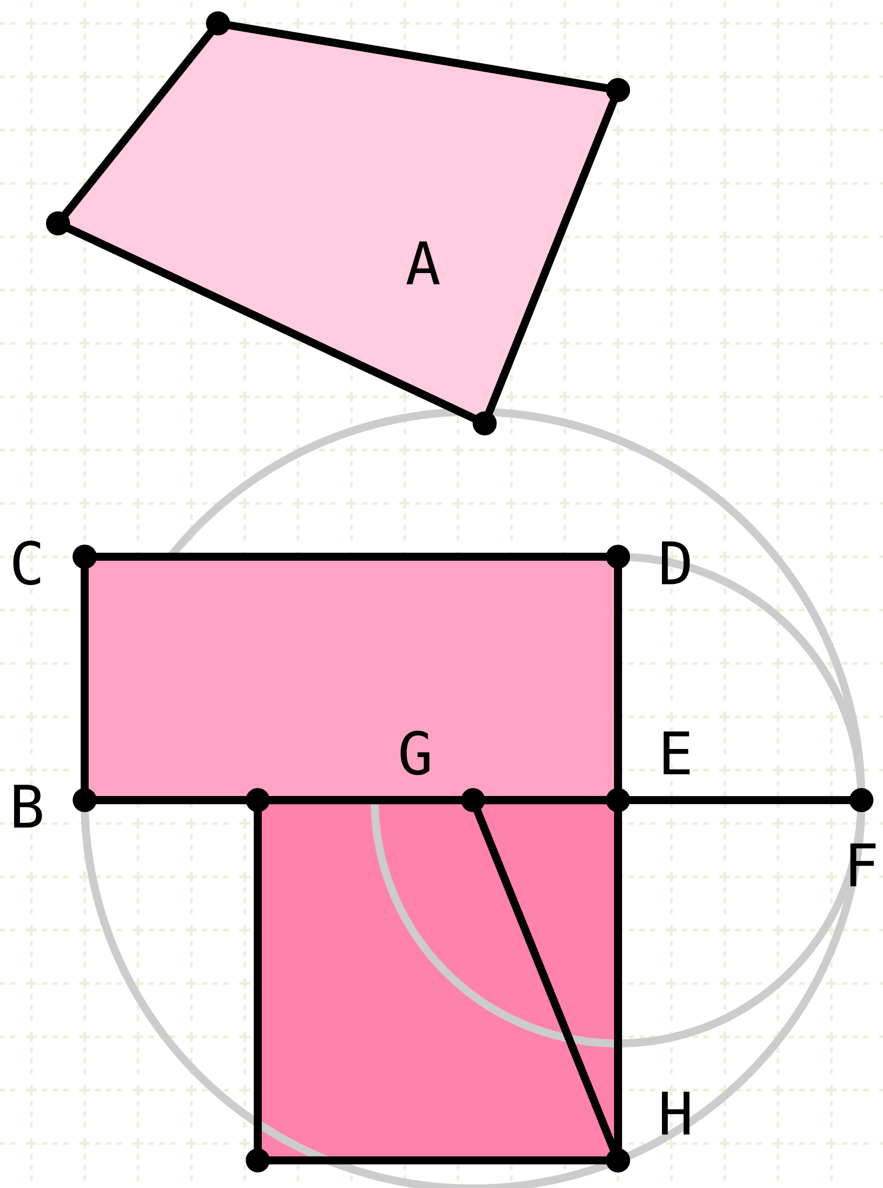
Euclid's Elements

Book II

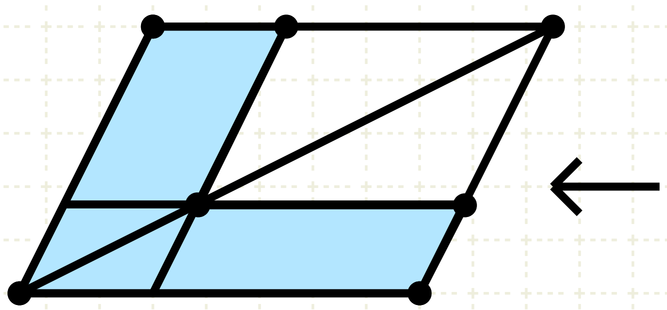


It is a remarkable fact in the history of geometry, that the Elements of Euclid, written two thousand years ago, are still regarded by many as the best introduction to the mathematical sciences.

Florian Cajori,
A History of Mathematics (1893)

Definitions:

Any rectangular parallelogram is said to be contained by the two straight lines containing the right angle.



And in any parallelogrammic area let any one whatever of the parallelograms about its diameter with the two complements be called a gnomon.

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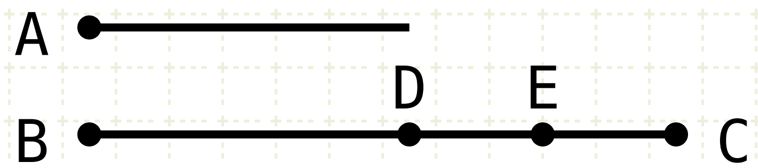
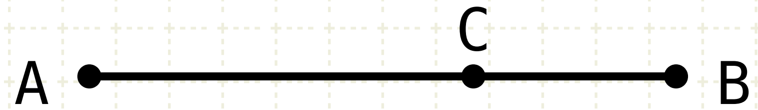
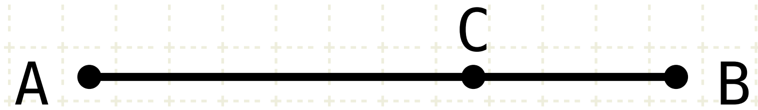
1 

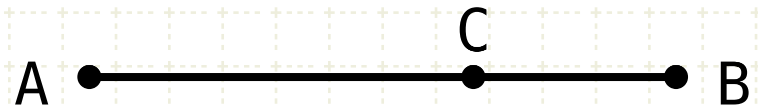
$A \cdot BC = A \cdot BD + A \cdot DE + A \cdot EC$

2 

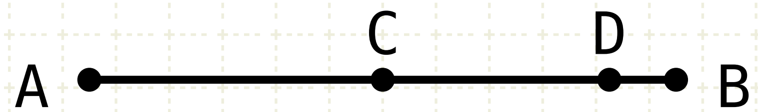
$AB^2 = AB \cdot AC + AB \cdot BC$

3 

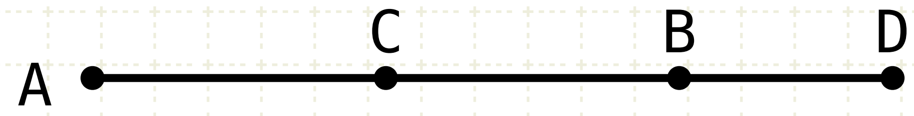
$AB \cdot CB = AC \cdot CB + CB^2$

4 

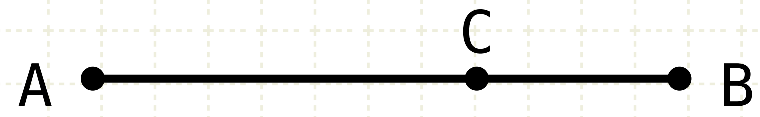
$AB^2 = AC^2 + CB^2 + 2 \cdot AC \cdot CB$

5 

$AD \cdot DB + CD^2 = CB^2$

6 

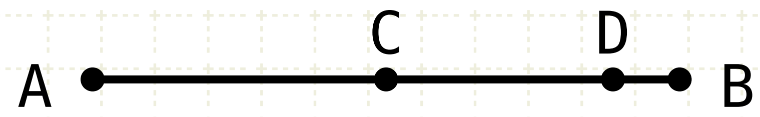
$AD \cdot DB + CB^2 = CD^2$

7 

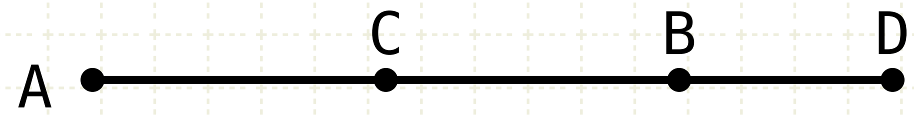
$AB^2 + BC^2 = AC^2 + 2 \cdot AB \cdot BC$

8 

$4 \cdot AB \cdot BC + AC^2 = (AB+BC)^2$

9 

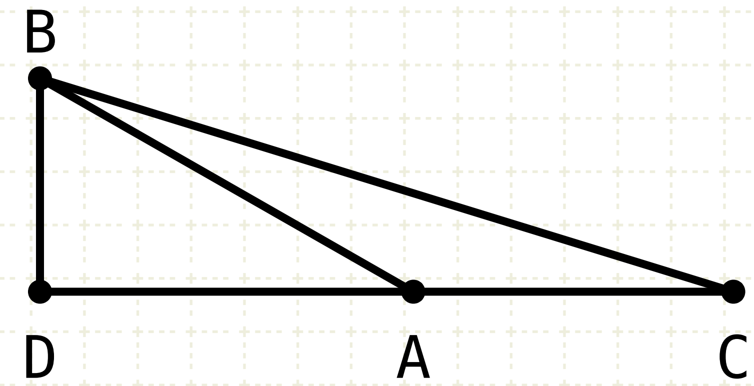
$AD^2 + DB^2 = 2 \cdot (AC^2 + CD^2)$

10 

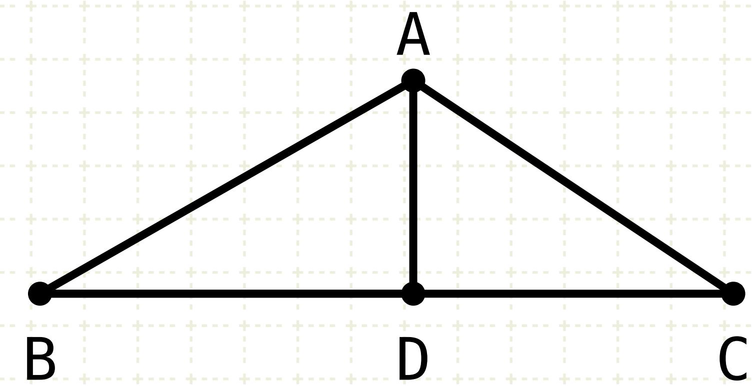
$AD^2 + DB^2 = 2 \cdot (AC^2 + CD^2)$

11 

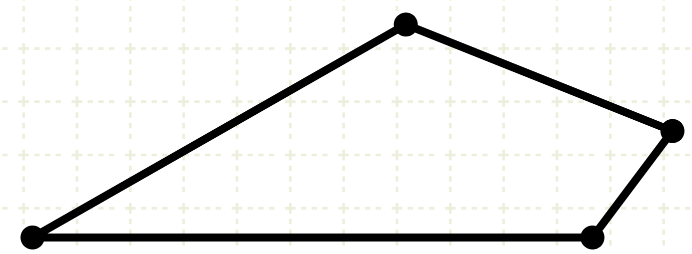
Find H. $AB \cdot BH = AH^2$

12 

Cosine Law. $BC^2 = AB^2+AC^2+2 \cdot AD \cdot AC$

13 

Cosine Law. $AC^2 = AB^2+BC^2-2 \cdot BD \cdot BC$

14 

Find square of polygon

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Proposition 14 of Book II

To construct a square equal to a given rectilineal figure.

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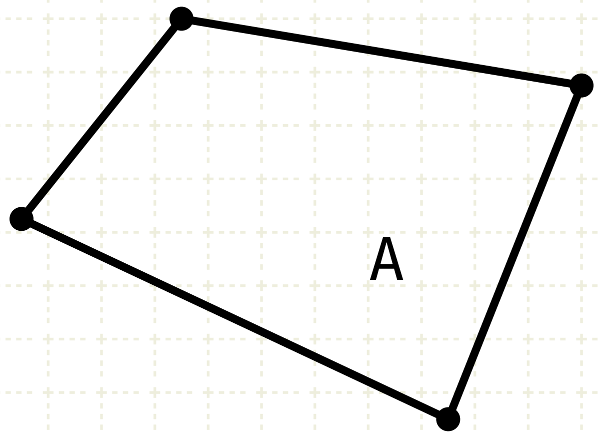
Proposition 14 of Book II

To construct a square equal to a given rectilineal figure.



Construction

Let A be the given rectilinear figure.

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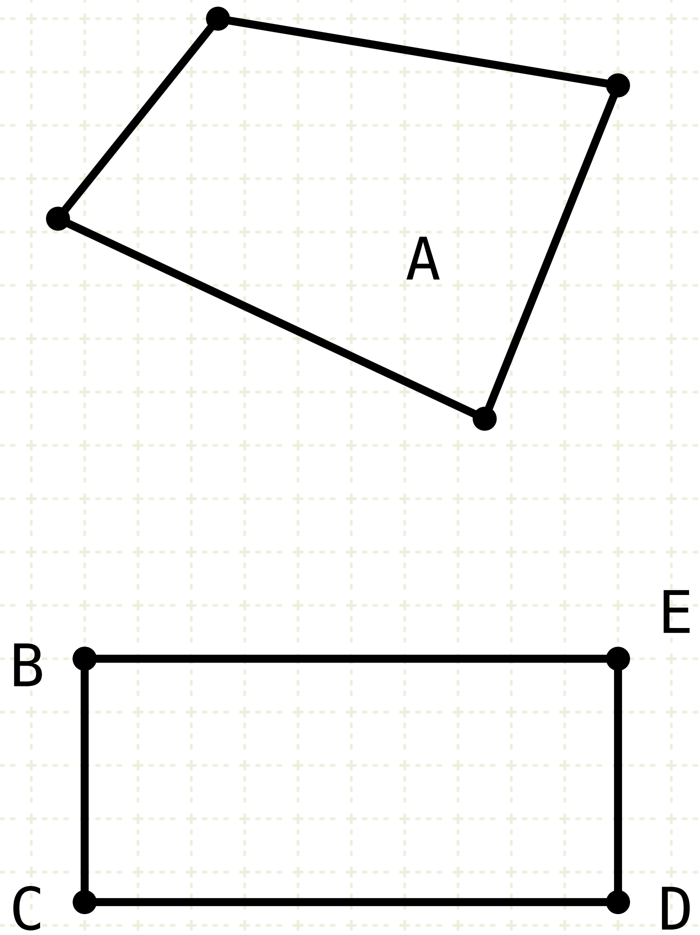

□A = □BD

Construction

Let A be the given rectilinear figure.

Copy A to a rectangle (I·45)

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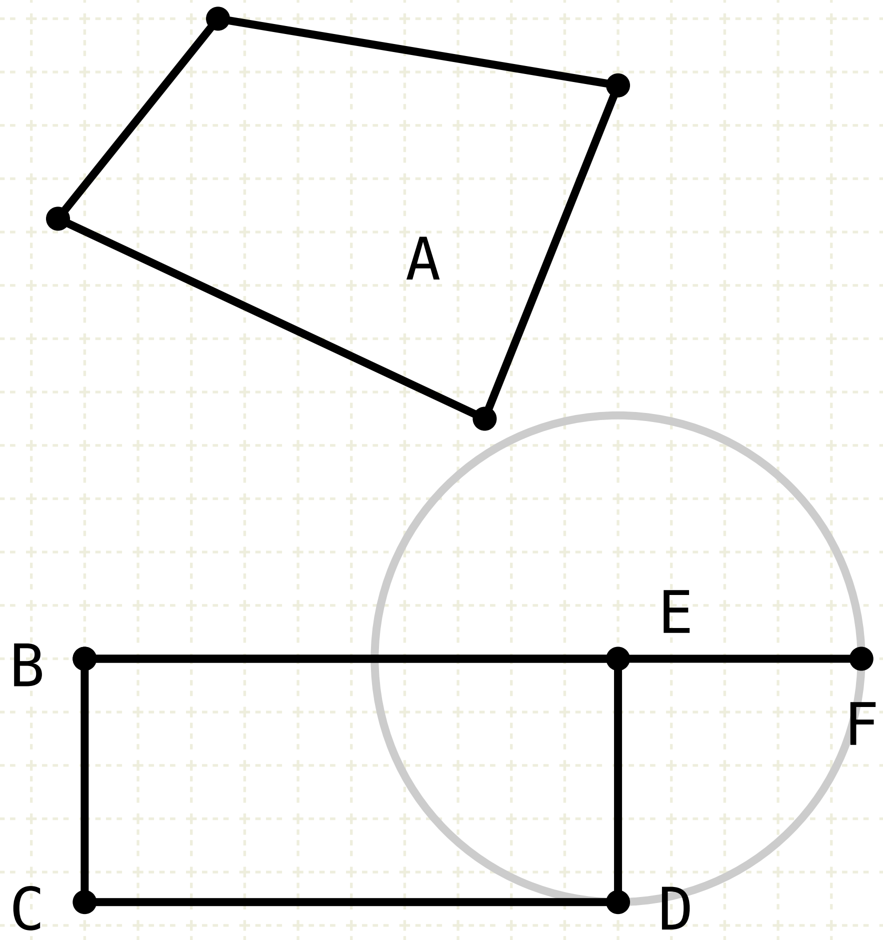
Proposition 14 of Book II

To construct a square equal to a given rectilineal figure.



□A = □BD

EF = BD

Construction

Let A be the given rectilinear figure.

Copy A to a rectangle (I·45)

If BE does not equal ED, and if BE is the larger, then extend BE to F, where EF equals ED

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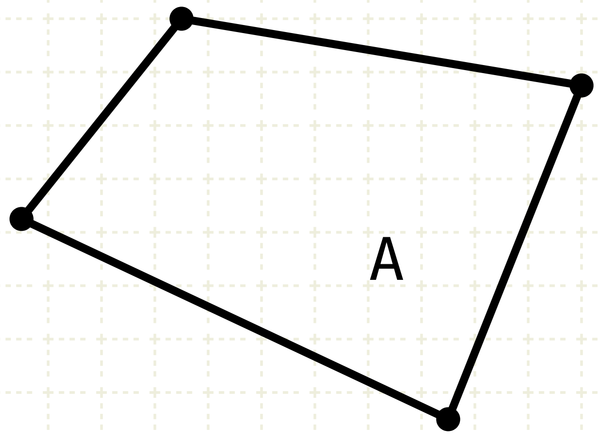
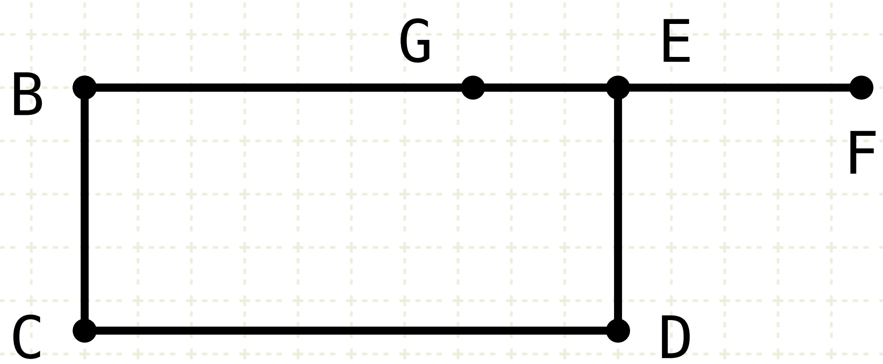


$\square A = \square BD$

$EF = BD$

$BG = GF$



Construction

Let A be the given rectilinear figure.

Copy A to a rectangle (I·45)

If BE does not equal ED, and if BE is the larger, then extend BE to F, where EF equals ED

Bisect BF (and label it point G)

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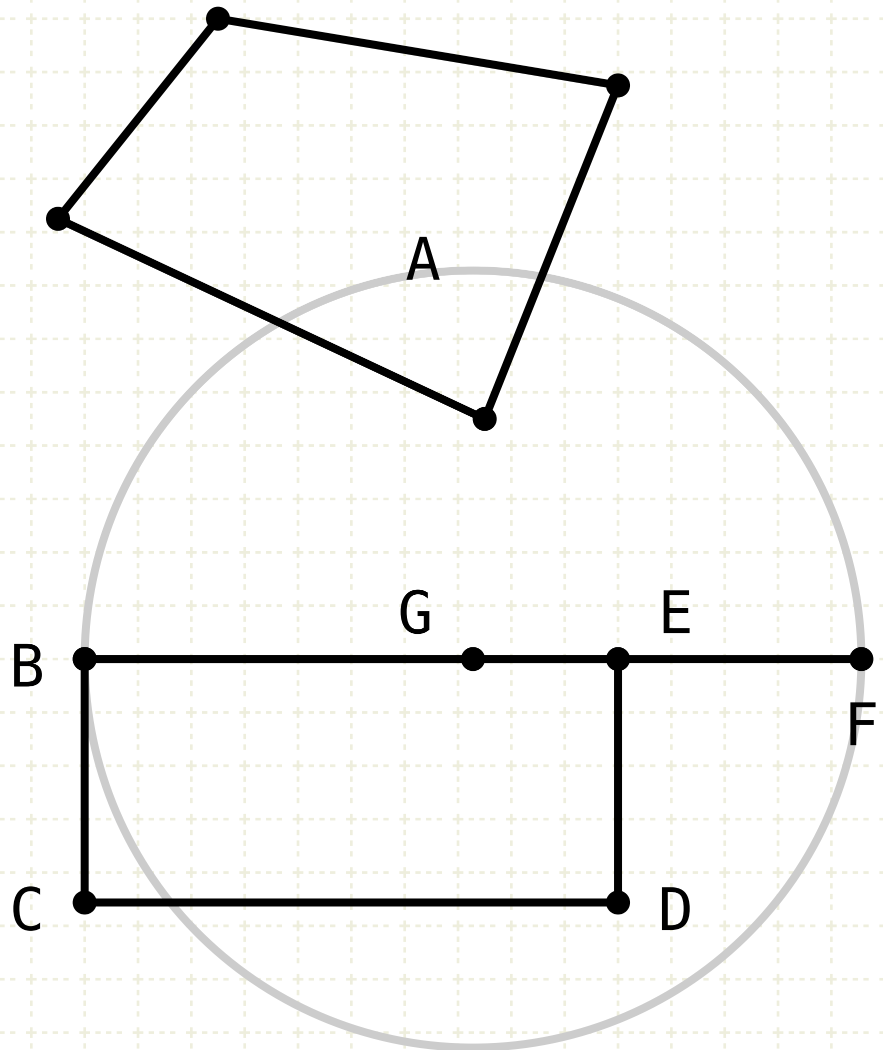

□A = □BD
EF = BD
BG = GF

Construction

Let A be the given rectilinear figure.

Copy A to a rectangle (I·45)

If BE does not equal ED, and if BE is the larger, then extend BE to F, where EF equals ED

Bisect BF (and label it point G)

Draw a circle with G as the center and GF as the radii

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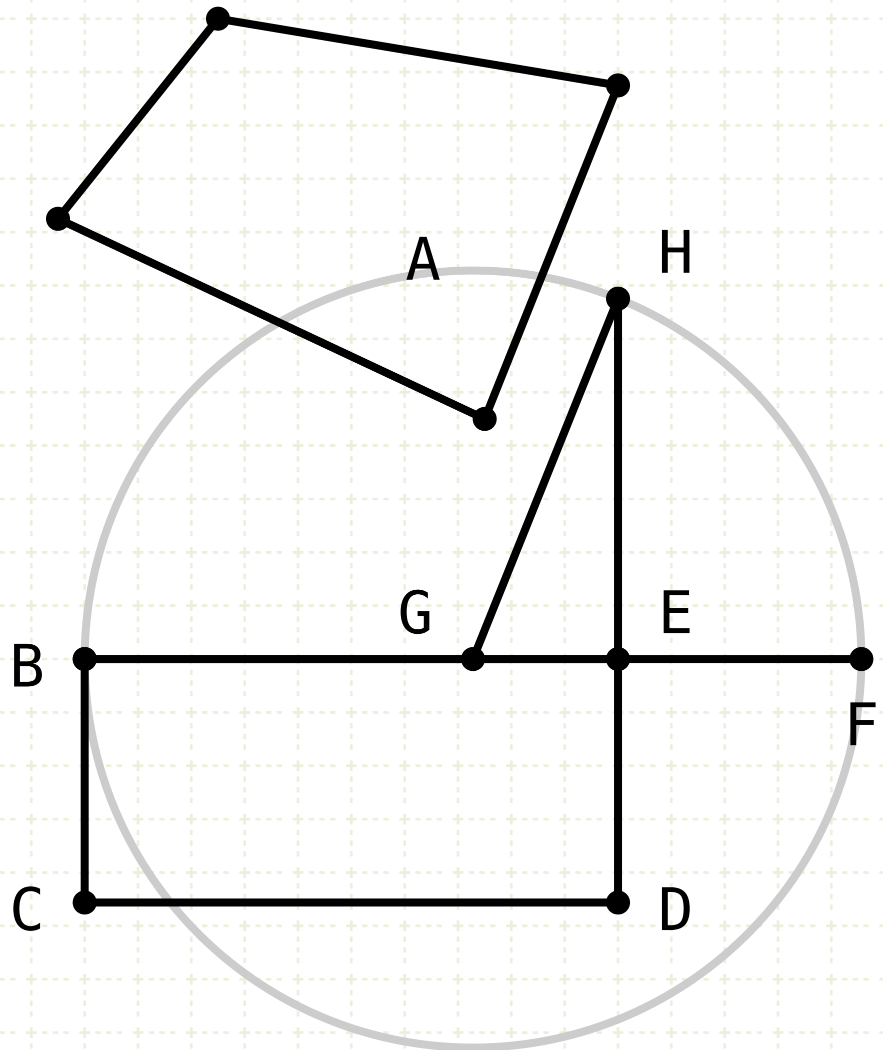
Proposition 14 of Book II

To construct a square equal to a given rectilineal figure.



□A = □BD

EF = BD

BG = GF

GH = GF

Construction

Let A be the given rectilinear figure.

Copy A to a rectangle (I·45)

If BE does not equal ED, and if BE is the larger, then

extend BE to F, where EF equals ED

Bisect BF (and label it point G)

Draw a circle with G as the center and GF as the radii

Extend DE to intersect with the circle at point H, and let GH be joined

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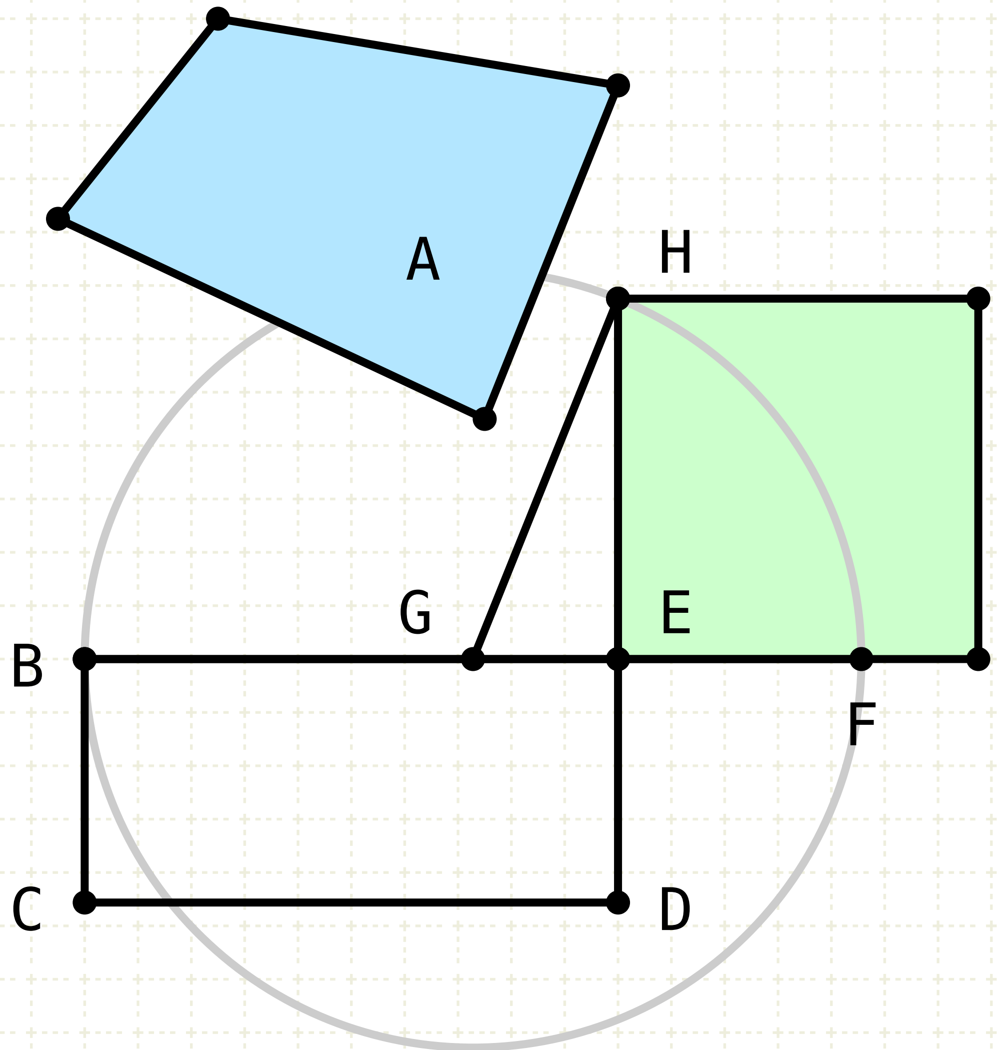
To construct a square equal to a given rectilineal figure.



$\square A = \square BD$

$EF = BD$

$BG = GF$

$GH = GF$

$\square HE = \square A$

Construction

Let A be the given rectilinear figure.

Copy A to a rectangle (I·45)

If BE does not equal ED, and if BE is the larger, then

extend BE to F, where EF equals ED

Bisect BF (and label it point G)

Draw a circle with G as the center and GF as the radii

Extend DE to intersect with the circle at point H, and let GH be joined

The square on HE is equal in area to figure A

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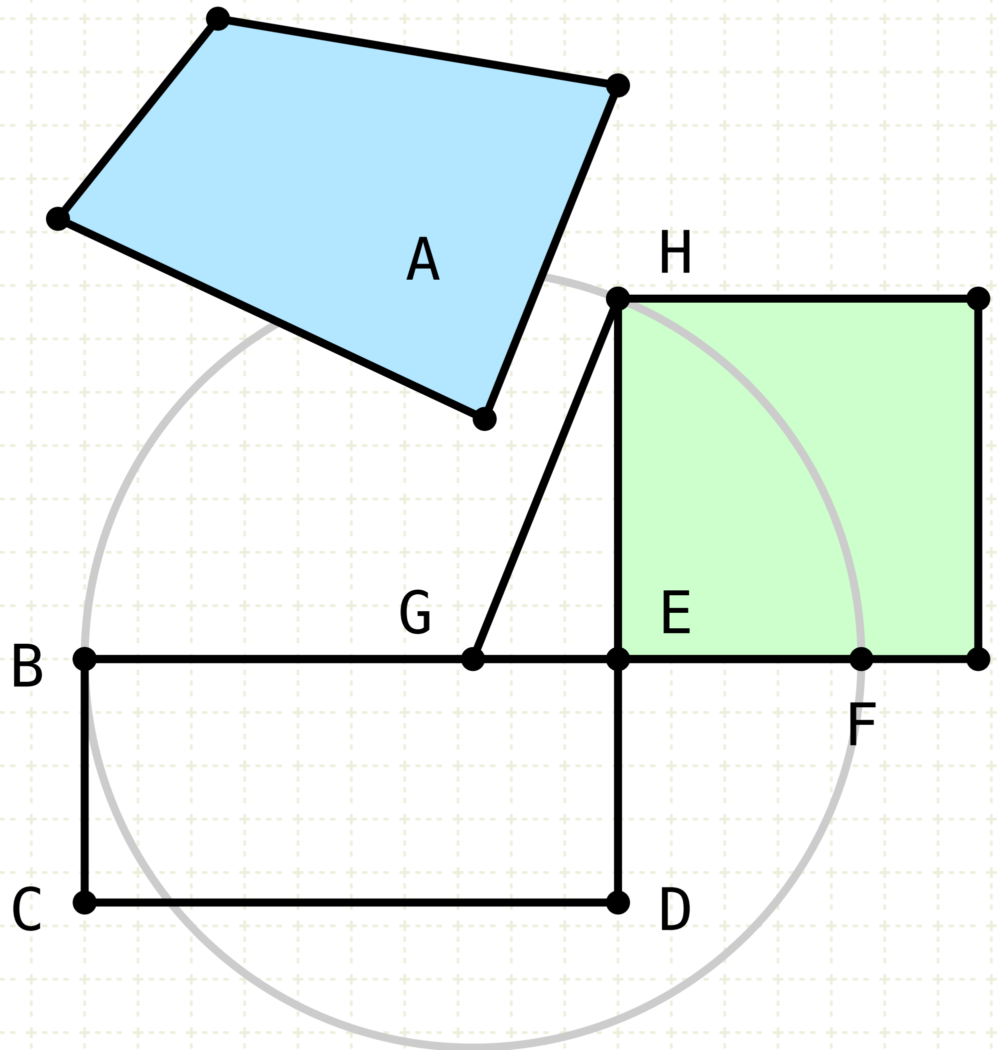

Proof:

□A = □BD
EF = BD
BG = GF
GH = GF

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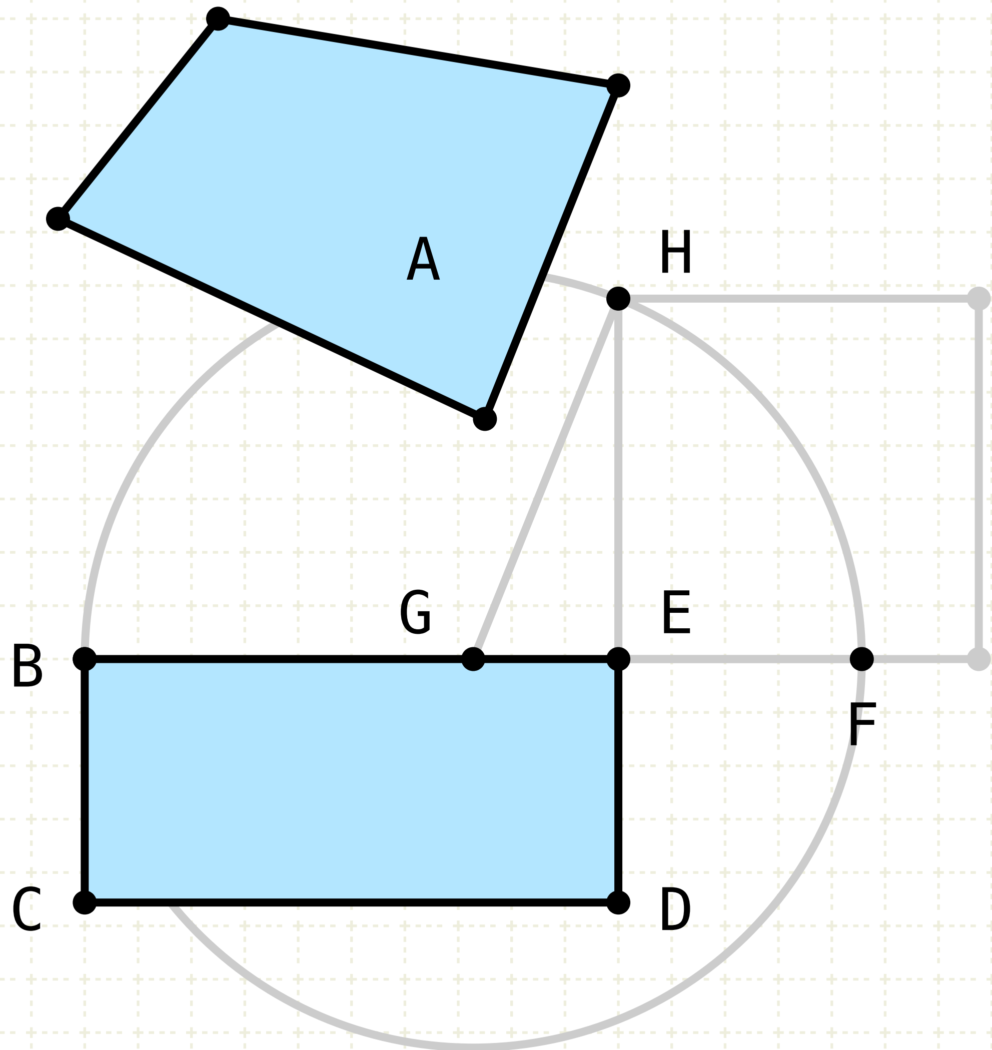

□A = □BD
EF = BD
BG = GF
GH = GF

Proof:

Polygon A equals the polygon BD by construction

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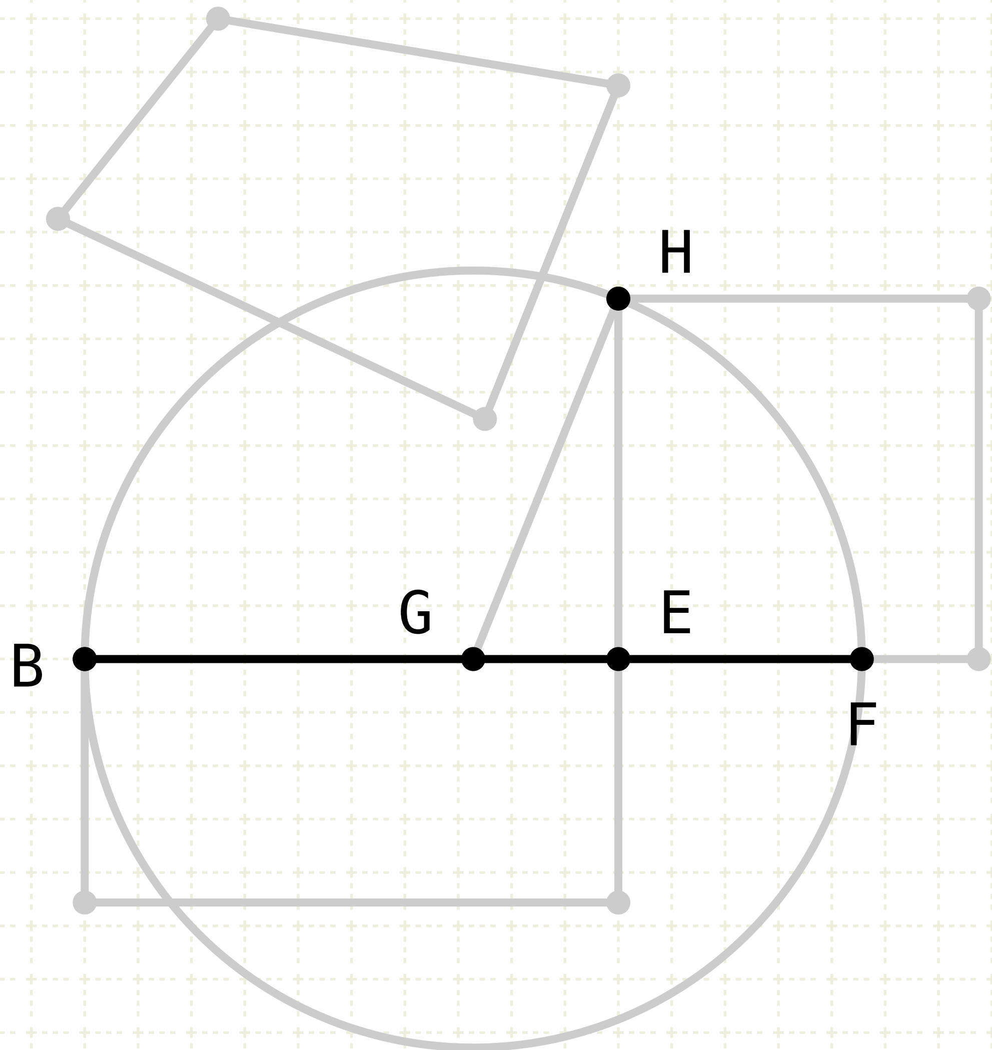

□A = □BD

EF = BD

BG = GF

GH = GF

BE·EF + EG² = GF²

Proof:

Polygon A equals the polygon BD by construction

Line BF is divided into equal (G) and unequal segments (E), thus the rectangle formed by BE,EF plus the square of EG is equal to the square on GF (II·5)

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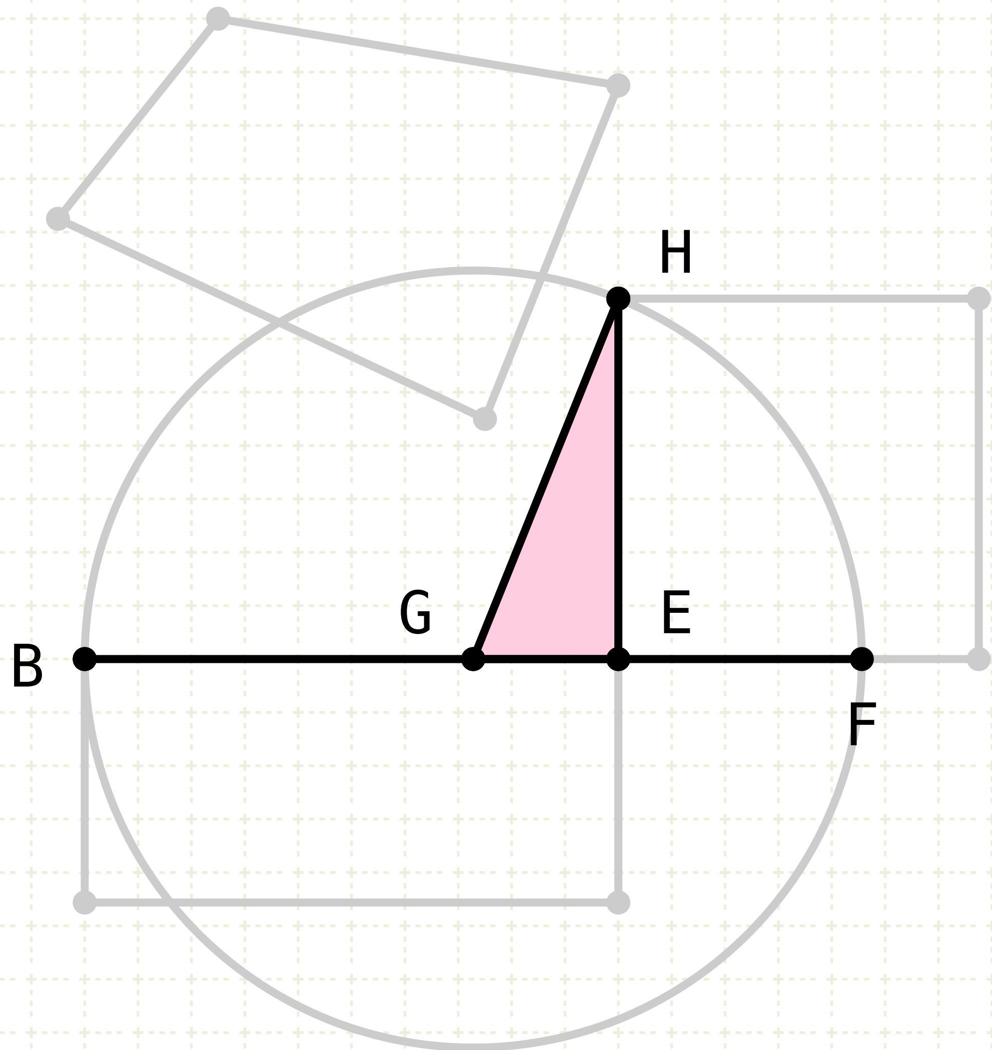
To construct a square equal to a given rectilineal figure.



$\square A = \square BD$

$EF = BD$

$BG = GF$

$GH = GF$

$BE \cdot EF + EG^2 = GF^2$

$GH^2 = EG^2 + EH^2$

$GF^2 = EG^2 + EH^2$

Proof:

Polygon A equals the polygon BD by construction

Line BF is divided into equal (G) and unequal segments (E), thus the rectangle formed by BE,EF plus the square of EG is equal to the square on GF (II·5)

Since GHE is a right triangle, and GH is equal to GF, the square of GF is equal to the sum of the squares on EG and GH (I·47)

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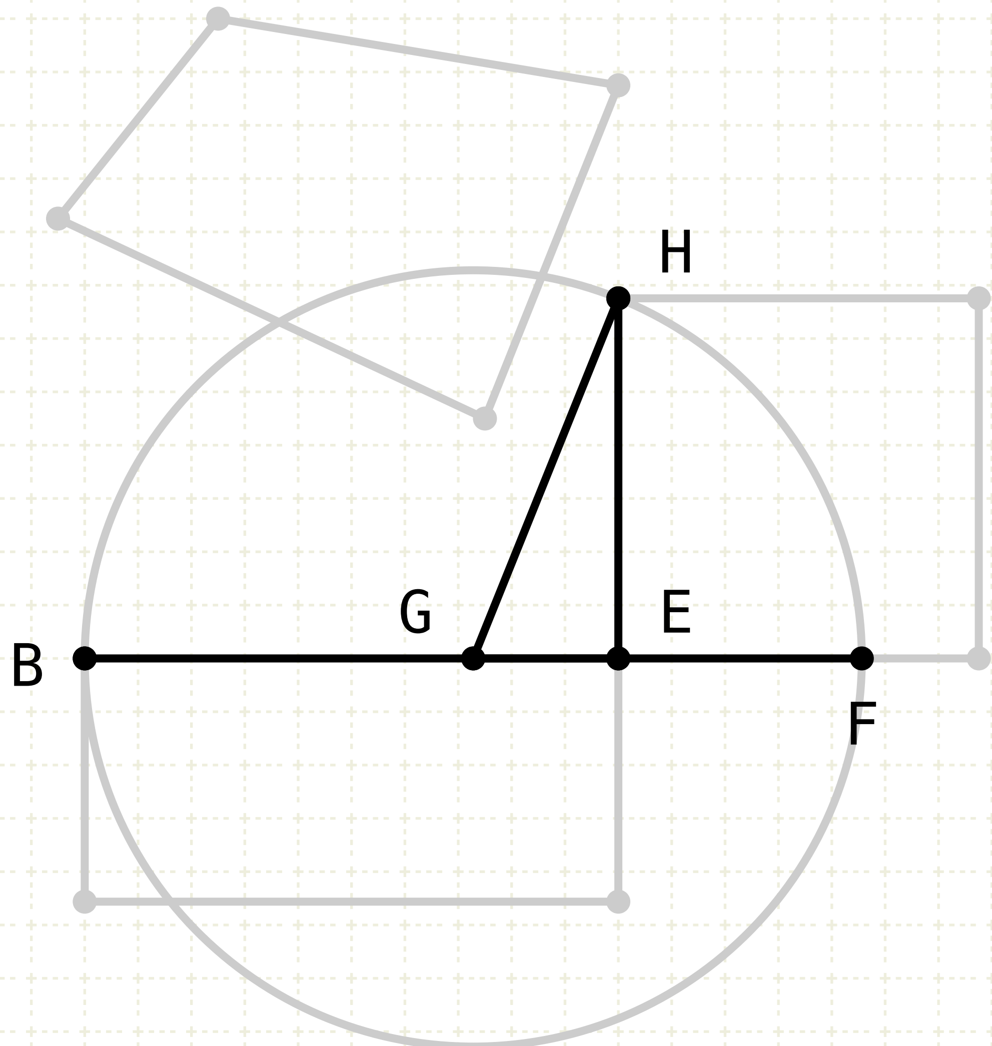
To construct a square equal to a given rectilineal figure.



□A = □BD

EF = BD

BG = GF

GH = GF

$BE \cdot EF + EG^2 = GF^2$

$GH^2 = EG^2 + EH^2$

$GF^2 = EG^2 + EH^2$

$BE \cdot EF + EG^2 = EG^2 + EH^2$

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Polygon A equals the polygon BD by construction

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Thus the rectangle formed by BE,EF plus the square of EG is equal to the sum of the squares on EG and GH

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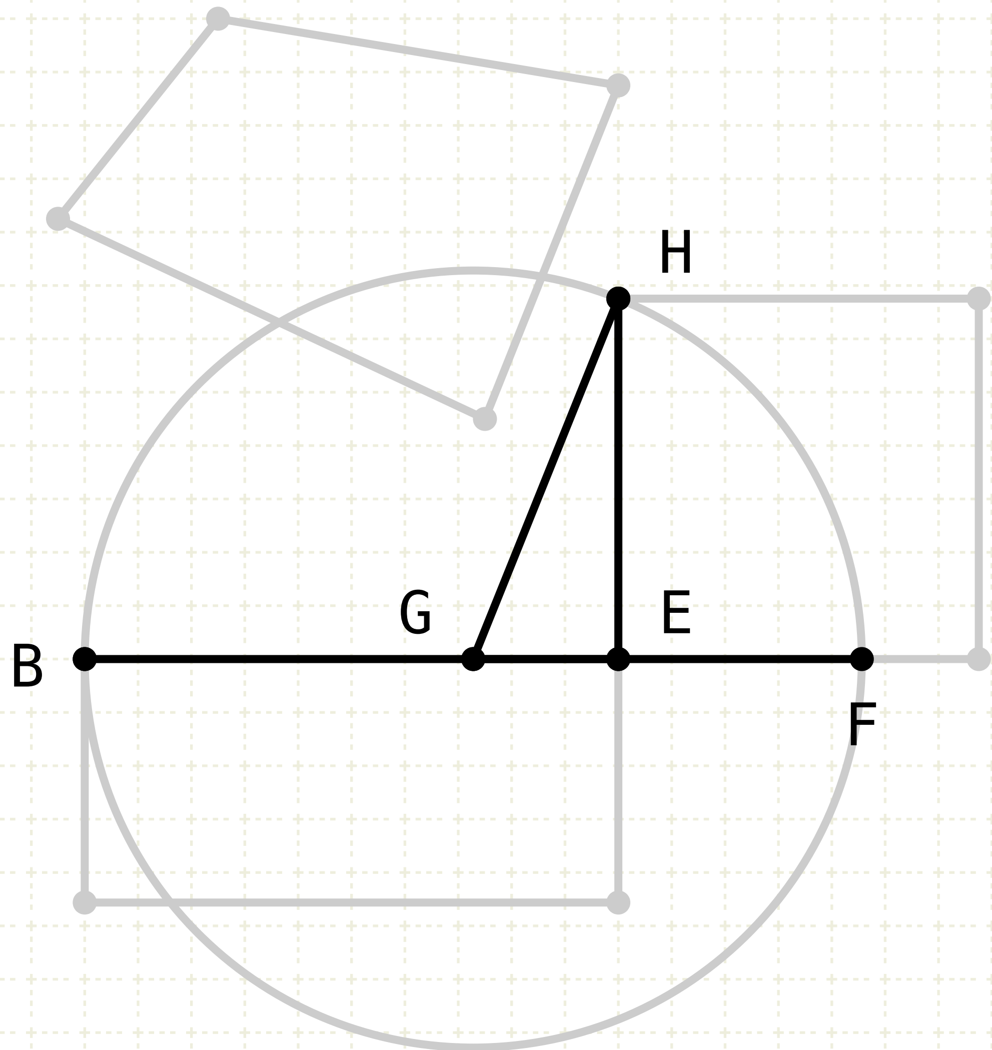

$$\square A = \square BD$$
$$EF = BD$$
$$BG = GF$$
$$GH = GF$$
$$BE \cdot EF + EG^2 = GF^2$$
$$GH^2 = EG^2 + EH^2$$
$$GF^2 = EG^2 + EH^2$$
$$BE \cdot EF + EG^2 = EG^2 + EH^2$$
$$BE \cdot EF = EH^2$$

Proof:

Polygon A equals the polygon BD by construction

Line BF is divided into equal (G) and unequal segments (E), thus the rectangle formed by BE,EF plus the square of EG is equal to the square on GF (II·5)

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Thus the rectangle formed by BE,EF plus the square of EG is equal to the sum of the squares on EG and GH

Subtracting EG from both sides of the equality, gives BE,EF equals the square of EH

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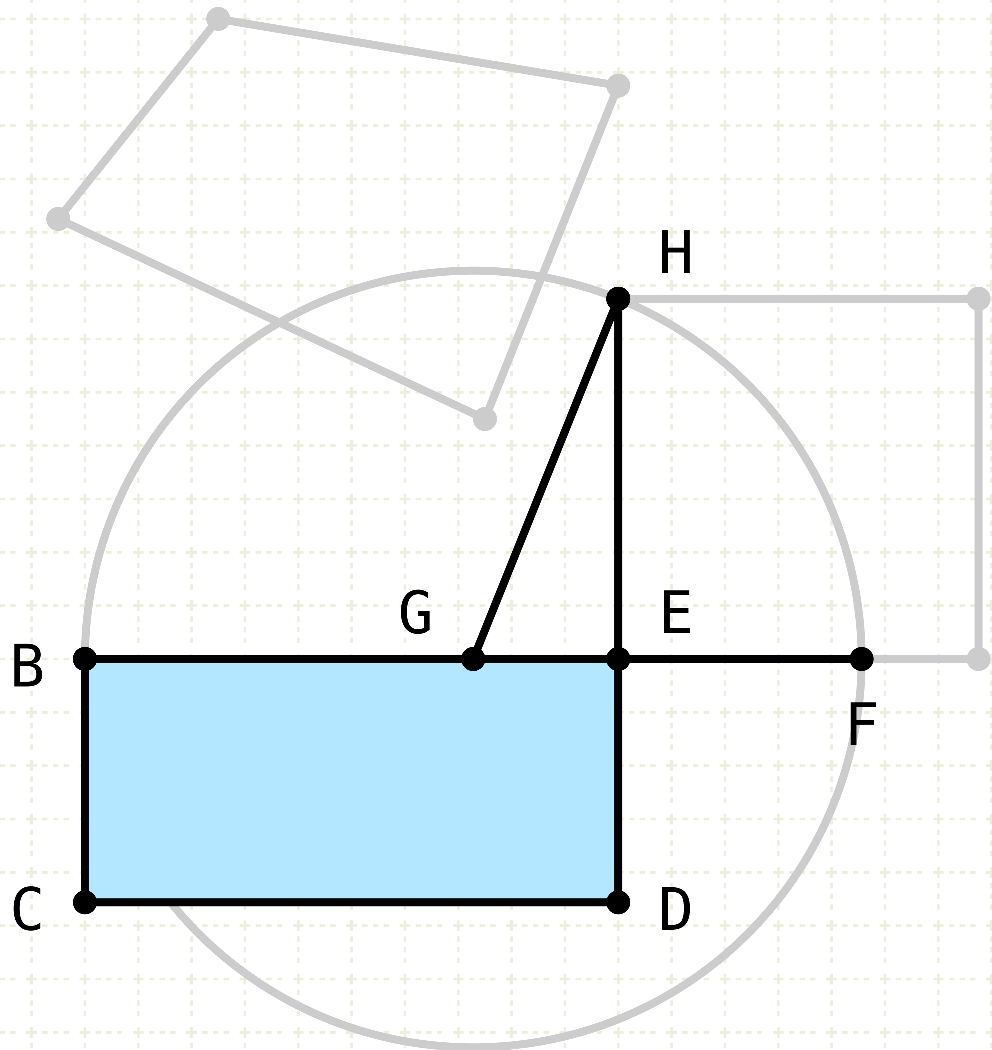
To construct a square equal to a given rectilineal figure.



$\square A = \square BD$

$EF = BD$

$BG = GF$

$GH = GF$

$BE \cdot EF + EG^2 = GF^2$

$GH^2 = EG^2 + EH^2$

$GF^2 = EG^2 + EH^2$

$BE \cdot EF + EG^2 = EG^2 + EH^2$

$BE \cdot EF = EH^2$

$BE \cdot EF = \square BD$

Proof:

Polygon A equals the polygon BD by construction

Line BF is divided into equal (G) and unequal segments (E), thus the rectangle formed by BE,EF plus the square of EG is equal to the square on GF (II·5)

Since GHE is a right triangle, and GH is equal to GF, the square of GF is equal to the sum of the squares on EG and GH (I·47)

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Subtracting EG from both sides of the equality, gives BE,EF equals the square of EH

The rectangle formed by BE,EF is BD, since EF equals ED

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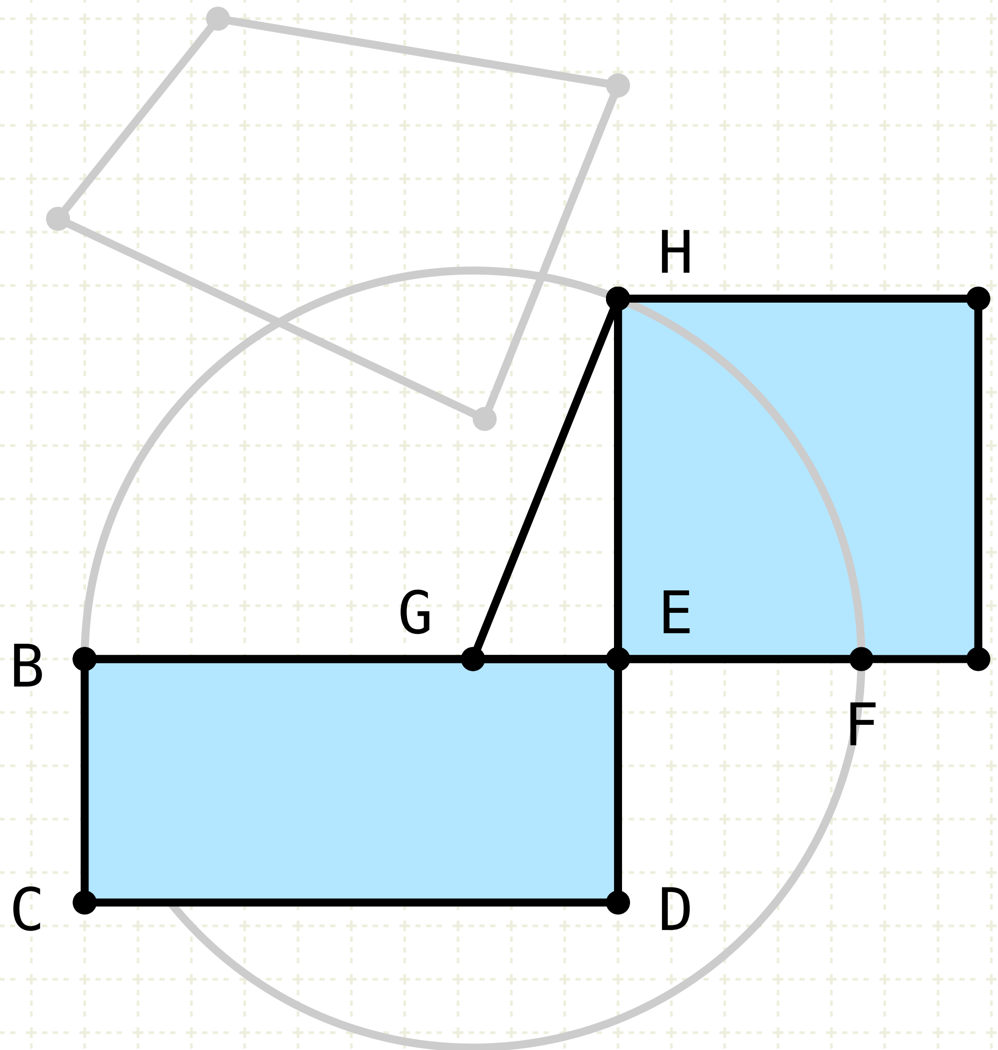
Proposition 14 of Book II

To construct a square equal to a given rectilineal figure.



$\square A = \square BD$

$EF = BD$

$BG = GF$

$GH = GF$

$BE \cdot EF + EG^2 = GF^2$

$GH^2 = EG^2 + EH^2$

$GF^2 = EG^2 + EH^2$

$BE \cdot EF + EG^2 = EG^2 + EH^2$

$BE \cdot EF = EH^2$

$BE \cdot EF = \square BD$

$\square BD = EH^2$

Proof:

Polygon A equals the polygon BD by construction

Line BF is divided into equal (G) and unequal segments (E), thus the rectangle formed by BE,EF plus the square of EG is equal to the square on GF (II·5)

Since GHE is a right triangle, and GH is equal to GF, the square of GF is equal to the sum of the squares on EG and GH (I·47)

Thus the rectangle formed by BE,EF plus the square of EG is equal to the sum of the squares on EG and GH

Subtracting EG from both sides of the equality, gives BE,EF equals the square of EH

The rectangle formed by BE,EF is BD, since EF equals ED

Therefore BD equals the square on EH

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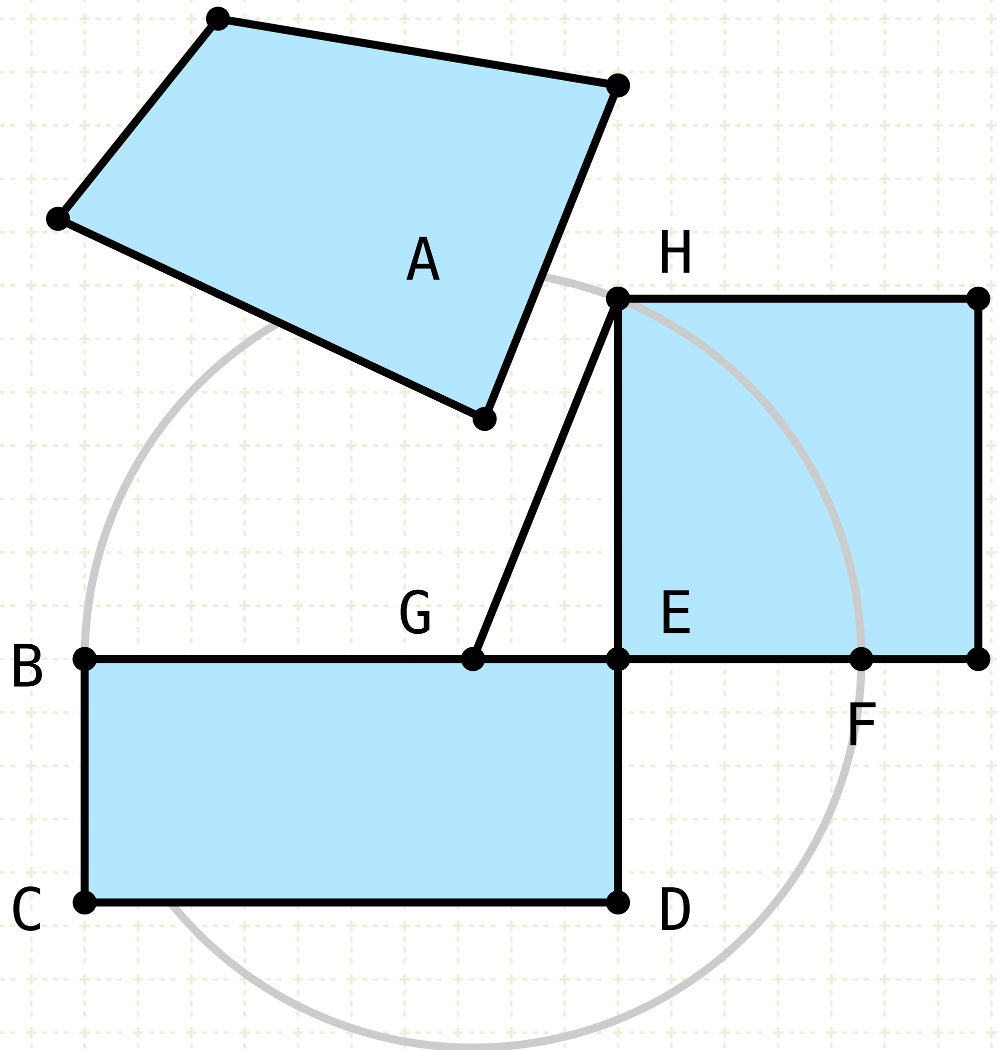

$\square A = \square BD$

$EF = BD$

$BG = GF$

$GH = GF$

$BE \cdot EF + EG^2 = GF^2$

$GH^2 = EG^2 + EH^2$

$GF^2 = EG^2 + EH^2$

$BE \cdot EF + EG^2 = EG^2 + EH^2$

$BE \cdot EF = EH^2$

$BE \cdot EF = \square BD$

$\square BD = EH^2$

$\square A = EH^2$

Proof:

Polygon A equals the polygon BD by construction

Line BF is divided into equal (G) and unequal segments (E), thus the rectangle formed by BE,EF plus the square of EG is equal to the square on GF (II·5)

Since GHE is a right triangle, and GH is equal to GF, the square of GF is equal to the sum of the squares on EG and GH (I·47)

Thus the rectangle formed by BE,EF plus the square of EG is equal to the sum of the squares on EG and GH

Subtracting EG from both sides of the equality, gives BE,EF equals the square of EH

The rectangle formed by BE,EF is BD, since EF equals ED

Therefore BD equals the square on EH

Therefore polygon 'A' equals the square on EH

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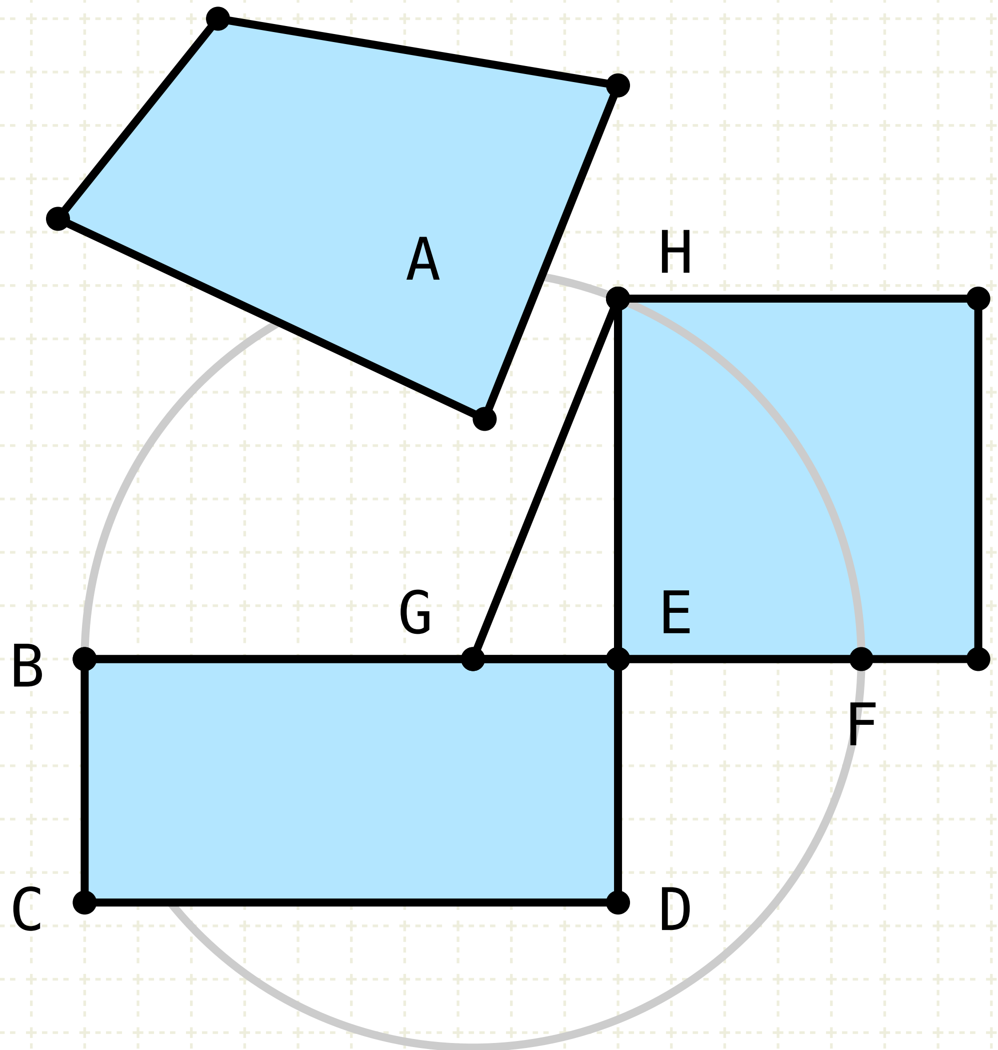
Proposition 14 of Book II

To construct a square equal to a given rectilineal figure.



$\square A = \square BD$
$EF = BD$
$BG = GF$
$GH = GF$
$BE \cdot EF + EG^2 = GF^2$
$GH^2 = EG^2 + EH^2$
$GF^2 = EG^2 + EH^2$
$BE \cdot EF + EG^2 = EG^2 + EH^2$
$BE \cdot EF = EH^2$
$BE \cdot EF = \square BD$
$\square BD = EH^2$
$\square A = EH^2$

Proof:

Polygon A equals the polygon BD by construction

Line BF is divided into equal (G) and unequal segments (E), thus the rectangle formed by BE,EF plus the square of EG is equal to the square on GF (II·5)

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Thus the rectangle formed by BE,EF plus the square of EG is equal to the sum of the squares on EG and GH

Subtracting EG from both sides of the equality, gives BE,EF equals the square of EH

The rectangle formed by BE,EF is BD, since EF equals ED

Therefore BD equals the square on EH

Therefore polygon 'A' equals the square on EH

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