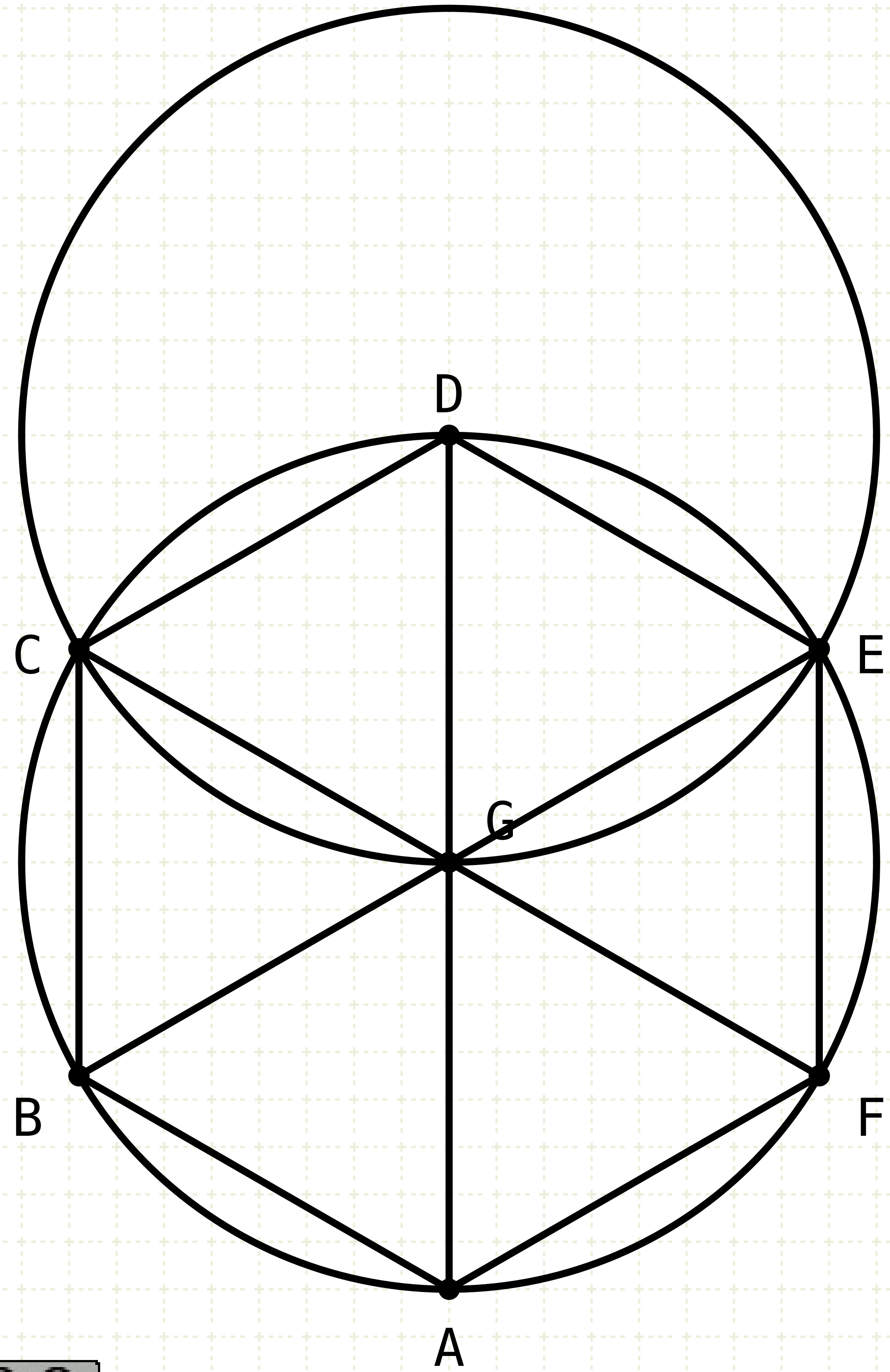
Euclid's Elements

Book IV



Philosophy (nature) is written in that great book which ever is before our eyes -- I mean the universe -- but we cannot understand it if we do not first learn the language and grasp the symbols in which it is written. The book is written in mathematical language, and the symbols are triangles, circles and other geometrical figures, without whose help it is impossible to comprehend a single word of it - without which one wanders in vain through a dark labyrinth.

Galileo Galilei

Proposition 11 of Book IV

In a given circle to inscribe an equilateral and equiangular pentagon.

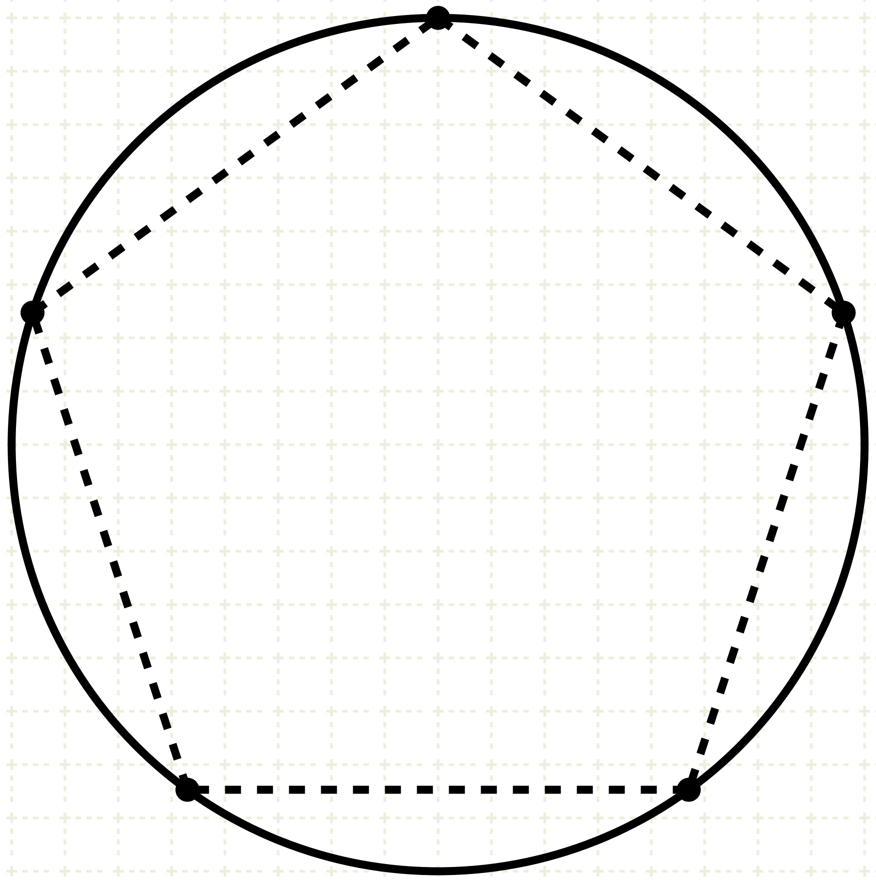
Table of Contents, Chapter 4

Proposition 11 of Book IV

In a given circle to inscribe an equilateral and equiangular pentagon.

Proposition 11 of Book IV

In a given circle to inscribe an equilateral and equiangular pentagon.

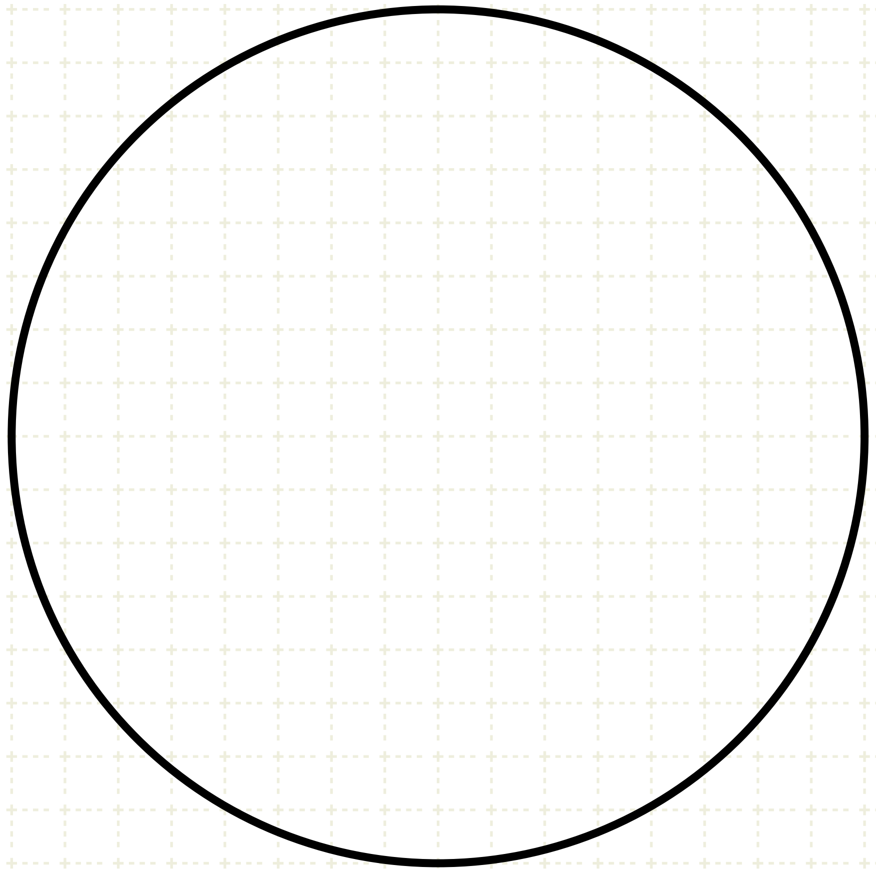


In other words

Construct a pentagon in a circle, where all lines and angles are equal

Proposition 11 of Book IV

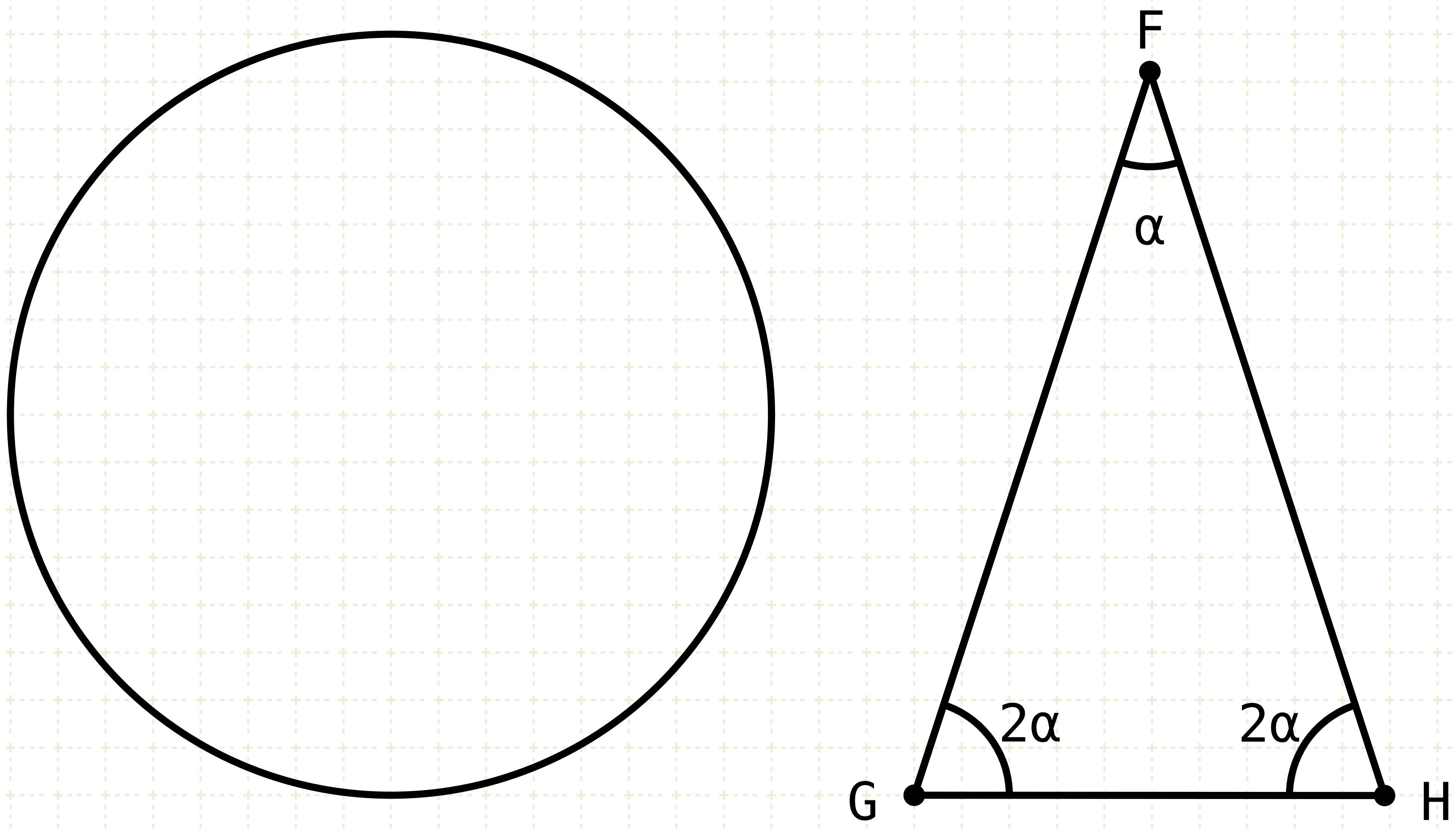
In a given circle to inscribe an equilateral and equiangular pentagon.



Construction

Proposition 11 of Book IV

In a given circle to inscribe an equilateral and equiangular pentagon.

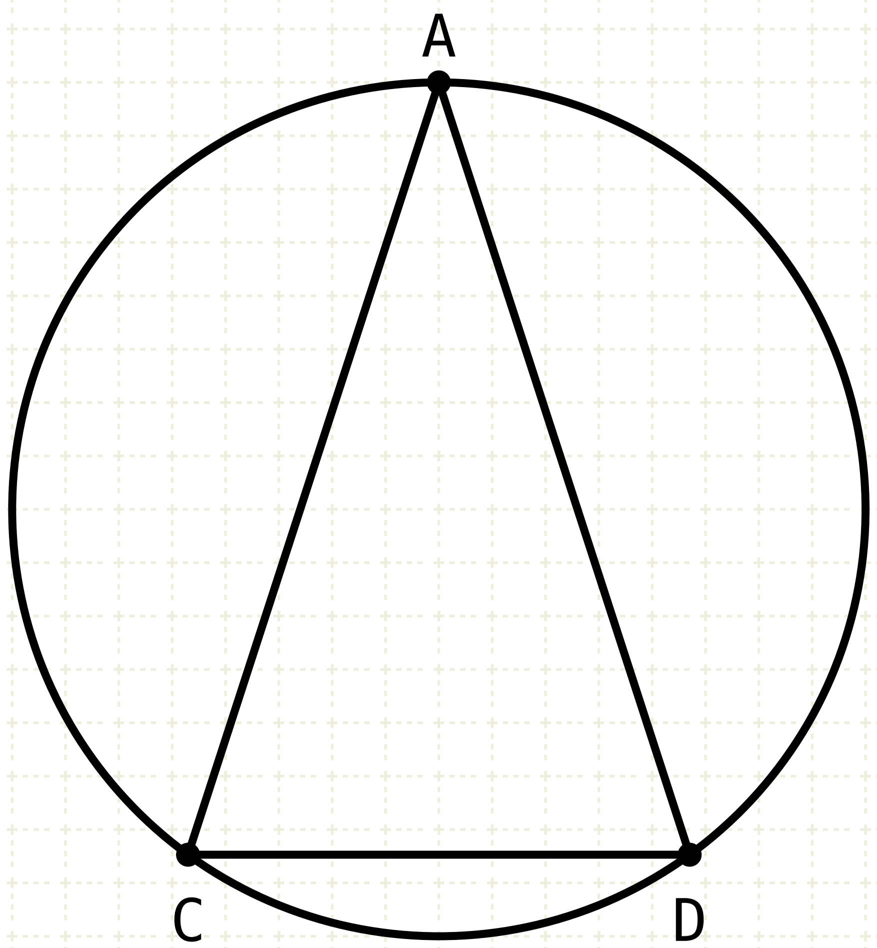


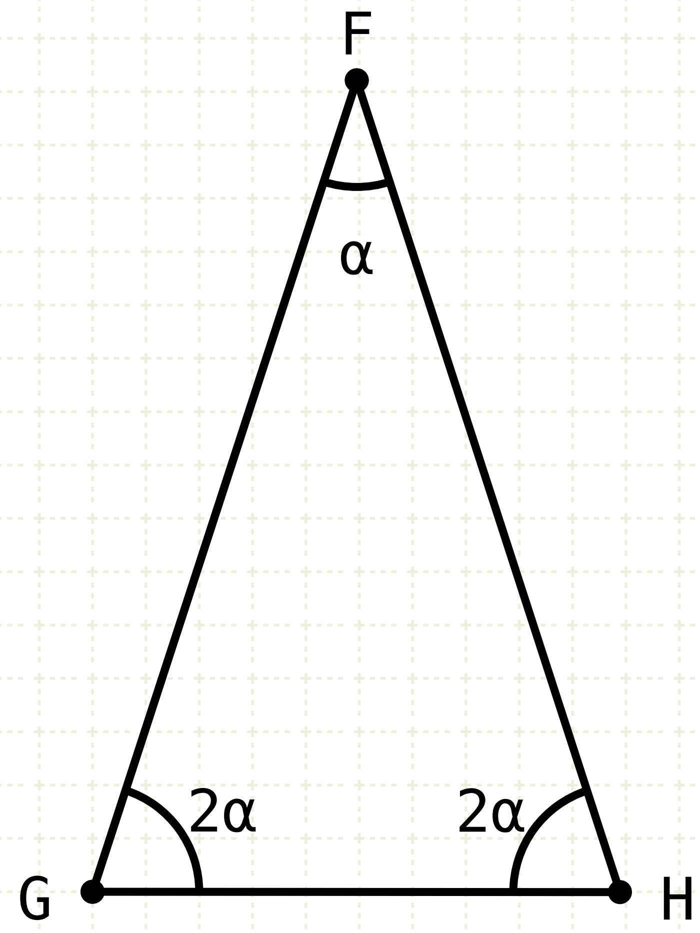
Construction

Draw an isosceles triangle FGH such that the angles at G and H are twice the angle at F (IV·10)

Proposition 11 of Book IV

In a given circle to inscribe an equilateral and equiangular pentagon.





$\angle A = 2\angle C = 2\angle D$

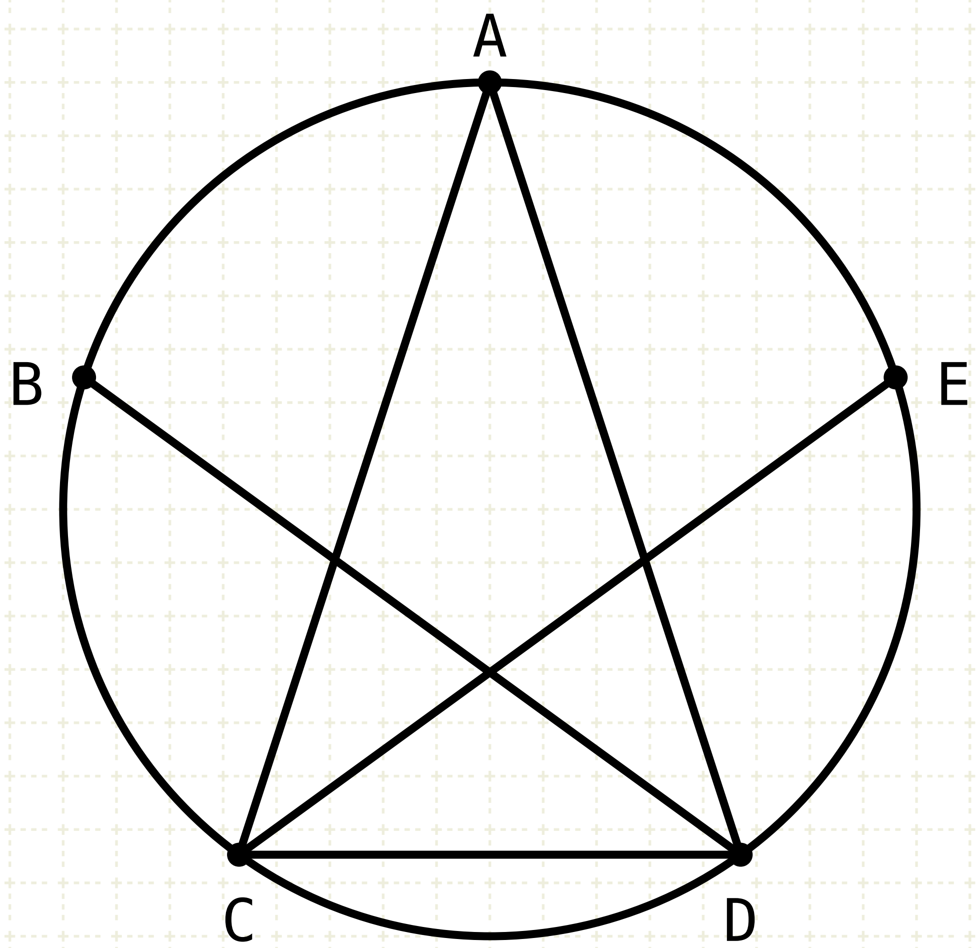
Construction

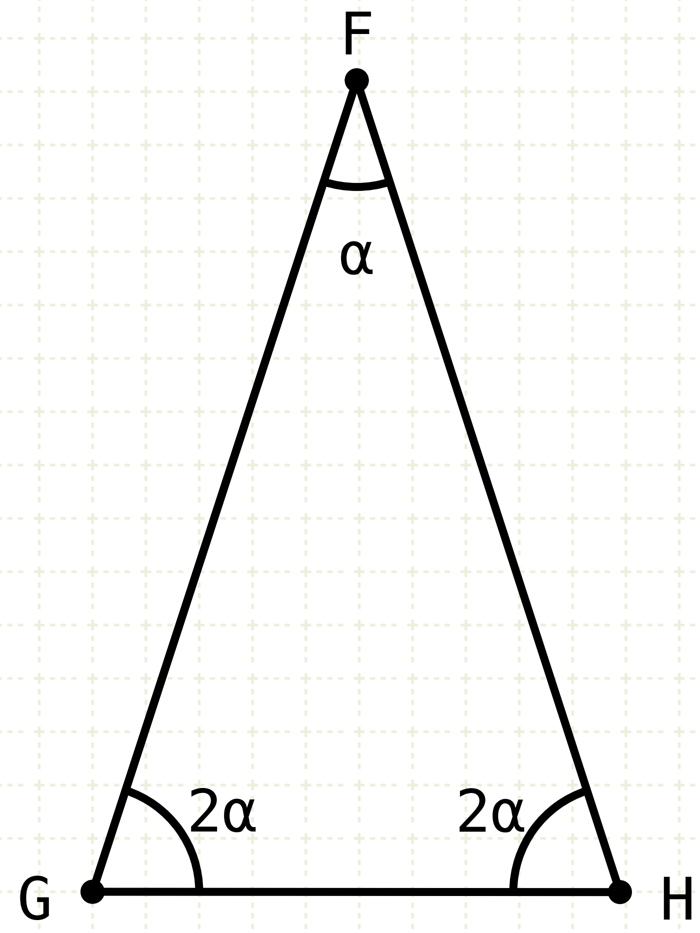
Draw an isosceles triangle FGH such that the angles at G and H are twice the angle at F (IV·10)

Copy this triangle into the circle (IV·2)

Proposition 11 of Book IV

In a given circle to inscribe an equilateral and equiangular pentagon.





Construction

Draw an isosceles triangle FGH such that the angles at G and H are twice the angle at F (IV·10)

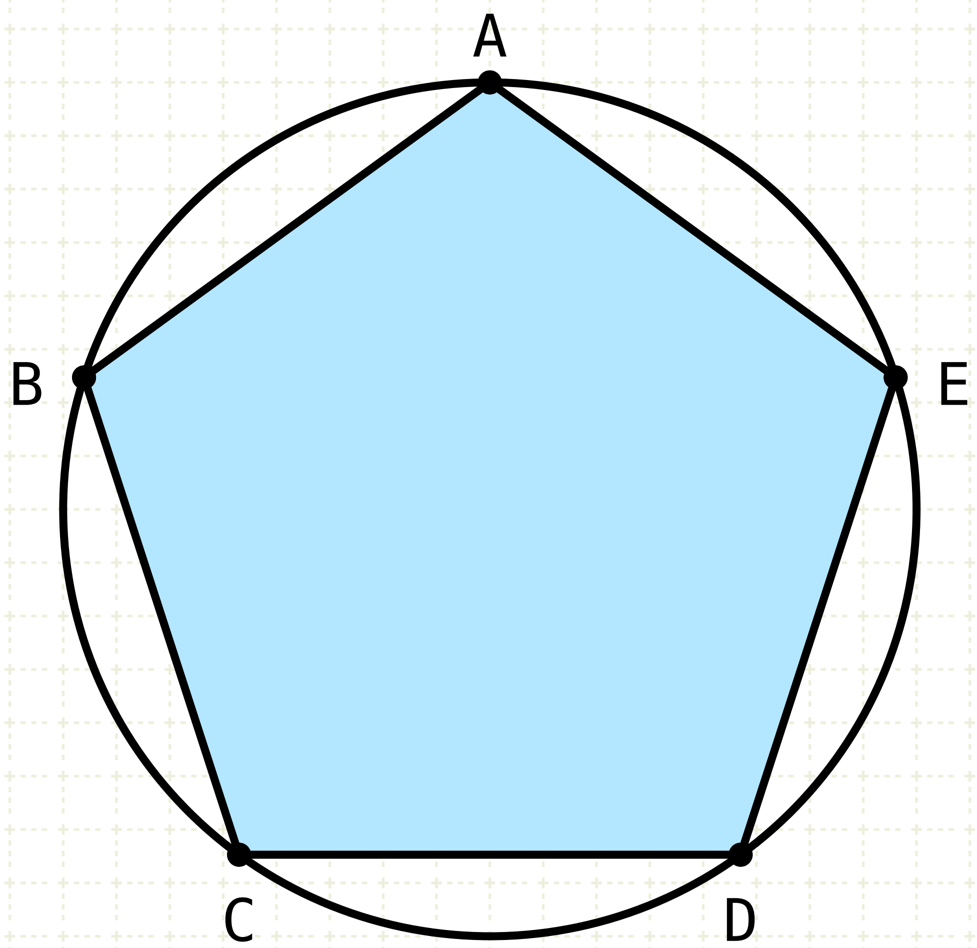
Copy this triangle into the circle (IV·2)

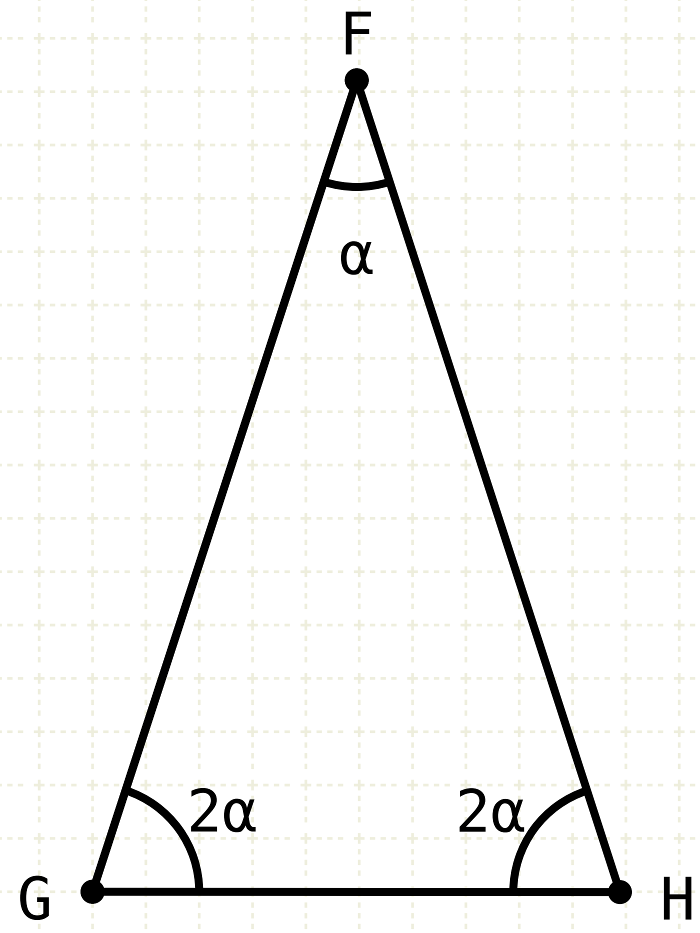
Bisect the angles at C and D with lines CE and DB (I·9)

$\angle A = 2\angle C = 2\angle D$

Proposition 11 of Book IV

In a given circle to inscribe an equilateral and equiangular pentagon.





Construction

Draw an isosceles triangle FGH such that the angles at G and H are twice the angle at F (IV·10)

Copy this triangle into the circle (IV·2)

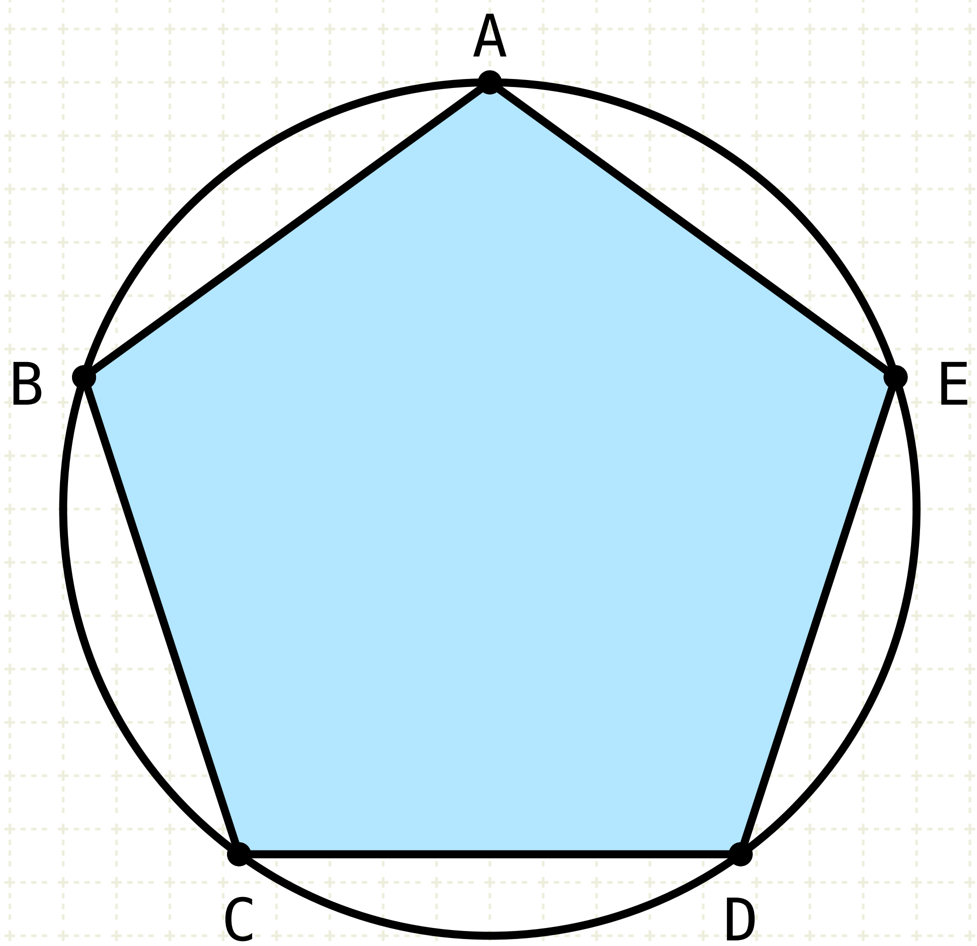
Bisect the angles at C and D with lines CE and DB (I·9)

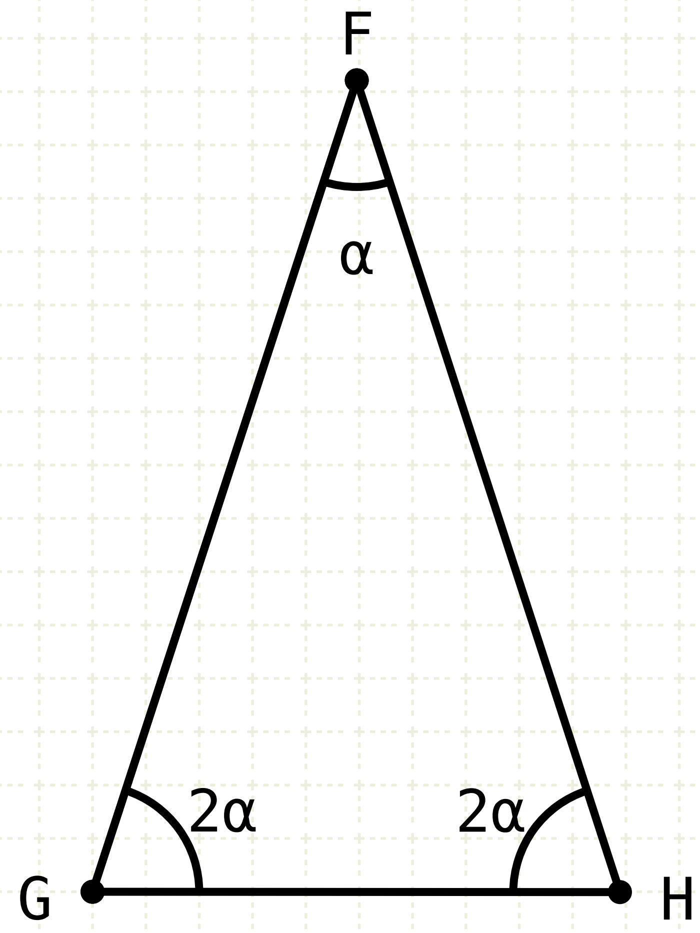
The pentagon ABCDE is equilateral and equiangular

$\angle A = 2\angle C = 2\angle D$

Proposition 11 of Book IV

In a given circle to inscribe an equilateral and equiangular pentagon.



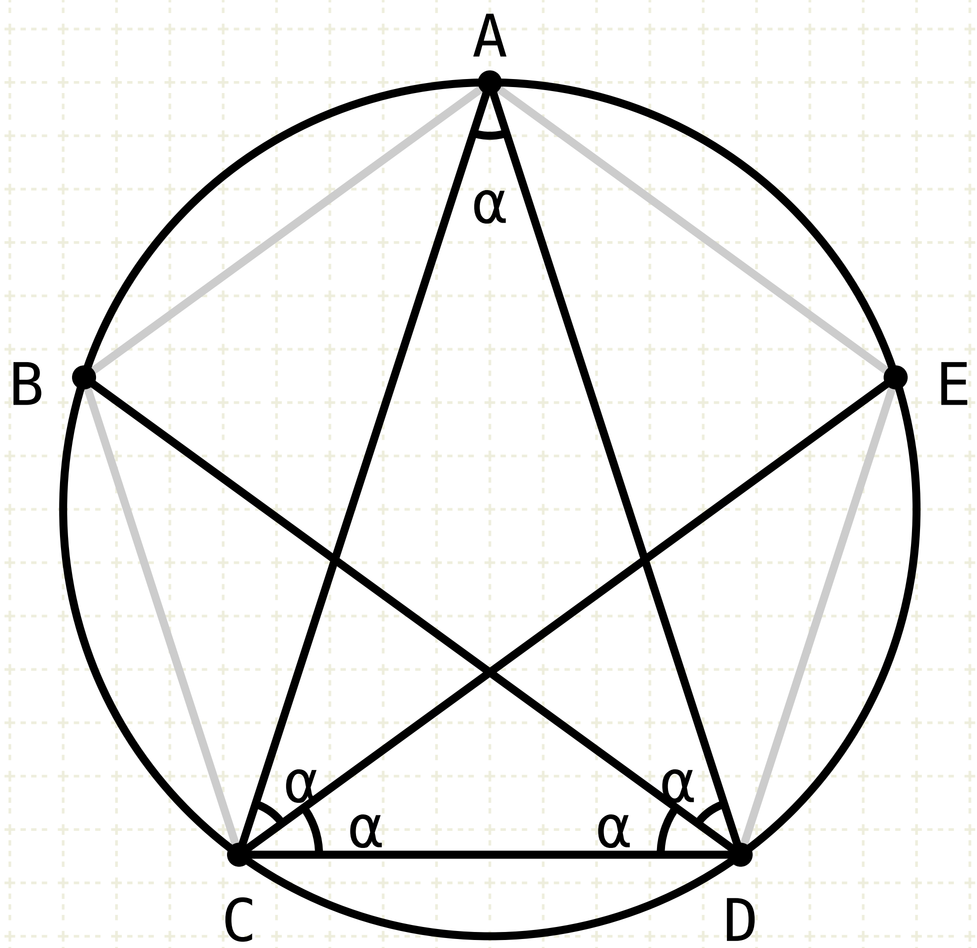


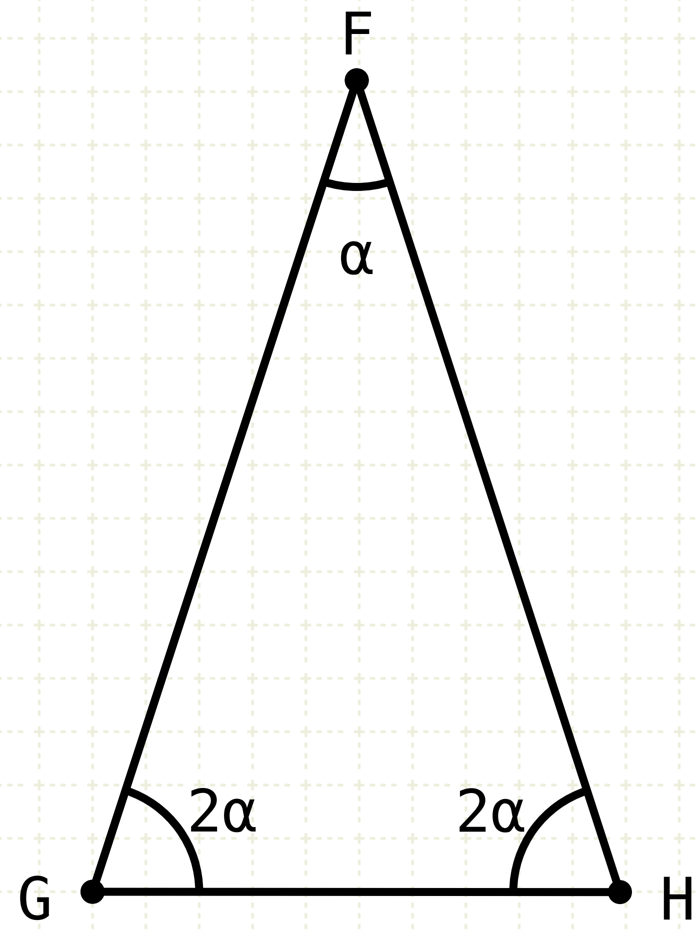
Proof

$\angle A = 2\angle C = 2\angle D$

Proposition 11 of Book IV

In a given circle to inscribe an equilateral and equiangular pentagon.





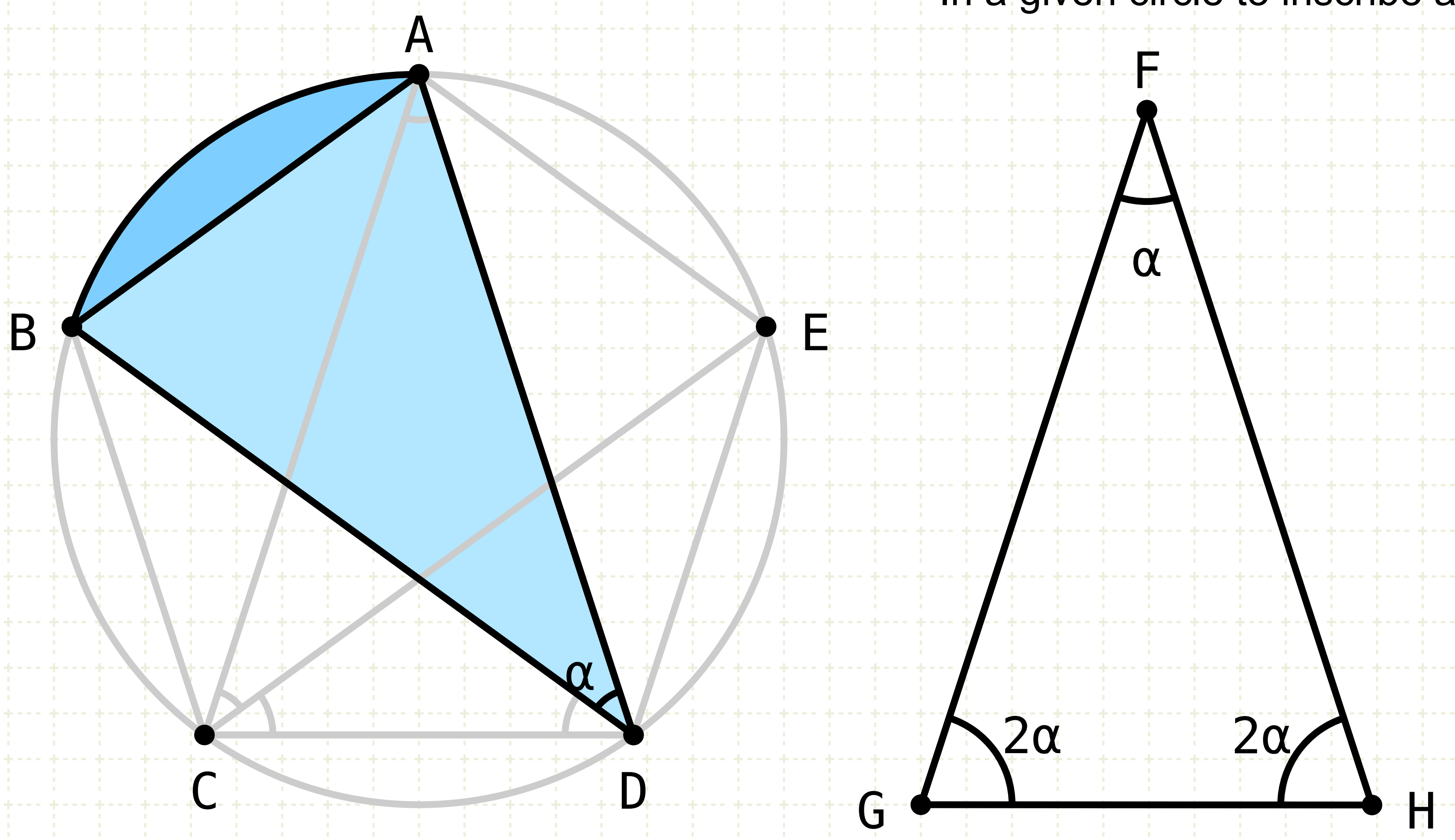
$\angle A = 2\angle C = 2\angle D$

Proof

Because C and D are half of A, and they have been bisected, the angles CAD, ADB, BDC, ECD, ACE are all equal (III·26)

Proposition 11 of Book IV

In a given circle to inscribe an equilateral and equiangular pentagon.



$\angle A = 2\angle C = 2\angle D$

↺AB =

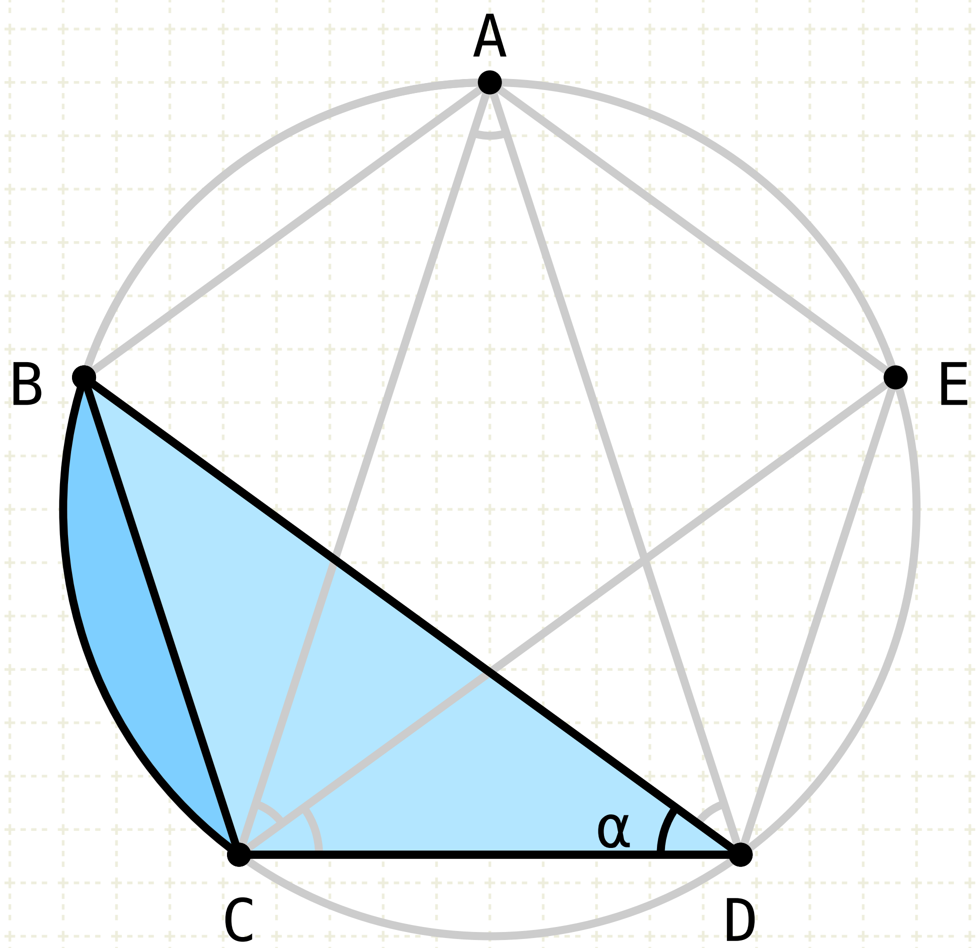
Proof

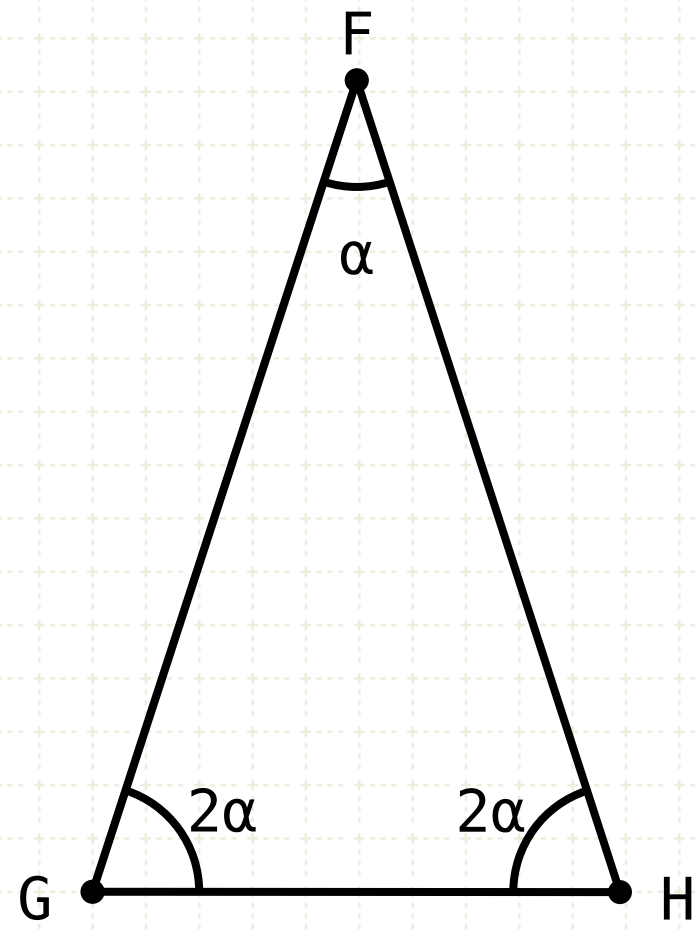
Because C and D are half of A, and they have been bisected, the angles CAD, ADB, BDC, ECD, ACE are all equal (III·26)

Equal angles subtend equal circumferences so the arcs AB, BC, CD, DE, and EA are all equal

Proposition 11 of Book IV

In a given circle to inscribe an equilateral and equiangular pentagon.





Proof

Because C and D are half of A, and they have been bisected, the angles CAD, ADB, BDC, ECD, ACE are all equal (III·26)

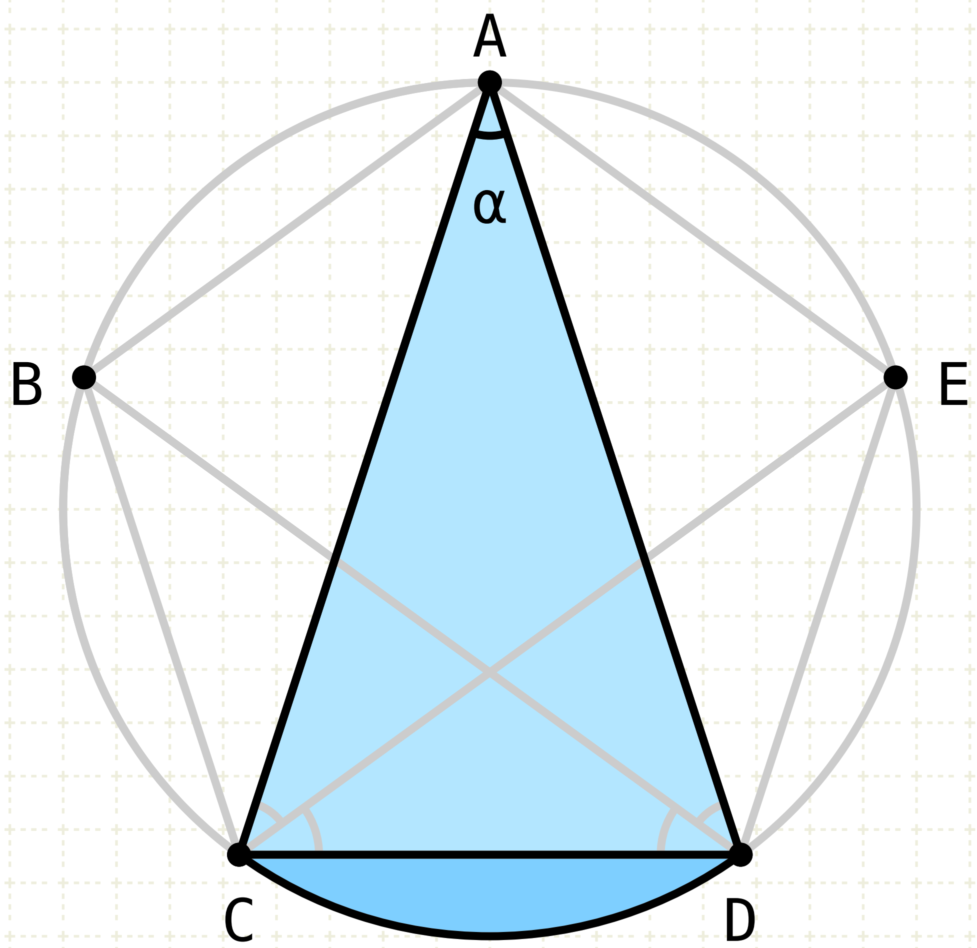
Equal angles subtend equal circumferences so the arcs AB, BC, CD, DE, and EA are all equal

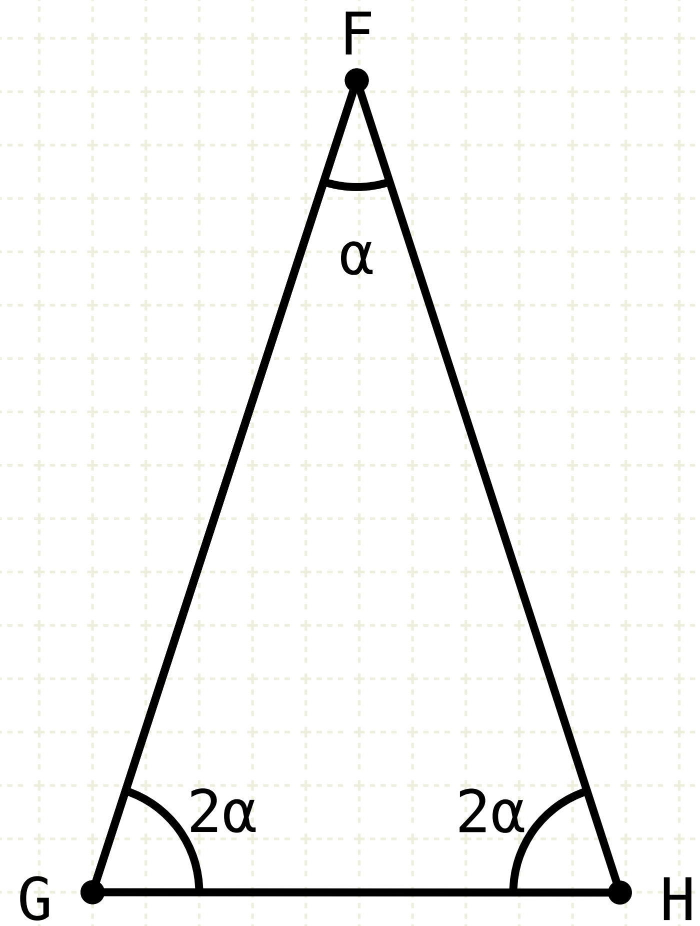
$\angle A = 2\angle C = 2\angle D$

↺AB = ↺BC =

Proposition 11 of Book IV

In a given circle to inscribe an equilateral and equiangular pentagon.





Proof

Because C and D are half of A, and they have been bisected, the angles CAD, ADB, BDC, ECD, ACE are all equal (III·26)

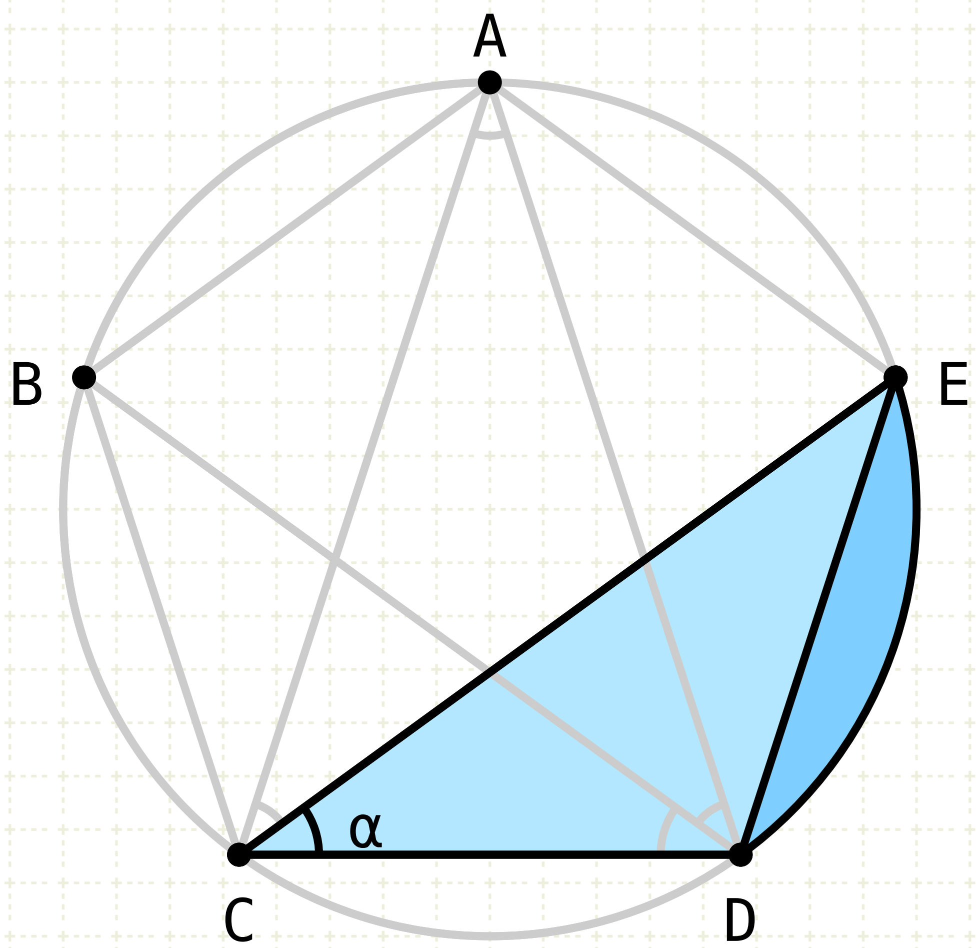
Equal angles subtend equal circumferences so the arcs AB, BC, CD, DE, and EA are all equal

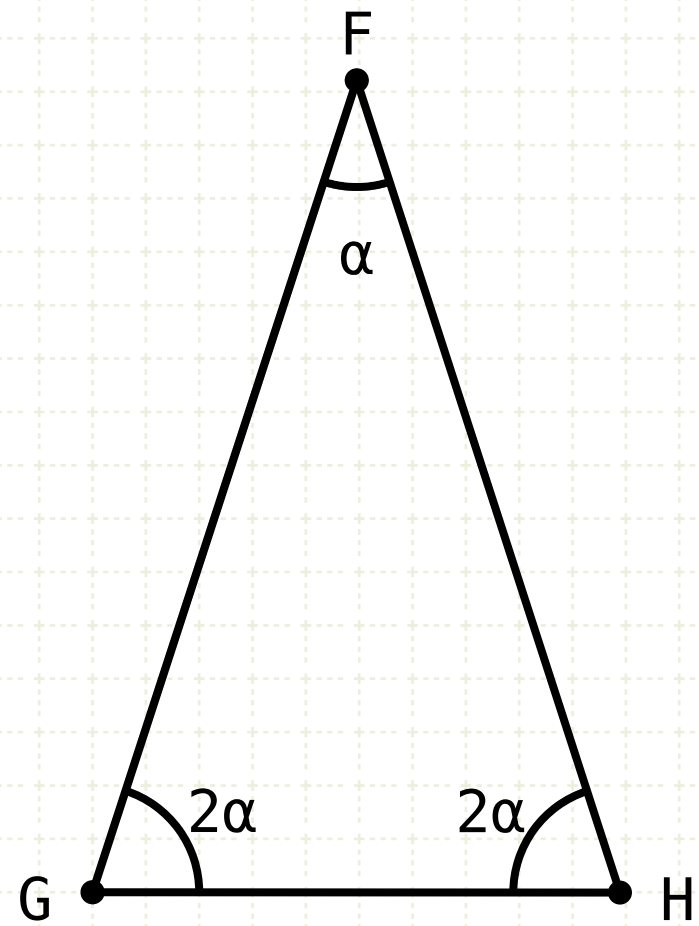
$\angle A = 2\angle C = 2\angle D$

↺AB = ↺BC = ↺CD =

Proposition 11 of Book IV

In a given circle to inscribe an equilateral and equiangular pentagon.





Proof

Because C and D are half of A, and they have been bisected, the angles CAD, ADB, BDC, ECD, ACE are all equal (III·26)

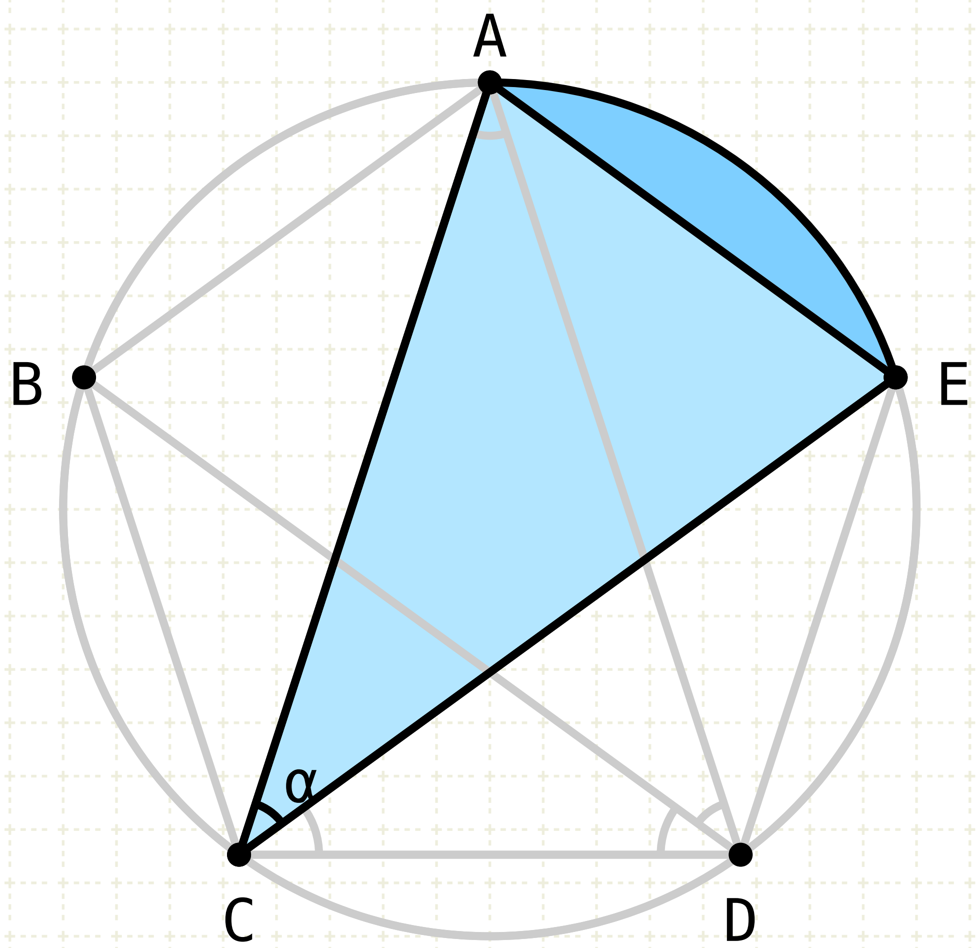
Equal angles subtend equal circumferences so the arcs AB, BC, CD, DE, and EA are all equal

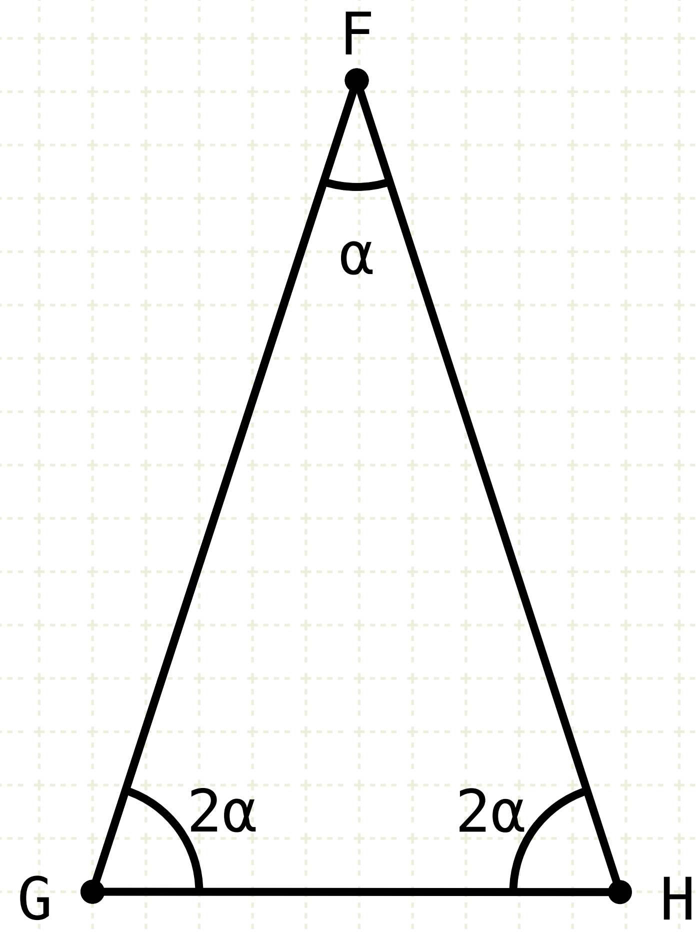
$\angle A = 2\angle C = 2\angle D$

$\circlearrowleft AB = \circlearrowleft BC = \circlearrowleft CD = \circlearrowleft DE =$

Proposition 11 of Book IV

In a given circle to inscribe an equilateral and equiangular pentagon.





Proof

Because C and D are half of A, and they have been bisected, the angles CAD, ADB, BDC, ECD, ACE are all equal (III·26)

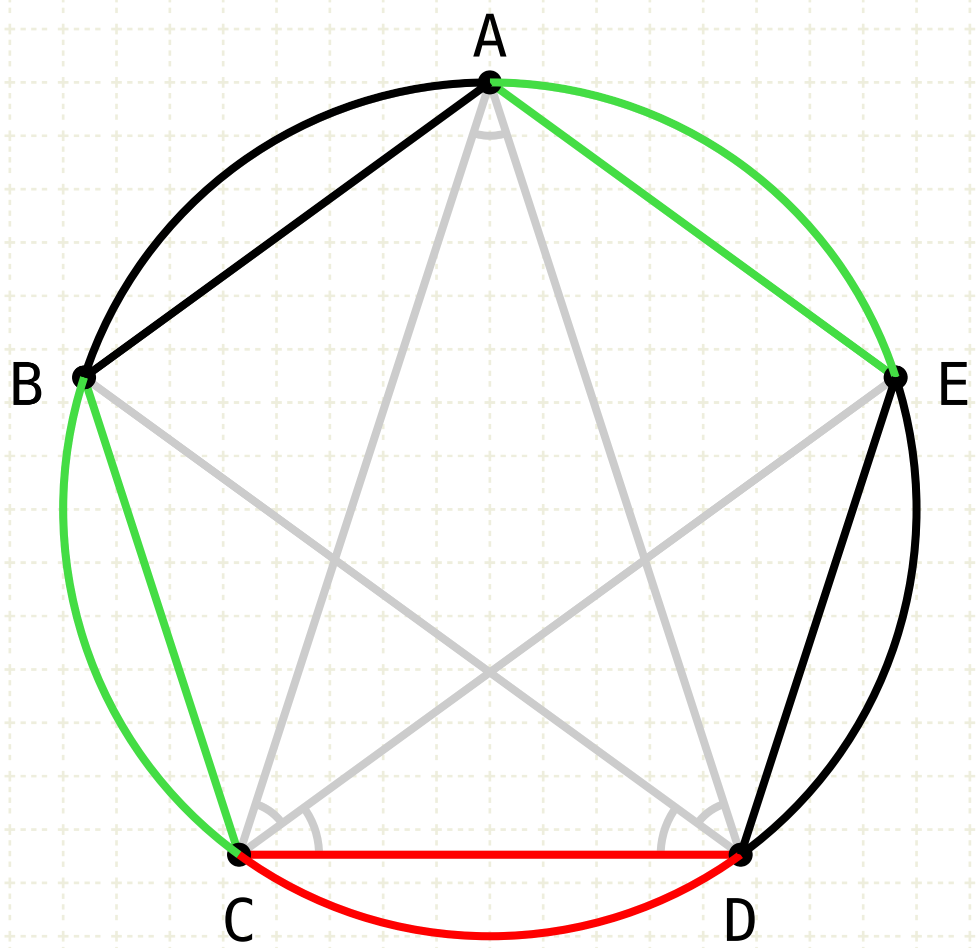
Equal angles subtend equal circumferences so the arcs AB, BC, CD, DE, and EA are all equal

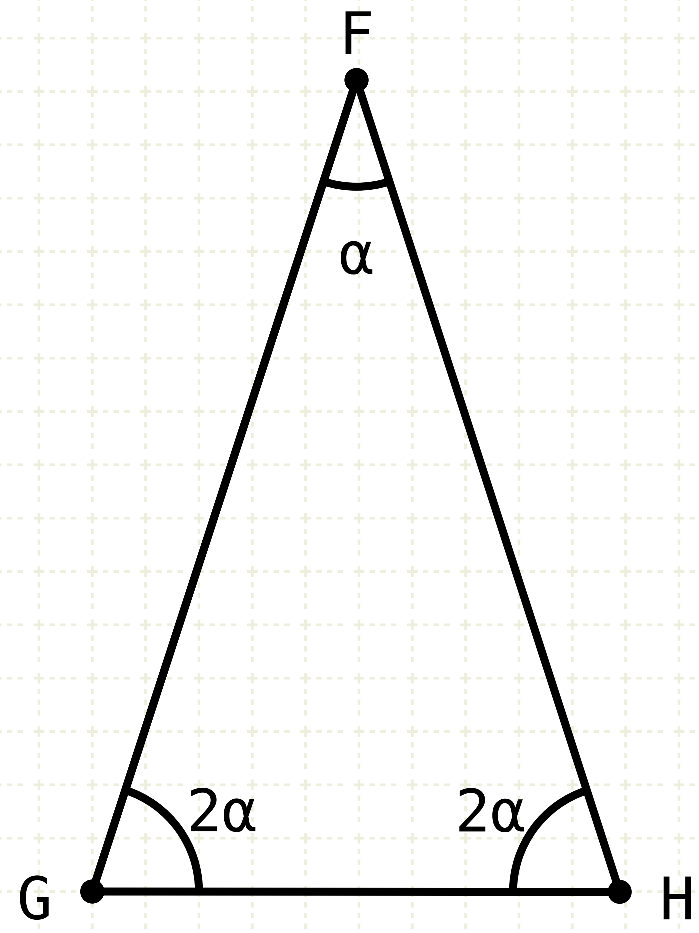
$\angle A = 2\angle C = 2\angle D$

$\overset{\frown}{AB} = \overset{\frown}{BC} = \overset{\frown}{CD} = \overset{\frown}{DE} = \overset{\frown}{EA}$

Proposition 11 of Book IV

In a given circle to inscribe an equilateral and equiangular pentagon.





Proof

Because C and D are half of A, and they have been bisected, the angles CAD, ADB, BDC, ECD, ACE are all equal (III·26)

Equal angles subtend equal circumferences so the arcs AB, BC, CD, DE, and EA are all equal

If the circumferences are equal, so are the lines subtending the circumferences (III·29)

Therefore the pentagon is equilateral

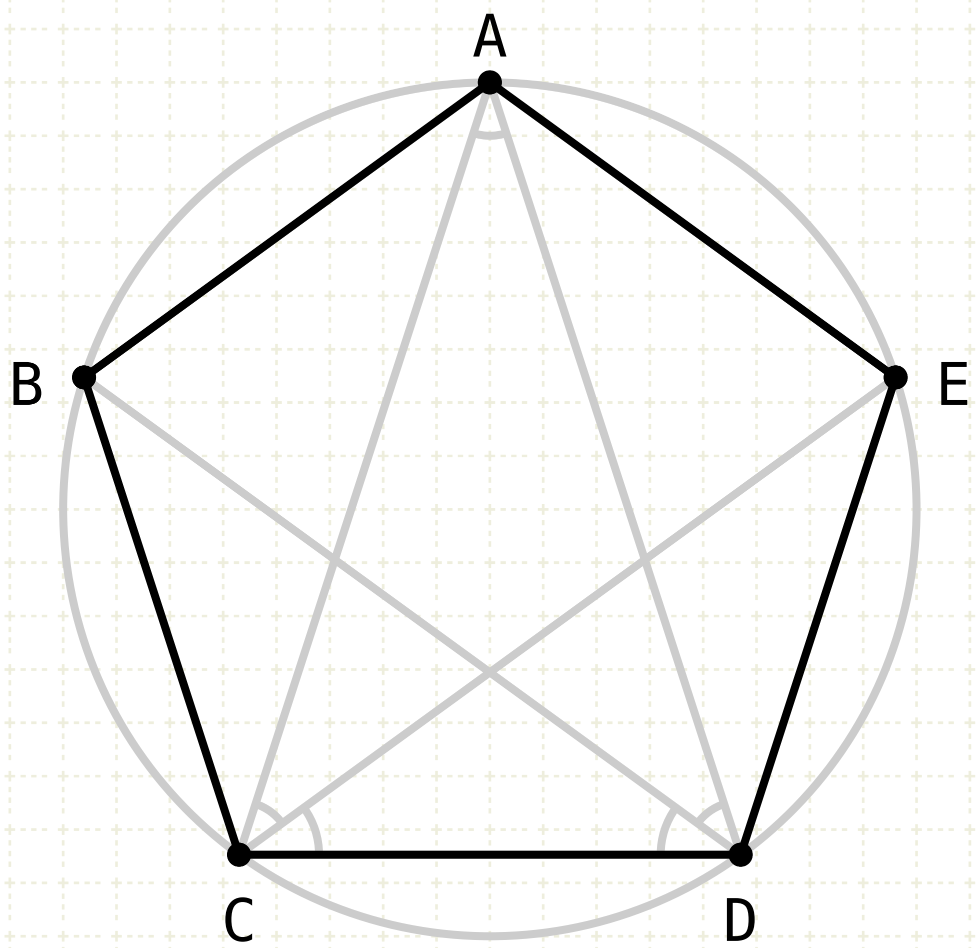
$\angle A = 2\angle C = 2\angle D$

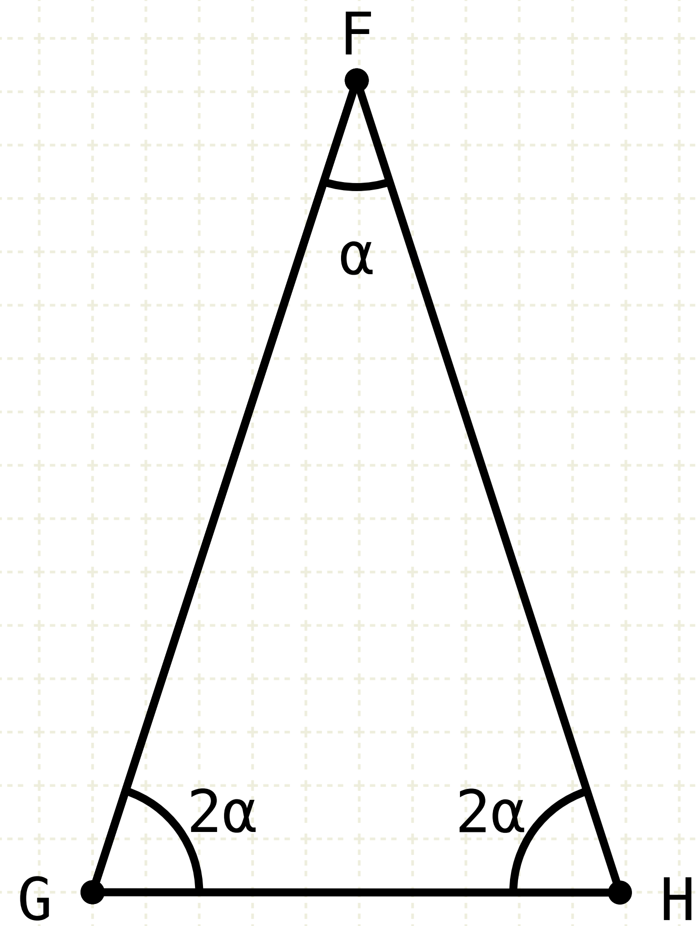
↺AB = ↺BC = ↺CD = ↺DE = ↺EA

AB = BC = CD = DE = EA

Proposition 11 of Book IV

In a given circle to inscribe an equilateral and equiangular pentagon.





Proof

Because C and D are half of A, and they have been bisected, the angles CAD, ADB, BDC, ECD, ACE are all equal (III·26)

Equal angles subtend equal circumferences so the arcs AB, BC, CD, DE, and EA are all equal

If the circumferences are equal, so are the lines subtending the circumferences (III·29)

Therefore the pentagon is equilateral

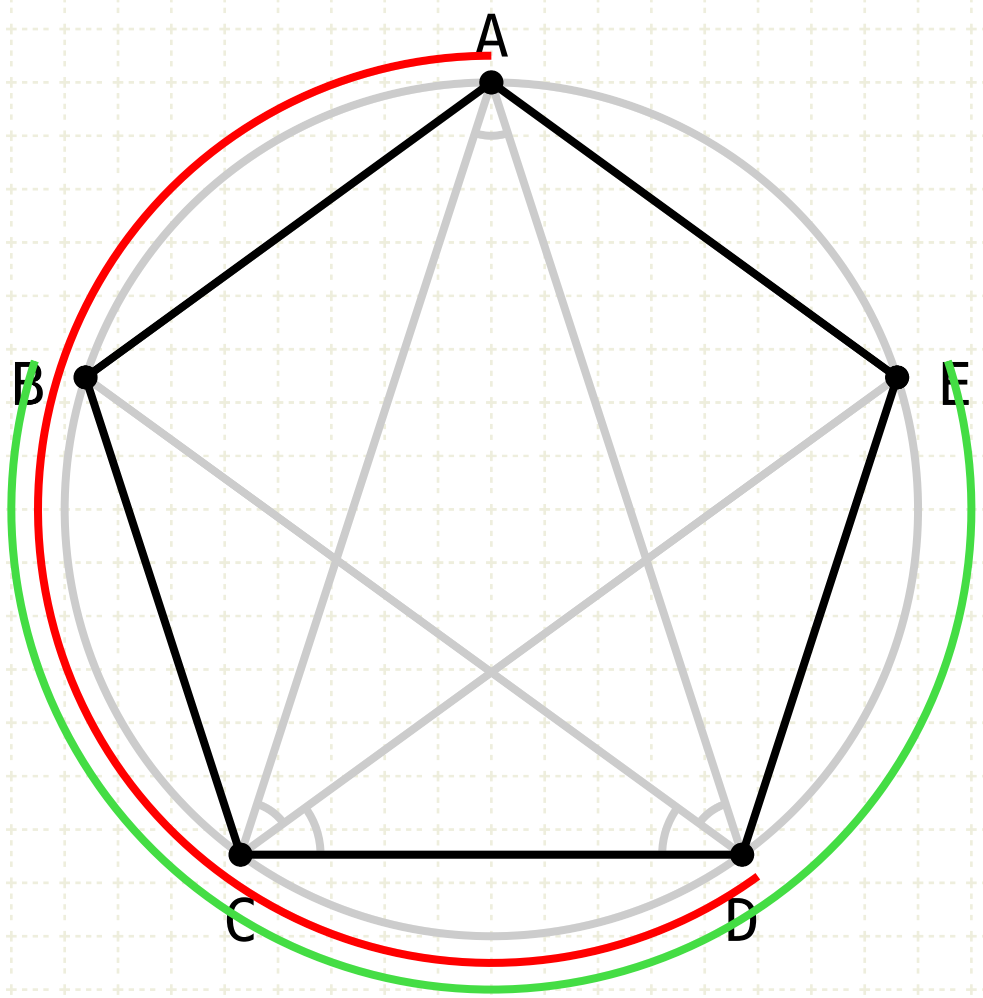
$\angle A = 2\angle C = 2\angle D$

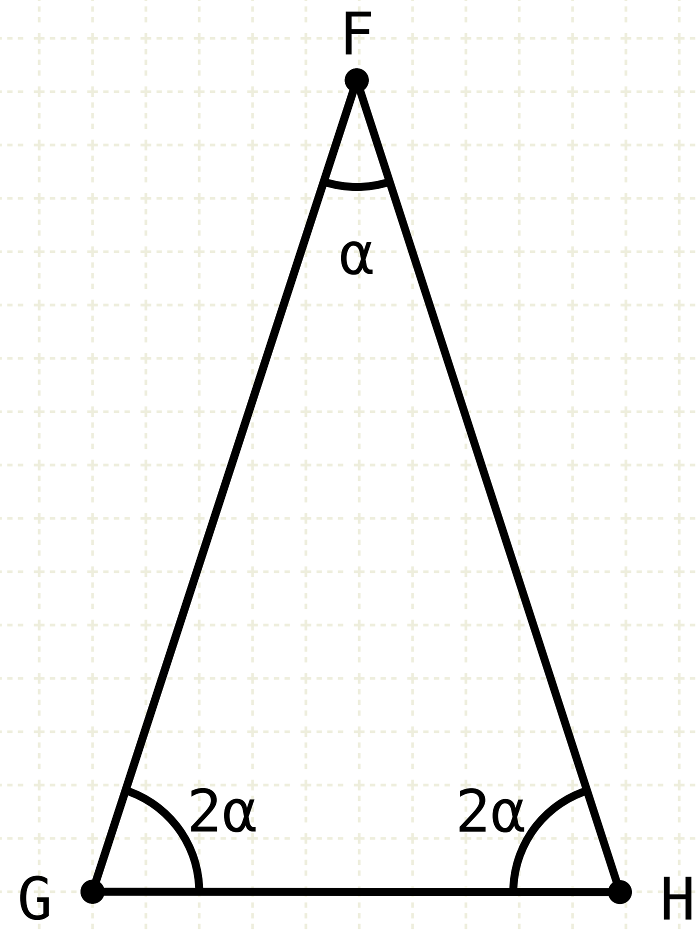
$\overset{\frown}{AB} = \overset{\frown}{BC} = \overset{\frown}{CD} = \overset{\frown}{DE} = \overset{\frown}{EA}$

$AB = BC = CD = DE = EA$

Proposition 11 of Book IV

In a given circle to inscribe an equilateral and equiangular pentagon.





Proof

Because C and D are half of A, and they have been bisected, the angles CAD, ADB, BDC, ECD, ACE are all equal (III·26)

Equal angles subtend equal circumferences so the arcs AB, BC, CD, DE, and EA are all equal

If the circumferences are equal, so are the lines subtending the circumferences (III·29)

Therefore the pentagon is equilateral

Because each circumference is equal, if the arcs BC,CD are added to AB and DE respectively, it follows that the circumference ABCD is equal to the circumference BCDE

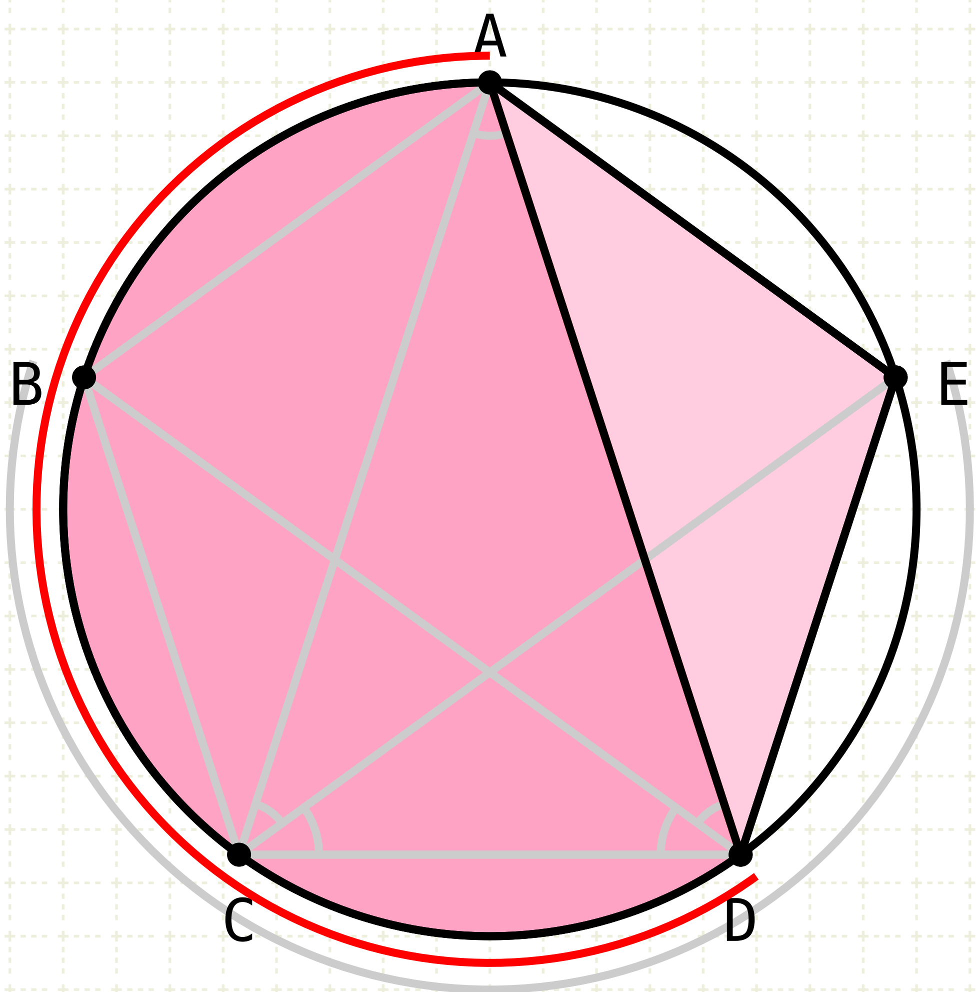
$\angle A = 2\angle C = 2\angle D$

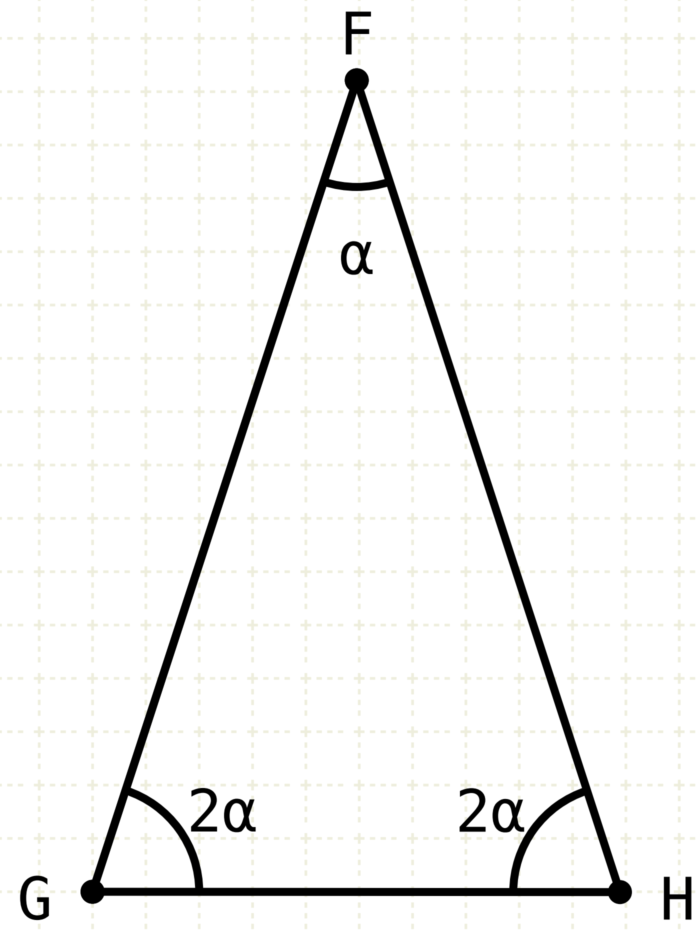
↺AB = ↺BC = ↺CD = ↺DE = ↺EA

AB = BC = CD = DE = EA

Proposition 11 of Book IV

In a given circle to inscribe an equilateral and equiangular pentagon.





Proof

Because C and D are half of A, and they have been bisected, the angles CAD, ADB, BDC, ECD, ACE are all equal (III·26)

Equal angles subtend equal circumferences so the arcs AB, BC, CD, DE, and EA are all equal

If the circumferences are equal, so are the lines subtending the circumferences (III·29)

Therefore the pentagon is equilateral

Because each circumference is equal, if the arcs BC,CD are added to AB and DE respectively, it follows that the circumference ABCD is equal to the circumference BCDE

Thus, since equal angles stand on equal circumferences the angles AED and BAE are equal (III·27)

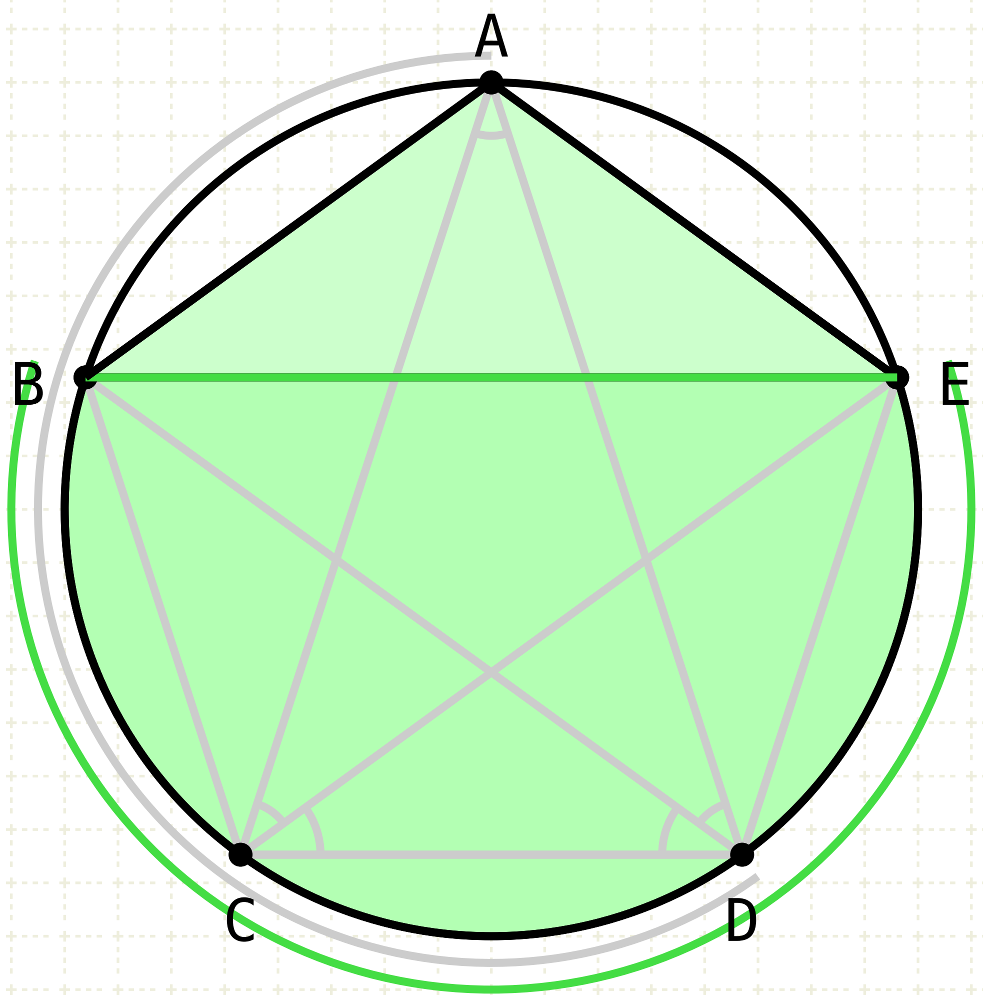
$\angle A = 2\angle C = 2\angle D$

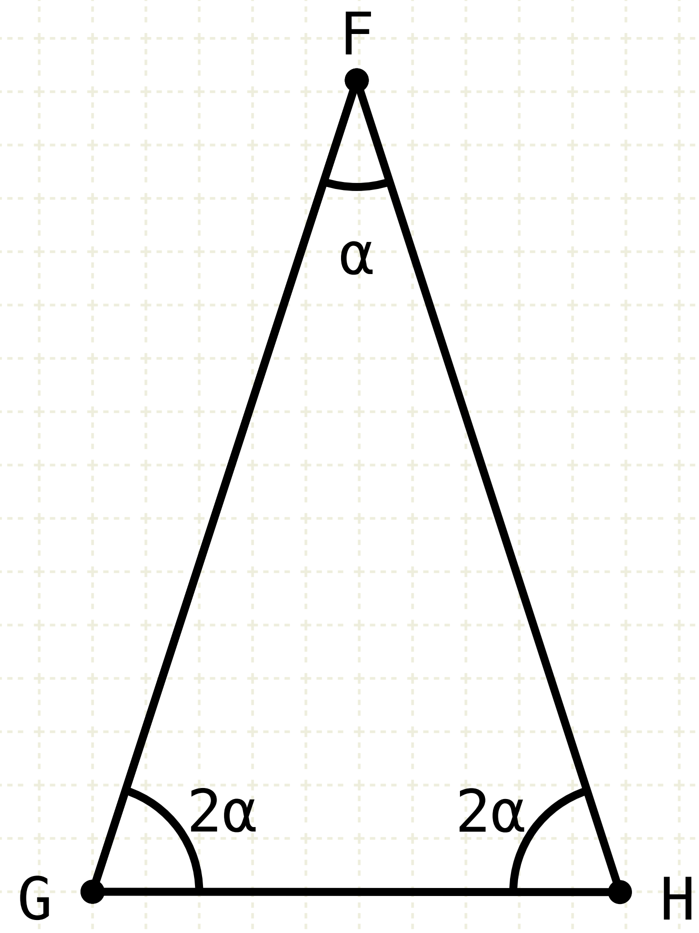
↺AB = ↺BC = ↺CD = ↺DE = ↺EA

AB = BC = CD = DE = EA

Proposition 11 of Book IV

In a given circle to inscribe an equilateral and equiangular pentagon.





Proof

Because C and D are half of A, and they have been bisected, the angles CAD, ADB, BDC, ECD, ACE are all equal (III·26)

Equal angles subtend equal circumferences so the arcs AB, BC, CD, DE, and EA are all equal

If the circumferences are equal, so are the lines subtending the circumferences (III·29)

Therefore the pentagon is equilateral

Because each circumference is equal, if the arcs BC,CD are added to AB and DE respectively, it follows that the circumference ABCD is equal to the circumference BCDE

Thus, since equal angles stand on equal circumferences the angles AED and BAE are equal (III·27)

$\angle A = 2\angle C = 2\angle D$

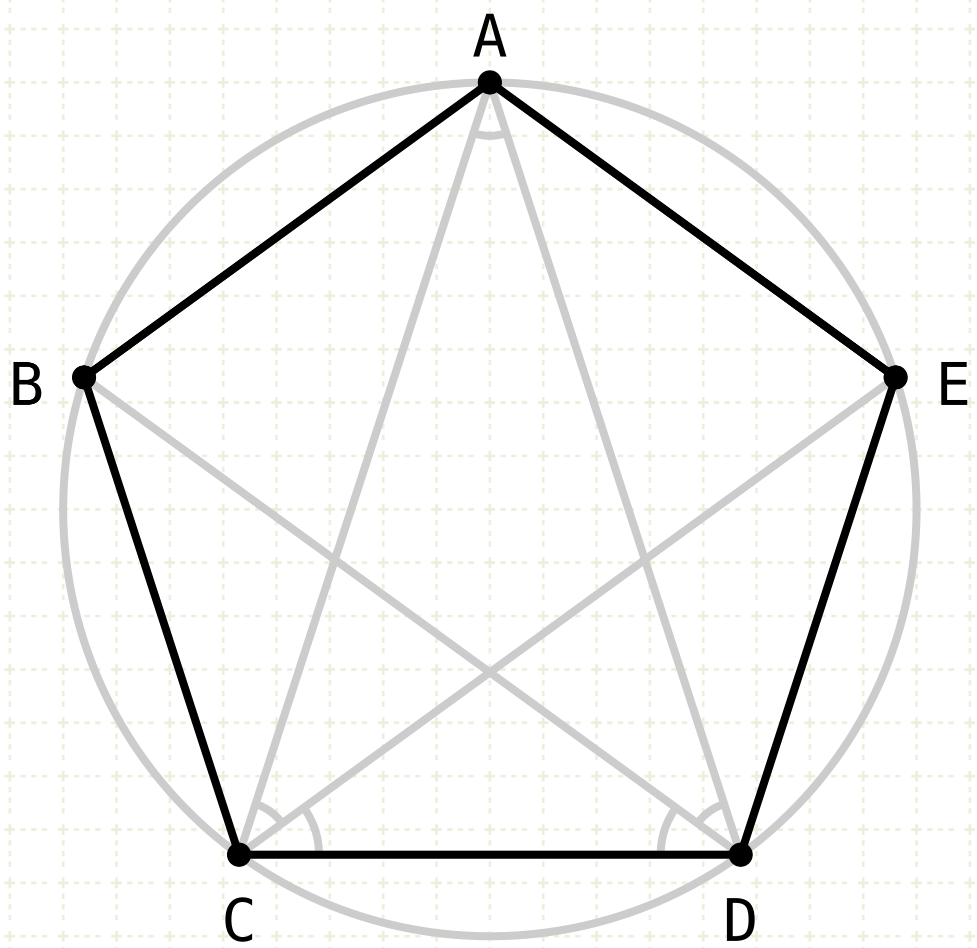
↺AB = ↺BC = ↺CD = ↺DE = ↺EA

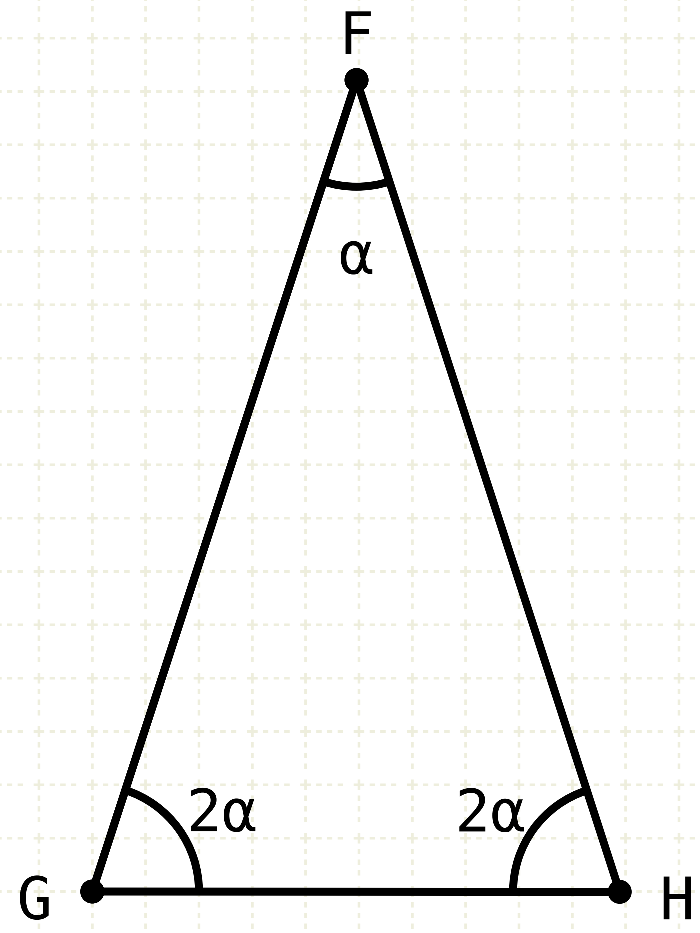
AB = BC = CD = DE = EA

$\angle BAE = \angle AED$

Proposition 11 of Book IV

In a given circle to inscribe an equilateral and equiangular pentagon.





$\angle A = 2\angle C = 2\angle D$

$\circlearrowleft AB = \circlearrowleft BC = \circlearrowleft CD = \circlearrowleft DE = \circlearrowleft EA$

$AB = BC = CD = DE = EA$

$\angle BAE = \angle AED$

$\angle BAE = \angle AED = \angle EDC = \angle DCB = \angle CBA$

Proof

Because C and D are half of A, and they have been bisected, the angles CAD, ADB, BDC, ECD, ACE are all equal (III·26)

Equal angles subtend equal circumferences so the arcs AB, BC, CD, DE, and EA are all equal

If the circumferences are equal, so are the lines subtending the circumferences (III·29)

Therefore the pentagon is equilateral

Because each circumference is equal, if the arcs BC,CD are added to AB and DE respectively, it follows that the circumference ABCD is equal to the circumference BCDE

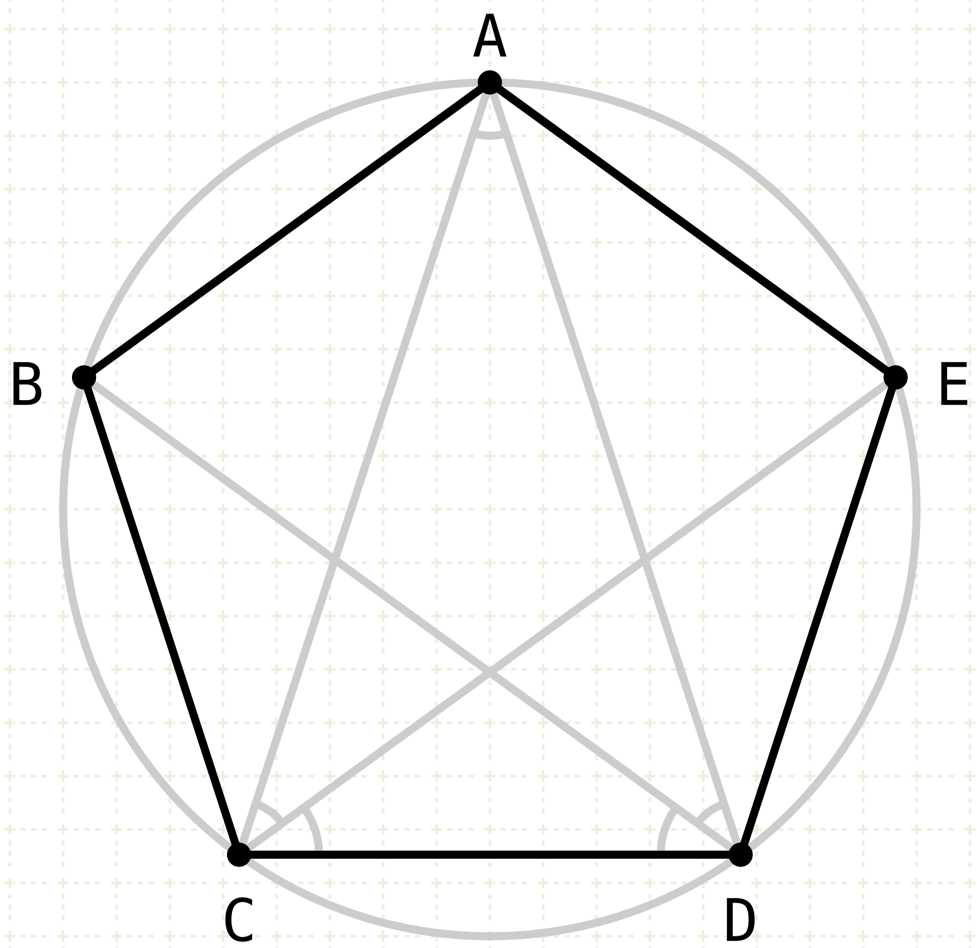
Thus, since equal angles stand on equal circumferences the angles AED and BAE are equal (III·27)

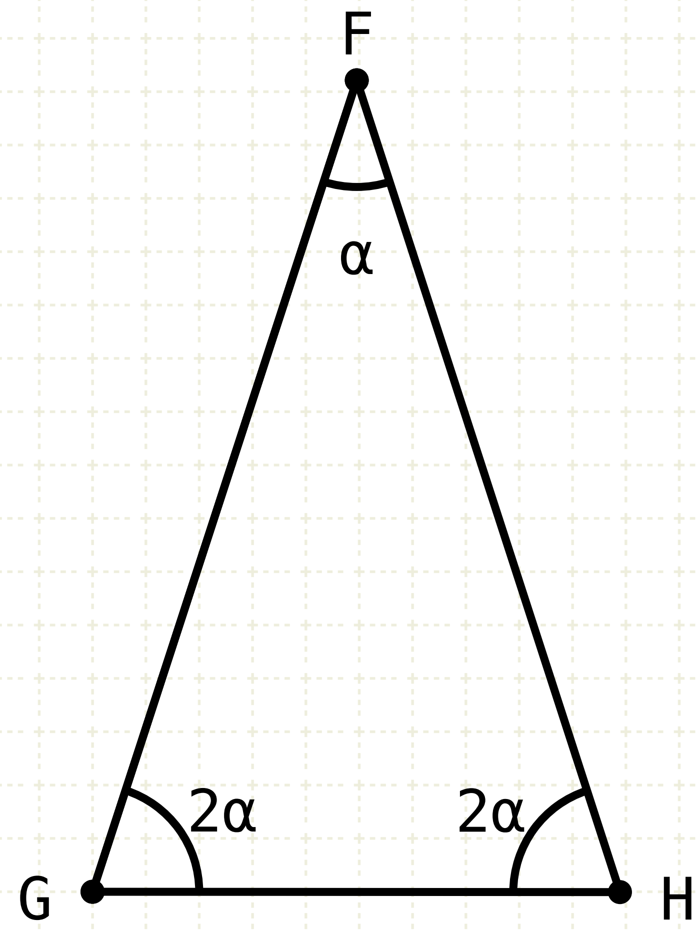
Similarly, all the angles of the pentagon can be shown to be equal

Therefore, the pentagon is equiangular

Proposition 11 of Book IV

In a given circle to inscribe an equilateral and equiangular pentagon.





Proof

Because C and D are half of A, and they have been bisected, the angles CAD, ADB, BDC, ECD, ACE are all equal (III·26)

Equal angles subtend equal circumferences so the arcs AB, BC, CD, DE, and EA are all equal

If the circumferences are equal, so are the lines subtending the circumferences (III·29)

Therefore the pentagon is equilateral

Because each circumference is equal, if the arcs BC,CD are added to AB and DE respectively, it follows that the circumference ABCD is equal to the circumference BCDE

Thus, since equal angles stand on equal circumferences the angles AED and BAE are equal (III·27)

Similarly, all the angles of the pentagon can be shown to be equal

Therefore, the pentagon is equiangular

Thus a regular pentagon has been drawn

$\angle A = 2\angle C = 2\angle D$

$\circlearrowleft AB = \circlearrowleft BC = \circlearrowleft CD = \circlearrowleft DE = \circlearrowleft EA$

$AB = BC = CD = DE = EA$

$\angle BAE = \angle AED$

$\angle BAE = \angle AED = \angle EDC = \angle DCB = \angle CBA$

Youtube Videos

https://www.youtube.com/c/SandyBultena