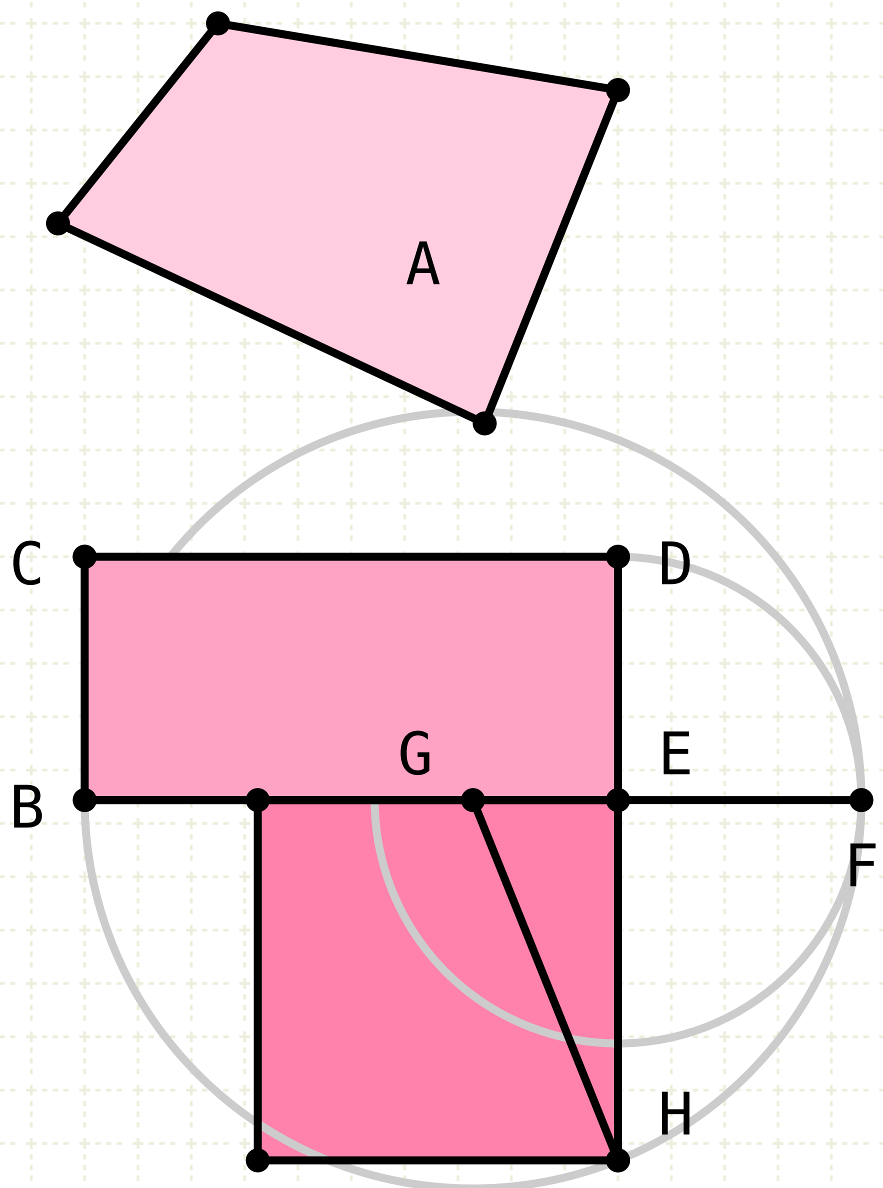
Euclid's Elements

Book II

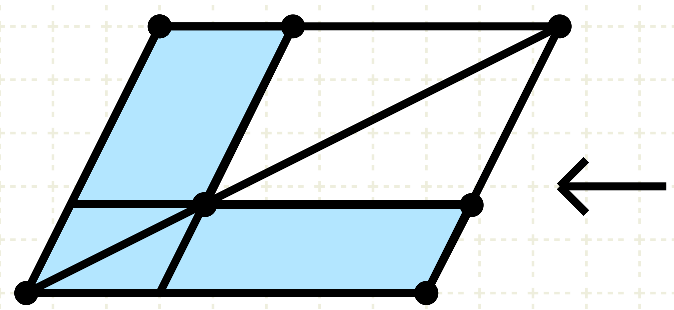


It is a remarkable fact in the history of geometry, that the Elements of Euclid, written two thousand years ago, are still regarded by many as the best introduction to the mathematical sciences.

Florian Cajori,

A History of Mathematics (1893)

Definitions:



Any rectangular parallelogram is said to be contained by the two straight lines containing the right angle.

And in any parallelogrammic area let any one whatever of the parallelograms about its diameter with the two complements be called a gnomon.

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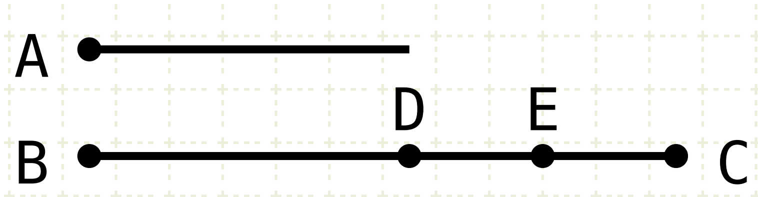
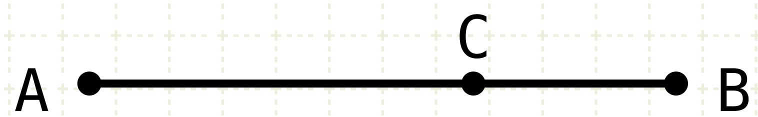


$A \cdot BC = A \cdot BD + A \cdot DE + A \cdot EC$

2

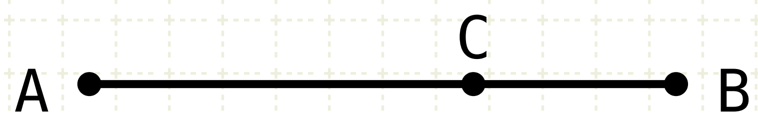


$AB^2 = AB \cdot AC + AB \cdot BC$

3

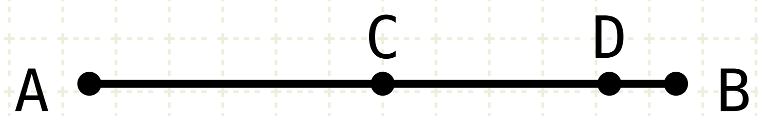
$AB \cdot CB = AC \cdot CB + CB^2$

4



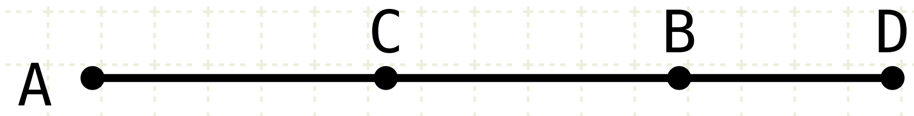
$AB^2 = AC^2 + CB^2 + 2 \cdot AC \cdot CB$

5



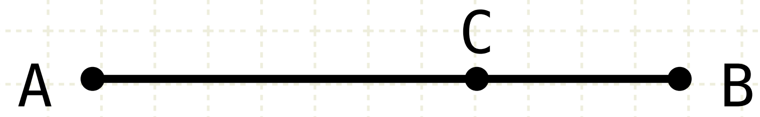
$AD \cdot DB + CD^2 = CB^2$

6



$AD \cdot DB + CB^2 = CD^2$

7

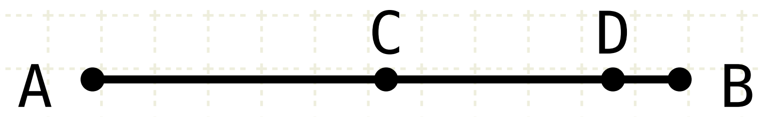


$AB^2 + BC^2 = AC^2 + 2 \cdot AB \cdot BC$

8

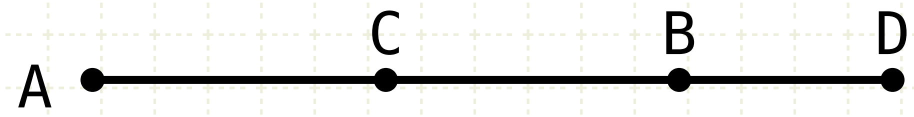
$4 \cdot AB \cdot BC + AC^2 = (AB+BC)^2$

9



$AD^2 + DB^2 = 2 \cdot (AC^2 + CD^2)$

10



$AD^2 + DB^2 = 2 \cdot (AC^2 + CD^2)$

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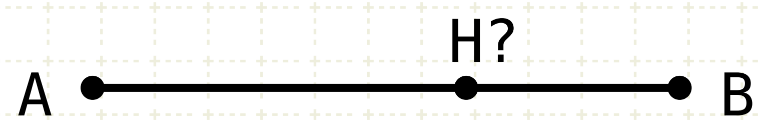
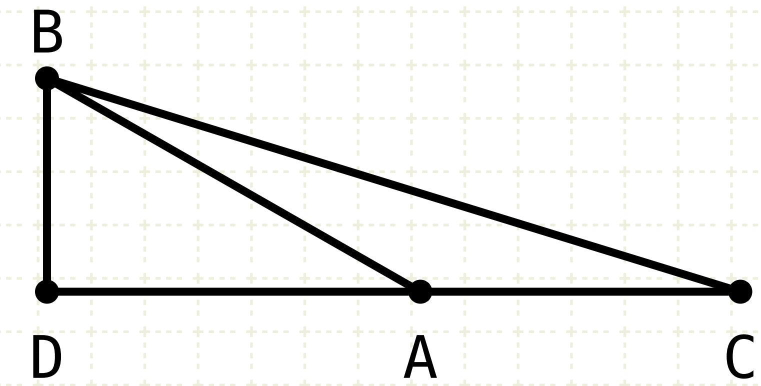


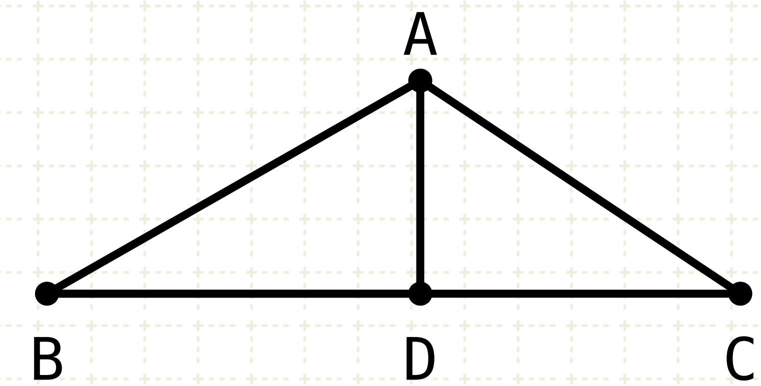
Find H. $AB \cdot BH = AH^2$

12



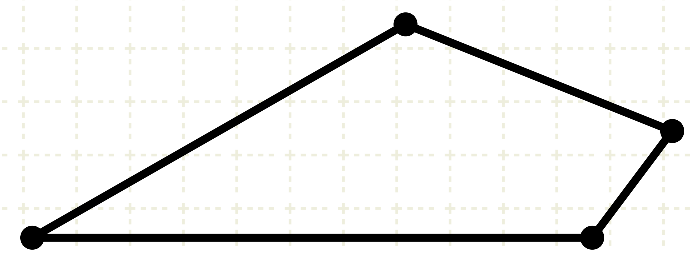
Cosine Law. $BC^2 = AB^2+AC^2+2 \cdot AD \cdot AC$

13



Cosine Law. $AC^2 = AB^2+BC^2-2 \cdot BD \cdot BC$

14



Find square of polygon

Proposition 10 of Book II

If a straight line be bisected, and a straight line be added to it in a straight line, the square on the whole with the added straight line and the square on the added straight line both together are double of the square on the half and of the square described on the straight line made up of the half and the added straight line as on one straight line.

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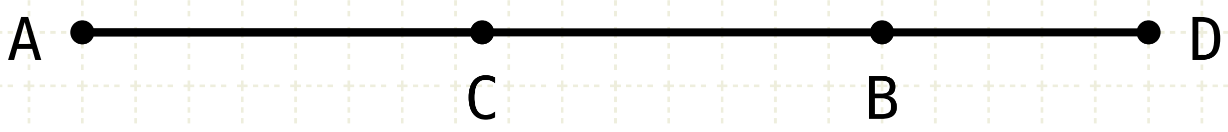
AC = CB, AC,CB,BD = AD

In other words

Let AB be a straight line, bisected at point C, and extended to an arbitrary point D



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$AC = CB,\ AC, CB, BD = AD$

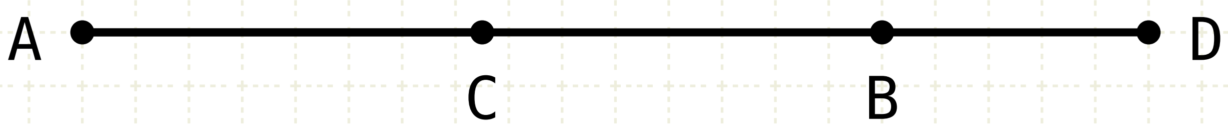
$AD^2 + DB^2$

$= 2 \cdot (AC^2 + CD^2)$

In other words

Let AB be a straight line, bisected at point C, and extended to an arbitrary point D

The sum of the squares of AD and DB is equal to twice the sum of the squares of AC and DC



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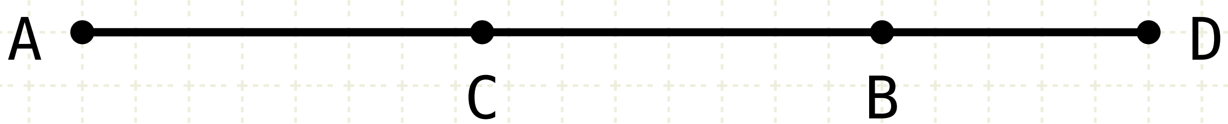

In other words

Let AB be a straight line, bisected at point C, and extended to an arbitrary point D

The sum of the squares of AD and DB is equal to twice the sum of the squares of AC and DC

Construction:

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If a straight line be bisected, and a straight line be added to it in a straight line, the square on the whole with the added straight line and the square on the added straight line both together are double of the square on the half and of the square described on the straight line made up of the half and the added straight line as on one straight line.

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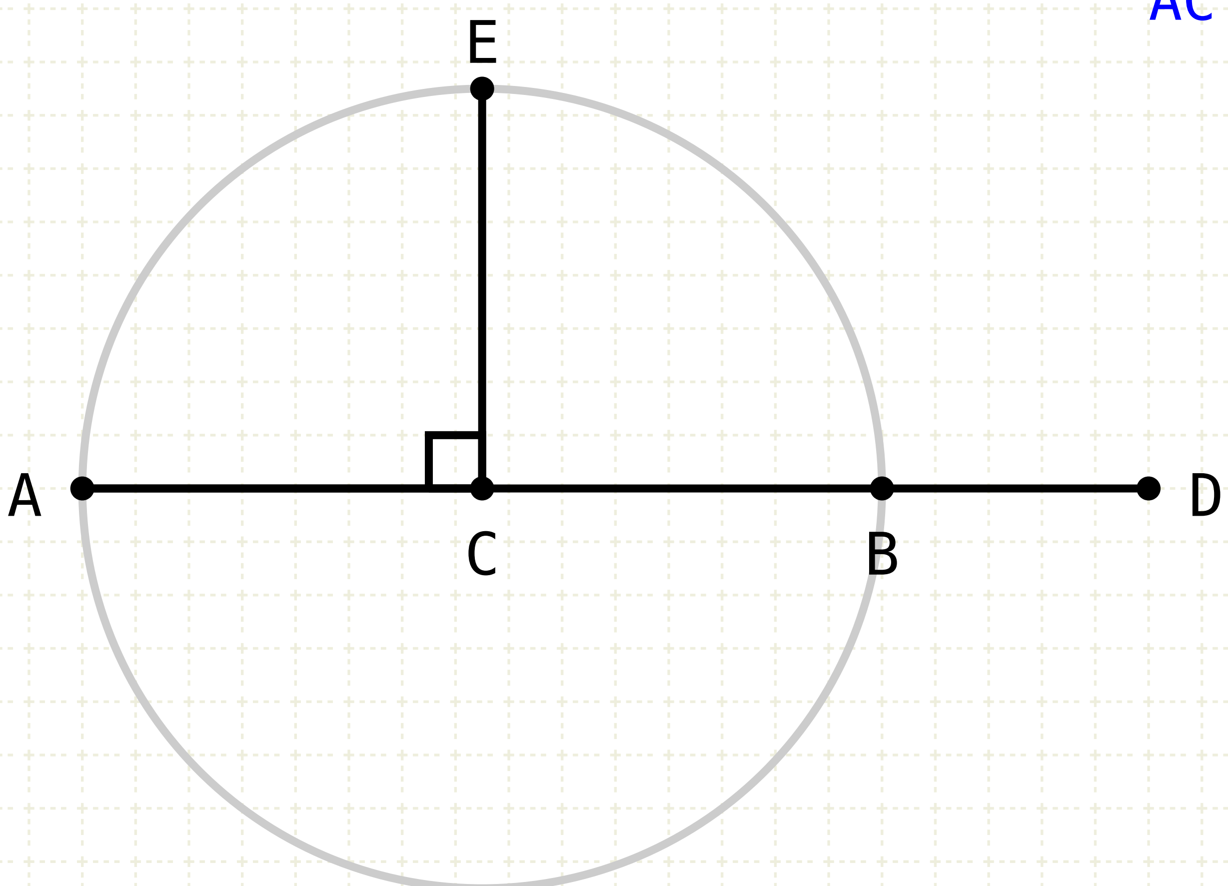

AC = CE
CB = CE

In other words

Let AB be a straight line, bisected at point C, and extended to an arbitrary point D

The sum of the squares of AD and DB is equal to twice the sum of the squares of AC and DC

Construction:

Draw a line perpendicular to AB through point C (I·11), and make its length equal to AC or CB (I·3)

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If a straight line be bisected, and a straight line be added to it in a straight line, the square on the whole with the added straight line and the square on the added straight line both together are double of the square on the half and of the square described on the straight line made up of the half and the added straight line as on one straight line.

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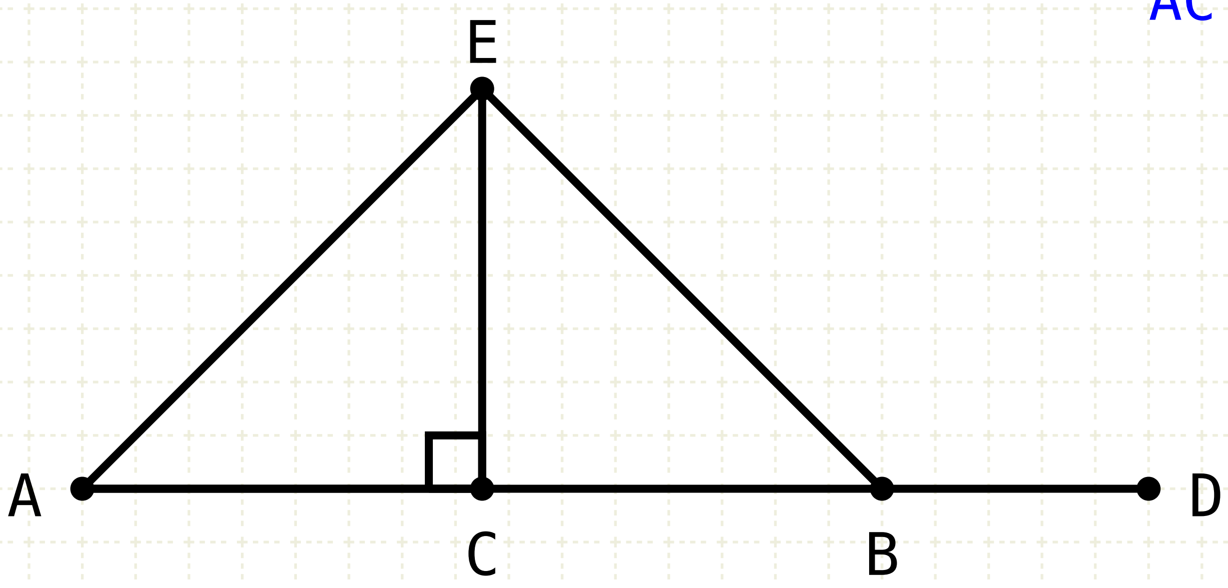

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Construction:

Draw a line perpendicular to AB through point C (I·11), and make its length equal to AC or CB (I·3)

Connect AE and EB

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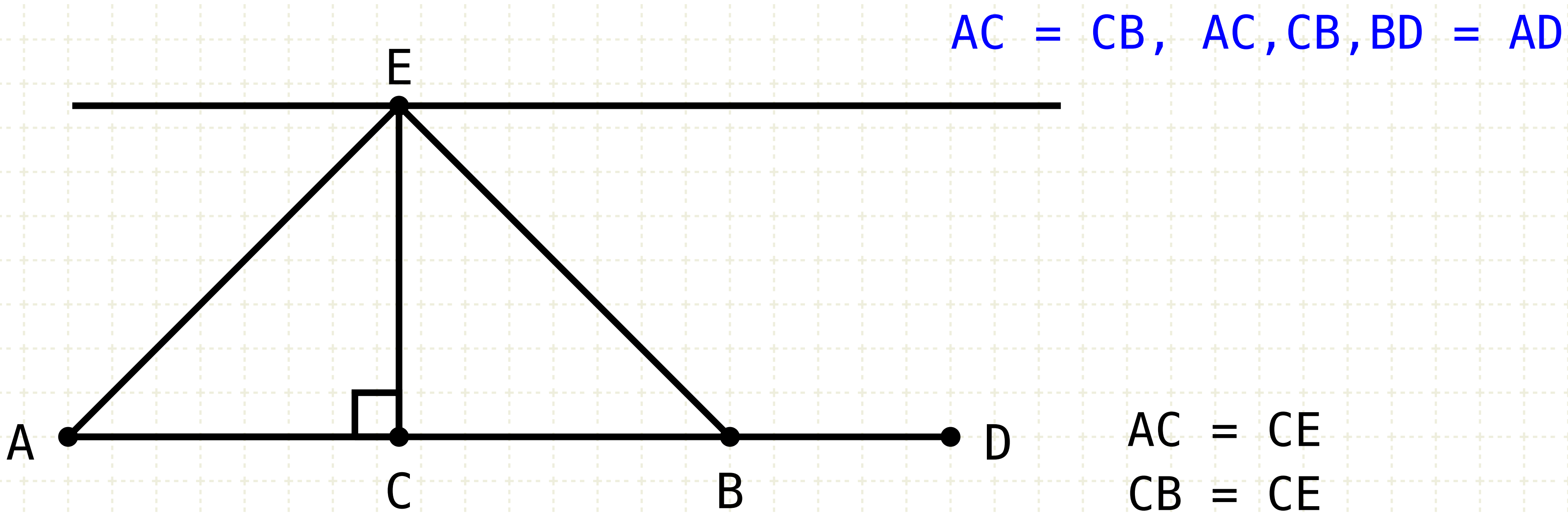
If a straight line be bisected, and a straight line be added to it in a straight line, the square on the whole with the added straight line and the square on the added straight line both together are double of the square on the half and of the square described on the straight line made up of the half and the added straight line as on one straight line.



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Construction:

Draw a line perpendicular to AB through point C (I·11), and make its length equal to AC or CB (I·3)

Connect AE and EB

Draw a line parallel to AD through point E (I·31)

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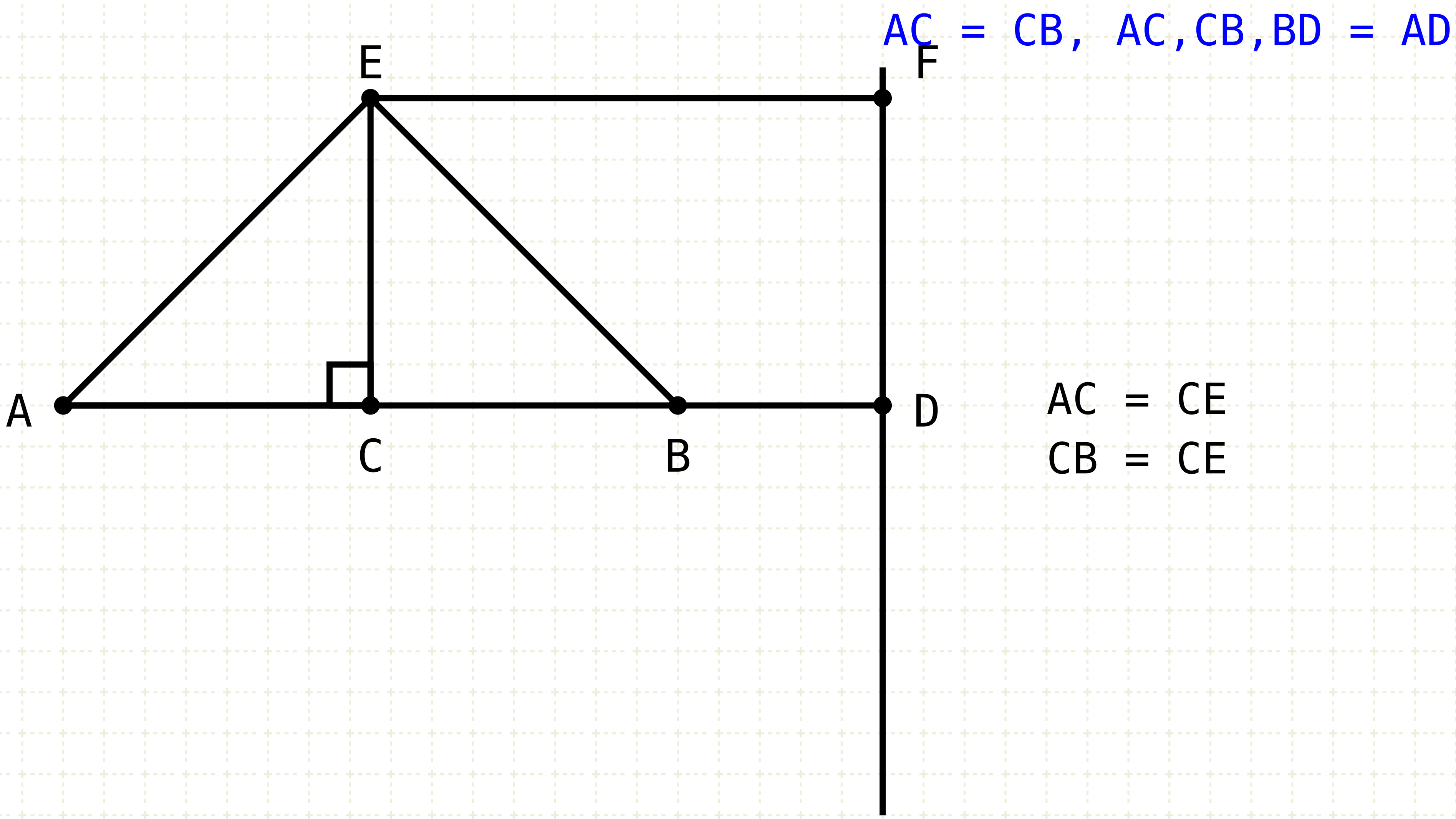
If a straight line be bisected, and a straight line be added to it in a straight line, the square on the whole with the added straight line and the square on the added straight line both together are double of the square on the half and of the square described on the straight line made up of the half and the added straight line as on one straight line.



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Construction:

Draw a line perpendicular to AB through point C (I·11), and make its length equal to AC or CB (I·3)

Connect AE and EB

Draw a line parallel to AD through point E (I·31)

Draw a line parallel to EC through point D (I·31)

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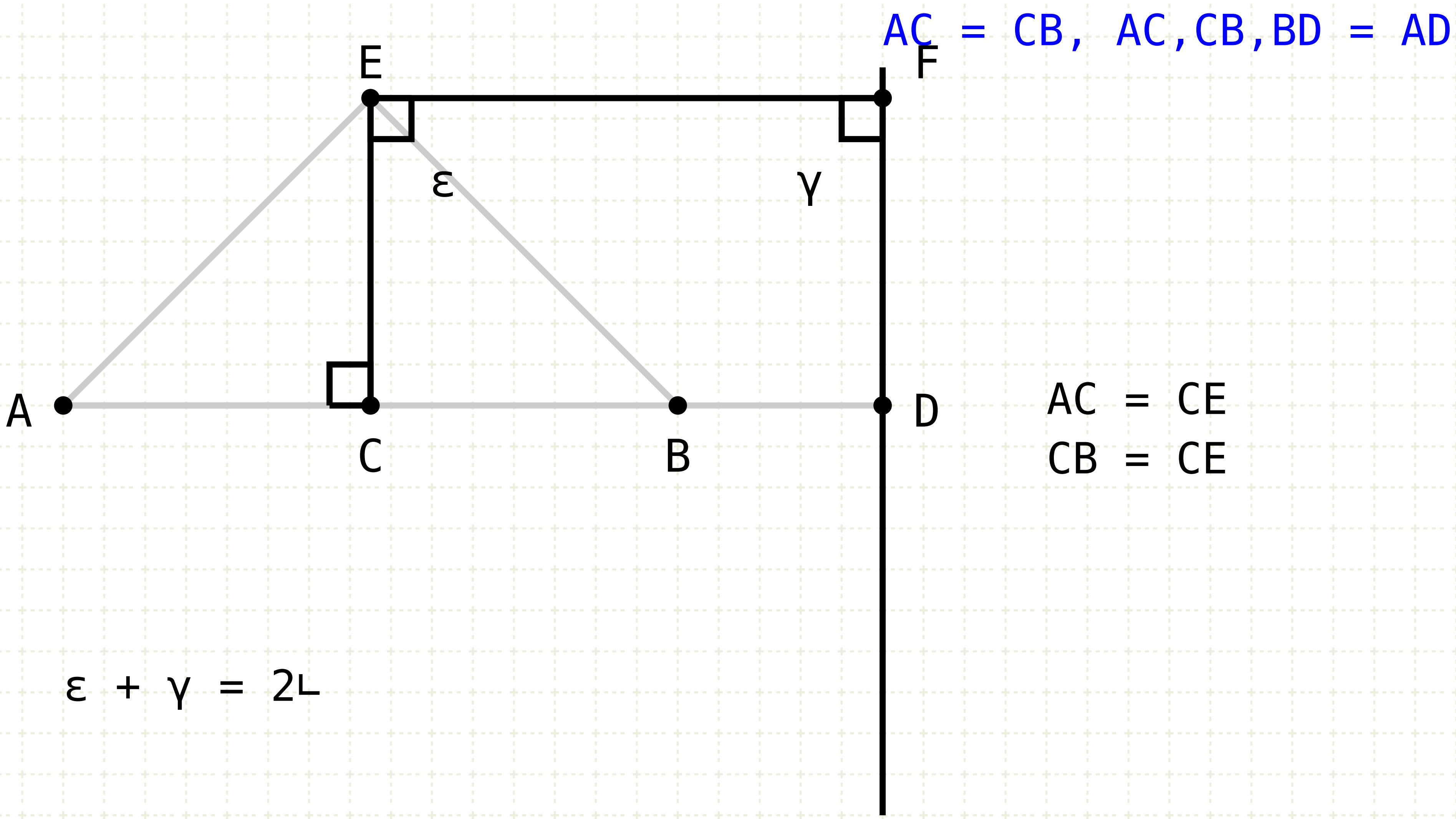

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Construction:

Draw a line perpendicular to AB through point C (I·11), and make its length equal to AC or CB (I·3)

Connect AE and EB

Draw a line parallel to AD through point E (I·31)

Draw a line parallel to EC through point D (I·31)

Since EF crosses two parallel lines (EC and FD), then the sum of the interior angles is two right angles (FEC and EFD)

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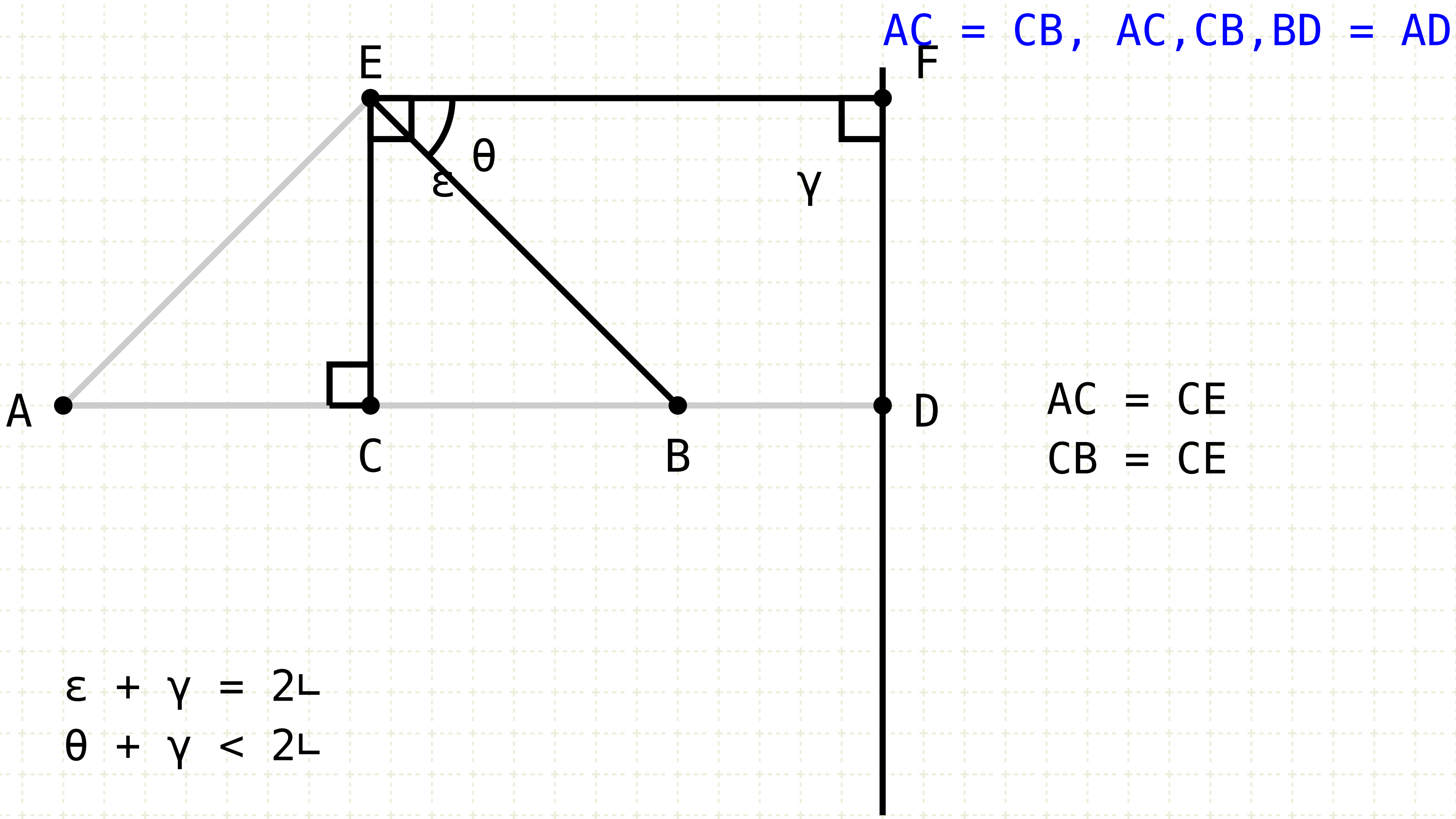

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Since EF crosses two parallel lines (EC and FD), then the sum of the interior angles is two right angles (FEC and EFD)

Thus the angles BEF and EFD sum to less than two right angles, and from proposition 5, EB and FD will intersect. Label this intersection G

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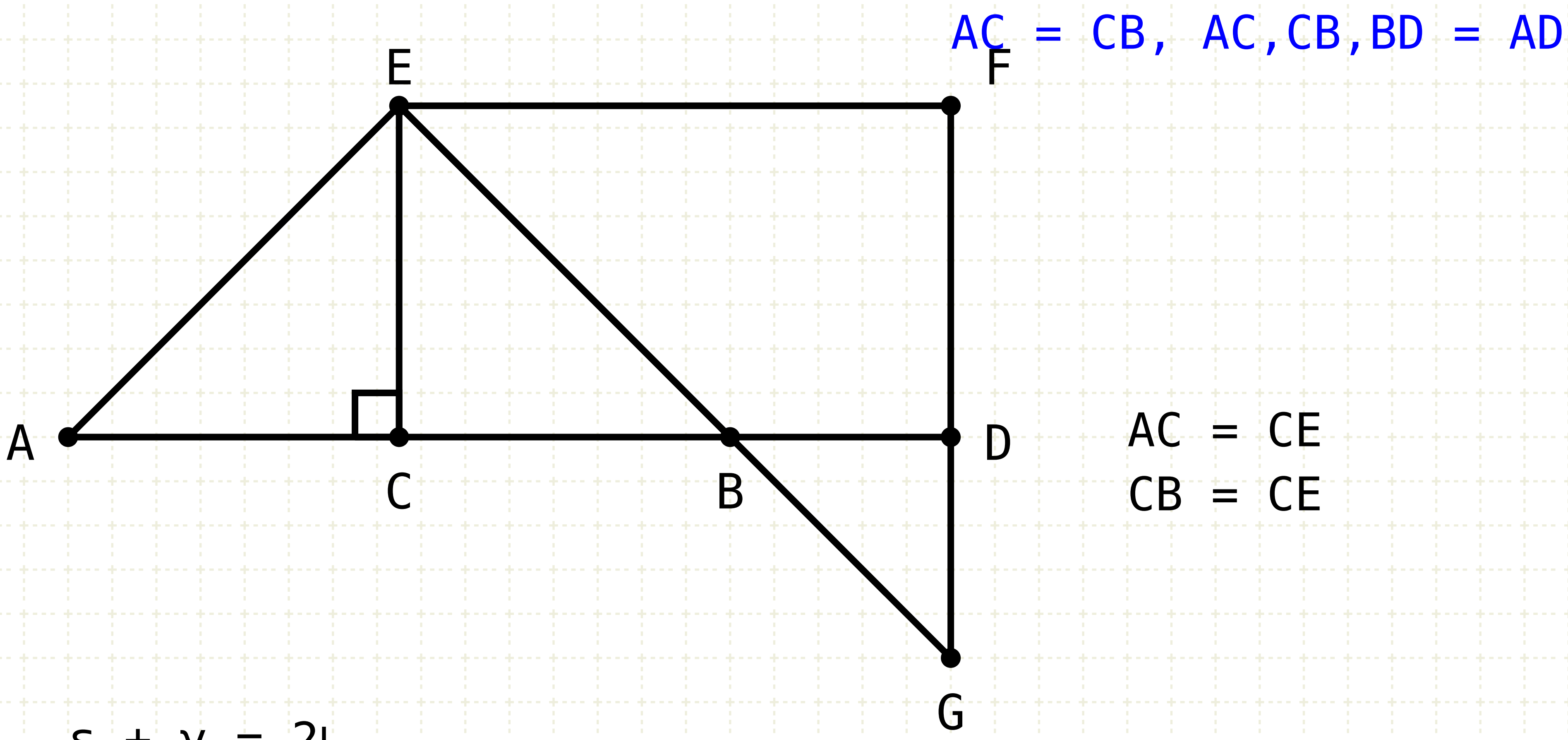

AC = CE
CB = CE

$\varepsilon + \gamma = 2\llcorner$
$\theta + \gamma < 2\llcorner$

In other words

Let AB be a straight line, bisected at point C, and extended to an arbitrary point D

The sum of the squares of AD and DB is equal to twice the sum of the squares of AC and DC

Construction:

Draw a line perpendicular to AB through point C (I·11), and make its length equal to AC or CB (I·3)

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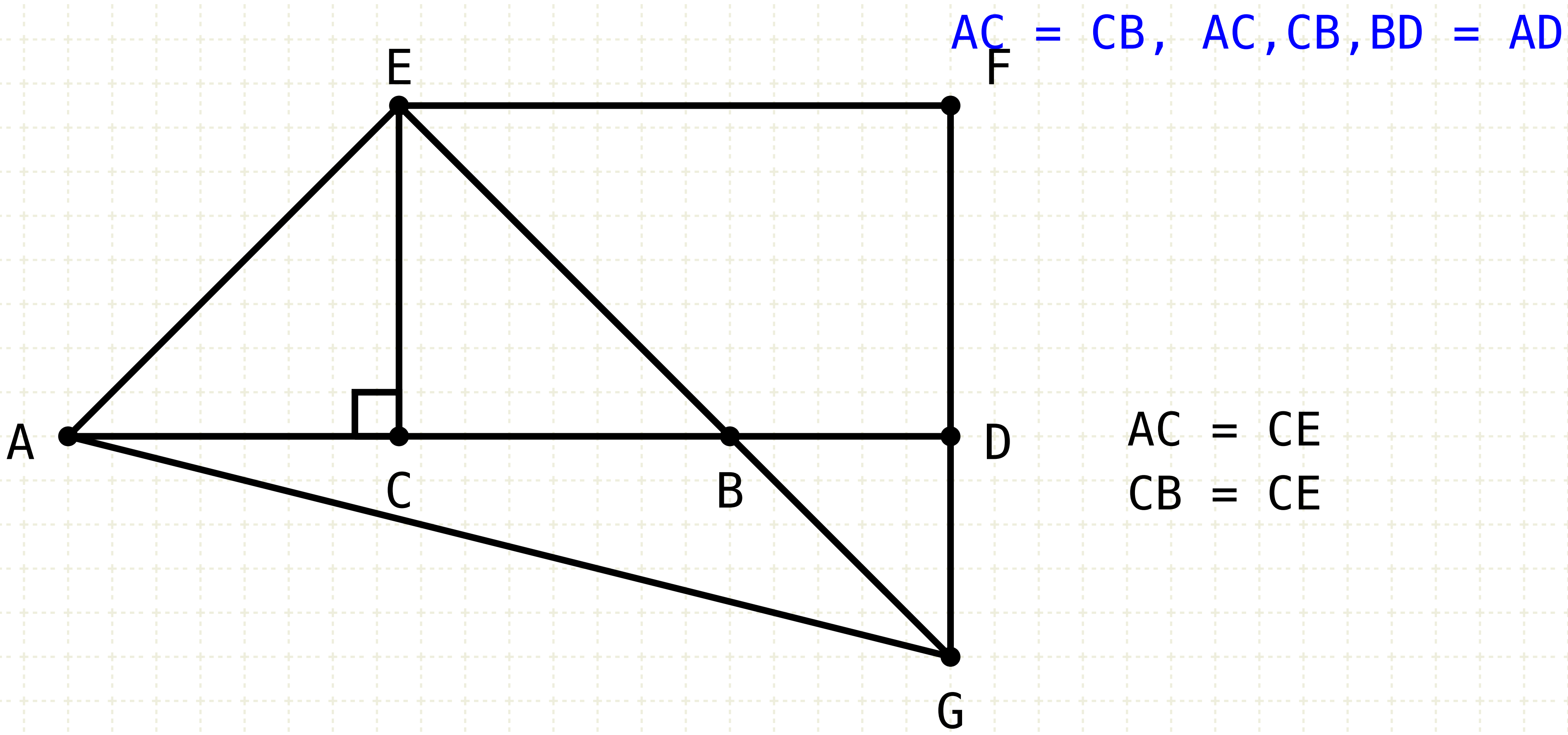

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Draw a line perpendicular to AB through point C (I·11), and make its length equal to AC or CB (I·3)

Connect AE and EB

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Draw line from A to G

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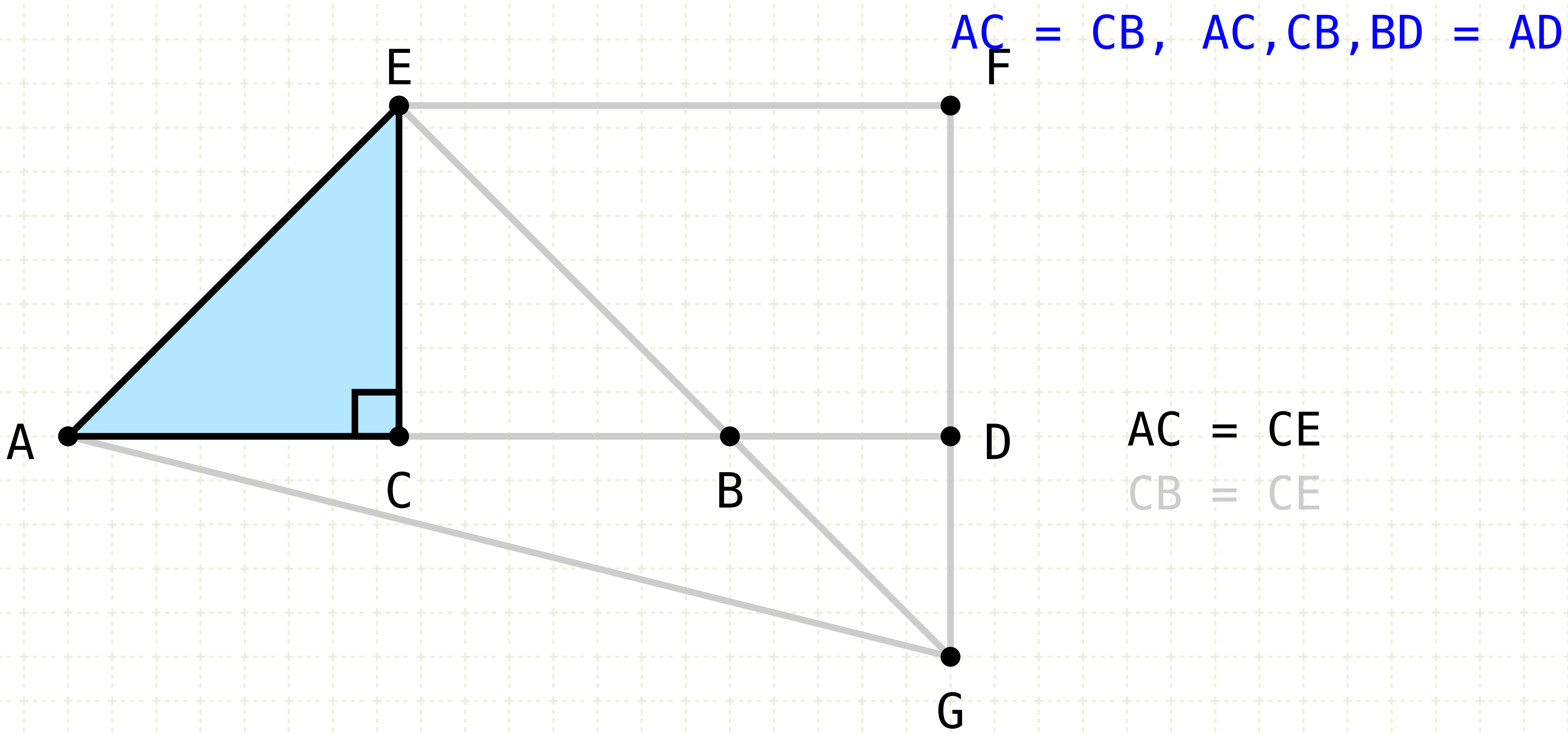

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Let AB be a straight line, bisected at point C, and extended to an arbitrary point D

The sum of the squares of AD and DB is equal to twice the sum of the squares of AC and DC

Proof

Triangle AEC is a right angle triangle, and AC and CE are equal, therefore it is an isosceles triangle

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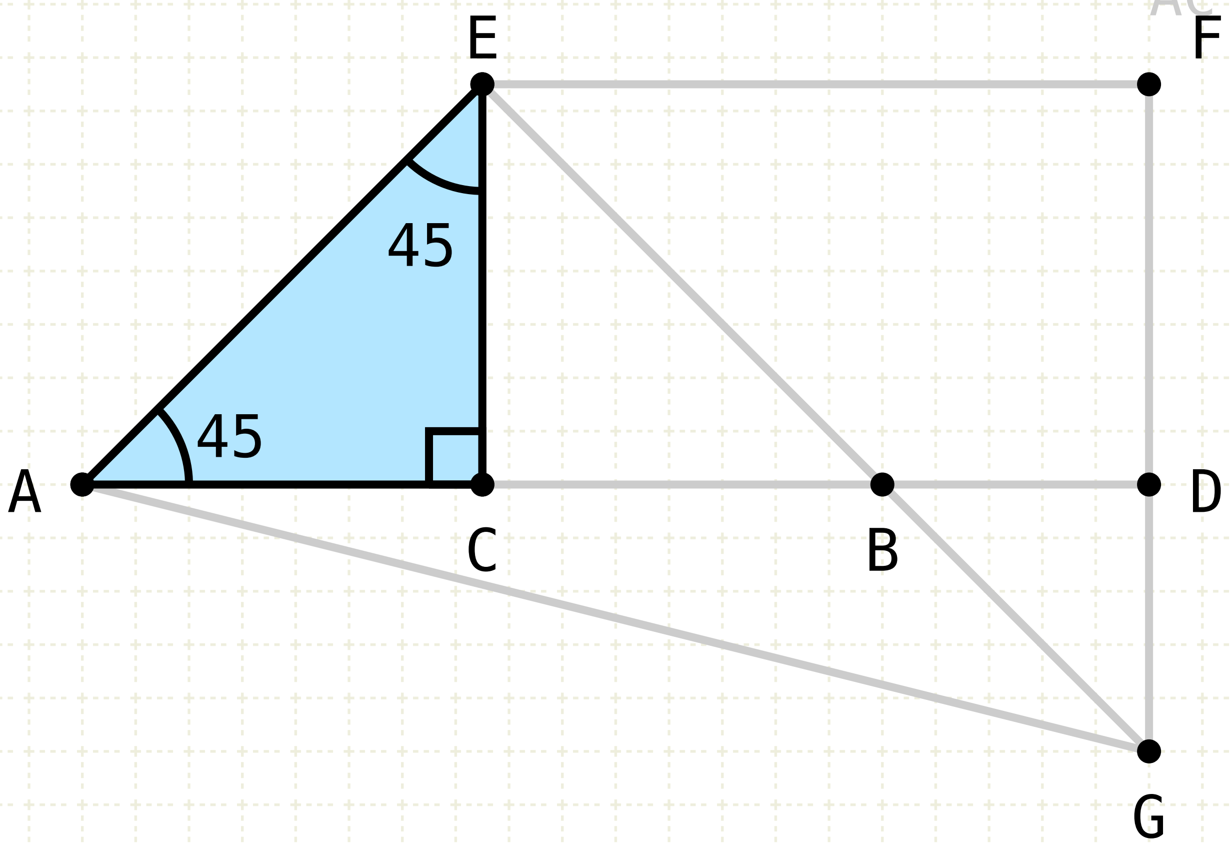
If a straight line be bisected, and a straight line be added to it in a straight line, the square on the whole with the added straight line and the square on the added straight line both together are double of the square on the half and of the square described on the straight line made up of the half and the added straight line as on one straight line.



AC = CB, AC,CB,BD = AD

∠EAC = ∠CEA = 45

AC = CE

CB = CE

In other words

Let AB be a straight line, bisected at point C, and extended to an arbitrary point D

The sum of the squares of AD and DB is equal to twice the sum of the squares of AC and DC

Proof

Triangle AEC is a right angle triangle, and AC and CE are equal, therefore it is an isosceles triangle

Since the sum of the angles in a triangle equals two right angles (I·32), and ACE is a right angle, then the two base angles (being equal (I·5)) each equal one half a right angle (45 degrees)

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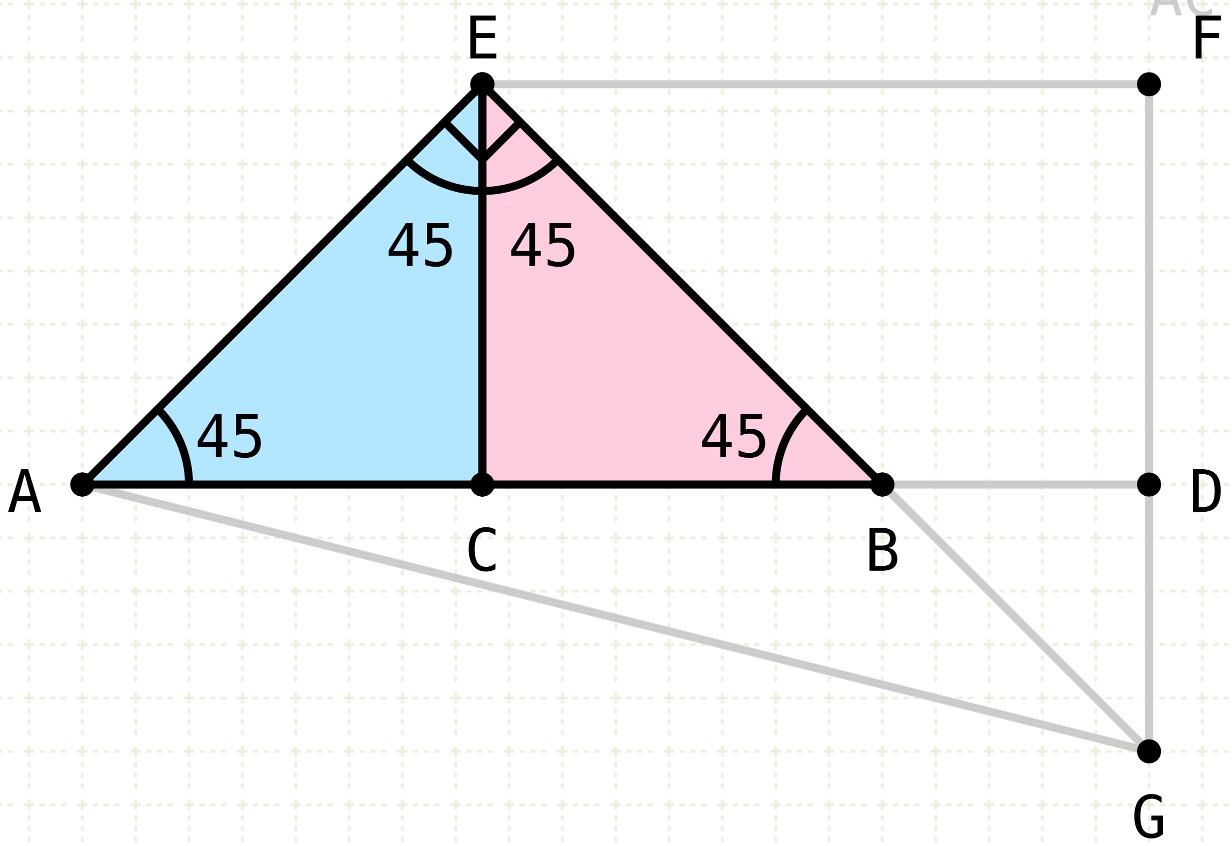

AC = CB, AC,CB,BD = AD

∠EAC = ∠CEA = 45

∠CEB = ∠CBE = 45

AC = CE

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By the same reason, angles CEB and CBE are each half a right angle, which makes AEB a right angle

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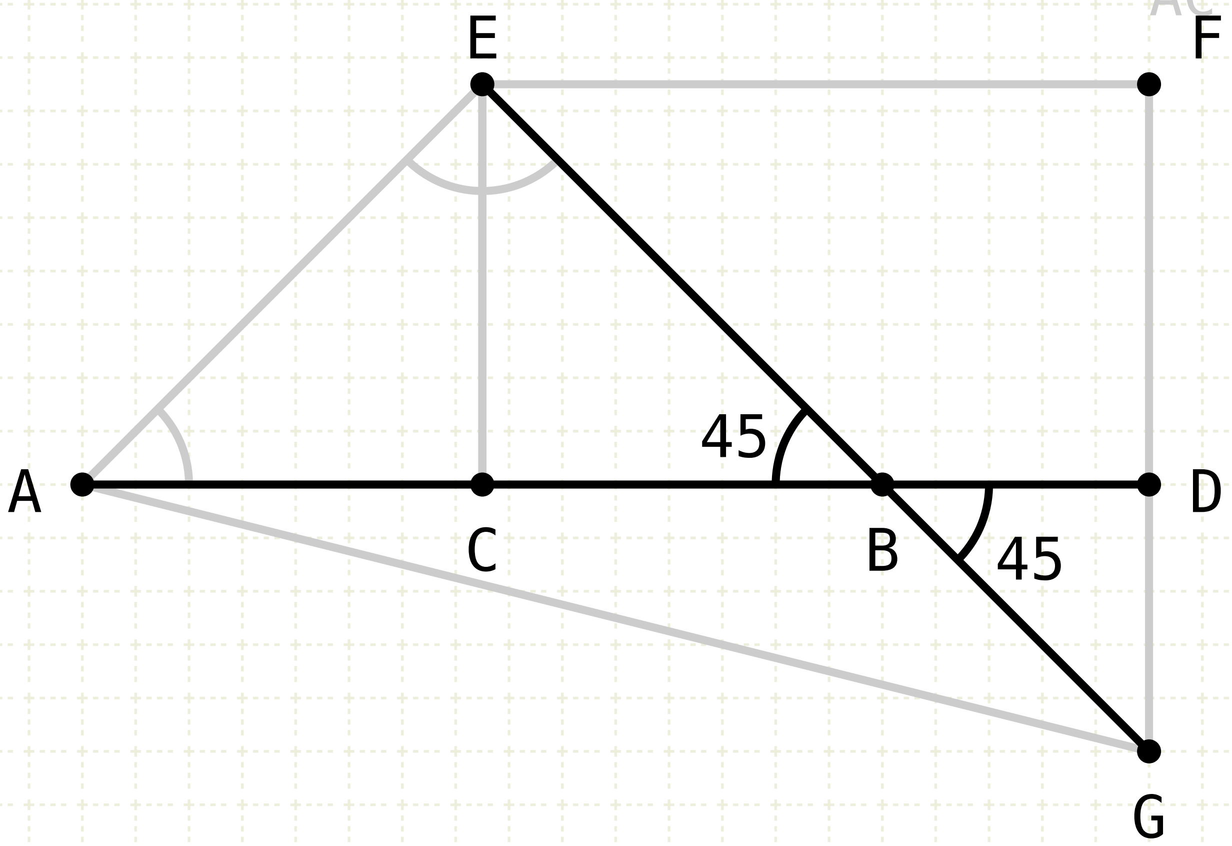

AC = CB, AC,CB,BD = AD

∠EAC = ∠CEA = 45

∠CEB = ∠CBE = 45

∠CBE = ∠GBD = 45

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The angle CBE and GBD are equal (I·15)

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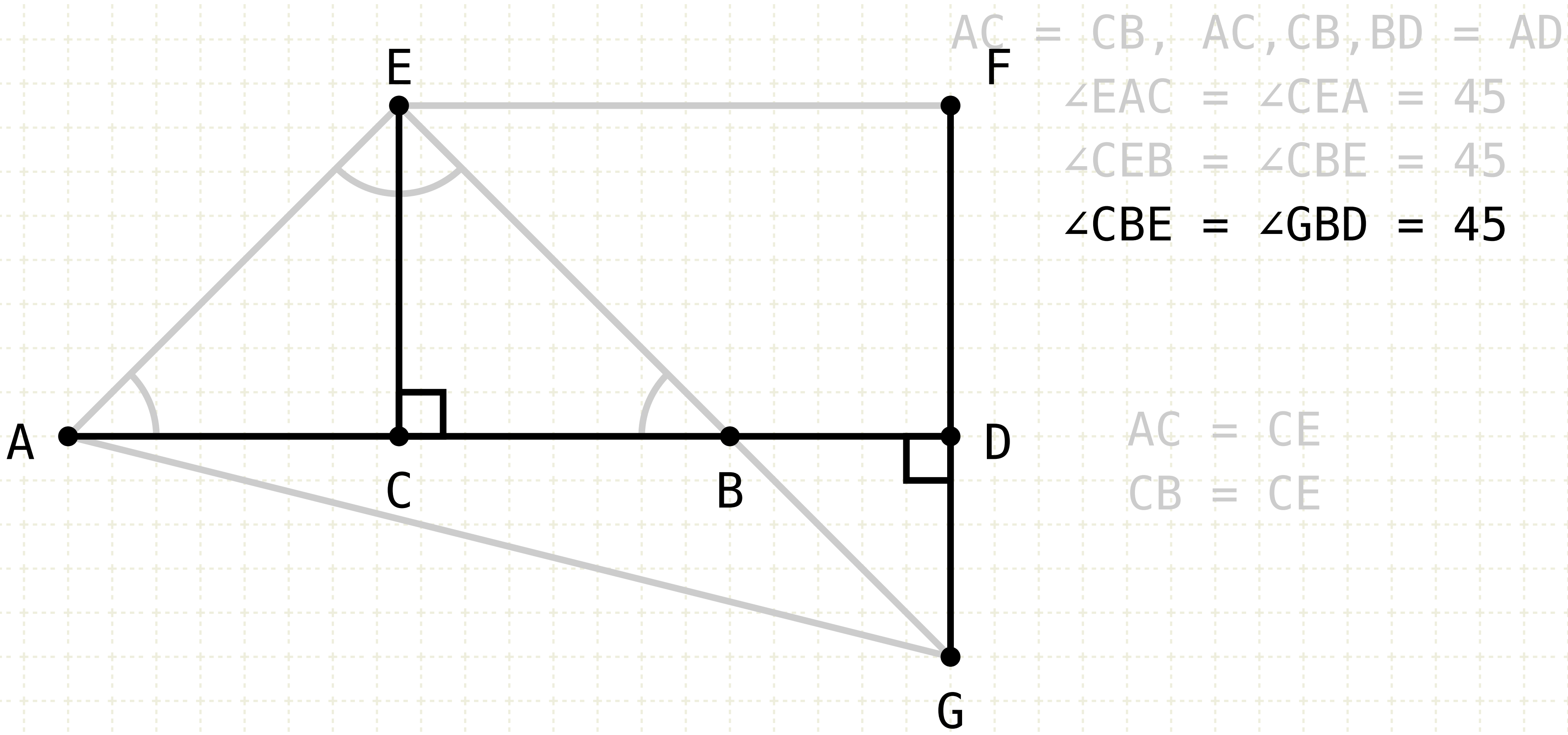

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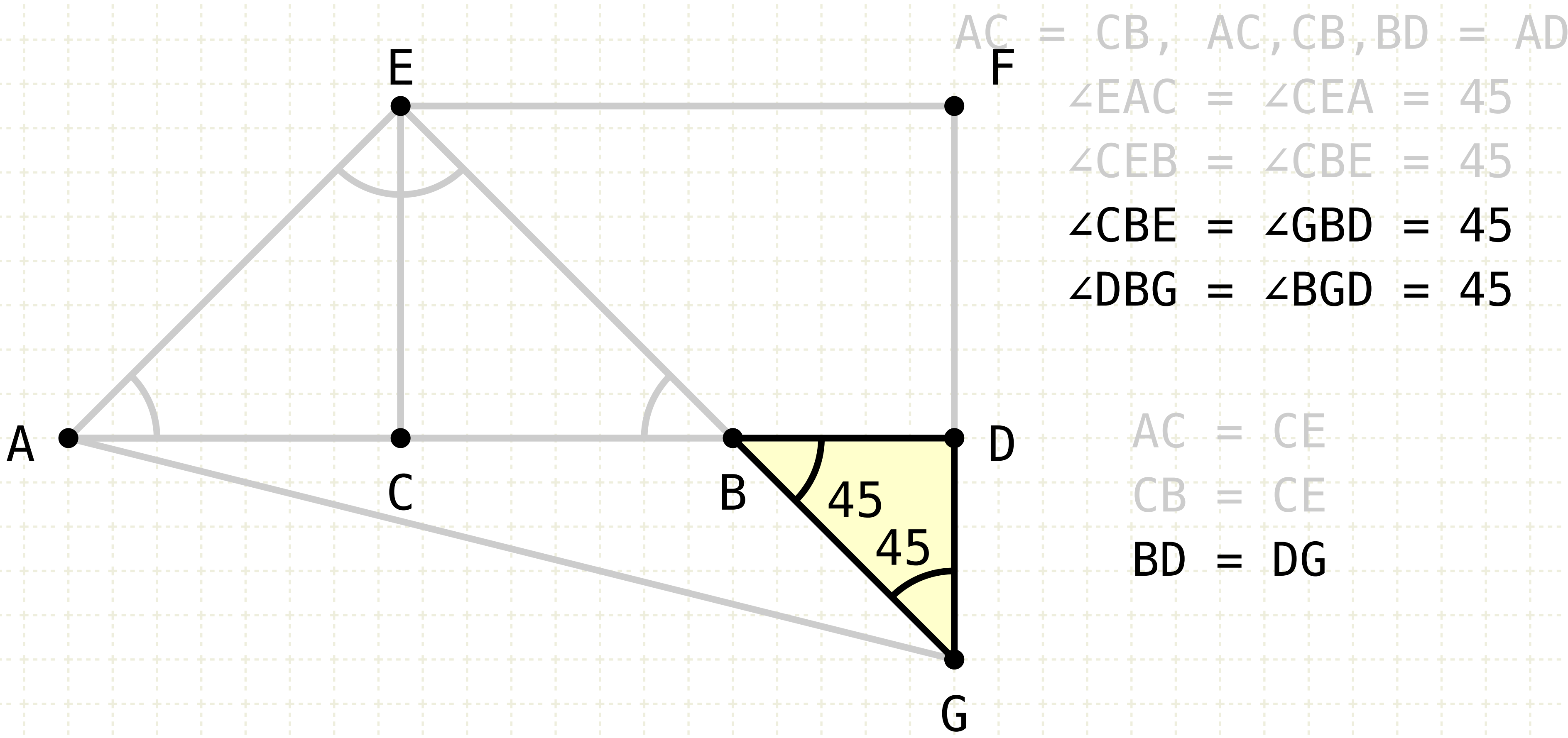
By the same reason, angles CEB and CBE are each half a right angle, which makes AEB a right angle

The angle CBE and GBD are equal (I·15)

EC and FG are parallel, thus opposite and interior angles are equal (I·29), therefore BDG is a right angle

Proposition 10 of Book II

If a straight line be bisected, and a straight line be added to it in a straight line, the square on the whole with the added straight line and the square on the added straight line both together are double of the square on the half and of the square described on the straight line made up of the half and the added straight line as on one straight line.



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By the same reason, angles CEB and CBE are each half a right angle, which makes AEB a right angle

The angle CBE and GBD are equal (I·15)

EC and FG are parallel, thus opposite and interior angles are equal (I·29), therefore BDG is a right angle

The angle DBG is one half a right angle, therefore BGD is one half a right angle (I·32), so BD equals DG (I·6)

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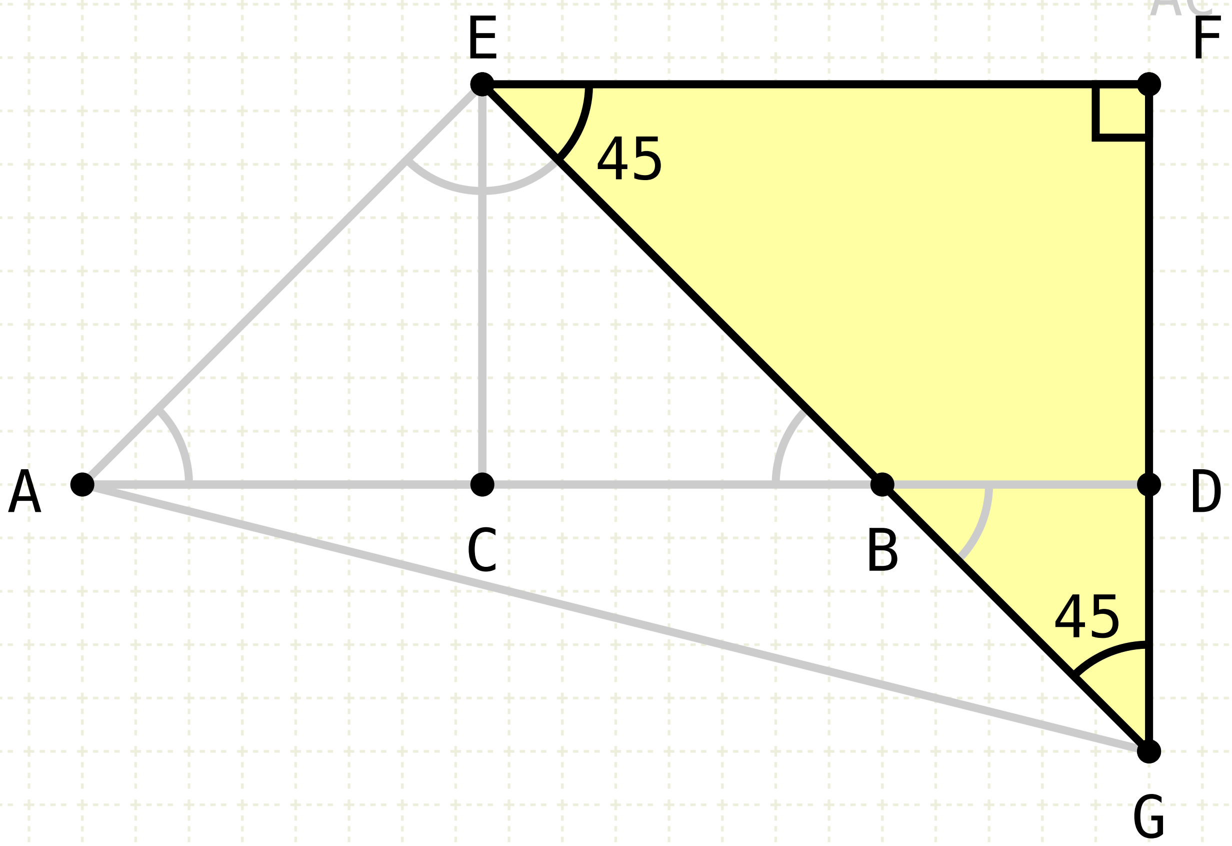
If a straight line be bisected, and a straight line be added to it in a straight line, the square on the whole with the added straight line and the square on the added straight line both together are double of the square on the half and of the square described on the straight line made up of the half and the added straight line as on one straight line.



$AC = CB, \quad AC, CB, BD = AD$

$\angle EAC = \angle CEA = 45$

$\angle CEB = \angle CBE = 45$

$\angle CBE = \angle GBD = 45$

$\angle DBG = \angle BGD = 45$

$\angle FEG = \angle EGF = 45$

$AC = CE$

$CB = CE$

$BD = DG$

$GF = EF = CD$

In other words

Let AB be a straight line, bisected at point C, and extended to an arbitrary point D

The sum of the squares of AD and DB is equal to twice the sum of the squares of AC and DC

Proof

Triangle AEC is a right angle triangle, and AC and CE are equal, therefore it is an isosceles triangle

Since the sum of the angles in a triangle equals two right angles (I·32), and ACE is a right angle, then the two base angles (being equal (I·5)) each equal one half a right angle (45 degrees)

By the same reason, angles CEB and CBE are each half a right angle, which makes AEB a right angle

The angle CBE and GBD are equal (I·15)

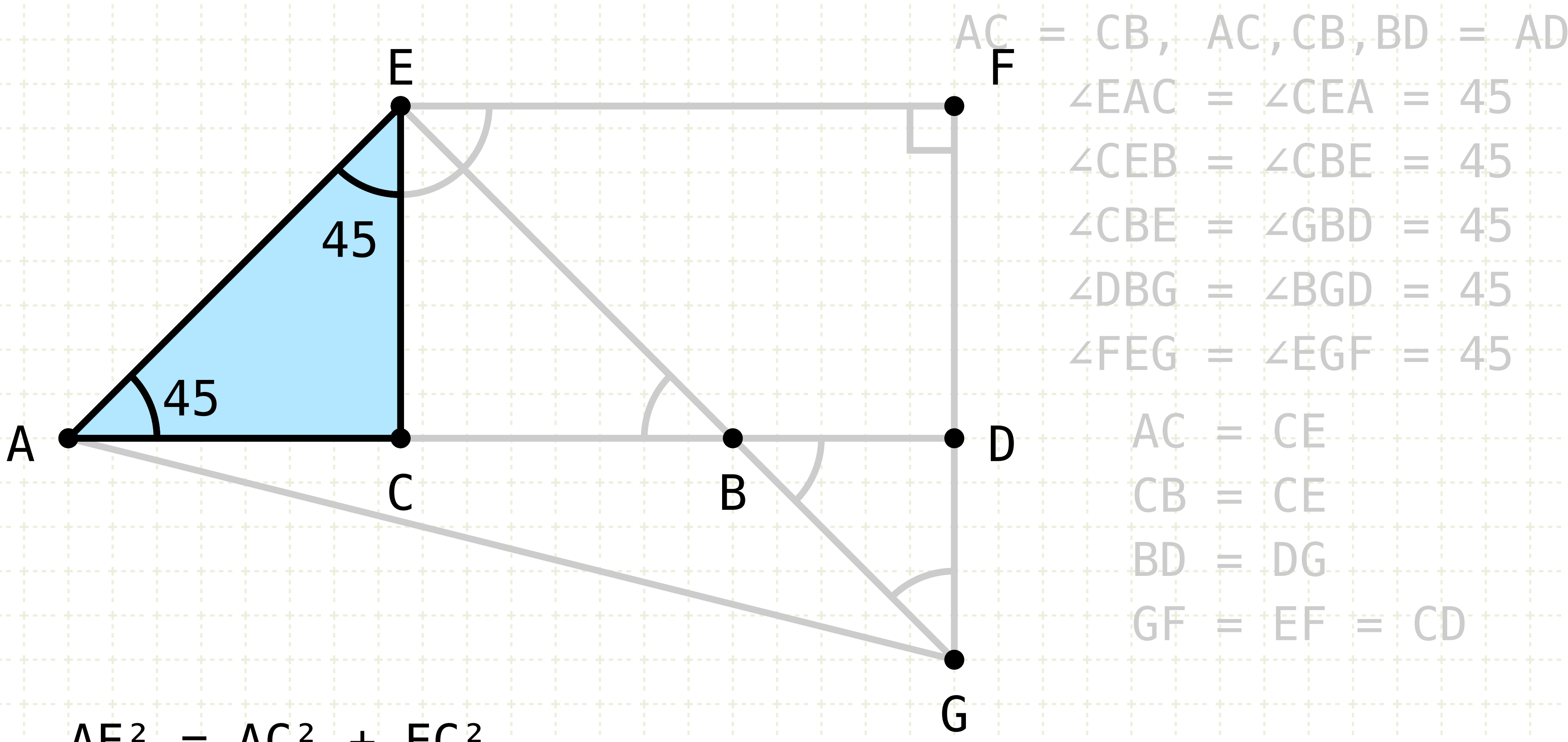
EC and FG are parallel, thus opposite and interior angles are equal (I·29), therefore BDG is a right angle

The angle DBG is one half a right angle, therefore BGD is one half a right angle (I·32), so BD equals DG (I·6)

Using the same logic, FEG is also an isosceles triangle, and GF equals EF

Proposition 10 of Book II

If a straight line be bisected, and a straight line be added to it in a straight line, the square on the whole with the added straight line and the square on the added straight line both together are double of the square on the half and of the square described on the straight line made up of the half and the added straight line as on one straight line.



$$AE^2 = AC^2 + EC^2$$

In other words

Let AB be a straight line, bisected at point C, and extended to an arbitrary point D

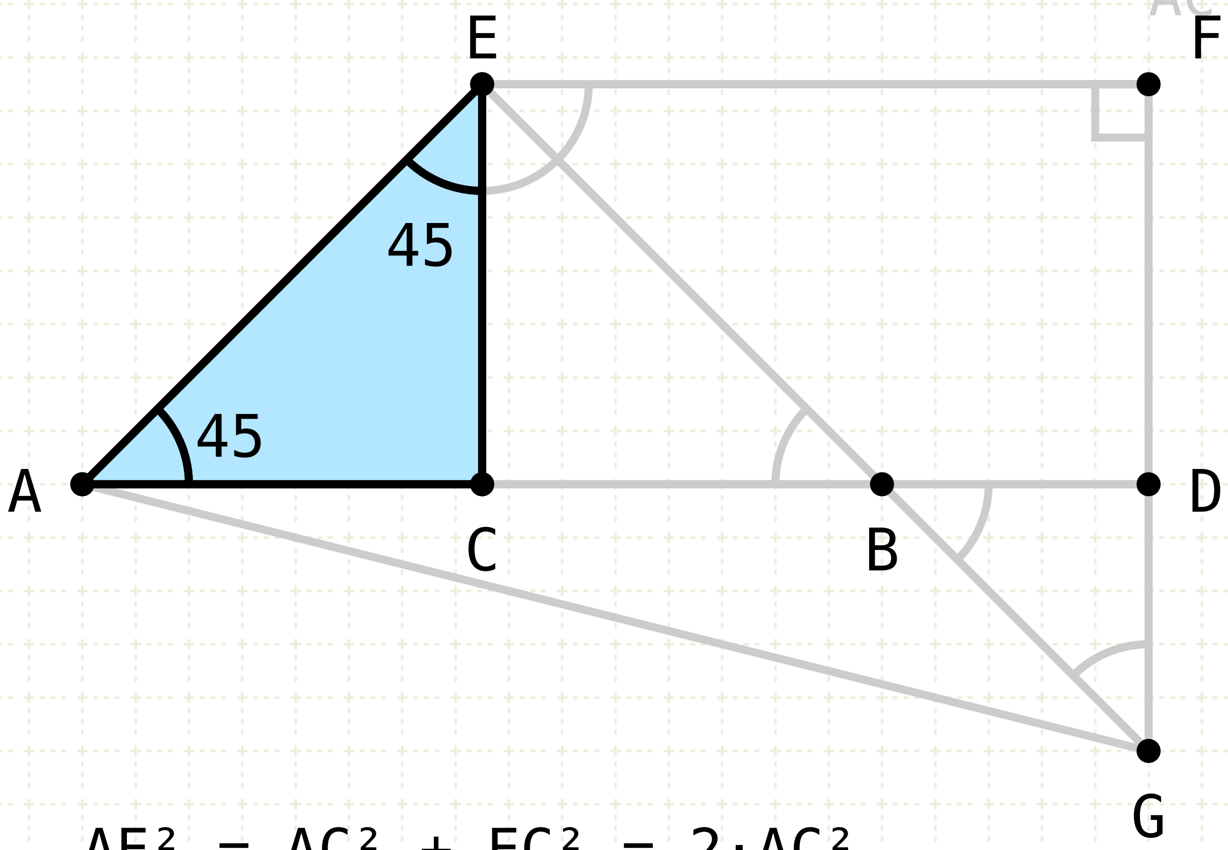
The sum of the squares of AD and DB is equal to twice the sum of the squares of AC and DC

Proof

The triangle AEC is a right angle, thus the square on AE equals the sum of the squares of AC and AE

Proposition 10 of Book II

If a straight line be bisected, and a straight line be added to it in a straight line, the square on the whole with the added straight line and the square on the added straight line both together are double of the square on the half and of the square described on the straight line made up of the half and the added straight line as on one straight line.



AC = CB, AC,CB,BD = AD

∠EAC = ∠CEA = 45

∠CEB = ∠CBE = 45

∠CBE = ∠GBD = 45

∠DBG = ∠BGD = 45

∠FEG = ∠EGF = 45

AC = CE

CB = CE

BD = DG

GF = EF = CD

$AE^2 = AC^2 + EC^2 = 2 \cdot AC^2$

In other words

Let AB be a straight line, bisected at point C, and extended to an arbitrary point D

The sum of the squares of AD and DB is equal to twice the sum of the squares of AC and DC

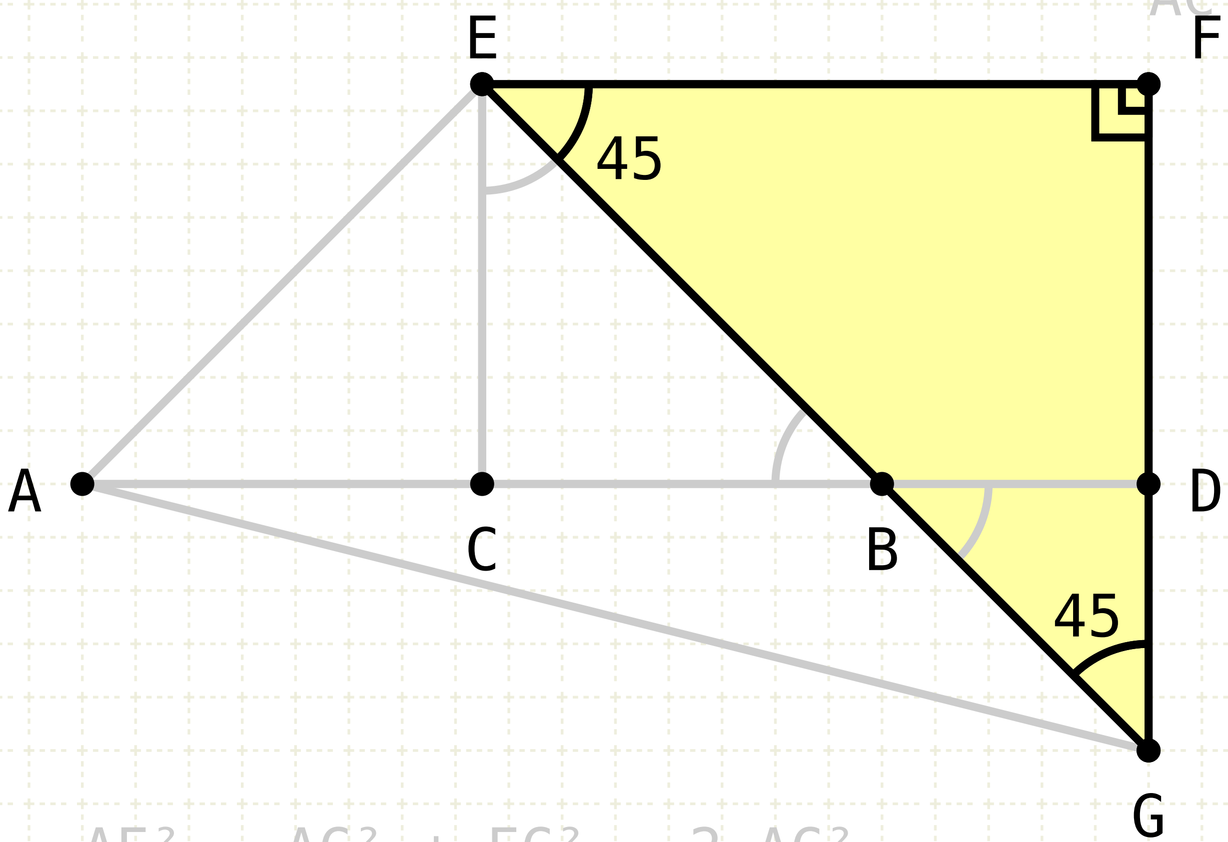
Proof

The triangle AEC is a right angle, thus the square on AE equals the sum of the squares of AC and AE

Since AC equals CE, the sum of the squares of AC and CE equals twice the square of AC

Proposition 10 of Book II

If a straight line be bisected, and a straight line be added to it in a straight line, the square on the whole with the added straight line and the square on the added straight line both together are double of the square on the half and of the square described on the straight line made up of the half and the added straight line as on one straight line.



$AC = CB,\ AC,CB,BD = AD$

$\angle EAC = \angle CEA = 45$

$\angle CEB = \angle CBE = 45$

$\angle CBE = \angle GBD = 45$

$\angle DBG = \angle BGD = 45$

$\angle FEG = \angle EGF = 45$

$AC = CE$

$CB = CE$

$BD = DG$

$GF = EF = CD$

$AE^2 = AC^2 + EC^2 = 2 \cdot AC^2$

$EG^2 = EF^2 + FG^2 = 2 \cdot CD^2$

In other words

Let AB be a straight line, bisected at point C, and extended to an arbitrary point D

The sum of the squares of AD and DB is equal to twice the sum of the squares of AC and DC

Proof

The triangle AEC is a right angle, thus the square on AE equals the sum of the squares of AC and AE

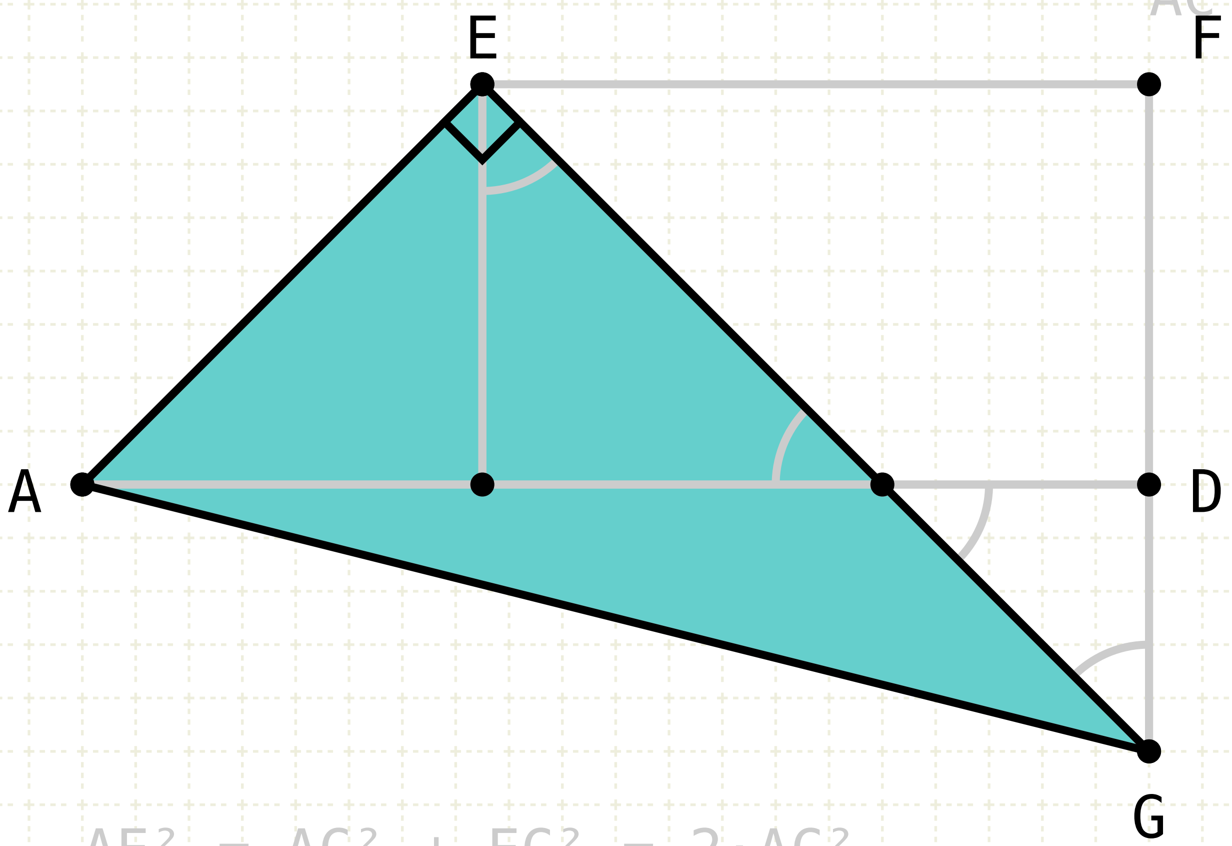
Since AC equals CE, the sum of the squares of AC and CE equals twice the square of AC

The triangle EGF is a right angle, thus the square on EG equals the sum of the squares of EF and FG (I·47)

EF equals FG, and EF equals CD (I·34) the sum of the squares of EF and FG equals twice the square of CD

Proposition 10 of Book II

If a straight line be bisected, and a straight line be added to it in a straight line, the square on the whole with the added straight line and the square on the added straight line both together are double of the square on the half and of the square described on the straight line made up of the half and the added straight line as on one straight line.



$AC = CB, \quad AC, CB, BD = AD$

$\angle EAC = \angle CEA = 45$

$\angle CEB = \angle CBE = 45$

$\angle CBE = \angle GBD = 45$

$\angle DBG = \angle BGD = 45$

$\angle FEG = \angle EGF = 45$

$AC = CE$

$CB = CE$

$BD = DG$

$GF = EF = CD$

$AE^2 = AC^2 + EC^2 = 2 \cdot AC^2$

$EG^2 = EF^2 + FG^2 = 2 \cdot CD^2$

$AG^2 = AE^2 + EG^2$

In other words

Let AB be a straight line, bisected at point C, and extended to an arbitrary point D

The sum of the squares of AD and DB is equal to twice the sum of the squares of AC and DC

Proof

The triangle AEC is a right angle, thus the square on AE equals the sum of the squares of AC and AE

Since AC equals CE, the sum of the squares of AC and CE equals twice the square of AC

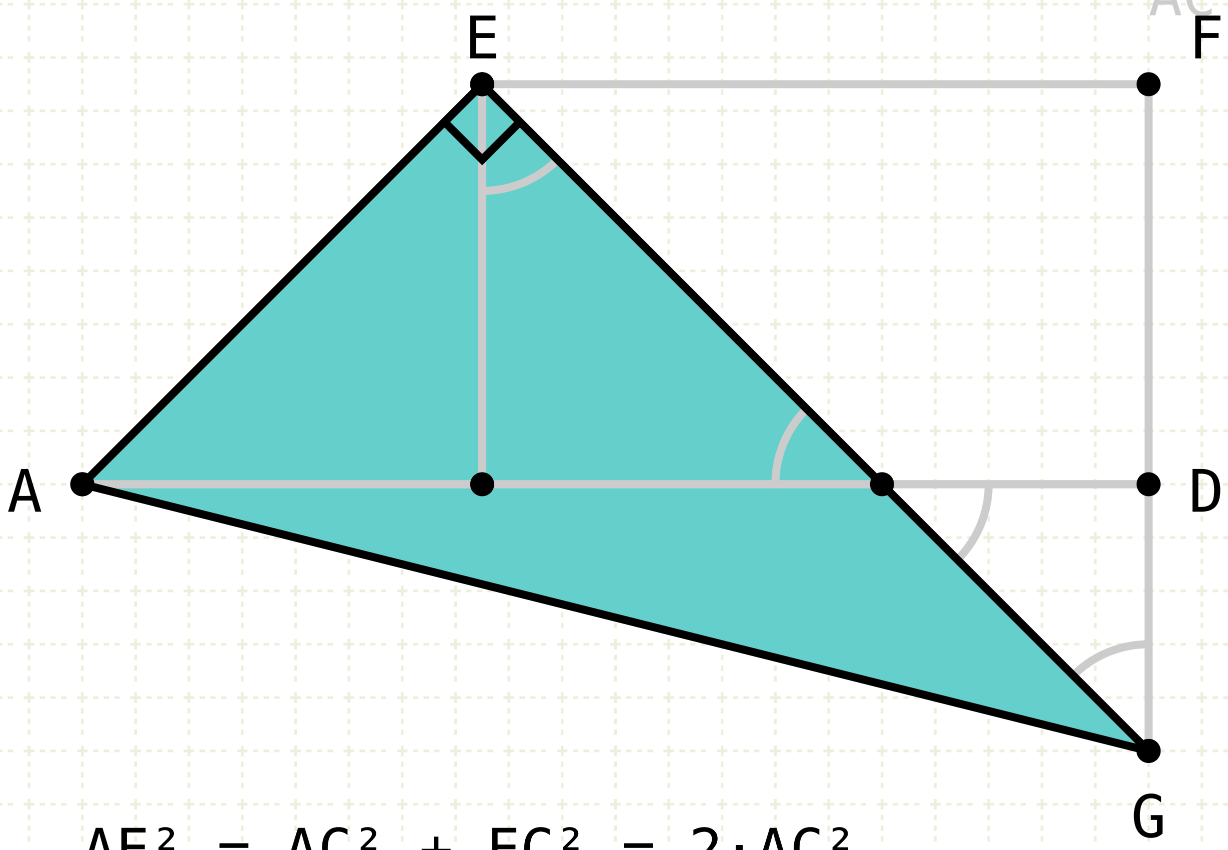
The triangle EGF is a right angle, thus the square on EG equals the sum of the squares of EF and FG (I·47)

EF equals FG, and EF equals CD (I·34) the sum of the squares of EF and FG equals twice the square of CD

The triangle AGE is a right angle, thus the square on AG equals the sum of the squares of AE and EG

Proposition 10 of Book II

If a straight line be bisected, and a straight line be added to it in a straight line, the square on the whole with the added straight line and the square on the added straight line both together are double of the square on the half and of the square described on the straight line made up of the half and the added straight line as on one straight line.



AC = CB, AC,CB,BD = AD

$\angle EAC = \angle CEA = 45$

$\angle CEB = \angle CBE = 45$

$\angle CBE = \angle GBD = 45$

$\angle DBG = \angle BGD = 45$

$\angle FEG = \angle EGF = 45$

AC = CE

CB = CE

BD = DG

GF = EF = CD

$AE^2 = AC^2 + EC^2 = 2 \cdot AC^2$

$EG^2 = EF^2 + FG^2 = 2 \cdot CD^2$

$AG^2 = AE^2 + EG^2$

$AG^2 = 2 \cdot AC^2 + 2 \cdot CD^2 = 2(AC^2 + CD^2)$

In other words

Let AB be a straight line, bisected at point C, and extended to an arbitrary point D

The sum of the squares of AD and DB is equal to twice the sum of the squares of AC and DC

Proof

The triangle AEC is a right angle, thus the square on AE equals the sum of the squares of AC and AE

Since AC equals CE, the sum of the squares of AC and CE equals twice the square of AC

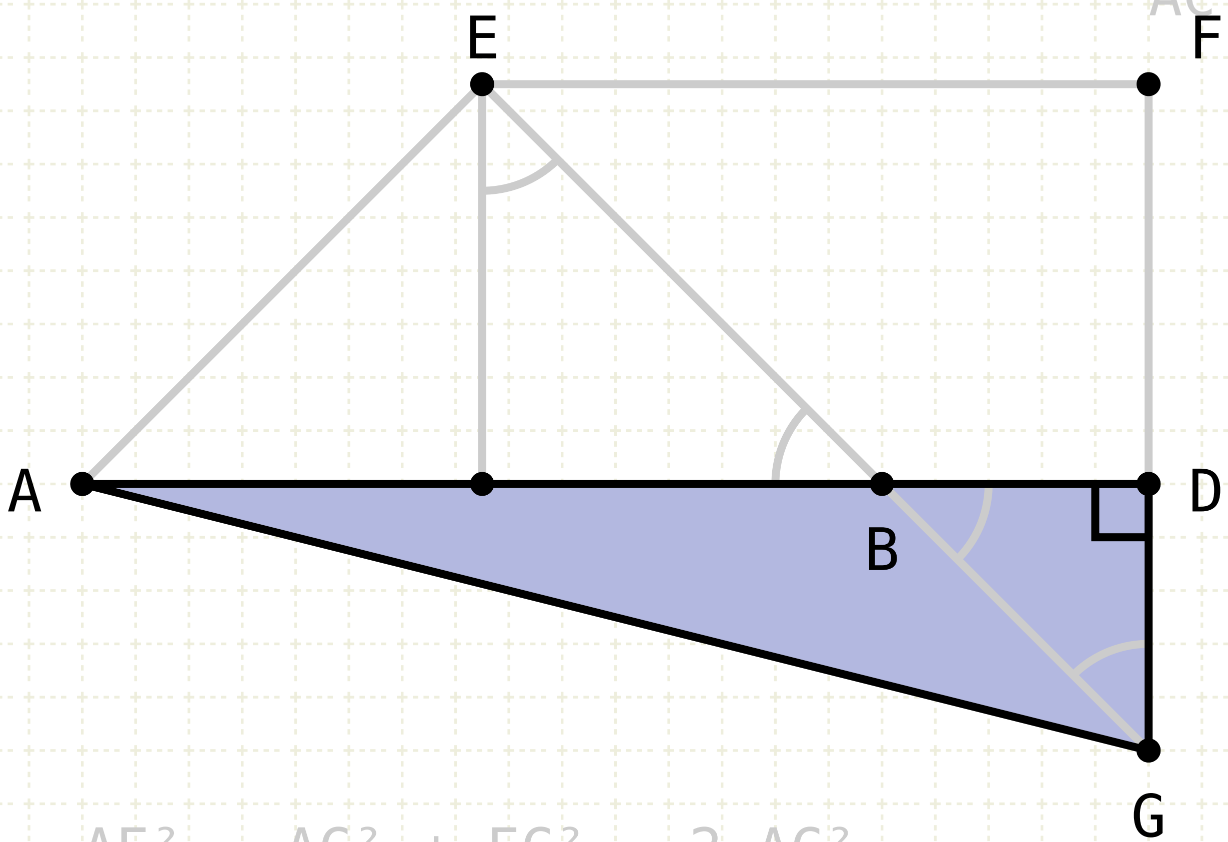
The triangle EGF is a right angle, thus the square on EG equals the sum of the squares of EF and FG (I·47)

EF equals FG, and EF equals CD (I·34) the sum of the squares of EF and FG equals twice the square of CD

The triangle AGE is a right angle, thus the square on AG equals the sum of the squares of AE and EG

Proposition 10 of Book II

If a straight line be bisected, and a straight line be added to it in a straight line, the square on the whole with the added straight line and the square on the added straight line both together are double of the square on the half and of the square described on the straight line made up of the half and the added straight line as on one straight line.



$AC = CB,\ AC,CB,BD = AD$

$\angle EAC = \angle CEA = 45$

$\angle CEB = \angle CBE = 45$

$\angle CBE = \angle GBD = 45$

$\angle DBG = \angle BGD = 45$

$\angle FEG = \angle EGF = 45$

$AC = CE$

$CB = CE$

$BD = DG$

$GF = EF = CD$

$AE^2 = AC^2 + EC^2 = 2 \cdot AC^2$

$EG^2 = EF^2 + FG^2 = 2 \cdot CD^2$

$AG^2 = AE^2 + EG^2$

$AG^2 = 2 \cdot AC^2 + 2 \cdot CD^2 = 2(AC^2 + CD^2)$

$AG^2 = AD^2 + DG^2$

In other words

Let AB be a straight line, bisected at point C, and extended to an arbitrary point D

The sum of the squares of AD and DB is equal to twice the sum of the squares of AC and DC

Proof

The triangle AEC is a right angle, thus the square on AE equals the sum of the squares of AC and AE

Since AC equals CE, the sum of the squares of AC and CE equals twice the square of AC

The triangle EGF is a right angle, thus the square on EG equals the sum of the squares of EF and FG (I·47)

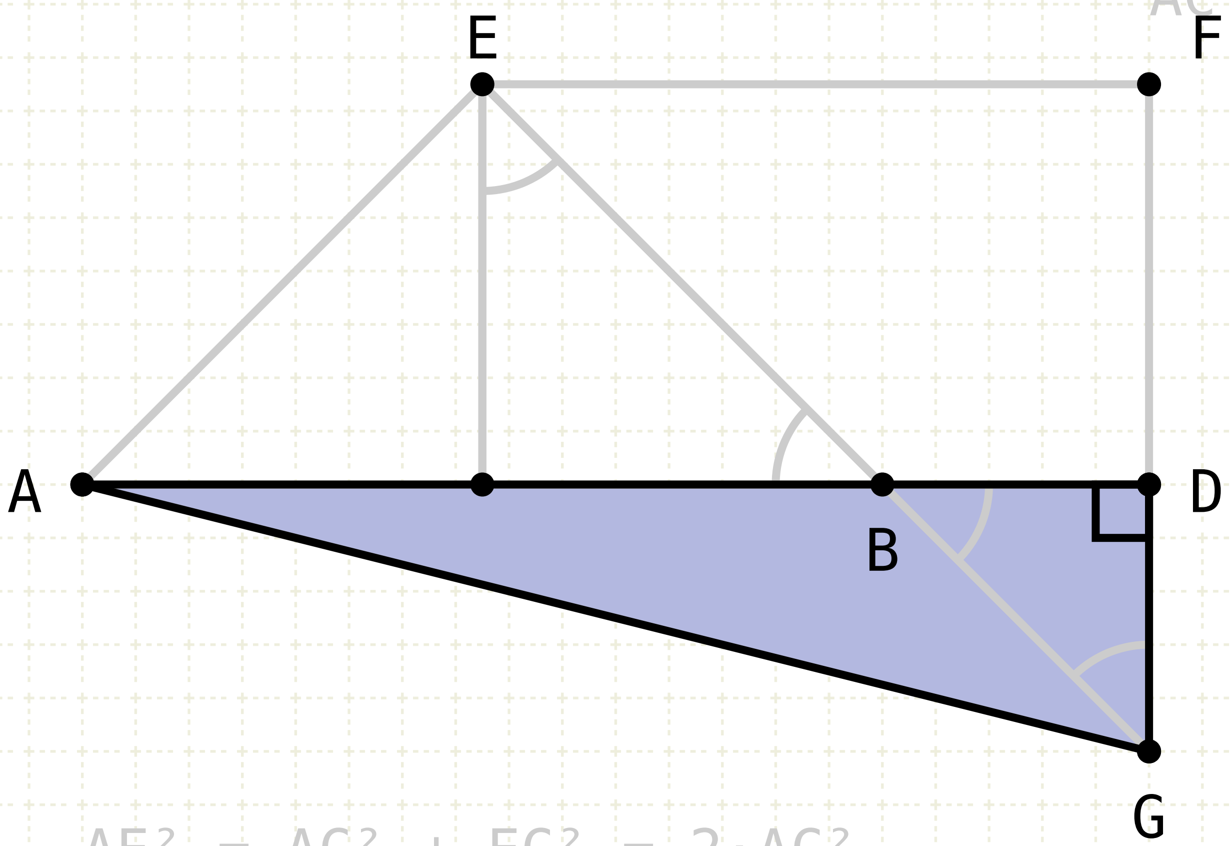
EF equals FG, and EF equals CD (I·34) the sum of the squares of EF and FG equals twice the square of CD

The triangle AGE is a right angle, thus the square on AG equals the sum of the squares of AE and EG

The triangle AGD is a right angle, thus the square on AG equals the sum of the squares of AD and DG

Proposition 10 of Book II

If a straight line be bisected, and a straight line be added to it in a straight line, the square on the whole with the added straight line and the square on the added straight line both together are double of the square on the half and of the square described on the straight line made up of the half and the added straight line as on one straight line.



$AC = CB,\ AC, CB, BD = AD$

$\angle EAC = \angle CEA = 45$

$\angle CEB = \angle CBE = 45$

$\angle CBE = \angle GBD = 45$

$\angle DBG = \angle BGD = 45$

$\angle FEG = \angle EGF = 45$

$AC = CE$

$CB = CE$

$BD = DG$

$GF = EF = CD$

$AE^2 = AC^2 + EC^2 = 2 \cdot AC^2$

$EG^2 = EF^2 + FG^2 = 2 \cdot CD^2$

$AG^2 = AE^2 + EG^2$

$AG^2 = 2 \cdot AC^2 + 2 \cdot CD^2 = 2(AC^2 + CD^2)$

$AG^2 = AD^2 + DG^2$

$AG^2 = AD^2 + DB^2$

In other words

Let AB be a straight line, bisected at point C, and extended to an arbitrary point D

The sum of the squares of AD and DB is equal to twice the sum of the squares of AC and DC

Proof

The triangle AEC is a right angle, thus the square on AE equals the sum of the squares of AC and AE

Since AC equals CE, the sum of the squares of AC and CE equals twice the square of AC

The triangle EGF is a right angle, thus the square on EG equals the sum of the squares of EF and FG (I·47)

EF equals FG, and EF equals CD (I·34) the sum of the squares of EF and FG equals twice the square of CD

The triangle AGE is a right angle, thus the square on AG equals the sum of the squares of AE and EG

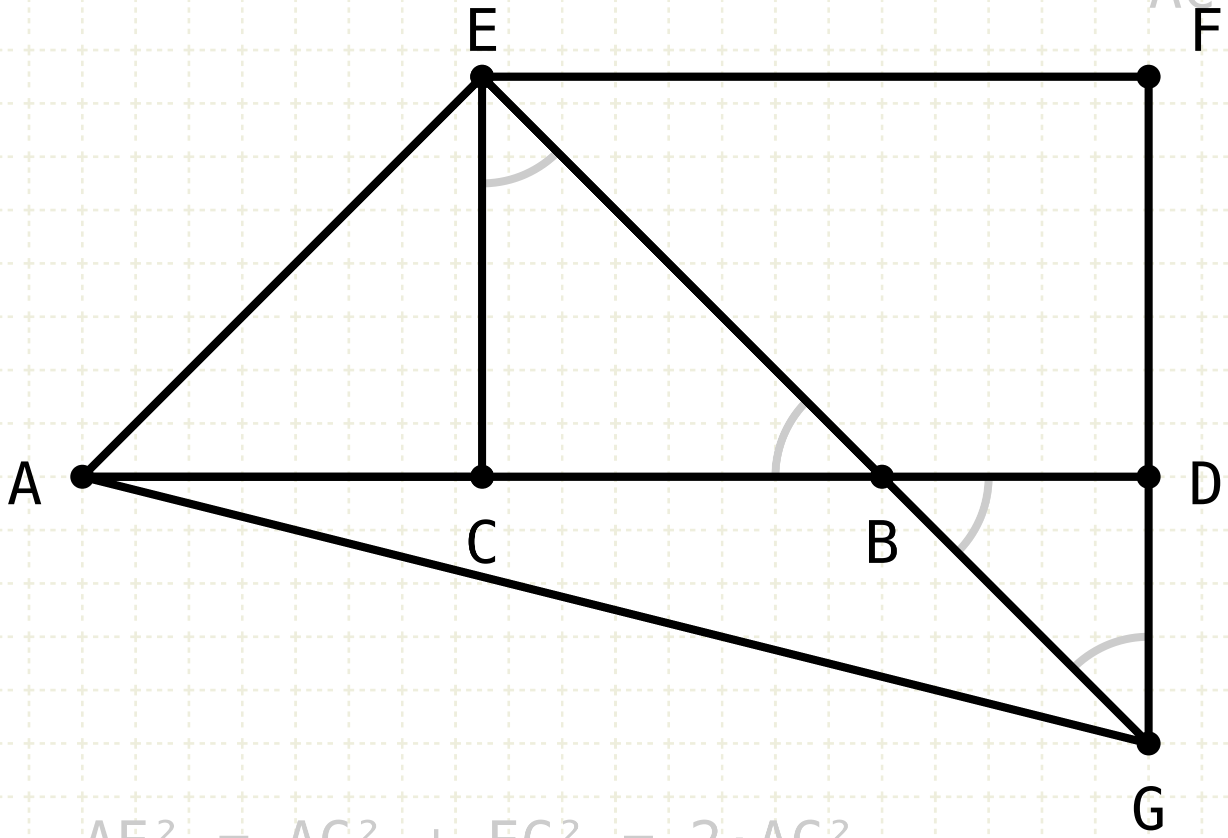
The triangle AGD is a right angle, thus the square on AG equals the sum of the squares of AD and DG

But DG equals DB, so the square of AG is the sum of the squares of AD and DB

Proposition 10 of Book II

If a straight line be bisected, and a straight line be added to it in a straight line, the square on the whole with the added straight line and the square on the added straight line both together are double of the square on the half and of the square described on the straight line made up of the half and the added straight line as on one straight line.

$AC = CB,\ AC, CB, BD = AD$



$\angle EAC = \angle CEA = 45$

$\angle CEB = \angle CBE = 45$

$\angle CBE = \angle GBD = 45$

$\angle DBG = \angle BGD = 45$

$\angle FEG = \angle EGF = 45$

$AC = CE$

$CB = CE$

$BD = DG$

$GF = EF = CD$

$AE^2 = AC^2 + EC^2 = 2 \cdot AC^2$

$EG^2 = EF^2 + FG^2 = 2 \cdot CD^2$

$AG^2 = AE^2 + EG^2$

$AG^2 = 2 \cdot AC^2 + 2 \cdot CD^2 = 2(AC^2 + CD^2)$

$AG^2 = AD^2 + DG^2$

$AG^2 = AD^2 + DB^2$

$AD^2 + DB^2 = 2(AC^2 + CD^2)$

In other words

Let AB be a straight line, bisected at point C, and extended to an arbitrary point D

The sum of the squares of AD and DB is equal to twice the sum of the squares of AC and DC

Proof

The triangle AEC is a right angle, thus the square on AE equals the sum of the squares of AC and AE

Since AC equals CE, the sum of the squares of AC and CE equals twice the square of AC

The triangle EGF is a right angle, thus the square on EG equals the sum of the squares of EF and FG (I·47)

EF equals FG, and EF equals CD (I·34) the sum of the squares of EF and FG equals twice the square of CD

The triangle AGE is a right angle, thus the square on AG equals the sum of the squares of AE and EG

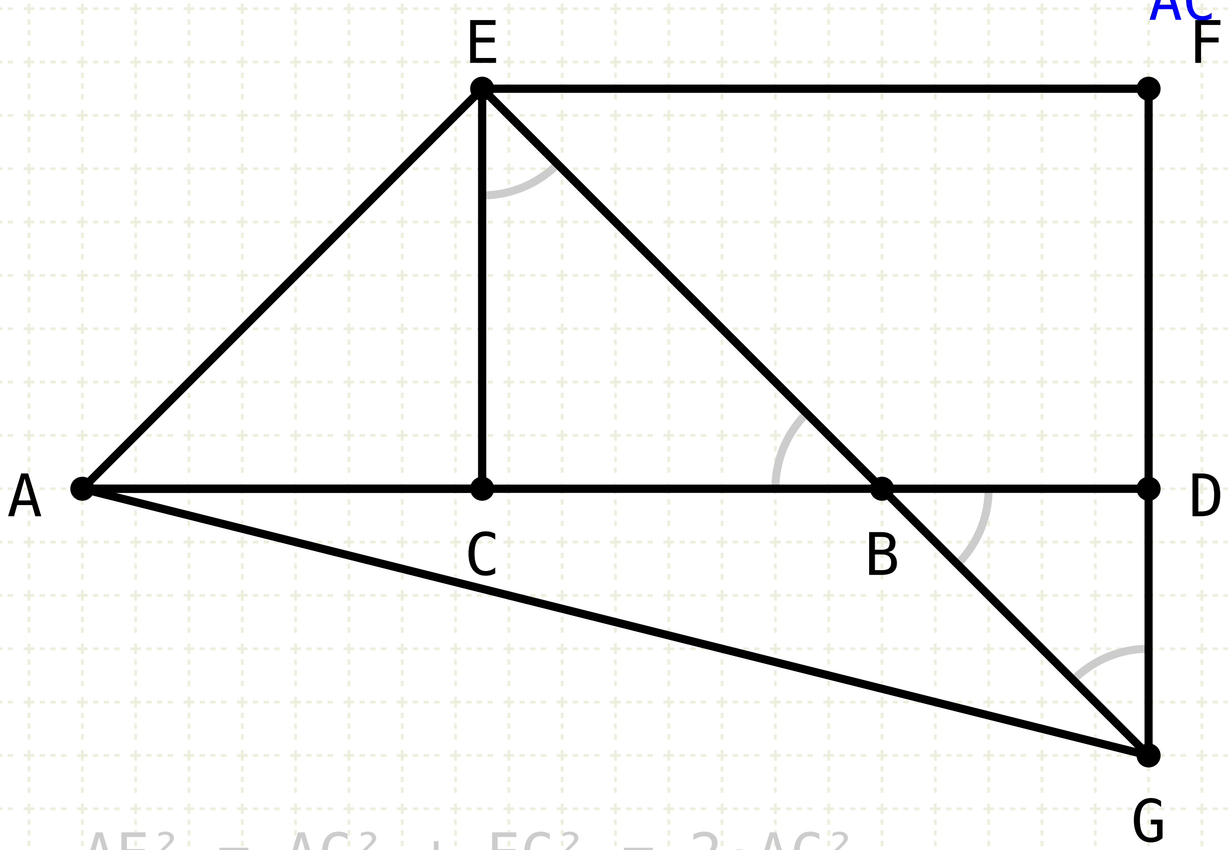
The triangle AGD is a right angle, thus the square on AG equals the sum of the squares of AD and DG

But DG equals DB, so the square of AG is the sum of the squares of AD and DB

Rearranging the equalities gives the original postulate

Proposition 10 of Book II

If a straight line be bisected, and a straight line be added to it in a straight line, the square on the whole with the added straight line and the square on the added straight line both together are double of the square on the half and of the square described on the straight line made up of the half and the added straight line as on one straight line.



$AC = CB,\ AC, CB, BD = AD$

$\angle EAC = \angle CEA = 45$

$\angle CEB = \angle CBE = 45$

$\angle CBE = \angle GBD = 45$

$\angle DBG = \angle BGD = 45$

$\angle FEG = \angle EGF = 45$

$AC = CE$

$CB = CE$

$BD = DG$

$GF = EF = CD$

$AE^2 = AC^2 + EC^2 = 2 \cdot AC^2$

$EG^2 = EF^2 + FG^2 = 2 \cdot CD^2$

$AG^2 = AE^2 + EG^2$

$AG^2 = 2 \cdot AC^2 + 2 \cdot CD^2 = 2(AC^2 + CD^2)$

$AG^2 = AD^2 + DG^2$

$AG^2 = AD^2 + DB^2$

$$AD^2 + DB^2 = 2(AC^2 + CD^2)$$

In other words

Let AB be a straight line, bisected at point C, and extended to an arbitrary point D

The sum of the squares of AD and DB is equal to twice the sum of the squares of AC and DC

Proof

The triangle AEC is a right angle, thus the square on AE equals the sum of the squares of AC and AE

Since AC equals CE, the sum of the squares of AC and CE equals twice the square of AC

The triangle EGF is a right angle, thus the square on EG equals the sum of the squares of EF and FG (I·47)

EF equals FG, and EF equals CD (I·34) the sum of the squares of EF and FG equals twice the square of CD

The triangle AGE is a right angle, thus the square on AG equals the sum of the squares of AE and EG

The triangle AGD is a right angle, thus the square on AG equals the sum of the squares of AD and DG

But DG equals DB, so the square of AG is the sum of the squares of AD and DB

Rearranging the equalities gives the original postulate

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