Euclid's Elements

Book VII

Definitions:

1   A unit is that by virtue of which each of the things that exist is called one

2   A number is a multitude composed of units. (not one)

3   A number is part of a number, the less of the greater, when it measures the greater

11   A prime number is that which is measured by a unit alone.

12   Numbers prime to one another are those which are measured by a unit alone as a common measure

15   A number is said to multiply a number when that which is multiplied is added to itself as many times as there are units in the other, and thus some number is produced.

20   Numbers are proportional when the first is the same multiple, or the same part, or the same parts, of the second that the third is of the fourth.

As long as algebra and geometry have been separated, their progress have been slow and their uses limited; but when these two sciences have been united, they have lent each mutual forces, and have marched together towards perfection.

Joseph-Louis Lagrange
(1736 to 1813)

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Proposition 27 of Book VII

If two numbers be prime to one another, and each by multiplying itself make a certain number, the products will be prime to one another; and, if the original numbers by multiplying the products make certain numbers, the latter will also be prime to one another (and this is always the case with the extremes).

Proposition 27 of Book VII

If two numbers be prime to one another, and each by multiplying itself make a certain number, the products will be prime to one another; and, if the original numbers by multiplying the products make certain numbers, the latter will also be prime to one another (and this is always the case with the extremes).

A

B

E

C

D
F

$$\texttt{gcd(A,B) = 1}$$
$$\texttt{C = A × A = A}^2$$
$$\texttt{D = A × C = A}^3$$
$$\texttt{E = B × B = B}^2$$
$$\texttt{F = B × E = B}^3$$

In other words

Let A and B be relatively prime

Proposition 27 of Book VII

If two numbers be prime to one another, and each by multiplying itself make a certain number, the products will be prime to one another; and, if the original numbers by multiplying the products make certain numbers, the latter will also be prime to one another (and this is always the case with the extremes).

A

B

E

C

D   F

$$gcd(A,B) = 1$$
$$C = A \times A = A^2$$
$$D = A \times C = A^3$$
$$E = B \times B = B^2$$
$$F = B \times E = B^3$$

$$gcd(C,E) = 1, \quad gcd(A^2, B^2) = 1$$
$$gcd(D,F) = 1, \quad gcd(A^3, B^3) = 1$$

In other words

Let A and B be relatively prime

Then A^2 and B^2 are relatively prime, and A^3, B^3 are also relatively prime
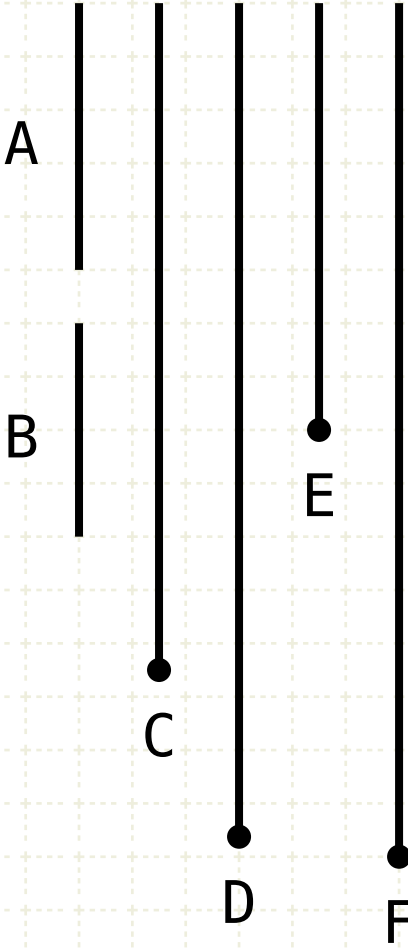
Proposition 27 of Book VII

If two numbers be prime to one another, and each by multiplying itself make a certain number, the products will be prime to one another; and, if the original numbers by multiplying the products make certain numbers, the latter will also be prime to one another (and this is always the case with the extremes).

A

B

C

E

D

F

```
gcd(A,B) = 1
C = A × A = A²
D = A × C = A³
E = B × B = B²
F = B × E = B³
```

In other words

Let A and B be relatively prime

Then A^2 and B^2 are relatively prime, and A^3, B^3 are also relatively prime
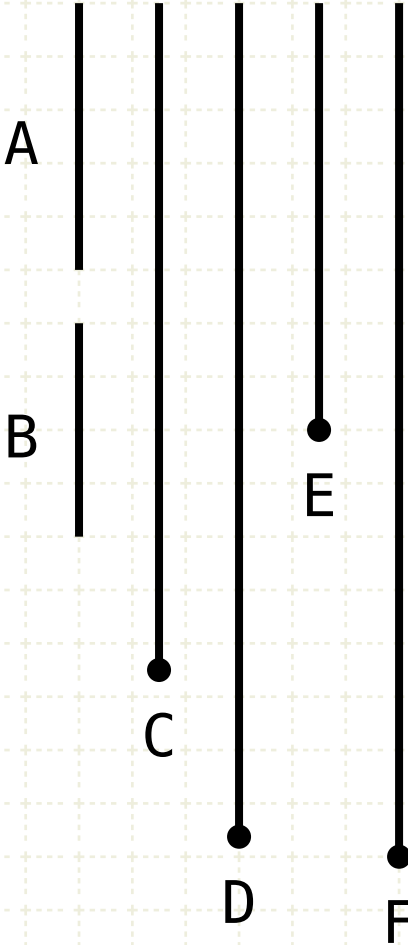
Proof

Proposition 27 of Book VII

If two numbers be prime to one another, and each by multiplying itself make a certain number, the products will be prime to one another; and, if the original numbers by multiplying the products make certain numbers, the latter will also be prime to one another (and this is always the case with the extremes).

A

B

C

E

D   F

$$\mathtt{gcd(A,B) = 1}$$
$$\mathtt{C = A \times A = A^2}$$
$$\mathtt{D = A \times C = A^3}$$
$$\mathtt{E = B \times B = B^2}$$
$$\mathtt{F = B \times E = B^3}$$

$$\mathtt{gcd(A^2,B) = 1}$$
$$\mathtt{gcd(C,B) = 1}$$

In other words

Let A and B be relatively prime

Then A² and B² are relatively prime, and A³, B³ are also relatively prime

Proof

Since A and B are prime to one another, then A², and B are prime to one another (VII·25), thus C and B are prime to one another

Proposition 27 of Book VII

If two numbers be prime to one another, and each by multiplying itself make a certain number, the products will be prime to one another; and, if the original numbers by multiplying the products make certain numbers, the latter will also be prime to one another (and this is always the case with the extremes).

A

B

E

C

D   F

$$\text{gcd(A,B) = 1}$$
$$C = A \times A = A^2$$
$$D = A \times C = A^3$$
$$E = B \times B = B^2$$
$$F = B \times E = B^3$$

$$\text{gcd}(A^2,B) = 1$$
$$\text{gcd(C,B) = 1}$$
$$\text{gcd(C,B}^2\text{)= 1}$$
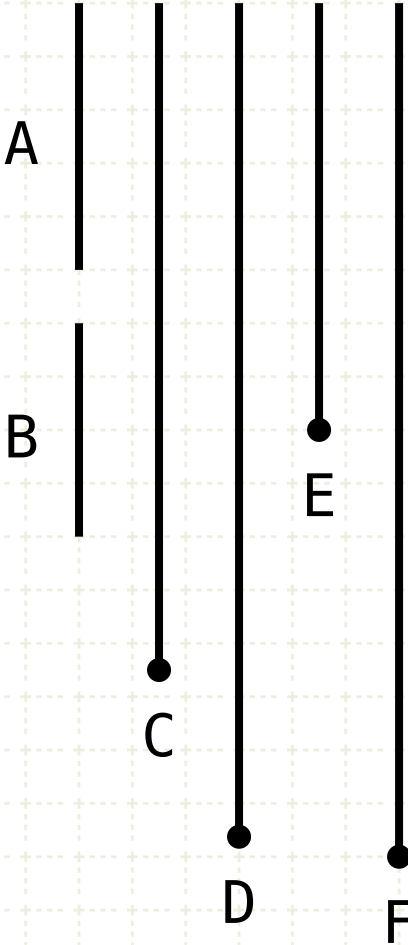$$\text{gcd(C,E) = 1} \rightarrow \text{gcd}(A^2,B^2)=1$$

In other words

Let A and B be relatively prime

Then A^2 and B^2 are relatively prime, and A^3, B^3 are also relatively prime

Proof

Since A and B are prime to one another, then A^2, and B are prime to one another (VII·25), thus C and B are prime to one another

Since C,B are prime to one another, and E is B times B, C and E are prime to one another (VII·25)

Proposition 27 of Book VII

If two numbers be prime to one another, and each by multiplying itself make a certain number, the products will be prime to one another; and, if the original numbers by multiplying the products make certain numbers, the latter will also be prime to one another (and this is always the case with the extremes).

A

B

E

C

D   F

$$gcd(A,B) = 1$$
$$C = A \times A = A^2$$
$$D = A \times C = A^3$$
$$E = B \times B = B^2$$
$$F = B \times E = B^3$$

$$gcd(A^2,B) = 1$$
$$gcd(C,B) = 1$$
$$gcd(C,B^2) = 1$$
$$gcd(C,E) = 1 \rightarrow gcd(A^2,B^2)=1$$
$$gcd(A,B^2) = 1$$
$$gcd(A,E) = 1$$

In other words

Let A and B be relatively prime

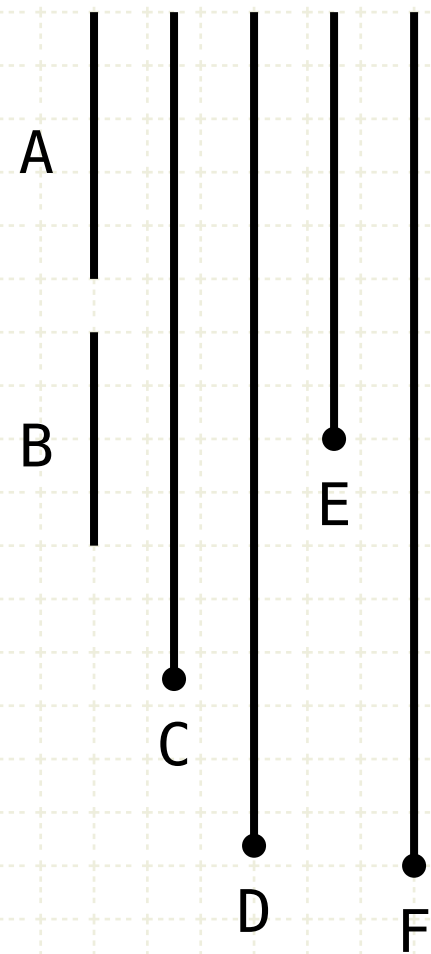Then A^2 and B^2 are relatively prime, and A^3, B^3 are also relatively prime
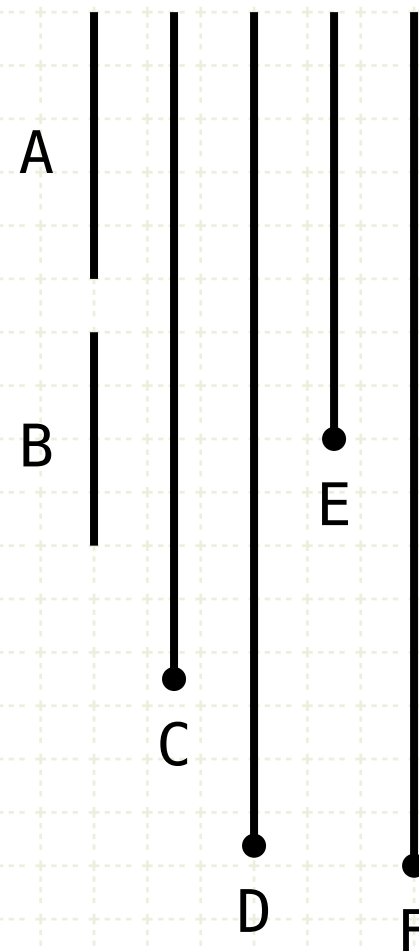
Proof

Since A and B are prime to one another, then A^2, and B are prime to one another (VII·25), thus C and B are prime to one another

Since C,B are prime to one another, and E is B times B, C and E are prime to one another (VII·25)

Since A,B are prime to one another, and E is B times B, A and E are prime to one another (VII·25)

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If two numbers be prime to one another, and each by multiplying itself make a certain number, the products will be prime to one another; and, if the original numbers by multiplying the products make certain numbers, the latter will also be prime to one another (and this is always the case with the extremes).

A

B

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D F

$$\gcd(A,B) = 1$$
$$C = A \times A = A^2$$
$$D = A \times C = A^3$$
$$E = B \times B = B^2$$
$$F = B \times E = B^3$$

$$\gcd(A^2,B) = 1$$
$$\gcd(C,B) = 1$$
$$\gcd(C,B^2) = 1$$
$$\gcd(C,E) = 1 \rightarrow \gcd(A^2,B^2)=1$$
$$\gcd(A,B^2) = 1$$
$$\gcd(A,E) = 1$$
$$\gcd(A \times C, B \times E) = 1$$

In other words

Let A and B be relatively prime

Then A^2 and B^2 are relatively prime, and A^3, B^3 are also relatively prime

Proof

Since A and B are prime to one another, then A^2, and B are prime to one another (VII·25), thus C and B are prime to one another
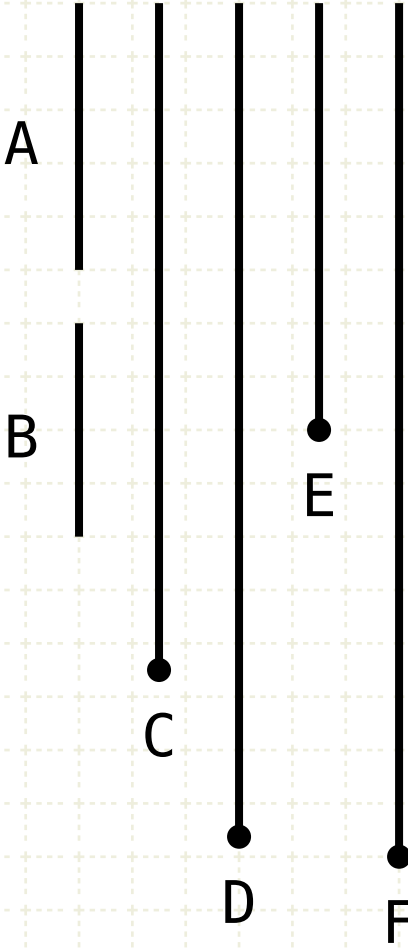
Since C,B are prime to one another, and E is B times B, C and E are prime to one another (VII·25)

Since A,B are prime to one another, and E is B times B, A and E are prime to one another (VII·25)

Since A and C are prime to the two numbers B and E, therefore the product of A and C will be prime to the product B and E (VII·26)

Proposition 27 of Book VII

If two numbers be prime to one another, and each by multiplying itself make a certain number, the products will be prime to one another; and, if the original numbers by multiplying the products make certain numbers, the latter will also be prime to one another (and this is always the case with the extremes).

A

B

C

D   E   F

$$gcd(A,B) = 1$$
$$C = A \times A = A^2$$
$$D = A \times C = A^3$$
$$E = B \times B = B^2$$
$$F = B \times E = B^3$$

$$gcd(A^2,B) = 1$$
$$gcd(C,B) = 1$$
$$gcd(C,B^2) = 1$$
$$gcd(C,E) = 1 \rightarrow gcd(A^2,B^2)=1$$
$$gcd(A,B^2) = 1$$
$$gcd(A,E) = 1$$
$$gcd(A \times C, B \times E) = 1$$
$$gcd(D,F) = 1$$
$$gcd(A^3,B^3) = 1$$

In other words

Let A and B be relatively prime

Then A^2 and B^2 are relatively prime, and A^3, B^3 are also relatively prime

Proof

Since A and B are prime to one another, then A^2, and B are prime to one another (VII·25), thus C and B are prime to one another

Since C,B are prime to one another, and E is B times B, C and E are prime to one another (VII·25)

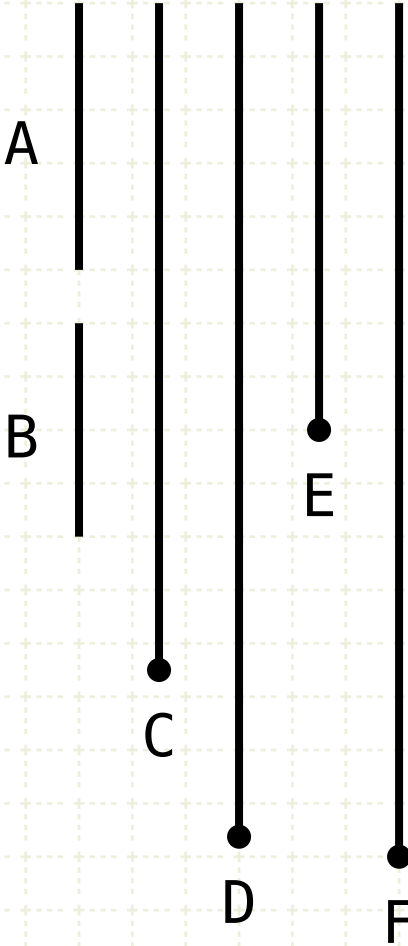Since A,B are prime to one another, and E is B times B, A and E are prime to one another (VII·25)
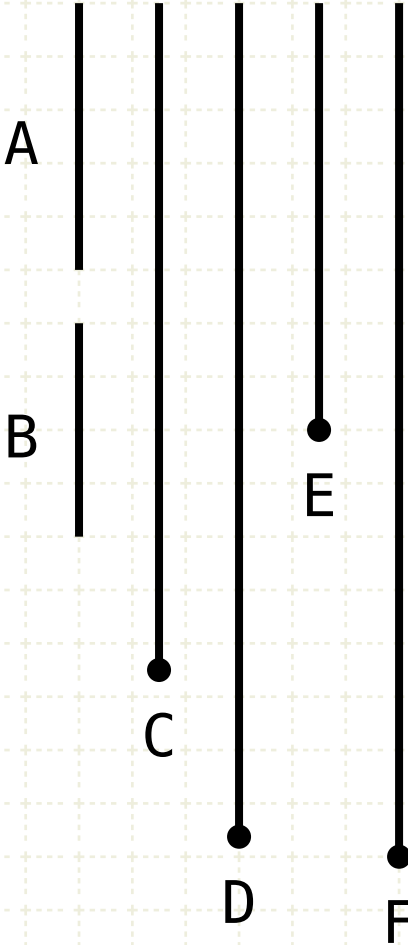
Since A and C are prime to the two numbers B and E, therefore the product of A and C will be prime to the product B and E (VII·26)

Therefore D and F are prime to one another

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Proposition 27 of Book VII

If two numbers be prime to one another, and each by multiplying itself make a certain number, the products will be prime to one another; and, if the original numbers by multiplying the products make certain numbers, the latter will also be prime to one another (and this is always the case with the extremes).

A

B

C

D

E

F

$$\texttt{gcd(A,B) = 1}$$
$$\texttt{C = A × A = A}^2$$
$$\texttt{D = A × C = A}^3$$
$$\texttt{E = B × B = B}^2$$
$$\texttt{F = B × E = B}^3$$

$$\texttt{gcd(A}^2\texttt{,B) = 1}$$
$$\texttt{gcd(C,B) = 1}$$
$$\texttt{gcd(C,B}^2\texttt{)= 1}$$
$$\texttt{gcd(C,E) = 1 → gcd(A}^2\texttt{,B}^2\texttt{)=1}$$
$$\texttt{gcd(A,B}^2\texttt{)= 1}$$
$$\texttt{gcd(A,E) = 1}$$
$$\texttt{gcd(A×C,B×E) = 1}$$
$$\texttt{gcd(D,F) = 1}$$
$$\texttt{gcd(A}^3\texttt{,B}^3\texttt{)= 1}$$

In other words

Let A and B be relatively prime

Then A^2 and B^2 are relatively prime, and A^3, B^3 are also relatively prime

Proof

Since A and B are prime to one another, then A^2, and B are prime to one another (VII·25), thus C and B are prime to one another

Since C,B are prime to one another, and E is B times B, C and E are prime to one another (VII·25)

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