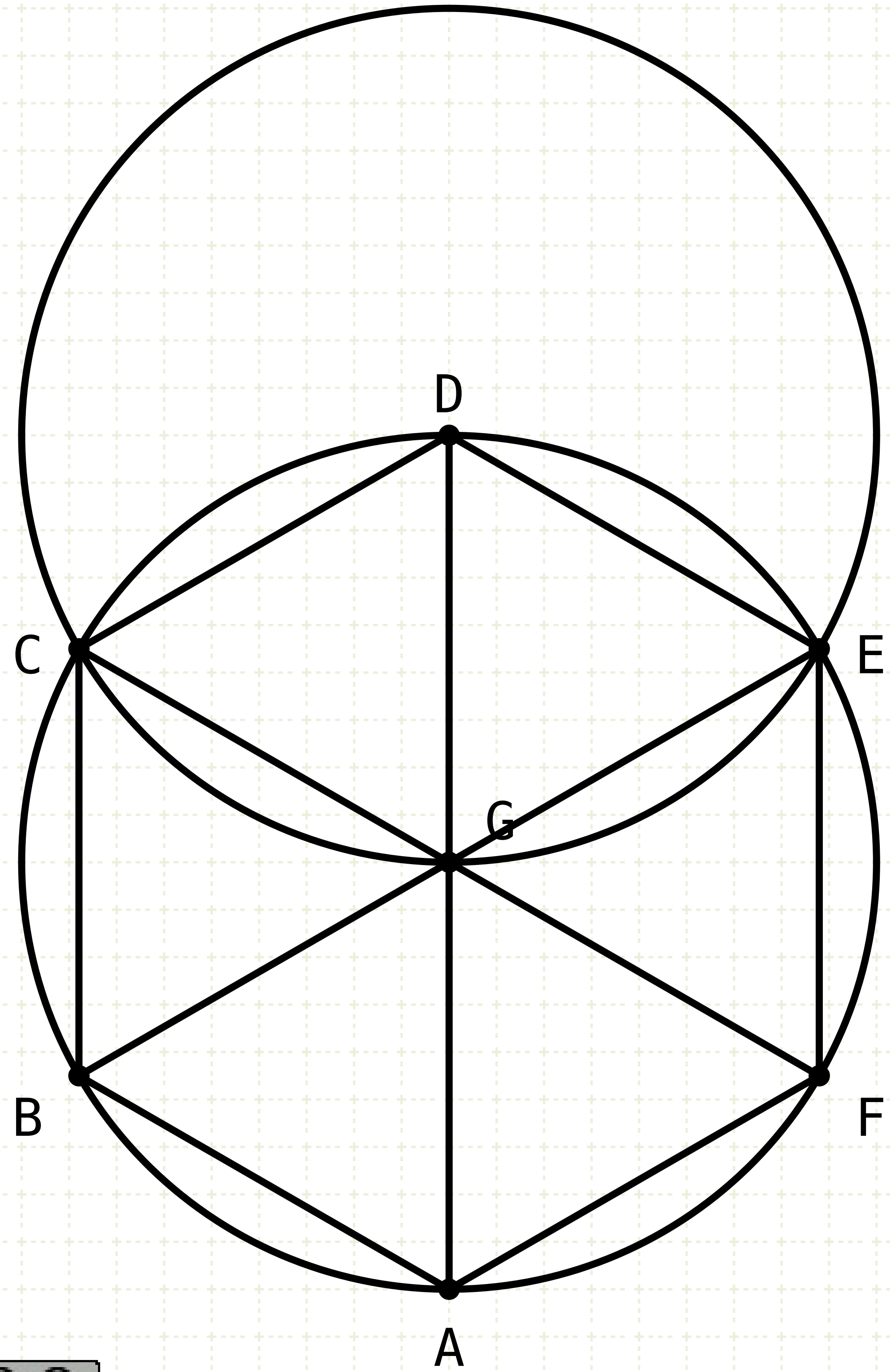
Euclid's Elements

Book IV



Philosophy (nature) is written in that great book which ever is before our eyes -- I mean the universe -- but we cannot understand it if we do not first learn the language and grasp the symbols in which it is written. The book is written in mathematical language, and the symbols are triangles, circles and other geometrical figures, without whose help it is impossible to comprehend a single word of it - without which one wanders in vain through a dark labyrinth.

Galileo Galilei

Proposition 3 of Book IV

About a given circle to circumscribe a triangle equiangular with a given triangle.

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Proposition 3 of Book IV

About a given circle to circumscribe a triangle equiangular with a given triangle.

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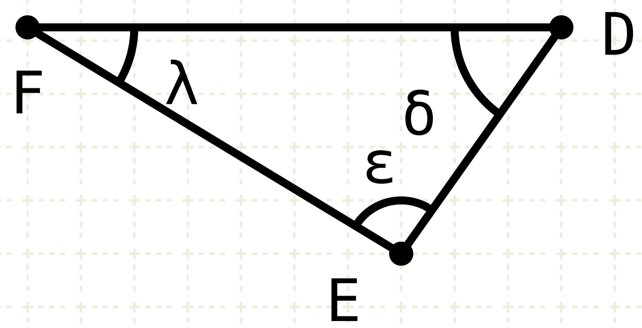
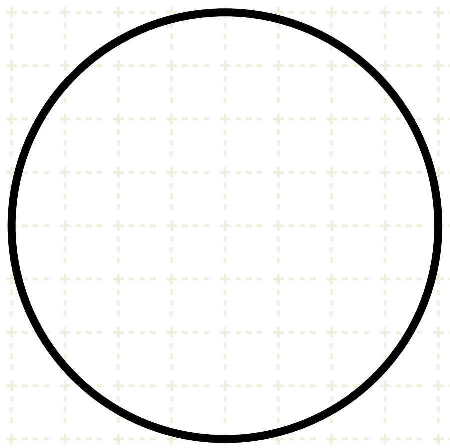




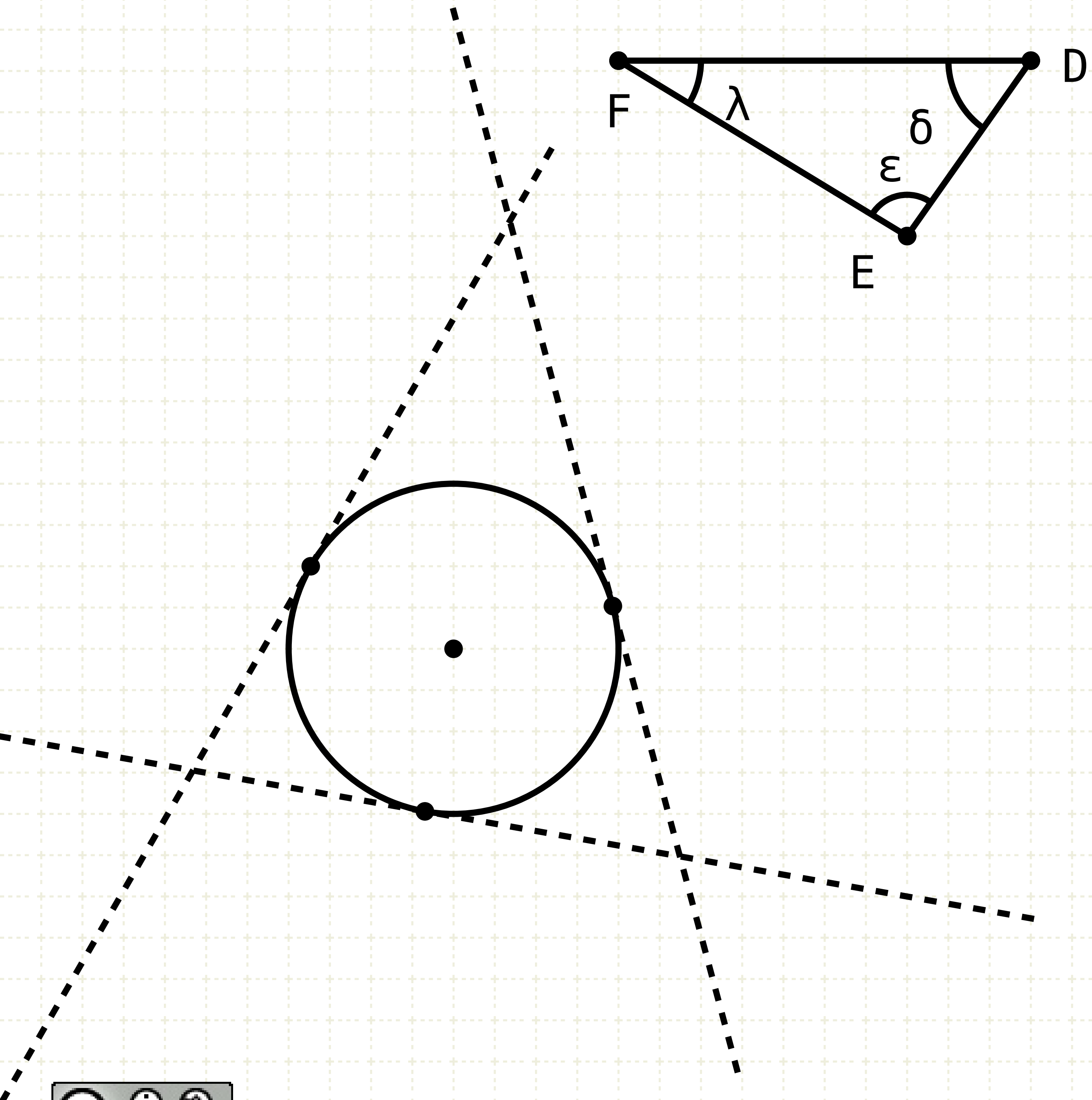
In other words

Given a circle and a triangle DEF:

Draw a triangle circumscribing the circle, where the angles in the new triangle equal the angles in triangle DEF

Proposition 3 of Book IV

About a given circle to circumscribe a triangle equiangular with a given triangle.



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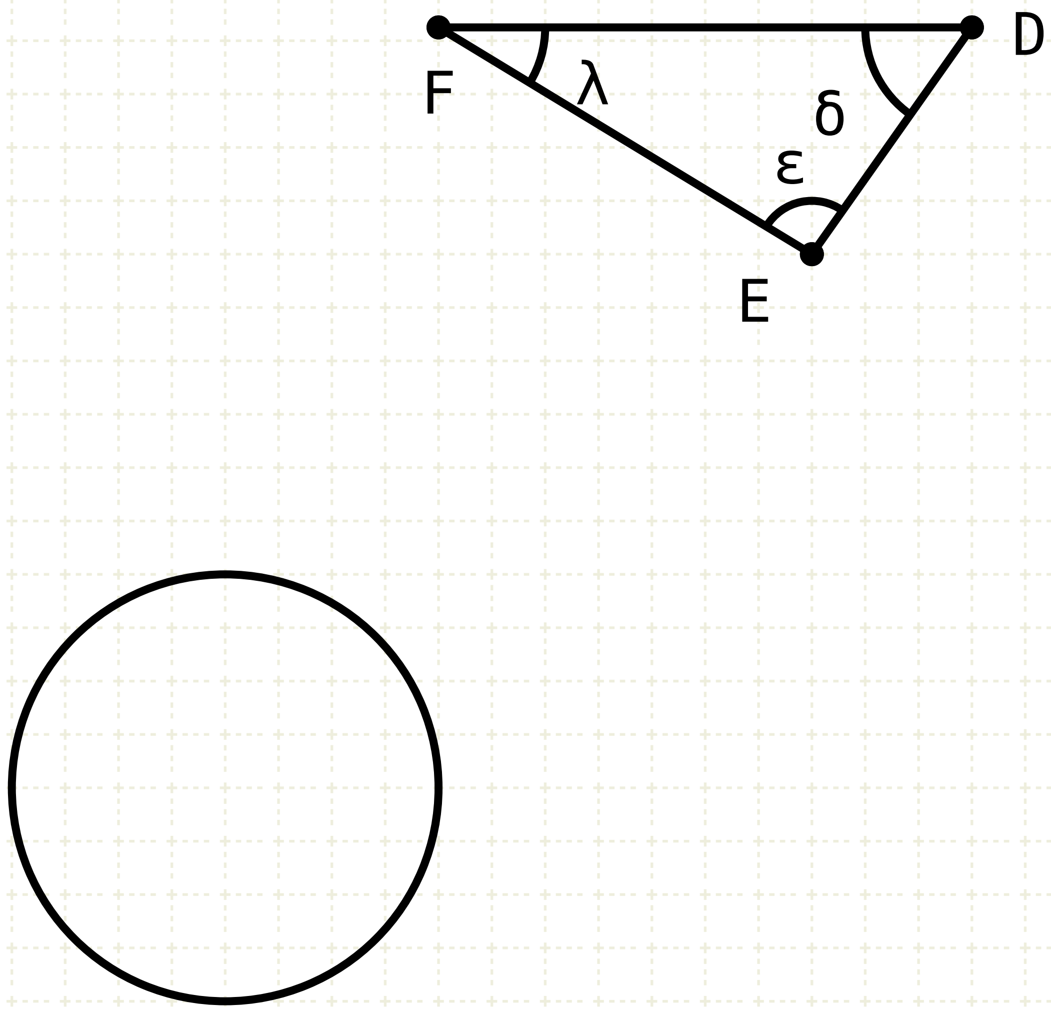
Construction



Proposition 3 of Book IV

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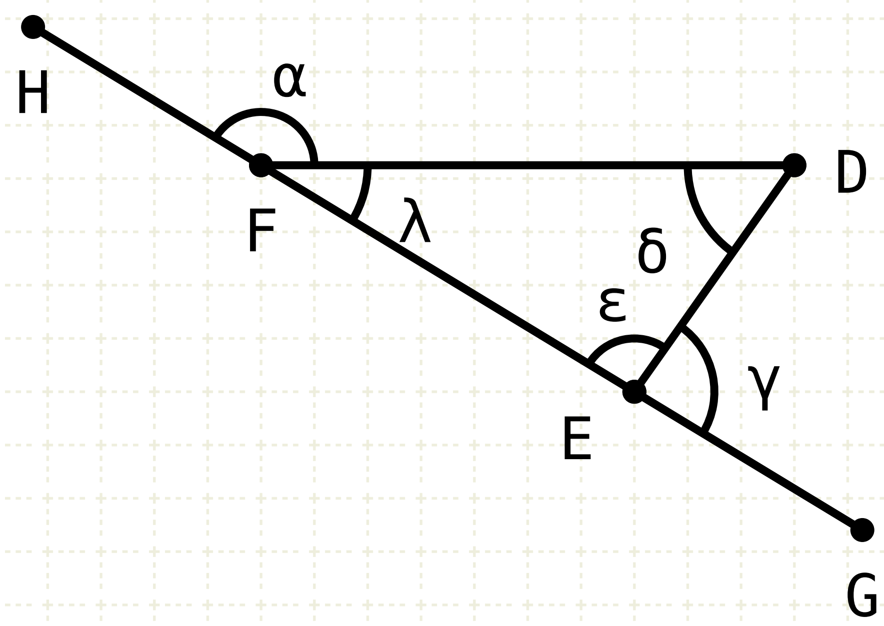
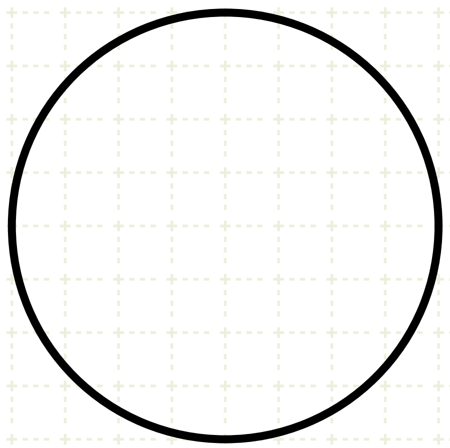


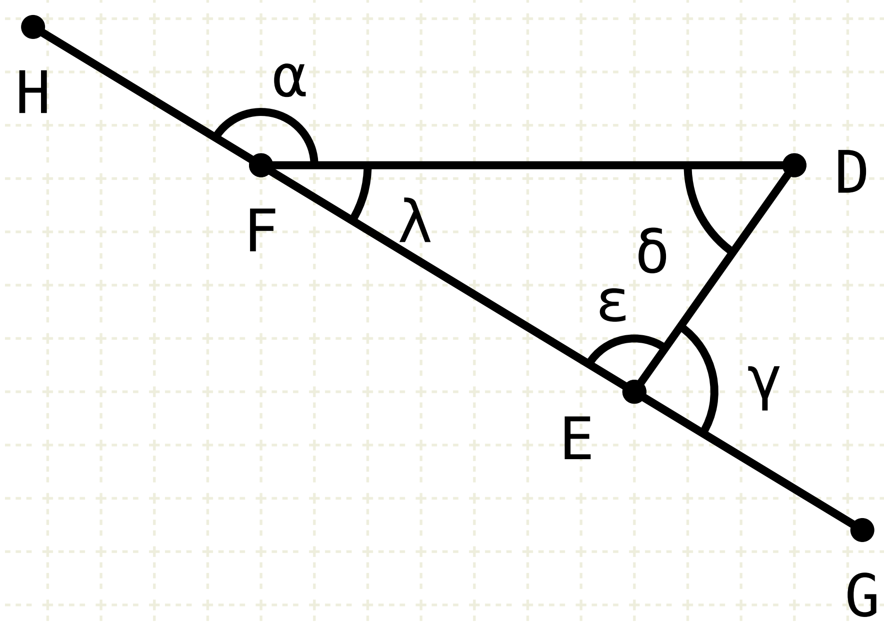


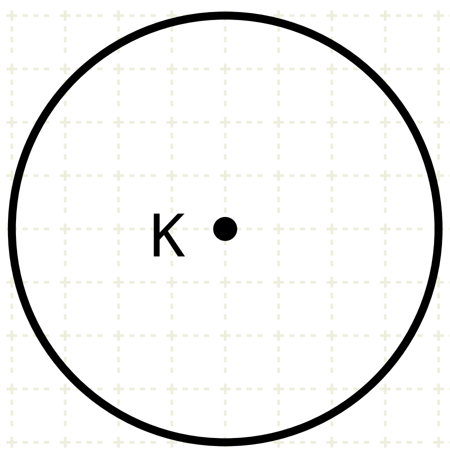
Construction

Extend the base of the triangle to points H and G

Proposition 3 of Book IV

About a given circle to circumscribe a triangle equiangular with a given triangle.





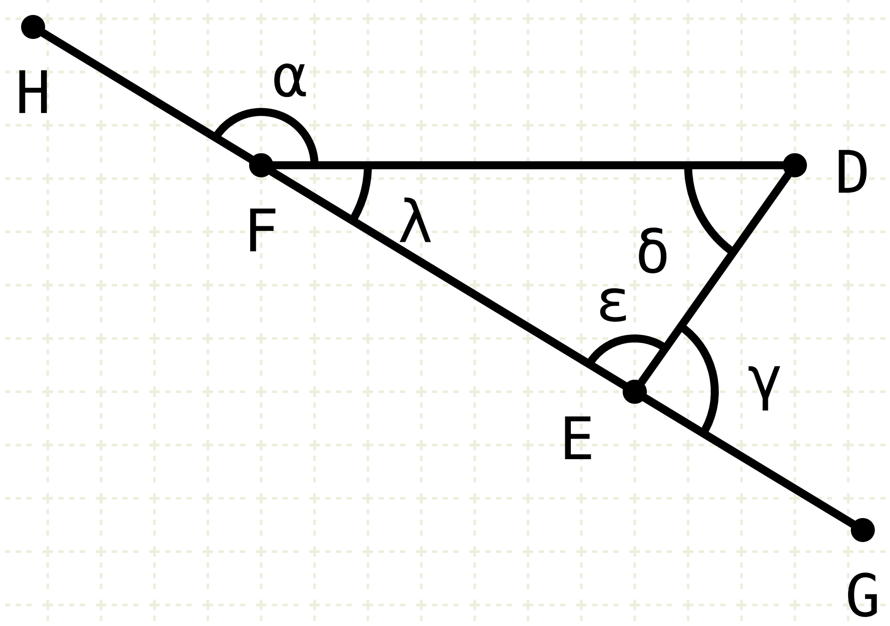
Construction

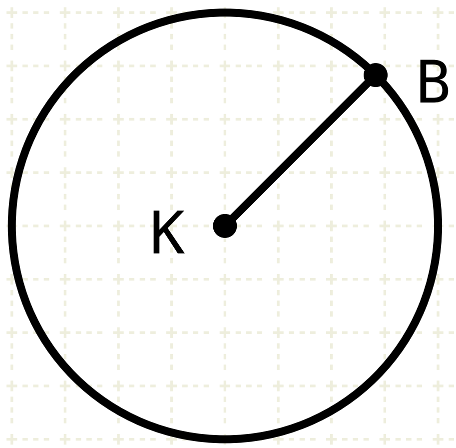
Extend the base of the triangle to points H and G

Find the centre of the circle K (III·1)

Proposition 3 of Book IV

About a given circle to circumscribe a triangle equiangular with a given triangle.





Construction

Extend the base of the triangle to points H and G

Find the centre of the circle K (III·1)

Let the line KB be drawn at random

Proposition 3 of Book IV

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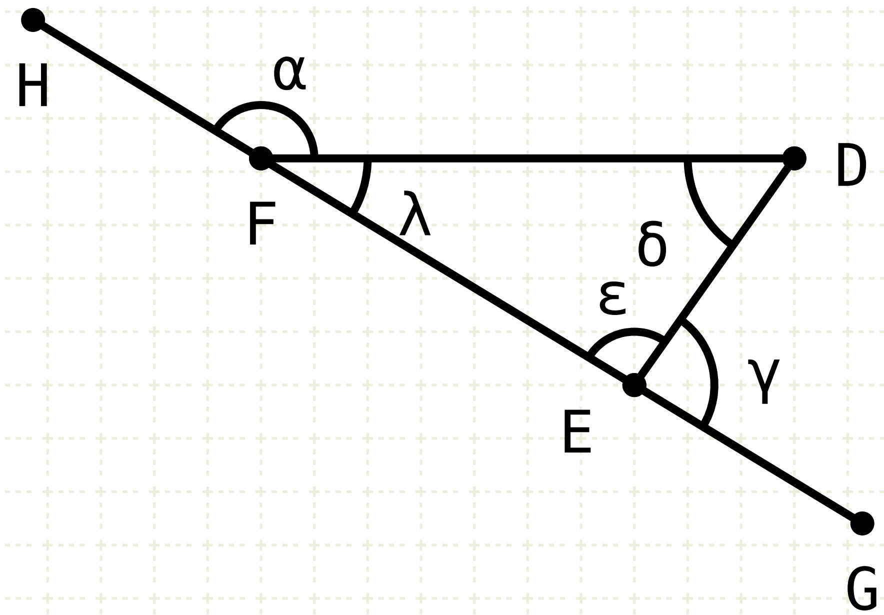
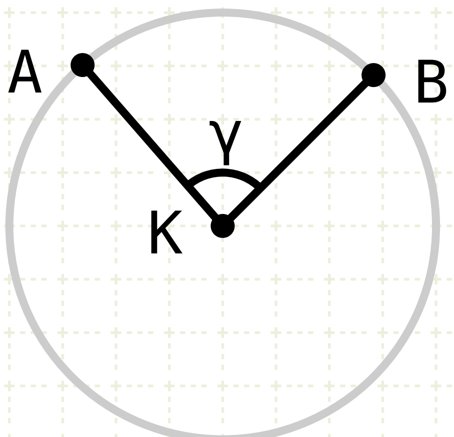




Construction

Extend the base of the triangle to points H and G

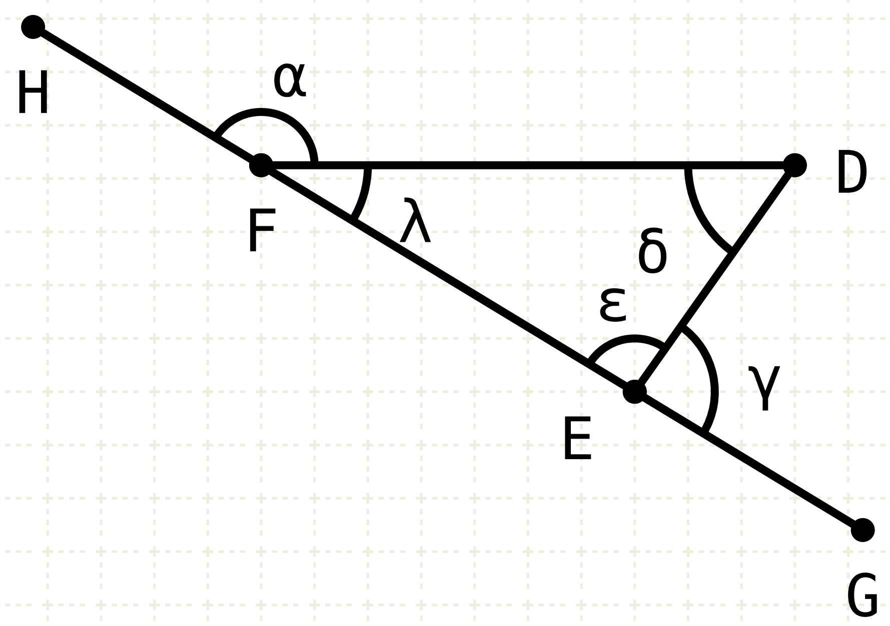
Find the centre of the circle K (III·1)

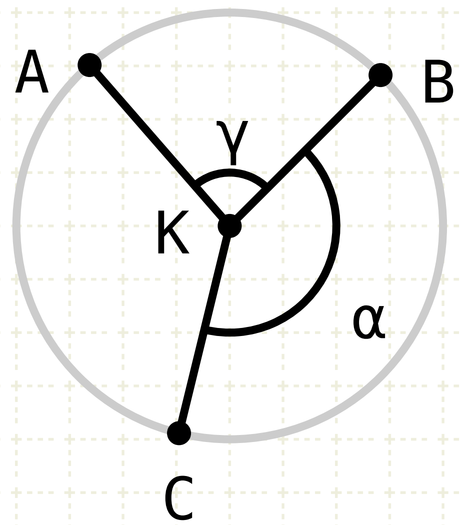
Let the line KB be drawn at random

Copy angle γ to line KB at the point K (I·23)

Proposition 3 of Book IV

About a given circle to circumscribe a triangle equiangular with a given triangle.





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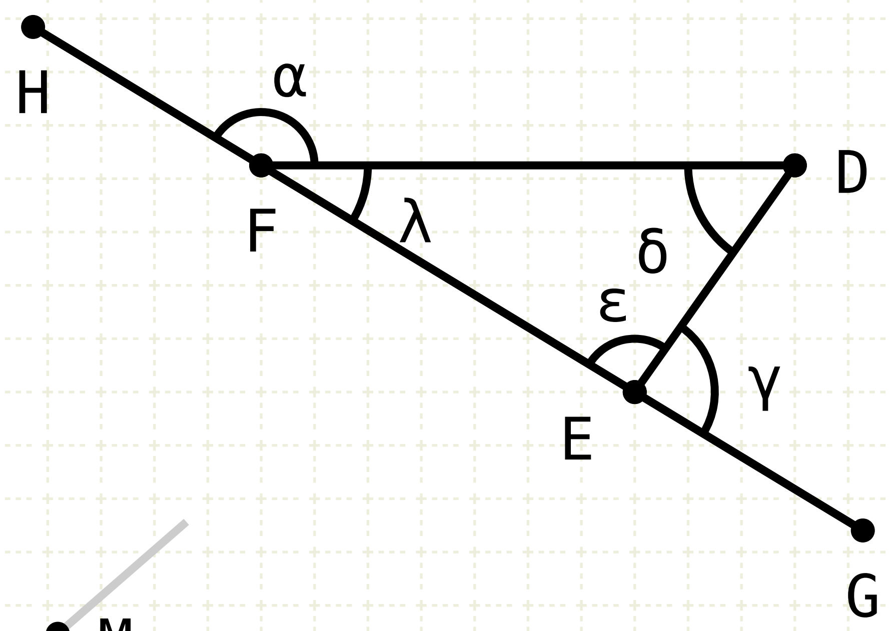
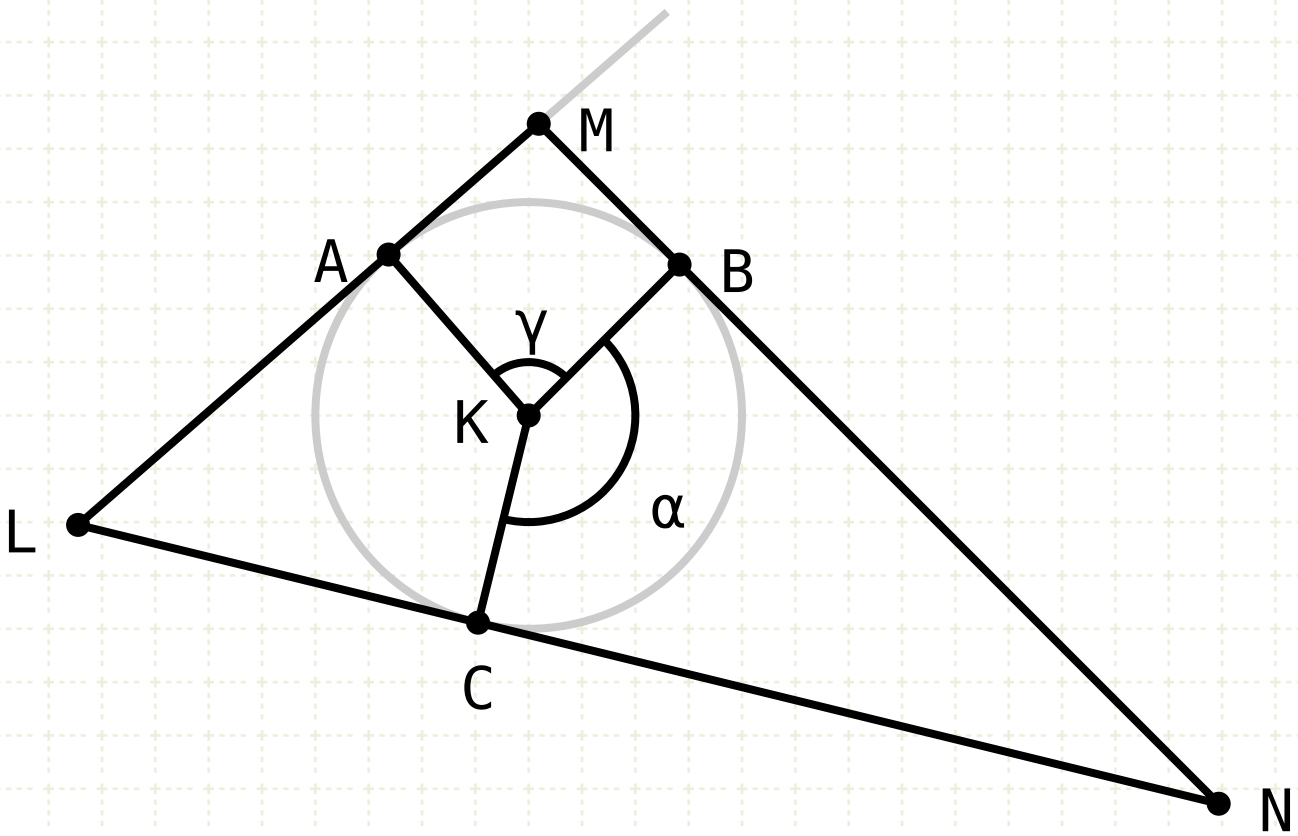




Construction

Extend the base of the triangle to points H and G

Find the centre of the circle K (III·1)

Let the line KB be drawn at random

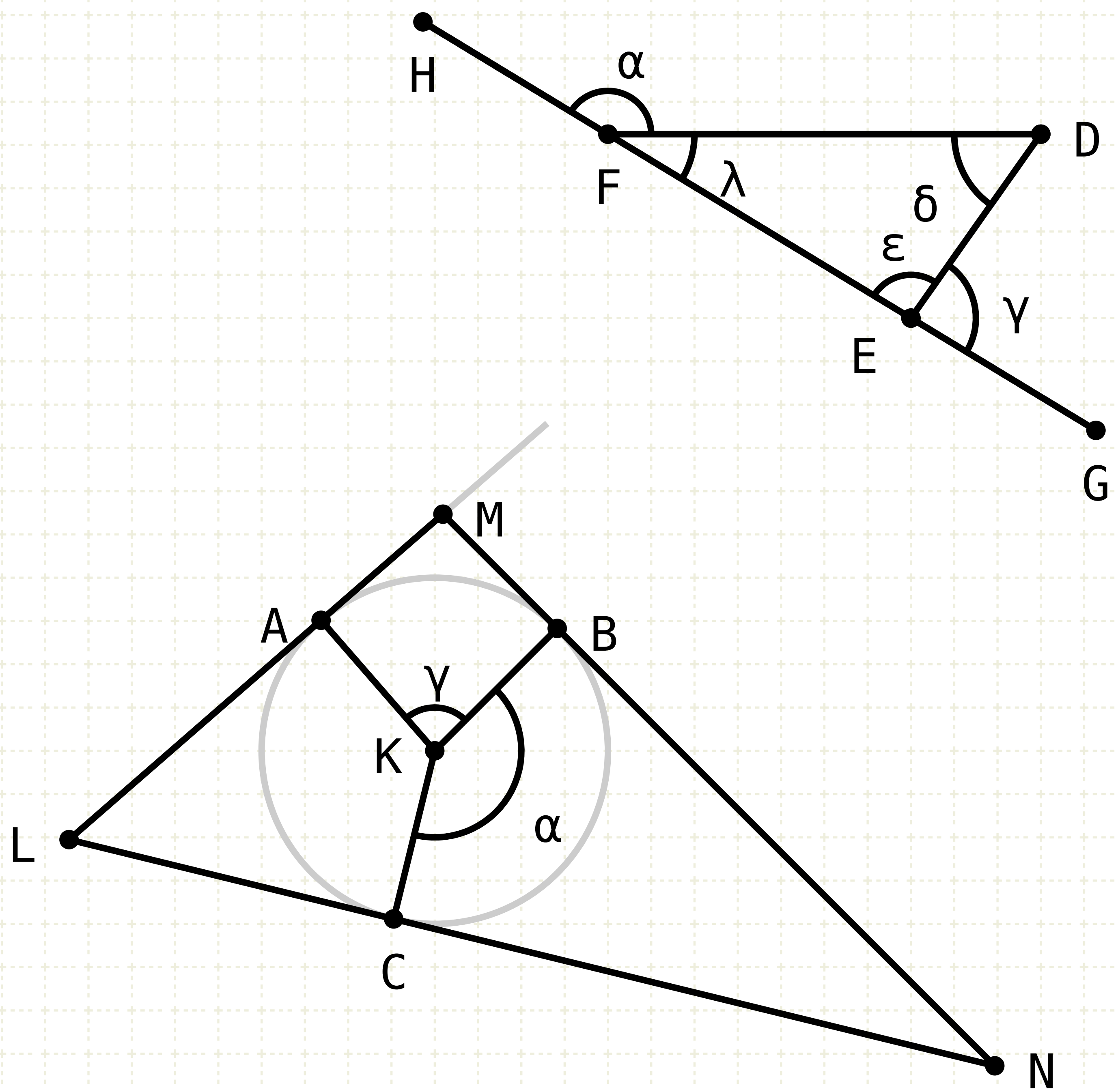
Copy angle γ to line KB at the point K (I·23)

Copy angle α to line KB at the point K (I·23)

Draw lines LM, MN, NL such that the touch the circle at points A, B and C respectively (III·16)

Proposition 3 of Book IV

About a given circle to circumscribe a triangle equiangular with a given triangle.



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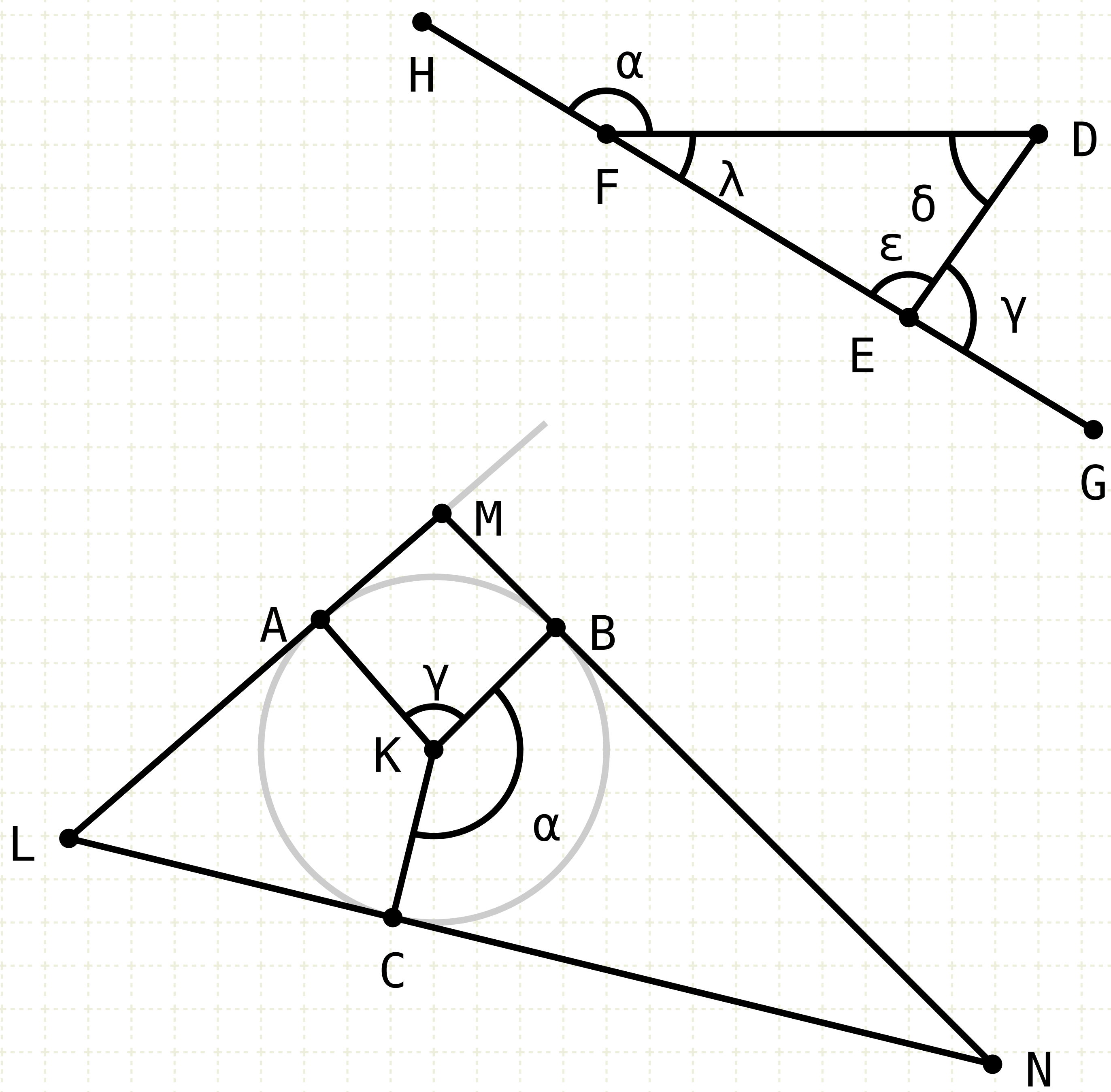
Triangle LMN is equi-angular to DEF

Proposition 3 of Book IV

About a given circle to circumscribe a triangle equiangular with a given triangle.

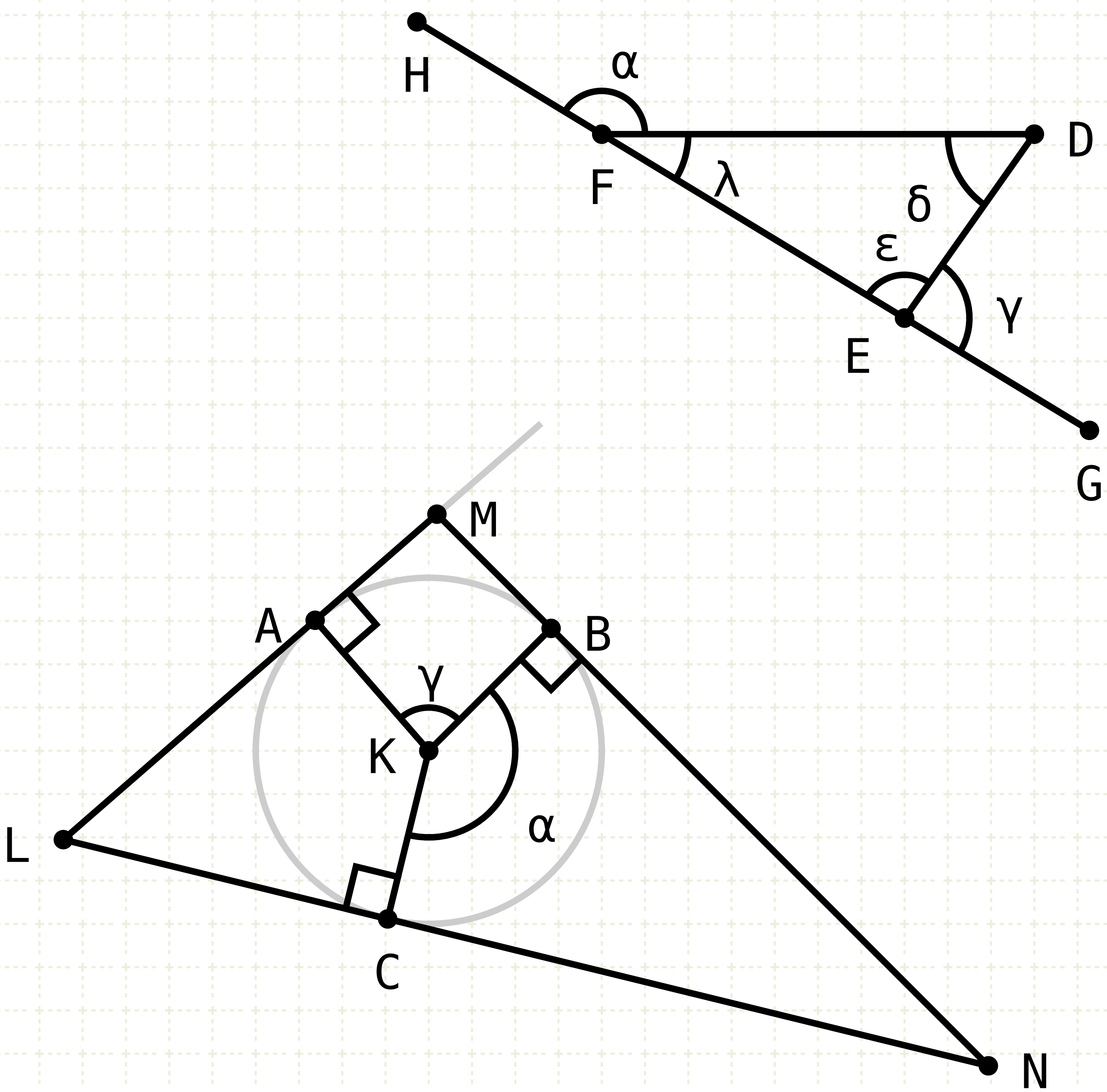


Triangle LMN is equi-angular to DEF

Proof

Proposition 3 of Book IV

About a given circle to circumscribe a triangle equiangular with a given triangle.



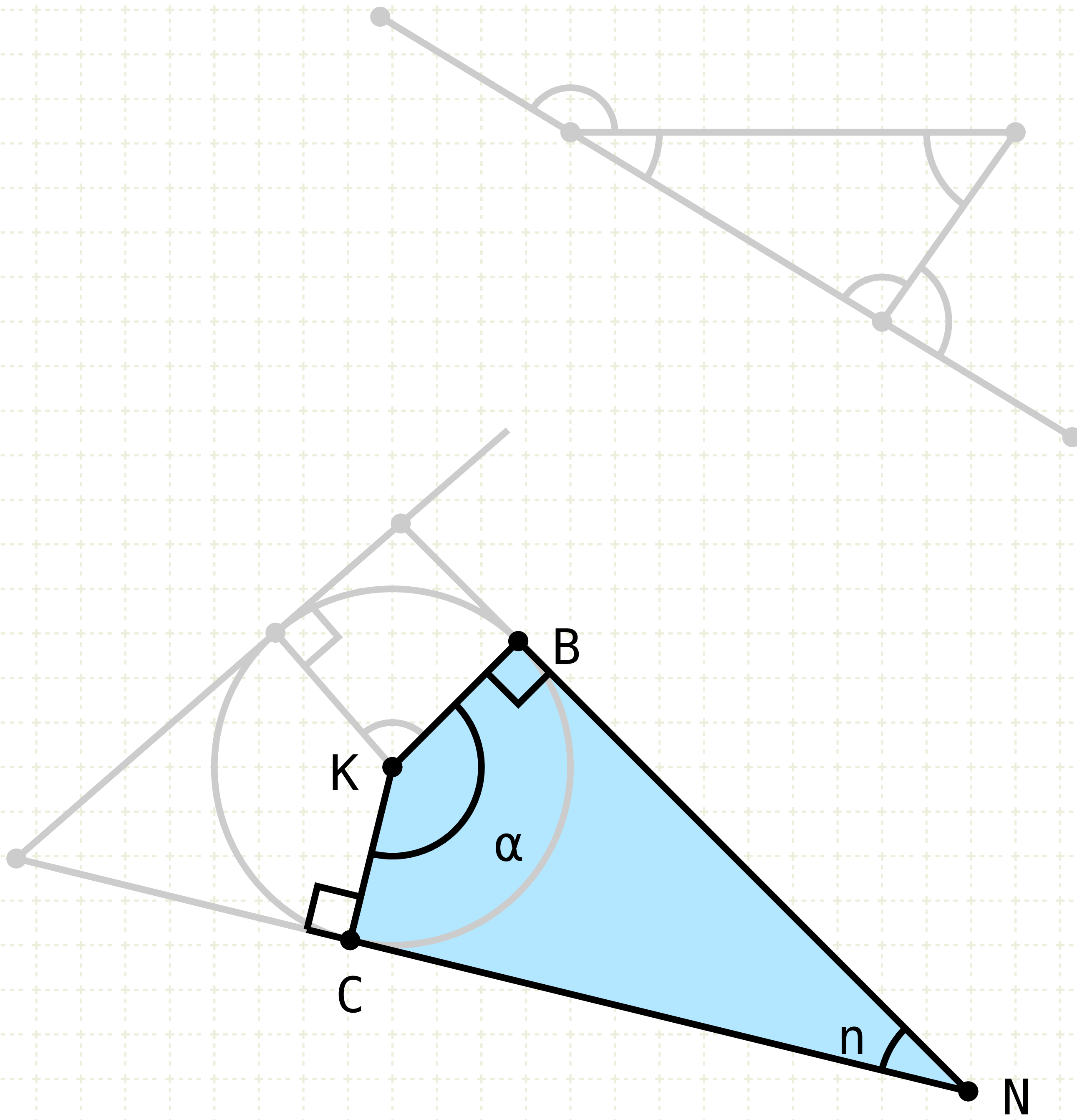
Triangle LMN is equi-angular to DEF

Proof

The sides of the triangle touch the circle, and the lines KA, KB and KC all pass through the centre of the circle, therefore the angles KCL, KAM and KBN are all right (III·18)

Proposition 3 of Book IV

About a given circle to circumscribe a triangle equiangular with a given triangle.



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Consider the quadilateral CKBN.

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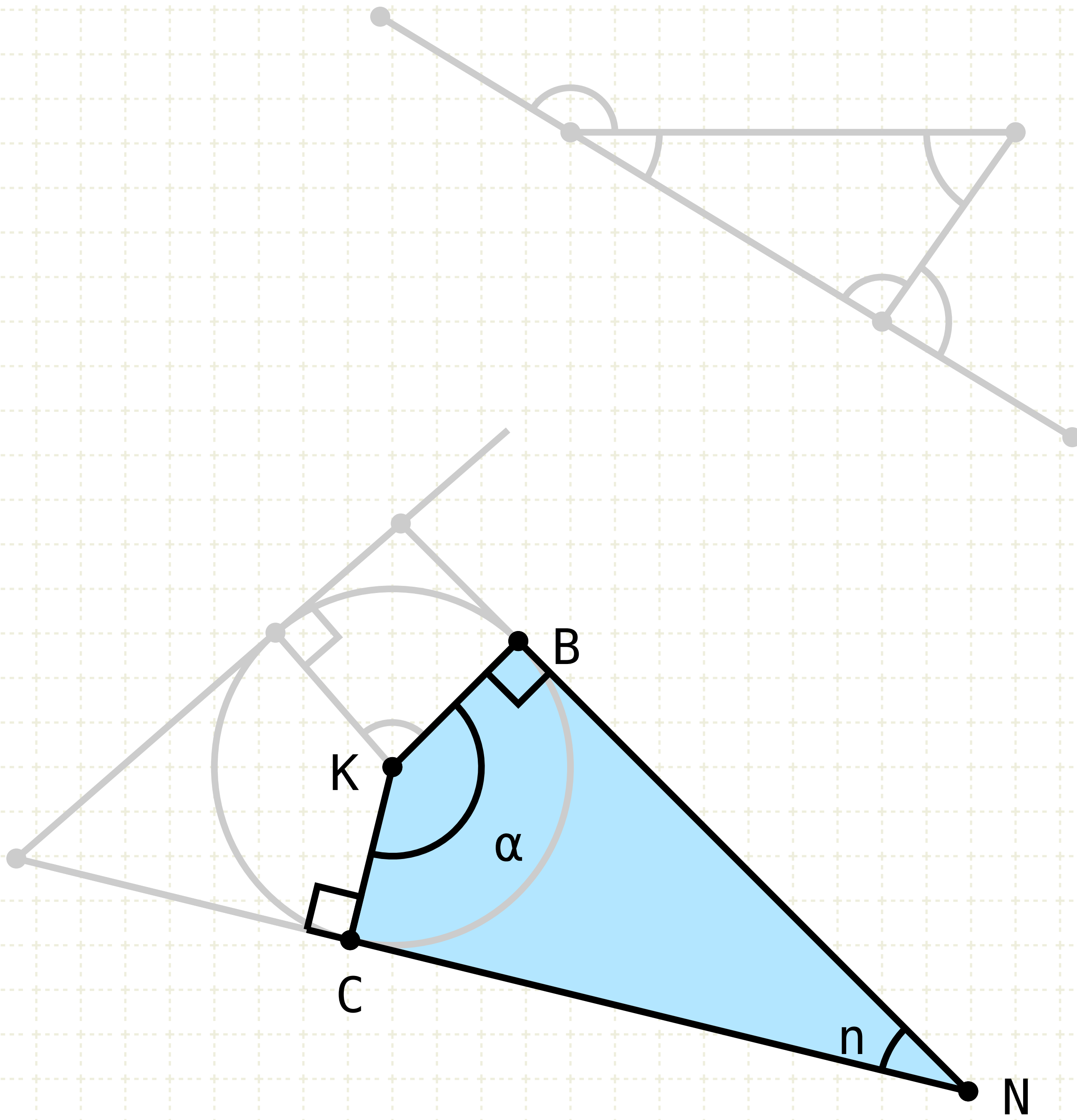

$\alpha + n = 2 \cdot \llcorner$

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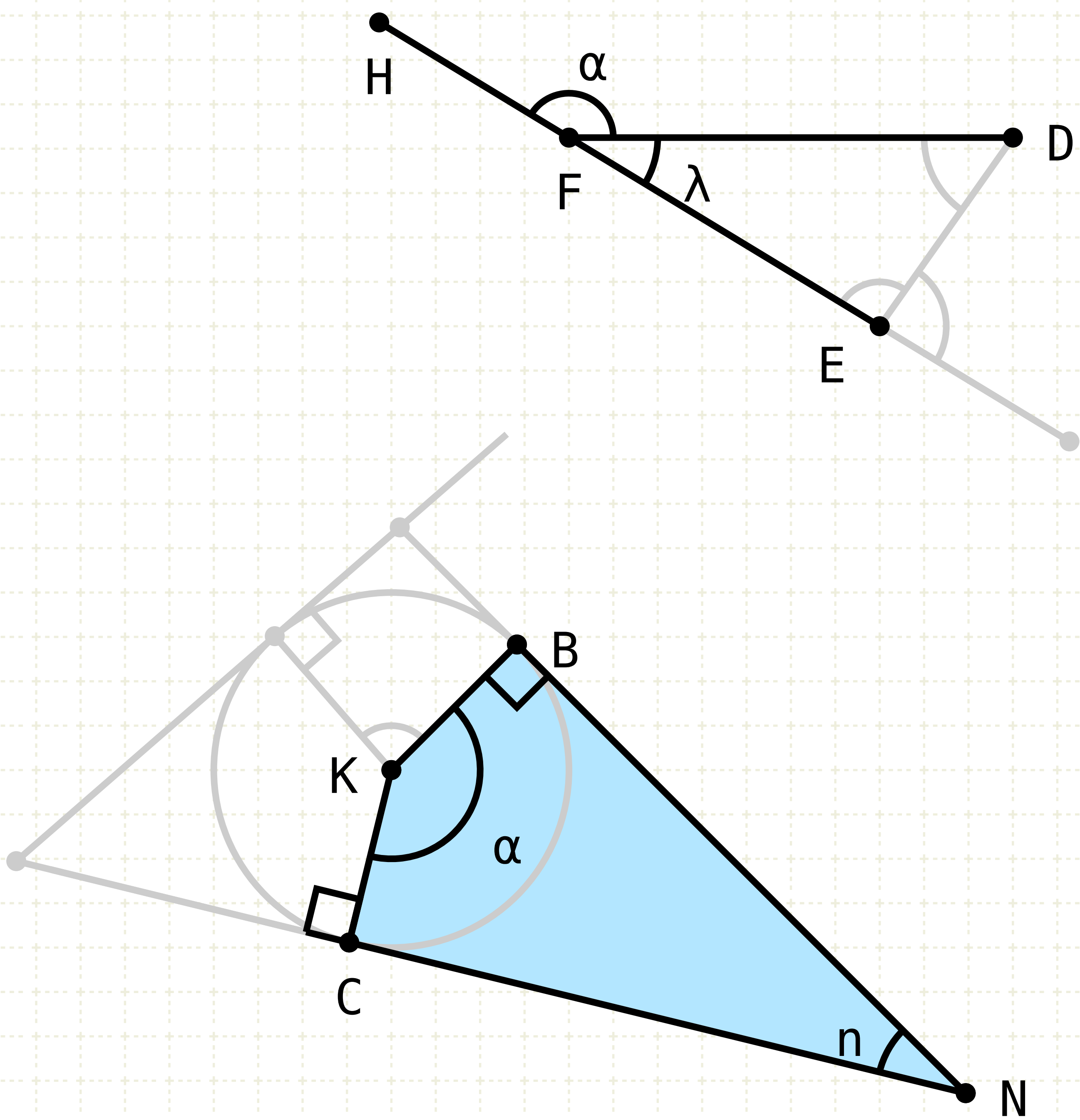
Consider the quadilateral CKBN.

The sum of all the angles is equal to four right angles, where KCN and KBN are right, thus the angles BKC (α) and BNC (n) are equal to two right angles

Proposition 3 of Book IV

About a given circle to circumscribe a triangle equiangular with a given triangle.



$$\alpha + n = 2 \cdot \llcorner$$
$$\alpha + \lambda = 2 \cdot \llcorner$$

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Proposition 3 of Book IV

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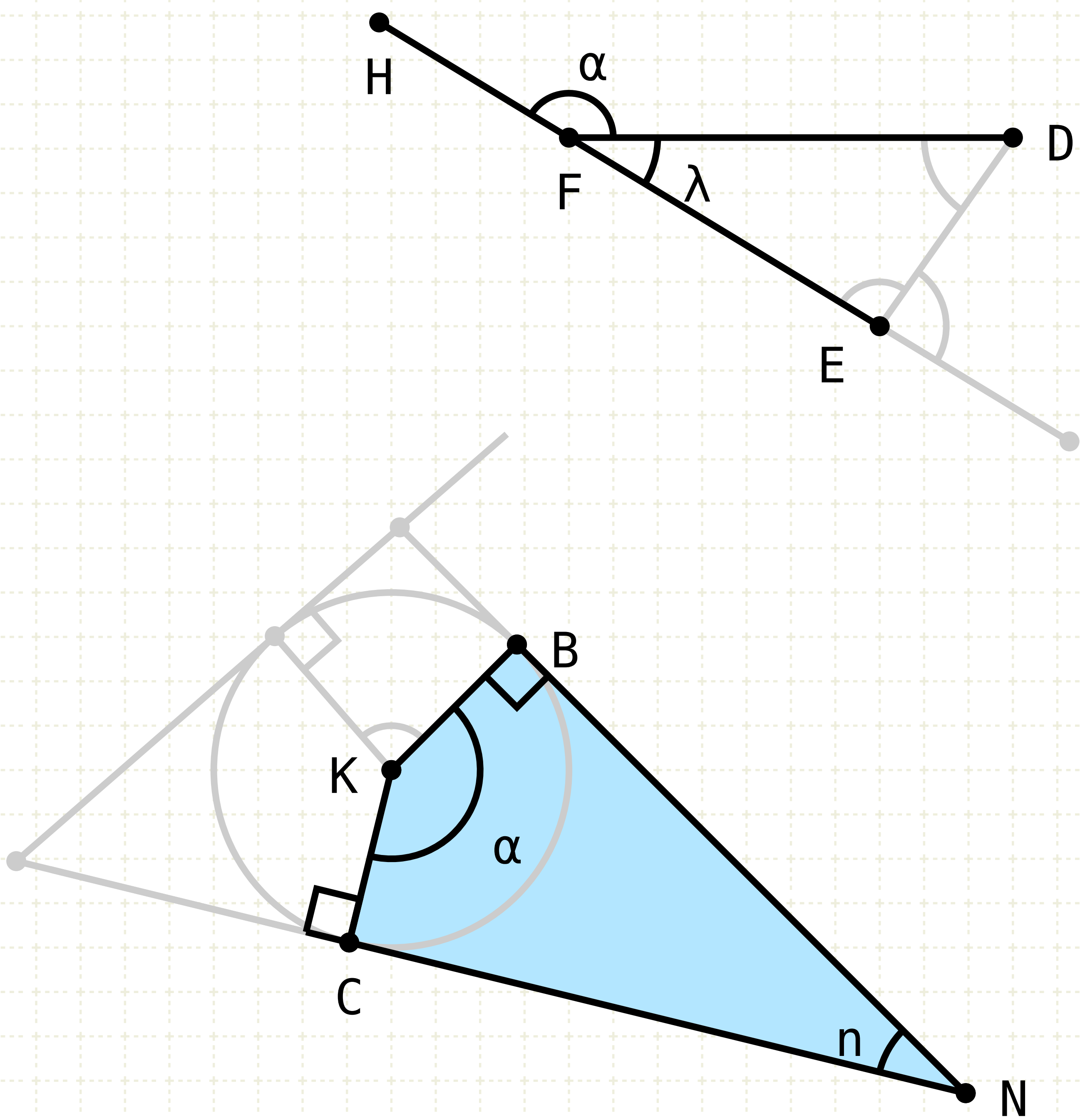

$\alpha + n = 2 \cdot ∟$

$\alpha + \lambda = 2 \cdot ∟$

$\therefore n = \lambda$

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Thus, angle n equals λ

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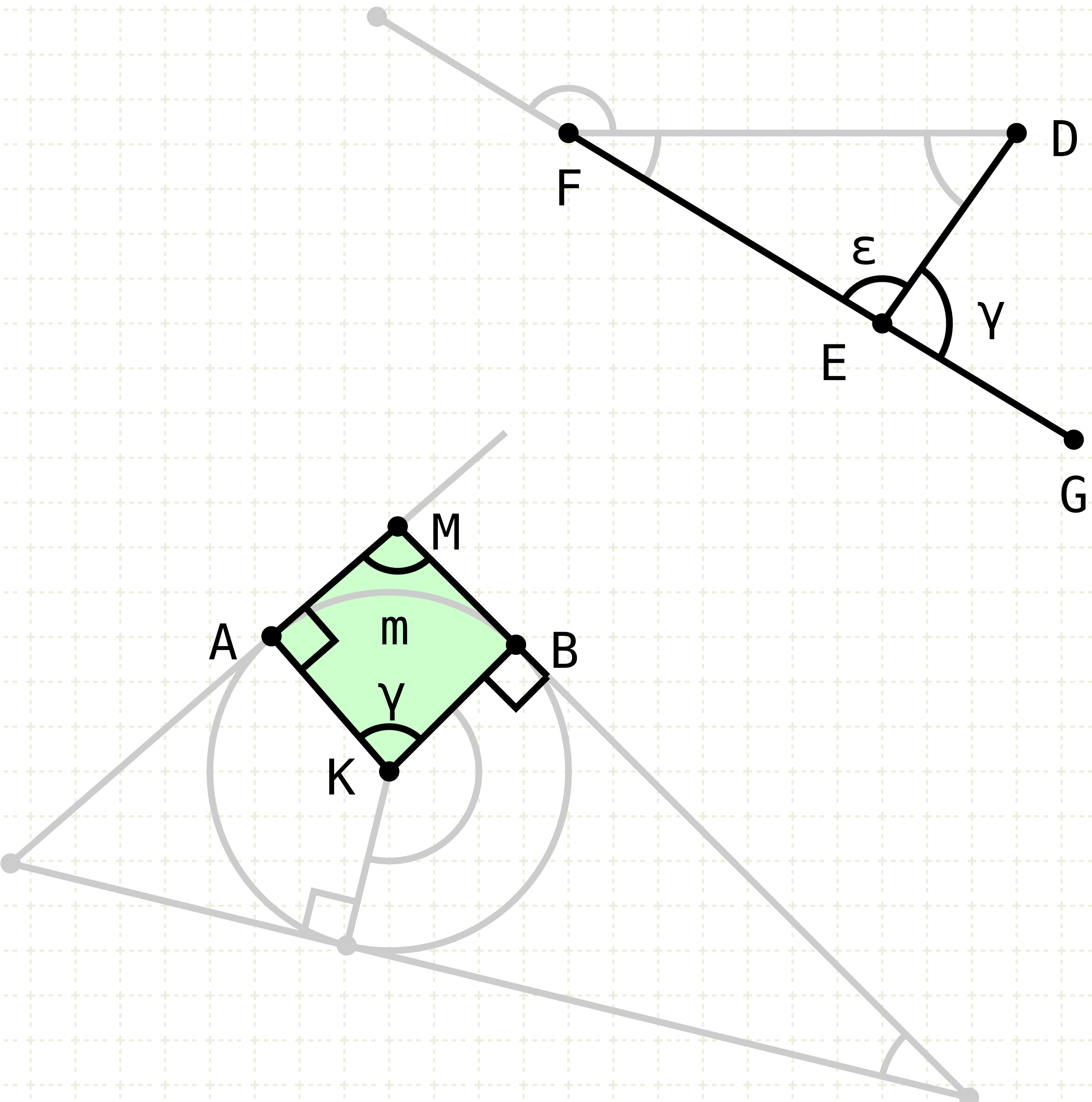

$$\alpha + n = 2\cdot\llcorner$$

$$\alpha + \lambda = 2\cdot\llcorner$$

$$\therefore n = \lambda$$

$$\gamma + \varepsilon = 2\cdot\llcorner$$

$$\gamma + m = 2\cdot\llcorner$$

$$\therefore m = \varepsilon$$

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Similarly, it can be shown that the angle m equals ε

Proposition 3 of Book IV

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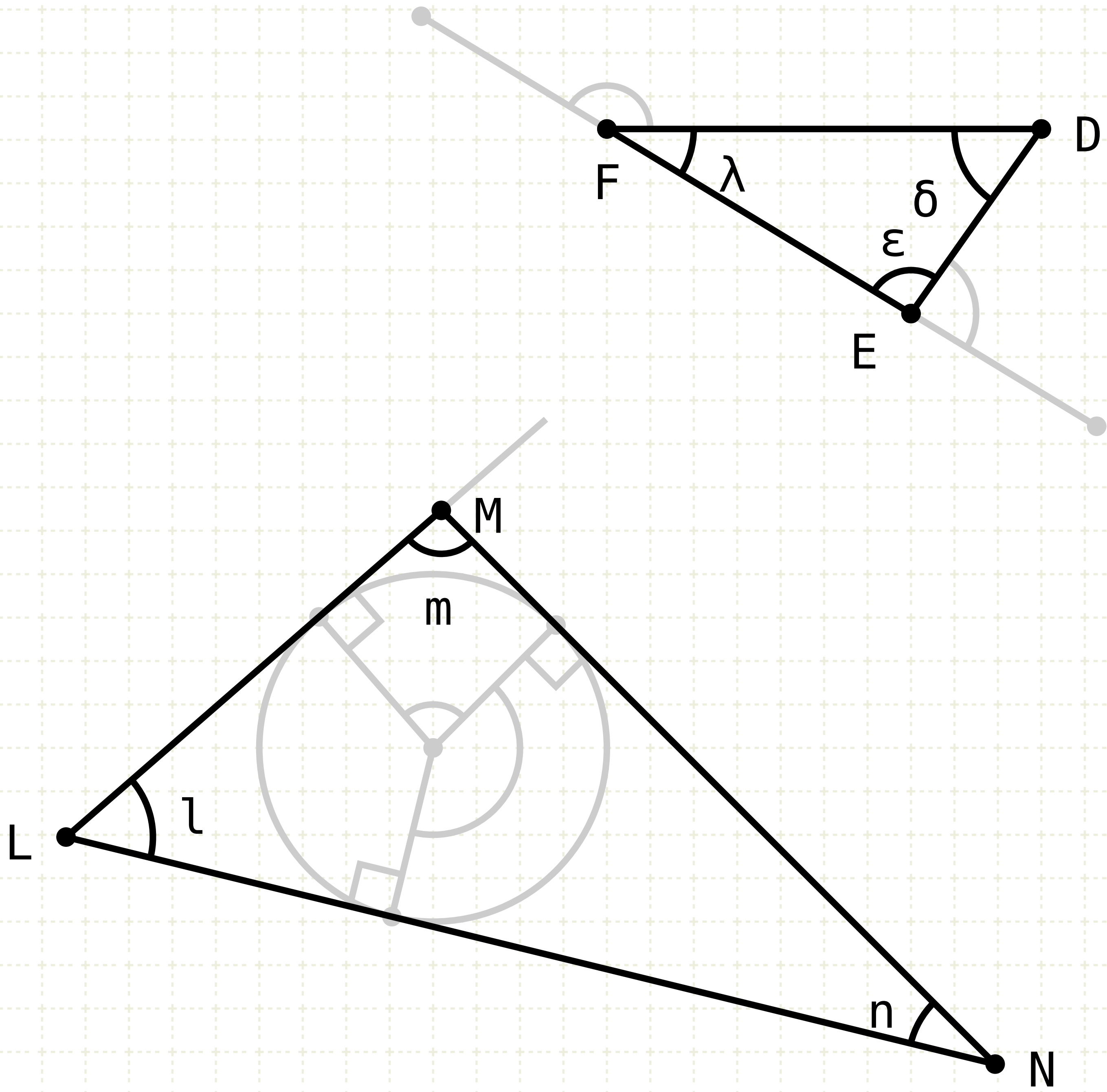

$$\alpha + n = 2\cdot\llcorner$$
$$\alpha + \lambda = 2\cdot\llcorner$$
$$\therefore n = \lambda$$
$$\gamma + \varepsilon = 2\cdot\llcorner$$
$$\gamma + m = 2\cdot\llcorner$$
$$\therefore m = \varepsilon$$
$$\varepsilon + \lambda + \delta = 2\cdot\llcorner$$
$$m + n + l = 2\cdot\llcorner$$
$$\therefore l = \delta$$

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Similarly, it can be shown that the angle m equals ε

Since the sum of all angles in a triangle is equal to two right angles, and the sum of m,n is equal to the sum of ε,λ (I·32)

… the angle l is equal to δ

Proposition 3 of Book IV

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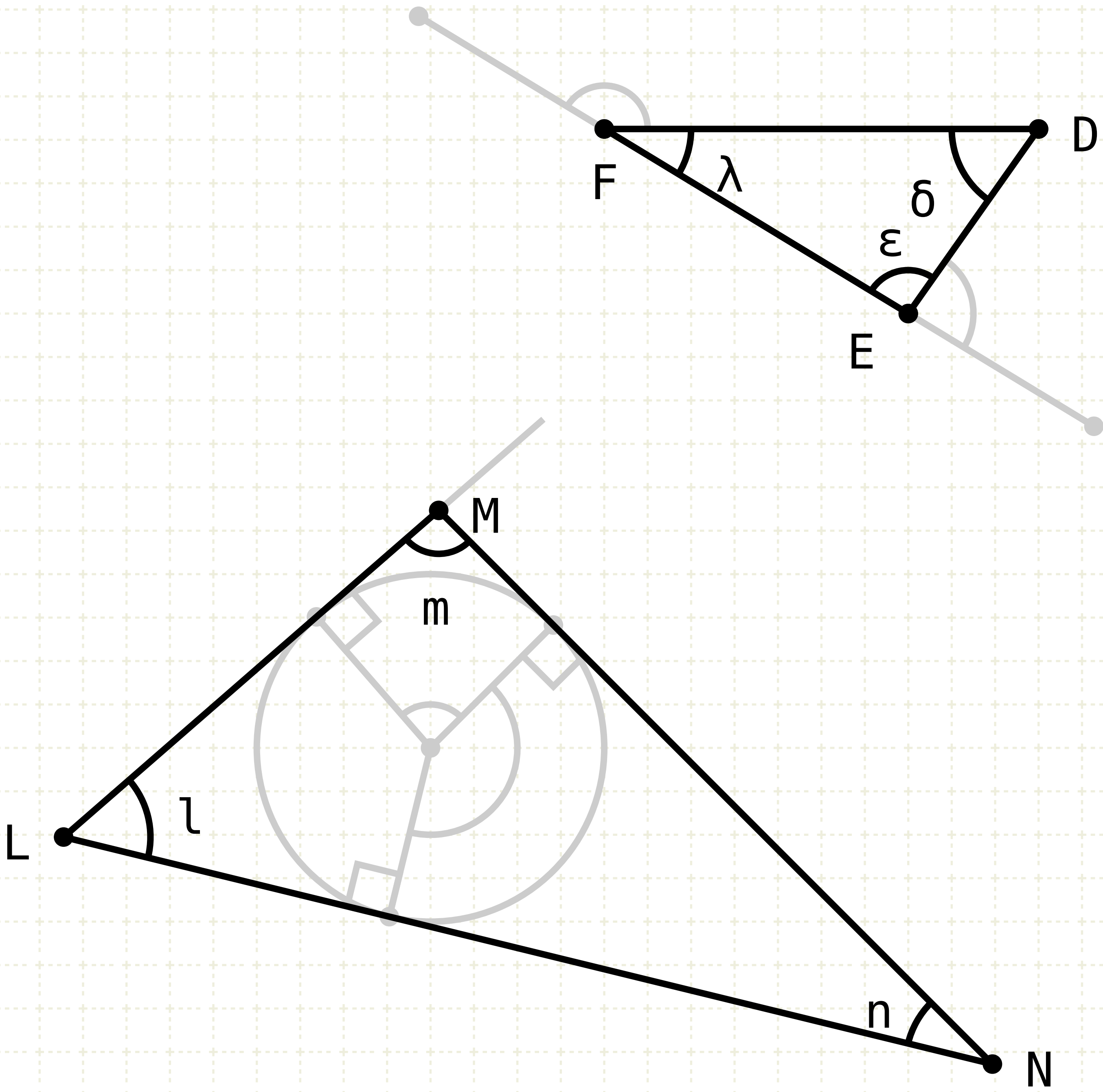

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$$\therefore n = \lambda$$
$$\gamma + \varepsilon = 2\cdot\llcorner$$
$$\gamma + m = 2\cdot\llcorner$$
$$\therefore m = \varepsilon$$
$$\varepsilon + \lambda + \delta = 2\cdot\llcorner$$
$$m + n + l = 2\cdot\llcorner$$
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