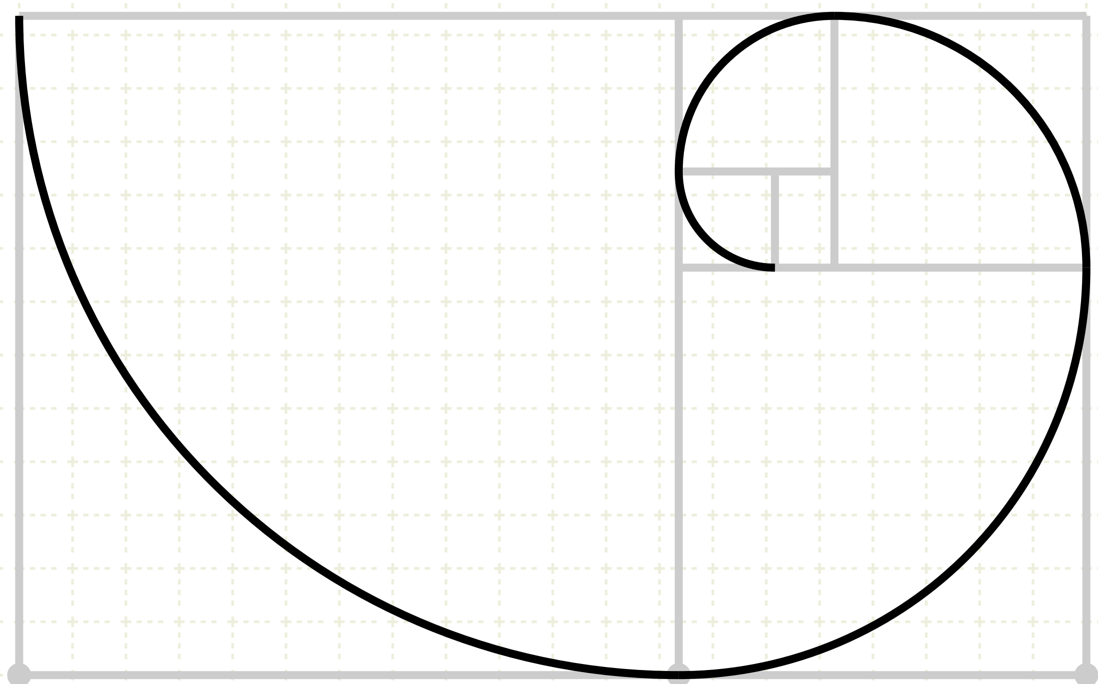
Euclid's Elements

Book VI



One can state, without exaggeration, that the observation of and the search for similarities and differences are the basis of all human knowledge.

Alfred Nobel

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Proposition 16 of Book VI

If four straight lines be proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means; and, if the rectangle contained by the extremes be equal to the rectangle contained by the means, the four straight lines will be proportional.

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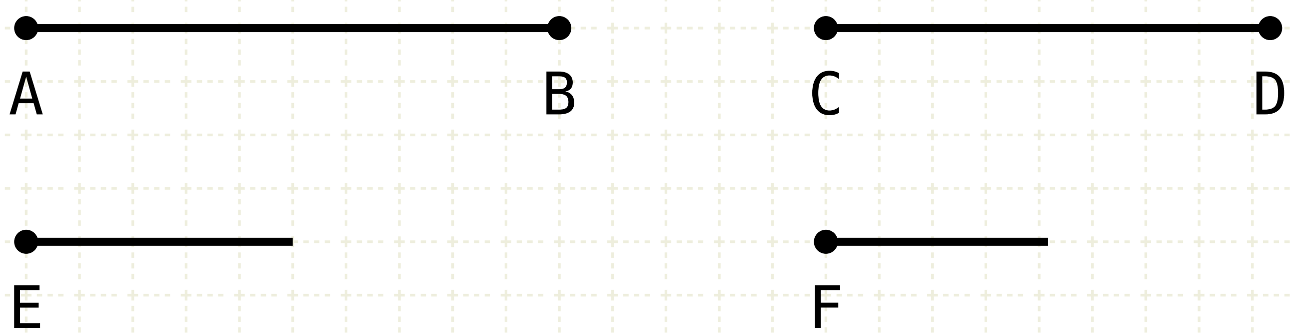


$AB:CD = E:F$
$\rightarrow AB \cdot F = CD \cdot E$

$AB \cdot F = E \cdot CD$
$\rightarrow AB:CD = E:F$

In other words

Given four lines AB, CD, E and F that are proportional, AB is to CD as E is to F

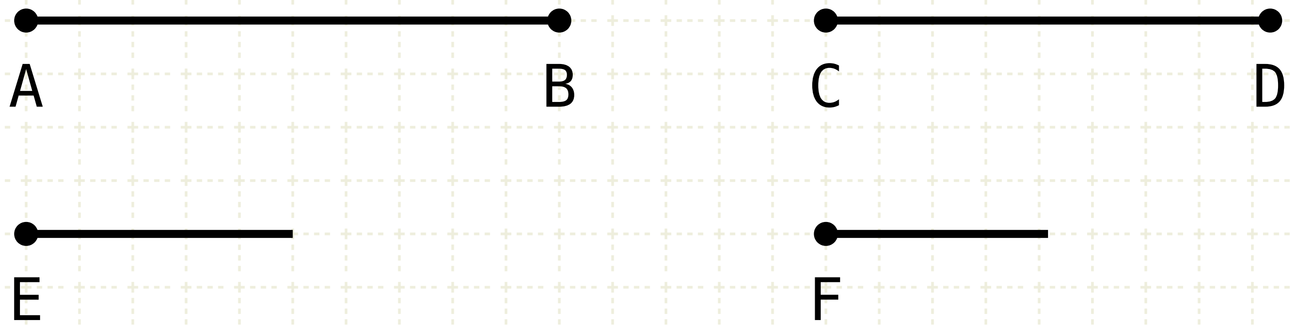
Then the product of AB, F is equal to the product CD, E

And the inverse

Proposition 16 of Book VI

If four straight lines be proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means; and, if the rectangle contained by the extremes be equal to the rectangle contained by the means, the four straight lines will be proportional.

Proof (Part 1)



AB:CD = E:F

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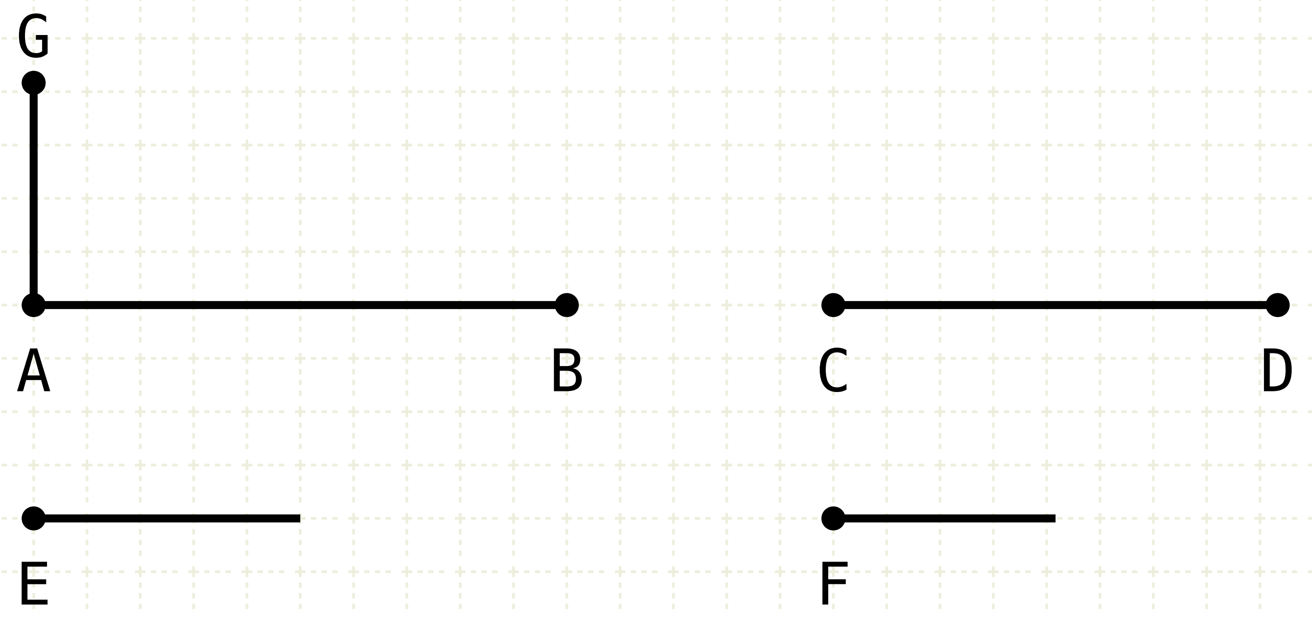

AB:CD = E:F

AG = F, AG ⊥ AB

Proof (Part 1)

Copy the line F to line AB, perpendicular to line AB

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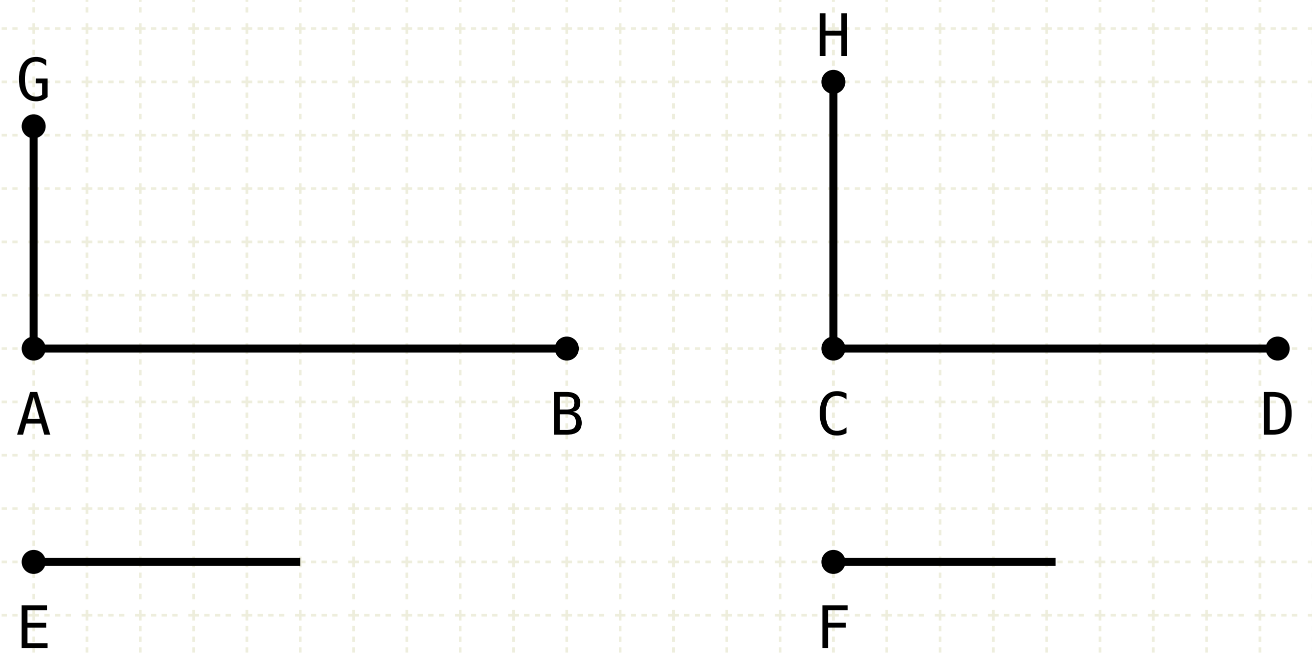
If four straight lines be proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means; and, if the rectangle contained by the extremes be equal to the rectangle contained by the means, the four straight lines will be proportional.



Proof (Part 1)

Copy the line F to line AB, perpendicular to line AB

Copy the line E to line CD, perpendicular to line CD

AB:CD = E:F

AG = F, AG ⊥ AB

CH = E, CH ⊥ CD

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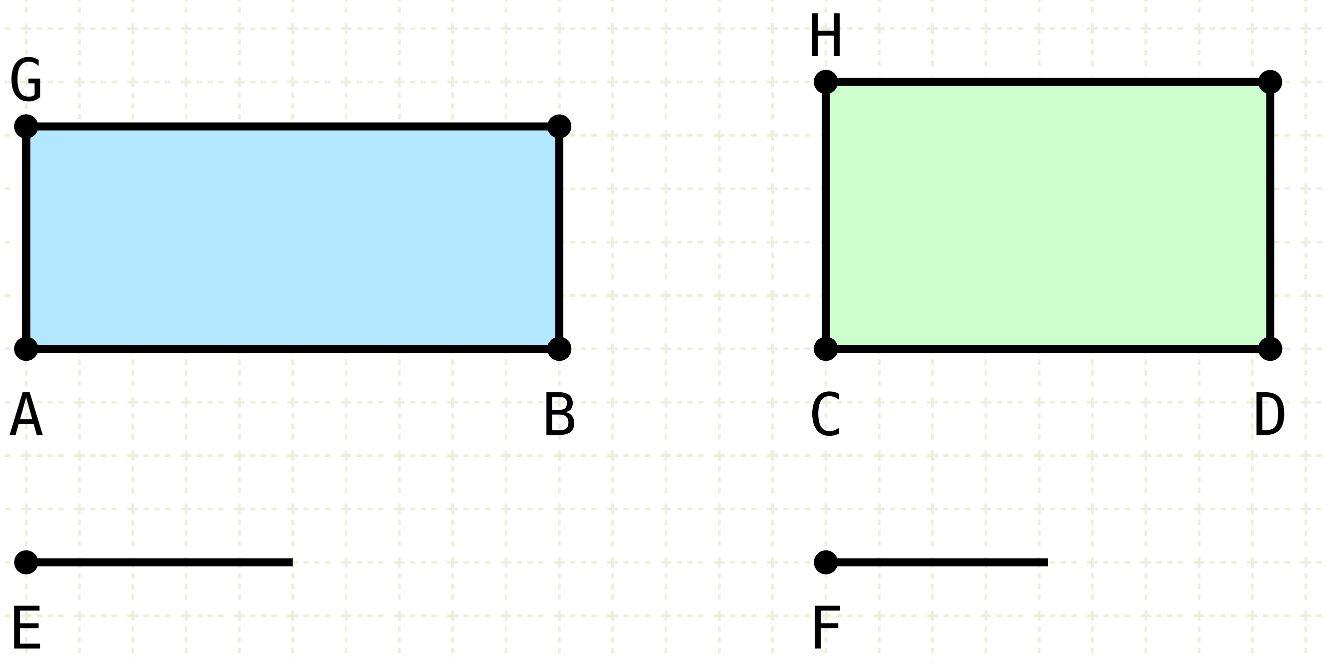
If four straight lines be proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means; and, if the rectangle contained by the extremes be equal to the rectangle contained by the means, the four straight lines will be proportional.



Proof (Part 1)

Copy the line F to line AB, perpendicular to line AB

Copy the line E to line CD, perpendicular to line CD

Finish the rectangles

$AB:CD = E:F$

$AG = F,\ AG \perp AB$

$CH = E,\ CH \perp CD$

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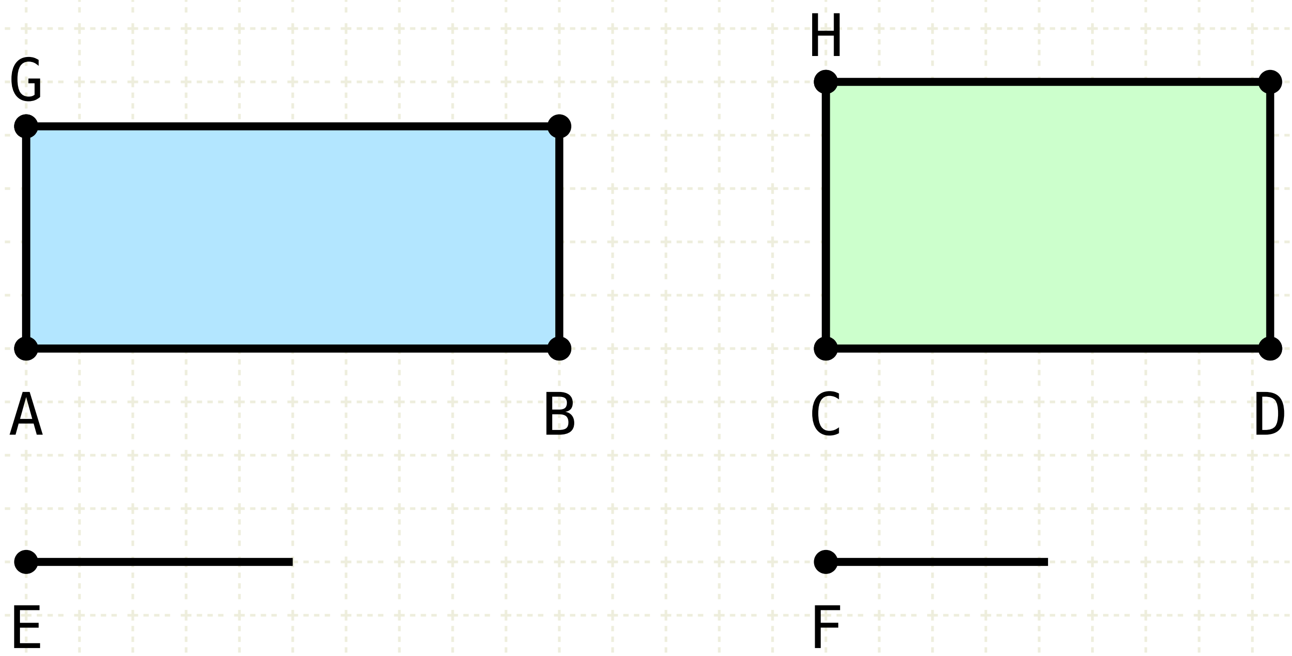
If four straight lines be proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means; and, if the rectangle contained by the extremes be equal to the rectangle contained by the means, the four straight lines will be proportional.



AB:CD = E:F
AG = F, AG ⊥ AB
CH = E, CH ⊥ CD
AB:CD = CH:AG

Proof (Part 1)

Copy the line F to line AB, perpendicular to line AB

Copy the line E to line CD, perpendicular to line CD

Finish the rectangles

Thus the sides of the rectangles (parallelograms) are inversely proportional

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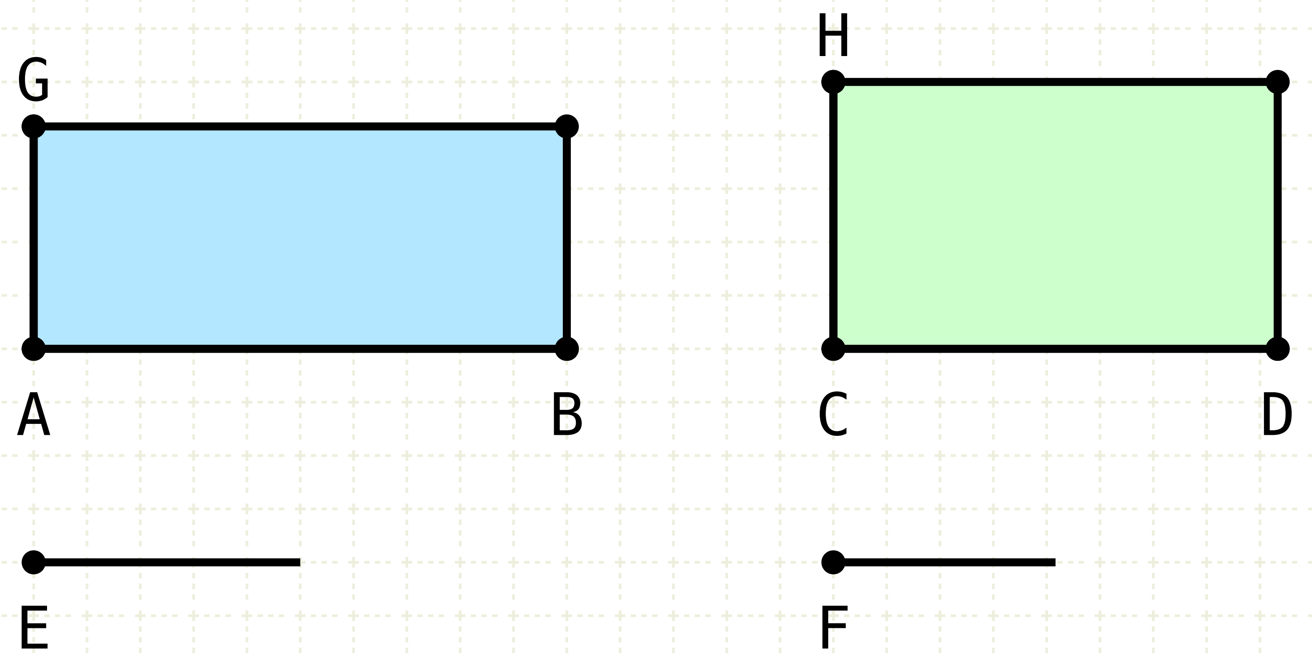

AB:CD = E:F

AG = F, AG ⊥ AB

CH = E, CH ⊥ CD

AB:CD = CH:AG

□BG = □DH

Proof (Part 1)

Copy the line F to line AB, perpendicular to line AB

Copy the line E to line CD, perpendicular to line CD

Finish the rectangles

Thus the sides of the rectangles (parallelograms) are inversely proportional

Rectangles BG and DH are equiangular parallelograms where the sides are reciprocally proportional around the equal angle

Therefore, the rectangles BG and DH are equal (VI·14)

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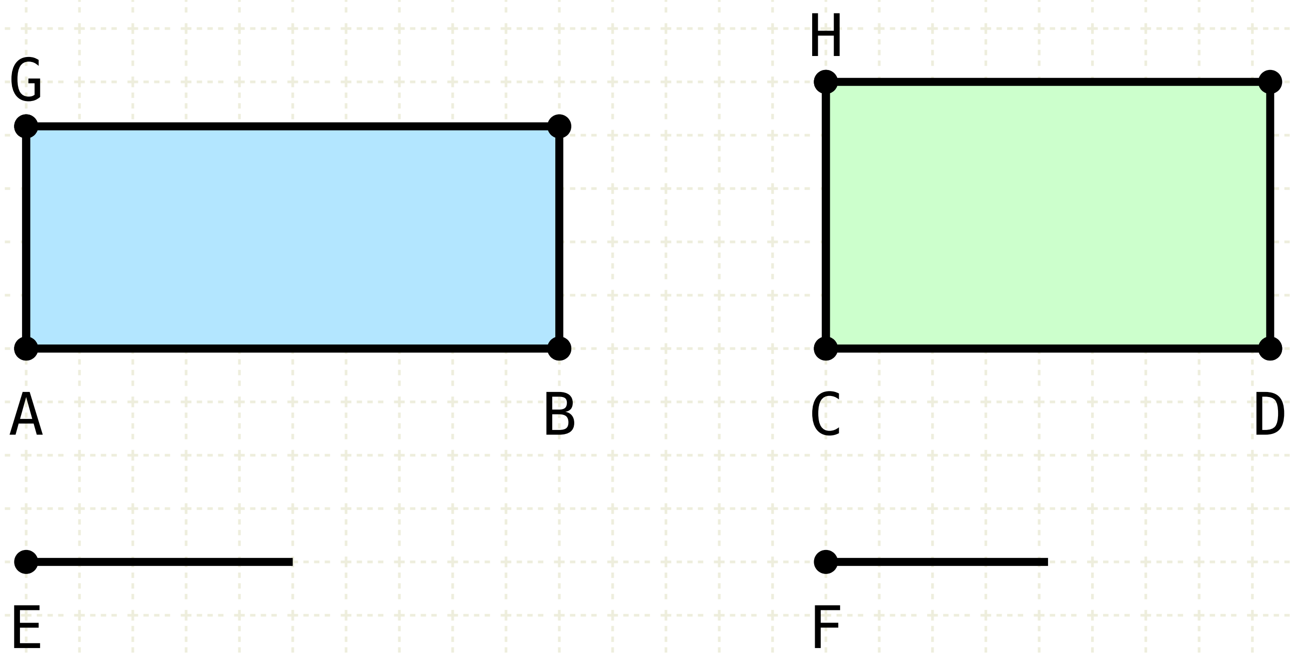

AB:CD = E:F

AG = F, AG ⊥ AB

CH = E, CH ⊥ CD

AB:CD = CH:AG

□BG = □DH

□BG = AB×F

□DH = CD×E

Proof (Part 1)

Copy the line F to line AB, perpendicular to line AB

Copy the line E to line CD, perpendicular to line CD

Finish the rectangles

Thus the sides of the rectangles (parallelograms) are inversely proportional

Rectangles BG and DH are equiangular parallelograms where the sides are reciprocally proportional around the equal angle

Therefore, the rectangles BG and DH are equal (VI·14)

And since AG equals F, and CH equals E, then the rectangles BG and DH are equal to the rectangles formed from AB,F and CD,E

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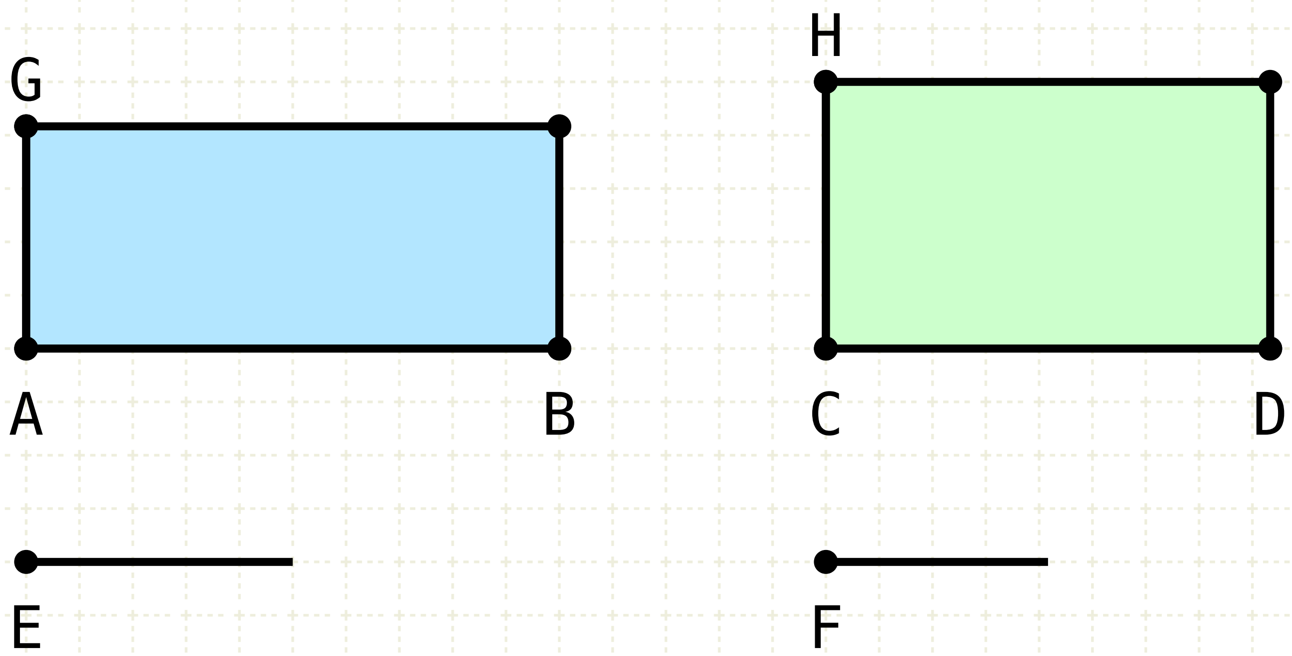

$AB:CD = E:F$

$AG = F,\ AG \perp AB$

$CH = E,\ CH \perp CD$

$AB:CD = CH:AG$

$\square BG = \square DH$

$\square BG = AB \times F$

$\square DH = CD \times E$

$AB \times F = CD \times E$

Proof (Part 1)

Copy the line F to line AB, perpendicular to line AB

Copy the line E to line CD, perpendicular to line CD

Finish the rectangles

Thus the sides of the rectangles (parallelograms) are inversely proportional

Rectangles BG and DH are equiangular parallelograms where the sides are reciprocally proportional around the equal angle

Therefore, the rectangles BG and DH are equal (VI·14)

And since AG equals F, and CH equals E, then the rectangles BG and DH are equal to the rectangles formed from AB,F and CD,E

Thus, the rectangle formed by AB.F is equal the rectangle formed by CD,E

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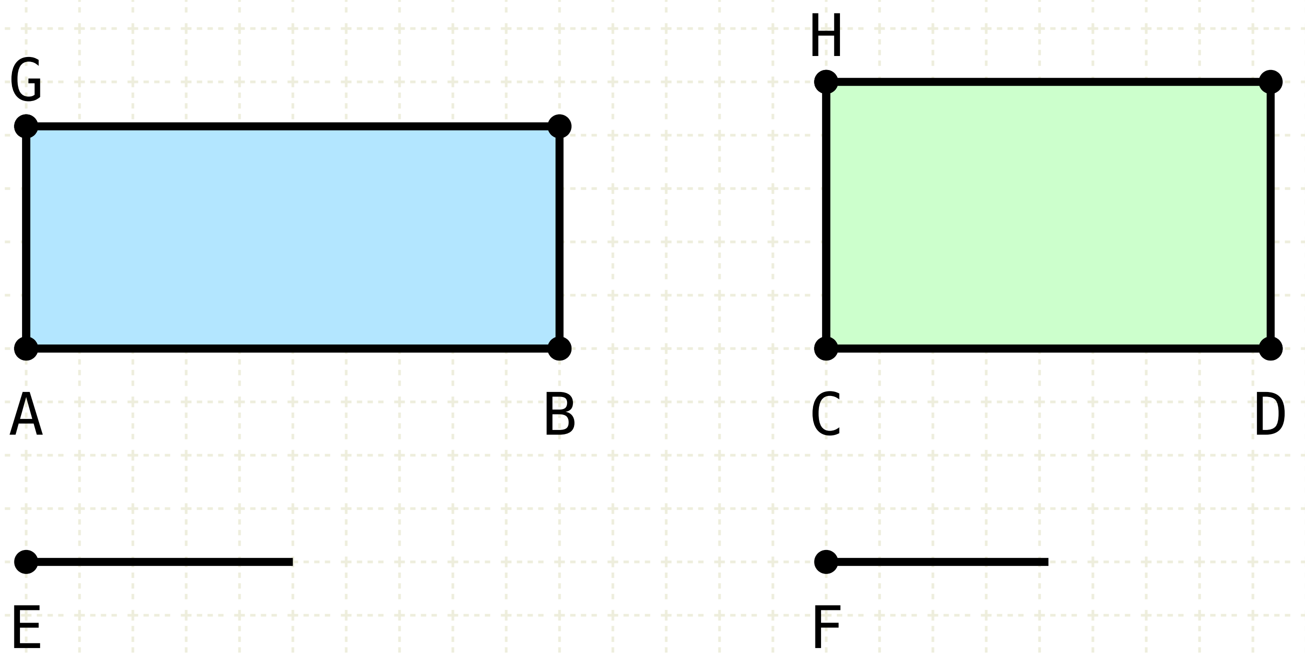

$AB \times F = CD \times E$

$AG = F,\ AG \perp AB$

$CH = E,\ CH \perp CD$

Proof (Part 2)

Copy the line F to line AB, perpendicular to line AB

Copy the line E to line CD, perpendicular to line CD

Finish the rectangles

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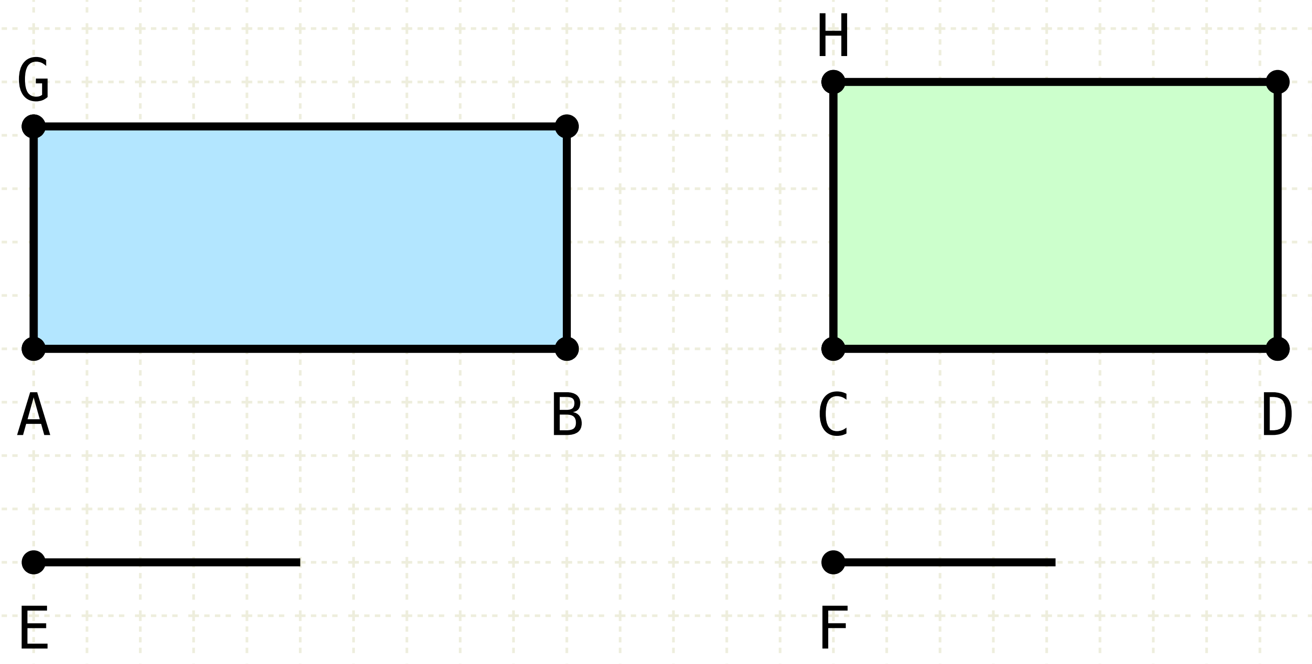

$AB \times F = CD \times E$

$AG = F,\ AG \perp AB$

$CH = E,\ CH \perp CD$

$\square BG = AB \times F$

$\square DH = CD \times E$

$\square BG = \square DH$

Proof (Part 2)

Copy the line F to line AB, perpendicular to line AB

Copy the line E to line CD, perpendicular to line CD

Finish the rectangles

Since AG,F are equal, and CH,E are equal, the rectangles BG and DH are equiangular, the areas are also equal

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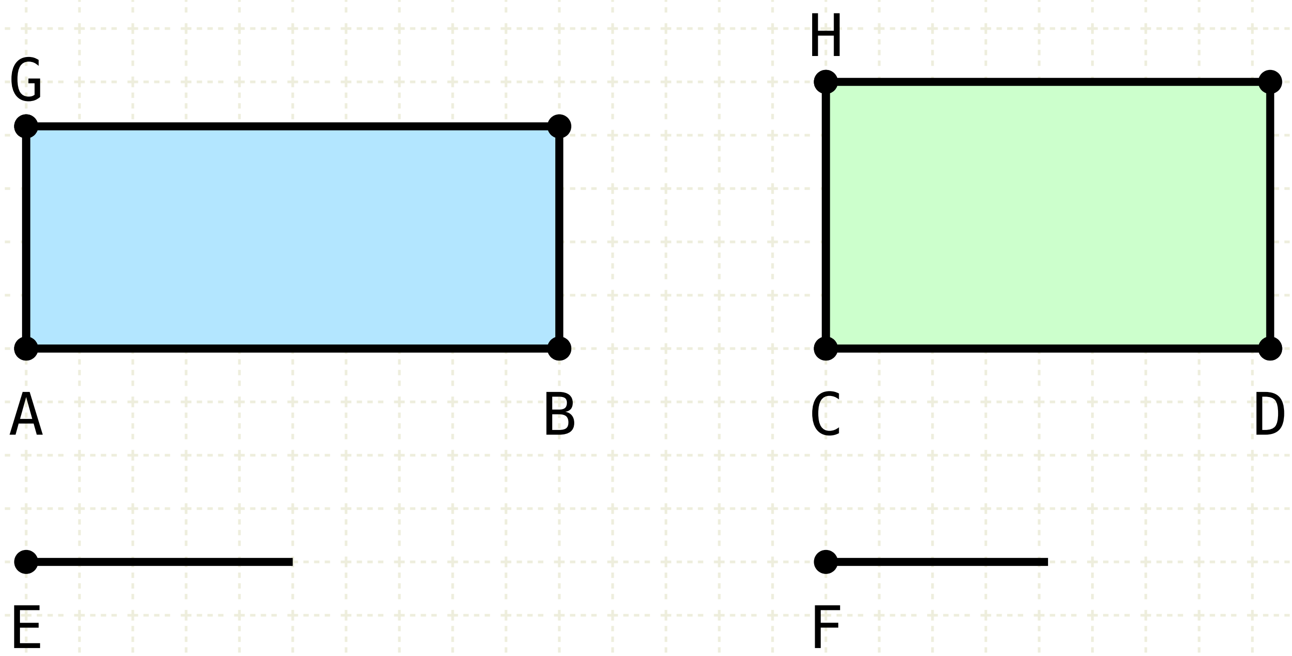

$AB \times F = CD \times E$

$AG = F,\ AG \perp AB$

$CH = E,\ CH \perp CD$

$\square BG = AB \times F$

$\square DH = CD \times E$

$\square BG = \square DH$

$AB : CD = CH : AG$

Proof (Part 2)

Copy the line F to line AB, perpendicular to line AB

Copy the line E to line CD, perpendicular to line CD

Finish the rectangles

Since AG,F are equal, and CH,E are equal, the rectangles BG and DH are equiangular, the areas are also equal

Equal and equiangular parallelograms the sides about the equal are reciprocally proportional (VI·14)

Therefore AB is to CD as CH is to AG

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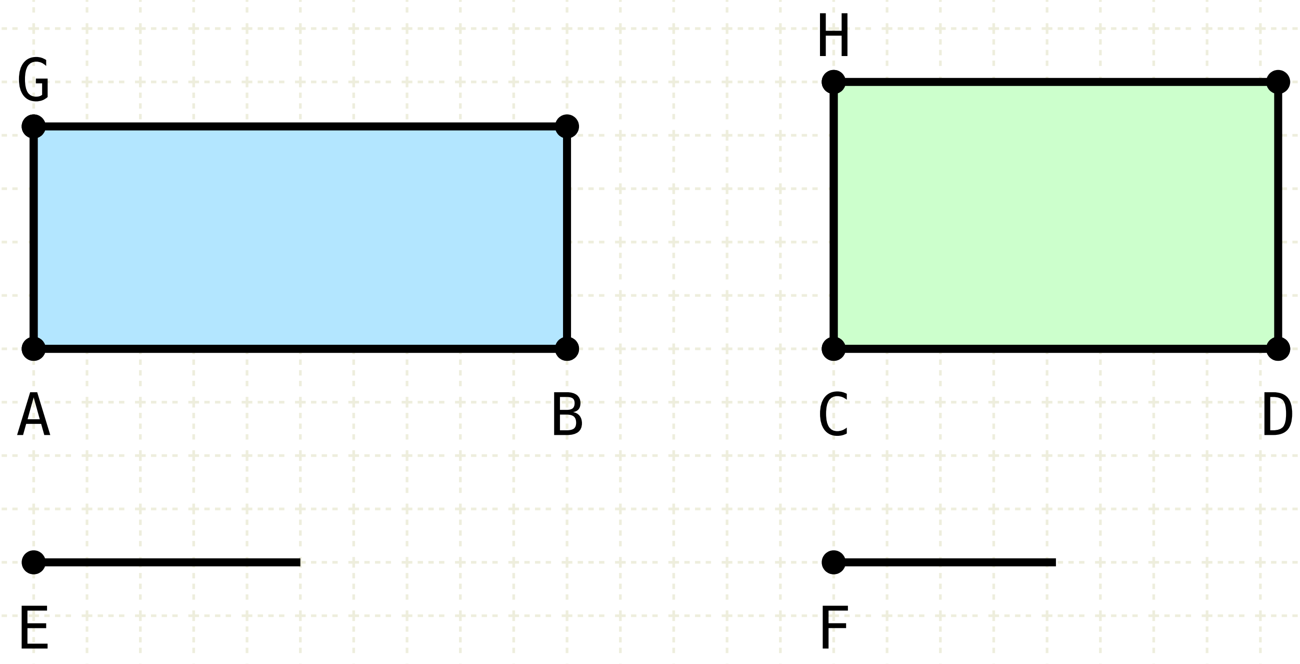

$AB \times F = CD \times E$

$AG = F,\ AG \perp AB$

$CH = E,\ CH \perp CD$

$\square BG = AB \times F$

$\square DH = CD \times E$

$\square BG = \square DH$

$AB : CD = CH : AG$

$AB : CD = E : F$

Proof (Part 2)

Copy the line F to line AB, perpendicular to line AB

Copy the line E to line CD, perpendicular to line CD

Finish the rectangles

Since AG,F are equal, and CH,E are equal, the rectangles BG and DH are equiangular, the areas are also equal

Equal and equiangular parallelograms the sides about the equal are reciprocally proportional (VI·14)

Therefore AB is to CD as CH is to AG

But CH,E are equal, and AG,F are equal, therefore AB is to CD as E is to F

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