

Euclid's Elements

Book III



A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)



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3	A line through the centre of a circle bisects a chord, and vice versa	11	Point of contact between two internal circles, and their centres, are collinear	19	If line touches a circle, then the centre of the circle lies on a line perpendicular to the original
4	A line not through the centre of a circle does not bisect a chord	12	Point of contact between two external circles, and their centres, are collinear	20	The angle at the centre of a circle is twice that from an angle from the circumference
5	If two circles cut one another, they will not have the same center	13	A circle does not touch a circle at more points than one, whether it touch it internally or externally.	21	In a circle the angles in the same segment are equal to one another
6	If two circles touch one another, they will not have the same center	14	In a circle equal straight lines are equally distant from the centre, and those which are equally distant from the centre are equal to one another.	22	The opposite angles of quadrilaterals in circles are equal to two right angles
7	Consider two lines from a point inside a circle to the edge, the longer one will be the one closest to the longest part of the diameter passing through the original point	15	The longest line in a circle is its diameter, shorter the farther away from the diameter	23	On the same straight line there cannot be constructed two similar and unequal segments of circles on the same side
8	Consider two lines from a point outside a circle to the edge, the line closest to the centre will be longer on the concave side and shorter on the convex side	16	A line on the circle, perpendicular to the diameter, lies outside the circle	24	Similar segments of circles on equal straight lines are equal to one another



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| 27 | In equal circles angles standing on equal circumferences are equal to one another | | |
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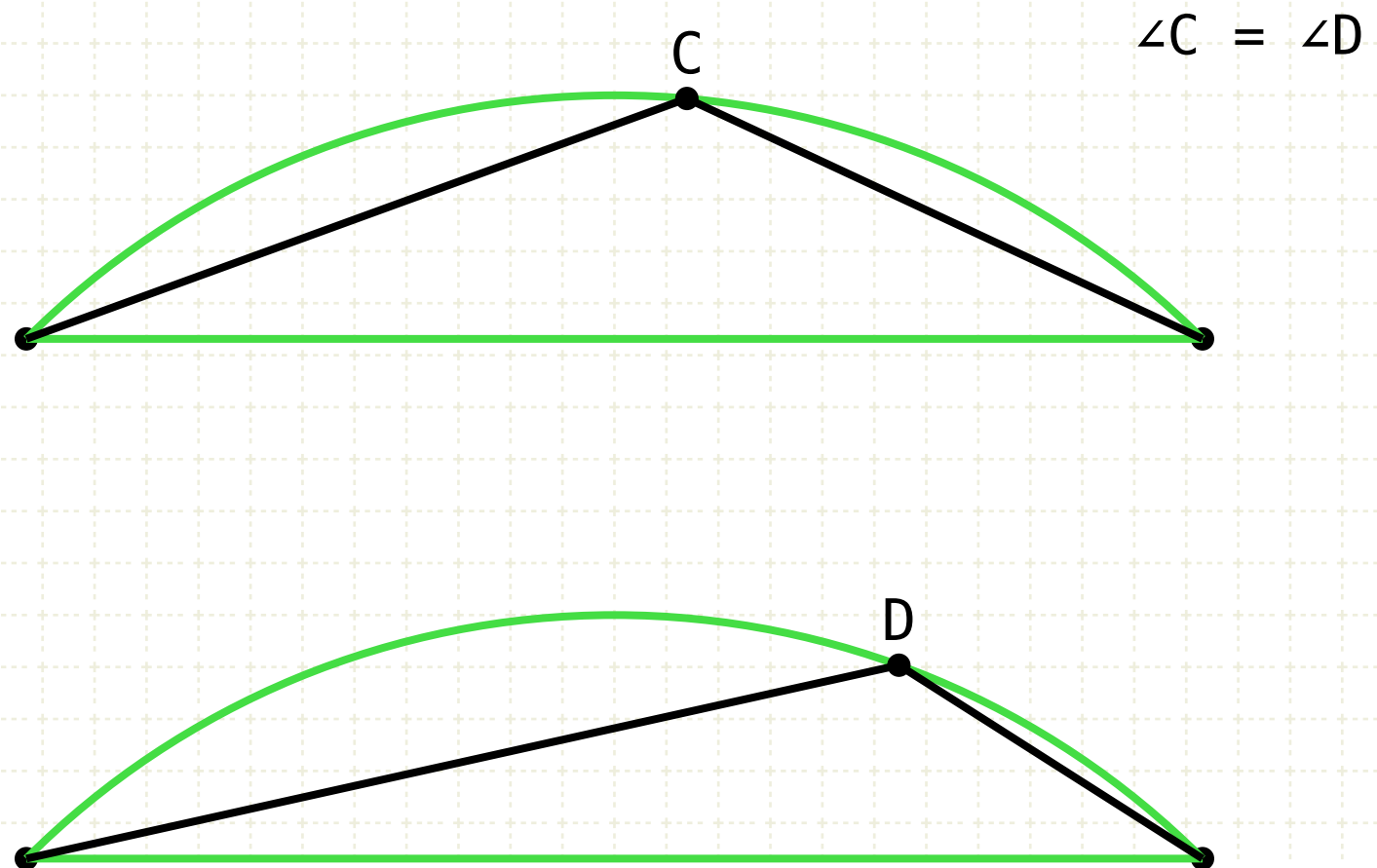
Proposition 23 of Book III

On the same straight line there cannot be constructed two similar and unequal segments of circles on the same side



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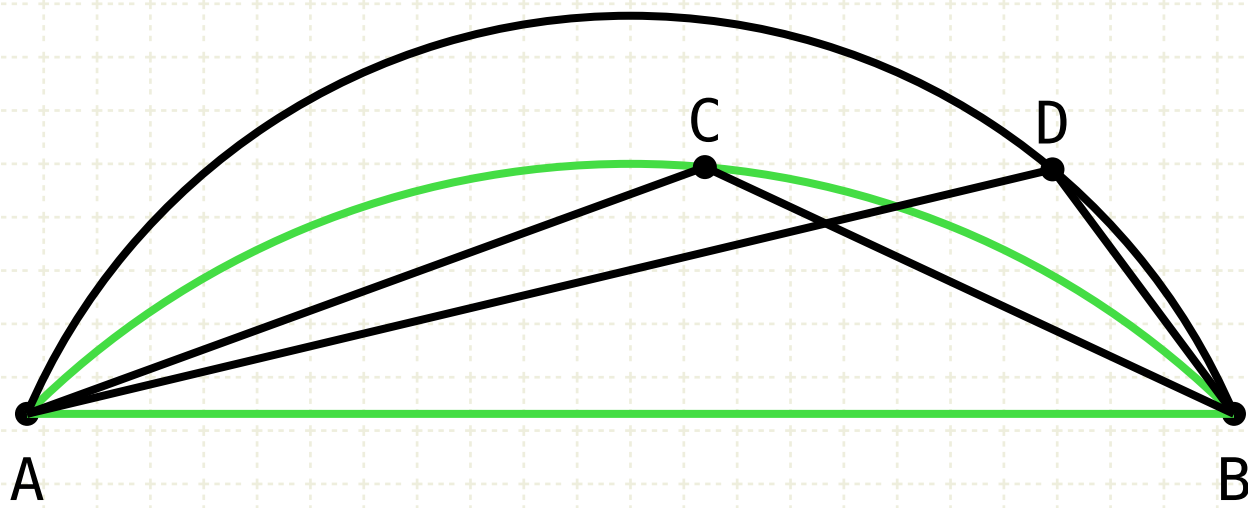


Definition - Similar segments of circles

'Similar segments of circles' are those which admit equal angles, or in which the angles are equal to one another

Proposition 23 of Book III

On the same straight line there cannot be constructed two similar and unequal segments of circles on the same side



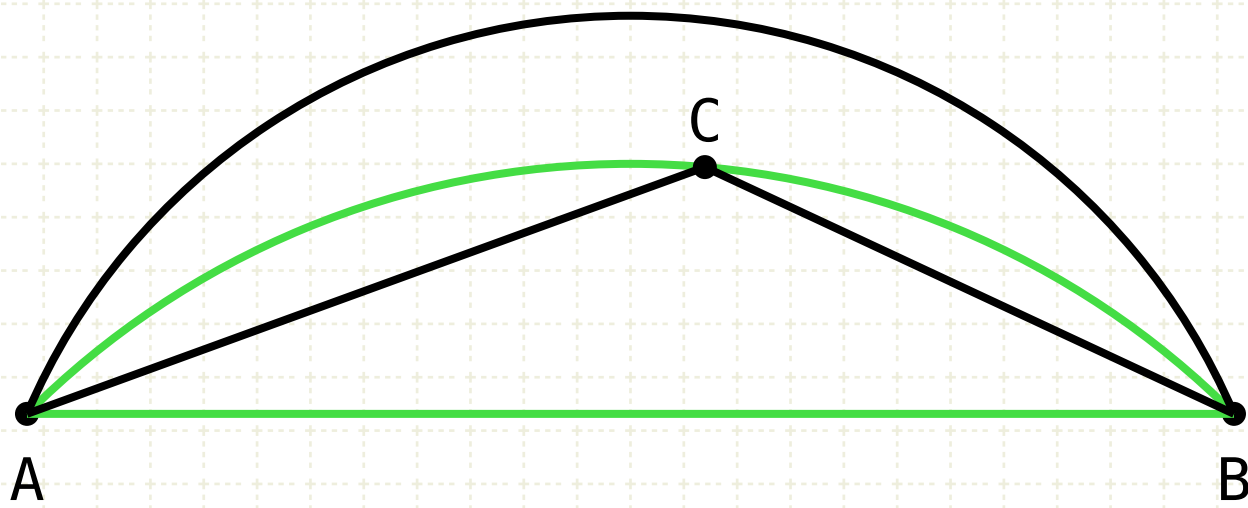
In other words

It is impossible to construct a figure (as shown) where

- the two segments of the circle are not equal, and
- any angle C on the inner circle is equal to any angle D on the outer circle

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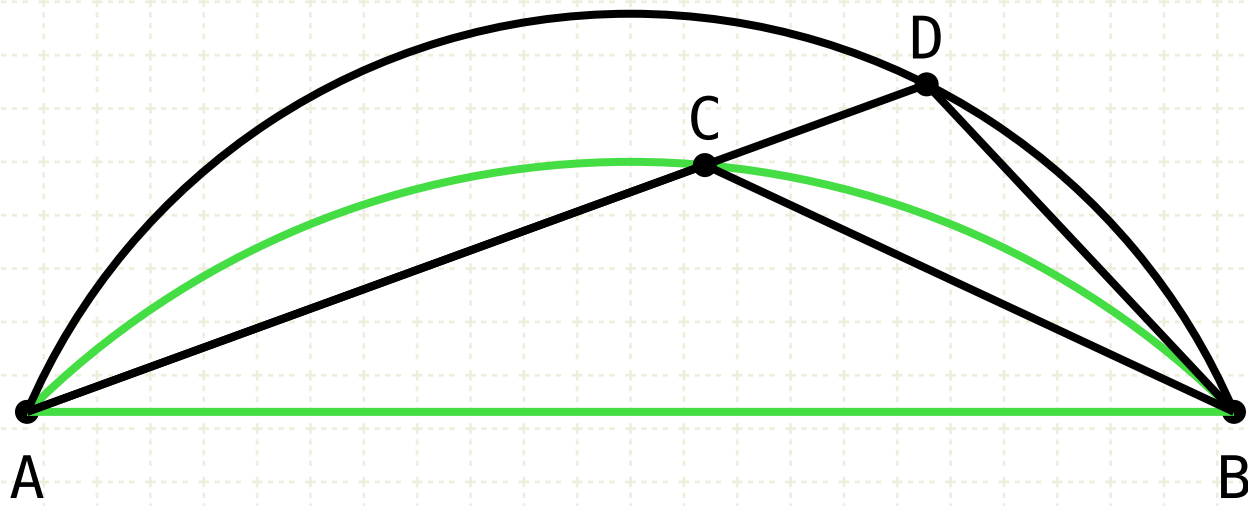
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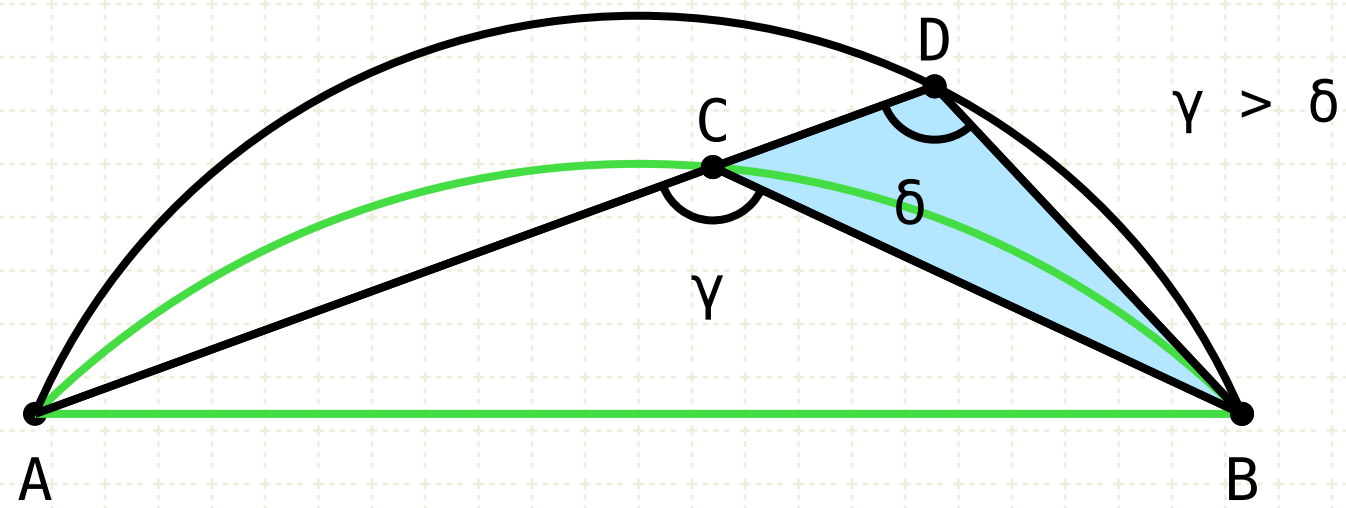
- the two segments of the circle are not equal, and
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Proof

Extend line AC such that it intersects the larger circle at point D, and draw a line from D to B

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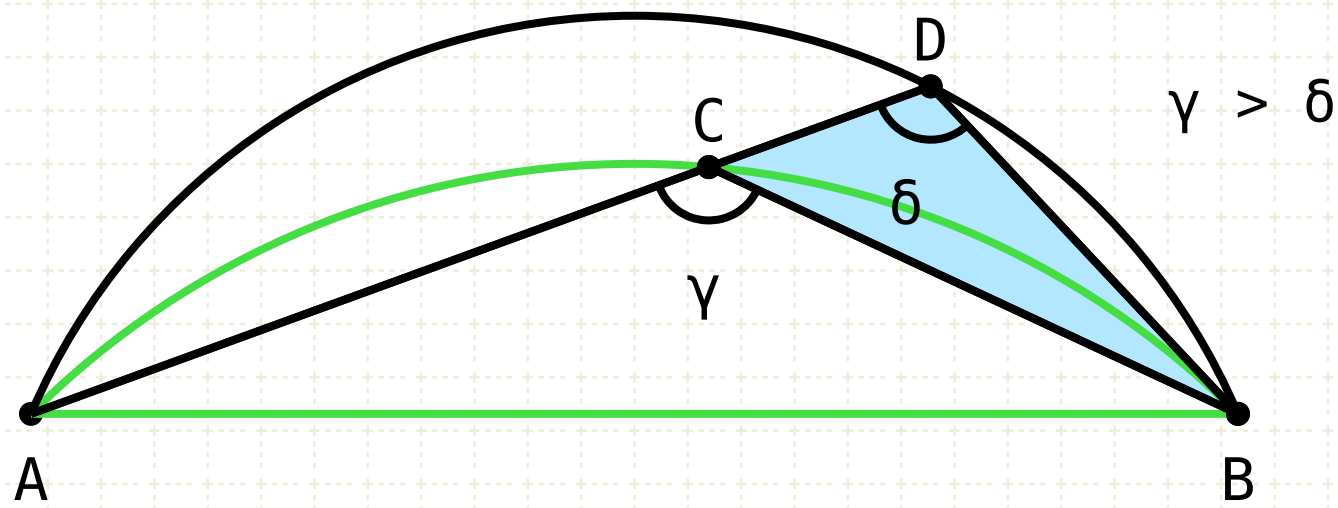
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Consider the angles ACB (γ) and CDB (δ)

The angle γ is exterior to the triangle CDB, and thus is larger than any interior/opposite angle (I-16)

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The angle γ is exterior to the triangle CDB, and thus is larger than any interior/opposite angle (I·16)

THUS, the two circle segments cannot be similar, because γ cannot be both equal and greater than δ

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