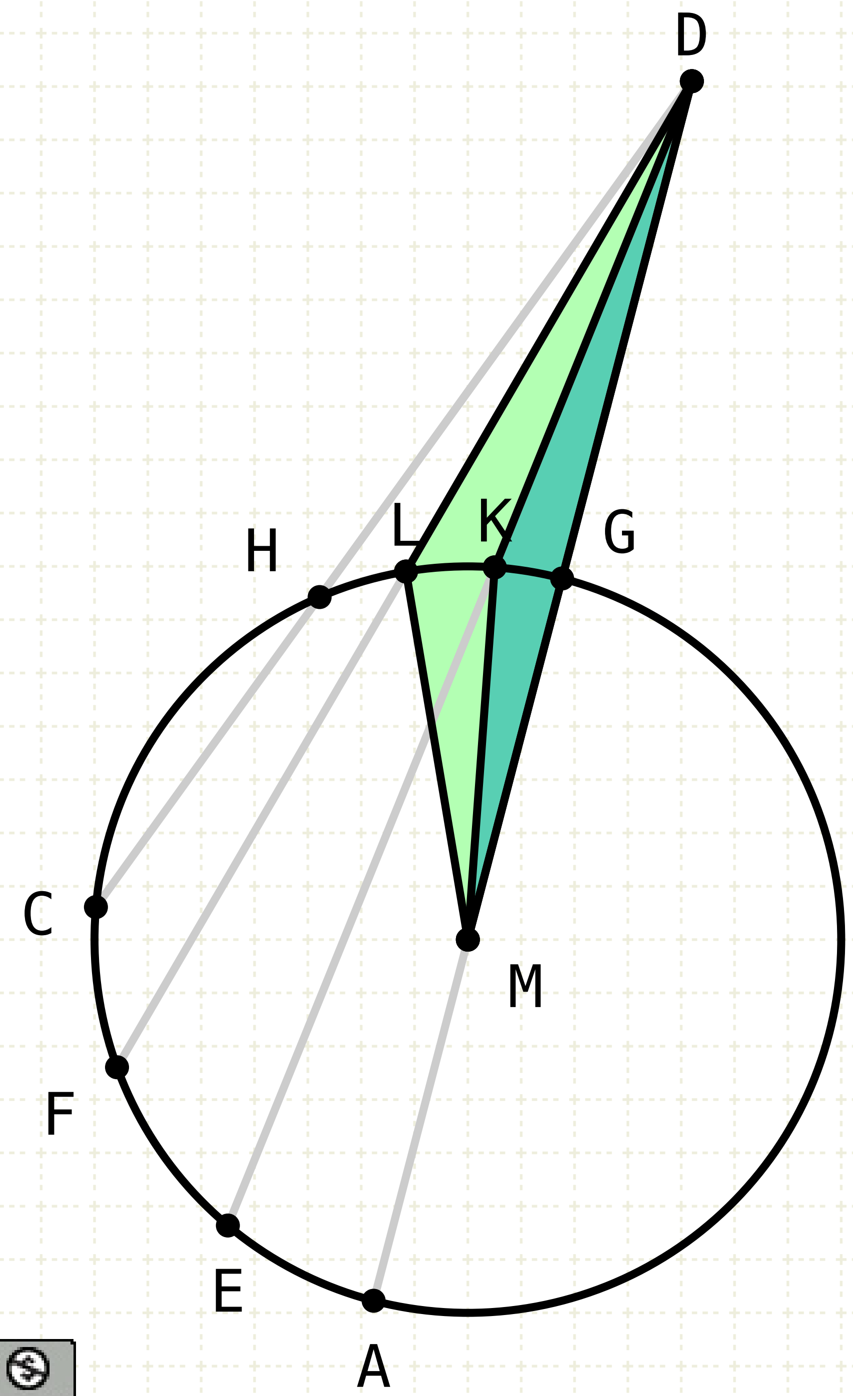
If a point be taken outside a circle and from the point straight lines be drawn through to the circle, one of which is through the centre and the others are drawn at random, then, of the straight lines which fall on the concave circumference, that through the centre is greatest, while of the rest the nearer to that through the centre is always greater than the more remote, but, of the straight lines falling on the convex circumference, that between the point and the diameter is least, while of the rest the nearer to the least is always less than the more remote, and only two equal straight lines will fall on the circle from the point, one on each side of the least.



DK + KM > DM

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Proof (part 2)

Consider triangle DKM, DK plus KM is greater than DM (I·20)

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Subtract GM from both sides of the inequality gives DK is greater than DG

Consider the two triangles DKM and DLM

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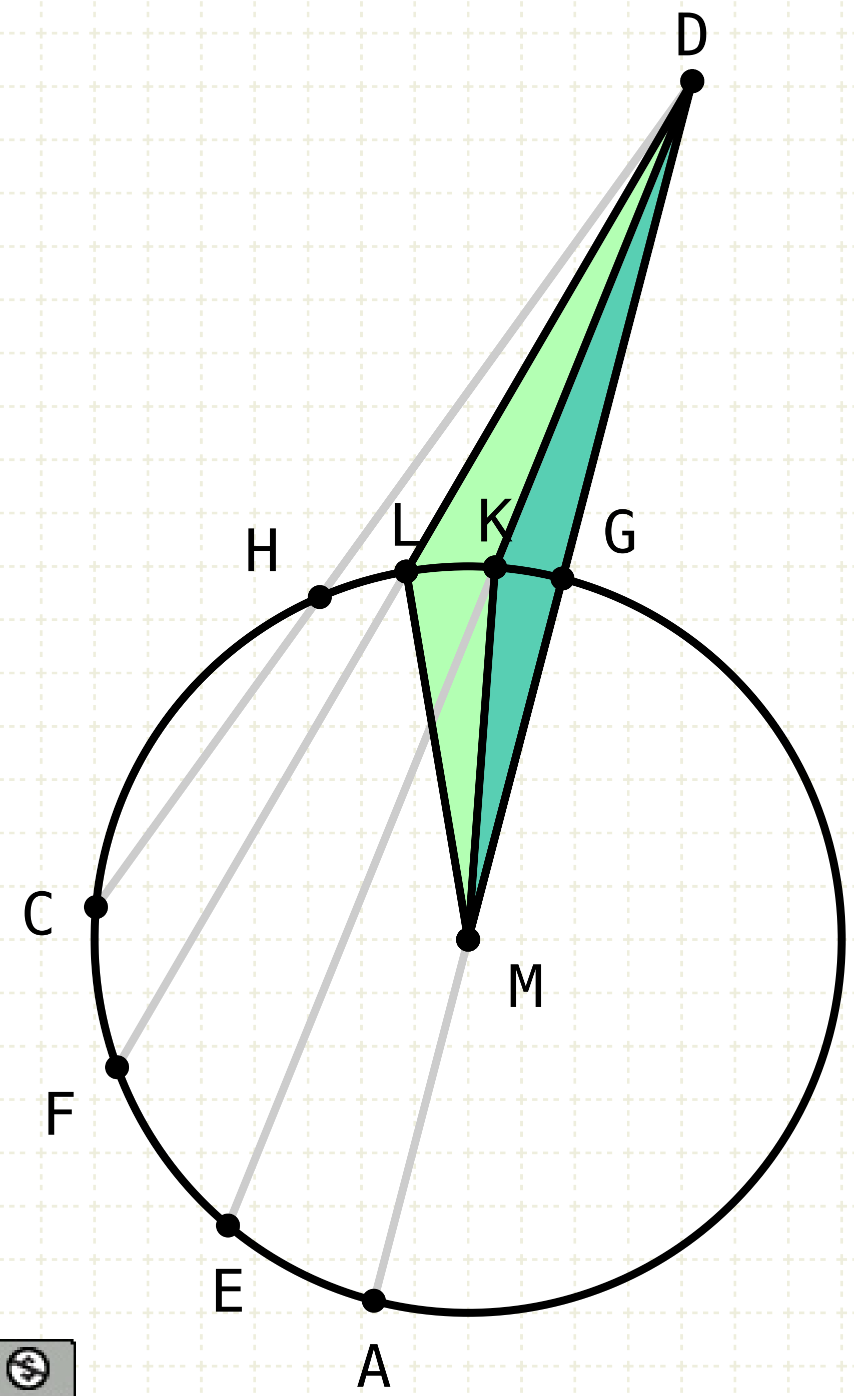
If a point be taken outside a circle and from the point straight lines be drawn through to the circle, one of which is through the centre and the others are drawn at random, then, of the straight lines which fall on the concave circumference, that through the centre is greatest, while of the rest the nearer to that through the centre is always greater than the more remote, but, of the straight lines falling on the convex circumference, that between the point and the diameter is least, while of the rest the nearer to the least is always less than the more remote, and only two equal straight lines will fall on the circle from the point, one on each side of the least.



$DK + KM > DM$

$DK + KM > DG + GM$

$DK > DG$

$DL + LM > DK + KM$

Proof (part 2)

Consider triangle DKM, DK plus KM is greater than DM (I·20)

But KM is equal to GM

Subtract GM from both sides of the inequality gives DK is greater than DG

Consider the two triangles DKM and DLM

The two lines DK and KM are wholly within the triangle DLM, therefore the sum of DK,KM is less than the sum of DL,LM (I·21)

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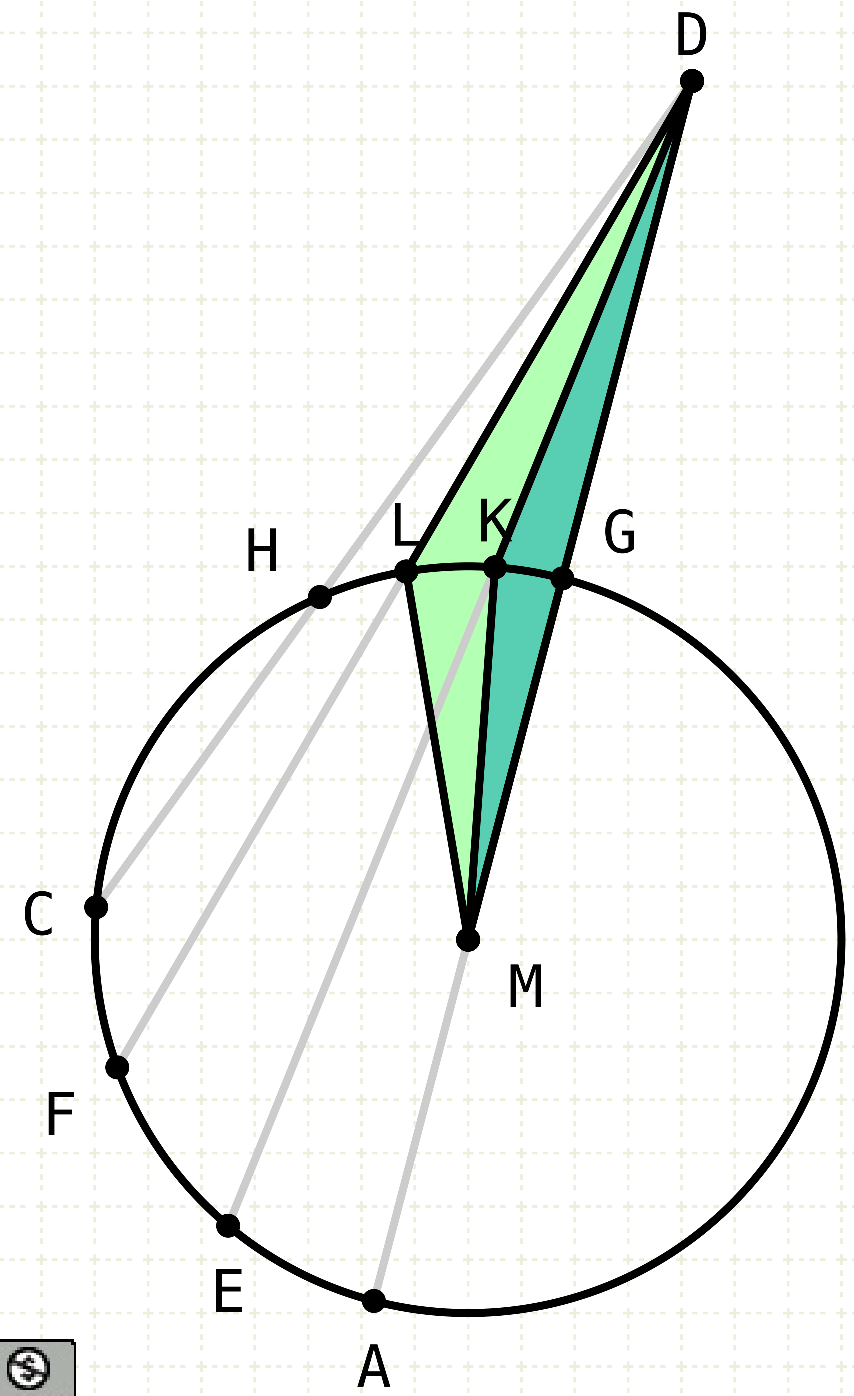
If a point be taken outside a circle and from the point straight lines be drawn through to the circle, one of which is through the centre and the others are drawn at random, then, of the straight lines which fall on the concave circumference, that through the centre is greatest, while of the rest the nearer to that through the centre is always greater than the more remote, but, of the straight lines falling on the convex circumference, that between the point and the diameter is least, while of the rest the nearer to the least is always less than the more remote, and only two equal straight lines will fall on the circle from the point, one on each side of the least.



$DK + KM > DM$

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$DL + LM > DK + KM$

$DL > DK$

Proof (part 2)

Consider triangle DKM, DK plus KM is greater than DM (I·20)

But KM is equal to GM

Subtract GM from both sides of the inequality gives DK is greater than DG

Consider the two triangles DKM and DLM

The two lines DK and KM are wholly within the triangle DLM, therefore the sum of DK,KM is less than the sum of DL,LM (I·21)

But LM is equal to KM

Subtract KM from both sides of the inequality gives DL is greater than DK

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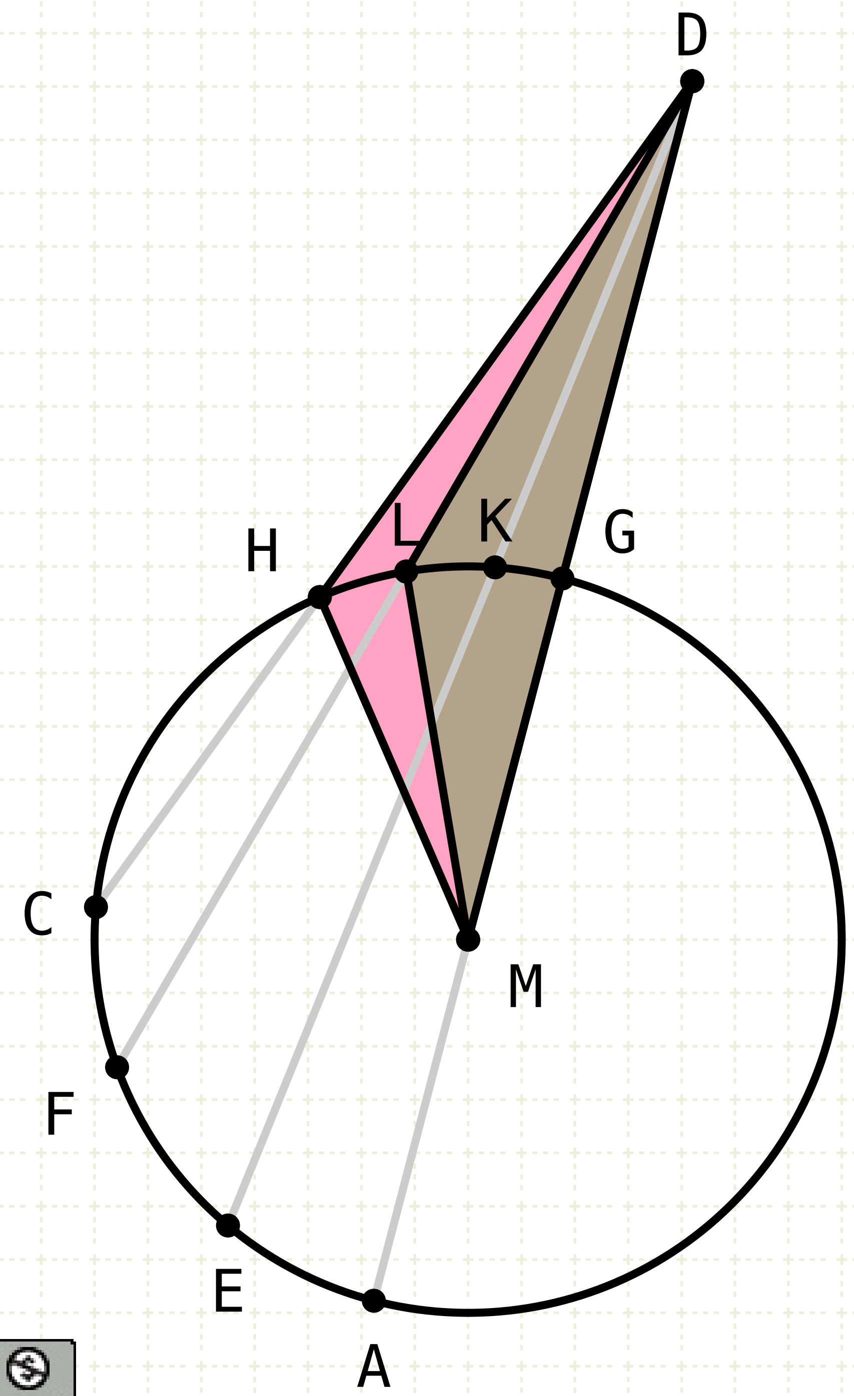
If a point be taken outside a circle and from the point straight lines be drawn through to the circle, one of which is through the centre and the others are drawn at random, then, of the straight lines which fall on the concave circumference, that through the centre is greatest, while of the rest the nearer to that through the centre is always greater than the more remote, but, of the straight lines falling on the convex circumference, that between the point and the diameter is least, while of the rest the nearer to the least is always less than the more remote, and only two equal straight lines will fall on the circle from the point, one on each side of the least.



DK + KM > DM

DK + KM > DG + GM

DK > DG

DL + LM > DK + KM

DL > DK

DH > DL

Proof (part 2)

Consider triangle DKM, DK plus KM is greater than DM (I·20)

But KM is equal to GM

Subtract GM from both sides of the inequality gives DK is greater than DG

Consider the two triangles DKM and DLM

The two lines DK and KM are wholly within the triangle DLM, therefore the sum of DK,KM is less than the sum of DL,LM (I·21)

But LM is equal to KM

Subtract KM from both sides of the inequality gives DL is greater than DK

Using the same logic, we have DH greater than DL

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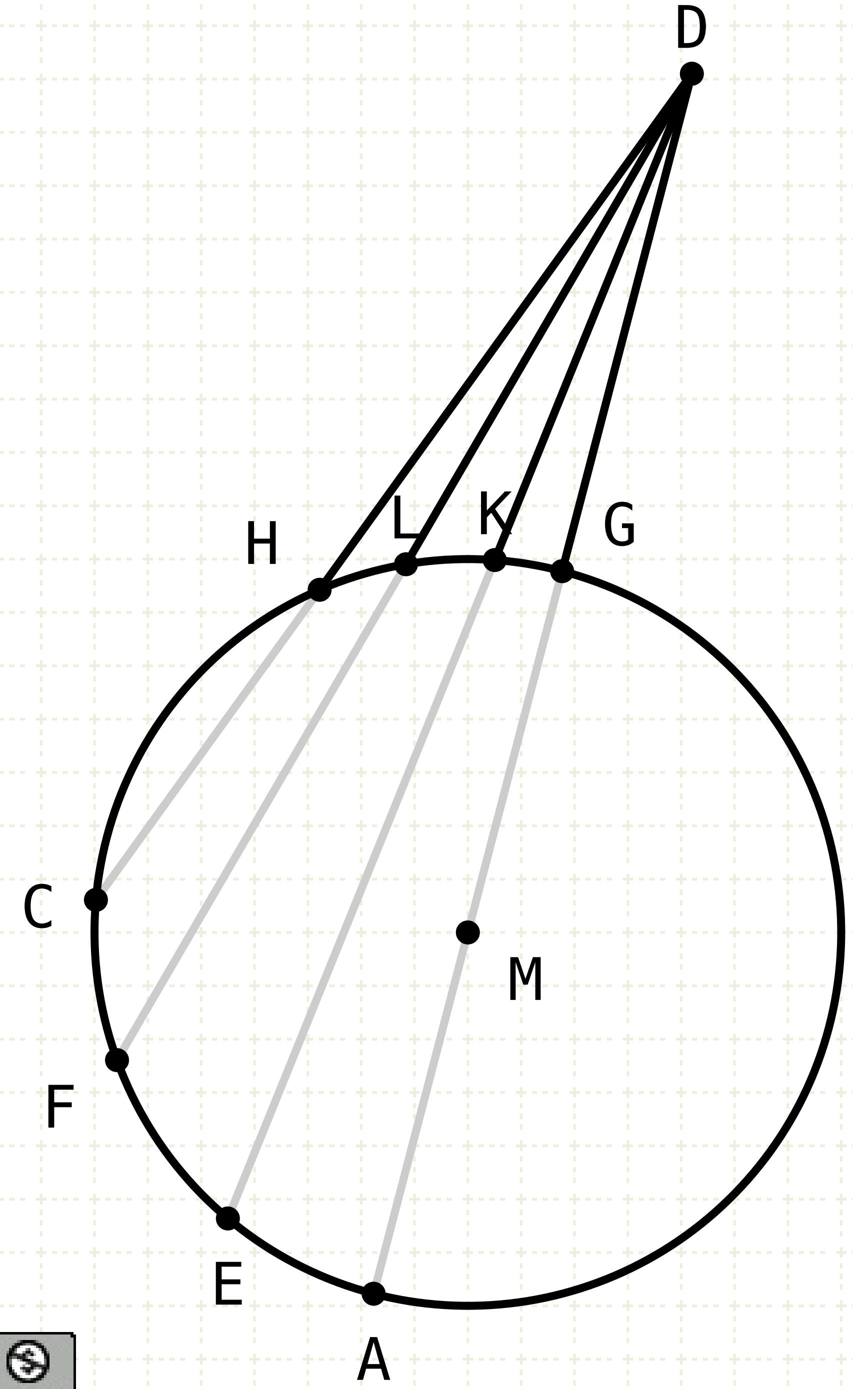

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$DL > DK$

$DH > DL$

$DG < DK < DL < DH$

Proof (part 2)

Consider triangle DKM, DK plus KM is greater than DM (I·20)

But KM is equal to GM

Subtract GM from both sides of the inequality gives DK is greater than DG

Consider the two triangles DKM and DLM

The two lines DK and KM are wholly within the triangle DLM, therefore the sum of DK,KM is less than the sum of DL,LM (I·21)

But LM is equal to KM

Subtract KM from both sides of the inequality gives DL is greater than DK

Using the same logic, we have DH greater than DL

Proposition 8 of Book III

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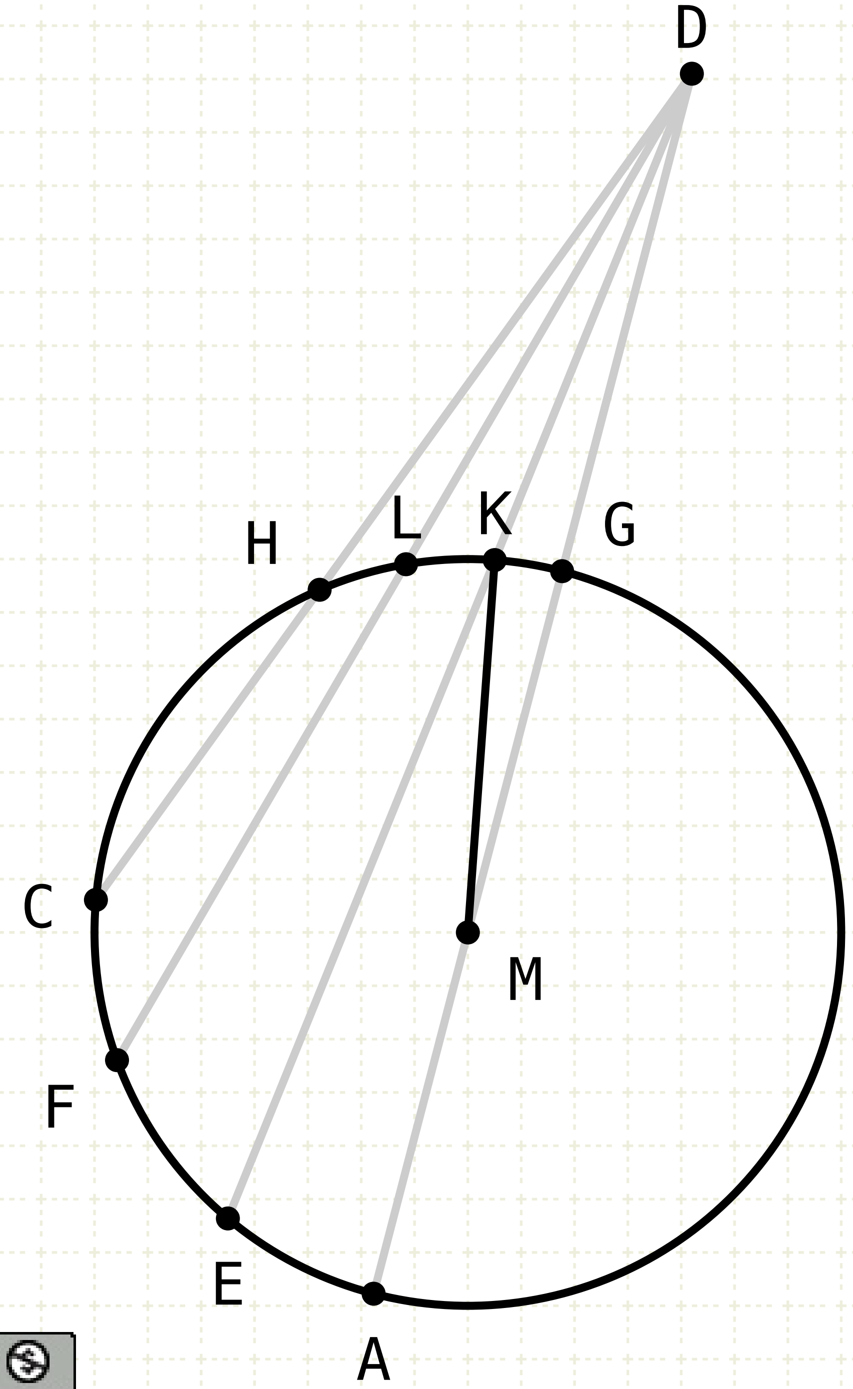

Proof (part 3)

Construct a line MB such that the angle DMK equals DMB, and draw the line DB

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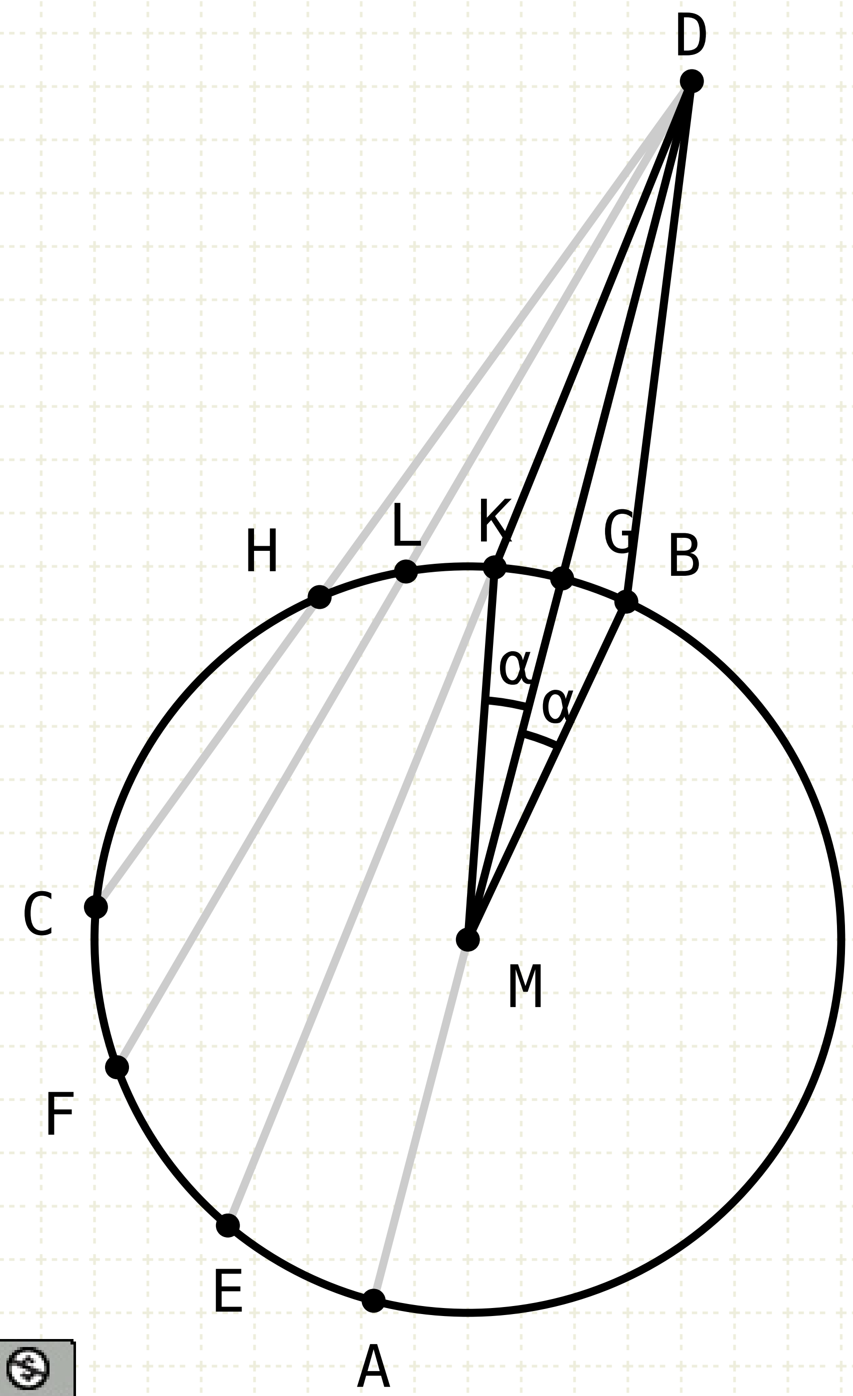

Proof (part 3)

Construct a line MB such that the angle DMK equals DMB, and draw the line DB

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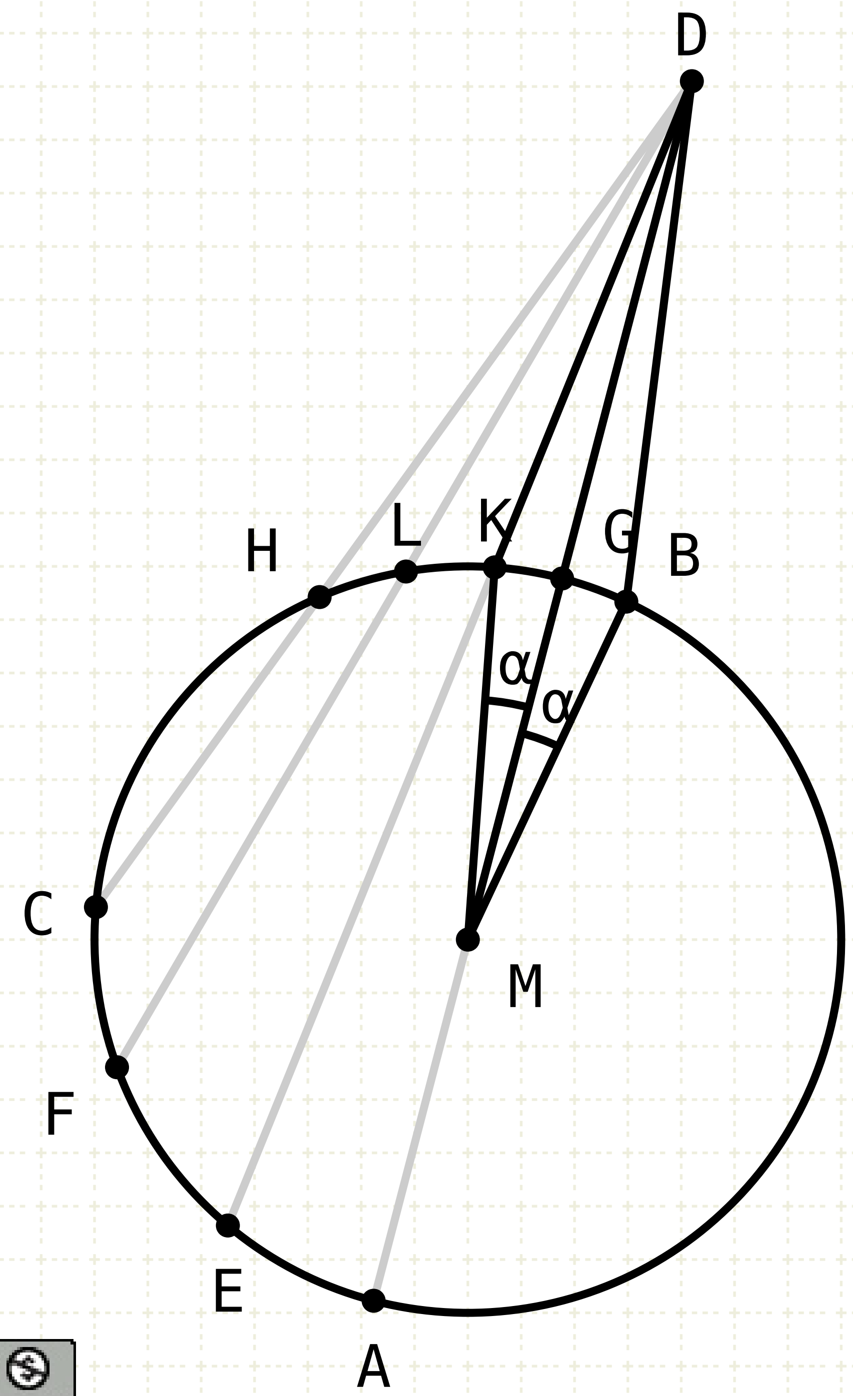

$\Delta DKM \equiv \Delta DBM$

$DK = DB$

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MK equals MB (radii of the same circle) and MD is common to both, so with two triangles with side-angle-side SAS equal, the triangles are equal and therefore KD equals BD (I·4)

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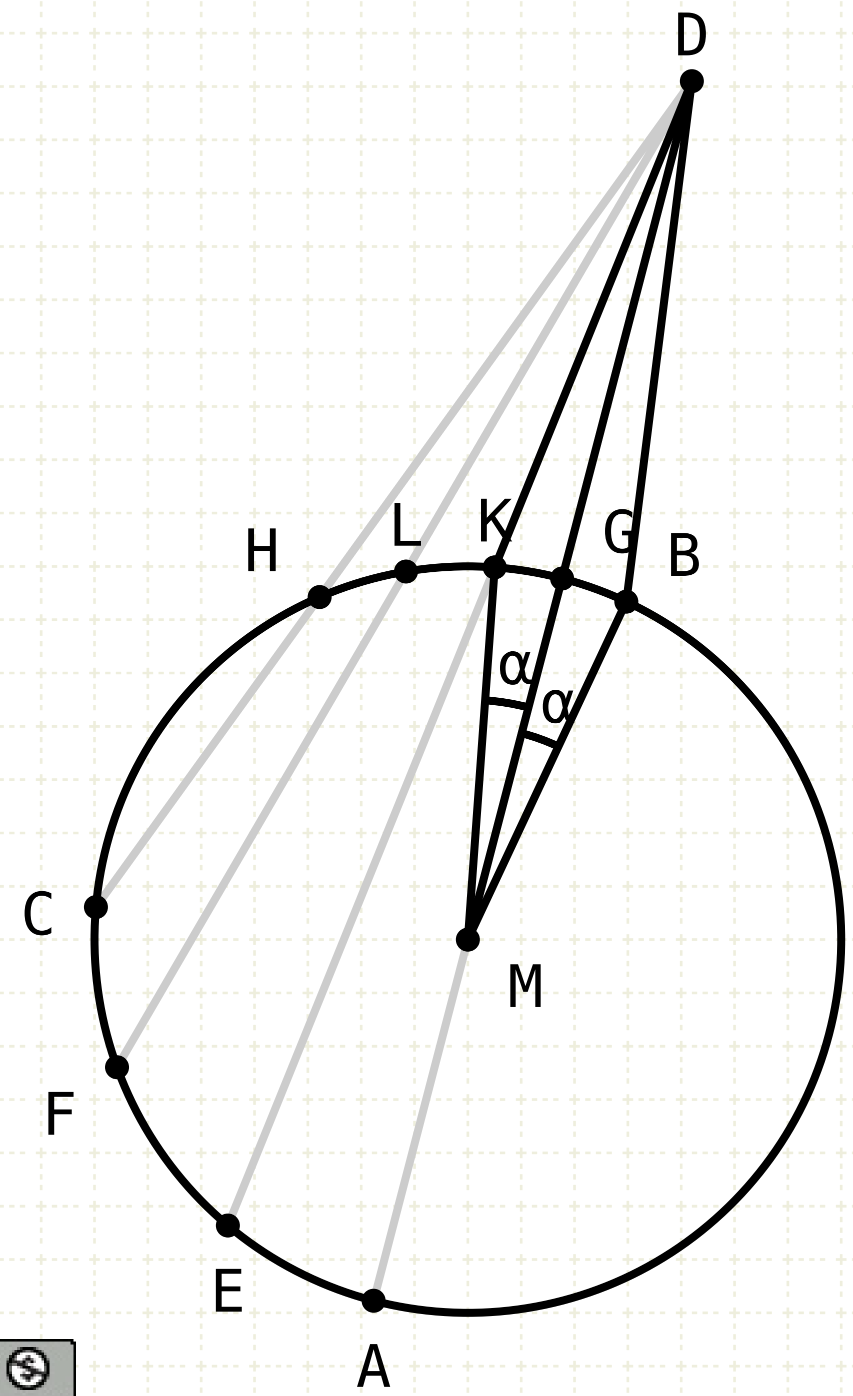

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There is no other line that can fall from D to the circle equal in length to DK and DB

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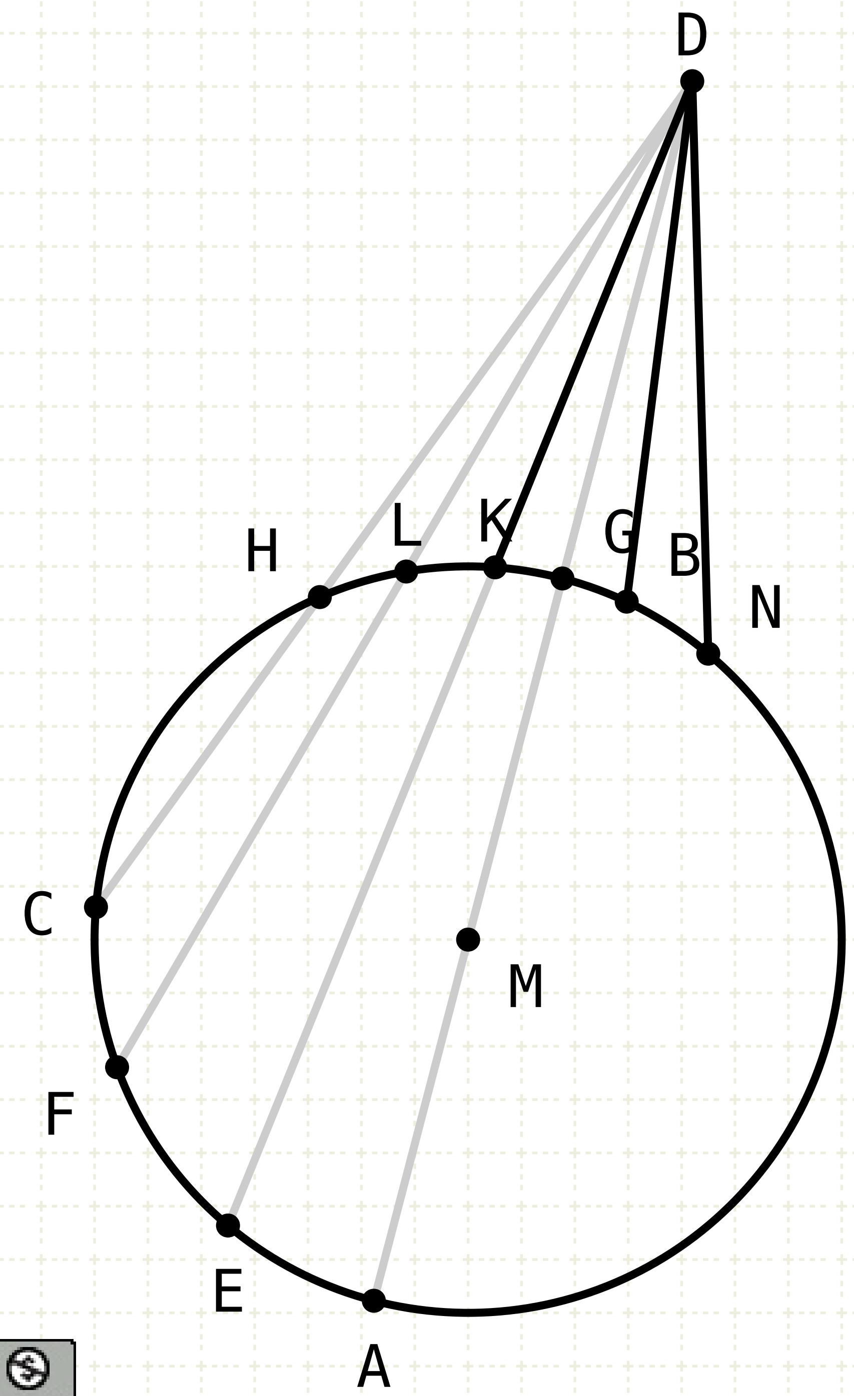

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Assume…

$DN = DK$

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Proof by contradiction:

Assume a line DN exists, equal in length to DK

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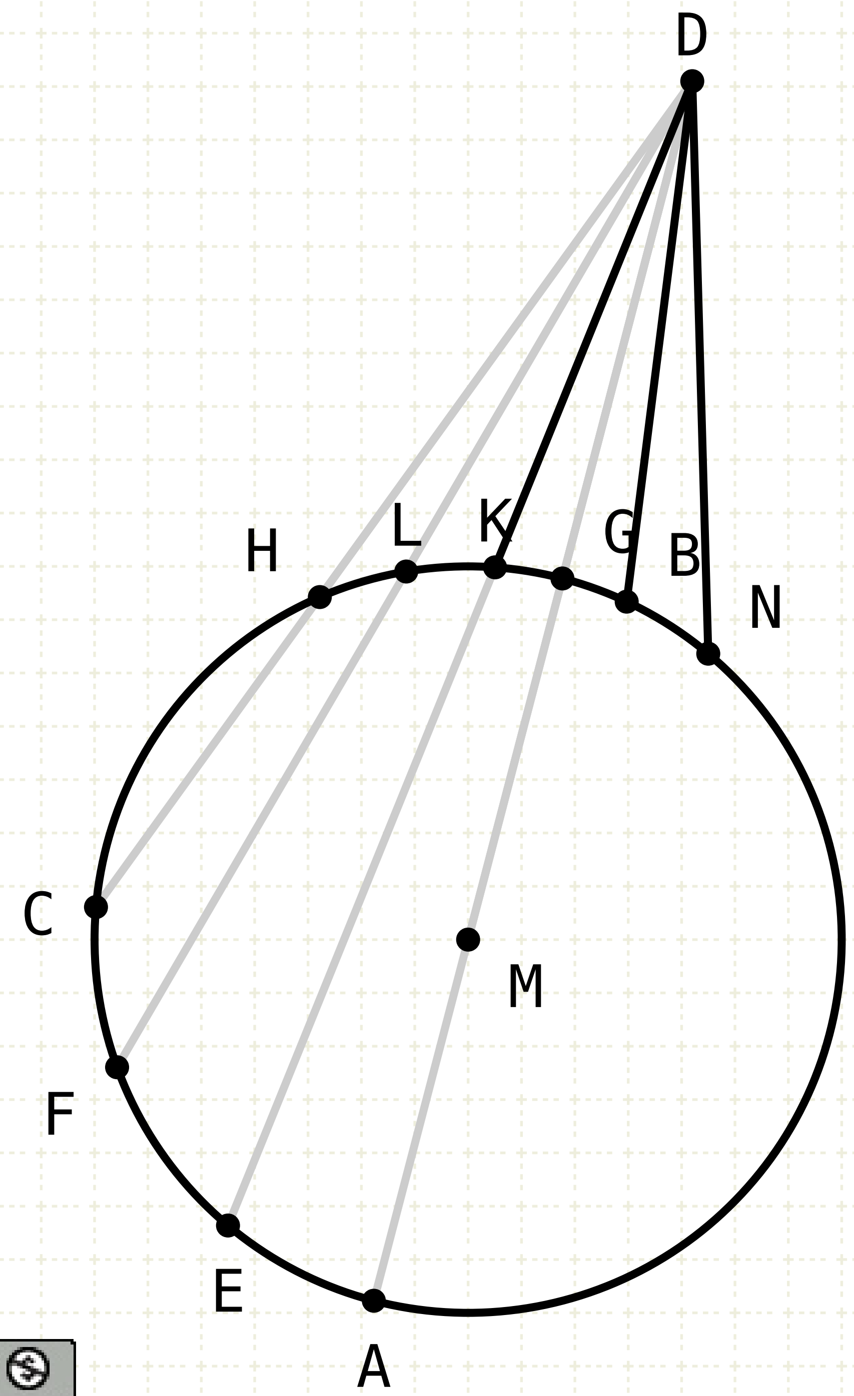

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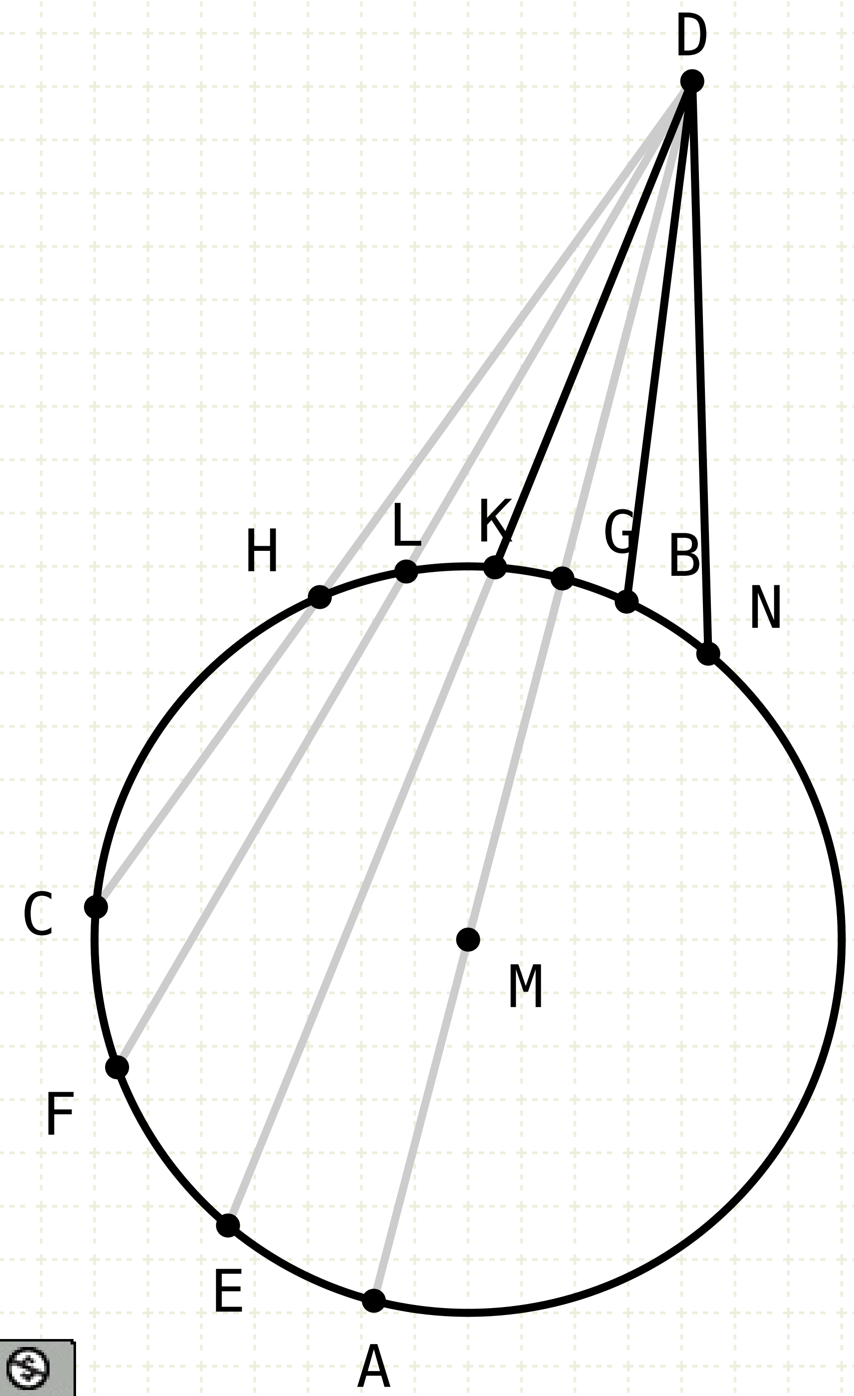

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Assume...

$DN = DK$

$DN = DB$ ✗

$DB < DN$

Proof (part 3)

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There is no other line that can fall from D to the circle equal in length to DK and DB

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But, according to the second part of this proposition, DB, being closer to DG, is smaller than DN, which contradicts the original statement

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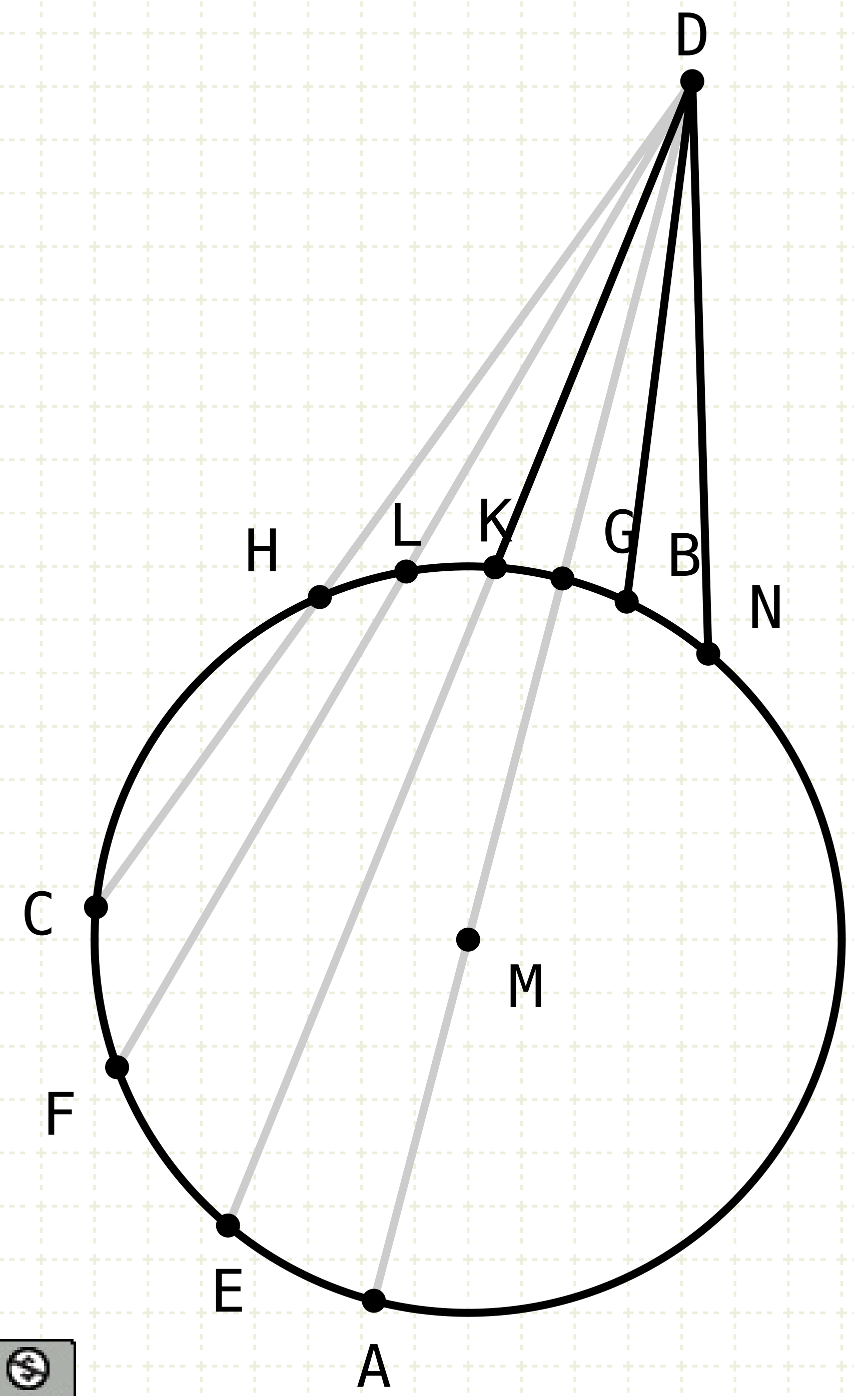

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Therefore there are only two lines of equal length from D to the circle circumference

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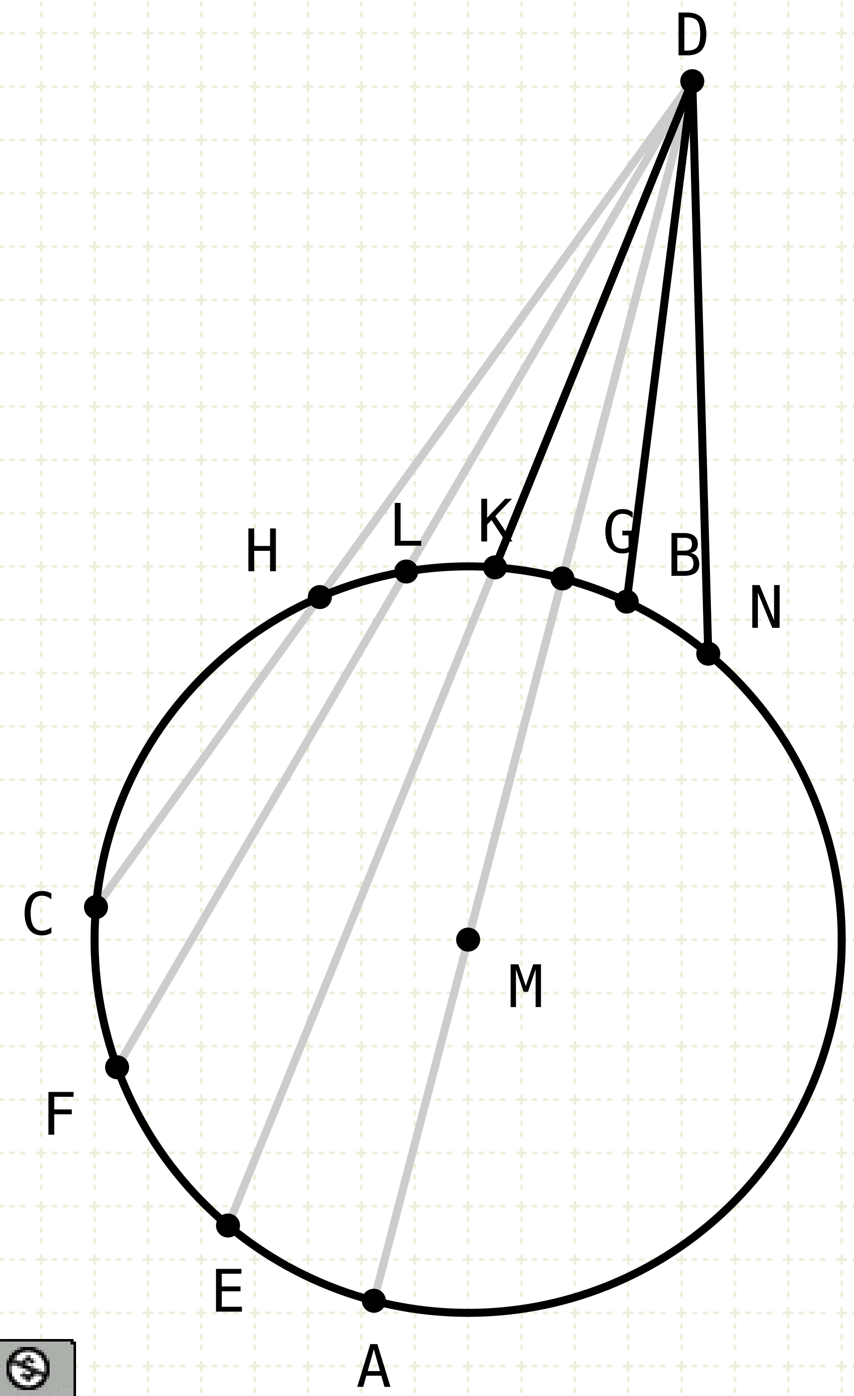

$\Delta DKM \equiv \Delta DBM$

$DK = DB$

Assume...

$DN = DK$

$DN = DB$ ✗

$DB < DN$

$DN \neq DK$

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