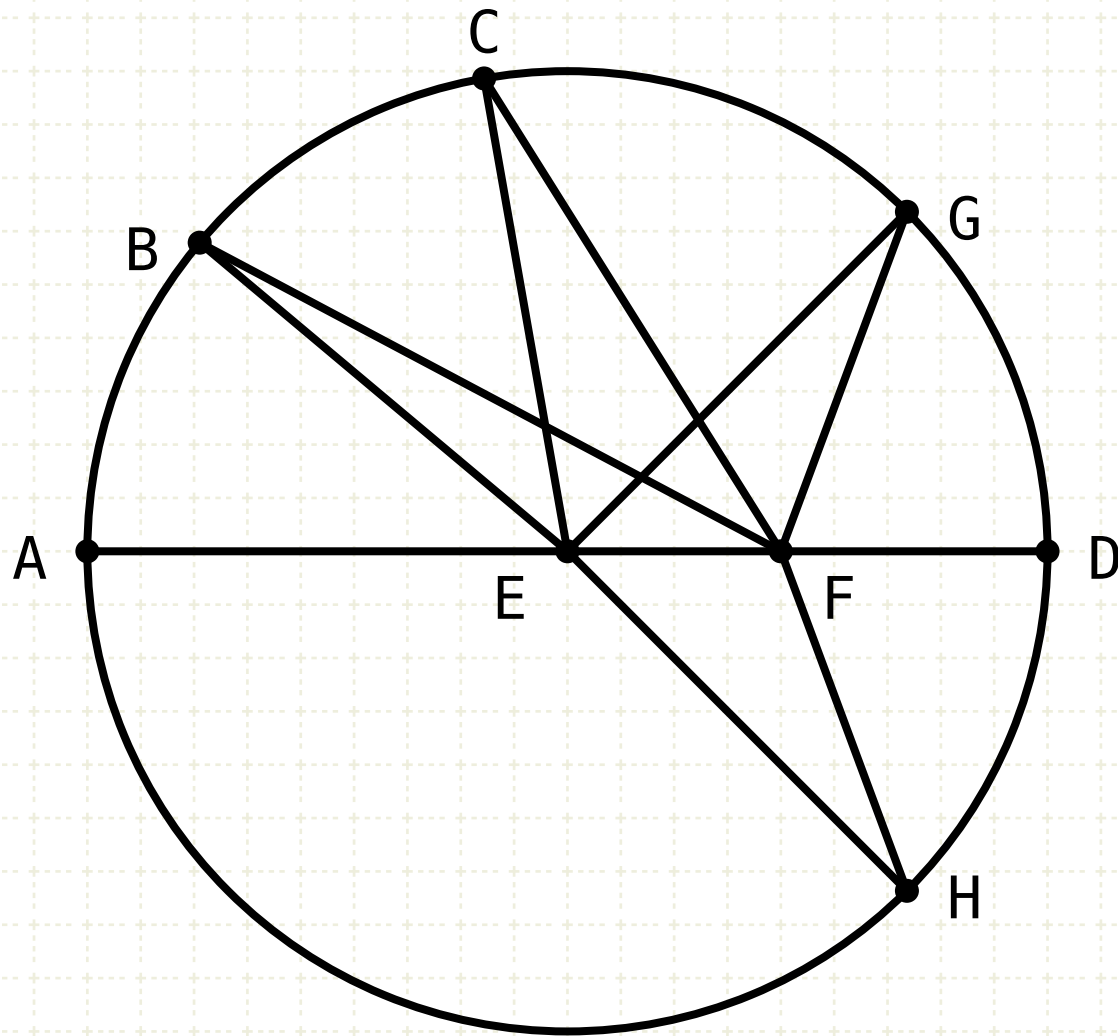


# Euclid's Elements

## Book III



*A circle is a round straight line with a hole in the middle.*

**Mark Twain**

quoting a schoolchild in "-English as She Is Taught-"

*If people stand in a circle long enough, they'll eventually begin to dance.*

**George Carlin, Napalm and Silly Putty (2001)**



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3	A line through the centre of a circle bisects a chord, and vice versa	11	Point of contact between two internal circles, and their centres, are collinear	19	If line touches a circle, then the centre of the circle lies on a line perpendicular to the original
4	A line not through the centre of a circle does not bisect a chord	12	Point of contact between two external circles, and their centres, are collinear	20	The angle at the centre of a circle is twice that from an angle from the circumference
5	If two circles cut one another, they will not have the same center	13	A circle does not touch a circle at more points than one, whether it touch it internally or externally.	21	In a circle the angles in the same segment are equal to one another
6	If two circles touch one another, they will not have the same center	14	In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another.	22	The opposite angles of quadrilaterals in circles are equal to two right angles
7	Consider two lines from a point inside a circle to the edge, the longer one will be the one closest to the longest part of the diameter passing through the original point	15	The longest line in a circle is its diameter, shorter the farther away from the diameter	23	On the same straight line there cannot be constructed two similar and unequal segments of circles on the same side
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## Table of Contents, Chapter 3

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## Proposition 32 of Book III

If a straight line touch a circle, and from the point of contact there be drawn across, in the circle, a straight line cutting the circle, the angles which it makes with the tangent will be equal to the angles in the alternate segments of the circle.

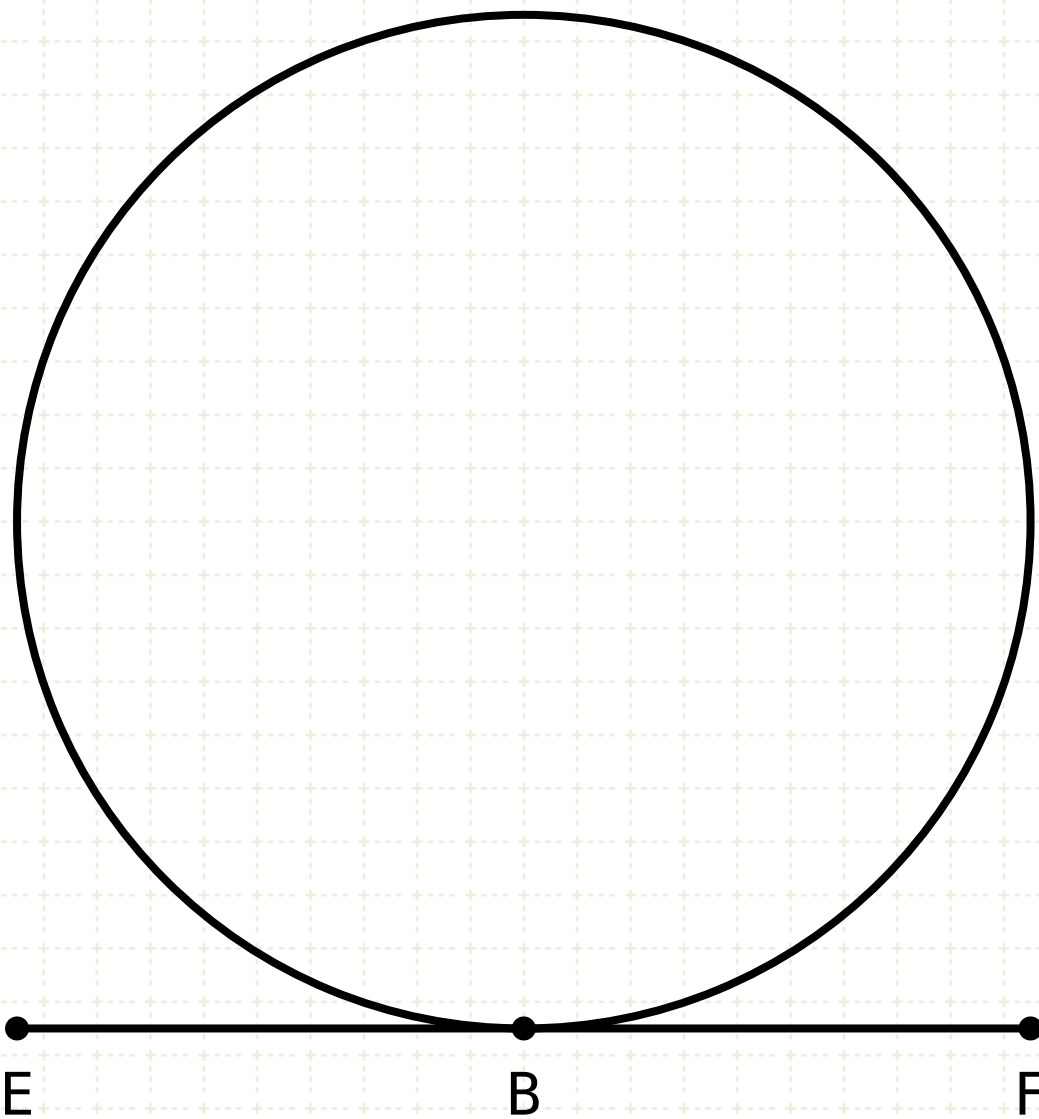


## Proposition 32 of Book III

If a straight line touch a circle, and from the point of contact there be drawn across, in the circle, a straight line cutting the circle, the angles which it makes with the tangent will be equal to the angles in the alternate segments of the circle.

### In other words

Let EF be a line that touches a circle at point B





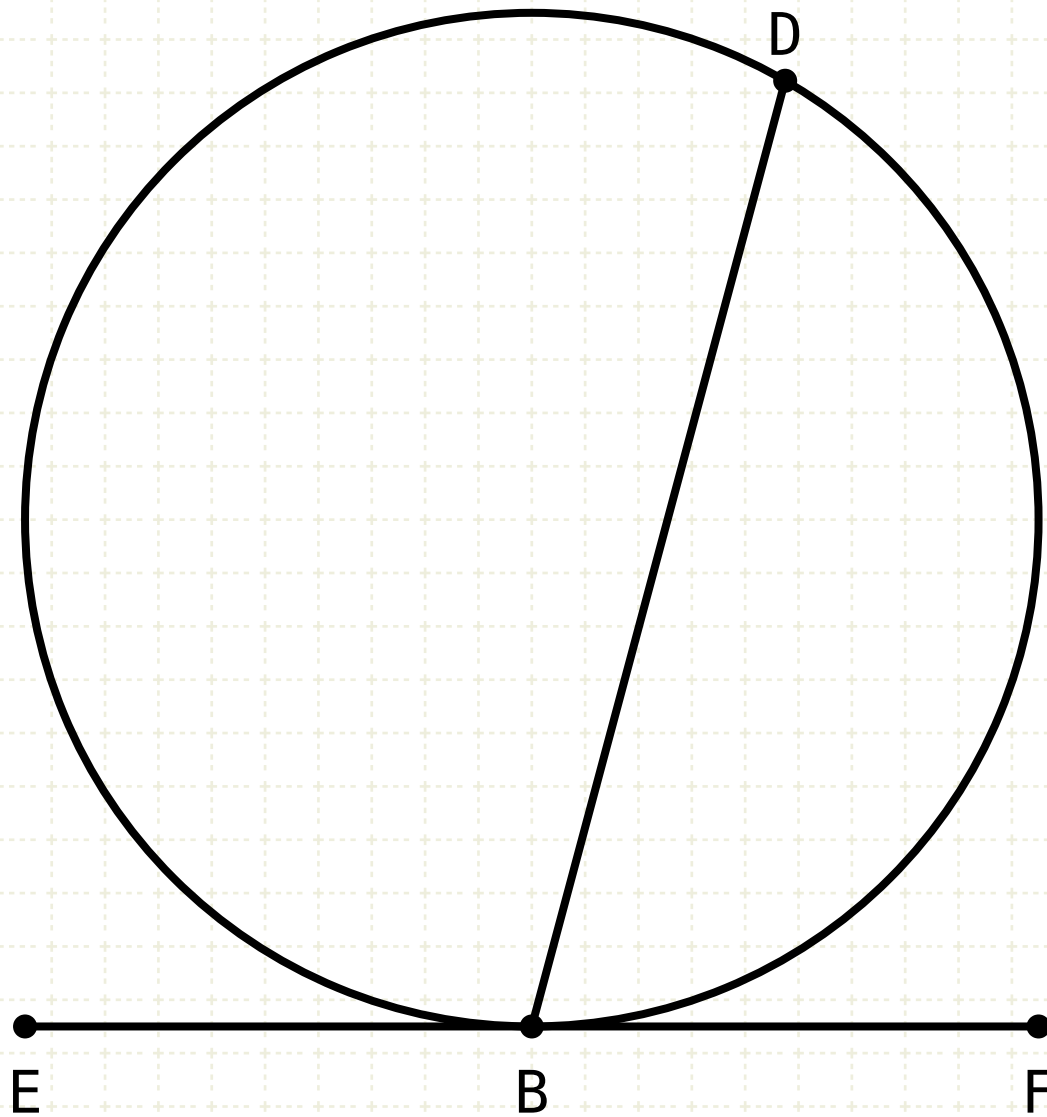
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### In other words

Let EF be a line that touches a circle at point B

Let an arbitrary line cut the circle from B to D



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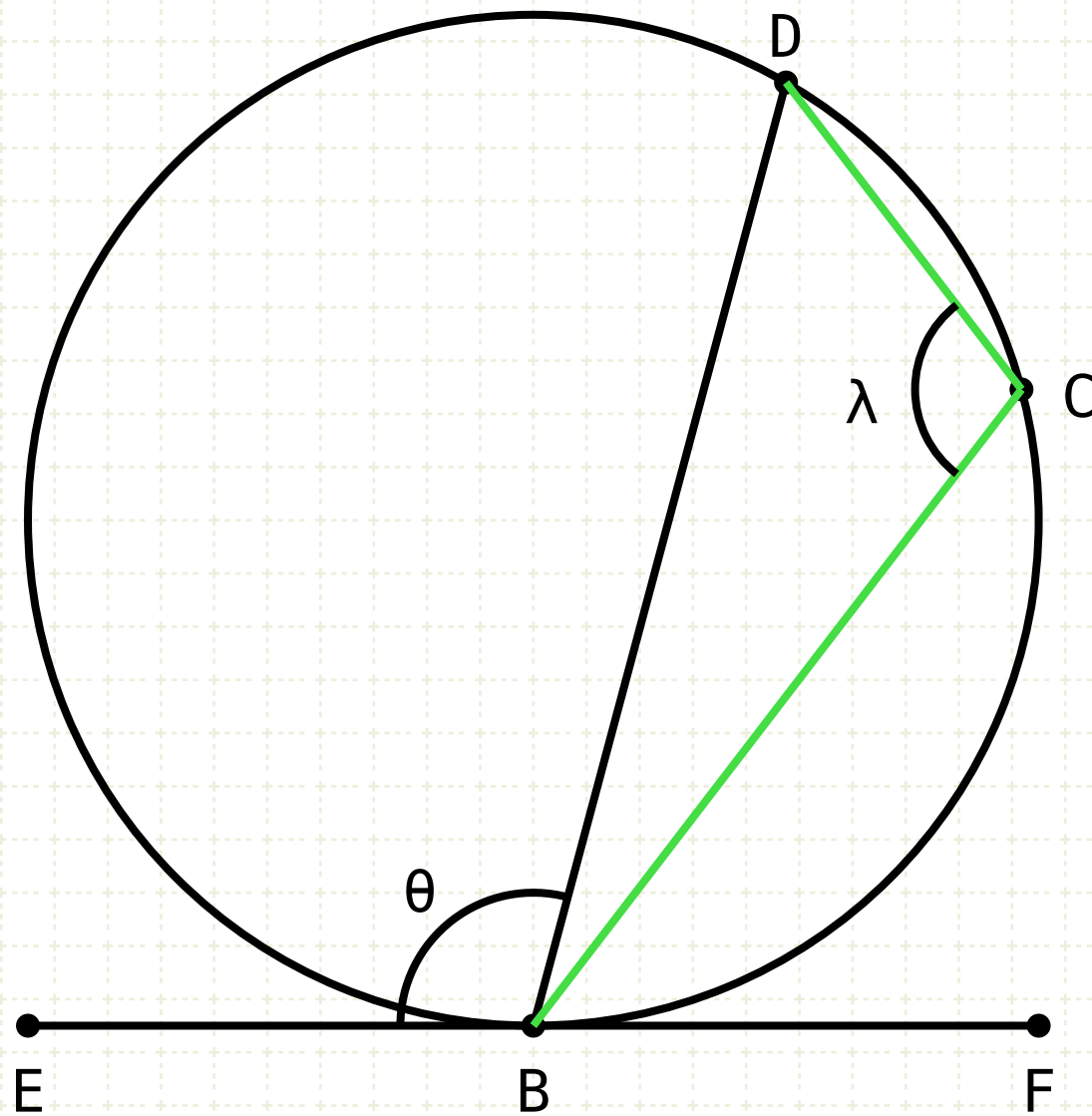
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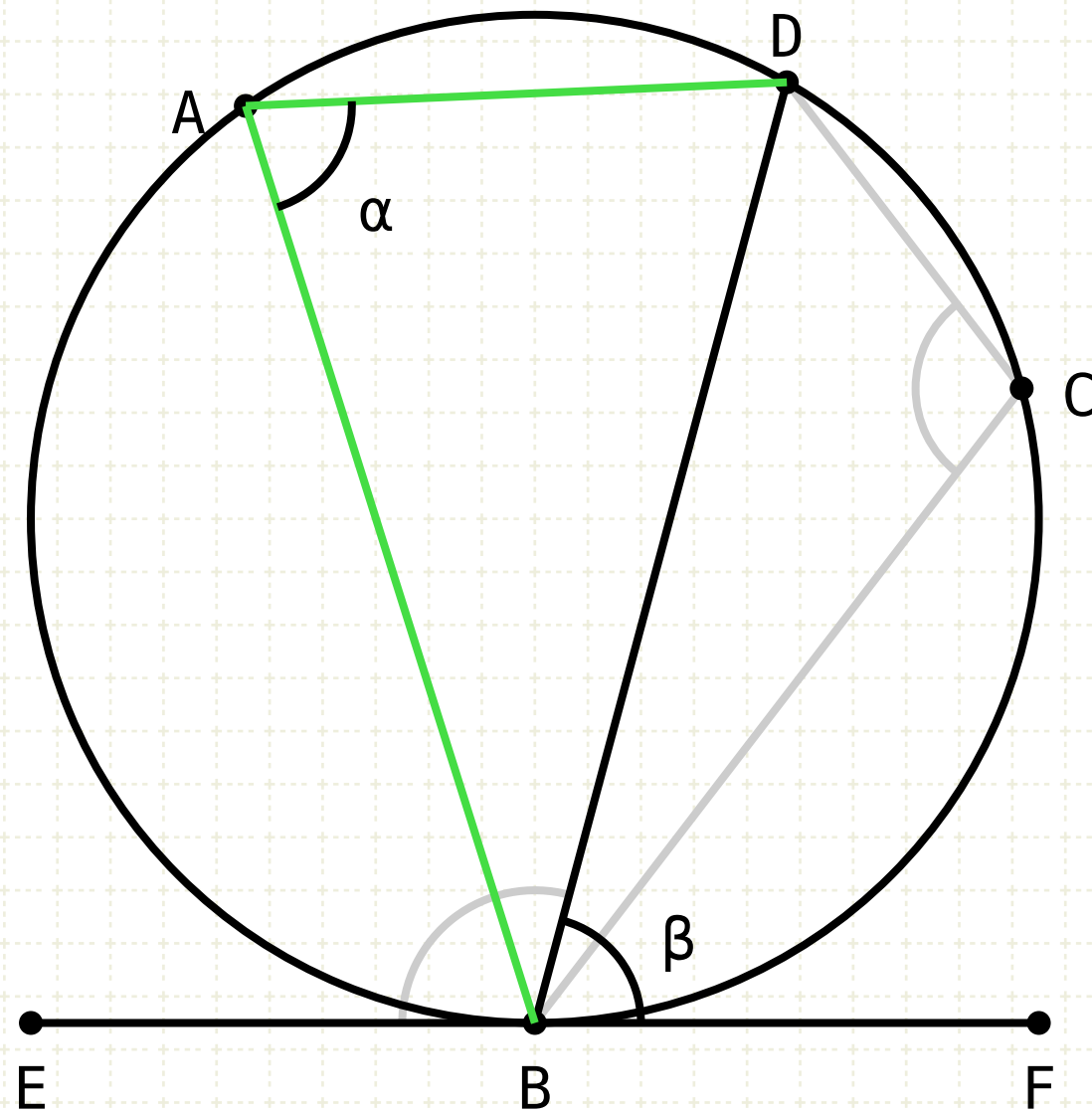
Let an arbitrary line cut the circle from B to D

Then, the angle EBD will be equal to the angle in the alternate segment DCB



## Proposition 32 of Book III

If a straight line touch a circle, and from the point of contact there be drawn across, in the circle, a straight line cutting the circle, the angles which it makes with the tangent will be equal to the angles in the alternate segments of the circle.



$$\theta = \lambda$$
$$\alpha = \beta$$

### In other words

Let  $EF$  be a line that touches a circle at point  $B$

Let an arbitrary line cut the circle from  $B$  to  $D$

Then, the angle  $EBD$  will be equal to the angle in the alternate segment  $DCB$

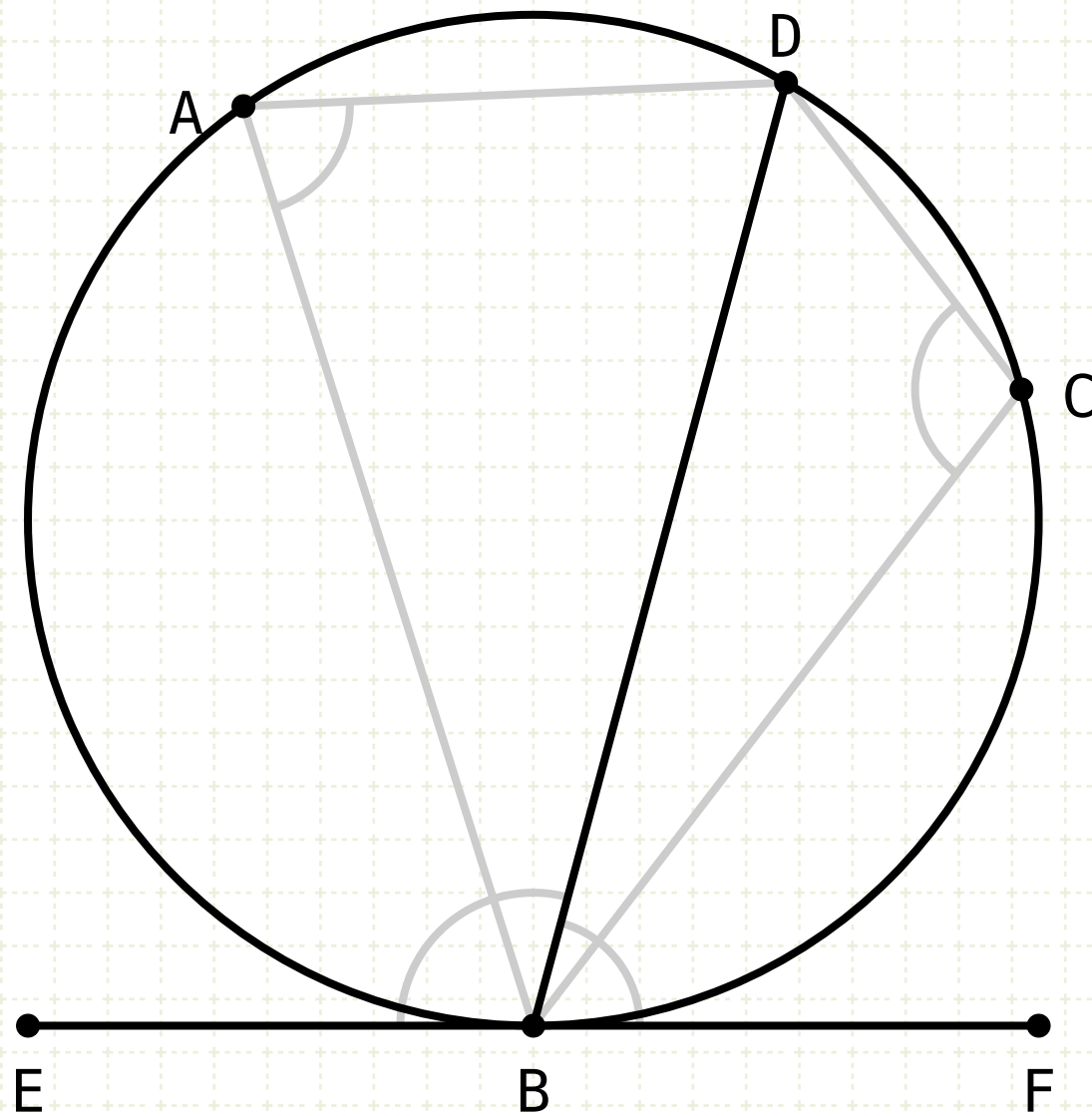
Conversely, angle  $DBF$  will be equal to the segment angle  $DAB$



## Proposition 32 of Book III

If a straight line touch a circle, and from the point of contact there be drawn across, in the circle, a straight line cutting the circle, the angles which it makes with the tangent will be equal to the angles in the alternate segments of the circle.

### Proof



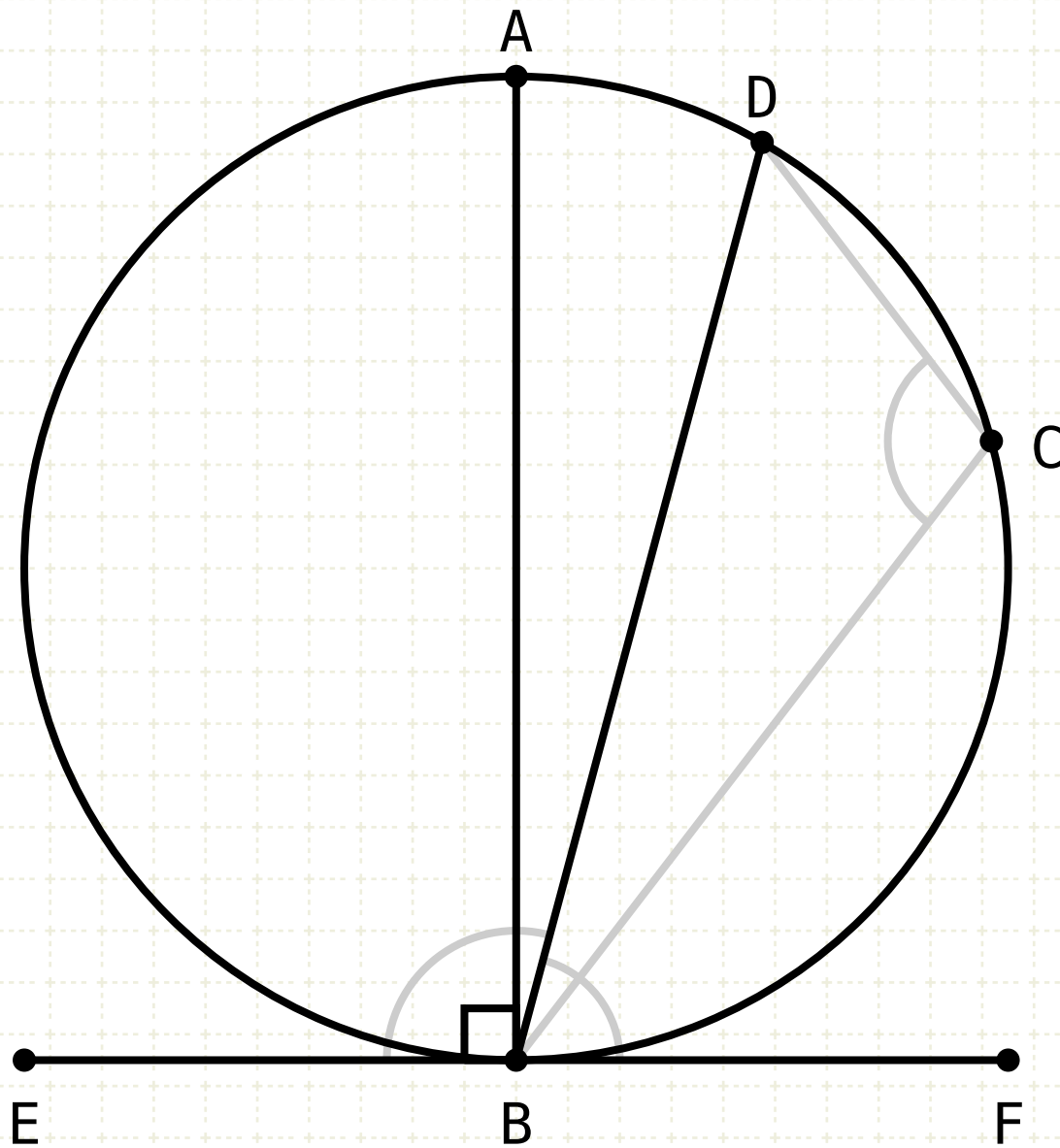
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If a straight line touch a circle, and from the point of contact there be drawn across, in the circle, a straight line cutting the circle, the angles which it makes with the tangent will be equal to the angles in the alternate segments of the circle.

### Proof

Draw the line BA such that it is perpendicular to EF

$$\angle EBA = \angle ABF = L$$



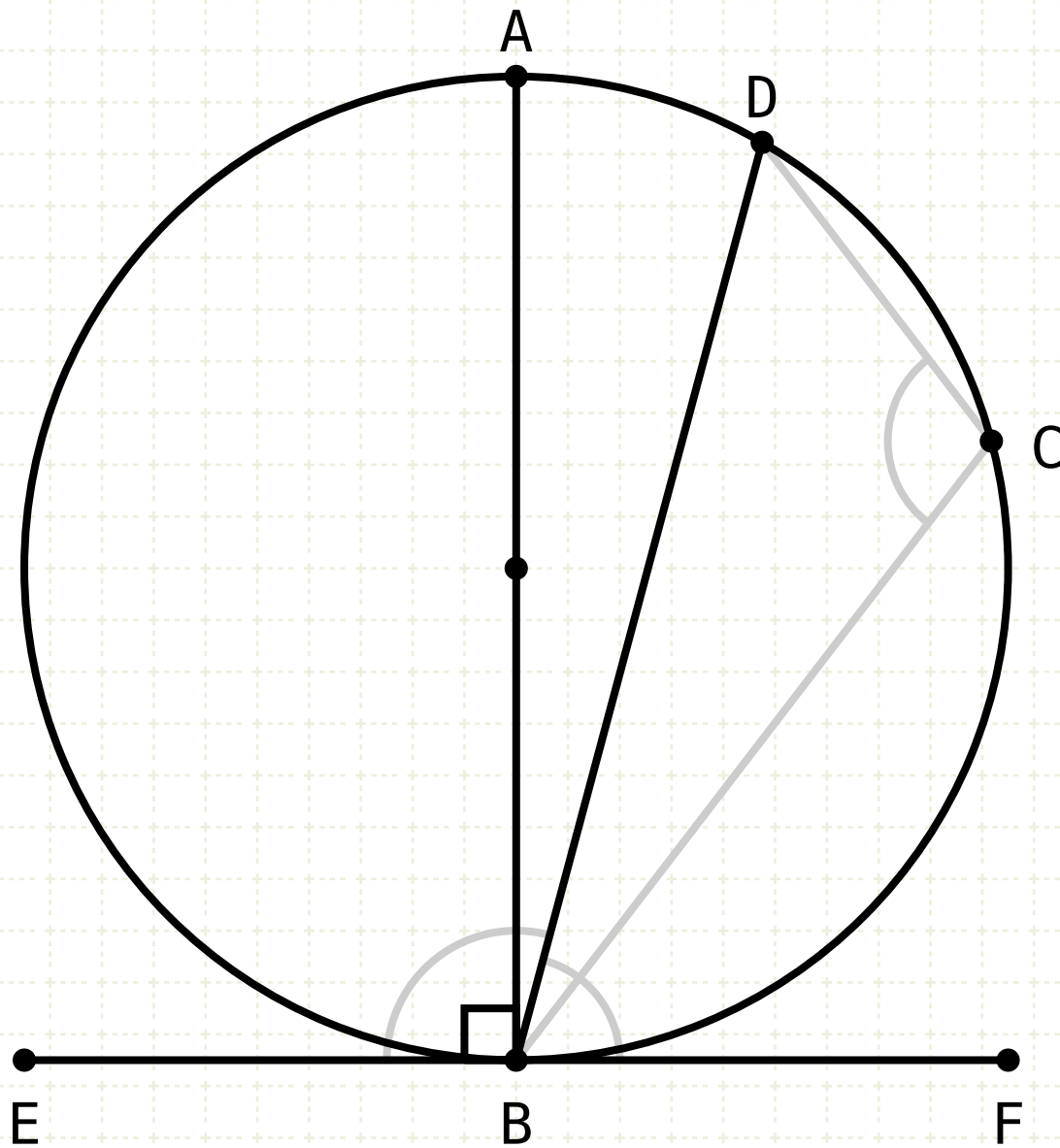
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### Proof

Draw the line BA such that it is perpendicular to EF

Since EF touches the circle at B, and BA is perpendicular to EF, BA is a diameter of the circle (III·19)



$$\angle EBA = \angle ABF = \angle ACB$$

AB = diameter

## Proposition 32 of Book III

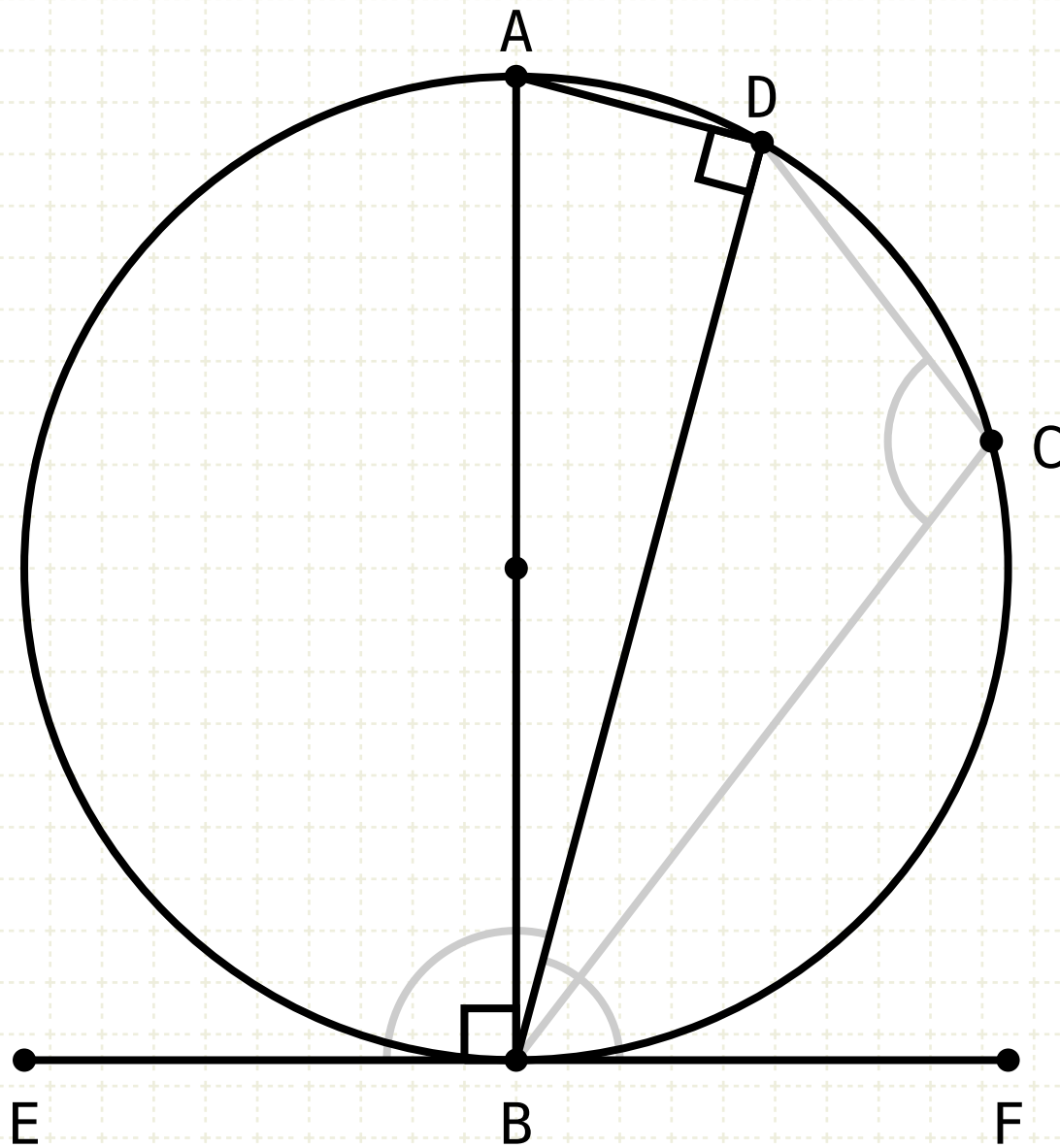
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### Proof

Draw the line BA such that it is perpendicular to EF

Since EF touches the circle at B, and BA is perpendicular to EF, BA is a diameter of the circle (III·19)

Thus ADB is a semicircle, and the angle ADB is right (III·31)



$$\angle EBA = \angle ABF = L$$

AB = diameter

$$\angle ADB = L$$

## Proposition 32 of Book III

If a straight line touch a circle, and from the point of contact there be drawn across, in the circle, a straight line cutting the circle, the angles which it makes with the tangent will be equal to the angles in the alternate segments of the circle.

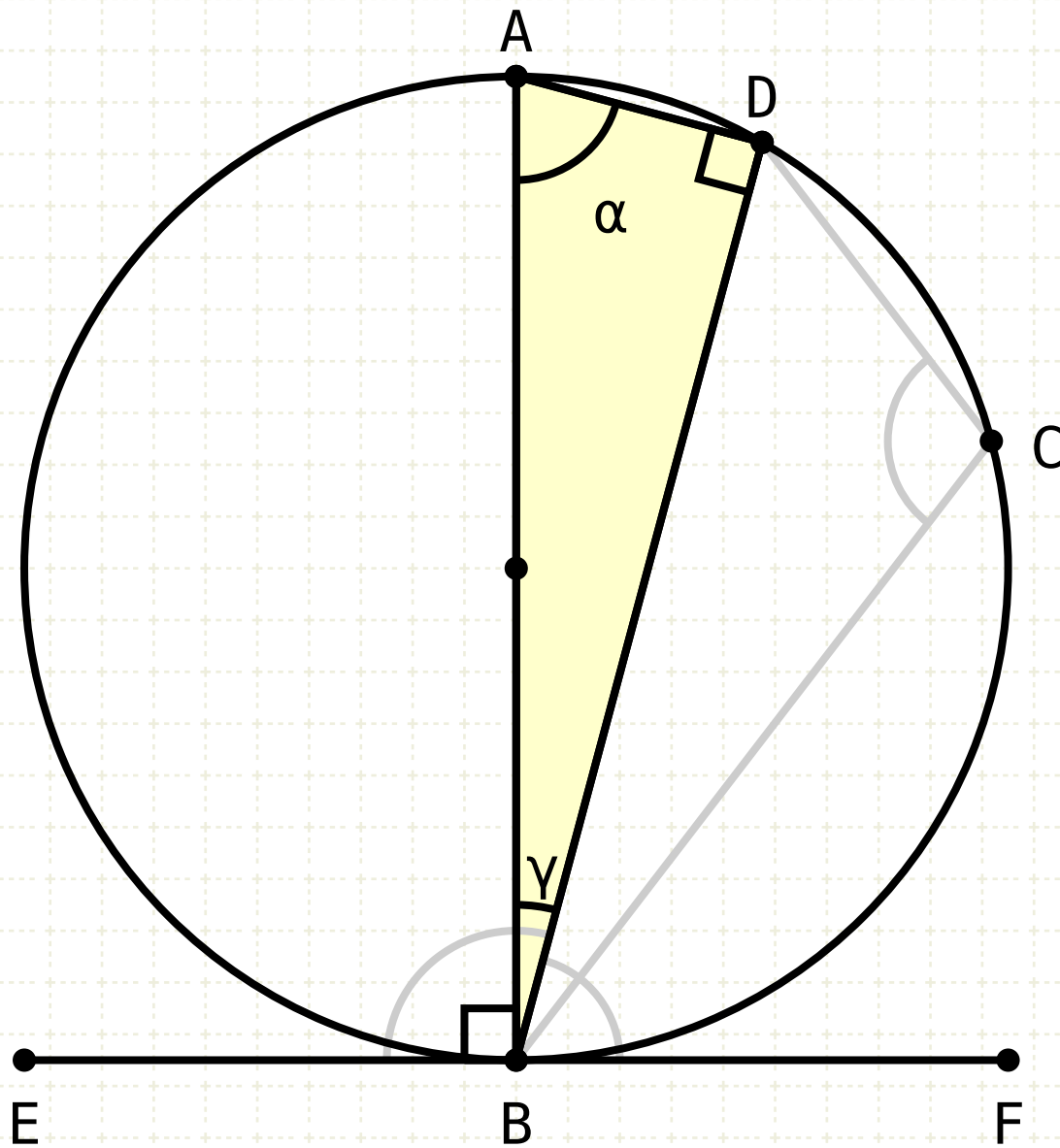
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The remaining angles of the triangle ABC ( $\gamma, \alpha$ ) sum to one right angle (I·32)



$$\begin{aligned}\angle EBA &= \angle ABF = L \\ AB &= \text{diameter} \\ \angle ADB &= L \\ \alpha + \gamma &= L\end{aligned}$$



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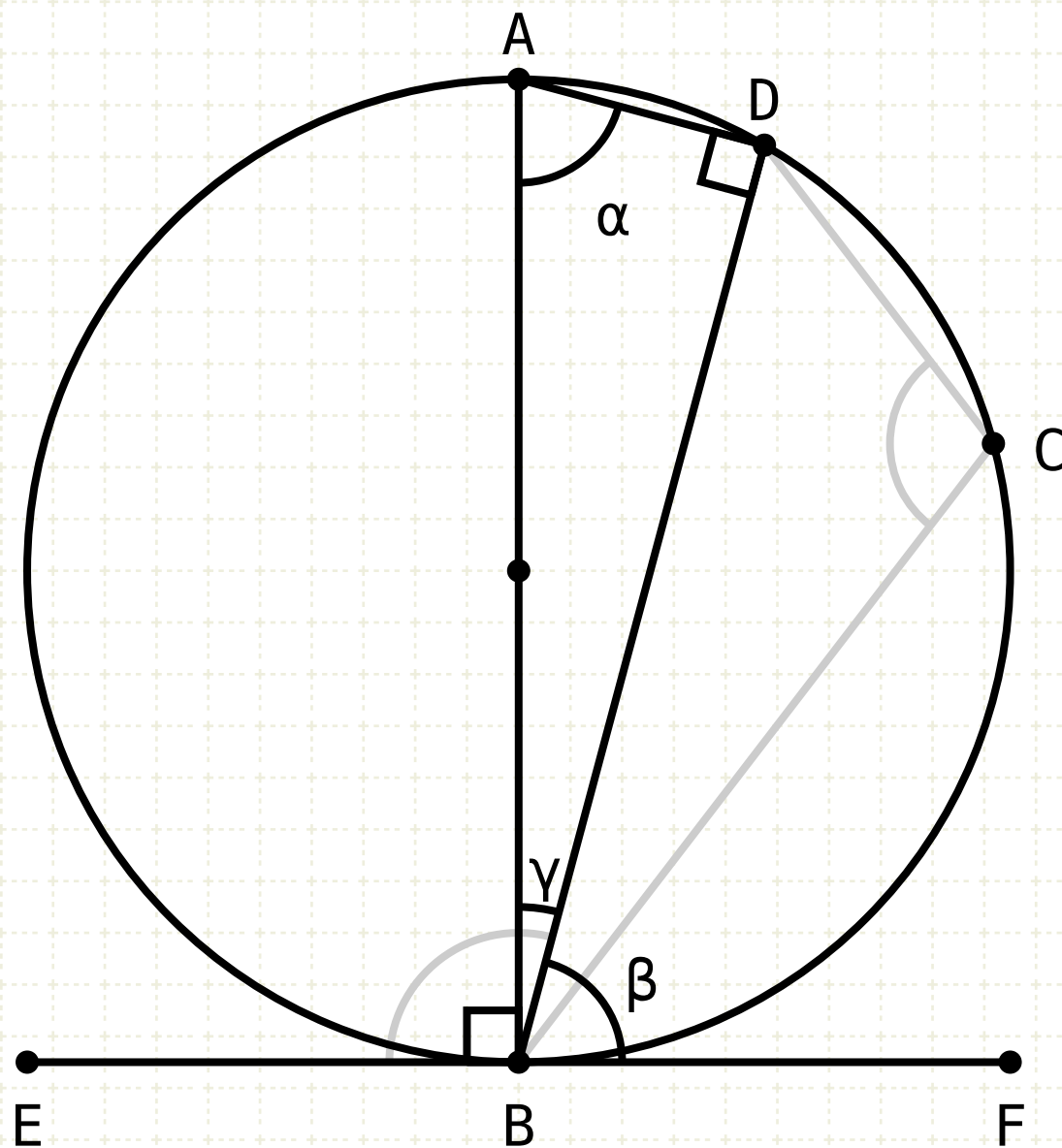
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Angle ABF is right, which is equal to  $\gamma$  plus  $\beta$ , therefore  $\alpha$  is equal to  $\beta$



$$\angle EBA = \angle ABF = L$$

$$AB = \text{diameter}$$

$$\angle ADB = L$$

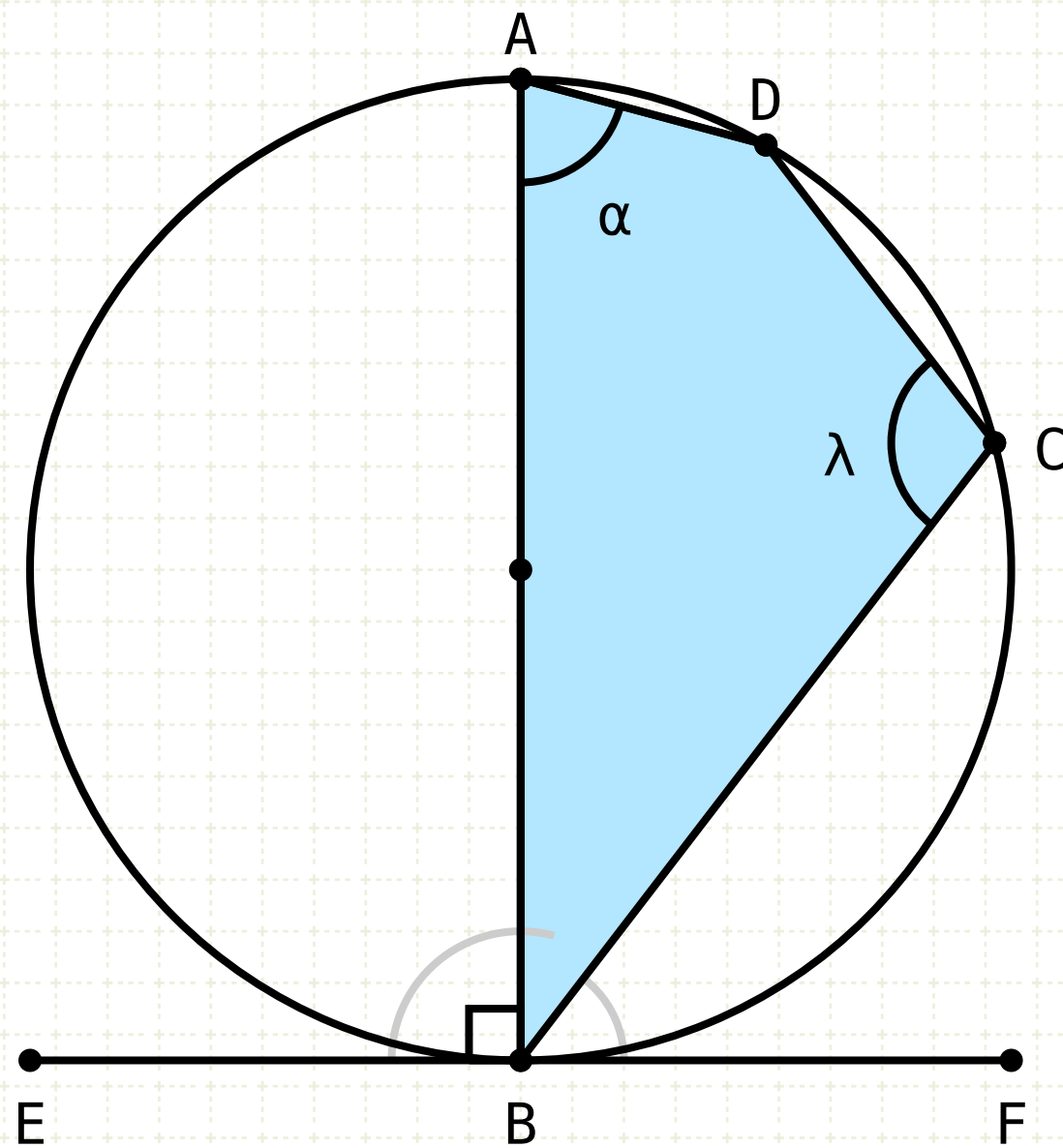
$$\alpha + \gamma = L$$

$$\beta + \gamma = L$$

$$\alpha = \beta$$

## Proposition 32 of Book III

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$$\angle EBA = \angle ABF = L$$

$$AB = \text{diameter}$$

$$\angle ADB = L$$

$$\alpha + \gamma = L$$

$$\beta + \gamma = L$$

$$\alpha = \beta$$

$$\alpha + \lambda = 2L$$

## Proof

Draw the line BA such that it is perpendicular to EF

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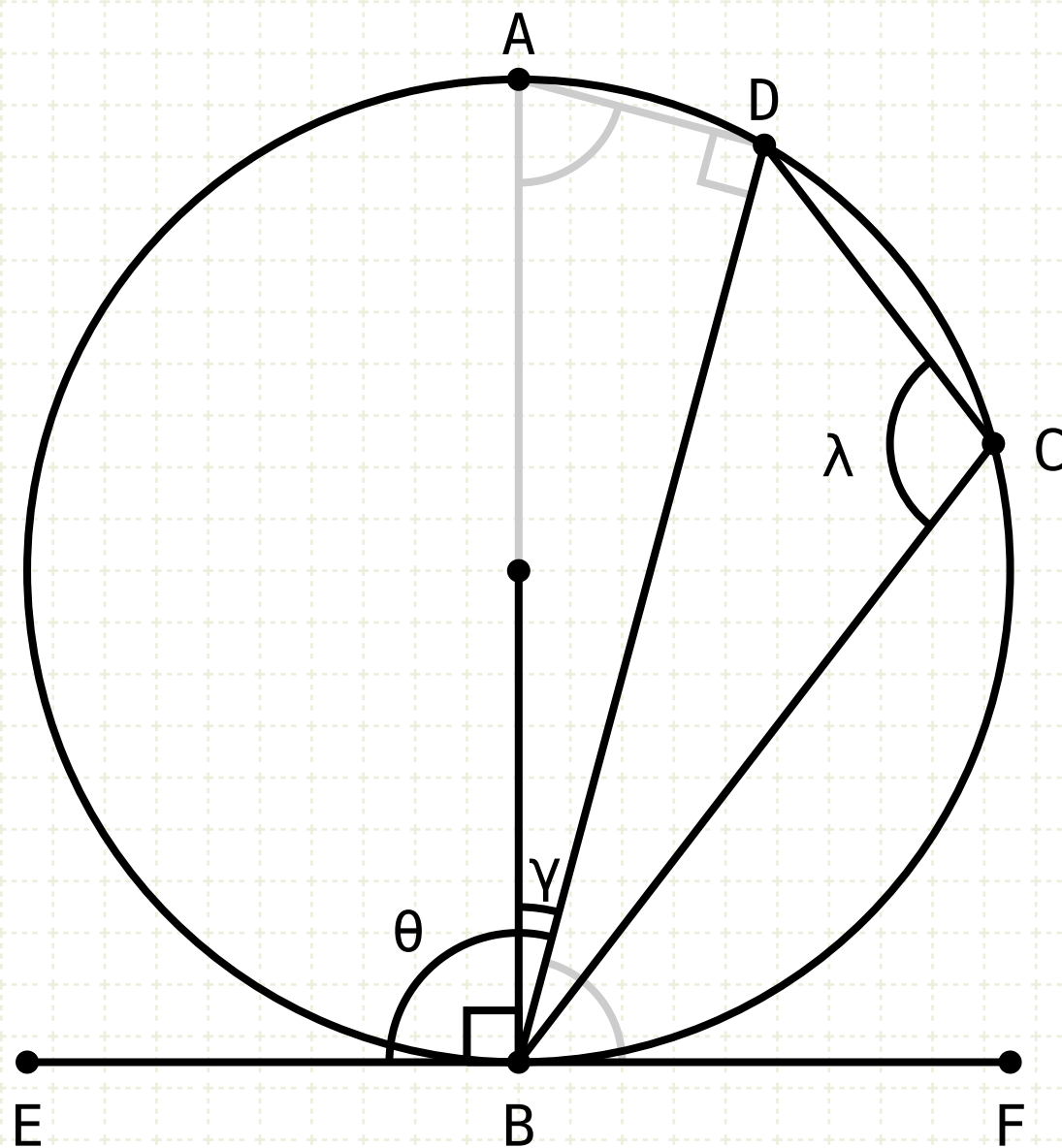
The remaining angles of the triangle ABC ( $\gamma, \alpha$ ) sum to one right angle (I·32)

Angle ABF is right, which is equal to  $\gamma$  plus  $\beta$ , therefore  $\alpha$  is equal to  $\beta$

In the quadrilateral ABCD, the angles at A and C sum to two right angles (III·22)

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$$\angle EBA = \angle ABF = L$$

AB = diameter

$$\angle ADB = L$$

$$\alpha + \gamma \equiv \mathbb{L}$$

$$\beta + \gamma = L$$

$$\alpha = \beta$$

$$\alpha + \lambda = 2L$$

$$\lambda = 2L - \alpha$$

# Proof

Draw the line BA such that it is perpendicular to EF

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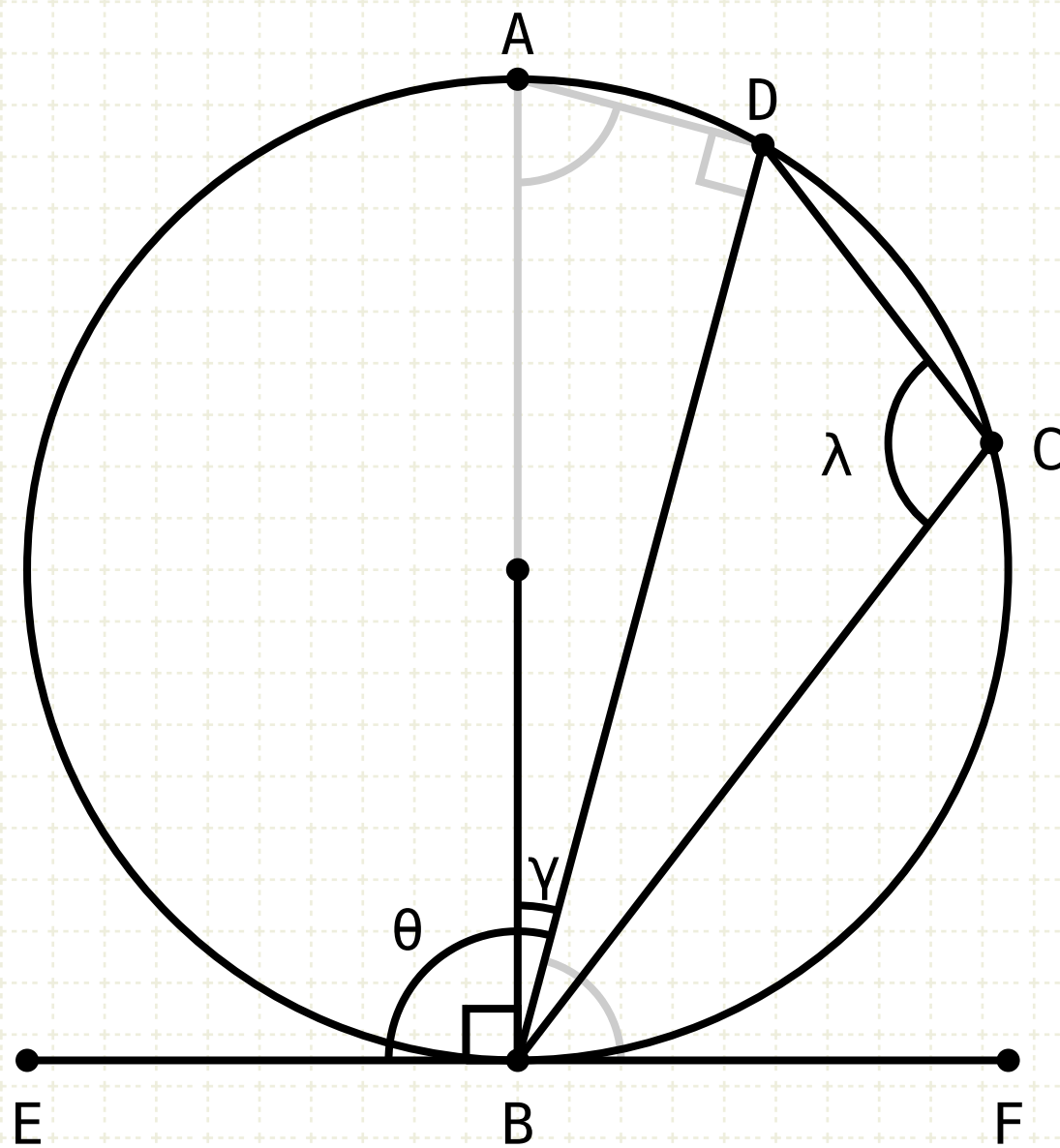
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$$\angle EBA = \angle ABF = L$$

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$$\angle ADB = L$$

$$\alpha + \gamma = L$$

$$\beta + \gamma = L$$

$$\alpha = \beta$$

$$\alpha + \lambda = 2L$$

$$\lambda = 2L - \alpha$$

$$\alpha = L - \gamma$$

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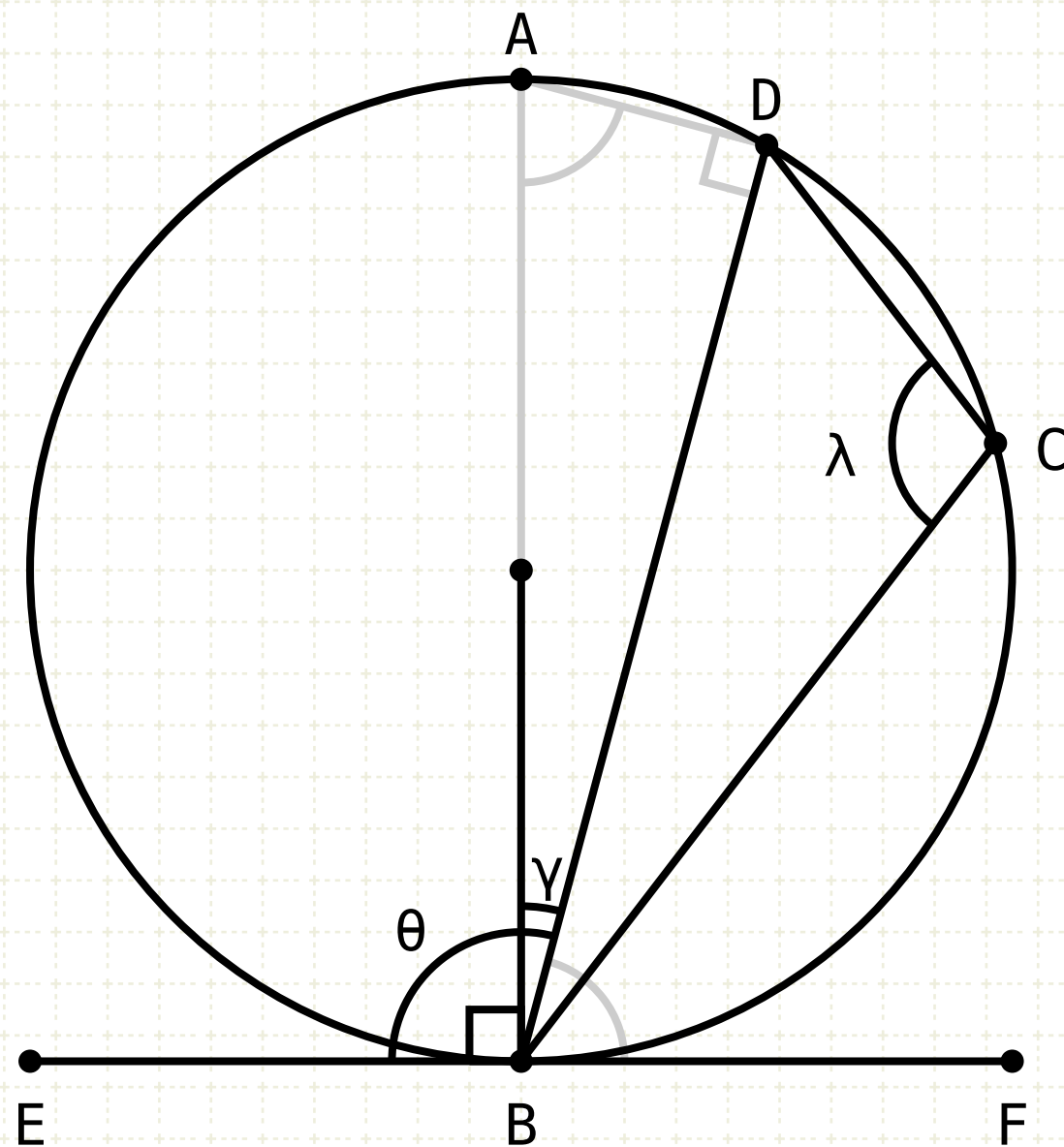
If  $\alpha, \lambda$  equals two right angles,  
and  $\alpha, \gamma$  equals one right angle,





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$$\alpha + \gamma = L$$

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$$\alpha + \lambda = 2L$$

$$\lambda = 2L - \alpha$$

$$\alpha = L - \gamma$$

$$\lambda = 2L - L + \gamma$$

$$\lambda = L + \gamma$$

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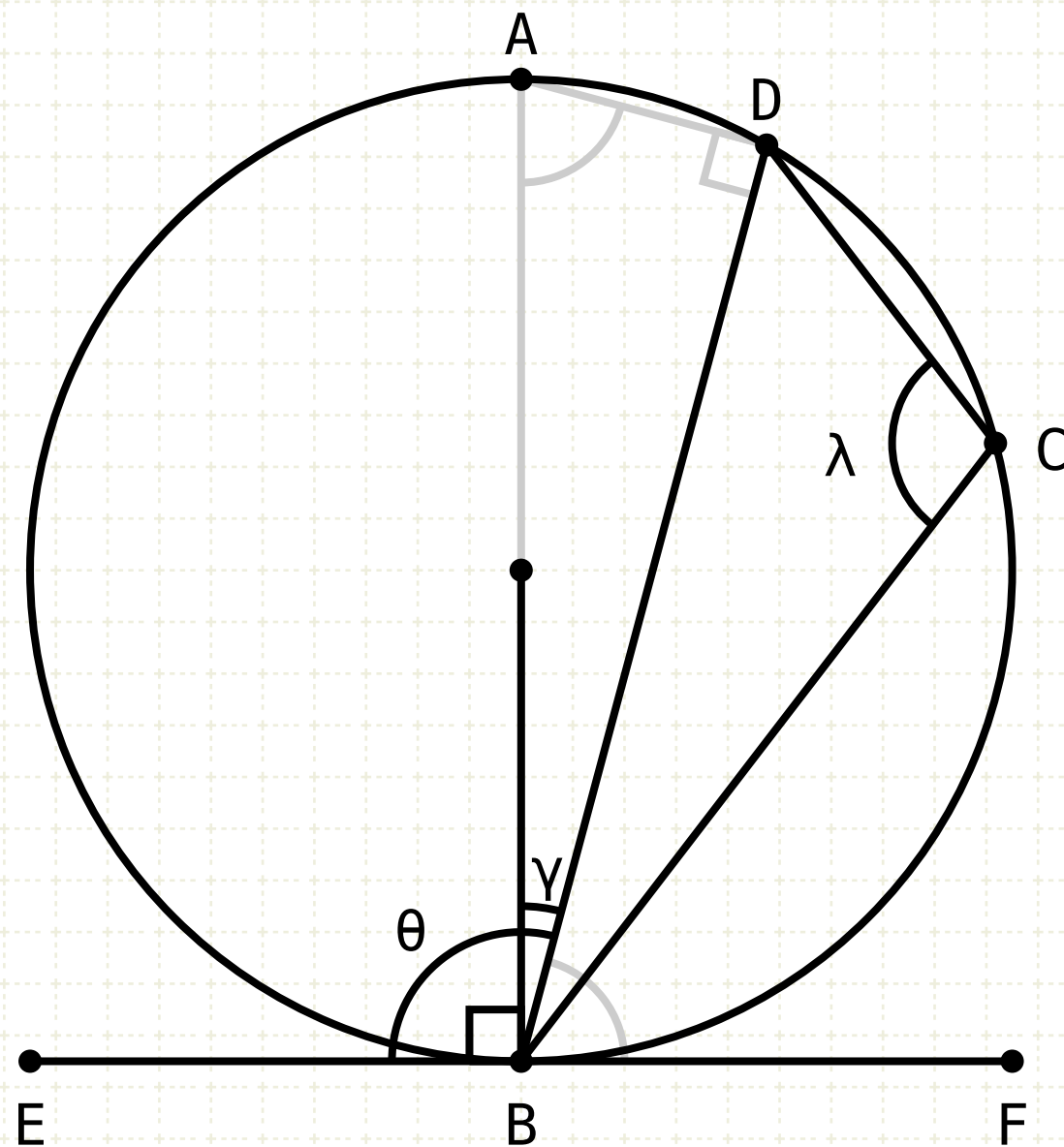
and  $\alpha, \gamma$  equals one right angle,

then  $\lambda$  equals a right angle plus  $\gamma$



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$$\alpha + \lambda = 2L$$

$$\lambda = 2L - \alpha$$

$$\alpha = L - \gamma$$

$$\lambda = 2L - L + \gamma$$

$$\lambda = L + \gamma$$

$$\theta = L + \gamma$$

$$\therefore \theta = \lambda$$

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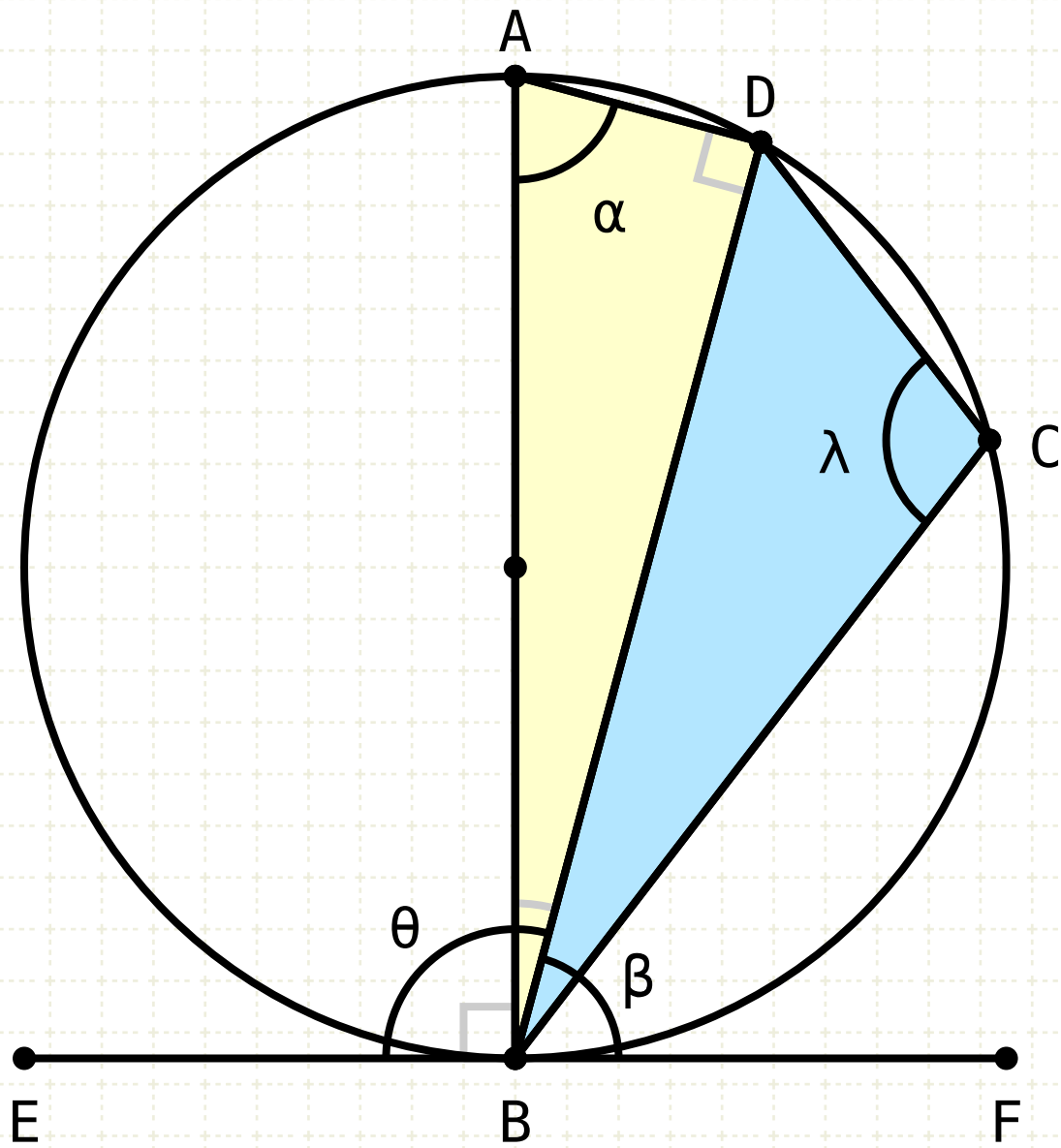
and  $\alpha, \gamma$  equals one right angle,

then  $\lambda$  equals a right angle plus  $\gamma$

But angle EBD ( $\theta$ ) equals one right angle plus  $\gamma$ , thus  $\theta$  equals  $\gamma$

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