

# Euclid's Elements

## Book VI

*One can state, without exaggeration, that the observation of and the search for similarities and differences are the basis of all human knowledge.*

**Alfred Nobel**



# Table of Contents, Chapter 6

1	If the height of two triangles are equal, then the ratio of the areas is equal to the ratio of the bases	7	If two triangles have one angle equal to one angle, and the sides about other angles are proportional, and the remaining angles either both less or both not less than a right angle, then triangles will be equiangular	14	In equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional; and vice versa
2	If a line cuts a triangle, parallel to its base, it will cut the sides of the triangle proportionally			15	In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and vice versa
3	If an angle of a triangle is bisected and the straight line cutting the angle also cuts the base, the segments of the base will have the same ratio as the remaining sides of the triangle	8	If in a right-angled triangle a perpendicular be drawn from the right angle to the base, the triangles adjoining the perpendicular are similar both to the whole and to one another	16	If four straight lines are proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means, and vice versa
4	If two triangles have equal angles, then the sides opposite the equal angles are proportional, as well, the sides of the triangles on either side of the equal angles are also proportional	9	From a given straight line to cut off a given fraction	17	If three straight lines are proportional, the rectangle contained by the extremes is equal to the square on the mean; and vice versa
5	If two triangles have proportional sides, the triangles will be equiangular	10	To cut a given uncut straight line similarly to a given cut straight line	18	On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure
6	If two triangles have one angle equal to one angle and the sides about the equal angles are proportional, then the triangles will be equiangular	11	To two given straight lines to find a third proportional	19	Similar triangles are to one another in the duplicate ratio of the corresponding sides
		12	To three given straight lines to find a fourth proportional		
		13	To two given straight lines to find a mean proportional		



# Table of Contents, Chapter 3

20	<b>Similar polygons are divided into the same number of similar triangles, which have the same ratio as the wholes, and the polygons have duplicate ratios to their corresponding sides</b>	26	If from a parallelogram a similar parallelogram with a common angle is subtracted, it is about the same diameter as the original	31	In right-angled triangles the figure on the side subtending the right angle is equal to the similar and similarly described figures on the sides containing the right angle
21	Figures which are similar to the same rectilineal figure are also similar to one another	27	Of all the parallelograms applied to the same straight line and deficient by parallelogrammic figures similar to a parallelogram drawn on half the said line, the largest will be one that is drawn on half of the straight line and is similar to the defect		
22	If four straight lines are proportional, similar rectilineal figures will also be proportional; and vice versa	28	To a given straight line, apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one		
23	Equiangular parallelograms have to one another the ratio compounded of the ratios of their sides	29	To a given straight line, apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one		
24	In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another	30	To cut a finite straight line in extreme ratio		
25	To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure				



## Proposition 20 of Book VI

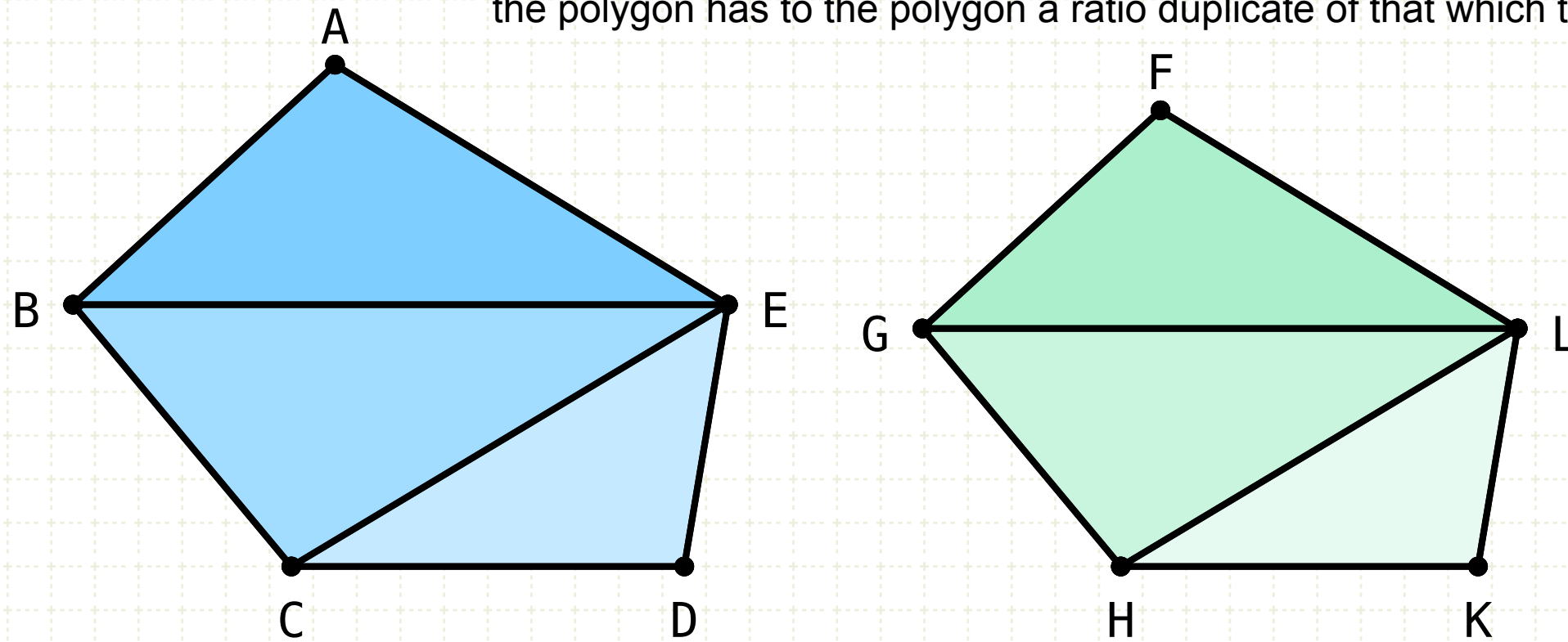
Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.





## Proposition 20 of Book VI

Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.



$$ABCDE \sim FGHLK$$

$$\triangle ABE \sim \triangle FGL$$

$$\triangle BEC \sim \triangle GLH$$

$$\triangle ECD \sim \triangle LHK$$

$$\begin{aligned} \triangle ABE : \triangle FGL &= \triangle BEC : \triangle LHK \\ &= \triangle ECD : \triangle LHK = ABCDE : FGHLK \end{aligned}$$

$$ABCDE : FGHLK = (AB : FG)^2$$

### In other words

If there are two polygons which are similar (equal angles, sides proportional),

then the polygons can be divided into an equal number of similar triangles

And the ratio of all the triangles are equal, which is also equal to the ratio of the original polygons

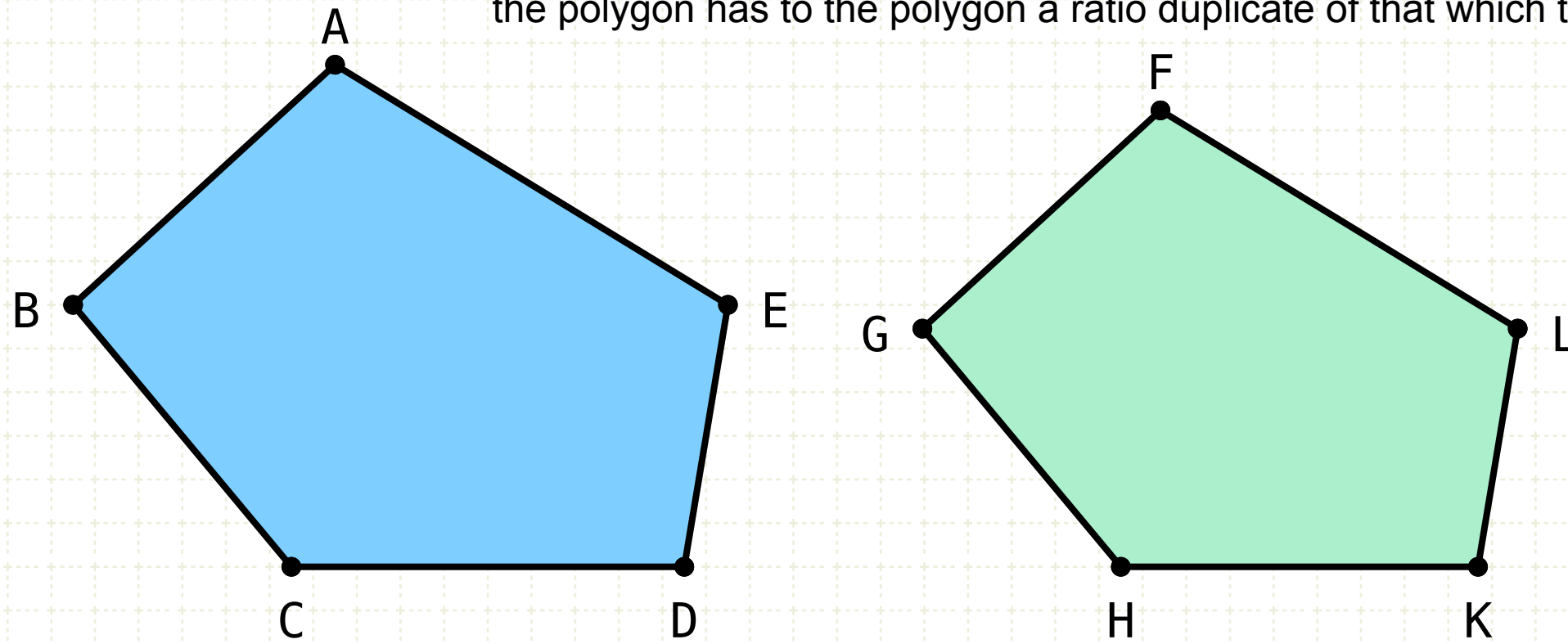
And finally, the ratio of the areas of the polygons is in duplicate ratio to the ratio of the sides



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Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.

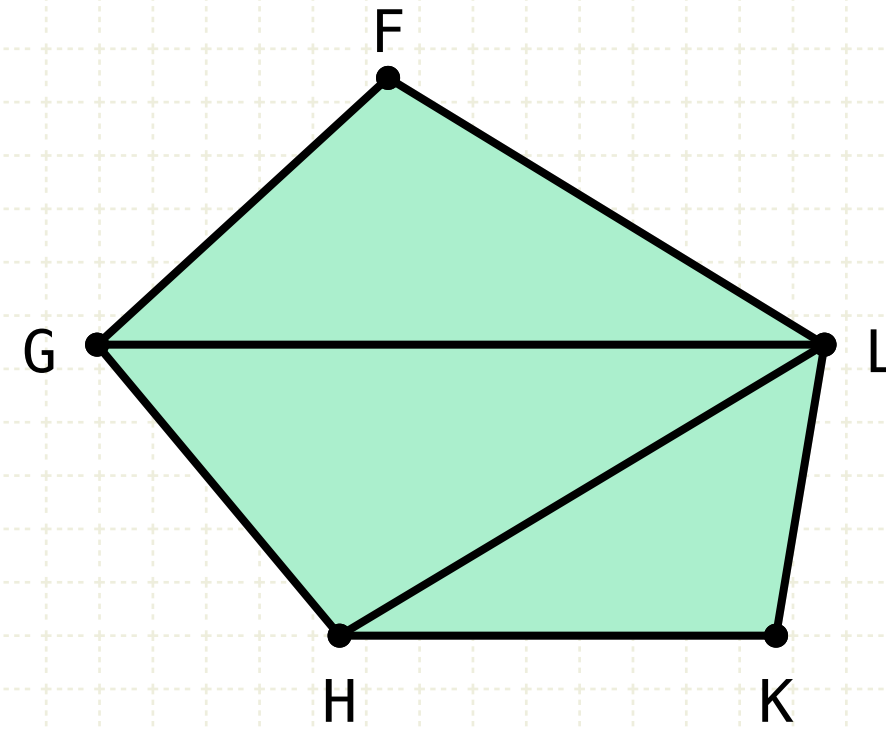
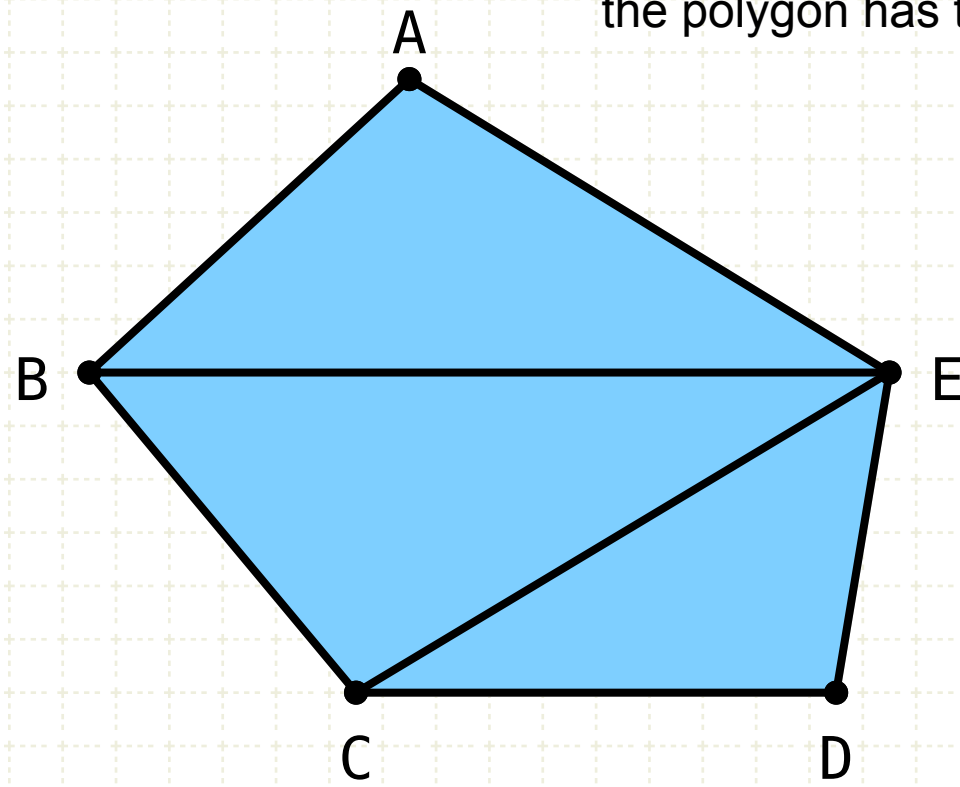
### Proof - Similar Triangles



$ABCDE \sim FGHLK$

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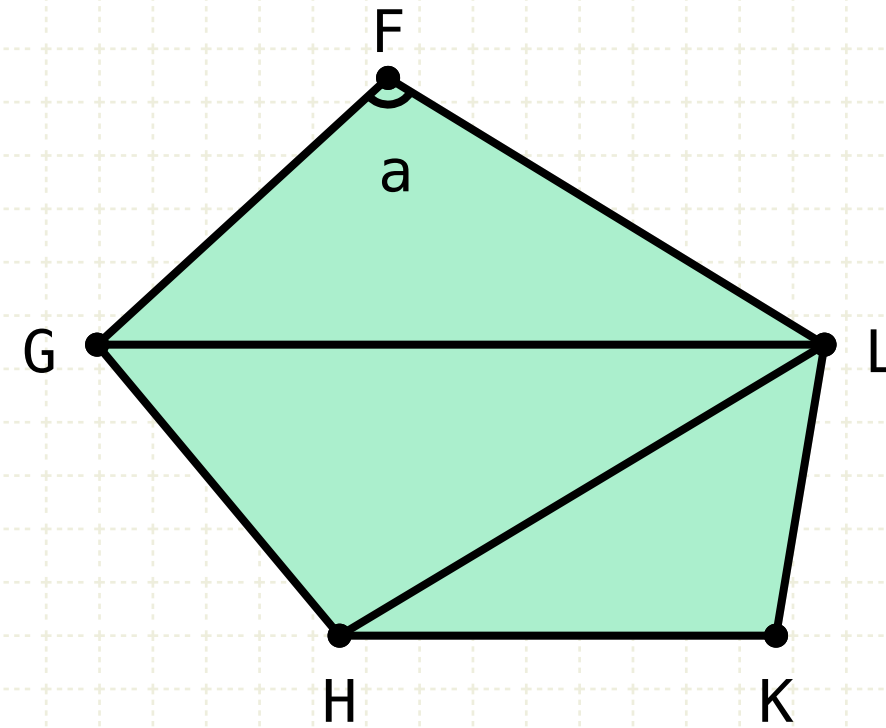
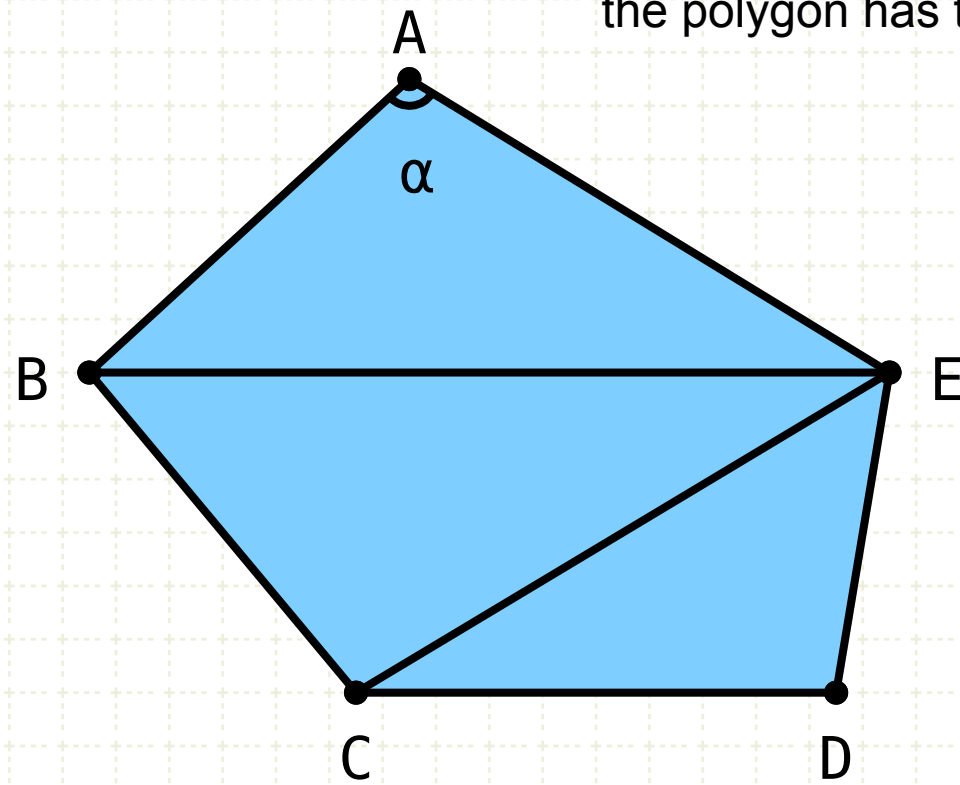
### Proof - Similar Triangles

Draw lines BE, EC, GL and LH

$ABCDE \sim FGHLK$

## Proposition 20 of Book VI

Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.



$ABCDE \sim FGHLK$

$a = \alpha$

$AB:AE = FG:FL$

### Proof - Similar Triangles

Draw lines BE, EC, GL and LH

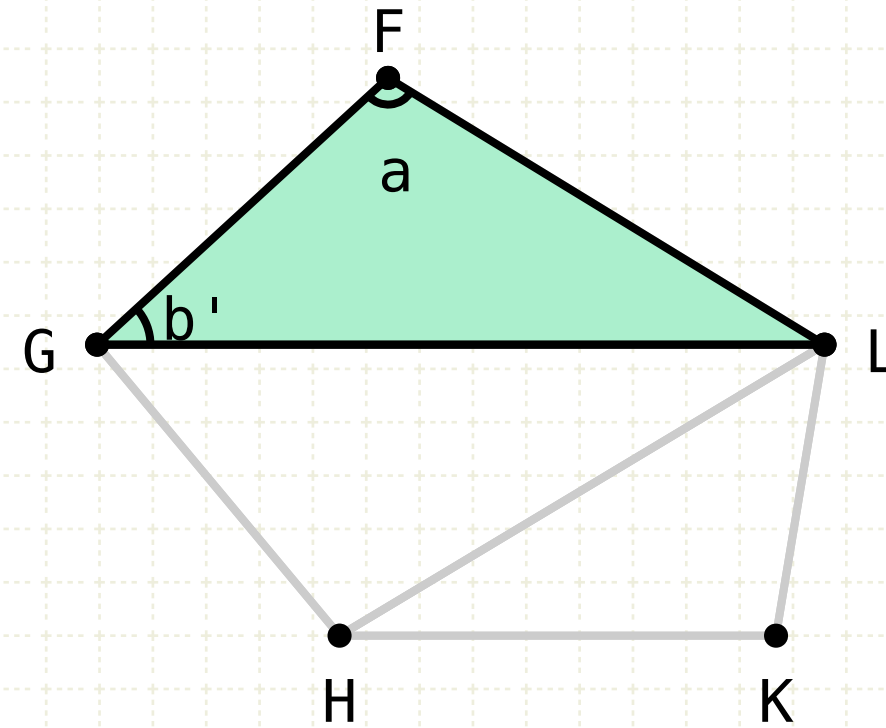
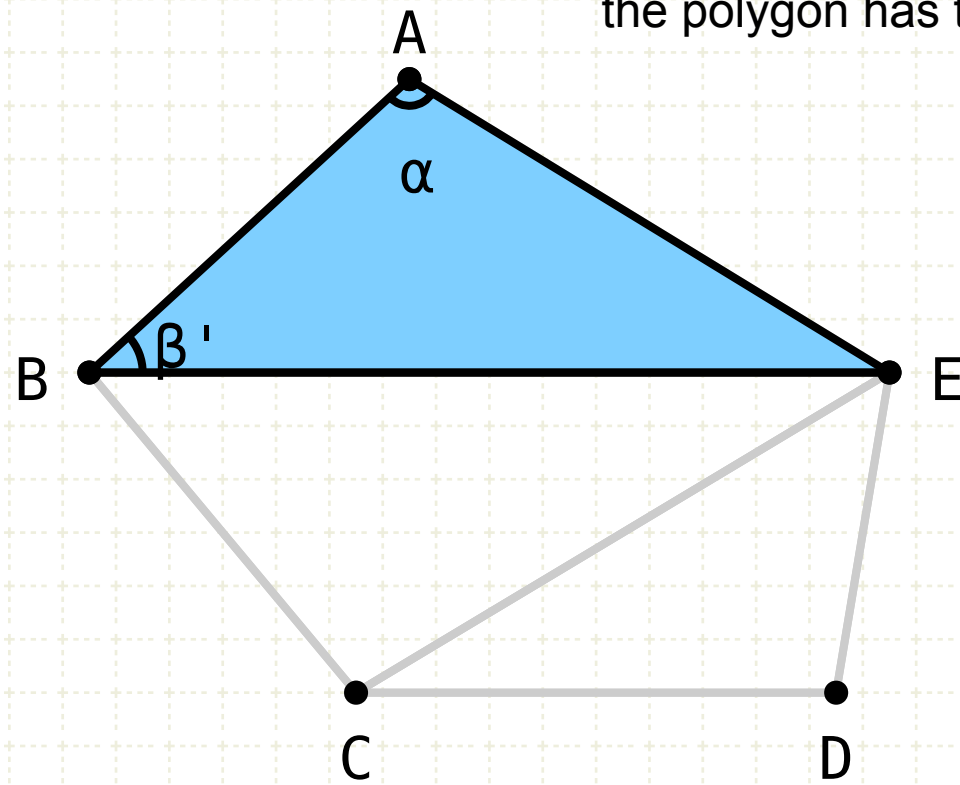
The two polygons are similar, therefore angle BAE is equal to angle GFL,

and the ratio of BA to AE is equal to the ratio GF to FL  
(VI·Def·1)



## Proposition 20 of Book VI

Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.



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$$\triangle ABE \sim \triangle FGL, \quad b' = \beta'$$

### Proof - Similar Triangles

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The two polygons are similar, therefore angle BAE is equal to angle GFL,

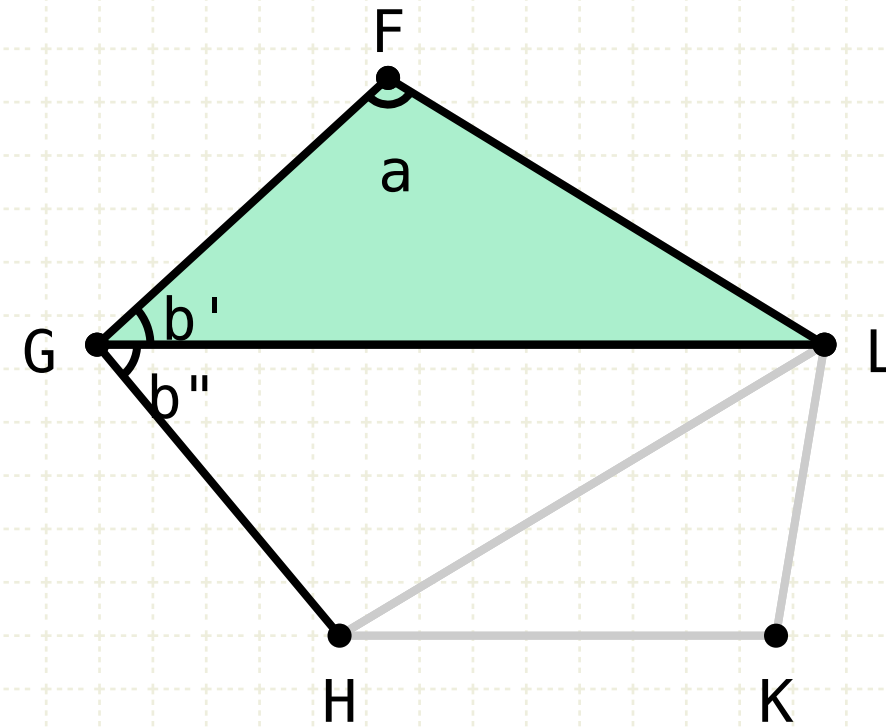
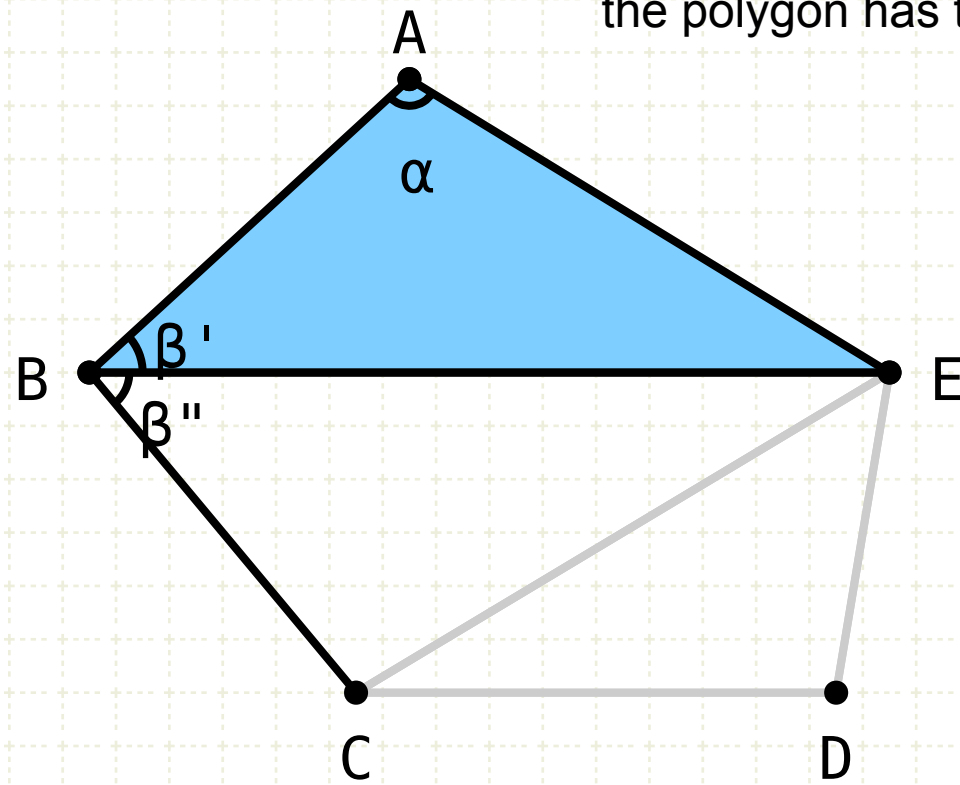
and the ratio of BA to AE is equal to the ratio GF to FL  
(VI-Def-1)

Since ABE, FGL are two triangles with one equal angle and the sides about the equal angle are proportional, the two triangles are equiangular (VI-6) and similar (VI-4), (VI-Def-1)

So angle ABE is equal to angle FGL

# Proposition 20 of Book VI

Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.



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$\triangle ABE \sim \triangle FGL, b' = \beta'$

$b = b' + b'' = \beta = \beta' + \beta''$

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## Proof - Similar Triangles

Draw lines BE, EC, GL and LH

The two polygons are similar, therefore angle BAE is equal to angle GFL,

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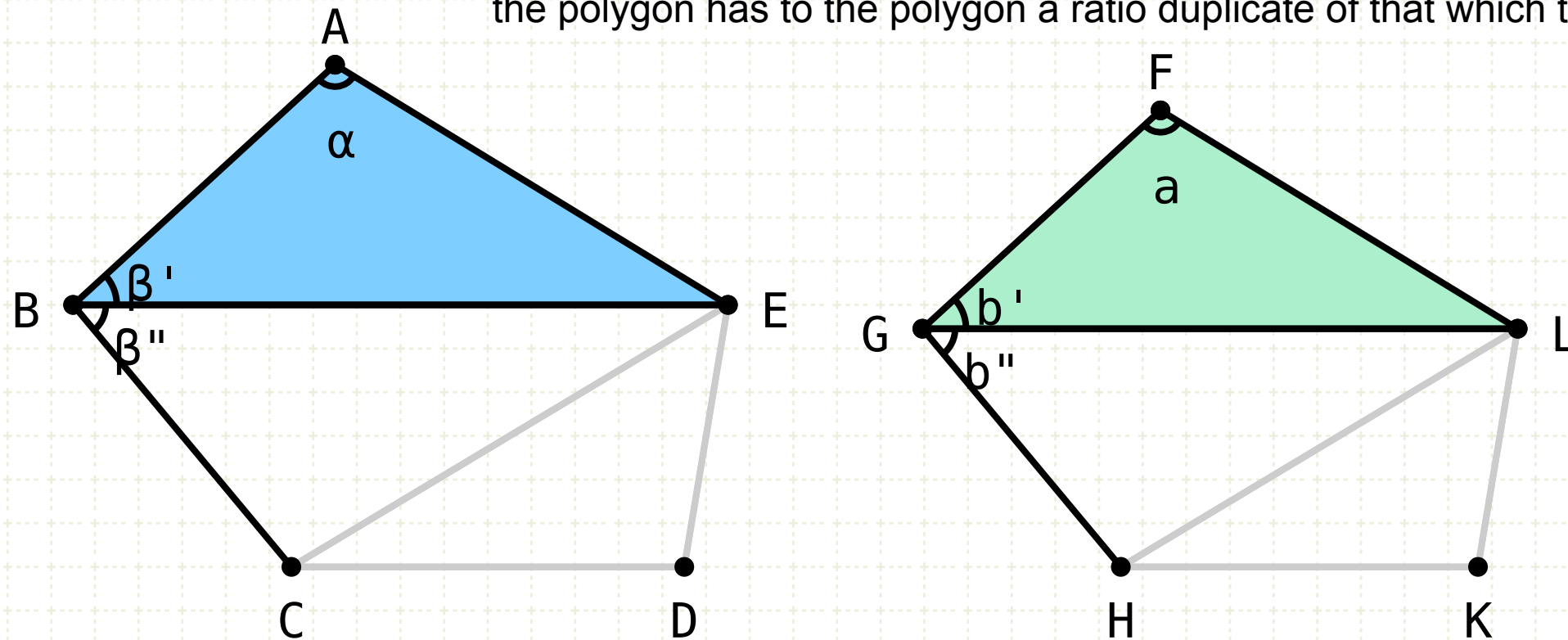
Since ABE, FGL are two triangles with one equal angle and the sides about the equal angle are proportional, the two triangles are equiangular (VI-6) and similar (VI-4), (VI-Def-1)

So angle ABE is equal to angle FGL

But angle ABC is equal to angle FGH because the polygons are similar, therefore the angles EBC and LGH are also equal

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$$BE:AB = GL:FG$$

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The two polygons are similar, therefore angle BAE is equal to angle GFL,

and the ratio of BA to AE is equal to the ratio GF to FL (VI-Def-1)

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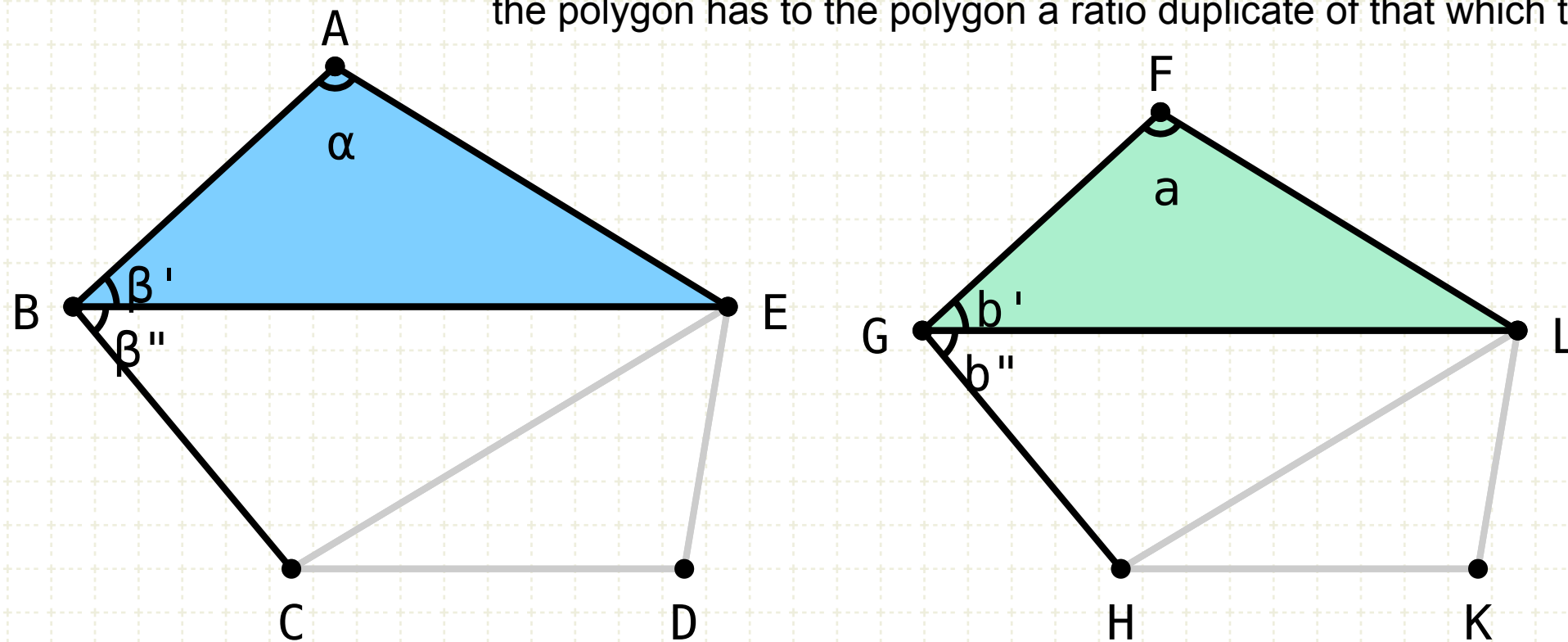
But angle ABC is equal to angle FGH because the polygons are similar, therefore the angles EBC and LGH are also equal

Because the triangle ABE is similar to the triangle FGL, BE is to AB as GL is to GF



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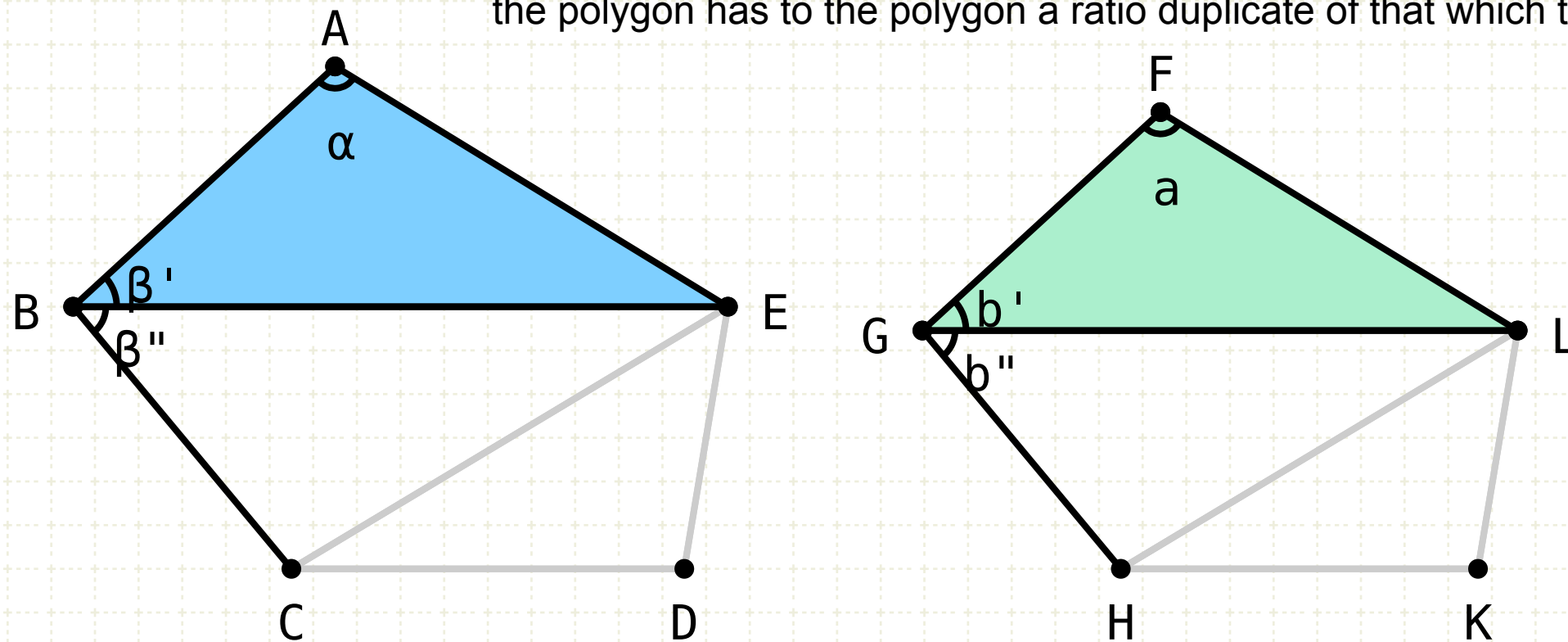
Again, because the polygons are similar, AB is to BC as FG is to GH





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Draw lines BE, EC, GL and LH

The two polygons are similar, therefore angle BAE is equal to angle GFL,

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Since ABE, FGL are two triangles with one equal angle and the sides about the equal angle are proportional, the two triangles are equiangular (VI-6) and similar (VI-4), (VI-Def-1)

So angle ABE is equal to angle FGL

But angle ABC is equal to angle FGH because the polygons are similar, therefore the angles EBC and LGH are also equal

Because the triangle ABE is similar to the triangle FGL, BE is to AB as GL is to GF

Again, because the polygons are similar, AB is to BC as FG is to GH

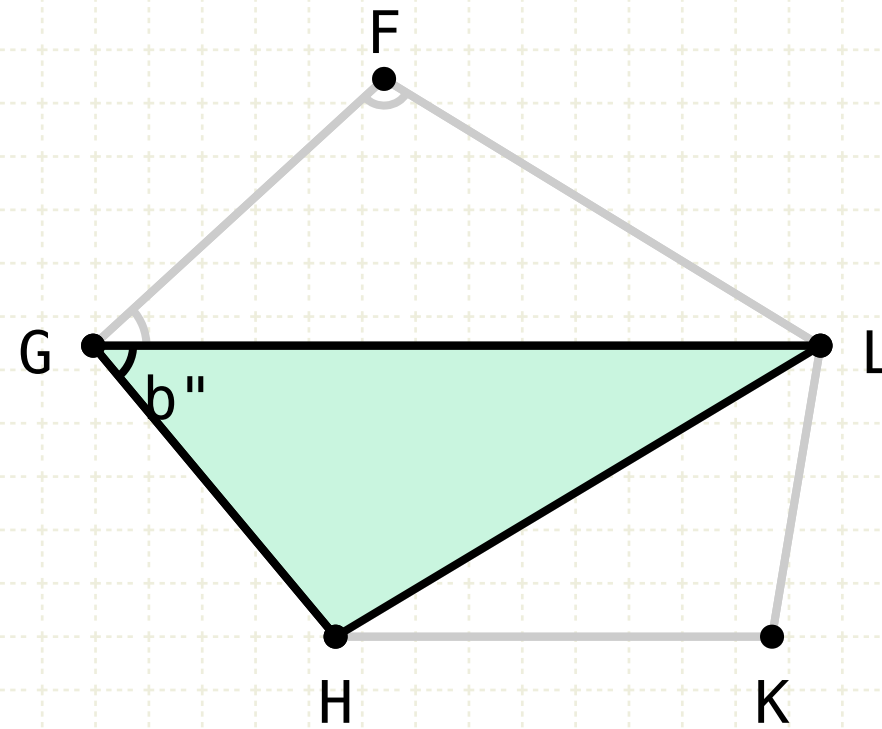
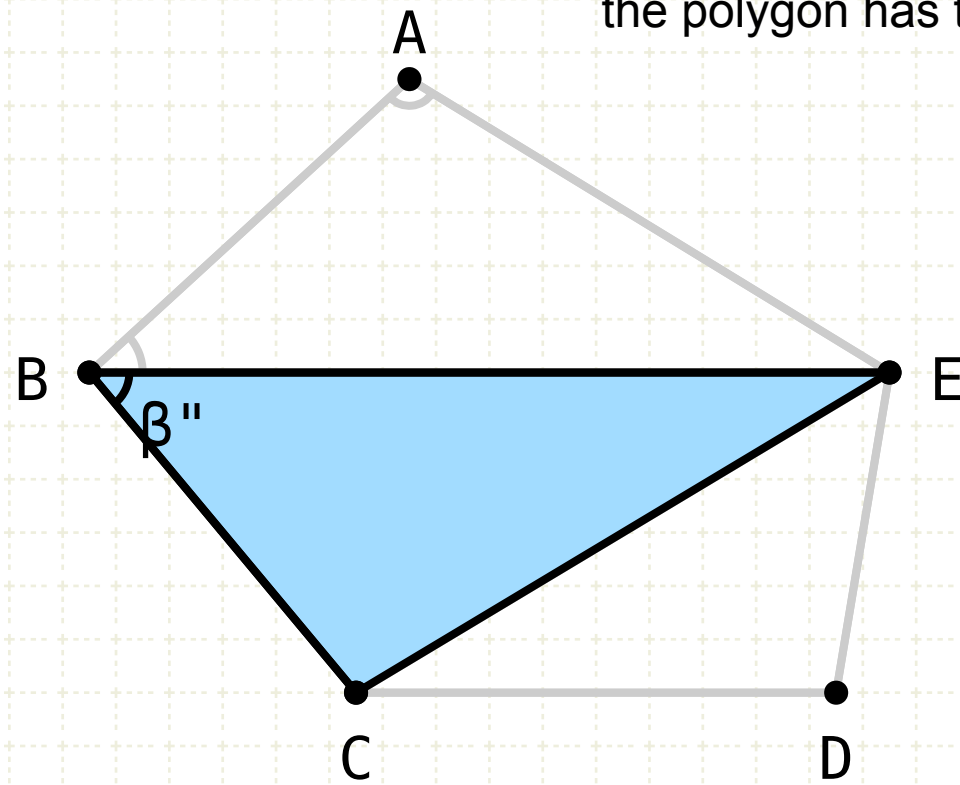
therefore BE is to BC as GL is to GH (V-22)





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$$\triangle BEC \sim \triangle LGK$$

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Again, because the polygons are similar, AB is to BC as FG is to GH

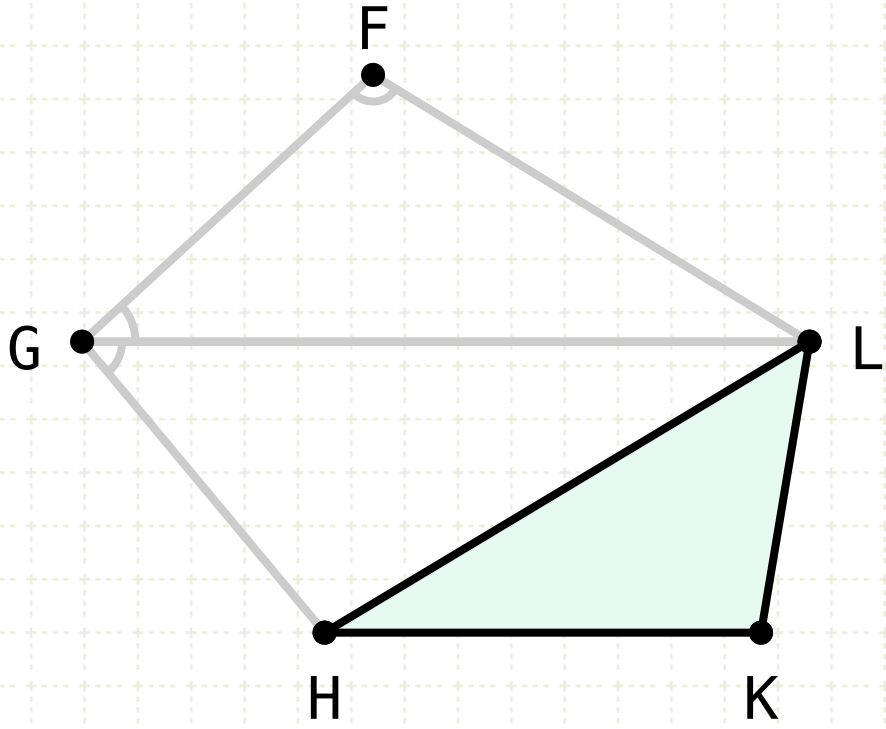
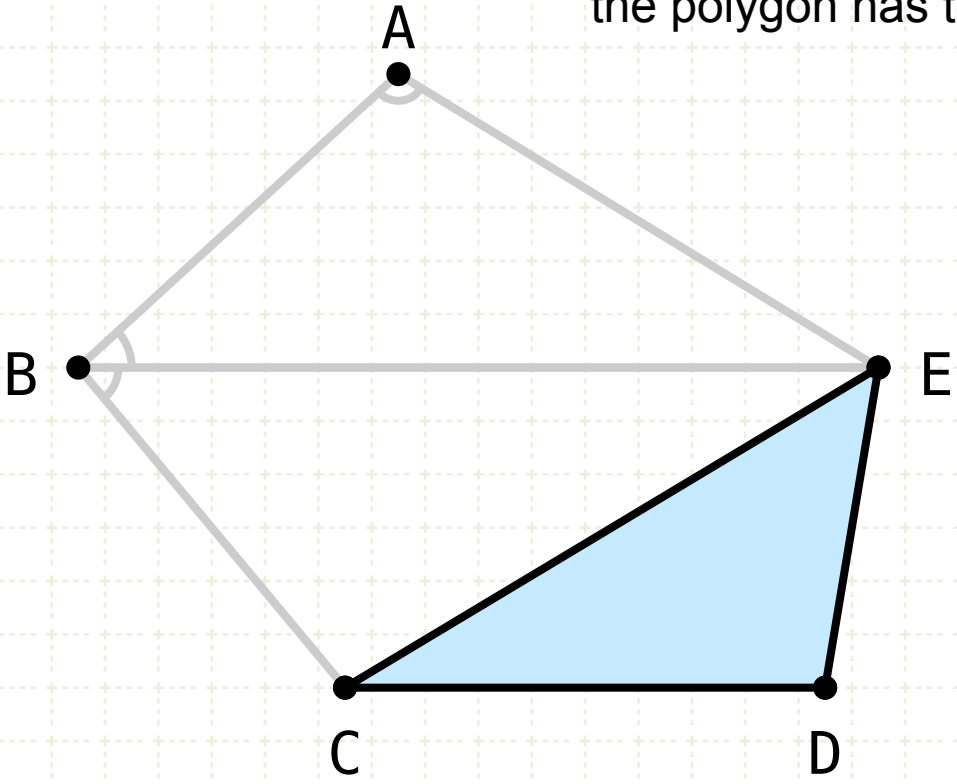
therefore BE is to BC as GL is to GH (V-22)

Since BEC, LGH are two triangles with one equal angle and the sides about the equal angle are proportional, the two triangles are equiangular (VI-6) and similar (VI-4), (VI-Def-1)



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## Proof - Similar Triangles

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Because the triangle ABE is similar to the triangle FGL, BE is to AB as GL is to GF

Again, because the polygons are similar, AB is to BC as FG is to GH

therefore BE is to BC as GL is to GH (V-22)

Since BEC, LGH are two triangles with one equal angle and the sides about the equal angle are proportional, the two triangles are equiangular (VI-6) and similar (VI-4), (VI-Def-1)

For the same reasons, the triangle ECD is similar to the triangle LHK

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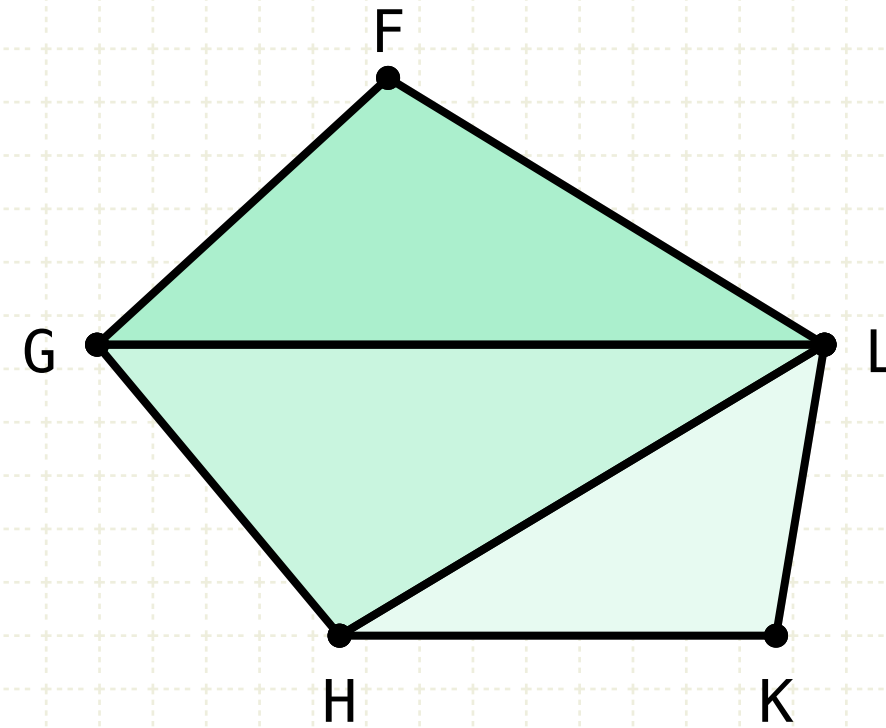
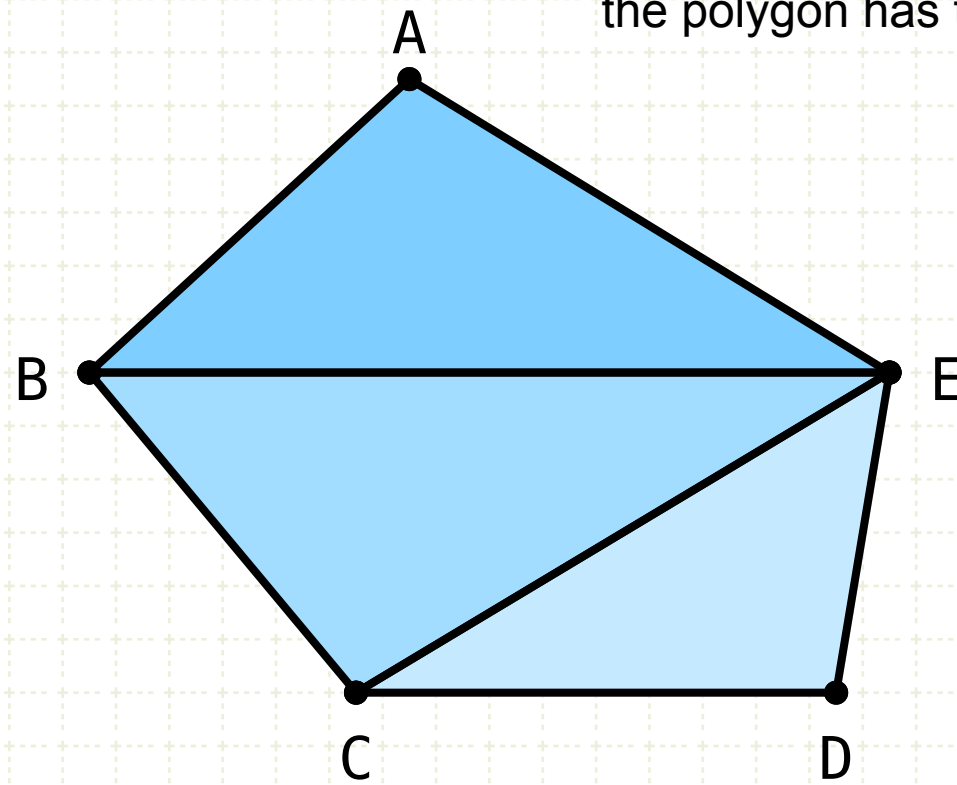
$$\triangle BEC \sim \triangle LGK$$

$$\triangle ECD \sim \triangle LHK$$



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Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.



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$\triangle BEC \sim \triangle LGK$

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## Proof - Similar Triangles

Draw lines BE, EC, GL and LH

The two polygons are similar, therefore angle BAE is equal to angle GFL,

and the ratio of BA to AE is equal to the ratio GF to FL (VI-Def-1)

Since ABE, FGL are two triangles with one equal angle and the sides about the equal angle are proportional, the two triangles are equiangular (VI-6) and similar (VI-4), (VI-Def-1)

So angle ABE is equal to angle FGL

But angle ABC is equal to angle FGH because the polygons are similar, therefore the angles EBC and LGH are also equal

Because the triangle ABE is similar to the triangle FGL, BE is to AB as GL is to GF

Again, because the polygons are similar, AB is to BC as FG is to GH

therefore BE is to BC as GL is to GH (V-22)

Since BEC, LGH are two triangles with one equal angle and the sides about the equal angle are proportional, the two triangles are equiangular (VI-6) and similar (VI-4), (VI-Def-1)

For the same reasons, the triangle ECD is similar to the triangle LHK

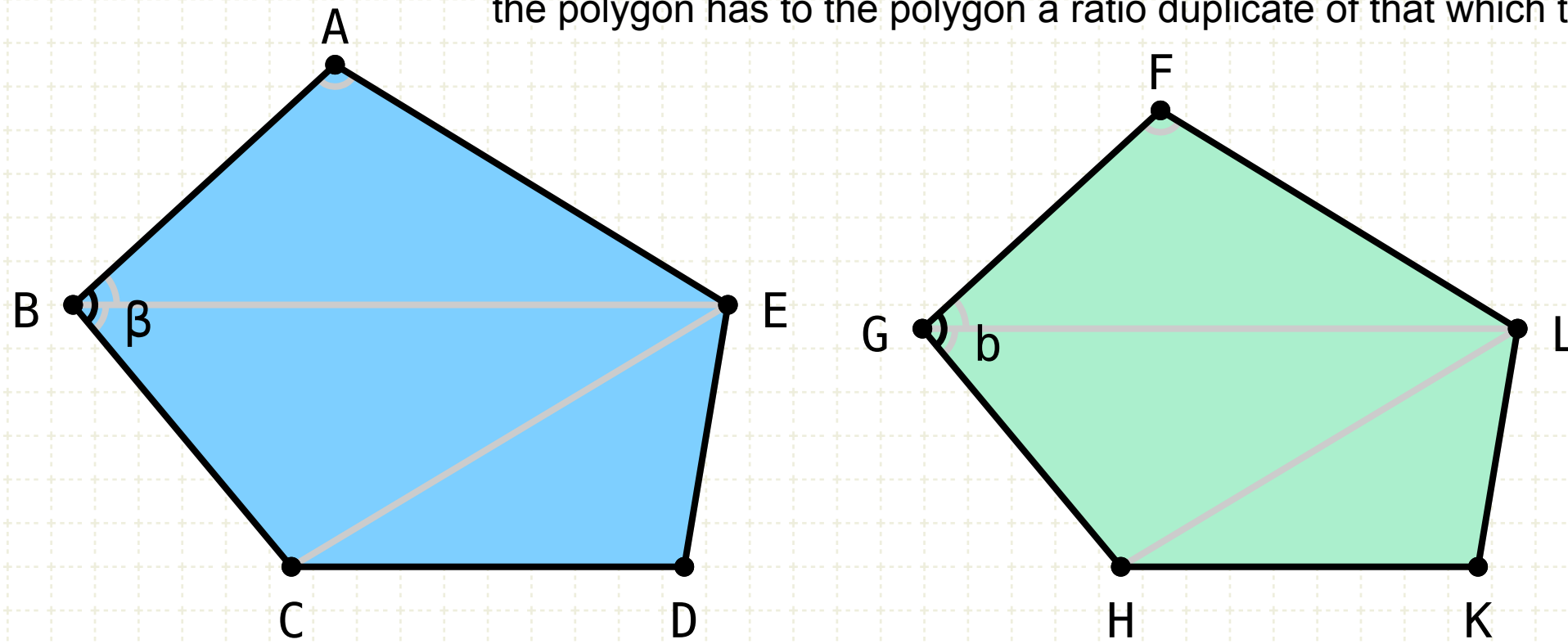
So the two similar polygon has been divided into similar triangles, equal in multitude



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### Proof - Duplicate Ratio to Sides



$$ABCDE \sim FGHLK$$

$$b = \beta$$

$$b' = \beta'$$

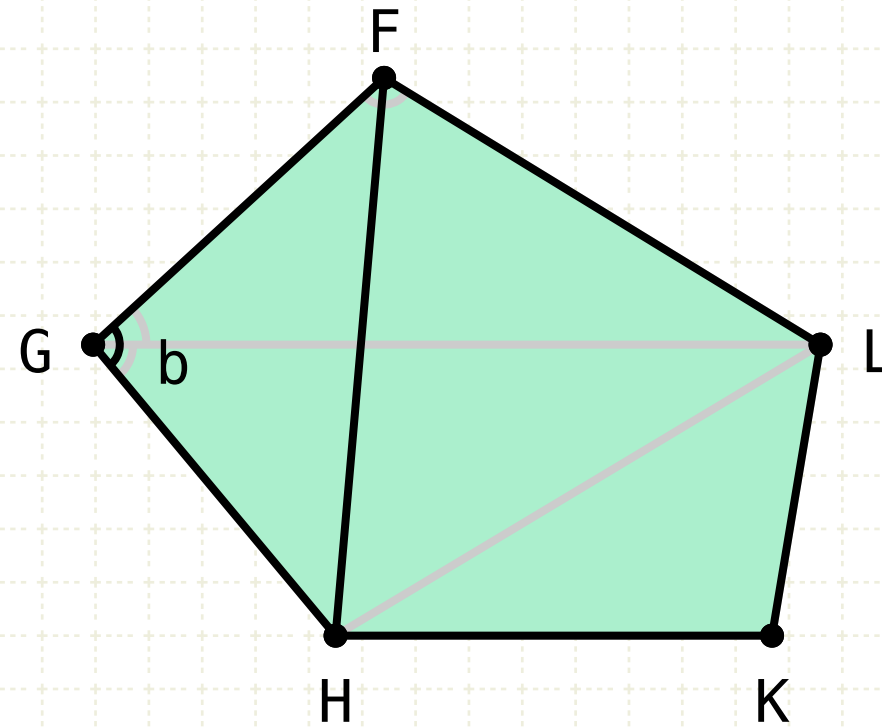
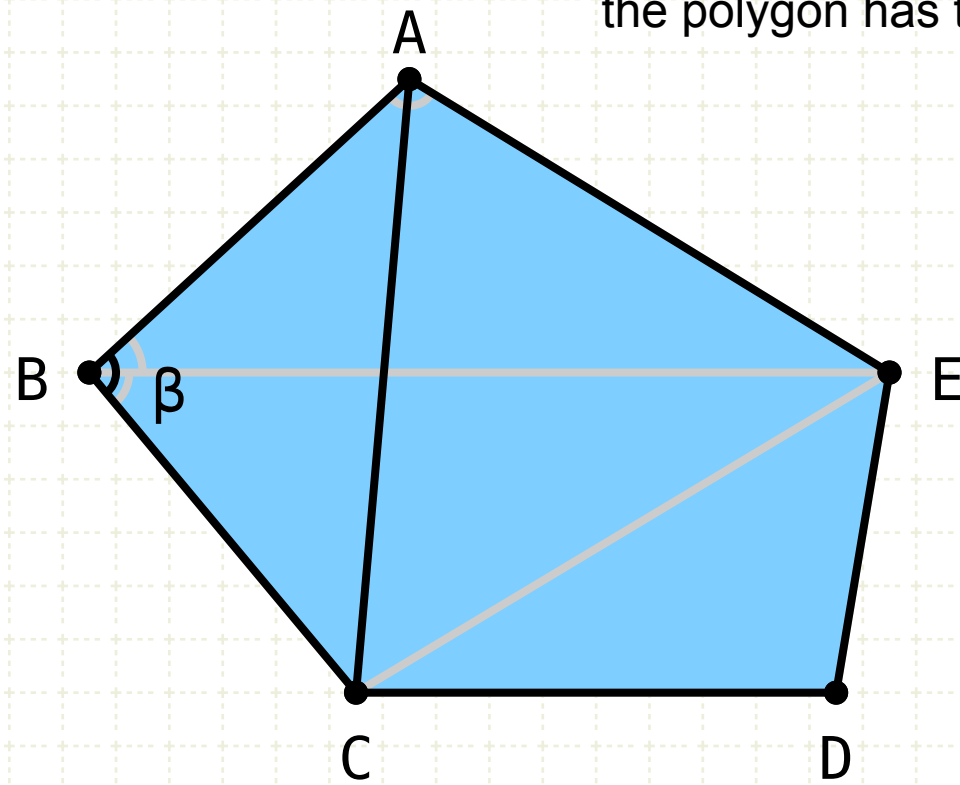
$$AB:BC = FG:GH$$





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Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.



### Proof - Duplicate Ratio to Sides

Draw AC, FH

$$ABCDE \sim FGHLK$$

$$b = \beta$$

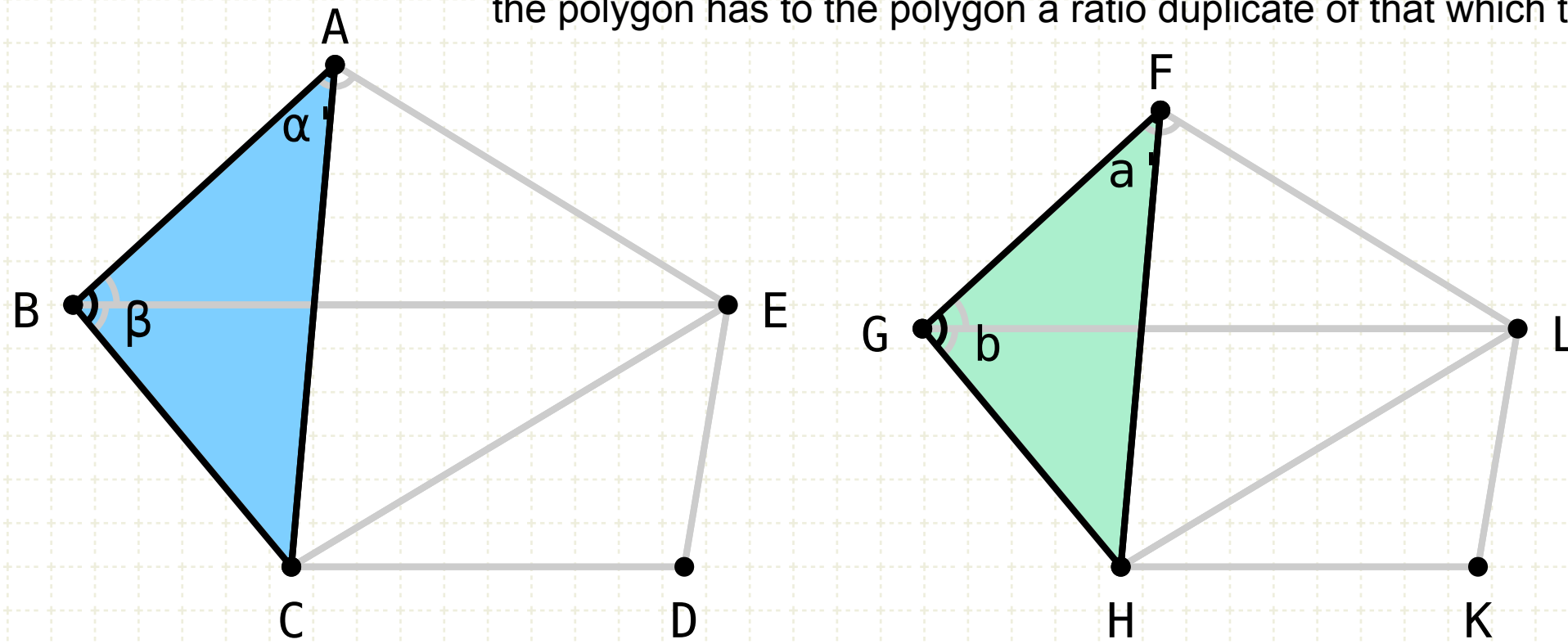
$$b' = \beta'$$

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Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.



### Proof - Duplicate Ratio to Sides

Draw AC, FH

Triangle ABC is similar to FGH (VI·6)

$$ABCDE \sim FGHLK$$

$$b = \beta$$

$$b' = \beta'$$

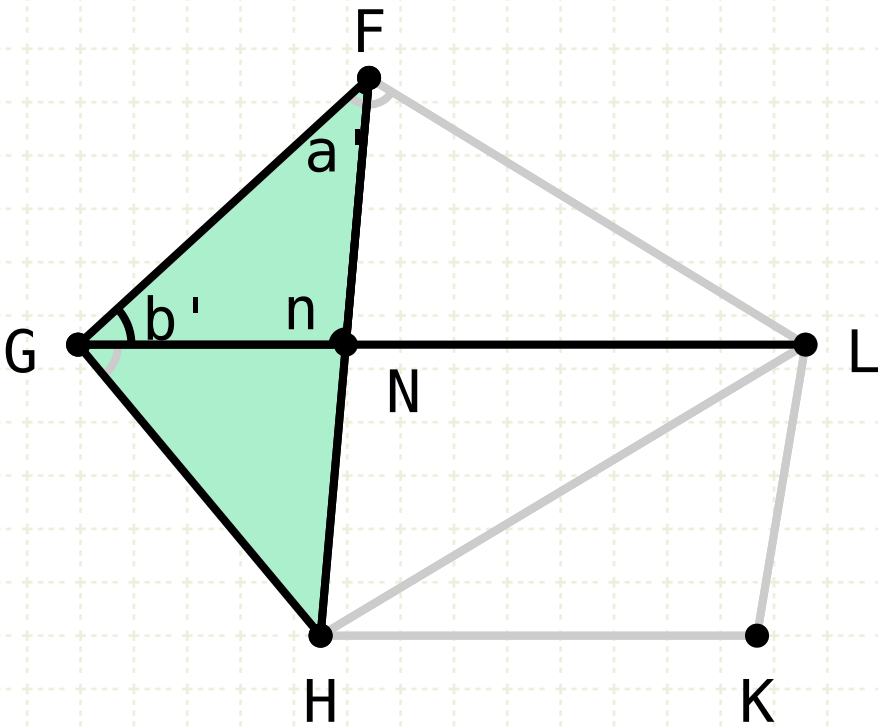
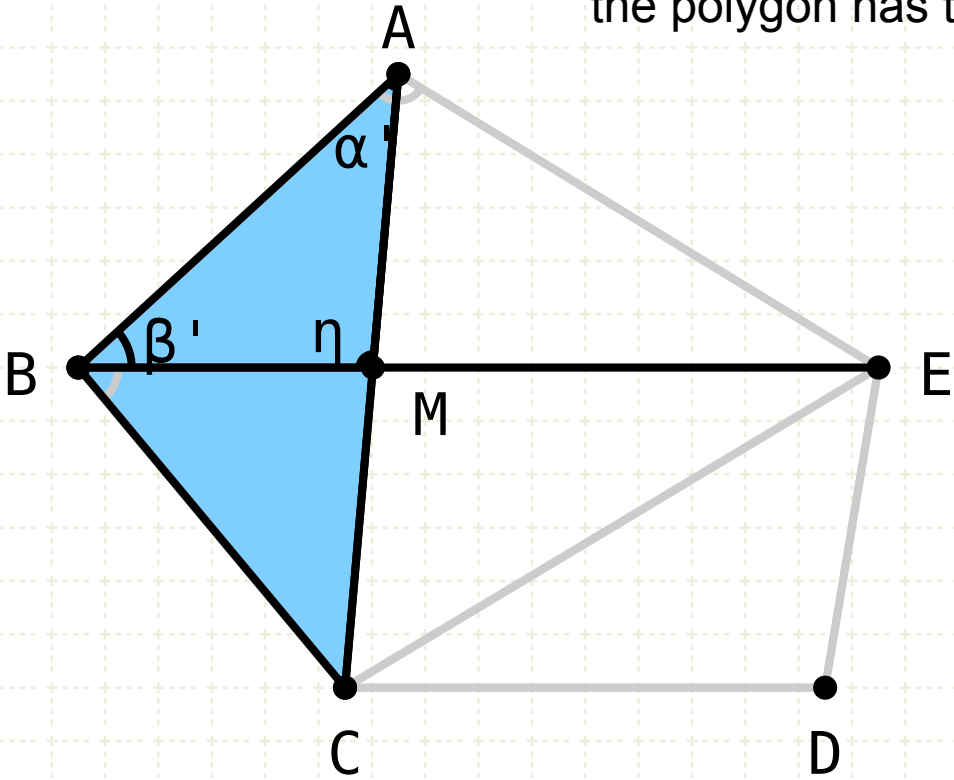
$$AB:BC = FG:GH$$

$$a' = \alpha'$$



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Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.



## Proof - Duplicate Ratio to Sides

Draw AC, FH

Triangle ABC is similar to FGH (VI-6)

Since the angles ABM and FGN are equal, and the angles BAM and GFN are equal, the angles AMB and FNG are also equal (I-32)

$$ABCDE \sim FGHLK$$

$$b = \beta$$

$$b' = \beta'$$

$$AB:BC = FG:GH$$

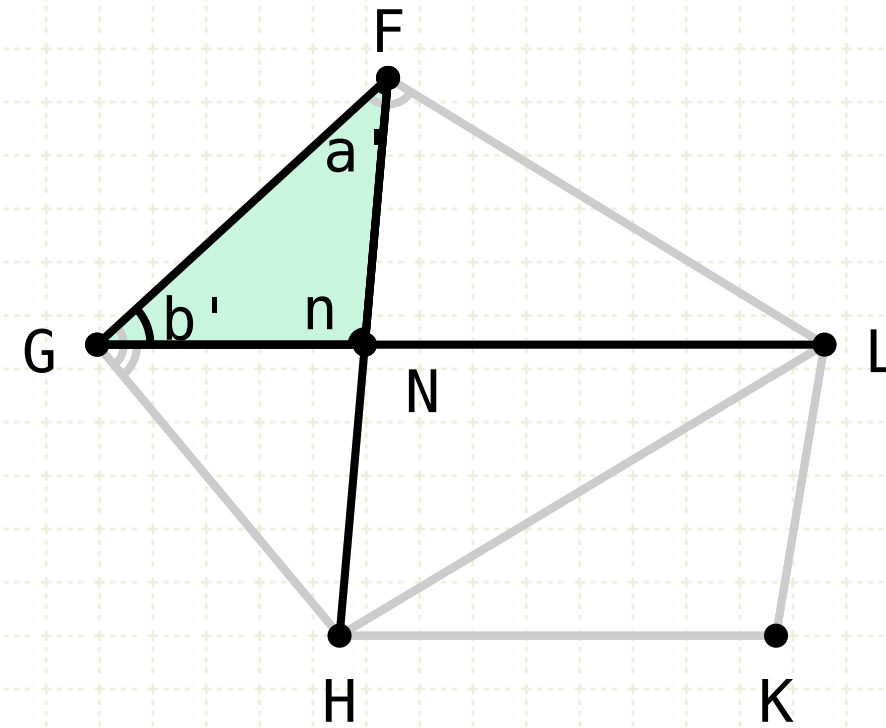
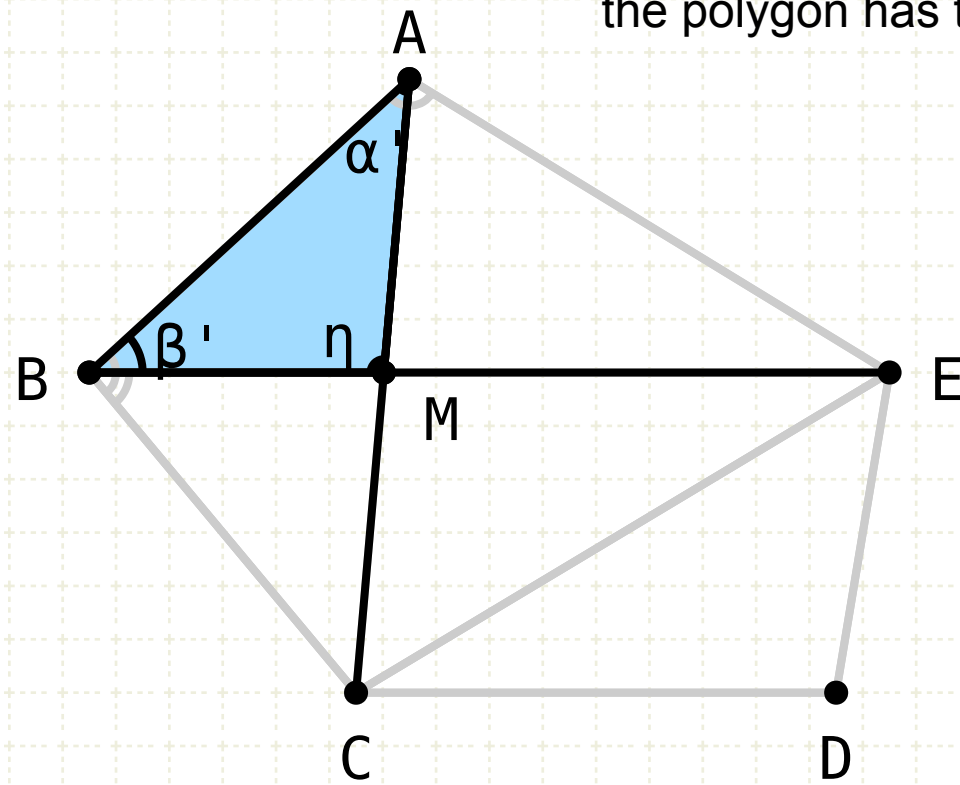
$$a' = \alpha'$$

$$n = \eta$$



# Proposition 20 of Book VI

Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.



## Proof - Duplicate Ratio to Sides

Draw AC, FH

Triangle ABC is similar to FGH (VI-6)

Since the angles ABM and FGN are equal, and the angles BAM and GFN are equal, the angles AMB and FNG are also equal (I-32)

Therefore triangle ABM is equiangular with triangle FGN and similar, so the ratio of AM to MB is equal to FN to NG

$$ABCDE \sim FGHLK$$

$$b = \beta$$

$$b' = \beta'$$

$$AB:BC = FG:GH$$

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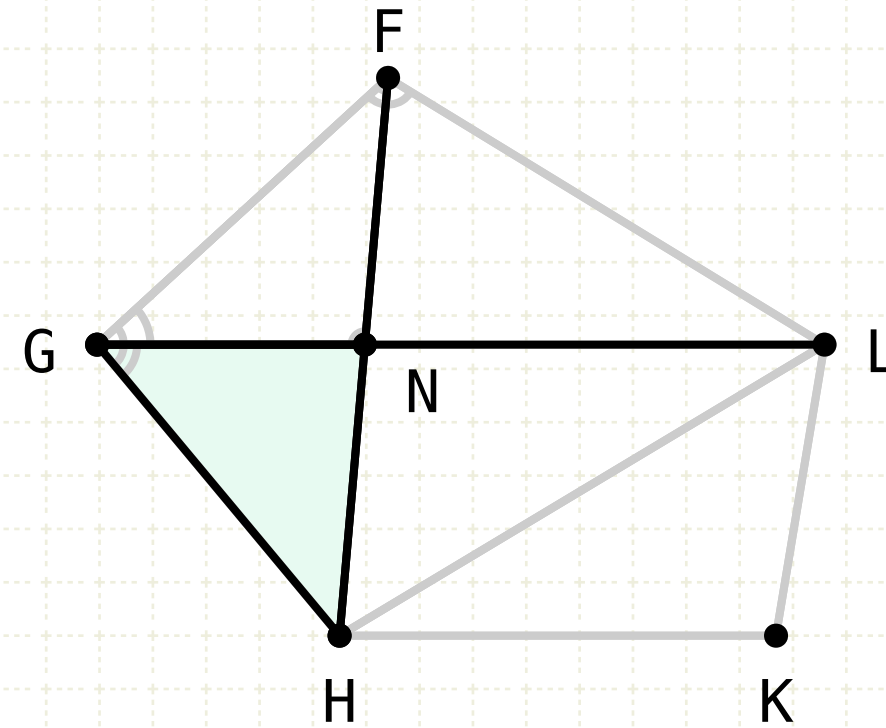
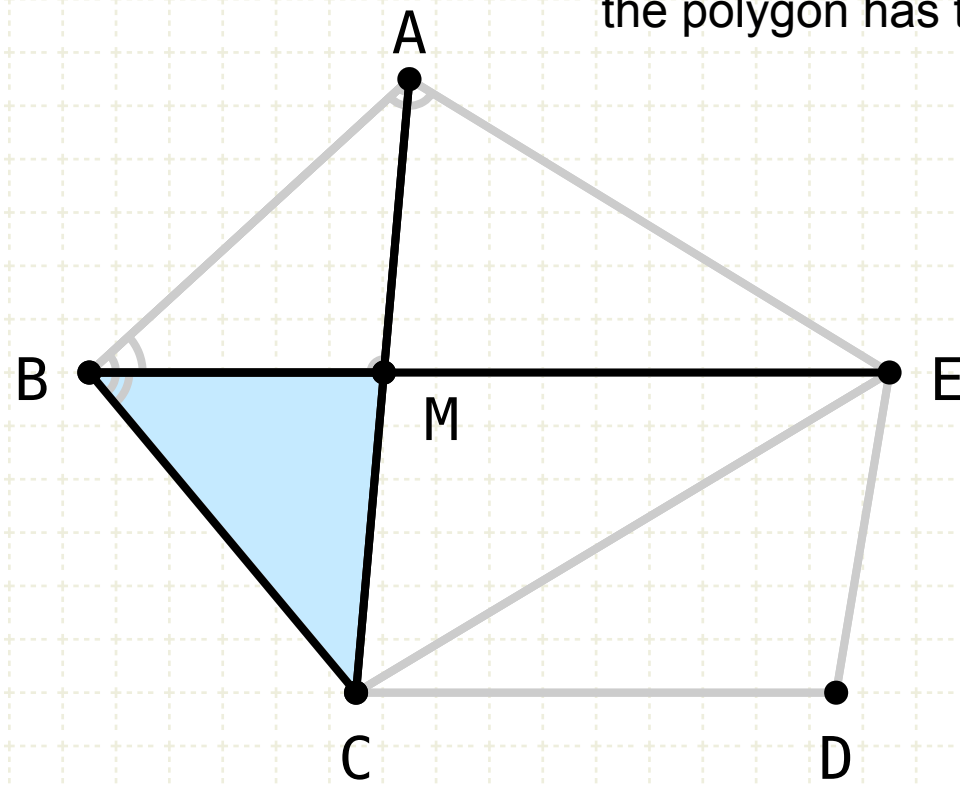
$$n = \eta$$

$$AM:BM = FN:NG$$



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Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.



## Proof - Duplicate Ratio to Sides

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Triangle ABC is similar to FGH (VI-6)

Since the angles ABM and FGN are equal, and the angles BAM and GFN are equal, the angles AMB and FNG are also equal (I-32)

Therefore triangle ABM is equiangular with triangle FGN and similar, so the ratio of AM to MB is equal to FN to NG

Similarly we can prove that triangle BMC is also equiangular with triangle GNH, thus BM is to MC as NG is to NH

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$$a' = \alpha'$$

$$n = \eta$$

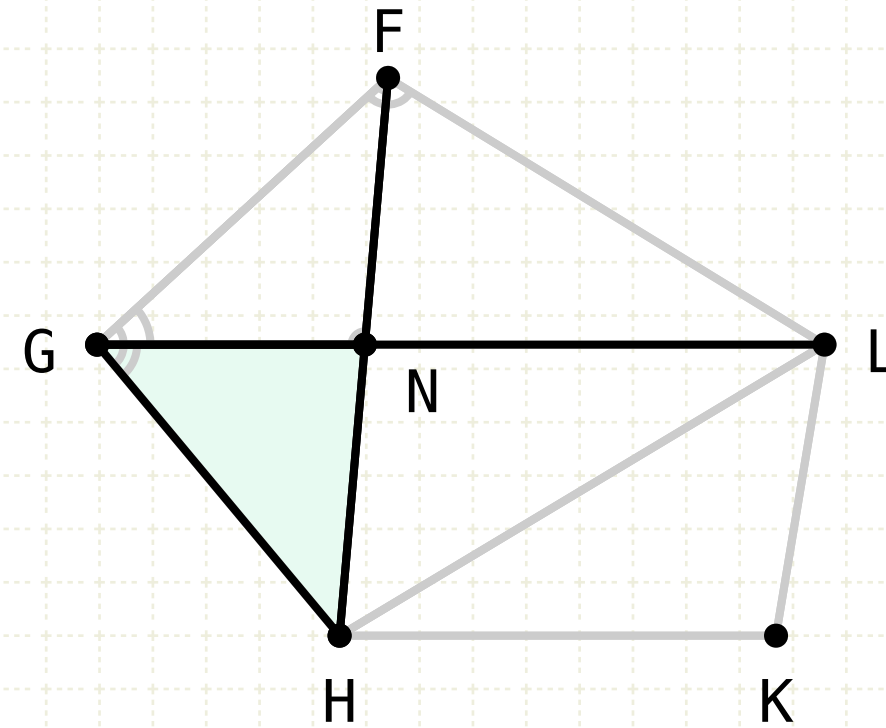
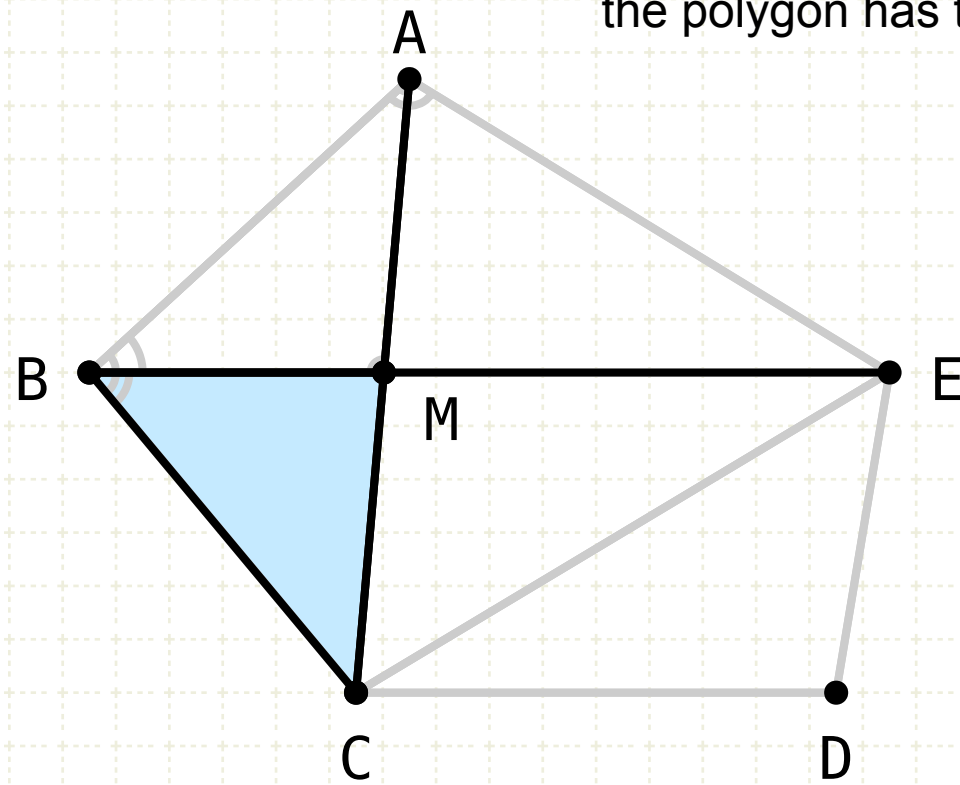
$$AM:BM = FN:NG$$

$$BM:MC = NG:NH$$



# Proposition 20 of Book VI

Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.



## Proof - Duplicate Ratio to Sides

Draw AC, FH

Triangle ABC is similar to FGH (VI-6)

Since the angles ABM and FGN are equal, and the angles BAM and GFN are equal, the angles AMB and FNG are also equal (I-32)

Therefore triangle ABM is equiangular with triangle FGN and similar, so the ratio of AM to MB is equal to FN to NG

Similarly we can prove that triangle BMC is also equiangular with triangle GNH, thus BM is to MC as NG is to NH

In addition, ex aequali, AM is to MC as is FN to NH

$$ABCDE \sim FGHLK$$

$$b = \beta$$

$$b' = \beta'$$

$$AB:BC = FG:GH$$

$$a' = \alpha'$$

$$n = \eta$$

$$AM:BM = FN:NG$$

$$BM:MC = NG:NH$$

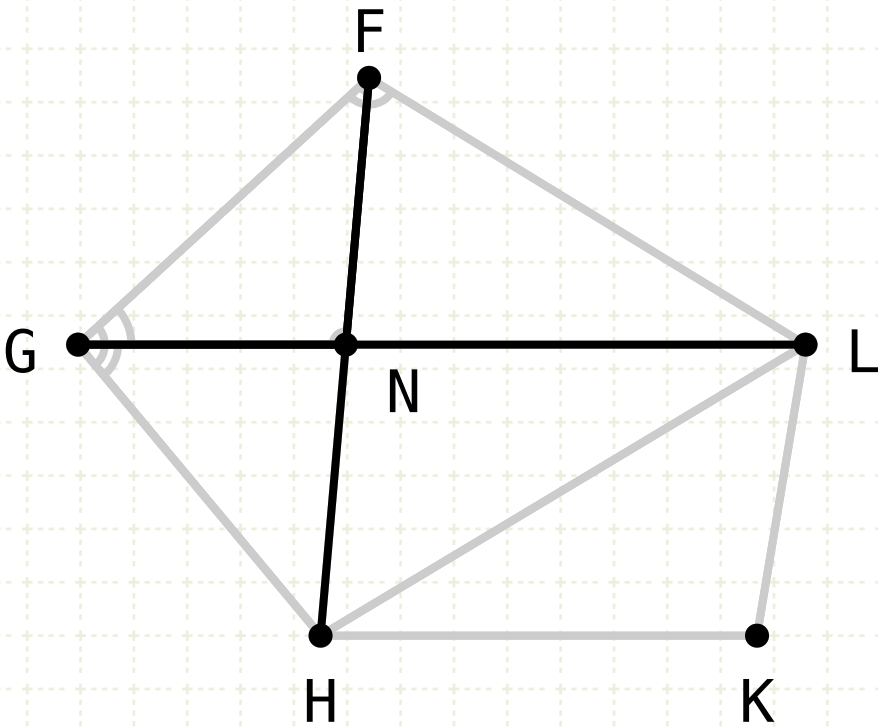
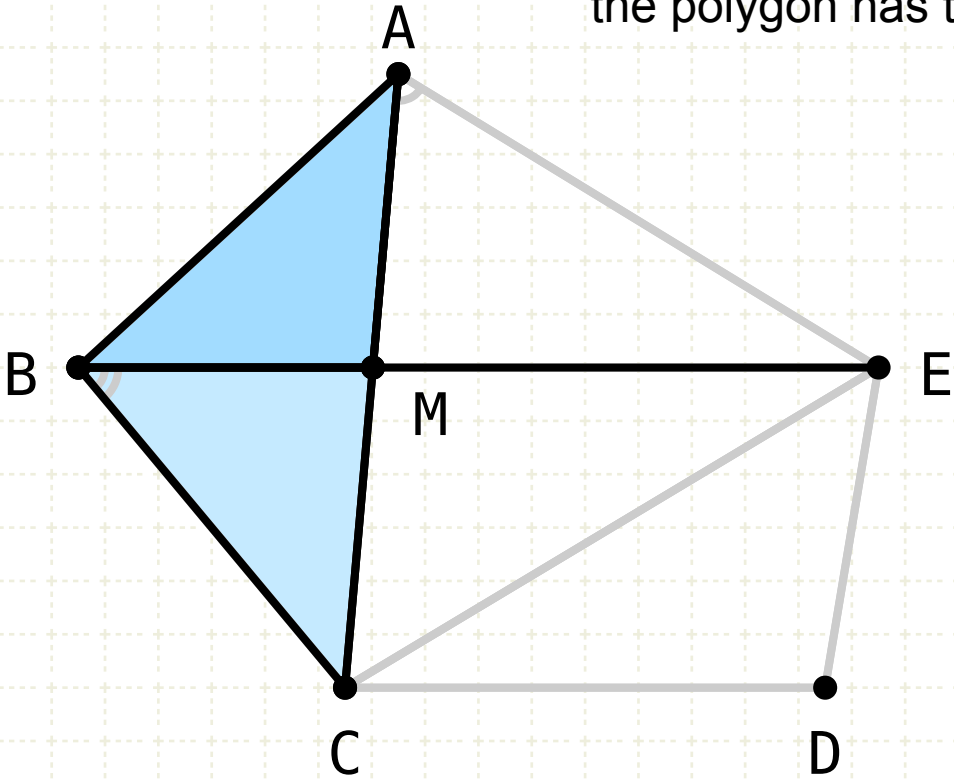
$$AM:MC = FN:NH$$





# Proposition 20 of Book VI

Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.



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Similarly we can prove that triangle BMC is also equiangular with triangle GNL, thus BM is to MC as NG is to NH

In addition, ex aequali, AM is to MC as is FN to NH

But, the ratio of AM to MC is equal to the ratio of the triangles ABM to MBC (VI-1)

$$ABCDE \sim FGHLK$$

$$b = \beta$$

$$b' = \beta'$$

$$AB:BC = FG:GH$$

$$a' = \alpha'$$

$$n = \eta$$

$$AM:BM = FN:NG$$

$$BM:MC = NG:NH$$

$$AM:MC = FN:NH$$

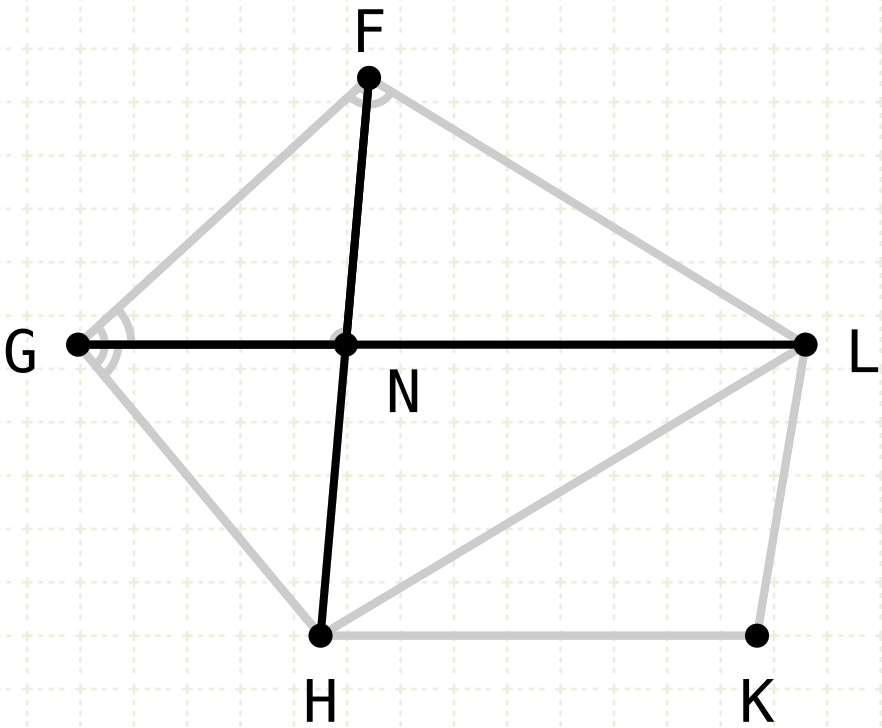
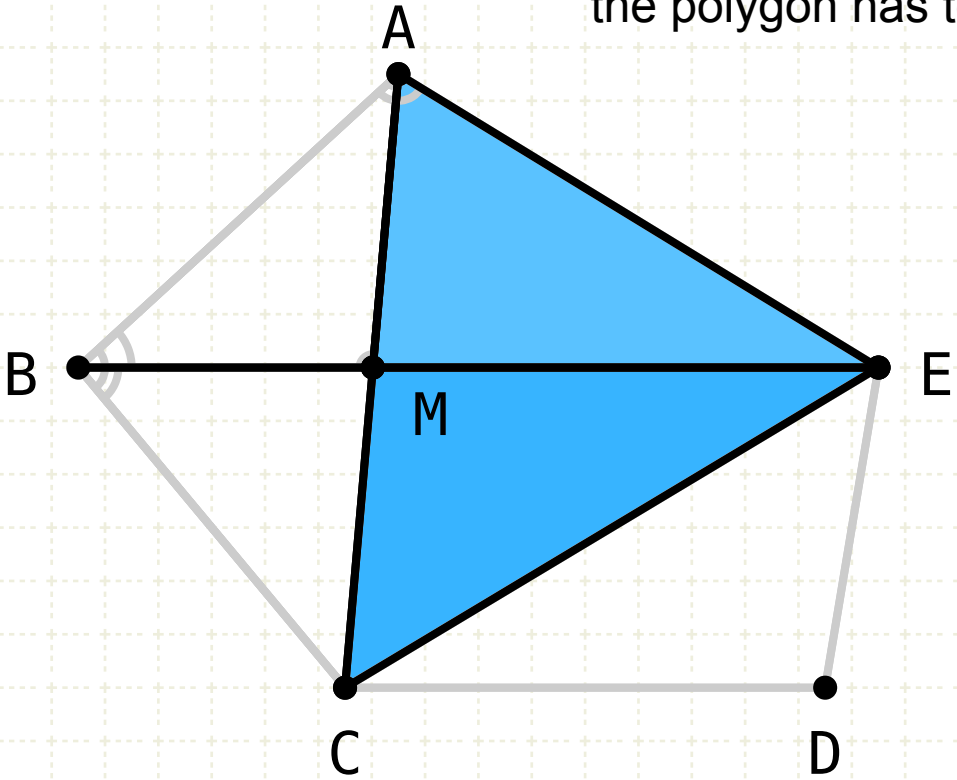
$$AM:MC$$

$$= \Delta ABM:\Delta MBC$$



# Proposition 20 of Book VI

Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.



## Proof - Duplicate Ratio to Sides

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Similarly we can prove that triangle BMC is also equiangular with triangle GNL, thus BM is to MC as NG is to NH

In addition, ex aequali, AM is to MC as is FN to NH

But, the ratio of AM to MC is equal to the ratio of the triangles ABM to MBC (VI-1)

Likewise, the ratio of AM to MC is equal to the ratio of the triangles AME to EMC (VI-1)

$$ABCDE \sim FGHLK$$

$$b = \beta$$

$$b' = \beta'$$

$$AB:BC = FG:GH$$

$$a' = \alpha'$$

$$n = \eta$$

$$AM:BM = FN:NG$$

$$BM:MC = NG:NH$$

$$AM:MC = FN:NH$$

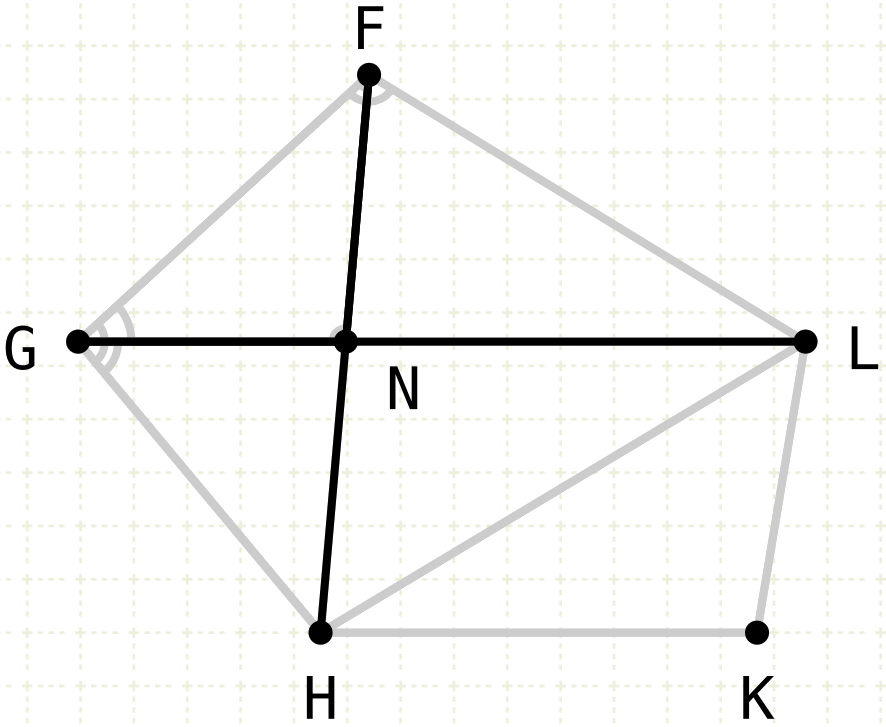
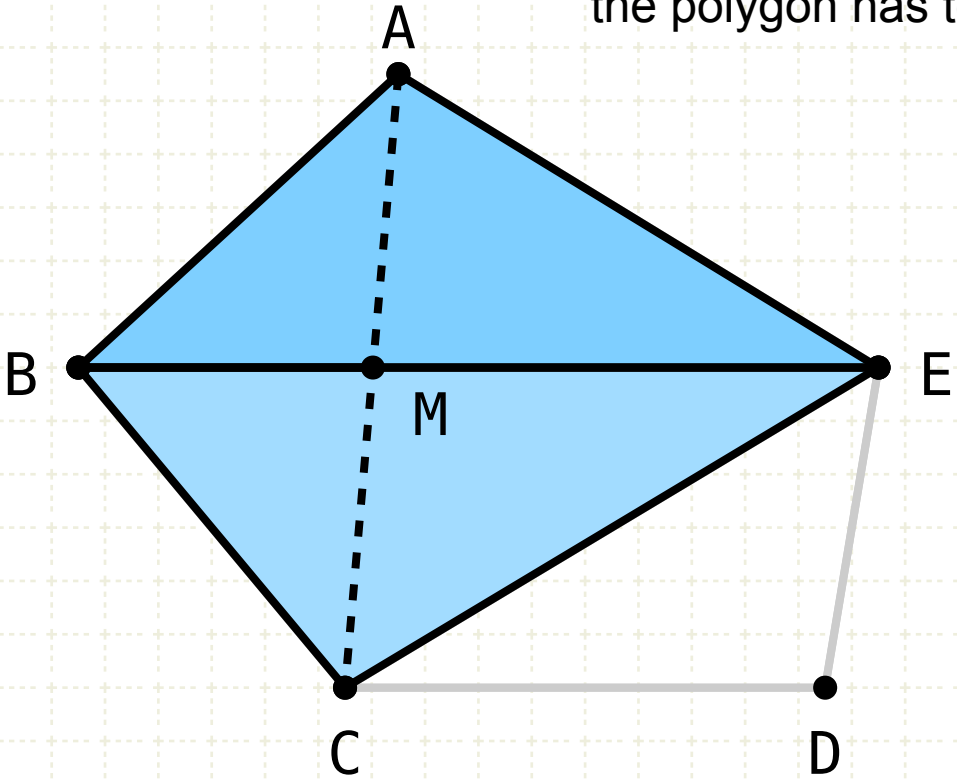
$$AM:MC$$

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# Proposition 20 of Book VI

Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.



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Similarly we can prove that triangle BMC is also equiangular with triangle GNH, thus BM is to MC as NG is to NH

In addition, ex aequali, AM is to MC as is FN to NH

But, the ratio of AM to MC is equal to the ratio of the triangles ABM to MBC (VI·1)

Likewise, the ratio of AM to MC is equal to the ratio of the triangles AME to EMC (VI·1)

If two ratios are equal, so is this the ratio of the sum of antecedents to the sum of consequents (V·12), so AME to EMC is equal to ABE to BCE

$$ABCDE \sim FGHL$$

$$b = \beta$$

$$b' = \beta'$$

$$AB:BC = FG:GH$$

$$a' = \alpha'$$

$$n = \eta$$

$$AM:BM = FN:NG$$

$$BM:MC = NG:NH$$

$$AM:MC = FN:NH$$

$$AM:MC$$

$$= \Delta ABM:\Delta MBC = \Delta AME:\Delta EMC$$

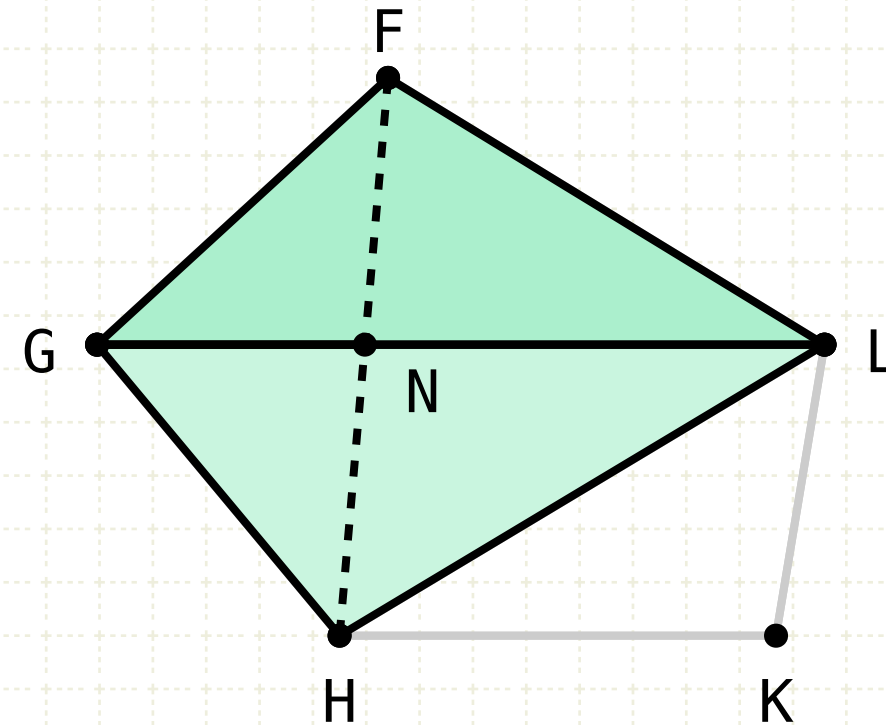
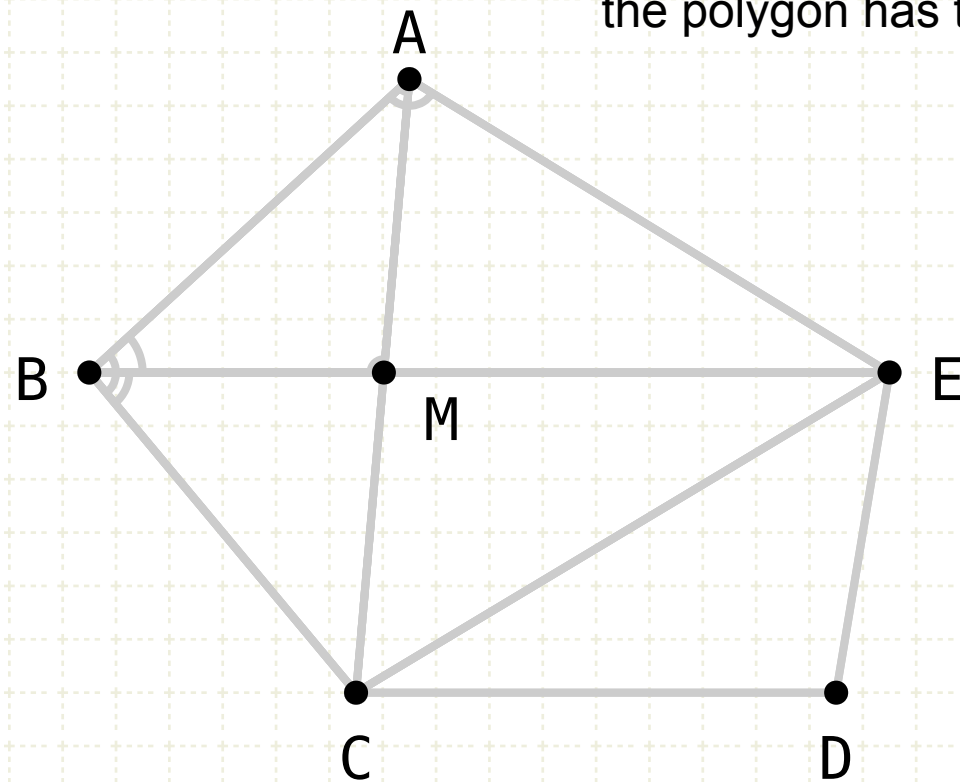
$$= (\Delta ABM+\Delta AME) : (\Delta MBC+\Delta EMC)$$

$$= \Delta ABE:\Delta BCE$$



# Proposition 20 of Book VI

Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.



## Proof - Duplicate Ratio to Sides

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In addition, ex aequali, AM is to MC as is FN to NH

But, the ratio of AM to MC is equal to the ratio of the triangles ABM to MBC (VI-1)

Likewise, the ratio of AM to MC is equal to the ratio of the triangles AME to EMC (VI-1)

If two ratios are equal, so is this the ratio of the sum of antecedents to the sum of consequents (V-12), so AME to EMC is equal to ABE to BCE

Similarly, FN to NH is equal to the triangles FGL to GHL

$$ABCDE \sim FGHLK$$

$$b = \beta$$

$$b' = \beta'$$

$$AB:BC = FG:GH$$

$$a' = \alpha'$$

$$n = \eta$$

$$AM:BM = FN:NG$$

$$BM:MC = NG:NH$$

$$AM:MC = FN:NH$$

$$AM:MC$$

$$= \Delta ABM:\Delta MBC = \Delta AME:\Delta EMC$$

$$= (\Delta ABM+\Delta AME) : (\Delta MBC+\Delta EMC)$$

$$= \Delta ABE:\Delta BCE$$

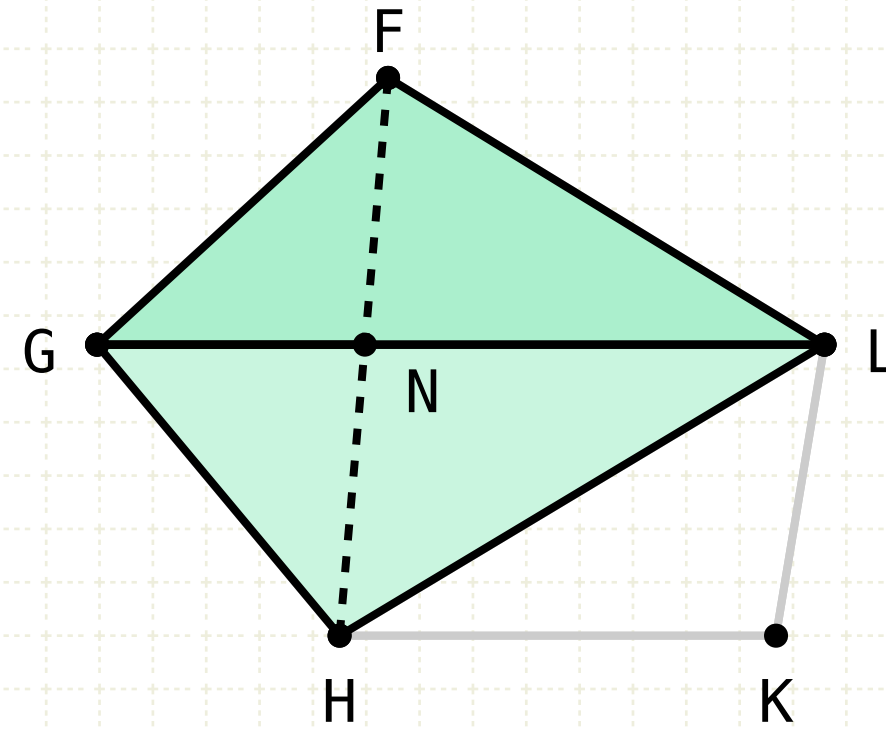
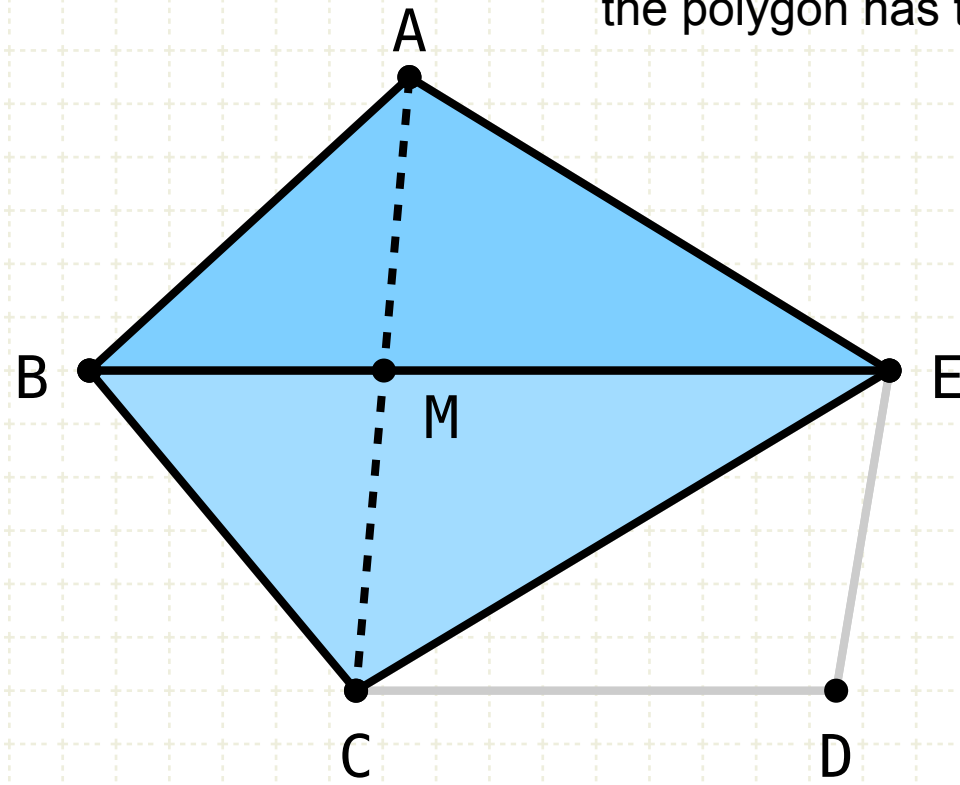
$$FN:NH = \Delta FGL:\Delta GHL$$





## Proposition 20 of Book VI

Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.



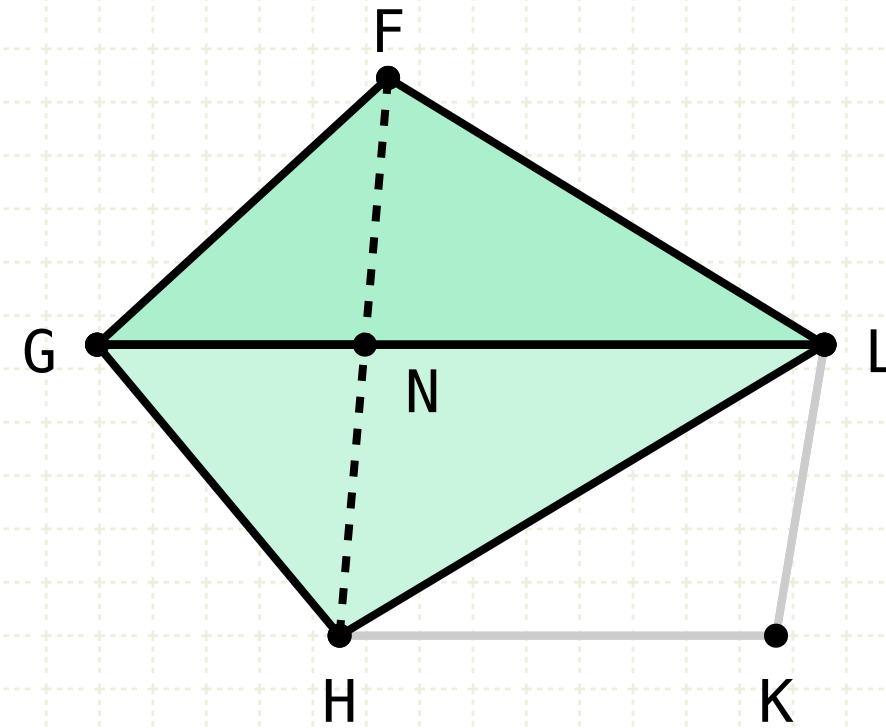
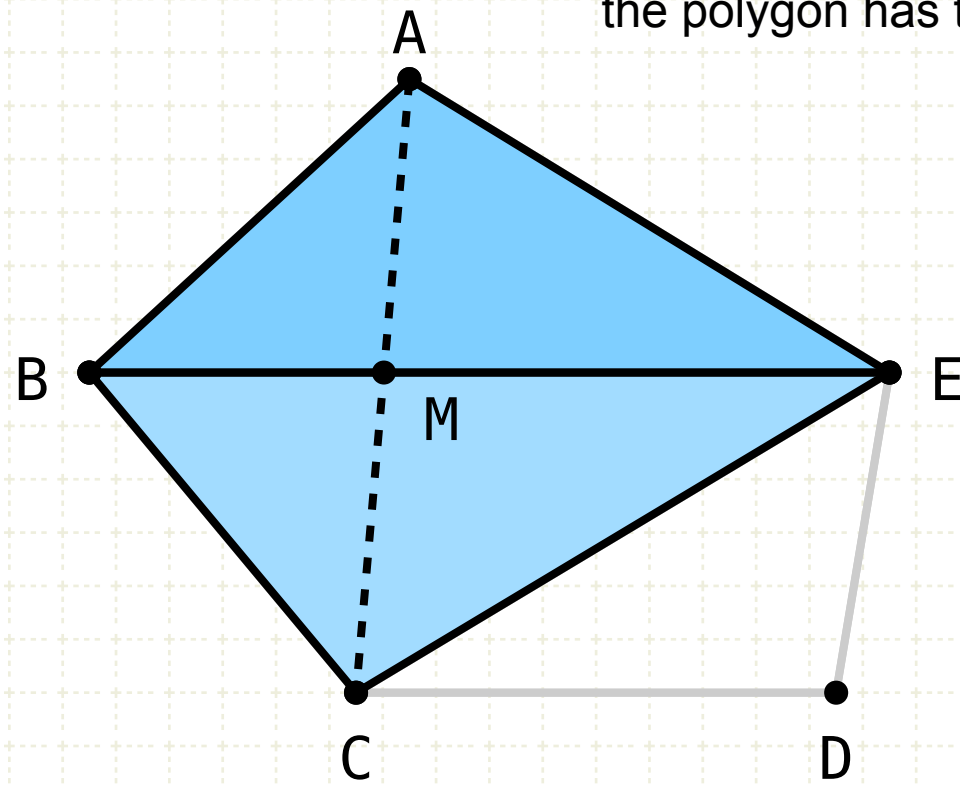
### Proof - Duplicate Ratio to Sides

$$\begin{aligned} ABCDE &\sim FGHLK \\ AB:BC &= FG:GH \\ AM:MC &= FN:NH \\ AM:MC &= \triangle ABE:\triangle BCE \\ FN:NH &= \triangle FGL:\triangle GHL \end{aligned}$$



## Proposition 20 of Book VI

Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.



### Proof - Duplicate Ratio to Sides

Substitute the triangle ratios for the ratios AM to MC and FN to NH with the result that the ratios of the triangles ABC to BCE equals the ratio of the triangles FGL to GHL

$$ABCDE \sim FGHLK$$

$$AB:BC = FG:GH$$

$$AM:MC = FN:NH$$

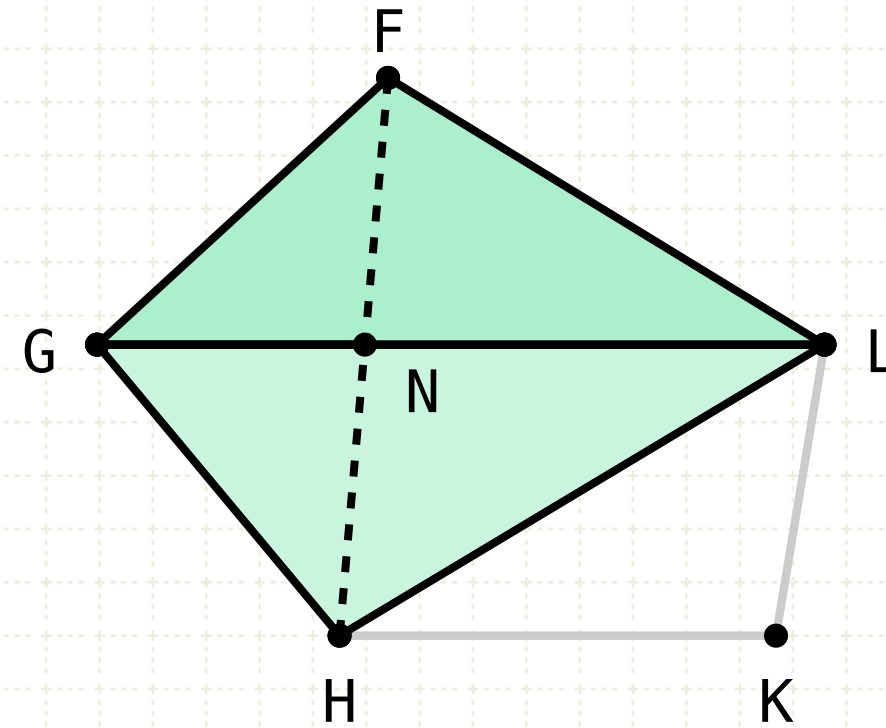
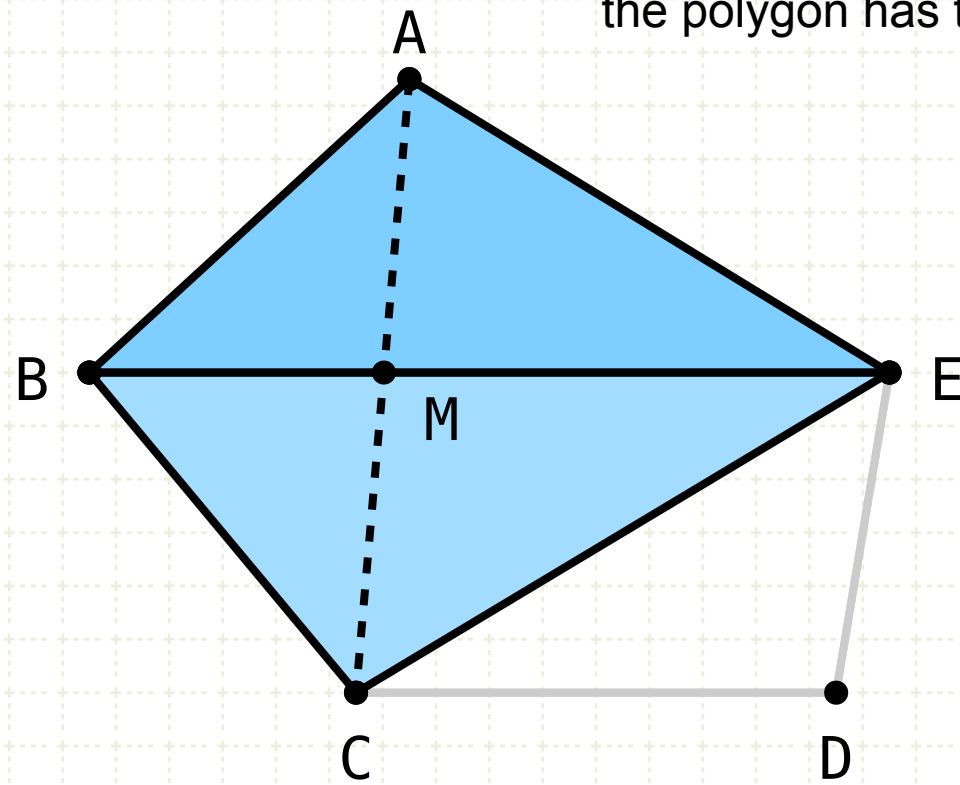
$$AM:MC = \triangle ABE:\triangle BCE$$

$$FN:NH = \triangle FGL:\triangle GHL$$

$$\triangle ABE:\triangle BCE = \triangle FGL:\triangle GHL$$

## Proposition 20 of Book VI

Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.



### Proof - Duplicate Ratio to Sides

Substitute the triangle ratios for the ratios AM to MC and FN to NH with the result that the ratios of the triangles ABE to BCE equals the ratio of the triangles FGL to GHL

And alternately, the ratio of triangles ABE to FGL is equal to the ratio BCE to GHL

$$ABCDE \sim FGHLK$$

$$AB:BC = FG:GH$$

$$AM:MC = FN:NH$$

$$AM:MC = \triangle ABE:\triangle BCE$$

$$FN:NH = \triangle FGL:\triangle GHL$$

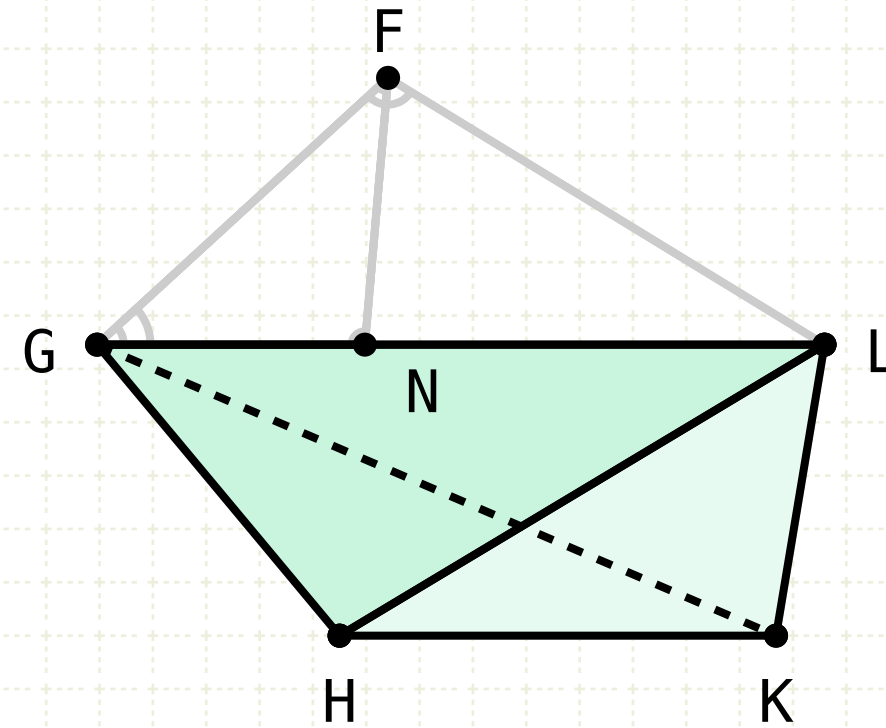
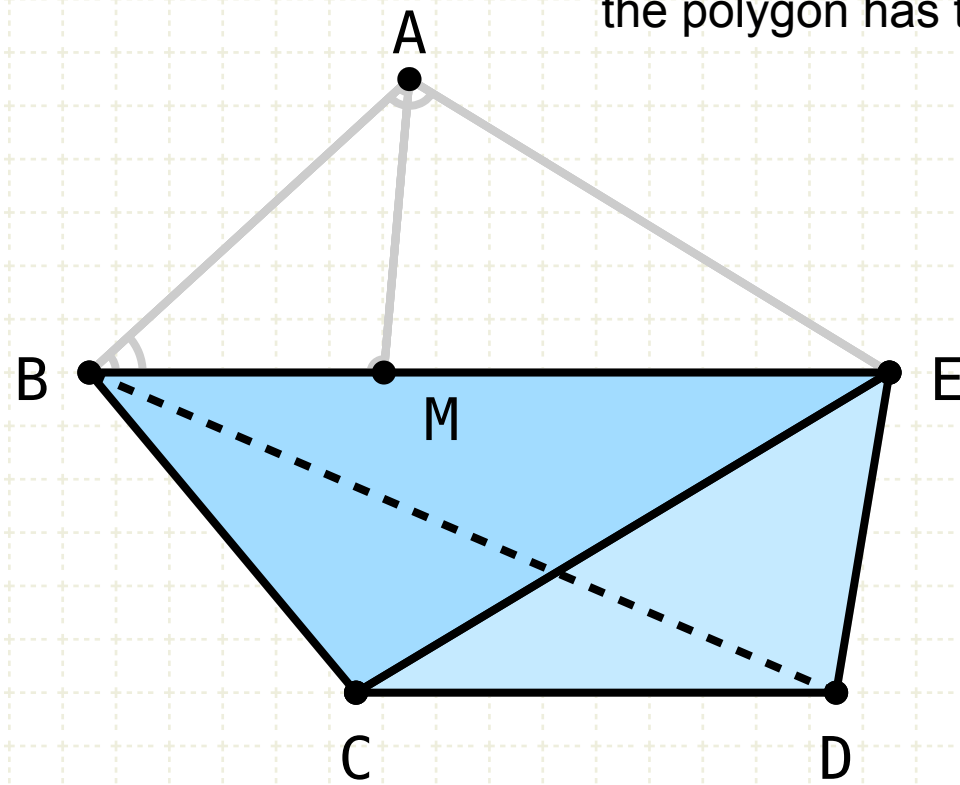
$$\triangle ABE:\triangle BCE = \triangle FGL:\triangle GHL$$

$$\triangle ABE:\triangle FGL = \triangle BCE:\triangle GHL$$



# Proposition 20 of Book VI

Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.



## Proof - Duplicate Ratio to Sides

Substitute the triangle ratios for the ratios AM to MC and FN to NH with the result that the ratios of the triangles ABC to BCE equals the ratio of the triangles FGL to GHL

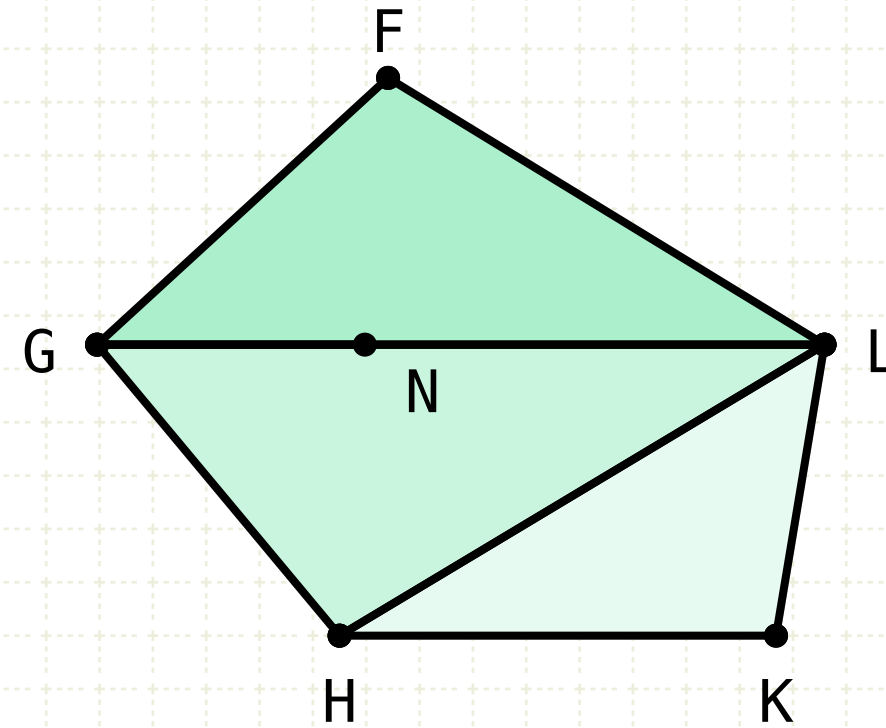
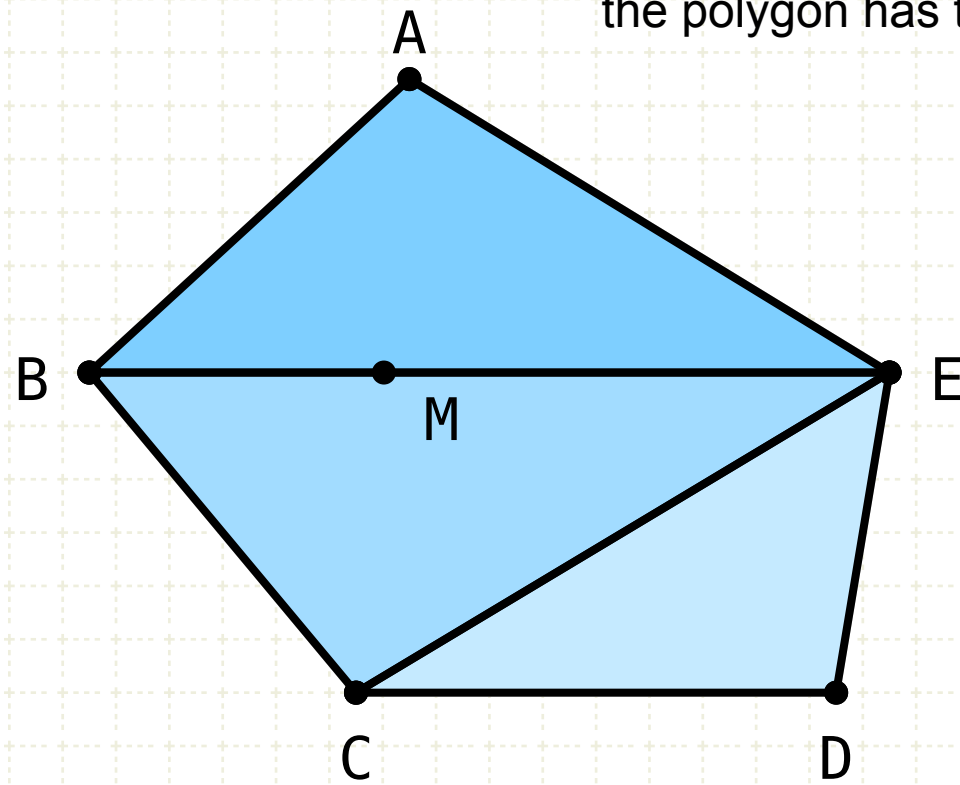
And alternately, the ratio of triangles ABE to FGL is equal to the ratio BCE to GHL

Similarly, we can show that if the lines BD and GK are drawn, the ratio of the triangles BCE to GHL is equal to the ratio of the triangles ECD to LHK.

$$\begin{aligned} ABCDE &\sim FGHLK \\ AB:BC &= FG:GH \\ AM:MC &= FN:NH \\ AM:MC &= \triangle ABE:\triangle BCE \\ FN:NH &= \triangle FGL:\triangle GHL \\ \triangle ABE:\triangle BCE &= \triangle FGL:\triangle GHL \\ \triangle ABE:\triangle FGL &= \triangle BCE:\triangle GHL \\ \triangle BCE:\triangle GHL &= \triangle ECD:\triangle LHK \end{aligned}$$

## Proposition 20 of Book VI

Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.



### Proof - Duplicate Ratio to Sides

Substitute the triangle ratios for the ratios AM to MC and FN to NH with the result that the ratios of the triangles ABE to BCE equals the ratio of the triangles FGL to GHL

And alternately, the ratio of triangles ABE to FGL is equal to the ratio BCE to GHL

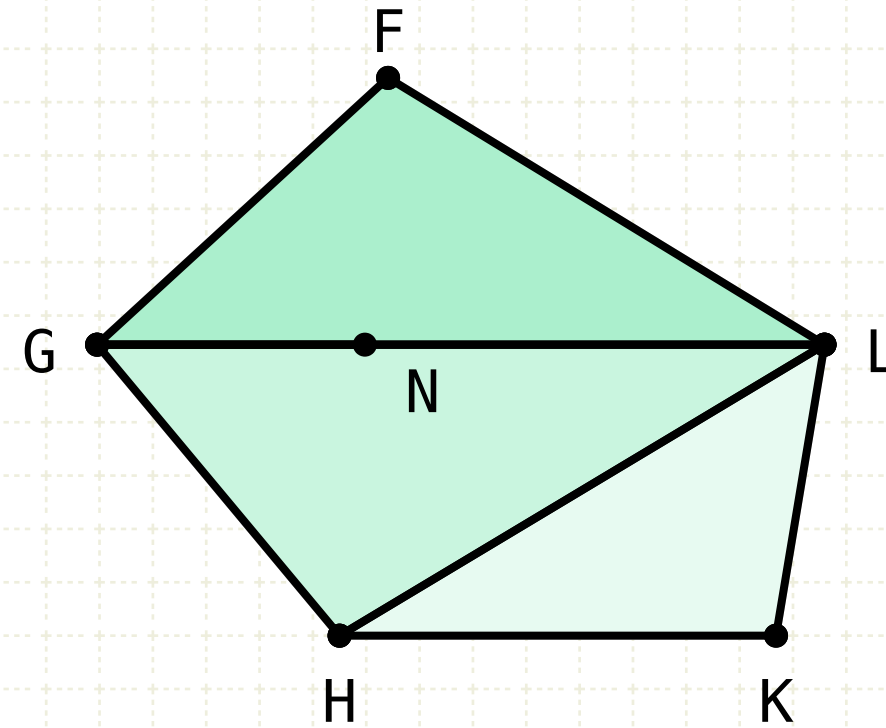
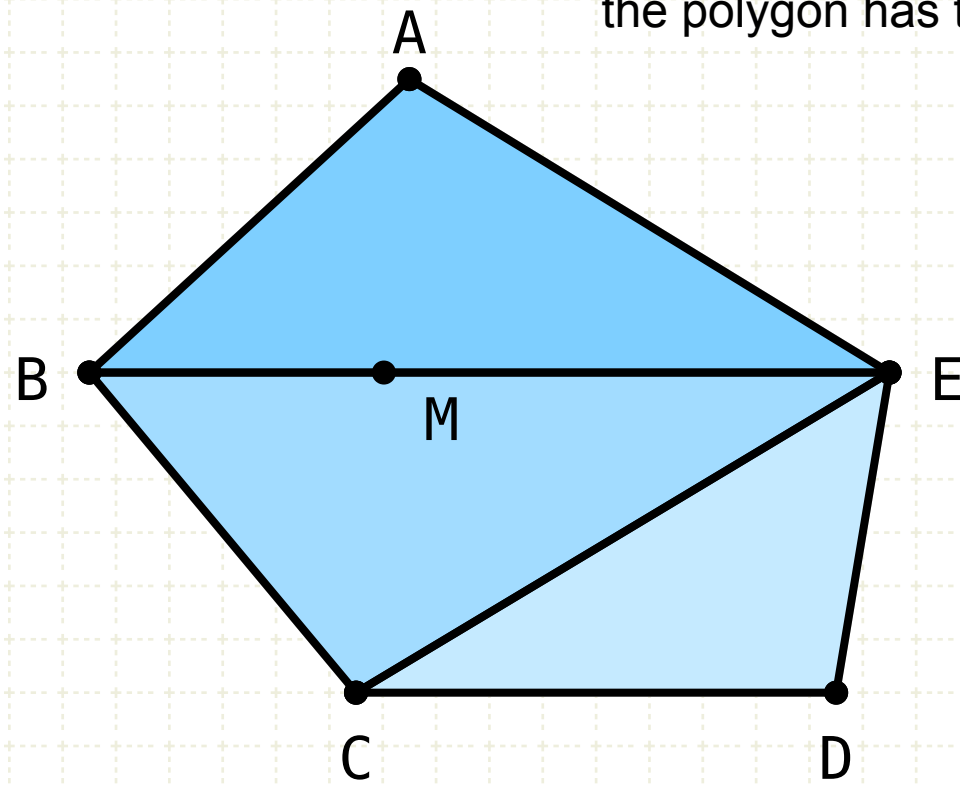
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$$\begin{aligned} ABCDE &\sim FGHLK \\ AB:BC &= FG:GH \\ AM:MC &= FN:NH \\ AM:MC &= \triangle ABE:\triangle BCE \\ FN:NH &= \triangle FGL:\triangle GHL \\ \triangle ABE:\triangle BCE &= \triangle FGL:\triangle GHL \\ \triangle ABE:\triangle FGL &= \triangle BCE:\triangle GHL \\ \triangle BCE:\triangle GHL &= \triangle ECD:\triangle LHK \end{aligned}$$



# Proposition 20 of Book VI

Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.



## Proof - Duplicate Ratio to Sides

Substitute the triangle ratios for the ratios AM to MC and FN to NH with the result that the ratios of the triangles ABC to BCE equals the ratio of the triangles FGL to GHL

And alternately, the ratio of triangles ABE to FGL is equal to the ratio BCE to GHL

Similarly, we can show that if the lines BD and GK are drawn, the ratio of the triangles BCE to GHL is equal to the ratio of the triangles ECD to LHK.

If a number of ratios are equal, then they are also equal to the ratio of the sums of the antecedents to the sums of the consequents (V.12)

$$ABCDE \sim FGHLK$$

$$AB:BC = FG:GH$$

$$AM:MC = FN:NH$$

$$AM:MC = \triangle ABE:\triangle BCE$$

$$FN:NH = \triangle FGL:\triangle GHL$$

$$\triangle ABE:\triangle BCE = \triangle FGL:\triangle GHL$$

$$\triangle ABE:\triangle FGL = \triangle BCE:\triangle GHL$$

$$\triangle BCE:\triangle GHL = \triangle ECD:\triangle LHK$$

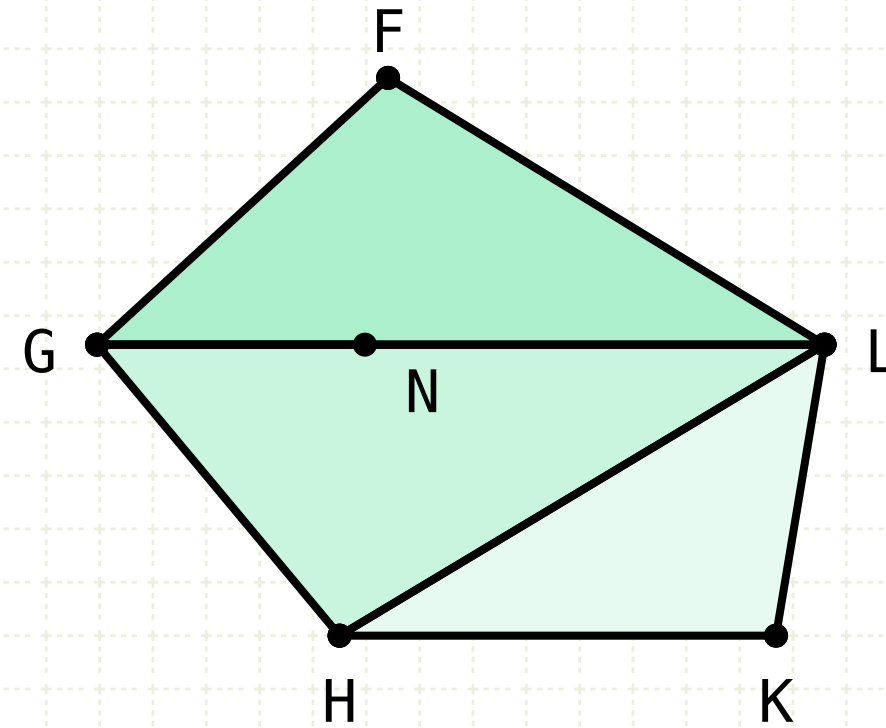
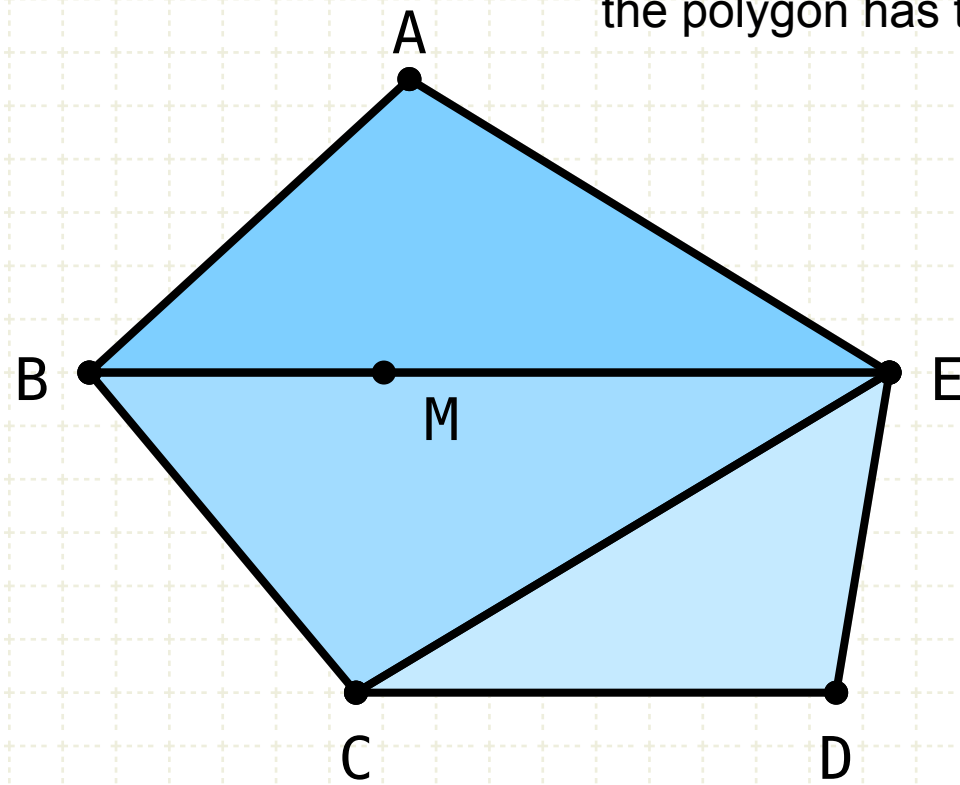
$$= (\triangle ABE + \triangle BCE + \triangle ECD) : (\triangle FGL + \triangle GHL + \triangle LHK)$$

$$\triangle ABE:\triangle FGL = ABCDE:FGHLK$$



## Proposition 20 of Book VI

Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.



### Proof - Duplicate Ratio to Sides

Substitute the triangle ratios for the ratios AM to MC and FN to NH with the result that the ratios of the triangles ABC to BCE equals the ratio of the triangles FGL to GHL

And alternately, the ratio of triangles ABE to FGL is equal to the ratio BCE to GHL

Similarly, we can show that if the lines BD and GK are drawn, the ratio of the triangles BCE to GHL is equal to the ratio of the triangles ECD to LHK.

If a number of ratios are equal, then they are also equal to the ratio of the sums of the antecedents to the sums of the consequents (V·12)

But the ratio of the triangles ABE to FGL is a duplicate ratio of the sides AB to FG (VI·19)

$$ABCDE \sim FGHLK$$

$$AB:BC = FG:GH$$

$$AM:MC = FN:NH$$

$$AM:MC = \triangle ABE:\triangle BCE$$

$$FN:NH = \triangle FGL:\triangle GHL$$

$$\triangle ABE:\triangle BCE = \triangle FGL:\triangle GHL$$

$$\triangle ABE:\triangle FGL = \triangle BCE:\triangle GHL$$

$$\triangle BCE:\triangle GHL = \triangle ECD:\triangle LHK$$

$$= (\triangle ABE + \triangle BCE + \triangle ECD) : (\triangle FGL + \triangle GHL + \triangle LHK)$$

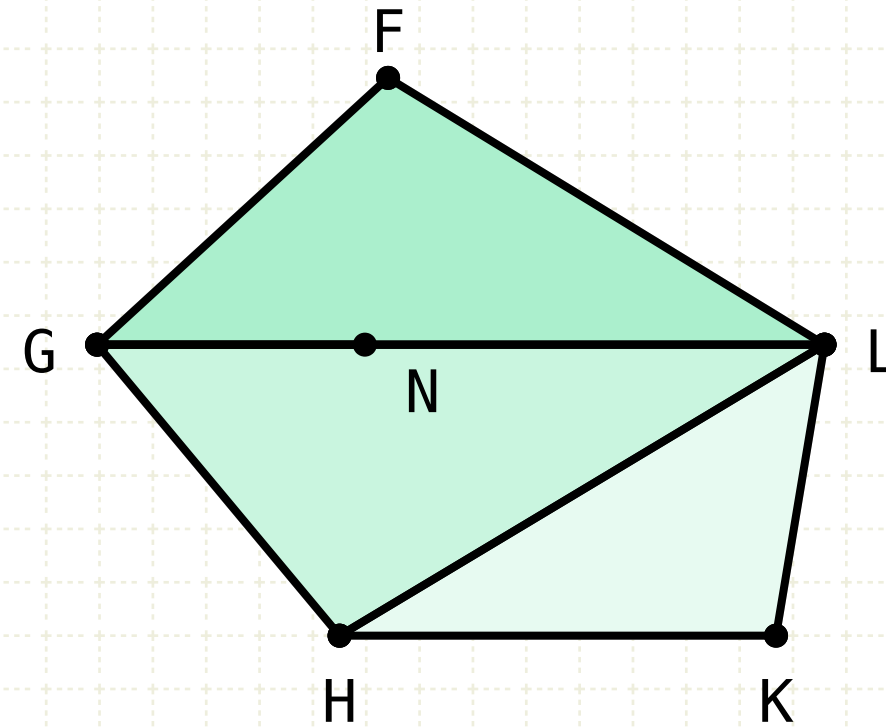
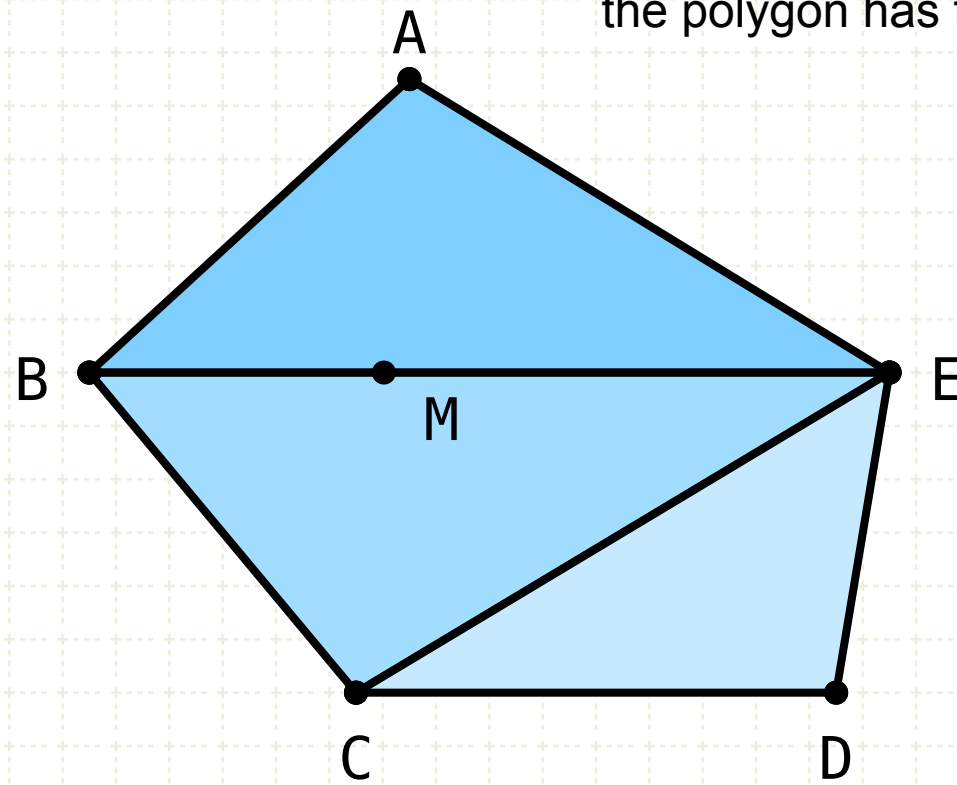
$$\triangle ABE:\triangle FGL = ABCDE:FGHLK$$

$$\triangle ABE:\triangle FGL = (AB:FG)^2$$



## Proposition 20 of Book VI

Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.



### Proof - Duplicate Ratio to Sides

Substitute the triangle ratios for the ratios AM to MC and FN to NH with the result that the ratios of the triangles ABE to BCE equals the ratio of the triangles FGL to GHL

And alternately, the ratio of triangles ABE to FGL is equal to the ratio BCE to GHL

Similarly, we can show that if the lines BD and GK are drawn, the ratio of the triangles BCE to GHL is equal to the ratio of the triangles ECD to LHK.

If a number of ratios are equal, then they are also equal to the ratio of the sums of the antecedents to the sums of the consequents (V·12)

But the ratio of the triangles ABE to FGL is a duplicate ratio of the sides AB to FG (VI·19)

Therefore, the ratio of the two polygons ABCDE and FGHLK is a duplicate ratio of the sides AB to FG

$$ABCDE \sim FGHLK$$

$$AB:BC = FG:GH$$

$$AM:MC = FN:NH$$

$$AM:MC = \triangle ABE:\triangle BCE$$

$$FN:NH = \triangle FGL:\triangle GHL$$

$$\triangle ABE:\triangle BCE = \triangle FGL:\triangle GHL$$

$$\triangle ABE:\triangle FGL = \triangle BCE:\triangle GHL$$

$$\triangle BCE:\triangle GHL = \triangle ECD:\triangle LHK$$

$$= (\triangle ABE + \triangle BCE + \triangle ECD) : (\triangle FGL + \triangle GHL + \triangle LHK)$$

$$\triangle ABE:\triangle FGL = ABCDE:FGHLK$$

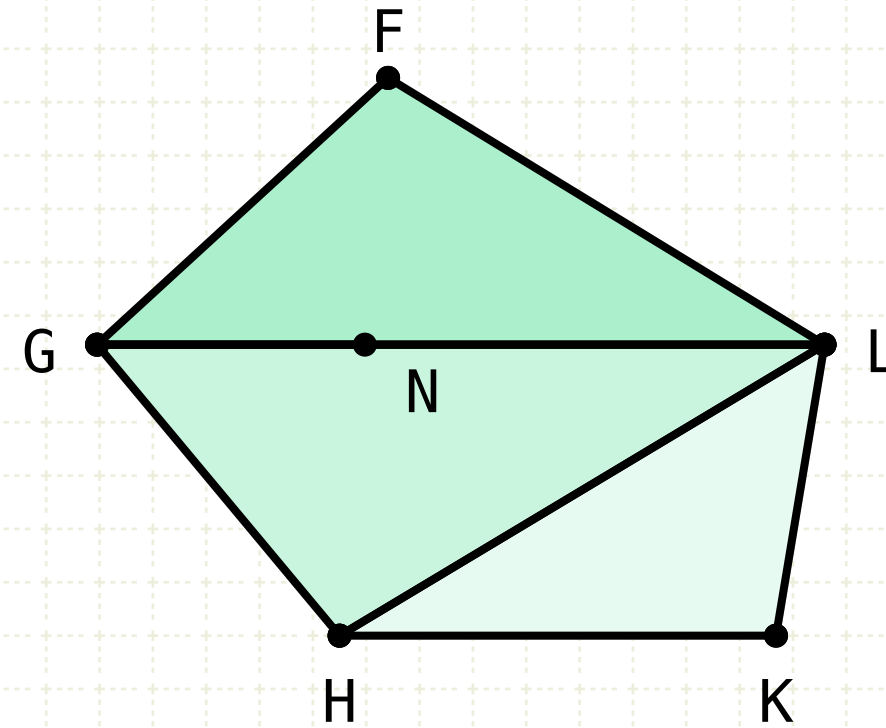
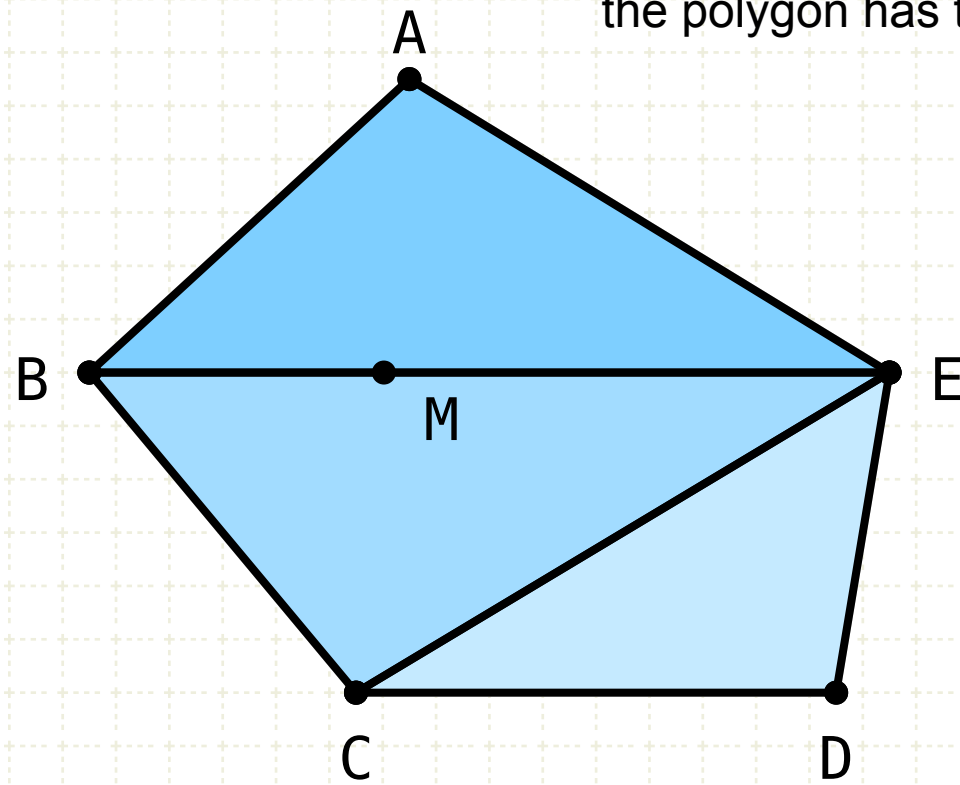
$$\triangle ABE:\triangle FGL = (AB:FG)^2$$

$$ABCDE:FGHLK = (AB:FG)^2$$



## Proposition 20 of Book VI

Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.



$$ABCDE \sim FGHLK$$

$$AB:BC = FG:GH$$

$$AM:MC = FN:NH$$

$$AM:MC = \triangle ABE:\triangle BCE$$

$$FN:NH = \triangle FGL:\triangle GHL$$

$$\triangle ABE:\triangle BCE = \triangle FGL:\triangle GHL$$

$$\triangle ABE:\triangle FGL = \triangle BCE:\triangle GHL$$

$$\triangle BCE:\triangle GHL = \triangle ECD:\triangle LHK$$

$$= (\triangle ABE + \triangle BCE + \triangle ECD) : (\triangle FGL + \triangle GHL + \triangle LHK)$$

$$\triangle ABE:\triangle FGL = ABCDE:FGHLK$$

$$\triangle ABE:\triangle FGL = (AB:FG)^2$$

$$ABCDE:FGHLK = (AB:FG)^2$$

### Proof - Duplicate Ratio to Sides

Substitute the triangle ratios for the ratios AM to MC and FN to NH with the result that the ratios of the triangles ABE to BCE equals the ratio of the triangles FGL to GHL

And alternately, the ratio of triangles ABE to FGL is equal to the ratio BCE to GHL

Similarly, we can show that if the lines BD and GK are drawn, the ratio of the triangles BCE to GHL is equal to the ratio of the triangles ECD to LHK.

If a number of ratios are equal, then they are also equal to the ratio of the sums of the antecedents to the sums of the consequents (V·12)

But the ratio of the triangles ABE to FGL is a duplicate ratio of the sides AB to FG (VI·19)

Therefore, the ratio of the two polygons ABCDE and FGHLK is a duplicate ratio of the sides AB to FG

Thus, it has been shown that the ratio of the two polygons is a duplicate ratio of their respective sides





# Youtube Videos

<https://www.youtube.com/c/SandyBultena>

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