

# Euclid's Elements

## Book I

*If Euclid did not kindle your youthful enthusiasm, you were not born to be a scientific thinker.*

Albert Einstein



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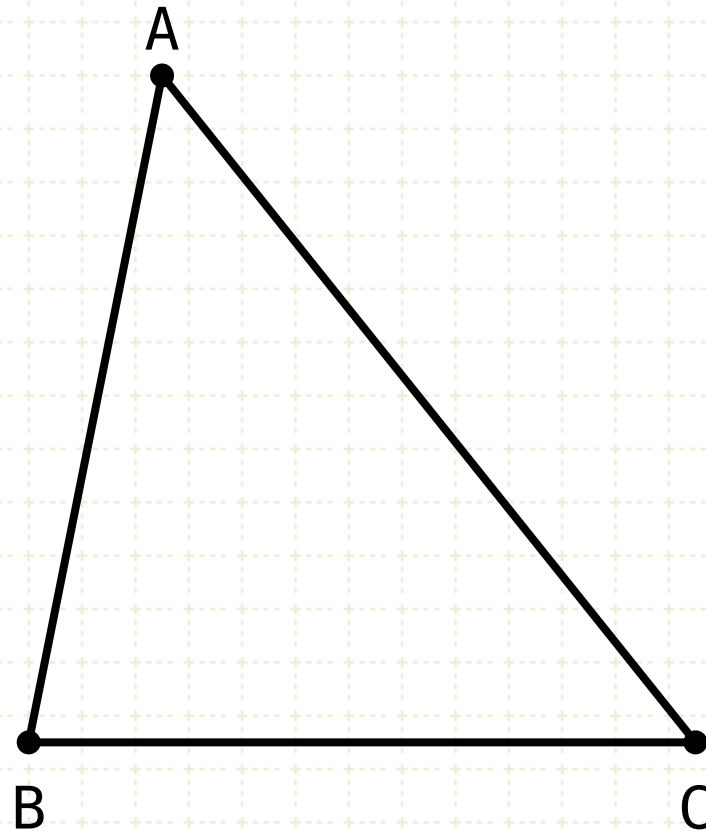
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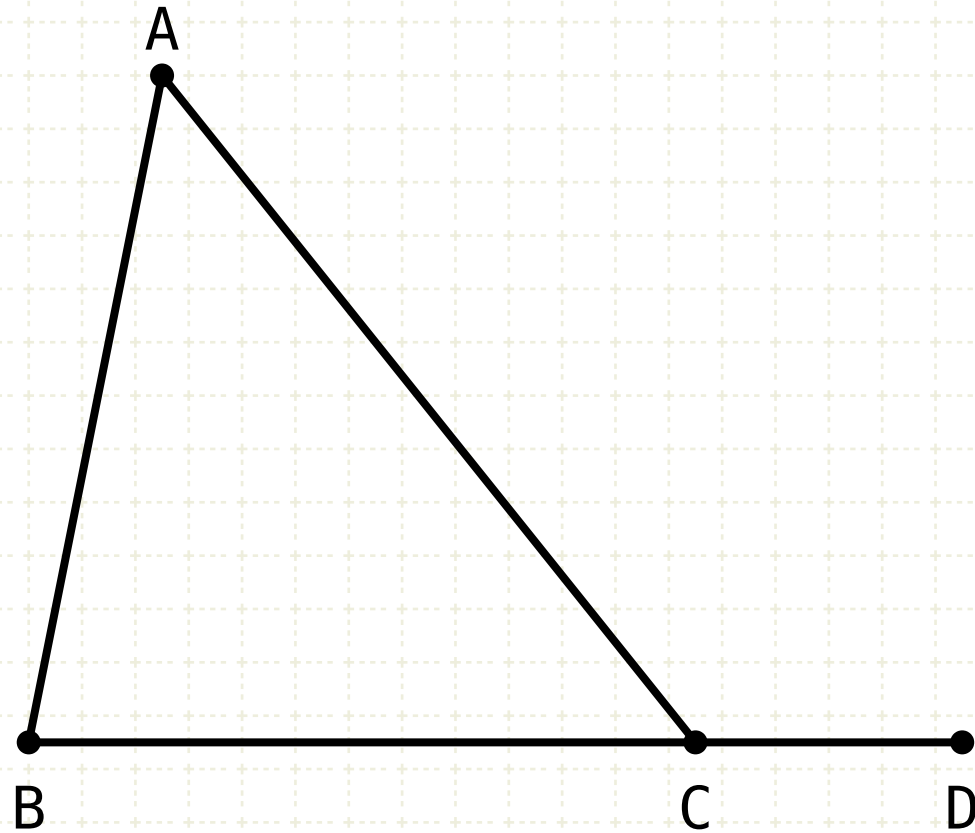
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Start with a triangle ABC



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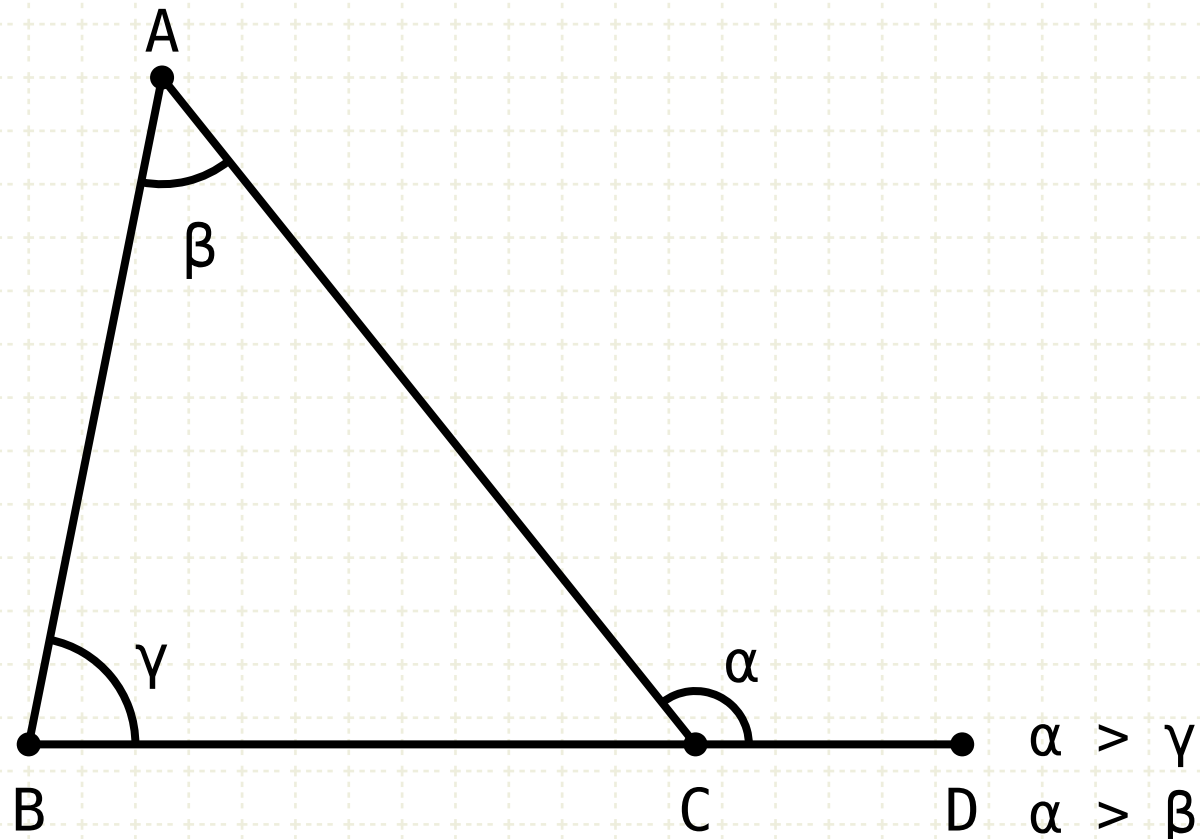
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Extend line BC to point D



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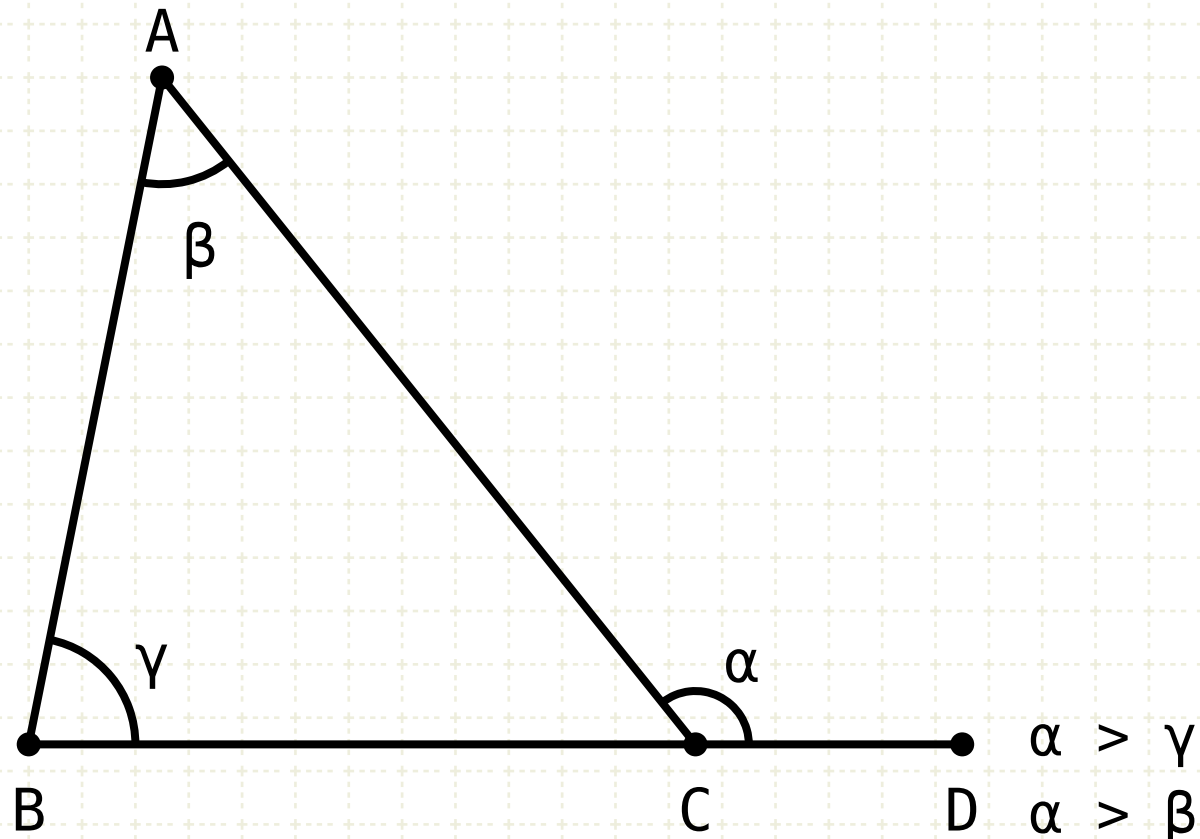
Extend line BC to point D

The angle ACD is larger than either ABC or CAB

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## Proof



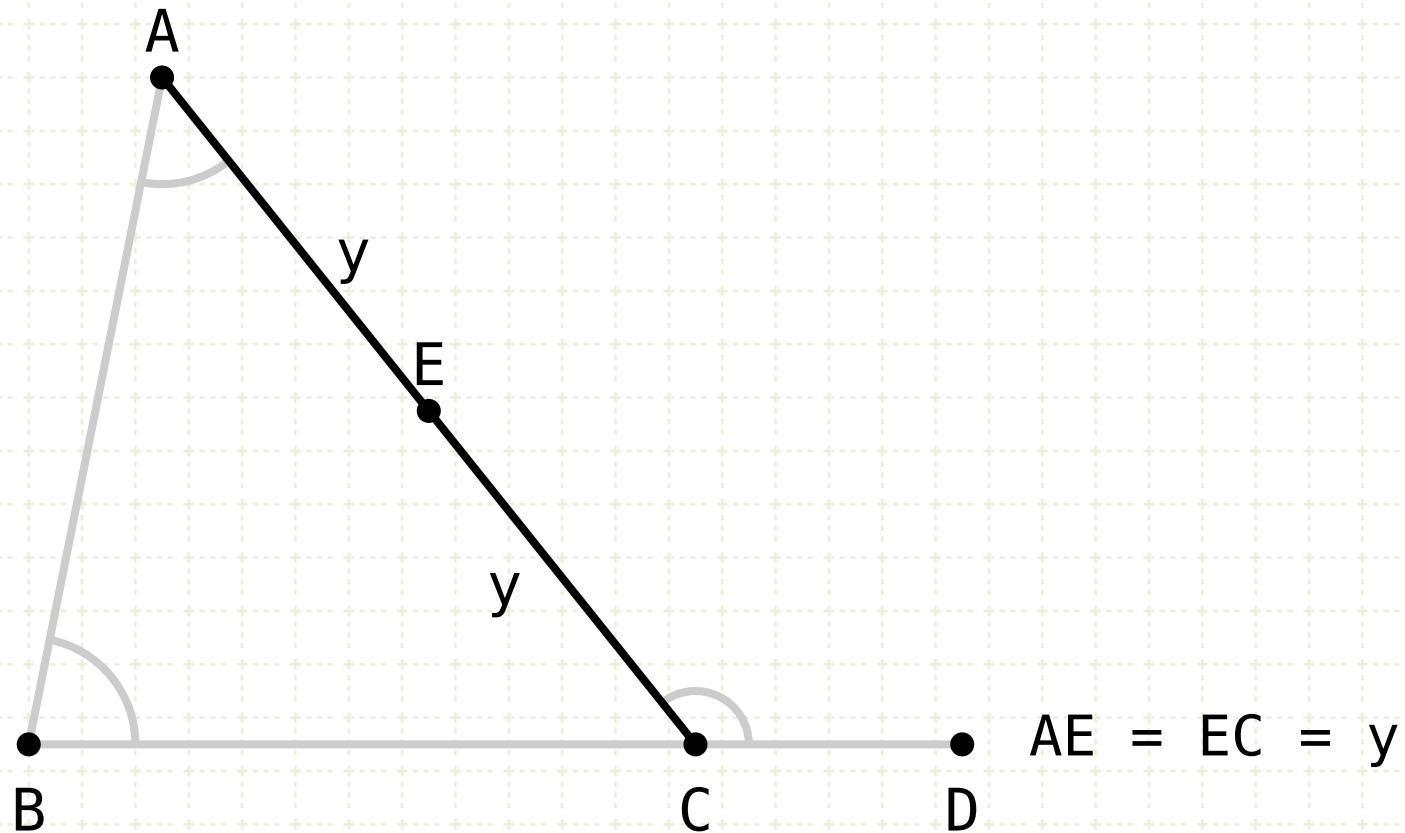


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Bisect line AC at point E (I·10)



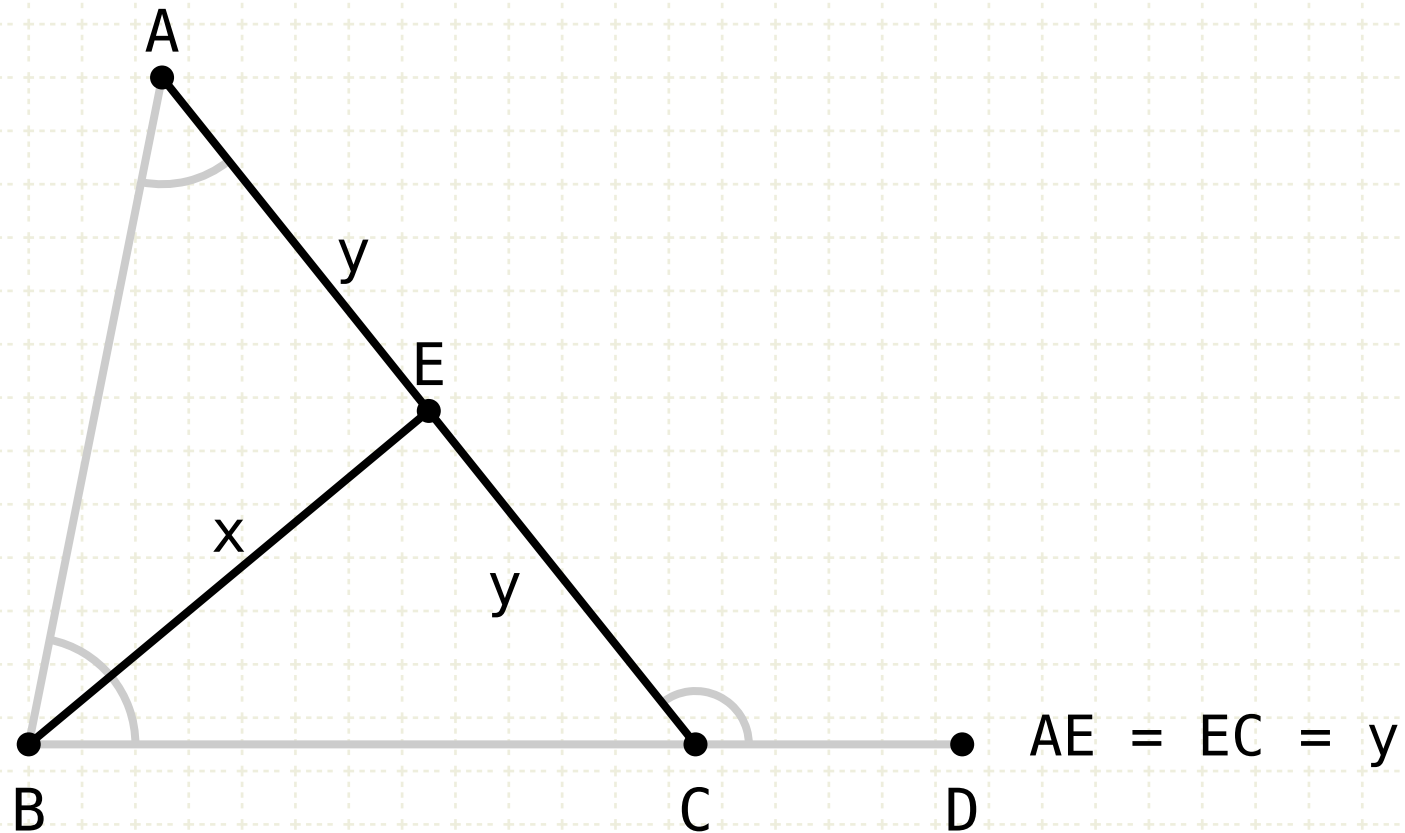
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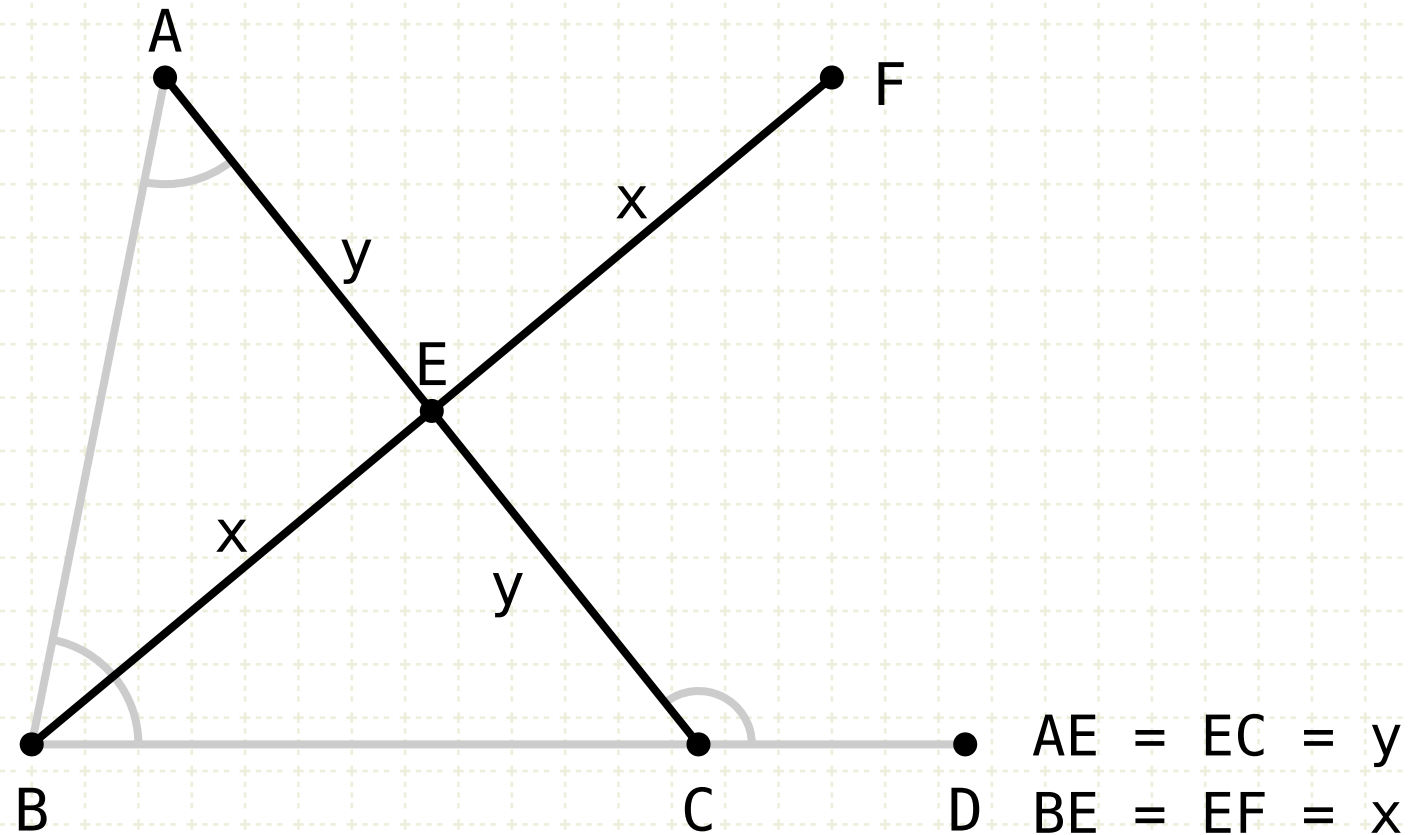
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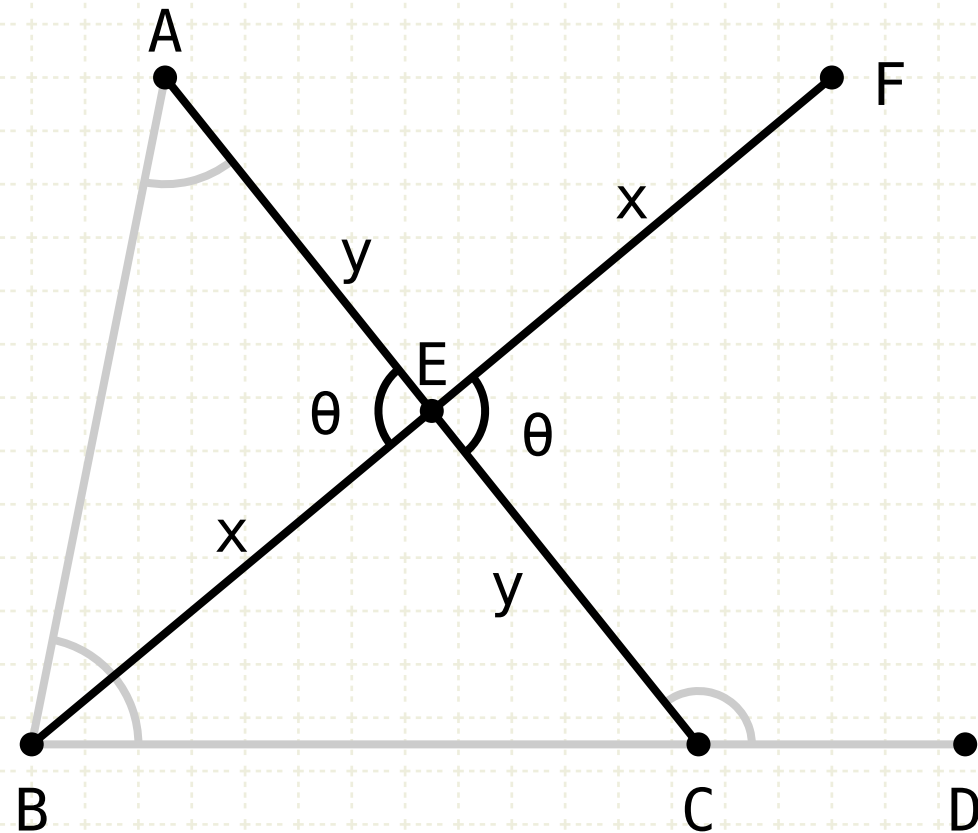
Bisect line  $AC$  at point  $E$  (I·10)

Create line segment  $BE$

Extend line  $BE$  to line  $F$ , where  $EF$  equals  $BE$

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In any triangle, if one of the sides is produced, then the exterior angle is greater than either of the interior and opposite angles.



$$\begin{aligned} &AE = EC = y \\ &BE = EF = x \\ &\angle AEB = \angle CEF = \theta \end{aligned}$$

## Proof

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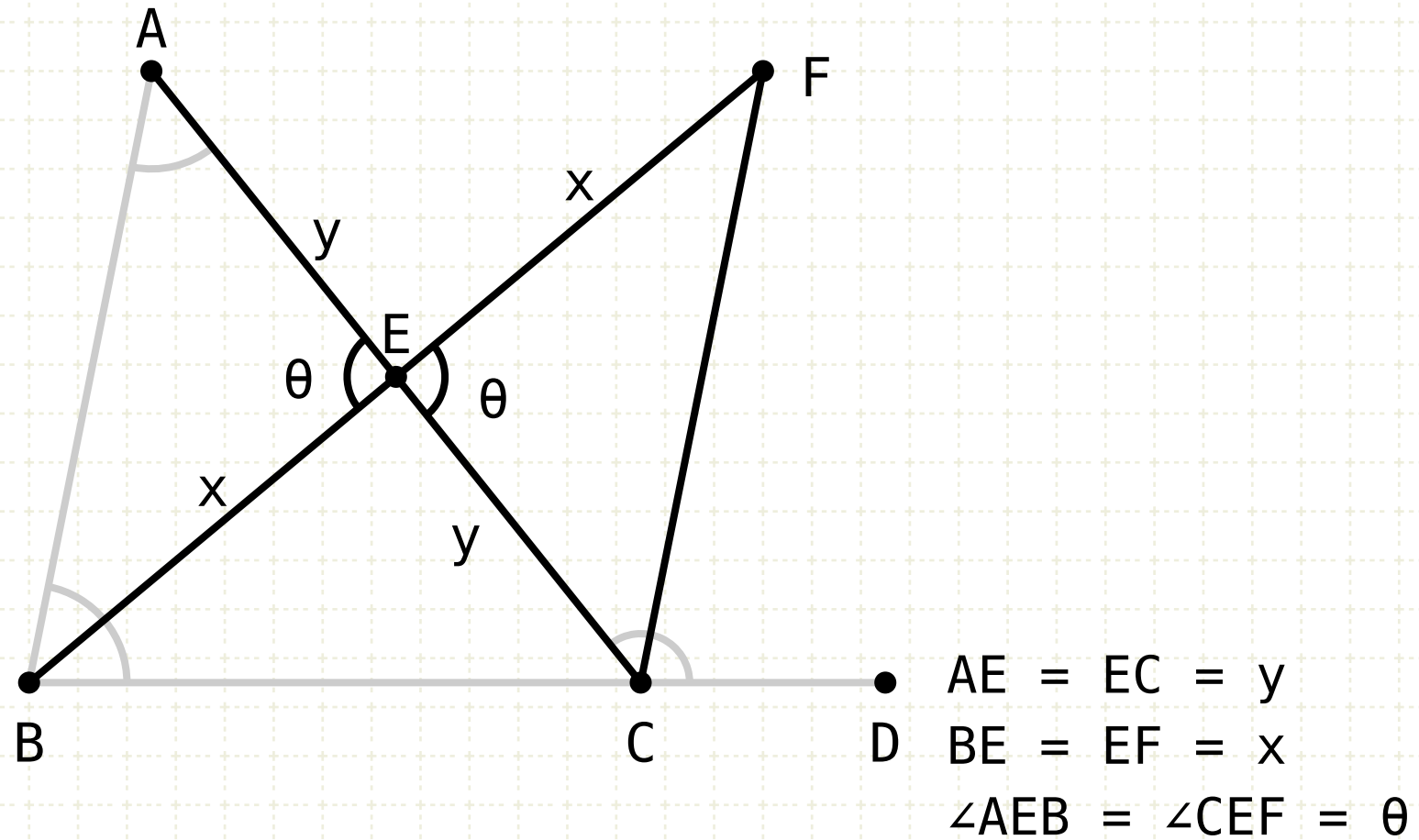
Create line segment BE

Extend line BE to line F, where EF equals BE

Angles AEB and CEF are vertical to each other, hence they are equal (I·15)

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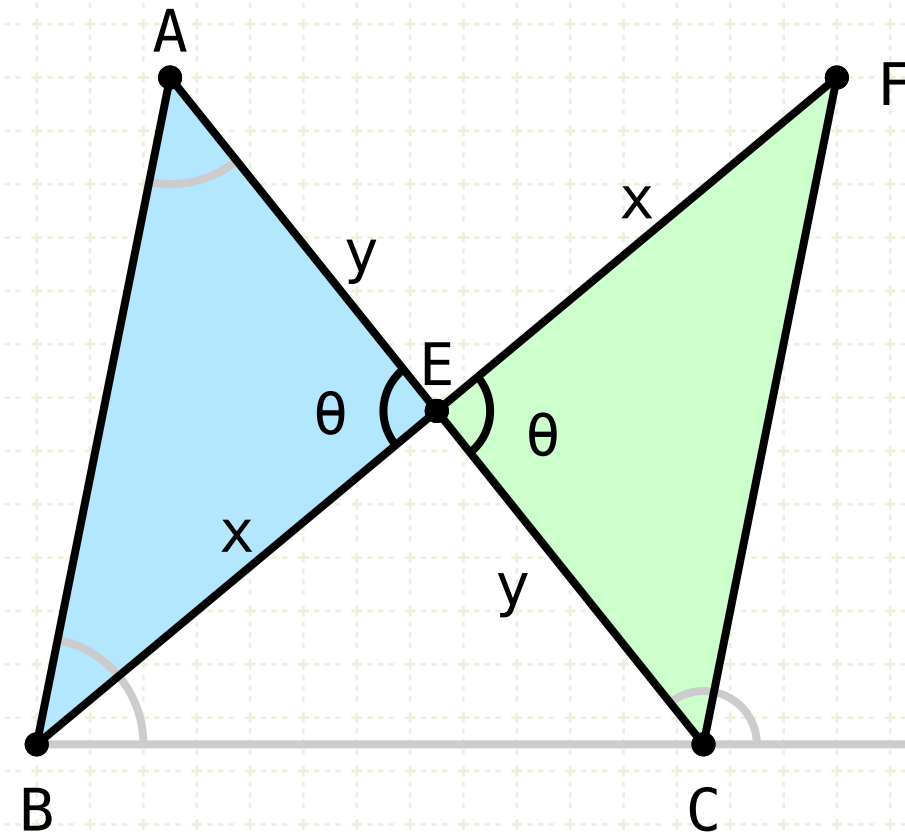
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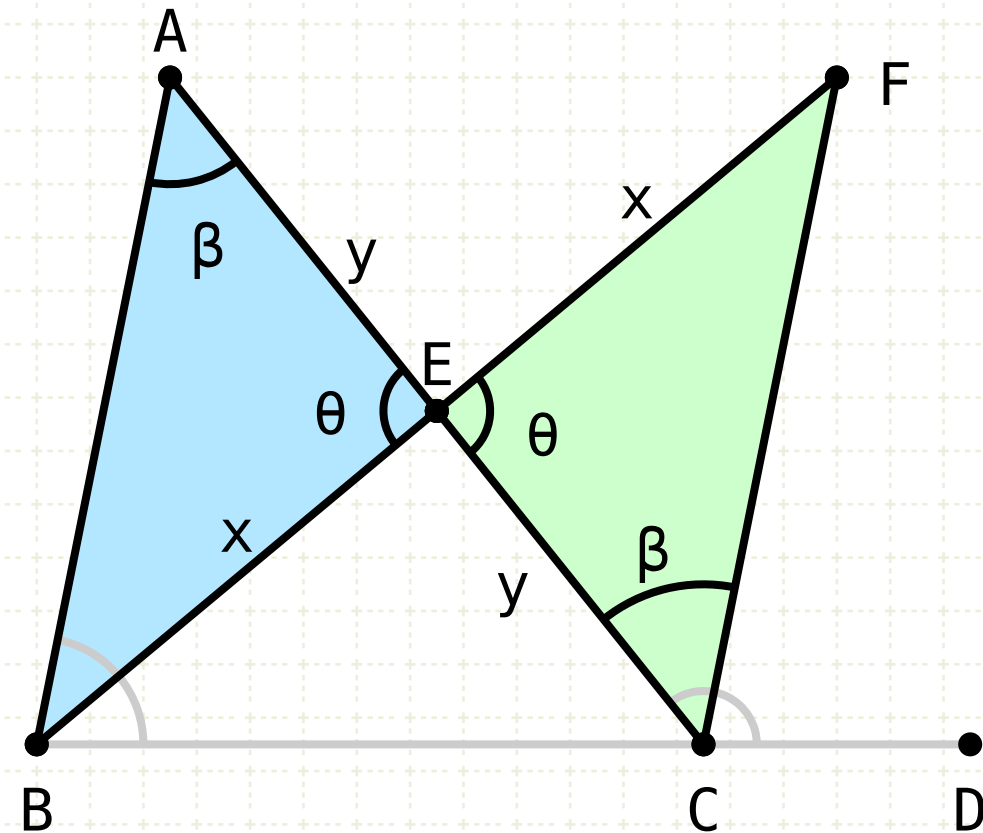
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Triangles ABE and FEC are equivalent since they have two equal sides, with an equal angle AEB and FEC



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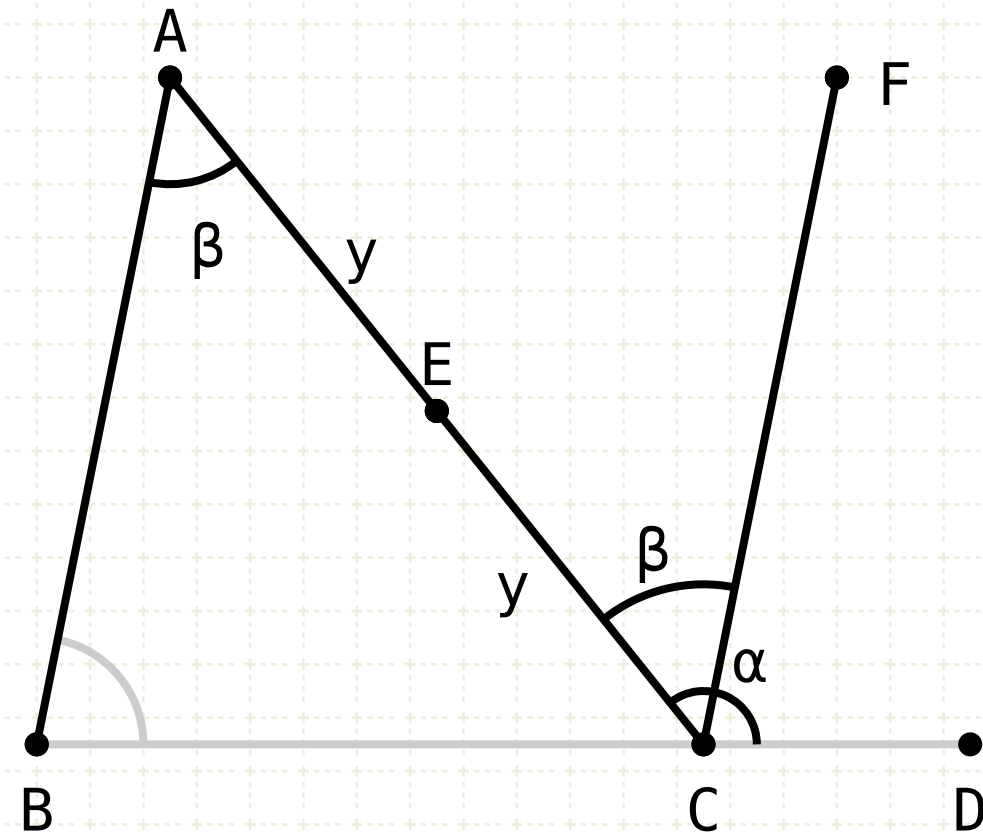
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Triangles ABE and FEC are equivalent since they have two equal sides, with an equal angle AEB and FEC

Thus, angles BAE and ECF are equal (I·4)

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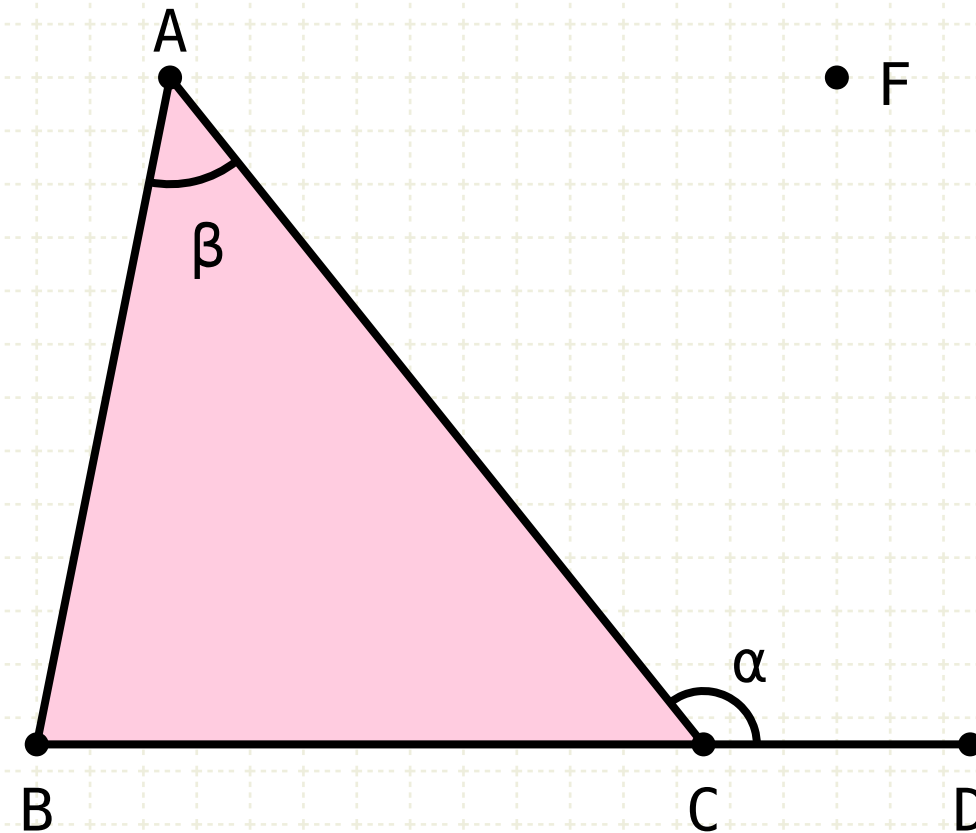
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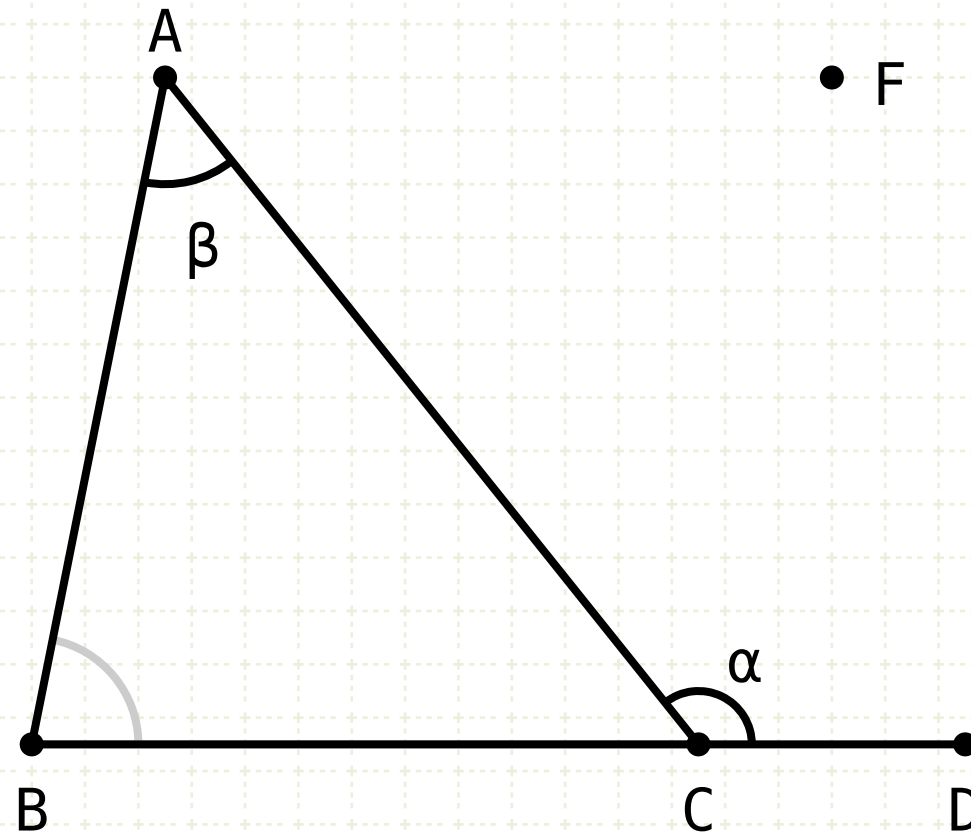
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Thus it has been shown that the exterior angle ACD is larger than the interior angle BAC

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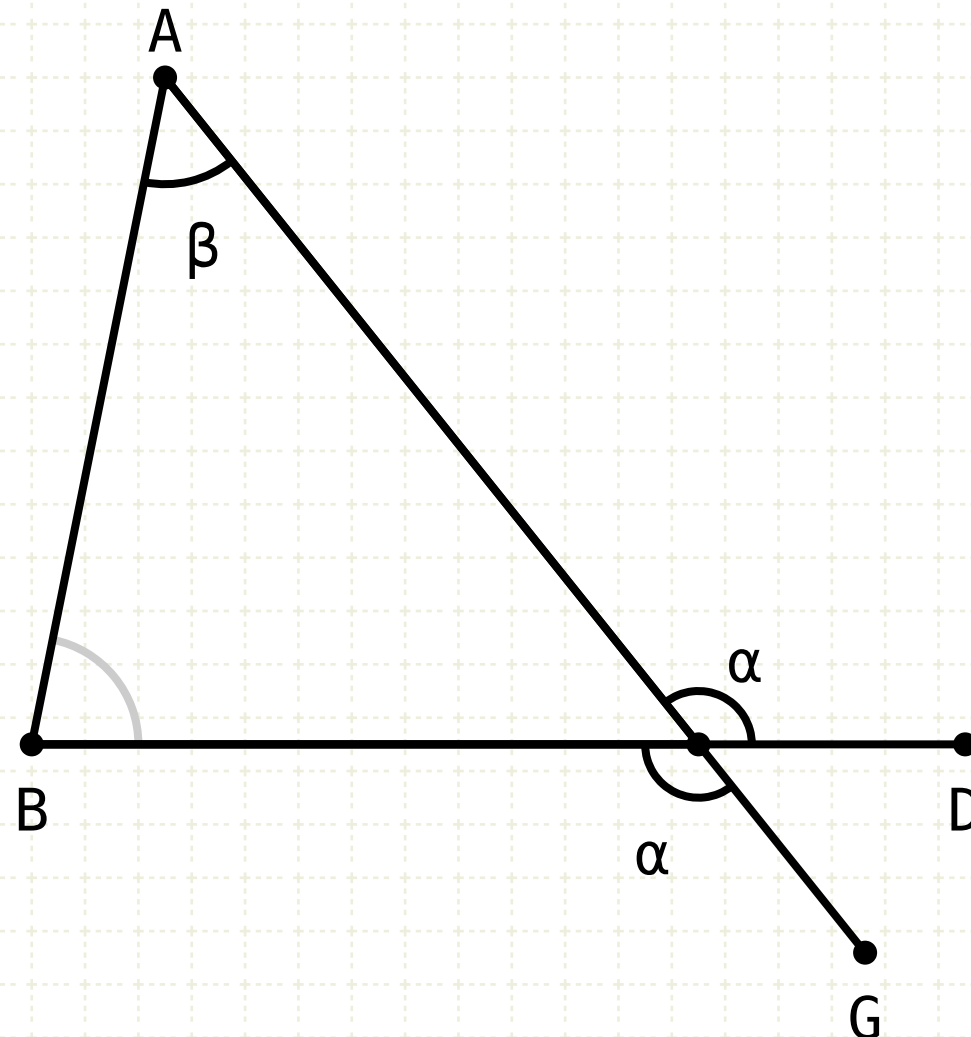
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Using the same method as before, we can prove that angle BCG is greater than ABC



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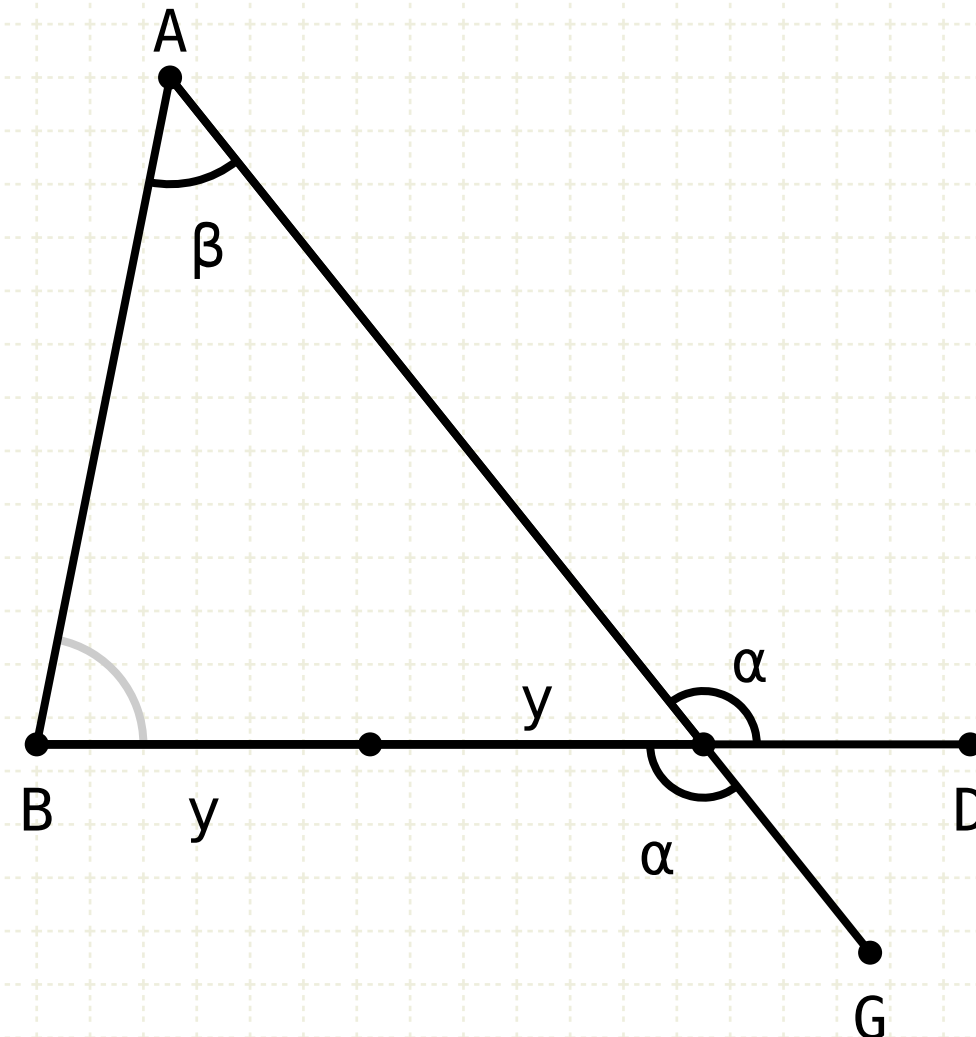
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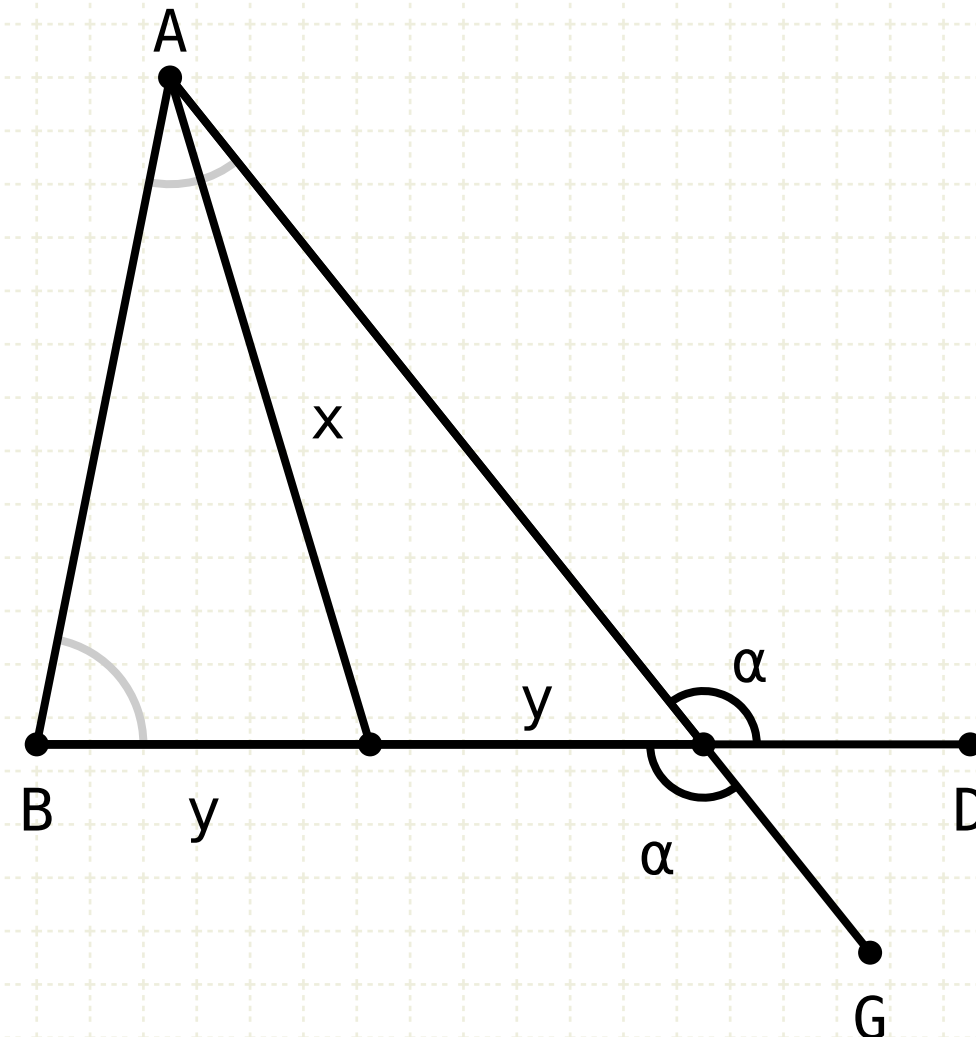
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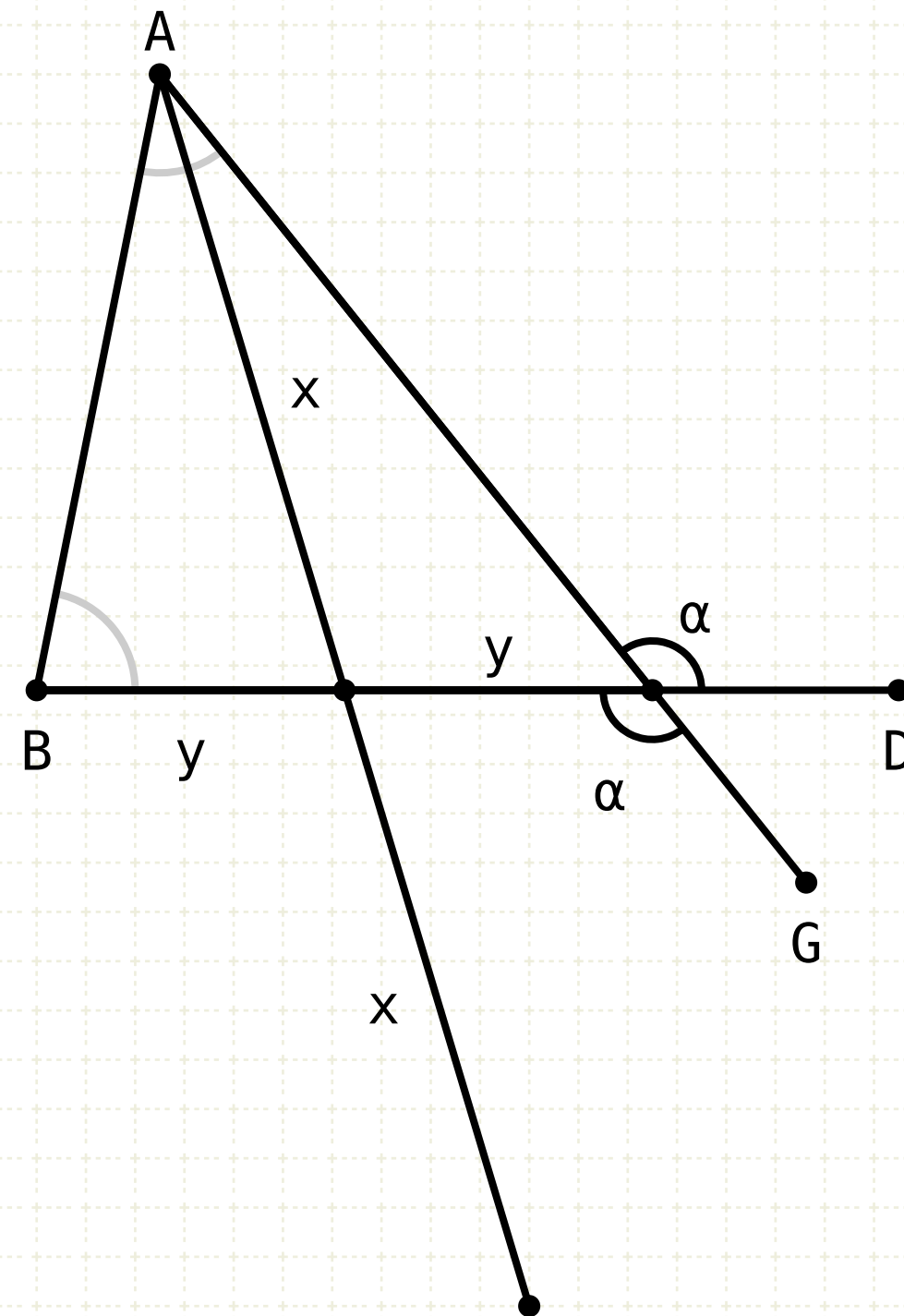
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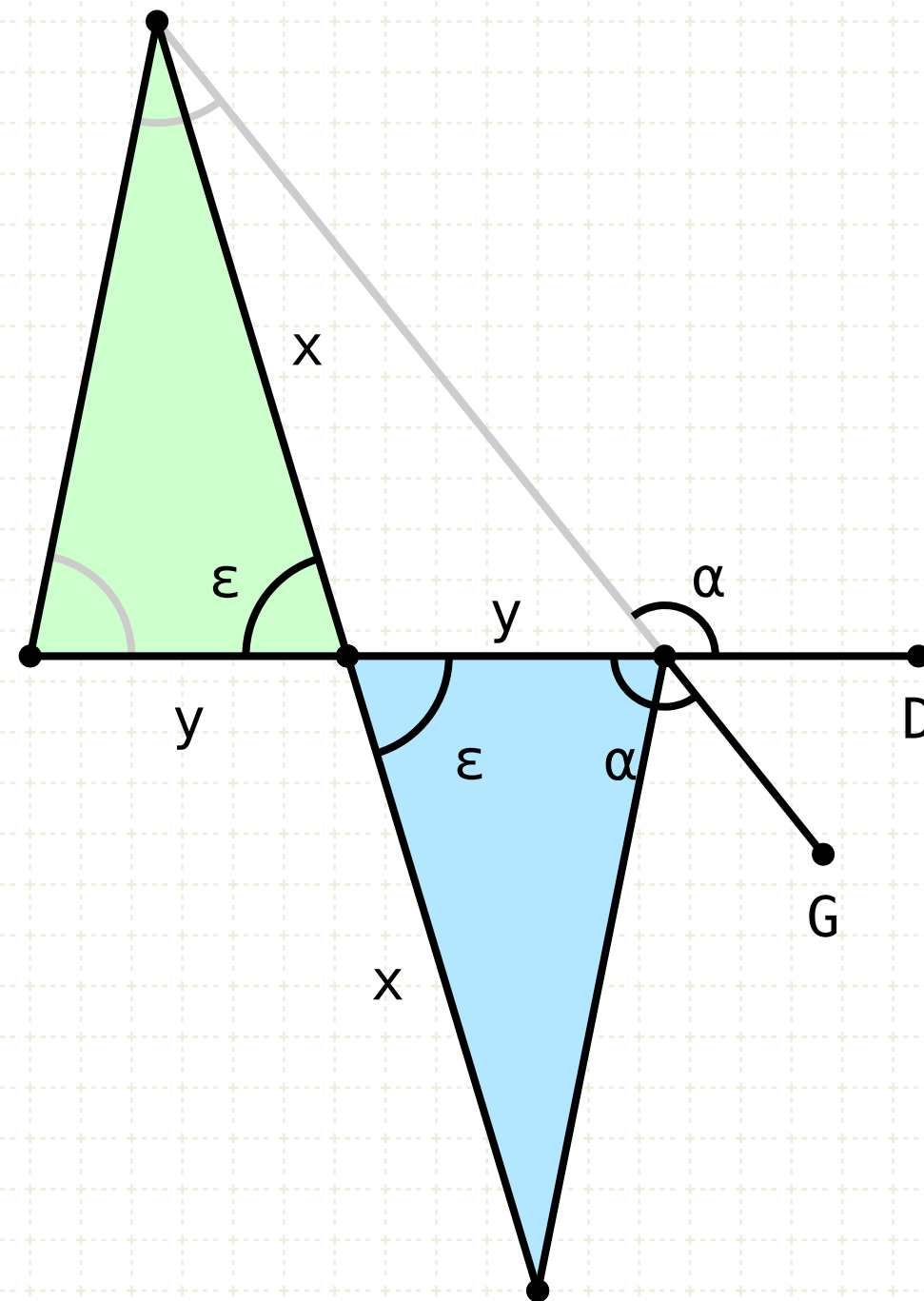
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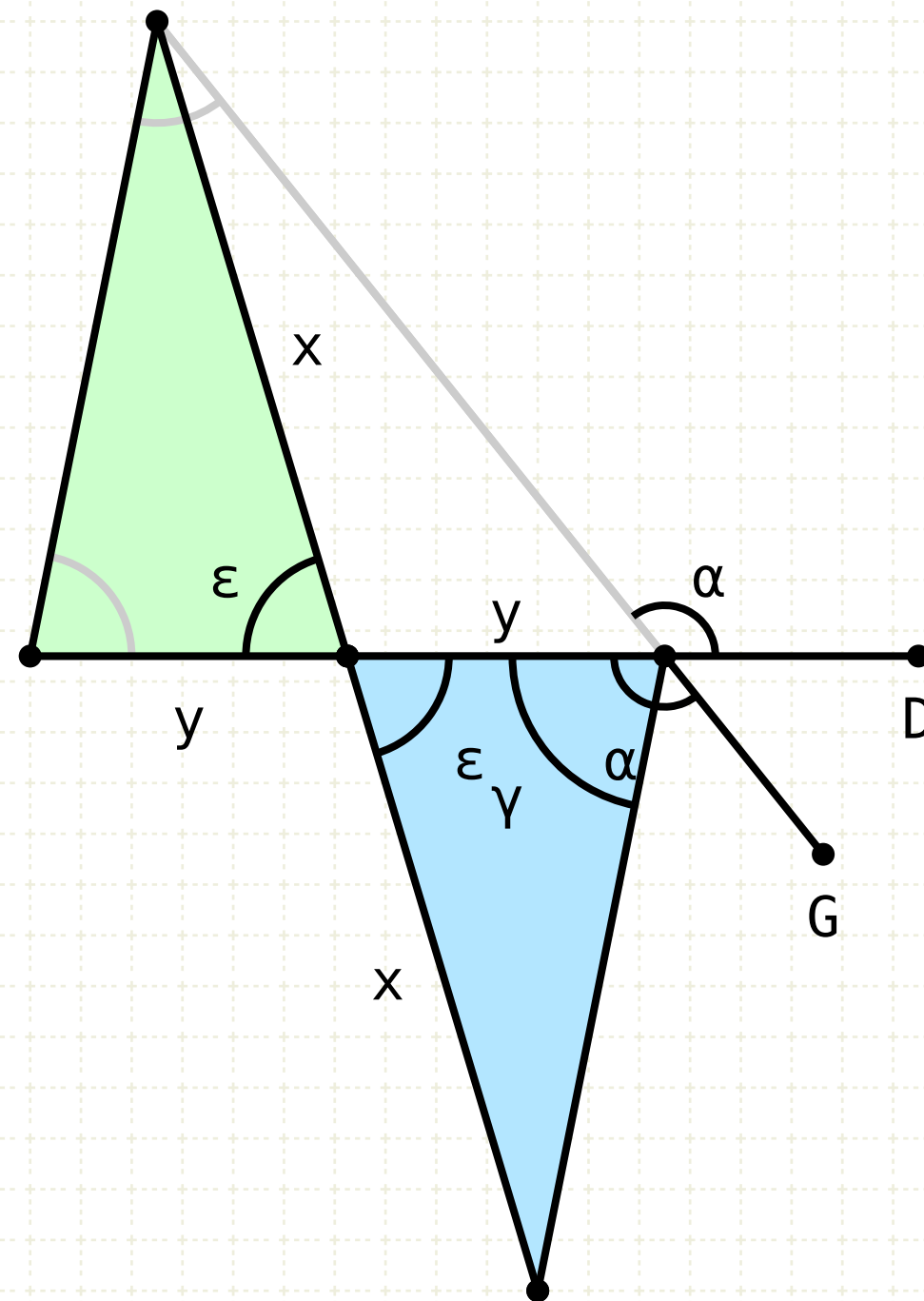
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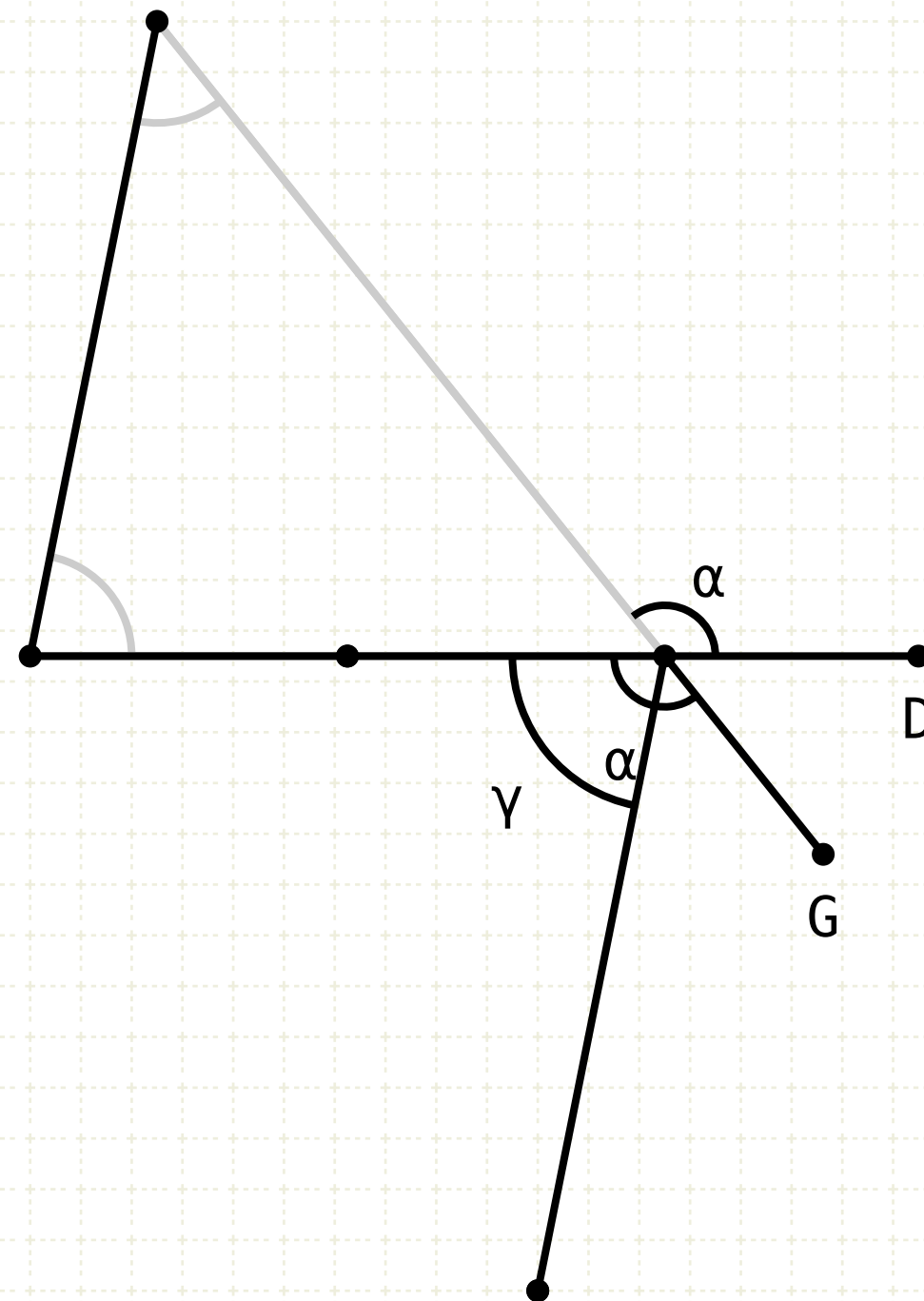
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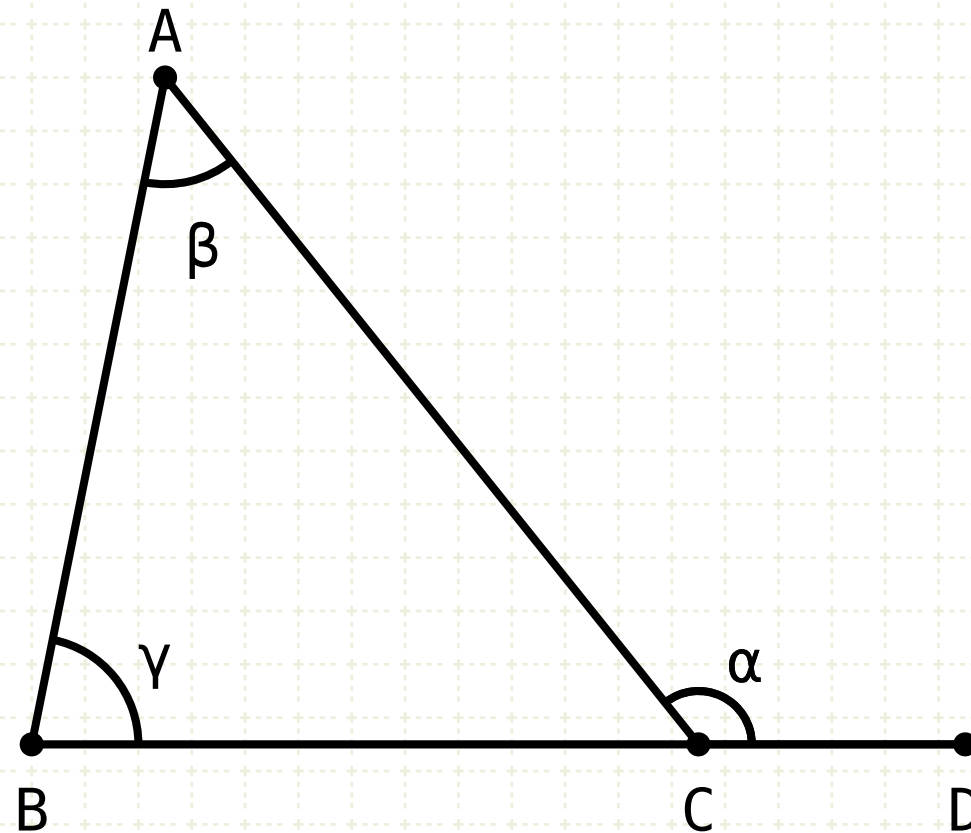
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