

Euclid's Elements

Book V



Proportions are what makes the old Greek temples classic in their beauty. They are like huge blocks, from which the air has been literally hewn out between the columns.

$$AB:C = DE:F$$

$$BG:C = EH:F$$

$$AG:C = DH:F$$

Arne Jacobsen



Table of Contents, Chapter 5

1	$n \cdot X + n \cdot Y = n \cdot (X + Y)$	11	if $A:B = C:D$ and $C:D = E:F$ then $A:B = E:F$	20	if $A:B = D:E$, $B:C = E:F$ and if $A > C$, then $D > F$
2	if $n \cdot C + m \cdot C = k \cdot C$ then $n \cdot F + m \cdot F = k \cdot F$	12	if $A:B = C:D = E:F$ then $(A+C+E):(B+D+F) = A:B$	21	if $A:B = E:F$, $B:C = D:E$ and if $A > C$, then $D > F$
3	if $E=m \cdot (n \cdot B)$ and $G=m \cdot (n \cdot D)$ then $E=k \cdot B$ and $G=k \cdot B$	13	if $A:B = C:D$ and $C:D > E:F$ then $A:B > E:F$	22	if $A:B = D:E$, $B:C = E:F$ then $A:C = D:F$
4	if $A:B=C:D$ then $(p \cdot A):(q \cdot B)=(p \cdot C):(q \cdot D)$	14	if $A:B = C:D$ and $A > C$ then $B > D$	23	if $A:B = E:F$, $B:C = D:E$ then $A:C = D:F$
5	$n \cdot X - n \cdot Y = n \cdot (X - Y)$	15	if $A = n \cdot C$ and $B = n \cdot D$ then $A:B = C:D$	24	if $A:C = D:F$, $B:C = E:F$ then $(A+B):C = (D+E):F$
6	if $n \cdot E - m \cdot E = k \cdot E$ then $n \cdot F - m \cdot F = k \cdot F$	16	if $A:B = C:D$ then $A:C = B:D$	25	if $A:B = C:D$ and $A > B, C, D$ and $D < A, B, C$ then $(A+D) > (B+C)$
7	if $A = B \neq C$ then $A:C = B:C$ and $C:A = C:B$	17	if $(A+B):B = (C+D):D$ then $A:B = C:D$		
8	if $A > B \neq D$ then $A:D > B:D$ and $D:A < D:B$	18	if $A:B = C:D$ then $(A+B):B = (C+D):D$		
9	if $A:C = B:C$, or $C:A = C:B$ then $A = B$	19	if $(A+C):(B+D) = C:D$ then $(A+C):(B+D) = A:B$		
10	if $A:C > B:C$, or $A:C < B:C$ then $A > B$, or $A < C$, respectively				



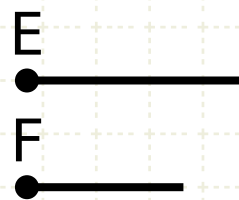
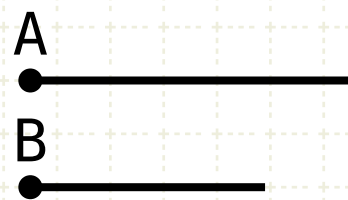
Proposition 11 of Book V

Ratios which are the same with the same ratio are also the same with one another.



Proposition 11 of Book V

Ratios which are the same with the same ratio are also the same with one another.



In other words

If A is to B as C is to D, and C is to D as E is to F then ...

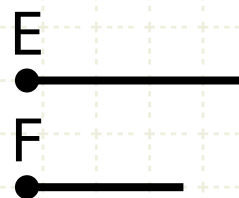
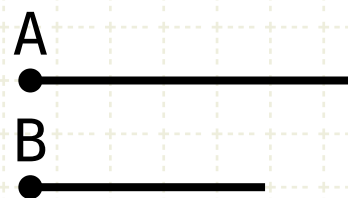
... A is to B as E is to F

$$\begin{aligned} A:B &= C:D \\ C:D &= E:F \\ \rightarrow A:B &= E:F \end{aligned}$$



Proposition 11 of Book V

Ratios which are the same with the same ratio are also the same with one another.



In other words

If A is to B as C is to D, and C is to D as E is to F then ...

... A is to B as E is to F

Proof

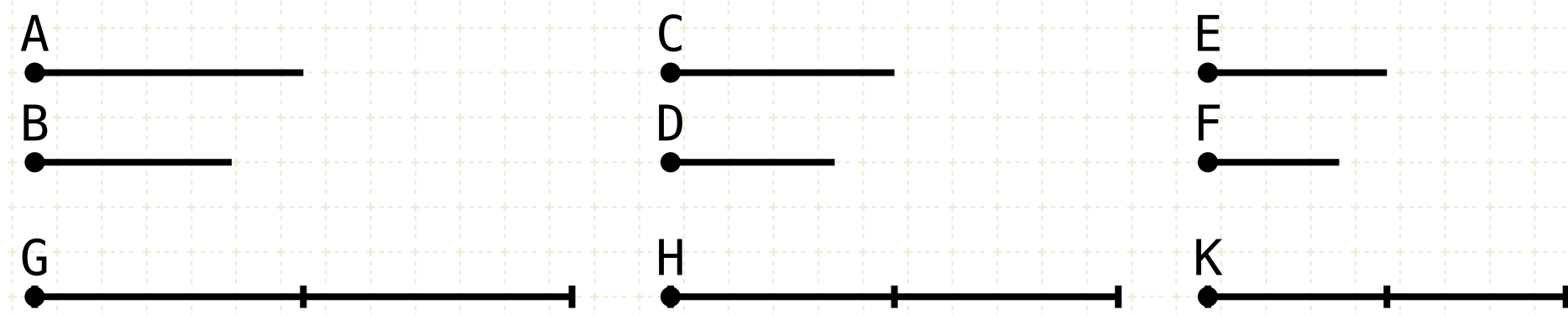
$$A:B = C:D$$

$$C:D = E:F$$



Proposition 11 of Book V

Ratios which are the same with the same ratio are also the same with one another.



$$\begin{aligned} A:B &= C:D \\ C:D &= E:F \\ G &= p \cdot A \\ H &= p \cdot C \\ K &= p \cdot E \end{aligned}$$

In other words

If A is to B as C is to D, and C is to D as E is to F then ...
... A is to B as E is to F

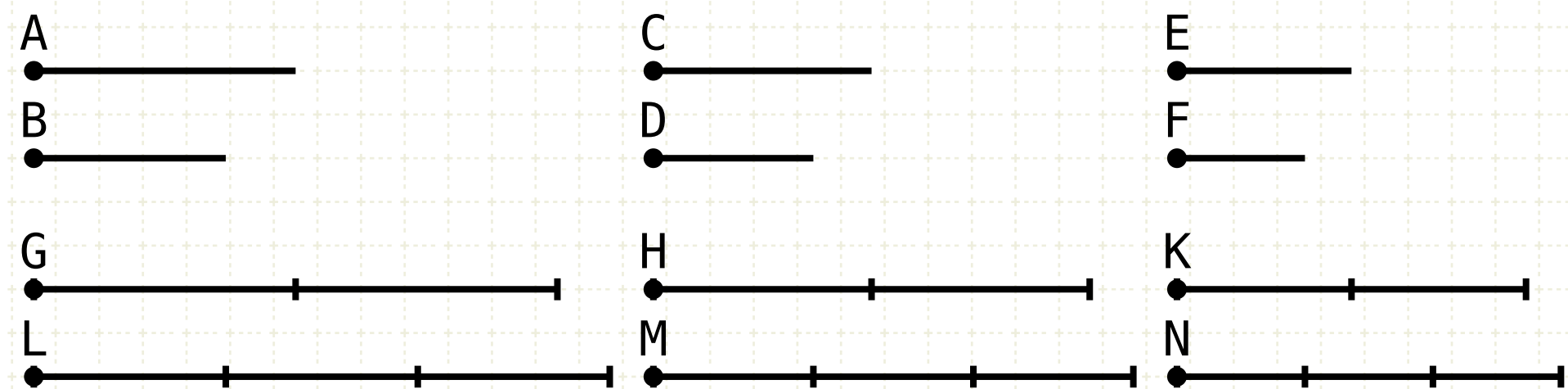
Proof

Let G,H,K be equimultiples of A,C and E



Proposition 11 of Book V

Ratios which are the same with the same ratio are also the same with one another.



$A:B = C:D$
 $C:D = E:F$
 $G = p \cdot A$
 $H = p \cdot C$
 $K = p \cdot E$
 $L = q \cdot B$
 $M = q \cdot D$
 $N = q \cdot F$

In other words

If A is to B as C is to D, and C is to D as E is to F then ...

... A is to B as E is to F

Proof

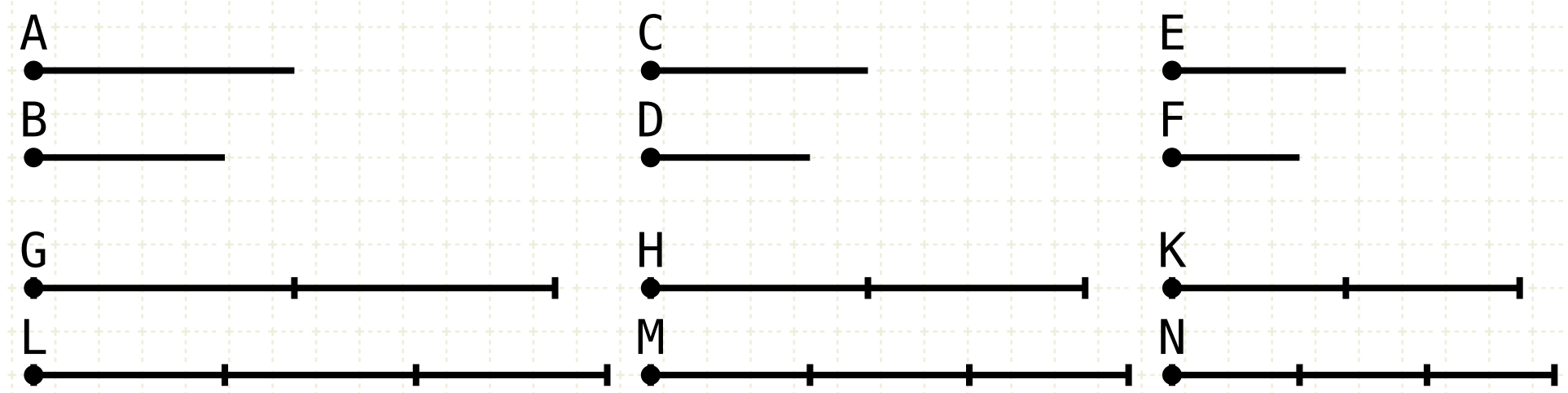
Let G,H,K be equimultiples of A,C and E

Let L,M,N be equimultiples of B,D and F



Proposition 11 of Book V

Ratios which are the same with the same ratio are also the same with one another.



$A:B = C:D$
 $C:D = E:F$
 $G = p \cdot A$
 $H = p \cdot C$
 $K = p \cdot E$
 $L = q \cdot B$
 $M = q \cdot D$
 $N = q \cdot F$

$G \geq L \rightarrow H \geq M$

In other words

If A is to B as C is to D, and C is to D as E is to F then ...
... A is to B as E is to F

Proof

Let G,H,K be equimultiples of A,C and E
Let L,M,N be equimultiples of B,D and F

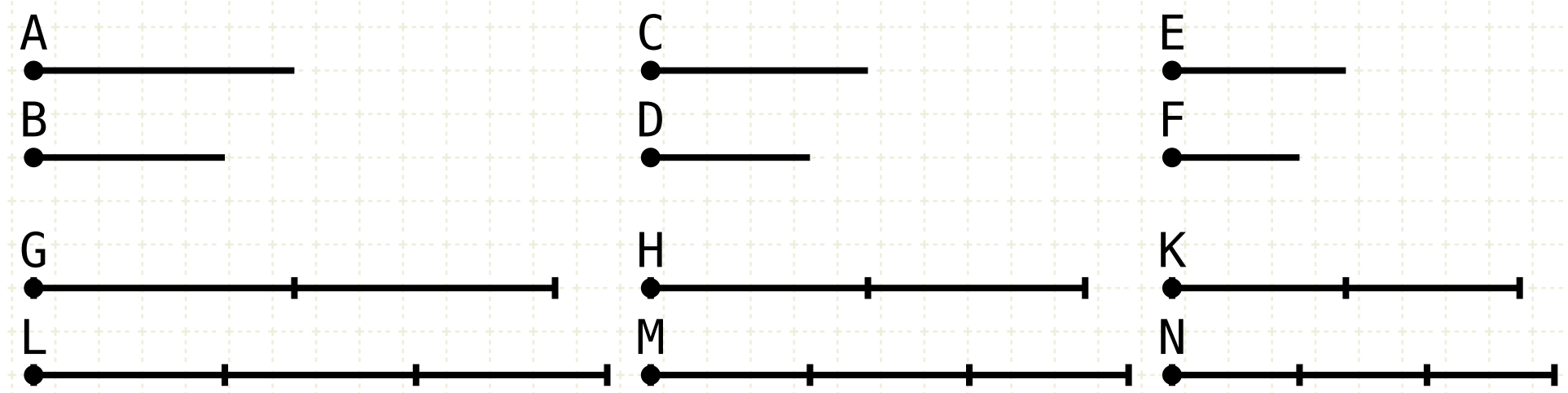
Since the ratios A to B and C to D are equal, and since equimultiples G,H and L,M have been taken...

If G is less than L, then H is less than M, etc (V.def 5)



Proposition 11 of Book V

Ratios which are the same with the same ratio are also the same with one another.



$A : B = C : D$
 $C : D = E : F$
 $G = p \cdot A$
 $H = p \cdot C$
 $K = p \cdot E$
 $L = q \cdot B$
 $M = q \cdot D$
 $N = q \cdot F$

$G \geq L \rightarrow H \geq M$
 $H \geq M \rightarrow K \geq N$

In other words

If A is to B as C is to D, and C is to D as E is to F then ...
... A is to B as E is to F

Proof

Let G,H,K be equimultiples of A,C and E
Let L,M,N be equimultiples of B,D and F

Since the ratios A to B and C to D are equal, and since equimultiples G,H and L,M have been taken...

If G is less than L, then H is less than M, etc (V.def 5)

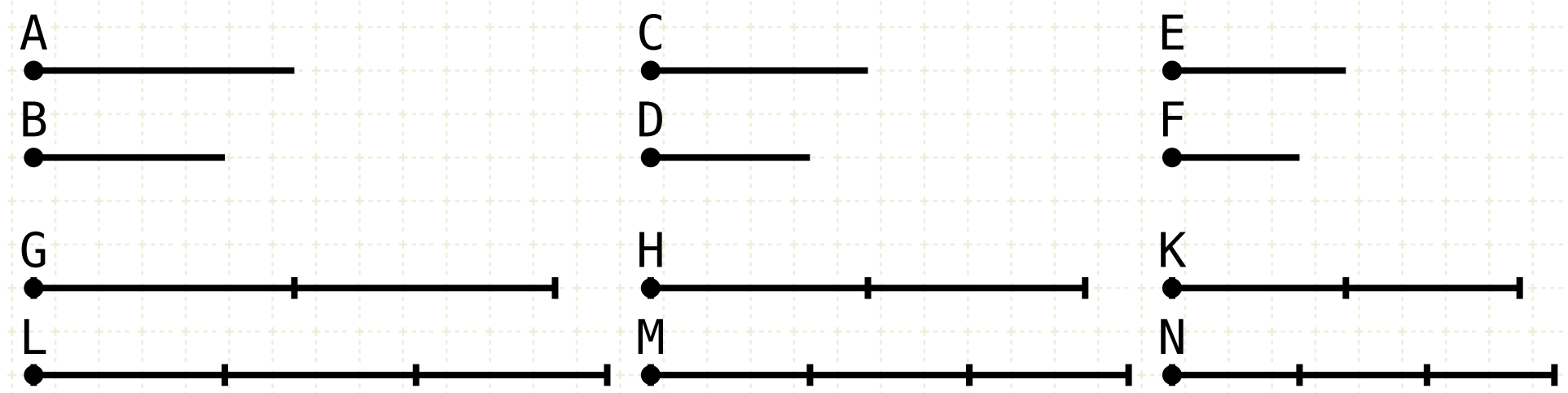
Since the ratios C to D and E to F are equal, and since equimultiples H,K and M,N have been taken...

If H is less than M, then K is less than N, etc (V.def 5)



Proposition 11 of Book V

Ratios which are the same with the same ratio are also the same with one another.



$A : B = C : D$
 $C : D = E : F$
 $G = p \cdot A$
 $H = p \cdot C$
 $K = p \cdot E$
 $L = q \cdot B$
 $M = q \cdot D$
 $N = q \cdot F$

$G \geq L \rightarrow H \geq M$
 $H \geq M \rightarrow K \geq N$
 $G \geq L \rightarrow K \geq N$

In other words

If A is to B as C is to D, and C is to D as E is to F then ...
... A is to B as E is to F

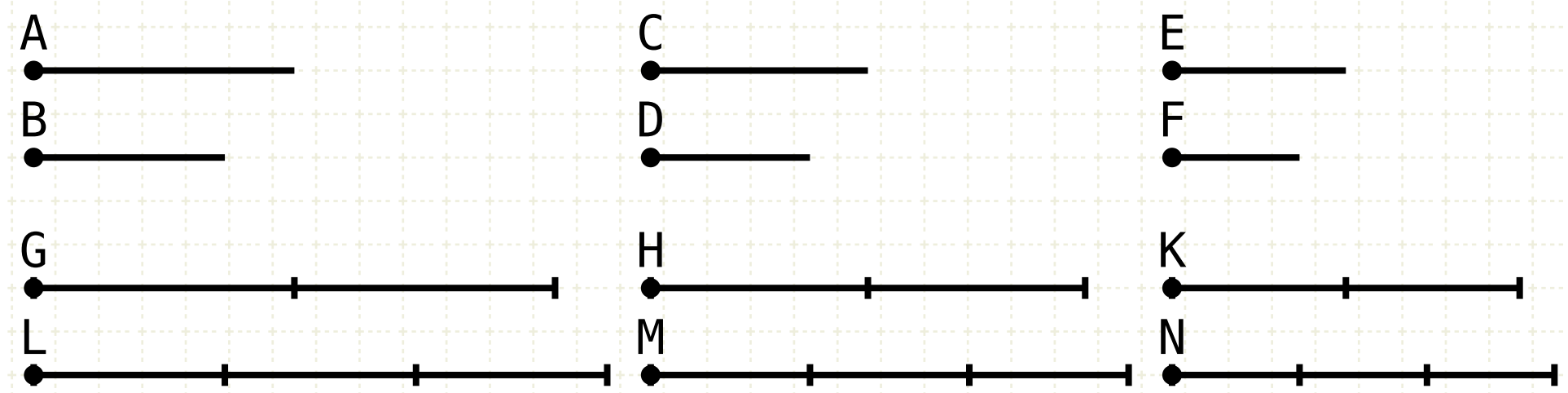
Proof

Let G,H,K be equimultiples of A,C and E
Let L,M,N be equimultiples of B,D and F
Since the ratios A to B and C to D are equal, and since equimultiples G,H and L,M have been taken...
If G is less than L, then H is less than M, etc (V.def 5)
Since the ratios C to D and E to F are equal, and since equimultiples H,K and M,N have been taken...
If H is less than M, then K is less than N, etc (V.def 5)
Therefore, if G is greater than L than that H is greater than M, and K is greater than N, then...
... If G is greater than L, then K is greater than N, etc.



Proposition 11 of Book V

Ratios which are the same with the same ratio are also the same with one another.



$$A : B = C : D$$

$$C : D = E : F$$

$$G = p \cdot A$$

$$H = p \cdot C$$

$$K = p \cdot E$$

$$L = q \cdot B$$

$$M = q \cdot D$$

$$N = q \cdot F$$

$$G \geq L \rightarrow H \geq M$$

$$H \geq M \rightarrow K \geq N$$

$$G \geq L \rightarrow K \geq N$$

$$p \cdot A \geq q \cdot B \rightarrow p \cdot E \geq q \cdot F$$

In other words

If A is to B as C is to D, and C is to D as E is to F then ...

... A is to B as E is to F

Proof

Let G,H,K be equimultiples of A,C and E

Let L,M,N be equimultiples of B,D and F

Since the ratios A to B and C to D are equal, and since equimultiples G,H and L,M have been taken...

If G is less than L, then H is less than M, etc (V.def 5)

Since the ratios C to D and E to F are equal, and since equimultiples H,K and M,N have been taken...

If H is less than M, then K is less than N, etc (V.def 5)

Therefore, if G is greater than L than that H is greater than M, and K is greater than N, then...

... If G is greater than L, then K is greater than N, etc.

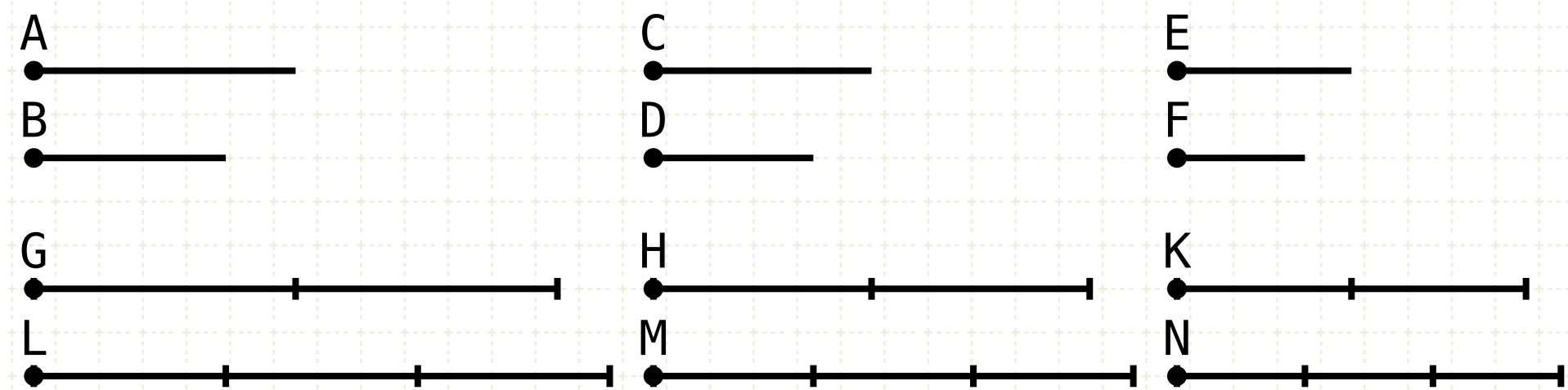
If G is greater/equal/less than than L, means that K is greater/equal/less than N, ...

... and G,K are equimultiples of A,E and L,N are equimultiples of B,F



Proposition 11 of Book V

Ratios which are the same with the same ratio are also the same with one another.



$$A:B = C:D$$

$$C:D = E:F$$

$$G = p \cdot A$$

$$H = p \cdot C$$

$$K = p \cdot E$$

$$L = q \cdot B$$

$$M = q \cdot D$$

$$N = q \cdot F$$

$$G \geq L \rightarrow H \geq M$$

$$H \geq M \rightarrow K \geq N$$

$$G \geq L \rightarrow K \geq N$$

$$p \cdot A \geq q \cdot B \rightarrow p \cdot E \geq q \cdot F$$

$$A:B = E:F$$

In other words

If A is to B as C is to D, and C is to D as E is to F then ...

... A is to B as E is to F

Proof

Let G,H,K be equimultiples of A,C and E

Let L,M,N be equimultiples of B,D and F

Since the ratios A to B and C to D are equal, and since equimultiples G,H and L,M have been taken...

If G is less than L, then H is less than M, etc (V.def 5)

Since the ratios C to D and E to F are equal, and since equimultiples H,K and M,N have been taken...

If H is less than M, then K is less than N, etc (V.def 5)

Therefore, if G is greater than L than that H is greater than M, and K is greater than N, then...

... If G is greater than L, then K is greater than N, etc.

If G is greater/equal/less than than L, means that K is greater/equal/less than N, ...

... and G,K are equimultiples of A,E and L,N are equimultiples of B,F

A is to B as E is to F (V.def 5)



Youtube Videos

<https://www.youtube.com/c/SandyBultena>

Copyright © 2019 by Sandy Bultena.



Except where otherwise noted, this work is licensed under
<http://creativecommons.org/licenses/by-nc/3.0>