CS 601 - Advanced Algorithms
Gale Shaply Termination Assignment

Gale Shaply Termination Algorithm:

```
Initialize each person to be free.
while(some man is free and hasn't proposed to every woman) {
        m= Choose such a man
        w = 1st women on m's list to whom m has not yet proposed
        if (w is free)
                assign m and w to be engaged
        else if (w prefers m to her fiance m')
                assign m and w to be engaged and m' to be free
        else
                w rejects m
}
```

Given Men, Women and their preference lists:

NAME	1ST	2ND	3RD	4TH	5TH
Victor	A	B	C	D	E
Wyatt	B	C	D	A	E
Xavier	C	D	A	B	E
Yancey	D	A	B	C	E
Zeus	A	B	C	D	E

NAME	1ST	2ND	3RD	4TH	5TH
Amy	W	X	Y	Z	V
Bertha	X	Y	Z	V	W
Clare	Y	Z	V	W	X
Diane	Z	V	W	X	Y
Erika	V	W	X	Y	Z

Algorithm Tracing:
Initially all men and women are free
freeMen = [V, W, X, Y, Z]
stablePairs = {} // empty

numberOfProposals = 0

Each and every men propose to all the women according to preference list:

'V' proposes to **'A'**, as **'A**' is the first preference and is free it gets engaged
Then, freeMen = **[W, X, Y, Z],** stablePairs = **{(V,A)}**
numberOfProposals = **1**

'W' proposes to **'B'**, as **'B**' is the first preference and is free it gets engaged
Then,freeMen = **[X, Y, Z],** stablePairs = **{(V,A), (W, B)}**
numberOfProposals = **2**

'X' proposes to **'C'**, as **'C'** is the first preference and is free it gets engaged
Then, freeMen = **[Y, Z],** stablePairs = **{(V,A), (W, B), (X, C)}**
numberOfProposals = **3**

'Y' proposes to **'D'**, as **'D'** is the first preference and is free it gets engaged
Then, freeMen = **[Z],** stablePairs = **{(V,A), (W, B), (X, C), (Y, D)}**
numberOfProposals = **4**

'Z' proposes to **'A'**, but as **'A'** is not free and **'A' prefers 'Z'** more than **'V'**
Therefore **'V' gets free and 'Z' engages with 'A'**
Then, freeMen = **[V],** stablePairs = **{(W, B), (X, C), (Y, D), (Z, A)}**
numberOfProposals = **5**

NAME	**1ST**	**2ND**	**3RD**	**4TH**	**5TH**
Victor	A	B	C	D	E
Wyatt	**B**	C	D	A	E
Xavier	**C**	D	A	B	E
Yancey	**D**	A	B	C	E
Zeus	**A**	B	C	D	E

NAME	**1ST**	**2ND**	**3RD**	**4TH**	**5TH**
Amy	W	X	Y	**Z**	V
Bertha	X	Y	Z	V	**W**
Clare	Y	Z	V	W	**X**

Diane	Z	V	W	X	**Y**
Erika	V	W	X	Y	Z

'V' proposes to its **next preference 'B'**, but as **'B'** is not free and **'B' prefers 'V'** more than **'W'**
Therefore **'W' gets free** and **'V' engages with 'B'**
Then, freeMen = **[W],** stablePairs = **{(X, C), (Y, D), (Z, A), (V, B)}**
numberOfProposals = **6**

'W' proposes to its **next preference 'C**', but as **'C'** is not free and **'C' prefers 'W'** more than **'X'**
Therefore **'X' gets free and 'W' engages with 'C'**
Then, freeMen = **[X],**stablePairs = **{(Y, D), (Z, A), (V, B), (W, C)}**
numberOfProposals = **7**

'X' proposes to its **next preference 'D'**, but as **'D'** is not free and **'D' prefers 'X'** more than **'Y'**
Therefore **'Y' gets free and 'X' engages with 'D'**
Then, freeMen = **[Y],** stablePairs = **{(Z, A), (V, B), (W, C), (X, D)}**
numberOfProposals = **8**

'Y' proposes to its **next preference 'A'**, but as **'A'** is not free and **'A' prefers 'Y'** more than **'Z'**
Therefore **'Z' gets free and 'Y' engages with 'A'**
Then, freeMen = **[Z],** stablePairs = **{(V, B), (W, C), (X, D), (Y, A)}**
numberOfProposals = **9**

'Z' proposes to its **next preference 'B'**, but as **'B'** is not free and **'B' prefers 'Z'** more than **'V'**
Therefore **'V' gets free and 'Z' engages with 'B'**
Then, freeMen = **[V],** stablePairs = **{(W, C), (X, D), (Y, A), (Z, B)}**
numberOfProposals = **10**

NAME	**1ST**	**2ND**	**3RD**	**4TH**	**5TH**
Victor	A	B	C	D	E
Wyatt	B	**C**	D	A	E
Xavier	C	**D**	A	B	E

Yancey	D	**A**	B	C	E
Zeus	A	**B**	C	D	E

NAME	**1ST**	**2ND**	**3RD**	**4TH**	**5TH**
Amy	W	X	**Y**	Z	V
Bertha	X	Y	**Z**	V	W
Clare	Y	Z	V	**W**	X
Diane	Z	V	W	**X**	Y
Erika	V	W	X	Y	Z

'V' proposes to its **next preference 'C'**, but as **'C'** is not free and **'C' prefers 'V'** more than **'W'**
Therefore **'W' gets free and 'V' engages with 'C'**
Then, freeMen = **[W],** stablePairs = **{(X, D), (Y, A), (Z, B), (V, C)}**
numberOfProposals = **11**

'W' proposes to its **next preference 'D'**, but as **'D'** is not free and **'D' prefers 'W'** more than **'X'**
Therefore **'X' gets free and 'W' engages with 'D'**
Then, freeMen = **[X],** stablePairs = **{(Y, A), (Z, B), (V, C), (W, D)}**
numberOfProposals = **12**

'X' proposes to its **next preference 'A'**, but as **'A'** is not free and **'A' prefers 'X'** more than **'Y'**
Therefore **'Y' gets free and 'X' engages with 'A'**
Then, freeMen = **[Y],** stablePairs = **{(Z, B), (V, C), (W, D), (X, A)}**
numberOfProposals = **13**

'Y' proposes to its **next preference 'B'**, but as **'B'** is not free and **'B' prefers 'Y'** more than **'Z'**
Therefore **'Z' gets free and 'Y' engages with 'B'**
Then, freeMen = **[Z]**, stablePairs = **{(V, C), (W, D), (X, A), (Y, B)}**
numberOfProposals = **14**

'Z' proposes to its **next preference 'C'**, but as **'C'** is not free and **'C' prefers 'Z'** more than **'V'**
Therefore **'V' gets free and 'Z' engages with 'C'**
Then, freeMen = **[V],** stablePairs = **{(W, D), (X, A), (Y, B), (Z, C)}**

numberOfProposals = **15**

NAME	1ST	2ND	3RD	4TH	5TH
Victor	A	B	C	D	E
Wyatt	B	C	**D**	A	E
Xavier	C	D	**A**	B	E
Yancey	D	A	**B**	C	E
Zeus	A	B	**C**	D	E

NAME	1ST	2ND	3RD	4TH	5TH
Amy	W	**X**	Y	Z	V
Bertha	X	**Y**	Z	V	W
Clare	Y	**Z**	V	W	X
Diane	Z	V	**W**	X	Y
Erika	V	W	X	Y	Z

'V' proposes to its **next preference 'D'**, but as **'D'** is not free and **'D' prefers 'V'** more than **'W'**
Therefore **'W' gets free and 'V' engages with 'D'**
Then, freeMen = **[W],** stablePairs = **{(X, A), (Y, B), (Z, C), (V, D)}**
numberOfProposals = **16**

'W' proposes to its **next preference 'A'**, but as **'A'** is not free and **'A' prefers 'W'** more than **'X'**
Therefore **'X' gets free and 'W' engages with 'X'**
Then, freeMen = **[X],** stablePairs = **{(Y, B), (Z, C), (V, D), (W, A)}**
numberOfProposals = **17**

'X' proposes to its **next preference 'B'**, but as **'B'** is not free and **'B' prefers 'X'** more than **'Y'**
Therefore **'Y' gets free and 'X' engages with 'B'**
Then, freeMen = **[Y],** stablePairs = **{(Z, C), (V, D), (W, A), (X, B)}**
numberOfProposals = **18**

'Y' proposes to its **next preference 'C'**, but as **'C'** is not free and **'C' prefers 'Y'** more than **'Z'**
Therefore **'Z' gets free and 'Y' engages with 'C'**

Then, freeMen = **[Z],** stablePairs = **{(V, D), (W, A), (X, B), (Y, C)}**
numberOfProposals = **19**

'Z' proposes to its **next preference 'D'**, but as **'D'** is not free and **'D' prefers 'Z'** more than **'V'**
Therefore **'V' gets free and 'Z' engages with 'D'**
Then, freeMen = **[V],** stablePairs = **{(W, A), (X, B), (Y, C), (Z, D)}**
numberOfProposals = **20**

'V' proposes to its **next preference 'E'**, as **'E'** is free it **gets engaged**
freeMen = **[]**
stablePairs = **{(W, A), (X, B), (Y, C), (Z, D), (V, E)}**
numberOfProposals = **21**

NAME	**1ST**	**2ND**	**3RD**	**4TH**	**5TH**
Victor	A	B	C	D	**E**
Wyatt	B	C	D	**A**	E
Xavier	C	D	A	**B**	E
Yancey	D	A	B	**C**	E
Zeus	A	B	C	**D**	E

NAME	**1ST**	**2ND**	**3RD**	**4TH**	**5TH**
Amy	**W**	X	Y	Z	V
Bertha	**X**	Y	Z	V	W
Clare	**Y**	Z	V	W	X
Diane	**Z**	V	W	X	Y
Erika	**V**	W	X	Y	Z

Therefore, now there are no free men, these are the final stable pairs obtained

Stable Pairs: **{(W, A), (X, B), (Y, C), (Z, D), (V, E)}**

Total number of proposals made(numberOfProposals) = **21 (which is: n(n-1) +1 = 5(4) + 1 = 21)**