# Euclid's Elements Book IV

Philosophy (nature) is written in that great book which ever is before our eyes -- I mean the universe -- but we cannot understand it if we do not first learn the language and grasp the symbols in which it is written. The book is written in mathematical language, and the symbols are triangles, circles and other geometrical figures, without whose help it is impossible to comprehend a single word of it - without which one wanders in vain through a dark labyrinth.

Galileo Galilei



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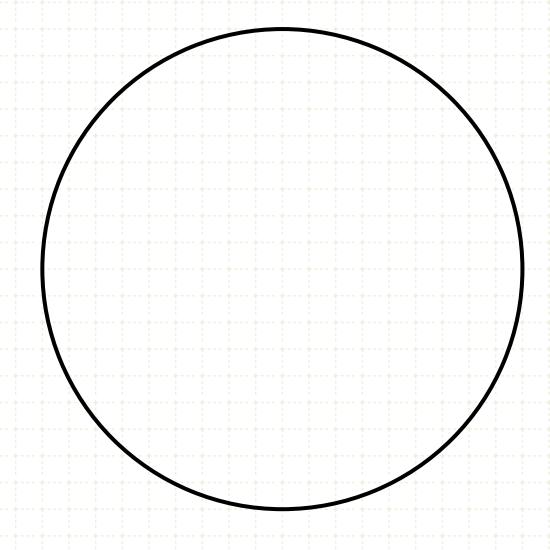




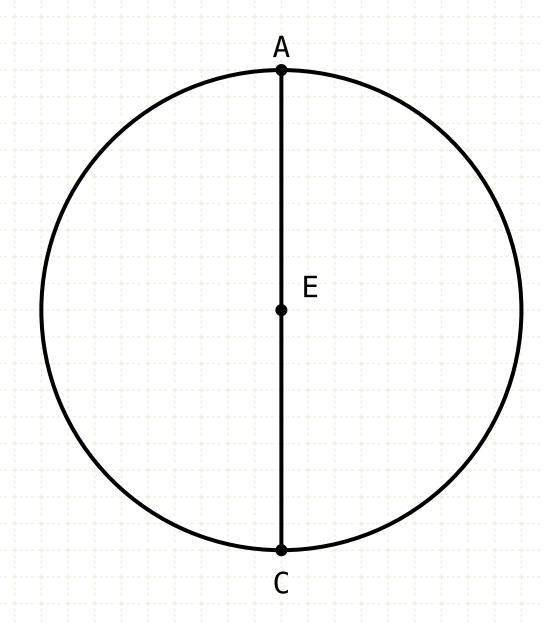
### In other words

Given a circle, draw a square ABCD within the circle





### Construction



### Construction

Draw a diameter AC through the centre of the circle E

# D

### **Proposition 6 of Book IV**

In a given circle to inscribe a square.

### Construction

Draw a diameter AC through the centre of the circle E

Draw a diameter BD, perpendicular to AC, through the centre of the circle E



# D

### **Proposition 6 of Book IV**

In a given circle to inscribe a square.

### Construction

Draw a diameter AC through the centre of the circle E

Draw a diameter BD, perpendicular to AC, through the centre of the circle E

Draw lines AB, BC, BC, DA

ABCD is a square



# D

### **Proposition 6 of Book IV**

In a given circle to inscribe a square.

### Construction

Draw a diameter AC through the centre of the circle E

Draw a diameter BD perpendicular to AC, through the centre

Draw a diameter BD, perpendicular to AC, through the centre of the circle E

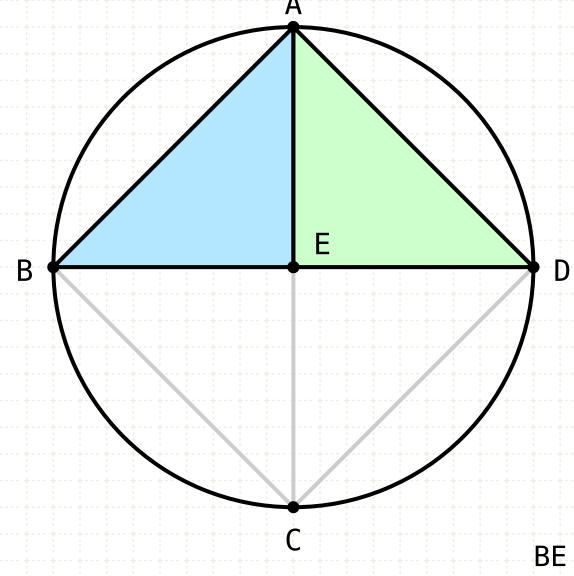
Draw lines AB, BC, BC, DA

ABCD is a square

### **Proof**



In a given circle to inscribe a square.



BE = ED ∠AEB = ∠AED = ∟ AE is common

### Construction

Draw a diameter AC through the centre of the circle E

Draw a diameter BD, perpendicular to AC, through the centre of the circle E

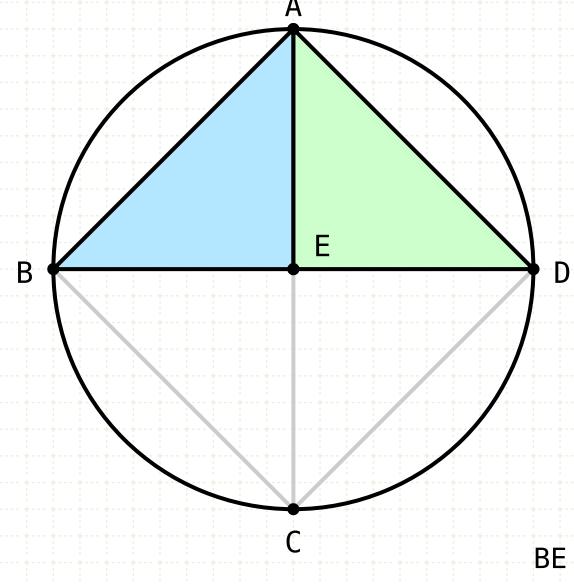
Draw lines AB, BC, BC, DA ABCD is a square

### **Proof**

The two triangles ABE and AED are equal, since they have a side (BE,ED), angle (AEB = AED =  $\bot$ ), side (AE) equal (I·4)



In a given circle to inscribe a square.



### Construction

Draw a diameter AC through the centre of the circle E

Draw a diameter BD, perpendicular to AC, through the centre of the circle E

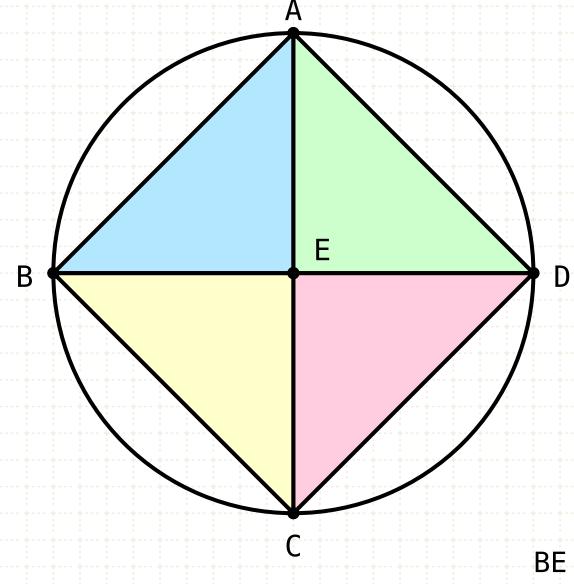
Draw lines AB, BC, BC, DA ABCD is a square

### **Proof**

The two triangles ABE and AED are equal, since they have a side (BE,ED), angle (AEB = AED =  $\bot$ ), side (AE) equal (I·4) Hence AB equals AD



In a given circle to inscribe a square.



### Construction

Draw a diameter AC through the centre of the circle E

Draw a diameter BD, perpendicular to AC, through the centre of the circle E

Draw lines AB, BC, BC, DA ABCD is a square

### **Proof**

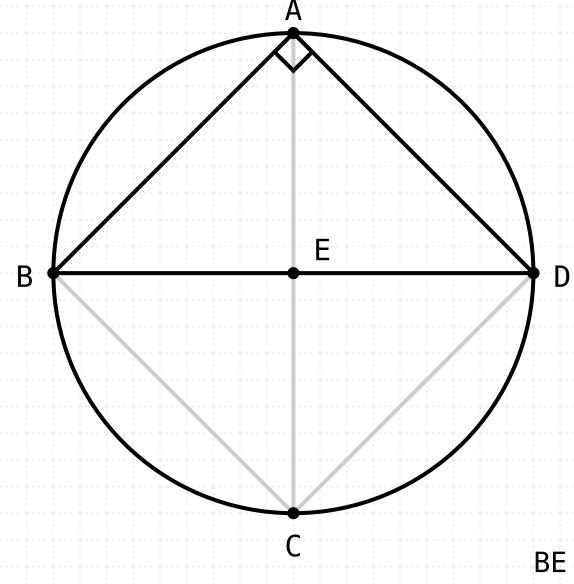
The two triangles ABE and AED are equal, since they have a side (BE,ED), angle (AEB = AED =  $\bot$ ), side (AE) equal (I·4)

Hence AB equals AD

Similarly, it can be shown that AD is equal to CD, and CD equals CB, hence ABCD is an equilateral



In a given circle to inscribe a square.



### Construction

Draw a diameter AC through the centre of the circle E

Draw a diameter BD, perpendicular to AC, through the centre of the circle E

Draw lines AB, BC, BC, DA ABCD is a square

### **Proof**

The two triangles ABE and AED are equal, since they have a side (BE,ED), angle (AEB = AED =  $\bot$ ), side (AE) equal (I·4)

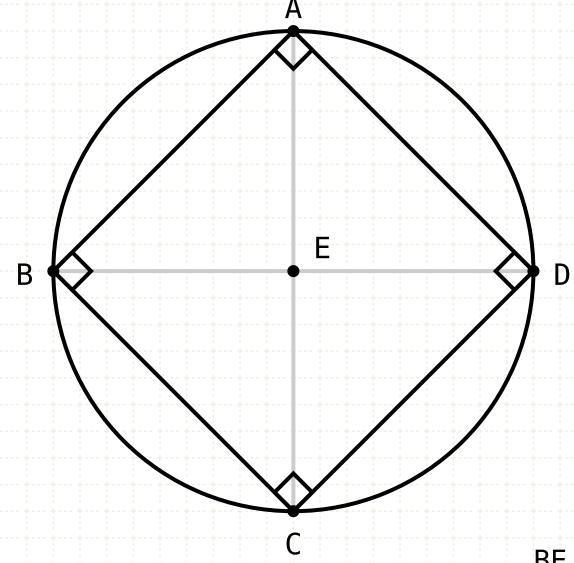
Hence AB equals AD

Similarly, it can be shown that AD is equal to CD, and CD equals CB, hence ABCD is an equilateral

BD is the diameter of the circle, and the angle of a semi-circle is a right angle (III-31), therefore the angle BAD is right



In a given circle to inscribe a square.



### Construction

Draw a diameter AC through the centre of the circle E

Draw a diameter BD, perpendicular to AC, through the centre of the circle E

Draw lines AB, BC, BC, DA ABCD is a square

### **Proof**

The two triangles ABE and AED are equal, since they have a side (BE,ED), angle (AEB = AED =  $\bot$ ), side (AE) equal (I·4)

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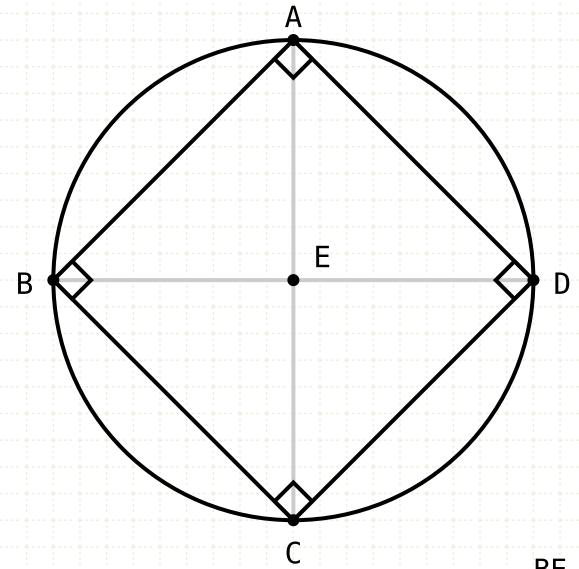
Similarly, it can be shown that AD is equal to CD, and CD equals CB, hence ABCD is an equilateral

BD is the diameter of the circle, and the angle of a semi-circle is a right angle (III-31), therefore the angle BAD is right

Similarly, it can be shown that all the angles touching the circles are right



In a given circle to inscribe a square.



### Construction

Draw a diameter AC through the centre of the circle E

Draw a diameter BD, perpendicular to AC, through the centre of the circle E

Draw lines AB, BC, BC, DA

ABCD is a square

### **Proof**

The two triangles ABE and AED are equal, since they have a side (BE,ED), angle (AEB = AED =  $\bot$ ), side (AE) equal (I·4)

Hence AB equals AD

Similarly, it can be shown that AD is equal to CD, and CD equals CB, hence ABCD is an equilateral

BD is the diameter of the circle, and the angle of a semi-circle is a right angle (III·31), therefore the angle BAD is right

Similarly, it can be shown that all the angles touching the circles are right

A quadilateral with all angles right angles is, by definition, a square



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