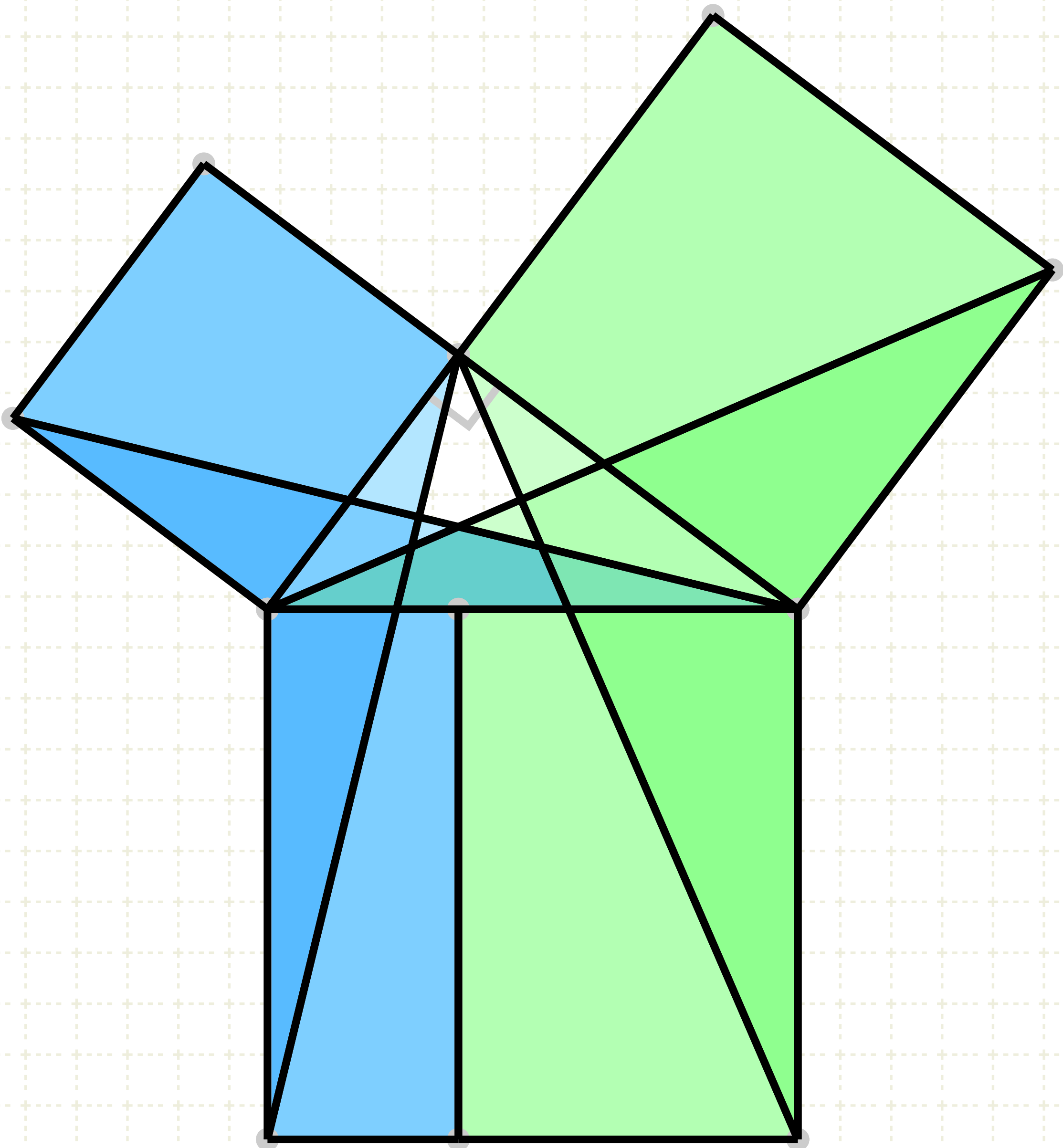
Euclid's Elements

Book I



If Euclid did not kindle your youthful enthusiasm, you were not born to be a scientific thinker.

Albert Einstein

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Proposition 45 of Book I

To construct a parallelogram equal to a given rectilinear figure in a given rectilinear angle.

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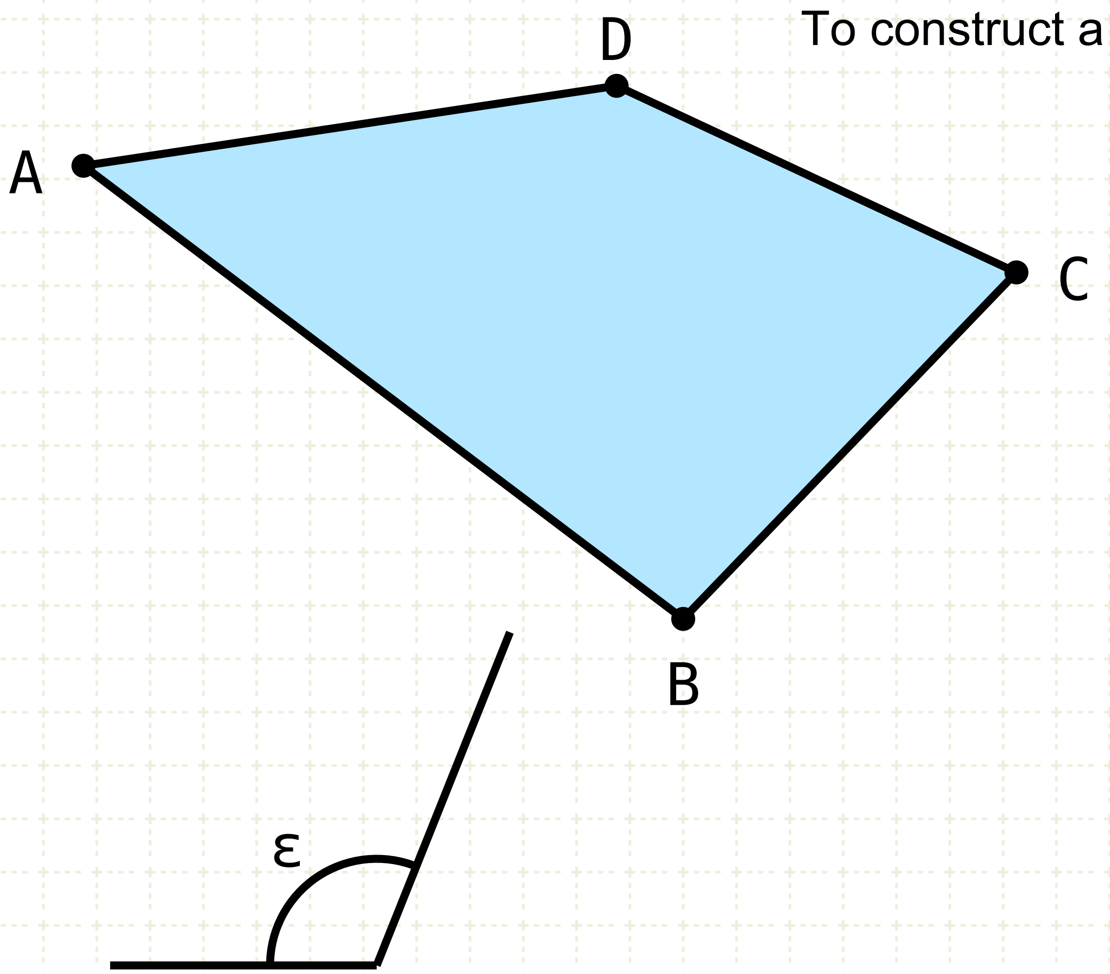
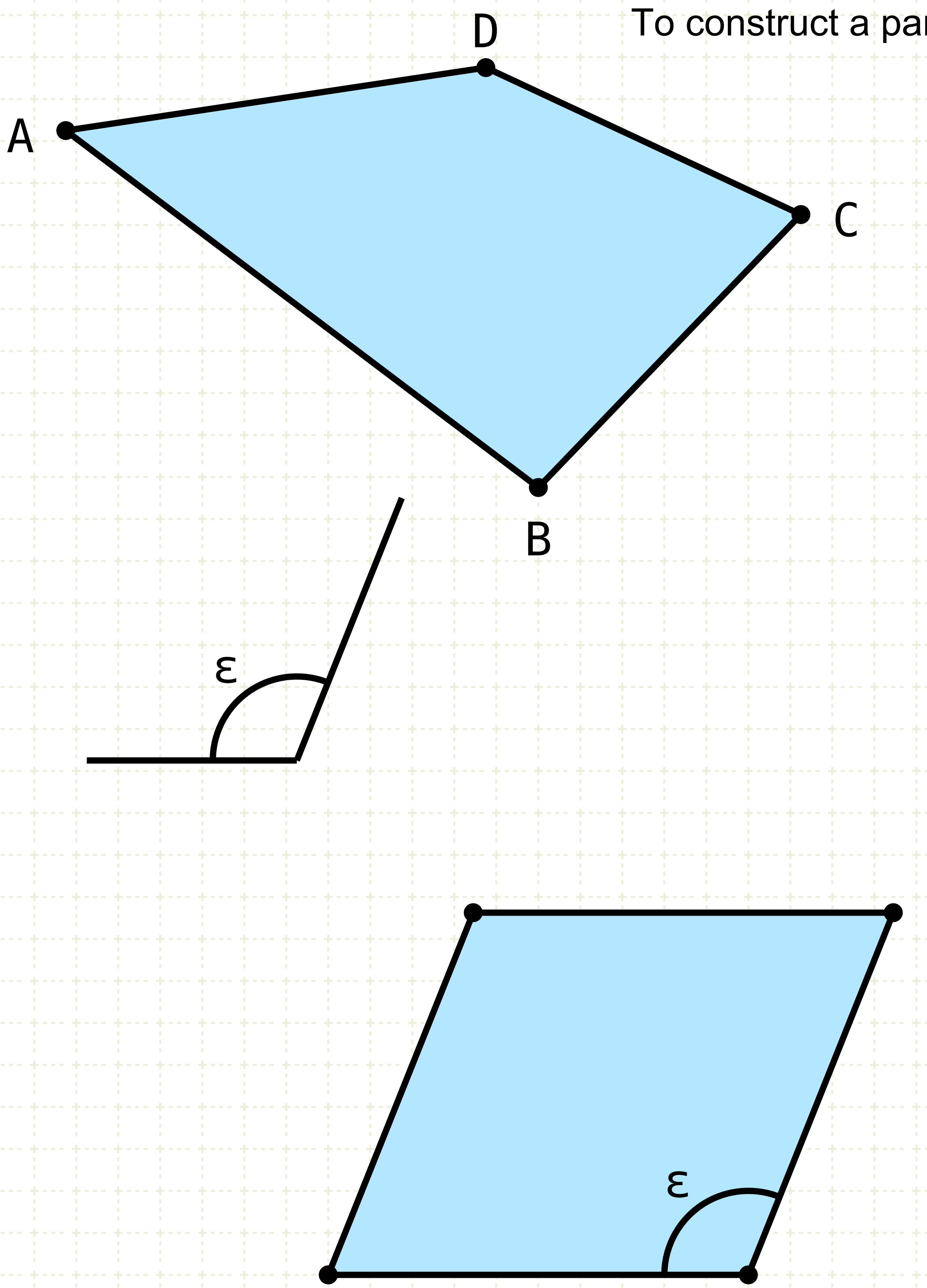


In other words

Start with a given rectilinear figure ABCD and a given angle ε

Proposition 45 of Book I

To construct a parallelogram equal to a given rectilinear figure in a given rectilinear angle.



In other words

Start with a given rectilinear figure ABCD and a given angle ε

Create a parallelogram with an angle ε, such that it is equal in area to the polygon ABCD

Proposition 45 of Book I

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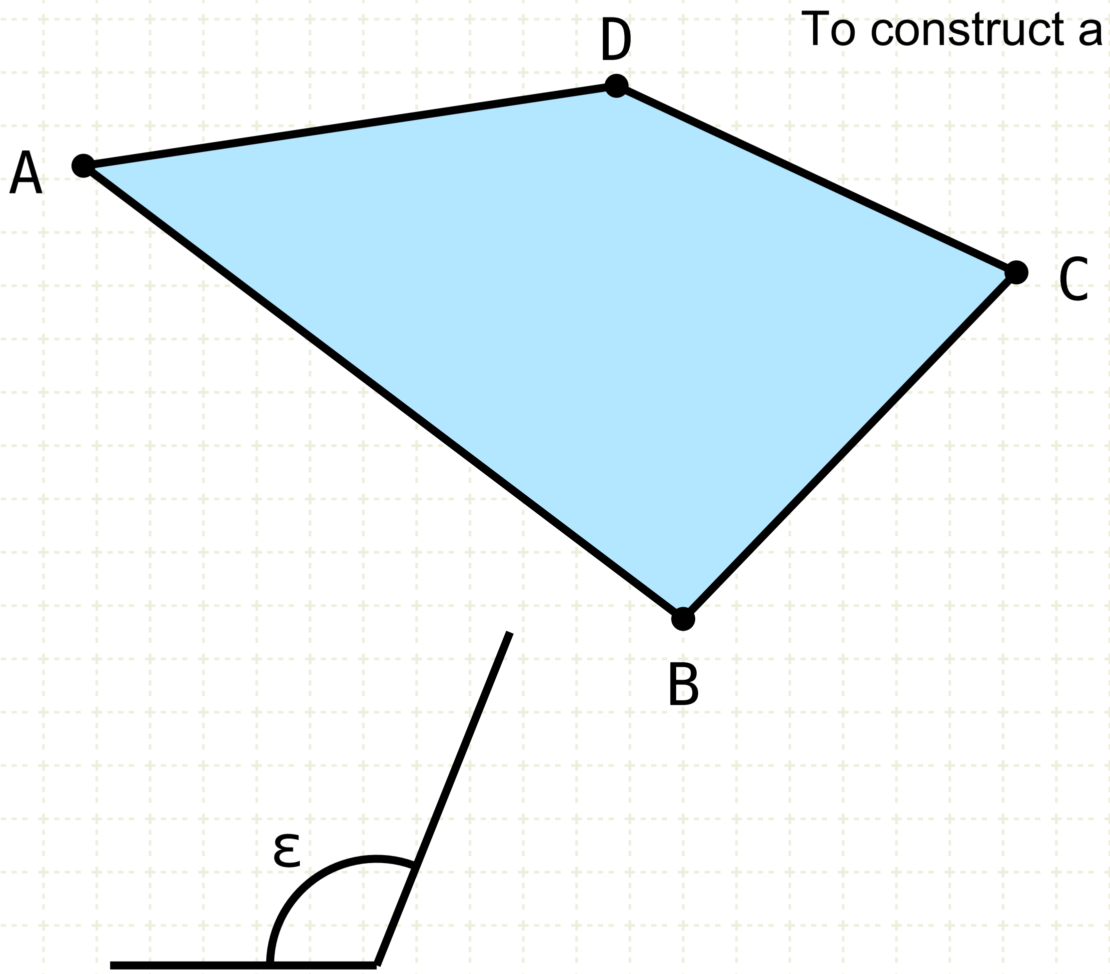


In other words

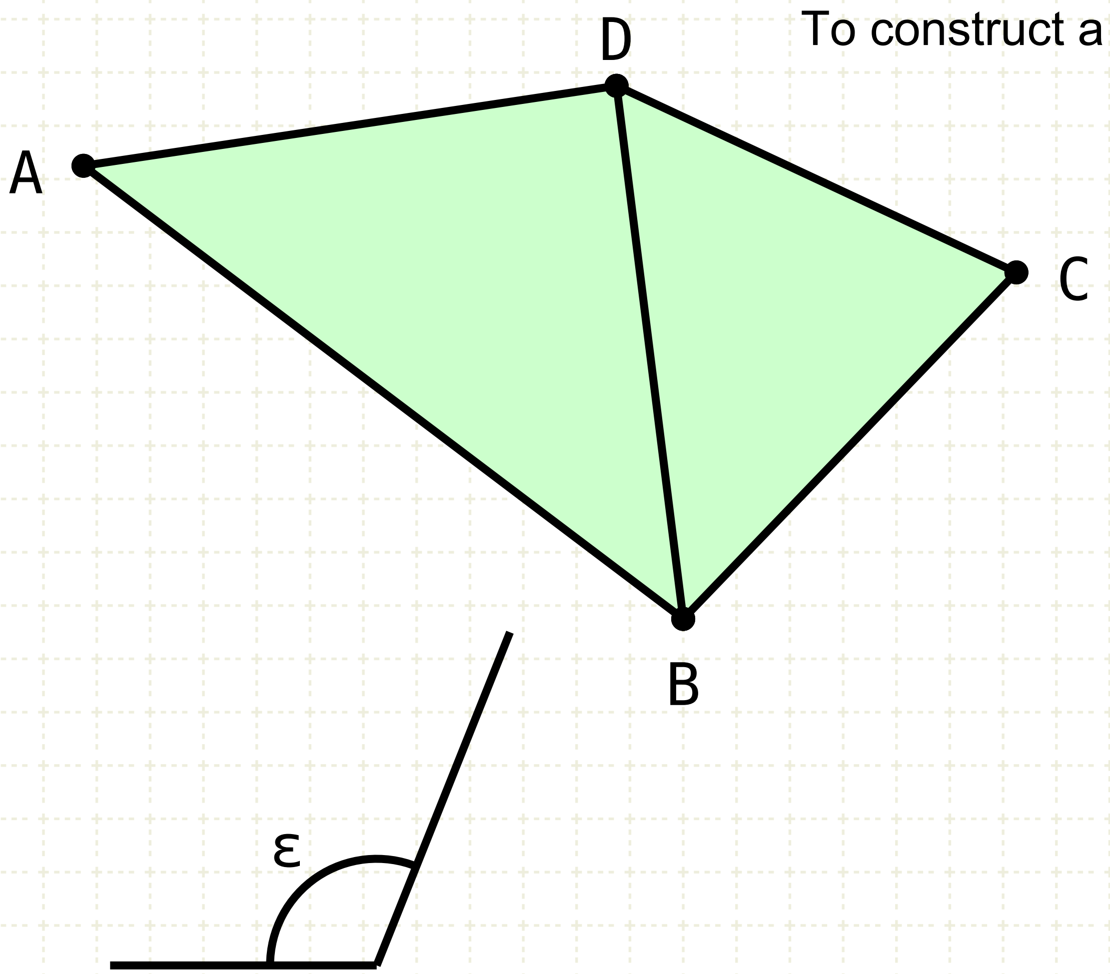
Start with a given rectilinear figure ABCD and a given angle ε

Create a parallelogram with an angle ε, such that it is equal in area to the polygon ABCD

Construction

Proposition 45 of Book I

To construct a parallelogram equal to a given rectilinear figure in a given rectilinear angle.



ABCD = ΔABD + ΔDBC

In other words

Start with a given rectilinear figure ABCD and a given angle ε

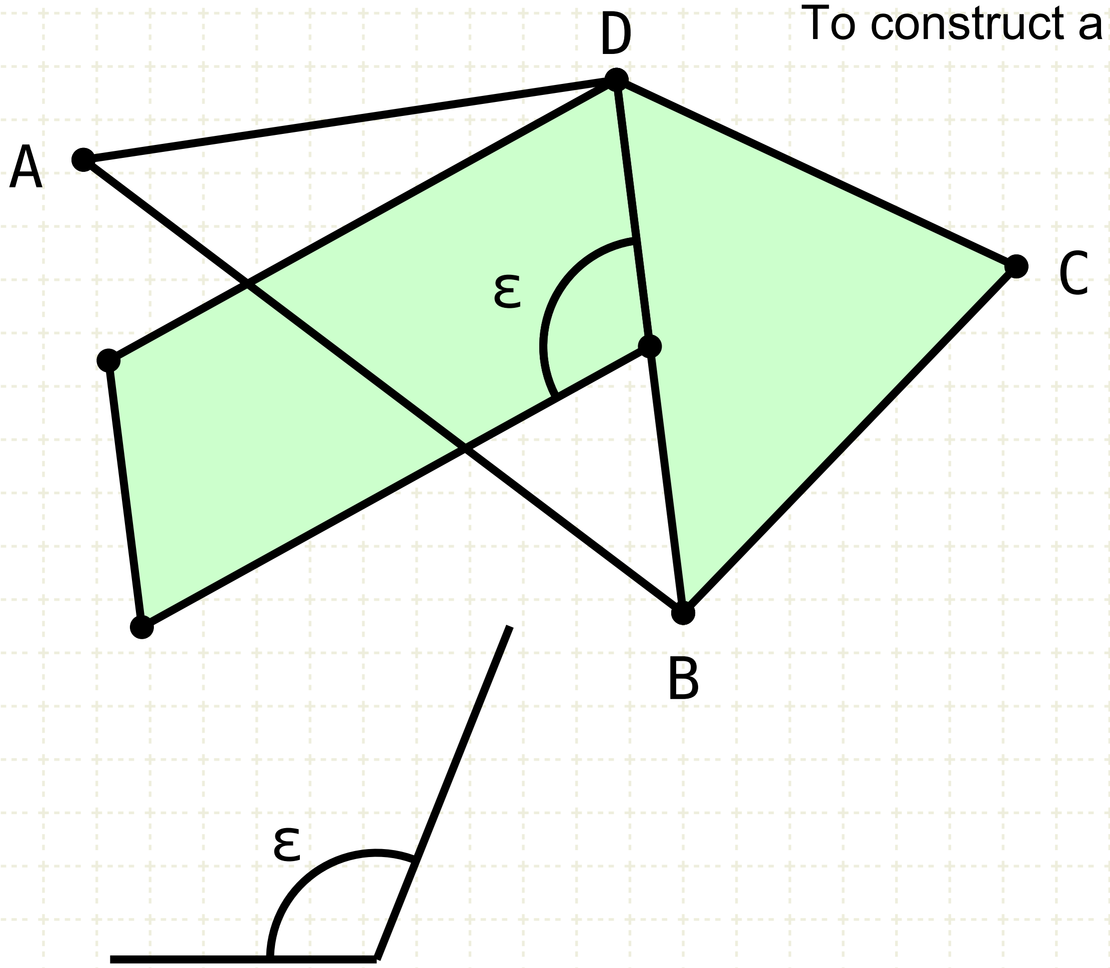
Create a parallelogram with an angle ε, such that it is equal in area to the polygon ABCD

Construction

Draw line DB, creating two triangles

Proposition 45 of Book I

To construct a parallelogram equal to a given rectilinear figure in a given rectilinear angle.



$$ABCD = \Delta ABD + \Delta DBC$$

In other words

Start with a given rectilinear figure ABCD and a given angle ε

Create a parallelogram with an angle ε, such that it is equal in area to the polygon ABCD

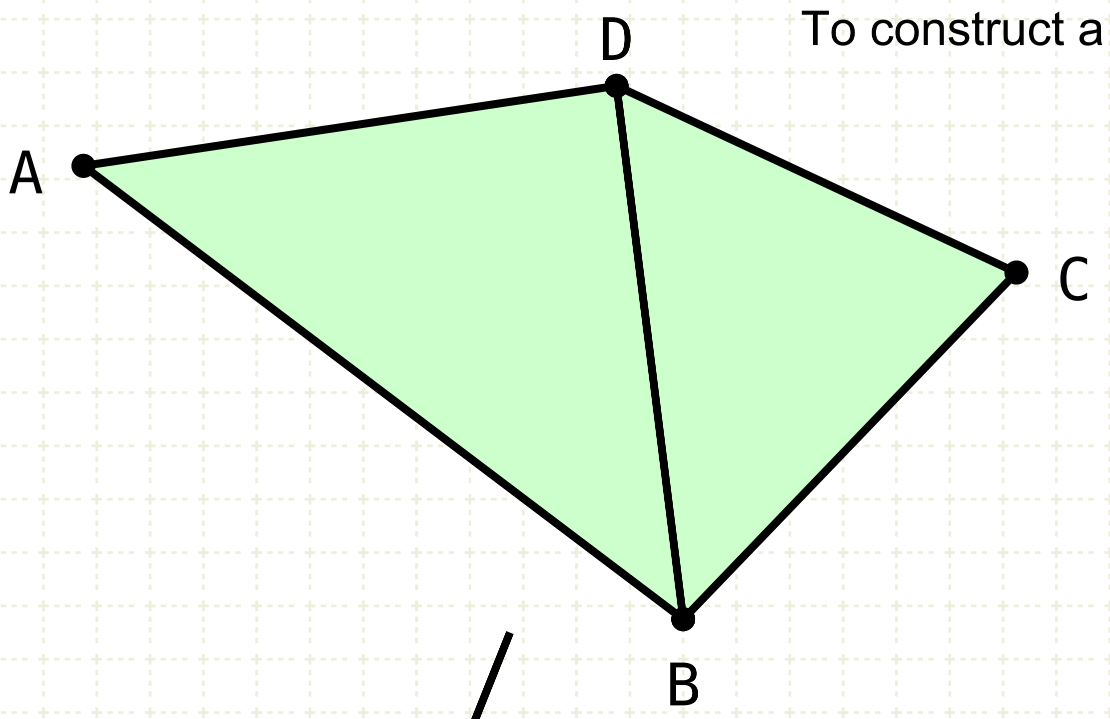
Construction

Draw line DB, creating two triangles

Create a parallelogram equal to triangle ABD, with angle ε (I·42)

Proposition 45 of Book I

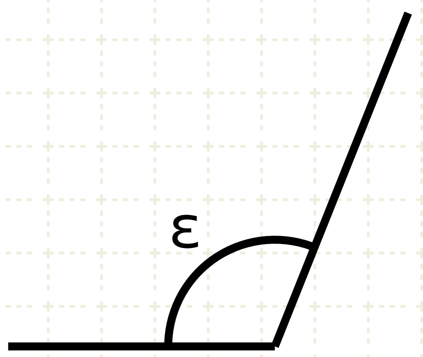
To construct a parallelogram equal to a given rectilinear figure in a given rectilinear angle.

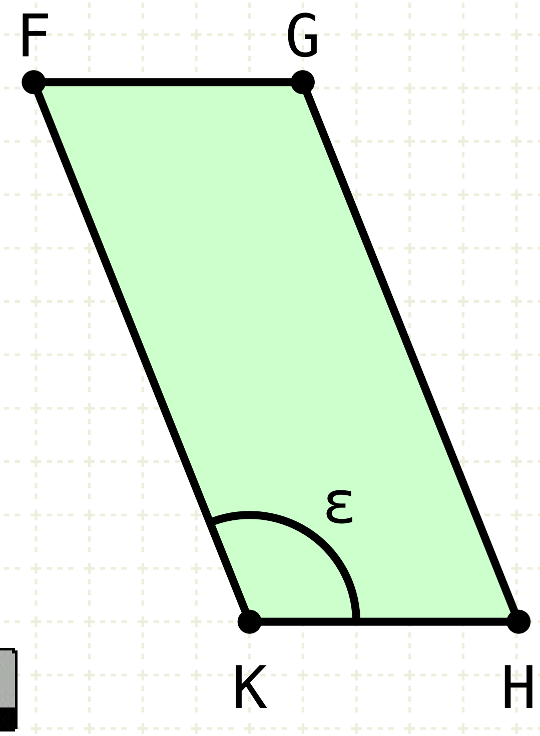


$ABCD = \Delta ABD + \Delta DBC$

$\angle FKH = \varepsilon$

$\Delta ADB = \square FGHK$





In other words

Start with a given rectilinear figure ABCD and a given angle ε

Create a parallelogram with an angle ε, such that it is equal in area to the polygon ABCD

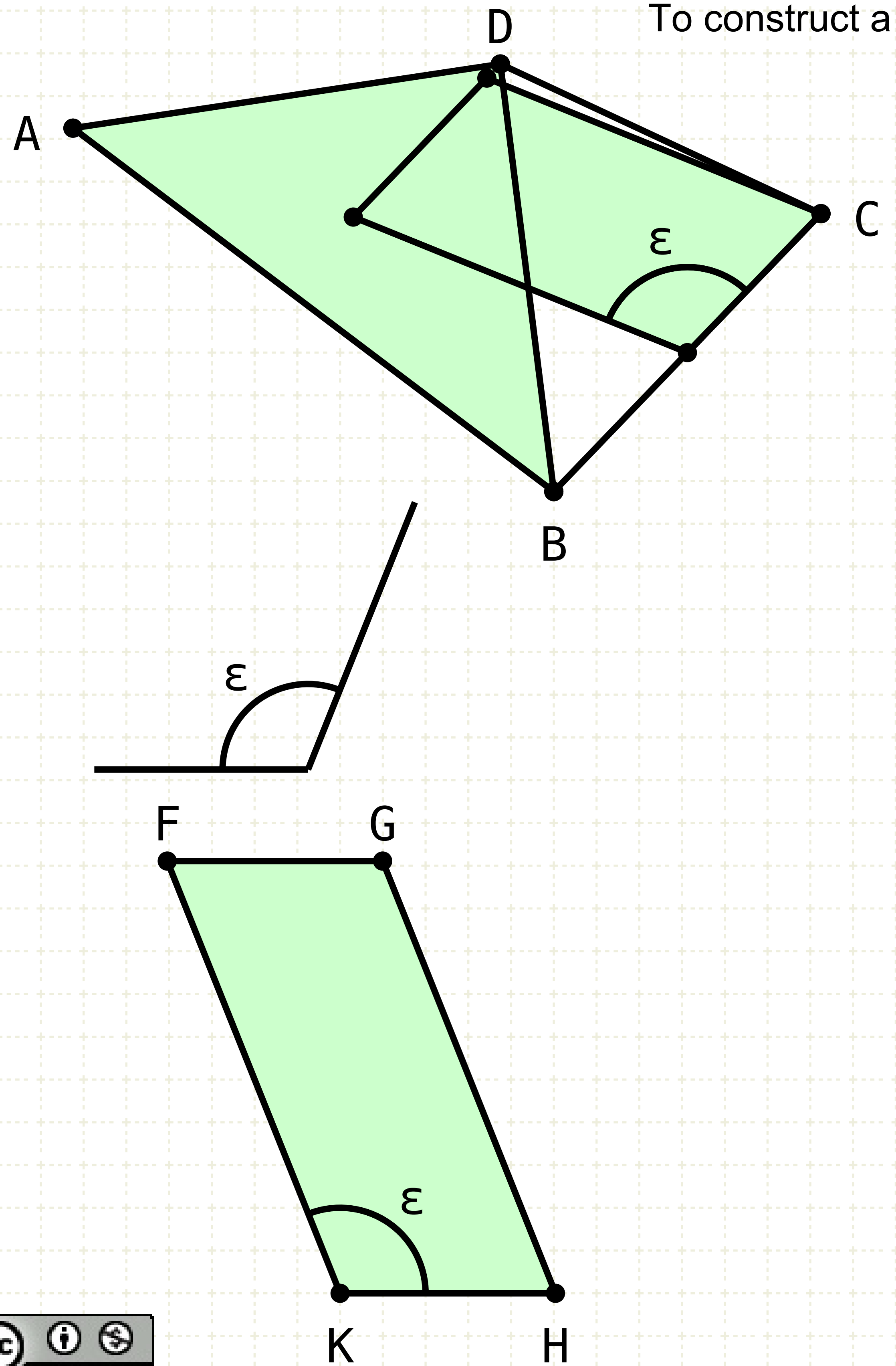
Construction

Draw line DB, creating two triangles

Create a parallelogram equal to triangle ABD, with angle ε (I·42)

Proposition 45 of Book I

To construct a parallelogram equal to a given rectilinear figure in a given rectilinear angle.



$ABCD = \Delta ABD + \Delta DBC$

$\angle FKH = \varepsilon$

$\Delta ADB = \square FGHK$

In other words

Start with a given rectilinear figure ABCD and a given angle ε

Create a parallelogram with an angle ε, such that it is equal in area to the polygon ABCD

Construction

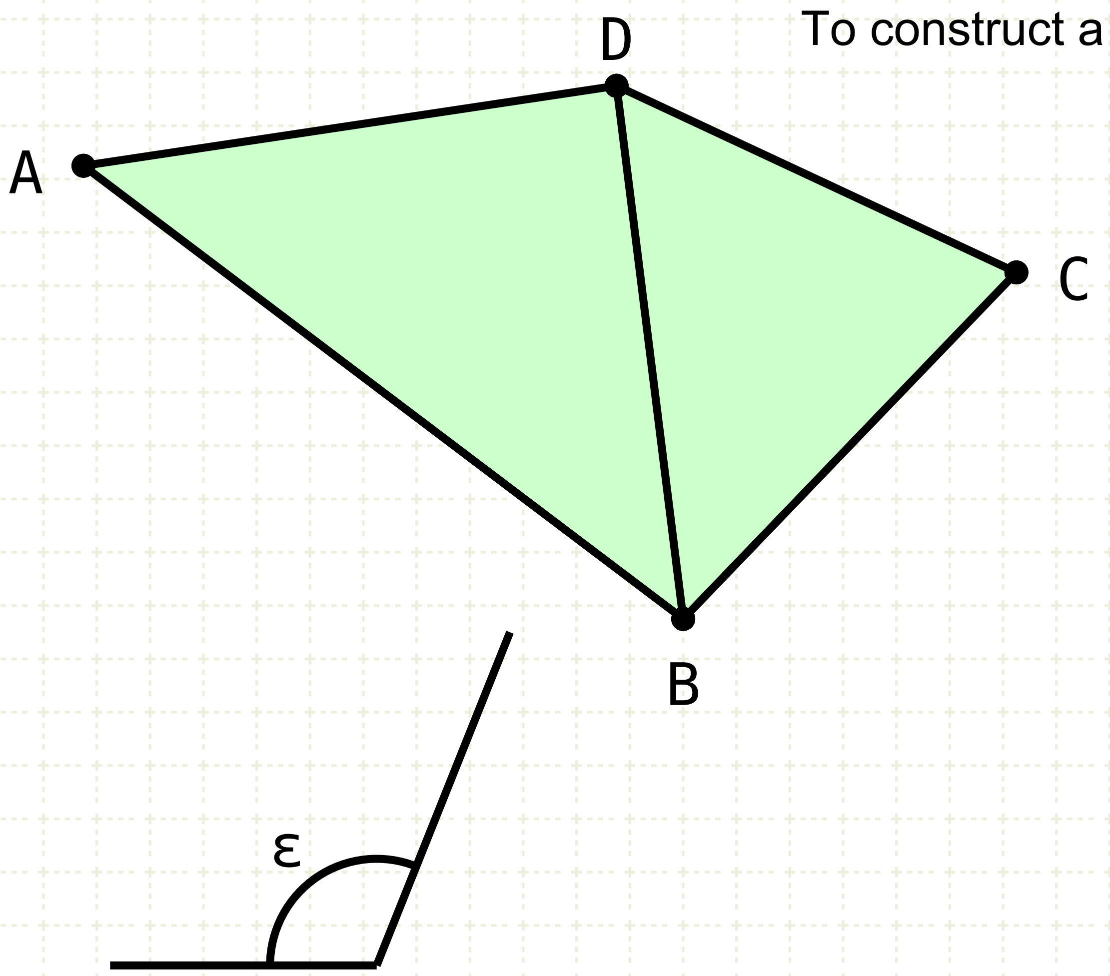
Draw line DB, creating two triangles

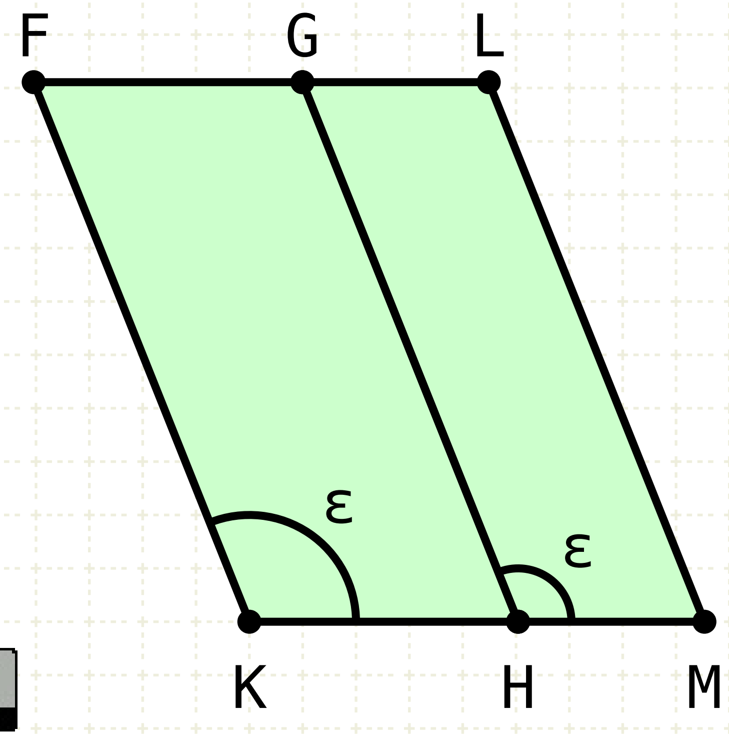
Create a parallelogram equal to triangle ABD, with angle ε (I·42)

Create a parallelogram equal to triangle DBC, with angle ε (I·42)

Proposition 45 of Book I

To construct a parallelogram equal to a given rectilinear figure in a given rectilinear angle.





$ABCD = \Delta ABD + \Delta DBC$

$\angle FKH = \varepsilon$

$\Delta ADB = \square FGHK$

$\angle GHM = \varepsilon$

$\Delta DBC = \square GHLM$

In other words

Start with a given rectilinear figure ABCD and a given angle ε

Create a parallelogram with an angle ε, such that it is equal in area to the polygon ABCD

Construction

Draw line DB, creating two triangles

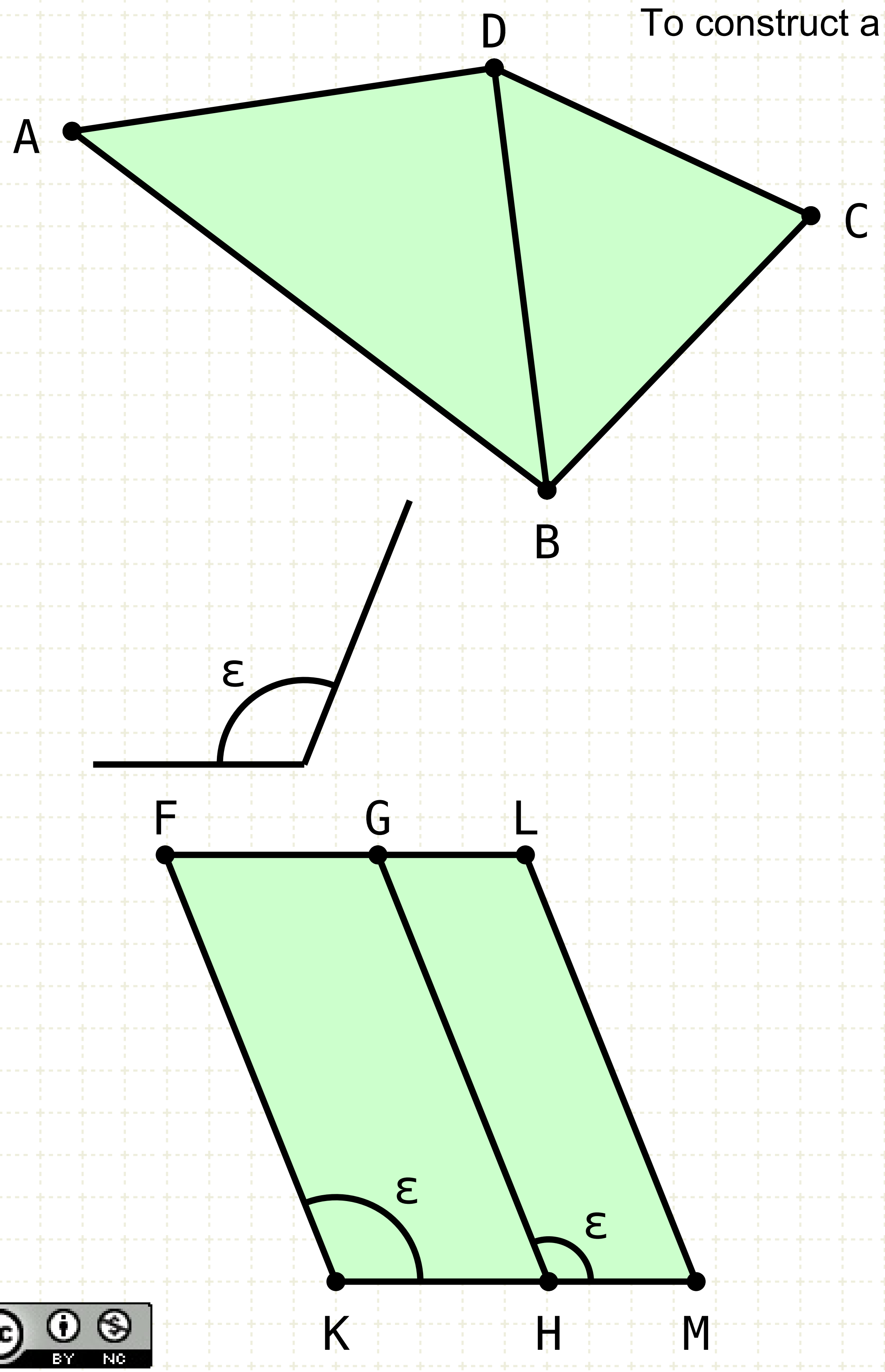
Create a parallelogram equal to triangle ABD, with angle ε (I·42)

Create a parallelogram equal to triangle DBC, with angle ε (I·42)

Create a parallelogram, equal to triangle DBC, on side GH (I·44)

Proposition 45 of Book I

To construct a parallelogram equal to a given rectilinear figure in a given rectilinear angle.



$ABCD = \triangle ABD + \triangle DBC$

$\angle FKH = \varepsilon$

$\triangle ADB = \square FGHK$

$\angle GHM = \varepsilon$

$\triangle DBC = \square GHLM$

$\square FLMK = ABCD$

In other words

Start with a given rectilinear figure ABCD and a given angle ε

Create a parallelogram with an angle ε, such that it is equal in area to the polygon ABCD

Construction

Draw line DB, creating two triangles

Create a parallelogram equal to triangle ABD, with angle ε (I·42)

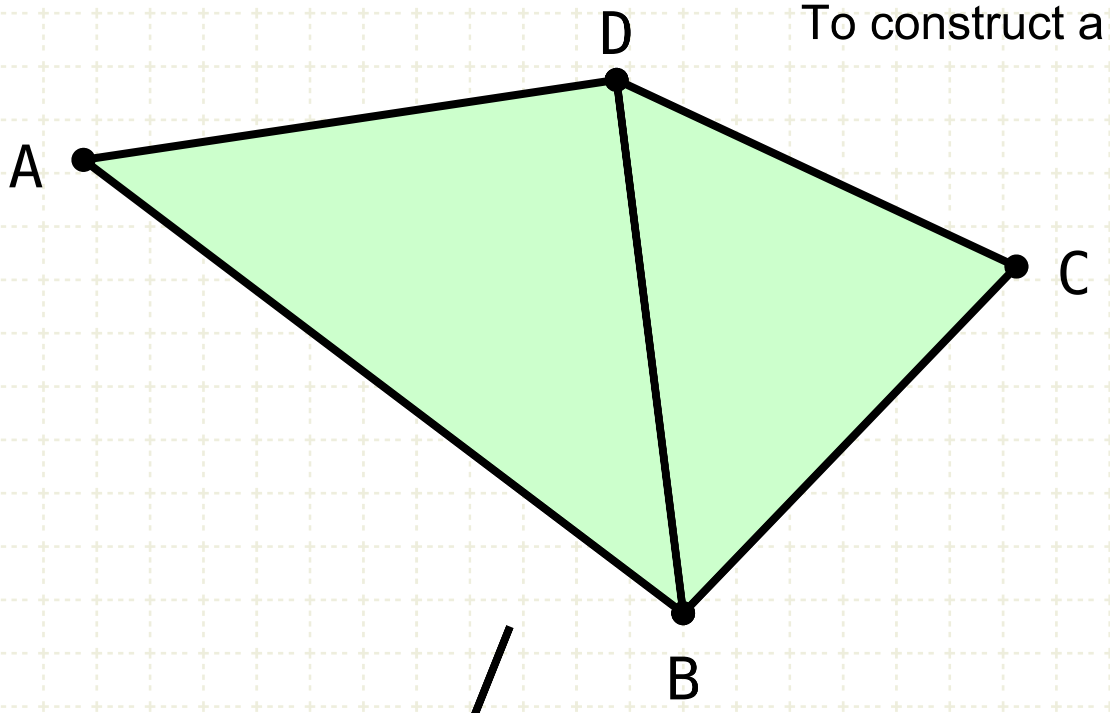
Create a parallelogram equal to triangle DBC, with angle ε (I·42)

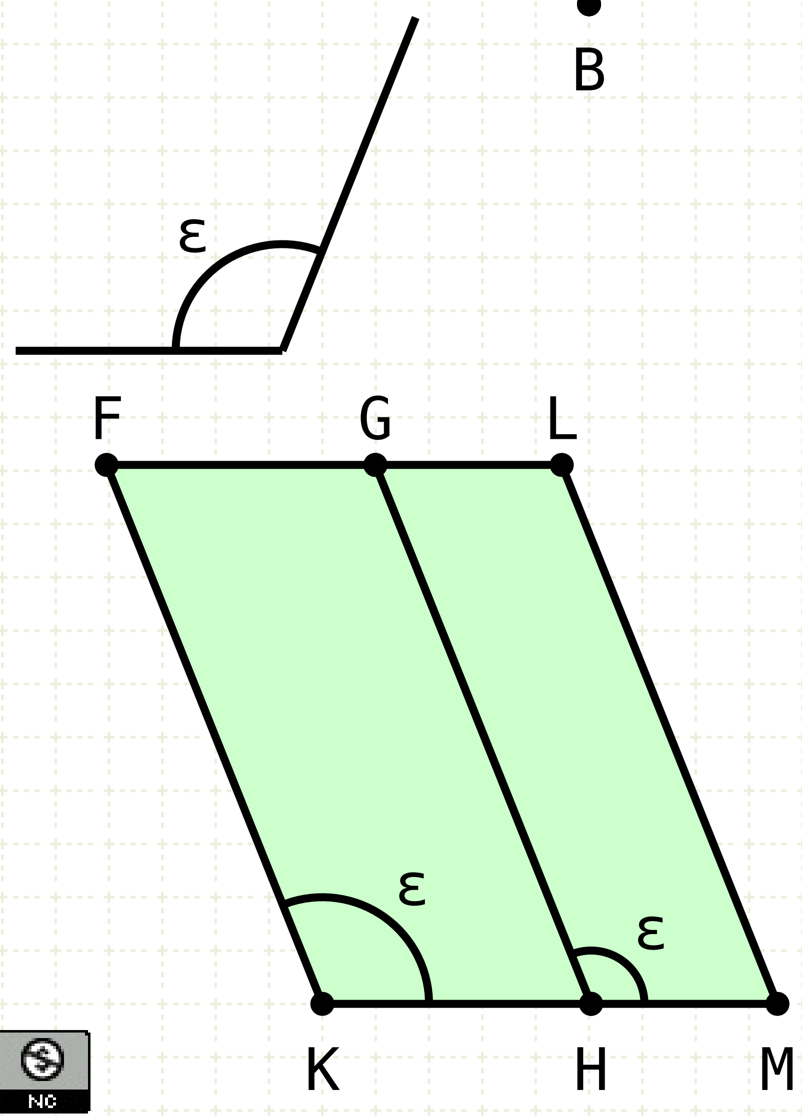
Create a parallelogram, equal to triangle DBC, on side GH (I·44)

FLMK is a parallelogram, and it's area is equal to the polygon ABCD

Proposition 45 of Book I

To construct a parallelogram equal to a given rectilinear figure in a given rectilinear angle.





Proof:

$ABCD = \Delta ABD + \Delta DBC$

$\angle FKH = \varepsilon$

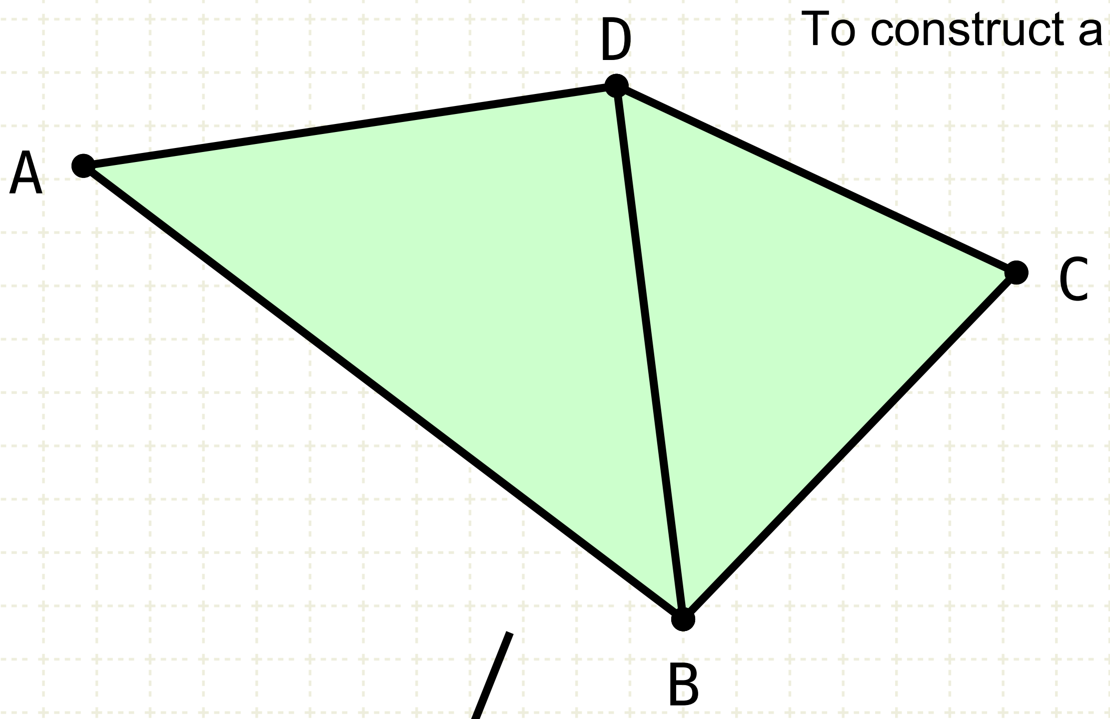
$\Delta ADB = \square FGHK$

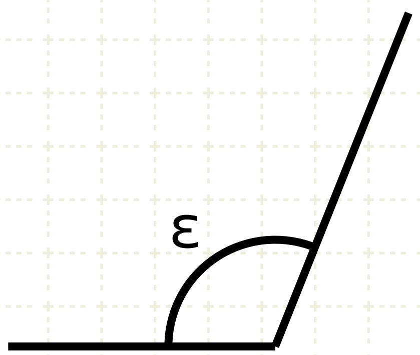
$\angle GHM = \varepsilon$

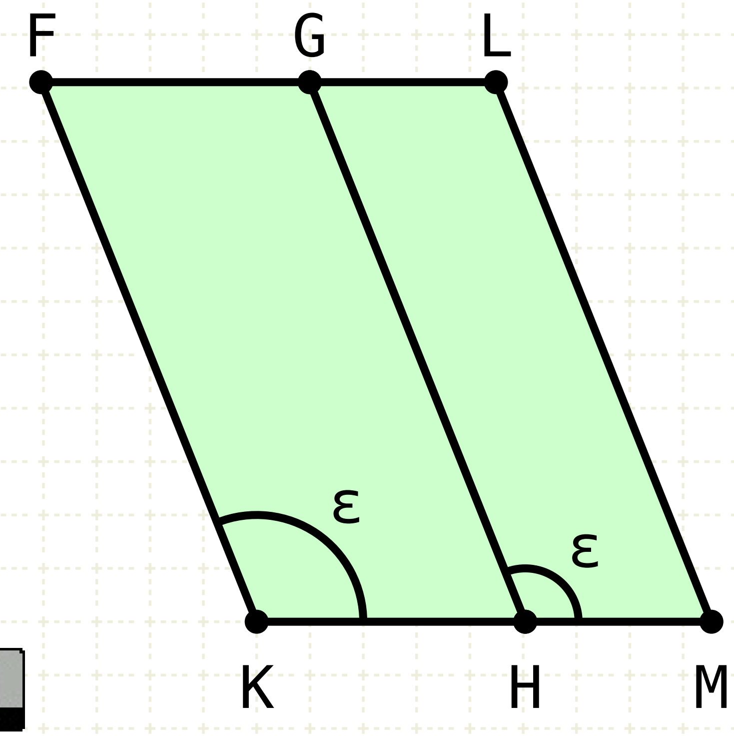
$\Delta DBC = \square GHLM$

Proposition 45 of Book I

To construct a parallelogram equal to a given rectilinear figure in a given rectilinear angle.







ABCD = ΔABD + ΔDBC

∠FKH = ε

ΔADB = □FGHK

∠GHM = ε

ΔDBC = □GHLM

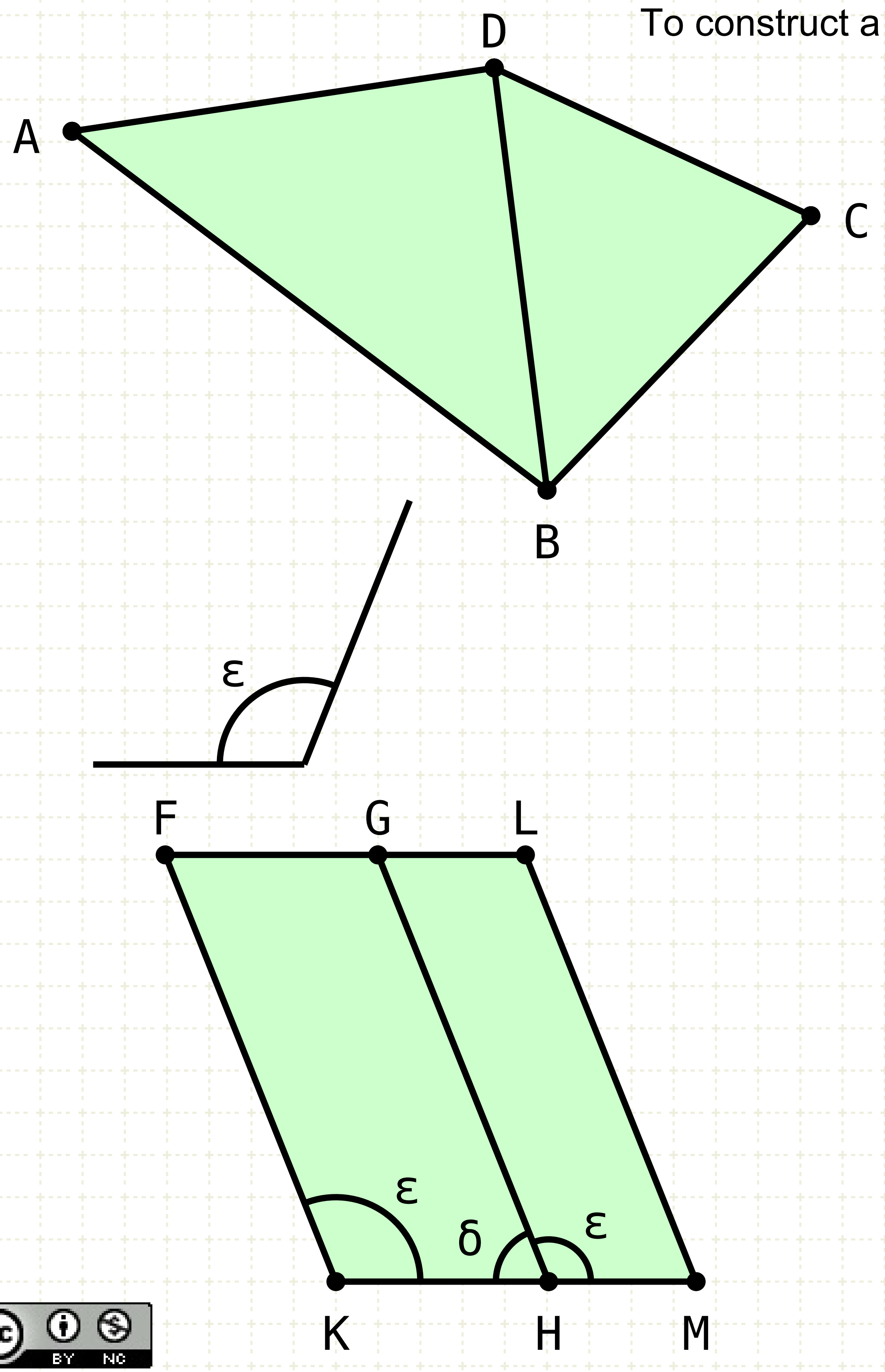
FK ∥ GH, FG ∥ KH

Proof:

FGHK is a parallelogram by construction so its sides are parallel

Proposition 45 of Book I

To construct a parallelogram equal to a given rectilinear figure in a given rectilinear angle.



ABCD = ΔABD + ΔDBC

∠FKH = ε

ΔADB = □FGHK

∠GHM = ε

ΔDBC = □GHLM

FK ∥ GH, FG ∥ KH

ε + δ = 2∟

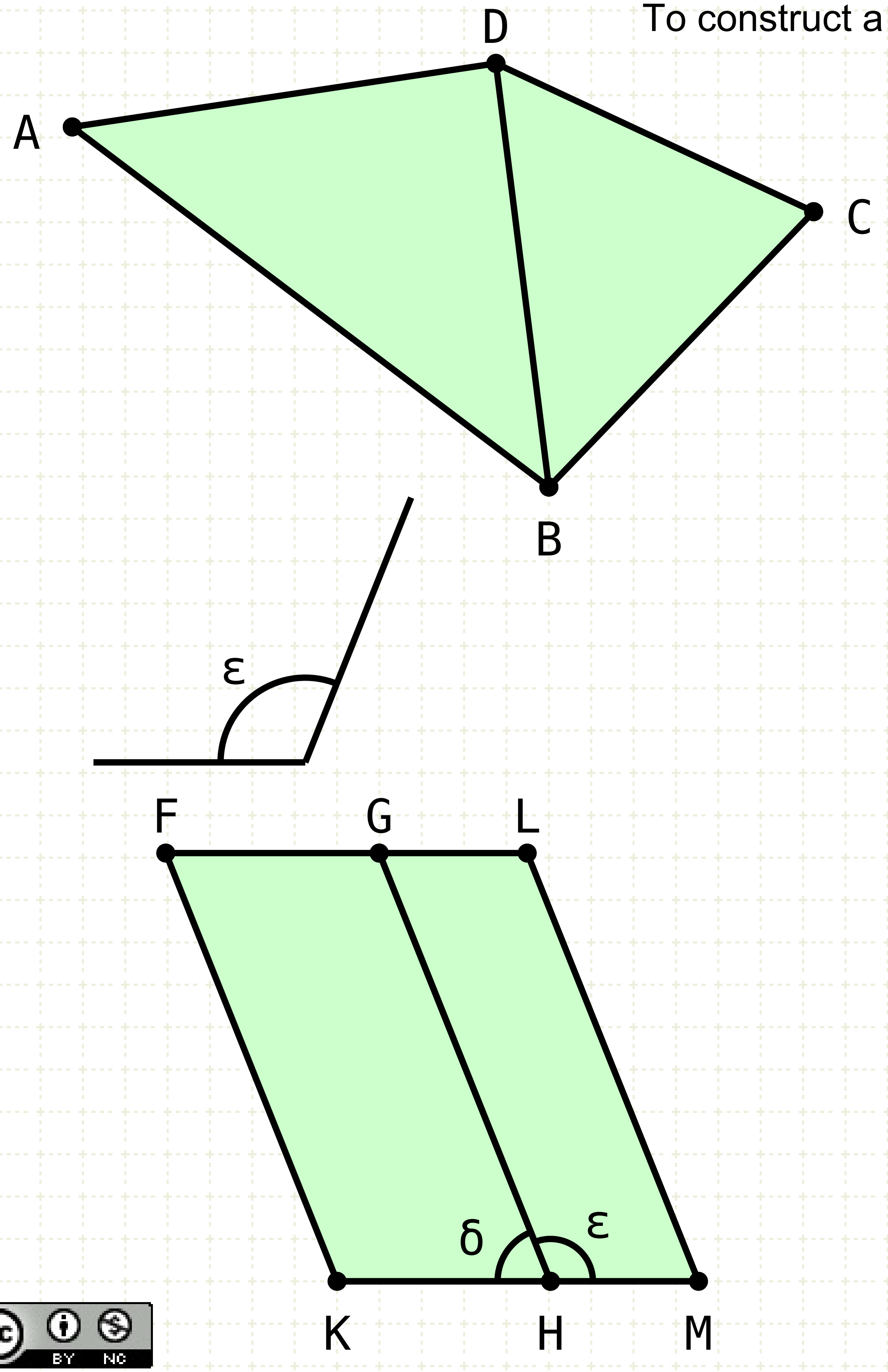
Proof:

FGHK is a parallelogram by construction so its sides are parallel

Since FK and GH are parallel lines, angles FKH and GHK sum to two right angles (I·29)

Proposition 45 of Book I

To construct a parallelogram equal to a given rectilinear figure in a given rectilinear angle.



$ABCD = \Delta ABD + \Delta DBC$

$\angle FKH = \varepsilon$

$\Delta ADB = \square FGHK$

$\angle GHM = \varepsilon$

$\Delta DBC = \square GHLM$

$FK \parallel GH,\ FG \parallel KH$

$\varepsilon + \delta = 2\llcorner$

$KH,HM = KM$

Proof:

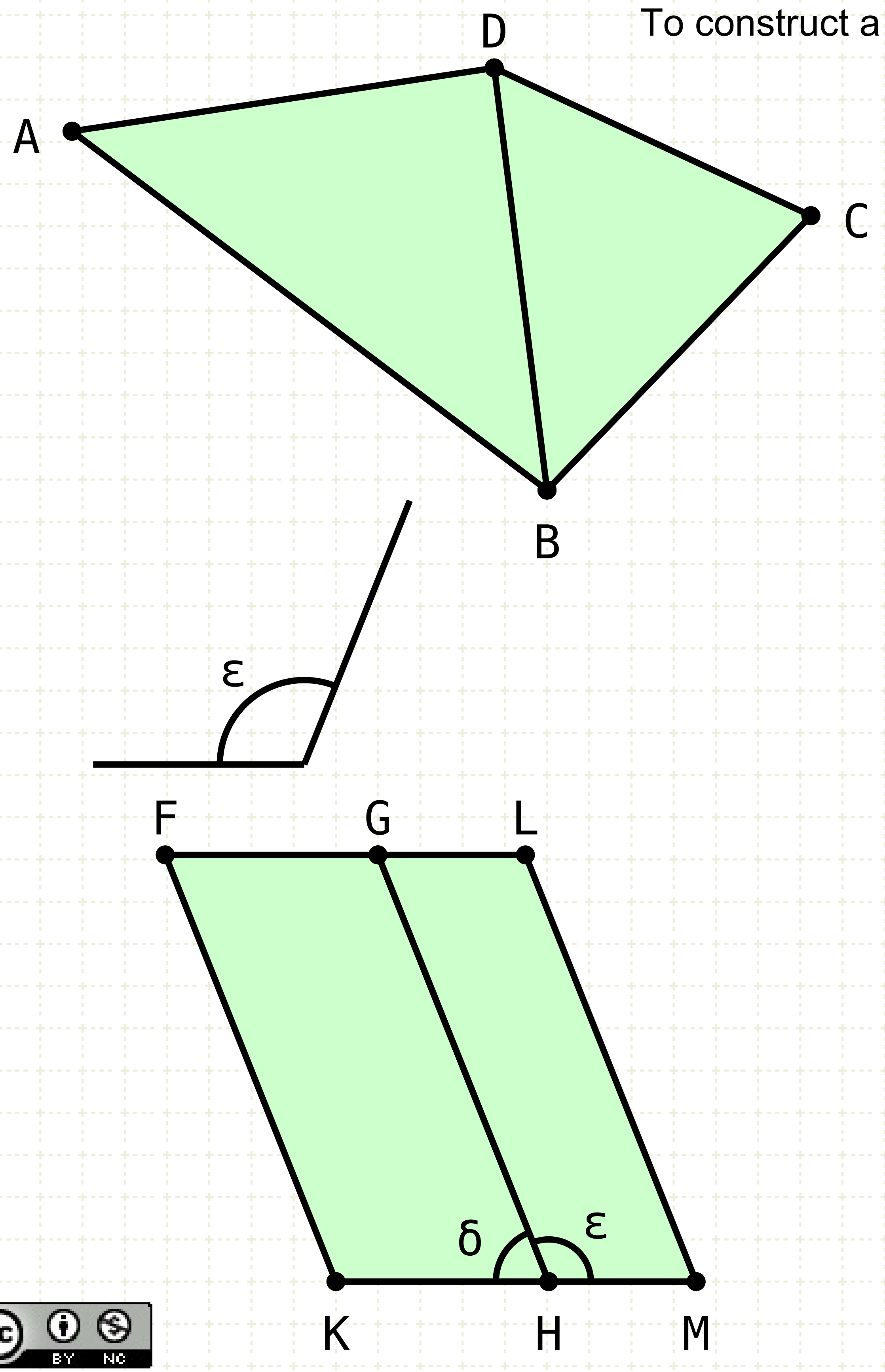
FGHK is a parallelogram by construction so its sides are parallel

Since FK and GH are parallel lines, angles FKH and GHK sum to two right angles (I·29)

Since angles GHK and GHM sum to two right angles, KH is in a straight line with HM (I·14)

Proposition 45 of Book I

To construct a parallelogram equal to a given rectilinear figure in a given rectilinear angle.



ABCD = ΔABD + ΔDBC

∠FKH = ε

ΔADB = □FGHK

∠GHM = ε

ΔDBC = □GHLM

FK ∥ GH, FG ∥ KH

ε + δ = 2∟

KH,HM = KM

KM ∥ FG

Proof:

FGHK is a parallelogram by construction so its sides are parallel

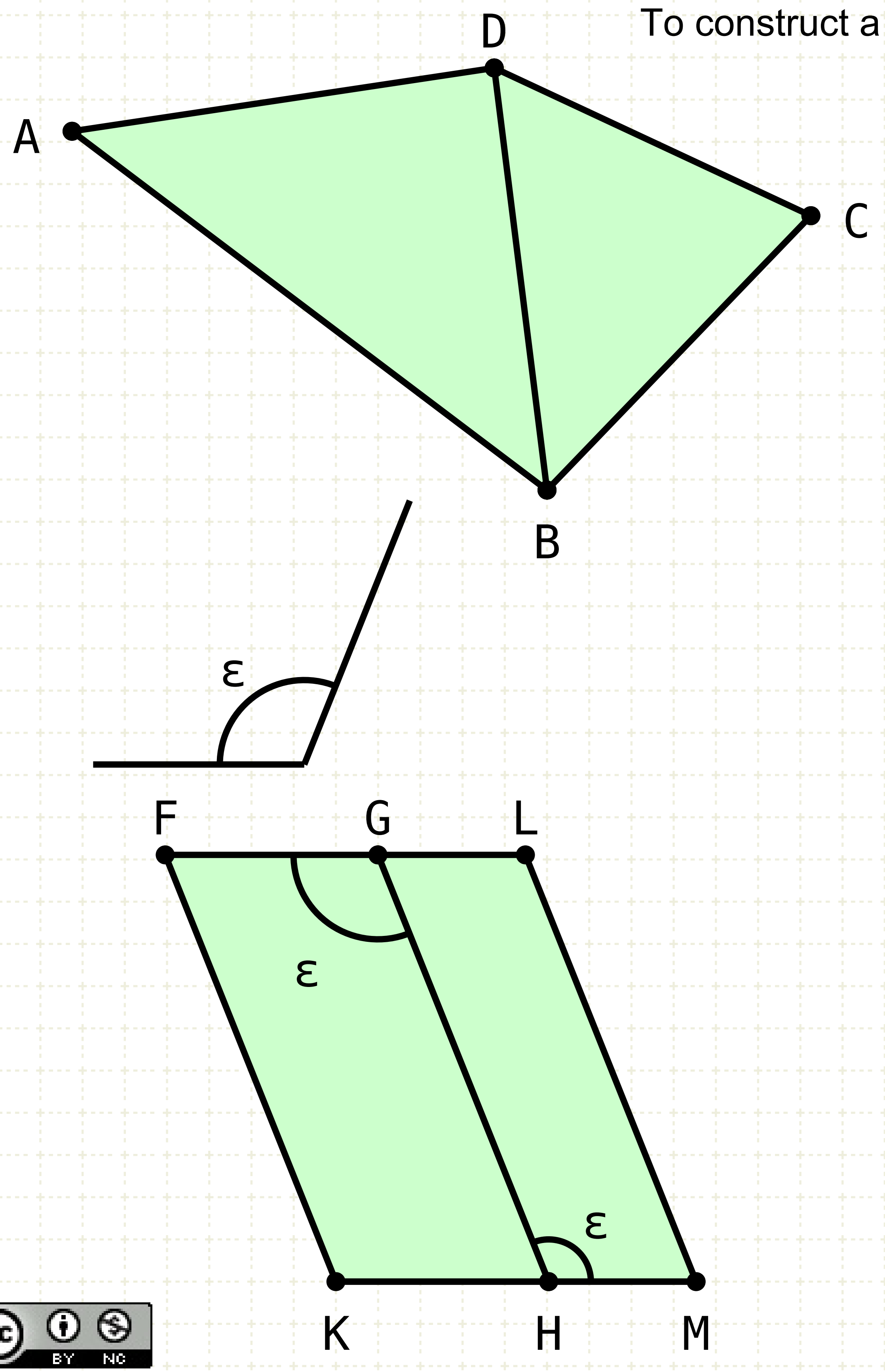
Since FK and GH are parallel lines, angles FKH and GHK sum to two right angles (I·29)

Since angles GHK and GHM sum to two right angles, KH is in a straight line with HM (I·14)

Lines KM and FG are parallel since FGHK is a parallelogram

Proposition 45 of Book I

To construct a parallelogram equal to a given rectilinear figure in a given rectilinear angle.



ABCD = ΔABD + ΔDBC

∠FKH = ε

ΔADB = □FGHK

∠GHM = ε

ΔDBC = □GHLM

FK ∥ GH, FG ∥ KH

ε + δ = 2∟

KH,HM = KM

KM ∥ FG

Proof:

FGHK is a parallelogram by construction so its sides are parallel

Since FK and GH are parallel lines, angles FKH and GHK sum to two right angles (I·29)

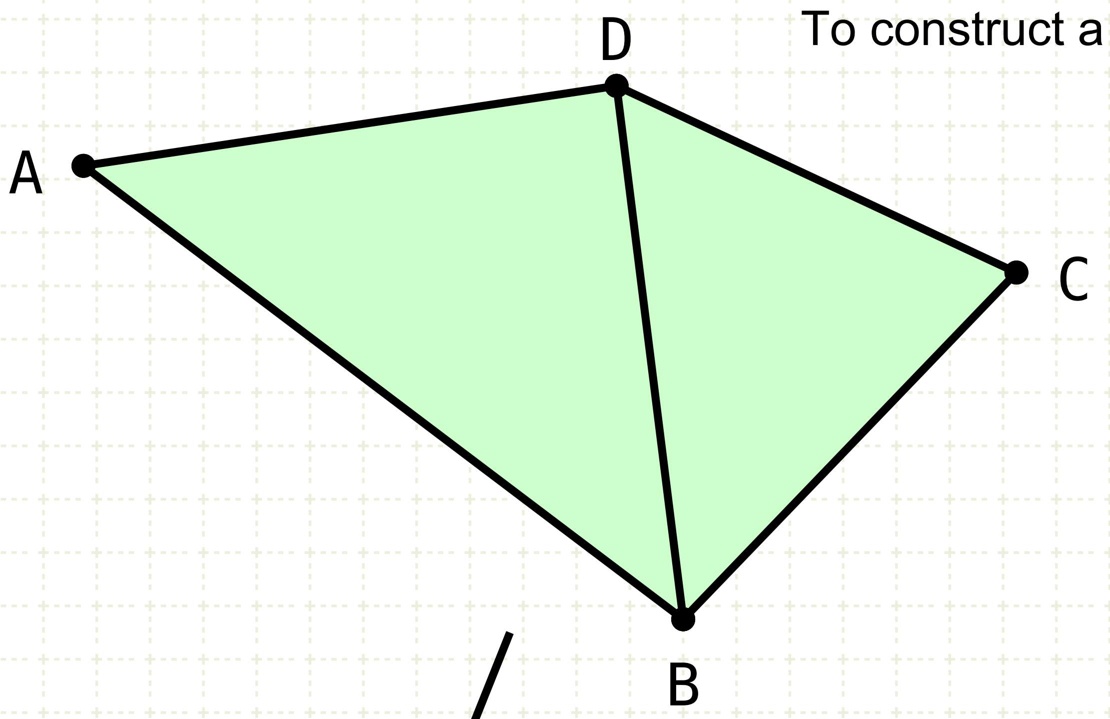
Since angles GHK and GHM sum to two right angles, KH is in a straight line with HM (I·14)

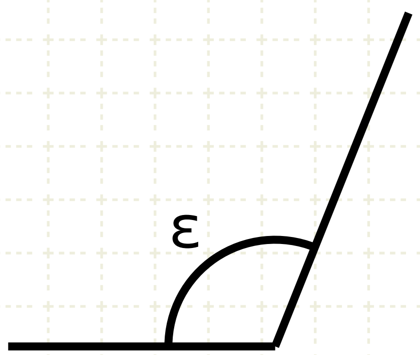
Lines KM and FG are parallel since FGHK is a parallelogram

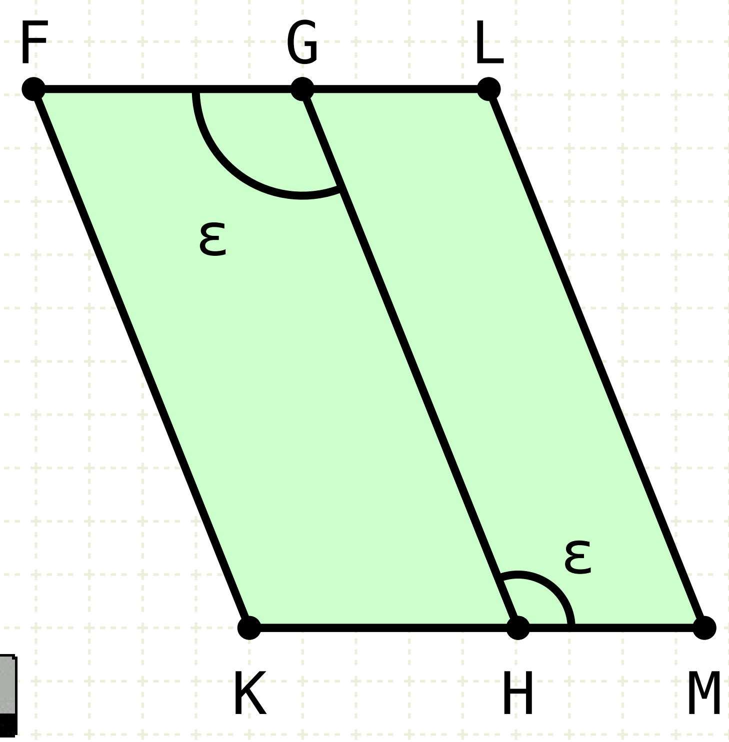
Thus, the alternate angles (HGF and GHM) are equal (I·29)

Proposition 45 of Book I

To construct a parallelogram equal to a given rectilinear figure in a given rectilinear angle.







$ABCD = \Delta ABD + \Delta DBC$

$\angle FKH = \varepsilon$

$\Delta ADB = \square FGHK$

$\angle GHM = \varepsilon$

$\Delta DBC = \square GHLM$

$FK \parallel GH,\ FG \parallel KH$

$\varepsilon + \delta = 2\llcorner$

$KH, HM = KM$

$KM \parallel FG$

$GL \parallel HM,\ GH \parallel LM$

Proof:

FGHK is a parallelogram by construction so its sides are parallel

Since FK and GH are parallel lines, angles FKH and GHK sum to two right angles (I·29)

Since angles GHK and GHM sum to two right angles, KH is in a straight line with HM (I·14)

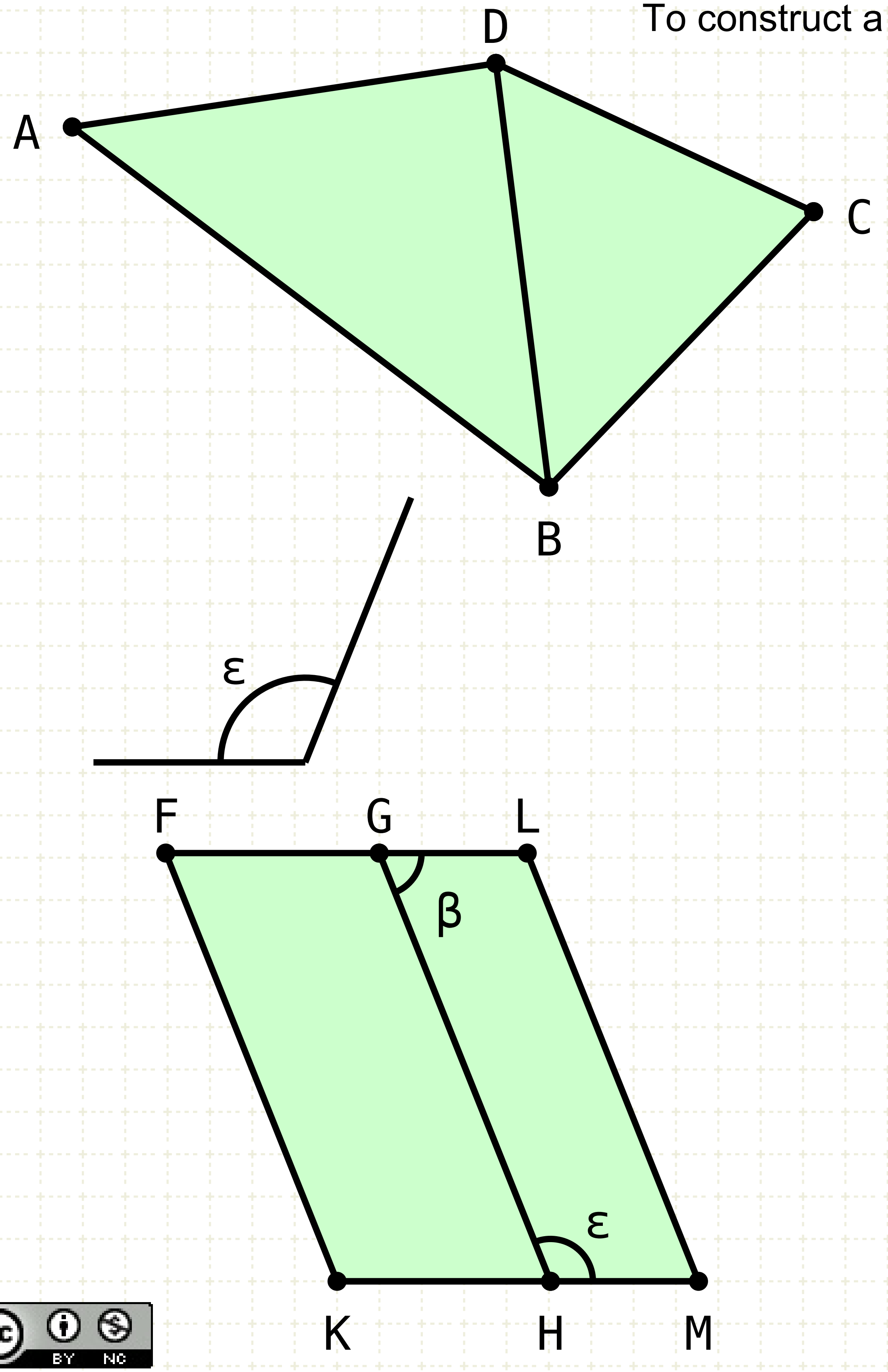
Lines KM and FG are parallel since FGHK is a parallelogram

Thus, the alternate angles (HGF and GHM) are equal (I·29)

GLMH is a parallelogram by construction, so all its sides are parallel

Proposition 45 of Book I

To construct a parallelogram equal to a given rectilinear figure in a given rectilinear angle.



$ABCD = \Delta ABD + \Delta DBC$

$\angle FKH = \varepsilon$

$\Delta ADB = \square FGHK$

$\angle GHM = \varepsilon$

$\Delta DBC = \square GHLM$

$FK \parallel GH,\ FG \parallel KH$

$\varepsilon + \delta = 2\llcorner$

$KH, HM = KM$

$KM \parallel FG$

$GL \parallel HM,\ GH \parallel LM$

$\varepsilon + \beta = 2\llcorner$

Proof:

FGHK is a parallelogram by construction so its sides are parallel

Since FK and GH are parallel lines, angles FKH and GHK sum to two right angles (I·29)

Since angles GHK and GHM sum to two right angles, KH is in a straight line with HM (I·14)

Lines KM and FG are parallel since FGHK is a parallelogram

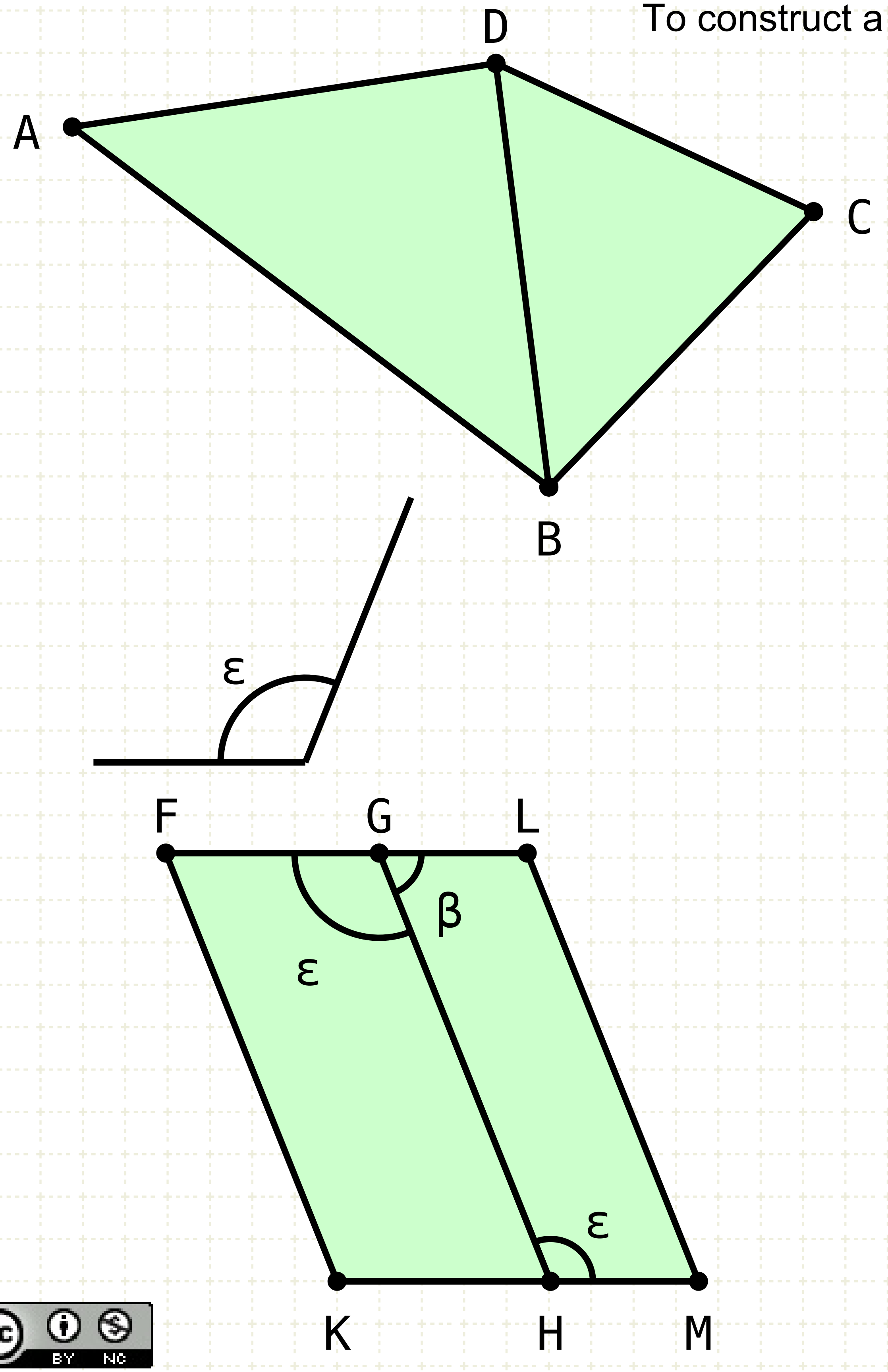
Thus, the alternate angles (HGF and GHM) are equal (I·29)

GLMH is a parallelogram by construction, so all its sides are parallel

Angles LGH and GHM sum to two right angles I·29

Proposition 45 of Book I

To construct a parallelogram equal to a given rectilinear figure in a given rectilinear angle.



$ABCD = \Delta ABD + \Delta DBC$

$\angle FKH = \varepsilon$

$\Delta ADB = \square FGHK$

$\angle GHM = \varepsilon$

$\Delta DBC = \square GHLM$

$FK \parallel GH,\ FG \parallel KH$

$\varepsilon + \delta = 2\llcorner$

$KH, HM = KM$

$KM \parallel FG$

$GL \parallel HM,\ GH \parallel LM$

$\varepsilon + \beta = 2\llcorner$

$FG, GL = FL$

Proof:

FGHK is a parallelogram by construction so its sides are parallel

Since FK and GH are parallel lines, angles FKH and GHK sum to two right angles (I·29)

Since angles GHK and GHM sum to two right angles, KH is in a straight line with HM (I·14)

Lines KM and FG are parallel since FGHK is a parallelogram

Thus, the alternate angles (HGF and GHM) are equal (I·29)

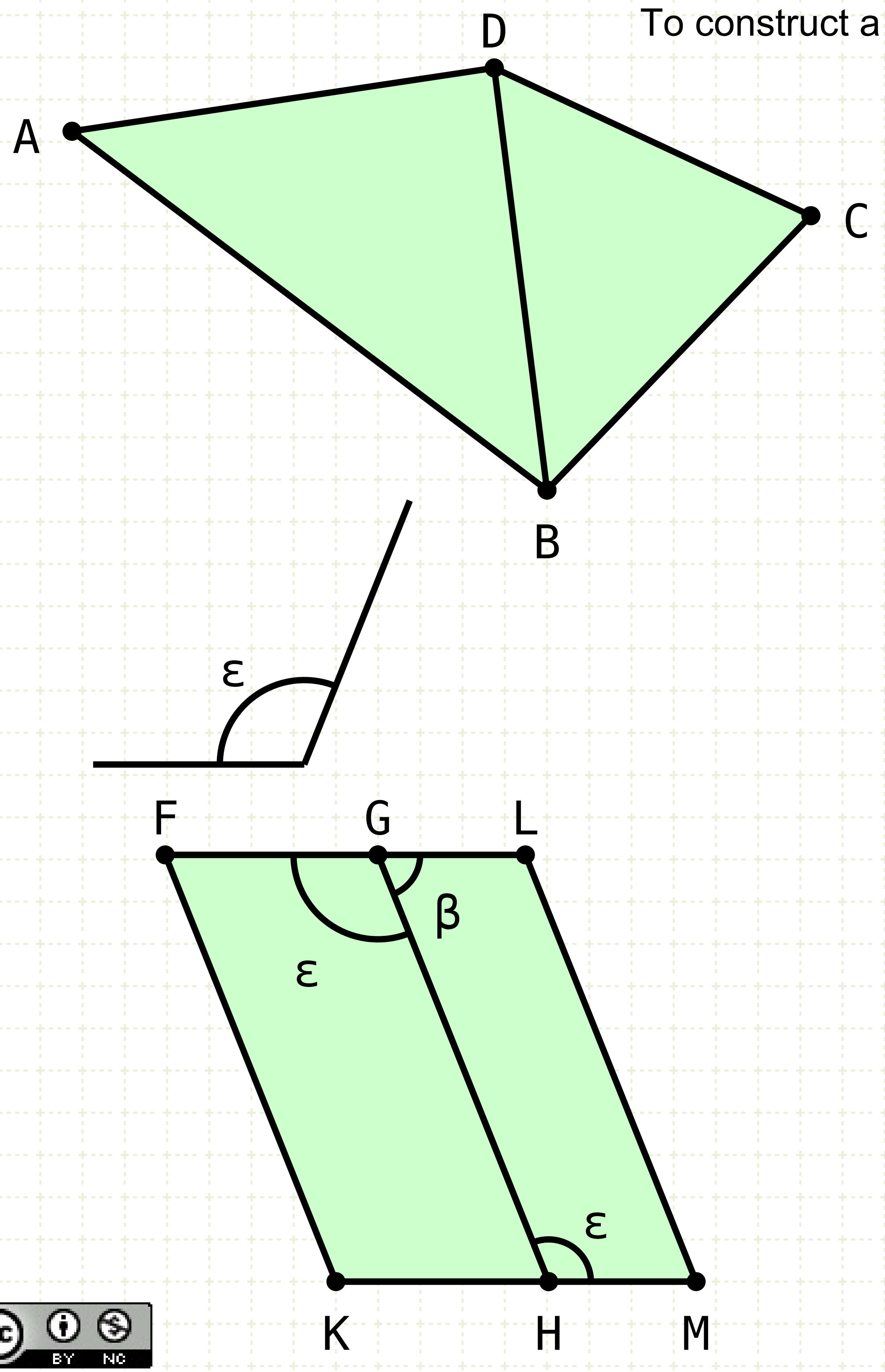
GLMH is a parallelogram by construction, so all its sides are parallel

Angles LGH and GHM sum to two right angles I·29

Since angles FGH and LGH sum to two right angles, FG is in a straight line with GL (I·14)

Proposition 45 of Book I

To construct a parallelogram equal to a given rectilinear figure in a given rectilinear angle.



ABCD = ΔABD + ΔDBC

∠FKH = ε

ΔADB = □FGHK

∠GHM = ε

ΔDBC = □GHLM

FK ∥ GH, FG ∥ KH

ε + δ = 2∟

KH,HM = KM

KM ∥ FG

GL ∥ HM, GH ∥ LM

ε + β = 2∟

FG,GL = FL

FK ∥ LM

Proof:

FGHK is a parallelogram by construction so its sides are parallel

Since FK and GH are parallel lines, angles FKH and GHK sum to two right angles (I·29)

Since angles GHK and GHM sum to two right angles, KH is in a straight line with HM (I·14)

Lines KM and FG are parallel since FGHK is a parallelogram

Thus, the alternate angles (HGF and GHM) are equal (I·29)

GLMH is a parallelogram by construction, so all its sides are parallel

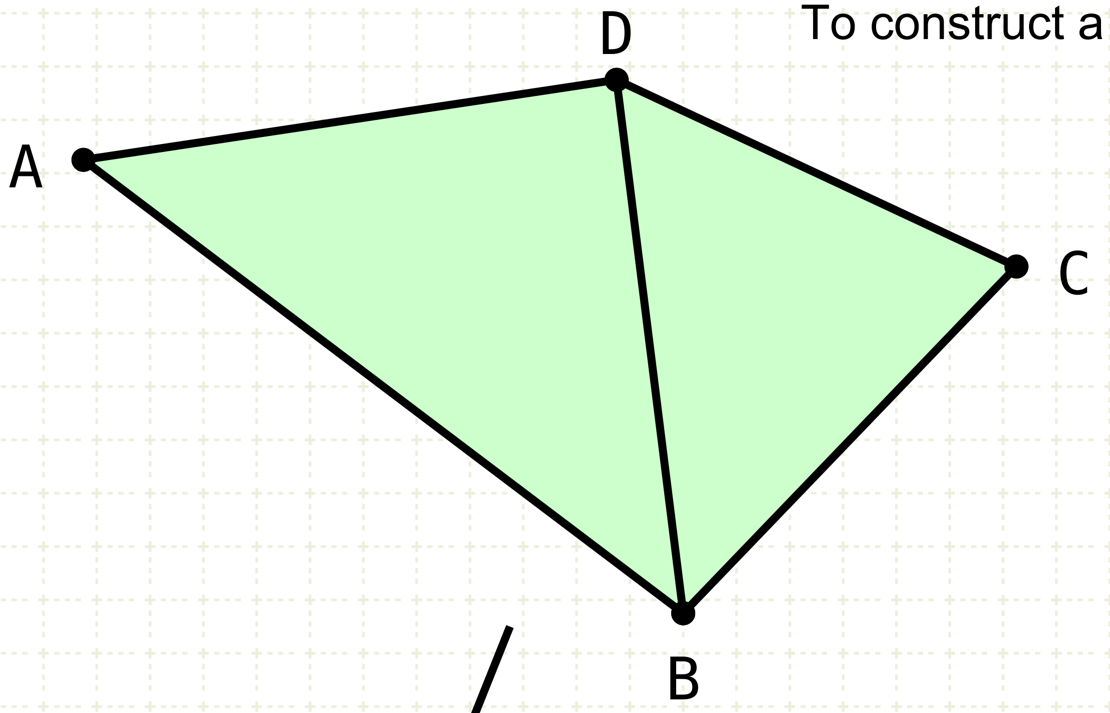
Angles LGH and GHM sum to two right angles I·29

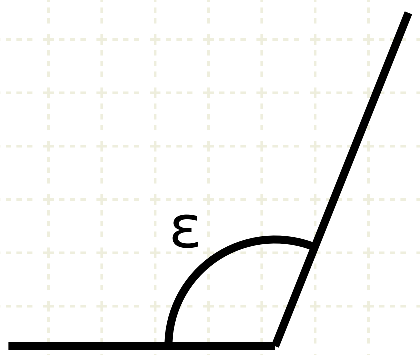
Since angles FGH and LGH sum to two right angles, FG is in a straight line with GL (I·14)

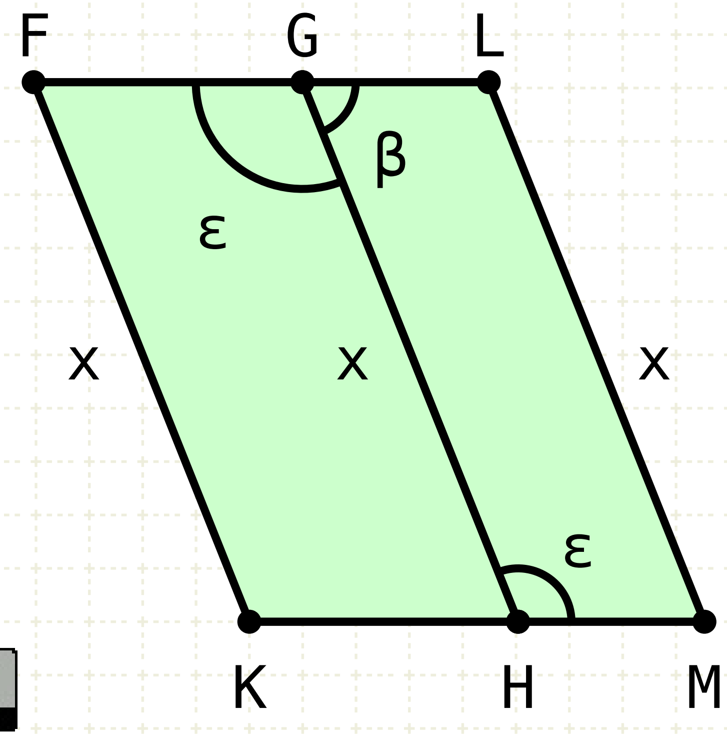
Lines FK and GH are parallel to each other, and lines GH and LM are parallel to each other, therefore lines FK and LM are parallel (I·30)

Proposition 45 of Book I

To construct a parallelogram equal to a given rectilinear figure in a given rectilinear angle.







$ABCD = \Delta ABD + \Delta DBC$

$\angle FKH = \varepsilon$

$\Delta ADB = \square FGHK$

$\angle GHM = \varepsilon$

$\Delta DBC = \square GHLM$

$FK \parallel GH,\ FG \parallel KH$

$\varepsilon + \delta = 2\llcorner$

$KH, HM = KM$

$KM \parallel FG$

$GL \parallel HM,\ GH \parallel LM$

$\varepsilon + \beta = 2\llcorner$

$FG, GL = FL$

$FK \parallel LM$

$GK = GH = LM = x$

Proof:

FGHK is a parallelogram by construction so its sides are parallel

Since FK and GH are parallel lines, angles FKH and GHK sum to two right angles (I·29)

Since angles GHK and GHM sum to two right angles, KH is in a straight line with HM (I·14)

Lines KM and FG are parallel since FGHK is a parallelogram

Thus, the alternate angles (HGF and GHM) are equal (I·29)

GLMH is a parallelogram by construction, so all its sides are parallel

Angles LGH and GHM sum to two right angles I·29

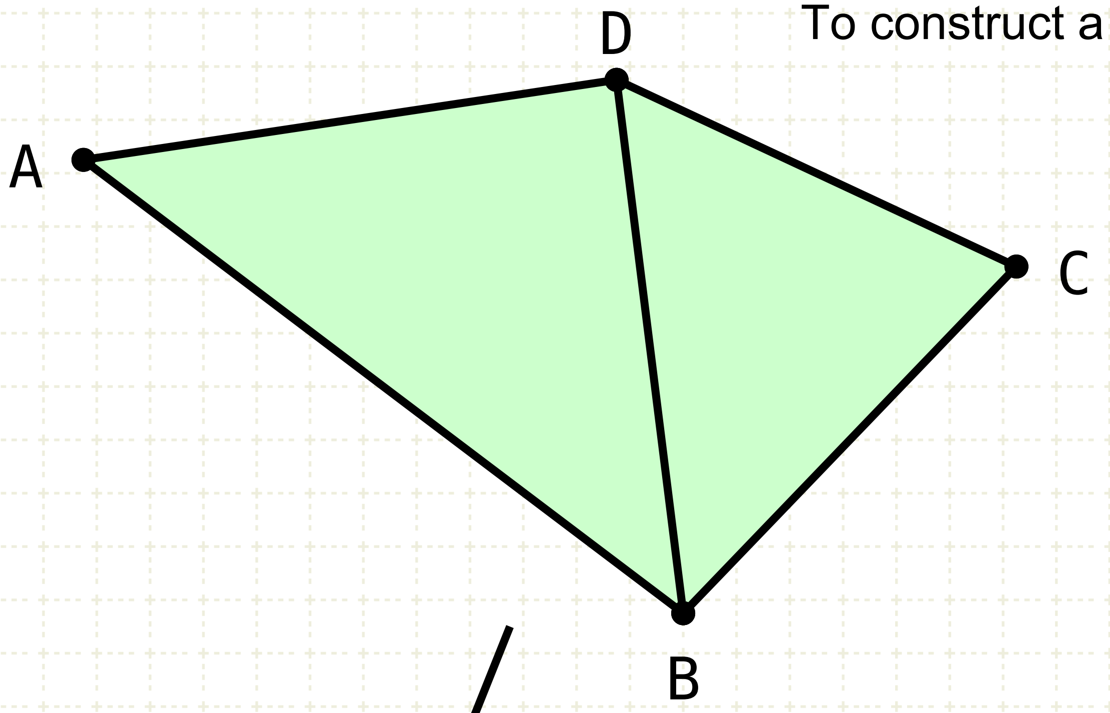
Since angles FGH and LGH sum to two right angles, FG is in a straight line with GL (I·14)

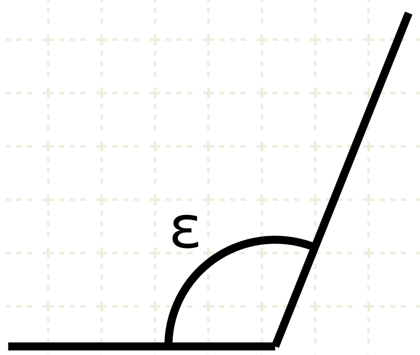
Lines FK and GH are parallel to each other, and lines GH and LM are parallel to each other, therefore lines FK and LM are parallel (I·30)

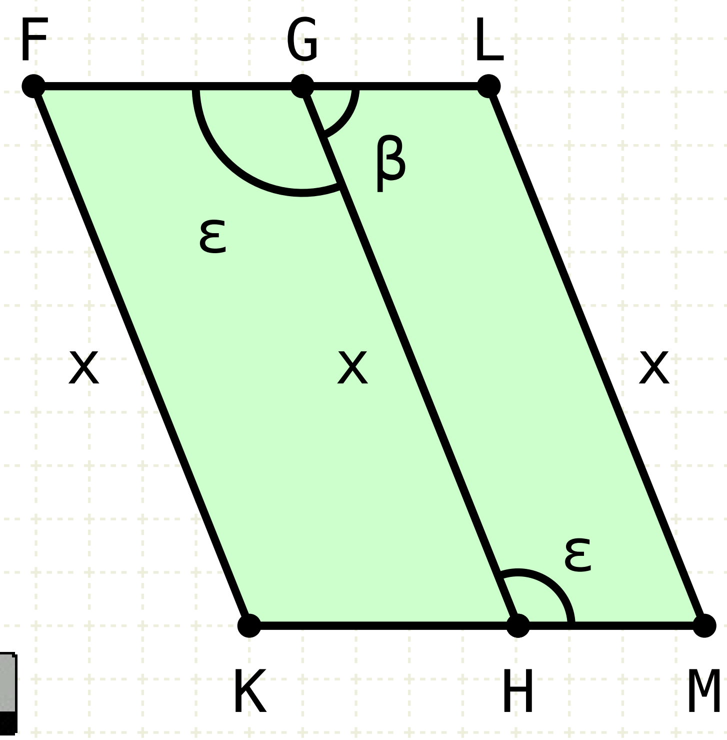
By their construction, lines GK and GH and LM are all of equal length

Proposition 45 of Book I

To construct a parallelogram equal to a given rectilinear figure in a given rectilinear angle.







ABCD = ΔABD + ΔDBC

∠FKH = ε

ΔADB = □FGHK

∠GHM = ε

ΔDBC = □GHLM

FK ∥ GH, FG ∥ KH

ε + δ = 2∟

KH,HM = KM

KM ∥ FG

GL ∥ HM, GH ∥ LM

ε + β = 2∟

FG,GL = FL

FK ∥ LM

GK = GH = LM = x

FL = KM   FL ∥ KM

Proof:

FGHK is a parallelogram by construction so its sides are parallel

Since FK and GH are parallel lines, angles FKH and GHK sum to two right angles (I·29)

Since angles GHK and GHM sum to two right angles, KH is in a straight line with HM (I·14)

Lines KM and FG are parallel since FGHK is a parallelogram

Thus, the alternate angles (HGF and GHM) are equal (I·29)

GLMH is a parallelogram by construction, so all its sides are parallel

Angles LGH and GHM sum to two right angles I·29

Since angles FGH and LGH sum to two right angles, FG is in a straight line with GL (I·14)

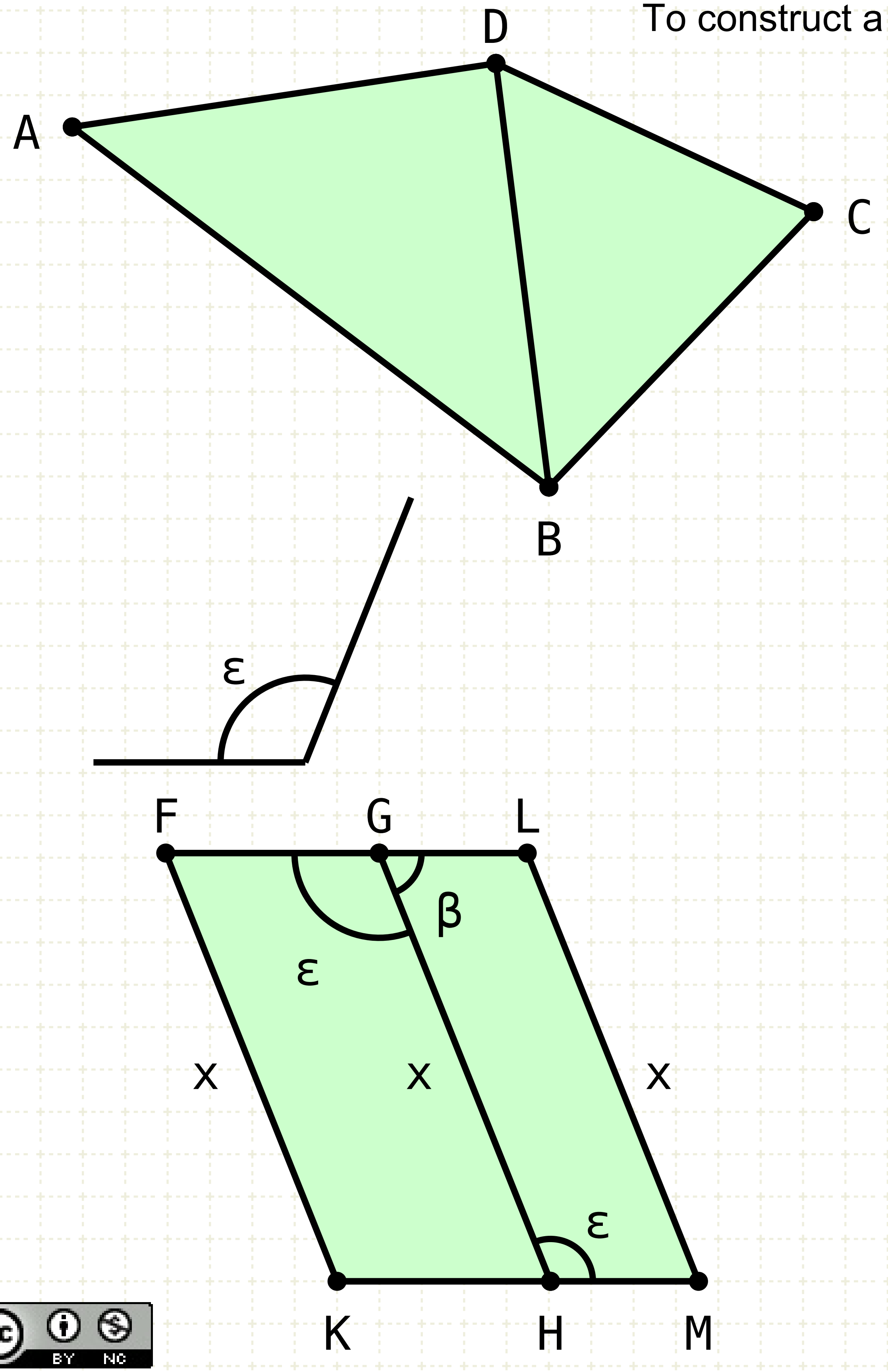
Lines FK and GH are parallel to each other, and lines GH and LM are parallel to each other, therefore lines FK and LM are parallel (I·30)

By their construction, lines GK and GH and LM are all of equal length

Lines joined to the extremities of equal parallel lines are themselves equal and parallel (I·33)

Proposition 45 of Book I

To construct a parallelogram equal to a given rectilinear figure in a given rectilinear angle.



$ABCD = \Delta ABD + \Delta DBC$

$\angle FKH = \varepsilon$

$\Delta ADB = \square FGHK$

$\angle GHM = \varepsilon$

$\Delta DBC = \square GHLM$

$FK \parallel GH,\ FG \parallel KH$

$\varepsilon + \delta = 2\llcorner$

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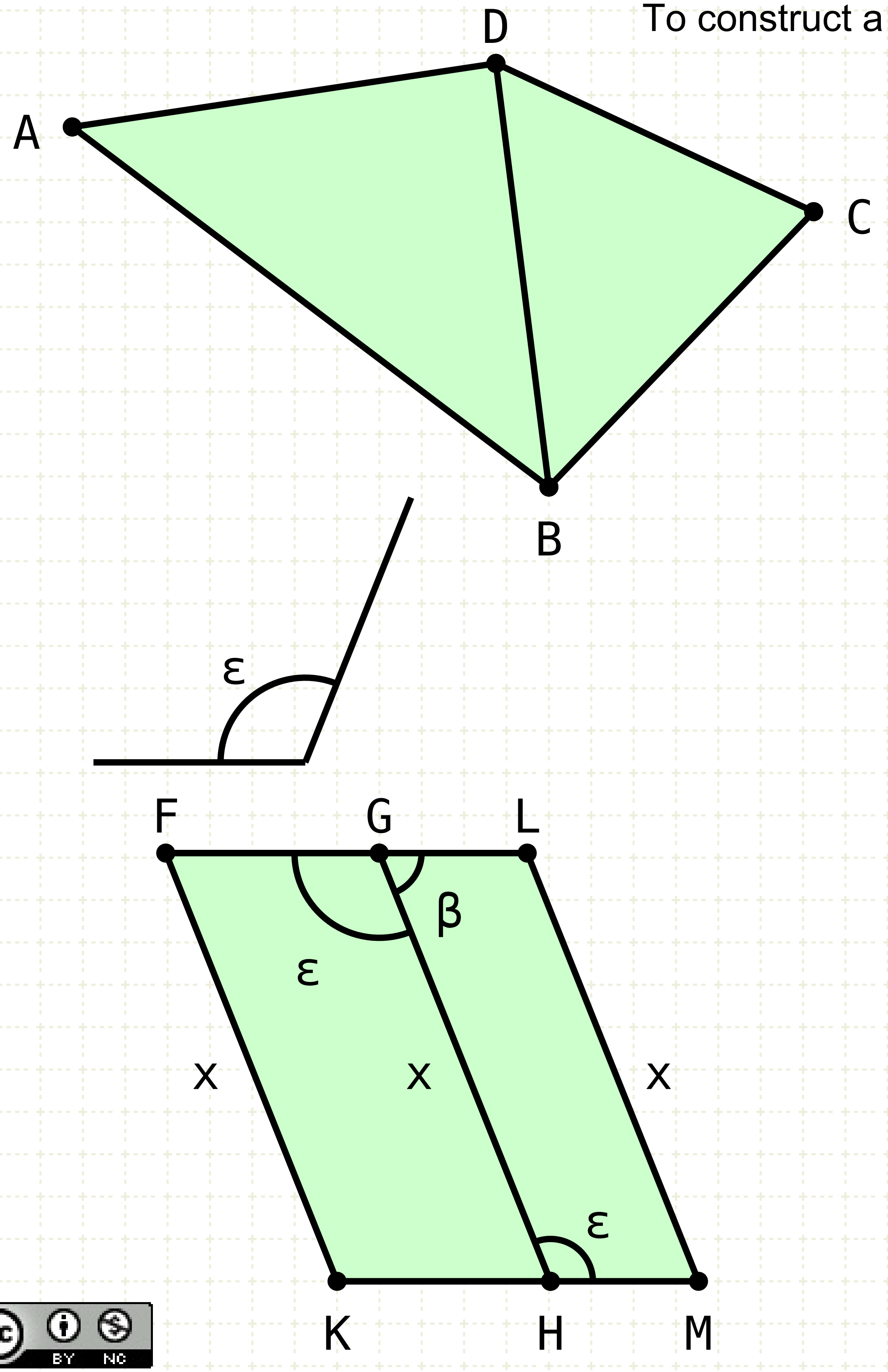

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Proposition 45 of Book I

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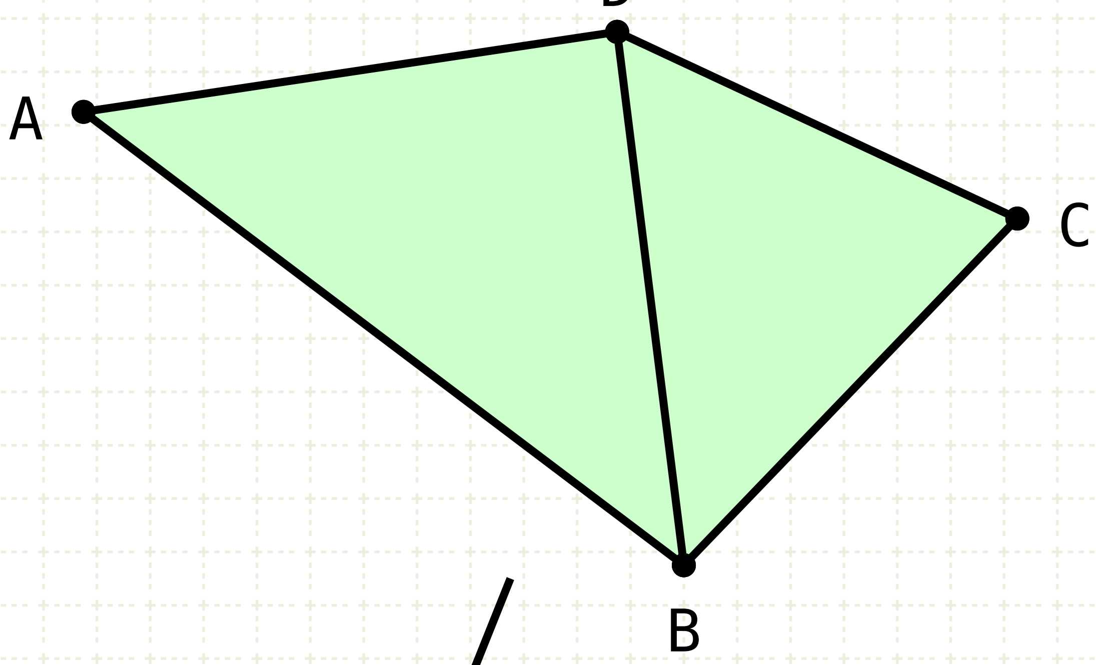

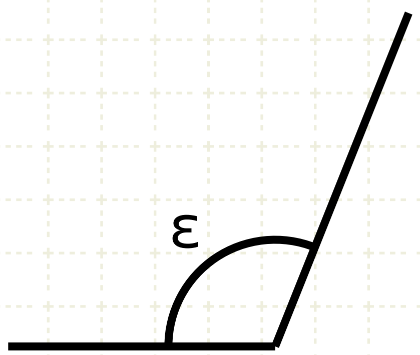

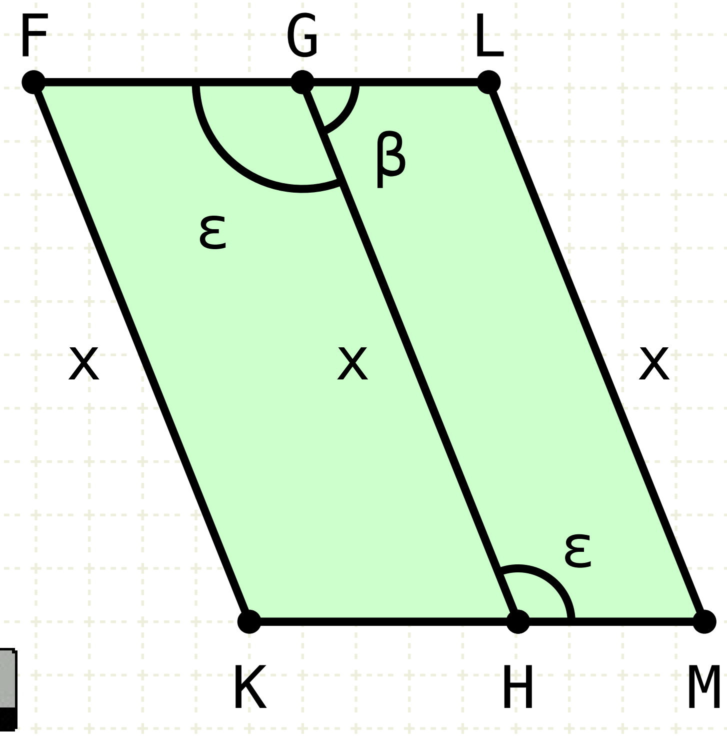

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