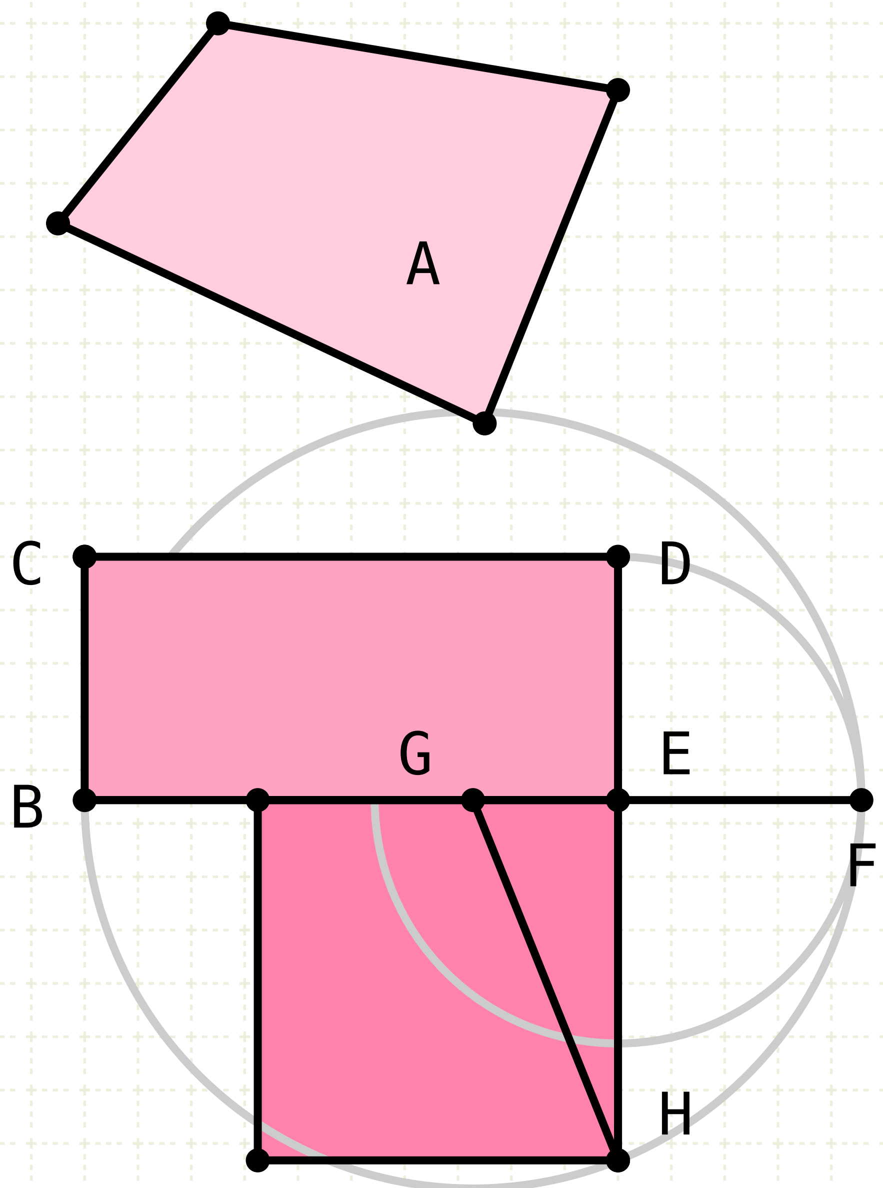
Euclid's Elements

Book II



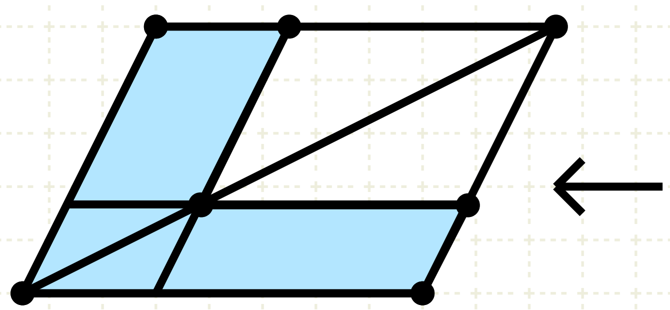
It is a remarkable fact in the history of geometry, that the Elements of Euclid, written two thousand years ago, are still regarded by many as the best introduction to the mathematical sciences.

Florian Cajori,

A History of Mathematics (1893)

Definitions:

Any rectangular parallelogram is said to be contained by the two straight lines containing the right angle.



And in any parallelogrammic area let any one whatever of the parallelograms about its diameter with the two complements be called a gnomon.

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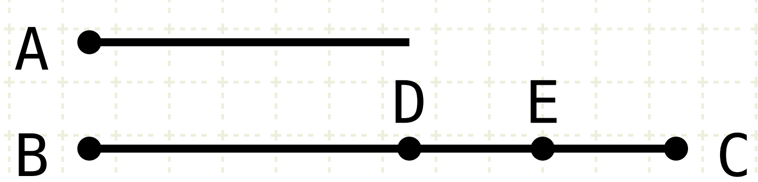
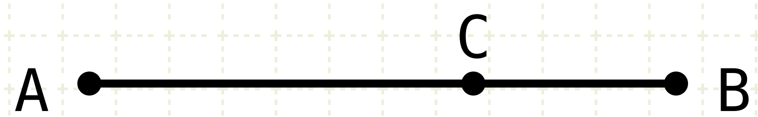


$A \cdot BC = A \cdot BD + A \cdot DE + A \cdot EC$

2

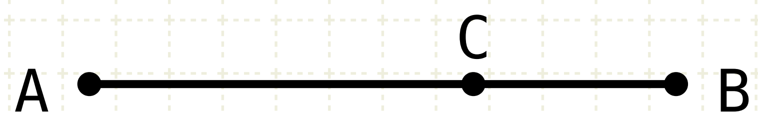


$AB^2 = AB \cdot AC + AB \cdot BC$

3

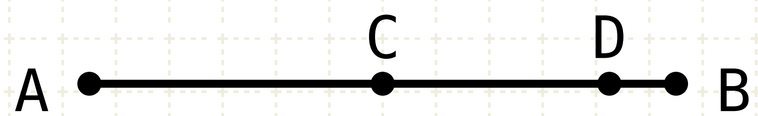
$AB \cdot CB = AC \cdot CB + CB^2$

4



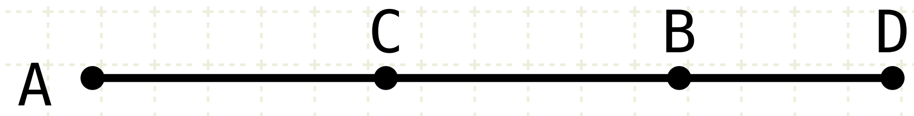
$AB^2 = AC^2 + CB^2 + 2 \cdot AC \cdot CB$

5



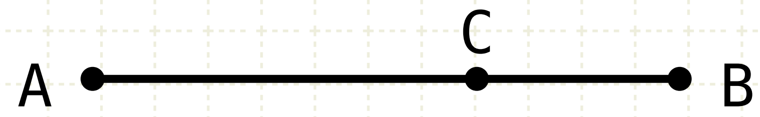
$AD \cdot DB + CD^2 = CB^2$

6



$AD \cdot DB + CB^2 = CD^2$

7

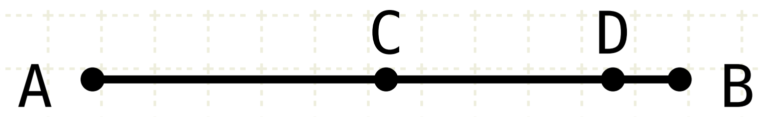


$AB^2 + BC^2 = AC^2 + 2 \cdot AB \cdot BC$

8

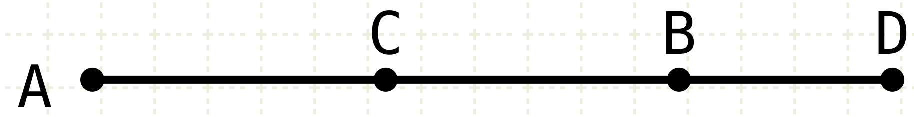
$4 \cdot AB \cdot BC + AC^2 = (AB+BC)^2$

9



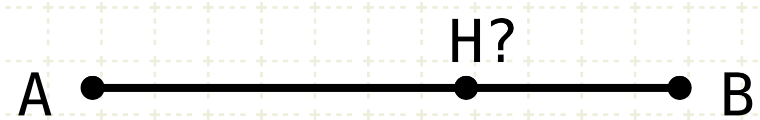
$AD^2 + DB^2 = 2 \cdot (AC^2 + CD^2)$

10



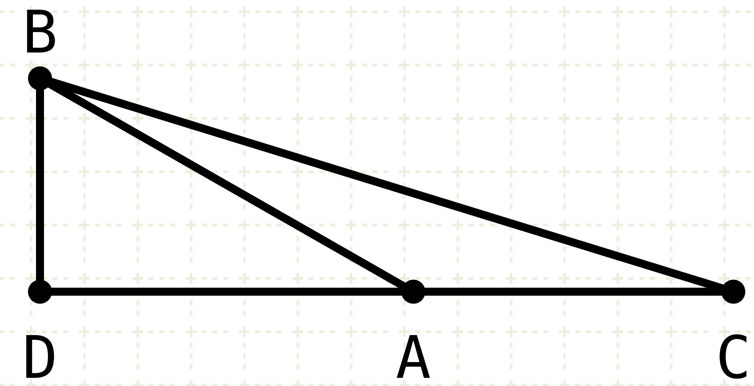
$AD^2 + DB^2 = 2 \cdot (AC^2 + CD^2)$

11



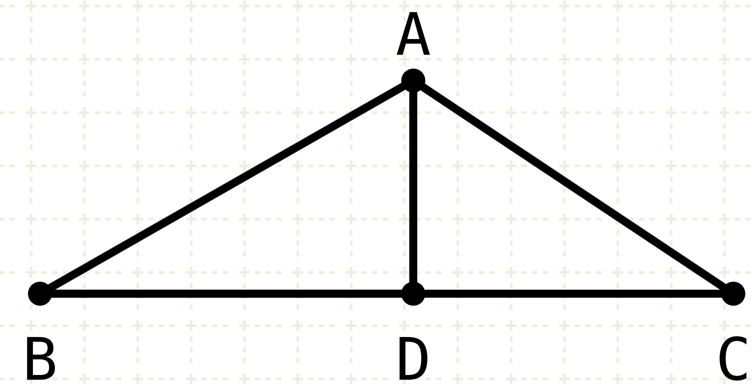
Find H. $\mathbf{AB \cdot BH = AH^2}$

12



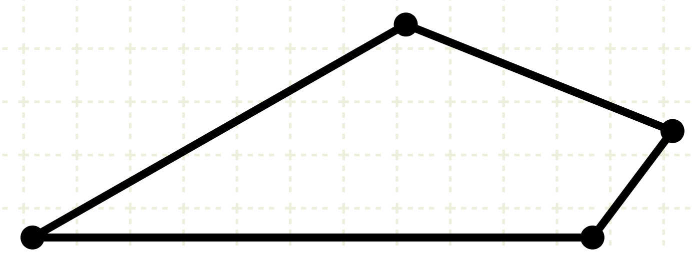
Cosine Law. $BC^2 = AB^2+AC^2+2 \cdot AD \cdot AC$

13



Cosine Law. $AC^2 = AB^2+BC^2-2 \cdot BD \cdot BC$

14



Find square of polygon

Proposition 11 of Book II

To cut a given straight line so that the rectangle contained by the whole and one of the segments is equal to the square on the remaining segment.

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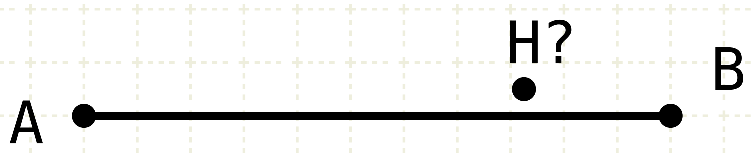


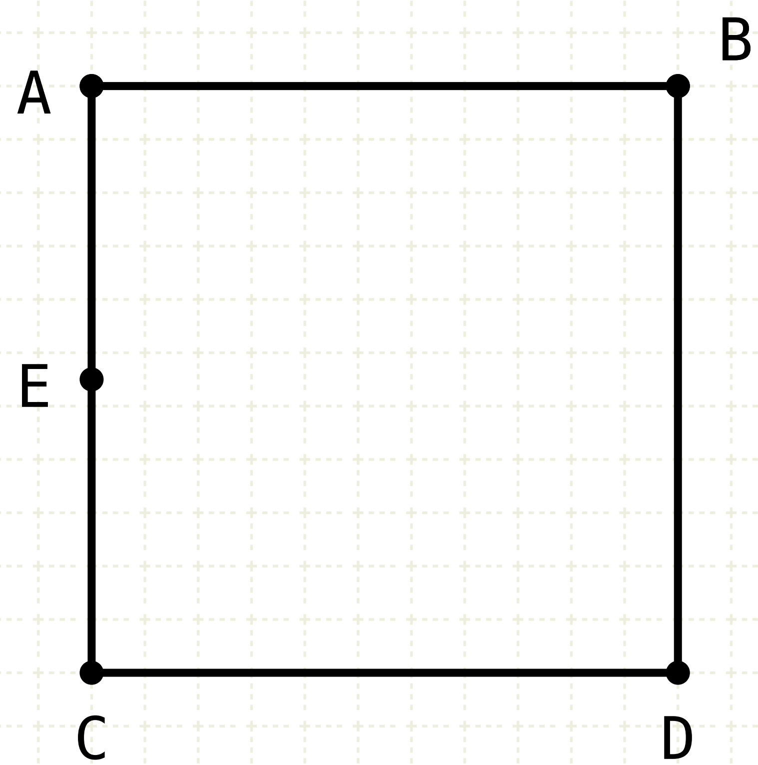
$$AB \cdot BH = AH^2$$

In other words

Find the point H on the line AB such that the rectangle formed by AB and BH is equal to the square on AH

Proposition 11 of Book II

To cut a given straight line so that the rectangle contained by the whole and one of the segments is equal to the square on the remaining segment.



AE = EC

In other words

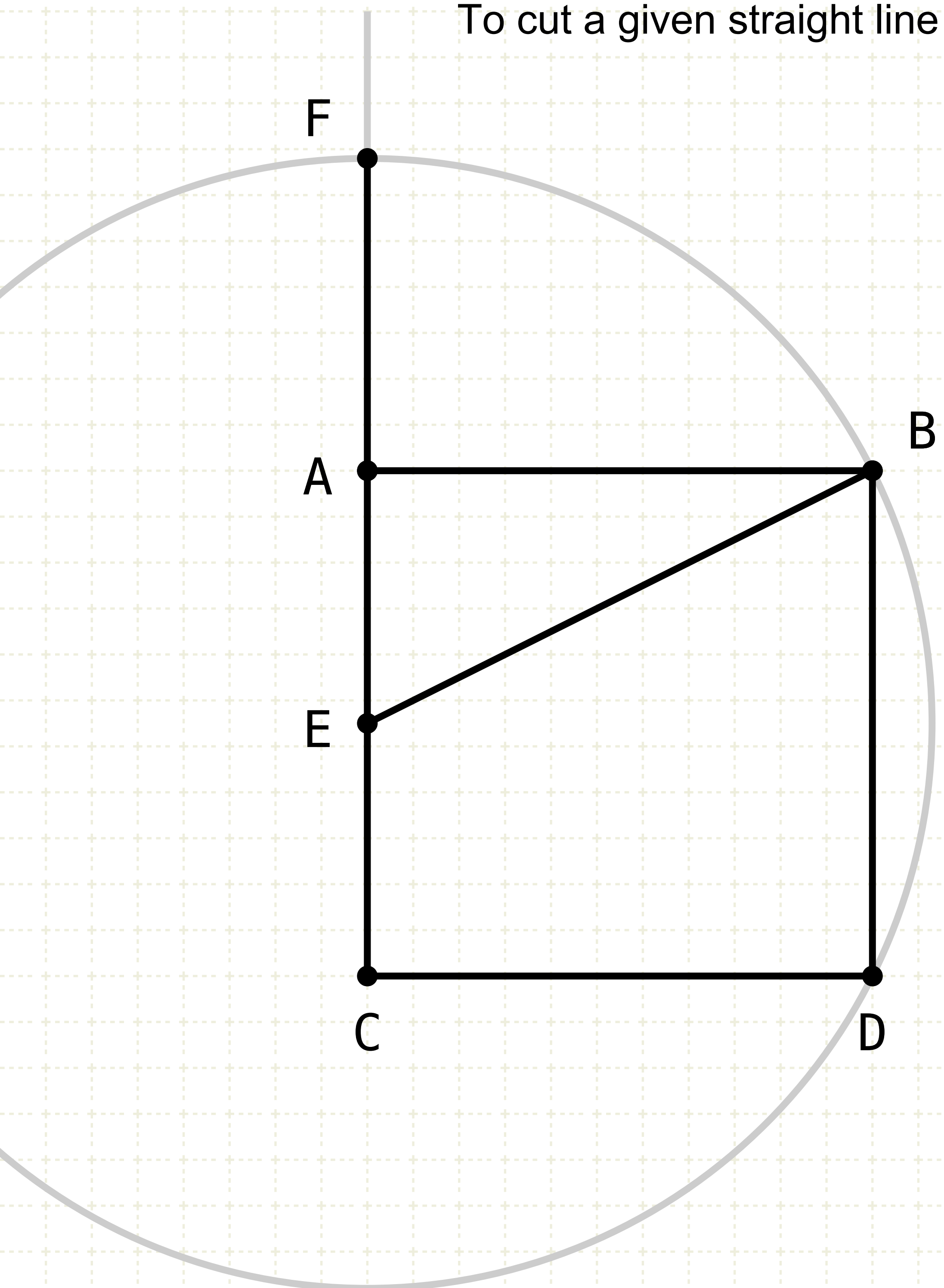
Find the point H on the line AB such that the rectangle formed by AB and BH is equal to the square on AH

Construction

Draw a square ABCD on AB (I·46), and bisect AC (I·10) at point E

Proposition 11 of Book II

To cut a given straight line so that the rectangle contained by the whole and one of the segments is equal to the square on the remaining segment.



AE = EC
EF = EB

In other words

Find the point H on the line AB such that the rectangle formed by AB and BH is equal to the square on AH

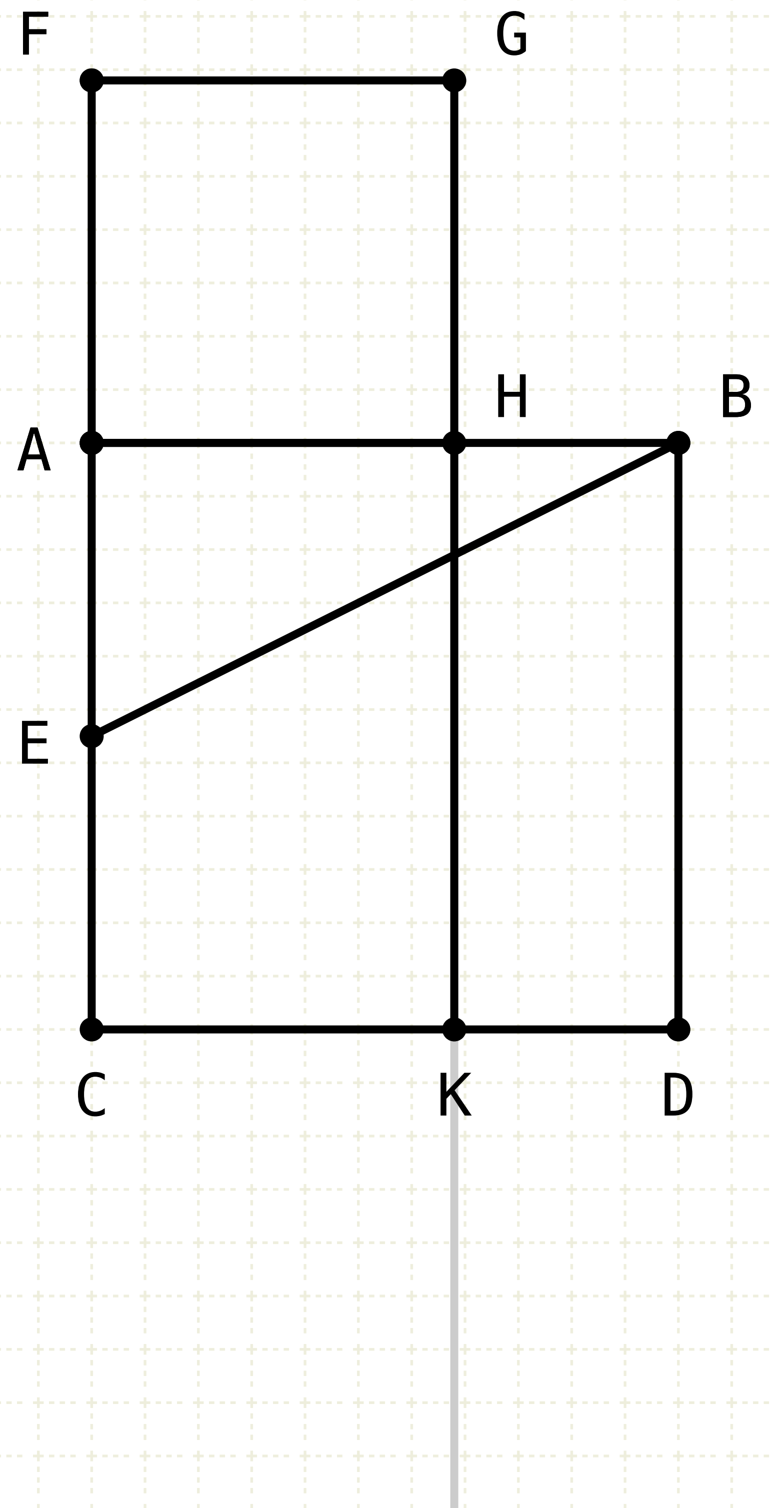
Construction

Draw a square ABCD on AB (I·46), and bisect AC (I·10) at point E

Let EB be joined, and extend CA to F such that EF equals AB

Proposition 11 of Book II

To cut a given straight line so that the rectangle contained by the whole and one of the segments is equal to the square on the remaining segment.



AE = EC
EF = EB
FA = AH

In other words

Find the point H on the line AB such that the rectangle formed by AB and BH is equal to the square on AH

Construction

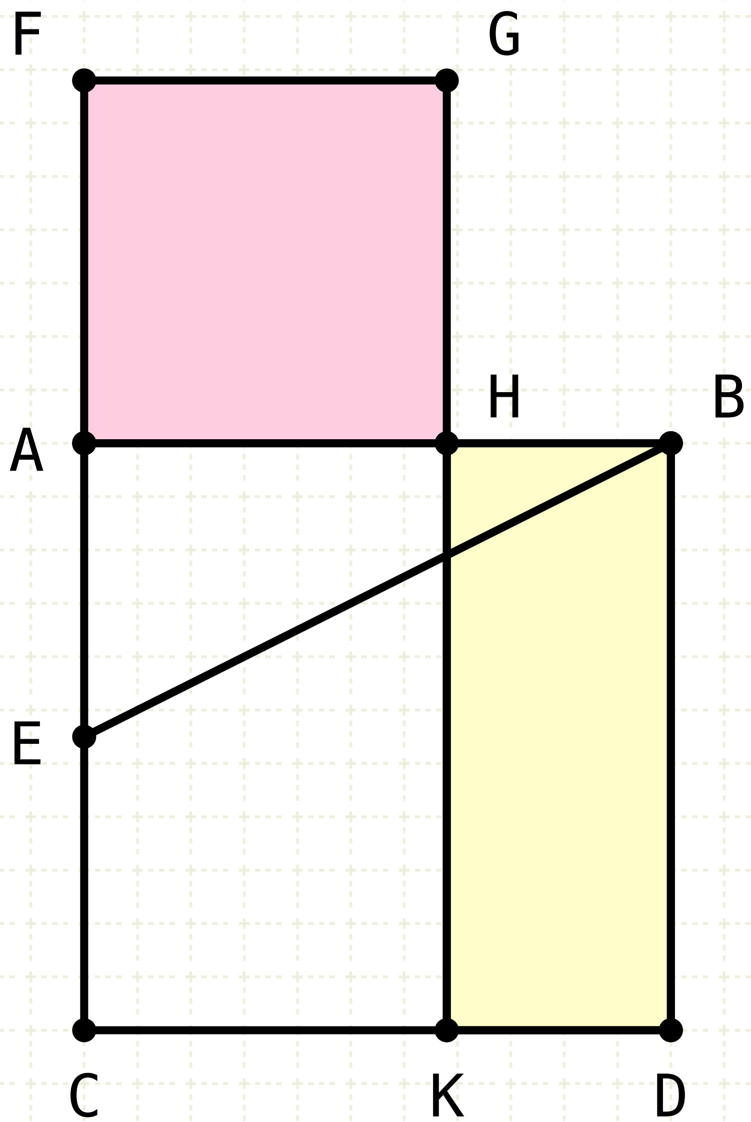
Draw a square ABCD on AB (I·46), and bisect AC (I·10) at point E

Let EB be joined, and extend CA to F such that EF equals AB

Draw a square FAGH on FA, and extend GH to line CD at point K

Proposition 11 of Book II

To cut a given straight line so that the rectangle contained by the whole and one of the segments is equal to the square on the remaining segment.



AE = EC

EF = EB

FA = AH

AB·BH = AH·AH

In other words

Find the point H on the line AB such that the rectangle formed by AB and BH is equal to the square on AH

Construction

Draw a square ABCD on AB (I·46), and bisect AC (I·10) at point E

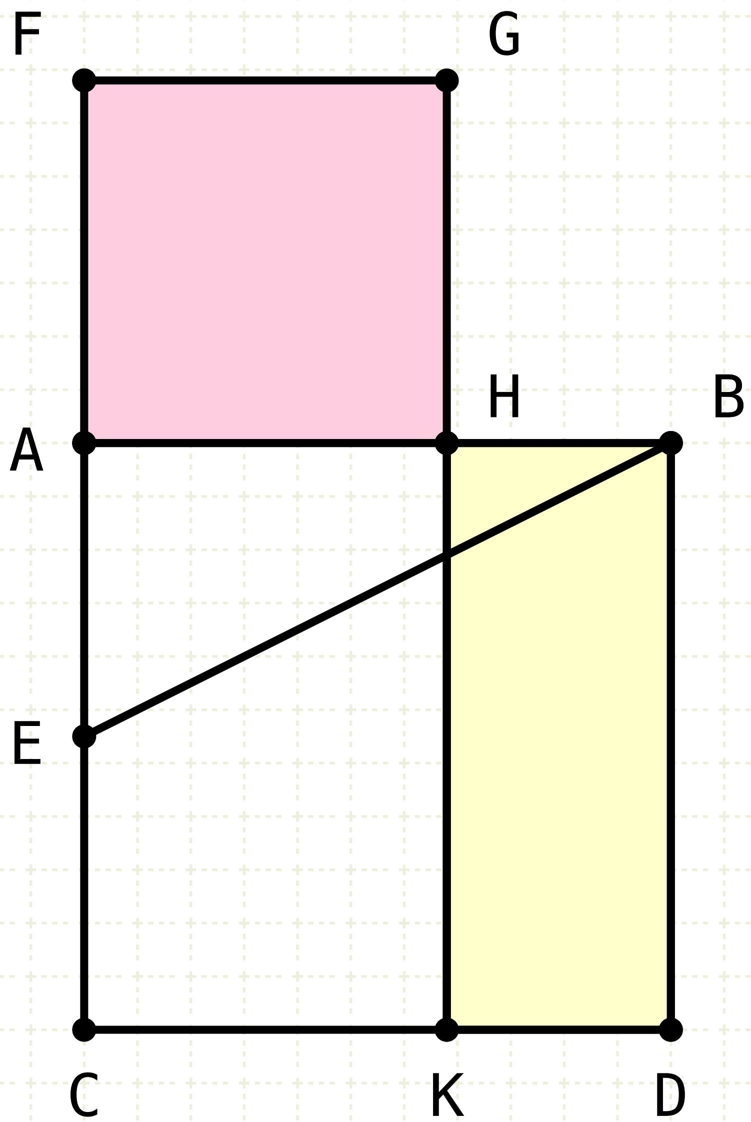
Let EB be joined, and extend CA to F such that EF equals AB

Draw a square FAGH on FA, and extend GH to line CD at point K

The point H has been defined such that FH equals HD

Proposition 11 of Book II

To cut a given straight line so that the rectangle contained by the whole and one of the segments is equal to the square on the remaining segment.



$AE = EC$

$EF = EB$

$FA = AH$

$AB \cdot BH = AH \cdot AH$

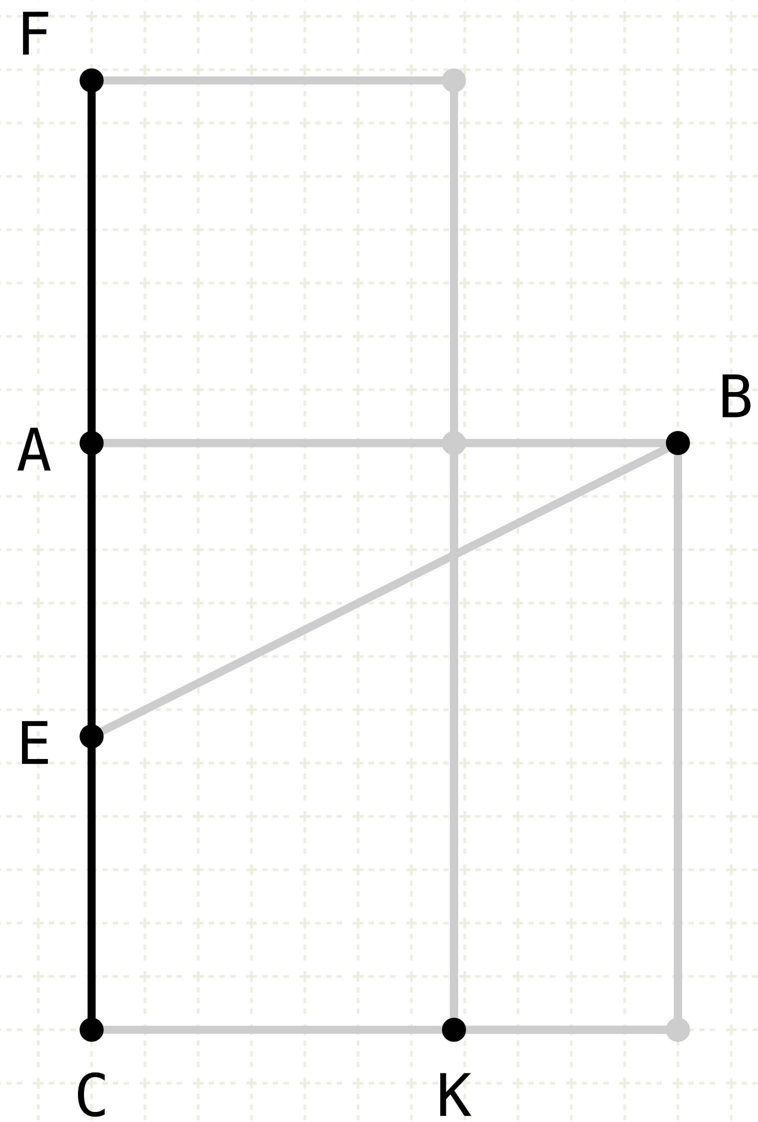
In other words

Find the point H on the line AB such that the rectangle formed by AB and BH is equal to the square on AH

Proof

Proposition 11 of Book II

To cut a given straight line so that the rectangle contained by the whole and one of the segments is equal to the square on the remaining segment.



$AE = EC$

$EF = EB$

$FA = AH$

$CF \cdot AF + AE^2 = EF^2$

In other words

Find the point H on the line AB such that the rectangle formed by AB and BH is equal to the square on AH

Proof

From proposition 6 (II·6), if we have a bisected line, and an addition to that line, then the extended line CF times the extension AF plus the square on AE is equal to the square on EF

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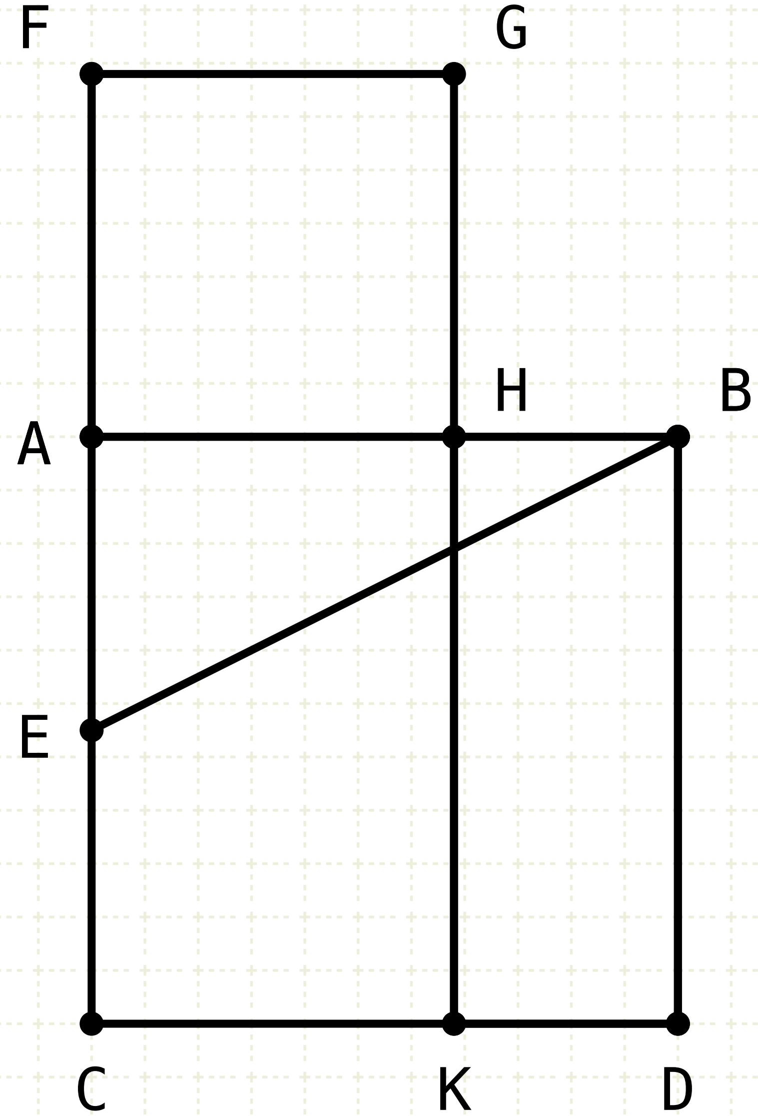
To cut a given straight line so that the rectangle contained by the whole and one of the segments is equal to the square on the remaining segment.



AE = EC

EF = EB

FA = AH

$CF \cdot AF + AE^2 = EF^2$

$CF \cdot AF + AE^2 = EB^2$

In other words

Find the point H on the line AB such that the rectangle formed by AB and BH is equal to the square on AH

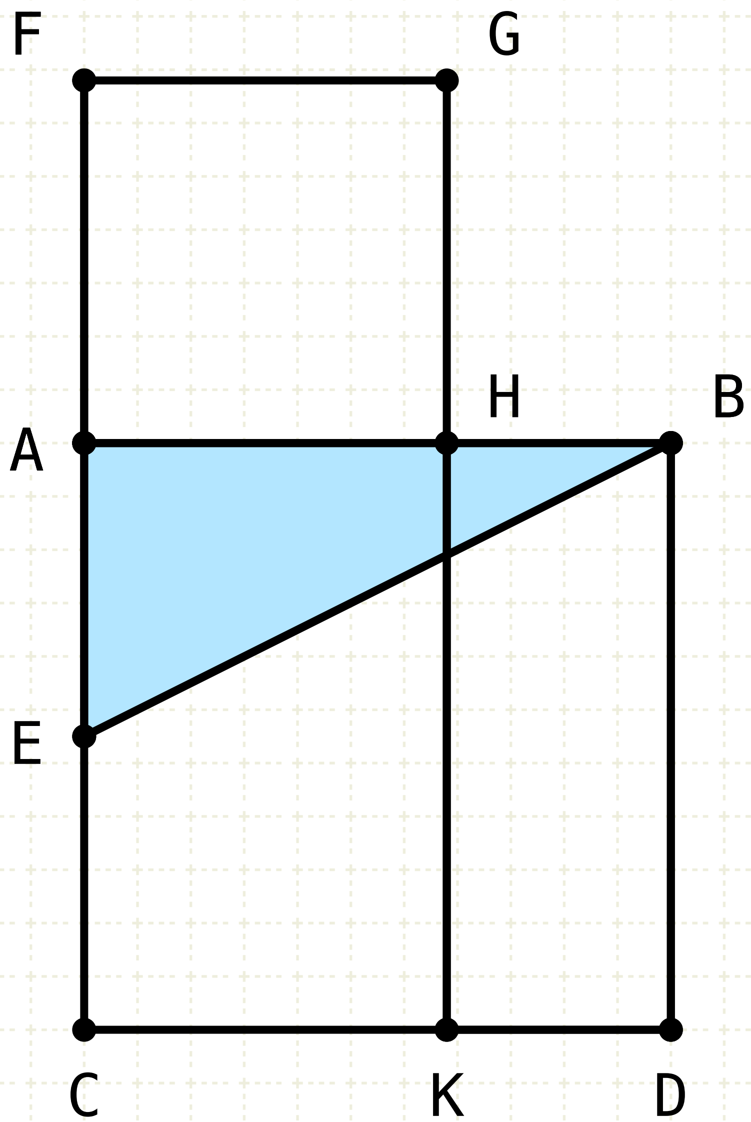
Proof

From proposition 6 (II·6), if we have a bisected line, and an addition to that line, then the extended line CF times the extension AF plus the square on AE is equal to the square on EF

But EB equals EF

Proposition 11 of Book II

To cut a given straight line so that the rectangle contained by the whole and one of the segments is equal to the square on the remaining segment.



$AE = EC$
$EF = EB$
$FA = AH$

$CF \cdot AF + AE^2 = EF^2$
$CF \cdot AF + AE^2 = EB^2$
$AB^2 + AE^2 = EB^2$

In other words

Find the point H on the line AB such that the rectangle formed by AB and BH is equal to the square on AH

Proof

From proposition 6 (II·6), if we have a bisected line, and an addition to that line, then the extended line CF times the extension AF plus the square on AE is equal to the square on EF

But EB equals EF

Triangle AEB is right angled, thus the square on AB plus the square on AE equals the square on EB (I·47)

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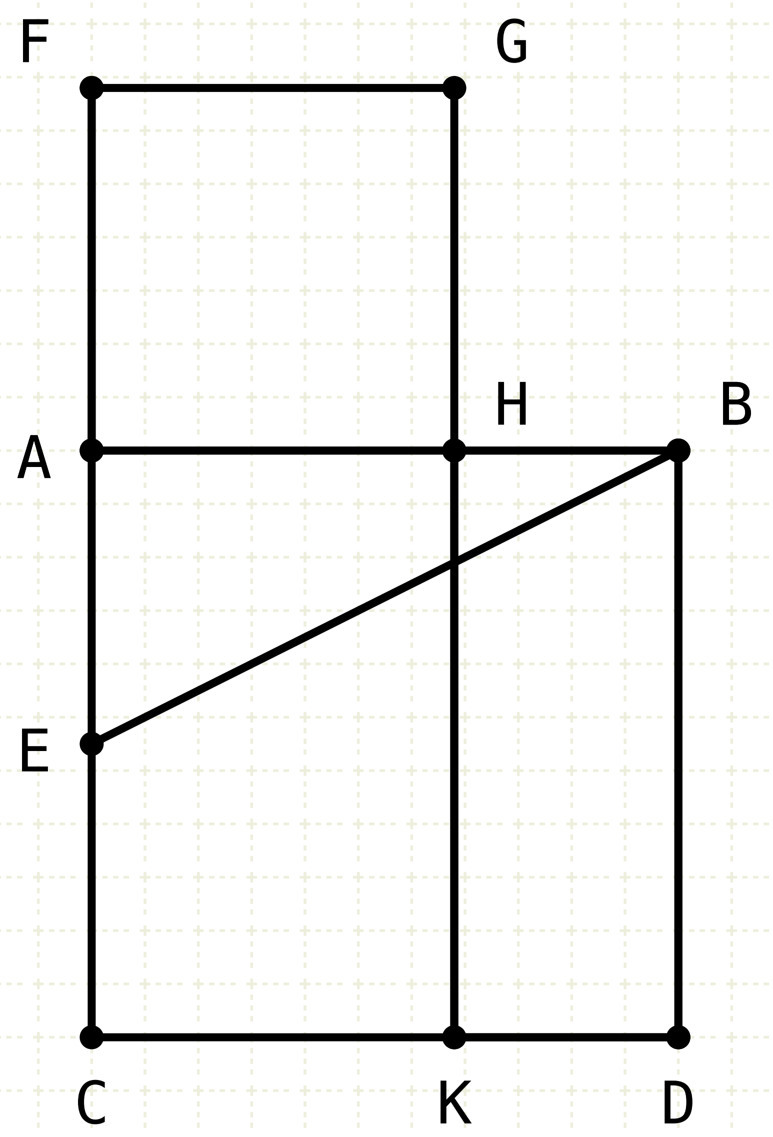
To cut a given straight line so that the rectangle contained by the whole and one of the segments is equal to the square on the remaining segment.



$AE = EC$
$EF = EB$
$FA = AH$

$CF \cdot AF + AE^2 = EF^2$
$CF \cdot AF + AE^2 = EB^2$
$AB^2 + AE^2 = EB^2$
$AB^2 = CF \cdot AF$

In other words

Find the point H on the line AB such that the rectangle formed by AB and BH is equal to the square on AH

Proof

From proposition 6 (II·6), if we have a bisected line, and an addition to that line, then the extended line CF times the extension AF plus the square on AE is equal to the square on EF

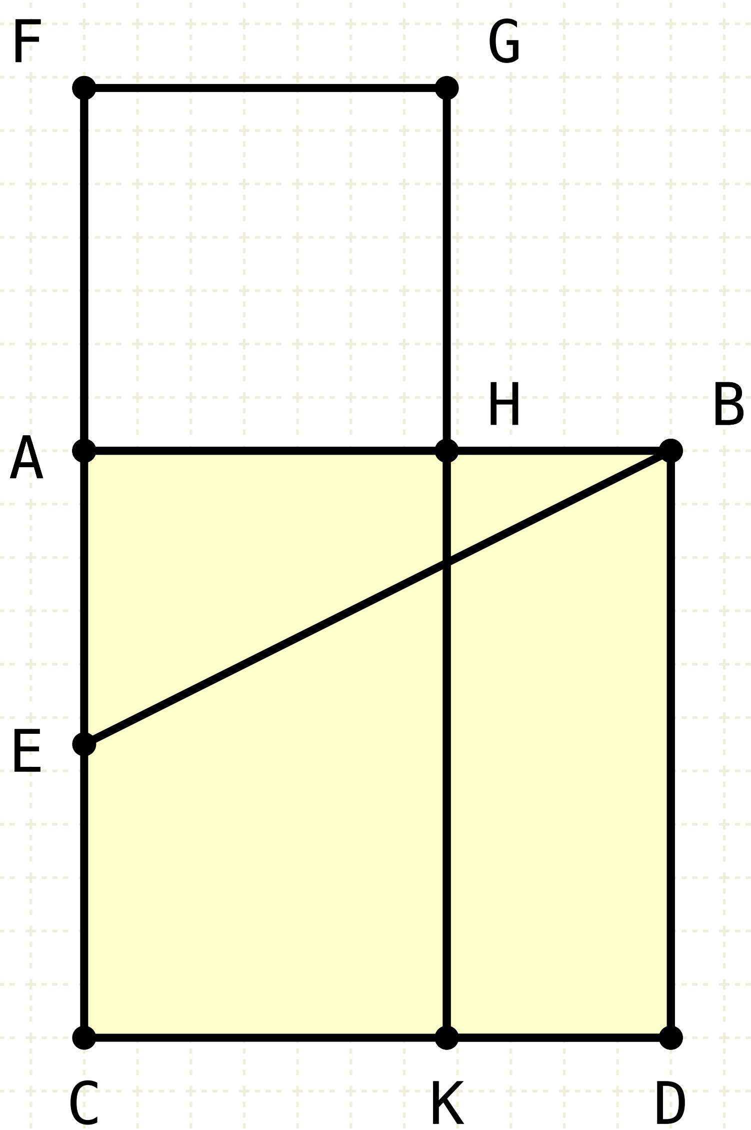
But EB equals EF

Triangle AEB is right angled, thus the square on AB plus the square on AE equals the square on EB (I·47)

By comparing the equalities, we see that the square of AB is equal to the rectangle formed by CF and AF

Proposition 11 of Book II

To cut a given straight line so that the rectangle contained by the whole and one of the segments is equal to the square on the remaining segment.



$AE = EC$

$EF = EB$

$FA = AH$

$CF \cdot AF + AE^2 = EF^2$

$CF \cdot AF + AE^2 = EB^2$

$AB^2 + AE^2 = EB^2$

$AB^2 = CF \cdot AF$

$AB^2 = \square AD$

In other words

Find the point H on the line AB such that the rectangle formed by AB and BH is equal to the square on AH

Proof

From proposition 6 (II·6), if we have a bisected line, and an addition to that line, then the extended line CF times the extension AF plus the square on AE is equal to the square on EF

But EB equals EF

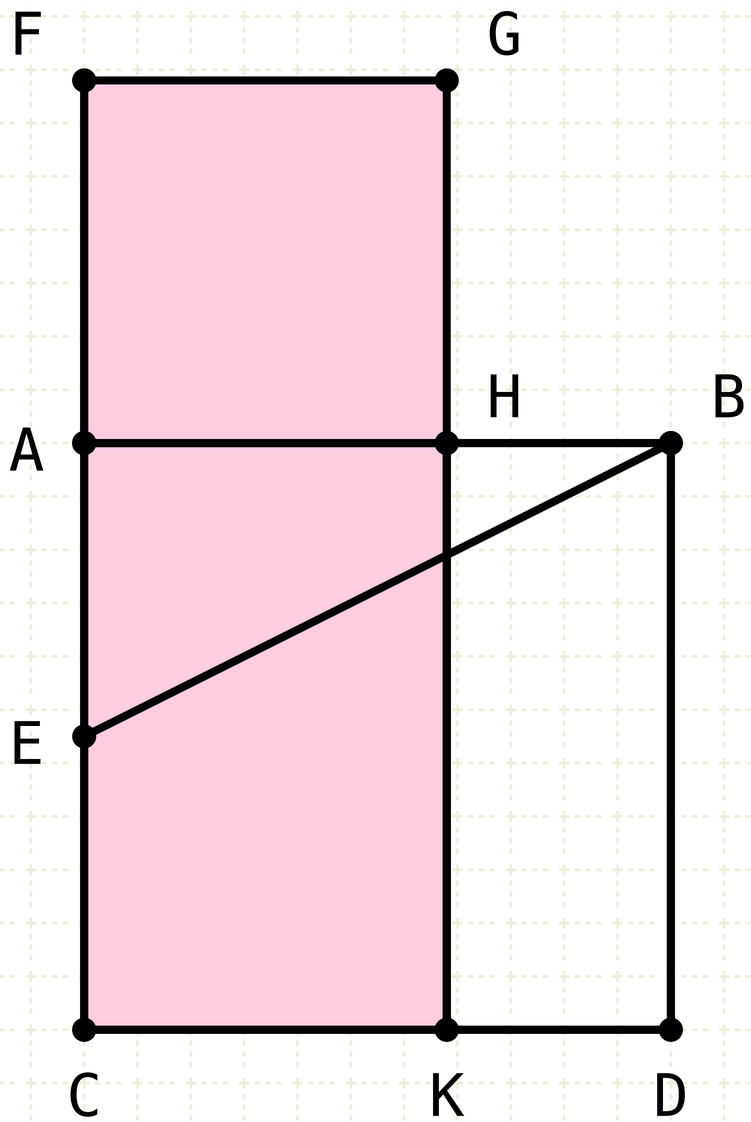
Triangle AEB is right angled, thus the square on AB plus the square on AE equals the square on EB (I·47)

By comparing the equalities, we see that the square of AB is equal to the rectangle formed by CF and AF

The square of AB is the the rectangle AD

Proposition 11 of Book II

To cut a given straight line so that the rectangle contained by the whole and one of the segments is equal to the square on the remaining segment.



$AE = EC$
$EF = EB$
$FA = AH$

$CF \cdot AF + AE^2 = EF^2$
$CF \cdot AF + AE^2 = EB^2$
$AB^2 + AE^2 = EB^2$
$AB^2 = CF \cdot AF$
$AB^2 = \square AD$
$CF \cdot AF = \square FK$

In other words

Find the point H on the line AB such that the rectangle formed by AB and BH is equal to the square on AH

Proof

From proposition 6 (II·6), if we have a bisected line, and an addition to that line, then the extended line CF times the extension AF plus the square on AE is equal to the square on EF

But EB equals EF

Triangle AEB is right angled, thus the square on AB plus the square on AE equals the square on EB (I·47)

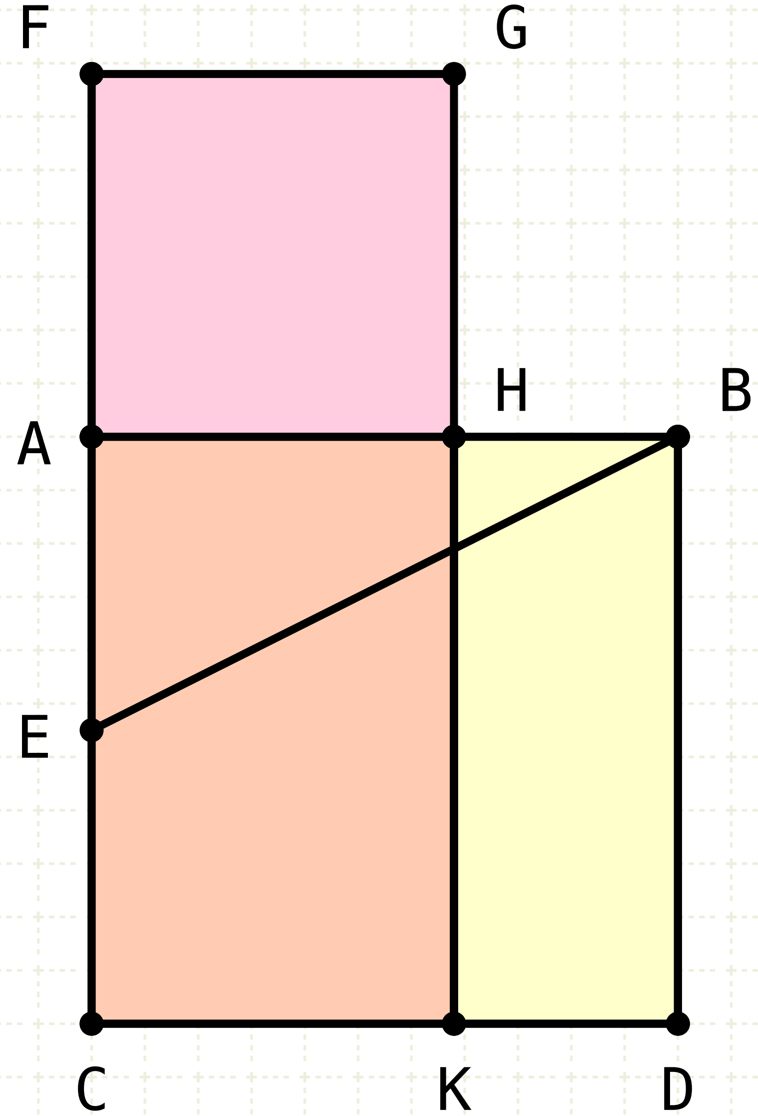
By comparing the equalities, we see that the square of AB is equal to the rectangle formed by CF and AF

The square of AB is the the rectangle AD

The rectangle CF,AF is the rectangle FK, since AF equal AH

Proposition 11 of Book II

To cut a given straight line so that the rectangle contained by the whole and one of the segments is equal to the square on the remaining segment.



$AE = EC$
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$CF \cdot AF + AE^2 = EF^2$
$CF \cdot AF + AE^2 = EB^2$
$AB^2 + AE^2 = EB^2$
$AB^2 = CF \cdot AF$
$AB^2 = \square AD$
$CF \cdot AF = \square FK$
$\square AD = \square FK$

In other words

Find the point H on the line AB such that the rectangle formed by AB and BH is equal to the square on AH

Proof

From proposition 6 (II·6), if we have a bisected line, and an addition to that line, then the extended line CF times the extension AF plus the square on AE is equal to the square on EF

But EB equals EF

Triangle AEB is right angled, thus the square on AB plus the square on AE equals the square on EB (I·47)

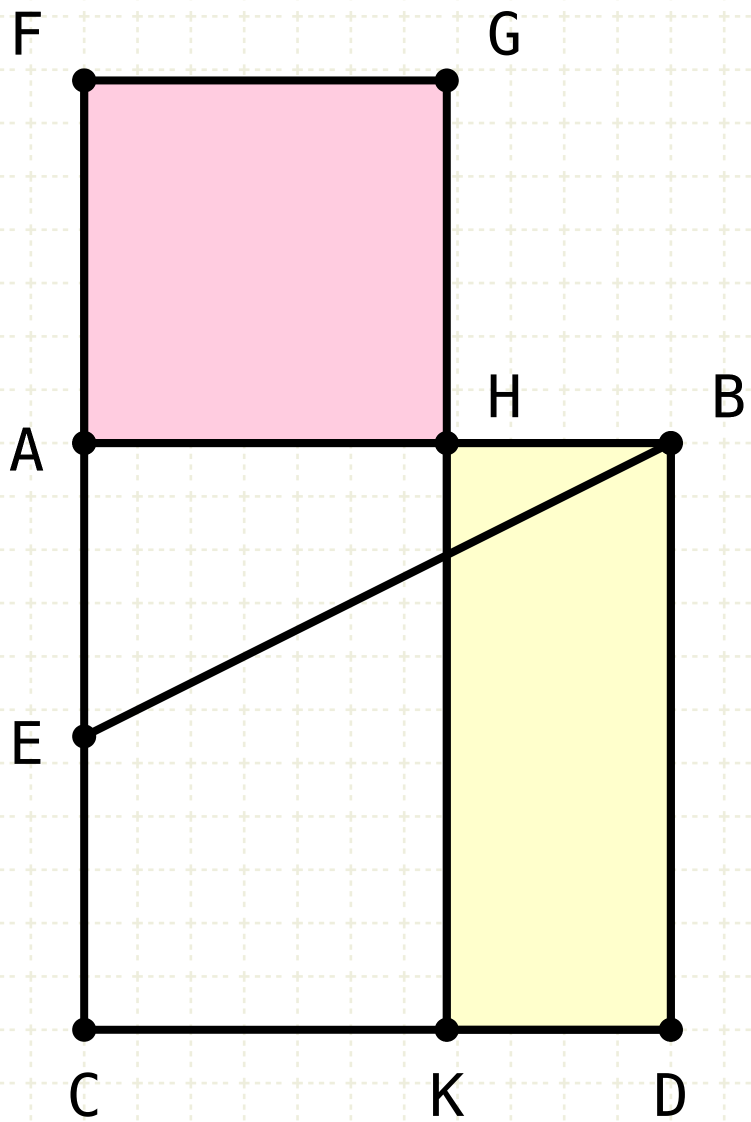
By comparing the equalities, we see that the square of AB is equal to the rectangle formed by CF and AF

The square of AB is the the rectangle AD

The rectangle CF,AF is the rectangle FK, since AF equal AH

Proposition 11 of Book II

To cut a given straight line so that the rectangle contained by the whole and one of the segments is equal to the square on the remaining segment.



$AE = EC$

$EF = EB$

$FA = AH$

$CF \cdot AF + AE^2 = EF^2$

$CF \cdot AF + AE^2 = EB^2$

$AB^2 \quad + AE^2 = EB^2$

$AB^2 = CF \cdot AF$

$AB^2 = \square AD$

$CF \cdot AF = \square FK$

$\square AD = \square FK$

$\square FH = \square HD$

In other words

Find the point H on the line AB such that the rectangle formed by AB and BH is equal to the square on AH

Proof

From proposition 6 (II·6), if we have a bisected line, and an addition to that line, then the extended line CF times the extension AF plus the square on AE is equal to the square on EF

But EB equals EF

Triangle AEB is right angled, thus the square on AB plus the square on AE equals the square on EB (I·47)

By comparing the equalities, we see that the square of AB is equal to the rectangle formed by CF and AF

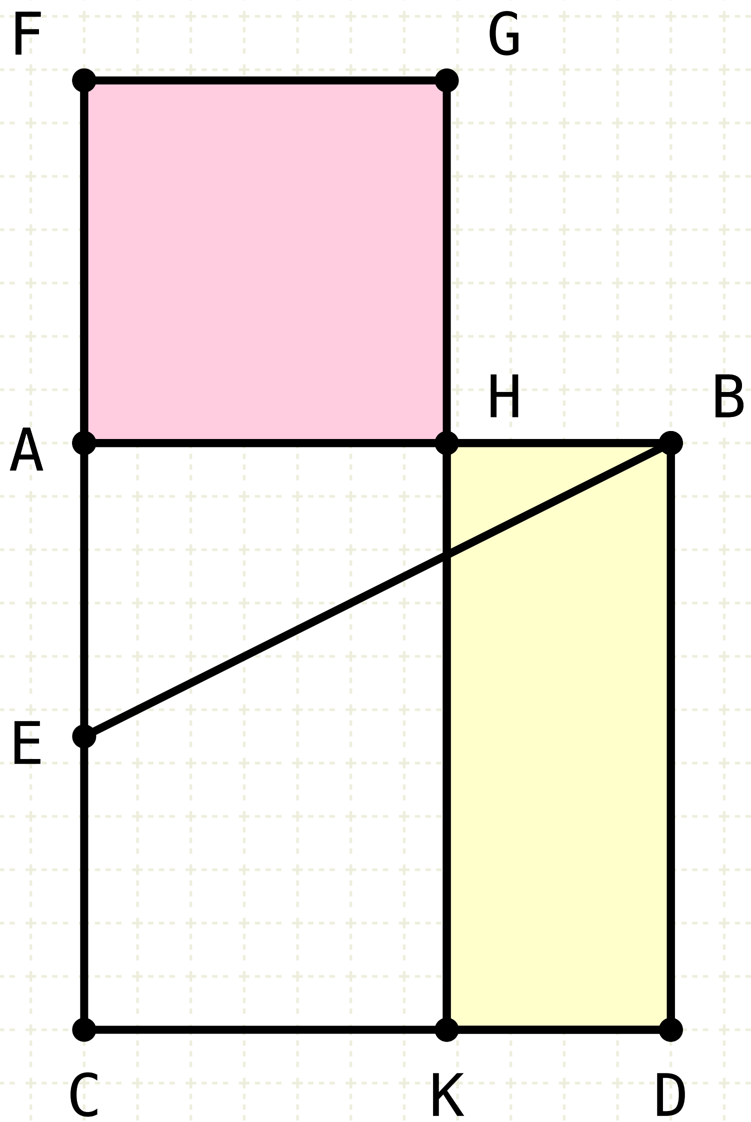
The square of AB is the the rectangle AD

The rectangle CF,AF is the rectangle FK, since AF equal AH

Subtract AK from both sides of the equality, and FH equals HD

Proposition 11 of Book II

To cut a given straight line so that the rectangle contained by the whole and one of the segments is equal to the square on the remaining segment.



$AE = EC$
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$FA = AH$

$CF \cdot AF + AE^2 = EF^2$
$CF \cdot AF + AE^2 = EB^2$
$AB^2 + AE^2 = EB^2$
$AB^2 = CF \cdot AF$
$AB^2 = \square AD$
$CF \cdot AF = \square FK$
$\square AD = \square FK$
$\square FH = \square HD$
$AH \cdot AH = AB \cdot BH$

In other words

Find the point H on the line AB such that the rectangle formed by AB and BH is equal to the square on AH

Proof

From proposition 6 (II·6), if we have a bisected line, and an addition to that line, then the extended line CF times the extension AF plus the square on AE is equal to the square on EF

But EB equals EF

Triangle AEB is right angled, thus the square on AB plus the square on AE equals the square on EB (I·47)

By comparing the equalities, we see that the square of AB is equal to the rectangle formed by CF and AF

The square of AB is the the rectangle AD

The rectangle CF,AF is the rectangle FK, since AF equal AH

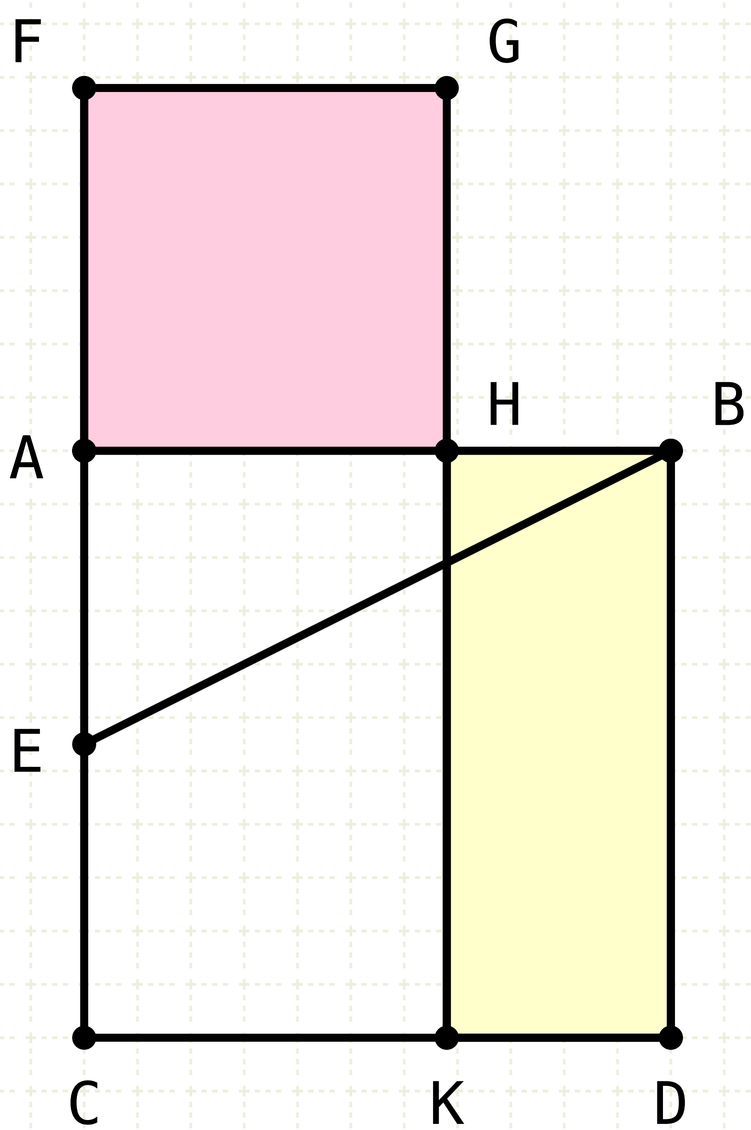
Subtract AK from both sides of the equality, and FH equals HD

But FH is formed as the square on AH, and HD is the rectangle formed by AB,BH since AB equals BD

Thus AH squared is equal to AB times BH

Proposition 11 of Book II

To cut a given straight line so that the rectangle contained by the whole and one of the segments is equal to the square on the remaining segment.



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$CF \cdot AF + AE^2 = EF^2$
$CF \cdot AF + AE^2 = EB^2$
$AB^2 + AE^2 = EB^2$
$AB^2 = CF \cdot AF$
$AB^2 = \square AD$
$CF \cdot AF = \square FK$
$\square AD = \square FK$
$\square FH = \square HD$
$AH \cdot AH = AB \cdot BH$

In other words

Find the point H on the line AB such that the rectangle formed by AB and BH is equal to the square on AH

Proof

From proposition 6 (II·6), if we have a bisected line, and an addition to that line, then the extended line CF times the extension AF plus the square on AE is equal to the square on EF

But EB equals EF

Triangle AEB is right angled, thus the square on AB plus the square on AE equals the square on EB (I·47)

By comparing the equalities, we see that the square of AB is equal to the rectangle formed by CF and AF

The square of AB is the the rectangle AD

The rectangle CF,AF is the rectangle FK, since AF equal AH

Subtract AK from both sides of the equality, and FH equals HD

But FH is formed as the square on AH, and HD is the rectangle formed by AB,BH since AB equals BD

Thus AH squared is equal to AB times BH

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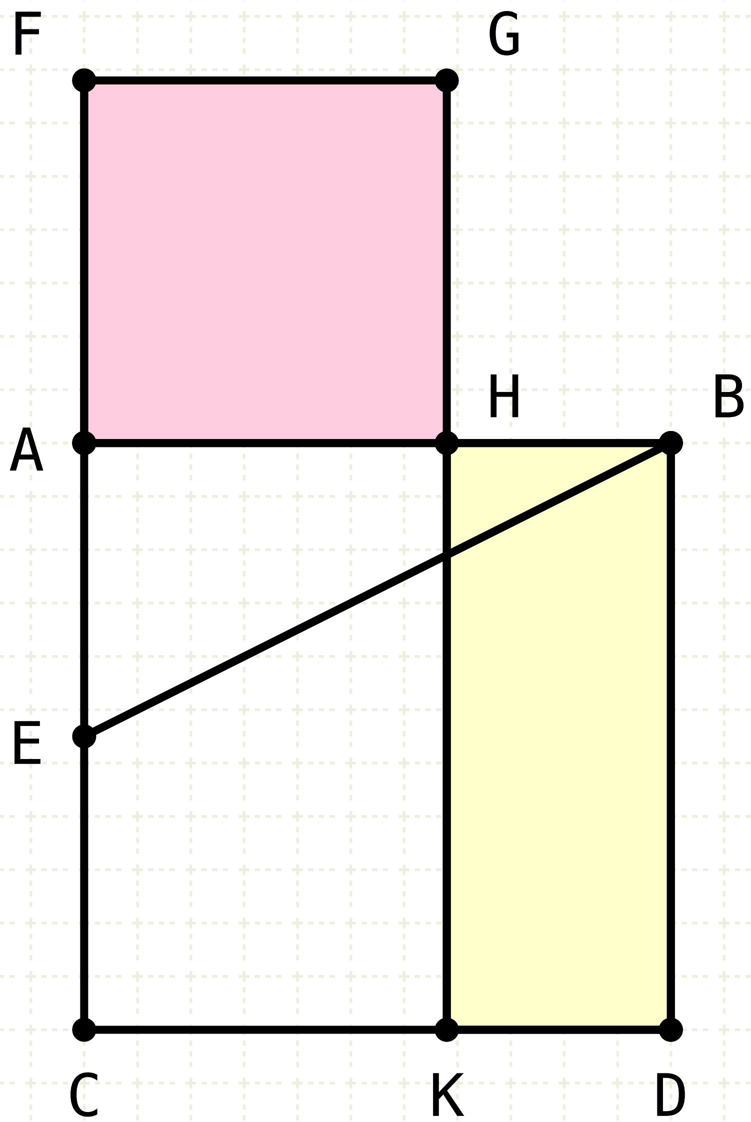

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$CF \cdot AF + AE^2 = EF^2$
$CF \cdot AF + AE^2 = EB^2$
$AB^2 + AE^2 = EB^2$
$AB^2 = CF \cdot AF$
$AB^2 = \square AD$
$CF \cdot AF = \square FK$
$\square AD = \square FK$
$\square FH = \square HD$
$AH \cdot AH = AB \cdot BH$

In other words

Find the point H on the line AB such that the rectangle formed by AB and BH is equal to the square on AH

Golden Ratio

The golden ratio is defined as

$$a/b = (a+b)/a, \text{ where } a>b$$

Since AB is equal to AH + BH, this proposition finds H such that

$$AH \cdot AH = (AH + BH) \cdot BH$$

or, the golden ratio…

$$AH/BH = (AH + BH)/AH$$

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