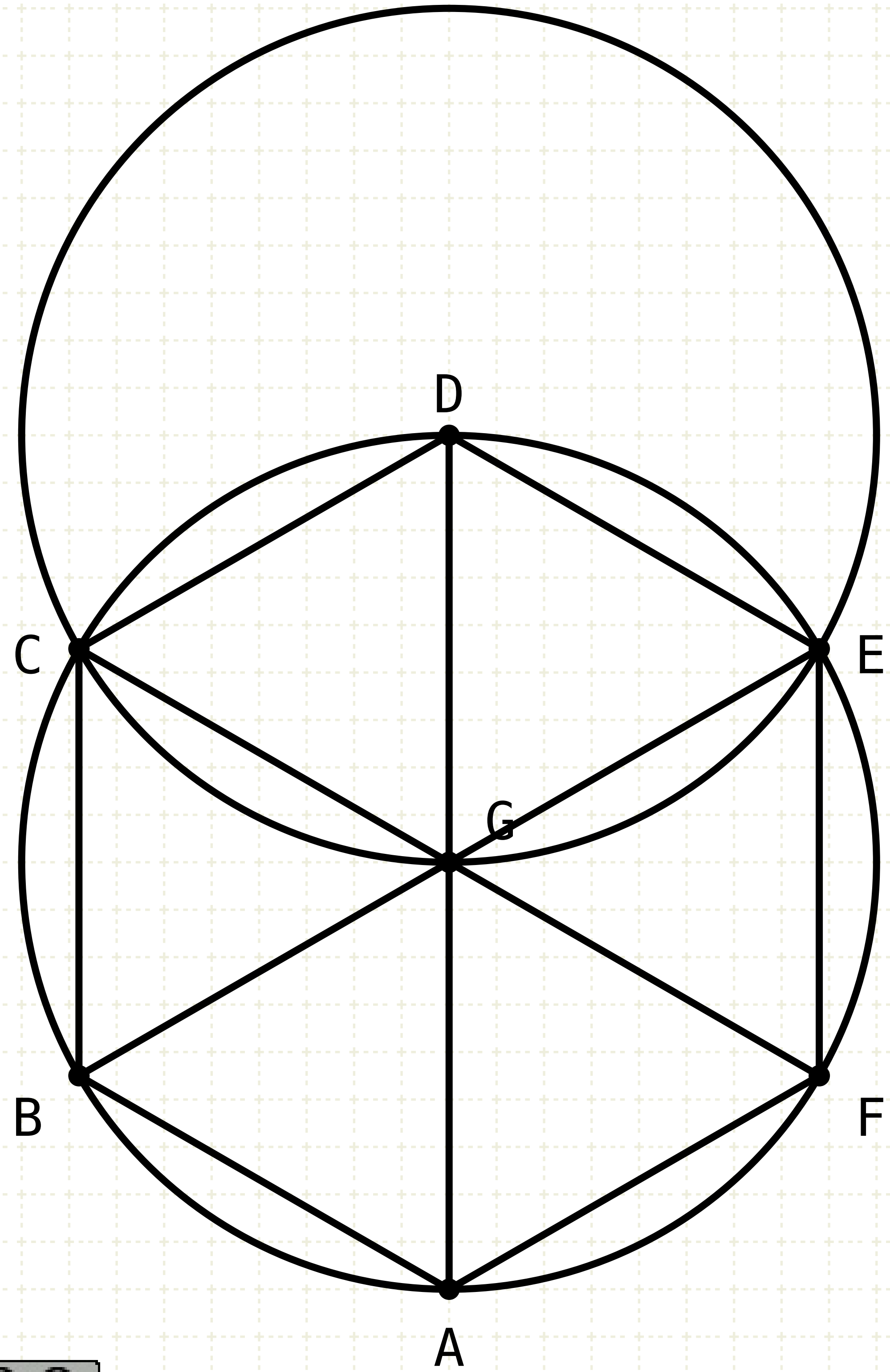
Euclid's Elements

Book IV



Philosophy (nature) is written in that great book which ever is before our eyes -- I mean the universe -- but we cannot understand it if we do not first learn the language and grasp the symbols in which it is written. The book is written in mathematical language, and the symbols are triangles, circles and other geometrical figures, without whose help it is impossible to comprehend a single word of it - without which one wanders in vain through a dark labyrinth.

Galileo Galilei

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Proposition 12 of Book IV

About a given circle to circumscribe an equilateral and equiangular pentagon.

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Proposition 12 of Book IV

About a given circle to circumscribe an equilateral and equiangular pentagon.

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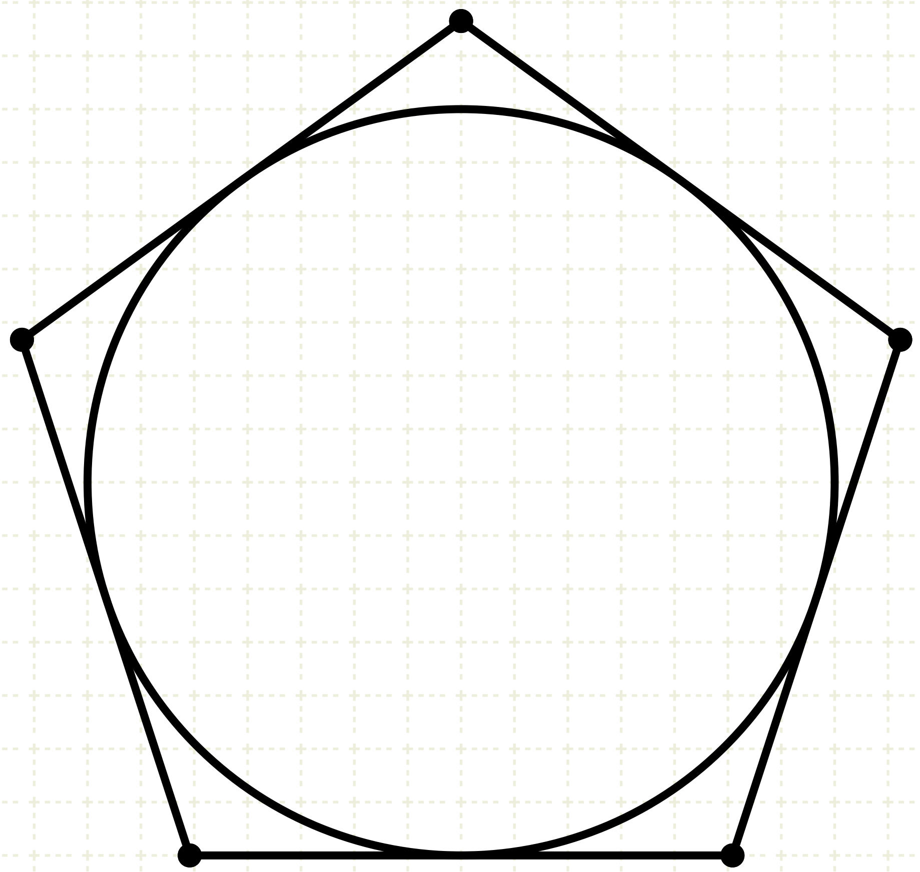
Proposition 12 of Book IV

About a given circle to circumscribe an equilateral and equiangular pentagon.



In other words

Construct a pentagon on the outside of a circle, where all lines and angles are equal

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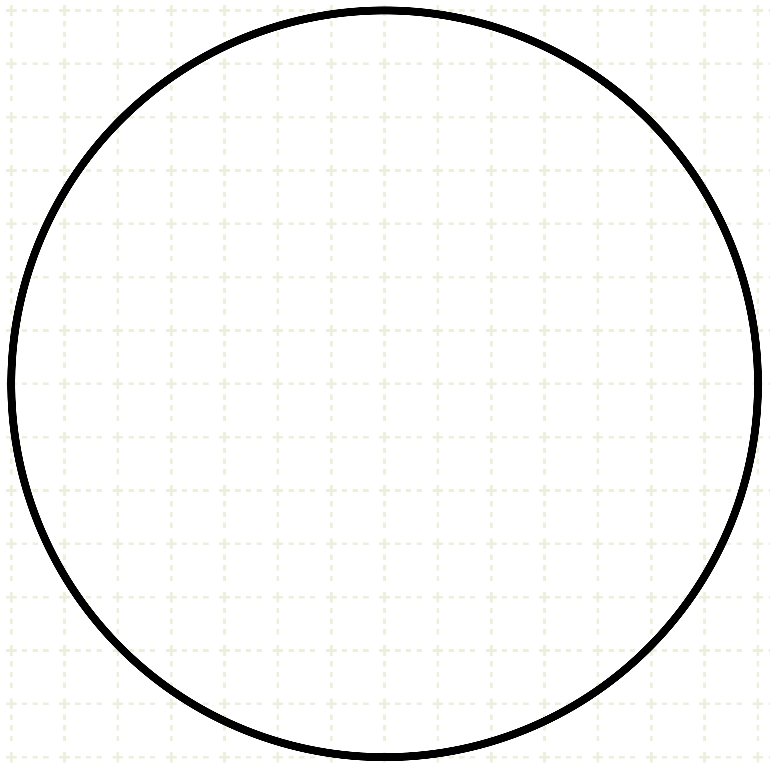
Proposition 12 of Book IV

About a given circle to circumscribe an equilateral and equiangular pentagon.



Construction

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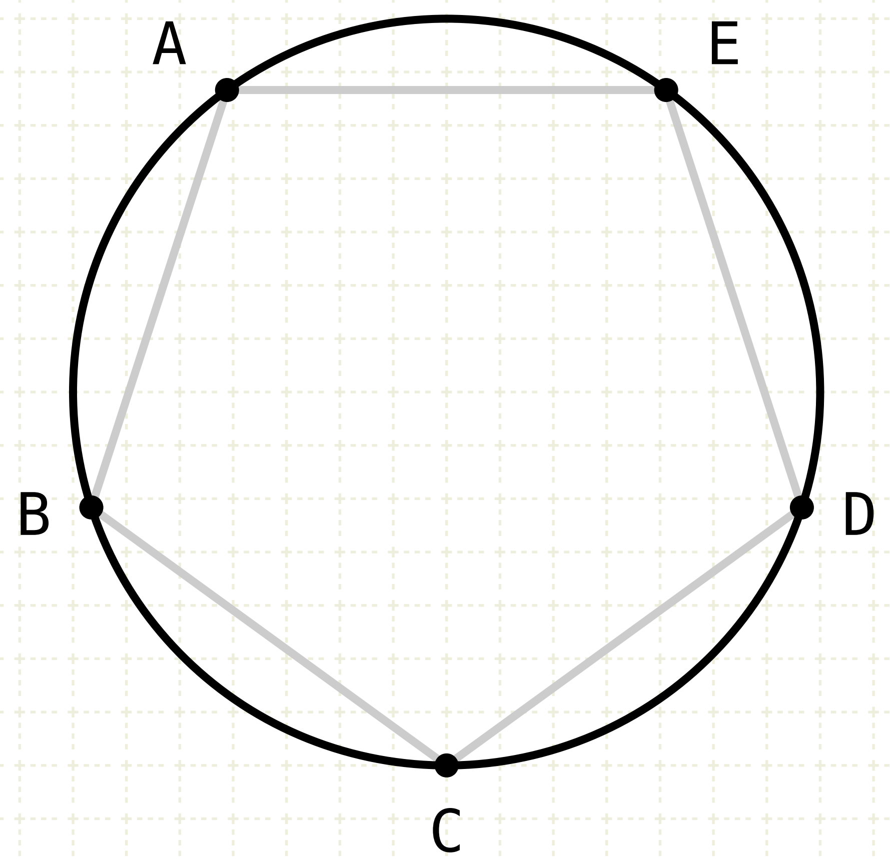
Proposition 12 of Book IV

About a given circle to circumscribe an equilateral and equiangular pentagon.



↺AB = ↺BC = ↺CD = ↺EA

Construction

Construct a pentagon in the circle, so that the AB, BC, CD, DE, EA circumferences are equal (IV·11)

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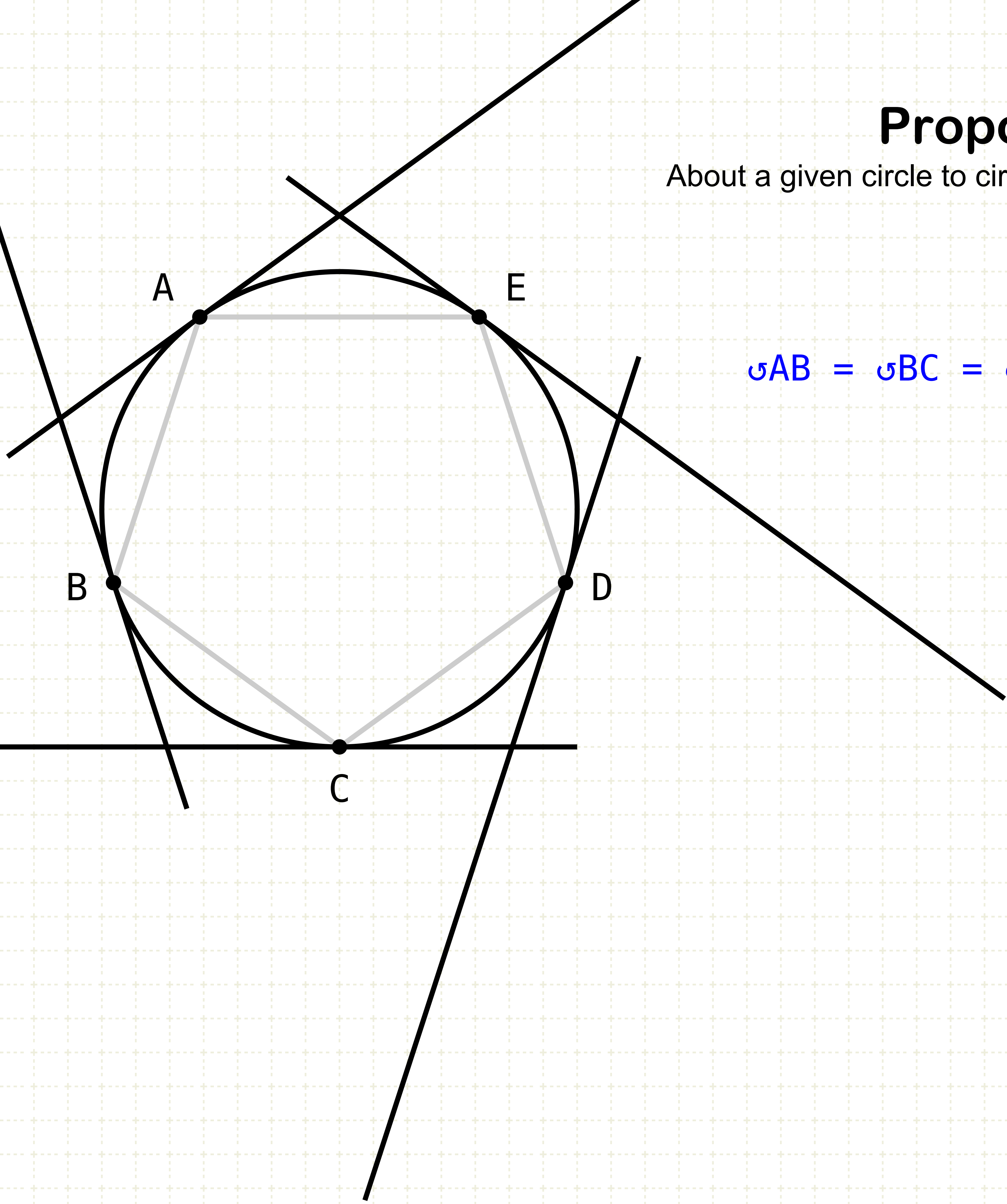
Proposition 12 of Book IV

About a given circle to circumscribe an equilateral and equiangular pentagon.



↺AB = ↺BC = ↺CD = ↺EA

Construction

Construct a pentagon in the circle, so that the AB, BC, CD, DE, EA circumferences are equal (IV·11)

Draw lines from each point A, B, C, D, E, just touching the circle (III·16)

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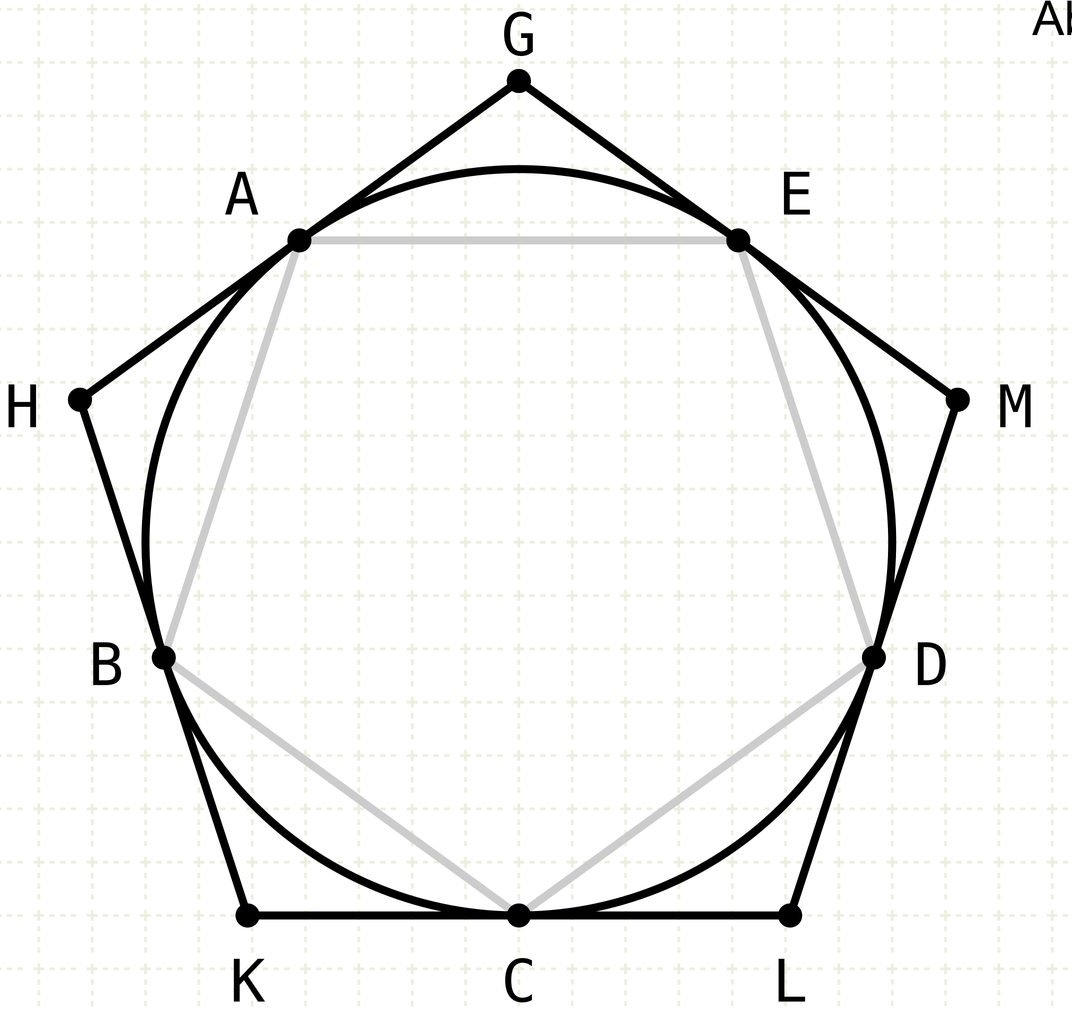
Proposition 12 of Book IV

About a given circle to circumscribe an equilateral and equiangular pentagon.



↺AB = ↺BC = ↺CD = ↺EA

Construction

Construct a pentagon in the circle, so that the AB, BC, CD, DE, EA circumferences are equal (IV·11)

Draw lines from each point A, B, C, D, E, just touching the circle (III·16)

Label the intersection points G, H, K, L and M

The pentagon GHKLM is equilateral and equiangular

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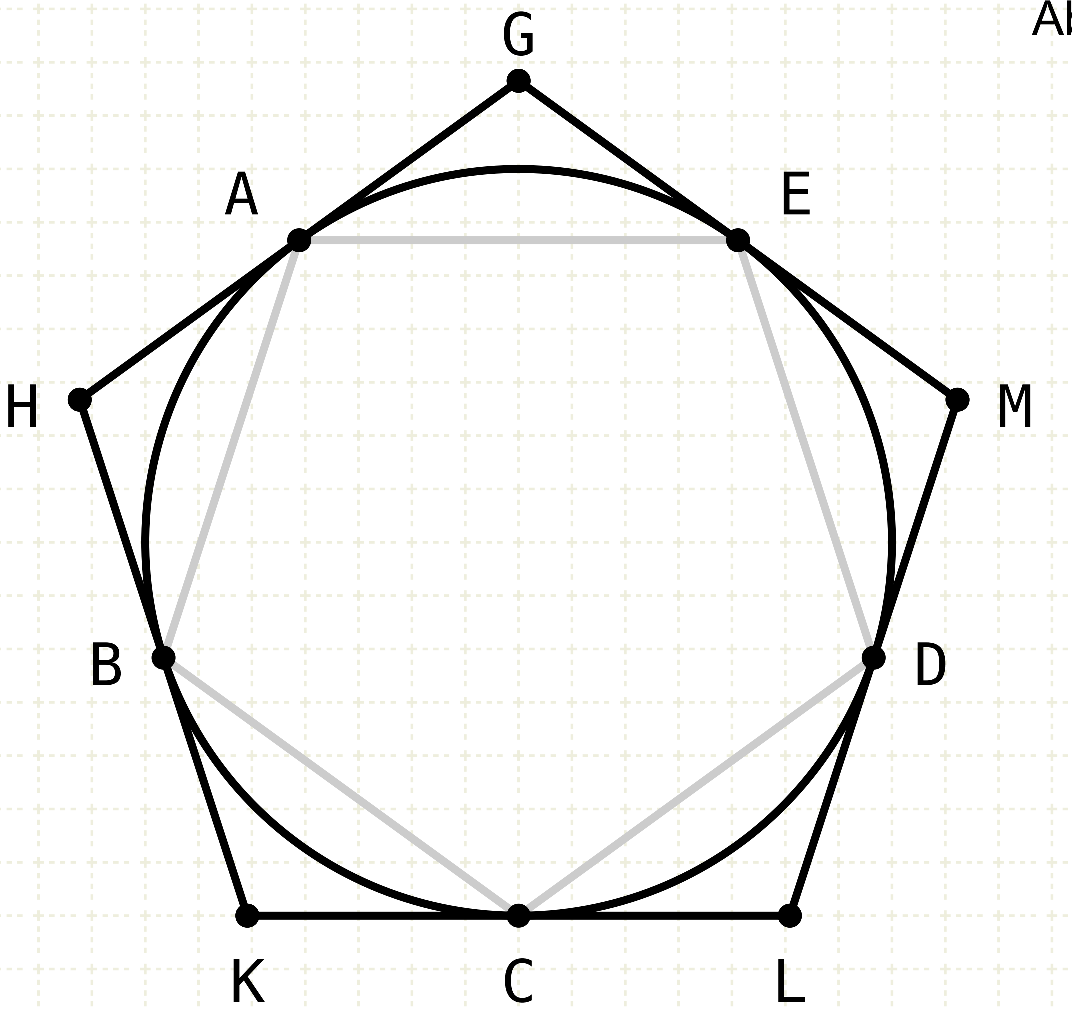
Proposition 12 of Book IV

About a given circle to circumscribe an equilateral and equiangular pentagon.



Proof

↺AB = ↺BC = ↺CD = ↺EA

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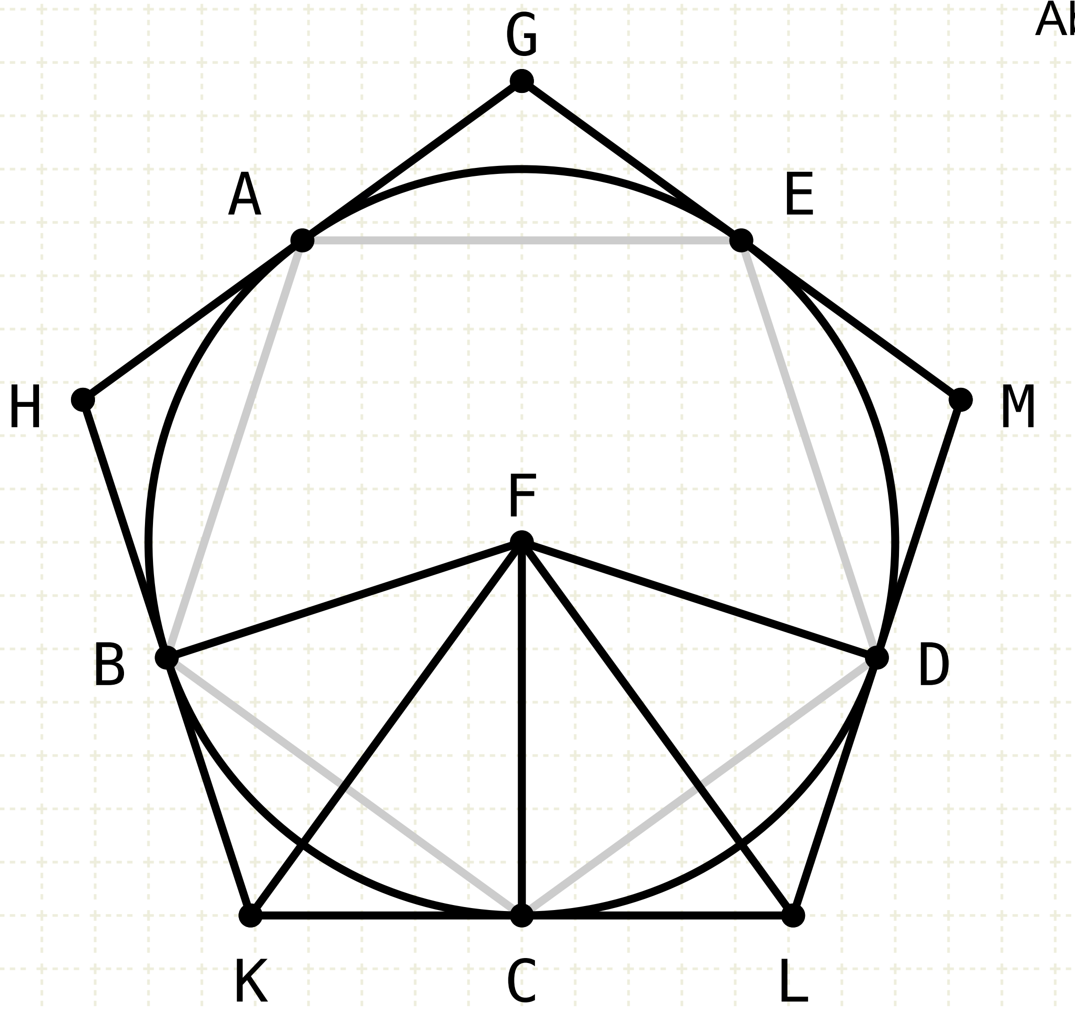
Proposition 12 of Book IV

About a given circle to circumscribe an equilateral and equiangular pentagon.



↺AB = ↺BC = ↺CD = ↺EA

Proof

Find and label the centre of the circle F (III·1)

Draw lines FB FK FC FL FD

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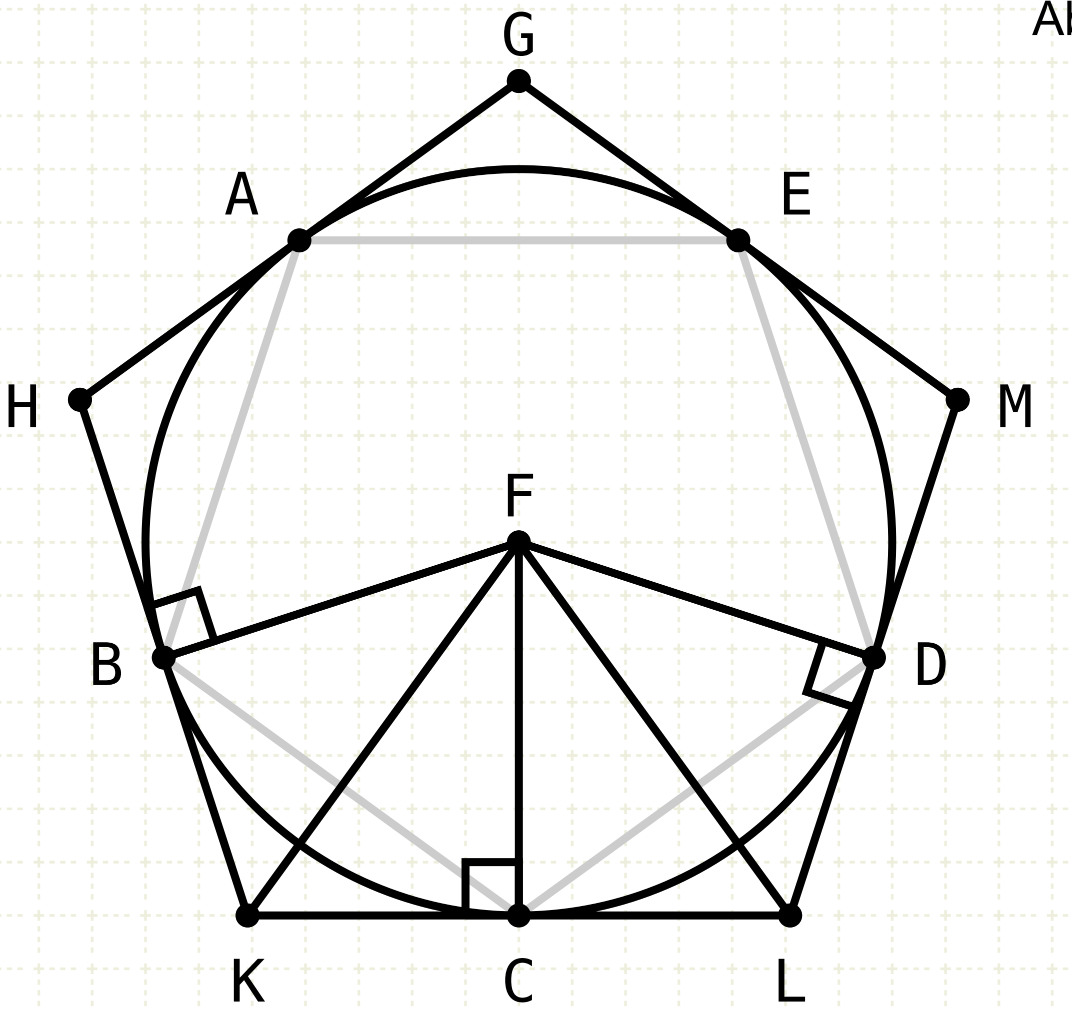
Proposition 12 of Book IV

About a given circle to circumscribe an equilateral and equiangular pentagon.



↺AB = ↺BC = ↺CD = ↺EA

FB = FC = FD

Proof

Find and label the centre of the circle F (III·1)

Draw lines FB FK FC FL FD

Since HK, KL and LM touch the circle at points B, C, D, and since the lines FB, FC, FD are drawn from the centre of the circle, the angles at B, C and D are right angles (III·18)

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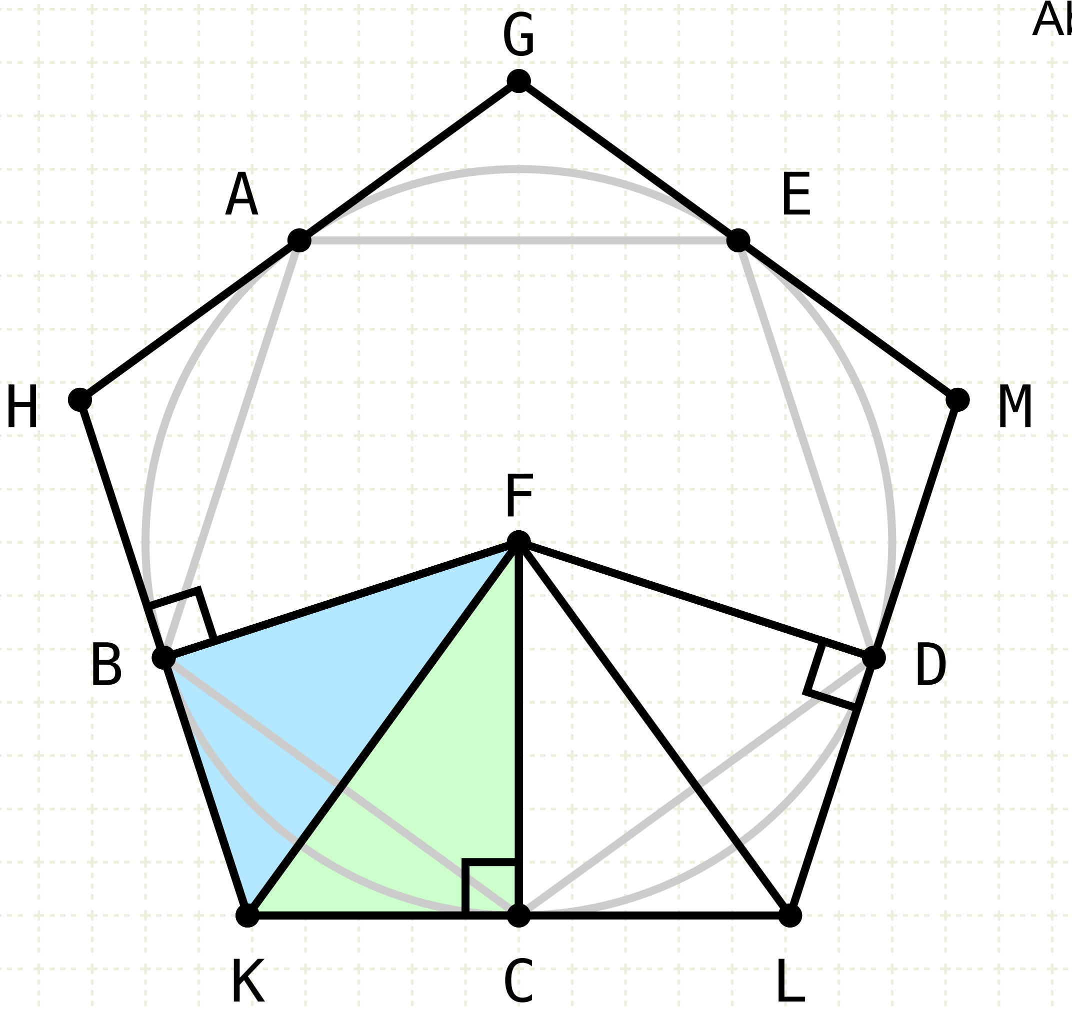
Proposition 12 of Book IV

About a given circle to circumscribe an equilateral and equiangular pentagon.



↺AB = ↺BC = ↺CD = ↺EA

$FB = FC = FD$

$FK^2 = FC^2 + KC^2$

$FK^2 = FB^2 + BK^2$

Proof

Find and label the centre of the circle F (III·1)

Draw lines FB FK FC FL FD

Since HK, KL and LM touch the circle at points B, C, D, and since the lines FB, FC, FD are drawn from the centre of the circle, the angles at B, C and D are right angles (III·18)

Using pythagoras' theorem (I·47), the square of FK is equal to the sum of the squares FC,CK and it is also equal to the sum of the the squares FB,BK

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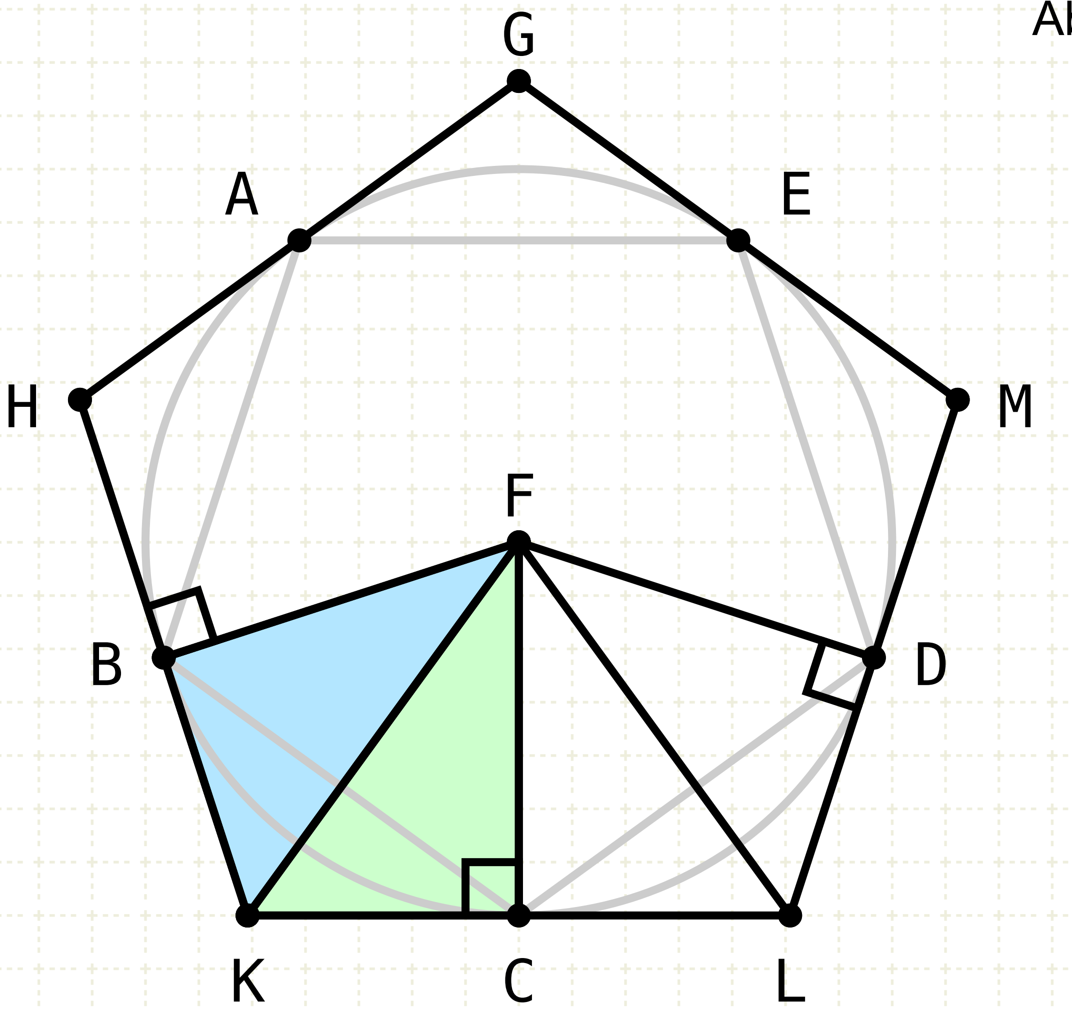
Proposition 12 of Book IV

About a given circle to circumscribe an equilateral and equiangular pentagon.



↺AB = ↺BC = ↺CD = ↺EA

$FB = FC = FD$

$FK^2 = FC^2 + KC^2$

$FK^2 = FB^2 + BK^2$

$BK = KC$

Proof

Find and label the centre of the circle F (III·1)

Draw lines FB FK FC FL FD

Since HK, KL and LM touch the circle at points B, C, D, and since the lines FB, FC, FD are drawn from the centre of the circle, the angles at B, C and D are right angles (III·18)

Using pythagoras' theorem (I·47), the square of FK is equal to the sum of the squares FC,CK and it is also equal to the sum of the the squares FB,BK

Since FC equals FB, then the square of KC equals the square of BK, or KC equals BK

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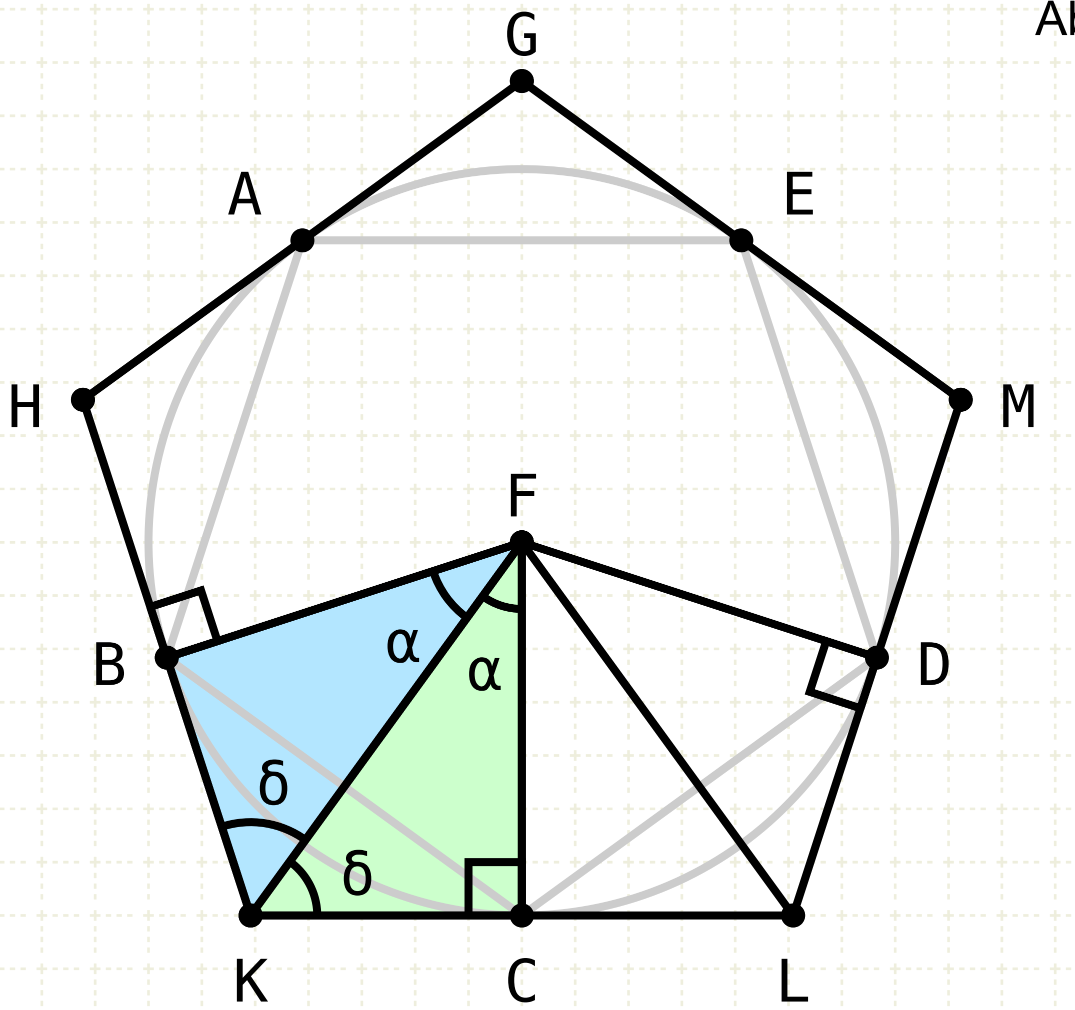
About a given circle to circumscribe an equilateral and equiangular pentagon.



↺AB = ↺BC = ↺CD = ↺EA

$FB = FC = FD$

$FK^2 = FC^2 + KC^2$

$FK^2 = FB^2 + BK^2$

$BK = KC$

$\angle BFK = \angle CFK$

$\angle FKB = \angle FKC$

Proof

Find and label the centre of the circle F (III·1)

Draw lines FB FK FC FL FD

Since HK, KL and LM touch the circle at points B, C, D, and since the lines FB, FC, FD are drawn from the centre of the circle, the angles at B, C and D are right angles (III·18)

Using pythagoras' theorem (I·47), the square of FK is equal to the sum of the squares FC,CK and it is also equal to the sum of the the squares FB,BK

Since FC equals FB, then the square of KC equals the square of BK, or KC equals BK

BF equals BC, BK equals KC, and FK is common, therefore the triangles FBK and FBC are equivalent((SSS) (I·8)

Thus angles BFK and KFC are equal

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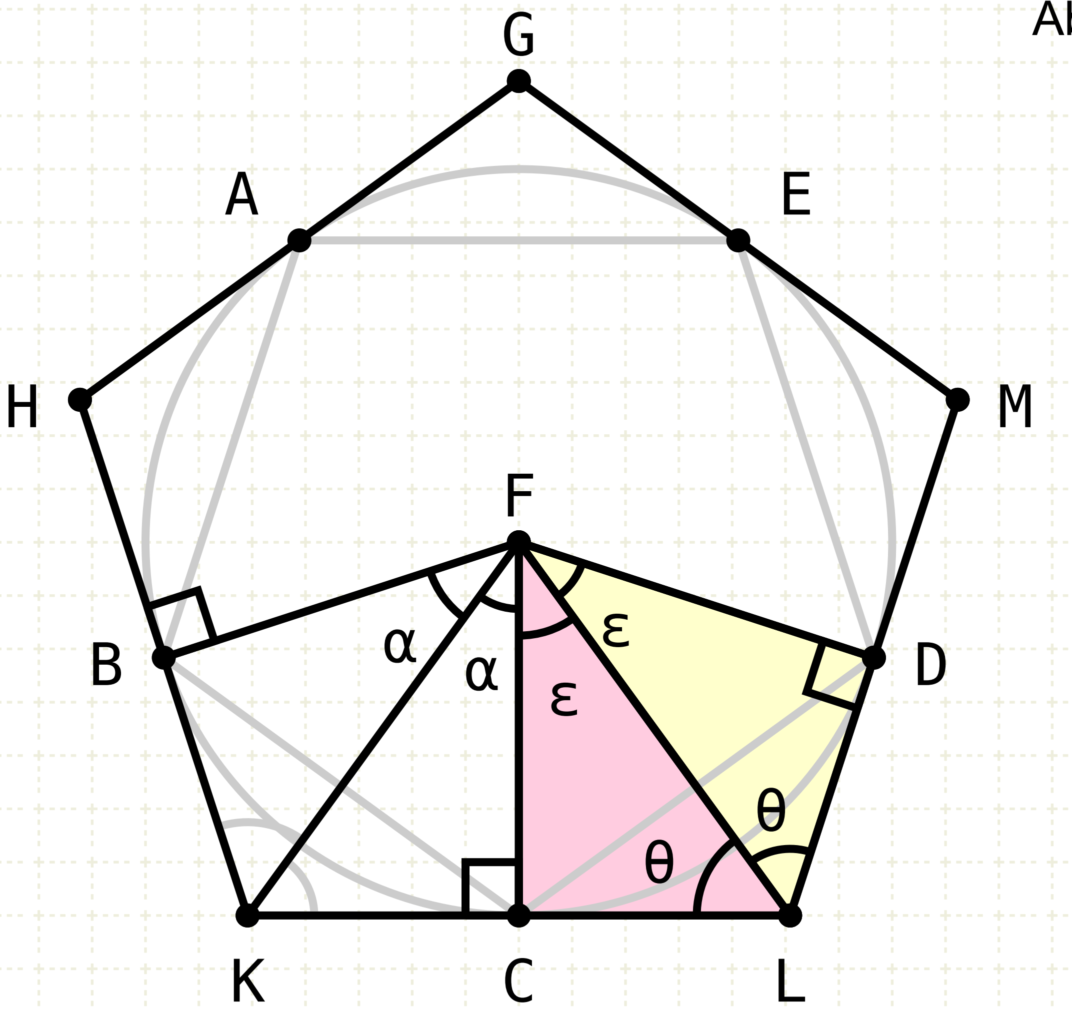
Proposition 12 of Book IV

About a given circle to circumscribe an equilateral and equiangular pentagon.



$\circlearrowleft AB = \circlearrowleft BC = \circlearrowleft CD = \circlearrowleft EA$

$FB = FC = FD$

$FK^2 = FC^2 + KC^2$

$FK^2 = FB^2 + BK^2$

$BK = KC$

$\angle BFK = \angle CFK$

$\angle FKB = \angle FKC$

$\angle CFL = \angle LFD$

Proof

Find and label the centre of the circle F (III·1)

Draw lines FB FK FC FL FD

Since HK, KL and LM touch the circle at points B, C, D, and since the lines FB, FC, FD are drawn from the centre of the circle, the angles at B, C and D are right angles (III·18)

Using pythagoras' theorem (I·47), the square of FK is equal to the sum of the squares FC,CK and it is also equal to the sum of the the squares FB,BK

Since FC equals FB, then the square of KC equals the square of BK, or KC equals BK

BF equals BC, BK equals KC, and FK is common, therefore the triangles FBK and FBC are equivalent((SSS) (I·8)

Thus angles BFK and KFC are equal

Similarly, angles CFL and LFD are equal

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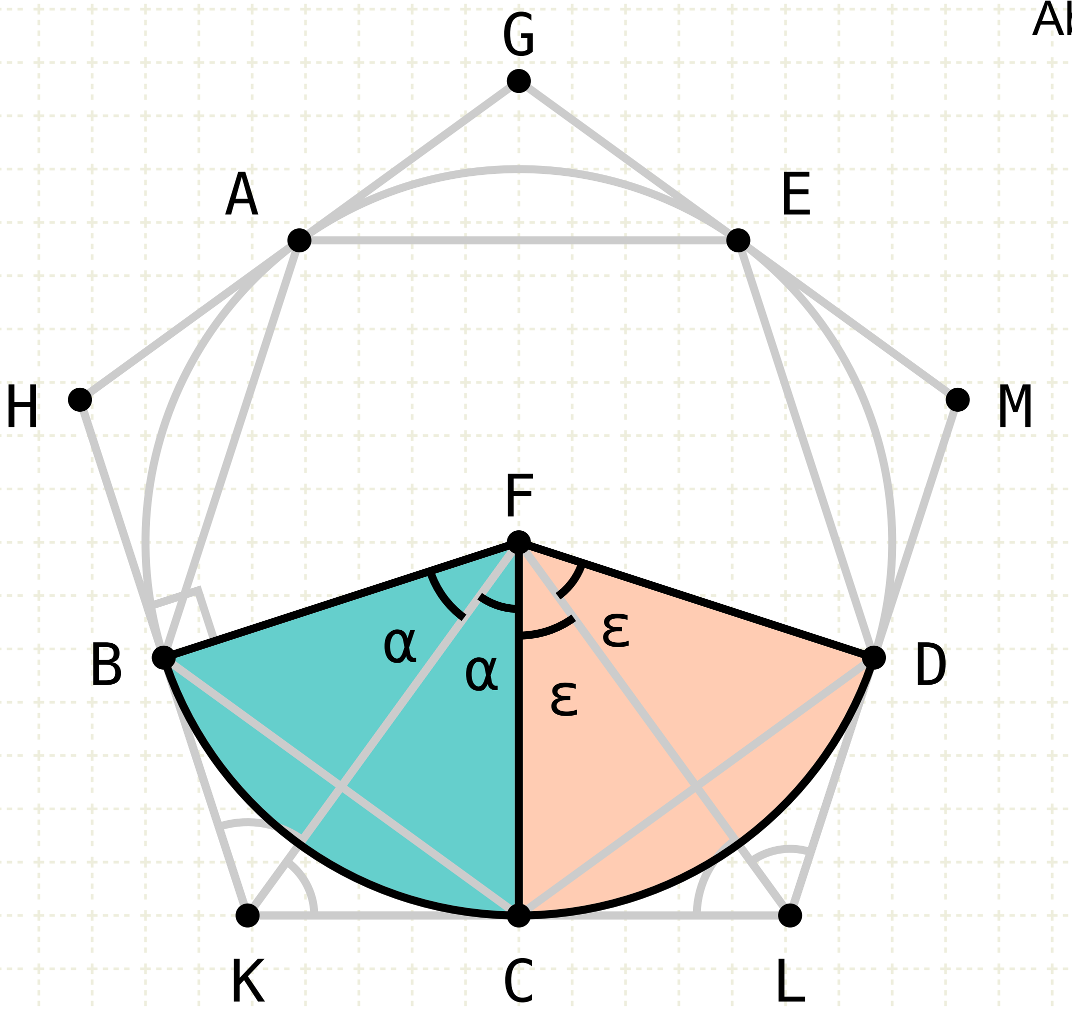
About a given circle to circumscribe an equilateral and equiangular pentagon.



↺AB = ↺BC = ↺CD = ↺EA

$FB = FC = FD$

$FK^2 = FC^2 + KC^2$

$FK^2 = FB^2 + BK^2$

$BK = KC$

$\angle BFK = \angle CFK$

$\angle FKB = \angle FKC$

$\angle CFL = \angle LFD$

$\angle BFC = \angle CFD$

$\alpha = \varepsilon$

Proof

Find and label the centre of the circle F (III·1)

Draw lines FB FK FC FL FD

Since HK, KL and LM touch the circle at points B, C, D, and since the lines FB, FC, FD are drawn from the centre of the circle, the angles at B, C and D are right angles (III·18)

Using pythagoras' theorem (I·47), the square of FK is equal to the sum of the squares FC,CK and it is also equal to the sum of the the squares FB,BK

Since FC equals FB, then the square of KC equals the square of BK, or KC equals BK

BF equals BC, BK equals KC, and FK is common, therefore the triangles FBK and FBC are equivalent((SSS) (I·8)

Thus angles BFK and KFC are equal

Similarly, angles CFL and LFD are equal

Since the circumference BC, CD are equal, so are the angles subtending them, angles BFC equals CFD (III·27)

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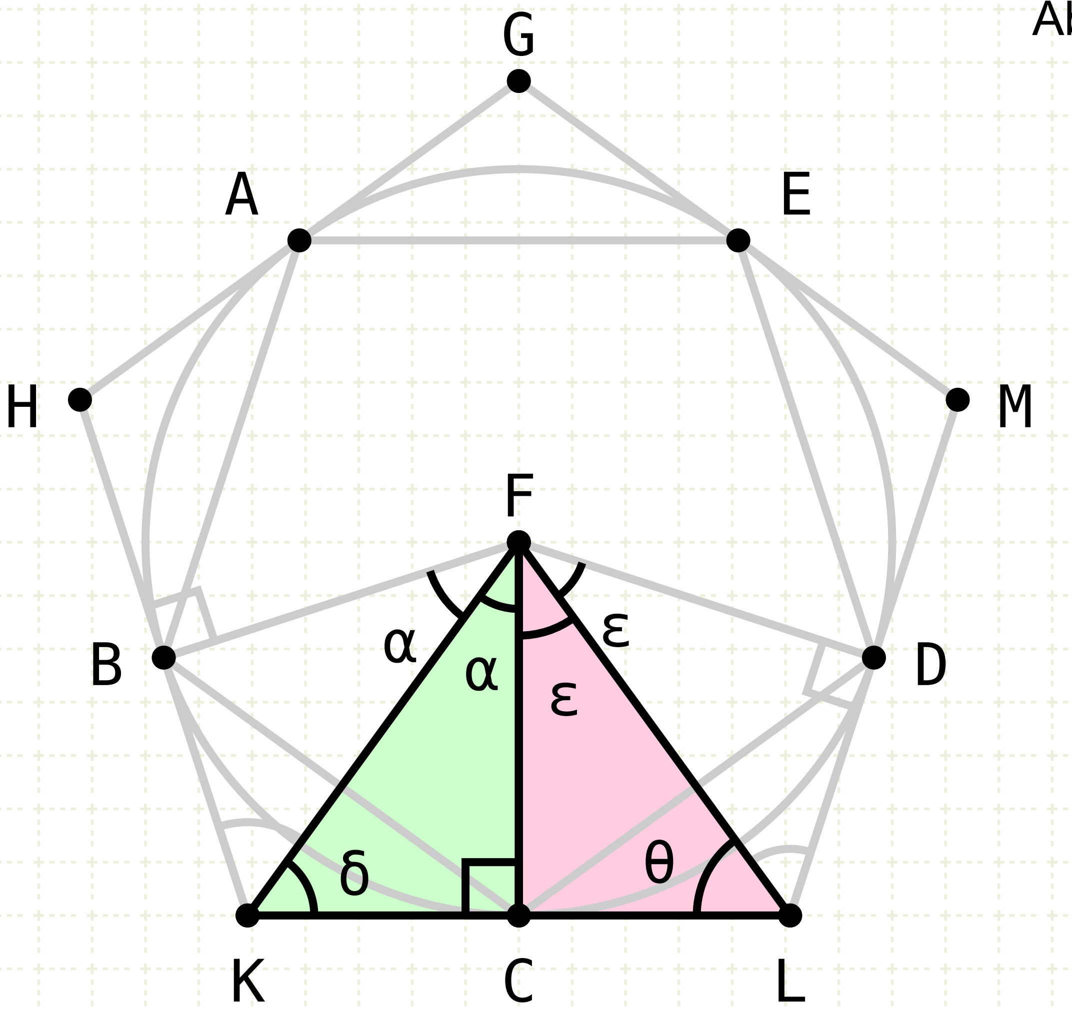
About a given circle to circumscribe an equilateral and equiangular pentagon.



$\circlearrowleft AB = \circlearrowleft BC = \circlearrowleft CD = \circlearrowleft EA$

$FB = FC = FD$

$FK^2 = FC^2 + KC^2$

$FK^2 = FB^2 + BK^2$

$BK = KC$

$\angle BFK = \angle CFK$

$\angle FKB = \angle FKC$

$\angle CFL = \angle LFD$

$\angle BFC = \angle CFD$

$\alpha = \varepsilon$

$KC = CL$

$KL = 2 \cdot KC$

$\delta = \theta$

Proof

Find and label the centre of the circle F (III·1)

Draw lines FB FK FC FL FD

Since HK, KL and LM touch the circle at points B, C, D, and since the lines FB, FC, FD are drawn from the centre of the circle, the angles at B, C and D are right angles (III·18)

Using pythagoras' theorem (I·47), the square of FK is equal to the sum of the squares FC,CK and it is also equal to the sum of the the squares FB,BK

Since FC equals FB, then the square of KC equals the square of BK, or KC equals BK

BF equals BC, BK equals KC, and FK is common, therefore the triangles FBK and FBC are equivalent((SSS) (I·8)

Thus angles BFK and KFC are equal

Similarly, angles CFL and LFD are equal

Since the circumference BC, CD are equal, so are the angles subtending them, angles BFC equals CFD (III·27)

Triangles FKC and FCL have one side and two angles equal, therefore they are equivalent (I·26), and KC equals CL, or KL equals twice KC and the angles FKC and FLC are equal

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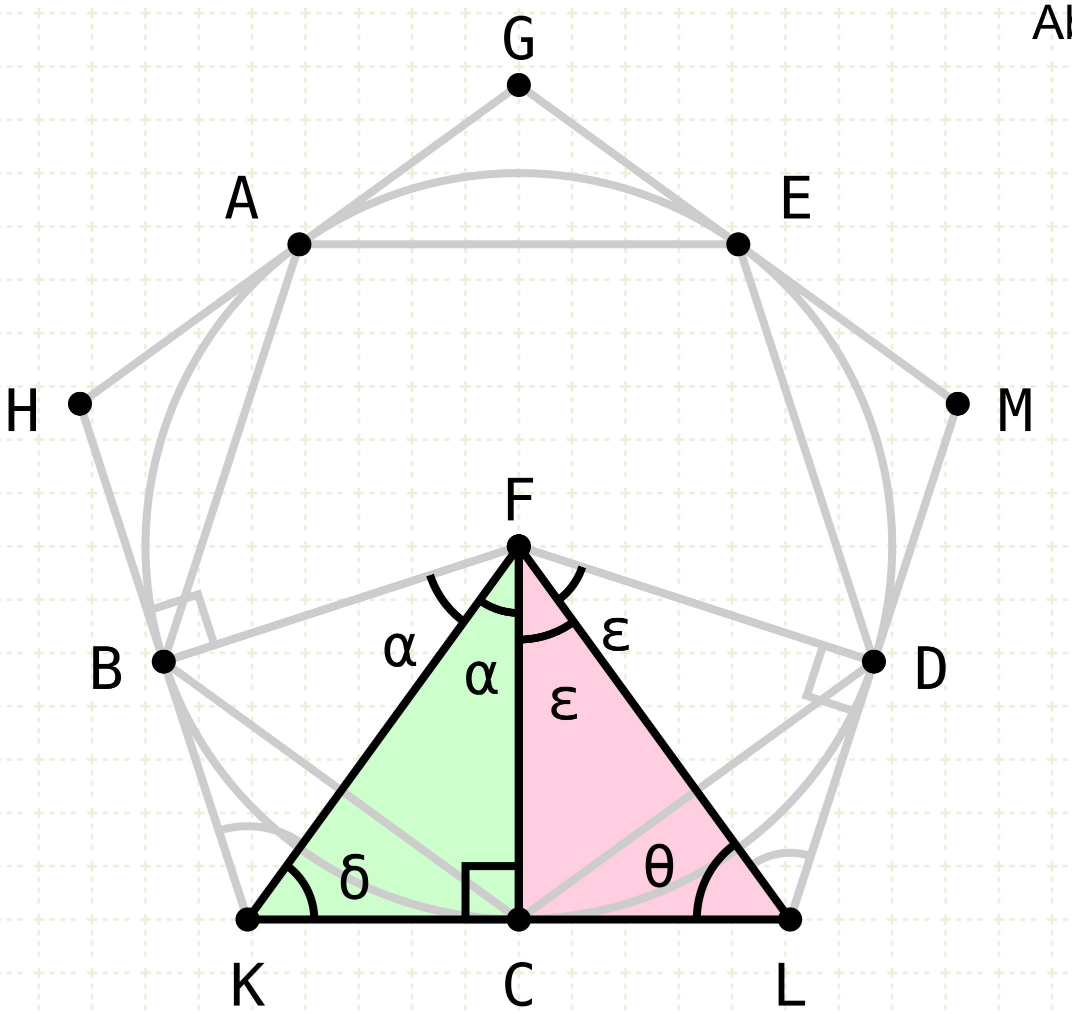
About a given circle to circumscribe an equilateral and equiangular pentagon.



↺AB = ↺BC = ↺CD = ↺EA

$FB = FC = FD$

$FK^2 = FC^2 + KC^2$

$FK^2 = FB^2 + BK^2$

$BK = KC$

$\angle BFK = \angle CFK$

$\angle FKB = \angle FKC$

$\angle CFL = \angle LFD$

$\angle BFC = \angle CFD$

$\alpha = \varepsilon$

$KC = CL$

$KL = 2 \cdot KC$

$\delta = \theta$

$HK = 2 \cdot BK = 2 \cdot KC$

$\therefore HK = KL$

Proof

Find and label the centre of the circle F (III·1)

Draw lines FB FK FC FL FD

Since HK, KL and LM touch the circle at points B, C, D, and since the lines FB, FC, FD are drawn from the centre of the circle, the angles at B, C and D are right angles (III·18)

Using pythagoras' theorem (I·47), the square of FK is equal to the sum of the squares FC,CK and it is also equal to the sum of the the squares FB,BK

Since FC equals FB, then the square of KC equals the square of BK, or KC equals BK

BF equals BC, BK equals KC, and FK is common, therefore the triangles FBK and FBC are equivalent((SSS) (I·8)

Thus angles BFK and KFC are equal

Similarly, angles CFL and LFD are equal

Since the circumference BC, CD are equal, so are the angles subtending them, angles BFC equals CFD (III·27)

Triangles FKC and FCL have one side and two angles equal, therefore they are equivalent (I·26), and KC equals CL, or KL equals twice KC and the angles FKC and FLC are equal

Similarly, it can be shown that HK is twice BK, and since BK equals KC, HK equals KL

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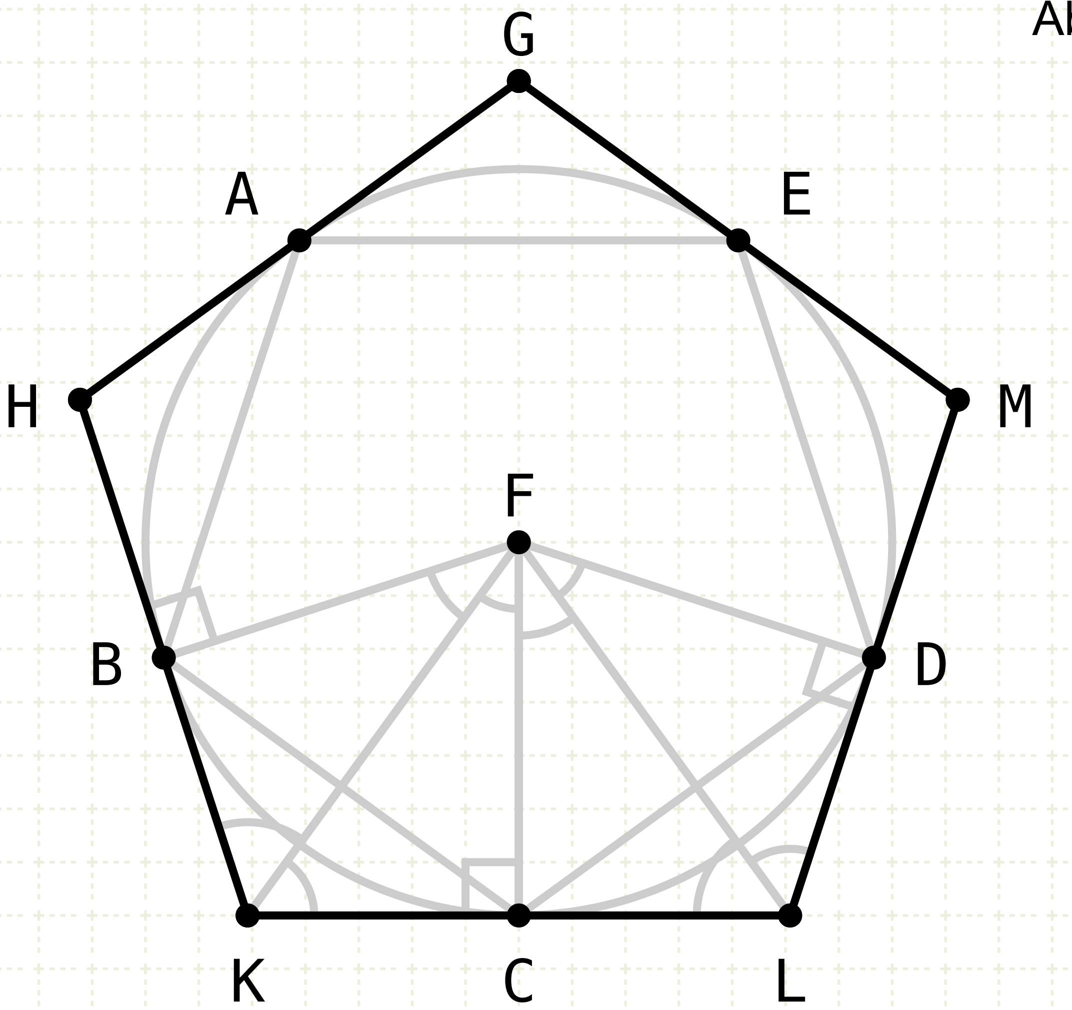
About a given circle to circumscribe an equilateral and equiangular pentagon.



$\circlearrowleft AB = \circlearrowleft BC = \circlearrowleft CD = \circlearrowleft EA$

$FB = FC = FD$

$FK^2 = FC^2 + KC^2$

$FK^2 = FB^2 + BK^2$

$BK = KC$

$\angle BFK = \angle CFK$

$\angle FKB = \angle FKC$

$\angle CFL = \angle LFD$

$\angle BFC = \angle CFD$

$\alpha = \varepsilon$

$KC = CL$

$KL = 2 \cdot KC$

$\delta = \theta$

$HK = 2 \cdot BK = 2 \cdot KC$

$\therefore HK = KL$

$HK = KL = LM = MG = GH$

Proof

Find and label the centre of the circle F (III·1)

Draw lines FB FK FC FL FD

Since HK, KL and LM touch the circle at points B, C, D, and since the lines FB, FC, FD are drawn from the centre of the circle, the angles at B, C and D are right angles (III·18)

Using pythagoras' theorem (I·47), the square of FK is equal to the sum of the squares FC,CK and it is also equal to the sum of the the squares FB,BK

Since FC equals FB, then the square of KC equals the square of BK, or KC equals BK

BF equals BC, BK equals KC, and FK is common, therefore the triangles FBK and FBC are equivalent((SSS) (I·8)

Thus angles BFK and KFC are equal

Similarly, angles CFL and LFD are equal

Since the circumference BC, CD are equal, so are the angles subtending them, angles BFC equals CFD (III·27)

Triangles FKC and FCL have one side and two angles equal, therefore they are equivalent (I·26), and KC equals CL, or KL equals twice KC and the angles FKC and FLC are equal

Similarly, it can be shown that HK is twice BK, and since BK equals KC, HK equals KL

Applying the same logic to the other sides of the pentagon proves that the pentagon is equilateral

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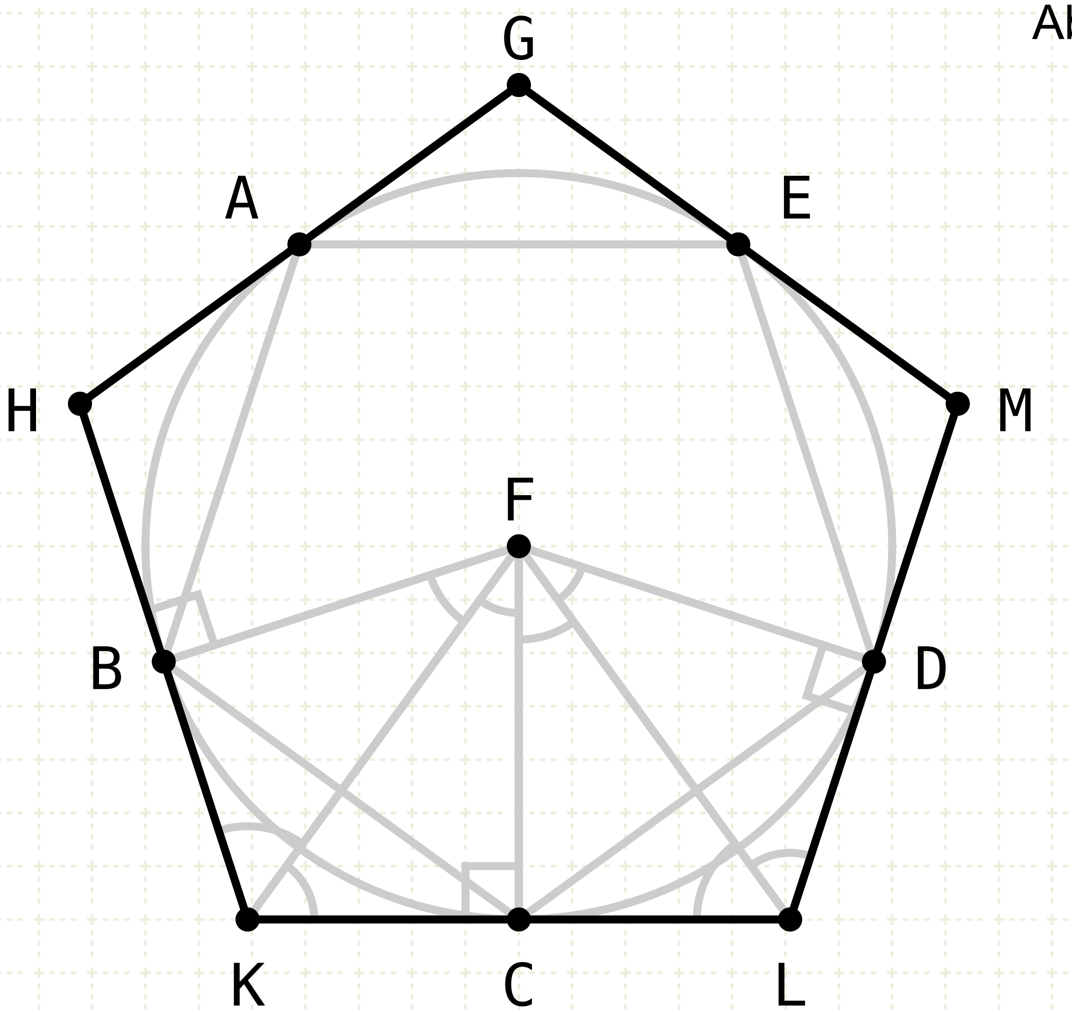
Proposition 12 of Book IV

About a given circle to circumscribe an equilateral and equiangular pentagon.



$↺AB = ↺BC = ↺CD = ↺EA$

$HK = KL = LM = MG = GH$

The pentagon has been proven to be equilateral

It is also equiangular

Proof (cont.)

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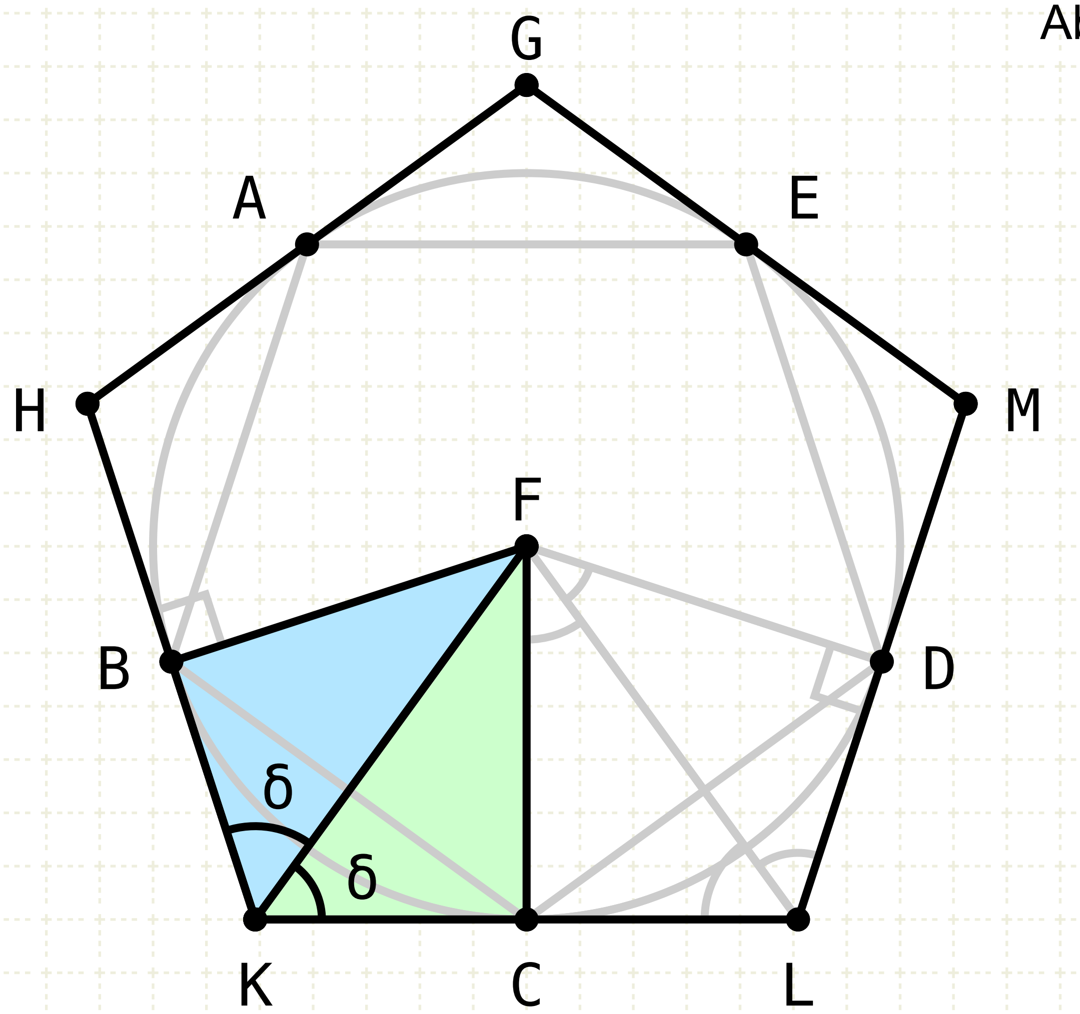
Proposition 12 of Book IV

About a given circle to circumscribe an equilateral and equiangular pentagon.



↺AB = ↺BC = ↺CD = ↺EA

HK = KL = LM = MG = GH

∠HKL = 2δ

The pentagon has been proven to be equilateral

It is also equiangular

Proof (cont.)

Triangles BFK and FKC have been proven to be equivalent, therefore the angles FKB and FKC are equal

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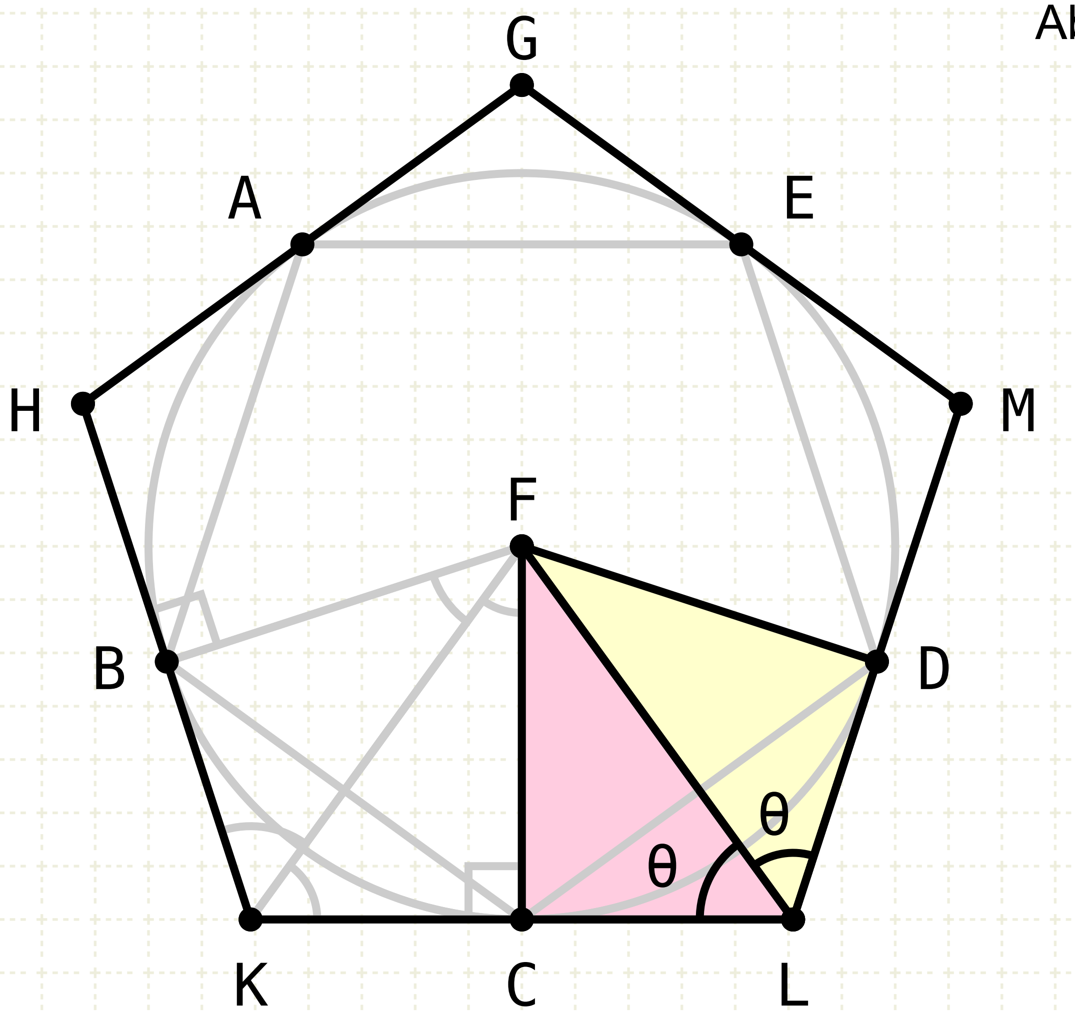
Proposition 12 of Book IV

About a given circle to circumscribe an equilateral and equiangular pentagon.



$\circlearrowleft AB = \circlearrowleft BC = \circlearrowleft CD = \circlearrowleft EA$

$HK = KL = LM = MG = GH$

$\angle HKL = 2\delta$

$\angle KLM = 2\theta$

The pentagon has been proven to be equilateral

It is also equiangular

Proof (cont.)

Triangles BFK and FKC have been proven to be equivalent, therefore the angles FKB and FKC are equal

Triangles FCL and FDL have been proven to be equal, therefore the angles FLC and FLD are equal

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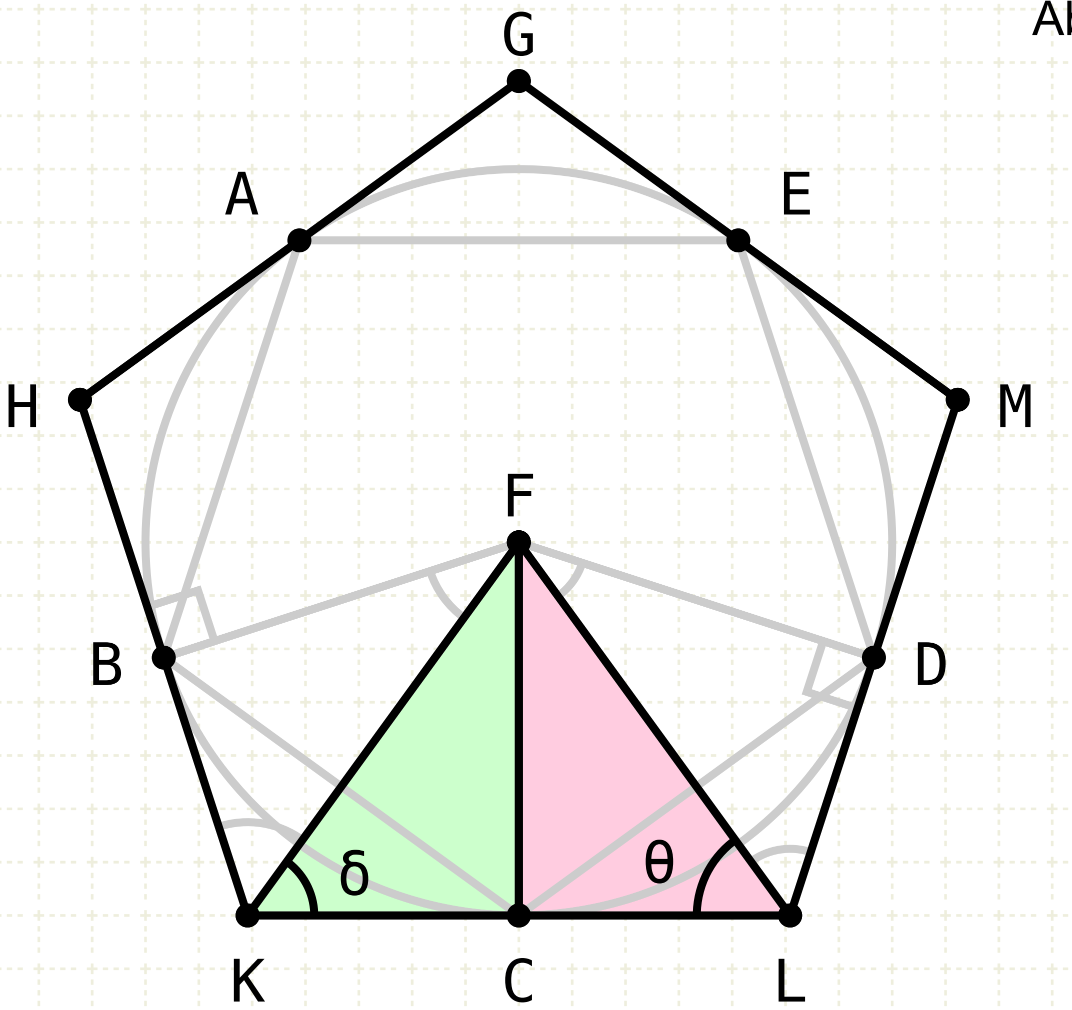
Proposition 12 of Book IV

About a given circle to circumscribe an equilateral and equiangular pentagon.



$\circlearrowleft AB = \circlearrowleft BC = \circlearrowleft CD = \circlearrowleft EA$

$HK = KL = LM = MG = GH$

$\angle HKL = 2\delta$

$\angle KLM = 2\theta$

$\delta = \theta$

The pentagon has been proven to be equilateral

It is also equiangular

Proof (cont.)

Triangles BFK and FKC have been proven to be equivalent, therefore the angles FKB and FKC are equal

Triangles FCL and FDL have been proven to be equal, therefore the angles FLC and FLD are equal

Triangles FKC and FCL have been proven to be equal, therefore the angles FKC and FLC are equal

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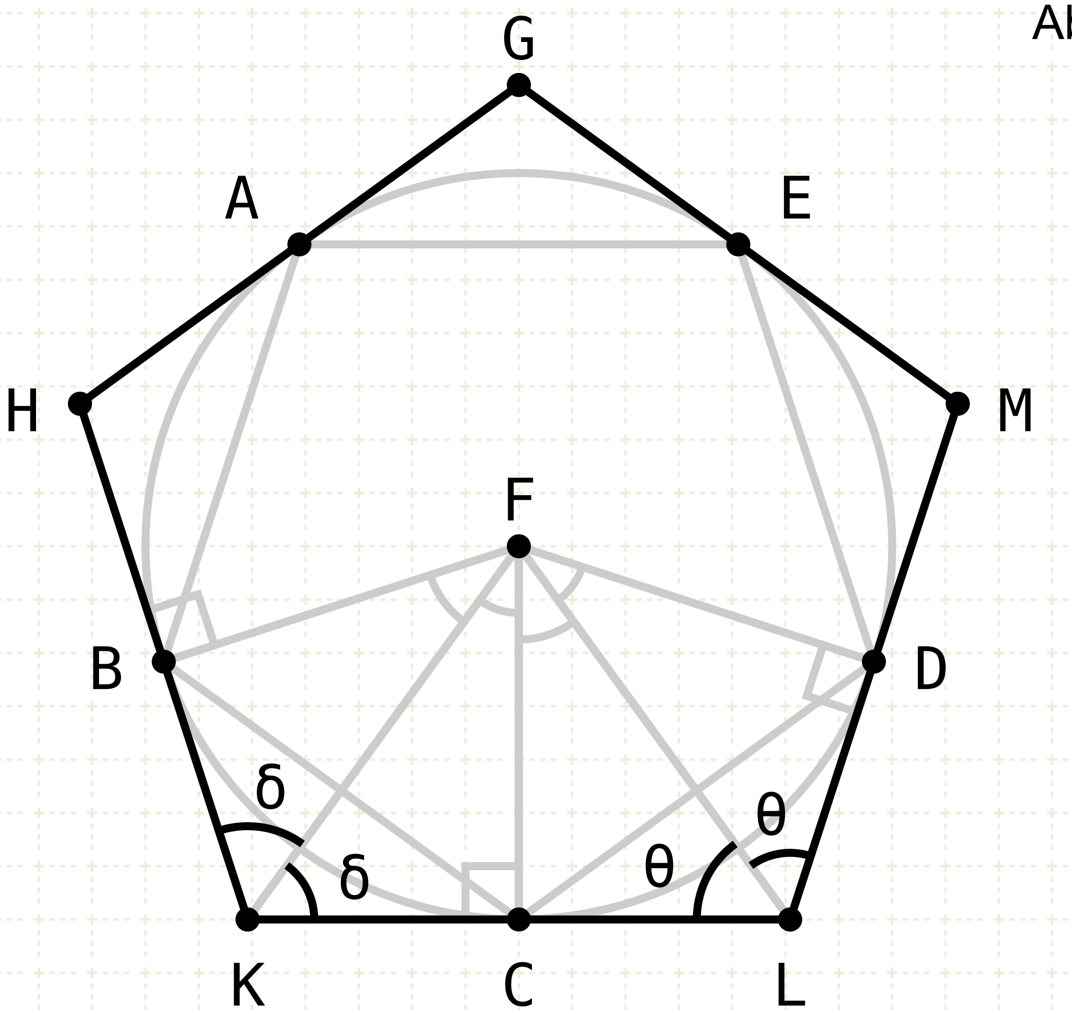
Proposition 12 of Book IV

About a given circle to circumscribe an equilateral and equiangular pentagon.



↺AB = ↺BC = ↺CD = ↺EA

HK = KL = LM = MG = GH

∠HKL = 2δ

∠KLM = 2θ

δ = θ

∠HKL = ∠FLC

The pentagon has been proven to be equilateral

It is also equiangular

Proof (cont.)

Triangles BFK and FKC have been proven to be equivalent, therefore the angles FKB and FKC are equal

Triangles FCL and FDL have been proven to be equal, therefore the angles FLC and FLD are equal

Triangles FKC and FCL have been proven to be equal, therefore the angles FKC and FLC are equal

Therefore, angles HKL and FLC are equal

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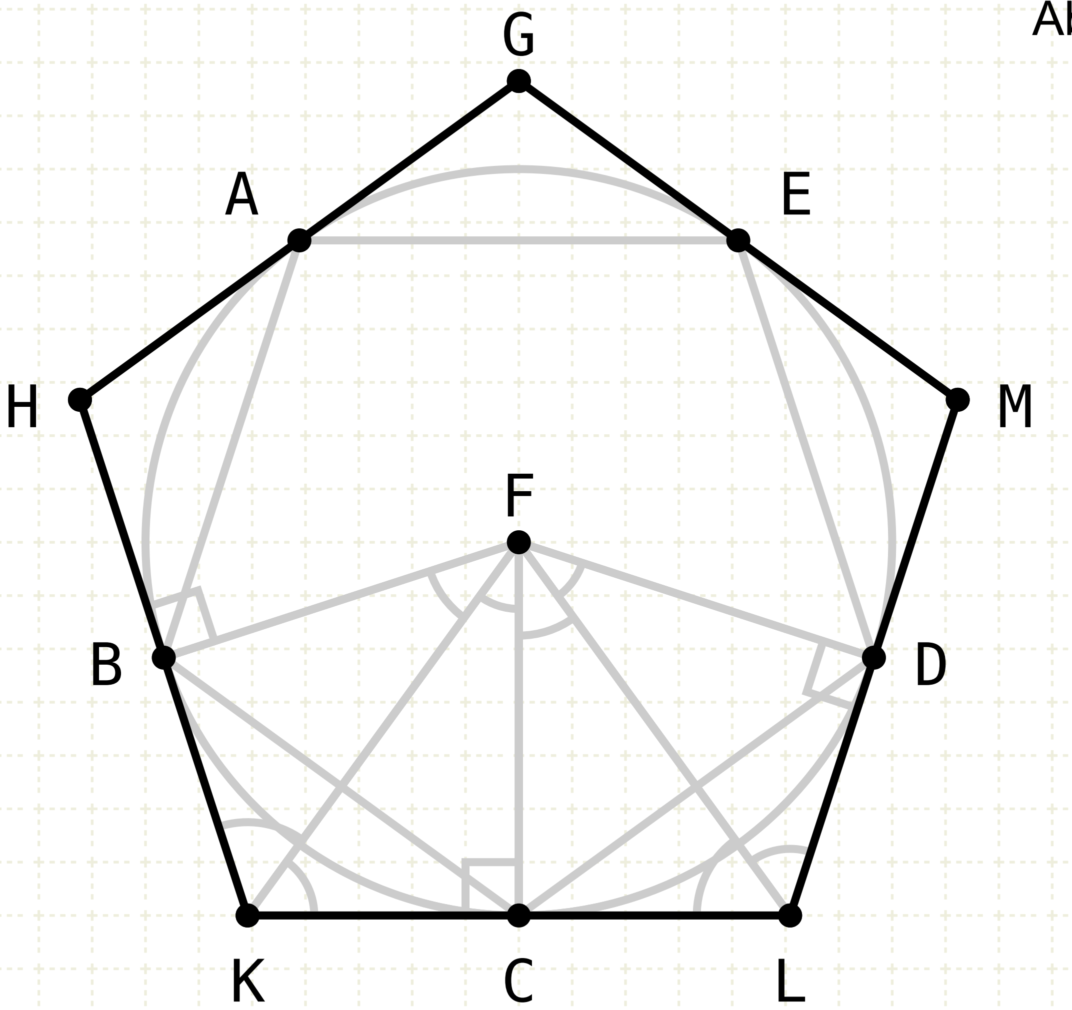
Proposition 12 of Book IV

About a given circle to circumscribe an equilateral and equiangular pentagon.



$\circlearrowleft AB = \circlearrowleft BC = \circlearrowleft CD = \circlearrowleft EA$

$HK = KL = LM = MG = GH$

$\angle HKL = 2\delta$

$\angle KLM = 2\theta$

$\delta = \theta$

$\angle HKL = \angle FLC$

$\angle GHK = \angle HKL = \angle KLM$
$= \angle LMG = \angle MGH$

The pentagon has been proven to be equilateral

It is also equiangular

Proof (cont.)

Triangles BFK and FKC have been proven to be equivalent, therefore the angles FKB and FKC are equal

Triangles FCL and FDL have been proven to be equal, therefore the angles FLC and FLD are equal

Triangles FKC and FCL have been proven to be equal, therefore the angles FKC and FLC are equal

Therefore, angles HKL and FLC are equal

Using the same logic, it can be shown that all the angle are equal, hence the pentagon is equiangular

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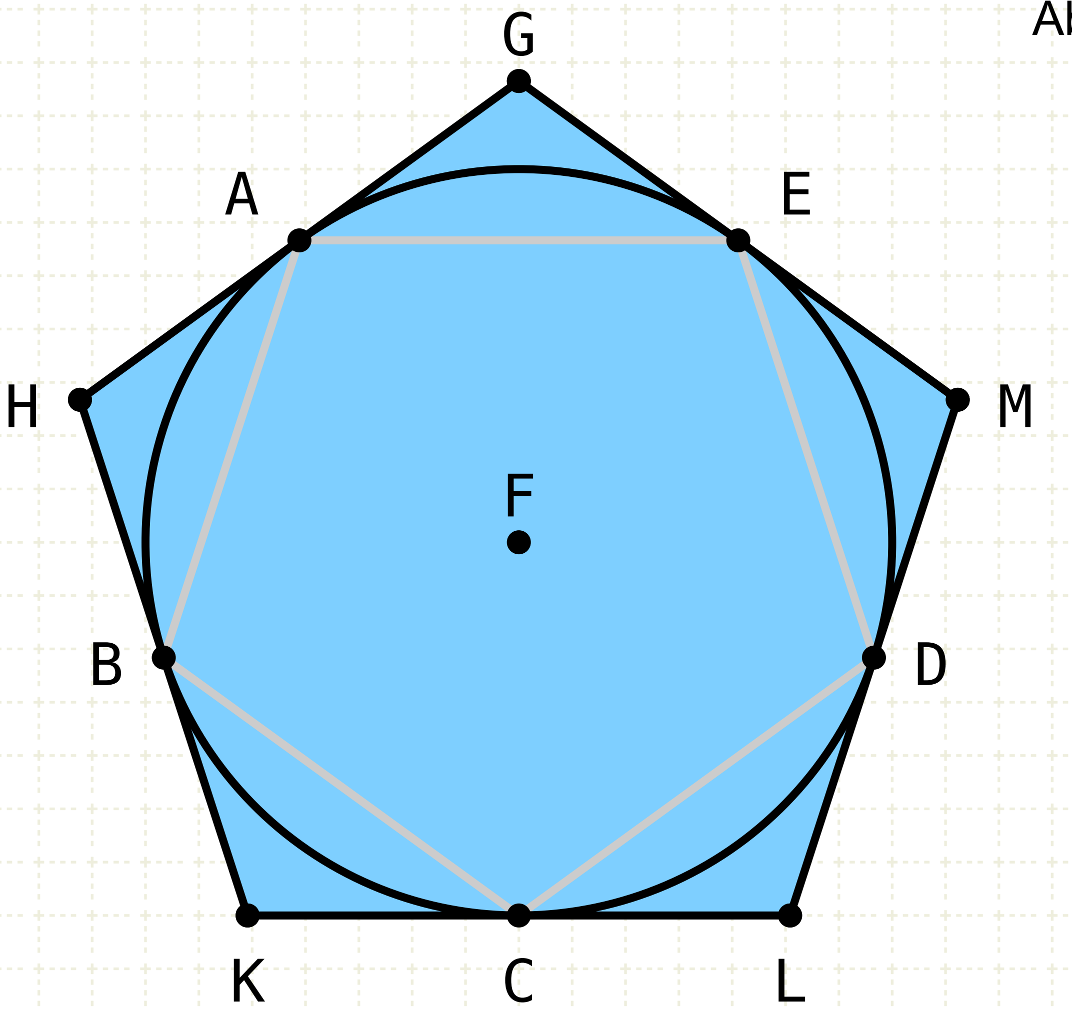
Proposition 12 of Book IV

About a given circle to circumscribe an equilateral and equiangular pentagon.



$$\circlearrowleft AB = \circlearrowleft BC = \circlearrowleft CD = \circlearrowleft EA$$

$$HK = KL = LM = MG = GH$$

$$\angle HKL = 2\delta$$

$$\angle KLM = 2\theta$$

$$\delta = \theta$$

$$\angle HKL = \angle FLC$$

$$\angle GHK = \angle HKL = \angle KLM$$
$$= \angle LMG = \angle MGH$$

The pentagon has been proven to be equilateral

It is also equiangular

Proof (cont.)

Triangles BFK and FKC have been proven to be equivalent, therefore the angles FKB and FKC are equal

Triangles FCL and FDL have been proven to be equal, therefore the angles FLC and FLD are equal

Triangles FKC and FCL have been proven to be equal, therefore the angles FKC and FLC are equal

Therefore, angles HKL and FLC are equal

Using the same logic, it can be shown that all the angle are equal, hence the pentagon is equiangular

Thus GHKLM is a regular pentagon

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