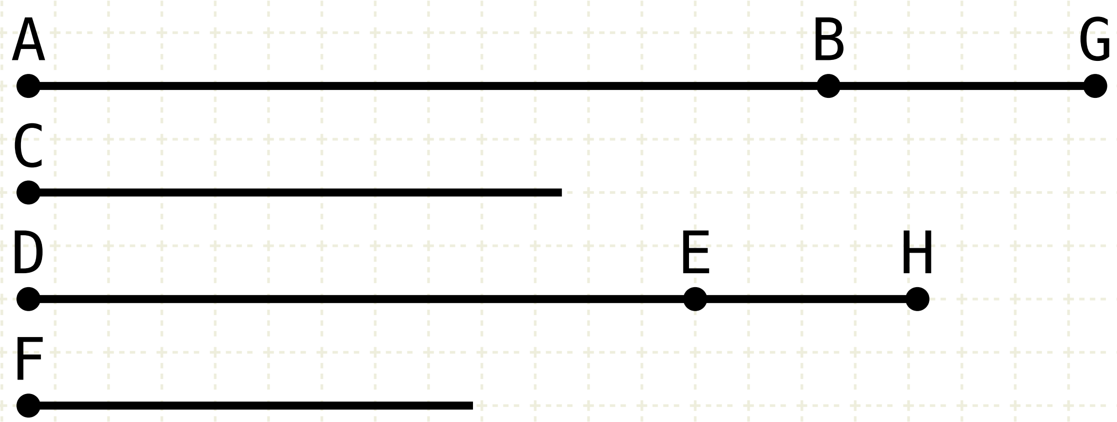
Euclid's Elements

Book V



AB:C = DE:F
BG:C = EH:F

AG:C = DH:F

Proportions are what makes the old Greek temples classic in their beauty. They are like huge blocks, from which the air has been literally hewn out between the columns.

Arne Jacobsen

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Proposition 7 of Book V

Equal magnitudes have the same ratio, as also has the same to equal magnitudes

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Equal magnitudes have the same ratio, as also has the same to equal magnitudes

Definitions

5. Magnitudes are said to BE IN THE SAME RATIO, the first to the second and the third to the fourth, when, if any equimultiples whatever be taken of the first and third, and any equimultiples whatever of the second and fourth, the former equimultiples alike exceed, are alike equal to, or alike fall short of, the latter equimultiples respectively taken in corresponding order

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The ratio A:B = C:D if, for any number 'p' and 'q'
  if pA > qB then pC > qD
  if pA < qB then pC < qD
  if pA = qB then pC = qD

if   A:B = C:D
then pA >=< qB → pC >=< qD
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Proposition 7 of Book V

Equal magnitudes have the same ratio, as also has the same to equal magnitudes

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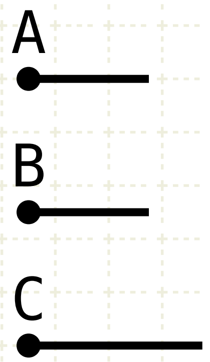

$A = B \neq C$

In other words

Let A,B be equal, and C not equal

Proposition 7 of Book V

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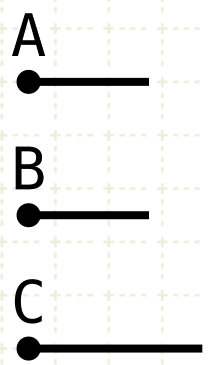

$A = B \neq C$

$A:C = B:C$

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Let A,B be equal, and C not equal

Then the ratio of A to C is the same as the ratio of B to C

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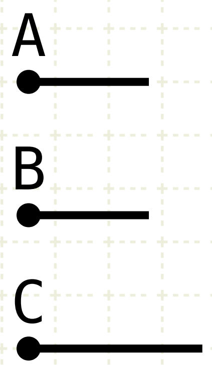

$A = B \neq C$

$A:C = B:C$

$C:A = C:B$

In other words

Let A,B be equal, and C not equal

Then the ratio of A to C is the same as the ratio of B to C

And the ratio of C to A is the same as the ratio of C to B

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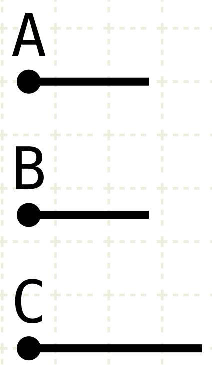

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Proof

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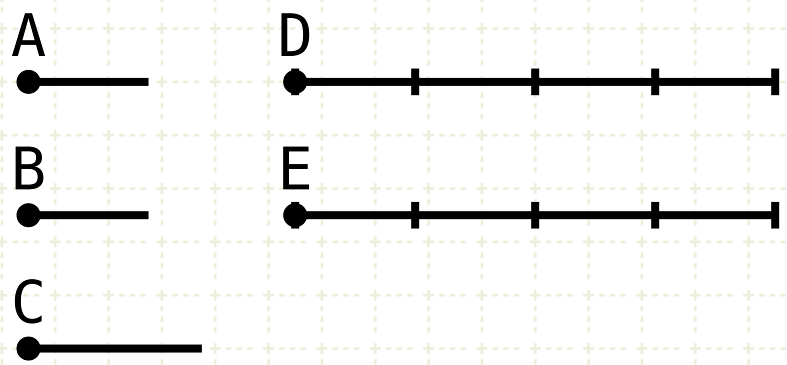

$A = B \neq C$

$D = nA$
$E = nB$

In other words

Let A,B be equal, and C not equal

Then the ratio of A to C is the same as the ratio of B to C

And the ratio of C to A is the same as the ratio of C to B

Proof

Let equimultiples D,E, be taken of magnitudes A,B

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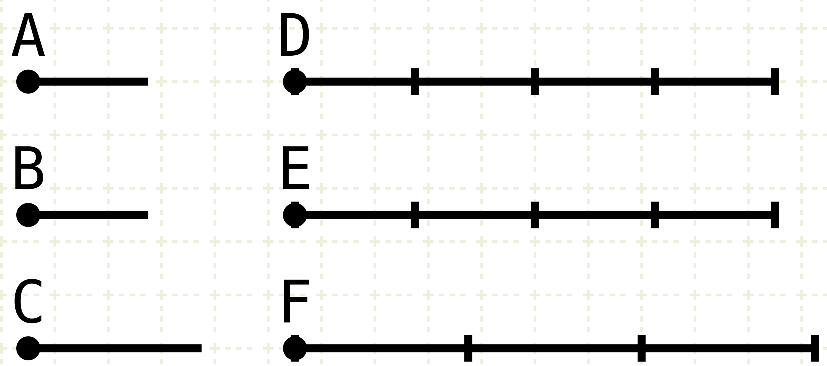

$A = B \neq C$

$D = nA$
$E = nB$
$F = mC$

In other words

Let A,B be equal, and C not equal

Then the ratio of A to C is the same as the ratio of B to C

And the ratio of C to A is the same as the ratio of C to B

Proof

Let equimultiples D,E, be taken of magnitudes A,B

And let another multiple F be taken of C

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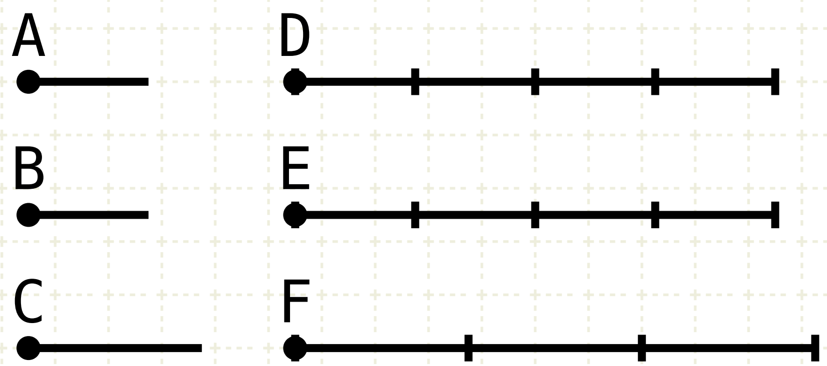

$A = B \neq C$

$D = nA$

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$D = E$

In other words

Let A,B be equal, and C not equal

Then the ratio of A to C is the same as the ratio of B to C

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Proof

Let equimultiples D,E, be taken of magnitudes A,B

And let another multiple F be taken of C

Since D is the same multiple of A that E is of B, and A is equal to B, then D is equal to E

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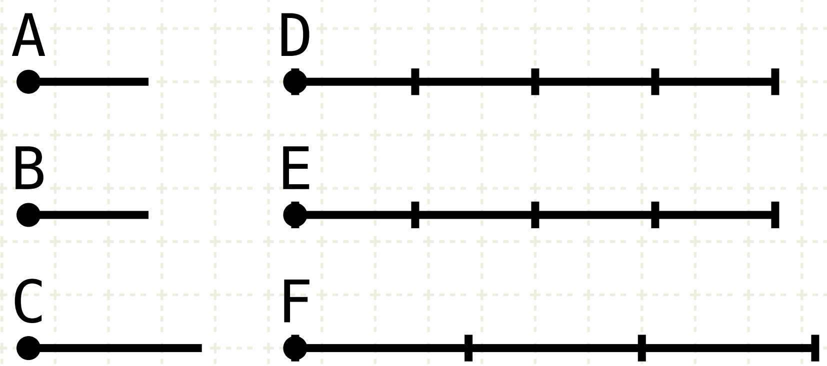


$A = B \neq C$

$D = nA$

$E = nB$

$F = mC$

$D = E$

$D >=< F \rightarrow E >=< F$

$nA >=< mC \rightarrow nB >=< mC$

In other words

Let A,B be equal, and C not equal

Then the ratio of A to C is the same as the ratio of B to C

And the ratio of C to A is the same as the ratio of C to B

Proof

Let equimultiples D,E, be taken of magnitudes A,B

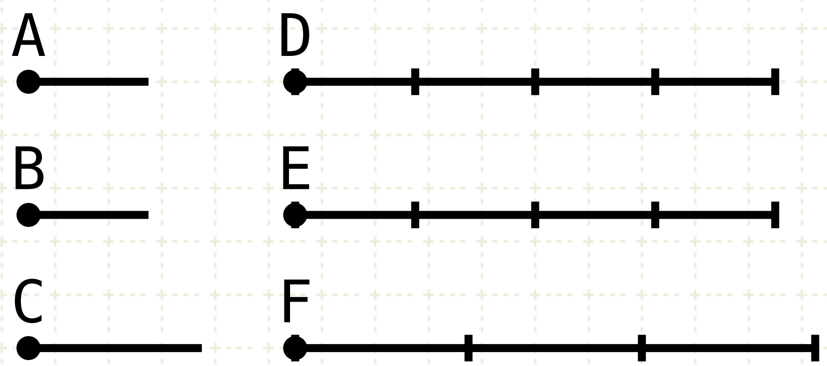
And let another multiple F be taken of C

Since D is the same multiple of A that E is of B, and A is equal to B, then D is equal to E

F is another arbitrary magnitude, so if D is greater than F, then so E is also greater than F, and so on

Proposition 7 of Book V

Equal magnitudes have the same ratio, as also has the same to equal magnitudes



$A = B \neq C$

$D = nA$

$E = nB$

$F = mC$

$D = E$

$D \gtreqless F \rightarrow E \gtreqless F$

$nA \gtreqless mC \rightarrow nB \gtreqless mC$

$A:C = B:C$

In other words

Let A,B be equal, and C not equal

Then the ratio of A to C is the same as the ratio of B to C

And the ratio of C to A is the same as the ratio of C to B

Proof

Let equimultiples D,E, be taken of magnitudes A,B

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Since D is the same multiple of A that E is of B, and A is equal to B, then D is equal to E

F is another arbitrary magnitude, so if D is greater than F, then so E is also greater than F, and so on

This is the requirement for equal ratios, so…

… A is to C as B is to C (V Def 5)

Proposition 7 of Book V

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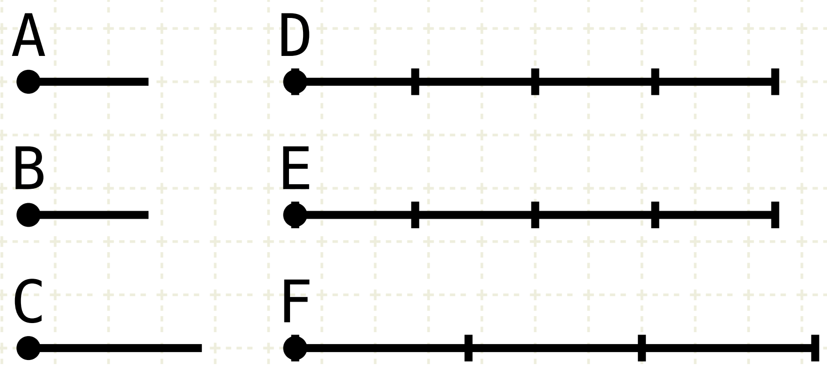

A = B ≠ C

D = nA
E = nB
F = mC
D = E

D >=< F → E >=< F
nA >=< mC → nB >=< mC
A:C = B:C

F >=< D → F >=< E
mC >=< nA → mC >=< nB

In other words

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Then the ratio of A to C is the same as the ratio of B to C

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… A is to C as B is to C (V Def 5)

Going back to D is equal to E, and F being an arbitrary magnitude…

If F is greater than D, then so F is also greater than E, and so on

Proposition 7 of Book V

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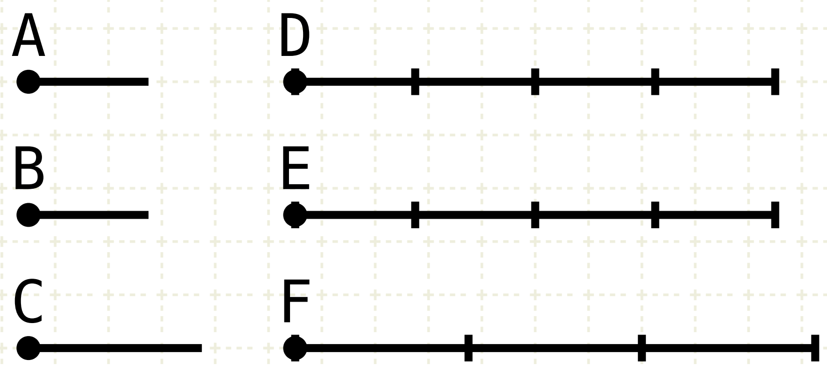

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D >=< F → E >=< F
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A:C = B:C

F >=< D → F >=< E
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C:A = C:B

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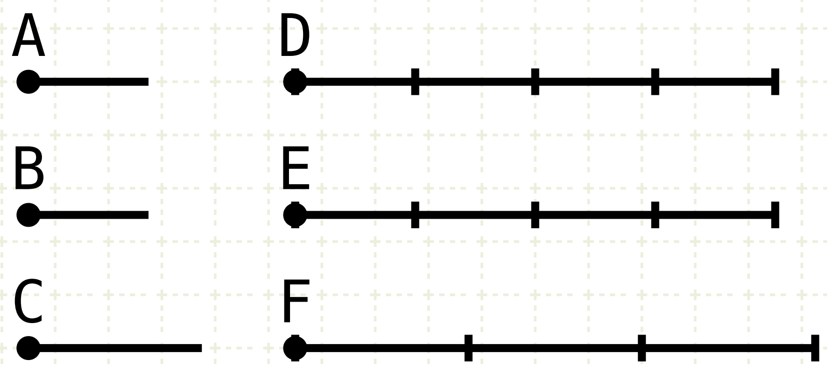

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$D = nA$
$E = nB$
$F = mC$
$D = E$

$D \ {>}{=}{<}\ F \rightarrow E \ {>}{=}{<}\ F$
$nA \ {>}{=}{<}\ mC \rightarrow nB \ {>}{=}{<}\ mC$
$A:C = B:C$

$F \ {>}{=}{<}\ D \rightarrow F \ {>}{=}{<}\ E$
$mC \ {>}{=}{<}\ nA \rightarrow mC \ {>}{=}{<}\ nB$
$C:A = C:B$

In other words

Let A,B be equal, and C not equal

Then the ratio of A to C is the same as the ratio of B to C

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