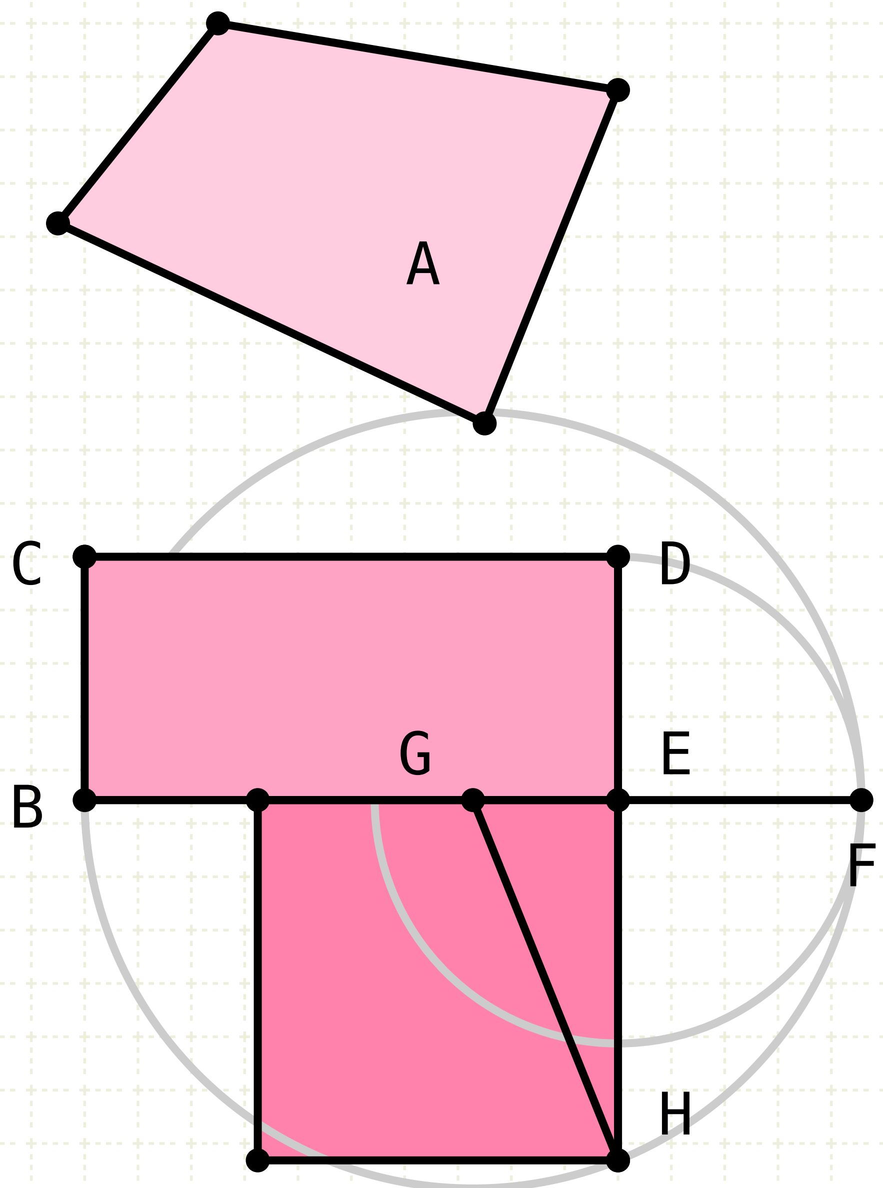
Euclid's Elements

Book II



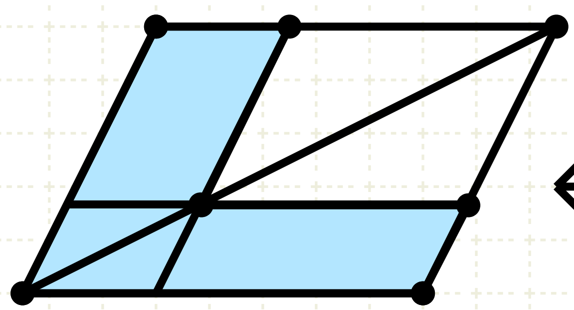
It is a remarkable fact in the history of geometry, that the Elements of Euclid, written two thousand years ago, are still regarded by many as the best introduction to the mathematical sciences.

Florian Cajori,

A History of Mathematics (1893)

Definitions:

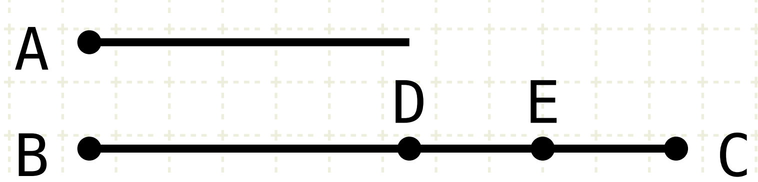
Any rectangular parallelogram is said to be contained by the two straight lines containing the right angle.



And in any parallelogrammic area let any one whatever of the parallelograms about its diameter with the two complements be called a gnomon.

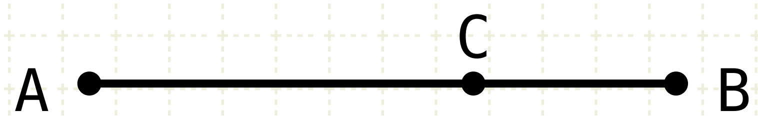
Table of Contents, Chapter 2

1



$A \cdot BC = A \cdot BD + A \cdot DE + A \cdot EC$

2

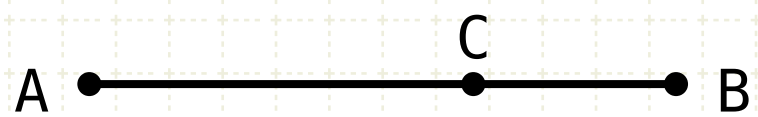


$AB^2 = AB \cdot AC + AB \cdot BC$

3

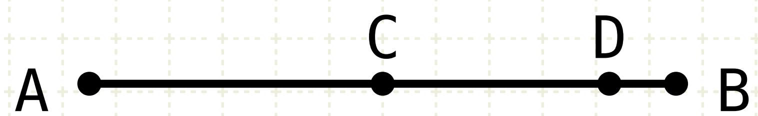
$AB \cdot CB = AC \cdot CB + CB^2$

4



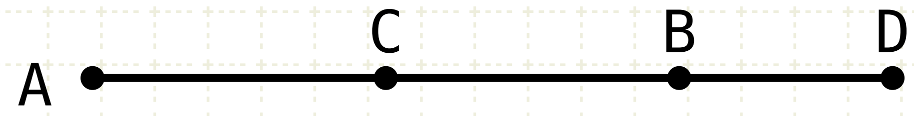
$AB^2 = AC^2 + CB^2 + 2 \cdot AC \cdot CB$

5



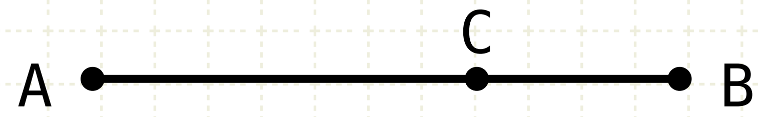
$AD \cdot DB + CD^2 = CB^2$

6



$AD \cdot DB + CB^2 = CD^2$

7

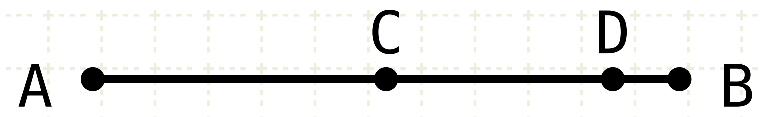


$AB^2 + BC^2 = AC^2 + 2 \cdot AB \cdot BC$

8

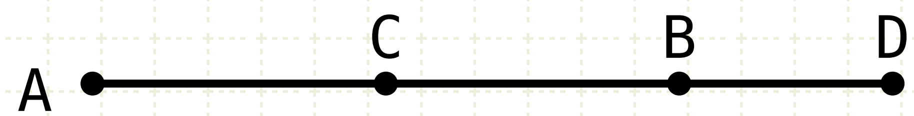
$4 \cdot AB \cdot BC + AC^2 = (AB+BC)^2$

9



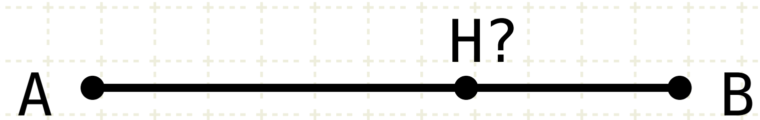
$\mathbf{AD^2 + DB^2 = 2 \cdot (AC^2 + CD^2)}$

10



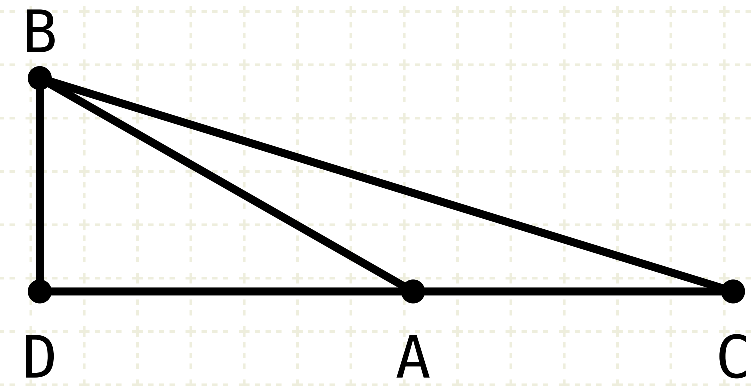
$AD^2 + DB^2 = 2 \cdot (AC^2 + CD^2)$

11



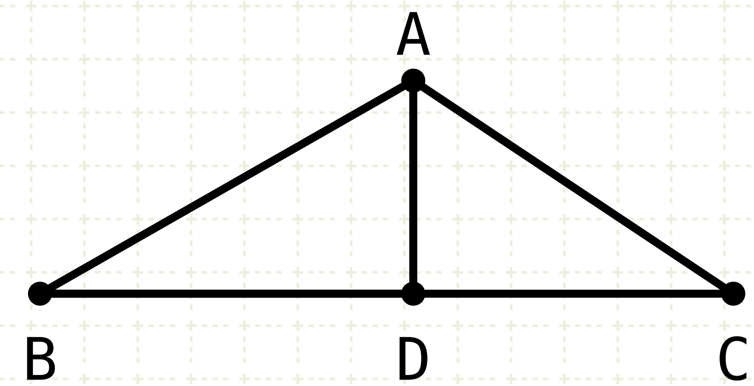
Find H. $AB \cdot BH = AH^2$

12



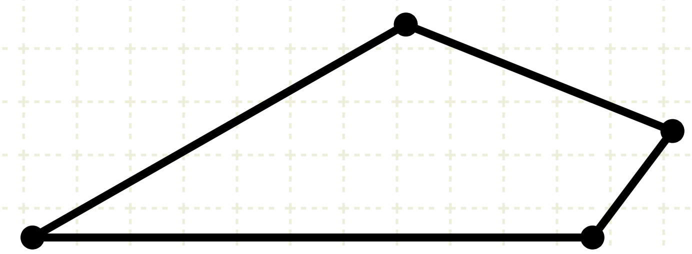
Cosine Law. $BC^2 = AB^2+AC^2+2 \cdot AD \cdot AC$

13



Cosine Law. $AC^2 = AB^2+BC^2-2 \cdot BD \cdot BC$

14



Find square of polygon

Proposition 9 of Book II

If a straight line be cut into equal and unequal segments, the squares on the unequal segments of the whole are double of the square on the half and of the square on the straight line between the points of section.

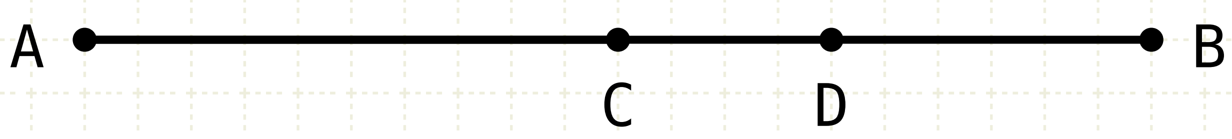
Proposition 9 of Book II

If a straight line be cut into equal and unequal segments, the squares on the unequal segments of the whole are double of the square on the half and of the square on the straight line between the points of section.

AC = CB, AC,CD,DB = AB

In other words

Let AB be a straight line, bisected at point C, and cut at an arbitrary point D



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If a straight line be cut into equal and unequal segments, the squares on the unequal segments of the whole are double of the square on the half and of the square on the straight line between the points of section.

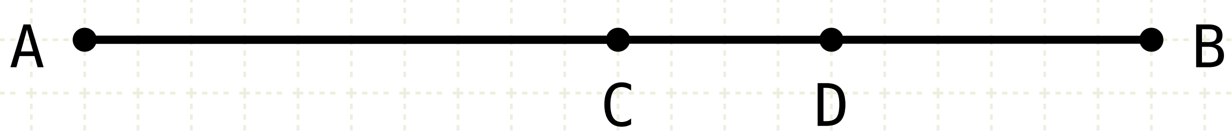
$AC = CB,\ AC, CD, DB = AB$

$AD^2 + DB^2 = 2 \cdot (AC^2 + CD^2)$

In other words

Let AB be a straight line, bisected at point C, and cut at an arbitrary point D

The sum of the squares of AD and DB is equal to twice the sum of the squares of AC and DC



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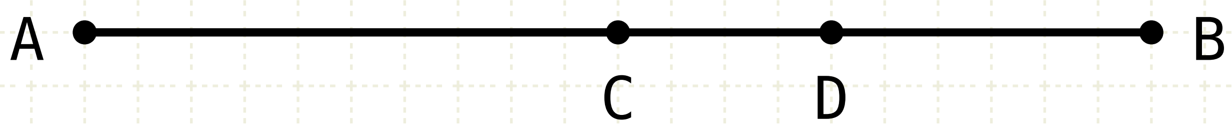
$AC = CB,\ AC, CD, DB = AB$

$AD^2 + DB^2 = 2 \cdot (AC^2 + CD^2)$

$(x+y)^2 + (x-y)^2$
$= 2(x^2 + y^2)$



In other words

Let AB be a straight line, bisected at point C, and cut at an arbitrary point D

The sum of the squares of AD and DB is equal to twice the sum of the squares of AC and DC

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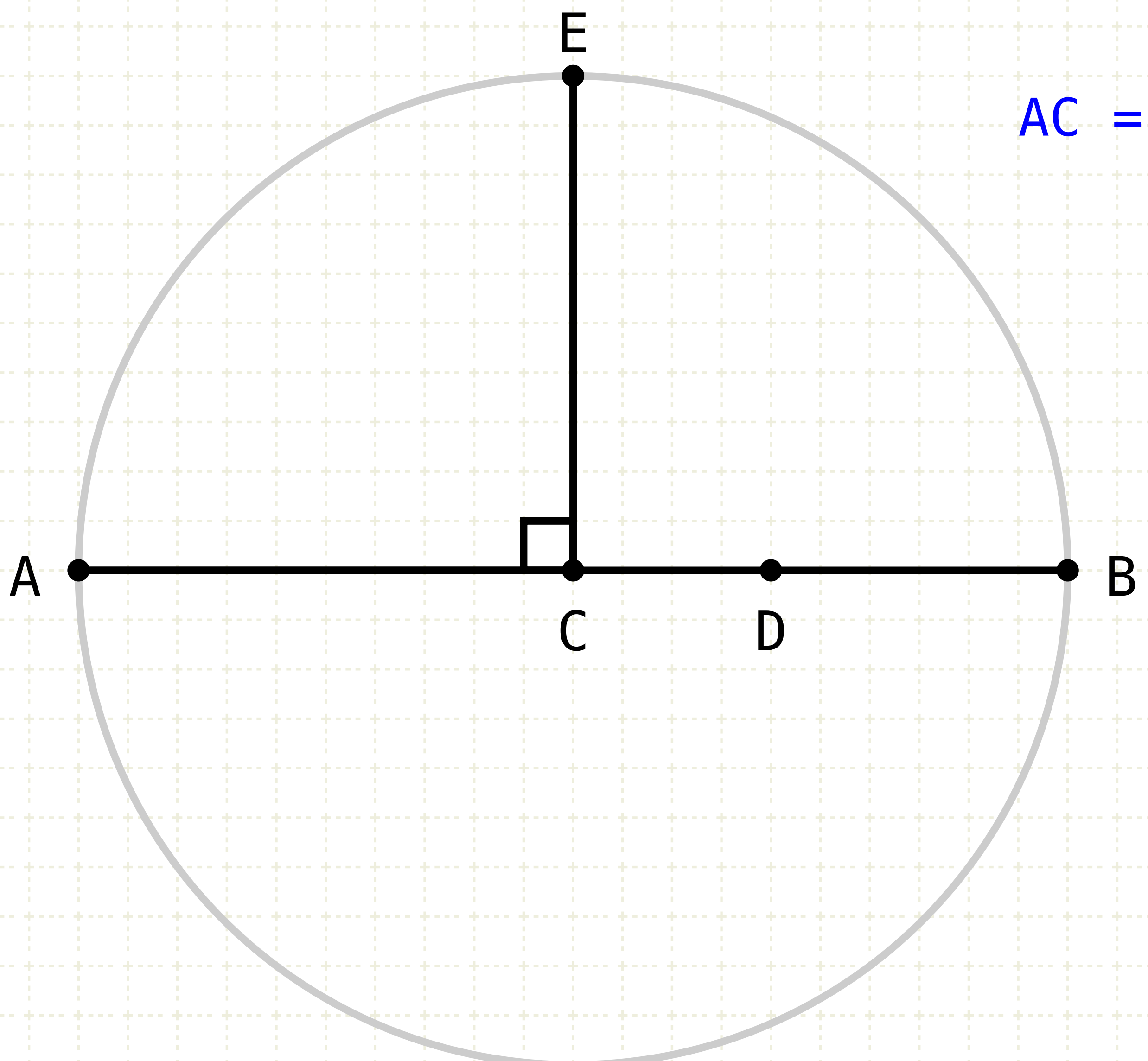

AC = CB, AC,CD,DB = AB

AC = CE
CB = CE

Construction:

Draw a line perpendicular to AB through point C (I·11), and make its length equal to AC or CB (I·3)

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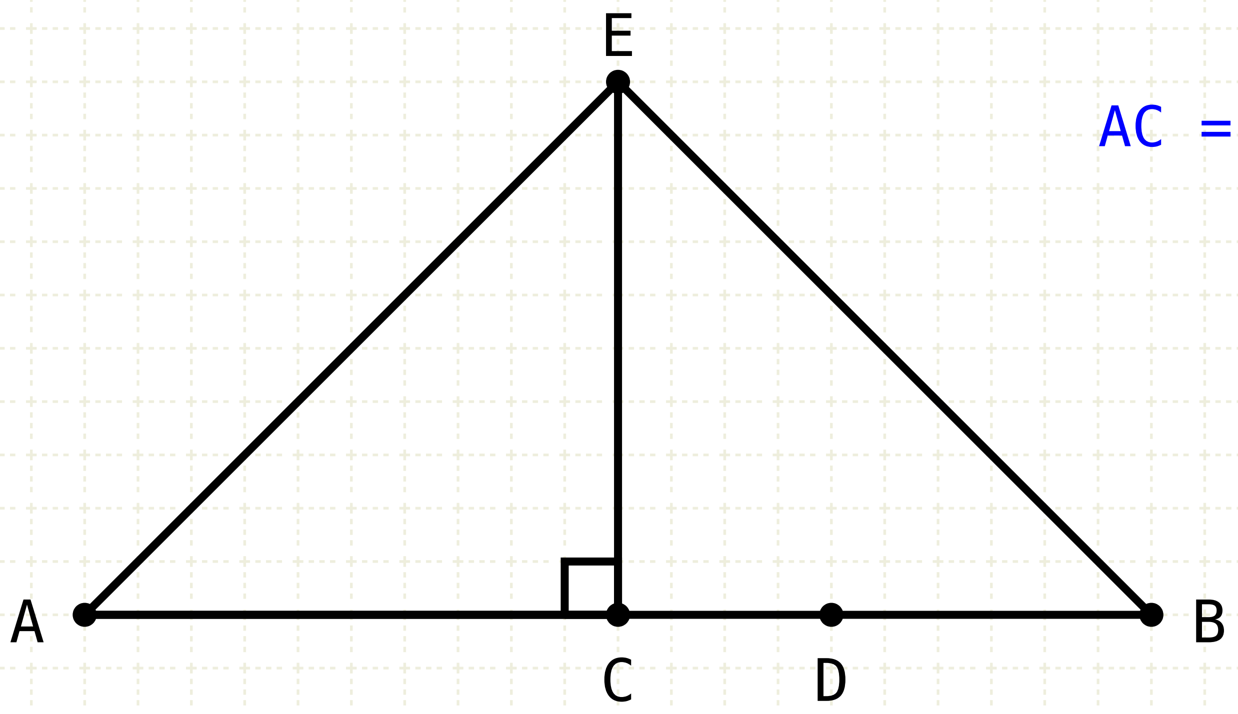

AC = CB, AC,CD,DB = AB

AC = CE
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Construction:

Draw a line perpendicular to AB through point C (I·11), and make its length equal to AC or CB (I·3)

Connect AE and EB

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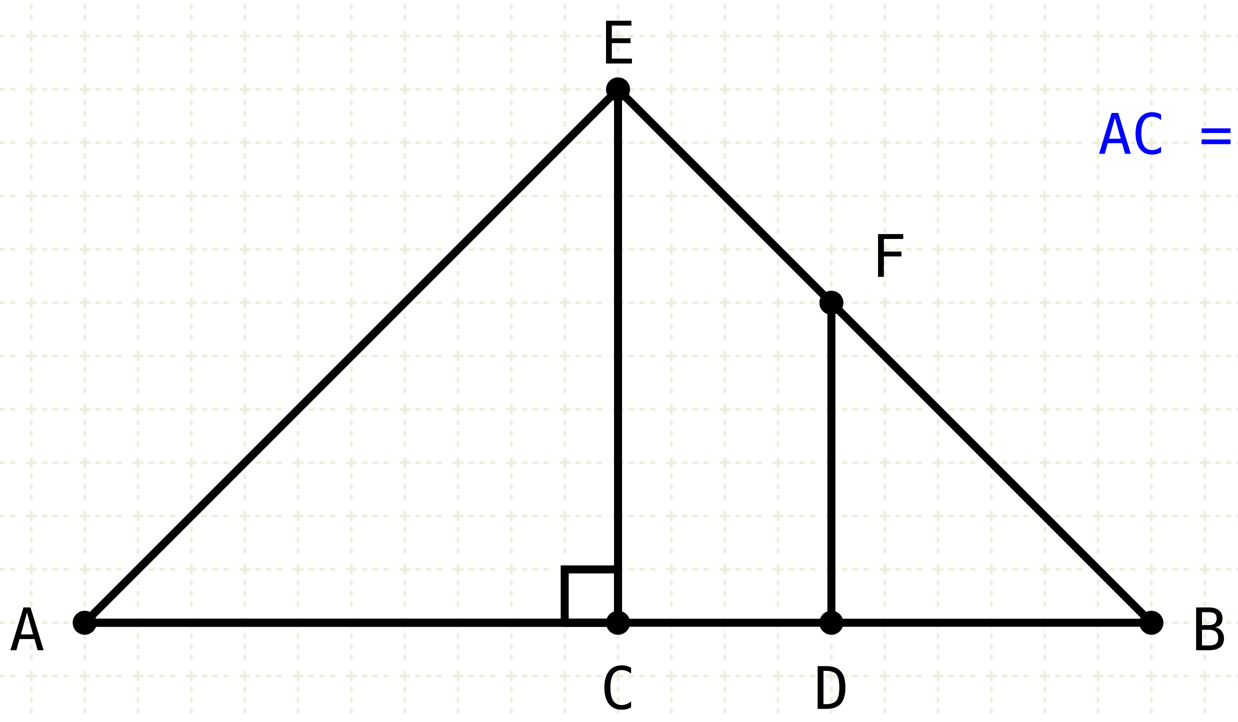

AC = CB, AC,CD,DB = AB

AC = CE
CB = CE

Construction:

Draw a line perpendicular to AB through point C (I·11), and make its length equal to AC or CB (I·3)

Connect AE and EB

Draw a line parallel to EC through point D

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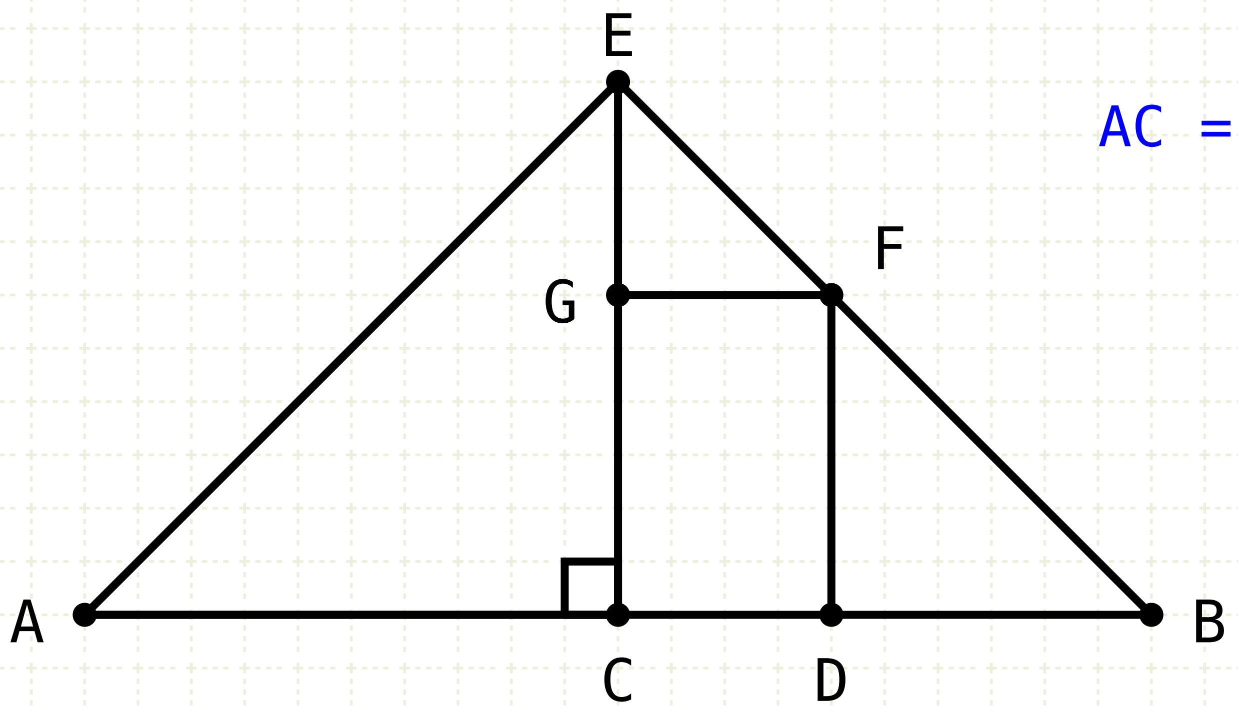

AC = CB, AC,CD,DB = AB

AC = CE
CB = CE

Construction:

Draw a line perpendicular to AB through point C (I·11), and make its length equal to AC or CB (I·3)

Connect AE and EB

Draw a line parallel to EC through point D

Draw a line parallel to AB through point F

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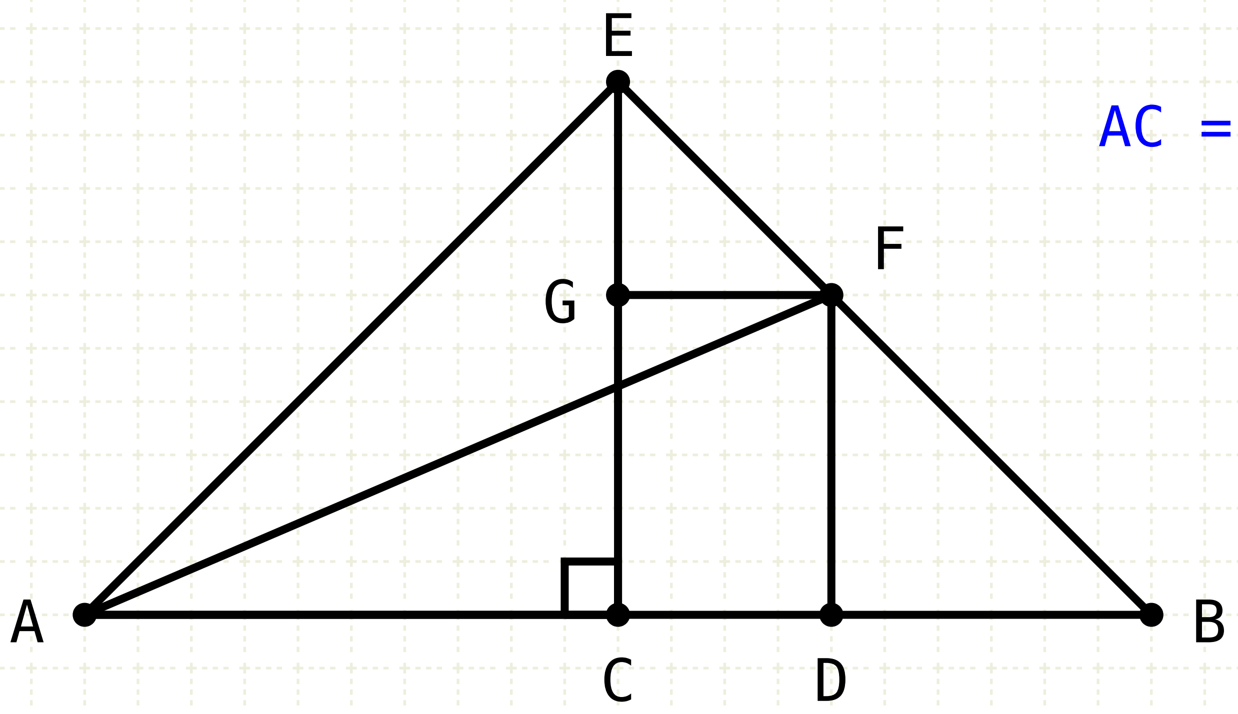

AC = CB, AC,CD,DB = AB

AC = CE
CB = CE

Construction:

Draw a line perpendicular to AB through point C (I·11), and make its length equal to AC or CB (I·3)

Connect AE and EB

Draw a line parallel to EC through point D

Draw a line parallel to AB through point F

Join AF

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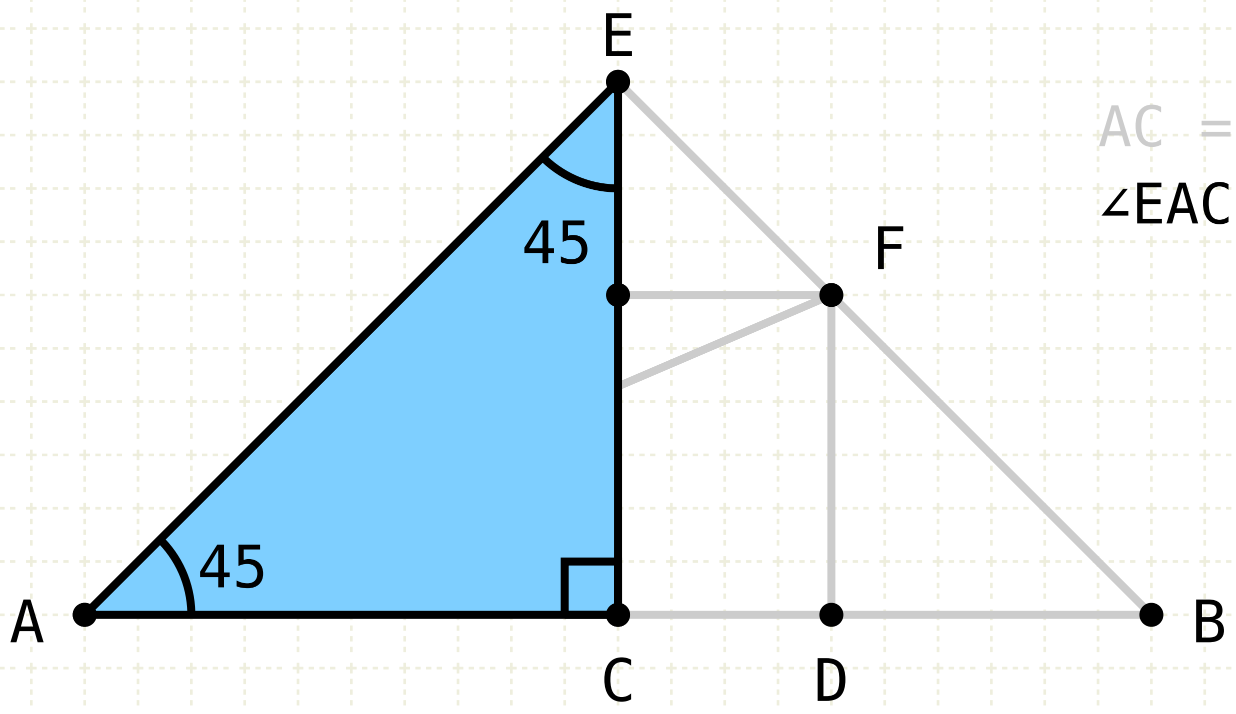

$AC = CB$, $AC, CD, DB = AB$

$\angle EAC = \angle CEA = 45$

$AC = CE$

$CB = CE$

Proof

Triangle AEC is a right angle triangle, and AC and CE are equal, therefore it is an isosceles triangle

Since the sum of the angles in a triangle equals two right angles (I·32), and ACE is a right angle, then the two base angles (being equal (I·5)) each equal one half a right angle (45 degrees)

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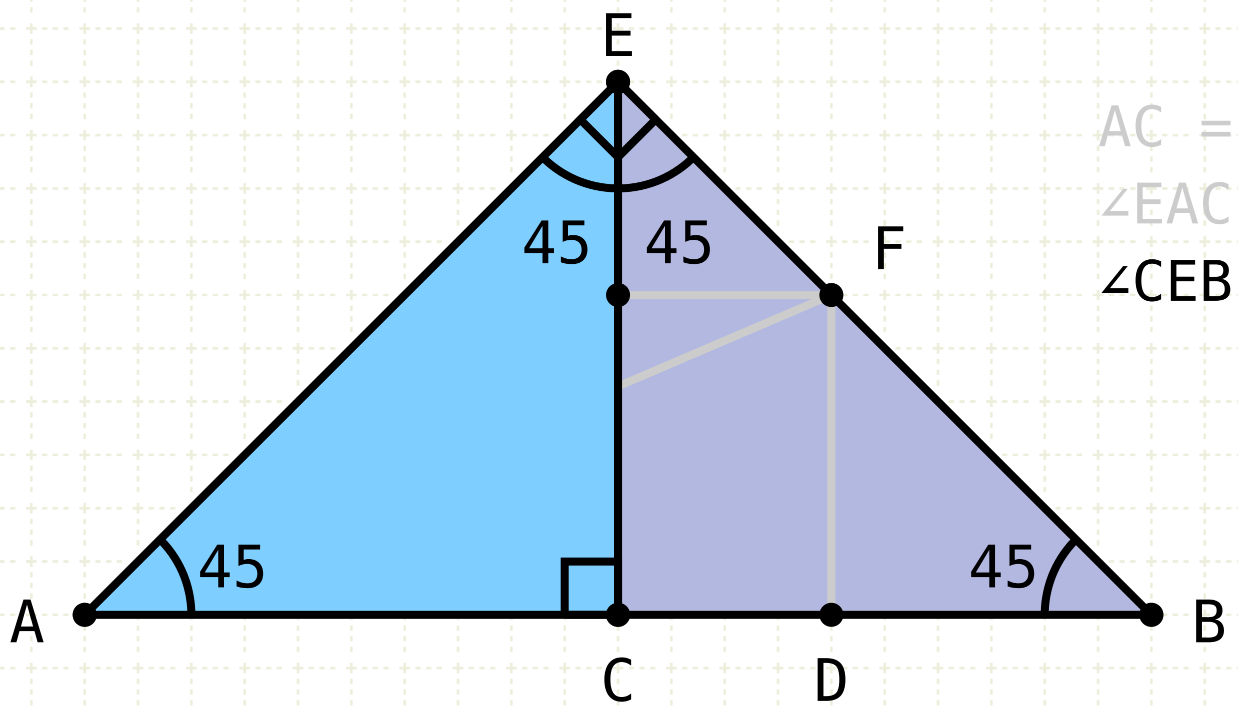

AC = CB, AC,CD,DB = AB

∠EAC = ∠CEA = 45

∠CEB = ∠CBE = 45

AC = CE

CB = CE

Proof

Triangle AEC is a right angle triangle, and AC and CE are equal, therefore it is an isosceles triangle

Since the sum of the angles in a triangle equals two right angles (I·32), and ACE is a right angle, then the two base angles (being equal (I·5)) each equal one half a right angle (45 degrees)

By the same reason, angles CEB and CBE are each half a right angle, which makes AEB a right angle

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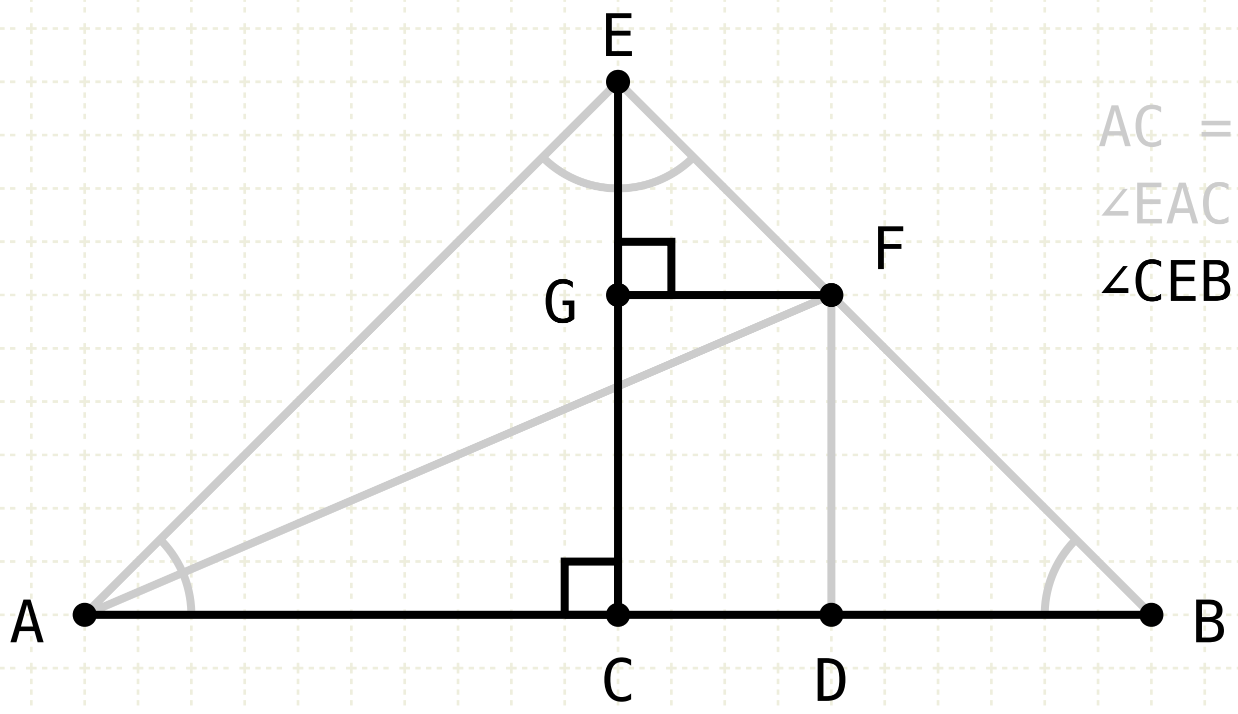

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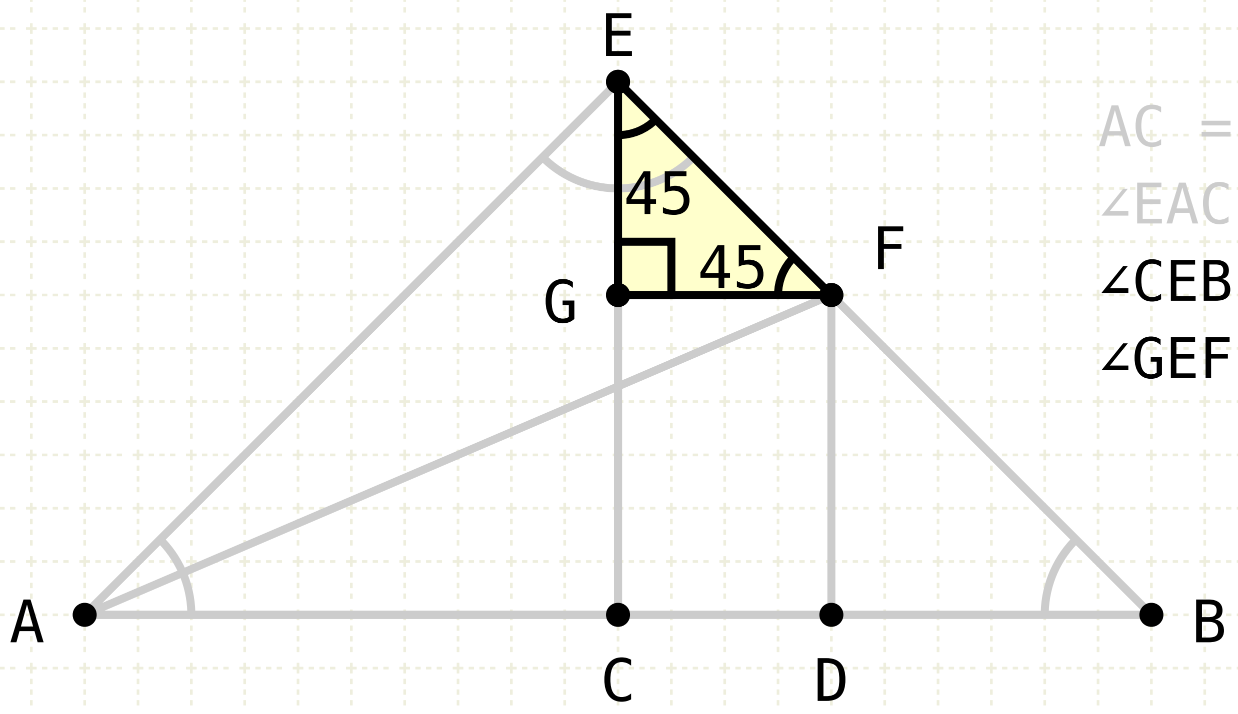
By the same reason, angles CEB and CBE are each half a right angle, which makes AEB a right angle

Since AB and GF are parallel, and CE intersects them, the opposite and interior angles are equal (I·29), so EGF is a right angle

Proposition 9 of Book II

If a straight line be cut into equal and unequal segments, the squares on the unequal segments of the whole are double of the square on the half and of the square on the straight line between the points of section.



AC = CB, AC,CD,DB = AB
∠EAC = ∠CEA = 45
∠CEB = ∠CBE = 45
∠GEF = ∠EFG = 45

AC = CE
CB = CE
EG = GF

Proof

Triangle AEC is a right angle triangle, and AC and CE are equal, therefore it is an isosceles triangle

Since the sum of the angles in a triangle equals two right angles (I·32), and ACE is a right angle, then the two base angles (being equal (I·5)) each equal one half a right angle (45 degrees)

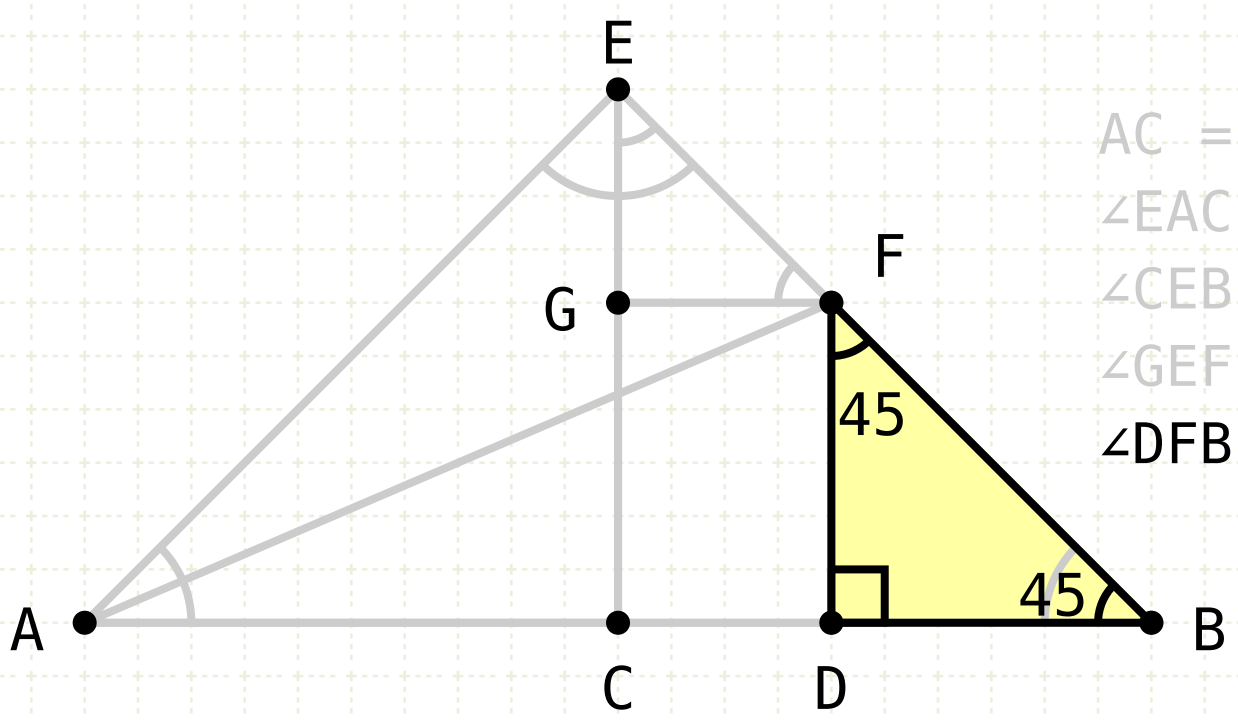
By the same reason, angles CEB and CBE are each half a right angle, which makes AEB a right angle

Since AB and GF are parallel, and CE intersects them, the opposite and interior angles are equal (I·29), so EGF is a right angle

The angle EFG is one half a right angle (I·32), and since two angles are equal, EGF is isosceles (I·6), so EG equals GF

Proposition 9 of Book II

If a straight line be cut into equal and unequal segments, the squares on the unequal segments of the whole are double of the square on the half and of the square on the straight line between the points of section.



AC = CB, AC,CD,DB = AB
∠EAC = ∠CEA = 45
∠CEB = ∠CBE = 45
∠GEF = ∠EFG = 45
∠DFB = ∠FBD = 45

AC = CE
CB = CE
EG = GF
DB = FD

Proof

Triangle AEC is a right angle triangle, and AC and CE are equal, therefore it is an isosceles triangle

Since the sum of the angles in a triangle equals two right angles (I·32), and ACE is a right angle, then the two base angles (being equal (I·5)) each equal one half a right angle (45 degrees)

By the same reason, angles CEB and CBE are each half a right angle, which makes AEB a right angle

Since AB and GF are parallel, and CE intersects them, the opposite and interior angles are equal (I·29), so EGF is a right angle

The angle EFG is one half a right angle (I·32), and since two angles are equal, EGF is isosceles (I·6), so EG equals GF

Using the same logic, FDB is also an isosceles triangle, and DB equals FD

Proposition 9 of Book II

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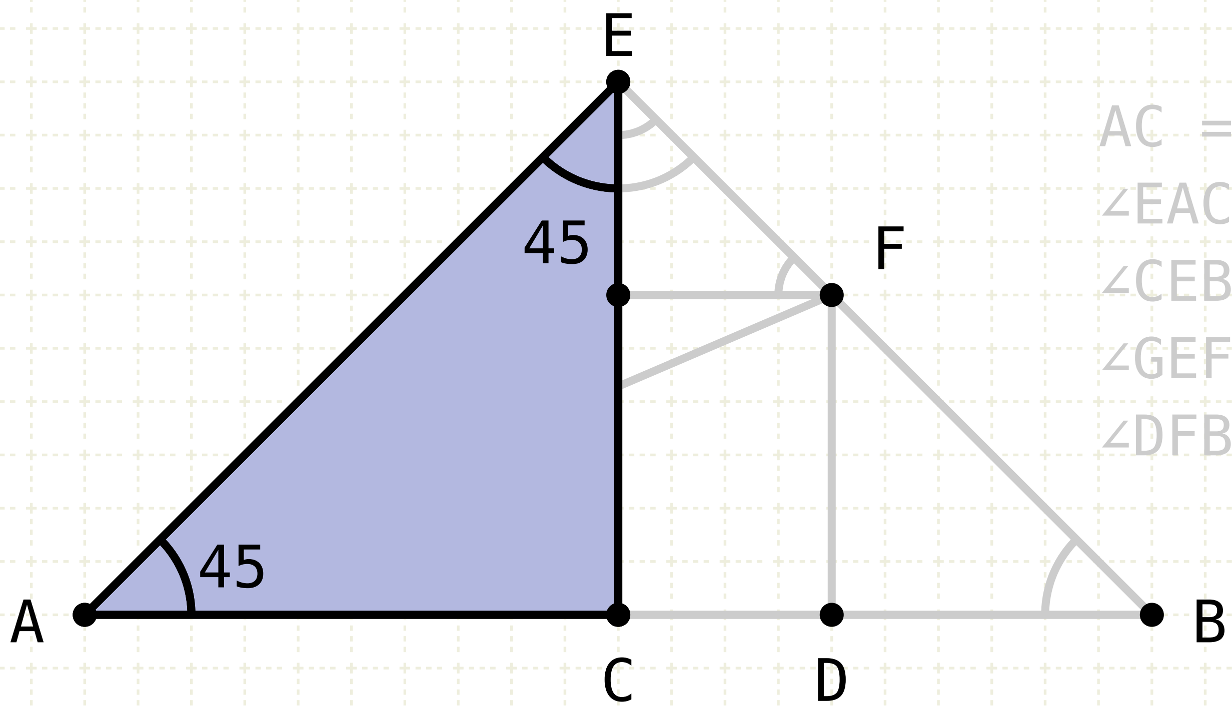


$AC = CB,\ AC, CD, DB = AB$

$\angle EAC = \angle CEA = 45$

$\angle CEB = \angle CBE = 45$

$\angle GEF = \angle EFG = 45$

$\angle DFB = \angle FBD = 45$

$AC = CE$

$CB = CE$

$EG = GF$

$DB = FD$

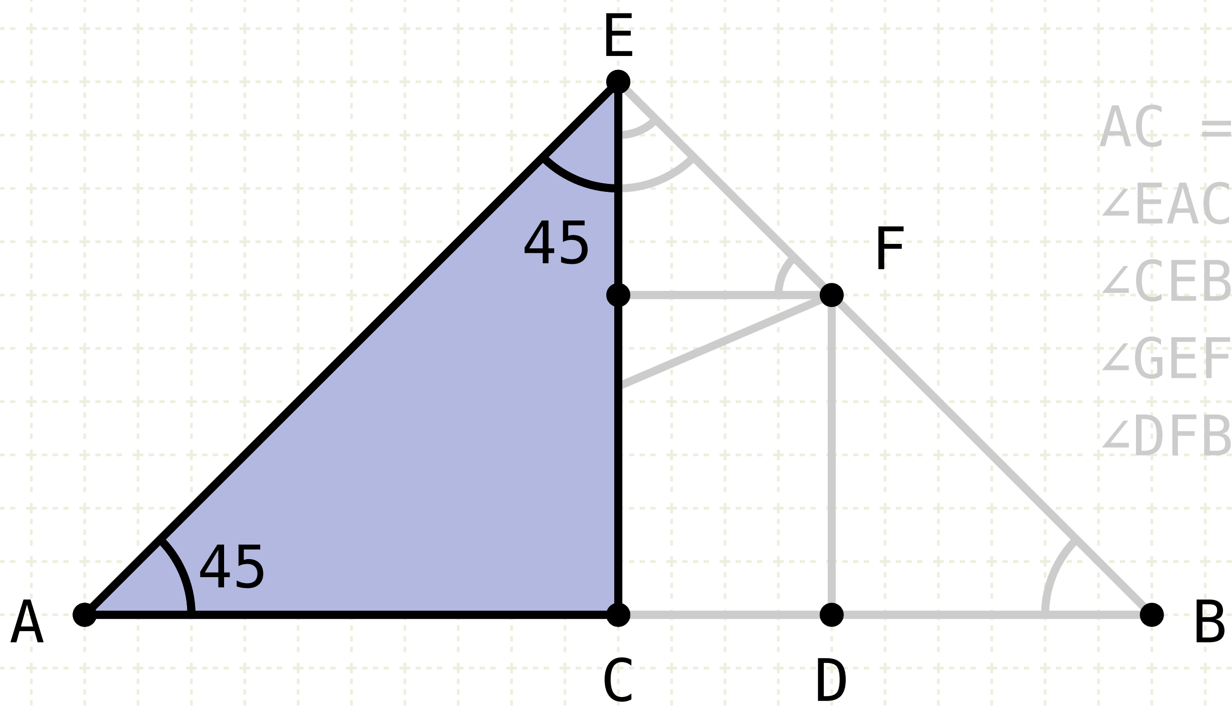
$$AE^2 = AC^2 + EC^2$$

Proof

The triangle AEC is a right angle, thus the square on AE equals the sum of the squares of AC and AE

Proposition 9 of Book II

If a straight line be cut into equal and unequal segments, the squares on the unequal segments of the whole are double of the square on the half and of the square on the straight line between the points of section.



AC = CB, AC,CD,DB = AB

∠EAC = ∠CEA = 45

∠CEB = ∠CBE = 45

∠GEF = ∠EFG = 45

∠DFB = ∠FBD = 45

AC = CE

CB = CE

EG = GF

DB = FD

$$AE^2 = AC^2 + EC^2 = 2 \cdot AC^2$$

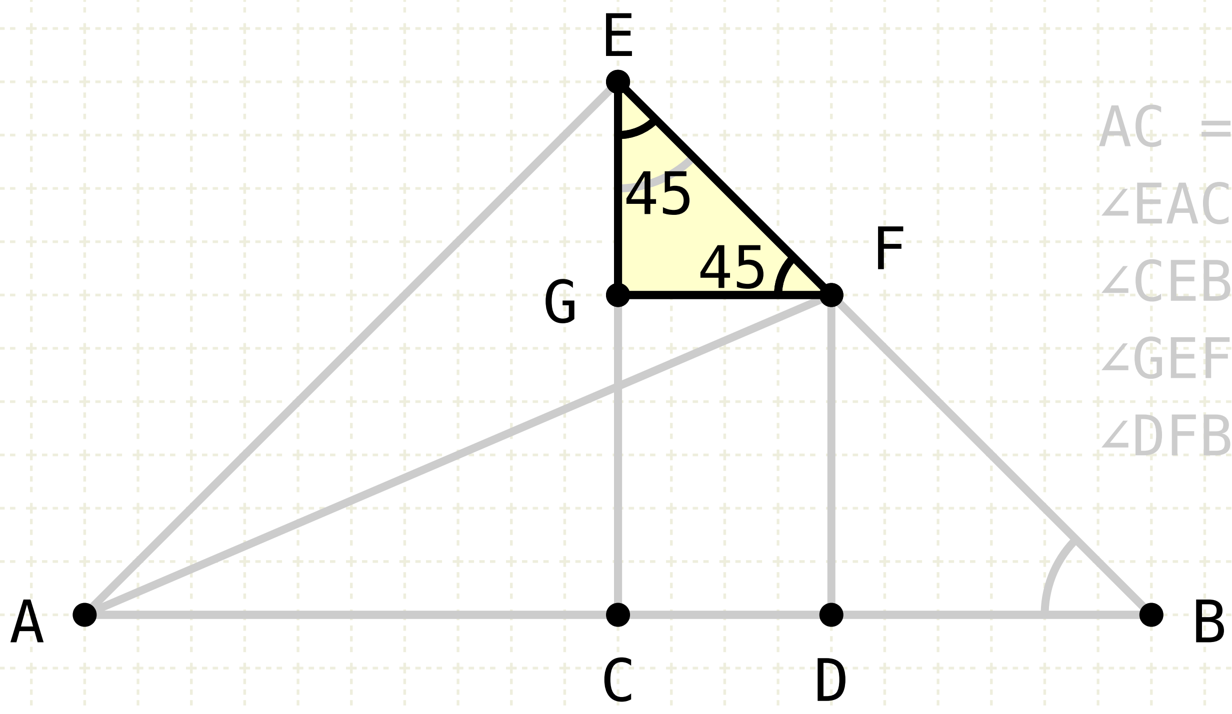
Proof

The triangle AEC is a right angle, thus the square on AE equals the sum of the squares of AC and AE

Since AC equals CE, the sum of the squares of AC and CE equals twice the square of AC

Proposition 9 of Book II

If a straight line be cut into equal and unequal segments, the squares on the unequal segments of the whole are double of the square on the half and of the square on the straight line between the points of section.



$AC = CB,\ AC, CD, DB = AB$

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$\angle GEF = \angle EFG = 45$

$\angle DFB = \angle FBD = 45$

$AC = CE$

$CB = CE$

$EG = GF$

$DB = FD$

$AE^2 = AC^2 + EC^2 = 2 \cdot AC^2$

$EF^2 = EG^2 + GF^2 = 2 \cdot GF^2$

Proof

The triangle AEC is a right angle, thus the square on AE equals the sum of the squares of AC and AE

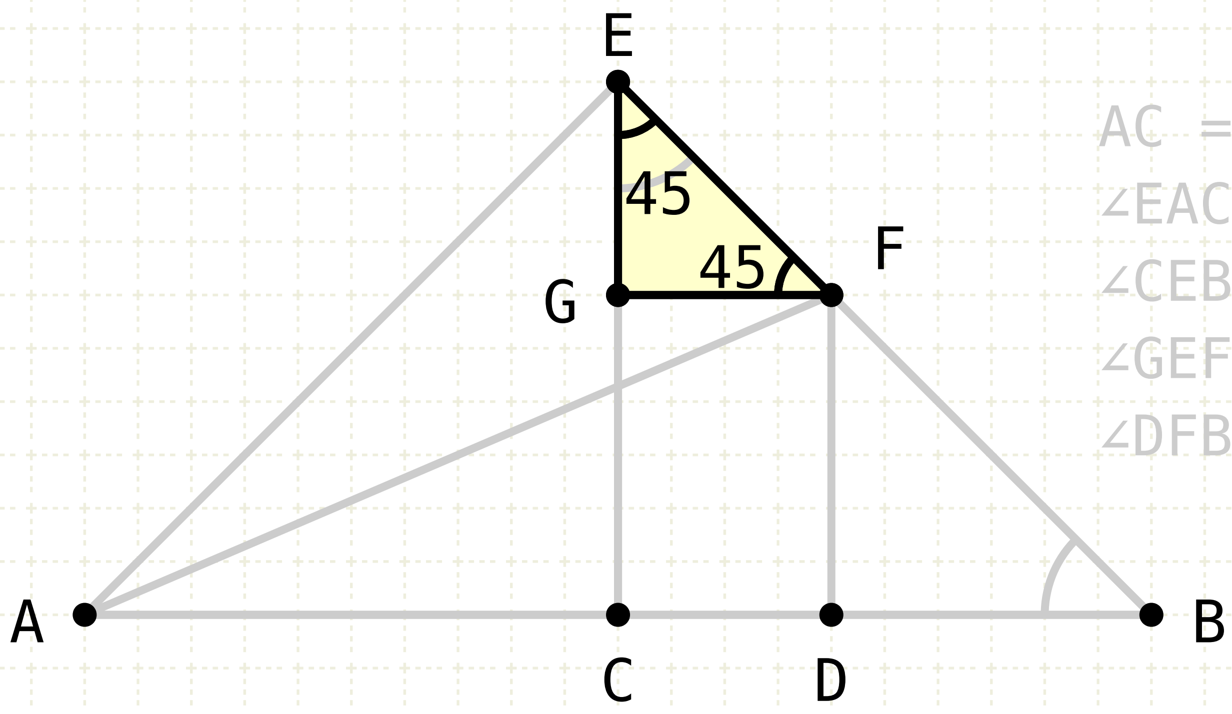
Since AC equals CE, the sum of the squares of AC and CE equals twice the square of AC

Since EG equals GF, the sum of the squares of EG and GF equals twice the square of GF

The triangle EGF is a right angle, thus the square on EF equals the sum of the squares of EG and GF

Proposition 9 of Book II

If a straight line be cut into equal and unequal segments, the squares on the unequal segments of the whole are double of the square on the half and of the square on the straight line between the points of section.



$AC = CB,\ AC, CD, DB = AB$

$\angle EAC = \angle CEA = 45$

$\angle CEB = \angle CBE = 45$

$\angle GEF = \angle EFG = 45$

$\angle DFB = \angle FBD = 45$

$AC = CE$

$CB = CE$

$EG = GF$

$DB = FD$

$AE^2 = AC^2 + EC^2 = 2 \cdot AC^2$

$EF^2 = EG^2 + GF^2 = 2 \cdot GF^2$

$EF^2 = 2 \cdot CD^2$

Proof

The triangle AEC is a right angle, thus the square on AE equals the sum of the squares of AC and AE

Since AC equals CE, the sum of the squares of AC and CE equals twice the square of AC

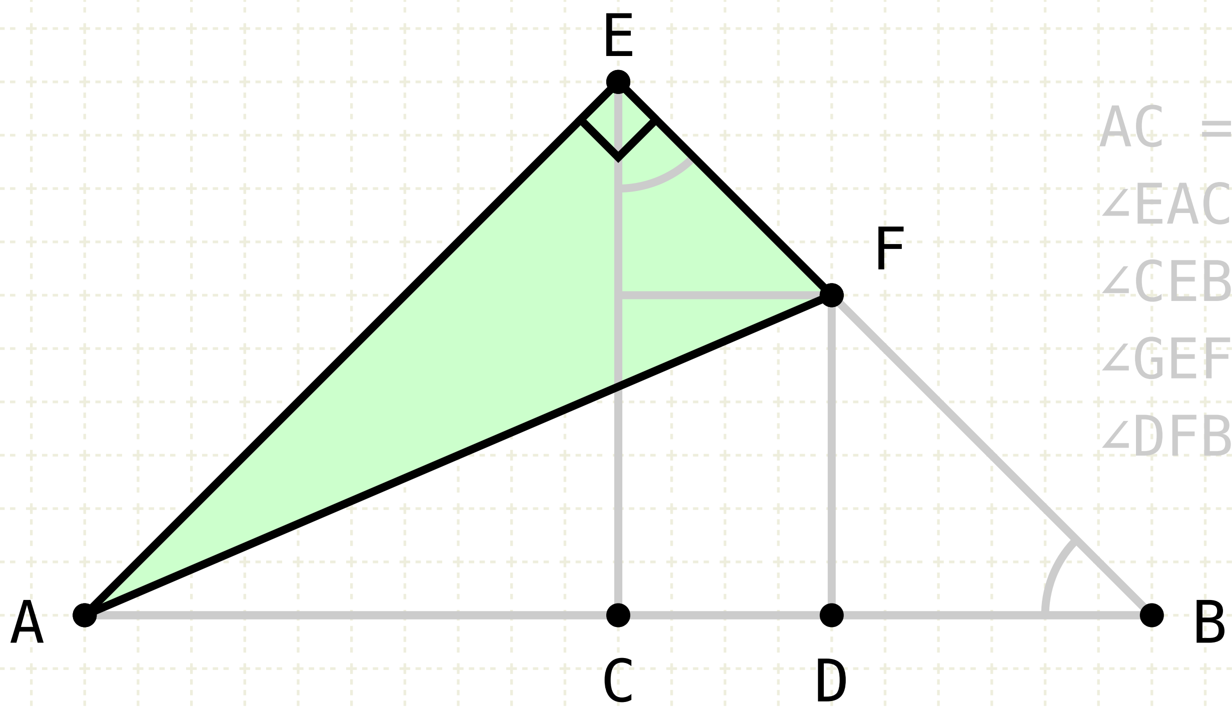
Since EG equals GF, the sum of the squares of EG and GF equals twice the square of GF

The triangle EGF is a right angle, thus the square on EF equals the sum of the squares of EG and GF

GF equals CD (I·34), thus the square on EF equals twice the sum of CD

Proposition 9 of Book II

If a straight line be cut into equal and unequal segments, the squares on the unequal segments of the whole are double of the square on the half and of the square on the straight line between the points of section.



AC = CB, AC,CD,DB = AB

$\angle EAC = \angle CEA = 45$

$\angle CEB = \angle CBE = 45$

$\angle GEF = \angle EFG = 45$

$\angle DFB = \angle FBD = 45$

AC = CE

CB = CE

EG = GF

DB = FD

$AE^2 = AC^2 + EC^2 = 2 \cdot AC^2$

$EF^2 = EG^2 + GF^2 = 2 \cdot GF^2$

$EF^2 = 2 \cdot CD^2$

$AF^2 = AE^2 + EF^2$

Proof

The triangle AEC is a right angle, thus the square on AE equals the sum of the squares of AC and AE

Since AC equals CE, the sum of the squares of AC and CE equals twice the square of AC

Since EG equals GF, the sum of the squares of EG and GF equals twice the square of GF

The triangle EGF is a right angle, thus the square on EF equals the sum of the squares of EG and GF

GF equals CD (I·34), thus the square on EF equals twice the sum of CD

The triangle EAF is a right angle, thus the square on AF equals the sum of the squares of AE and EF

Proposition 9 of Book II

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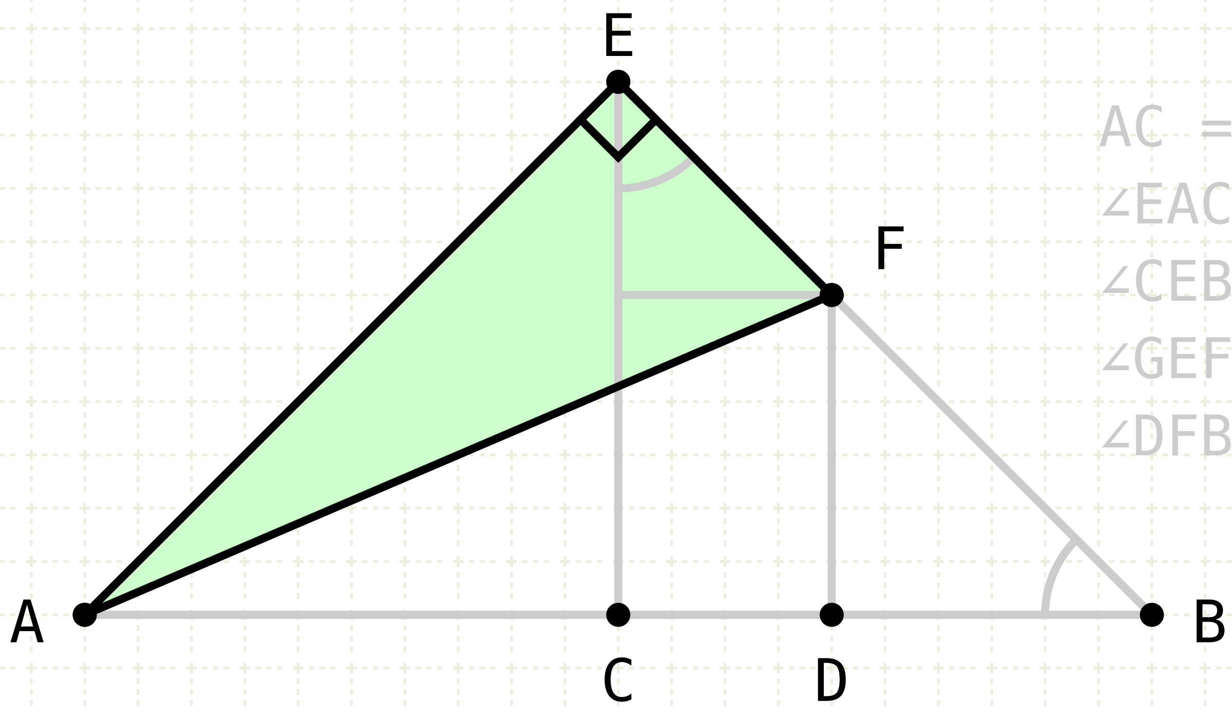

$AC = CB,\ AC, CD, DB = AB$

$\angle EAC = \angle CEA = 45$

$\angle CEB = \angle CBE = 45$

$\angle GEF = \angle EFG = 45$

$\angle DFB = \angle FBD = 45$

$AC = CE$

$CB = CE$

$EG = GF$

$DB = FD$

$AE^2 = AC^2 + EC^2 = 2 \cdot AC^2$

$EF^2 = EG^2 + GF^2 = 2 \cdot GF^2$

$EF^2 = 2 \cdot CD^2$

$AF^2 = AE^2 + EF^2$

$AF^2 = 2 \cdot AC^2 + 2 \cdot CD^2 = 2(AC^2 + CD^2)$

Proof

The triangle AEC is a right angle, thus the square on AE equals the sum of the squares of AC and AE

Since AC equals CE, the sum of the squares of AC and CE equals twice the square of AC

Since EG equals GF, the sum of the squares of EG and GF equals twice the square of GF

The triangle EGF is a right angle, thus the square on EF equals the sum of the squares of EG and GF

GF equals CD (I·34), thus the square on EF equals twice the sum of CD

The triangle EAF is a right angle, thus the square on AF equals the sum of the squares of AE and EF

Proposition 9 of Book II

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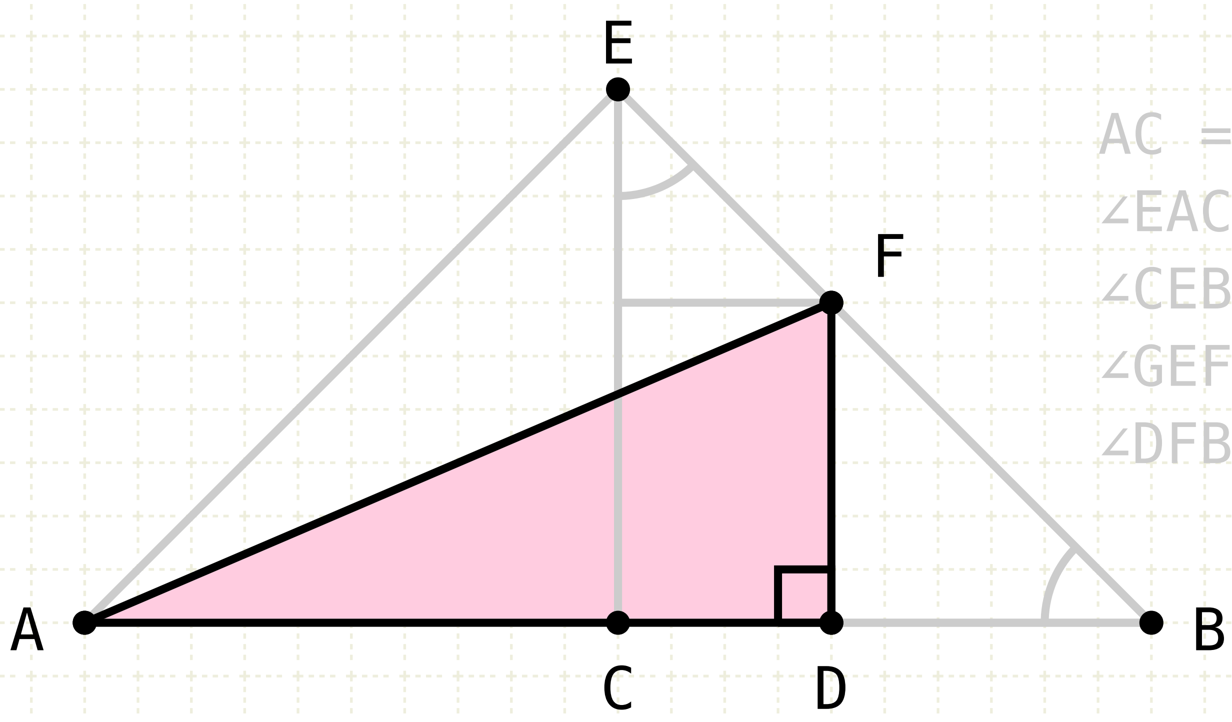


$AC = CB,\ AC, CD, DB = AB$

$\angle EAC = \angle CEA = 45$

$\angle CEB = \angle CBE = 45$

$\angle GEF = \angle EFG = 45$

$\angle DFB = \angle FBD = 45$

$AC = CE$

$CB = CE$

$EG = GF$

$DB = FD$

$AE^2 = AC^2 + EC^2 = 2 \cdot AC^2$

$EF^2 = EG^2 + GF^2 = 2 \cdot GF^2$

$EF^2 = 2 \cdot CD^2$

$AF^2 = AE^2 + EF^2$

$AF^2 = 2 \cdot AC^2 + 2 \cdot CD^2 = 2(AC^2 + CD^2)$

$AF^2 = AD^2 + DF^2$

Proof

The triangle AEC is a right angle, thus the square on AE equals the sum of the squares of AC and AE

Since AC equals CE, the sum of the squares of AC and CE equals twice the square of AC

Since EG equals GF, the sum of the squares of EG and GF equals twice the square of GF

The triangle EGF is a right angle, thus the square on EF equals the sum of the squares of EG and GF

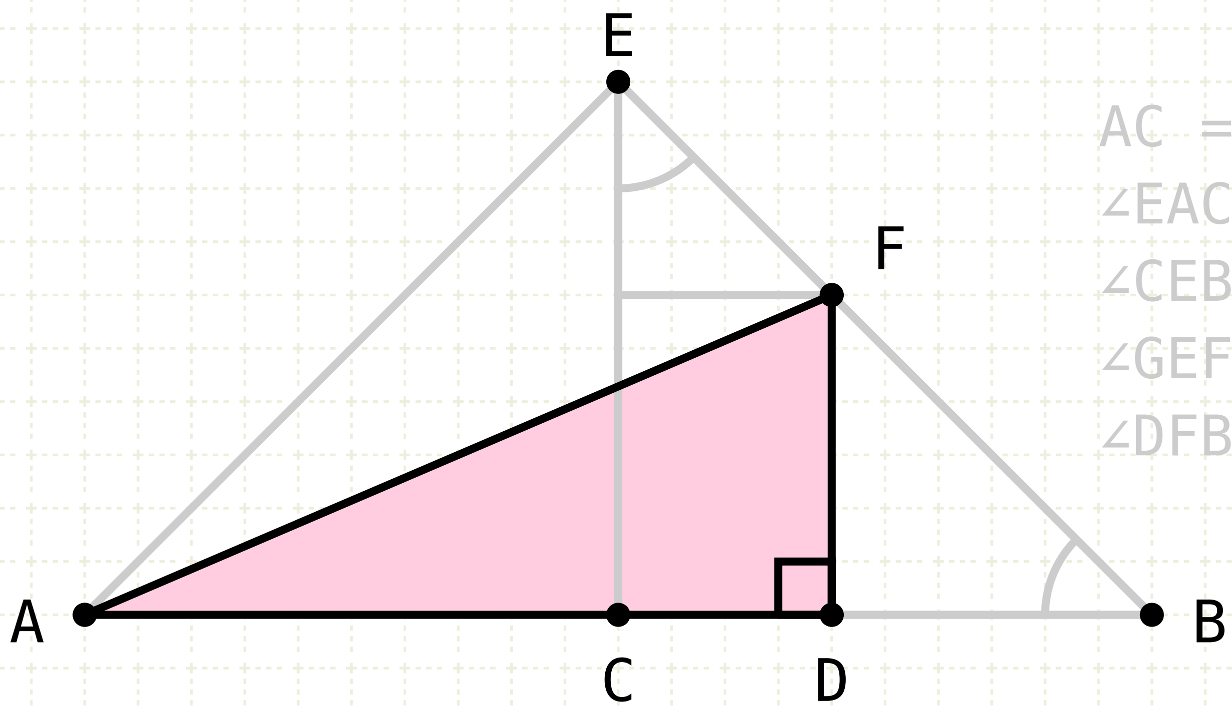
GF equals CD (I·34), thus the square on EF equals twice the sum of CD

The triangle EAF is a right angle, thus the square on AF equals the sum of the squares of AE and EF

The triangle FAD is a right angle, thus the square on AF equals the sum of the squares of AD and DF

Proposition 9 of Book II

If a straight line be cut into equal and unequal segments, the squares on the unequal segments of the whole are double of the square on the half and of the square on the straight line between the points of section.



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The triangle FAD is a right angle, thus the square on AF equals the sum of the squares of AD and DF

But DF equals DB, so the square of AF is the sum of the squares of AD and DB

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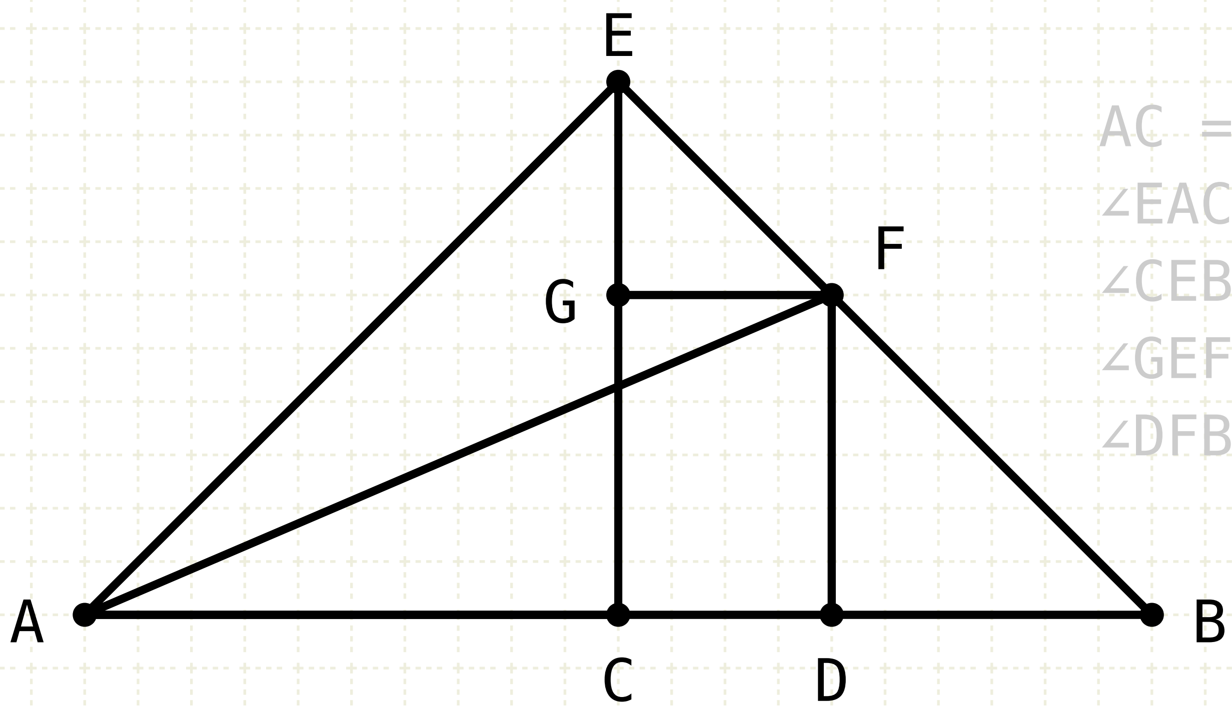

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Rearranging the equalities gives the original postulate

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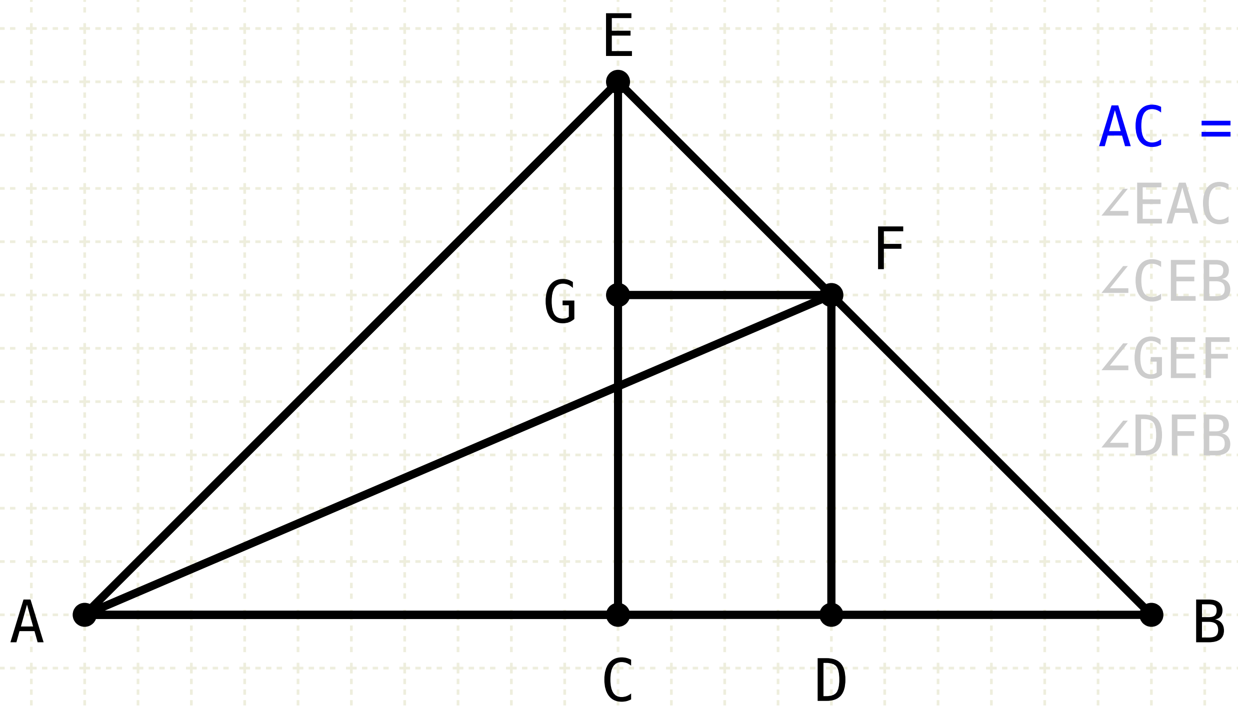

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