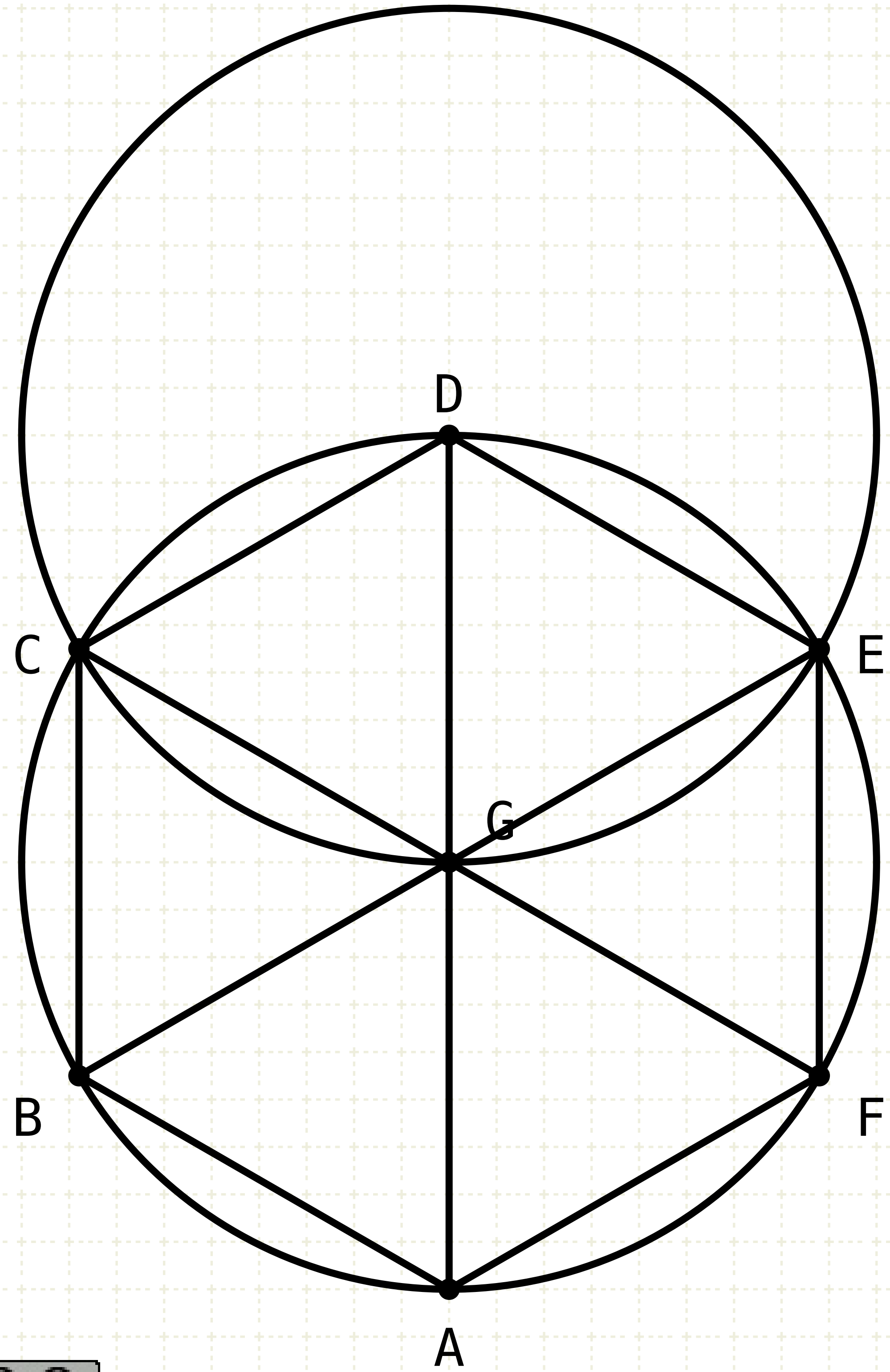
# Euclid's Elements

# Book IV




Philosophy (nature) is written in that great book which ever is before our eyes -- I mean the universe -- but we cannot understand it if we do not first learn the language and grasp the symbols in which it is written. The book is written in mathematical language, and the symbols are triangles, circles and other geometrical figures, without whose help it is impossible to comprehend a single word of it - without which one wanders in vain through a dark labyrinth.

**Galileo Galilei**

# Proposition 14 of Book IV

About a given pentagon, which is equilateral and equiangular, to circumscribe a circle.

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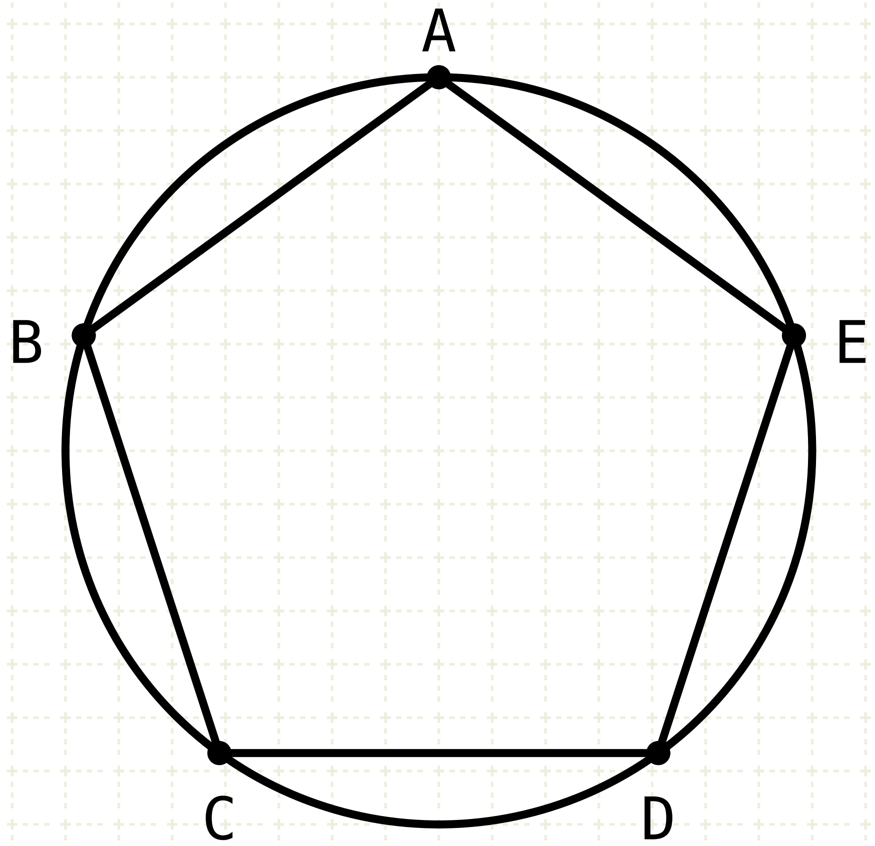
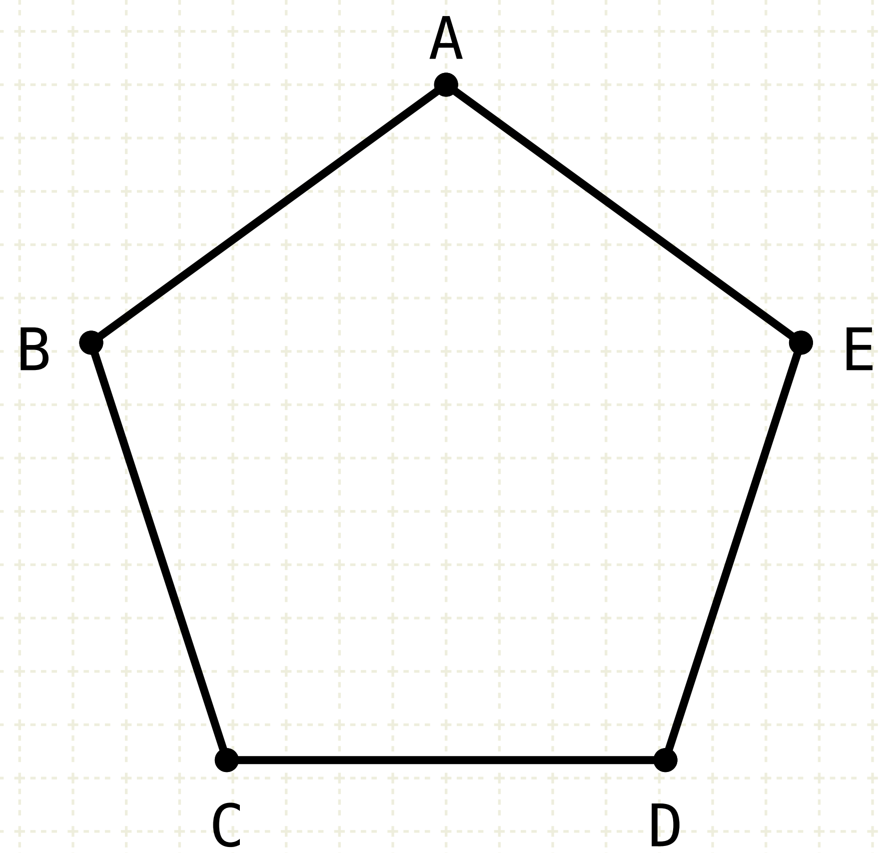



## In other words

Given a pentagon draw a circle on the outside, where the circle passes through the vertices of the pentagon

# Proposition 14 of Book IV

About a given pentagon, which is equilateral and equiangular, to circumscribe a circle.




## Construction

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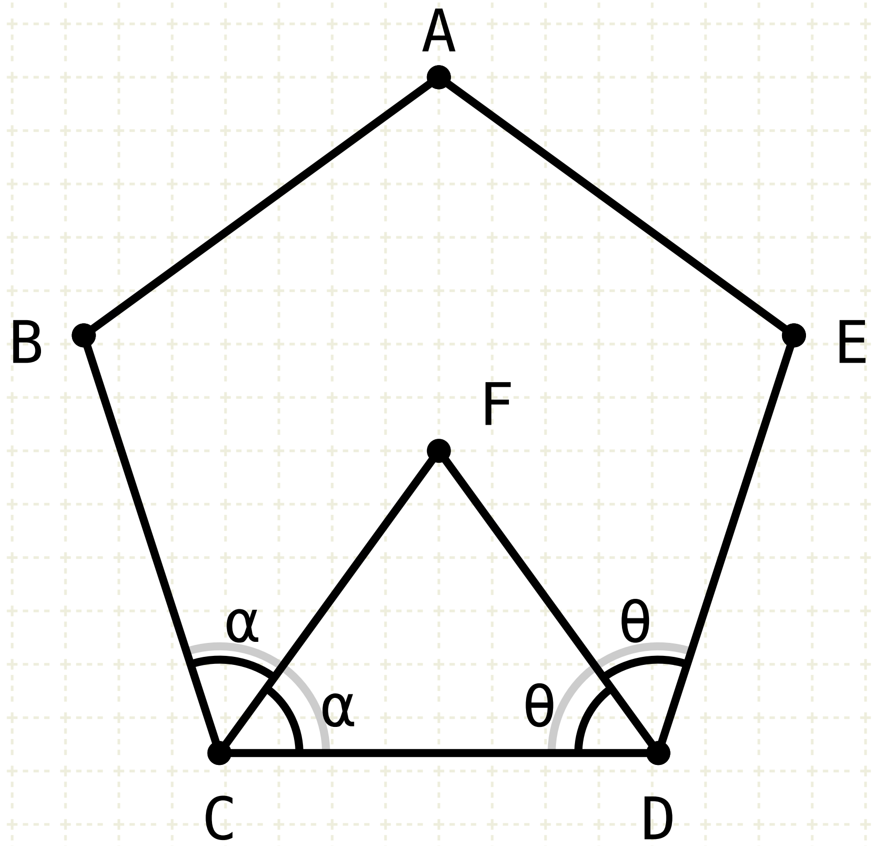


$2\alpha = 2\theta$

$\angle BCF = \angle FCD = \alpha$

$\angle CDF = \angle FDE = \theta$

## Construction

Start with a equilateral, equiangular pentagon

Bisect the angles BCD and CDE by the lines CF and DF

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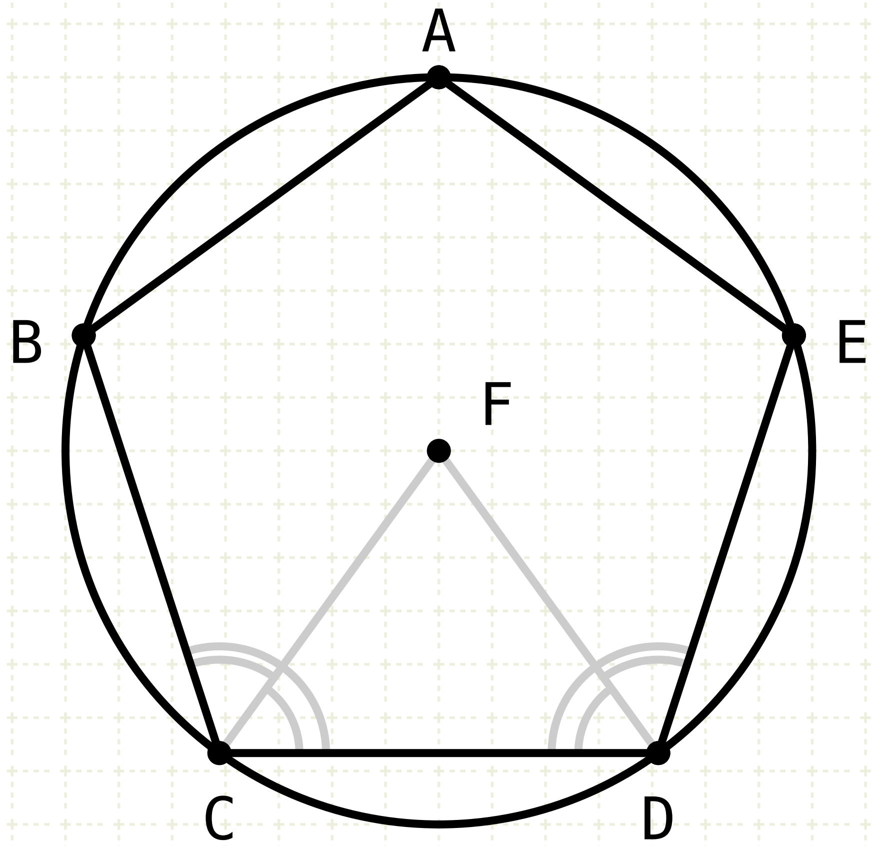
About a given pentagon, which is equilateral and equiangular, to circumscribe a circle.




$2\alpha = 2\theta$

$\angle BCF = \angle FCD = \alpha$

$\angle CDF = \angle FDE = \theta$

## Construction

Start with a equilateral, equiangular pentagon

Bisect the angles BCD and CDE by the lines CF and DF

Draw a circle with F as the centre, and FC as the radius

The circle circumscribes the pentagon

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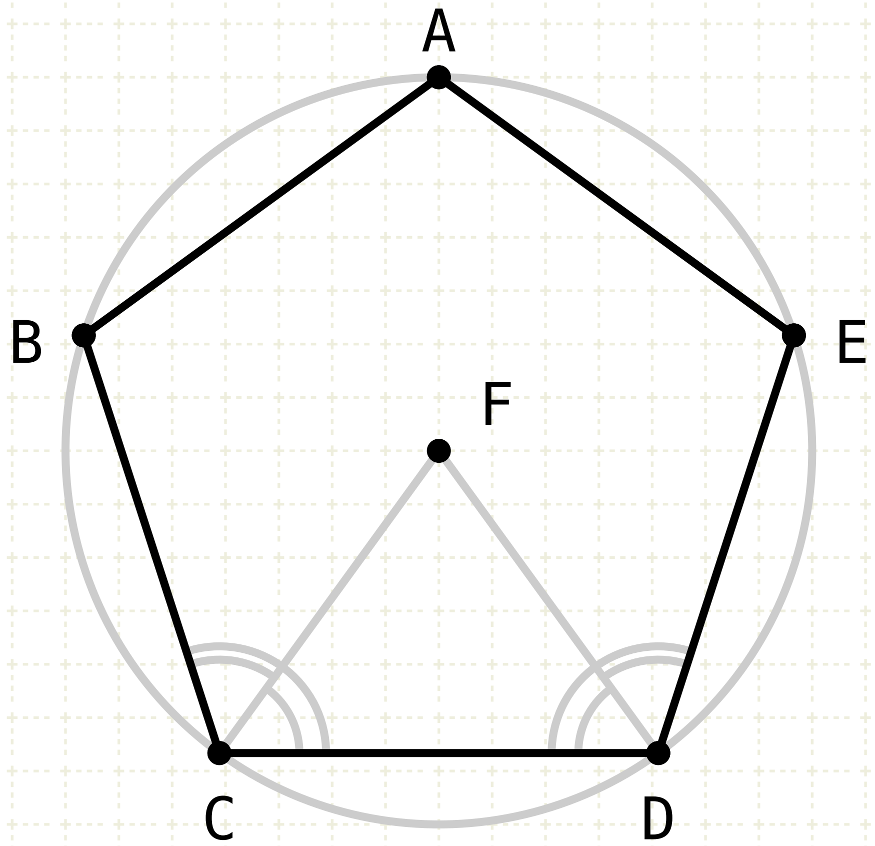
About a given pentagon, which is equilateral and equiangular, to circumscribe a circle.




## Proof

$2\alpha = 2\theta$

$\angle BCF = \angle FCD = \alpha$

$\angle CDF = \angle FDE = \theta$

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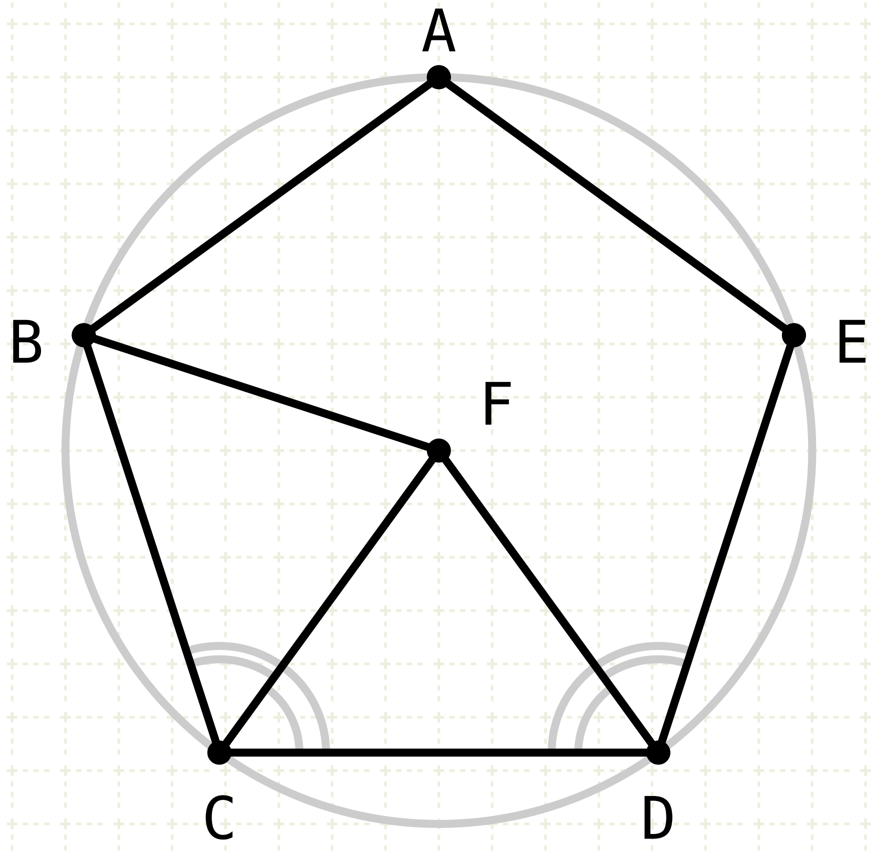


$2\alpha = 2\theta$

$\angle BCF = \angle FCD = \alpha$

$\angle CDF = \angle FDE = \theta$

## Proof

Draw line BF

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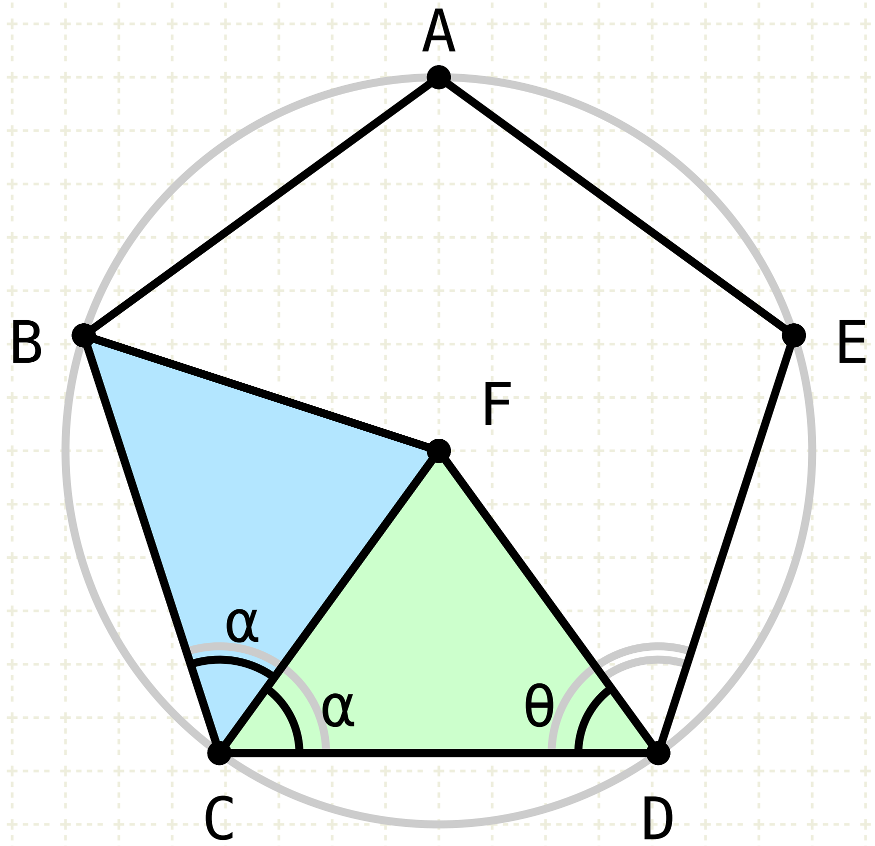


$2\alpha = 2\theta$

$\angle BCF = \angle FCD = \alpha$

$\angle CDF = \angle FDE = \theta$

## Proof

Draw line BF

Since BC is equal to CD, FC is common, and the angles BCF and FCD are equal the two triangles BFC and FCD are equivalent (SAS) (I·4)

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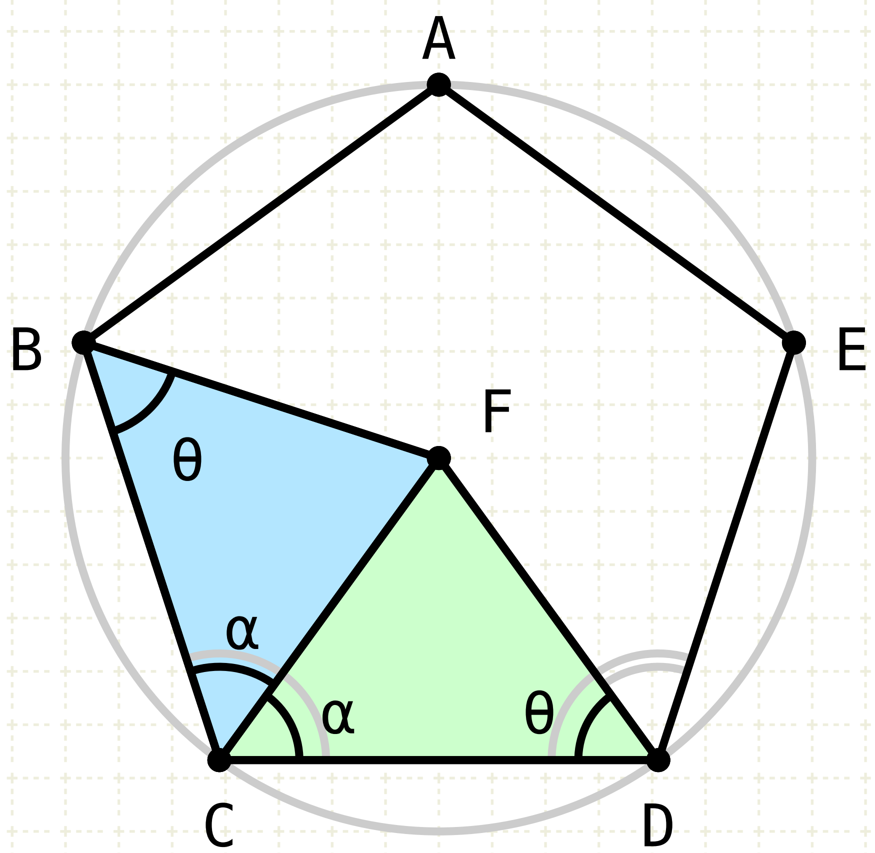
About a given pentagon, which is equilateral and equiangular, to circumscribe a circle.




$2\alpha = 2\theta$

$\angle BCF = \angle FCD = \alpha$

$\angle CDF = \angle FDE = \theta$

$\angle CBF = \angle CDF = \theta$

## Proof

Draw line BF

Since BC is equal to CD, FC is common, and the angles BCF and FCD are equal the two triangles BFC and FCD are equivalent (SAS) (I·4)

Therefore the angles CBF and CDF are equal

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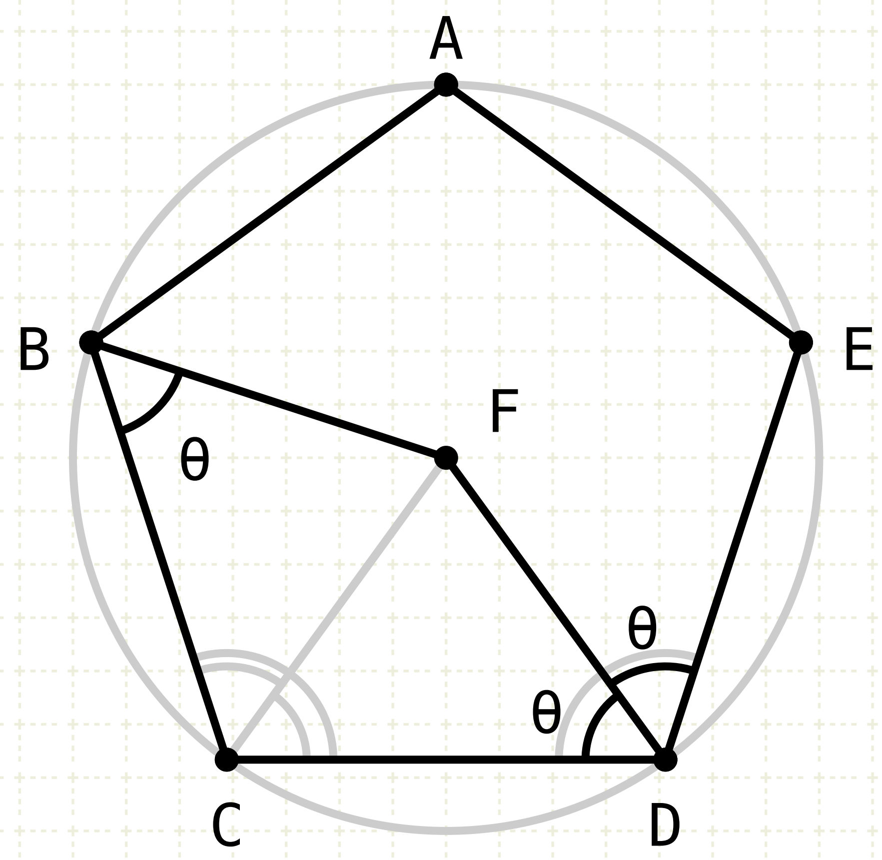


$2\alpha = 2\theta$

$\angle BCF = \angle FCD = \alpha$

$\angle CDF = \angle FDE = \theta$

$\angle CBF = \angle CDF = \theta$

$\angle CDE = \angle ABC = 2\theta$

## Proof

Draw line BF

Since BC is equal to CD, FC is common, and the angles BCF and FCD are equal the two triangles BFC and FCD are equivalent (SAS) (I·4)

Therefore the angles CBF and CDF are equal

The pentagon is equiangular, hence angles ABC and CDE are equal

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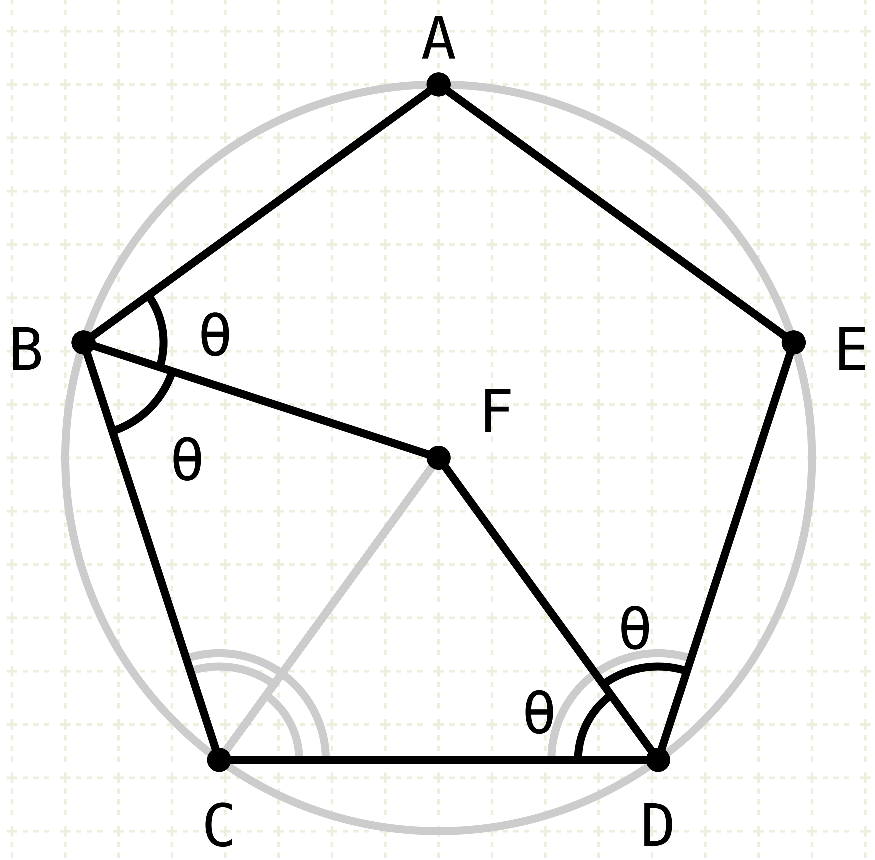
About a given pentagon, which is equilateral and equiangular, to circumscribe a circle.




$2\alpha = 2\theta$
$\angle BCF = \angle FCD = \alpha$
$\angle CDF = \angle FDE = \theta$

$\angle CBF = \angle CDF = \theta$
$\angle CDE = \angle ABC = 2\theta$
$\therefore \angle ABF = \angle CBF = \theta$

## Proof

Draw line BF

Since BC is equal to CD, FC is common, and the angles BCF and FCD are equal the two triangles BFC and FCD are equivalent (SAS) (I·4)

Therefore the angles CBF and CDF are equal

The pentagon is equiangular, hence angles ABC and CDE are equal

Angle CBF (θ) is equal to FDC (θ), and FDC is half of CDE (2θ), therefore BF bisects the angle ABC

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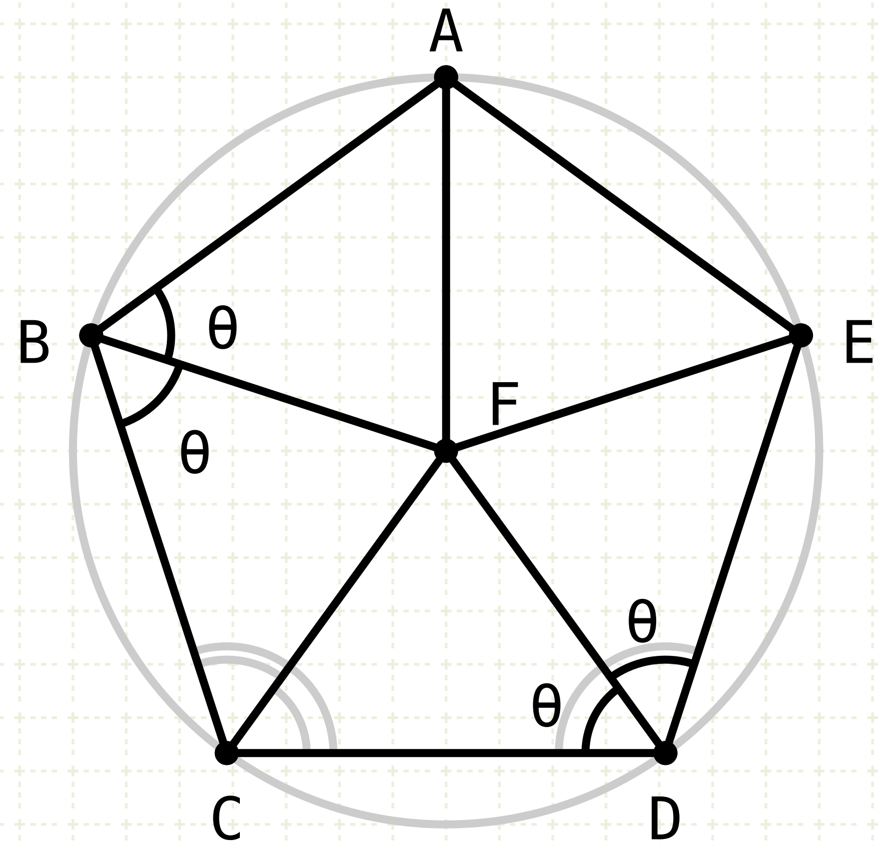
About a given pentagon, which is equilateral and equiangular, to circumscribe a circle.




$$2\alpha = 2\theta$$
$$\angle BCF = \angle FCD = \alpha$$
$$\angle CDF = \angle FDE = \theta$$

$$\angle CBF = \angle CDF = \theta$$
$$\angle CDE = \angle ABC = 2\theta$$
$$\therefore \angle ABF = \angle CBF = \theta$$
$$\angle BAF = \angle FAE$$
$$\angle AEF = \angle FED$$

## Proof

Draw line BF

Since BC is equal to CD, FC is common, and the angles BCF and FCD are equal the two triangles BFC and FCD are equivalent (SAS) (I·4)

Therefore the angles CBF and CDF are equal

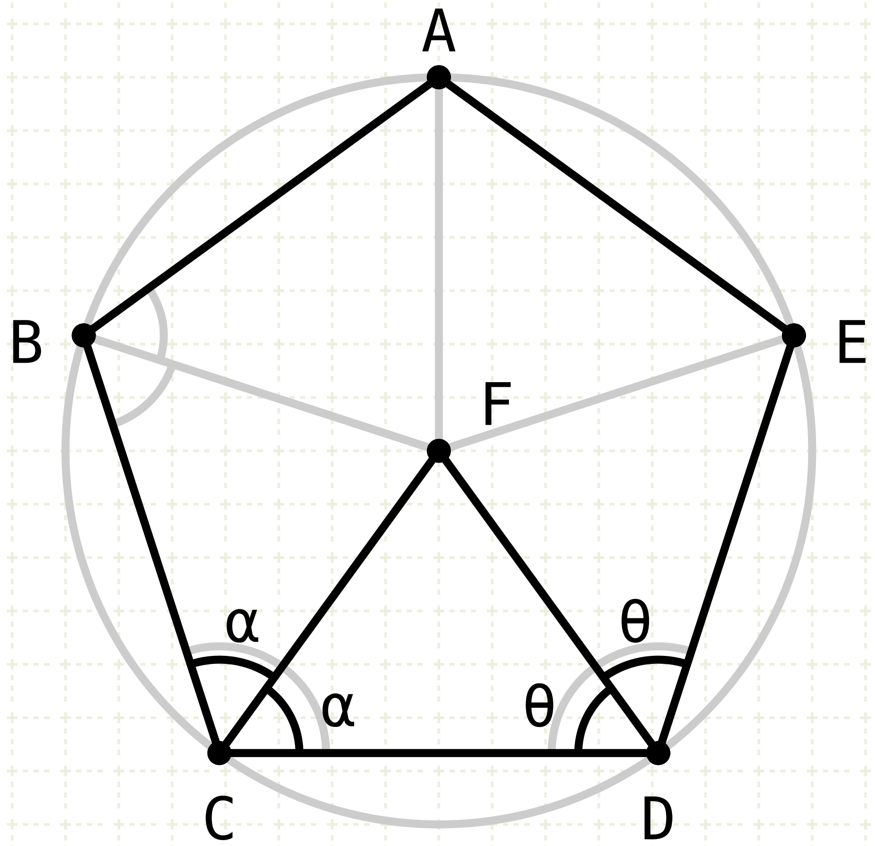
The pentagon is equiangular, hence angles ABC and CDE are equal

Angle CBF (θ) is equal to FDC (θ), and FDC is half of CDE (2θ), therefore BF bisects the angle ABC

Similarly, it can be shown that AF and EF bisect the angles BAE and AED respectively

# Proposition 14 of Book IV

About a given pentagon, which is equilateral and equiangular, to circumscribe a circle.




$2\alpha = 2\theta$

$\angle BCF = \angle FCD = \alpha$

$\angle CDF = \angle FDE = \theta$

$\angle CBF = \angle CDF = \theta$

$\angle CDE = \angle ABC = 2\theta$

$\therefore\ \angle ABF = \angle CBF = \theta$

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## Proof

Draw line BF

Since BC is equal to CD, FC is common, and the angles BCF and FCD are equal the two triangles BFC and FCD are equivalent (SAS) (I·4)

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The pentagon is equiangular, hence angles ABC and CDE are equal

Angle CBF (θ) is equal to FDC (θ), and FDC is half of CDE (2θ), therefore BF bisects the angle ABC

Similarly, it can be shown that AF and EF bisect the angles BAE and AED respectively

The pentagon is equiangular, hence angles BCD and CDE are equal

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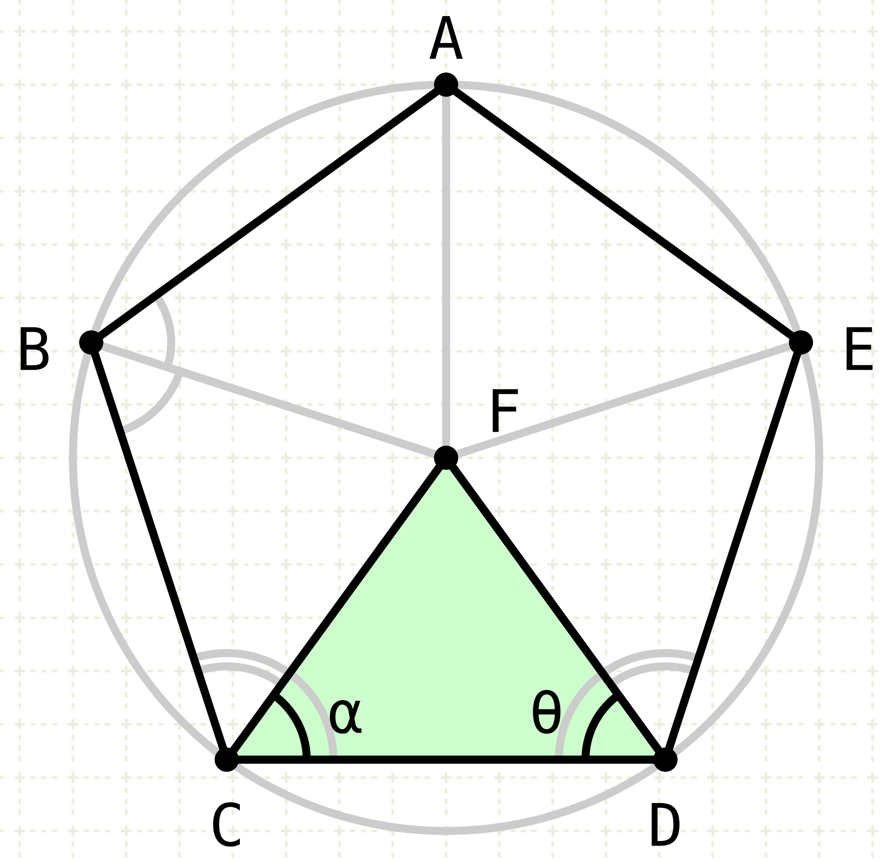
About a given pentagon, which is equilateral and equiangular, to circumscribe a circle.




$2\alpha = 2\theta$
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$\angle CDE = \angle ABC = 2\theta$
$\therefore \angle ABF = \angle CBF = \theta$
$\angle BAF = \angle FAE$
$\angle AEF = \angle FED$

$2\alpha = 2\theta$
$\alpha = \theta$
$FC = FD$

## Proof

Draw line BF

Since BC is equal to CD, FC is common, and the angles BCF and FCD are equal the two triangles BFC and FCD are equivalent (SAS) (I·4)

Therefore the angles CBF and CDF are equal

The pentagon is equiangular, hence angles ABC and CDE are equal

Angle CBF (θ) is equal to FDC (θ), and FDC is half of CDE (2θ), therefore BF bisects the angle ABC

Similarly, it can be shown that AF and EF bisect the angles BAE and AED respectively

The pentagon is equiangular, hence angles BCD and CDE are equal

Therefore angles FCD and FDC are also equal, and the triangle FCD is an isosceles triangle

Thus, FC and FD are equal (I·6)

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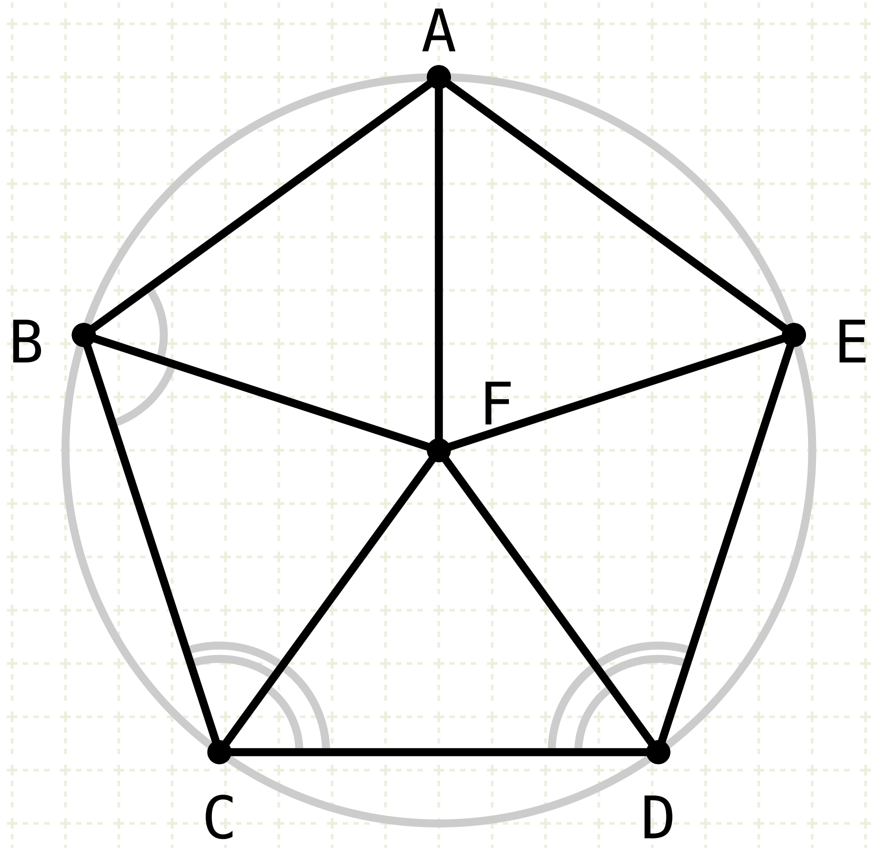
About a given pentagon, which is equilateral and equiangular, to circumscribe a circle.




$2\alpha = 2\theta$

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$\angle CBF = \angle CDF = \theta$

$\angle CDE = \angle ABC = 2\theta$

$\therefore \angle ABF = \angle CBF = \theta$

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$2\alpha = 2\theta$

$\alpha = \theta$

$FC = FD$

$FA = FB = FC = FD = FE$

## Proof

Draw line BF

Since BC is equal to CD, FC is common, and the angles BCF and FCD are equal the two triangles BFC and FCD are equivalent (SAS) (I·4)

Therefore the angles CBF and CDF are equal

The pentagon is equiangular, hence angles ABC and CDE are equal

Angle CBF (θ) is equal to FDC (θ), and FDC is half of CDE (2θ), therefore BF bisects the angle ABC

Similarly, it can be shown that AF and EF bisect the angles BAE and AED respectively

The pentagon is equiangular, hence angles BCD and CDE are equal

Therefore angles FCD and FDC are also equal, and the triangle FCD is an isosceles triangle

Thus, FC and FD are equal (I·6)

Similarly, it can be shown that the lines FA, FB, FC, FD, and FE are all equal

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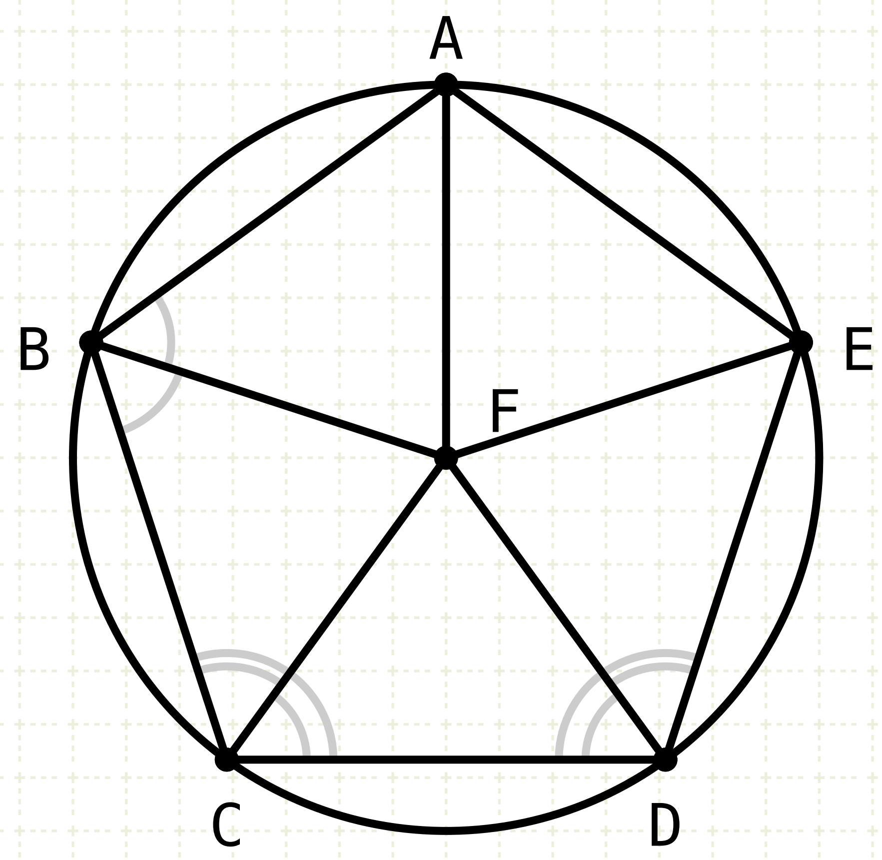
About a given pentagon, which is equilateral and equiangular, to circumscribe a circle.

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Since BC is equal to CD, FC is common, and the angles BCF and FCD are equal the two triangles BFC and FCD are equivalent (SAS) (I·4)

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The pentagon is equiangular, hence angles ABC and CDE are equal

Angle CBF (θ) is equal to FDC (θ), and FDC is half of CDE (2θ), therefore BF bisects the angle ABC

Similarly, it can be shown that AF and EF bisect the angles BAE and AED respectively

The pentagon is equiangular, hence angles BCD and CDE are equal

Therefore angles FCD and FDC are also equal, and the triangle FCD is an isosceles triangle

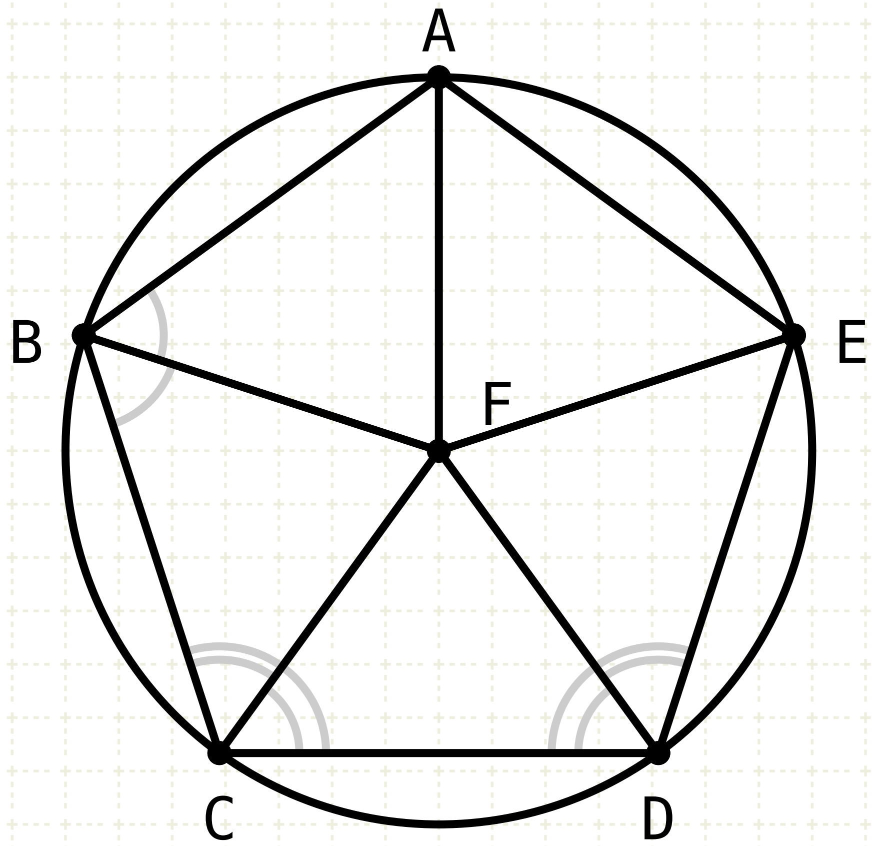
Thus, FC and FD are equal (I·6)

Similarly, it can be shown that the lines FA, FB, FC, FD, and FE are all equal

A circle drawn with a centre at F, and a radius of FA will pass through all the points A, B, C, D and E

# Proposition 14 of Book IV

About a given pentagon, which is equilateral and equiangular, to circumscribe a circle.

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$2\alpha = 2\theta$
$\alpha = \theta$
$FC = FD$
$FA = FB = FC = FD = FE$

## Proof

Draw line BF

Since BC is equal to CD, FC is common, and the angles BCF and FCD are equal the two triangles BFC and FCD are equivalent (SAS) (I·4)

Therefore the angles CBF and CDF are equal

The pentagon is equiangular, hence angles ABC and CDE are equal

Angle CBF (θ) is equal to FDC (θ), and FDC is half of CDE (2θ), therefore BF bisects the angle ABC

Similarly, it can be shown that AF and EF bisect the angles BAE and AED respectively

The pentagon is equiangular, hence angles BCD and CDE are equal

Therefore angles FCD and FDC are also equal, and the triangle FCD is an isosceles triangle

Thus, FC and FD are equal (I·6)

Similarly, it can be shown that the lines FA, FB, FC, FD, and FE are all equal

A circle drawn with a centre at F, and a radius of FA will pass through all the points A, B, C, D and E

Thus we have circumscribed the pentagon with a circle

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