## Euclid's Elements

### Book I

If Euclid did not kindle your youthful enthusiasm, you were not born to be a scientific thinker.

Albert Einstein

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## Proposition 10 of Book I To bisect a given finite straight line.



## Proposition 10 of Book I To bisect a given finite straight line.

### In other words

Start with a line segment AB



To bisect a given finite straight line.

### In other words

Start with a line segment AB and cut it in half





To bisect a given finite straight line.

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### Construction:



## A = B

### Proposition 10 of Book I

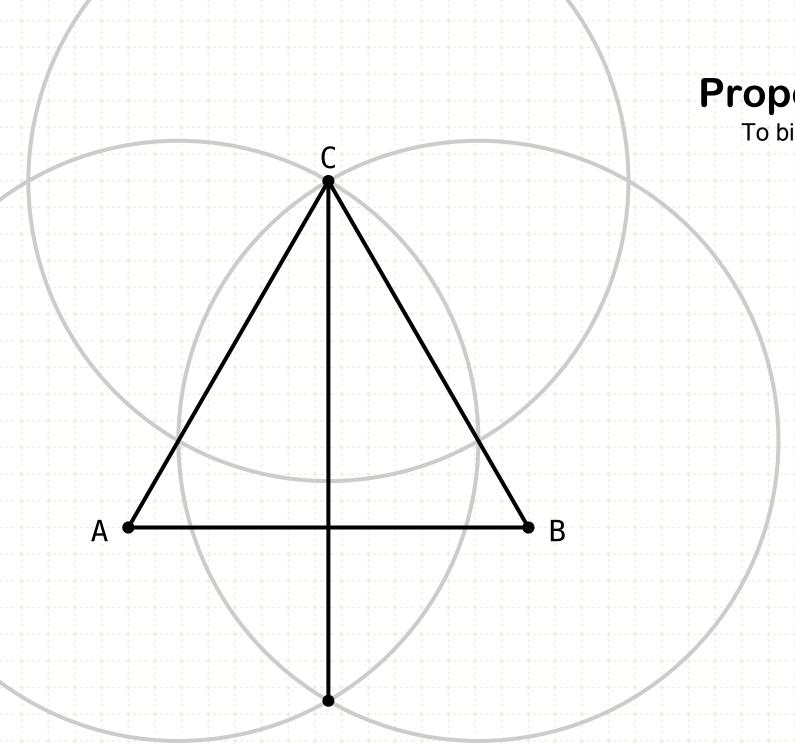
To bisect a given finite straight line.

### In other words

Start with a line segment AB and cut it in half

### Construction:

Construct an equilateral triangle on AB and label the vertex C (I·1)



To bisect a given finite straight line.

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Start with a line segment AB and cut it in half

### Construction:

Construct an equilateral triangle on AB and label the vertex C (I·1)

Bisect angle ACB, and extend line past the line segment AB (I-9)

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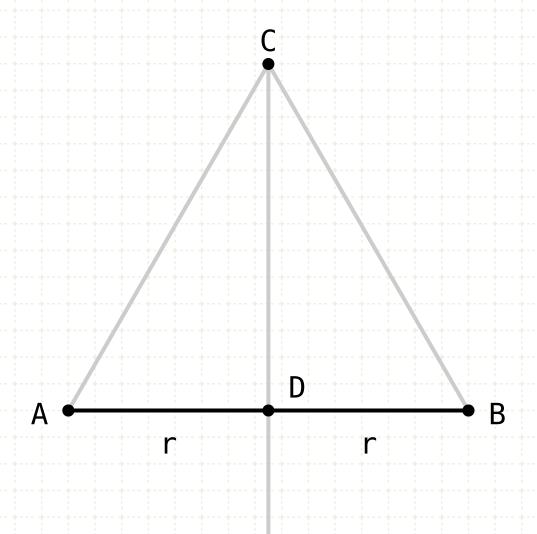
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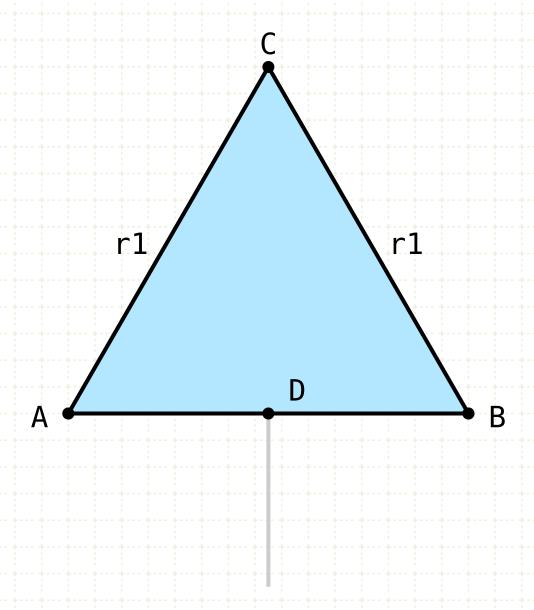
### Construction:

Construct an equilateral triangle on AB and label the vertex C (I·1)

Bisect angle ACB, and extend line past the line segment AB (I-9)

Line AD is equal to line DB

To bisect a given finite straight line.



AC = CB

### In other words

Start with a line segment AB and cut it in half

### Construction:

Construct an equilateral triangle on AB and label the vertex C (I·1)

Bisect angle ACB, and extend line past the line segment AB (I-9)

Line AD is equal to line DB

### **Proof**

AC equals BC since they are sides of an equilateral triangle

# 

### Proposition 10 of Book I

To bisect a given finite straight line.

AC = CB

 $\angle ACD = \angle BCD = \alpha$ 

### In other words

Start with a line segment AB and cut it in half

### Construction:

Construct an equilateral triangle on AB and label the vertex C (I·1)

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### **Proof**

AC equals BC since they are sides of an equilateral triangle Angle ACD equals BCD since we bisected angle ACB

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### **Proposition 10 of Book I**

To bisect a given finite straight line.

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### Construction:

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### **Proof**

AC equals BC since they are sides of an equilateral triangle Angle ACD equals BCD since we bisected angle ACB Since the two triangles ACD and CDB have two equal sides, and an equal angle between them,

# 

### **Proposition 10 of Book I**

To bisect a given finite straight line.

$$AC = CB$$
  
 $\angle ACD = \angle BCD = \alpha$   
 $AD = DB = r$ 

### In other words

Start with a line segment AB and cut it in half

### Construction:

Construct an equilateral triangle on AB and label the vertex C (I·1)

Bisect angle ACB, and extend line past the line segment AB (I-9)

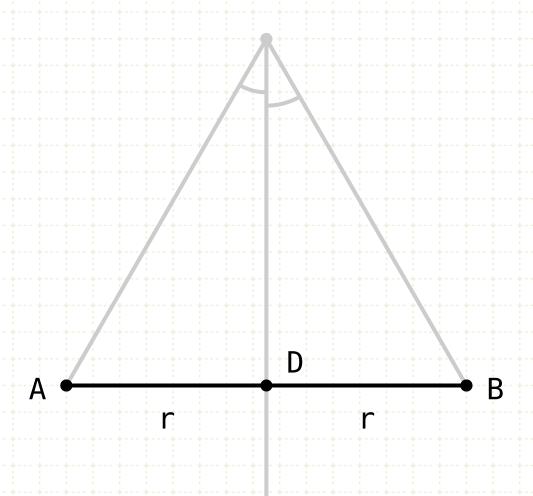
Line AD is equal to line DB

### **Proof**

AC equals BC since they are sides of an equilateral triangle Angle ACD equals BCD since we bisected angle ACB Since the two triangles ACD and CDB have two equal sides, and an equal angle between them,

then the third side of each triangle is equal (I-4)

To bisect a given finite straight line.



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