5.3 Exercises

1. You can use the bisection algorithm to estimate the value of some constants to a specified accuracy.

   (A) Find a function that has $\sqrt{2}$ as a root, but does not explicitly have $\sqrt{2}$ in its definition (otherwise you would need to know $\sqrt{2}$ to define it).

   (B) Use the bisection method on your function with tolerance 10^{-12} and sufficient iterations for convergence to estimate $\sqrt{2}$.

2. Suppose that a projectile is launched with initial velocity v_0. The equations for the coordinates of the arrow $(x(t), y(t))$ at time t seconds after launch is given by:

$$x(t) = v_0 \frac{\sqrt{2}}{2} t$$
$$y(t) = -\frac{1}{2} g t^2 + v_0 \frac{\sqrt{2}}{2} t$$

   Where g is the acceleration near the Earth's surface due to Earth's gravity, $g \approx 9.81 m/s^2$. The arrow strikes the ground some time later (located at $y = 0$) 300m away. Use a root finding algorithm of your choice to help you answer the following, accurate to 3 decimal places:

   (A) What time did the arrow hit the ground?

   (B) What was the initial speed v_0 of the arrow?

3. You can use Newton's method to estimate the reciprocal of a number a, $\frac{1}{a}$ without performing any divisions. You can do this with the funtion $f(x) = a - \frac{1}{x}$.

   (A) Prove by hand from the definition of Newton's that for the given function f above no divisions are necessary to calculate x_{n+1} from x_n.

   (B) With a tolerance of 10^{-14} and initial guess of $x_0 = 0.1$, use the above to calculate the reciprocal of 12.