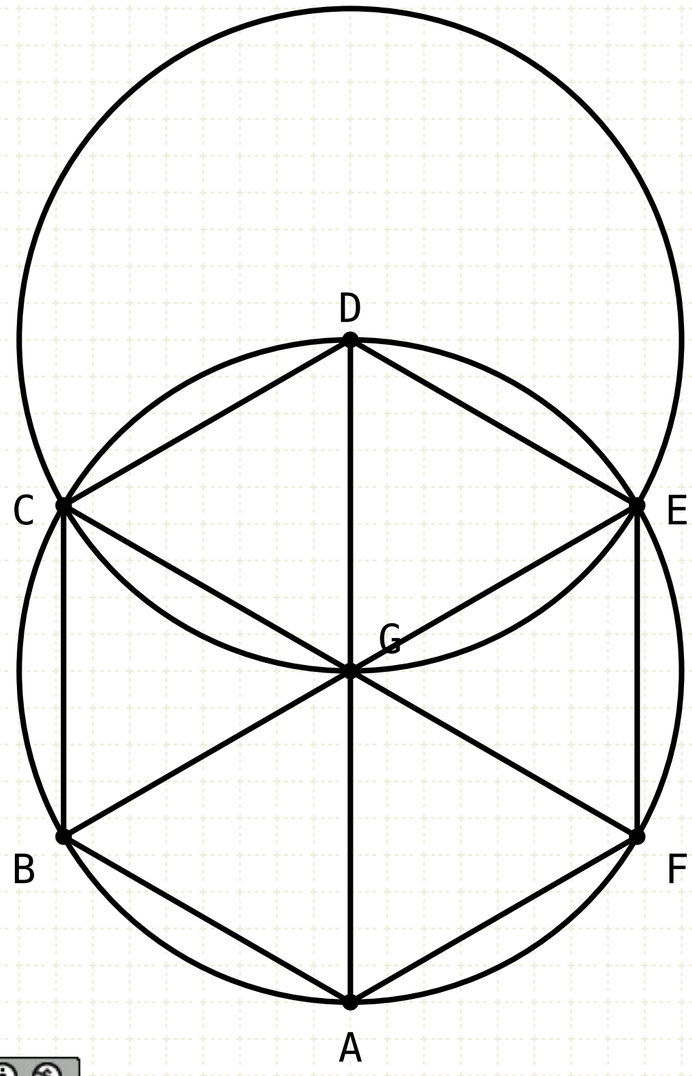
Euclid's Elements

Book IV



Philosophy (nature) is written in that great book which ever is before our eyes -- I mean the universe -- but we cannot understand it if we do not first learn the language and grasp the symbols in which it is written. The book is written in mathematical language, and the symbols are triangles, circles and other geometrical figures, without whose help it is impossible to comprehend a single word of it - without which one wanders in vain through a dark labyrinth.

Galileo Galilei

Proposition 9 of Book IV

About a given square, to circumscribe a circle.

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Proposition 9 of Book IV

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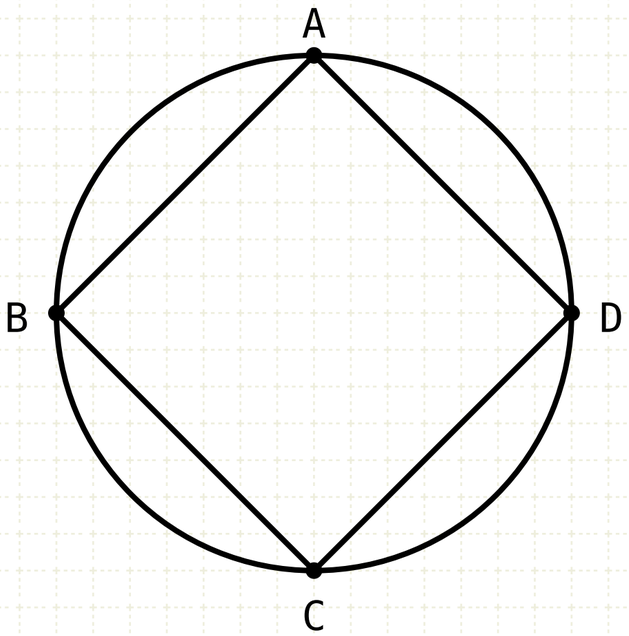
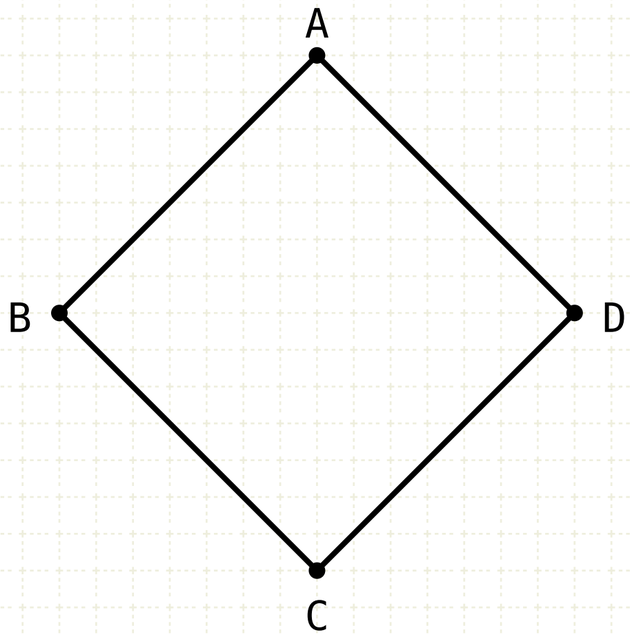


In other words

Given a square ABCD, draw a circle outside the square, going through points A,B,C and D

Proposition 9 of Book IV

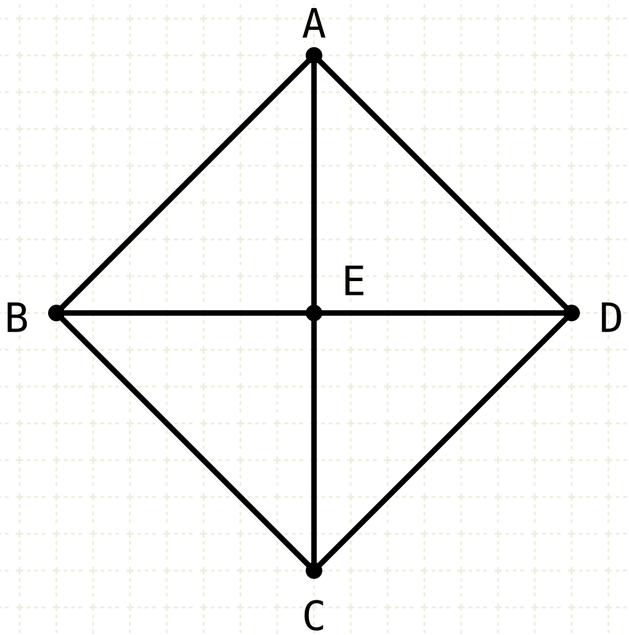
About a given square, to circumscribe a circle.



Construction

Proposition 9 of Book IV

About a given square, to circumscribe a circle.



Construction

Draw lines AC and BD, and label the intersection point E

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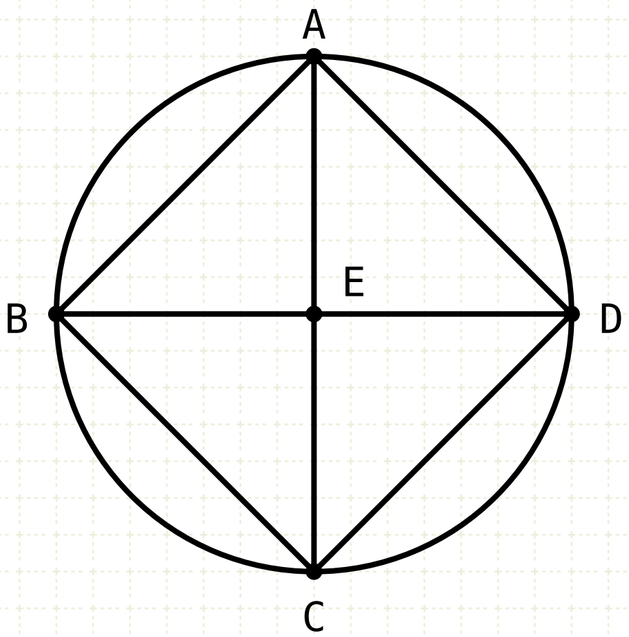

Construction

Draw lines AC and BD, and label the intersection point E

Draw a circle with centre E, and radius AE

This circle circumscribes the square

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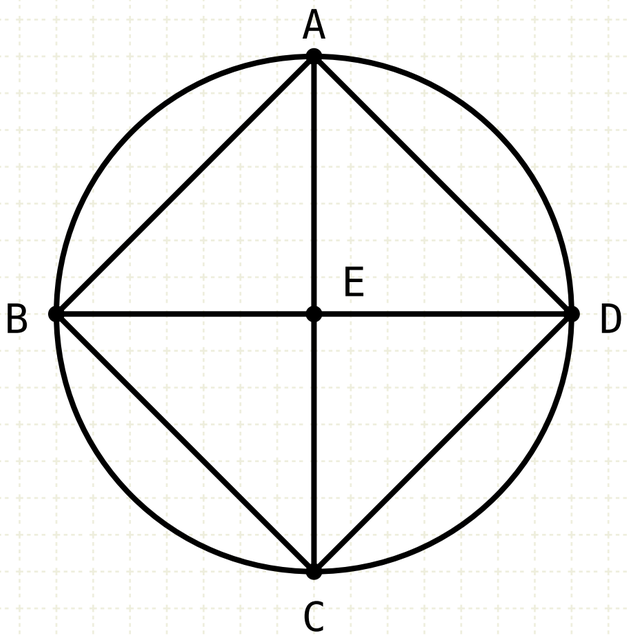

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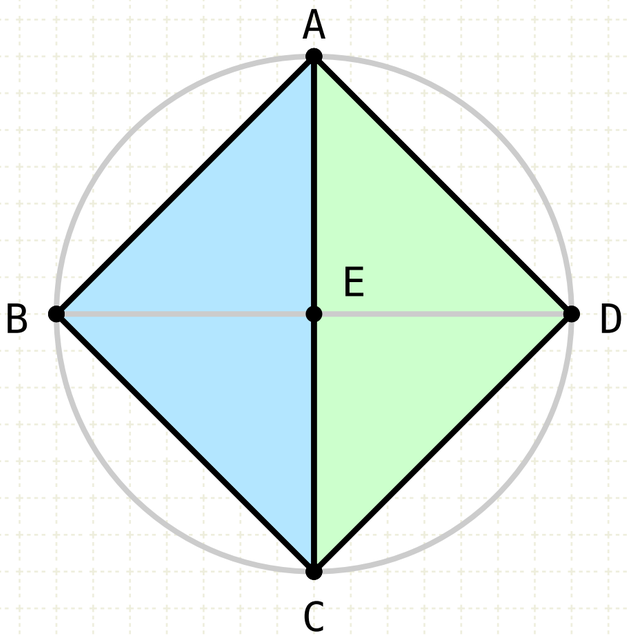

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Consider the two triangles ABC and ADC

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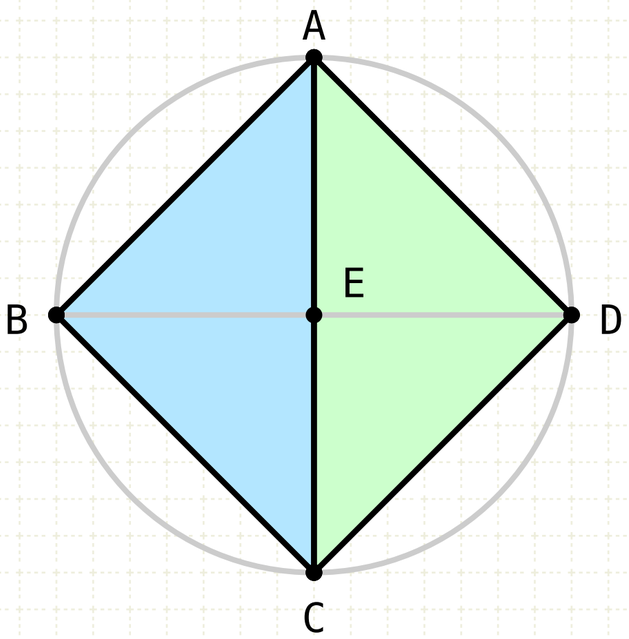

$\Delta ABC \equiv \Delta ADC$

Construction

Draw lines AC and BD, and label the intersection point E

Draw a circle with centre E, and radius AE

This circle circumscribes the square

Proof

Consider the two triangles ABC and ADC

AB equals AD, and AC is common to both triangles

Given that ABCD is a square, the bases BC and CD are also equal, so the triangles are equal in all respects (SSS) (I·8)

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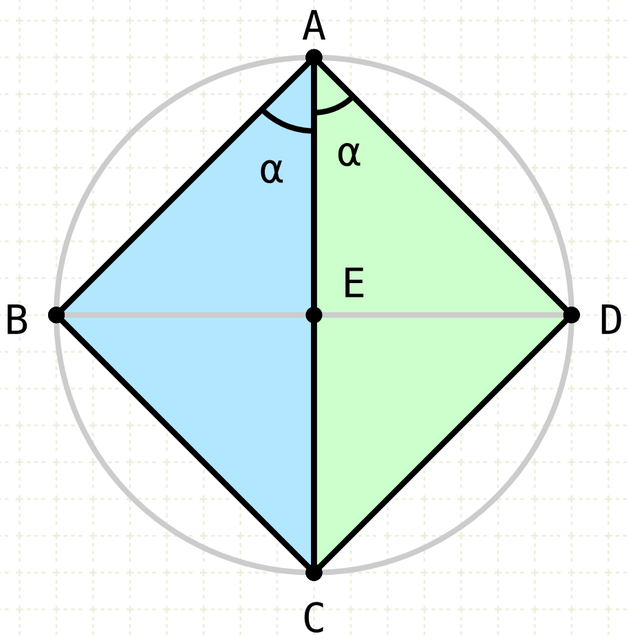

$\Delta ABC \equiv \Delta ADC$

$\alpha = \frac{1}{2} \angle A$

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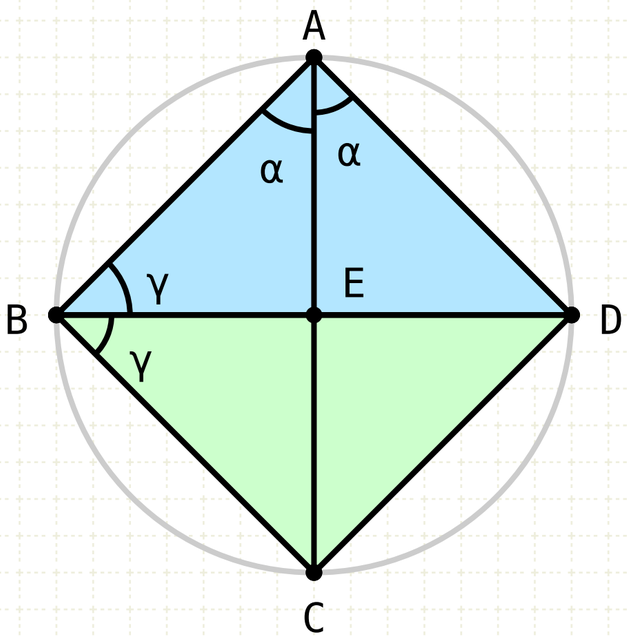

$\Delta ABC \equiv \Delta ADC$

$\alpha = \frac{1}{2} \angle A$

$\gamma = \frac{1}{2} \angle B$

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Draw lines AC and BD, and label the intersection point E

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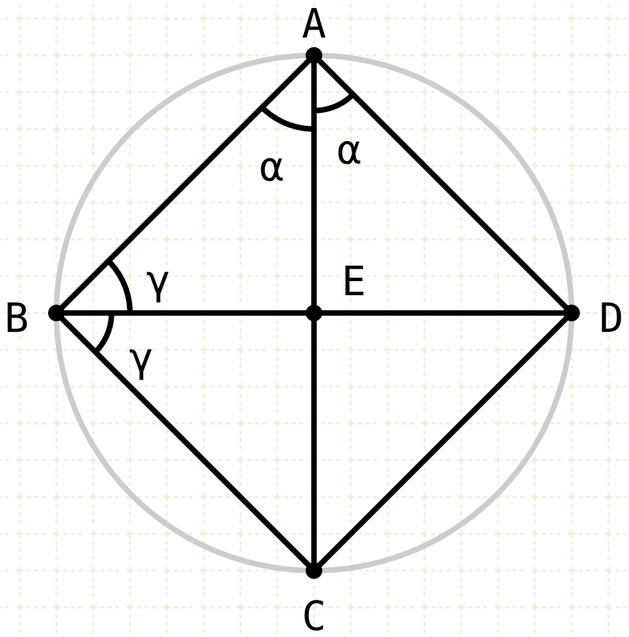

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$\alpha = \frac{1}{2} \angle A$

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$\angle A = \angle B \therefore \alpha = \gamma$

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Since angle A is equal to angle B, and α and γ are half of A and B respectively, α is equal to γ

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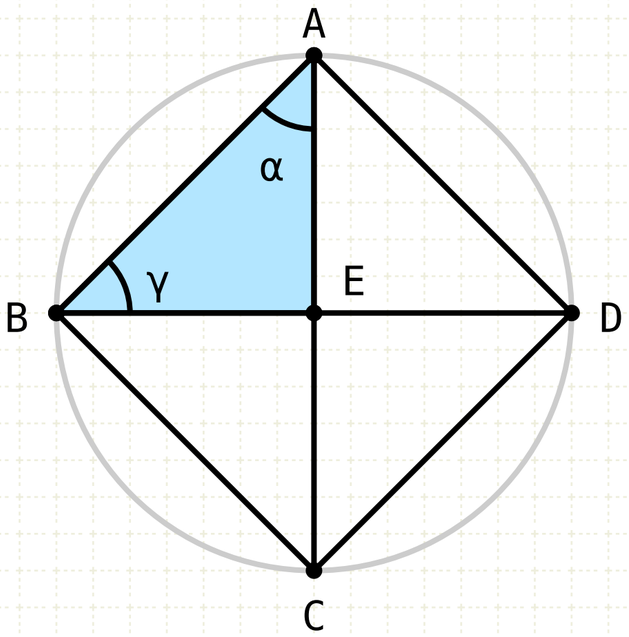

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$AE = BE$

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The triangle ABE is an isosceles triangle, and therefore AE equals BE (I·6)

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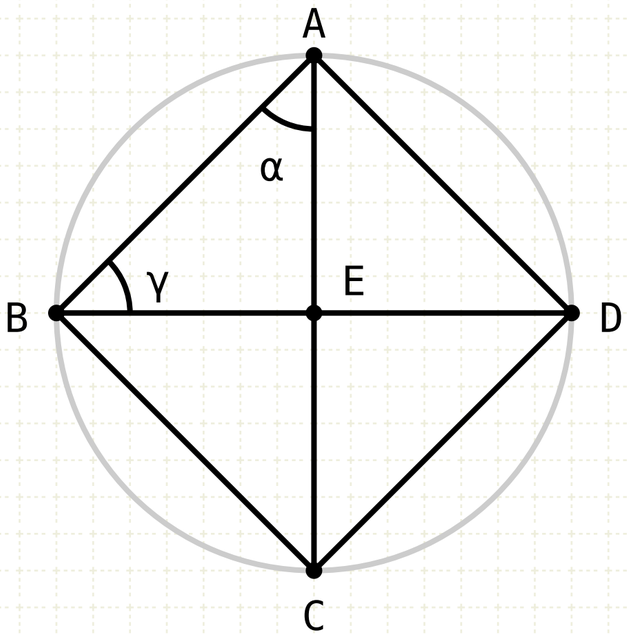

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$AE = BE$

$AE = BE = CE = DE$

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Using the same methods, it can be shown that CE and DE are also equal to AE and BE

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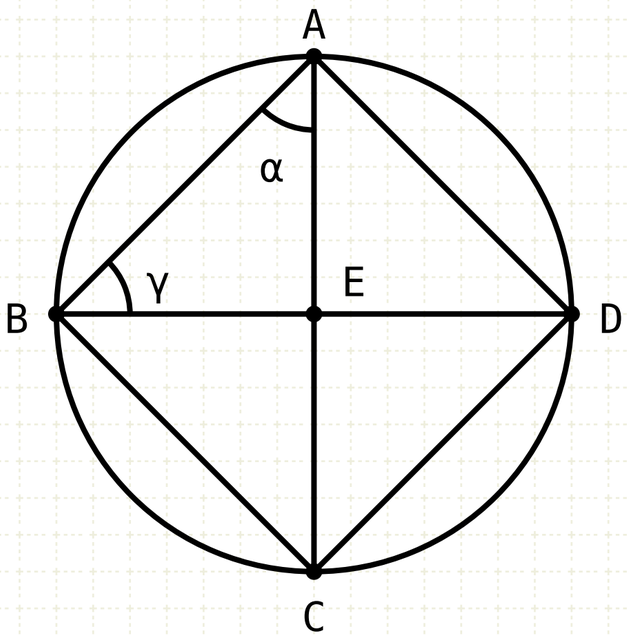

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The triangle ABE is an isosceles triangle, and therefore AE equals BE (I·6)

Using the same methods, it can be shown that CE and DE are also equal to AE and BE

Thus, a circle with the centre at E, with radius AE will pass through the points A,B,C and D

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