

Euclid's Elements

Book VII

Definitions:

- 1 A unit is that by virtue of which each of the things that exist is called one
- 2 A number is a multitude composed of units. (not one)
- 3 A number is part of a number, the less of the greater, when it measures the greater
- 11 A prime number is that which is measured by a unit alone.
- 12 Numbers prime to one another are those which are measured by a unit alone as a common measure
- 15 A number is said to multiply a number when that which is multiplied is added to itself as many times as there are units in the other, and thus some number is produced.
- 20 Numbers are proportional when the first is the same multiple, or the same part, or the same parts, of the second that the third is of the fourth.

As long as algebra and geometry have been separated, their progress have been slow and their uses limited; but when these two sciences have been united, they have lent each mutual forces, and have marched together towards perfection.

Joseph-Louis Lagrange
(1736 to 1813)



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1	Determine if two numbers are relatively prime	10	If $B = (p/q) \cdot A$ and $D = (p/q) \cdot C$, and If $B = (r/s) \cdot D$, then $A = (r/s) \cdot C$	21	If A,B are relatively prime, then A,B are the smallest whole numbers that can be used to describe the ratio A:B
2	Find the greatest common divisor for two numbers	11	If $A:B = C:D$, then $(A-C):(B-D) = A:B$	22	If A,B are the smallest whole numbers that can be used to describe the ratio A:B, then A,B are relatively prime
3	Find the largest common divisor for three numbers	12	If $A:B = C:D$, then $(A+C):(B+C) = A:B$	23	If A,B are relatively prime and if $A = n \cdot C$, then B,C are relatively prime
4	Given two natural numbers, A and B, either B is part of A, or there exists a natural number (a part) that can measure both A and B	13	If $A:B = C:D$, then $A:C = B:D$	24	If A,C are relatively prime and B,C are relatively prime then the $A \times B$ is relatively prime to C
5	If $B = (1/q) \cdot A$ and $D = (1/q) \cdot C$, then $(B+D) = (1/q) \cdot (A+C)$	14	If $A:B = D:E$ and $B:C = E:F$, then $A:C = D:F$	25	If A,B are relatively prime then A^2, B are relatively prime
6	If $B = (p/q) \cdot A$ and $D = (p/q) \cdot C$, then $(B+D) = (p/q) \cdot (A+C)$	15	If $B = i \cdot 1$ and $E = i \cdot D$, and if $D = j \cdot 1$ then $E = j \cdot B$	26	If A is relatively prime to C and D, and if B is also relatively prime to C and D, then $A \times B$ is relatively prime to $C \times D$
7	If $B = A/q$ and $D = C/q$, $B > D$, then $(B-D) = (A-C)/q$	16	$A \times B = B \times A$	27	If A,B are relatively prime, then A^2, B^2 are relatively prime, and A^3, B^3 are relatively prime, and so on
8	If $B = (p/q) \cdot A$ and $D = (p/q) \cdot C$, $B > D$, then $(B-D) = (p/q) \cdot (A-C)$	17	If $D = A \times B$ and $E = A \times C$ then $D:E = B:C$		
9	If $B = (1/q) \cdot A$ and $D = (1/q) \cdot C$, and If $B = (r/s) \cdot D$, then $A = (r/s) \cdot C$	18	If $D = B \times A$ and $E = C \times A$ then $D:E = B:C$		
		19	If $A:B = C:D$ then $A \times D = B \times C$ If $A \times D = B \times C$ then $A:B = C:D$		
		20	Given the ratio A:B and C,D are the smallest numbers such that $A:B = C:D$ then $A = n \cdot C$ and $B = n \cdot D$		



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| 29 | If A is prime, and $B \neq n \cdot A$, then A,B are relatively prime | 38 | If $A = (1/c) \cdot B$ and $C = c \cdot 1$ then $A = n \cdot C$ |
| 30 | If $C = A \times B$ and $C = i \cdot D$ where D is prime, then either $A = j \cdot D$ or $B = j \cdot D$ | 39 | Find the smallest number that has the fractions $1/a, 1/b, 1/c$ |
| 31 | If $A = B \times C$, then $A = j \cdot D$ where D is prime | | |
| 32 | If A is a number then it is either prime, or $A = j \cdot D$ where D is prime | | |
| 33 | Find the smallest numbers X,Y,Z where the ratio X:Y:Z is equal to the given ratio A:B:C | | |
| 34 | Find the lowest common denominator of 2 numbers | | |
| 35 | If E is the lowest common denominator of A,B, and if $C = n \cdot A = m \cdot B$, then $C = i \cdot E$ | | |
| 36 | Find the least common multiple of 3 numbers | | |



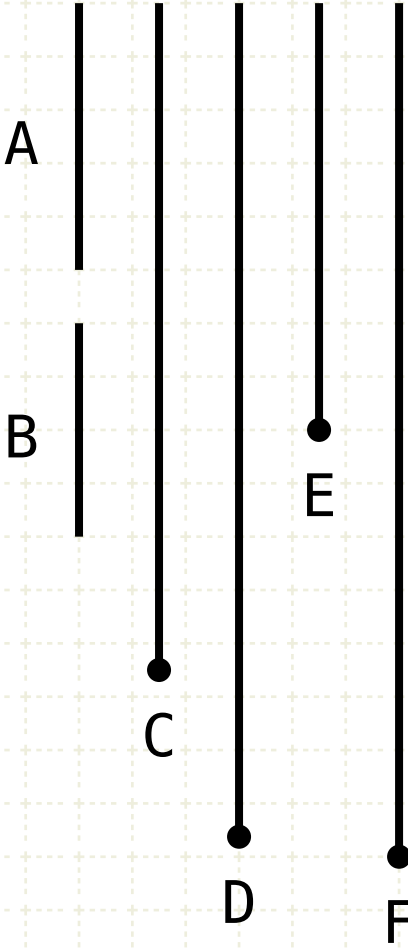
Proposition 27 of Book VII

If two numbers be prime to one another, and each by multiplying itself make a certain number, the products will be prime to one another; and, if the original numbers by multiplying the products make certain numbers, the latter will also be prime to one another (and this is always the case with the extremes).



Proposition 27 of Book VII

If two numbers be prime to one another, and each by multiplying itself make a certain number, the products will be prime to one another; and, if the original numbers by multiplying the products make certain numbers, the latter will also be prime to one another (and this is always the case with the extremes).



$$\begin{aligned} \gcd(A, B) &= 1 \\ C &= A \times A = A^2 \\ D &= A \times C = A^3 \\ E &= B \times B = B^2 \\ F &= B \times E = B^3 \end{aligned}$$

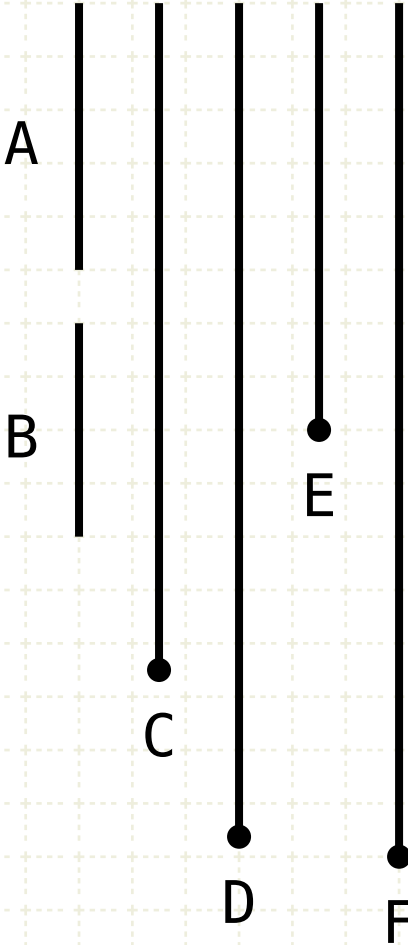
In other words

Let A and B be relatively prime



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If two numbers be prime to one another, and each by multiplying itself make a certain number, the products will be prime to one another; and, if the original numbers by multiplying the products make certain numbers, the latter will also be prime to one another (and this is always the case with the extremes).



$$\gcd(A, B) = 1$$

$$C = A \times A = A^2$$

$$D = A \times C = A^3$$

$$E = B \times B = B^2$$

$$F = B \times E = B^3$$

$$\gcd(C, E) = 1, \quad \gcd(A^2, B^2) = 1$$

$$\gcd(D, F) = 1, \quad \gcd(A^3, B^3) = 1$$

In other words

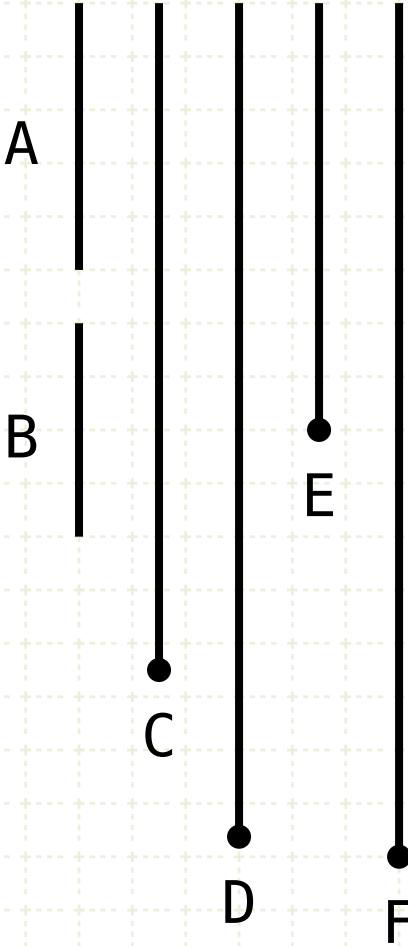
Let A and B be relatively prime

Then A^2 and B^2 are relatively prime, and A^3 , B^3 are also relatively prime



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If two numbers be prime to one another, and each by multiplying itself make a certain number, the products will be prime to one another; and, if the original numbers by multiplying the products make certain numbers, the latter will also be prime to one another (and this is always the case with the extremes).



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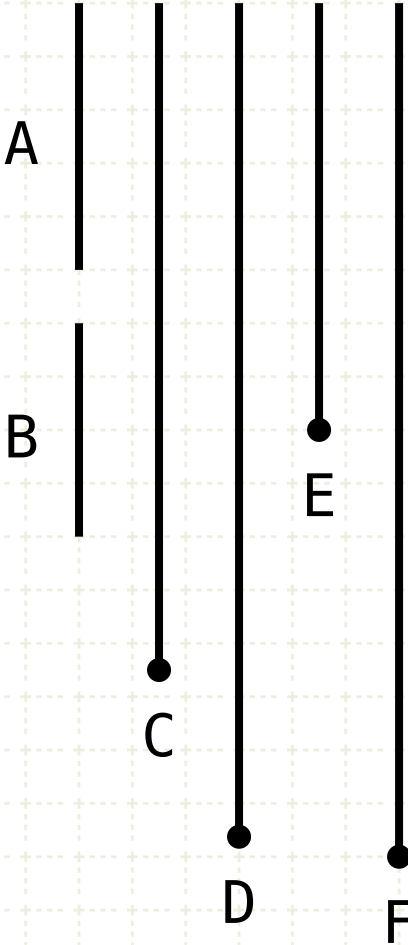
Let A and B be relatively prime
Then A^2 and B^2 are relatively prime, and A^3 , B^3 are also relatively prime

Proof



Proposition 27 of Book VII

If two numbers be prime to one another, and each by multiplying itself make a certain number, the products will be prime to one another; and, if the original numbers by multiplying the products make certain numbers, the latter will also be prime to one another (and this is always the case with the extremes).



$$\begin{aligned} \gcd(A, B) &= 1 \\ C &= A \times A = A^2 \\ D &= A \times C = A^3 \\ E &= B \times B = B^2 \\ F &= B \times E = B^3 \end{aligned}$$

$$\begin{aligned} \gcd(A^2, B) &= 1 \\ \gcd(C, B) &= 1 \end{aligned}$$

In other words

Let A and B be relatively prime
Then A^2 and B^2 are relatively prime, and A^3 , B^3 are also relatively prime

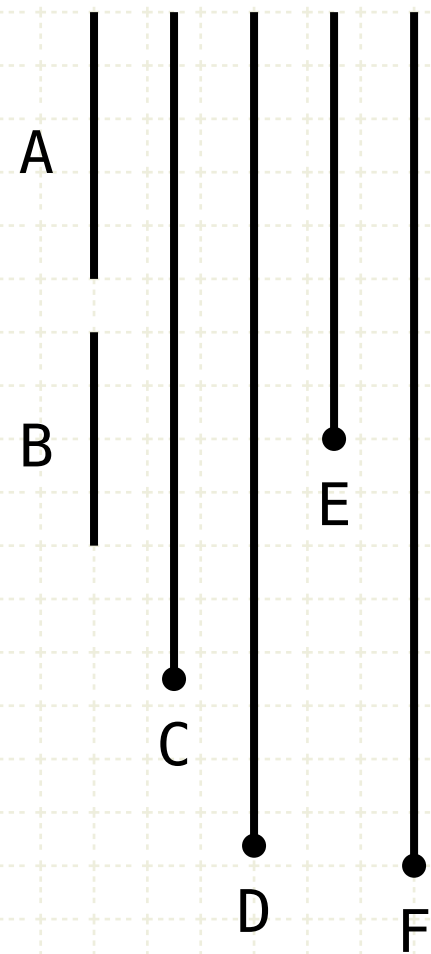
Proof

Since A and B are prime to one another, then A^2 , and B are prime to one another (VII-25), thus C and B are prime to one another



Proposition 27 of Book VII

If two numbers be prime to one another, and each by multiplying itself make a certain number, the products will be prime to one another; and, if the original numbers by multiplying the products make certain numbers, the latter will also be prime to one another (and this is always the case with the extremes).



$$\gcd(A, B) = 1$$

$$C = A \times A = A^2$$

$$D = A \times C = A^3$$

$$E = B \times B = B^2$$

$$F = B \times E = B^3$$

$$\gcd(A^2, B) = 1$$

$$\gcd(C, B) = 1$$

$$\gcd(C, B^2) = 1$$

$$\gcd(C, E) = 1 \rightarrow \gcd(A^2, B^2) = 1$$

In other words

Let A and B be relatively prime

Then A^2 and B^2 are relatively prime, and A^3 , B^3 are also relatively prime

Proof

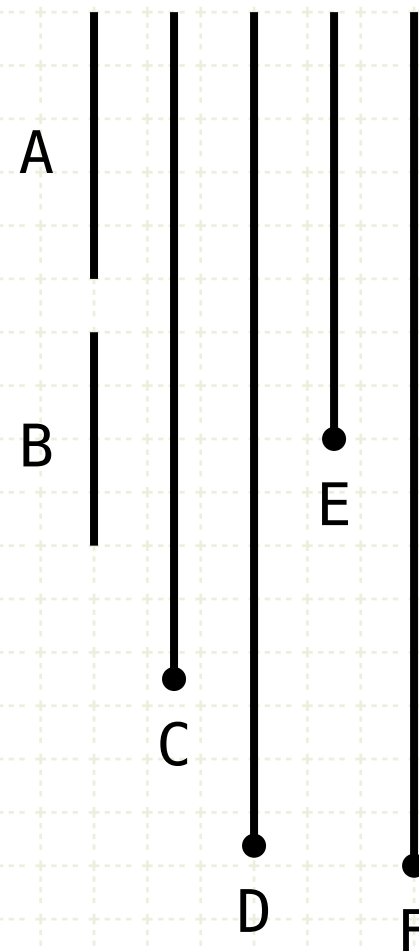
Since A and B are prime to one another, then A^2 , and B are prime to one another (VII-25), thus C and B are prime to one another

Since C,B are prime to one another, and E is B times B, C and E are prime to one another (VII-25)



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If two numbers be prime to one another, and each by multiplying itself make a certain number, the products will be prime to one another; and, if the original numbers by multiplying the products make certain numbers, the latter will also be prime to one another (and this is always the case with the extremes).



$$\gcd(A, B) = 1$$

$$C = A \times A = A^2$$

$$D = A \times C = A^3$$

$$E = B \times B = B^2$$

$$F = B \times E = B^3$$

$$\gcd(A^2, B) = 1$$

$$\gcd(C, B) = 1$$

$$\gcd(C, B^2) = 1$$

$$\gcd(C, E) = 1 \rightarrow \gcd(A^2, B^2) = 1$$

$$\gcd(A, B^2) = 1$$

$$\gcd(A, E) = 1$$

In other words

Let A and B be relatively prime

Then A^2 and B^2 are relatively prime, and A^3 , B^3 are also relatively prime

Proof

Since A and B are prime to one another, then A^2 , and B are prime to one another (VII-25), thus C and B are prime to one another

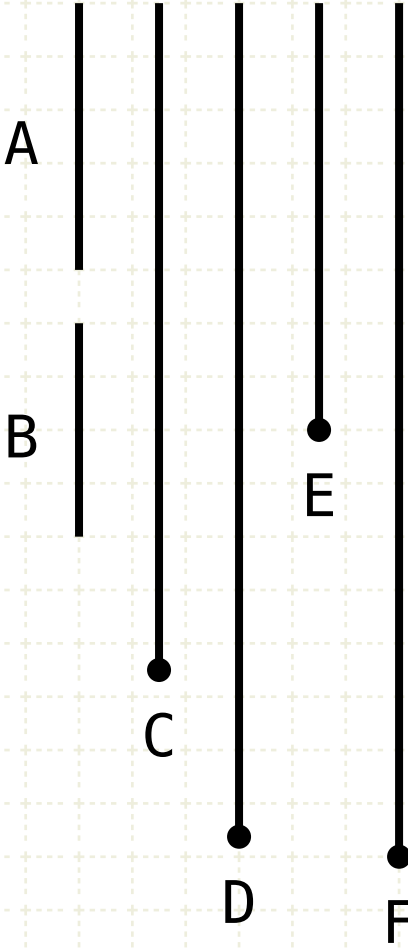
Since C,B are prime to one another, and E is B times B, C and E are prime to one another (VII-25)

Since A,B are prime to one another, and E is B times B, A and E are prime to one another (VII-25)



Proposition 27 of Book VII

If two numbers be prime to one another, and each by multiplying itself make a certain number, the products will be prime to one another; and, if the original numbers by multiplying the products make certain numbers, the latter will also be prime to one another (and this is always the case with the extremes).



$$\gcd(A, B) = 1$$

$$C = A \times A = A^2$$

$$D = A \times C = A^3$$

$$E = B \times B = B^2$$

$$F = B \times E = B^3$$

$$\gcd(A^2, B) = 1$$

$$\gcd(C, B) = 1$$

$$\gcd(C, B^2) = 1$$

$$\gcd(C, E) = 1 \rightarrow \gcd(A^2, B^2) = 1$$

$$\gcd(A, B^2) = 1$$

$$\gcd(A, E) = 1$$

$$\gcd(A \times C, B \times E) = 1$$

In other words

Let A and B be relatively prime

Then A^2 and B^2 are relatively prime, and A^3 , B^3 are also relatively prime

Proof

Since A and B are prime to one another, then A^2 , and B are prime to one another (VII·25), thus C and B are prime to one another

Since C, B are prime to one another, and E is B times B, C and E are prime to one another (VII·25)

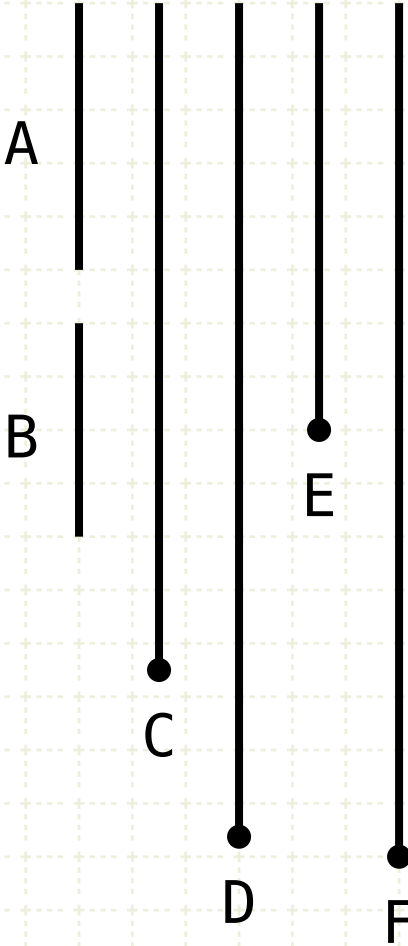
Since A, B are prime to one another, and E is B times B, A and E are prime to one another (VII·25)

Since A and C are prime to the two numbers B and E, therefore the product of A and C will be prime to the product B and E (VII·26)



Proposition 27 of Book VII

If two numbers be prime to one another, and each by multiplying itself make a certain number, the products will be prime to one another; and, if the original numbers by multiplying the products make certain numbers, the latter will also be prime to one another (and this is always the case with the extremes).



$$\gcd(A, B) = 1$$

$$C = A \times A = A^2$$

$$D = A \times C = A^3$$

$$E = B \times B = B^2$$

$$F = B \times E = B^3$$

$$\gcd(A^2, B) = 1$$

$$\gcd(C, B) = 1$$

$$\gcd(C, B^2) = 1$$

$$\gcd(C, E) = 1 \rightarrow \gcd(A^2, B^2) = 1$$

$$\gcd(A, B^2) = 1$$

$$\gcd(A, E) = 1$$

$$\gcd(A \times C, B \times E) = 1$$

$$\gcd(D, F) = 1$$

$$\gcd(A^3, B^3) = 1$$

In other words

Let A and B be relatively prime

Then A^2 and B^2 are relatively prime, and A^3 , B^3 are also relatively prime

Proof

Since A and B are prime to one another, then A^2 , and B are prime to one another (VII·25), thus C and B are prime to one another

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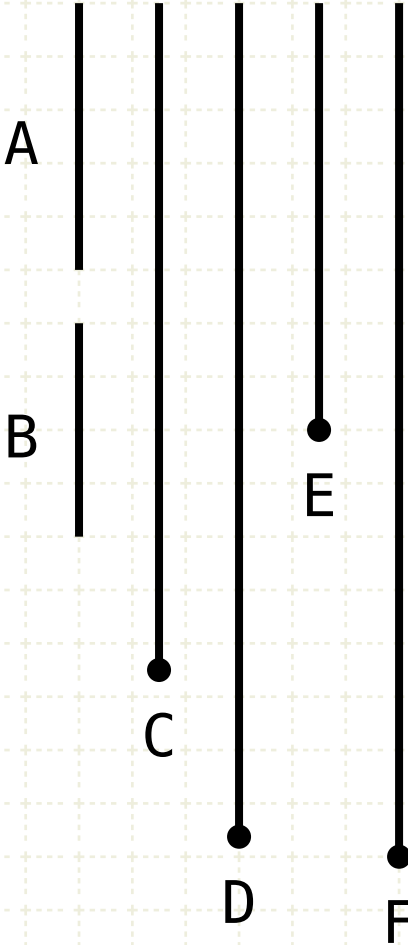
Since A and C are prime to the two numbers B and E, therefore the product of A and C will be prime to the product B and E (VII·26)

Therefore D and F are prime to one another



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$$\begin{aligned} \gcd(A^2, B) &= 1 \\ \gcd(C, B) &= 1 \\ \gcd(C, B^2) &= 1 \\ \gcd(C, E) &= 1 \rightarrow \gcd(A^2, B^2) = 1 \\ \gcd(A, B^2) &= 1 \\ \gcd(A, E) &= 1 \\ \gcd(A \times C, B \times E) &= 1 \\ \gcd(D, F) &= 1 \\ \gcd(A^3, B^3) &= 1 \end{aligned}$$

In other words

Let A and B be relatively prime
Then A^2 and B^2 are relatively prime, and A^3 , B^3 are also relatively prime

Proof

Since A and B are prime to one another, then A^2 , and B are prime to one another (VII·25), thus C and B are prime to one another
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Therefore D and F are prime to one another



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