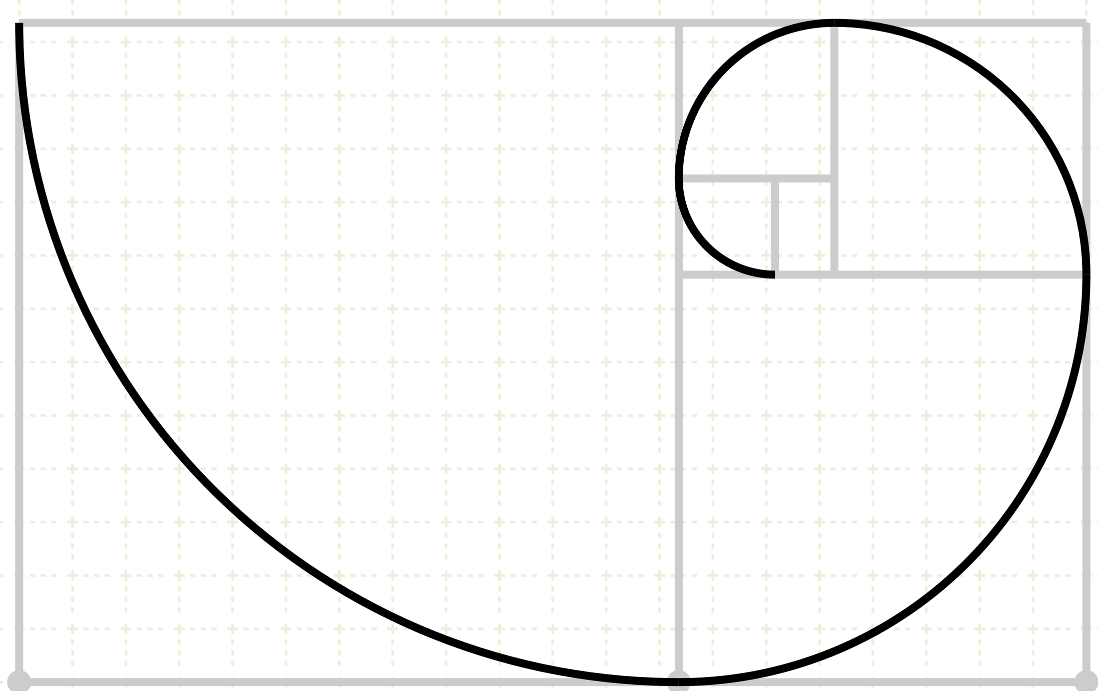
Euclid's Elements

Book VI



One can state, without exaggeration, that the observation of and the search for similarities and differences are the basis of all human knowledge.

Alfred Nobel

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Proposition 22 of Book VI

If four straight lines be proportional, the rectilineal figures similar and similarly described upon them will also be proportional;
and if the rectilineal figures similar and similarly described upon them be proportional,
the straight lines will themselves also be proportional

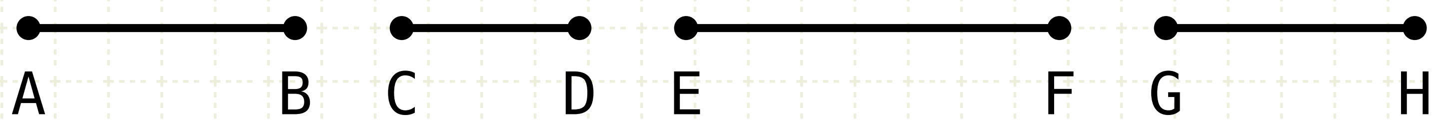
Proposition 22 of Book VI

If four straight lines be proportional, the rectilineal figures similar and similarly described upon them will also be proportional;
and if the rectilineal figures similar and similarly described upon them be proportional,
the straight lines will themselves also be proportional

In other words

If the line segments AB, CD, EF, GH are proportional, then if similar polygons are drawn on AB,CD and other similar polygons are drawn on EF,GH, these polygons will also be similar

And vice-versa



AB:CD = EF:GH

Proposition 22 of Book VI

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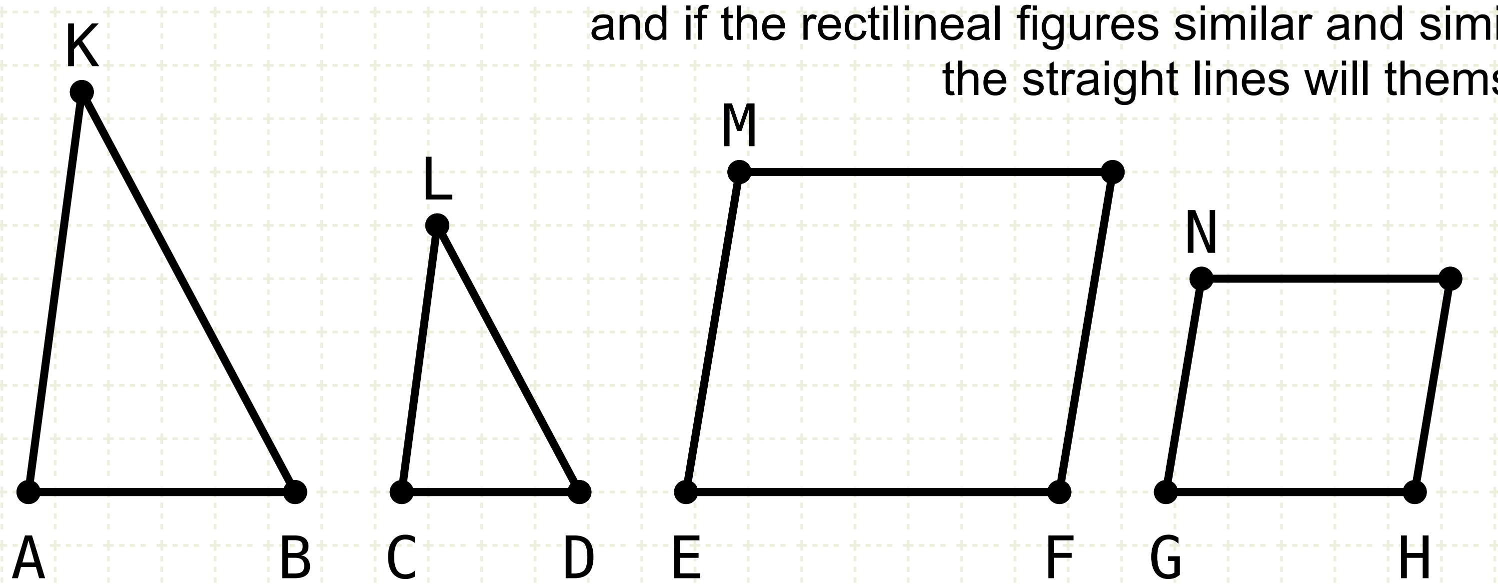

In other words

If the line segments AB, CD, EF, GH are proportional, then if similar polygons are drawn on AB,CD and other similar polygons are drawn on EF,GH, these polygons will also be similar

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AB:CD = EF:GH

ΔKAB:ΔLCD = □MF:□NH

Proposition 22 of Book VI

If four straight lines be proportional, the rectilineal figures similar and similarly described upon them will also be proportional;
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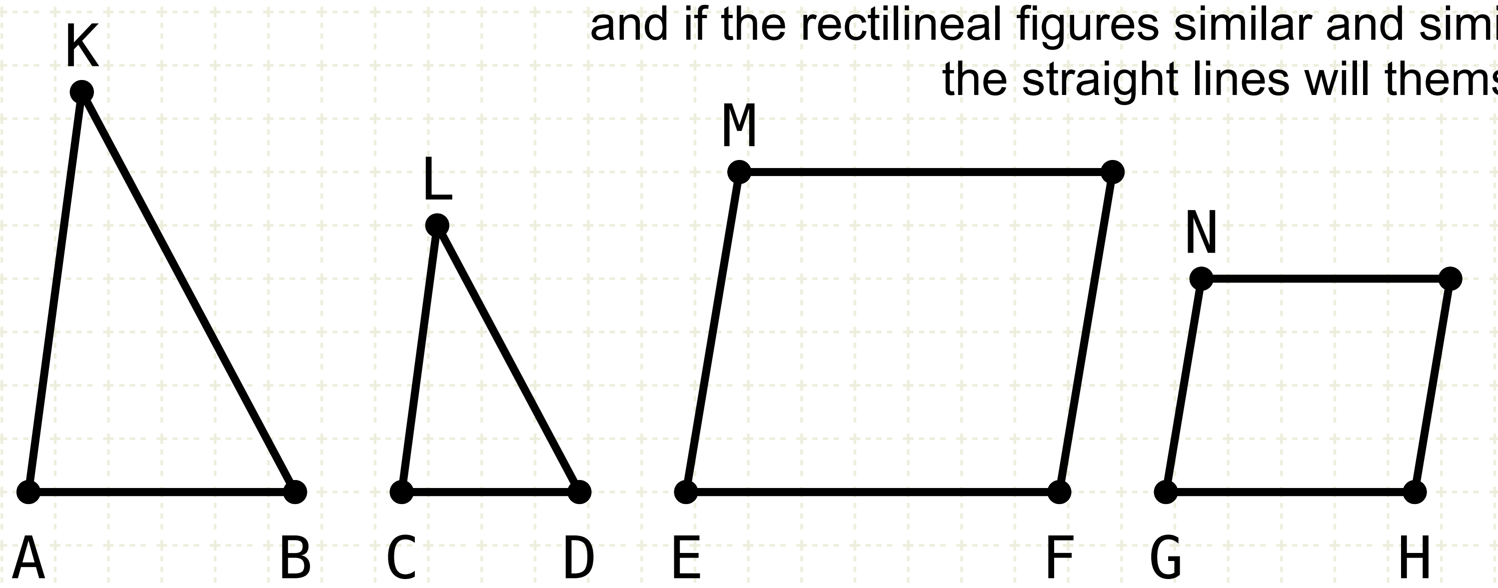

Proof - Part 1

AB:CD = EF:GH

Proposition 22 of Book VI

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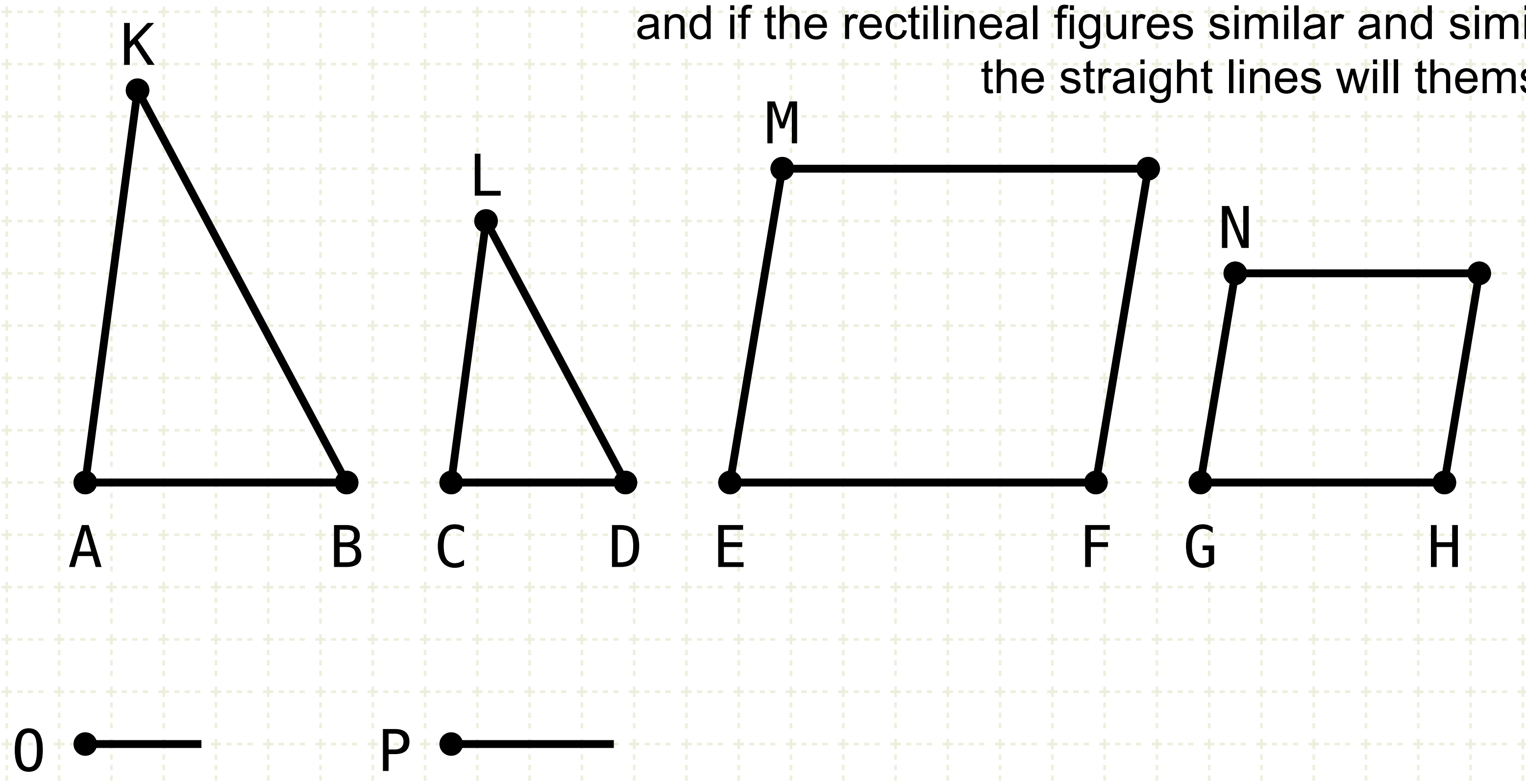

Proof - Part 1

Create a third proportional (O) to AB, CD, and another third proportional (P) to EF, GH (VI·11)

AB:CD = EF:GH
AB:CD = CD:O
EF:GH = GH:P

Proposition 22 of Book VI

If four straight lines be proportional, the rectilineal figures similar and similarly described upon them will also be proportional;
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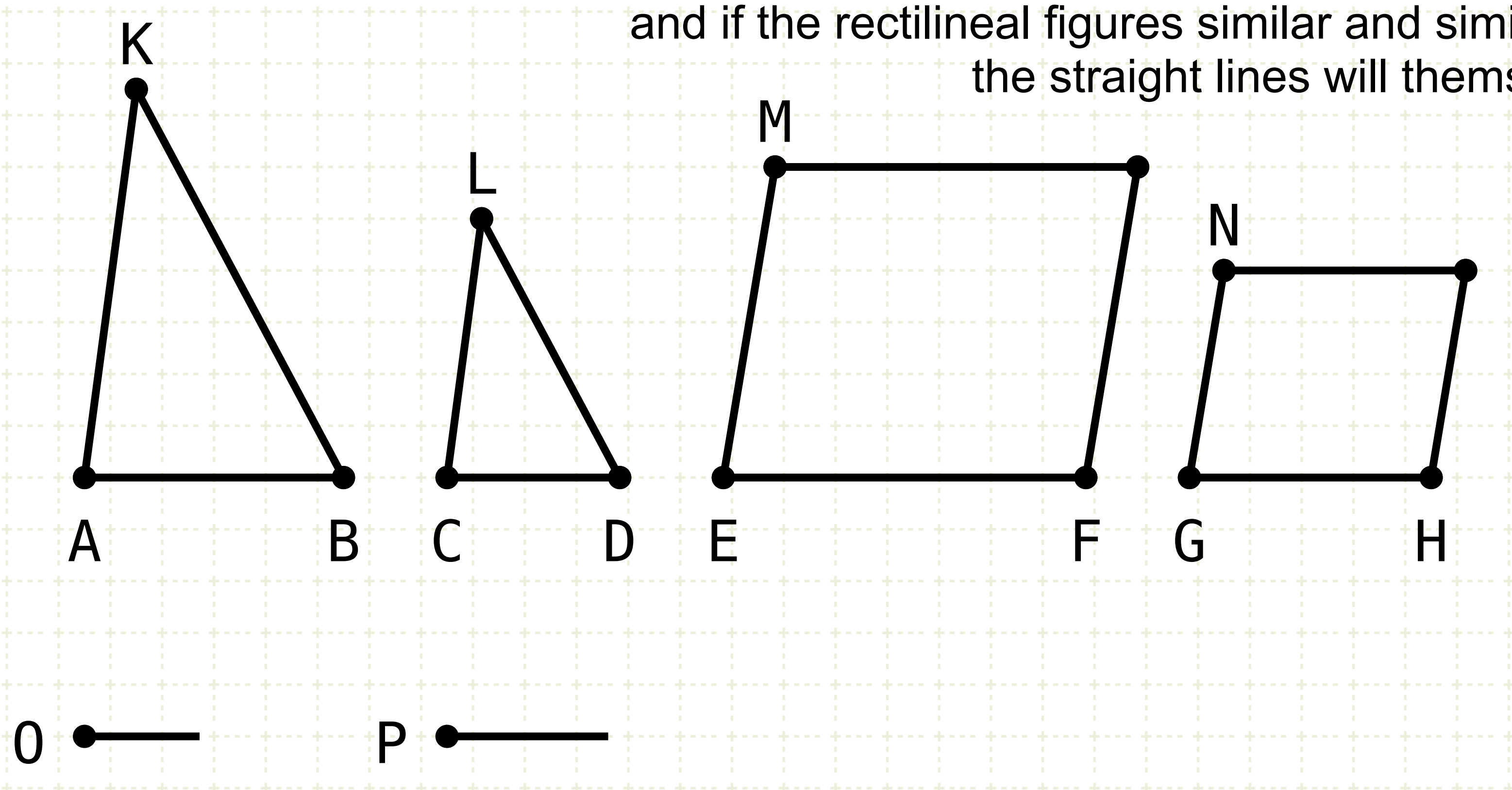

Proof - Part 1

Create a third proportional (O) to AB, CD, and another third proportional (P) to EF, GH (VI·11)

Thus the ratios CD to O and GH to P are also equal

AB:CD = EF:GH
AB:CD = CD:O
EF:GH = GH:P
CD:O = GH:P

Proposition 22 of Book VI

If four straight lines be proportional, the rectilineal figures similar and similarly described upon them will also be proportional;
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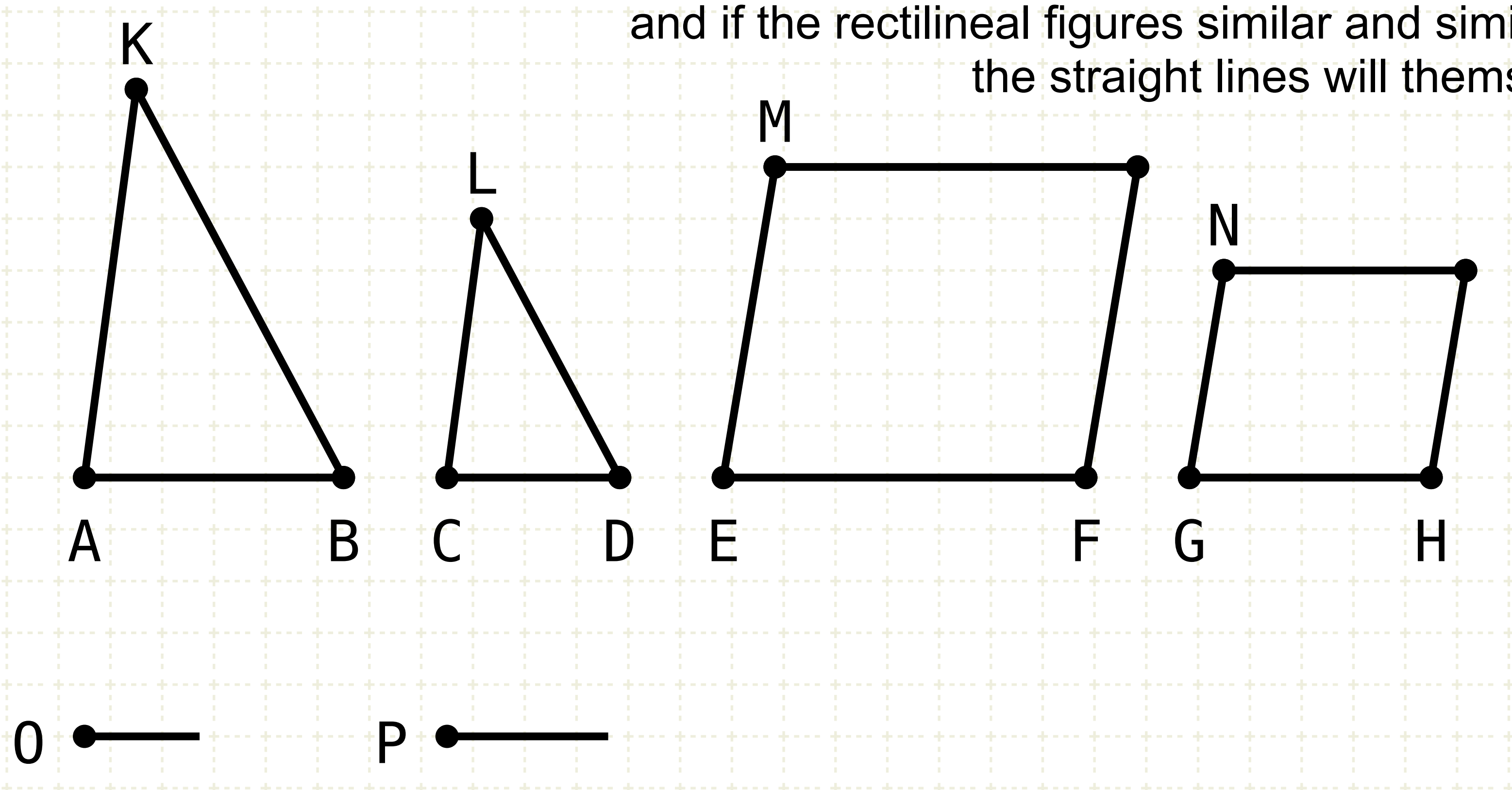

Proof - Part 1

Create a third proportional (O) to AB, CD, and another third proportional (P) to EF, GH (VI·11)

Thus the ratios CD to O and GH to P are also equal

Therefore, ex aequali, AB is to O as EF is to P (V·22)

AB:CD = EF:GH
AB:CD = CD:O
EF:GH = GH:P
CD:O = GH:P
AB:O = EF:P

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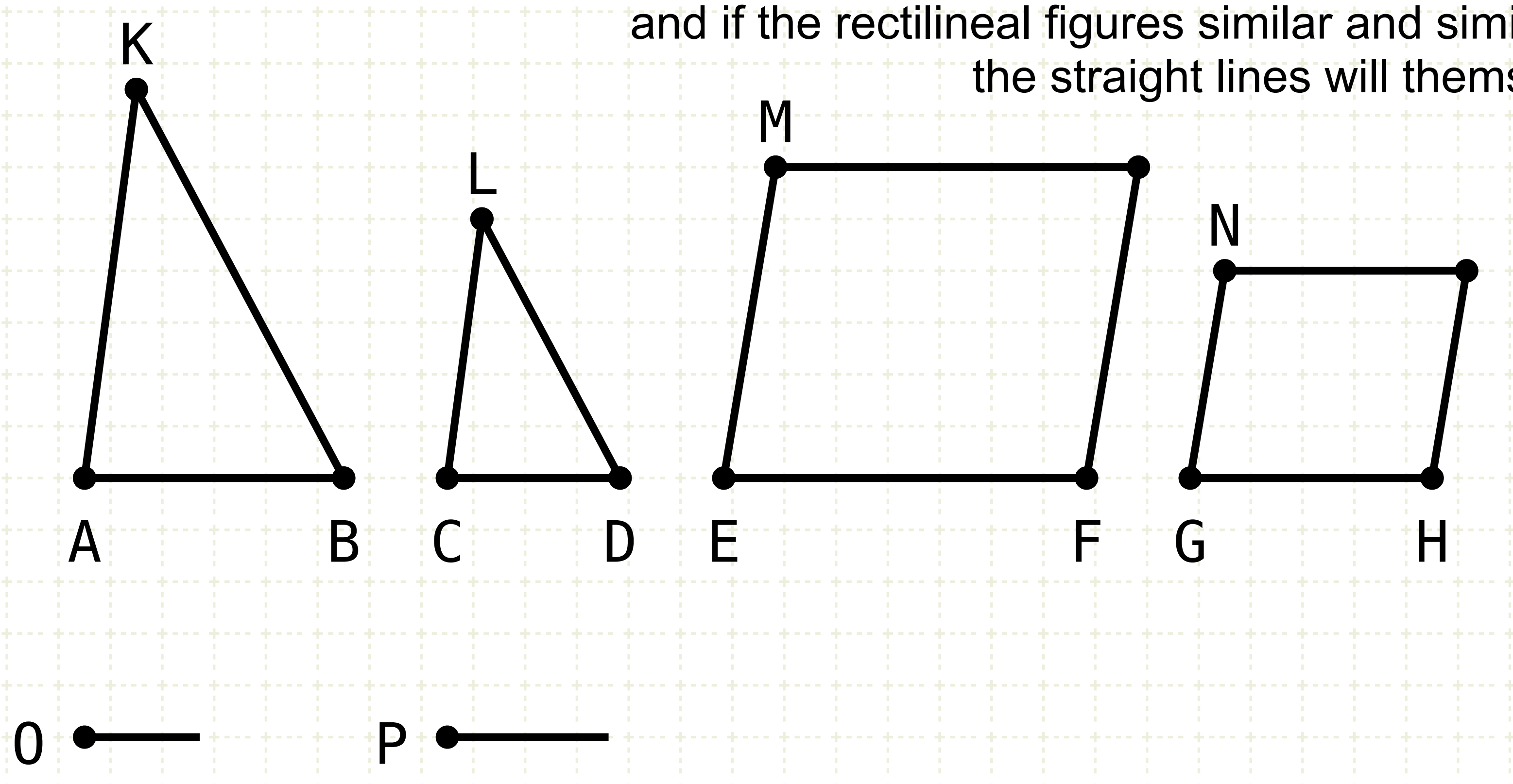

AB:CD = EF:GH
AB:CD = CD:O
EF:GH = GH:P
CD:O = GH:P
AB:O = EF:P
ΔKAB:ΔLCD = AB:O

Proof - Part 1

Create a third proportional (O) to AB, CD, and another third proportional (P) to EF, GH (VI·11)

Thus the ratios CD to O and GH to P are also equal

Therefore, ex aequali, AB is to O as EF is to P (V·22)

The ratios of similar triangles is equal to the duplicate ratio of their sides about an equal angle (VI·19.Por), so KAB is to LCD as AB is to O

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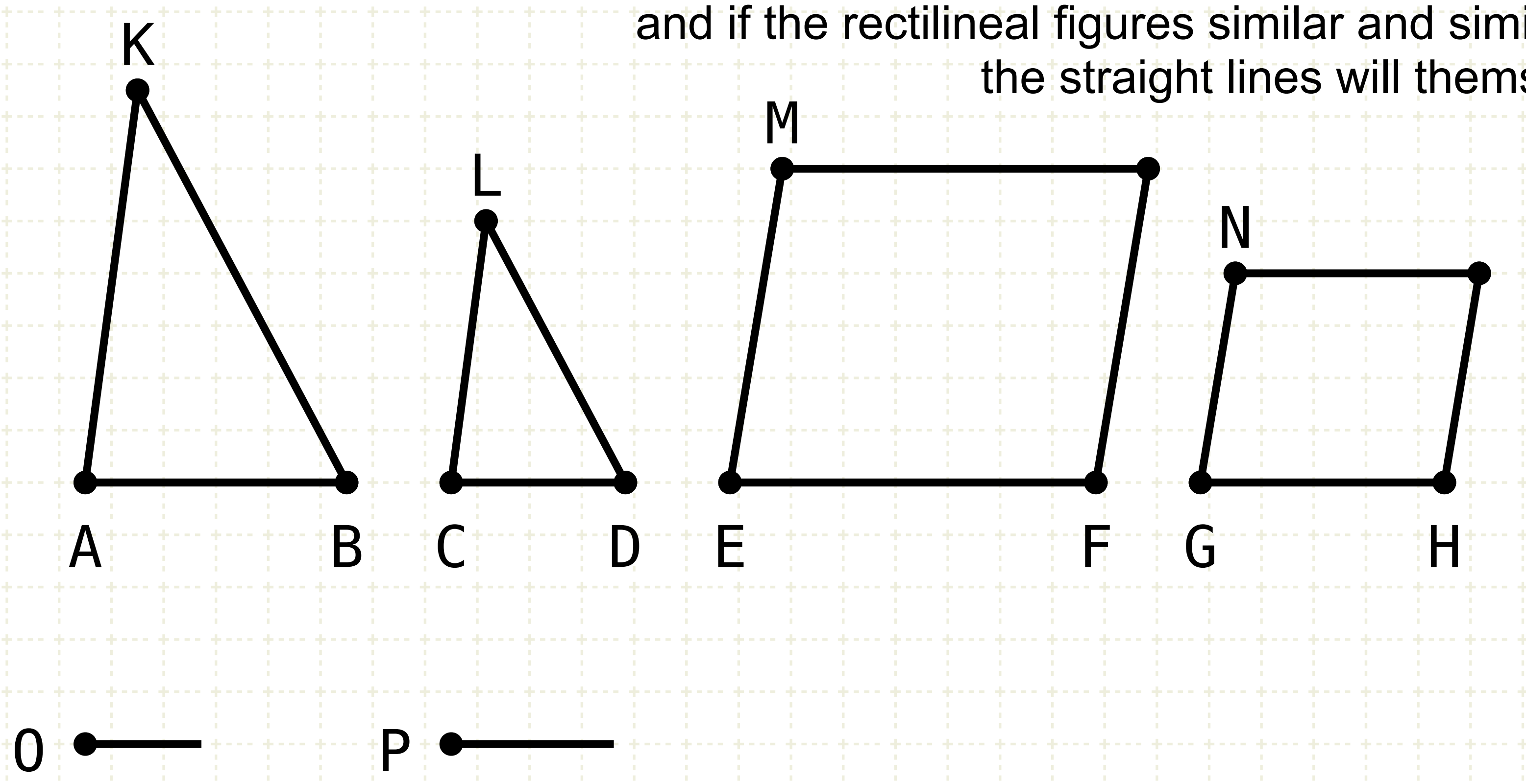

Proof - Part 1

Create a third proportional (O) to AB, CD, and another third proportional (P) to EF, GH (VI·11)

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The ratios of similar triangles is equal to the duplicate ratio of their sides about an equal angle (VI·19.Por), so KAB is to LCD as AB is to O

Likewise, MF is to NH as EF is to P

AB:CD = EF:GH

AB:CD = CD:O

EF:GH = GH:P

CD:O = GH:P

AB:O = EF:P

ΔKAB:ΔLCD = AB:O

□MF:□NH = EF:P

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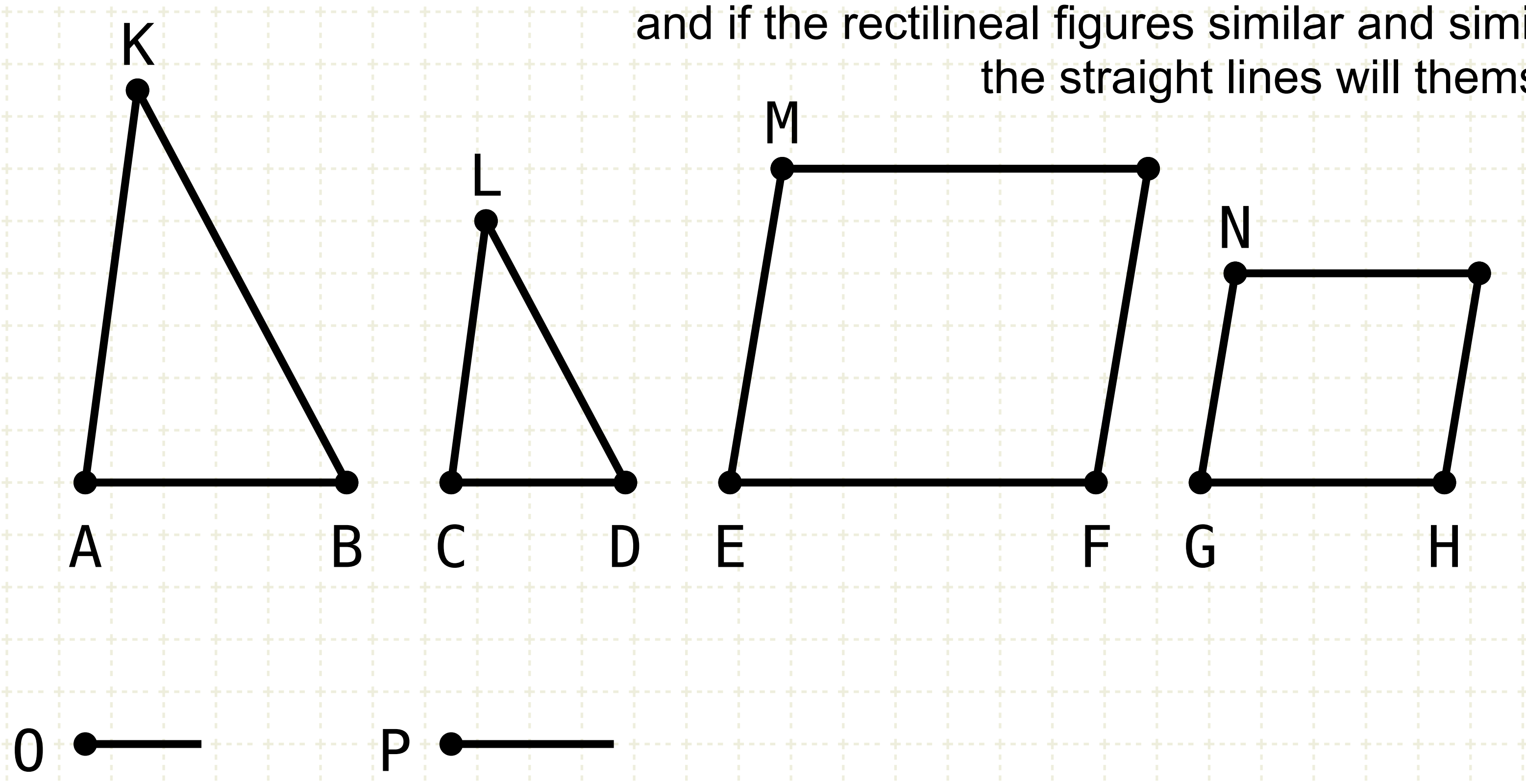

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Create a third proportional (O) to AB, CD, and another third proportional (P) to EF, GH (VI·11)

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Likewise, MF is to NH as EF is to P

Therefore, KAB is to LCD as MF is to NH (V·11)

AB:CD = EF:GH
AB:CD = CD:O
EF:GH = GH:P
CD:O = GH:P
AB:O = EF:P
ΔKAB:ΔLCD = AB:O
□MF:□NH = EF:P
ΔKAB:ΔLCD = □MF:□NH

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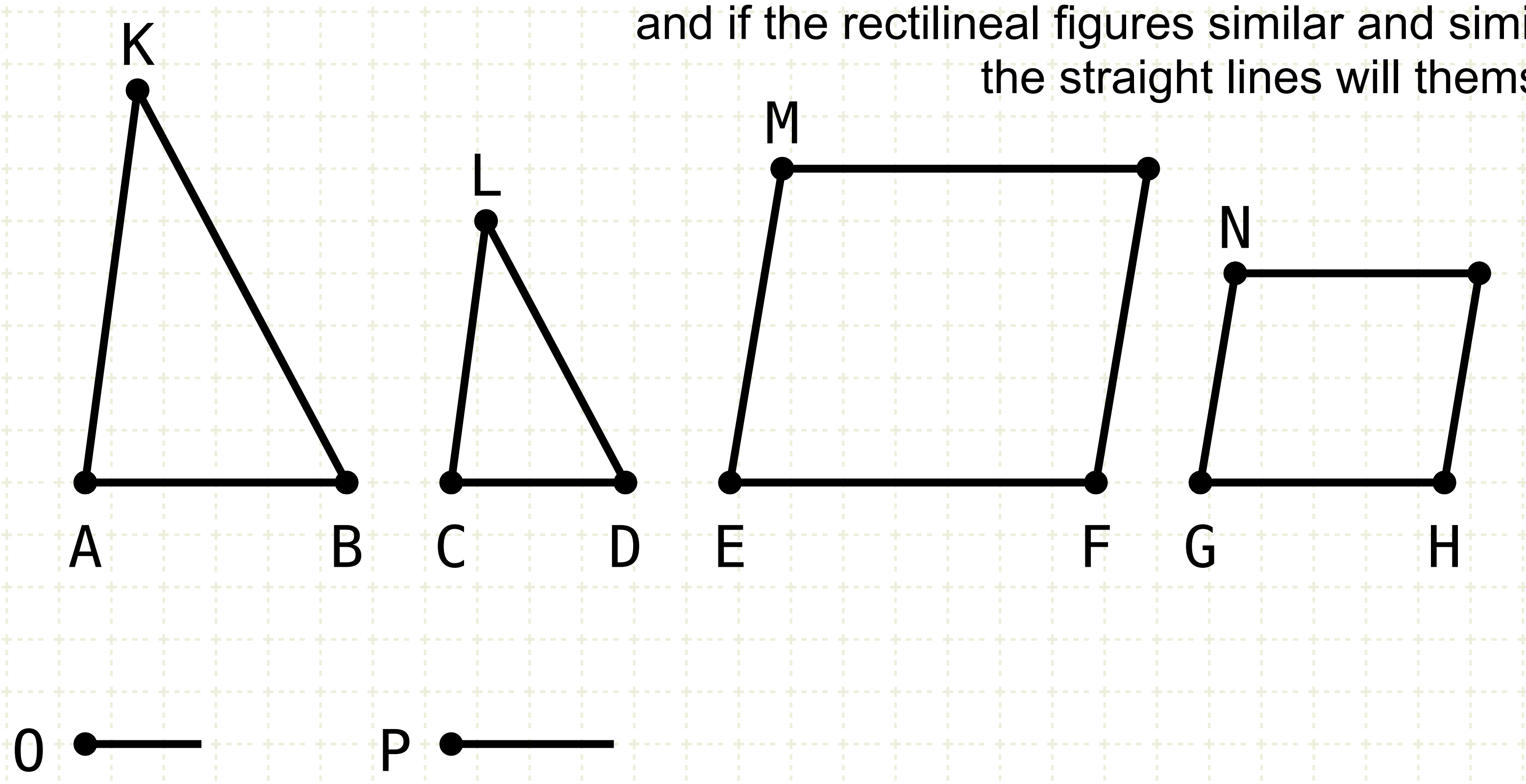

AB:CD = EF:GH
AB:CD = CD:O
EF:GH = GH:P
CD:O = GH:P
AB:O = EF:P
ΔKAB:ΔLCD = AB:O
□MF:□NH = EF:P
ΔKAB:ΔLCD = □MF:□NH

Proof - Part 1

Create a third proportional (O) to AB, CD, and another third proportional (P) to EF, GH (VI·11)

Thus the ratios CD to O and GH to P are also equal

Therefore, ex aequali, AB is to O as EF is to P (V·22)

The ratios of similar triangles is equal to the duplicate ratio of their sides about an equal angle (VI·19.Por), so KAB is to LCD as AB is to O

Likewise, MF is to NH as EF is to P

Therefore, KAB is to LCD as MF is to NH (V·11)

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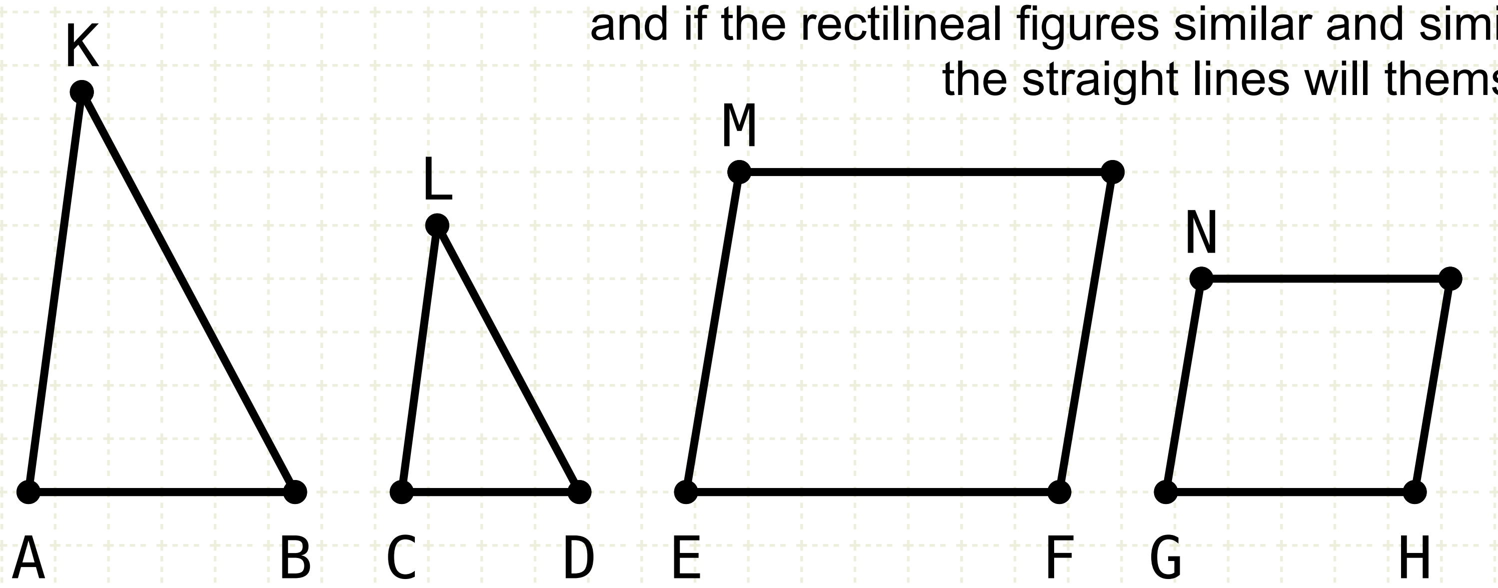

In other words - Part 2

If KAB is to LCD as MF is to NH, then AB is to CD as EF is to GH

$\Delta KAB : \Delta LCD = \square MF : \square NH$

$\rightarrow AB : CD = EF : GH$

Proposition 22 of Book VI

If four straight lines be proportional, the rectilineal figures similar and similarly described upon them will also be proportional;
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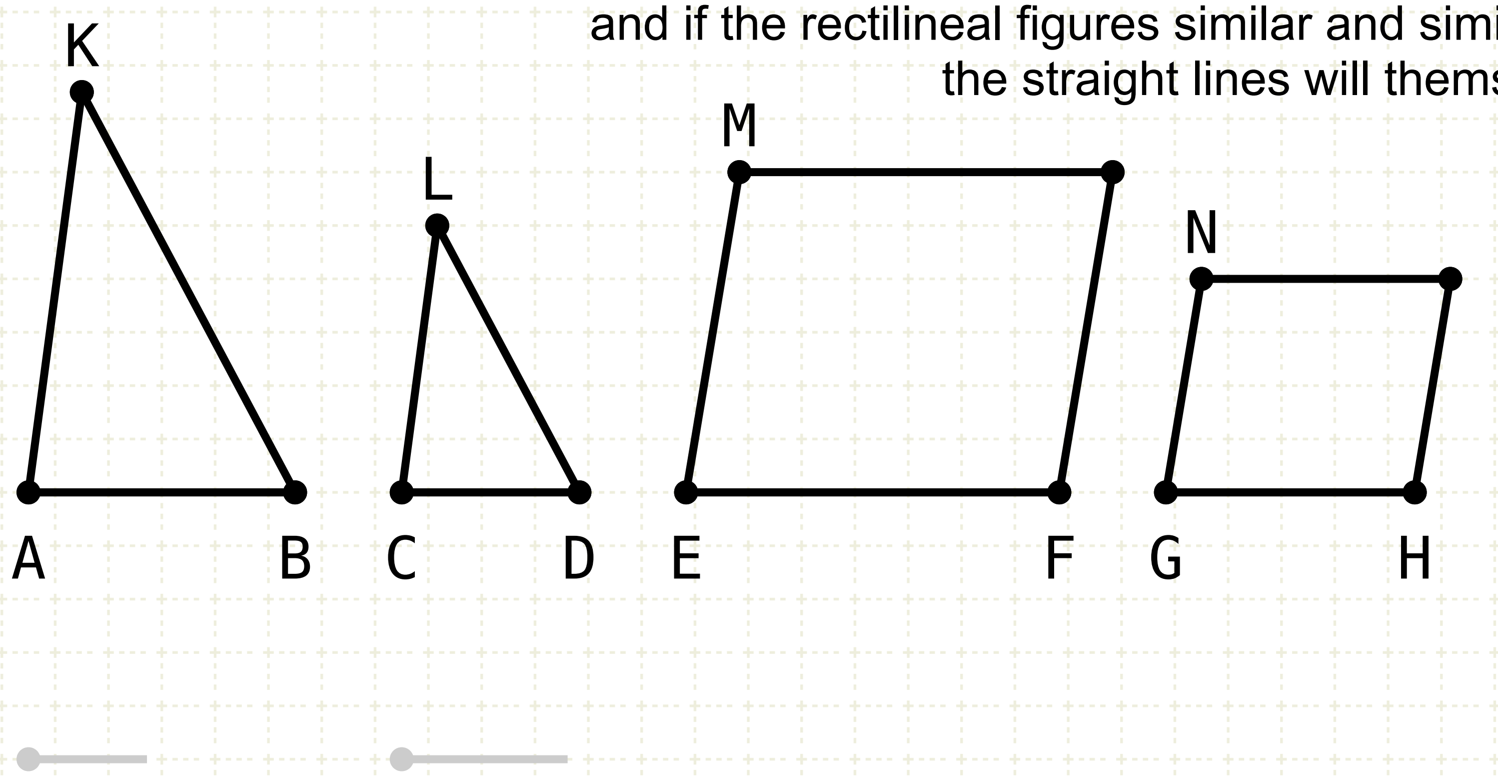

In other words - Part 2

If KAB is to LCD as MF is to NH, then AB is to CD as EF is to GH

Proof by Contradiction

ΔKAB:ΔLCD = □MF:□NH

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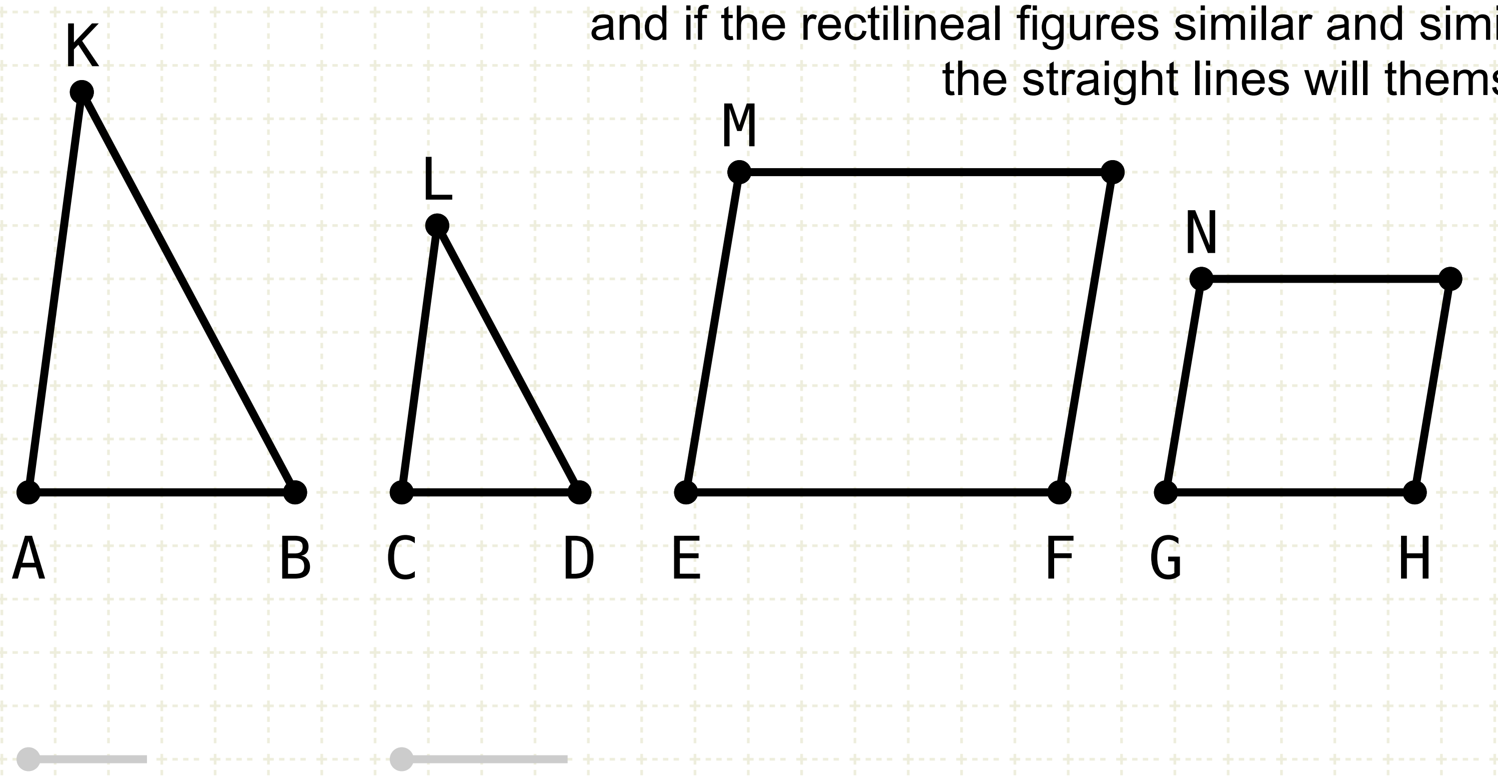

In other words - Part 2

If KAB is to LCD as MF is to NH, then AB is to CD as EF is to GH

Proof by Contradiction

Assume that the ratio EF to GH is not equal to the ratio AB to CD

$\Delta KAB : \Delta LCD = \square MF : \square NH$

$AB : CD \neq EF : GH$

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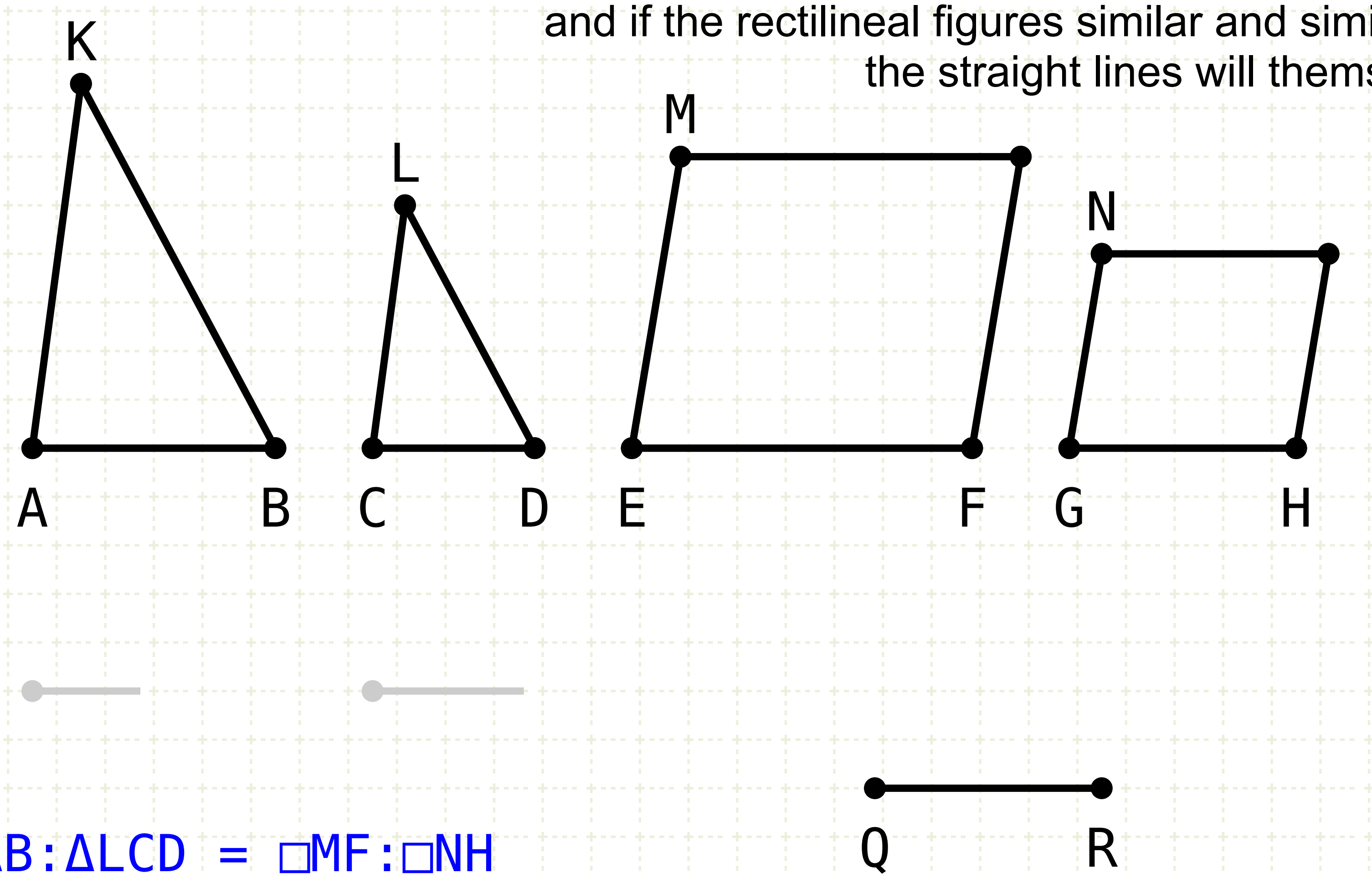

$\Delta KAB : \Delta LCD = \square MF : \square NH$

$AB:CD \neq EF:GH$

$AB:CD = EF:QR$

In other words - Part 2

If KAB is to LCD as MF is to NH, then AB is to CD as EF is to GH

Proof by Contradiction

Assume that the ratio EF to GH is not equal to the ratio AB to CD

Define QR such that EF to QR is equal to AB to CD

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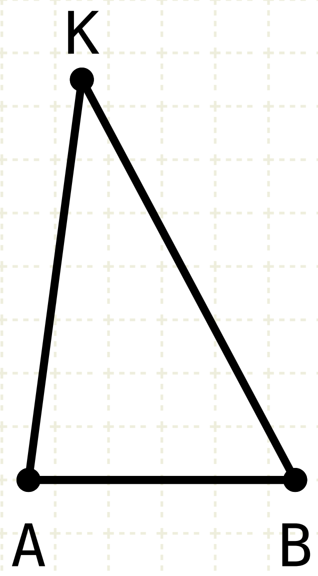

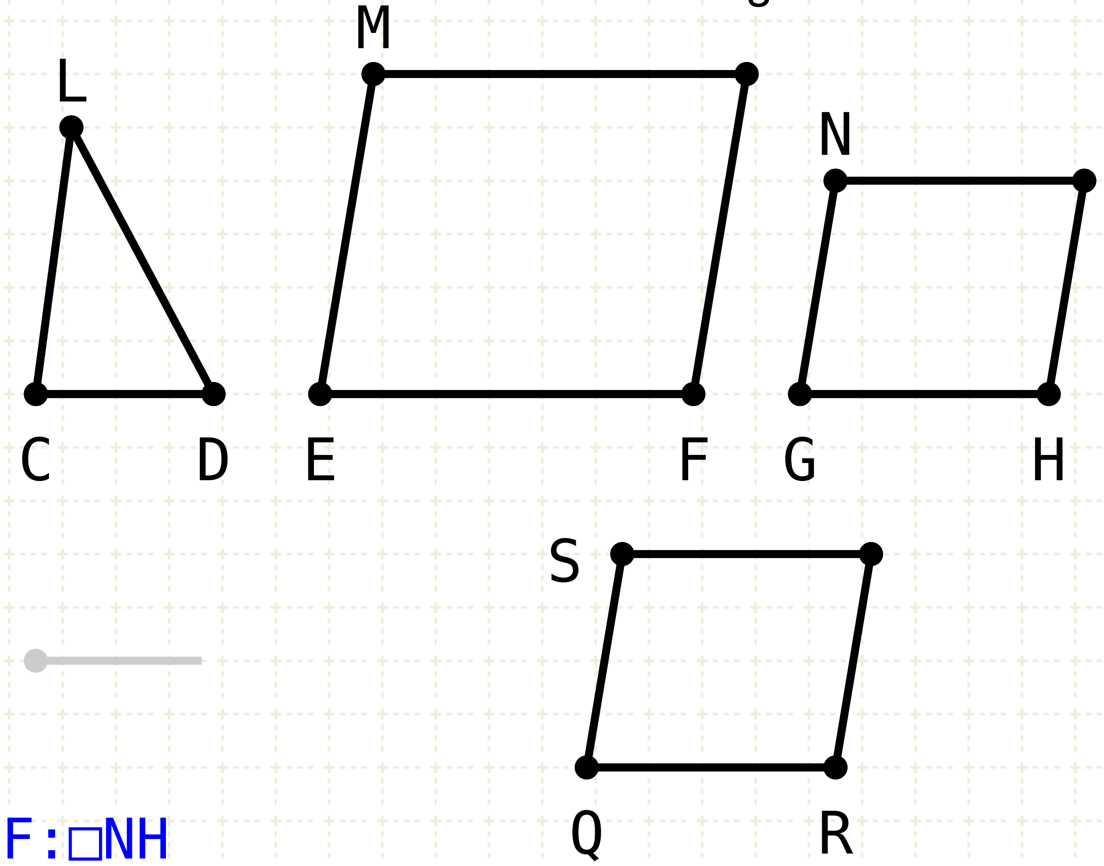

$\Delta KAB:\Delta LCD = \square MF:\square NH$

$AB:CD \neq EF:GH$

$AB:CD = EF:QR$

$\square SR \sim \square NH$

In other words - Part 2

If KAB is to LCD as MF is to NH, then AB is to CD as EF is to GH

Proof by Contradiction

Assume that the ratio EF to GH is not equal to the ratio AB to CD

Define QR such that EF to QR is equal to AB to CD

Draw SR similar to either MF or NH (VI·18)

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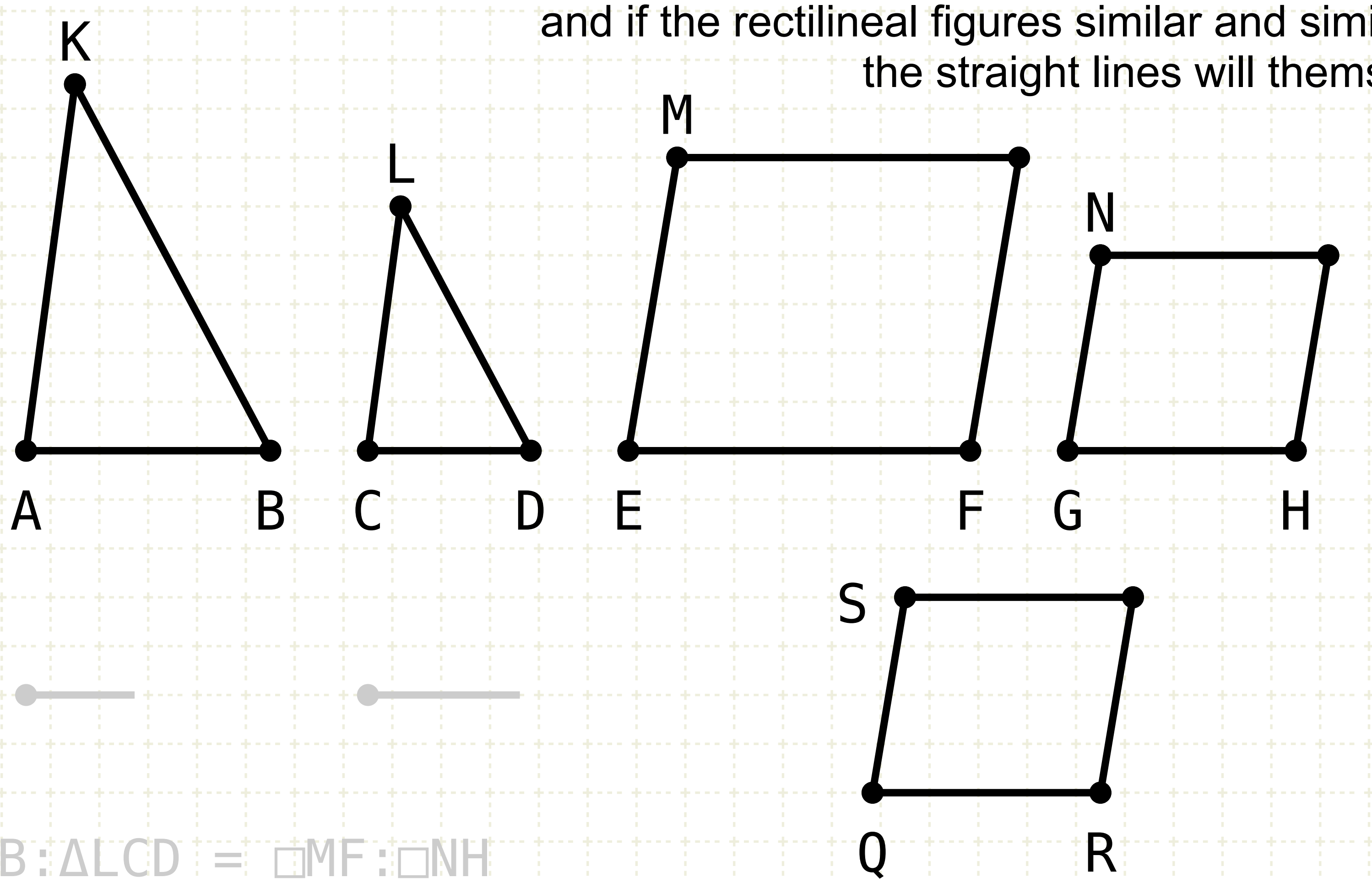

ΔKAB:ΔLCD = □MF:□NH

AB:CD ≠ EF:GH

AB:CD = EF:QR

ΔKAB:ΔLCD = □MF:□SR

□SR ~ □NH

In other words - Part 2

If KAB is to LCD as MF is to NH, then AB is to CD as EF is to GH

Proof by Contradiction

Assume that the ratio EF to GH is not equal to the ratio AB to CD

Define QR such that EF to QR is equal to AB to CD

Draw SR similar to either MF or NH (VI·18)

Since AB is to CD as EF is to QR, then KAB is to LCD, so is MF to SR

Proposition 22 of Book VI

If four straight lines be proportional, the rectilineal figures similar and similarly described upon them will also be proportional;
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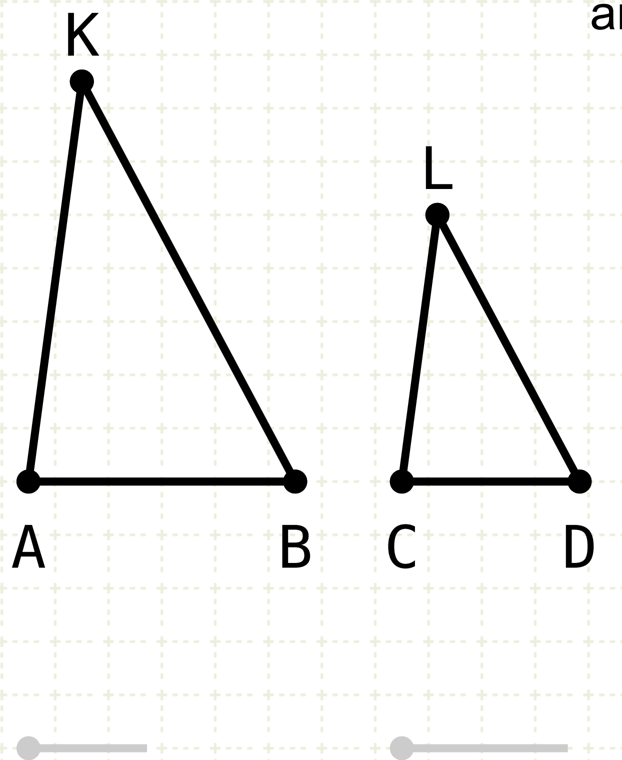

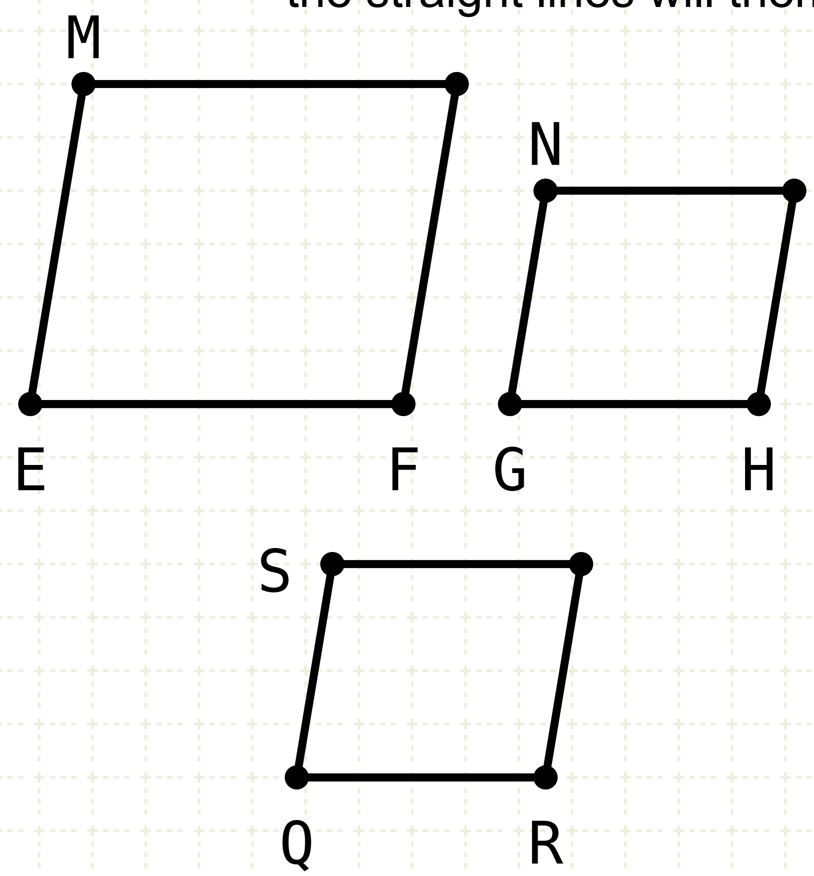

ΔKAB:ΔLCD = □MF:□NH

AB:CD ≠ EF:GH

AB:CD = EF:QR

ΔKAB:ΔLCD = □MF:□SR

□MF:□NH = □MF:□SR

□SR ~ □NH

In other words - Part 2

If KAB is to LCD as MF is to NH, then AB is to CD as EF is to GH

Proof by Contradiction

Assume that the ratio EF to GH is not equal to the ratio AB to CD

Define QR such that EF to QR is equal to AB to CD

Draw SR similar to either MF or NH (VI·18)

Since AB is to CD as EF is to QR, then KAB is to LCD, so is MF to SR

But MF to NH is also equal to KAB to LCD, therefore MF is to SR so is MF to NH (V·11)

Proposition 22 of Book VI

If four straight lines be proportional, the rectilineal figures similar and similarly described upon them will also be proportional;
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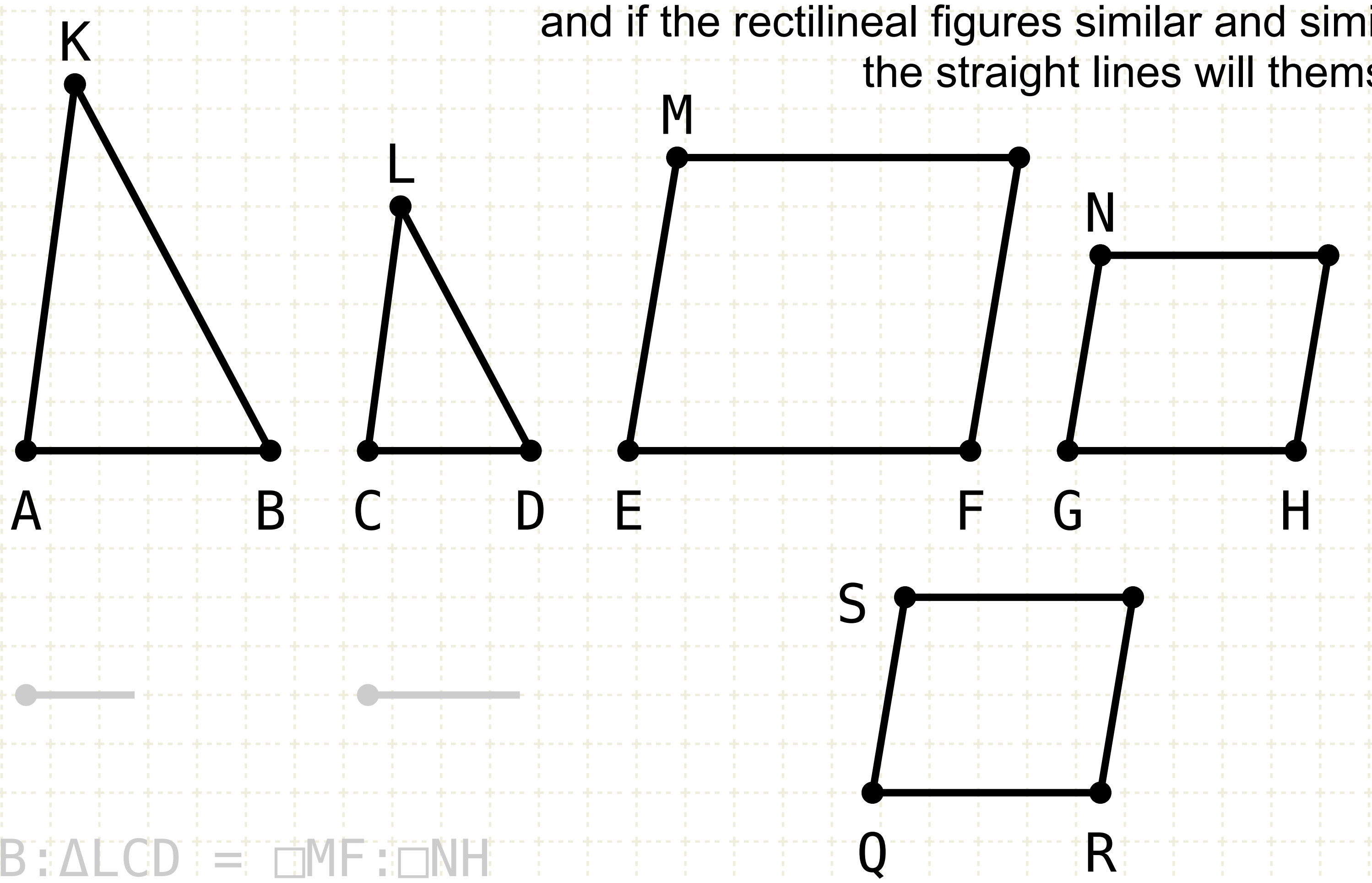

□SR ~ □NH

ΔKAB:ΔLCD = □MF:□NH

AB:CD ≠ EF:GH

AB:CD = EF:QR

ΔKAB:ΔLCD = □MF:□SR

□MF:□NH = □MF:□SR

□NH = □SR

In other words - Part 2

If KAB is to LCD as MF is to NH, then AB is to CD as EF is to GH

Proof by Contradiction

Assume that the ratio EF to GH is not equal to the ratio AB to CD

Define QR such that EF to QR is equal to AB to CD

Draw SR similar to either MF or NH (VI·18)

Since AB is to CD as EF is to QR, then KAB is to LCD, so is MF to SR

But MF to NH is also equal to KAB to LCD, therefore MF is to SR so is MF to NH (V·11)

MF has the same ratio to SR as it does to NH, therefore SR is equal to NH (V·9)

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If four straight lines be proportional, the rectilineal figures similar and similarly described upon them will also be proportional;
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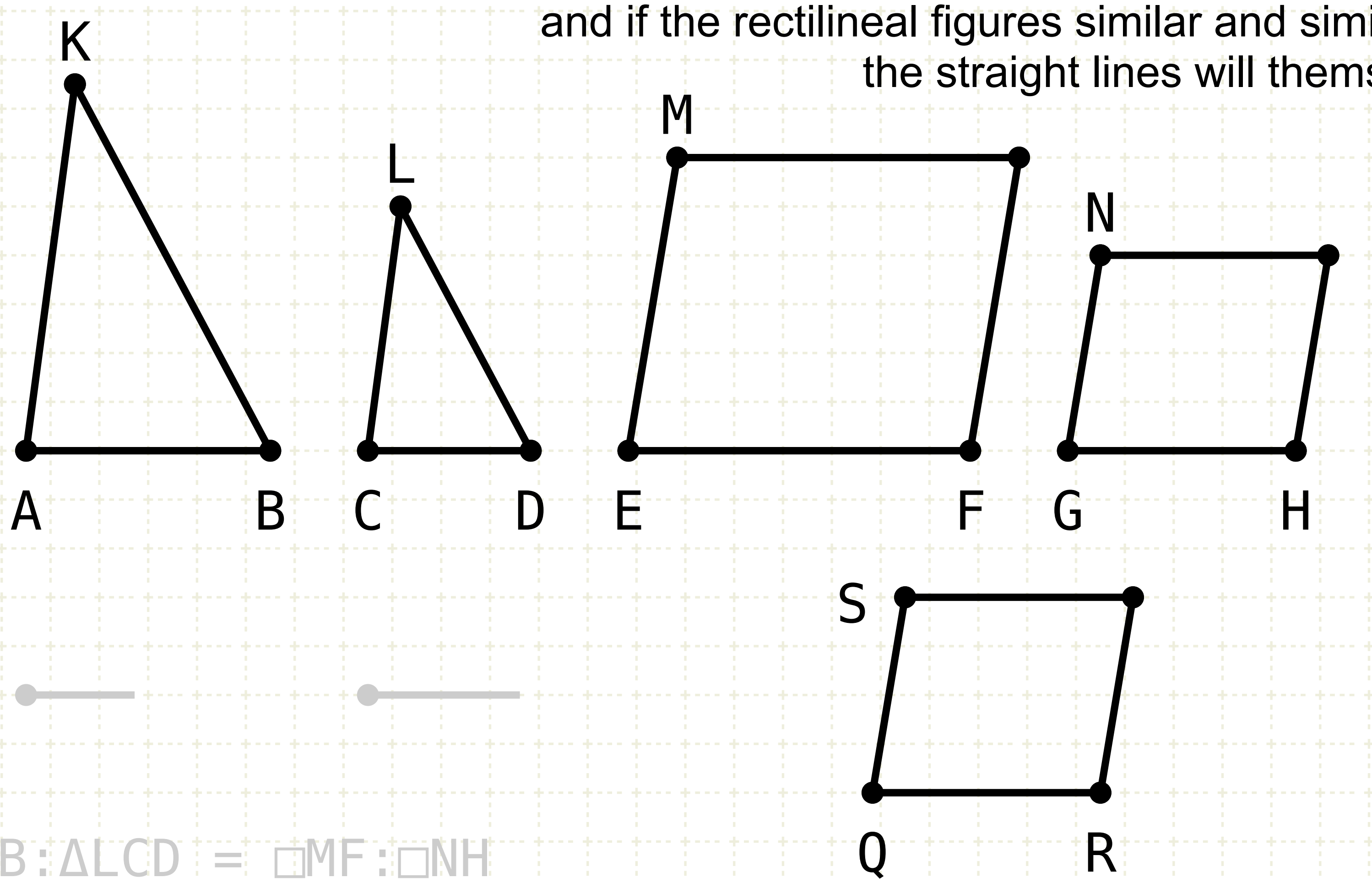

□SR ~ □NH

ΔKAB:ΔLCD = □MF:□NH

AB:CD ≠ EF:GH

AB:CD = EF:QR

ΔKAB:ΔLCD = □MF:□SR

□MF:□NH = □MF:□SR

□NH = □SR

QH = GR

In other words - Part 2

If KAB is to LCD as MF is to NH, then AB is to CD as EF is to GH

Proof by Contradiction

Assume that the ratio EF to GH is not equal to the ratio AB to CD

Define QR such that EF to QR is equal to AB to CD

Draw SR similar to either MF or NH (VI·18)

Since AB is to CD as EF is to QR, then KAB is to LCD, so is MF to SR

But MF to NH is also equal to KAB to LCD, therefore MF is to SR so is MF to NH (V·11)

MF has the same ratio to SR as it does to NH, therefore SR is equal to NH (V·9)

But NH and SR are similar, therefore GH = QR

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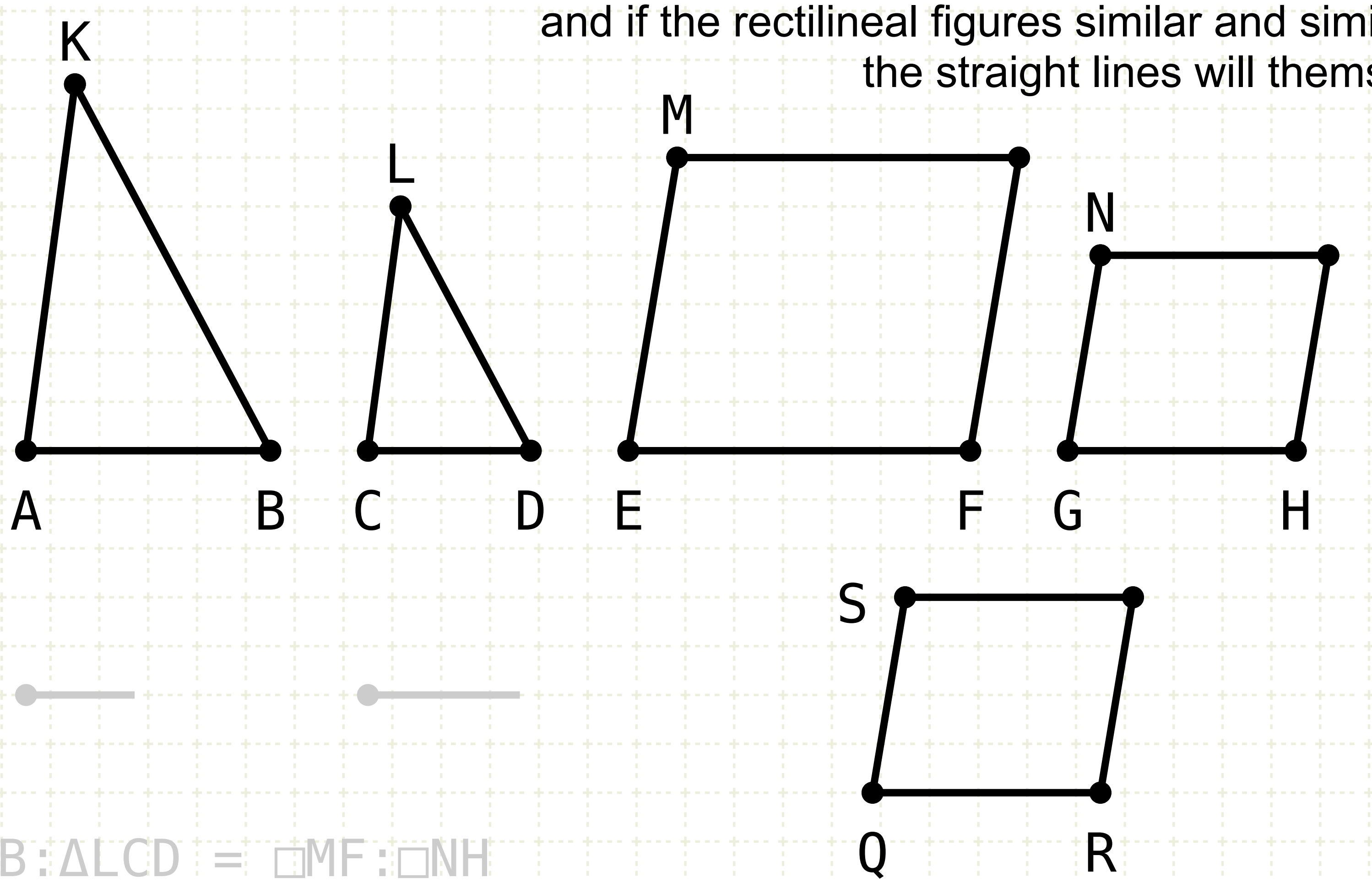

□SR ~ □NH

ΔKAB:ΔLCD = □MF:□NH

AB:CD ≠ EF:GH

AB:CD = EF:QR

ΔKAB:ΔLCD = □MF:□SR

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□NH = □SR

QH = GR

AB:CD = EF:GH

In other words - Part 2

If KAB is to LCD as MF is to NH, then AB is to CD as EF is to GH

Proof by Contradiction

Assume that the ratio EF to GH is not equal to the ratio AB to CD

Define QR such that EF to QR is equal to AB to CD

Draw SR similar to either MF or NH (VI·18)

Since AB is to CD as EF is to QR, then KAB is to LCD, so is MF to SR

But MF to NH is also equal to KAB to LCD, therefore MF is to SR so is MF to NH (V·11)

MF has the same ratio to SR as it does to NH, therefore SR is equal to NH (V·9)

But NH and SR are similar, therefore GH = QR

If AB is to CD as EF is to QR, and QR is equal to GH, then AB is to CE as EF is to GH

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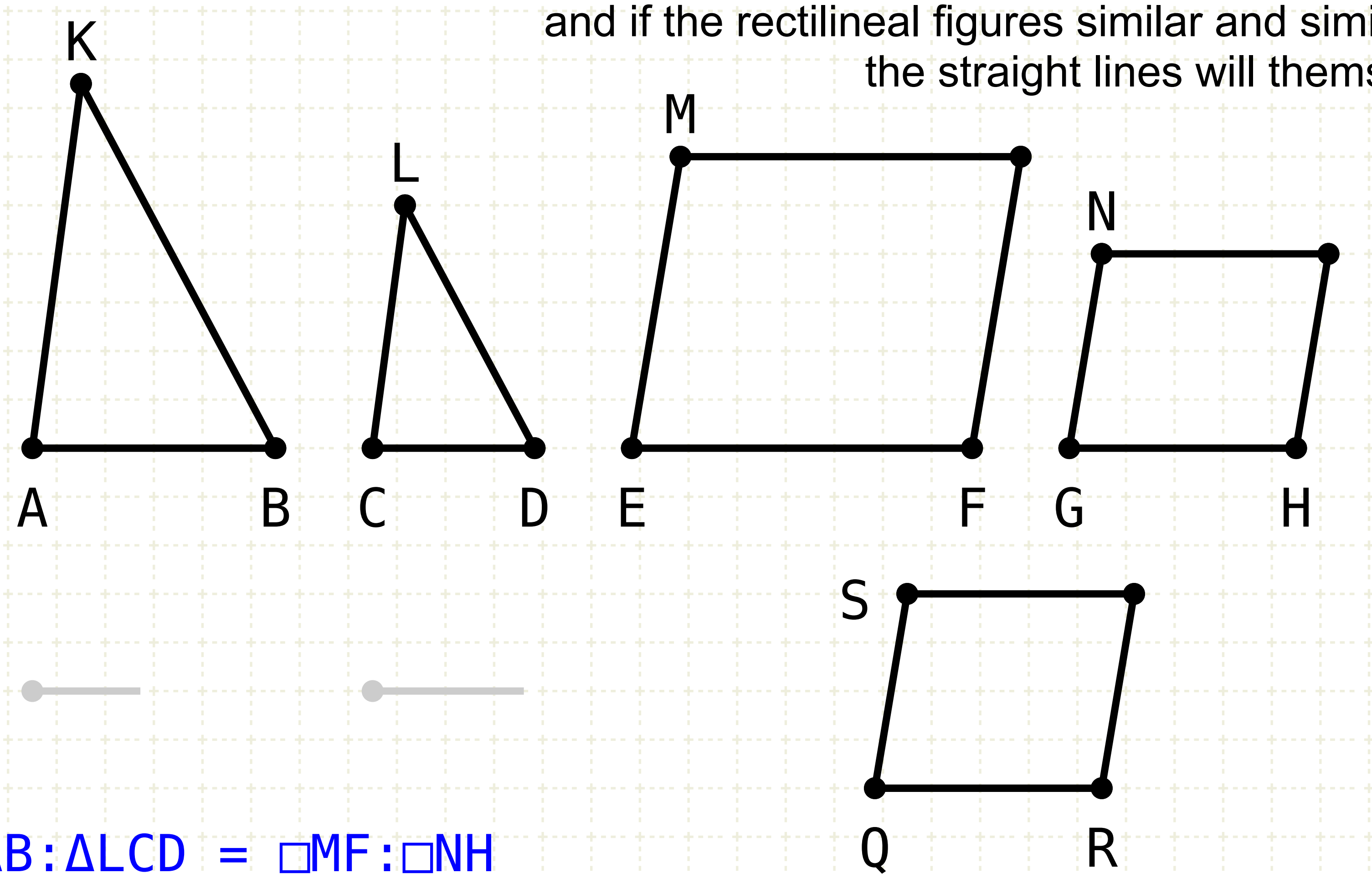

$\Delta KAB : \Delta LCD = \square MF : \square NH$

$AB : CD \neq EF : GH$

$AB : CD = EF : QR$

$\Delta KAB : \Delta LCD = \square MF : \square SR$

$\square MF : \square NH = \square MF : \square SR$

$\square NH = \square SR$

$QH = GR$

$\square SR \sim \square NH$

$AB : CD = EF : GH$

In other words - Part 2

If KAB is to LCD as MF is to NH, then AB is to CD as EF is to GH

Proof by Contradiction

Assume that the ratio EF to GH is not equal to the ratio AB to CD

Define QR such that EF to QR is equal to AB to CD

Draw SR similar to either MF or NH (VI·18)

Since AB is to CD as EF is to QR, then KAB is to LCD, so is MF to SR

But MF to NH is also equal to KAB to LCD, therefore MF is to SR so is MF to NH (V·11)

MF has the same ratio to SR as it does to NH, therefore SR is equal to NH (V·9)

But NH and SR are similar, therefore GH = QR

If AB is to CD as EF is to QR, and QR is equal to GH, then AB is to CE as EF is to GH

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