

Euclid's Elements

Book III



A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)



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4	A line not through the centre of a circle does not bisect a chord	12	Point of contact between two external circles, and their centres, are collinear	19	If line touches a circle, then the centre of the circle lies on a line perpendicular to the original	
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6	If two circles touch one another, they will not have the same center	14	In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another.	21	In a circle the angles in the same segment are equal to one another	
7	Consider two lines from a point inside a circle to the edge, the longer one will be the one closest to the longest part of the diameter passing through the original point	15	The longest line in a circle is its diameter, shorter the farther away from the diameter	22	The opposite angles of quadrilaterals in circles are equal to two right angles	
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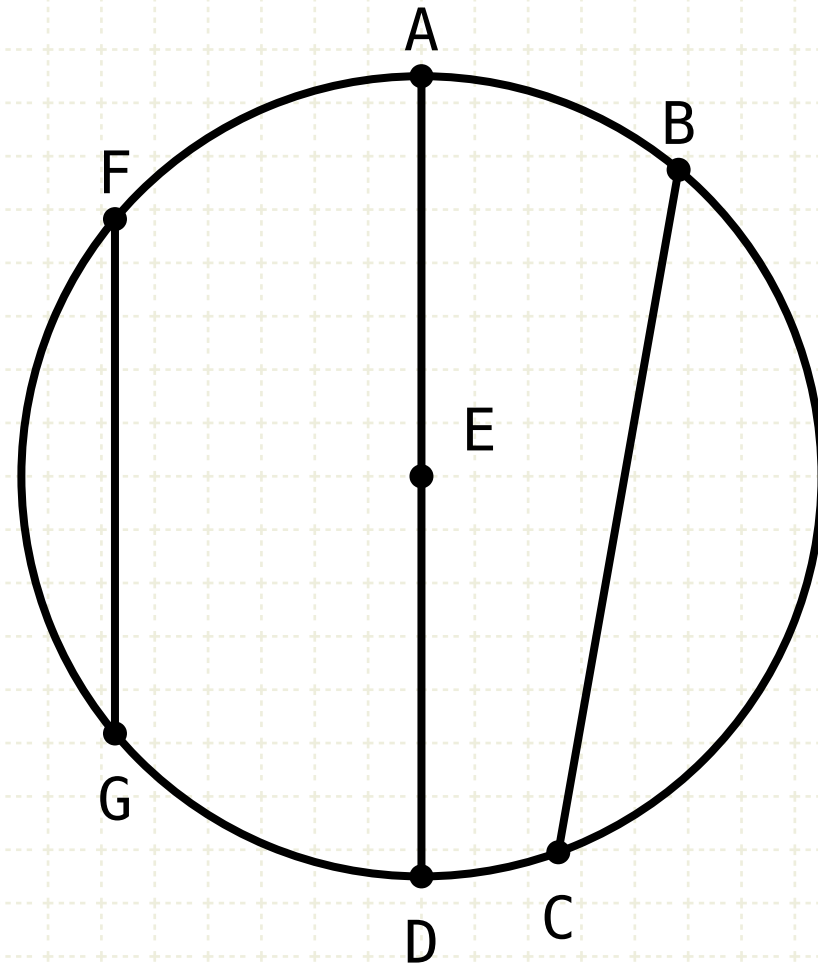
Proposition 15 of Book III

Of straight lines in a circle the diameter is greatest, and of the rest the nearer to the centre is always greater than the more remote.



Proposition 15 of Book III

Of straight lines in a circle the diameter is greatest, and of the rest the nearer to the centre is always greater than the more remote.



$$AD > BC > FG$$

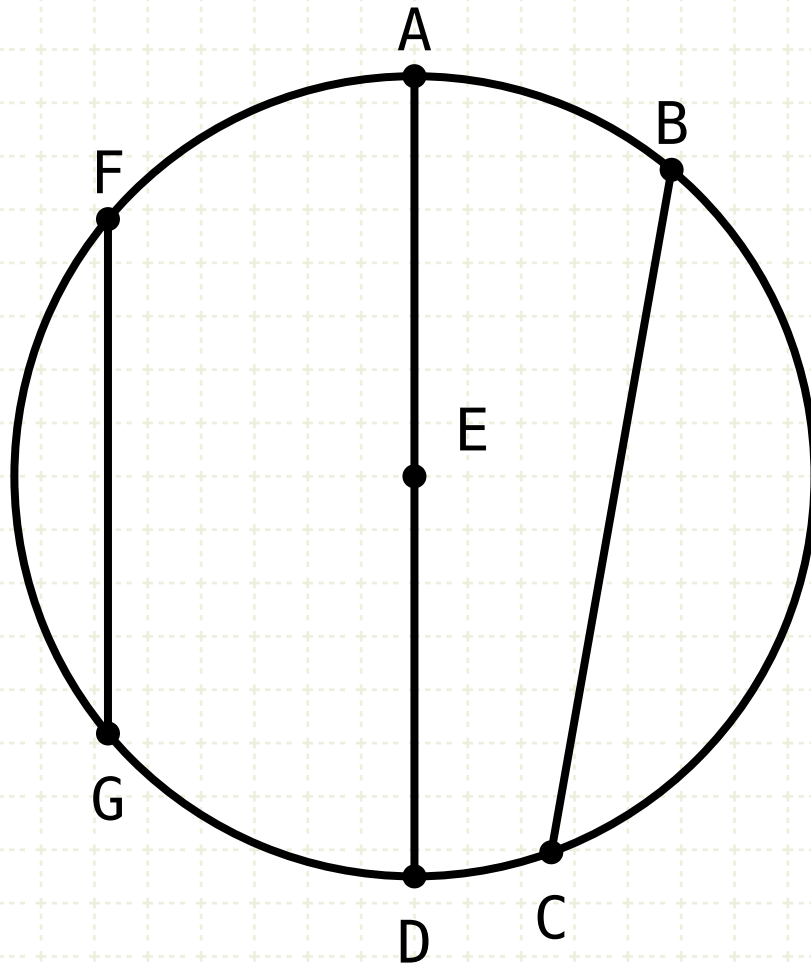
In other words

The line AD is larger than BC, since it passes through the centre of the circle, and line BC will be larger than FG since it is closer to the centre of the circle

Proposition 15 of Book III

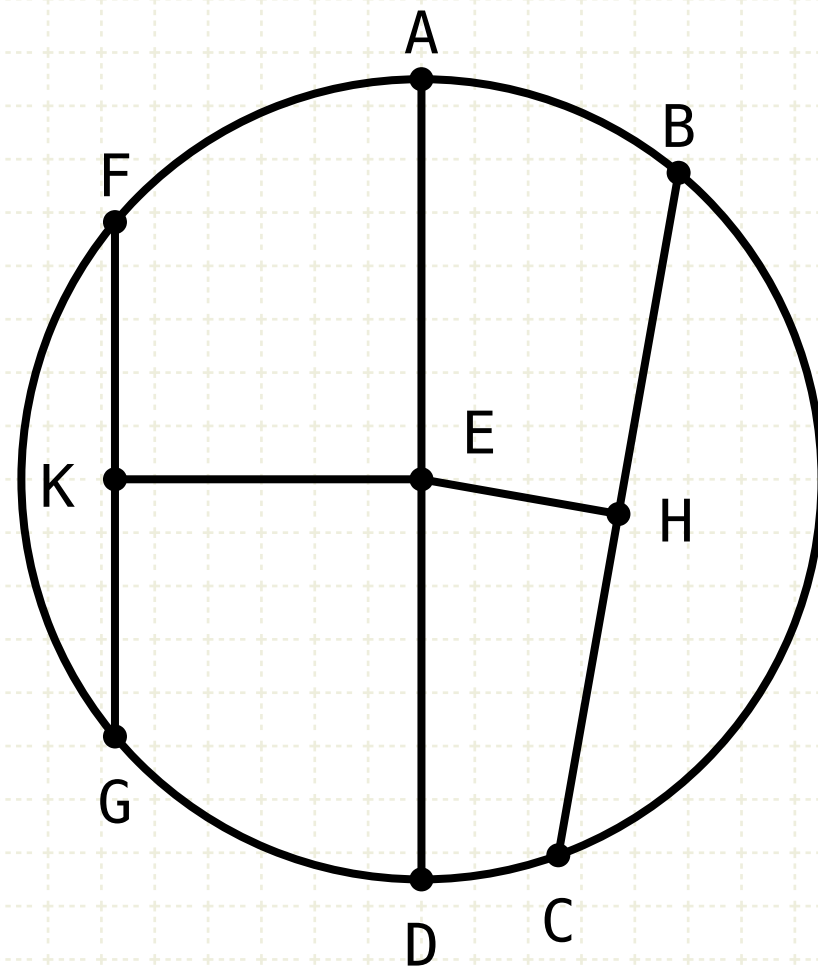
Of straight lines in a circle the diameter is greatest, and of the rest the nearer to the centre is always greater than the more remote.

Proof



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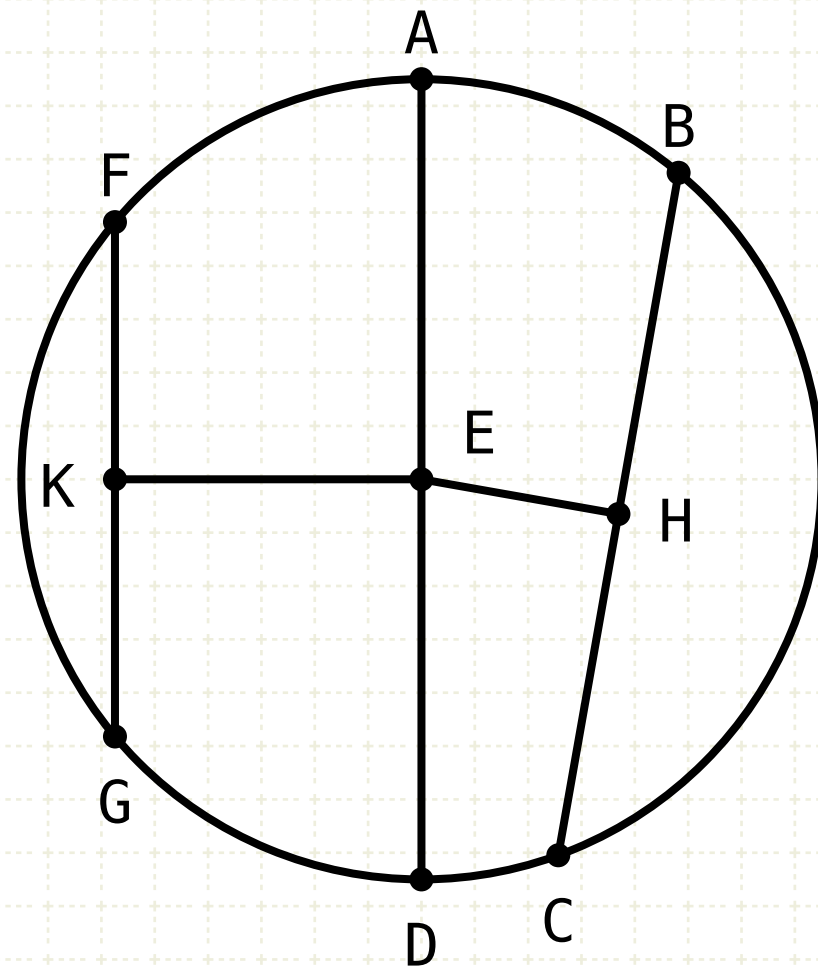


Proof

Draw lines from the centre of the circle perpendicular to the lines BC and FG

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Of straight lines in a circle the diameter is greatest, and of the rest the nearer to the centre is always greater than the more remote.



$$EH < EK$$

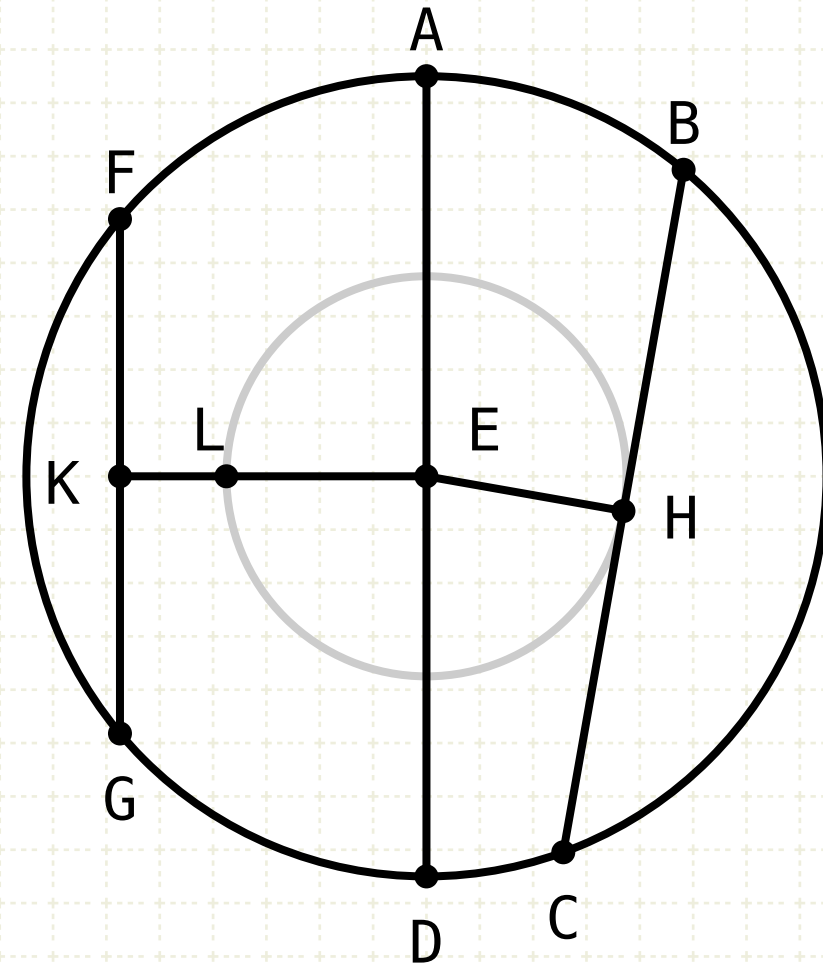
Proof

Draw lines from the centre of the circle perpendicular to the lines BC and FG

Since BC is nearer to centre E than line FG, EH is less than EK (by definition)

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Of straight lines in a circle the diameter is greatest, and of the rest the nearer to the centre is always greater than the more remote.



$$EH < EK$$
$$EL = EH$$

Proof

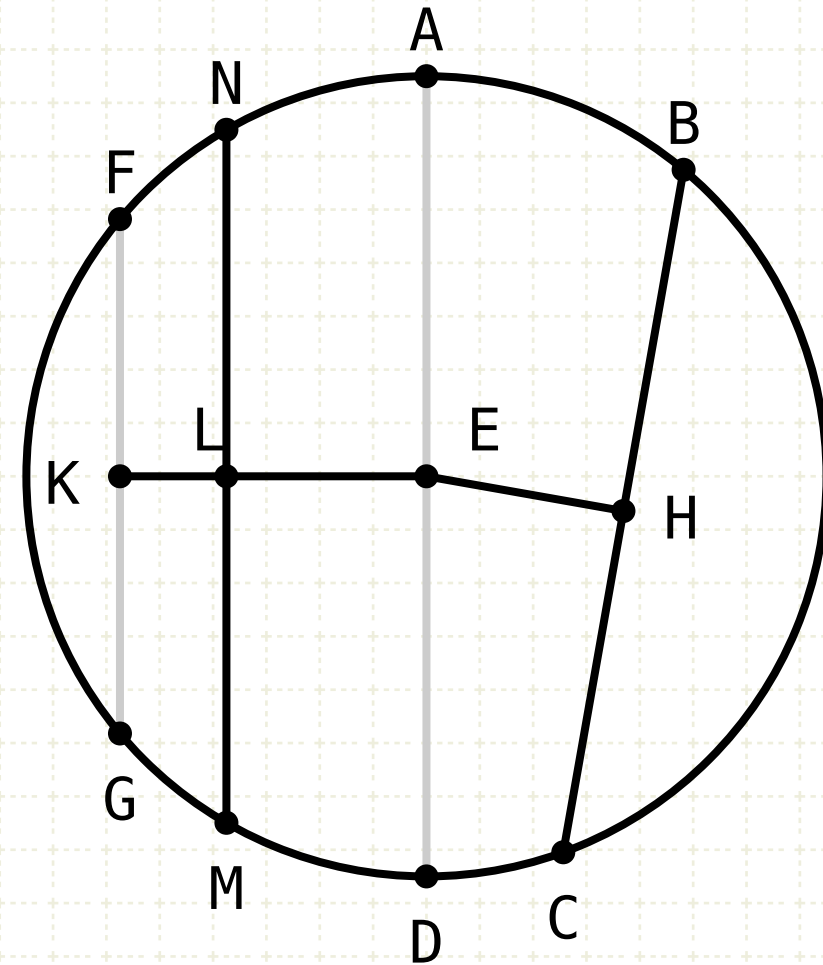
Draw lines from the centre of the circle perpendicular to the lines BC and FG

Since BC is nearer to centre E than line FG, EH is less than EK (by definition)

Define a point L on line EK, such that LE equals EH

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Of straight lines in a circle the diameter is greatest, and of the rest the nearer to the centre is always greater than the more remote.



$$\begin{aligned} EH &< EK \\ EL &= EH \\ MN &= BC \end{aligned}$$

Proof

Draw lines from the centre of the circle perpendicular to the lines BC and FG

Since BC is nearer to centre E than line FG, EH is less than EK (by definition)

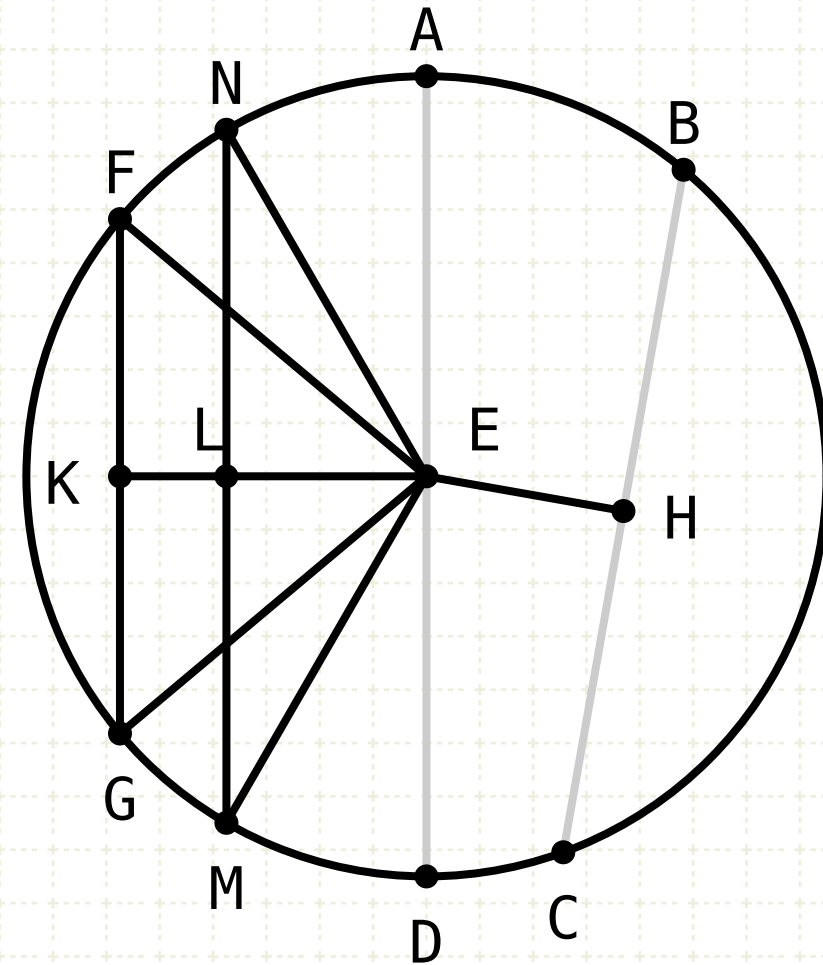
Define a point L on line EK, such that LE equals EH

Draw a line perpendicular to EL, touching the circle at points M and N

The lines BC and MN are equal (III·14)

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Of straight lines in a circle the diameter is greatest, and of the rest the nearer to the centre is always greater than the more remote.



$$EH < EK$$

$$EL = EH$$

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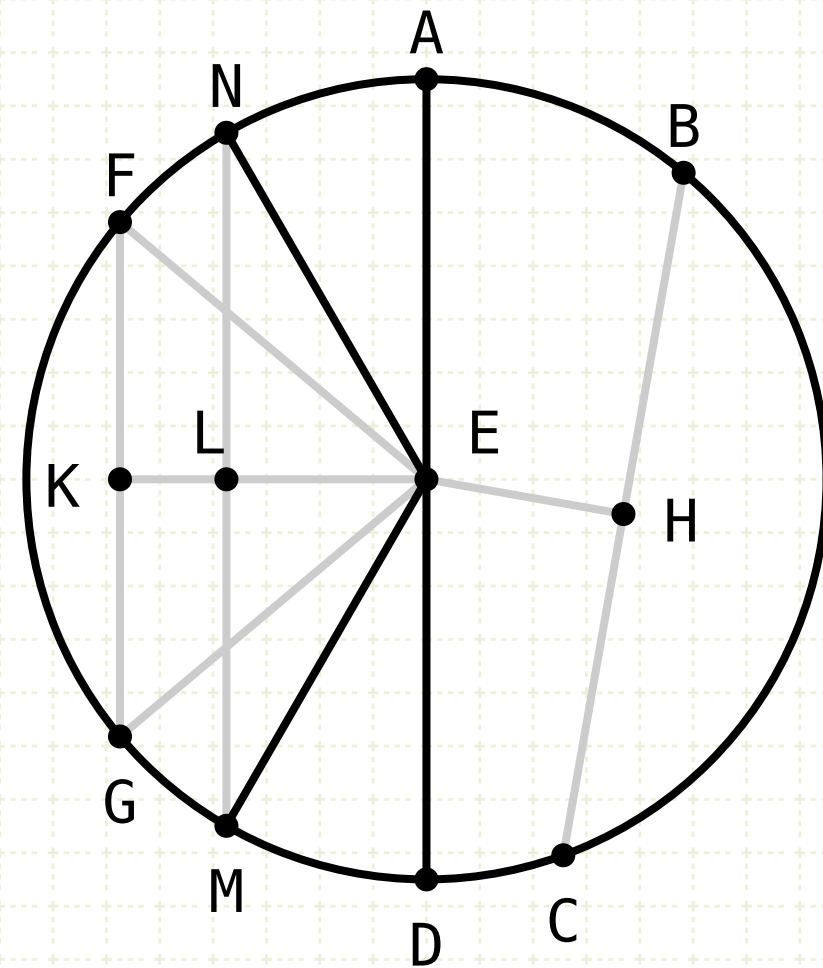
Draw a line perpendicular to EL, touching the circle at points M and N

The lines BC and MN are equal (III·14)

Draw the lines FE, GE, ME and NE

Proposition 15 of Book III

Of straight lines in a circle the diameter is greatest, and of the rest the nearer to the centre is always greater than the more remote.



$$EH < EK$$

$$EL = EH$$

$$MN = BC$$

$$AE + ED = AD = ME + EN$$

Proof

Draw lines from the centre of the circle perpendicular to the lines BC and FG

Since BC is nearer to centre E than line FG, EH is less than EK (by definition)

Define a point L on line EK, such that LE equals EH

Draw a line perpendicular to EL, touching the circle at points M and N

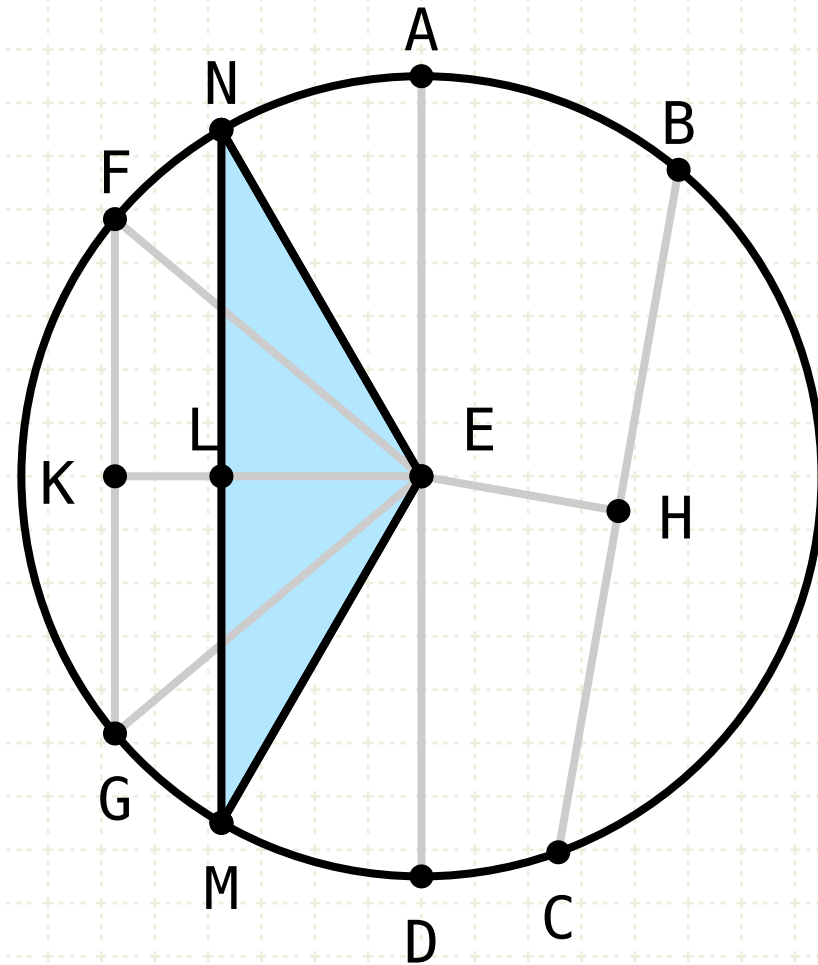
The lines BC and MN are equal (III·14)

Draw the lines FE, GE, ME and NE

Since EA equals EM, and ED equals EN (all radii), then the sum of EA and ED equals the sum of EM and EN

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$$EH < EK$$

$$EL = EH$$

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$$AE + ED = AD = ME + EN$$

$$MN < ME + EN$$

$$\therefore MN < AD$$

Proof

Draw lines from the centre of the circle perpendicular to the lines BC and FG

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Define a point L on line EK, such that LE equals EH

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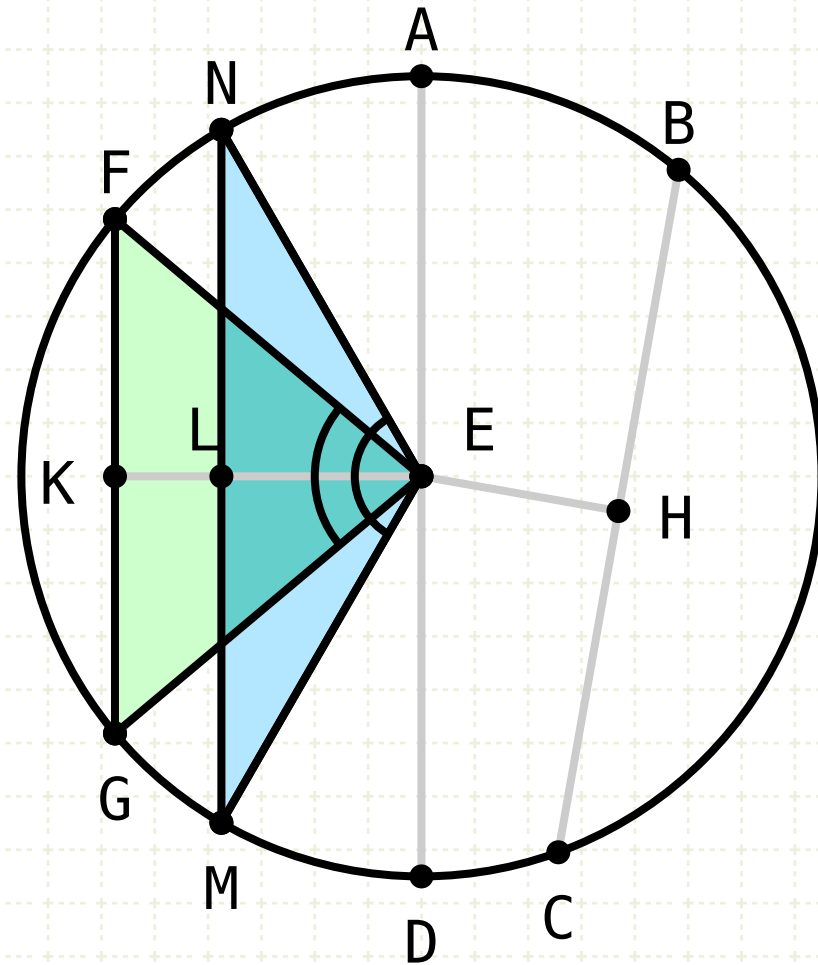
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One side of a triangle is less than the sum of the two other sides (I·20), thus MN is less than ME, EN, or MN is less than AD

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Of straight lines in a circle the diameter is greatest, and of the rest the nearer to the centre is always greater than the more remote.



$$\begin{aligned}EH &< EK \\EL &= EH \\MN &= BC \\AE+ED &= AD = ME+EN \\MN &< ME+EN \\\therefore MN &< AD \\ME &= FE \\NE &= GE \\\angle MEN &> \angle FEG \\\therefore FG &< MN\end{aligned}$$

Proof

Draw lines from the centre of the circle perpendicular to the lines BC and FG

Since BC is nearer to centre E than line FG, EH is less than EK (by definition)

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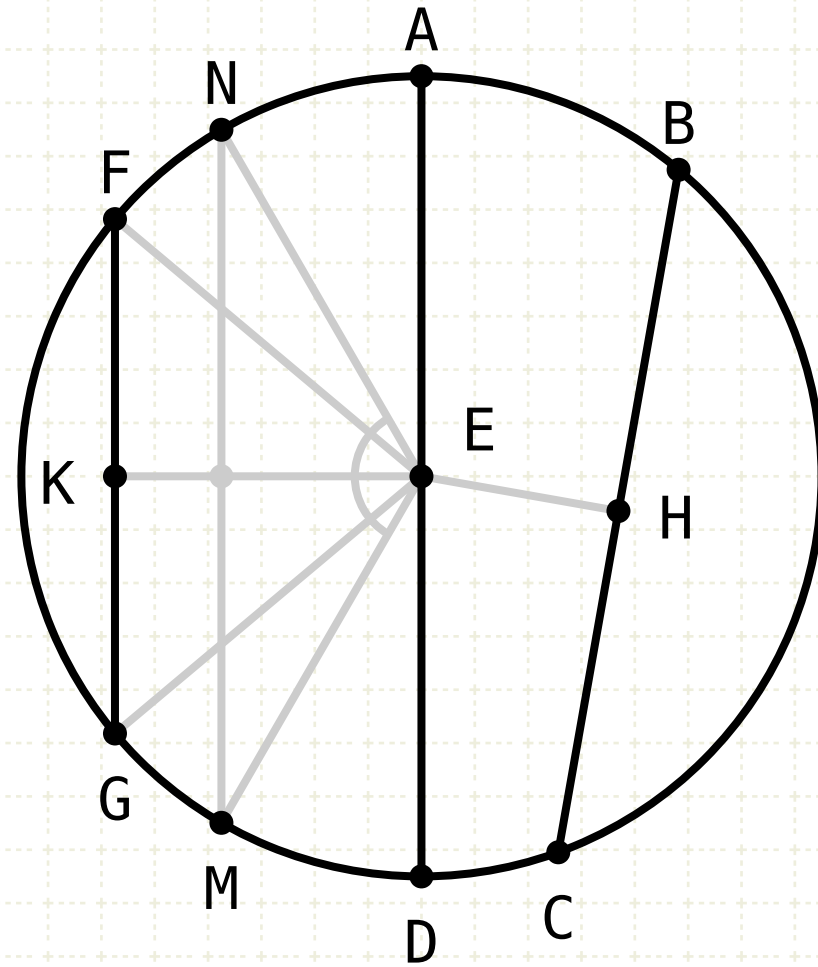
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One side of a triangle is less than the sum of the two other sides (I·20), thus MN is less than ME,EN, or MN is less than AD

ME equals FE, NE equals GE, the angle MEN is greater than FEG, therefore the base MN is greater than the base FG (I·24)

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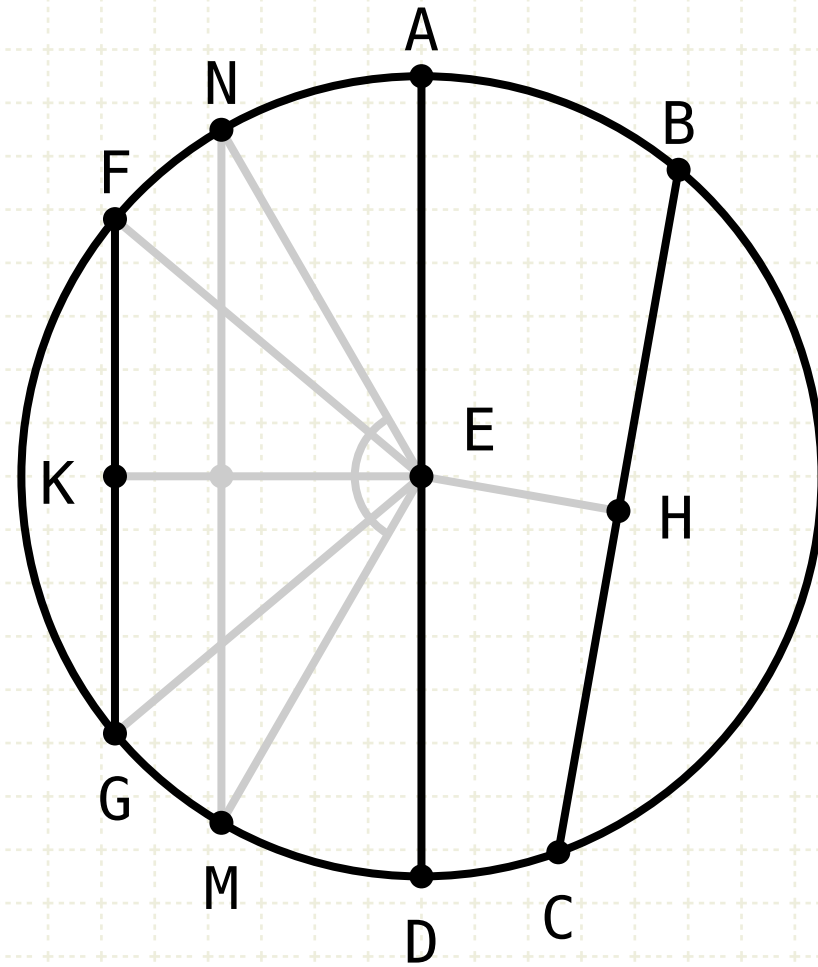
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Putting it all together gives FG is less than BC, is less than AD, or in other words, the further away from the centre, the smaller the line

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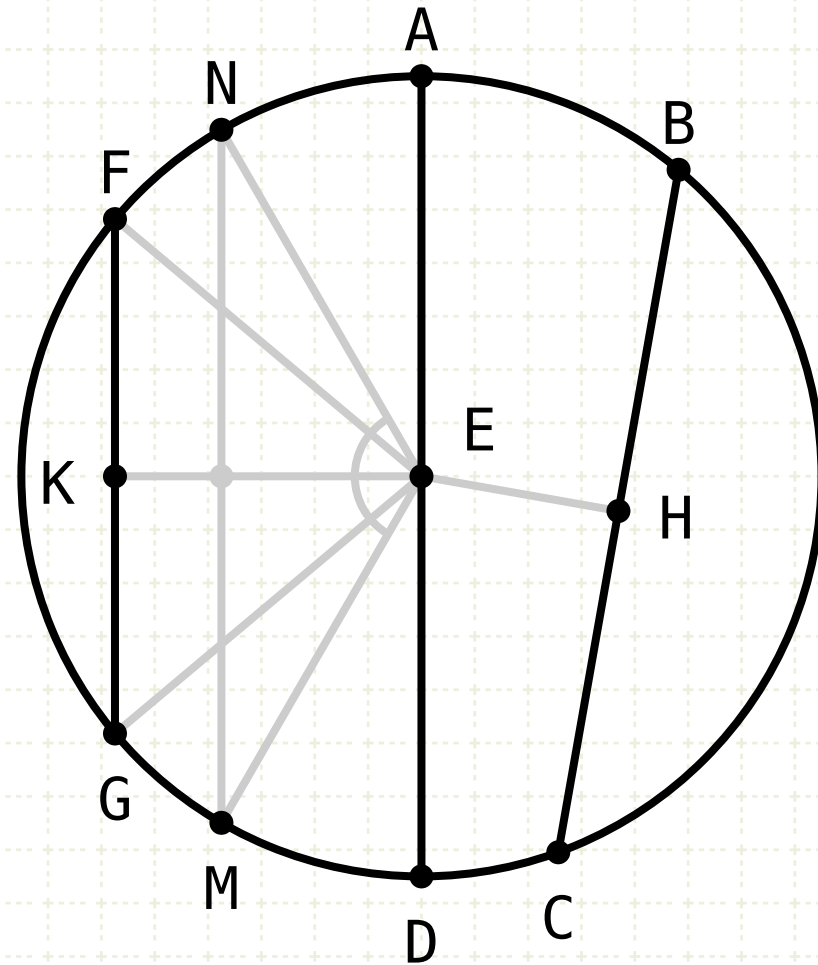
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