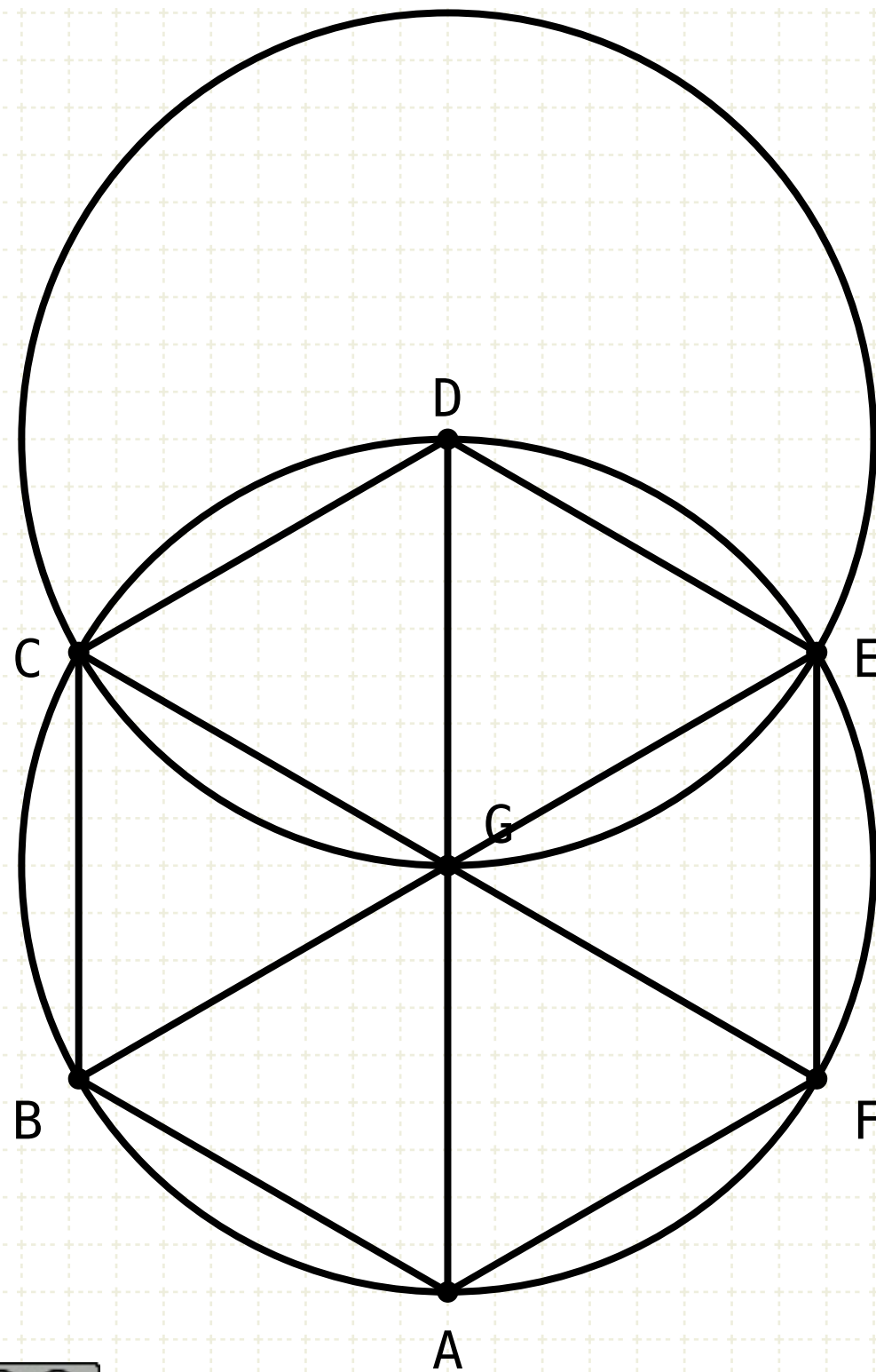


# Euclid's Elements

## Book IV



Philosophy (nature) is written in that great book which ever is before our eyes -- I mean the universe -- but we cannot understand it if we do not first learn the language and grasp the symbols in which it is written. The book is written in mathematical language, and the symbols are triangles, circles and other geometrical figures, without whose help it is impossible to comprehend a single word of it - without which one wanders in vain through a dark labyrinth.

**Galileo Galilei**



# Proposition 2 of Book IV

In a given circle to inscribe a triangle equiangular with a given triangle.



# Table of Contents, Chapter 4

1	Fit a given straight line into a given circle, if the line is less than the diameter	11	In a given circle to inscribe an equilateral and equiangular pentagon
2	<b>In a given circle to inscribe a triangle equiangular with a given triangle</b>	12	About a given circle to circumscribe an equilateral and equiangular pentagon
3	About a given circle to circumscribe a triangle equiangular with a given triangle	13	In a given pentagon, which is equilateral and equiangular, to inscribe a circle
4	In a given triangle, to inscribe a circle	14	About a given pentagon, which is equilateral and equiangular, to circumscribe a circle
5	About a given triangle to circumscribe a circle	15	In a given circle to inscribe an equilateral and equiangular hexagon
6	In a given circle to inscribe a square	16	In a given circle to inscribe a fifteen angled figure which shall be both equilateral and equiangular
7	About a given circle to circumscribe a square		
8	In a given square, to inscribe a circle		
9	About a given square, to circumscribe a circle		
10	To construct an isosceles triangle having each of the angles at the base double of the remaining one		



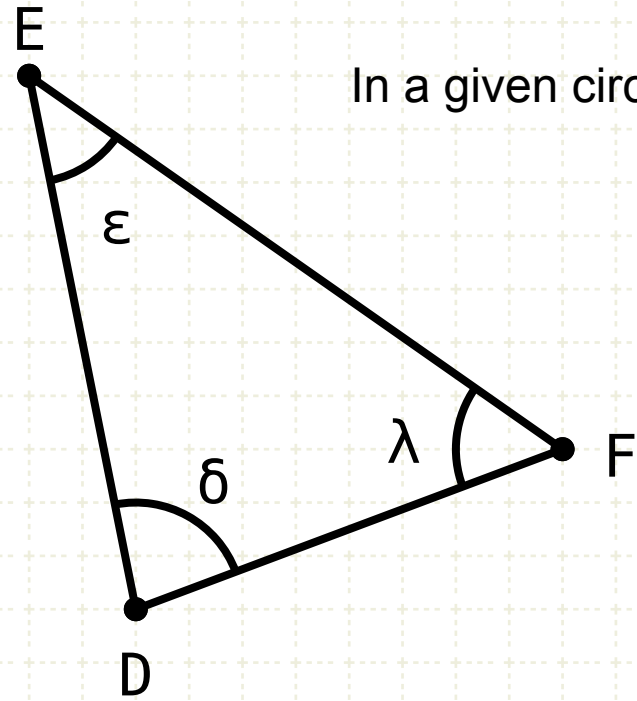
# Proposition 2 of Book IV

In a given circle to inscribe a triangle equiangular with a given triangle.



## Proposition 2 of Book IV

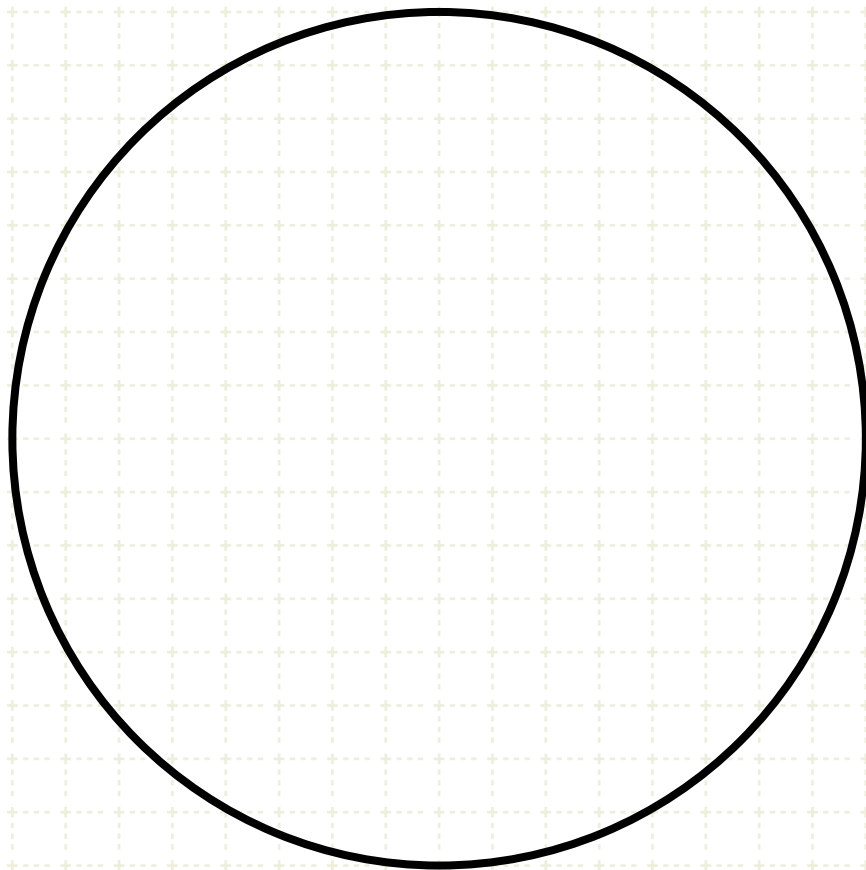
In a given circle to inscribe a triangle equiangular with a given triangle.



### In other words

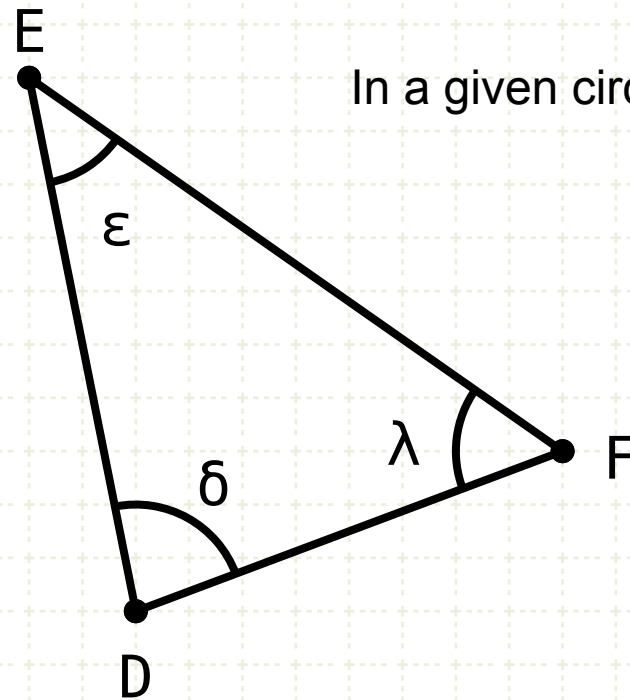
Given a circle and a triangle DEF:

Draw a triangle within the circle, where the angles in the new triangle equal the angles in triangle DEF



## Proposition 2 of Book IV

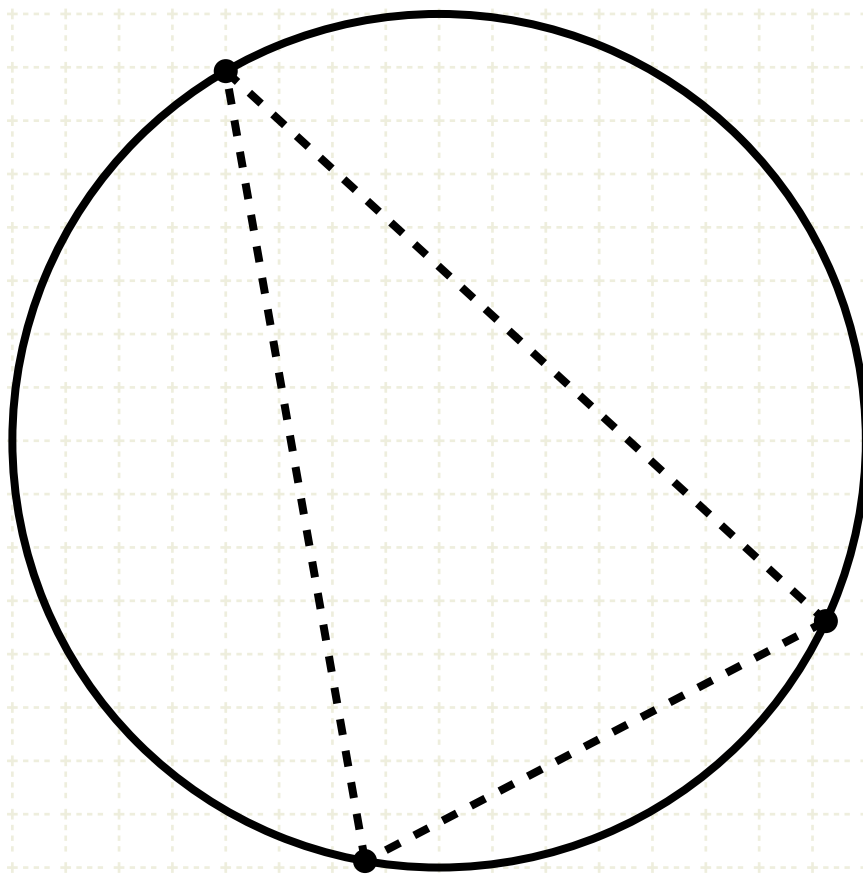
In a given circle to inscribe a triangle equiangular with a given triangle.



### In other words

Given a circle and a triangle DEF:

Draw a triangle within the circle, where the angles in the new triangle equal the angles in triangle DEF

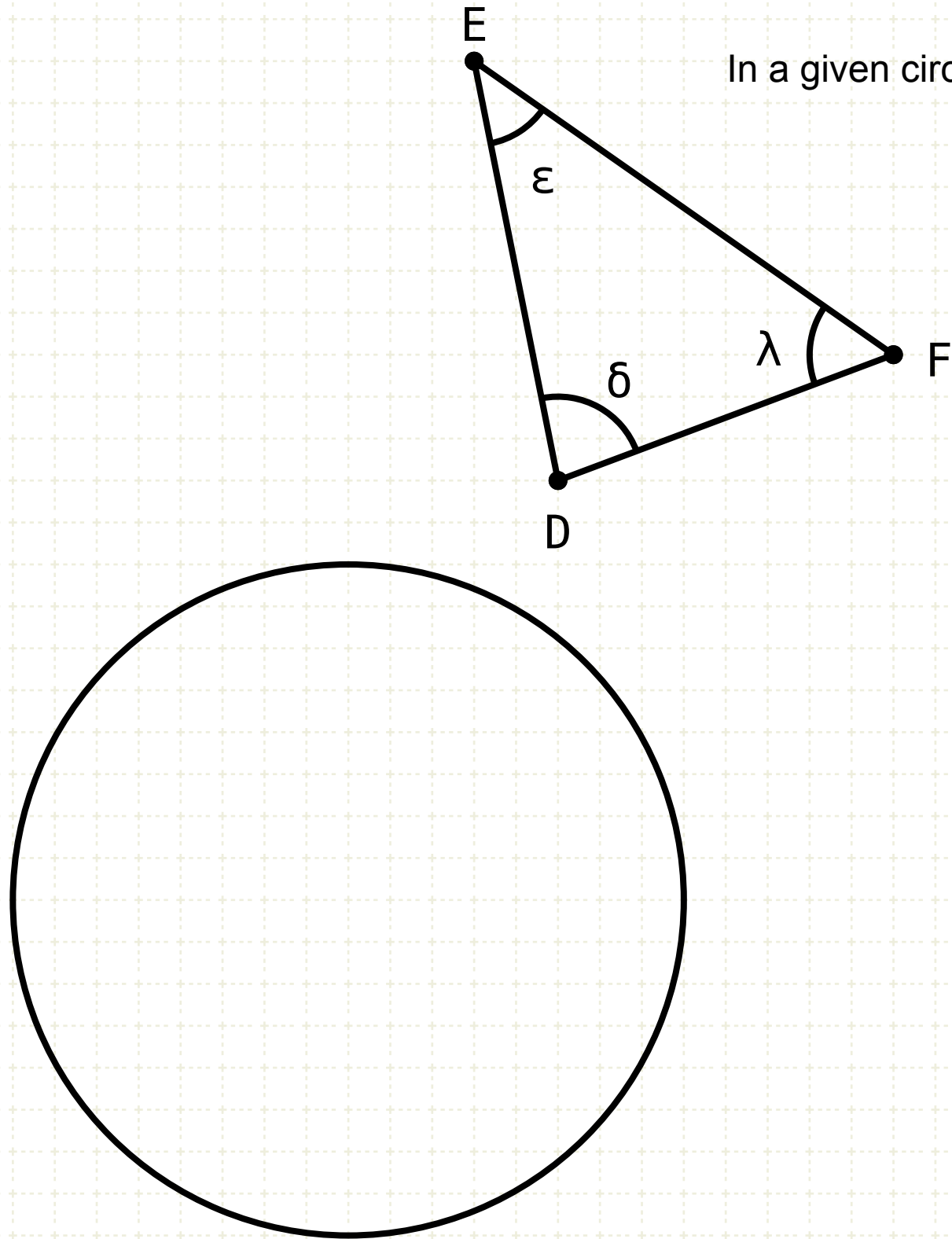




## Proposition 2 of Book IV

In a given circle to inscribe a triangle equiangular with a given triangle.

### Construction

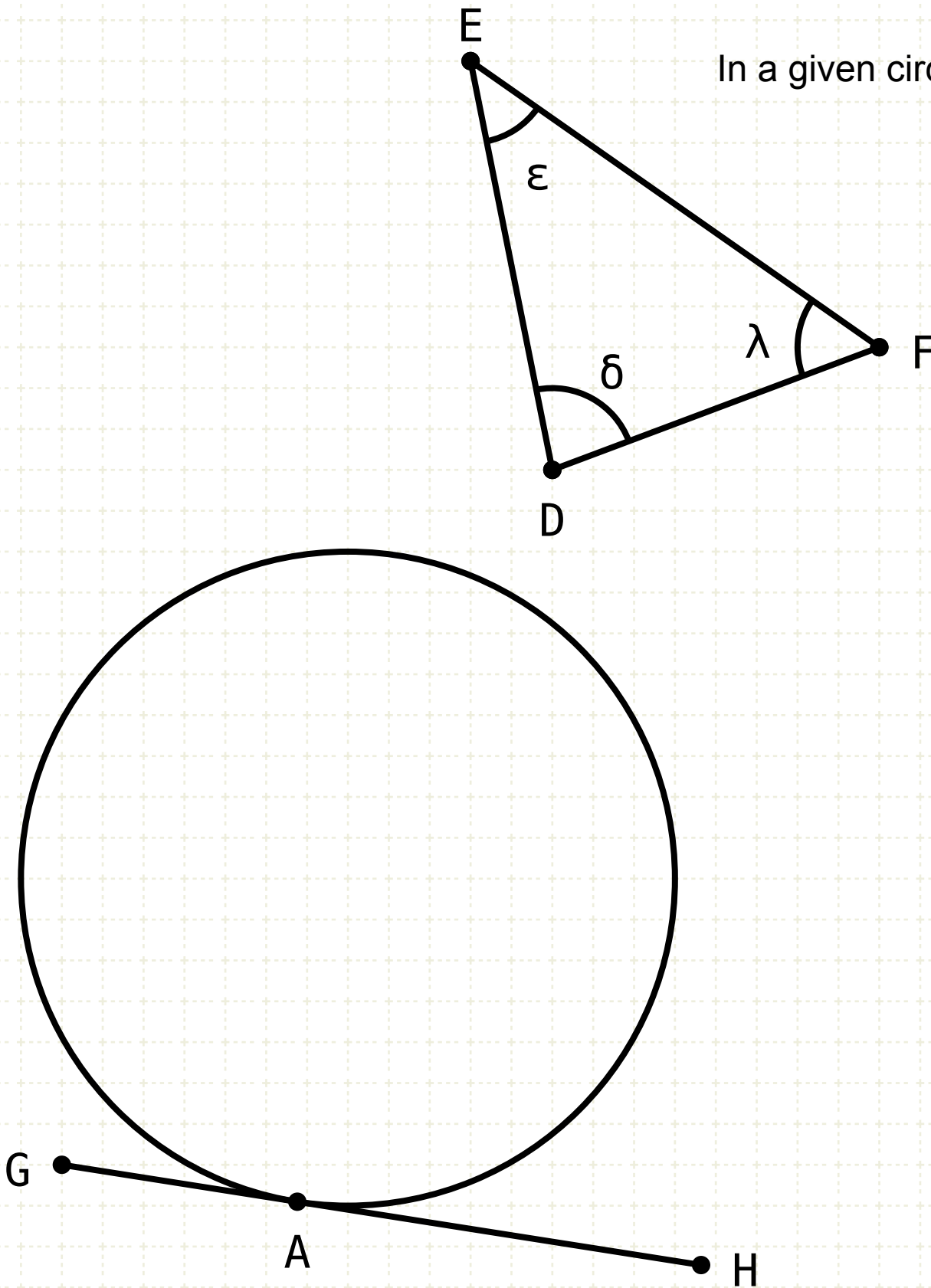


## Proposition 2 of Book IV

In a given circle to inscribe a triangle equiangular with a given triangle.

### Construction

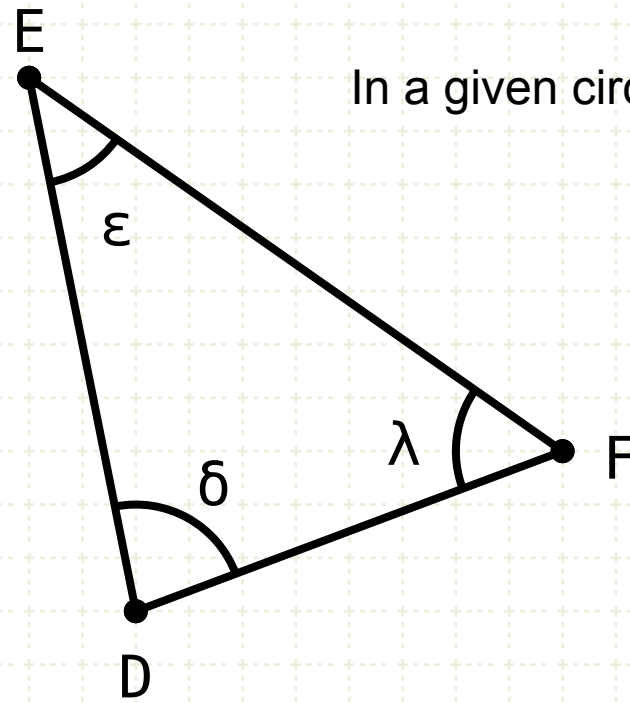
Draw a line GH touching the circle at point A (III·16)





## Proposition 2 of Book IV

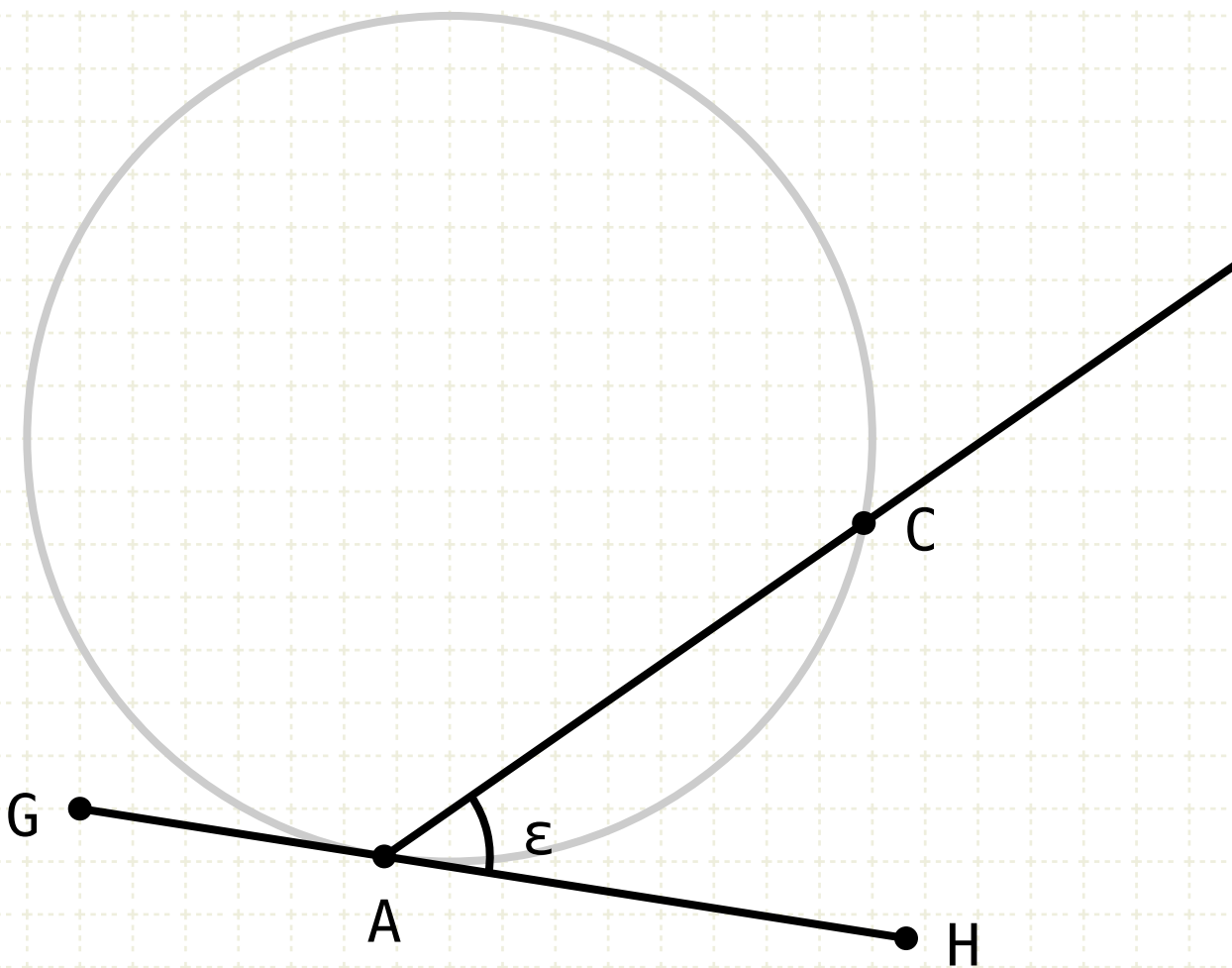
In a given circle to inscribe a triangle equiangular with a given triangle.



### Construction

Draw a line  $GH$  touching the circle at point  $A$  (III·16)

Copy the angle  $\epsilon$  to line  $GH$ , at point  $A$  (I·23)



## Proposition 2 of Book IV

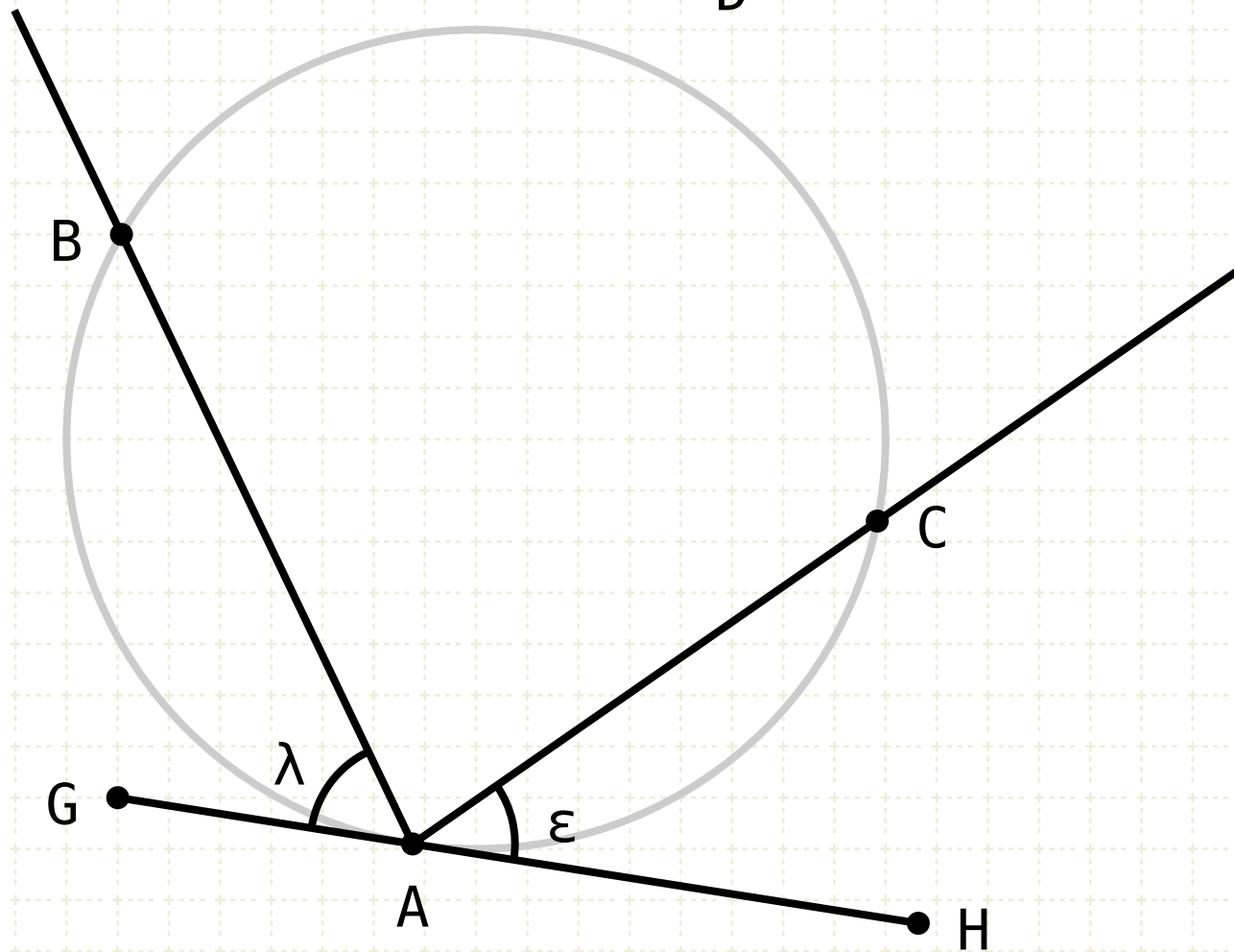
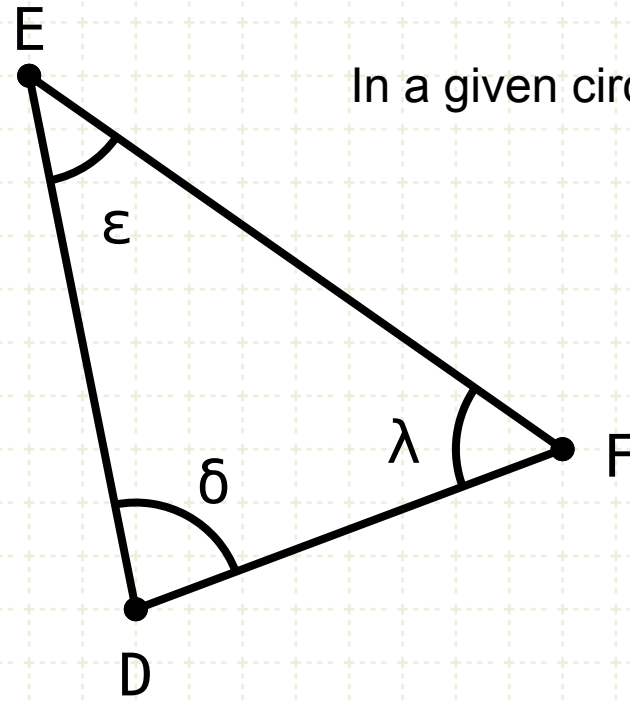
In a given circle to inscribe a triangle equiangular with a given triangle.

### Construction

Draw a line GH touching the circle at point A (III·16)

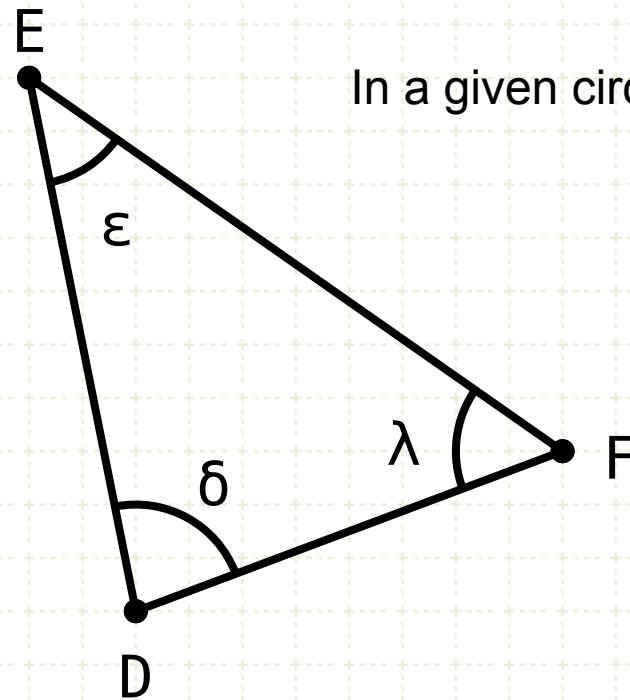
Copy the angle  $\varepsilon$  to line GH, at point A (I·23)

Copy the angle  $\lambda$  to line GH, at point A (I·23)



## Proposition 2 of Book IV

In a given circle to inscribe a triangle equiangular with a given triangle.



### Construction

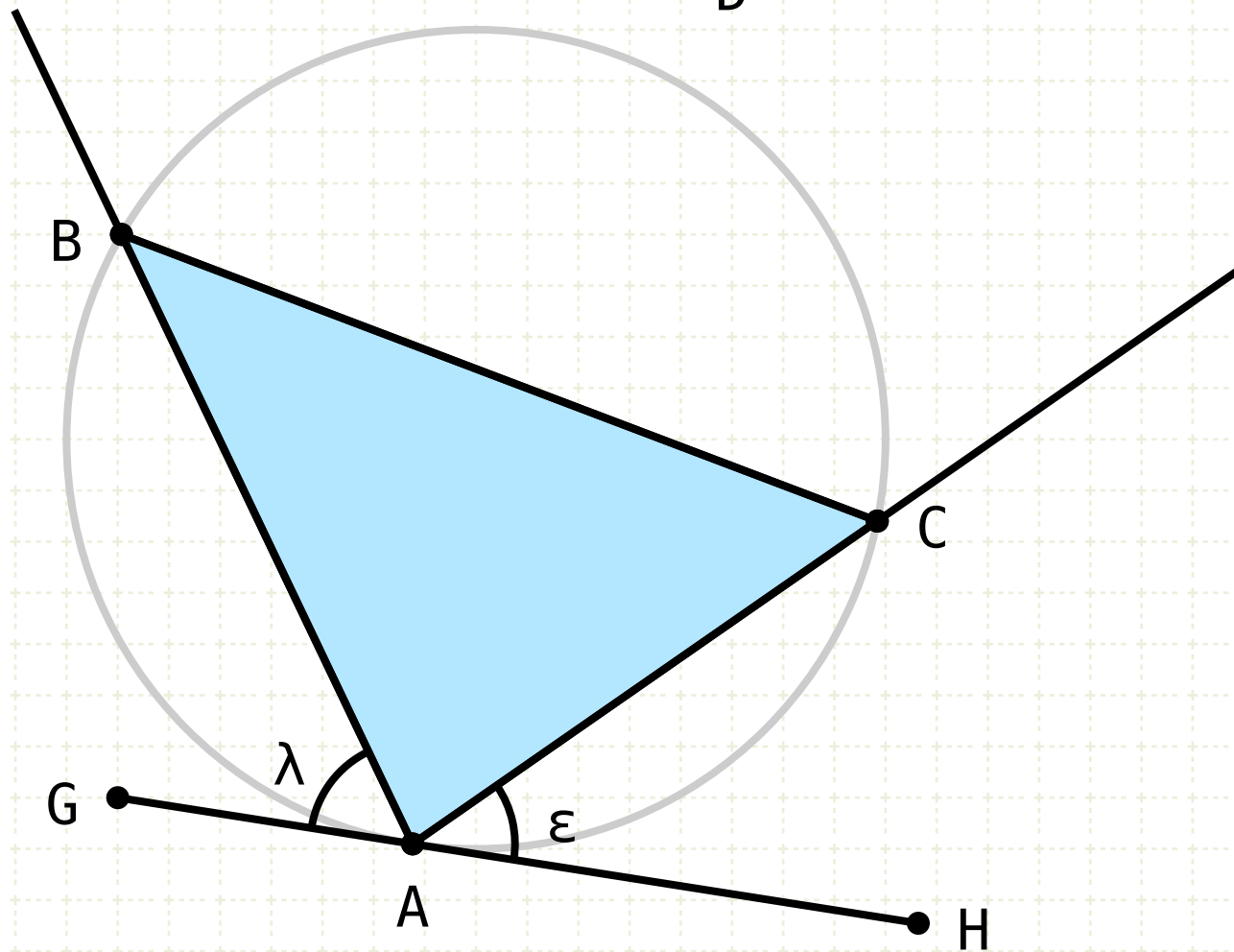
Draw a line  $GH$  touching the circle at point  $A$  (III·16)

Copy the angle  $\varepsilon$  to line  $GH$ , at point  $A$  (I·23)

Copy the angle  $\lambda$  to line  $GH$ , at point  $A$  (I·23)

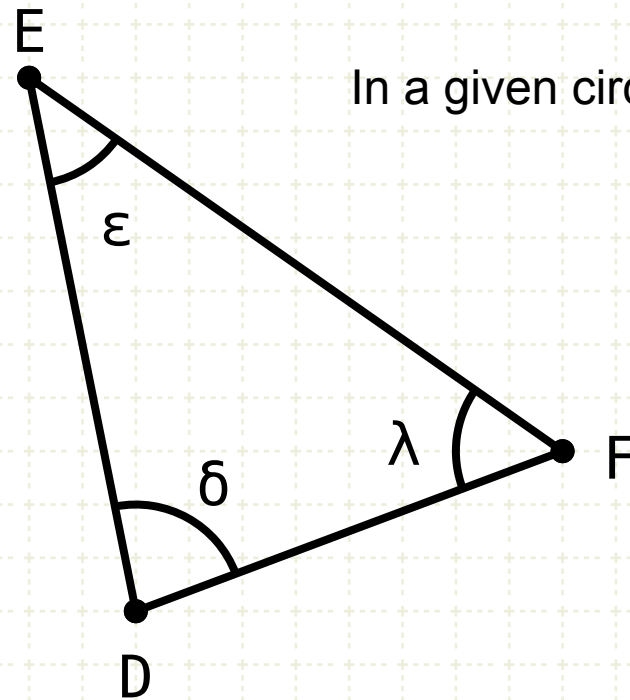
Connect  $B$  and  $C$  with a straight line

The resulting triangle (circumscribed by the circle) is equiangular to the original triangle  $DEF$



## Proposition 2 of Book IV

In a given circle to inscribe a triangle equiangular with a given triangle.



### Construction

Draw a line GH touching the circle at point A (III·16)

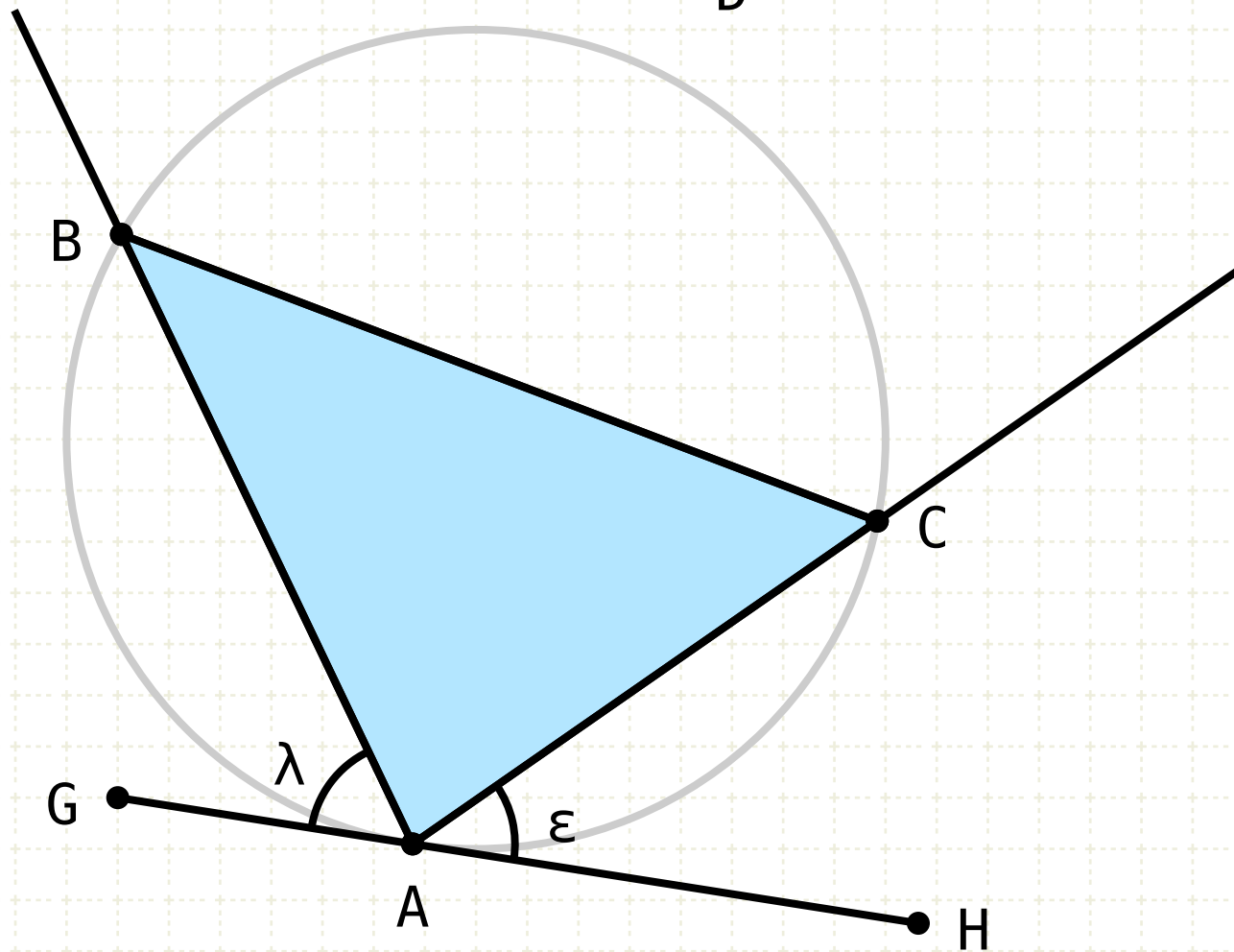
Copy the angle  $\epsilon$  to line GH, at point A (I·23)

Copy the angle  $\lambda$  to line GH, at point A (I·23)

Connect B and C with a straight line

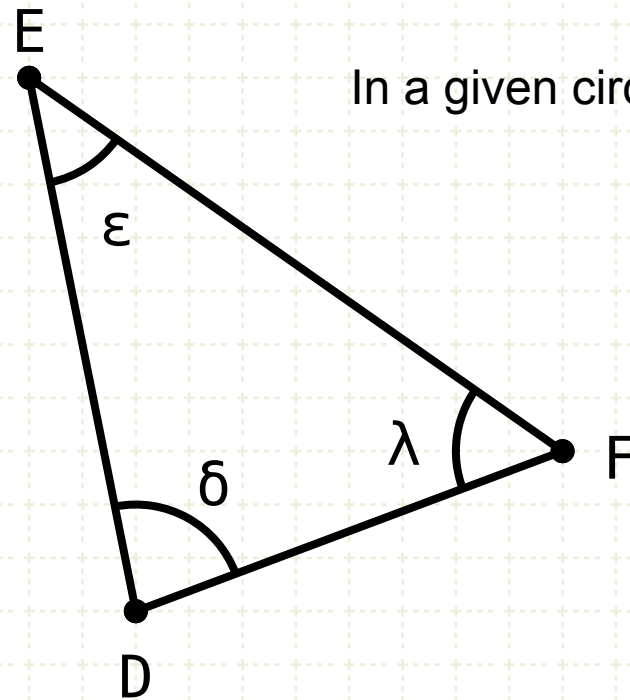
The resulting triangle (circumscribed by the circle) is equiangular to the original triangle DEF

### Proof



## Proposition 2 of Book IV

In a given circle to inscribe a triangle equiangular with a given triangle.



### Construction

Draw a line GH touching the circle at point A (III·16)

Copy the angle  $\epsilon$  to line GH, at point A (I·23)

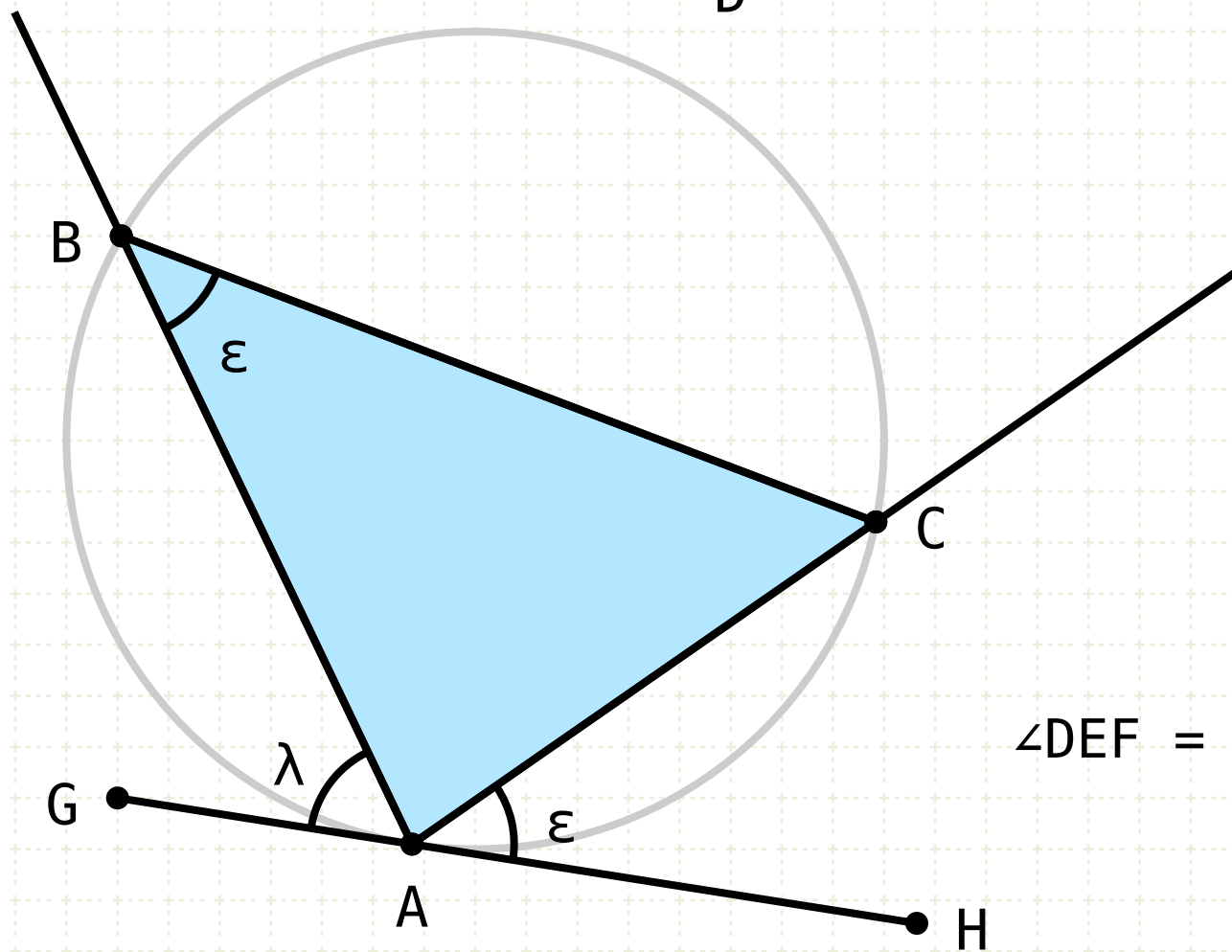
Copy the angle  $\lambda$  to line GH, at point A (I·23)

Connect B and C with a straight line

The resulting triangle (circumscribed by the circle) is equiangular to the original triangle DEF

### Proof

Since GH touches the circle at A, and AC cuts the circle, the angle in the alternate segment of the circle (HAC) equals the angle between GH and AC (III·32)

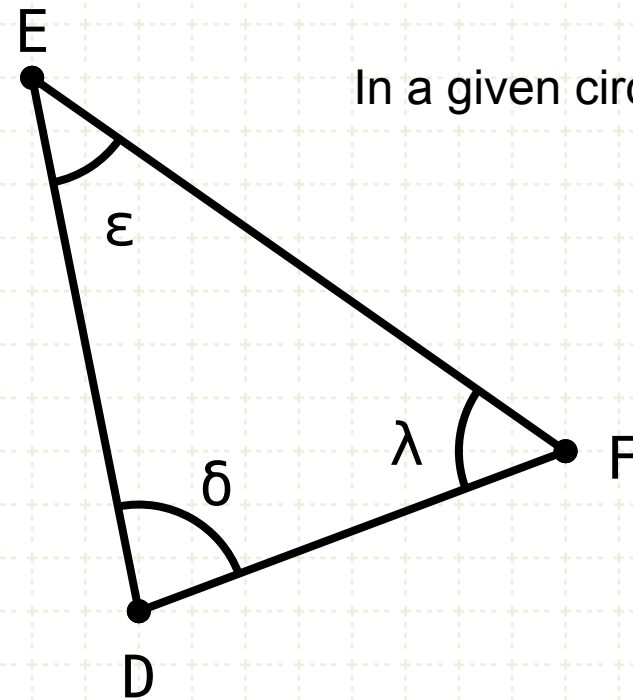


$$\angle DEF = \angle CAH = \angle ABC = \epsilon$$



## Proposition 2 of Book IV

In a given circle to inscribe a triangle equiangular with a given triangle.



### Construction

Draw a line GH touching the circle at point A (III·16)

Copy the angle  $\epsilon$  to line GH, at point A (I·23)

Copy the angle  $\lambda$  to line GH, at point A (I·23)

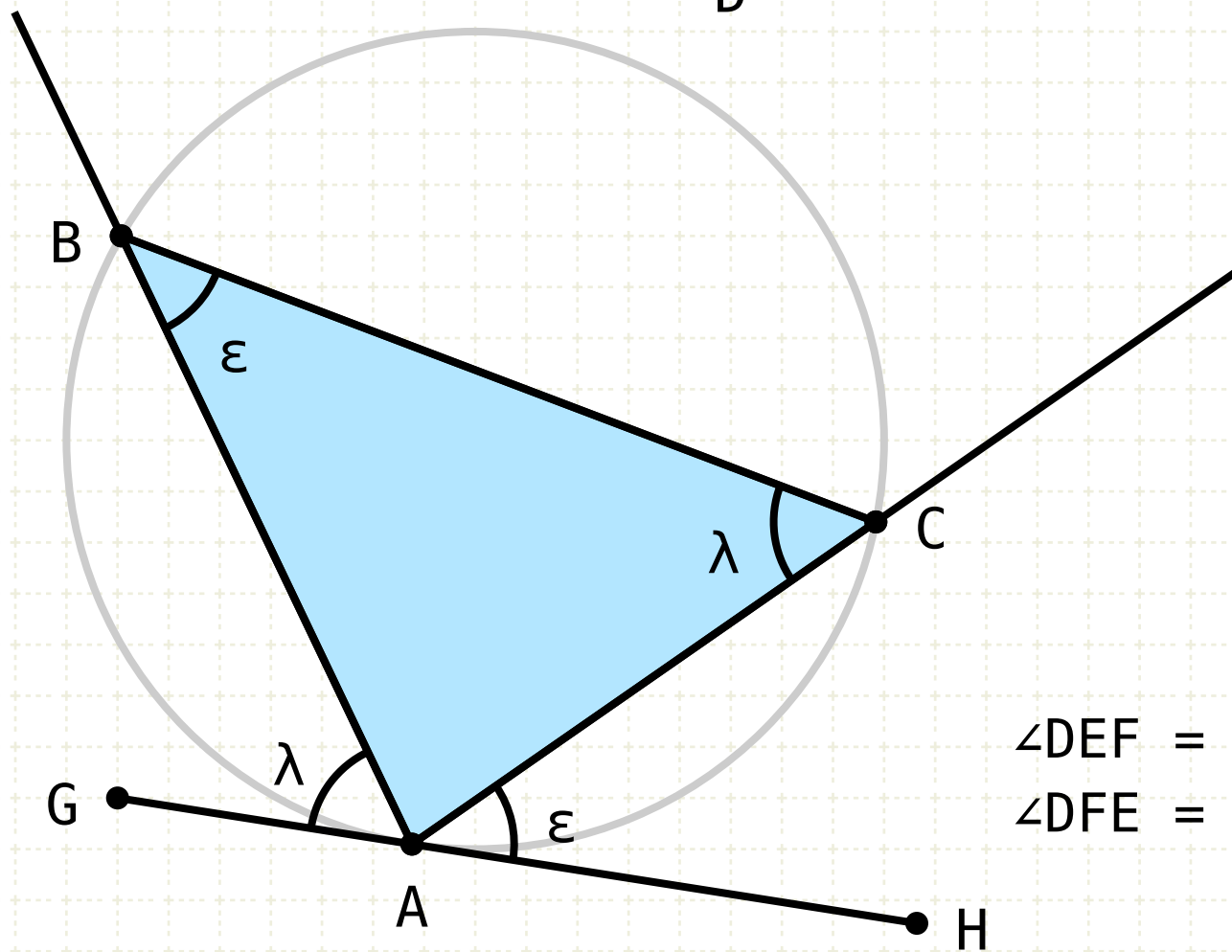
Connect B and C with a straight line

The resulting triangle (circumscribed by the circle) is equiangular to the original triangle DEF

### Proof

Since GH touches the circle at A, and AC cuts the circle, the angle in the alternate segment of the circle (HAC) equals the angle between GH and AC (III·32)

Similarly for angles GAB and BCA

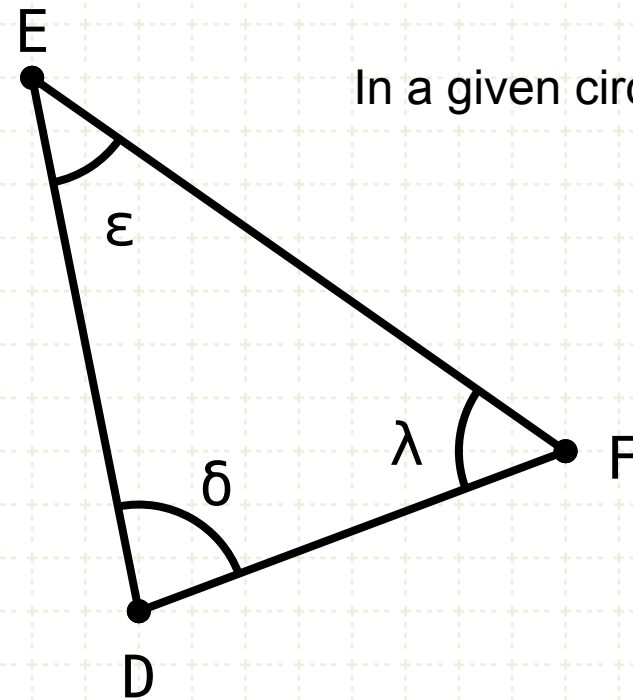


$$\begin{aligned}\angle DEF &= \angle CAH = \angle ABC = \epsilon \\ \angle DFE &= \angle GAB = \angle BCA = \lambda\end{aligned}$$



## Proposition 2 of Book IV

In a given circle to inscribe a triangle equiangular with a given triangle.



### Construction

Draw a line GH touching the circle at point A (III·16)

Copy the angle  $\varepsilon$  to line GH, at point A (I·23)

Copy the angle  $\lambda$  to line GH, at point A (I·23)

Connect B and C with a straight line

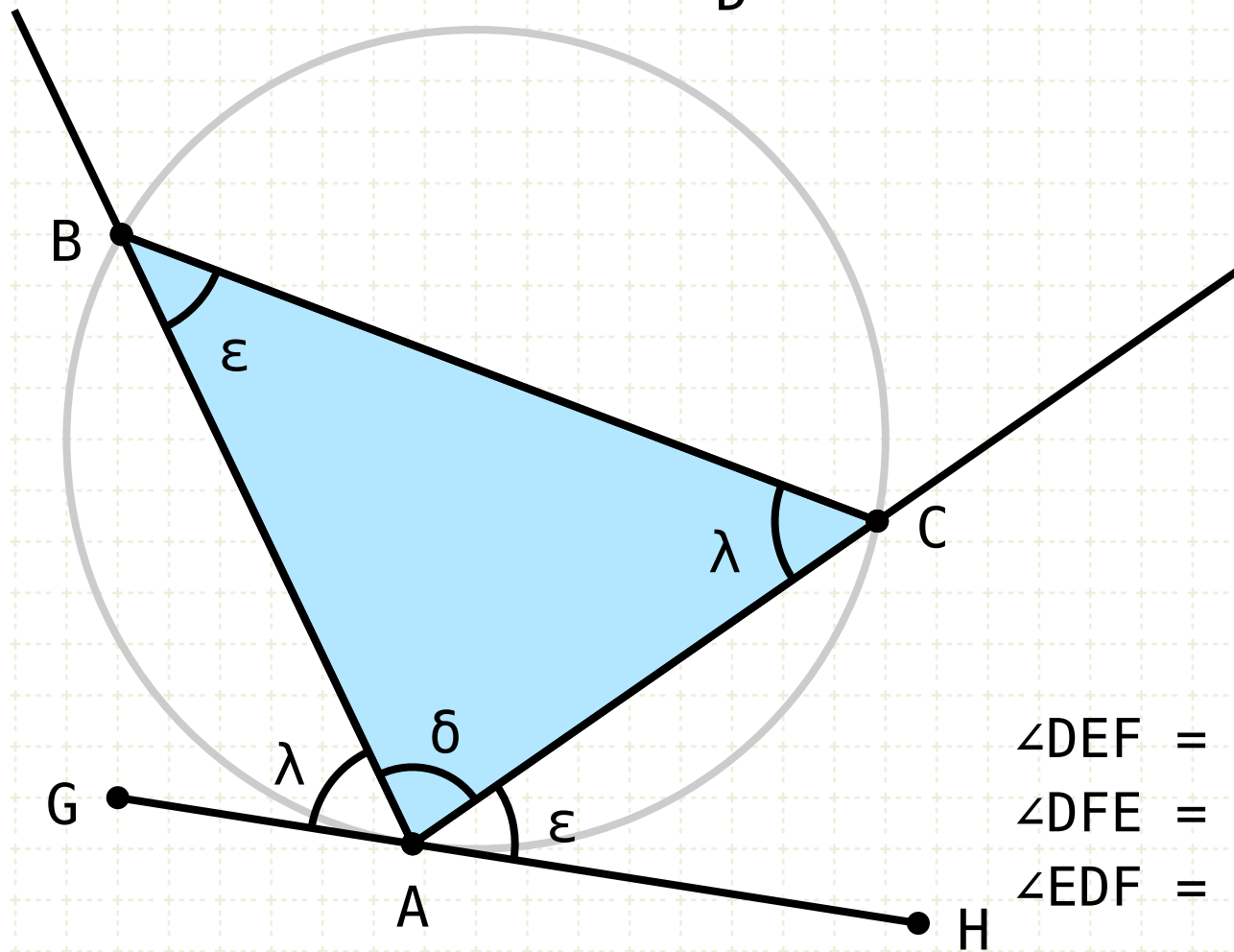
The resulting triangle (circumscribed by the circle) is equiangular to the original triangle DEF

### Proof

Since GH touches the circle at A, and AC cuts the circle, the angle in the alternate segment of the circle (HAC) equals the angle between GH and AC (III·32)

Similarly for angles GAB and BCA

And finally, the sum of all angles in both triangles equals two right angles, it follows that the remaining angle is equal to EDF



$$\begin{aligned}\angle DEF &= \angle CAH = \angle ABC = \varepsilon \\ \angle DFE &= \angle GAB = \angle BCA = \lambda \\ \angle EDF &= \angle BAC = \delta\end{aligned}$$

# Youtube Videos

<https://www.youtube.com/c/SandyBultena>

*Copyright © 2019 by Sandy Bultena.*



Except where otherwise noted, this work is licensed under  
<http://creativecommons.org/licenses/by-nc/3.0>