Fall 2019

1. Give a context free grammar generating the following language over $\Sigma = \{0,1\}$:

$$\{0^m 1^n 0^k : k \geq m;\ m,n,k \geq 0\}$$

$\Rightarrow$ **CFG:** $G = (V, \Sigma, R, S)$

$V = \{S, A, B\}$

$\Sigma = \{0, 1\}$

$S = S.$

$R = \{ S \to 0SB0 \mid A \mid \varepsilon,$

$A \to 1A \mid \varepsilon,$

$B \to 0B \mid \varepsilon$

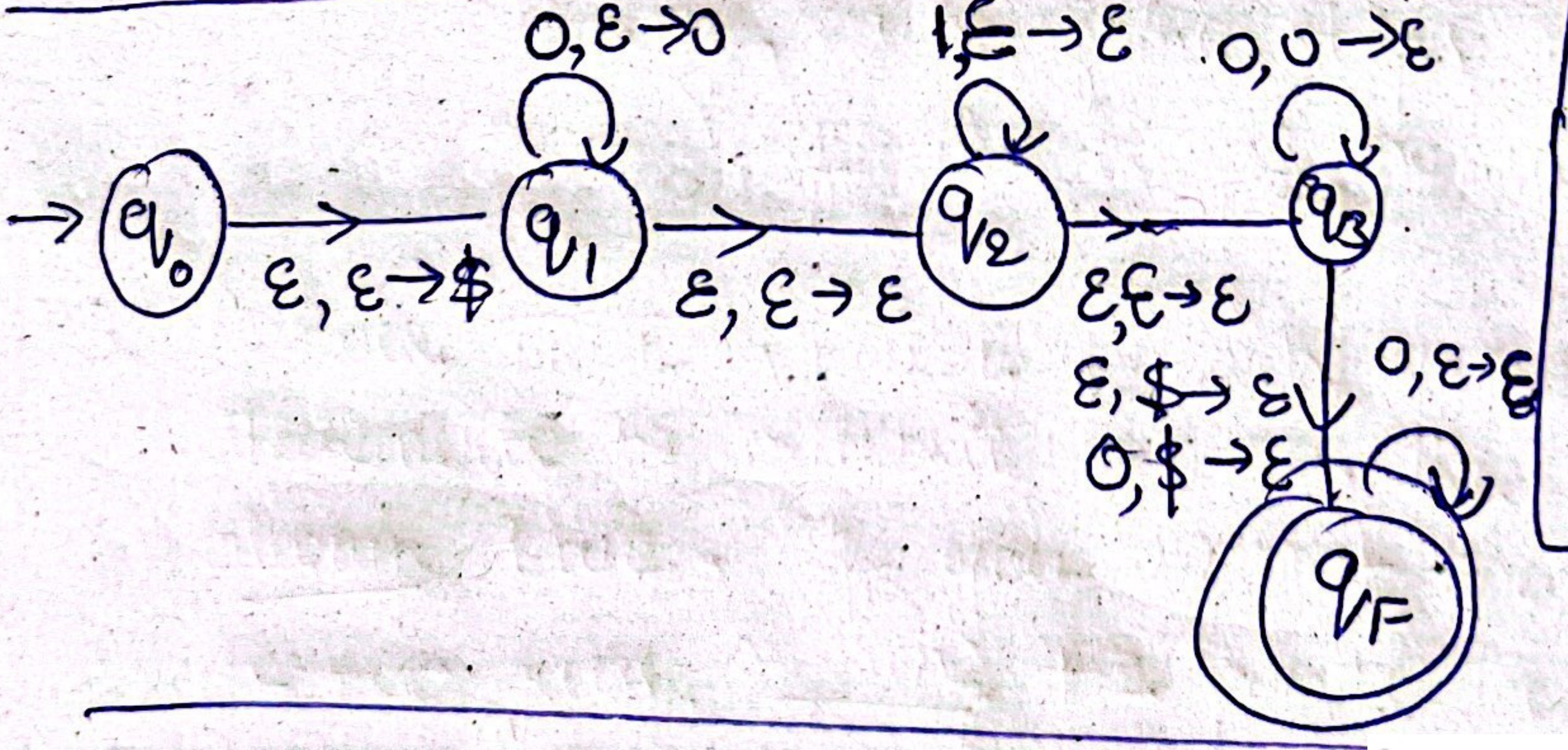
$\}$

or $R = \{ S \to 0S0 \mid T,\ T \to 1T \mid U,\ U \to 0U \mid \varepsilon \}$

001000 or 010

$S \to 0SB0$
$\to 00SB0B0$
$\to 00AB0B0$
$\to 001AB0B0$
$\to 001\varepsilon 0\varepsilon 0\varepsilon 0$
$\to 001000$

$S \to 0SB0$
$\to 01AB0$
$\to 01\varepsilon\varepsilon 0$
$\to 010$



2. A Hamiltonian circuit in an undirected graph is a cycle that visits each node exactly once.
A cycle in a graph is a non-empty path in which the only repeated node is the first and last.

Consider the following problem:

HAMCIRCUIT = { V, E : G=(V, E) is an undirected graph containing a Hamiltonian Circuit }

Show that HAMCIRCUIT ∈ NP.

⇒ HAMCIRCUIT is a decision problem

<u>Certificate</u> : Path $V_0 - V_1 - V_2 - \dots - V_{n-1} - V_0$

<u>Verifier</u> : 1. Go through path to ensure every vertex is listed only once except the first and last vertex.
$O(n) \cdot O(n) = O(n^2)$

2. Run breadth-first-Search on graph G to ensure every vertex in the graph is contained in the certificate path.
$O(n+m)$

3. Run breadth-first-Search on graph G starting at the first vertex listed in the certificate and continuing with the subsequent vertices in the certificate to verify the path to all of the vertices is valid.
$O(n+m)$

4. If steps 1-3 pass, accept; otherwise reject.

Time complexity is $O(n^2)$, which is polynomial.

02

HC is in NP

The certificate c is obviously the circuit. Without loss of generality, we can assume that any circuit starts and ends at the first vertex v_1, so there is no need to include v_1 in the certificate. Now,

HAMILTONIAN-CIRCUIT-VERIFY(G,c) =

Check that each element of c is a vertex in G.
Check that each element in c is distinct.
Check that $|c| = n-1$.
Check that v_1 is not in the certificate.
Check that $\{v_1, c_1\}$ is an edge in V.
Check that $\{c_{n-1}, v_1\}$ is an edge in V.
For each c_i and c_{i+1} in c, $1 \le i \le n-2$
 Check that $\{c_i, c_{i+1}\}$ is an edge in V,
 Clearly, each of the steps needed to verify the certificate is polynomially bounded.

3. Answer TRUE or FALSE for each of the following statements to indicate whether the conclusion is always true. If you do not know the answer, do not guess. Scoring: +2 points for correct answer; 0 point for no answer; -1 point for wrong answer.

a) If $A \le_p \overline{B}$ and $B \in$ co-NP, then $A \in$ NP.
⇒ True

b) If $A \le_p B$ and $A \in$ NP-Complete, then $B \in$ NP-hard.
⇒ True

c) If $A \le B$ and B is not decidable, then A is not acceptable.

⇒ False

doubt

d). If $A \leq B$ and $B \in P$, then A is acceptable

$\Rightarrow$ True

doubt

e) If $A \leq_p B$ is $B \in NP$, then $A \in EXPTIME$.

$\Rightarrow$ True

f) If $A \leq_p B$ and $B \in NP$ - Complete, then $A \in P$.

$\Rightarrow$ False

g) If $A \leq_p B$ is and B is decidable, then A is decidable.

$\Rightarrow$ True

doubt

h) If $A \leq_p B$ and $B \in NP$- Complete, then $A \in NP$ - Complete

$\Rightarrow$ True

doubt

i) If $A \leq B$ and B is co- acceptable, then A is co-acceptable.

$\Rightarrow$ True

j) If $A \leq B$ and A is not co - acceptable, then B is not decidable.

$\Rightarrow$ True