6.5 Exercises

1. We are going to use the finite difference formula $f'(\bar{x}) \approx af(\bar{x}+2h) + bf(\bar{x}+h) + cf(\bar{x}) + df(\bar{x}-h) + ef(\bar{x}-2h)$ to estimate an unknown function value from data. The constants are given below.

$$a = -\frac{1}{12h}, b = \frac{2}{3h}, c = 0, d = -\frac{2}{3h}, e = \frac{1}{12h}$$

You are given the following population data for a small town:

Year (x)	1880	1890	1900	1910	1920
Population ($f(x)$)	362	391	?	420	490

We would like to use this data and our formula to estimate the unknown population in the year 1900.

(A) If we wanted to use our formula with this data to estimate $f'(1900)$, what would h have to be?

(B) Use the formula to estimate $f'(1900)$

(C) Add $f'(1900)$ to $f(1890)$ or subtract $f'(1900)$ from $f(1910)$ to approximate $f(1900)$

2. Consider the following ODE:

$$y'(x) = 3y(x) + 2$$

And suppose that we have an initial value of $y(0) = 2$.

(A) Substitute in by hand a forward difference approximation $y'(x) \approx \frac{y(x+h)-y(x)}{h}$ to get an approximate solution $y(x+h) = ...$ to the ODE.

(B) Using your approximation above with $h = 0.1$ and the given initial value, find $y(0.1), y(0.2)$ and $y(0.3)$. You can do this with code or by hand.