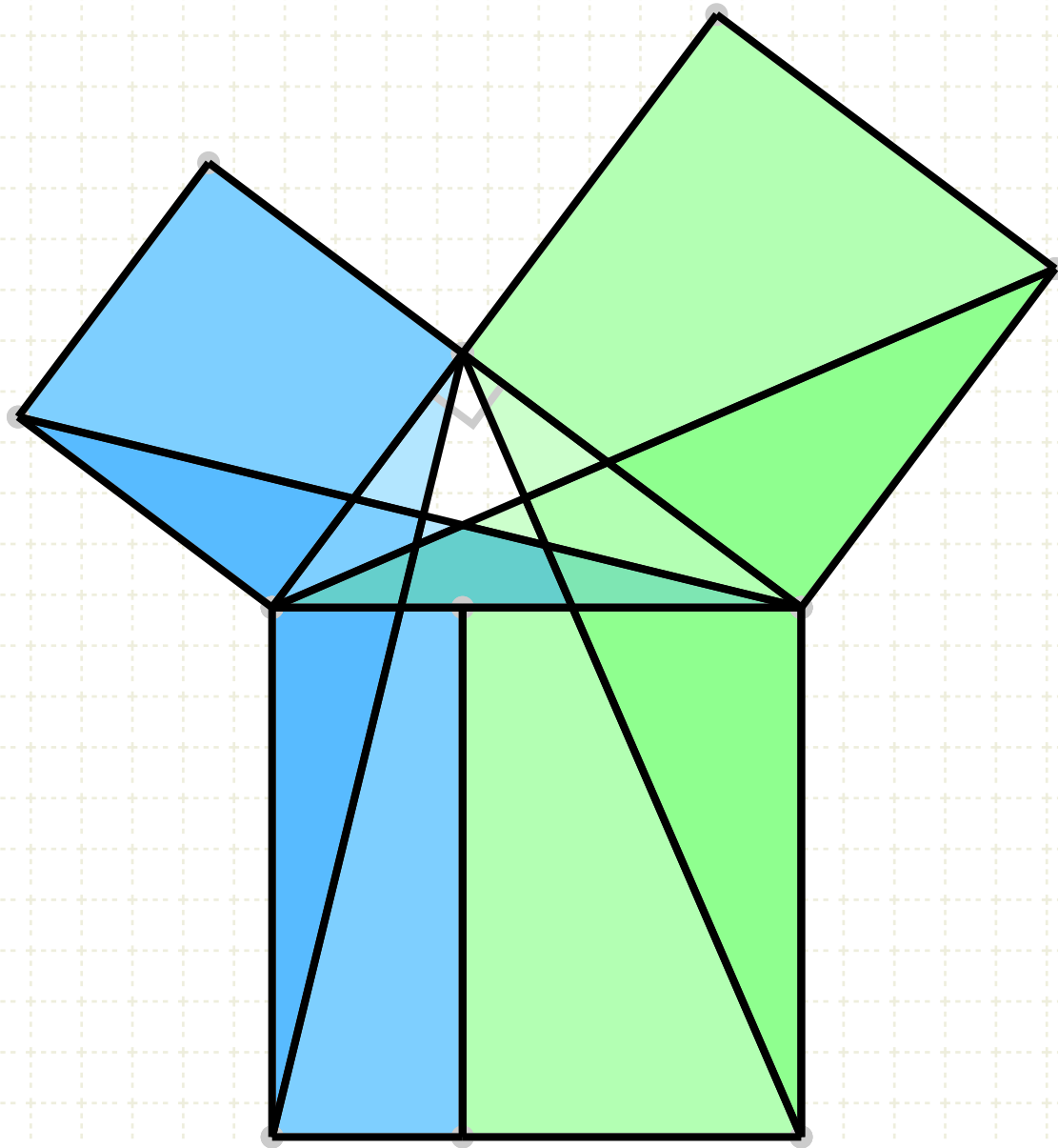


# Euclid's Elements

## Book I

*If Euclid did not kindle your youthful enthusiasm, you were not born to be a scientific thinker.*

Albert Einstein



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48	Inverse Pythagoras' theorem



# Proposition 9 of Book I

To bisect a given rectilinear angle.

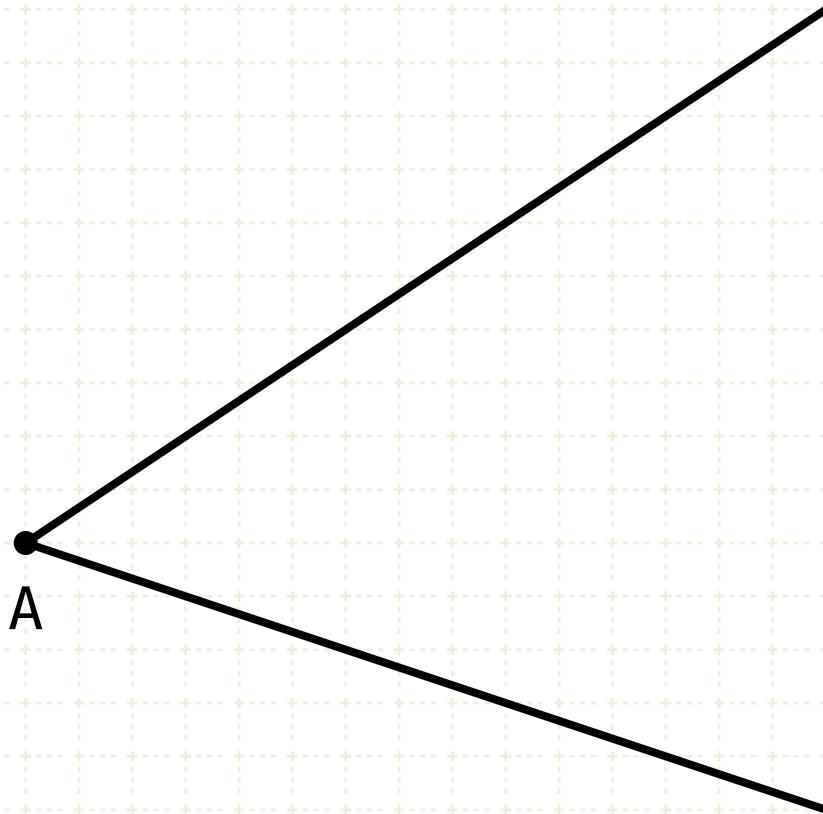


# Proposition 9 of Book I

To bisect a given rectilinear angle.

## In other words

Start with two straight lines joined at a single point



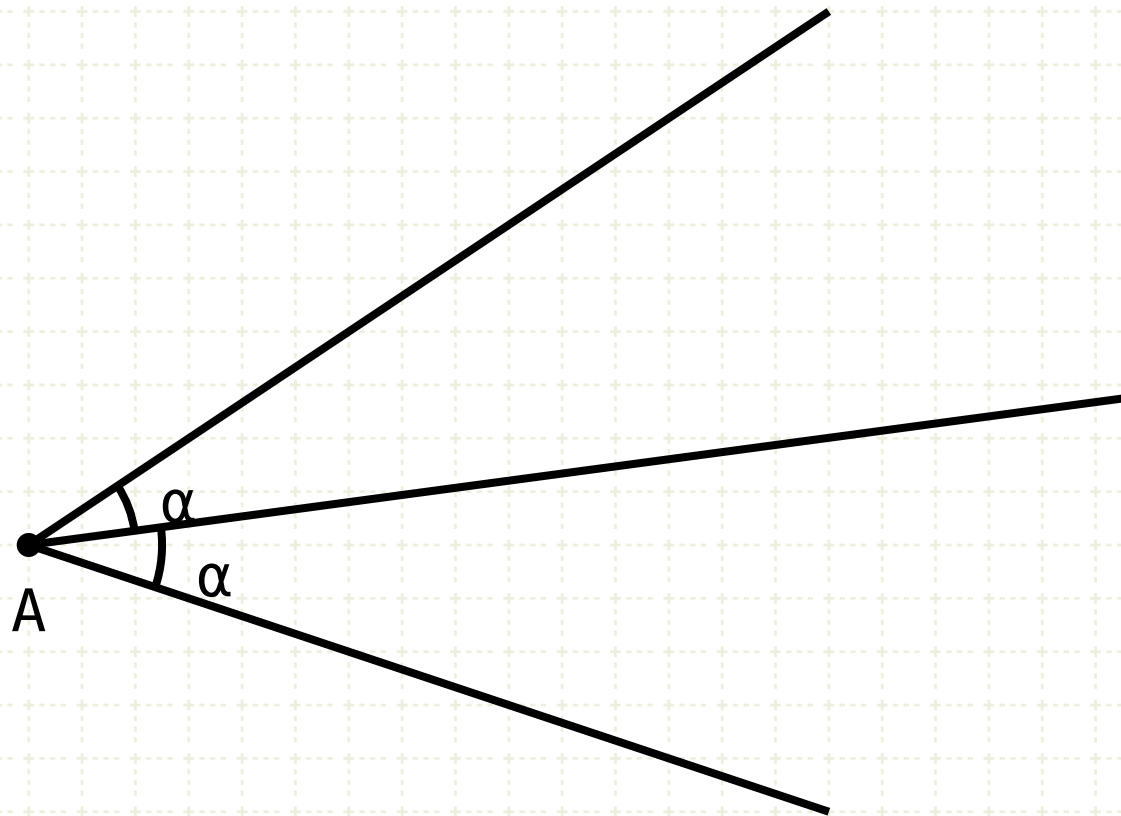
# Proposition 9 of Book I

To bisect a given rectilinear angle.

## In other words

Start with two straight lines joined at a single point

Divide the resulting angle into two, using only a straight edge and compass

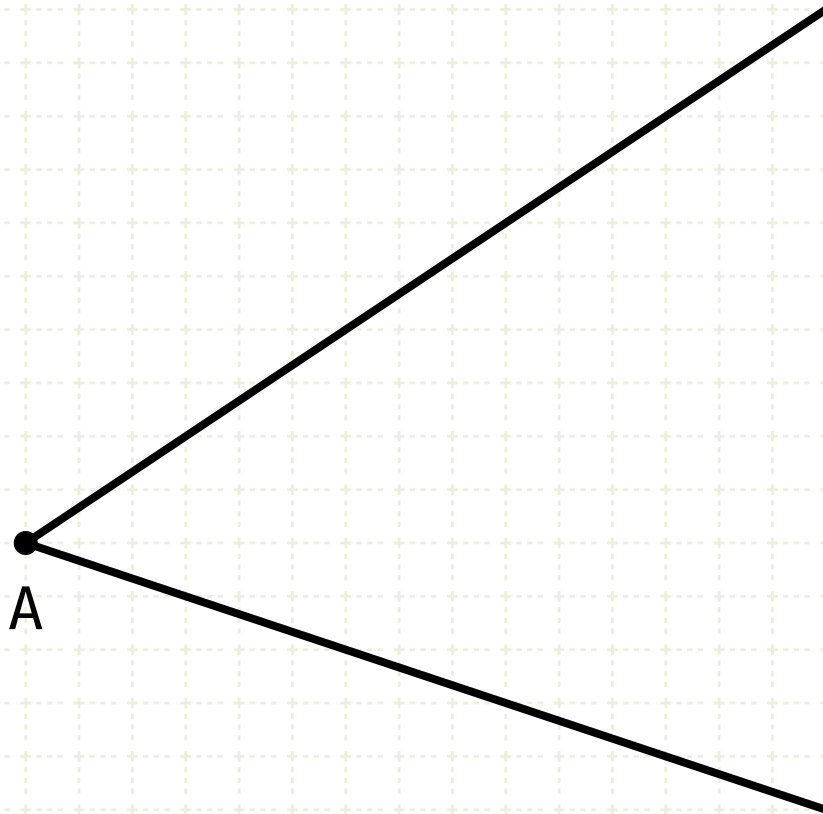




# Proposition 9 of Book I

To bisect a given rectilinear angle.

Construction:

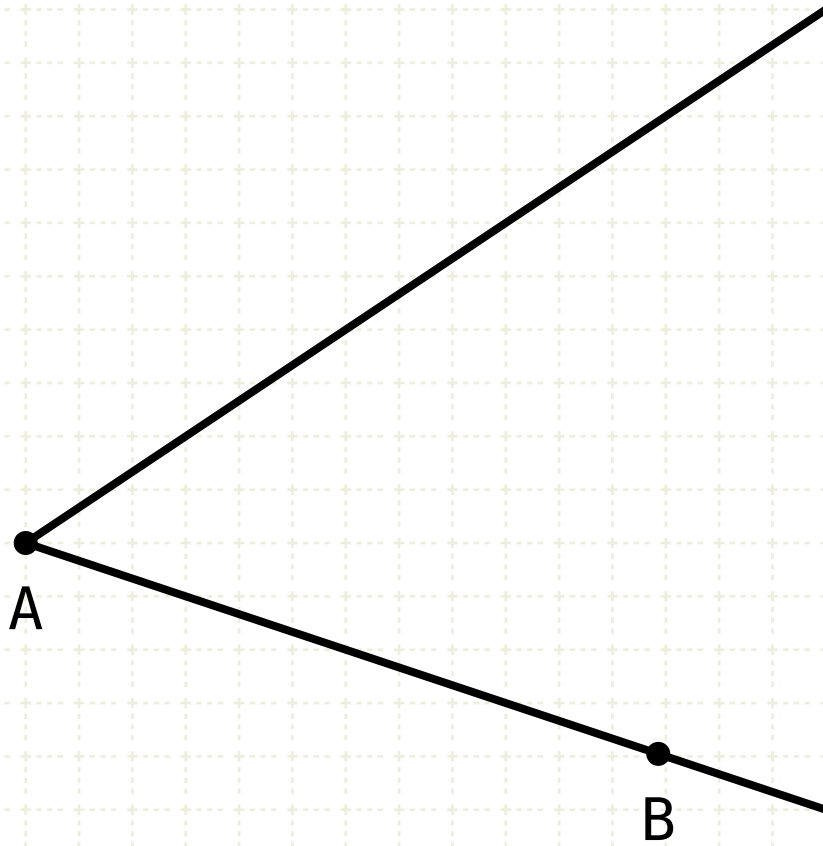


# Proposition 9 of Book I

To bisect a given rectilinear angle.

## Construction:

Pick an arbitrary point B on one of the lines, and construct another point C on the other line, such that AB and AC are equal



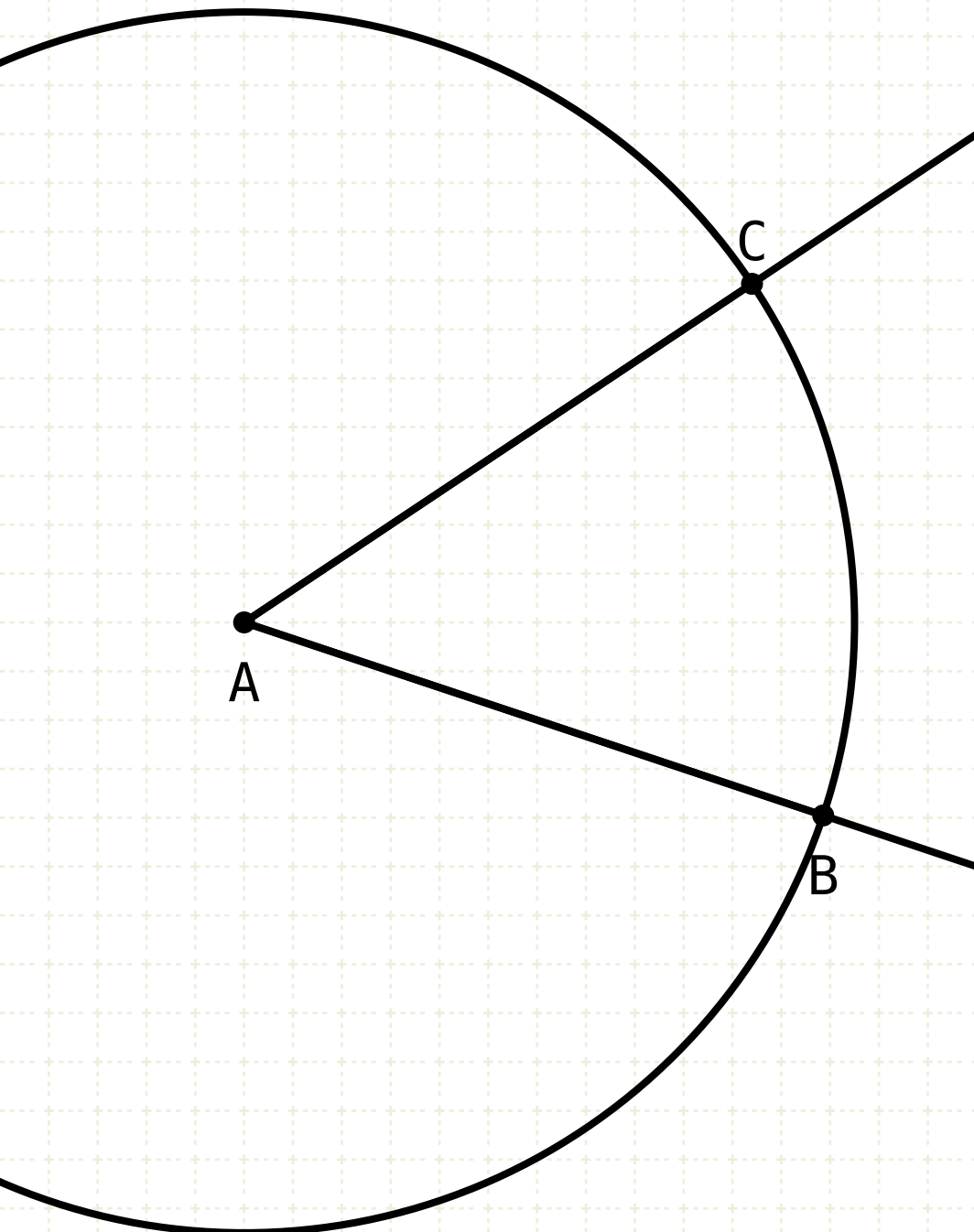


# Proposition 9 of Book I

To bisect a given rectilinear angle.

## Construction:

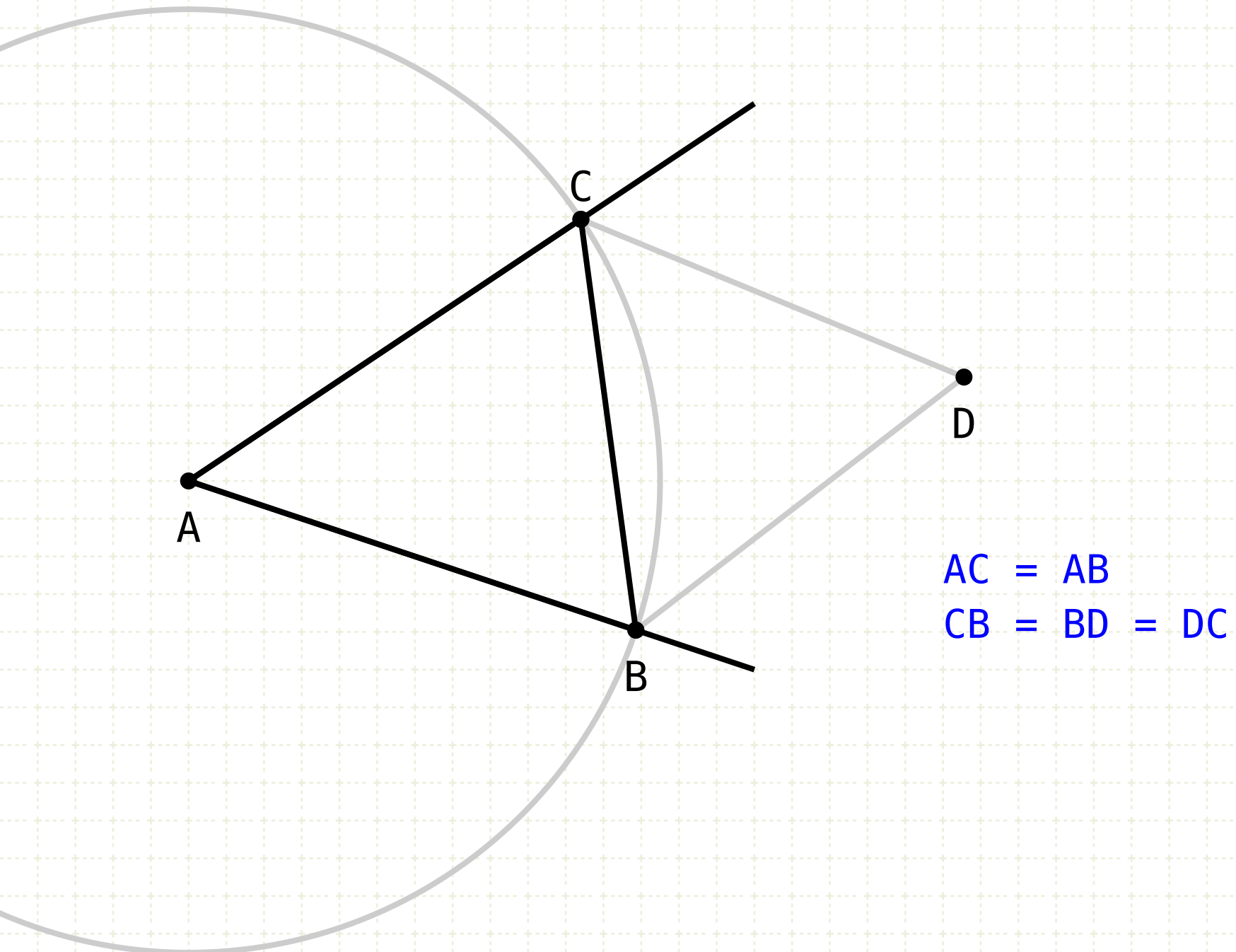
Pick an arbitrary point B on one of the lines, and construct another point C on the other line, such that AB and AC are equal



$$AC = AB$$

# Proposition 9 of Book I

To bisect a given rectilinear angle.



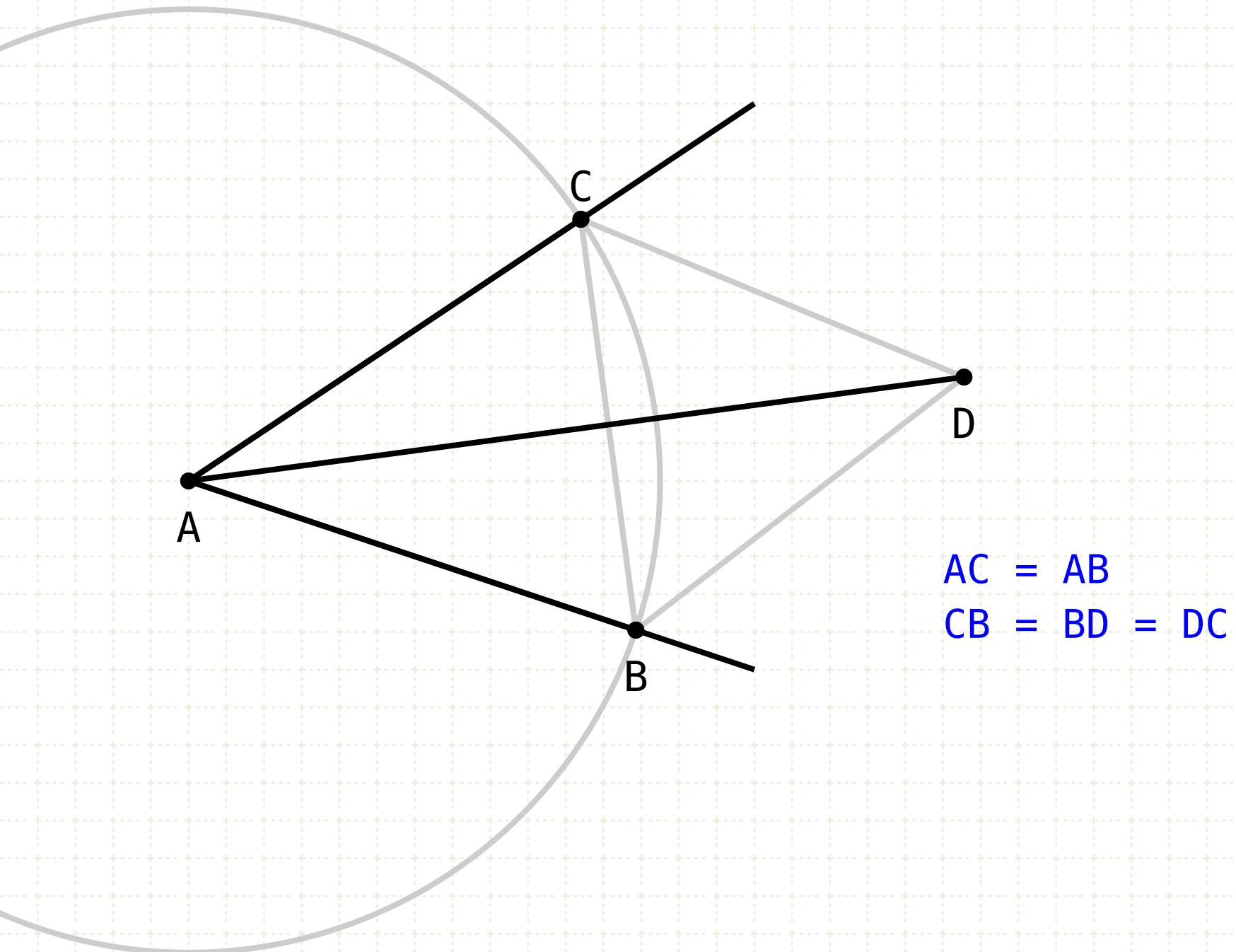
## Construction:

Pick an arbitrary point B on one of the lines, and construct another point C on the other line, such that AB and AC are equal

Construct an equilateral triangle on line AC, and label the vertex D (I.1)

# Proposition 9 of Book I

To bisect a given rectilinear angle.



## Construction:

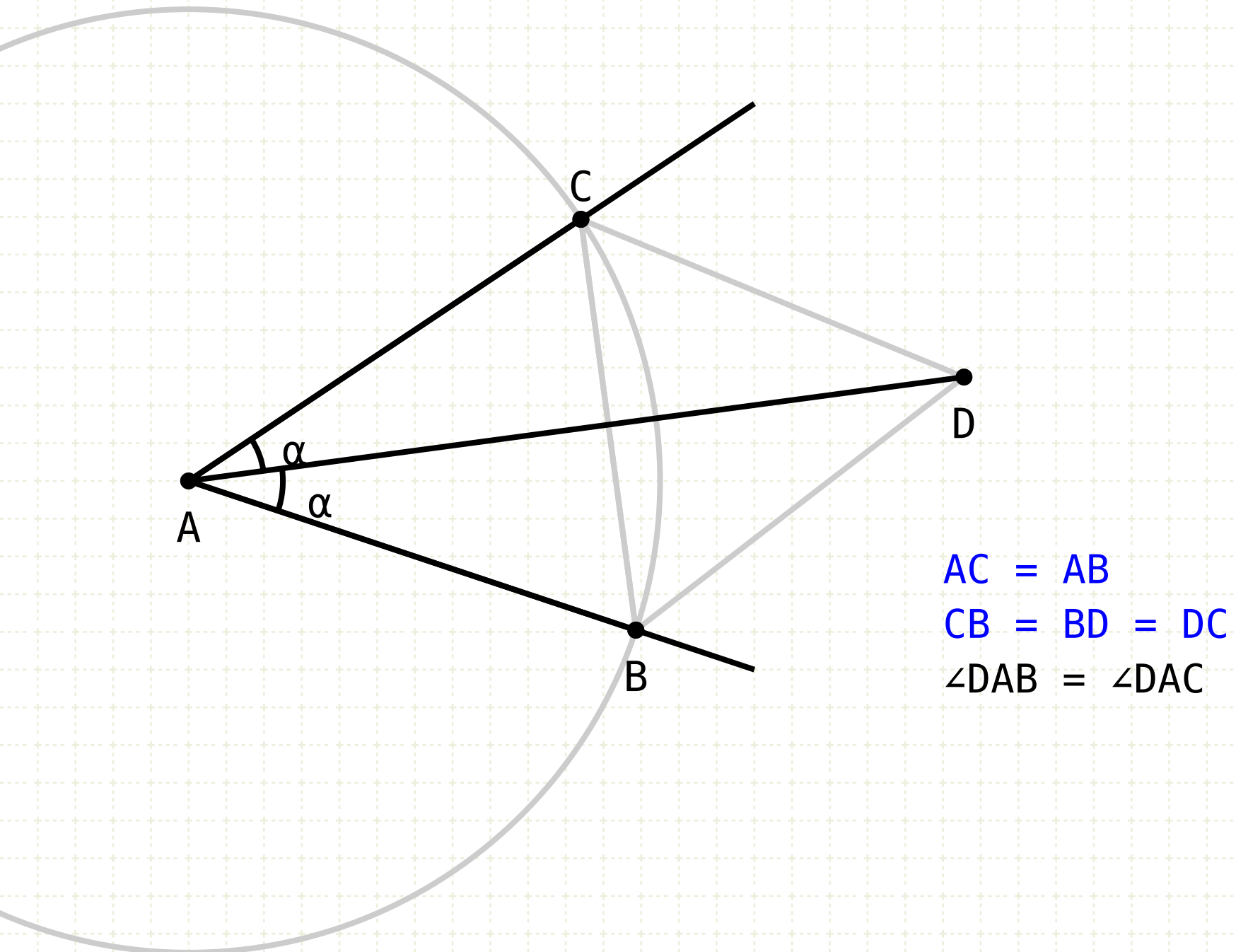
Pick an arbitrary point B on one of the lines, and construct another point C on the other line, such that AB and AC are equal

Construct an equilateral triangle on line AC, and label the vertex D (I.1)

Create a line between points A and D

# Proposition 9 of Book I

To bisect a given rectilinear angle.



## Construction:

Pick an arbitrary point B on one of the lines, and construct another point C on the other line, such that AB and AC are equal

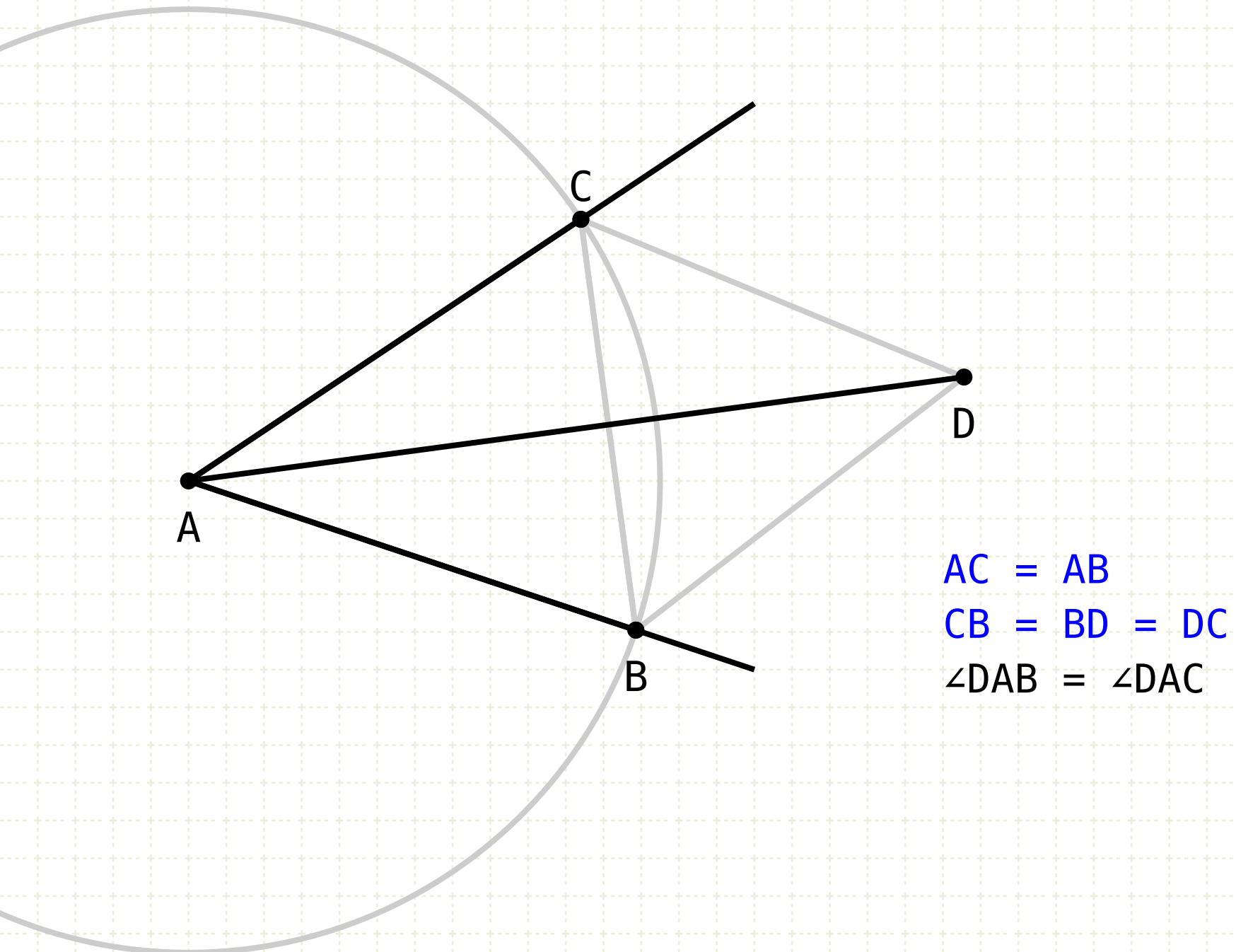
Construct an equilateral triangle on line AC, and label the vertex D (I.1)

Create a line between points A and D

Line AD bisects the angle CAB

# Proposition 9 of Book I

To bisect a given rectilinear angle.



## Construction:

Pick an arbitrary point B on one of the lines, and construct another point C on the other line, such that AB and AC are equal

Construct an equilateral triangle on line AC, and label the vertex D (I.1)

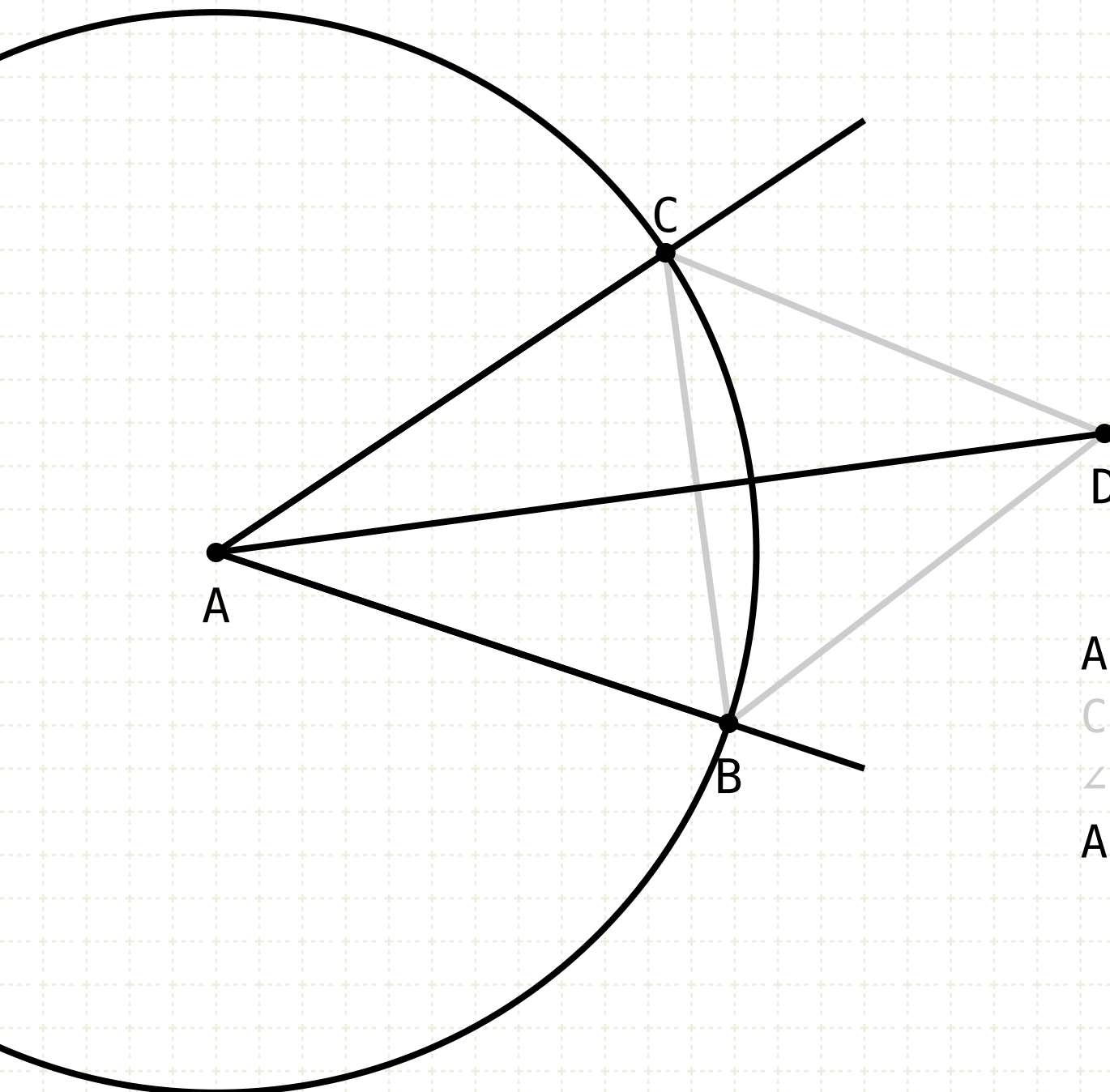
Create a line between points A and D

Line AD bisects the angle CAB

## Proof

# Proposition 9 of Book I

To bisect a given rectilinear angle.



$$AC = AB$$

$$CB = BD = DC$$

$$\angle DAB = \angle DAC$$

$$AB = AC$$

## Construction:

Pick an arbitrary point B on one of the lines, and construct another point C on the other line, such that AB and AC are equal

Construct an equilateral triangle on line AC, and label the vertex D (I.1)

Create a line between points A and D

Line AD bisects the angle CAB

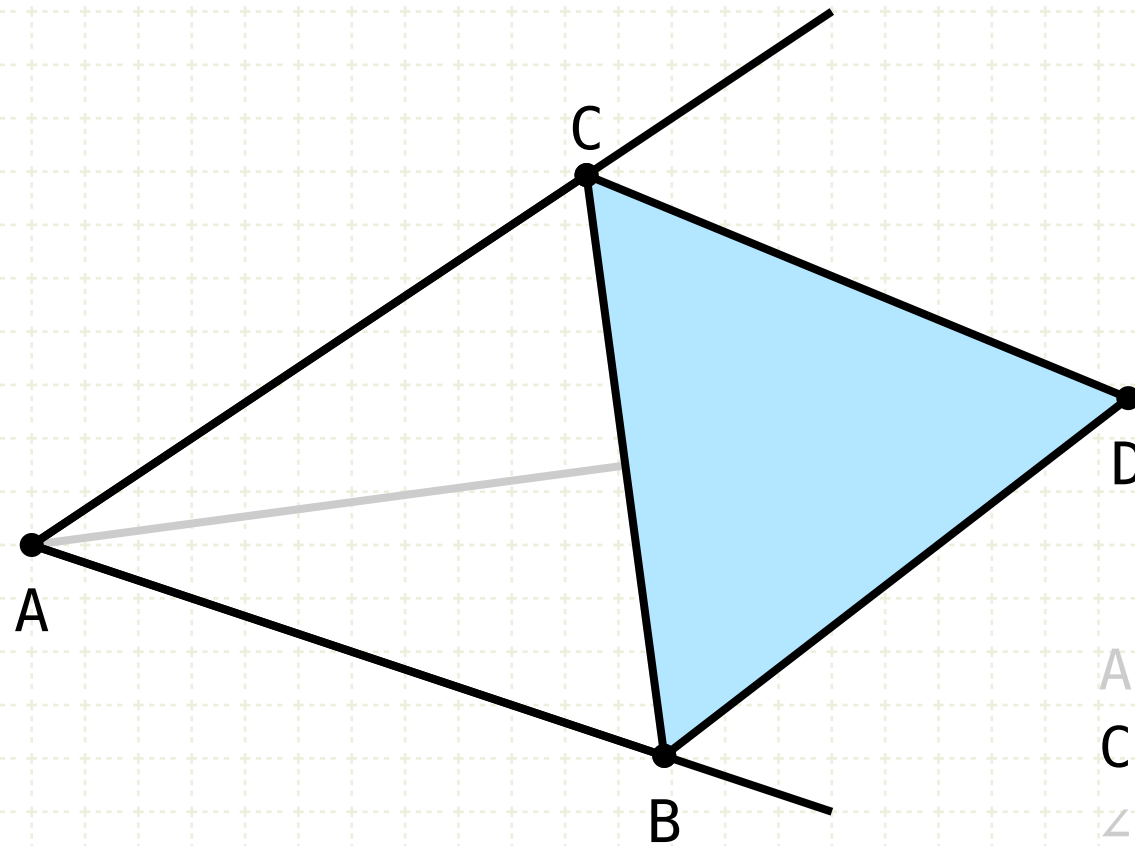
## Proof

Points B and C are equi-distance from point A since they are the radii of the same circle



# Proposition 9 of Book I

To bisect a given rectilinear angle.



$$\begin{aligned}AC &= AB \\CB &= BD = DC \\ \angle DAB &= \angle DAC \\AB &= AC \\DB &= DC\end{aligned}$$

## Construction:

Pick an arbitrary point B on one of the lines, and construct another point C on the other line, such that AB and AC are equal

Construct an equilateral triangle on line AC, and label the vertex D (I.1)

Create a line between points A and D

Line AD bisects the angle CAB

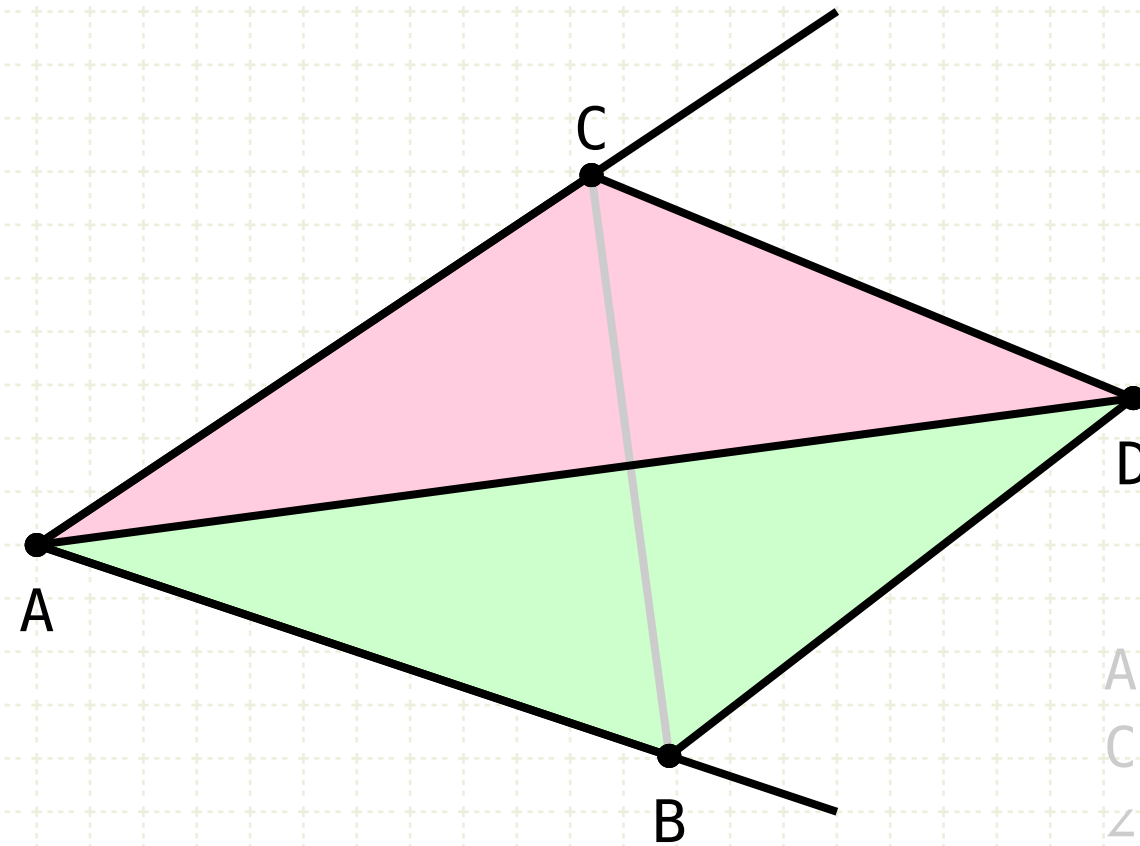
## Proof

Points B and C are equi-distance from point A since they are the radii of the same circle

Points B and C are equi-distance from point D since they are sides of an equilateral triangle

# Proposition 9 of Book I

To bisect a given rectilinear angle.



$$\begin{aligned}AC &= AB \\CB &= BD = DC \\ \angle DAB &= \angle DAC \\AB &= AC \\DB &= DC\end{aligned}$$

## Construction:

Pick an arbitrary point B on one of the lines, and construct another point C on the other line, such that AB and AC are equal

Construct an equilateral triangle on line AC, and label the vertex D (I·1)

Create a line between points A and D

Line AD bisects the angle CAB

## Proof

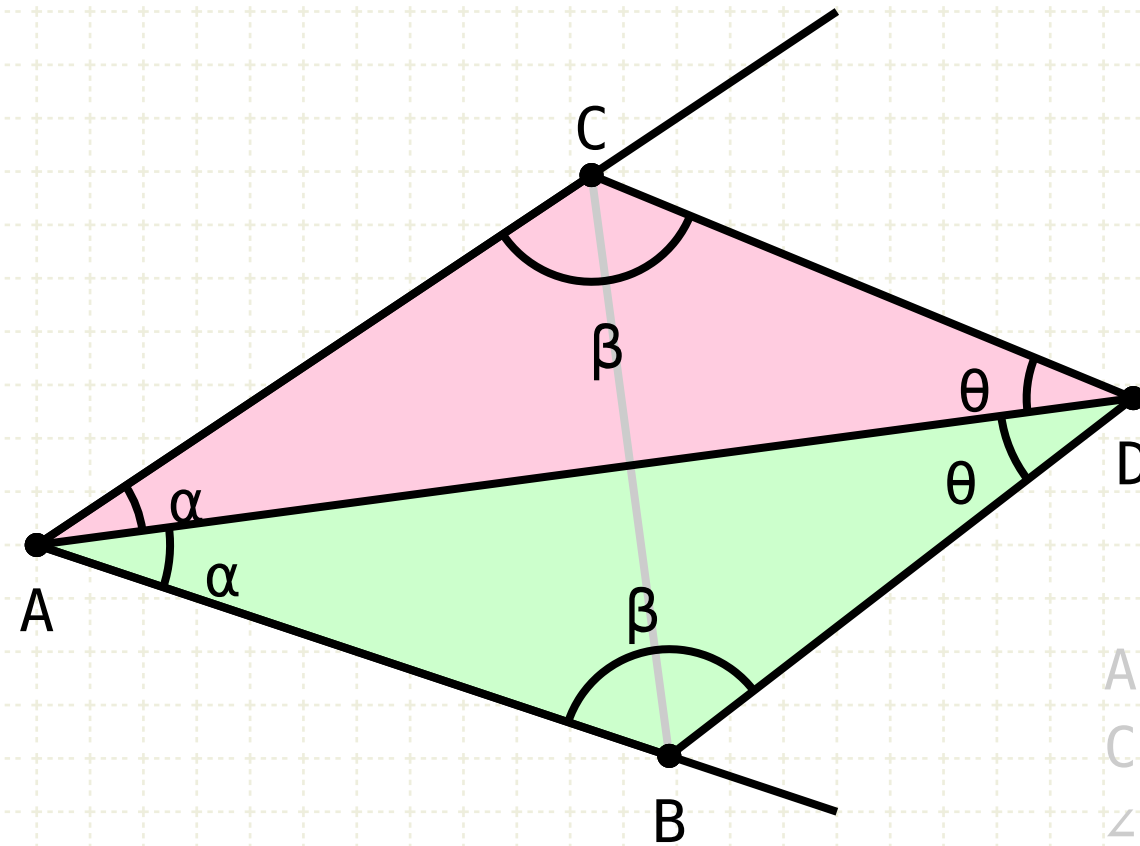
Points B and C are equi-distance from point A since they are the radii of the same circle

Points B and C are equi-distance from point D since they are sides of an equilateral triangle

Triangle ACD and ABD are congruent because they have three equal sides (I·8)

# Proposition 9 of Book I

To bisect a given rectilinear angle.



$$AC = AB$$

$$CB = BD = DC$$

$$\angle DAB = \angle DAC$$

$$AB = AC$$

$$DB = DC$$

$$\angle CAD = \angle DAB = \alpha$$

$$\angle ACD = \angle ABD = \beta$$

$$\angle CDA = \angle BDA = \theta$$

## Construction:

Pick an arbitrary point B on one of the lines, and construct another point C on the other line, such that AB and AC are equal

Construct an equilateral triangle on line AC, and label the vertex D (I·1)

Create a line between points A and D

Line AD bisects the angle CAB

## Proof

Points B and C are equi-distance from point A since they are the radii of the same circle

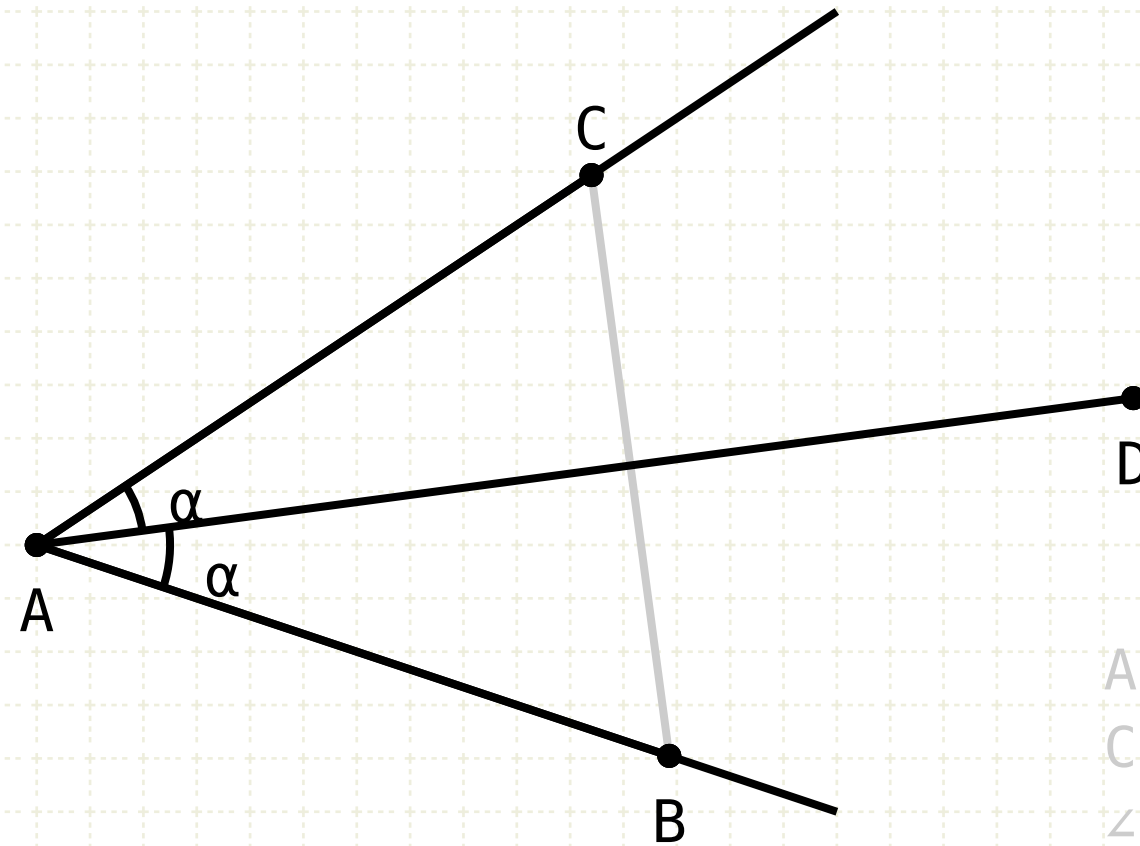
Points B and C are equi-distance from point D since they are sides of an equilateral triangle

Triangle ACD and ABD are congruent because they have three equal sides (I·8)

Hence, the angles are congruent as well

# Proposition 9 of Book I

To bisect a given rectilinear angle.



$$AC = AB$$

$$CB = BD = DC$$

$$\angle DAB = \angle DAC$$

$$AB = AC$$

$$DB = DC$$

$$\angle CAD = \angle DAB = \alpha$$

$$\angle ACD = \angle ABD = \beta$$

$$\angle CDA = \angle BDA = \theta$$

$$\angle CAB = 2 \times \alpha$$

## Construction:

Pick an arbitrary point B on one of the lines, and construct another point C on the other line, such that AB and AC are equal

Construct an equilateral triangle on line AC, and label the vertex D (I·1)

Create a line between points A and D

Line AD bisects the angle CAB

## Proof

Points B and C are equi-distance from point A since they are the radii of the same circle

Points B and C are equi-distance from point D since they are sides of an equilateral triangle

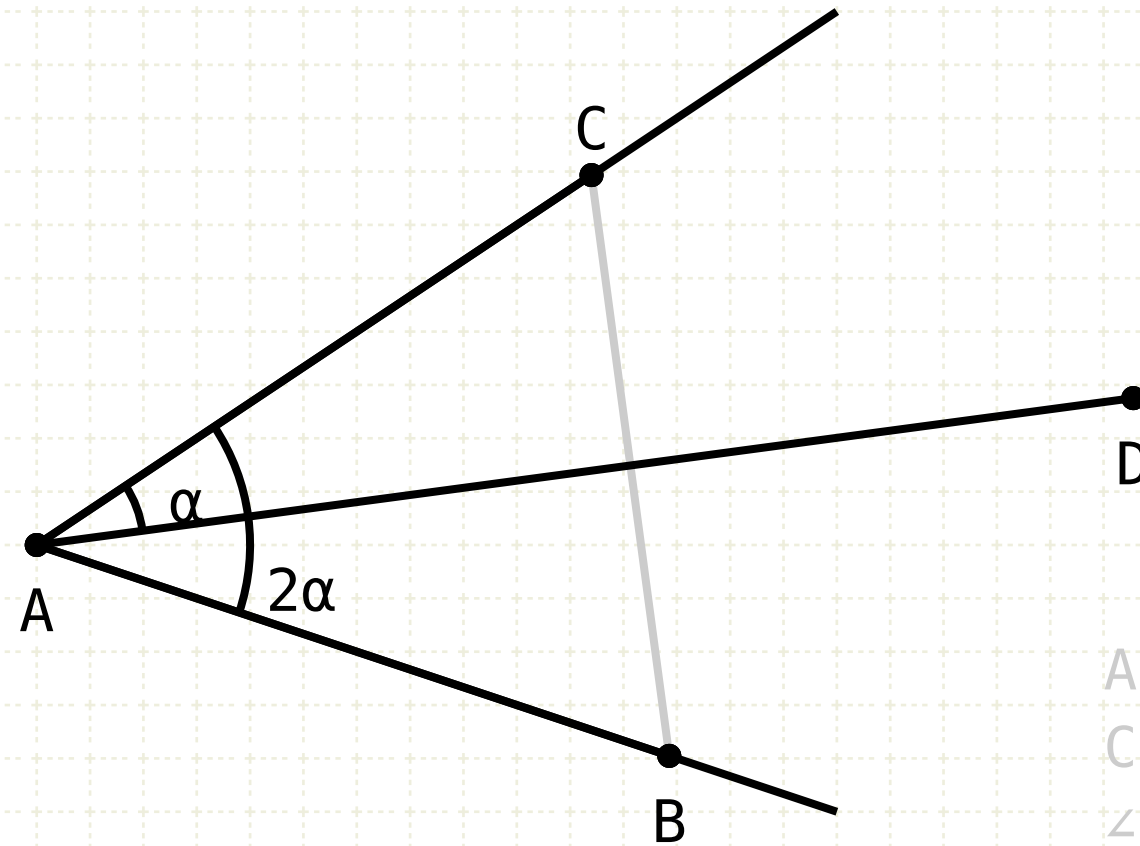
Triangle ACD and ABD are congruent because they have three equal sides (I·8)

Hence, the angles are congruent as well

Angle CAB is equal to twice angle CAD

# Proposition 9 of Book I

To bisect a given rectilinear angle.



$$\begin{aligned}AC &= AB \\CB &= BD = DC \\ \angle DAB &= \angle DAC \\AB &= AC \\DB &= DC \\ \angle CAD &= \angle DAB = \alpha \\ \angle ACD &= \angle ABD = \beta \\ \angle CDA &= \angle BDA = \theta \\ \angle CAB &= 2 \times \alpha \\ \angle CAD &= \frac{1}{2} \angle CAB\end{aligned}$$

## Construction:

Pick an arbitrary point B on one of the lines, and construct another point C on the other line, such that AB and AC are equal

Construct an equilateral triangle on line AC, and label the vertex D (I·1)

Create a line between points A and D

Line AD bisects the angle CAB

## Proof

Points B and C are equi-distance from point A since they are the radii of the same circle

Points B and C are equi-distance from point D since they are sides of an equilateral triangle

Triangle ACD and ABD are congruent because they have three equal sides (I·8)

Hence, the angles are congruent as well

Angle CAB is equal to twice angle CAD

Or angle CAD is half the angle CAB



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