

Euclid's Elements

Book III



A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)



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6	If two circles touch one another, they will not have the same center	14	In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another.	22	The opposite angles of quadrilaterals in circles are equal to two right angles
7	Consider two lines from a point inside a circle to the edge, the longer one will be the one closest to the longest part of the diameter passing through the original point	15	The longest line in a circle is its diameter, shorter the farther away from the diameter	23	On the same straight line there cannot be constructed two similar and unequal segments of circles on the same side
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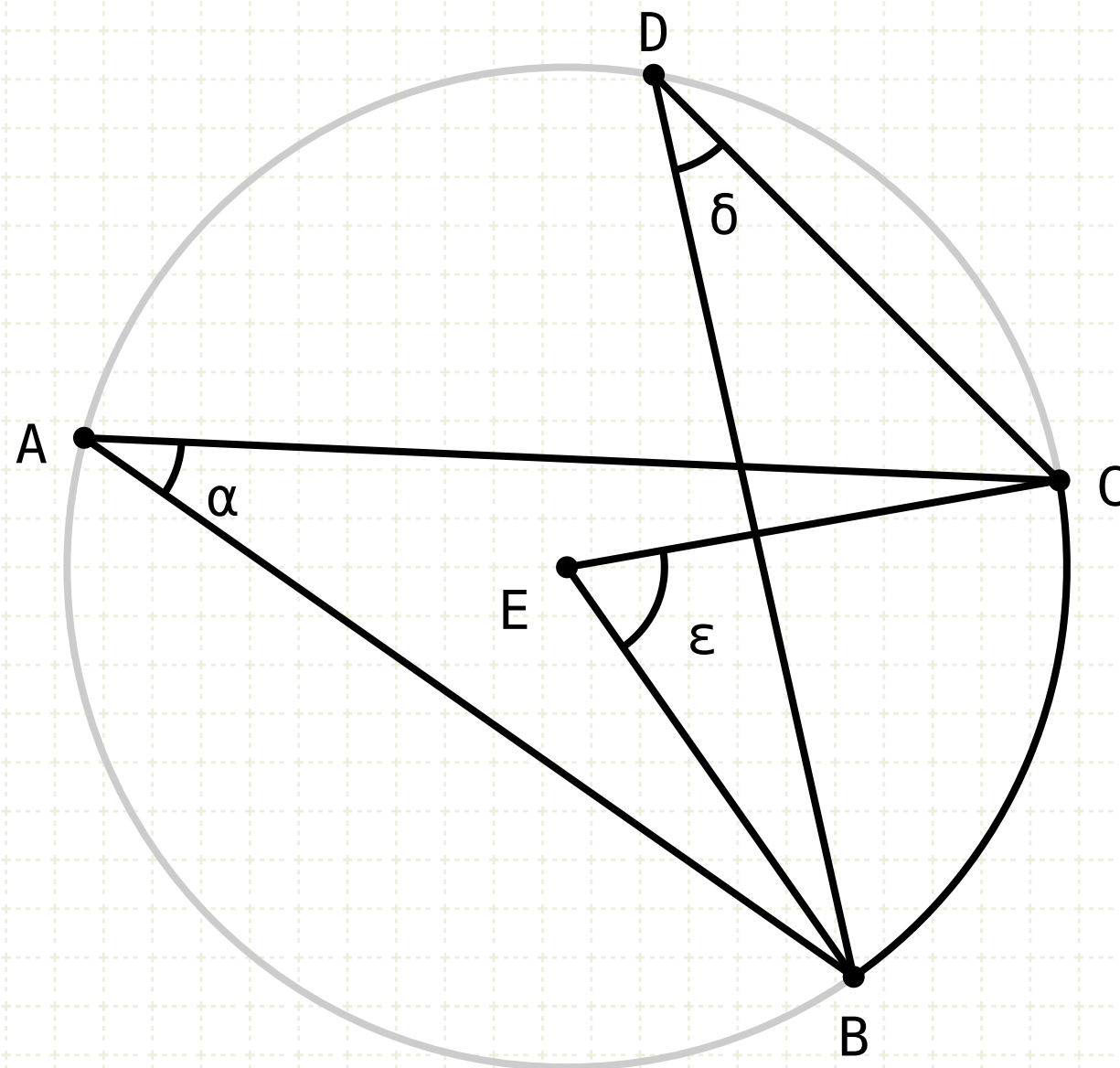
Proposition 20 of Book III

In a circle the angle at the centre is double of the angle at the circumference, when the angles have the same circumference as base.



Proposition 20 of Book III

In a circle the angle at the centre is double of the angle at the circumference, when the angles have the same circumference as base.



E is the centre of the circle

$$\varepsilon = 2\alpha$$

$$\varepsilon = 2\delta$$

In other words

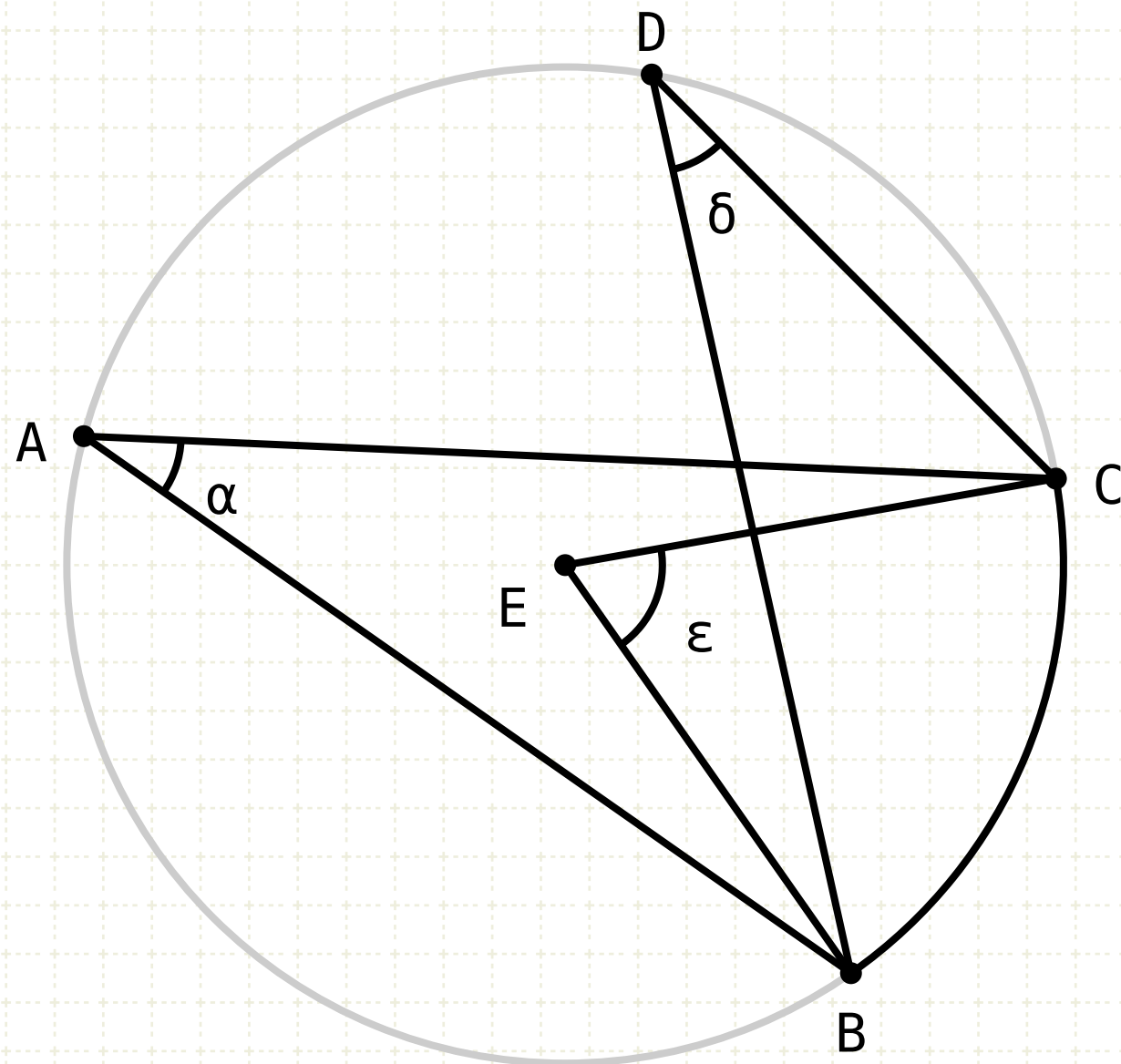
If E is the centre of a circle, and the arc BC the base of the angle BEC (ε) then ε will be double...

... any angle drawn from the circumference of the circle with the BC as its base,

... angle BAC (α) and angle BDC (δ) for example,

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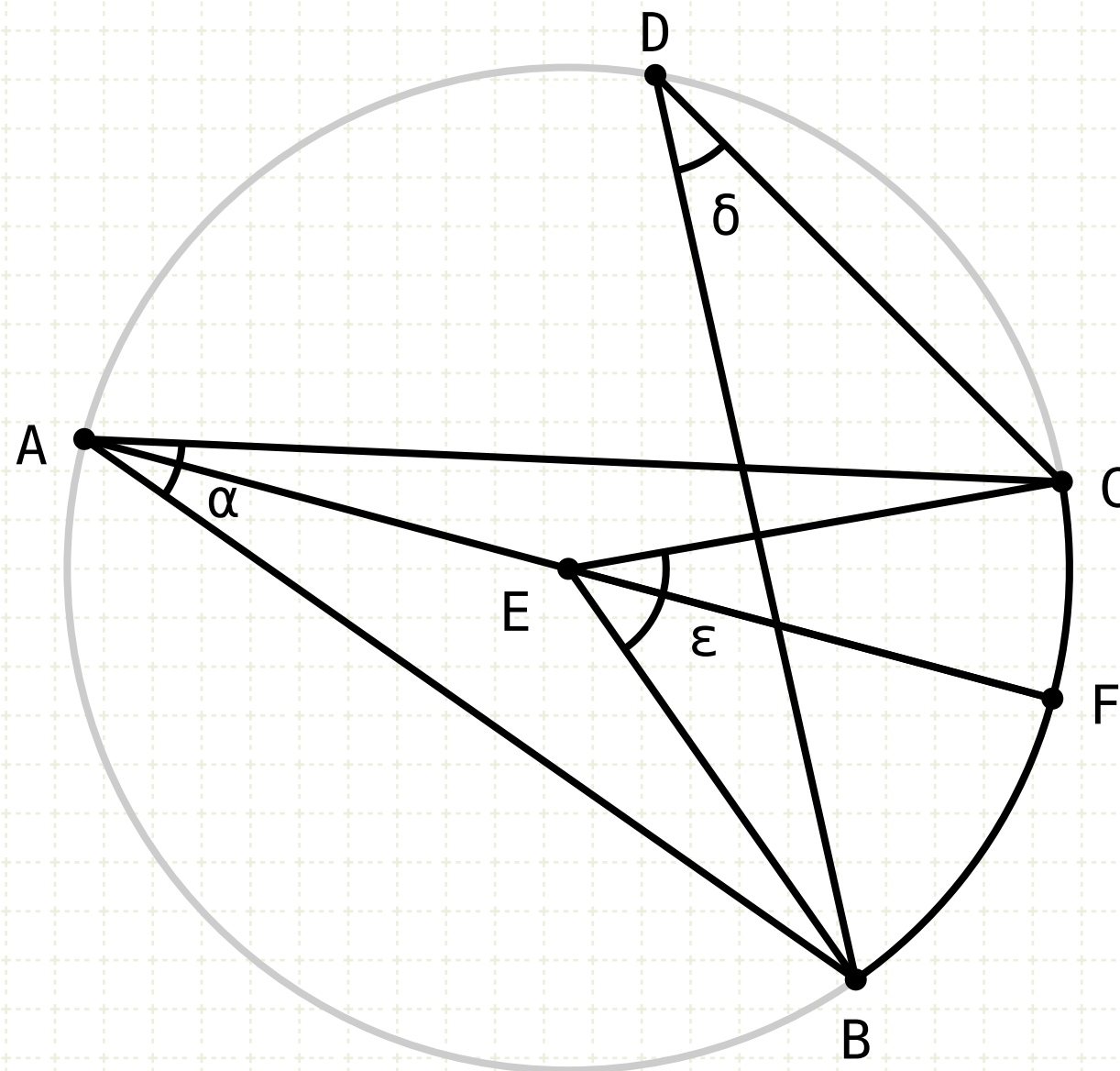
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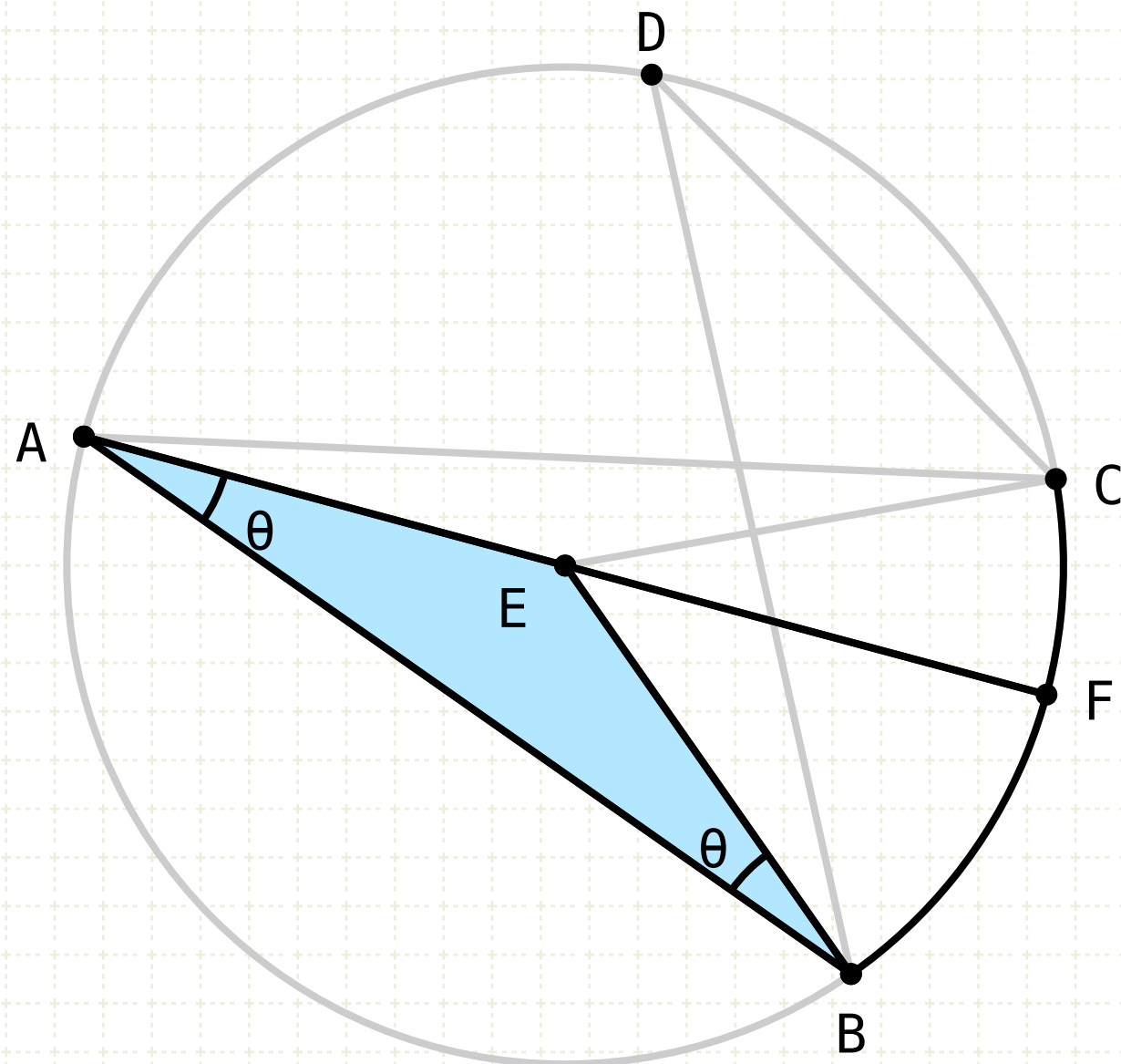
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Draw the line from A to E and extend it to point F

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In a circle the angle at the centre is double of the angle at the circumference, when the angles have the same circumference as base.



E is the centre of the circle

$$\angle EAB = \angle EBA = \theta$$

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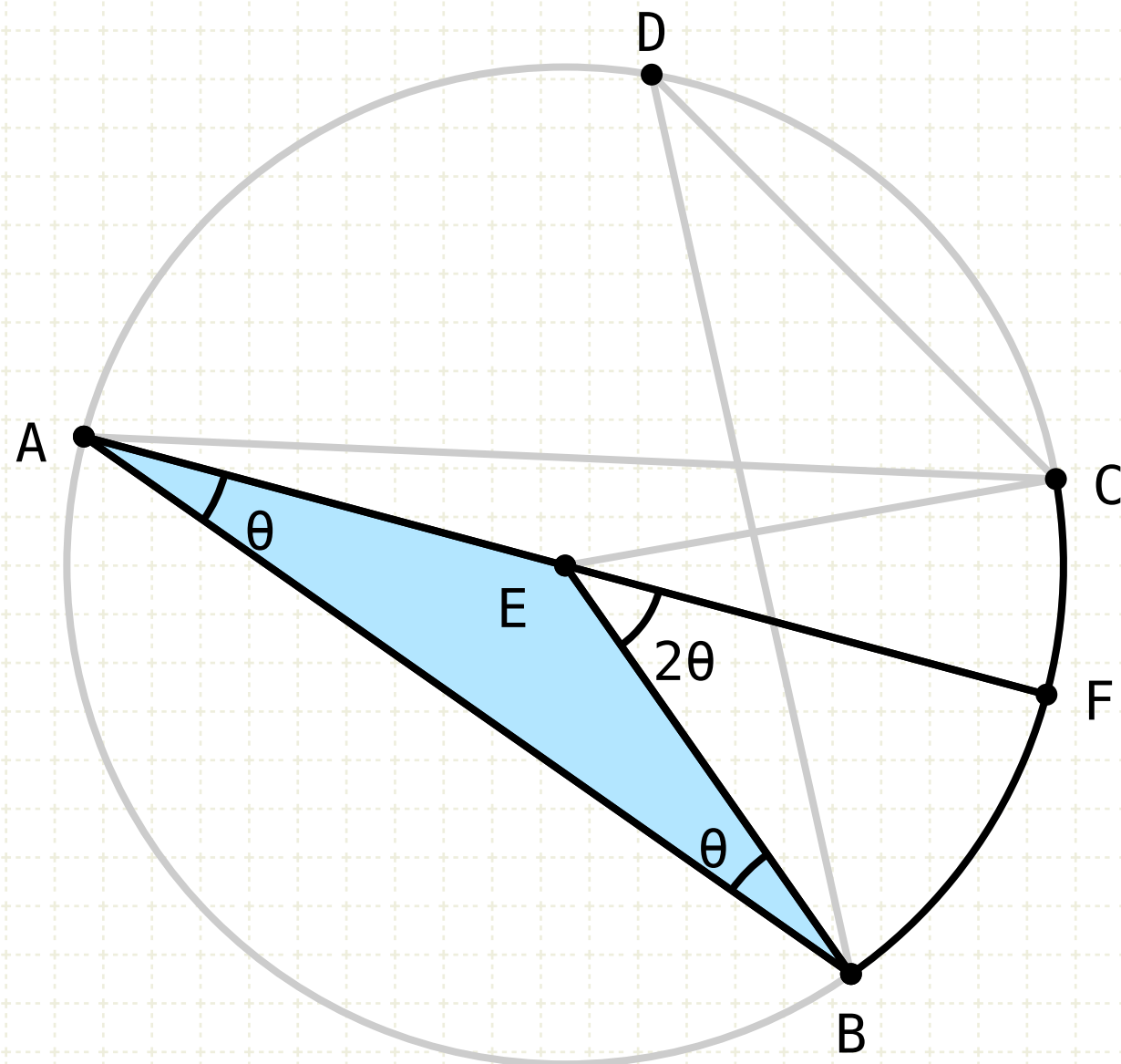
Proof

Draw the line from A to E and extend it to point F

Since AE and BE are equal, triangle ABE is isosceles, and its base angles EAB and EBA are equal (I-5)

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$$\angle FEB = 2\theta$$

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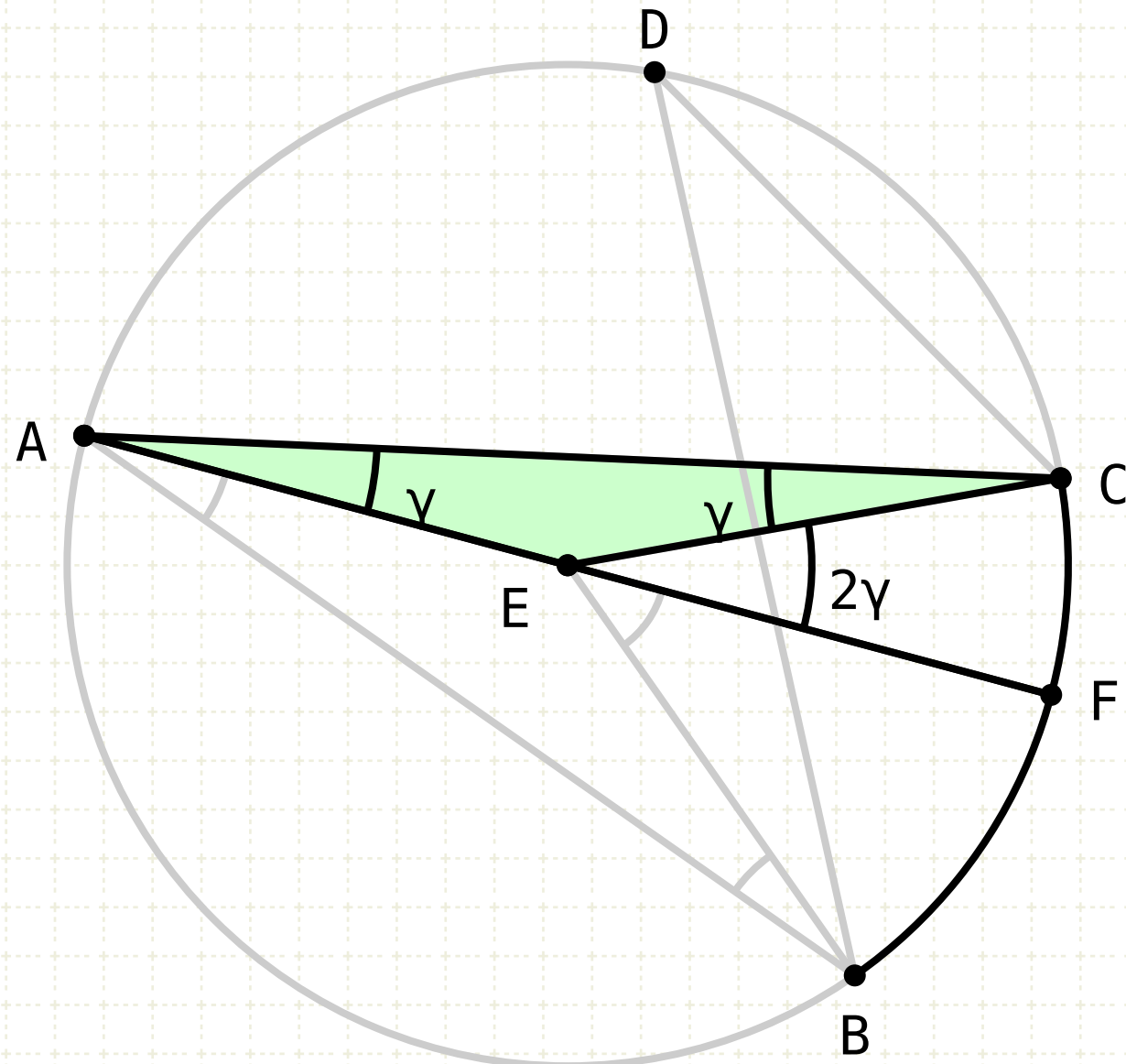
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$$\angle EAC = \angle ECA = \gamma$$

$$\angle FEC = 2\gamma$$

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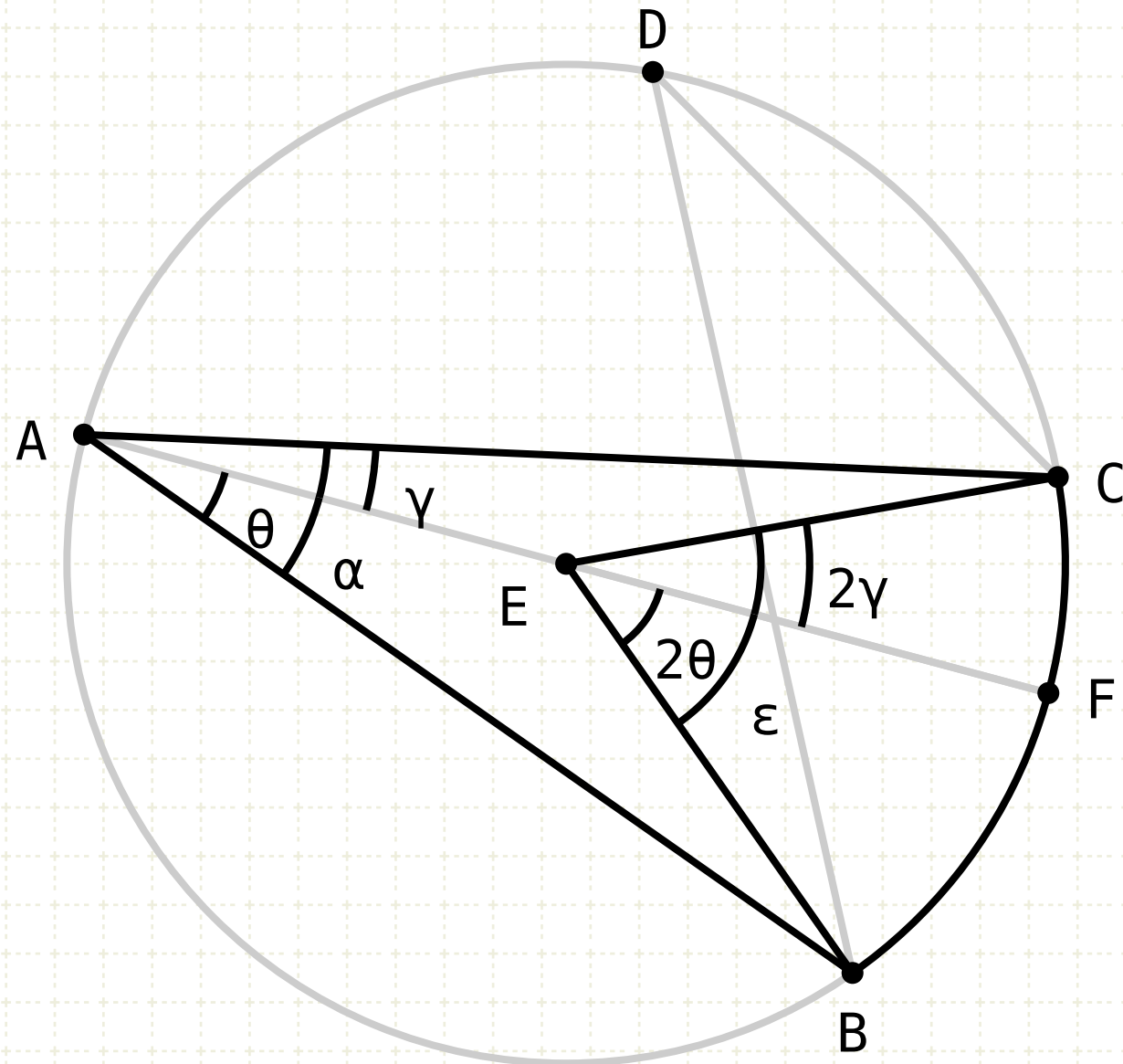
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Similarly, it can be shown that angle FEC is twice angle EAC

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$$\angle EAB = \angle EBA = \theta$$

$$\angle FEB = 2\theta$$

$$\angle EAC = \angle ECA = \gamma$$

$$\angle FEC = 2\gamma$$

$$\angle BAC = \alpha$$

$$\alpha = \gamma + \theta$$

$$\angle BEC = \epsilon$$

$$\epsilon = 2\gamma + 2\theta$$

$$\epsilon = 2\alpha$$

In other words

If E is the centre of a circle, and the arc BC the base of the angle BEC (ϵ) then ϵ will be double...

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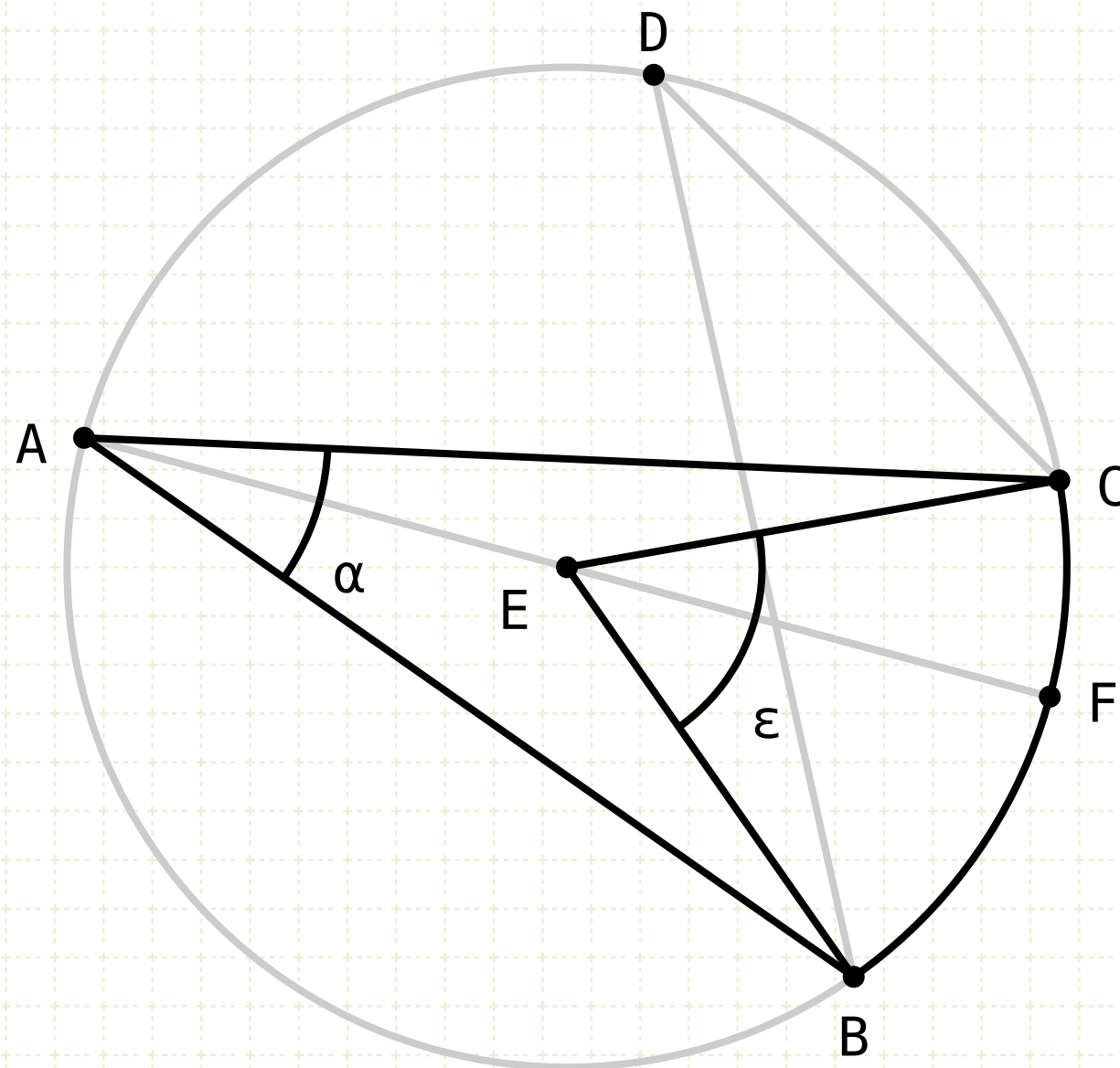
Similarly, it can be shown that angle FEC is twice angle EAC

Now, angle BAC (α) is equal to the sum of θ and γ , and the angle BEC (ϵ) is equal to the sum of 2θ and 2γ , thus giving us BEC is twice BAC



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Proof

Draw the line from A to E and extend it to point F

Since AE and BE are equal, triangle ABE is isosceles, and its base angles EAB and EBA are equal (I-5)

The exterior angle FEB is equal to the sum of the opposite interior angles (I-32) so FEB equals twice the angle BAE

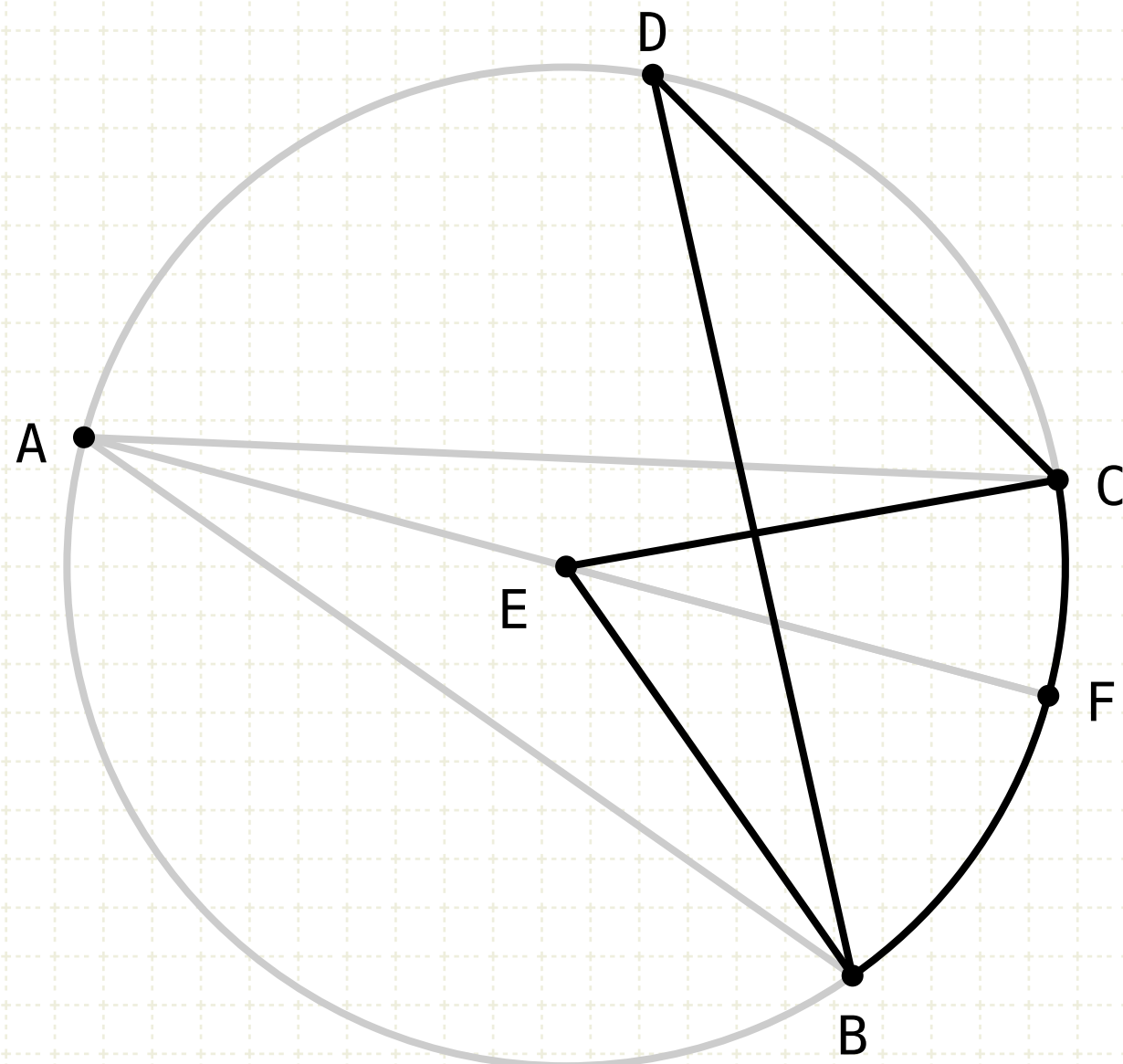
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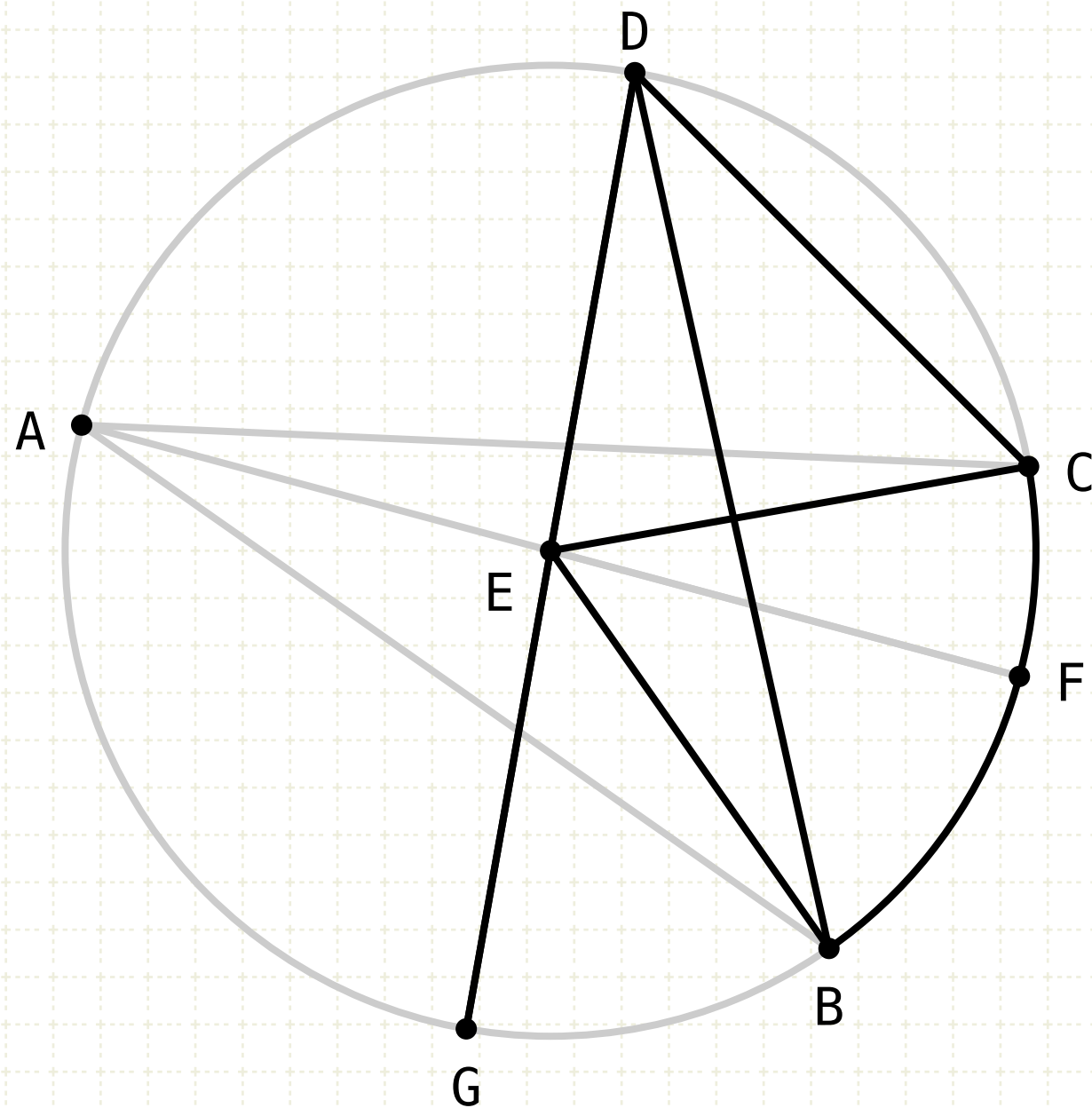
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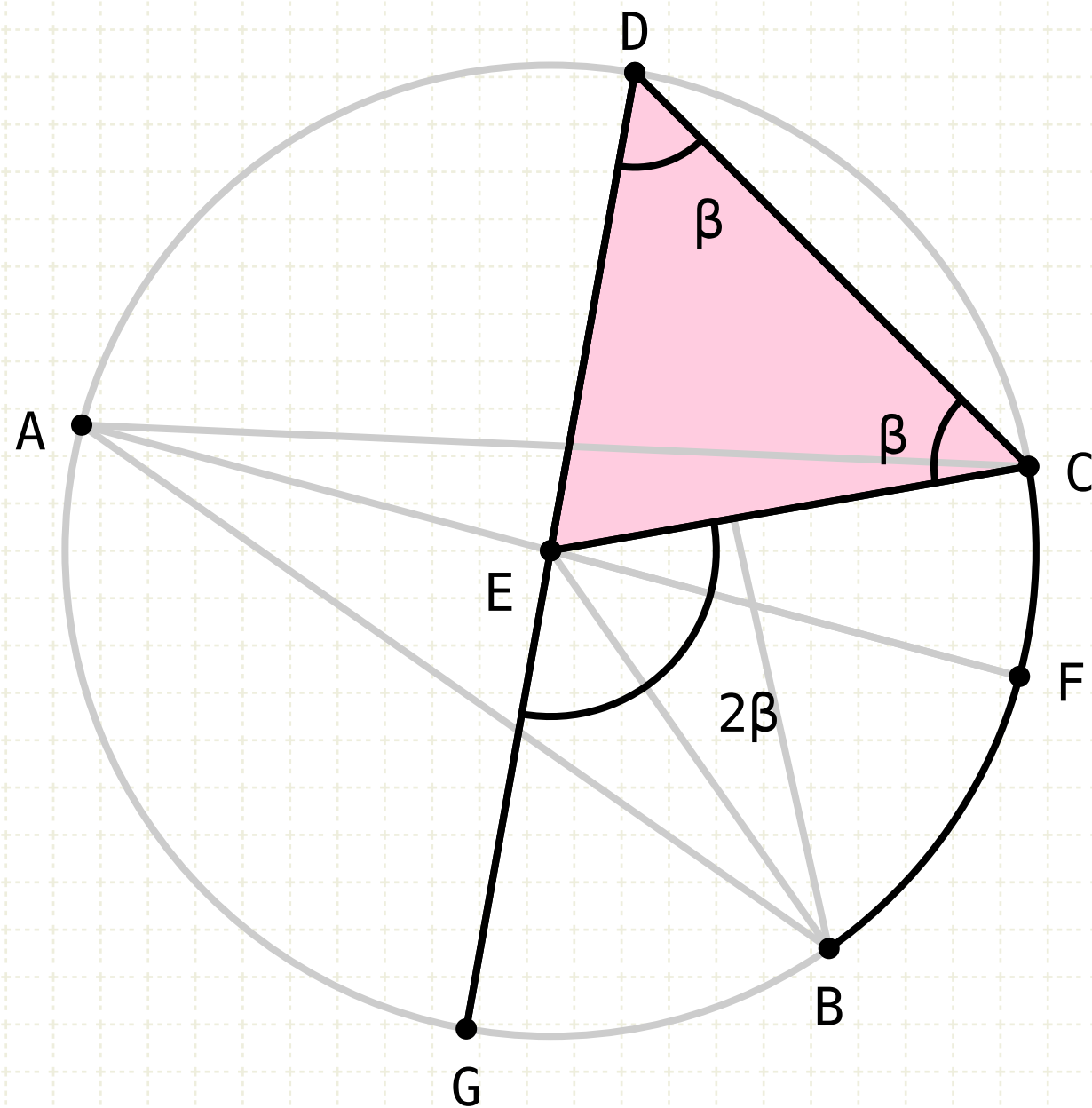
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Proof

Draw a line from DE and extend to G

Proposition 20 of Book III

In a circle the angle at the centre is double of the angle at the circumference, when the angles have the same circumference as base.



E is the centre of the circle

$$\varepsilon = 2\alpha$$

$$\angle EDC = \angle DCE = \beta$$

$$\angle GEC = 2\beta$$

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If E is the centre of a circle, and the arc BC the base of the angle BEC (ε) then ε will be double...

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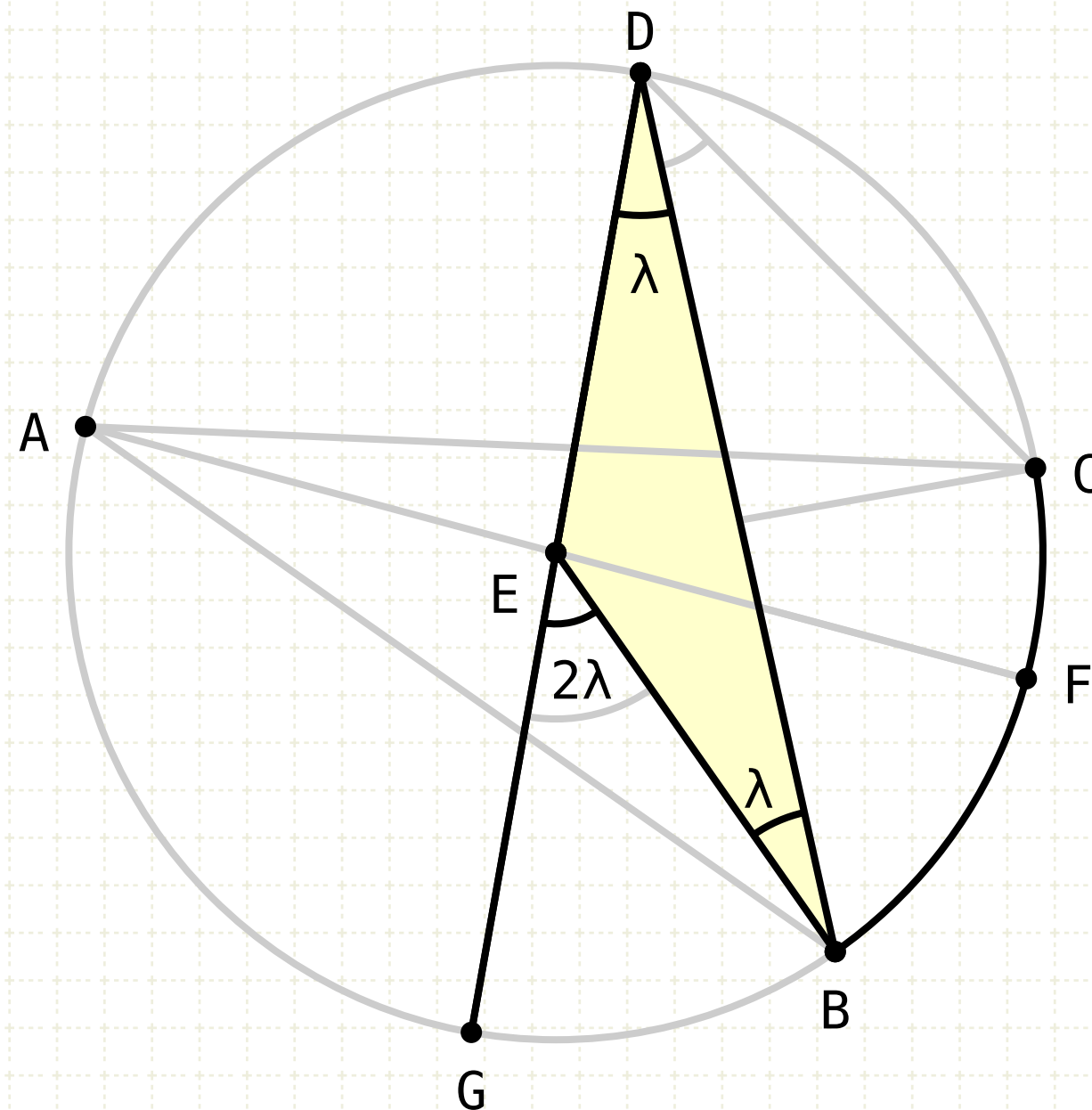
Proof

Draw a line from DE and extend to G

Using the same arguments as before, it can be seen that GEC is twice EDC

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In a circle the angle at the centre is double of the angle at the circumference, when the angles have the same circumference as base.



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$$\angle EDB = \angle DBE = \lambda$$

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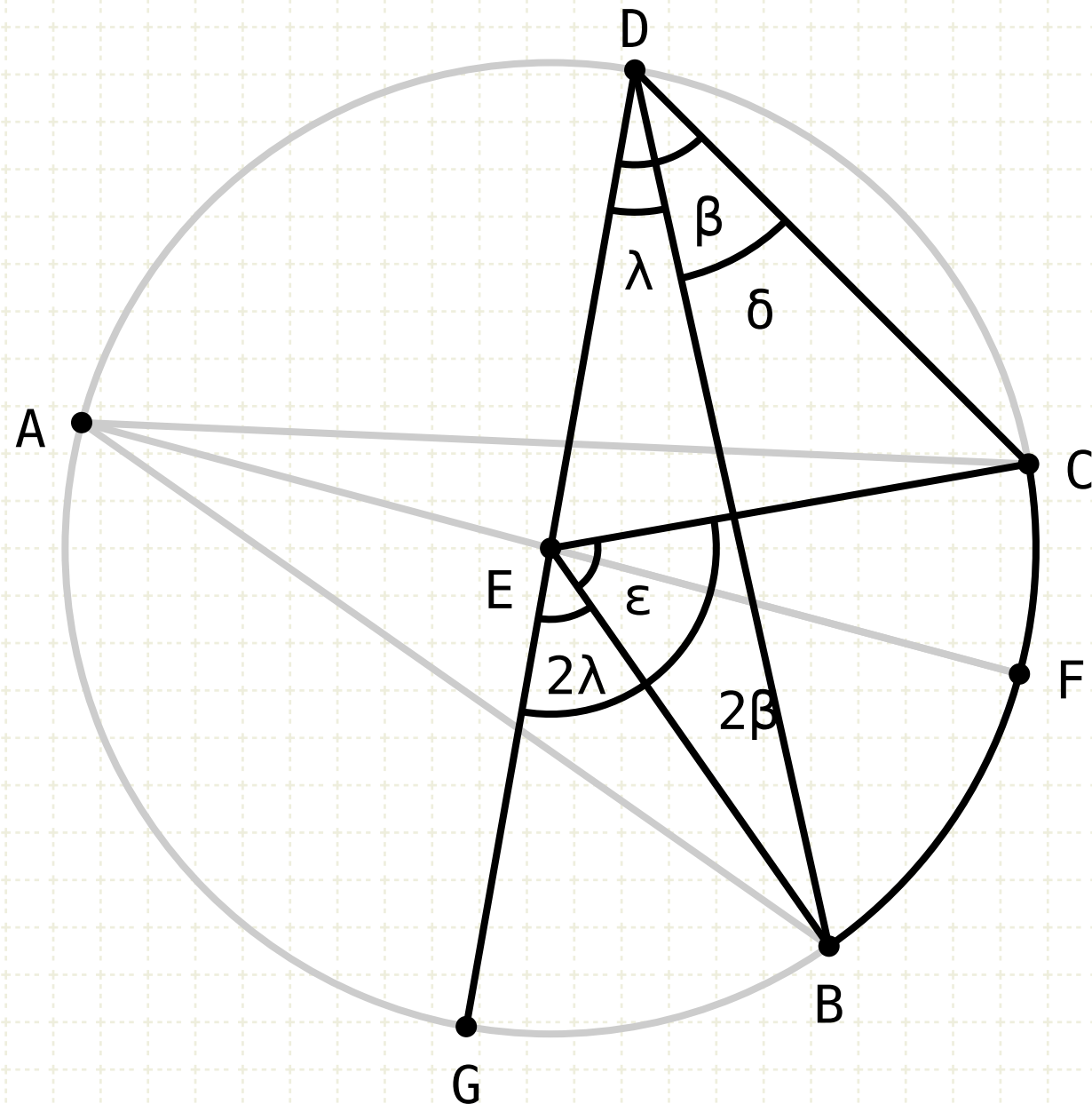
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It can also be seen that GEB is twice EDB

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$$\angle CDB = \delta$$

$$\delta = \beta - \lambda$$

$$\angle BEC = \epsilon$$

$$\epsilon = 2\beta - 2\lambda$$

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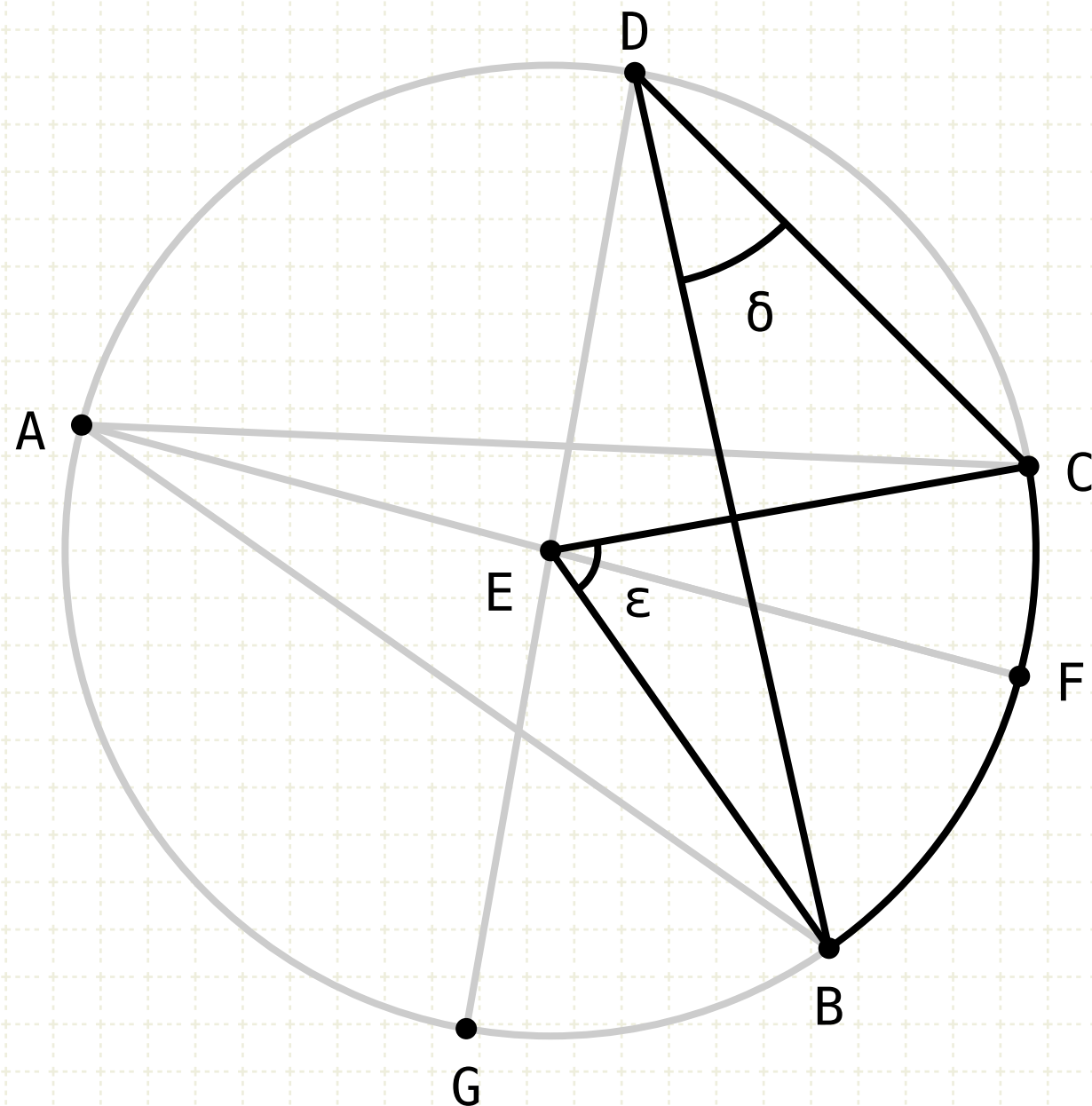
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It can also be seen that GEB is twice EDB

Thus the angle CDB (δ) is the difference between CDE (β) and BDE (λ) and angle CEB (ϵ) is the difference between CEG (2β) and BEG (2λ)

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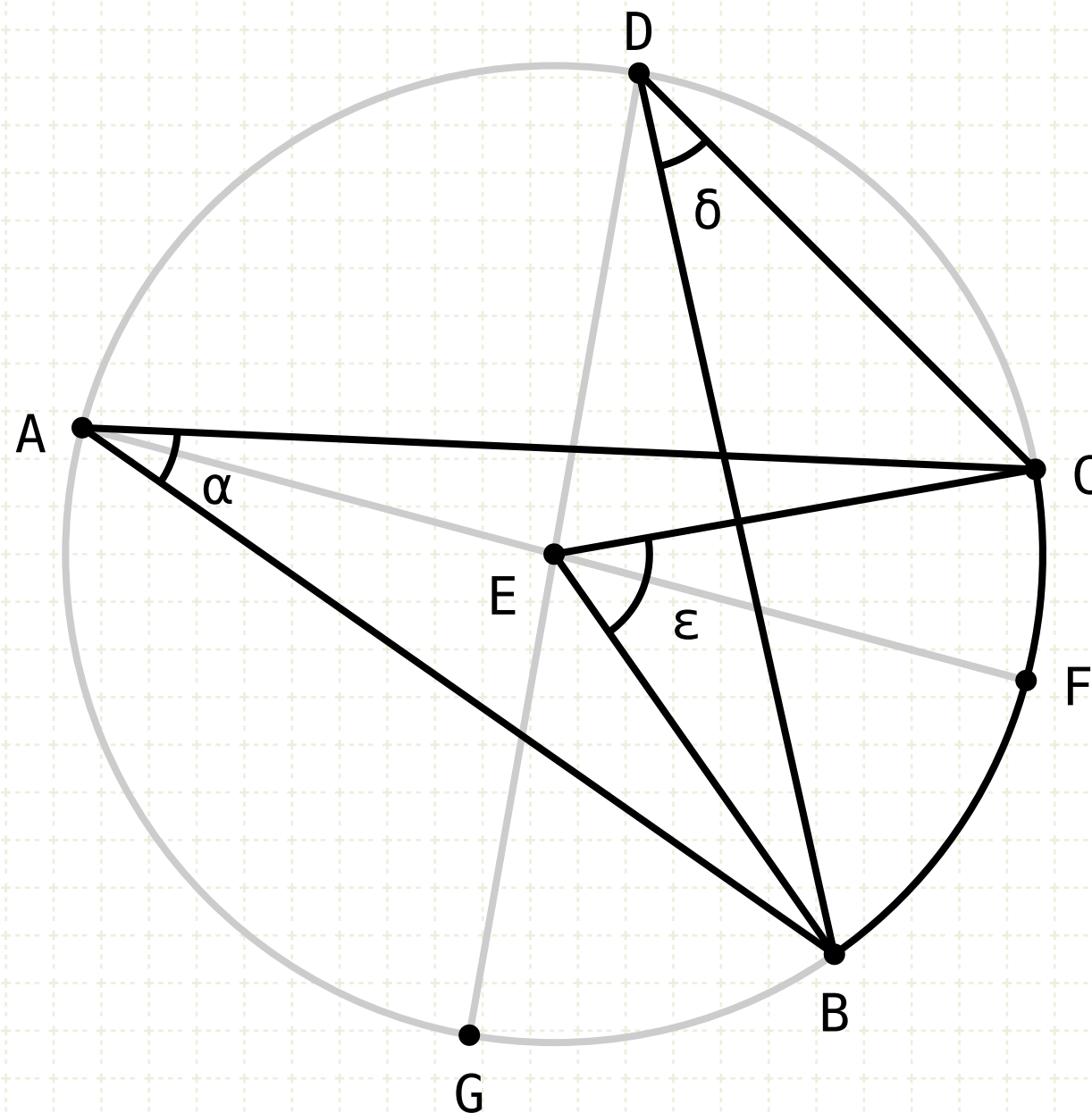
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Thus, angle BEC (ε) is twice BDC (δ)

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