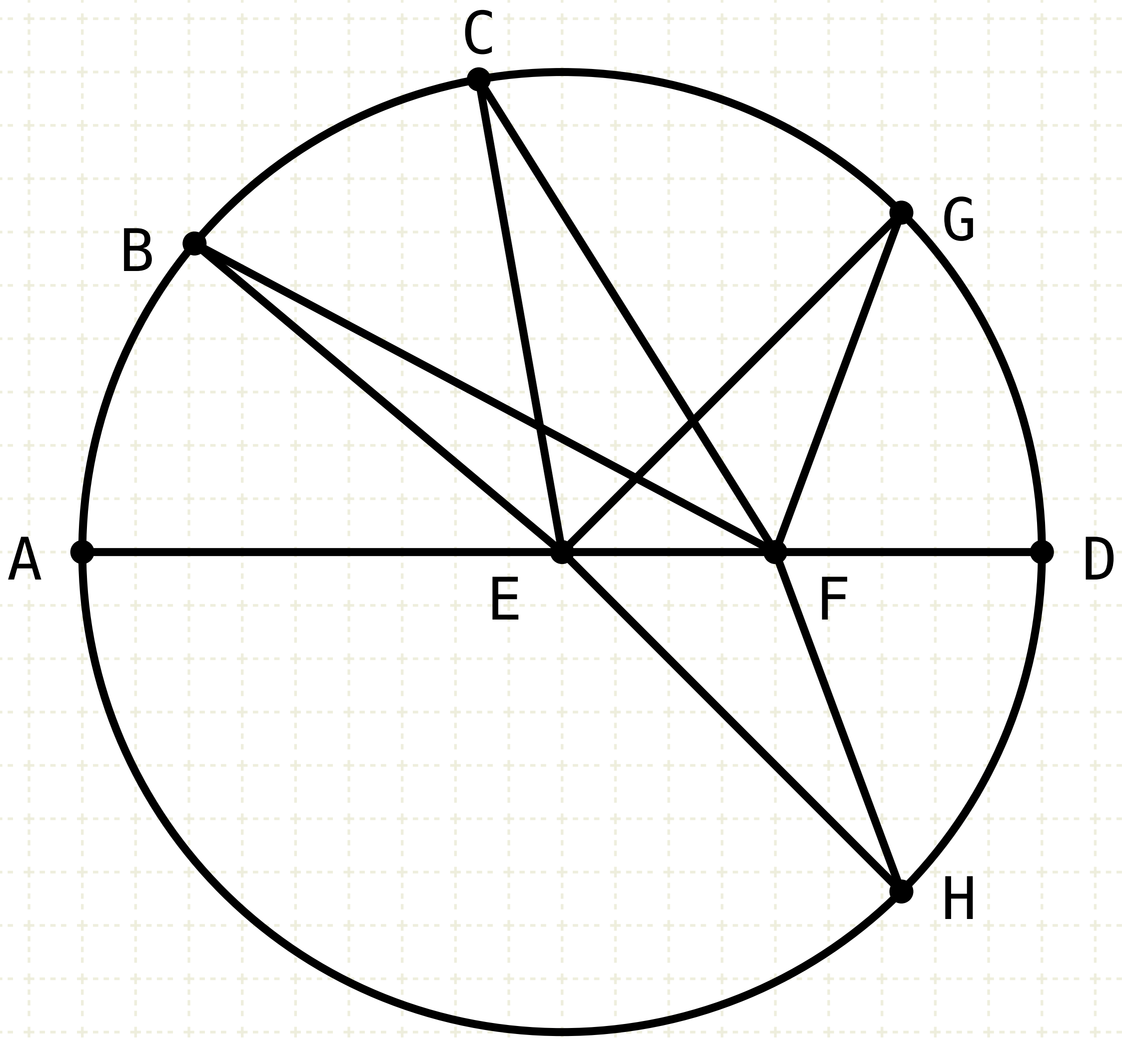
Euclid's Elements

Book III



A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)

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Proposition 20 of Book III

In a circle the angle at the centre is double of the angle at the circumference, when the angles have the same circumference as base.

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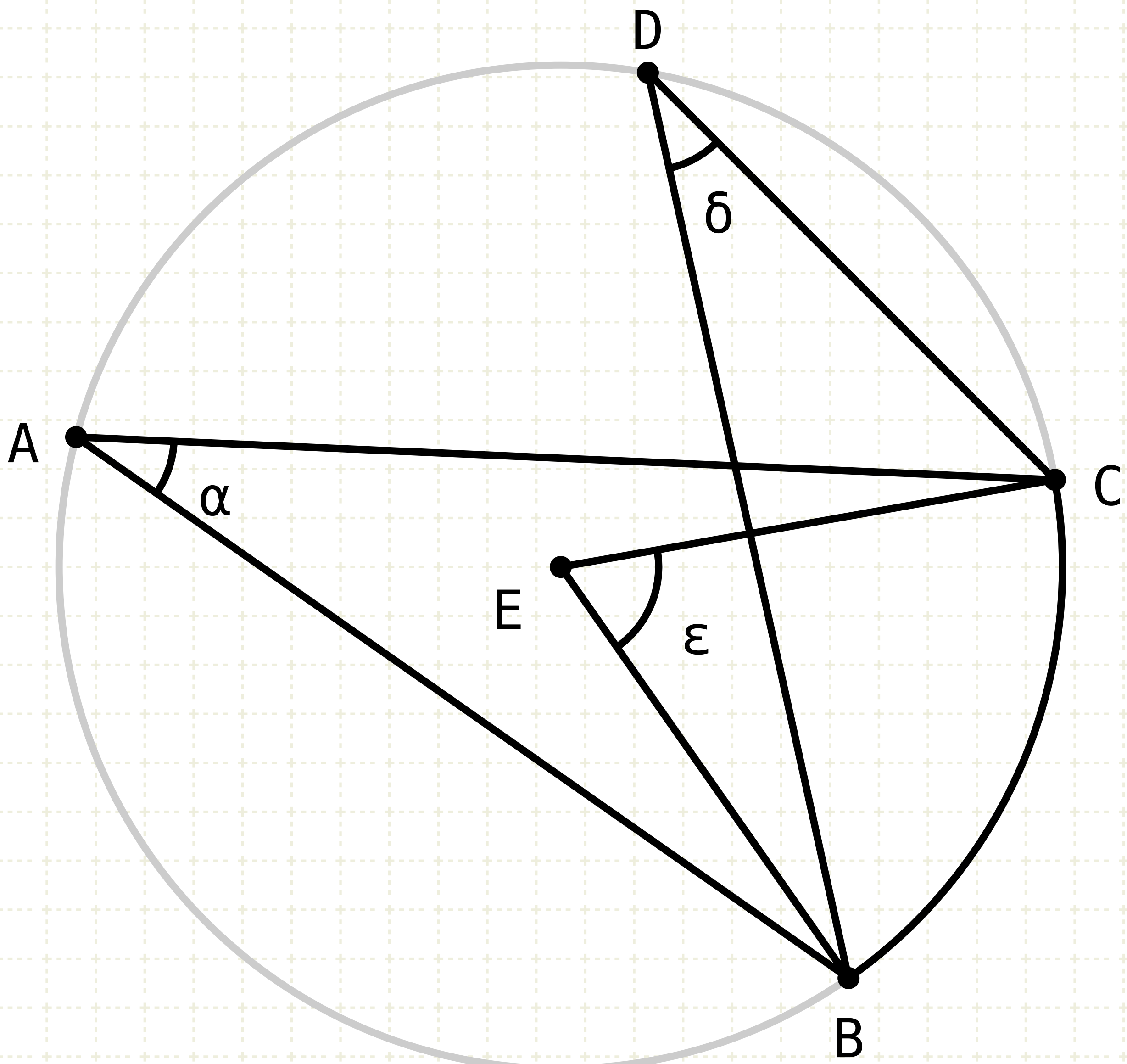

E is the centre of the circle

$\varepsilon = 2\alpha$

$\varepsilon = 2\delta$

In other words

If E is the centre of a circle, and the arc BC the base of the angle BEC (ε) then ε will be double…

… any angle drawn from the circumference of the circle with the BC as its base,

… angle BAC (α) and angle BDC (δ) for example,

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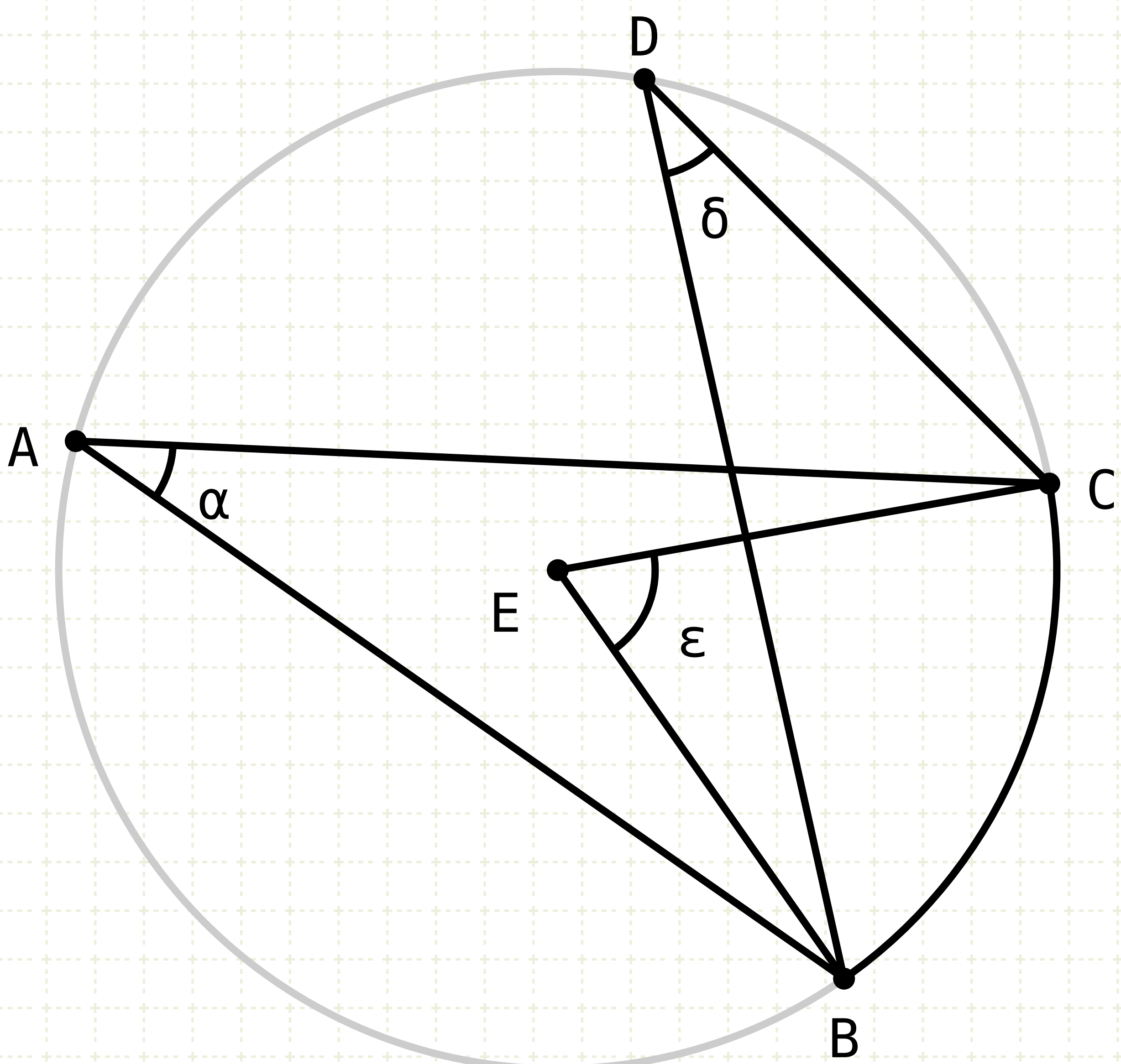

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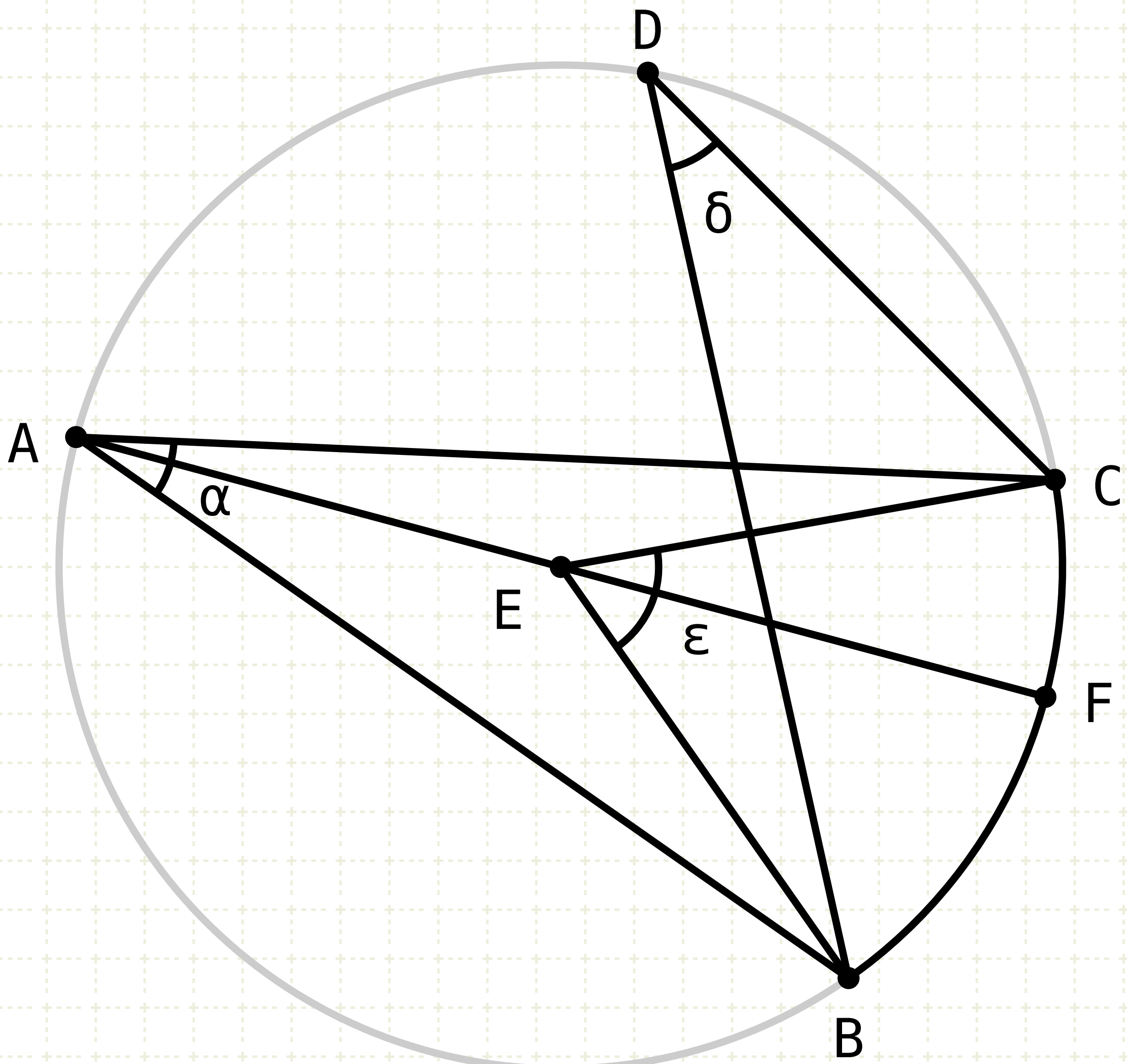

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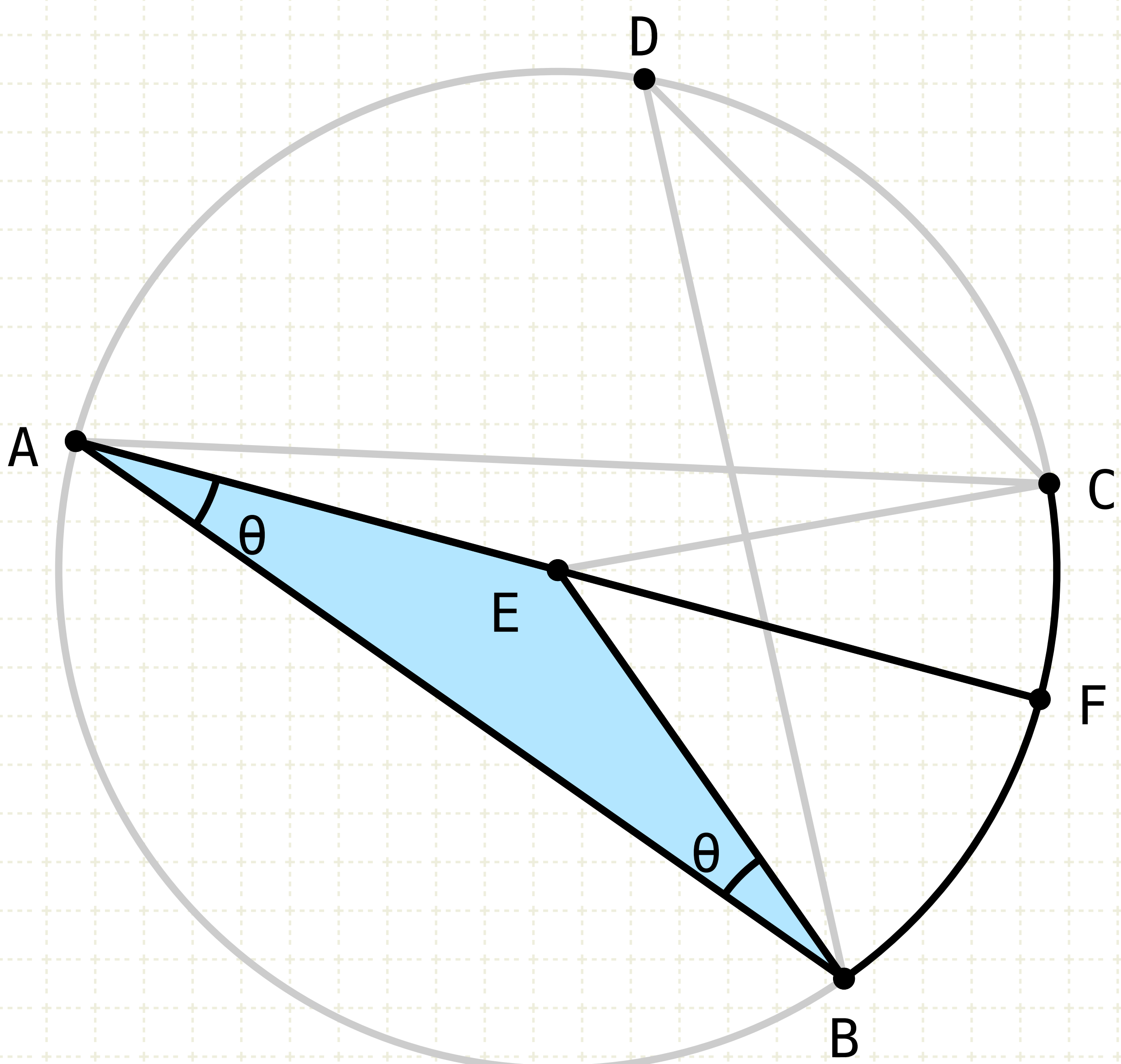

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∠EAB = ∠EBA = θ

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Since AE and BE are equal, triangle ABE is isosceles, and its base angles EAB and EBA are equal (I·5)

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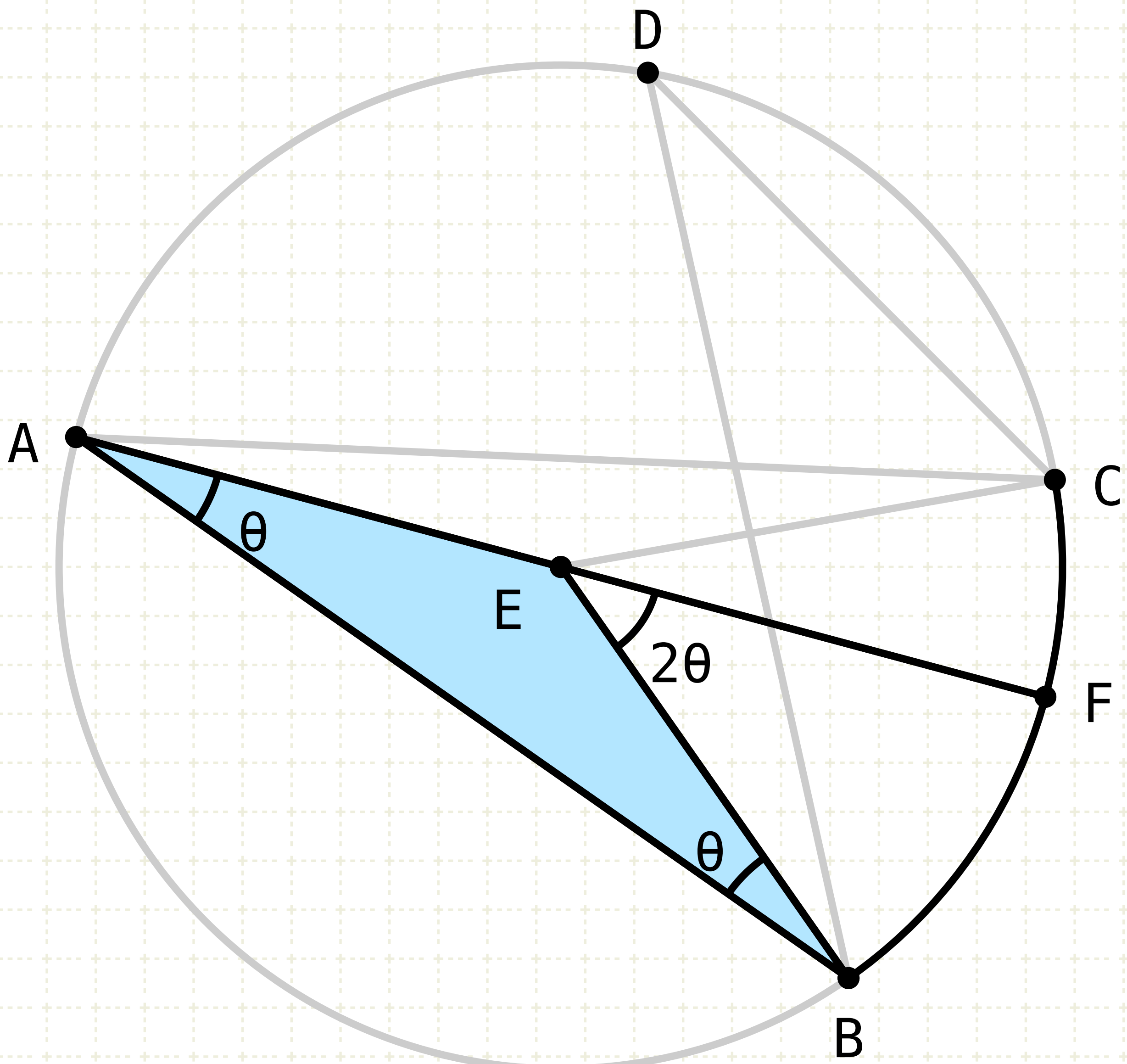

E is the centre of the circle

$\angle EAB = \angle EBA = \theta$

$\angle FEB = 2\theta$

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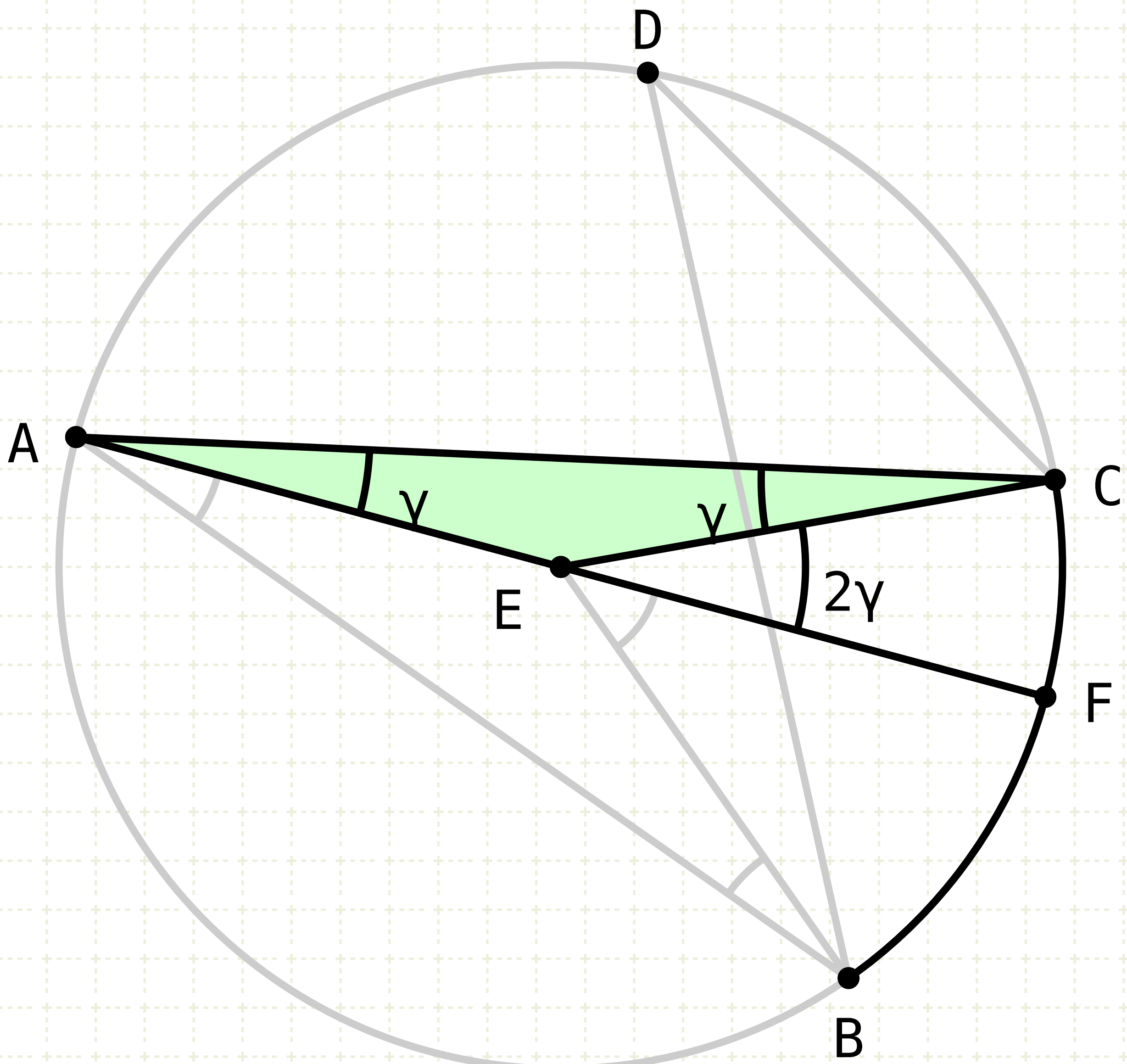

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$\angle EAC = \angle ECA = \gamma$

$\angle FEC = 2\gamma$

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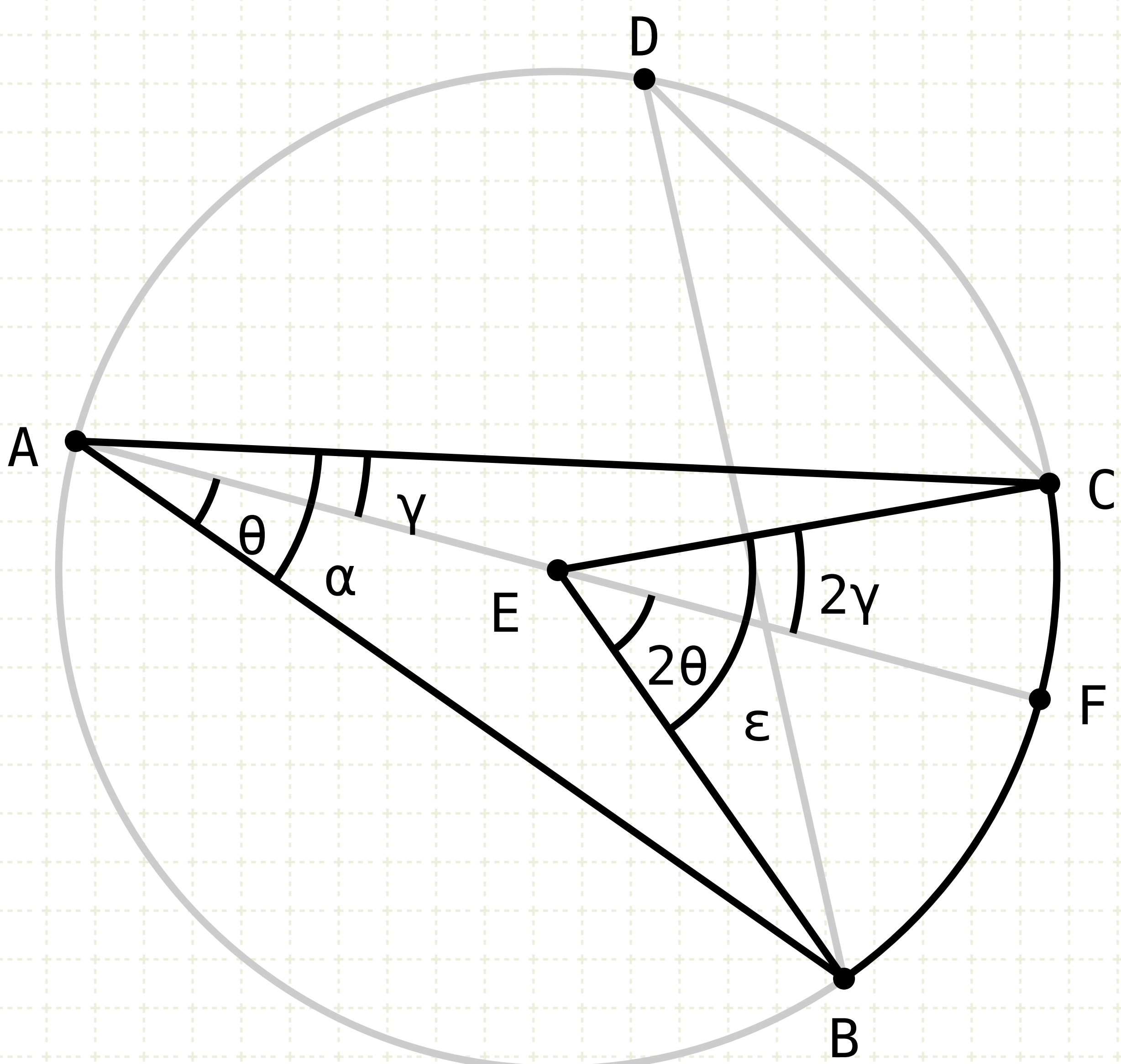

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$\angle EAB = \angle EBA = \theta$
$\angle FEB = 2\theta$
$\angle EAC = \angle ECA = \gamma$
$\angle FEC = 2\gamma$

$\angle BAC = \alpha$
$\alpha = \gamma + \theta$

$\angle BEC = \varepsilon$
$\varepsilon = 2\gamma + 2\theta$

$\varepsilon = 2\alpha$

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Now, angle BAC (α) is equal to the sum of θ and γ, and the angle BEC (ε) is equal to the sum of 2θ and 2γ, thus giving us BEC is twice BAC

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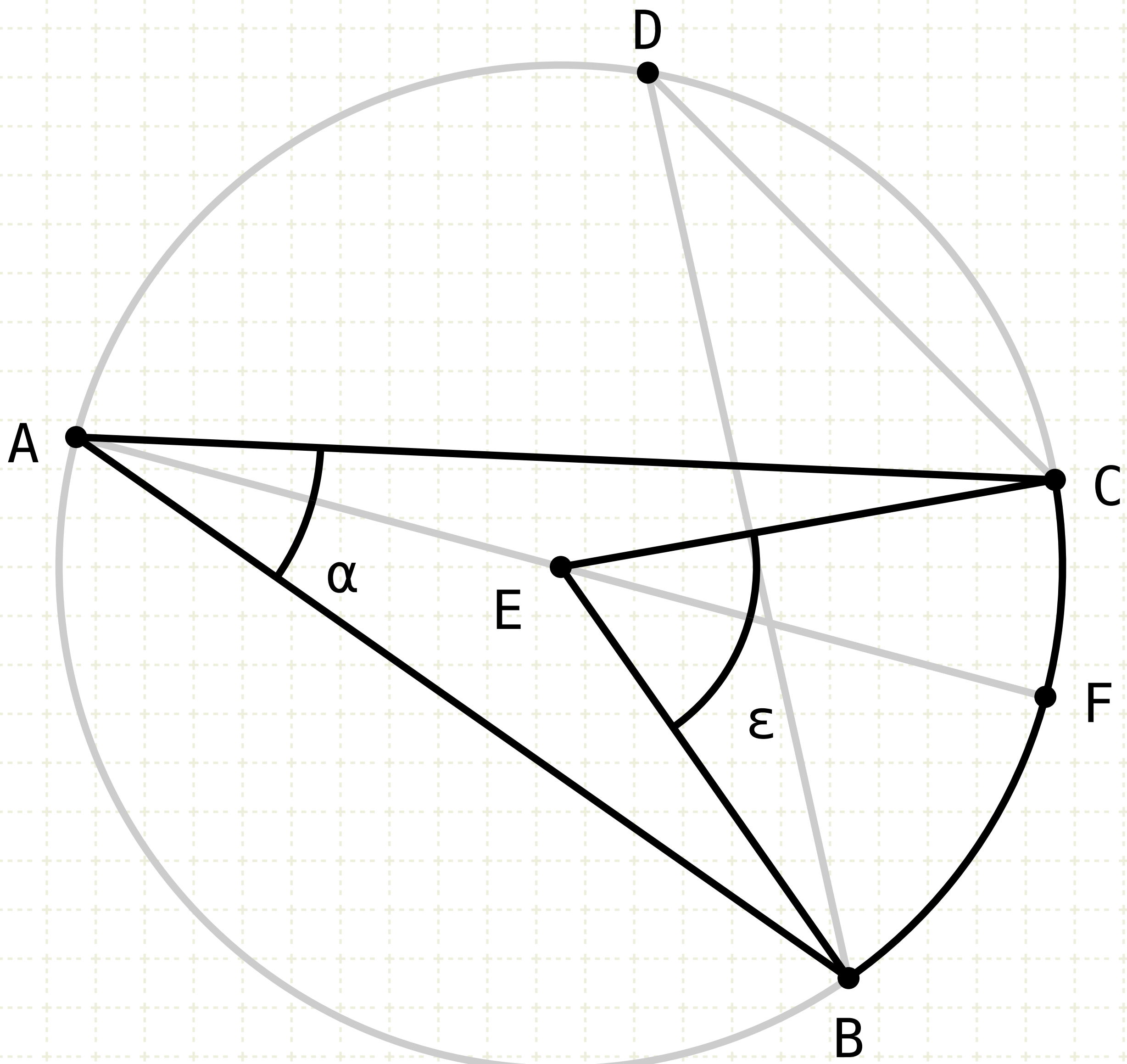

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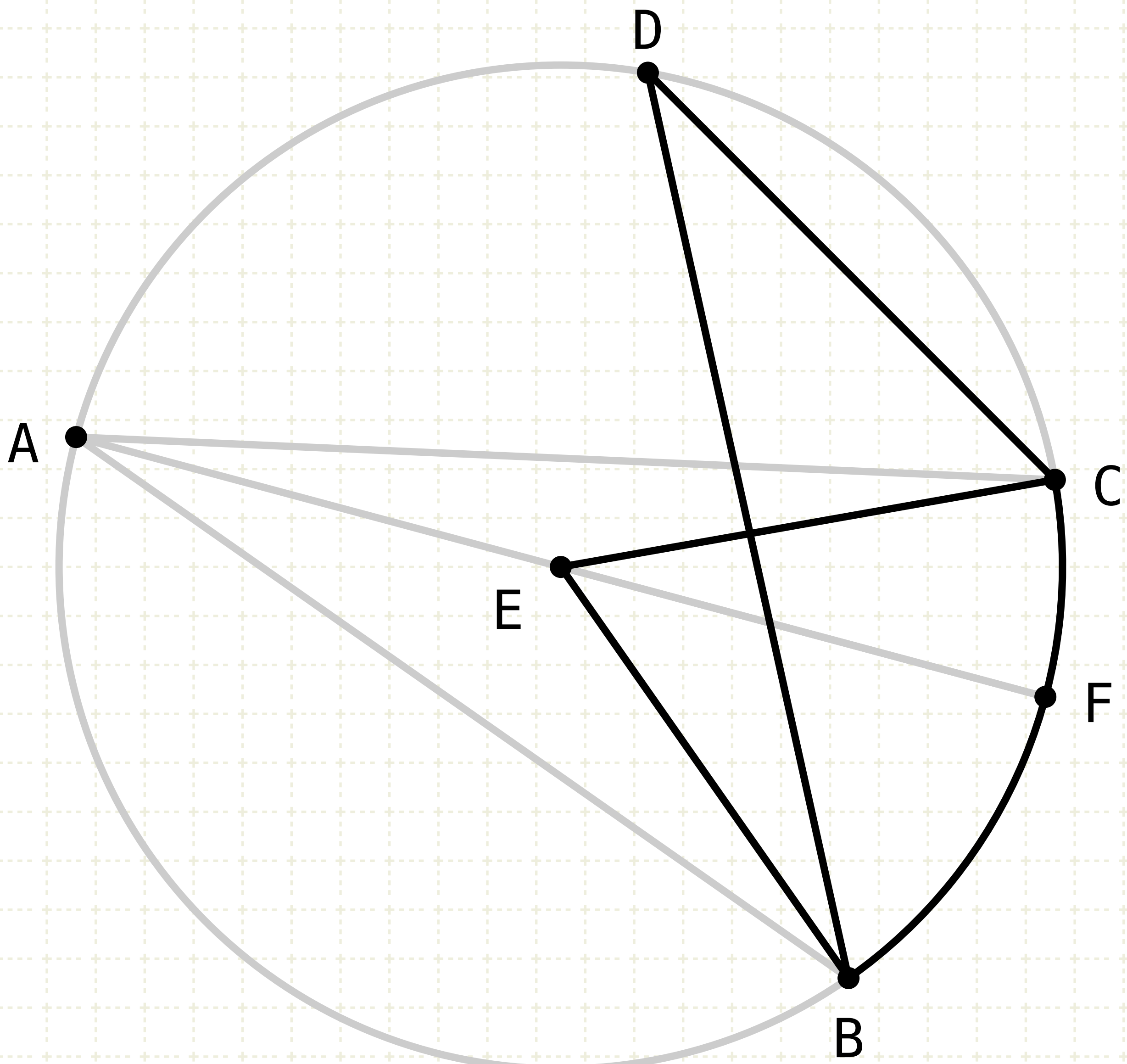

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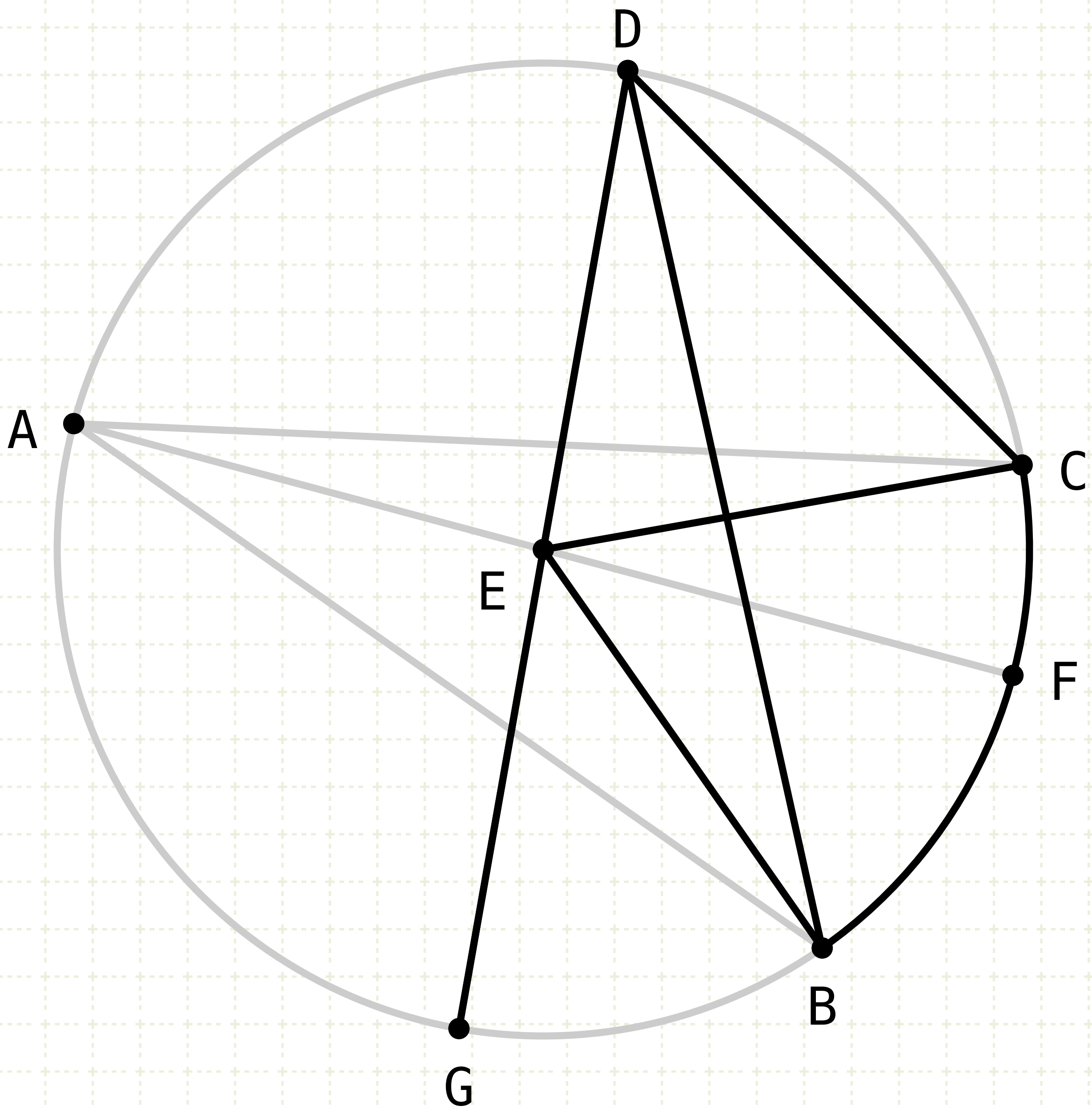

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Draw a line from DE and extend to G

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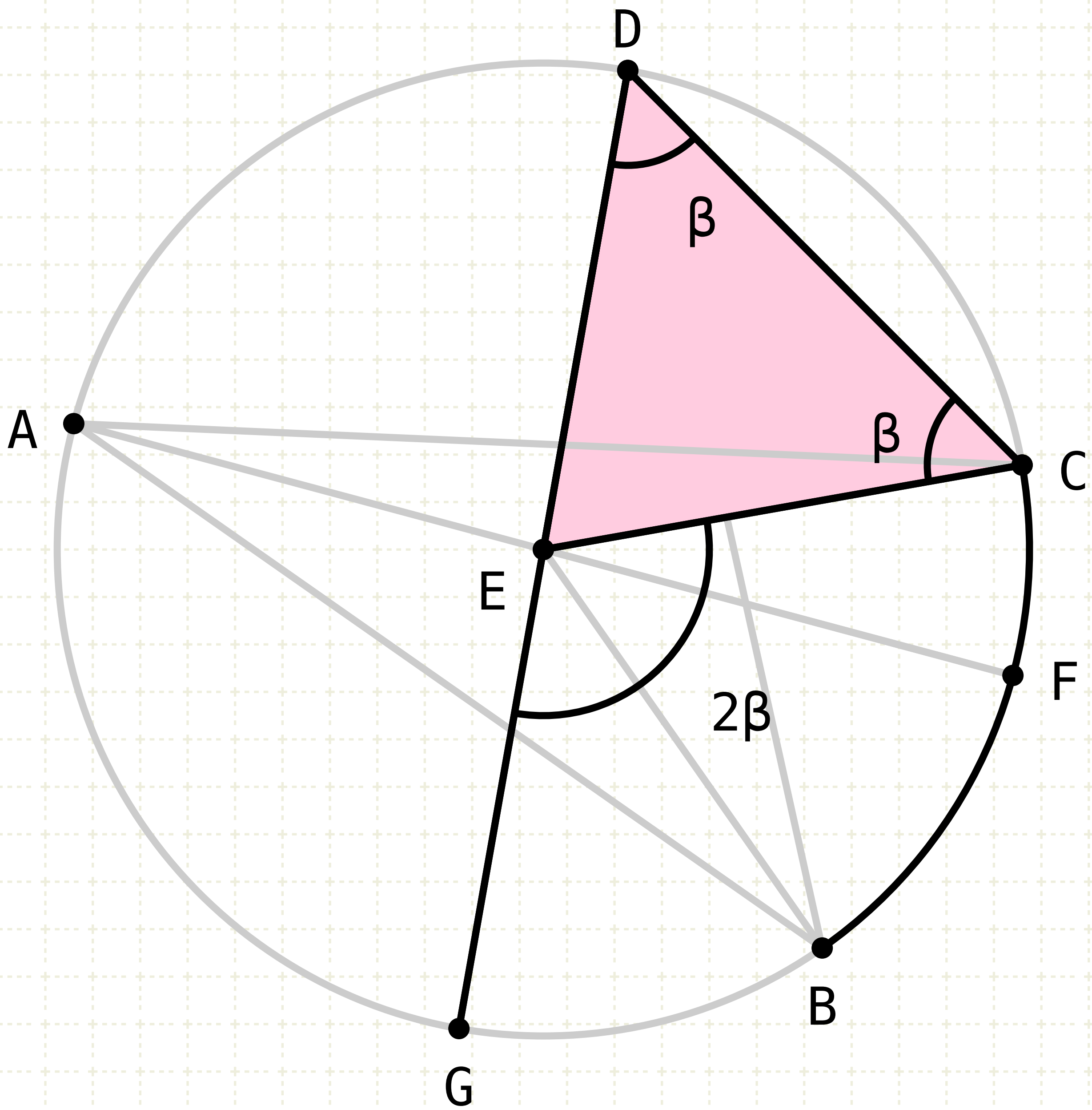

E is the centre of the circle

$\varepsilon = 2\alpha$

$\angle EDC = \angle DCE = \beta$
$\angle GEC = 2\beta$

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Using the same arguments as before, it can be seen that GEC is twice EDC

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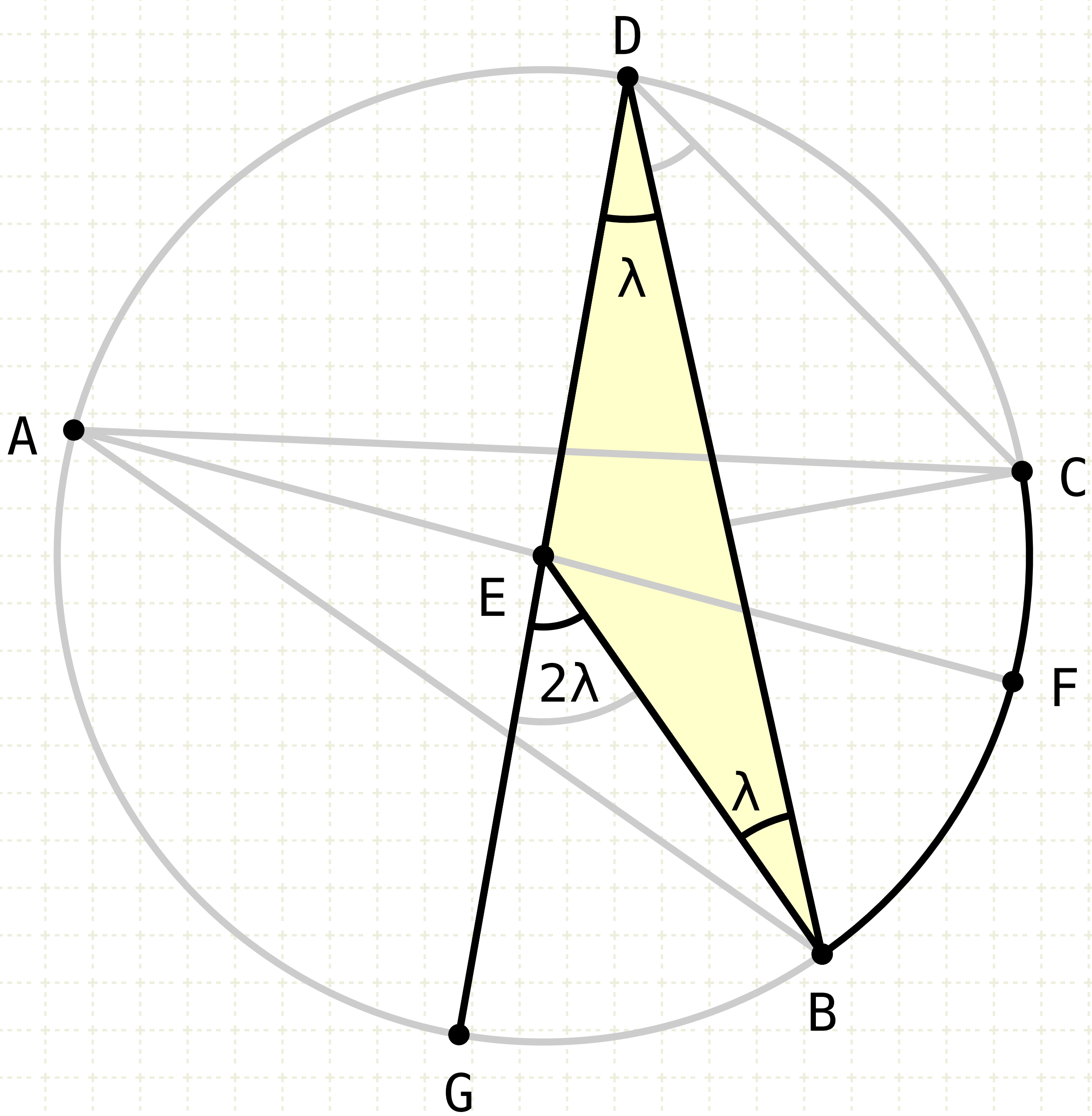

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$\angle EDB = \angle DBE = \lambda$

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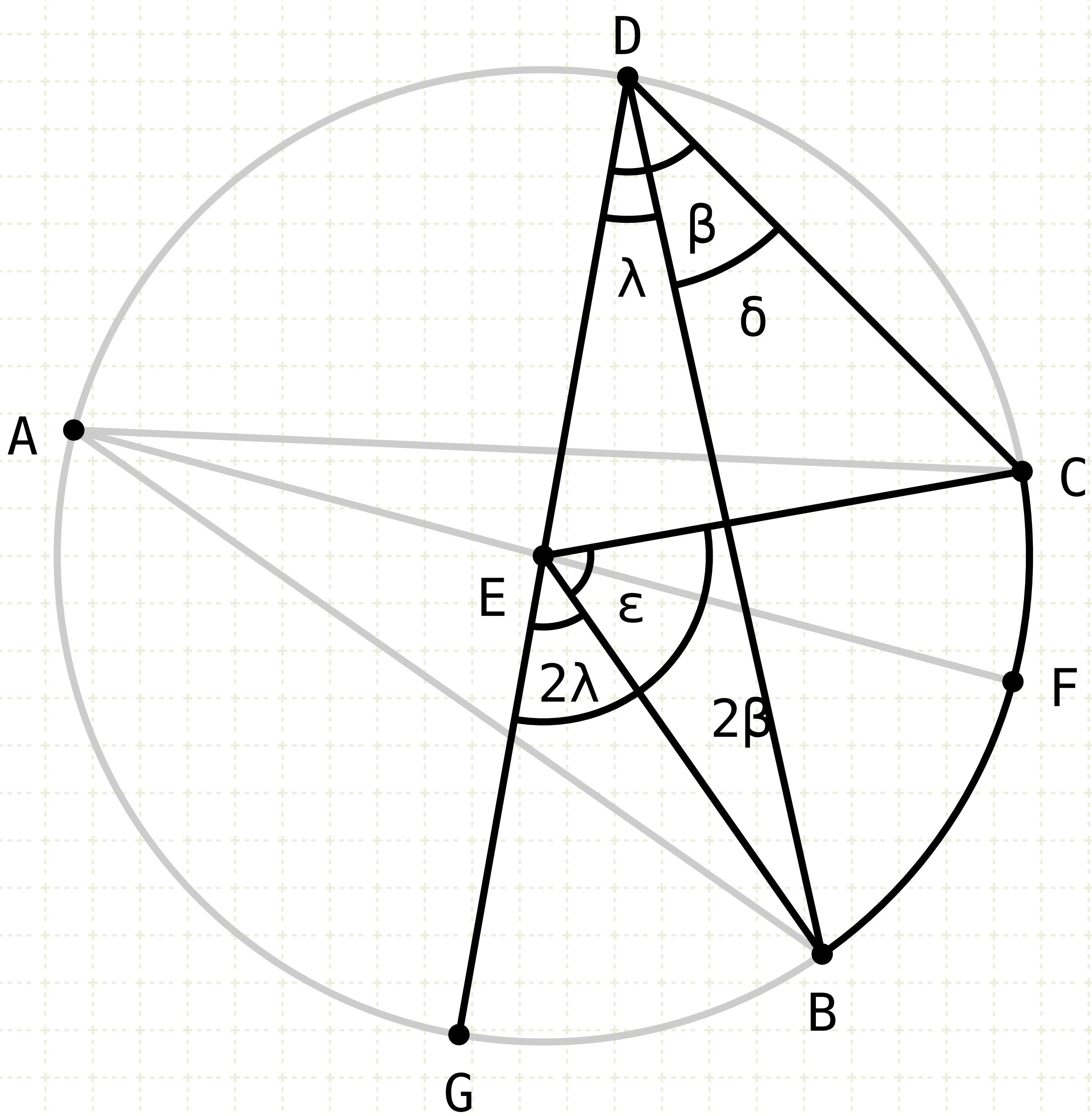

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$$\angle CDB = \delta$$
$$\delta = \beta - \lambda$$

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Draw a line from DE and extend to G

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Thus the angle CDB (δ) is the difference between CDE (β) and BDE (λ) and angle CEB (ε) is the difference between CEG (2β) and BEG (2λ)

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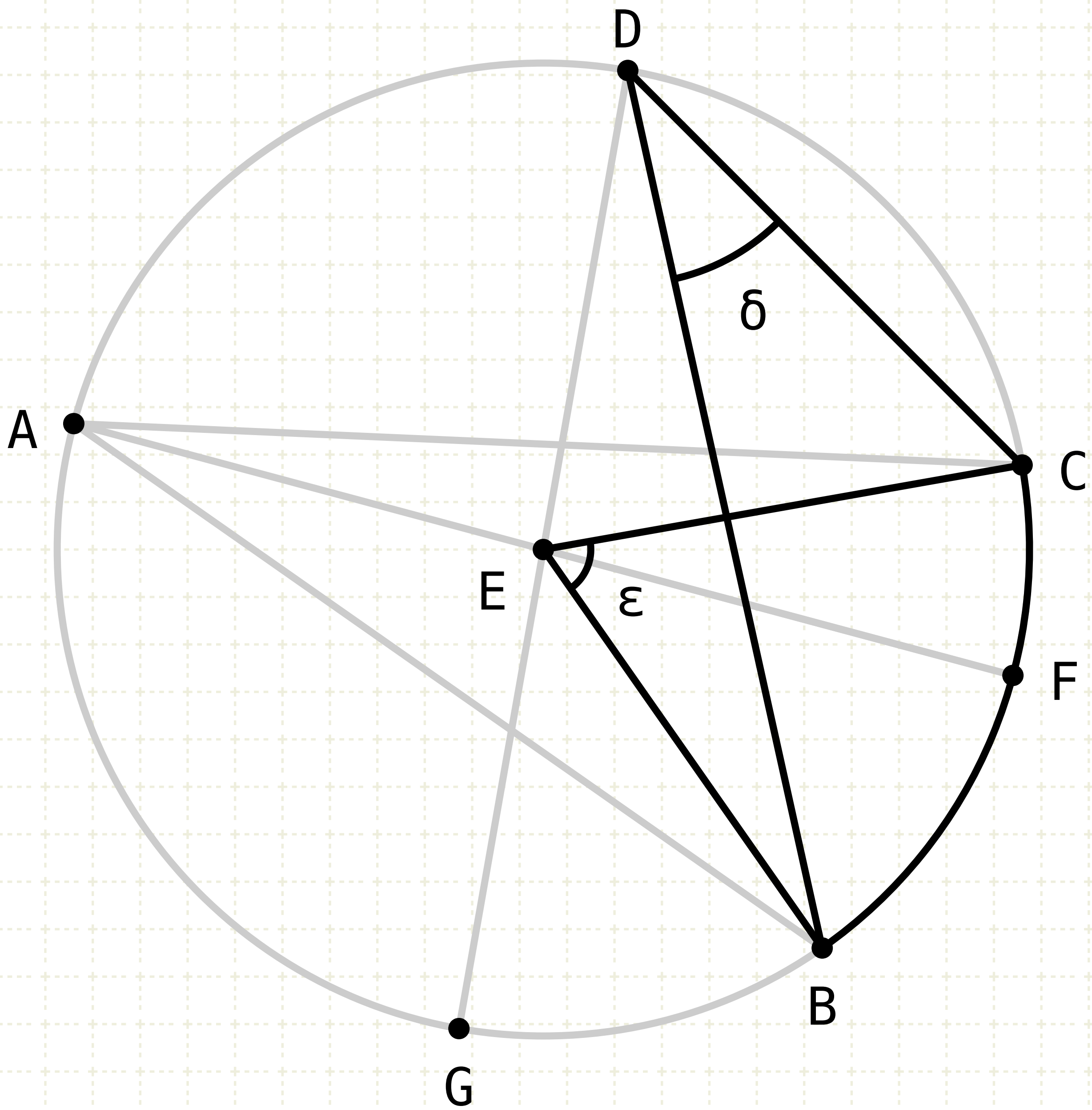

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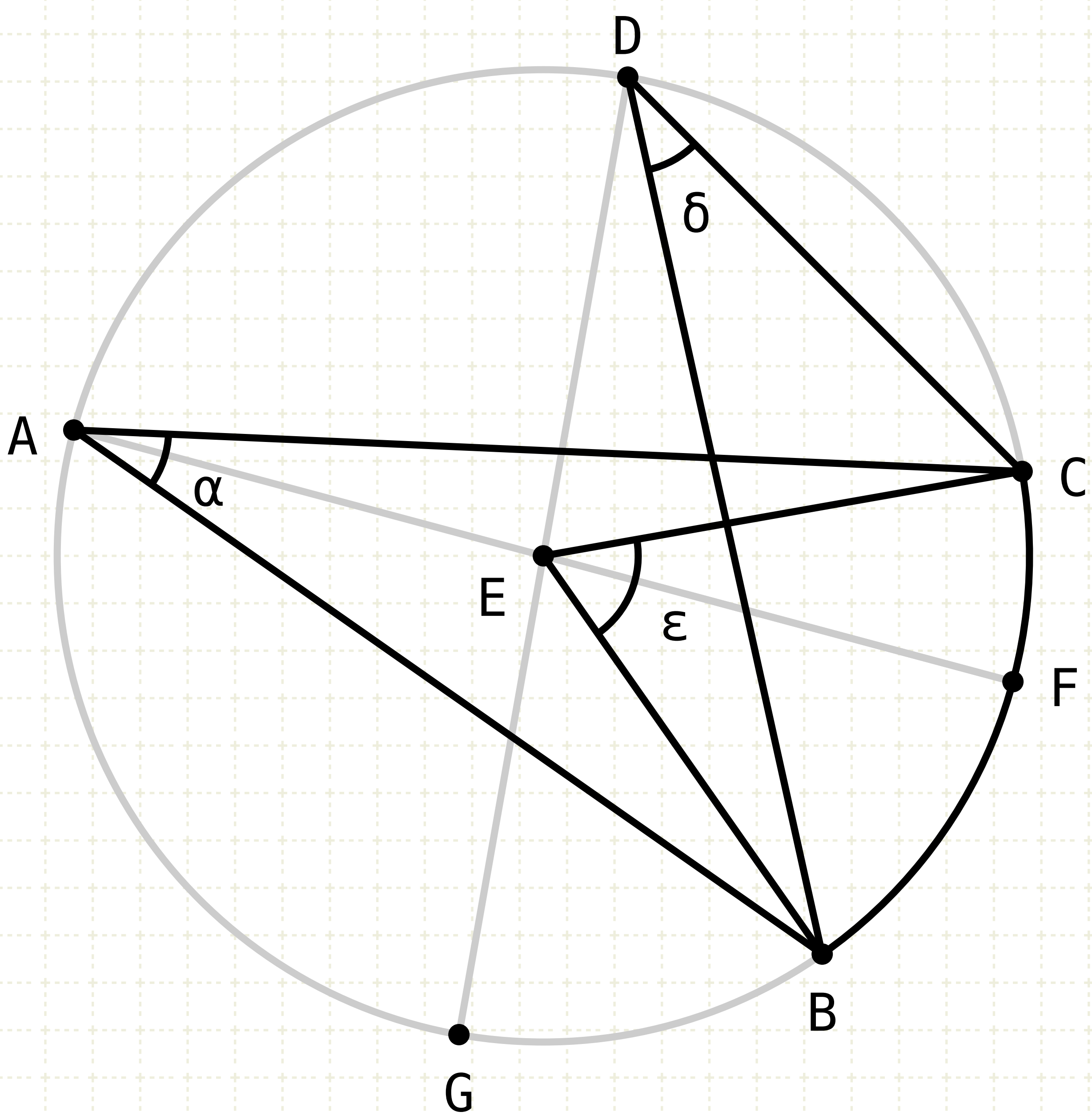

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