

# Euclid's Elements

## Book III



*A circle is a round straight line with a hole in the middle.*

**Mark Twain**

quoting a schoolchild in "-English as She Is Taught-"

*If people stand in a circle long enough, they'll eventually begin to dance.*

**George Carlin, Napalm and Silly Putty (2001)**



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2	A chord of a circle always lies inside the circle	10	A circle does not cut a circle at more points than two	18	If line touches a circle, then it is perpendicular to the diameter that touches that point
3	A line through the centre of a circle bisects a chord, and vice versa	11	Point of contact between two internal circles, and their centres, are collinear	19	If line touches a circle, then the centre of the circle lies on a line perpendicular to the original
4	A line not through the centre of a circle does not bisect a chord	12	Point of contact between two external circles, and their centres, are collinear	20	The angle at the centre of a circle is twice that from an angle from the circumference
5	If two circles cut one another, they will not have the same center	13	A circle does not touch a circle at more points than one, whether it touch it internally or externally.	21	In a circle the angles in the same segment are equal to one another
6	If two circles touch one another, they will not have the same center	14	In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another.	22	The opposite angles of quadrilaterals in circles are equal to two right angles
7	Consider two lines from a point inside a circle to the edge, the longer one will be the one closest to the longest part of the diameter passing through the original point	15	The longest line in a circle is its diameter, shorter the farther away from the diameter	23	On the same straight line there cannot be constructed two similar and unequal segments of circles on the same side
8	Consider two lines from a point outside a circle to the edge, the line closest to the centre will be longer on the concave side and shorter on the convex side	16	A line on the circle, perpendicular to the diameter, lies outside the circle	24	Similar segments of circles on equal straight lines are equal to one another



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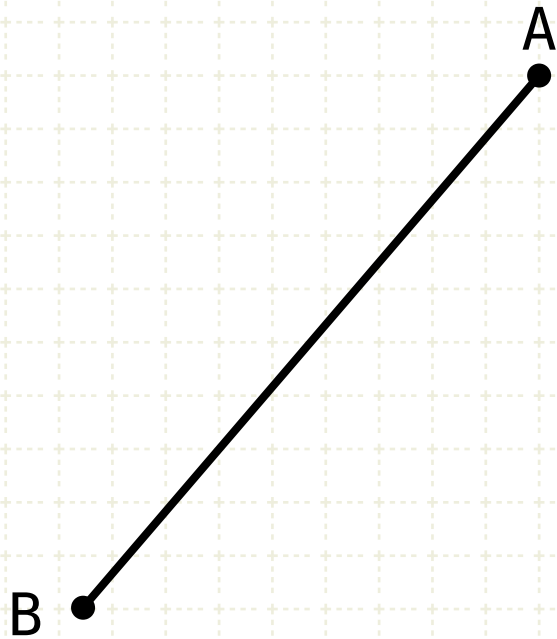
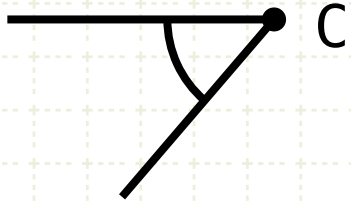
# Proposition 33 of Book III

On a given straight line to describe a segment of a circle admitting an angle equal to a given rectilineal angle.



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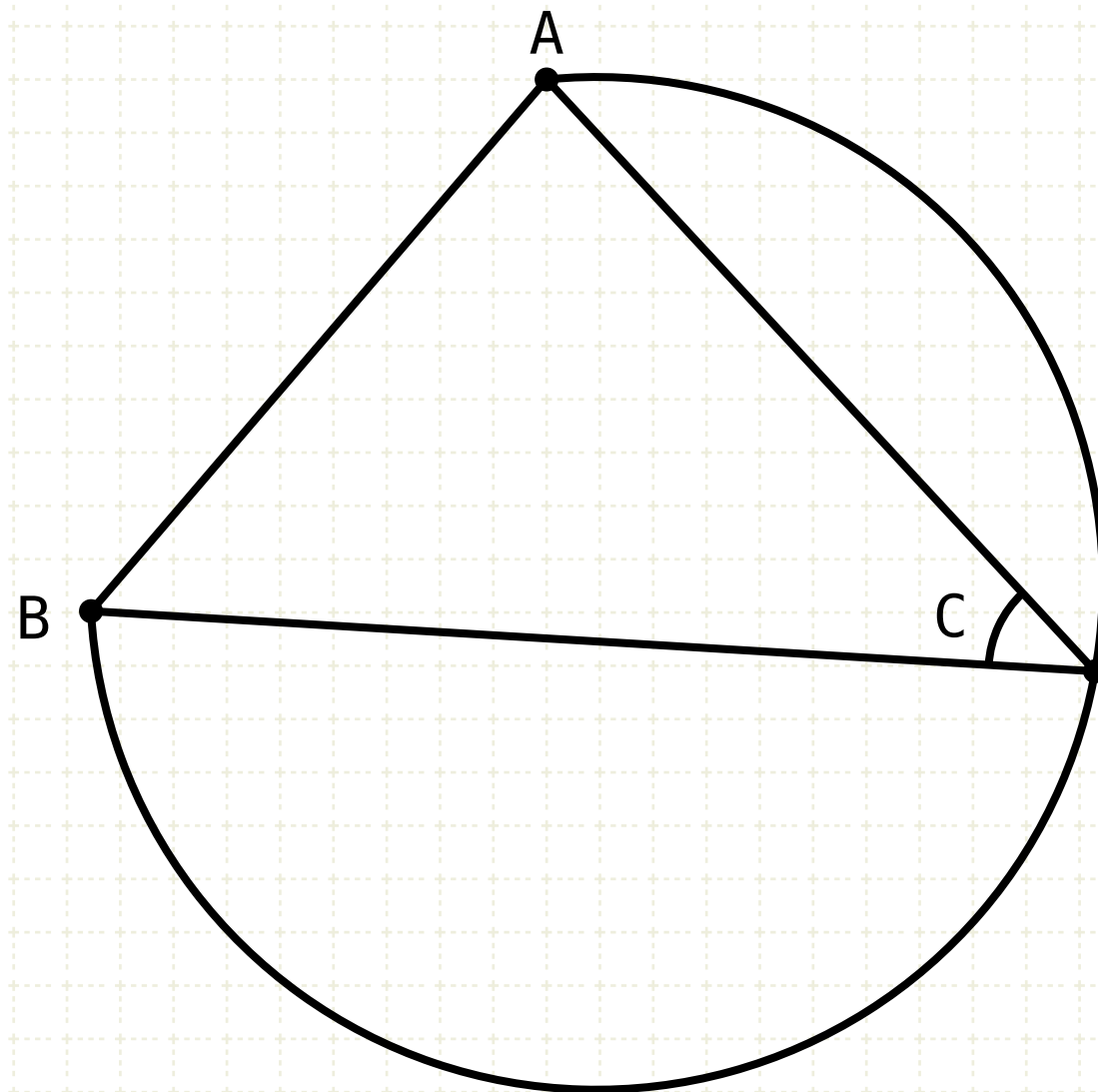
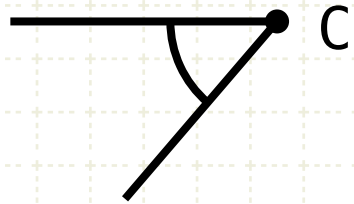
### In other words

Given an angle C, and a line segment AB...



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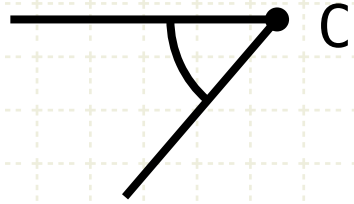
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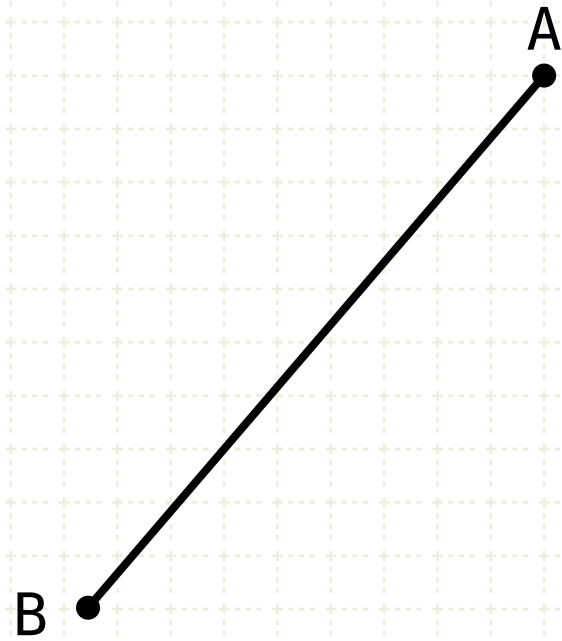
... construct a circle segment such that the angle within the segment is equal to C

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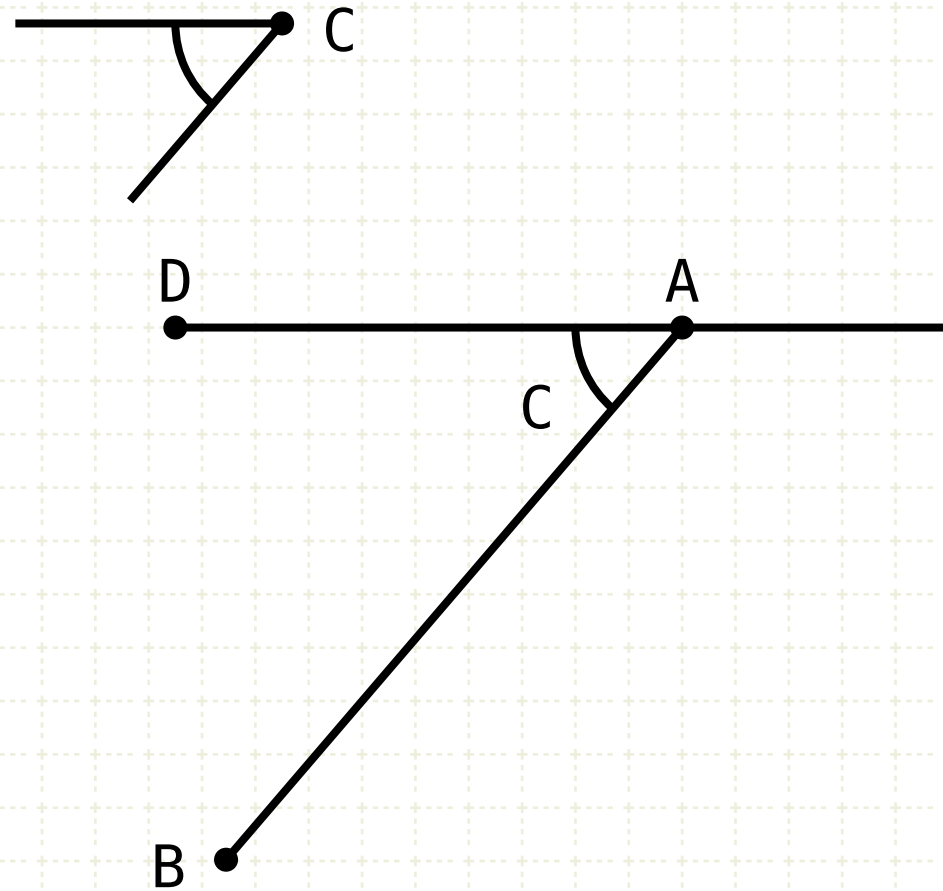


## Construction (Acute Angle)



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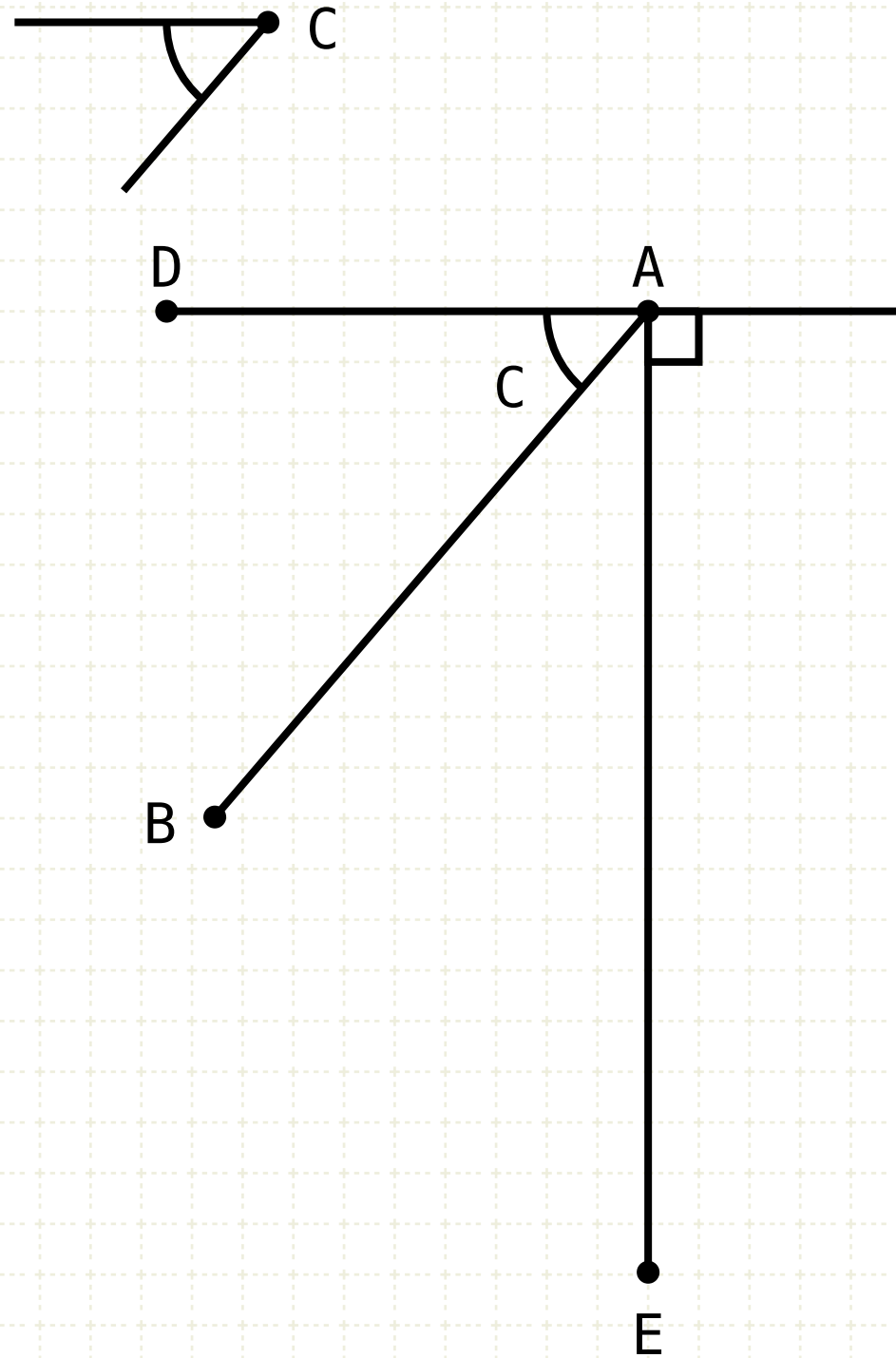
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Copy the angle C to the line AB, at point A



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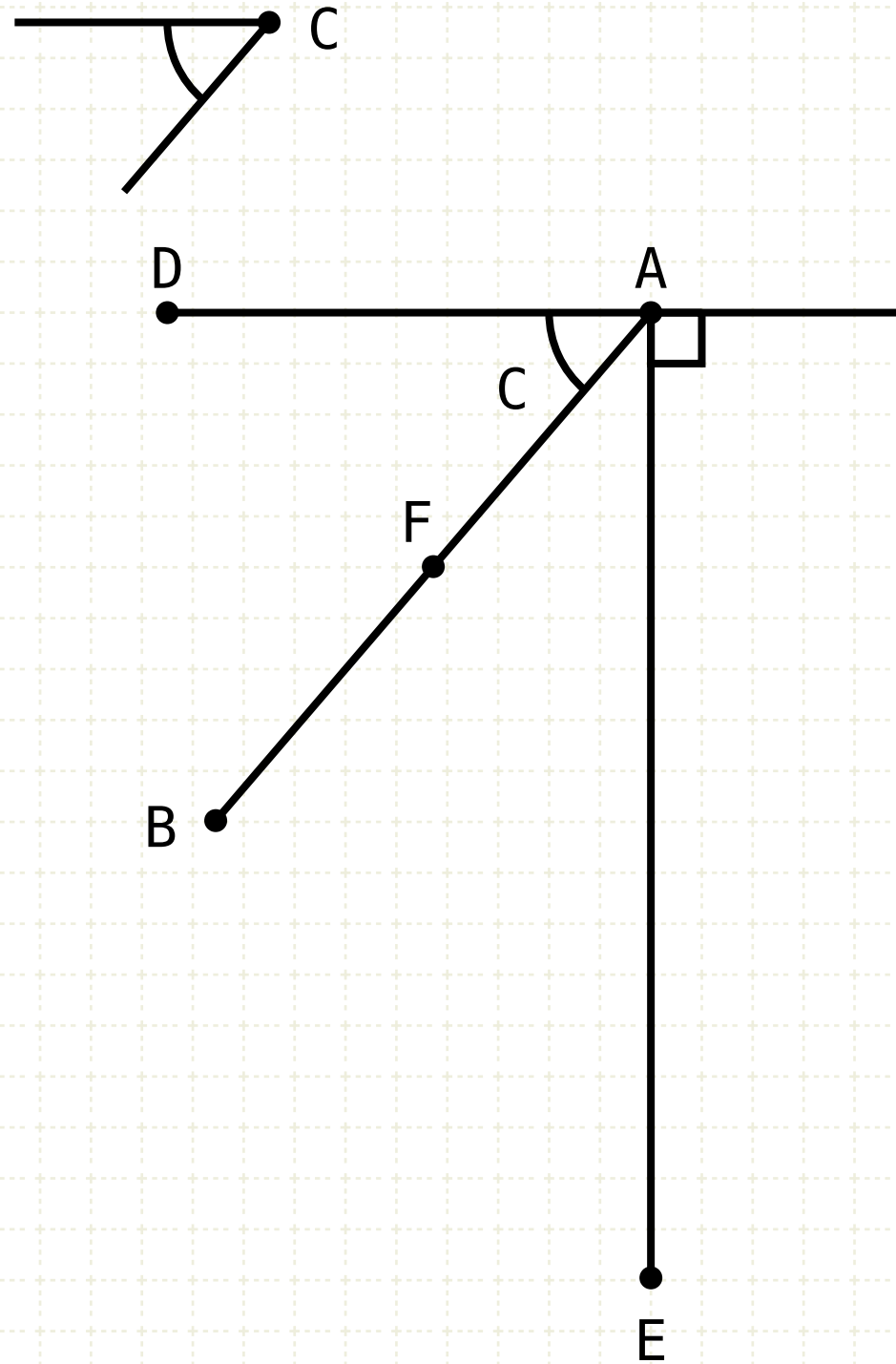
## Construction (Acute Angle)

Copy the angle C to the line AB, at point A

Draw a line perpendicular to AD, from point A

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$$BF = FA$$

## Construction (Acute Angle)

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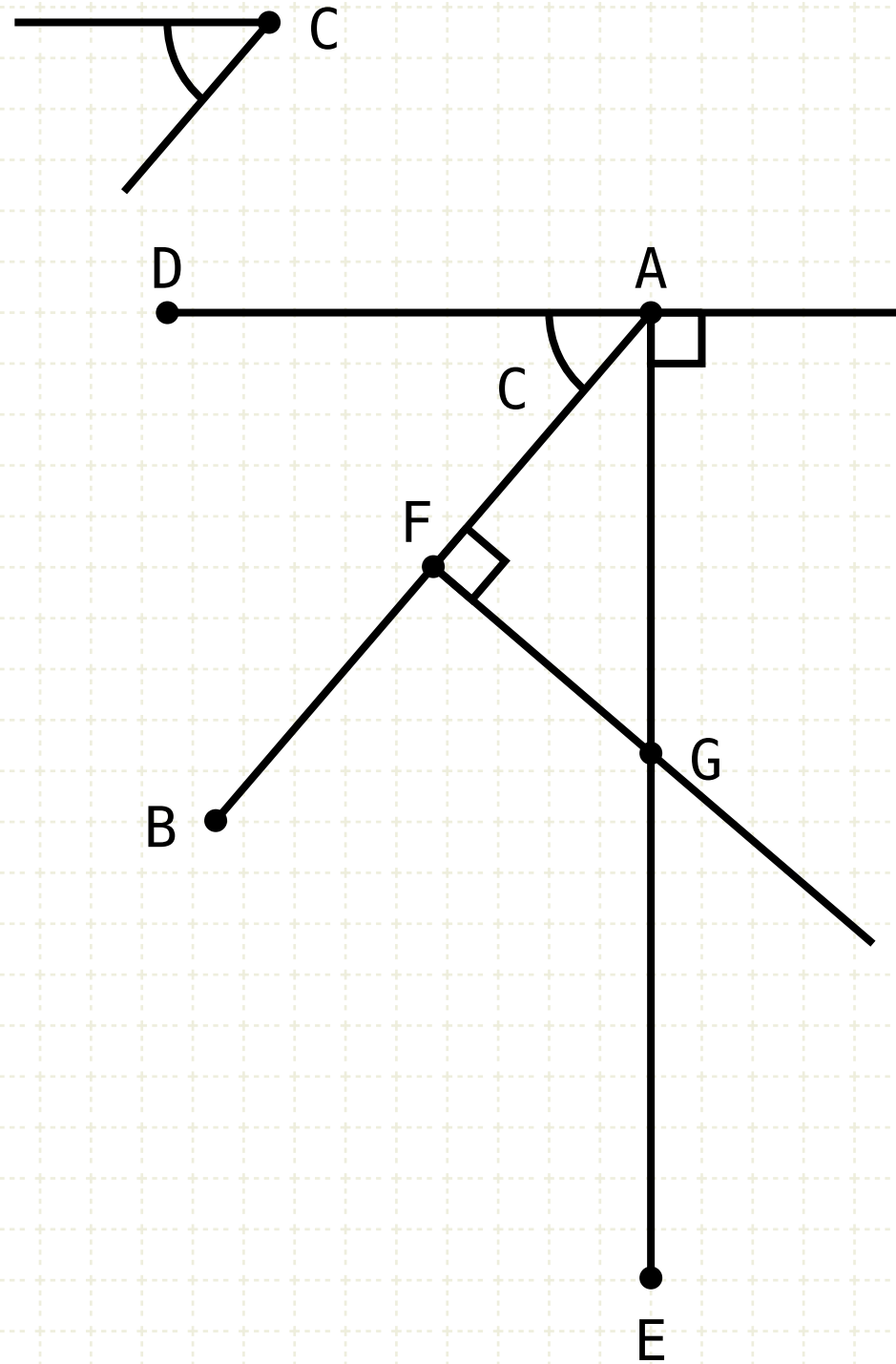
Draw a line perpendicular to AD, from point A

Bisect line AB at point F



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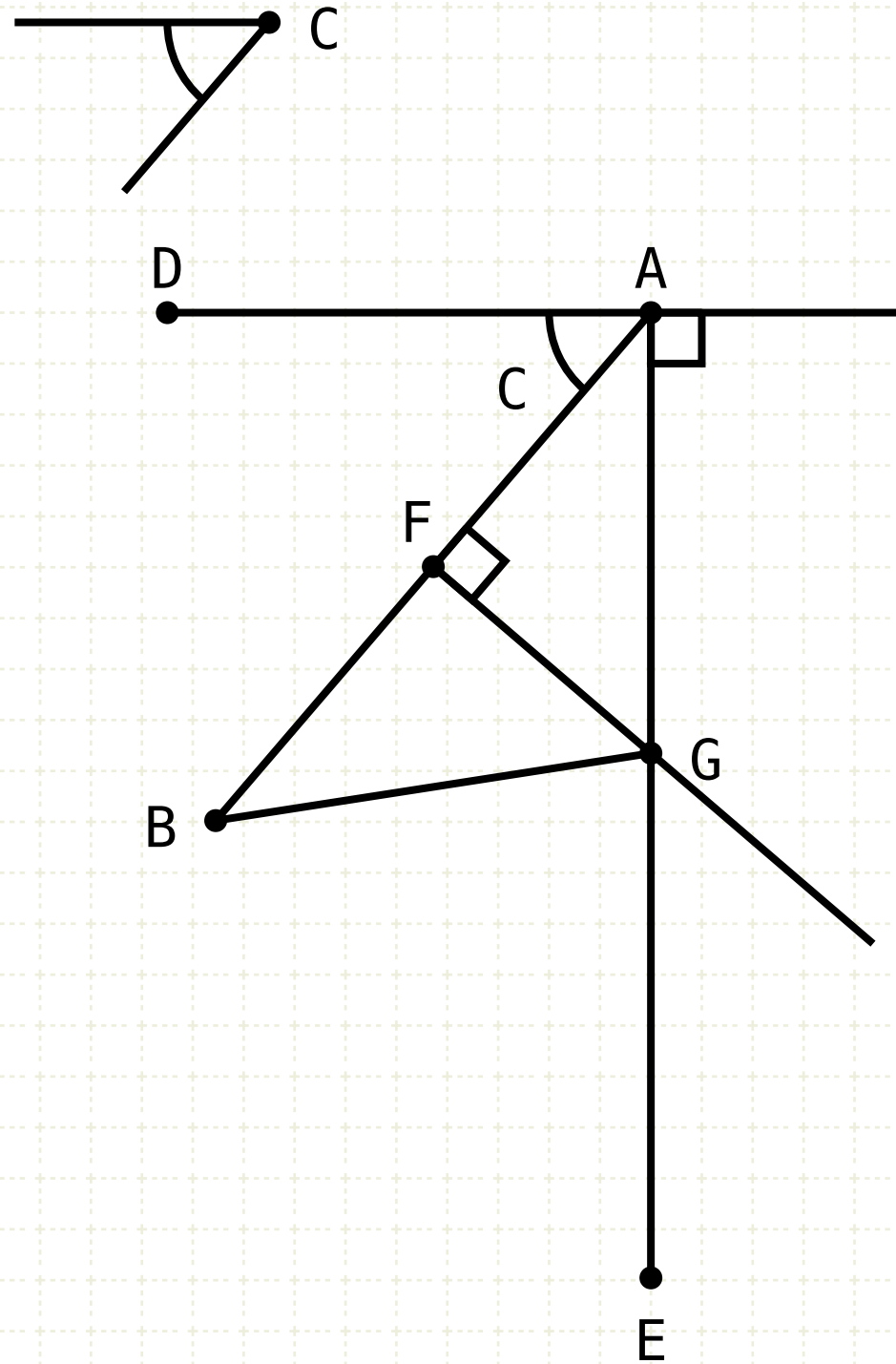
Bisect line AB at point F

Draw a line FG perpendicular to AB from point F, where G is the intersection between this line and AE



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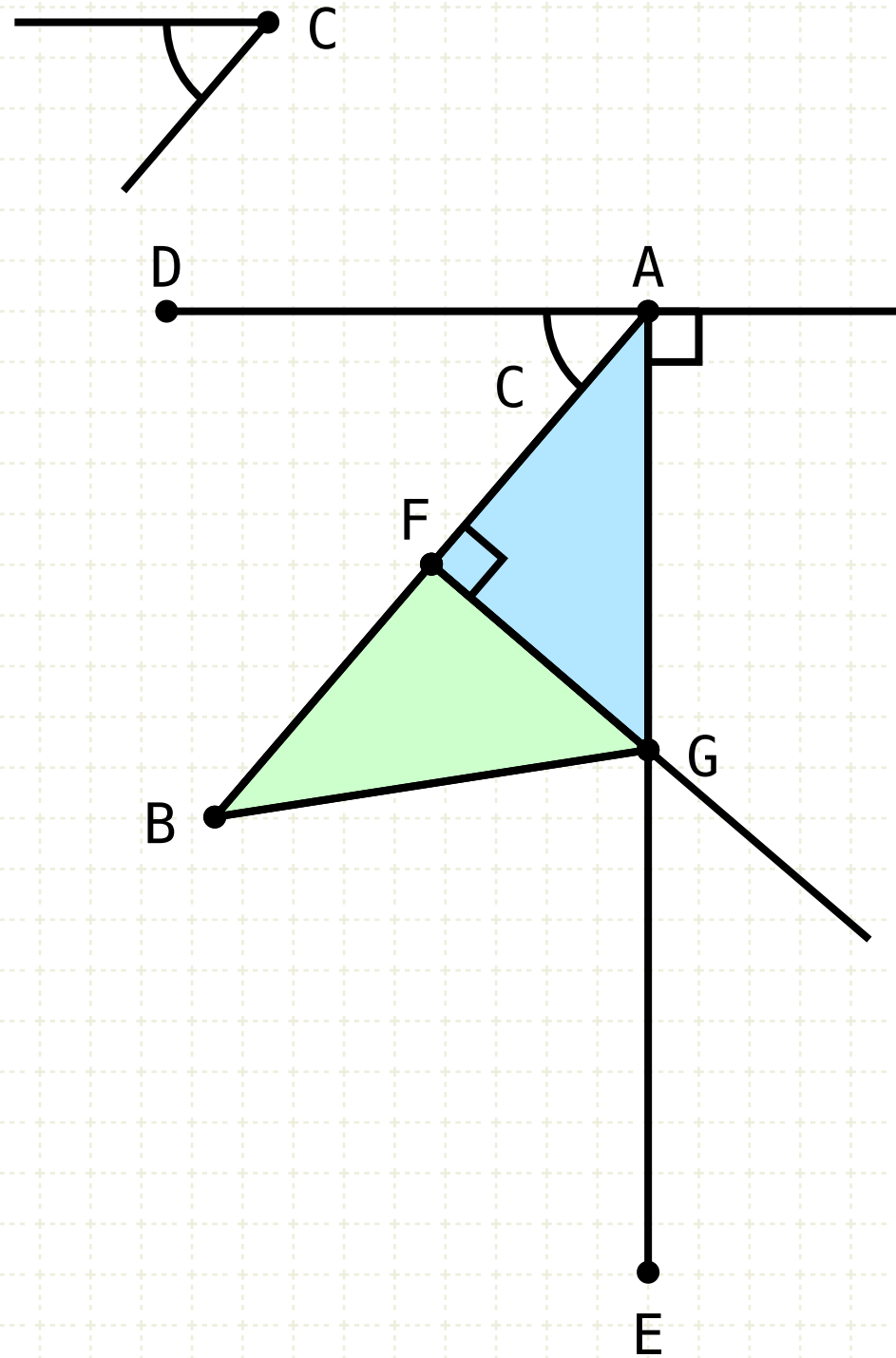
Bisect line AB at point F

Draw a line FG perpendicular to AB from point F, where G is the intersection between this line and AE

Draw line BG

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$$BF = FA$$

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### Construction (Acute Angle)

Copy the angle C to the line AB, at point A

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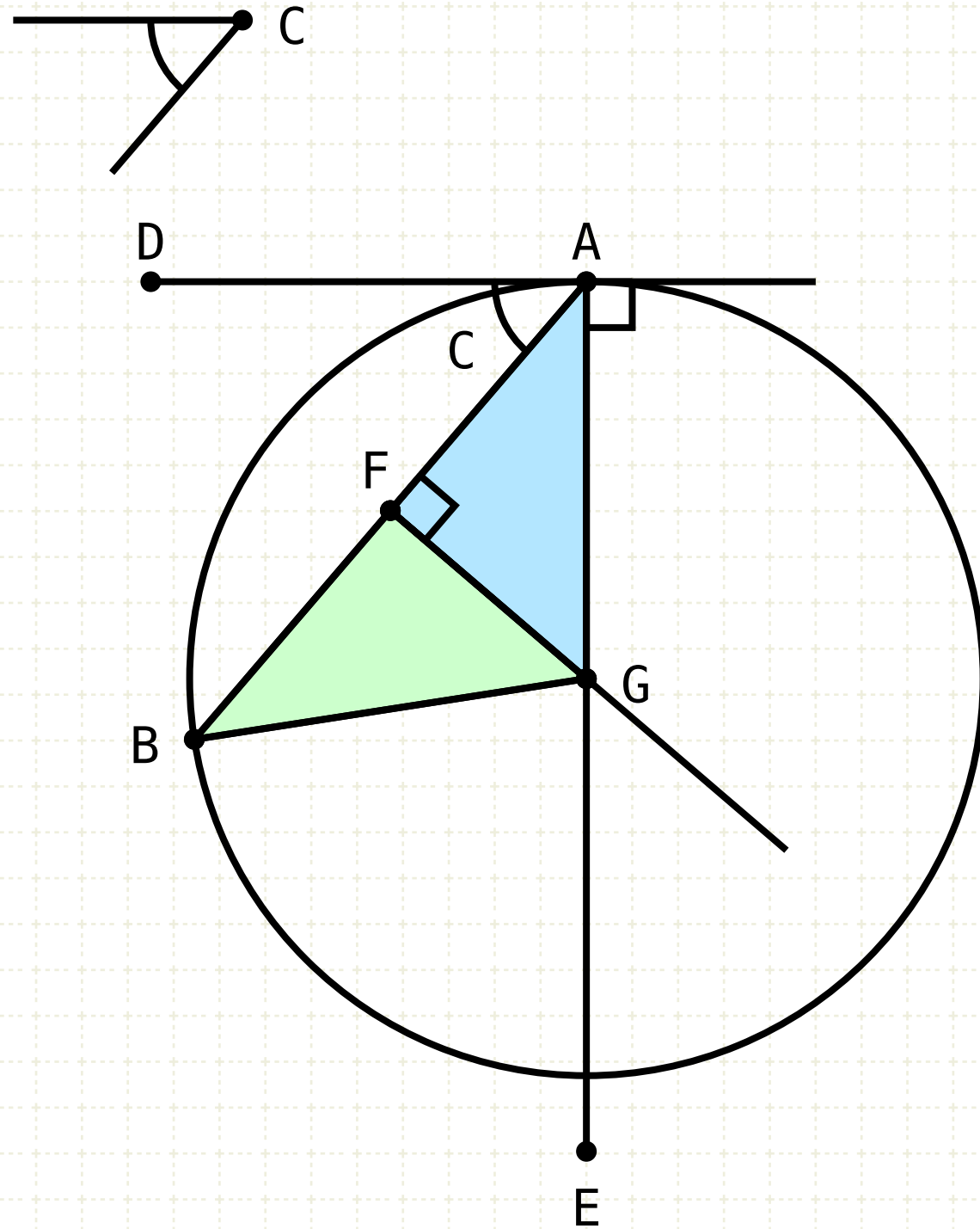
Draw a line FG perpendicular to AB from point F, where G is the intersection between this line and AE

Draw line BG

Since BF equals FA and FG is common, and the angles AFG equals BFG, then the two triangles are equal (I-4), and the lines BG and AG are equal

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## Construction (Acute Angle)

Copy the angle C to the line AB, at point A

Draw a line perpendicular to AD, from point A

Bisect line AB at point F

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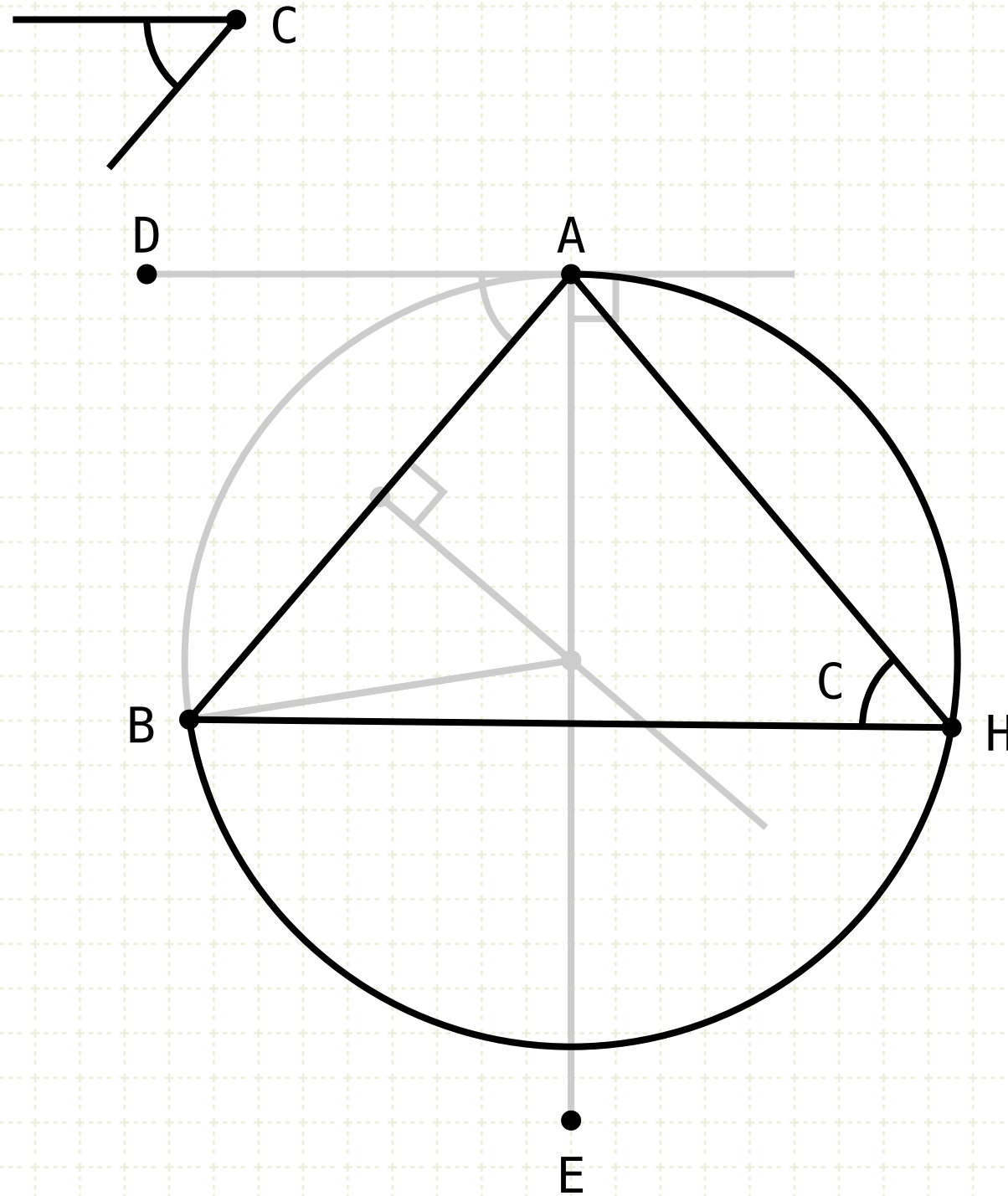
Since BF equals FA and FG is common, and the angles AFG equals BFG, then the two triangles are equal (I-4), and the lines BG and AG are equal

Thus, drawing a circle with centre G and radius AG will pass through points A and B



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$$BF = FA$$
$$BG = AG$$

## Construction (Acute Angle)

Copy the angle C to the line AB, at point A

Draw a line perpendicular to AD, from point A

Bisect line AB at point F

Draw a line  $FG$  perpendicular to  $AB$  from point  $F$ , where  $G$  is the intersection between this line and  $AE$

Draw line BG

Since BF equals FA and FG is common, and the angles AFG equals BFG, then the two triangles are equal (I-4), and the lines BG and AG are equal

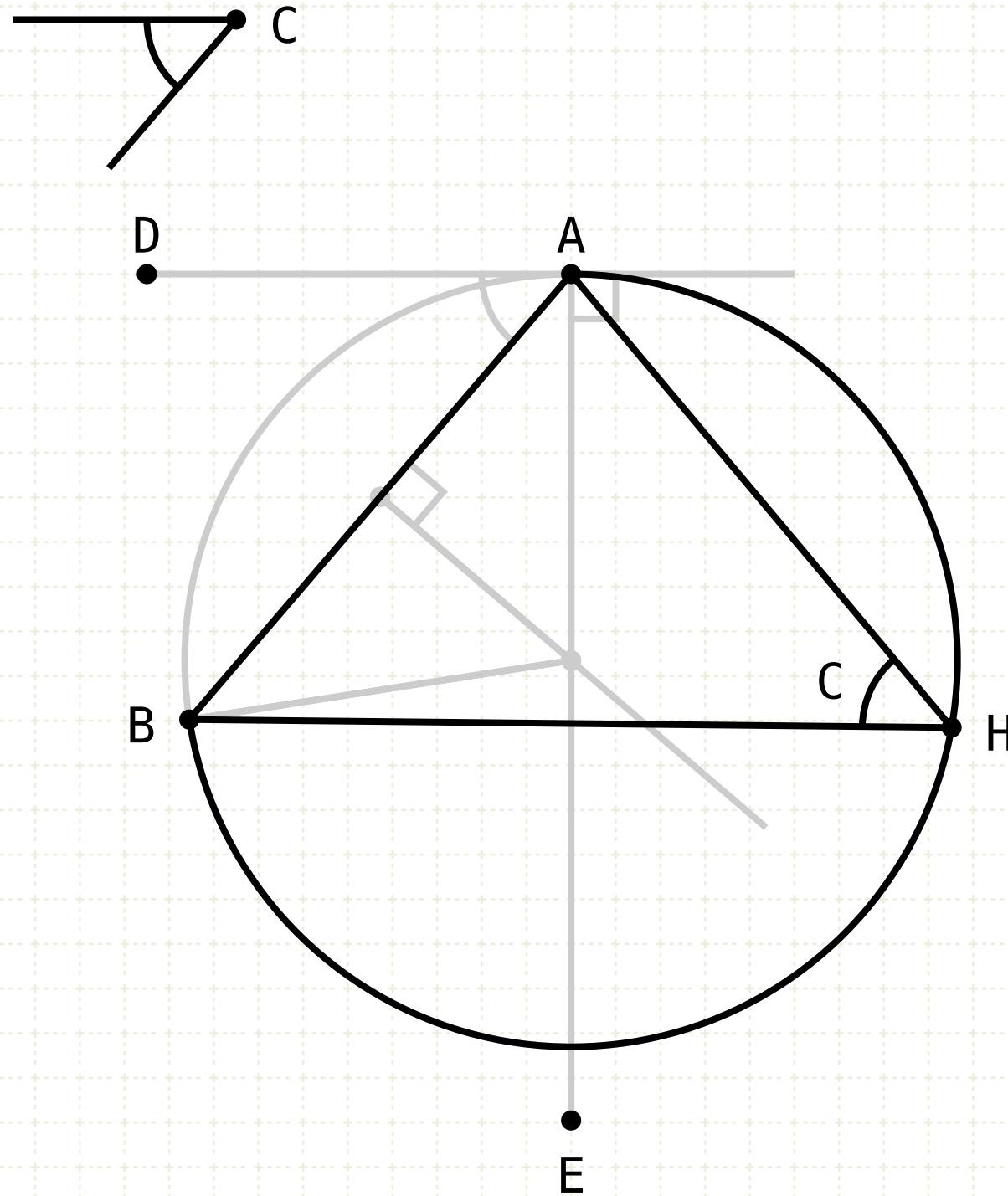
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The circle segment BHA contains the angle C



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Draw line BG

Since BF equals FA and FG is common, and the angles AFG equals BFG, then the two triangles are equal (I-4), and the lines BG and AG are equal

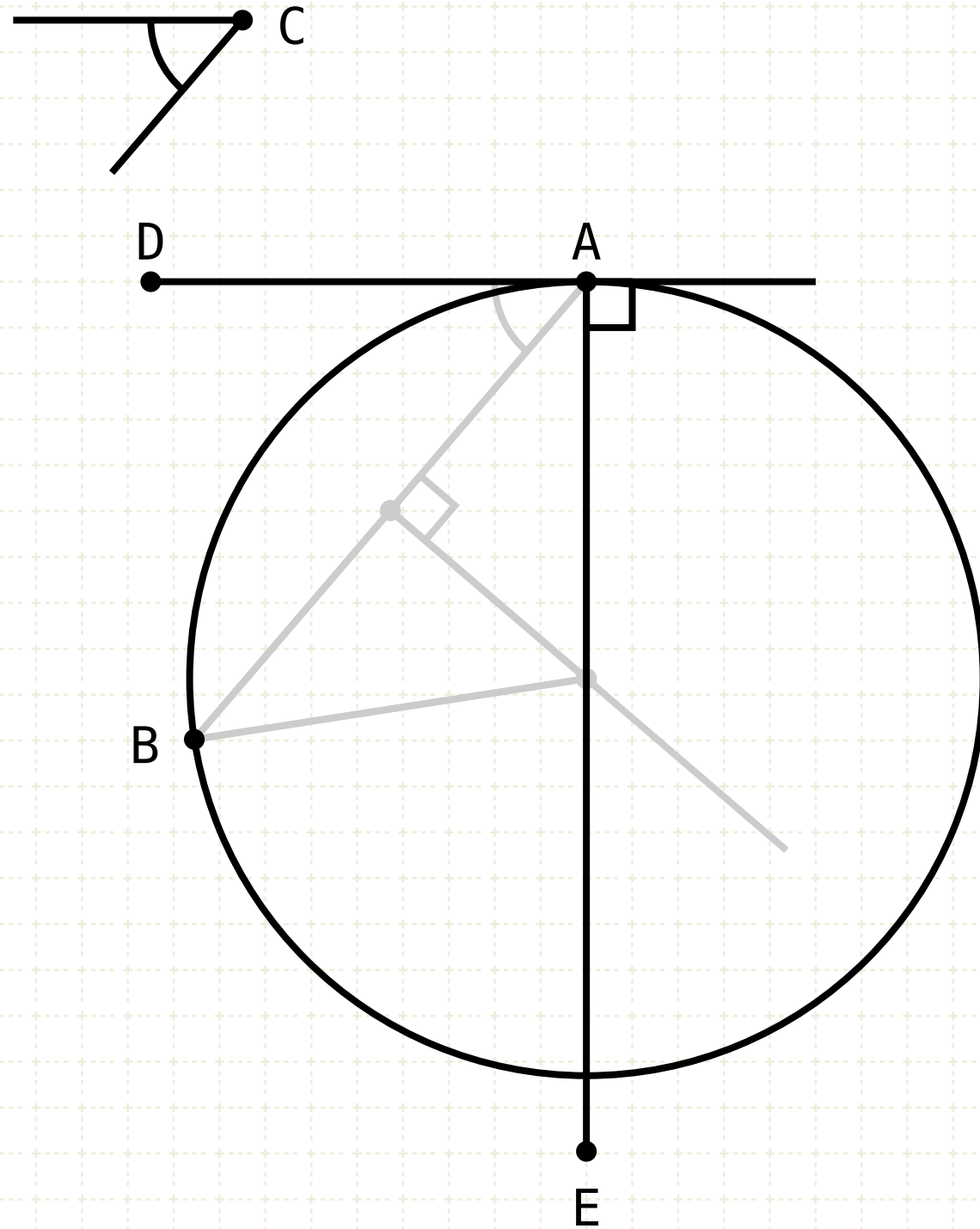
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## Proof

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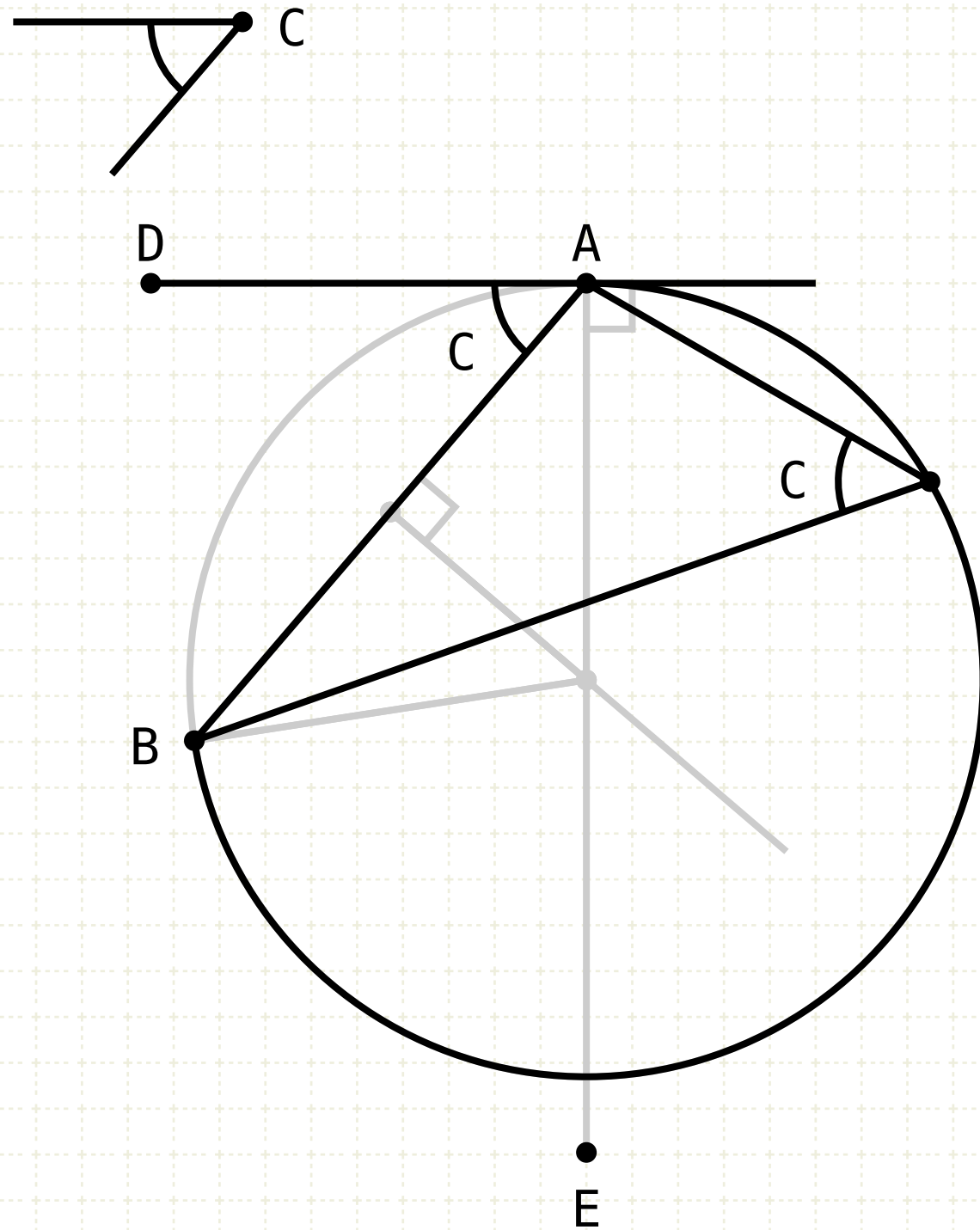
## Proof

The line AD is at the extremity of the circle diameter, and is at right angles to the diameter, thus the line AD touches the circle (III·16)



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Copy the angle C to the line AB, at point A

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## Proof

The line AD is at the extremity of the circle diameter, and is at right angles to the diameter, thus the line AD touches the circle (III·16)

Since AD touches the circle, the angle DAB equals the angle in the opposite circle segment (III·32)

The angle DAB is equal to C by construction, so thus the angle in the segment BEA equals C



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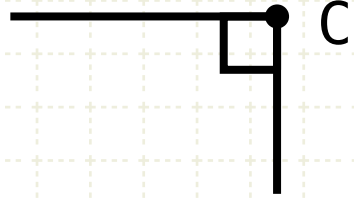
## Construction (Right Angle)





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## Construction (Right Angle)

A



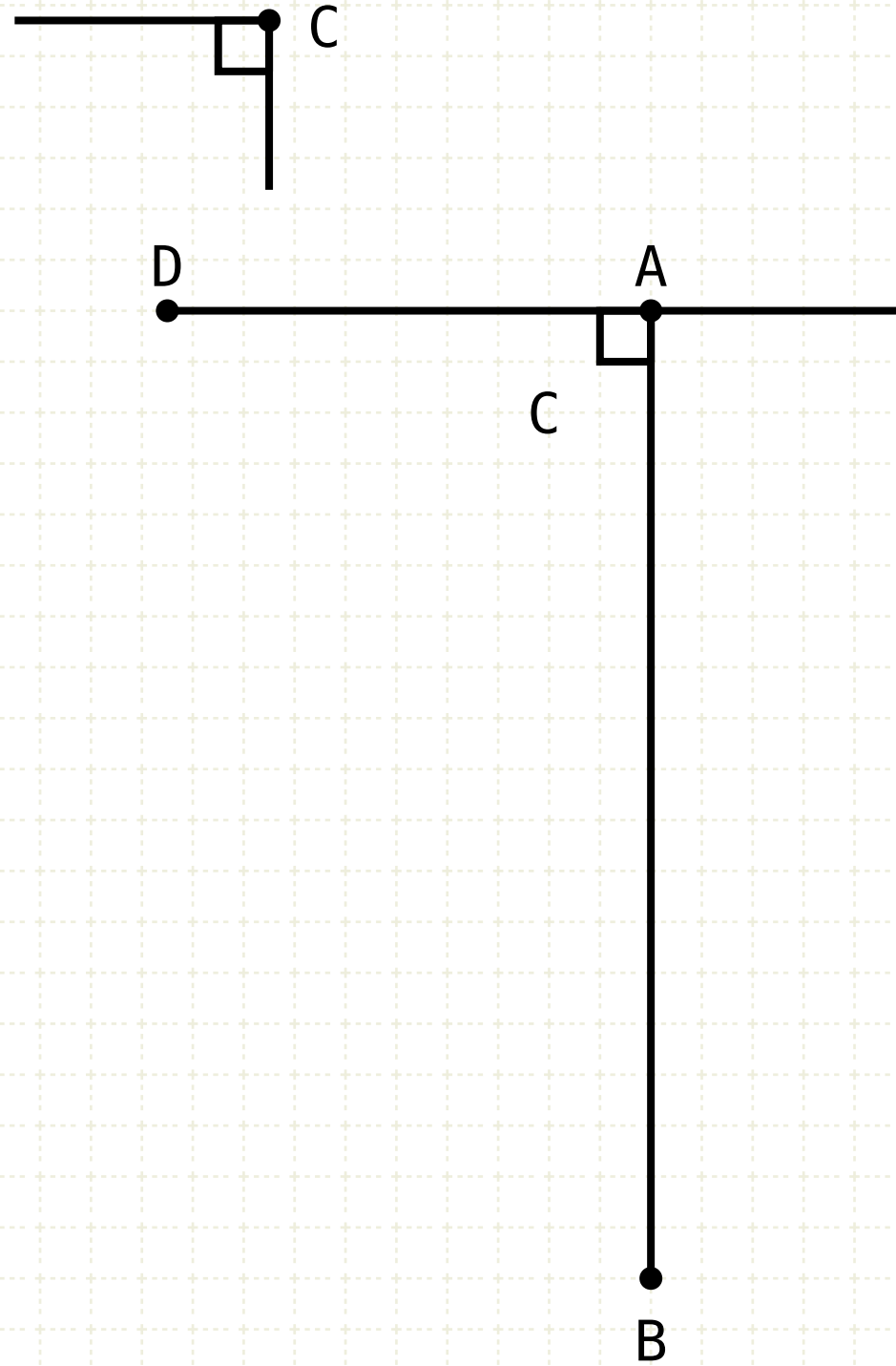
B





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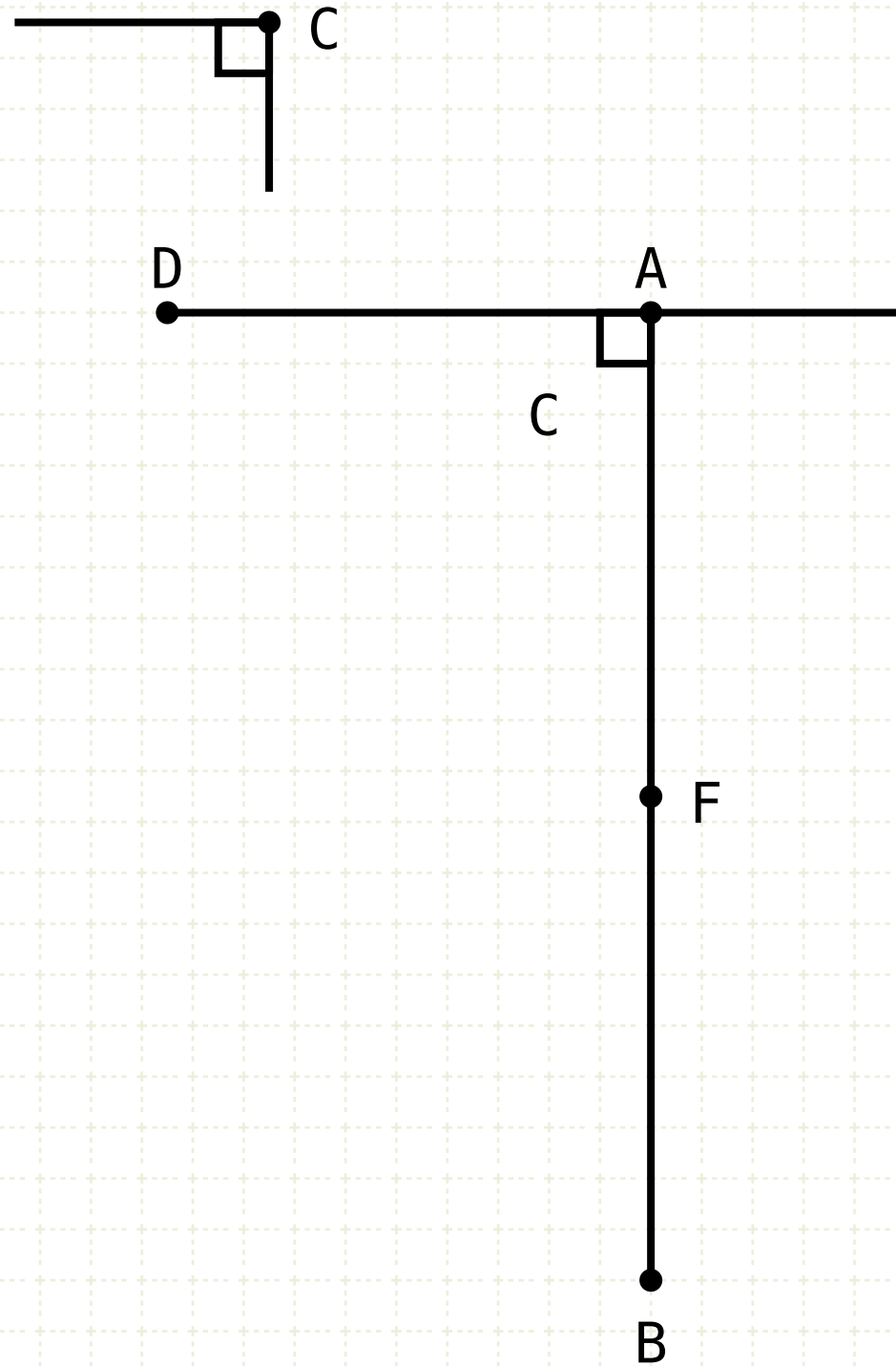


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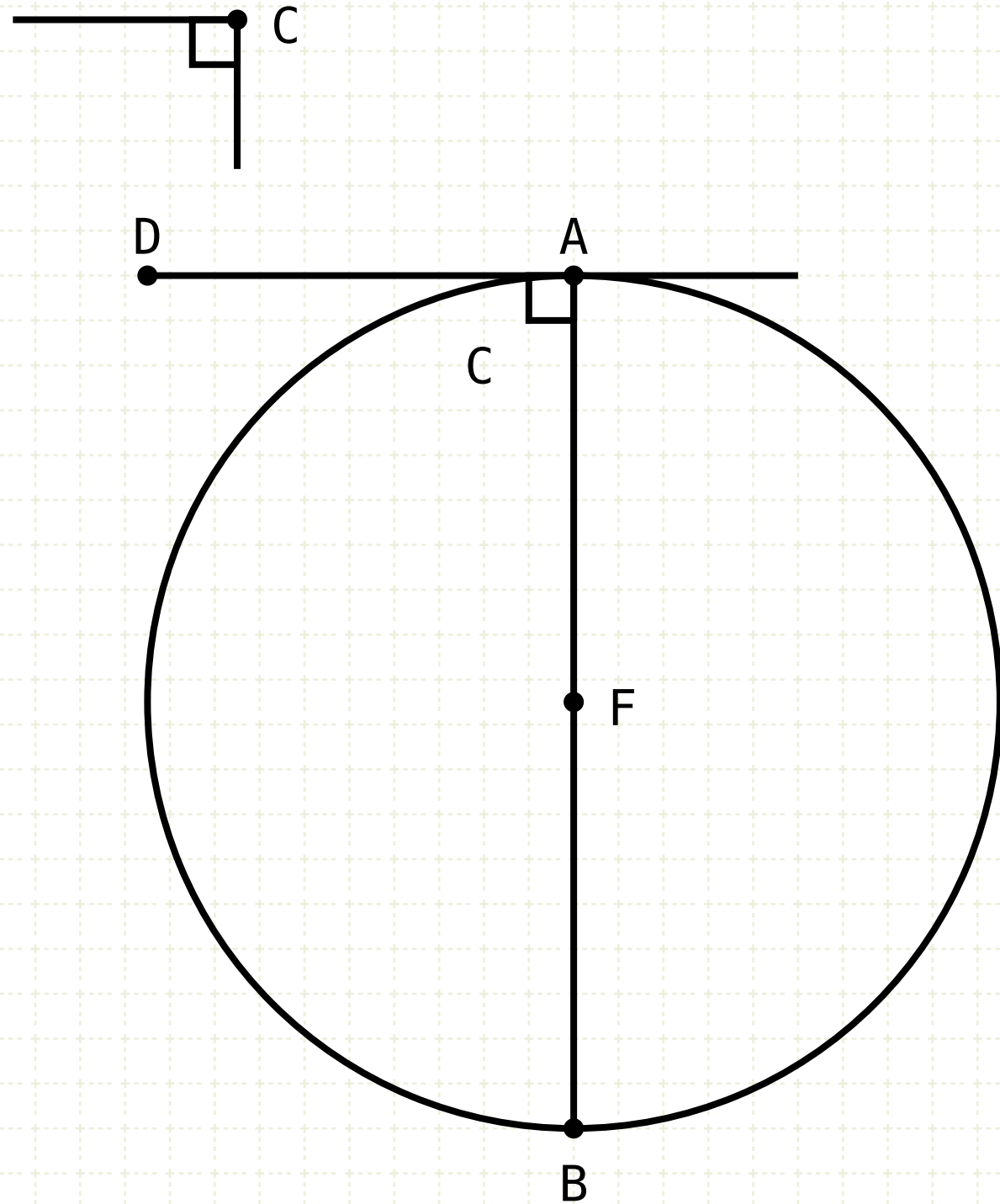
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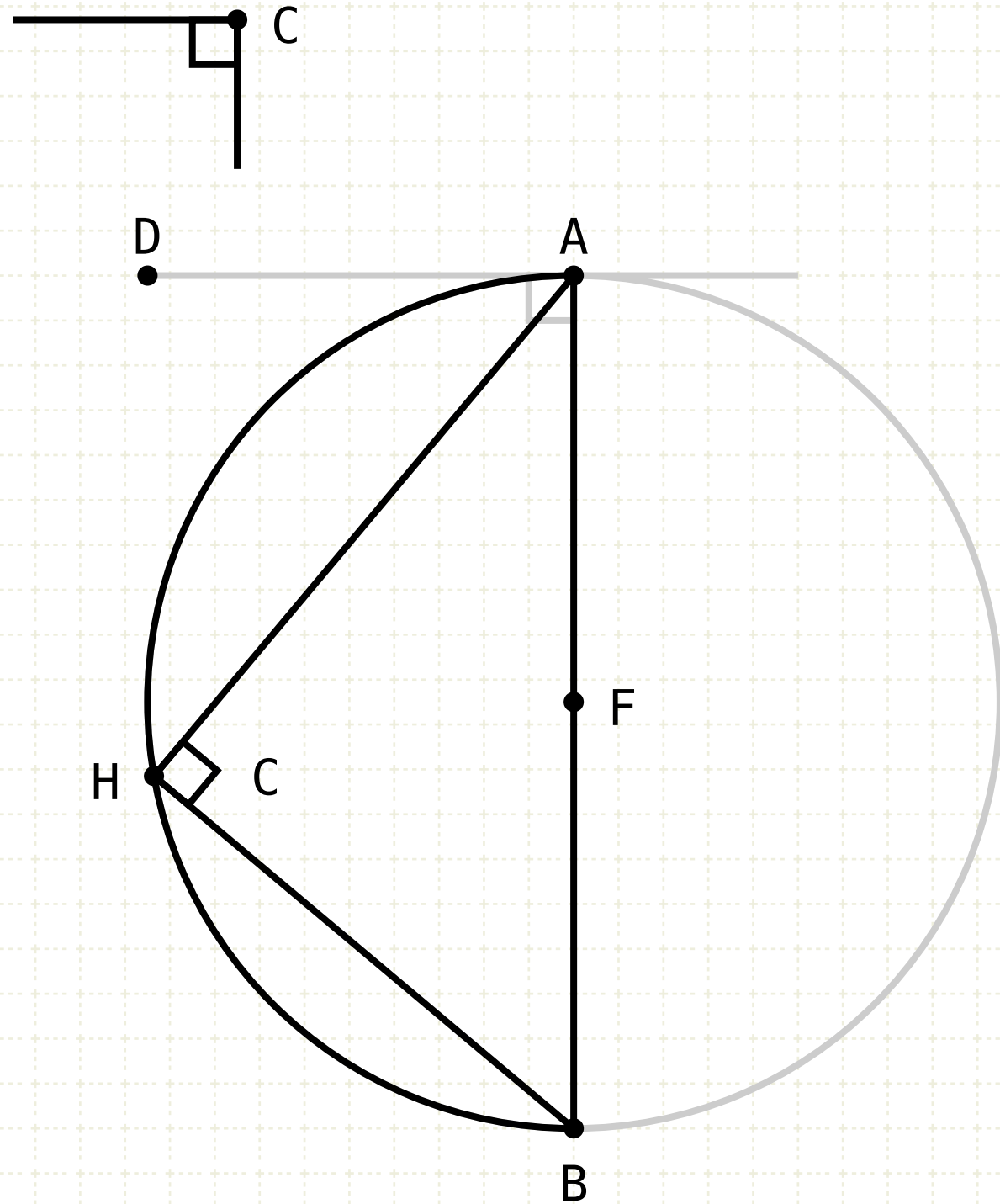
Copy the angle C to the line AB, at point A

Bisect line AB at point F

Drawing a circle with centre F and radius AF will pass through points A and B

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## Construction (Right Angle)

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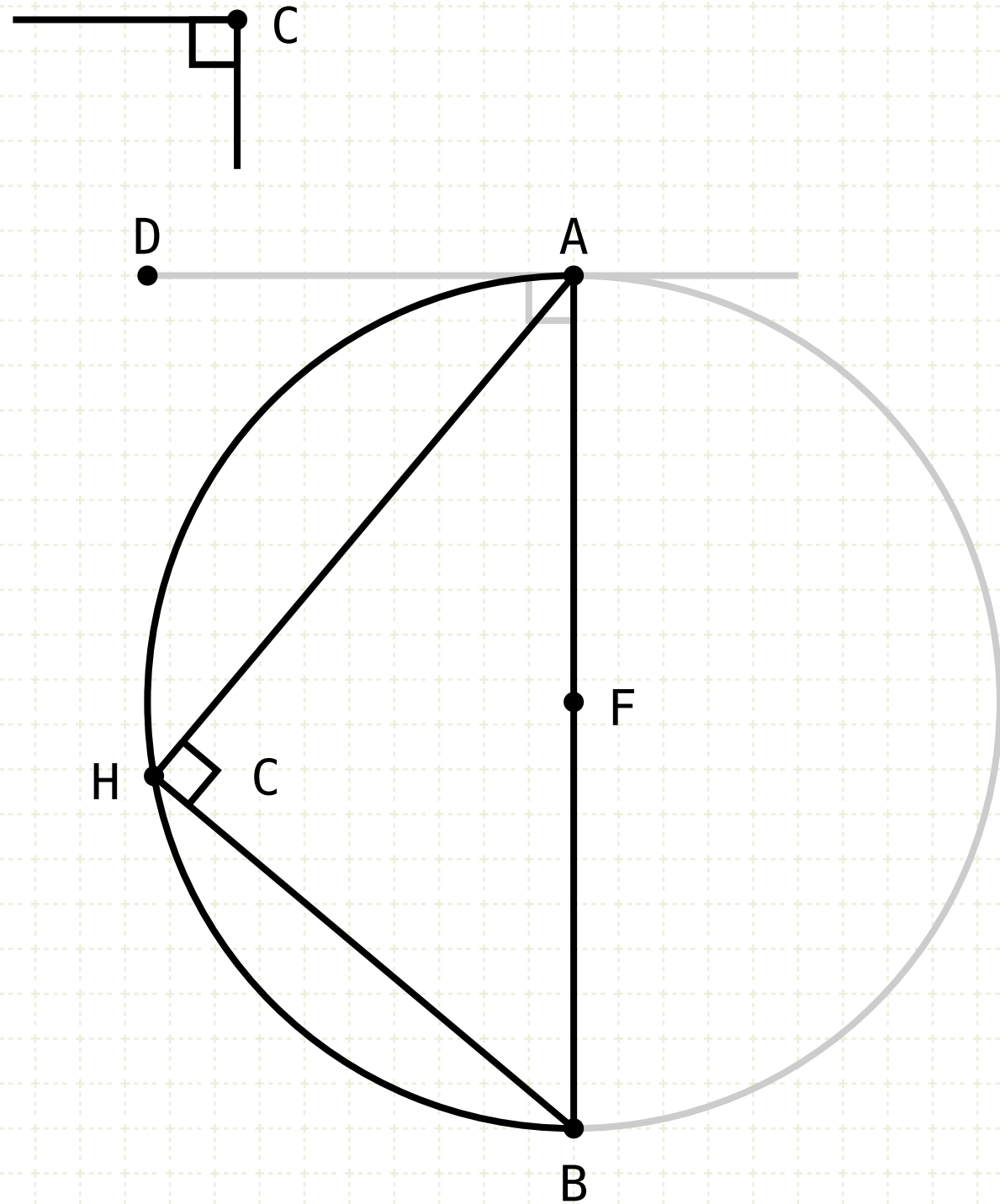
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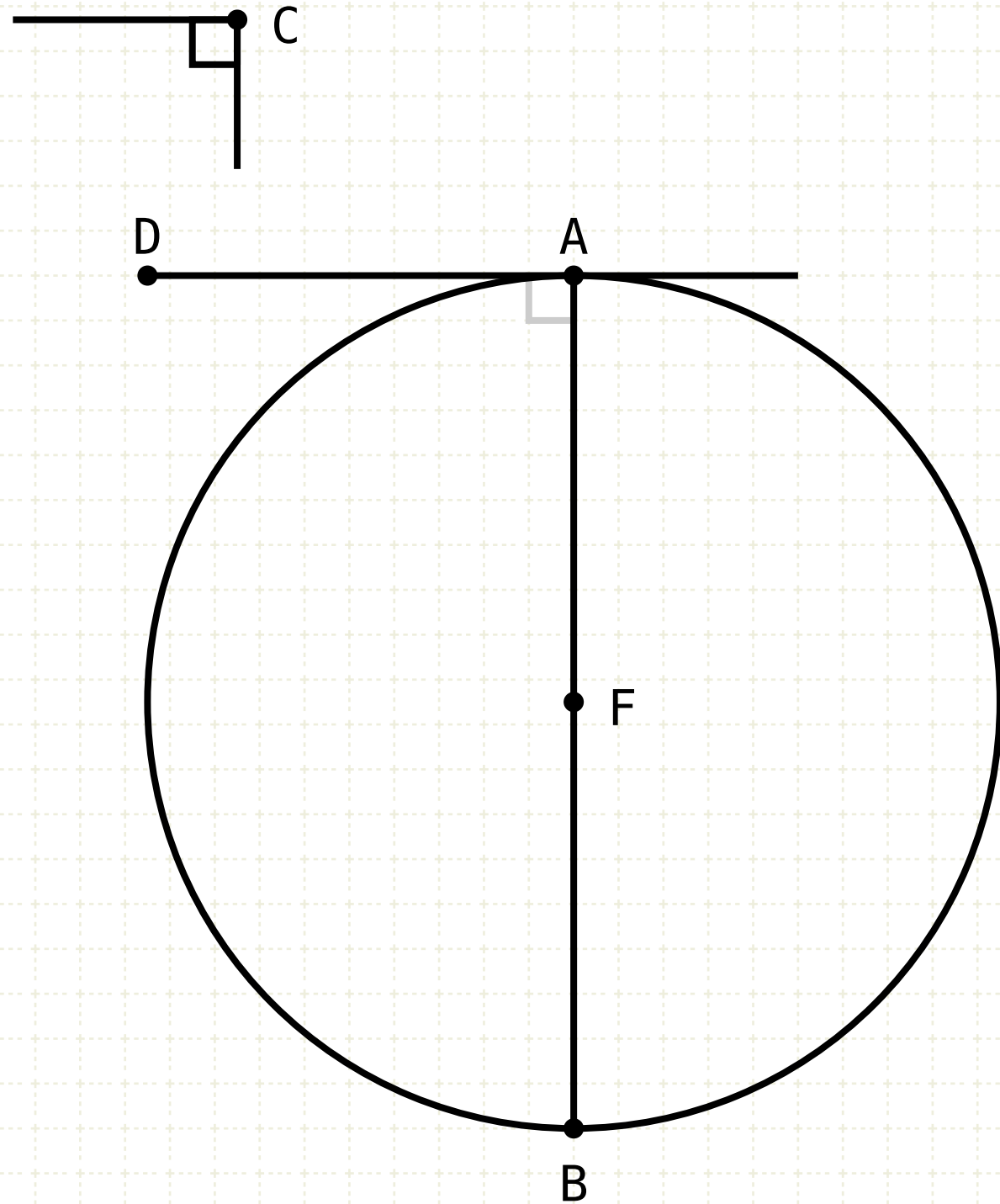
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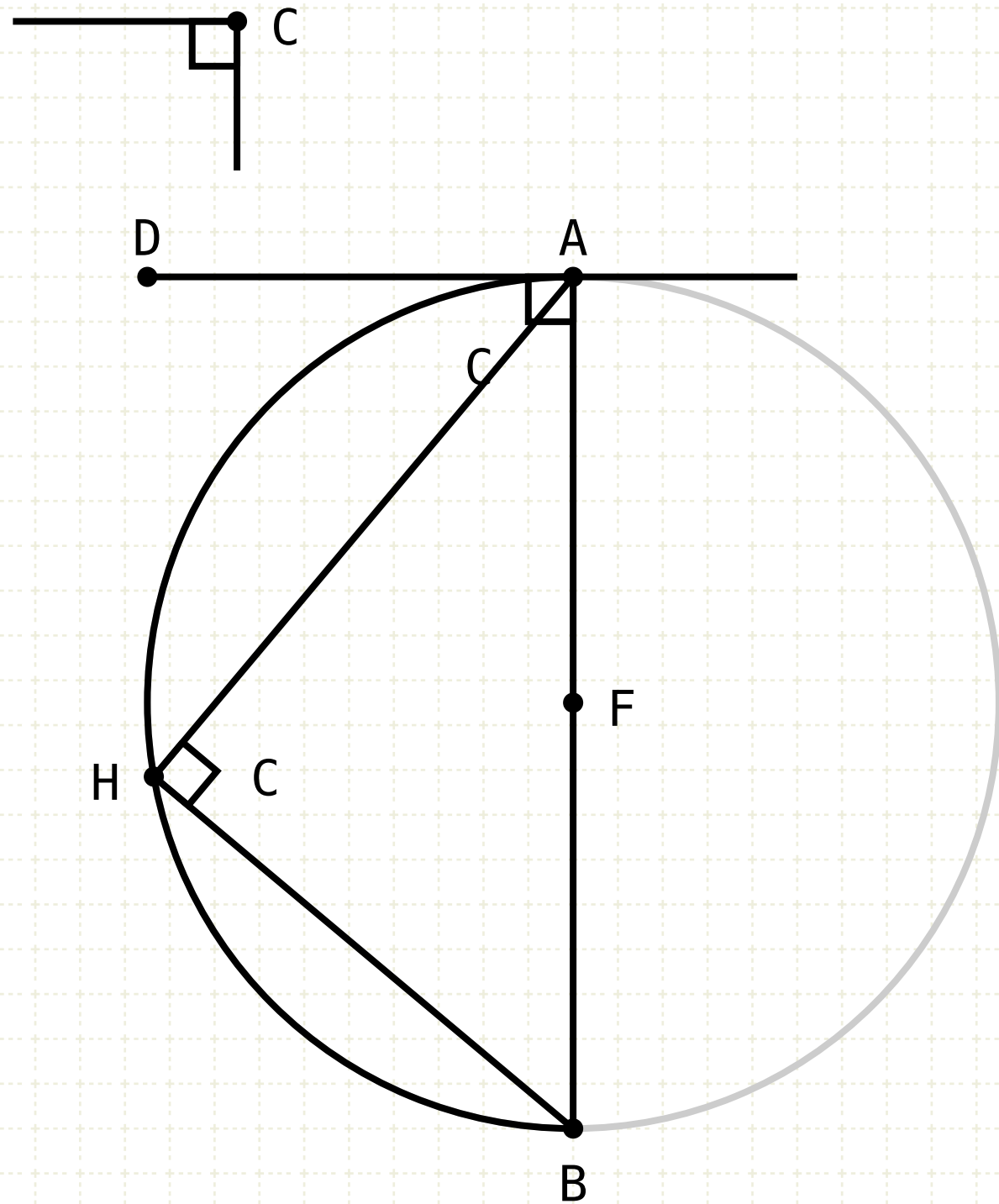
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The line AD is at the extremity of the circle diameter, and is at right angles to the diameter, thus the line AD touches the circle (III·16)



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## Proof

The line AD is at the extremity of the circle diameter, and is at right angles to the diameter, thus the line AD touches the circle (III·16)

The angle ACB is right, because it is in a semi-circle (III·31), which is equal to the angle C

# Proposition 33 of Book III

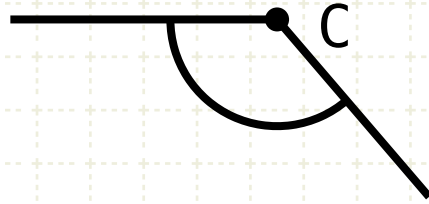
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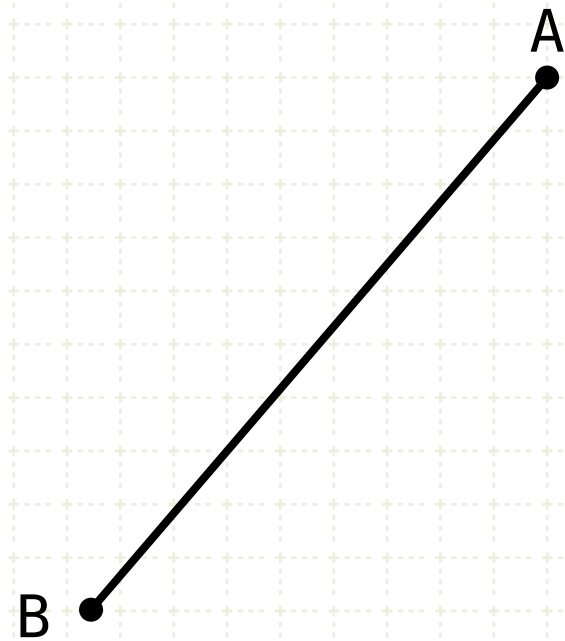


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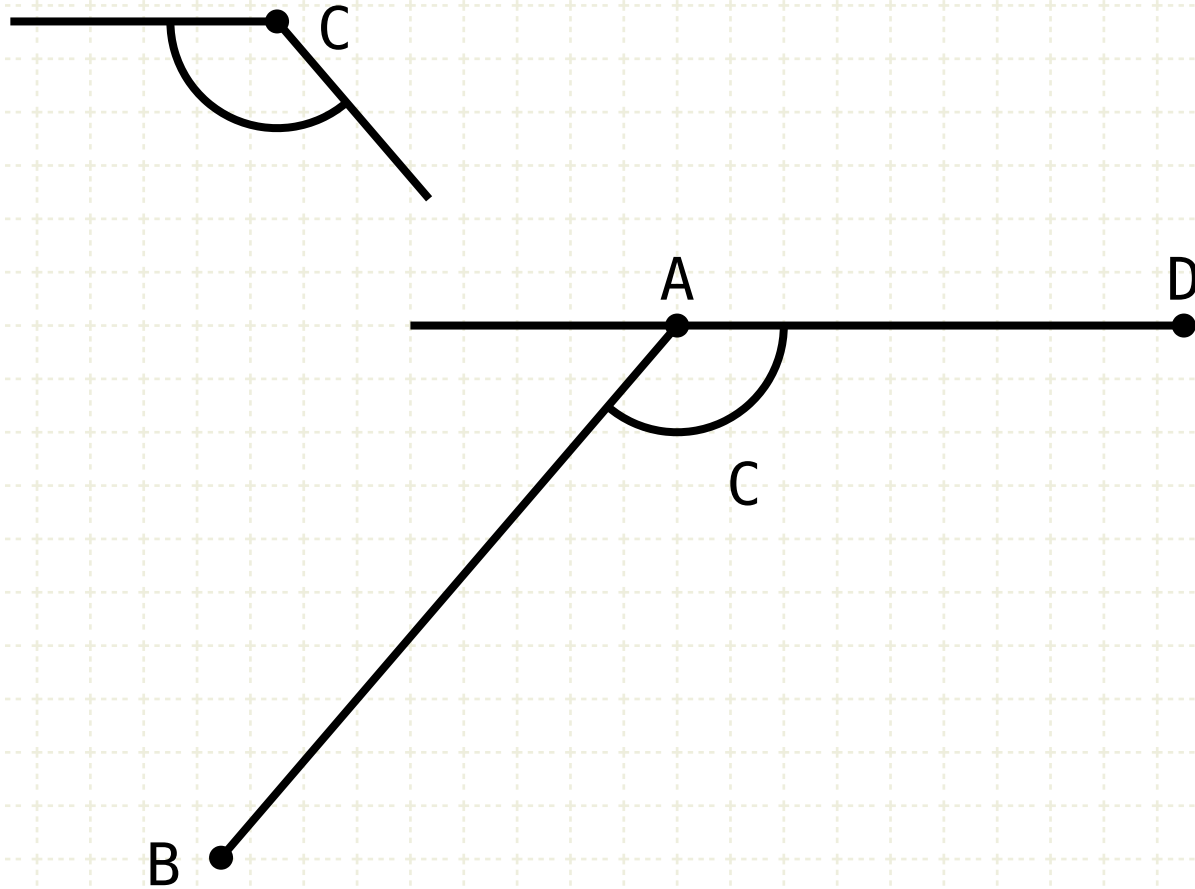


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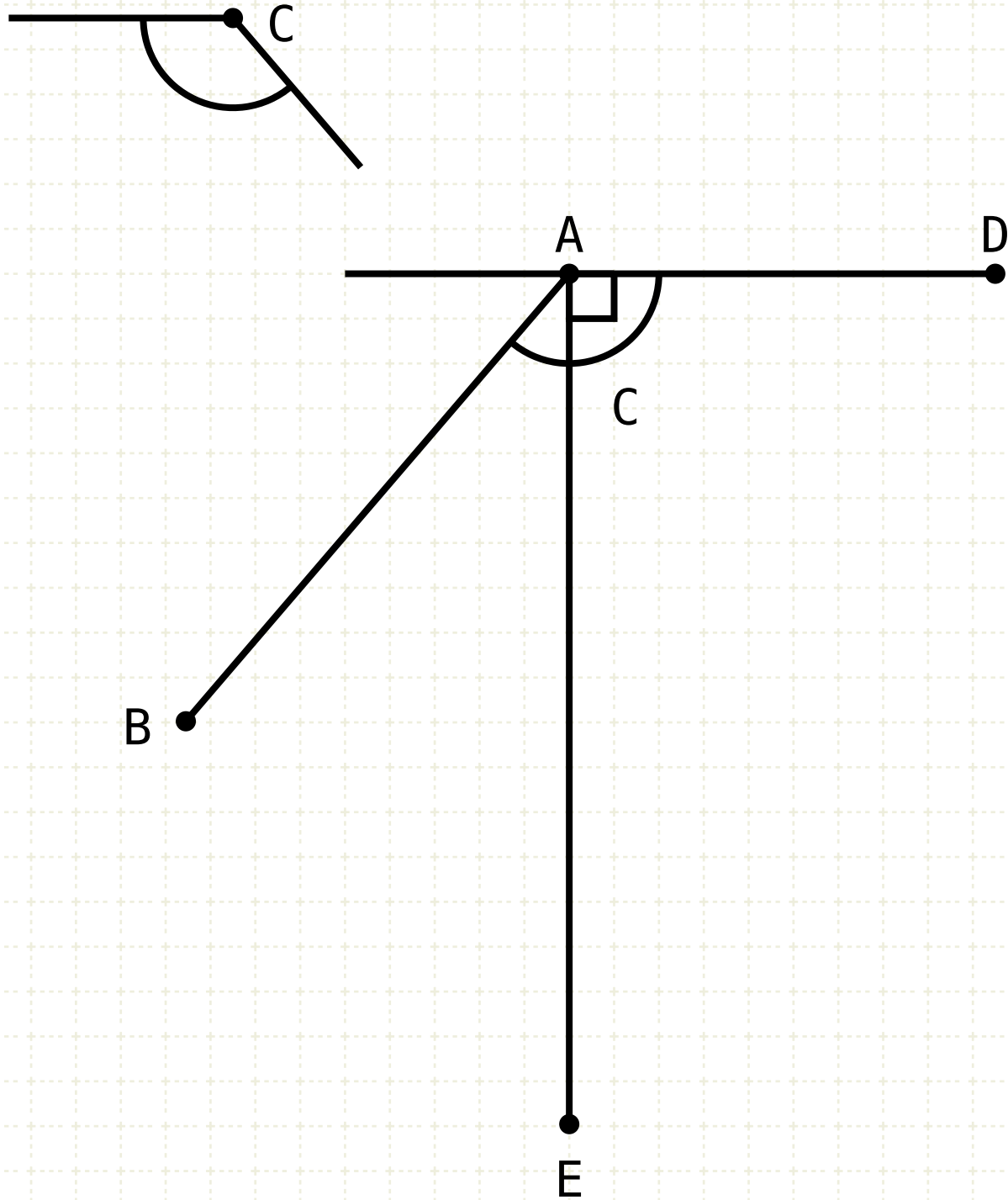


## Construction (Obtuse Angle)

Copy the angle  $C$  to the line  $AB$ , at point  $A$

# Proposition 33 of Book III

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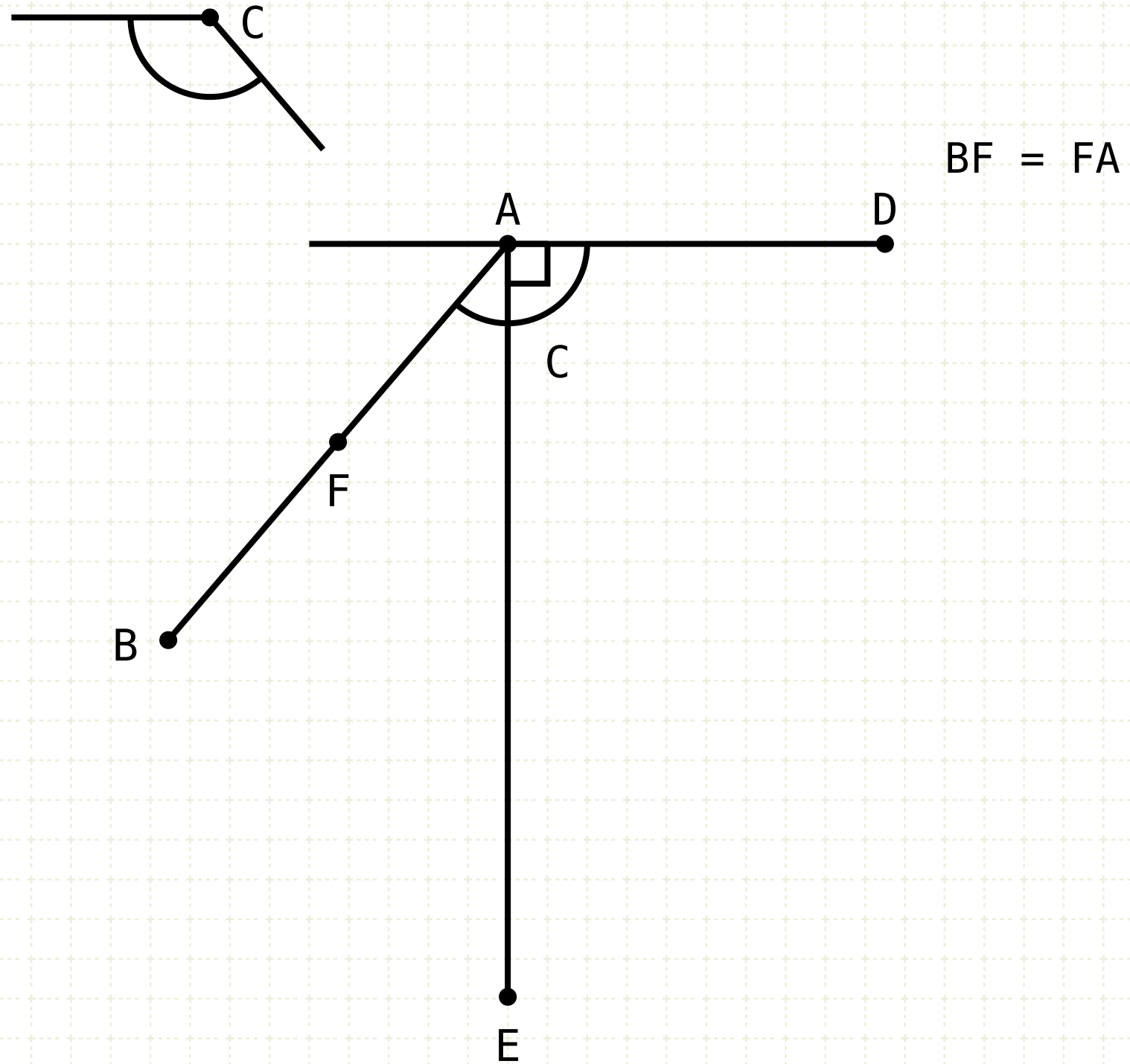
## Construction (Obtuse Angle)

Copy the angle  $C$  to the line  $AB$ , at point  $A$

Draw a line perpendicular to  $AD$ , from point  $A$

# Proposition 33 of Book III

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## Construction (Obtuse Angle)

Copy the angle C to the line AB, at point A

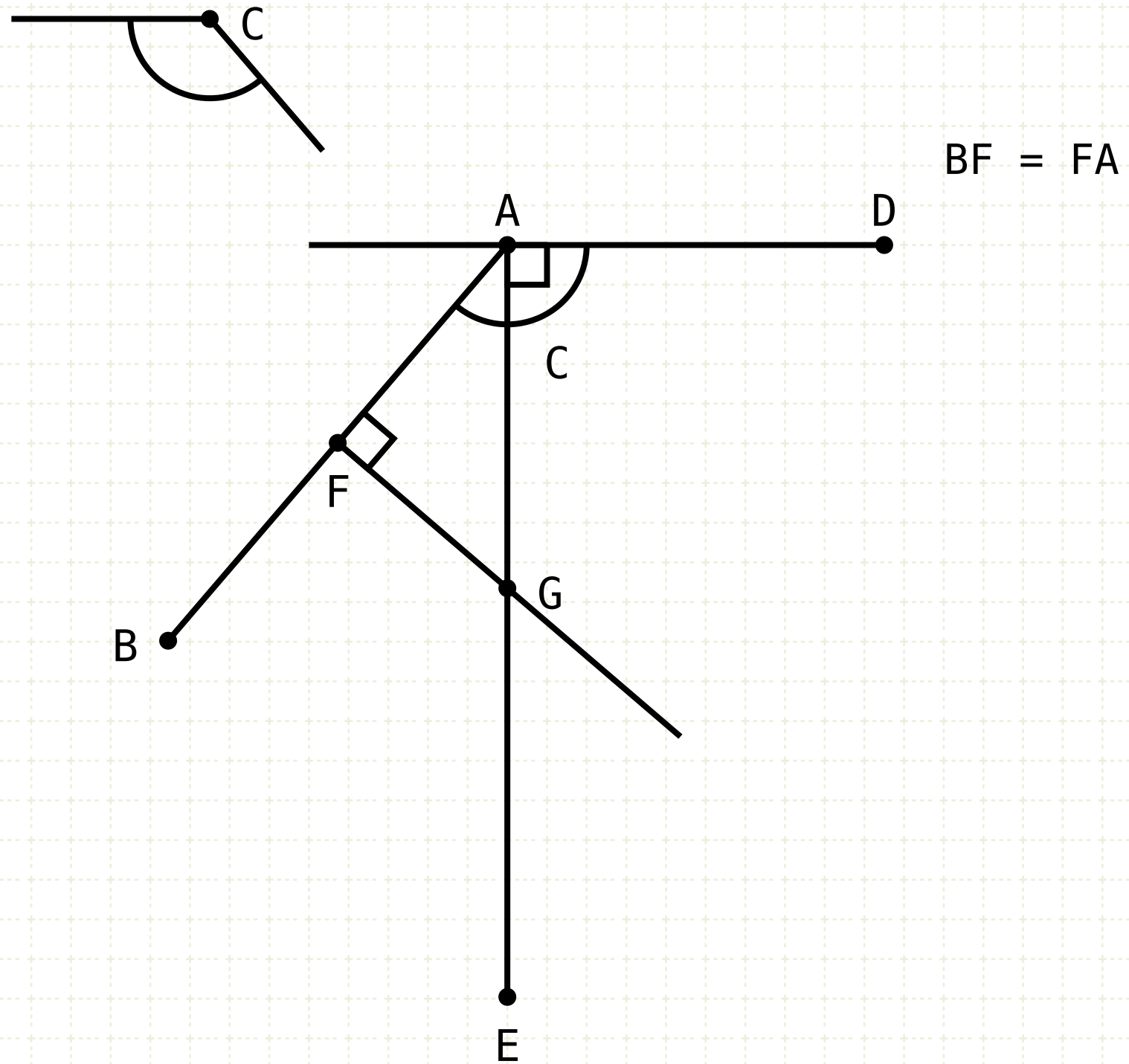
Draw a line perpendicular to AD, from point A

Bisect line AB at point F



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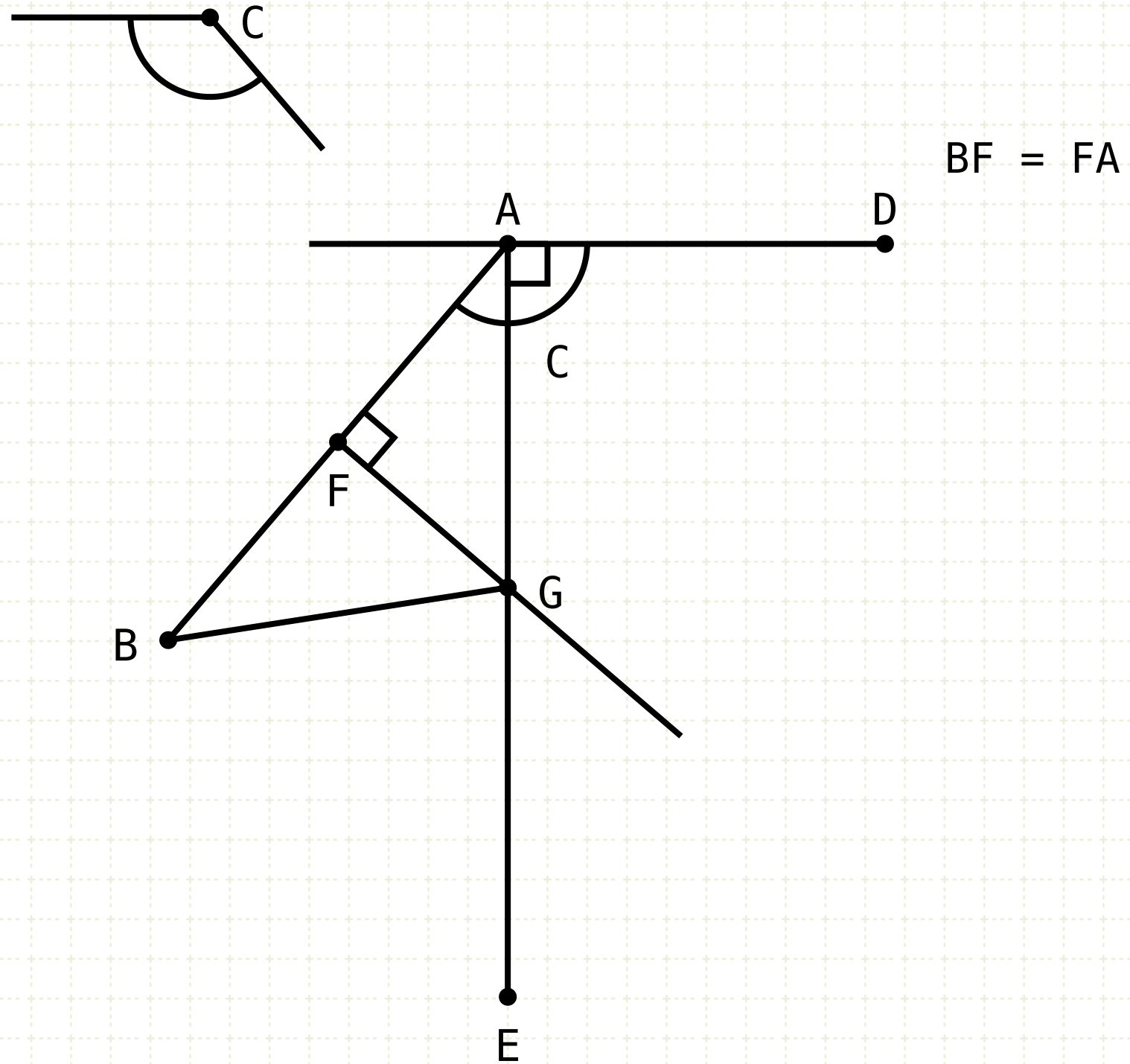
Draw a line perpendicular to AD, from point A

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Draw a line FG perpendicular to AB from point F, where G is the intersection between this line and AE

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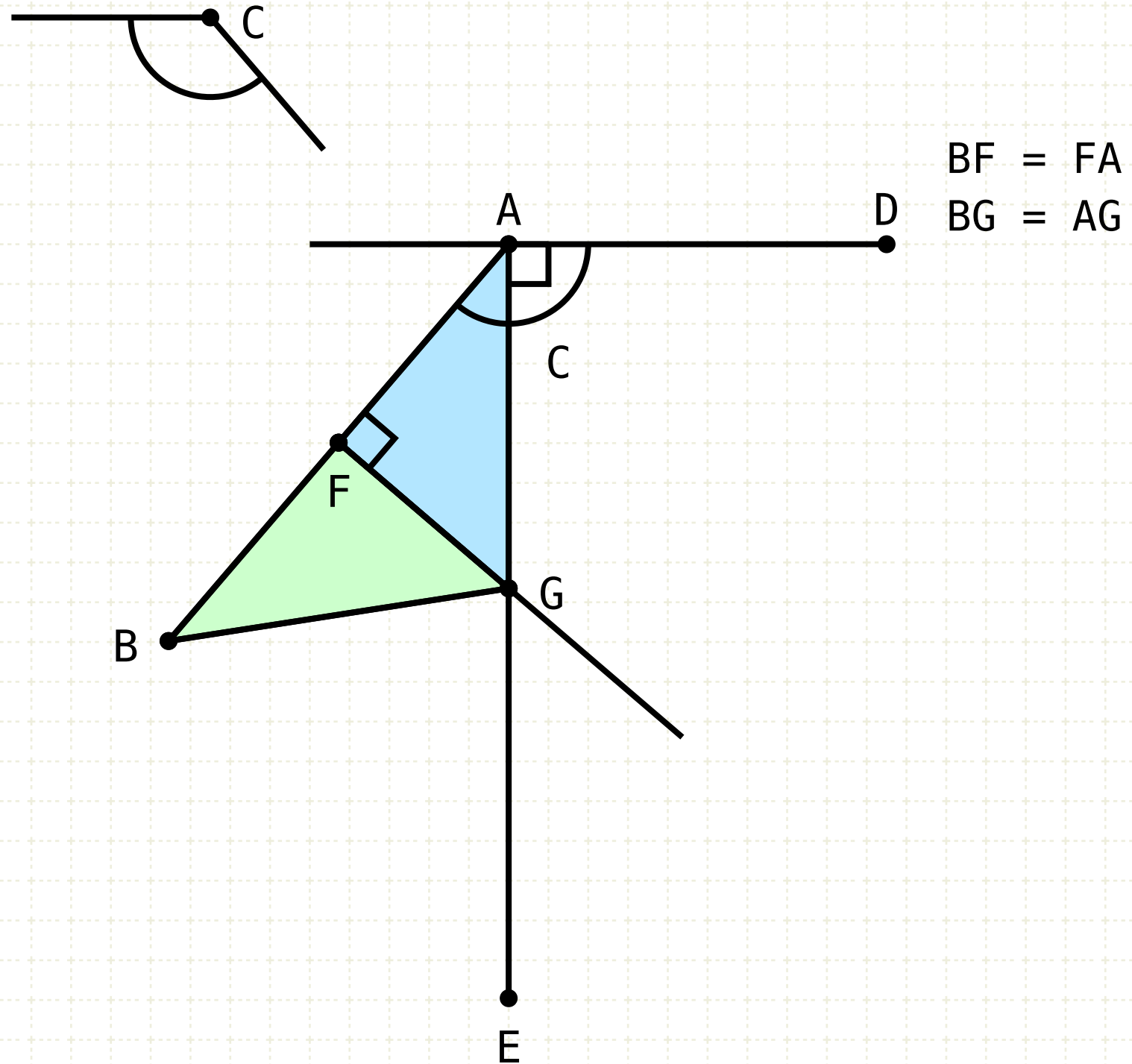
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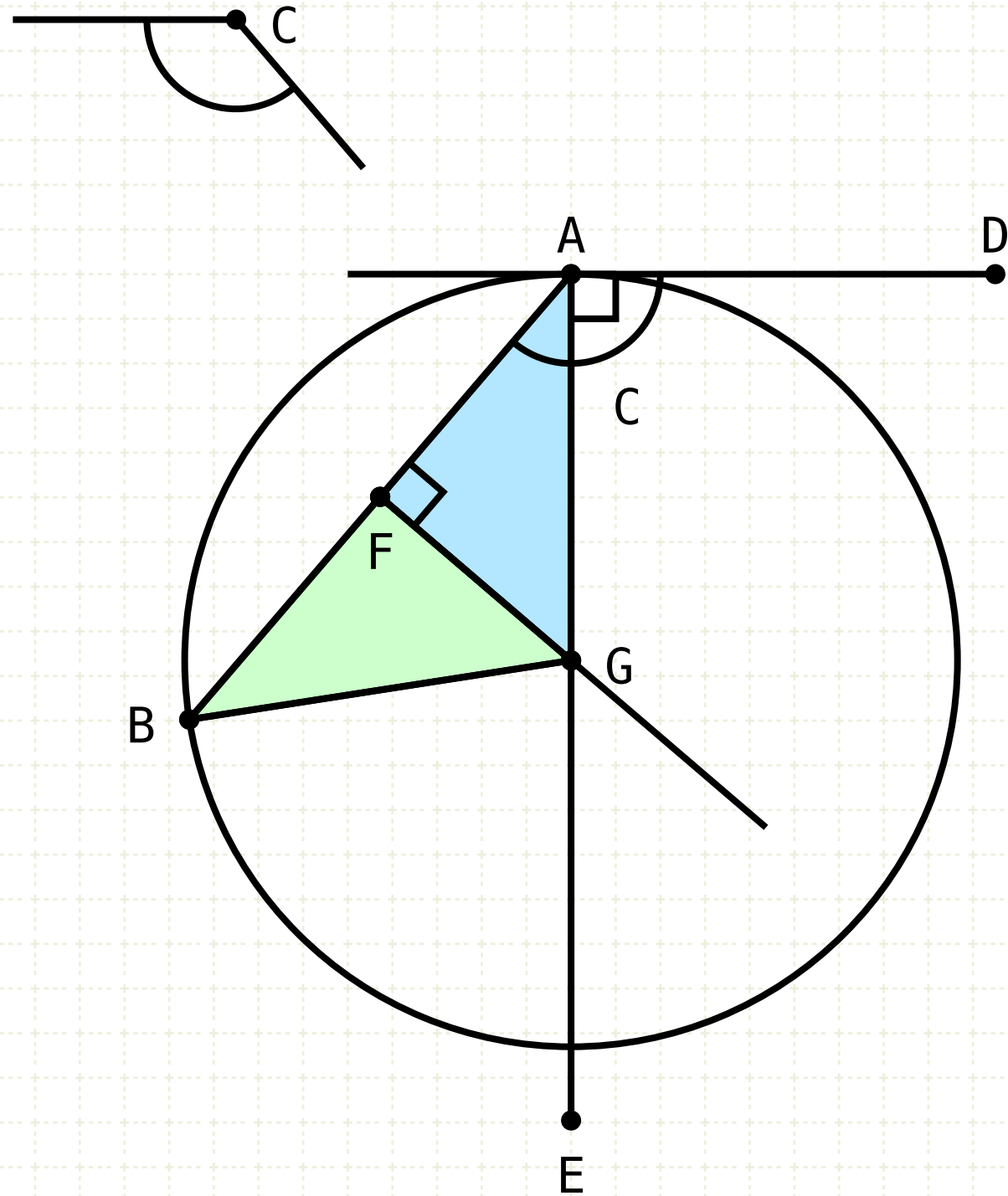
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Since BF equals FA and FG is common, and the angles AFG equals BFG, then the two triangles are equal (I-4), and the lines BG and AG are equal

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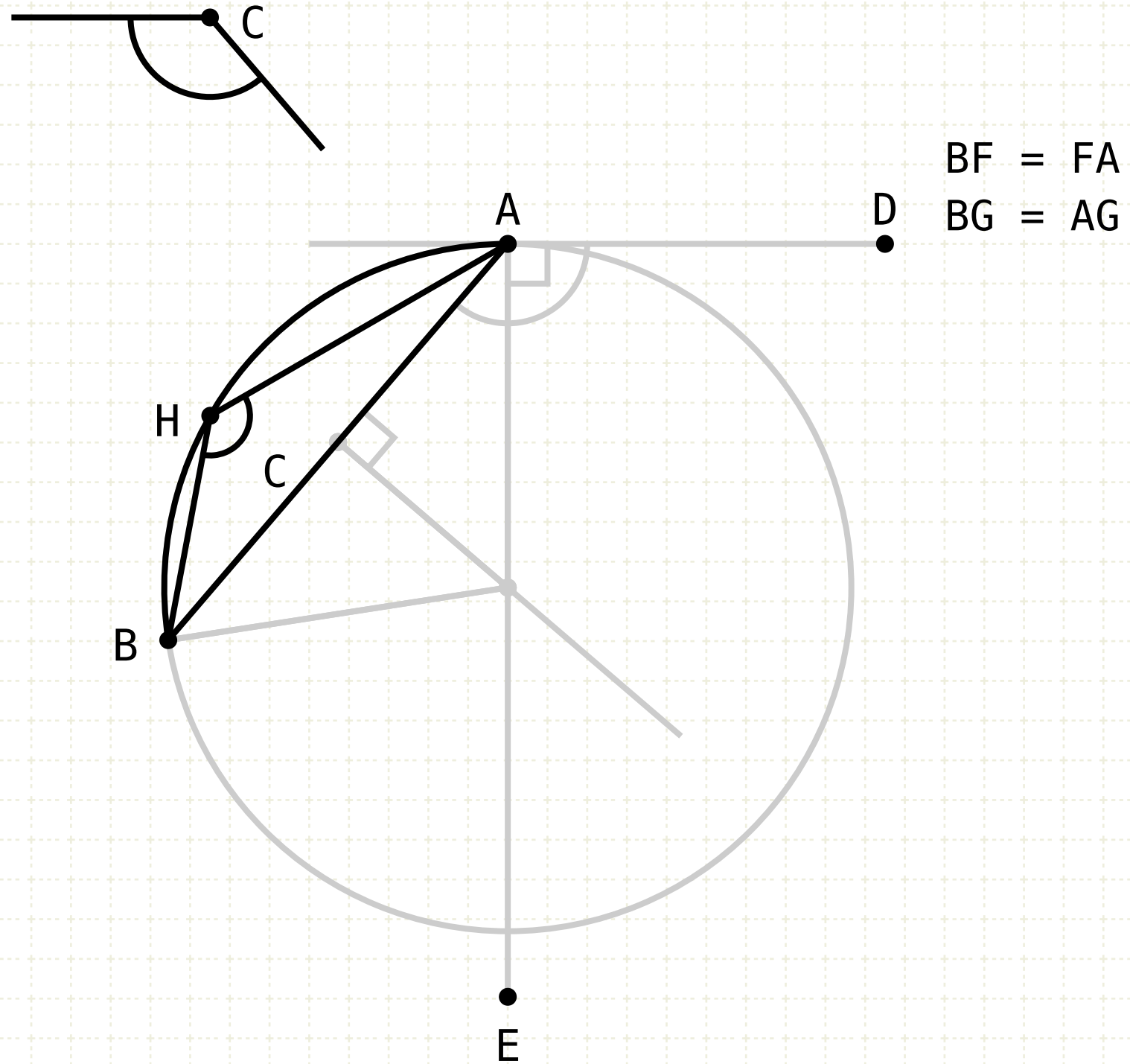
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Thus, drawing a circle with centre G and radius AG will pass through points A and B



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## Construction (Obtuse Angle)

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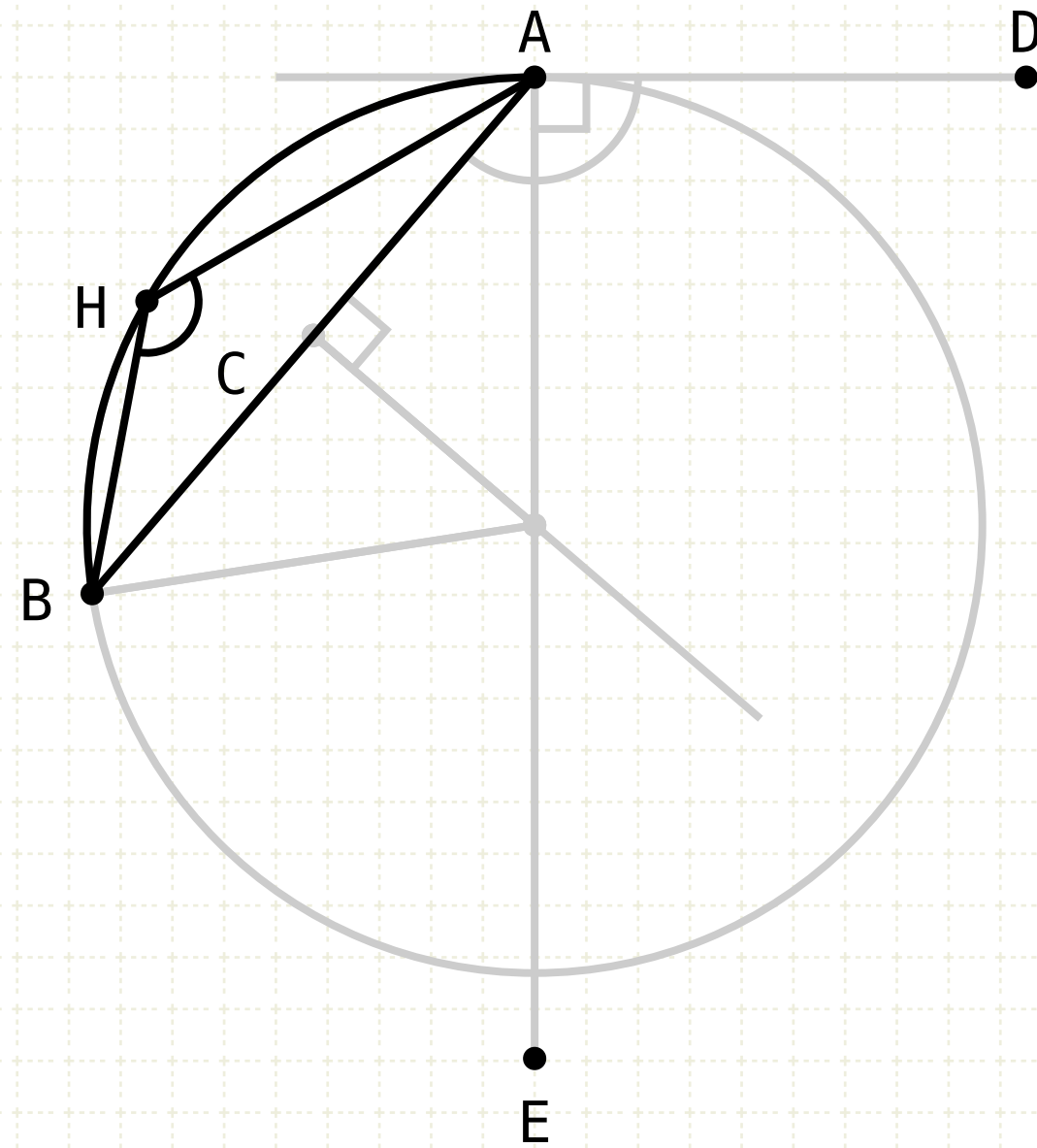
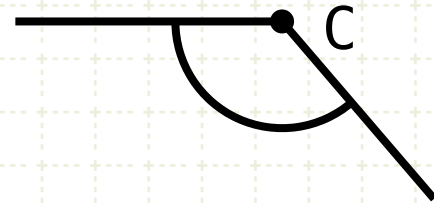
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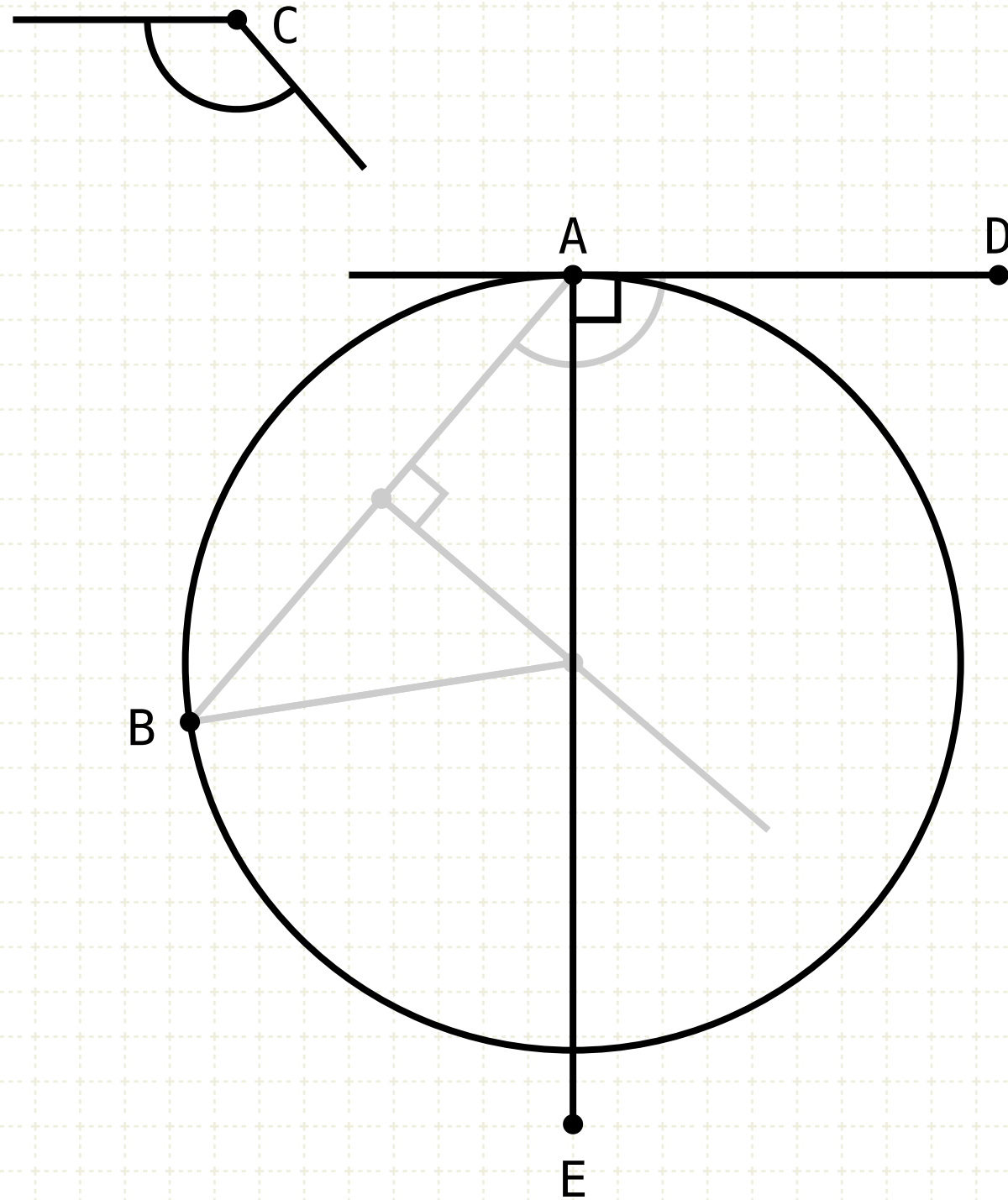
$$BG = AG$$

## Proof



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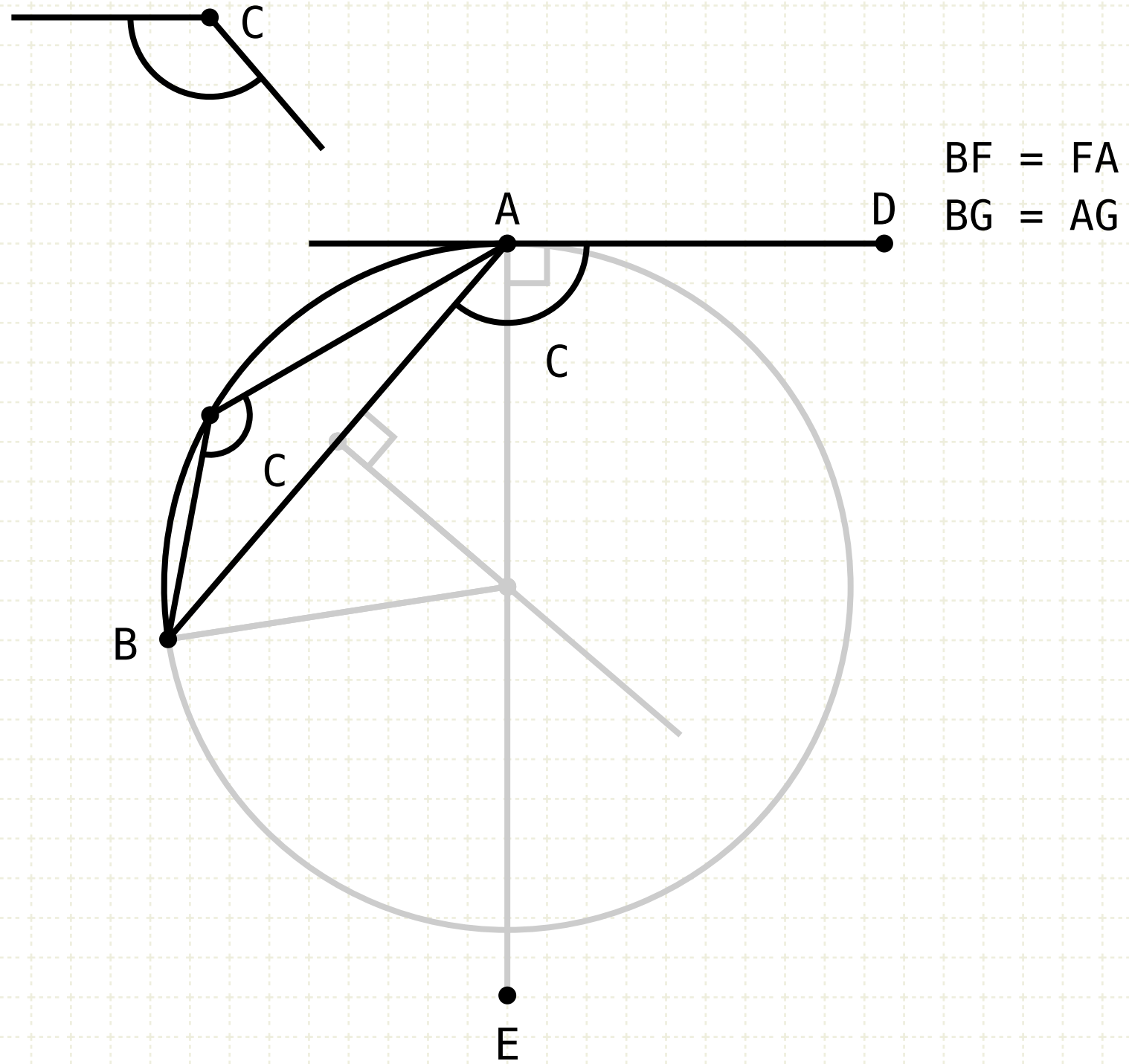
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## Proof

The line AD is at the extremity of the circle diameter, and is at right angles to the diameter, thus the line AD touches the circle (III·16)

## Proposition 33 of Book III

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# Proof

The line AD is at the extremity of the circle diameter, and is at right angles to the diameter, thus the line AD touches the circle (III·16)

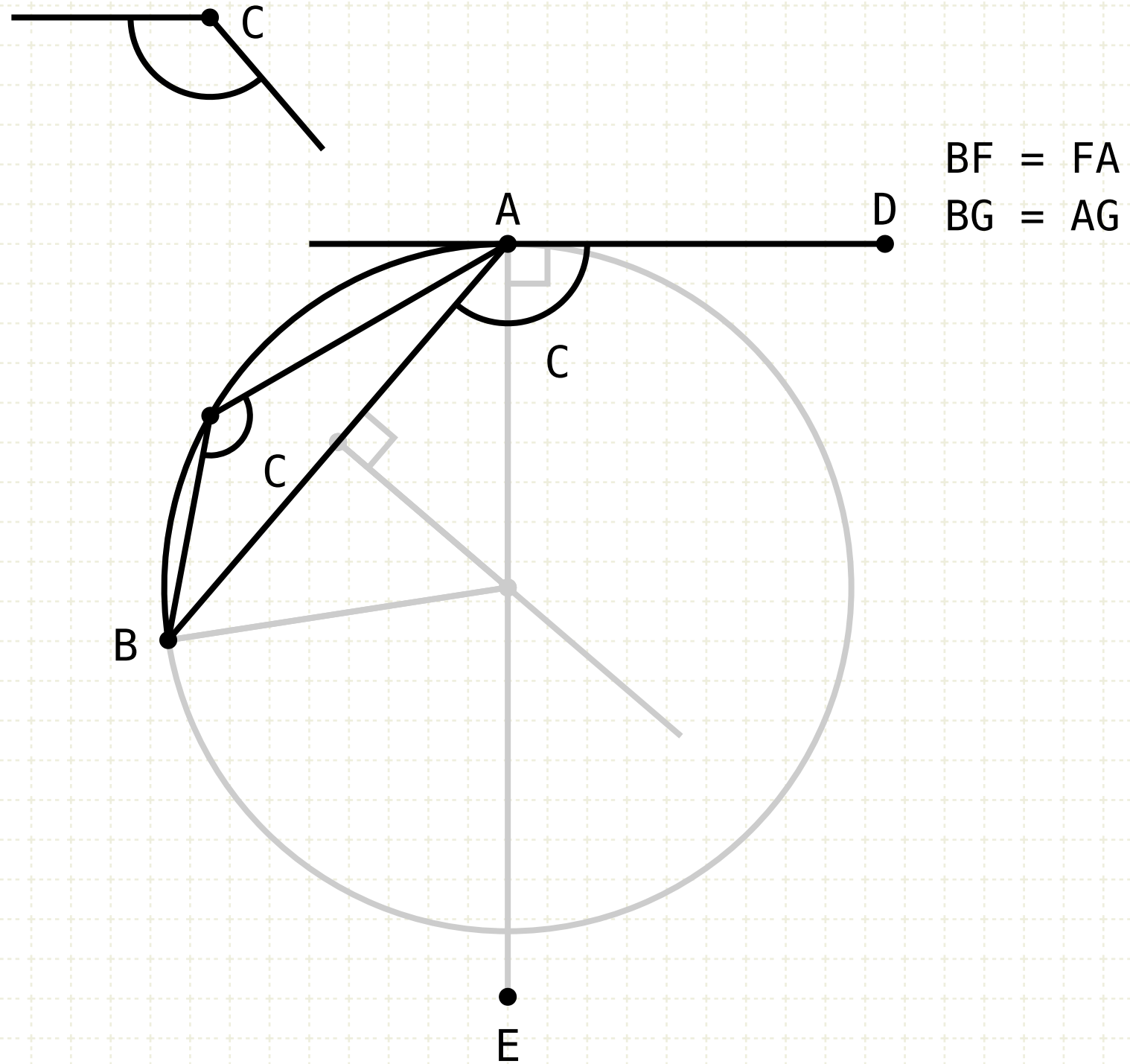
Since AD touches the circle, the angle DAB equals the angle in the opposite circle segment (III-32)

The angle DAB is equal to C by construction, so thus the angle in the segment BEA equals C



## Proposition 33 of Book III

On a given straight line to describe a segment of a circle admitting an angle equal to a given rectilineal angle.



# Proof

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