

Euclid's Elements

Book III



A circle is a round straight line with a hole in the middle.

Mark Twain

quoting a schoolchild in "-English as She Is Taught-"

If people stand in a circle long enough, they'll eventually begin to dance.

George Carlin, Napalm and Silly Putty (2001)



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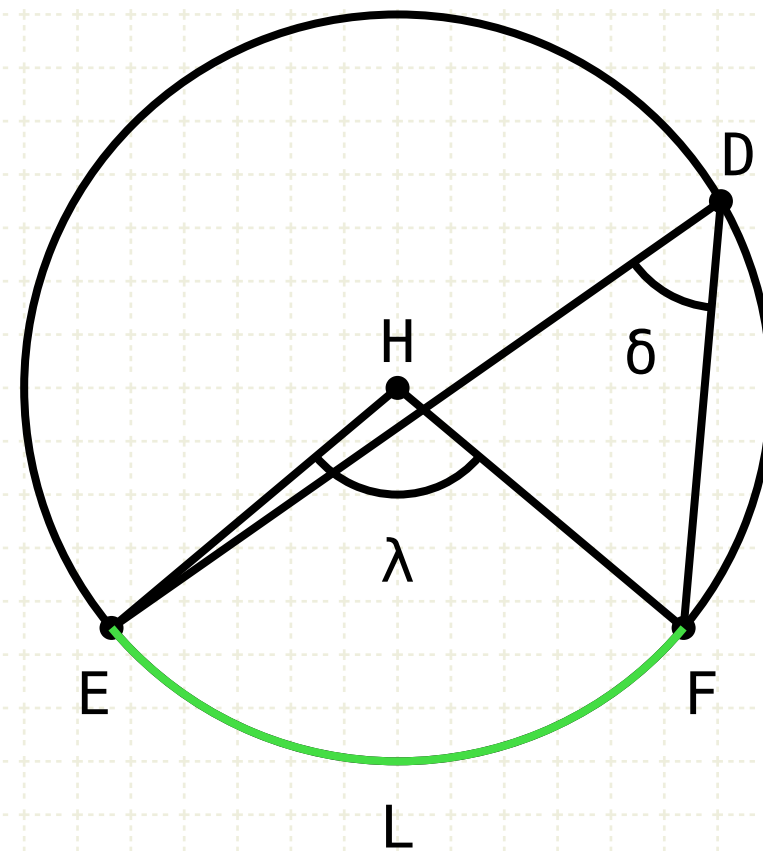
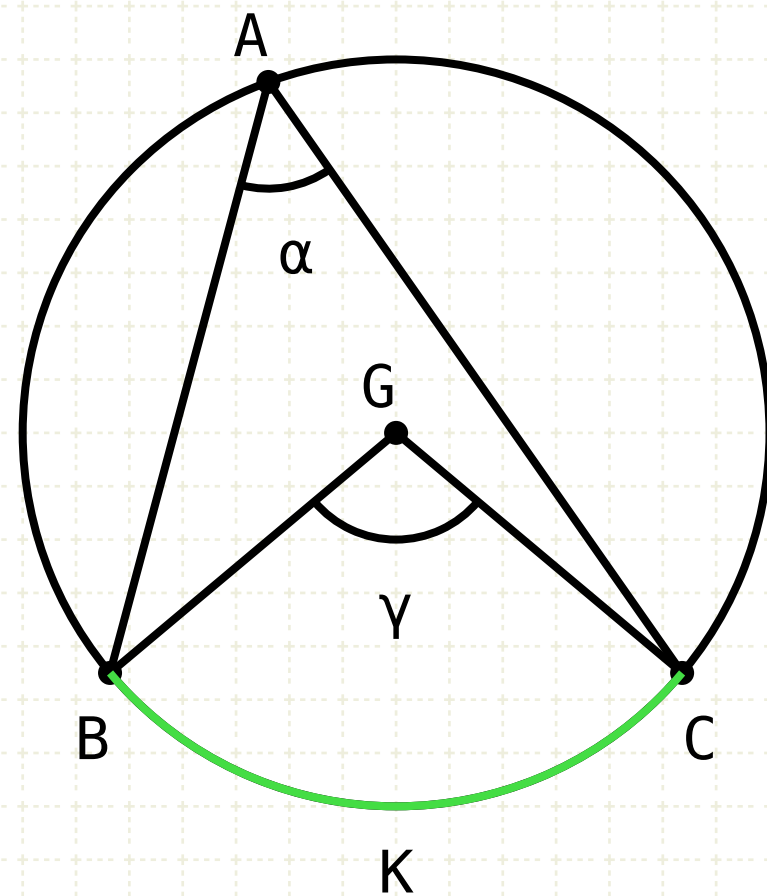
Proposition 26 of Book III

In equal circles equal angles stand on equal circumferences, whether they stand at the centres or at the circumferences.



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$$\sphericalangle ABC = \sphericalangle EDF$$

$$\alpha = \delta$$

$$\gamma = \lambda$$

$$\text{arc } K = \text{arc } L$$

In other words

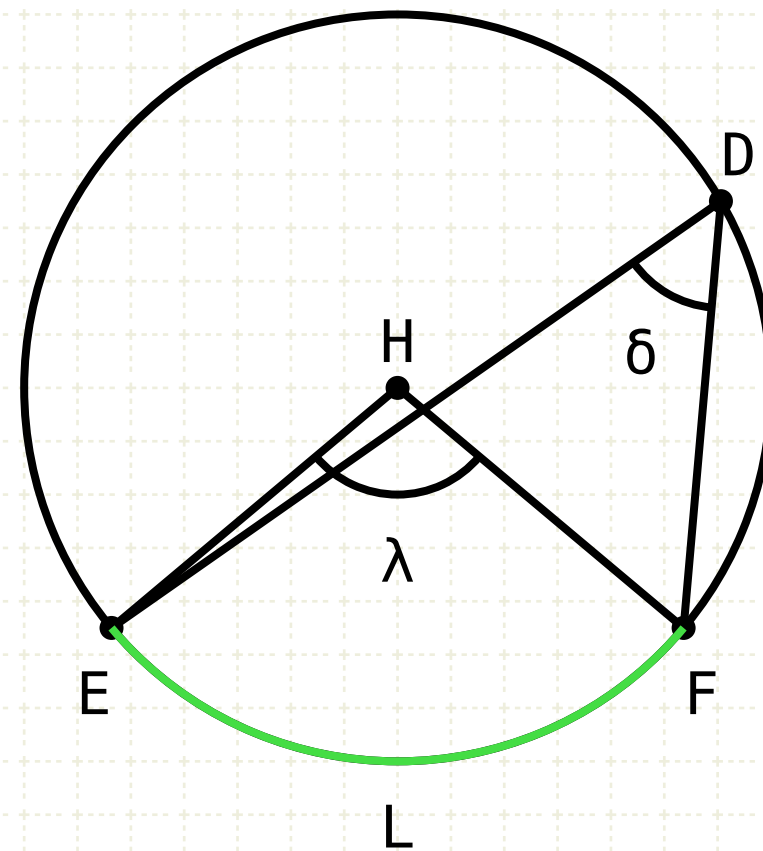
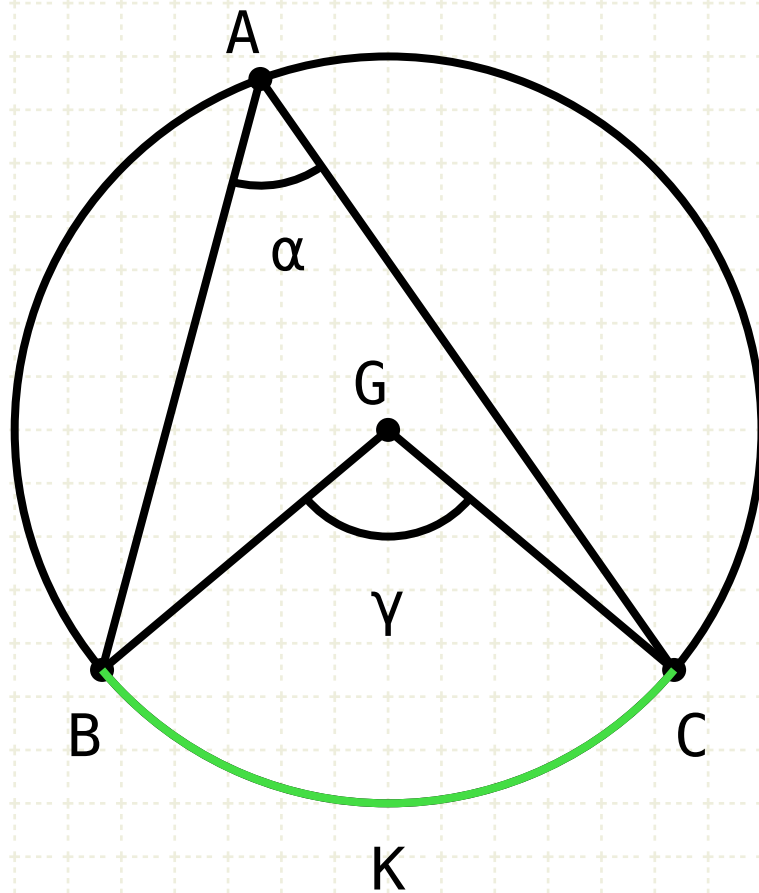
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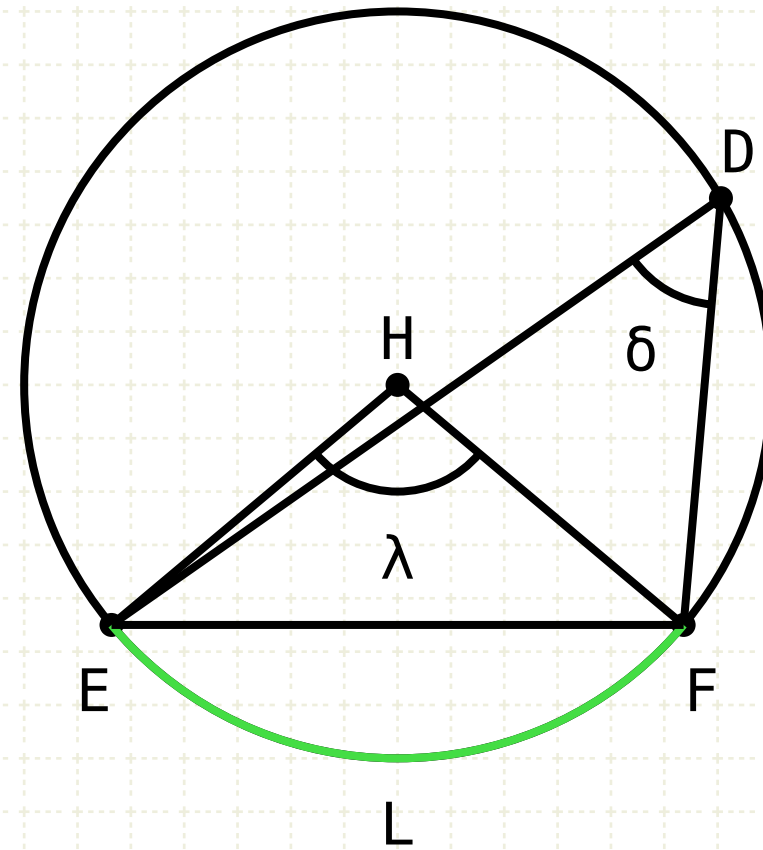
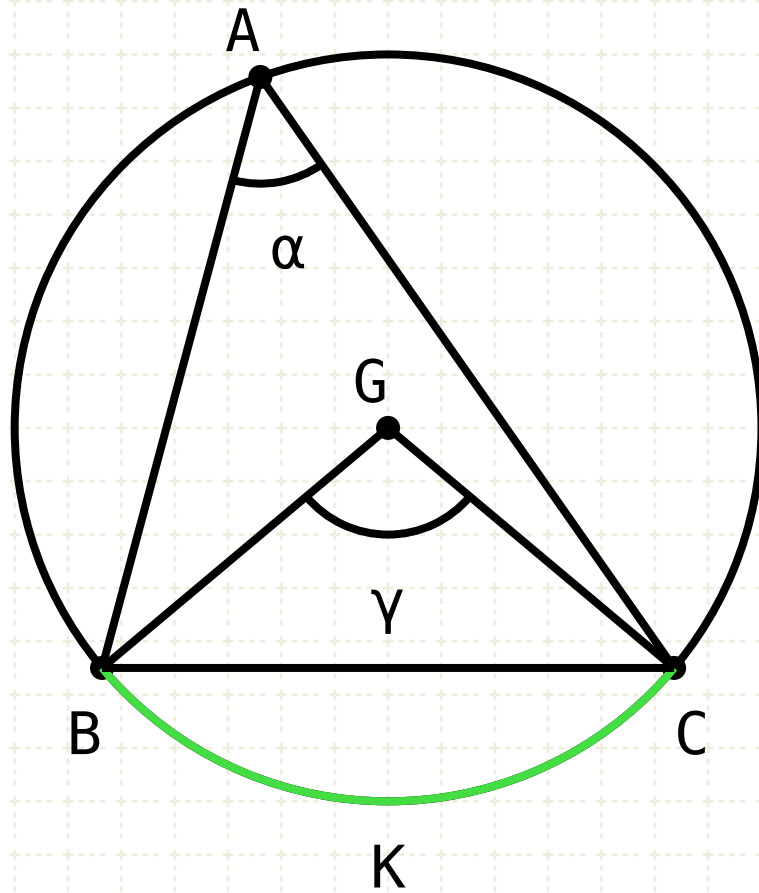
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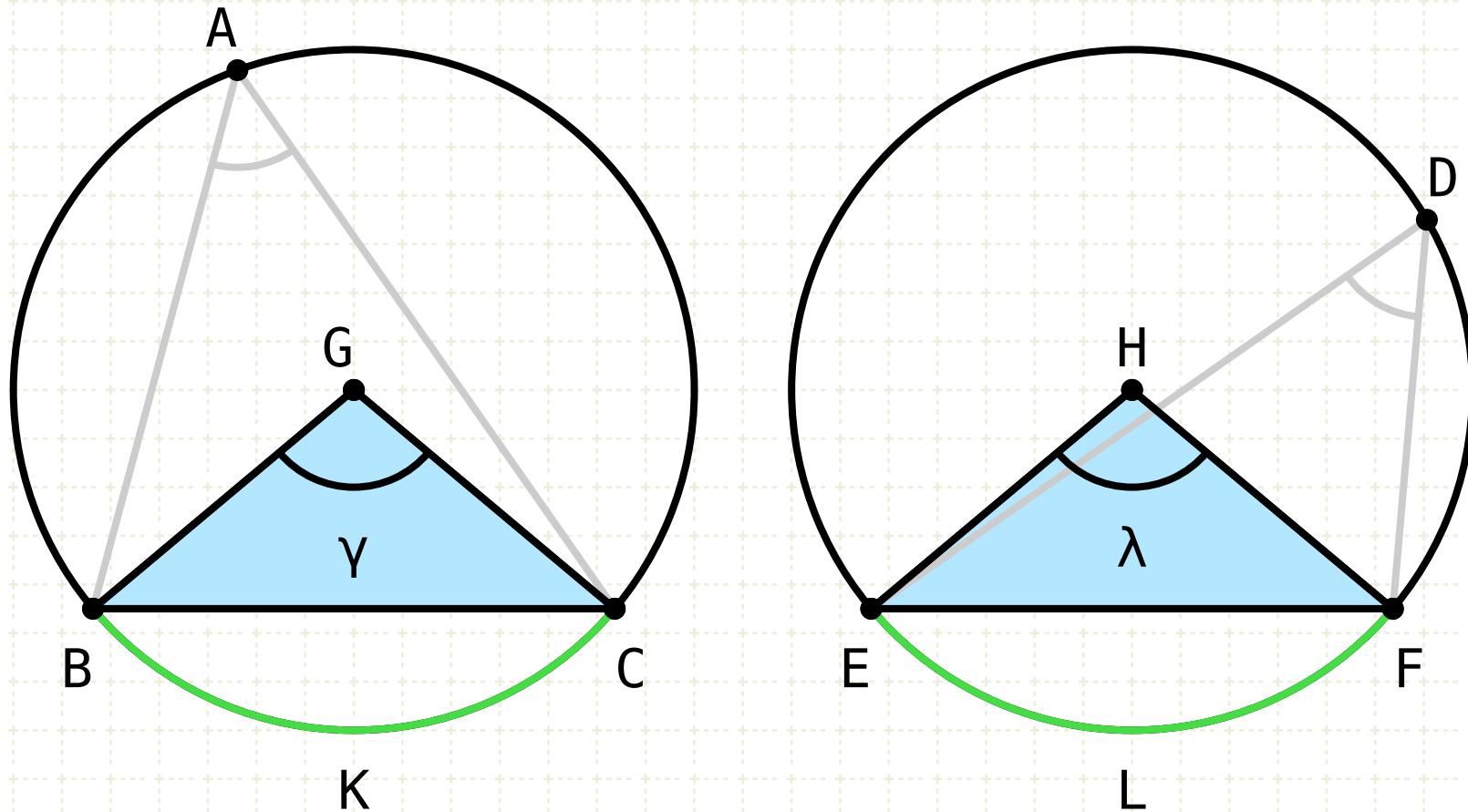
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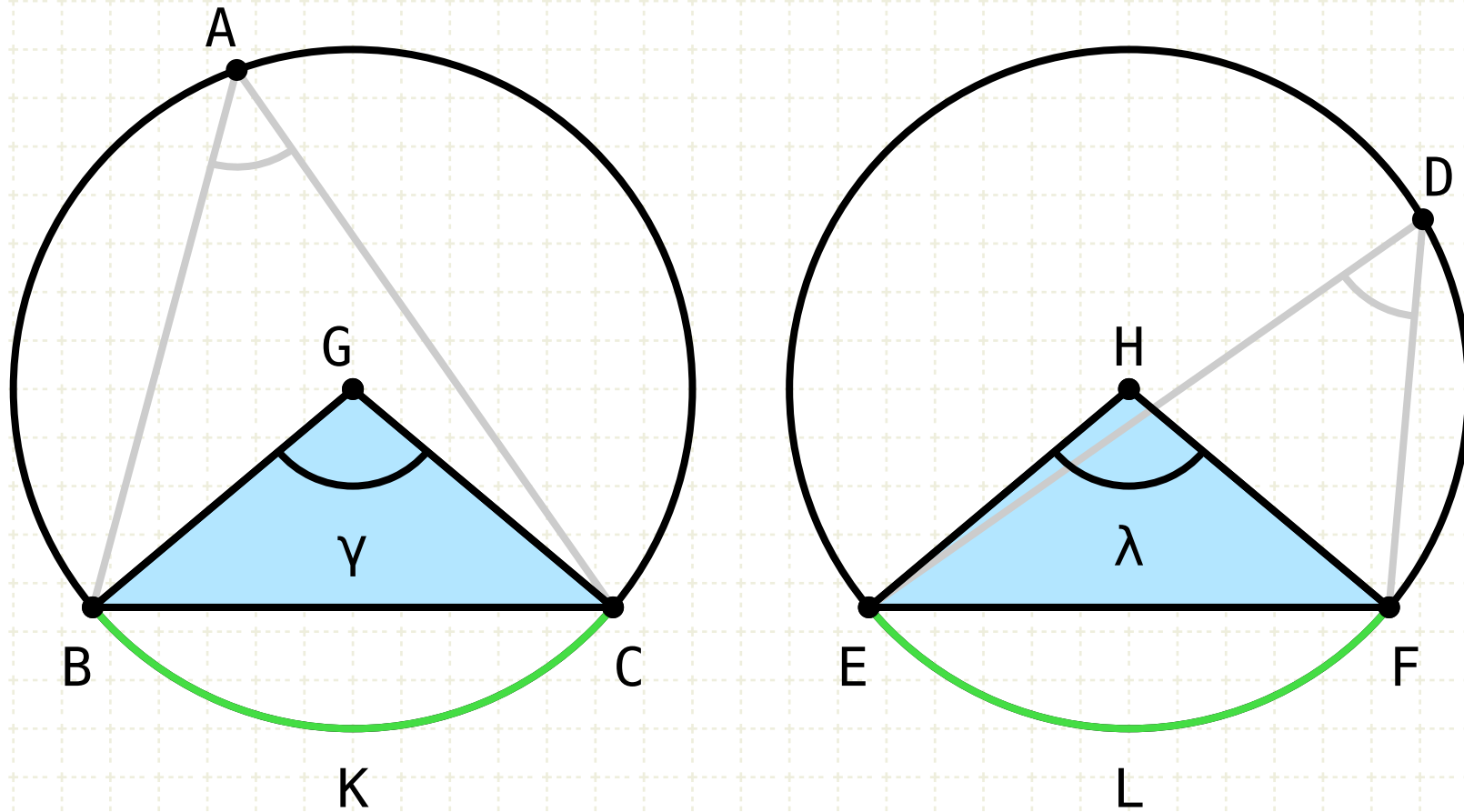
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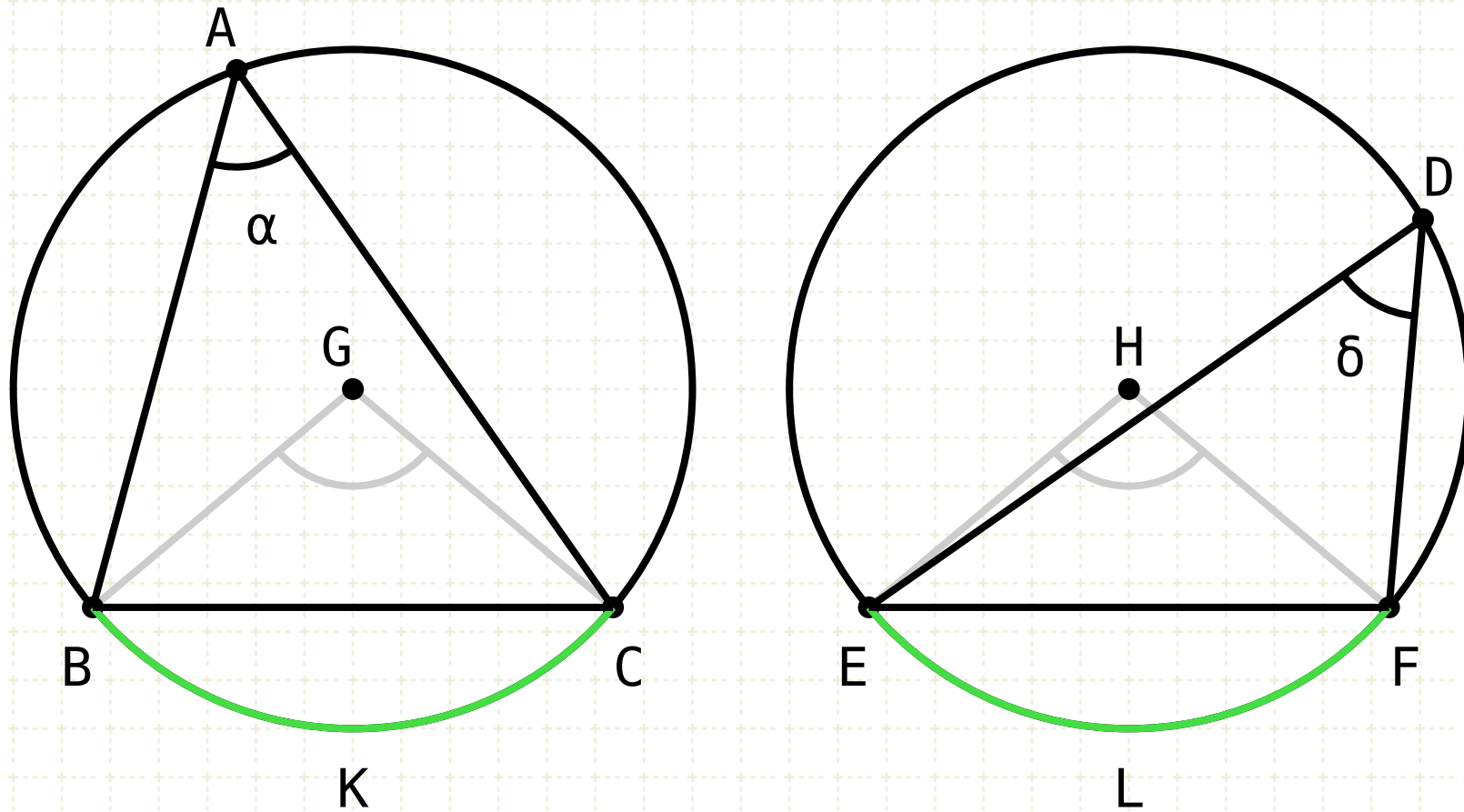
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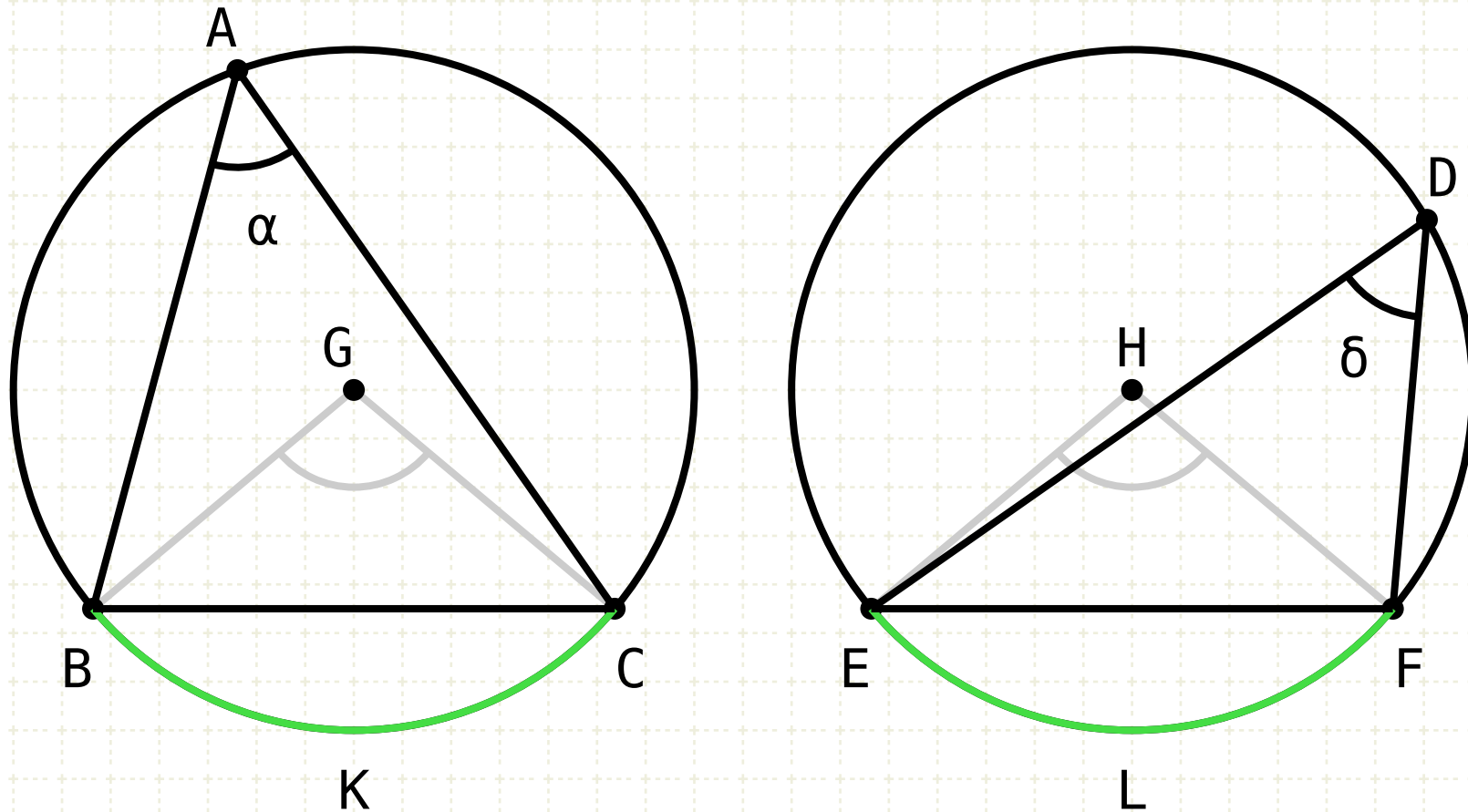
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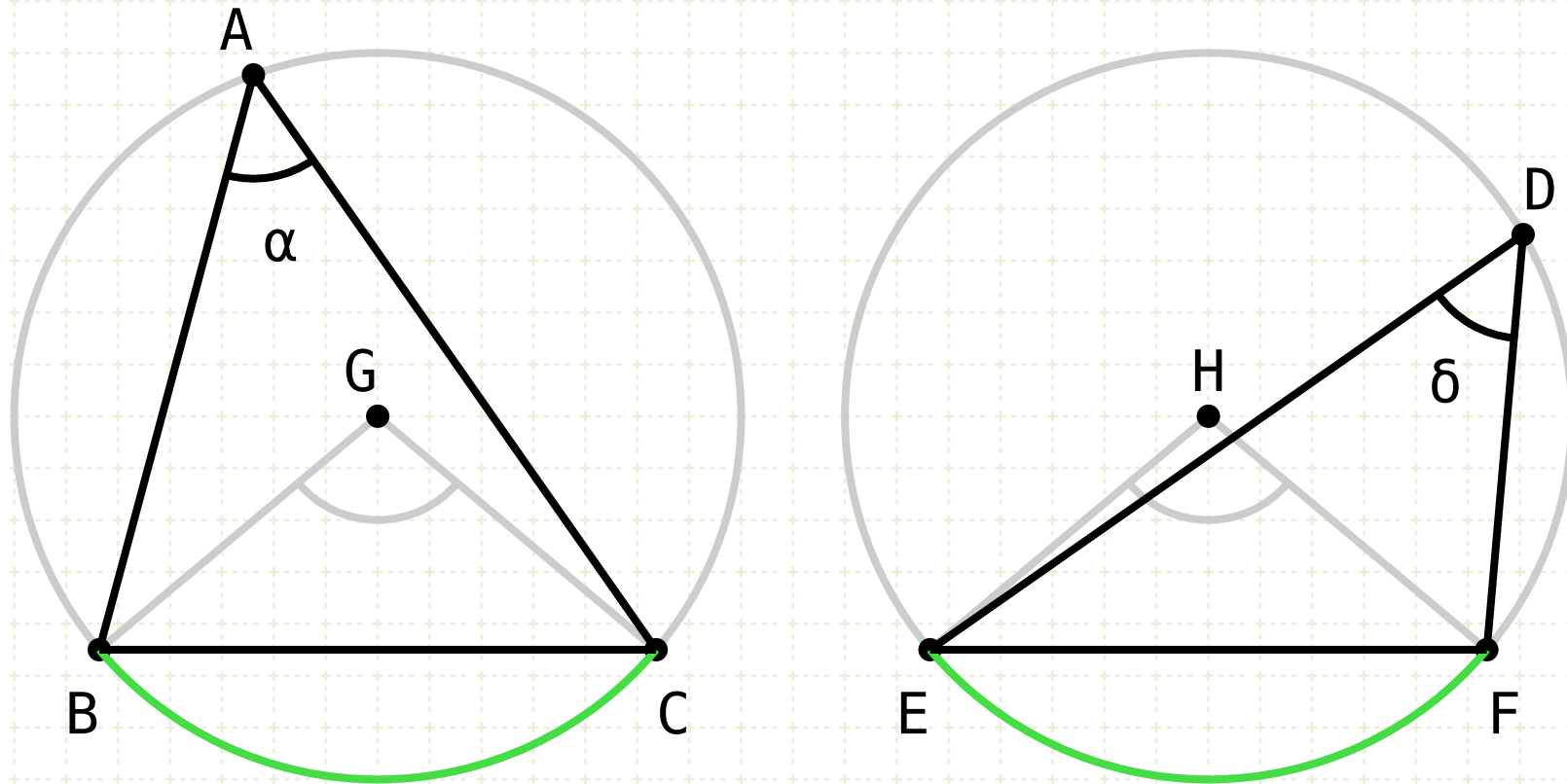
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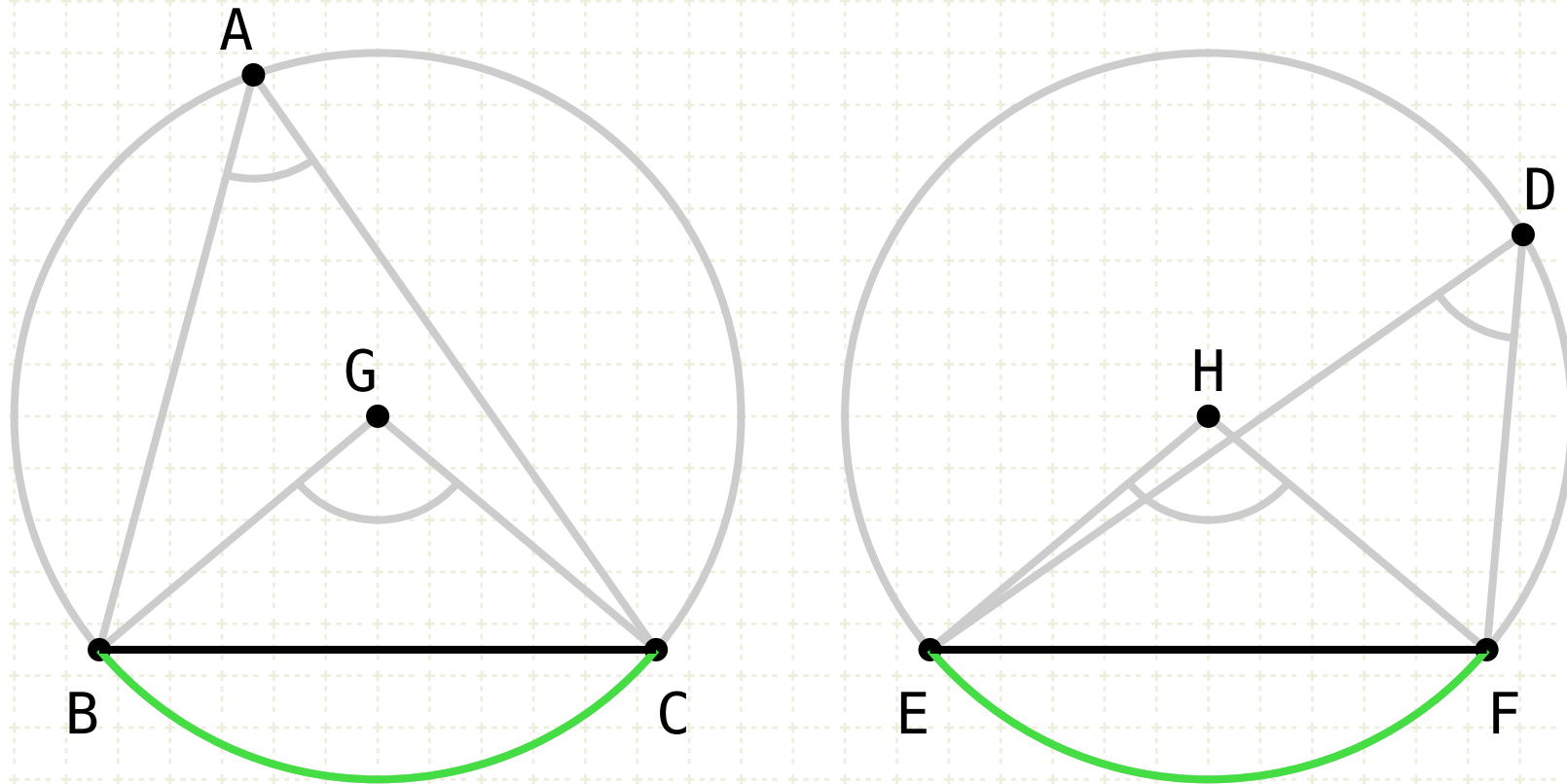
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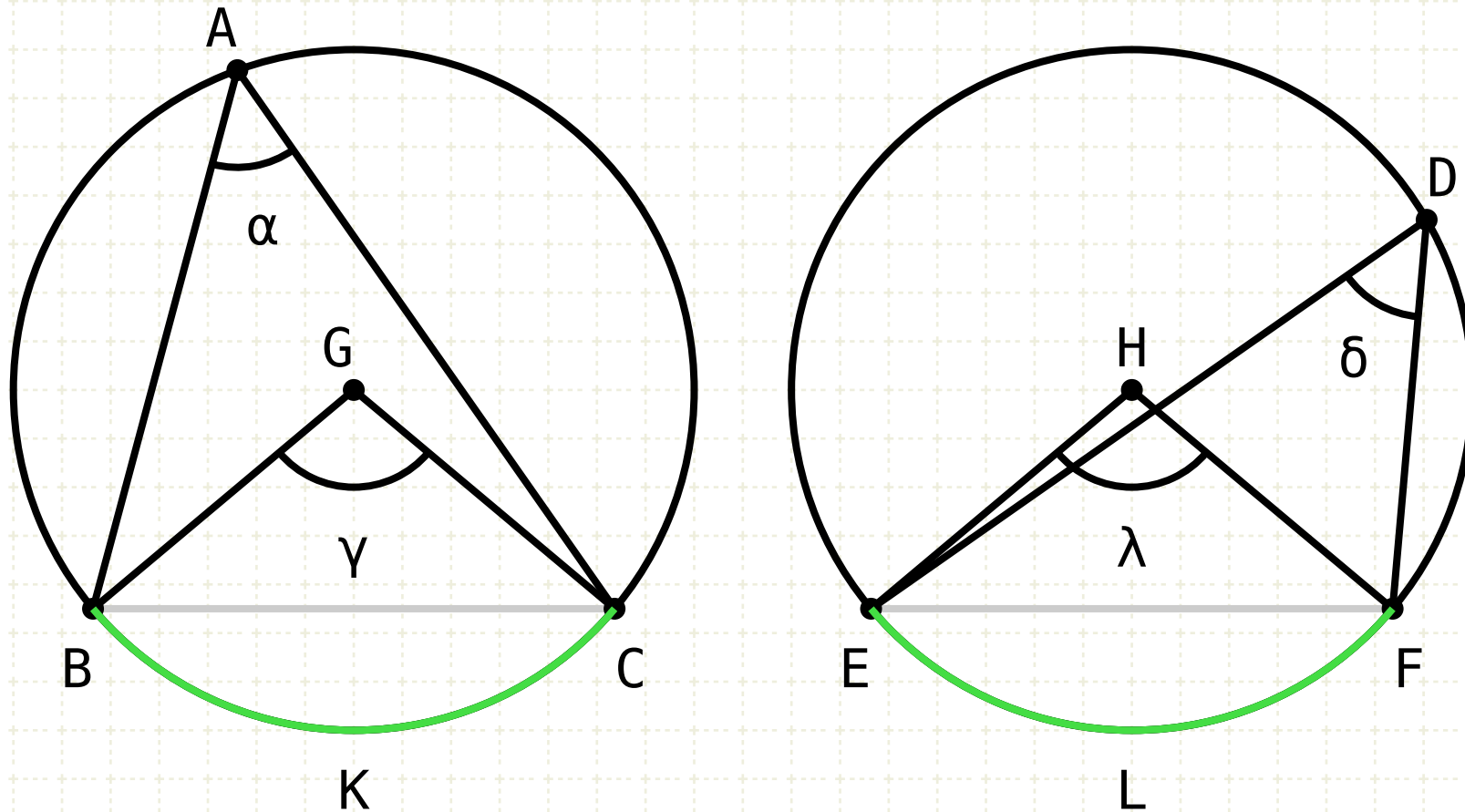
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The circles are equal (by definition), and since the circle segments are equal, the difference between the two (the circumferences BC and EF) are also equal



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