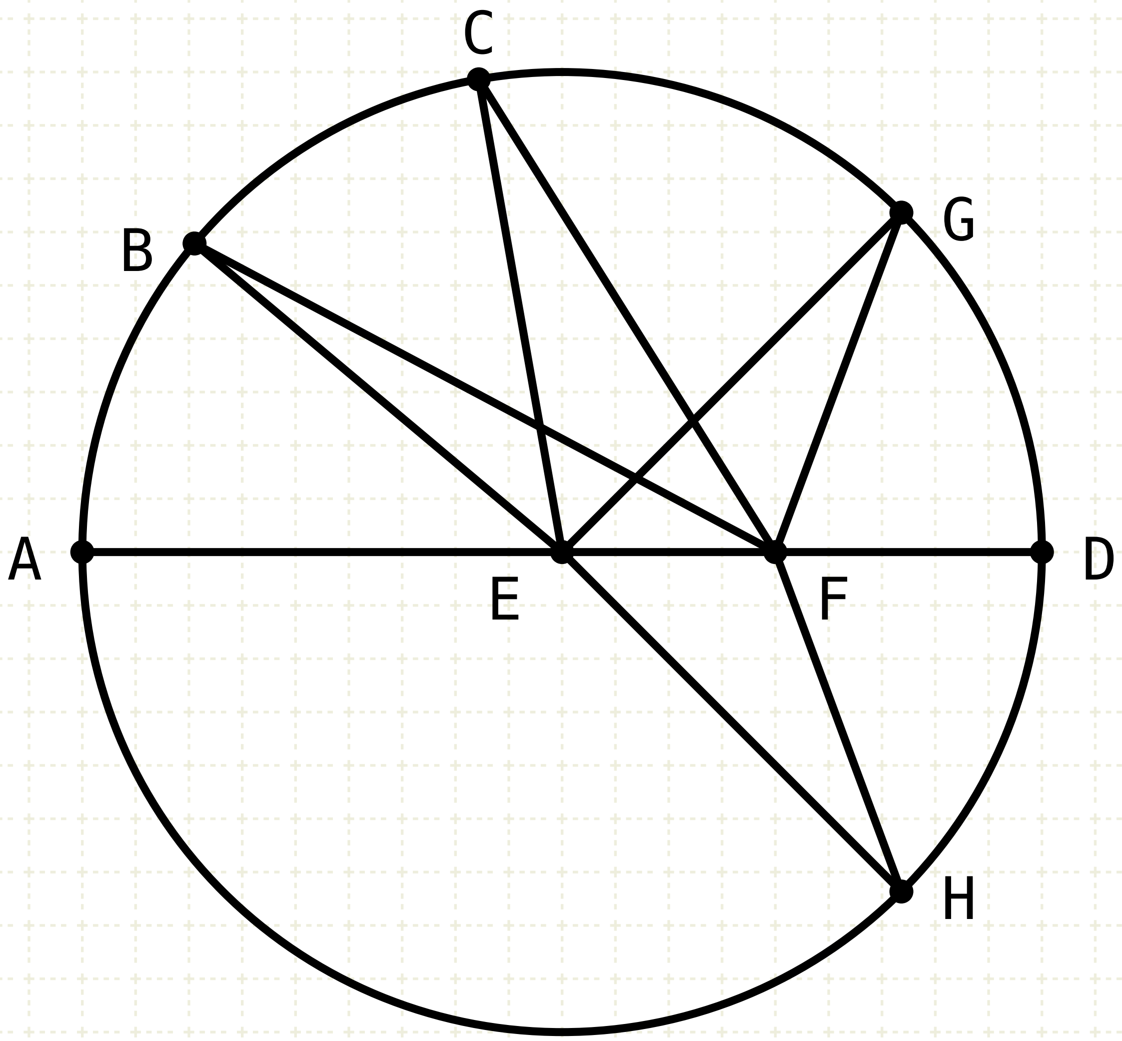
Euclid's Elements

Book III



A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)

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Proposition 32 of Book III

If a straight line touch a circle, and from the point of contact there be drawn across, in the circle, a straight line cutting the circle, the angles which it makes with the tangent will be equal to the angles in the alternate segments of the circle.

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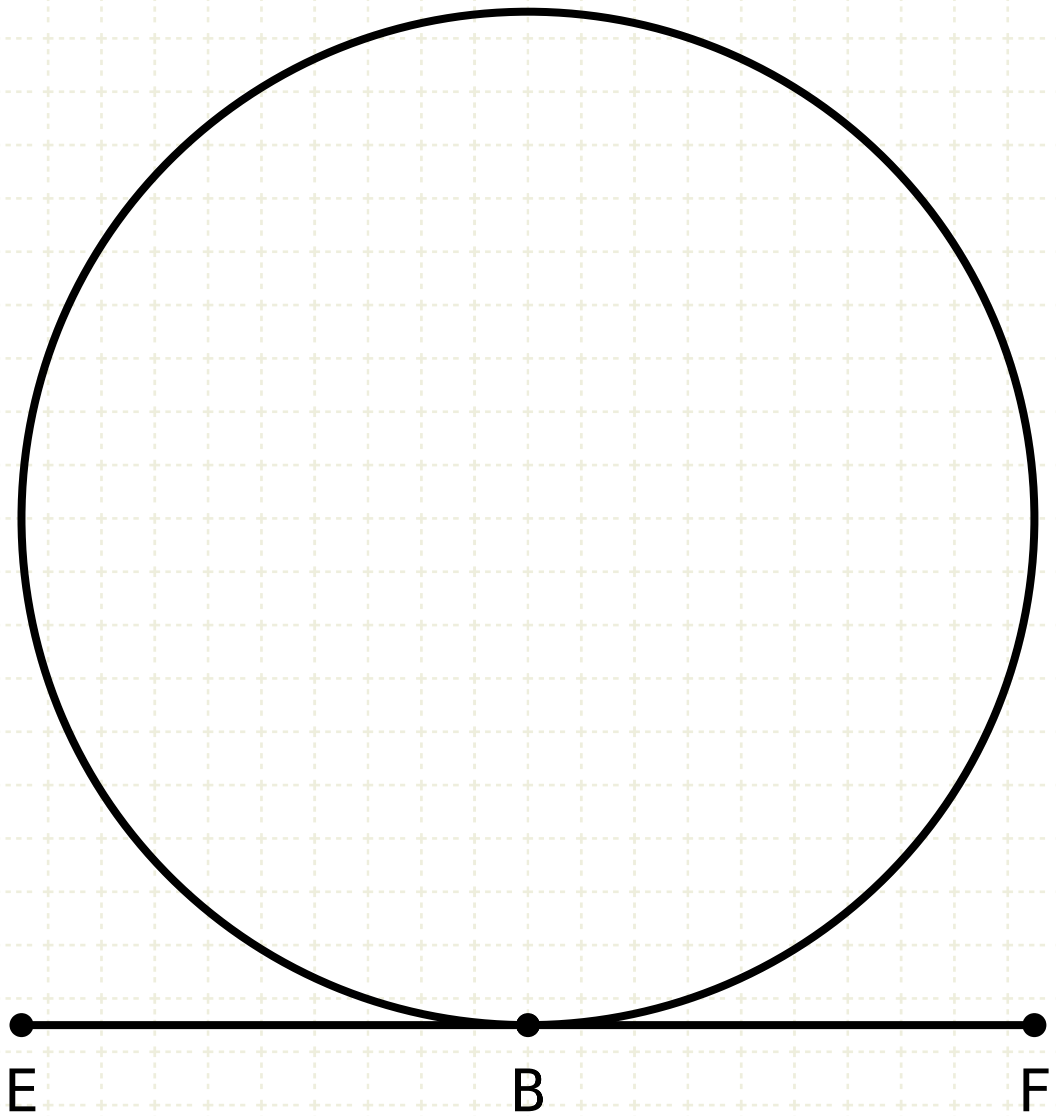

In other words

Let EF be a line that touches a circle at point B

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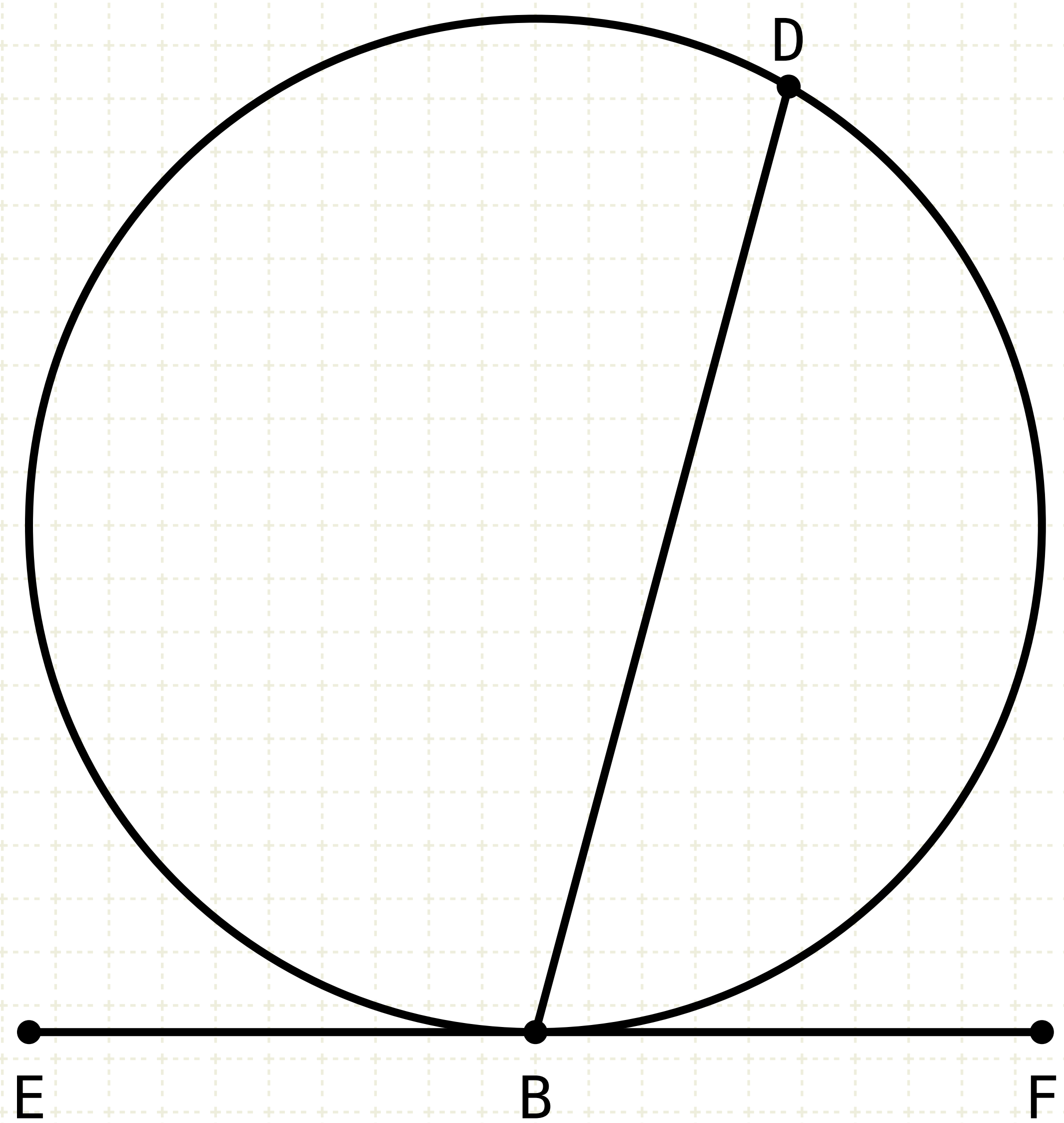

In other words

Let EF be a line that touches a circle at point B

Let an arbitrary line cut the circle from B to D

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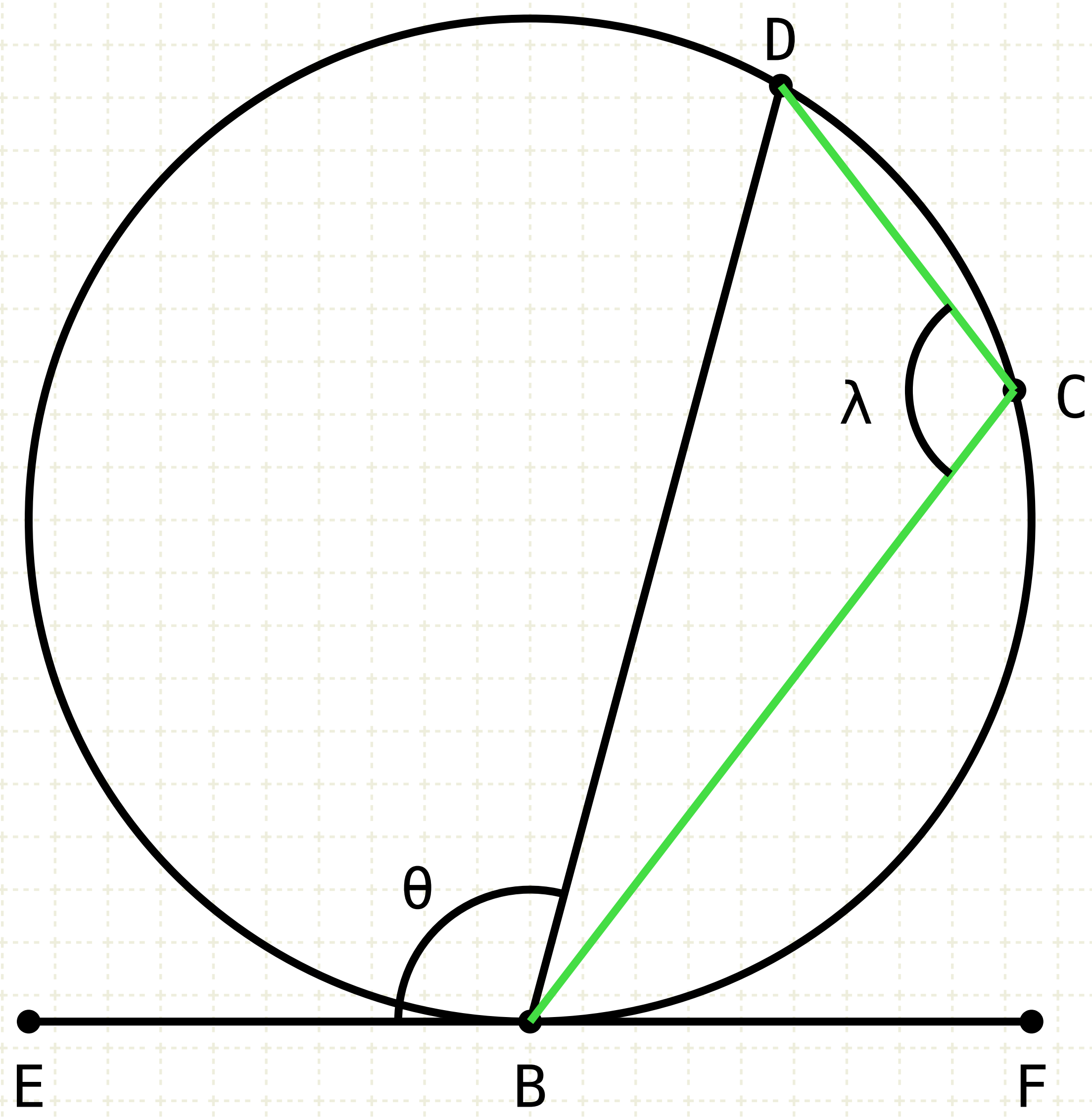

$\theta = \lambda$

In other words

Let EF be a line that touches a circle at point B

Let an arbitrary line cut the circle from B to D

Then, the angle EBD will be equal to the angle in the alternate segment DCB

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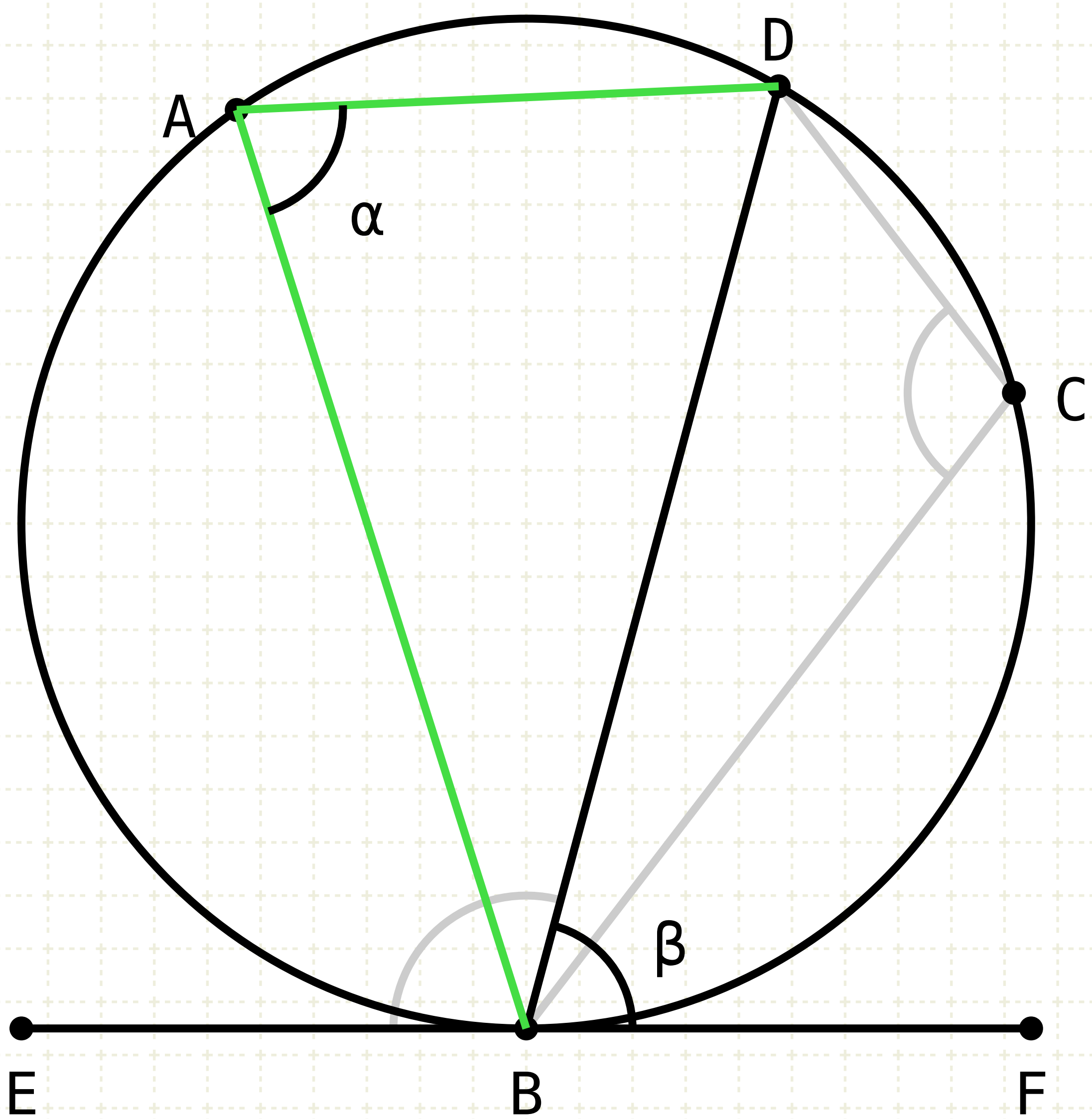


$\theta = \lambda$

$\alpha = \beta$

In other words

Let EF be a line that touches a circle at point B

Let an arbitrary line cut the circle from B to D

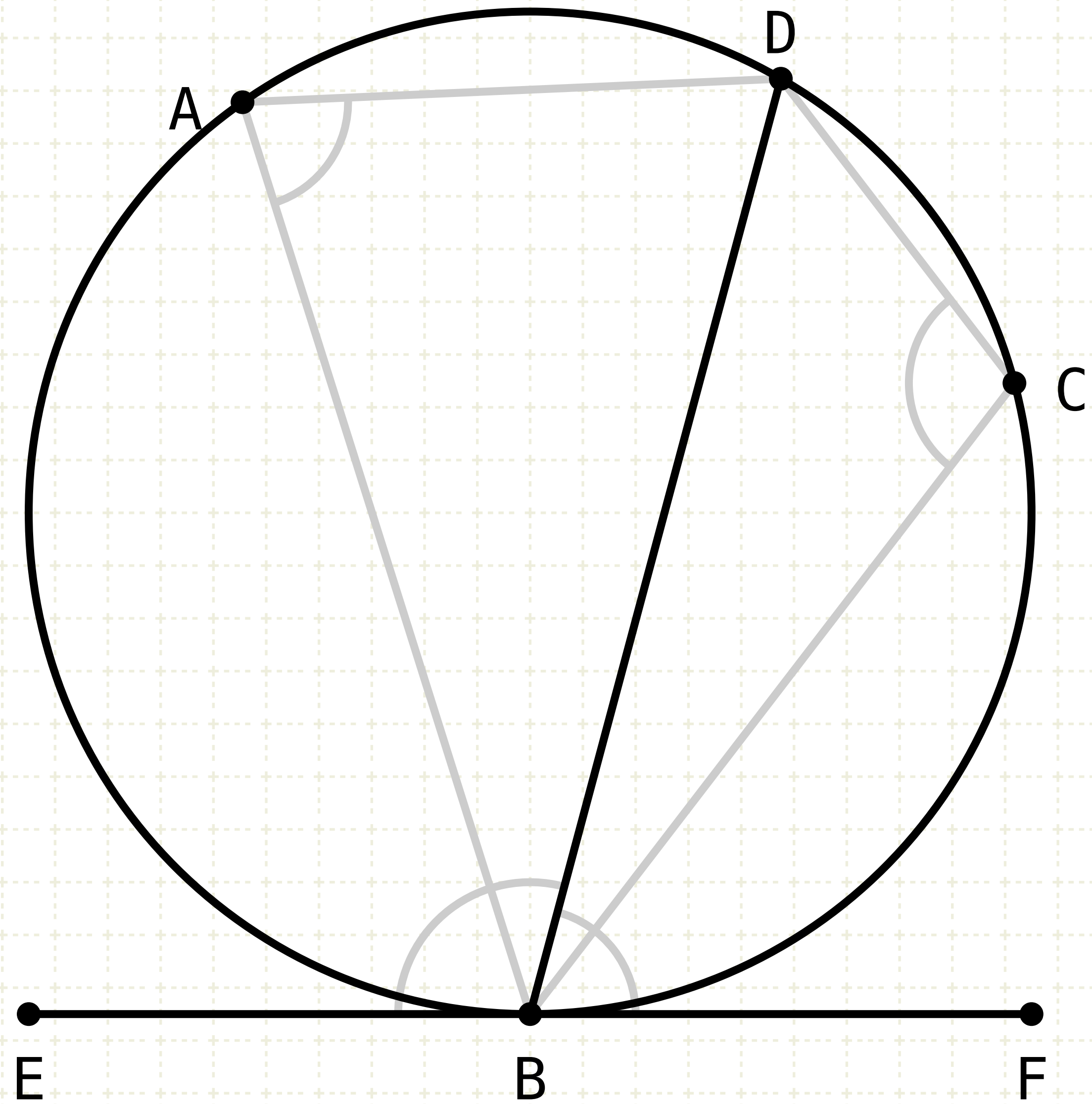
Then, the angle EBD will be equal to the angle in the alternate segment DCB

Conversely, angle DBF will be equal to the segment angle DAB

Proposition 32 of Book III

If a straight line touch a circle, and from the point of contact there be drawn across, in the circle, a straight line cutting the circle, the angles which it makes with the tangent will be equal to the angles in the alternate segments of the circle.

Proof



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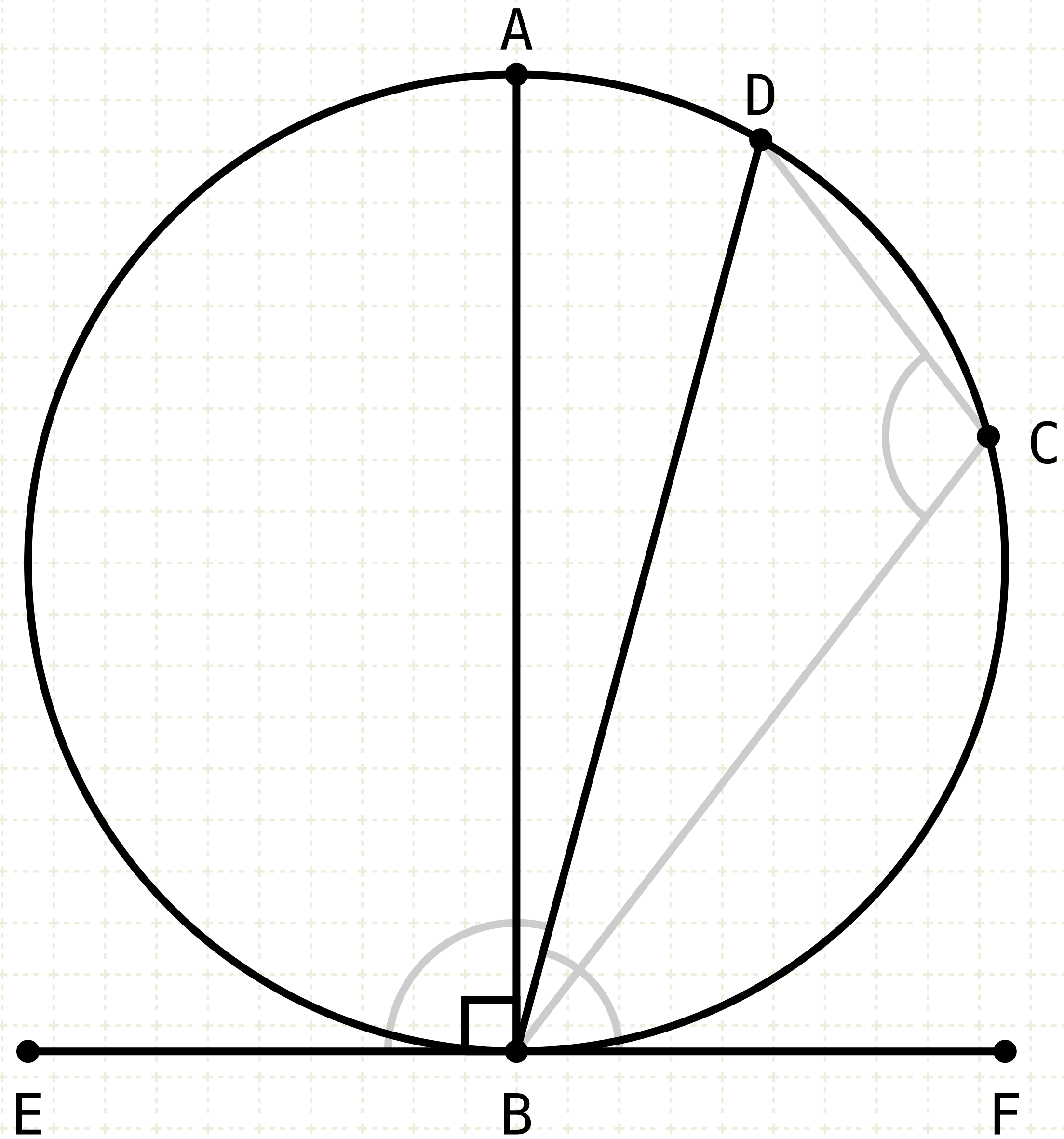

$\angle EBA = \angle ABF = ∟$

Proof

Draw the line BA such that it is perpendicular to EF

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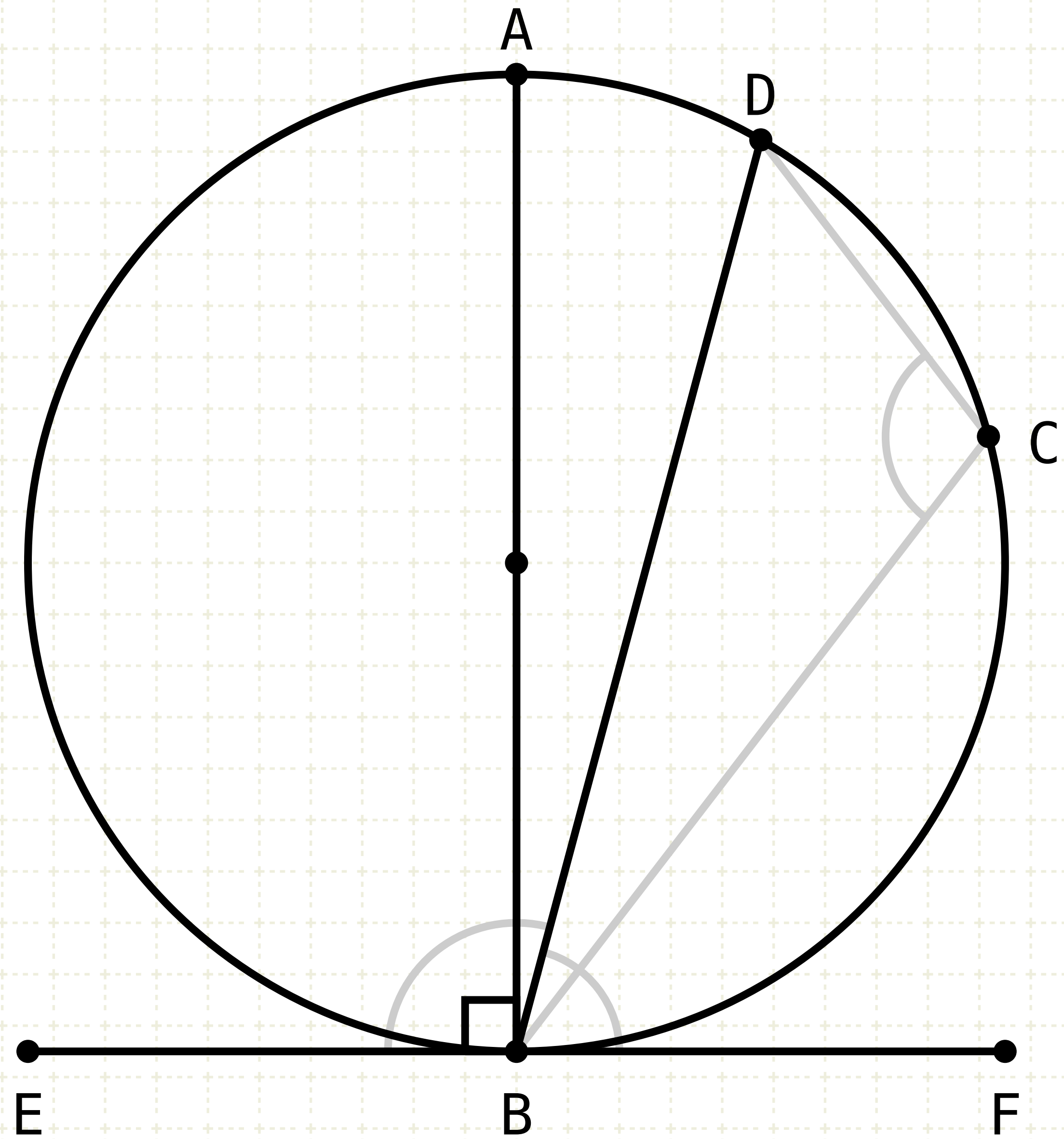

$\angle EBA = \angle ABF = ⦜$

$AB = \text{diameter}$

Proof

Draw the line BA such that it is perpendicular to EF

Since EF touches the circle at B, and BA is perpendicular to EF, BA is a diameter of the circle (III·19)

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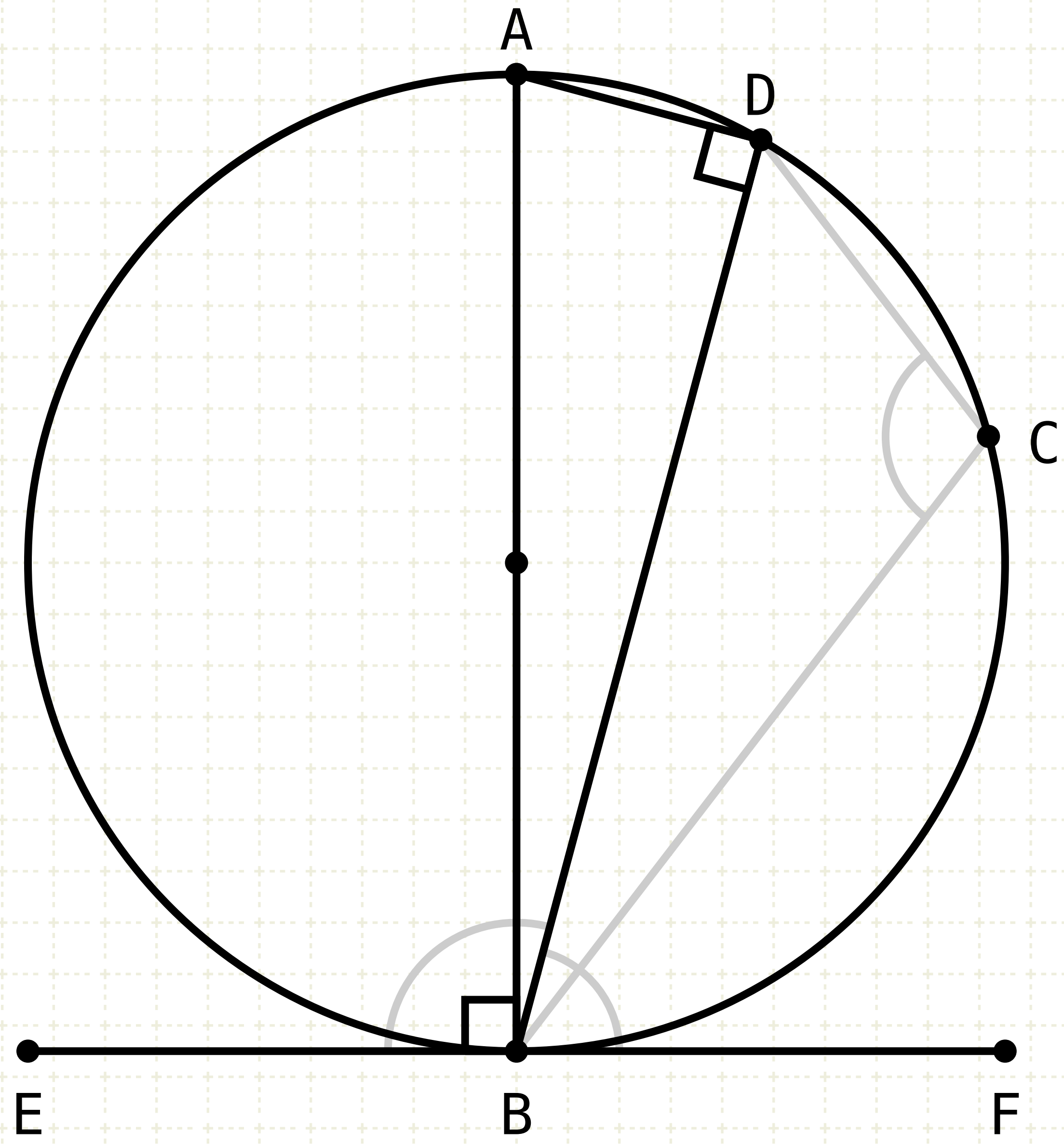

∠EBA = ∠ABF = ∟

AB = diameter

∠ADB = ∟

Proof

Draw the line BA such that it is perpendicular to EF

Since EF touches the circle at B, and BA is perpendicular to EF, BA is a diameter of the circle (III·19)

Thus ADB is a semicircle, and the angle ADB is right (III·31)

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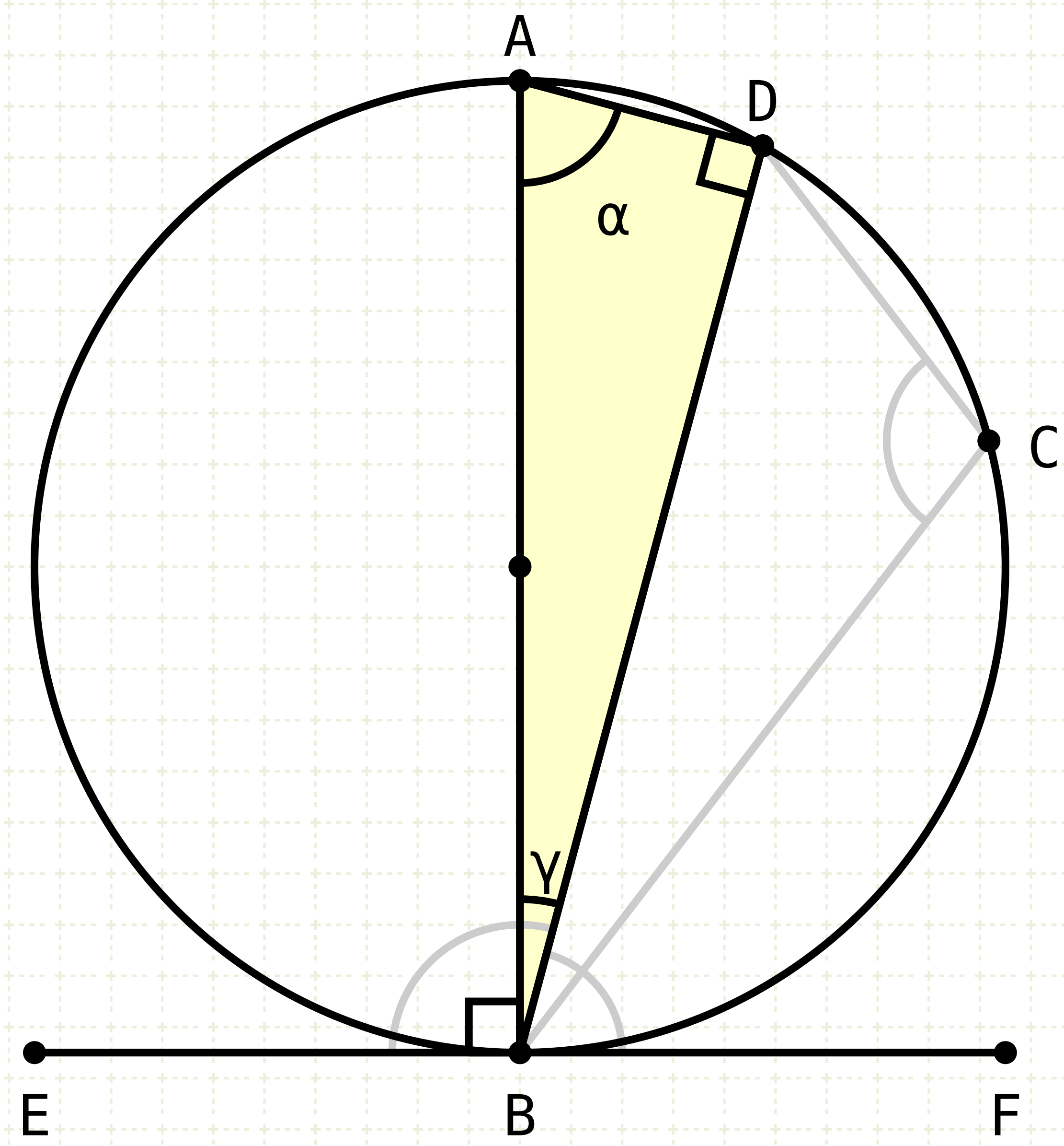

∠EBA = ∠ABF = ∟

AB = diameter

∠ADB = ∟

α + γ = ∟

Proof

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Since EF touches the circle at B, and BA is perpendicular to EF, BA is a diameter of the circle (III·19)

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The remaining angles of the triangle ABC (γ,α) sum to one right angle (I·32)

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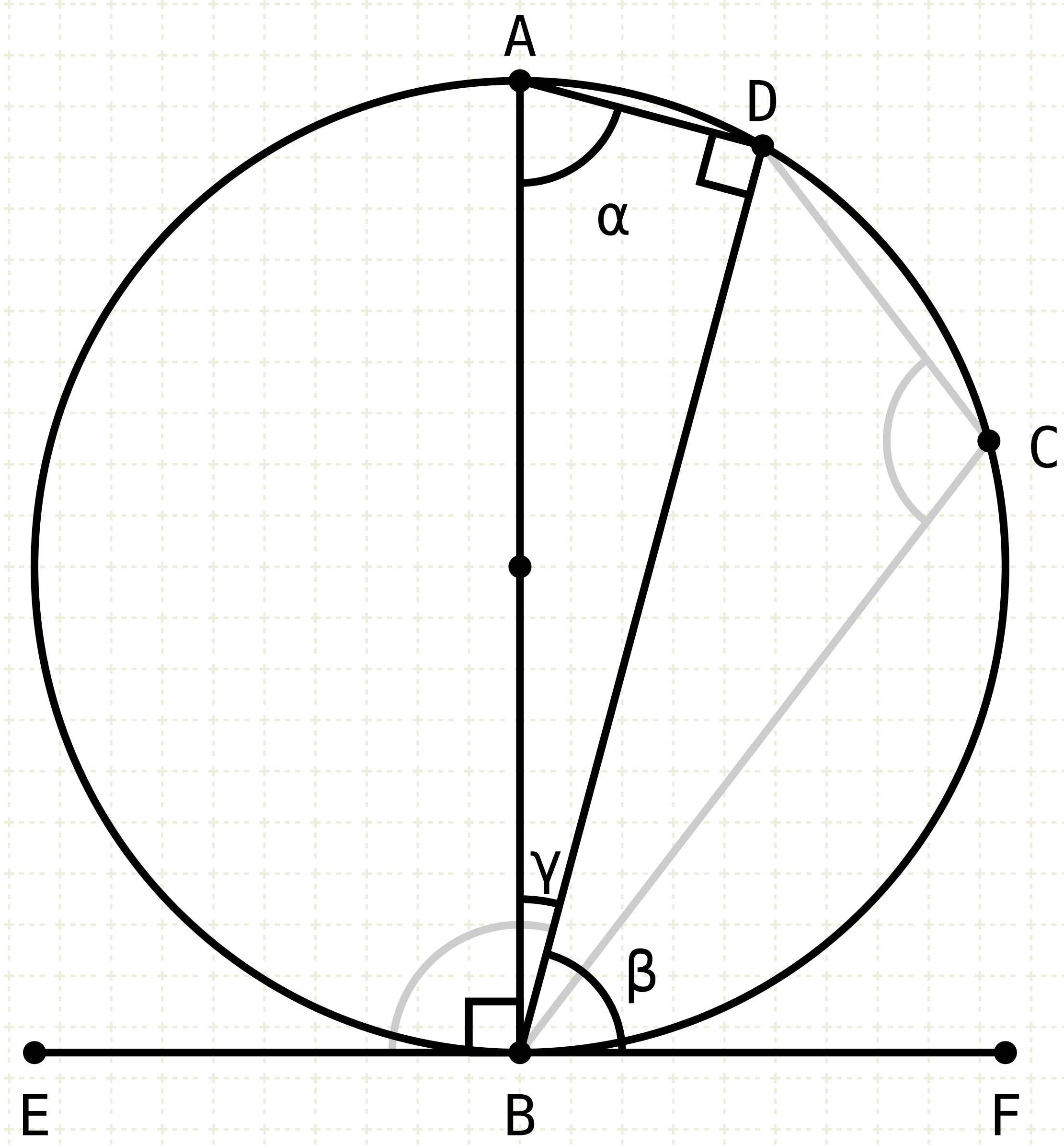

$\angle EBA = \angle ABF = ∟$

$AB = \text{diameter}$

$\angle ADB = ∟$

$\alpha + \gamma = ∟$

$\beta + \gamma = ∟$

$\alpha = \beta$

Proof

Draw the line BA such that it is perpendicular to EF

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Angle ABF is right, which is equal to γ plus β, therefore α is equal to β

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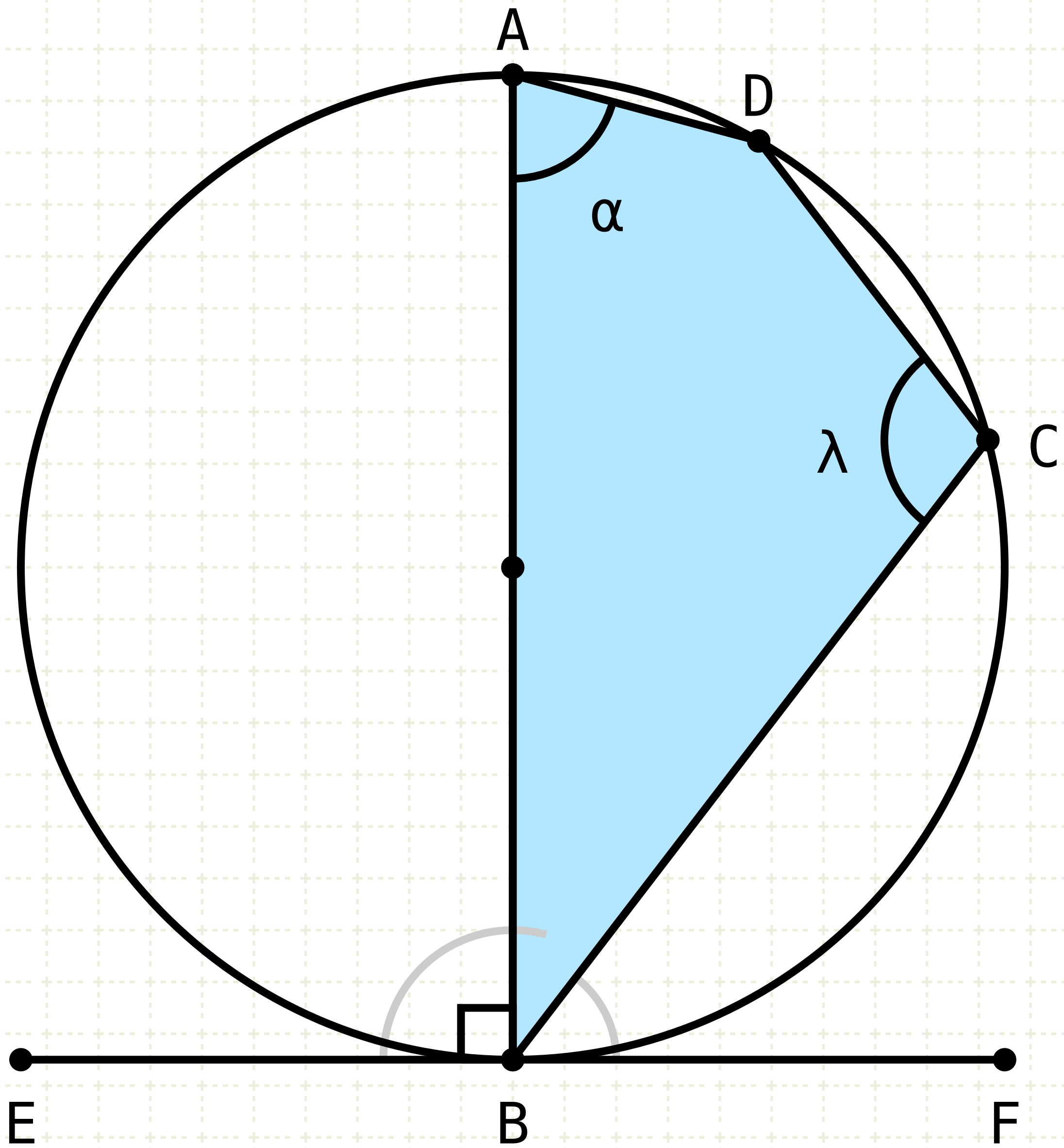

$\angle EBA = \angle ABF = \llcorner$

$AB = \text{diameter}$

$\angle ADB = \llcorner$

$\alpha + \gamma = \llcorner$

$\beta + \gamma = \llcorner$

$\alpha = \beta$

$\alpha + \lambda = 2\llcorner$

Proof

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In the quadilateral ABCD, the angles at A and C sum to two right angles (III·22)

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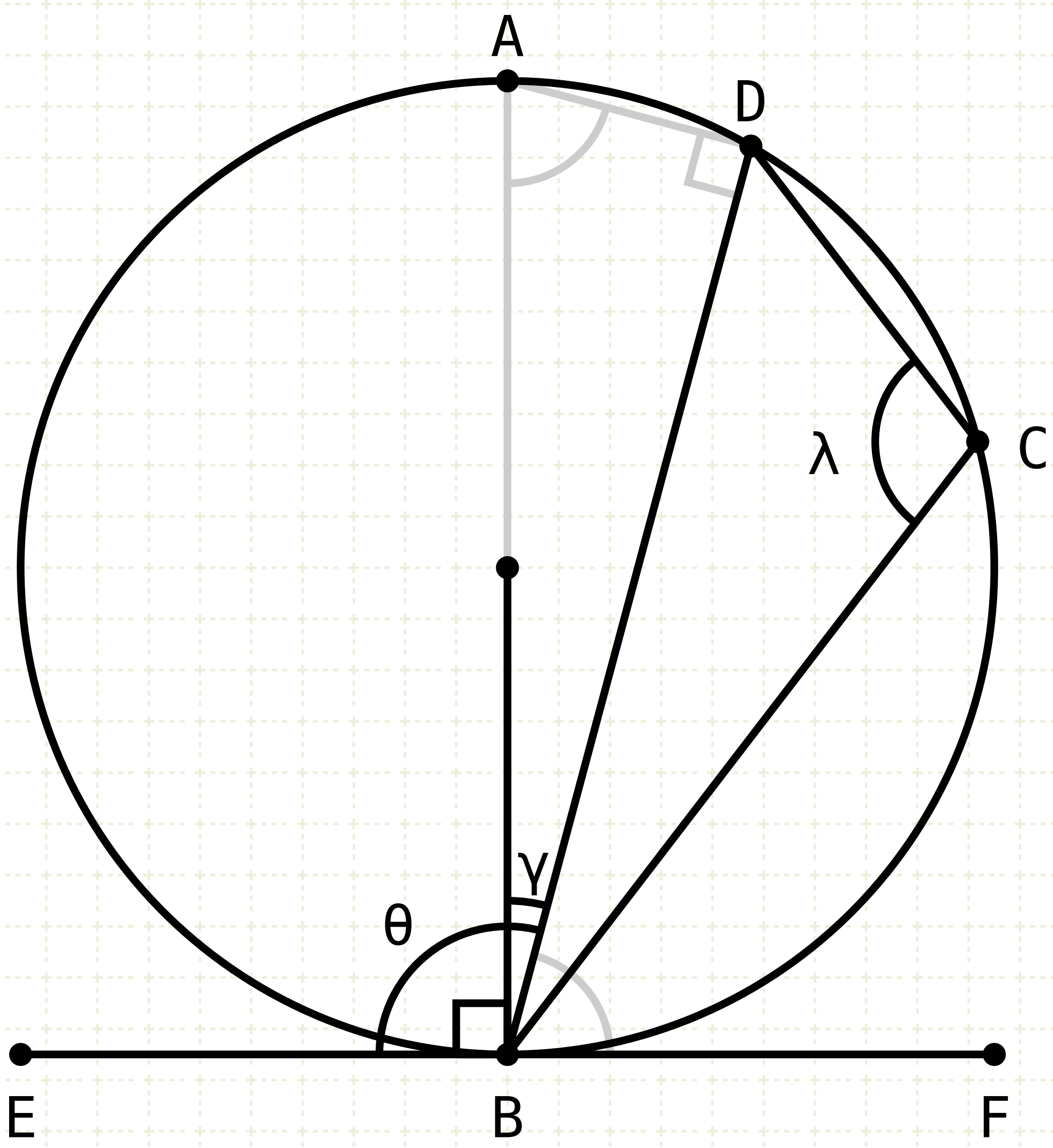

$\angle EBA = \angle ABF = \llcorner$

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$\angle ADB = \llcorner$

$\alpha + \gamma = \llcorner$

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$\alpha = \beta$

$\alpha + \lambda = 2\llcorner$

$\lambda = 2\llcorner - \alpha$

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If α,λ equals two right angles,

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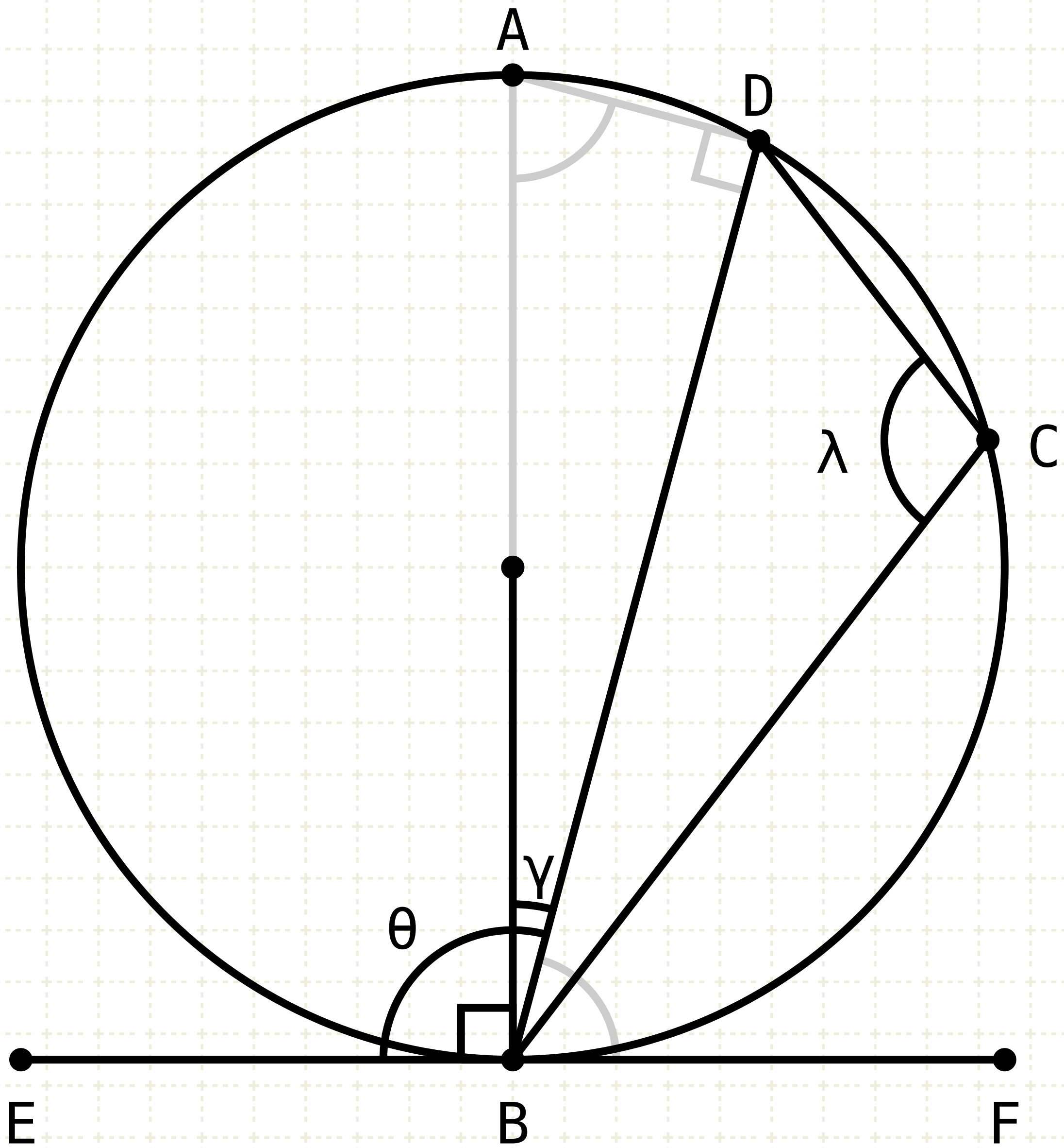

$\angle EBA = \angle ABF = ∟$
$AB = \text{diameter}$
$\angle ADB = ∟$
$\alpha + \gamma = ∟$
$\beta + \gamma = ∟$
$\alpha = \beta$

$\alpha + \lambda = 2∟$
$\lambda = 2∟ - \alpha$
$\alpha = ∟ - \gamma$

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If α,λ equals two right angles,

and α,γ equals one right angle,

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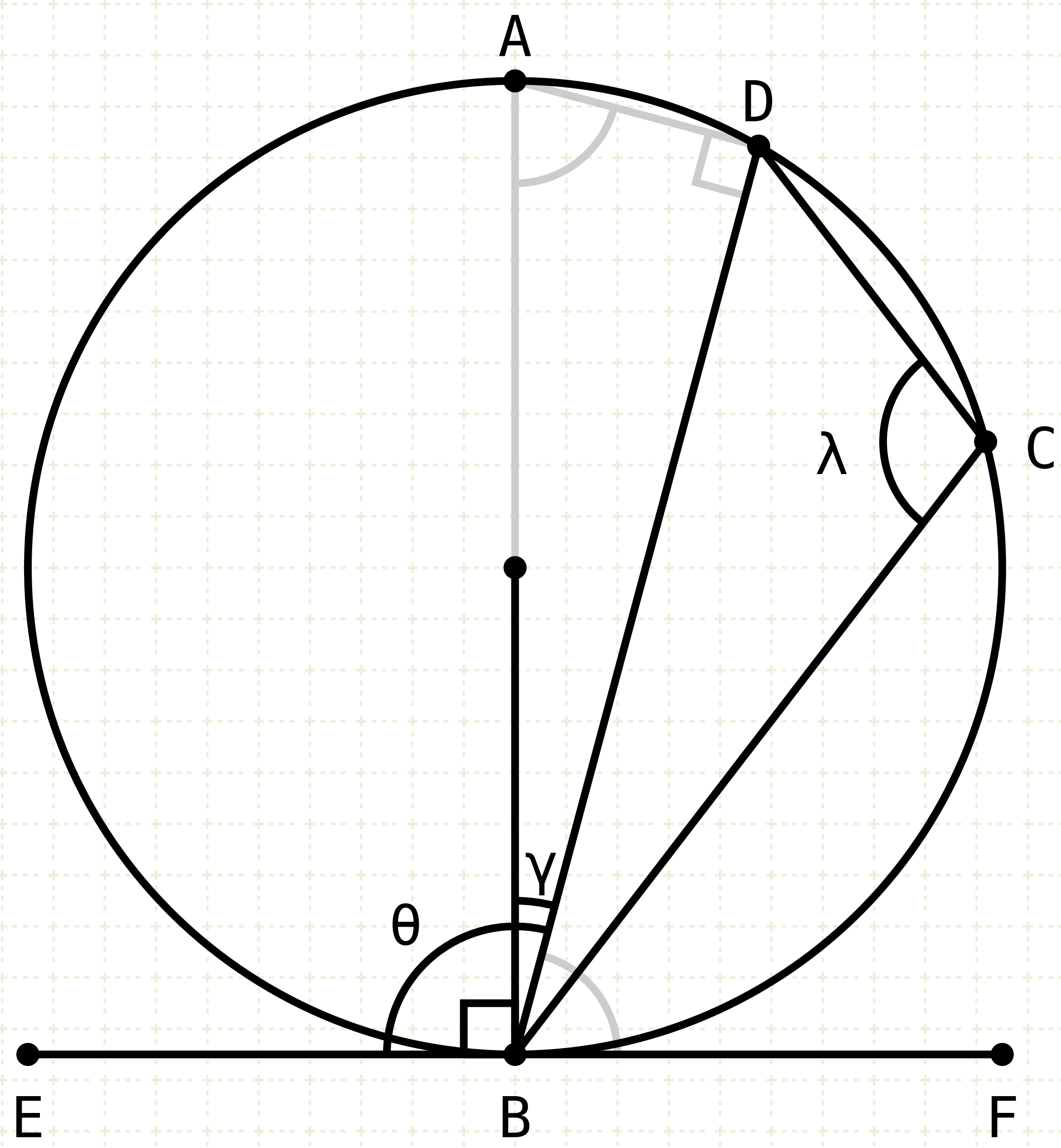

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$\angle ADB = \llcorner$

$\alpha + \gamma = \llcorner$

$\beta + \gamma = \llcorner$

$\alpha = \beta$

$\alpha + \lambda = 2\llcorner$

$\lambda = 2\llcorner - \alpha$

$\alpha = \llcorner - \gamma$

$\lambda = 2\llcorner - \llcorner + \gamma$

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then λ equals a right angle plus γ

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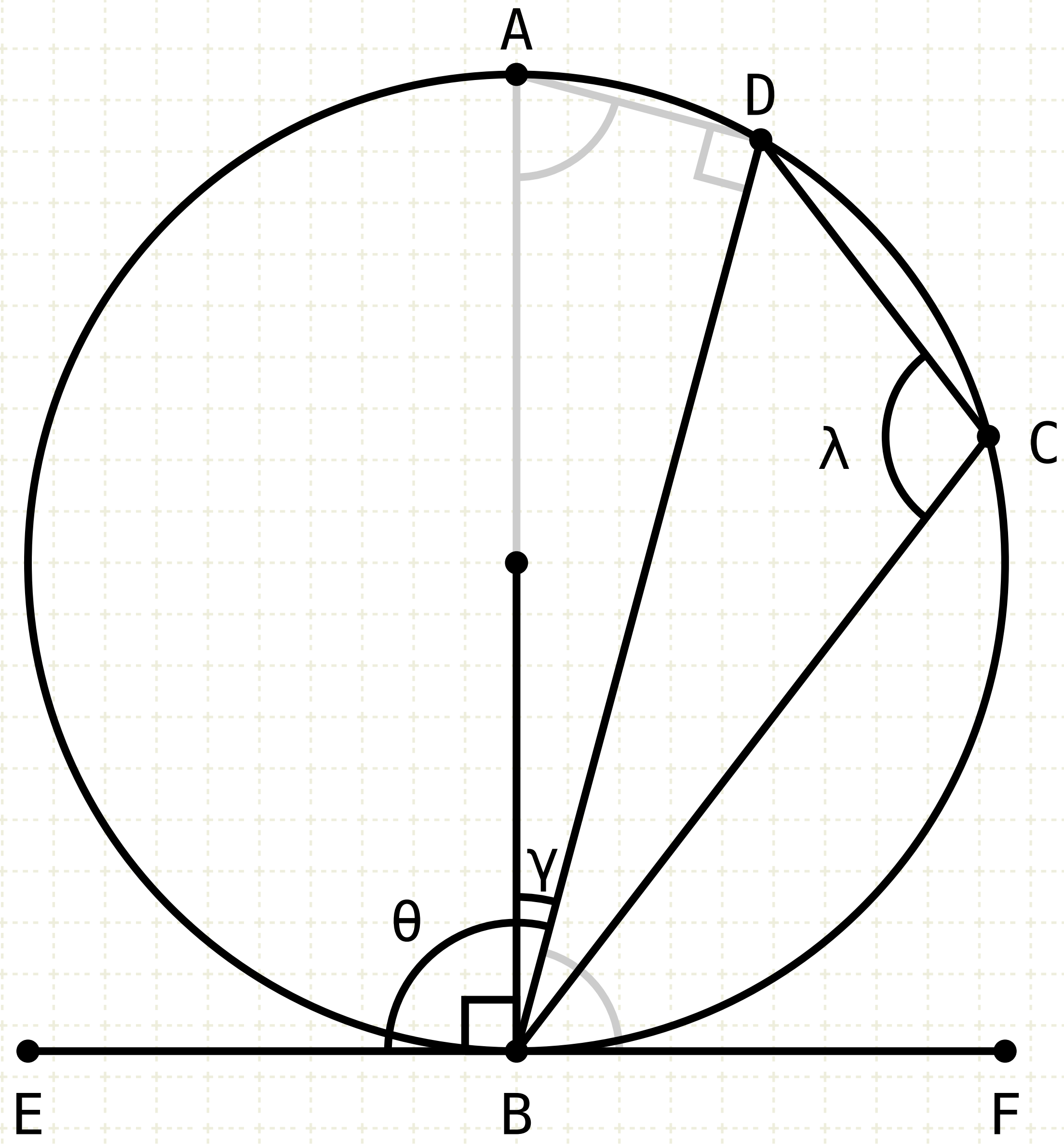

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$\alpha + \gamma = \llcorner$
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$\alpha + \lambda = 2\llcorner$
$\lambda = 2\llcorner - \alpha$
$\alpha = \llcorner - \gamma$
$\lambda = 2\llcorner - \llcorner + \gamma$
$\lambda = \llcorner + \gamma$
$\theta = \llcorner + \gamma$
$\therefore\ \theta = \lambda$

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then λ equals a right angle plus γ

But angle EBD (θ) equals one right angle plus γ, thus θ equals γ

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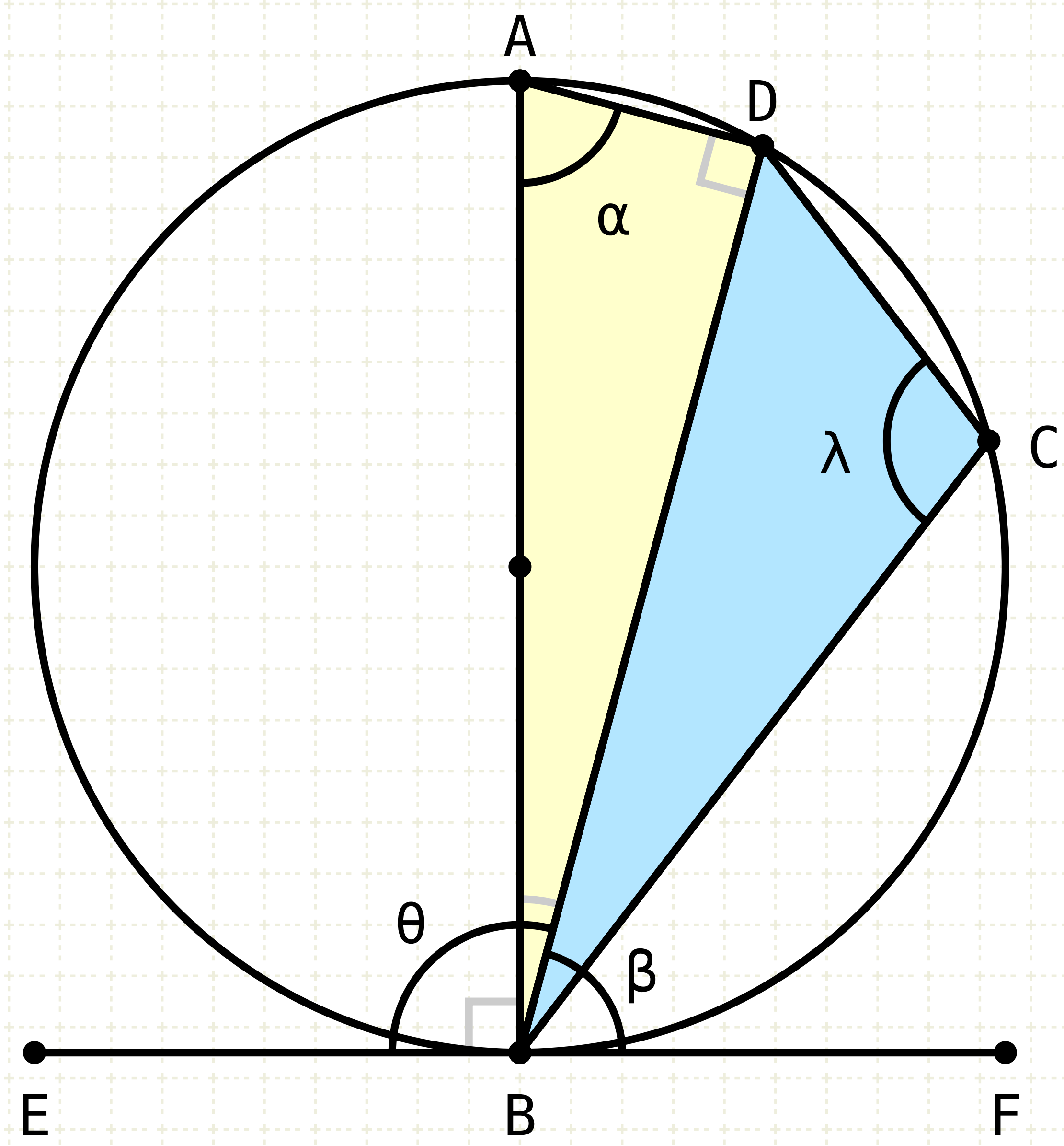

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$\lambda = \llcorner + \gamma$

$\theta = \llcorner + \gamma$

$\therefore \theta = \lambda$

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