

# Euclid's Elements

## Book VII

### Definitions:

- 1 A unit is that by virtue of which each of the things that exist is called one
- 2 A number is a multitude composed of units. (not one)
- 3 A number is part of a number, the less of the greater, when it measures the greater
- 11 A prime number is that which is measured by a unit alone.
- 12 Numbers prime to one another are those which are measured by a unit alone as a common measure
- 15 A number is said to multiply a number when that which is multiplied is added to itself as many times as there are units in the other, and thus some number is produced.
- 20 Numbers are proportional when the first is the same multiple, or the same part, or the same parts, of the second that the third is of the fourth.

*As long as algebra and geometry have been separated, their progress have been slow and their uses limited; but when these two sciences have been united, they have lent each mutual forces, and have marched together towards perfection.*

**Joseph-Louis Lagrange**  
**(1736 to 1813)**



# Table of Contents, Chapter 7

1	Determine if two numbers are relatively prime	10	If $B = (p/q) \cdot A$ and $D = (p/q) \cdot C$ , and If $B = (r/s) \cdot D$ , then $A = (r/s) \cdot C$	21	If A,B are relatively prime, then A,B are the smallest whole numbers that can be used to describe the ratio A:B
2	Find the greatest common divisor for two numbers	11	If $A:B = C:D$ , then $(A-C):(B-D) = A:B$	22	If A,B are the smallest whole numbers that can be used to describe the ratio A:B, then A,B are relatively prime
3	Find the largest common divisor for three numbers	12	If $A:B = C:D$ , then $(A+C):(B+C) = A:B$	23	If A,B are relatively prime and if $A = n \cdot C$ , then B,C are relatively prime
4	Given two natural numbers, A and B, either B is part of A, or there exists a natural number (a part) that can measure both A and B	13	If $A:B = C:D$ , then $A:C = B:D$	24	If A,C are relatively prime and B,C are relatively prime then the $A \times B$ is relatively prime to C
5	If $B = (1/q) \cdot A$ and $D = (1/q) \cdot C$ , then $(B+D) = (1/q) \cdot (A+C)$	14	If $A:B = D:E$ and $B:C = E:F$ , then $A:C = D:F$	25	If A,B are relatively prime then $A^2, B$ are relatively prime
6	If $B = (p/q) \cdot A$ and $D = (p/q) \cdot C$ , then $(B+D) = (p/q) \cdot (A+C)$	15	If $B = i \cdot 1$ and $E = i \cdot D$ , and if $D = j \cdot 1$ then $E = j \cdot B$	26	If A is relatively prime to C and D, and if B is also relatively prime to C and D, then $A \times B$ is relatively prime to $C \times D$
7	If $B = A/q$ and $D = C/q$ , $B > D$ , then $(B-D) = (A-C)/q$	16	$A \times B = B \times A$	27	If A,B are relatively prime, then $A^2, B^2$ are relatively prime, and $A^3, B^3$ are relatively prime, and so on
8	If $B = (p/q) \cdot A$ and $D = (p/q) \cdot C$ , $B > D$ , then $(B-D) = (p/q) \cdot (A-C)$	17	If $D = A \times B$ and $E = A \times C$ then $D:E = B:C$		
9	If $B = (1/q) \cdot A$ and $D = (1/q) \cdot C$ , and If $B = (r/s) \cdot D$ , then $A = (r/s) \cdot C$	18	If $D = B \times A$ and $E = C \times A$ then $D:E = B:C$		
		19	If $A:B = C:D$ then $A \times D = B \times C$ If $A \times D = B \times C$ then $A:B = C:D$		
		20	Given the ratio A:B and C,D are the smallest numbers such that $A:B = C:D$ then $A = n \cdot C$ and $B = n \cdot D$		



# Table of Contents, Chapter 7

- 28 If A,B are relatively prime, then A,(A+B) are relatively prime
- 29 If A is prime, and  $B \neq n \cdot A$ , then A,B are relatively prime
- 30 If  $C = A \times B$  and  $C = i \cdot D$  where D is prime, then either  $A = j \cdot D$  or  $B = j \cdot D$
- 31 If  $A = B \times C$ , then  $A = j \cdot D$  where D is prime
- 32 If A is a number then it is either prime, or  $A = j \cdot D$  where D is prime
- 33 Find the smallest numbers X,Y,Z where the ratio X:Y:Z is equal to the given ratio A:B:C
- 34 Find the lowest common denominator of 2 numbers
- 35 If E is the lowest common denominator of A,B, and if  $C = n \cdot A = m \cdot B$ , then  $C = i \cdot E$
- 36 Find the least common multiple of 3 numbers

- 37 If  $A = p \cdot B$ , then  $A = q \cdot C$  where  $C = p \cdot 1$
- 38 If  $A = (1/c) \cdot B$  and  $C = c \cdot 1$  then  $A = n \cdot C$
- 39 Find the smallest number that has the fractions  $1/a, 1/b, 1/c$



# Proposition 37 of Book VII

If a number be measured by any number, the number which is measured will have a part called by the same name as the measuring number



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$$A = c \cdot B$$

## In other words

If A is measured by B, then ...





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$$A = c \cdot B$$
$$A = b \cdot C$$
$$C = (1/b)A$$

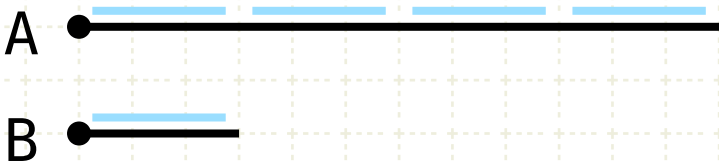
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## Proof



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If a number be measured by any number, the number which is measured will have a part called by the same name as the measuring number

A 

B 

D 

$$A = c \cdot B$$

$$D = 1$$

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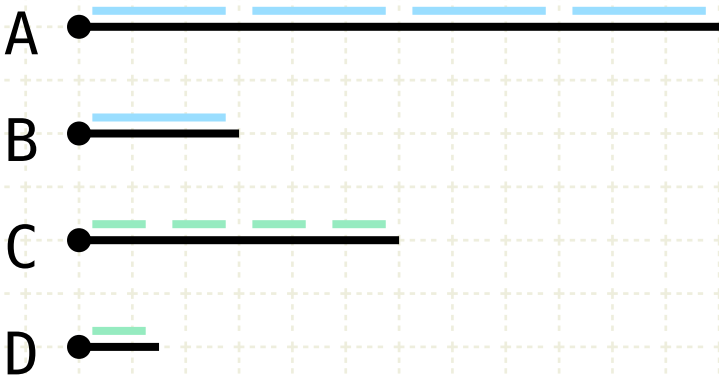
Let the unit measure be D





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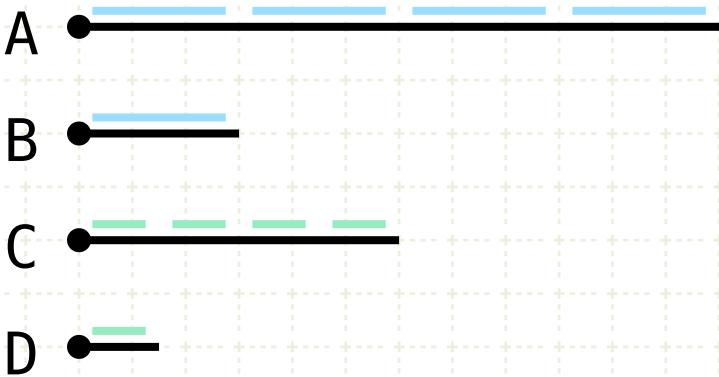
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Let the unit measure be D  
Let C have as many units that is equal to the number of times that B measures A



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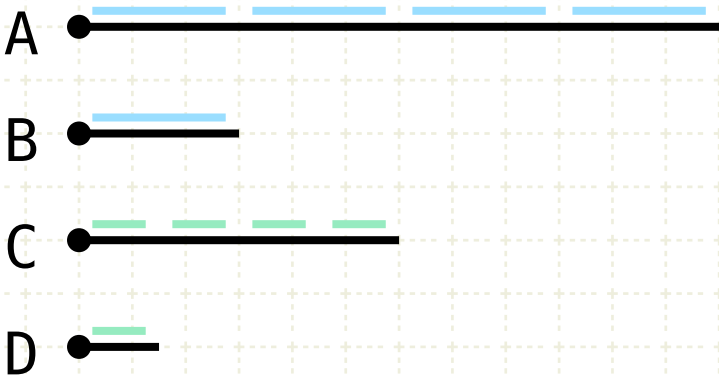
## Proof

Let the unit measure be D  
Let C have as many units that is equal to the number of times that B measures A  
B measures A according to the units in C, and the unit D also measures the number C according to the units in it...



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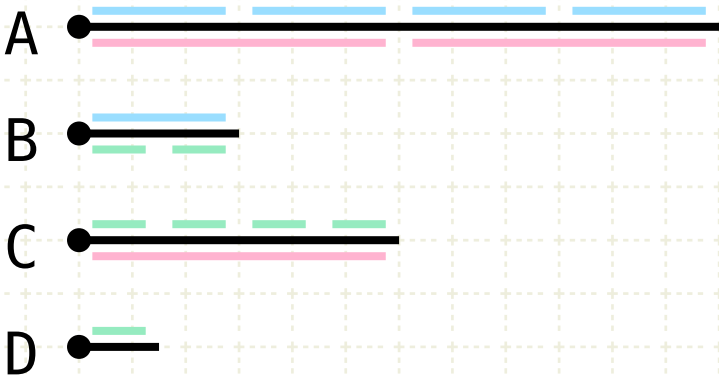
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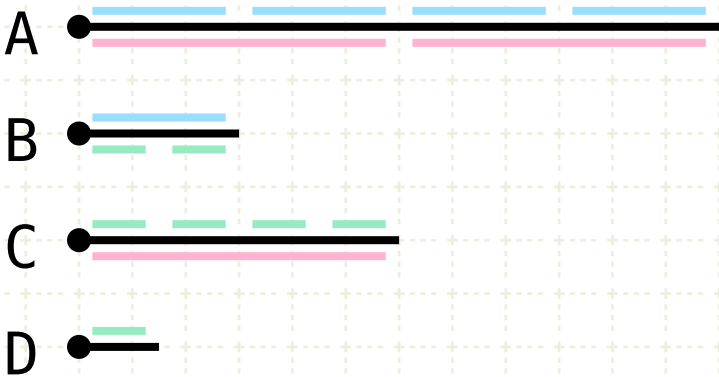
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Consequently, D measures the number B the same number of times as C measures A (VII-15)



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If A is measured by B, then ...

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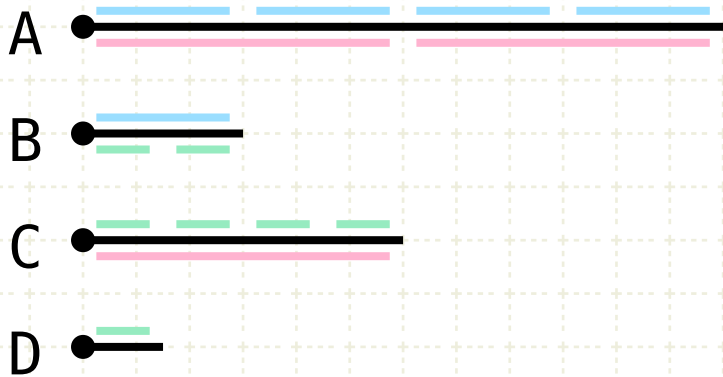
So whatever part D is of the number B, so is the part C of A





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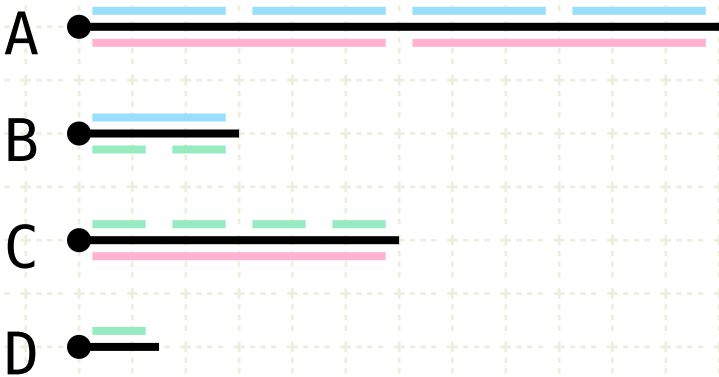
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But the unit D is a part of the number B called by the same name as it



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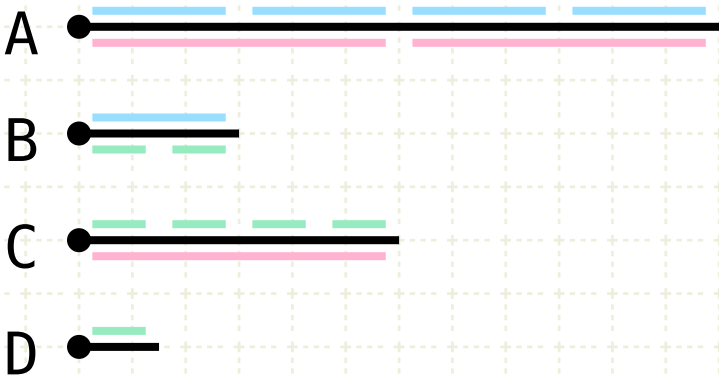
But the unit D is a part of the number B called by the same name as it

Therefore A has a part C, which is called by the same number as B



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