2.11 Exercises

1. Write a Python program that defines functions equivalent to $f(x) = x^2 - 6x + 2$ and $g(x) = -2x$. Use them to test whether or not $f(g(2)) = g(f(2))$.

2. Let $A = [a_1, a_2, ..., a_n]$ be a list of numbers. Define the geometric mean $GM(A)$ of the list as follows:

$$\begin{aligned} GM(A) &:= (\Pi_{i=1}^{n} a_i)^{\frac{1}{n}} \\ &= \sqrt[n]{a_1 \cdot a_2 \cdots a_n} \end{aligned}$$

   As an example, if $B = [1, 2, 3]$ then $GM(B) = \sqrt[3]{1 \cdot 2 \cdot 3} = \sqrt[3]{6} \approx 1.817$.

   Write a Python function called GM that calculates the geometric mean of a given list of numbers.

3. Using list comprehension, make a list of all the numbers from 1 to 10,000 (inclusive) that are divisible by 5 or 8. Your list should end up having 3000 entries.

4. Let $f(x)$ be a function. Define the new function $f^{(n)}(x)$ which applies f to an input x n times. In other words:

$$f^{(n)}(x) := \underbrace{f \circ f \cdots \circ f}_{n \text{ times}}(x)$$

   Write/code a function called fn which takes in a function f, a natural number n, and an input x and then applies f to x n times.

   For an example and a check, if you give your program the function $f(x) = x^2$ and $n = 3$ then it should calculate $f^{(3)}(x) = f(f(f(x))) = ((x^2)^2)^2$ for a given x.

5. Plot the function $f(x) = e^{-x}\cos(2\pi x)$ on the interval $[0, 5]$. Include your code and the generated plot.