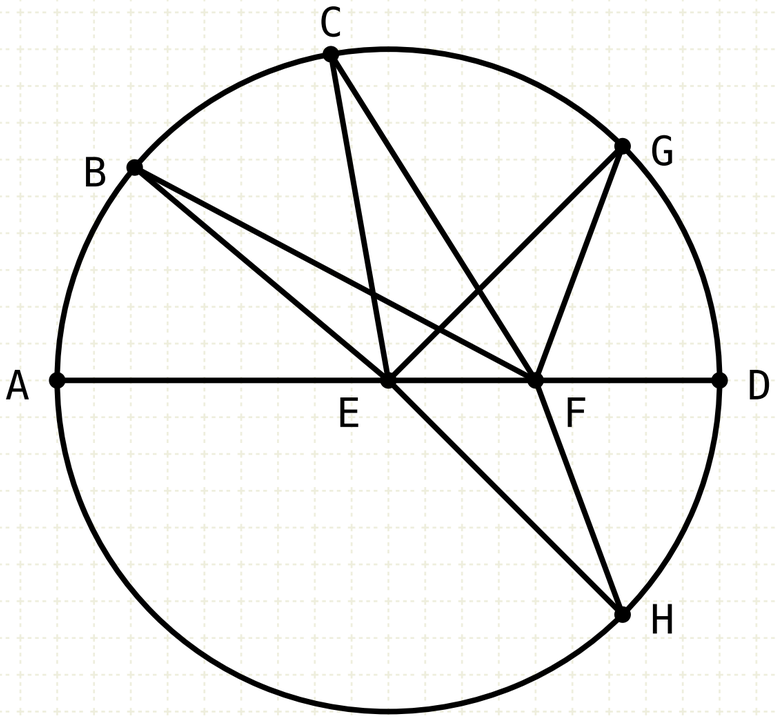
Euclid's Elements

Book III



A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)

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Proposition 23 of Book III

On the same straight line there cannot be constructed two similar and unequal segments of circles on the same side

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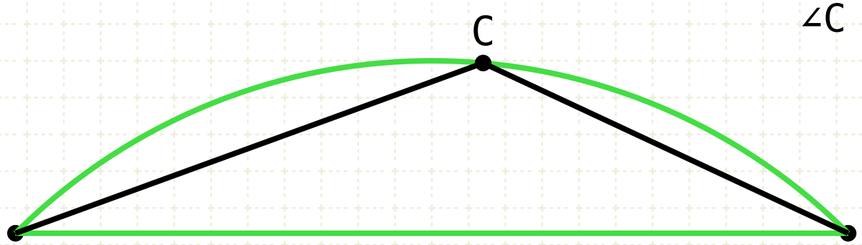
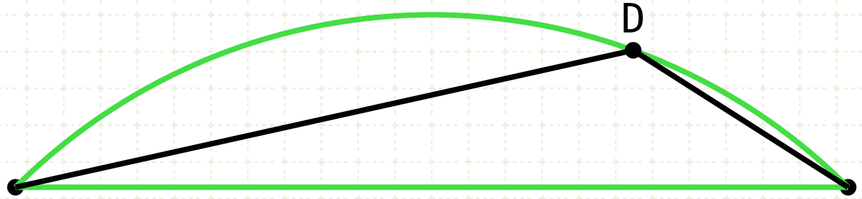


∠C = ∠D

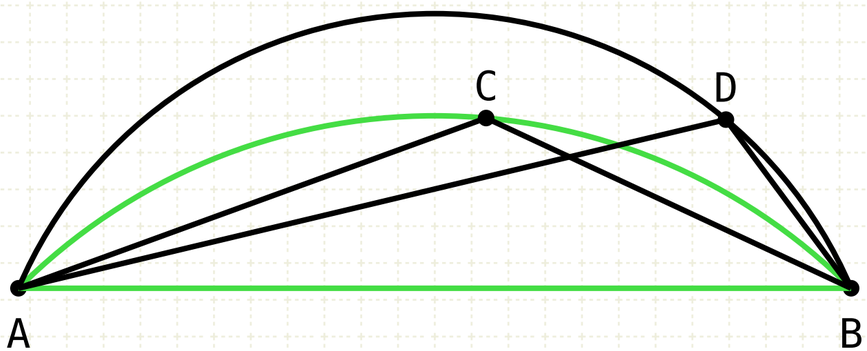


Definition - Similar segments of circles

'Similar segments of circles' are those which admit equal angles, or in which the angles are equal to one another

Proposition 23 of Book III

On the same straight line there cannot be constructed two similar and unequal segments of circles on the same side



In other words

It is impossible to construct a figure (as shown) where

- the two segments of the circle are not equal, and
- any angle C on the inner circle is equal to any angle D on the outer circle

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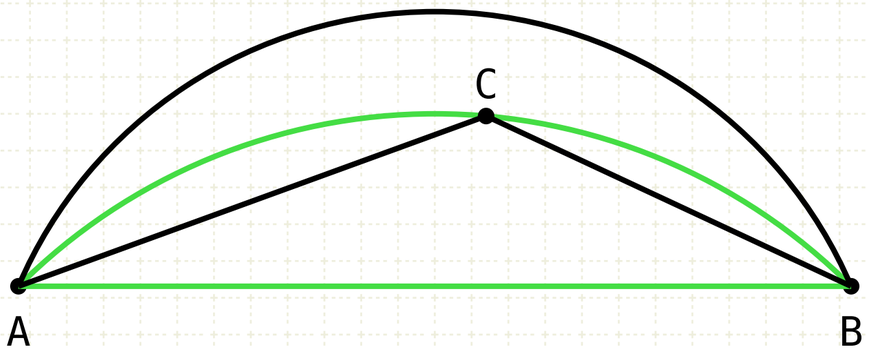

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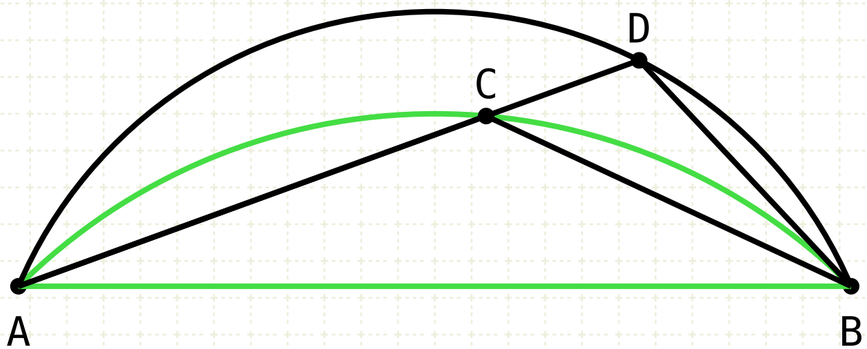

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Proof

Extend line AC such that it intersects the larger circle at point D, and draw a line from D to B

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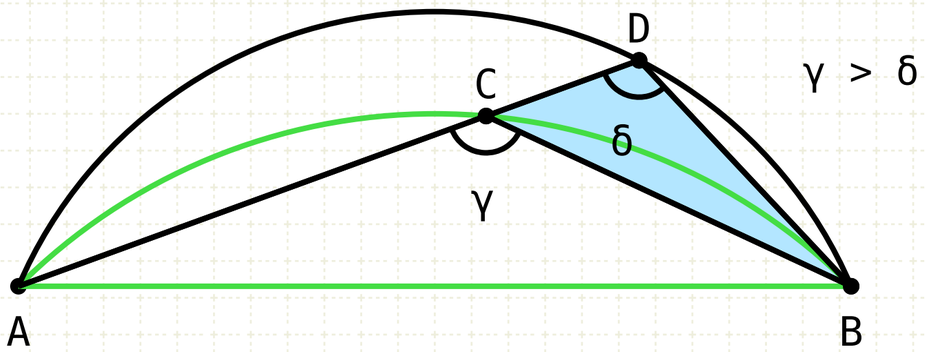

γ > δ

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Consider the angles ACB (γ) and CDB (δ)

The angle γ is exterior to the triangle CDB, and thus is larger than any interior/opposite angle (I·16)

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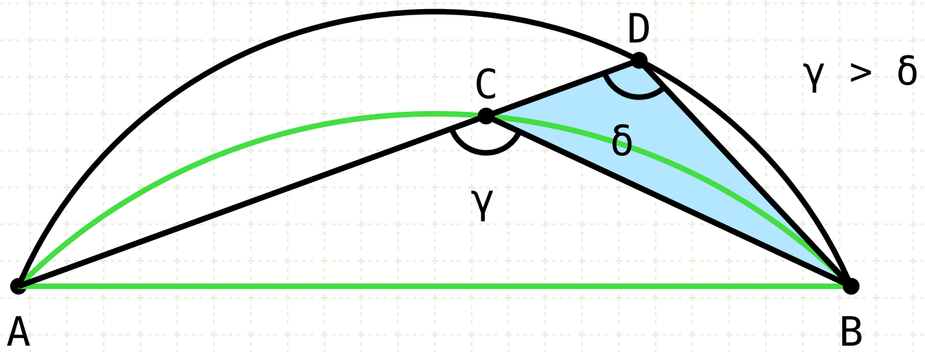

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THUS, the two circle segments cannot be similar, because γ cannot be both equal and greater than δ

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