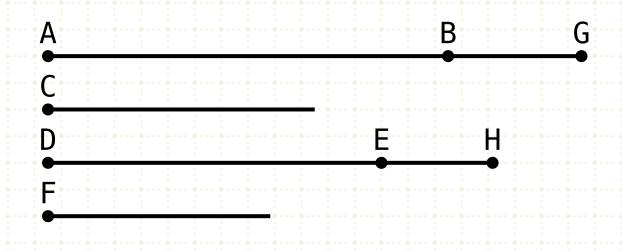
Euclid's Elements

Book V



AB:C = DE:F

BG:C = EH:F

AG:C = DH:F

Proportions are what makes the old Greek temples classic in their beauty. They are like huge blocks, from which the air has been literally hewn out between the columns.

Arne Jacobsen



Table of Contents, Chapter 5

$$1 \quad n \cdot X + n \cdot Y = n \cdot (X + Y)$$

2 if
$$n \cdot C + m \cdot C = k \cdot C$$
 then
 $n \cdot F + m \cdot F = k \cdot F$

- 3 if E=m·(n·B) and G=m·(n·D) then E=k·B and G=k·B
- 4 if A:B=C:D then $(p\cdot A):(q\cdot B)=(p\cdot C):(q\cdot D)$
- 5 $n \cdot X n \cdot Y = n \cdot (X Y)$
- 6 if $n \cdot E m \cdot E = k \cdot E$ then $n \cdot F m \cdot F = k \cdot F$
- 7 if $A = B \neq C$ then A:C = B:C and C:A = C:B
- 8 if A > B ≠ D then A:D > B:D and D:A < D:B
- 9 if A:C = B:C, or C:A = C:B then A = B
- 10 if A:C > B:C, or A:C < B:C then A > B, or A < C, respectively

- 11 if A:B = C:D and C:D = E:F then A:B = E:F
- 12 if A:B = C:D = E:F then (A+C+E):(B+D+F) = A:B
- 13 if A:B = C:D and C:D > E:F then A:B > E:F
- 14 if A:B = C:D and A > C then B > D
- 15 if $A = n \cdot C$ and $B = n \cdot D$ then A:B = C:D
- 16 if A:B = C:D then A:C = B:D
- 17 if (A+B):B = (C+D):D then A:B = C:D
- 18 if A:B = C:D then (A+B):B = (C+D):D
- 19 if (A+C):(B+D) = C:D then (A+C):(B+D) = A:B

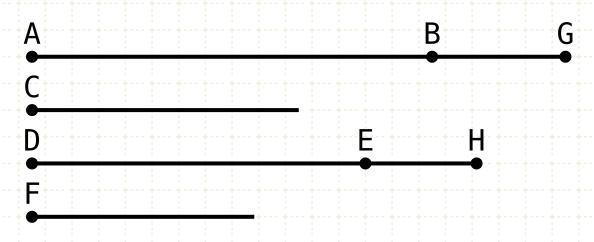
- 20 if A:B = D:E, B:C = E:F and if A > C, then D > F
- 21 if A:B = E:F, B:C = D:E and if A > C, then D > F
- 22 if A:B = D:E, B:C = E:F then A:C = D:F
- 23 if A:B = E:F, B:C = D:E then A:C = D:F
- 24 if A:C = D:F, B:C = E:F then (A+B):C = (D+E):F
- 25 if A:B = C:D and A > B,C,D and D < A,B,C then (A+D) > (B+C)



If a first magnitude have to a second the same ratio as a third has to a fourth, and also a fifth have to the second the same ratio as a sixth to the fourth, the first and fifth added together will have to the second the same ratio as the third and sixth have to the fourth.



If a first magnitude have to a second the same ratio as a third has to a fourth, and also a fifth have to the second the same ratio as a sixth to the fourth, the first and fifth added together will have to the second the same ratio as the third and sixth have to the fourth.



AB:C = DE:F BG:C = EH:F

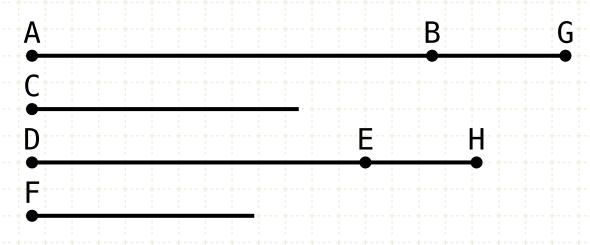
→ (AB+BG):C = (DE+EH):F
AG:C = DH:F

In other words

Let AB to C be as DE to F, and let BG to C be as EH to F
Then AG (sum of AB,BG) will be to C as DH (sum of DE,EH) is to F



If a first magnitude have to a second the same ratio as a third has to a fourth, and also a fifth have to the second the same ratio as a sixth to the fourth, the first and fifth added together will have to the second the same ratio as the third and sixth have to the fourth.



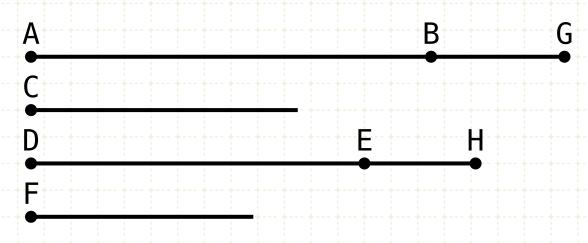
AB:C = DE:F BG:C = EH:F

In other words

Let AB to C be as DE to F, and let BG to C be as EH to F
Then AG (sum of AB,BG) will be to C as DH (sum of DE,EH) is to F

Proof

If a first magnitude have to a second the same ratio as a third has to a fourth, and also a fifth have to the second the same ratio as a sixth to the fourth, the first and fifth added together will have to the second the same ratio as the third and sixth have to the fourth.



AB:C = DE:F

BG:C = EH:F

C:BG = F:EH

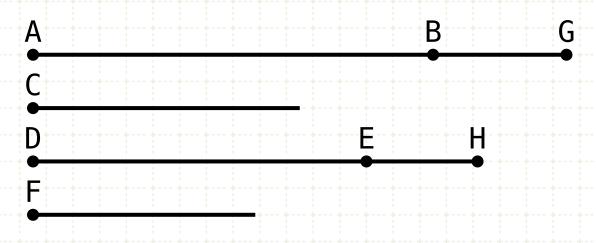
In other words

Let AB to C be as DE to F, and let BG to C be as EH to F
Then AG (sum of AB,BG) will be to C as DH (sum of DE,EH) is to F

Proof

Since BG is to C, so is EH to F, therefore inversely, as C is to BG as F is to EH

If a first magnitude have to a second the same ratio as a third has to a fourth, and also a fifth have to the second the same ratio as a sixth to the fourth, the first and fifth added together will have to the second the same ratio as the third and sixth have to the fourth.



AB:C = DE:F BG:C = EH:F

C:BG = F:EH AB:BG = DE:EH

In other words

Let AB to C be as DE to F, and let BG to C be as EH to F
Then AG (sum of AB,BG) will be to C as DH (sum of DE,EH) is to F

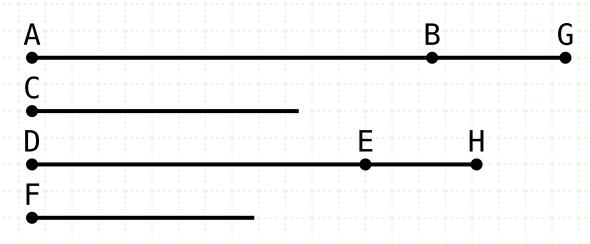
Proof

Since BG is to C, so is EH to F, therefore inversely, as C is to BG as F is to EH

Since AB is to C as DE is to F, and C is to BG as F is to EH, Then AB is to BG so is DE to EH (V·22)



If a first magnitude have to a second the same ratio as a third has to a fourth, and also a fifth have to the second the same ratio as a sixth to the fourth, the first and fifth added together will have to the second the same ratio as the third and sixth have to the fourth.



AB:C = DE:FBG:C = EH:F

C:BG = F:EH

AB:BG = DE:EH

(AB+BG):BG = (DE+EH):EH

AG:BG = DH:EH

In other words

Let AB to C be as DE to F, and let BG to C be as EH to F
Then AG (sum of AB,BG) will be to C as DH (sum of DE,EH) is to F

Proof

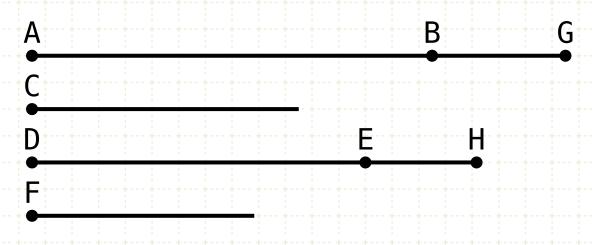
Since BG is to C, so is EH to F, therefore inversely, as C is to BG as F is to EH

Since AB is to C as DE is to F, and C is to BG as F is to EH, Then AB is to BG so is DE to EH (V·22)

And, since magnitudes are proportional SEPARANDO, they will also be proportional COMPONENDO; therefore, as AG is to BG, so is DH to EH (V·18)



If a first magnitude have to a second the same ratio as a third has to a fourth, and also a fifth have to the second the same ratio as a sixth to the fourth, the first and fifth added together will have to the second the same ratio as the third and sixth have to the fourth.



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AB:C = DE:F

BG:C = EH:F
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C:BG = F:EH

AB:BG = DE:EH

(AB+BG):BG = (DE+EH):EH

AG:BG = DH:EH

AG:C = DH:F

In other words

Let AB to C be as DE to F, and let BG to C be as EH to F
Then AG (sum of AB,BG) will be to C as DH (sum of DE,EH) is to F

Proof

Since BG is to C, so is EH to F, therefore inversely, as C is to BG as F is to EH

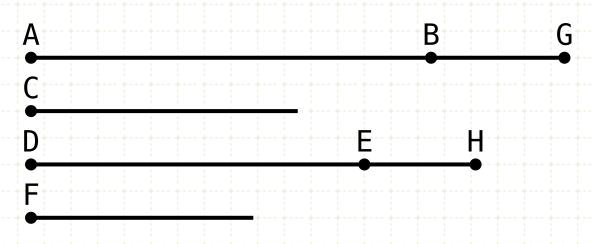
Since AB is to C as DE is to F, and C is to BG as F is to EH, Then AB is to BG so is DE to EH (V·22)

And, since magnitudes are proportional SEPARANDO, they will also be proportional COMPONENDO; therefore, as AG is to BG, so is DH to EH (V·18)

But also, as BG is to C so EH is to F; therefore EX AEQUALI, as AG is to C, so DH is to F (V·22)



If a first magnitude have to a second the same ratio as a third has to a fourth, and also a fifth have to the second the same ratio as a sixth to the fourth, the first and fifth added together will have to the second the same ratio as the third and sixth have to the fourth.



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AB:C = DE:F
BG:C = EH:F
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C:BG = F:EH

AB:BG = DE:EH

(AB+BG):BG = (DE+EH):EH

AG:BG = DH:EH

AG:C = DH:F

In other words

Let AB to C be as DE to F, and let BG to C be as EH to F
Then AG (sum of AB,BG) will be to C as DH (sum of DE,EH) is to F

Proof

Since BG is to C, so is EH to F, therefore inversely, as C is to BG as F is to EH

Since AB is to C as DE is to F, and C is to BG as F is to EH, Then AB is to BG so is DE to EH (V·22)

And, since magnitudes are proportional SEPARANDO, they will also be proportional COMPONENDO; therefore, as AG is to BG, so is DH to EH (V·18)

But also, as BG is to C so EH is to F; therefore EX AEQUALI, as AG is to C, so DH is to F (V·22)



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