Euclid's Elements
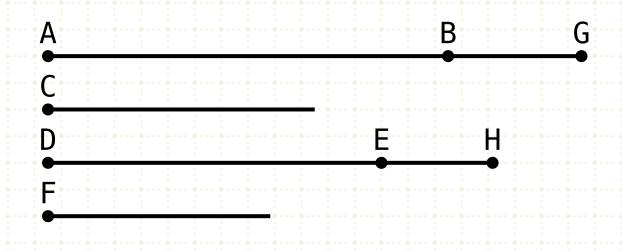
Book V

Proportions are what makes the old Greek temples classic in their beauty. They are like huge blocks, from which the air has been literally hewn out between the columns.

Arne Jacobsen

AB:C = DE:F

BG:C = EH:F

AG:C = DH:F

Table of Contents, Chapter 5

Proposition 10 of Book V

Of magnitudes which have a ratio to the same, that which has a greater ratio is greater; and that to which the same has a greater ratio is less.

Proposition 10 of Book V

Of magnitudes which have a ratio to the same, that which has a greater ratio is greater; and that to which the same has a greater ratio is less.

A

B

C

In other words

Let the ratio of A to C be greater than the ratio B to C

Then A is greater than B

$A:C > B:C \rightarrow A > B$

$C:B > C:A \rightarrow B < A$

Proposition 10 of Book V

Of magnitudes which have a ratio to the same, that which has a greater ratio is greater; and that to which the same has a greater ratio is less.

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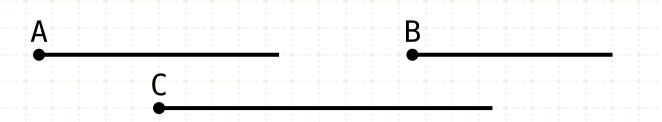Then A is greater than B

Proof

$$A:C > B:C$$

Proposition 10 of Book V

Of magnitudes which have a ratio to the same, that which has a greater ratio is greater; and that to which the same has a greater ratio is less.

A

B

C

$$A:C > B:C$$
$$A = B \rightarrow A:C = B:C$$
$$A < B \rightarrow A:C < B:C$$

In other words

Let the ratio of A to C be greater than the ratio B to C

Then A is greater than B

Proof

If A is not greater than B, then A either equals B or is less than B

If A equals B then the ratios A to C and B to C would be equal (V·7)

If A is less than B then the ratio A to C would be less than B to C (V·8)

Proposition 10 of Book V

Of magnitudes which have a ratio to the same, that which has a greater ratio is greater; and that to which the same has a greater ratio is less.

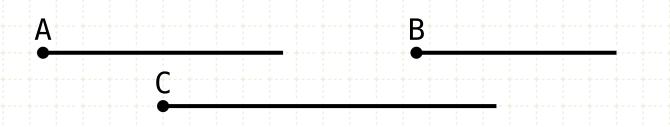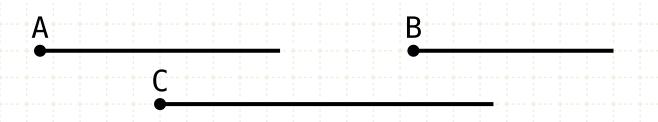A _____

B _____

C _____

$$A:C > B:C$$
$$A = B \rightarrow A:C = B:C$$
$$A < B \rightarrow A:C < B:C$$
$$But\ A:C > B:C \therefore A > B$$

In other words

Let the ratio of A to C be greater than the ratio B to C
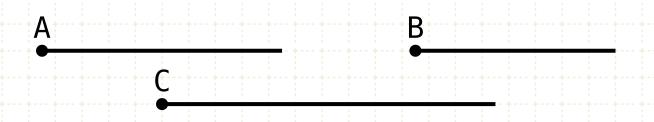
Then A is greater than B

Proof

If A is not greater than B, then A either equals B or is less than B

If A equals B then the ratios A to C and B to C would be equal (V·7)

If A is less than B then the ratio A to C would be less than B to C (V·8)

But A to C IS greater than B to C, therefore A is greater than B

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Proposition 10 of Book V

Of magnitudes which have a ratio to the same, that which has a greater ratio is greater; and that to which the same has a greater ratio is less.

A
B
C

In other words

Let the ratio of A to C be greater than the ratio B to C

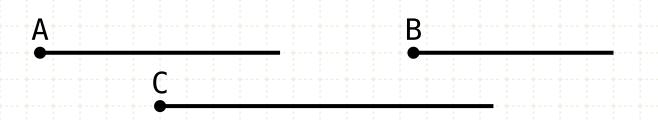Then A is greater than B

Proof

If A is not greater than B, then A either equals B or is less than B

If A equals B then the ratios A to C and B to C would be equal (V·7)

If A is less than B then the ratio A to C would be less than B to C (V·8)

But A to C IS greater than B to C, therefore A is greater than B

```
A:C > B:C
A = B → A:C = B:C
A < B → A:C < B:C
But A:C > B:C ∴ A > B
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C:B > C:A

Proposition 10 of Book V

Of magnitudes which have a ratio to the same, that which has a greater ratio is greater; and that to which the same has a greater ratio is less.

A

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In other words

Let the ratio of A to C be greater than the ratio B to C

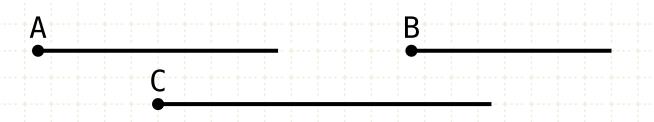Then A is greater than B

Proof

If A is not greater than B, then A either equals B or is less than B

If A equals B then the ratios A to C and B to C would be equal (V·7)

If A is less than B then the ratio A to C would be less than B to C (V·8)

But A to C IS greater than B to C, therefore A is greater than B

If B is not less than A, then B either equals A or is greater than A

If B equals A then the ratios C to B and C to A would be equal (V·7)

If B is greater than A then the ratio C to B would be less than C to A (V·8)

$A:C > B:C$
$A = B \rightarrow A:C = B:C$
$A < B \rightarrow A:C < B:C$
$But\ A:C > B:C \therefore A > B$

$C:B > C:A$
$B = A \rightarrow C:B = C:A$
$B > A \rightarrow C:B < C:A$

Proposition 10 of Book V

Of magnitudes which have a ratio to the same, that which has a greater ratio is greater; and that to which the same has a greater ratio is less.

A ●————————————

B ●————————————

C ●————————————

$A:C > B:C$

$A = B \rightarrow A:C = B:C$

$A < B \rightarrow A:C < B:C$

$But\ A:C > B:C \therefore A > B$

$C:B > C:A$

$B = A \rightarrow C:B = C:A$

$B > A \rightarrow C:B < C:A$

$But\ C:B > C:A \therefore B < A$

In other words

Let the ratio of A to C be greater than the ratio B to C

Then A is greater than B

Proof

If A is not greater than B, then A either equals B or is less than B

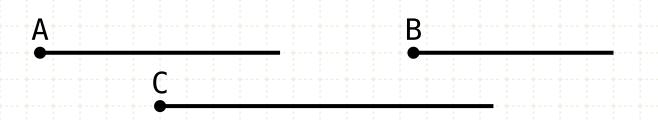If A equals B then the ratios A to C and B to C would be equal (V·7)

If A is less than B then the ratio A to C would be less than B to C (V·8)

But A to C IS greater than B to C, therefore A is greater than B

If B is not less than A, then B either equals A or is greater than A

If B equals A then the ratios C to B and C to A would be equal (V·7)

If B is greater than A then the ratio C to B would be less than C to A (V·8)

But C to B IS greater than C to A, therefore B is less than A

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