

Euclid's Elements

Book I

If Euclid did not kindle your youthful enthusiasm, you were not born to be a scientific thinker.

Albert Einstein

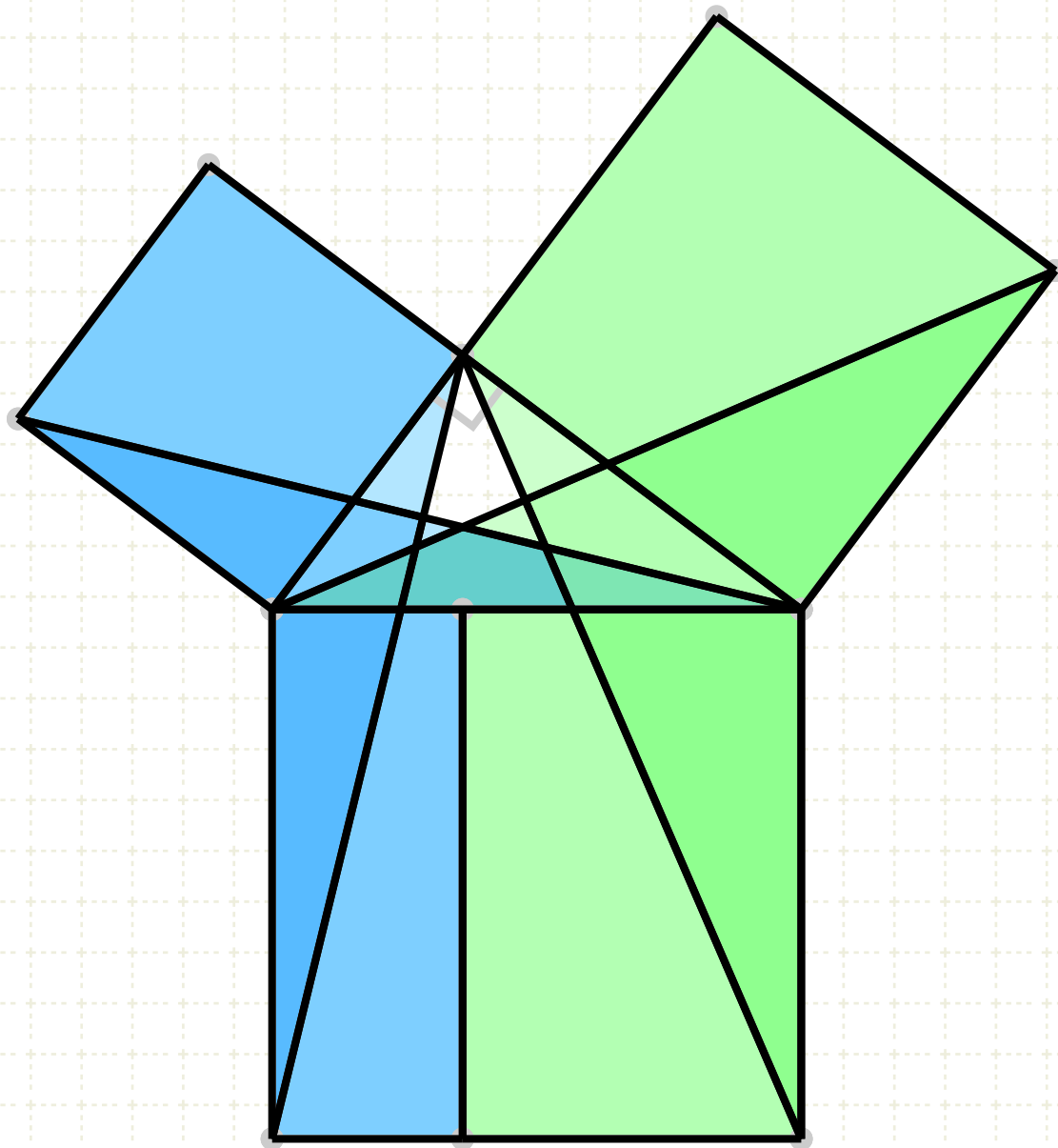


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Proposition 37 of Book I

Triangles which are on equal bases and in the same parallels equal one another.



Proposition 37 of Book I

Triangles which are on equal bases and in the same parallels equal one another.

In other words

Triangles with equal base and height have the same area



Proposition 37 of Book I

Triangles which are on equal bases and in the same parallels equal one another.

In other words

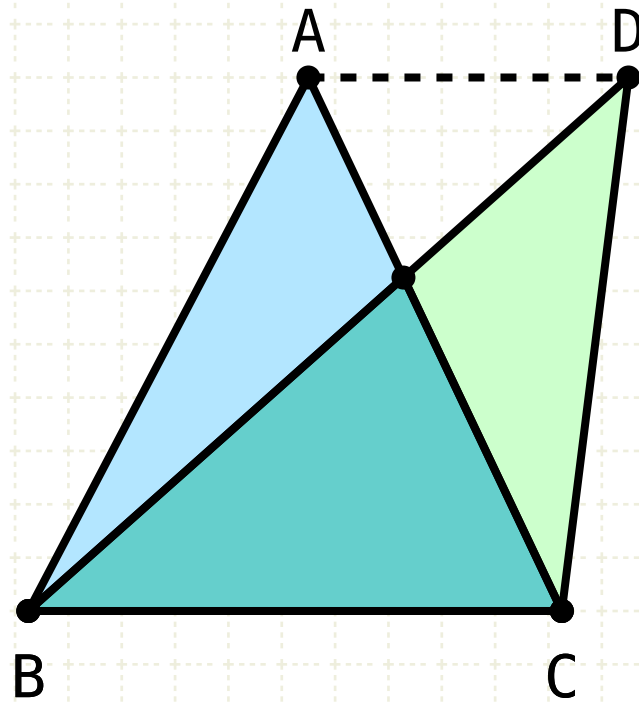
Given two parallel lines

AD || BC



Proposition 37 of Book I

Triangles which are on equal bases and in the same parallels equal one another.



In other words

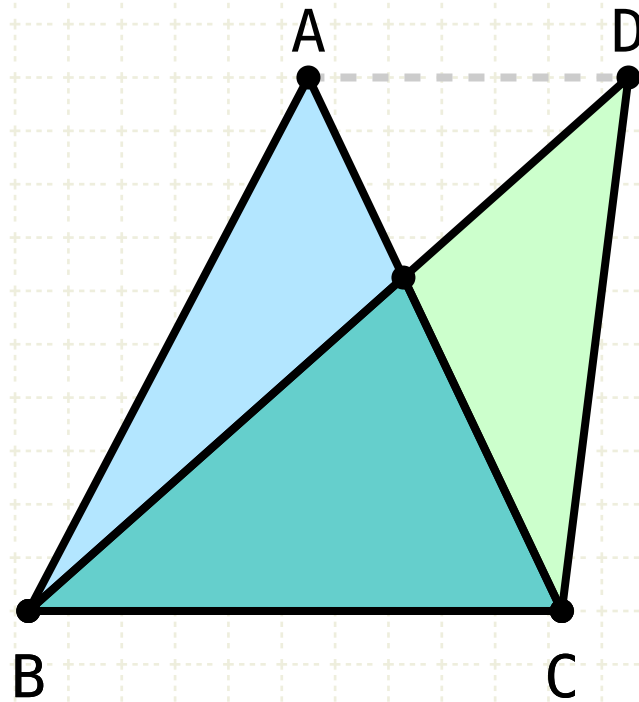
Given two parallel lines

Let ABC and DBC be triangles on the same base BC, and on the same parallels BC and AD

$AD \parallel BC$

Proposition 37 of Book I

Triangles which are on equal bases and in the same parallels equal one another.



In other words

Given two parallel lines

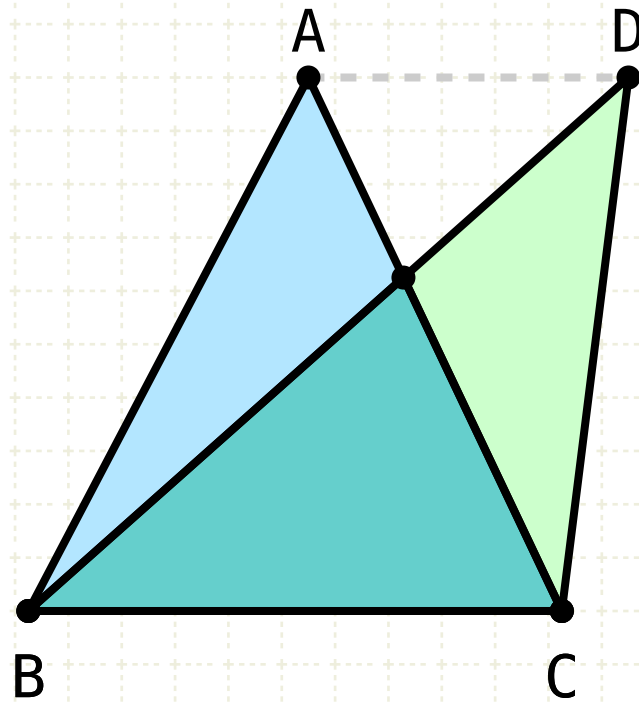
Let ABC and DBC be triangles on the same base BC, and on the same parallels BC and AD

The areas of ABC and DBC are equal

$$AD \parallel BC$$
$$\triangle ABC = \triangle DBC$$

Proposition 37 of Book I

Triangles which are on equal bases and in the same parallels equal one another.



$AD \parallel BC$

In other words

Given two parallel lines

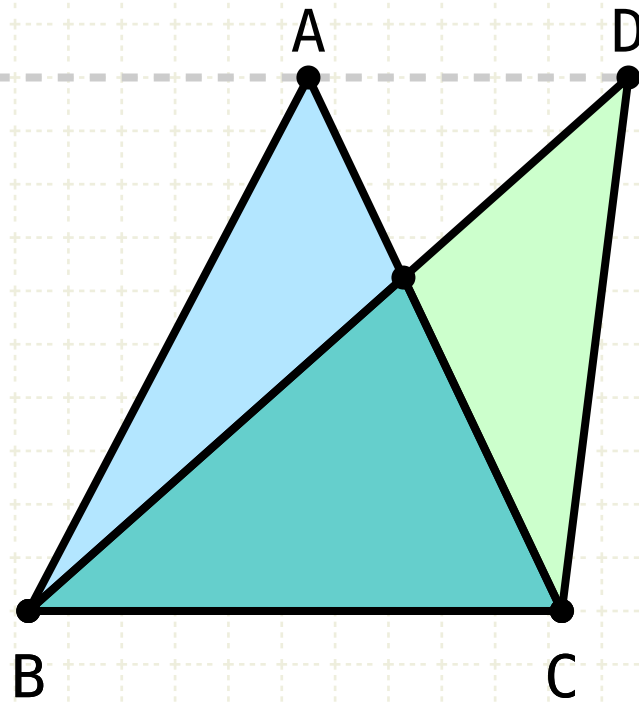
Let ABC and DBC be triangles on the same base BC, and on the same parallels BC and AD

The areas of ABC and DBC are equal

Proof

Proposition 37 of Book I

Triangles which are on equal bases and in the same parallels equal one another.



$AD \parallel BC$

In other words

Given two parallel lines

Let ABC and DBC be triangles on the same base BC, and on the same parallels BC and AD

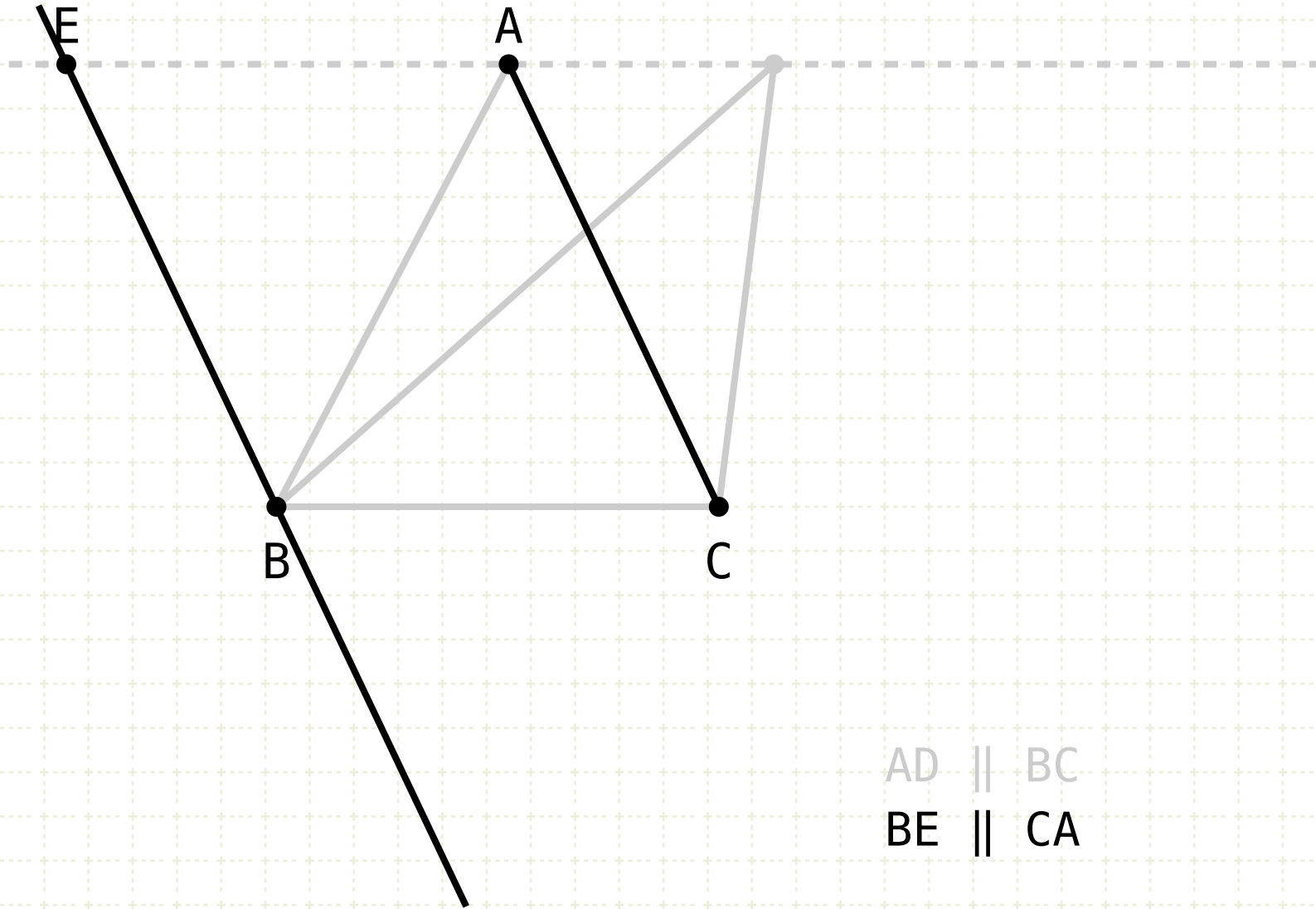
The areas of ABC and DBC are equal

Proof

Extend line AD

Proposition 37 of Book I

Triangles which are on equal bases and in the same parallels equal one another.



AD \parallel BC
BE \parallel CA

In other words

Given two parallel lines

Let ABC and DBC be triangles on the same base BC, and on the same parallels BC and AD

The areas of ABC and DBC are equal

Proof

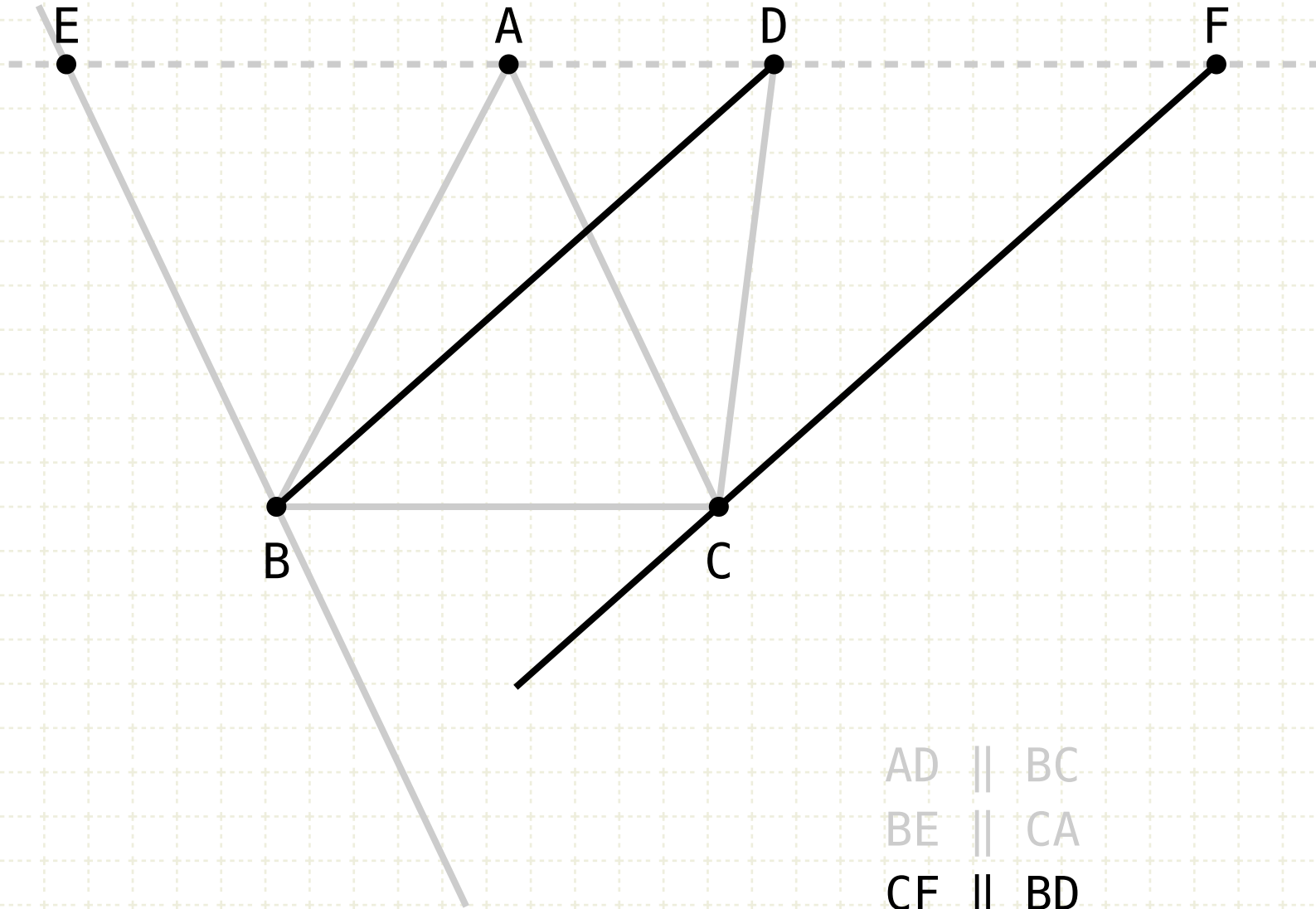
Extend line AD

Draw BE parallel to CA (I-31)



Proposition 37 of Book I

Triangles which are on equal bases and in the same parallels equal one another.



In other words

Given two parallel lines

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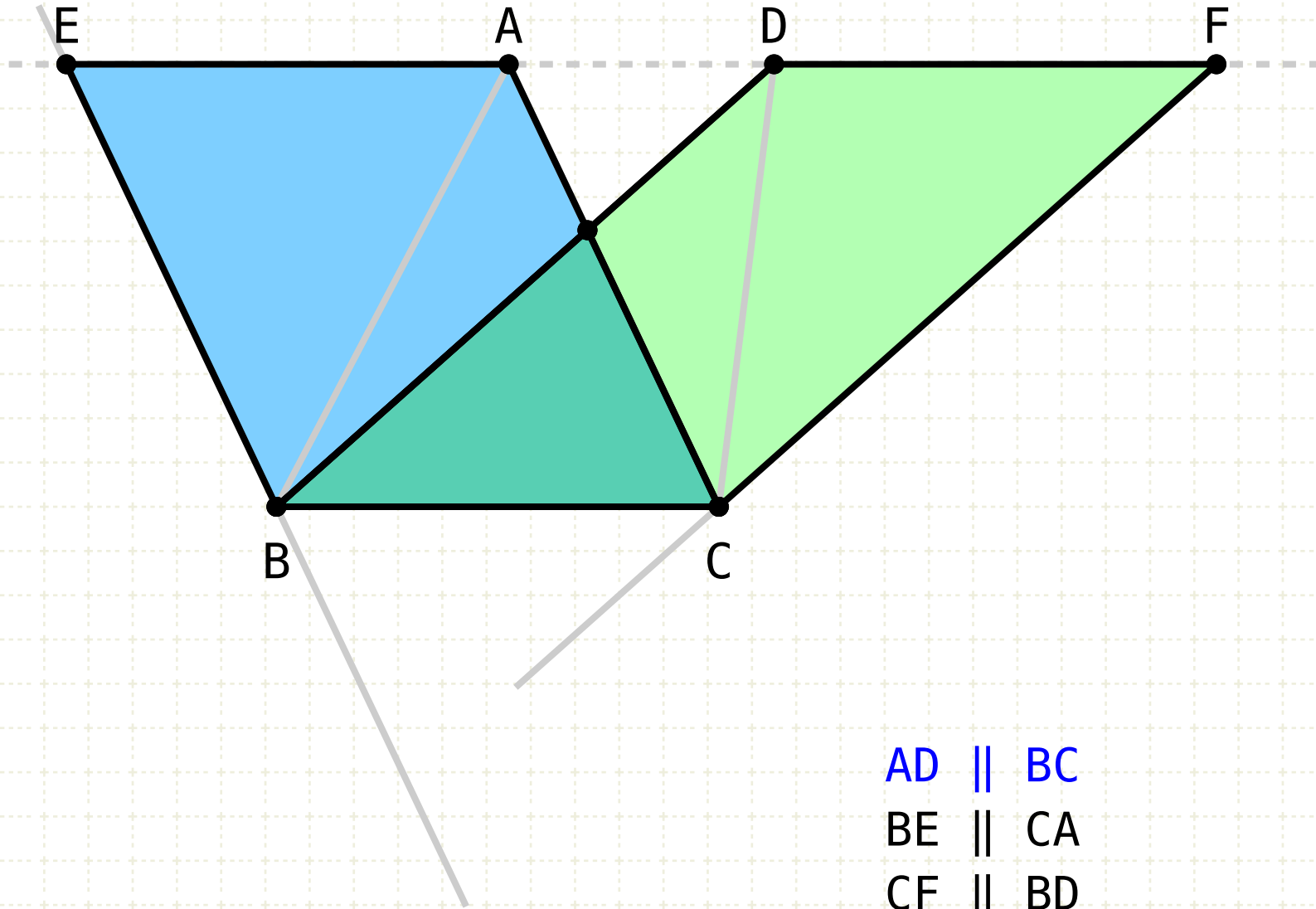
Draw BE parallel to CA (I-31)

Draw CF parallel to BD (I-31)



Proposition 37 of Book I

Triangles which are on equal bases and in the same parallels equal one another.



$AD \parallel BC$
 $BE \parallel CA$
 $CF \parallel BD$
 $EBCA = DBCF$

In other words

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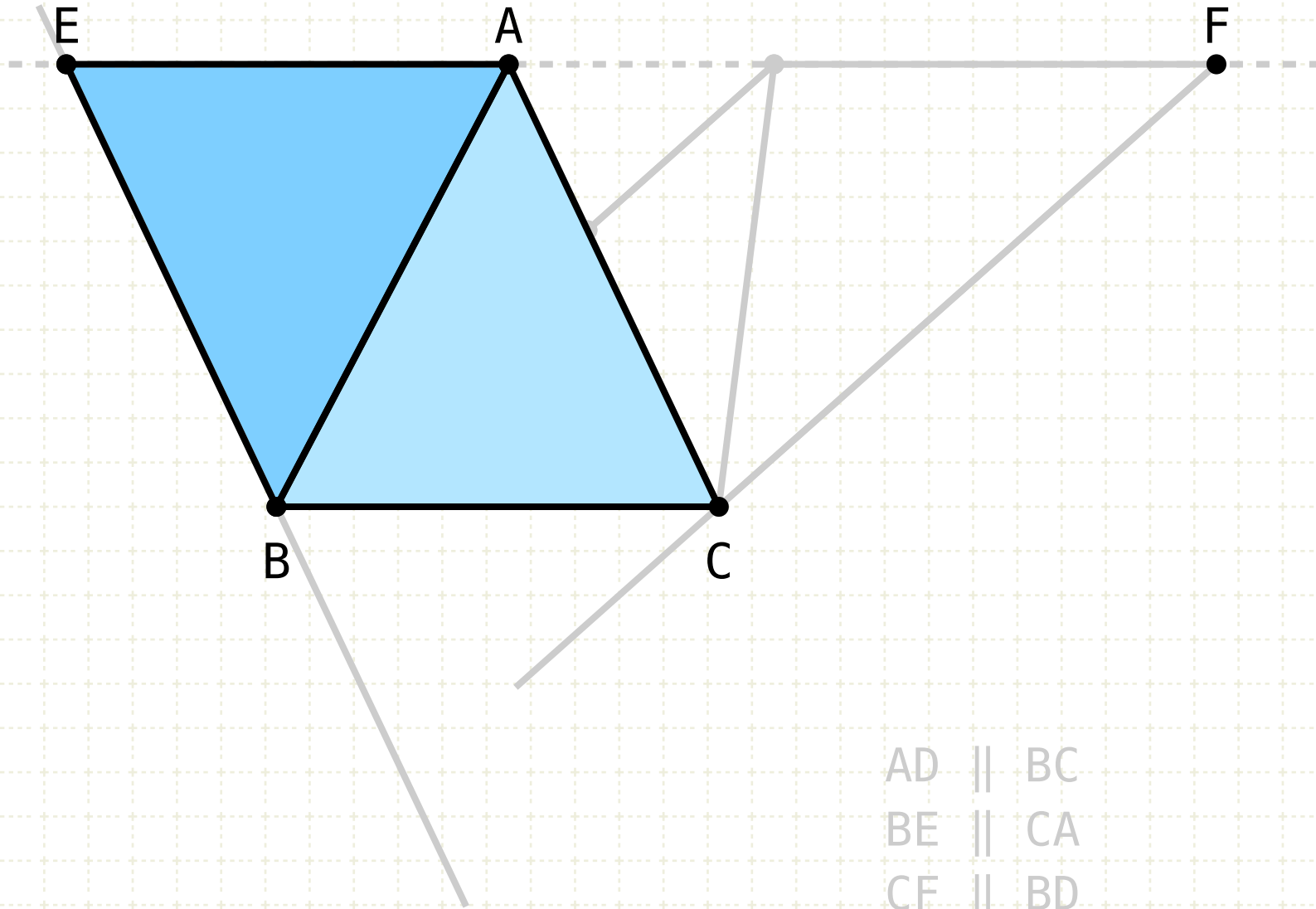
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The parallelograms EBCA and DBCF are equal (I-35)



Proposition 37 of Book I

Triangles which are on equal bases and in the same parallels equal one another.



$$\begin{aligned} AD &\parallel BC \\ BE &\parallel CA \\ CF &\parallel BD \\ EBCA &= DBCF \\ \Delta ABC &= \frac{1}{2} EBCA \end{aligned}$$

In other words

Given two parallel lines

Let ABC and DBC be triangles on the same base BC, and on the same parallels BC and AD

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Proof

Extend line AD

Draw BE parallel to CA (I·31)

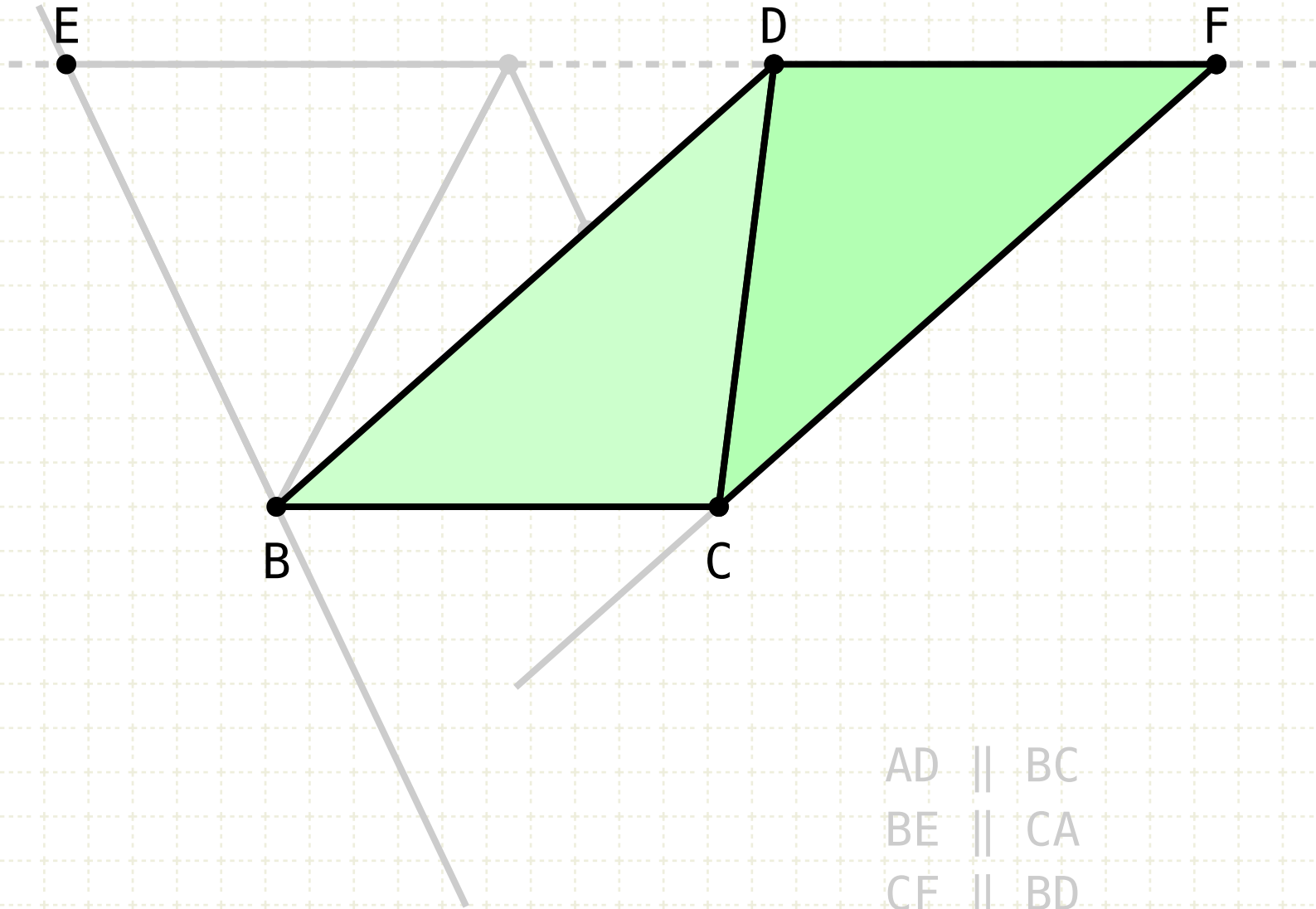
Draw CF parallel to BD (I·31)

The parallelograms EBCA and DBCF are equal (I·35)

The triangle ABC is half the area of EBCA since line AB bisects the parallelogram (I·34)

Proposition 37 of Book I

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$$\begin{aligned} AD &\parallel BC \\ BE &\parallel CA \\ CF &\parallel BD \\ EBCA &= DBCF \\ \triangle ABC &= \frac{1}{2} EBCA \\ \triangle DBC &= \frac{1}{2} DBCF \end{aligned}$$

In other words

Given two parallel lines

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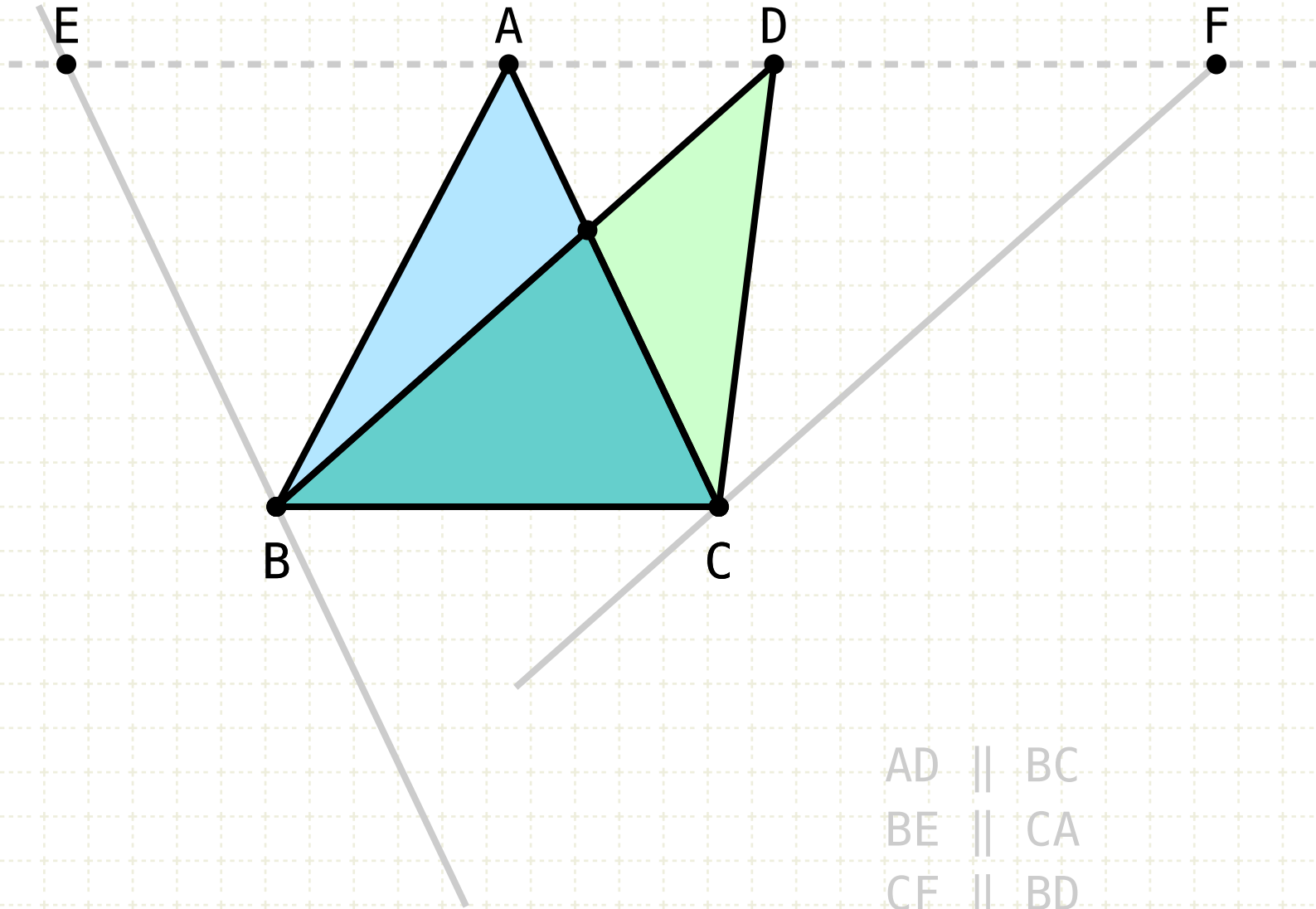
The triangle ABC is half the area of EBCA since line AB bisects the parallelogram (I·34)

The triangle DBC is half the area of DBCF since line DC bisects the parallelogram (I·34)



Proposition 37 of Book I

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$AD \parallel BC$

$BE \parallel CA$

$CF \parallel BD$

$EBCA = DBCF$

$\Delta ABC = \frac{1}{2} EBCA$

$\Delta DBC = \frac{1}{2} DBCF$

$\Delta ABC = \Delta DBC$

In other words

Given two parallel lines

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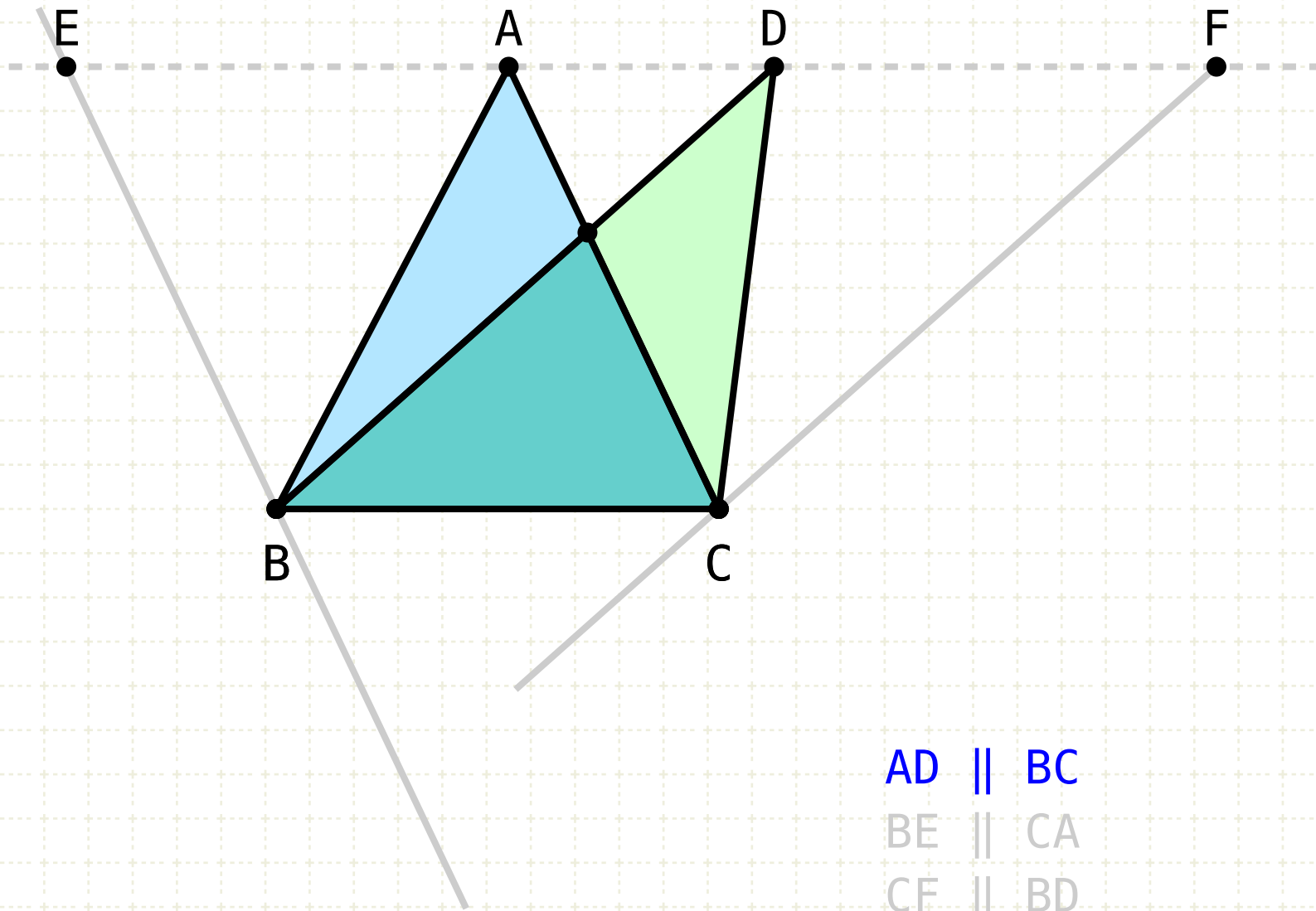
The triangle DBC is half the area of DBCF since line DC bisects the parallelogram (I-34)

Half of equals are equal, so ABC equals DBC



Proposition 37 of Book I

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