CS 6901 Capstone Exam Theory Spring 2015

Choose any 2 of the 3 problems.

1). Give the state diagram for a DFA that recognizes the language:

$$L = \{w: w \text{ has prefix } 01 \text{ and suffix } 10\}$$

2). Show that the collection of decidable languages is closed under the operation of

a. Concatenation.

b. Kleene closure.

3). Answer **TRUE** or **FALSE** for each of the following statement to indicate whether the conclusion is **always true**. If you do not know the answer, do not guess.

Scoring: +2 points for correct answer; 0 point for no answer; -1 point for wrong answer.

a. If $A \leq B$ and B is not decidable, then A is not decidable.

b. If $A \leq B$ and B is decidable, then A is decidable.

c. If $A \leq B$ and B is Turing recognizable, then A is Turing recognizable.

d. If $A \leq B$ and B is not Turing recognizable, then A is not Turing recognizable.

e. If $A \leq B$ and B is a regular language, then A is a regular language.

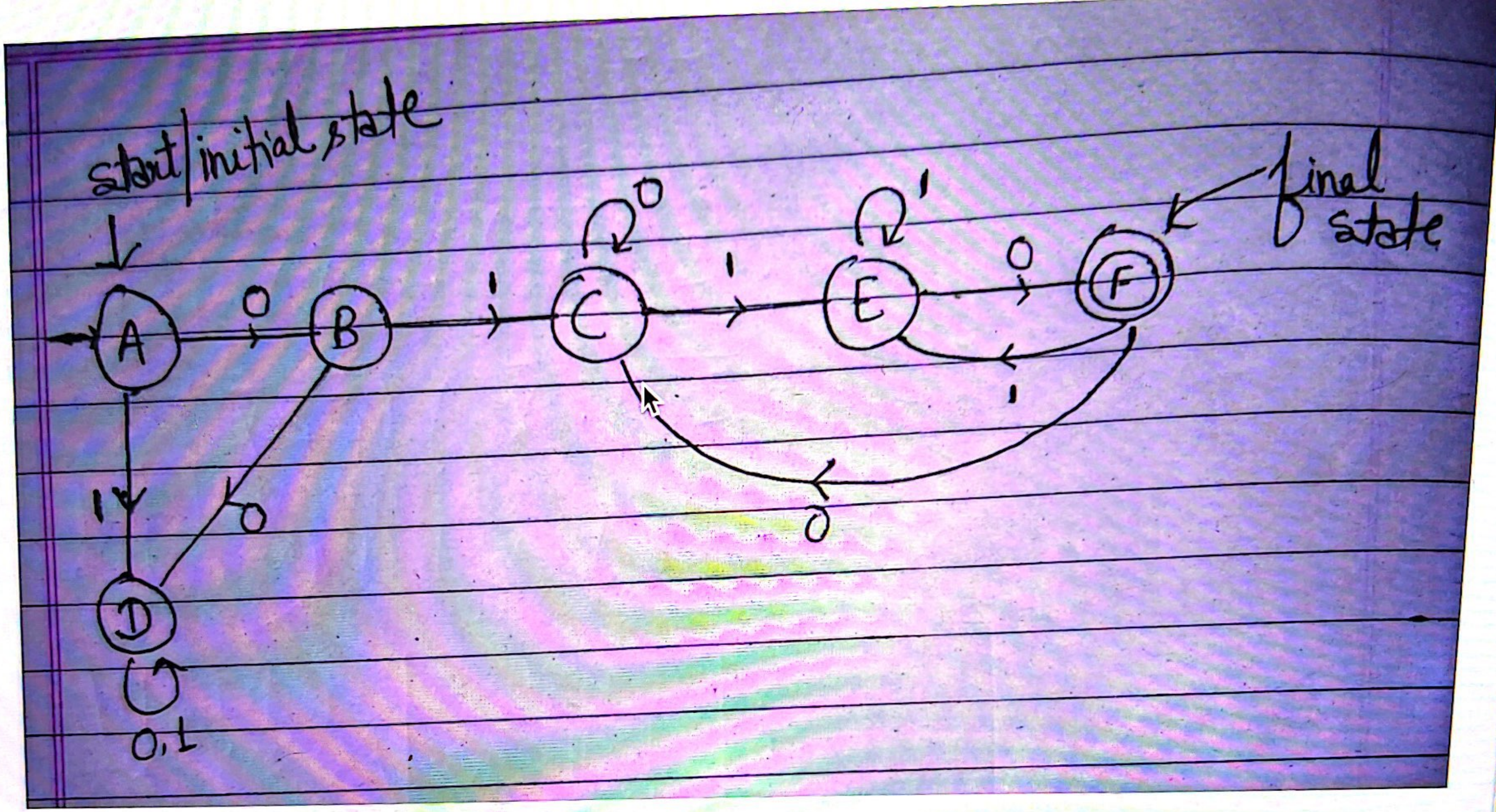
f. If $A \leq B$ and $B \leq C$, then $A \leq C$.

g. If A is Turing-recognizable, and $A \leq \bar{A}$, then A is decidable.

h. If $A \leq_p B$ and $B \in NP$, then $A \in NP$.

i. If a problem cannot be solved in polynomial-time, then it is NP-complete.

j. If $A \leq_p B$ and B is NP-Complete, then A is in NP.



Questions viewed by other students

Q: 1) Give the state diagram for a DFA that recognizes thelanguage: L = {w: w has prefix 01 and suffix 10}.

A: See answer

2). Show that the collection of decidable languages is closed under the operation of

a. Concatenation.

b. Kleene closure.

Expert Answer

Anonymous answered this
7 answers

Here, Decidable languag is "Recursive Language" and Undecidable language is "not recusive language".

Closed means either accept or reject .

And here is the proof that decidable language is closed under Concatenation and kleene closure

Concatenation: Let L1 and L2 be two decidable languages. By definition there are deciders M1 and M2 such that L(M1) = L1 and L(M2) = L2

We construct the following nondeterministic 3-tape Turing machine M

1 On input x

2 Nondeterministically split the input string into two parts x = w1w2 and copy w1 on second tape and w2 on the third tape.

3 On the second tape run M1 on w1

4 If M1 accepted then continue with step 5, else M rejects

5 On the third tape run M2 on w2

My

Anonymous answered this
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3 On the second tape run M1 on w1

4 If M1 accepted then continue with step 5, else M rejects

5 On the third tape run M2 on w2

6 If M2 accepted then M accepts else M rejects

Now M is surely a nondeterministic decider because both M1 and M2 are deciders and L(M) = L1 .L2

Any 3-tape nondeterministic decider is equivalent to some single tape deterministic decider. Hence we have a decider for the concatenation of L1 and L2

Kleene closure: Let L1 be a decidable language. By definition there is a decider M1 such that L(M1) = L1

We construct the following nondeterministic 2-tape Turing machine M

My

View a

Any 3-tape nondeterministic decider is equivalent to some single tape deterministic decider. Hence we have a decider for the concatenation of L1 and L2

Kleene closure: Let L1 be a decidable language. By definition there is a decider M1 such that L(M1) = L1

We construct the following nondeterministic 2-tape Turing machine M

1 On input x

2 Nondeterministically select a nonempty left-most part of the input x which has not been read yet and copy it on the second tape

3 On the second tape run M1 on the present string

4 If M1 accepted and the whole input x was processed, then M accepts. If M1 accepted and some suffix of x still has to be processed then clean the second tape and continue with step 2. If M1 rejected then M rejects

Now M is surely a nondeterministic decider because M1 is a decider and L(M) = L * 1

Any 2-tape nondeterministic decider is equivalent to some single tape deterministic decider. Hence we have a decider for the Kleene star of L1

So Regular , CFL,CSL,and REC are closed under concatenation and Kleene closure.

Practice with similar questions

Q: 2). Show that the collection of decidable languages is closedunder the operation ofa. Concatenation.b. Kleene closure.

A: See answer

langs. under set difference.

If $A \leq B$ and B is not decidable, then A is not decidable.
F

If $A \leq B$ and B is decidable, then A is decidable.
T

If $A \leq B$ and B is turing recognizable, then A is turing recog.
T

If $A \leq B$ and B is not turing recognizable, then A is not turing recognizable.
F

If $A \leq B$ and B is regular lang., then A is a regular lang.
F

If $A \leq B$ and $B \leq C$, then $A \leq C$.
T

(g) If A is Turing-recognizable, and $A \le \overline{A}$,
then A is decidable.

T

(h) If $A \le_p B$ and $B \in NP$, then $A \in NP$.

T

(i) If a problem can't be solved in polynomial-time, then it is NP-complete.

F

(j) If $A \le_p B$ and B is NP-complete, then A is in NP.