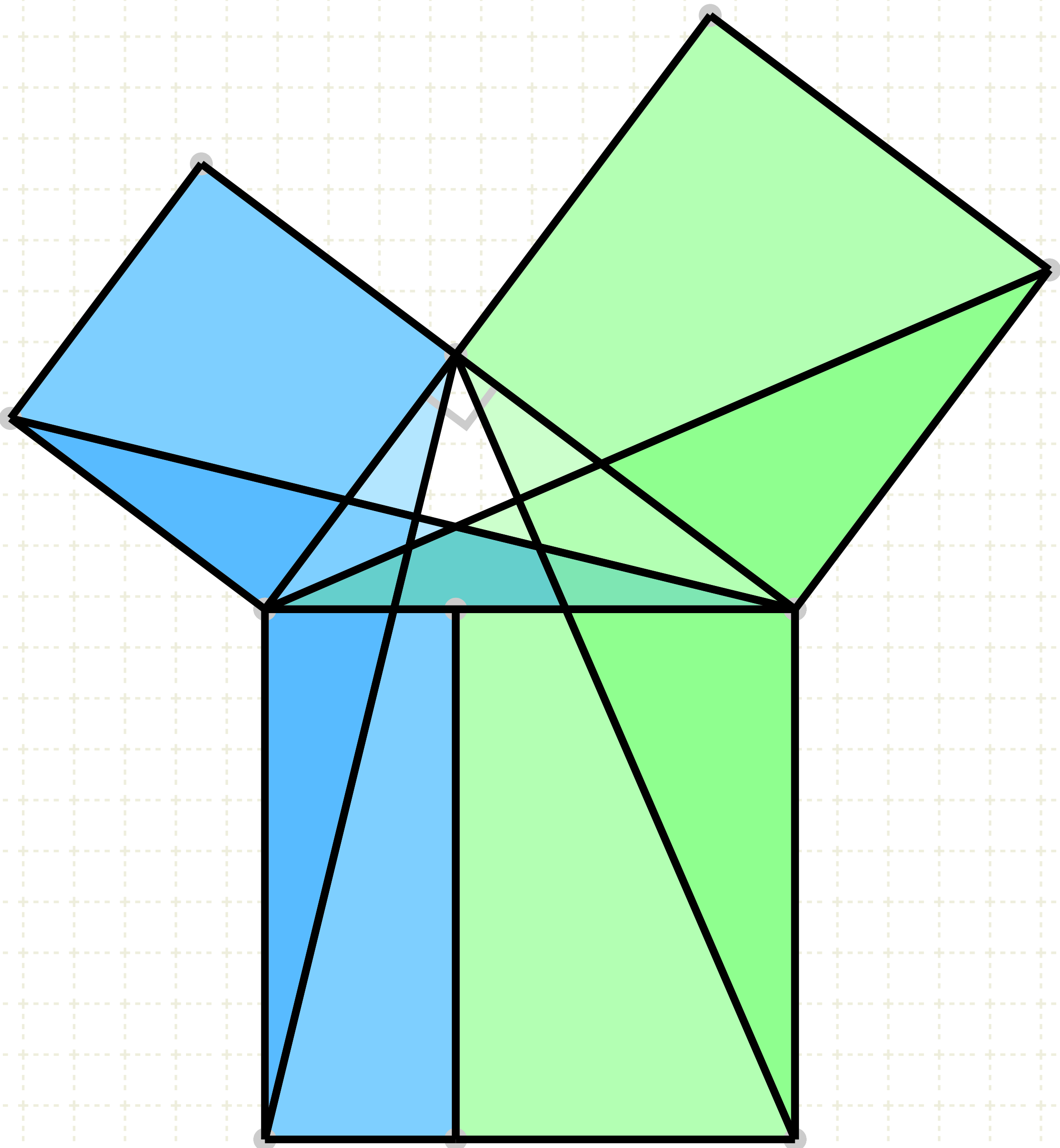
Euclid's Elements

Book I



If Euclid did not kindle your youthful enthusiasm, you were not born to be a scientific thinker.

Albert Einstein

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Proposition 27 of Book I

If a straight line falling on two straight lines makes the alternate angles equal to one another, then the straight lines are parallel to one another.

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Definition - Parallel Lines

Parallel straight lines are straight lines which, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction.

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Definition - Alternate Angles

If a line intersect two straight lines AB and CD at the points E and F, then the pairs of alternate angles are: AEF (α), DFE (δ) and CFE (γ), BEF (β)

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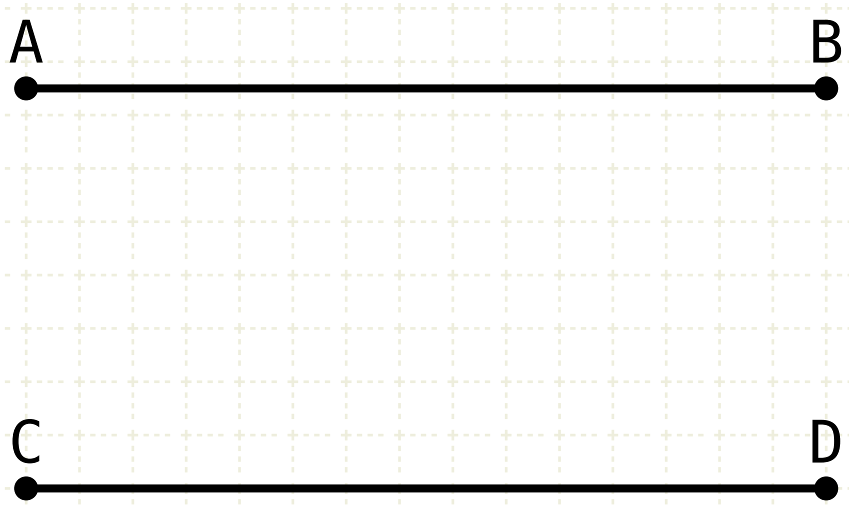

In other words

Start with two straight lines AB and CD

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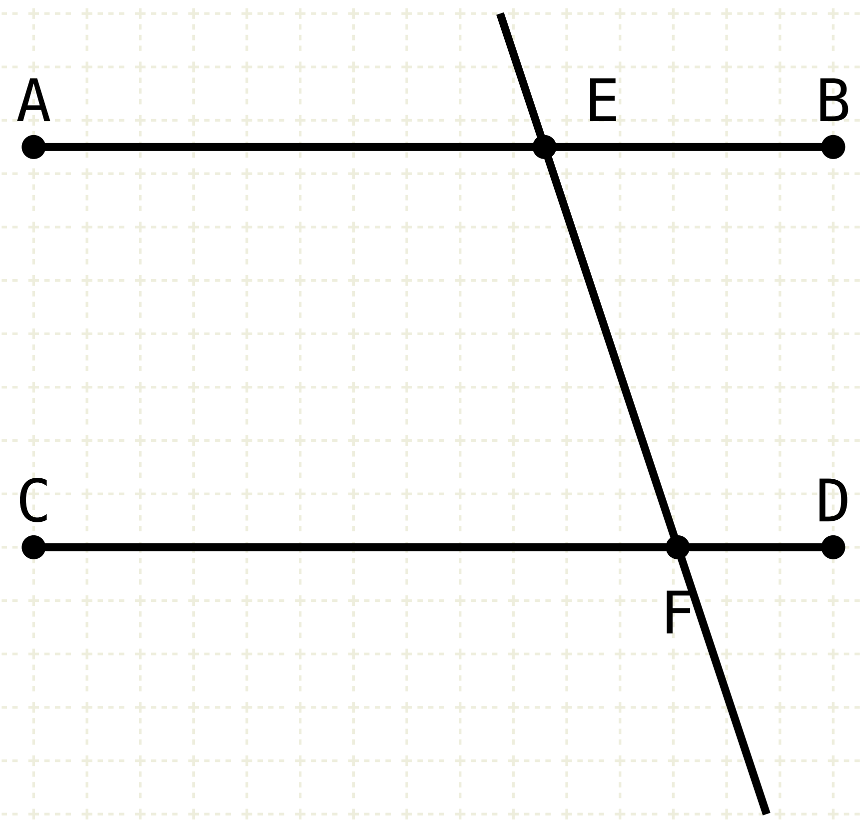

In other words

Start with two straight lines AB and CD

Construct a third line such that it intersects lines AB and CD at points E and F

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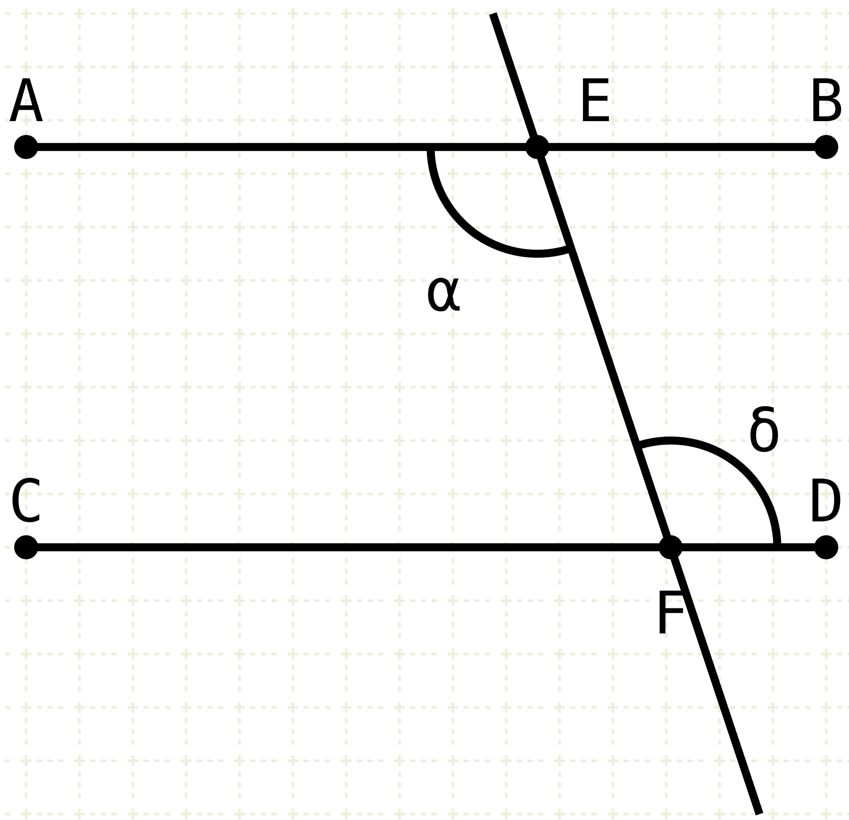
If a straight line falling on two straight lines makes the alternate angles equal to one another, then the straight lines are parallel to one another.

$$\text{if } \alpha = \delta$$
$$\Rightarrow AB \parallel CD$$



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Start with two straight lines AB and CD

Construct a third line such that it intersects lines AB and CD at points E and F

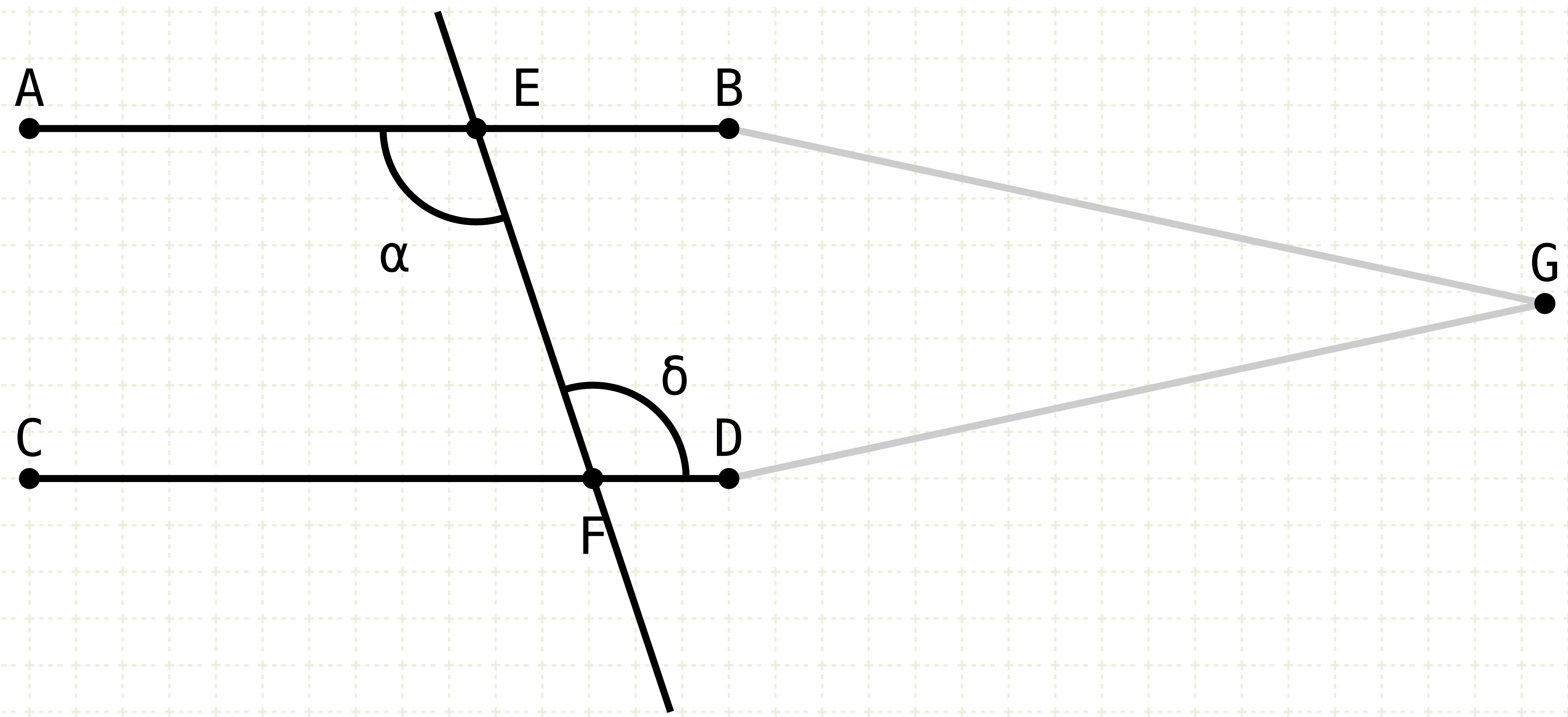
If angles AEF and EFD are equal, then the lines are parallel

Proposition 27 of Book I

If a straight line falling on two straight lines makes the alternate angles equal to one another, then the straight lines are parallel to one another.

$\alpha = \delta$

$\exists\ \Delta EFG$



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Proof by Contradiction

Assume that the lines intersect at point G

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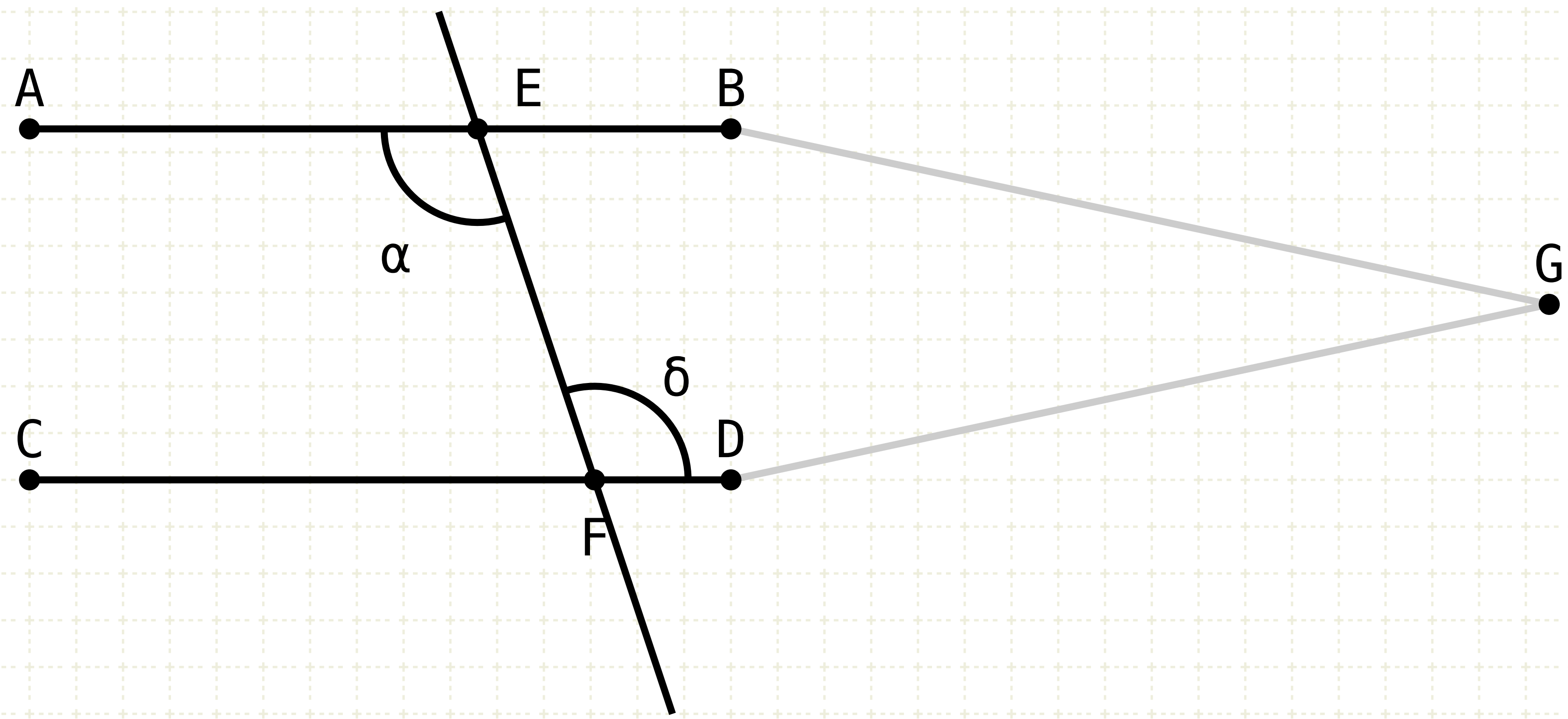

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Then angle AEF is an exterior angle to the triangle EFG, which means that angle AEF is larger than angle EFG (I·16)

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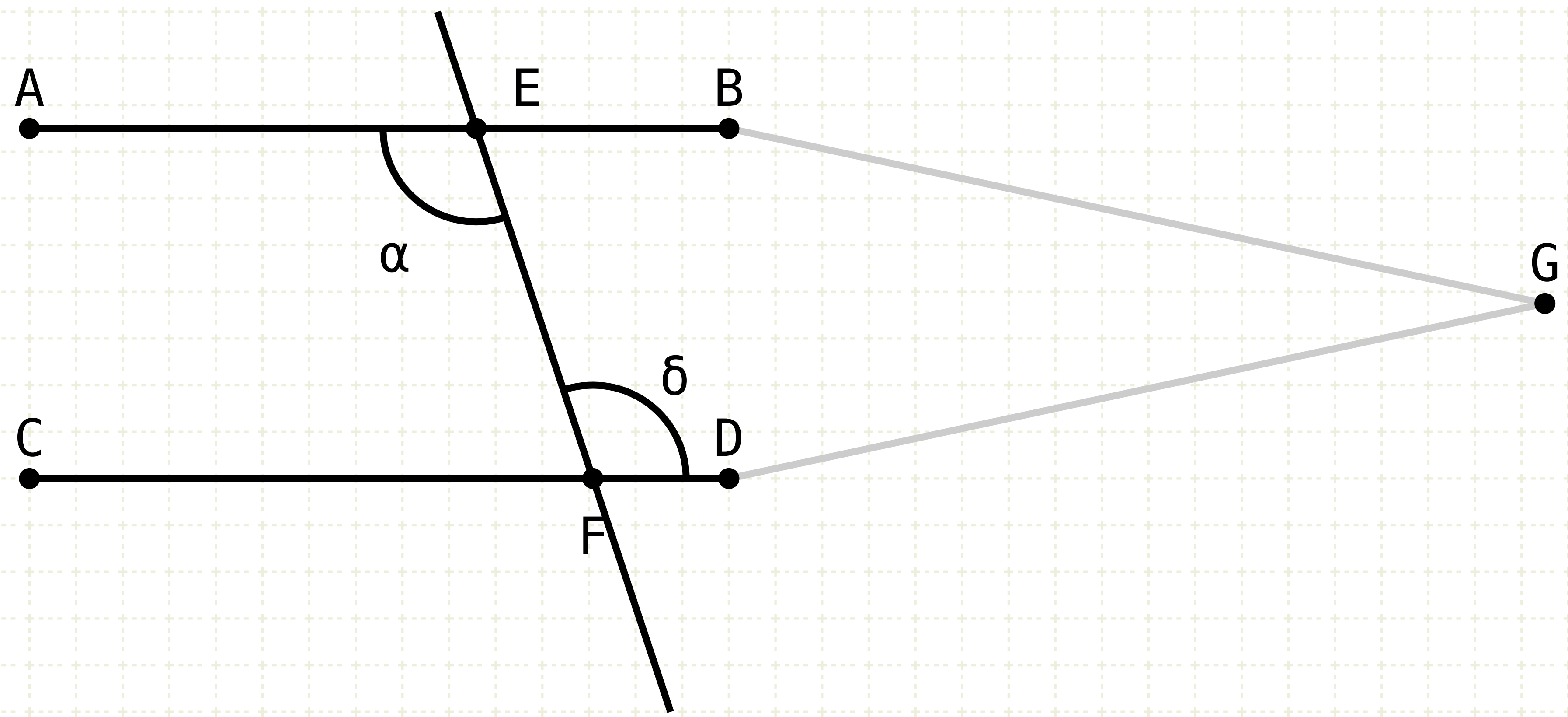

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But AEF equals EFG, so there is a contradiction

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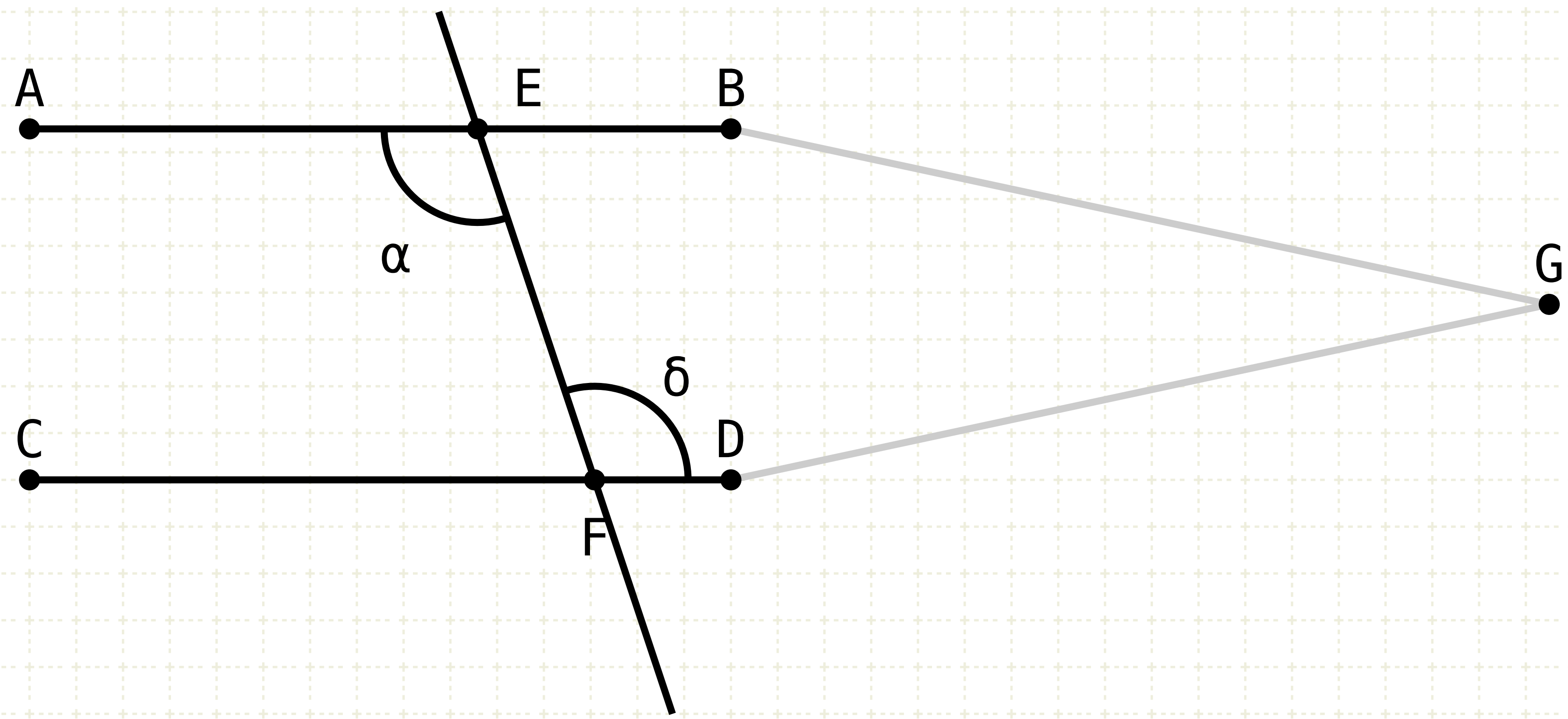

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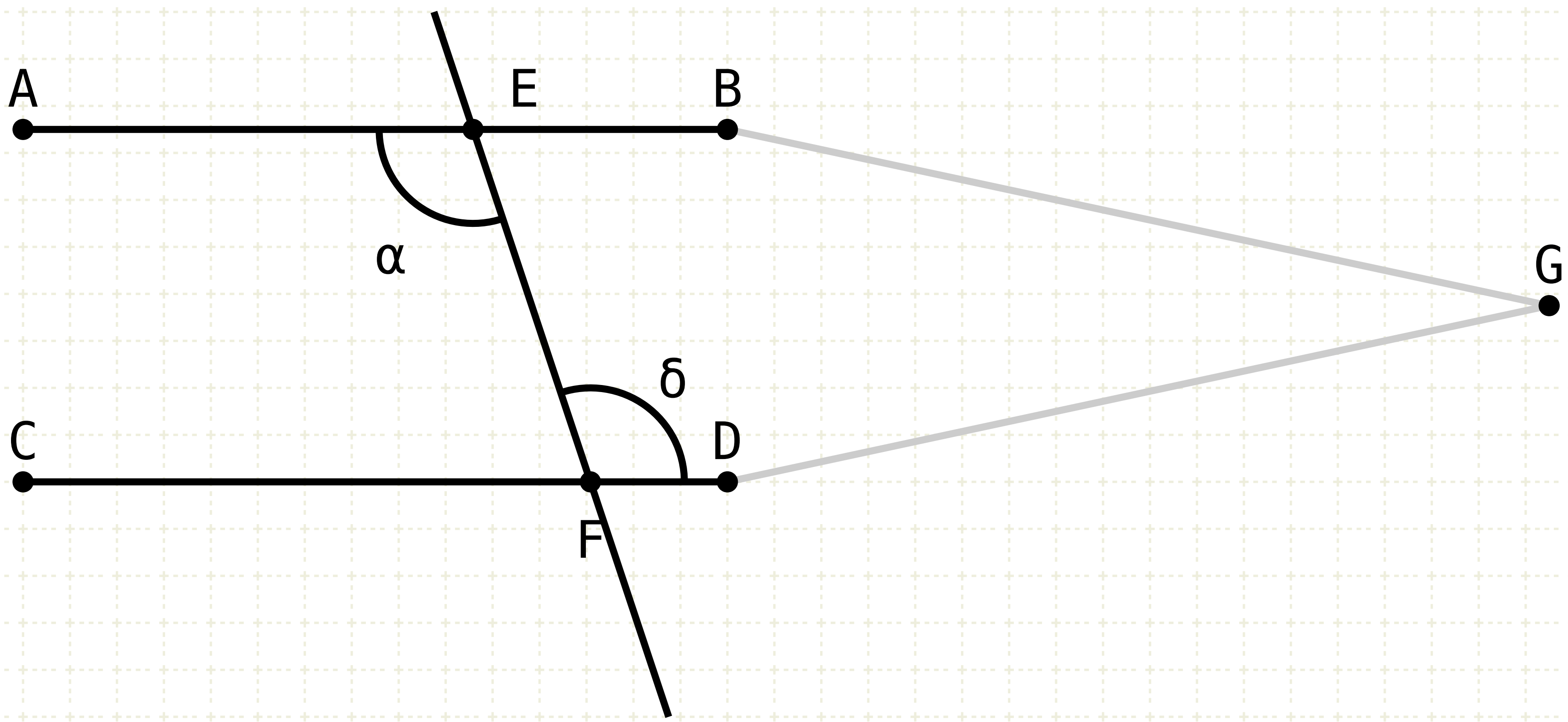

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The two lines can never meet at point G, and are therefore parallel

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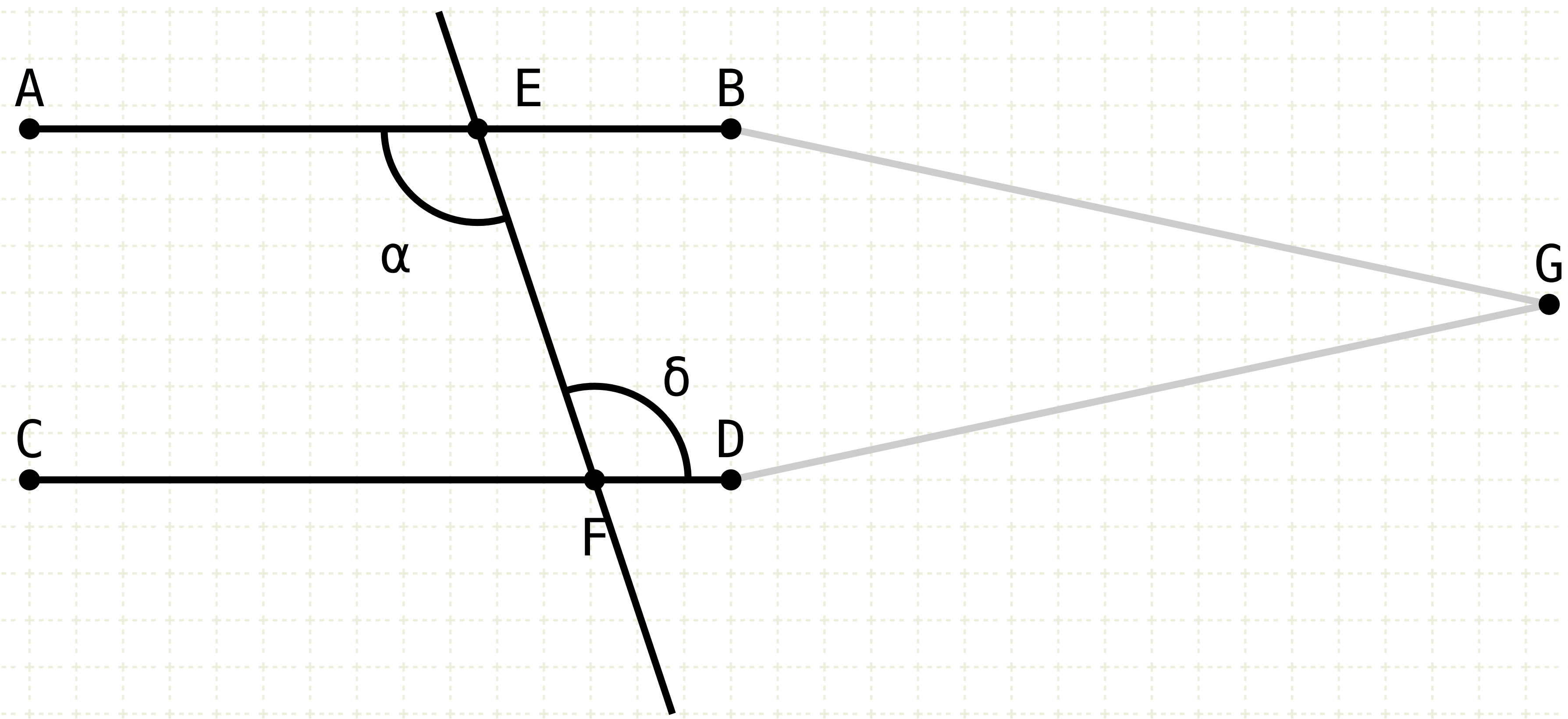

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