Euclid's Elements

Book VII

Definitions:

1 A unit is that by virtue of which each of the things that exist is called one

2 A number is a multitude composed of units. (not one)

3 A number is part of a number, the less of the greater, when it measures the greater

11 A prime number is that which is measured by a unit alone.

12 Numbers prime to one another are those which are measured by a unit alone as a common measure

15 A number is said to multiply a number when that which is multiplied is added to itself as many times as there are units in the other, and thus some number is produced.

20 Numbers are proportional when the first is the same multiple, or the same part, or the same parts, of the second that the third is of the fourth.

As long as algebra and geometry have been separated, their progress have been slow and their uses limited; but when these two sciences have been united, they have lent each mutual forces, and have marched together towards perfection.

Joseph-Louis Lagrange
(1736 to 1813)

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Proposition 8 of Book VII

If a number be that parts of a number that a number subracted is of a number subtracted, the remainder will also be the same parts of the remainder that the whole is of the whole.

Proposition 8 of Book VII

If a number be that parts of a number that a number subracted is of a number subtracted, the remainder will also be the same parts of the remainder that the whole is of the whole.

$$b = (p/q)\cdot a$$

$$d = (p/q)\cdot c$$

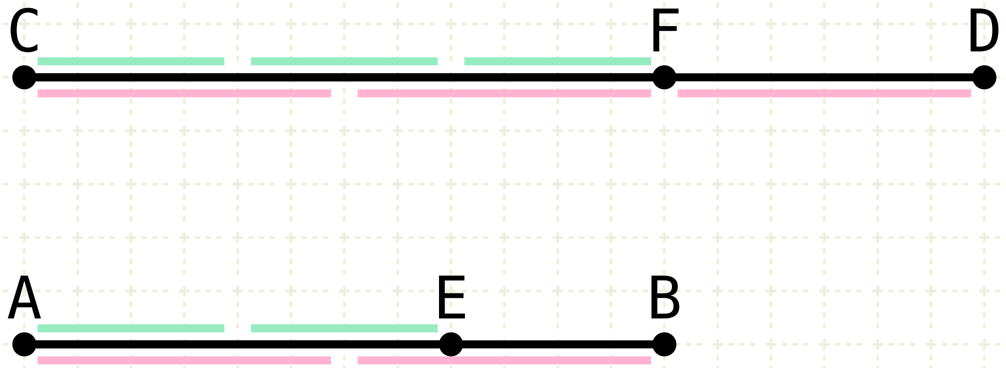
$$\rightarrow (b-d) = (p/q)\cdot(a-c)$$

In other words

If b is the same fraction of a as d is to c, then the result of d subtracted from b will also be the same fraction of the result of c subtracted from a

Proposition 8 of Book VII

If a number be that parts of a number that a number subracted is of a number subtracted, the remainder will also be the same parts of the remainder that the whole is of the whole.



$$AB = (p/q)CD$$
$$AE = (p/q)CF$$

In other words

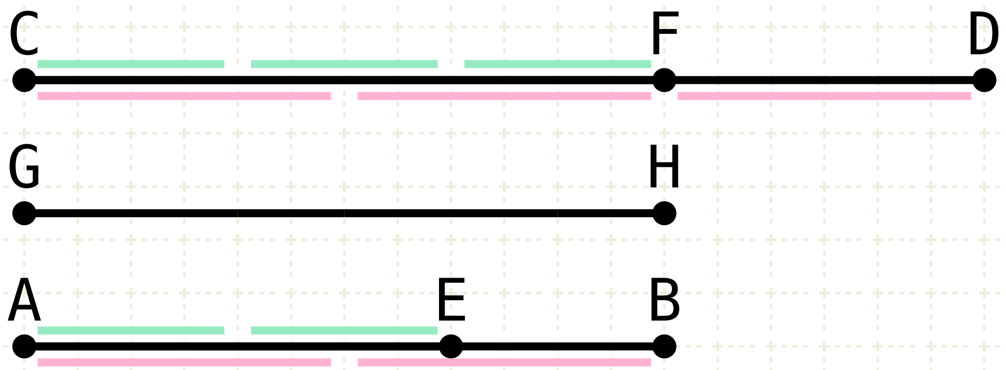
If b is the same fraction of a as d is to c, then the result of d subtracted from b will also be the same fraction of the result of c subtracted from a

Proof

Let the number AB be parts (fractions) of CD, and let AE be the same parts (fractions) of CF

Proposition 8 of Book VII

If a number be that parts of a number that a number subracted is of a number subtracted, the remainder will also be the same parts of the remainder that the whole is of the whole.



AB = (p/q)CD

AE = (p/q)CF

GH = AB

In other words

If b is the same fraction of a as d is to c, then the result of d subtracted from b will also be the same fraction of the result of c subtracted from a

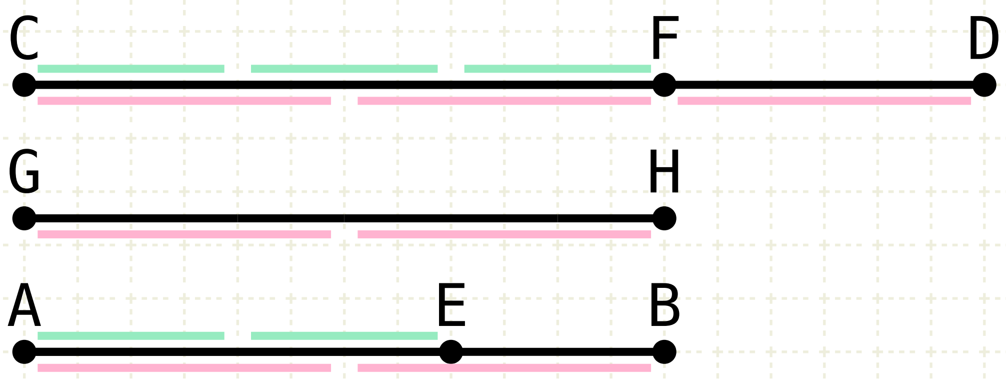
Proof

Let the number AB be parts (fractions) of CD, and let AE be the same parts (fractions) of CF

Make GH equal to AB

Proposition 8 of Book VII

If a number be that parts of a number that a number subracted is of a number subtracted, the remainder will also be the same parts of the remainder that the whole is of the whole.



AB = (p/q)CD

AE = (p/q)CF

GH = AB

GH = (p/q)CD

In other words

If b is the same fraction of a as d is to c, then the result of d subtracted from b will also be the same fraction of the result of c subtracted from a

Proof

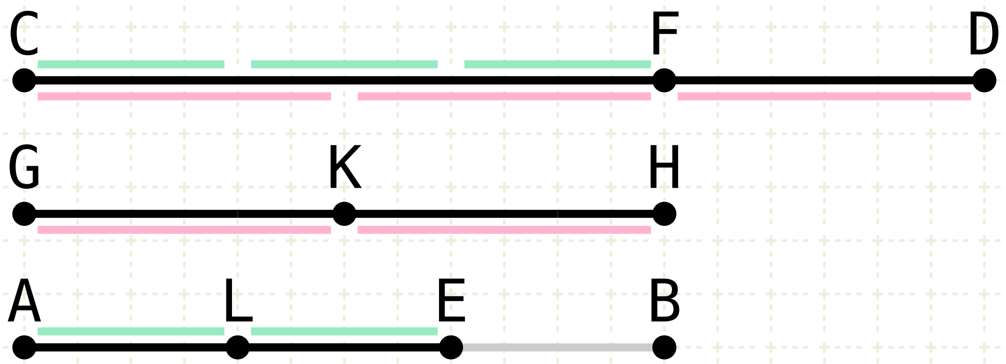
Let the number AB be parts (fractions) of CD, and let AE be the same parts (fractions) of CF

Make GH equal to AB

AE is the same parts of CF that GH is of CD

Proposition 8 of Book VII

If a number be that parts of a number that a number subracted is of a number subtracted, the remainder will also be the same parts of the remainder that the whole is of the whole.



AB = (p/q)CD

AE = (p/q)CF

GH = AB

GH = (p/q)CD

GK = KH = (1/q)CD

AL = LE = (1/q)CF

In other words

If b is the same fraction of a as d is to c, then the result of d subtracted from b will also be the same fraction of the result of c subtracted from a

Proof

Let the number AB be parts (fractions) of CD, and let AE be the same parts (fractions) of CF

Make GH equal to AB

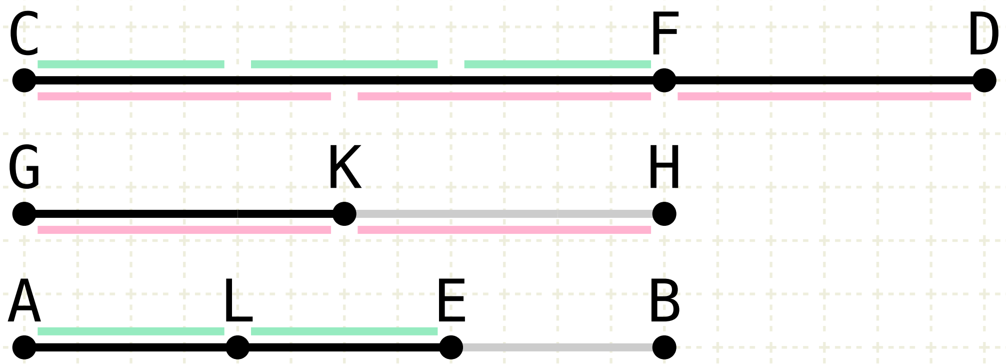
AE is the same parts of CF that GH is of CD

Divide GH and AE into the number of parts of CD, or …

… divide GH into sections where each section is equal to one part of CD, and divide AE into sections where each section is equal to one part of CF

Proposition 8 of Book VII

If a number be that parts of a number that a number subracted is of a number subtracted, the remainder will also be the same parts of the remainder that the whole is of the whole.



AB = (p/q)CD

AE = (p/q)CF

GH = AB

GH = (p/q)CD

GK = KH = (1/q)CD

AL = LE = (1/q)CF

GK > AL

In other words

If b is the same fraction of a as d is to c, then the result of d subtracted from b will also be the same fraction of the result of c subtracted from a

Proof

Let the number AB be parts (fractions) of CD, and let AE be the same parts (fractions) of CF

Make GH equal to AB

AE is the same parts of CF that GH is of CD

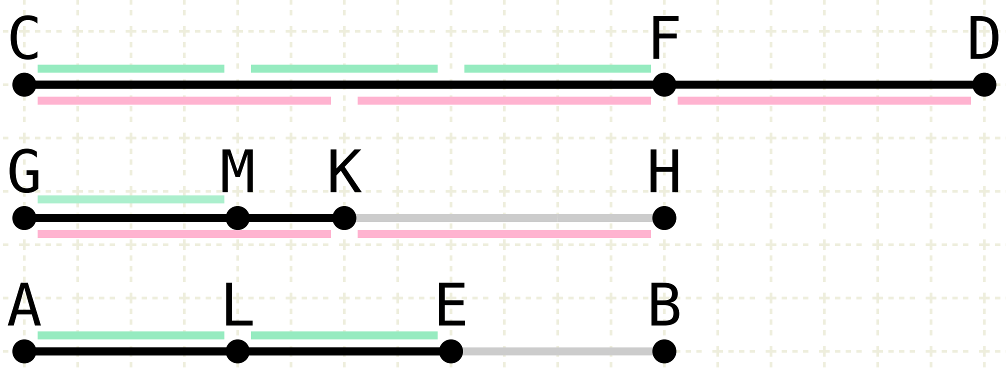
Divide GH and AE into the number of parts of CD, or …

… divide GH into sections where each section is equal to one part of CD, and divide AE into sections where each section is equal to one part of CF

Since AL is the same part of CF that GK is of CD, and CD is greater than CF, then GK is greater than AL

Proposition 8 of Book VII

If a number be that parts of a number that a number subracted is of a number subtracted, the remainder will also be the same parts of the remainder that the whole is of the whole.



AB = (p/q)CD

AE = (p/q)CF

GH = AB

GH = (p/q)CD

GK = KH = (1/q)CD

AL = LE = (1/q)CF

GK > AL

GM = AL = (1/q)CF

In other words

If b is the same fraction of a as d is to c, then the result of d subtracted from b will also be the same fraction of the result of c subtracted from a

Proof

Let the number AB be parts (fractions) of CD, and let AE be the same parts (fractions) of CF

Make GH equal to AB

AE is the same parts of CF that GH is of CD

Divide GH and AE into the number of parts of CD, or …

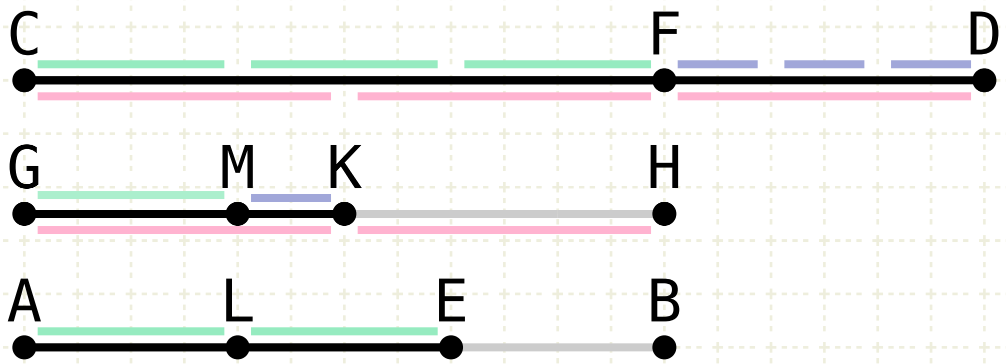
… divide GH into sections where each section is equal to one part of CD, and divide AE into sections where each section is equal to one part of CF

Since AL is the same part of CF that GK is of CD, and CD is greater than CF, then GK is greater than AL

Let GM equal to AL

Proposition 8 of Book VII

If a number be that parts of a number that a number subracted is of a number subtracted, the remainder will also be the same parts of the remainder that the whole is of the whole.



AB = (p/q)CD

AE = (p/q)CF

GH = AB

GH = (p/q)CD

GK = KH = (1/q)CD

AL = LE = (1/q)CF

GK > AL

GM = AL = (1/q)CF

MK = GK - GM = (1/q)(CD - CF) = (1/q)FD

In other words

If b is the same fraction of a as d is to c, then the result of d subtracted from b will also be the same fraction of the result of c subtracted from a

Proof

Let the number AB be parts (fractions) of CD, and let AE be the same parts (fractions) of CF

Make GH equal to AB

AE is the same parts of CF that GH is of CD

Divide GH and AE into the number of parts of CD, or …

… divide GH into sections where each section is equal to one part of CD, and divide AE into sections where each section is equal to one part of CF

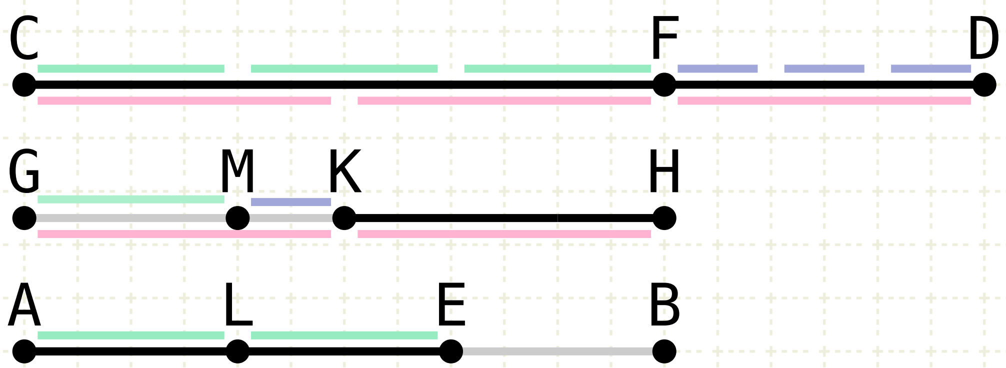
Since AL is the same part of CF that GK is of CD, and CD is greater than CF, then GK is greater than AL

Let GM equal to AL

Now GK is the same part of CD that GM is of CF, therefore MK is also the same part of FD (VII·7)

Proposition 8 of Book VII

If a number be that parts of a number that a number subracted is of a number subtracted, the remainder will also be the same parts of the remainder that the whole is of the whole.



AB = (p/q)CD

AE = (p/q)CF

GH = AB

GH = (p/q)CD

GK = KH = (1/q)CD

AL = LE = (1/q)CF

GK > AL

GM = AL = (1/q)CF

MK = GK - GM = (1/q)(CD - CF) = (1/q)FD

KH > LE

In other words

If b is the same fraction of a as d is to c, then the result of d subtracted from b will also be the same fraction of the result of c subtracted from a

Proof

Let the number AB be parts (fractions) of CD, and let AE be the same parts (fractions) of CF

Make GH equal to AB

AE is the same parts of CF that GH is of CD

Divide GH and AE into the number of parts of CD, or …

… divide GH into sections where each section is equal to one part of CD, and divide AE into sections where each section is equal to one part of CF

Since AL is the same part of CF that GK is of CD, and CD is greater than CF, then GK is greater than AL

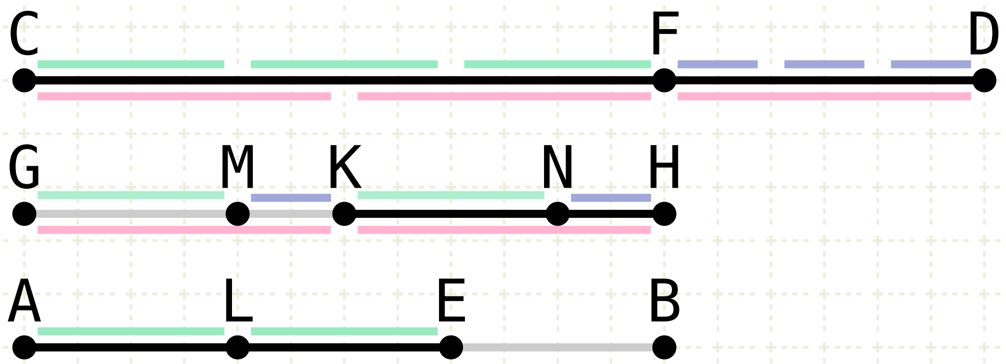
Let GM equal to AL

Now GK is the same part of CD that GM is of CF, therefore MK is also the same part of FD (VII·7)

Since LE is the same part of CF that KH is of CD, and CD is greater than CF, then KH is greater than LE

Proposition 8 of Book VII

If a number be that parts of a number that a number subracted is of a number subtracted, the remainder will also be the same parts of the remainder that the whole is of the whole.



AB = (p/q)CD

AE = (p/q)CF

GH = AB

GH = (p/q)CD

GK = KH = (1/q)CD

AL = LE = (1/q)CF

GK > AL

GM = AL = (1/q)CF

MK = GK - GM = (1/q)(CD - CF) = (1/q)FD

KH > LE

KN = LE = (1/q)CF

In other words

If b is the same fraction of a as d is to c, then the result of d subtracted from b will also be the same fraction of the result of c subtracted from a

Proof

Let the number AB be parts (fractions) of CD, and let AE be the same parts (fractions) of CF

Make GH equal to AB

AE is the same parts of CF that GH is of CD

Divide GH and AE into the number of parts of CD, or …

… divide GH into sections where each section is equal to one part of CD, and divide AE into sections where each section is equal to one part of CF

Since AL is the same part of CF that GK is of CD, and CD is greater than CF, then GK is greater than AL

Let GM equal to AL

Now GK is the same part of CD that GM is of CF, therefore MK is also the same part of FD (VII·7)

Since LE is the same part of CF that KH is of CD, and CD is greater than CF, then KH is greater than LE

Let KN equal to LE

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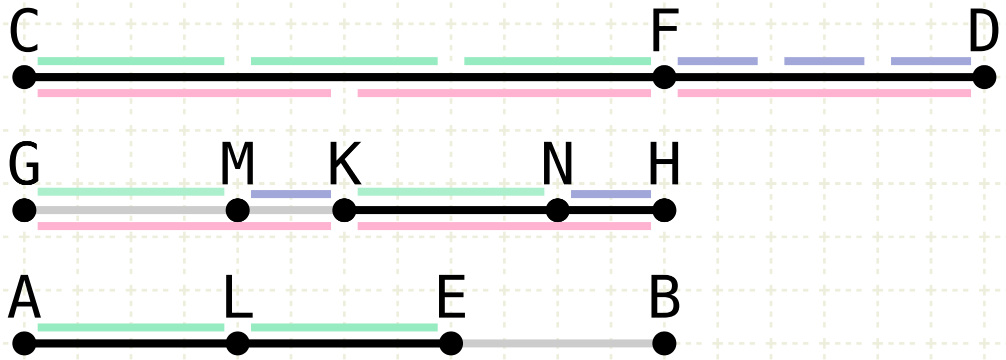

$AB = (p/q)CD$

$AE = (p/q)CF$

$GH = AB$

$GH = (p/q)CD$

$GK = KH = (1/q)CD$

$AL = LE = (1/q)CF$

$GK > AL$

$GM = AL = (1/q)CF$

$MK = GK - GM = (1/q)(CD - CF) = (1/q)FD$

$KH > LE$

$KN = LE = (1/q)CF$

$NH = KH - KN = (1/q)(CD - CF) = (1/q)FD$

In other words

If b is the same fraction of a as d is to c, then the result of d subtracted from b will also be the same fraction of the result of c subtracted from a

Proof

Let the number AB be parts (fractions) of CD, and let AE be the same parts (fractions) of CF

Make GH equal to AB

AE is the same parts of CF that GH is of CD

Divide GH and AE into the number of parts of CD, or …

… divide GH into sections where each section is equal to one part of CD, and divide AE into sections where each section is equal to one part of CF

Since AL is the same part of CF that GK is of CD, and CD is greater than CF, then GK is greater than AL

Let GM equal to AL

Now GK is the same part of CD that GM is of CF, therefore MK is also the same part of FD (VII·7)

Since LE is the same part of CF that KH is of CD, and CD is greater than CF, then KH is greater than LE

Let KN equal to LE

Now KH is the same part of CD that KN is of CF, therefore NH is also the same part of FD (VII·7)

Proposition 8 of Book VII

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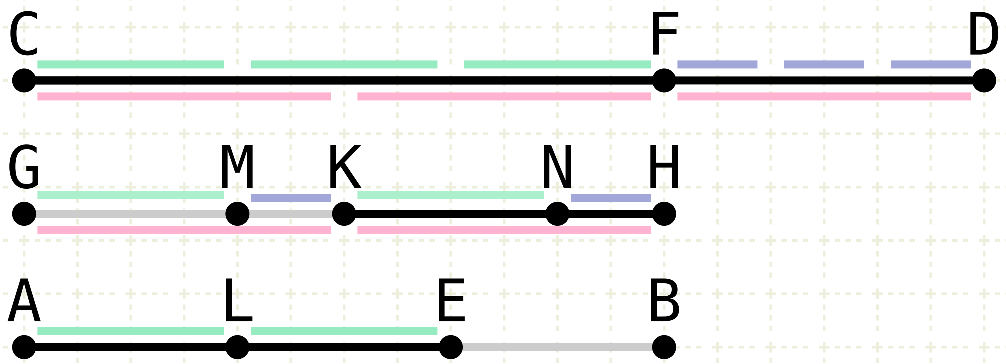

$AB = (p/q)CD$

$AE = (p/q)CF$

$GH = AB = (p/q)CD$

$GK = KH = (1/q)CD$

$GM = KN = AL = LE = (1/q)CF$

$MK = NH = (1/q)FD$

In other words

If b is the same fraction of a as d is to c, then the result of d subtracted from b will also be the same fraction of the result of c subtracted from a

Proof (cont.)

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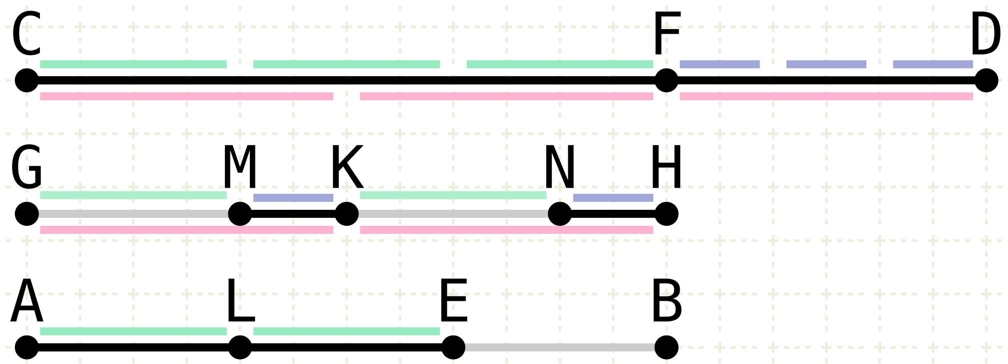


$AB = (p/q)CD$

$AE = (p/q)CF$

$GH = AB = (p/q)CD$

$GK = KH = (1/q)CD$

$GM = KN = AL = LE = (1/q)CF$

$MK = NH = (1/q)FD$

$MK + NH + \ldots = (p/q)FD$

$GK + KH + \ldots = (p/q)CD$

In other words

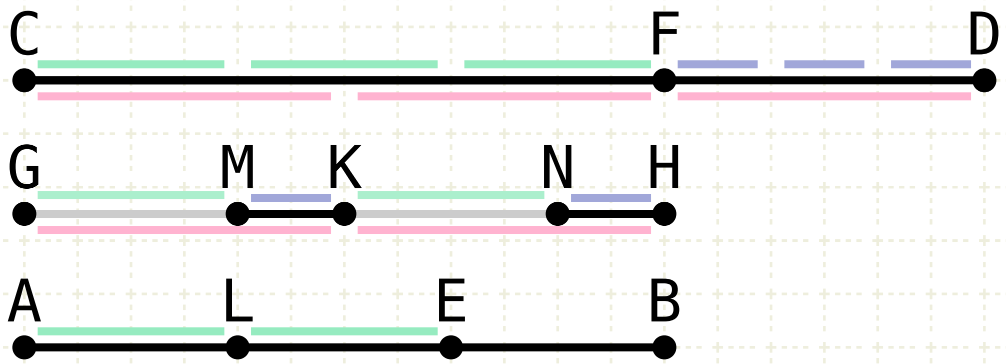
If b is the same fraction of a as d is to c, then the result of d subtracted from b will also be the same fraction of the result of c subtracted from a

Proof (cont.)

MK and NH are each equal to one part of FD, so the sum of them is equal to the same number of parts of FD as GH is of CD

Proposition 8 of Book VII

If a number be that parts of a number that a number subracted is of a number subtracted, the remainder will also be the same parts of the remainder that the whole is of the whole.

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AB = (p/q)CD
AE = (p/q)CF
GH = AB = (p/q)CD
GK = KH = (1/q)CD
GM = KN = AL = LE = (1/q)CF
MK = NH = (1/q)FD
MK + NH + … = (p/q)FD
GK + KH + … = (p/q)CD
AB - AL - LE = EB
GH - GM - KN = MK + NH
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In other words

If b is the same fraction of a as d is to c, then the result of d subtracted from b will also be the same fraction of the result of c subtracted from a

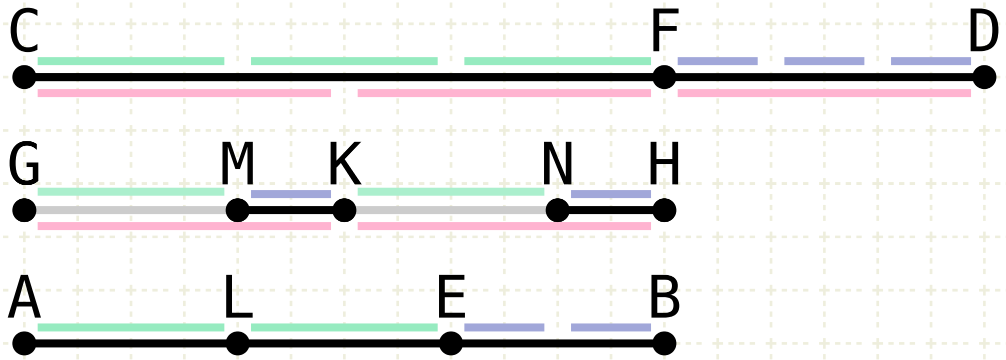
Proof (cont.)

MK and NH are each equal to one part of FD, so the sum of them is equal to the same number of parts of FD as GH is of CD

EB is the remainder of AB minus AL and LE, MK plus NH is the remainder of GH minus GM and KN

Proposition 8 of Book VII

If a number be that parts of a number that a number subracted is of a number subtracted, the remainder will also be the same parts of the remainder that the whole is of the whole.



AB = (p/q)CD

AE = (p/q)CF

GH = AB = (p/q)CD

GK = KH = (1/q)CD

GM = KN = AL = LE = (1/q)CF

MK = NH = (1/q)FD

MK + NH + … = (p/q)FD

GK + KH + … = (p/q)CD

AB - AL - LE = EB

GH - GM - KN = MK + NH

MK + NH = EB

In other words

If b is the same fraction of a as d is to c, then the result of d subtracted from b will also be the same fraction of the result of c subtracted from a

Proof (cont.)

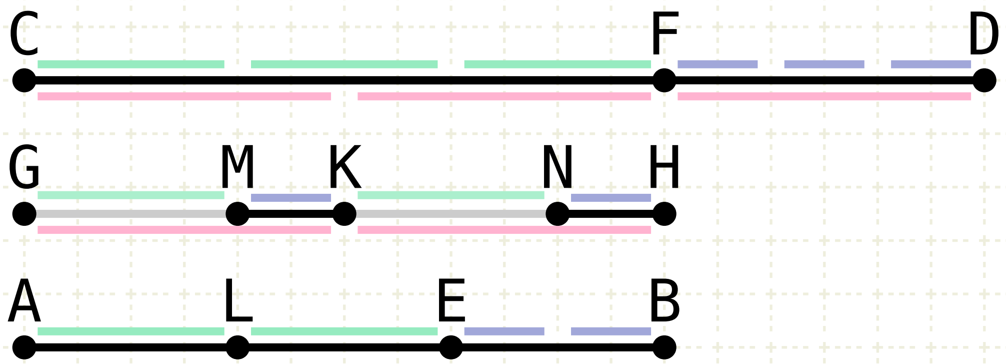
MK and NH are each equal to one part of FD, so the sum of them is equal to the same number of parts of FD as GH is of CD

EB is the remainder of AB minus AL and LE, MK plus NH is the remainder of GH minus GM and KN

But GM and KN equal AL and LE respectively, and GH is equal to AB, so the remainder MK,NH is equal to the remainder EB

Proposition 8 of Book VII

If a number be that parts of a number that a number subracted is of a number subtracted, the remainder will also be the same parts of the remainder that the whole is of the whole.



AB = (p/q)CD

AE = (p/q)CF

GH = AB = (p/q)CD

GK = KH = (1/q)CD

GM = KN = AL = LE = (1/q)CF

MK = NH = (1/q)FD

MK + NH + … = (p/q)FD

GK + KH + … = (p/q)CD

AB - AL - LE = EB

GH - GM - KN = MK + NH

MK + NH = EB

EB = (p/q)FD

In other words

If b is the same fraction of a as d is to c, then the result of d subtracted from b will also be the same fraction of the result of c subtracted from a

Proof (cont.)

MK and NH are each equal to one part of FD, so the sum of them is equal to the same number of parts of FD as GH is of CD

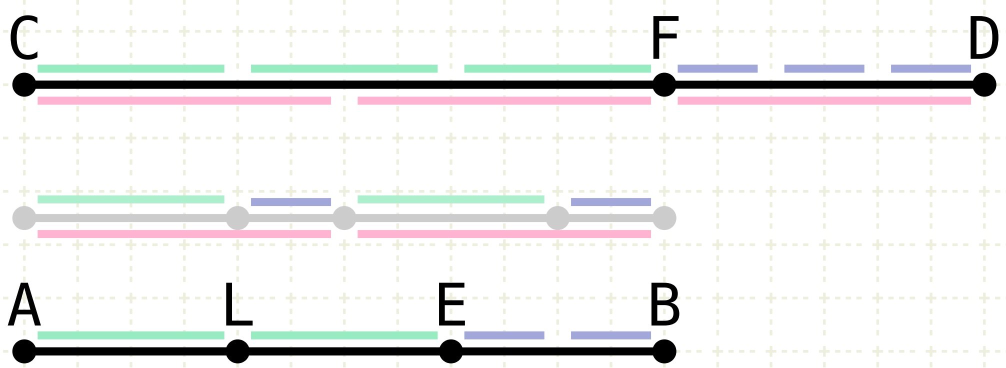
EB is the remainder of AB minus AL and LE, MK plus NH is the remainder of GH minus GM and KN

But GM and KN equal AL and LE respectively, and GH is equal to AB, so the remainder MK,NH is equal to the remainder EB

Therefore, EB also equals the same number of parts of FD as AB is of CD and AE is of CF

Proposition 8 of Book VII

If a number be that parts of a number that a number subracted is of a number subtracted, the remainder will also be the same parts of the remainder that the whole is of the whole.



$AB = (p/q)CD$

$AE = (p/q)CF$

$GH = AB = (p/q)CD$

$GK = KH = (1/q)CD$

$GM = KN = AL = LE = (1/q)CF$

$MK = NH = (1/q)FD$

$MK + NH + \ldots = (p/q)FD$

$GK + KH + \ldots = (p/q)CD$

$AB - AL - LE = EB$

$GH - GM - KN = MK + NH$

$MK + NH = EB$

$EB = (p/q)FD$

$AB - AE = (p/q)(CD - CF)$

In other words

If b is the same fraction of a as d is to c, then the result of d subtracted from b will also be the same fraction of the result of c subtracted from a

Proof (cont.)

MK and NH are each equal to one part of FD, so the sum of them is equal to the same number of parts of FD as GH is of CD

EB is the remainder of AB minus AL and LE, MK plus NH is the remainder of GH minus GM and KN

But GM and KN equal AL and LE respectively, and GH is equal to AB, so the remainder MK,NH is equal to the remainder EB

Therefore, EB also equals the same number of parts of FD as AB is of CD and AE is of CF

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