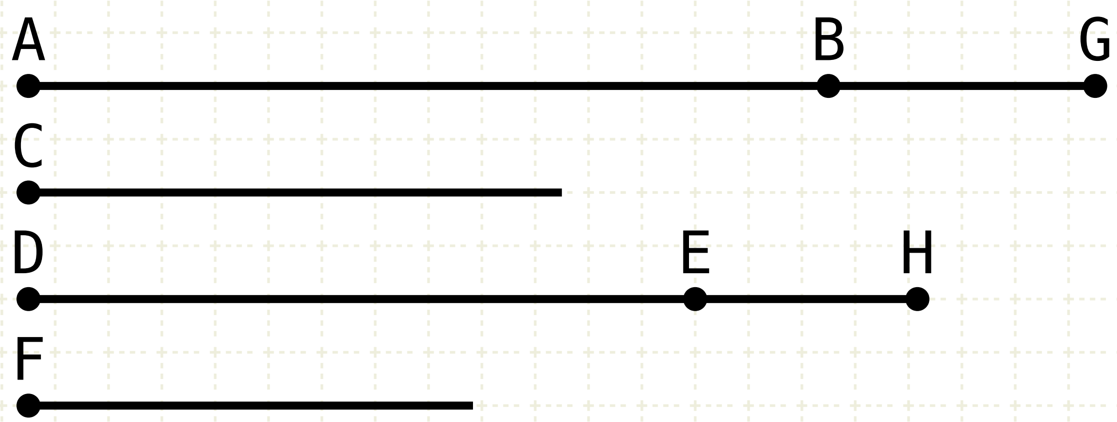
Euclid's Elements

Book V



$AB:C = DE:F$
$BG:C = EH:F$

$AG:C = DH:F$

Proportions are what makes the old Greek temples classic in their beauty. They are like huge blocks, from which the air has been literally hewn out between the columns.

Arne Jacobsen

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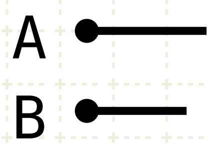
Proposition 4 of Book V

If a first magnitude have to a second the same ratio as a third to a fourth, any equimultiples whatever of the first and third will also have the same ratio to any equimultiples whatever of the second and fourth respectively, taken in corresponding order.

Proposition 4 of Book V

If a first magnitude have to a second the same ratio as a third to a fourth, any equimultiples whatever of the first and third will also have the same ratio to any equimultiples whatever of the second and fourth respectively, taken in corresponding order.



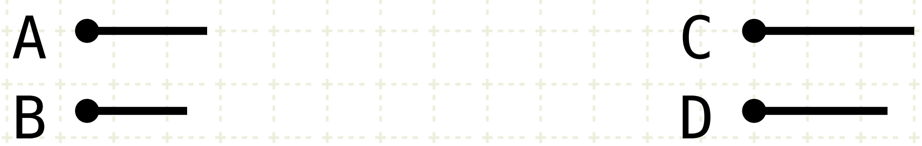
A and B have a ratio (A:B) if there exists a 'p' and 'q' such that $p \cdot A > B$, and $A < q \cdot B$

Definitions

3. A RATIO is a sort of relation in respect of size between two magnitudes of the same kind
4. Magnitudes are said to HAVE A RATIO to one another which are capable, when multiplied, of exceeding one another

Proposition 4 of Book V

If a first magnitude have to a second the same ratio as a third to a fourth, any equimultiples whatever of the first and third will also have the same ratio to any equimultiples whatever of the second and fourth respectively, taken in corresponding order.

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A and B have a ratio (A:B) if there
exists a 'p' and 'q' such that
p·A > B, and A < q·B

The ratio A:B = C:D if, for any number 'p' and 'q'
  if p·A > q·B then p·C > q·D
  if p·A < q·B then p·C < q·D
  if p·A = q·B then p·C = q·D

  p·A >=< q·B → p·C >=< q·D

if   A:B = C:D
then p·A >=< q·B → p·C >=< q·D
```

Definitions

3. A RATIO is a sort of relation in respect of size between two magnitudes of the same kind
4. Magnitudes are said to HAVE A RATIO to one another which are capable, when multiplied, of exceeding one another
5. Magnitudes are said to BE IN THE SAME RATIO, the first to the second and the third to the fourth, when, if any equimultiples whatever be taken of the first and third, and any equimultiples whatever of the second and fourth, the former equimultiples alike exceed, are alike equal to, or alike fall short of, the latter equimultiples respectively taken in corresponding order

Proposition 4 of Book V

If a first magnitude have to a second the same ratio as a third to a fourth, any equimultiples whatever of the first and third will also have the same ratio to any equimultiples whatever of the second and fourth respectively, taken in corresponding order.

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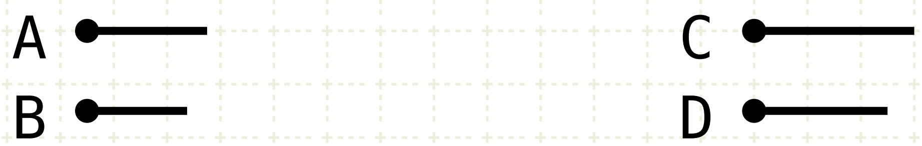

In other words

Let the ratio of A to B be the same ratio C to D

$$A:B = C:D$$

$$pA >=< qB \rightarrow pC >=< qD$$

Proposition 4 of Book V

If a first magnitude have to a second the same ratio as a third to a fourth, any equimultiples whatever of the first and third will also have the same ratio to any equimultiples whatever of the second and fourth respectively, taken in corresponding order.

In other words

Let the ratio of A to B be the same ratio C to D

Draw lines E and F that are equimultiple to A and C

$$A:B = C:D$$
$$pA >=< qB \rightarrow pC >=< qD$$

$$E = i \cdot A$$
$$F = i \cdot C$$

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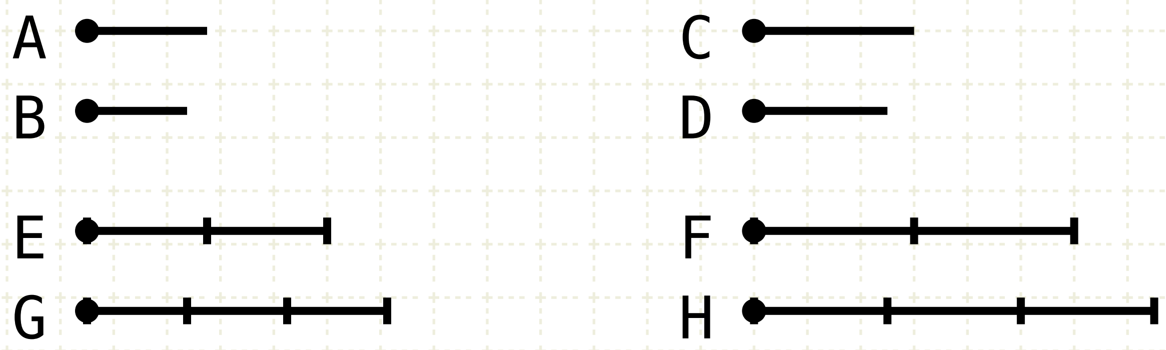

In other words

Let the ratio of A to B be the same ratio C to D

Draw lines E and F that are equimultiple to A and C

Draw lines G and H that are equimultiple to B and D

$$A:B = C:D$$
$$pA >=< qB \rightarrow pC >=< qD$$

$$E = i \cdot A$$
$$F = i \cdot C$$
$$G = j \cdot B$$
$$H = j \cdot D$$

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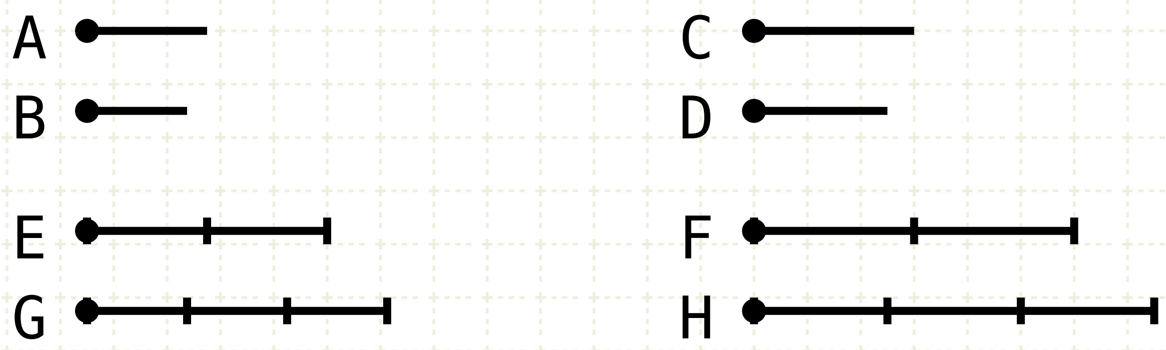

In other words

Let the ratio of A to B be the same ratio C to D

Draw lines E and F that are equimultiple to A and C

Draw lines G and H that are equimultiple to B and D

The ratio E to G is equal to the ratio F to H

$$A:B = C:D$$
$$pA >=< qB \rightarrow pC >=< qD$$

$$E = i \cdot A$$
$$F = i \cdot C$$
$$G = j \cdot B$$
$$H = j \cdot D$$

$$E:G = F:H$$

… or if $A:B = C:D$

then $i \cdot A : j \cdot B = i \cdot C : j \cdot D$

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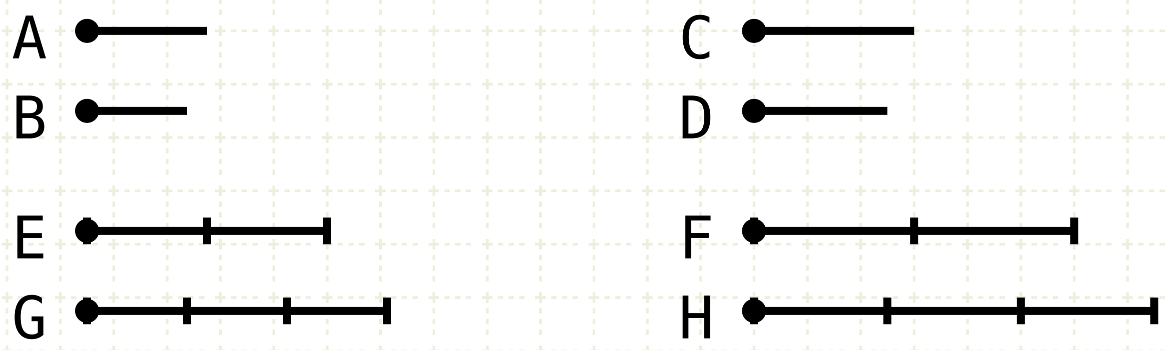


In other words

Let the ratio of A to B be the same ratio C to D

Draw lines E and F that are equimultiple to A and C

Draw lines G and H that are equimultiple to B and D

The ratio E to G is equal to the ratio F to H

Proof

A:B = C:D

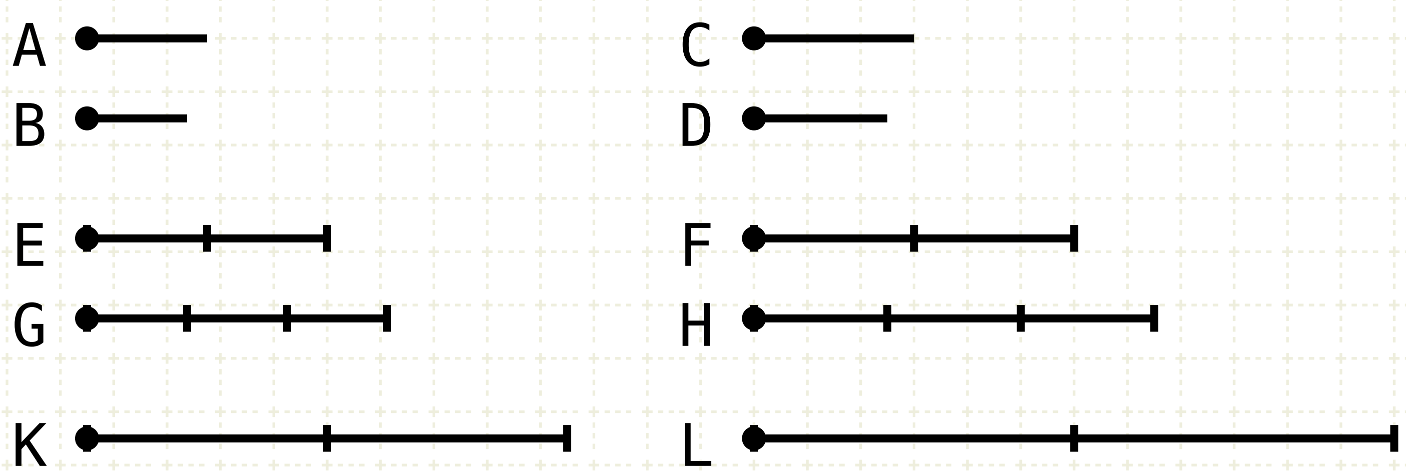
pA >=< qB → pC >=< qD

E = i·A, F = i·C

G = j·B, H = j·D

Proposition 4 of Book V

If a first magnitude have to a second the same ratio as a third to a fourth, any equimultiples whatever of the first and third will also have the same ratio to any equimultiples whatever of the second and fourth respectively, taken in corresponding order.



$A:B = C:D$

$pA \gtreqless qB \rightarrow pC \gtreqless qD$

$E = i \cdot A,\ F = i \cdot C$

$G = j \cdot B,\ H = j \cdot D$

$K = mE,\ L = mF$

In other words

Let the ratio of A to B be the same ratio C to D

Draw lines E and F that are equimultiple to A and C

Draw lines G and H that are equimultiple to B and D

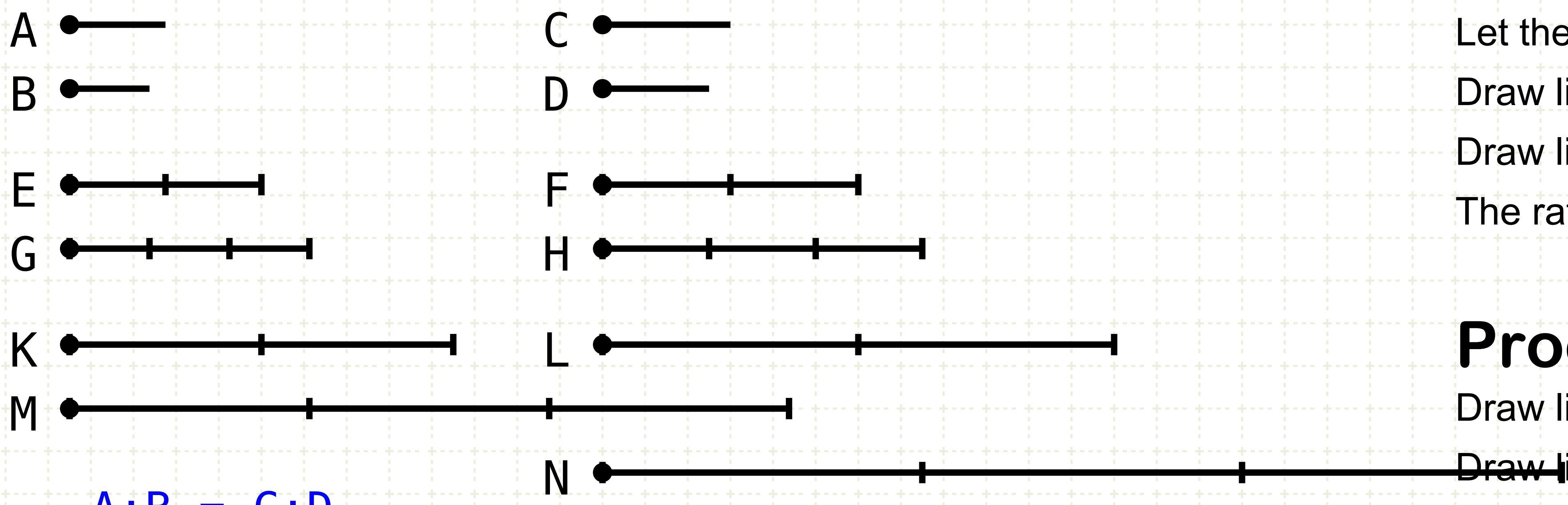
The ratio E to G is equal to the ratio F to H

Proof

Draw lines K and L that are equimultiple to E and F

Proposition 4 of Book V

If a first magnitude have to a second the same ratio as a third to a fourth, any equimultiples whatever of the first and third will also have the same ratio to any equimultiples whatever of the second and fourth respectively, taken in corresponding order.



$A:B = C:D$

$pA \gtreqless qB \rightarrow pC \gtreqless qD$

$E = i \cdot A,\ F = i \cdot C$

$G = j \cdot B,\ H = j \cdot D$

$K = mE,\ L = mF$

$M = nG,\ N = nH$

In other words

Let the ratio of A to B be the same ratio C to D

Draw lines E and F that are equimultiple to A and C

Draw lines G and H that are equimultiple to B and D

The ratio E to G is equal to the ratio F to H

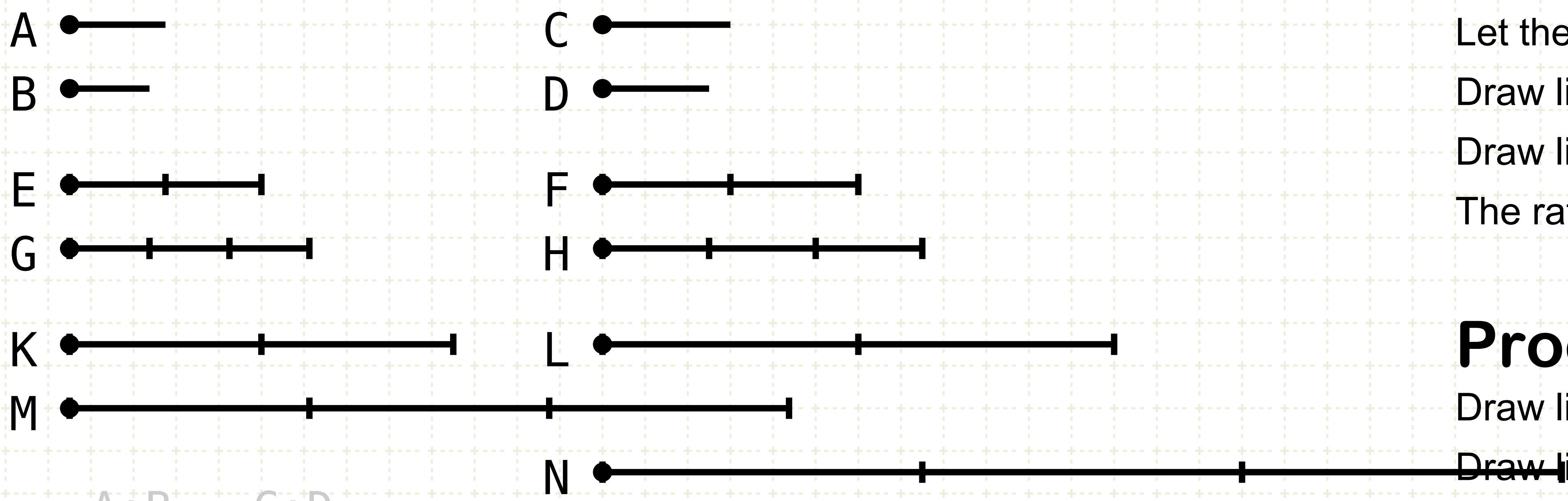
Proof

Draw lines K and L that are equimultiple to E and F

Draw lines M and N that are equimultiple to G and H

Proposition 4 of Book V

If a first magnitude have to a second the same ratio as a third to a fourth, any equimultiples whatever of the first and third will also have the same ratio to any equimultiples whatever of the second and fourth respectively, taken in corresponding order.



$A:B = C:D$

$pA \gtreqless qB \rightarrow pC \gtreqless qD$

$E = i \cdot A,\ F = i \cdot C$

$G = j \cdot B,\ H = j \cdot D$

$K = mE,\ L = mF$

$M = nG,\ N = nH$

$K = pA,\ L = pC$

In other words

Let the ratio of A to B be the same ratio C to D

Draw lines E and F that are equimultiple to A and C

Draw lines G and H that are equimultiple to B and D

The ratio E to G is equal to the ratio F to H

Proof

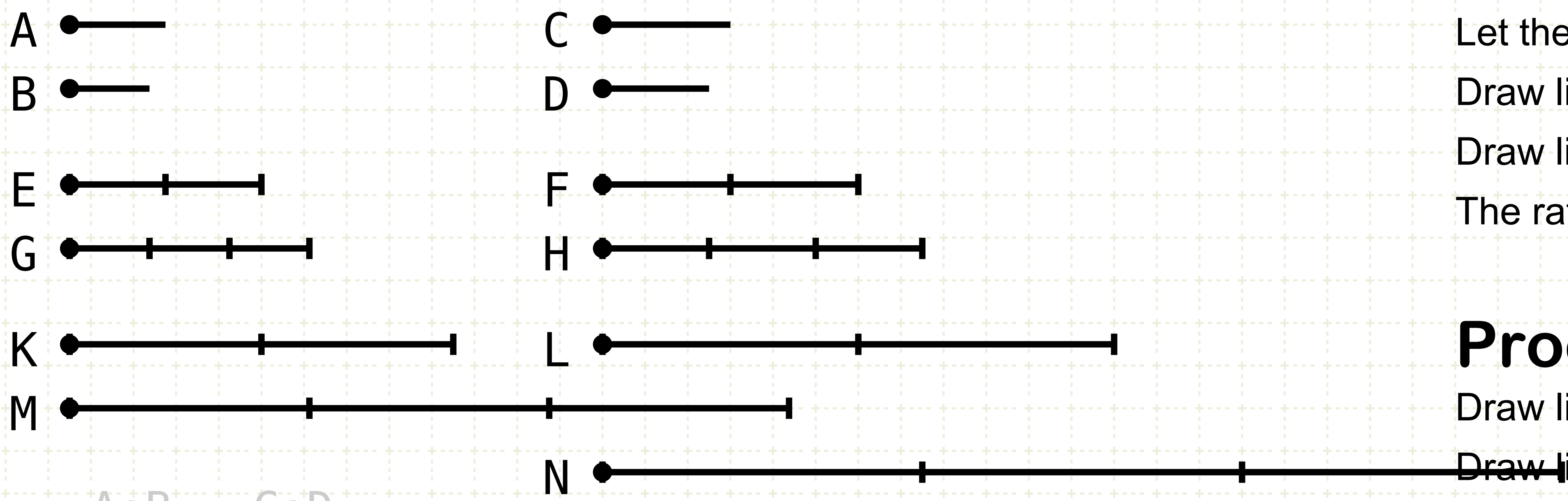
Draw lines K and L that are equimultiple to E and F

Draw lines M and N that are equimultiple to G and H

Since E and F are the same multiple of A and C, and K and L are the same multiple of E and F, then K and L are also the same multiple of A and C (V·3)

Proposition 4 of Book V

If a first magnitude have to a second the same ratio as a third to a fourth, any equimultiples whatever of the first and third will also have the same ratio to any equimultiples whatever of the second and fourth respectively, taken in corresponding order.



$A:B = C:D$

$pA >=< qB \rightarrow pC >=< qD$

$E = i \cdot A,\ F = i \cdot C$

$G = j \cdot B,\ H = j \cdot D$

$K = mE,\ L = mF$

$M = nG,\ N = nH$

$K = pA,\ L = pC$

$M = qB,\ N = qD$

In other words

Let the ratio of A to B be the same ratio C to D

Draw lines E and F that are equimultiple to A and C

Draw lines G and H that are equimultiple to B and D

The ratio E to G is equal to the ratio F to H

Proof

Draw lines K and L that are equimultiple to E and F

Draw lines M and N that are equimultiple to G and H

Since E and F are the same multiple of A and C, and K and L are the same multiple of E and F, then K and L are also the same multiple of A and C (V·3)

Likewise M and N are equimultiples of B and D (V·3)

Proposition 4 of Book V

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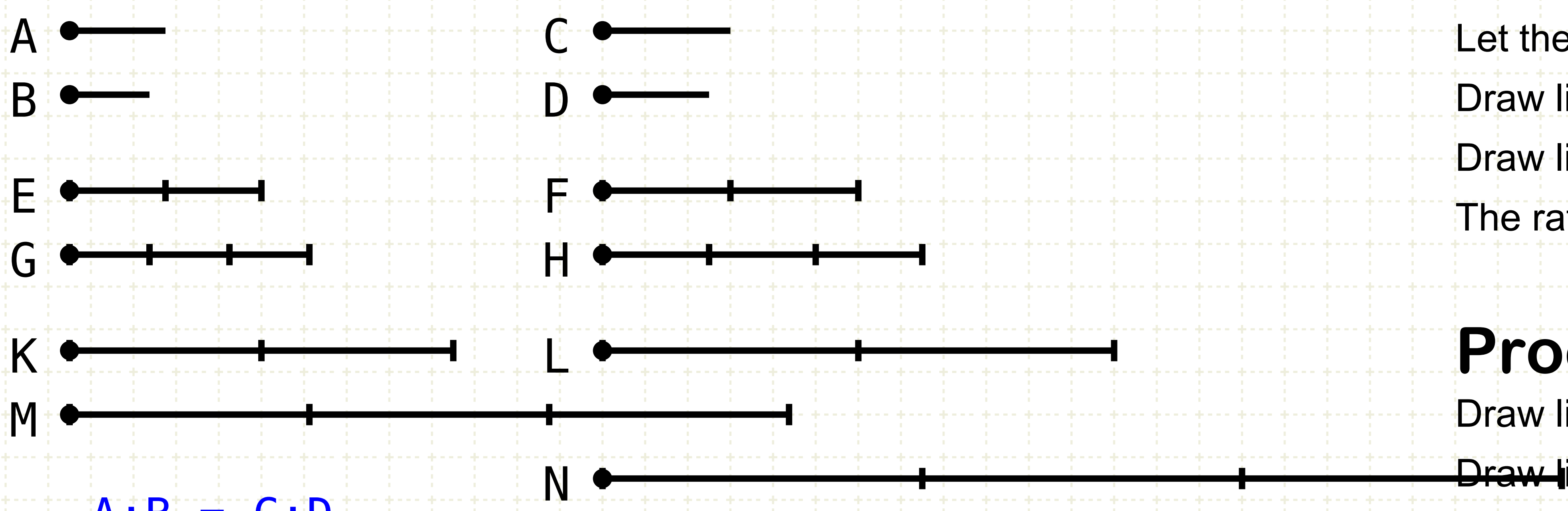

A:B = C:D

pA >=< qB → pC >=< qD   K >=< M → L >=< N

E = i·A, F = i·C
G = j·B, H = j·D

K = mE, L = mF
M = nG, N = nH

K = pA, L = pC
M = qB, N = qD

In other words

Let the ratio of A to B be the same ratio C to D

Draw lines E and F that are equimultiple to A and C

Draw lines G and H that are equimultiple to B and D

The ratio E to G is equal to the ratio F to H

Proof

Draw lines K and L that are equimultiple to E and F

Draw lines M and N that are equimultiple to G and H

Since E and F are the same multiple of A and C, and K and L are the same multiple of E and F, then K and L are also the same multiple of A and C (V·3)

Likewise M and N are equimultiples of B and D (V·3)

Because K and L are equimultiples of A,C, and M,N are equimultiples of B,D, then if K exceeds M, L exceeds N, etc (V def 5)

Proposition 4 of Book V

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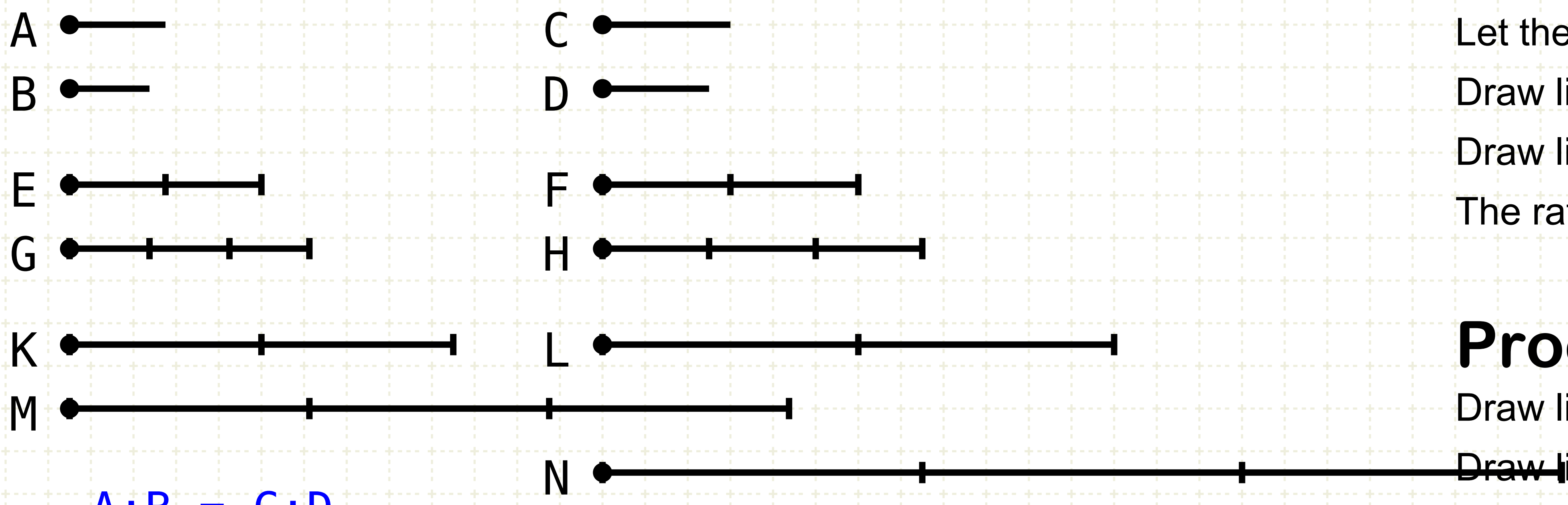

$A:B = C:D$

$pA >=< qB \rightarrow pC >=< qD \quad K >=< M \rightarrow L >=< N$

$E = i\cdot A,\ F = i\cdot C \quad mE >=< nG \rightarrow mF >=< nH$

$G = j\cdot B,\ H = j\cdot D$

$K = mE,\ L = mF$

$M = nG,\ N = nH$

$K = pA,\ L = pC$

$M = qB,\ N = qD$

In other words

Let the ratio of A to B be the same ratio C to D

Draw lines E and F that are equimultiple to A and C

Draw lines G and H that are equimultiple to B and D

The ratio E to G is equal to the ratio F to H

Proof

Draw lines K and L that are equimultiple to E and F

Draw lines M and N that are equimultiple to G and H

Since E and F are the same multiple of A and C, and K and L are the same multiple of E and F, then K and L are also the same multiple of A and C (V·3)

Likewise M and N are equimultiples of B and D (V·3)

Because K and L are equimultiples of A,C, and M,N are equimultiples of B,D, then if K exceeds M, L exceeds N, etc (V def 5)

K,L are equimultiples of E,F and M,N are equimultiples of G,H

Proposition 4 of Book V

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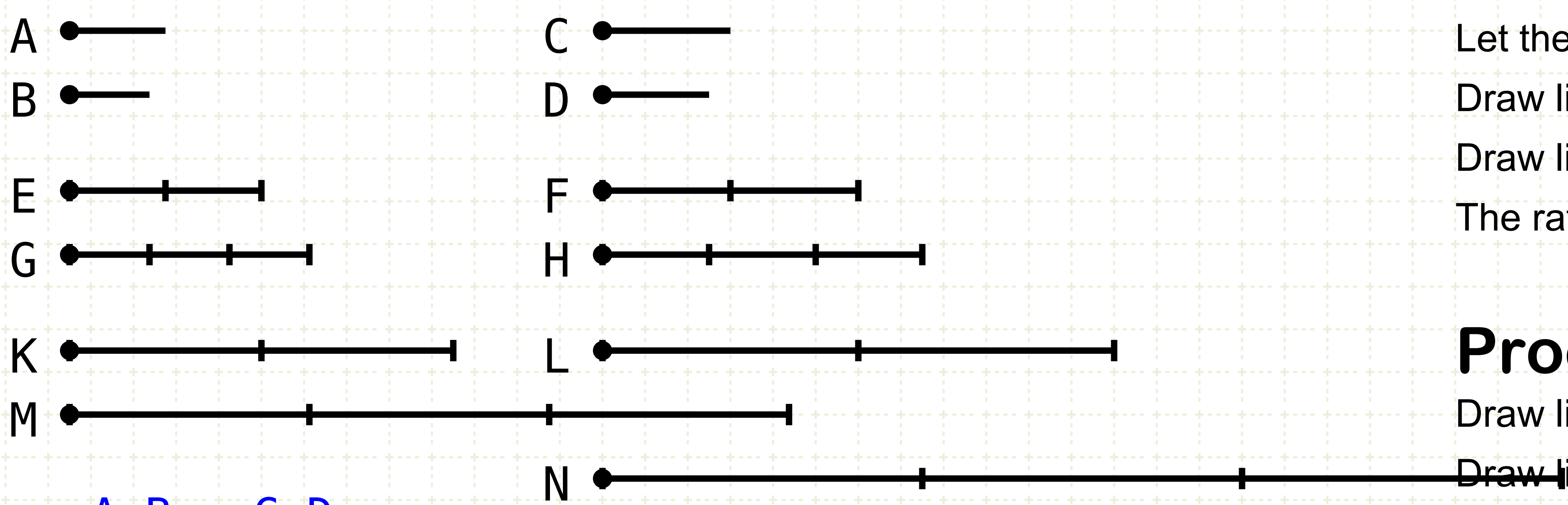

A:B = C:D

pA >=< qB → pC >=< qD   K >=< M → L >=< N

E = i·A, F = i·C
G = j·B, H = j·D

mE >=< nG → mF >=< nH

E:G = F:H

K = mE, L = mF
M = nG, N = nH

K = pA, L = pC
M = qB, N = qD

In other words

Let the ratio of A to B be the same ratio C to D

Draw lines E and F that are equimultiple to A and C

Draw lines G and H that are equimultiple to B and D

The ratio E to G is equal to the ratio F to H

Proof

Draw lines K and L that are equimultiple to E and F

Draw lines M and N that are equimultiple to G and H

Since E and F are the same multiple of A and C, and K and L are the same multiple of E and F, then K and L are also the same multiple of A and C (V·3)

Likewise M and N are equimultiples of B and D (V·3)

Because K and L are equimultiples of A,C, and M,N are equimultiples of B,D, then if K exceeds M, L exceeds N, etc (V def 5)

K,L are equimultiples of E,F and M,N are equimultiples of G,H

Therefore, E is to G as F is to H (V def 5)

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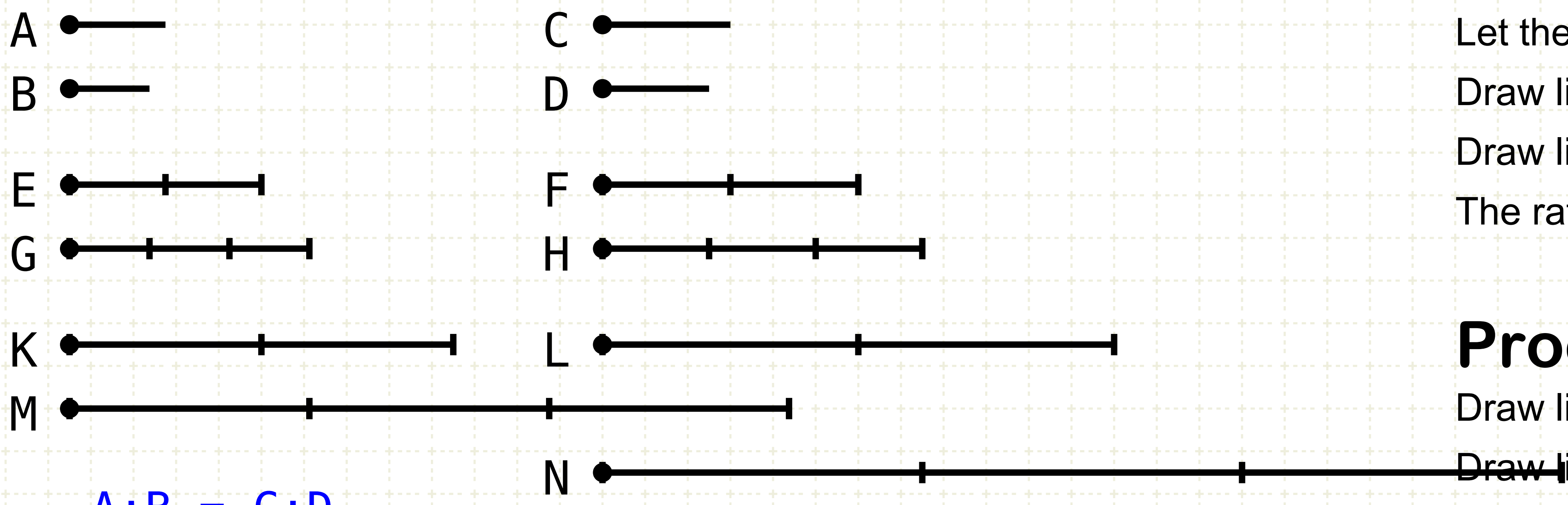

$A:B = C:D$

$pA \gtreqless qB \rightarrow pC \gtreqless qD \quad K \gtreqless M \rightarrow L \gtreqless N$

$E = i \cdot A,\ F = i \cdot C$

$G = j \cdot B,\ H = j \cdot D$

$mE \gtreqless nG \rightarrow mF \gtreqless nH$

$E:G = F:H$

$(i \cdot A):(j \cdot B) = (i \cdot C):(j \cdot D)$

$K = mE,\ L = mF$

$M = nG,\ N = nH$

$K = pA,\ L = pC$

$M = qB,\ N = qD$

In other words

Let the ratio of A to B be the same ratio C to D

Draw lines E and F that are equimultiple to A and C

Draw lines G and H that are equimultiple to B and D

The ratio E to G is equal to the ratio F to H

Proof

Draw lines K and L that are equimultiple to E and F

Draw lines M and N that are equimultiple to G and H

Since E and F are the same multiple of A and C, and K and L are the same multiple of E and F, then K and L are also the same multiple of A and C (V·3)

Likewise M and N are equimultiples of B and D (V·3)

Because K and L are equimultiples of A,C, and M,N are equimultiples of B,D, then if K exceeds M, L exceeds N, etc (V def 5)

K,L are equimultiples of E,F and M,N are equimultiples of G,H

Therefore, E is to G as F is to H (V def 5)

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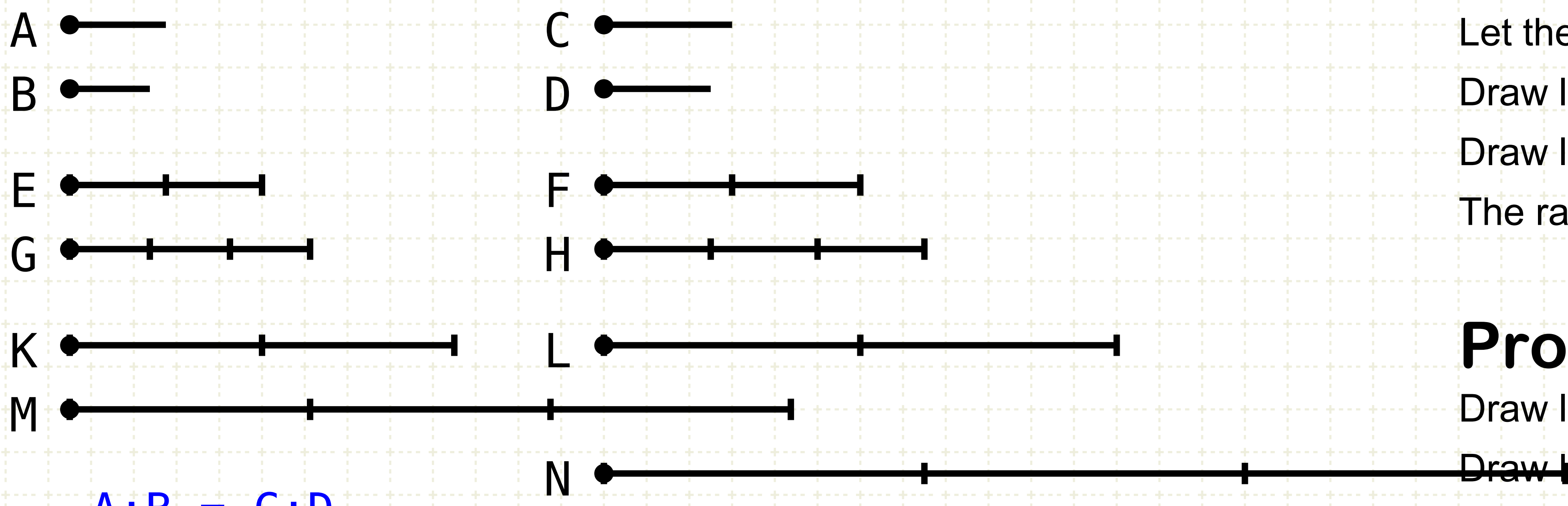

$A:B = C:D$

$pA \gtreqless qB \rightarrow pC \gtreqless qD \quad K \gtreqless M \rightarrow L \gtreqless N$

$E = i \cdot A,\ F = i \cdot C$

$G = j \cdot B,\ H = j \cdot D$

$mE \gtreqless nG \rightarrow mF \gtreqless nH$

$E:G = F:H$

$(i \cdot A):(j \cdot B) = (i \cdot C):(j \cdot D)$

$K = mE,\ L = mF$

$M = nG,\ N = nH$

$K = pA,\ L = pC$

$M = qB,\ N = qD$

In other words

Let the ratio of A to B be the same ratio C to D

Draw lines E and F that are equimultiple to A and C

Draw lines G and H that are equimultiple to B and D

The ratio E to G is equal to the ratio F to H

Proof

Draw lines K and L that are equimultiple to E and F

Draw lines M and N that are equimultiple to G and H

Since E and F are the same multiple of A and C, and K and L are the same multiple of E and F, then K and L are also the same multiple of A and C (V·3)

Likewise M and N are equimultiples of B and D (V·3)

Because K and L are equimultiples of A,C, and M,N are equimultiples of B,D, then if K exceeds M, L exceeds N, etc (V def 5)

K,L are equimultiples of E,F and M,N are equimultiples of G,H

Therefore, E is to G as F is to H (V def 5)

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