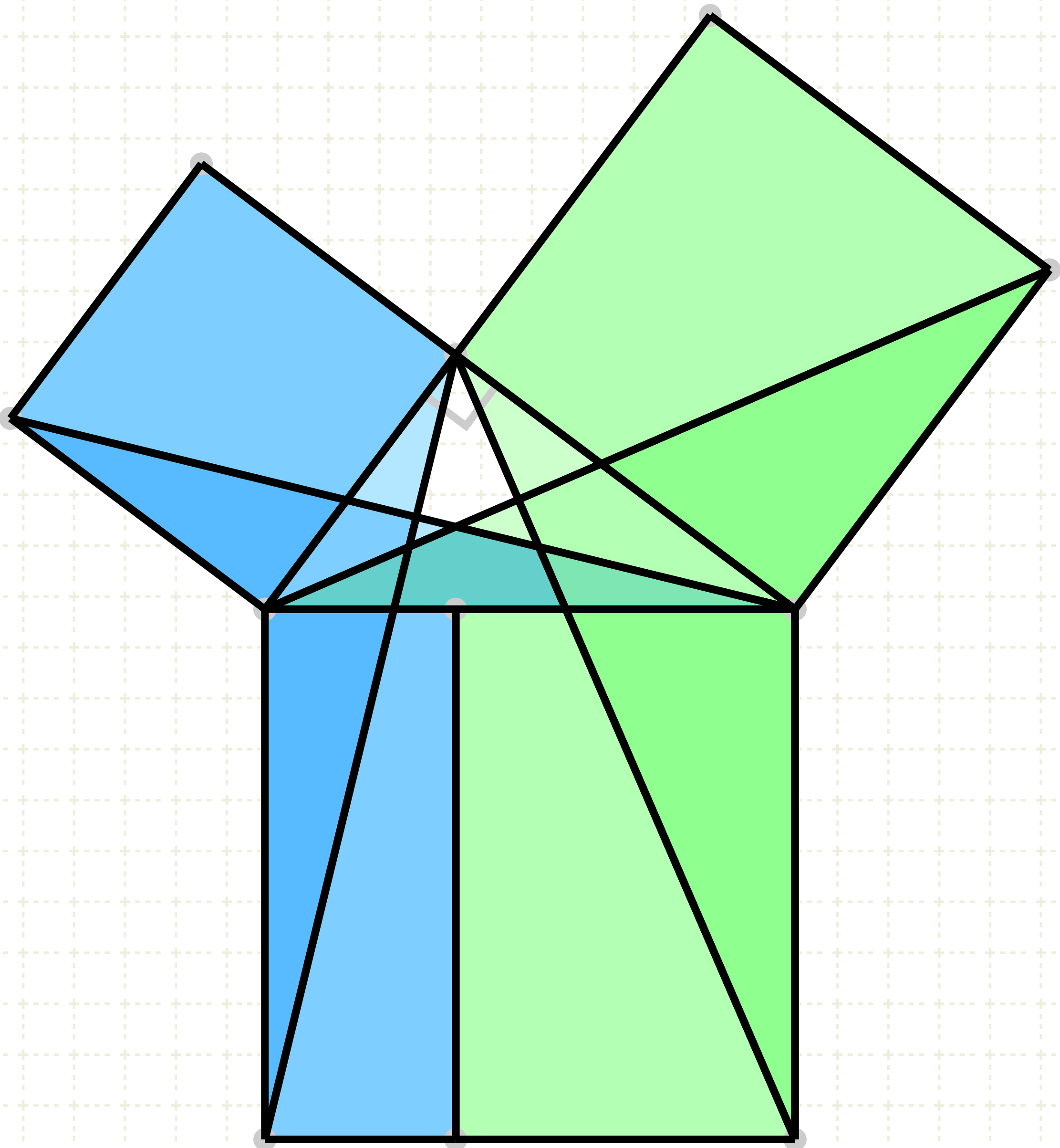
Euclid's Elements

Book I



If Euclid did not kindle your youthful enthusiasm, you were not born to be a scientific thinker.

Albert Einstein

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Proposition 6 of Book I

If two angles of a triangle are equal, then the sides opposite them will be equal.

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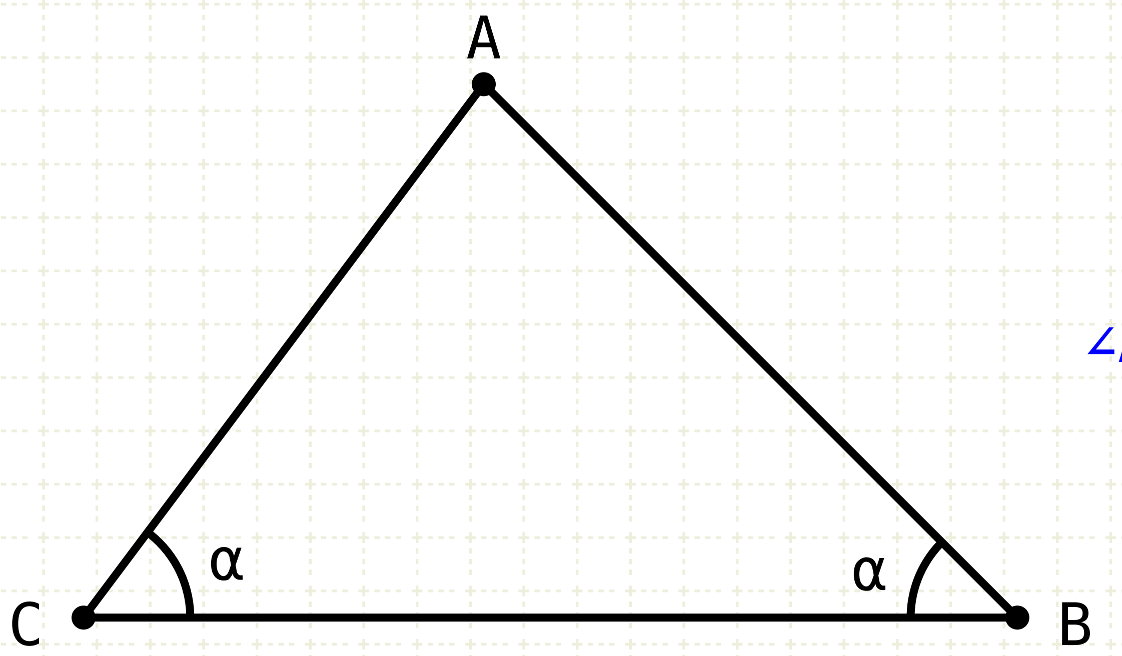
If two angles of a triangle are equal, then the sides opposite them will be equal.

In other words

Start with a triangle with equal base angles



$\angle ACB = \angle ABC$

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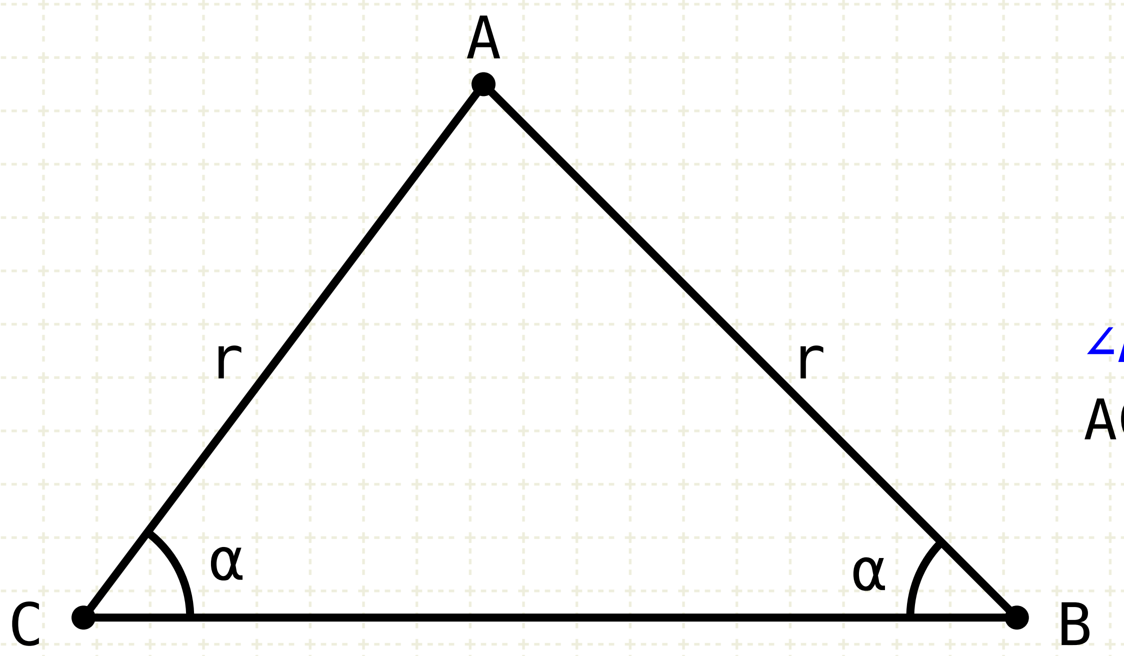

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$AC = AB$

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Then the sides opposite the equal angles are equal

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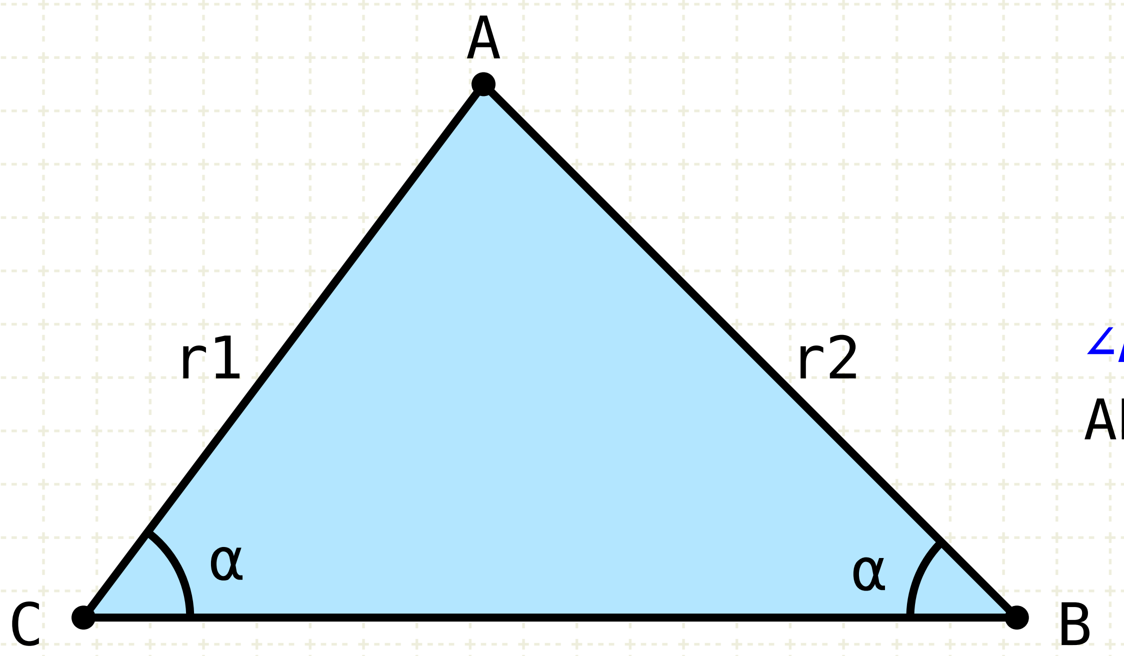

$\angle ACB = \angle ABC$

$AB > AC$, $(r2 > r1)$

In other words

Start with a triangle with equal base angles

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Proof by contradiction

Assume that the sides are not equal, and demonstrate that this leads to a logical inconsistency

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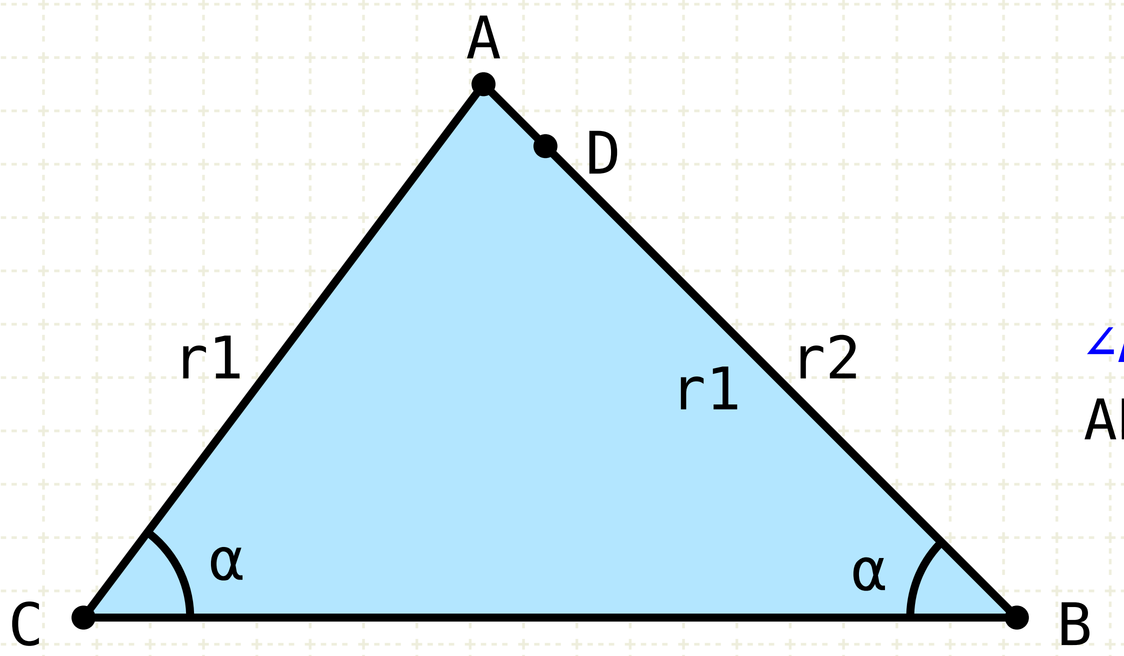

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AB > AC, (r2 > r1)

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Use the method from Propositions 2 and 3 to find a point D such that BD equals AC

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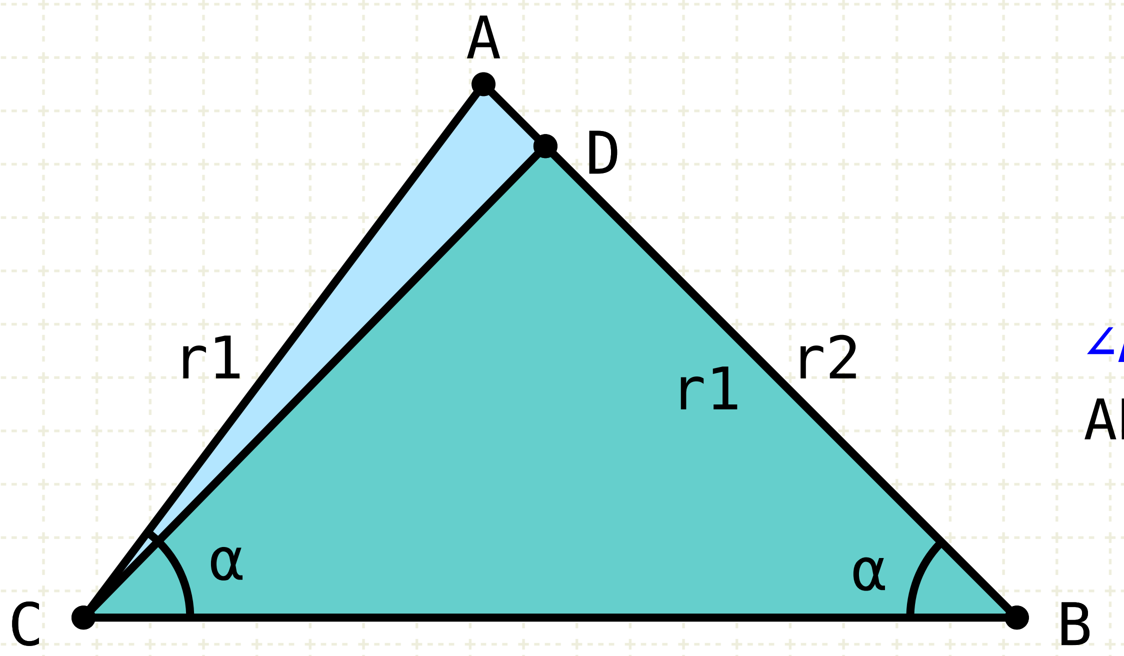

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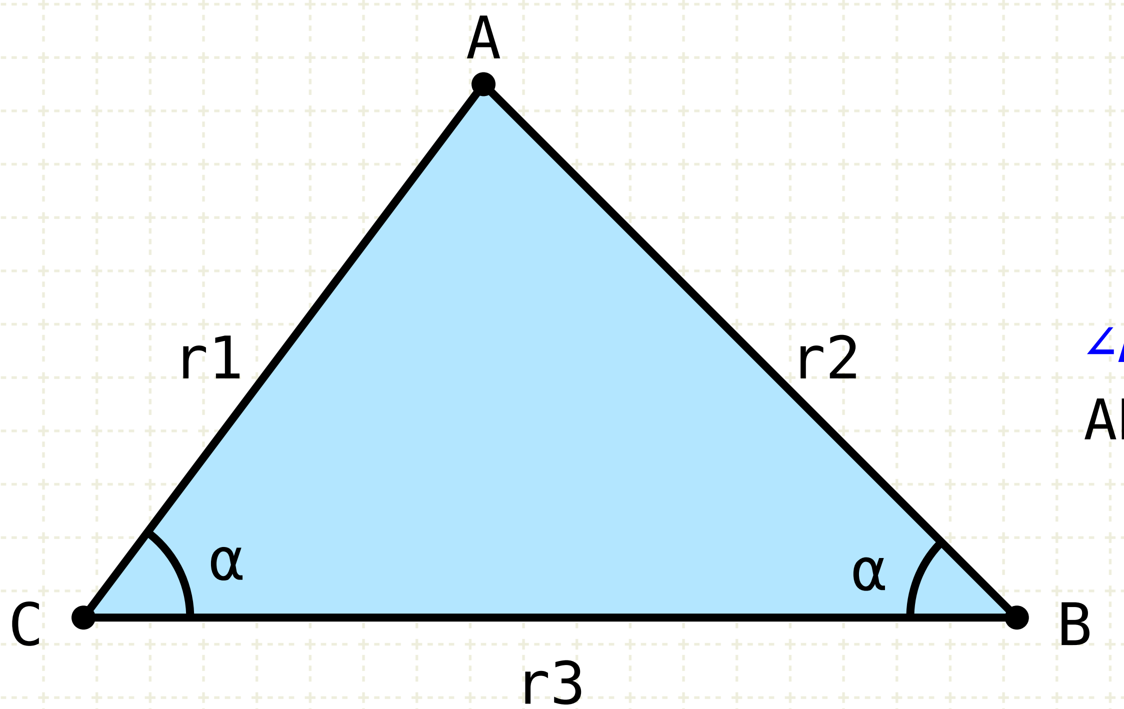

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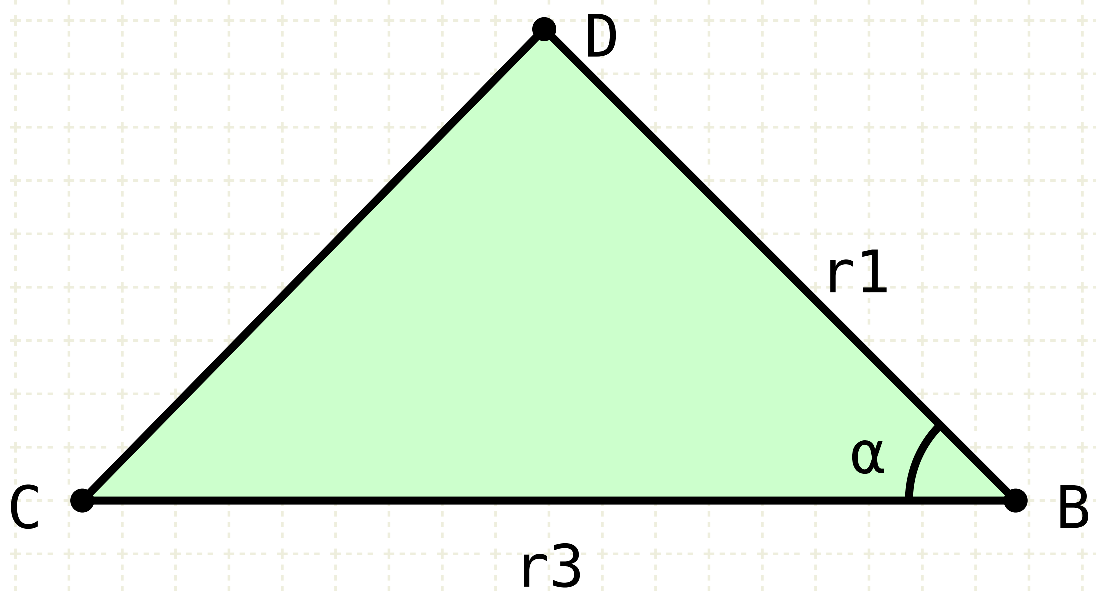

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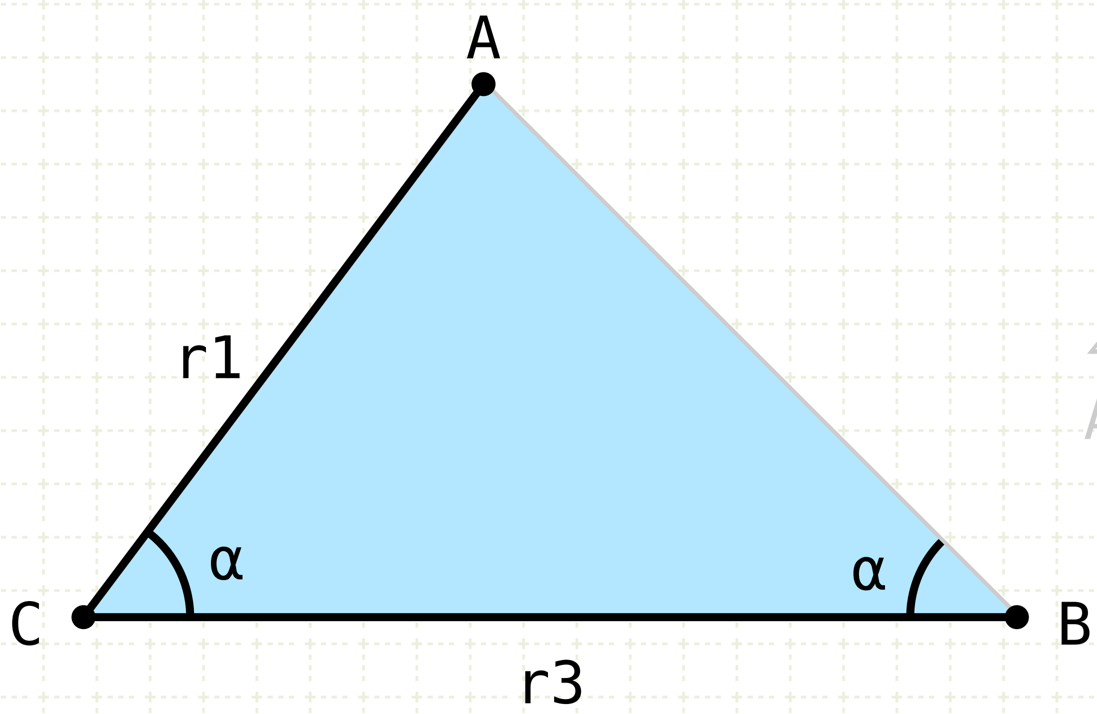


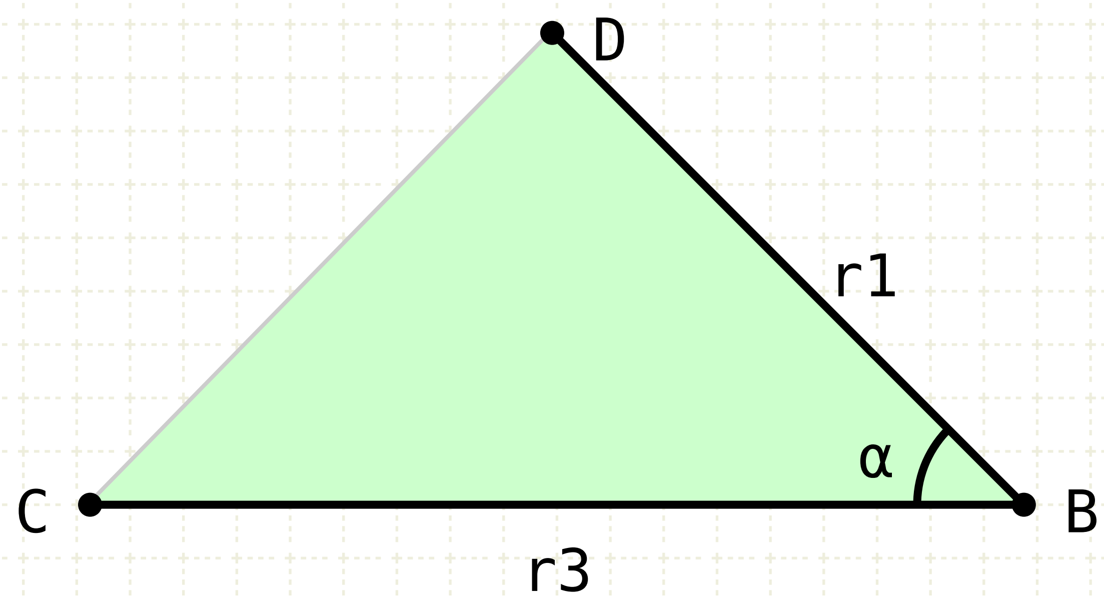
∠ACB = ∠ABC

AB > AC, (r2 > r1)

BD = AC = r1

BD=r1 ∠DBC=α BC=r3

AC=r1 ∠ACB=α BC=r3



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Let's move DCB to a different spot so we can see more clearly

Since two sides and the angle between are the same for both triangles, then all the sides and angles are equal (I·4)

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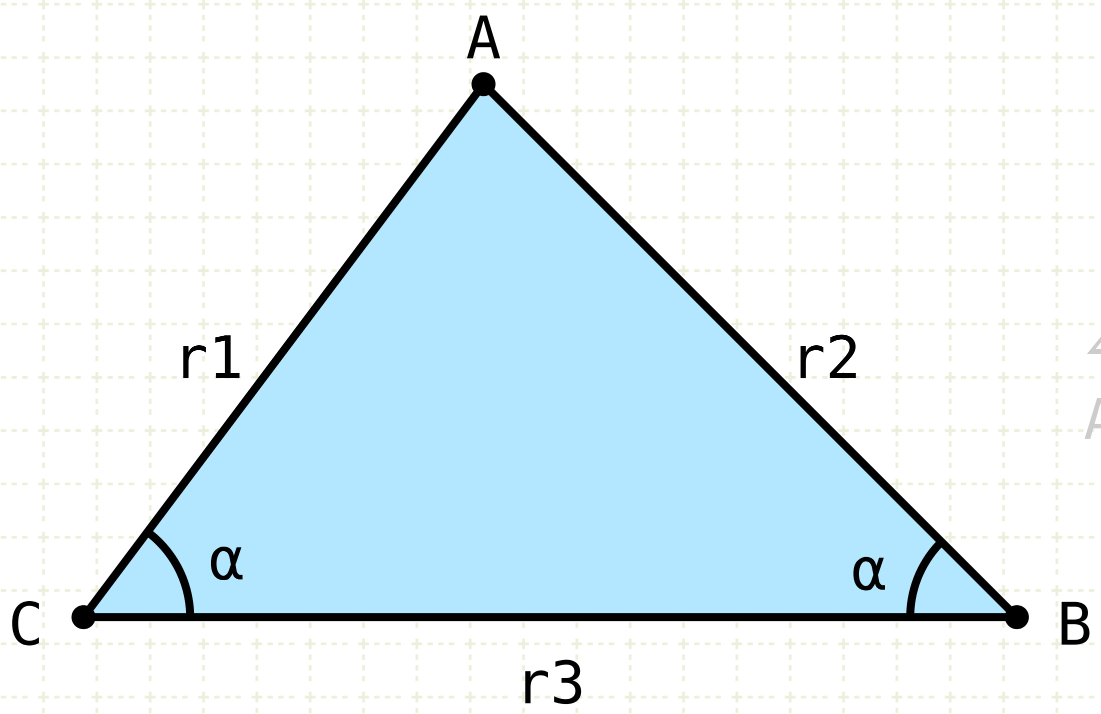


$\angle ACB = \angle ABC$

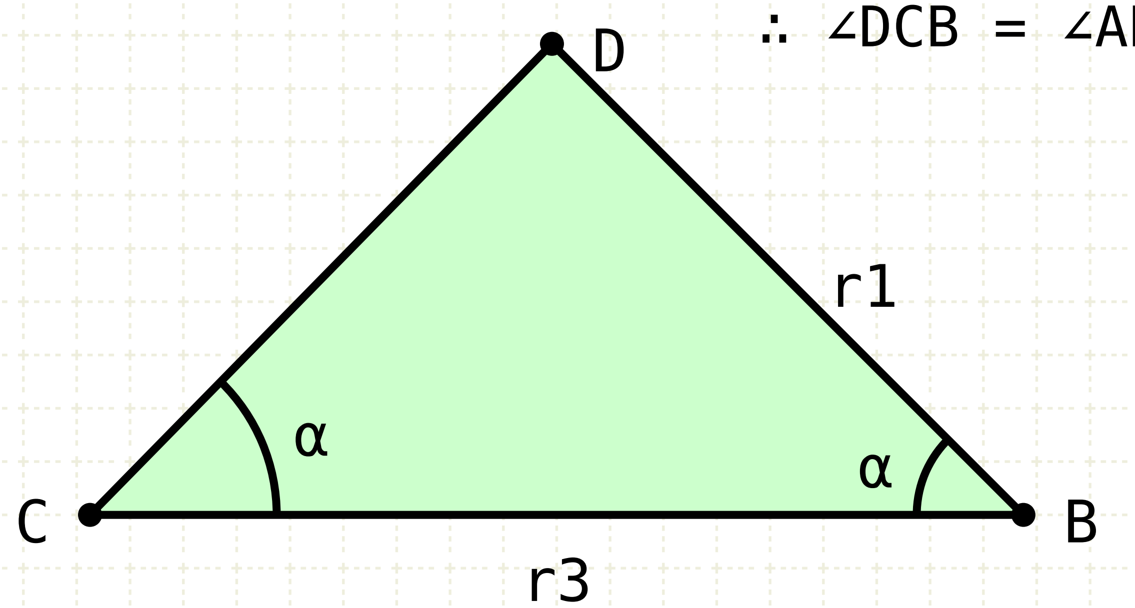
$AB > AC, \ (r2 > r1)$

$BD = AC = r1$

$BD=r1 \ \angle DBC=\alpha \ BC=r3$

$AC=r1 \ \angle ACB=\alpha \ BC=r3$

$\therefore \angle DCB = \angle ABC = \alpha$



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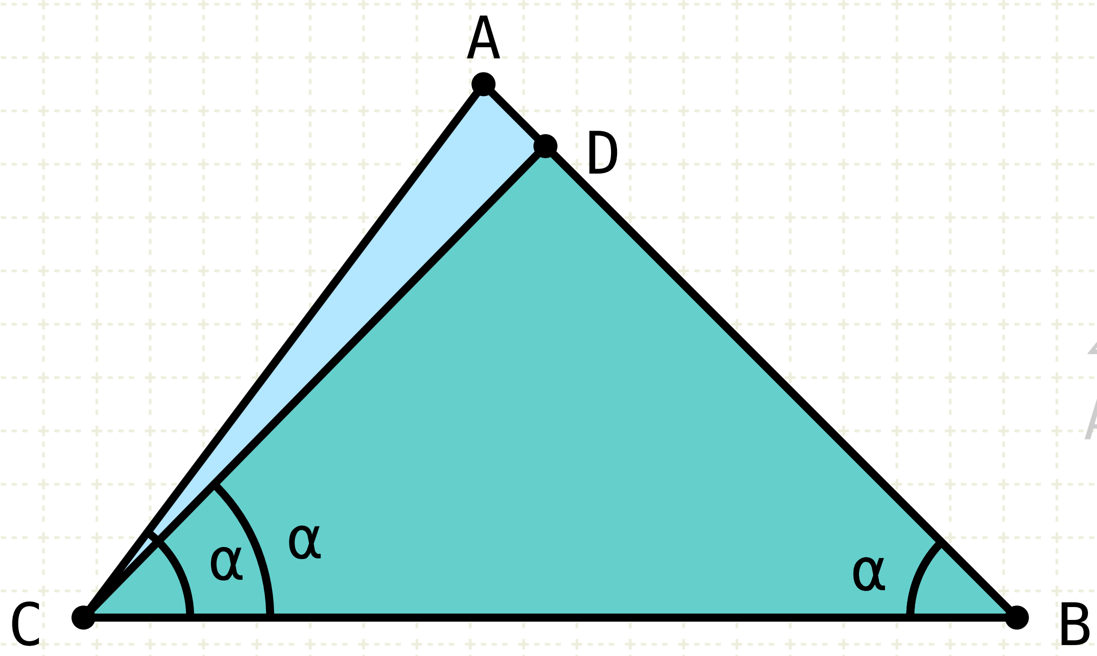

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$BD = AC = r1$

$BD=r1\ \angle DBC=\alpha\ BC=r3$

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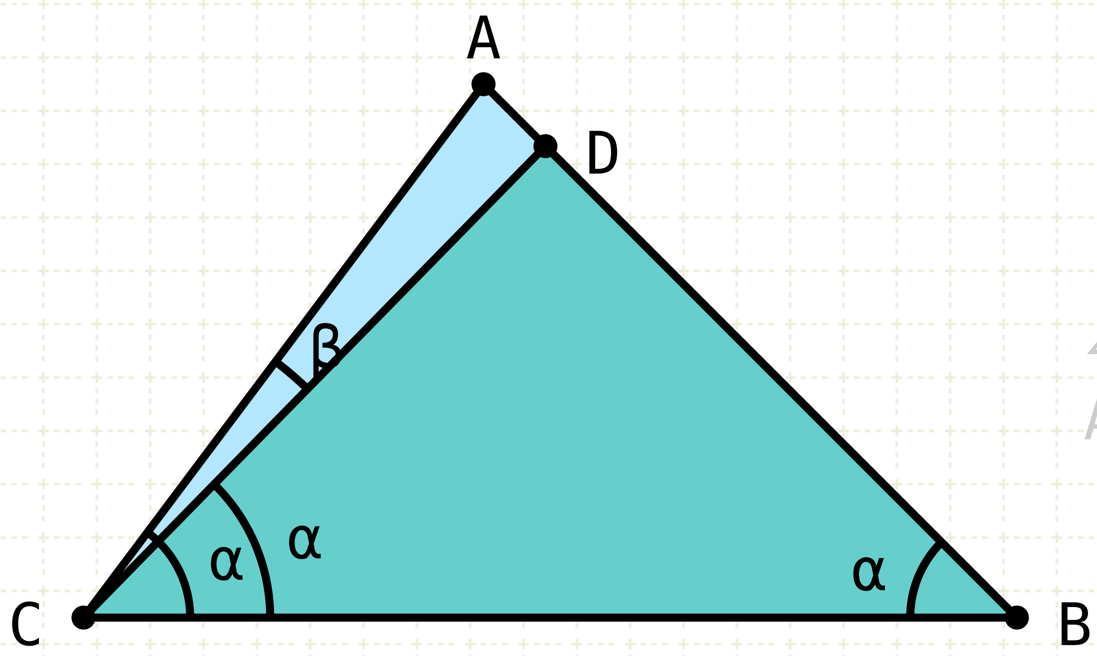

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let $\angle ACD = \beta$

$\Rightarrow \beta + \alpha = \alpha$

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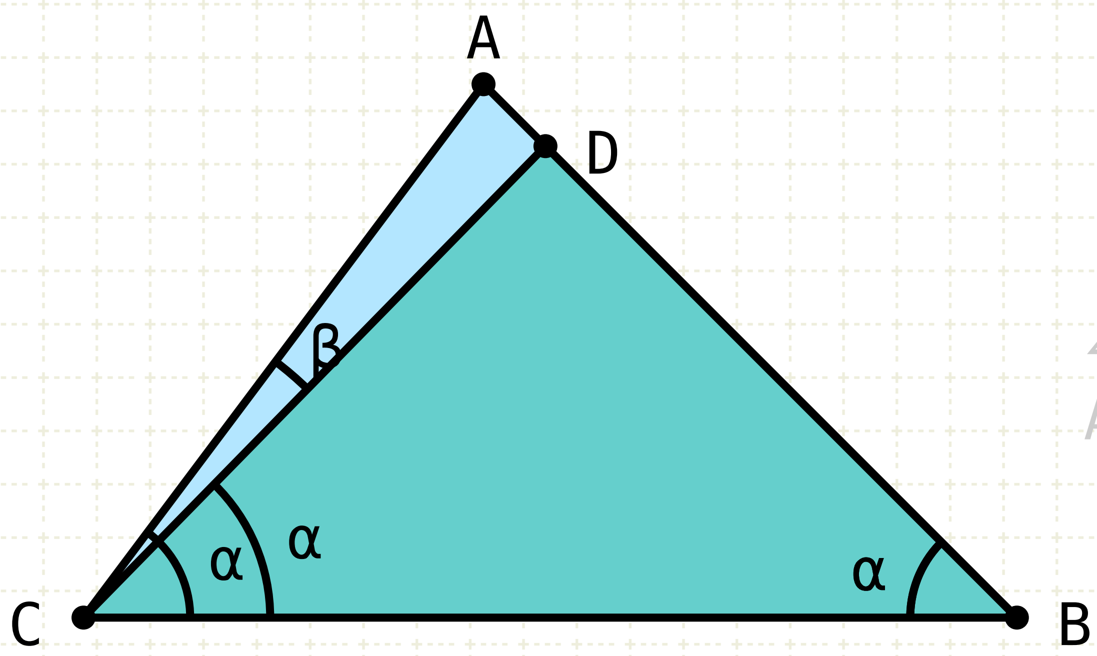

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AB > AC, (r2 > r1)
BD = AC = r1

BD=r1 ∠DBC=α BC=r3
AC=r1 ∠ACB=α BC=r3
∴ ∠DCB = ∠ABC = α
let ∠ACD = β
=> β + α = α ✗

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Let's move DCB to a different spot so we can see more clearly

Since two sides and the angle between are the same for both triangles, then all the sides and angles are equal (I·4)

We now have an angle α which is equal to α plus β

This leaves us with a violation of the common notion 5 that the whole is greater than the part

… unless β is zero!

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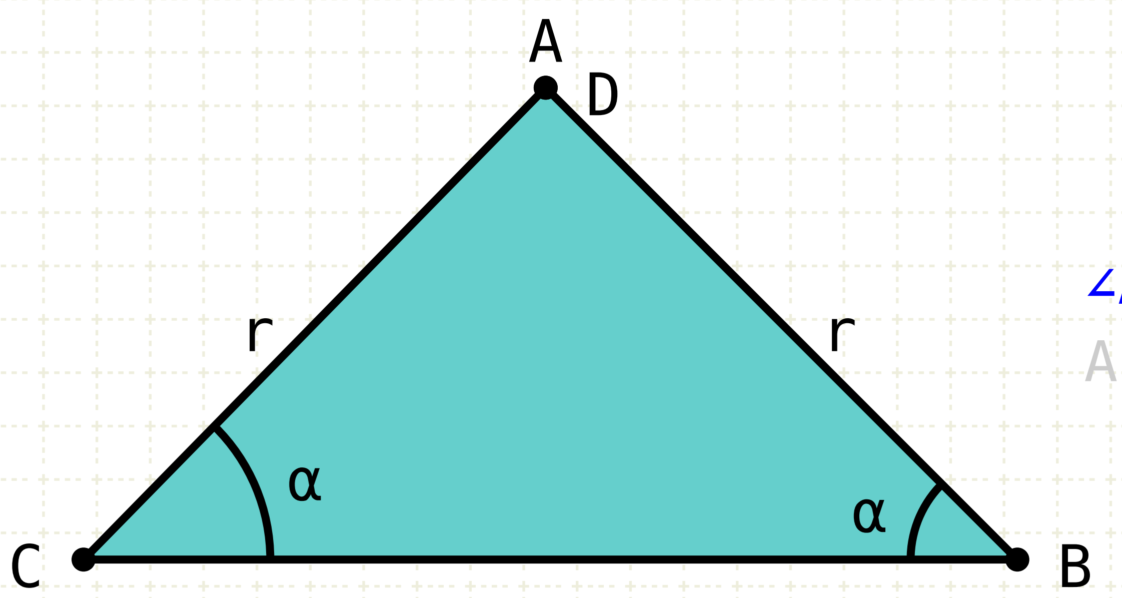

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$\therefore\ \angle DCB = \angle ABC = \alpha$

$\text{let } \angle ACD = \beta$

$\Rightarrow \beta + \alpha = \alpha \quad ✗$

$A = D$

$AC = DB = r$

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… unless β is zero!

This implies that D is concurrent with A, and that the two sides of the triangle are equal

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