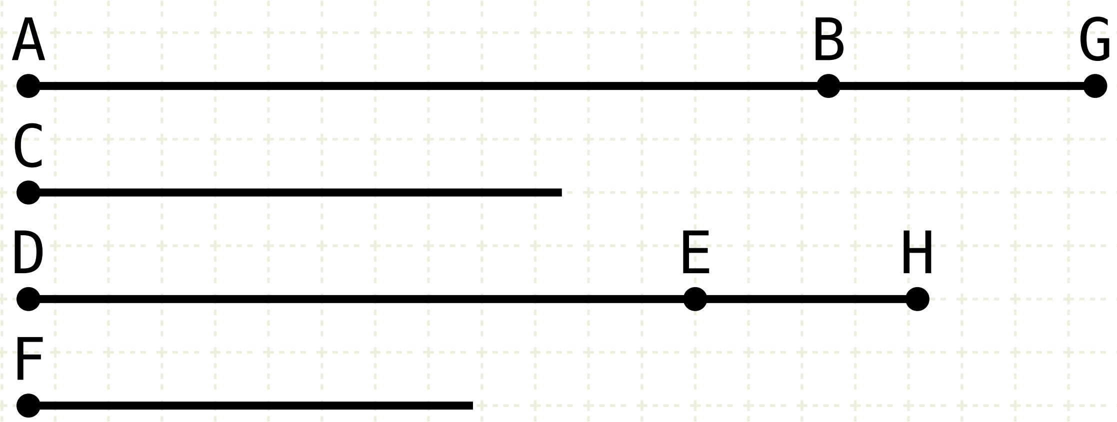
Euclid's Elements

Book V



AB:C = DE:F
BG:C = EH:F

AG:C = DH:F

Proportions are what makes the old Greek temples classic in their beauty. They are like huge blocks, from which the air has been literally hewn out between the columns.

Arne Jacobsen

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Proposition 8 of Book V

Of unequal magnitudes, the greater has to the same a greater ratio than the less has; and the same has to the less a greater ratio than it has to the greater

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Of unequal magnitudes, the greater has to the same a greater ratio than the less has; and the same has to the less a greater ratio than it has to the greater

Definitions

4. Magnitudes are said to HAVE A RATIO to one another which are capable, when multiplied, of exceeding one another

```
A and B have a ratio (A:B) if there
exists a 'p' and 'q' such that
pA > B, and A < qB
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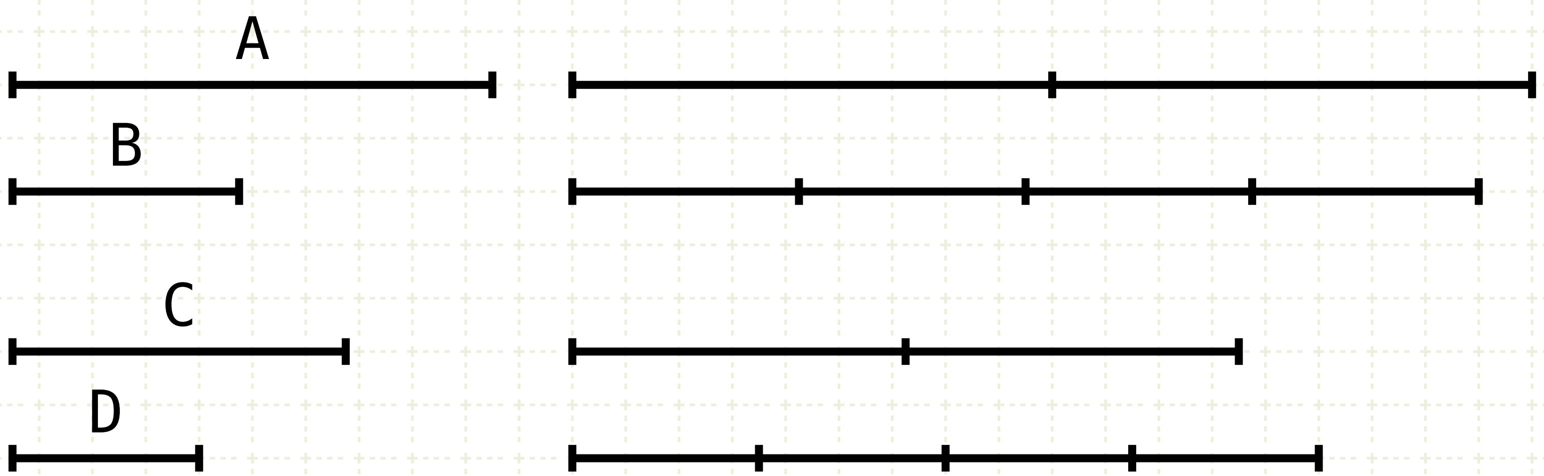


```
If   n·A >  m·B
and  n·C <= m·D
then A:B > C:D

Example:
compare 180:85 to 125:70
2×180 > 4×85
2×125 < 4×70
∴ 180:85 > 125:70
```

Definitions

7. When, of the equimultiples, the multiple of the first magnitude exceeds the multiple of the second, but the multiple of the third does not exceed the multiple of the fourth, then the first is said to have a greater ratio to the second than the third has to the fourth.

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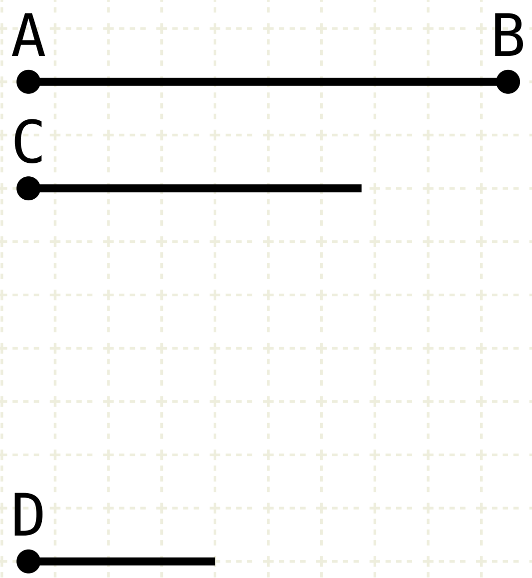

$AB > C \neq D$

In other words

Let AB be greater than C and let D be an arbitrary magnitude

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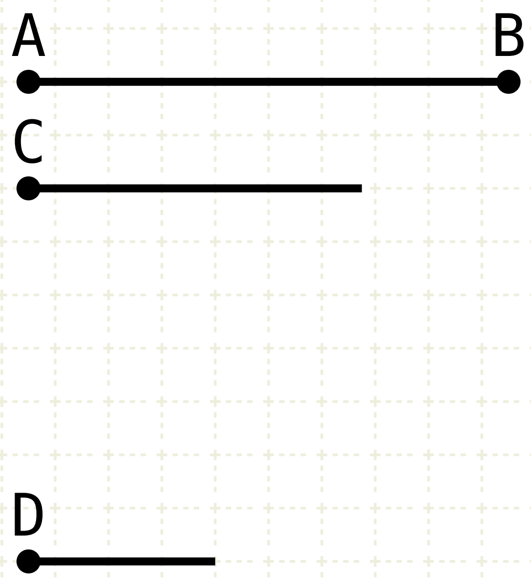

In other words

Let AB be greater than C and let D be an arbitrary magnitude

Then the ratio of AB to D is greater than the ratio of C to D

Then the ratio of D to AB is less than the ratio of D to C

$AB > C \neq D$

$AB:D > C:D$

$D:AB < D:C$

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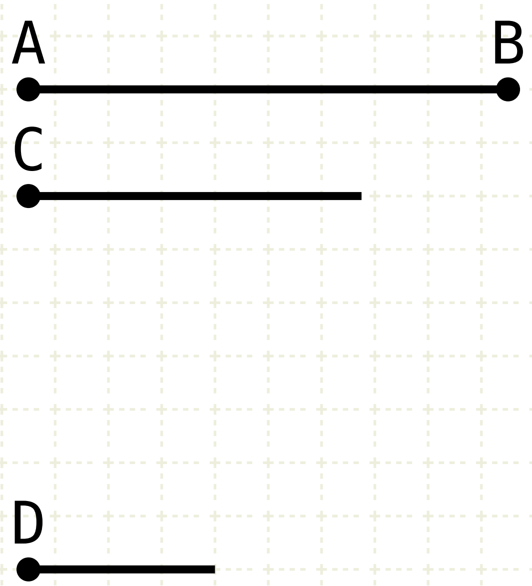

AB > C ≠ D

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Let AB be greater than C and let D be an arbitrary magnitude

Then the ratio of AB to D is greater than the ratio of C to D

Then the ratio of D to AB is less than the ratio of D to C

Proof

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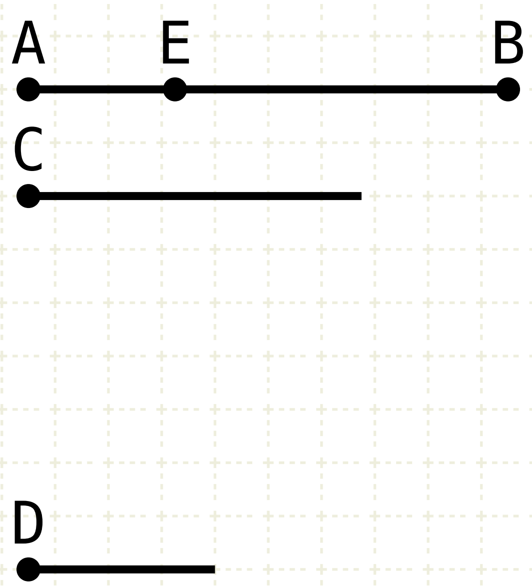

AB > C ≠ D

EB = C

In other words

Let AB be greater than C and let D be an arbitrary magnitude

Then the ratio of AB to D is greater than the ratio of C to D

Then the ratio of D to AB is less than the ratio of D to C

Proof

First, let EB be equal to C. Then the lesser of AE,EB can be multiplied by a number such that it is larger than D (V def.4)

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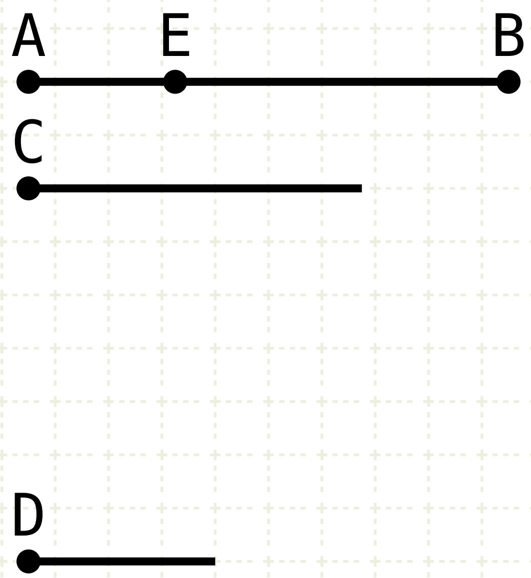

AB > C ≠ D

EB = C

AE < EB

In other words

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Then the ratio of AB to D is greater than the ratio of C to D

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Proof

First, let EB be equal to C. Then the lesser of AE,EB can be multiplied by a number such that it is larger than D (V def.4)

CASE 1: AE < EB

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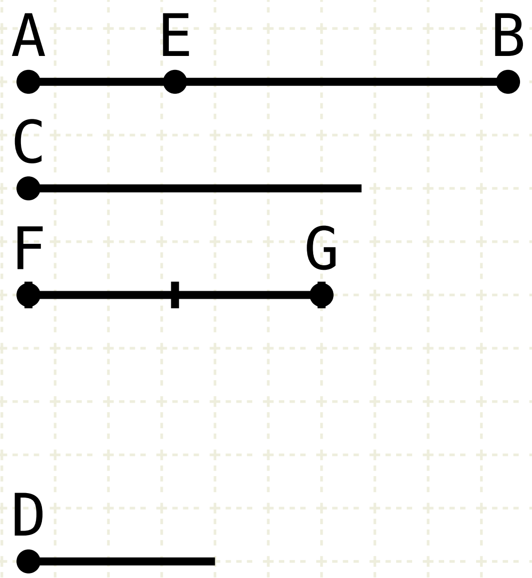

$$AB > C \neq D$$
$$EB = C$$
$$AE < EB$$
$$FG = n \cdot AE > D$$

In other words

Let AB be greater than C and let D be an arbitrary magnitude

Then the ratio of AB to D is greater than the ratio of C to D

Then the ratio of D to AB is less than the ratio of D to C

Proof

First, let EB be equal to C. Then the lesser of AE,EB can be multiplied by a number such that it is larger than D (V def.4)

CASE 1: AE < EB

Define a line FG such that is is a multiple of AE, AND it is larger than D

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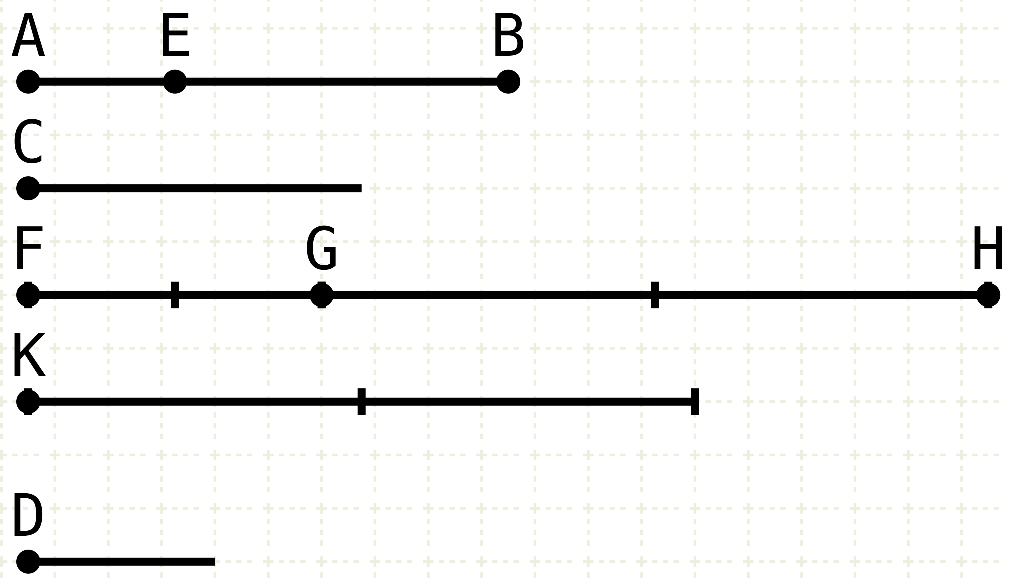

$AB > C \neq D$

$EB = C$

$AE < EB$

$FG = n \cdot AE > D$

$GH = n \cdot EB$

$K = n \cdot C$

In other words

Let AB be greater than C and let D be an arbitrary magnitude

Then the ratio of AB to D is greater than the ratio of C to D

Then the ratio of D to AB is less than the ratio of D to C

Proof

First, let EB be equal to C. Then the lesser of AE,EB can be multiplied by a number such that it is larger than D (V def.4)

CASE 1: AE < EB

Define a line FG such that is is a multiple of AE, AND it is larger than D

Using the same multiple, define line GH to be the same multiple of EB, and K to be the same multiple of C

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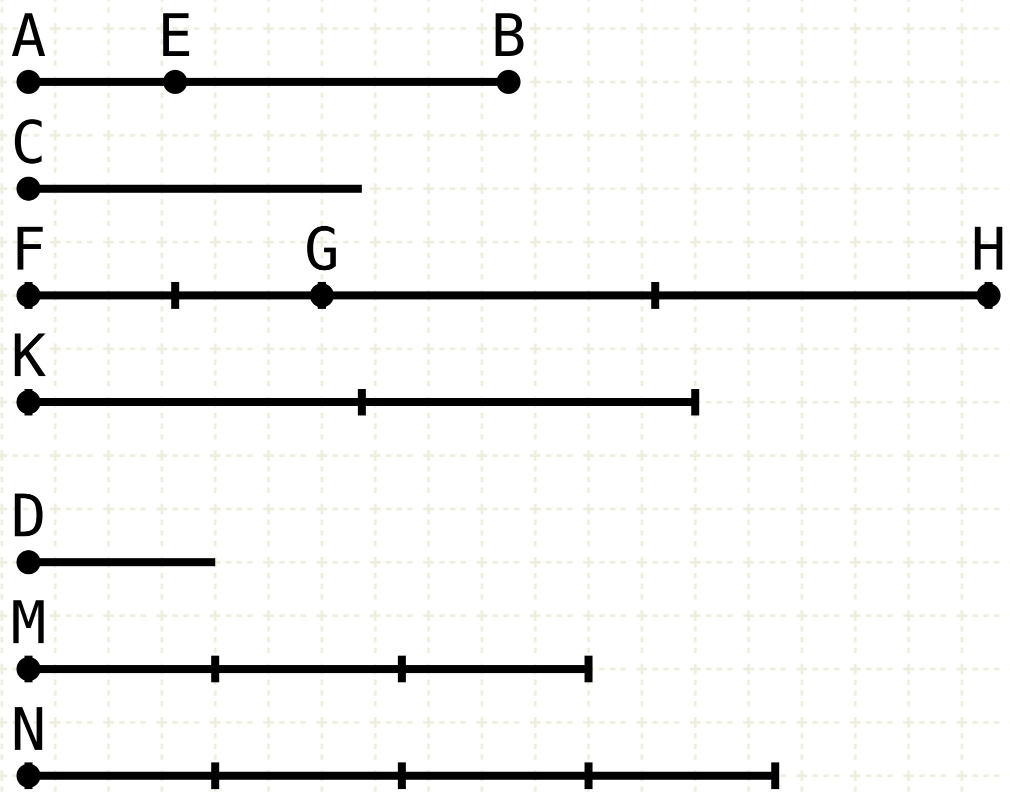

$AB > C \neq D$

$EB = C$

$AE < EB$

$FG = n \cdot AE > D$

$GH = n \cdot EB$

$K = n \cdot C$

$M = (j-1) \cdot D \leq K$

$N = j \cdot D > K$

In other words

Let AB be greater than C and let D be an arbitrary magnitude

Then the ratio of AB to D is greater than the ratio of C to D

Then the ratio of D to AB is less than the ratio of D to C

Proof

First, let EB be equal to C. Then the lesser of AE,EB can be multiplied by a number such that it is larger than D (V def.4)

CASE 1: AE < EB

Define a line FG such that is is a multiple of AE, AND it is larger than D

Using the same multiple, define line GH to be the same multiple of EB, and K to be the same multiple of C

Define lines M and N such that

* M is one less multiple than N,
* M is less than or equal to K
* and N is greater than K

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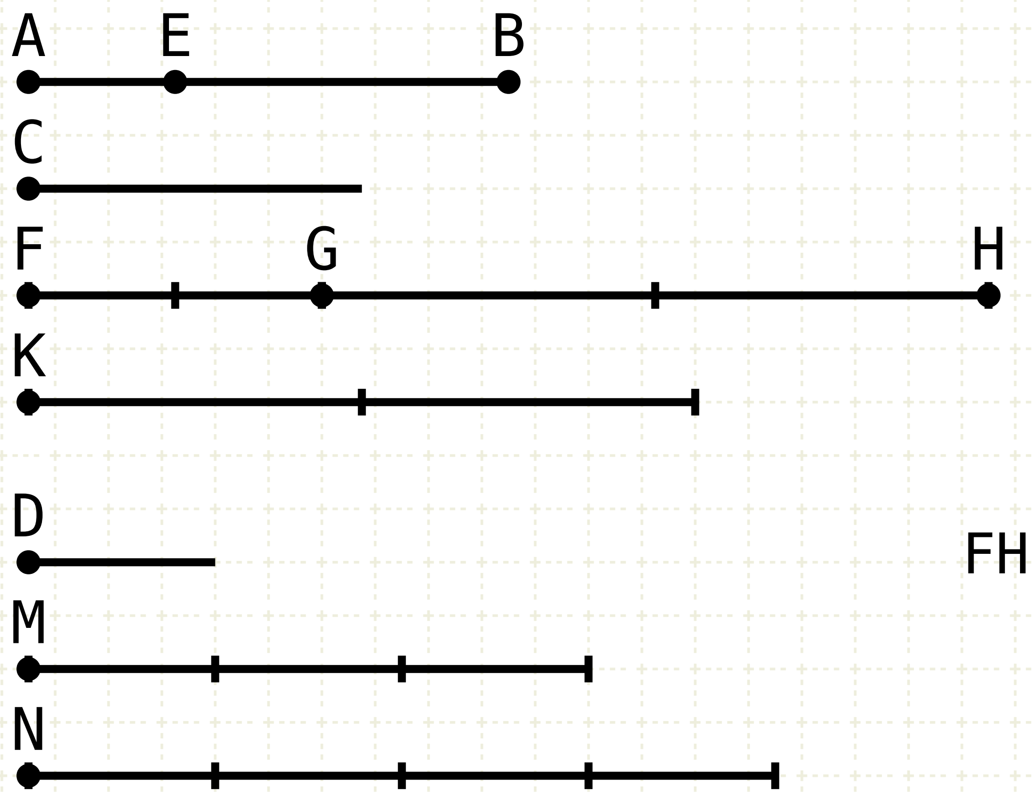
Of unequal magnitudes, the greater has to the same a greater ratio than the less has; and the same has to the less a greater ratio than it has to the greater



$FH = n \cdot AB$

$AB > C \neq D$

$EB = C$

$AE < EB$

$FG = n \cdot AE > D$

$GH = n \cdot EB$

$K = n \cdot C$

$M = (j-1) \cdot D \leq K$

$N = j \cdot D > K$

Proof

Case 1: AE < EB

Since FG is the same multiple of AE as GH is to EB, then sum of FG,GH (FH) is the same multiple of the sum AE,EB (AB) (V·1)

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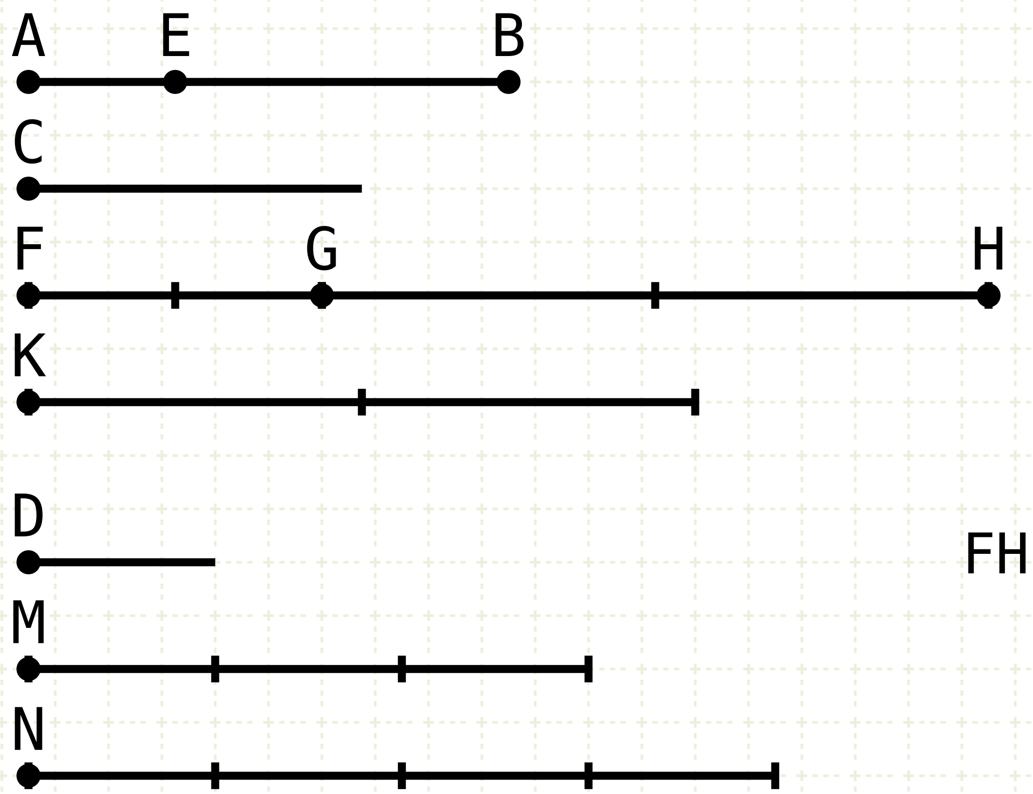

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$K = n \cdot C$

$M = (j-1) \cdot D \leq K$

$N = j \cdot D > K$

Proof

Case 1: AE < EB

Since FG is the same multiple of AE as GH is to EB, then sum of FG,GH (FH) is the same multiple of the sum AE,EB (AB) (V·1)

But FG is the same multiple of AE that K is of C, therefore FH is the same multiple of AB that K is of C.

Therefore FH and K are equimultiples of AB and C.

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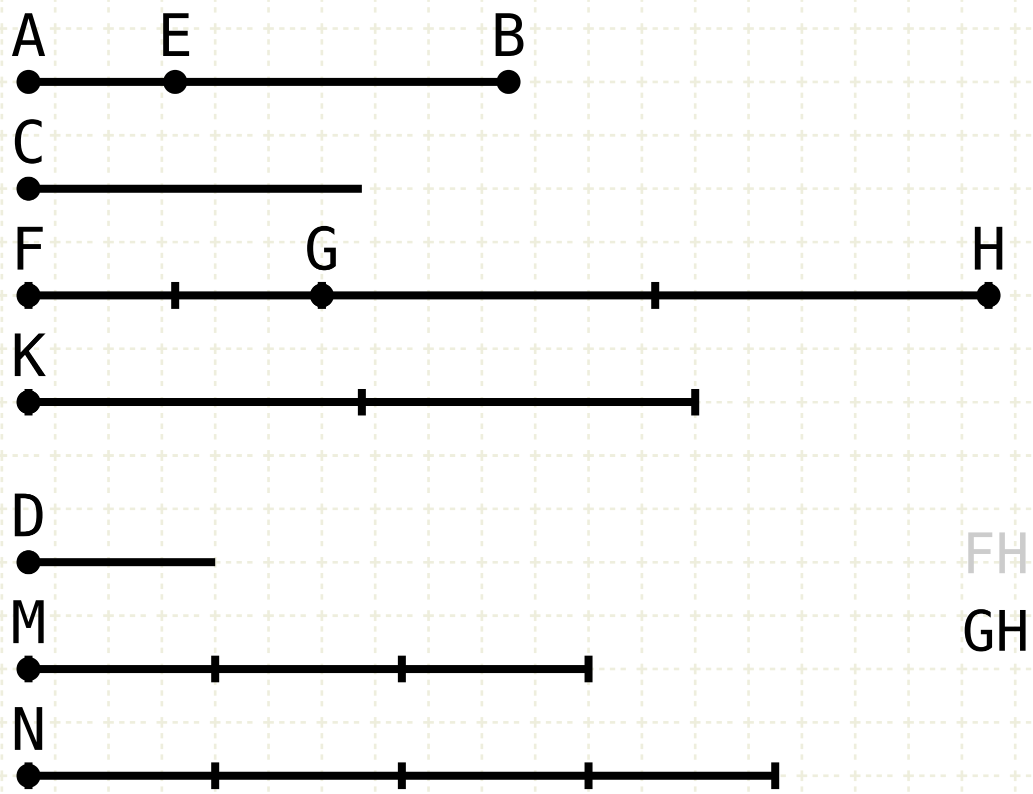

FH = n·AB

GH = K

AB > C ≠ D

EB = C

AE < EB

FG = n·AE > D

GH = n·EB

K = n·C

M = (j-1)·D ≤ K

N = j·D > K

Proof

Case 1: AE < EB

Since FG is the same multiple of AE as GH is to EB, then sum of FG,GH (FH) is the same multiple of the sum AE,EB (AB) (V·1)

But FG is the same multiple of AE that K is of C, therefore FH is the same multiple of AB that K is of C.

Therefore FH and K are equimultiples of AB and C.

Since GH and K are equimultiples of EB and C, and EB and C are equal, GH equals K

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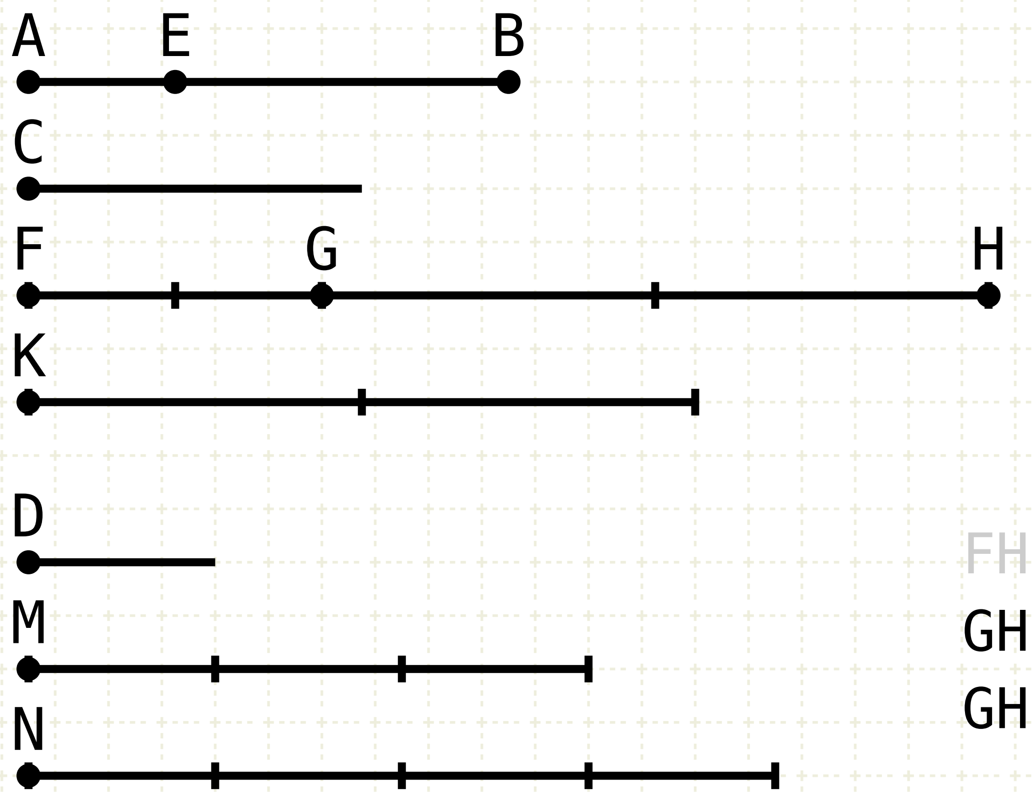

$FH = n \cdot AB$

$GH = K$

$GH \geq M$

$AB > C \neq D$

$EB = C$

$AE < EB$

$FG = n \cdot AE > D$

$GH = n \cdot EB$

$K = n \cdot C$

$M = (j-1) \cdot D \leq K$

$N = j \cdot D > K$

Proof

Case 1: AE < EB

Since FG is the same multiple of AE as GH is to EB, then sum of FG,GH (FH) is the same multiple of the sum AE,EB (AB) (V·1)

But FG is the same multiple of AE that K is of C, therefore FH is the same multiple of AB that K is of C.

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Since GH and K are equimultiples of EB and C, and EB and C are equal, GH equals K

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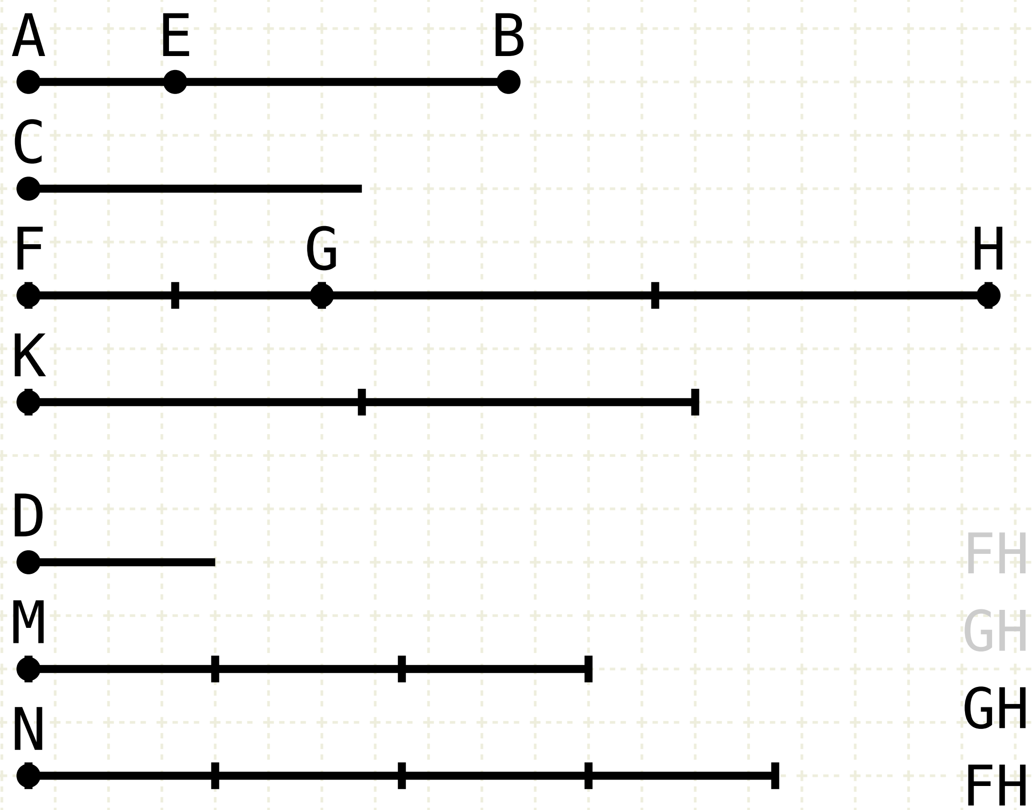

$FH = n \cdot AB$

$GH = K$

$GH \geq M$

$FH = FG+GH > D + M$

$AB > C \neq D$

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Since GH equals K, and K is greater than M, GH is greater than M

FG is greater than D, GH is greater than or equal to M, so the sum of FG,GH is greater than the sum D,M

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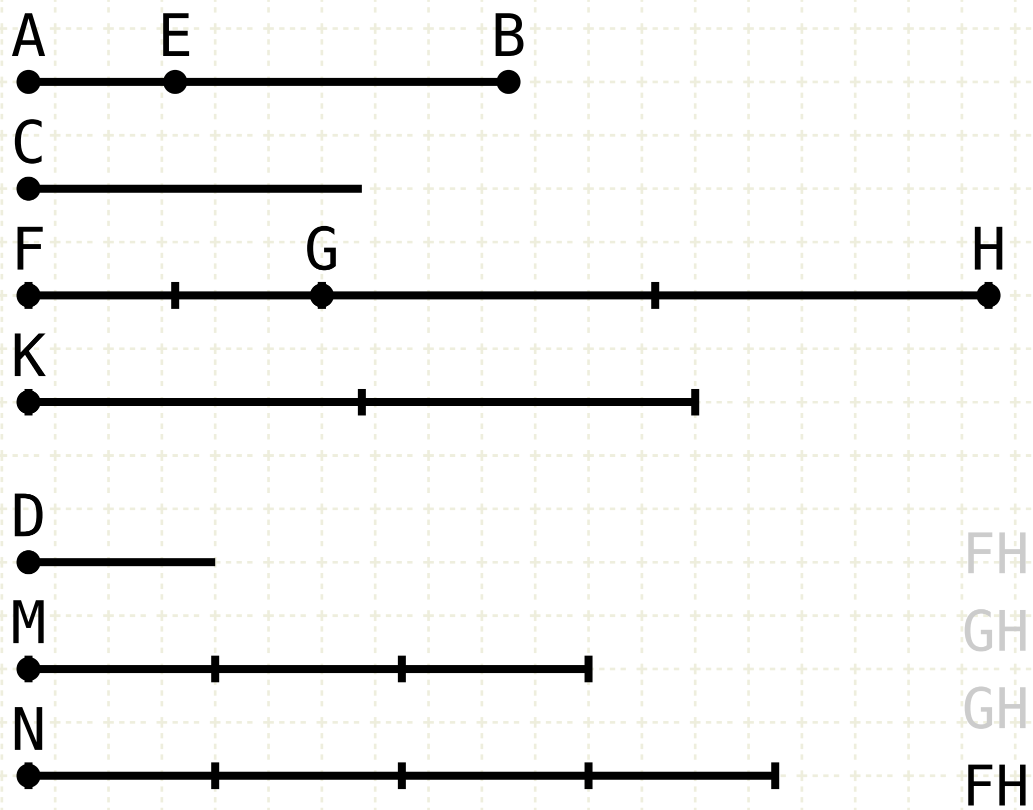

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$GH \geq M$

$FH = FG+GH > D + M$

$D+M = N$

$FH > N$

Proof

Case 1: AE < EB

Since FG is the same multiple of AE as GH is to EB, then sum of FG,GH (FH) is the same multiple of the sum AE,EB (AB) (V·1)

But FG is the same multiple of AE that K is of C, therefore FH is the same multiple of AB that K is of C.

Therefore FH and K are equimultiples of AB and C.

Since GH and K are equimultiples of EB and C, and EB and C are equal, GH equals K

Since GH equals K, and K is greater than M, GH is greater than M

FG is greater than D, GH is greater than or equal to M, so the sum of FG,GH is greater than the sum D,M

But M is defined as one less multiple of D than N, so the sum of D,M is equal to N

Thus FH is greater than N

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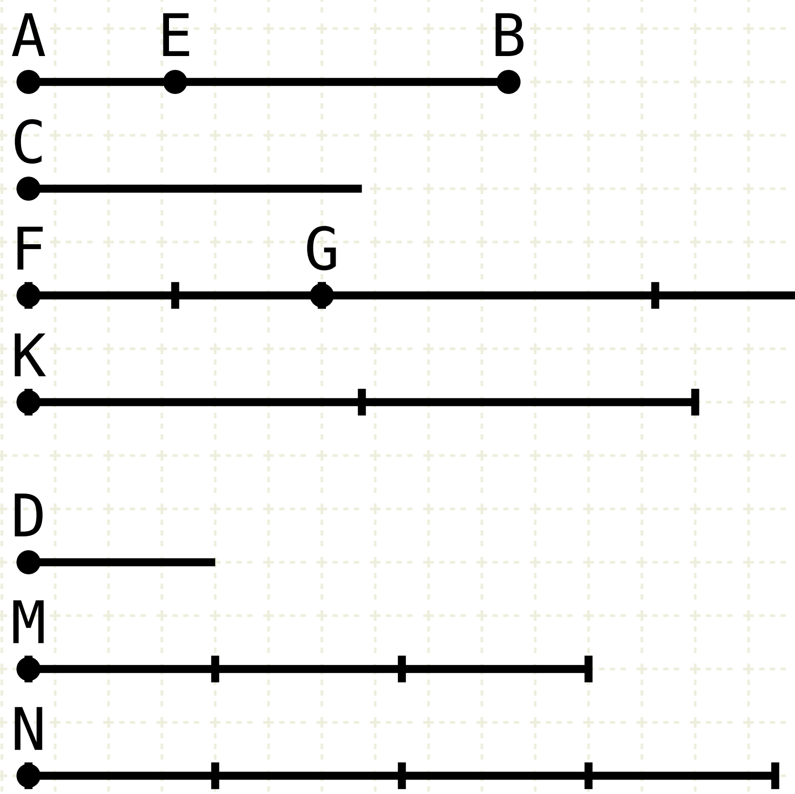

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$FH = n \cdot AB$

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$GH \geq M$

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$D+M = N$

$FH > N$

$K < N$

Proof

Case 1: AE < EB

Since FG is the same multiple of AE as GH is to EB, then sum of FG,GH (FH) is the same multiple of the sum AE,EB (AB) (V·1)

But FG is the same multiple of AE that K is of C, therefore FH is the same multiple of AB that K is of C.

Therefore FH and K are equimultiples of AB and C.

Since GH and K are equimultiples of EB and C, and EB and C are equal, GH equals K

Since GH equals K, and K is greater than M, GH is greater than M

FG is greater than D, GH is greater than or equal to M, so the sum of FG,GH is greater than the sum D,M

But M is defined as one less multiple of D than N, so the sum of D,M is equal to N

Thus FH is greater than N

But N is defined to be greater than K

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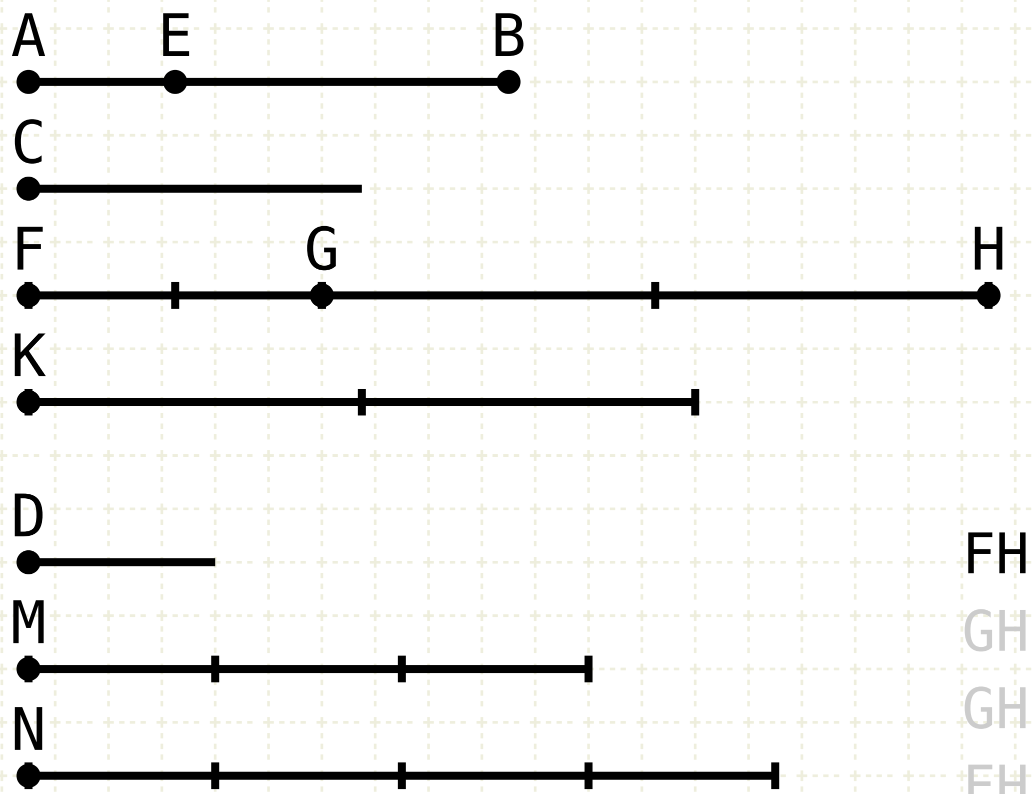

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$N = j \cdot D > K$

$FH = n \cdot AB$

$GH = K$

$GH \geq M$

$FH = FG + GH > D + M$

$D + M = N$

$FH > N$

$K < N$

$n \cdot AB > j \cdot D, \ n \cdot C < j \cdot D$

Proof

Case 1: AE < EB

Since FG is the same multiple of AE as GH is to EB, then sum of FG,GH (FH) is the same multiple of the sum AE,EB (AB) (V·1)

But FG is the same multiple of AE that K is of C, therefore FH is the same multiple of AB that K is of C.

Therefore FH and K are equimultiples of AB and C.

Since GH and K are equimultiples of EB and C, and EB and C are equal, GH equals K

Since GH equals K, and K is greater than M, GH is greater than M

FG is greater than D, GH is greater than or equal to M, so the sum of FG,GH is greater than the sum D,M

But M is defined as one less multiple of D than N, so the sum of D,M is equal to N

Thus FH is greater than N

But N is defined to be greater than K

Rewriting the resulting inequalities as the appropriate multiples of AB,C and D, we have the definition used to describe inequalities of ratios (V def.7)

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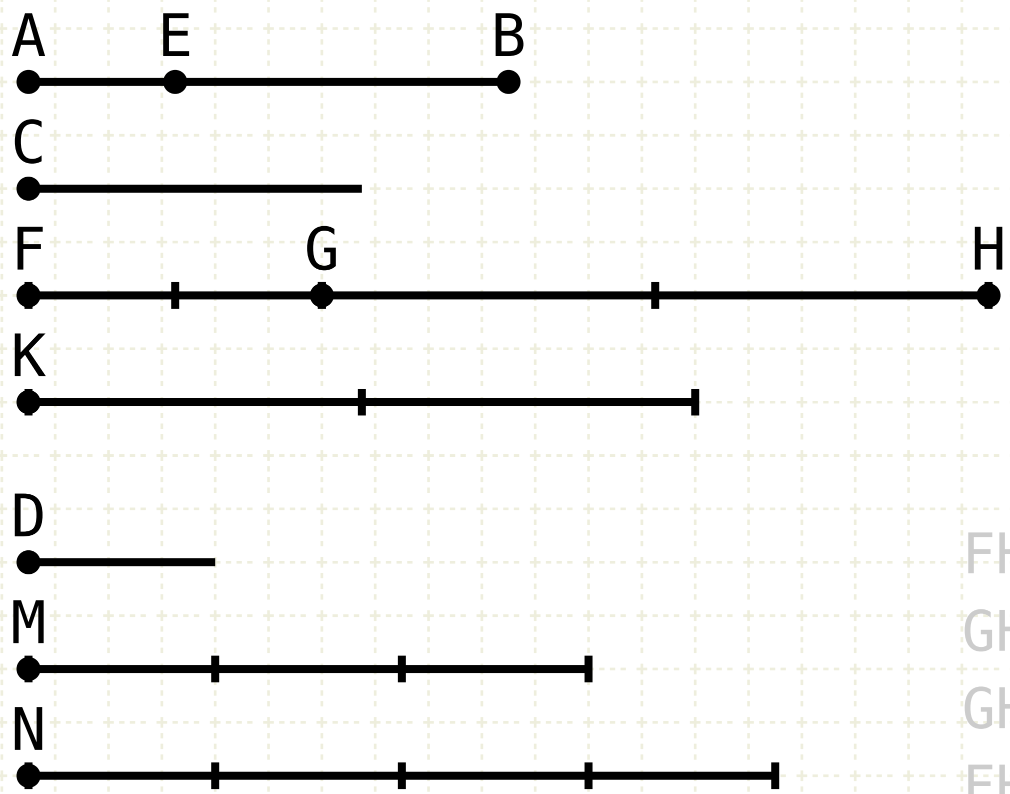

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$K = n \cdot C$

$M = (j-1) \cdot D \leq K$

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$FH = n \cdot AB$

$GH = K$

$GH \geq M$

$FH = FG + GH > D + M$

$D + M = N$

$FH > N$

$K < N$

$n \cdot AB > j \cdot D, \; n \cdot C < j \cdot D$

$AB:D > C:D$

Proof

Case 1: AE < EB

Since FG is the same multiple of AE as GH is to EB, then sum of FG,GH (FH) is the same multiple of the sum AE,EB (AB) (V·1)

But FG is the same multiple of AE that K is of C, therefore FH is the same multiple of AB that K is of C.

Therefore FH and K are equimultiples of AB and C.

Since GH and K are equimultiples of EB and C, and EB and C are equal, GH equals K

Since GH equals K, and K is greater than M, GH is greater than M

FG is greater than D, GH is greater than or equal to M, so the sum of FG,GH is greater than the sum D,M

But M is defined as one less multiple of D than N, so the sum of D,M is equal to N

Thus FH is greater than N

But N is defined to be greater than K

Rewriting the resulting inequalities as the appropriate multiples of AB,C and D, we have the definition used to describe inequalities of ratios (V def.7)

Thus the ratio AB to D is larger than the ratio C to D

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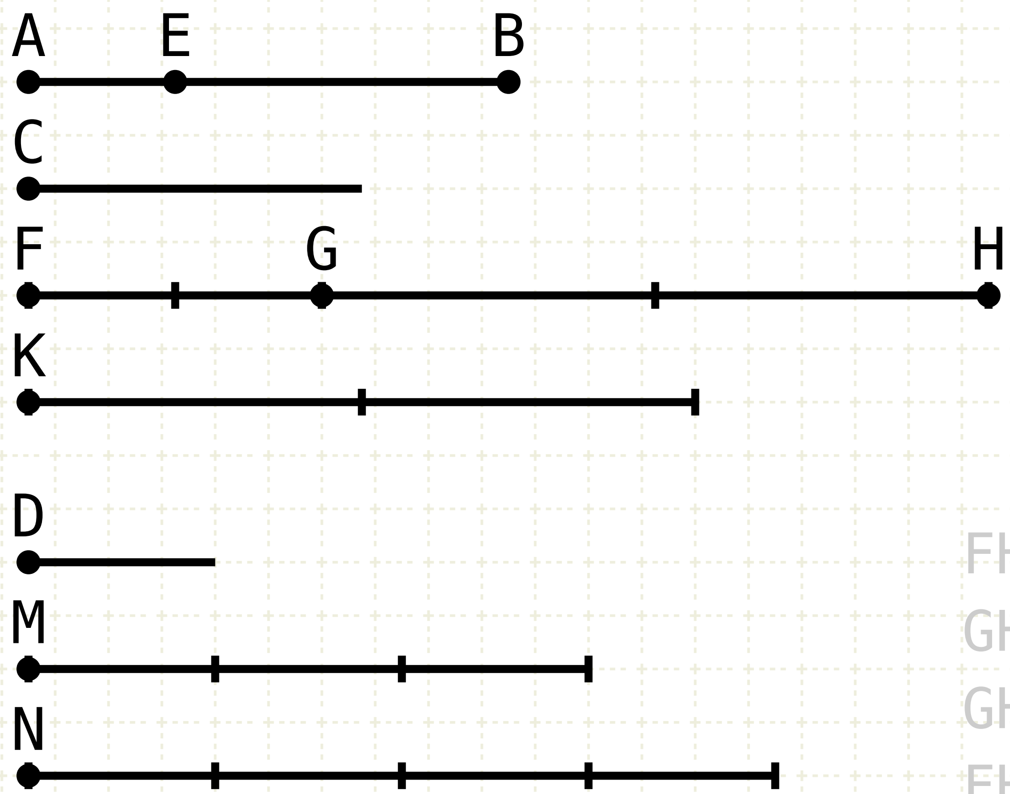

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$FH = n \cdot AB$

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$GH \geq M$

$FH = FG+GH > D + M$

$D+M = N$

$FH > N$

$K < N$

$n \cdot AB > j \cdot D, \ n \cdot C < j \cdot D$

$AB:D > C:D$

Proof

Case 1: AE < EB

Since FG is the same multiple of AE as GH is to EB, then sum of FG,GH (FH) is the same multiple of the sum AE,EB (AB) (V·1)

But FG is the same multiple of AE that K is of C, therefore FH is the same multiple of AB that K is of C.

Therefore FH and K are equimultiples of AB and C.

Since GH and K are equimultiples of EB and C, and EB and C are equal, GH equals K

Since GH equals K, and K is greater than M, GH is greater than M

FG is greater than D, GH is greater than or equal to M, so the sum of FG,GH is greater than the sum D,M

But M is defined as one less multiple of D than N, so the sum of D,M is equal to N

Thus FH is greater than N

But N is defined to be greater than K

Rewriting the resulting inequalities as the appropriate multiples of AB,C and D, we have the definition used to describe inequalities of ratios (V def.7)

Thus the ratio AB to D is larger than the ratio C to D

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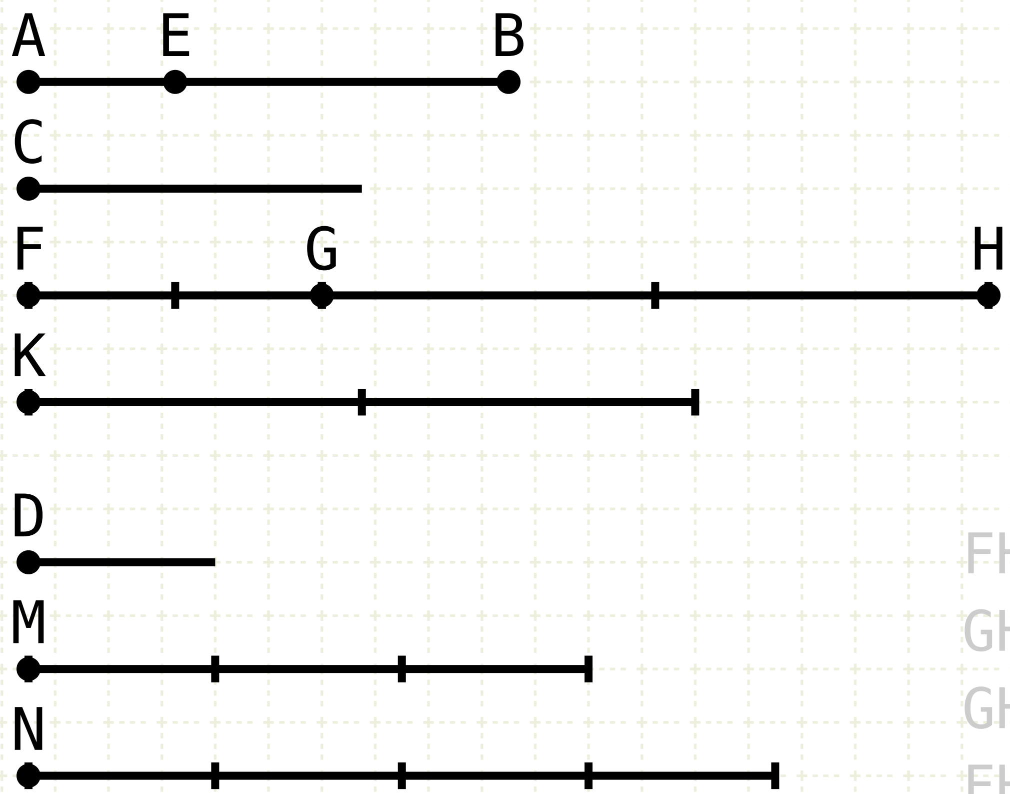

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$FH > N$

$K < N$

$n \cdot AB > j \cdot D, \; n \cdot C < j \cdot D$

$AB:D > C:D$

$j \cdot D > n \cdot C, \; j \cdot D < n \cdot AB$

$D:C > D:AB$

Proof

Case 1: AE < EB

Since FG is the same multiple of AE as GH is to EB, then sum of FG,GH (FH) is the same multiple of the sum AE,EB (AB) (V·1)

But FG is the same multiple of AE that K is of C, therefore FH is the same multiple of AB that K is of C.

Therefore FH and K are equimultiples of AB and C.

Since GH and K are equimultiples of EB and C, and EB and C are equal, GH equals K

Since GH equals K, and K is greater than M, GH is greater than M

FG is greater than D, GH is greater than or equal to M, so the sum of FG,GH is greater than the sum D,M

But M is defined as one less multiple of D than N, so the sum of D,M is equal to N

Thus FH is greater than N

But N is defined to be greater than K

Rewriting the resulting inequalities as the appropriate multiples of AB,C and D, we have the definition used to describe inequalities of ratios (V def.7)

Thus the ratio AB to D is larger than the ratio C to D

Similarly, reversing the comparison operators, it can be seen that the ratio D to C is greater than D to AB

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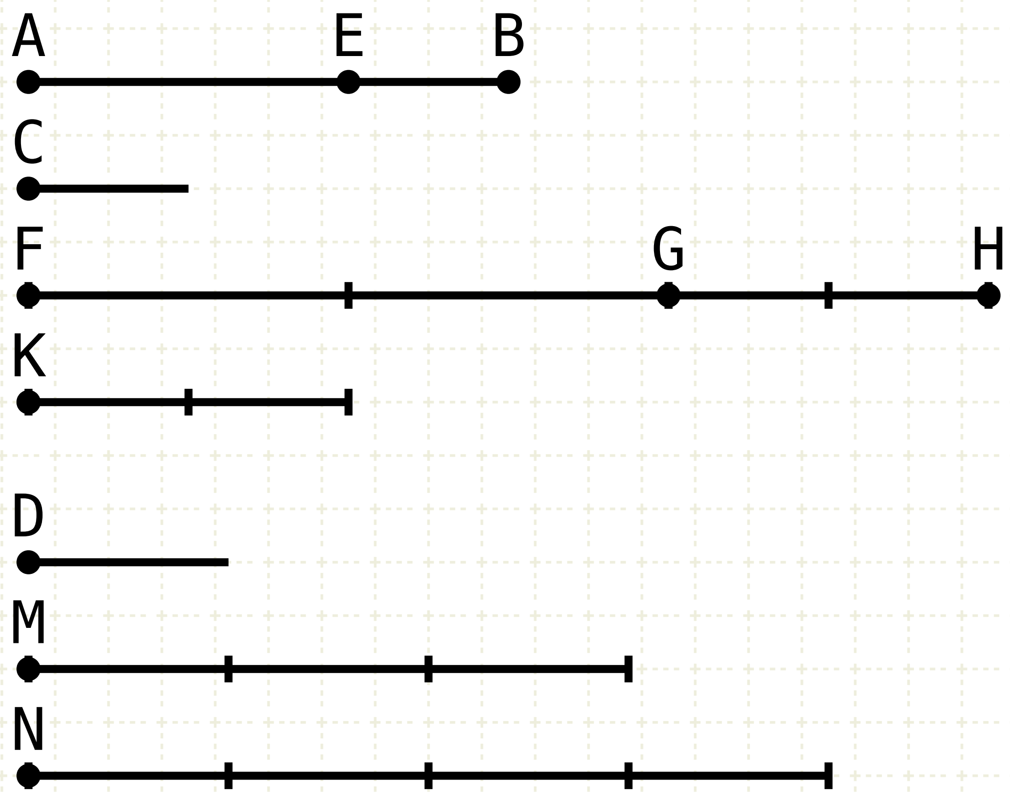
Proposition 8 of Book V

Of unequal magnitudes, the greater has to the same a greater ratio than the less has; and the same has to the less a greater ratio than it has to the greater



$AB > C$

$EB = C$

$AE > EB$

$GH = n \cdot EB > D$

$FG = n \cdot AE$

$K = n \cdot C$

$M = (j-1) \cdot D \leq FG$

$N = j \cdot D > FG$

Proof

First, let EB be equal to C. Then the lesser of AE,EB can be multiplied by a number such that it is larger than D (V def.4)

CASE 2: AE > EB

Define a line GH such that is is a multiple of EB, AND it is larger than D

Using the same multiple, define line KG to be the same multiple of AE, and K to be the same multiple of C

Define lines M and N such that

* M is one less multiple than N,
* M is less than or equal to FG
* and N is greater than FG

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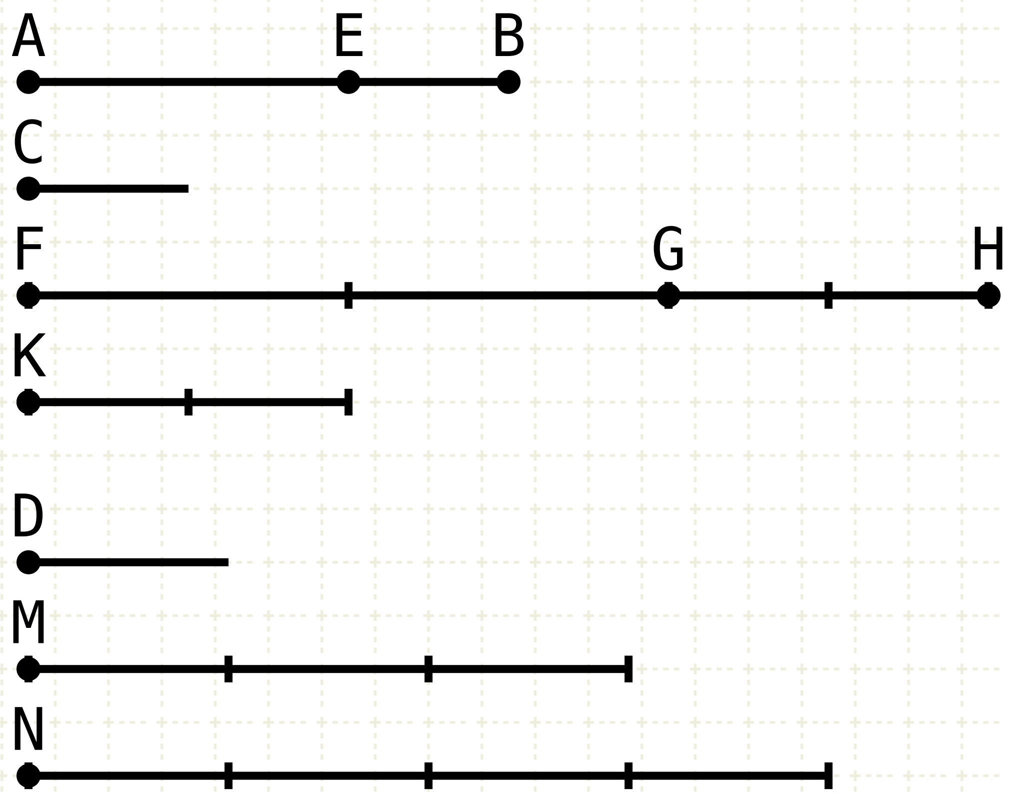

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$N = j \cdot D > FG$

Proof

Case 2: AE > EB

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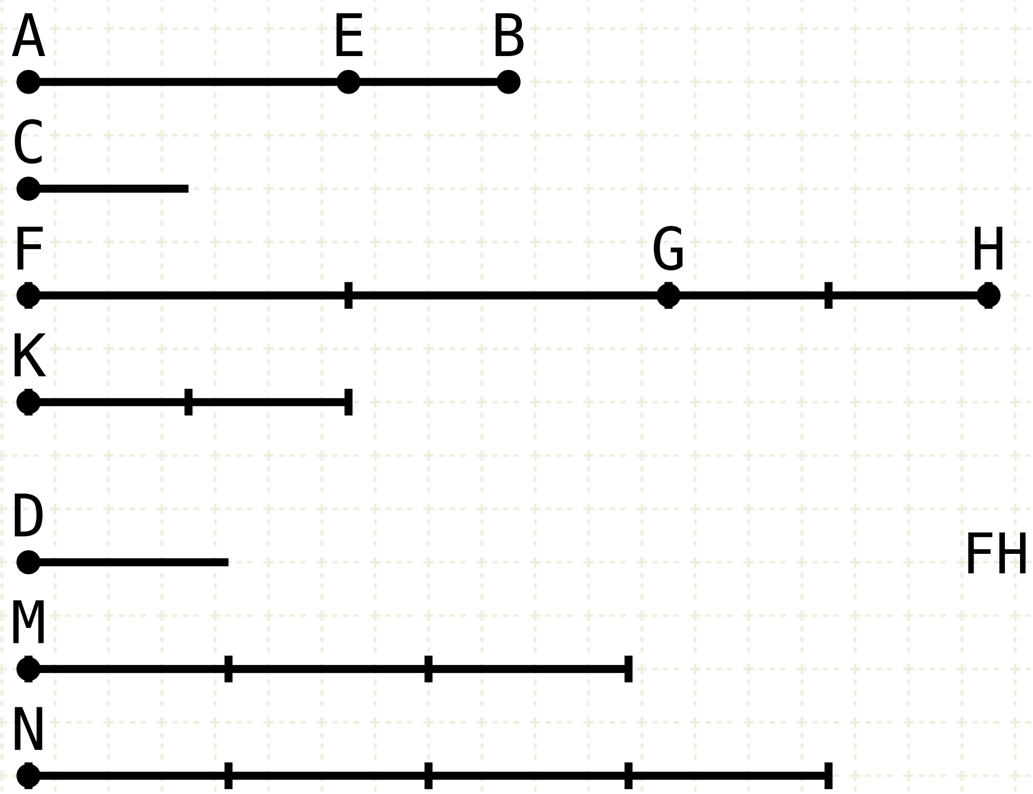

$FH = n \cdot AB$

$AB > C$

$EB = C$

$AE > EB$

$GH = n \cdot EB > D$

$FG = n \cdot AE$

$K = n \cdot C$

$M = (j-1) \cdot D \leq FG$

$N = j \cdot D > FG$

Proof

Case 2: AE > EB

FH,K are equimultiples of AB and C

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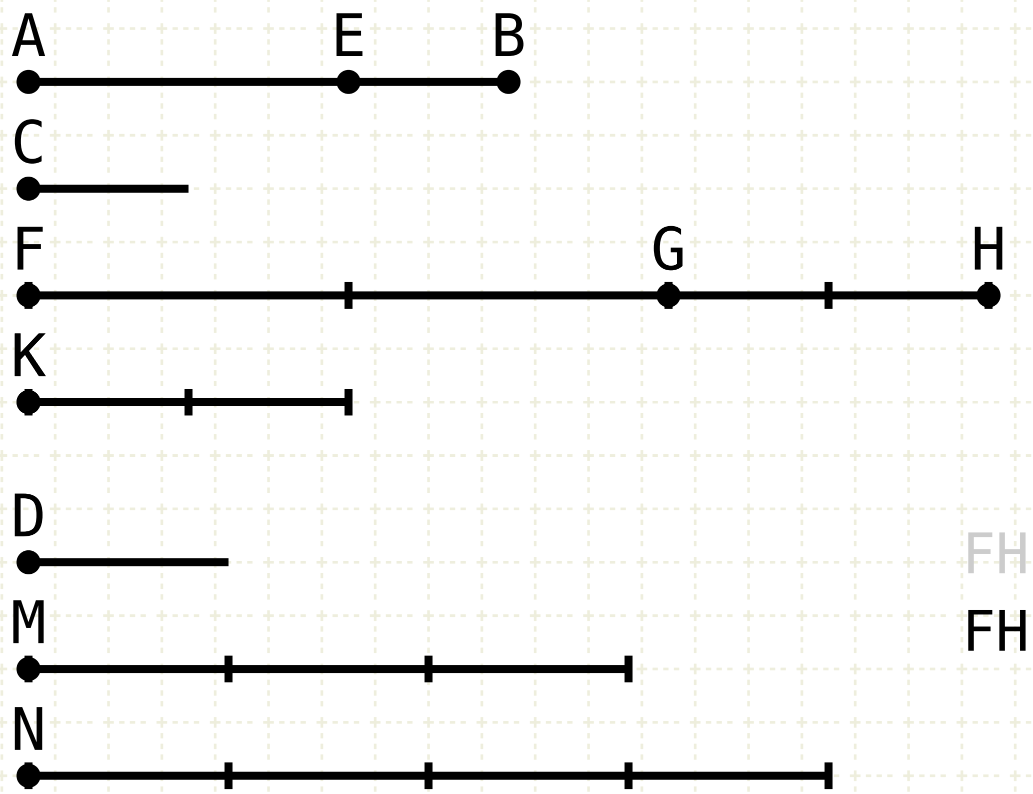
Proposition 8 of Book V

Of unequal magnitudes, the greater has to the same a greater ratio than the less has; and the same has to the less a greater ratio than it has to the greater



$FH = n \cdot AB$

$FH > D+M \rightarrow FH > N$

$AB > C$

$EB = C$

$AE > EB$

$GH = n \cdot EB > D$

$FG = n \cdot AE$

$K = n \cdot C$

$M = (j-1) \cdot D \leq FG$

$N = j \cdot D > FG$

Proof

Case 2: AE > EB

FH,K are equimultiples of AB and C

The whole of FH is greater than the sum of D and M, which is equal to N

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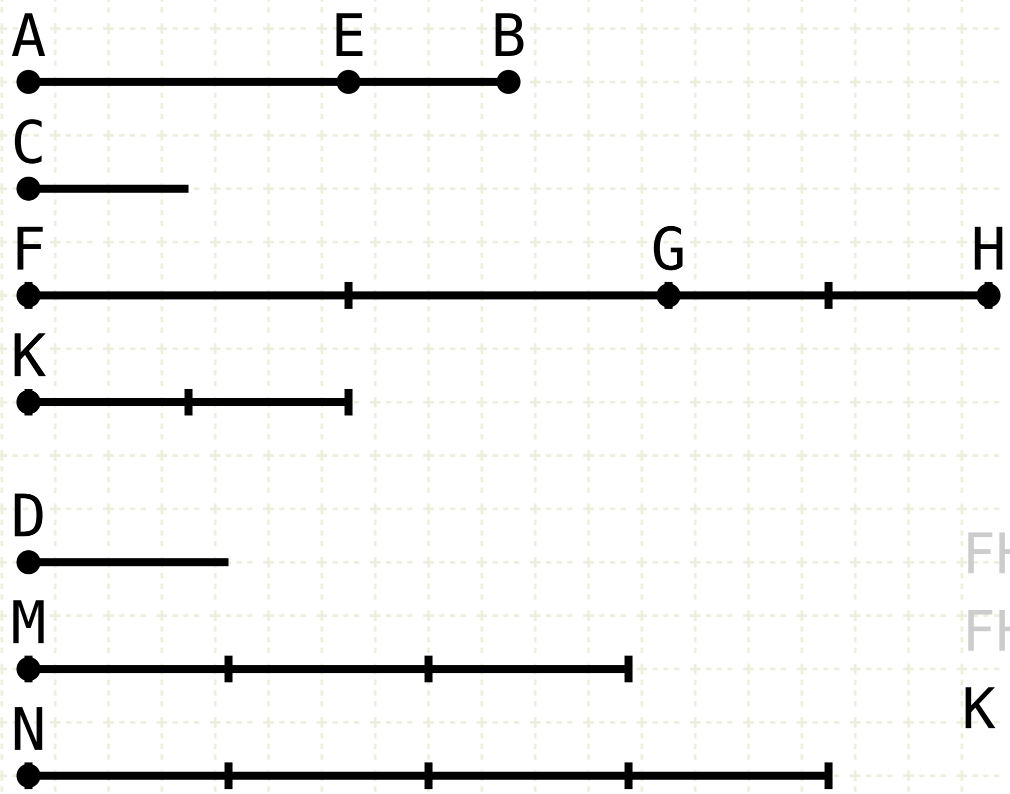
Proposition 8 of Book V

Of unequal magnitudes, the greater has to the same a greater ratio than the less has; and the same has to the less a greater ratio than it has to the greater



$FH = n \cdot AB$

$FH > D+M \rightarrow FH > N$

$K = GH$

$AB > C$

$EB = C$

$AE > EB$

$GH = n \cdot EB > D$

$FG = n \cdot AE$

$K = n \cdot C$

$M = (j-1) \cdot D \leq FG$

$N = j \cdot D > FG$

Proof

Case 2: AE > EB

FH,K are equimultiples of AB and C

The whole of FH is greater than the sum of D and M, which is equal to N

K is equal to GH

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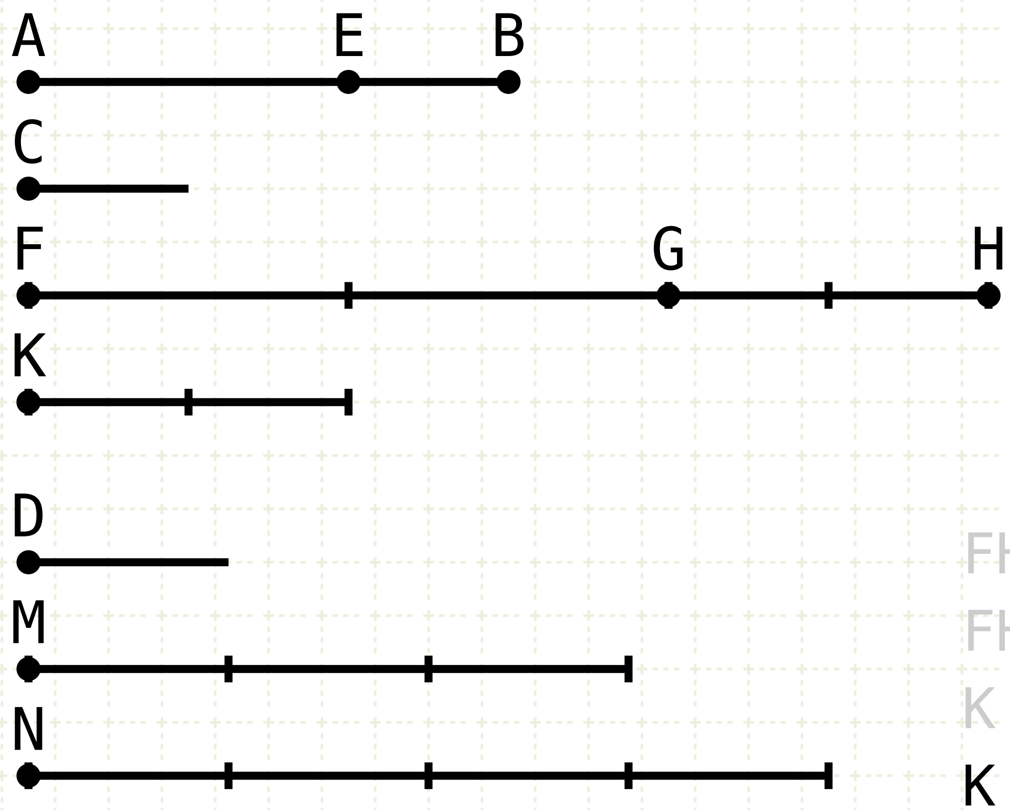

FH = n·AB

FH > D+M → FH > N

K = GH

K = GH

GH < FG ≤ N

K < N

AB > C

EB = C

AE > EB

GH = n·EB > D

FG = n·AE

K = n·C

M = (j-1)·D ≤ FG

N = j·D > FG

Proof

Case 2: AE > EB

FH,K are equimultiples of AB and C

The whole of FH is greater than the sum of D and M, which is equal to N

K is equal to GH

But K is equal to GH, and GH is less than FG which is less than N thus K is less than N

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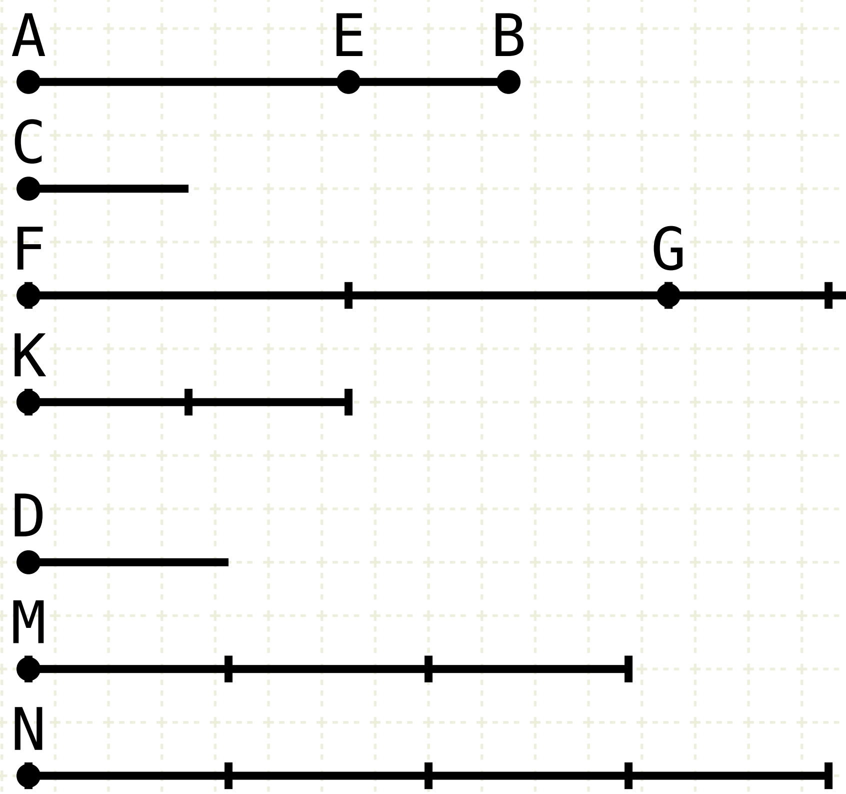
Of unequal magnitudes, the greater has to the same a greater ratio than the less has; and the same has to the less a greater ratio than it has to the greater



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$K = n \cdot C$

$M = (j-1) \cdot D \leq FG$

$N = j \cdot D > FG$

$FH = n \cdot AB$

$FH > D+M \rightarrow FH > N$

$K = GH$

$K = GH$

$GH < FG \leq N$

$K < N$

$n \cdot AB > j \cdot D$

$n \cdot C < j \cdot D$

$AB:D > C:D$

Proof

Case 2: AE > EB

FH,K are equimultiples of AB and C

The whole of FH is greater than the sum of D and M, which is equal to N

K is equal to GH

But K is equal to GH, and GH is less than FG which is less than N thus K is less than N

Therefore we have the inequalities that prove the relationships between the ratios

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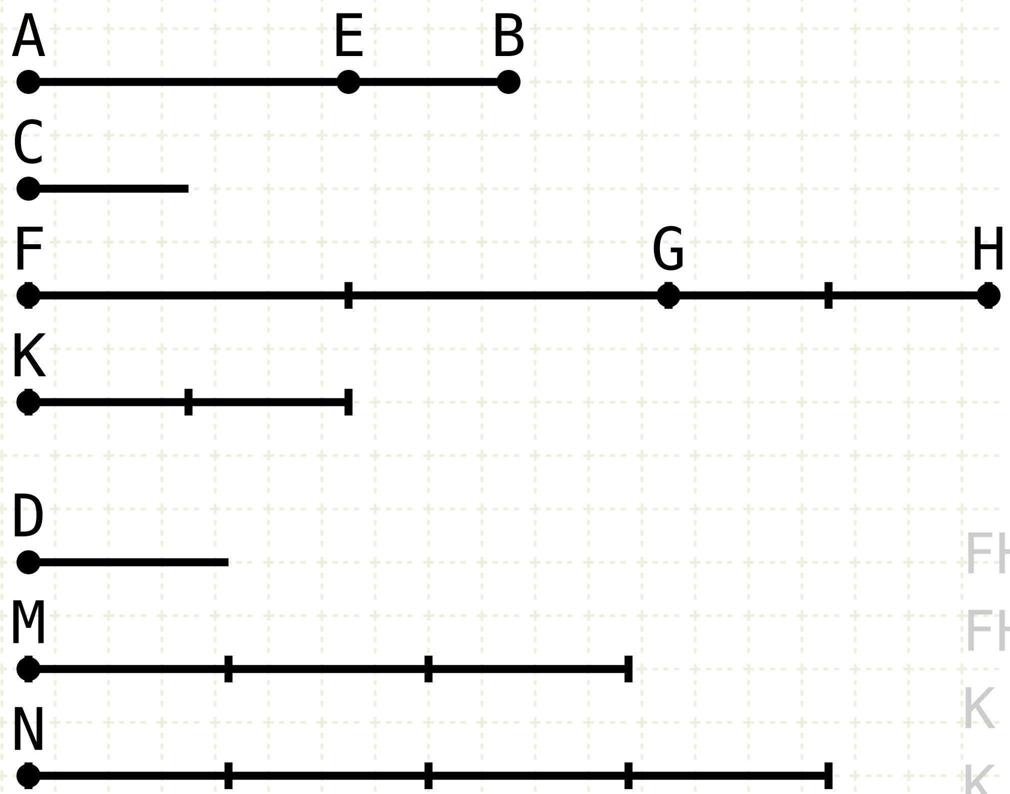
Of unequal magnitudes, the greater has to the same a greater ratio than the less has; and the same has to the less a greater ratio than it has to the greater



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$K = n \cdot C$

$M = (j-1) \cdot D \leq FG$

$N = j \cdot D > FG$

$FH = n \cdot AB$

$FH > D+M \rightarrow FH > N$

$K = GH$

$K = GH$

$GH < FG \leq N$

$K < N$

$n \cdot AB > j \cdot D$

$n \cdot C < j \cdot D$

$AB:D > C:D$

Proof

Case 2: AE > EB

FH,K are equimultiples of AB and C

The whole of FH is greater than the sum of D and M, which is equal to N

K is equal to GH

But K is equal to GH, and GH is less than FG which is less than N thus K is less than N

Therefore we have the inequalities that prove the relationships between the ratios

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