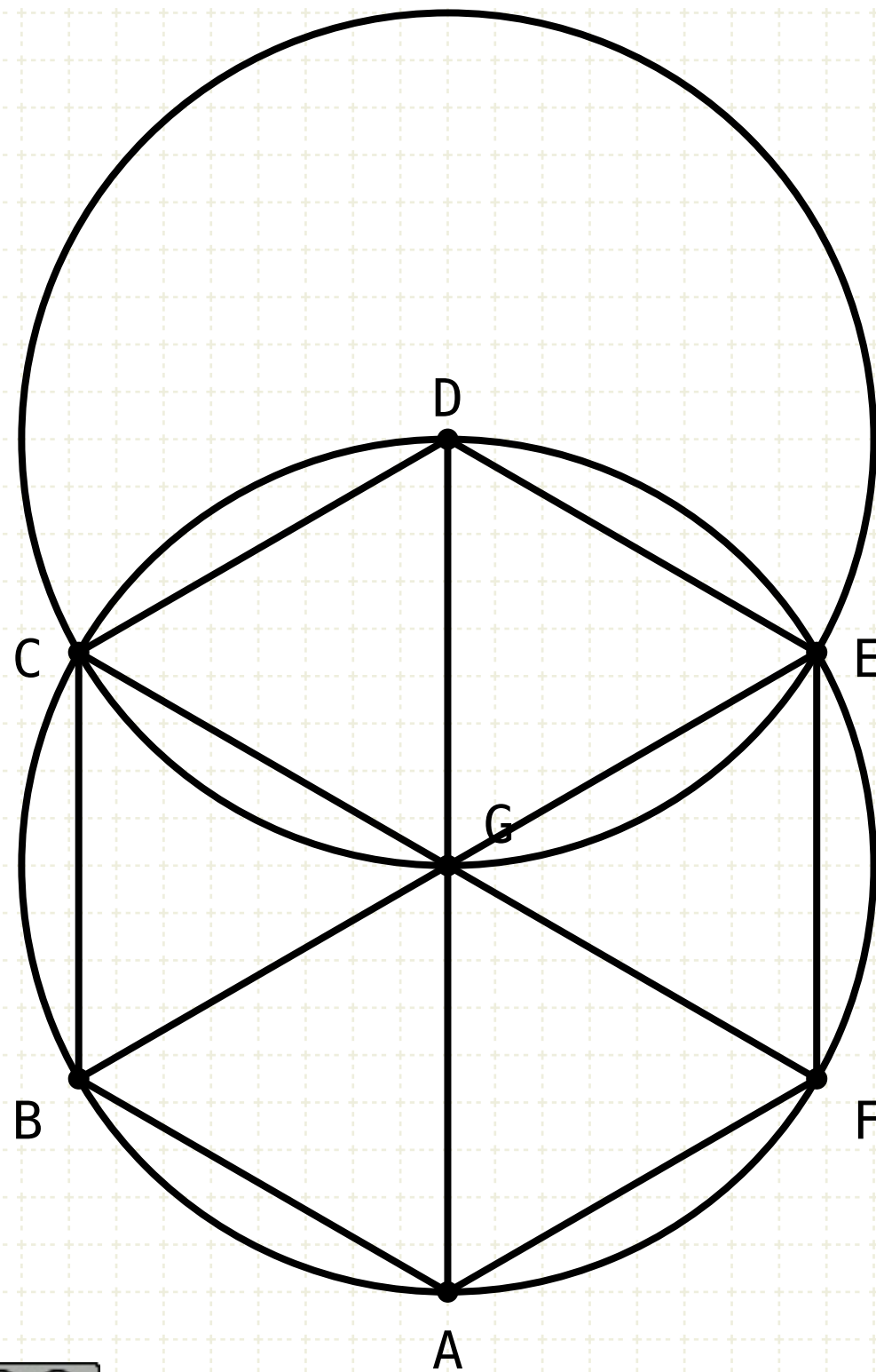


Euclid's Elements

Book IV



Philosophy (nature) is written in that great book which ever is before our eyes -- I mean the universe -- but we cannot understand it if we do not first learn the language and grasp the symbols in which it is written. The book is written in mathematical language, and the symbols are triangles, circles and other geometrical figures, without whose help it is impossible to comprehend a single word of it - without which one wanders in vain through a dark labyrinth.

Galileo Galilei



Proposition 9 of Book IV

About a given square, to circumscribe a circle.



Table of Contents, Chapter 4

1	Fit a given straight line into a given circle, if the line is less than the diameter	11	In a given circle to inscribe an equilateral and equiangular pentagon
2	In a given circle to inscribe a triangle equiangular with a given triangle	12	About a given circle to circumscribe an equilateral and equiangular pentagon
3	About a given circle to circumscribe a triangle equiangular with a given triangle	13	In a given pentagon, which is equilateral and equiangular, to inscribe a circle
4	In a given triangle, to inscribe a circle	14	About a given pentagon, which is equilateral and equiangular, to circumscribe a circle
5	About a given triangle to circumscribe a circle	15	In a given circle to inscribe an equilateral and equiangular hexagon
6	In a given circle to inscribe a square	16	In a given circle to inscribe a fifteen angled figure which shall be both equilateral and equiangular
7	About a given circle to circumscribe a square		
8	In a given square, to inscribe a circle		
9	About a given square, to circumscribe a circle		
10	To construct an isosceles triangle having each of the angles at the base double of the remaining one		



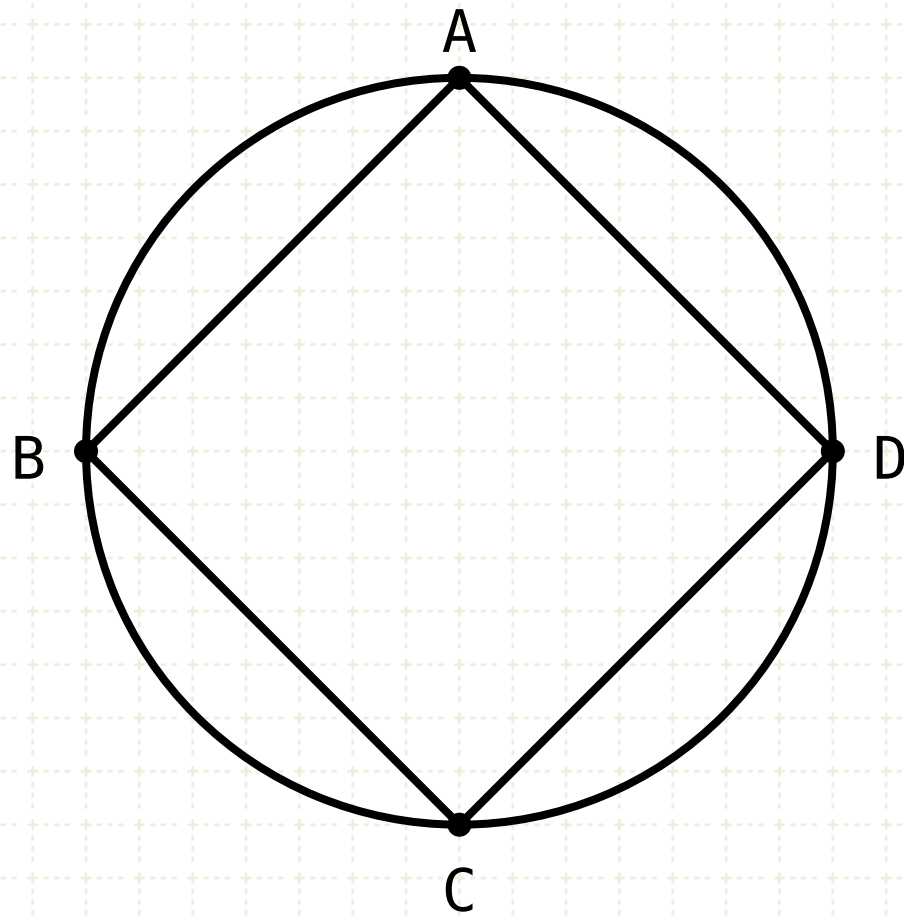
Proposition 9 of Book IV

About a given square, to circumscribe a circle.



Proposition 9 of Book IV

About a given square, to circumscribe a circle.



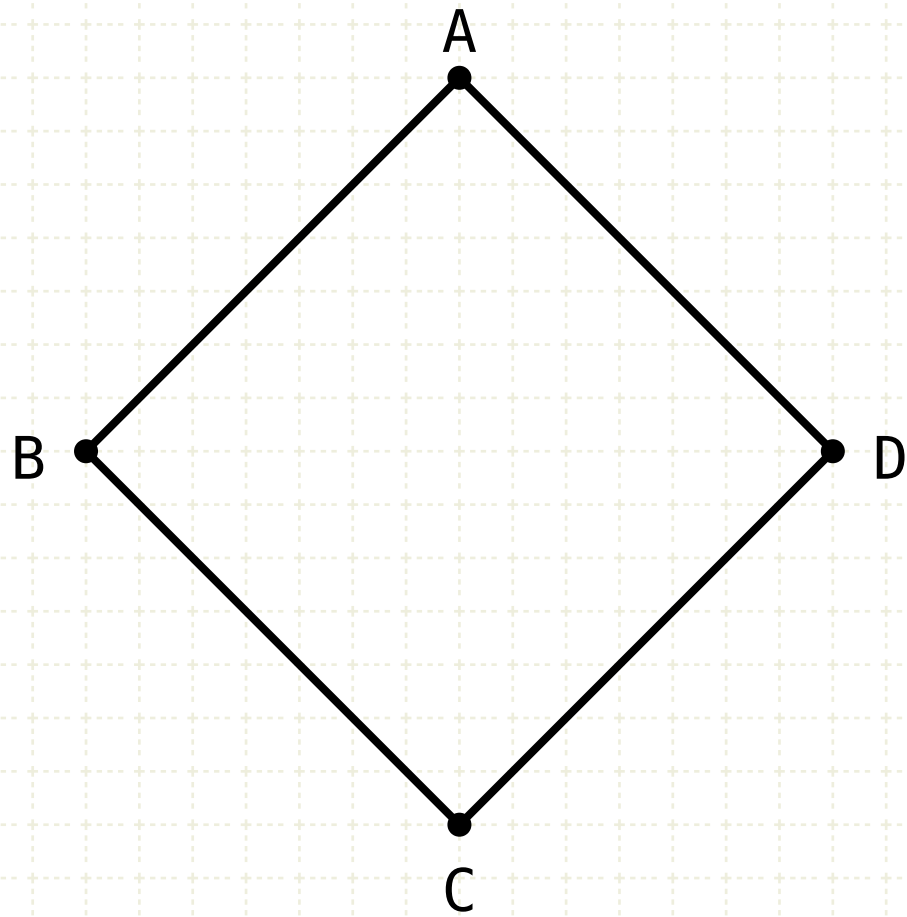
In other words

Given a square ABCD, draw a circle outside the square, going through points A,B,C and D

Proposition 9 of Book IV

About a given square, to circumscribe a circle.

Construction

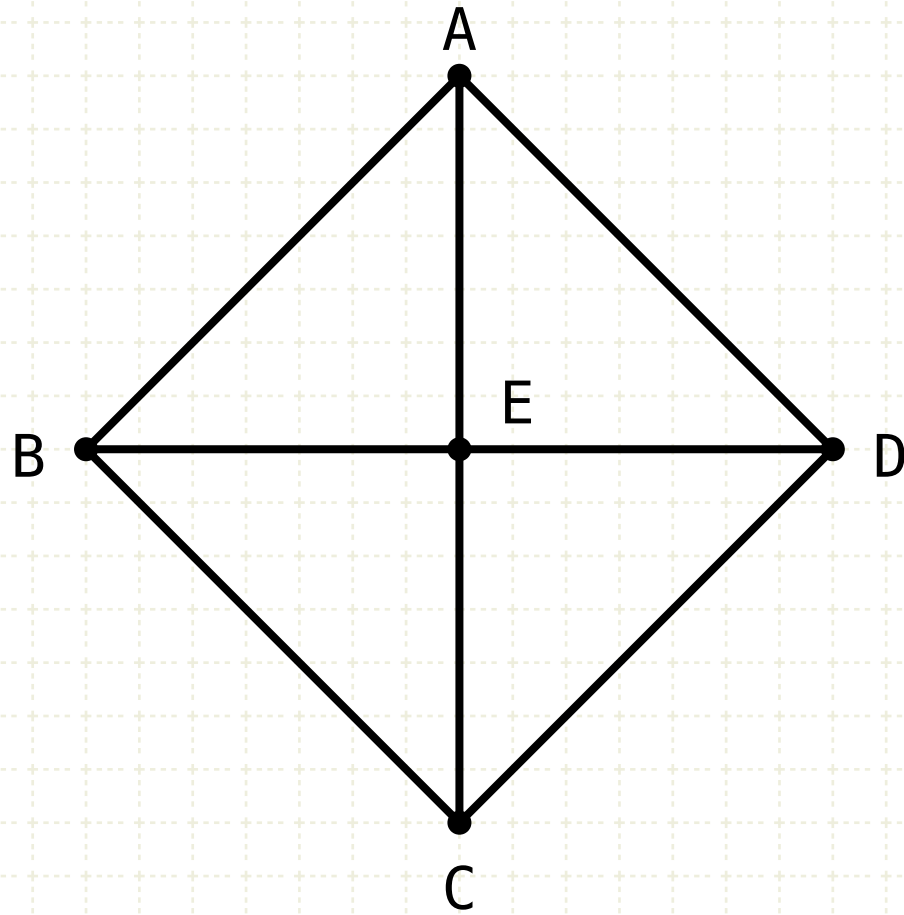


Proposition 9 of Book IV

About a given square, to circumscribe a circle.

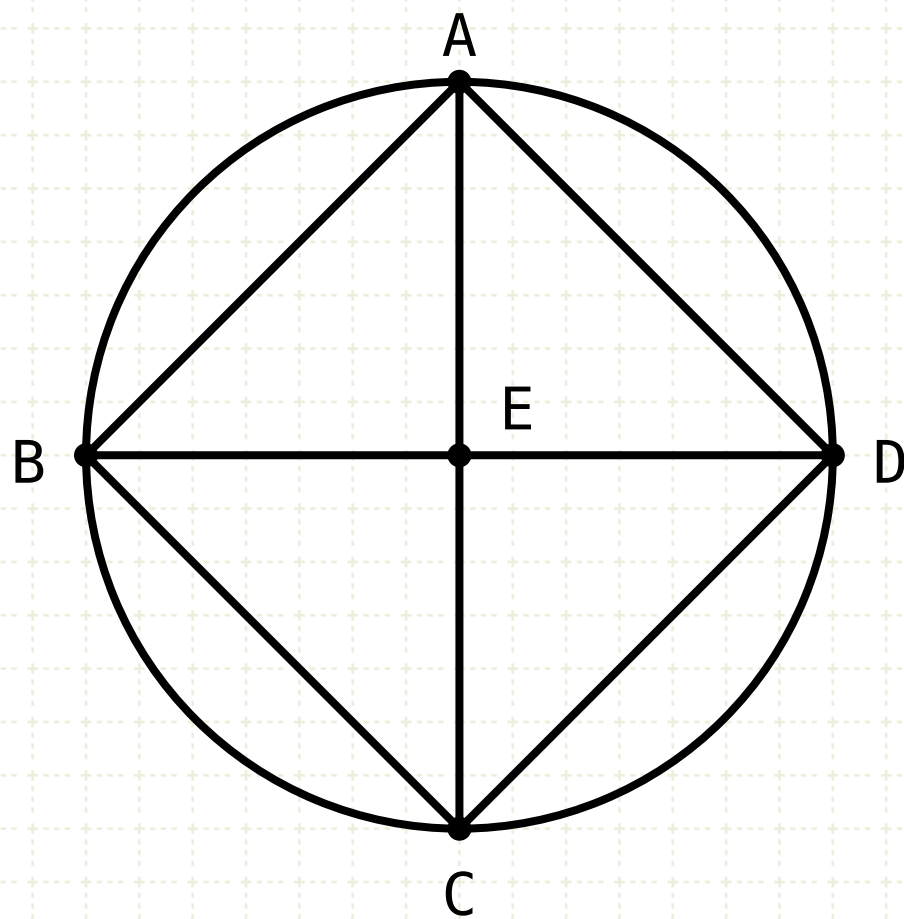
Construction

Draw lines AC and BD, and label the intersection point E



Proposition 9 of Book IV

About a given square, to circumscribe a circle.



Construction

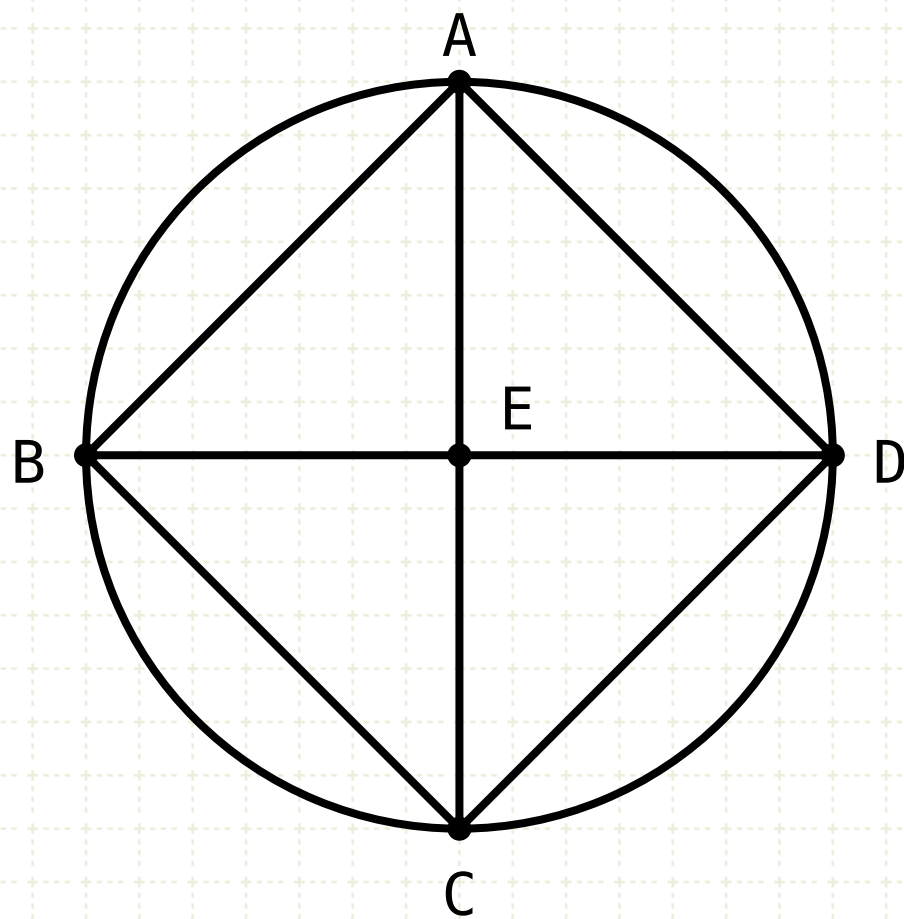
Draw lines AC and BD, and label the intersection point E

Draw a circle with centre E, and radius AE

This circle circumscribes the square

Proposition 9 of Book IV

About a given square, to circumscribe a circle.



Construction

Draw lines AC and BD, and label the intersection point E

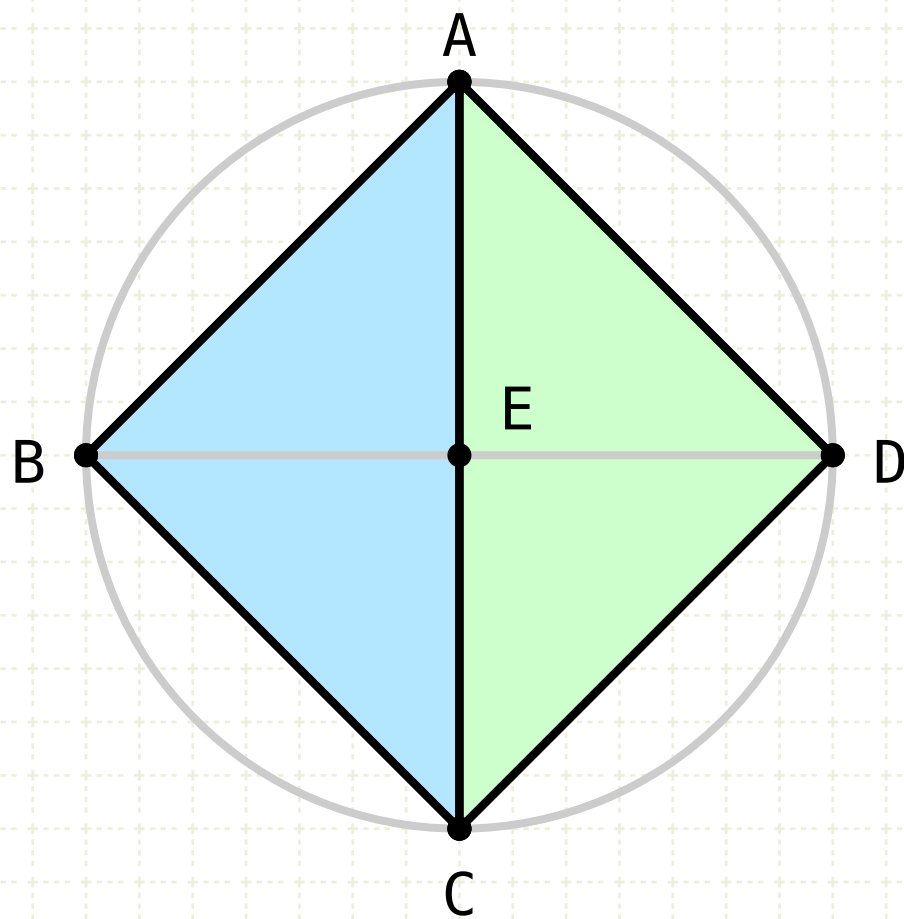
Draw a circle with centre E, and radius AE

This circle circumscribes the square

Proof

Proposition 9 of Book IV

About a given square, to circumscribe a circle.



Construction

Draw lines AC and BD, and label the intersection point E

Draw a circle with centre E, and radius AE

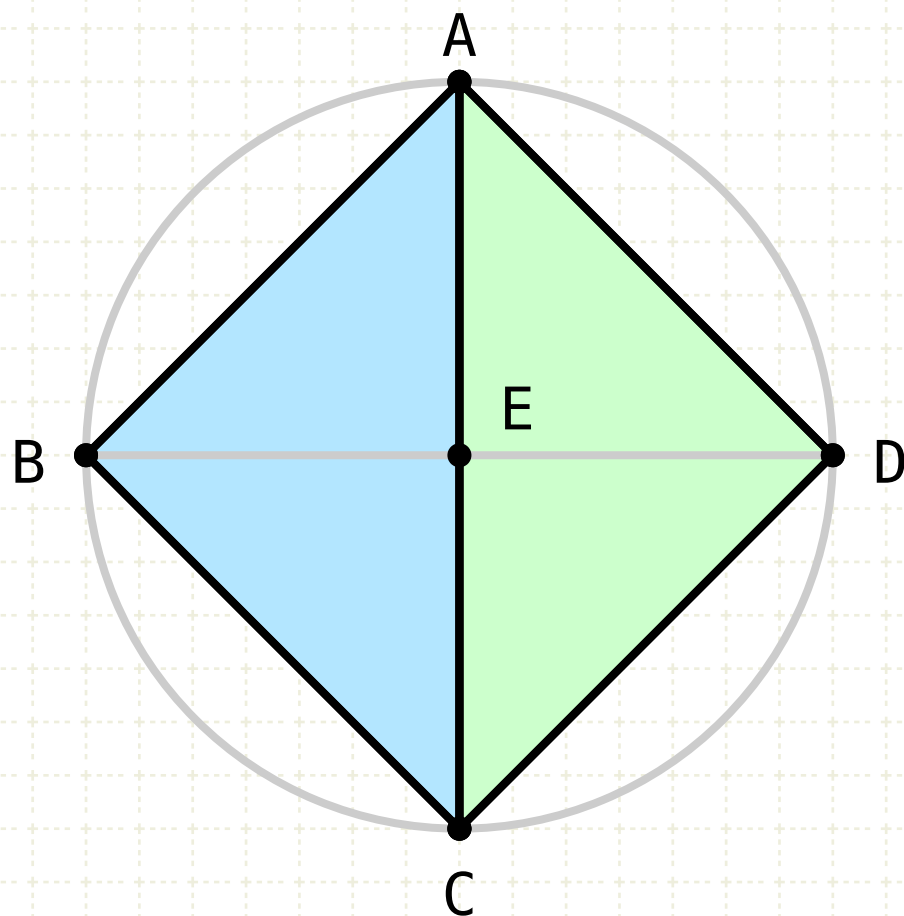
This circle circumscribes the square

Proof

Consider the two triangles ABC and ADC

Proposition 9 of Book IV

About a given square, to circumscribe a circle.



$$\triangle ABC \equiv \triangle ADC$$

Construction

Draw lines AC and BD, and label the intersection point E

Draw a circle with centre E, and radius AE

This circle circumscribes the square

Proof

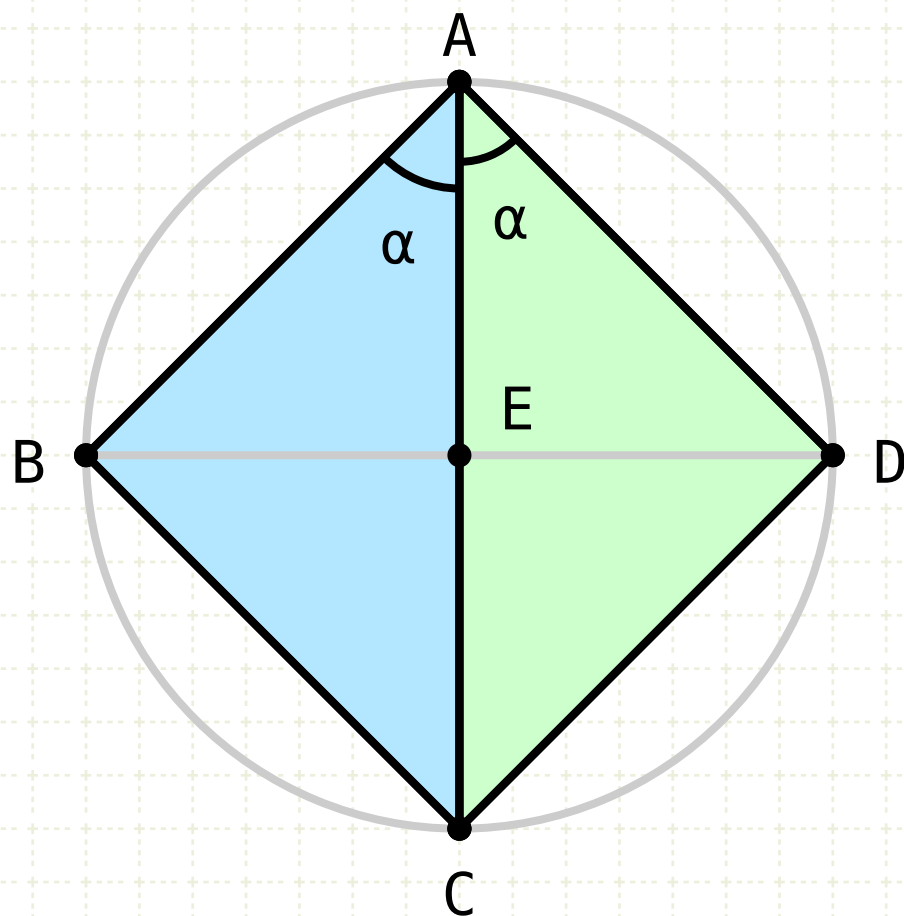
Consider the two triangles ABC and ADC

AB equals AD, and AC is common to both triangles

Given that ABCD is a square, the bases BC and CD are also equal, so the triangles are equal in all respects (SSS) (I·8)

Proposition 9 of Book IV

About a given square, to circumscribe a circle.



$$\triangle ABC \equiv \triangle ADC$$

$$\alpha = \frac{1}{2} \angle A$$

Construction

Draw lines AC and BD, and label the intersection point E

Draw a circle with centre E, and radius AE

This circle circumscribes the square

Proof

Consider the two triangles ABC and ADC

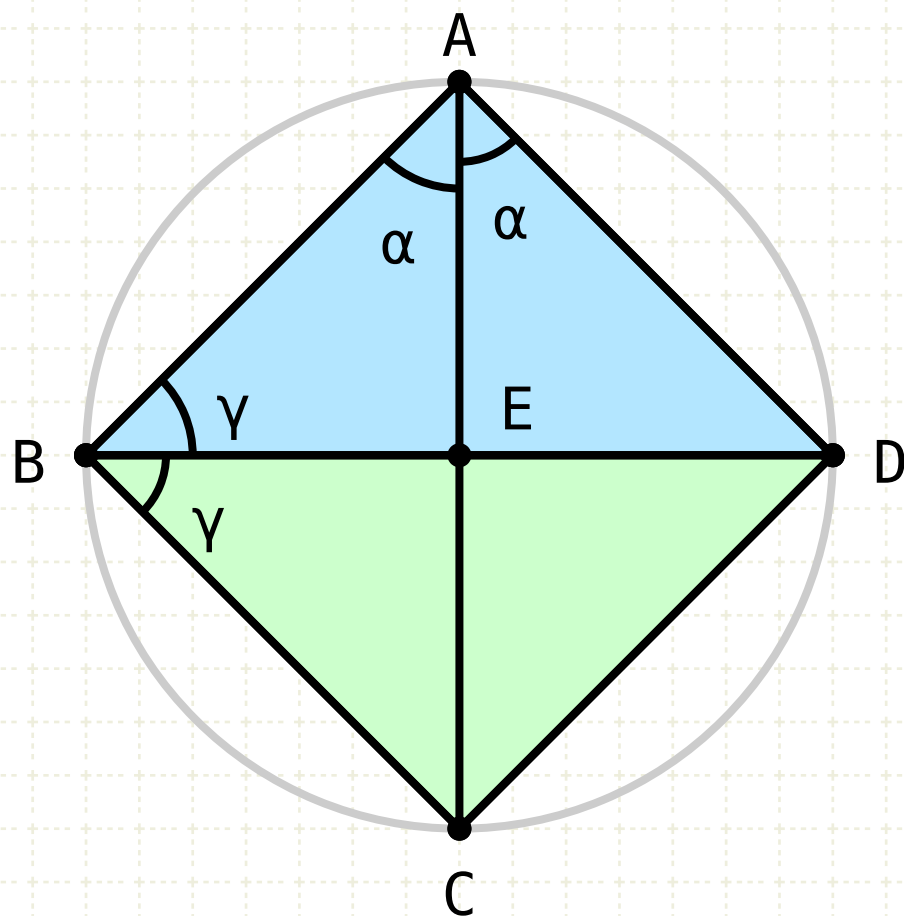
AB equals AD, and AC is common to both triangles

Given that ABCD is a square, the bases BC and CD are also equal, so the triangles are equal in all respects (SSS) (I·8)

Thus, the angles BAC and DAC are equal, and angle A is bisected

Proposition 9 of Book IV

About a given square, to circumscribe a circle.



$$\begin{aligned}\triangle ABC &\equiv \triangle ADC \\ \alpha &= \frac{1}{2} \angle A \\ \gamma &= \frac{1}{2} \angle B\end{aligned}$$

Construction

Draw lines AC and BD, and label the intersection point E

Draw a circle with centre E, and radius AE

This circle circumscribes the square

Proof

Consider the two triangles ABC and ADC

AB equals AD, and AC is common to both triangles

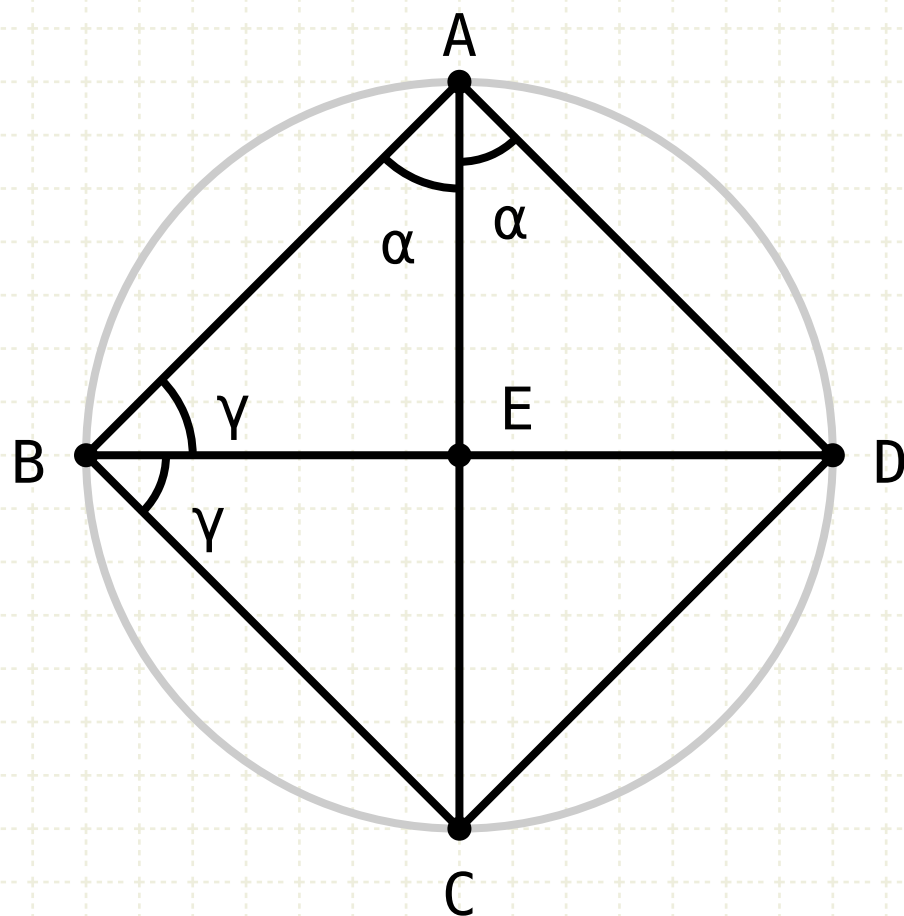
Given that ABCD is a square, the bases BC and CD are also equal, so the triangles are equal in all respects (SSS) (I·8)

Thus, the angles BAC and DAC are equal, and angle A is bisected

Similarly, the angles ABD and CBD are equal

Proposition 9 of Book IV

About a given square, to circumscribe a circle.



$$\triangle ABC \equiv \triangle ADC$$

$$\alpha = \frac{1}{2} \angle A$$

$$\gamma = \frac{1}{2} \angle B$$

$$\angle A = \angle B \therefore \alpha = \gamma$$

Construction

Draw lines AC and BD, and label the intersection point E

Draw a circle with centre E, and radius AE

This circle circumscribes the square

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Consider the two triangles ABC and ADC

AB equals AD, and AC is common to both triangles

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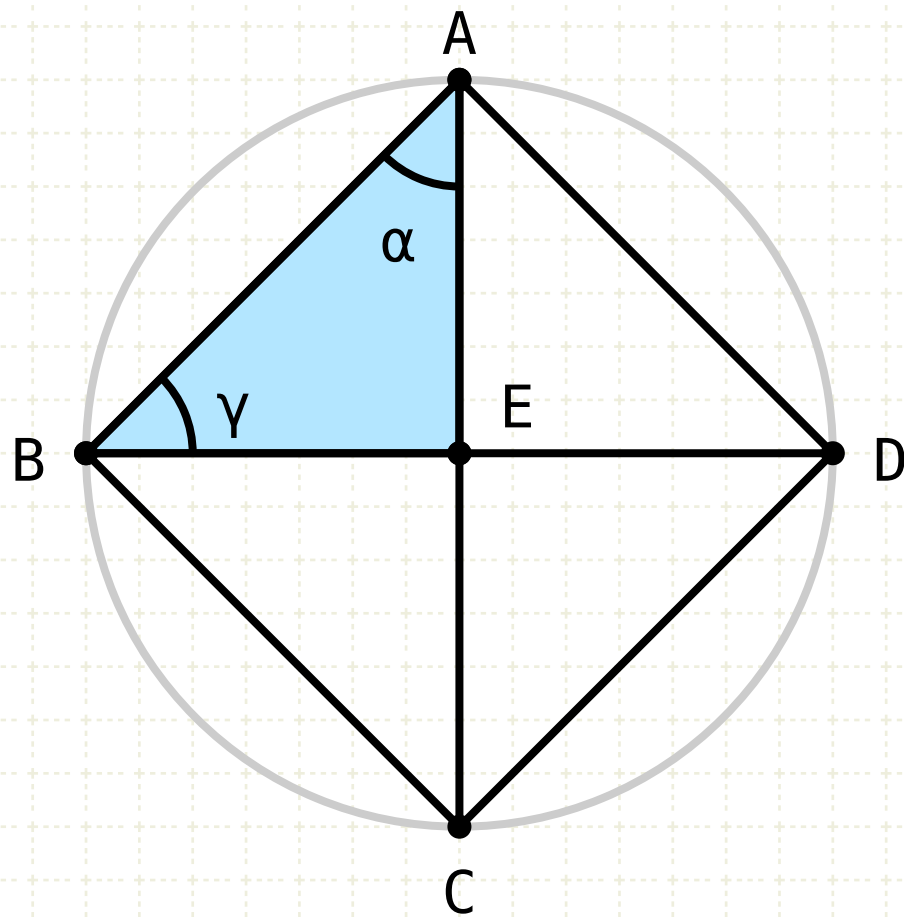
Thus, the angles BAC and DAC are equal, and angle A is bisected

Similarly, the angles ABD and CBD are equal

Since angle A is equal to angle B, and α and γ are half of A and B respectively, α is equal to γ

Proposition 9 of Book IV

About a given square, to circumscribe a circle.



$$\triangle ABC \equiv \triangle ADC$$

$$\alpha = \frac{1}{2} \angle A$$

$$\gamma = \frac{1}{2} \angle B$$

$$\angle A = \angle B \therefore \alpha = \gamma$$

$$AE = BE$$

Construction

Draw lines AC and BD, and label the intersection point E

Draw a circle with centre E, and radius AE

This circle circumscribes the square

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Consider the two triangles ABC and ADC

AB equals AD, and AC is common to both triangles

Given that ABCD is a square, the bases BC and CD are also equal, so the triangles are equal in all respects (SSS) (I·8)

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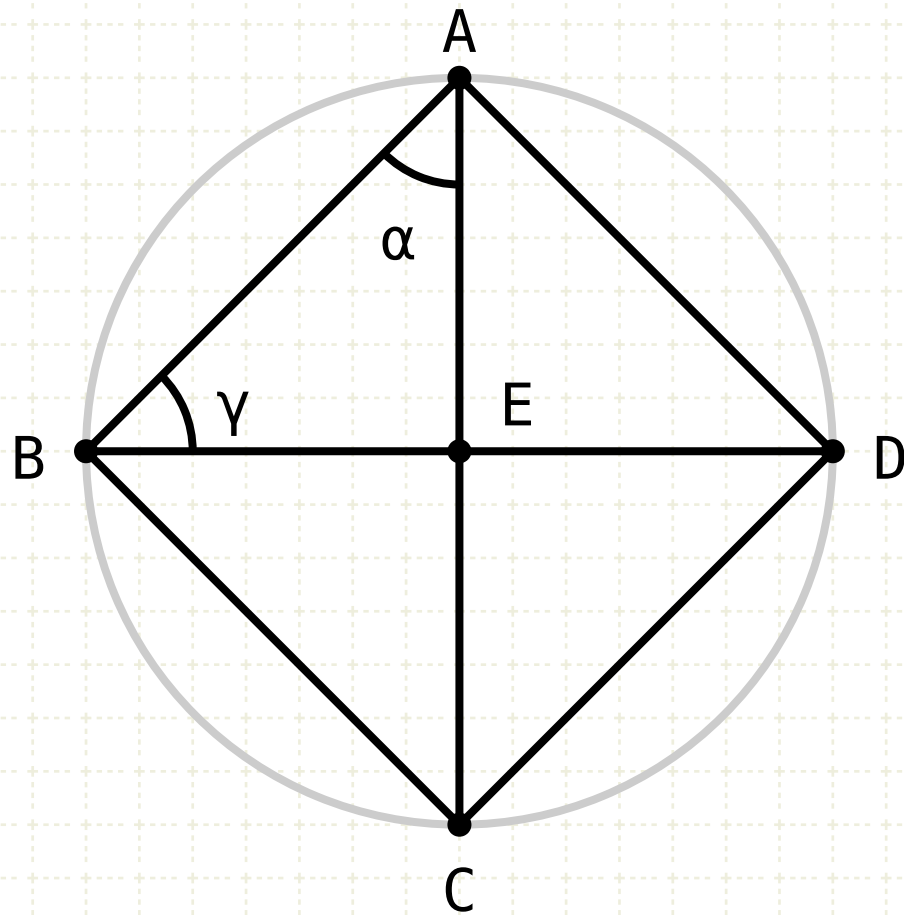
Similarly, the angles ABD and CBD are equal

Since angle A is equal to angle B, and α and γ are half of A and B respectively, α is equal to γ

The triangle ABE is an isosceles triangle, and therefore AE equals BE (I·6)

Proposition 9 of Book IV

About a given square, to circumscribe a circle.



$$\triangle ABC \equiv \triangle ADC$$

$$\alpha = \frac{1}{2} \angle A$$

$$\gamma = \frac{1}{2} \angle B$$

$$\angle A = \angle B \therefore \alpha = \gamma$$

$$AE = BE$$

$$AE = BE = CE = DE$$

Construction

Draw lines AC and BD, and label the intersection point E

Draw a circle with centre E, and radius AE

This circle circumscribes the square

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Consider the two triangles ABC and ADC

AB equals AD, and AC is common to both triangles

Given that ABCD is a square, the bases BC and CD are also equal, so the triangles are equal in all respects (SSS) (I·8)

Thus, the angles BAC and DAC are equal, and angle A is bisected

Similarly, the angles ABD and CBD are equal

Since angle A is equal to angle B, and α and γ are half of A and B respectively, α is equal to γ

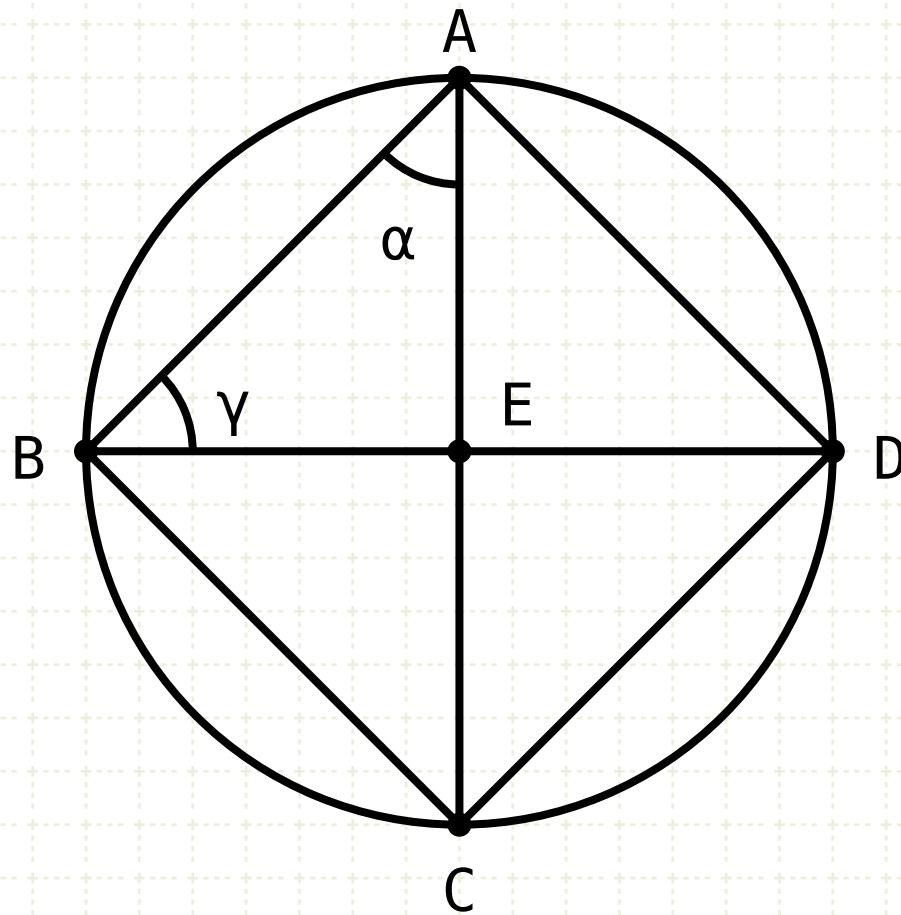
The triangle ABE is an isosceles triangle, and therefore AE equals BE (I·6)

Using the same methods, it can be shown that CE and DE are also equal to AE and BE



Proposition 9 of Book IV

About a given square, to circumscribe a circle.



$$\triangle ABC \equiv \triangle ADC$$

$$\alpha = \frac{1}{2} \angle A$$

$$\gamma = \frac{1}{2} \angle B$$

$$\angle A = \angle B \therefore \alpha = \gamma$$

$$AE = BE$$

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Draw lines AC and BD, and label the intersection point E

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Consider the two triangles ABC and ADC

AB equals AD, and AC is common to both triangles

Given that ABCD is a square, the bases BC and CD are also equal, so the triangles are equal in all respects (SSS) (I·8)

Thus, the angles BAC and DAC are equal, and angle A is bisected

Similarly, the angles ABD and CBD are equal

Since angle A is equal to angle B, and α and γ are half of A and B respectively, α is equal to γ

The triangle ABE is an isosceles triangle, and therefore AE equals BE (I·6)

Using the same methods, it can be shown that CE and DE are also equal to AE and BE

Thus, a circle with the centre at E, with radius AE will pass through the points A,B,C and D



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