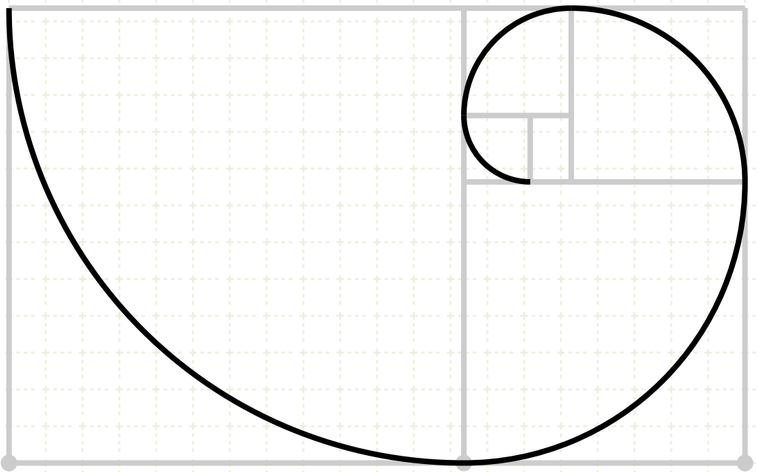
Euclid's Elements

Book VI



One can state, without exaggeration, that the observation of and the search for similarities and differences are the basis of all human knowledge.

Alfred Nobel

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Proposition 28 of Book VI

To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect

Proposition 28 of Book VI

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In other words

Given a straight line AB and

Proposition 28 of Book VI

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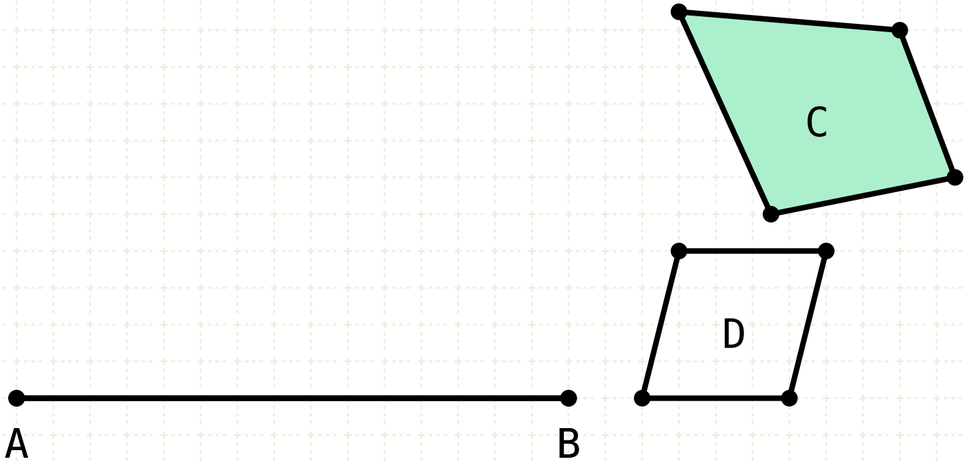

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Given a straight line AB and

Let C be a rectilineal figure and D be a parallelogram

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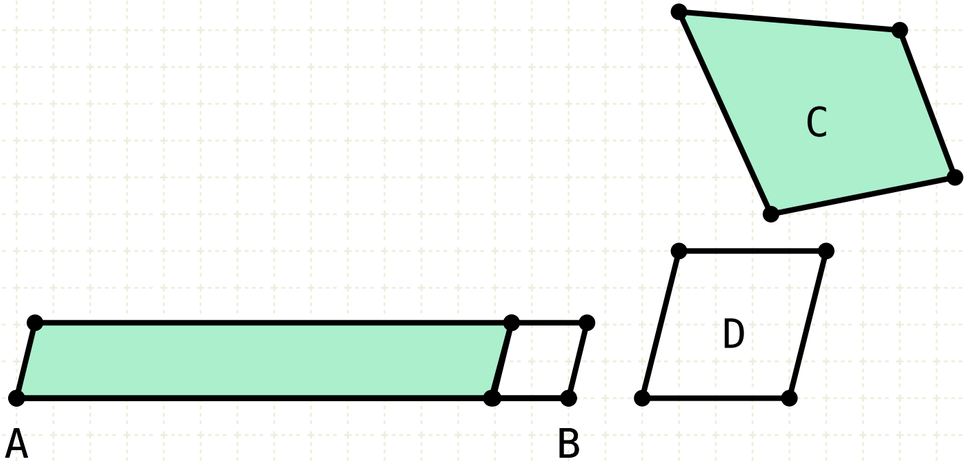

In other words

Given a straight line AB and

Let C be a rectilineal figure and D be a parallelogram

We want to draw a parallelogram on AB such that …

* If a parallelogram similar to parallelogram D is removed, then…
* the remainder is equal in area to C

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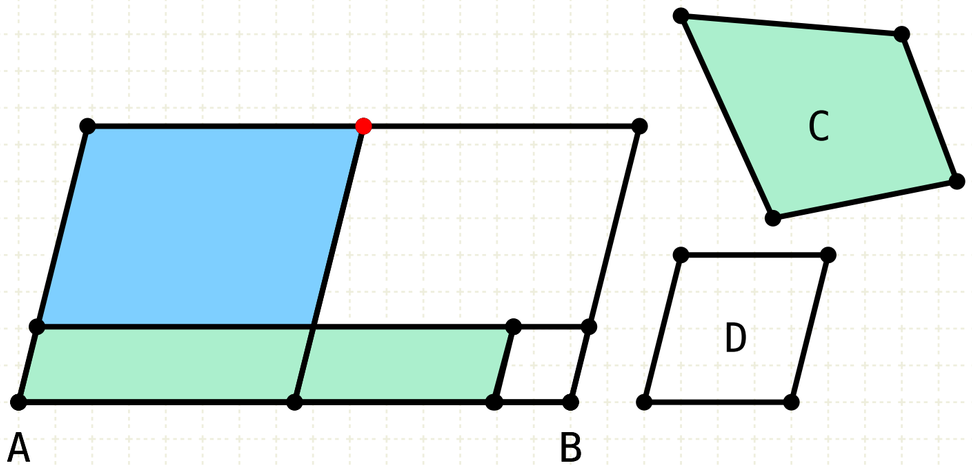

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Let C be a rectilineal figure and D be a parallelogram

We want to draw a parallelogram on AB such that …

* If a parallelogram similar to parallelogram D is removed, then…
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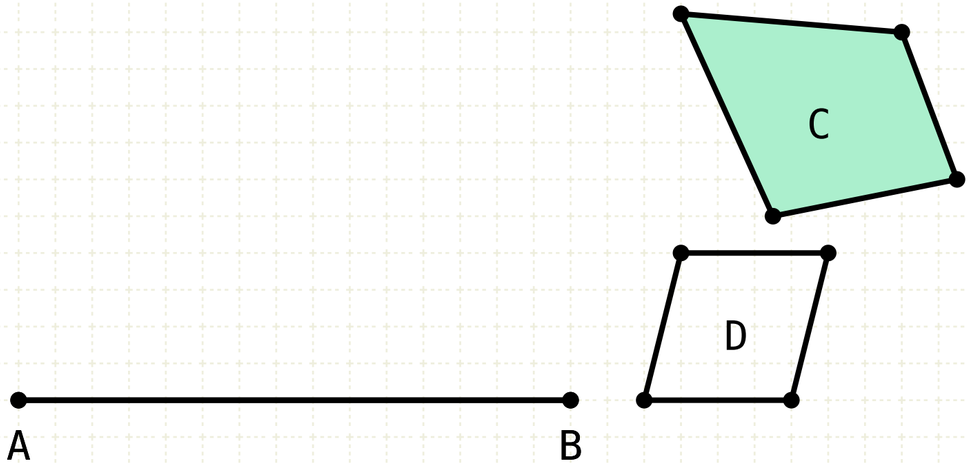
Note that:

* the area of C cannot be greater than the area of half the parallelogram on the line AB

Proposition 28 of Book VI

To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



Construction

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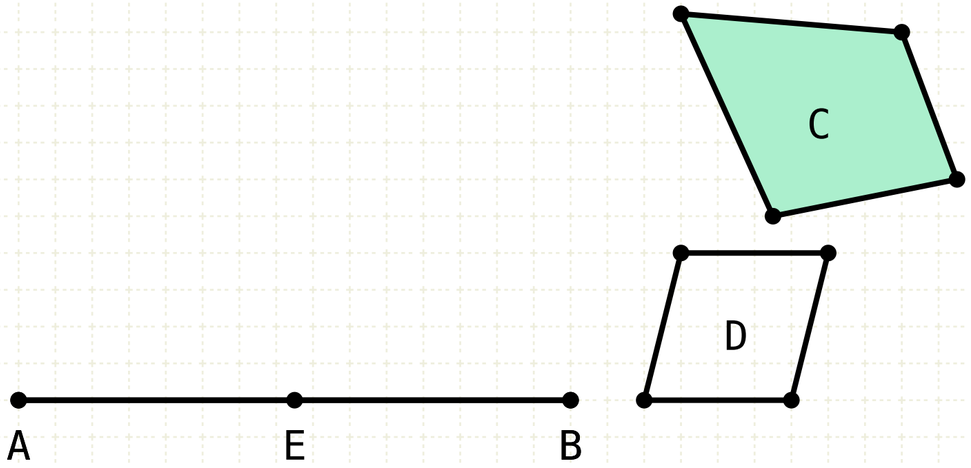

Construction

Bisect the line AB at point E

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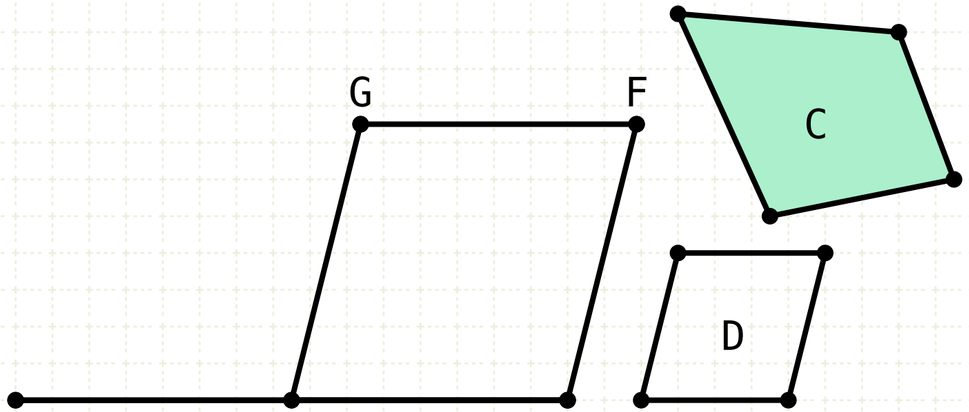

Construction

Bisect the line AB at point E

Create a parallelogram similar to D on line EB (VI·18)

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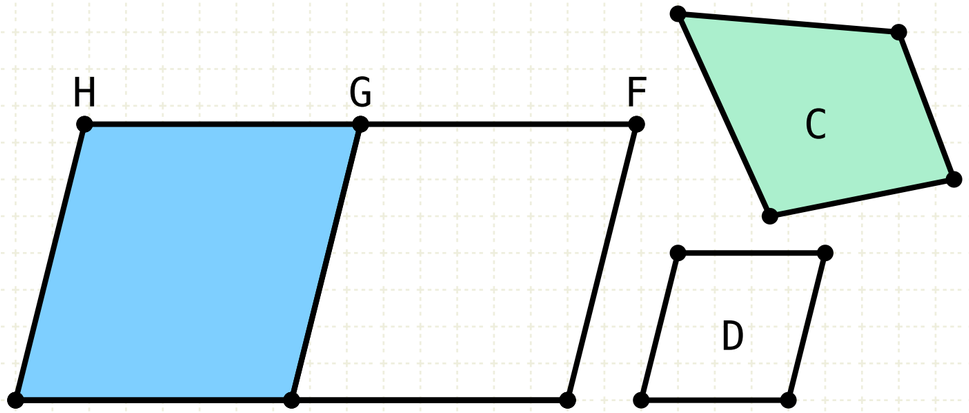

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Bisect the line AB at point E

Create a parallelogram similar to D on line EB (VI·18)

Let the parallelogram AG be completed

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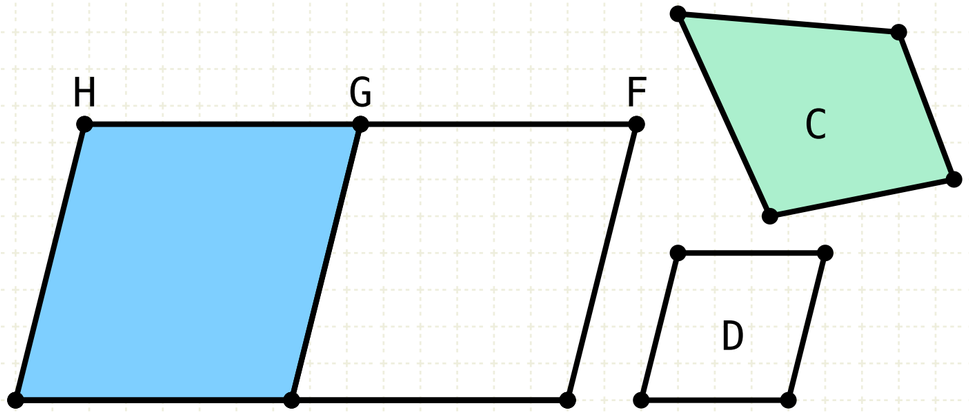

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Create a parallelogram similar to D on line EB (VI·18)

Let the parallelogram AG be completed

If AG is equal in size to C, then we are finished, otherwise …
HE is greater than C

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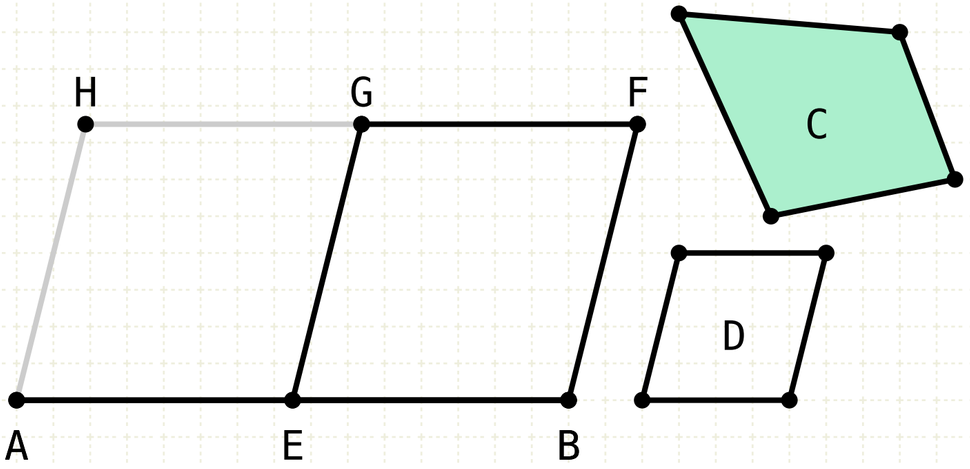

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GB ~ D
If HE > C
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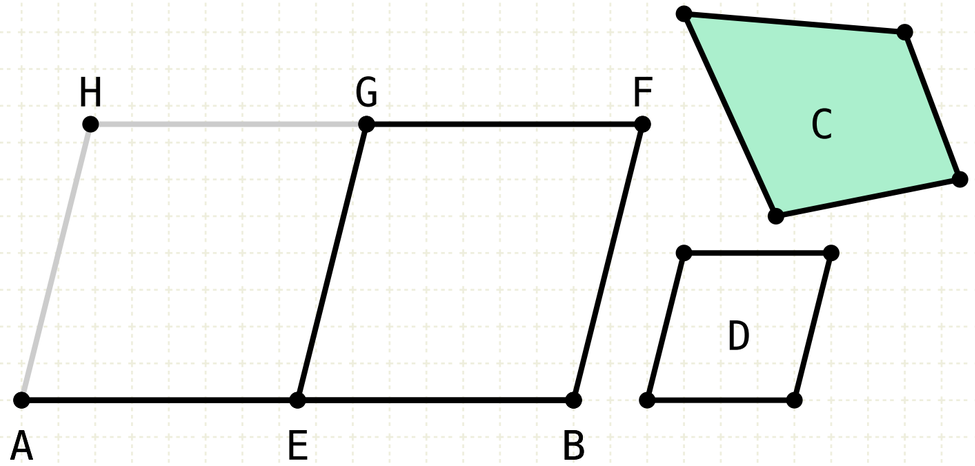

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Let KLMN be constructed such that it is equal to the the area of GB minus the area of C, and is similar to D (VI·25)

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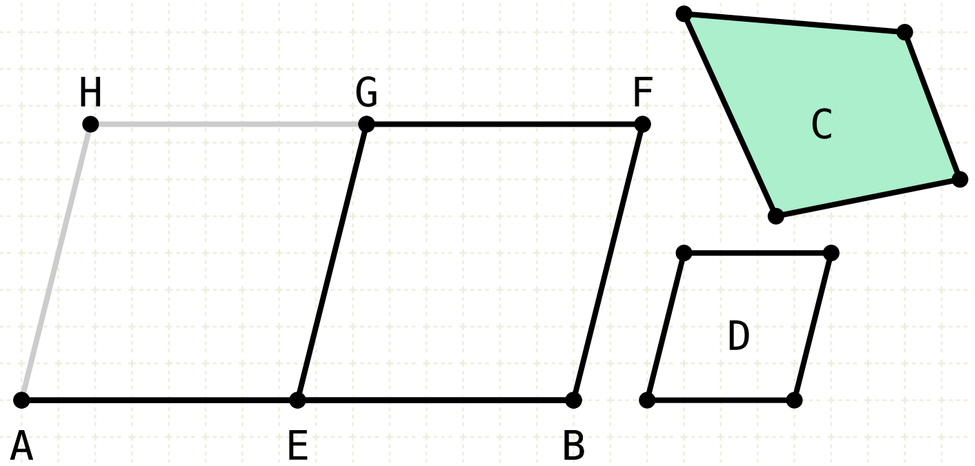

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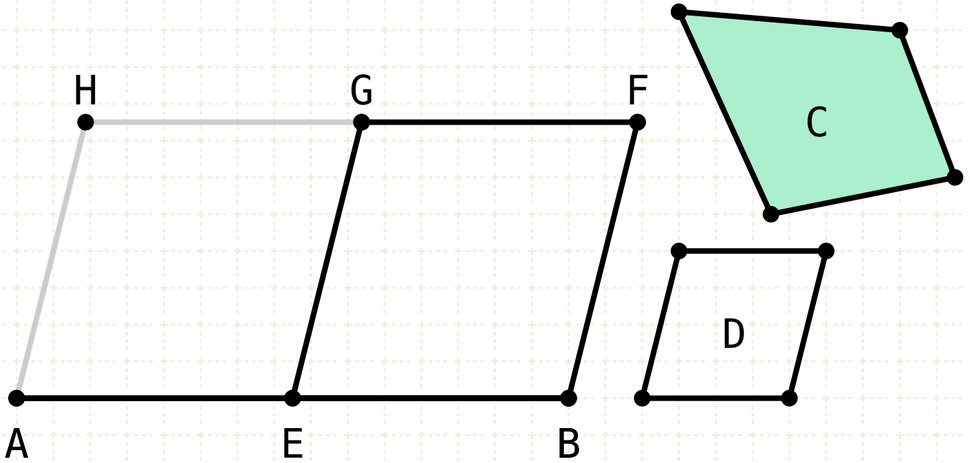

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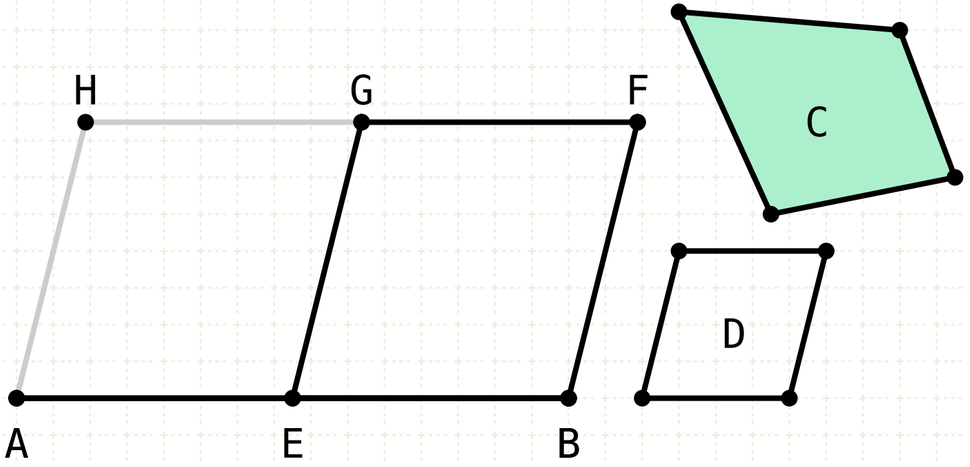

Let KLMN be constructed…

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Copy the rectilineal figure C to a parallelogram on line EB, with an inner angle of GEB

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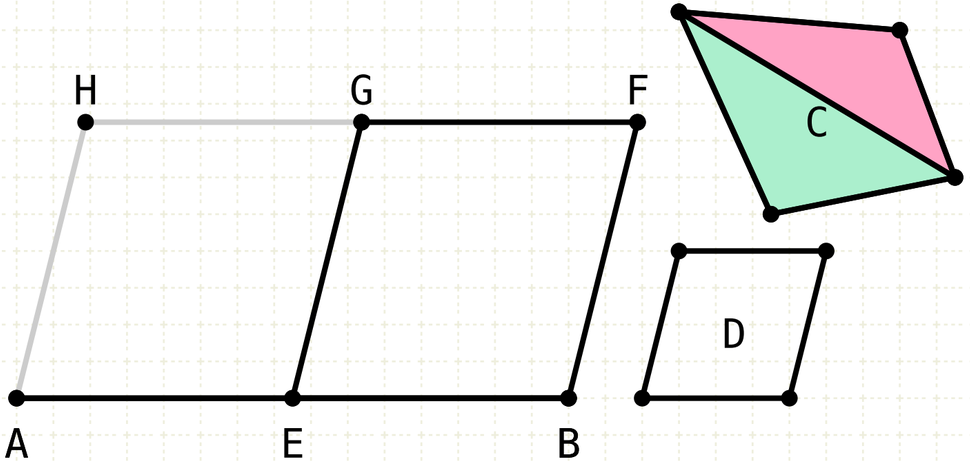

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Copy the rectilineal figure C to a parallelogram on line EB, with an inner angle of GEB

* Split C into two triangles C1 and C2

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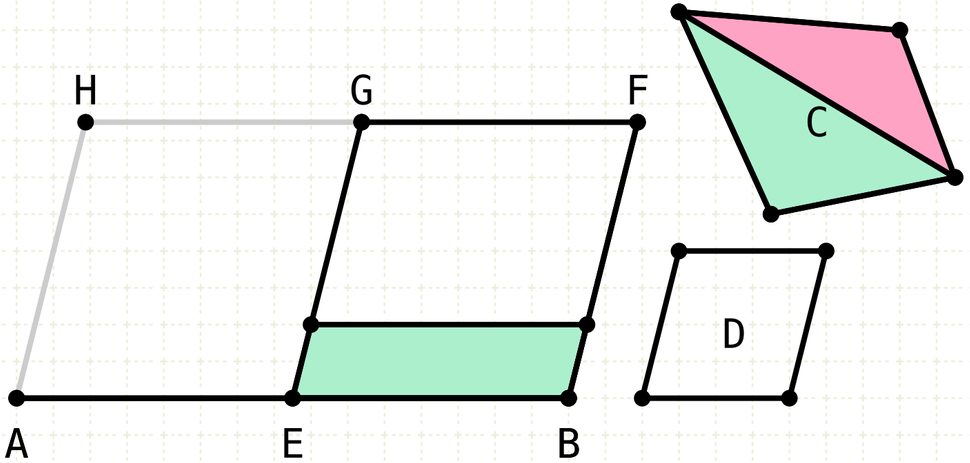

Let KLMN be constructed…

… such that it is equal to the area of GB minus the area of C, and is similar to D

Copy the rectilineal figure C to a parallelogram on line EB, with an inner angle of GEB

* Split C into two triangles C1 and C2
* Construct a parallelogram on the base EB such that it is equal in area to the triangle C1 (I·44)

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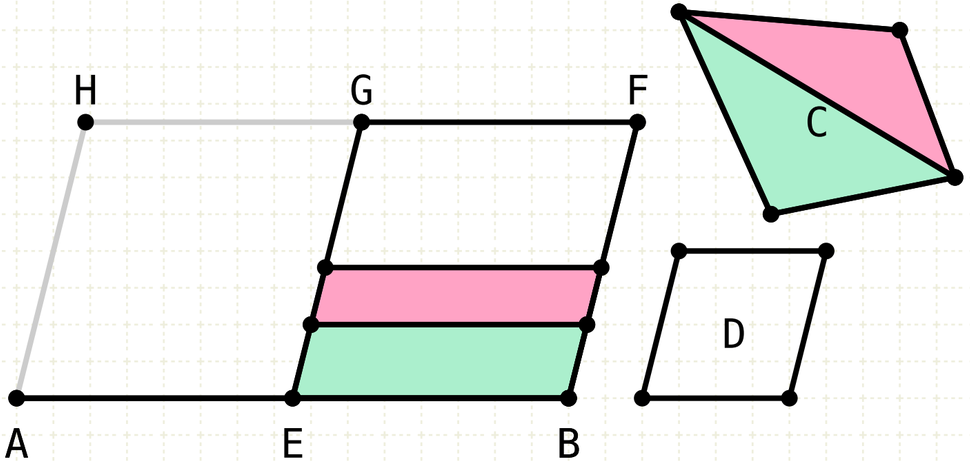

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Copy the rectilineal figure C to a parallelogram on line EB, with an inner angle of GEB

* Split C into two triangles C1 and C2
* Construct a parallelogram on the base EB such that it is equal in area to the triangle C1 (I·44)
* Construct a parallelogram on the top of the previous parallelogram such that it is equal in area to the triangle C2 (I·44)

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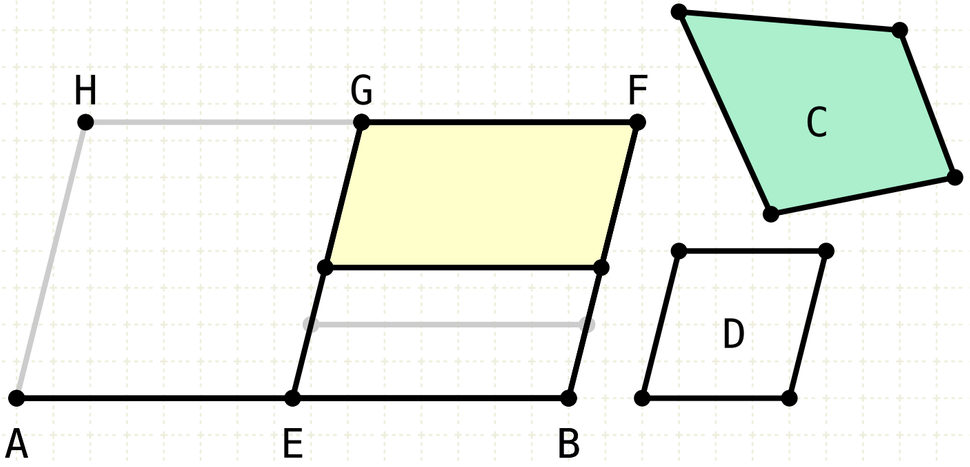

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Copy the rectilineal figure C to a parallelogram on line EB, with an inner angle of GEB

* Split C into two triangles C1 and C2
* Construct a parallelogram on the base EB such that it is equal in area to the triangle C1 (I·44)
* Construct a parallelogram on the top of the previous parallelogram such that it is equal in area to the triangle C2 (I·44)

What is left over in the parallelogram EF is now equal to the area of EF minus the area of C

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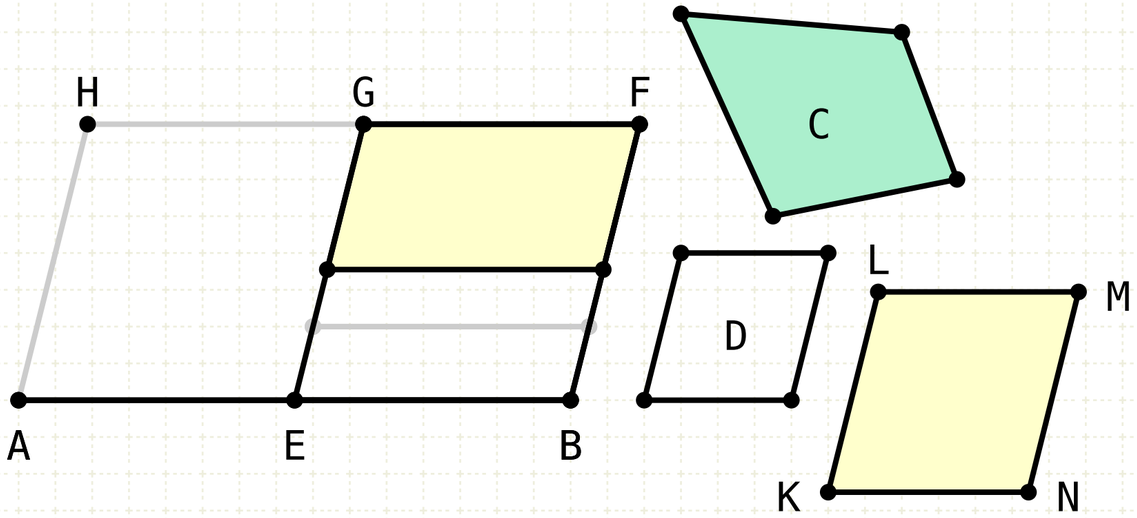

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* Construct a parallelogram on the top of the previous parallelogram such that it is equal in area to the triangle C2 (I·44)

What is left over in the parallelogram EF is now equal to the area of EF minus the area of C

Now, copy this new polygon to KLMN, which is similar to the polygon D (VI·25)

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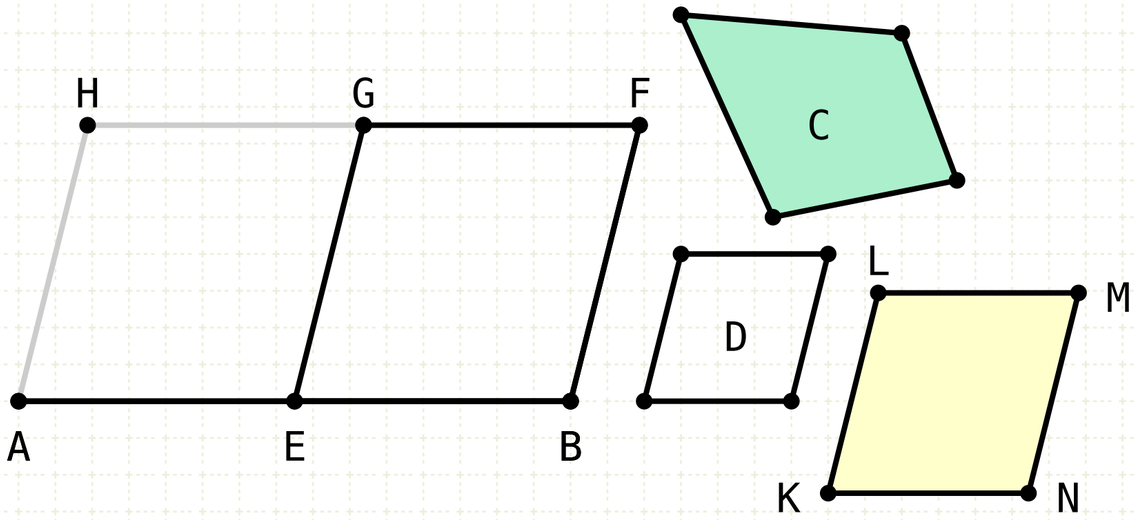
To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



Construction

Bisect the line AB at point E

Create a parallelogram similar to D on line EB (VI·18)

Let the parallelogram AG be completed

If AG is equal in size to C, then we are finished, otherwise …
HE is greater than C

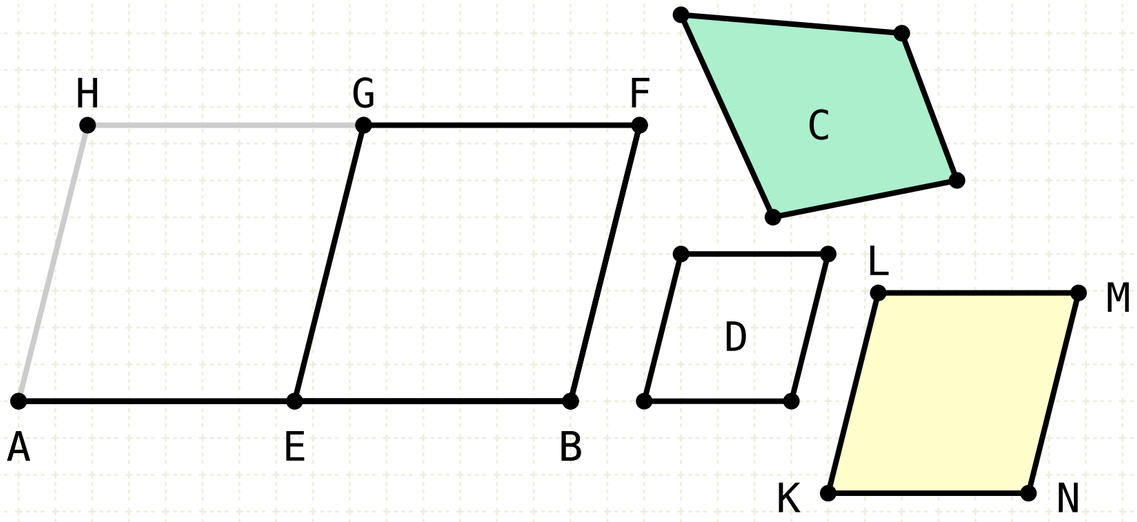
HE is equal to GB, therefore GB is also greater than C

Let KLMN be constructed such that it is equal to the the area of GB minus the area of C, and is similar to D (VI·25)

HE = GB
GB ~ D
If HE > C
GB > C
KM + C = GB

Proposition 28 of Book VI

To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



HE = GB
GB ~ D
If HE > C
GB > C
KM + C = GB
KM ~ D
D ~ GB
∴ KM ~ GB

Construction

Bisect the line AB at point E

Create a parallelogram similar to D on line EB (VI·18)

Let the parallelogram AG be completed

If AG is equal in size to C, then we are finished, otherwise …
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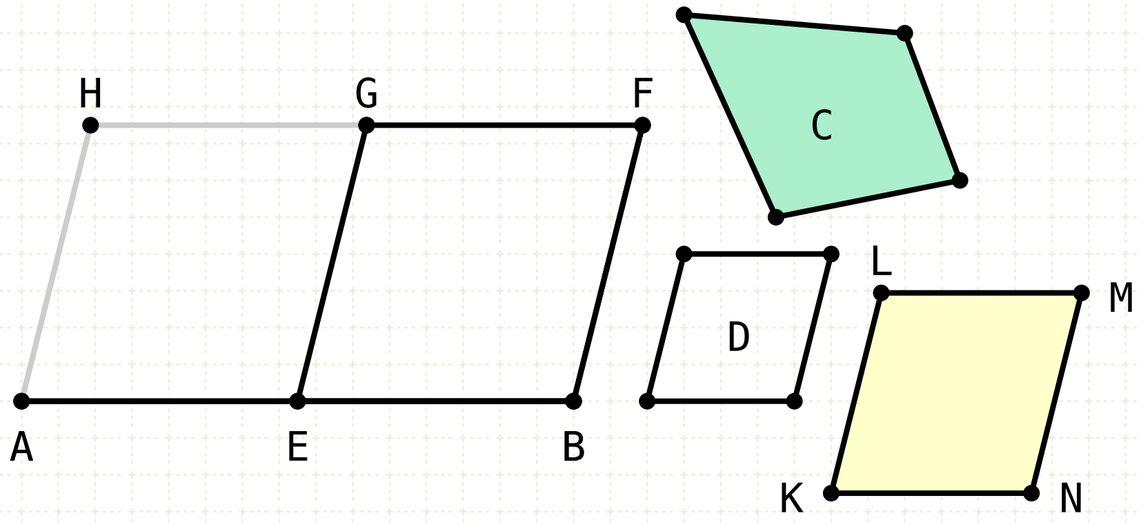
Let KLMN be constructed such that it is equal to the the area of GB minus the area of C, and is similar to D (VI·25)

KLMN is similar to D, which is also similar to GB, therefore KLMN is similar to GB (VI·21)

Proposition 28 of Book VI

To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



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Bisect the line AB at point E

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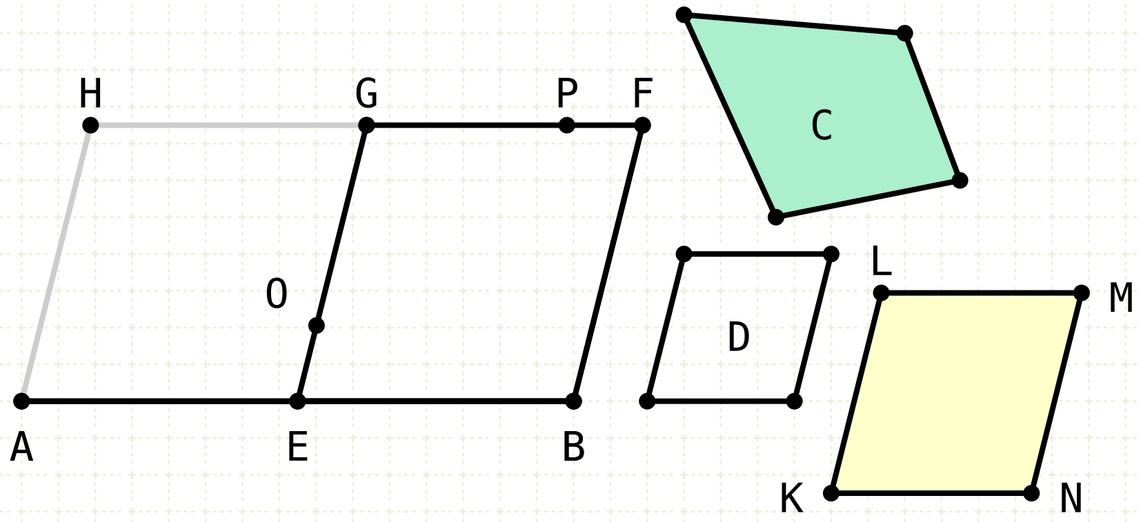
Let KLMN be constructed such that it is equal to the the area of GB minus the area of C, and is similar to D (VI·25)

KLMN is similar to D, which is also similar to GB, therefore KLMN is similar to GB (VI·21)

Since KM is smaller than GB, and since they are similar, LM is less than GF and LK is less than GE

Proposition 28 of Book VI

To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



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KLMN is similar to D, which is also similar to GB, therefore KLMN is similar to GB (VI·21)

Since KM is smaller than GB, and since they are similar, LM is less than GF and LK is less than GE

Copy line KL to GE, and LM to GF

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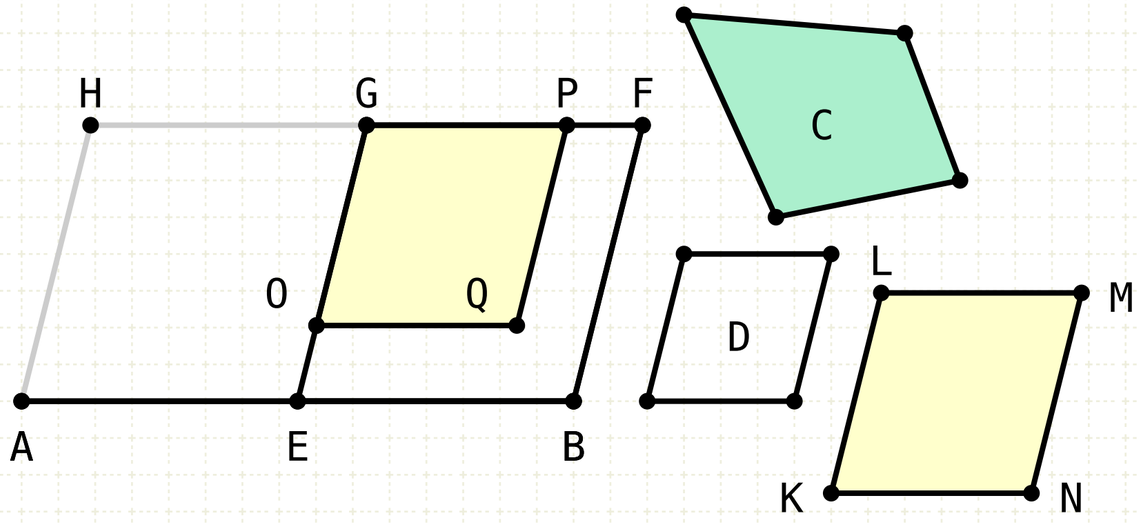
To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



HE = GB
GB ~ D
If HE > C
GB > C
KM + C = GB
KM ~ D
D ~ GB
∴ KM ~ GB
GQ = KM

Construction

Bisect the line AB at point E

Create a parallelogram similar to D on line EB (VI·18)

Let the parallelogram AG be completed

If AG is equal in size to C, then we are finished, otherwise … HE is greater than C

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KLMN is similar to D, which is also similar to GB, therefore KLMN is similar to GB (VI·21)

Since KM is smaller than GB, and since they are similar, LM is less than GF and LK is less than GE

Copy line KL to GE, and LM to GF

Complete the parallelogram OGPQ

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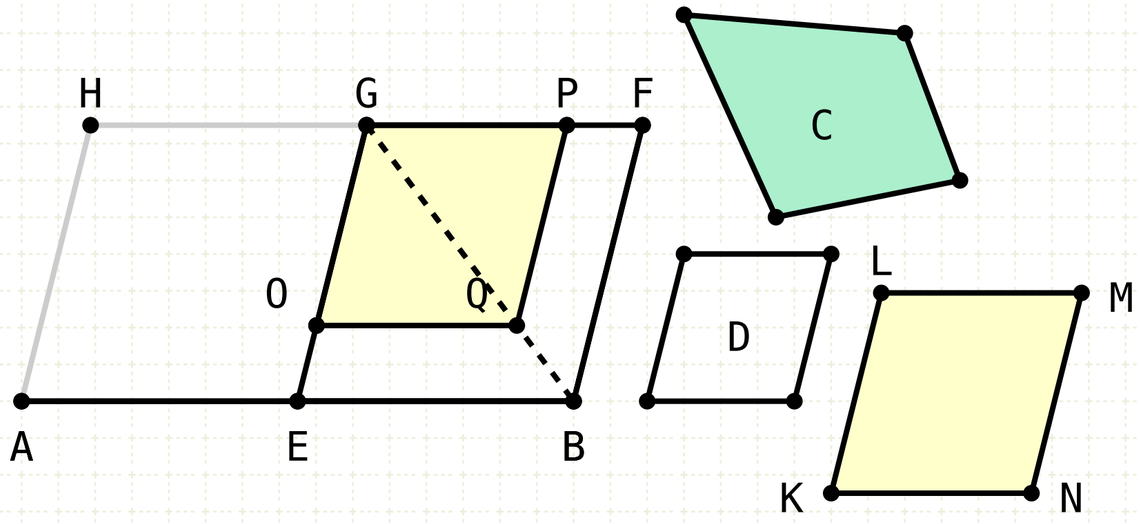
To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



HE = GB

GB ~ D

If HE > C

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D ~ GB

∴ KM ~ GB

GQ = KM

GQ ~ KM ~ D ~ GB

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Since KM is smaller than GB, and since they are similar, LM is less than GF and LK is less than GE

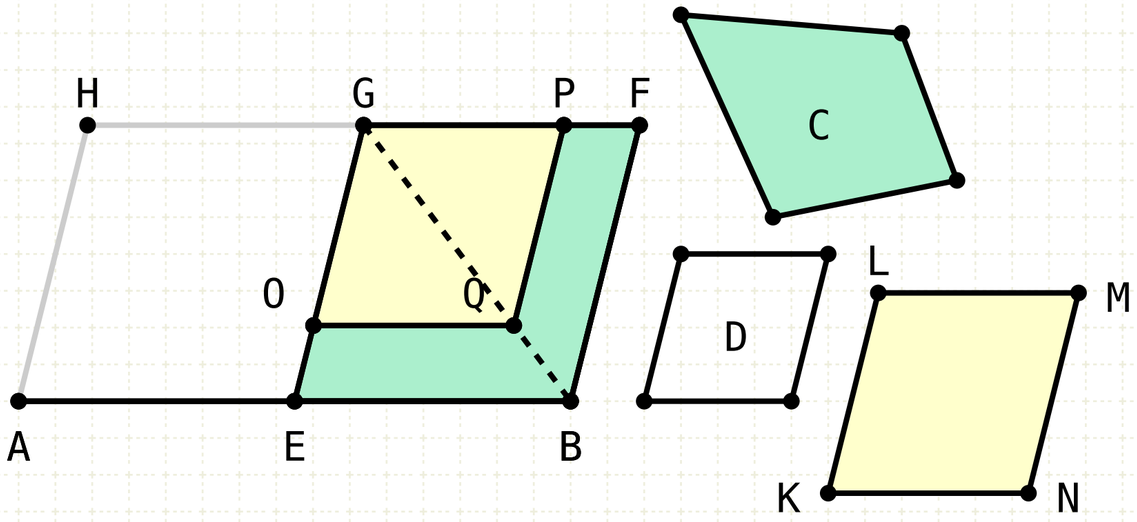
Copy line KL to GE, and LM to GF

Complete the parallelogram OGPQ

Since GB is similar to KM, so is GQ similar to GB (VI·21), thus the points Q and B lie on the same diagonal (VI·26)

Proposition 28 of Book VI

To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



HE = GB

GB ~ D

If HE > C

GB > C

KM + C = GB

KM ~ D

D ~ GB

∴ KM ~ GB

GQ = KM

GQ ~ KM ~ D ~ GB

OEBFQP = C

Construction

Bisect the line AB at point E

Create a parallelogram similar to D on line EB (VI·18)

Let the parallelogram AG be completed

If AG is equal in size to C, then we are finished, otherwise … HE is greater than C

HE is equal to GB, therefore GB is also greater than C

Let KLMN be constructed such that it is equal to the the area of GB minus the area of C, and is similar to D (VI·25)

KLMN is similar to D, which is also similar to GB, therefore KLMN is similar to GB (VI·21)

Since KM is smaller than GB, and since they are similar, LM is less than GF and LK is less than GE

Copy line KL to GE, and LM to GF

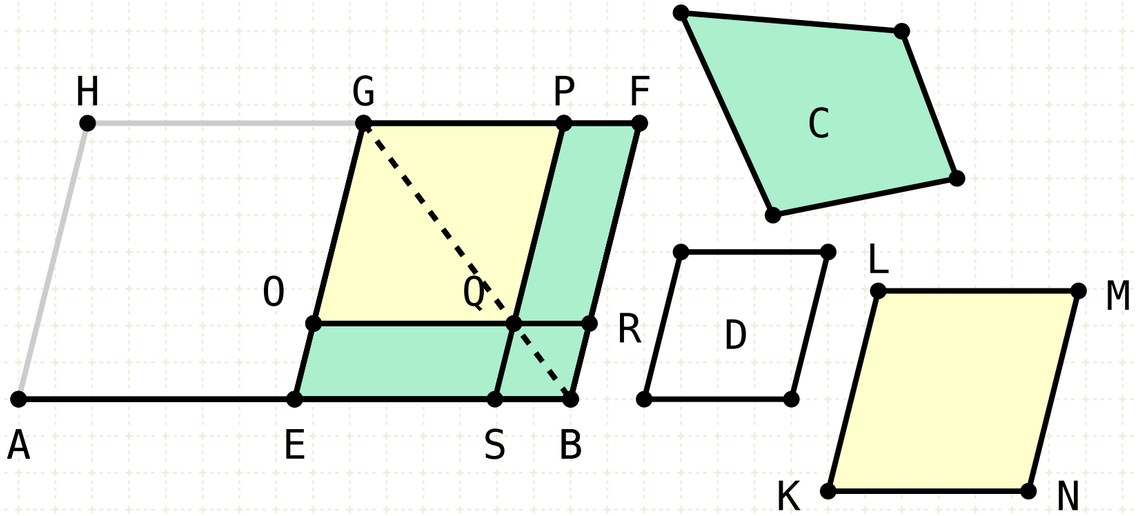
Complete the parallelogram OGPQ

Since GB is similar to KM, so is GQ similar to GB (VI·21), thus the points Q and B lie on the same diagonal (VI·26)

GB is equal to sum of KM and C, and since GQ is equal to KM, the remaining gnomon (OEBFPQ) is equal in area to C

Proposition 28 of Book VI

To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



Construction (cont.)

extend OQ to R, and PQ to S

HE = GB

GB ~ D

If HE > C

GB > C

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KM ~ D

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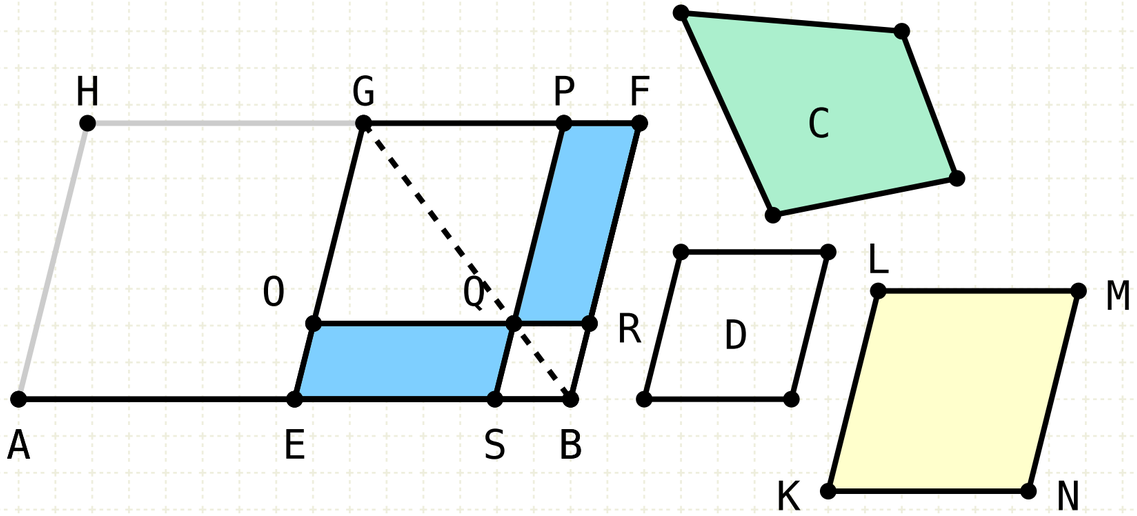

Construction (cont.)

extend OQ to R, and PQ to S

The parallelograms PR and OS are equal

HE = GB

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GB > C

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KM ~ D

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GQ = KM

GQ ~ KM ~ D ~ GB

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PR = OS

Proposition 28 of Book VI

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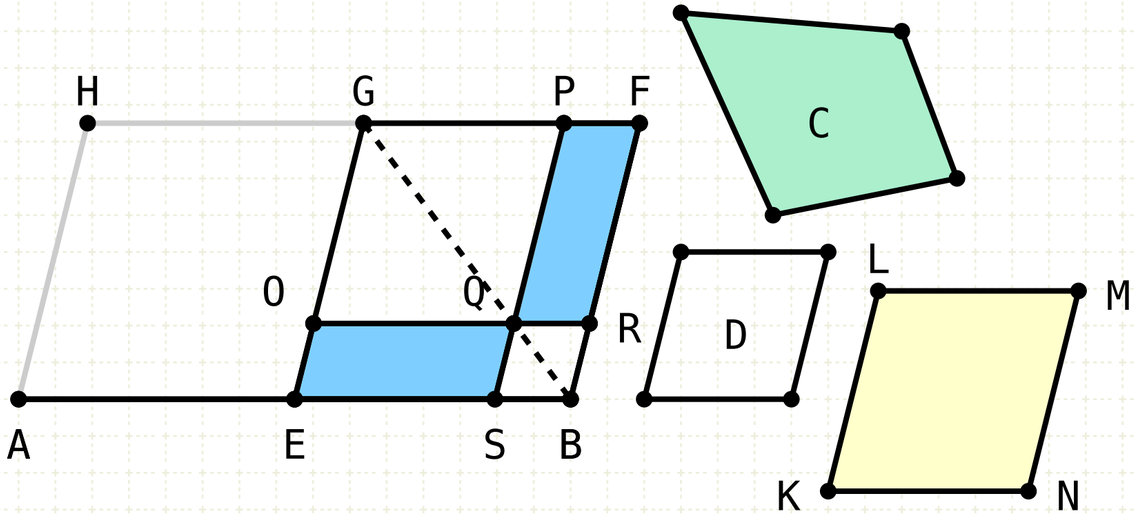

Construction (cont.)

extend OQ to R, and PQ to S

The parallelograms PR and OS are equal

If QB is added to PR and OS, then OB and PB are equal

HE = GB

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If HE > C

GB > C

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GQ = KM

GQ ~ KM ~ D ~ GB

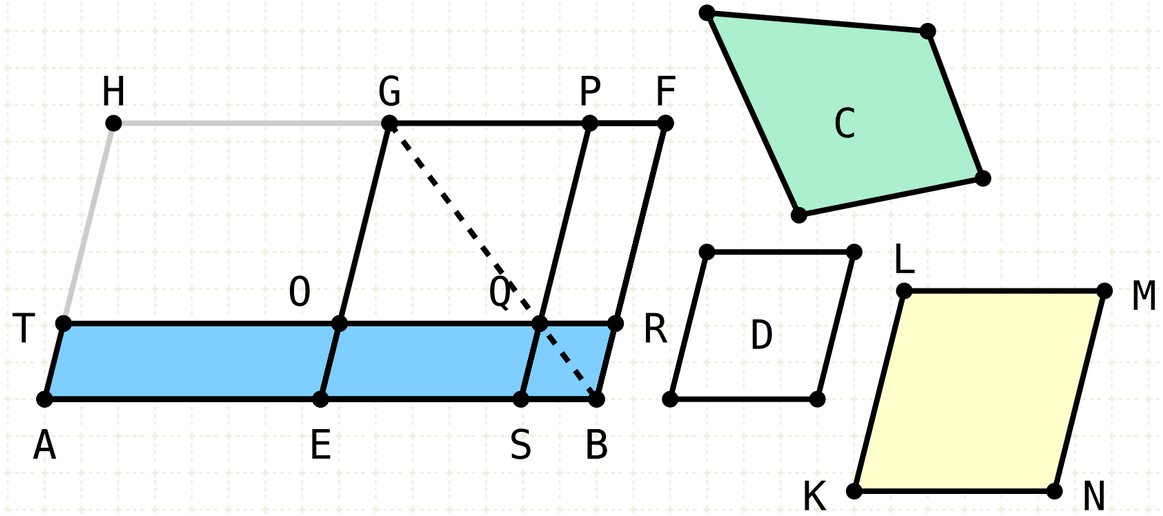
OEBFQP = C

PR = OS

PB = OB

Proposition 28 of Book VI

To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



Construction (cont.)

extend OQ to R, and PQ to S

The parallelograms PR and OS are equal

If QB is added to PR and OS, then OB and PB are equal

OB is equal to TE, since AE and EB are equal (I·36)

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OEBFQP = C

PR = OS

PB = OB

TE = OB

Proposition 28 of Book VI

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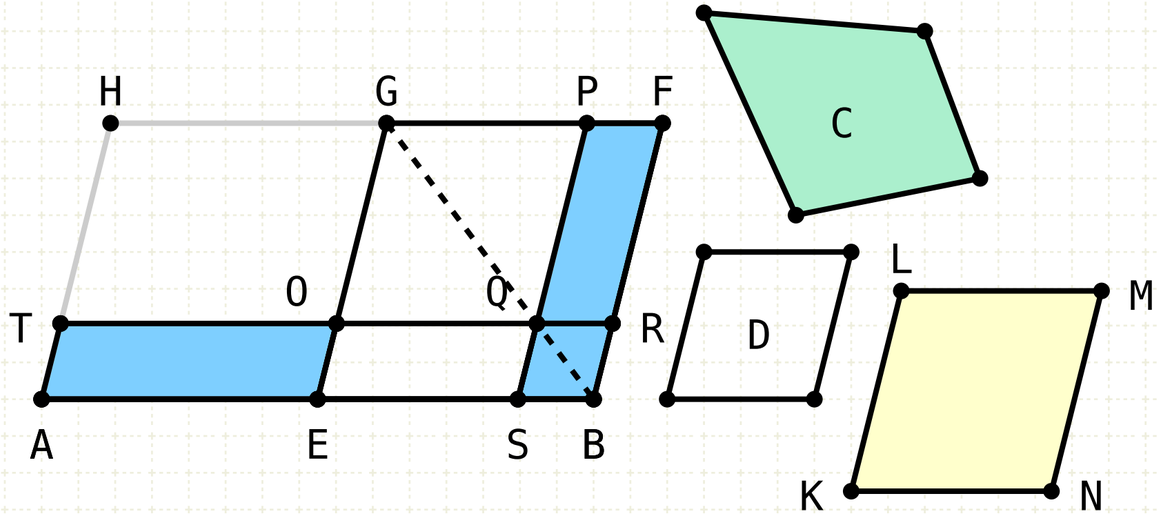

Construction (cont.)

extend OQ to R, and PQ to S

The parallelograms PR and OS are equal

If QB is added to PR and OS, then OB and PB are equal

OB is equal to TE, since AE and EB are equal (I·36)

PB equals OB, TE is equal to PB

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Proposition 28 of Book VI

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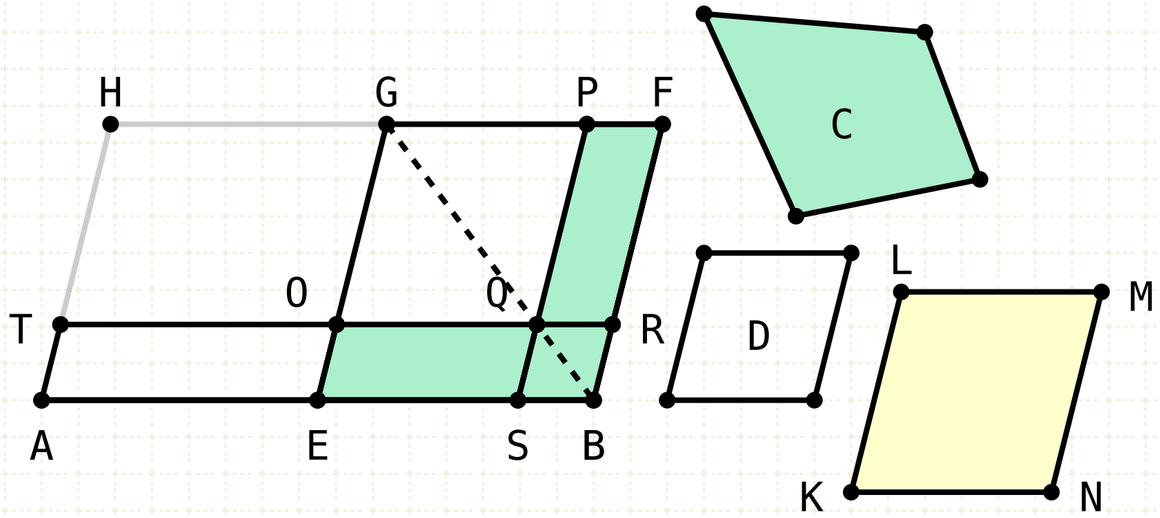

Construction (cont.)

extend OQ to R, and PQ to S

The parallelograms PR and OS are equal

If QB is added to PR and OS, then OB and PB are equal

OB is equal to TE, since AE and EB are equal (I·36)

PB equals OB, TE is equal to PB

But PB, or TE, added together with OS is the gnomon that is equal to the area of C

HE = GB

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D ~ GB

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GQ = KM

GQ ~ KM ~ D ~ GB

OEBFQP = C

PR = OS

PB = OB

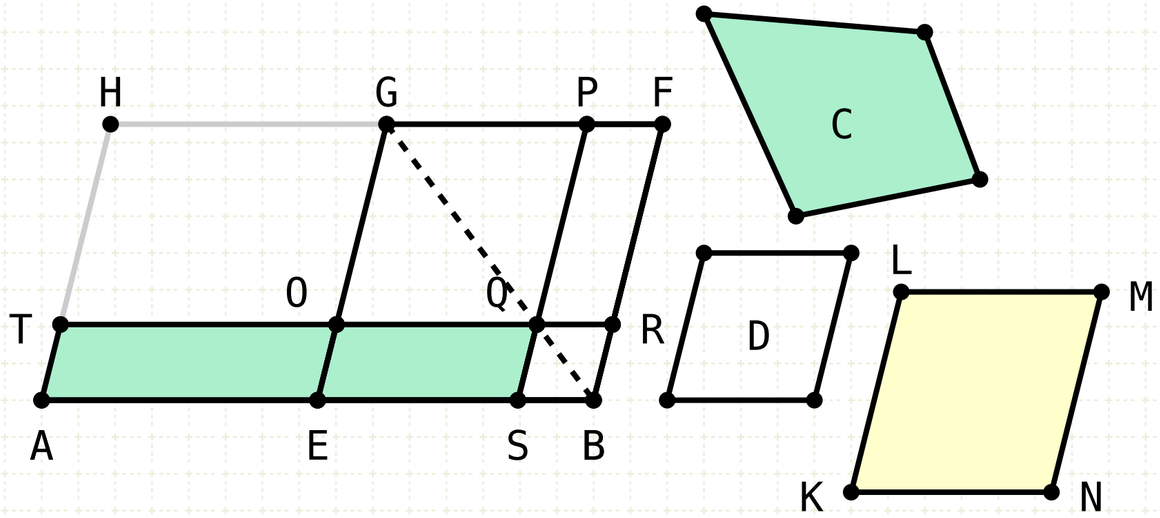
TE = OB

TE = PB

OS + PB = C

Proposition 28 of Book VI

To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



Construction (cont.)

extend OQ to R, and PQ to S

The parallelograms PR and OS are equal

If QB is added to PR and OS, then OB and PB are equal

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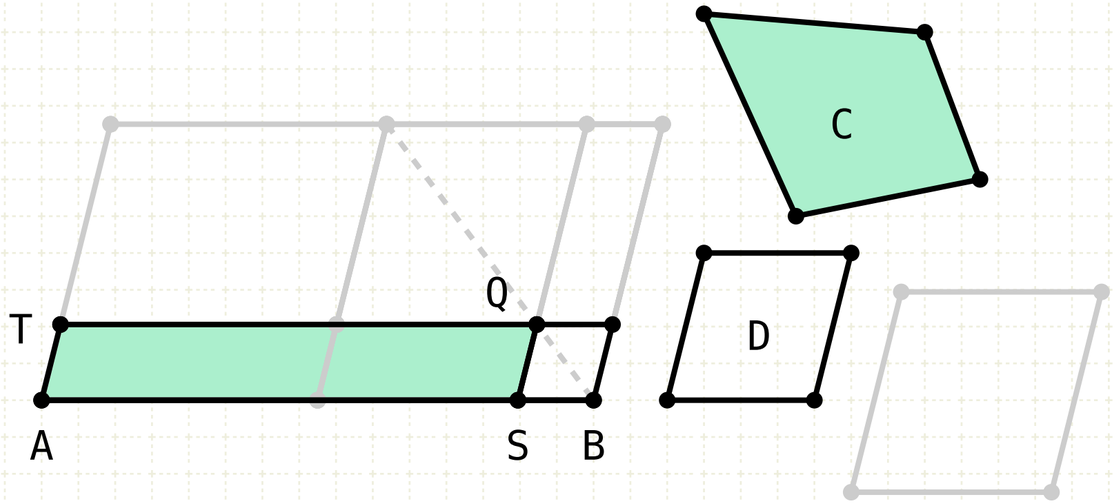
GQ ~ KM ~ D ~ GB
OEBFQP = C
PR = OS
PB = OB
TE = OB
TE = PB
OS + PB = C
OS + TE = C

Proposition 28 of Book VI

To a given straight line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect



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GQ = KM

GQ ~ KM ~ D ~ GB

OEBFQP = C

PR = OS

PB = OB

TE = OB

TE = PB

OS + PB = C

OS + TE = C

TS = C

Construction (cont.)

extend OQ to R, and PQ to S

The parallelograms PR and OS are equal

If QB is added to PR and OS, then OB and PB are equal

OB is equal to TE, since AE and EB are equal (I·36)

PB equals OB, TE is equal to PB

But PB, or TE, added together with OS is the gnomon that is equal to the area of C

Thus TS is a parallelogram, drawn on AB, minus the parallelogram QB (which is similar to D), whose area equals the polygon C

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