

Euclid's Elements

Book I

If Euclid did not kindle your youthful enthusiasm, you were not born to be a scientific thinker.

Albert Einstein



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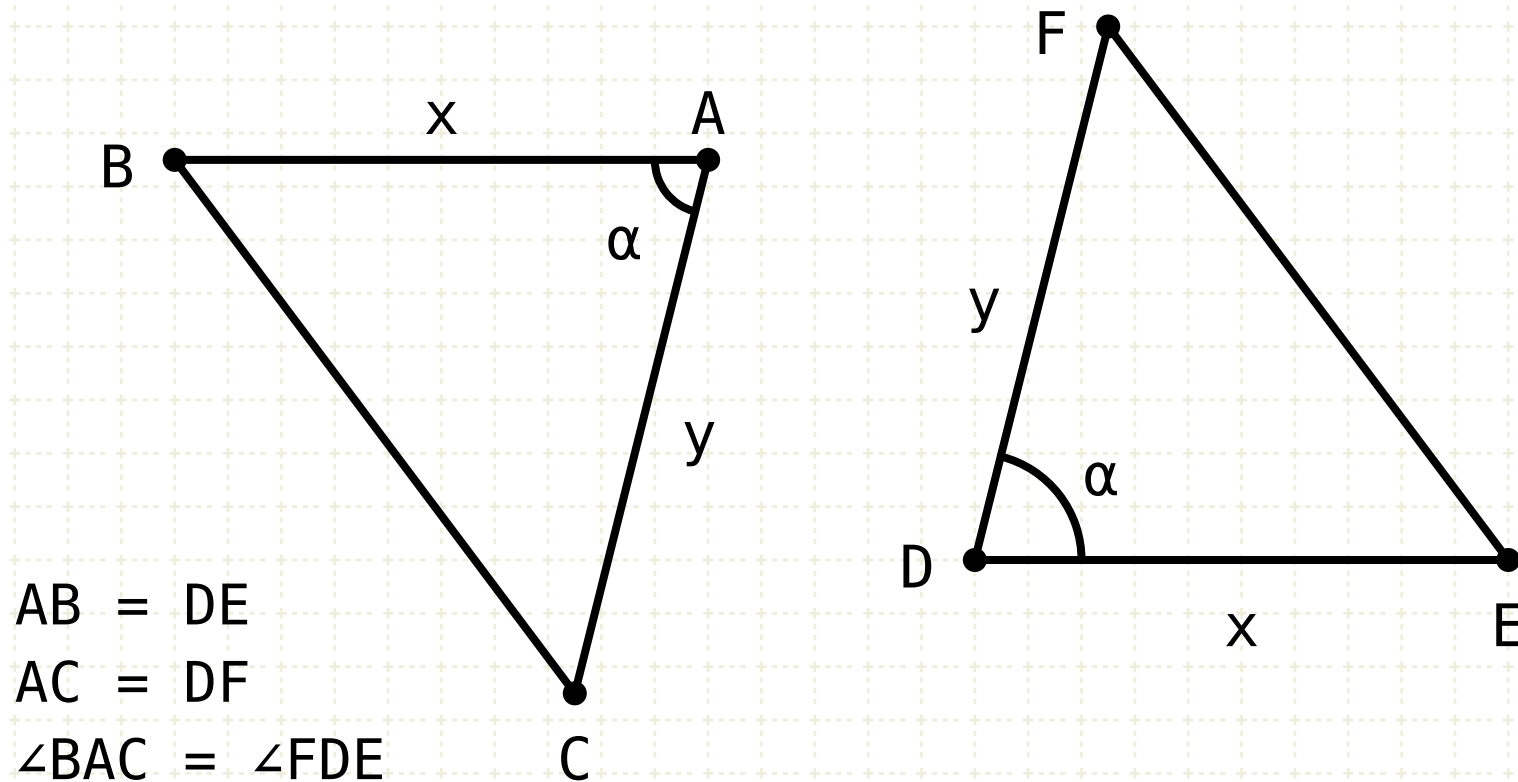
Proposition 4 of Book I

If two triangles have two sides equal to two sides respectively, and have the angles contained by the equal straight lines equal, then they also have the base equal to the base, the triangle equals the triangle, and the remaining angles equal the remaining angles respectively, namely those opposite the equal sides.



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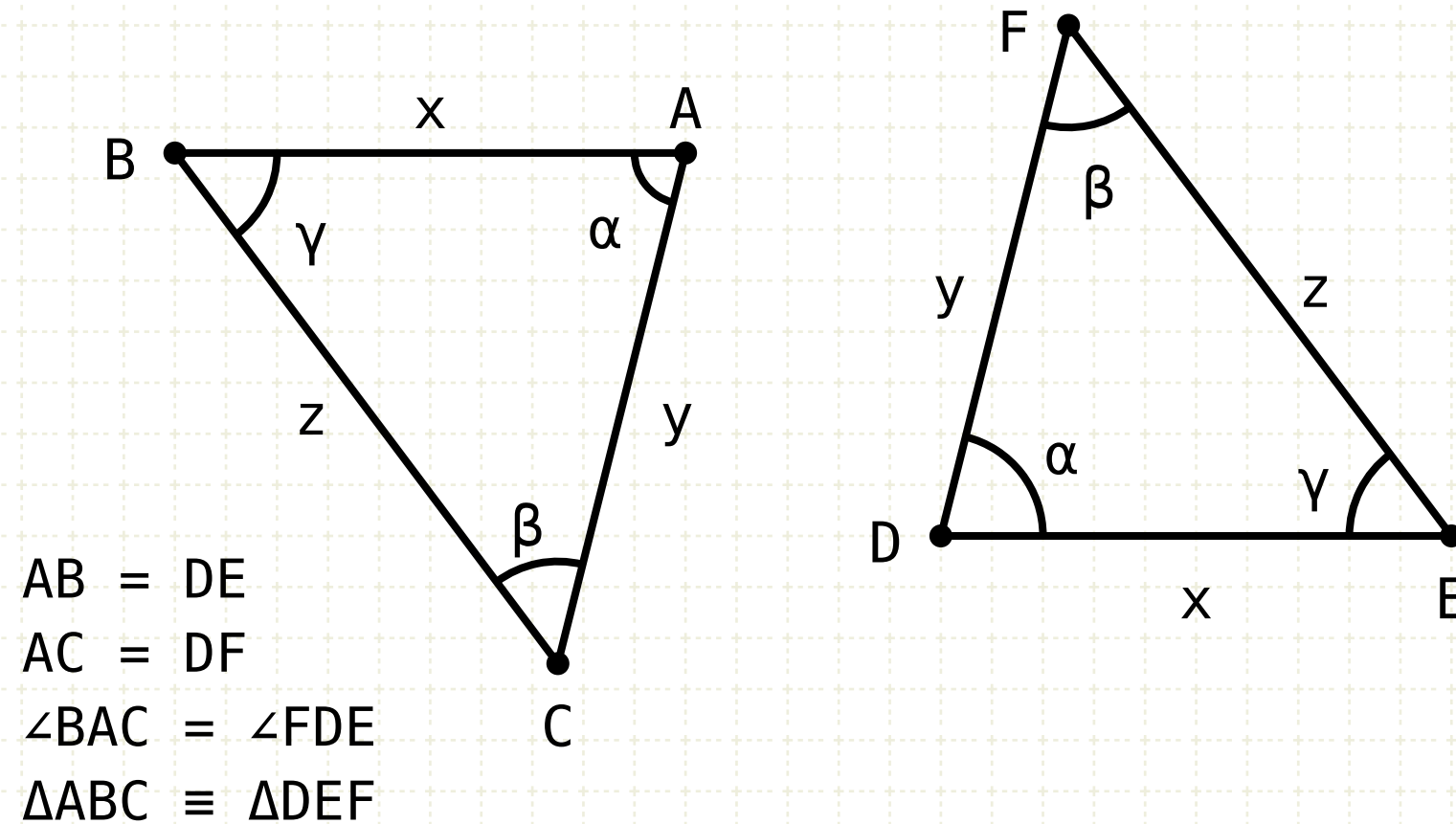


In other words

If two triangles have two sides which are equivalent, and if the angles between the two sides are also equivalent, (side-angle-side SAS)...

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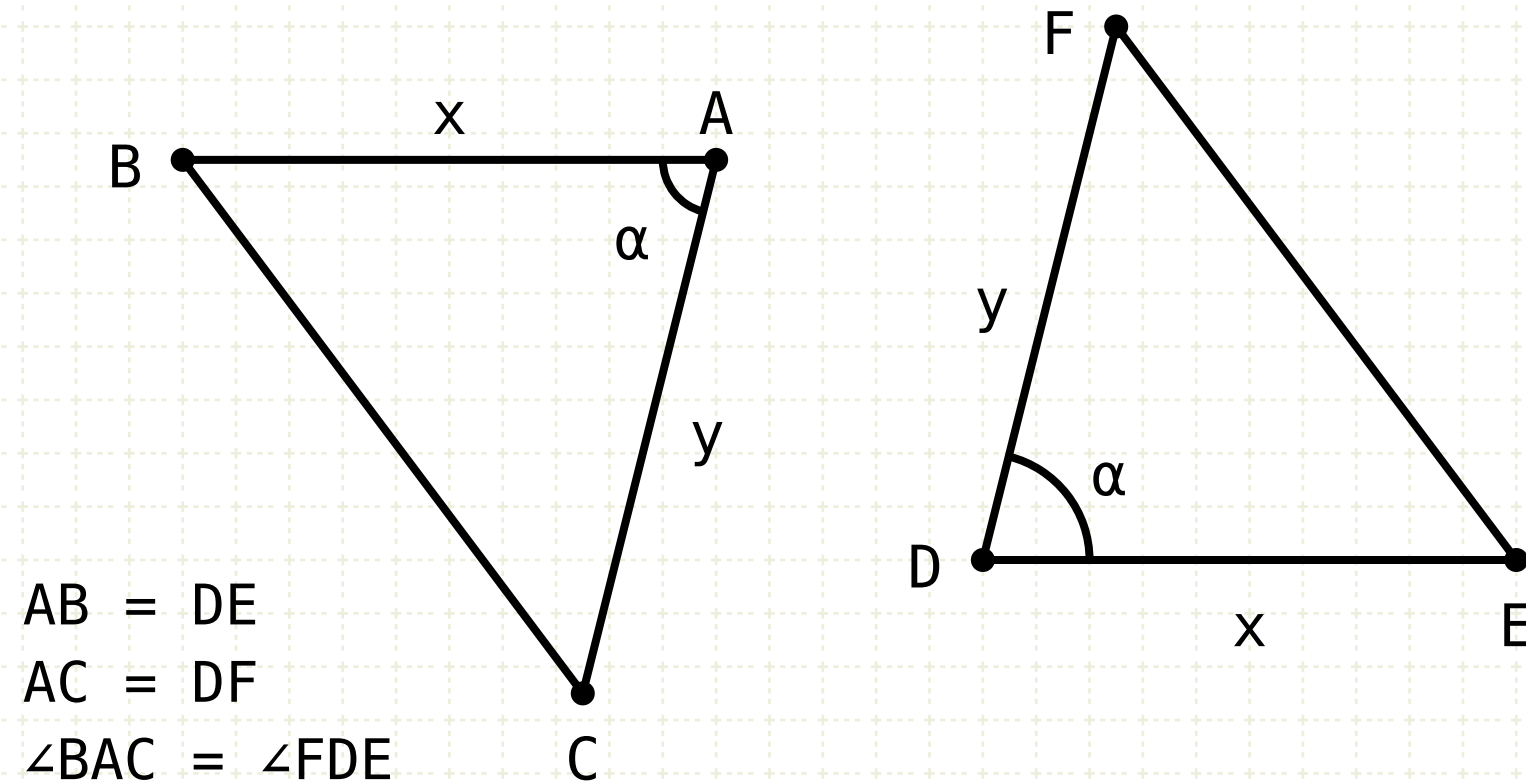
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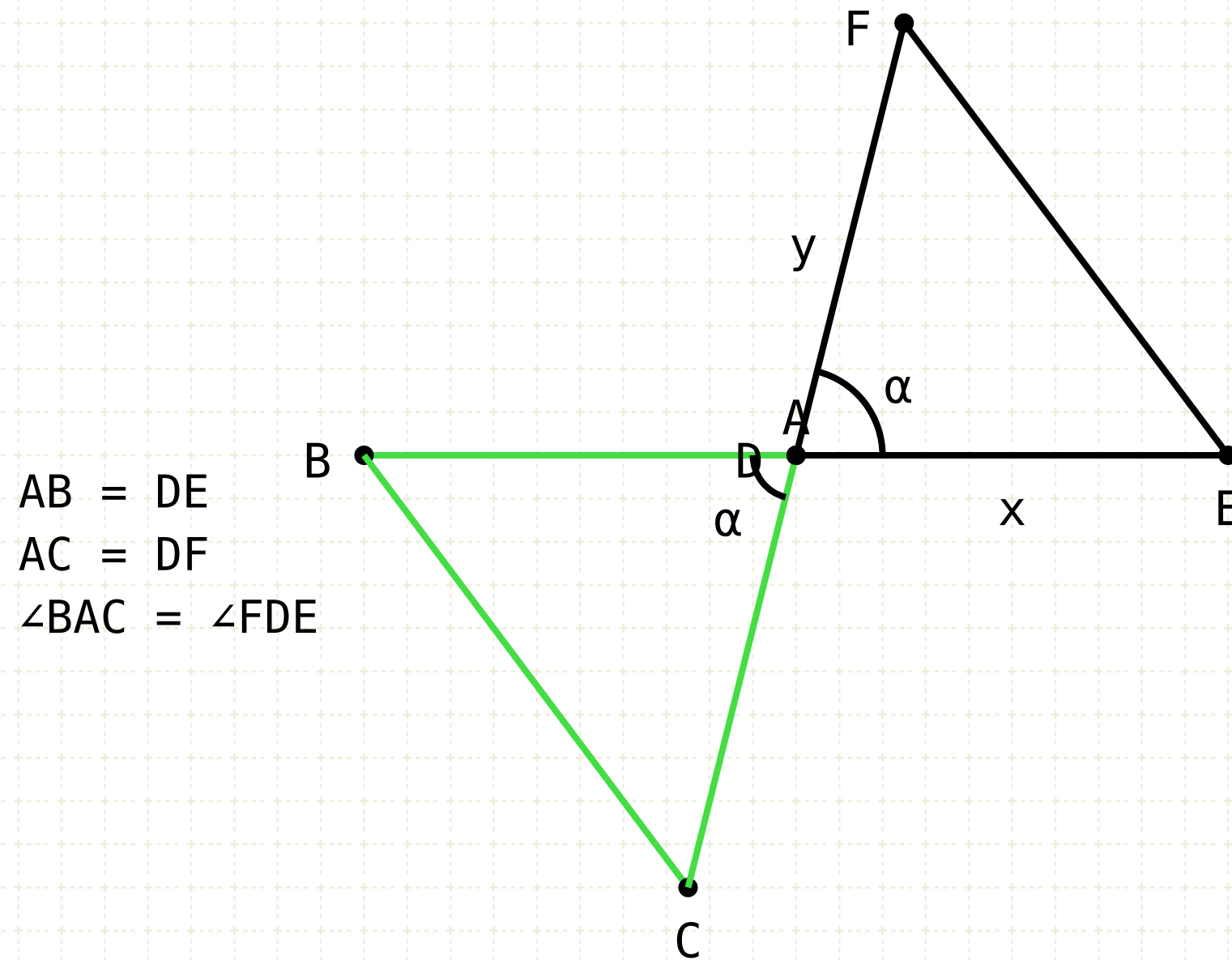
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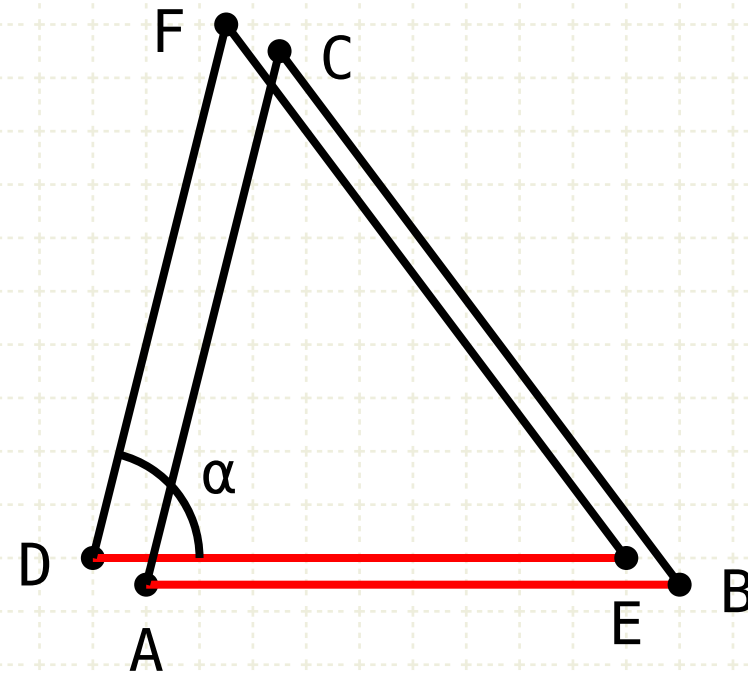
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Move triangle ABC such that point A coincides with point D

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$$\begin{aligned} AB &= DE \\ AC &= DF \\ \angle BAC &= \angle FDE \end{aligned}$$

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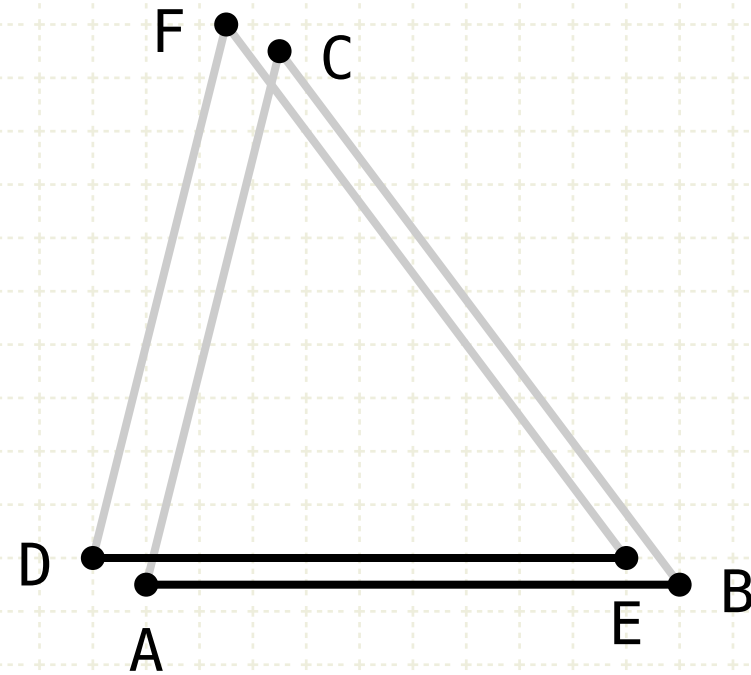
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Rotate the triangle so that line AB line coincides with DE.

... diagram is offset a bit so we can see more clearly

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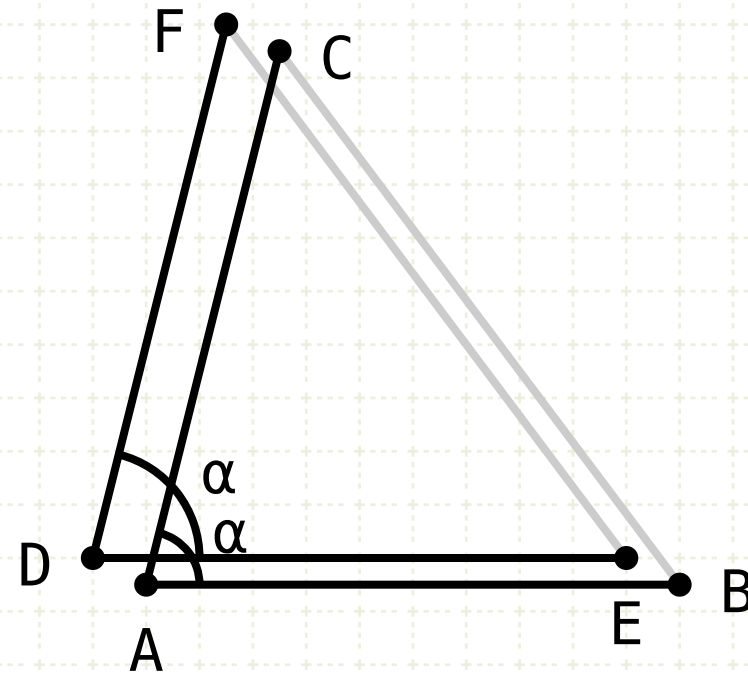
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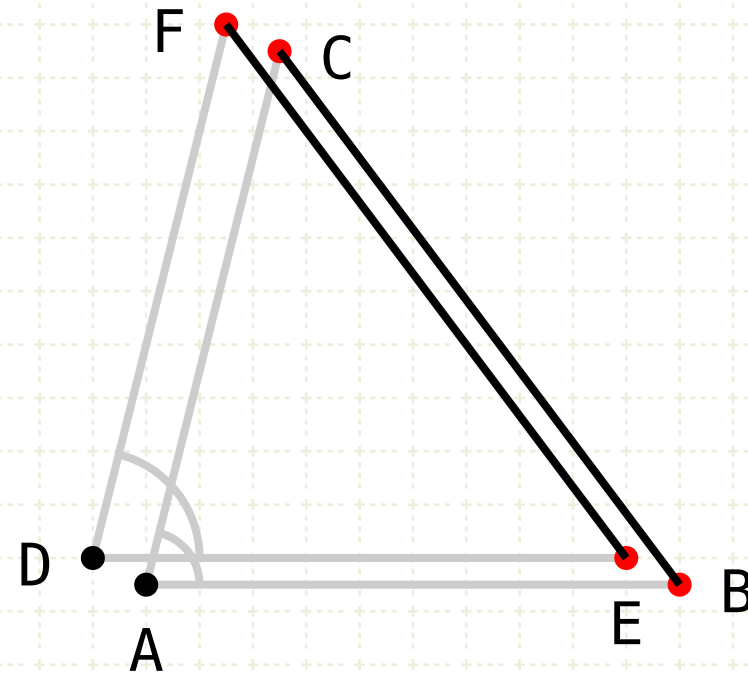
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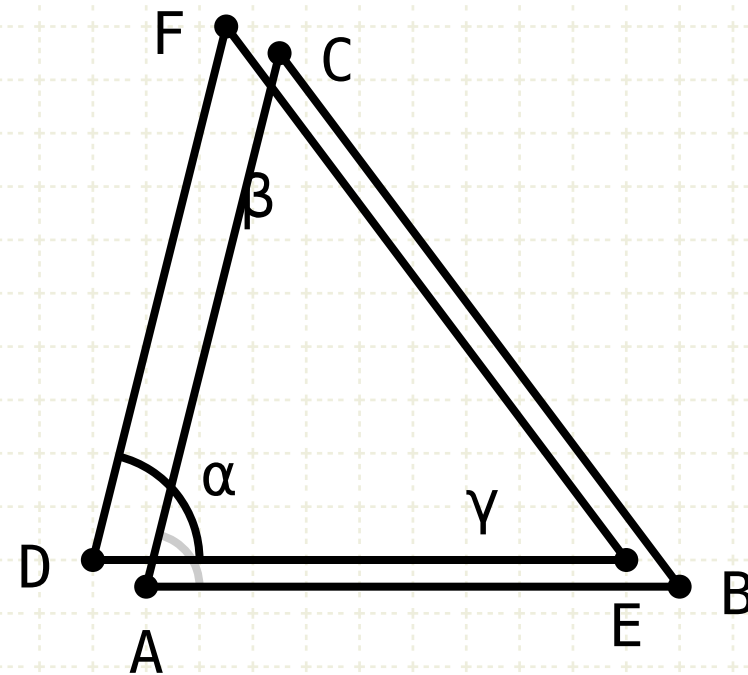
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From common notion 4, things which coincide with one another, equal one another



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