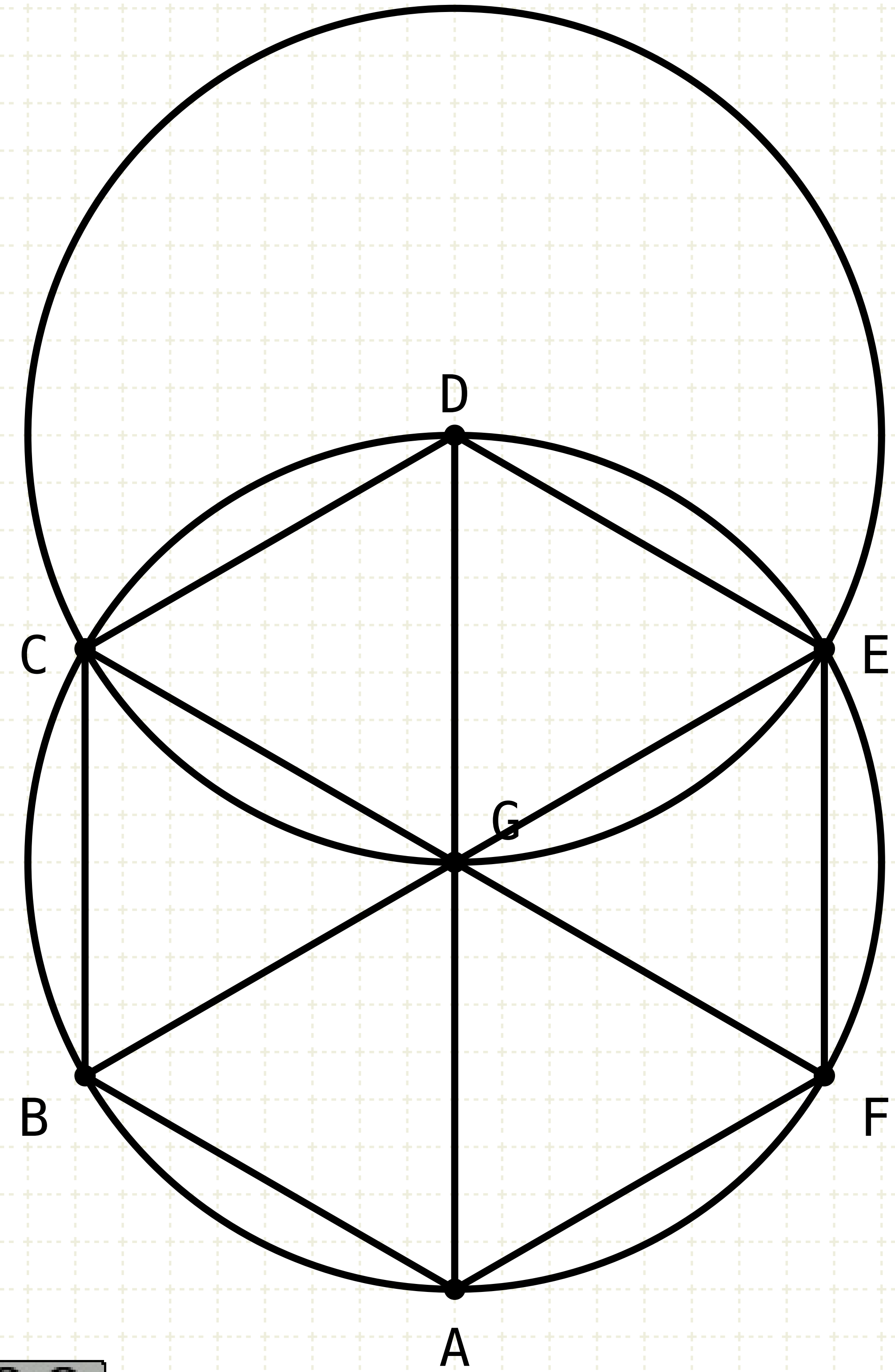
Euclid's Elements

Book IV



Philosophy (nature) is written in that great book which ever is before our eyes -- I mean the universe -- but we cannot understand it if we do not first learn the language and grasp the symbols in which it is written. The book is written in mathematical language, and the symbols are triangles, circles and other geometrical figures, without whose help it is impossible to comprehend a single word of it - without which one wanders in vain through a dark labyrinth.

Galileo Galilei

Proposition 13 of Book IV

In a given pentagon, which is equilateral and equiangular, to inscribe a circle.

Table of Contents, Chapter 4

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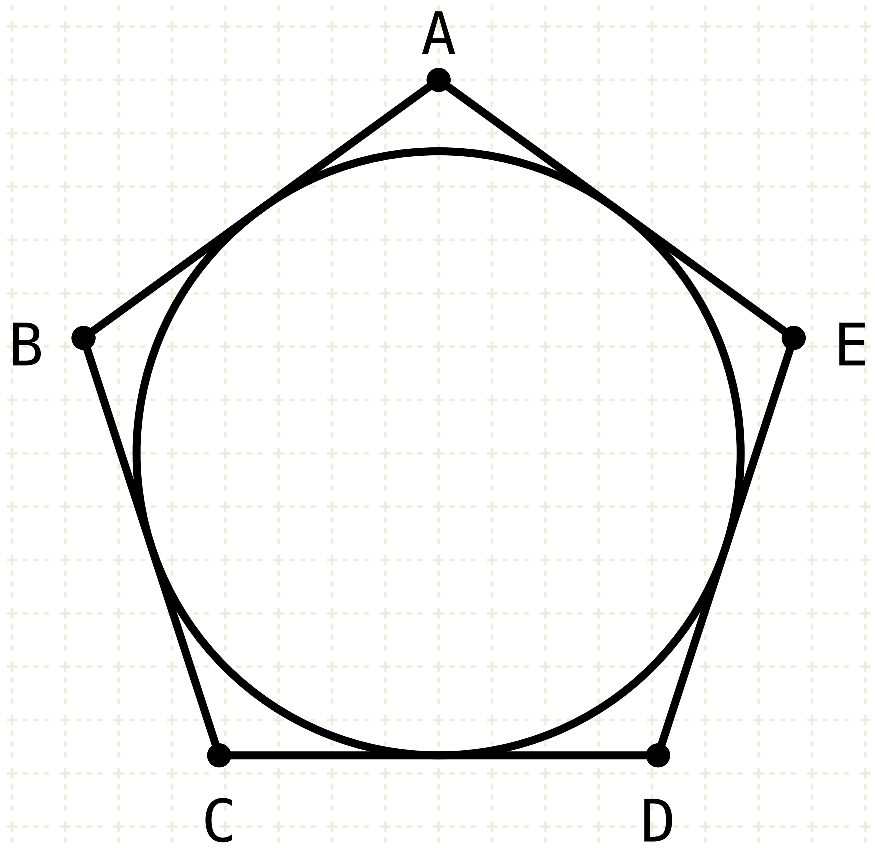

In other words

Given a pentagon draw a circle on the inside, where the sides of the pentagon touch the circle

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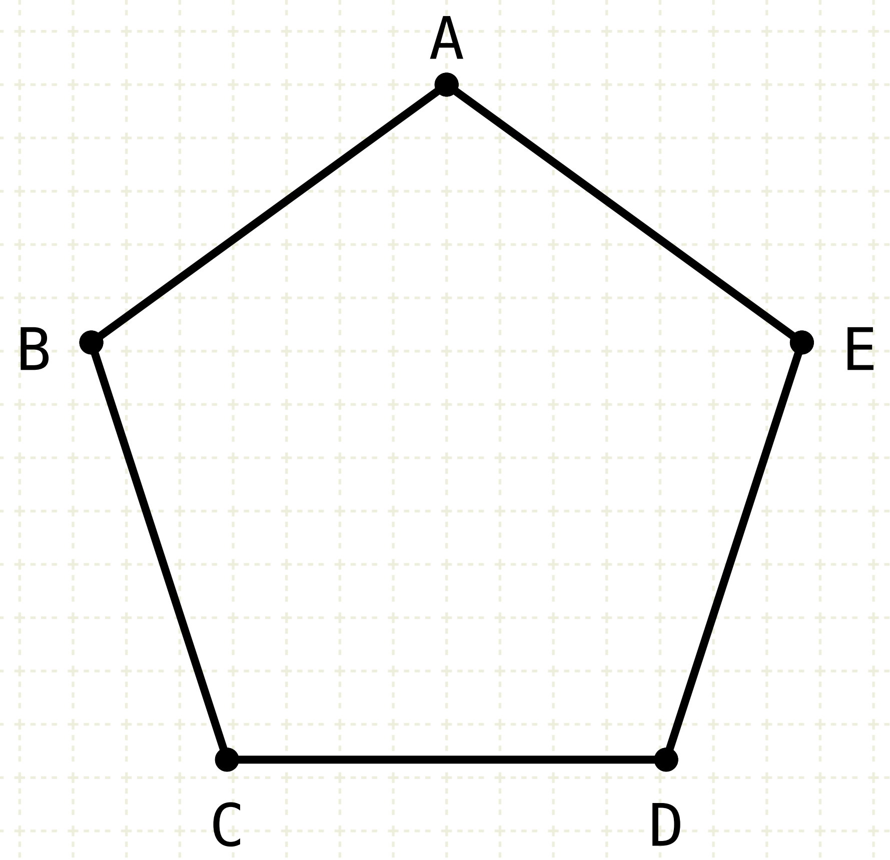
In a given pentagon, which is equilateral and equiangular, to inscribe a circle.



Construction

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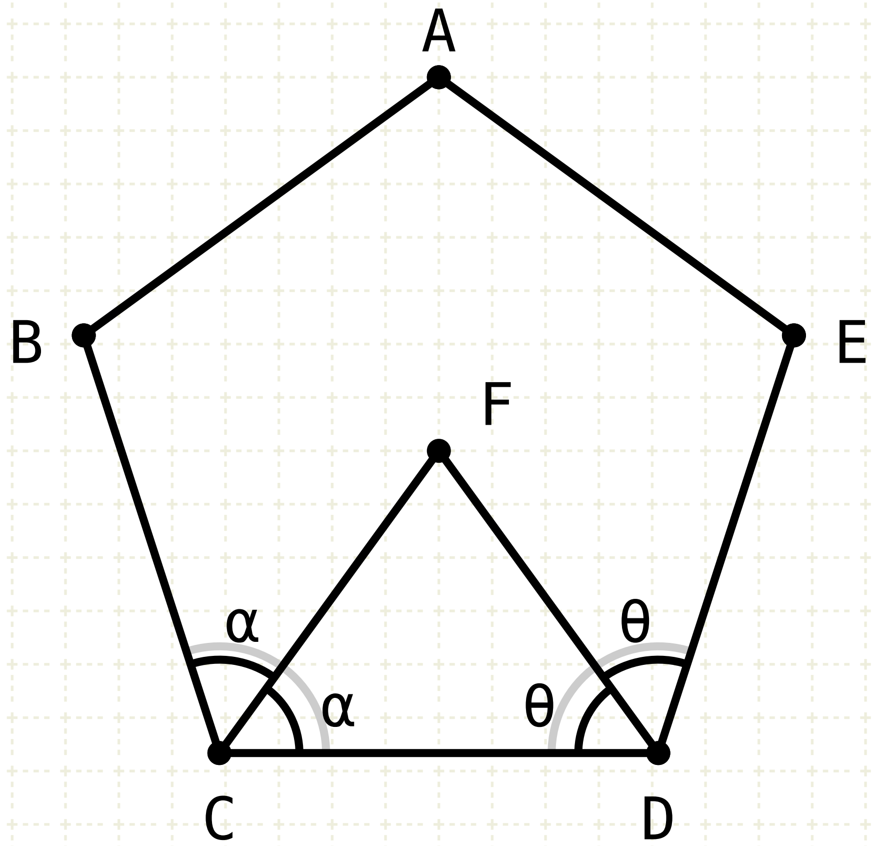

$2\alpha = 2\theta$

$\angle BCF = \angle FCD = \alpha$

$\angle CDF = \angle FDE = \theta$

Construction

Bisect the angles BCD and CDE by the lines CF and DF

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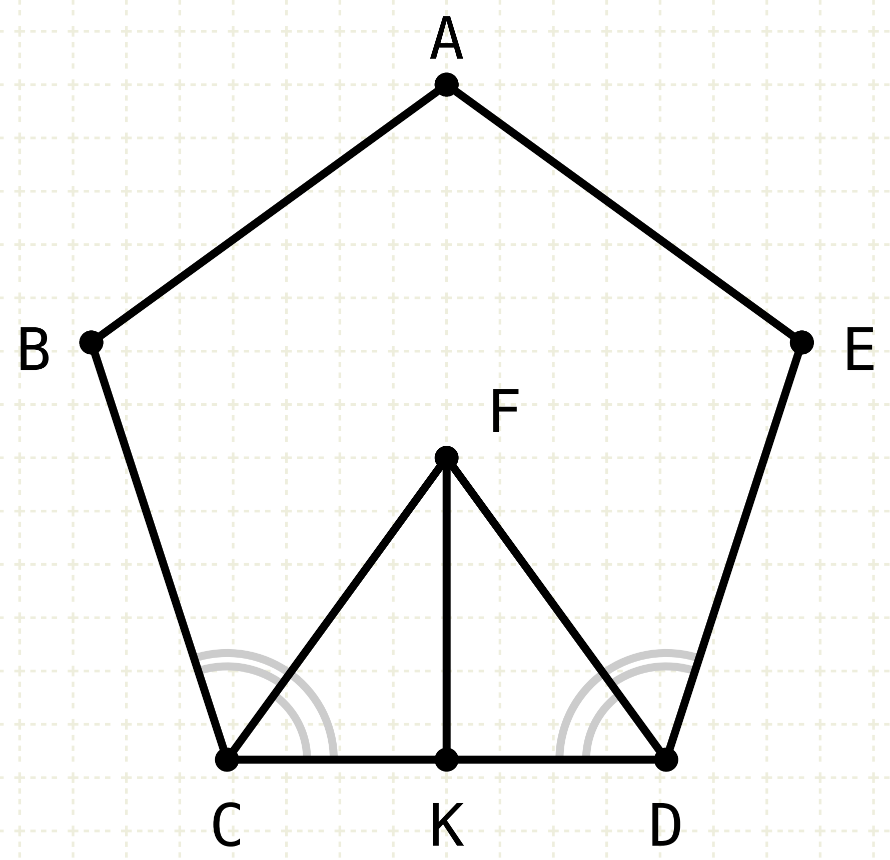

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Draw a perpendicular from point F to side CD

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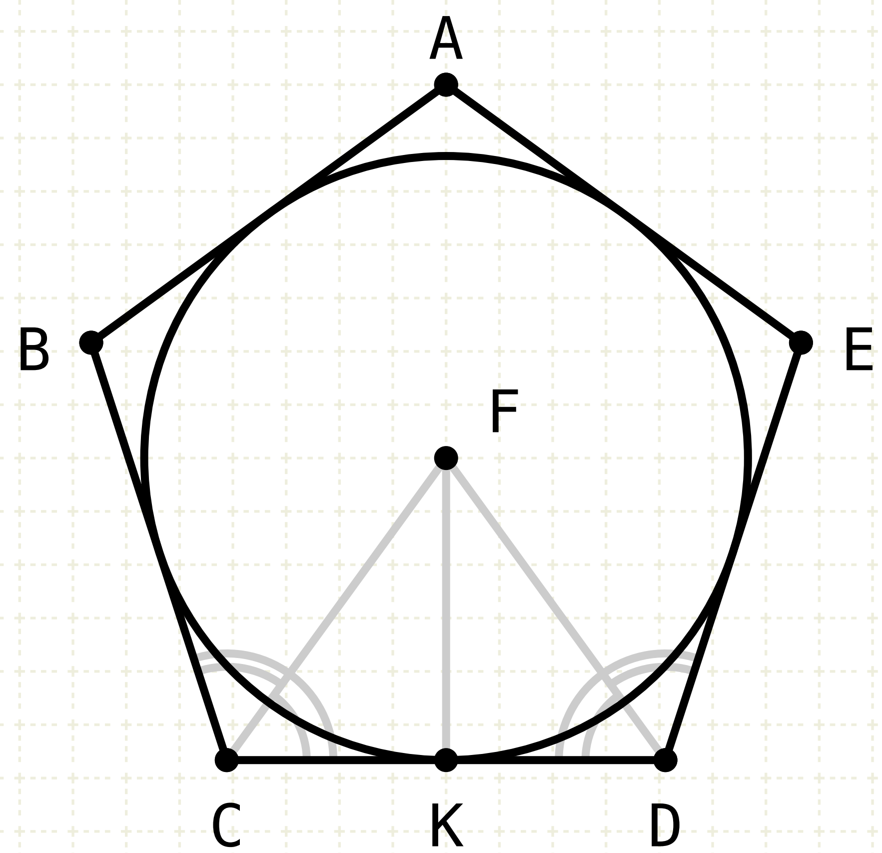

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Construction

Bisect the angles BCD and CDE by the lines CF and DF

Draw a perpendicular from point F to side CD

Draw a circle with F as the centre, and FK as the radius

The circle inscribes the pentagon

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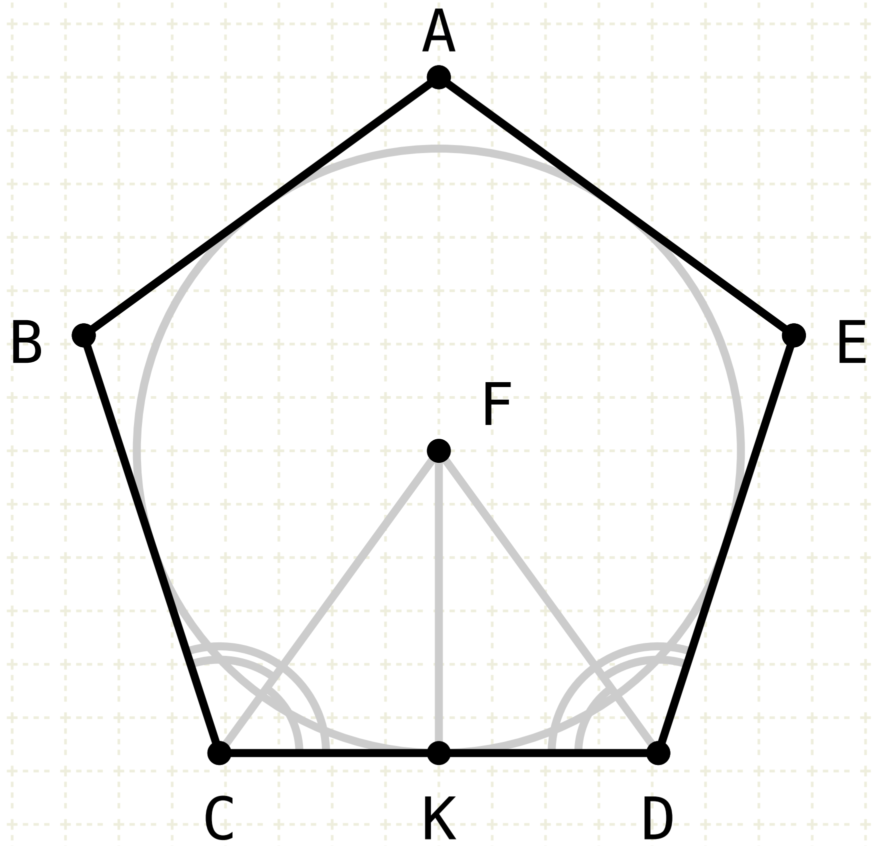
In a given pentagon, which is equilateral and equiangular, to inscribe a circle.



Proof

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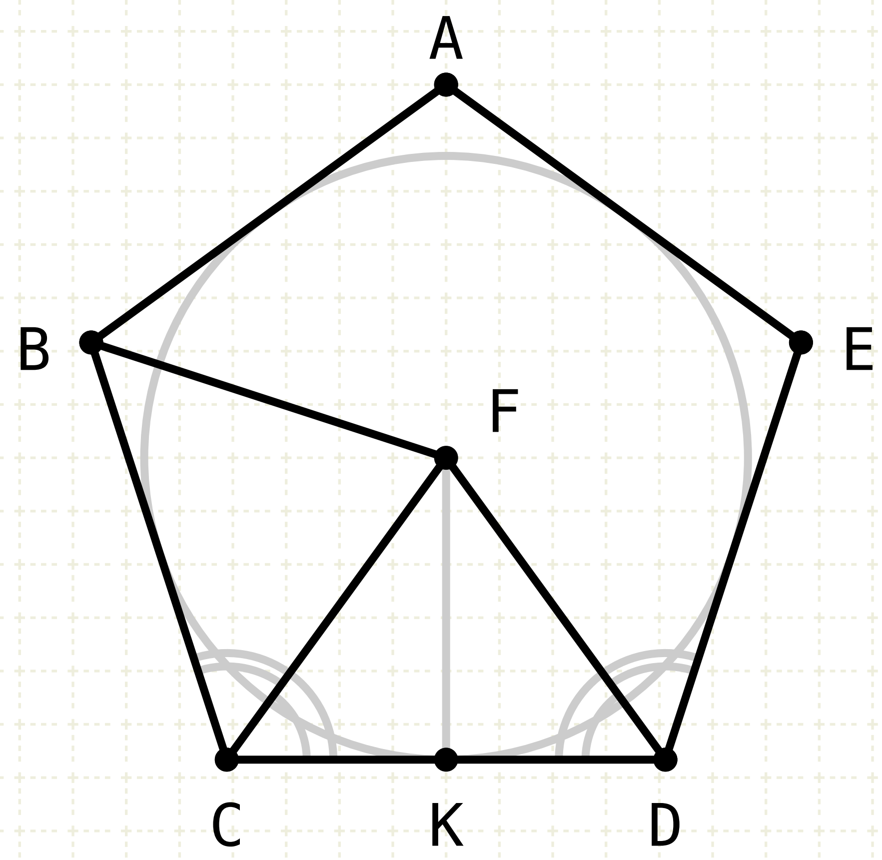

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Draw line BF

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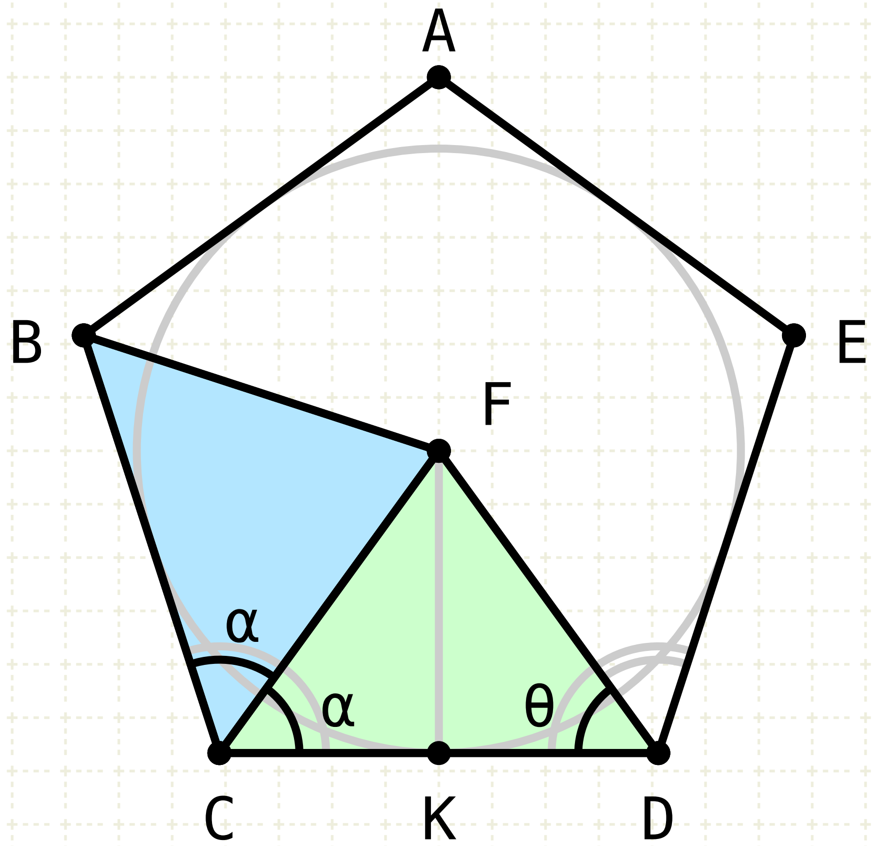

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Proof

Draw line BF

Since BC is equal to CD, FC is common, and the angles BCF and FCD are equal the two triangles BFC and FCD are equivalent (SAS) (I·4)

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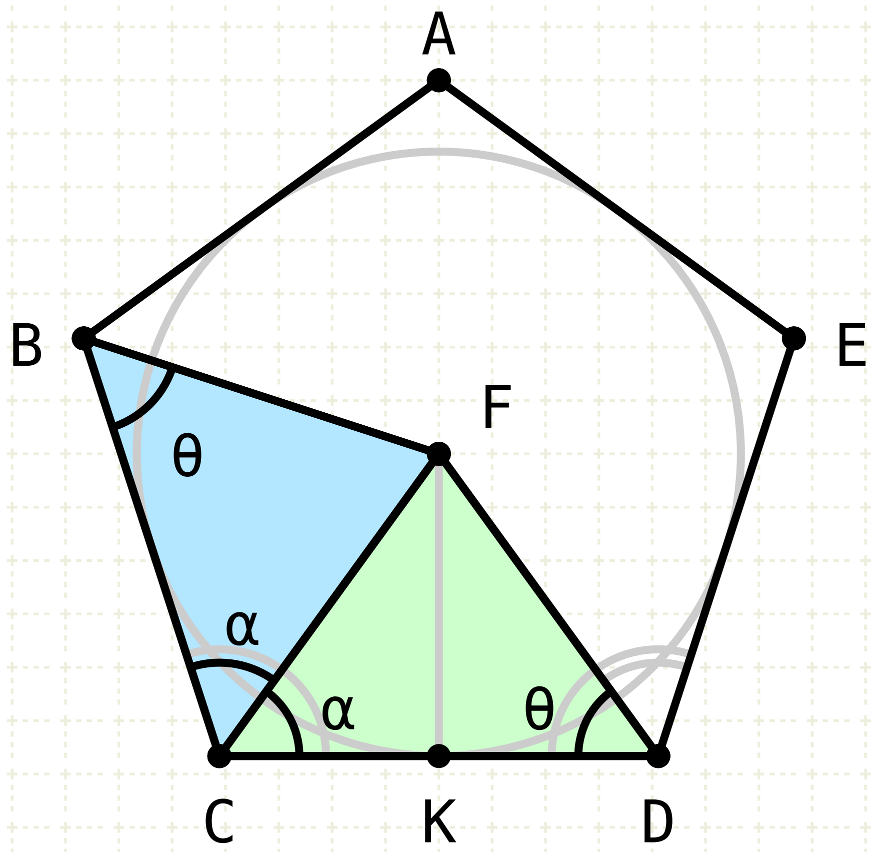

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Since BC is equal to CD, FC is common, and the angles BCF and FCD are equal the two triangles BFC and FCD are equivalent (SAS) (I·4)

Therefore the angles CBF and CDF are equal

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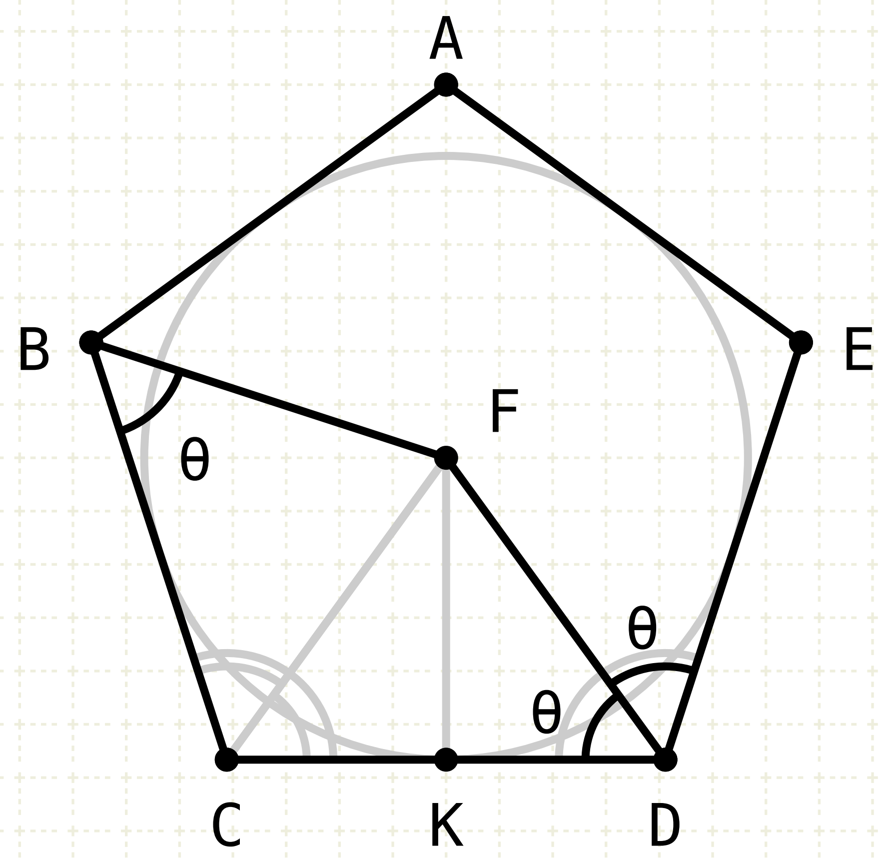

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$\angle BCF = \angle FCD = \alpha$

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$\angle CBF = \angle CDF = \theta$

$\angle CDE = \angle ABC = 2\theta$

Proof

Draw line BF

Since BC is equal to CD, FC is common, and the angles BCF and FCD are equal the two triangles BFC and FCD are equivalent (SAS) (I·4)

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The pentagon is equiangular, hence angles ABC and CDE are equal

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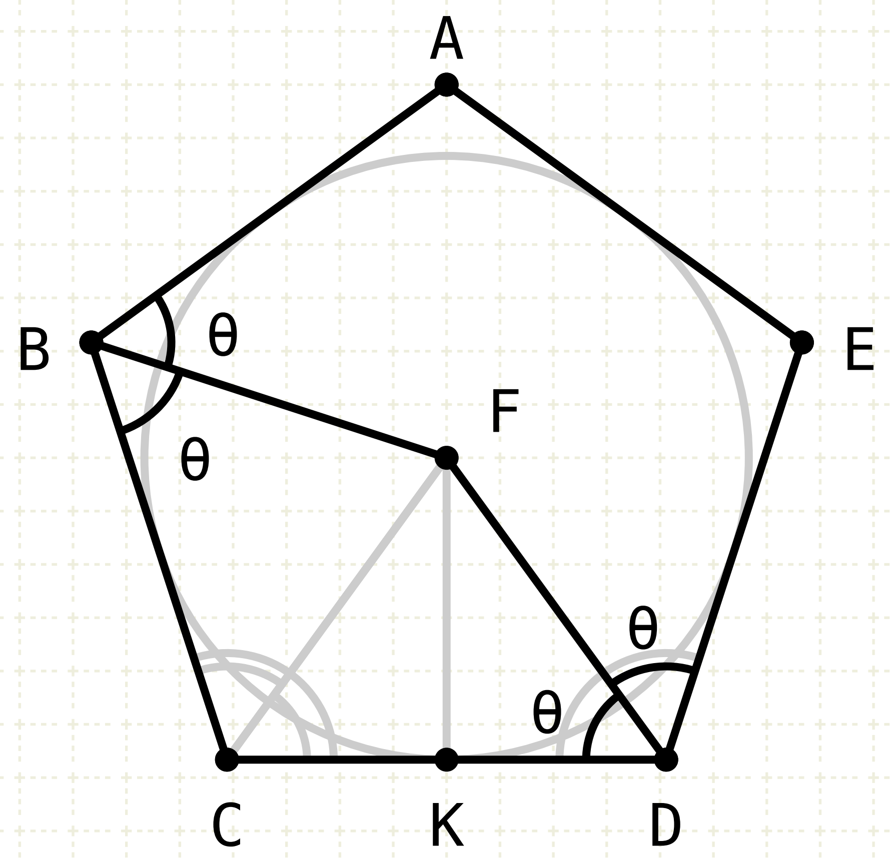

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The pentagon is equiangular, hence angles ABC and CDE are equal

Angle CBF (θ) is equal to FDC (θ), and FDC is half of CDE (2θ), therefore BF bisects the angle ABC

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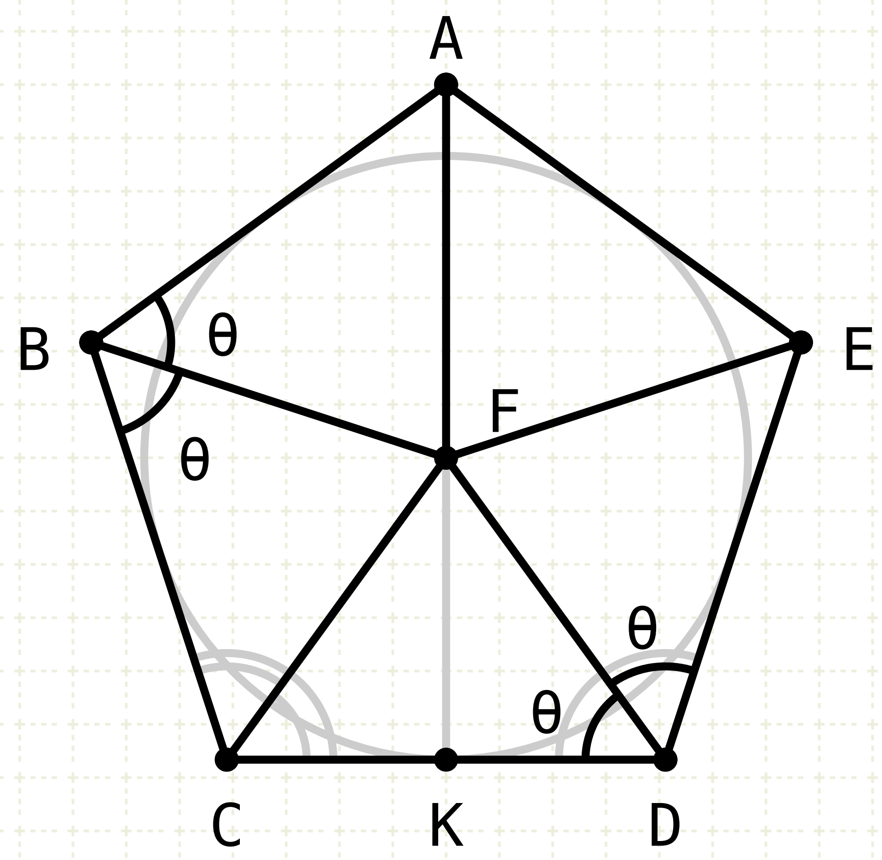

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The pentagon is equiangular, hence angles ABC and CDE are equal

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Similarly, it can be shown that AF and EF bisect the angles BAE and AED respectively

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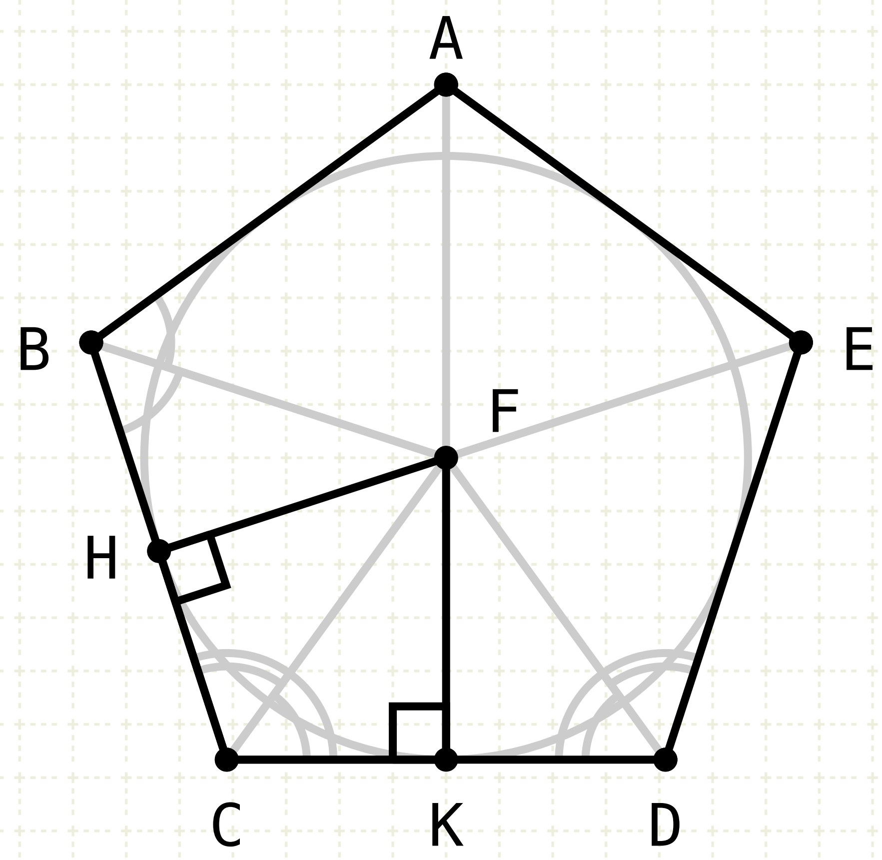

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Draw perpendiculars from F to line CD and BC

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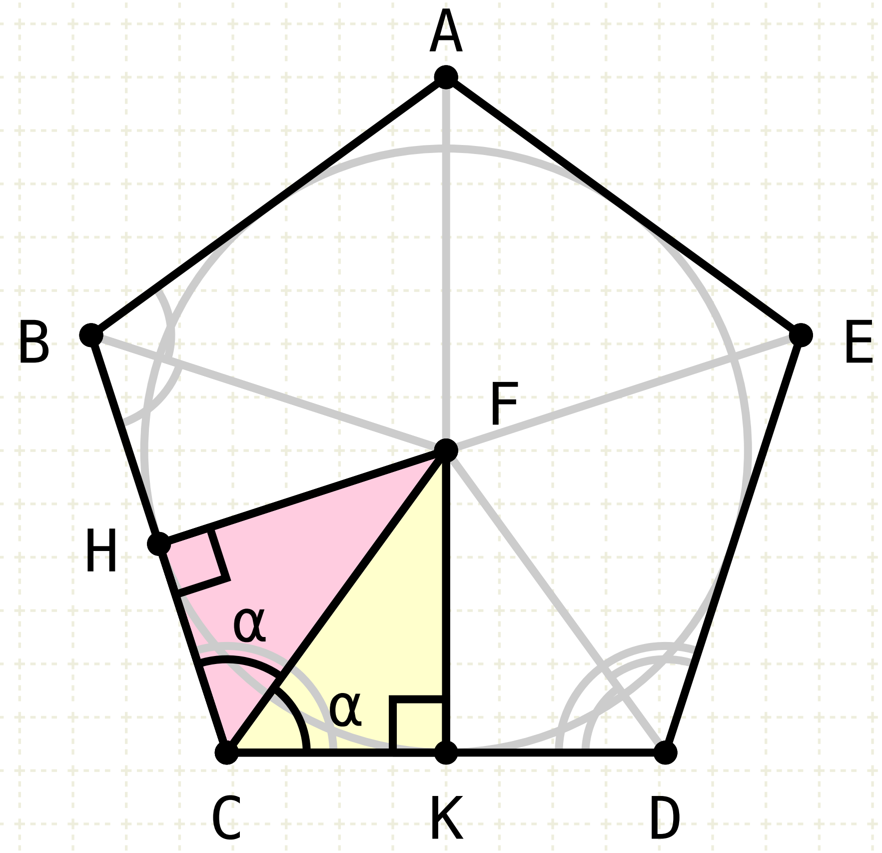

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$FH = FK$

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Draw perpendiculars from F to line CD and BC

Triangles HFC and FKC are right angled triangles with angles FCH and FCK equal and a common side FC

Therefore, the two triangles are equivalent (ASA) (I·26), and FH equals FK

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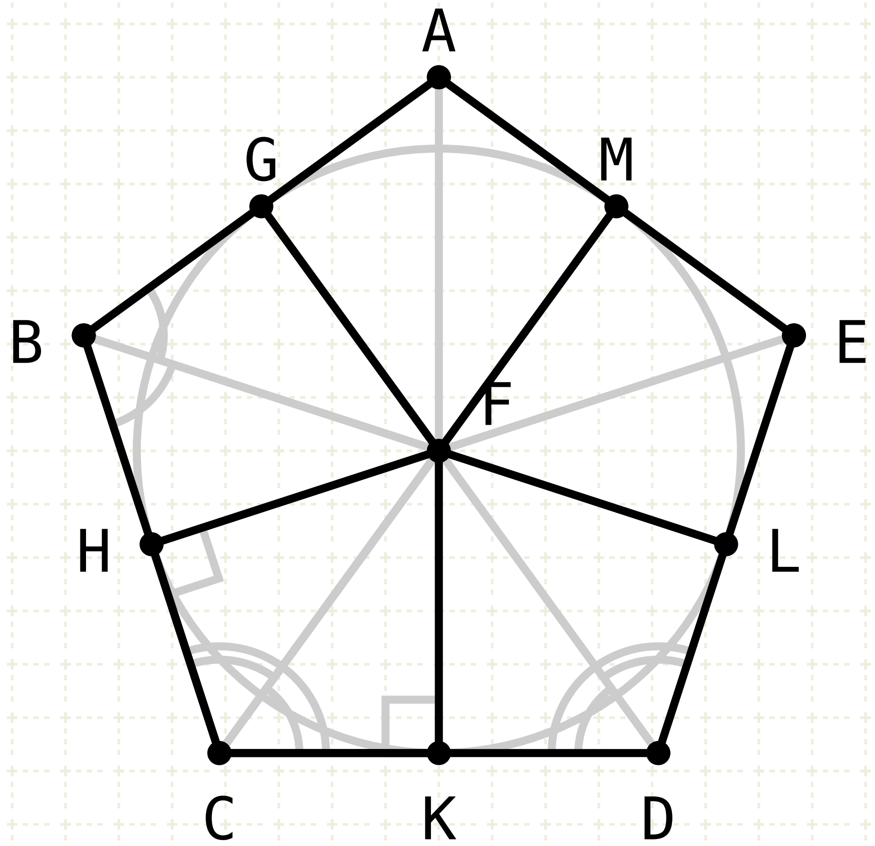

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$FH = FK$
$FH = FK = FL = FM = FG$

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The pentagon is equiangular, hence angles ABC and CDE are equal

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Triangles HFC and FKC are right angled triangles with angles FCH and FCK equal and a common side FC

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Similarly, it can be shown that perpendiculars drawn from F to the remaining sides of the pentagon are all equal

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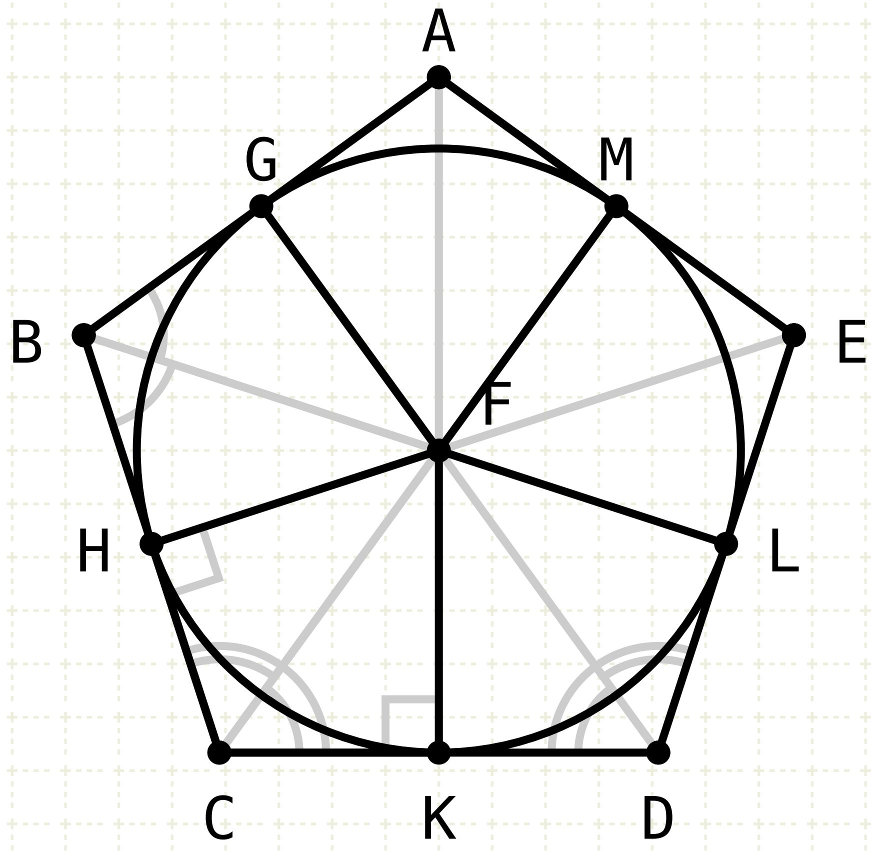

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Thus, a circle drawn with a centre at F, and a radius of FH will pass through all the points H, K, L, M and G

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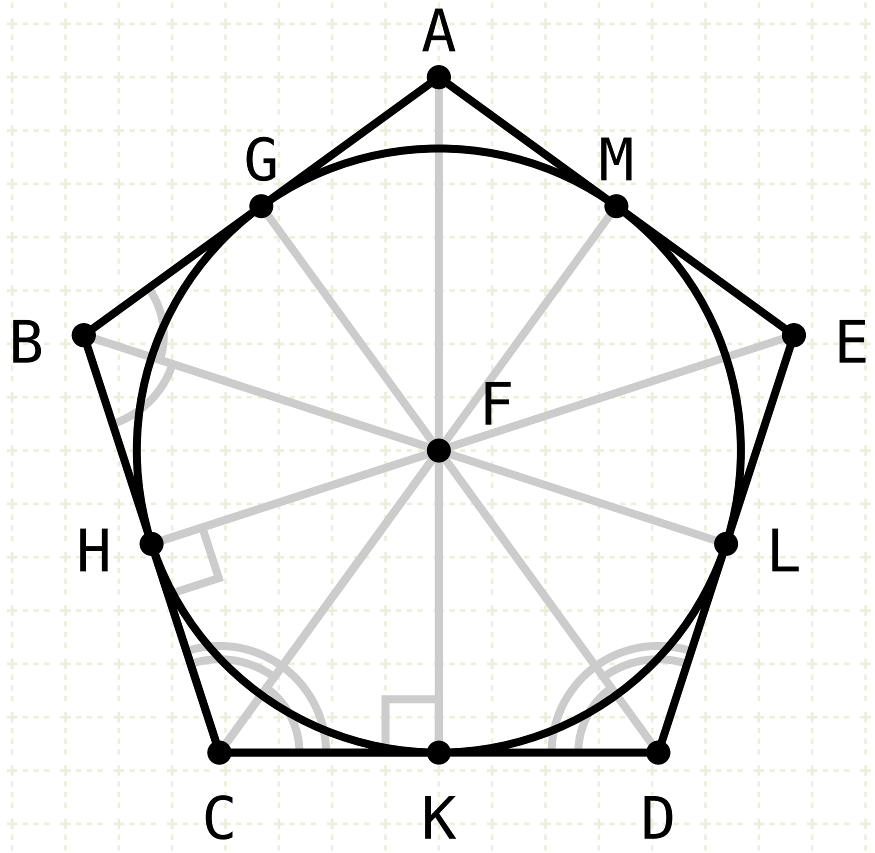

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Since the sides of the pentagon are at right angles to the radii, the pentagon touches the circle (III·16)

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