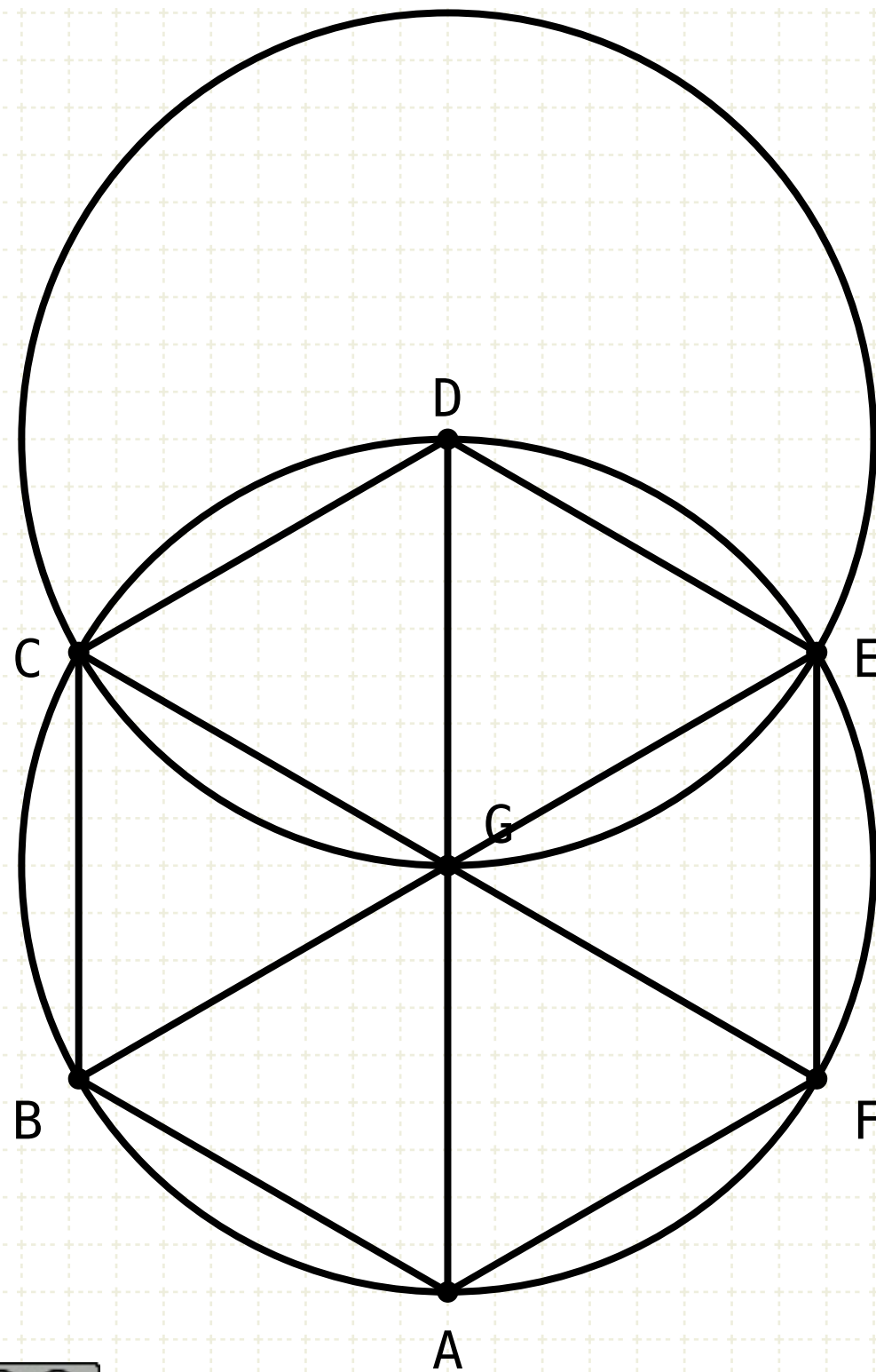


# Euclid's Elements

## Book IV



Philosophy (nature) is written in that great book which ever is before our eyes -- I mean the universe -- but we cannot understand it if we do not first learn the language and grasp the symbols in which it is written. The book is written in mathematical language, and the symbols are triangles, circles and other geometrical figures, without whose help it is impossible to comprehend a single word of it - without which one wanders in vain through a dark labyrinth.

**Galileo Galilei**



# Proposition 5 of Book IV

About a given triangle to circumscribe a circle.



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8	In a given square, to inscribe a circle		
9	About a given square, to circumscribe a circle		
10	To construct an isosceles triangle having each of the angles at the base double of the remaining one		



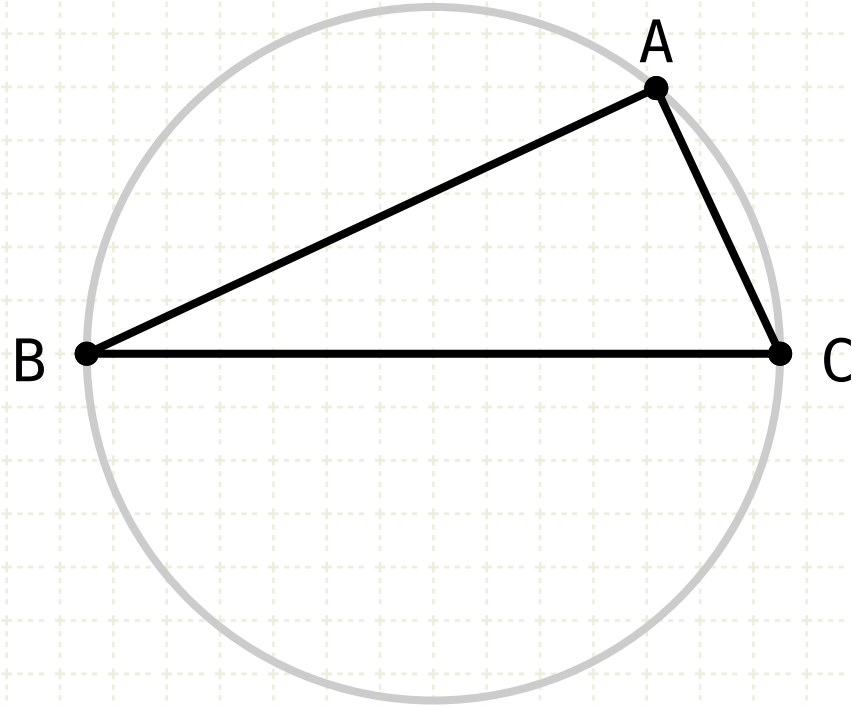
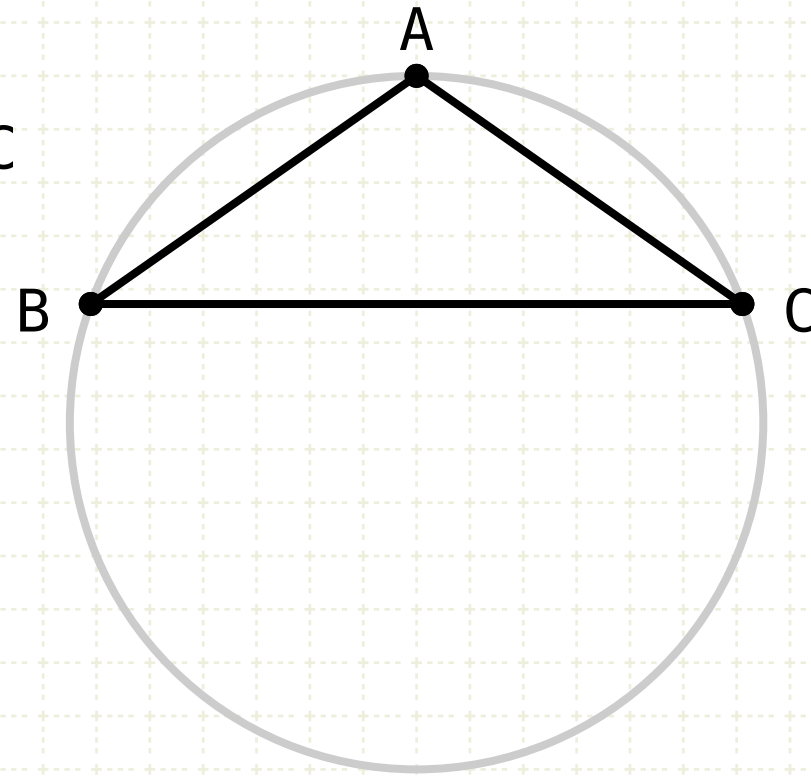
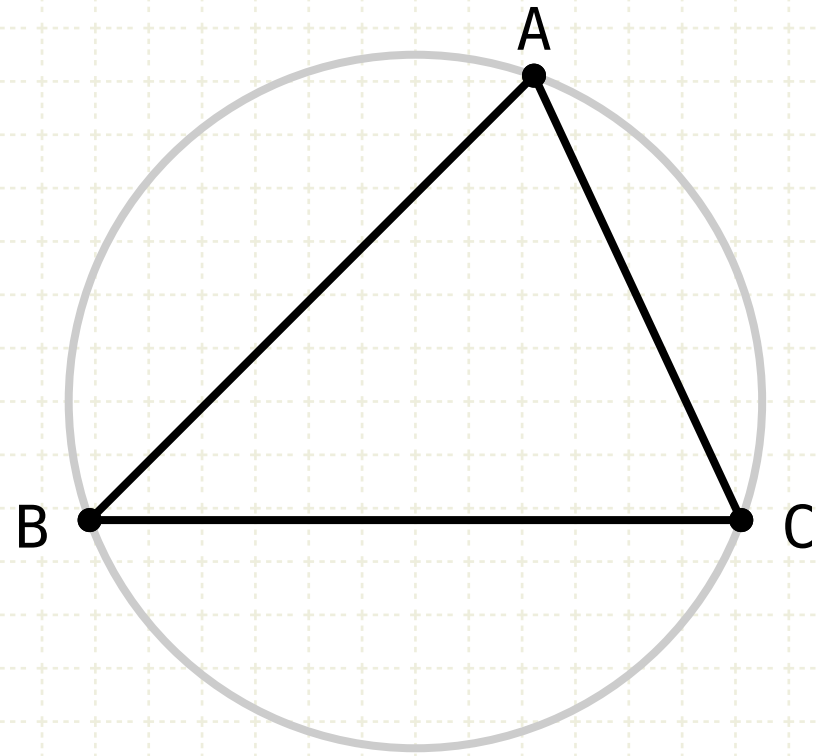
# Proposition 5 of Book IV

About a given triangle to circumscribe a circle.



## Proposition 5 of Book IV

About a given triangle to circumscribe a circle.



### In other words

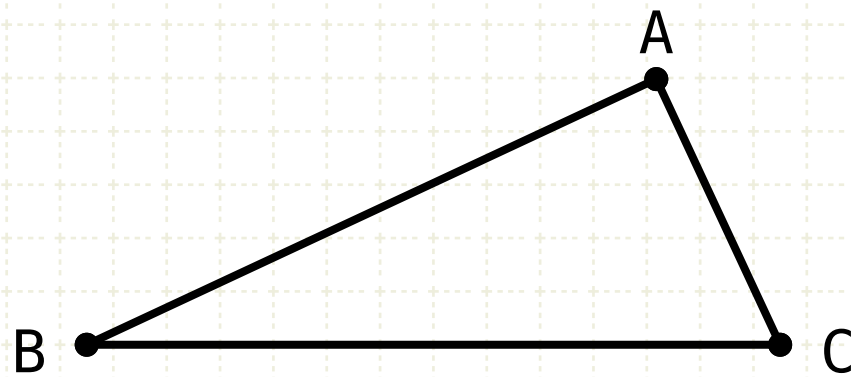
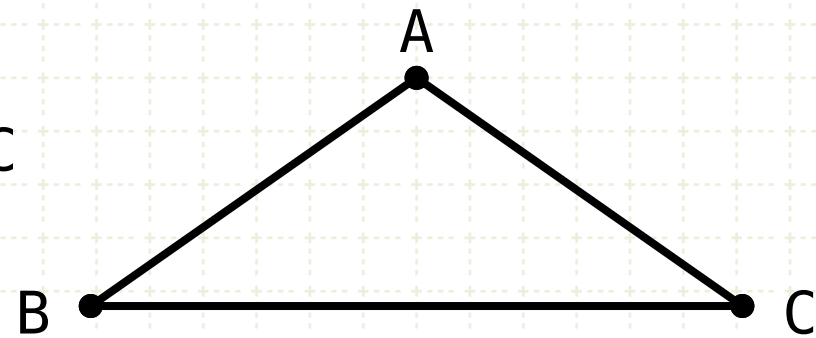
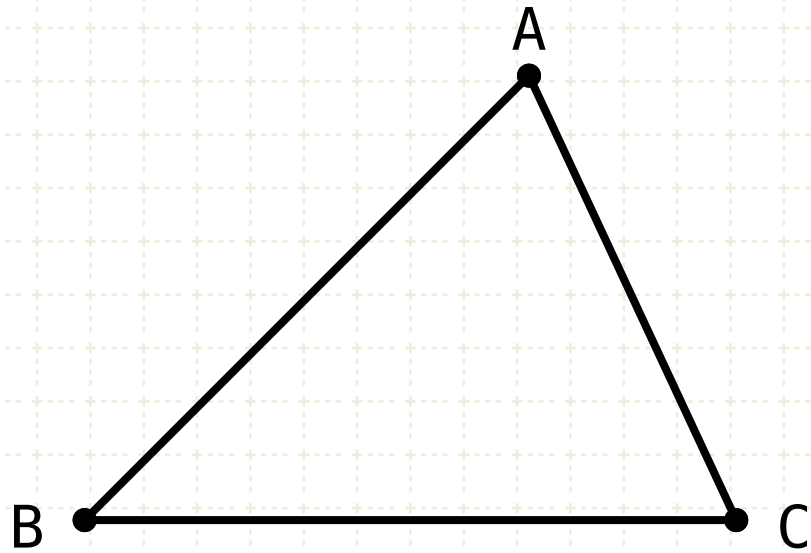
Given a triangle ABC

Draw a circle so that it passes through each vertex of the triangle

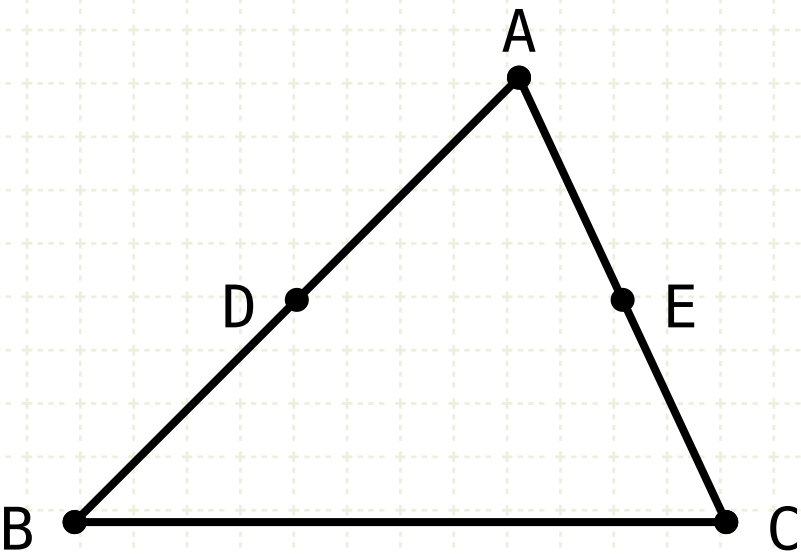
# Proposition 5 of Book IV

About a given triangle to circumscribe a circle.

## Construction





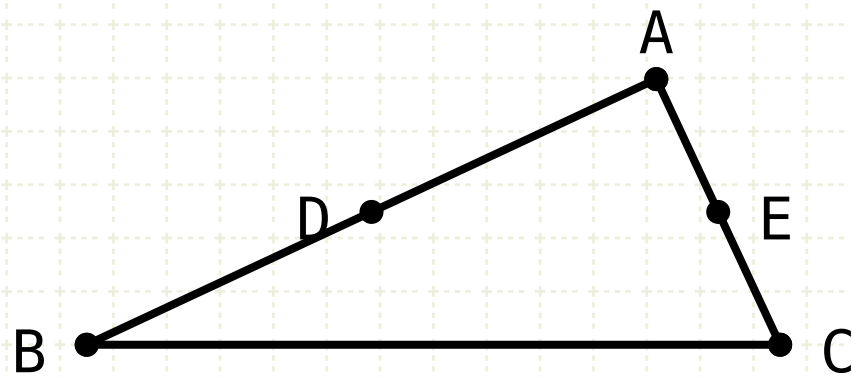
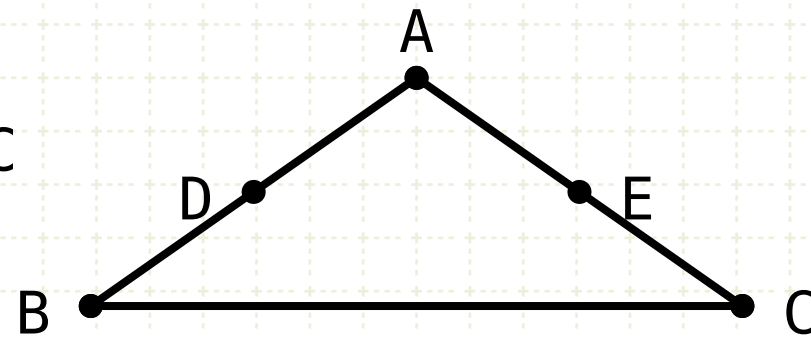


## Proposition 5 of Book IV

About a given triangle to circumscribe a circle.

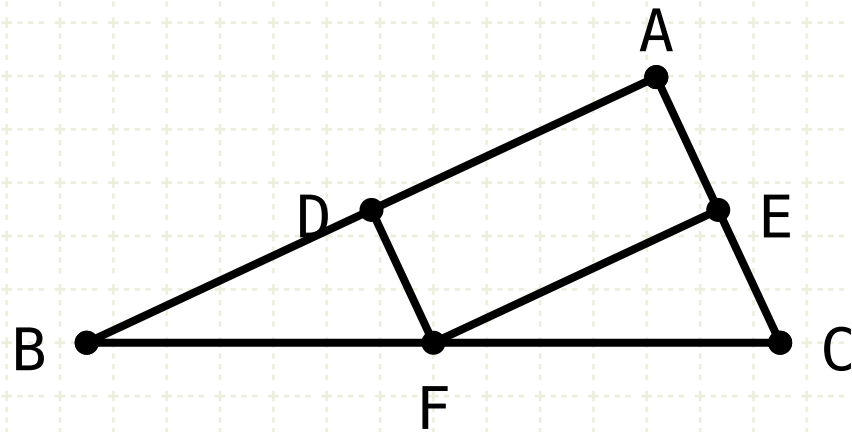
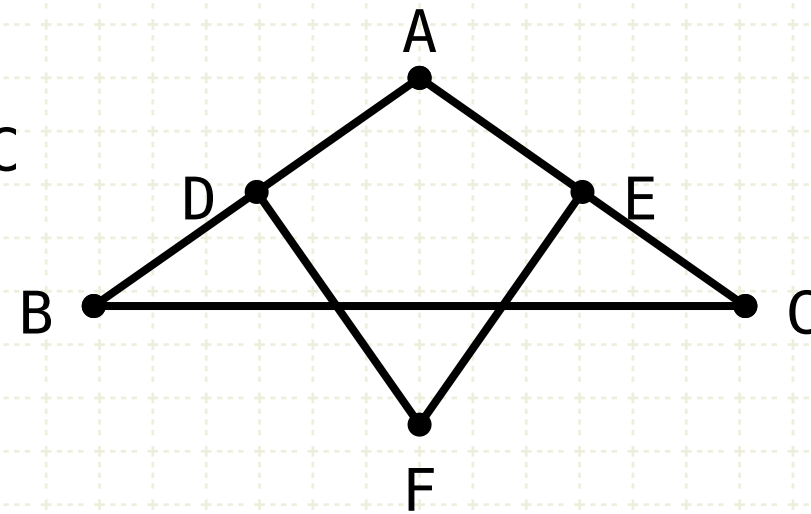
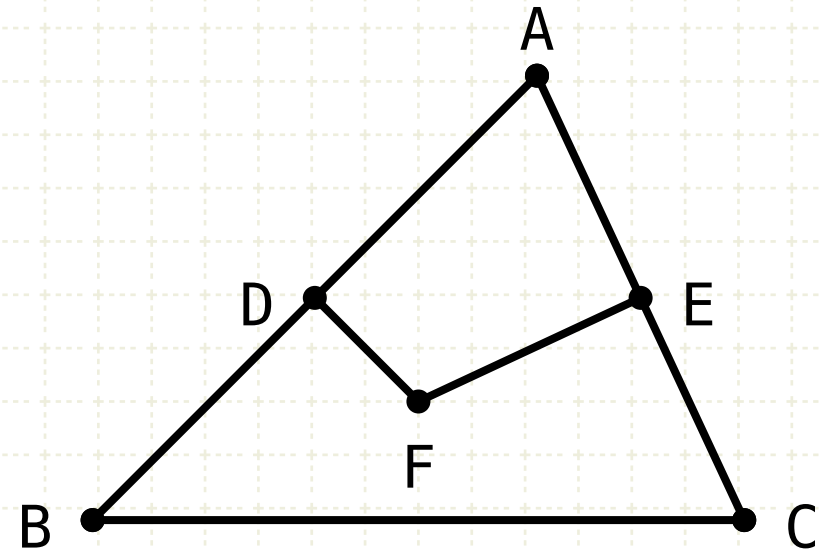
### Construction

Bisect line AB at point D and line AC at point E (I·10)



# Proposition 5 of Book IV

About a given triangle to circumscribe a circle.



## Construction

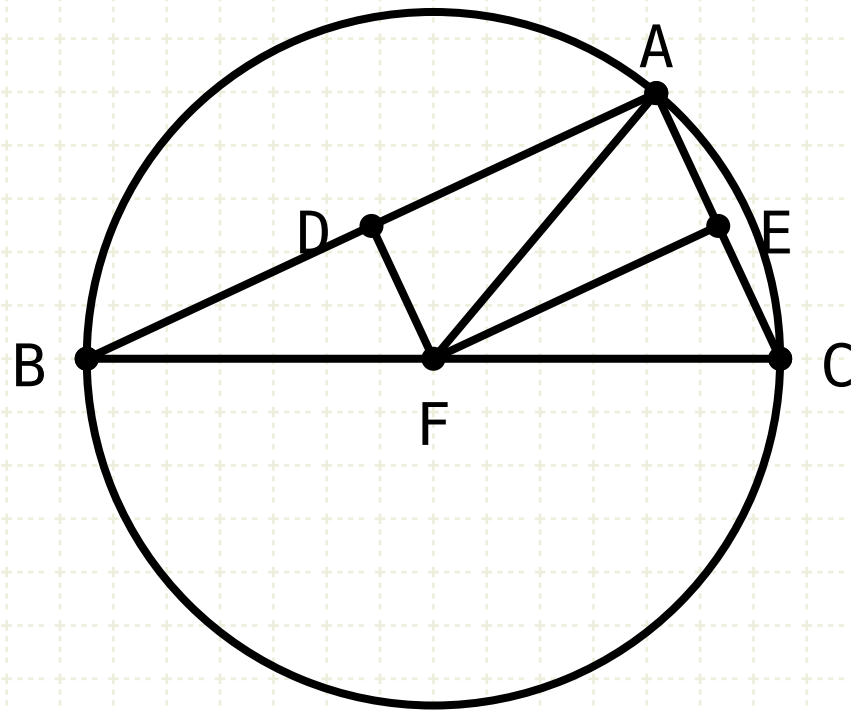
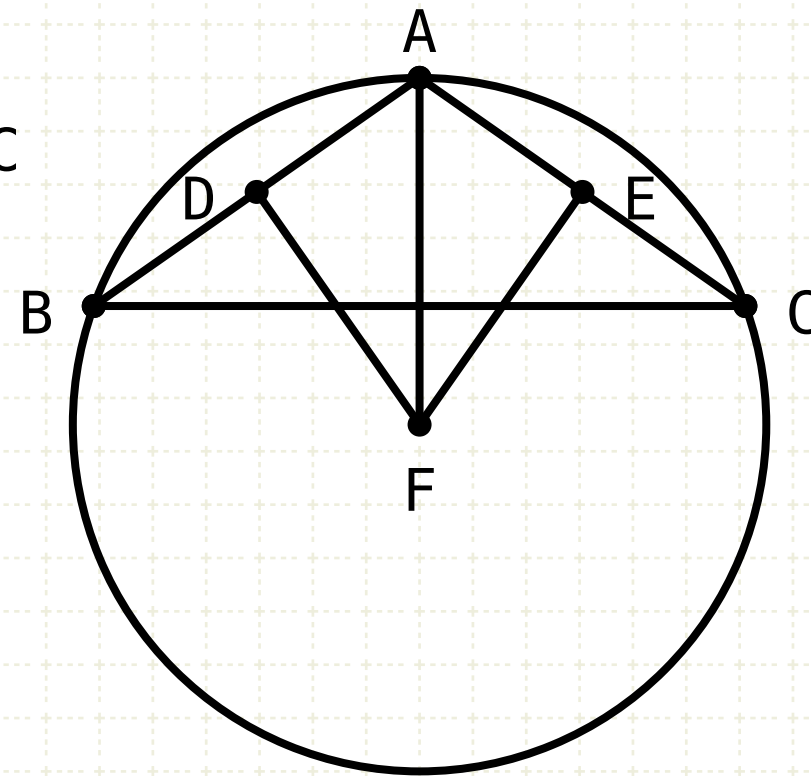
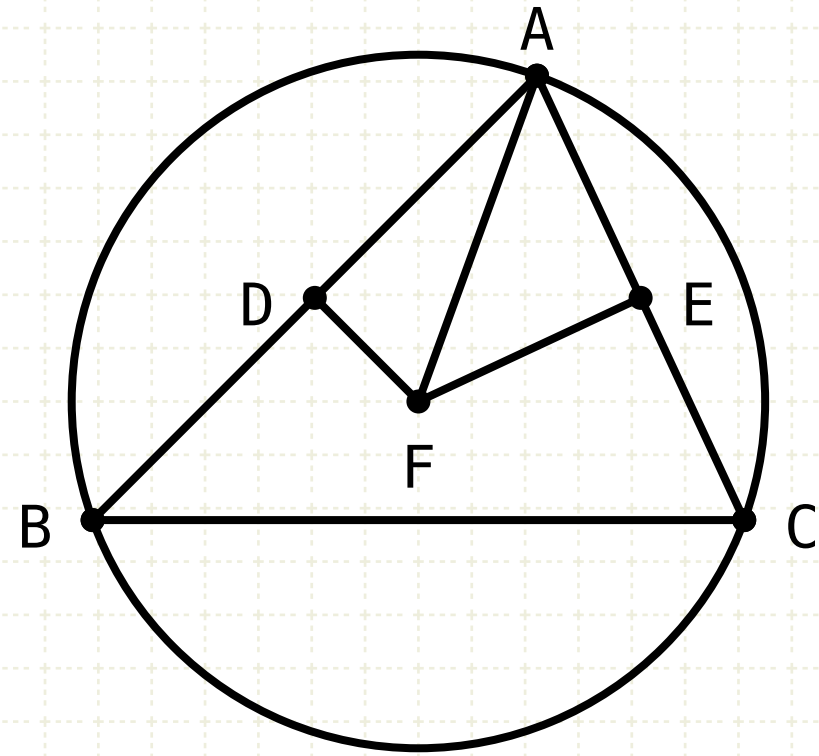
Bisect line AB at point D and line AC at point E (I·10)

Draw lines from the points D,E perpendicular to their respective sides of the triangle, intersecting at point F (I·11)



## Proposition 5 of Book IV

About a given triangle to circumscribe a circle.



### Construction

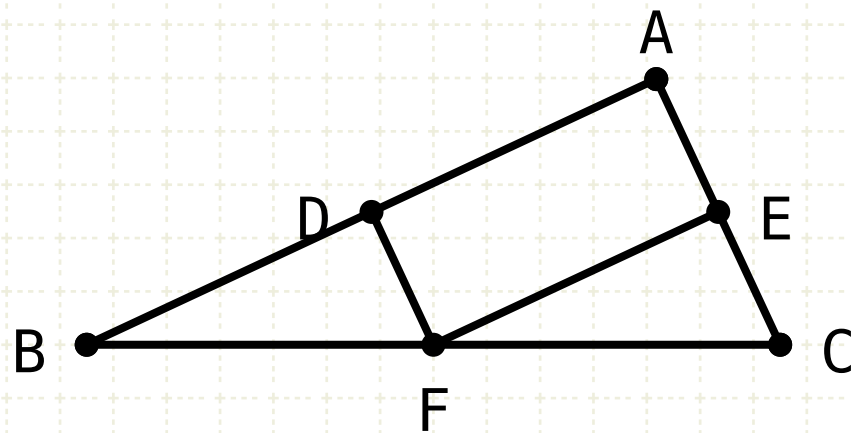
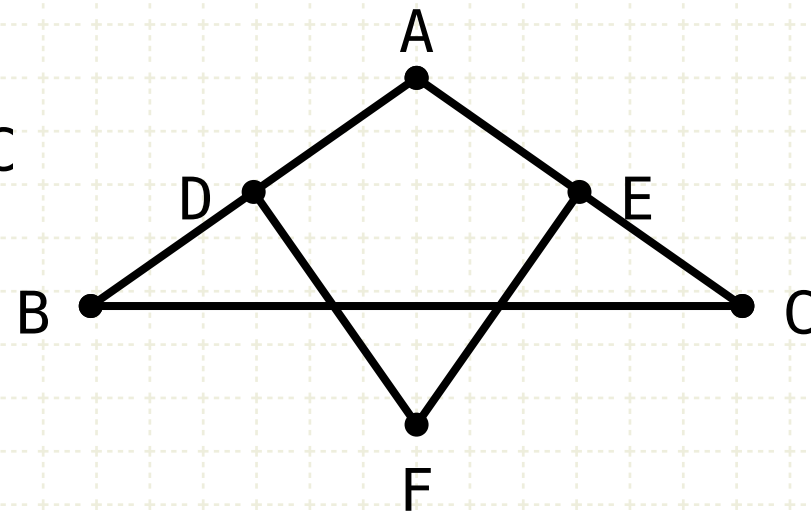
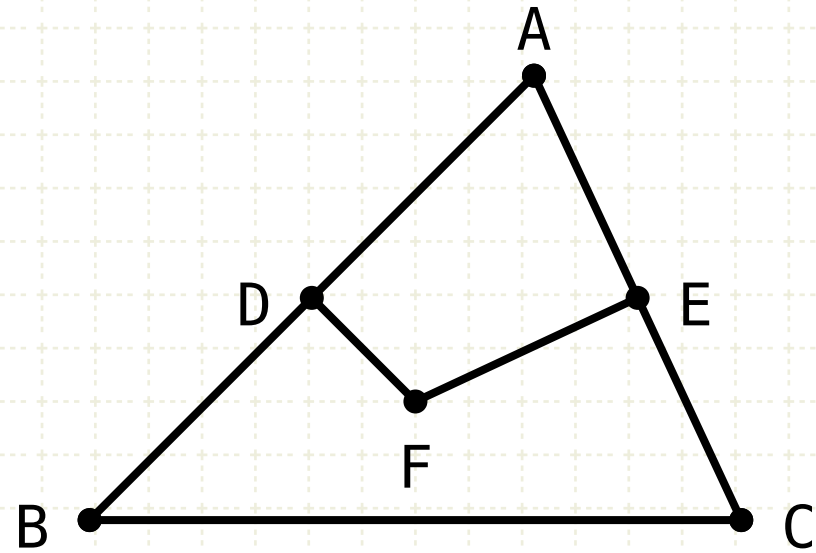
Bisect line AB at point D and line AC at point E (I·10)

Draw lines from the points D,E perpendicular to their respective sides of the triangle, intersecting at point F (I·11)

With F as the centre, and AF as the radius, it is possible to draw a circle that passes through each point A, B and C

# Proposition 5 of Book IV

About a given triangle to circumscribe a circle.



## Construction

Bisect line AB at point D and line AC at point E (I·10)

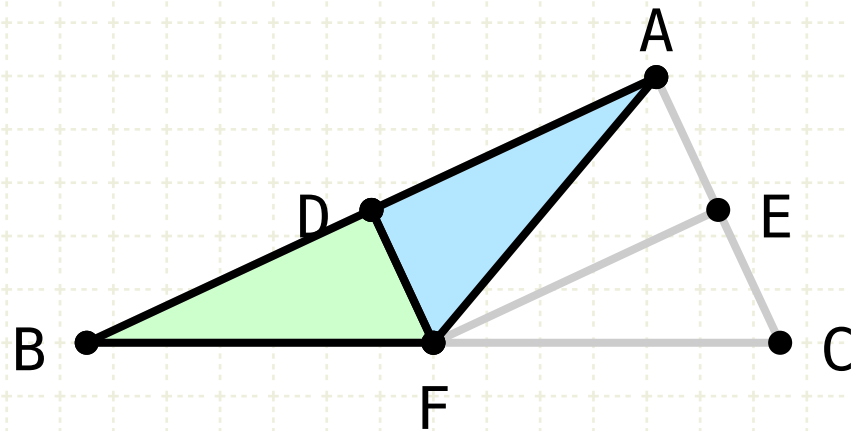
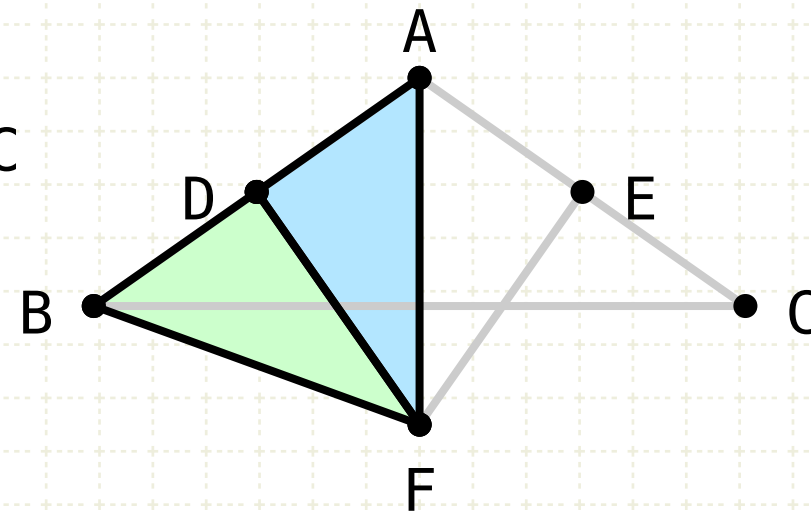
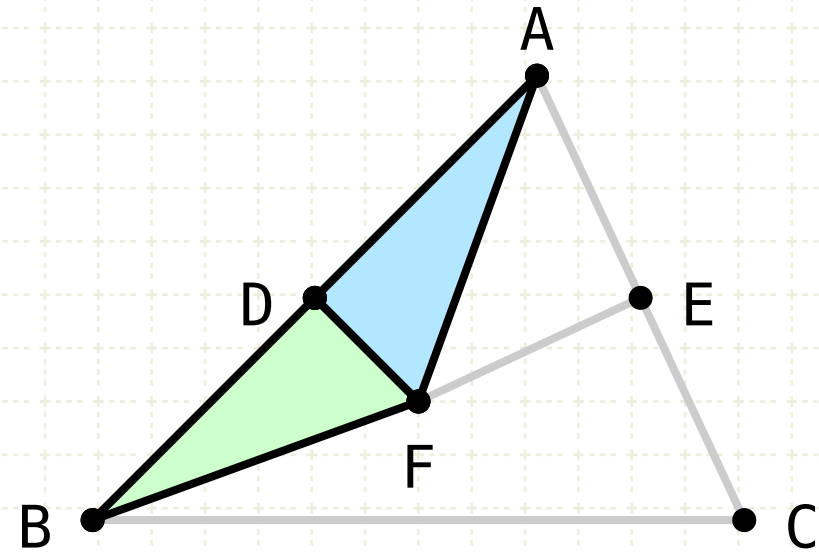
Draw lines from the points D,E perpendicular to their respective sides of the triangle, intersecting at point F (I·11)

With F as the centre, and AF as the radius, it is possible to draw a circle that passes through each point A, B and C

## Proof

# Proposition 5 of Book IV

About a given triangle to circumscribe a circle.



$BD = AD$   
 $\angle ADF = \angle BDF = \angle$   
 $DF$  is common

## Construction

Bisect line AB at point D and line AC at point E (I·10)

Draw lines from the points D,E perpendicular to their respective sides of the triangle, intersecting at point F (I·11)

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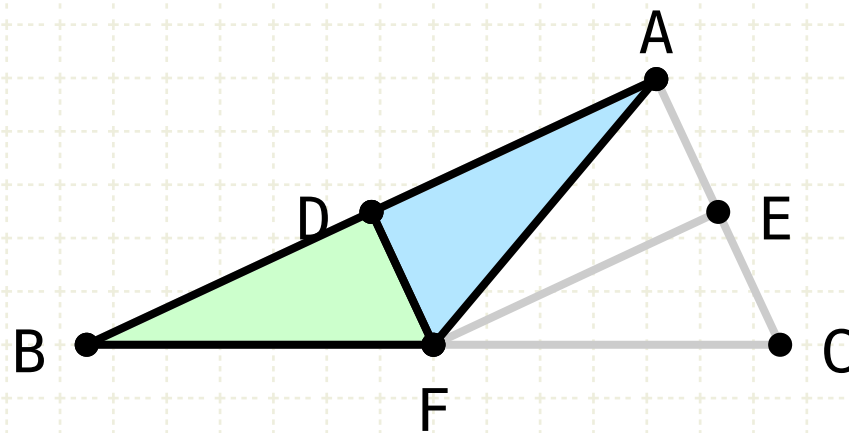
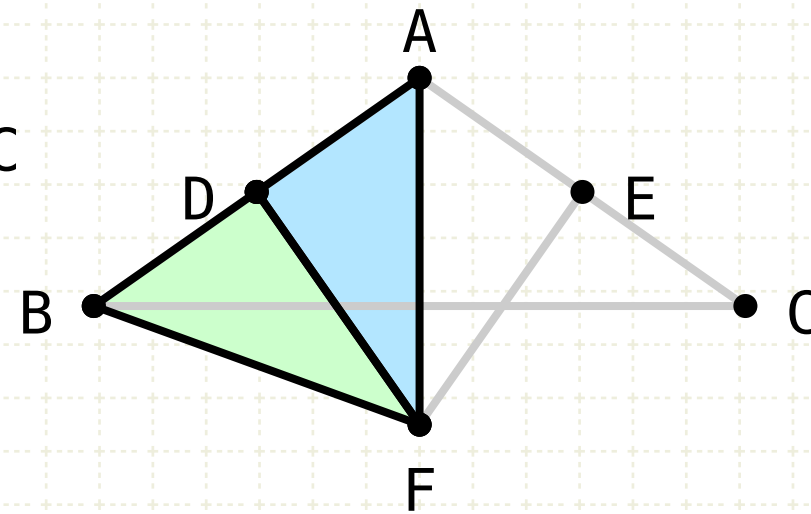
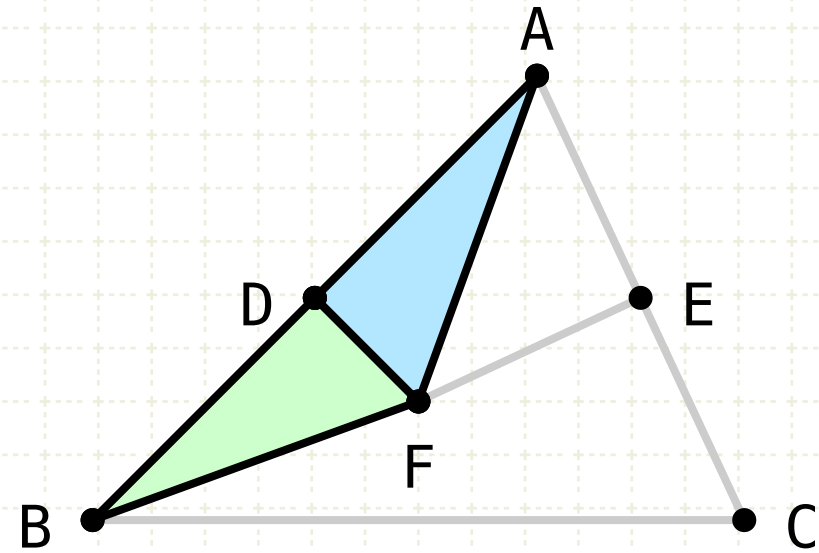
## Proof

Draw lines BF and AF

The two triangles ADF and BDF are equal in all respects, since they have a side (BD,AD), angle ( $\angle ADF = \angle BDF = \angle$ ), side (DF) equal (I·4)

# Proposition 5 of Book IV

About a given triangle to circumscribe a circle.



$BD = AD$   
 $\angle ADF = \angle BDF = \angle$   
 $DF$  is common  
 $\therefore BF = AF$

## Construction

Bisect line AB at point D and line AC at point E (I·10)

Draw lines from the points D,E perpendicular to their respective sides of the triangle, intersecting at point F (I·11)

With F as the centre, and AF as the radius, it is possible to draw a circle that passes through each point A, B and C

## Proof

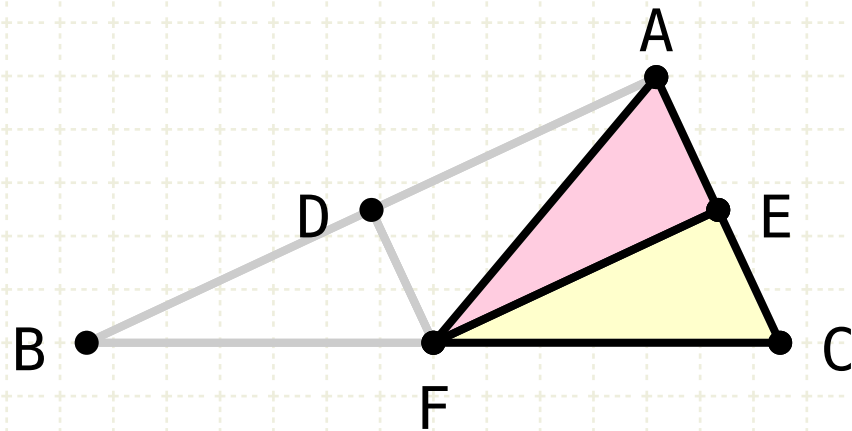
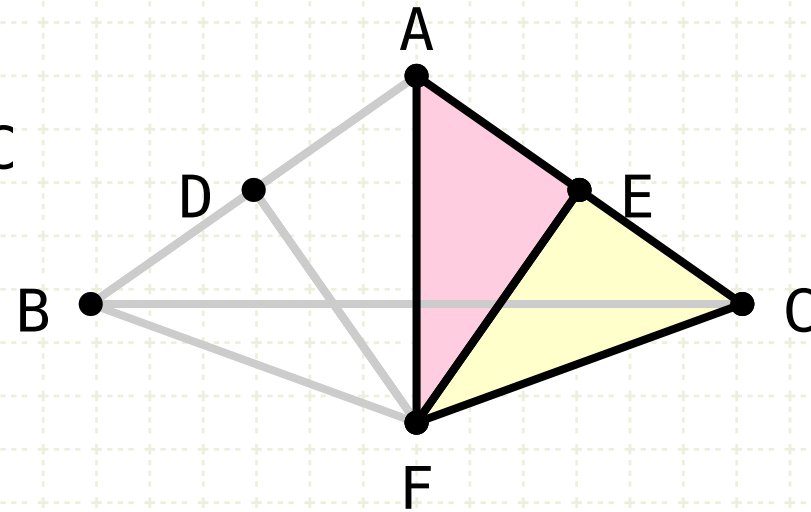
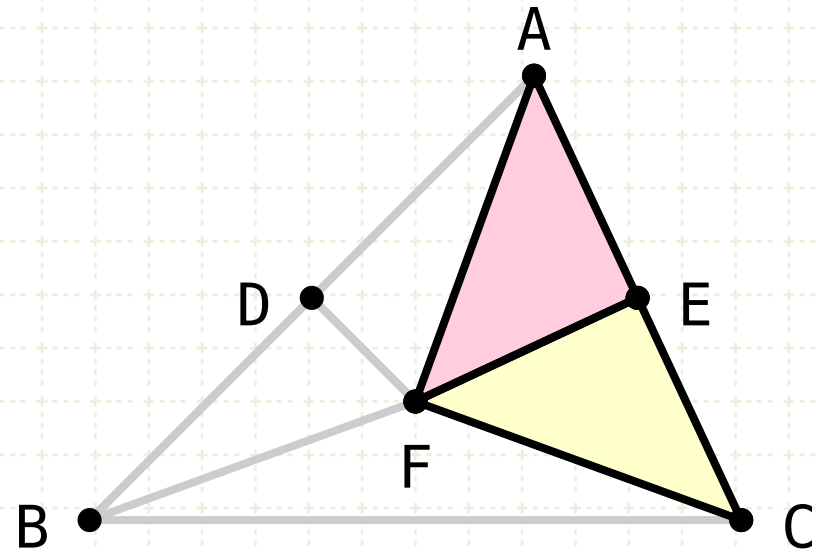
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Hence BF equals AF

# Proposition 5 of Book IV

About a given triangle to circumscribe a circle.



## Construction

Bisect line AB at point D and line AC at point E (I·10)

Draw lines from the points D,E perpendicular to their respective sides of the triangle, intersecting at point F (I·11)

With F as the centre, and AF as the radius, it is possible to draw a circle that passes through each point A, B and C

## Proof

Draw lines BF and AF

The two triangles ADF and BDF are equal in all respects, since they have a side (BD,AD), angle ( $\angle ADF = \angle BDF = 90^\circ$ ), side (DF) equal (I·4)

Hence BF equals AF

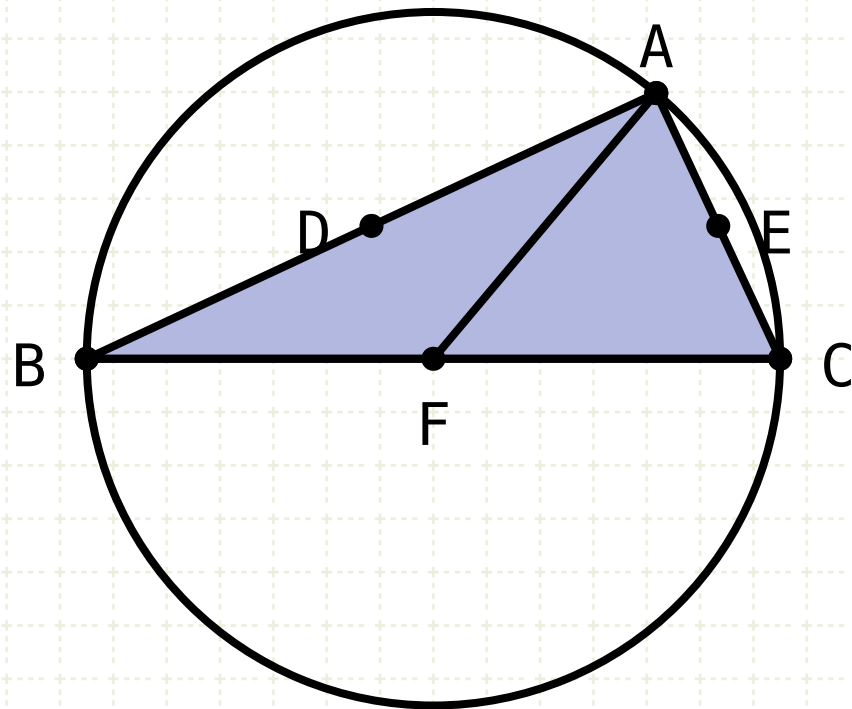
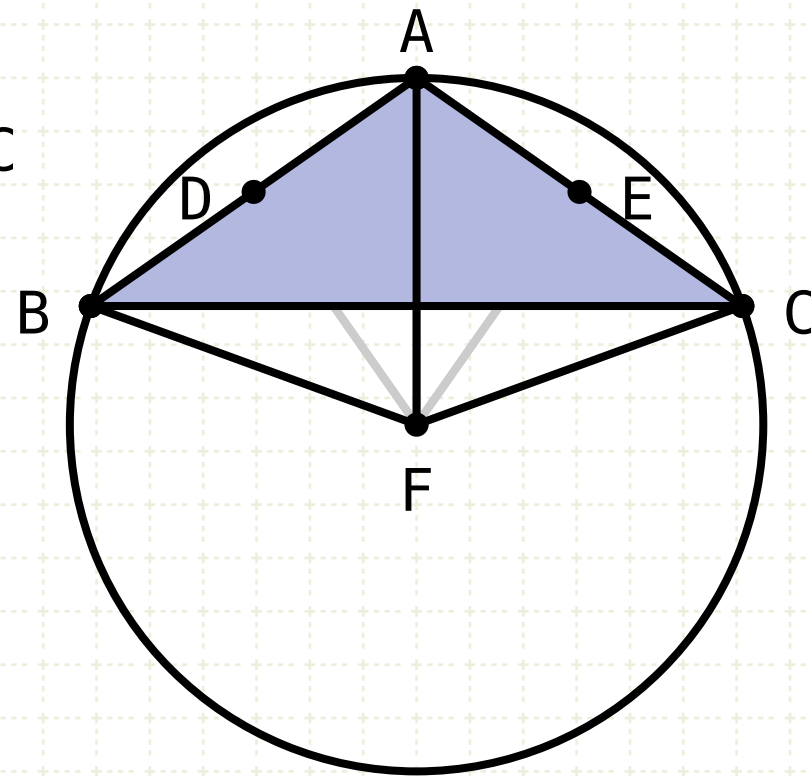
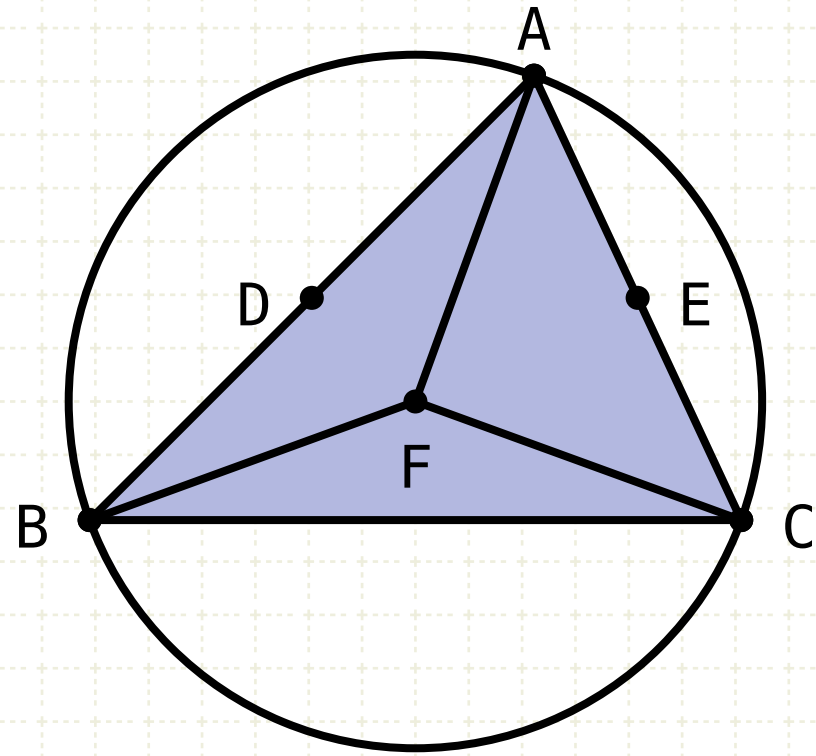
Similarly, it can be shown that AF is equal to CF

$$\begin{aligned} BD &= AD \\ \angle ADF &= \angle BDF = 90^\circ \\ DF &\text{ is common} \\ \therefore BF &= AF \\ AF &= CF \end{aligned}$$



# Proposition 5 of Book IV

About a given triangle to circumscribe a circle.



$$\begin{aligned} BD &= AD \\ \angle ADF &= \angle BDF = \text{L} \\ DF &\text{ is common} \\ \therefore BF &= AF \\ AF &= CF \\ BF &= AF = CF \end{aligned}$$

## Construction

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With F as the centre, and AF as the radius, it is possible to draw a circle that passes through each point A, B and C

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Hence BF equals AF

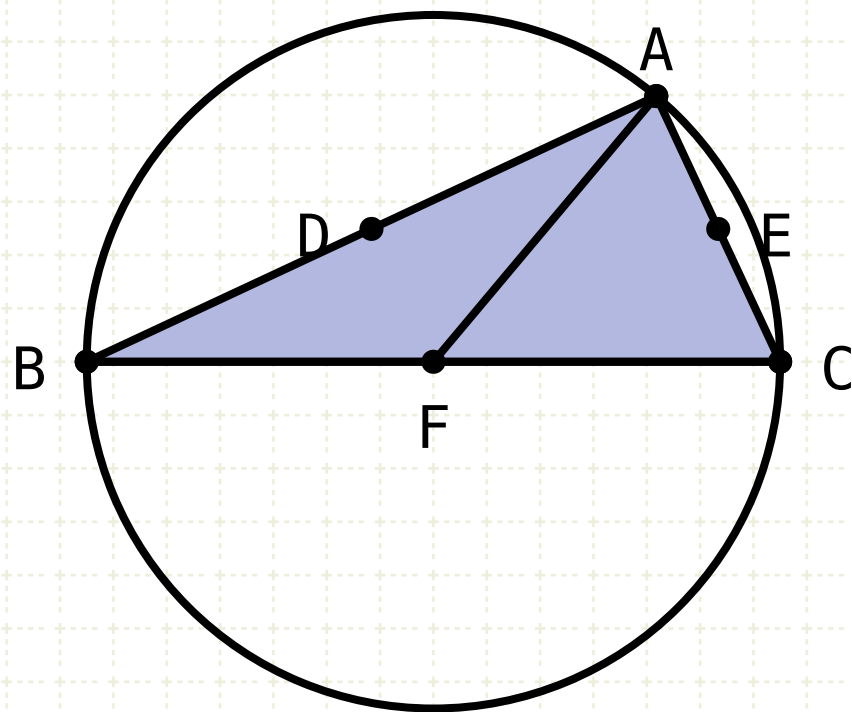
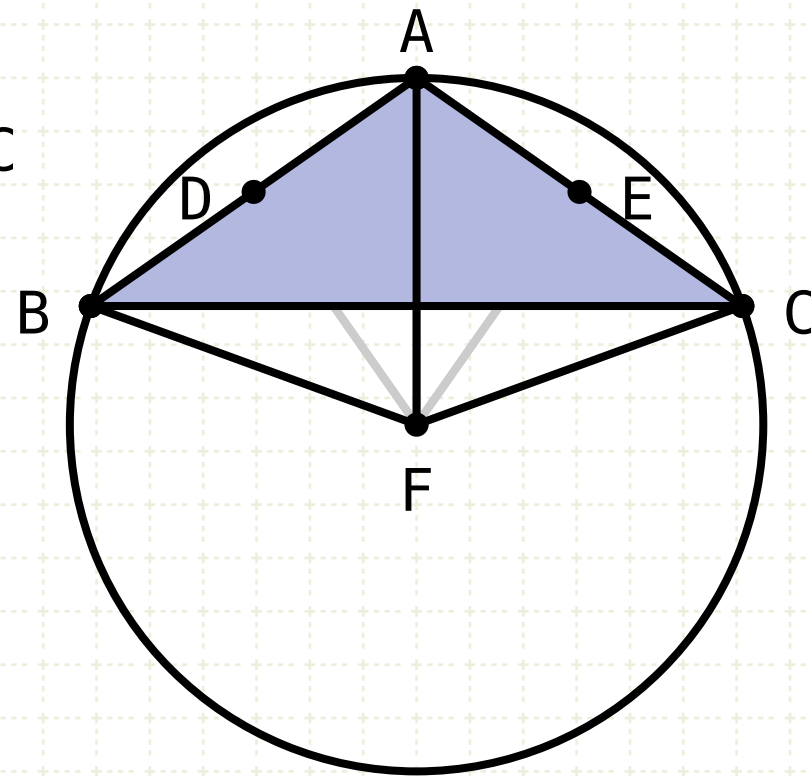
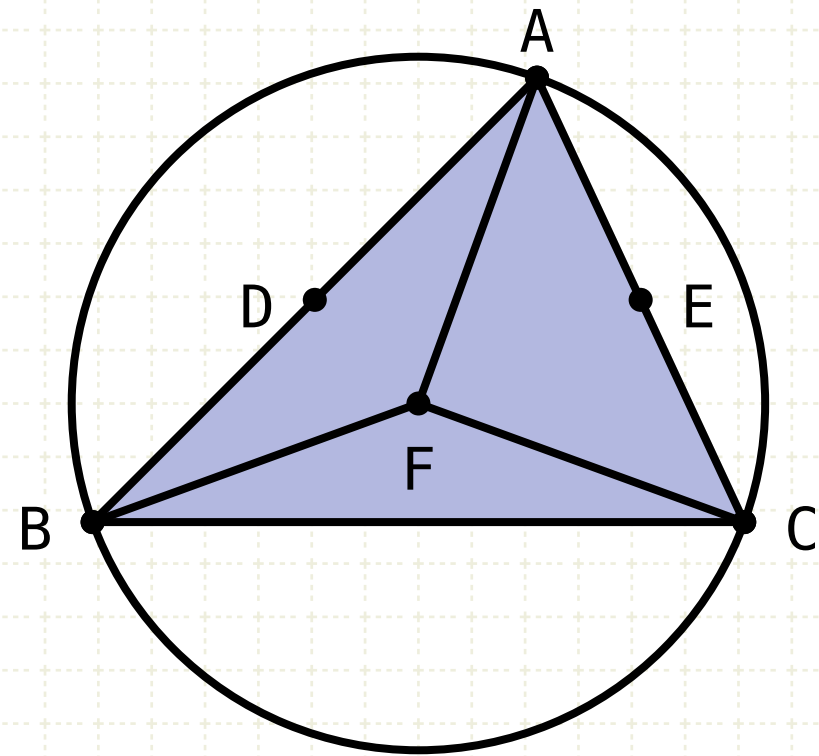
Similarly,it can be shown that AF is equal to CF

Since BF, AF and CF are all equal, a circle with the centre at F, with radius AF will pass through the points A, B, C



# Proposition 5 of Book IV

About a given triangle to circumscribe a circle.



$$\begin{aligned} BD &= AD \\ \angle ADF &= \angle BDF = \angle CEF \\ DF &\text{ is common} \\ \therefore BF &= AF \\ AF &= CF \\ BF &= AF = CF \end{aligned}$$

## Construction

Bisect line AB at point D and line AC at point E (I·10)

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The two triangles ADF and BDF are equal in all respects, since they have a side (BD,AD), angle ( $\angle ADF = \angle BDF = \angle$ ), side (DF) equal (I·4)

Hence BF equals AF

Similarly, it can be shown that AF is equal to CF

Since BF, AF and CF are all equal, a circle with the centre at F, with radius AF will pass through the points A, B, C

## Note: (III·31)

If the centre of the circle falls within the triangle, then the angle BAC is less than a right angle

If the centre of the circle falls on the line AC, then the angle BAC is less than a right angle

If the centre of the circle falls outside the triangle, then the angle BAC is greater than a right angle

# Youtube Videos

<https://www.youtube.com/c/SandyBultena>

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