8.4 Exercises

1. For a 2d vector $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ we have the *rotation matrix* that rotates x by an angle θ given by:

$$A(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Write a program that takes in a vector x and rotates it by θ.

2. The trace of a square $n \times n$ matrix A is defined by the sum of the diagonal entries:

$$tr(A) := \sum_{i=1}^{n} a_{ii}$$

Write a program that calculates the trace of a given matrix.

3. Let e_i denote the column vector in $\mathbb{R}^n$ with a 1 in the ith entry and 0 everywhere else, so that $e_1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$. You can find the inverse of a $n \times n$ matrix A by solving $Ax_i = e_i$ for $i = 1, ..., n$.

The inverse of the matrix will be:

$$A^{-1} = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \end{bmatrix}$$

Where the x_i's form the columns of A^{-1}.

Write a program to calculate the inverse of a matrix based on any of the matrix solving methods in this chapter. You may find the command `np.column_stack((col1,col2,...,coln))` useful in forming the final matrix.