

# Euclid's Elements

## Book V



*Proportions are what makes the old Greek temples classic in their beauty. They are like huge blocks, from which the air has been literally hewn out between the columns.*

$$AB:C = DE:F$$

$$BG:C = EH:F$$

$$AG:C = DH:F$$

**Arne Jacobsen**



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## Proposition 14 of Book V

If a first magnitude have to a second the same ratio as a third has to a fourth, and the first be greater than the third, the second will also be greater than the fourth; if equal, equal; and if less, less.



# Proposition 14 of Book V

If a first magnitude have to a second the same ratio as a third has to a fourth, and the first be greater than the third, the second will also be greater than the fourth; if equal, equal; and if less, less.



$A : B = C : D$

$A \geq C$

$\rightarrow B \geq D$

## In other words

If A is to B as C is to D, and A is greater than C ...

... then B is also greater than D



# Proposition 14 of Book V

If a first magnitude have to a second the same ratio as a third has to a fourth, and the first be greater than the third, the second will also be greater than the fourth; if equal, equal; and if less, less.



$A : B = C : D$

$A > C$

## In other words

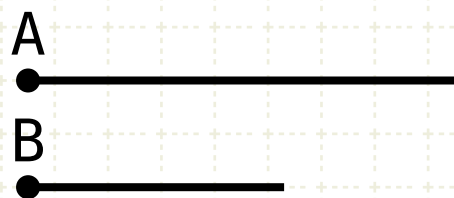
If A is to B as C is to D, and A is greater than C ...  
... then B is also greater than D

## Proof



# Proposition 14 of Book V

If a first magnitude have to a second the same ratio as a third has to a fourth, and the first be greater than the third, the second will also be greater than the fourth; if equal, equal; and if less, less.



$$A : B = C : D$$

$$A > C$$

$$A : B > C : D$$

## In other words

If A is to B as C is to D, and A is greater than C ...

... then B is also greater than D

## Proof

Since A is greater than C, and B is another arbitrary magnitude, then the ratio of A to B is larger than the ratio C to B (V·8)





# Proposition 14 of Book V

If a first magnitude have to a second the same ratio as a third has to a fourth, and the first be greater than the third, the second will also be greater than the fourth; if equal, equal; and if less, less.



$A : B = C : D$

$A > C$

$A : B > C : B$

$C : D > C : B$

## In other words

If A is to B as C is to D, and A is greater than C ...  
... then B is also greater than D

## Proof

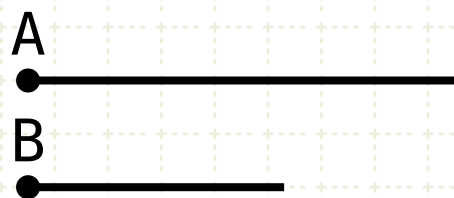
Since A is greater than C, and B is another arbitrary magnitude, then the ratio of A to B is larger than the ratio C to B (V·8)

But A is to B as C is to D, so the ratio of C to D is also greater than C is to B (V·13)



# Proposition 14 of Book V

If a first magnitude have to a second the same ratio as a third has to a fourth, and the first be greater than the third, the second will also be greater than the fourth; if equal, equal; and if less, less.



$$A : B = C : D$$

$$A > C$$

$$A : B > C : B$$

$$C : D > C : B$$

$$D < B$$

## In other words

If A is to B as C is to D, and A is greater than C ...

... then B is also greater than D

## Proof

Since A is greater than C, and B is another arbitrary magnitude, then the ratio of A to B is larger than the ratio C to B (V·8)

But A is to B as C is to D, so the ratio of C to D is also greater than C is to B (V·13)

If the C to D is greater than the ratio C to B, then D is less than B (V·10)





# Proposition 14 of Book V

If a first magnitude have to a second the same ratio as a third has to a fourth, and the first be greater than the third, the second will also be greater than the fourth; if equal, equal; and if less, less.



$A : B = C : D$

$A > C$

$A : B > C : B$

$C : D > C : B$

$D < B$

$B > D$

## In other words

If A is to B as C is to D, and A is greater than C ...  
... then B is also greater than D

## Proof

Since A is greater than C, and B is another arbitrary magnitude, then the ratio of A to B is larger than the ratio C to B (V·8)

But A is to B as C is to D, so the ratio of C to D is also greater than C is to B (V·13)

If the C to D is greater than the ratio C to B, then D is less than B (V·10)

Thus, if A is greater than C, B is greater than D



# Proposition 14 of Book V

If a first magnitude have to a second the same ratio as a third has to a fourth, and the first be greater than the third, the second will also be greater than the fourth; if equal, equal; and if less, less.



$A:B = C:D$

$A > C$

$A:B > C:B$

$C:D > C:B$

$D < B$

$B > D$

## In other words

If A is to B as C is to D, and A is greater than C ...  
... then B is also greater than D

## Proof

Since A is greater than C, and B is another arbitrary magnitude, then the ratio of A to B is larger than the ratio C to B (V·8)

But A is to B as C is to D, so the ratio of C to D is also greater than C is to B (V·13)

If the C to D is greater than the ratio C to B, then D is less than B (V·10)

Thus, if A is greater than C, B is greater than D



# Proposition 14 of Book V

If a first magnitude have to a second the same ratio as a third has to a fourth, and the first be greater than the third, the second will also be greater than the fourth; if equal, equal; and if less, less.



$A : B = C : D$

$A > C$

$A : B > C : B$

$C : D > C : B$

$D < B$

$B > D$

$A = C \rightarrow B = D$

$A < C \rightarrow B < D$

## In other words

If A is to B as C is to D, and A is greater than C ...  
... then B is also greater than D

## Proof

Since A is greater than C, and B is another arbitrary magnitude, then the ratio of A to B is larger than the ratio C to B (V·8)

But A is to B as C is to D, so the ratio of C to D is also greater than C is to B (V·13)

If the C to D is greater than the ratio C to B, then D is less than B (V·10)

Thus, if A is greater than C, B is greater than D

Similarly, if A is equal to C, then B is equal to D, and if A is less than C, then B is less than D



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