

Fall - 2014

doubt

1. Provide a context free grammar that generates the following language over  $\Sigma = \{0, 1\}$ :  $\{w = 0^* 1^* : |w| \text{ is odd}\}$

→  $L = \{ \epsilon, 0, 1, 011, 010, \dots \}$ .

$S \rightarrow AOB \mid AIB$

$A \rightarrow \epsilon \mid A00$

$B \rightarrow \epsilon \mid B11$

2. A clique

→ Since, clique is NP problem and every NP problem is easy to verify in polynomial time.

Here, given 3 clique is a decidable problem means in fact, problem statement is that a given graph  $G = \{V, E\}$

we determine whether given graph is 3 clique or not in polynomial time.



we can design any algorithm for  
detection of 3 clique like that

3clique (vertex  $V$ , edges  $E$ )  
{

// if graph has not 3 vertex then it's  
// not 3clique.

if (vertex.size() != 3)  
return false.

// if there are only 1 to 2, 2 to 3  
and 3 to 1 edges then true.

if (edges.size() != 3)  
return false.

if (!edges 1 to 2 or !edges 2 to 3  
or !edges 1 to 3)  
return false

return true

}

just like any algorithm we can detect



in polynomial time whether given graph is 3 clique or not so given problem is decidable. In fact every NP problem is decidable and clique is NP problem.

3. Answer the following.

a. Define the class NP.

→ The NP in NP class stands for Non-deterministic polynomial time. It is the collection of decision problems that can be solved by a non-deterministic machine in polynomial time.

Features :

- 1) The Solutions - of the NP class are hard to find since they are being solved by a non-deterministic machine but the solutions are easy to verify.
- 2) Problems of NP can be verified by a Turing machine in a polynomial time.



b) Show that the class NP is closed under Concatenation.

→ Let A and B be languages that are decided by NP-machines  $T_A$  and  $T_B$

Now we want to show that, there is a non-deterministic poly time decider  $T_{A \cup B}$  that decides concatenation of A and B.

The construction of  $T_{A \cup B}$  is as follows:

$T_{A \cup B}$  = "on input S :

- 1) Split S into  $S_1, S_2$  such that  $S = S_1 S_2$ .
- 2) Run the NP machine  $T_A$  on  $S_1$ . If  $T_A$  is rejected, then reject.
- 3) Else run  $T_B$  on  $S_2$ . If  $T_B$  is rejected, then reject.
- 4) Else accept.

The time taken by Step 1 is  $O(n)$  in a 2 ~~tape~~ tape Turing Machine. Thus, T is a poly-time non deterministic decider for  $A \cup B$



classmate

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Page \_\_\_\_\_

Thus, it is proved that NP is closed under concatenation.