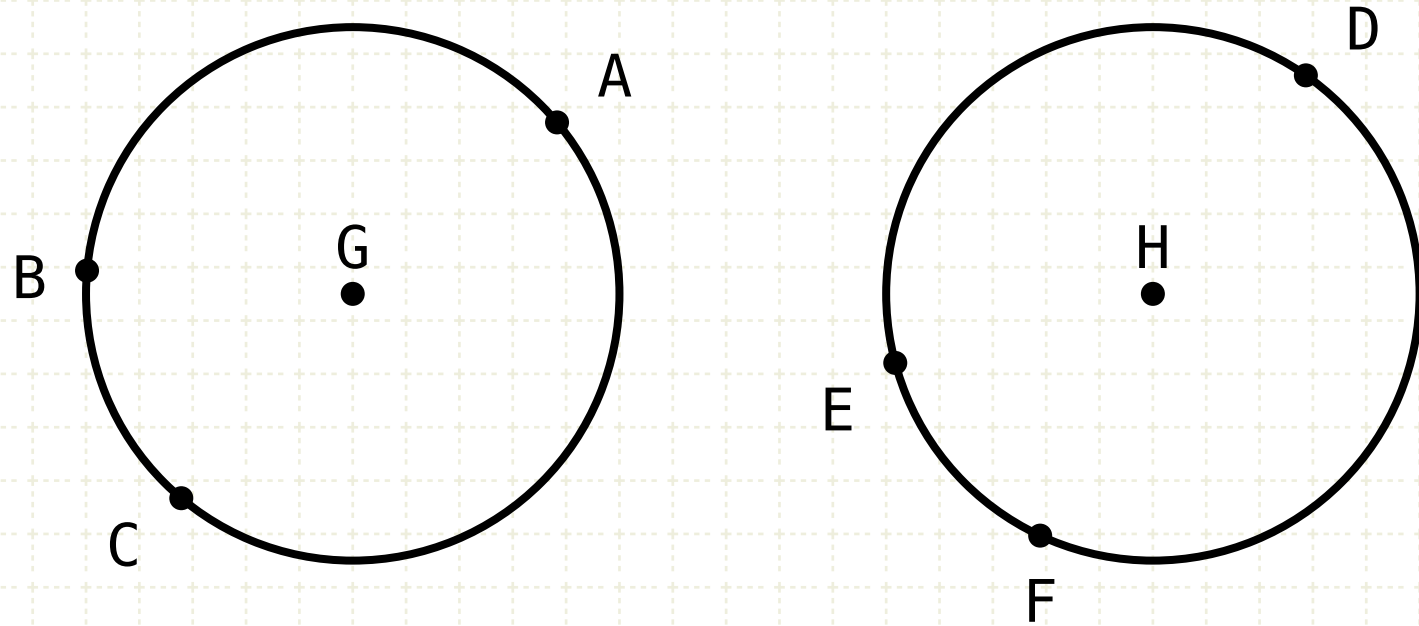


Proposition 33 of Book VI

In equal circles angles have the same ratio as the circumferences on which they stand, whether they stand at the centres or at the circumferences

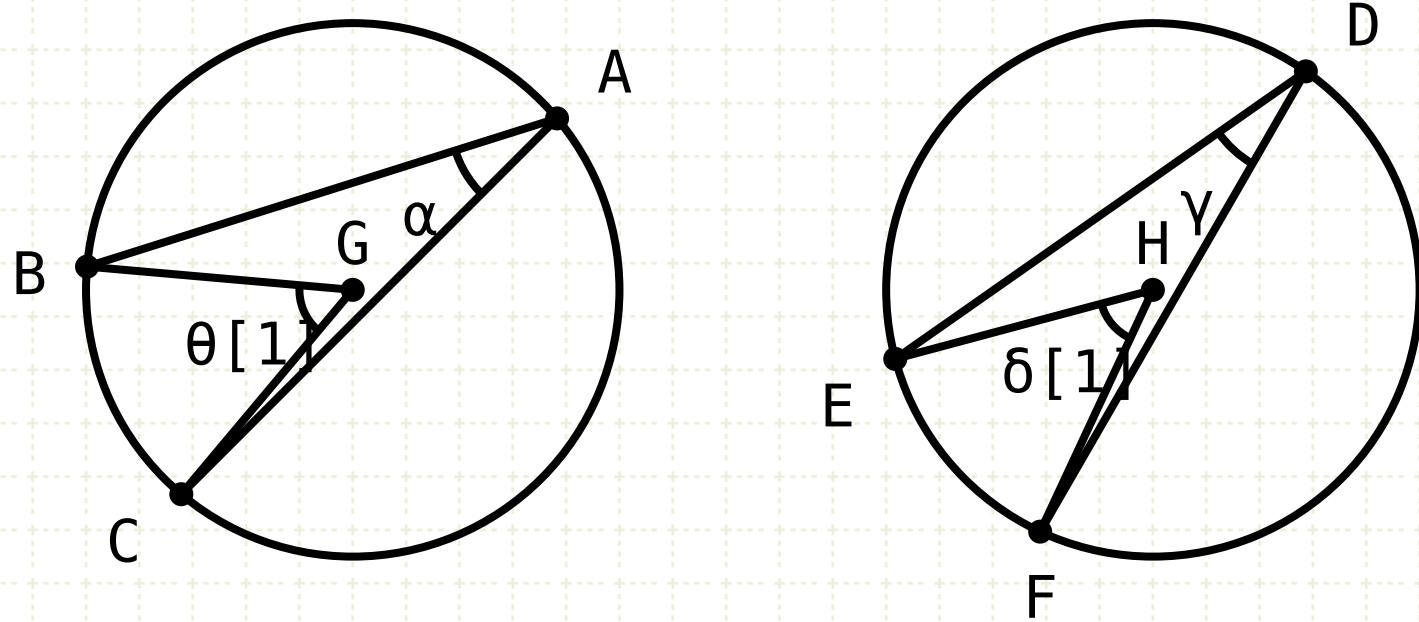


In other words

Given two circles ABC,DEF with centres G and H respectively

Proposition 33 of Book VI

In equal circles angles have the same ratio as the circumferences on which they stand, whether they stand at the centres or at the circumferences

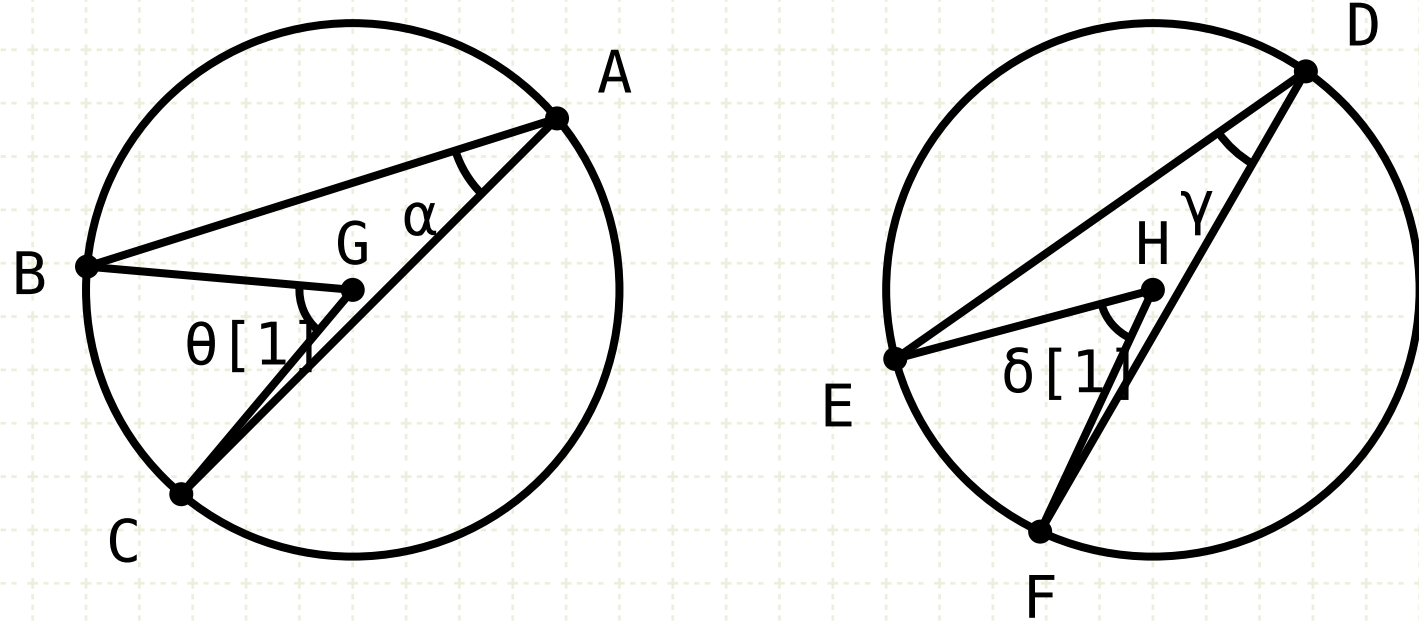


In other words

Given two circles ABC,DEF with centres G and H respectively
Let the angles BAC, BGC and EDF and EHF be drawn

Proposition 33 of Book VI

In equal circles angles have the same ratio as the circumferences on which they stand, whether they stand at the centres or at the circumferences



$$BC : EF = \theta_1 : \delta_1 = \alpha : \gamma$$

In other words

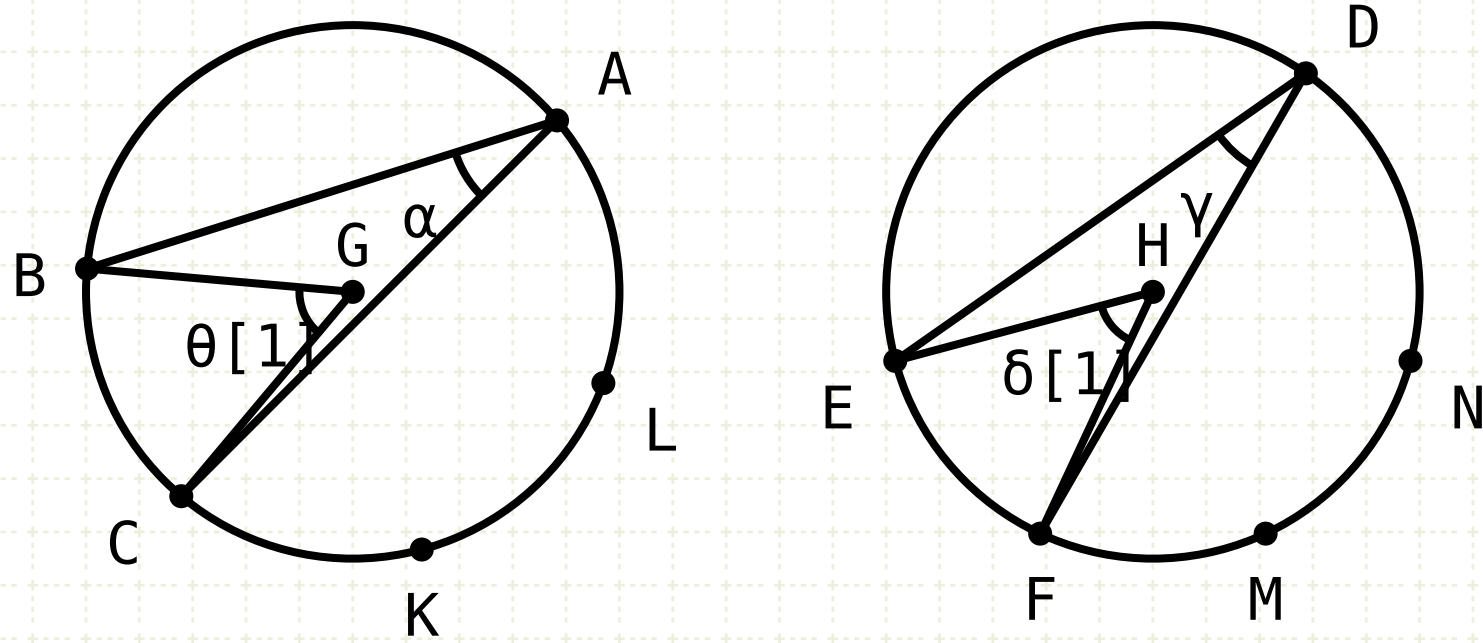
Given two circles ABC,DEF with centres G and H respectively

Let the angles BAC, BGC and EDF and EHF be drawn

Then the ratio of the circumferences BC to EF is equal to the ratios of BGC to EHF and BAC to EDF

Proposition 33 of Book VI

In equal circles angles have the same ratio as the circumferences on which they stand, whether they stand at the centres or at the circumferences



$$\begin{aligned} BC &= CK = KL \\ EF &= EM = MN \end{aligned}$$

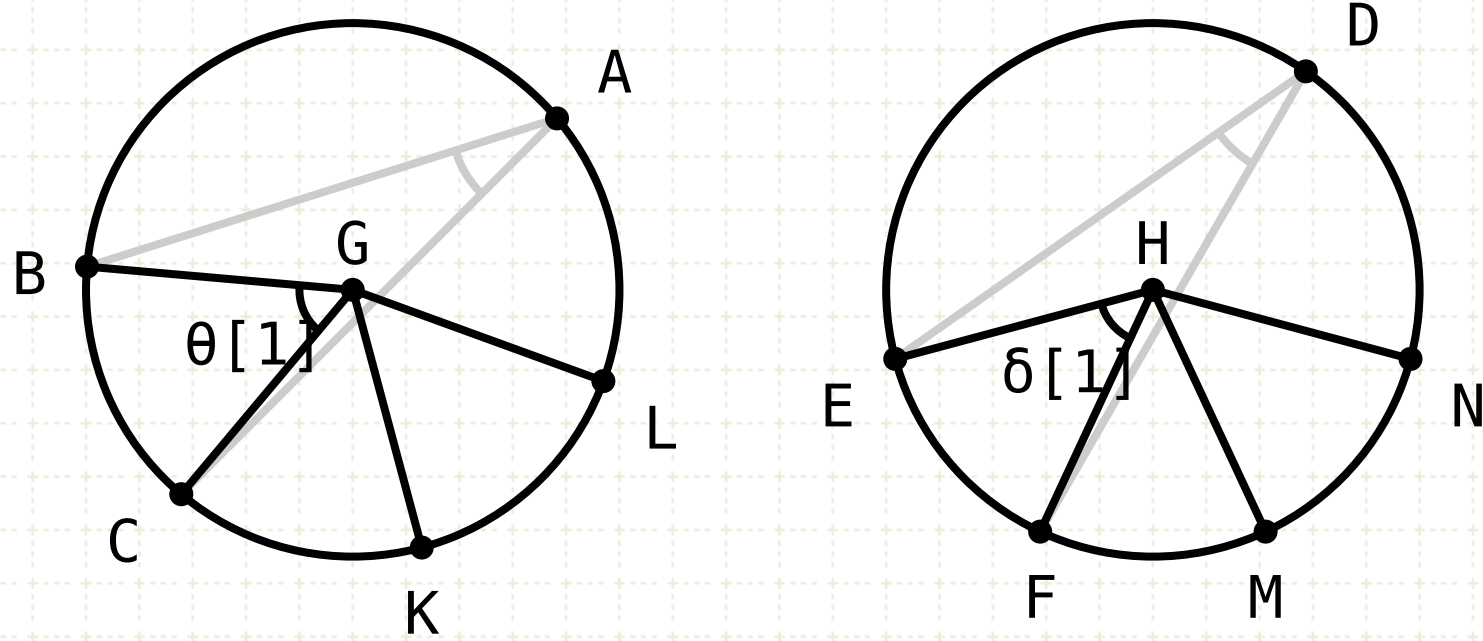
Proof

Create an number of consecutive circumferences CK, KL equal to BC

And create and number of consecutive circumferences FM, MN equal to EF

Proposition 33 of Book VI

In equal circles angles have the same ratio as the circumferences on which they stand, whether they stand at the centres or at the circumferences



Proof

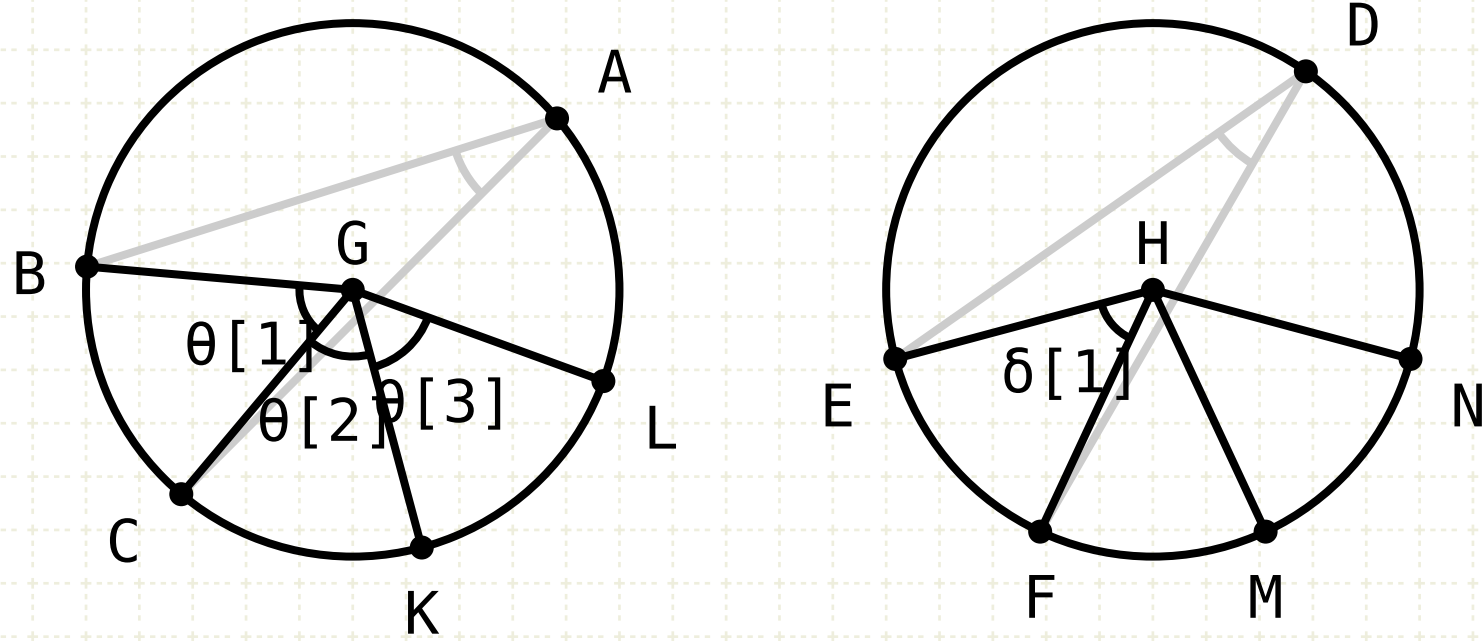
Create an number of consecutive circumferences CK, KL equal to BC

And create and number of consecutive circumferences FM, MN equal to EF

And draw the lines GK, GL, HM, HN

Proposition 33 of Book VI

In equal circles angles have the same ratio as the circumferences on which they stand, whether they stand at the centres or at the circumferences



$$\theta_1 = \theta_2 = \theta_3$$

Proof

Create an number of consecutive circumferences CK, KL equal to BC

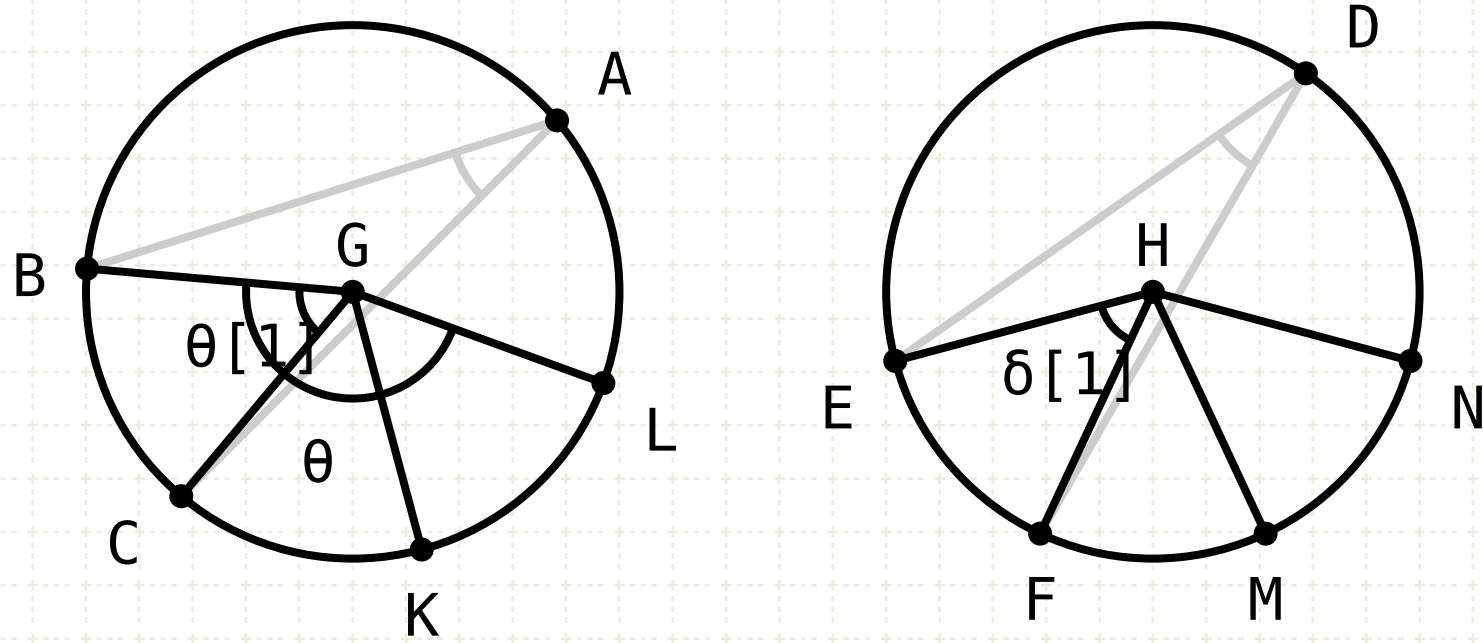
And create and number of consecutive circumferences FM, MN equal to EF

And draw the lines GK, GL, HM, HN

And since BC, CK, KL are equal, so are the angles BGC, CGK, KGL (III·27)

Proposition 33 of Book VI

In equal circles angles have the same ratio as the circumferences on which they stand, whether they stand at the centres or at the circumferences



$$\begin{aligned}\theta_1 &= \theta_2 = \theta_3 \\ BL &= n \cdot BC \\ \theta &= n \cdot \theta_1\end{aligned}$$

Proof

Create an number of consecutive circumferences CK, KL equal to BC

And create and number of consecutive circumferences FM, MN equal to EF

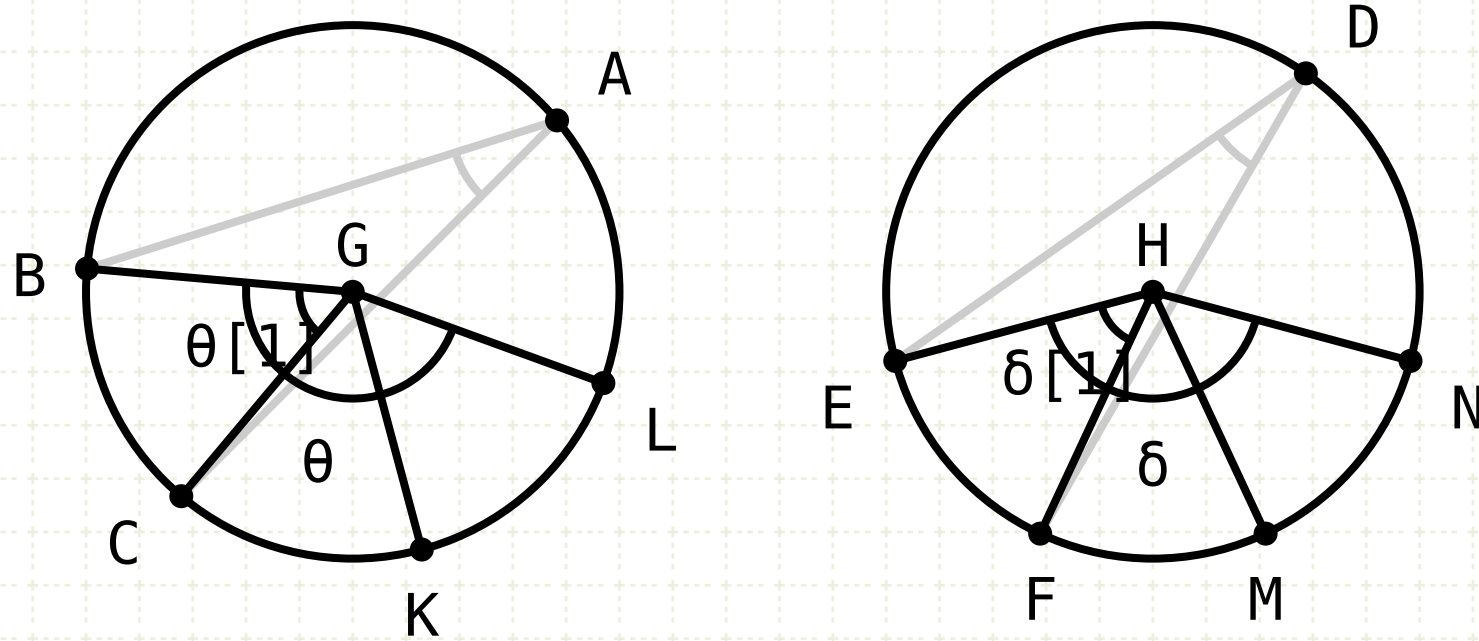
And draw the lines GK, GL, HM, HN

And since BC, CK, KL are equal, so are the angles BGC, CGK, KGL (III·27)

Thus, whatever multiple BL is of BC, it is the same multiple that BGL is to BGC

Proposition 33 of Book VI

In equal circles angles have the same ratio as the circumferences on which they stand, whether they stand at the centres or at the circumferences



$$\begin{aligned}\theta_1 &= \theta_2 = \theta_3 \\ BL &= n \cdot BC \\ \theta &= n \cdot \theta_1 \\ EN &= m \cdot EF \\ \delta &= m \cdot \delta_1\end{aligned}$$

Proof

Create an number of consecutive circumferences CK, KL equal to BC

And create and number of consecutive circumferences FM, MN equal to EF

And draw the lines GK, GL, HM, HN

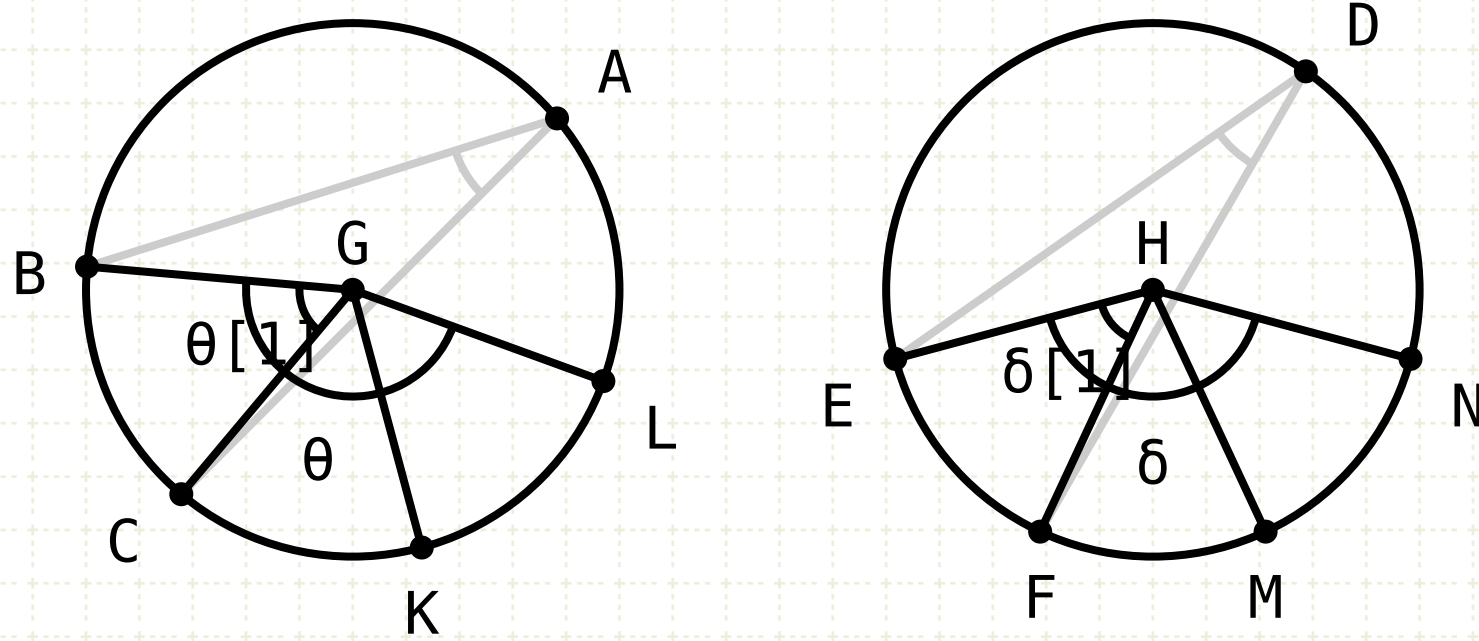
And since BC, CK, KL are equal, so are the angles BGC, CGK, KGL (III·27)

Thus, whatever multiple BL is of BC, it is the same multiple that BGL is to BGC

For the same reason, whatever multiple EN is of EF, it is the same multiple that EHN is to EHF

Proposition 33 of Book VI

In equal circles angles have the same ratio as the circumferences on which they stand, whether they stand at the centres or at the circumferences



$$\begin{aligned}\theta_1 &= \theta_2 = \theta_3 \\ BL &= n \cdot BC \\ \theta &= n \cdot \theta_1 \\ EN &= m \cdot EF \\ \delta &= m \cdot \delta_1 \\ BL &\Leftrightarrow EN \rightarrow \theta \Leftrightarrow \delta\end{aligned}$$

Proof

Create an number of consecutive circumferences CK, KL equal to BC

And create and number of consecutive circumferences FM, MN equal to EF

And draw the lines GK, GL, HM, HN

And since BC, CK, KL are equal, so are the angles BGC, CGK, KGL (III·27)

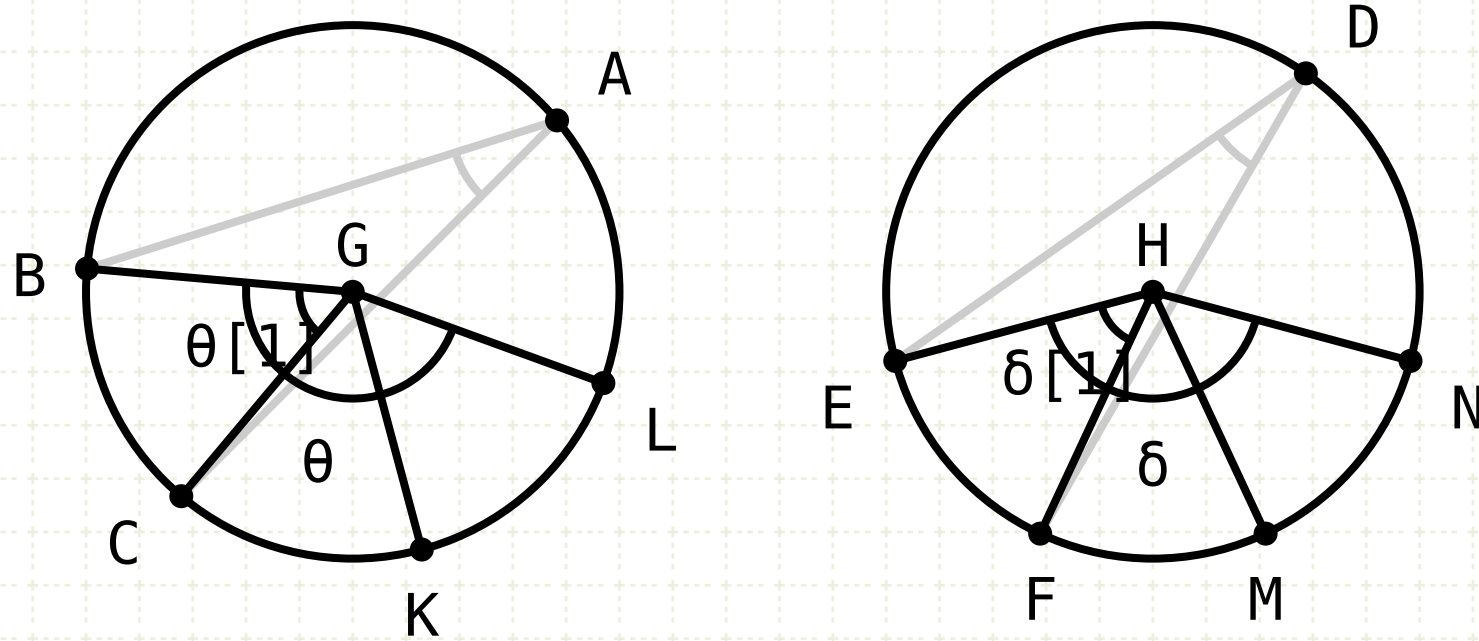
Thus, whatever multiple BL is of BC, it is the same multiple that BGL is to BGC

For the same reason, whatever multiple EN is of EF, it is the same multiple that EHN is to EHF

Equal circumferences on equal circles have equal angles (III·27), thus if BL is greater than EN, BGL will also be greater than EHN, if equal then equal, if less than, then less

Proposition 33 of Book VI

In equal circles angles have the same ratio as the circumferences on which they stand, whether they stand at the centres or at the circumferences



$$\theta_1 = \theta_2 = \theta_3$$

$$BL = n \cdot BC$$

$$\theta = n \cdot \theta_1$$

$$EN = m \cdot EF$$

$$\delta = m \cdot \delta_1$$

$$BL \Leftrightarrow EN \rightarrow \theta \Leftrightarrow \delta$$

$$n \cdot BC \Leftrightarrow m \cdot EF \rightarrow n \cdot \theta_1 \Leftrightarrow m \cdot \delta_1$$

Proof

Create an number of consecutive circumferences CK, KL equal to BC

And create and number of consecutive circumferences FM, MN equal to EF

And draw the lines GK, GL, HM, HN

And since BC, CK, KL are equal, so are the angles BGC, CGK, KGL (III·27)

Thus, whatever multiple BL is of BC, it is the same multiple that BGL is to BGC

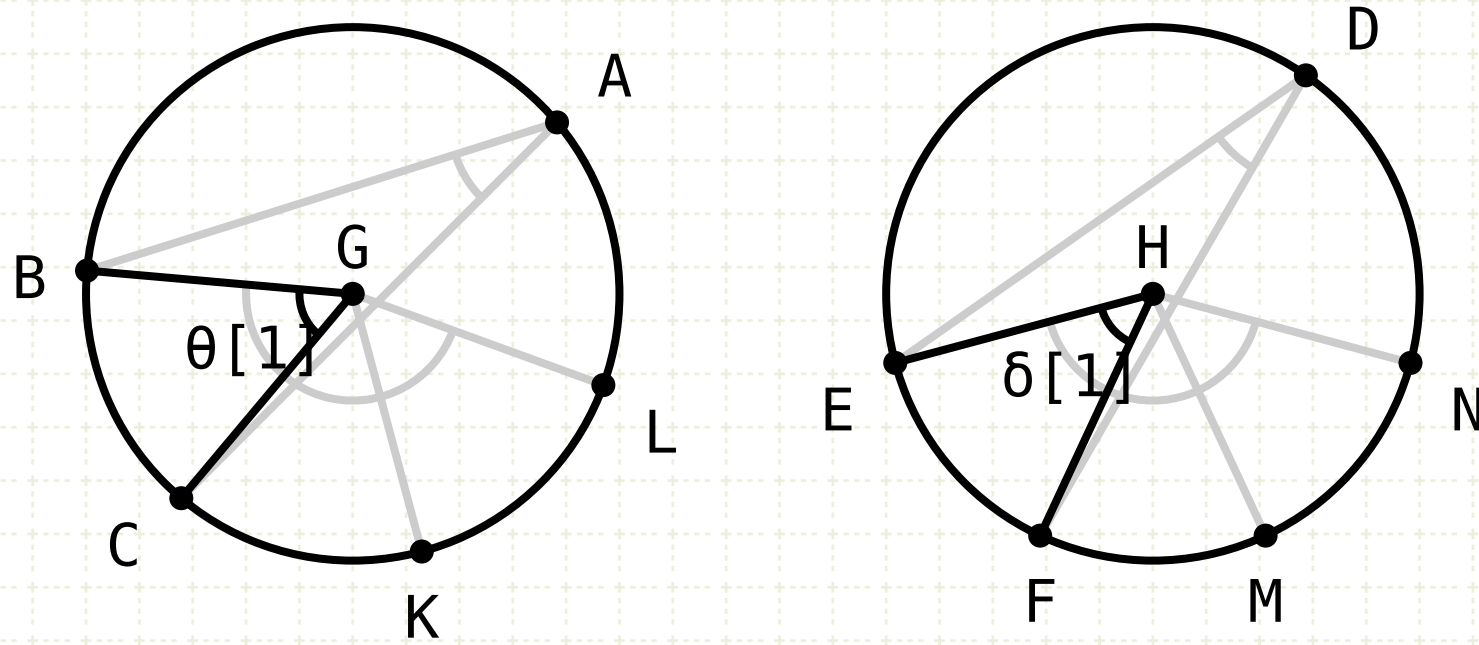
For the same reason, whatever multiple EN is of EF, it is the same multiple that EHN is to EHF

Equal circumferences on equal circles have equal angles (III·27), thus if BL is greater than EN, BGL will also be greater than EHN, if equal then equal, if less than, then less

But BL, EN are equimultiples of BGC, EHF, so substitute them in the previous equation

Proposition 33 of Book VI

In equal circles angles have the same ratio as the circumferences on which they stand, whether they stand at the centres or at the circumferences



$$\theta_1 = \theta_2 = \theta_3$$

$$BL = n \cdot BC$$

$$\theta = n \cdot \theta_1$$

$$EN = m \cdot EF$$

$$\delta = m \cdot \delta_1$$

$$BL \Leftrightarrow EN \rightarrow \theta \Leftrightarrow \delta$$

$$n \cdot BC \Leftrightarrow m \cdot EF \rightarrow n \cdot \theta_1 \Leftrightarrow m \cdot \delta_1$$

$$BC : EF = \theta_1 : \delta_1$$

Proof

Create an number of consecutive circumferences CK, KL equal to BC

And create and number of consecutive circumferences FM, MN equal to EF

And draw the lines GK, GL, HM, HN

And since BC, CK, KL are equal, so are the angles BGC, CGK, KGL (III·27)

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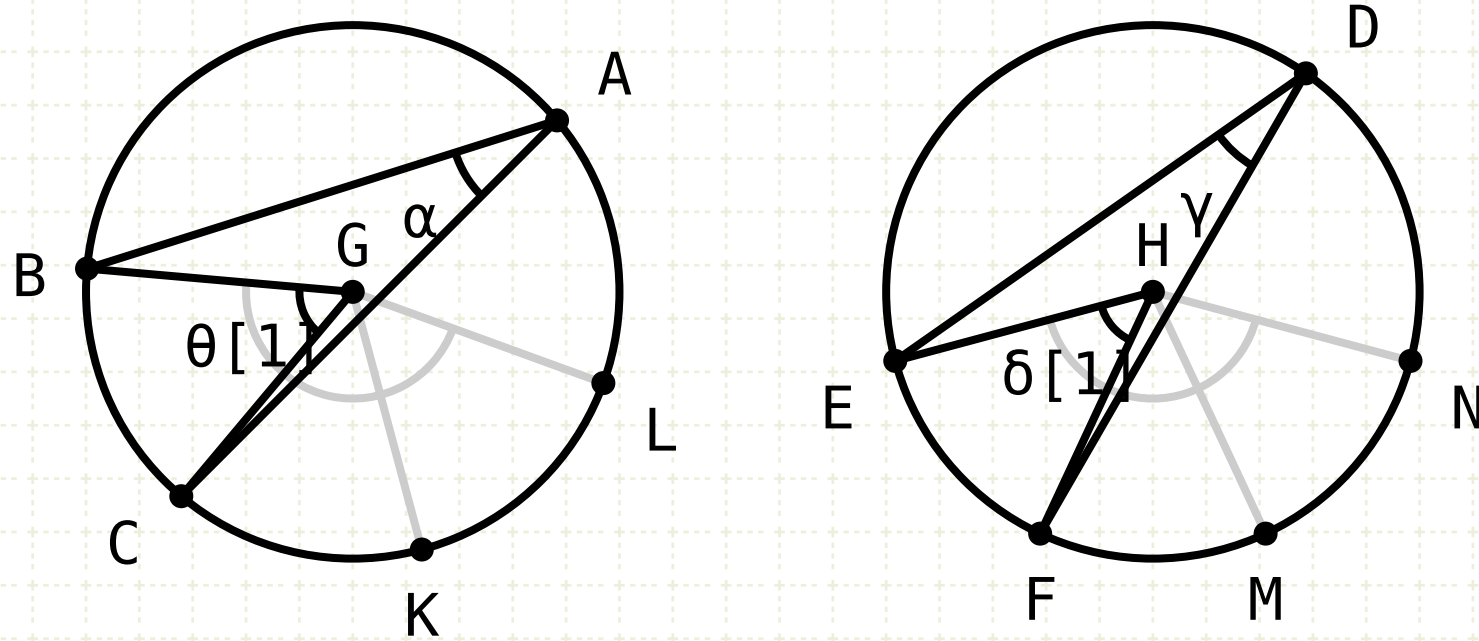
But BL, EN are equimultiples of BGC, EHF, so substitute them in the previous equation

Which in turn is the definition for equal ratios



Proposition 33 of Book VI

In equal circles angles have the same ratio as the circumferences on which they stand, whether they stand at the centres or at the circumferences



$$\theta_1 = \theta_2 = \theta_3$$

$$BL = n \cdot BC$$

$$\theta = n \cdot \theta_1$$

$$EN = m \cdot EF$$

$$\delta = m \cdot \delta_1$$

$$BL \Leftrightarrow EN \rightarrow \theta \Leftrightarrow \delta$$

$$n \cdot BC \Leftrightarrow m \cdot EF \rightarrow n \cdot \theta_1 \Leftrightarrow m \cdot \delta_1$$

$$BC : EF = \theta_1 : \delta_1$$

$$BC : EF = \alpha : \gamma$$

Proof

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And since BC, CK, KL are equal, so are the angles BGC, CGK, KGL (III·27)

Thus, whatever multiple BL is of BC, it is the same multiple that BGL is to BGC

For the same reason, whatever multiple EN is of EF, it is the same multiple that EHN is to EHF

Equal circumferences on equal circles have equal angles (III·27), thus if BL is greater than EN, BGL will also be greater than EHN, if equal then equal, if less than, then less

But BL, EN are equimultiples of BGC, EHF, so substitute them in the previous equation

Which in turn is the definition for equal ratios

And since BAC is half BGC, and EDF is half EHF, the ratio of BC, EF is also equal to the angles BAC, EHF



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