

Euclid's Elements

Book I

If Euclid did not kindle your youthful enthusiasm, you were not born to be a scientific thinker.

Albert Einstein



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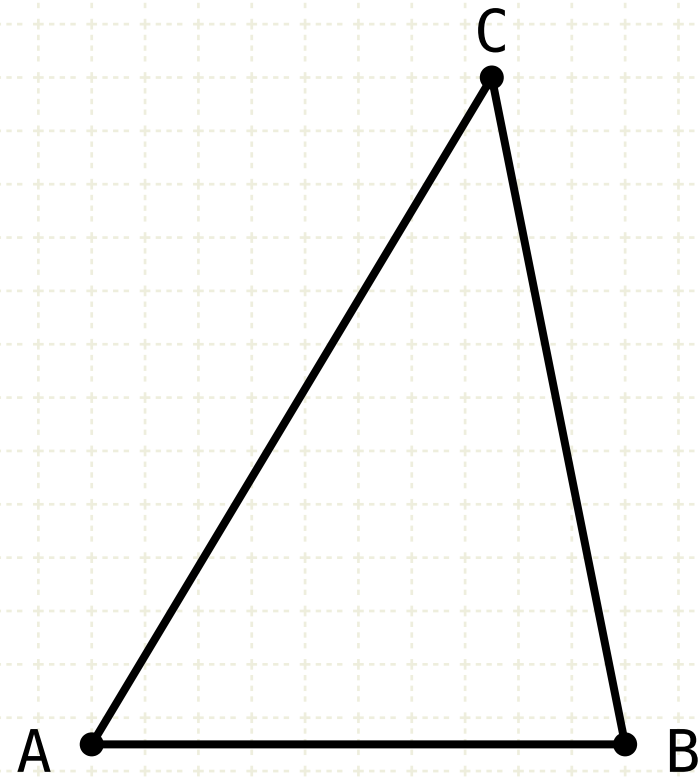
Proposition 8 of Book I

If two triangles have the two sides equal to two sides respectively, and also have the base equal to the base, then they also have the angles equal which are contained by the equal straight lines.

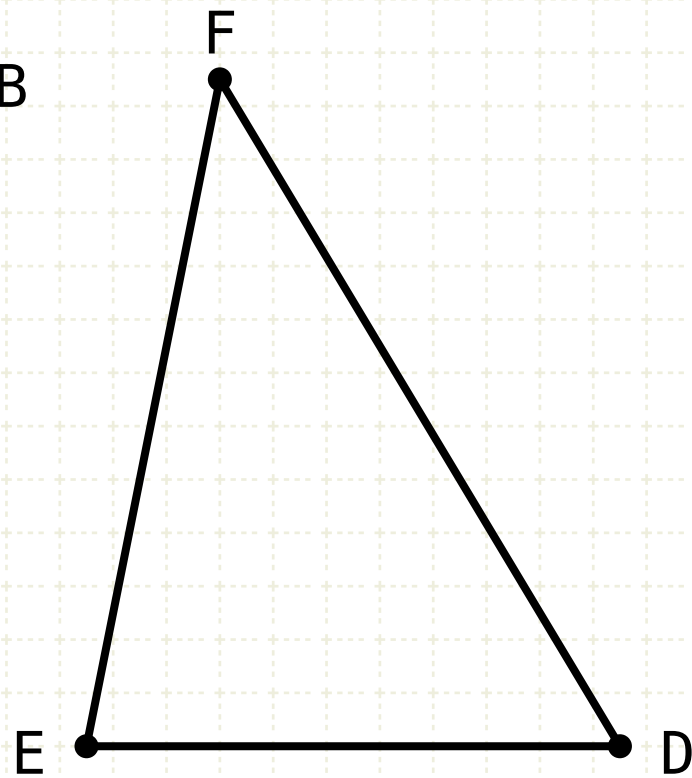


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$$\begin{aligned}CB &= EF \\AC &= DF \\AB &= ED\end{aligned}$$

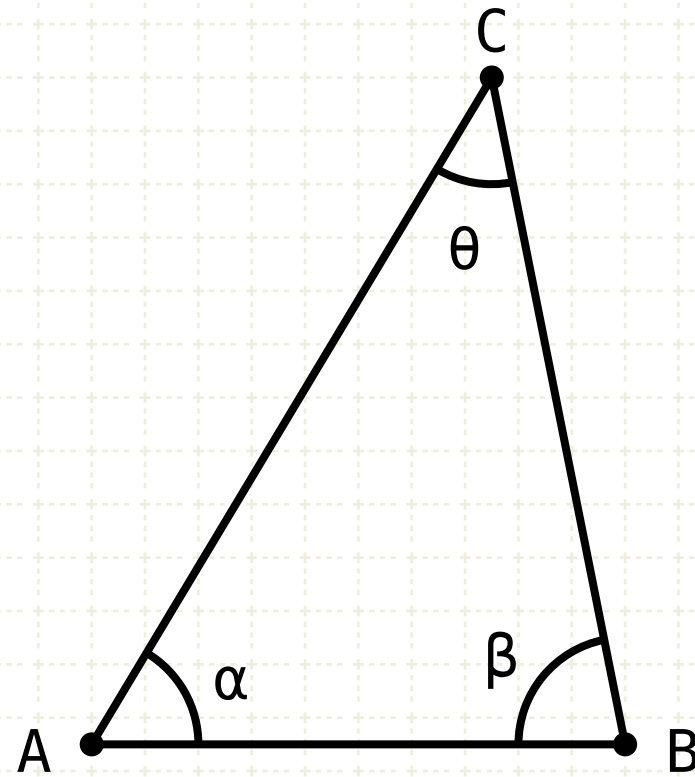


In other words...

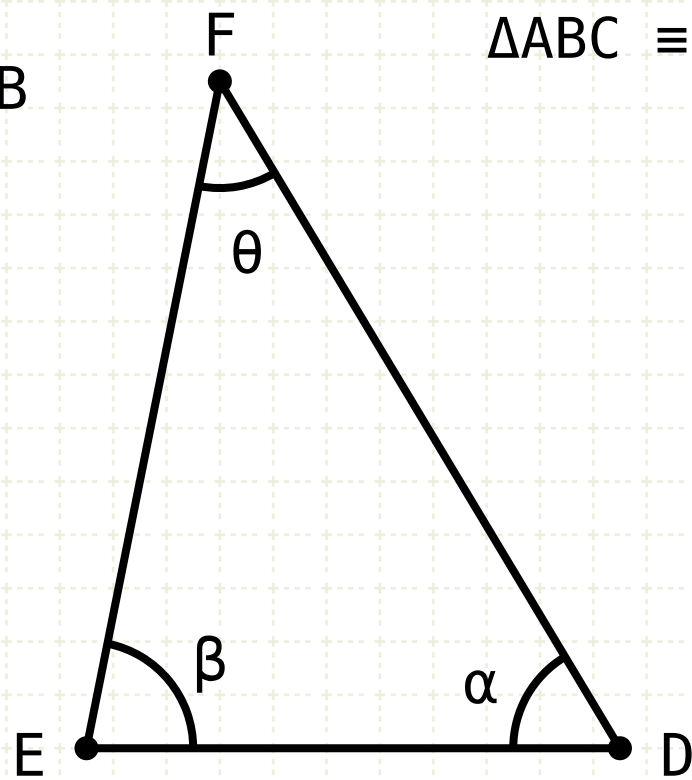
Given two triangles with three sides of one triangle equal to the three sides of the other triangle (SSS)

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If two triangles have the two sides equal to two sides respectively, and also have the base equal to the base, then they also have the angles equal which are contained by the equal straight lines.



$$\begin{aligned}CB &= EF \\AC &= DF \\AB &= ED \\ \Delta ABC &\equiv \Delta DEF\end{aligned}$$



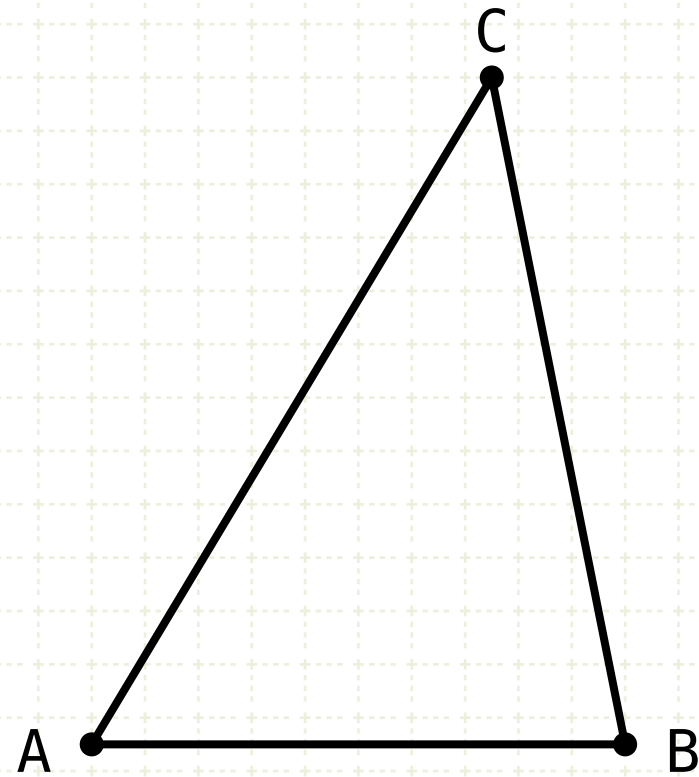
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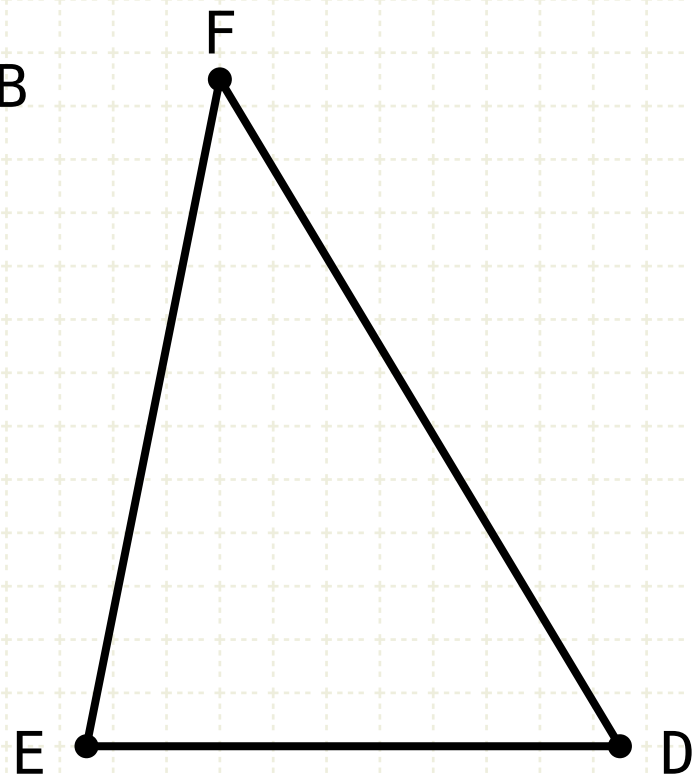
Then the two triangles are equivalent in all respects

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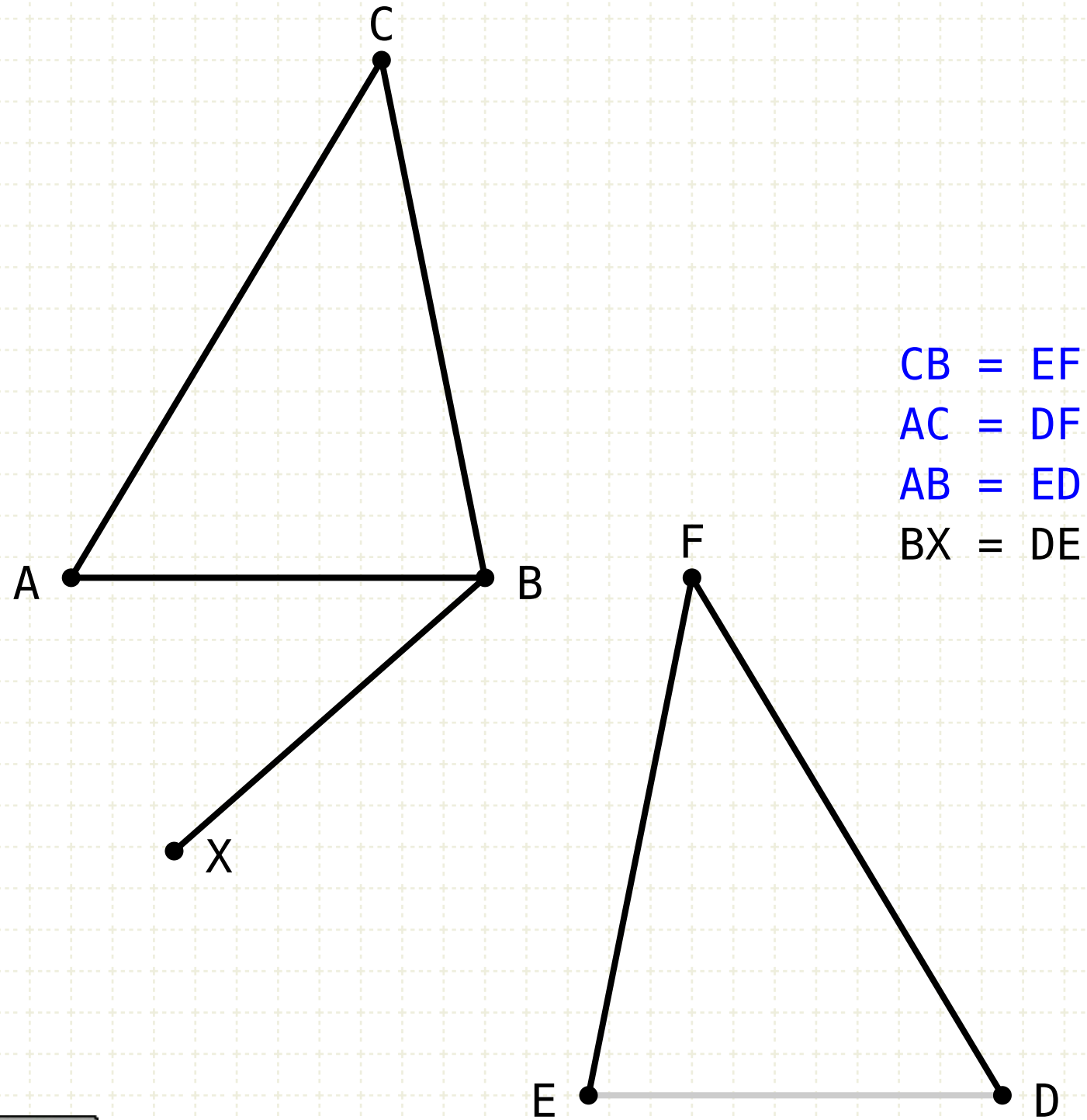
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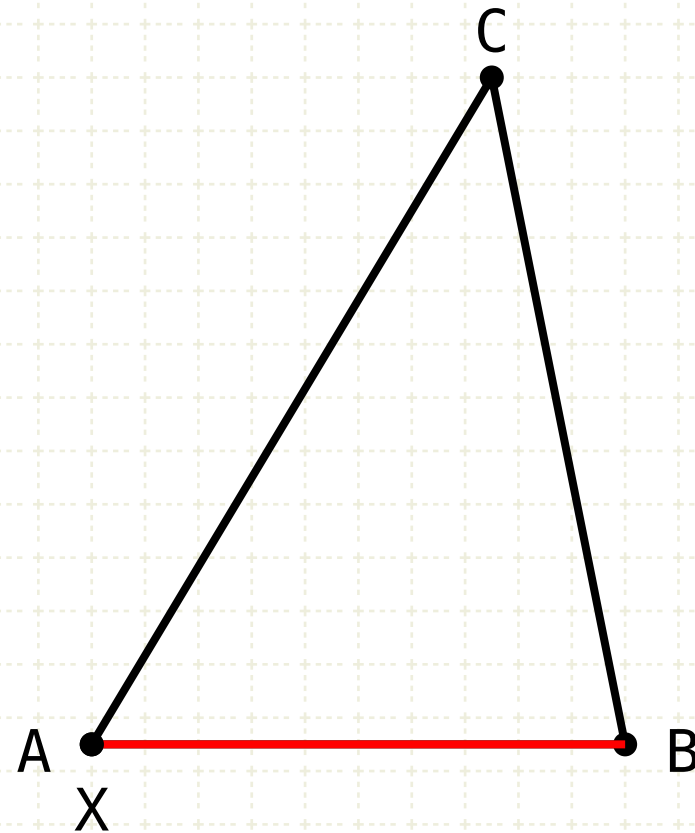
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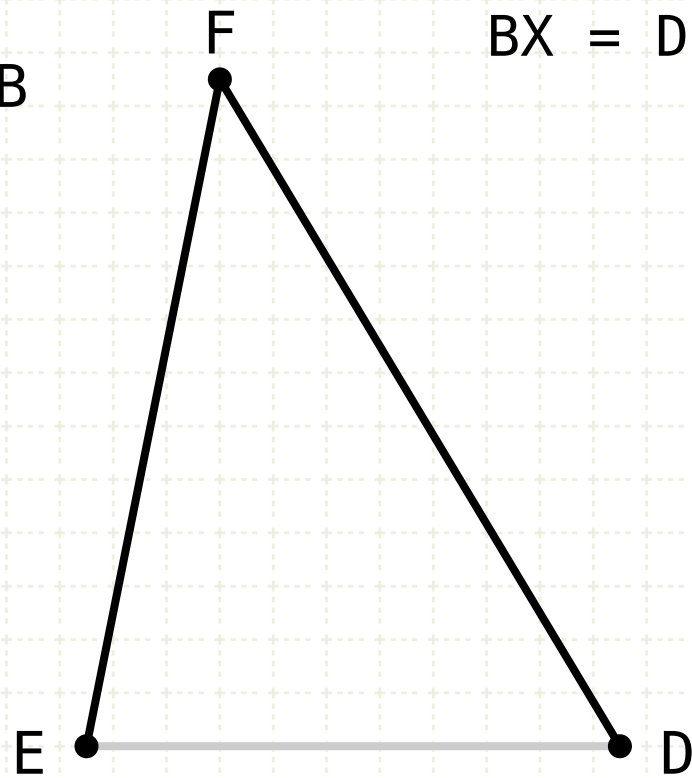
Construct line segment BX equal to DE at point B (I-2)

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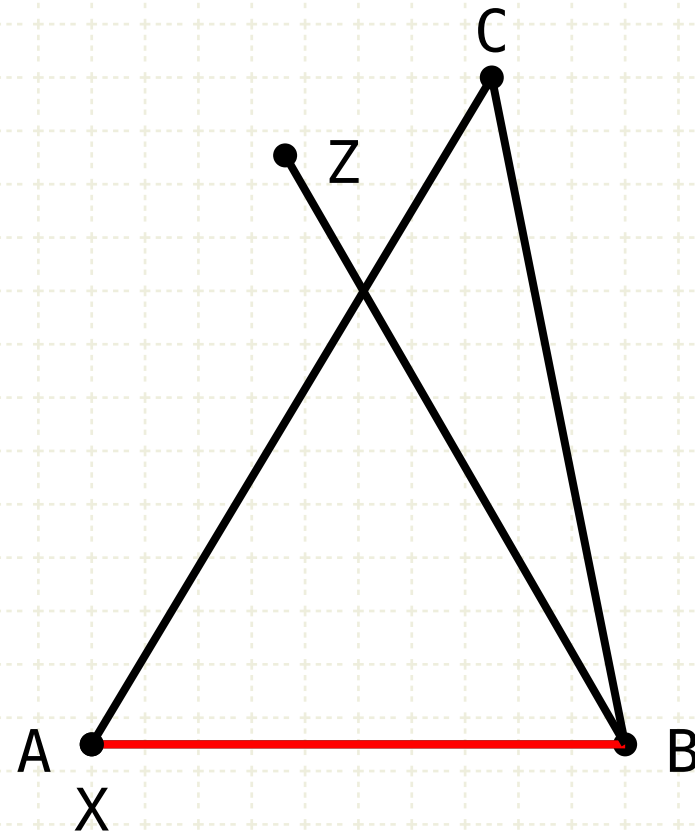
Proof

Construct line segment BX equal to DE at point B (I-2)

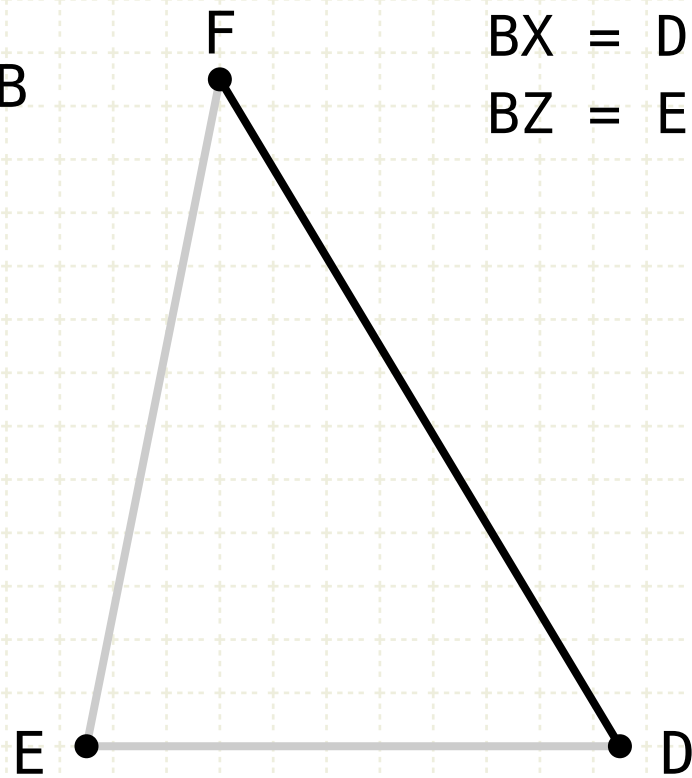
Align BX to AB. Since they are the same lengths, the endpoints are congruent

Proposition 8 of Book I

If two triangles have the two sides equal to two sides respectively, and also have the base equal to the base, then they also have the angles equal which are contained by the equal straight lines.



$$\begin{aligned}CB &= EF \\AC &= DF \\AB &= ED \\BX &= DE \\BZ &= EF\end{aligned}$$



In other words...

Given two triangles with three sides of one triangle equal to the three sides of the other triangle (SSS)

Then the two triangles are equivalent in all respects

Proof

Construct line segment BX equal to DE at point B (I·2)

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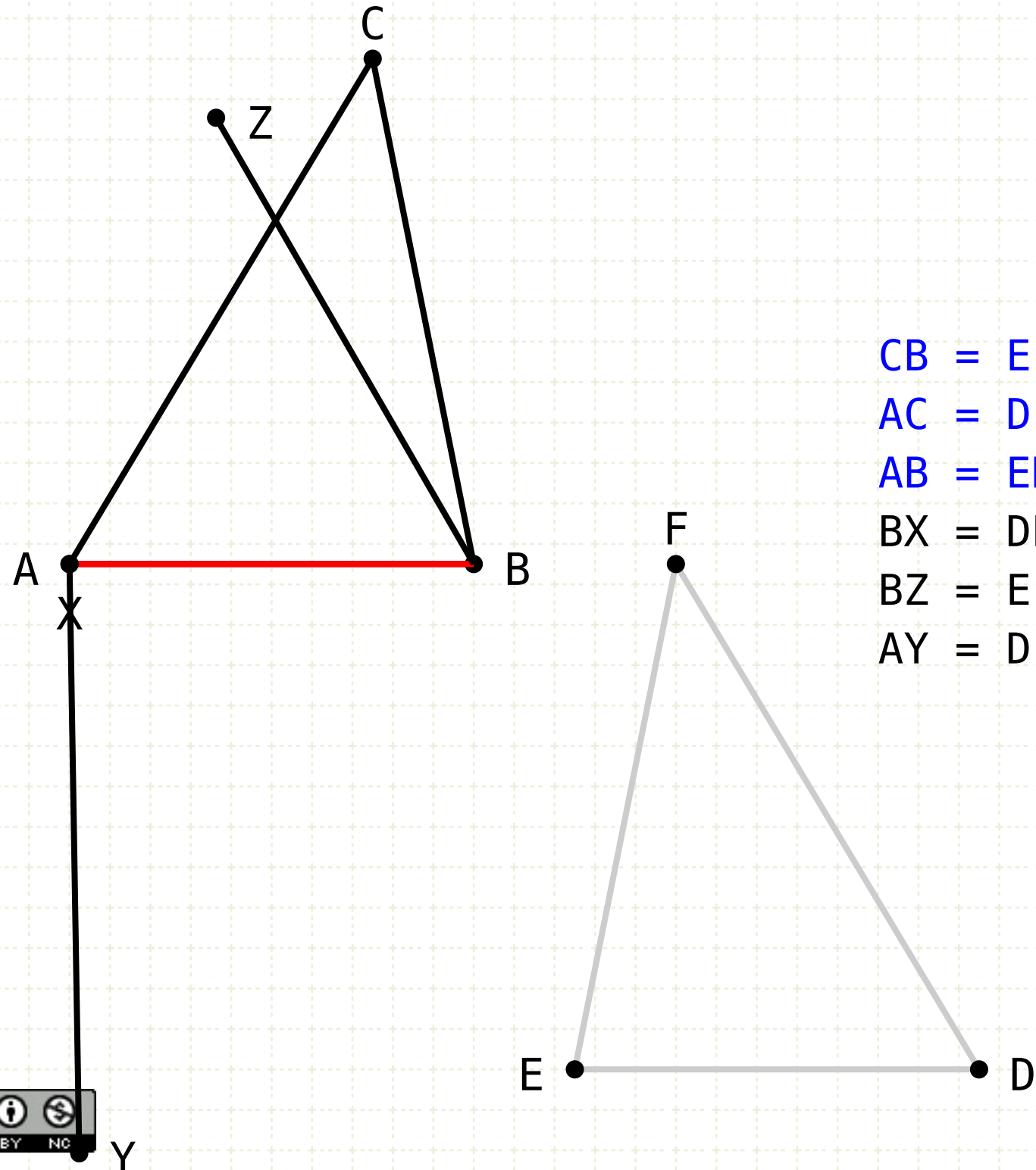
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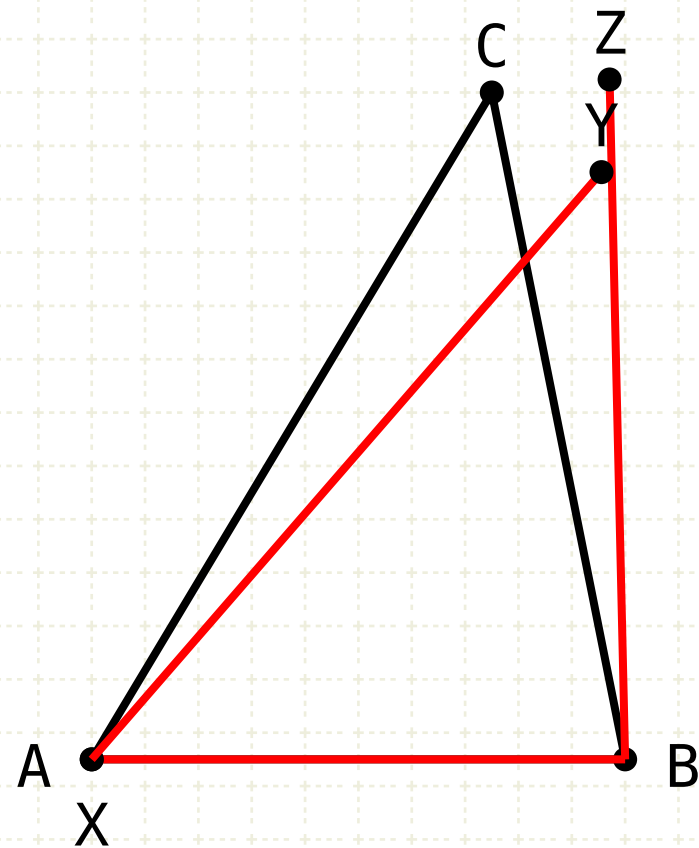
Construct line segment BZ equal to EF at point B (I·2)

Construct line segment AY equal to DF at point A (I·2)



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If two triangles have the two sides equal to two sides respectively, and also have the base equal to the base, then they also have the angles equal which are contained by the equal straight lines.



$$CB = EF$$

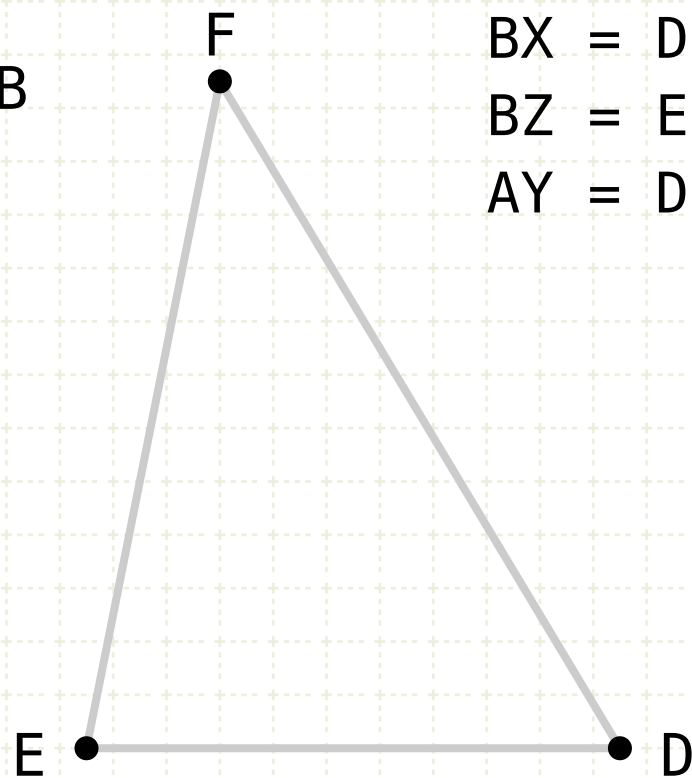
$$AC = DF$$

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$$BZ = EF$$

$$AY = DF$$



In other words...

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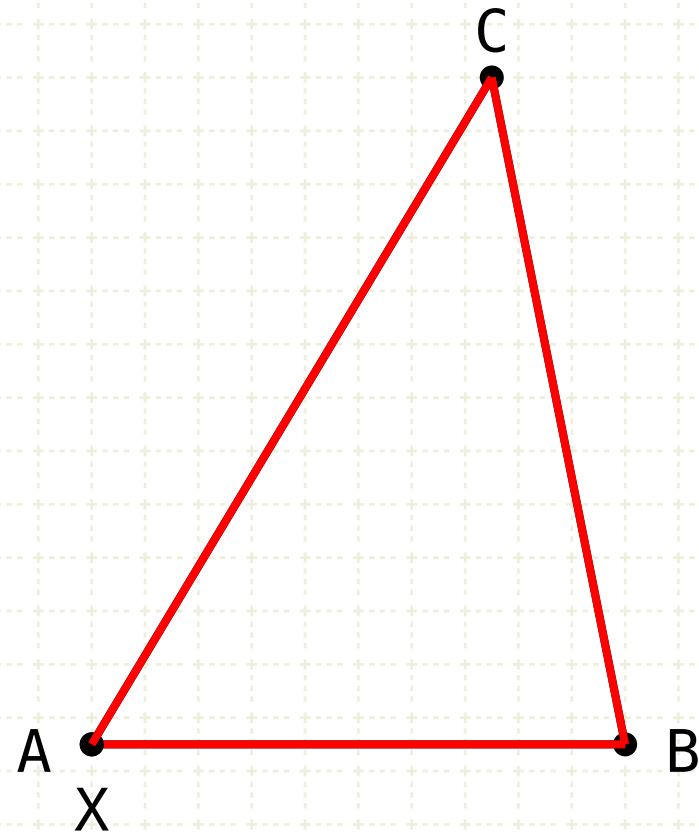
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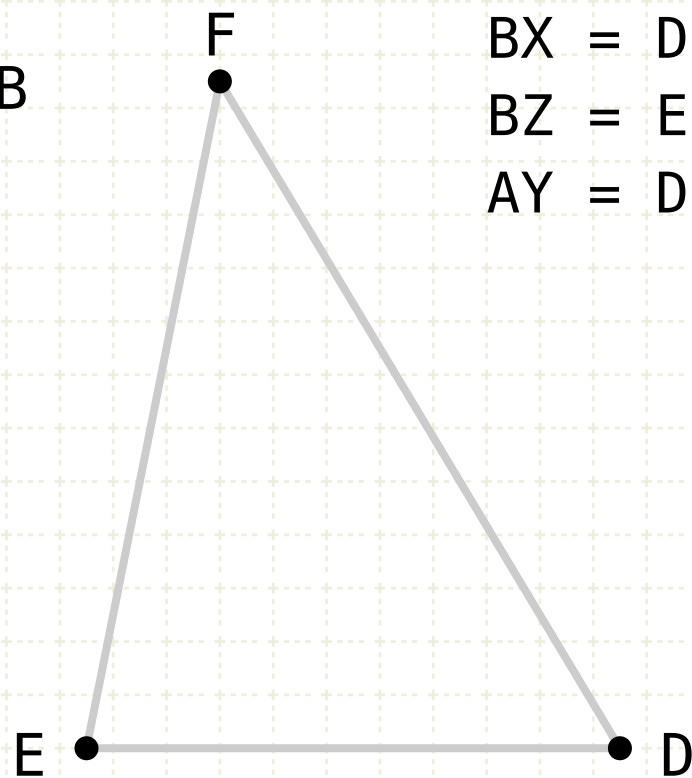
Where do ends of the lines BZ and AY meet?

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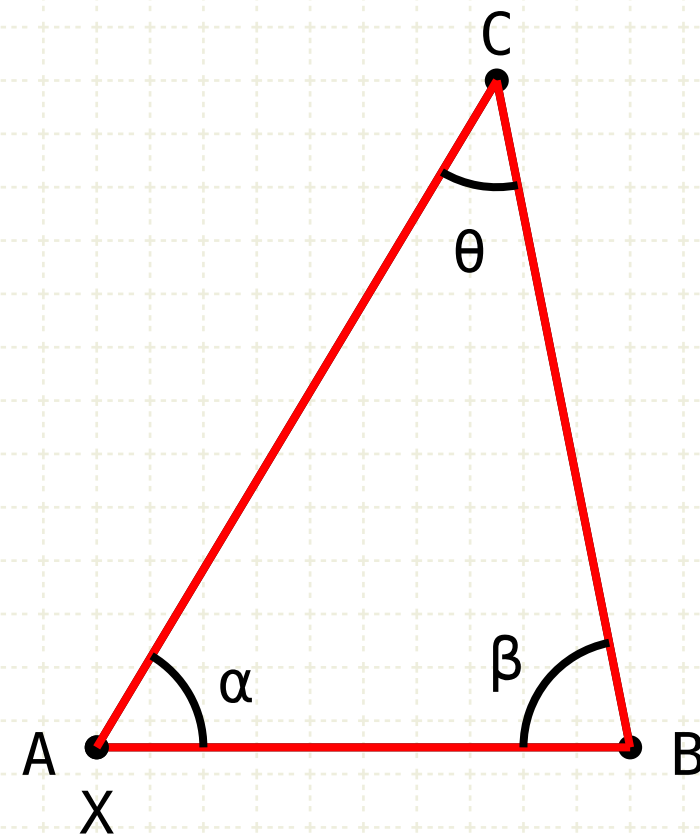
Construct line segment AY equal to DF at point A (I·2)

Where do ends of the lines BZ and AY meet?

They can only meet at one point, 'C' (I·7)

Proposition 8 of Book I

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$$CB = EF$$

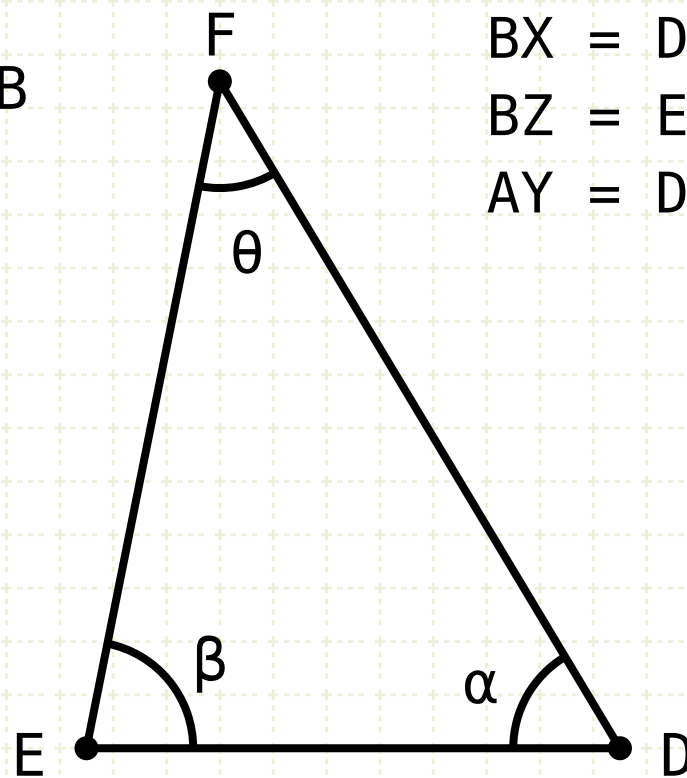
$$AC = DF$$

$$AB = ED$$

$$BX = DE$$

$$BZ = EF$$

$$AY = DF$$



In other words...

Given two triangles with three sides of one triangle equal to the three sides of the other triangle (SSS)

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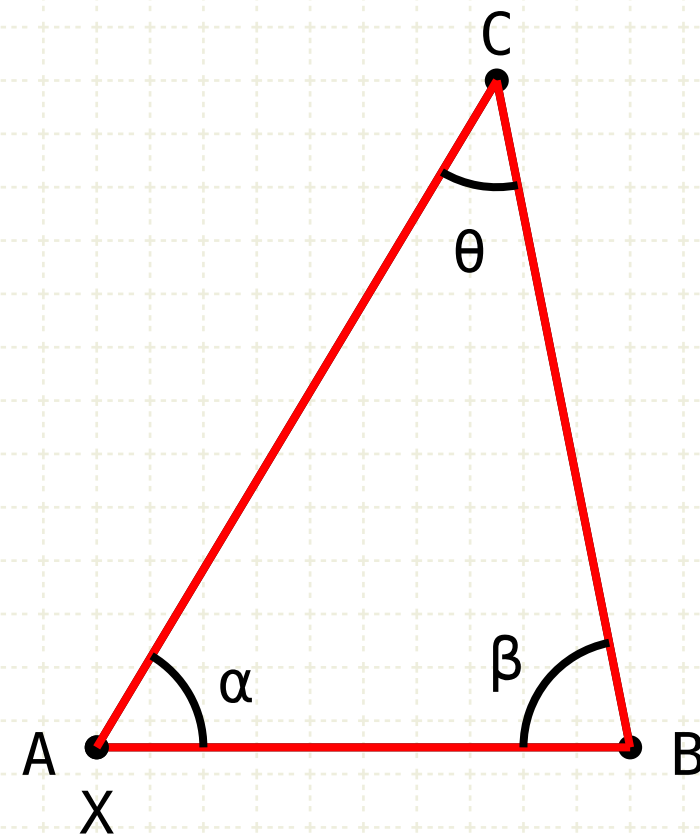
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Since all the endpoints of the lines are congruent, then the angles must also be congruent

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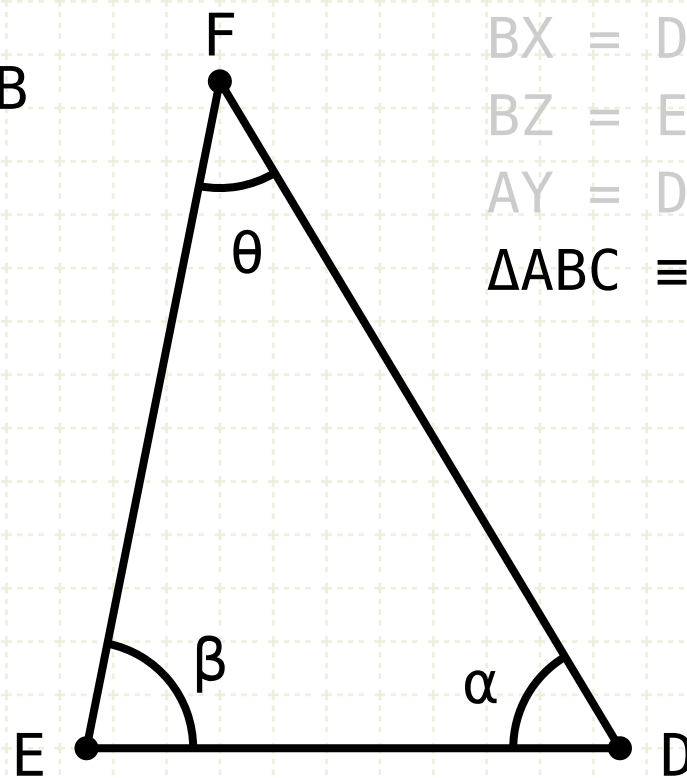
$$AB = ED$$

$$BX = DE$$

$$BZ = EF$$

$$AY = DF$$

$$\triangle ABC \equiv \triangle DEF$$



In other words...

Given two triangles with three sides of one triangle equal to the three sides of the other triangle (SSS)

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Since all the endpoints of the lines are congruent, then the angles must also be congruent

Thus the two triangles are equivalent

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