

Euclid's Elements

Book III



A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)



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6	If two circles touch one another, they will not have the same center	14	In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another.	22	The opposite angles of quadrilaterals in circles are equal to two right angles
7	Consider two lines from a point inside a circle to the edge, the longer one will be the one closest to the longest part of the diameter passing through the original point	15	The longest line in a circle is its diameter, shorter the farther away from the diameter	23	On the same straight line there cannot be constructed two similar and unequal segments of circles on the same side
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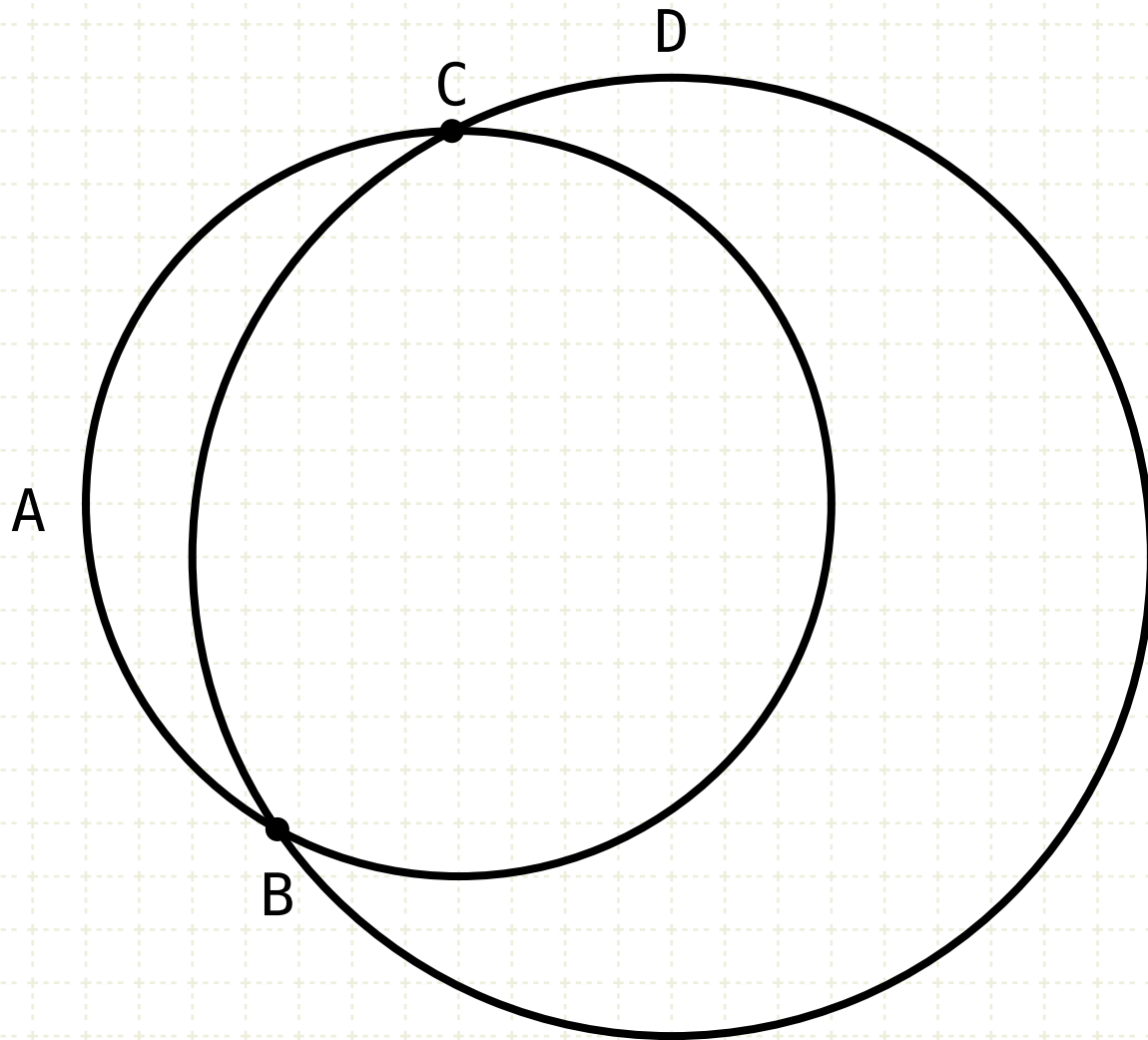
Proposition 5 of Book III

If two circles cut one another, they will not have the same center.



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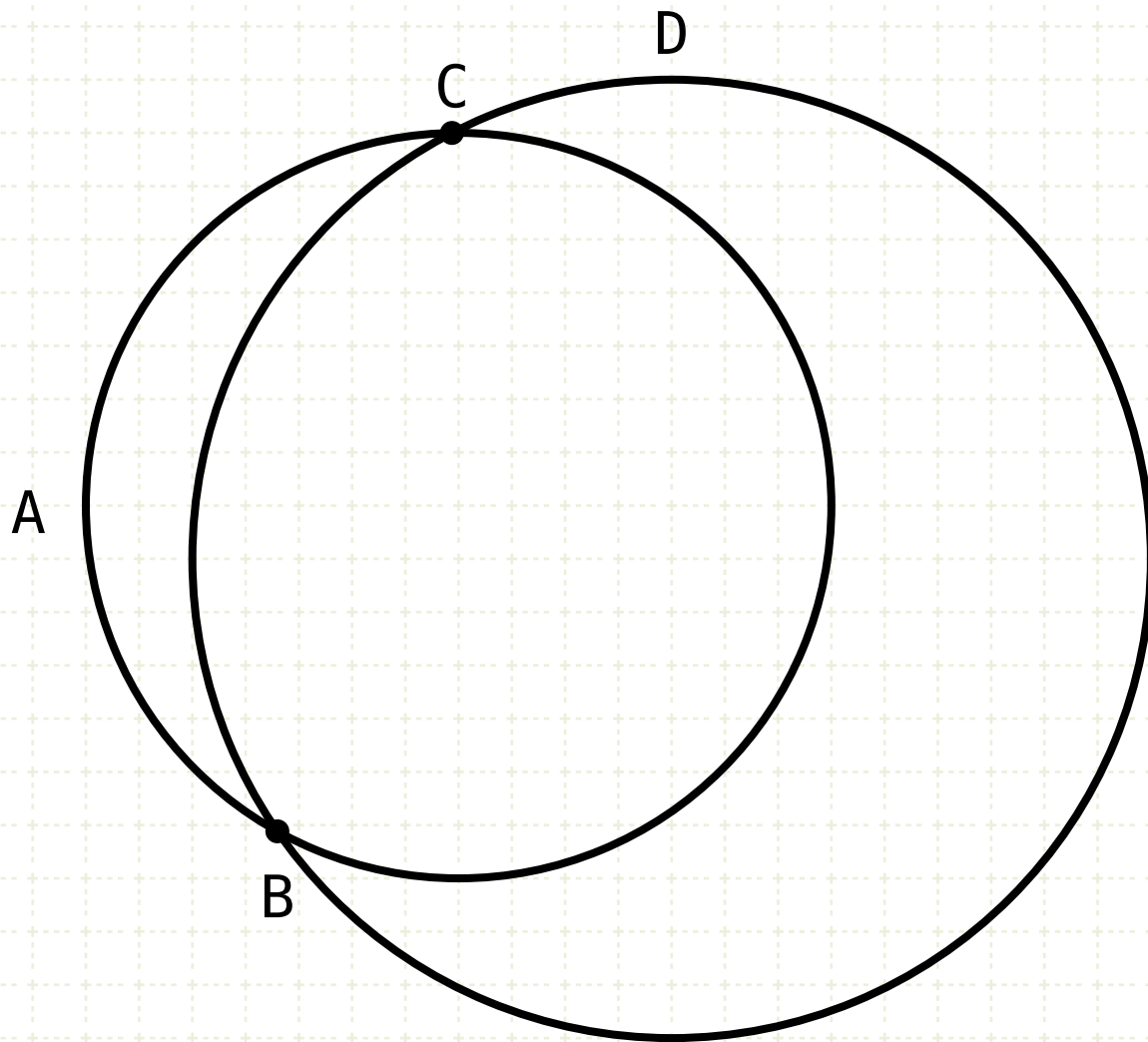
In other words

If two circles ABC, and BCD cut each other, then the center of the ABC is not the same as the center of BCD



Proposition 5 of Book III

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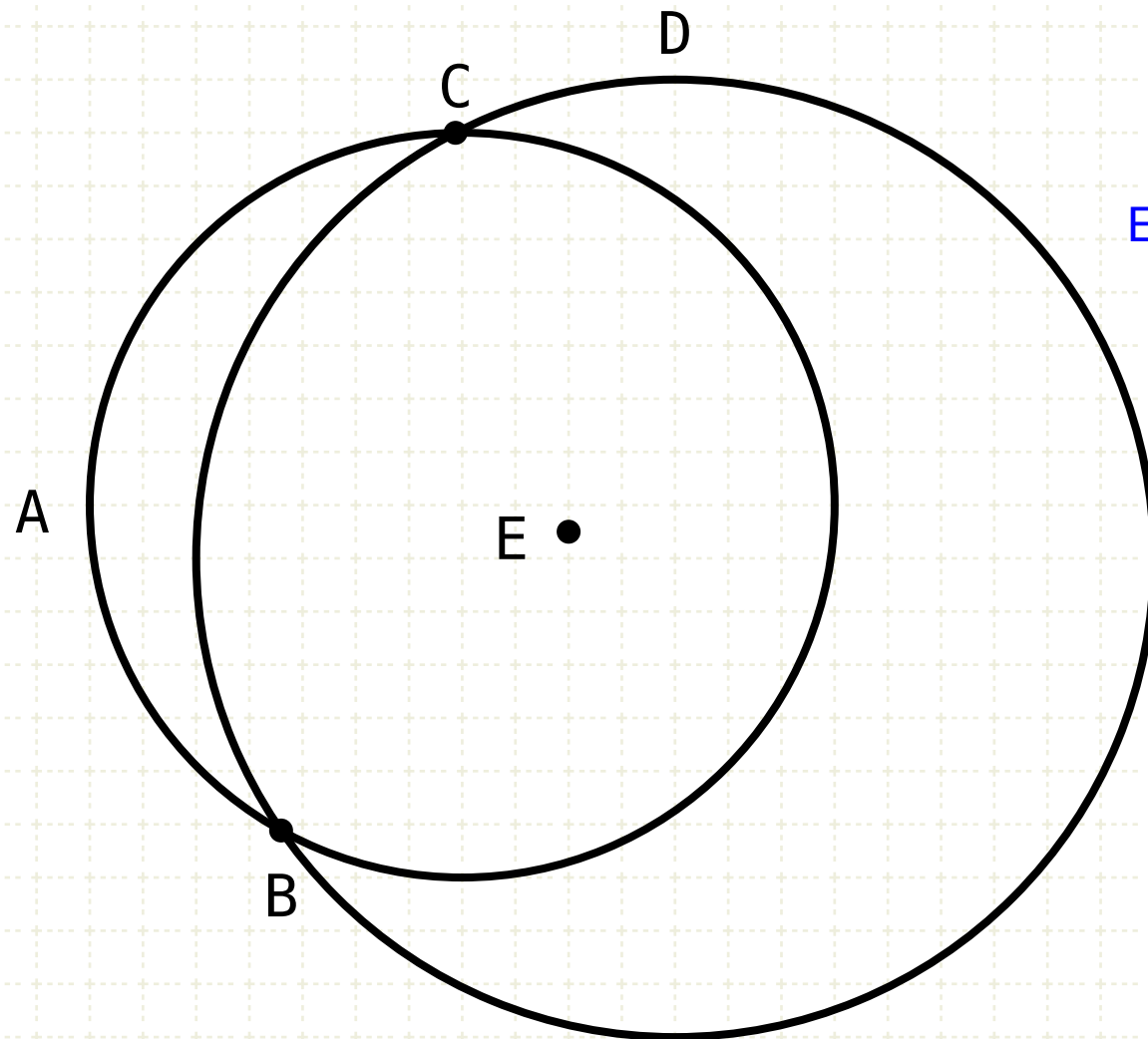
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Proof by contradiction



Proposition 5 of Book III

If two circles cut one another, they will not have the same center.



E is centre of both circles

In other words

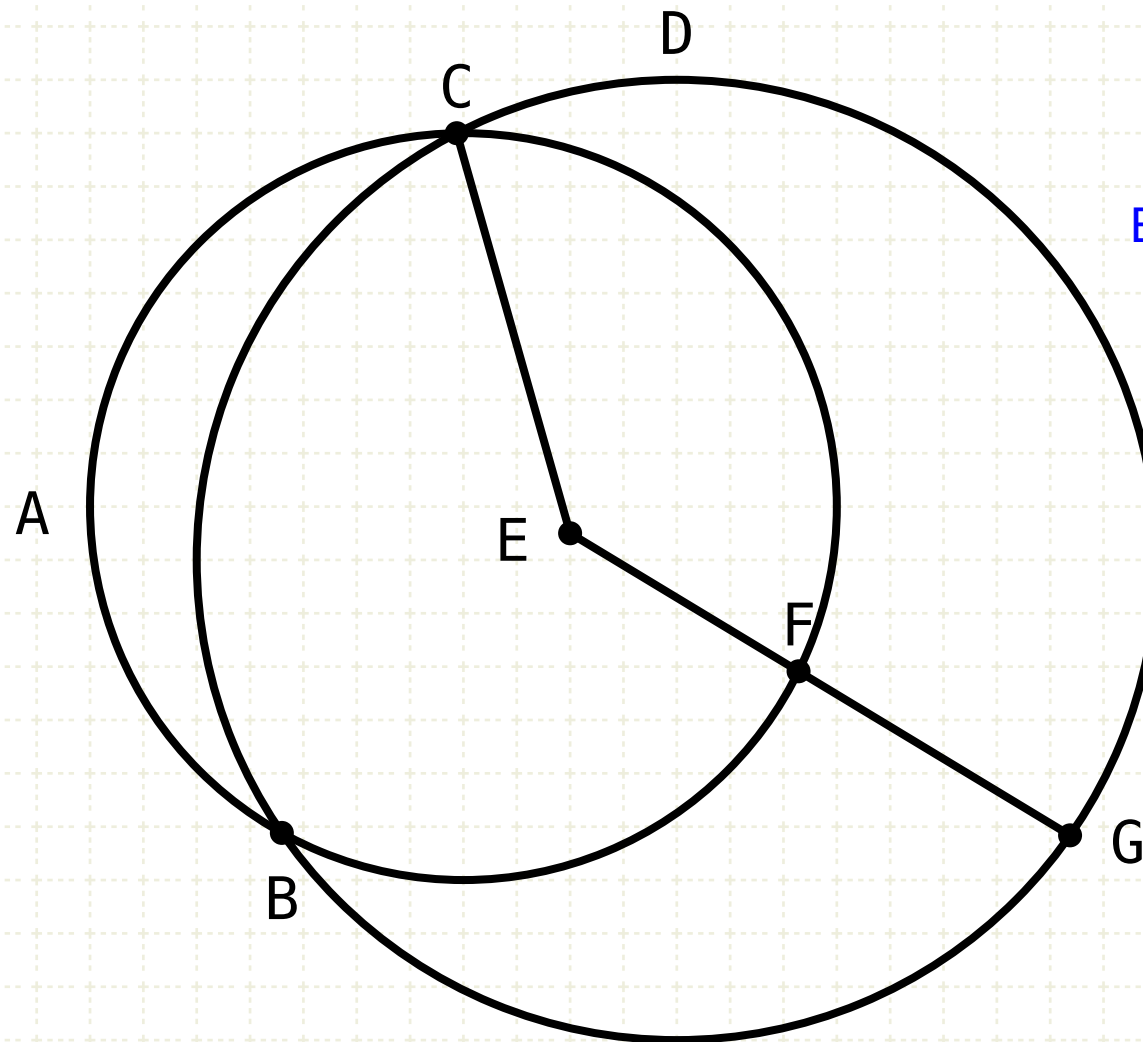
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Proof by contradiction

Assume that E is the center of both circles

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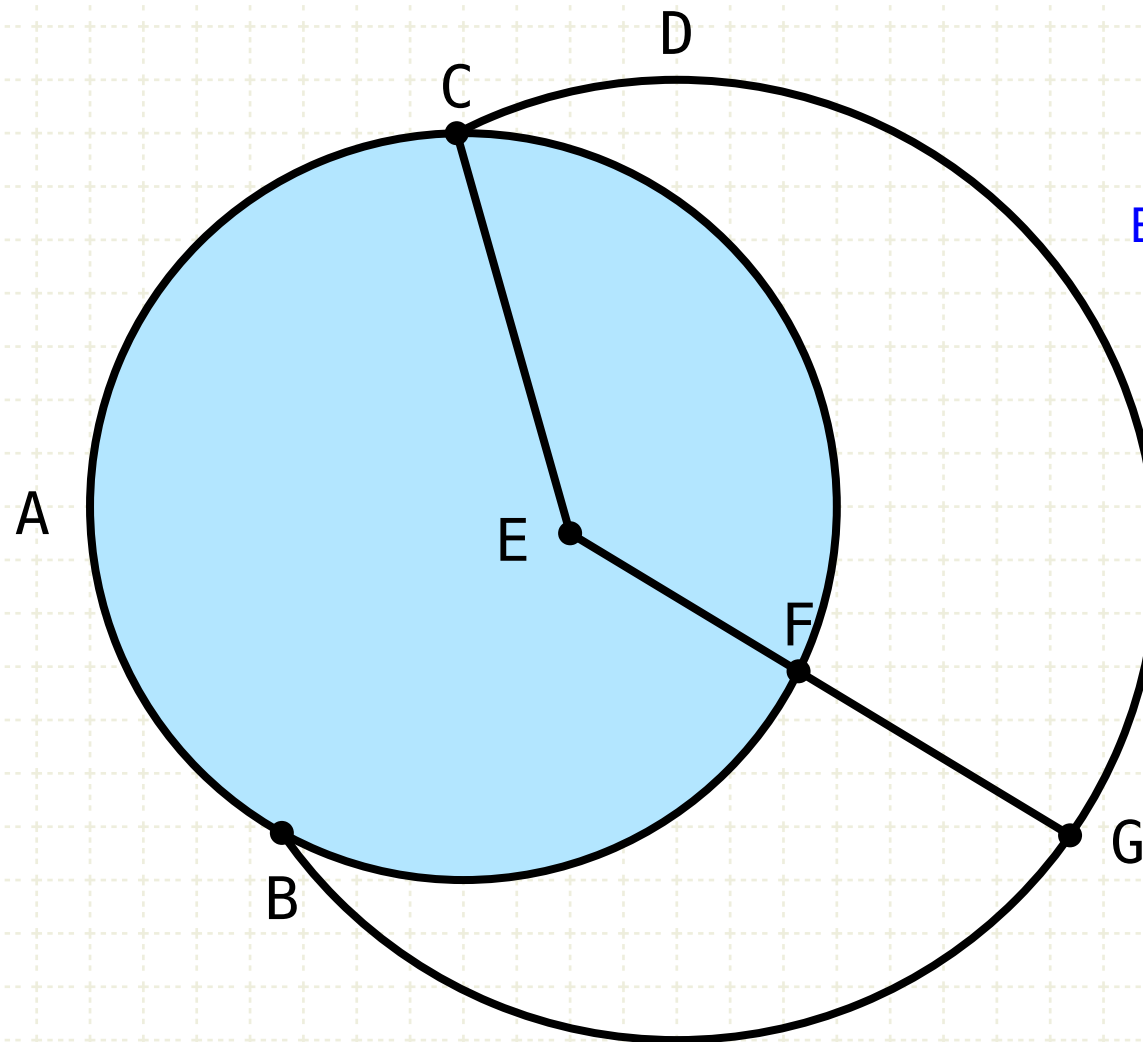
Proof by contradiction

Assume that E is the center of both circles

Join EC and draw a line EFG at random

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If two circles cut one another, they will not have the same center.



E is centre of both circles

$$CE = EF$$

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If two circles ABC, and BCD cut each other, then the center of the ABC is not the same as the center of BCD

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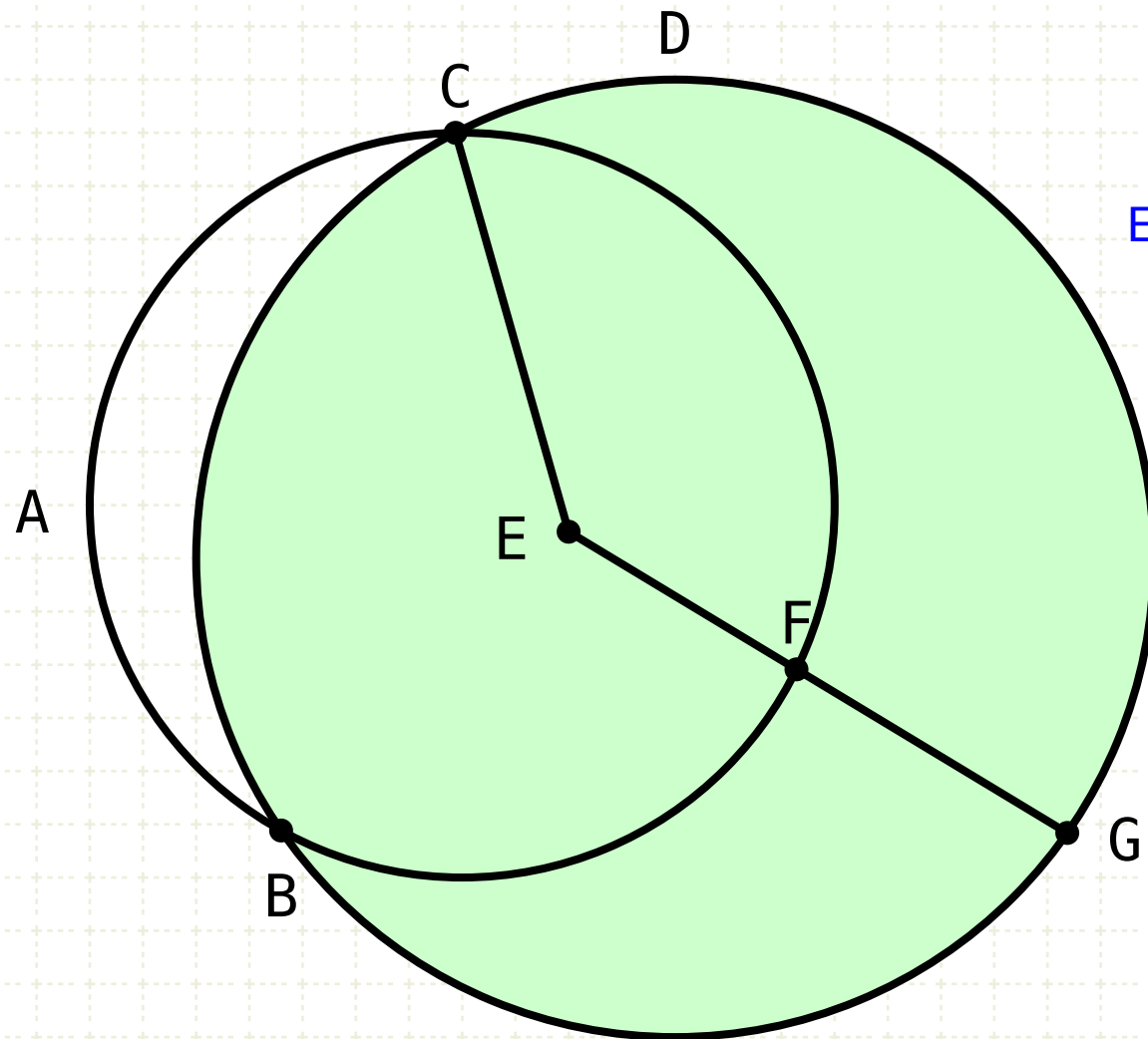
Assume that E is the center of both circles

Join EC and draw a line EFG at random

Since CE and EF are radii of the circle ABC, they are equal

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If two circles cut one another, they will not have the same center.



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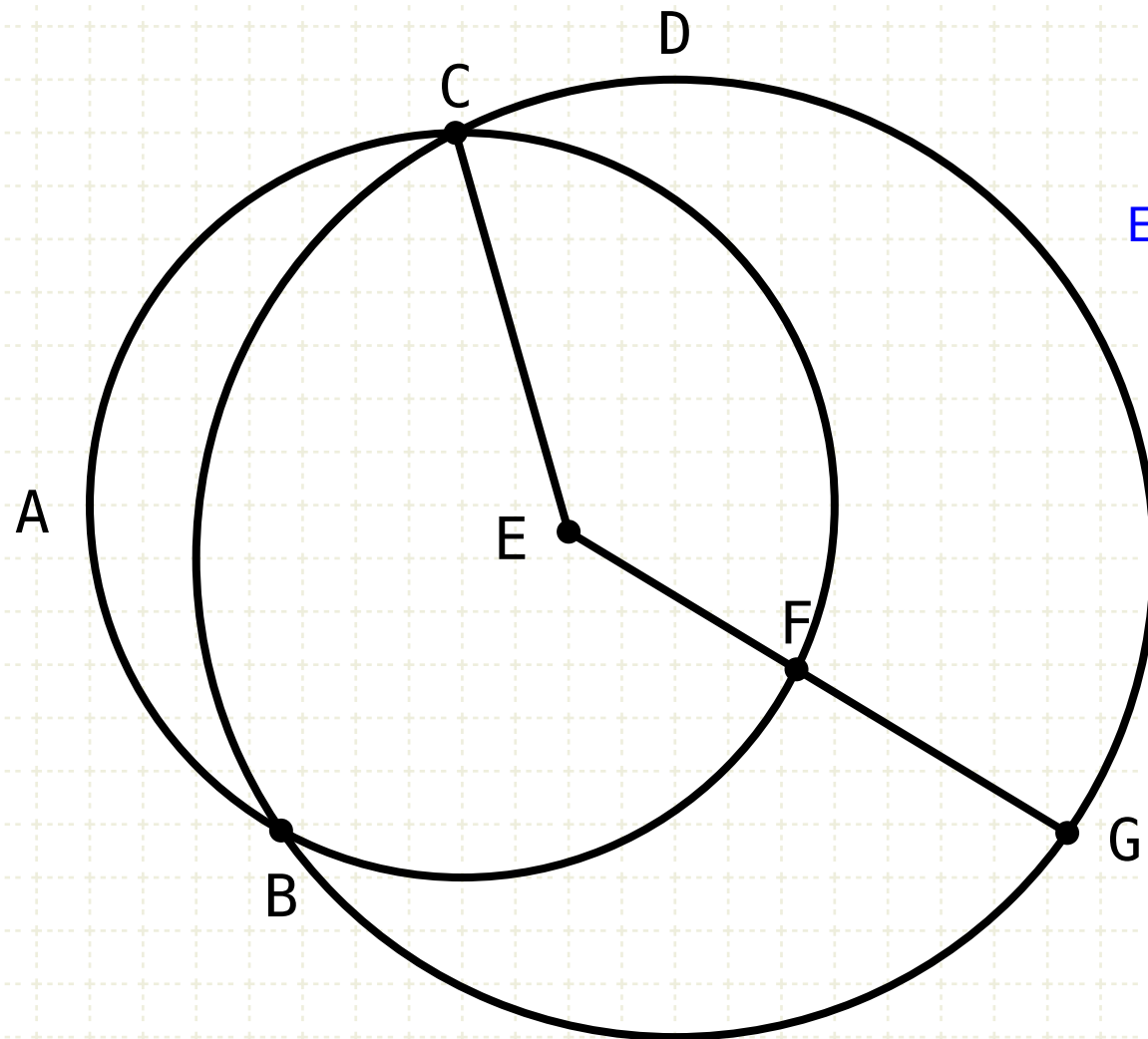
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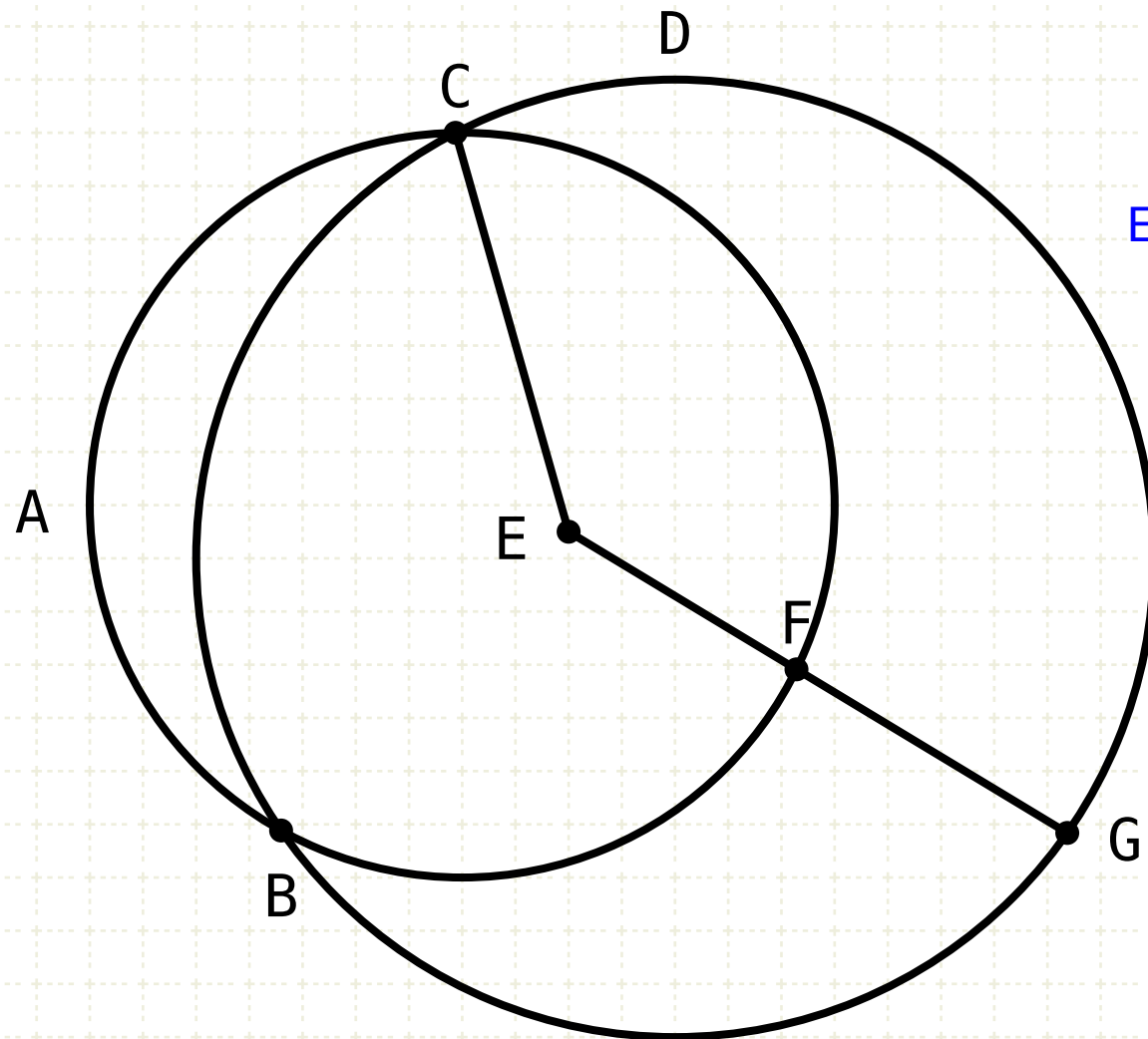
Since CE and EF are radii of the circle ABC, they are equal

Since CE and EG are radii of the circle BCD, they are equal

Since EF and EG are both equal to CE, they are equal to each other

Proposition 5 of Book III

If two circles cut one another, they will not have the same center.



E is centre of both circles

$$CE = EF$$

$$CE = EG$$

$$EF = EG$$

$$EF < EG$$

In other words

If two circles ABC, and BCD cut each other, then the center of the ABC is not the same as the center of BCD

Proof by contradiction

Assume that E is the center of both circles

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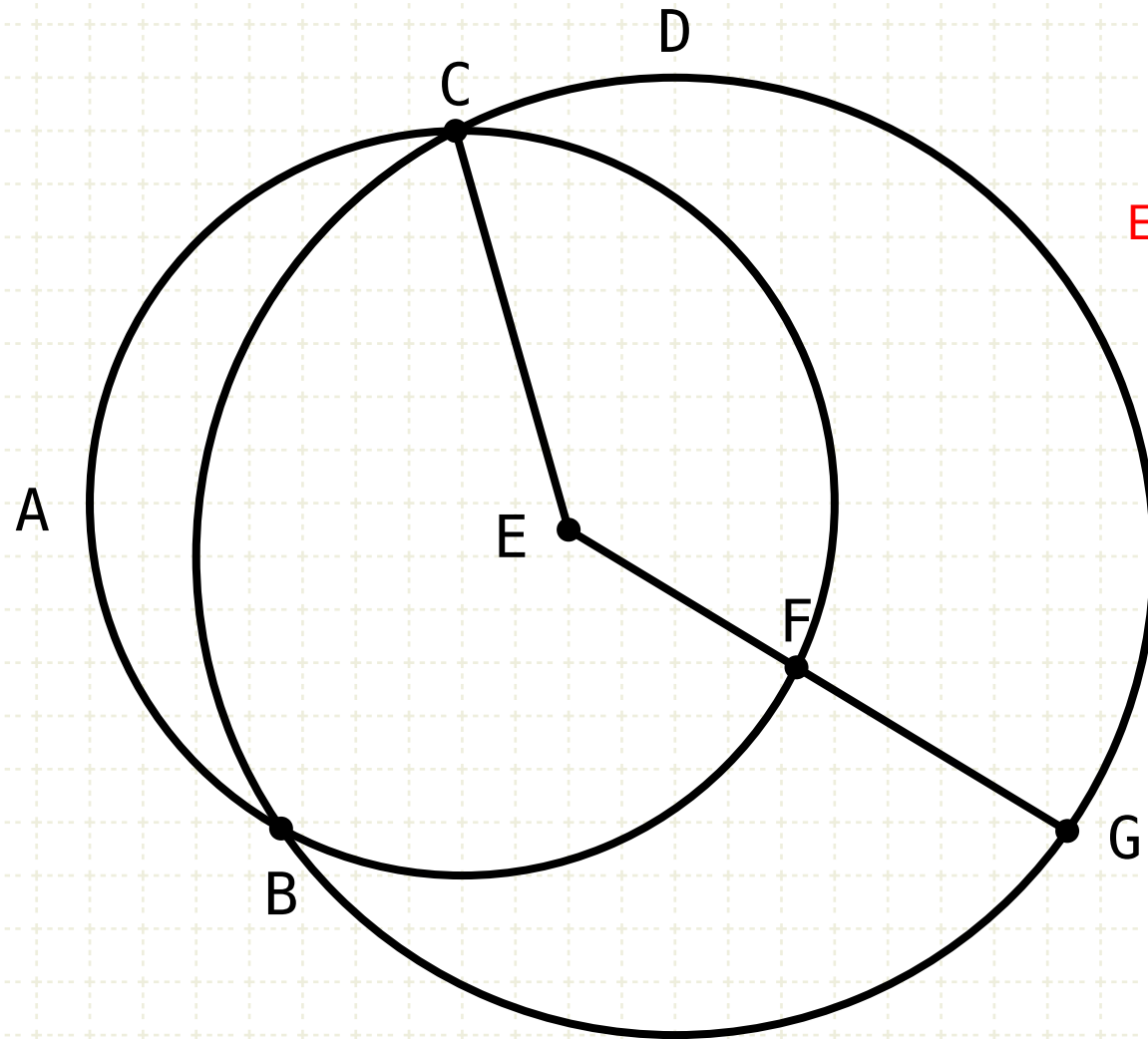
Since CE and EG are radii of the circle BCD, they are equal

Since EF and EG are both equal to CE, they are equal to each other

But EF is less than EG, which is inconsistent with the above statement

Proposition 5 of Book III

If two circles cut one another, they will not have the same center.



E is centre of both circles

$$\begin{aligned} CE &= EF \\ CE &= EG \\ EF &= EG \\ EF &< EG \end{aligned}$$

In other words

If two circles ABC, and BCD cut each other, then the center of the ABC is not the same as the center of BCD

Proof by contradiction

Assume that E is the center of both circles

Join EC and draw a line EFG at random

Since CE and EF are radii of the circle ABC, they are equal

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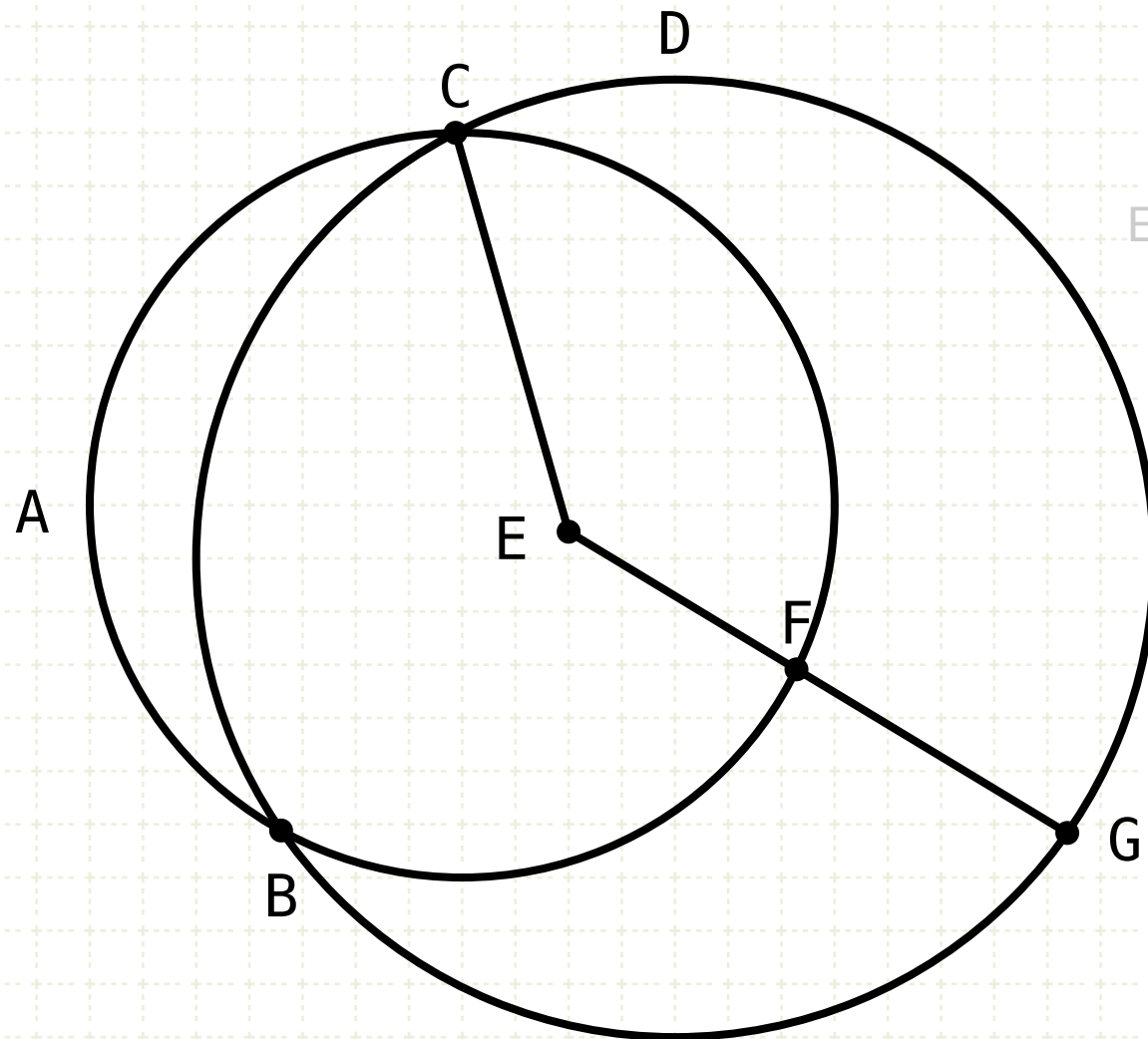
Since EF and EG are both equal to CE, they are equal to each other

But EF is less than EG, which is inconsistent with the above statement

Therefore E cannot be the center of both circles

Proposition 5 of Book III

If two circles cut one another, they will not have the same center.



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If two circles ABC, and BCD cut each other, then the center of the ABC is not the same as the center of BCD

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Assume that E is the center of both circles

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Since CE and EG are radii of the circle BCD, they are equal

Since EF and EG are both equal to CE, they are equal to each other

But EF is less than EG, which is inconsistent with the above statement

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