Spring 2021

Q1) Give context-free grammars generating the following languages over $\Sigma = \{0, 1, 2\}$ (10 points)

a) $\{0^n 1^m 2^k \mid n, m, k \geq 0, \text{ and } n = m \text{ or } m = k\}$

$0^i 1^i 2^i$

$\Rightarrow G = (V, \Sigma_1, R, S)$

$V = \{S, A, B, C, D\}$

$\Sigma_1 = \{0, 1, 2, \varepsilon\}$

$R = \{ S \rightarrow AC \mid DB$

$A \rightarrow 0A1 \mid \varepsilon,$

$B \rightarrow 1B2 \mid \varepsilon,$

$C \rightarrow 2C \mid \varepsilon,$

$D \rightarrow 0D \mid \varepsilon$

$\}$

$S \rightarrow AC$
$\rightarrow 0A1C$
$\rightarrow 012C$
$\rightarrow \underline{012}$

$S \rightarrow DB$
$\rightarrow 0DB$
$\rightarrow 01B2$
$\rightarrow 011B22$
$\rightarrow \underline{01^2 2^2}$

b) $\{0^n 1^m 2^k \mid n, m, k \geq 0 \text{ and } \underline{n+m = k}\}$

$\Rightarrow G = (V, \Sigma_1, R, S)$

$V = \{S, A\}$, $\Sigma_1 = \{0, 1, 2, \varepsilon\}$

$R = \{ S \rightarrow 0S2 \mid A,$

$A \rightarrow 1A2 \mid \varepsilon$

$\}$

$S \rightarrow 0S2$
$\rightarrow 0A2$
$\rightarrow 01A22$
$\rightarrow \underline{012^2}$

Q2. Consider $\Sigma = \{0, 1\}$:

a) (5 points) State the Pumping Lemma for regular languages.

⇒ Pumping Lemma:-

If A is a regular language, then there is a ~~small~~ number P (the pumping length) where if s is any string in A of length at least p, then s may be divided into three pieces, $s = xyz$, satisfying the following conditions:

1. for each $i \geq 0$, $xy^iz \in A$,
2. $|y| > 0$, and
3. $|xy| \leq P$

b) (15 points) Prove whether or not the following language is a regular language: $L = \{0^n 1^m \mid m < n\}$. If you choose to disprove, you need to apply the pumping lemma.

⇒ Let's assume L to be regular language.

Let pumping length = P. And $S = xyz$, where $|S| \geq P$, $S \in L$.

$$S = 0^{P+1} 1^P = \underbrace{0^a}_{x}\underbrace{0^b}_{y}\underbrace{0^1 1^P}_{z} \quad \text{where } a, b, P > 0.$$

where $a + b + 1 = P + 1$

$$\Rightarrow P = a + b$$

(i) $|y| = b > 0$

(ii) $|xy| = a + b = P \leq P$

(iii) $S_0 = xy^iz \ \forall \ i \geq 0$

Let $i = 0 \Rightarrow xz = 0^a 0^1 1^P$

$= 0^{a+1} 1^P$

$= 0^{a+1} 1^{a+b}$

Here $m \geq n$ as $b \geq 1$

$\therefore S_0 \notin L$

$\therefore$ It is a contradiction.

$\therefore$ L is not regular language.

(Or)

Let string (s) $= \underbrace{0000}_{x}\,\underbrace{1}_{}\underbrace{1}_{y}\underbrace{1}_{z}$

$xy^iz \; (i=2) \;=\; 0000\;11111$

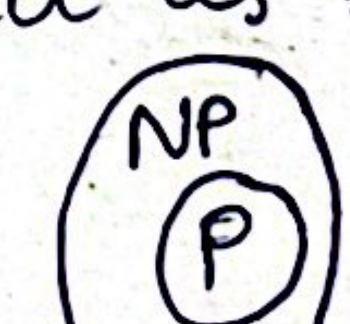
$= 0^4 1^5 \notin L \quad (4 \ngtr 5)$

So, there exists infinitely many cases like this. Hence our assumption is wrong. So language is not regular.

Q3. Answer the following questions. Please clearly explain each in detail and show all your work.

(a) How do you prove, in general, that a Problem X is in NP? Please give the steps and explain (4 points).

→ NP problems are generally act as superset of P problems represented as



* So, the problems which are solved in polynomial time are NP but viceversa not possible.
* But NP problems always verifies for a witness of a solution i.e it verifies the solution but cannot give solution (Generally Halt).