

1) a) $\{w: w \text{ contains the substring } 101\}$

101 as substring.

$$L = \{101, 0101, 11101, 0101, \dots\}$$

101

$$\Rightarrow R.E = (0+1)^* 101 (0+1)^*$$

b) $\{w: w \text{ contains at least three 0's}\}$

$$L = \{000, 1000, 01010, \dots\}$$

$$R.E = (0+1)^* 0 (0+1)^* 0 (0+1)^*$$

c) $\{w: w \text{ contains at most three 1's}\}$

$$L = \{00, 100, 0, 110, 111, 0100, \dots\}$$

$$R.E = 0^* 1 0^* 1 0^* 1 0^*$$

d) $\{w: |w| \geq 3\}$

$$L = \{000, 111, 1001, \dots\}$$

$$(0+1)^* (0+1) (0+1) (0+1)^*$$

e) $\{w: |w| \leq 3\}$

$$L = \{0, 00, 11, 01, 111, \dots\}$$

$$(0+1+\epsilon)(0+1+\epsilon)(0+1+\epsilon).$$

f: $\{w: w \text{ starts and ends with different symbols}\}$

$$L = \{10, 01, 001, 1001, \dots\}$$

$$0(0+1)^*1 + 1(0+1)^*0$$

(Doubt).

g: $\{w: \text{every odd position of } w \text{ is } 0\}$

$$L = \{\epsilon, 0, 01, 00, 010, 001, \dots\}$$

$$\epsilon + 0 + 01 + 00 + 010 + 001 + \dots$$

$$(0(0+1)^*)^*$$

$$\Rightarrow (0(1+0))^*(0+\epsilon) / \text{or } (0(0+1))^*$$

$\epsilon, 00, \dots$

h) $\{w: w \text{ does not contain exactly two } 1\text{'s}\}$

$$\epsilon + 0^*10^* + \cancel{0^*10^*10^*}$$

$$0^*10^*10^*10^*10^*$$

i) $\{w: \text{every } 0 \text{ in } w \text{ followed by two } 1\text{'s}\}$

$$(1^*0111^*)^*$$

j) $\{w: w \text{ starts with substring } 011 \text{ and contains the substring } 110\}$

$$011(0+1)^*110(0+1)^*$$

2)

Turing machine that decides the following language $\Sigma = \{0,1\}^*$

$L = \{w : w \text{ contains an even no. of occurrence of substring } 011 \text{ and } |w| \text{ is odd}\}$

