Analysis and Investigation over Periodic Points

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Introduction:

This report is aimed to research about the behaviors of tent map, and then use return map function and multiple approaches to learn about the fixed points and stable points of sine map.

In the analysis part, we calculate the stability of fixed points/n-cycles of the tent map up to 2 periods, and provide a solution to solve any period. And we use computer simulation to check our results.

In the investigation part, we programmed and tested the return map function. With multiple approaches, to learn n-cycles and fixed points of sine map.

In the code part, we provided all the codes used in the report.

Analysis:

Since tent map function equals to $T(x) = \begin{cases} 2 \cdot x \;\; when \;\; 0 \le x \le 1/2 \\ 2(1-x) \;\; when \frac{1}{2} \le x \le 1 \end{cases}$, we can use absolute value to deal with this function instead of using the piecewise function form, hence we can find out that $\mathrm{T(x)} = 1 - 2|\mathrm{x} - \frac{1}{2}|$.

First, we calculate the period 1, and we have $\mathrm{T(x)} = \mathrm{x}$, and after solving the equation, we have 2 solutions, which is $\mathrm{x} = 0 \;\; \mathrm{or} \;\; \mathrm{x} = \frac{2}{3}$.

And for period 2, we have $\mathrm{T(T(x))} = \mathrm{x}$, which have 4 solutions, and they are $\mathrm{x} = 0 \;\; \mathrm{or} \;\; \mathrm{x} = \frac{2}{5} \;\; \mathrm{or} \;\; \mathrm{x} = \frac{4}{5} \;\; \mathrm{or} \;\; \mathrm{x} = \frac{2}{3}$.

For period larger than 2, we need to solve the equation $T^n(x) = x$, this can be solved with computer or by hands. For example, we have $x = \frac{2}{9}, \frac{2}{7}, \frac{4}{9}, \frac{4}{7}, \frac{2}{3}, \frac{6}{7}, \frac{8}{9},$ as the solutions of period-4.

We can see that the simulation fits our calculation.

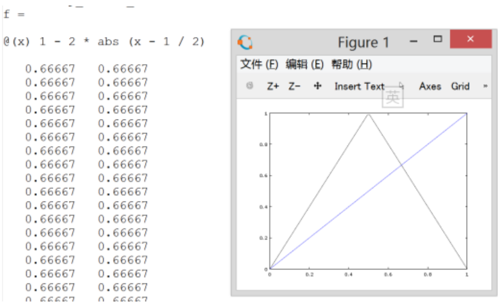
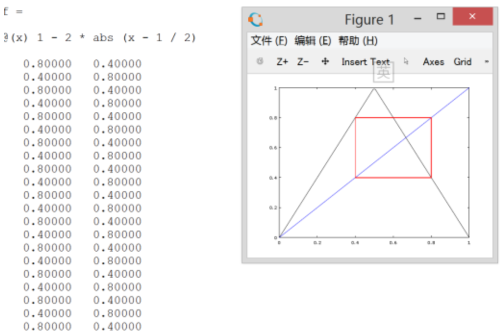


Figure 1 Computer Simulation of Period-1



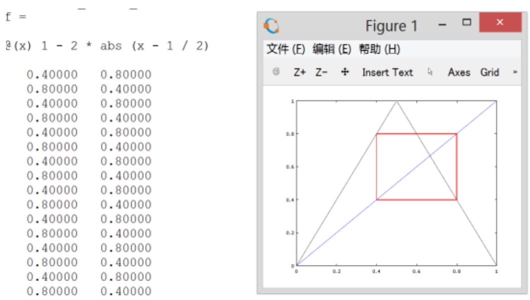


Figure 2 & 3 Computer Simulation of Period-2

Investigation:

The flow chart of return map is figure 4. And after testing our codes for λ from 0, and N, M equals to 40 and 50 for x(1-x^2), result (figure.5) is reasonable.

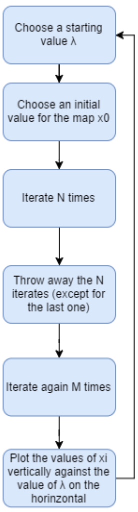


Figure 4 Flowchart

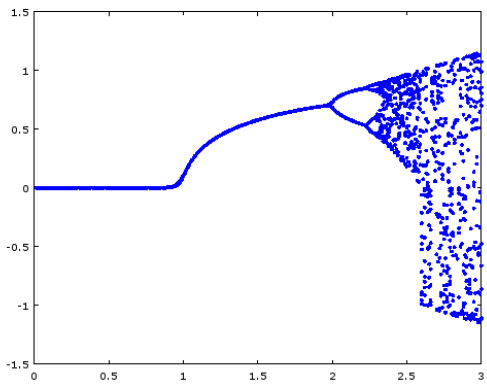


Figure 5 Return Map of x(1-x^2)

As for the sine map, we first use return map to analyze, λ from 0 to 1, we can get the figure of return map.

Which means, we can learn the fixed point from the figure under λ approximately equals 0.833266.

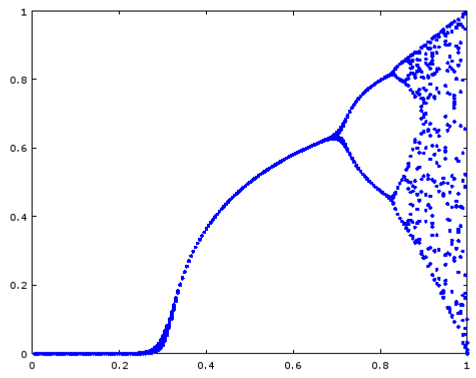


Figure 6. Return Map of Sin(pi*x)

And then using Newton or Bisection to find the solution for equation that allows sin(pi*x) * λ and x equals to each other. For λ = 1, we have x = 0.73648, and for λ = 0.35, we have x = 0.23792. When using these 2 ways to find solution, we can find out that Newton is faster and more accurate to locate solution than Bisection.

Using iterations plot to analyze this conclusion, for example, x0 equals to 0.6 and λ equals to 0.35, the stable point is 0.23792 which is the same as the return map and the results of Newton and Bisection.

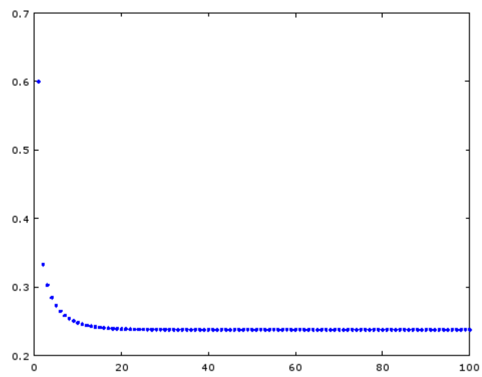


Figure 7 Iterations of λ = 0.35

And then we can try using cobweb to confirm our findings, for example, x0 equals to 0.6 and λ equals to 0.35, the stable point is 0.23792 which is the same as the return map and the results of Newton and Bisection and the iteration map.

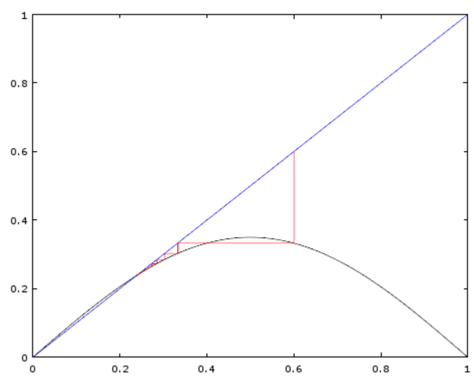


Figure 8 Cobweb of sine (λ=0.35)

After using these ways to investigate the fixed points of sine map, we can try to work out the fixed point become unstable. When derivation of sine map equals 1, we have x= sin(pi*x) * λ and (sin(pi*x) * λ)' =1, and we can solve x = 0, λ = 0.31831 or x = 0.645774, λ = 0.719962. These two points are clearly shown on Figure.6.

Next we compute the twice iterated function of sine map and see where the fixed point of a period - 2 limit cycle of sine map becomes unstable: x= sin(pi* sin(pi*x) * λ) * λ and (sin(pi*x) * λ)" =1, we have x = 0.820765, λ = 0.833266.

These conclusions were solved by Newton, and can be confirmed by cobweb and iterations.

By using this way, we can compute fourth iterated function or even more, but the results have become unclear on the return map.

In conclusion, by using Bisection or Newton, we can find out the points (n-cycle or fixed points) numerically. And by iterations and cobweb, we can analyze them geometrically. Return maps is a great way to put all λ on the plot.

Codes:

All codes are available in my dropbox:

https://www.dropbox.com/home/Chen%20Yuxuan%20(Sandy)%20-%20comp