

Euclid's Elements

Book III



A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)



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2	A chord of a circle always lies inside the circle	10	A circle does not cut a circle at more points than two	18	If line touches a circle, then it is perpendicular to the diameter that touches that point
3	A line through the centre of a circle bisects a chord, and vice versa	11	Point of contact between two internal circles, and their centres, are collinear	19	If line touches a circle, then the centre of the circle lies on a line perpendicular to the original
4	A line not through the centre of a circle does not bisect a chord	12	Point of contact between two external circles, and their centres, are collinear	20	The angle at the centre of a circle is twice that from an angle from the circumference
5	If two circles cut one another, they will not have the same center	13	A circle does not touch a circle at more points than one, whether it touch it internally or externally.	21	In a circle the angles in the same segment are equal to one another
6	If two circles touch one another, they will not have the same center	14	In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another.	22	The opposite angles of quadrilaterals in circles are equal to two right angles
7	Consider two lines from a point inside a circle to the edge, the longer one will be the one closest to the longest part of the diameter passing through the original point	15	The longest line in a circle is its diameter, shorter the farther away from the diameter	23	On the same straight line there cannot be constructed two similar and unequal segments of circles on the same side
8	Consider two lines from a point outside a circle to the edge, the line closest to the centre will be longer on the concave side and shorter on the convex side	16	A line on the circle, perpendicular to the diameter, lies outside the circle	24	Similar segments of circles on equal straight lines are equal to one another



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| 26 | In equal circles equal angles stand on equal circumferences | 35 | If two circle chords intersect, the segments on one multiplied together equals the segments of the other multiplied together |
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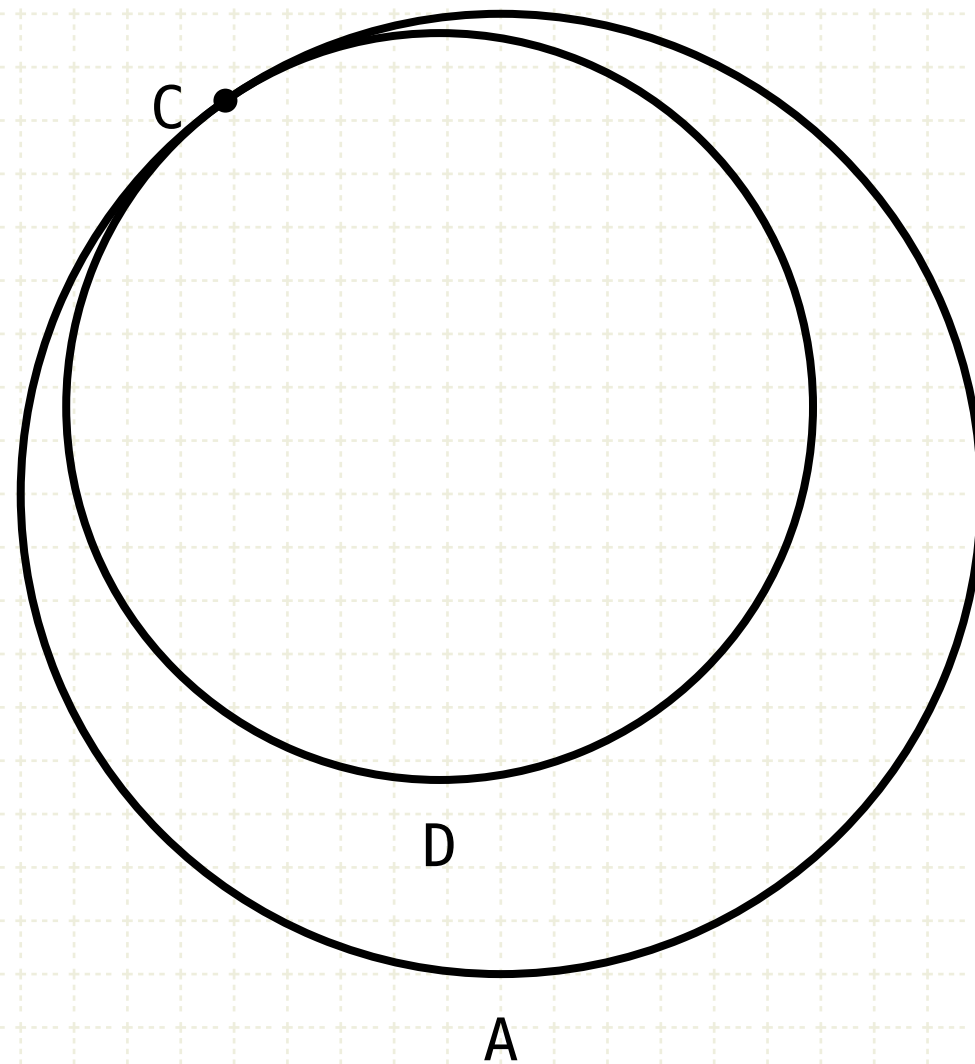
Proposition 6 of Book III

If two circles touch one another, they will not have the same center.



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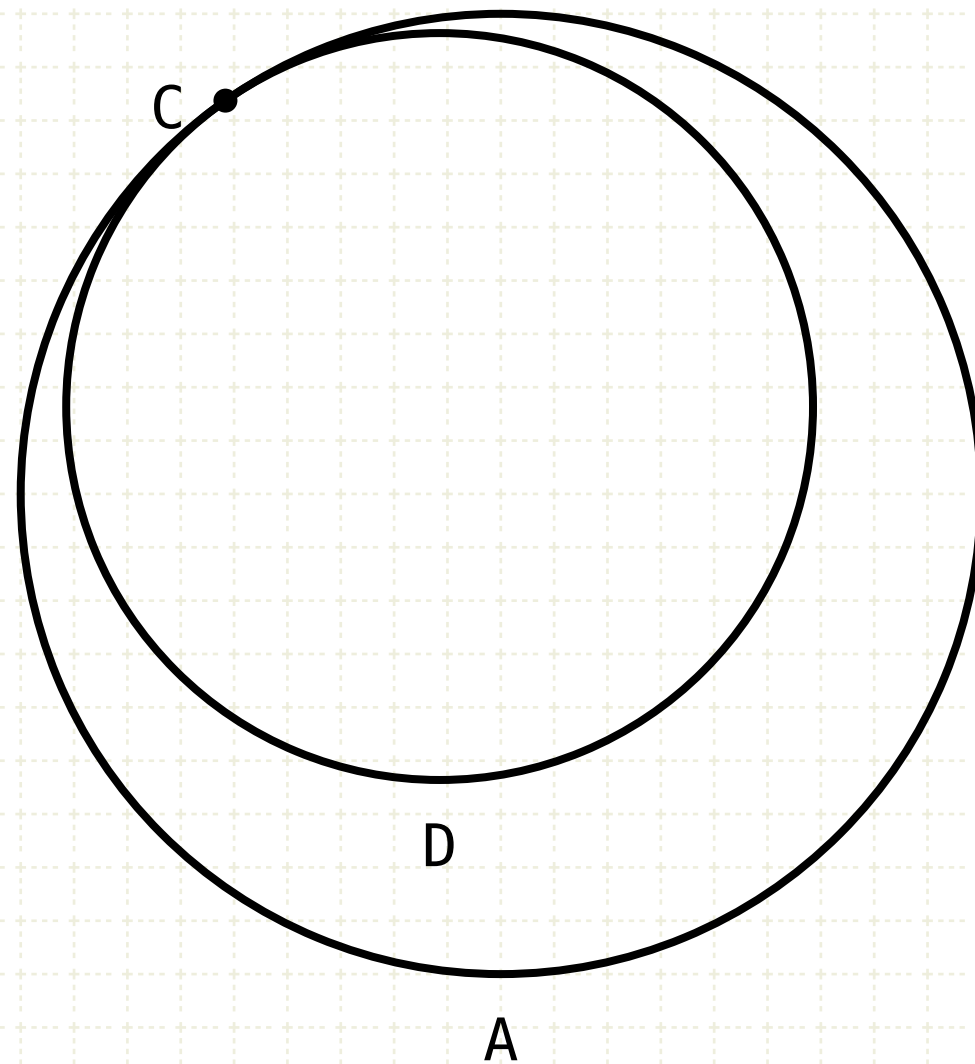
In other words

If two circles AC, and CD touch each other, then the center of the circle AC is not the same as the center of CD



Proposition 6 of Book III

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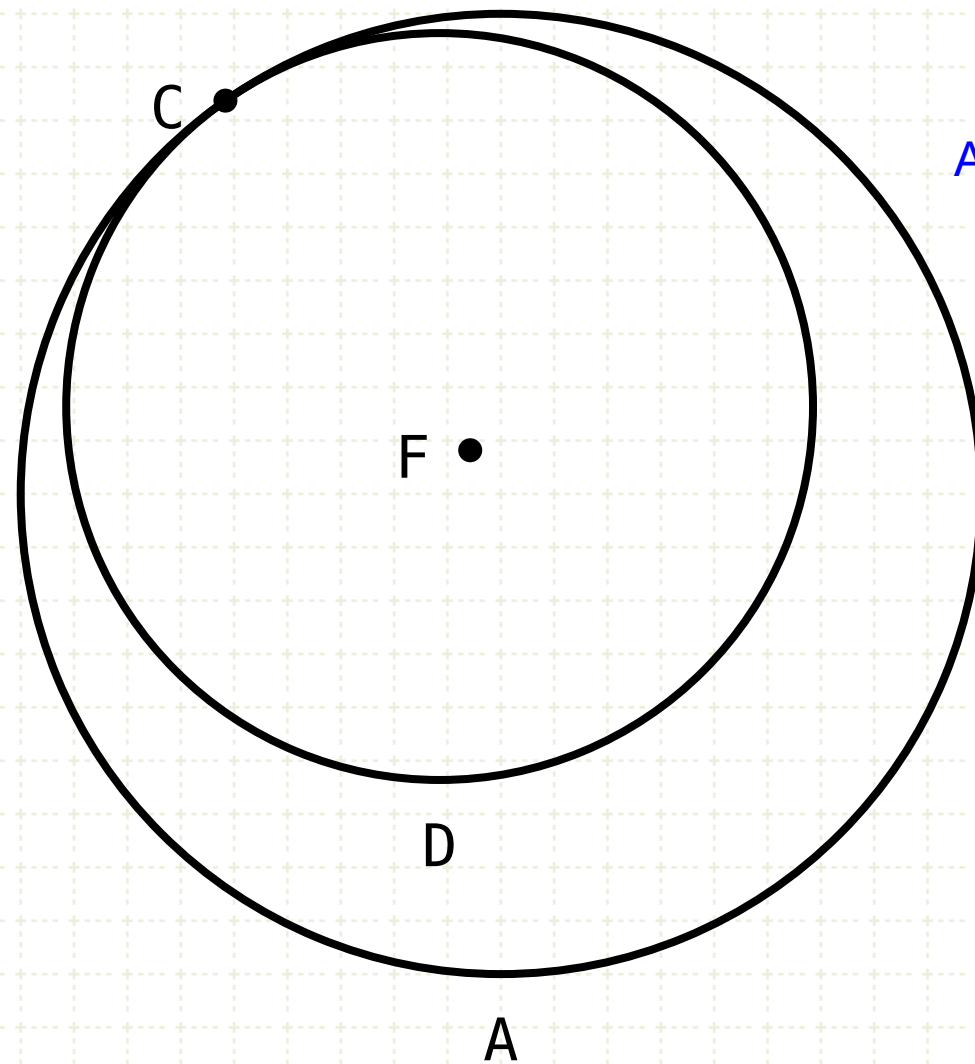
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Proof by contradiction



Proposition 6 of Book III

If two circles touch one another, they will not have the same center.



Assume F is centre of both circles

In other words

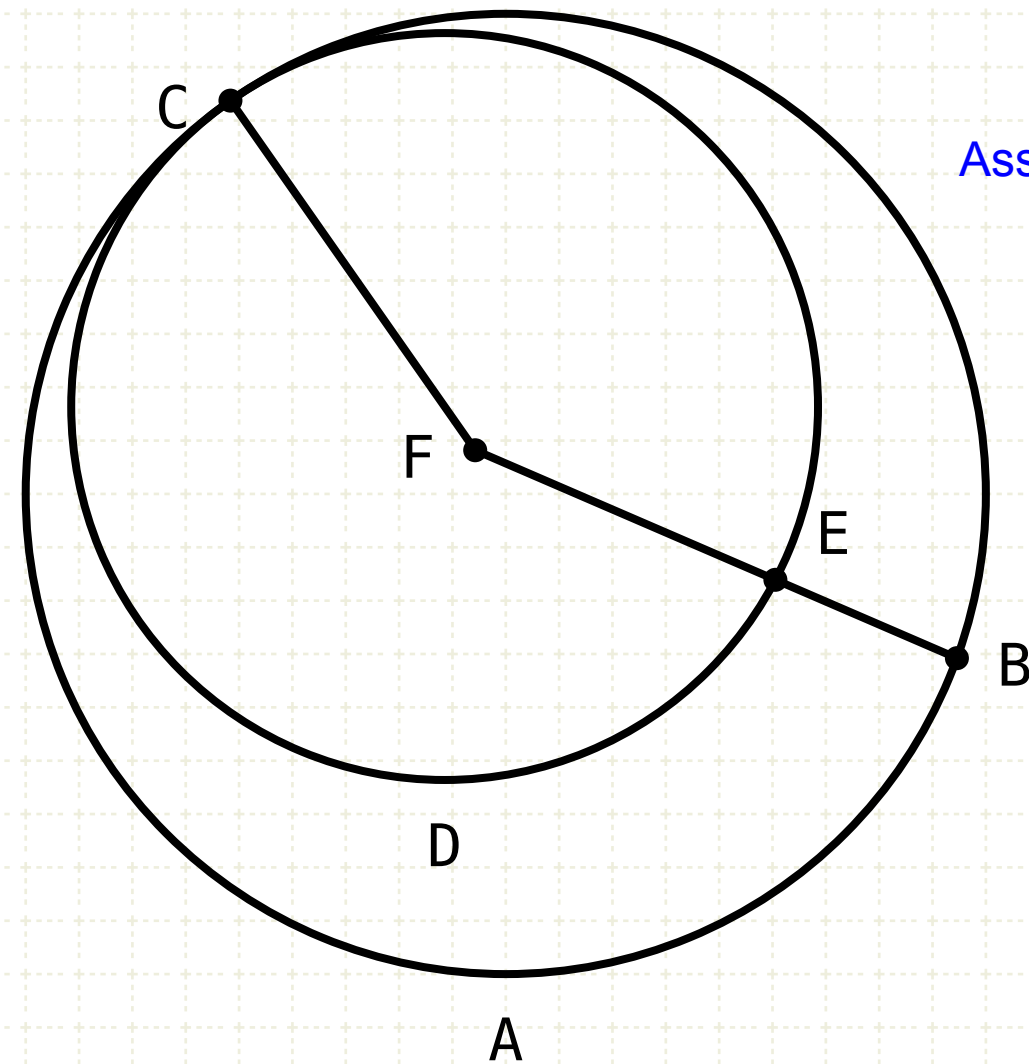
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Assume that F is the center of both circles

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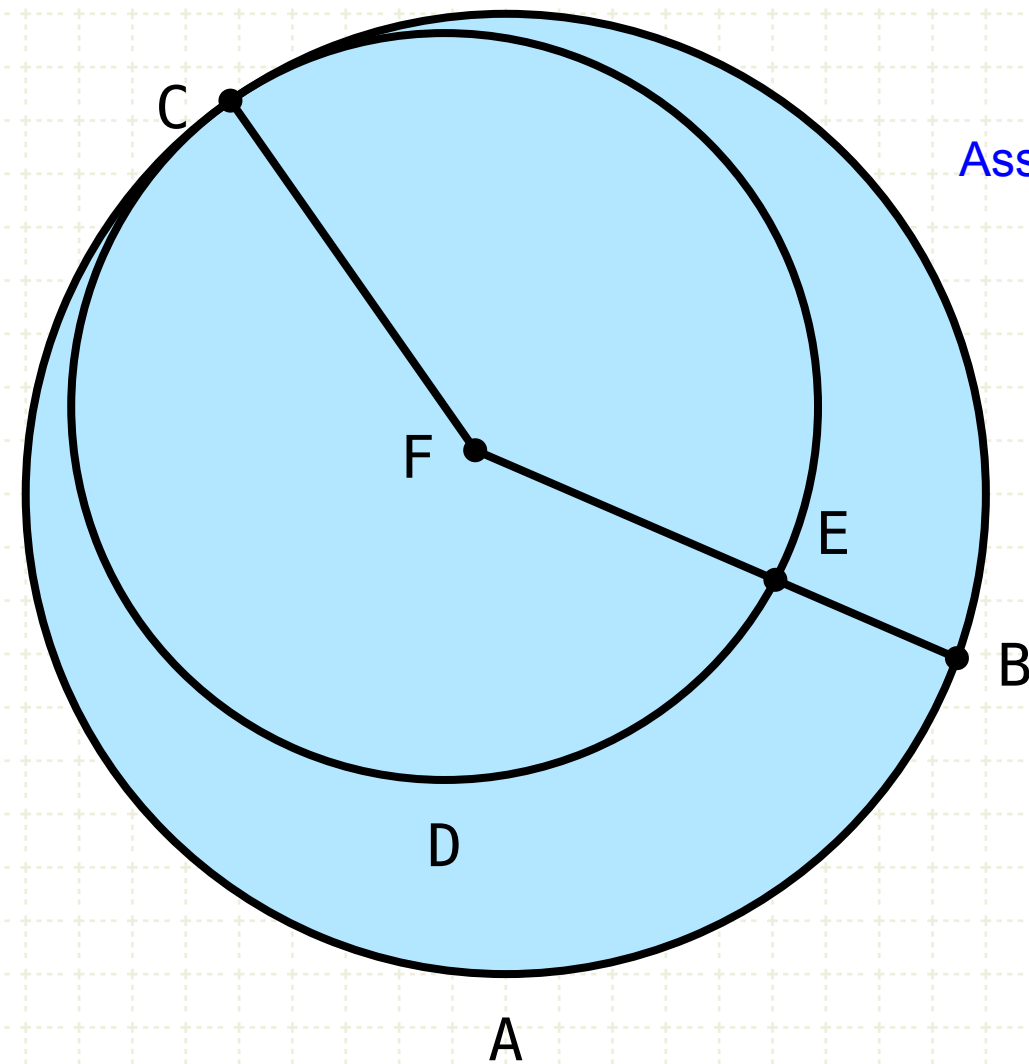
Proof by contradiction

Assume that F is the center of both circles

Join FC and draw a line FEB at random

Proposition 6 of Book III

If two circles touch one another, they will not have the same center.



Assume F is centre of both circles

$$FC = FB$$

In other words

If two circles AC, and CD touch each other, then the center of the circle AC is not the same as the center of CD

Proof by contradiction

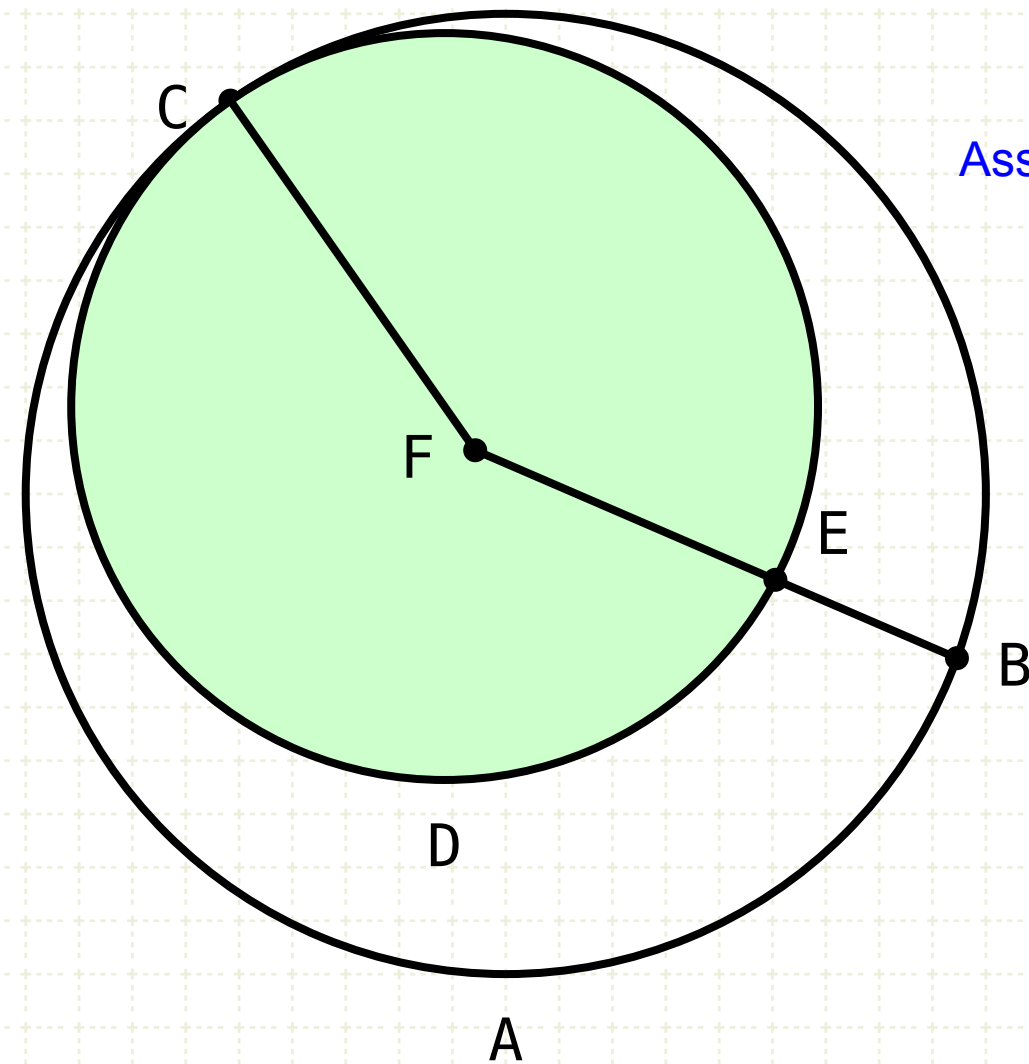
Assume that F is the center of both circles

Join FC and draw a line FEB at random

Since FC and FB are radii of the circle AC, they are equal

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If two circles touch one another, they will not have the same center.



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$$\begin{aligned} FC &= FB \\ FC &= FE \end{aligned}$$

In other words

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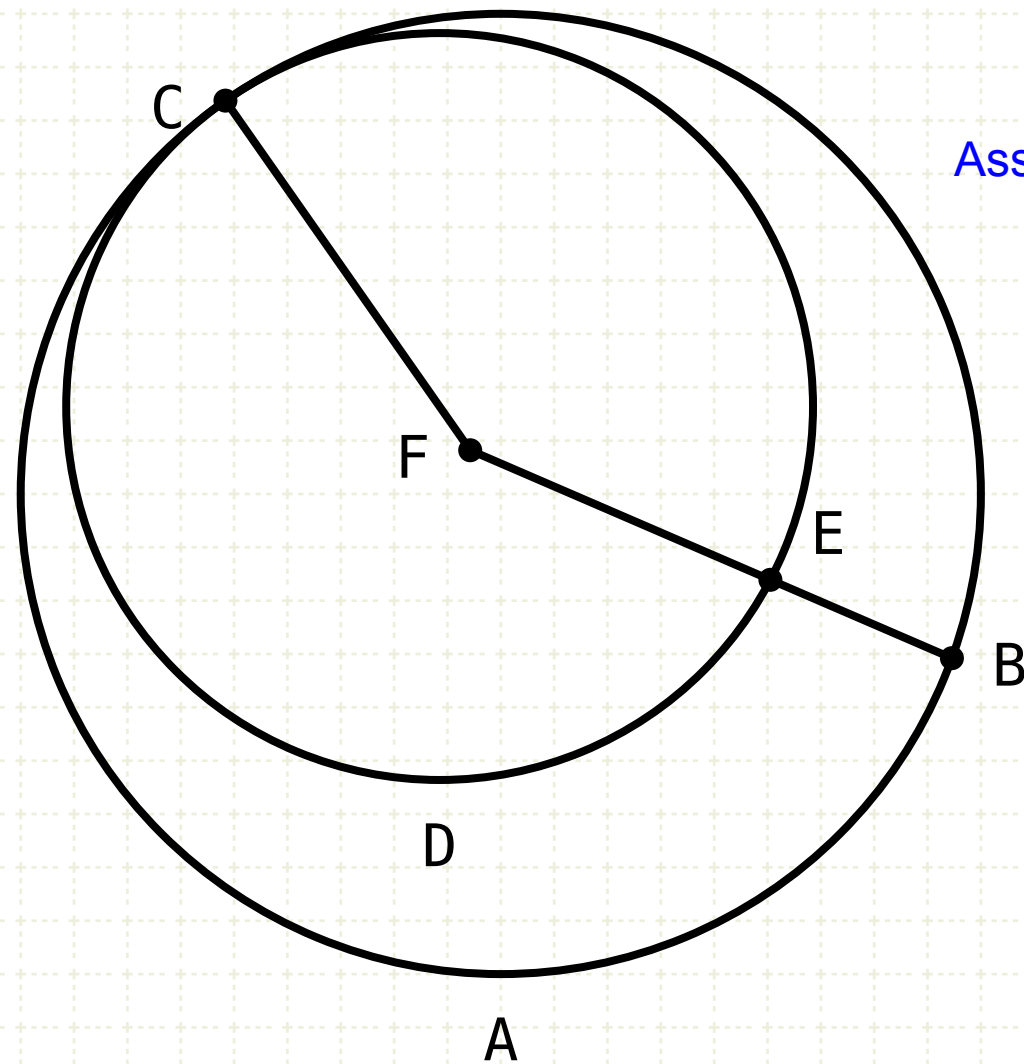
Join FC and draw a line FEB at random

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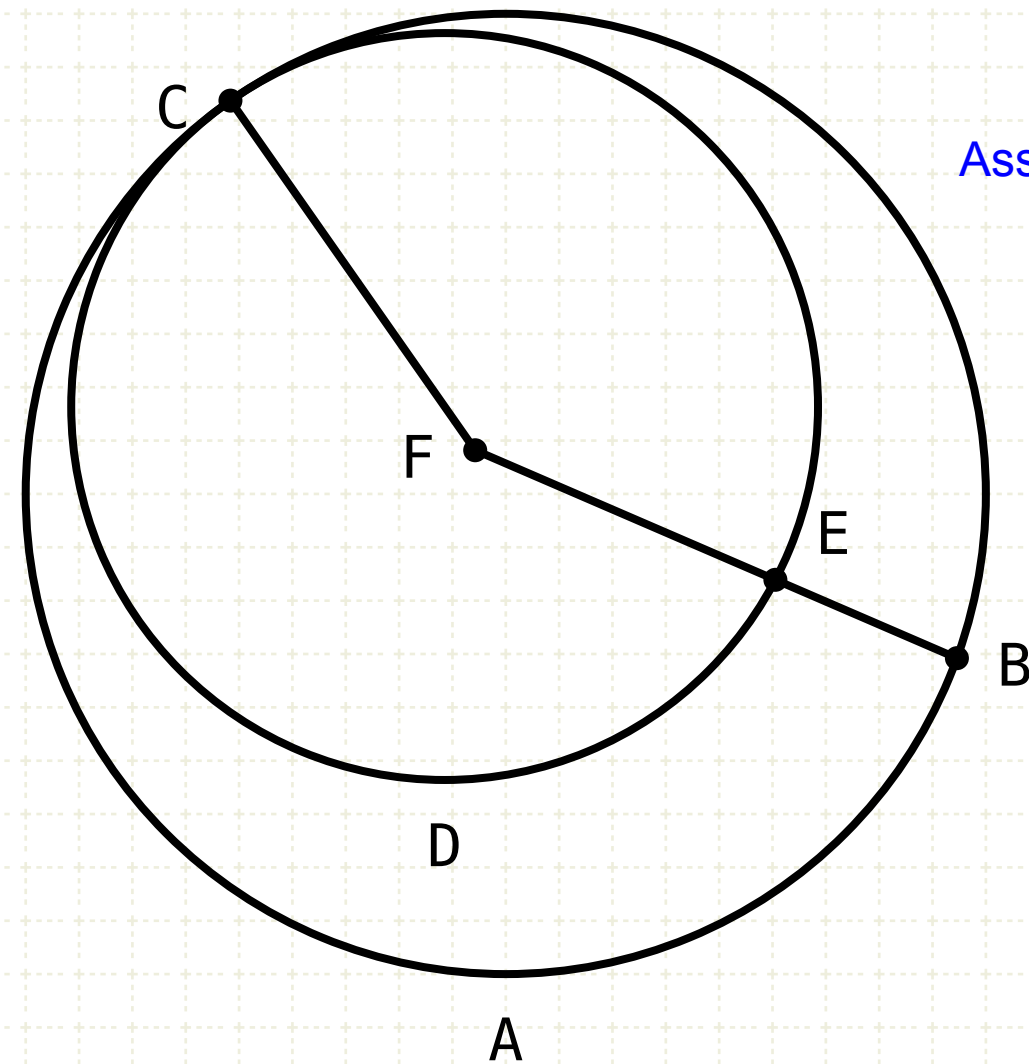
Since FC and FB are radii of the circle AC, they are equal

Since FC and FE are radii of the circle CD, they are equal

Since FB and FE are both equal to FC, they are equal to each other

Proposition 6 of Book III

If two circles touch one another, they will not have the same center.



Assume F is centre of both circles

$$FC = FB$$

$$FC = FE$$

$$FB = FE$$

$$FB > FE$$

In other words

If two circles AC, and CD touch each other, then the center of the circle AC is not the same as the center of CD

Proof by contradiction

Assume that F is the center of both circles

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Since FC and FB are radii of the circle AC, they are equal

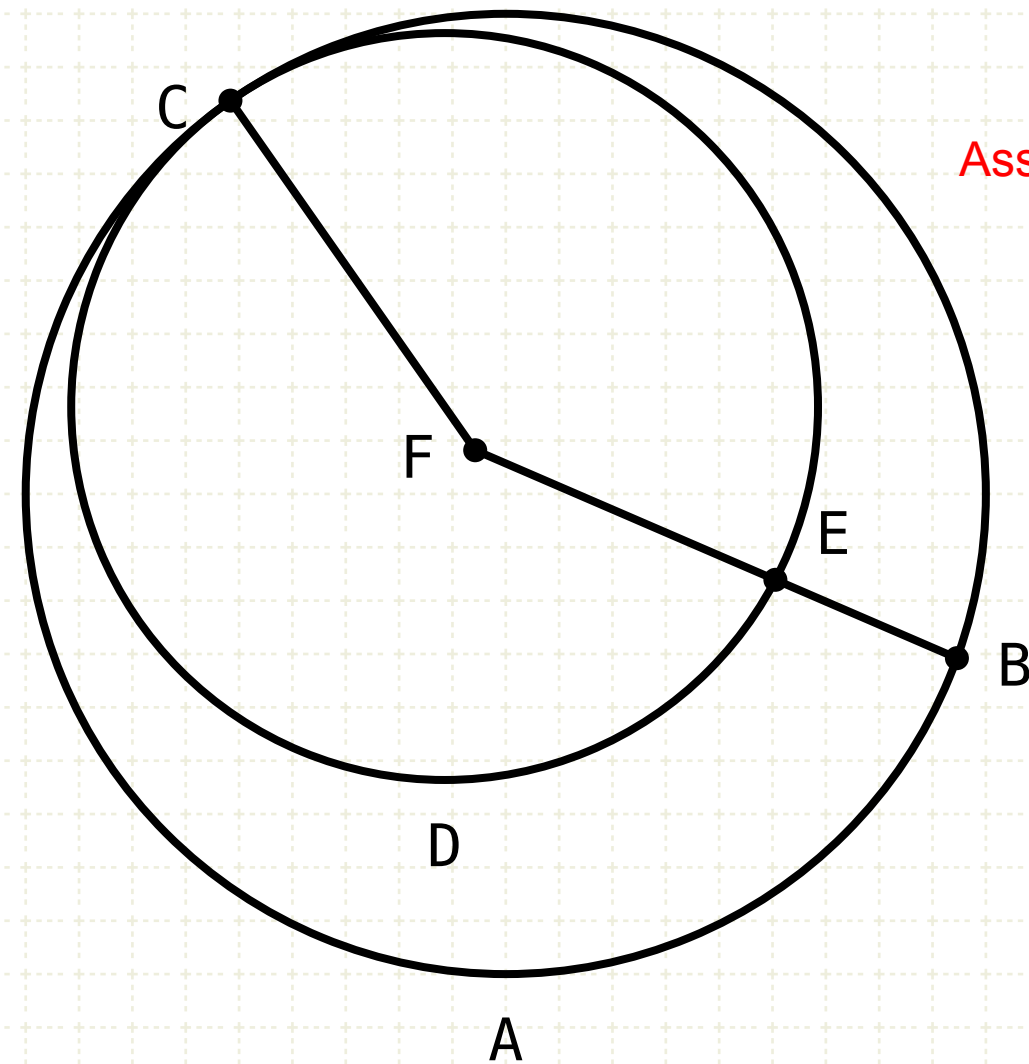
Since FC and FE are radii of the circle CD, they are equal

Since FB and FE are both equal to FC, they are equal to each other

But FE is less than FB, which is inconsistent with the above statement

Proposition 6 of Book III

If two circles touch one another, they will not have the same center.



Assume F is centre of both circles

$$FC = FB$$

$$FC = FE$$

$$FB = FE$$

$$FB > FE$$

In other words

If two circles AC, and CD touch each other, then the center of the circle AC is not the same as the center of CD

Proof by contradiction

Assume that F is the center of both circles

Join FC and draw a line FEB at random

Since FC and FB are radii of the circle AC, they are equal

Since FC and FE are radii of the circle CD, they are equal

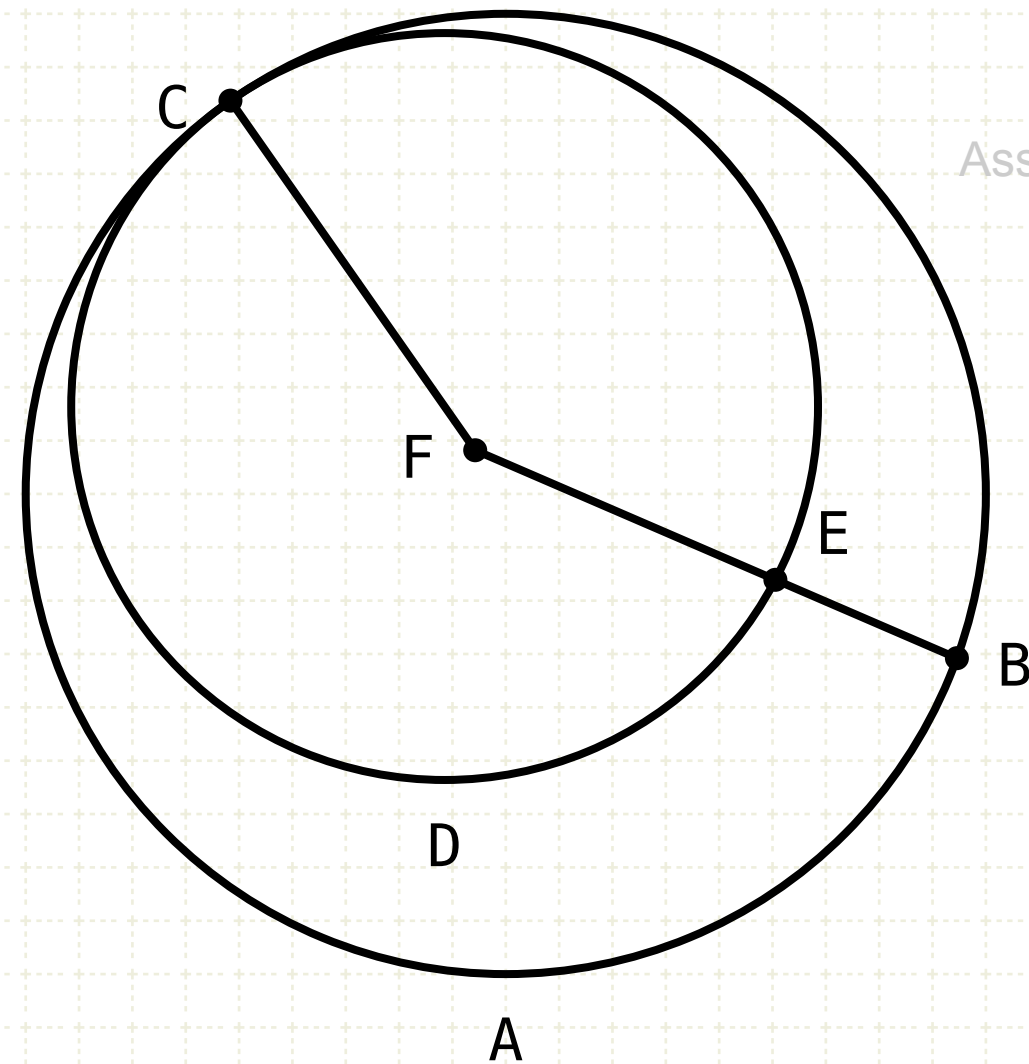
Since FB and FE are both equal to FC, they are equal to each other

But FE is less than FB, which is inconsistent with the above statement

Therefore F cannot be the center of both circles

Proposition 6 of Book III

If two circles touch one another, they will not have the same center.



Assume F is centre of both circles

$$FC = FB$$

$$FC = FE$$

$$FB = FE$$

$$FB > FE$$

F is not the centre of both circles

In other words

If two circles AC, and CD touch each other, then the center of the circle AC is not the same as the center of CD

Proof by contradiction

Assume that F is the center of both circles

Join FC and draw a line FEB at random

Since FC and FB are radii of the circle AC, they are equal

Since FC and FE are radii of the circle CD, they are equal

Since FB and FE are both equal to FC, they are equal to each other

But FE is less than FB, which is inconsistent with the above statement

Therefore F cannot be the center of both circles

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