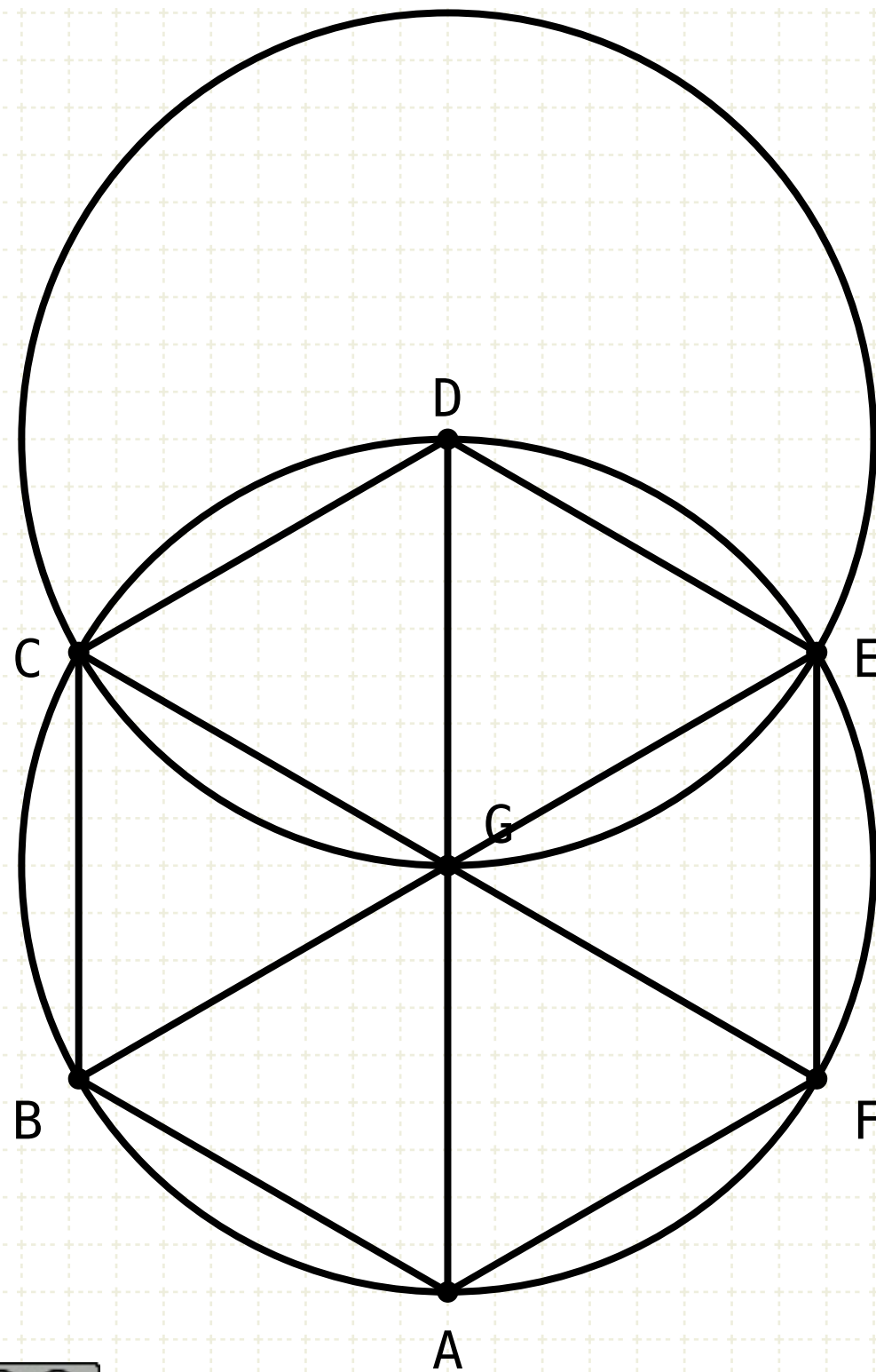


Euclid's Elements

Book IV



Philosophy (nature) is written in that great book which ever is before our eyes -- I mean the universe -- but we cannot understand it if we do not first learn the language and grasp the symbols in which it is written. The book is written in mathematical language, and the symbols are triangles, circles and other geometrical figures, without whose help it is impossible to comprehend a single word of it - without which one wanders in vain through a dark labyrinth.

Galileo Galilei



Proposition 13 of Book IV

In a given pentagon, which is equilateral and equiangular, to inscribe a circle.



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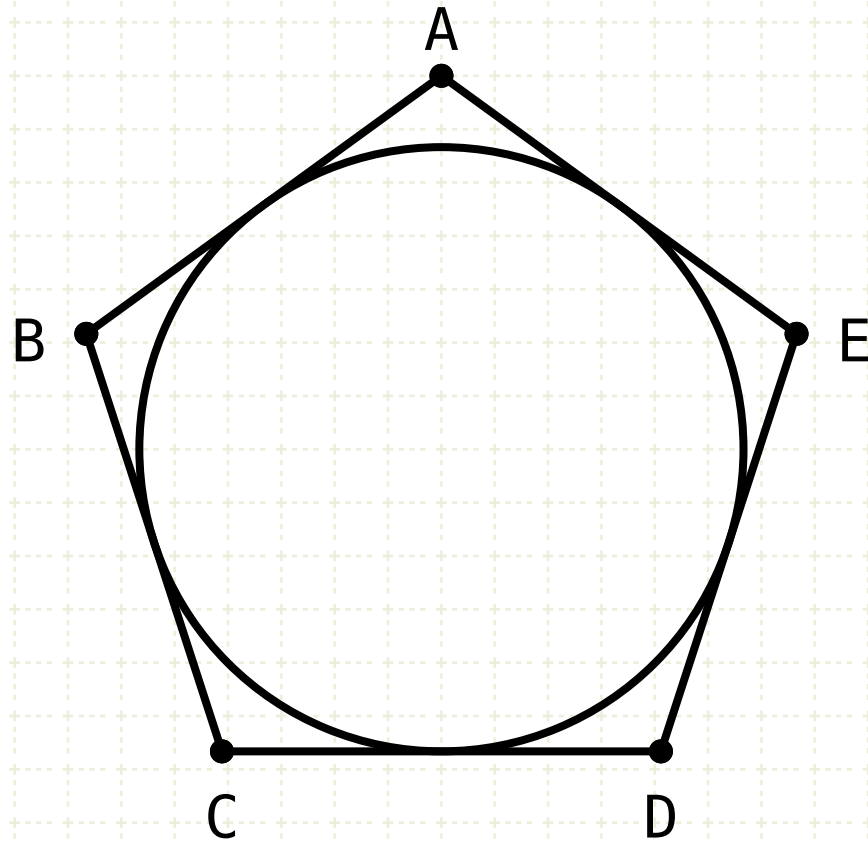
Proposition 13 of Book IV

In a given pentagon, which is equilateral and equiangular, to inscribe a circle.



Proposition 13 of Book IV

In a given pentagon, which is equilateral and equiangular, to inscribe a circle.

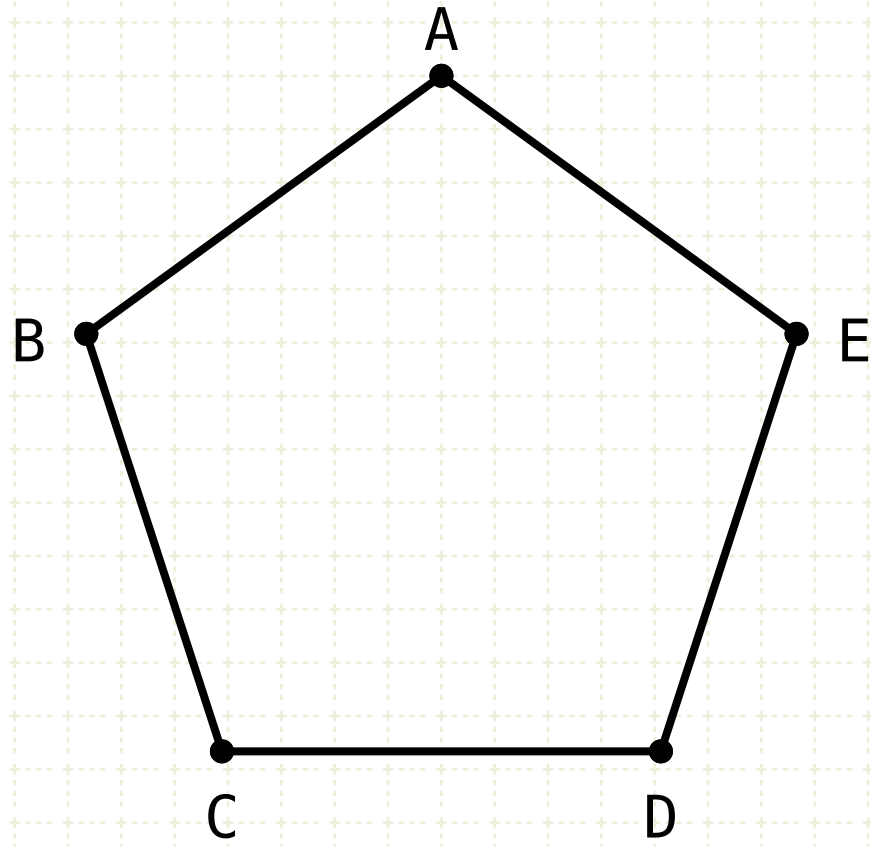


In other words

Given a pentagon draw a circle on the inside, where the sides of the pentagon touch the circle

Proposition 13 of Book IV

In a given pentagon, which is equilateral and equiangular, to inscribe a circle.

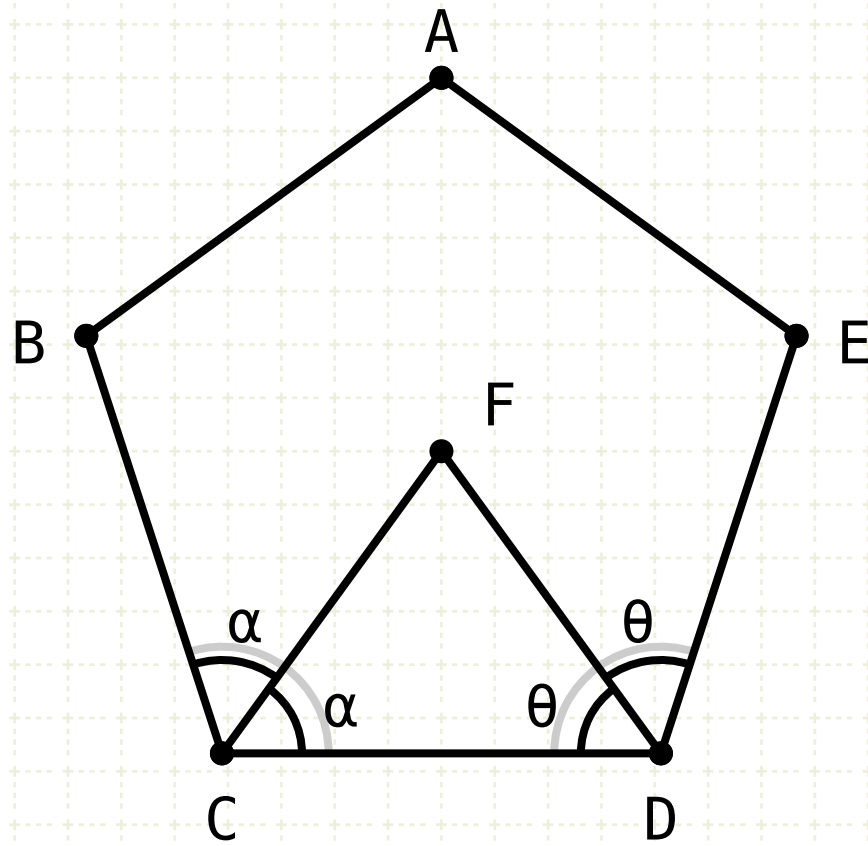


Construction



Proposition 13 of Book IV

In a given pentagon, which is equilateral and equiangular, to inscribe a circle.



$$2\alpha = 2\theta$$

$$\angle BCF = \angle FCD = \alpha$$

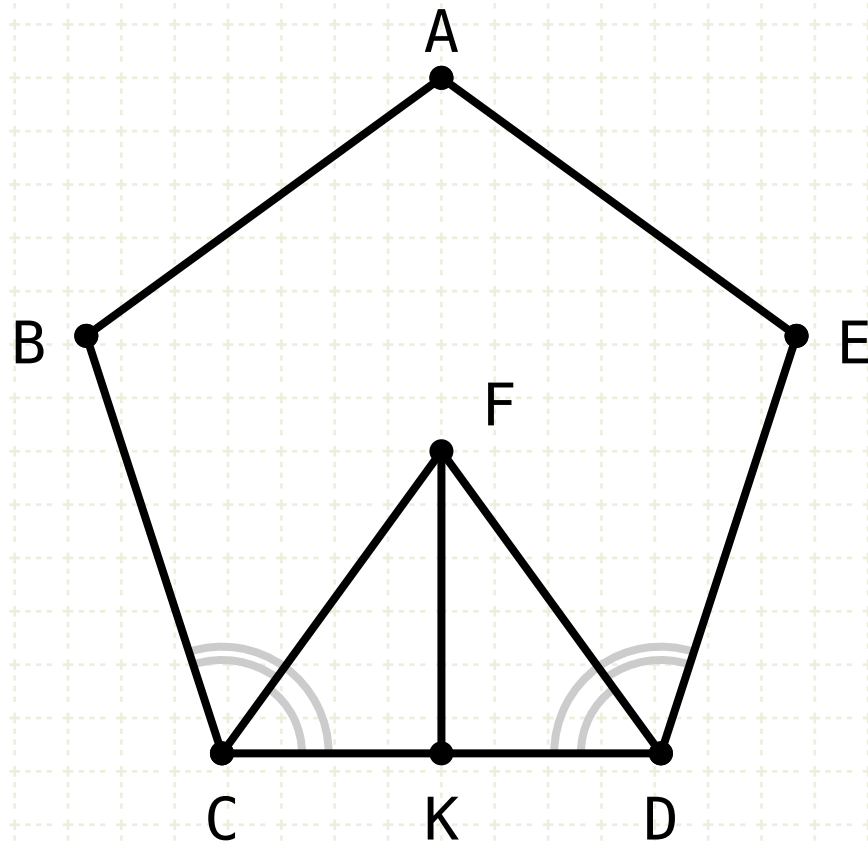
$$\angle CDF = \angle FDE = \theta$$

Construction

Bisect the angles BCD and CDE by the lines CF and DF

Proposition 13 of Book IV

In a given pentagon, which is equilateral and equiangular, to inscribe a circle.



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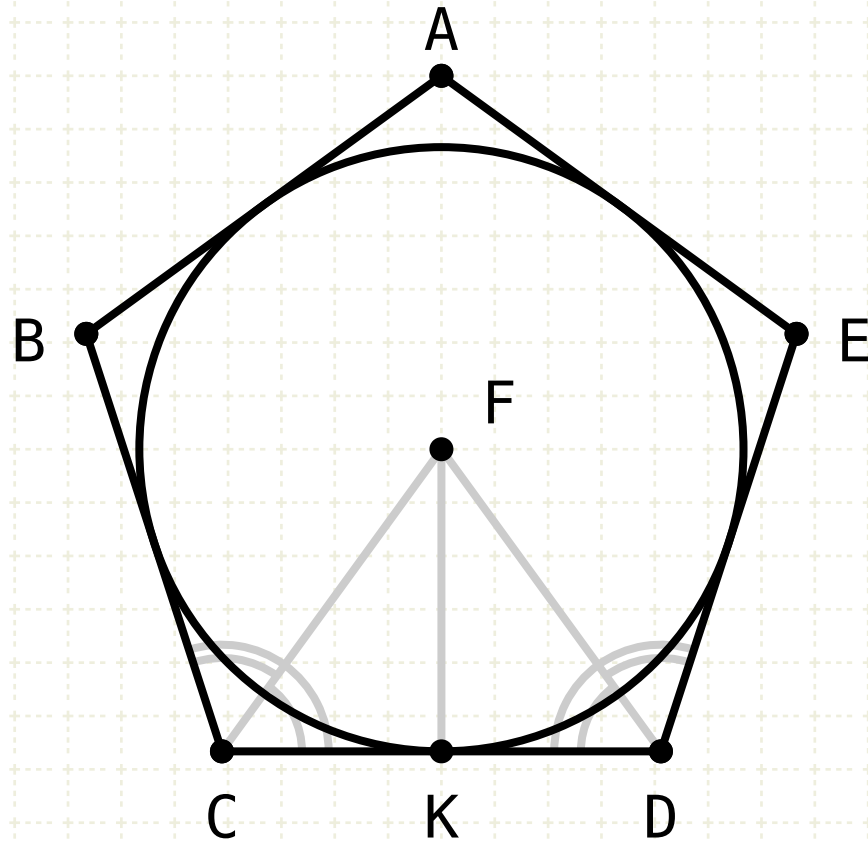
Construction

Bisect the angles BCD and CDE by the lines CF and DF

Draw a perpendicular from point F to side CD

Proposition 13 of Book IV

In a given pentagon, which is equilateral and equiangular, to inscribe a circle.



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Construction

Bisect the angles BCD and CDE by the lines CF and DF

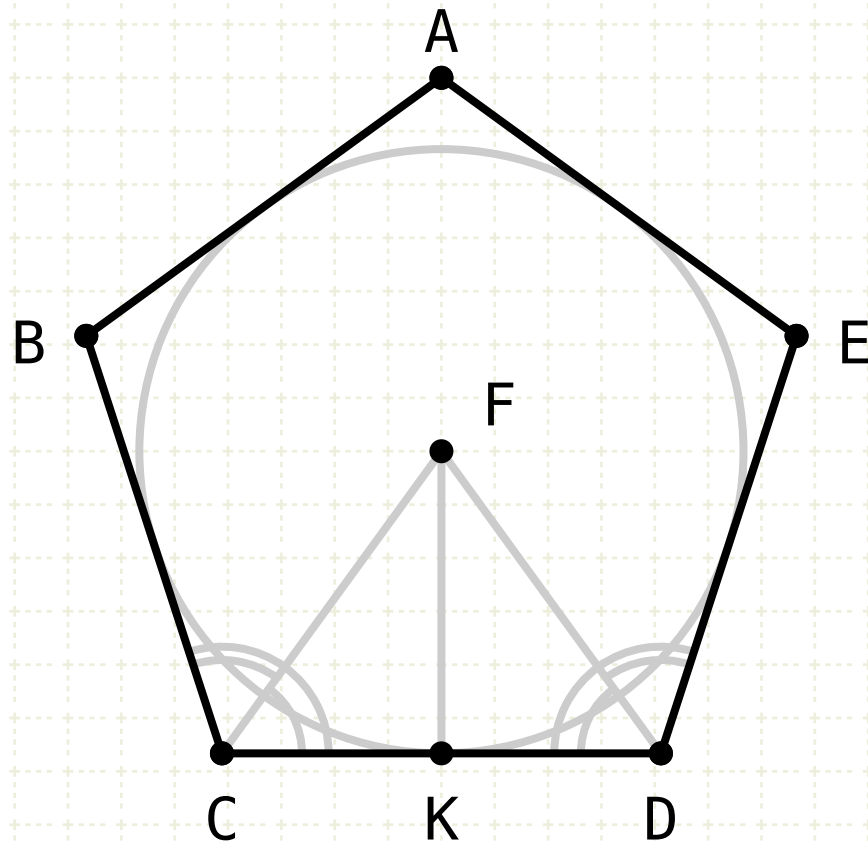
Draw a perpendicular from point F to side CD

Draw a circle with F as the centre, and FK as the radius

The circle inscribes the pentagon

Proposition 13 of Book IV

In a given pentagon, which is equilateral and equiangular, to inscribe a circle.

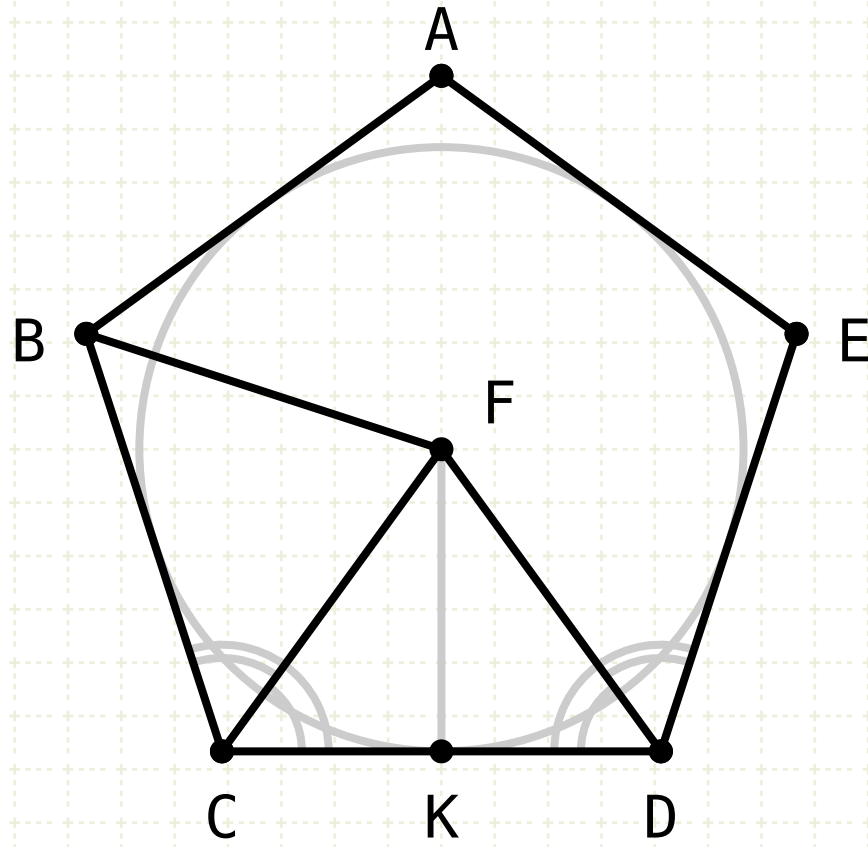


Proof

$$\begin{aligned} 2\alpha &= 2\theta \\ \angle BCF &= \angle FCD = \alpha \\ \angle CDF &= \angle FDE = \theta \end{aligned}$$

Proposition 13 of Book IV

In a given pentagon, which is equilateral and equiangular, to inscribe a circle.



$$2\alpha = 2\theta$$

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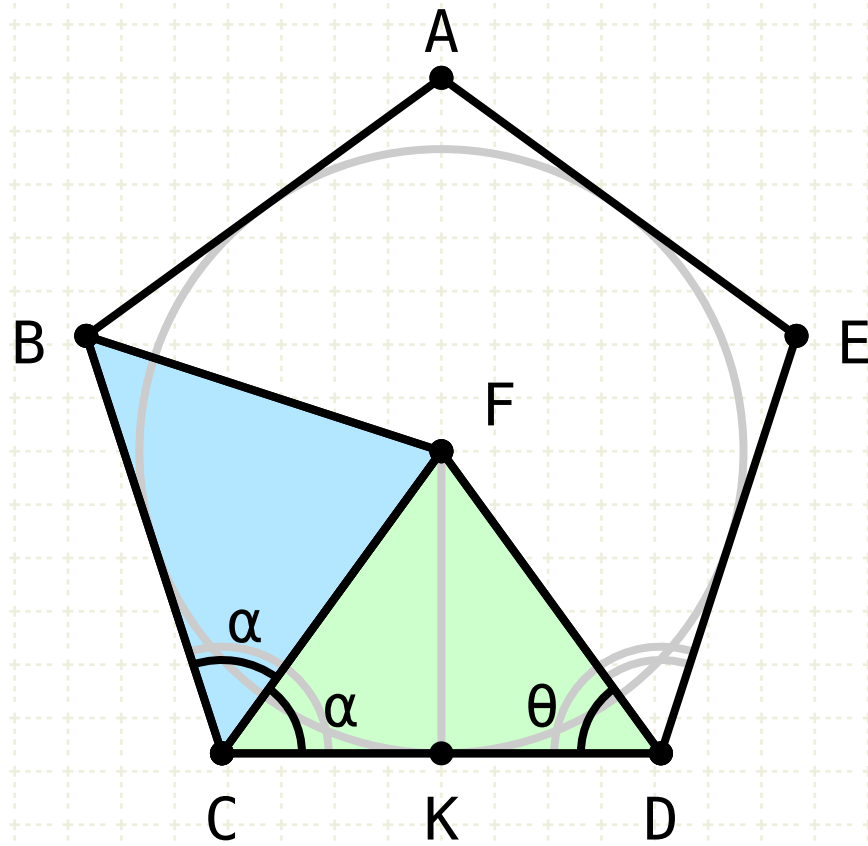
$$\angle CDF = \angle FDE = \theta$$

Proof

Draw line BF

Proposition 13 of Book IV

In a given pentagon, which is equilateral and equiangular, to inscribe a circle.



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$$\angle CDF = \angle FDE = \theta$$

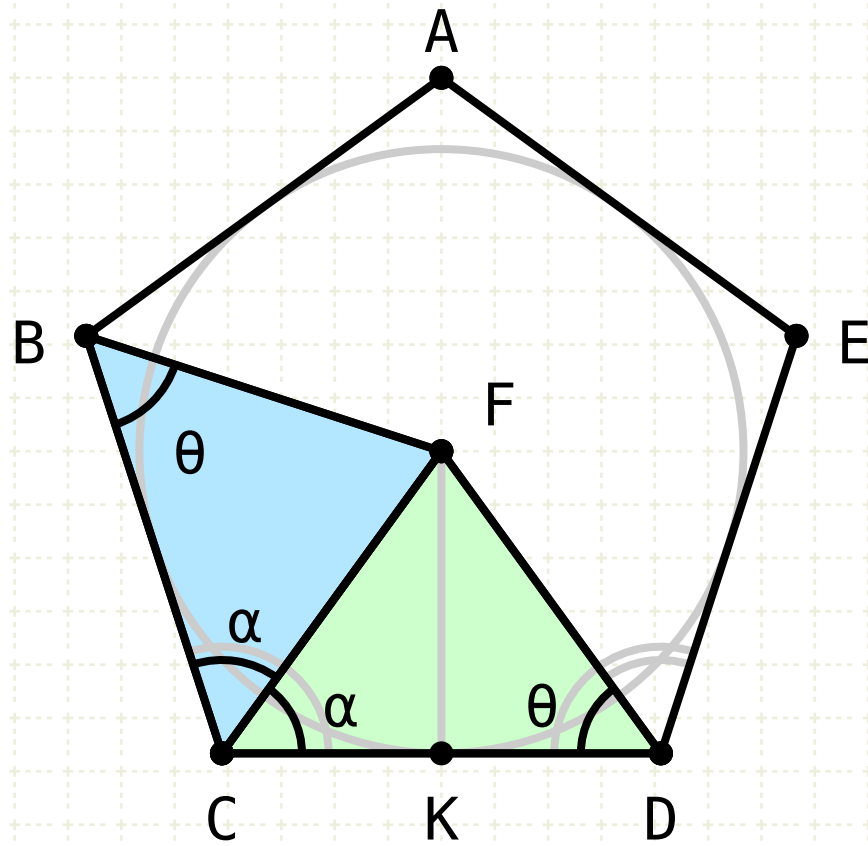
Proof

Draw line BF

Since BC is equal to CD, FC is common, and the angles BCF and FCD are equal the two triangles BFC and FCD are equivalent (SAS) (I·4)

Proposition 13 of Book IV

In a given pentagon, which is equilateral and equiangular, to inscribe a circle.



$$2\alpha = 2\theta$$

$$\angle BCF = \angle FCD = \alpha$$

$$\angle \text{CDF} = \angle \text{FDE} = \theta$$

$$\angle CBF = \angle CDF = \theta$$

Proof

Draw line BF

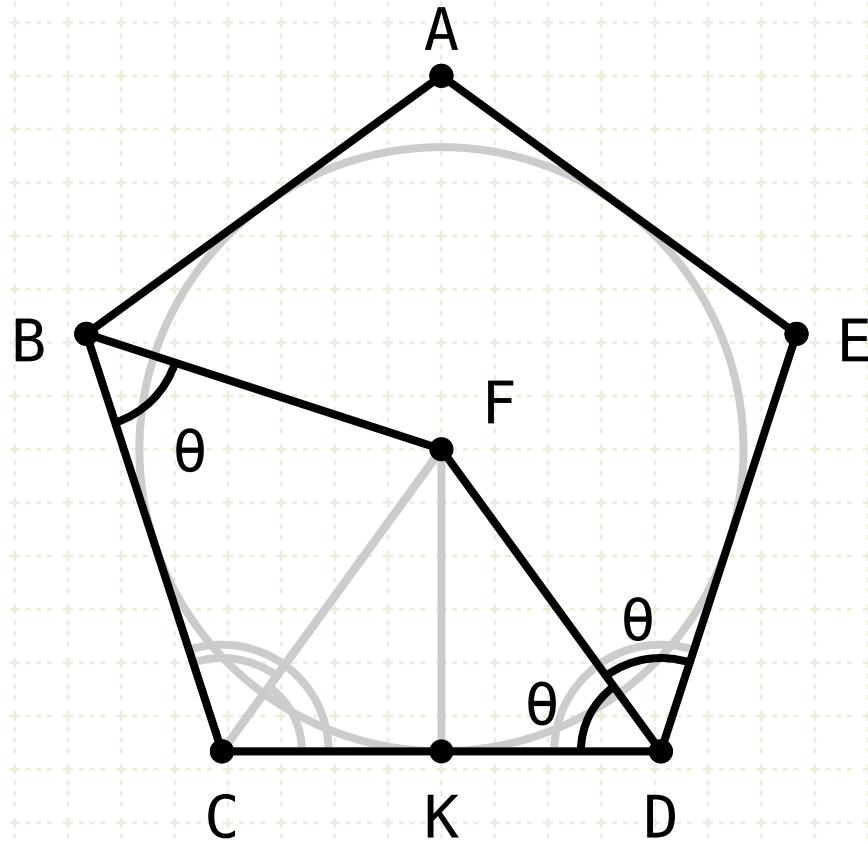
Since BC is equal to CD, FC is common, and the angles BCF and FCD are equal the two triangles BFC and FCD are equivalent (SAS) (I-4)

Therefore the angles CBF and CDF are equal



Proposition 13 of Book IV

In a given pentagon, which is equilateral and equiangular, to inscribe a circle.



$$2\alpha = 2\theta$$

$$\angle BCF = \angle FCD = \alpha$$

$$\angle CDF = \angle FDE = \theta$$

$$\angle CBF = \angle CDF = \theta$$

$$\angle CDE = \angle ABC = 2\theta$$

Proof

Draw line BF

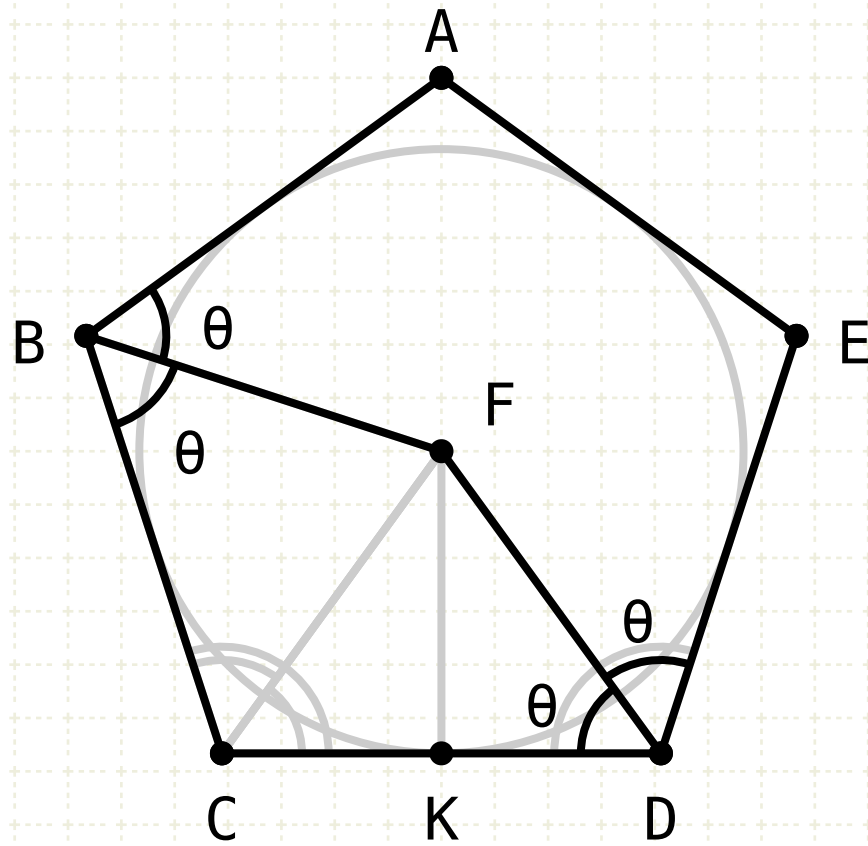
Since BC is equal to CD, FC is common, and the angles BCF and FCD are equal the two triangles BFC and FCD are equivalent (SAS) (I·4)

Therefore the angles CBF and CDF are equal

The pentagon is equiangular, hence angles ABC and CDE are equal

Proposition 13 of Book IV

In a given pentagon, which is equilateral and equiangular, to inscribe a circle.



$$2\alpha = 2\theta$$

$$\angle BCF = \angle FCD = \alpha$$

$$\angle CDF = \angle FDE = \theta$$

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$$\angle CDE = \angle ABC = 2\theta$$

$$\therefore \angle ABF = \angle CBF = \theta$$

Proof

Draw line BF

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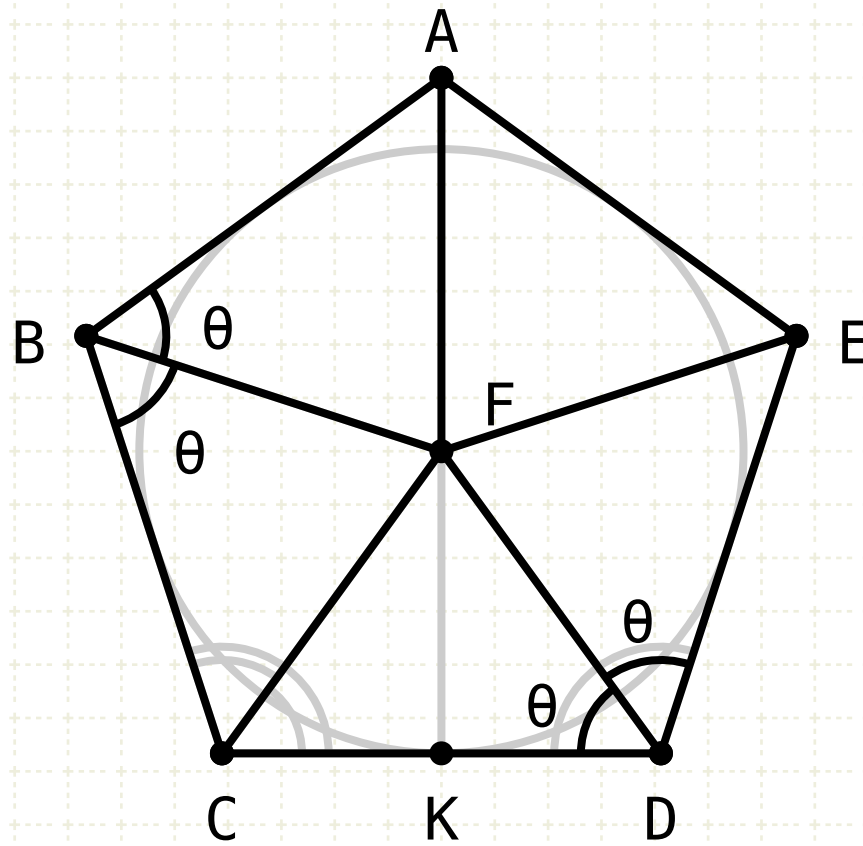
Therefore the angles CBF and CDF are equal

The pentagon is equiangular, hence angles ABC and CDE are equal

Angle CBF (θ) is equal to FDC (θ), and FDC is half of CDE (2θ), therefore BF bisects the angle ABC

Proposition 13 of Book IV

In a given pentagon, which is equilateral and equiangular, to inscribe a circle.



$$2\alpha = 2\theta$$

$$\angle BCF = \angle FCD = \alpha$$

$$\angle \text{CDF} = \angle \text{FDE} = \theta$$

$$\angle CBF = \angle CDF = \theta$$

$$\angle CDE = \angle ABC = 2\theta$$

$$\therefore \angle ABF = \angle CBF = \theta$$

$$\angle BAF = \angle FAE$$

$$\angle AEF = \angle FED$$

Proof

Draw line BF

Since BC is equal to CD, FC is common, and the angles BCF and FCD are equal the two triangles BFC and FCD are equivalent (SAS) (I-4)

Therefore the angles CBF and CDF are equal

The pentagon is equiangular, hence angles ABC and CDE are equal

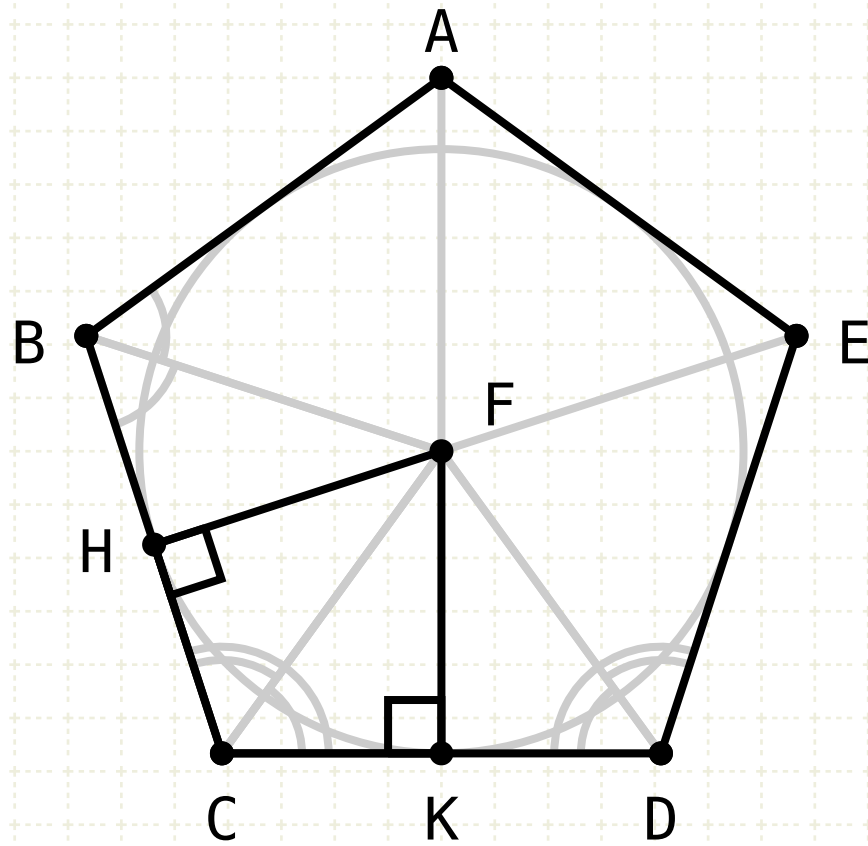
Angle CBF (θ) is equal to FDC (θ), and FDC is half of CDE (2θ), therefore BF bisects the angle ABC

Similarly, it can be shown that AF and EF bisect the angles BAE and AED respectively



Proposition 13 of Book IV

In a given pentagon, which is equilateral and equiangular, to inscribe a circle.



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Draw line BF

Since BC is equal to CD, FC is common, and the angles BCF and FCD are equal the two triangles BFC and FCD are equivalent (SAS) (I·4)

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The pentagon is equiangular, hence angles ABC and CDE are equal

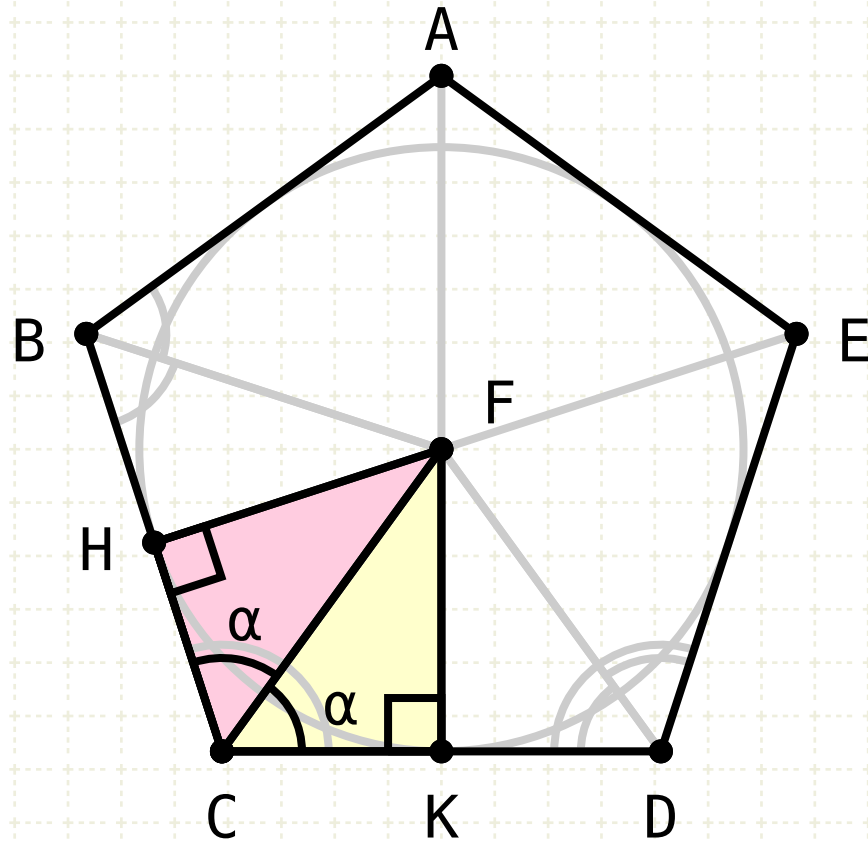
Angle CBF (θ) is equal to FDC (θ), and FDC is half of CDE (2θ), therefore BF bisects the angle ABC

Similarly, it can be shown that AF and EF bisect the angles BAE and AED respectively

Draw perpendiculars from F to line CD and BC

Proposition 13 of Book IV

In a given pentagon, which is equilateral and equiangular, to inscribe a circle.



$$\begin{aligned}
 2\alpha &= 2\theta \\
 \angle BCF &= \angle FCD = \alpha \\
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 \angle CDE &= \angle ABC = 2\theta \\
 \therefore \angle ABF &= \angle CBF = \theta \\
 \angle BAF &= \angle FAE \\
 \angle AEF &= \angle FED
 \end{aligned}$$

$$FH = FK$$

Proof

Draw line BF

Since BC is equal to CD, FC is common, and the angles BCF and FCD are equal the two triangles BFC and FCD are equivalent (SAS) (I·4)

Therefore the angles CBF and CDF are equal

The pentagon is equiangular, hence angles ABC and CDE are equal

Angle CBF (θ) is equal to FDC (θ), and FDC is half of CDE (2θ), therefore BF bisects the angle ABC

Similarly, it can be shown that AF and EF bisect the angles BAE and AED respectively

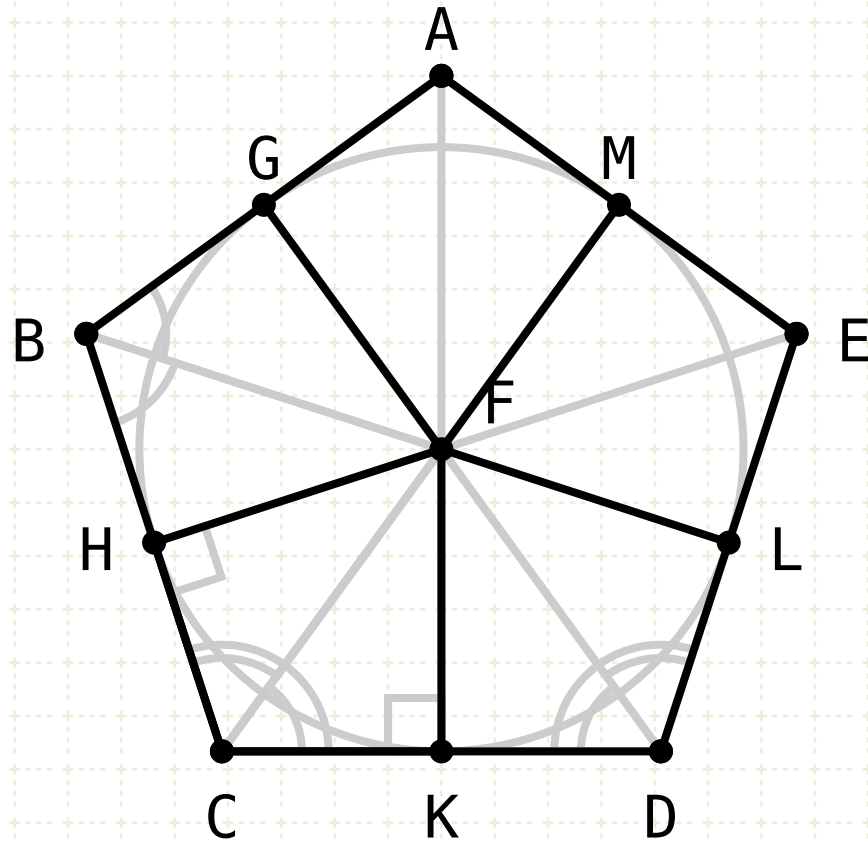
Draw perpendiculars from F to line CD and BC

Triangles HFC and FKC are right angled triangles with angles FCH and FCK equal and a common side FC

Therefore, the two triangles are equivalent (ASA) (I·26), and FH equals FK

Proposition 13 of Book IV

In a given pentagon, which is equilateral and equiangular, to inscribe a circle.



$$\begin{aligned}
 2\alpha &= 2\theta \\
 \angle BCF &= \angle FCD = \alpha \\
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 \angle BAF &= \angle FAE \\
 \angle AEF &= \angle FED
 \end{aligned}$$

$$FH = FK$$

$$FH = FK = FL = FM = FG$$

Proof

Draw line BF

Since BC is equal to CD, FC is common, and the angles BCF and FCD are equal the two triangles BFC and FCD are equivalent (SAS) (I·4)

Therefore the angles CBF and CDF are equal

The pentagon is equiangular, hence angles ABC and CDE are equal

Angle CBF (θ) is equal to FDC (θ), and FDC is half of CDE (2θ), therefore BF bisects the angle ABC

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Draw perpendiculars from F to line CD and BC

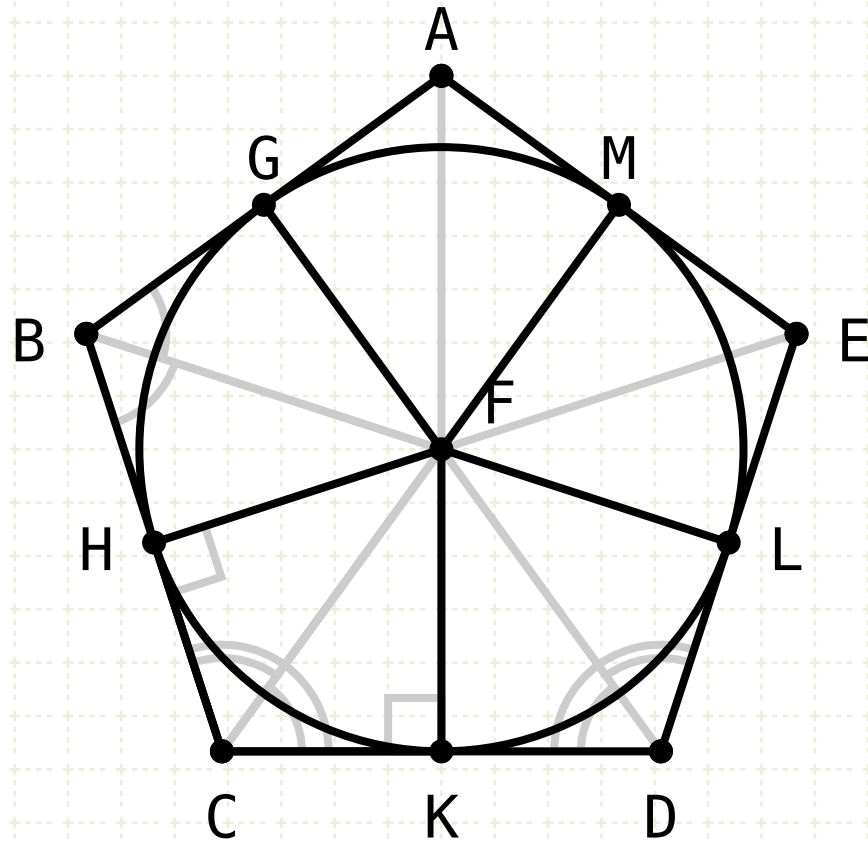
Triangles HFC and FKC are right angled triangles with angles FCH and FCK equal and a common side FC

Therefore, the two triangles are equivalent (ASA) (I·26), and FH equals FK

Similarly, it can be shown that perpendiculars drawn from F to the remaining sides of the pentagon are all equal

Proposition 13 of Book IV

In a given pentagon, which is equilateral and equiangular, to inscribe a circle.



$$2\alpha = 2\theta$$

$$\angle BCF = \angle FCD = \alpha$$

$$\angle CDF = \angle FDE = \theta$$

$$\angle CBF = \angle CDF = \theta$$

$$\angle CDE = \angle ABC = 2\theta$$

$$\therefore \angle ABF = \angle CBF = \theta$$

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$$FH = FK$$

$$FH = FK = FL = FM = FG$$

Proof

Draw line BF

Since BC is equal to CD, FC is common, and the angles BCF and FCD are equal the two triangles BFC and FCD are equivalent (SAS) (I·4)

Therefore the angles CBF and CDF are equal

The pentagon is equiangular, hence angles ABC and CDE are equal

Angle CBF (θ) is equal to FDC (θ), and FDC is half of CDE (2θ), therefore BF bisects the angle ABC

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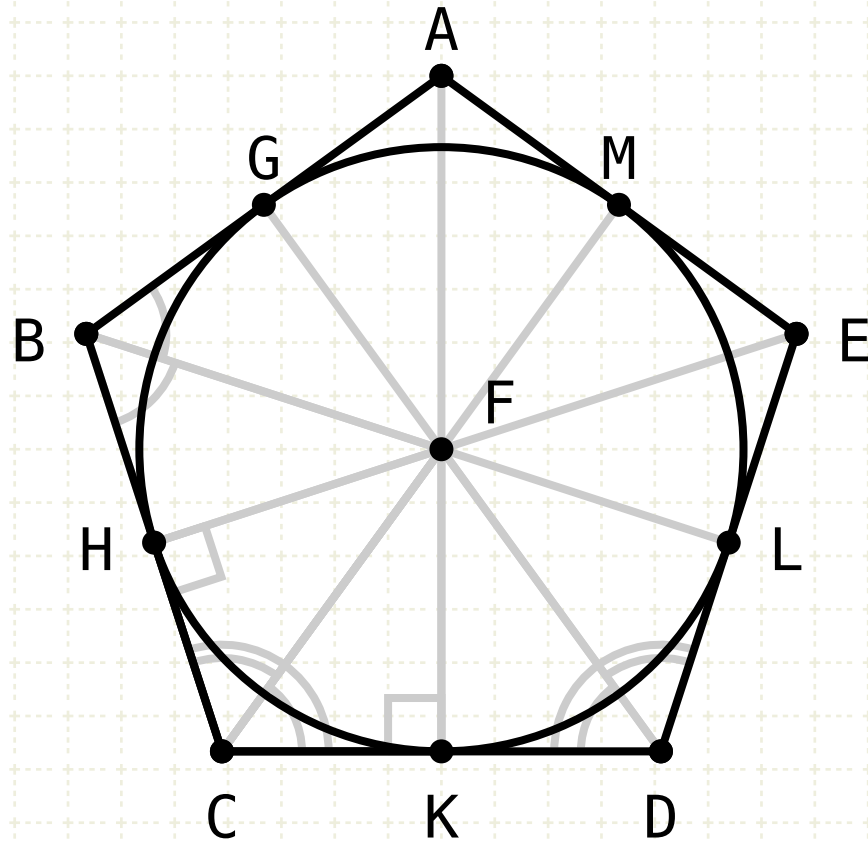
Therefore, the two triangles are equivalent (ASA) (I·26), and FH equals FK

Similarly, it can be shown that perpendiculars drawn from F to the remaining sides of the pentagon are all equal

Thus, a circle drawn with a centre at F, and a radius of FH will pass through all the points H, K, L, M and G

Proposition 13 of Book IV

In a given pentagon, which is equilateral and equiangular, to inscribe a circle.



$$\begin{aligned} 2\alpha &= 2\theta \\ \angle BCF &= \angle FCD = \alpha \\ \angle CDF &= \angle FDE = \theta \\ \angle CBF &= \angle CDF = \theta \\ \angle CDE &= \angle ABC = 2\theta \\ \therefore \angle ABF &= \angle CBF = \theta \\ \angle BAF &= \angle FAE \\ \angle AEF &= \angle FED \end{aligned}$$

$$FH = FK$$

$$FH = FK = FL = FM = FG$$

Proof

Draw line BF

Since BC is equal to CD, FC is common, and the angles BCF and FCD are equal the two triangles BFC and FCD are equivalent (SAS) (I·4)

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The pentagon is equiangular, hence angles ABC and CDE are equal

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Triangles HFC and FKC are right angled triangles with angles FCH and FCK equal and a common side FC

Therefore, the two triangles are equivalent (ASA) (I·26), and FH equals FK

Similarly, it can be shown that perpendiculars drawn from F to the remaining sides of the pentagon are all equal

Thus, a circle drawn with a centre at F, and a radius of FH will pass through all the points H, K, L, M and G

Since the sides of the pentagon are at right angles to the radii, the pentagon touches the circle (III·16)

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