

Euclid's Elements

Book VI

One can state, without exaggeration, that the observation of and the search for similarities and differences are the basis of all human knowledge.

Alfred Nobel



Table of Contents, Chapter 6

1	If the height of two triangles are equal, then the ratio of the areas is equal to the ratio of the bases	7	If two triangles have one angle equal to one angle, and the sides about other angles are proportional, and the remaining angles either both less or both not less than a right angle, then triangles will be equiangular	14	In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and vice versa
2	If a line cuts a triangle, parallel to its base, it will cut the sides of the triangle proportionally			15	In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and vice versa
3	If an angle of a triangle is bisected and the straight line cutting the angle also cuts the base, the segments of the base will have the same ratio as the remaining sides of the triangle	8	If in a right-angled triangle a perpendicular be drawn from the right angle to the base, the triangles adjoining the perpendicular are similar both to the whole and to one another	16	If four straight lines are proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means, and vice versa
4	If two triangles have equal angles, then the sides opposite the equal angles are proportional, as well, the sides of the triangles on either side of the equal angles are also proportional	9	From a given straight line to cut off a given fraction	17	If three straight lines are proportional, the rectangle contained by the extremes is equal to the square on the mean; and vice versa
5	If two triangles have proportional sides, the triangles will be equiangular	10	To cut a given uncut straight line similarly to a given cut straight line	18	On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure
6	If two triangles have one angle equal to one angle and the sides about the equal angles are proportional, then the triangles will be equiangular	11	To two given straight lines to find a third proportional	19	Similar triangles are to one another in the duplicate ratio of the corresponding sides
		12	To three given straight lines to find a fourth proportional		
		13	To two given straight lines to find a mean proportional		



Table of Contents, Chapter 3

20	Similar polygons are divided into the same number of similar triangles, which have the same ratio as the wholes, and the polygons have duplicate ratios to their corresponding sides	26	If from a parallelogram a similar parallelogram with a common angle is subtracted, it is about the same diameter as the original	31	In right-angled triangles the figure on the side subtending the right angle is equal to the similar and similarly described figures on the sides containing the right angle
21	Figures which are are similar to the same rectilineal figure are also similar to one another	27	Of all the parallelograms applied to the same straight line and deficient by parallelogrammic figures similar to a parallelogram drawn on half the said line, the largest will be one that is drawn on half of the straight line and is similar to the defect		
22	If four straight lines are proportional, similar rectilineal figures will also be proportional; and vice versa	28	To a given straight line, apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one		
23	Equiangular parallelograms have to one another the ratio compounded of the ratios of their sides	29	To a given straight line, apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one		
24	In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another	30	To cut a finite straight line in extreme ratio		
25	To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure				



Proposition 4 of Book VI

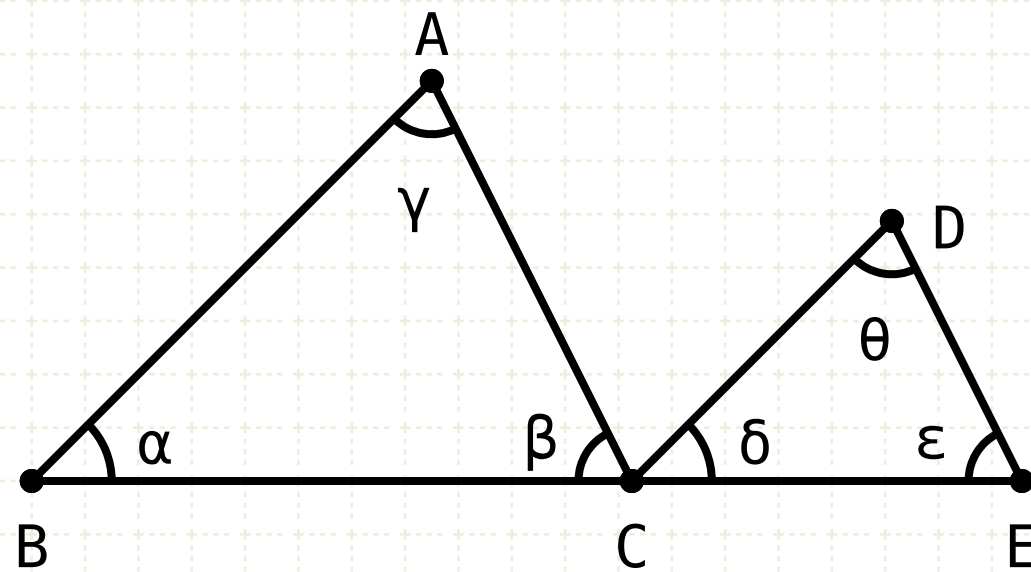
In equiangular triangles the sides about the equal angles are proportional, and those are corresponding sides which subtend the equal angles.



Proposition 4 of Book VI

In equiangular triangles the sides about the equal angles are proportional, and those are corresponding sides which subtend the equal angles.

$$\alpha = \delta, \beta = \varepsilon, \gamma = \theta$$



In other words

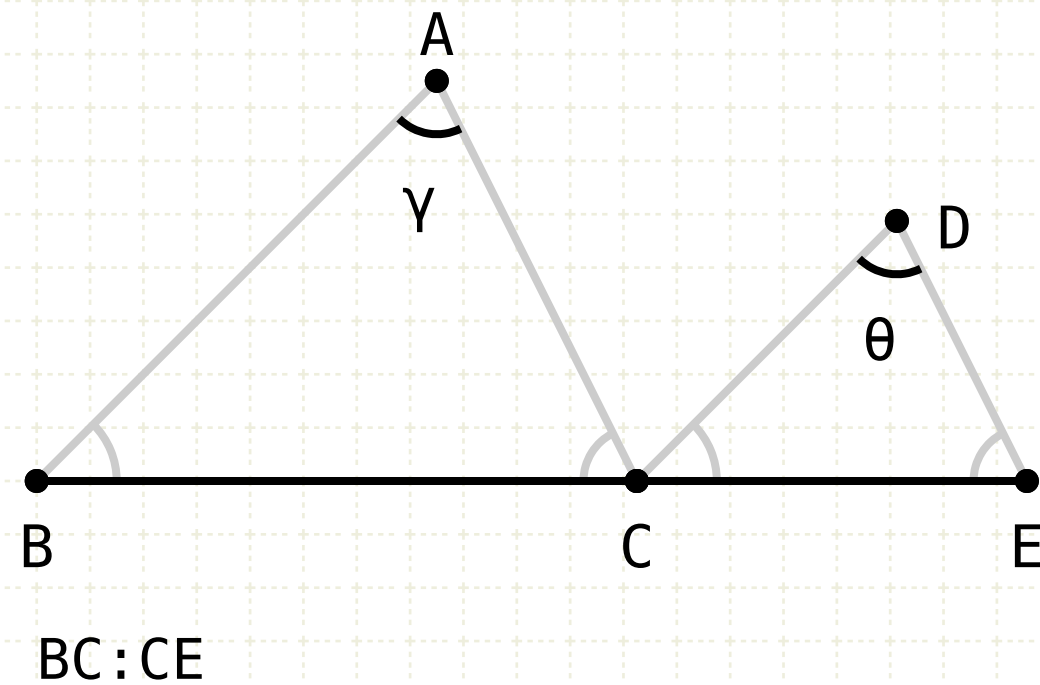
Given two equiangular triangles ABC and CDE, where the angle α equals angle δ , γ equals θ , and finally β equals ε , then...

The ratio of the sides (BC, CE) subtending the angles γ and θ is proportional to the ratio of the sides (AC, DE) subtending the angles α and δ , etc.

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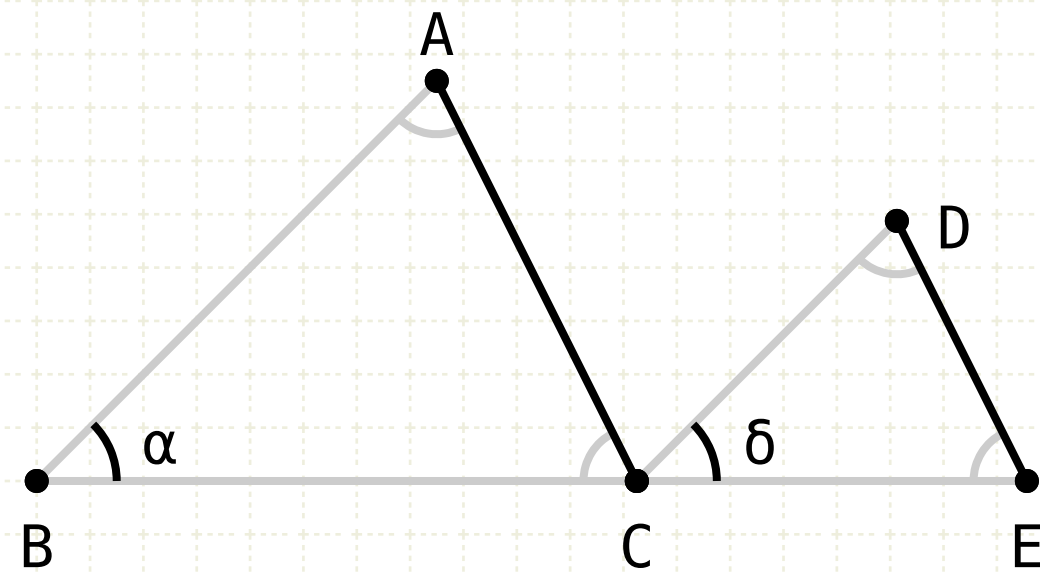
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$$BC : CE = AC : DE$$

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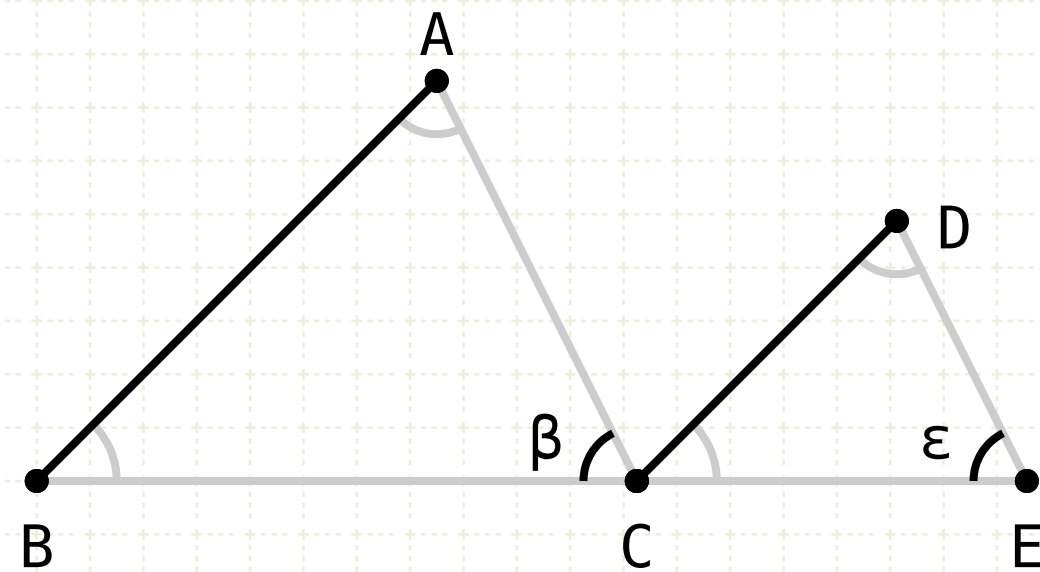
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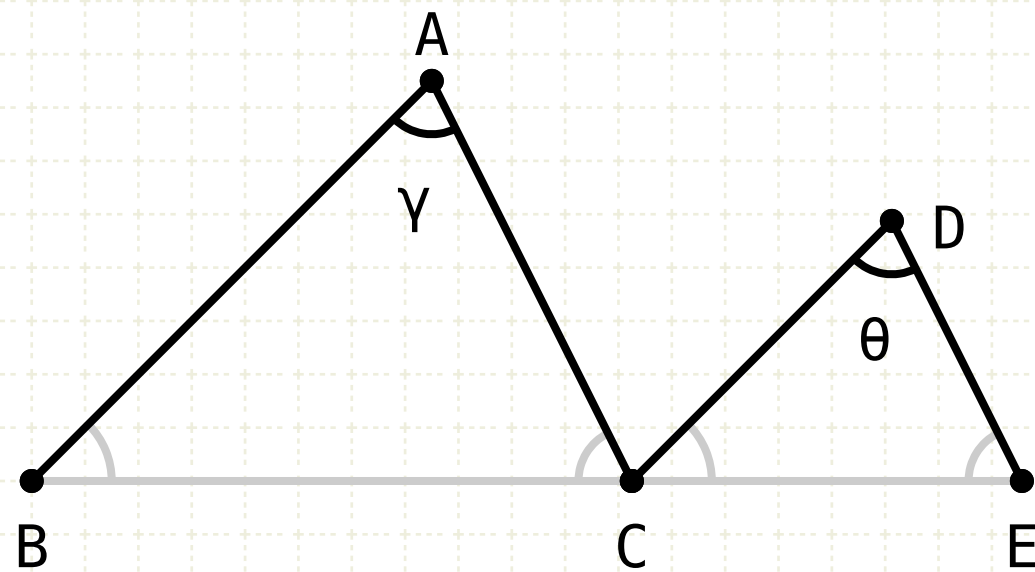
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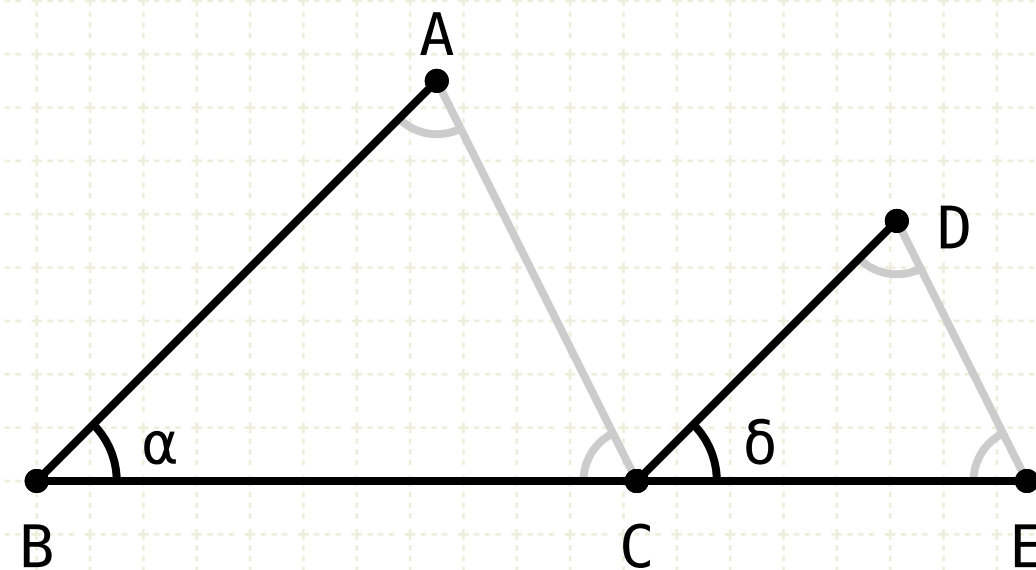
The ratio of the sides (BC, CE) subtending the angles γ and θ is proportional to the ratio of the sides (AC, DE) subtending the angles α and δ , etc.

Also, the ratio of the lines on either side of an angle in one triangle will be equal to the ratio of the lines in the other triangle which are on either side of the equal angle. AB to BC equals DE to CE, AC to BC equals DE to CE and AC to BC equals DE to CE

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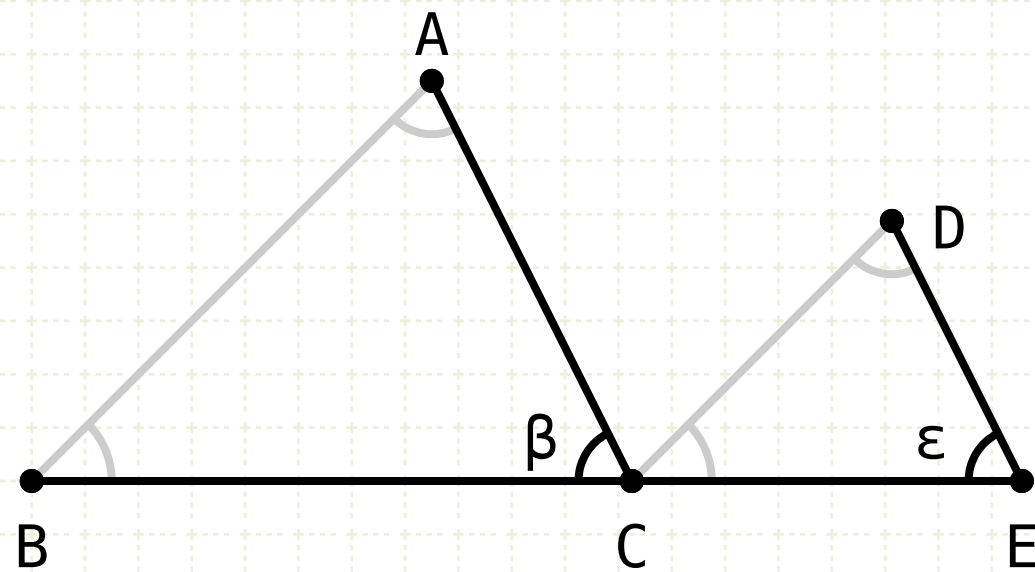
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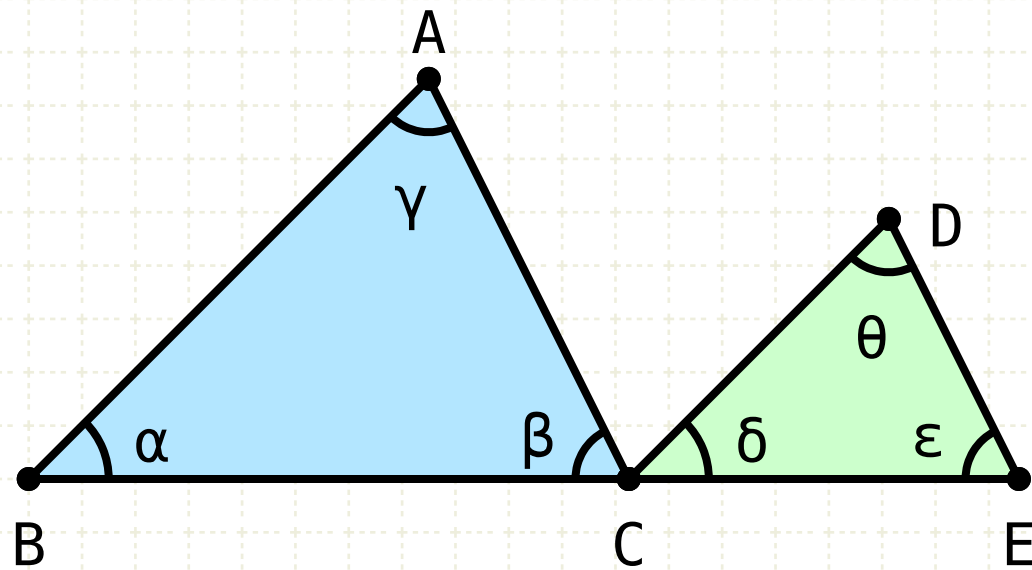
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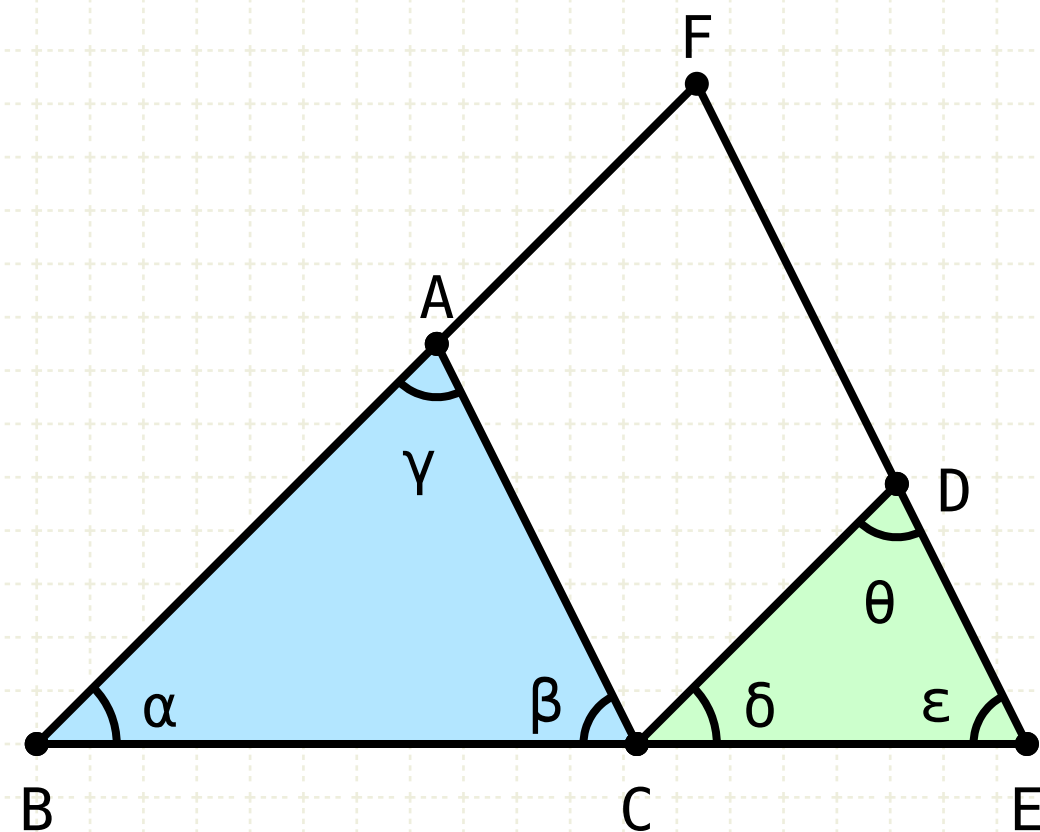
$$\alpha = \delta, \beta = \varepsilon, \gamma = \theta$$

Proof



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$$\alpha = \delta, \beta = \epsilon, \gamma = \theta$$

Proof

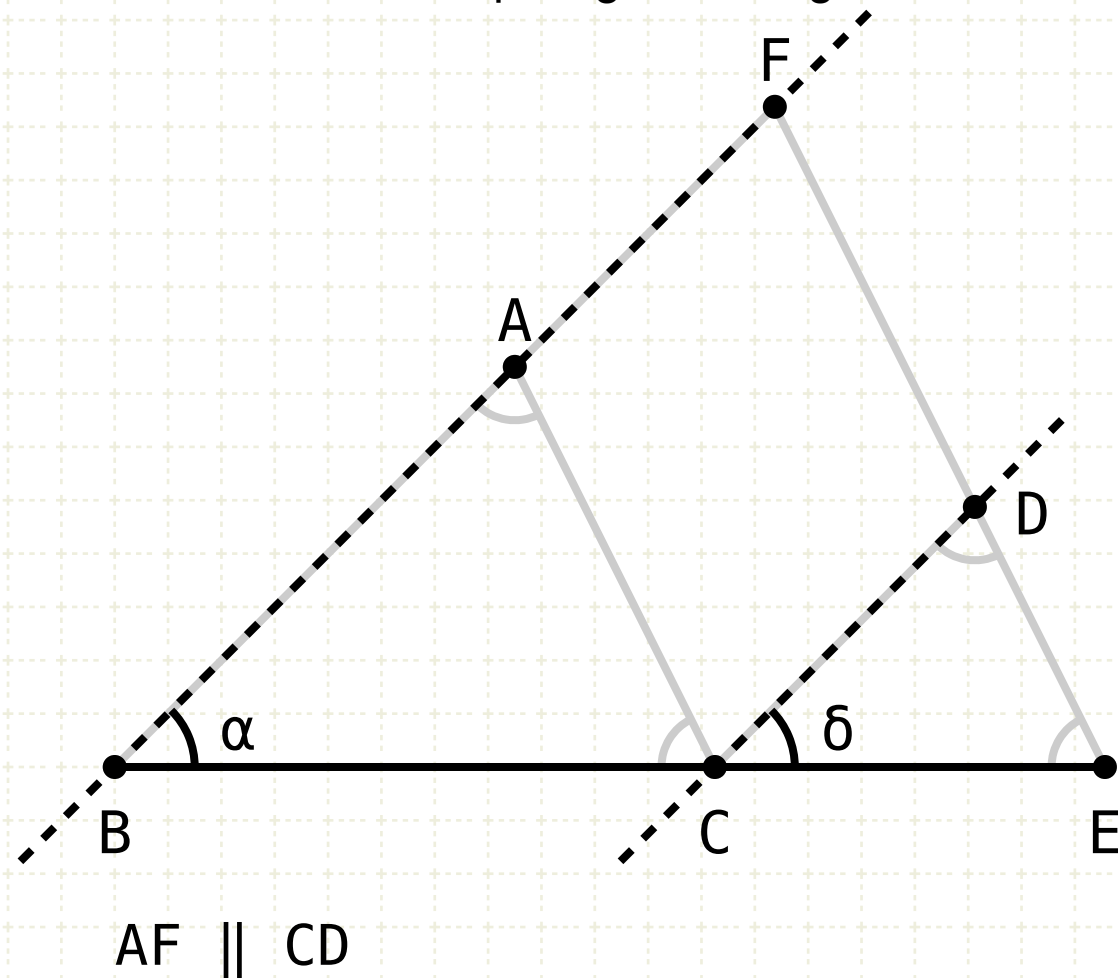
Since the sum of the angles α and β are less than two right angles (I .17), and the angles β and ϵ are equal, ...

... the sum of the angles α and ϵ are less than two right angles, thus the lines AB and DE will meet (at point F) if extended (I·Post·5)

$$\begin{aligned}\alpha + \beta &< 2 \cdot L \\ \beta &= \epsilon \\ \alpha + \epsilon &< 2 \cdot L\end{aligned}$$

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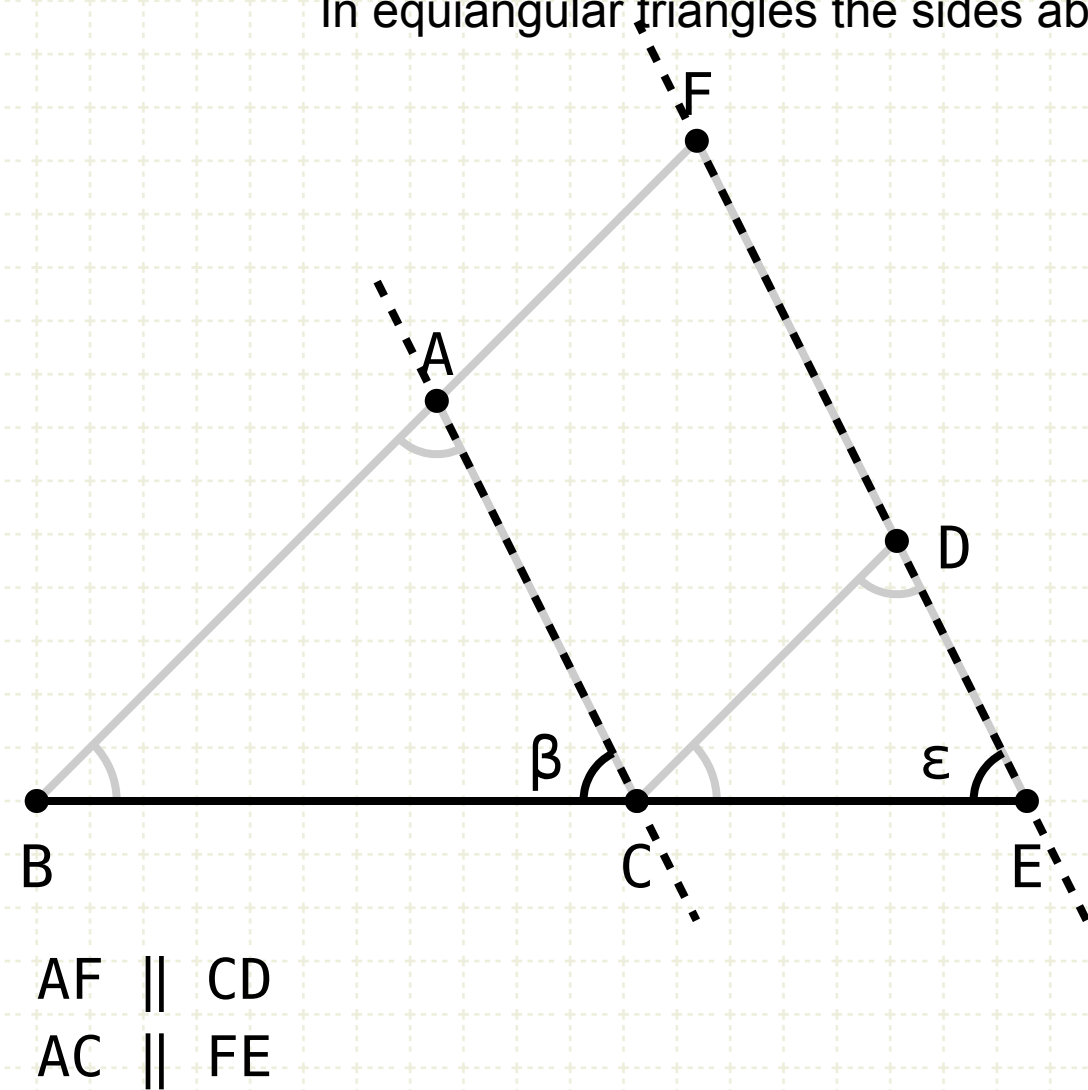
$$\alpha = \delta, \beta = \epsilon, \gamma = \theta$$

Proof

Since α equals δ , AF is parallel to CD (I-28)

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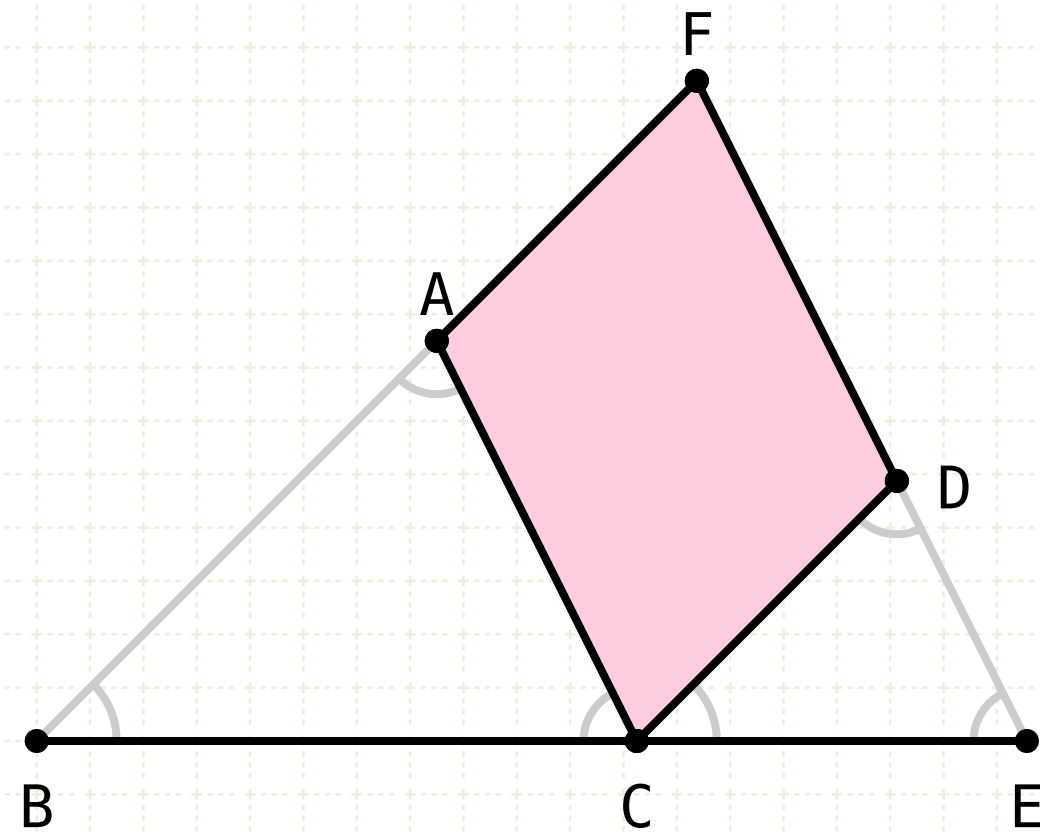
Proof

Since α equals δ , AF is parallel to CD (I·28)

Similarly, since β equals ϵ , AC is parallel to FE (I·28)

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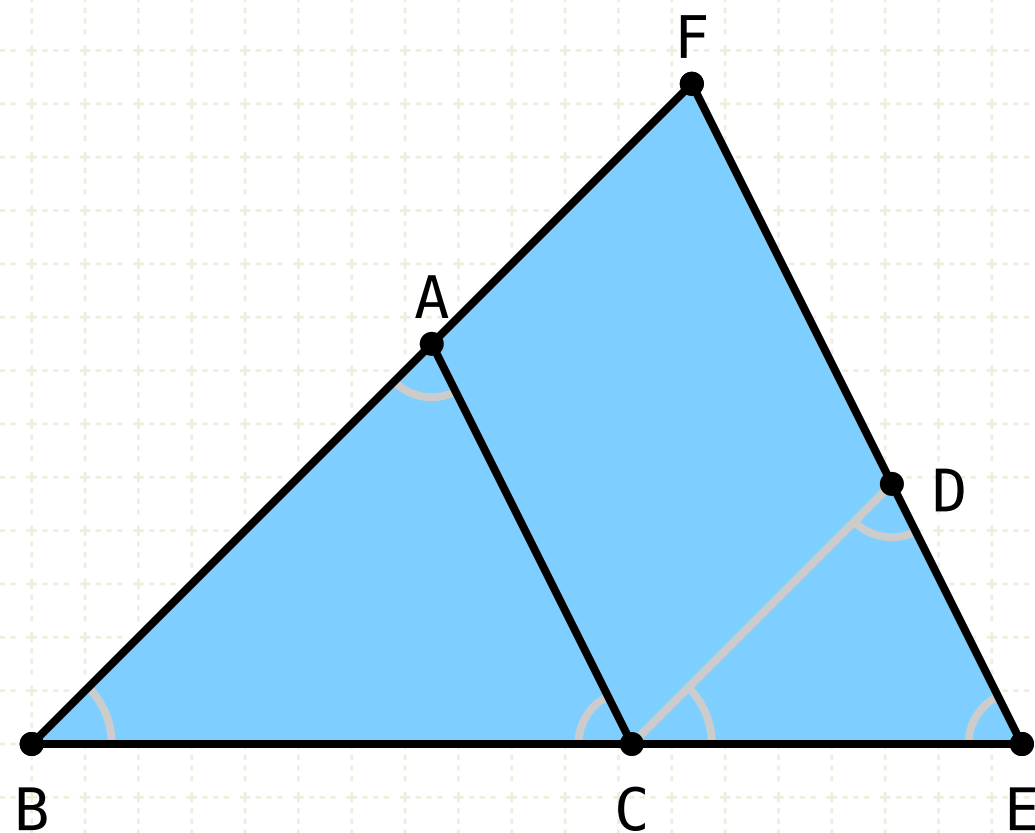
Similarly, since β equals ε , AC is parallel to FE (I·28)

Therefore FACD is a parallelogram, and AF, CD are equal as are AC, FD (I·34)

$$\begin{aligned} AF &\parallel CD \\ AC &\parallel FE \\ AF &= CD \\ AC &= FD \end{aligned}$$

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Since AC is parallel FE, the ratios AB to AF and BC to CE are equal (VI·2)

AF \parallel CD

AC \parallel FE

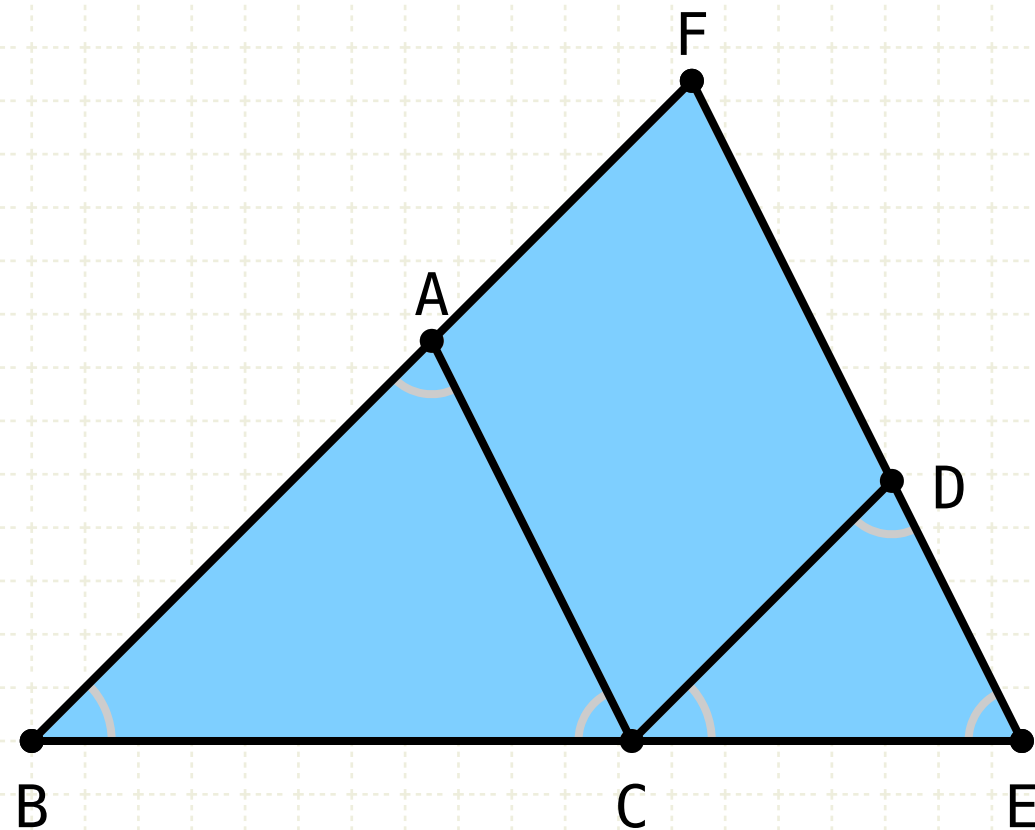
AF = CD

AC = FD

AB:AF = BC:CE

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Since AC is parallel FE, the ratios AB to AF and BC to CE are equal (VI·2)

But since AF equals CD, AB to CD is equal to BC to CE (V·16)

$$AF \parallel CD$$

$$AC \parallel FE$$

$$AF = CD$$

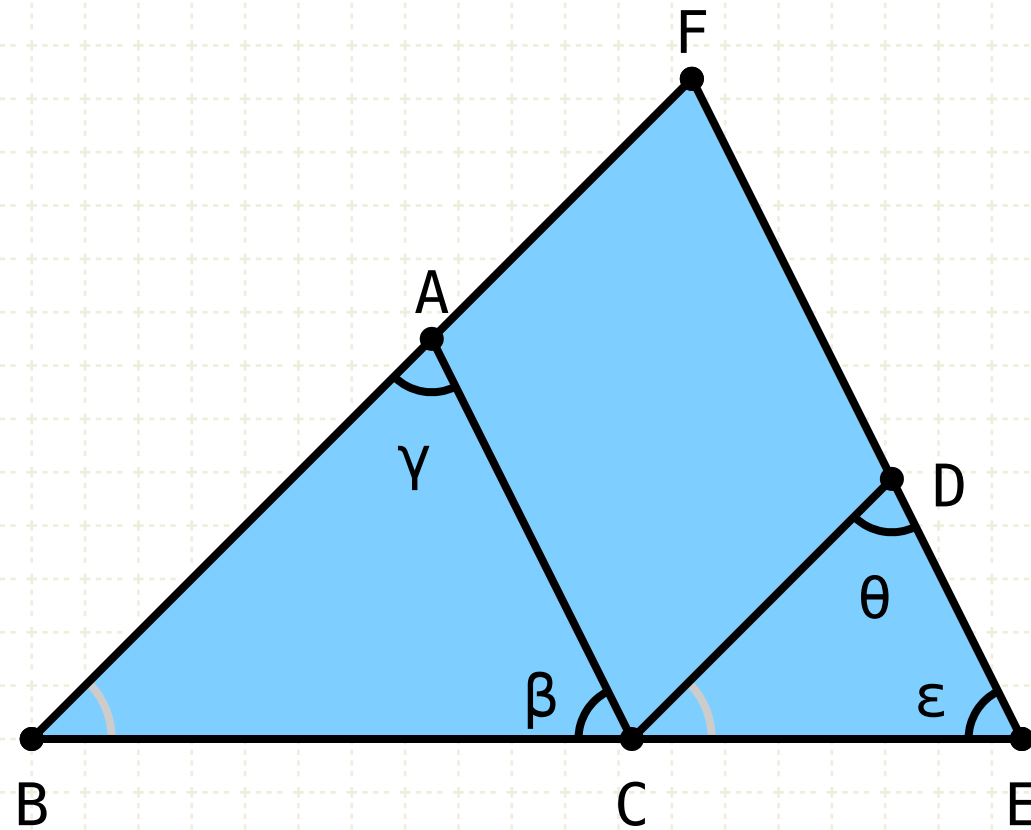
$$AC = FD$$

$$AB:AF = BC:CE$$

$$AB:CD = BC:CE$$

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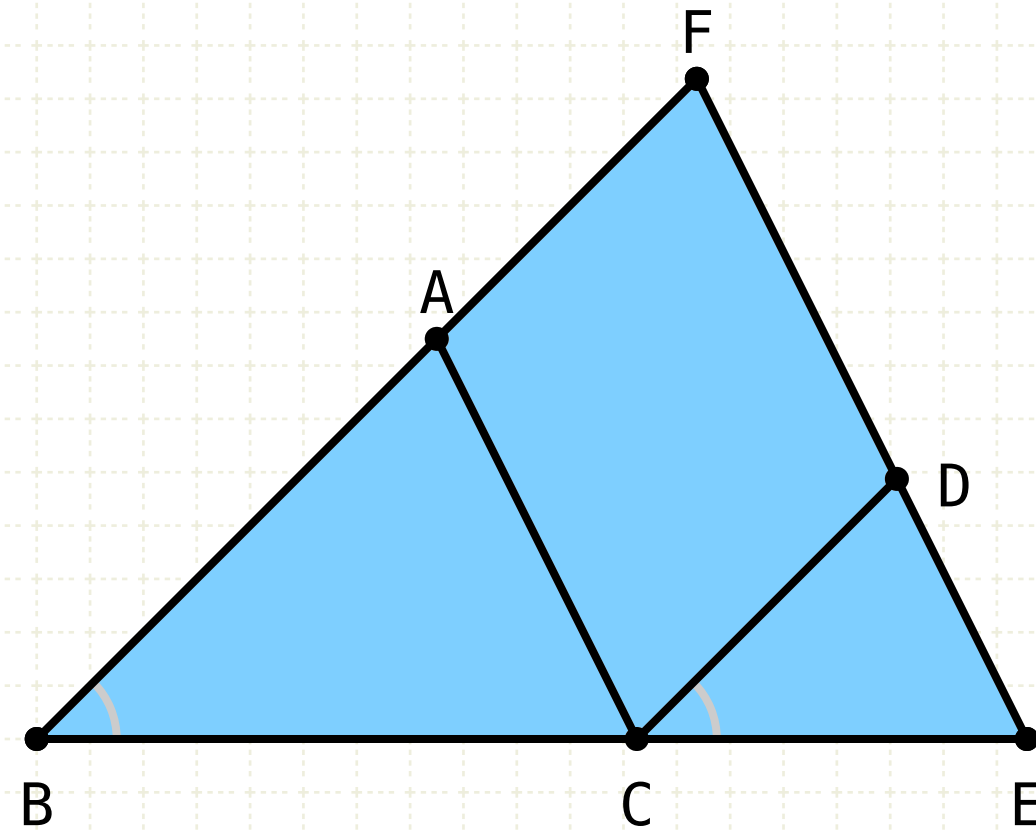
$$AB:AF = BC:CE$$

$$AB:CD = BC:CE$$



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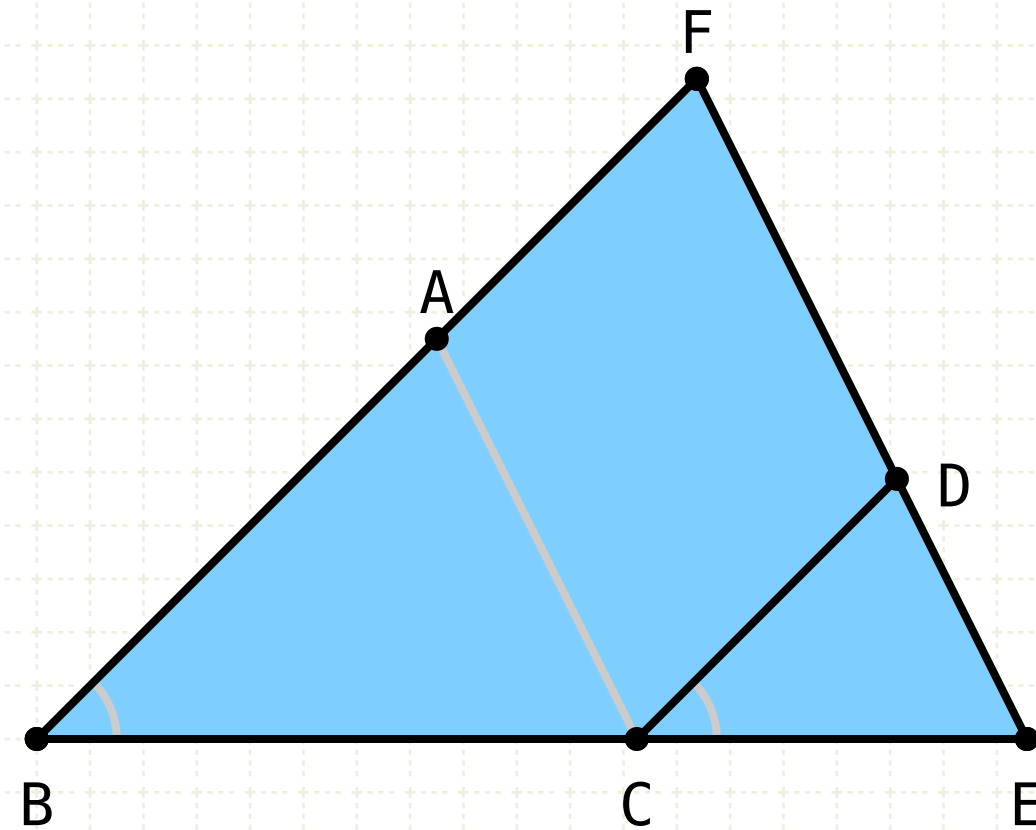
$$AB:AF = BC:CE$$

$$AB:CD = BC:CE$$

$$AB:BC = CD:CE$$

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But since AF equals CD, AB to CD is equal to BC to CE (V·16)

And alternately AB to BC is equal to CD to CE (V·16)

Since CD is parallel BF, therefore the ratios FD to DE and BC to CE are equal (VI·2)

$$AF \parallel CD$$

$$AC \parallel FE$$

$$AF = CD$$

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$$AB:AF = BC:CE$$

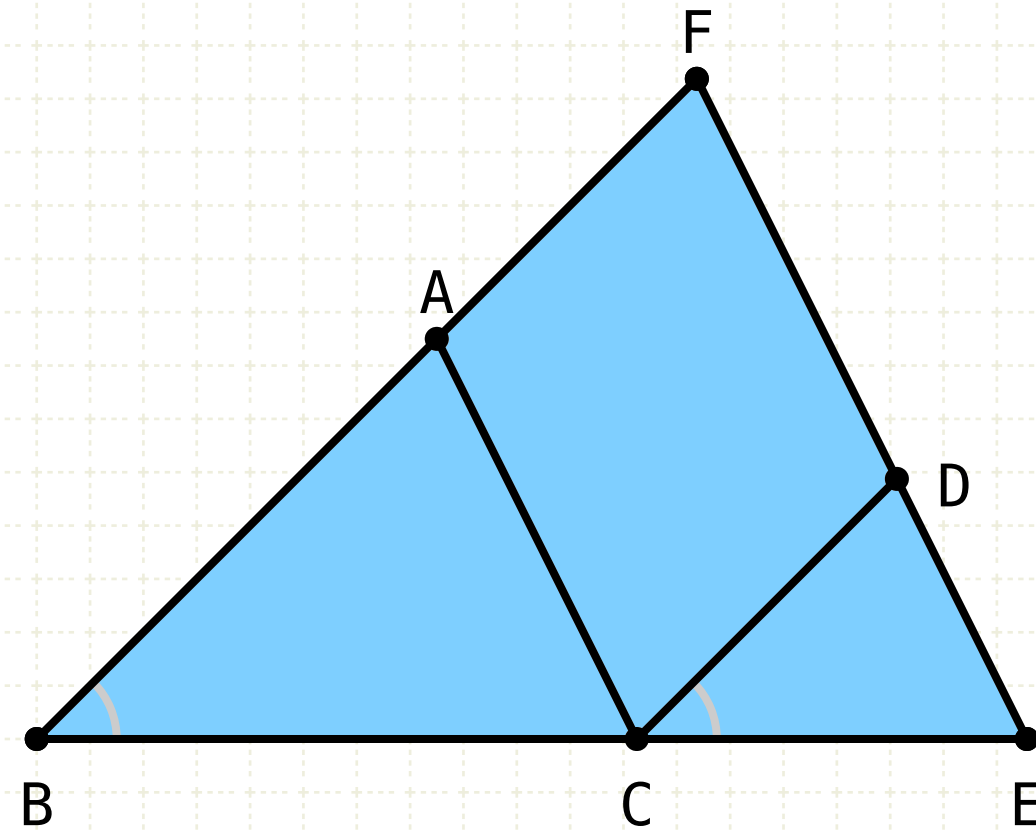
$$AB:CD = BC:CE$$

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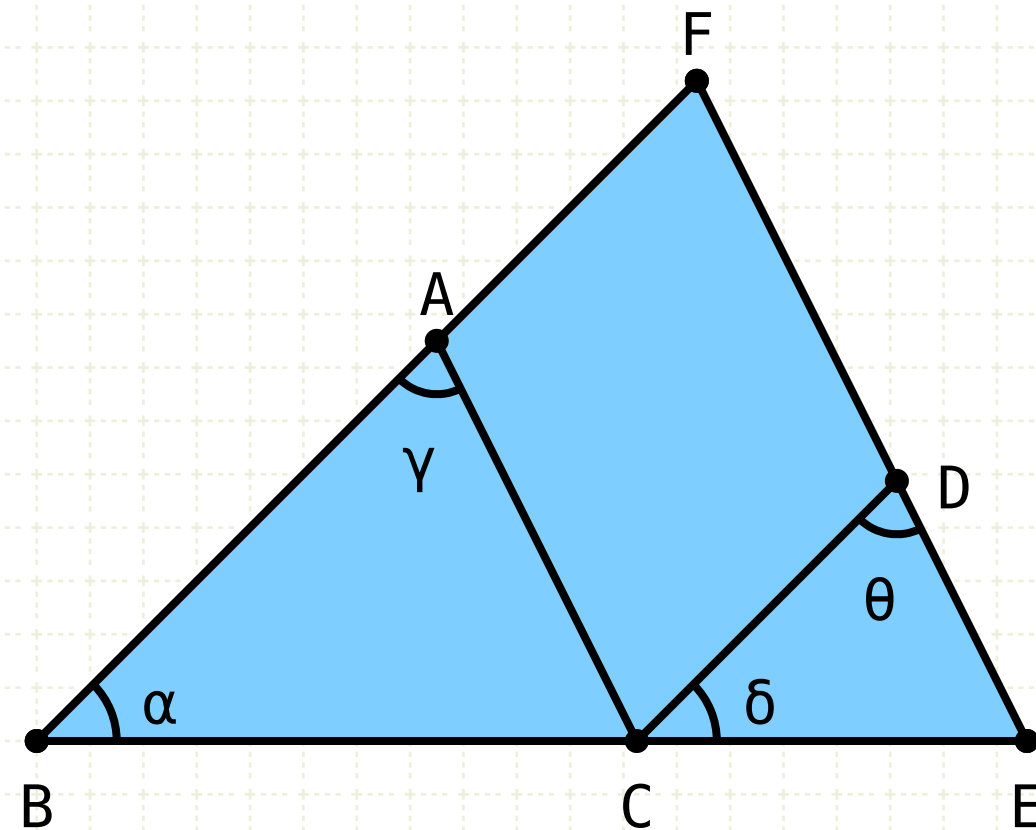
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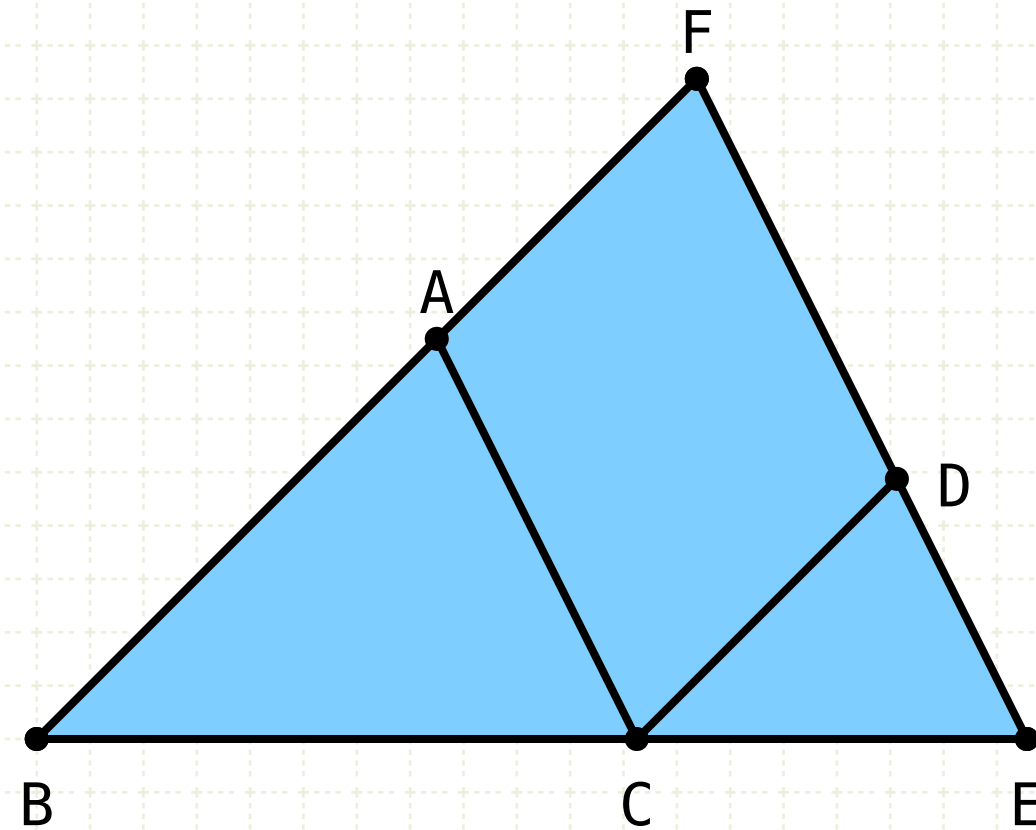
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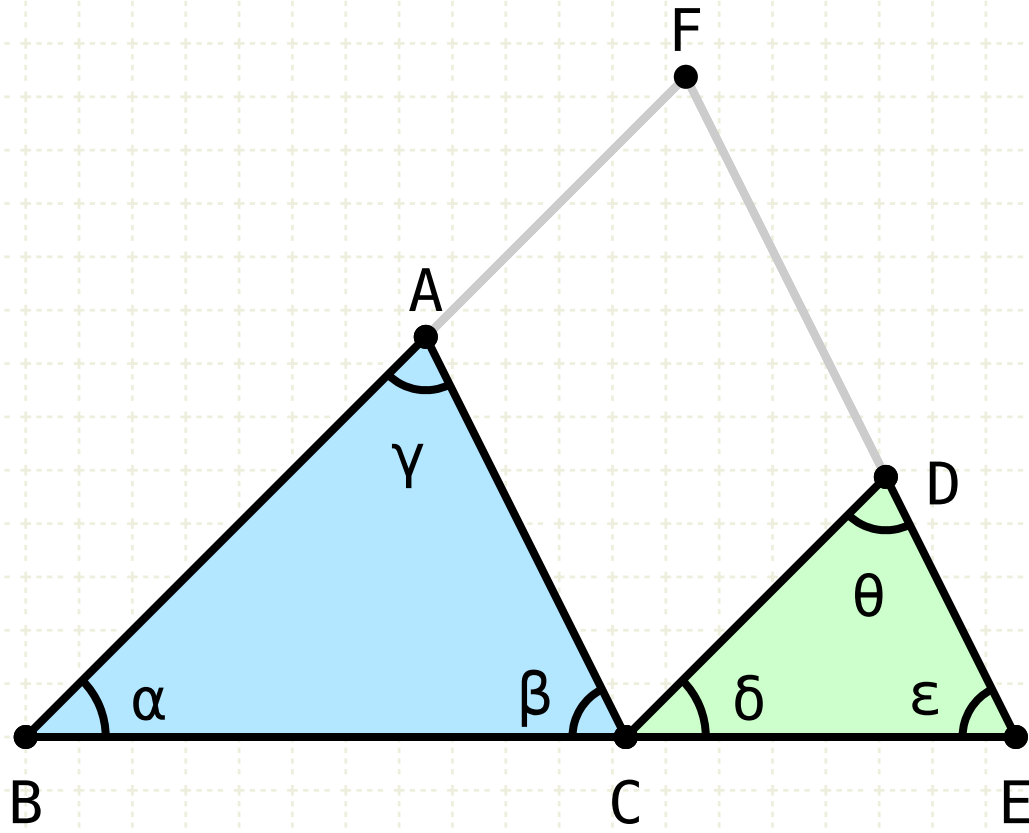
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But since AC equals FD, AC to DE is equal to BC to CE (V·16)

And alternately AC to BC is equal to DE to CE (V·16)

Since AB is to BC, so is CD to CE, and as BC is to AC, so is CE to DE, therefore AB is is to AC so is CD to DE (V·22)

$$AF \parallel CD$$

$$AC \parallel FE$$

$$AF = CD$$

$$AC = FD$$

$$AB:AF = BC:CE$$

$$AB:CD = BC:CE$$

$$AB:BC = CD:CE$$

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