# Euclid's Elements

# Book VII

#### **Definitions:**

- A unit is that by virtue of which each of the things that exist is called one
- 2 A number is a multitude composed of units. (not one)
- A number is part of a number, the less of the greater, when it measures the greater
- 11 A prime number is that which is measured by a unit alone.
- 12 Numbers prime to one another are those which are measured by a unit alone as a common measure
- A number is said to multiply a number when that which is multiplied is added to itself as many times as there are units in the other, and thus some number is produced.
- Numbers are proportional when the first is the same multiple, or the same part, or the same parts, of the second that the third is of the fourth.

As long as algebra and geometry have been separated, their progress have been slow and their uses limited; but when these two sciences have been united, they have lent each mutual forces, and have marched together towards perfection.

Joseph-Louis Lagrange (1736 to 1813)



### **Table of Contents, Chapter 7**

- 1 Determine if two numbers are relatively prime
- 2 Find the greatest common divisor for two numbers
- 3 Find the largest common divisor for three numbers
- 4 Given two natural numbers, A and B, either B is part of A, or there exists a natural number (a part) that can measure both A and B
- 5 If B =  $(1/q)\cdot A$  and D =  $(1/q)\cdot C$ , then  $(B+D) = (1/q)\cdot (A+C)$
- 6 If B =  $(p/q)\cdot A$  and D =  $(p/q)\cdot C$ , then  $(B+D) = (p/q)\cdot (A+C)$
- 7 If B = A/q and D = C/q, B>D, then (B-D) = (A-C)/q
- 8 If B =  $(p/q)\cdot A$  and D =  $(p/q)\cdot C$ , B>D, then  $(B-D) = (p/q)\cdot (A-C)$
- 9 If B = (1/q)·A and D = (1/q)·C, and If B = (r/s)·D, then A = (r/s)·C

- 10 If B =  $(p/q)\cdot A$  and D =  $(p/q)\cdot C$ , and If B =  $(r/s)\cdot D$ , then A =  $(r/s)\cdot C$
- 11 If A:B = C:D, then (A-C):(B-D) = A:B
- 12 If A:B = C:D, then (A+C):(B+C) = A:B
- 13 If A:B = C:D, then A:C = B:D
- 14 If A:B = D:E and B:C = E:F, then A:C = D:F
- 15 If B = i·1 and E = i·D, and if D = j·1 then E = j·B
- 16  $A \times B = B \times A$
- 17 If D = A × B and E = A × C then D:E = B:C
- 18 If D = B × A and E = C × A then D:E = B:C
- 19 If A:B = C:D then  $A \times D = B \times C$ If  $A \times D = B \times C$  then A:B = C:D
- 20 Given the ratio A:B and C,D are the smallest numbers such that A:B = C:D then A = n·C and B = n·D

- If A,B are relatively prime, then A,B are the smallest whole numbers that can be used to describe the ratio A:B
- 22 If A,B are the smallest whole numbers that can be used to describe the ratio A:B, then A,B are relatively prime
- 23 If A,B are relatively prime and if A = n·C, then B,C are relatively prime
- 24 If A,C are relatively prime and B,C are relatively prime then the A × B is relatively prime to C
- 25 If A,B are relatively prime then A<sup>2</sup>,B are relatively prime
- If A is relatively prime to C and D, and if B is also relatively prime to C and D, then A × B is relatively prime to C × D
- 27 If A,B are relatively prime, then A<sup>2</sup>,B<sup>2</sup> are relatively prime, and A<sup>3</sup>,B<sup>3</sup> are relatively prime, and so on



## **Table of Contents, Chapter 7**

- 28 If A,B are relatively prime, then A,(A+B) are relatively prime
- 29 If A is prime, and B ≠ n·A, then A,B are relatively prime
- 30 If C = A×B and C = i·D where D is prime, then either A = j·D or B = j·D
- 31 If  $A = B \times C$ , then  $A = j \cdot D$  where D is prime
- 32 If A is a number then it is either prime, or  $A = j \cdot D$  where D is prime
- Find the smallest numbers X,Y,Z where the ratio X:Y:Z is equal to the given ratio A:B:C
- 34 Find the lowest common denominator of 2 numbers
- 35 If E is the lowest common denominator of A,B, and if C = n ·A = m·B, then C = i·E
- 36 Find the least common multiple of 3 numbers

- If  $A = p \cdot B$ , then  $A = q \cdot C$  where  $C = p \cdot 1$
- 38 If  $A = (1/c) \cdot B$  and  $C = c \cdot 1$  then  $A = n \cdot C$
- Find the smallest number that has the fractions 1/a, 1/b, 1/c





In other words

Let A be a some number

Any number either is prime or is measured by some prime number

A •

#### In other words

Let A be a some number

Then A is either prime, or is measured by a prime number



#### Proof

A is prime

Proof

Let A be prime

A is prime

Proof

Let A be prime

Then we have shown that A is prime

Any number either is prime or is measured by some prime number



A is prime

or...

$$A = i \times j$$

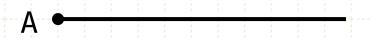
#### **Proof**

Let A be prime

Then we have shown that A is prime

Or, if A is not prime then it is a composite number

Any number either is prime or is measured by some prime number



A is prime

or...

$$A = i \times j$$

p is prime

$$A = s \cdot p$$

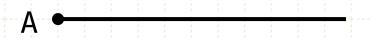
#### **Proof**

Let A be prime

Then we have shown that A is prime

Or, if A is not prime then it is a composite number If A is a composite number, then there is some prime number that measures it (VII-31).

Any number either is prime or is measured by some prime number



A is prime

or...

 $A = i \times j$ 

p is prime

 $A = s \cdot p$ 

#### **Proof**

Let A be prime

Then we have shown that A is prime

Or, if A is not prime then it is a composite number If A is a composite number, then there is some prime number that measures it (VII-31).

Therefore A is either a prime number, or is measured by a prime number

#### **Youtube Videos**

https://www.youtube.com/c/SandyBultena











Except where otherwise noted, this work is licensed under http://creativecommons.org/licenses/by-nc/3.0