Theory Exam

Answer **ANY TWO** of the following three questions:

1. A certain programming language *P* defines a comment as delimited by /# and #/. Let the alphabet $\Sigma = \{a, b, /, \#\}$ and let *C* be the set of all comments that begin with /#, end with #/, and contain no intervening #/. The shortest legal string in *L* is therefore /##/.

   a. (10 points) Give a deterministic finite automaton (DFA) that recognizes legal comments *C* in the language *P*.

   b. (10 points) Write a context-free grammar (CFG) that generates legal comments *C* in the language *P*.

2. Consider the language $L = \{\langle M\rangle \mid M \text{ is a Turing machine that accepts the string } w = 0011\}$.

   a. (5 points) Is *L* decidable or undecidable?

   b. (15 points) Prove your answer above using reducibility. You may assume that the following languages are known to be undecidable:

      $HALT_{TM} = \{\langle M, w\rangle : M \text{ is a Turing machine that halts on } w\}$
      $A_{TM} = \{\langle M, w\rangle : M \text{ is a Turing machine that accepts } w\}$

      You may not use Rice's Theorem.

3. For each decision problem listed below, answer:

   i. Is the problem in the class NP?

   ii. Is the problem NP-complete?

Scoring: each correct answer given is +2, each incorrect answers given is -1, no answer given is 0]
DO NOT GUESS!

   a. Given a graph *G*, does *G* contains a 3-clique?

      (a 3-clique is a subgraph of *G* that is fully connected or complete on 3 vertices)

   b. Given two integers *n* and *m*, are *n* and *m* relatively prime?

      (two integers are relatively prime if their greatest common divisor is 1)

   c. Given a graph *G* and a number *k*, is the largest clique in *G* of size *k*?

      (a clique is a subgraph of *G* that is a complete graph)

   d. Given a Boolean expression *E*, are there are <u>exactly</u> two truth assignments that satisfy *E*?

      (a Boolean expression is satisfiable if some assignment of variables makes it true)

   e. Given a set of students $N = \{s_1, s_2, \ldots, s_{|N|}\}$, a set of final exams $M = \{e_1, e_2, \ldots, e_{|M|}\}$, a mapping $f: N \to P(M)$ showing the specific subset of exams each student is taking, and a number *t* of possible time slots for the exams, is it possible to schedule the exams into the *t* time slots such that no student has two of his or her exams assigned to the same time slot?

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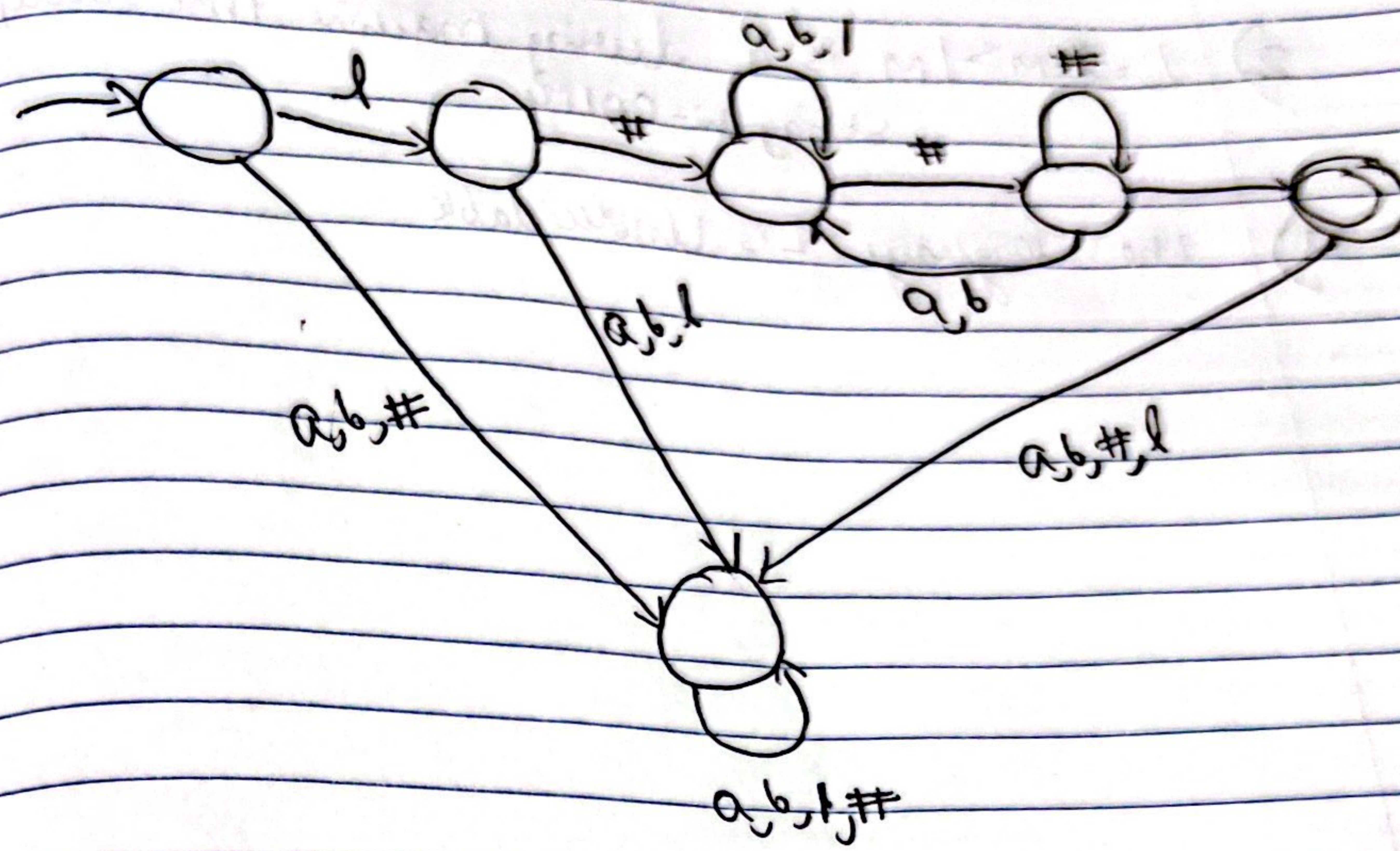
1)

a)



b) CFG :-

$R.E = /\#(a+b+/+\#^*a+\#^*b)^*\#/$

$G = (V, \Sigma, R, S)$

$R = \{ S \rightarrow /A/,$

$A \rightarrow \#B\#$

$B \rightarrow aB \mid bB \mid \varepsilon$

$\}$

$V = \{S, A, B\}$

$\Sigma = \{/, \#, a, b\}$

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2. $L = \{ \langle M \rangle \mid M$ is a Turing machine that accepts string $W = 0011 \}$

   a. L is <u>undecidable</u>.

   b. $f(M, W) =$

Construct a new TM M' as follows:

```
M'(x) =
    if(x == 0011){
        reject;
    }

    Run M on W;

    if(M accepts W){
        accept;
    }
    else{
        reject;
    }
```

Output M'

If M accepts W then M' will accept the string 0011 because $L(M') = \Sigma^*$. If M does not accept W then M' will not accept the string 0011 because $L(M') = \emptyset$. Therefore, a yes of A_{TM} maps to a yes of M' accepts 0011 and a no of A_{TM} maps to a no of M' accepts W. Since A_{TM} is undecidable, language L is also undecidable.

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8) Given that

$L = \{ \langle M \rangle \mid M$ is a turing machine that accepts $w = 0011. \}$

a) undecidable

b) Proof:

Let us assume L is decidable it means it halts on string $w = 0011$.

$HALT_{TM} = \{ \langle M, w \rangle \mid M$ halts on $w \}$

So, $HALT_{TM}$ halts on all strings including "0011".

$A_{TM} = \{ \langle M, w \rangle \mid M$ accepts $w \}$

A_{TM}, let M doesnot halt on w then it loops forever for all strings including "0011" which is contradiction.

So, M is undecidable.