Fall 2020

Q1. Answer the following questions. In all cases $\Sigma = \{a, b\}$

a) Let G be the grammar below:

$$S \rightarrow aS \mid aB$$
$$B \rightarrow bB \mid b$$

Let $L(G)$ be the language generated by grammar G. Either prove that $L(G)$ is regular by providing a regular expression that describes $L(G)$ or disprove it by applying the pumping lemma.

$\Rightarrow$ G: $S \rightarrow aS \mid aB$
$B \rightarrow bB \mid b$

$S \rightarrow aS$
$\rightarrow aaB$
$\rightarrow aabB$
$aabb$
$\rightarrow aab$

$$L = \{a^m b^n \mid m, n > 0\}$$

Let's assume that L is regular language. Let P = pumping length, $S = xyz$ where $|S| \geq P$, $S \in L$. Let $S = a^P b^P$.

Case I: $S = \underbrace{a^r}_{x} \underbrace{a^s}_{y} \underbrace{a^t b^P}_{z}$ where $P = r+s+t$ and $r, s, t, P > 0$.

(i) $|y| = s > 0$ ✓

(ii) $|xy| = r+s \leq P$
$r+s \leq r+s+t$ ✓ $(\because t > 0)$

(iii) Let $s = xy^iz$ $\forall i \geq 0$
$i = 0 \Rightarrow S_0 = a^{r+t} b^P$
$r+t > 0$ $\therefore S_0 \in L$ ✓
$P > 0$

Case II: $S = \underbrace{a^r a^s}_{x} \underbrace{a^t}_{y} \underbrace{b^P}_{z}$

(i) $|y| = t > 0$ ✓

(ii) $|xy| = r+s+t = P \leq P$ ✓

(iii) Let $S = xy^iz$ $\forall i \geq 0$
$r+s > 0$ $\therefore S_0 \in L$ ✓
$P > 0$

Case III : $s = \underbrace{a^r}_{x} \underbrace{a^s a^t}_{y} \underbrace{b^p}_{z}$.

(i) $|y| = s+t > 0$ ✓

(ii) $|xy| = r+s+t \leq p$

$p \leq p$ ✓

(iii) Let $s = xy^iz \ \forall i \geq 0$

$i = 0 \Rightarrow s = a^r b^p \in L$ ✓

Since we are not able to prove that L is not regular by proof by contradiction

$\boxed{\text{L is Regular language}}$

b) $S \rightarrow aSb \mid ab$. aabb

$L = \{a^n b^n \mid n > 0\}$

Let's assume L is regular language.

Let $s = a^p b^p$ where P = pumping length,

$|s| \geq p$; $s \in L$

Let $s = \underbrace{a^r}_{x} \underbrace{a^s}_{y} \underbrace{b^p}_{z}$ where $x, y, z > 0$

$r+s = p$

(i) $|y| = s > 0$ ✓

(ii) $|xy| \leq p \Rightarrow r+s = p \leq p$ ✓

(iii) $s = xy^iz \ \forall i \geq 0$

Let $i = 0 \Rightarrow s = a^r b^p$

$s \notin L$ as $p > r$ and $p \neq r$. ✗

∴ Our assumption is wrong.

∴ L is not Regular.

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Q2. Provide a context free grammar for each of the following languages. In all cases $\Sigma = \{0, 1\}$.

a) $L = \{ w \mid w = w^R$ and $|w|$ is even$\}$, where $w = w^R$ means w is palindrome (reads the same forward and backward).

⇒ Given $\Sigma = \{0, 1\}$

so, w example is $L = \{ \varepsilon, 00, 11, 0000, 0110, 1001, 1111, \ldots \}$

CFG, $G = (V, \Sigma, R, S)$ where

$V = \{S\}$

$\Sigma = \{0, 1, \varepsilon\}$

$R = \{ S \rightarrow \varepsilon \mid 0S0 \mid 1S1 \}$

Extra: $|w|$ is odd then

$R = \{ S \rightarrow 0S0 \mid 1S1 \mid 0 \mid 1 \}$

b) $L = \{ w \mid w$ starts and ends with the same symbol$\}$

⇒ CFG, $G = (V, \Sigma, R, S)$

$V = \{S, A, B\}$

$\Sigma = \{0, 1, \varepsilon\}$

$R = \{ S \rightarrow 0A0 \mid 0B0 \mid 1A1 \mid 1B1 \mid \varepsilon,$

$A \rightarrow 0A \mid \varepsilon,$

$B \rightarrow 1B \mid \varepsilon \}$

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[Extra:
w starts and ends with diff symbol]

$$R = \{ S \to 0A1 \mid 1A0 \mid 0B1 \mid 1B0,$$
$$A \to 0A \mid \varepsilon$$
$$B \to 1B \mid \varepsilon$$
$$\}$$

3) VC is NP

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