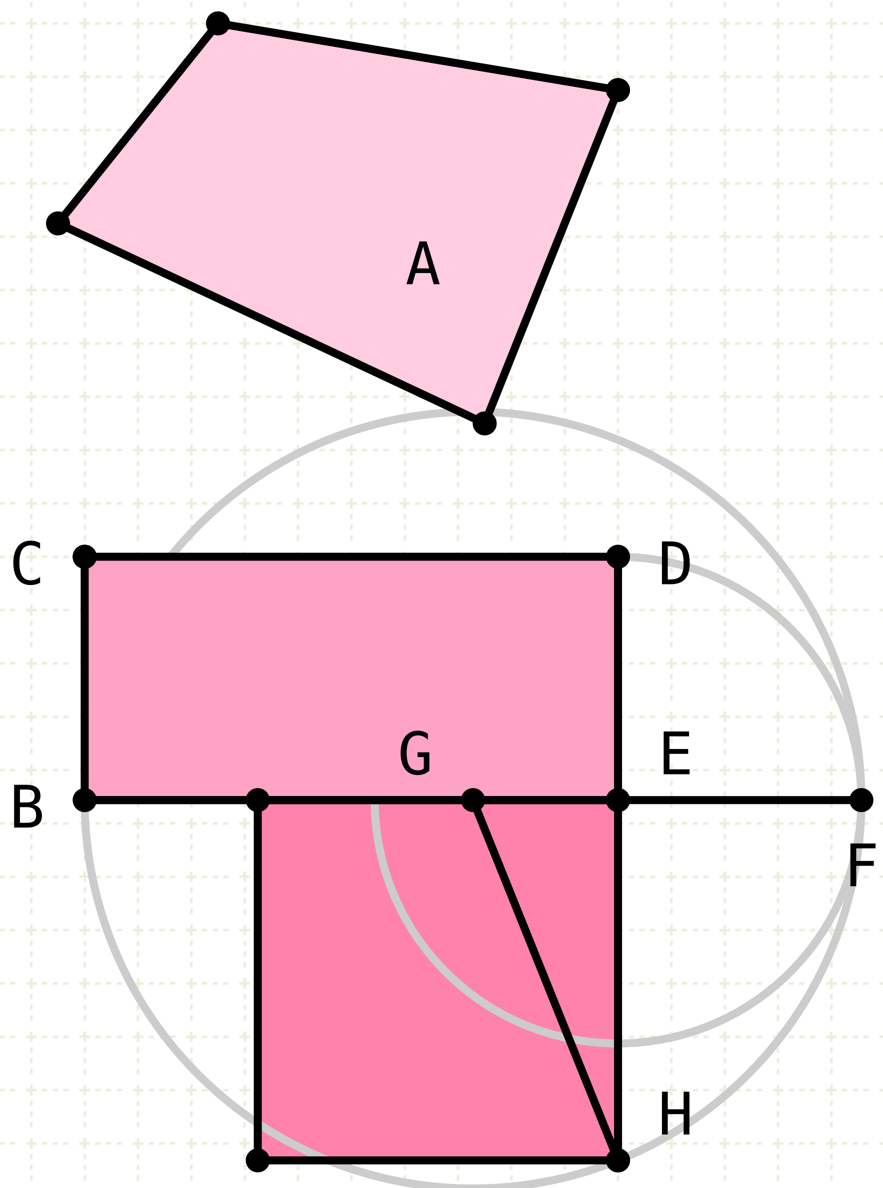
Euclid's Elements

Book II



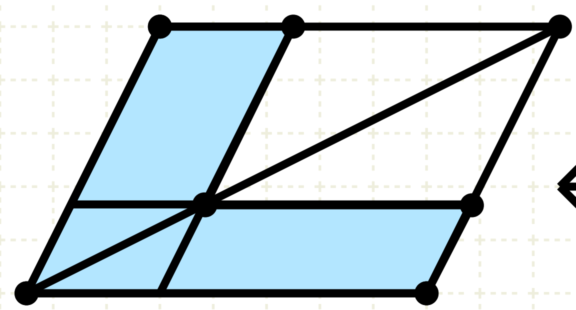
It is a remarkable fact in the history of geometry, that the Elements of Euclid, written two thousand years ago, are still regarded by many as the best introduction to the mathematical sciences.

Florian Cajori,

A History of Mathematics (1893)

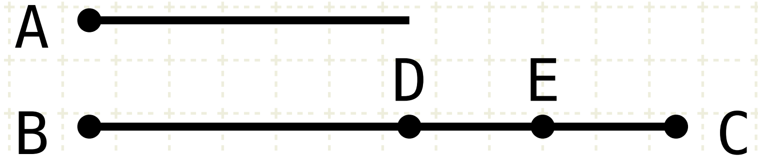
Definitions:

Any rectangular parallelogram is said to be contained by the two straight lines containing the right angle.

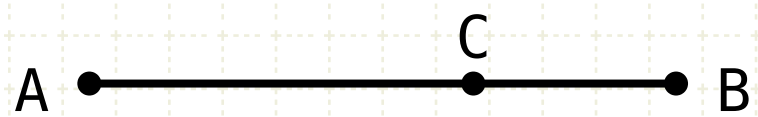


And in any parallelogrammic area let any one whatever of the parallelograms about its diameter with the two complements be called a gnomon.

Table of Contents, Chapter 2

1


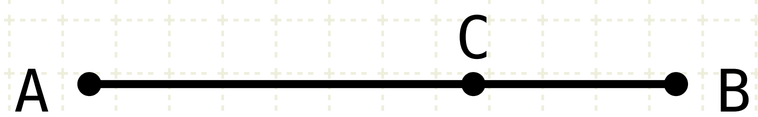
$A \cdot BC = A \cdot BD + A \cdot DE + A \cdot EC$

2


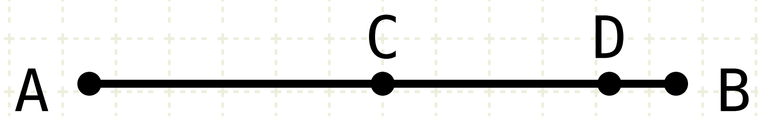
$AB^2 = AB \cdot AC + AB \cdot BC$

3

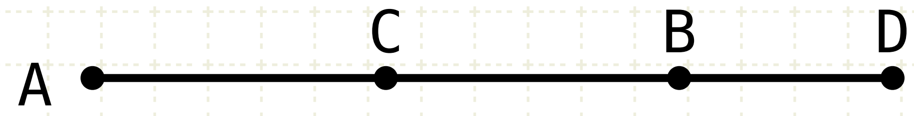

$AB \cdot CB = AC \cdot CB + CB^2$

4


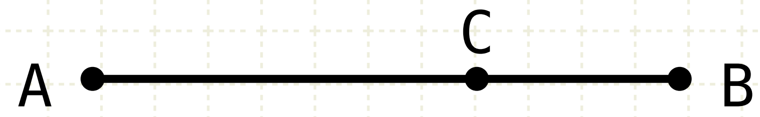
$\mathbf{AB^2 = AC^2 + CB^2 + 2 \cdot AC \cdot CB}$

5


$AD \cdot DB + CD^2 = CB^2$

6


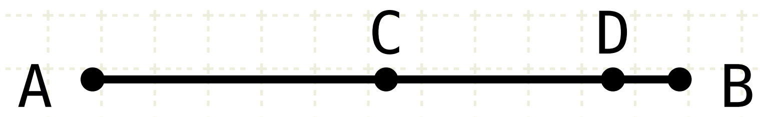
$AD \cdot DB + CB^2 = CD^2$

7


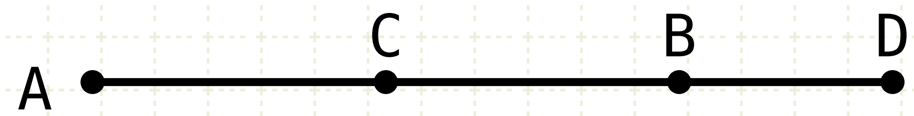
$AB^2 + BC^2 = AC^2 + 2 \cdot AB \cdot BC$

8

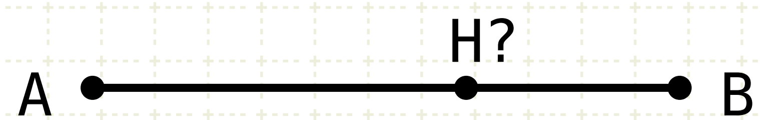

$4 \cdot AB \cdot BC + AC^2 = (AB+BC)^2$

9


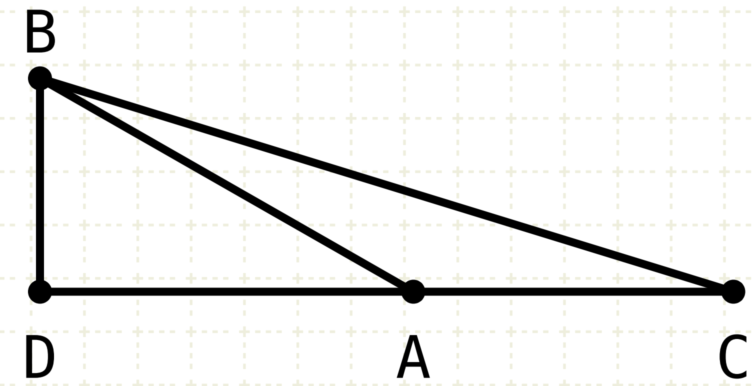
$AD^2 + DB^2 = 2 \cdot (AC^2 + CD^2)$

10


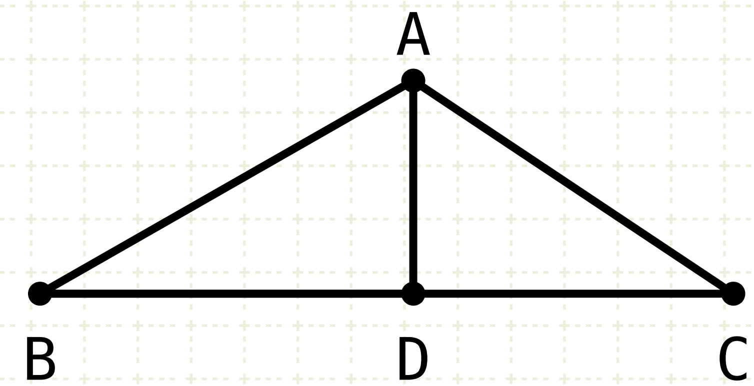
$AD^2 + DB^2 = 2 \cdot (AC^2 + CD^2)$

11


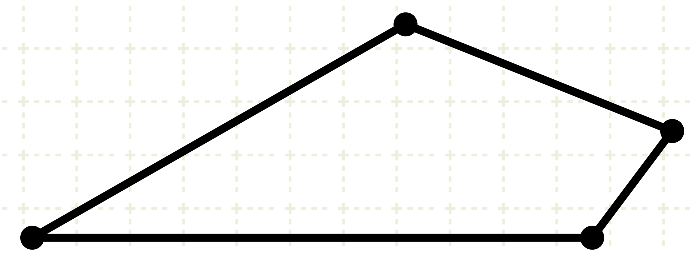
Find H. $AB \cdot BH = AH^2$

12


Cosine Law. $BC^2 = AB^2+AC^2+2 \cdot AD \cdot AC$

13


Cosine Law. $AC^2 = AB^2+BC^2-2 \cdot BD \cdot BC$

14


Find square of polygon

Proposition 4 of Book II

If a straight line be cut at random, the square on the whole is equal to the squares on the segments and twice the rectangle contained by the segments.

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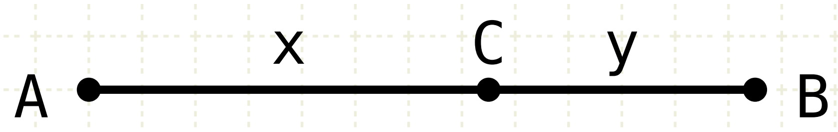

In other words

Let AB be a straight line, arbitrarily cut at point C

$AB = AC + CB$

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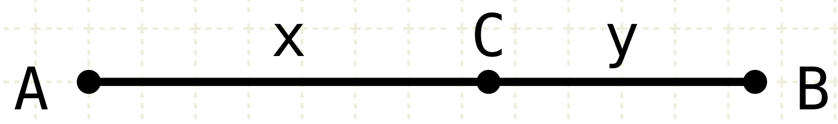
If a straight line be cut at random, the square on the whole is equal to the squares on the segments and twice the rectangle contained by the segments.



In other words

Let AB be a straight line, arbitrarily cut at point C

Then the square formed by line AB is equal in area to the sum of the squares formed by line CB and AC, plus twice the area of the rectangles formed by lines AC and CB

$AB = AC + CB$

$AB \cdot AB = AC \cdot AC + CB \cdot CB + 2 \cdot AC \cdot CB$

$(x+y)^2 = x^2 + y^2 + 2xy$

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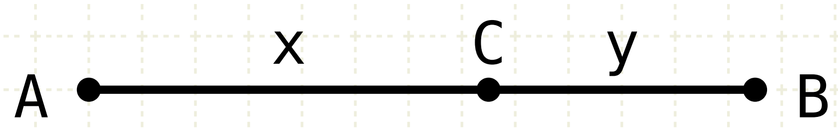

Proof:

$AB = AC + CB$

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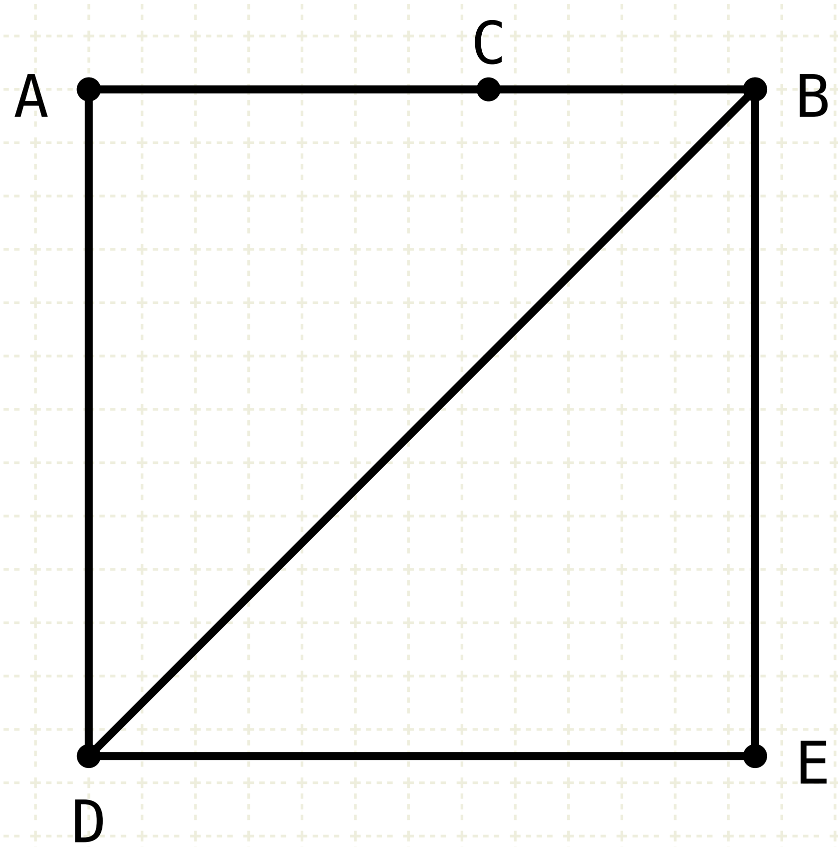

AB = BD

AB = AC + CB

Proof:

Draw a square ADEB on the line AB (I·46), and draw the diagonal BD

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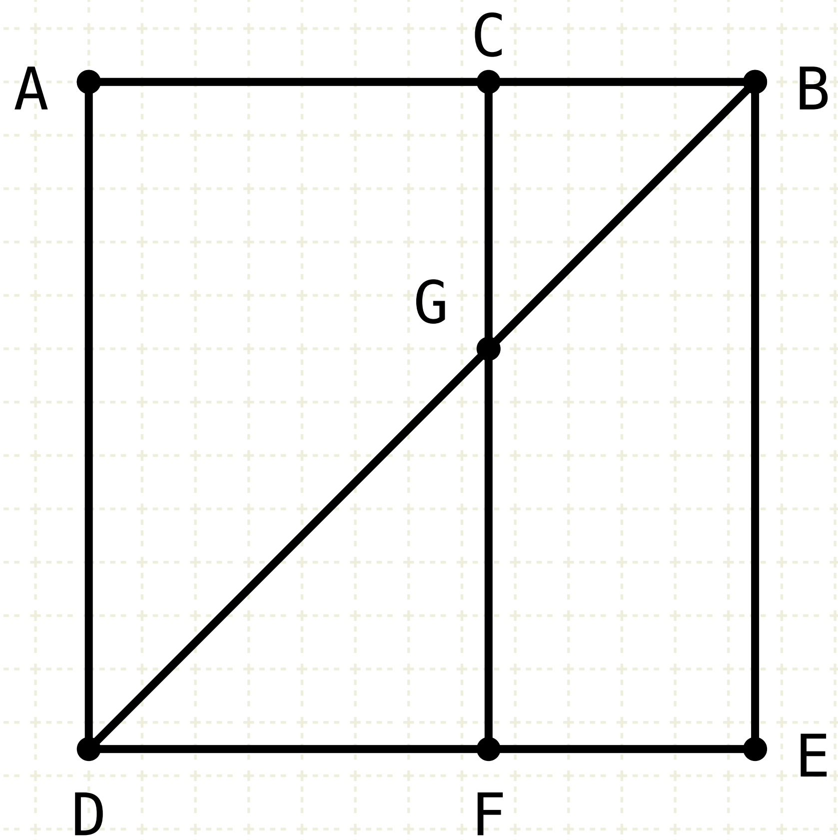

AB = BD

AB = AC + CB

Proof:

Draw a square ADEB on the line AB (I·46), and draw the diagonal BD

Draw a line CF parallel to either AD or BE (I·31), labelling the intersection with the diagonal as G.

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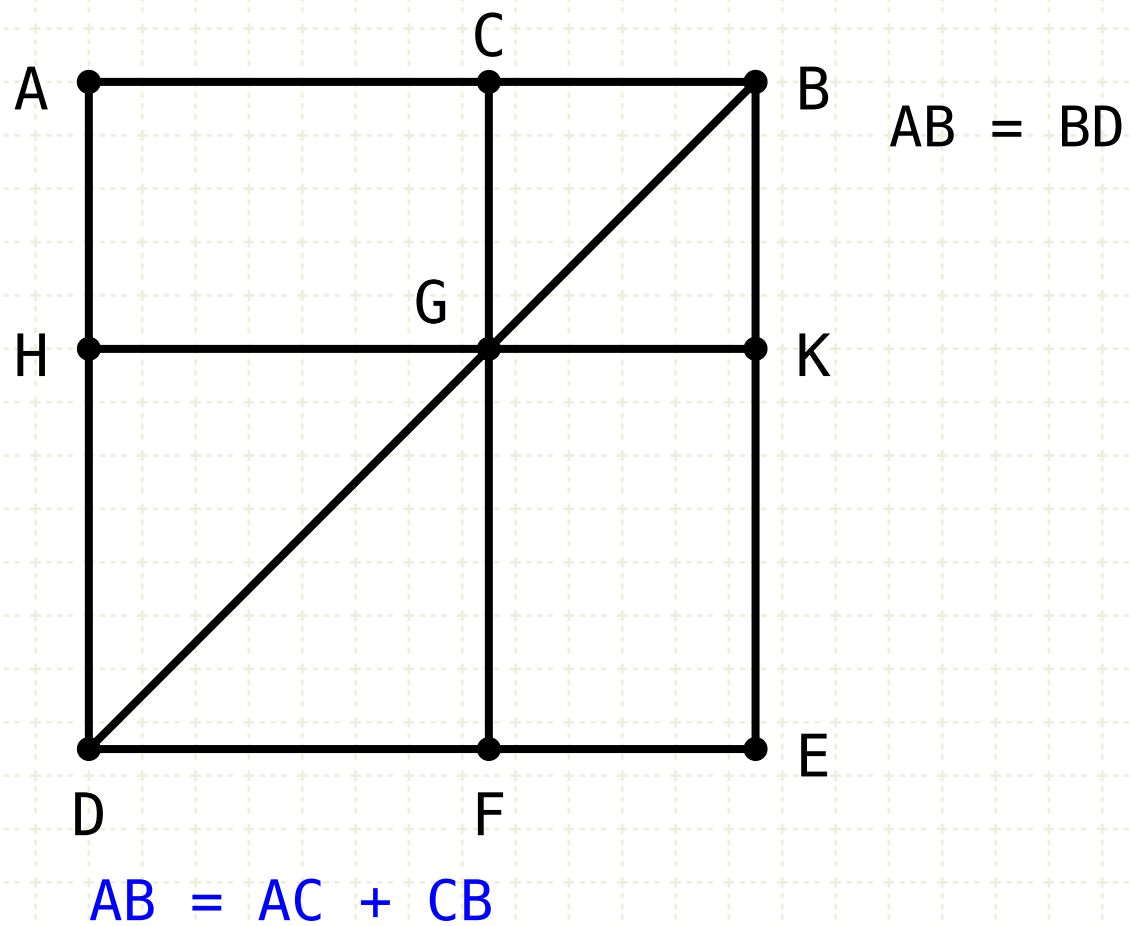

Proof:

Draw a square ADEB on the line AB (I·46), and draw the diagonal BD

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Draw a line parallel to AB through the point G (I·31).

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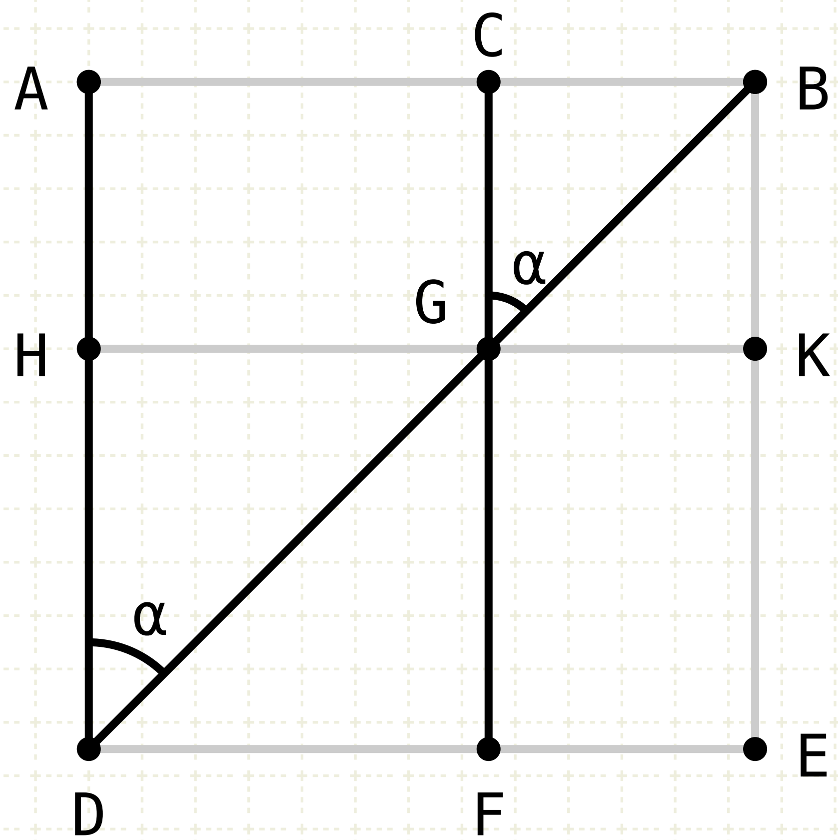

AB = BD
∠CGB = ∠ADB

AB = AC + CB

Proof:

Draw a square ADEB on the line AB (I·46), and draw the diagonal BD

Draw a line CF parallel to either AD or BE (I·31), labelling the intersection with the diagonal as G.

Draw a line parallel to AB through the point G (I·31).

Since BD crosses two parallel lines, (AD and CF), then the exterior angle is equal to the interior and opposite angle (I·29)

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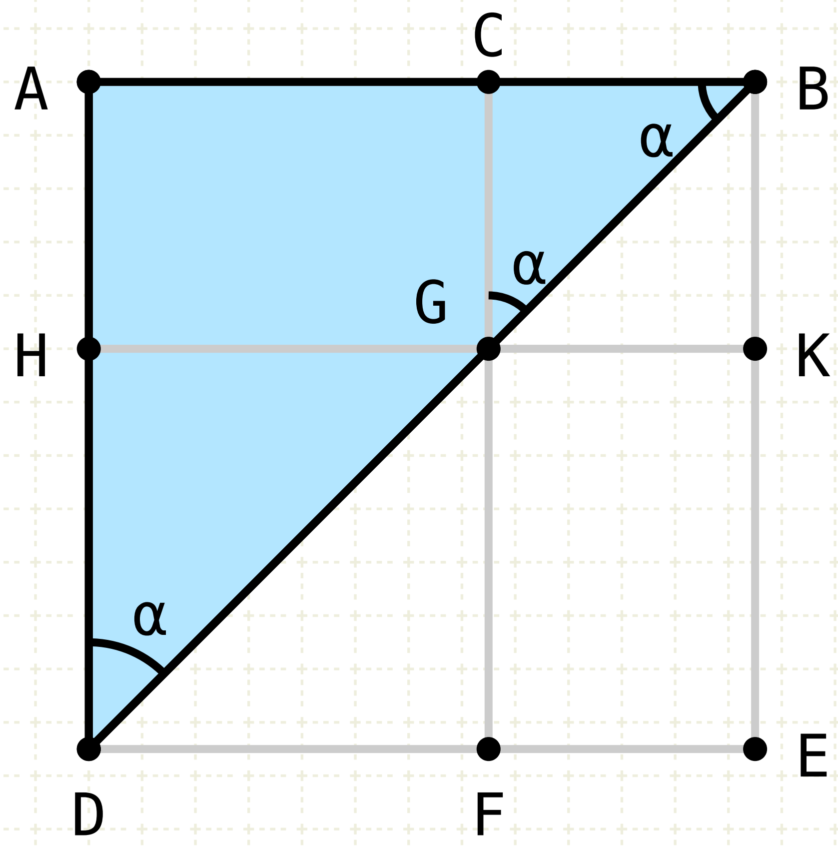

AB = BD

∠CGB = ∠ADB

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Since BD crosses two parallel lines, (AD and CF), then the exterior angle is equal to the interior and opposite angle (I·29)

Since triangle ABD is an isosceles triangle, then the angles at the base are also equal (I·5)

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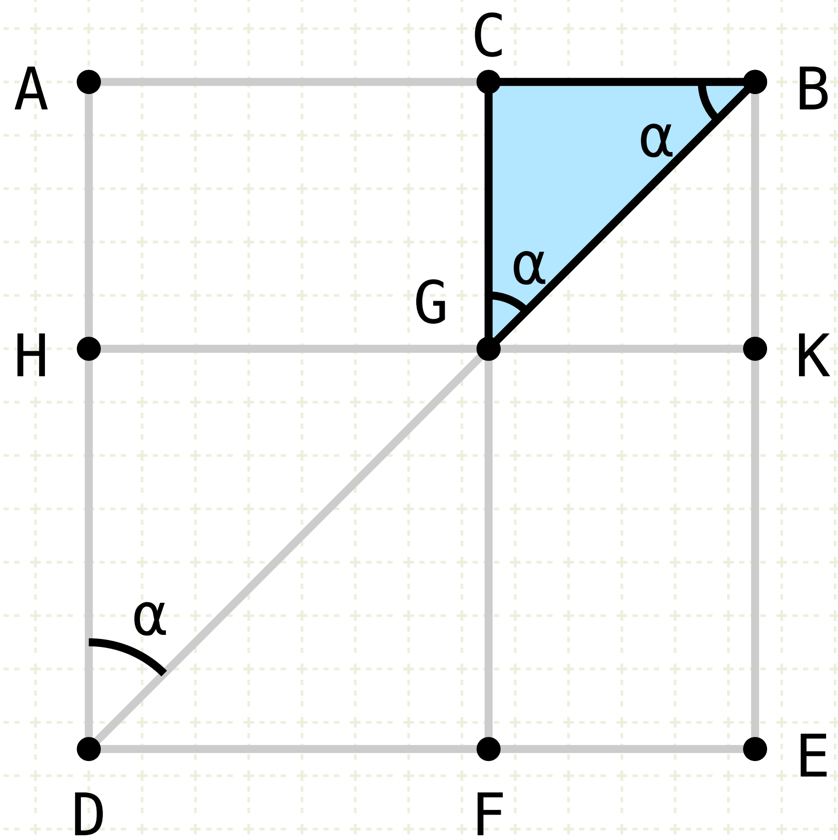

AB = BD
∠CGB = ∠ADB
∠ADB

AB = AC + CB
CG = CB

Proof:

Draw a square ADEB on the line AB (I·46), and draw the diagonal BD

Draw a line CF parallel to either AD or BE (I·31), labelling the intersection with the diagonal as G.

Draw a line parallel to AB through the point G (I·31).

Since BD crosses two parallel lines, (AD and CF), then the exterior angle is equal to the interior and opposite angle (I·29)

Since triangle ABD is an isosceles triangle, then the angles at the base are also equal (I·5)

Since triangle BCG has two equal angles, it is an isosceles triangle (I·6), therefore the sides of the triangle are equal

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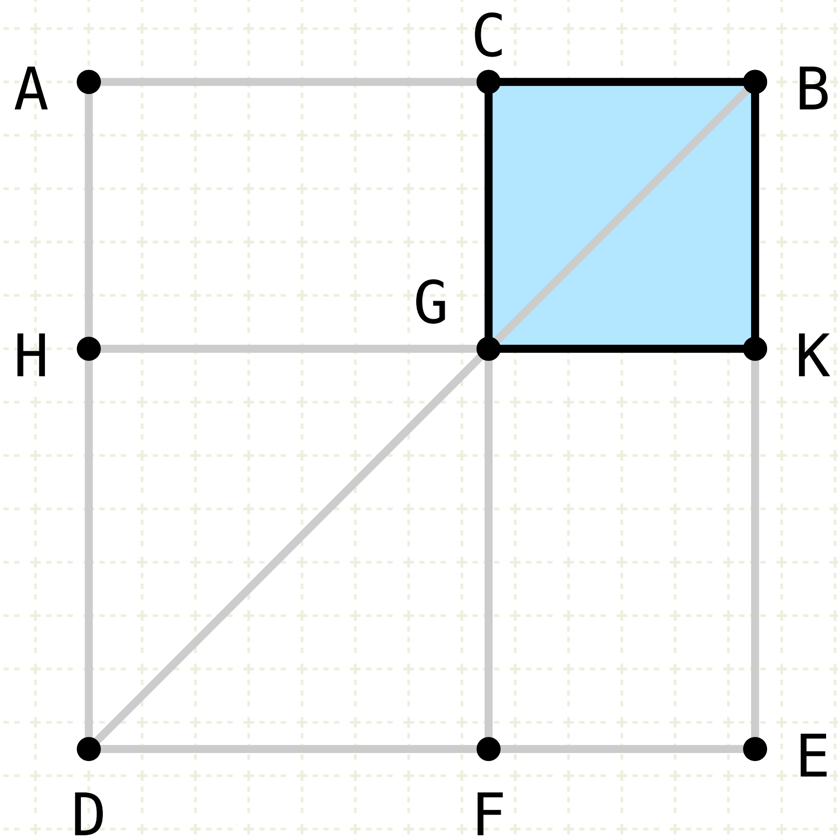

$AB = BD$

$\angle CGB = \angle ADB$

$\angle ADB$

$AB = AC + CB$

$CG = CB$

$CB \parallel GK,\ CG \parallel BK \quad \therefore CB = GK,\ CG = BK$

Proof:

Draw a square ADEB on the line AB (I·46), and draw the diagonal BD

Draw a line CF parallel to either AD or BE (I·31), labelling the intersection with the diagonal as G.

Draw a line parallel to AB through the point G (I·31).

Since BD crosses two parallel lines, (AD and CF), then the exterior angle is equal to the interior and opposite angle (I·29)

Since triangle ABD is an isosceles triangle, then the angles at the base are also equal (I·5)

Since triangle BCG has two equal angles, it is an isosceles triangle (I·6), therefore the sides of the triangle are equal

CB and GK are parallel,CG and BK are parallel, so the opposite sides equal on another (I·34), and since CB equals CG, all sides are equal, therefore CK is an equilateral

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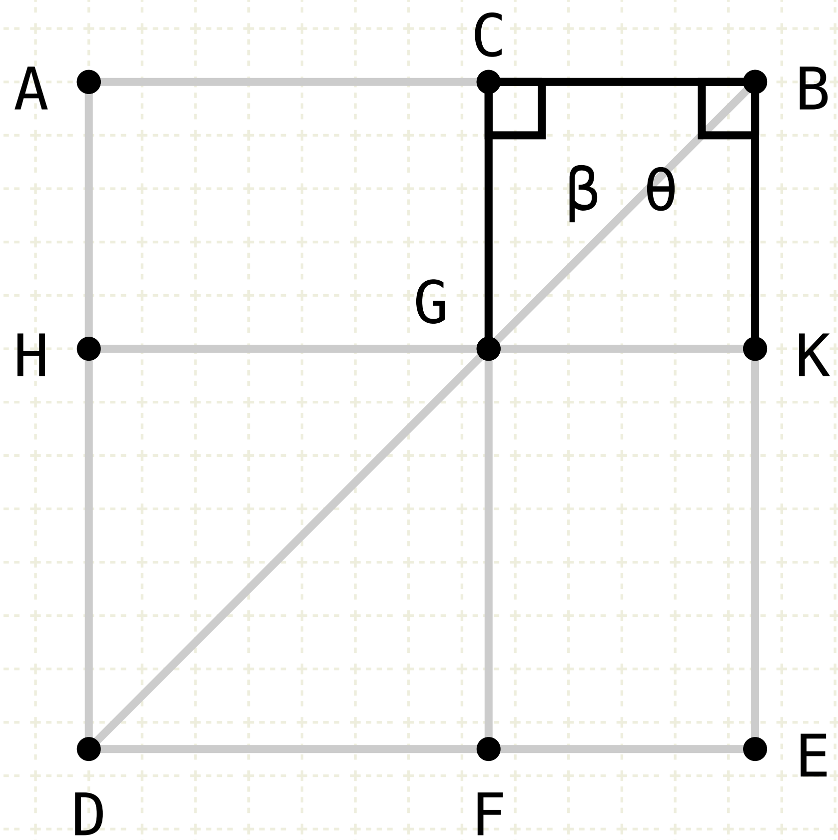


$AB = BD$

$\angle CGB = \angle ADB$

$\angle ADB$

$CBK = BCG = ∟$

$AB = AC + CB$

$CG = CB$

$CB \| GK,\ CG \| BK \quad \therefore CB = GK,\ CG = BK$

$\beta + \theta = ∟ + ∟, \quad \theta = ∟ \quad \therefore \beta = ∟$

Proof:

Draw a square ADEB on the line AB (I·46), and draw the diagonal BD

Draw a line CF parallel to either AD or BE (I·31), labelling the intersection with the diagonal as G.

Draw a line parallel to AB through the point G (I·31).

Since BD crosses two parallel lines, (AD and CF), then the exterior angle is equal to the interior and opposite angle (I·29)

Since triangle ABD is an isosceles triangle, then the angles at the base are also equal (I·5)

Since triangle BCG has two equal angles, it is an isosceles triangle (I·6), therefore the sides of the triangle are equal

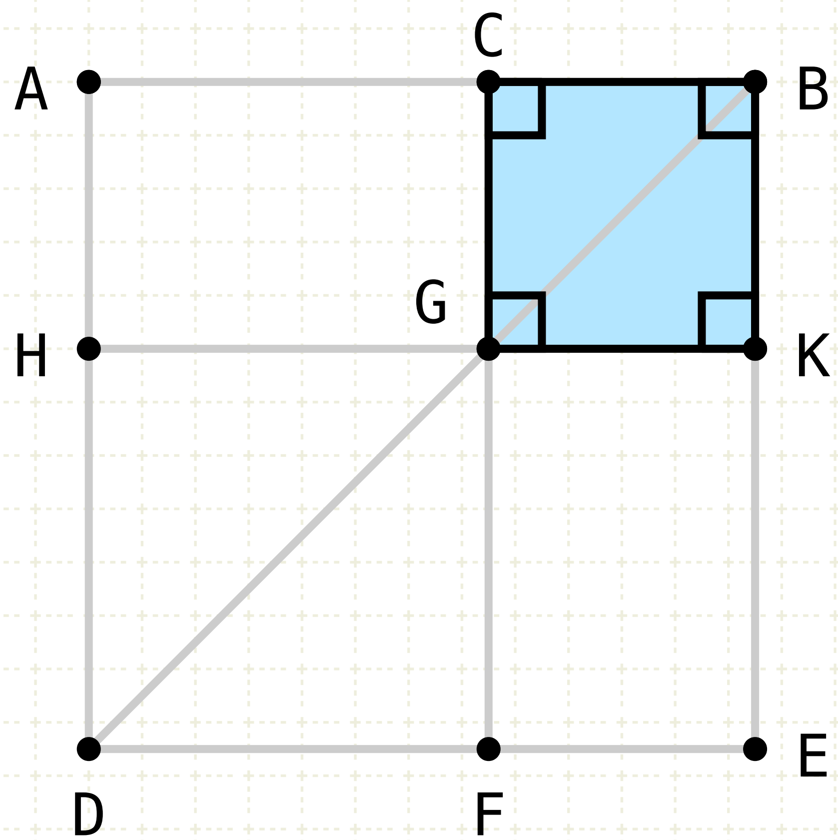
CB and GK are parallel,CG and BK are parallel, so the opposite sides equal on another (I·34), and since CB equals CG, all sides are equal, therefore CK is an equilateral

Since CG is parallel to BK, then the sum of the interior angles is two right angles (I·29)

We know that angle CBK is right, so then angle BCG is also right.

Proposition 4 of Book II

If a straight line be cut at random, the square on the whole is equal to the squares on the segments and twice the rectangle contained by the segments.



$AB = BD$

$\angle CGB = \angle ADB$

$\angle ADB$

$CBK = BCG = ∟$

$AB = AC + CB$

$CG = CB$

$\square CK = CB \cdot CB$

Proof:

Draw a square ADEB on the line AB (I·46), and draw the diagonal BD

Draw a line CF parallel to either AD or BE (I·31), labelling the intersection with the diagonal as G.

Draw a line parallel to AB through the point G (I·31).

Since BD crosses two parallel lines, (AD and CF), then the exterior angle is equal to the interior and opposite angle (I·29)

Since triangle ABD is an isosceles triangle, then the angles at the base are also equal (I·5)

Since triangle BCG has two equal angles, it is an isosceles triangle (I·6), therefore the sides of the triangle are equal

CB and GK are parallel,CG and BK are parallel, so the opposite sides equal on another (I·34), and since CB equals CG, all sides are equal, therefore CK is an equilateral

Since CG is parallel to BK, then the sum of the interior angles is two right angles (I·29)

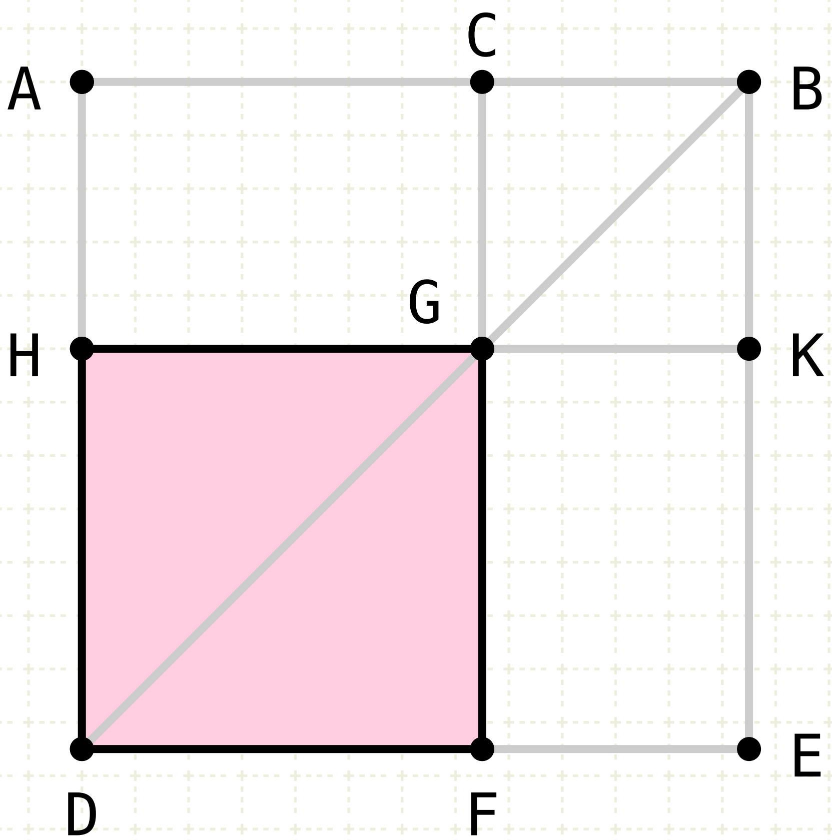
We know that angle CBK is right, so then angle BCG is also right.

The angles opposite one another are equal in a parallelogram, so the other two angles are right angles as well (I·34).

So CK is a square, equal to the square of CB

Proposition 4 of Book II

If a straight line be cut at random, the square on the whole is equal to the squares on the segments and twice the rectangle contained by the segments.



AB = AC + CB

CG = CB

□CK = CB·CB

□HF = AC·AC

Proof:

Draw a square ADEB on the line AB (I·46), and draw the diagonal BD

Draw a line CF parallel to either AD or BE (I·31), labelling the intersection with the diagonal as G.

Draw a line parallel to AB through the point G (I·31).

Similarly, HDFG is a square, equal to the square of AC

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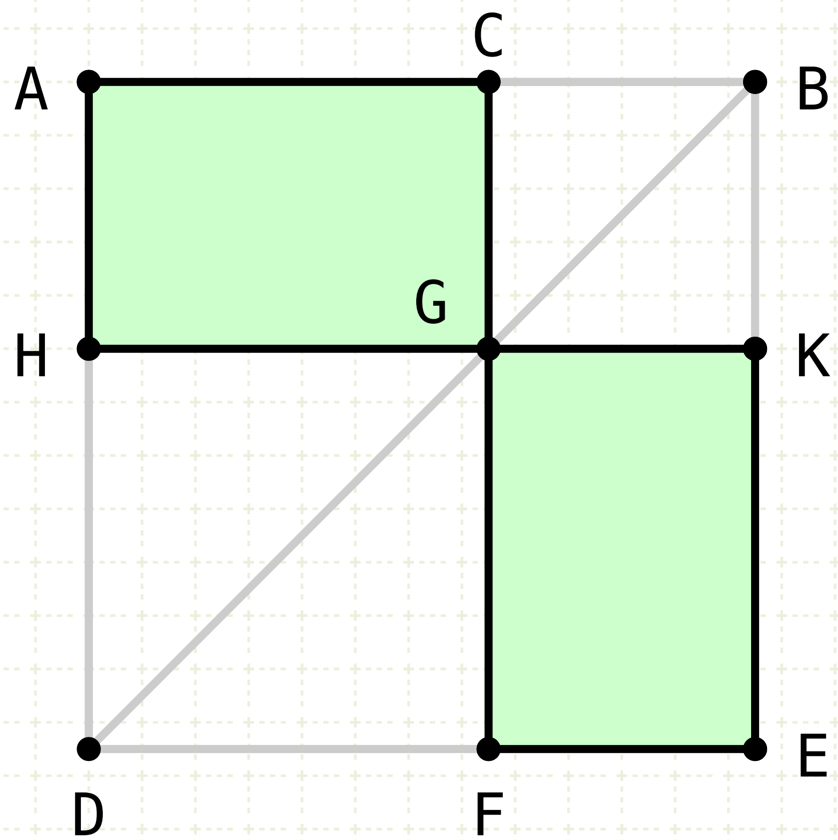
If a straight line be cut at random, the square on the whole is equal to the squares on the segments and twice the rectangle contained by the segments.



AB = AC + CB

CG = CB

□CK = CB·CB

□HF = AC·AC

□AG = □GE

□AG = AC·CG = AC·CB

Proof:

Draw a square ADEB on the line AB (I·46), and draw the diagonal BD

Draw a line CF parallel to either AD or BE (I·31), labelling the intersection with the diagonal as G.

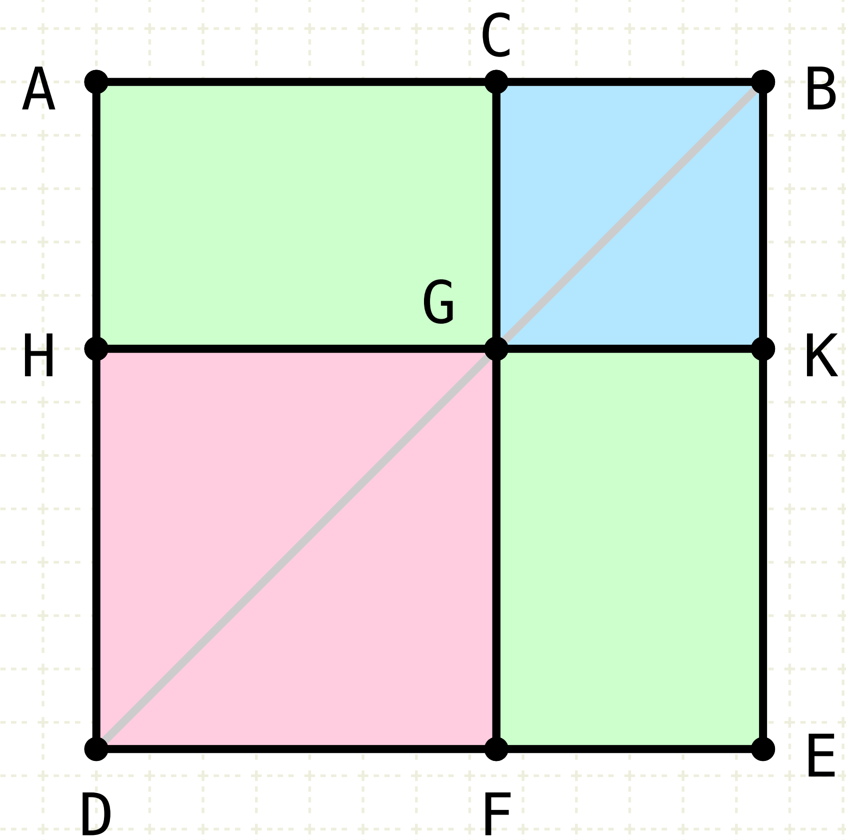
Draw a line parallel to AB through the point G (I·31).

Similarly, HDFG is a square, equal to the square of AC

Rectangles AG and GE are equal (complements of a parallelogram) (I·43), and are equal to the rectangle formed from lines AC and CB

Proposition 4 of Book II

If a straight line be cut at random, the square on the whole is equal to the squares on the segments and twice the rectangle contained by the segments.



$AB = AC + CB$

$CG = CB$

$\square CK = CB \cdot CB$

$\square HF = AC \cdot AC$

$\square AG = \square GE$

$\square AG = AC \cdot CG = AC \cdot CB$

$\square AE = \square CK + \square HF + \square AG + \square GE$

$AB \cdot AB = CB \cdot CB + AC \cdot AC + 2 \cdot AC \cdot CB$

Proof:

Draw a square ADEB on the line AB (I·46), and draw the diagonal BD

Draw a line CF parallel to either AD or BE (I·31), labelling the intersection with the diagonal as G.

Draw a line parallel to AB through the point G (I·31).

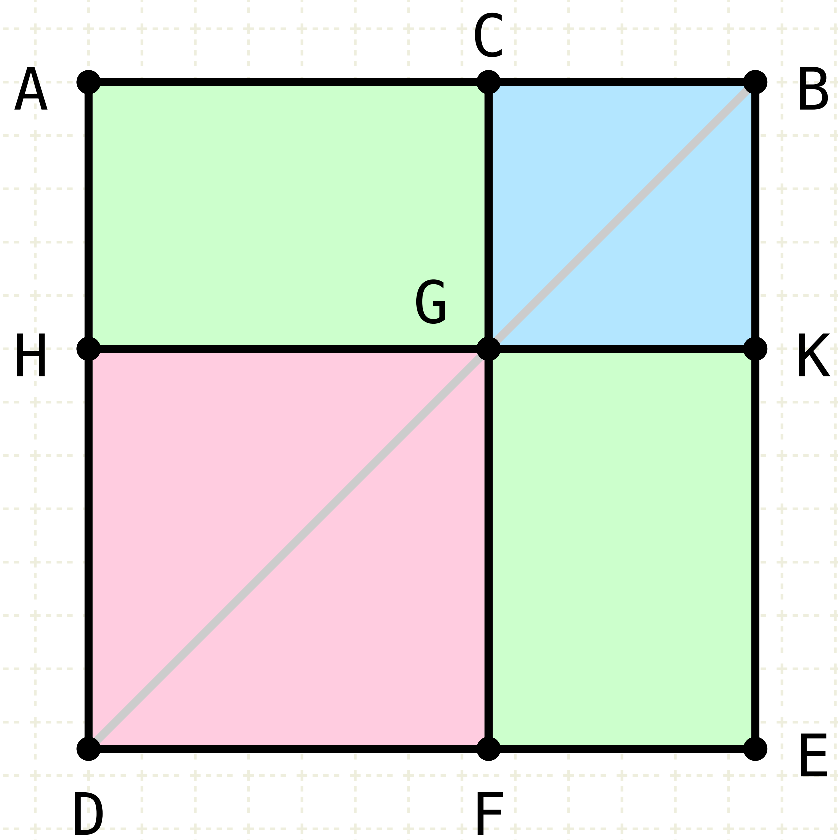
Similarly, HDFG is a square, equal to the square of AC

Rectangles AG and GE are equal (complements of a parallelogram) (I·43), and are equal to the rectangle formed from lines AC and CB

The sum of all the rectangles equals the square on AB

Proposition 4 of Book II

If a straight line be cut at random, the square on the whole is equal to the squares on the segments and twice the rectangle contained by the segments.



$AB = AC + CB$

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$\square AG = \square GE$

$\square AG = AC \cdot CG = AC \cdot CB$

$\square AE = \square CK + \square HF + \square AG + \square GE$

$AB \cdot AB = CB \cdot CB + AC \cdot AC + 2 \cdot AC \cdot CB$

Proof:

Draw a square ADEB on the line AB (I·46), and draw the diagonal BD

Draw a line CF parallel to either AD or BE (I·31), labelling the intersection with the diagonal as G.

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The sum of all the rectangles equals the square on AB

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