

Euclid's Elements

Book III



A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)



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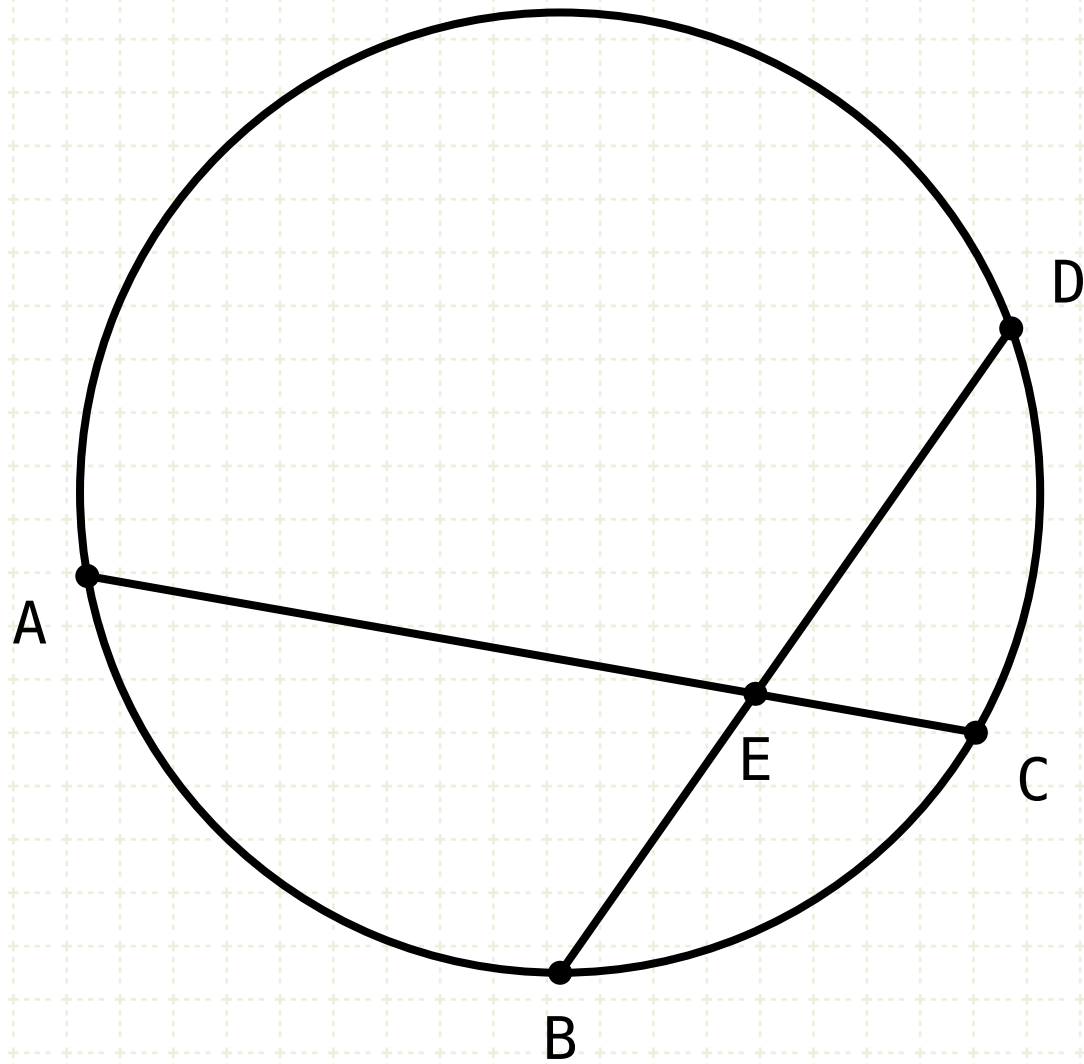
Proposition 4 of Book III

If in a circle two straight lines cut one another which are not through the center, they do not bisect one another.



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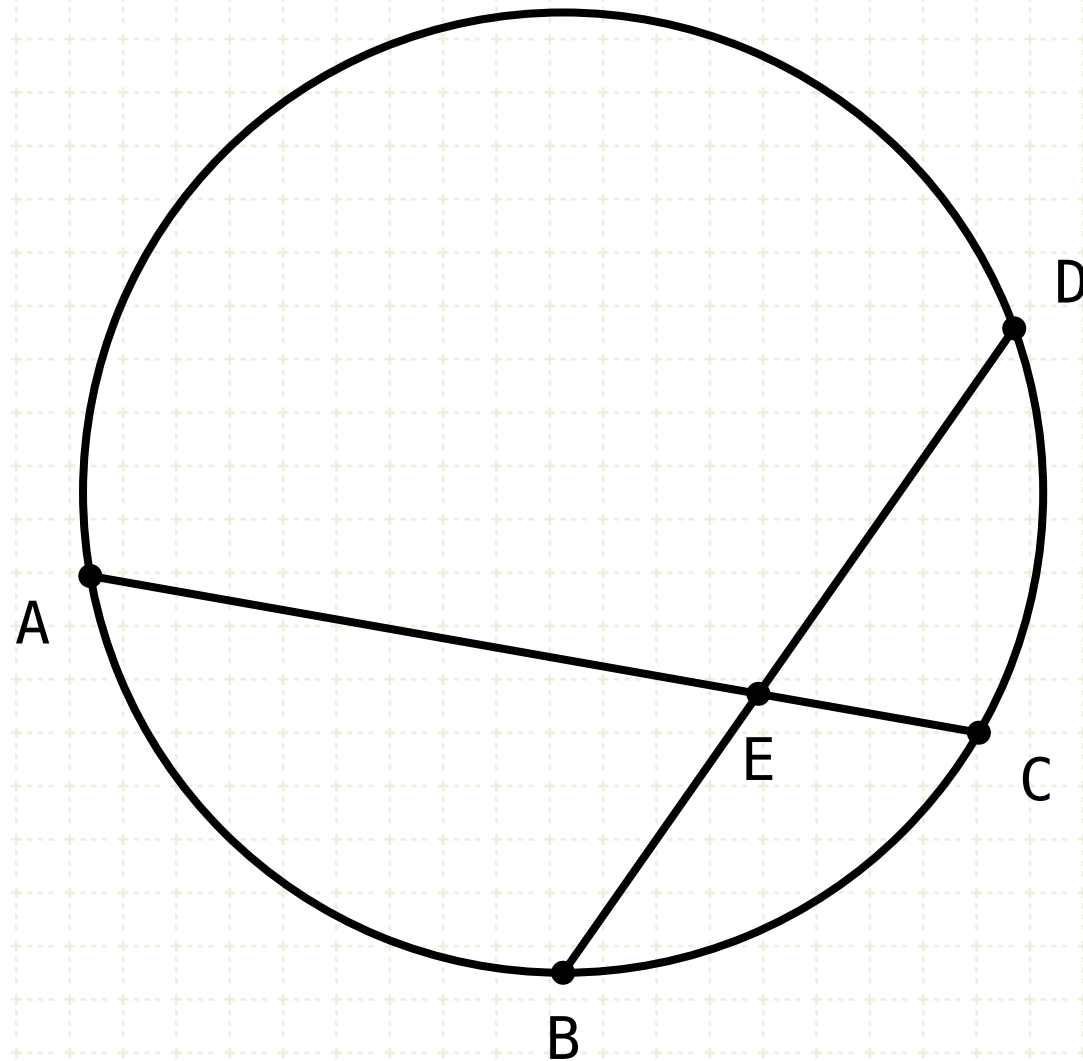
In other words

Let two straight lines AC and BD, not passing through the center of the circle, intersect at point E

- AC and BD do not bisect each other

Proposition 4 of Book III

If in a circle two straight lines cut one another which are not through the center, they do not bisect one another.



Assume ...

$$AE = EC, BE = ED$$

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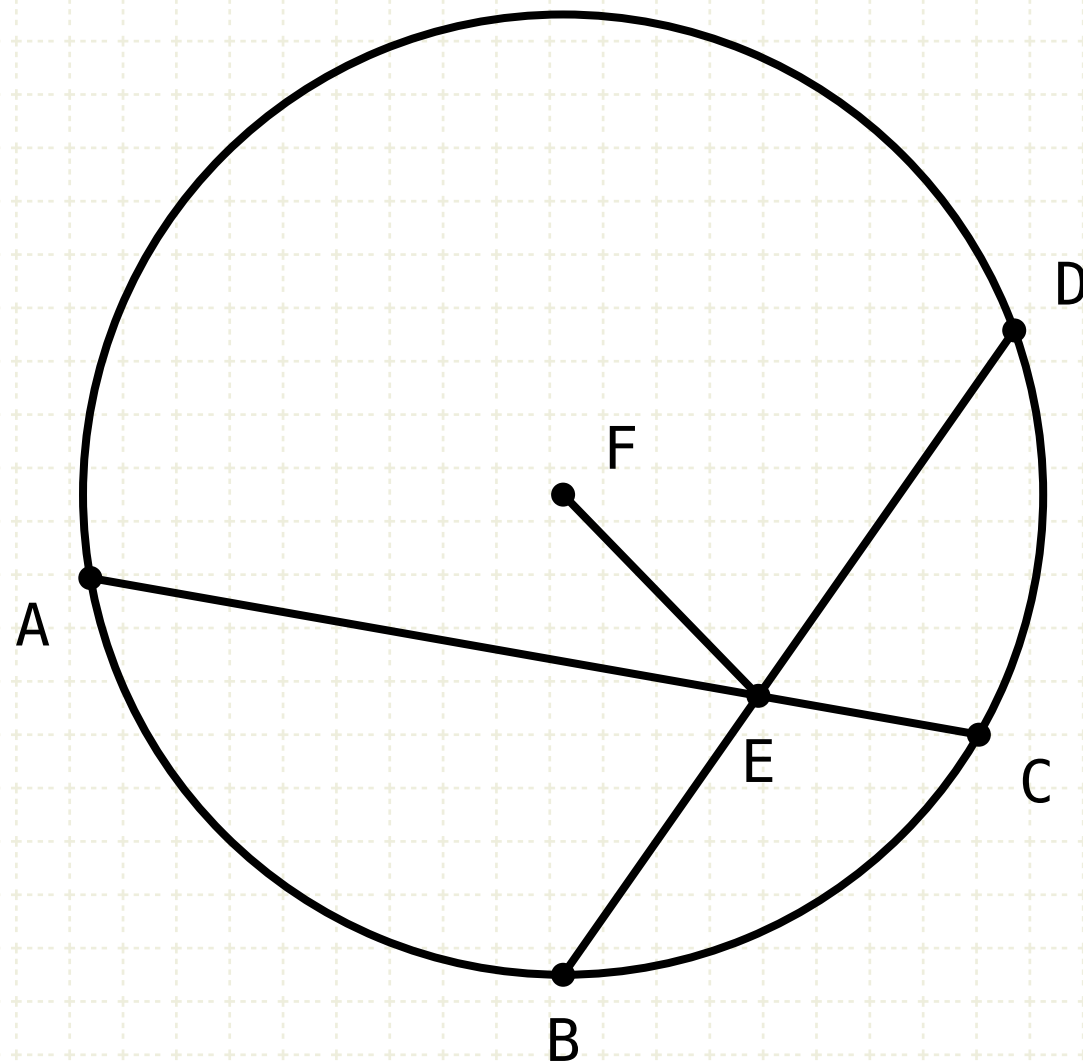
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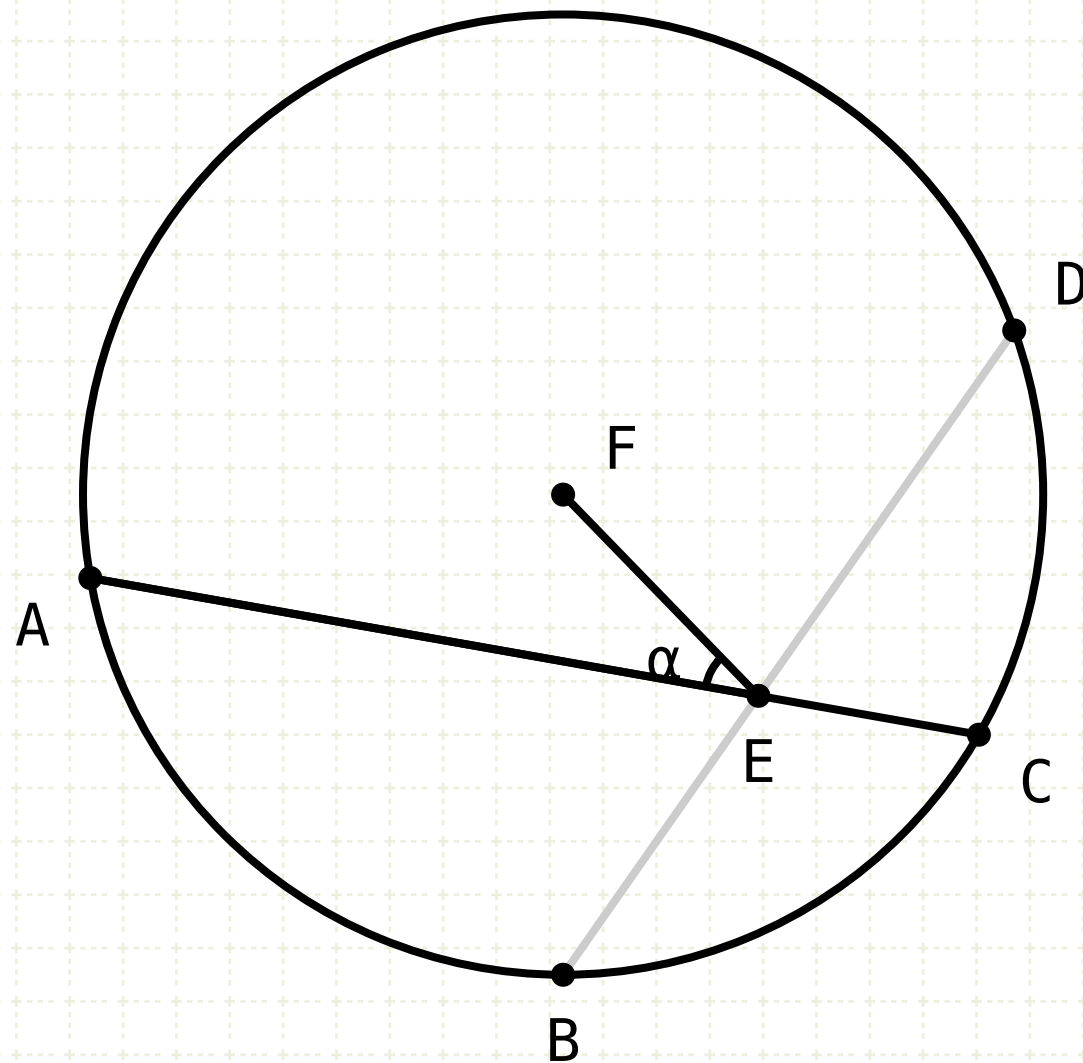
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Find (III·1) the center of the circle (F) and join FE



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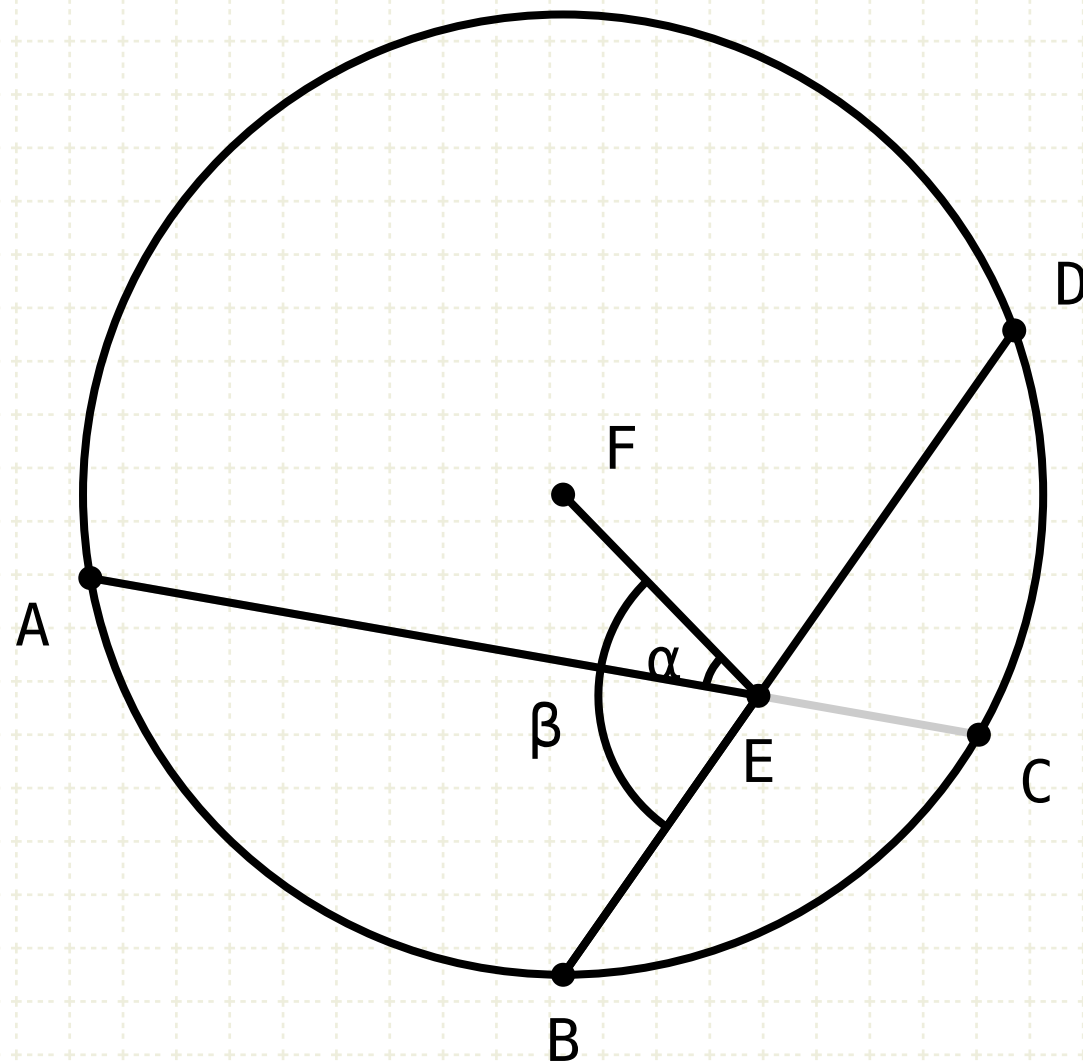
Find (III·1) the center of the circle (F) and join FE

Since FE is a line passing through the center of a circle, bisecting AC, a line not through the center, then the angle FEA (α) is right (III·3)



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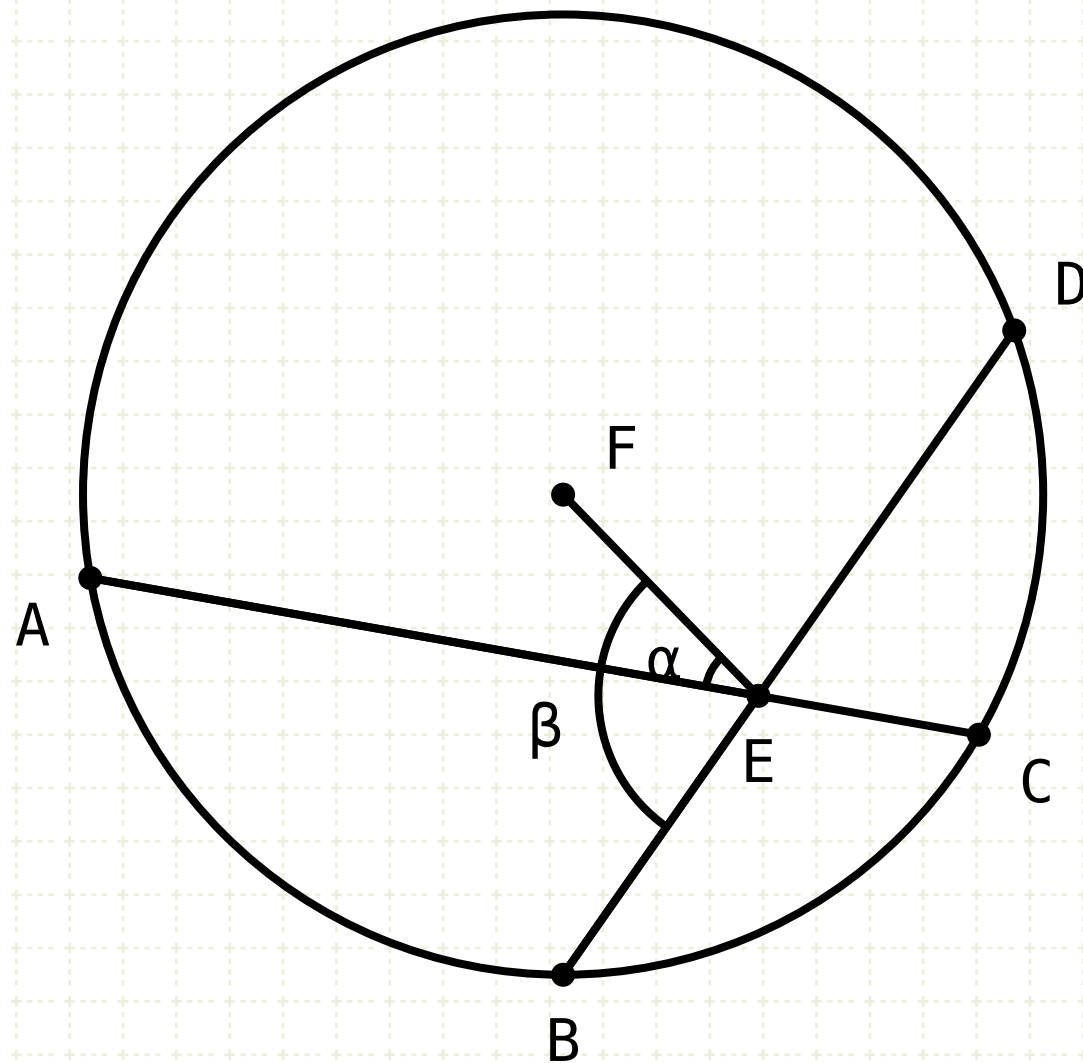
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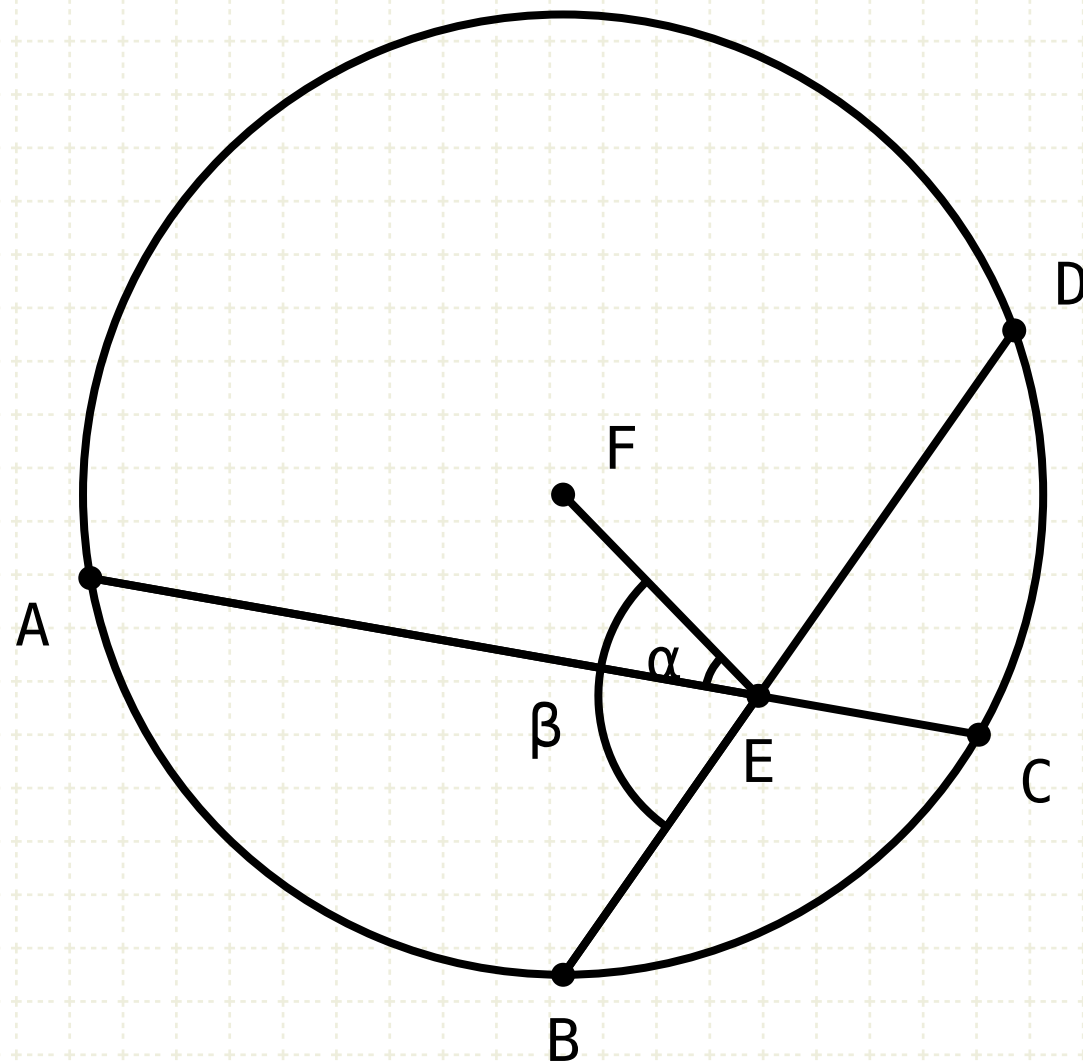
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If α and β are both right angles, they are equal to each other

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Assume ...

$$AE = EC, BE = ED$$

$$\alpha = \beta$$

$$\beta = \alpha$$

$$\alpha = \beta$$

$$\alpha < \beta$$

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Let two straight lines AC and BD, not passing through the center of the circle, intersect at point E

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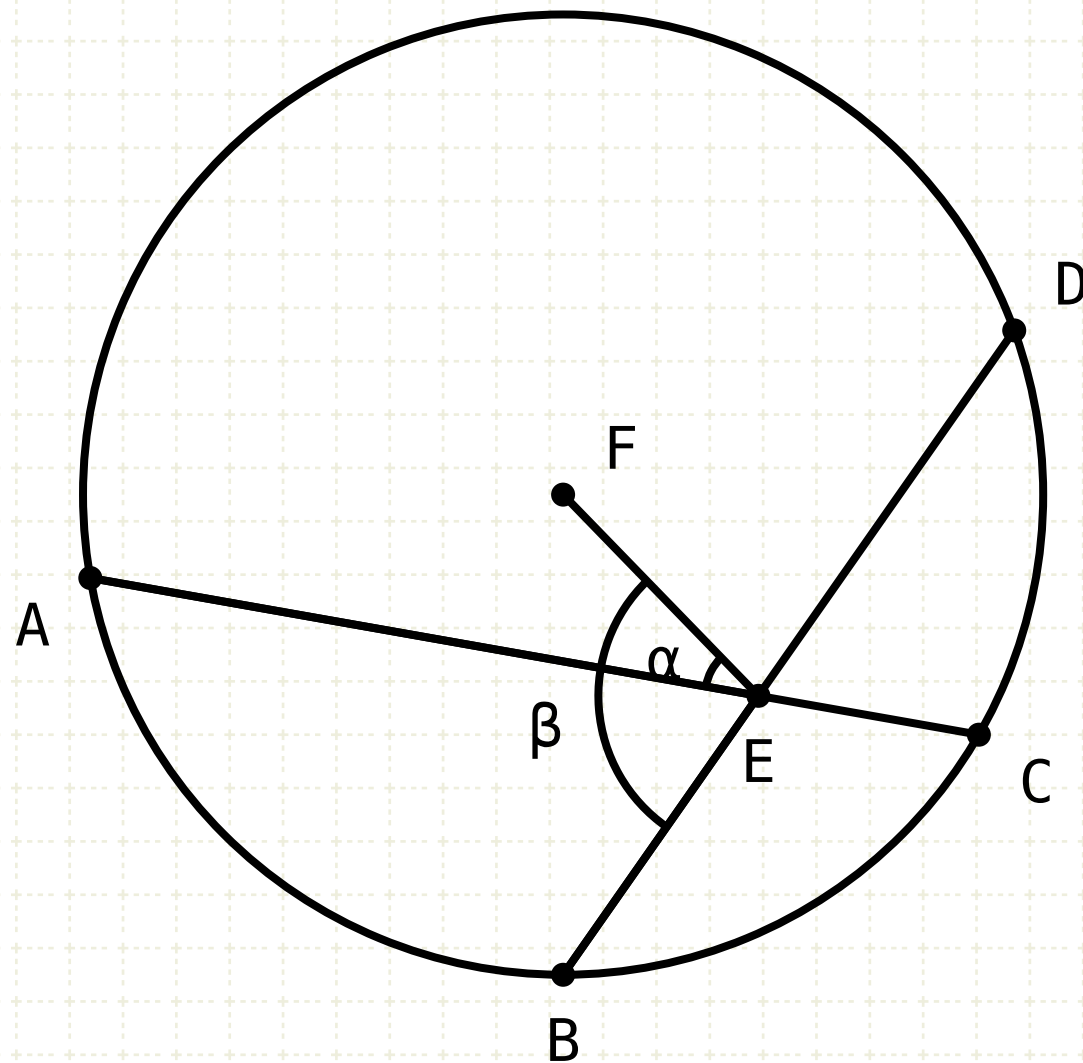
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But β is larger than α , which is inconsistent with the previous statement

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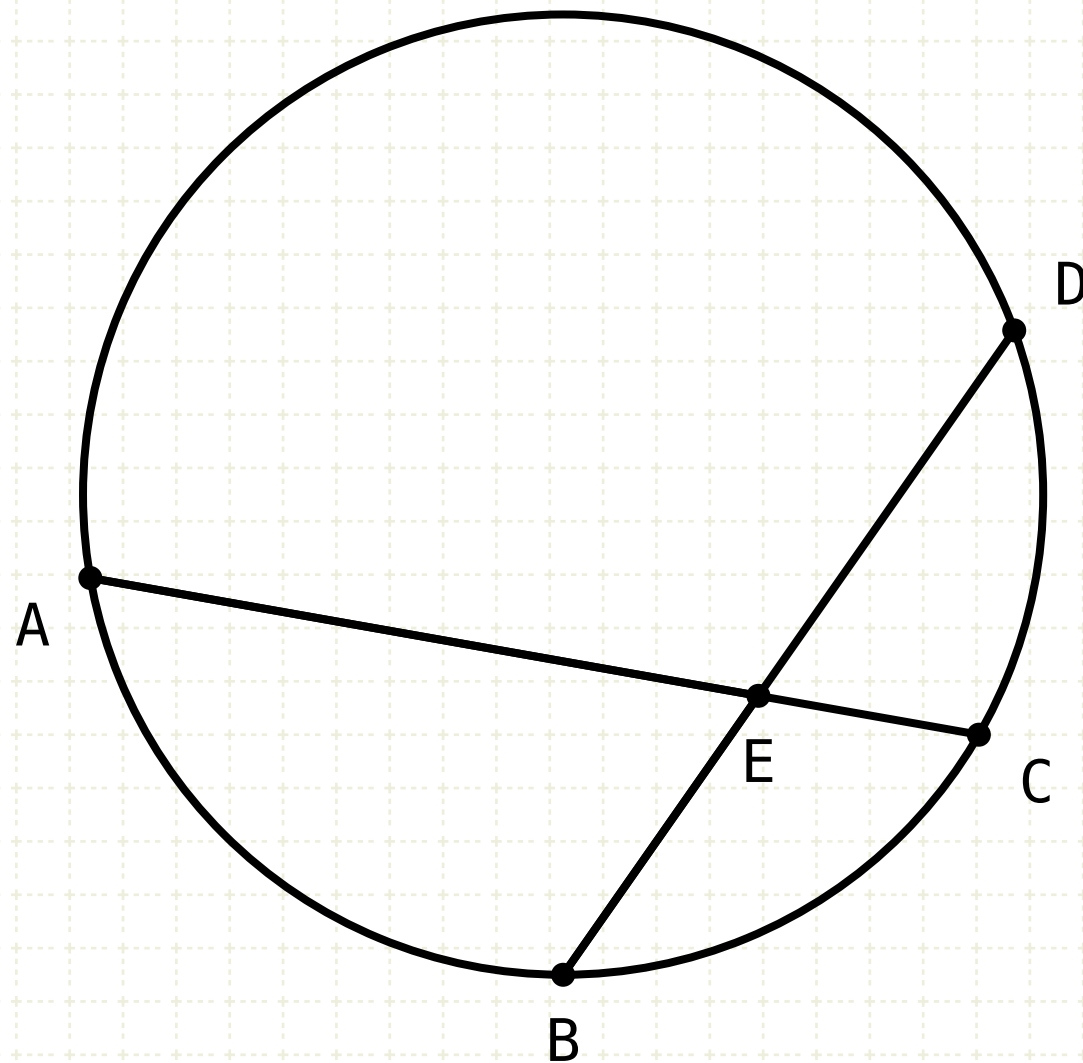
But β is larger than α , which is inconsistent with the previous statement

Hence lines AC and BD cannot both be bisected at point E



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$$AE \neq EC, BE \neq ED$$

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