

Euclid's Elements

Book III



A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)



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7	Consider two lines from a point inside a circle to the edge, the longer one will be the one closest to the longest part of the diameter passing through the original point	14	In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another.	23	On the same straight line there cannot be constructed two similar and unequal segments of circles on the same side
8	Consider two lines from a point outside a circle to the edge, the line closest to the centre will be longer on the concave side and shorter on the convex side	15	The longest line in a circle is its diameter, shorter the farther away from the diameter	24	Similar segments of circles on equal straight lines are equal to one another
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Proposition 10 of Book III

A circle does not cut a circle at more points than two.



Proposition 10 of Book III

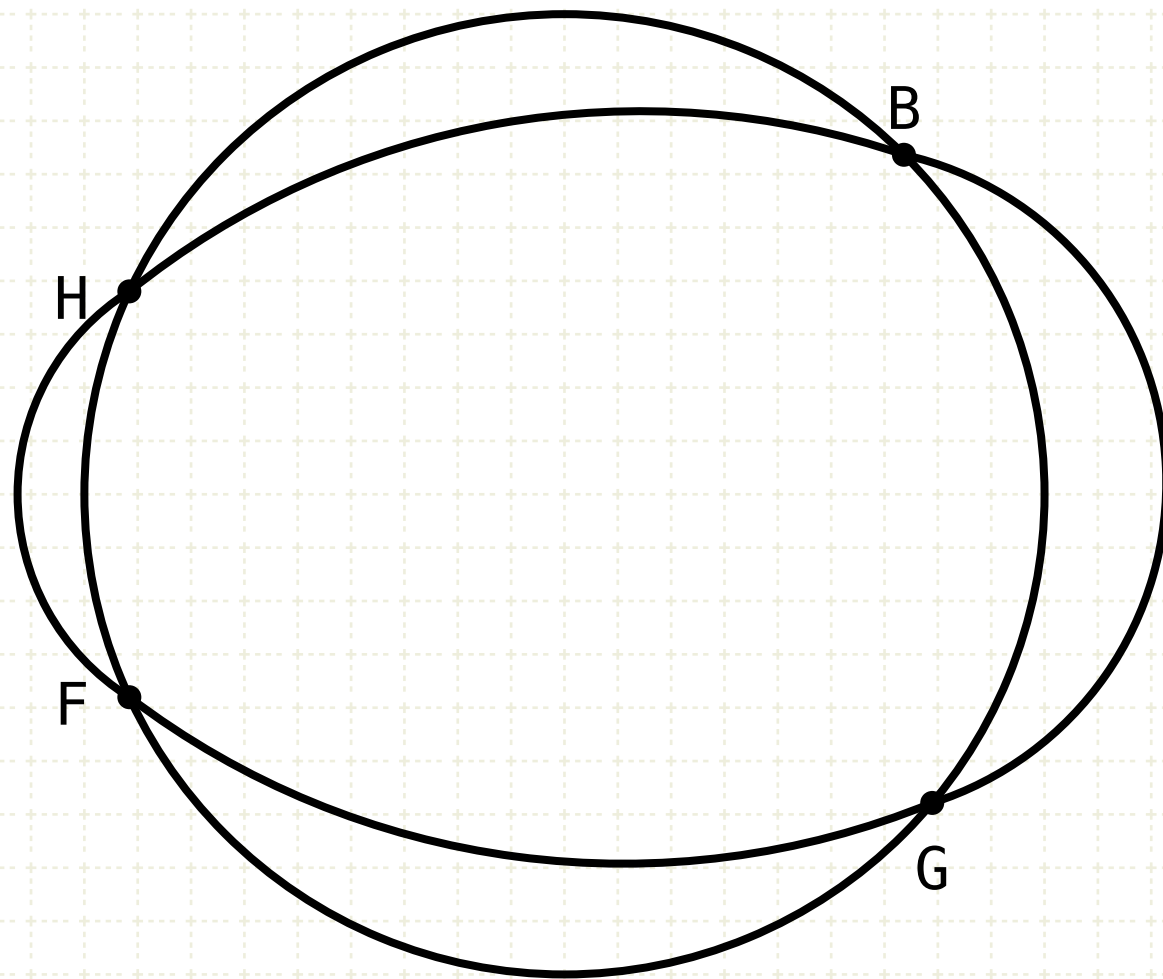
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Proof by Contradiction



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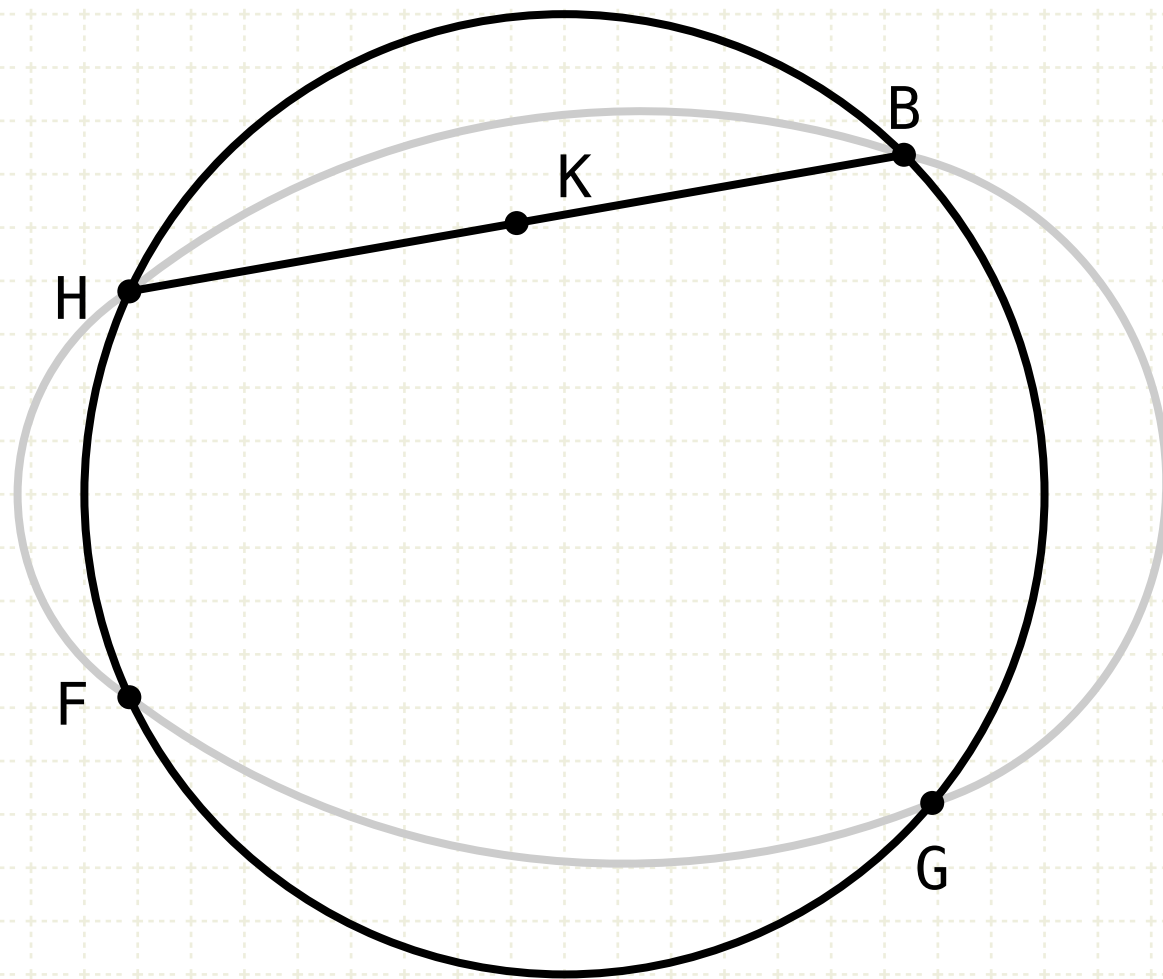
Proof by Contradiction

Assume we have two circles which intersect at 4 points



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$$HK = KB$$

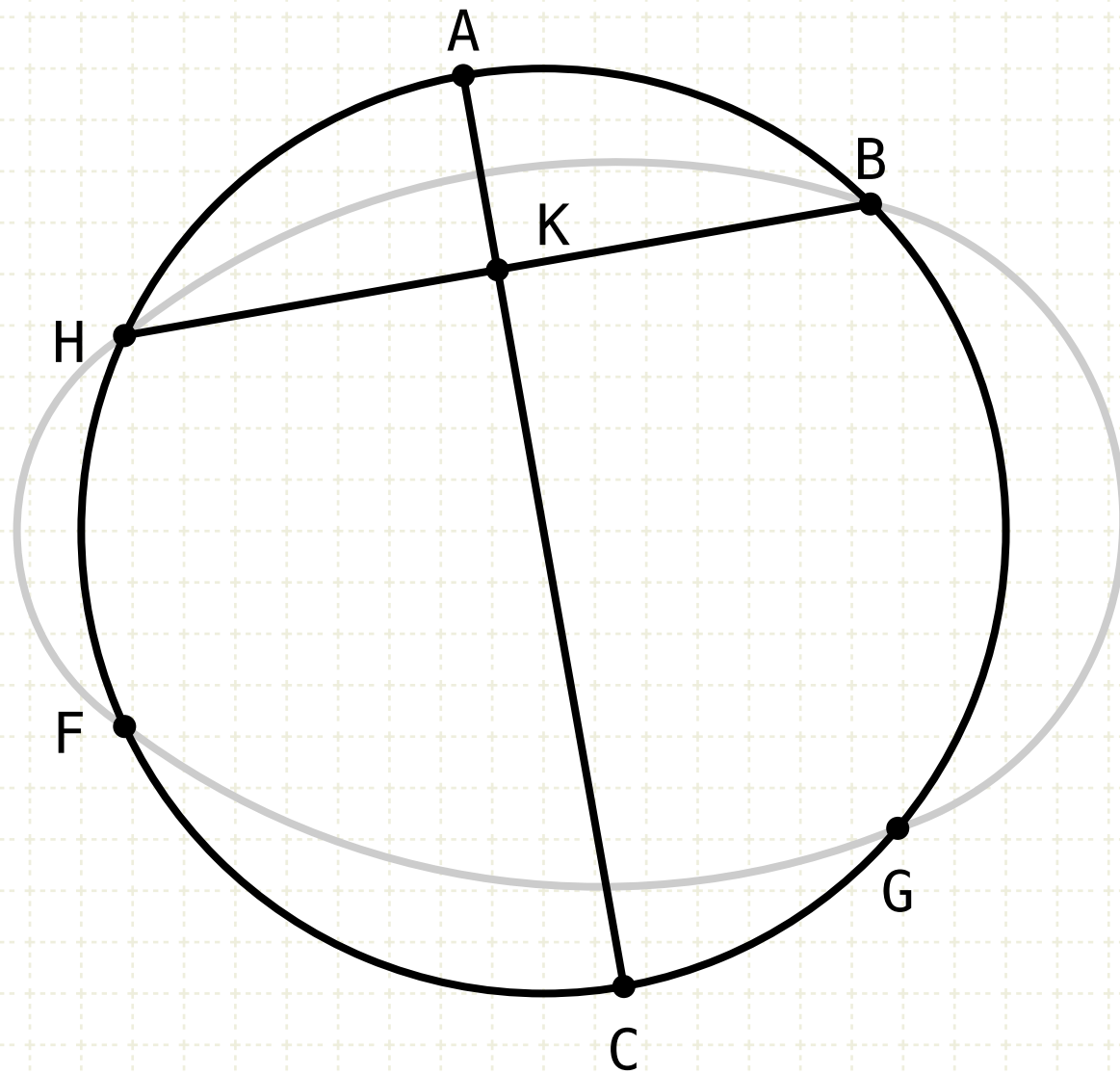
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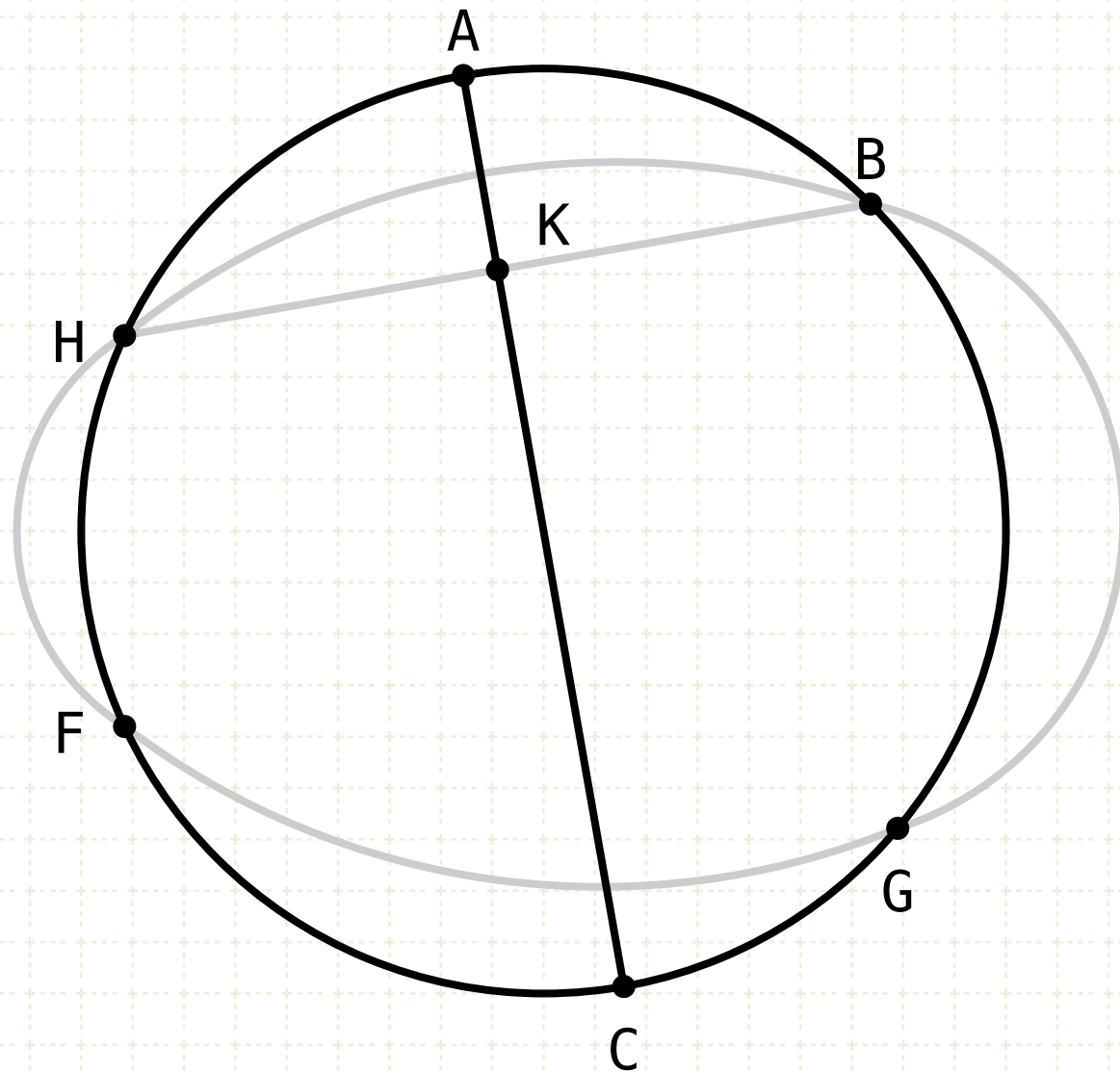
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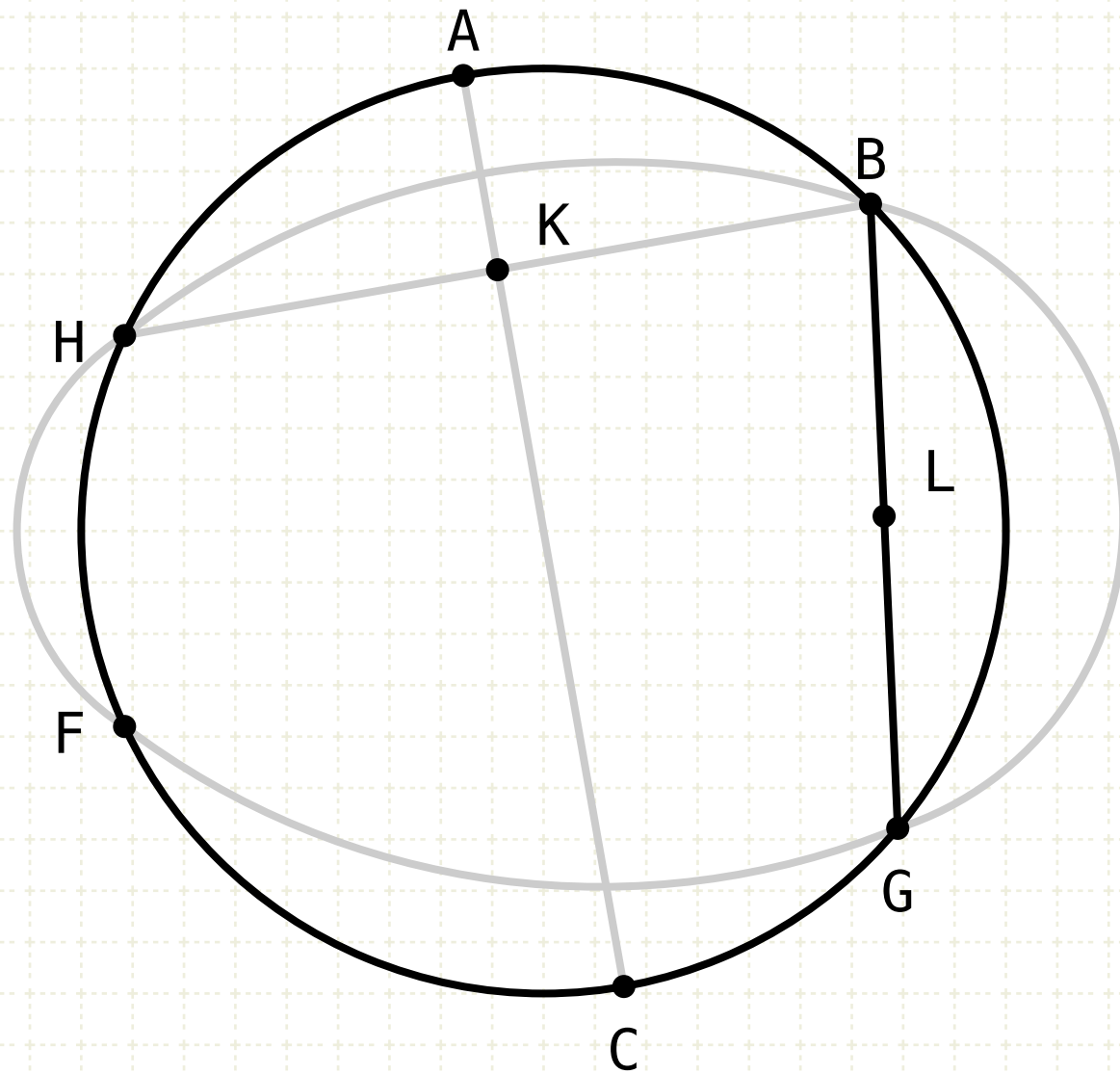
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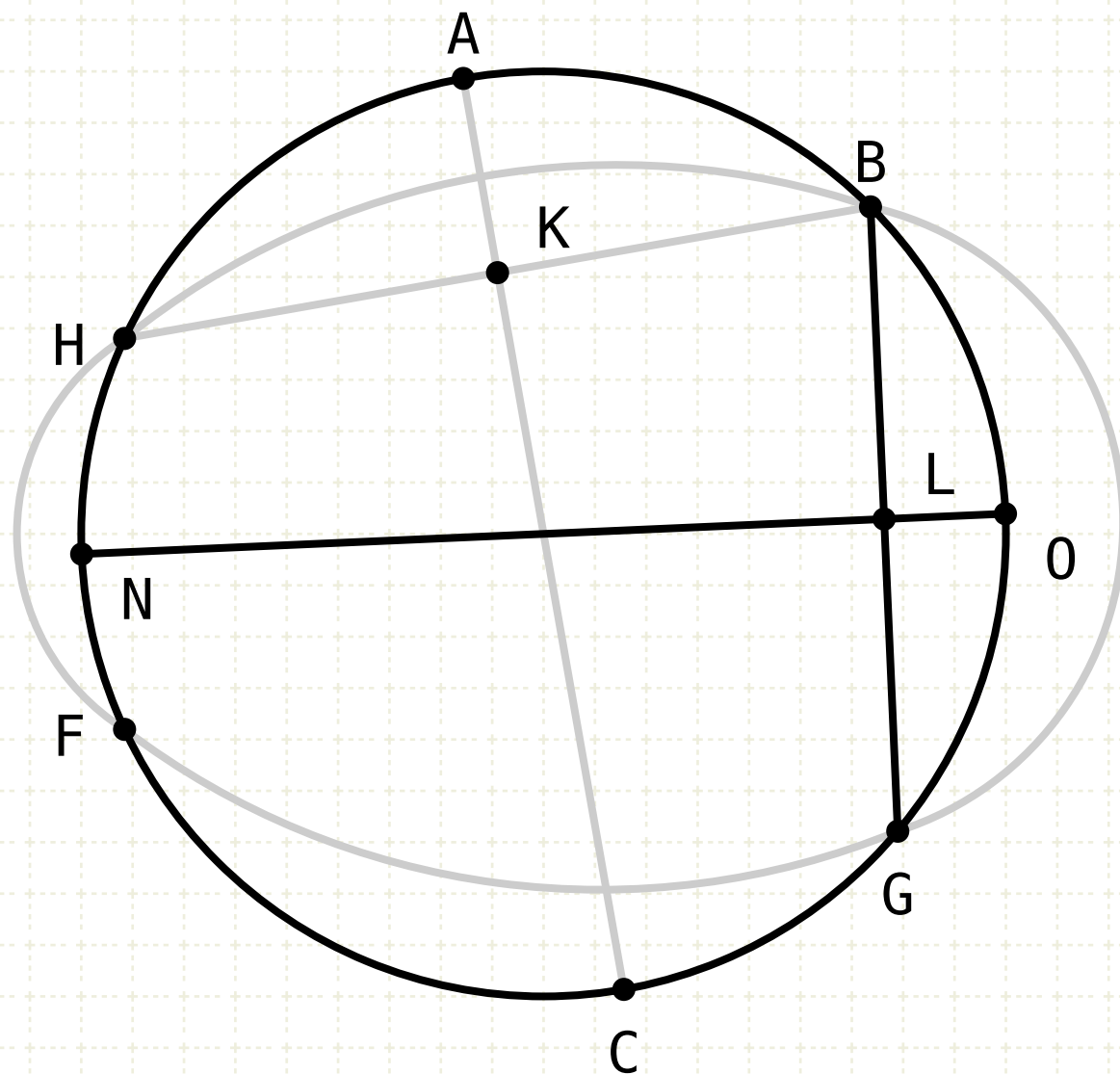
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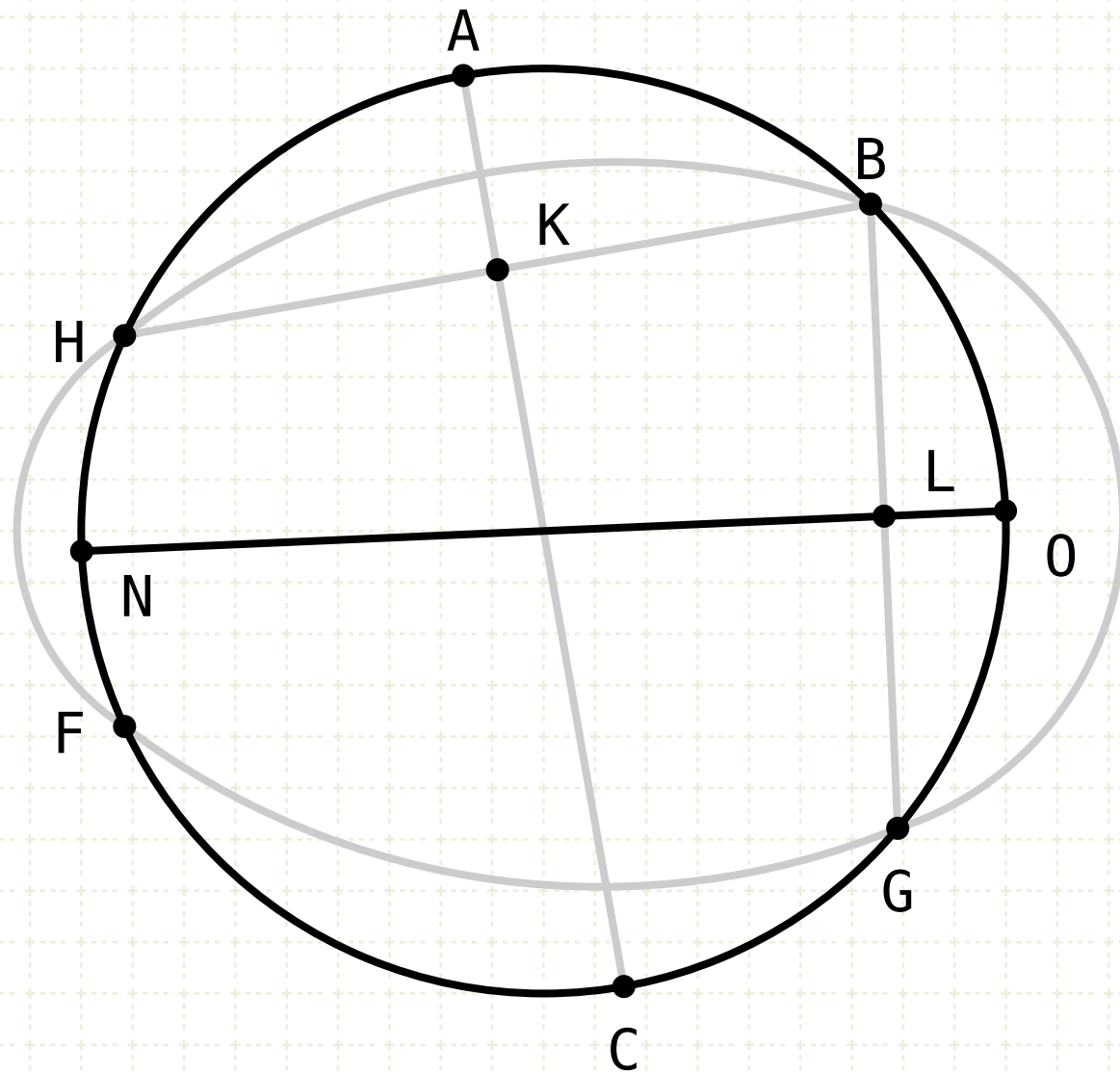
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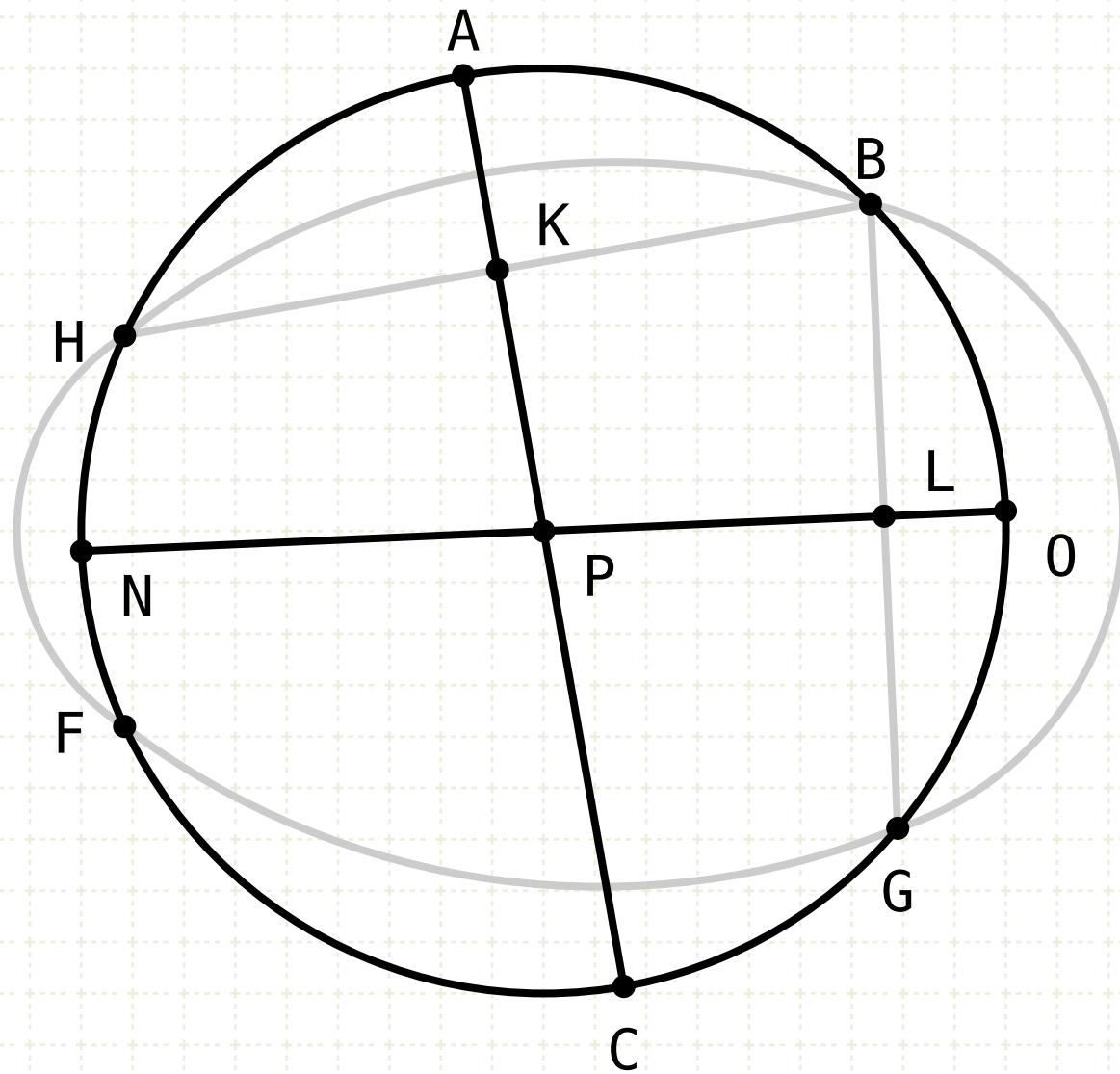
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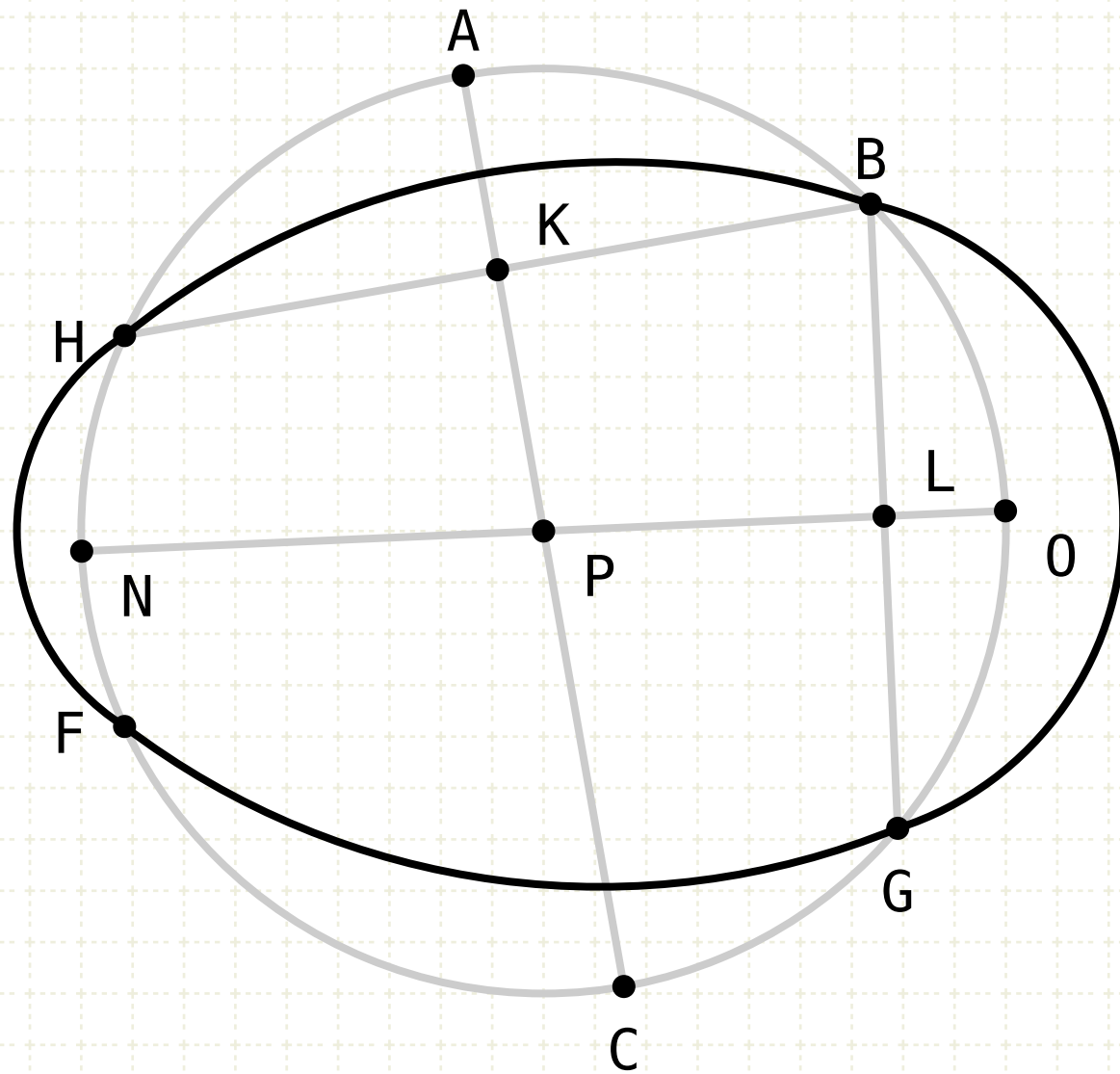
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From proposition III·1, we know that the centre of the circle ABC lies on the line NO

If the centre of the circle is on AC and NO, then the centre of the circle ABC must be point P

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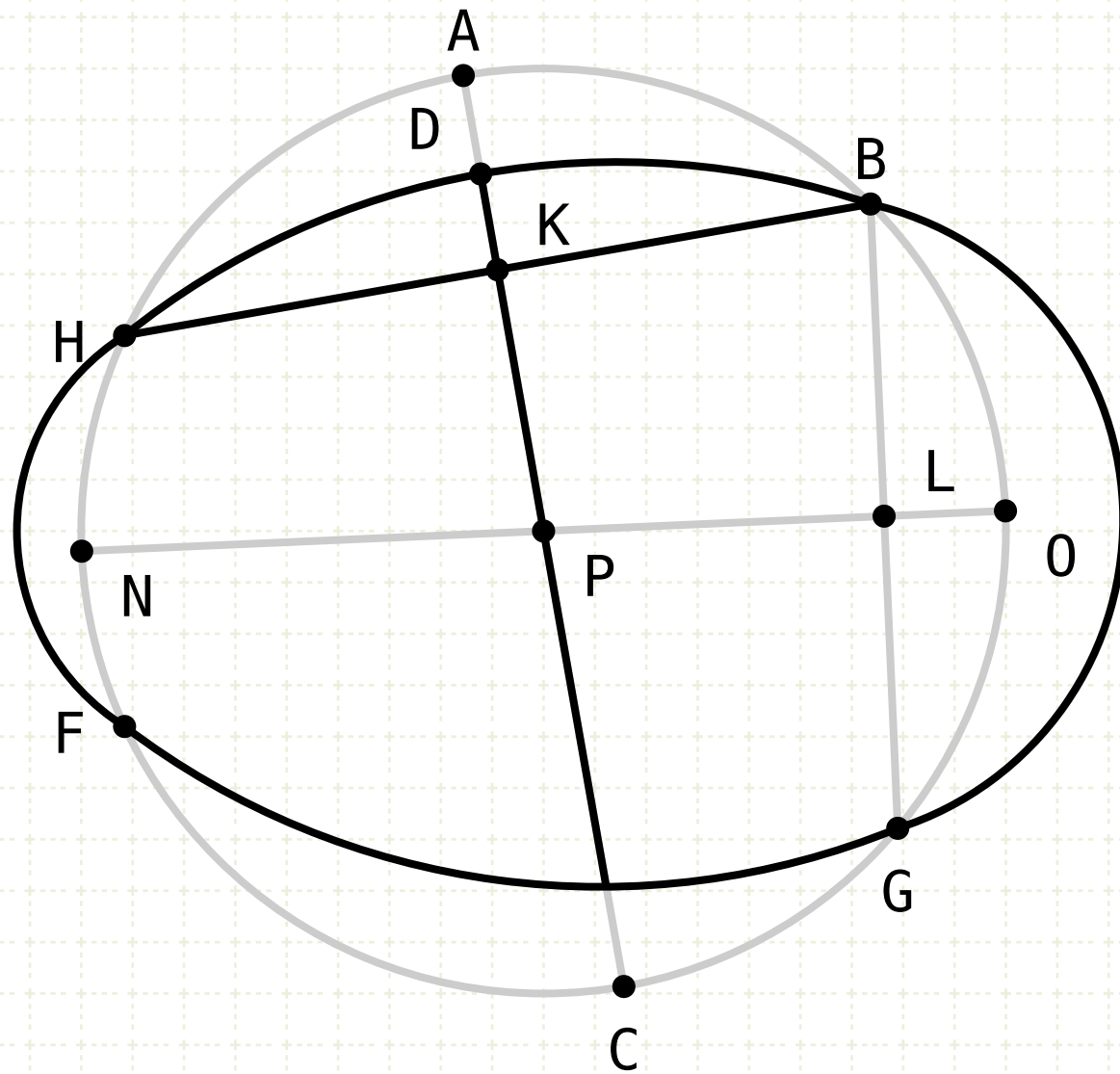
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Using the same logic, it can be demonstrated that P is also the centre of DEF

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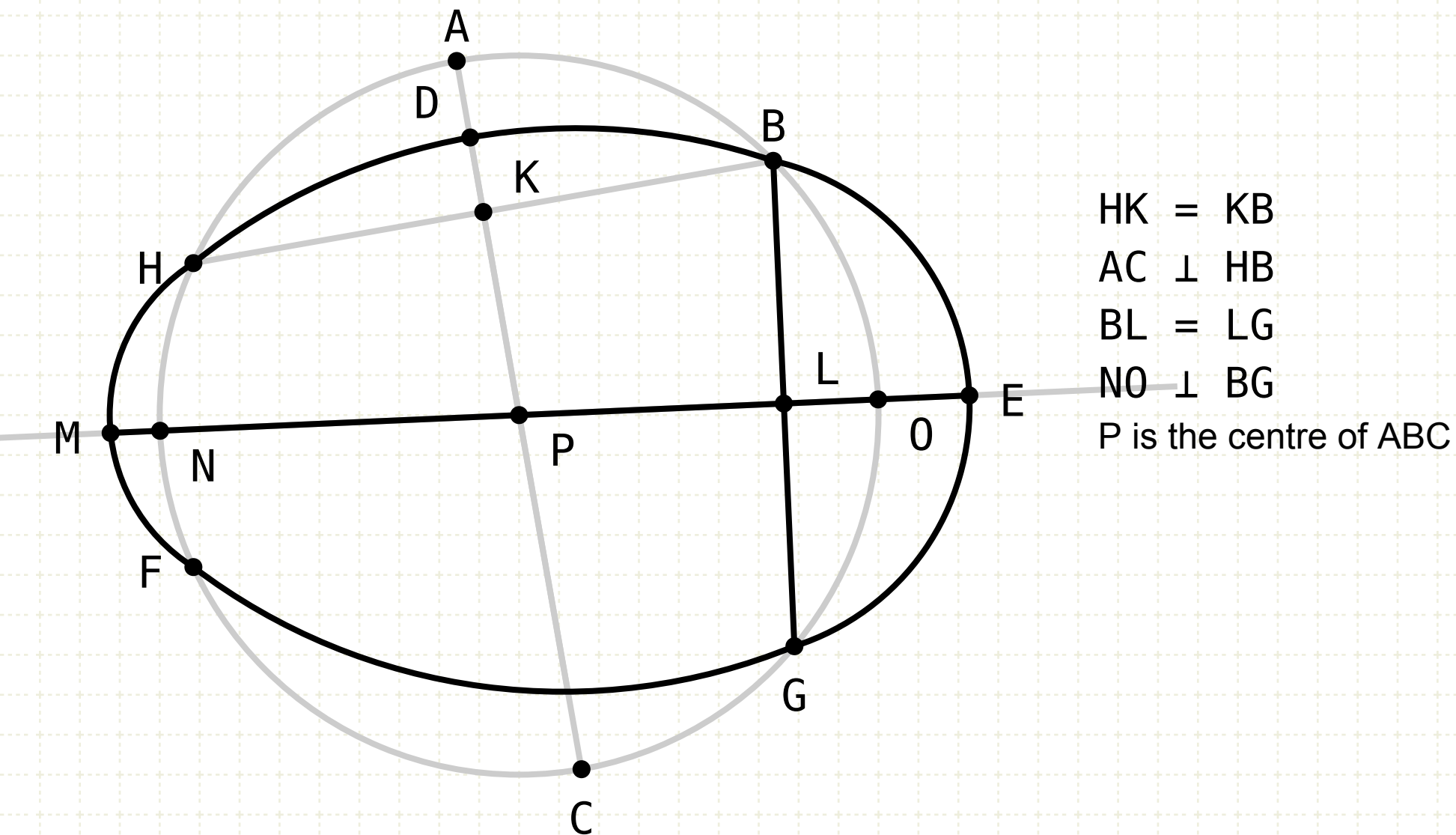
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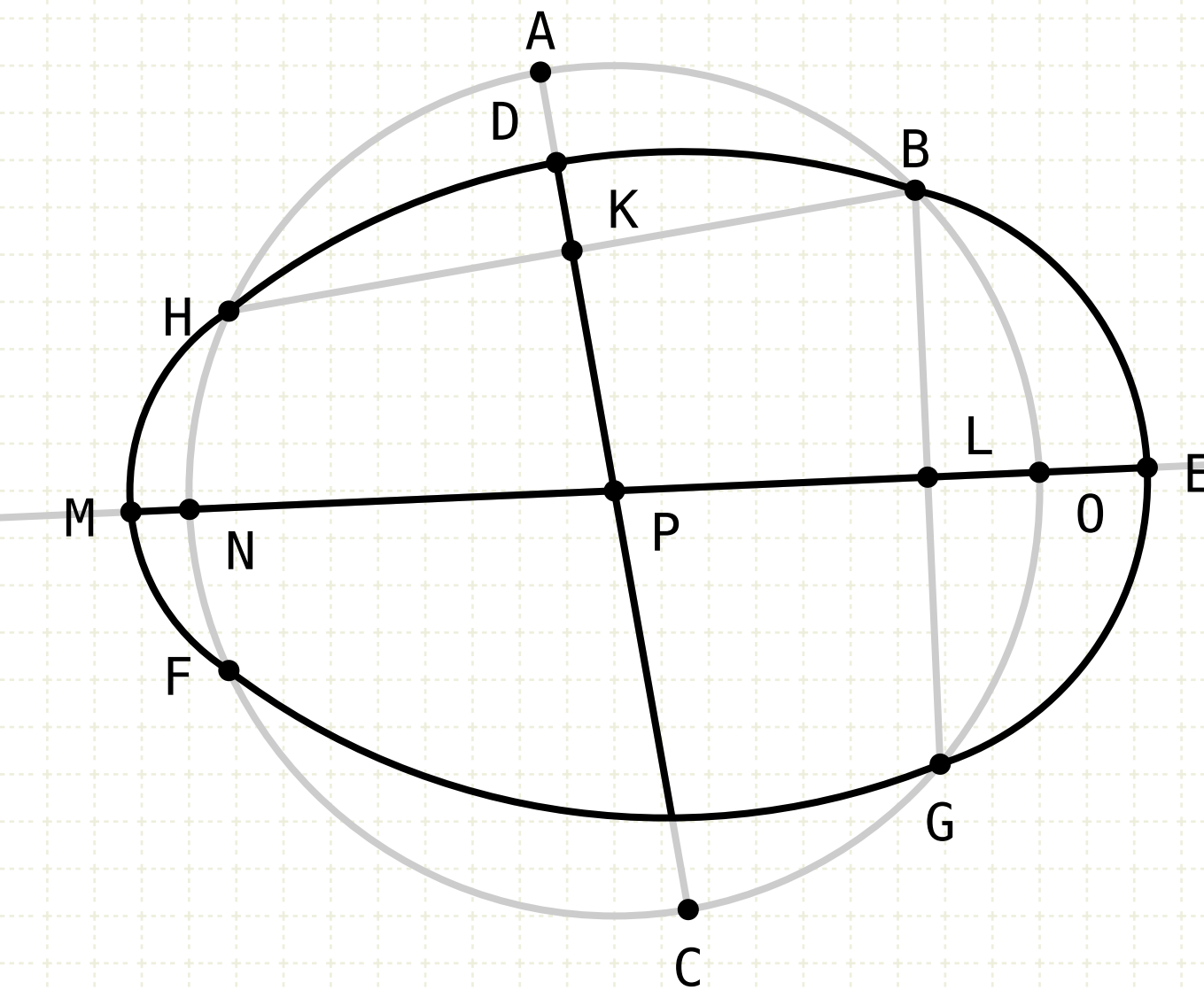
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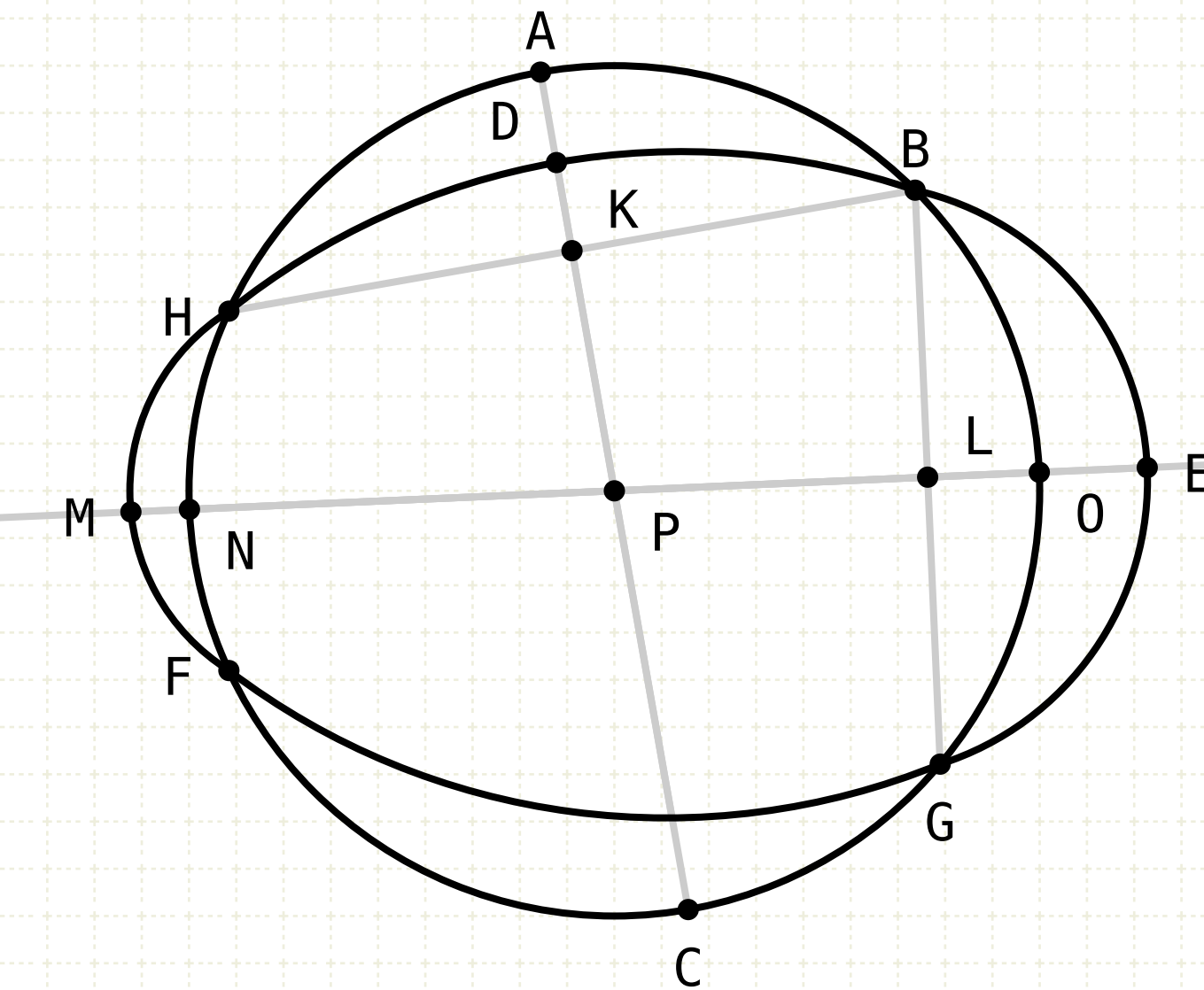
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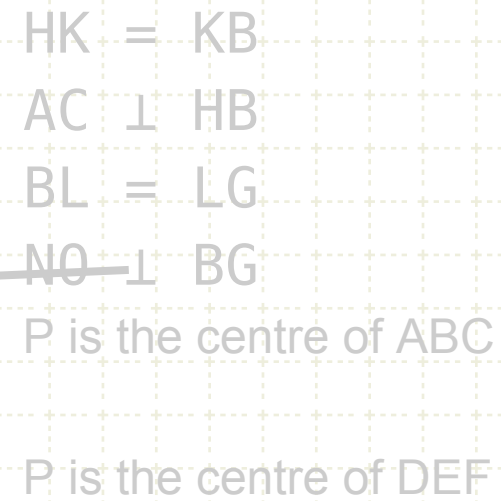
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But two intersecting circles cannot have the same centre (III·5), hence a contradiction



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Thus two circles cannot intersect at more than two points



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