Euclid's Elements

Book VII

Definitions:

1 A unit is that by virtue of which each of the things that exist is called one

2 A number is a multitude composed of units. (not one)

3 A number is part of a number, the less of the greater, when it measures the greater

11 A prime number is that which is measured by a unit alone.

12 Numbers prime to one another are those which are measured by a unit alone as a common measure

15 A number is said to multiply a number when that which is multiplied is added to itself as many times as there are units in the other, and thus some number is produced.

20 Numbers are proportional when the first is the same multiple, or the same part, or the same parts, of the second that the third is of the fourth.

As long as algebra and geometry have been separated, their progress have been slow and their uses limited; but when these two sciences have been united, they have lent each mutual forces, and have marched together towards perfection.

Joseph-Louis Lagrange (1736 to 1813)

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Proposition 9 of Book VII

If a number be a part of a number, and another be the same part of another, alternately also, whatever part or parts the first of the third, the same part, or the same parts, will the second also be of the fourth

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$$b = (1/q)a$$
$$d = (1/q)c$$
$$\text{if } b = (r/s)\cdot d \rightarrow a = (r/s)\cdot c$$

$$\frac{a}{b} = \frac{c}{d} \rightarrow \frac{a}{c} = \frac{b}{d}$$

In other words

If b is the same fraction of a as d is of c, and b is another fraction of d, then the fraction that b is of d, then a is the same fraction of c

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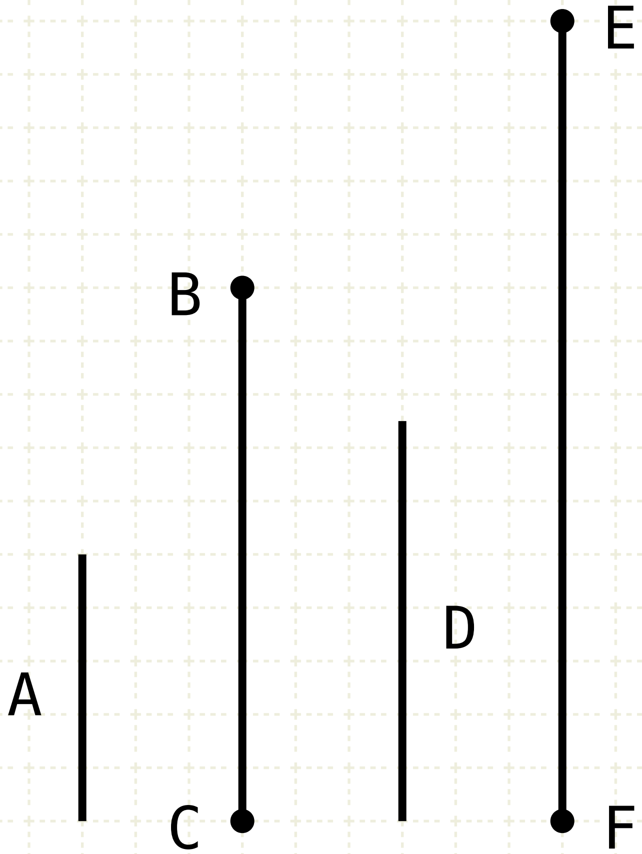

$A = (1/q)BC$

$D = (1/q)EF$

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Proof

Let A be a part of the number BC, and let D be the same part of EF

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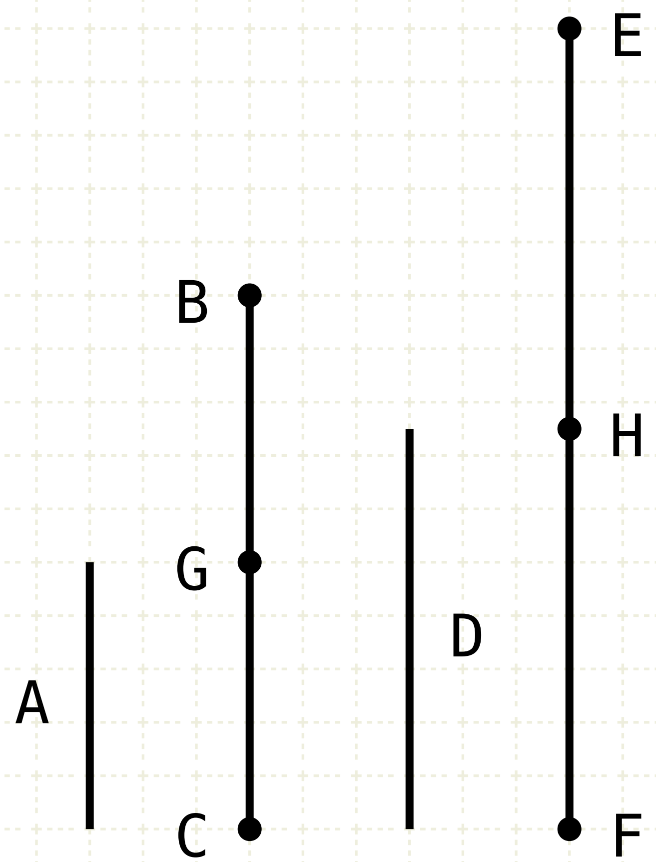

A = (1/q)BC
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BG = GC = A
EH = HF = D

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Let A be a part of the number BC, and let D be the same part of EF

Since A and D are the same part of BC,EF respectively, then there is an equal number of 'A' in BC as there is 'D' in EF

Divide BC into equal sections of 'A', and CF into equal sections of 'D'

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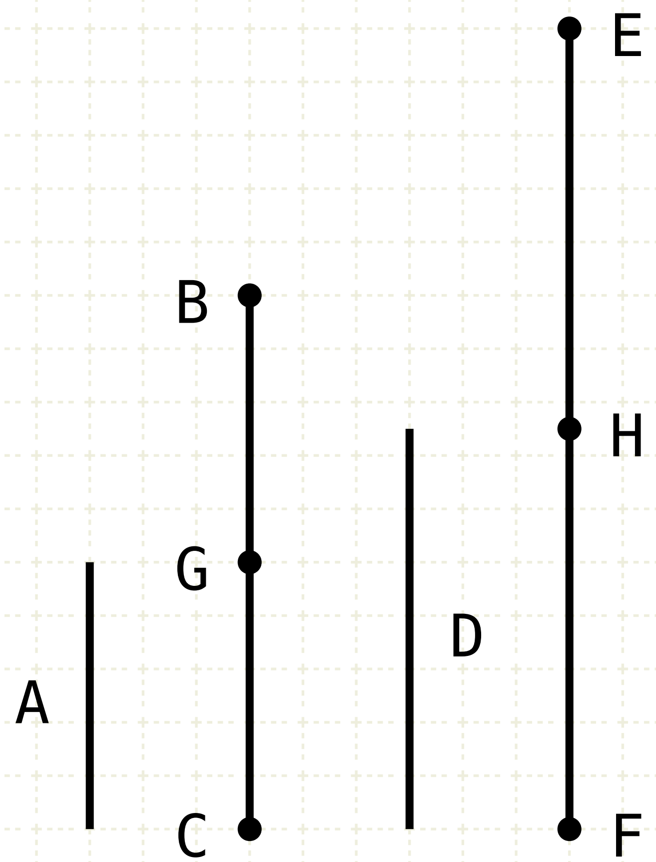

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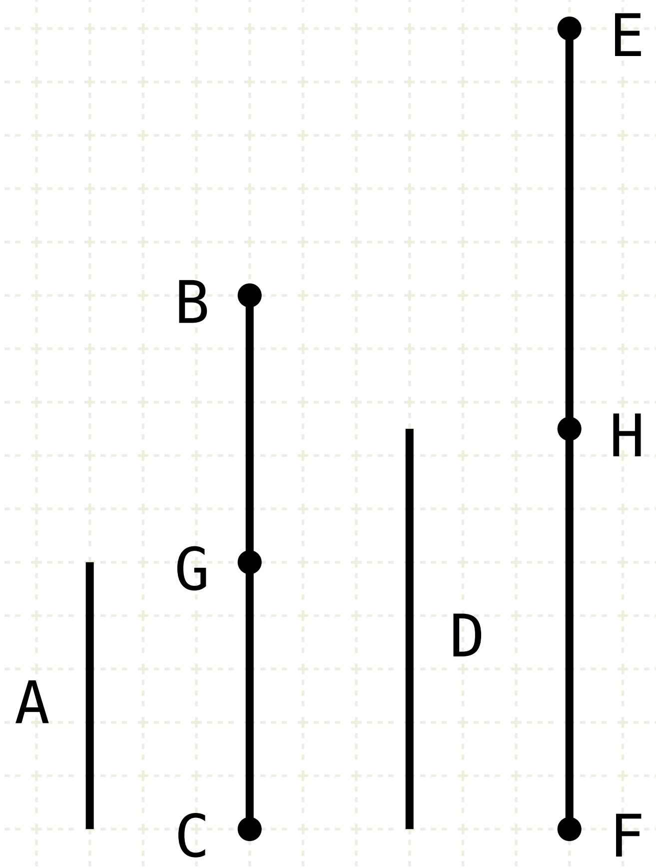

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Thus, since the number of parts (BG,GC) are equal to the number of parts (EH,HF), the sum of BG,GC is the same part or parts of the sum GC,HF as BG is to EH (VI·5, VI·6)

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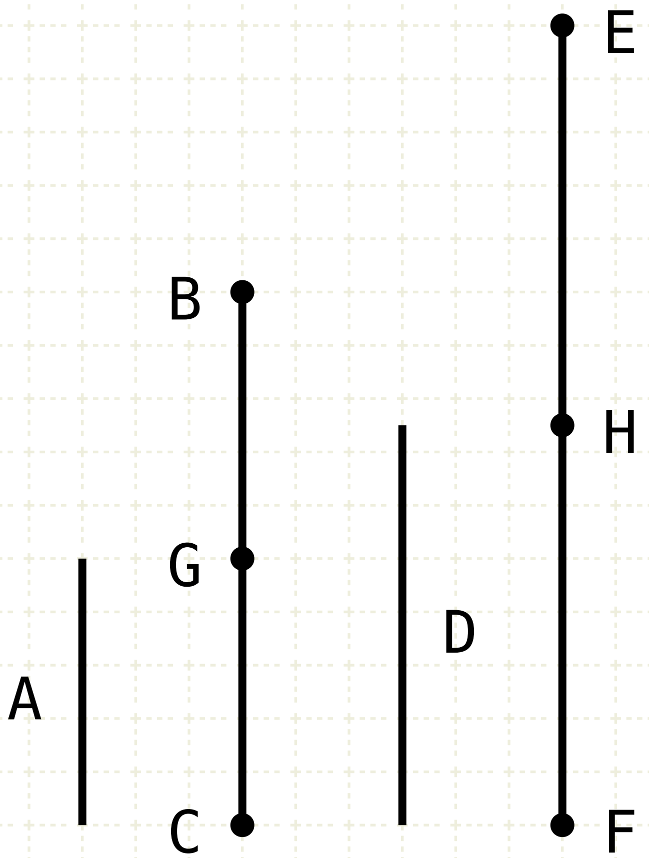

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BG is equal to A and EH is equal to D, therefore the part or parts of BG to EH is the same as A to D

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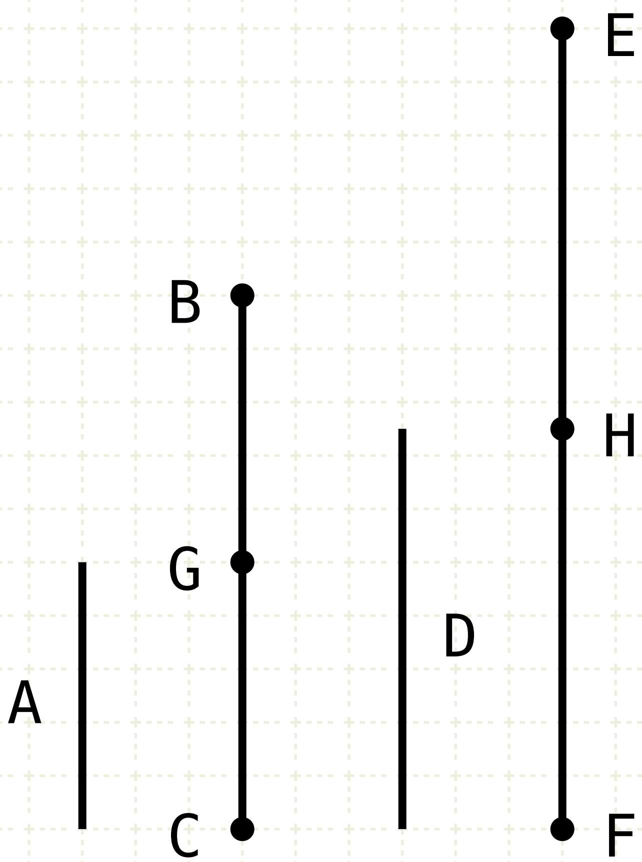

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BG is equal to A and EH is equal to D, therefore the part or parts of BG to EH is the same as A to D

And since BC is the same part or parts of EF as BG is to EH, BC is the part or parts of EF as A is of D

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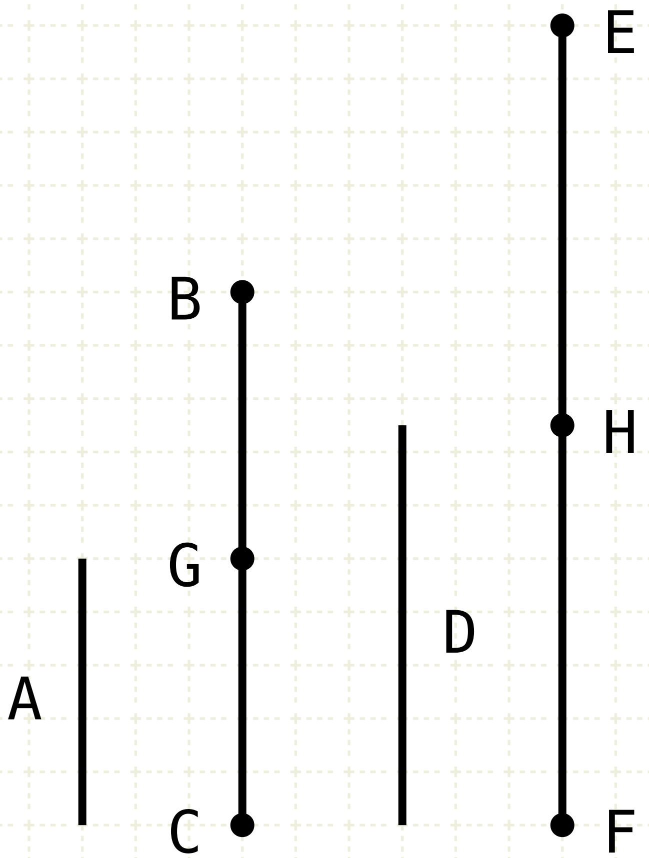

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