

Euclid's Elements

Book I

*If Euclid did not kindle your youthful enthusiasm, you
were not born to be a scientific thinker.*

Albert Einstein

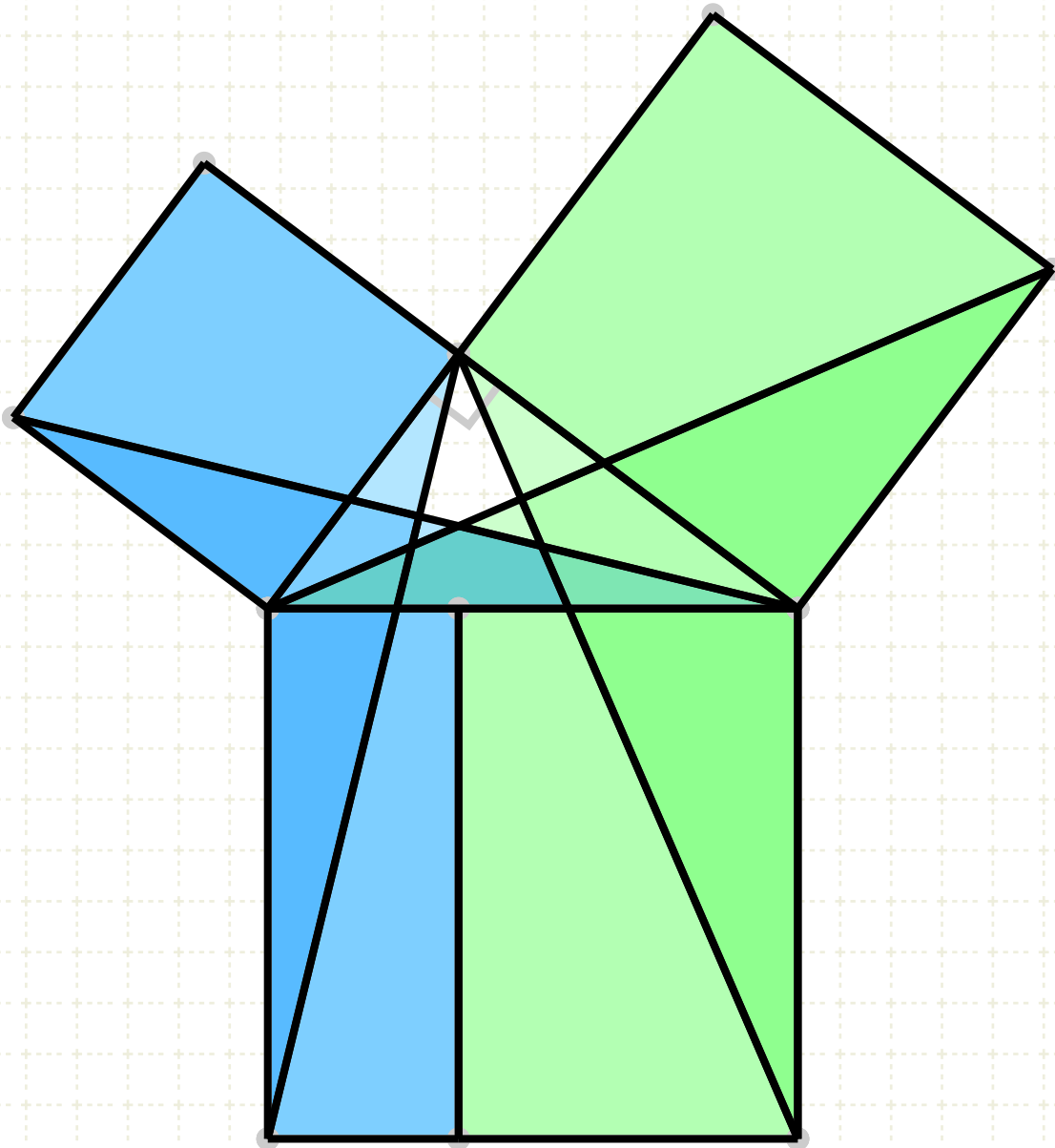


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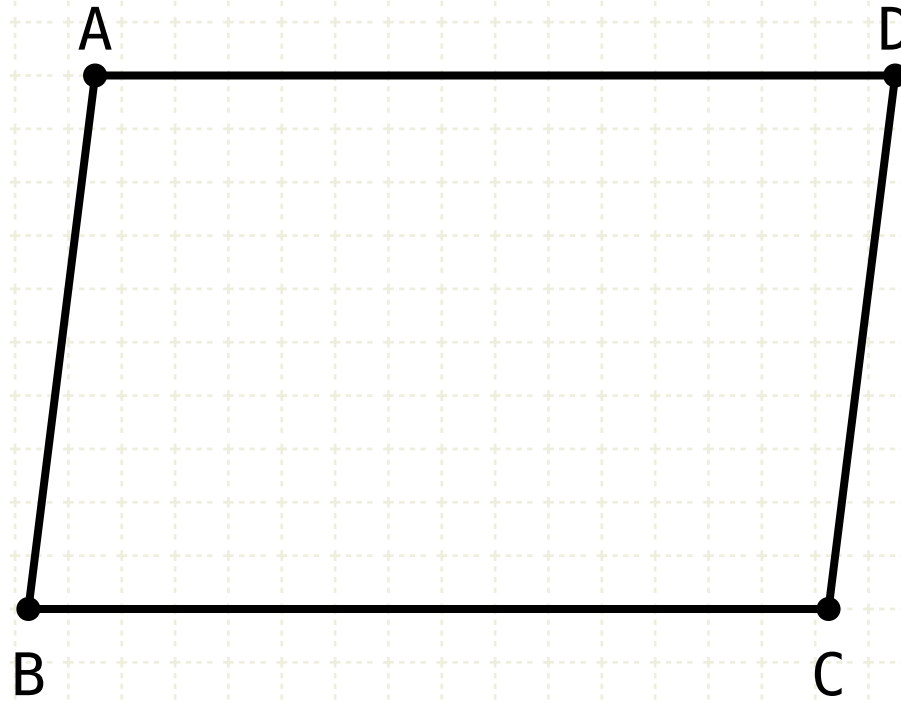
Proposition 43 of Book I

In any parallelogram the complements of the parallelograms about the diameter equal one another.



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In any parallelogram the complements of the parallelograms about the diameter equal one another.

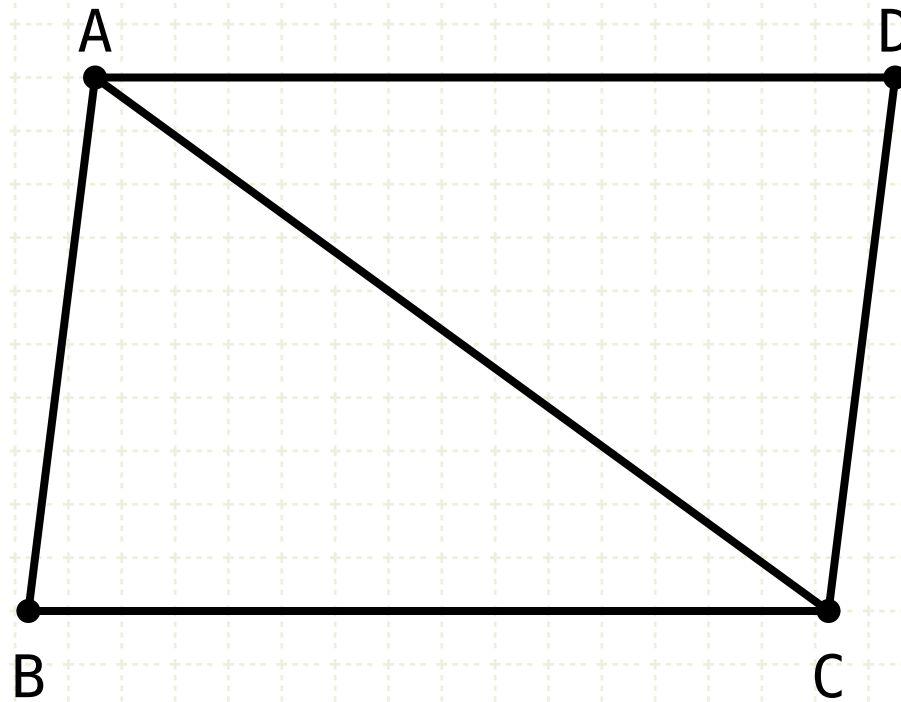


In other words

Given a parallelogram ABCD

Proposition 43 of Book I

In any parallelogram the complements of the parallelograms about the diameter equal one another.



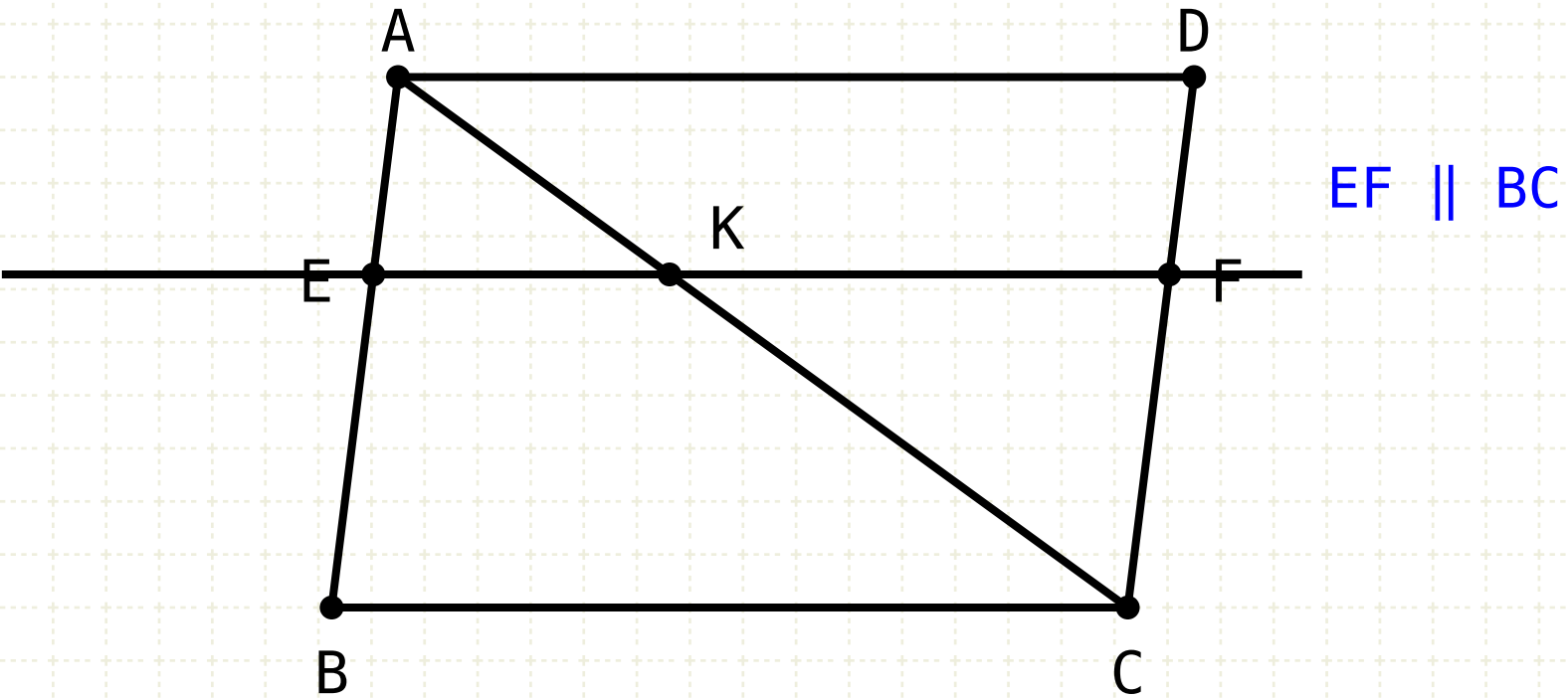
In other words

Given a parallelogram ABCD

With a diameter AC

Proposition 43 of Book I

In any parallelogram the complements of the parallelograms about the diameter equal one another.



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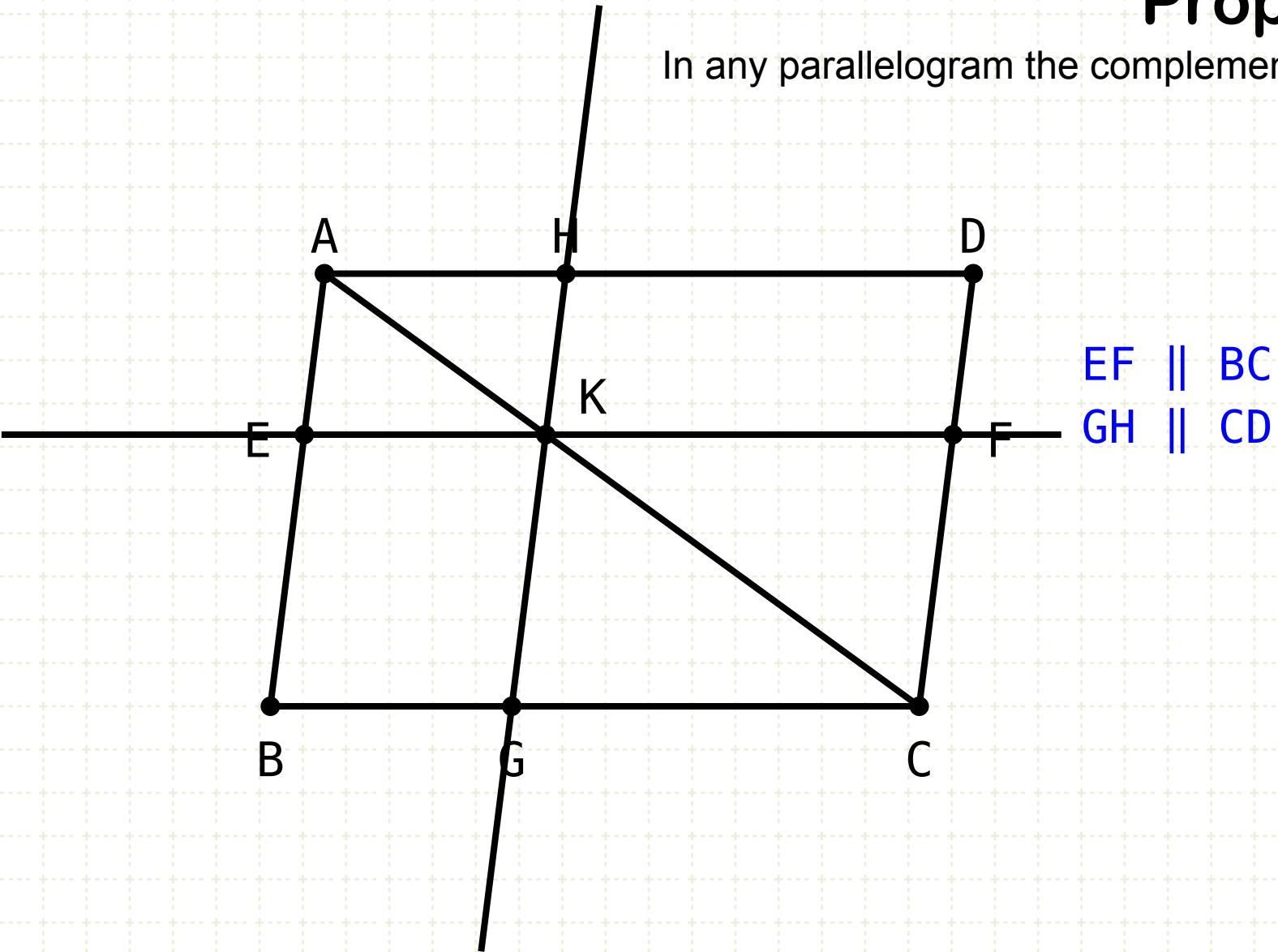
Given a parallelogram ABCD

With a diameter AC

And, from an arbitrary point E on line AB, draw a line parallel to AD intersecting the diagonal at point K

Proposition 43 of Book I

In any parallelogram the complements of the parallelograms about the diameter equal one another.



In other words

Given a parallelogram $ABCD$

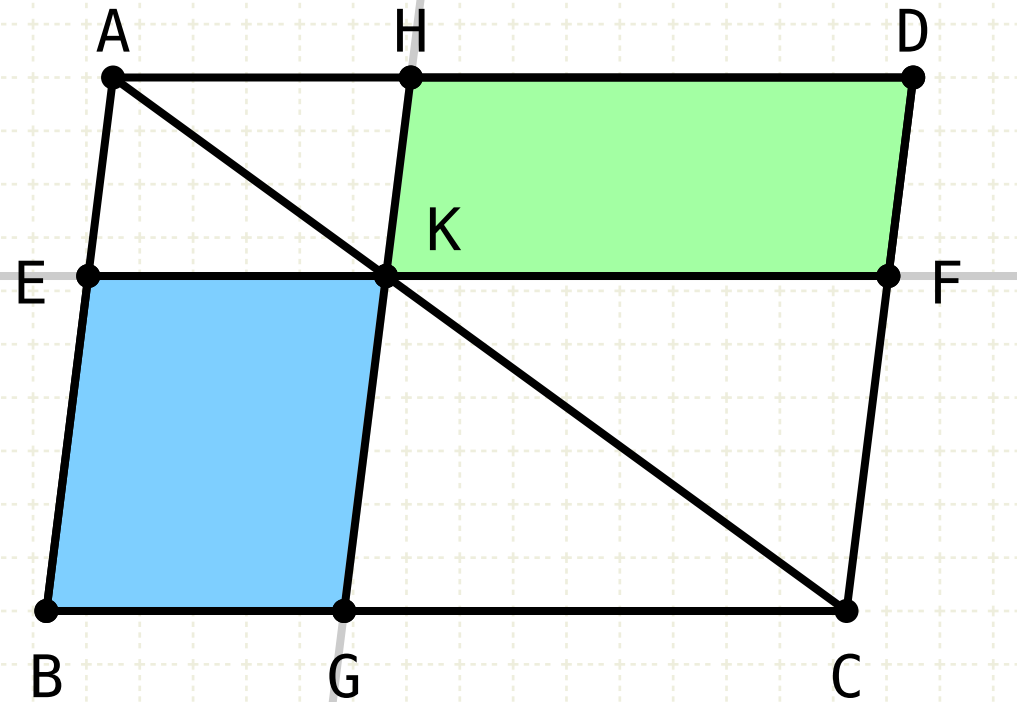
With a diameter AC

And, from an arbitrary point E on line AB , draw a line parallel to AD intersecting the diagonal at point K

And finally, draw a line parallel to AB through point K , intersecting the parallelogram at points G and H

Proposition 43 of Book I

In any parallelogram the complements of the parallelograms about the diameter equal one another.



$EF \parallel BC$
 $GH \parallel CD$
 $EBGK = HKFD$

In other words

Given a parallelogram ABCD

With a diameter AC

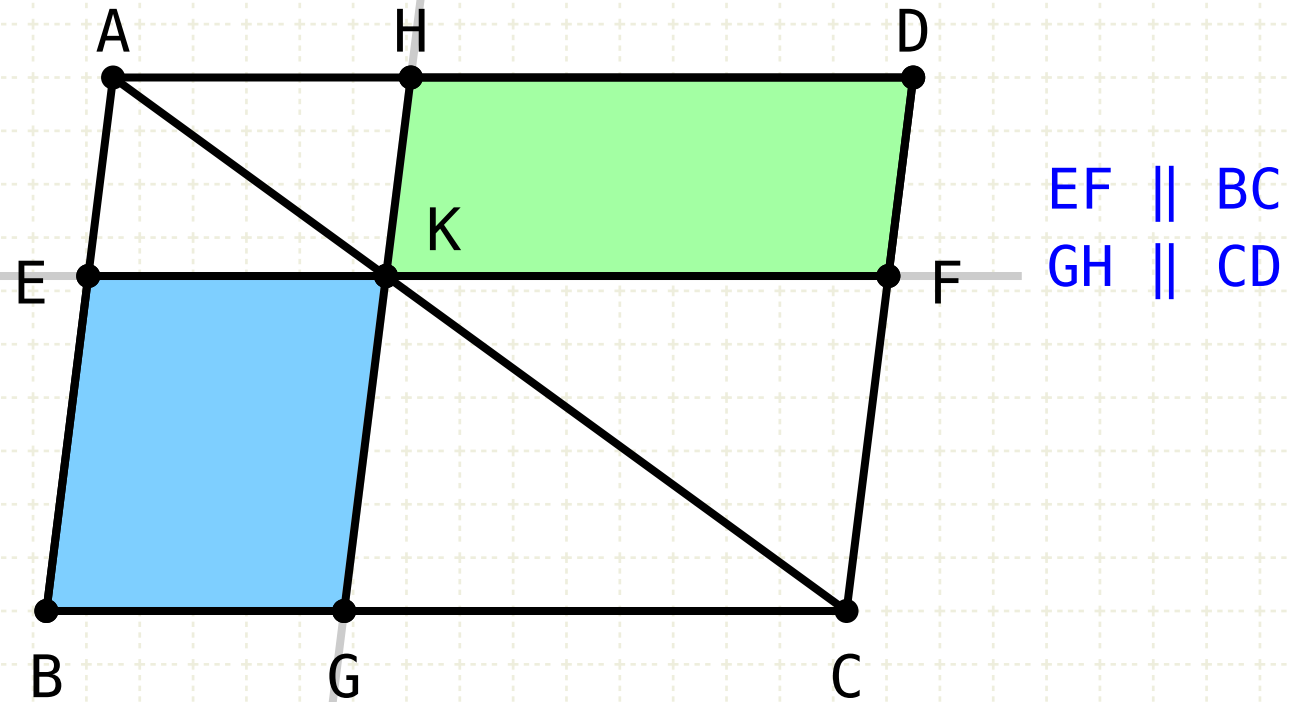
And, from an arbitrary point E on line AB, draw a line parallel to AD intersecting the diagonal at point K

And finally, draw a line parallel to AB through point K, intersecting the parallelogram at points G and H

Then the parallelograms EBGK and HKFD (complements of ABCD) and are equal

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In any parallelogram the complements of the parallelograms about the diameter equal one another.



In other words

Given a parallelogram ABCD

With a diameter AC

And, from an arbitrary point E on line AB, draw a line parallel to AD intersecting the diagonal at point K

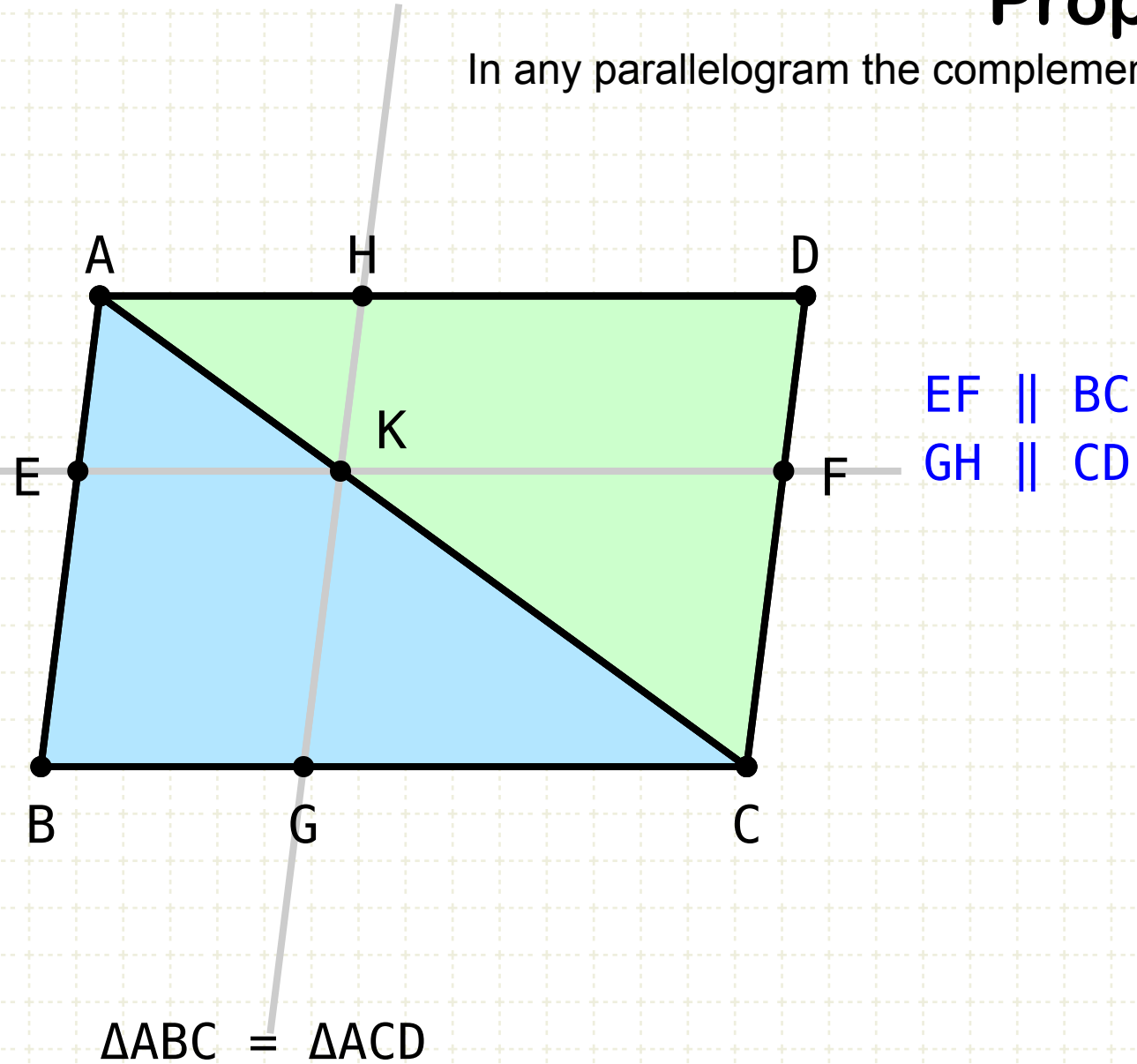
And finally, draw a line parallel to AB through point K, intersecting the parallelogram at points G and H

Then the parallelograms EBKG and HKFD (complements of ABCD) and are equal

Proof:

Proposition 43 of Book I

In any parallelogram the complements of the parallelograms about the diameter equal one another.



In other words

Given a parallelogram ABCD

With a diameter AC

And, from an arbitrary point E on line AB, draw a line parallel to AD intersecting the diagonal at point K

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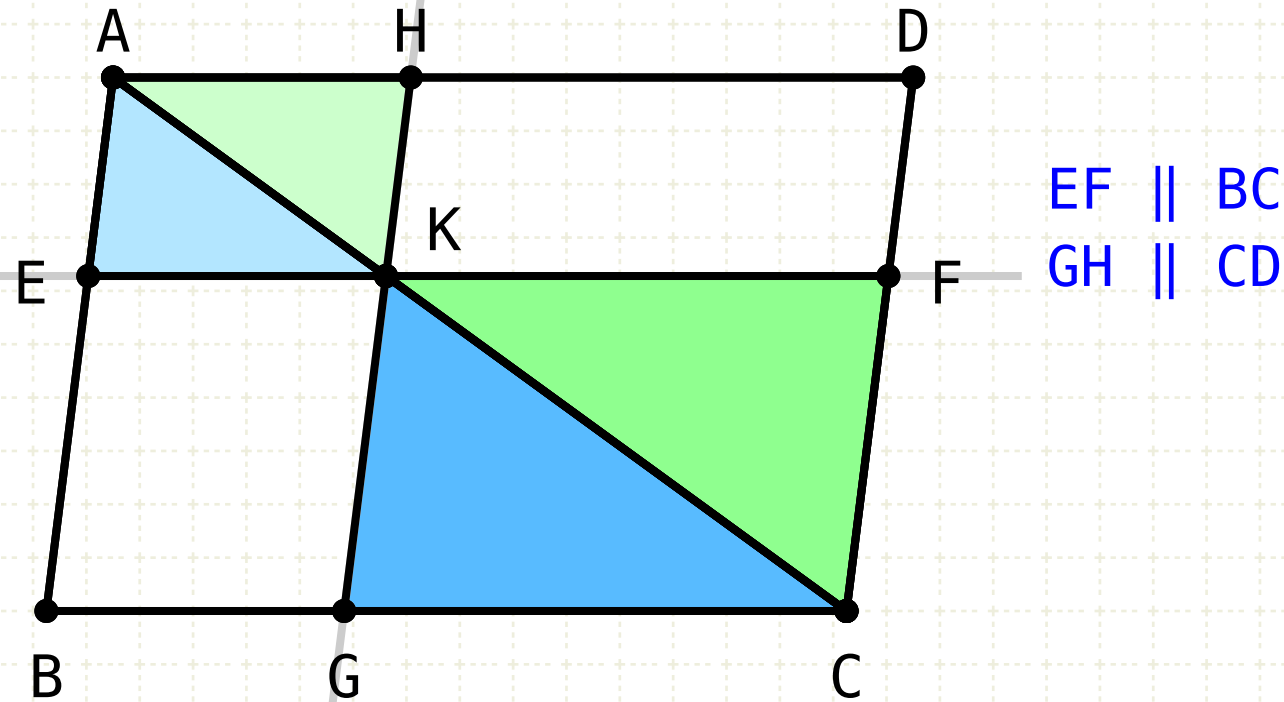
Then the parallelograms EBGK and HKFD (complements of ABCD) and are equal

Proof:

Since AC is the diameter of the parallelogram ABCD, triangles ABC and ACD are equal (I·34)

Proposition 43 of Book I

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And, from an arbitrary point E on line AB, draw a line parallel to AD intersecting the diagonal at point K

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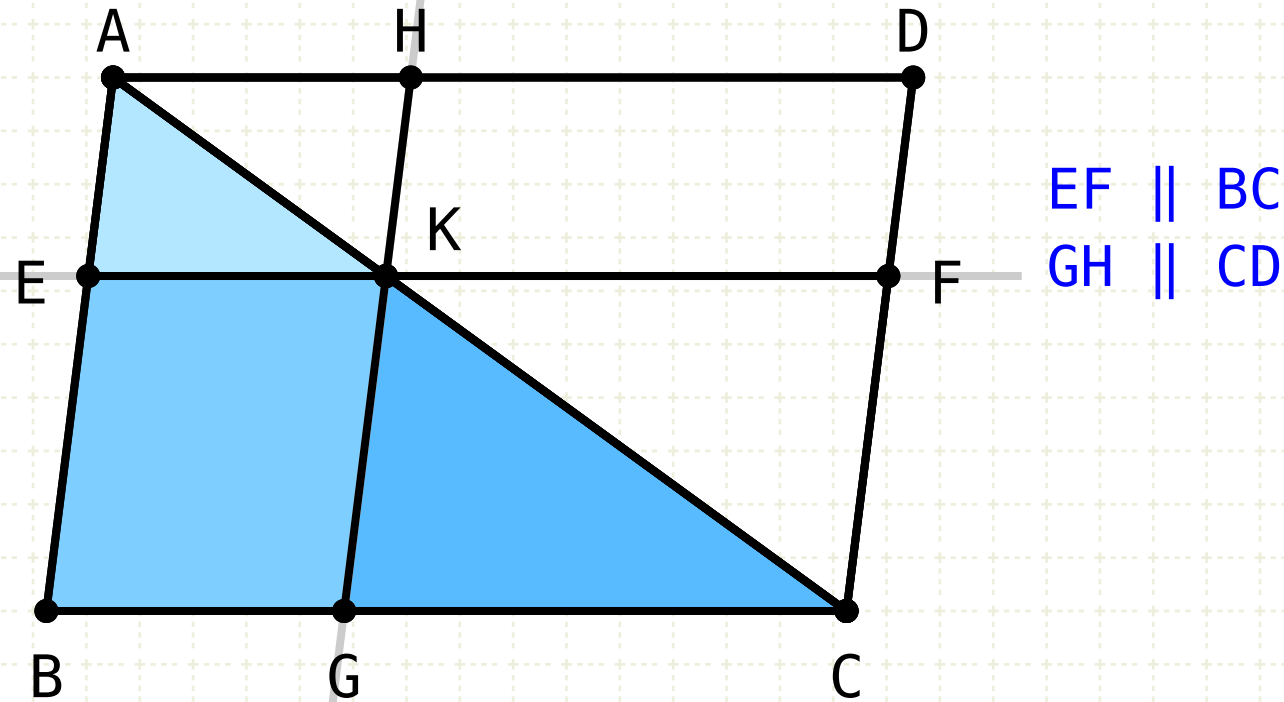
Proof:

Since AC is the diameter of the parallelogram ABCD, triangles ABC and ACD are equal (I·34)

By the same reasoning, triangles AEK and AHK are equal, as well as KGC and KCF (I·34)

Proposition 43 of Book I

In any parallelogram the complements of the parallelograms about the diameter equal one another.



$$\begin{aligned}\triangle ABC &= \triangle ACD \\ \triangle AEK &= \triangle AKH, \quad \triangle KGC = \triangle KCF \\ \triangle ABC &= \triangle AEK + \text{EBGK} + \triangle KGC\end{aligned}$$

In other words

Given a parallelogram ABCD

With a diameter AC

And, from an arbitrary point E on line AB, draw a line parallel to AD intersecting the diagonal at point K

And finally, draw a line parallel to AB through point K, intersecting the parallelogram at points G and H

Then the parallelograms EBGK and HKFD (complements of ABCD) and are equal

Proof:

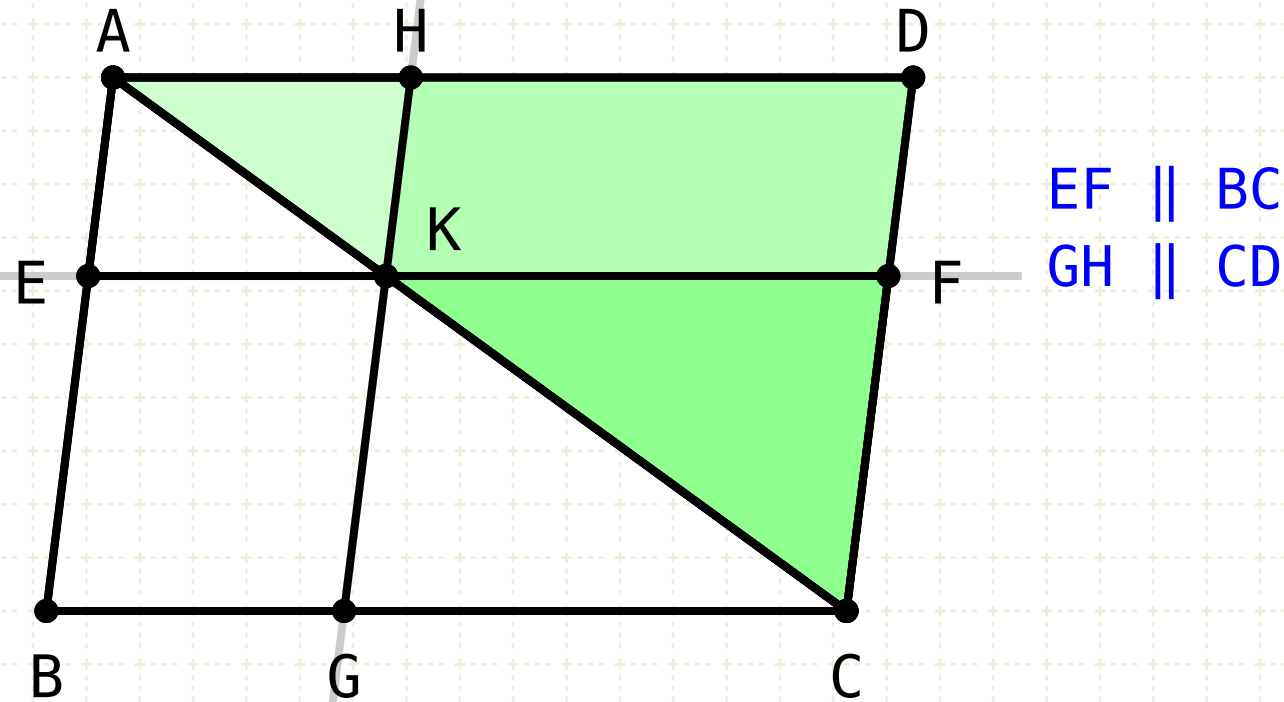
Since AC is the diameter of the parallelogram ABCD, triangles ABC and ACD are equal (I·34)

By the same reasoning, triangles AEK and AHK are equal, as well as KGC and KCF (I·34)

Triangle ABC is the sum of AED, EBGK and KGC

Proposition 43 of Book I

In any parallelogram the complements of the parallelograms about the diameter equal one another.



$$\begin{aligned}\triangle ABC &= \triangle ACD \\ \triangle AEK &= \triangle AKH, \quad \triangle KGC = \triangle KCF \\ \triangle ABC &= \triangle AEK + \text{EBKG} + \triangle KGC \\ \triangle ADC &= \triangle AKH + \text{HKFD} + \triangle KCF\end{aligned}$$

In other words

Given a parallelogram ABCD

With a diameter AC

And, from an arbitrary point E on line AB, draw a line parallel to AD intersecting the diagonal at point K

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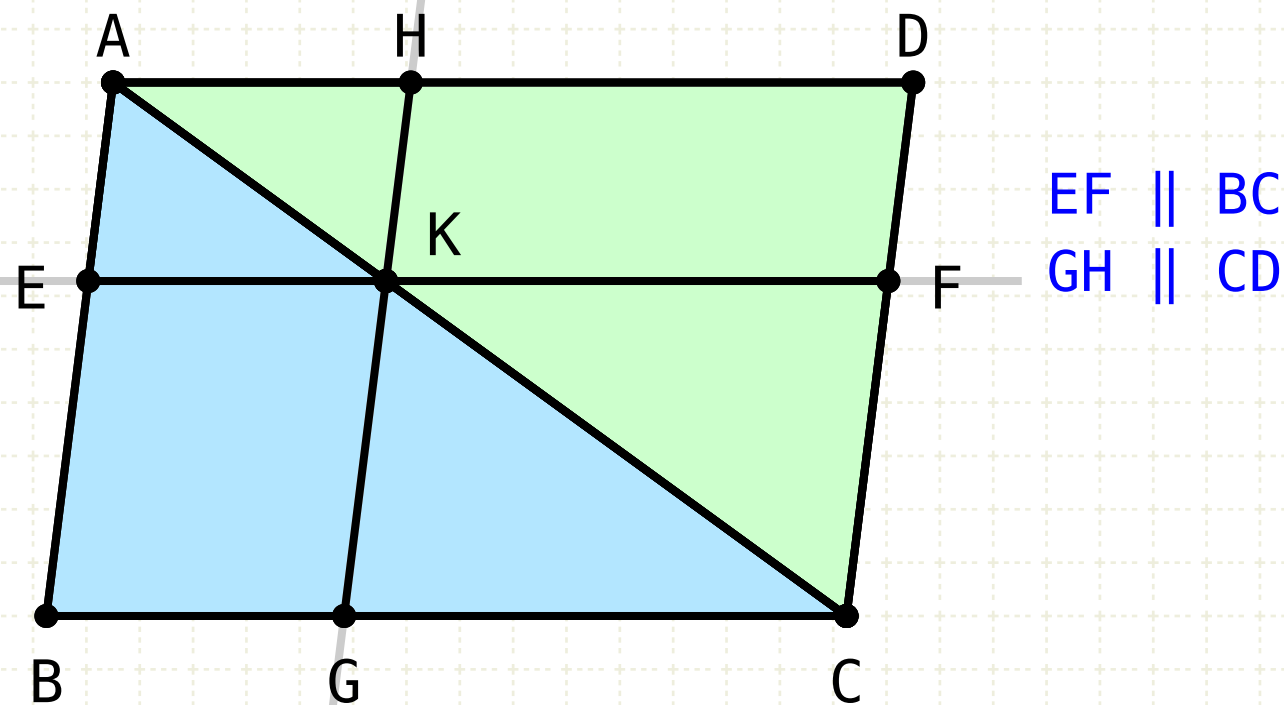
Triangle ABC is the sum of AED, EBGK and KGC

Triangle ADC is the sum of AKH, HKFD and KCF



Proposition 43 of Book I

In any parallelogram the complements of the parallelograms about the diameter equal one another.



$$\triangle ABC = \triangle ACD$$

$$\triangle AEK = \triangle AKH, \quad \triangle KGC = \triangle KCF$$

$$\triangle ABC = \triangle AEK + \text{EBGK} + \triangle KGC$$

$$\triangle ADC = \triangle AKH + \text{HKFD} + \triangle KCF$$

$$\triangle AKH + \text{HKFD} + \triangle KCF = \triangle AEK + \text{EBGK} + \triangle KGC$$

In other words

Given a parallelogram ABCD

With a diameter AC

And, from an arbitrary point E on line AB, draw a line parallel to AD intersecting the diagonal at point K

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Then the parallelograms EBGK and HKFD (complements of ABCD) and are equal

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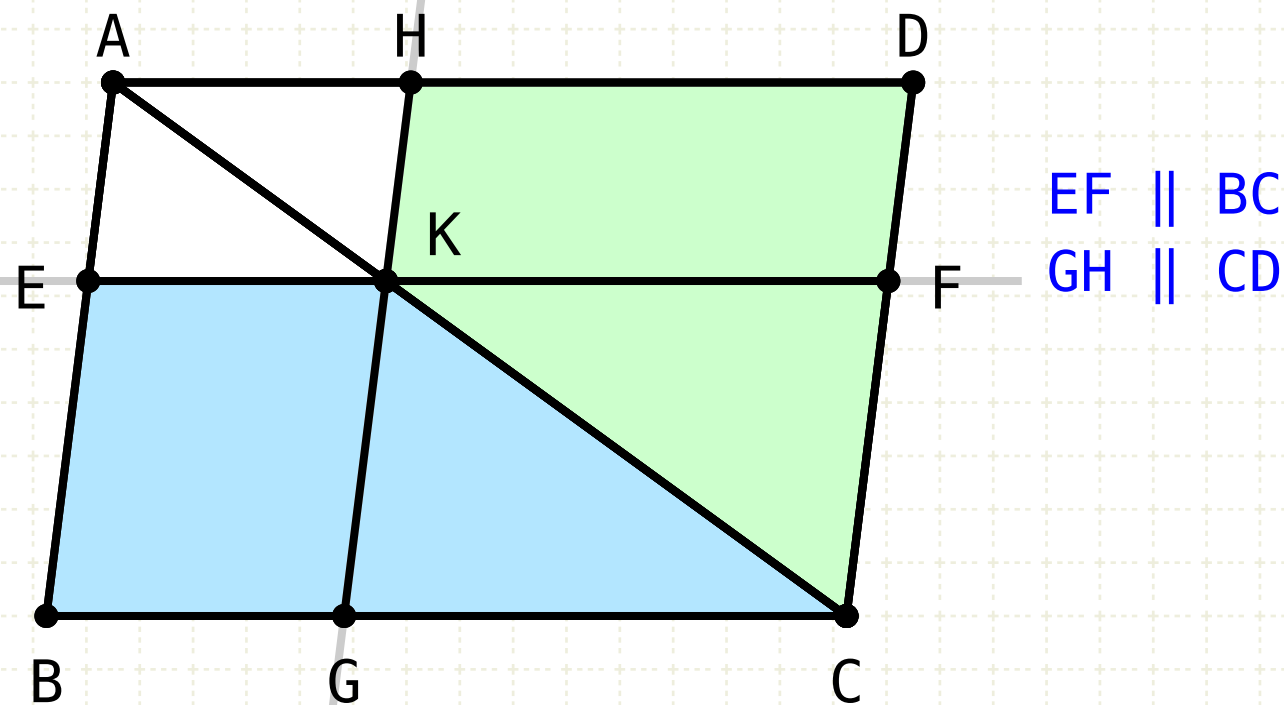
Triangle ADC is the sum of AKH, HKFD and KCF

Again, since ABC is equal to ABD, when the equal triangles are subtracted, we are left with the two complements EBGK and HKFD equal to each other



Proposition 43 of Book I

In any parallelogram the complements of the parallelograms about the diameter equal one another.



In other words

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And, from an arbitrary point E on line AB, draw a line parallel to AD intersecting the diagonal at point K

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Then the parallelograms EBGK and HKFD (complements of ABCD) and are equal

Proof:

Since AC is the diameter of the parallelogram ABCD, triangles ABC and ACD are equal (I·34)

By the same reasoning, triangles AEK and AHK are equal, as well as KGC and KCF (I·34)

Triangle ABC is the sum of AED, EBGK and KGC

Triangle ADC is the sum of AKH, HKFD and KCF

Again, since ABC is equal to ABD, when the equal triangles are subtracted, we are left with the two complements EBGK and HKFD equal to each other

$$\triangle ABC = \triangle ACD$$

$$\triangle AEK = \triangle AKH, \quad \triangle KGC = \triangle KCF$$

$$\triangle ABC = \triangle AEK + \text{EBGK} + \triangle KGC$$

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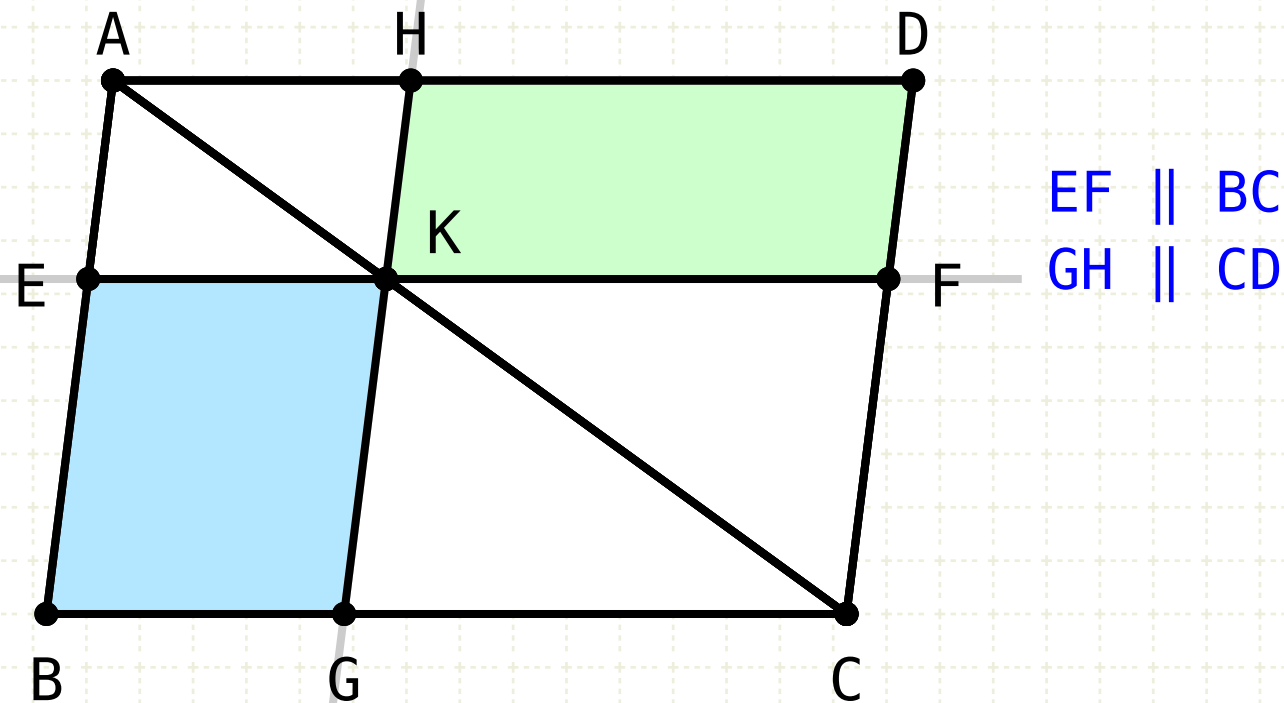
$$\triangle AKH + \text{HKFD} + \triangle KCF = \triangle AEK + \text{EBGK} + \triangle KGC$$

$$\text{HKFD} + \triangle KCF = \text{EBGK} + \triangle KGC$$



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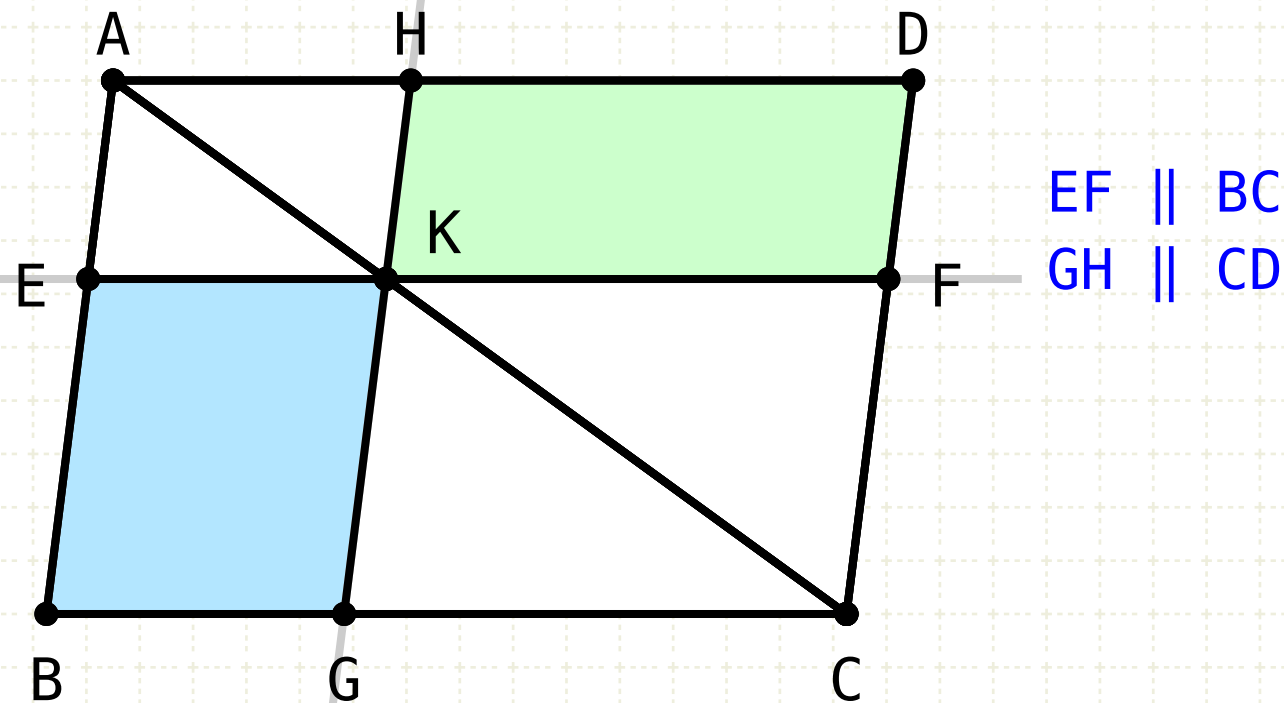
$$\text{HKFD} + \triangle KCF = \text{EBGK} + \triangle KGC$$

$$\text{HKFD} = \text{EBGK}$$



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 \triangle ADC &= \triangle AKH + \text{HKFD} + \triangle KCF \\
 \triangle AKH + \text{HKFD} + \triangle KCF &= \triangle AEK + \text{EBGK} + \triangle KGC \\
 \text{HKFD} + \triangle KCF &= \text{EBGK} + \triangle KGC \\
 \text{HKFD} &= \text{EBGK}
 \end{aligned}$$



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