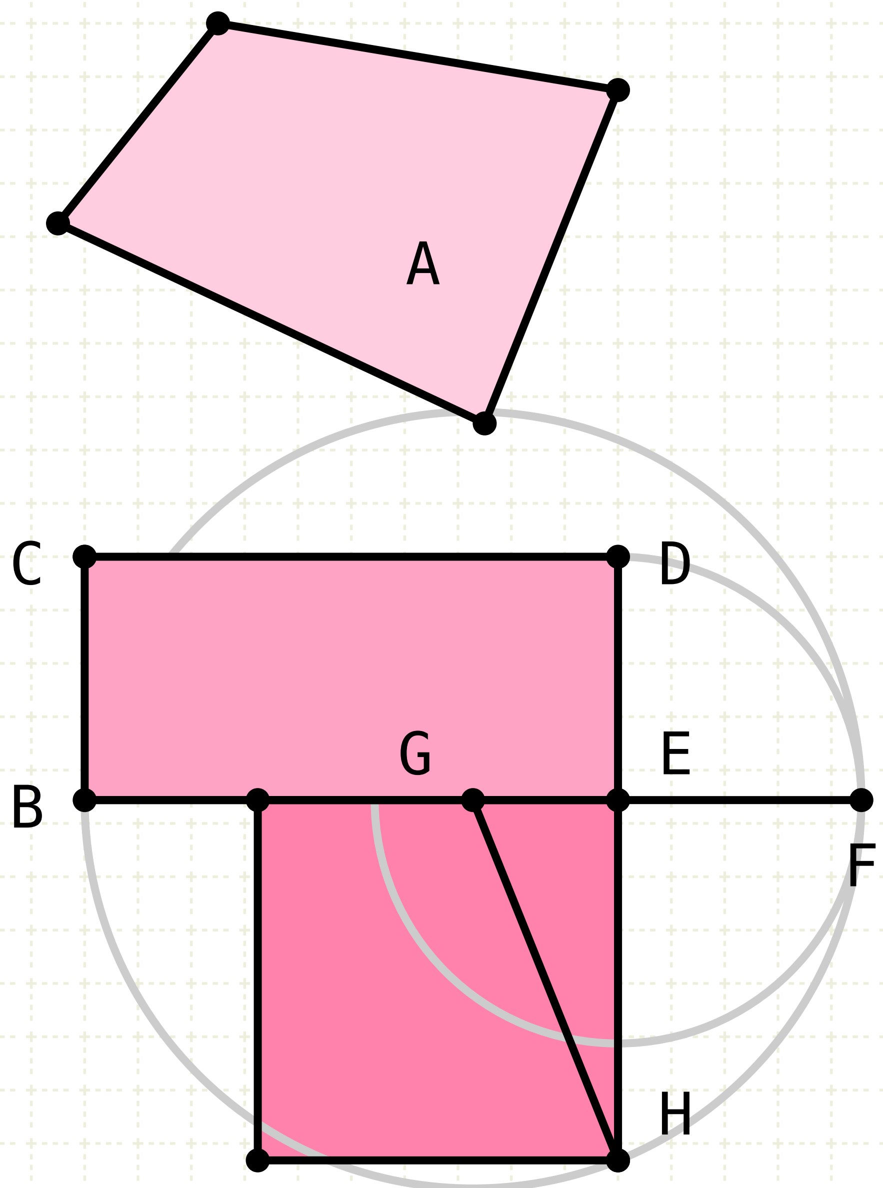
Euclid's Elements

Book II



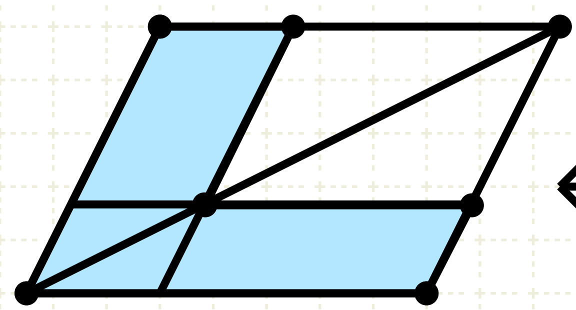
It is a remarkable fact in the history of geometry, that the Elements of Euclid, written two thousand years ago, are still regarded by many as the best introduction to the mathematical sciences.

Florian Cajori,

A History of Mathematics (1893)

Definitions:

Any rectangular parallelogram is said to be contained by the two straight lines containing the right angle.



And in any parallelogrammic area let any one whatever of the parallelograms about its diameter with the two complements be called a gnomon.

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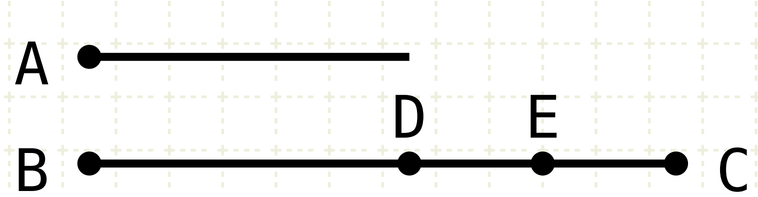
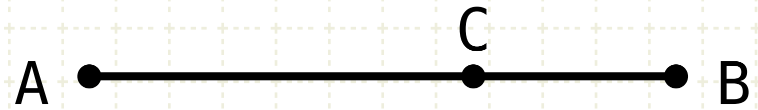


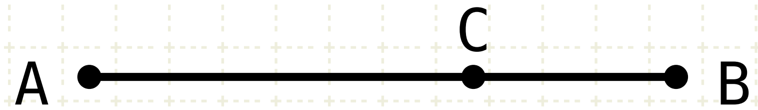
$A \cdot BC = A \cdot BD + A \cdot DE + A \cdot EC$

2



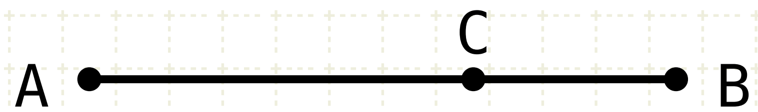
$AB^2 = AB \cdot AC + AB \cdot BC$

3



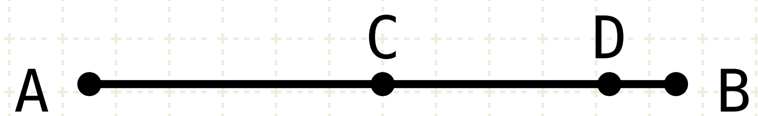
$AB \cdot CB = AC \cdot CB + CB^2$

4



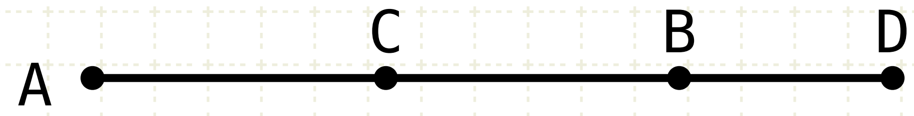
$AB^2 = AC^2 + CB^2 + 2 \cdot AC \cdot CB$

5



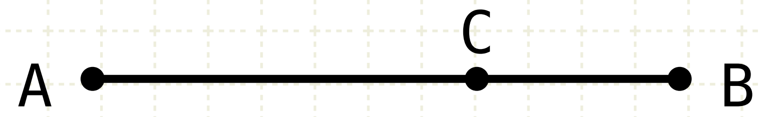
$AD \cdot DB + CD^2 = CB^2$

6



$AD \cdot DB + CB^2 = CD^2$

7

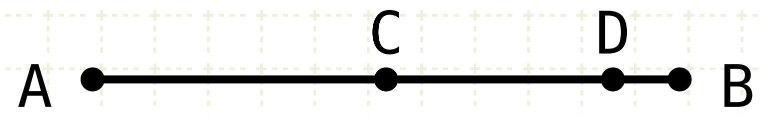


$AB^2 + BC^2 = AC^2 + 2 \cdot AB \cdot BC$

8

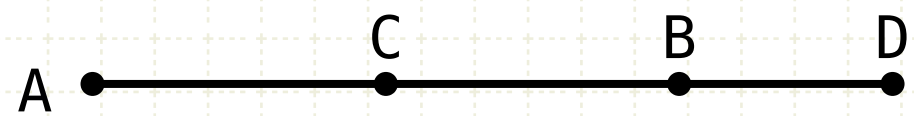
$4 \cdot AB \cdot BC + AC^2 = (AB+BC)^2$

9



$AD^2 + DB^2 = 2 \cdot (AC^2 + CD^2)$

10

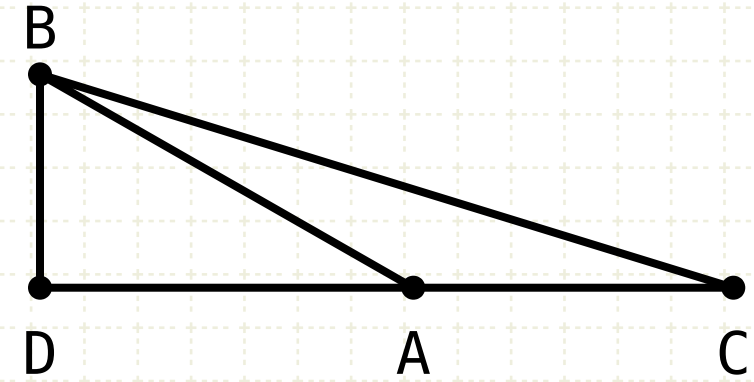


$AD^2 + DB^2 = 2 \cdot (AC^2 + CD^2)$

11

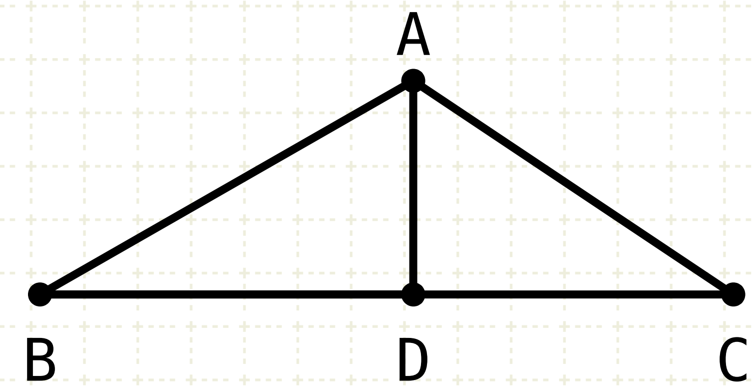
Find H. $AB \cdot BH = AH^2$

12



Cosine Law. $BC^2 = AB^2+AC^2+2 \cdot AD \cdot AC$

13



Cosine Law. $AC^2 = AB^2+BC^2-2 \cdot BD \cdot BC$

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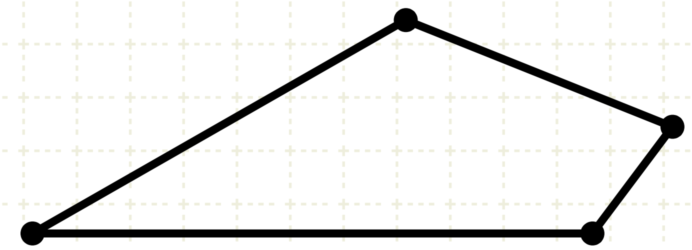

Find square of polygon

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Proposition 5 of Book II

If a straight line be cut into equal and unequal segments, the rectangle contained by the unequal segments of the whole together with the square on the straight line between the points of section is equal to the square on the half

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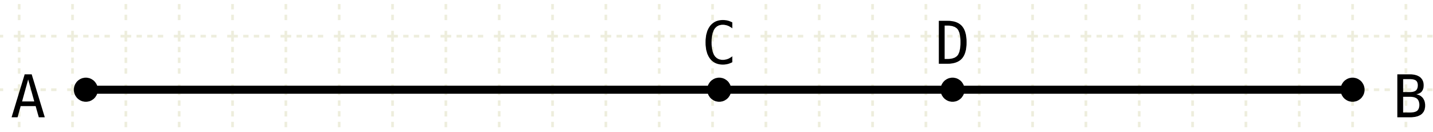

In other words

Let AB be a straight line, bisected at point C, and cut at an arbitrary point D

AC = CB, AD = AC+CD, DB = BC-CD

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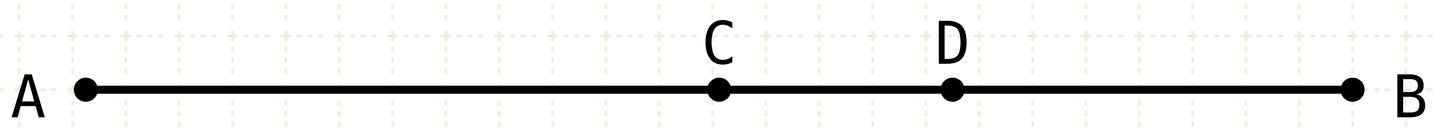

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Let AB be a straight line, bisected at point C, and cut at an arbitrary point D

The rectangle formed by the uneven segments (AD and DB) added to the square of the tiny segment CD, is equal to the half segment (CB) all squared.

$AC = CB, AD = AC+CD, DB = BC-CD$

$AD \cdot DB + CD \cdot CD = CB \cdot CB$

$AD \cdot DB = CB \cdot CB - CD \cdot CD$

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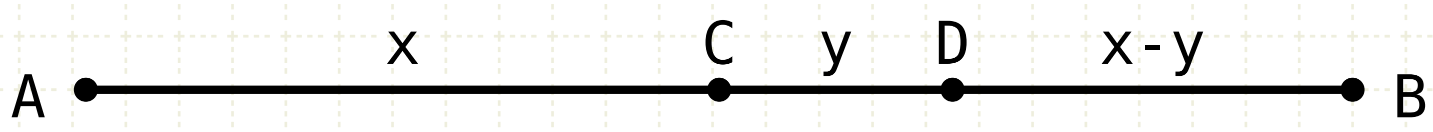

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$$AD \cdot DB + CD \cdot CD = CB \cdot CB$$

$$AD \cdot DB = CB \cdot CB - CD \cdot CD$$

$$(x+y) \cdot (x-y) = x^2 - y^2$$

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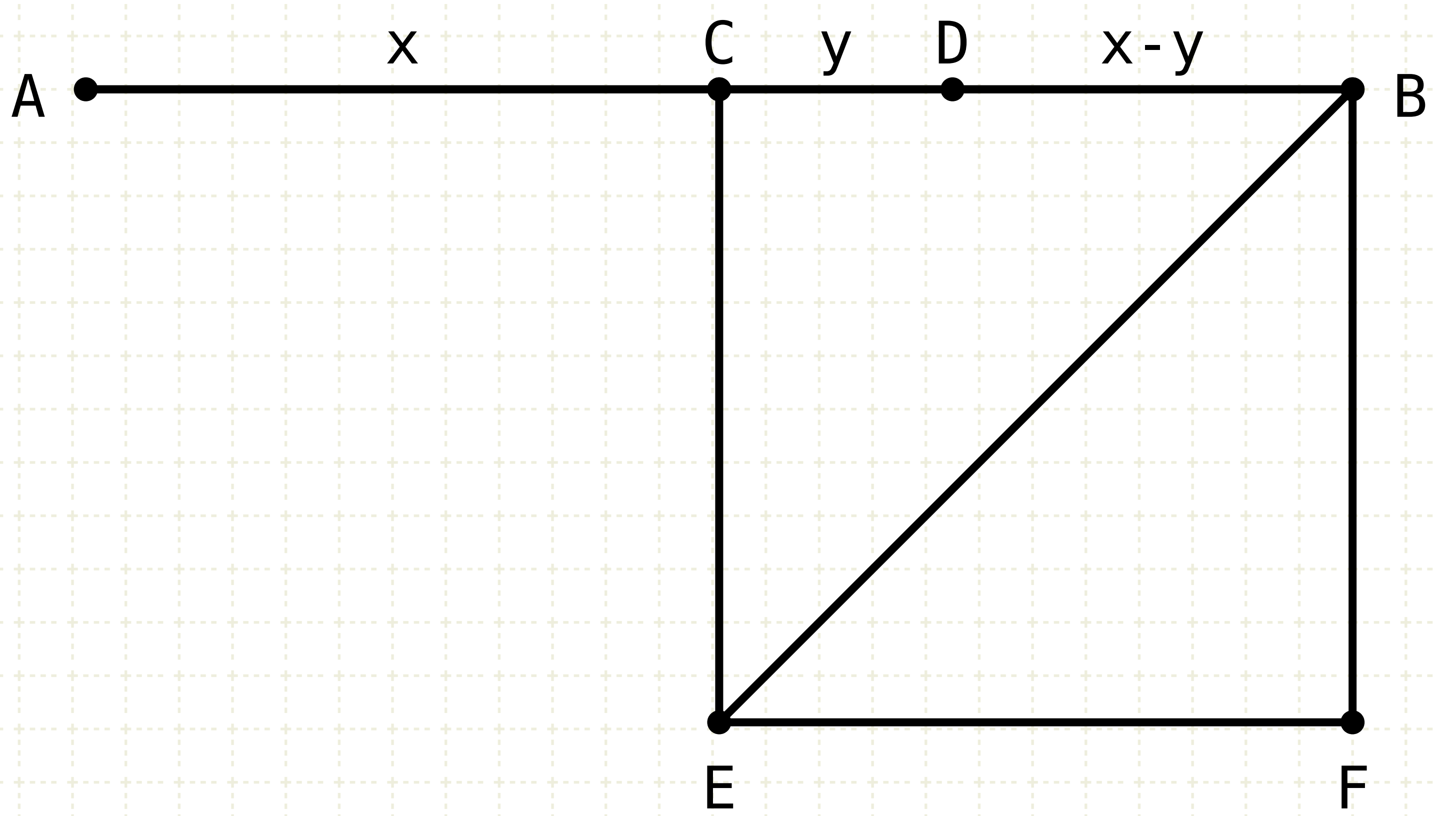

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Construction:

Draw a square CEFB on the line CB (I·46) and draw the diagonal BE

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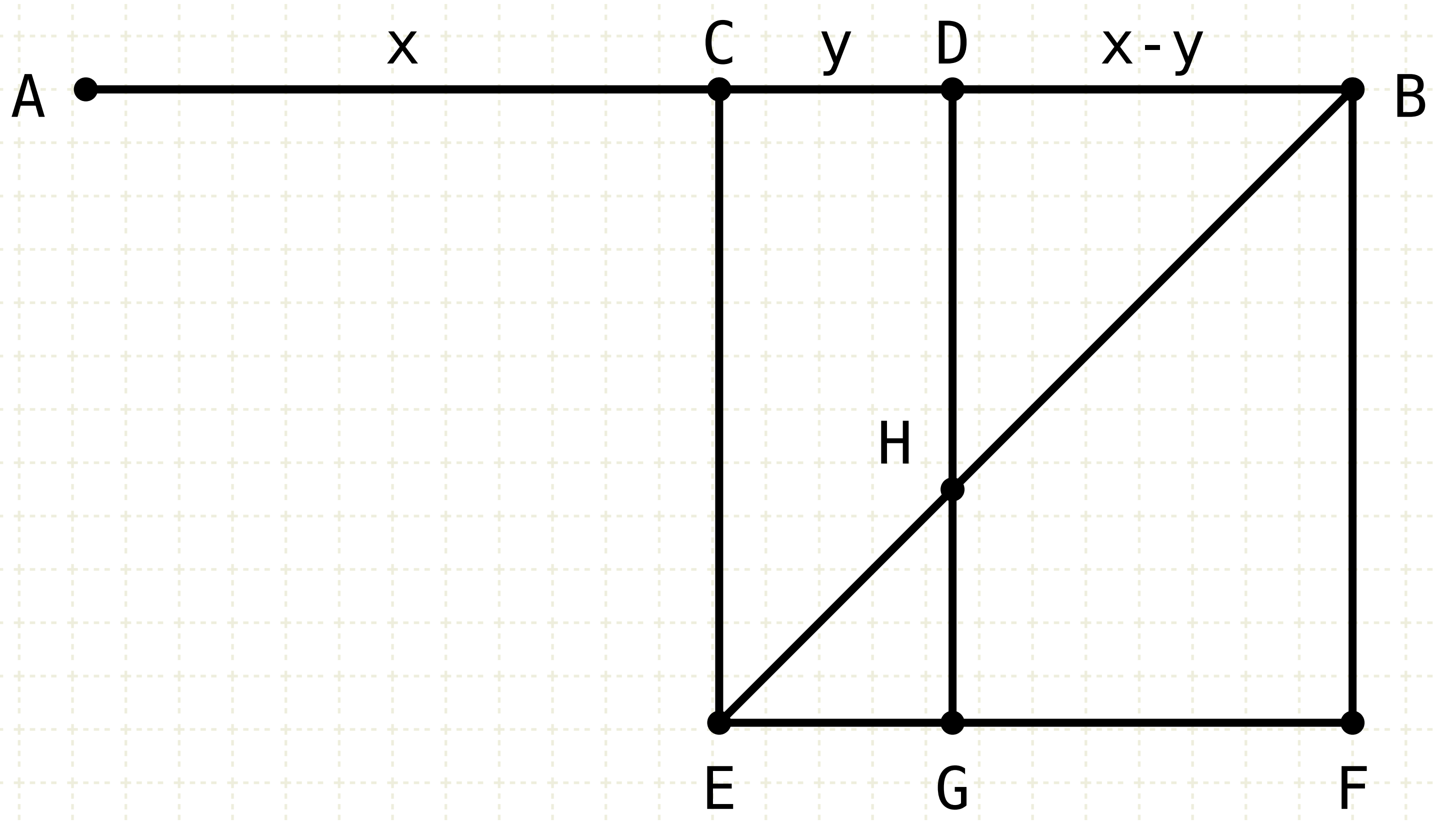

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Construction:

Draw a square CEFB on the line CB (I·46) and draw the diagonal BE

From point D, draw a line parallel to either CE or BF (I·31)

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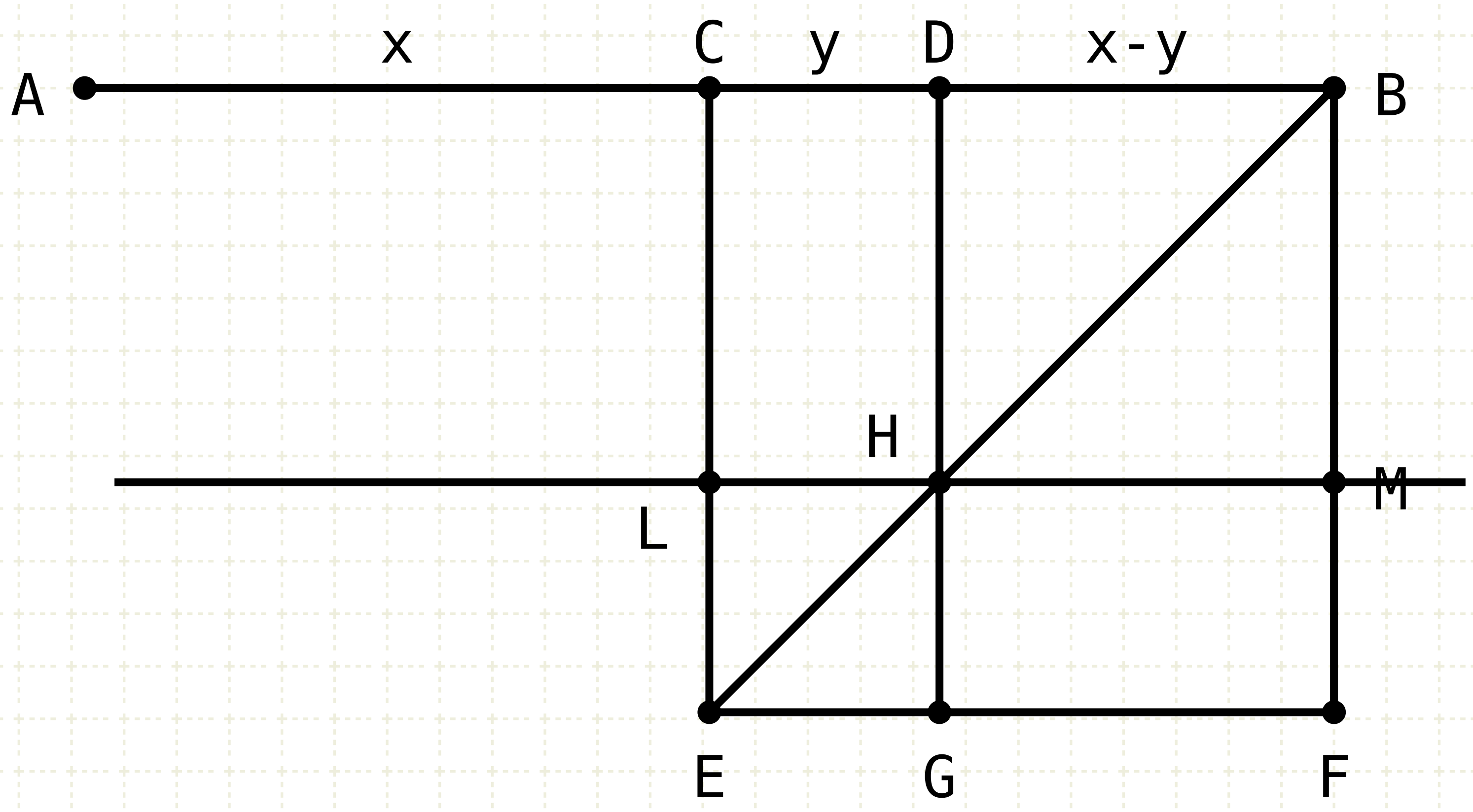

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From point D, draw a line parallel to either CE or BF (I·31)

From point H, draw a line parallel to either AB or EF (I·31)

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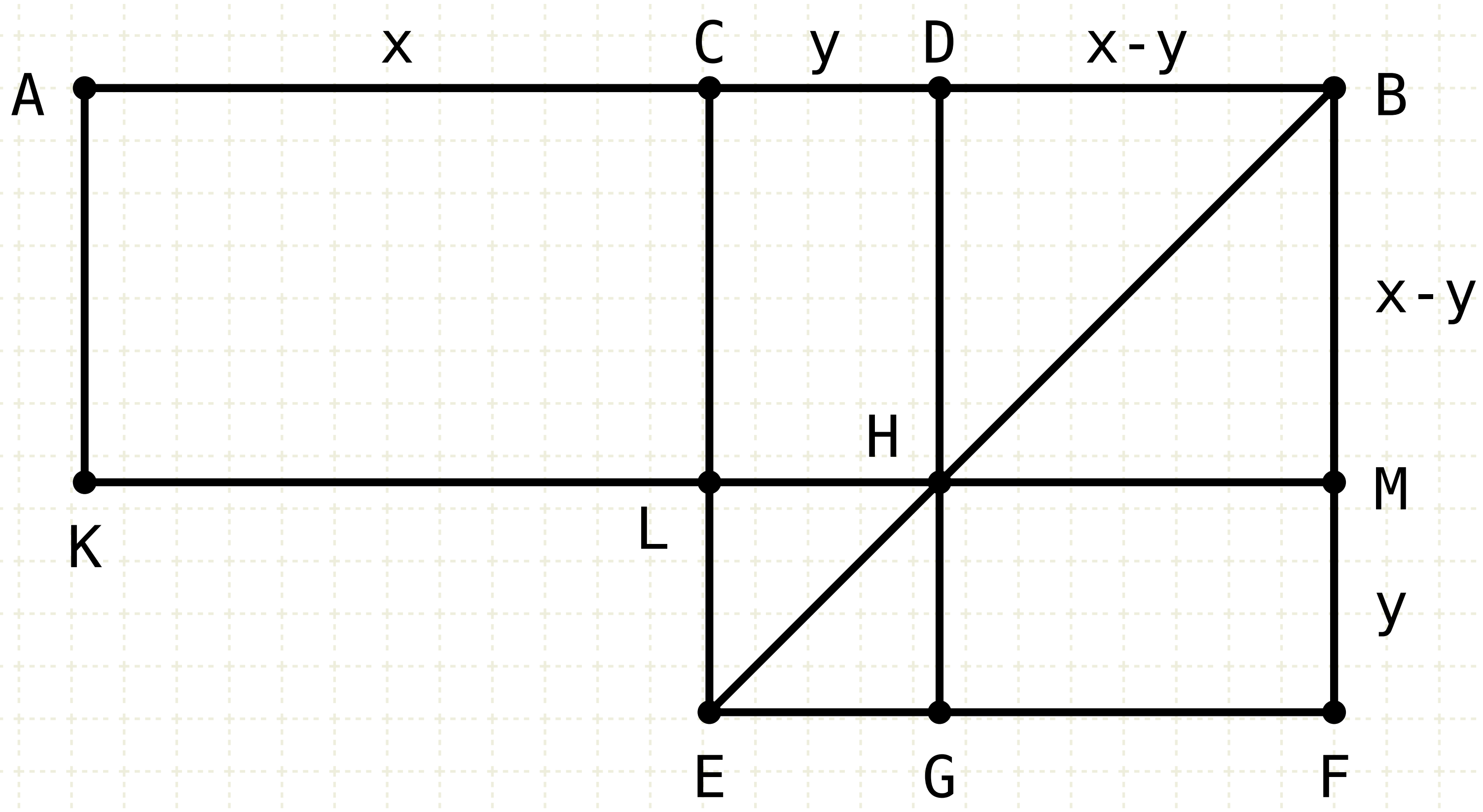

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From point H, draw a line parallel to either AB or EF (I·31)

From point A, draw a line parallel to either CL or BM (I·31)

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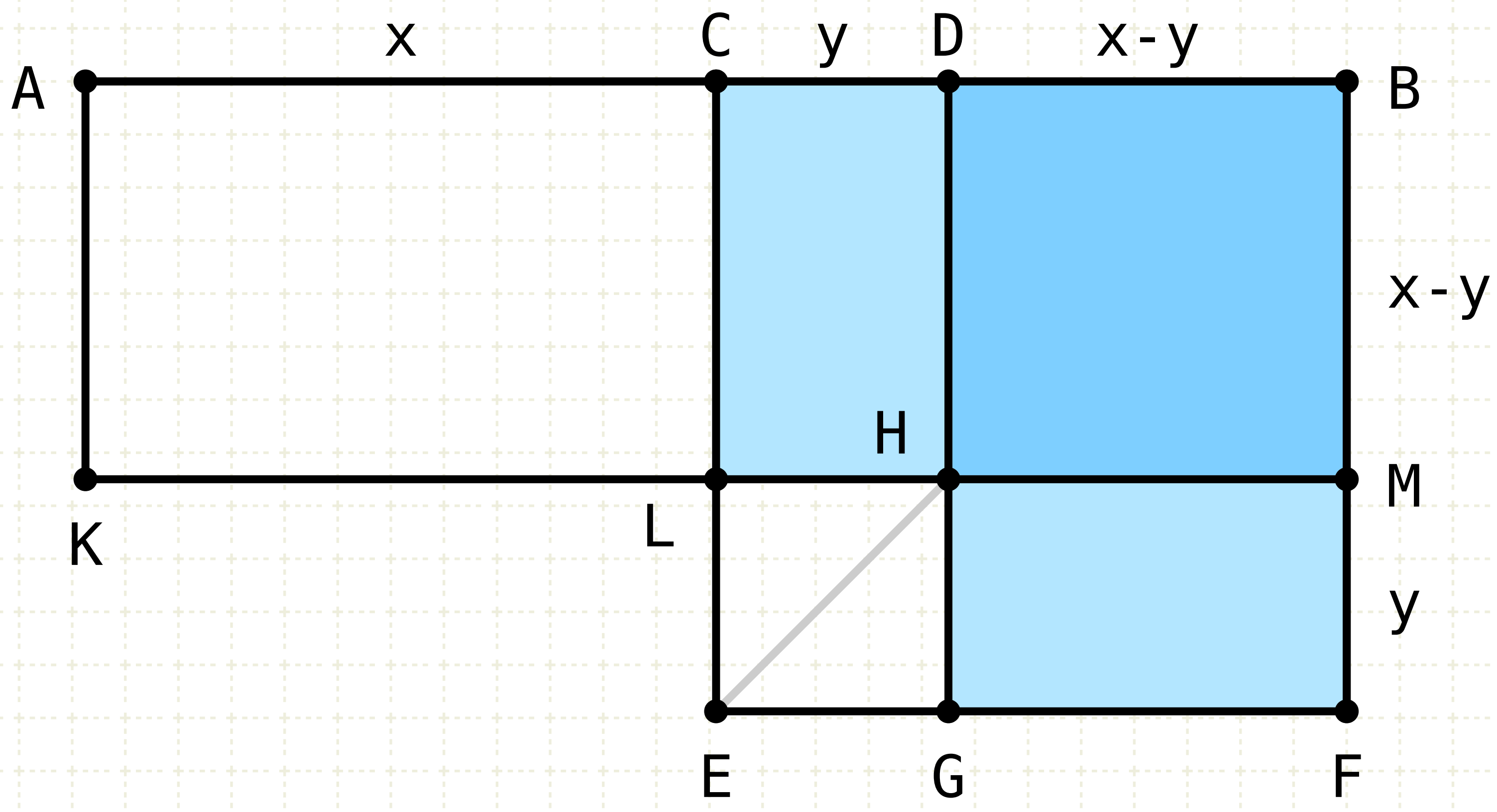

AC = CB, AD = AC+CD, DB = AC-CD

□CH = □HF ∴ □CM = □DF

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Proof:

The complements CH and HF are equal (I·43), and if we add the rectangle DM, then the rectangles CM and DF are equal

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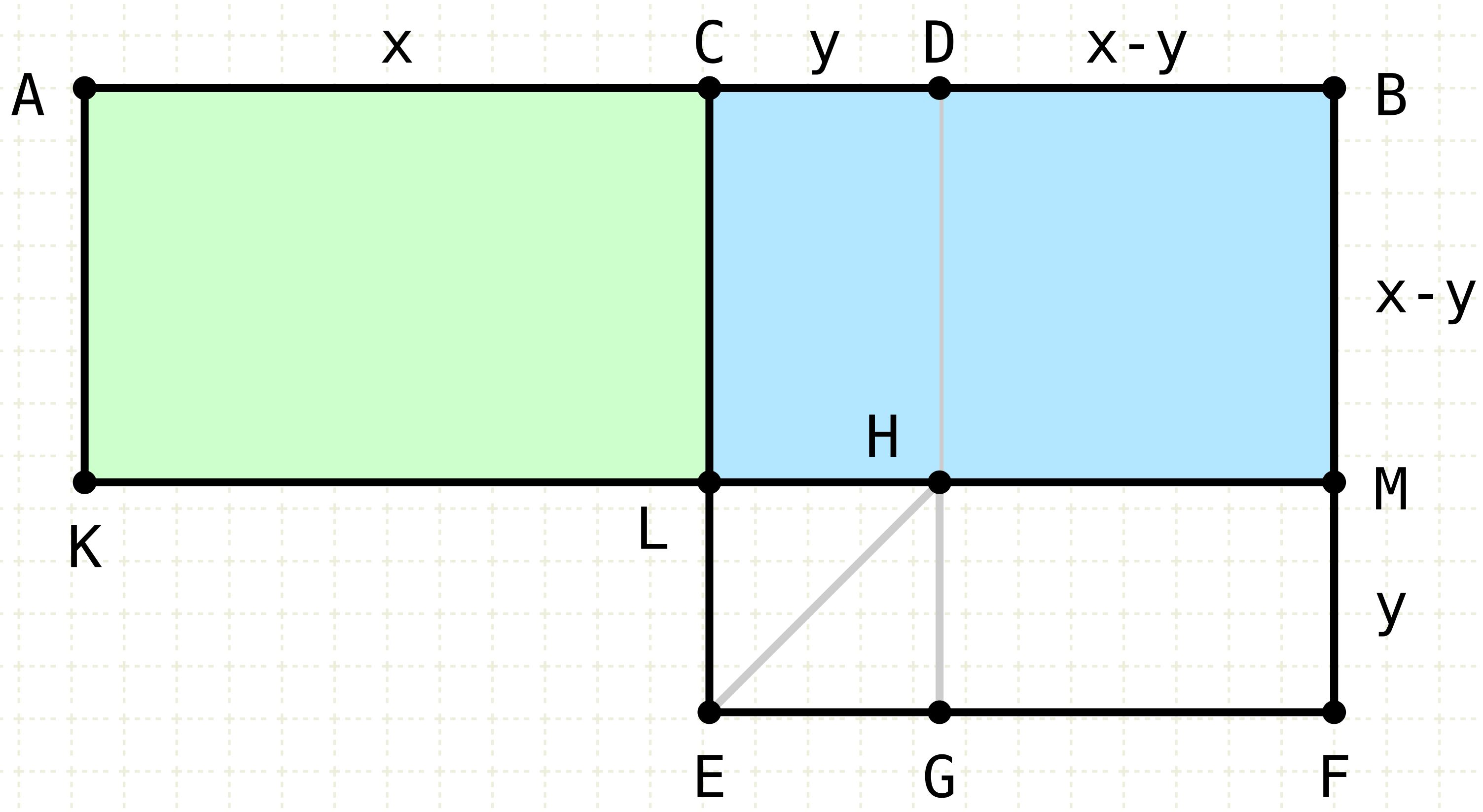

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The rectangles CM and AL are equal (I·36)

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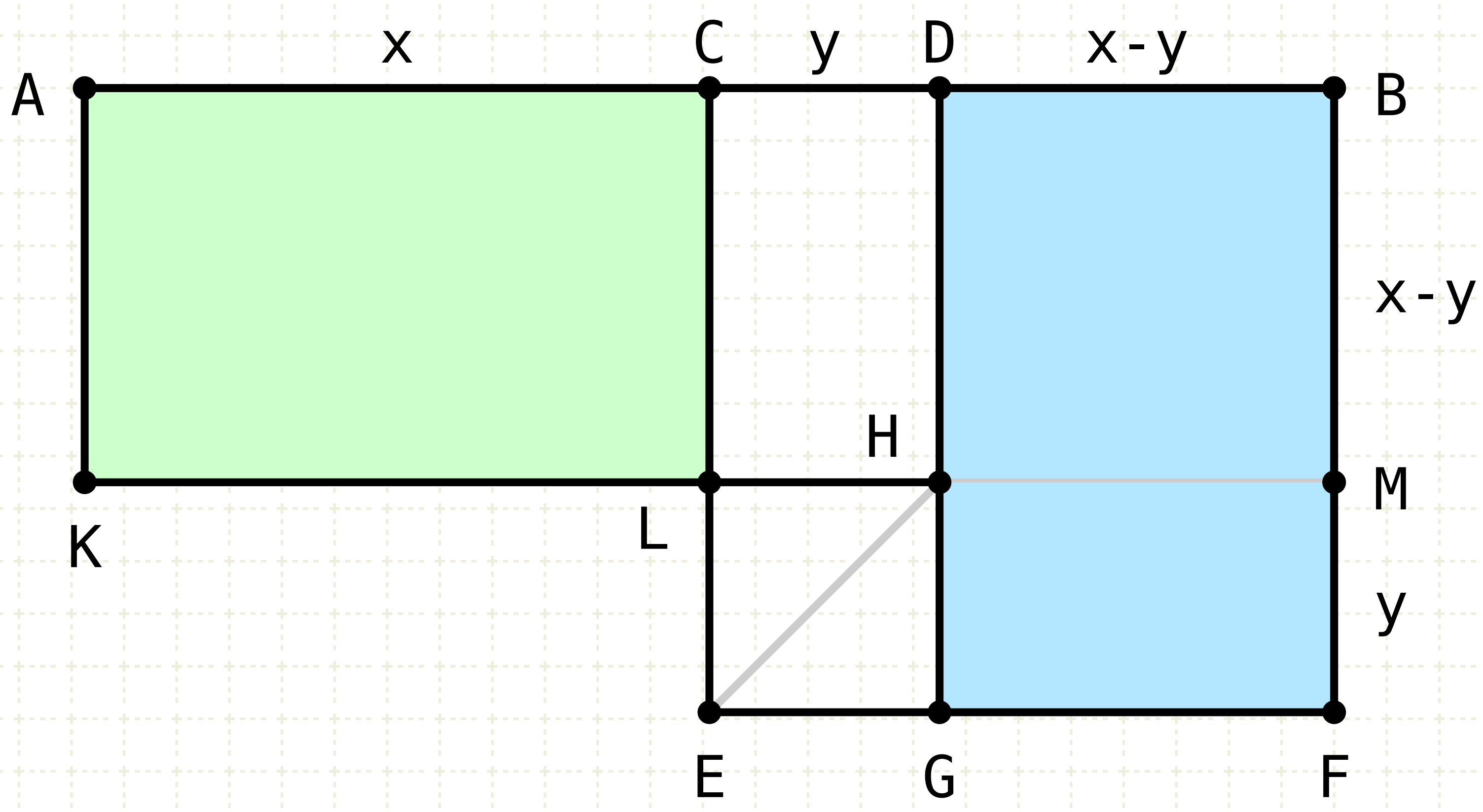

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The rectangles CM and AL are equal (I·36)

which means that AL and DF are also equal

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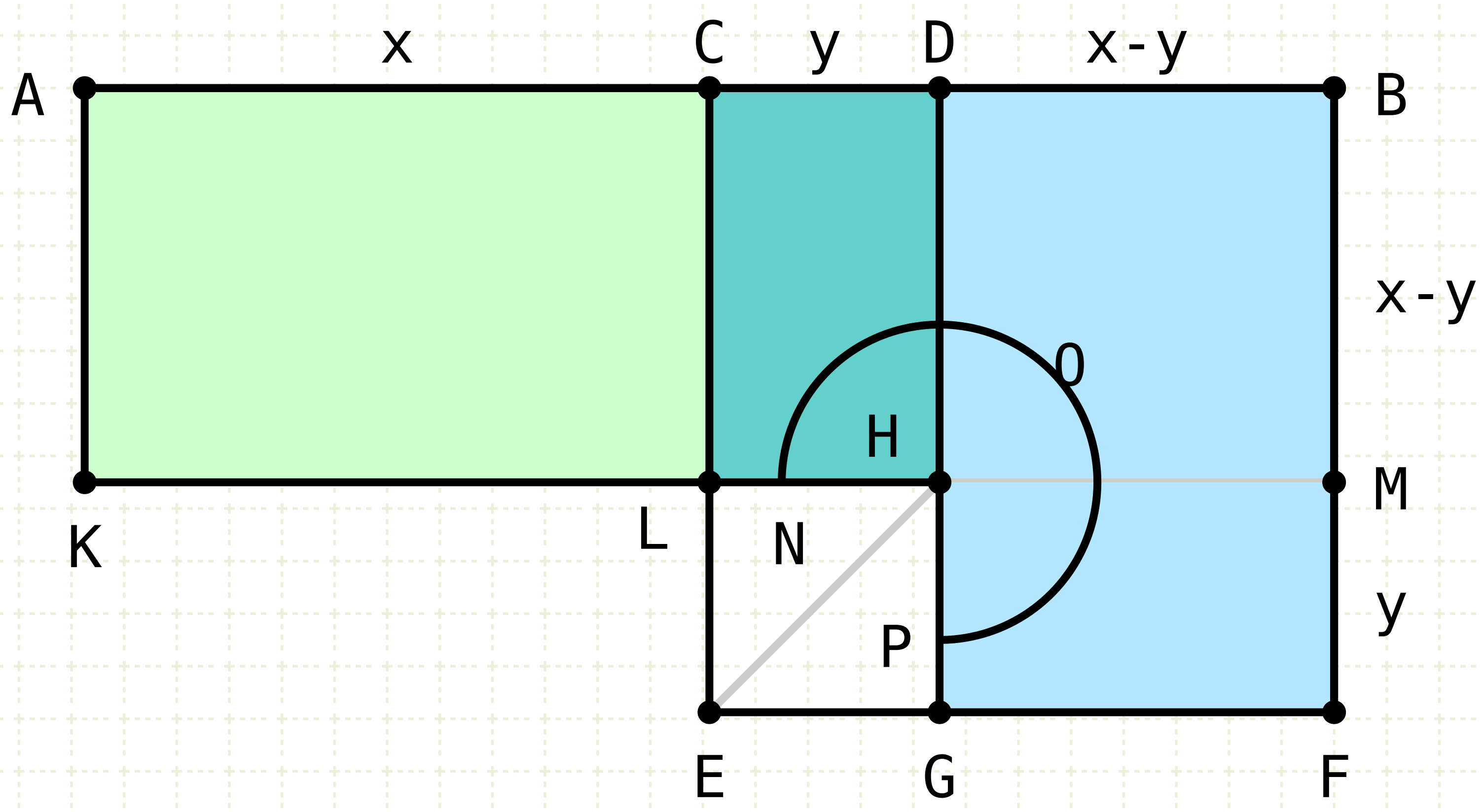

AC = CB, AD = AC+CD, DB = AC-CD

□CH = □HF ∴ □CM = □DF

□AL = □CM = □DF

□AH = NOP

In other words

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Let CH be added to each of AL and DF. Now AH is equal to gnomon NOP

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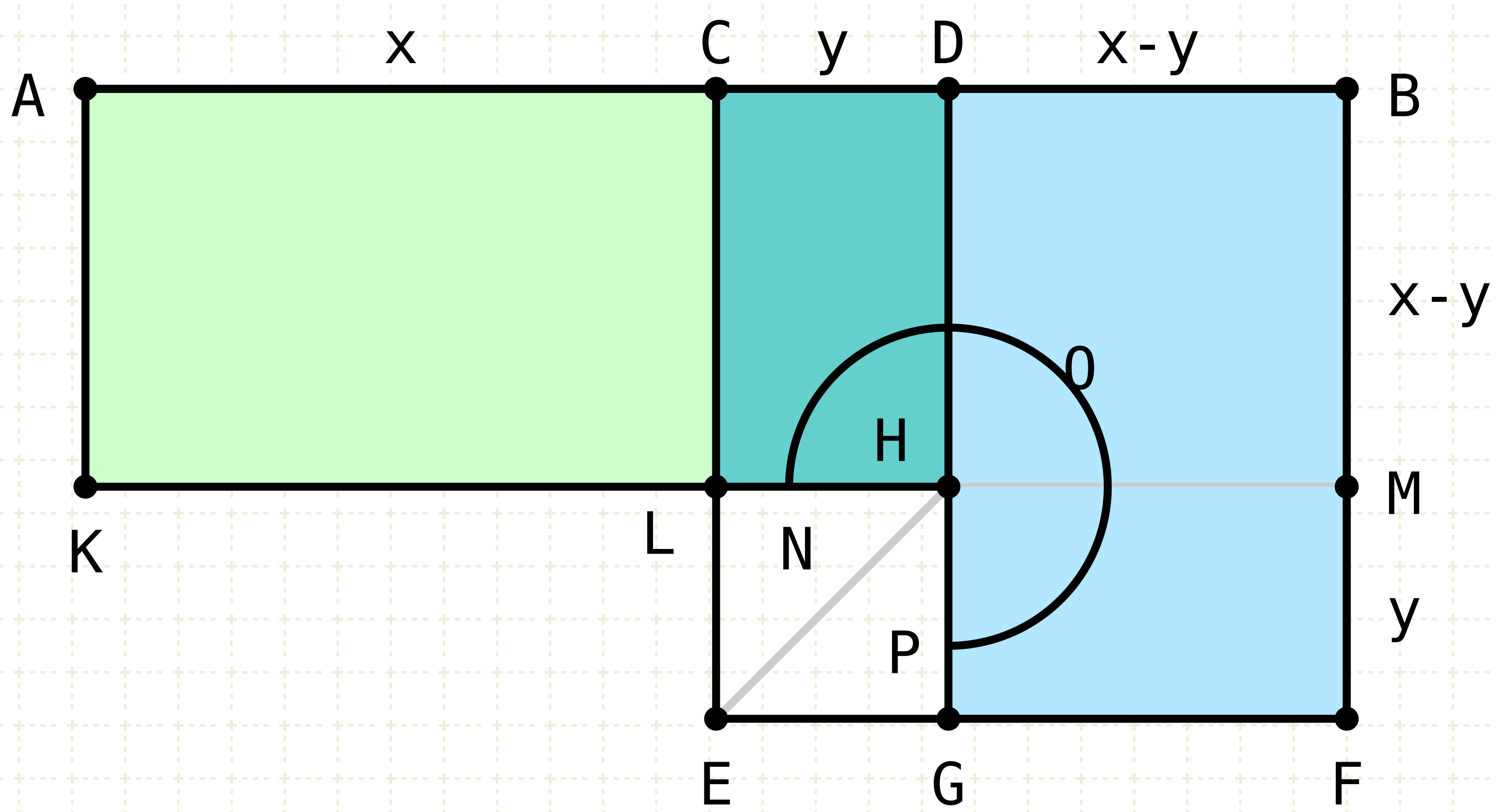

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□AL = □CM = □DF

□AH = NOP

DH = DB, □LG = CD·CD

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Let CH be added to each of AL and DF. Now AH is equal to gnomon NOP

For a proof showing that DM and LG are squares, see II·4

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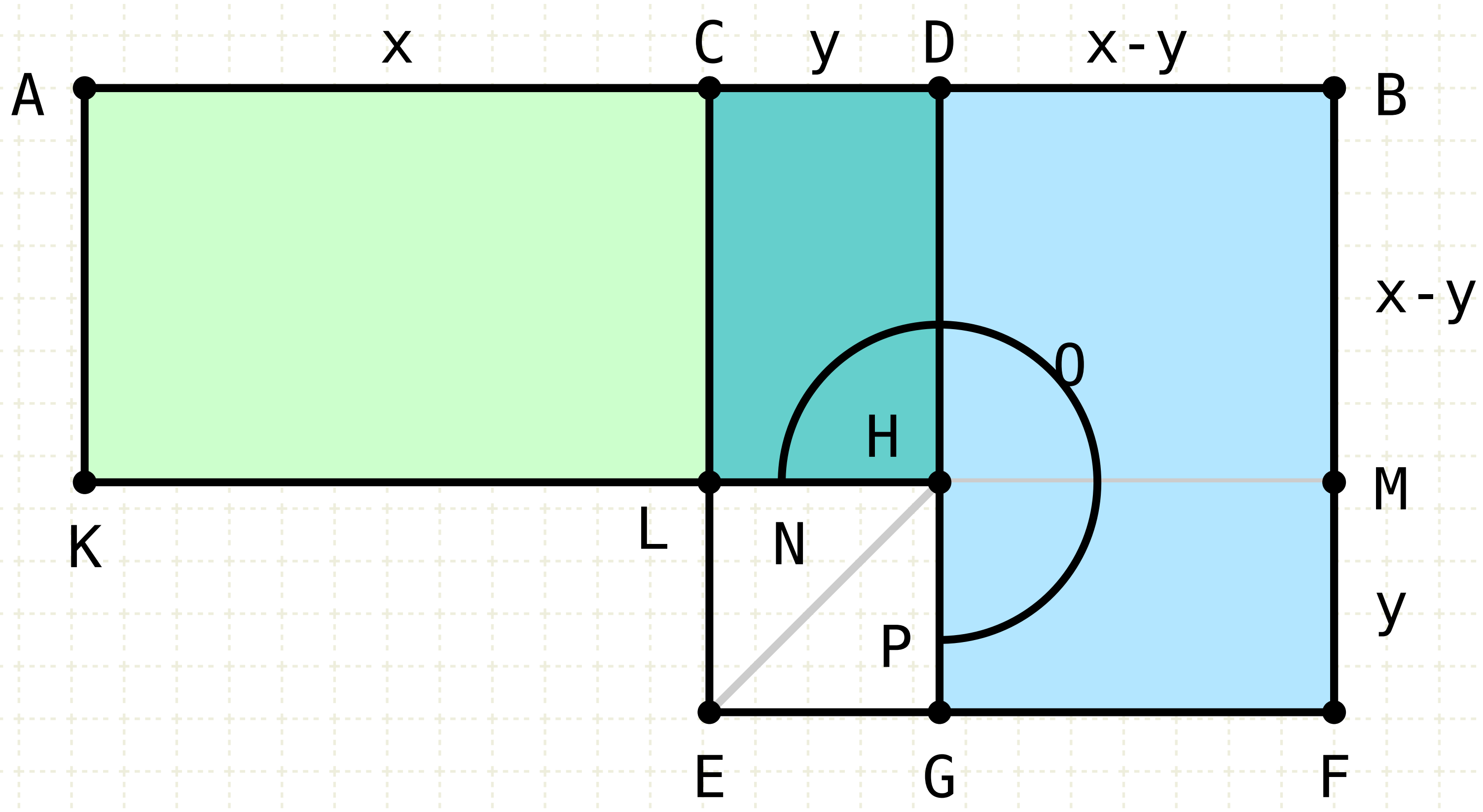

$AC = CB,\ AD = AC+CD,\ DB = AC-CD$

$\square CH = \square HF \quad \therefore \quad \square CM = \square DF$

$\square AL = \square CM = \square DF$

$\square AH = NOP$

$DH = DB,\ \square LG = CD \cdot CD$

$\square AH = AD \cdot DB = NOP$

In other words

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Let CH be added to each of AL and DF. Now AH is equal to gnomon NOP

For a proof showing that DM and LG are squares, see II·4

AH is equal to the rectangle formed by AD,DH, and also by AD,DB, therefore AD·DB is equal to the gnomon NOP

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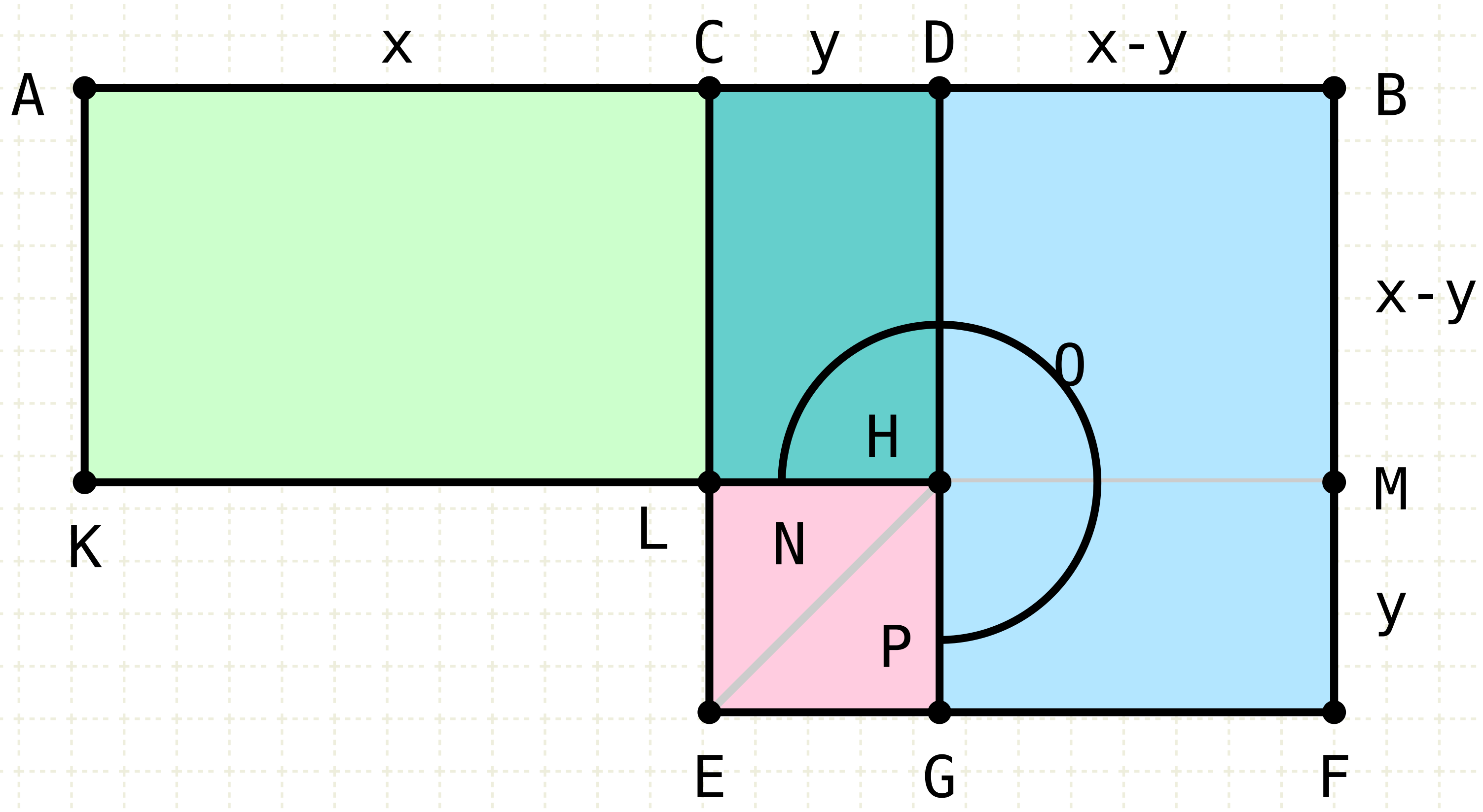

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□AL = □CM = □DF

□AH = NOP

DH = DB, □LG = CD·CD

□AH = AD·DB = NOP

AD·DB + CD·CD = NOP + □LG

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AH is equal to the rectangle formed by AD,DH, and also by AD,DB, therefore AD·DB is equal to the gnomon NOP

LG is equal to the square on CD, add it to both AH and NOP, retaining the equality

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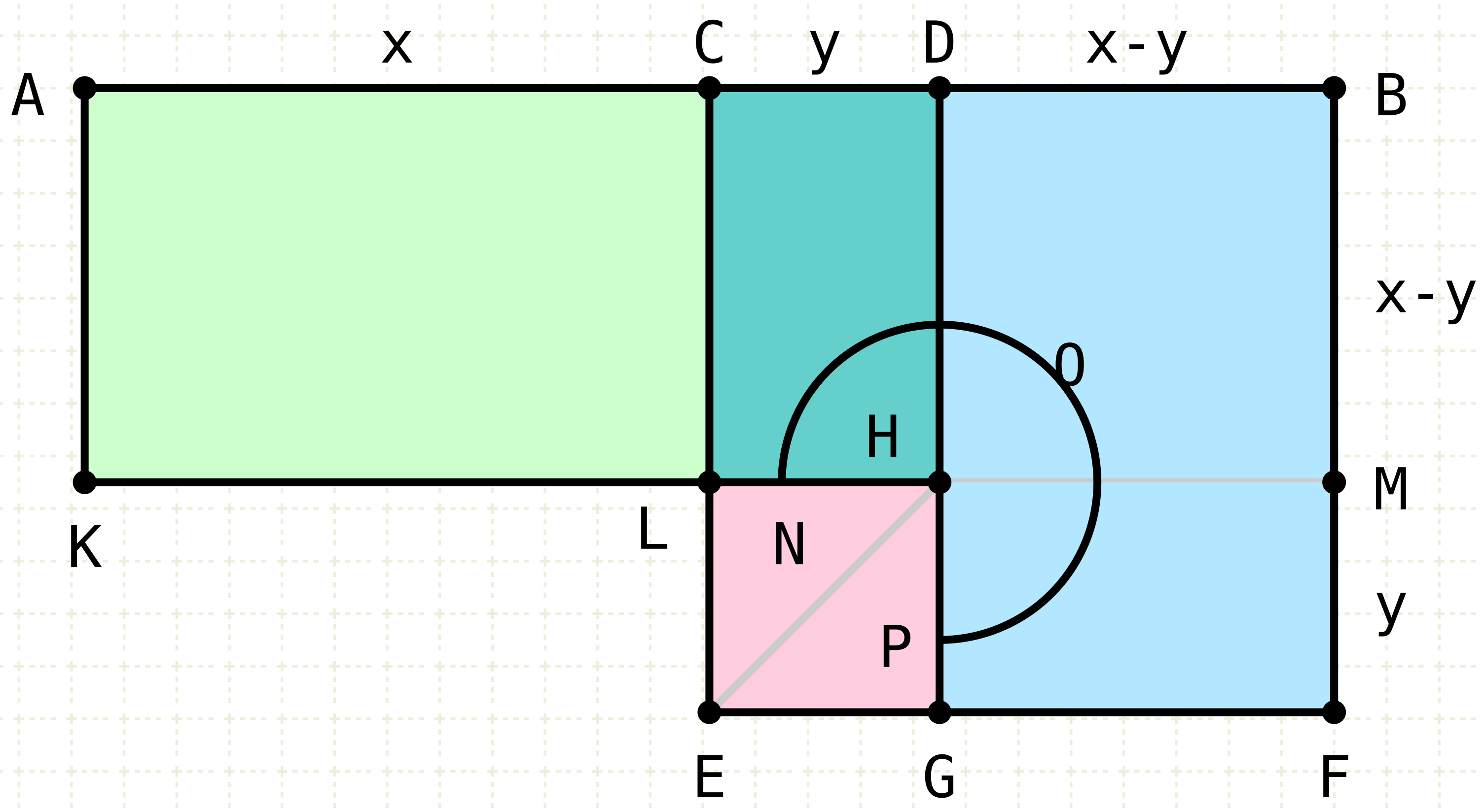

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$AD \cdot DB + CD \cdot CD = NOP + \square LG$

$AD \cdot DB + CD \cdot CD = CB \cdot CB$

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LG is equal to the square on CD, add it to both AH and NOP, retaining the equality

But CF is equal to the square on CB, which is also equal to the gnomon NOP added to the rectangle LG, we have demonstrated the proof for this postulate

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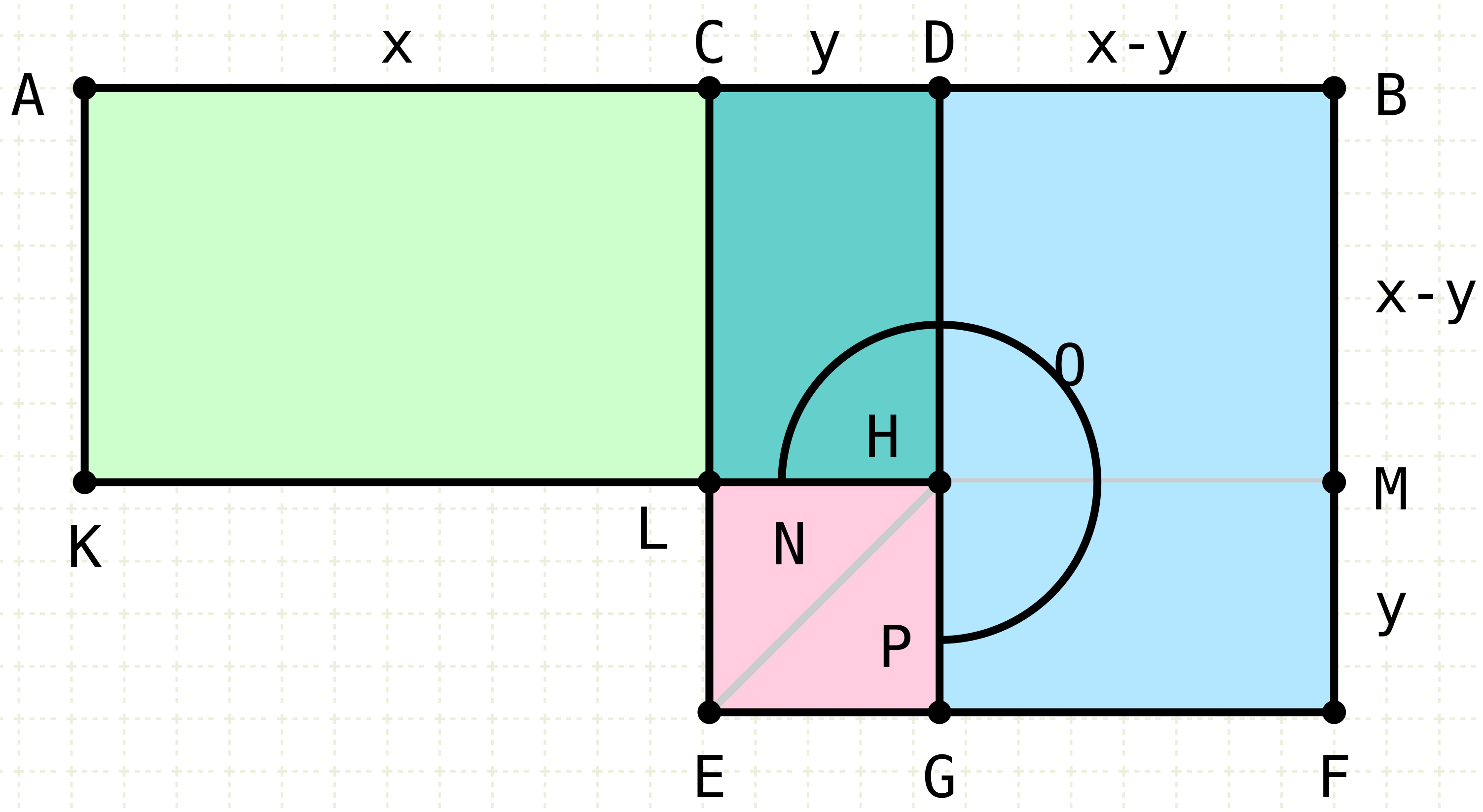

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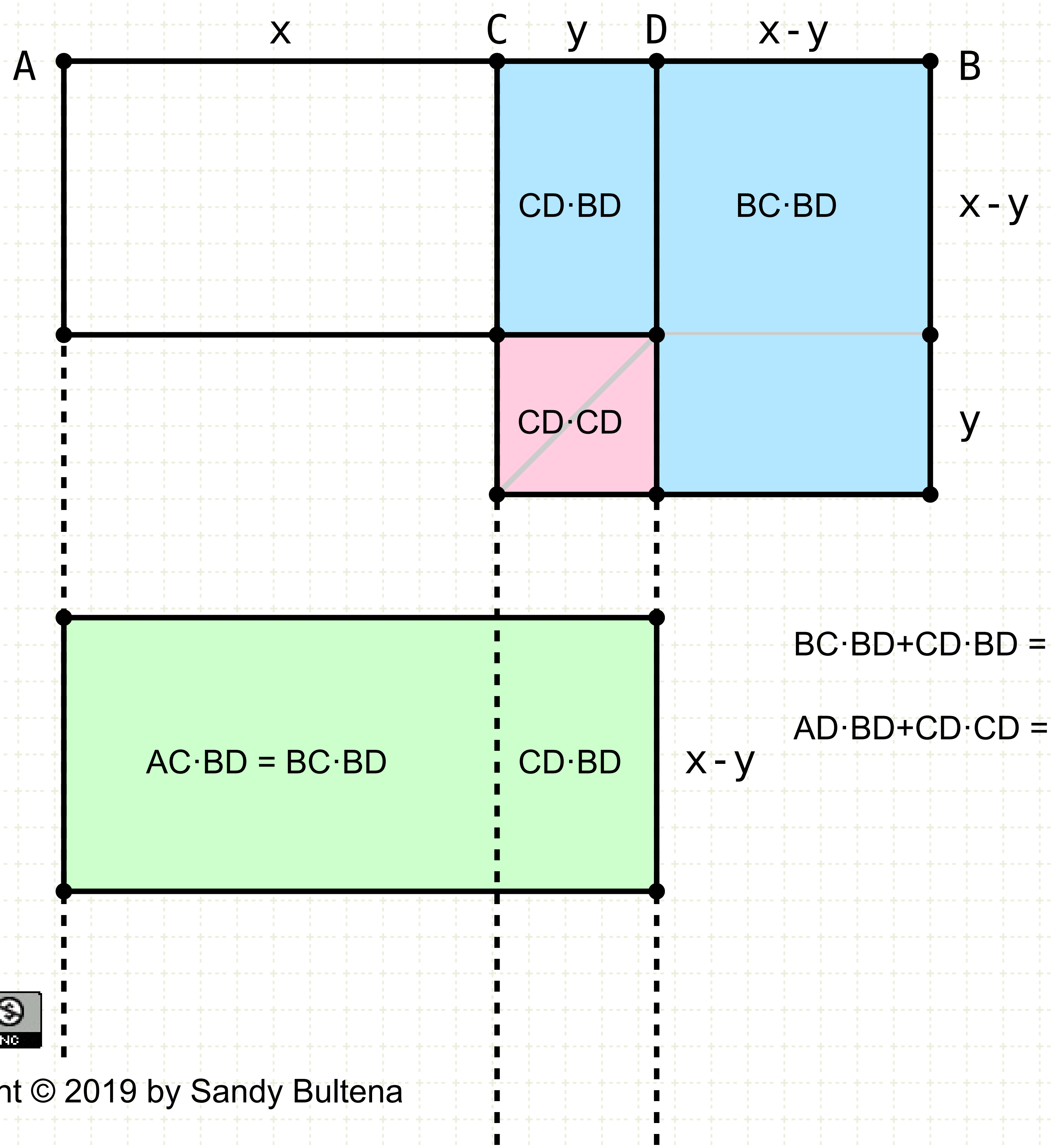

BC·BD+CD·BD = AD·BD

AD·BD+CD·CD = BC·BC

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Let CH be added to each of AL and DF. Now AH is equal to gnomon NOP

For a proof showing that DM and LG are squares, see II·4

AH is equal to the rectangle formed by AD,DH, and also by AD,DB, therefore AD·DB is equal to the gnomon NOP

LG is equal to the square on CD, add it to both AH and NOP, retaining the equality

But CF is equal to the square on CB, which is also equal to the gnomon NOP added to the rectangle LG, we have demonstrated the proof for this postulate

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