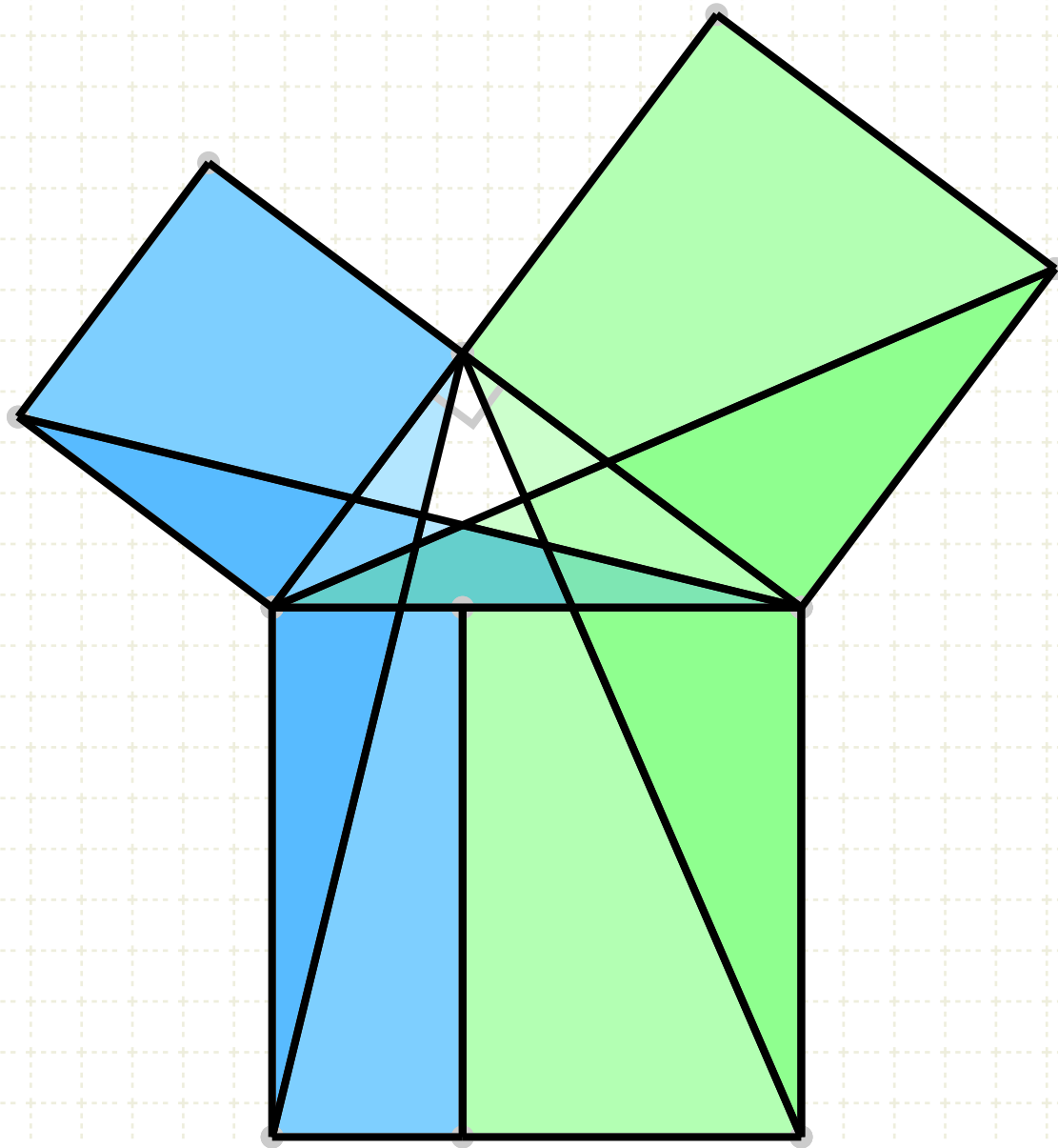


# Euclid's Elements

## Book I

*If Euclid did not kindle your youthful enthusiasm, you  
were not born to be a scientific thinker.*

Albert Einstein



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46	Construct a square
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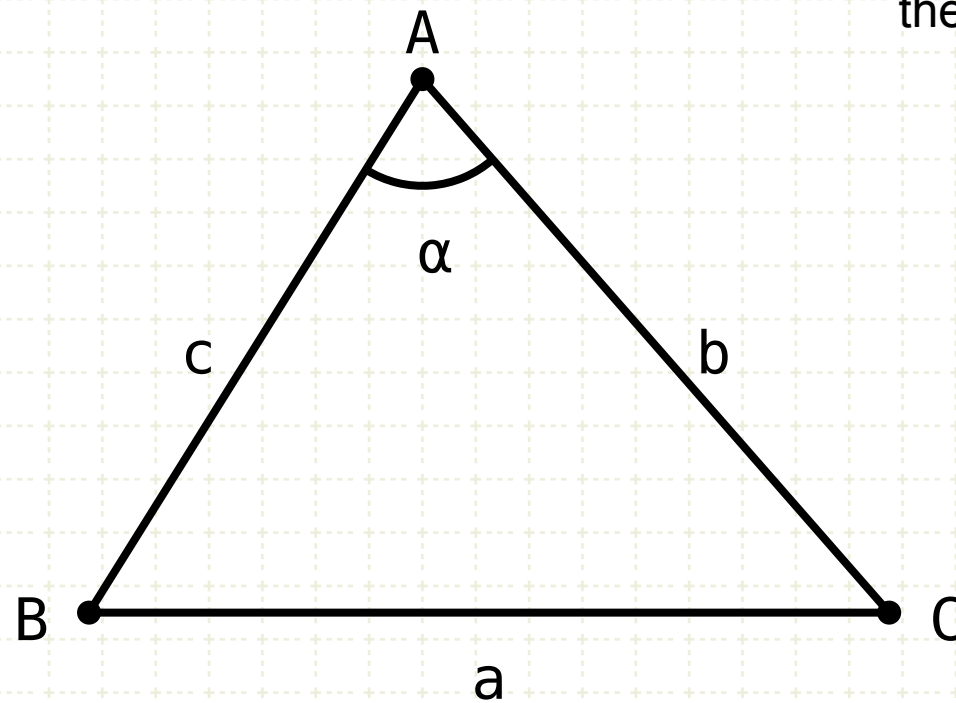
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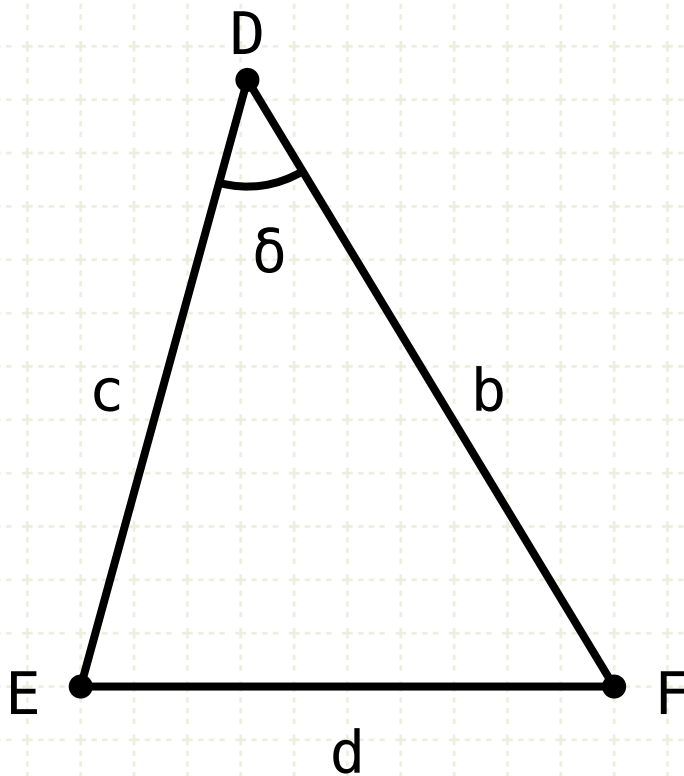
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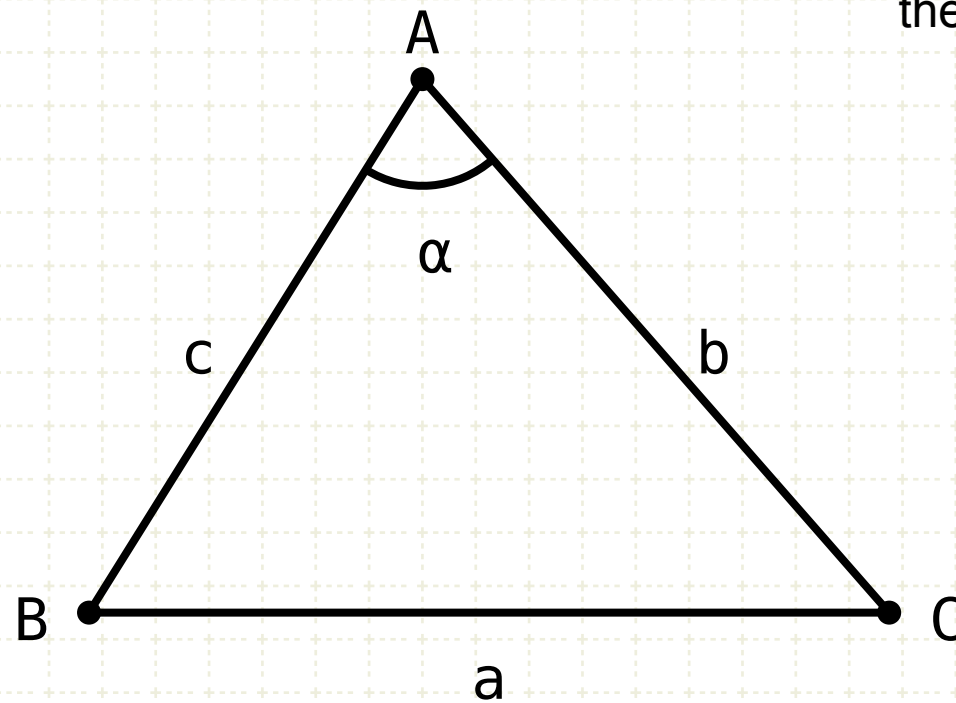
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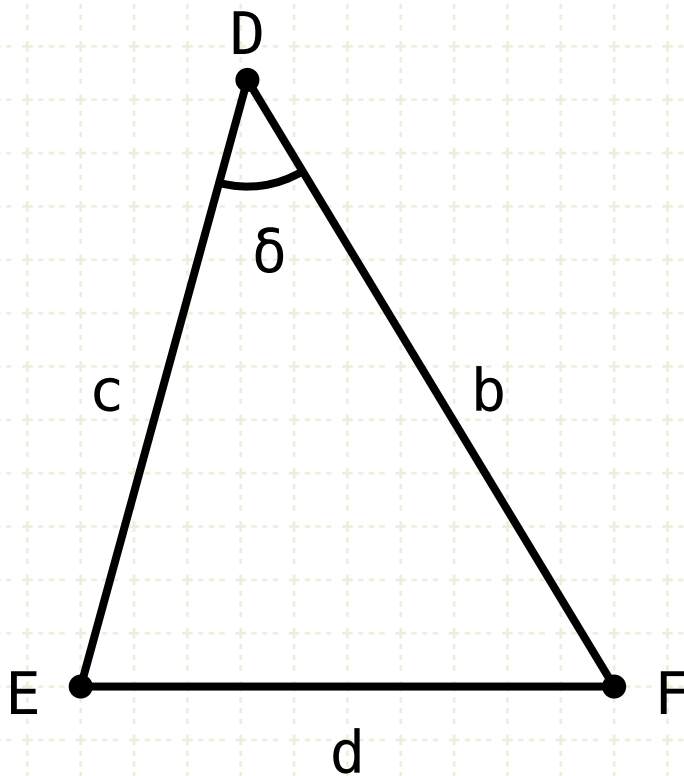
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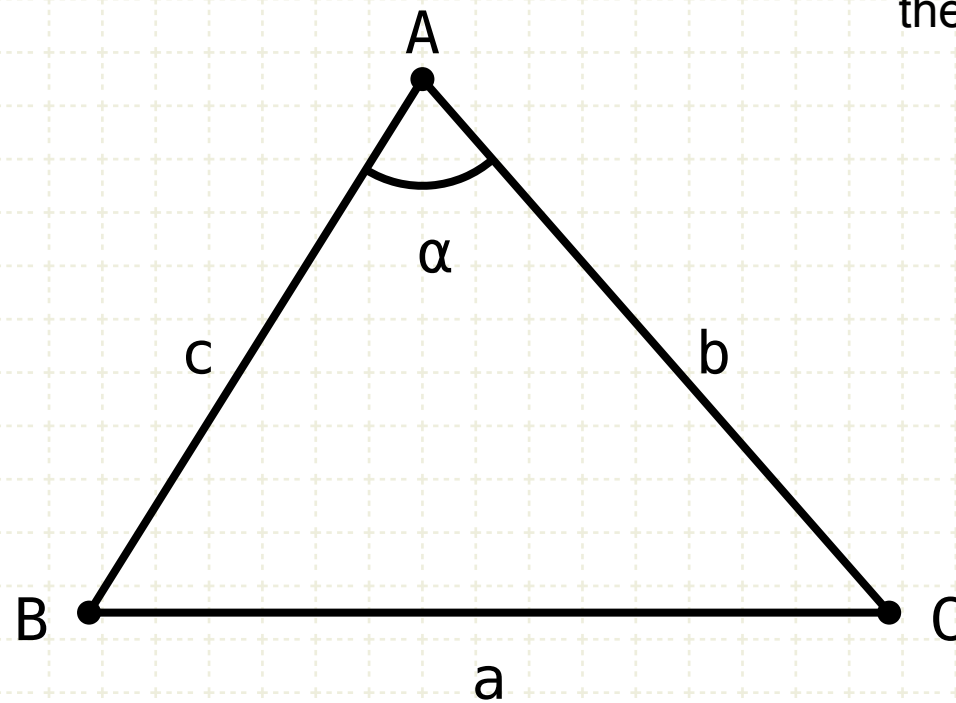
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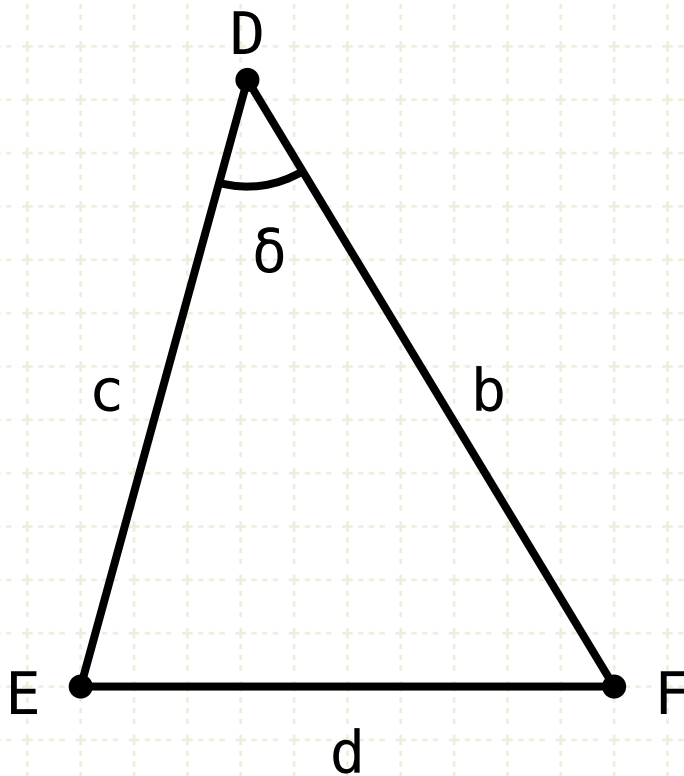
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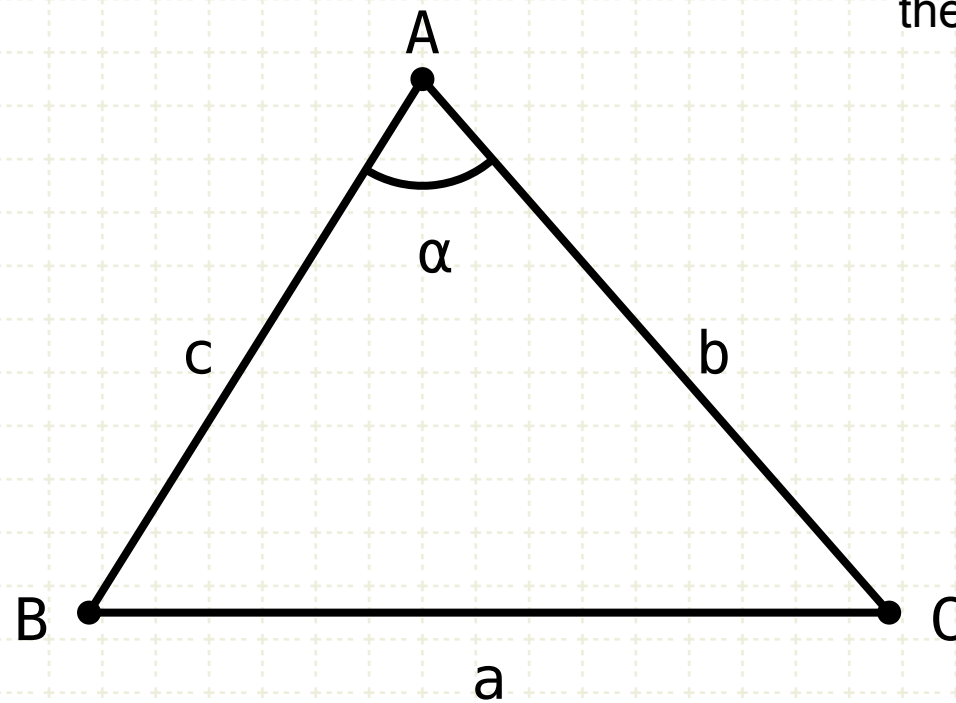
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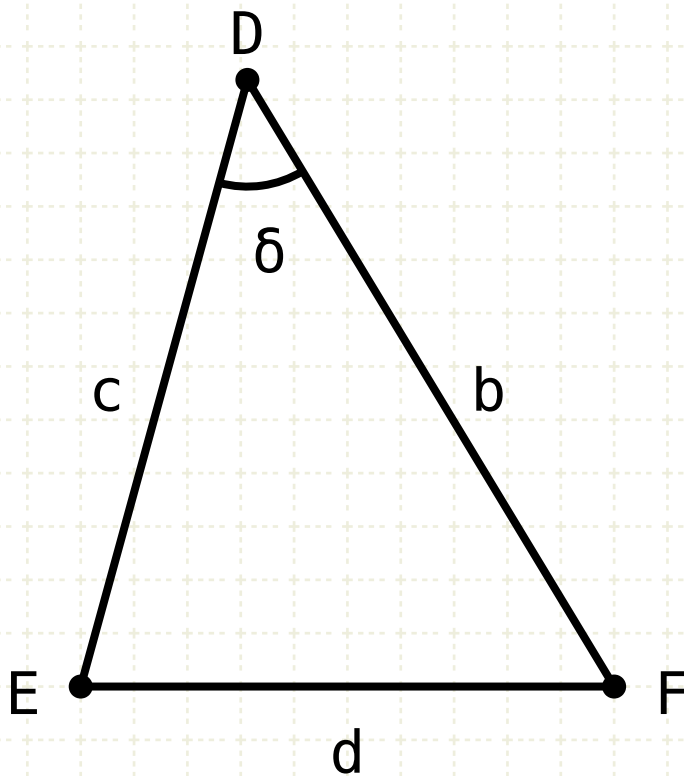
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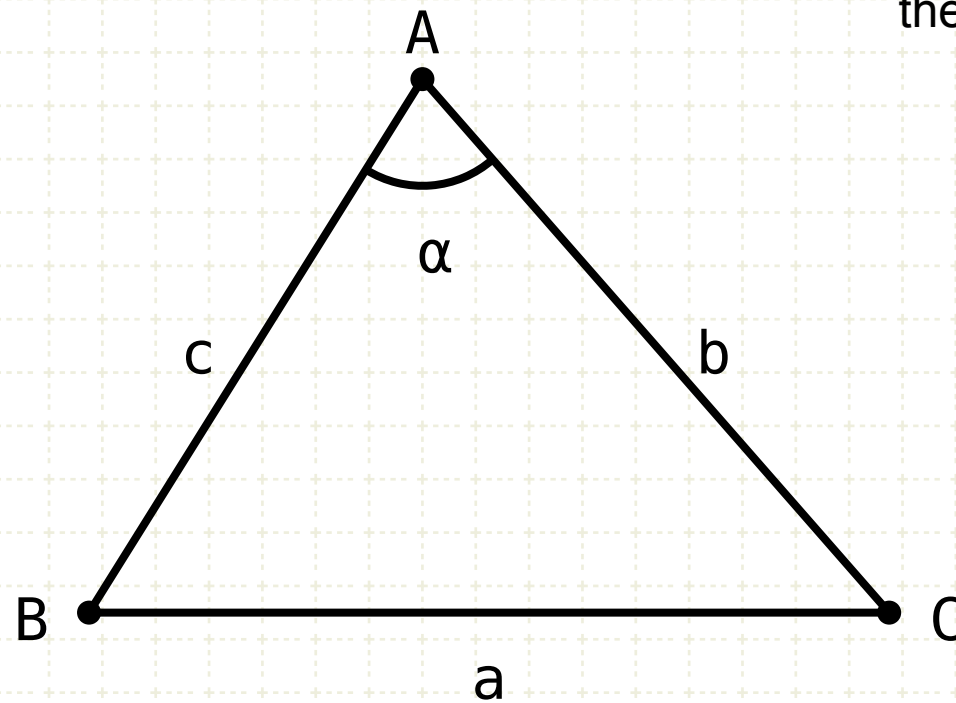
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$$BC > EF, a > d$$

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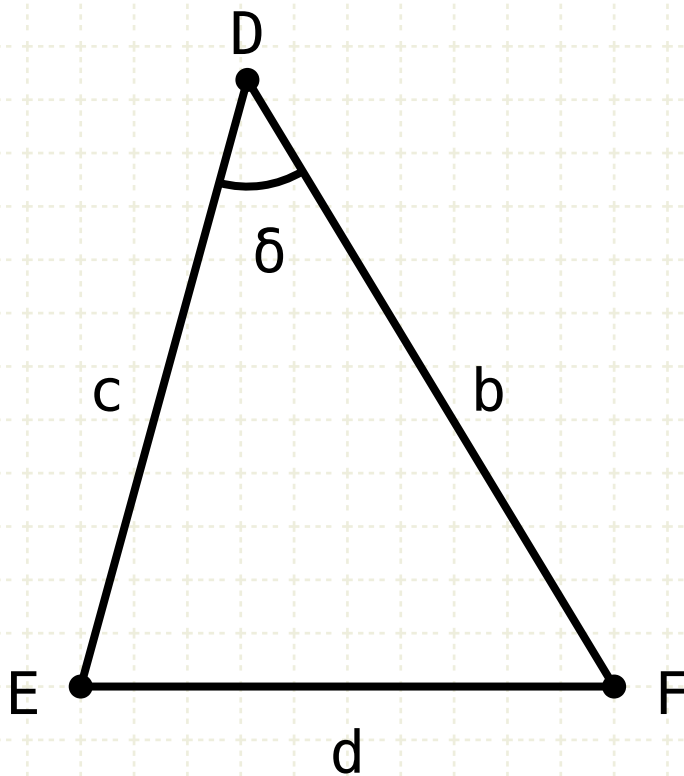
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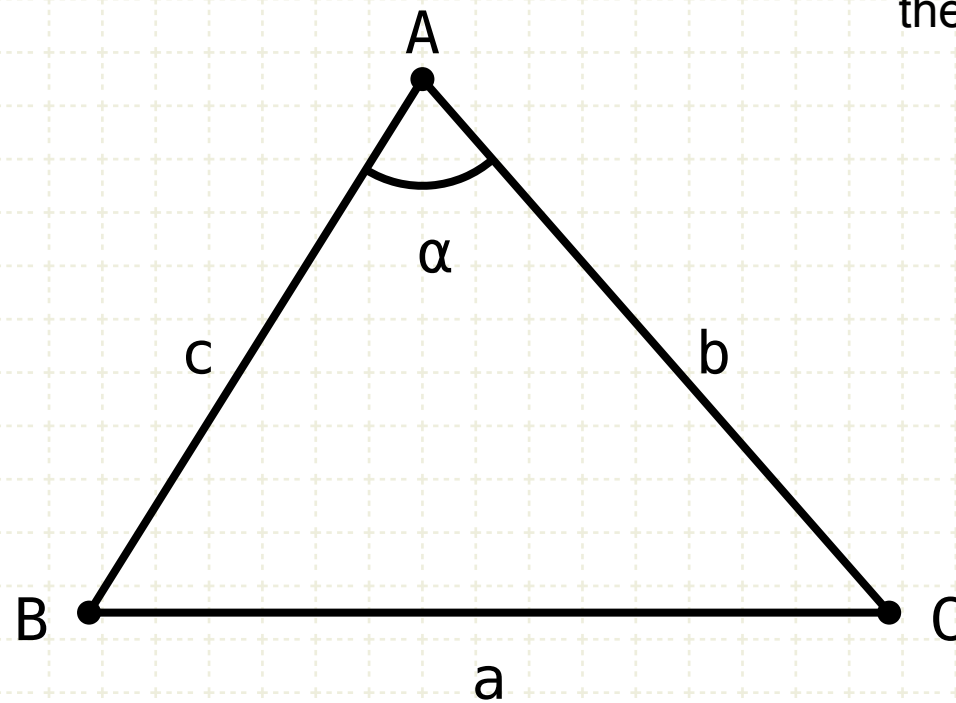
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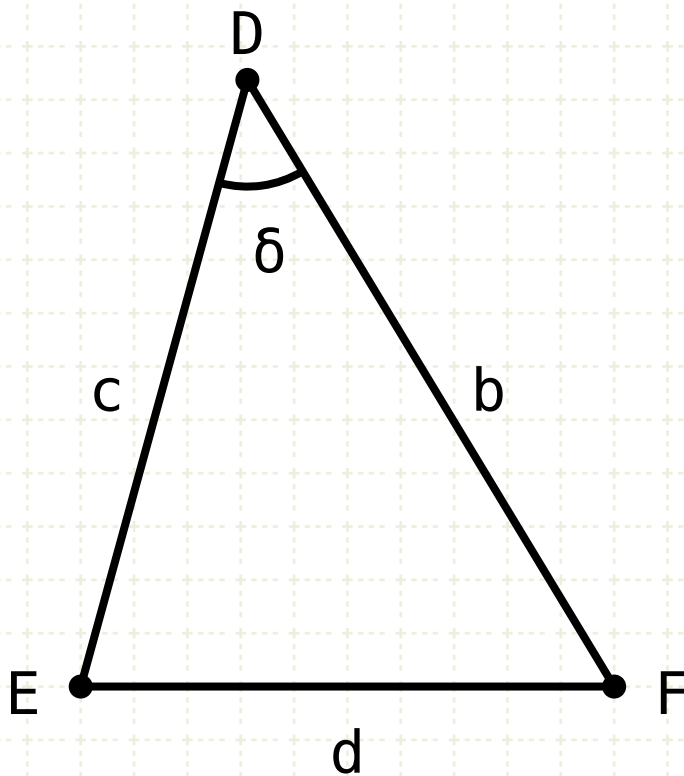
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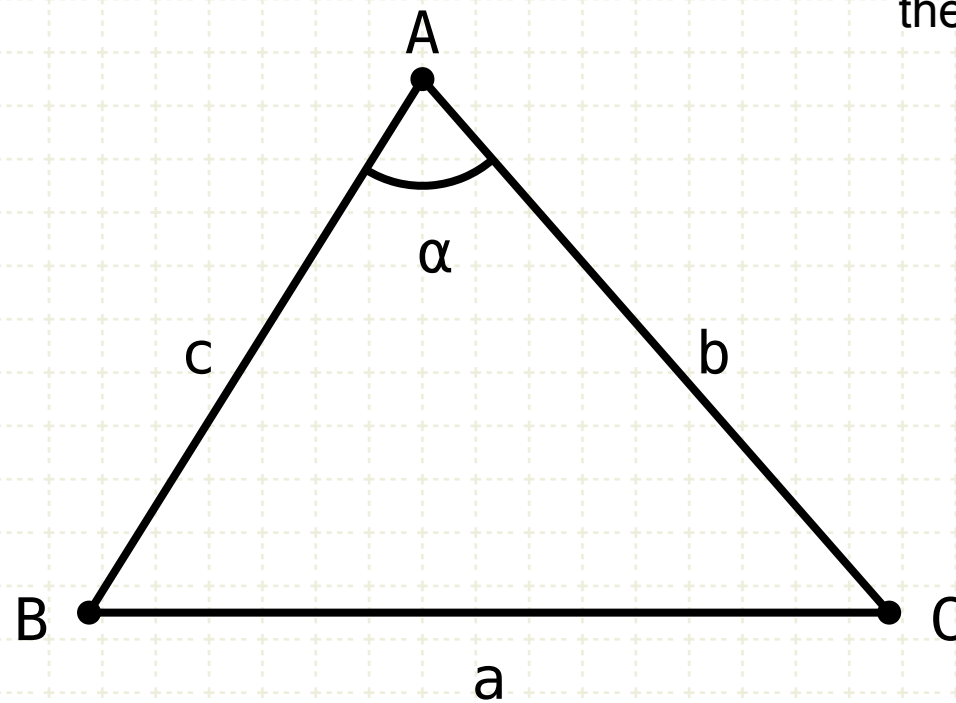
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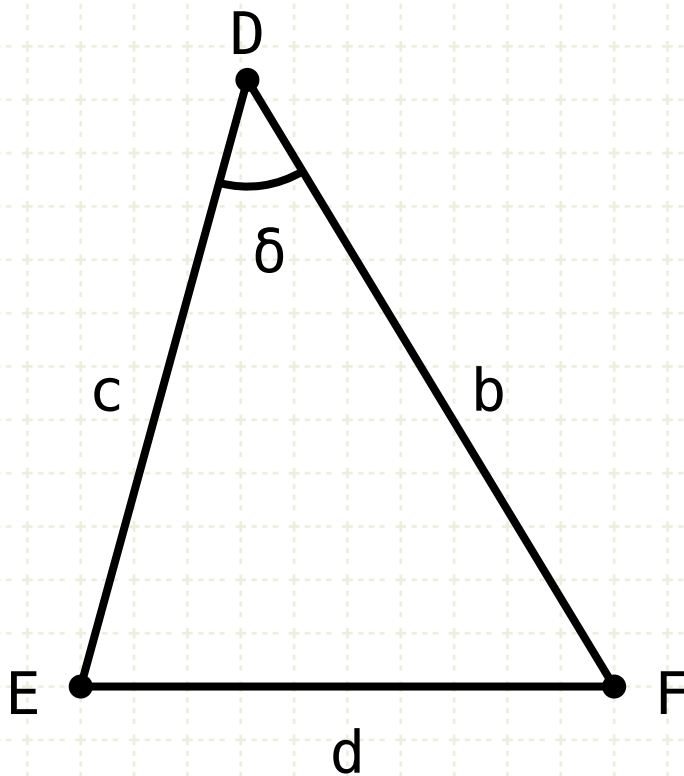
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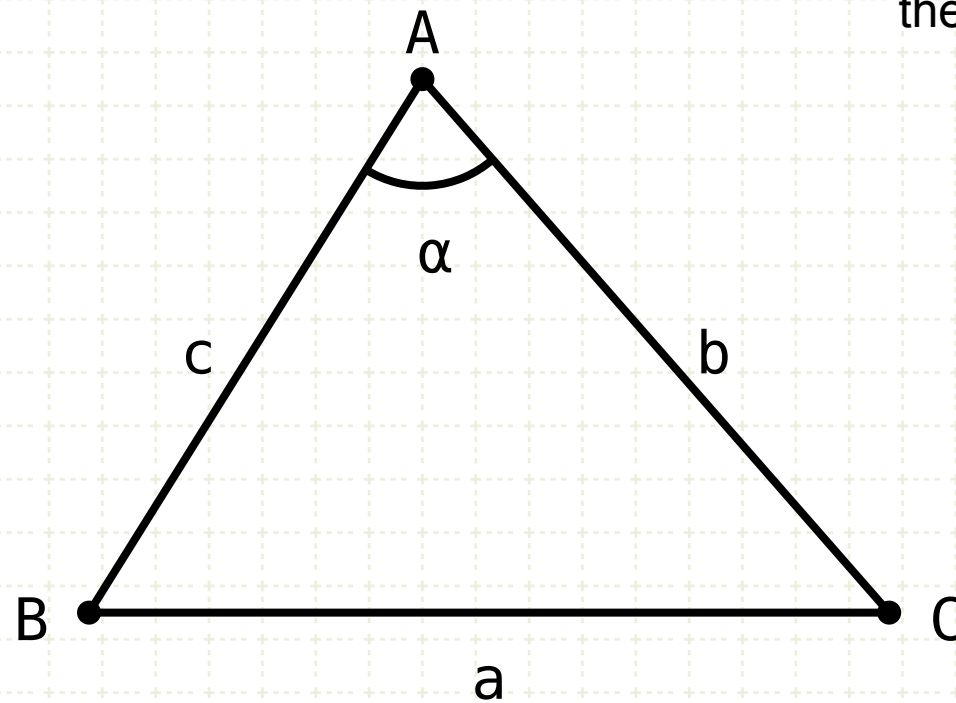
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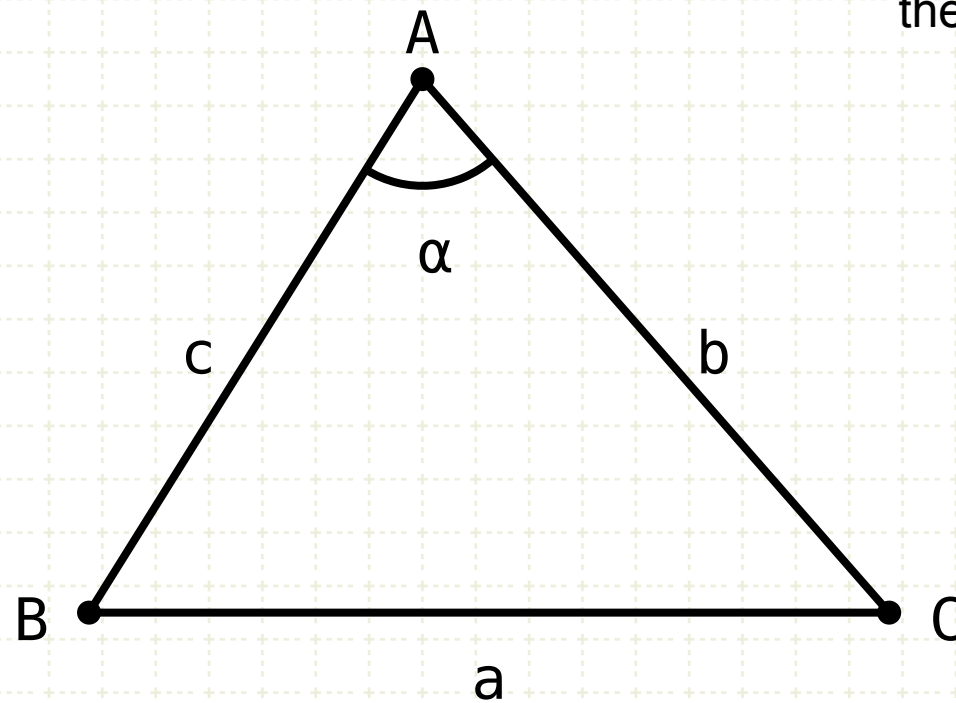
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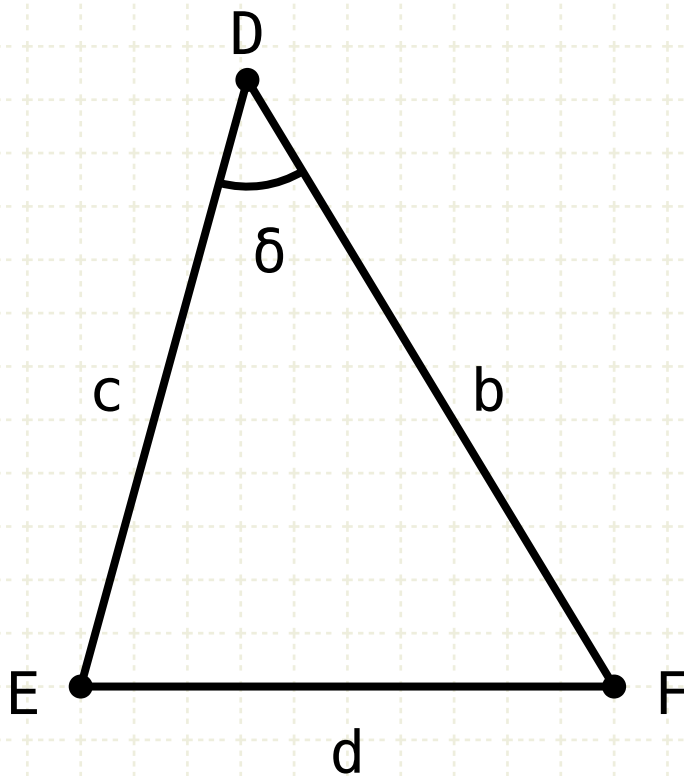
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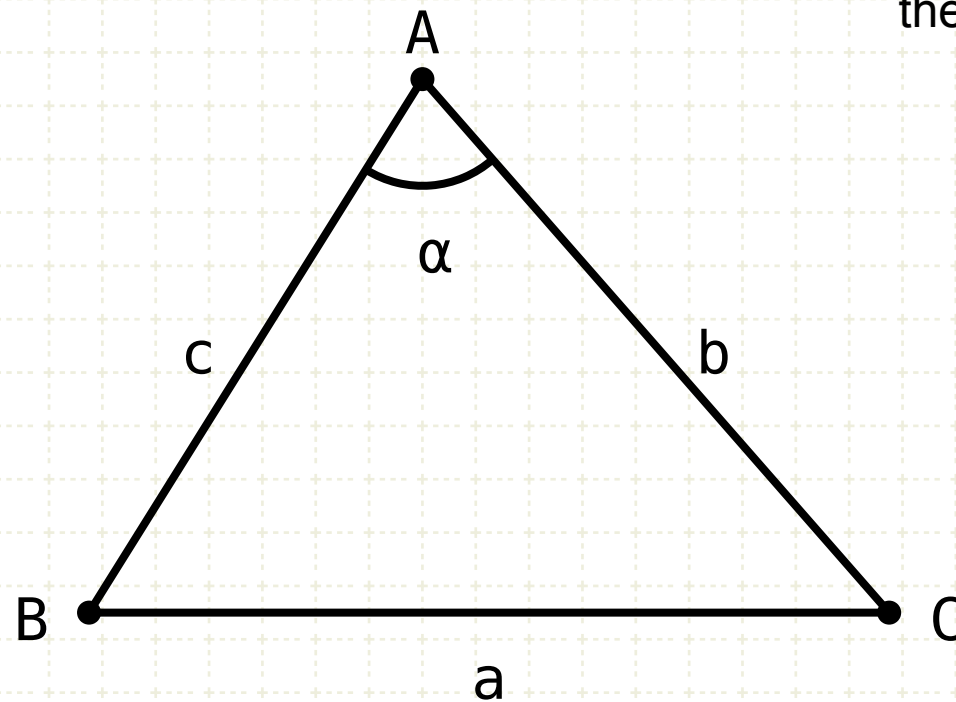
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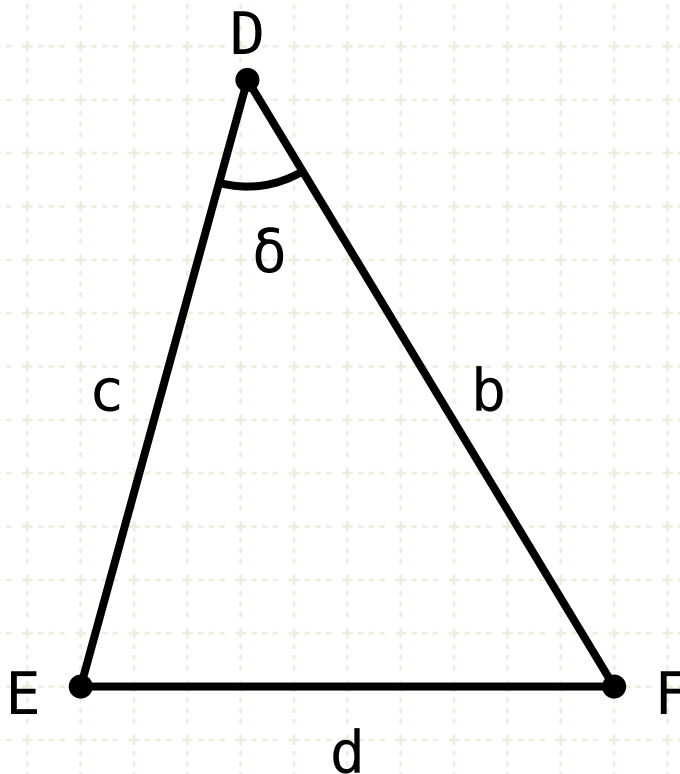
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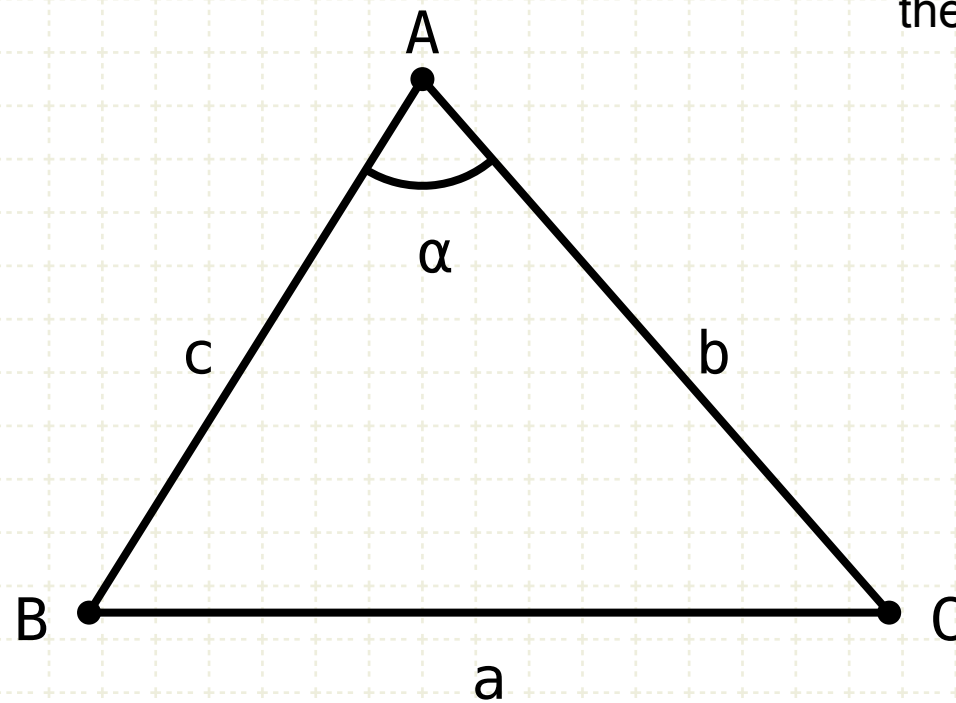
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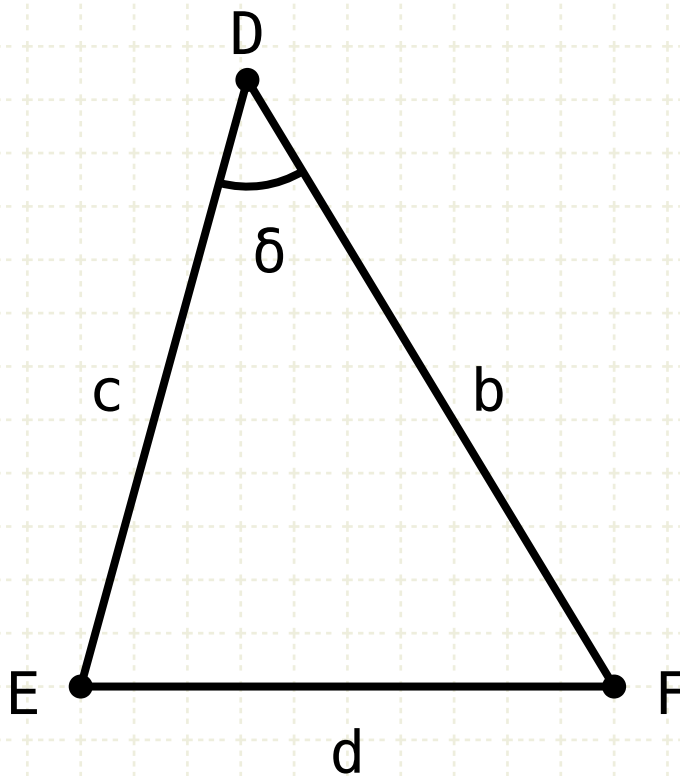
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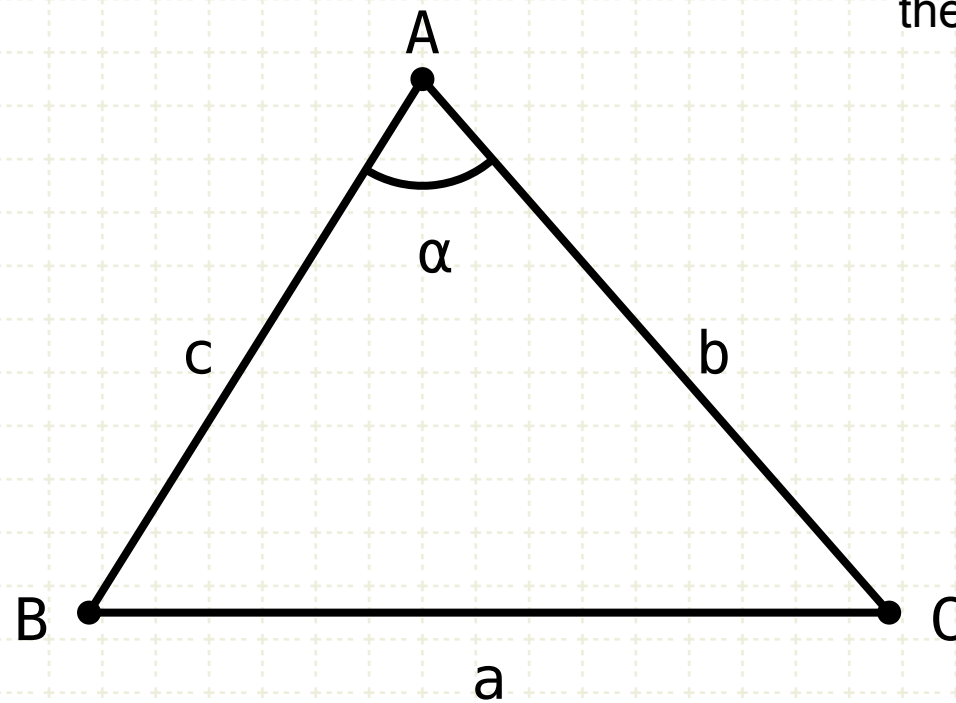
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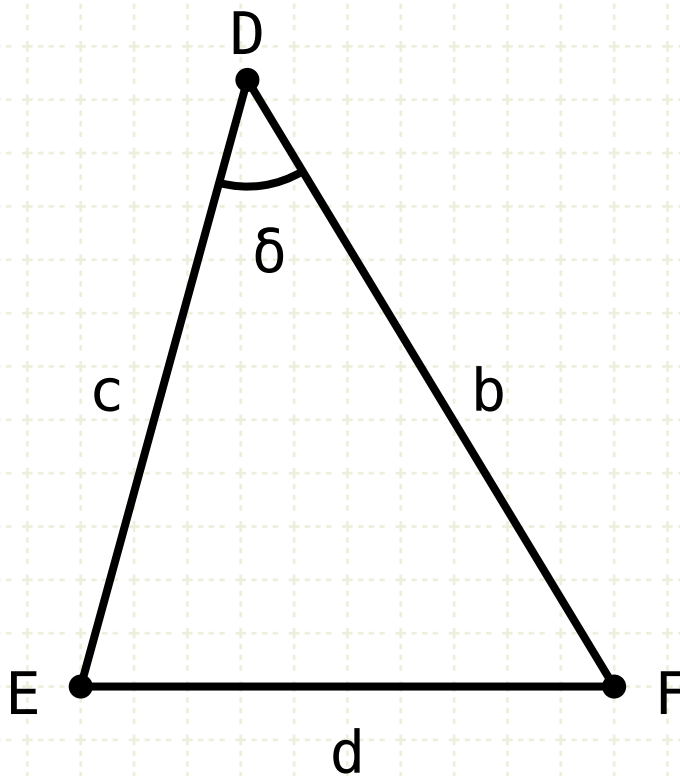


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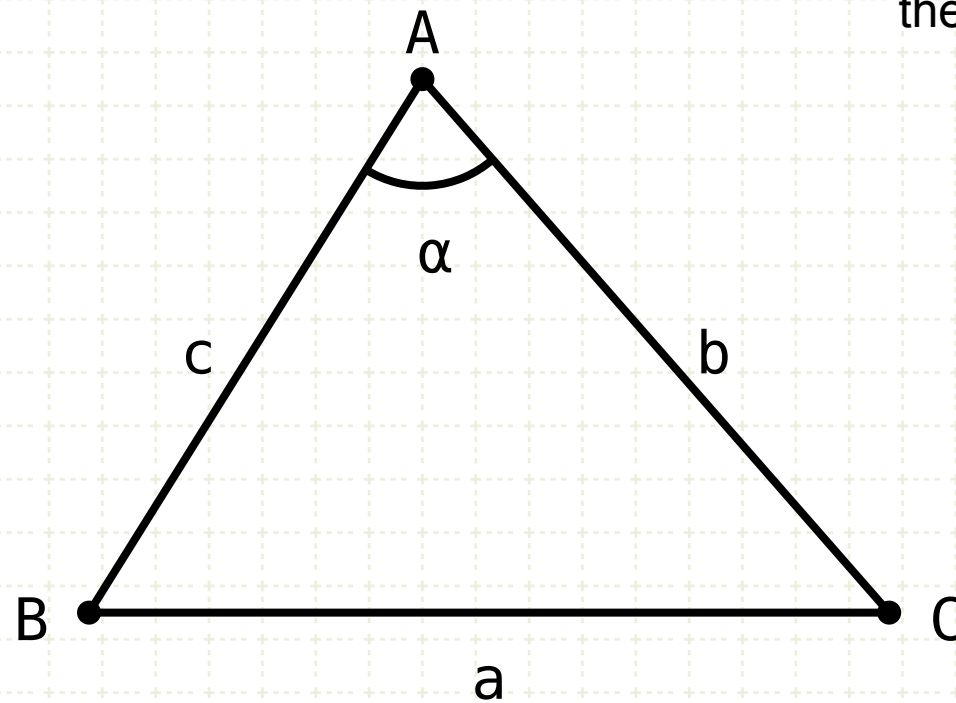
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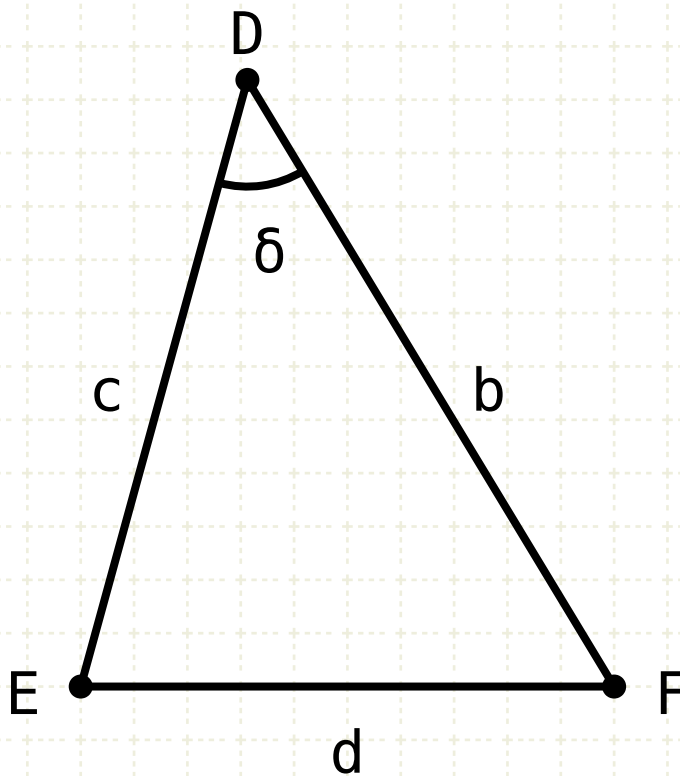


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