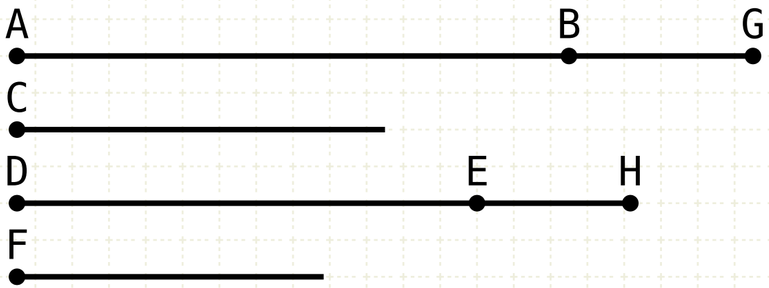
Euclid's Elements

Book V



AB:C = DE:F
BG:C = EH:F

AG:C = DH:F

Proportions are what makes the old Greek temples classic in their beauty. They are like huge blocks, from which the air has been literally hewn out between the columns.

Arne Jacobsen

Table of Contents, Chapter 5

Proposition 21 of Book V

If there be three magnitudes, and others equal to them in multitude, which taken two and two together are in the same ratio, and the proportion of them be perturbed, the, if EX AEQUALI the first magnitude is greater than the third, the fourth will also be greater than the sixth; if equal, equal; and if less, less

Proposition 21 of Book V

If there be three magnitudes, and others equal to them in multitude, which taken two and two together are in the same ratio, and the proportion of them be perturbed, the, if EX AEQUALI the first magnitude is greater than the third, the fourth will also be greater than the sixth; if equal, equal; and if less, less

```
ratio EX AEQUALI
  a:b = d:e
  b:c = e:f
→ a:c = d:f
ratio PERTURBED PROPORTION
  a:b = d:e
  b:c = f:d
→ a:c = f:e
```

Definitions

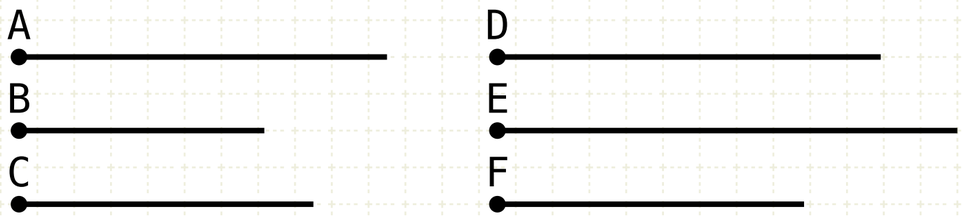
17. A ratio EX AEQUALI arises when, there being several magnitudes and another set equal to them in multitude which taken two and two are in the same proportion, as the first is to the last among the the first magnitudes, so is the first is to the last among the second magnitudes
18. A PERTURBED PROPORTION arises when, there being three magnitudes and another set equal to them in multitude, as antecedent is to consequent among the first magnitudes, so is antecedent to consequent among the second magnitudes, while, as the consequent is to a third among the first magnitudes, so is a third to the antecedent among the second magnitudes

Proposition 21 of Book V

If there be three magnitudes, and others equal to them in multitude, which taken two and two together are in the same ratio, and the proportion of them be perturbed, the, if EX AEQUALI the first magnitude is greater than the third, the fourth will also be greater than the sixth; if equal, equal; and if less, less

Proposition 21 of Book V

If there be three magnitudes, and others equal to them in multitude, which taken two and two together are in the same ratio, and the proportion of them be perturbed, the, if EX AEQUALI the first magnitude is greater than the third, the fourth will also be greater than the sixth; if equal, equal; and if less, less



In other words

Given two sets of numbers A,B,C and D,E,F where A is to B as E is to F, and where B is to C as D is to E (PERTURBED RATIO)

Then if A is greater than C, D will be greater than F, and if A is equal to C, D will be equal to F, etc

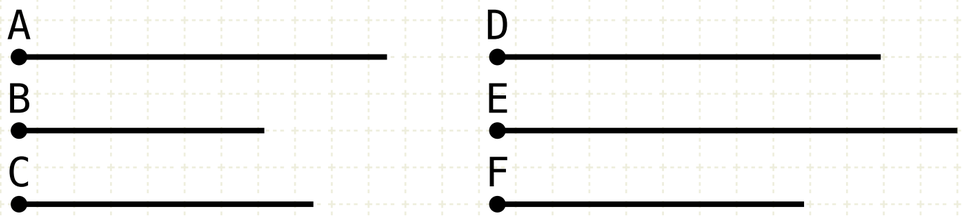
$$A:B = E:F$$

$$B:C = D:E$$

$$A >=< C \rightarrow D >=< F$$

Proposition 21 of Book V

If there be three magnitudes, and others equal to them in multitude, which taken two and two together are in the same ratio, and the proportion of them be perturbed, the, if EX AEQUALI the first magnitude is greater than the third, the fourth will also be greater than the sixth; if equal, equal; and if less, less



A:B = E:F

B:C = D:E

In other words

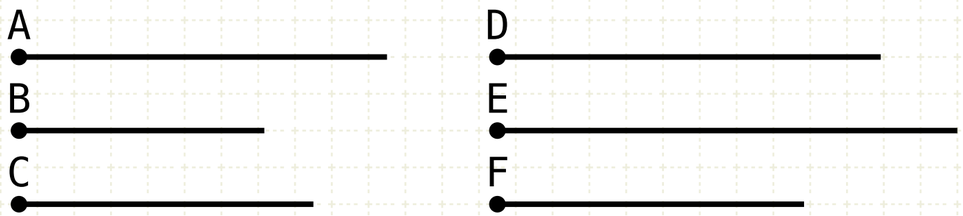
Given two sets of numbers A,B,C and D,E,F where A is to B as E is to F, and where B is to C as D is to E (PERTURBED RATIO)

Then if A is greater than C, D will be greater than F, and if A is equal to C, D will be equal to F, etc

Proof

Proposition 21 of Book V

If there be three magnitudes, and others equal to them in multitude, which taken two and two together are in the same ratio, and the proportion of them be perturbed, the, if EX AEQUALI the first magnitude is greater than the third, the fourth will also be greater than the sixth; if equal, equal; and if less, less



A:B = E:F

B:C = D:E

A > C

In other words

Given two sets of numbers A,B,C and D,E,F where A is to B as E is to F, and where B is to C as D is to E (PERTURBED RATIO)

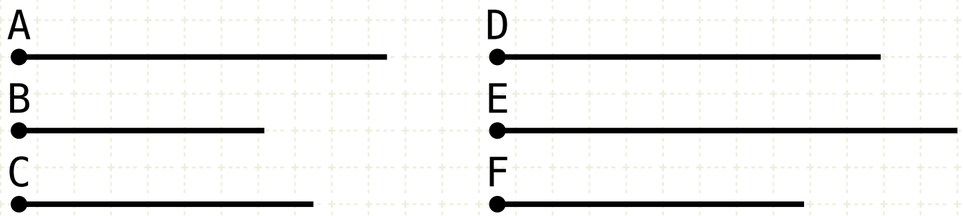
Then if A is greater than C, D will be greater than F, and if A is equal to C, D will be equal to F, etc

Proof

Let A be greater than C

Proposition 21 of Book V

If there be three magnitudes, and others equal to them in multitude, which taken two and two together are in the same ratio, and the proportion of them be perturbed, the, if EX AEQUALI the first magnitude is greater than the third, the fourth will also be greater than the sixth; if equal, equal; and if less, less



A:B = E:F

B:C = D:E

A > C

A:B > C:B

In other words

Given two sets of numbers A,B,C and D,E,F where A is to B as E is to F, and where B is to C as D is to E (PERTURBED RATIO)

Then if A is greater than C, D will be greater than F, and if A is equal to C, D will be equal to F, etc

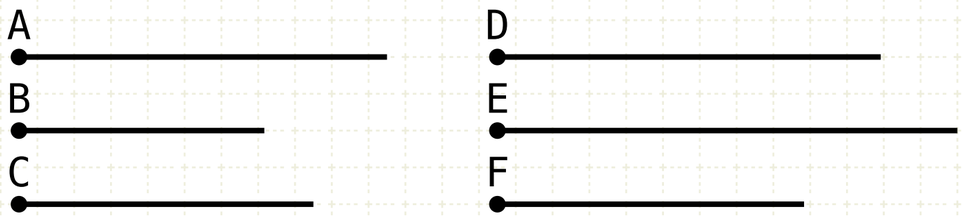
Proof

Let A be greater than C

If A is greater than C, when compared to another magnitude, in this case B, A will have a greater ratio to B than C will have to B (V·8)

Proposition 21 of Book V

If there be three magnitudes, and others equal to them in multitude, which taken two and two together are in the same ratio, and the proportion of them be perturbed, the, if EX AEQUALI the first magnitude is greater than the third, the fourth will also be greater than the sixth; if equal, equal; and if less, less



A:B = E:F

B:C = D:E

A > C

A:B > C:B

C:B = E:D

In other words

Given two sets of numbers A,B,C and D,E,F where A is to B as E is to F, and where B is to C as D is to E (PERTURBED RATIO)

Then if A is greater than C, D will be greater than F, and if A is equal to C, D will be equal to F, etc

Proof

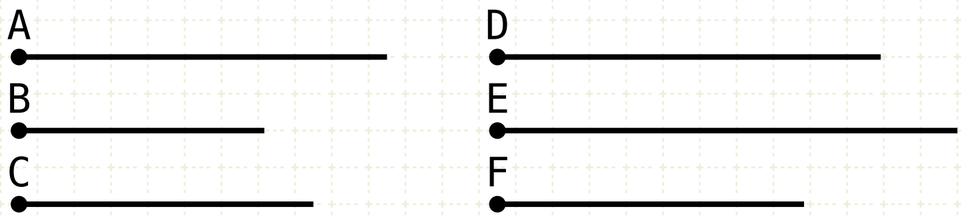
Let A be greater than C

If A is greater than C, when compared to another magnitude, in this case B, A will have a greater ratio to B than C will have to B (V·8)

Since the ratio B to C equals the ratio D to E, then inverse ratios (C to B and E to D) are also equal (V·13)

Proposition 21 of Book V

If there be three magnitudes, and others equal to them in multitude, which taken two and two together are in the same ratio, and the proportion of them be perturbed, the, if EX AEQUALI the first magnitude is greater than the third, the fourth will also be greater than the sixth; if equal, equal; and if less, less



A:B = E:F

B:C = D:E

A > C

A:B > C:B

C:B = E:D

A:B > E:D

In other words

Given two sets of numbers A,B,C and D,E,F where A is to B as E is to F, and where B is to C as D is to E (PERTURBED RATIO)

Then if A is greater than C, D will be greater than F, and if A is equal to C, D will be equal to F, etc

Proof

Let A be greater than C

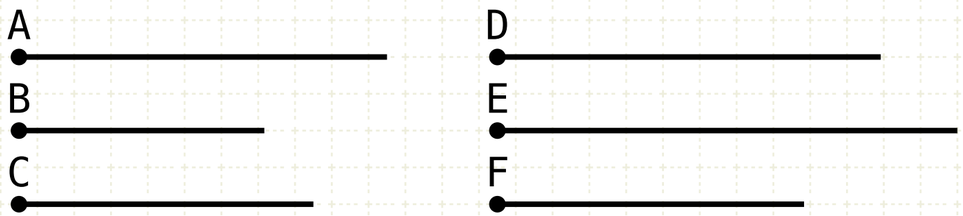
If A is greater than C, when compared to another magnitude, in this case B, A will have a greater ratio to B than C will have to B (V·8)

Since the ratio B to C equals the ratio D to E, then inverse ratios (C to B and E to D) are also equal (V·13)

Since A to B is greater than C to B, and C to B equals E to D, then A to B is greater than E to D

Proposition 21 of Book V

If there be three magnitudes, and others equal to them in multitude, which taken two and two together are in the same ratio, and the proportion of them be perturbed, the, if EX AEQUALI the first magnitude is greater than the third, the fourth will also be greater than the sixth; if equal, equal; and if less, less



A:B = E:F

B:C = D:E

A > C

A:B > C:B

C:B = E:D

A:B > E:D

E:F > E:D

In other words

Given two sets of numbers A,B,C and D,E,F where A is to B as E is to F, and where B is to C as D is to E (PERTURBED RATIO)

Then if A is greater than C, D will be greater than F, and if A is equal to C, D will be equal to F, etc

Proof

Let A be greater than C

If A is greater than C, when compared to another magnitude, in this case B, A will have a greater ratio to B than C will have to B (V·8)

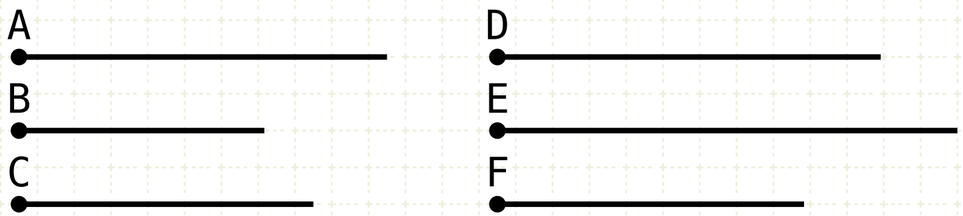
Since the ratio B to C equals the ratio D to E, then inverse ratios (C to B and E to D) are also equal (V·13)

Since A to B is greater than C to B, and C to B equals E to D, then A to B is greater than E to D

And since A to B equals E to F, then E to F is also greater than E to D

Proposition 21 of Book V

If there be three magnitudes, and others equal to them in multitude, which taken two and two together are in the same ratio, and the proportion of them be perturbed, the, if EX AEQUALI the first magnitude is greater than the third, the fourth will also be greater than the sixth; if equal, equal; and if less, less



A:B = E:F

B:C = D:E

A > C

A:B > C:B

C:B = E:D

A:B > E:D

E:F > E:D

D > F

In other words

Given two sets of numbers A,B,C and D,E,F where A is to B as E is to F, and where B is to C as D is to E (PERTURBED RATIO)

Then if A is greater than C, D will be greater than F, and if A is equal to C, D will be equal to F, etc

Proof

Let A be greater than C

If A is greater than C, when compared to another magnitude, in this case B, A will have a greater ratio to B than C will have to B (V·8)

Since the ratio B to C equals the ratio D to E, then inverse ratios (C to B and E to D) are also equal (V·13)

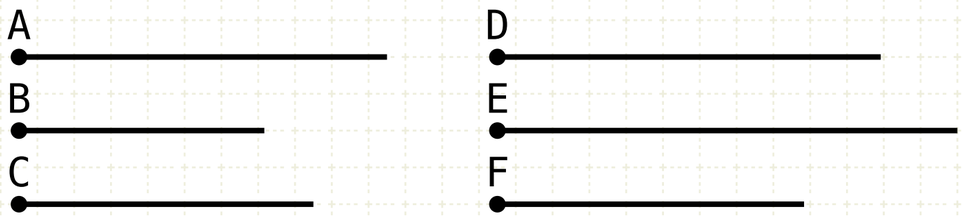
Since A to B is greater than C to B, and C to B equals E to D, then A to B is greater than E to D

And since A to B equals E to F, then E to F is also greater than E to D

If E to F is greater than E to D, then D is greater than F (V·10)

Proposition 21 of Book V

If there be three magnitudes, and others equal to them in multitude, which taken two and two together are in the same ratio, and the proportion of them be perturbed, the, if EX AEQUALI the first magnitude is greater than the third, the fourth will also be greater than the sixth; if equal, equal; and if less, less



A:B = E:F

B:C = D:E

A > C

A:B > C:B

C:B = E:D

A:B > E:D

E:F > E:D

D > F

In other words

Given two sets of numbers A,B,C and D,E,F where A is to B as E is to F, and where B is to C as D is to E (PERTURBED RATIO)

Then if A is greater than C, D will be greater than F, and if A is equal to C, D will be equal to F, etc

Proof

Let A be greater than C

If A is greater than C, when compared to another magnitude, in this case B, A will have a greater ratio to B than C will have to B (V·8)

Since the ratio B to C equals the ratio D to E, then inverse ratios (C to B and E to D) are also equal (V·13)

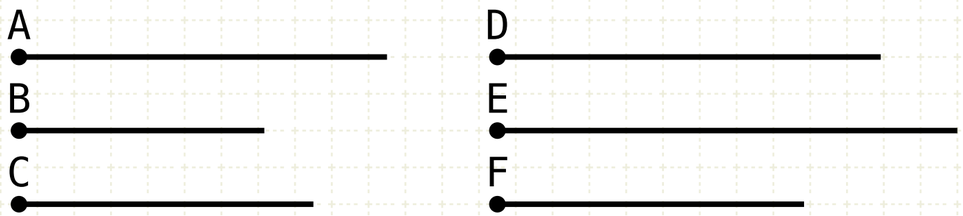
Since A to B is greater than C to B, and C to B equals E to D, then A to B is greater than E to D

And since A to B equals E to F, then E to F is also greater than E to D

If E to F is greater than E to D, then D is greater than F (V·10)

Proposition 21 of Book V

If there be three magnitudes, and others equal to them in multitude, which taken two and two together are in the same ratio, and the proportion of them be perturbed, the, if EX AEQUALI the first magnitude is greater than the third, the fourth will also be greater than the sixth; if equal, equal; and if less, less



A:B = E:F

B:C = D:E

A > C

A:B > C:B

C:B = E:D

A:B > E:D

E:F > E:D

D > F

A >=< C → D >=< F

In other words

Given two sets of numbers A,B,C and D,E,F where A is to B as E is to F, and where B is to C as D is to E (PERTURBED RATIO)

Then if A is greater than C, D will be greater than F, and if A is equal to C, D will be equal to F, etc

Proof

Let A be greater than C

If A is greater than C, when compared to another magnitude, in this case B, A will have a greater ratio to B than C will have to B (V·8)

Since the ratio B to C equals the ratio D to E, then inverse ratios (C to B and E to D) are also equal (V·13)

Since A to B is greater than C to B, and C to B equals E to D, then A to B is greater than E to D

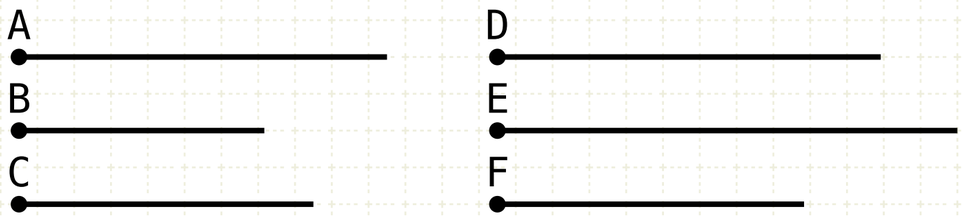
And since A to B equals E to F, then E to F is also greater than E to D

If E to F is greater than E to D, then D is greater than F (V·10)

Similarly, we can prove that if A is equal to C, then D is equal to F, and if less, less

Proposition 21 of Book V

If there be three magnitudes, and others equal to them in multitude, which taken two and two together are in the same ratio, and the proportion of them be perturbed, the, if EX AEQUALI the first magnitude is greater than the third, the fourth will also be greater than the sixth; if equal, equal; and if less, less



perturbed proportional

A:B = E:F

B:C = D:E

A > C

A:B > C:B

C:B = E:D

A:B > E:D

E:F > E:D

D > F

A >=< C → D >=< F

proportional

A:B = D:E

B:C = E:F

A > C

A:B > C:B

C:B = F:E

A:B > F:E

D:E > F:E

D > F

A >=< C → D >=< F

In other words

Given two sets of numbers A,B,C and D,E,F where A is to B as E is to F, and where B is to C as D is to E (PERTURBED RATIO)

Then if A is greater than C, D will be greater than F, and if A is equal to C, D will be equal to F, etc

Proof

Let A be greater than C

If A is greater than C, when compared to another magnitude, in this case B, A will have a greater ratio to B than C will have to B (V·8)

Since the ratio B to C equals the ratio D to E, then inverse ratios (C to B and E to D) are also equal (V·13)

Since A to B is greater than C to B, and C to B equals E to D, then A to B is greater than E to D

And since A to B equals E to F, then E to F is also greater than E to D

If E to F is greater than E to D, then D is greater than F (V·10)

Similarly, we can prove that if A is equal to C, then D is equal to F, and if less, less

Youtube Videos

https://www.youtube.com/c/SandyBultena