# Euclid's Elements

# Book VII

## Definitions:

1 A unit is that by virtue of which each of the things that exist is called one

2 A number is a multitude composed of units. (not one)

3 A number is part of a number, the less of the greater, when it measures the greater

11 A prime number is that which is measured by a unit alone.

12 Numbers prime to one another are those which are measured by a unit alone as a common measure

15 A number is said to multiply a number when that which is multiplied is added to itself as many times as there are units in the other, and thus some number is produced.

20 Numbers are proportional when the first is the same multiple, or the same part, or the same parts, of the second that the third is of the fourth.

*As long as algebra and geometry have been separated, their progress have been slow and their uses limited; but when these two sciences have been united, they have lent each mutual forces, and have marched together towards perfection.*

**Joseph-Louis Lagrange**
**(1736 to 1813)**

# Table of Contents, Chapter 7

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# Proposition 28 of Book VII

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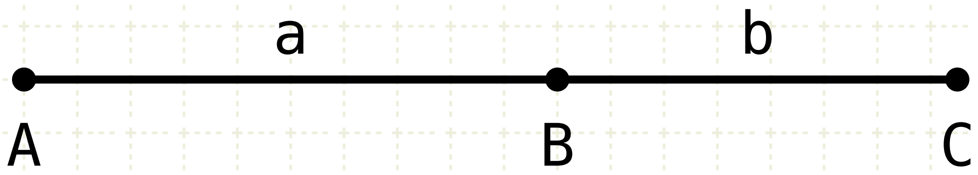


## In other words (part a)

Let AB and BC be relatively prime

$\gcd(AB, BC) = 1$ $\gcd(a, b) = 1$

$AC = AB + BC$

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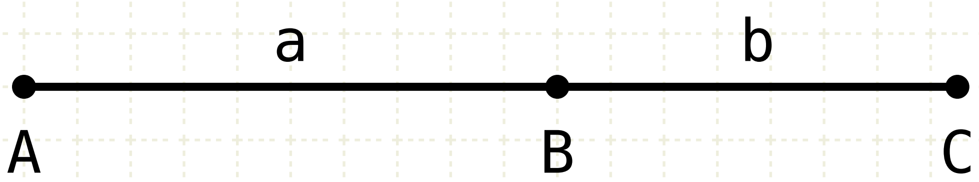


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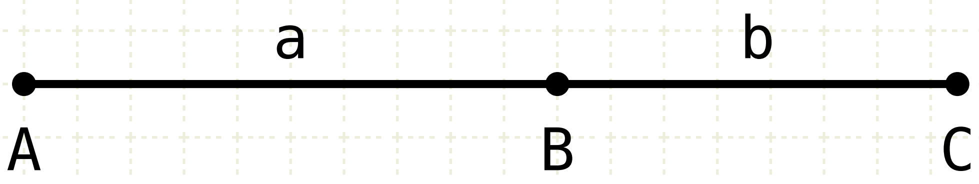


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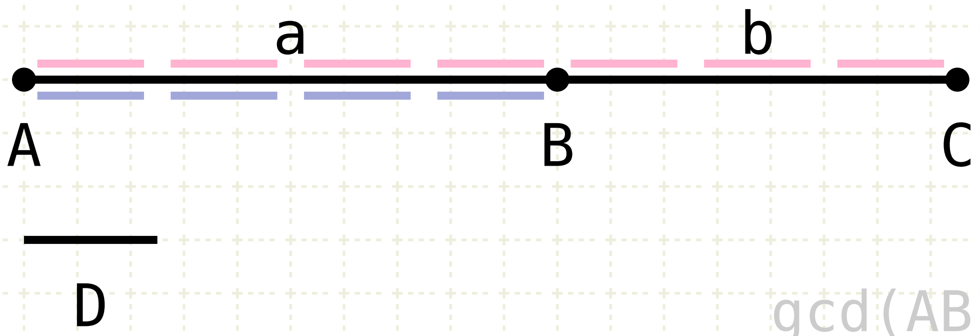


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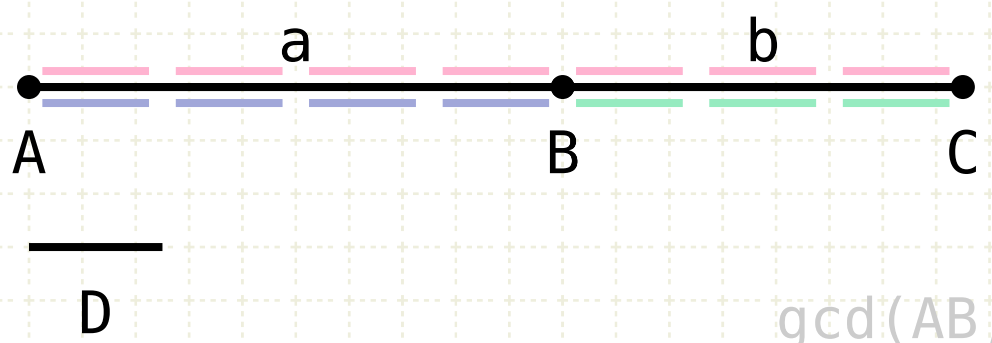


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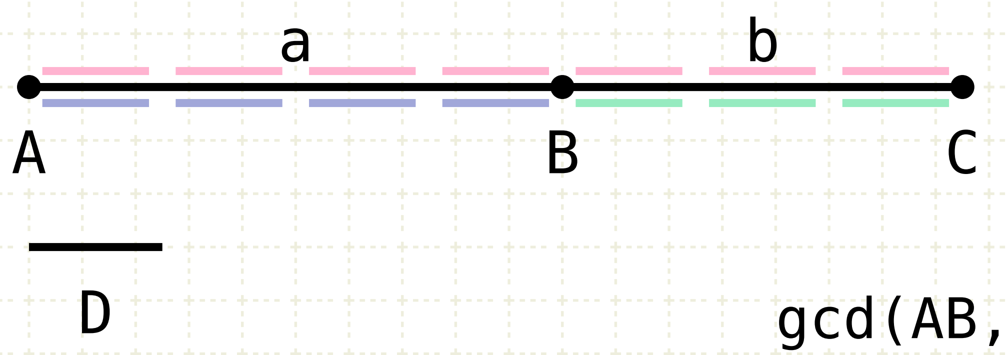


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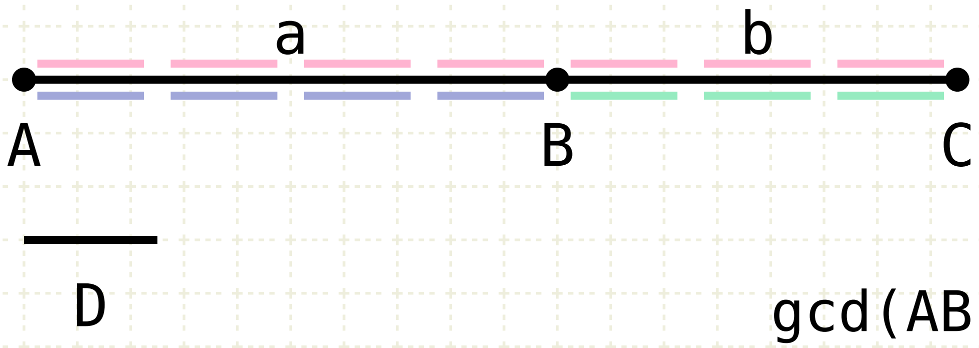


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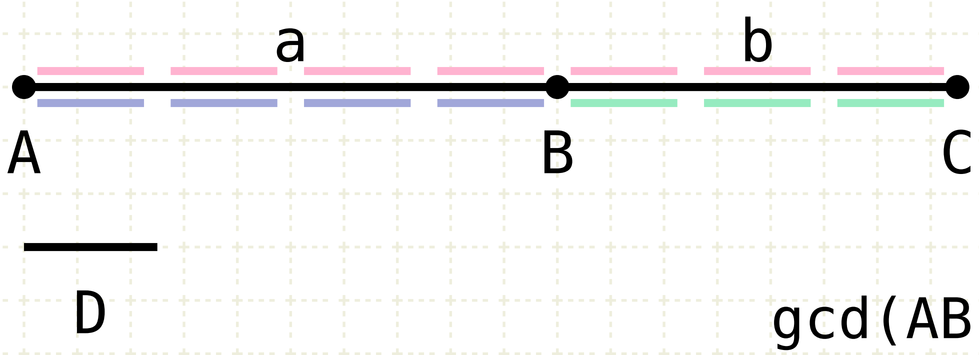


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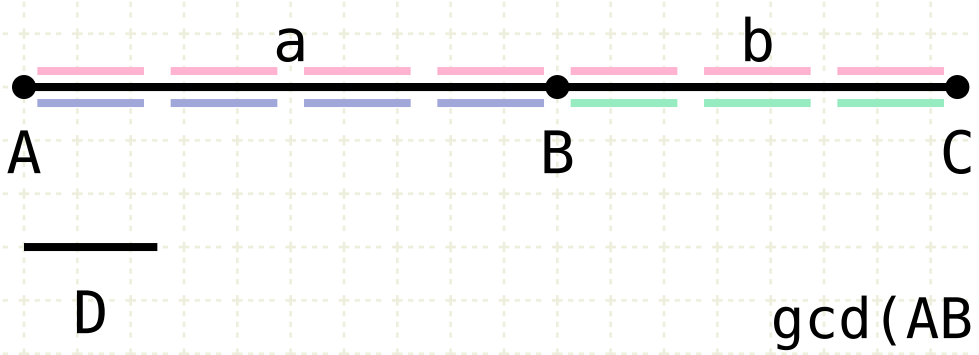


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Thus there is no number D that measures AB,AC, so they are prime to one another

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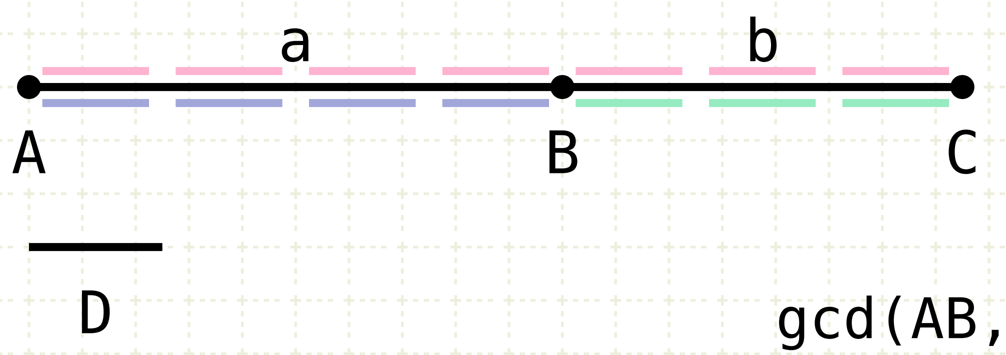


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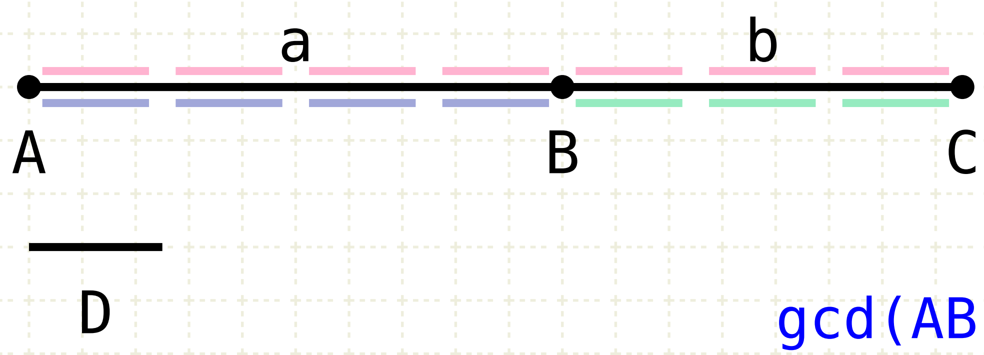


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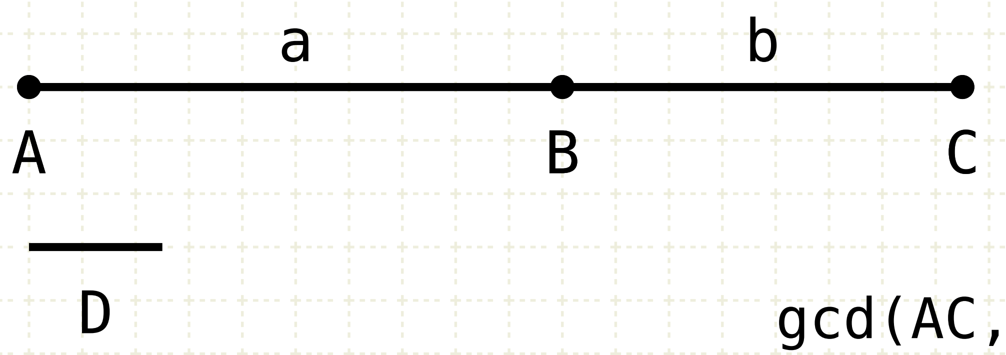


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Let AB and AC be relatively prime

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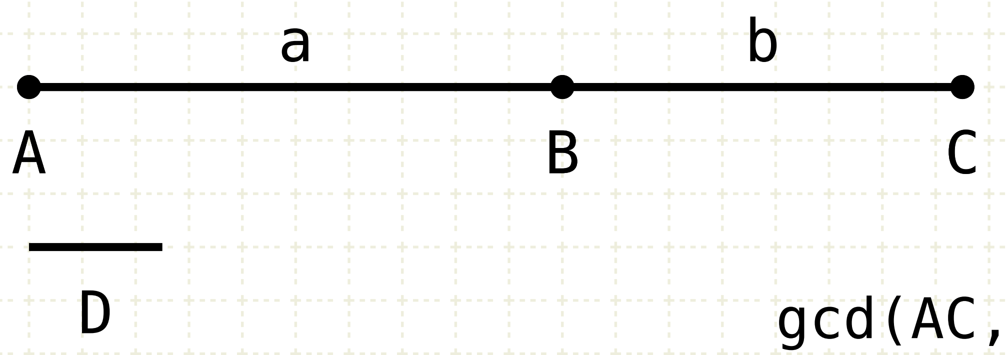


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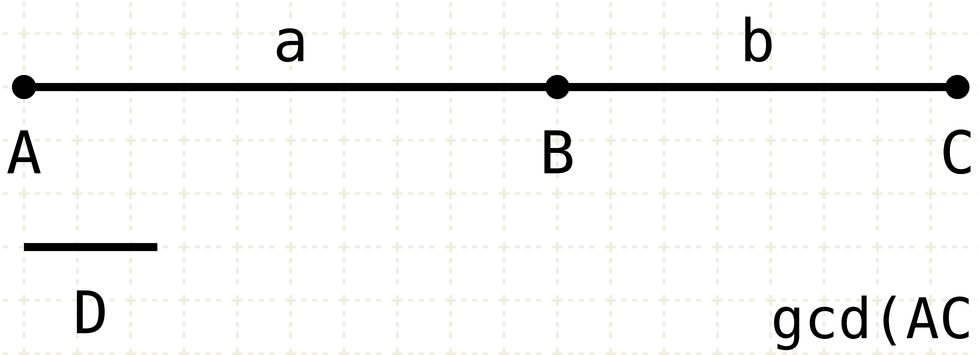


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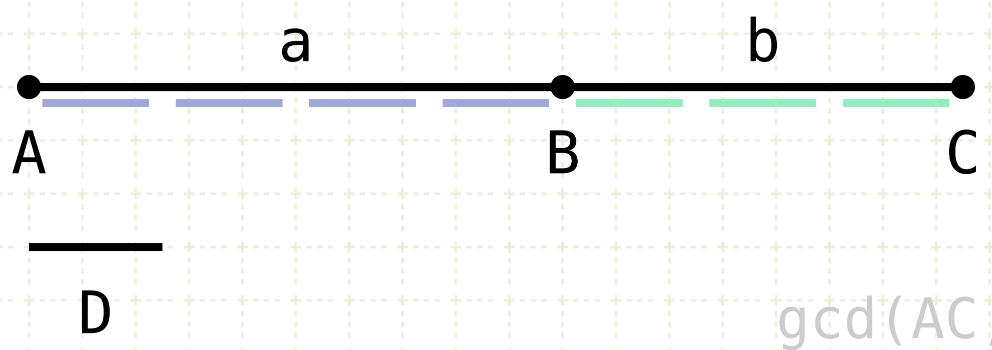


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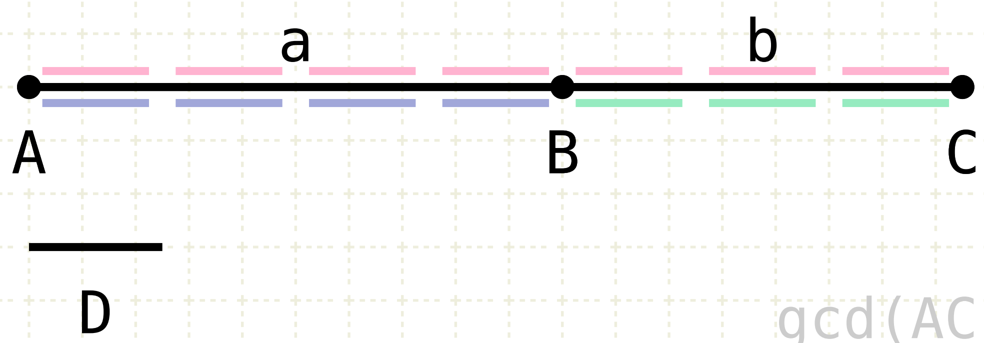


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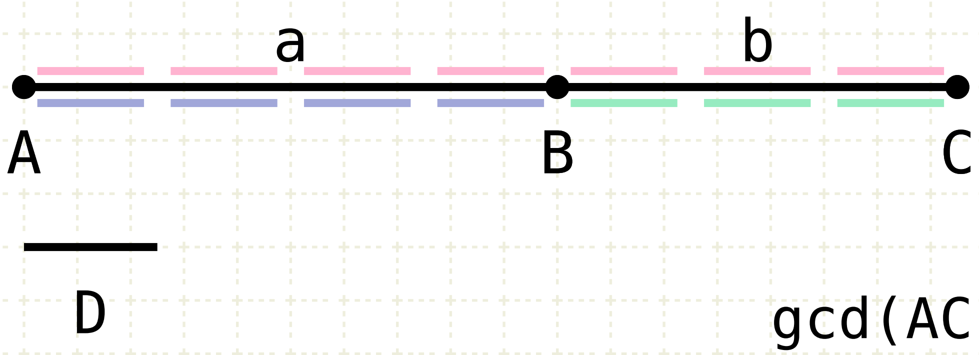


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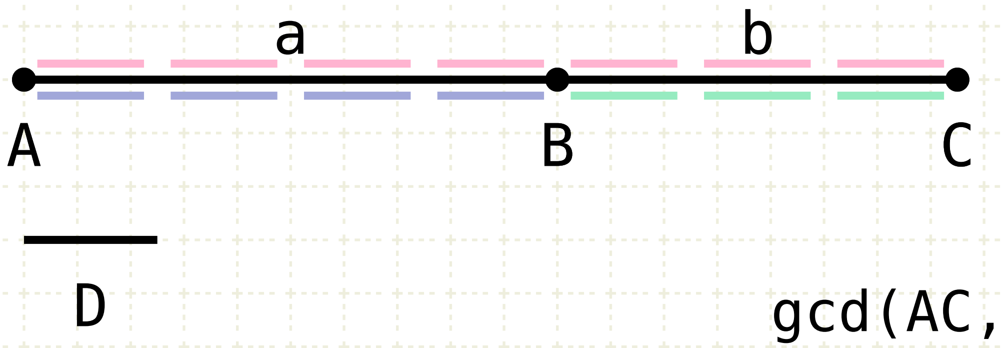


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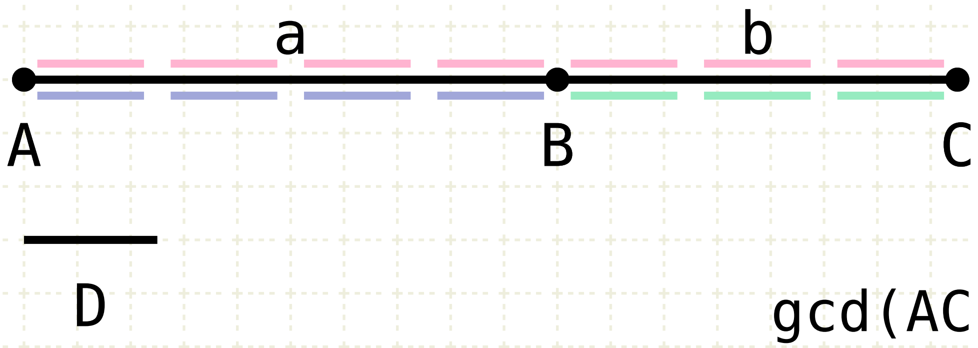


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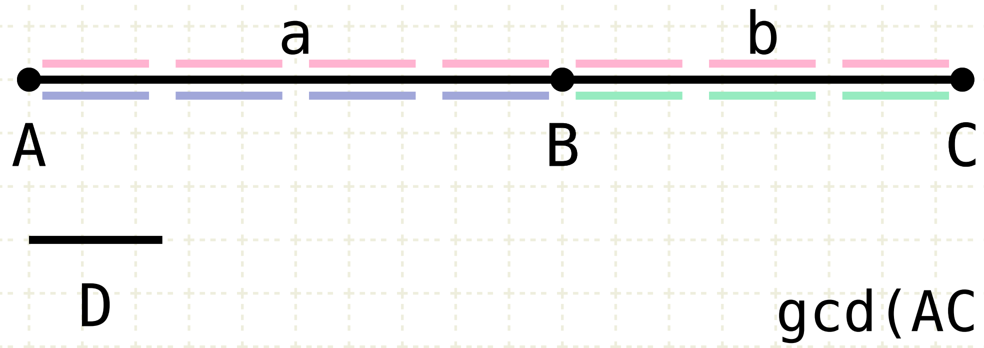


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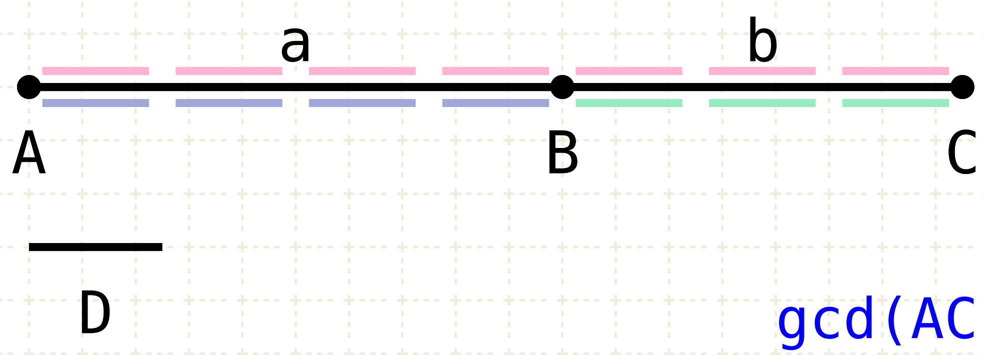


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