

Euclid's Elements

Book I

*If Euclid did not kindle your youthful enthusiasm, you
were not born to be a scientific thinker.*

Albert Einstein

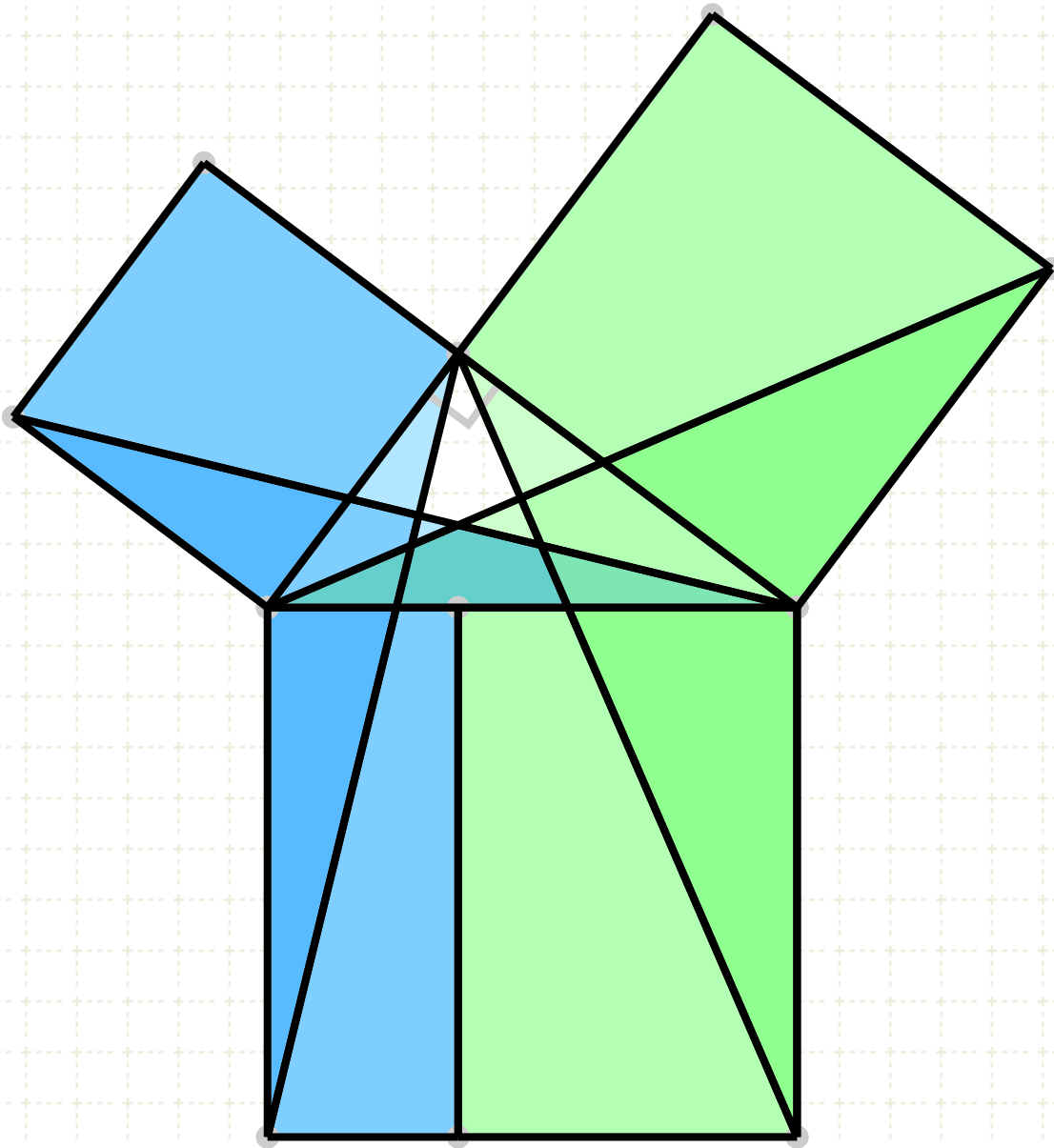


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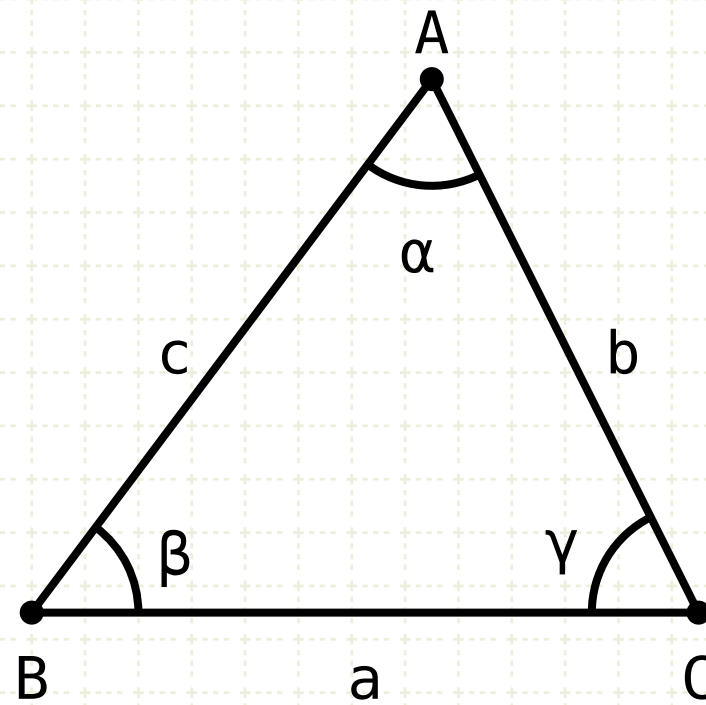
Proposition 20 of Book I

Any two sides of a triangle are together greater than the third side.



Proposition 20 of Book I

Any two sides of a triangle are together greater than the third side.

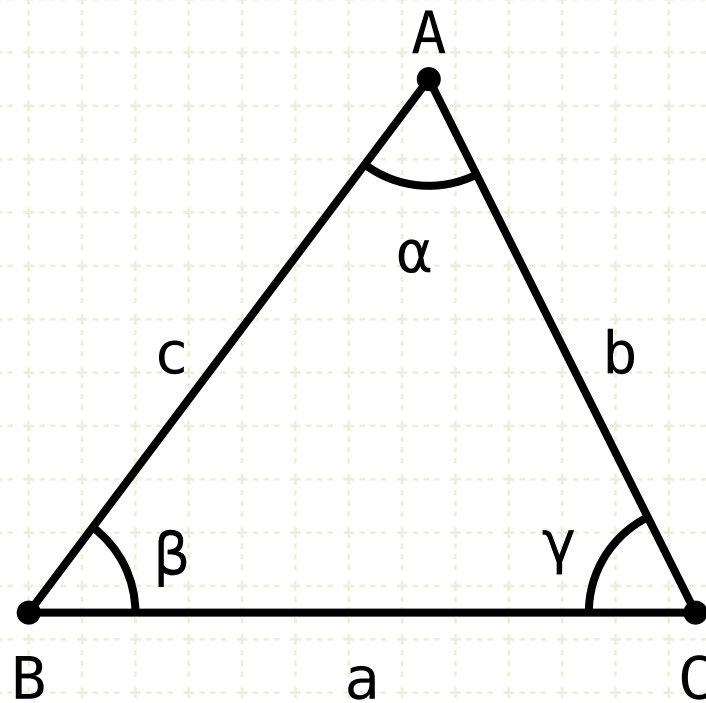


In other words

Given a triangle ABC

Proposition 20 of Book I

Any two sides of a triangle are together greater than the third side.



In other words

Given a triangle ABC

The sum of any two sides of the triangle is greater than the third

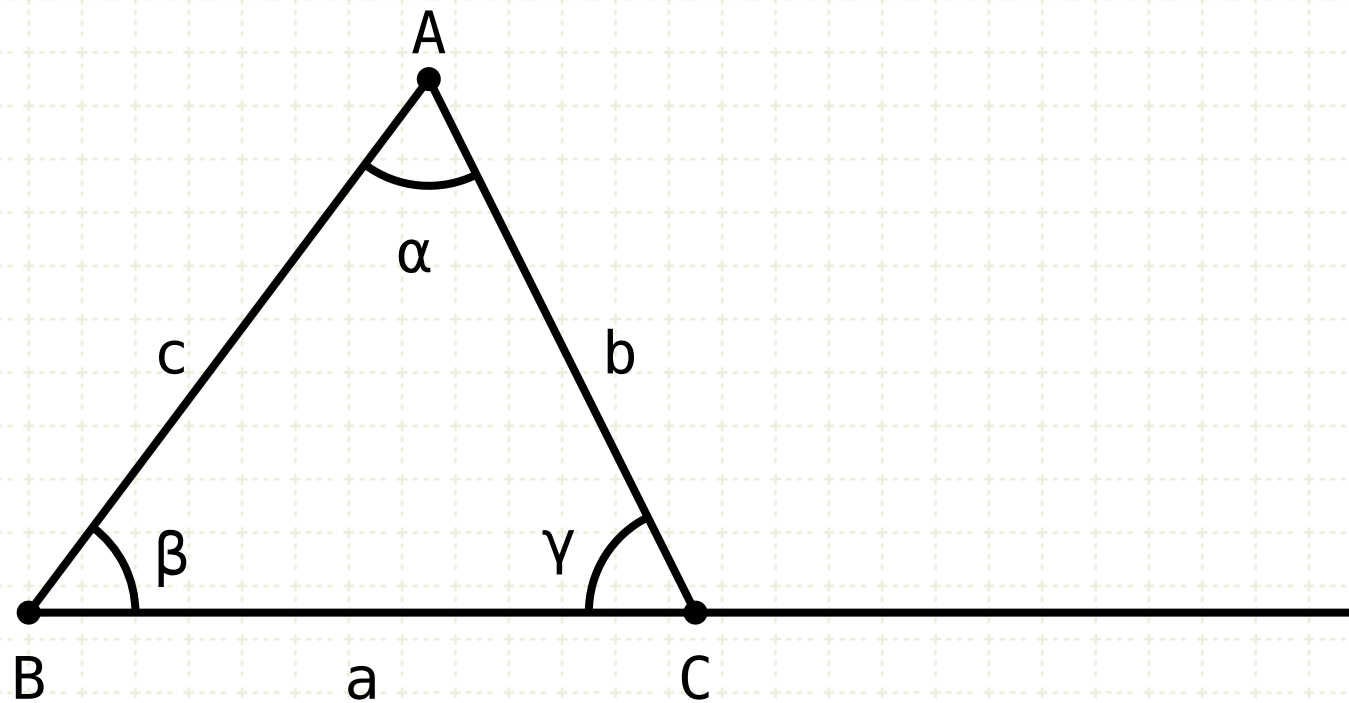
$$AC + AB > BC \quad (b + c > a)$$

$$BC + AC > AB \quad (a + b > c)$$

$$AB + BC > AC \quad (c + a > b)$$

Proposition 20 of Book I

Any two sides of a triangle are together greater than the third side.



In other words

Given a triangle ABC

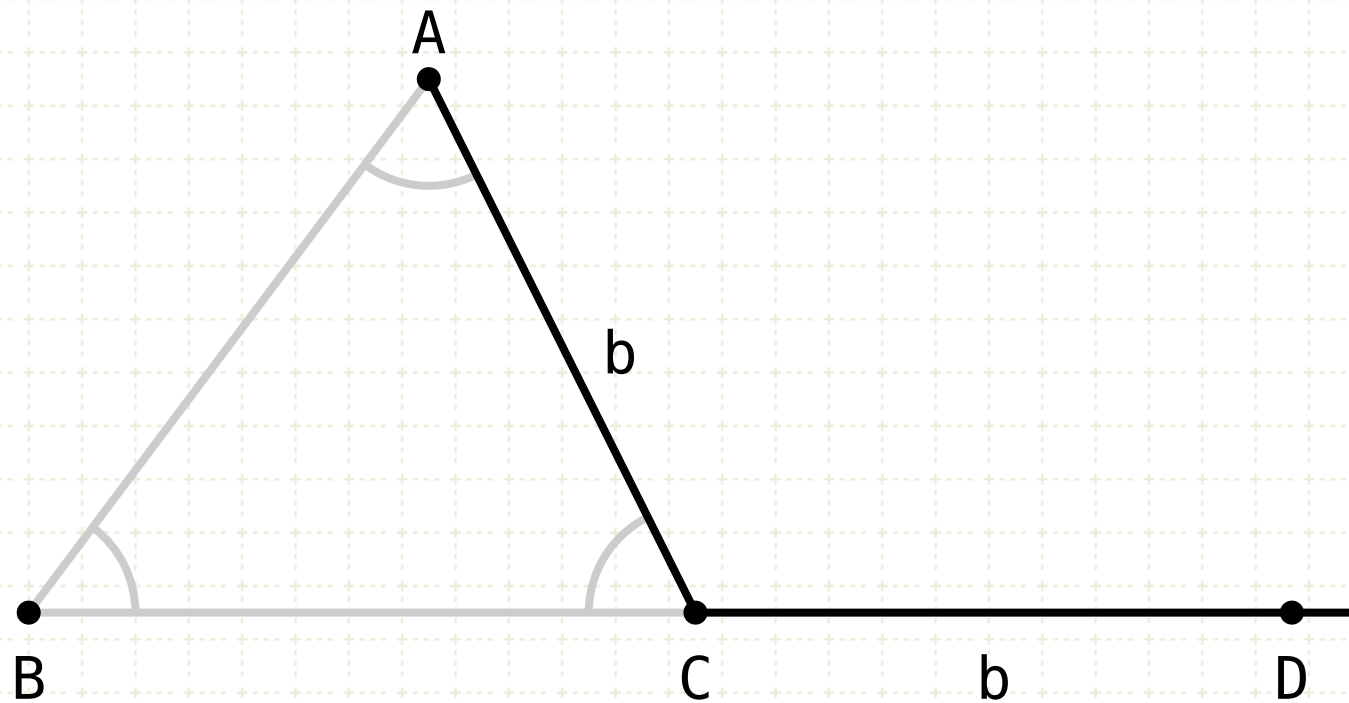
The sum of any two sides of the triangle is greater than the third

Proof

Extend BC

Proposition 20 of Book I

Any two sides of a triangle are together greater than the third side.



In other words

Given a triangle ABC

The sum of any two sides of the triangle is greater than the third

Proof

Extend BC

Define point D, such that CD equals AC

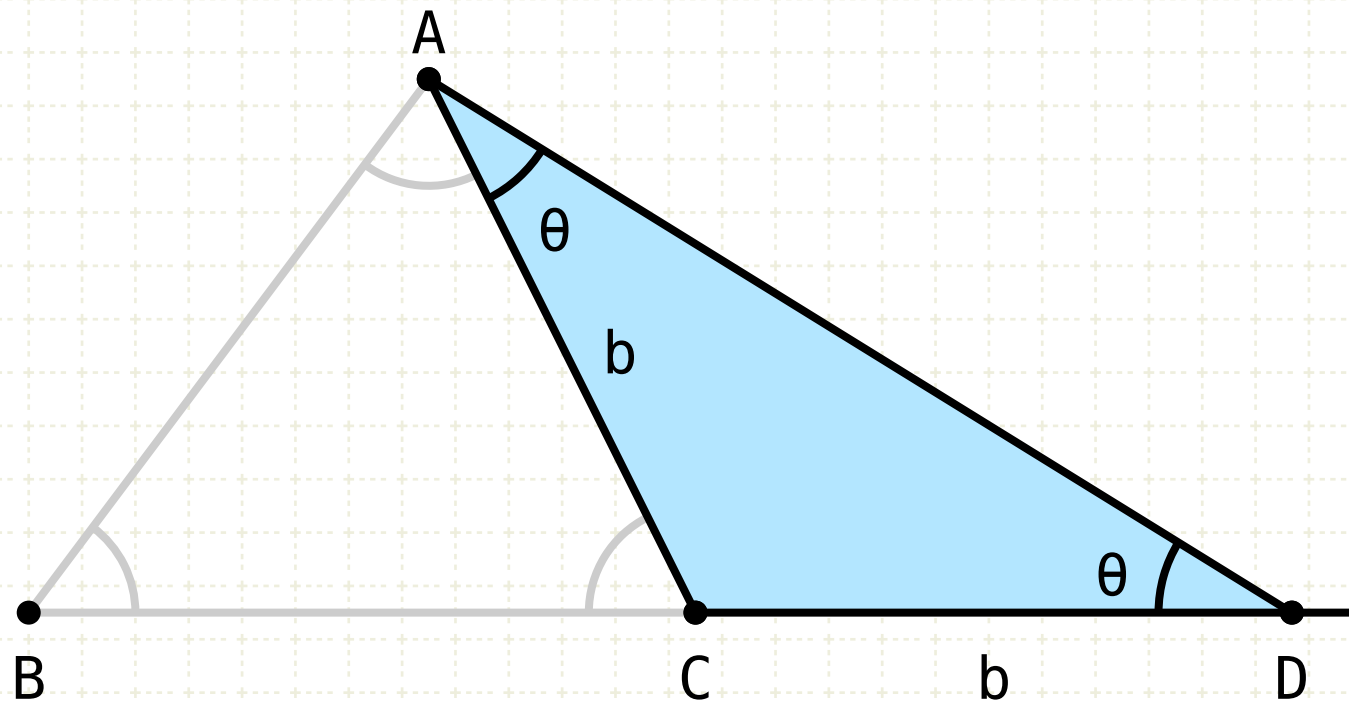
$$CD = AC$$

$$BD = BC + CD$$

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Proposition 20 of Book I

Any two sides of a triangle are together greater than the third side.



$$CD = AC$$

$$BD = BC + CD$$

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$$\angle CAD = \angle CDA = \theta$$

In other words

Given a triangle ABC

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Proof

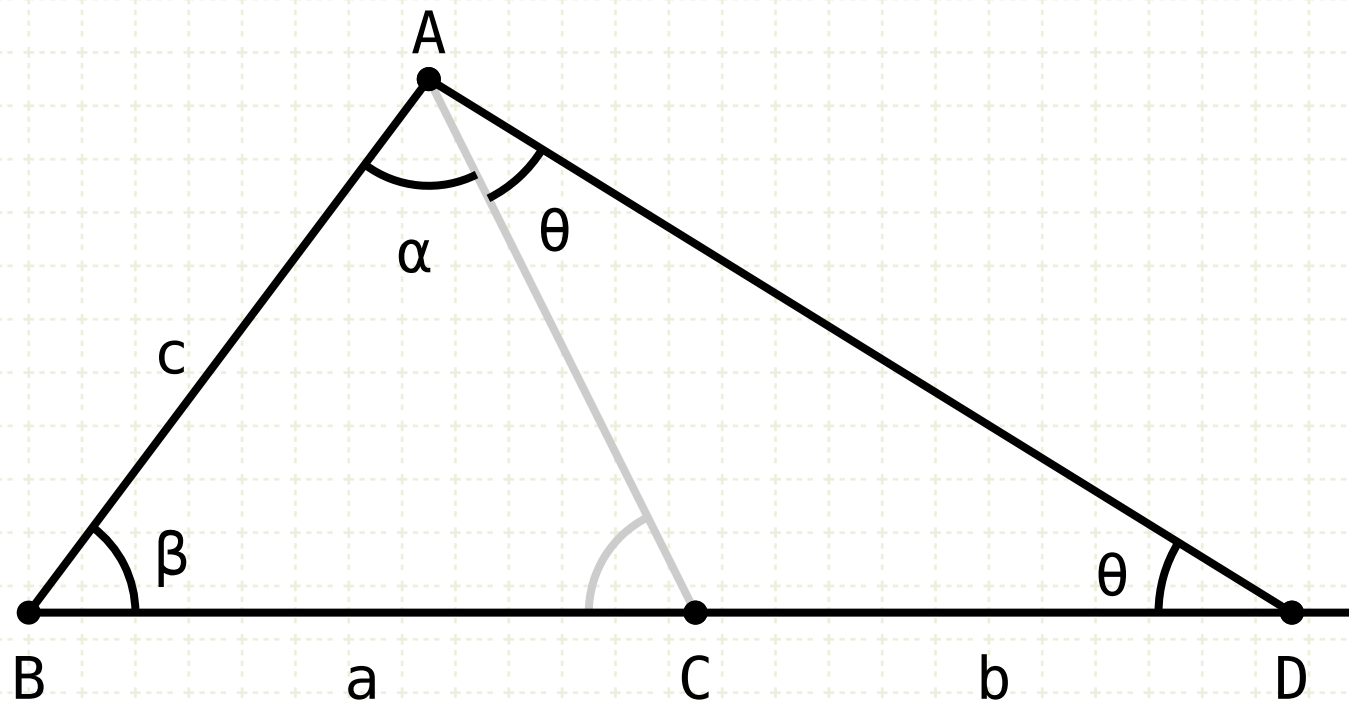
Extend BC

Define point D, such that CD equals AC

Create line AD, making the triangle ACD an isosceles triangle, thus the angles CAD and CDA are equal (I·5)

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$$CD = AC$$

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$$\angle CAD = \angle CDA = \theta$$

$$\alpha + \theta > \theta$$

$$\angle BAD > \angle BDA$$

$$\therefore BD > AB$$

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Define point D, such that CD equals AC

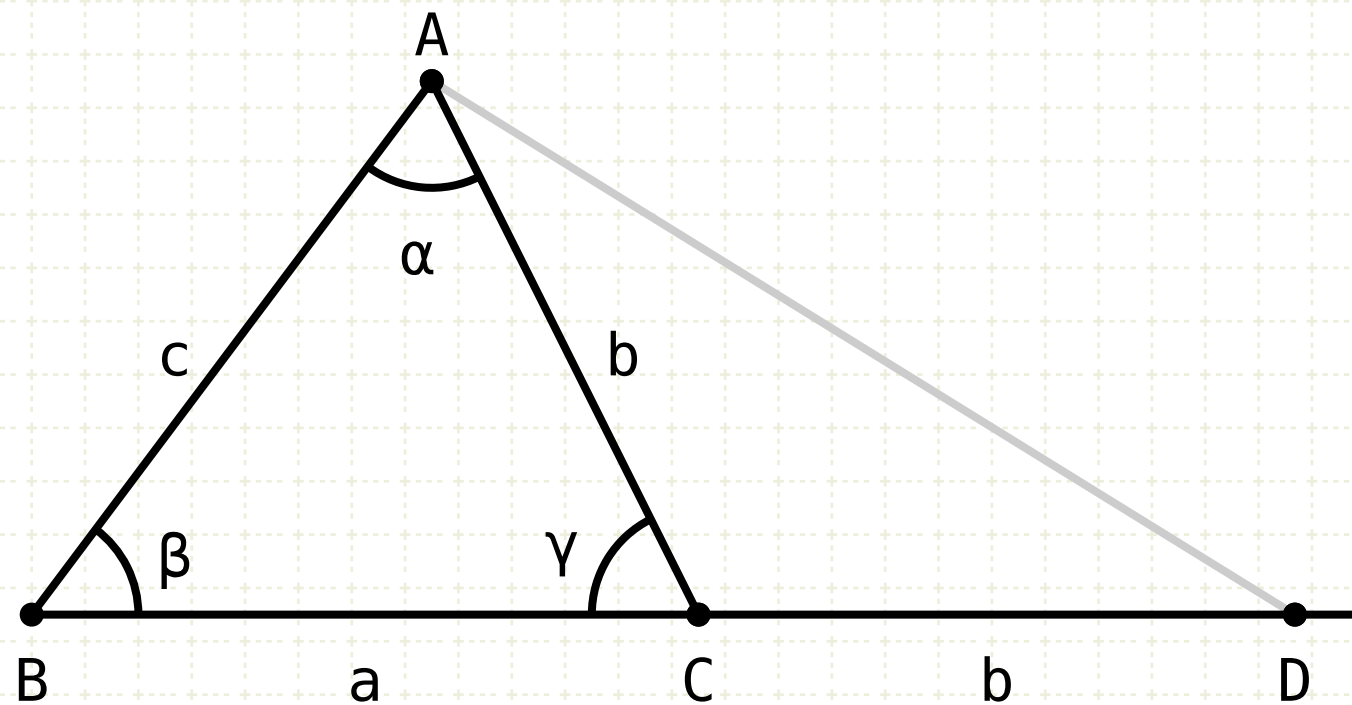
Create line AD, making the triangle ACD an isosceles triangle, thus the angles CAD and CDA are equal (I·5)

Consider triangle ABD

Angle BAD is obviously larger than angle BDA, thus length BD is larger than AB (I·18)

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Angle BAD is obviously larger than angle BDA, thus length BD is larger than AB (I·18)

But BD is the sum of BC and AC

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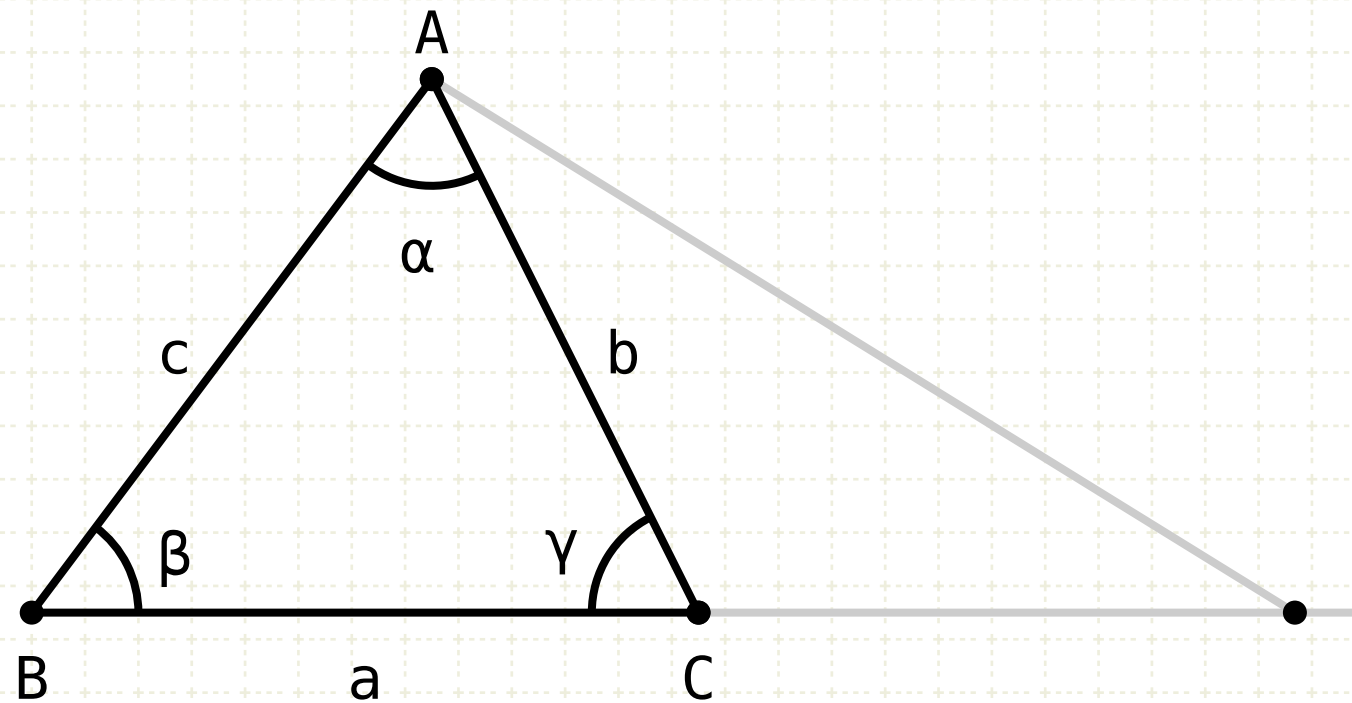
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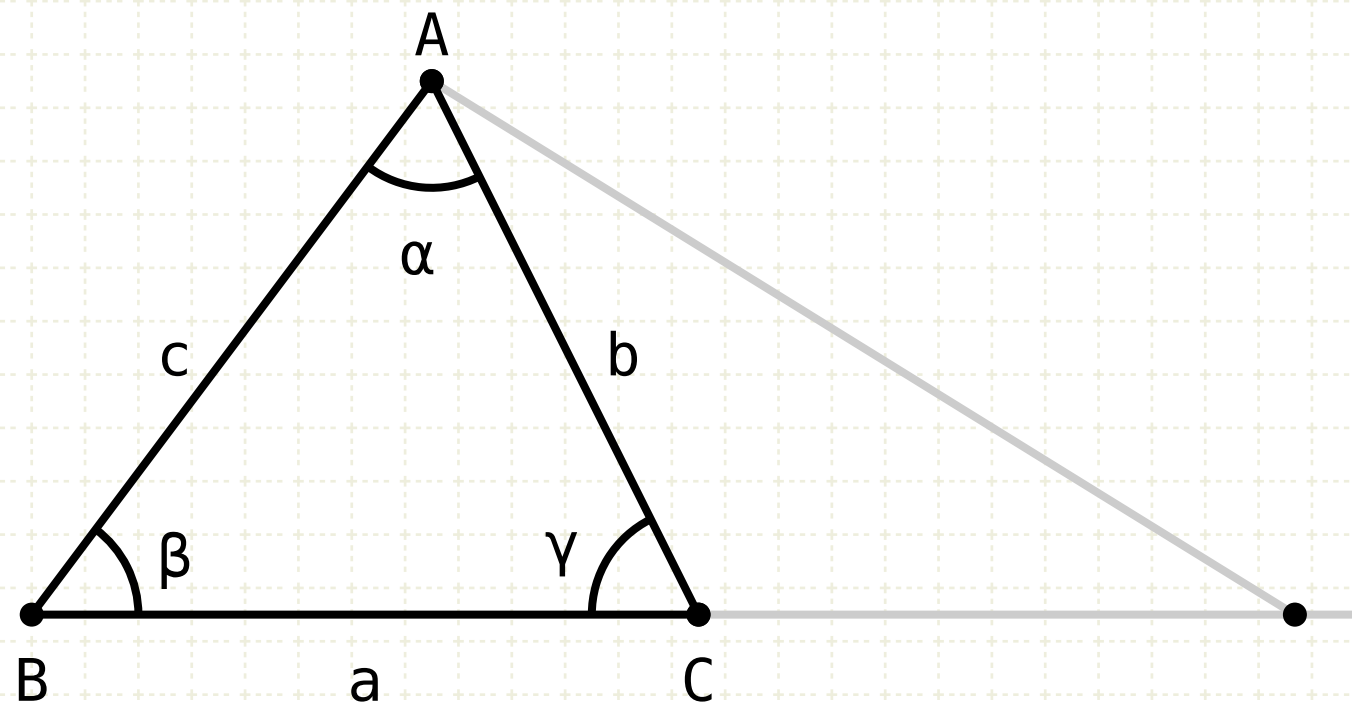
Angle BAD is obviously larger than angle BDA, thus length BD is larger than AB (I·18)

But BD is the sum of BC and AC

Thus BD, one side of a triangle, is less than the sum of the other two sides

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Consider triangle ABD

Angle BAD is obviously larger than angle BDA, thus length BD is larger than AB (I·18)

But BD is the sum of BC and AC

Thus BD, one side of a triangle, is less than the sum of the other two sides

This procedure can be used on any side of the triangle, hence the sum of two sides is always greater than the third



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