

Euclid's Elements

Book I

If Euclid did not kindle your youthful enthusiasm, you were not born to be a scientific thinker.

Albert Einstein



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Proposition 42 of Book I

To construct a parallelogram equal to a given triangle in a given rectilinear angle.



Proposition 42 of Book I

To construct a parallelogram equal to a given triangle in a given rectilinear angle.

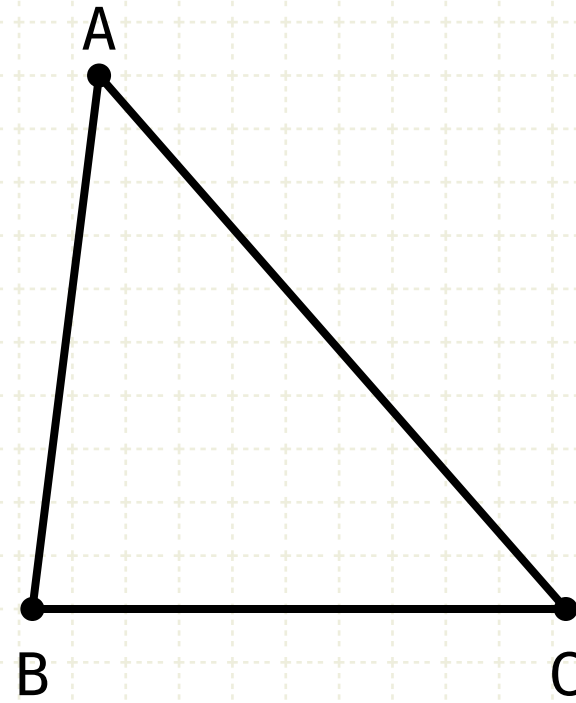
In other words

Create a parallelogram with a specific angle, whose area is equal to a given triangle



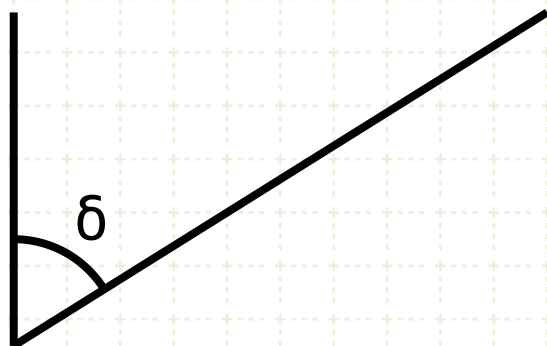
Proposition 42 of Book I

To construct a parallelogram equal to a given triangle in a given rectilinear angle.



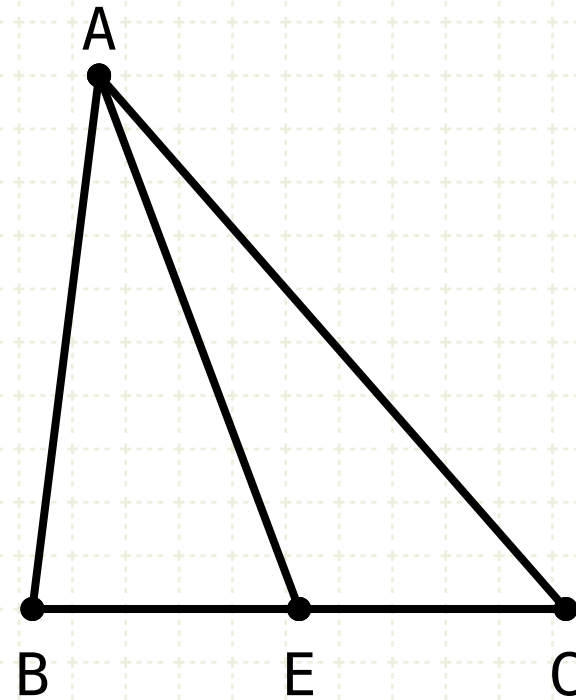
Construction

Start with triangle ABC and angle δ



Proposition 42 of Book I

To construct a parallelogram equal to a given triangle in a given rectilinear angle.

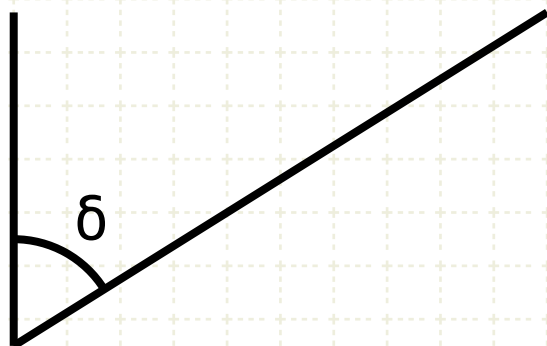


Construction

Start with triangle ABC and angle δ

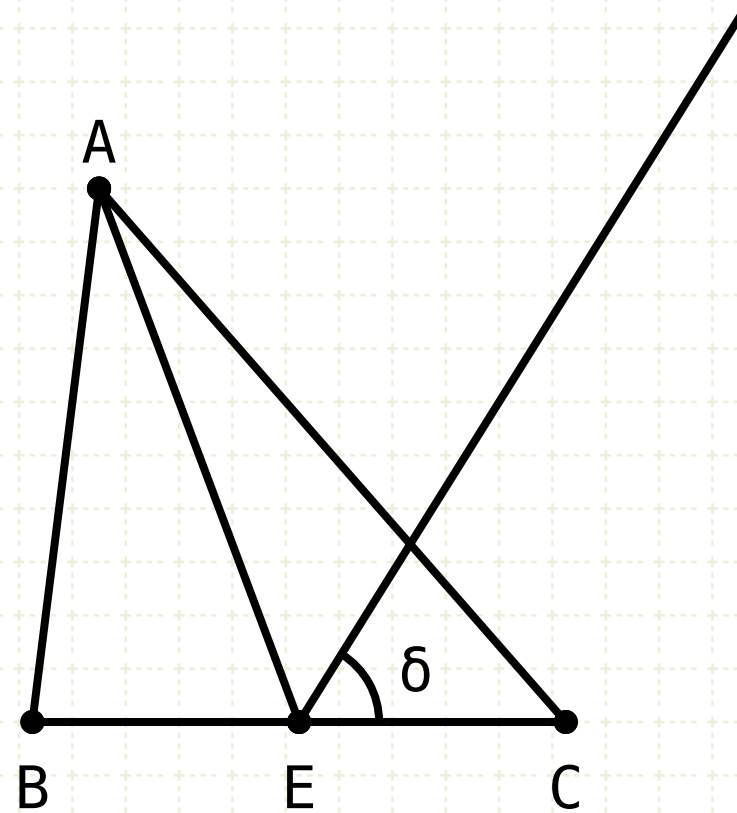
Bisect line BC at point E (I·10). Draw line AC

$$BE = EC$$



Proposition 42 of Book I

To construct a parallelogram equal to a given triangle in a given rectilinear angle.



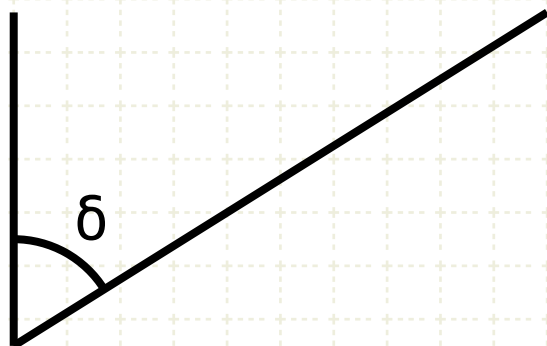
Construction

Start with triangle ABC and angle δ

Bisect line BC at point E (I·10). Draw line AC

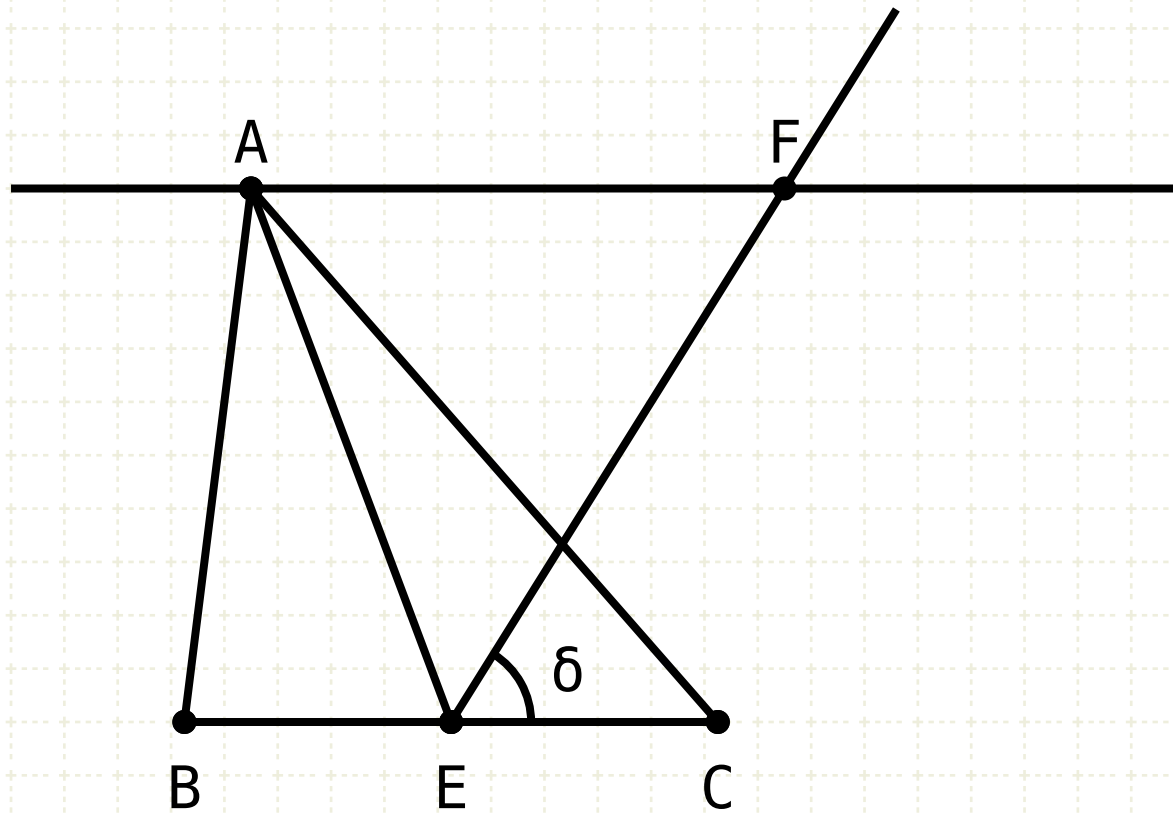
Copy angle δ onto line EC, with the vertex at point E (I·23)

$$BE = EC$$



Proposition 42 of Book I

To construct a parallelogram equal to a given triangle in a given rectilinear angle.



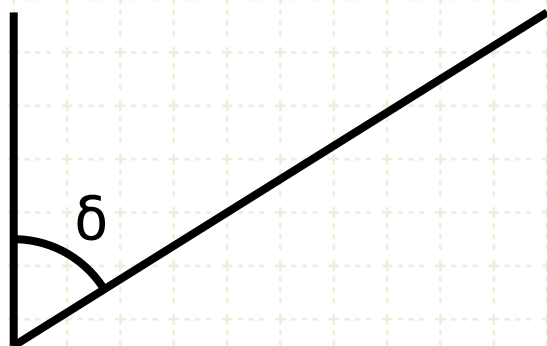
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Bisect line BC at point E (I·10). Draw line AC

Copy angle δ onto line EC, with the vertex at point E (I·23)

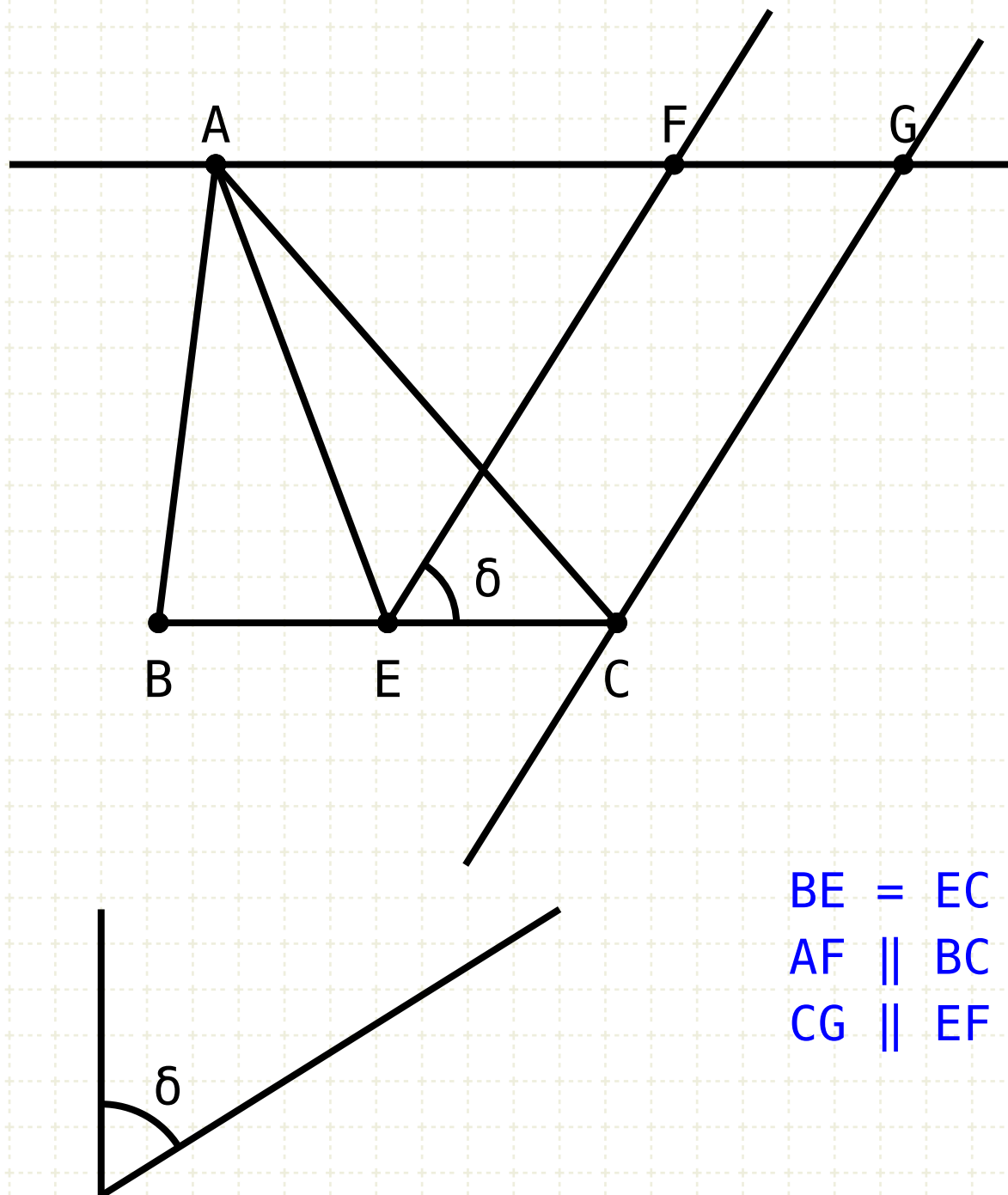
Draw a line AF, through A, parallel to BC (I·31)



$$\begin{aligned} BE &= EC \\ AF &\parallel BC \end{aligned}$$

Proposition 42 of Book I

To construct a parallelogram equal to a given triangle in a given rectilinear angle.



$$\begin{aligned} BE &= EC \\ AF &\parallel BC \\ CG &\parallel EF \end{aligned}$$

Construction

Start with triangle ABC and angle δ

Bisect line BC at point E (I·10). Draw line AC

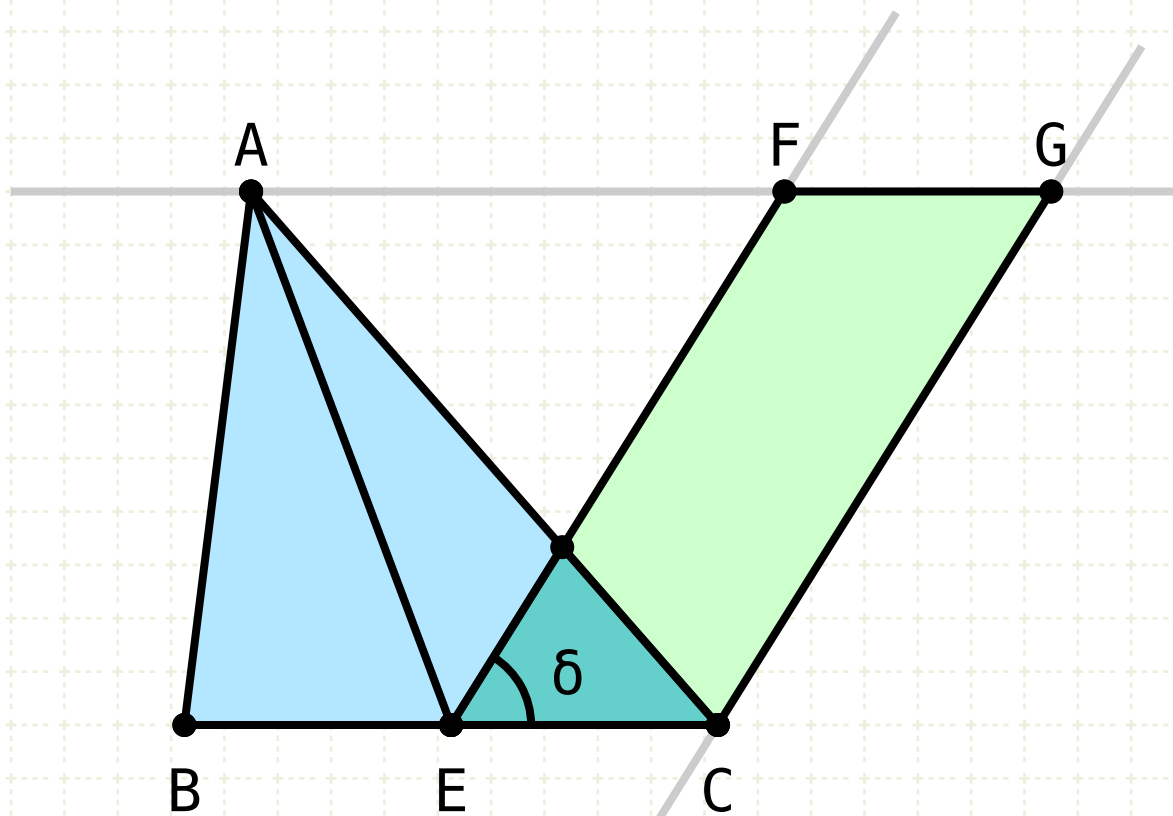
Copy angle δ onto line EC , with the vertex at point E (I·23)

Draw a line AF , through A , parallel to BC (I·31)

Draw a line CG , through C , parallel to EF (I·31)

Proposition 42 of Book I

To construct a parallelogram equal to a given triangle in a given rectilinear angle.



Construction

Start with triangle ABC and angle δ

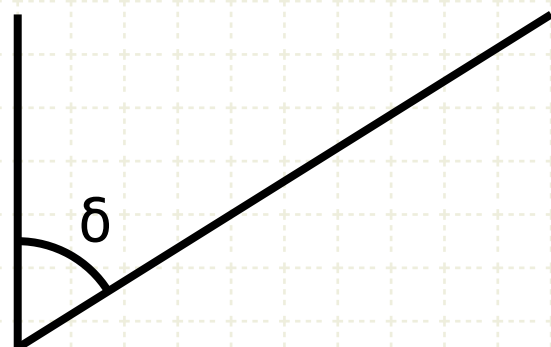
Bisect line BC at point E (I·10). Draw line AC

Copy angle δ onto line EC, with the vertex at point E (I·23)

Draw a line AF, through A, parallel to BC (I·31)

Draw a line CG, through C, parallel to EF (I·31)

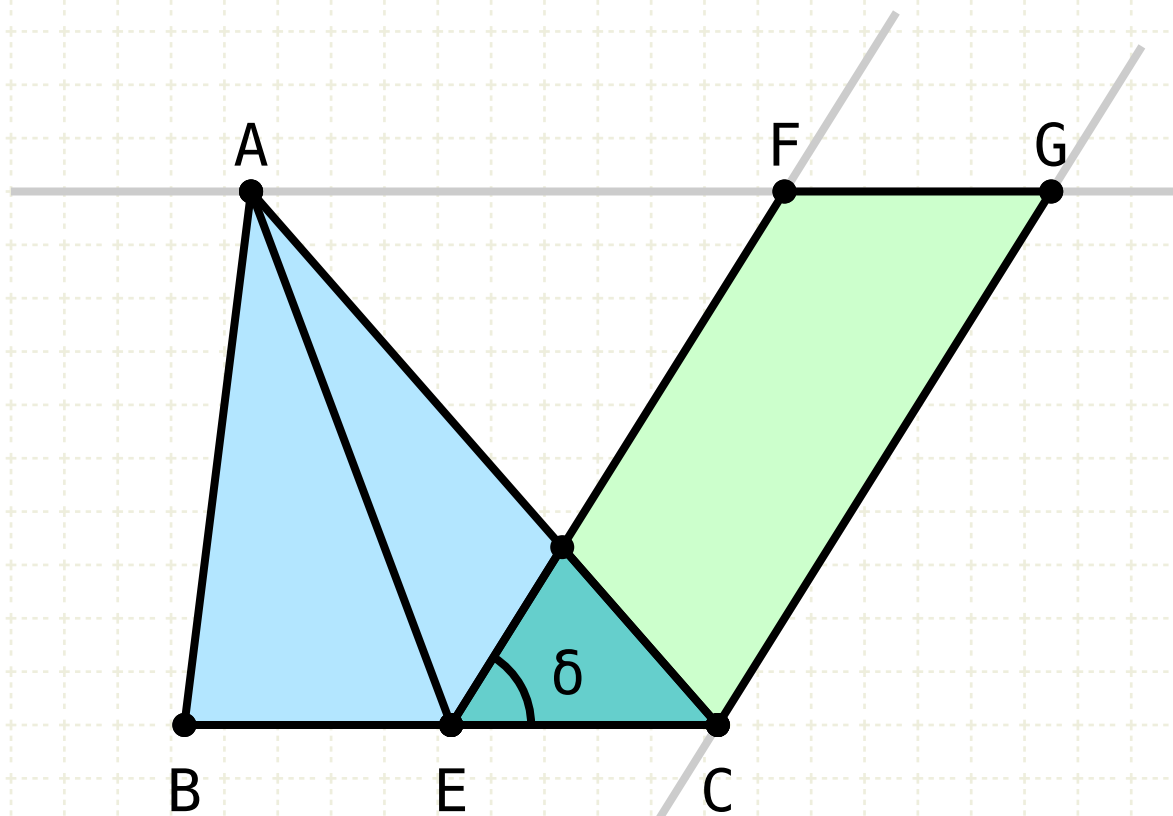
The parallelogram FECG is equal in area to the triangle ABC



$$\begin{aligned} BE &= EC \\ AF &\parallel BC \\ CG &\parallel EF \\ \Delta ABC &= FECG \end{aligned}$$

Proposition 42 of Book I

To construct a parallelogram equal to a given triangle in a given rectilinear angle.



Construction

Start with triangle ABC and angle δ

Bisect line BC at point E (I·10). Draw line AC

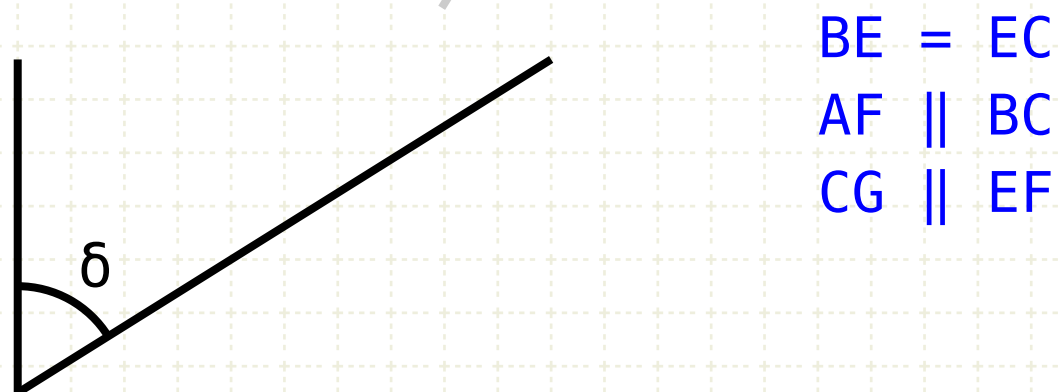
Copy angle δ onto line EC, with the vertex at point E (I·23)

Draw a line AF, through A, parallel to BC (I·31)

Draw a line CG, through C, parallel to EF (I·31)

The parallelogram FECG is equal in area to the triangle ABC

Proof:



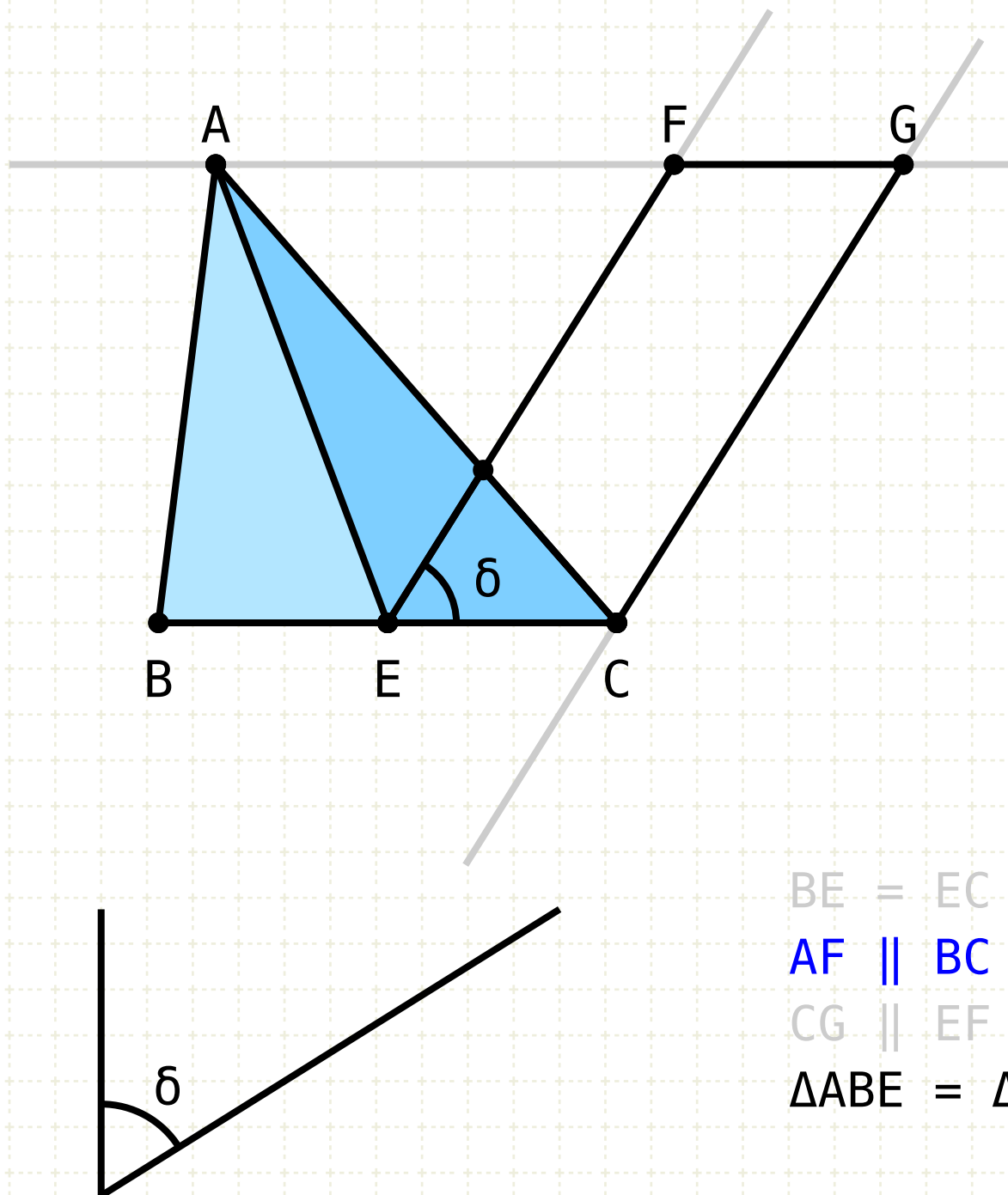
$$BE = EC$$

$$AF \parallel BC$$

$$CG \parallel EF$$

Proposition 42 of Book I

To construct a parallelogram equal to a given triangle in a given rectilinear angle.



$$\begin{aligned}
 BE &= EC \\
 AF &\parallel BC \\
 CG &\parallel EF \\
 \Delta ABE &= \Delta AEC = \frac{1}{2} \Delta ABC
 \end{aligned}$$

Construction

Start with triangle ABC and angle δ

Bisect line BC at point E (I·10). Draw line AC

Copy angle δ onto line EC, with the vertex at point E (I·23)

Draw a line AF, through A, parallel to BC (I·31)

Draw a line CG, through C, parallel to EF (I·31)

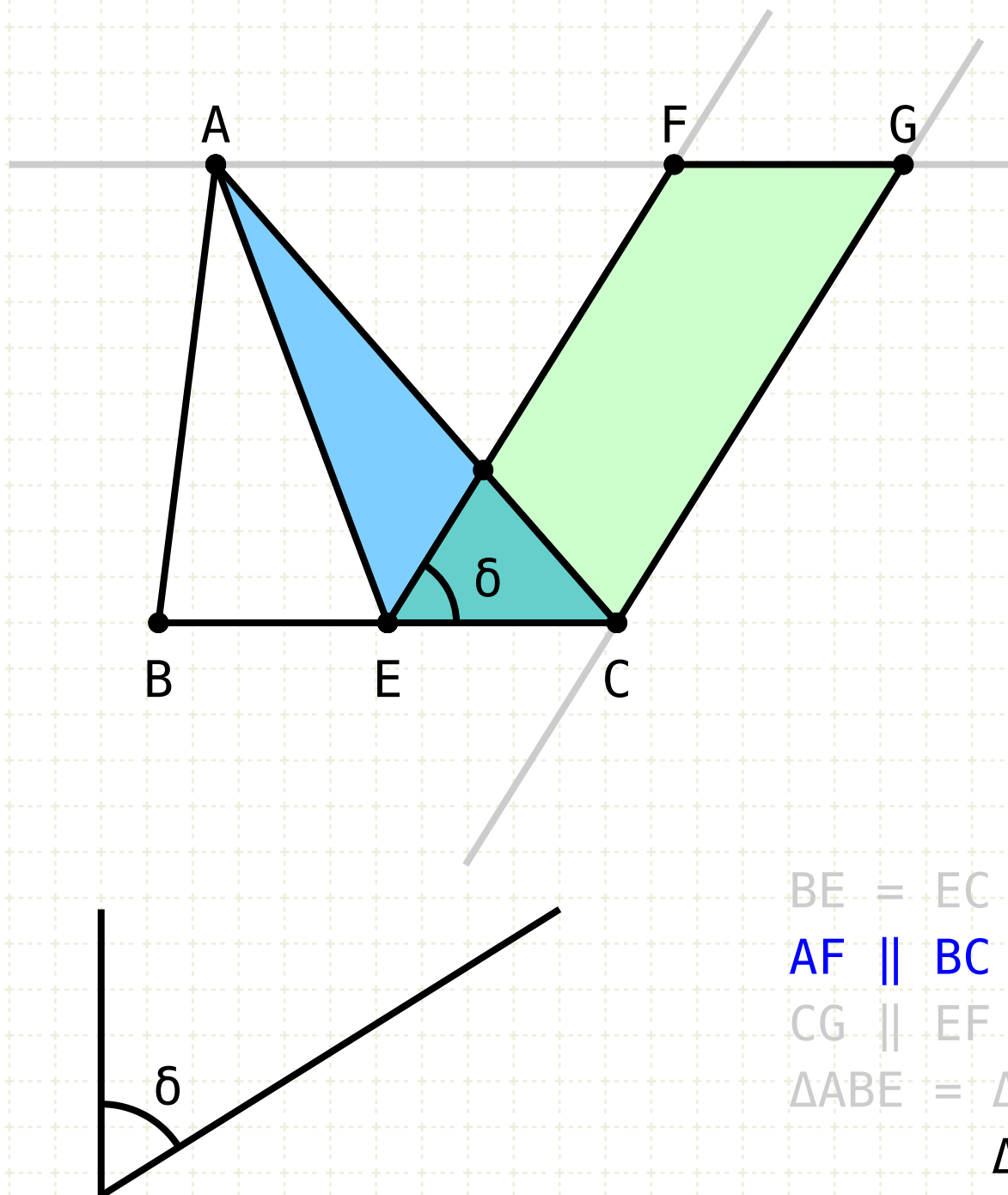
The parallelogram FECG is equal in area to the triangle ABC

Proof:

Triangle ABE and AEC have equal bases (BE and EC) and are on the same parallels, so their areas are equal (I·38)

Proposition 42 of Book I

To construct a parallelogram equal to a given triangle in a given rectilinear angle.



$$\begin{aligned} BE &= EC \\ AF &\parallel BC \\ CG &\parallel EF \\ \triangle ABE &= \triangle AEC = \frac{1}{2} \triangle ABC \\ \triangle AEC &= \frac{1}{2} \text{FECG} \end{aligned}$$

Construction

Start with triangle ABC and angle δ

Bisect line BC at point E (I·10). Draw line AC

Copy angle δ onto line EC, with the vertex at point E (I·23)

Draw a line AF, through A, parallel to BC (I·31)

Draw a line CG, through C, parallel to EF (I·31)

The parallelogram FECG is equal in area to the triangle ABC

Proof:

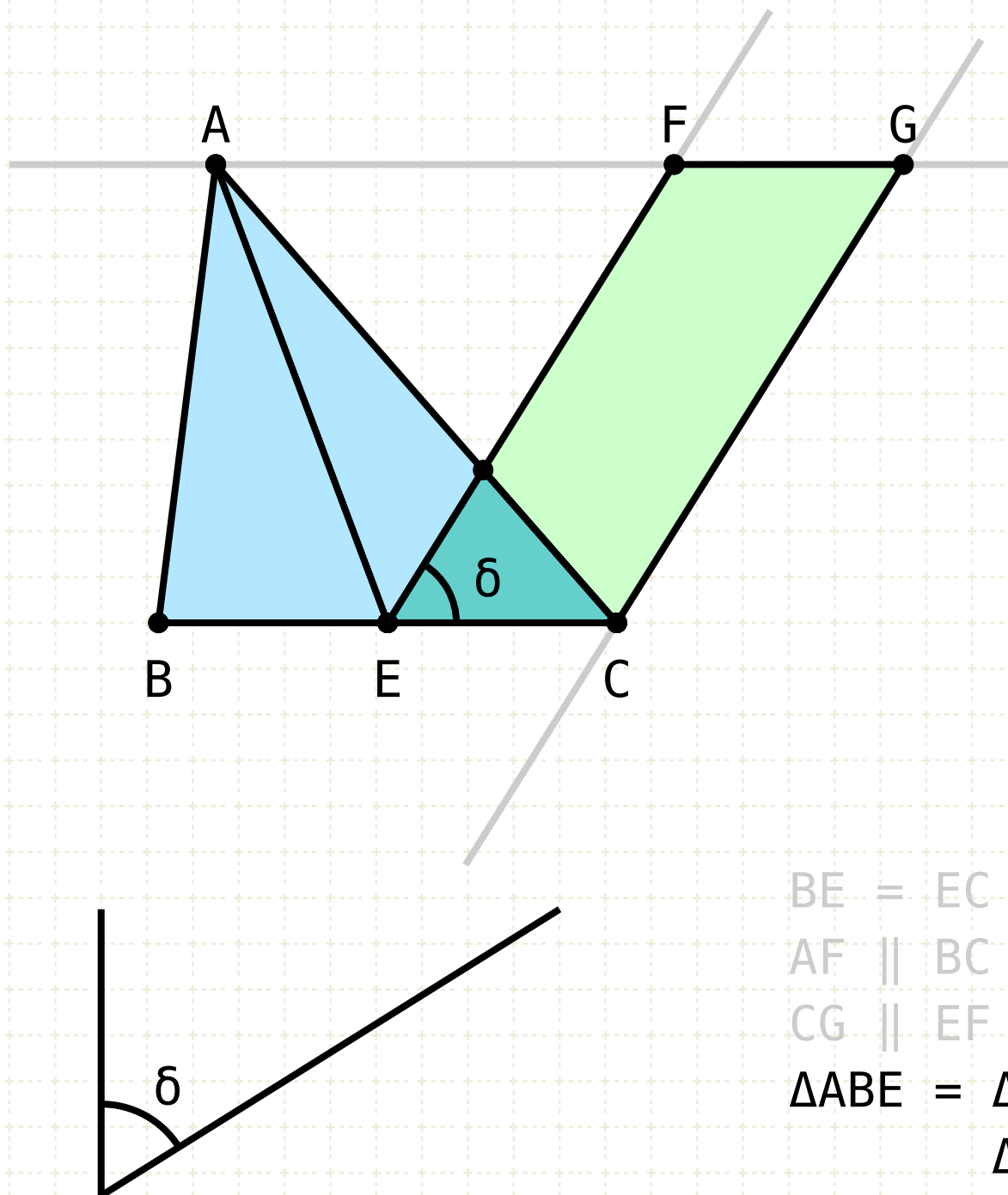
Triangle ABE and AEC have equal bases (BE and EC) and are on the same parallels, so their areas are equal (I·38)

Triangle AEC shares the same base as the parallelogram FECG, so it is half the area of the parallelogram (I·41)



Proposition 42 of Book I

To construct a parallelogram equal to a given triangle in a given rectilinear angle.



$$\begin{aligned}
 BE &= EC \\
 AF &\parallel BC \\
 CG &\parallel EF \\
 \Delta ABE &= \Delta AEC = \frac{1}{2} \Delta ABC \\
 \Delta AEC &= \frac{1}{2} \text{FECG} \\
 \text{FECG} &= \Delta ABC
 \end{aligned}$$

Construction

Start with triangle ABC and angle δ

Bisect line BC at point E (I·10). Draw line AC

Copy angle δ onto line EC, with the vertex at point E (I·23)

Draw a line AF, through A, parallel to BC (I·31)

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The parallelogram FECG is equal in area to the triangle ABC

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Triangle ABE and AEC have equal bases (BE and EC) and are on the same parallels, so their areas are equal (I·38)

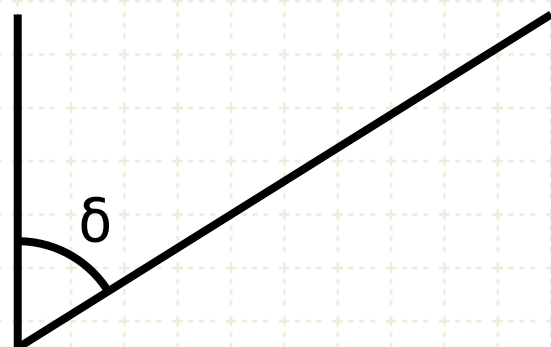
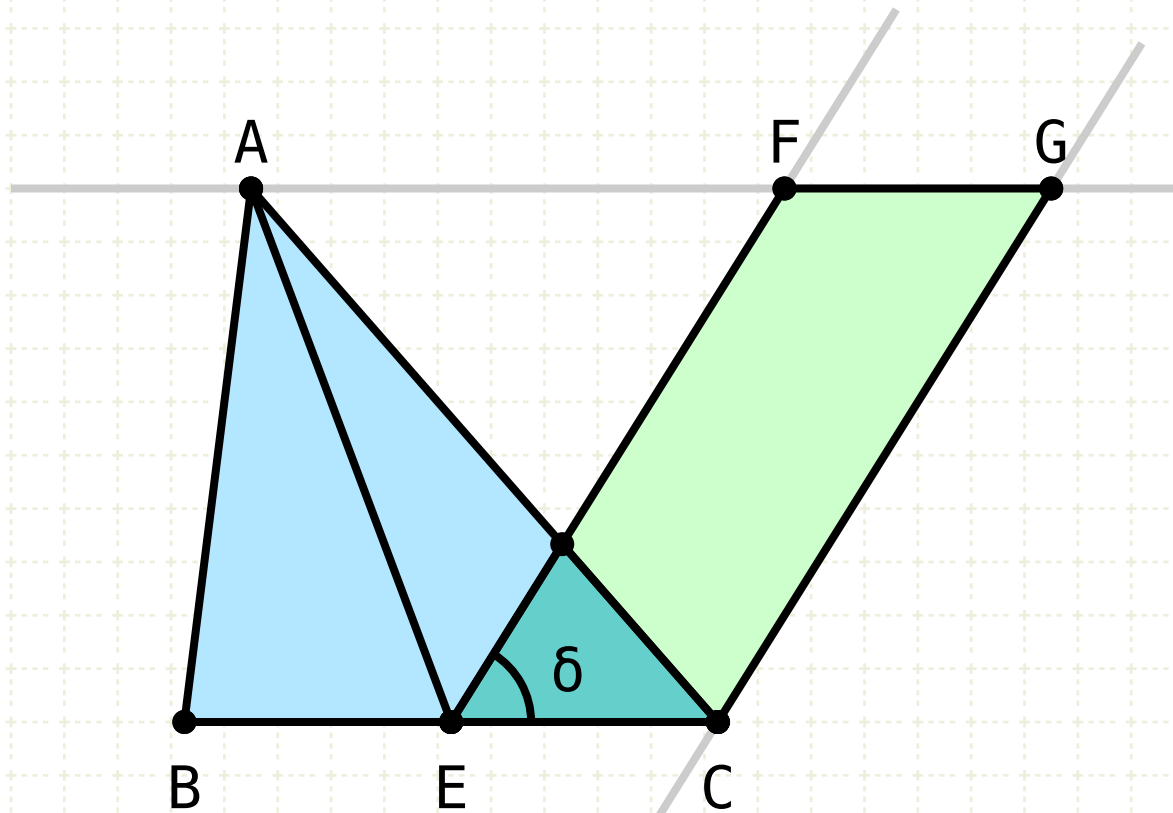
Triangle AEC shares the same base as the parallelogram FECG, so it is half the area of the parallelogram (I·41)

Hence, FECG is equal in area to ABC



Proposition 42 of Book I

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$$BE = EC$$

$$AF \parallel BC$$

$$CG \parallel EF$$

$$\triangle ABE = \triangle AEC = \frac{1}{2} \triangle ABC$$

$$\triangle AEC = \frac{1}{2} \text{FECG}$$

$$\text{FECG} = \triangle ABC$$

Construction

Start with triangle ABC and angle δ

Bisect line BC at point E (I·10). Draw line AC

Copy angle δ onto line EC, with the vertex at point E (I·23)

Draw a line AF, through A, parallel to BC (I·31)

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The parallelogram FECG is equal in area to the triangle ABC

Proof:

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Triangle AEC shares the same base as the parallelogram FECG, so it is half the area of the parallelogram (I·41)

Hence, FECG is equal in area to ABC



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