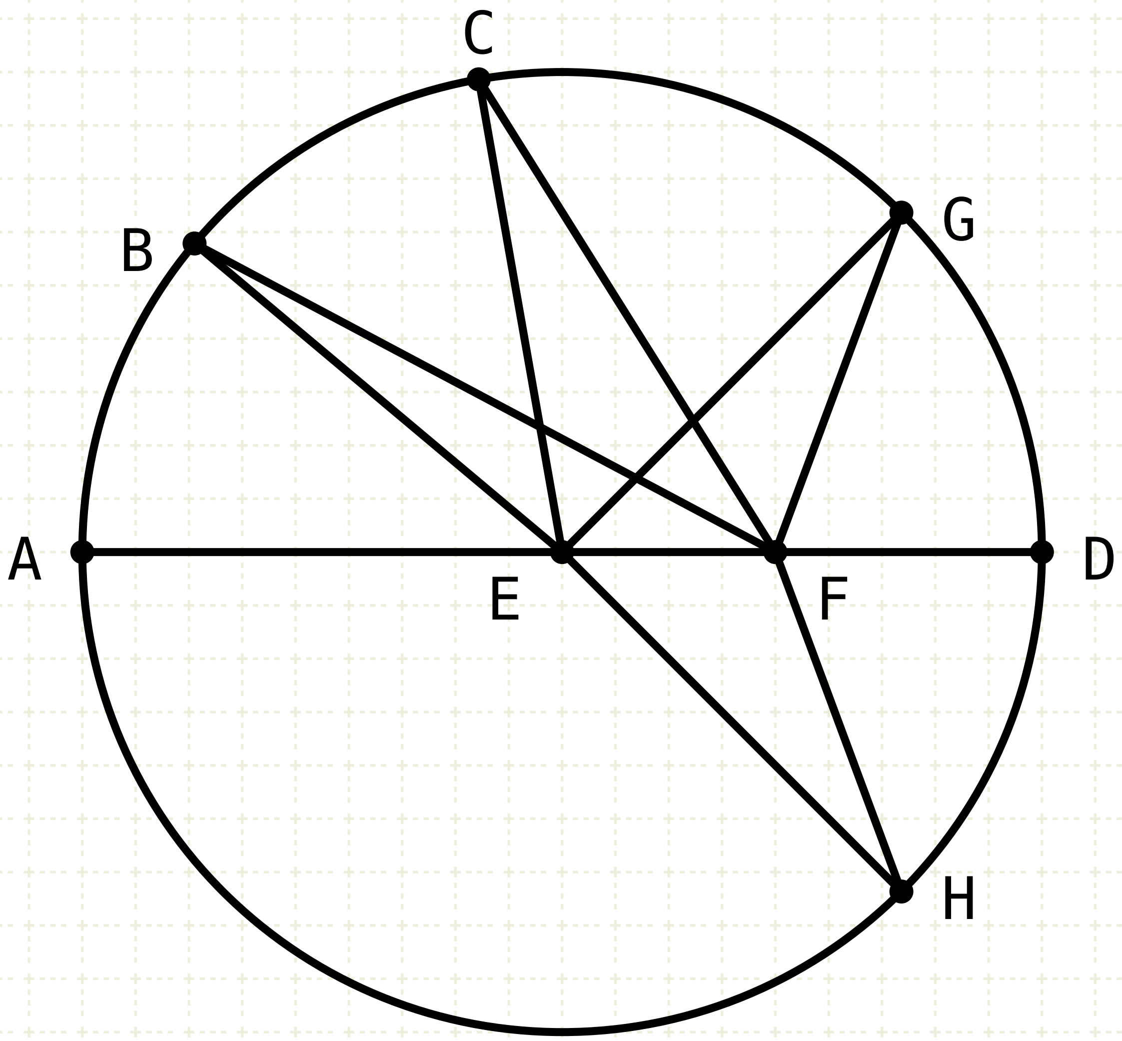
Euclid's Elements

Book III



A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)

Table of Contents, Chapter 3

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Proposition 15 of Book III

Of straight lines in a circle the diameter is greatest, and of the rest the nearer to the centre is always greater than the more remote.

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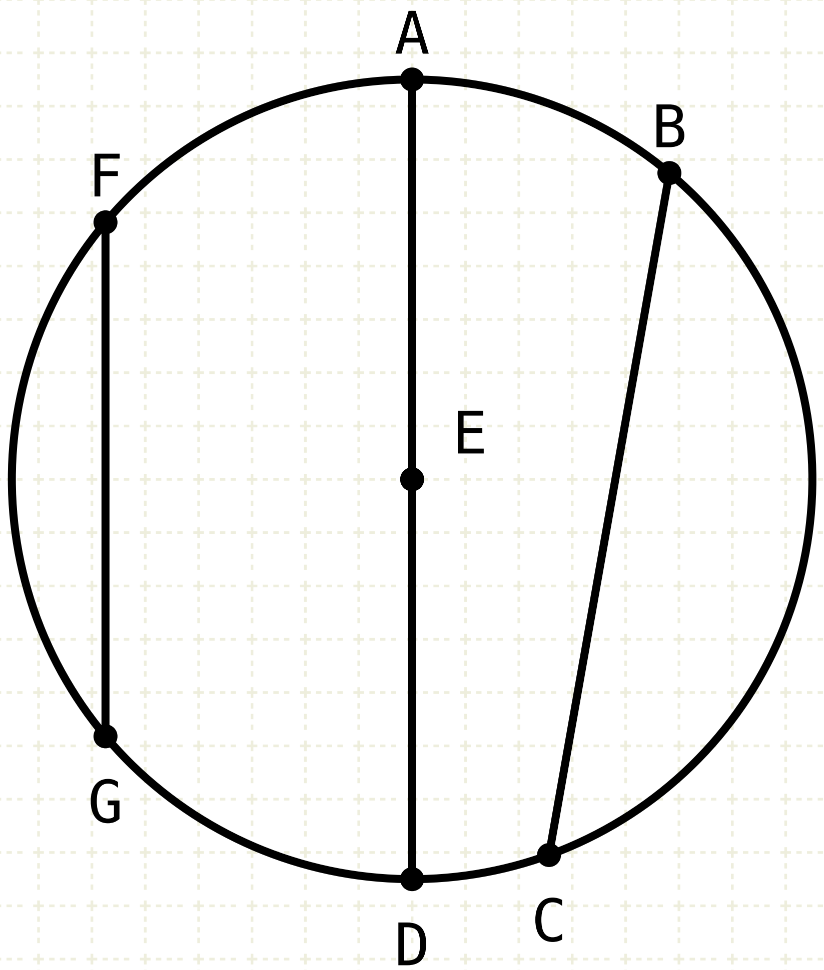


AD > BC > FG

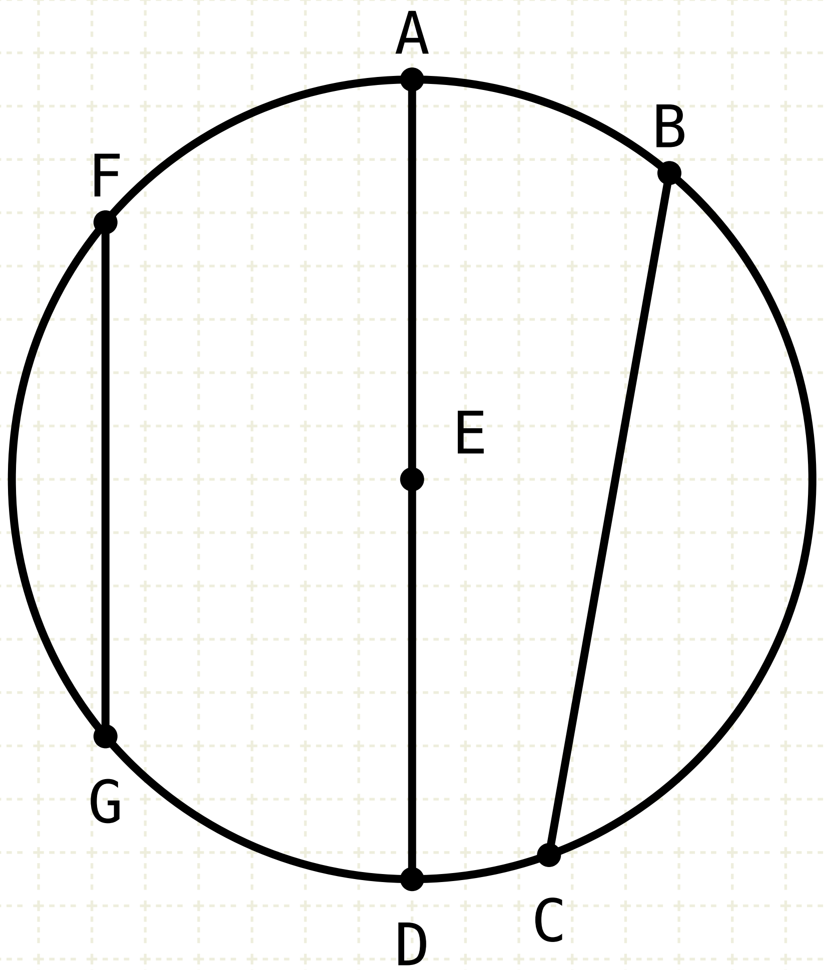
In other words

The line AD is larger than BC, since it passes through the centre of the circle, and line BC will be larger than FG since it is closer to the centre of the circle

Proposition 15 of Book III

Of straight lines in a circle the diameter is greatest, and of the rest the nearer to the centre is always greater than the more remote.

Proof



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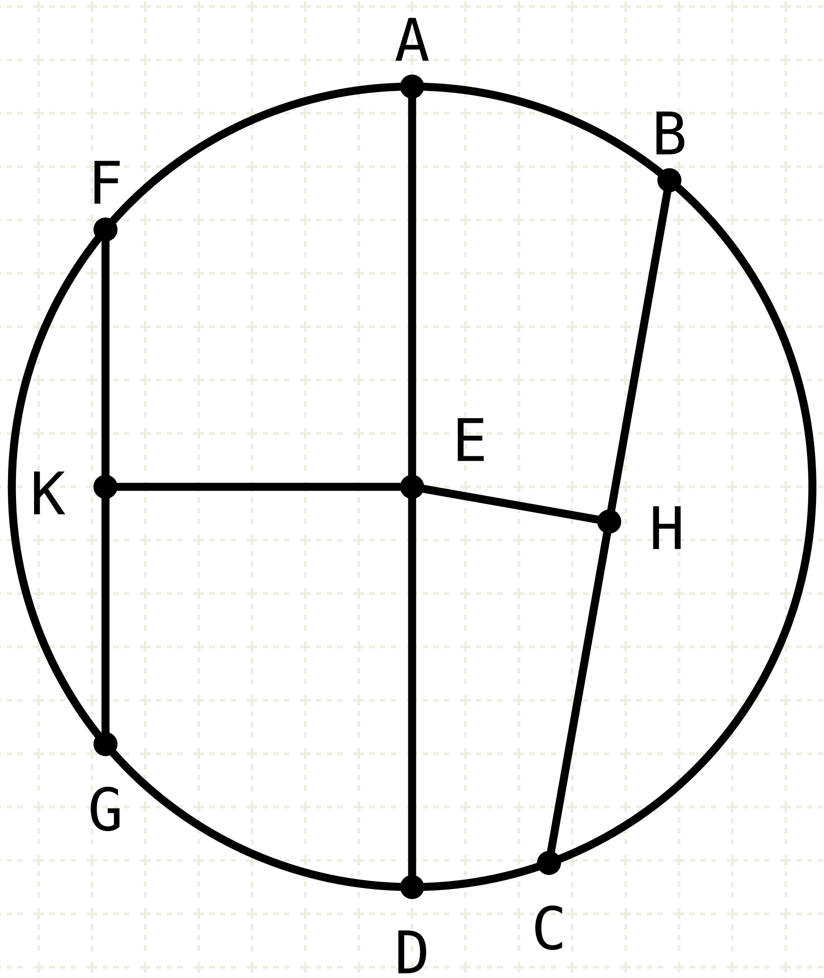

Proof

Draw lines from the centre of the circle perpendicular to the lines BC and FG

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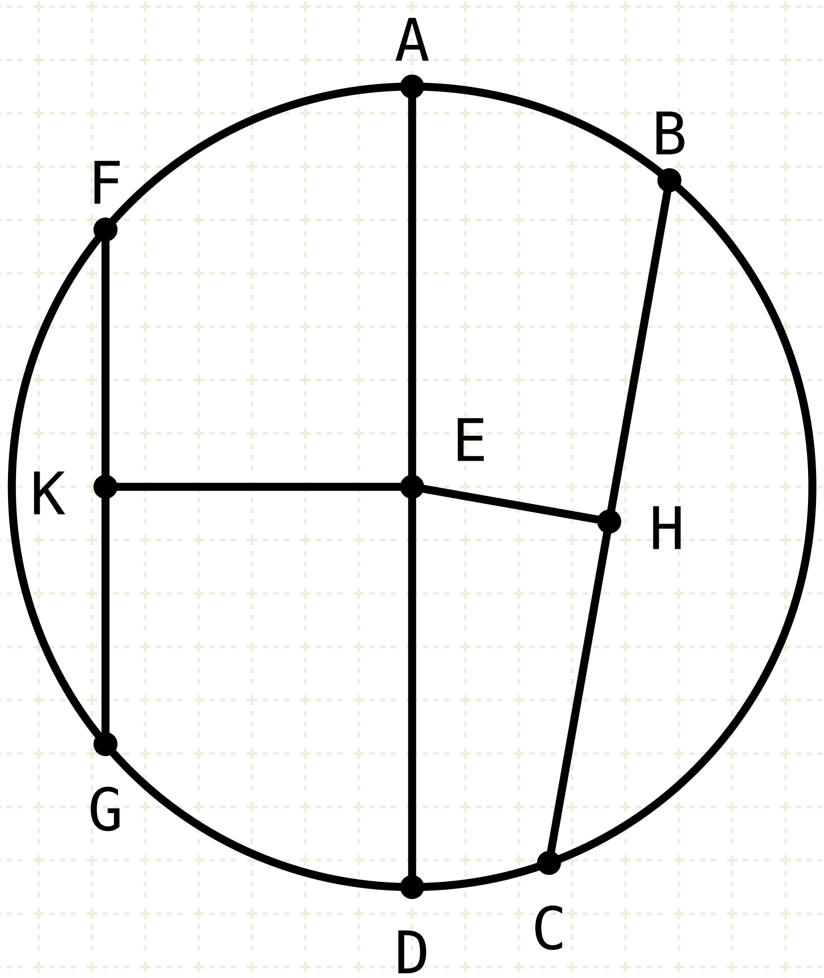

EH < EK

Proof

Draw lines from the centre of the circle perpendicular to the lines BC and FG

Since BC is nearer to centre E than line FG, EH is less than EK (by definition)

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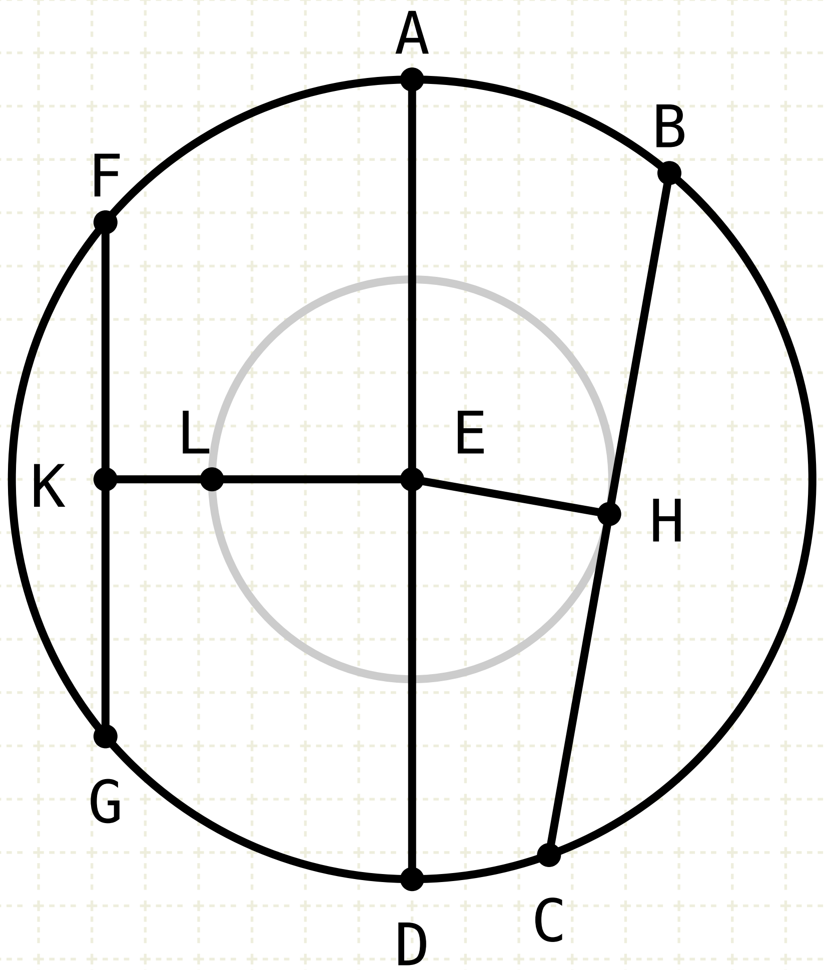

$EH < EK$

$EL = EH$

Proof

Draw lines from the centre of the circle perpendicular to the lines BC and FG

Since BC is nearer to centre E than line FG, EH is less than EK (by definition)

Define a point L on line EK, such that LE equals EH

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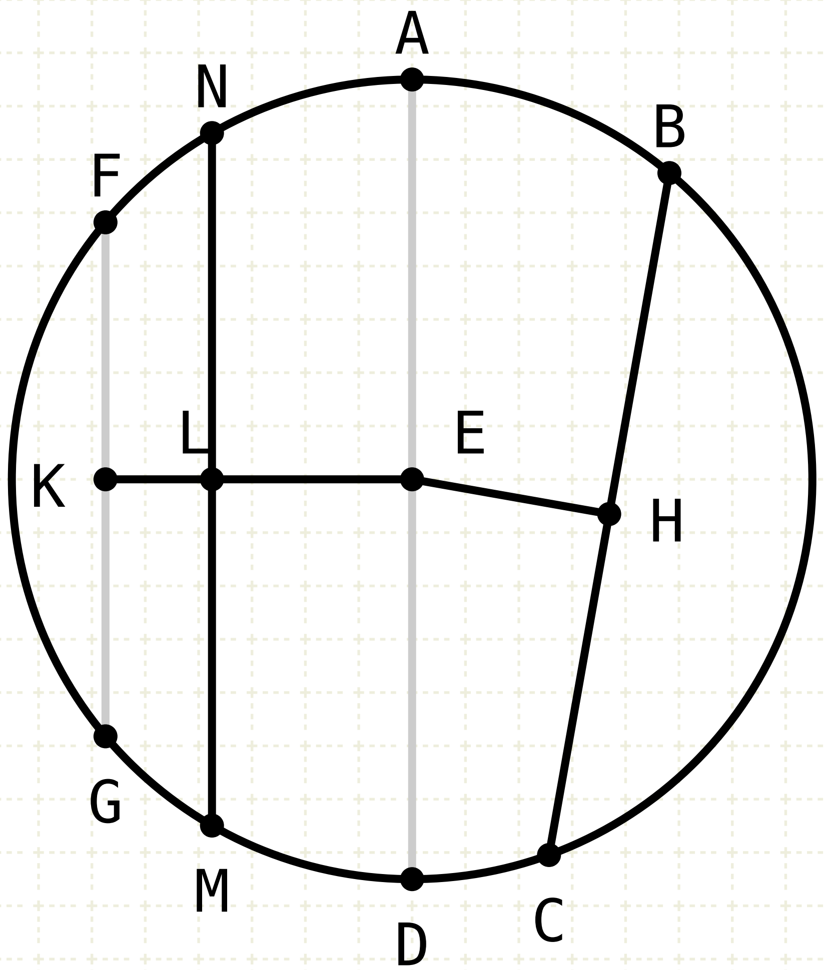

EH < EK

EL = EH

MN = BC

Proof

Draw lines from the centre of the circle perpendicular to the lines BC and FG

Since BC is nearer to centre E than line FG, EH is less than EK (by definition)

Define a point L on line EK, such that LE equals EH

Draw a line perpendicular to EL, touching the circle at points M and N

The lines BC and MN are equal (III·14)

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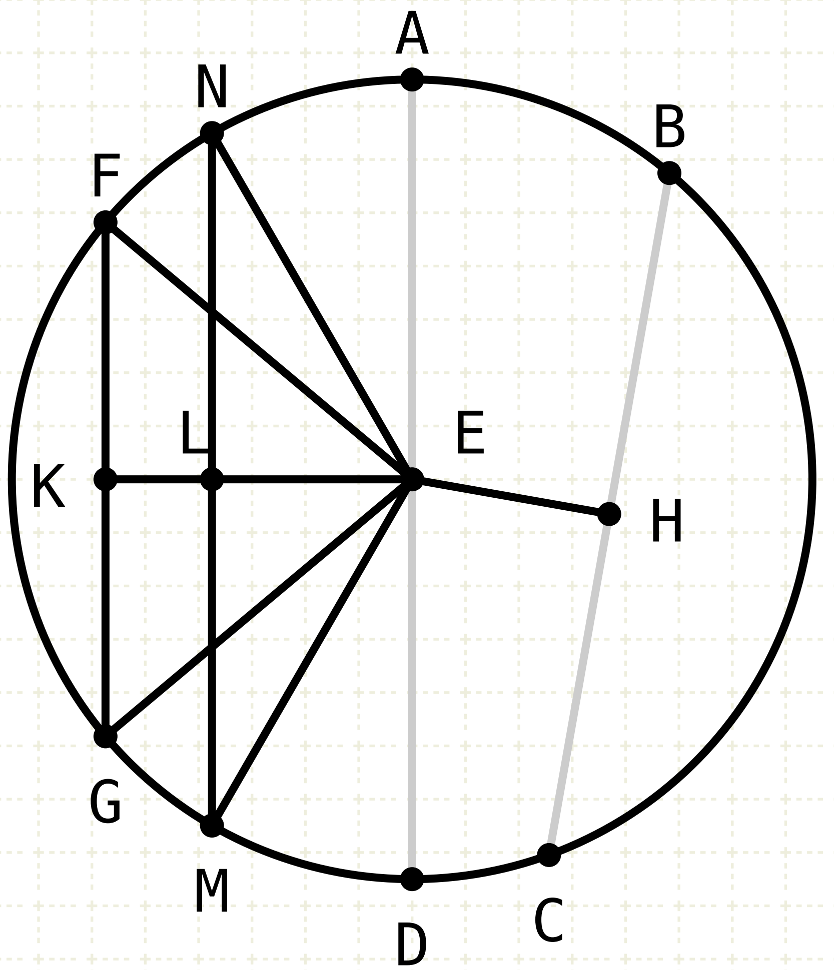

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Draw the lines FE, GE, ME and NE

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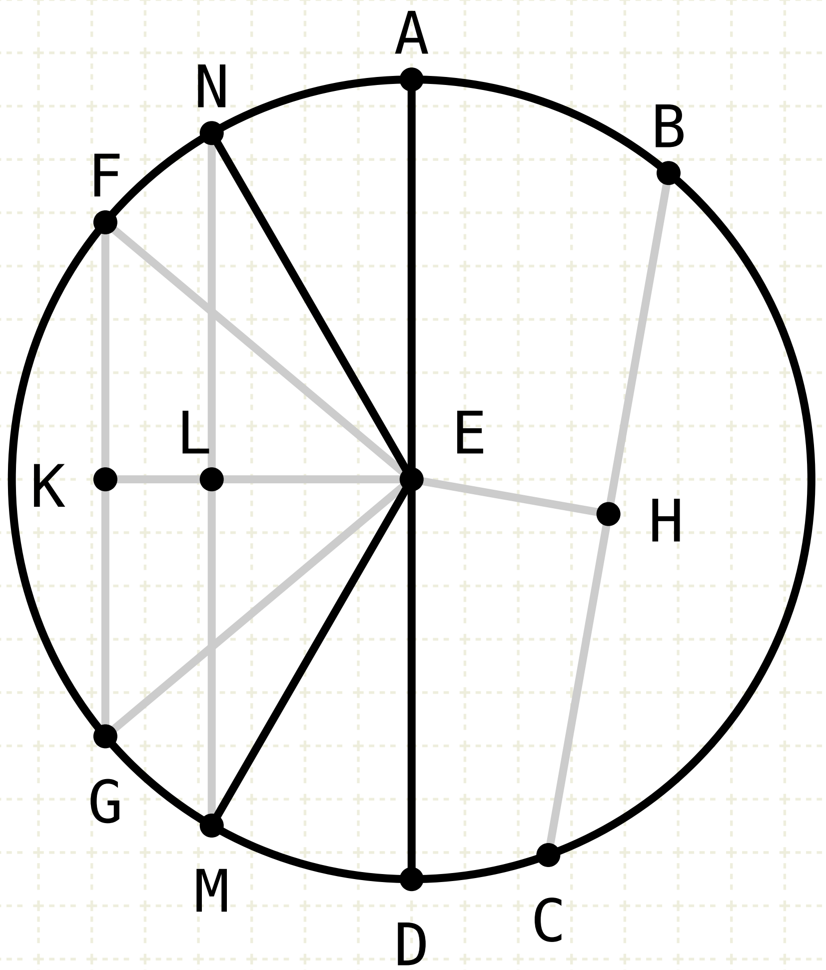

EH < EK

EL = EH

MN = BC

AE+ED = AD = ME+EN

Proof

Draw lines from the centre of the circle perpendicular to the lines BC and FG

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Since EA equals EM, and ED equals EN (all radii), then the sum of EA and ED equals the sum of EM and EN

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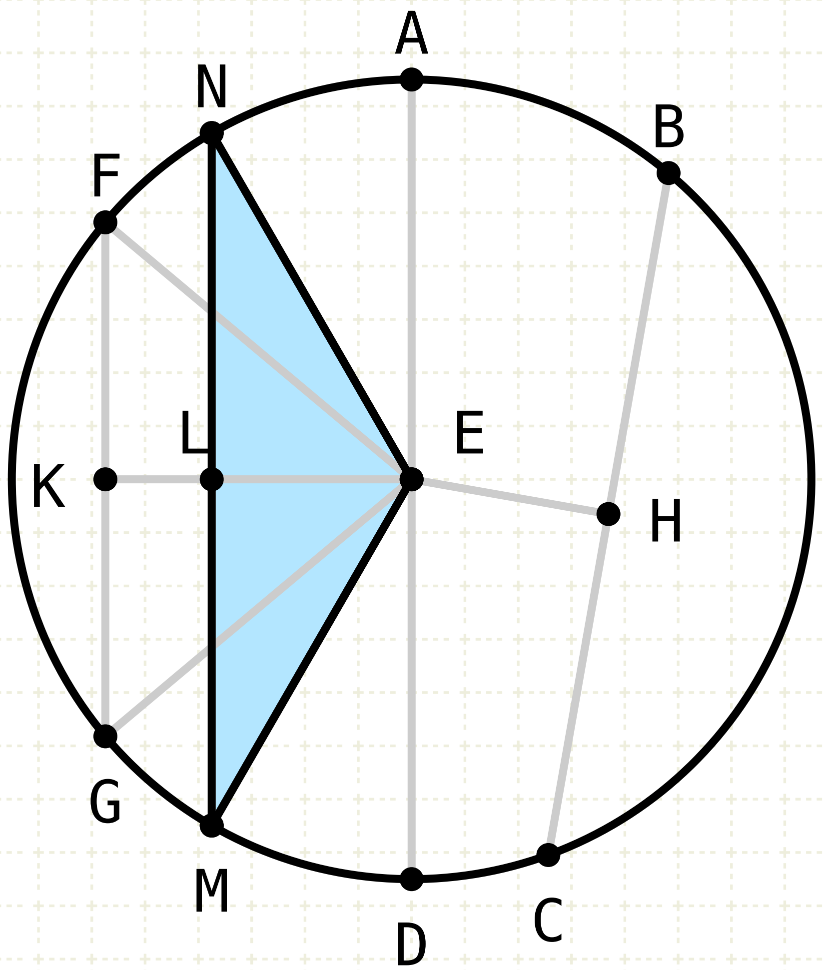

$EH < EK$

$EL = EH$

$MN = BC$

$AE+ED = AD = ME+EN$

$MN < ME+EN$

$\therefore MN < AD$

Proof

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Draw a line perpendicular to EL, touching the circle at points M and N

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Draw the lines FE, GE, ME and NE

Since EA equals EM, and ED equals EN (all radii), then the sum of EA and ED equals the sum of EM and EN

One side of a triangle is less than the sum of the two other sides (I·20), thus MN is less than ME,EN, or MN is less than AD

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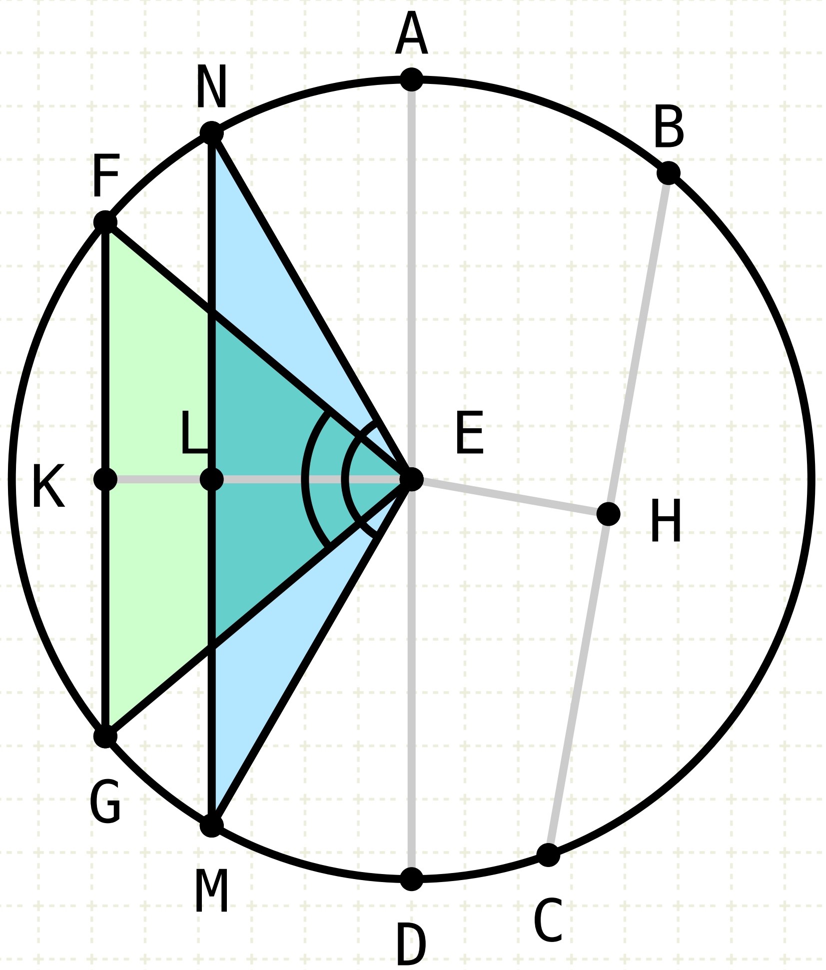

EH < EK

EL = EH

MN = BC

AE+ED = AD = ME+EN

MN < ME+EN

∴ MN < AD

ME = FE

NE = GE

∠MEN > ∠FEG

∴ FG < MN

Proof

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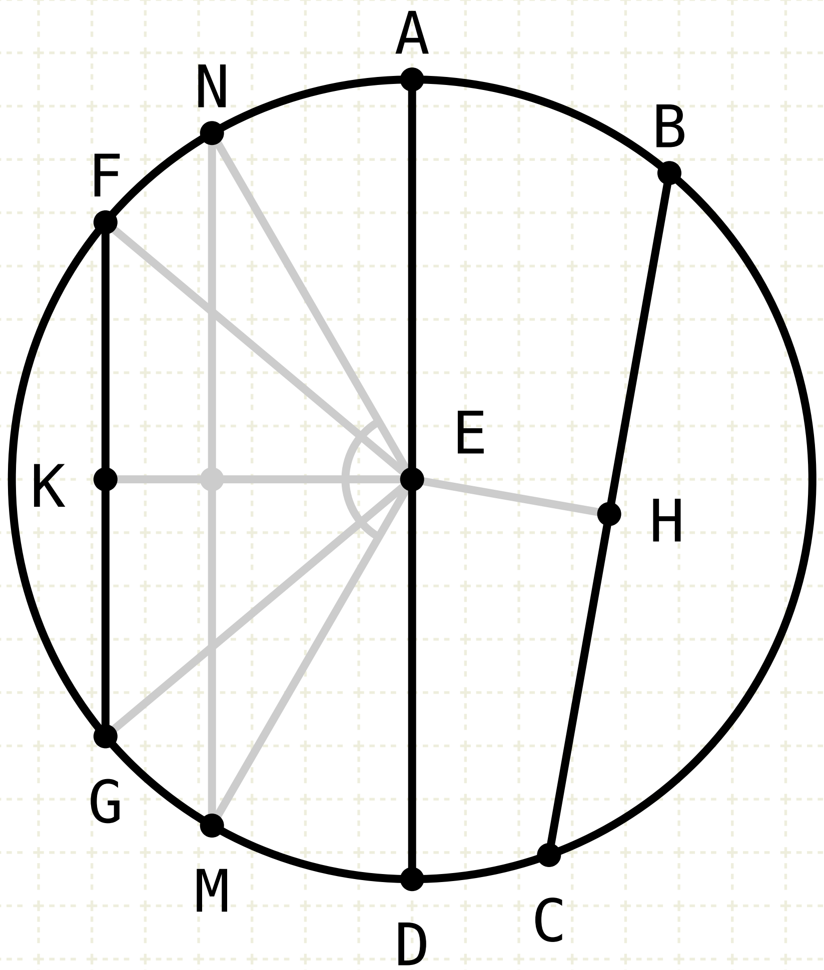

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$\angle MEN > \angle FEG$

$\therefore FG < MN$

$FG < MN < AD$

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Putting it all together gives FG is less than BC, is less than AD, or in other words, the further away from the centre, the smaller the line

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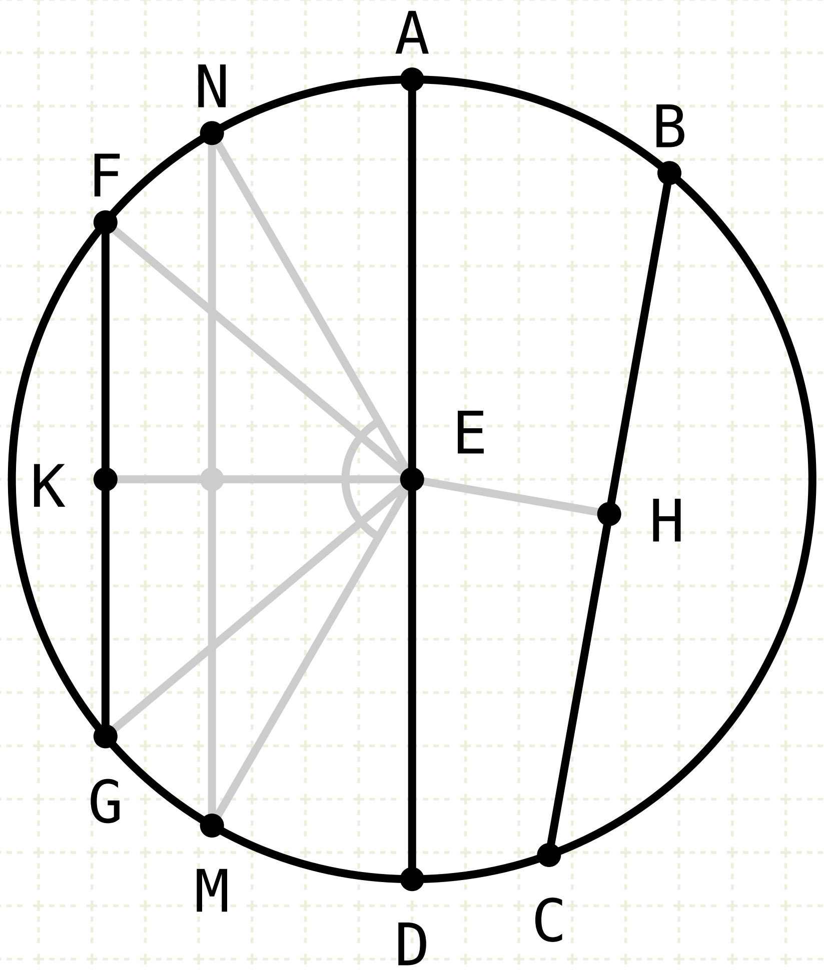

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NE = GE
∠MEN > ∠FEG
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FG < MN < AD
FG < BC < AD

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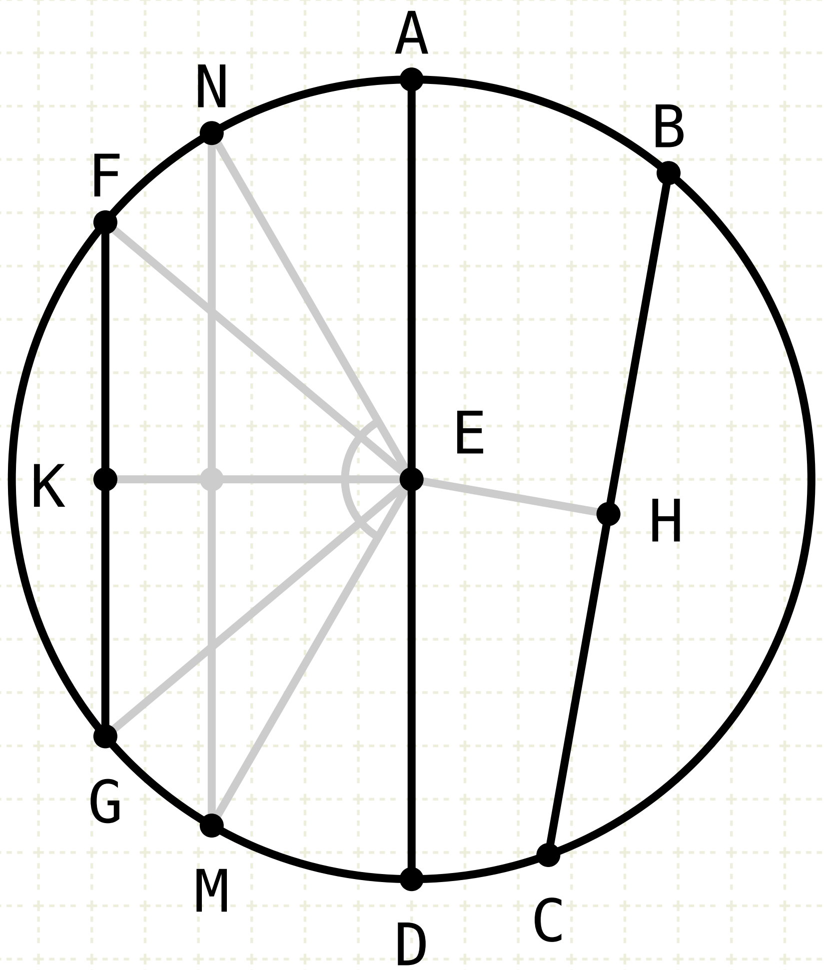

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$\angle MEN > \angle FEG$

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$FG < MN < AD$

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