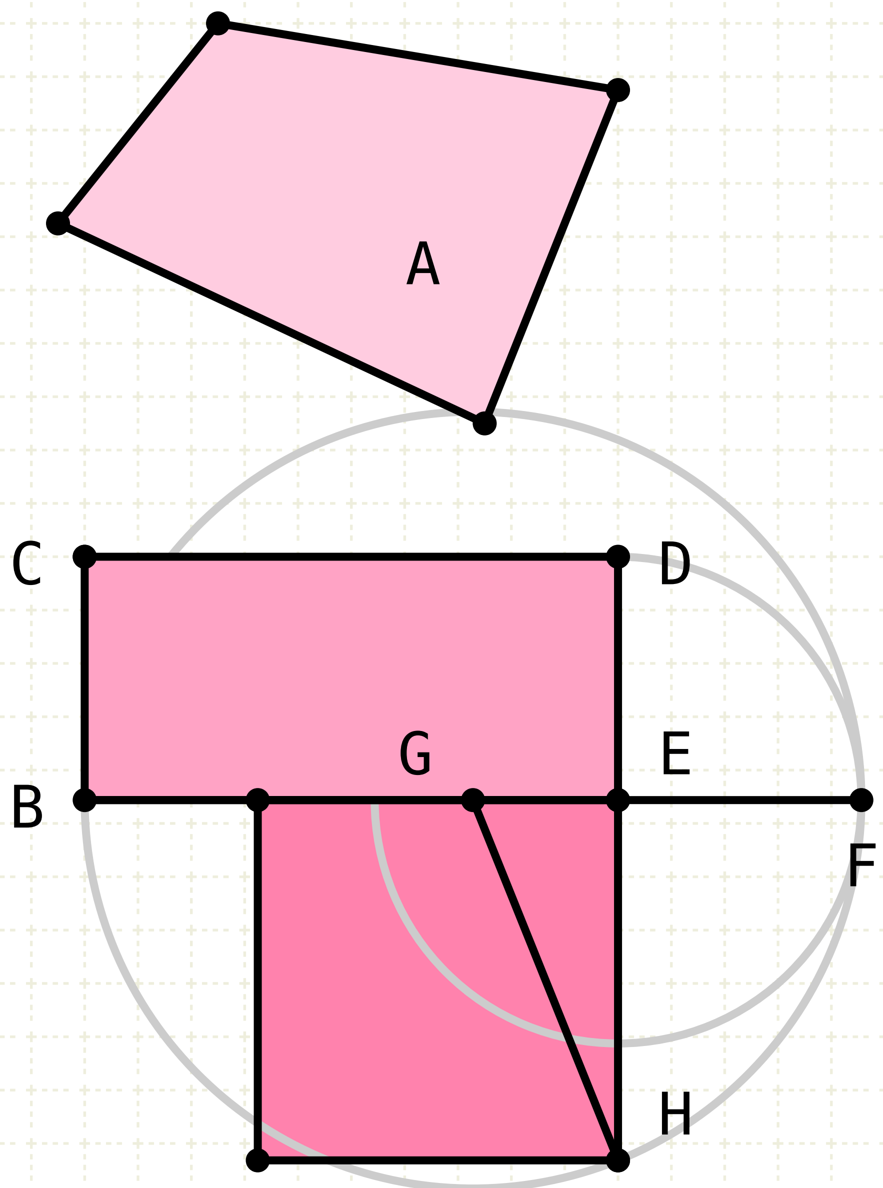
Euclid's Elements

Book II



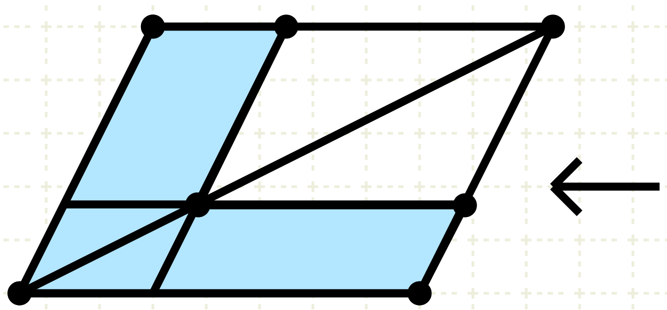
It is a remarkable fact in the history of geometry, that the Elements of Euclid, written two thousand years ago, are still regarded by many as the best introduction to the mathematical sciences.

Florian Cajori,

A History of Mathematics (1893)

Definitions:

Any rectangular parallelogram is said to be contained by the two straight lines containing the right angle.



And in any parallelogrammic area let any one whatever of the parallelograms about its diameter with the two complements be called a gnomon.

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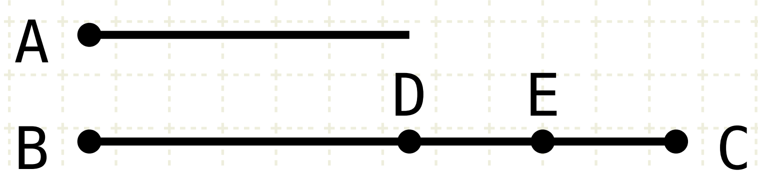
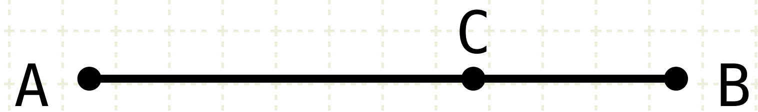


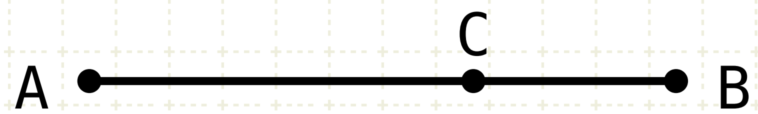
$A \cdot BC = A \cdot BD + A \cdot DE + A \cdot EC$

2



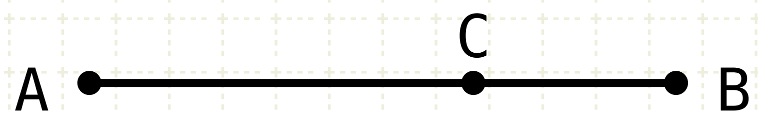
$AB^2 = AB \cdot AC + AB \cdot BC$

3



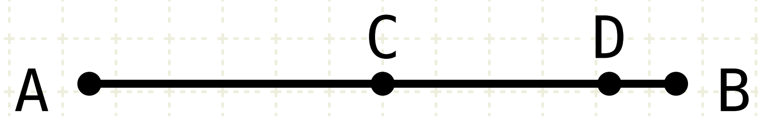
$AB \cdot CB = AC \cdot CB + CB^2$

4



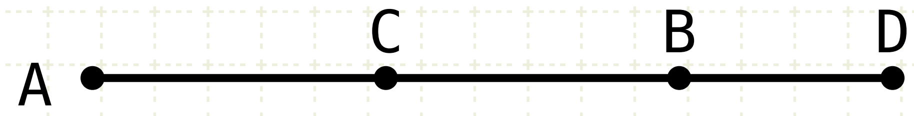
$AB^2 = AC^2 + CB^2 + 2 \cdot AC \cdot CB$

5



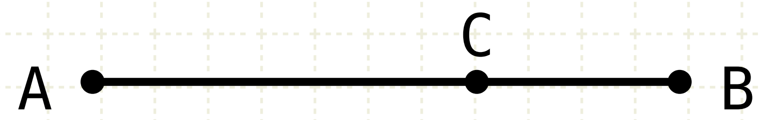
$AD \cdot DB + CD^2 = CB^2$

6



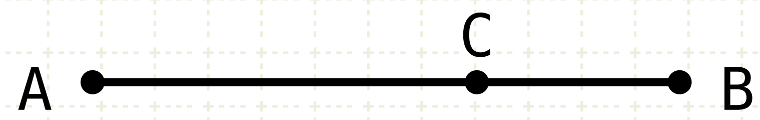
$AD \cdot DB + CB^2 = CD^2$

7



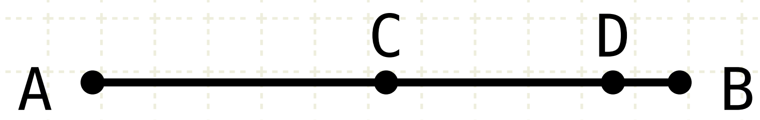
$AB^2 + BC^2 = AC^2 + 2 \cdot AB \cdot BC$

8



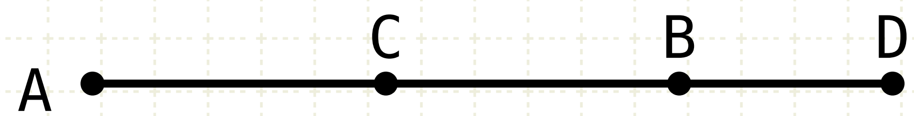
$4 \cdot AB \cdot BC + AC^2 = (AB+BC)^2$

9



$AD^2 + DB^2 = 2 \cdot (AC^2 + CD^2)$

10



$AD^2 + DB^2 = 2 \cdot (AC^2 + CD^2)$

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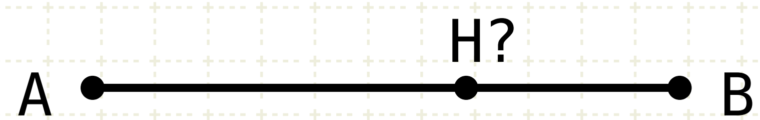
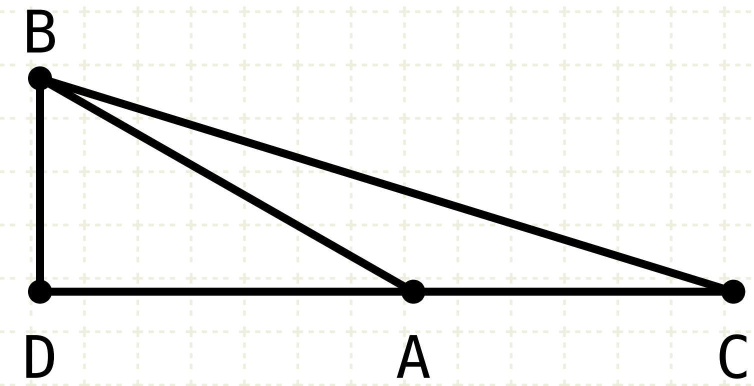


Find H. $AB \cdot BH = AH^2$

12



Cosine Law. $BC^2 = AB^2+AC^2+2 \cdot AD \cdot AC$

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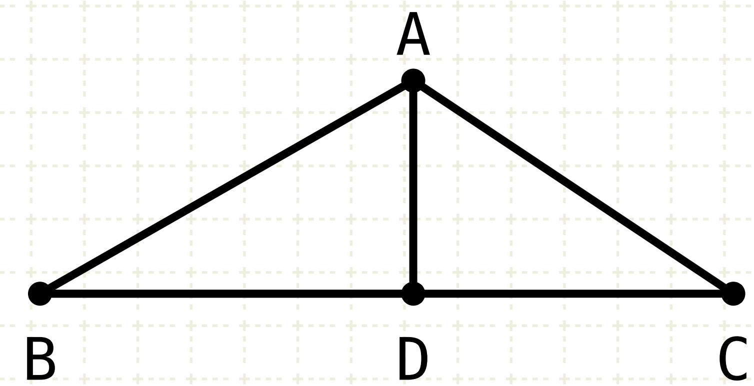
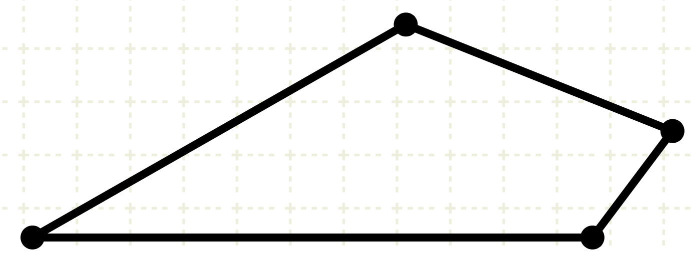


Cosine Law. $AC^2 = AB^2+BC^2-2 \cdot BD \cdot BC$

14



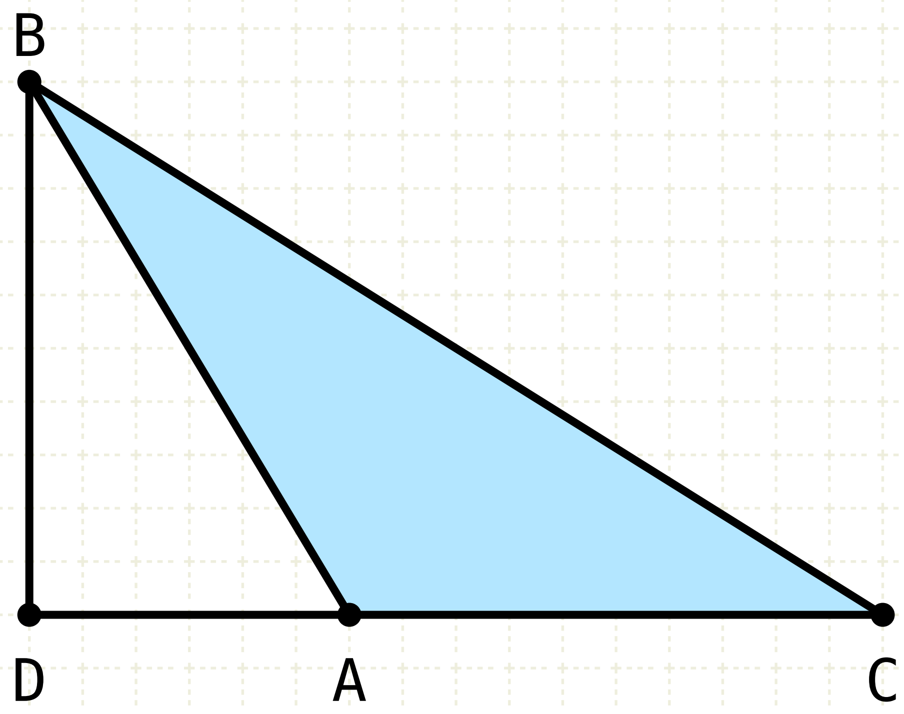
Find square of polygon

Proposition 12 of Book II

In obtuse-angled triangles the square on the side subtending the obtuse angle is greater than the squares on the sides containing the obtuse angle by twice the rectangle contained by one of the sides about the obtuse angle, namely that on which the perpendicular falls, and the straight line cut off outside by the perpendicular towards the obtuse angle.

Proposition 12 of Book II

In obtuse-angled triangles the square on the side subtending the obtuse angle is greater than the squares on the sides containing the obtuse angle by twice the rectangle contained by one of the sides about the obtuse angle, namely that on which the perpendicular falls, and the straight line cut off outside by the perpendicular towards the obtuse angle.



In other words

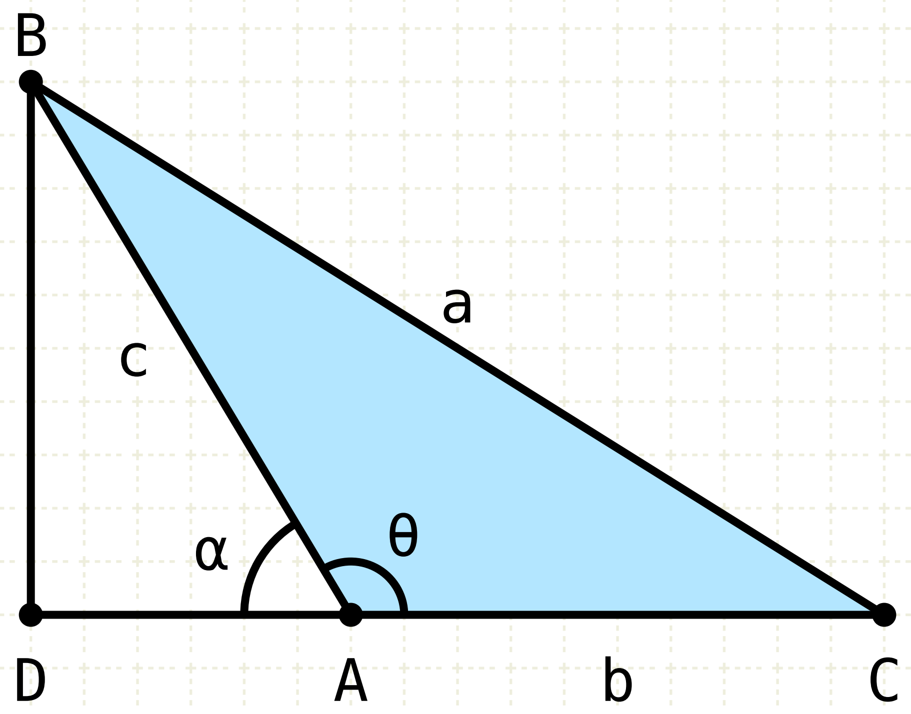
Given an obtuse triangle ABC, where the obtuse angle is opposite of BC. Extend the base AC to point D, where D is the intersection of the perpendicular from point B to the line AC.

The square of BC equals the square of AB and AC plus twice the rectangle formed by AC,AD

$$BC^2 = AB^2 + AC^2 + 2 \cdot AD \cdot AC$$

Proposition 12 of Book II

In obtuse-angled triangles the square on the side subtending the obtuse angle is greater than the squares on the sides containing the obtuse angle by twice the rectangle contained by one of the sides about the obtuse angle, namely that on which the perpendicular falls, and the straight line cut off outside by the perpendicular towards the obtuse angle.



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$$BC^2 = AB^2 + AC^2 + 2\cdot AD\cdot AC$$

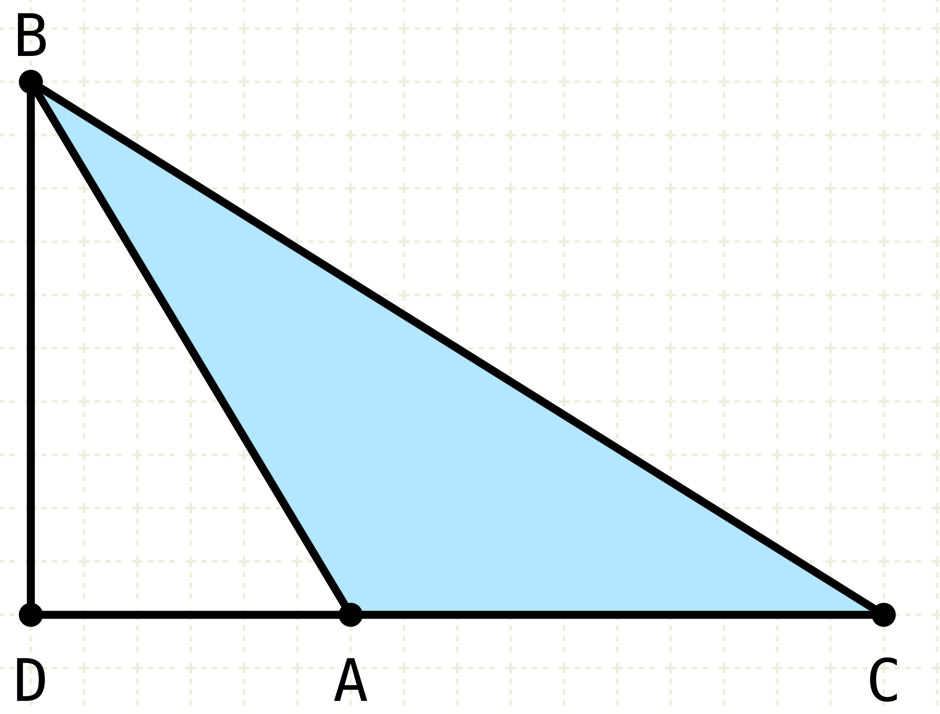
Or… the cosine law

$$BC=a,\ AB=c,\ AC=b,\ AD=c\cdot\cos(\alpha)$$
$$\cos(\alpha)=-\cos(\theta)\ \therefore\ AD=-c\cdot\cos(\theta)$$

$$BC^2 = AB^2 + AC^2 + 2\cdot AC\cdot AD$$
$$a^2 = c^2 + b^2 - 2\cdot b\cdot c\cdot\cos(\theta)$$

Proposition 12 of Book II

In obtuse-angled triangles the square on the side subtending the obtuse angle is greater than the squares on the sides containing the obtuse angle by twice the rectangle contained by one of the sides about the obtuse angle, namely that on which the perpendicular falls, and the straight line cut off outside by the perpendicular towards the obtuse angle.



$$DC^2 = DA^2 + AC^2 + 2 \cdot DA \cdot AC$$

In other words

Given an obtuse triangle ABC, where the obtuse angle is opposite of BC. Extend the base AC to point D, where D is the intersection of the perpendicular from point B to the line AC.

The square of BC equals the square of AB and AC plus twice the rectangle formed by AC,AD

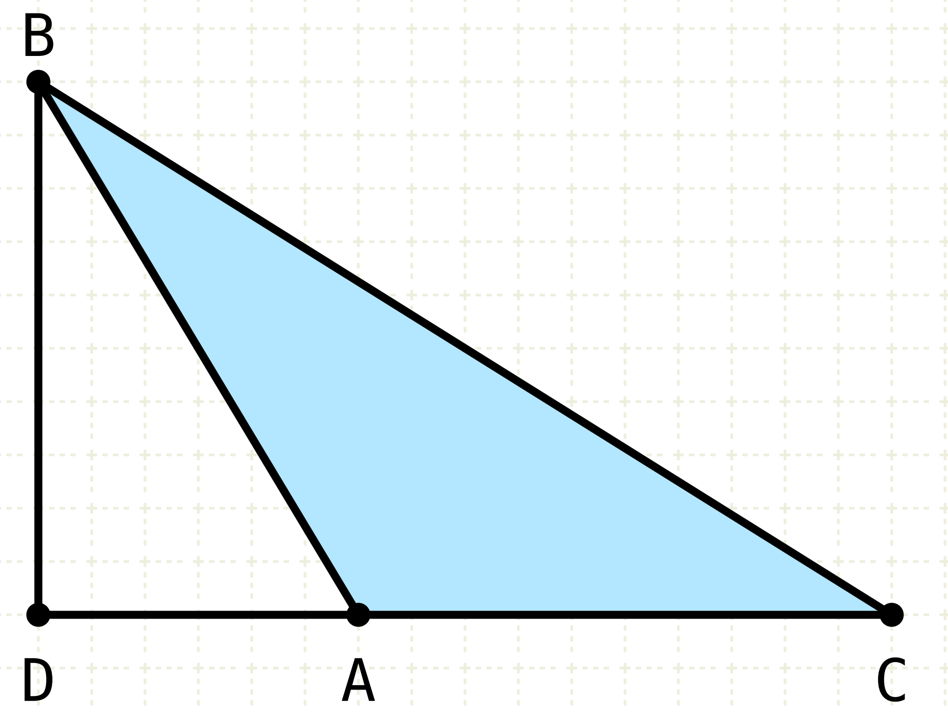
$$BC^2 = AB^2 + AC^2 + 2 \cdot AD \cdot AC$$

Proof

The line DC is cut at a point A, and thus the square of DC is equal to the squares of DA and AC plus twice the rectangle formed by DA and AC (II·4)

Proposition 12 of Book II

In obtuse-angled triangles the square on the side subtending the obtuse angle is greater than the squares on the sides containing the obtuse angle by twice the rectangle contained by one of the sides about the obtuse angle, namely that on which the perpendicular falls, and the straight line cut off outside by the perpendicular towards the obtuse angle.



$$DC^2 = DA^2 + AC^2 + 2 \cdot DA \cdot AC$$
$$(DC^2 + DB^2) = (DA^2 + DB^2) + AC^2 + 2 \cdot DA \cdot AC$$

In other words

Given an obtuse triangle ABC, where the obtuse angle is opposite of BC. Extend the base AC to point D, where D is the intersection of the perpendicular from point B to the line AC.

The square of BC equals the square of AB and AC plus twice the rectangle formed by AC,AD

$$BC^2 = AB^2 + AC^2 + 2 \cdot AD \cdot AC$$

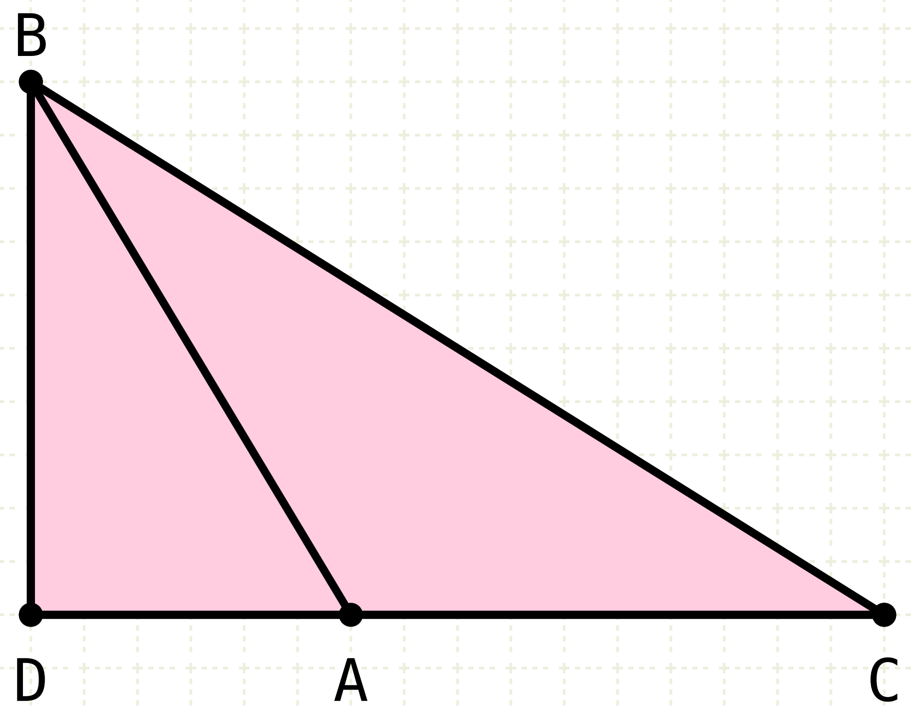
Proof

The line DC is cut at a point A, and thus the square of DC is equal to the squares of DA and AC plus twice the rectangle formed by DA and AC (II·4)

Add the square of DB to both sides of the equality

Proposition 12 of Book II

In obtuse-angled triangles the square on the side subtending the obtuse angle is greater than the squares on the sides containing the obtuse angle by twice the rectangle contained by one of the sides about the obtuse angle, namely that on which the perpendicular falls, and the straight line cut off outside by the perpendicular towards the obtuse angle.



$$DC^2 = DA^2 + AC^2 + 2 \cdot DA \cdot AC$$
$$(DC^2 + DB^2) = (DA^2 + DB^2) + AC^2 + 2 \cdot DA \cdot AC$$
$$DC^2 + DB^2 = BC^2$$

In other words

Given an obtuse triangle ABC, where the obtuse angle is opposite of BC. Extend the base AC to point D, where D is the intersection of the perpendicular from point B to the line AC.

The square of BC equals the square of AB and AC plus twice the rectangle formed by AC,AD

$$BC^2 = AB^2 + AC^2 + 2 \cdot AD \cdot AC$$

Proof

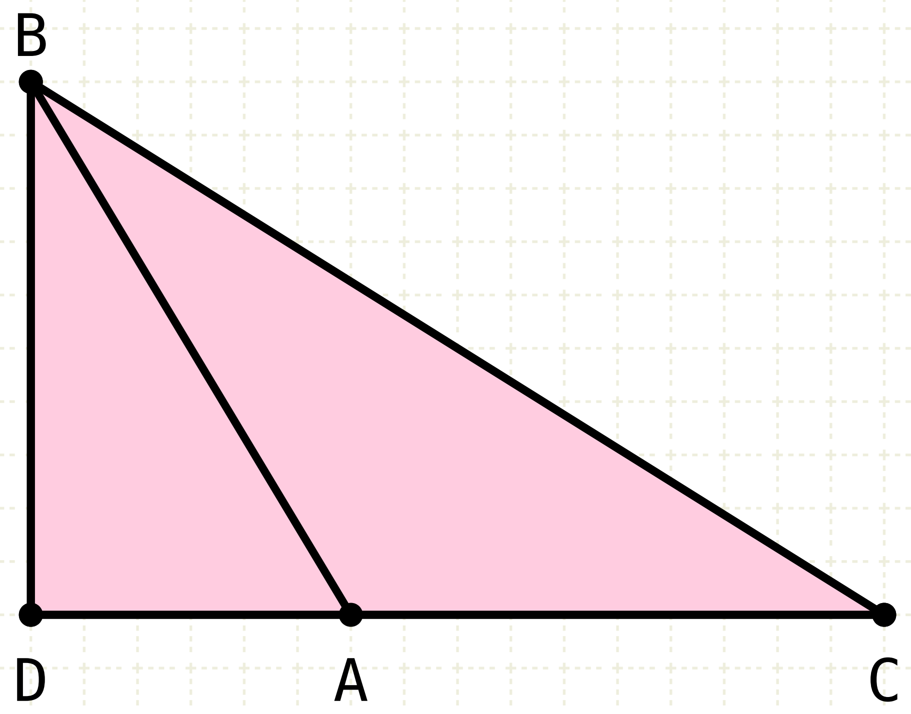
The line DC is cut at a point A, and thus the square of DC is equal to the squares of DA and AC plus twice the rectangle formed by DA and AC (II·4)

Add the square of DB to both sides of the equality

The squares of DC and DB equals the square of BC (I·47)

Proposition 12 of Book II

In obtuse-angled triangles the square on the side subtending the obtuse angle is greater than the squares on the sides containing the obtuse angle by twice the rectangle contained by one of the sides about the obtuse angle, namely that on which the perpendicular falls, and the straight line cut off outside by the perpendicular towards the obtuse angle.



$$DC^2 = DA^2 + AC^2 + 2 \cdot DA \cdot AC$$

$$(DC^2 + DB^2) = (DA^2 + DB^2) + AC^2 + 2 \cdot DA \cdot AC$$

$$DC^2 + DB^2 = BC^2$$

$$BC^2 = (DA^2 + DB^2) + AC^2 + 2 \cdot DA \cdot AC$$

In other words

Given an obtuse triangle ABC, where the obtuse angle is opposite of BC. Extend the base AC to point D, where D is the intersection of the perpendicular from point B to the line AC.

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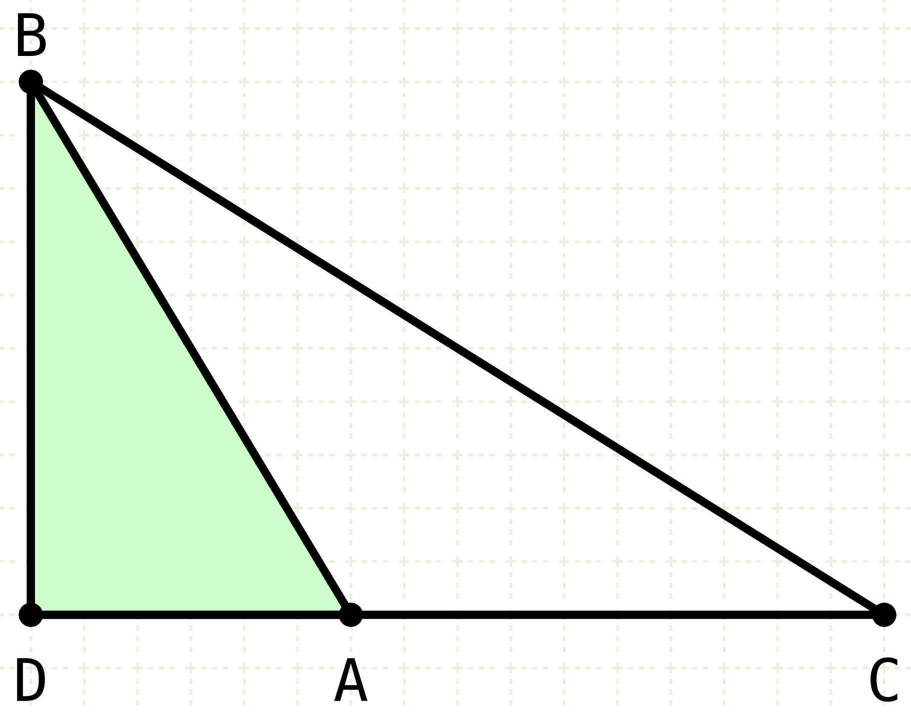

$$DC^2 = DA^2 + AC^2 + 2 \cdot DA \cdot AC$$
$$(DC^2 + DB^2) = (DA^2 + DB^2) + AC^2 + 2 \cdot DA \cdot AC$$
$$DC^2 + DB^2 = BC^2$$
$$BC^2 = (DA^2 + DB^2) + AC^2 + 2 \cdot DA \cdot AC$$
$$DA^2 + DB^2 = AB^2$$

In other words

Given an obtuse triangle ABC, where the obtuse angle is opposite of BC. Extend the base AC to point D, where D is the intersection of the perpendicular from point B to the line AC.

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$$BC^2 = AB^2 + AC^2 + 2 \cdot AD \cdot AC$$

Proof

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Add the square of DB to both sides of the equality

The squares of DC and DB equals the square of BC (I·47)

The squares of DA and DB equals the square of AB (I·47)

Proposition 12 of Book II

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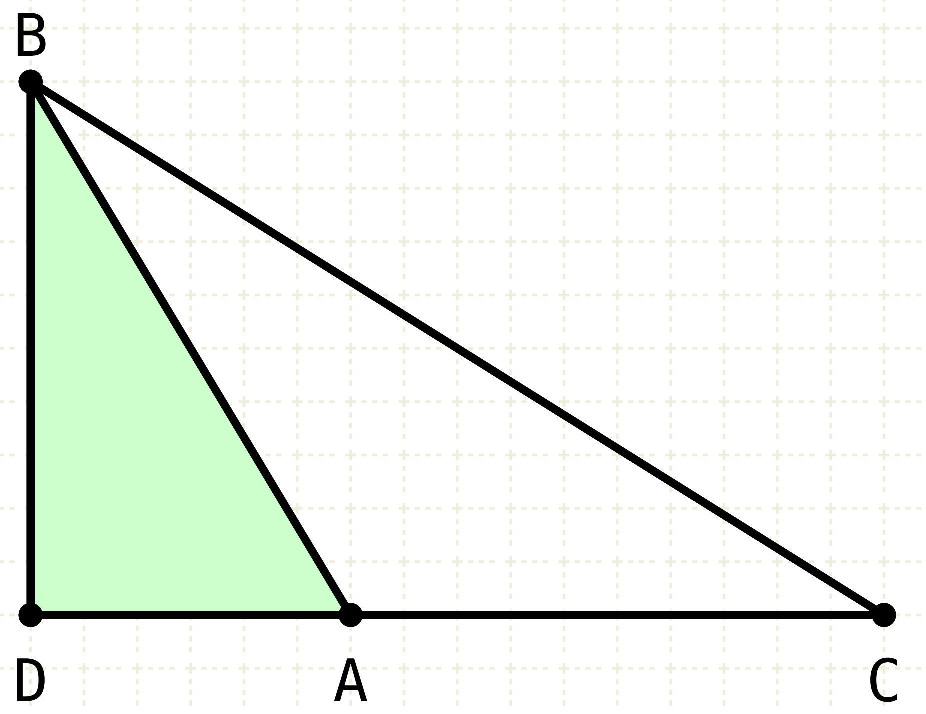

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The square of BC equals the square of AB and AC plus twice the rectangle formed by AC,AD

$$BC^2 = AB^2 + AC^2 + 2 \cdot AD \cdot AC$$

Proof

The line DC is cut at a point A, and thus the square of DC is equal to the squares of DA and AC plus twice the rectangle formed by DA and AC (II·4)

Add the square of DB to both sides of the equality

The squares of DC and DB equals the square of BC (I·47)

The squares of DA and DB equals the square of AB (I·47)

$$DC^2 = DA^2 + AC^2 + 2 \cdot DA \cdot AC$$
$$(DC^2 + DB^2) = (DA^2 + DB^2) + AC^2 + 2 \cdot DA \cdot AC$$
$$DC^2 + DB^2 = BC^2$$
$$BC^2 = (DA^2 + DB^2) + AC^2 + 2 \cdot DA \cdot AC$$
$$DA^2 + DB^2 = AB^2$$
$$BC^2 = AB^2 + AC^2 + 2 \cdot DA \cdot AC$$

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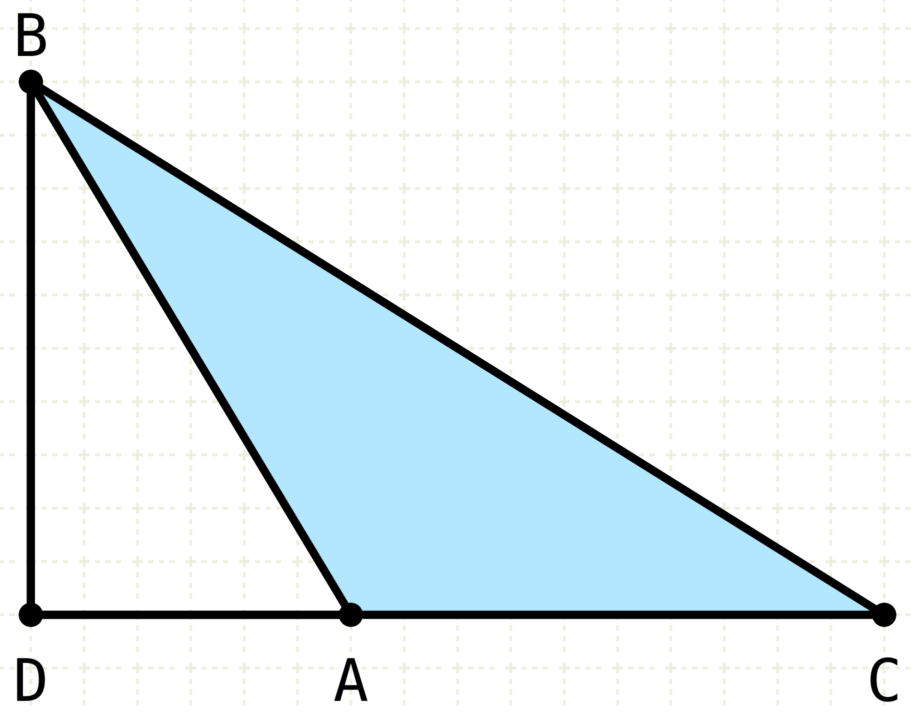

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The square of BC equals the square of AB and AC plus twice the rectangle formed by AC,AD

$$BC^2 = AB^2 + AC^2 + 2 \cdot AD \cdot AC$$

Proof

The line DC is cut at a point A, and thus the square of DC is equal to the squares of DA and AC plus twice the rectangle formed by DA and AC (II·4)

Add the square of DB to both sides of the equality

The squares of DC and DB equals the square of BC (I·47)

The squares of DA and DB equals the square of AB (I·47)

Thus the square of BC is equal to the sum of the squares of AB and AC, plus the rectangle formed by DA,AC

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