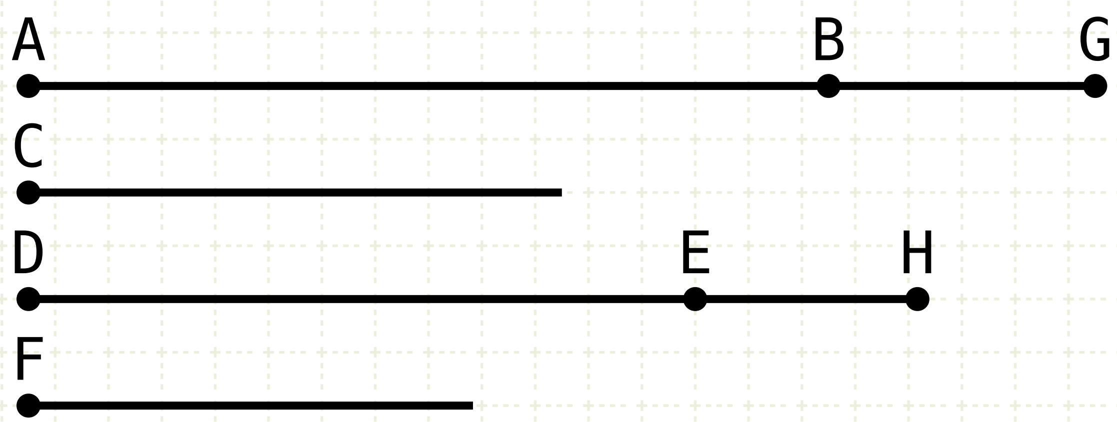
Euclid's Elements

Book V



$AB:C = DE:F$
$BG:C = EH:F$

$AG:C = DH:F$

Proportions are what makes the old Greek temples classic in their beauty. They are like huge blocks, from which the air has been literally hewn out between the columns.

Arne Jacobsen

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Proposition 22 of Book V

If there be any number of magnitudes whatever, and others equal to them in multitude, which taken two and two together are in the same ratio, they will also be in the same ratio EX AEQUALI

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ratio EX AEQUALI

$$a:b = d:e$$
$$b:c = e:f$$
$$\rightarrow a:c = d:f$$

Definitions

17. A ratio EX AEQUALI arises when, there being several magnitudes and another set equal to them in multitude which taken two and two are in the same proportion, as the first is to the last among the the first magnitudes, so is the first is to the last among the second magnitudes

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In other words

Given two sets of numbers A,B,C and D,E,F where A is to B as D is to E, and where B is to C as E is to F

Then they will also be in the same ratio EX AEQUALI (A is to C as D is to F)

$$A:B = D:E$$
$$B:C = E:F$$

$$\rightarrow A:C = D:F$$

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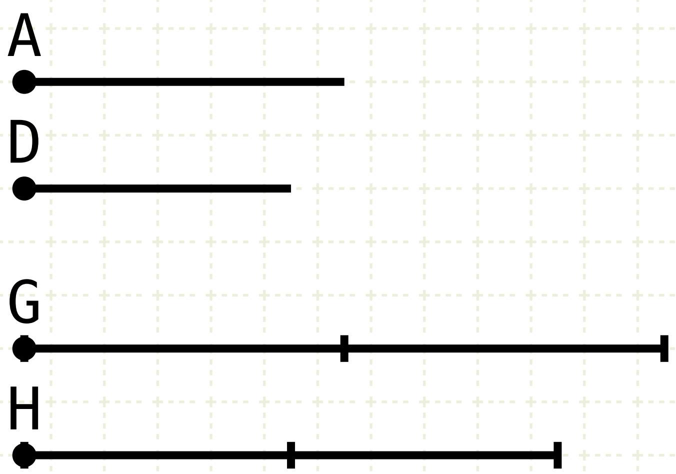

$$A:B = D:E$$
$$B:C = E:F$$

$$G = m\cdot A$$
$$H = m\cdot D$$

In other words

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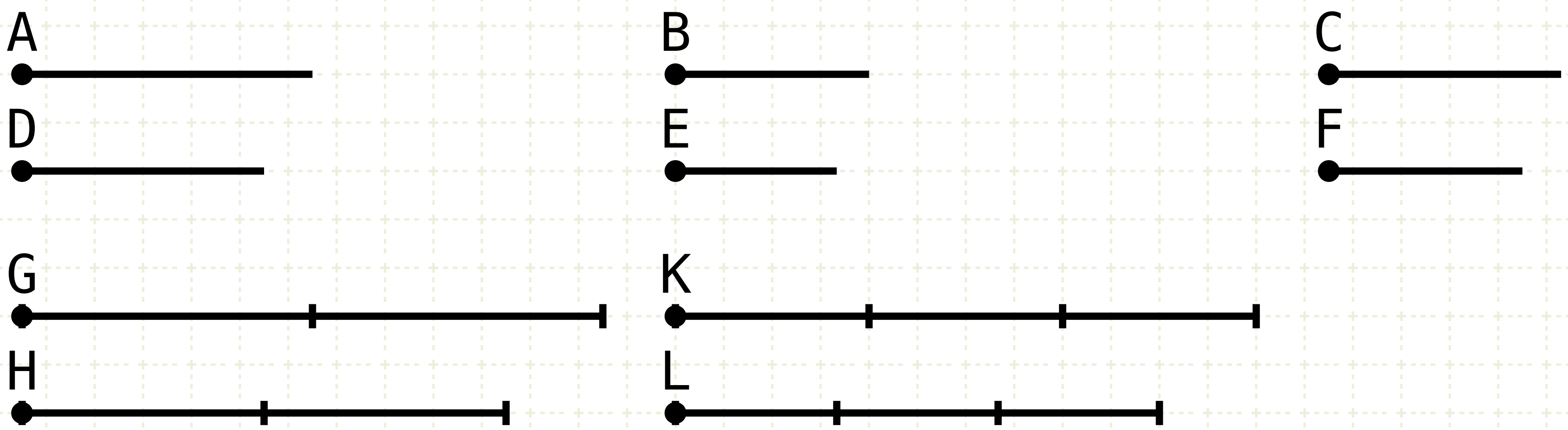
Proof

Let G,H be equimultiples of A and D

Proposition 22 of Book V

If there be any number of magnitudes whatever, and others equal to them in multitude, which taken two and two together are in the same ratio, they will also be in the same ratio EX AEQUALI



$A:B = D:E$

$B:C = E:F$

$G = m \cdot A$

$H = m \cdot D$

$K = n \cdot B$

$L = n \cdot E$

In other words

Given two sets of numbers A,B,C and D,E,F where A is to B as D is to E, and where B is to C as E is to F

Then they will also be in the same ratio EX AEQUALI (A is to C as D is to F)

Proof

Let G,H be equimultiples of A and D

Let K,L be equimultiples of B and E

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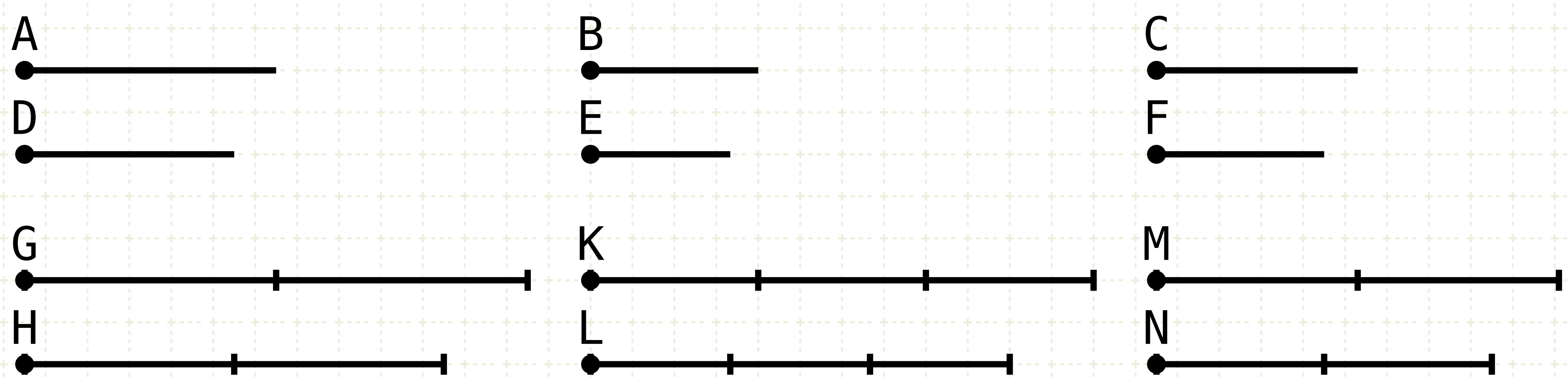

$$A:B = D:E$$
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$$H = m \cdot D$$
$$K = n \cdot B$$
$$L = n \cdot E$$
$$M = p \cdot C$$
$$N = p \cdot F$$

In other words

Given two sets of numbers A,B,C and D,E,F where A is to B as D is to E, and where B is to C as E is to F

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Proof

Let G,H be equimultiples of A and D

Let K,L be equimultiples of B and E

Let M,N be equimultiples of C and F

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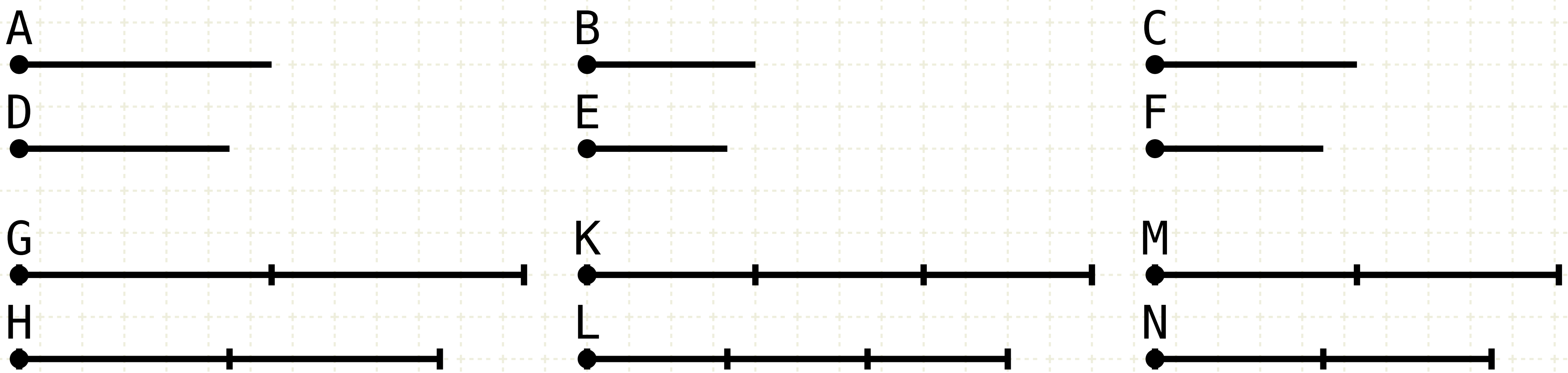

$A:B = D:E$

$B:C = E:F$

$m \cdot A : n \cdot B = m \cdot D : n \cdot E$

$G:K = H:L$

$G = m \cdot A$

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Given two sets of numbers A,B,C and D,E,F where A is to B as D is to E, and where B is to C as E is to F

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Proof

Let G,H be equimultiples of A and D

Let K,L be equimultiples of B and E

Let M,N be equimultiples of C and F

Since G,H are equimultiples of A,D and K,L are equimultiples of B,E and A is to D as B is to E, then G is to K as H is to L (V·4)

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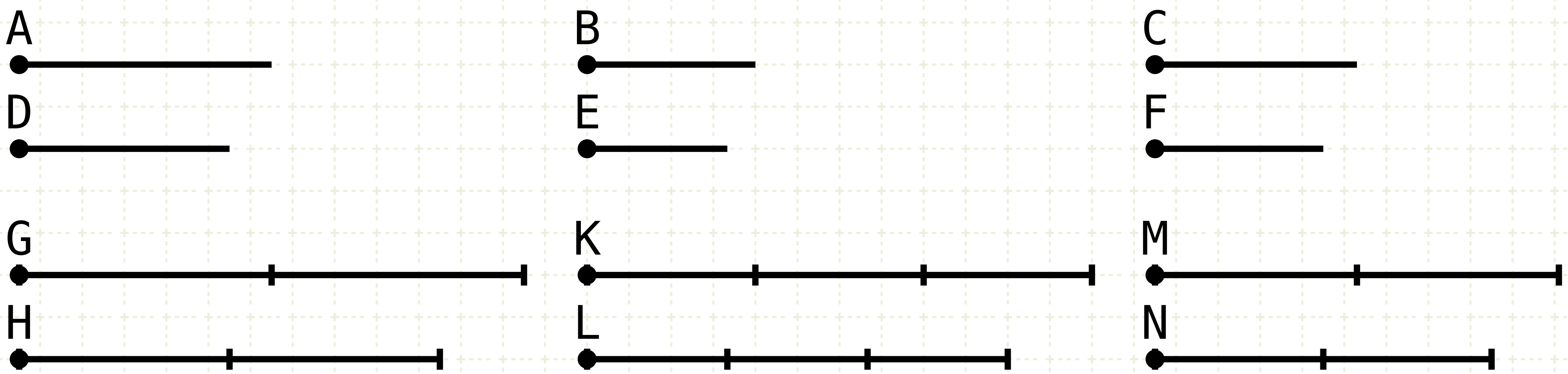

$A:B = D:E$

$B:C = E:F$

$m \cdot A : n \cdot B = m \cdot D : n \cdot E$

$G:K = H:L$

$K:M = L:N$

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In other words

Given two sets of numbers A,B,C and D,E,F where A is to B as D is to E, and where B is to C as E is to F

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Proof

Let G,H be equimultiples of A and D

Let K,L be equimultiples of B and E

Let M,N be equimultiples of C and F

Since G,H are equimultiples of A,D and K,L are equimultiples of B,E and A is to D as B is to E, then G is to K as H is to L (V·4)

Similarly it can be shown that K is to M, so is L to N

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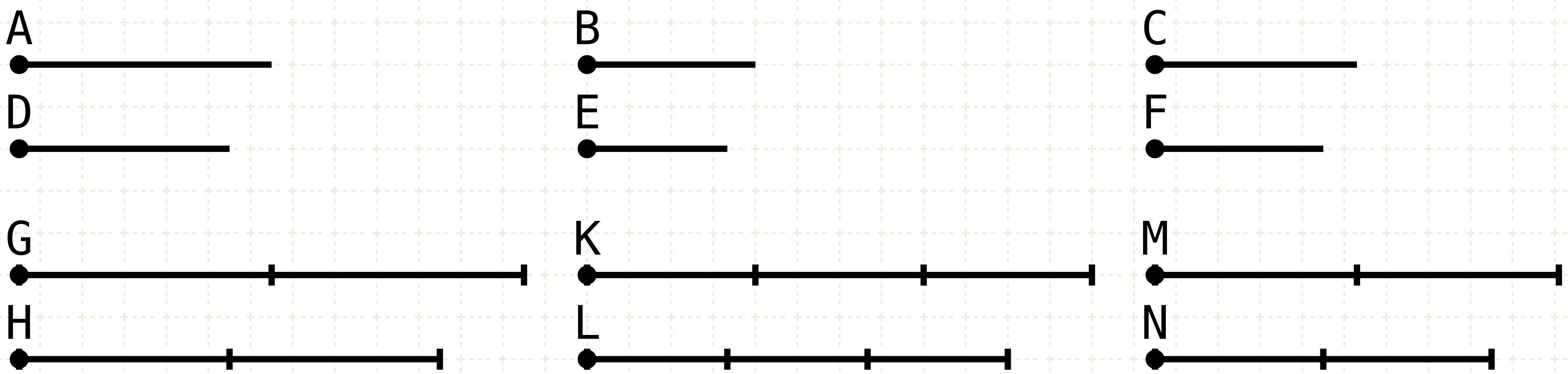

$A:B = D:E$

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$G = m \cdot A$

$H = m \cdot D$

$K = n \cdot B$

$L = n \cdot E$

$M = p \cdot C$

$N = p \cdot F$

$m \cdot A : n \cdot B = m \cdot D : n \cdot E$

$G:K = H:L$

$K:M = L:N$

$G >=< M \rightarrow H >=< N$

In other words

Given two sets of numbers A,B,C and D,E,F where A is to B as D is to E, and where B is to C as E is to F

Then they will also be in the same ratio EX AEQUALI (A is to C as D is to F)

Proof

Let G,H be equimultiples of A and D

Let K,L be equimultiples of B and E

Let M,N be equimultiples of C and F

Since G,H are equimultiples of A,D and K,L are equimultiples of B,E and A is to D as B is to E, then G is to K as H is to L (V·4)

Similarly it can be shown that K is to M, so is L to N

With three magnitudes G,K,M and another three magnitudes H,L,N are two by two equal in ratios, then if G is in excess of M, H is in excess of N, etc. (V·20)

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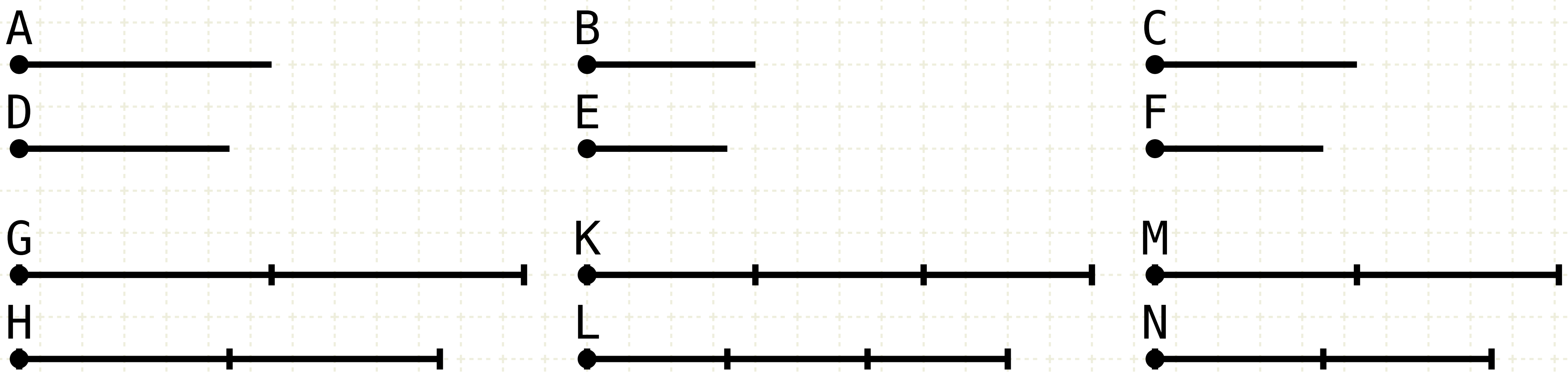

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$m \cdot A : n \cdot B = m \cdot D : n \cdot E$

$G:K = H:L$

$K:M = L:N$

$G >=< M \rightarrow H >=< N$

$m \cdot A >=< p \cdot C \rightarrow m \cdot D >=< p \cdot F$

$A:C = D:F$

In other words

Given two sets of numbers A,B,C and D,E,F where A is to B as D is to E, and where B is to C as E is to F

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Proof

Let G,H be equimultiples of A and D

Let K,L be equimultiples of B and E

Let M,N be equimultiples of C and F

Since G,H are equimultiples of A,D and K,L are equimultiples of B,E and A is to D as B is to E, then G is to K as H is to L (V·4)

Similarly it can be shown that K is to M, so is L to N

With three magnitudes G,K,M and another three magnitudes H,L,N are two by two equal in ratios, then if G is in excess of M, H is in excess of N, etc. (V·20)

G,H are equimultiples of A,D and M,N are chance equimultiples of C,F, therefore A is to C so is D to F (V·def·5)

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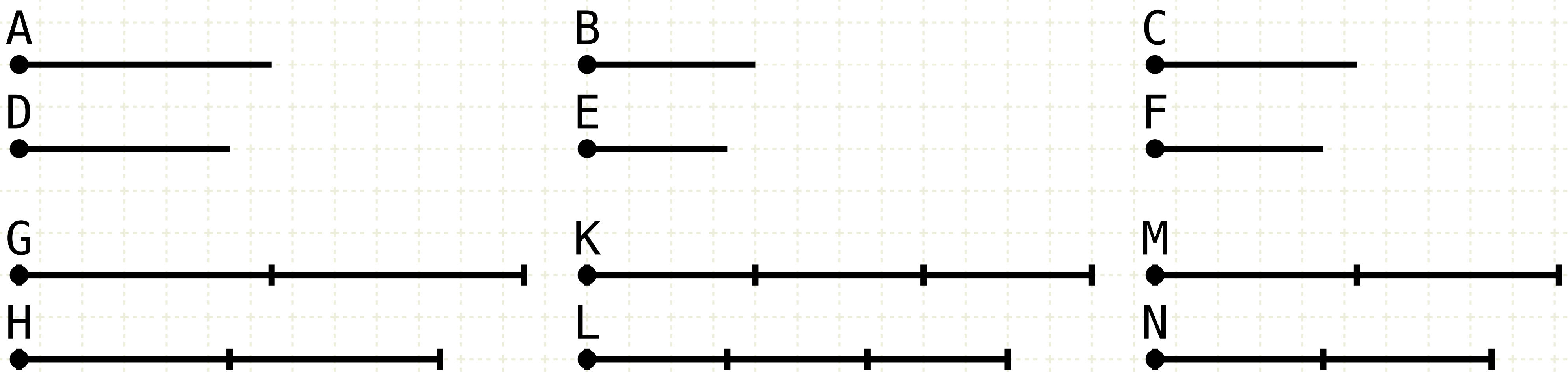

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$G:K = H:L$

$K:M = L:N$

$G >=< M \rightarrow H >=< N$

$m \cdot A >=< p \cdot C \rightarrow m \cdot D >=< p \cdot F$

$A:C = D:F$

In other words

Given two sets of numbers A,B,C and D,E,F where A is to B as D is to E, and where B is to C as E is to F

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Proof

Let G,H be equimultiples of A and D

Let K,L be equimultiples of B and E

Let M,N be equimultiples of C and F

Since G,H are equimultiples of A,D and K,L are equimultiples of B,E and A is to D as B is to E, then G is to K as H is to L (V·4)

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