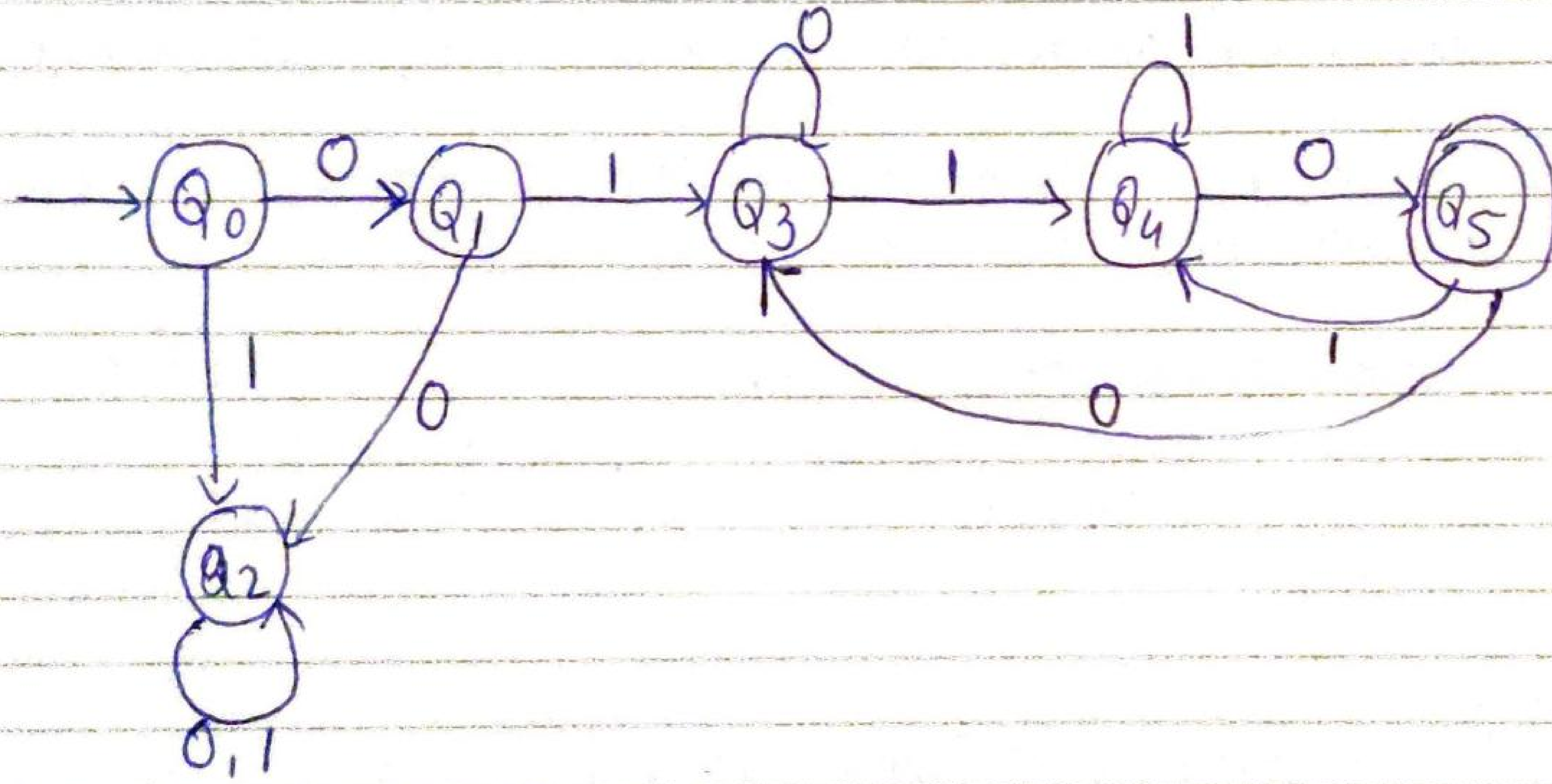
Spring 2015

Q.1 → Give the State diagram for a DFA that recognizes the language.

$L = \{ w : w \text{ has prefix } 01 \text{ and suffix } 10 \}$



Q.2 → Show that the collection of decidable languages is closed under the operation of
a) concatenation b) kleene closure.

→ ~~Let L1 and~~
Decidable language is "Recursive language" and undecidable language is "not recursive language".

closed means either accept or reject.

a) Concatenation :
. Let L_1 and L_2 be 2 decidable languages. By definition there are deciders M_1 and M_2 such that $L(M_1) = L_1$ and $L(M_2) = L_2$.

we construct the following nondeterministic 3-tape Turing machine M

1) on input x
2) Nondeterministically split the input string into 2 parts $x = w_1 w_2$ and copy w_1 on second tape and w_2 on the third tape.
3) on the second tape run M_1 on w_1
4) If M_1 accepted then continue with step 5, elose M rejects.
5) on the third tape run M_2 on w_2.
6) If M_2 accepted then M accepts else M rejects.

Now, M is surely a nondeterministic decider because both M_1 and M_2 are deciders and $L(M) = L_1 \cdot L_2$.

∴ Any 3-tape nondeterministic decider is equivalent to some single tape deterministic decider. Hence, we have a decider for the concatenation of L_1 and L_2.

b) Kleene closure:

Let L_1 be a decidable language. By definition there is a decider M_1 such that $L(M_1) = L_1$

we construct the following nondeterministic 2-tape Turing machine M:

1) on input x:
2) Nondeterministically select a nonempty left-most part of the input x which has not been read yet and copy it on the second tape.
3) on the second tape run M_1 on the present string.
4) If M_1 accepted and the whole input X was processed, then M accepts. If M_1 accepted and some suffix of x still has to be processed then clean the second tape and continue with step 2. if M_1 is rejected then M rejects.

Now M is ~~surer~~ surely a nondeterministic decider because M_1 is a decider and $L(M) = L^*1$.

Any 2-tape nondeterministic decider is equivalent to some single tape deterministic decider. Hence, we have a decider for the kleene closure of L_1.

Thus, collection of ~~deciadble~~ decidable languages is closed under concatenation and kleene closure.

ans from chegg (not sure).

Q.3 → Answer True / False for each of the following statement to indicate whether the conclusion is always true
(negative marking)

a) If $A \leq B$ and B is not decidable, then A is not decidable.

→ ~~False~~ True

b) If $A \leq B$ and B is decidable, then A is decidable

→ True

c) If $A \leq B$ and B is Turing recognizable, then A is Turing recognizable.

→ True.

d) If $A \leq B$ and B is not Turing recognizable then A is not turing recognizable.

→ True.

e) If $A \leq B$ and B is a ~~regual~~ regular language, then A is a regular language.

→ False

f) If $A \leq B$ and $B \leq C$, then $A \leq C$

→ True

g) If A is Turing recognizable and $A \leq \bar{A}$, then A is decidable.

→ True.

h) If $A \leq_p B$ and $B \in NP$, then $A \in NP$

→ True.

i) If a problem cannot be solved in polynomial time, then it is NP-complete.

→ ~~False~~ True

j) If $A \leq_p B$, and B is NP-complete, then A is in NP.

→ False.