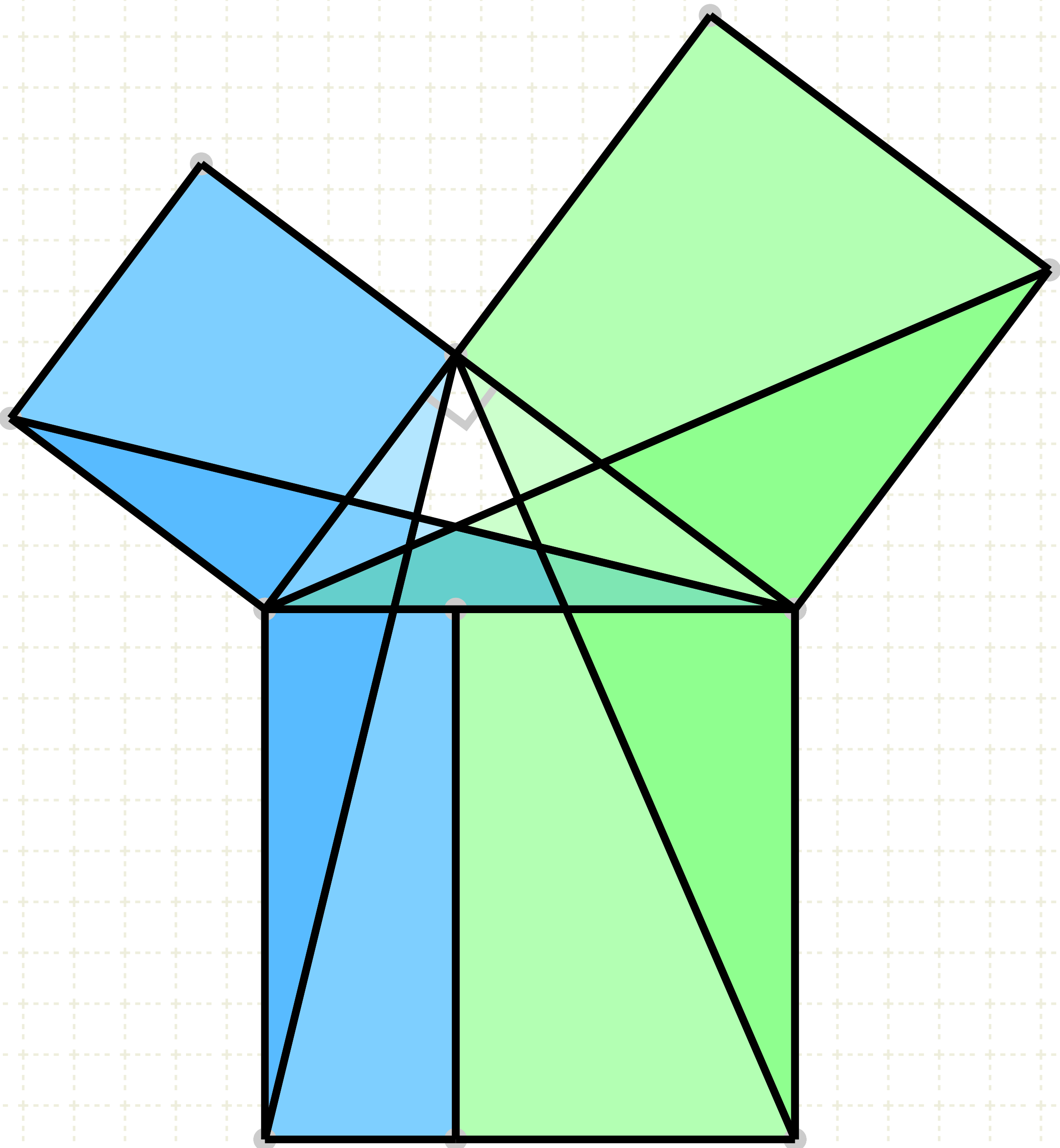
Euclid's Elements

Book I



If Euclid did not kindle your youthful enthusiasm, you were not born to be a scientific thinker.

Albert Einstein

Table of Contents, Chapter 1

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Proposition 4 of Book I

If two triangles have two sides equal to two sides respectively, and have the angles contained by the equal straight lines equal, then they also have the base equal to the base, the triangle equals the triangle, and the remaining angles equal the remaining angles respectively, namely those opposite the equal sides.

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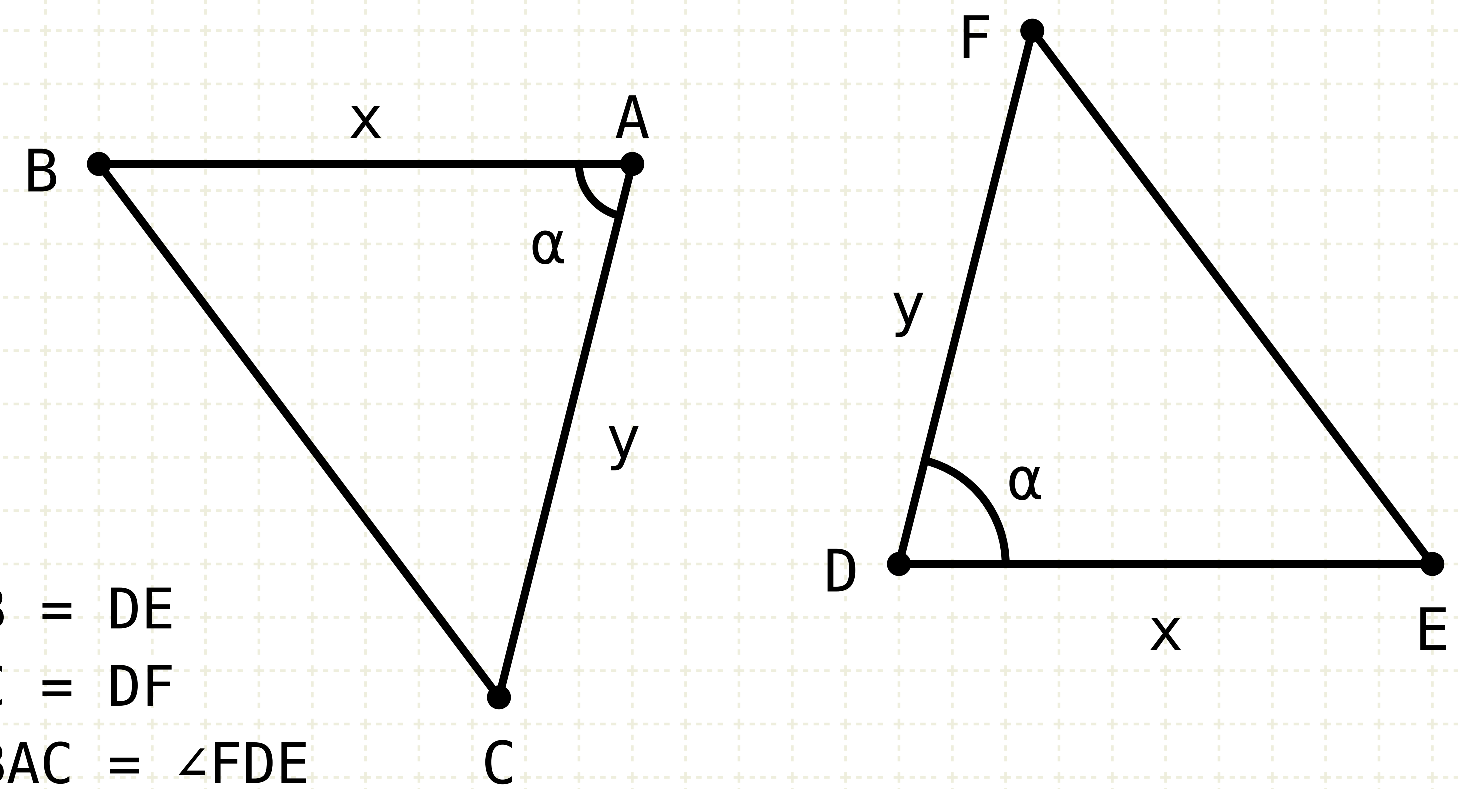

$AB = DE$
$AC = DF$
$\angle BAC = \angle FDE$

In other words

If two triangles have two sides which are equivalent, and if the angles between the two sides are also equivalent, (side-angle-side SAS)…

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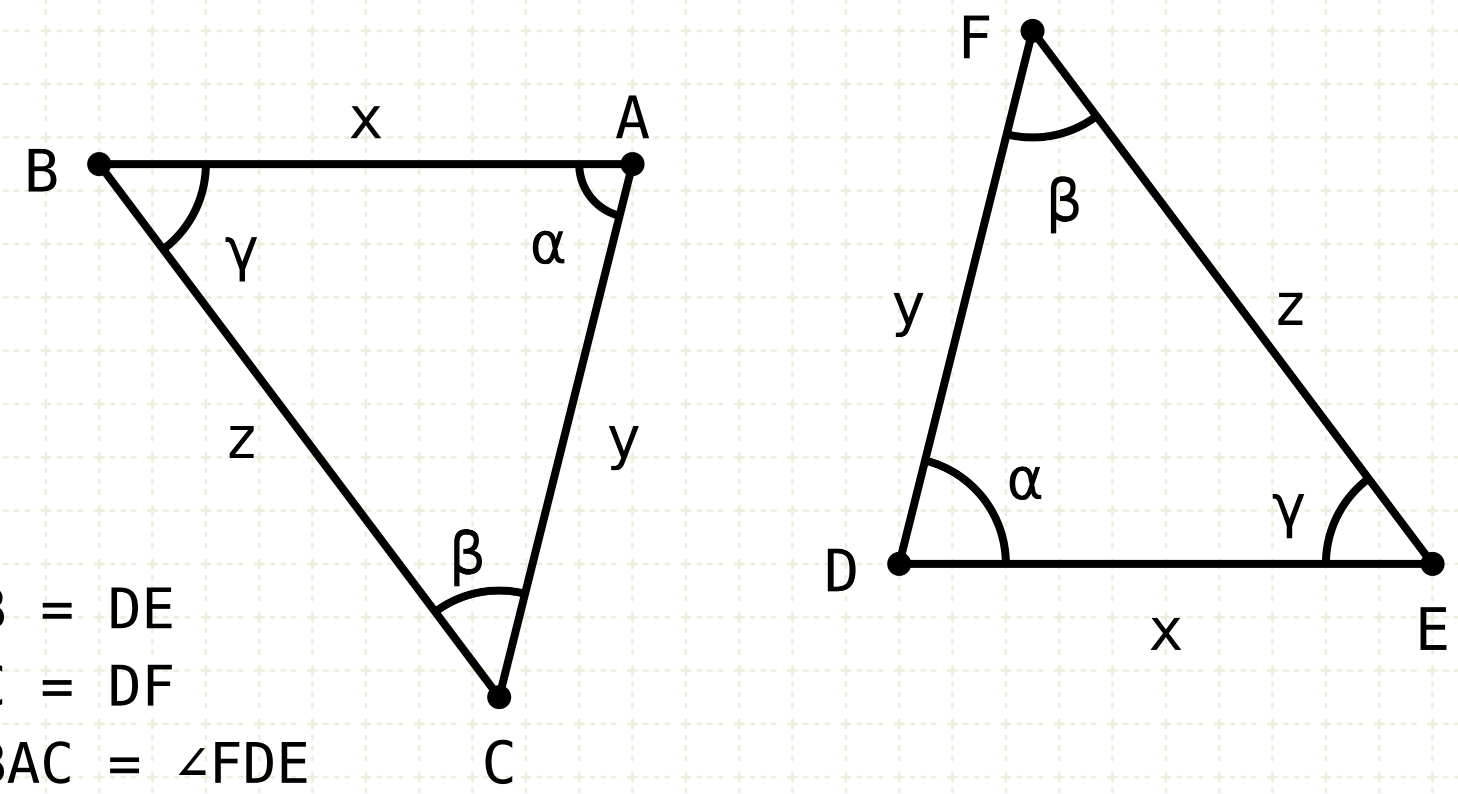

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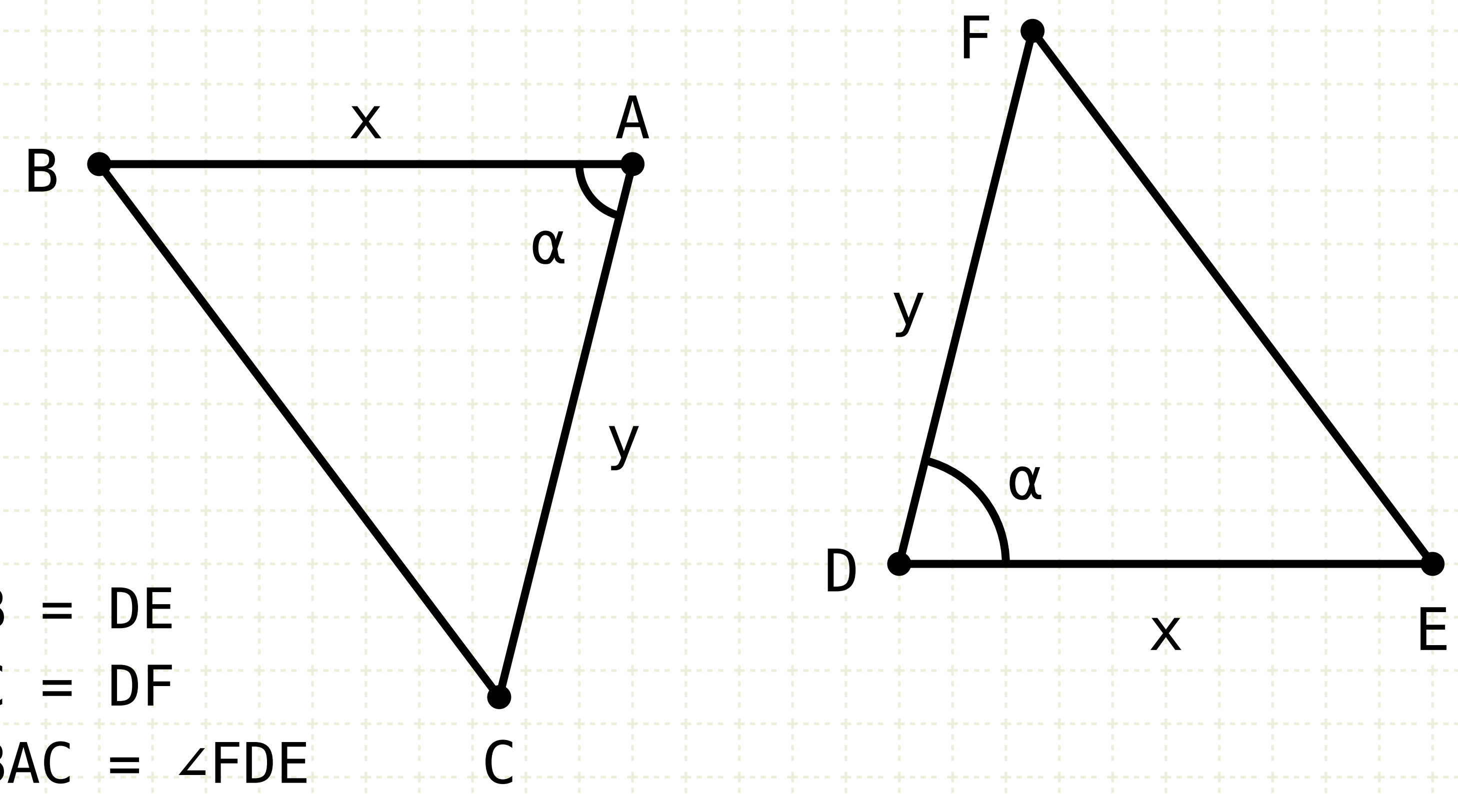

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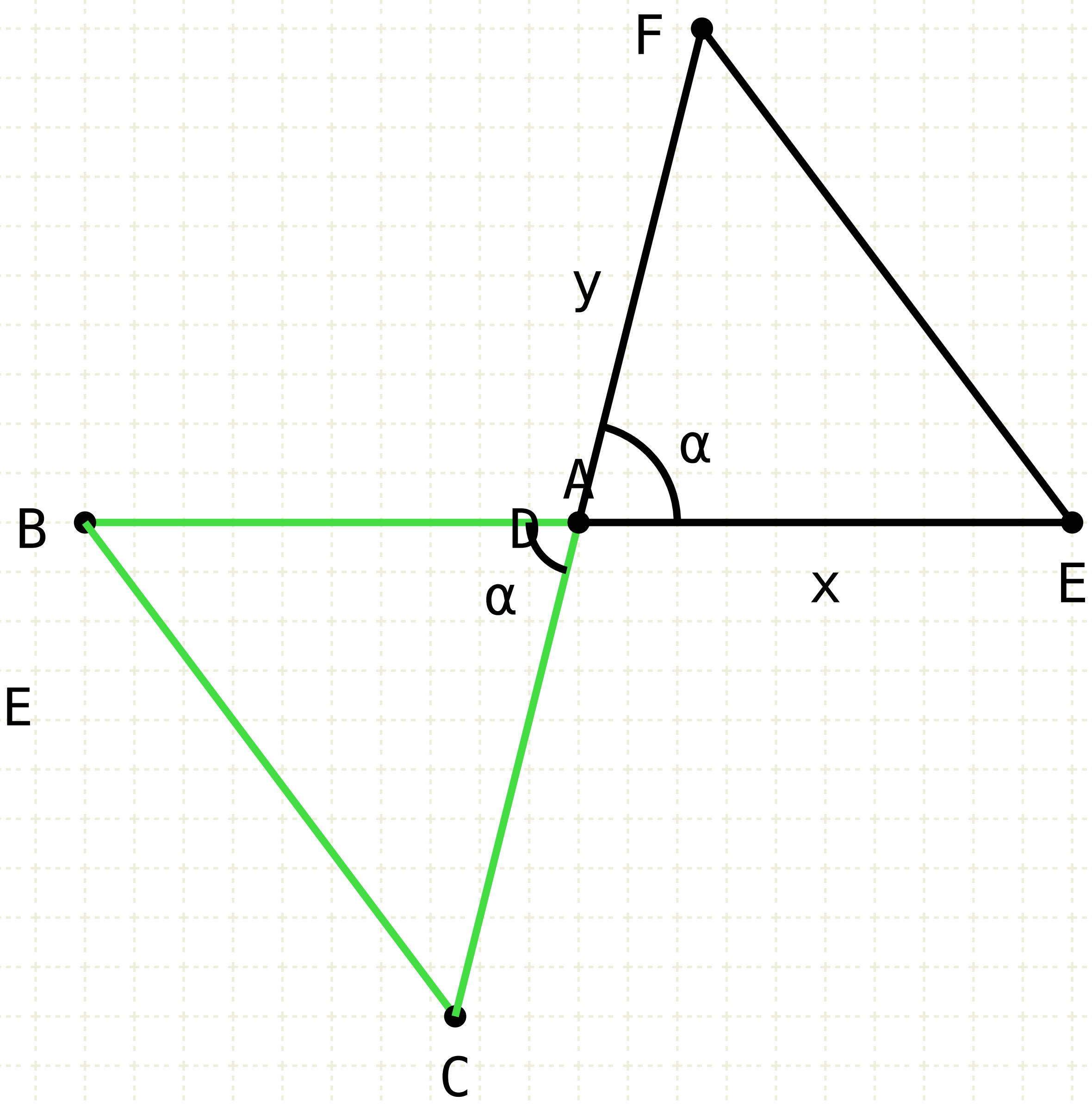

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If two triangles have two sides which are equivalent, and if the angles between the two sides are also equivalent, (side-angle-side SAS)…

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Proof

Move triangle ABC such that point A coincides with point D

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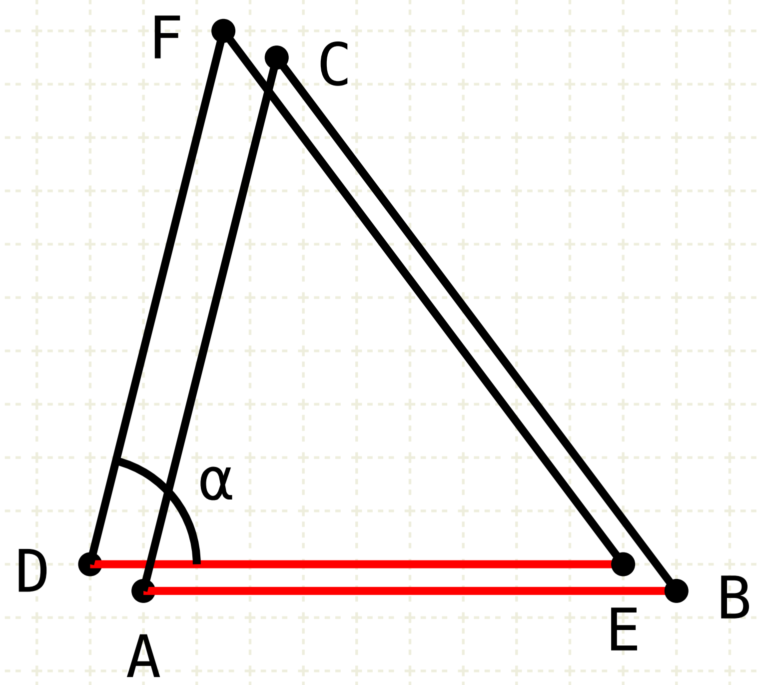

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Rotate the triangle so that line AB line coincides with DE.

… diagram is offset a bit so we can see more clearly

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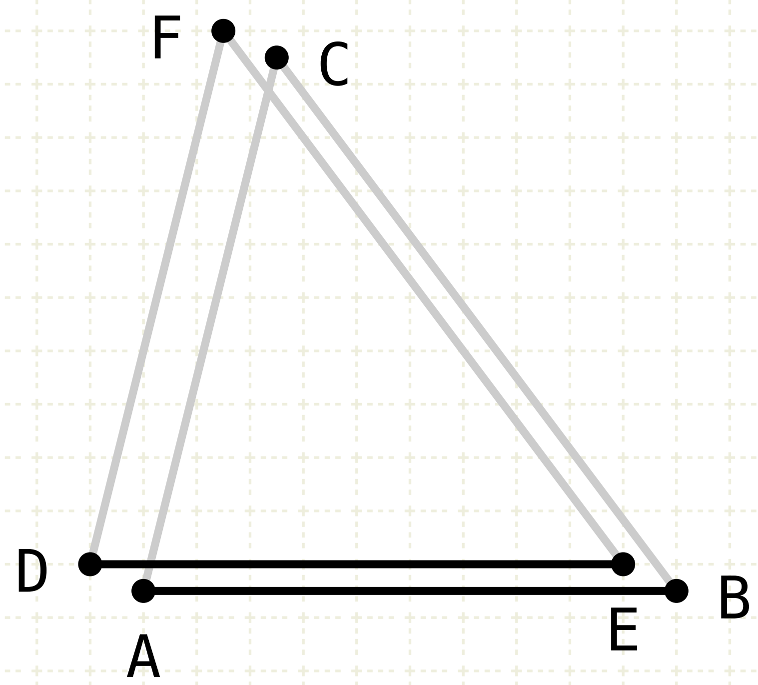

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Since lines AB and DE are the same lengths, the endpoints are congruent

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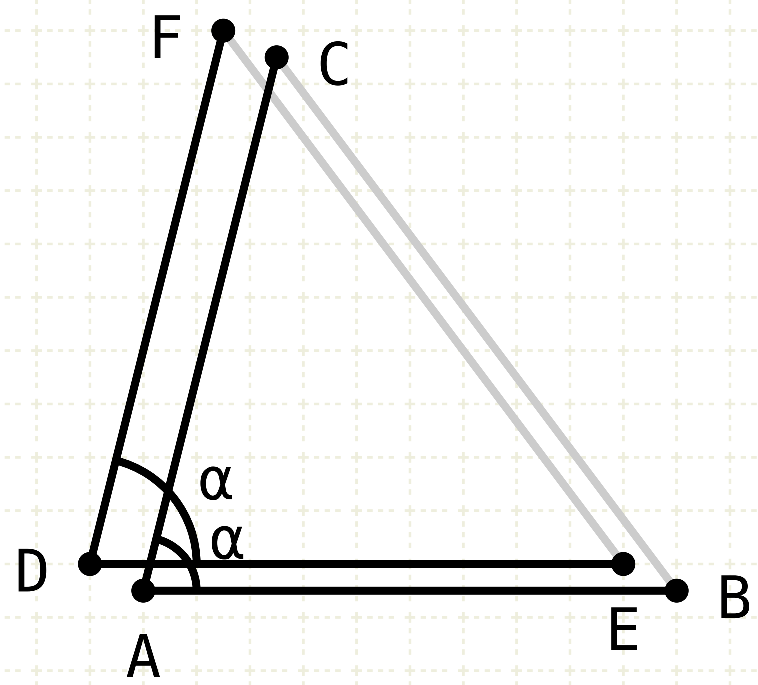

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C = F
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Line AC coincides with DF since they are the same length and the angles BAC and EDF are equal

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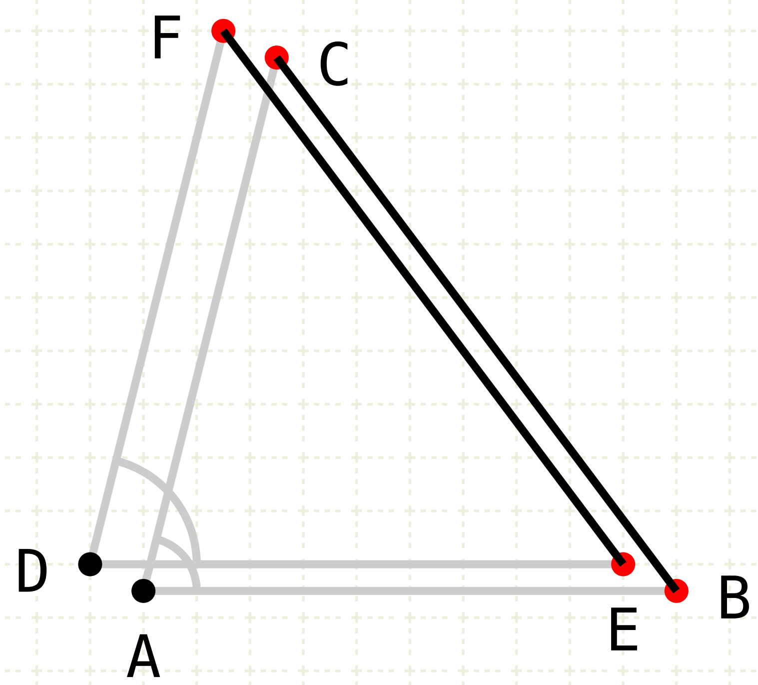

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Since the points B coincides with E and C with F, then the lines BC coincides with EF… based on the implicit understanding that there is only one straight path between two points

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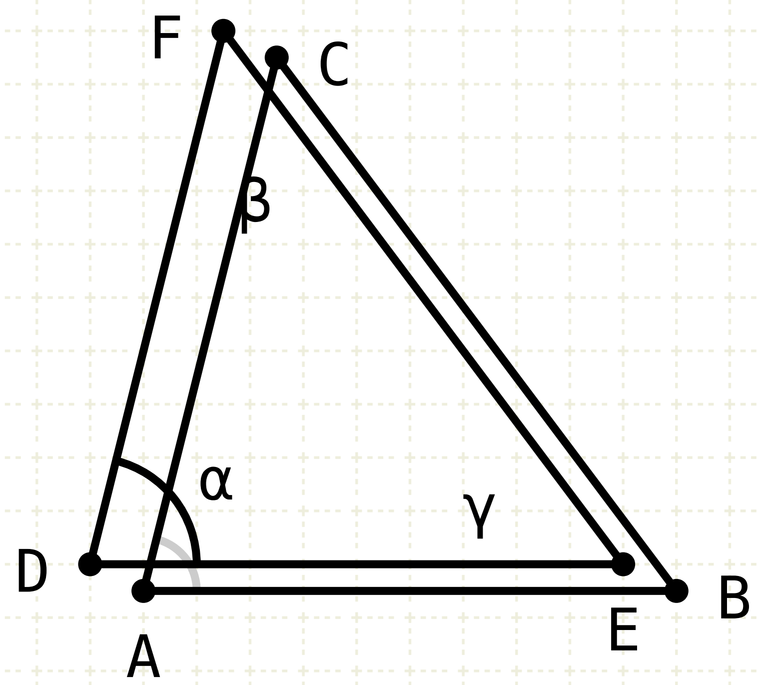

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From common notion 4, things which coincide with one another, equal one another

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