

Euclid's Elements

Book V



Proportions are what makes the old Greek temples classic in their beauty. They are like huge blocks, from which the air has been literally hewn out between the columns.

$$AB:C = DE:F$$

$$BG:C = EH:F$$

$$AG:C = DH:F$$

Arne Jacobsen



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7	if $A = B \neq C$ then $A:C = B:C$ and $C:A = C:B$	17	if $(A+B):B = (C+D):D$ then $A:B = C:D$		
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10	if $A:C > B:C$, or $A:C < B:C$ then $A > B$, or $A < C$, respectively				



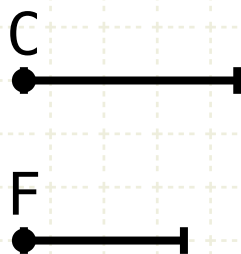
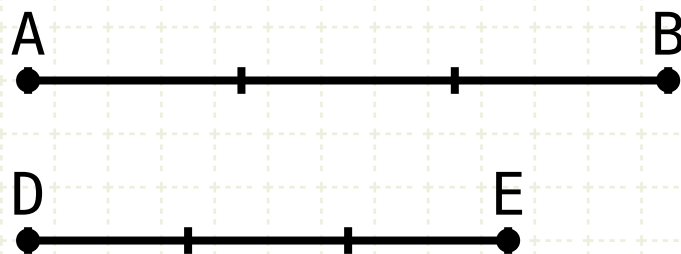
Proposition 15 of Book V

Parts have the same ratio as the same multiples of them taken in corresponding order



Proposition 15 of Book V

Parts have the same ratio as the same multiples of them taken in corresponding order



In other words

If AB is the same multiple of C as DE is of F ...
... then the ratio of AB to DE is the same as D is to F

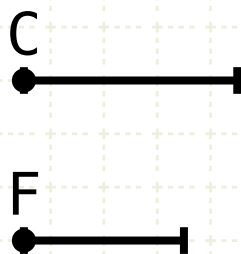
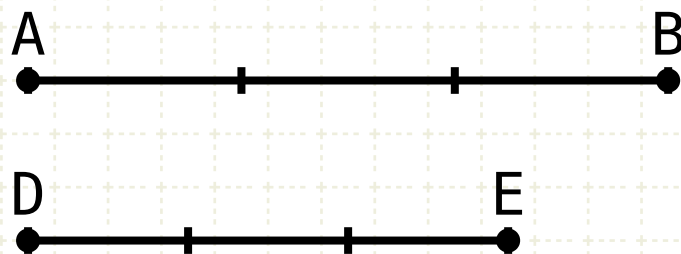
$$\begin{aligned} AB &= m \cdot C \\ DE &= m \cdot F \end{aligned}$$

$$AB:DE = C:F$$



Proposition 15 of Book V

Parts have the same ratio as the same multiples of them taken in corresponding order



$AB = m \cdot C$
 $DE = m \cdot F$

In other words

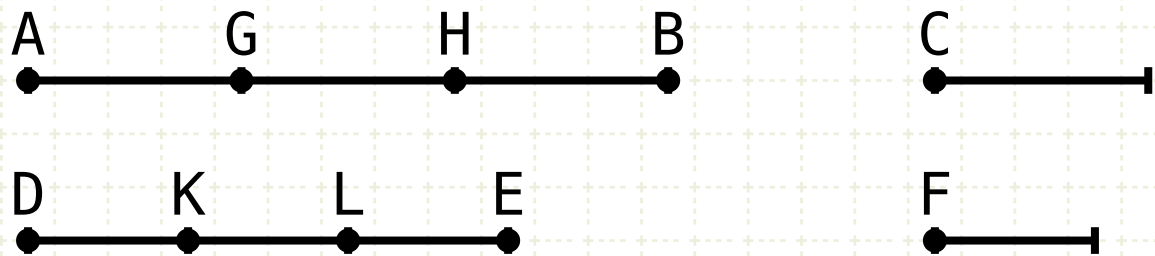
If AB is the same multiple of C as DE is of F ...
... then the ratio of AB to DE is the same as D is to F

Proof



Proposition 15 of Book V

Parts have the same ratio as the same multiples of them taken in corresponding order



$$AB = m \cdot C$$

$$DE = m \cdot F$$

$$AG = GH = HB = C$$

$$DK = KL = LE = F$$

In other words

If AB is the same multiple of C as DE is of F ...

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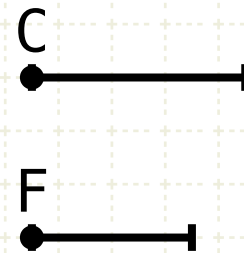
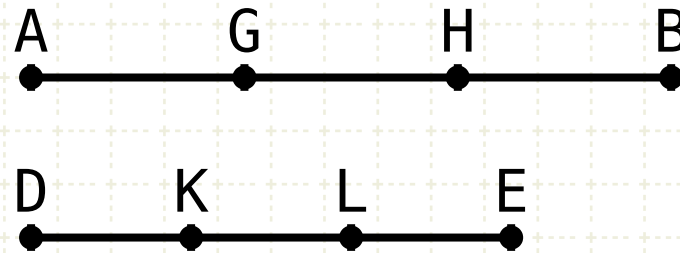
Proof

Let AB be divided into the magnitudes AG, GH, HB, where each is equal to C, and let DE be divided into magnitudes DK, KL, LE where each is equal to F



Proposition 15 of Book V

Parts have the same ratio as the same multiples of them taken in corresponding order



$$AB = m \cdot C$$

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$$AG = GH = HB = C$$

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$$AG:DK = GH:KL = HB:LE$$

In other words

If AB is the same multiple of C as DE is of F ...

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Proof

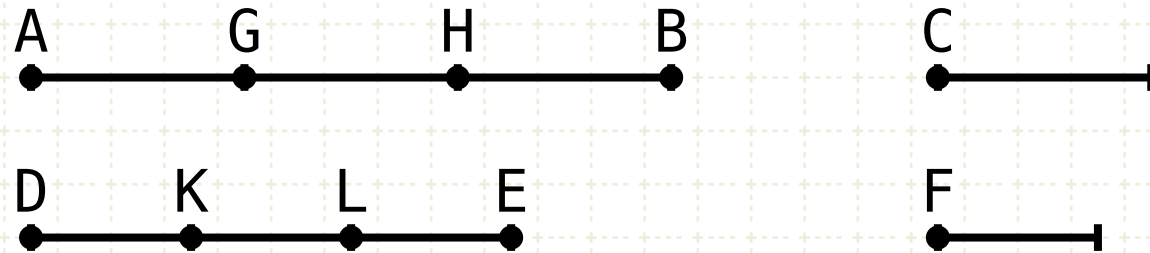
Let AB be divided into the magnitudes AG, GH, HB, where each is equal to C, and let DE be divided into magnitudes DK, KL, LE where each is equal to F

Since AG, GH, HB are equal and DK, KL, LE are equal
therefore AG is to DK as GH is to KL, as HB is to LE (V·7)



Proposition 15 of Book V

Parts have the same ratio as the same multiples of them taken in corresponding order



$$AB = m \cdot C$$

$$DE = m \cdot F$$

$$AG = GH = HB = C$$

$$DK = KL = LE = F$$

$$AG:DK = GH:KL = HB:LE$$

$$AG:DK = AB:DE$$

In other words

If AB is the same multiple of C as DE is of F ...

... then the ratio of AB to DE is the same as D is to F

Proof

Let AB be divided into the magnitudes AG, GH, HB, where each is equal to C, and let DE be divided into magnitudes DK, KL, LE where each is equal to F

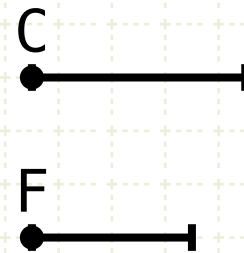
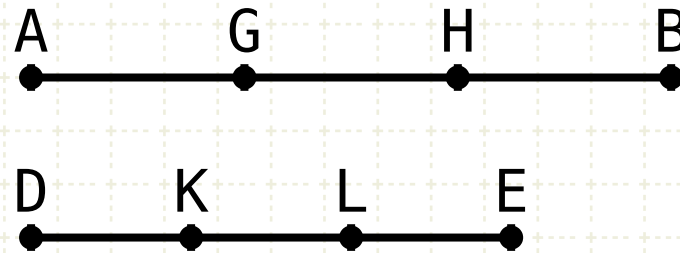
Since AG, GH, HB are equal and DK, KL, LE are equal therefore AG is to DK as GH is to KL, as HB is to LE (V·7)

Since all the ratios are equal, then the sum of the antecedents to the sum of the consequents will have the same ratio (V·12)



Proposition 15 of Book V

Parts have the same ratio as the same multiples of them taken in corresponding order



$$AB = m \cdot C$$

$$DE = m \cdot F$$

$$AG = GH = HB = C$$

$$DK = KL = LE = F$$

$$AG:DK = GH:KL = HB:LE$$

$$AG:DK = AB:DE$$

$$C:F = AB:DE$$

In other words

If AB is the same multiple of C as DE is of F ...

... then the ratio of AB to DE is the same as D is to F

Proof

Let AB be divided into the magnitudes AG, GH, HB, where each is equal to C, and let DE be divided into magnitudes DK, KL, LE where each is equal to F

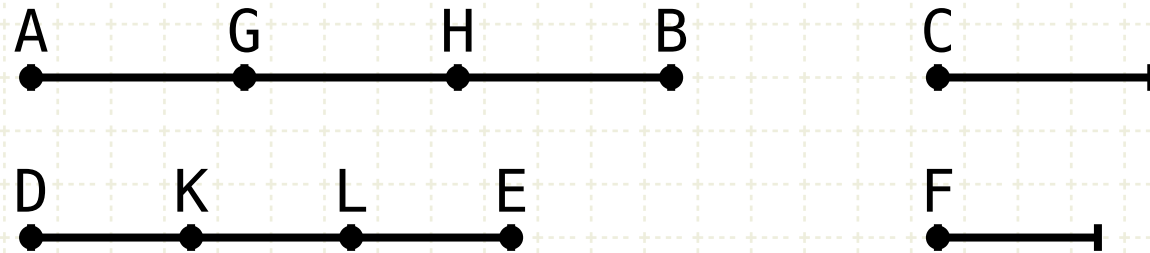
Since AG, GH, HB are equal and DK, KL, LE are equal therefore AG is to DK as GH is to KL, as HB is to LE (V·7)

Since all the ratios are equal, then the sum of the antecedents to the sum of the consequents will have the same ratio (V·12)

But AG is equal to C, and DK is equal to F, so the ratio of C to F is the same as the ratio of AB to DE

Proposition 15 of Book V

Parts have the same ratio as the same multiples of them taken in corresponding order



$$AB = m \cdot C$$

$$DE = m \cdot F$$

$$AG = GH = HB = C$$

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$$AG:DK = GH:KL = HB:LE$$

$$AG:DK = AB:DE$$

$$C:F = AB:DE$$

In other words

If AB is the same multiple of C as DE is of F ...

... then the ratio of AB to DE is the same as D is to F

Proof

Let AB be divided into the magnitudes AG, GH, HB, where each is equal to C, and let DE be divided into magnitudes DK, KL, LE where each is equal to F

Since AG, GH, HB are equal and DK, KL, LE are equal therefore AG is to DK as GH is to KL, as HB is to LE (V·7)

Since all the ratios are equal, then the sum of the antecedents to the sum of the consequents will have the same ratio (V·12)

But AG is equal to C, and DK is equal to F, so the ratio of C to F is the same as the ratio of AB to DE

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