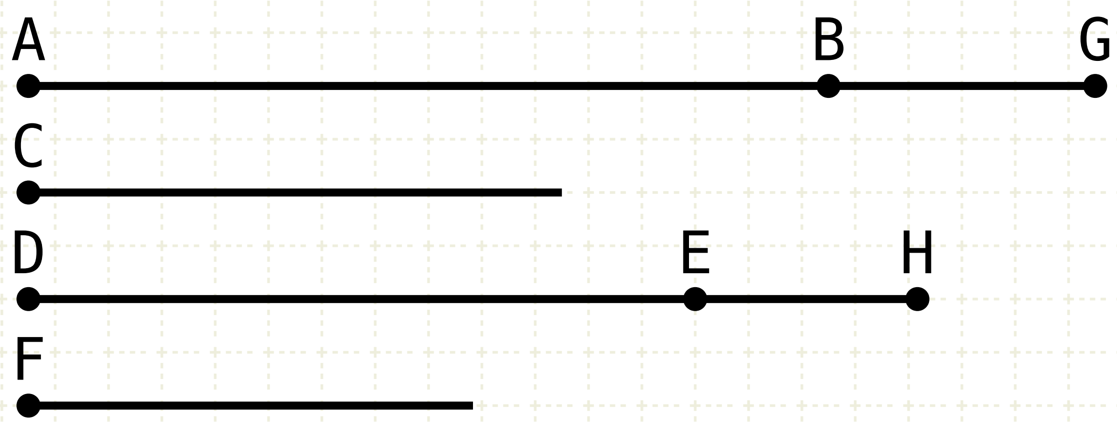
Euclid's Elements

Book V



AB:C = DE:F

BG:C = EH:F

AG:C = DH:F

Proportions are what makes the old Greek temples classic in their beauty. They are like huge blocks, from which the air has been literally hewn out between the columns.

Arne Jacobsen

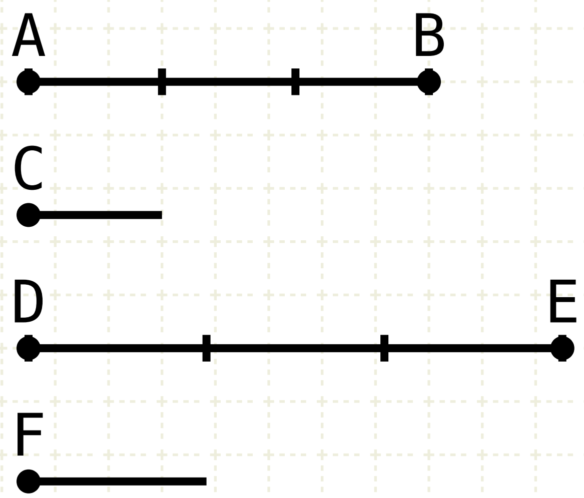
Table of Contents, Chapter 5

Proposition 2 of Book V

If a first magnitude be the same multiple of a second that a third is of a fourth, and a fifth also be the same multiple of the second that a sixth is of the fourth, the sum of the first and fifth will also be the same multiple of the second that the sum of the third and sixth is of the fourth

Proposition 2 of Book V

If a first magnitude be the same multiple of a second that a third is of a fourth, and a fifth also be the same multiple of the second that a sixth is of the fourth, the sum of the first and fifth will also be the same multiple of the second that the sum of the third and sixth is of the fourth



AB = first line
C = second line
DE = third line
F = fourth line

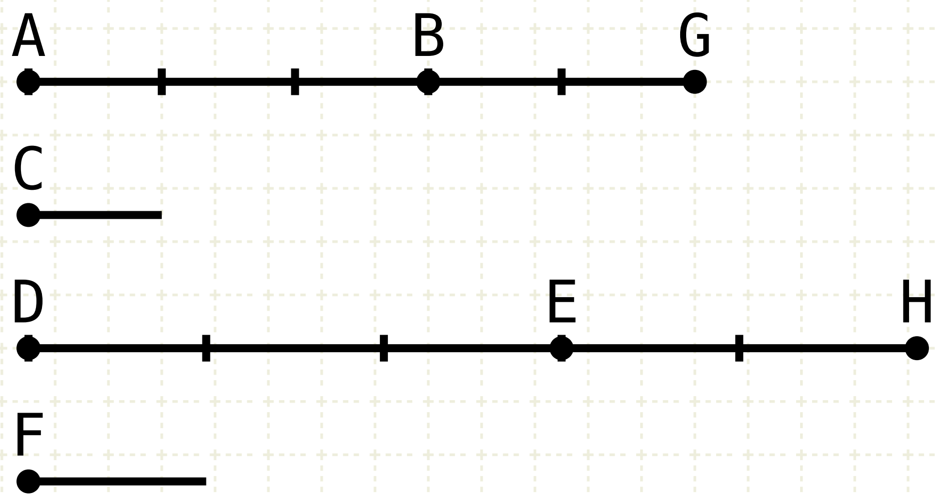
In other words

If we have two lines (AB and DE) that are equal multiples of two other lines (C and F respectively) and …

If $AB=n\cdot C$, $DE=n\cdot F$

Proposition 2 of Book V

If a first magnitude be the same multiple of a second that a third is of a fourth, and a fifth also be the same multiple of the second that a sixth is of the fourth, the sum of the first and fifth will also be the same multiple of the second that the sum of the third and sixth is of the fourth



AB = first line
C = second line
DE = third line
F = fourth line
BG = fifth line
EH = sixth line

In other words

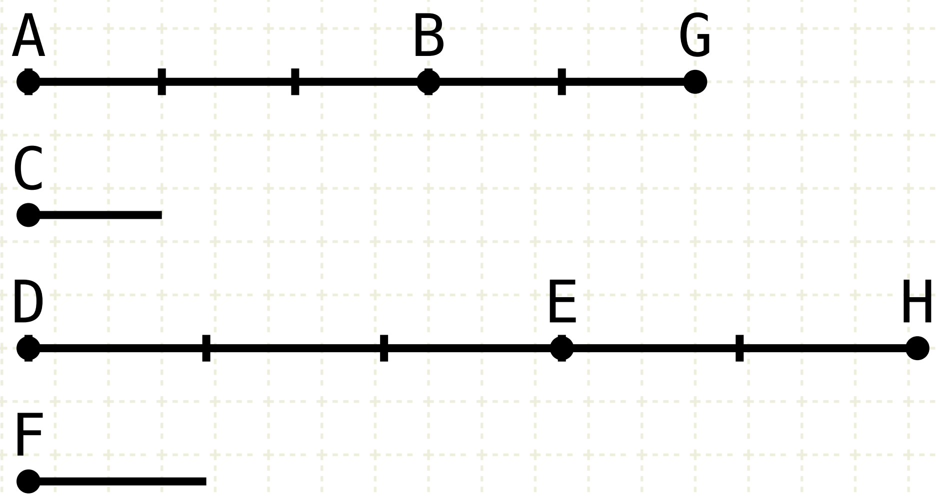
If we have two lines (AB and DE) that are equal multiples of two other lines (C and F respectively) and …

we have another two lines (BG and EH) that are also equal multiples of lines C and F, then…

If $AB=n \cdot C$, $DE=n \cdot F$
and $BG=m \cdot C$, $EH=m \cdot F$

Proposition 2 of Book V

If a first magnitude be the same multiple of a second that a third is of a fourth, and a fifth also be the same multiple of the second that a sixth is of the fourth, the sum of the first and fifth will also be the same multiple of the second that the sum of the third and sixth is of the fourth




```
AB = first line
C  = second line
DE = third line
F  = fourth line
BG = fifth line
EH = sixth line
```

In other words

If we have two lines (AB and DE) that are equal multiples of two other lines (C and F respectively) and …

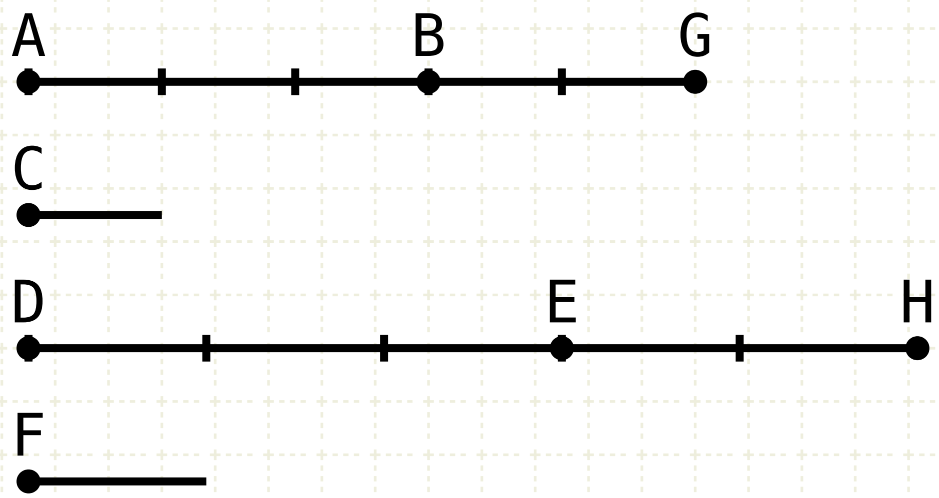
we have another two lines (BG and EH) that are also equal multiples of lines C and F, then…

the line AG will be the same multiplier of C as DH is to F

```
If   AB=n·C, DE=n·F
and  BG=m·C, EH=m·F
then AG=k·C and DH=k·F
```

Proposition 2 of Book V

If a first magnitude be the same multiple of a second that a third is of a fourth, and a fifth also be the same multiple of the second that a sixth is of the fourth, the sum of the first and fifth will also be the same multiple of the second that the sum of the third and sixth is of the fourth




```
AB = first line
C  = second line
DE = third line
F  = fourth line
BG = fifth line
EH = sixth line
```

In other words

If we have two lines (AB and DE) that are equal multiples of two other lines (C and F respectively) and …

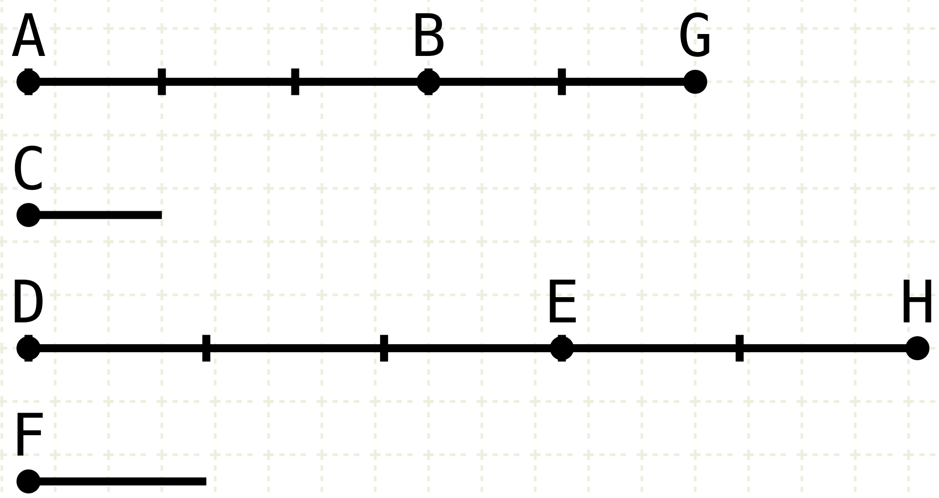
we have another two lines (BG and EH) that are also equal multiples of lines C and F, then…

the line AG will be the same multiplier of C as DH is to F

Proof

Proposition 2 of Book V

If a first magnitude be the same multiple of a second that a third is of a fourth, and a fifth also be the same multiple of the second that a sixth is of the fourth, the sum of the first and fifth will also be the same multiple of the second that the sum of the third and sixth is of the fourth



$AB = n \cdot C, \quad DE = n \cdot F$

AB = first line
C = second line
DE = third line
F = fourth line
BG = fifth line
EH = sixth line

In other words

If we have two lines (AB and DE) that are equal multiples of two other lines (C and F respectively) and …

we have another two lines (BG and EH) that are also equal multiples of lines C and F, then…

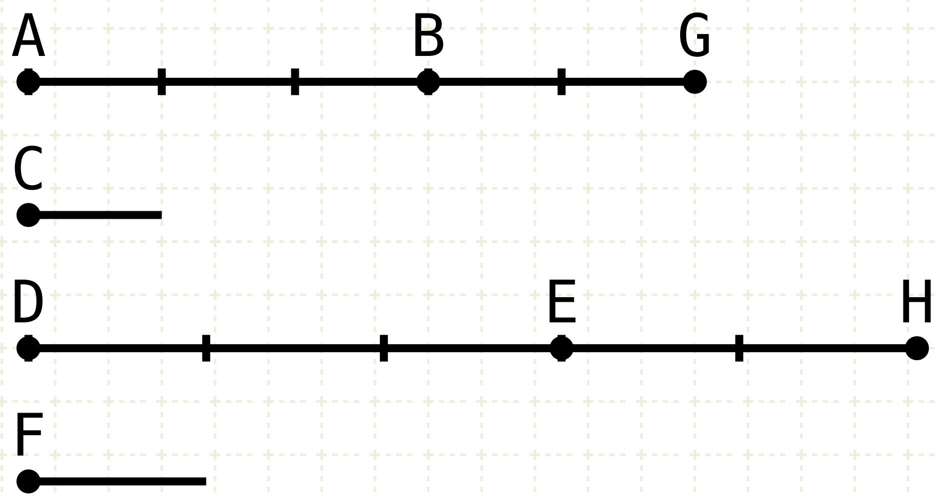
the line AG will be the same multiplier of C as DH is to F

Proof

Since AB and DE are the same multiples of C and F respectively, then there are the an equal number of magnitudes (line segments) in AB and DE

Proposition 2 of Book V

If a first magnitude be the same multiple of a second that a third is of a fourth, and a fifth also be the same multiple of the second that a sixth is of the fourth, the sum of the first and fifth will also be the same multiple of the second that the sum of the third and sixth is of the fourth



AB = first line
C = second line
DE = third line
F = fourth line
BG = fifth line
EH = sixth line

$AB = n \cdot C, \quad DE = n \cdot F$

$BG = m \cdot C, \quad EH = m \cdot F$

In other words

If we have two lines (AB and DE) that are equal multiples of two other lines (C and F respectively) and …

we have another two lines (BG and EH) that are also equal multiples of lines C and F, then…

the line AG will be the same multiplier of C as DH is to F

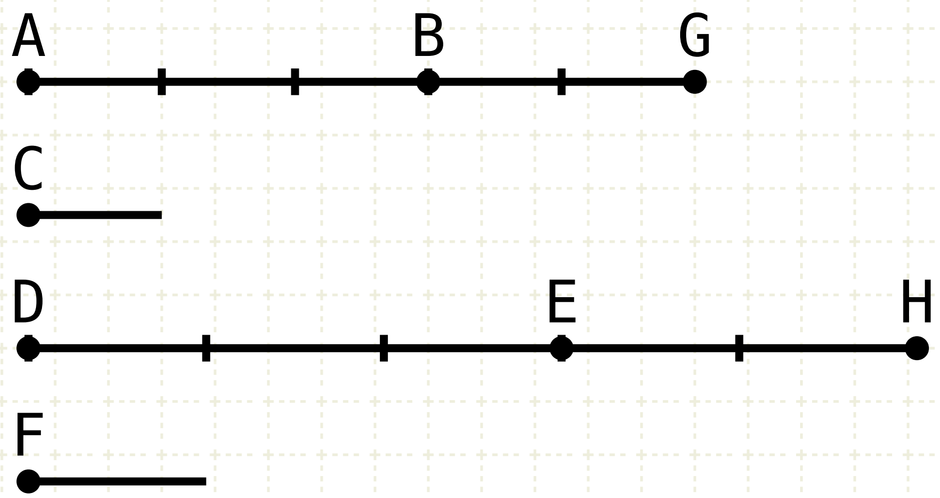
Proof

Since AB and DE are the same multiples of C and F respectively, then there are the an equal number of magnitudes (line segments) in AB and DE

For the same reason, there is an equal number of magnitudes in lines BG and EH

Proposition 2 of Book V

If a first magnitude be the same multiple of a second that a third is of a fourth, and a fifth also be the same multiple of the second that a sixth is of the fourth, the sum of the first and fifth will also be the same multiple of the second that the sum of the third and sixth is of the fourth



AB = first line
C = second line
DE = third line
F = fourth line
BG = fifth line
EH = sixth line

$AB = n \cdot C, \quad DE = n \cdot F$

$BG = m \cdot C, \quad EH = m \cdot F$

$AG = AB + BG = n \cdot C + m \cdot C = (n+m) \cdot C$

$DH = DE + EH = n \cdot F + m \cdot F = (n+m) \cdot F$

In other words

If we have two lines (AB and DE) that are equal multiples of two other lines (C and F respectively) and …

we have another two lines (BG and EH) that are also equal multiples of lines C and F, then…

the line AG will be the same multiplier of C as DH is to F

Proof

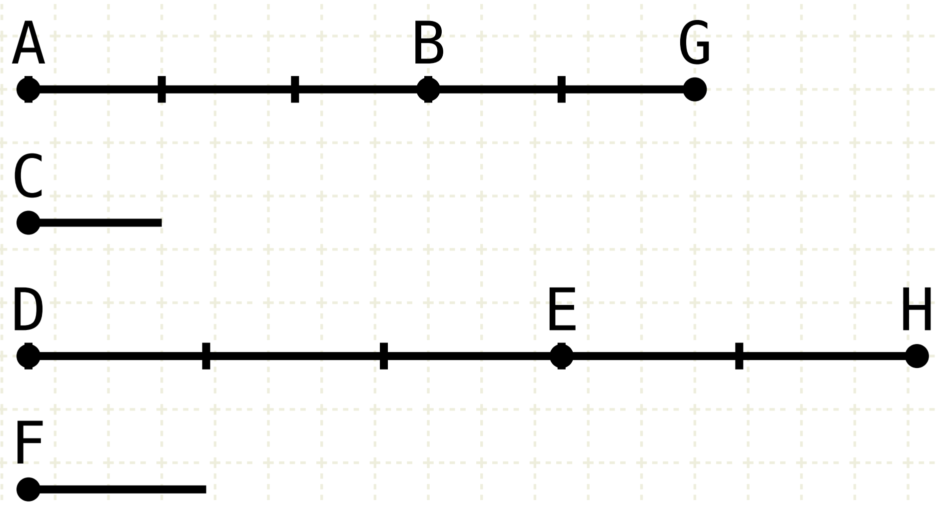
Since AB and DE are the same multiples of C and F respectively, then there are the an equal number of magnitudes (line segments) in AB and DE

For the same reason, there is an equal number of magnitudes in lines BG and EH

Thus, the total number of magnitudes of size C in line AG (m+n) is equal to the total number of magnitudes of size F in line DH (m+n)

Proposition 2 of Book V

If a first magnitude be the same multiple of a second that a third is of a fourth, and a fifth also be the same multiple of the second that a sixth is of the fourth, the sum of the first and fifth will also be the same multiple of the second that the sum of the third and sixth is of the fourth



AB = first line
C = second line
DE = third line
F = fourth line
BG = fifth line
EH = sixth line

$AB = n \cdot C, \quad DE = n \cdot F$
$BG = m \cdot C, \quad EH = m \cdot F$
$AG = AB + BG = n \cdot C + m \cdot C = (n+m) \cdot C$
$DH = DE + EH = n \cdot F + m \cdot F = (n+m) \cdot F$

In other words

If we have two lines (AB and DE) that are equal multiples of two other lines (C and F respectively) and …

we have another two lines (BG and EH) that are also equal multiples of lines C and F, then…

the line AG will be the same multiplier of C as DH is to F

Proof

Since AB and DE are the same multiples of C and F respectively, then there are the an equal number of magnitudes (line segments) in AB and DE

For the same reason, there is an equal number of magnitudes in lines BG and EH

Thus, the total number of magnitudes of size C in line AG (m+n) is equal to the total number of magnitudes of size F in line DH (m+n)

Or, AG is the same multiple of C that DG is of F

Youtube Videos

https://www.youtube.com/c/SandyBultena