CS101 Lecture 11: Data Representation: Binary Numbers

Number Systems
Binary Numbers

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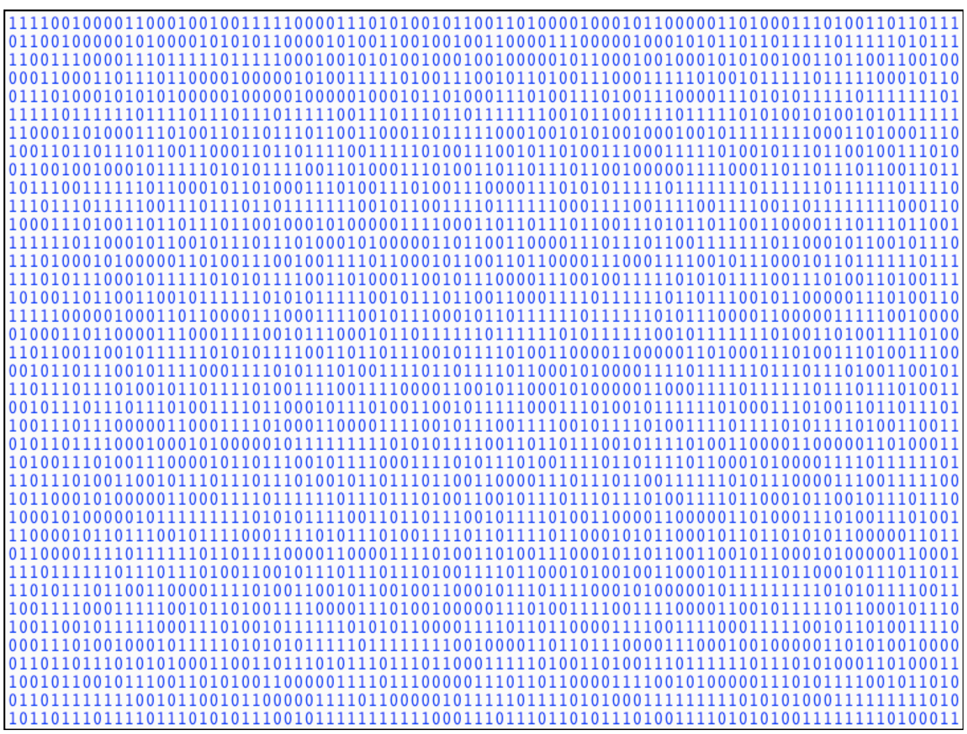
Computer Science



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!!! MATH WARNING !!!

TODAY'S LECTURE CONTAINS
TRACE AMOUNTS OF
ARITHMETIC AND ALGEBRA

PLEASE BE ADVISED THAT CALCULTORS
WILL BE ALLOWED ON THE QUIZ
(and that you probably won't need them)

Overview/Questions

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- What gives a number its value?
- What is a number system?
- I've heard that computers use binary numbers. What's a binary number?
- What kind of numbers do computers store and manipulate?

These are the numbers you know

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Natural Numbers
Zero and any number obtained by repeatedly adding one to it.

Examples: 100, 0, 45645, 32

Negative Numbers
A value less than 0, with a – sign

Examples: -24, -1, -45645, -32

These are the numbers you know

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Integers
A natural number, a negative number, zero

Examples: 249, 0, -45645, -32

Rational Numbers
An integer or the quotient of two integers

Examples: -249, -1, 0, 3/7, -2/5

Numbering Systems

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A numbering system assigns meaning to the position of the numeric symbols.

For example, consider this set of symbols:

642

What number is it? Why?

Numbering Systems

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It depends on the numbering system.

642 is 600 + 40 + 2 in **BASE 10**

The **base** of a number determines the number of digits (e.g. symbols) and the value of digit positions

Positional Notation

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Continuing with our example…

642 in base 10 *positional notation* is:

$$
\begin{array}{lclcl}
6 \times 10^2 & = & 6 \times 100 & = 600 & \\
+\ 4 \times 10^1 & = & 4 \times 10 & = 40 & \\
+\ 2 \times 10^0 & = & 2 \times 1 & = 2 & = 642 \text{ in base 10}
\end{array}
$$

This number is in base 10

The power indicates the position of the number

Positional Notation

Computer Science

$$642 = 6_3 * 10^2 + 4_2 * 10^1 + 2_1 * 10^0$$

B is the base

As a general form:

$$d_n * B^{n-1} + d_{n-1} * B^{n-2} + \ldots + d_1 * B^0$$

n is the number of digits in the number

d is the digit in the i^{th} position in the number

What Would Pooh Do?

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Binary Numbers

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Digital computers are made up of electronic circuits, which have exactly 2 states: on and off.

Computers use a numbering system which has exactly 2 symbols, representing on and off.

Binary Numbers

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Decimal is base 10 and has 10 digits:
0,1,2,3,4,5,6,7,8,9

Binary is base 2 and has 2, so we use only 2 symbols:
0,1

For a given base, valid numbers will only contain the digits in that base, which range from 0 up to (but not including) the base.

Binary Numbers and Computers

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A binary digit or bit can take on only these two values.

Low Voltage = 0
High Voltage = 1 all bits have 0 or 1

Binary numbers are built by concatenating a string of bits together.
Example: 10101010

Positional Notation: Binary

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Recall this general form:

$$d_n * B^{n-1} + d_{n-1} * B^{n-2} + \ldots + d_1 * B^0$$

The same can be applied to base-2 numbers:

$1011_{bin} = 1 * 2^3 + 0 * 2^2 + 1 * 2^1 + 1 * 2^0$

$1011_{bin} = (1 * 8) + (0 * 4) + (1 * 2) + (1 * 1)$

$1011_{bin} = 8 + 0 + 2 + 1 = 11_{dec}$

Converting Binary to Decimal

Computer Science

What is the decimal equivalent of the binary number 01101110?

(you try it! Work left-to-right)

Converting Binary to Decimal

Computer Science

What is the decimal equivalent of the binary number 01101110?

$$
\begin{array}{lcl}
\ 0 \times 2^7 &=& 0 \times 128 = 0 \\
+\ 1 \times 2^6 &=& 1 \times 64 = 64 \\
+\ 1 \times 2^5 &=& 1 \times 32 = 32 \\
+\ 0 \times 2^4 &=& 0 \times 16 = 0 \\
+\ 1 \times 2^3 &=& 1 \times 8 = 8 \\
+\ 1 \times 2^2 &=& 1 \times 4 = 4 \\
+\ 1 \times 2^1 &=& 1 \times 2 = 2 \\
+\ 0 \times 2^0 &=& 0 \times 1 = 0 \\
 & & = 110 \text{ (decimal)}
\end{array}
$$

Converting Binary to Decimal

Computer Science

Try another one. What is the decimal equivalent of the binary number 10101011?

(you try it! Work left-to-right)

Converting Binary to Decimal

Computer Science

Try another one. What is the decimal equivalent of the binary number 10101011?

$$
\begin{aligned}
& 1 \times 2^7 = 1 \times 128 = 128 \\
&+ 0 \times 2^6 = 0 \times 64 = 0 \\
&+ 1 \times 2^5 = 1 \times 32 = 32 \\
&+ 0 \times 2^4 = 0 \times 16 = 0 \\
&+ 1 \times 2^3 = 1 \times 8 = 8 \\
&+ 0 \times 2^2 = 0 \times 4 = 0 \\
&+ 1 \times 2^1 = 1 \times 2 = 2 \\
&+ 1 \times 2^0 = 1 \times 1 = 1 \\
& = 171 \text{ (decimal)}
\end{aligned}
$$

Converting from Decimal to Other Bases

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Algorithm (process) for converting number in base 10 to other bases

While (the quotient is not zero)
  Divide the decimal number by the new base*
  Make the remainder the next digit to the left in the answer
  Replace the original decimal number with the quotient

** Using whole number (integer) division only.*
Example: 3 / 2 gives us a quotient of 1 and a remainder 1

Converting Decimal to Binary

Computer Science

What is the binary equivalent of the decimal number 103?

103 / 2 = 51, remainder 1 → rightmost bit
51 / 2 = 25, remainder 1
25 / 2 = 12, remainder 1
12 / 2 = 6, remainder 0
6 / 2 = 3, remainder 0
3 / 2 = 1, remainder 1
1 / 2 = 0, remainder 1→ leftmost bit

$103_{dec} = 1\ 1\ 0\ 0\ 1\ 1\ 1_{bin}$

Converting Decimal to Binary

Computer Science

Now you try one. What is the binary equivalent of the decimal number 201?

Recall the algorithm:

While (the quotient is not zero)
    Divide the decimal number by the new base*
    Make the remainder the next digit to the left in the answer
    Replace the original decimal number with the quotient

Converting Decimal to Binary

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What is the binary equivalent of the decimal number 201?

201 / 2 = 100, remainder 1 → rightmost bit
100 / 2 = 50, remainder 0
50 / 2 = 25, remainder 0
25 / 2 = 12, remainder 1
12 / 2 = 6, remainder 0
6 / 2 = 3, remainder 0
3 / 2 = 1, remainder 1
1 / 2 = 0, remainder 1→ leftmost bit

$201_{dec} = 1\ 1\ 0\ 0\ 1\ 0\ 0\ 1_{bin}$

Bits, Bytes, Words

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Byte

8 bits – a common unit of computer memory.

Word

A computer word is a group of bits which are passed around together during computation.

The word length of the computer's processor is how many bits are grouped together.

- 8-bit machine (e.g. Nintendo Gameboy, 1989)
- 16-bit machine (e.g. Sega Genesis, 1989)
- 32-bit machines (e.g. Sony PlayStation, 1994)
- 64-bit machines (e.g. Nintendo 64, 1996)

Bytes, Kilobytes, Megabytes

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Data Usage Summary			
Messaging Unlimited			
	Used	198	
200MB OF DATA			
	Plan MB	200	
	MB Used	275	
5.	MB Billed at $15.00/200MB	75	15.00

1 Gigabyte (GB) = 1024MB, 1 Megabyte (MB) = 1024KB

A byte is 8 bits…
… enough to hold one character.

A kilobyte (KB) is 1024 bytes…
…enough to hold about one page of text.

A megabyte (MB) is 1024 kilobytes, or 1,048,576 bytes…
… about enough to hold a digital picture.

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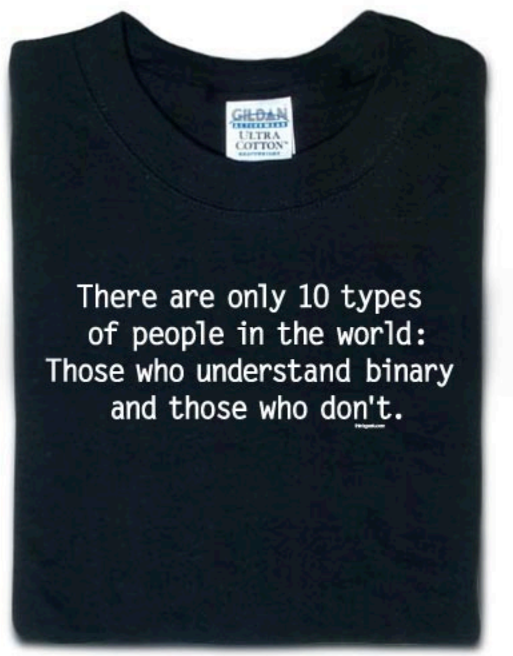

What You Learned Today

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- Encoding: Symbols Represent Values
- Number Systems
- Binary Numbers, Bits, and Bytes
- Algorithms: converting binary to decimal, converting decimal to binary

Announcements and To Do

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- Homework 1 due tonight!
- Readings:
  - Reed ch 5, pp 83-87, 89-90 (today)
  - Reed ch 2, pp 19-26 (Friday)
- Lab 2 on THURSDAY
  - Go to your scheduled lab time.
  - You must submit a document from the lab as evidence of attendance!

Want to learn more?

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If you've read this far, maybe you'd like to learn about other binary representations of other types of numbers?

Read about this on Wikipedia and we can discuss your questions:

- Two's complement (negative numbers)
- IEE754 (real numbers)