Euclid's Elements

Book VII

Definitions:

1 A unit is that by virtue of which each of the things that exist is called one

2 A number is a multitude composed of units. (not one)

3 A number is part of a number, the less of the greater, when it measures the greater

11 A prime number is that which is measured by a unit alone.

12 Numbers prime to one another are those which are measured by a unit alone as a common measure

15 A number is said to multiply a number when that which is multiplied is added to itself as many times as there are units in the other, and thus some number is produced.

20 Numbers are proportional when the first is the same multiple, or the same part, or the same parts, of the second that the third is of the fourth.

As long as algebra and geometry have been separated, their progress have been slow and their uses limited; but when these two sciences have been united, they have lent each mutual forces, and have marched together towards perfection.

Joseph-Louis Lagrange
(1736 to 1813)

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1 Determine if two numbers are relatively prime

2 Find the greatest common divisor for two numbers

3 Find the largest common divisor for three numbers

4 Given two natural numbers, A and B, either B is part of A, or there exists a natural number (a part) that can measure both A and B

5 If B = (1/q)·A and D = (1/q)·C, then (B+D) = (1/q)·(A+C)

6 If B = (p/q)·A and D = (p/q)·C, then (B+D) = (p/q)·(A+C)

7 If B = A/q and D = C/q, B>D, then (B-D) = (A-C)/q

8 If B = (p/q)·A and D = (p/q)·C, B>D, then (B-D) = (p/q)·(A-C)

9 If B = (1/q)·A and D = (1/q)·C, and If B = (r/s)·D, then A = (r/s)·C

10 If B = (p/q)·A and D = (p/q)·C, and If B = (r/s)·D, then A = (r/s)·C

11 If A:B = C:D, then (A-C):(B-D) = A:B

12 If A:B = C:D, then (A+C):(B+C) = A:B

13 If A:B = C:D, then A:C = B:D

14 If A:B = D:E and B:C = E:F, then A:C = D:F

15 If B = i·1 and E = i·D, and if D = j·1 then E = j·B

16 A × B = B × A

17 If D = A × B and E = A × C then D:E = B:C

18 If D = B × A and E = C × A then D:E = B:C

19 If A:B = C:D then A × D = B × C
If A × D = B × C then A:B = C:D

20 Given the ratio A:B and C,D are the smallest numbers such that A:B = C:D then A = n·C and B = n·D

21 If A,B are relatively prime, then A,B are the smallest whole numbers that can be used to describe the ratio A:B

22 If A,B are the smallest whole numbers that can be used to describe the ratio A:B, then A,B are relatively prime

23 If A,B are relatively prime and if A = n·C, then B,C are relatively prime

24 If A,C are relatively prime and B,C are relatively prime then the A × B is relatively prime to C

25 If A,B are relatively prime then A^2,B are relatively prime

26 If A is relatively prime to C and D, and if B is also relatively prime to C and D, then A × B is relatively prime to C × D

27 If A,B are relatively prime, then A^2,B^2 are relatively prime, and A^3,B^3 are relatively prime, and so on

Table of Contents, Chapter 7

28 If A,B are relatively prime, then A,(A+B) are relatively prime

29 If A is prime, and B ≠ n·A, then A,B are relatively prime

30 If C = A×B and C = i·D where D is prime, then either A = j·D or B = j·D

31 If A = B×C, then A = j·D where D is prime

32 If A is a number then it is either prime, or A = j·D where D is prime

33 Find the smallest numbers X,Y,Z where the ratio X:Y:Z is equal to the given ratio A:B:C

34 Find the lowest common denominator of 2 numbers

35 If E is the lowest common denominator of A,B, and if C = n ·A = m·B, then C = i·E

36 Find the least common multiple of 3 numbers

37 If A = p·B, then A = q·C where C = p·1

38 If A = (1/c)·B and C = c·1 then A = n·C

39 Find the smallest number that has the fractions 1/a, 1/b, 1/c

Proposition 7 of Book VII

If a number be that part of a number, which a number subtracted is of a number subracted, the remainder will also be the same part of the remainder that the whole is of the whole

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$$b = a/q$$

$$d = c/q$$

$$\rightarrow (b-d) = (a-c)/q$$

In other words

If b is the same fraction of a as d is to c, then the result of d subtracted from b will also be the same fraction of the result of c subtracted from a

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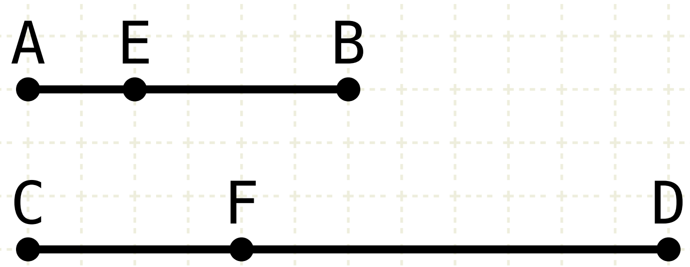

$AB = (1/q)CD$

$AE = (1/q)CF$

In other words

If b is the same fraction of a as d is to c, then the result of d subtracted from b will also be the same fraction of the result of c subtracted from a

Proof

Let the number AB be a part of CD, and let AE be the same part of CF

And let AE be subtracted from AB, and CF be subtracted from CF

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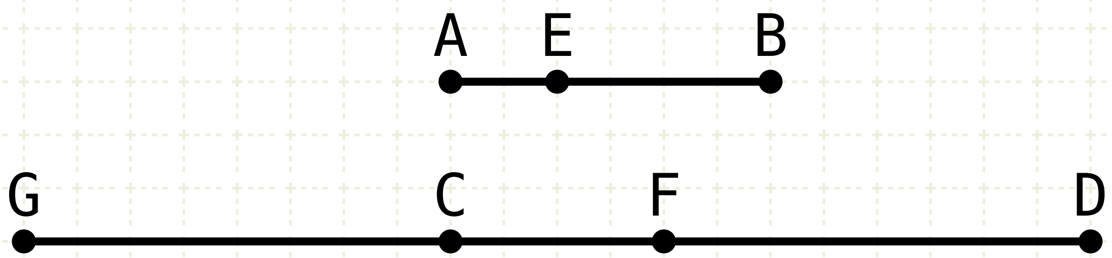

AB = (1/q)CD

AE = (1/q)CF

EB = (1/q)GC

In other words

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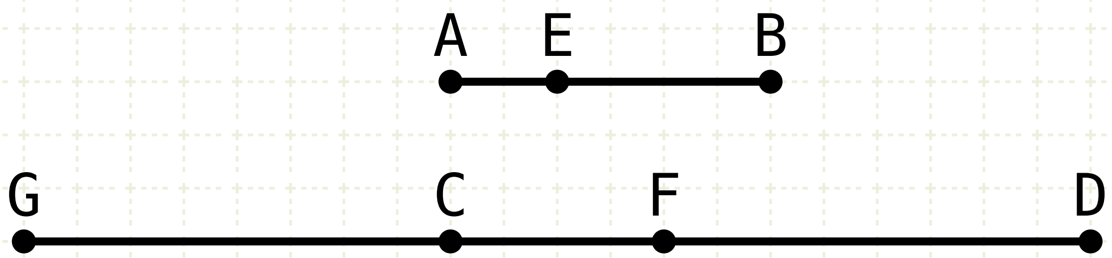

$AB = (1/q)CD$

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$AE + EB = (1/q)(GC + CF) = (1/q)GF$

$AB = (1/q)GF$

In other words

If b is the same fraction of a as d is to c, then the result of d subtracted from b will also be the same fraction of the result of c subtracted from a

Proof

Let the number AB be a part of CD, and let AE be the same part of CF

And let AE be subtracted from AB, and CF be subtracted from CF

Let EB be the same part of GC that AE is to CF

Since EB is the same part as CG as AE is of CF, the sum AB will be the same part of the sum GF (VII·5)

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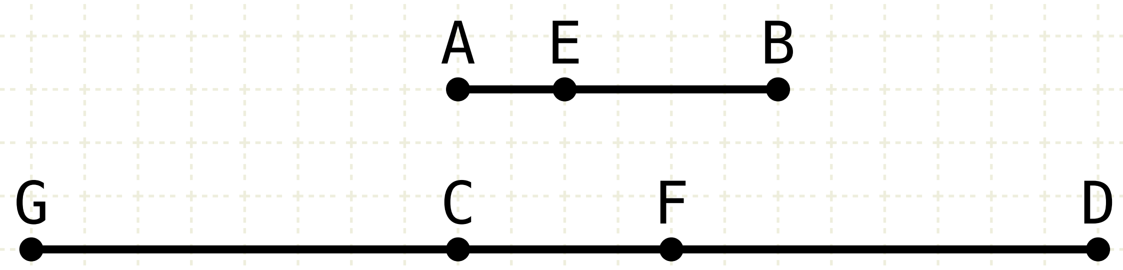

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Proof

Let the number AB be a part of CD, and let AE be the same part of CF

And let AE be subtracted from AB, and CF be subtracted from CF

Let EB be the same part of GC that AE is to CF

Since EB is the same part as CG as AE is of CF, the sum AB will be the same part of the sum GF (VII·5)

Whatever fraction AB is of GF, it is the same fraction of CD;

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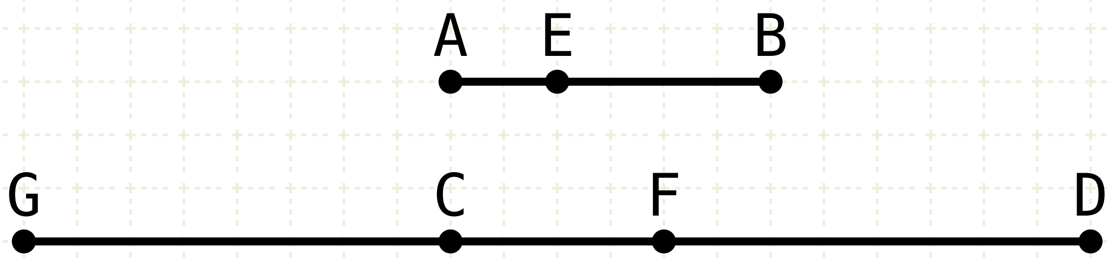

AB = (1/q)CD

AE = (1/q)CF

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AE + EB = (1/q)(GC + CF) = (1/q)GF

AB = (1/q)GF

GF = CD

In other words

If b is the same fraction of a as d is to c, then the result of d subtracted from b will also be the same fraction of the result of c subtracted from a

Proof

Let the number AB be a part of CD, and let AE be the same part of CF

And let AE be subtracted from AB, and CF be subtracted from CF

Let EB be the same part of GC that AE is to CF

Since EB is the same part as CG as AE is of CF, the sum AB will be the same part of the sum GF (VII·5)

Whatever fraction AB is of GF, it is the same fraction of CD;

Therefore GF is equal to CD

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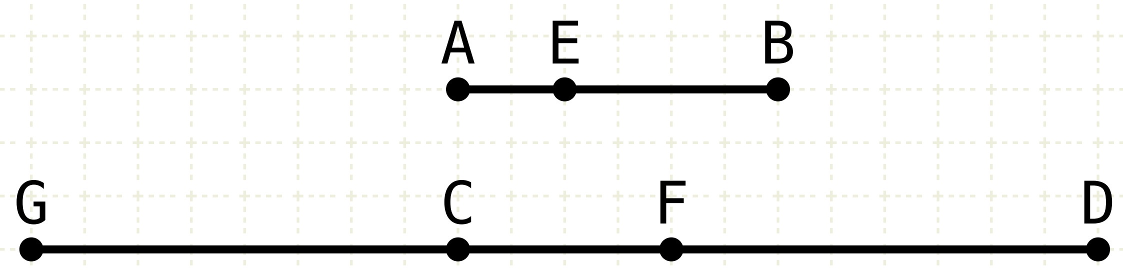

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$EB = (1/q)GC$

$AE + EB = (1/q)(GC + CF) = (1/q)GF$

$AB = (1/q)GF$

$GF = CD$

$GC = FD$

In other words

If b is the same fraction of a as d is to c, then the result of d subtracted from b will also be the same fraction of the result of c subtracted from a

Proof

Let the number AB be a part of CD, and let AE be the same part of CF

And let AE be subtracted from AB, and CF be subtracted from CF

Let EB be the same part of GC that AE is to CF

Since EB is the same part as CG as AE is of CF, the sum AB will be the same part of the sum GF (VII·5)

Whatever fraction AB is of GF, it is the same fraction of CD;

Therefore GF is equal to CD

Subtract CF from GF and FD, and the remainders are equal

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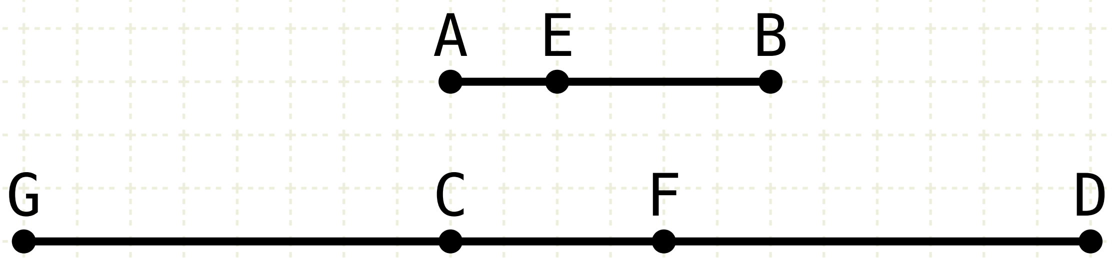

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$AB = (1/q)GF$

$GF = CD$

$GC = FD$

$EB = (1/q)FD$

In other words

If b is the same fraction of a as d is to c, then the result of d subtracted from b will also be the same fraction of the result of c subtracted from a

Proof

Let the number AB be a part of CD, and let AE be the same part of CF

And let AE be subtracted from AB, and CF be subtracted from CF

Let EB be the same part of GC that AE is to CF

Since EB is the same part as CG as AE is of CF, the sum AB will be the same part of the sum GF (VII·5)

Whatever fraction AB is of GF, it is the same fraction of CD;

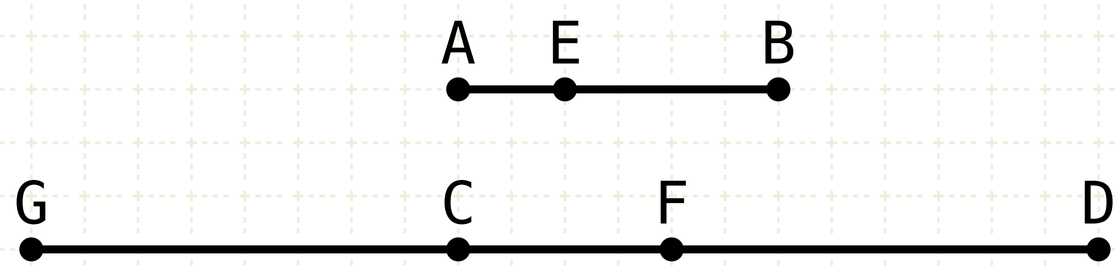
Therefore GF is equal to CD

Subtract CF from GF and FD, and the remainders are equal

Now, EB is the same part of GC that AE is of CF, and GC equals FD

Proposition 7 of Book VII

If a number be that part of a number, which a number subtracted is of a number subracted, the remainder will also be the same part of the remainder that the whole is of the whole



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$AB = (1/q)GF$

$GF = CD$

$GC = FD$

$EB = (1/q)FD$

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If b is the same fraction of a as d is to c, then the result of d subtracted from b will also be the same fraction of the result of c subtracted from a

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Let the number AB be a part of CD, and let AE be the same part of CF

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Let EB be the same part of GC that AE is to CF

Since EB is the same part as CG as AE is of CF, the sum AB will be the same part of the sum GF (VII·5)

Whatever fraction AB is of GF, it is the same fraction of CD;

Therefore GF is equal to CD

Subtract CF from GF and FD, and the remainders are equal

Now, EB is the same part of GC that AE is of CF, and GC equals FD

Therefore EB is the same part of FD that AE is of CF

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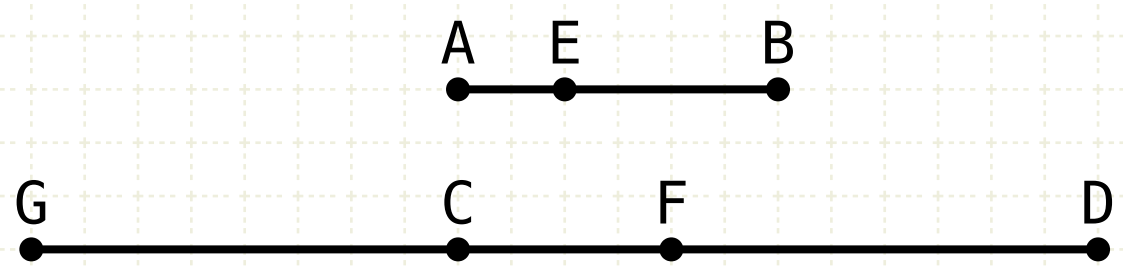

AB = (1/q)CD

AE = (1/q)CF

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AE + EB = (1/q)(GC + CF) = (1/q)GF

AB = (1/q)GF

GF = CD

GC = FD

EB = (1/q)FD

EB = (1/q)(CD - CF)

In other words

If b is the same fraction of a as d is to c, then the result of d subtracted from b will also be the same fraction of the result of c subtracted from a

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Let the number AB be a part of CD, and let AE be the same part of CF

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Let EB be the same part of GC that AE is to CF

Since EB is the same part as CG as AE is of CF, the sum AB will be the same part of the sum GF (VII·5)

Whatever fraction AB is of GF, it is the same fraction of CD;

Therefore GF is equal to CD

Subtract CF from GF and FD, and the remainders are equal

Now, EB is the same part of GC that AE is of CF, and GC equals FD

Therefore EB is the same part of FD that AE is of CF

FD is the remainder of CF subtracted from CD

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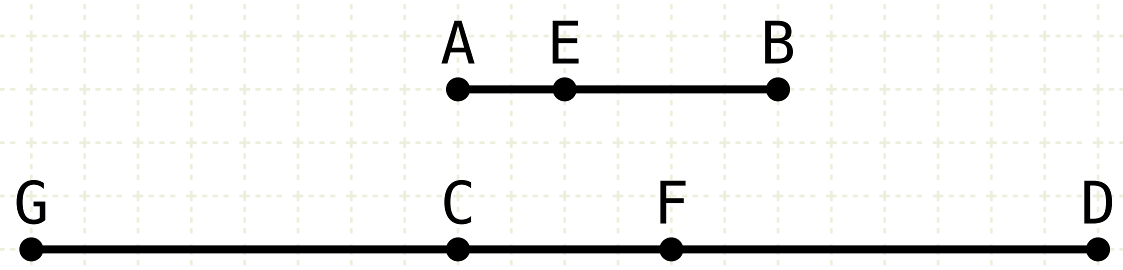

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AE + EB = (1/q)(GC + CF) = (1/q)GF

AB = (1/q)GF

GF = CD

GC = FD

EB = (1/q)FD

EB = (1/q)(CD - CF)

AB - AE = (1/q)(CD - CF)

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If b is the same fraction of a as d is to c, then the result of d subtracted from b will also be the same fraction of the result of c subtracted from a

Proof

Let the number AB be a part of CD, and let AE be the same part of CF

And let AE be subtracted from AB, and CF be subtracted from CF

Let EB be the same part of GC that AE is to CF

Since EB is the same part as CG as AE is of CF, the sum AB will be the same part of the sum GF (VII·5)

Whatever fraction AB is of GF, it is the same fraction of CD;

Therefore GF is equal to CD

Subtract CF from GF and FD, and the remainders are equal

Now, EB is the same part of GC that AE is of CF, and GC equals FD

Therefore EB is the same part of FD that AE is of CF

FD is the remainder of CF subtracted from CD

Finally, EB is the remainder of AE subtracted from AB

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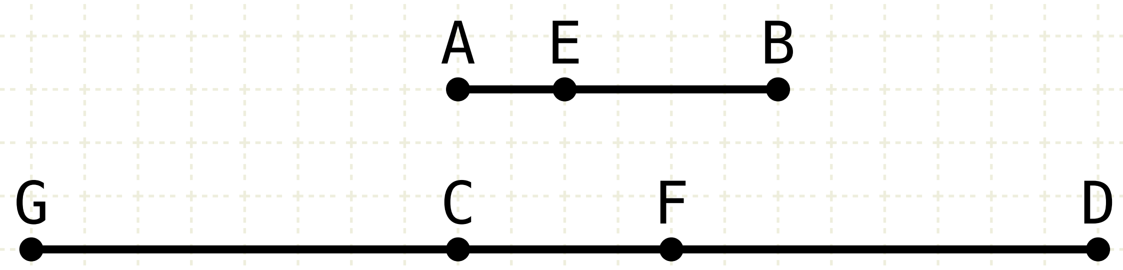

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AB = (1/q)GF
GF = CD
GC = FD
EB = (1/q)FD
EB = (1/q)(CD - CF)
AB - AE = (1/q)(CD - CF)

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If b is the same fraction of a as d is to c, then the result of d subtracted from b will also be the same fraction of the result of c subtracted from a

Proof

Let the number AB be a part of CD, and let AE be the same part of CF

And let AE be subtracted from AB, and CF be subtracted from CF

Let EB be the same part of GC that AE is to CF

Since EB is the same part as CG as AE is of CF, the sum AB will be the same part of the sum GF (VII·5)

Whatever fraction AB is of GF, it is the same fraction of CD;

Therefore GF is equal to CD

Subtract CF from GF and FD, and the remainders are equal

Now, EB is the same part of GC that AE is of CF, and GC equals FD

Therefore EB is the same part of FD that AE is of CF

FD is the remainder of CF subtracted from CD

Finally, EB is the remainder of AE subtracted from AB

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