Analyzing the Relationship Between Torque and Angular Acceleration Through Rotational Dynamics

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The goal of this lab is to use our knowledge and implementation of rotational dynamics in order to determine the relationship between applied torque and angular acceleration for a simple pulley system. To do this, we run a series of trials using LoggerPro, a Vernier Rotary Motion Sensor, and a Vernier "ultra pulley" system. Using these tools, we develop an understanding of how angular acceleration remains relatively constant over time. By altering the applied force (combined weight of the hanging mass and slotted masses) and the radii of pulleys, we can create a $log(\alpha)$ vs. $log(\tau)$ plot in order to illustrate the linear relationship between torque and angular acceleration. The values of slope and y-intercept are used to calculate the moment of inertia which is then compared to the theoretical value. Not only does this lab demonstrate the linear relationship between τ, α, and I, but it forces us to calculate uncertainties and interpret the consistency of our results.

I. INTRODUCTION

Rotational dynamics pertains to the study of how objects rotate and what triggers this rotation. Objects moving about fixed points in curved paths feel something known as torque. Torque is a measure of how much of a force causes an object to rotate, and is responsible for triggering angular accelerations. The formula to calculate torque is:

$$\tau = r\text{x}F = rFsin(\theta) = I\alpha$$

This is the equivalent of Newton's second law:

$$F = ma$$

Angular acceleration is the rate of change of the angular velocity. Knowing both the torque and angular acceleration, we can calculate the moment of inertia of an object. The moment of inertia is how much an object opposes the speed of rotation about an axis altered by applied torque.[1]. This lab uses the linear relationship between torque and angular acceleration to estimate a value of moment of inertia in a mass-pulley system. This experiment is necessary in order to recognize that torque, angular acceleration, and moment of inertia can be expressed in a logarithmic form that illustrates the linear relationship between these values.

II. PROCEDURE

In this lab, we use a Vernier "ultra pulley" system to measure the angular acceleration that results from different torque values. After gathering all necessary materials, measure the masses of the rod and two additional screw-on masses. Next, secure each mass the same distance away from the center of the rod and measure that distance. Finally, measure the radii of the three pulleys. When making your measurements, be sure to note the uncertainty in each case because you will need to use this for your analysis. Now that all measurements are complete, you can begin setting up the lab station for data collection.

Attach the Rotary Motion Sensor to the ring stand, and the swivel mount and ultra pulley to the sensor. Be sure to use a counterweight to make sure that the ring stand is stable. Next, connect the sensor to the Dig/Sonic port on the LabQuest device which is connected to the computer through a USB port. Now that everything is connnected, you can launch LoggerPro on the laptop. To set up data collection, select "Experiments", "Set Up Sensors","Lab Pro 1". A pop-up window will show up on the screen. From this menu, select "Choose Sensor", "Rotary Motion". Customize the graph display so that angle, angular velocity, and angular acceleration versus time graphs are shown on the screen. Prior to recording actual data, make sure you test your setup. To do this, wind the string around the first pulley on the Rotary Motion Sensor, attach a mass hanger through the loop in the string, and hang it over the ultra pulley. Once you release the mass, begin recording your data. The string will unwind and the rod will start spinning. Make sure to keep your distance from the apparatus because the rod can spin extremely fast and cause injury. Once you notice appropriate behavior in your graphs, you can begin your first set of data collection.

You are going to be varying the applied force (i.e. changing the mass attached to the pulley system) at different radii (i.e. changing pulleys). Choose a combination of the hanging mass with one or more additional slotted masses.

The combined mass will act as the applied force throughout the experiment. Wind the string around one of the three pulleys and attach the combined mass to the loop in the string. When you are ready, begin collecting your data on LoggerPro and release the mass. Once the rod finishes rotating or when the mass hits the ground, stop collecting data and export it as a .txt file. Repeat all of these steps for the other two pulleys. Take multiple trials for each mass so that you can take the best trial to use for your analysis.

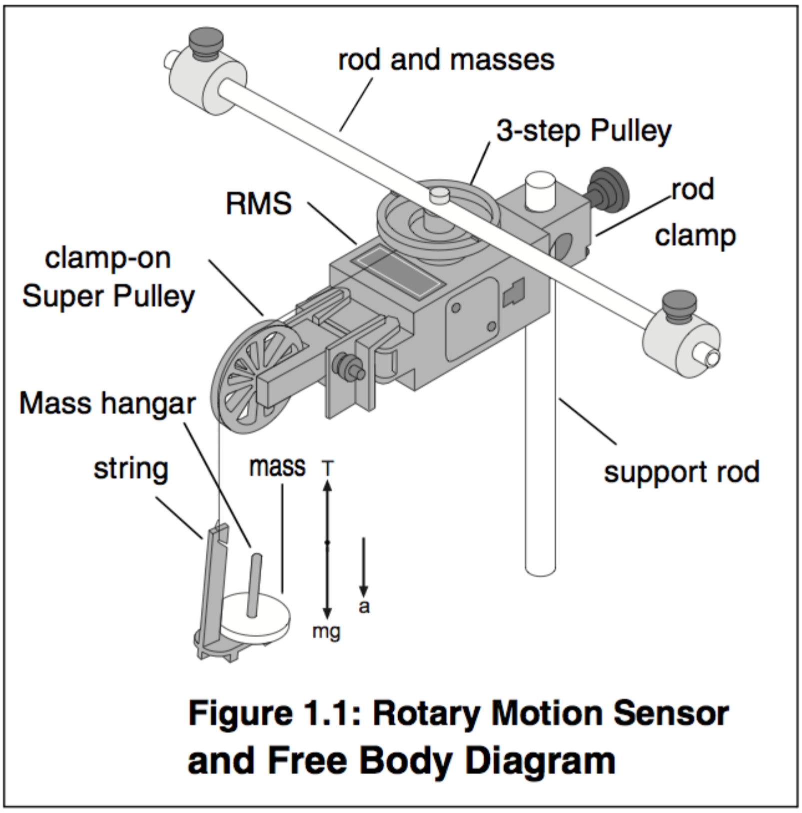

FIG. 1: Depicted above is the experimental setup for this lab. The rod, masses, mass hanger, string, and 3-step pulley are all shown. Refer to this figure when setting up your experiment.

III. DATA

This section includes three tables of all calculated values for each mass-pulley system. Table 1 includes all measured values for the rod and corresponding screw-on masses. Table 2 consists of: mass, radius, mean and standard deviation in the angular acceleration, as well as each torque value and its corresponding uncertainty. Table 3 consists of: $log(\tau)$, uncertainty in $log(\tau)$, $log(\alpha)$, and uncertainty in $log(\alpha)$.

Rod and Screw-on Mass Measurements			
Rod Mass (kg)	Rod Length (m)	Screw-on Mass (kg)	Distance of Screw-on Mass from Center of Rod (m)
0.042	0.382	0.080	0.184

TABLE I: This table includes length and mass information for the rod and screw-on masses. These values will be used later on to calculate the theoretical moment of inertia.

Mean, Standard Error, and Torque Values					
Pulley Radius (m)	Mass (kg)	Mean (rad/s^2)	Standard Error (rad/s^2)	Torque (Nm)	$\delta\tau$ (Nm)
0.005	0.15	1.524	0.0397	0.0074	0.0015
0.005	0.25	2.404	0.0524	0.0123	0.0020
0.005	0.55	4.826	0.0611	0.0270	0.0031
0.014	0.15	4.002	0.0990	0.0206	0.0027
0.014	0.25	6.416	0.115	0.0343	0.0026
0.014	0.55	13.087	0.160	0.0755	0.0031
0.024	0.15	6.218	0.0971	0.0353	0.0029
0.024	0.07	2.988	0.0993	0.0165	0.0026
0.024	0.05	2.176	0.0768	0.0118	0.0024

TABLE II: This table shows our mean and standard error in the mean for our angular acceleration values, torque, and uncertainties in the torque in each mass-pulley system. The mean and standard error values are calculated by using the numpy package in Python.

Logarithmic Values of Torque and Angular Acceleration			
$log(\tau)$	$\delta log(\tau)$	$log(\alpha)$	$\delta log(\alpha)$
-2.134	0.001	0.183	0.011
-1.912	0.001	0.381	0.009
-1.569	0.001	0.684	0.005
-1.687	0.001	0.602	0.011
-1.465	0.001	0.807	0.008
-1.122	0.001	1.117	0.005
-1.452	0.001	0.794	0.007
-1.783	0.001	0.475	0.014
-1.930	0.001	0.338	0.015

TABLE III: This table shows our logarithmic values of torque and angular acceleration as well as their corresponding uncertainties.

IV. ANALYSIS

The analytical goal of this lab is to experimentally determine the moment of inertia of our simple pulley system and compare it with theoretical expectations. We do this by determining the average angular acceleration and torque values for different mass-pulley systems and finding the slope of a $log(\alpha)$ vs. $log(\tau)$ plot.

We took three-five trials for each hanging mass-pulley combination to ensure that we had a good dataset to work with in our analysis. However, none of our data reflect constant angular accelerations throughout the entire measurement due to human error described in the Discussion section. In order to combat this, we "sliced" the good portions of our data, or the areas that exhibited constant angular acceleration. This is shown below in Figure 2.

Using the time range of 0 to 4 seconds, I plot angle, angular velocity, and angular acceleration vs. time graphs for each pulley. These plots are shown for pulley 1 below. The corresponding plots for pulleys 2 and 3 exhibit the same characteristics.

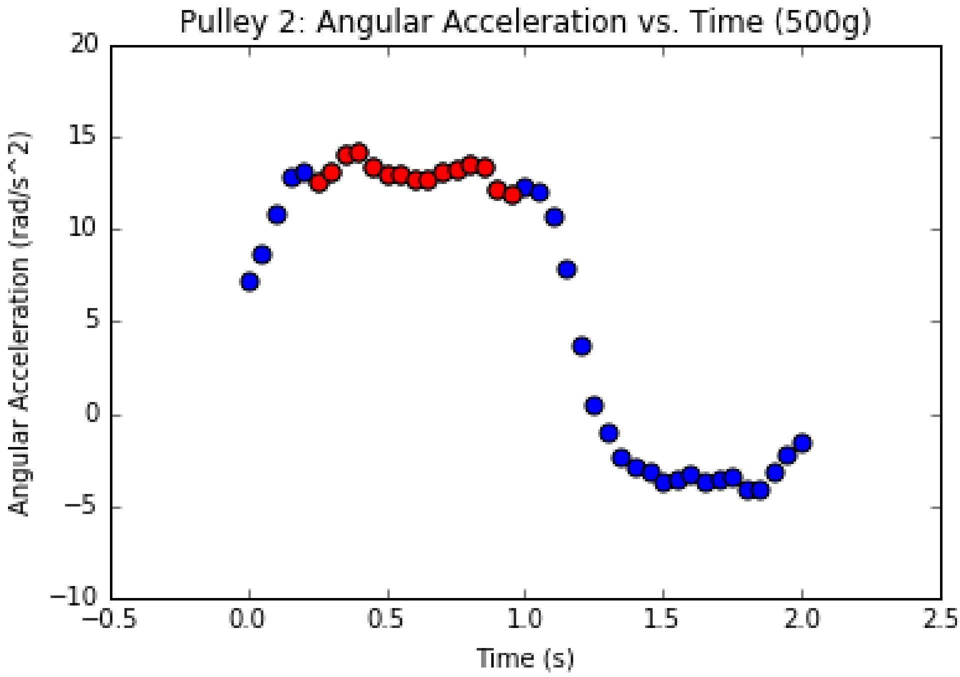

FIG. 2: Depicted above is the entire set of data for one of our trials for pulley 2 using a 500 g weight. The blue dots show all the data taken whereas the red shows the "good" data, or the data that appears to show a constant acceleration over a certain period of time.

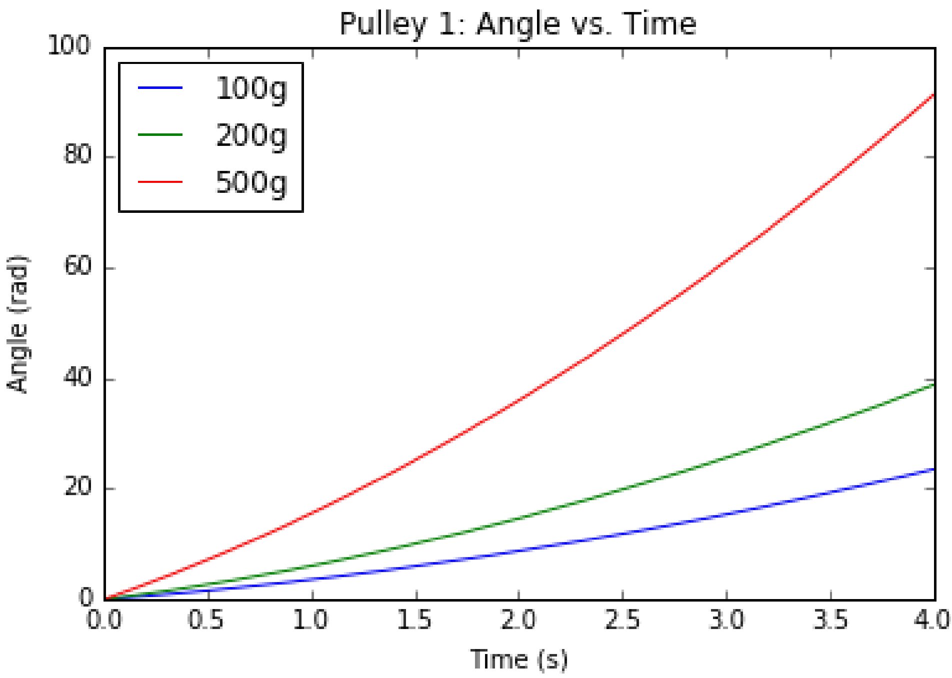

FIG. 3: Depicted above is an angle vs. time plot for three different masses on pulley 1. The graph exhibits an increasing exponential relationship over time.

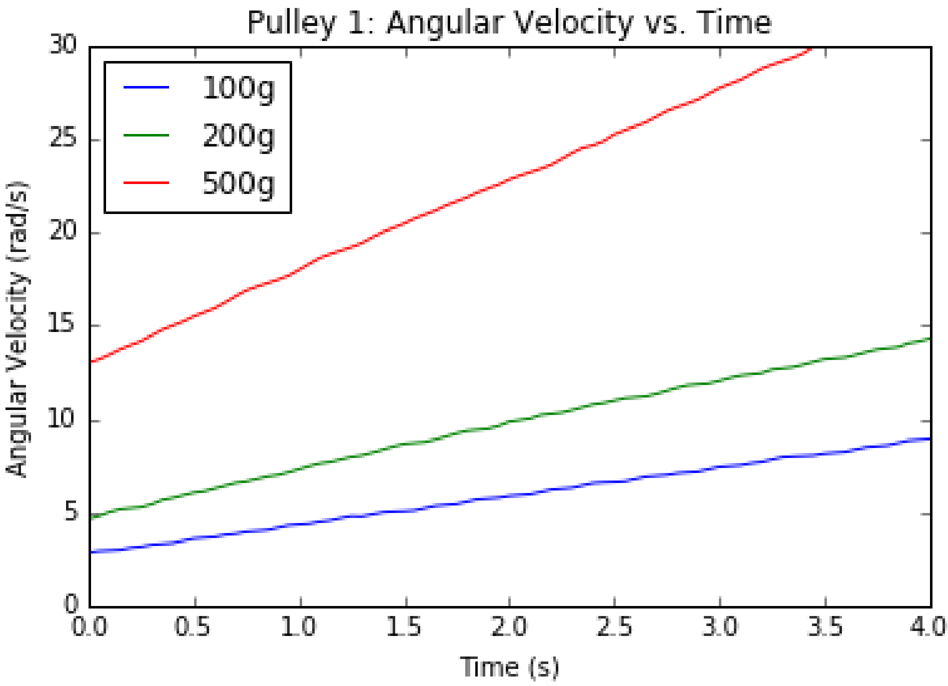

FIG. 4: Depicted above is a velocity vs. time plot for three different masses on pulley 1. The graph exhibits a positive constant slope over time.

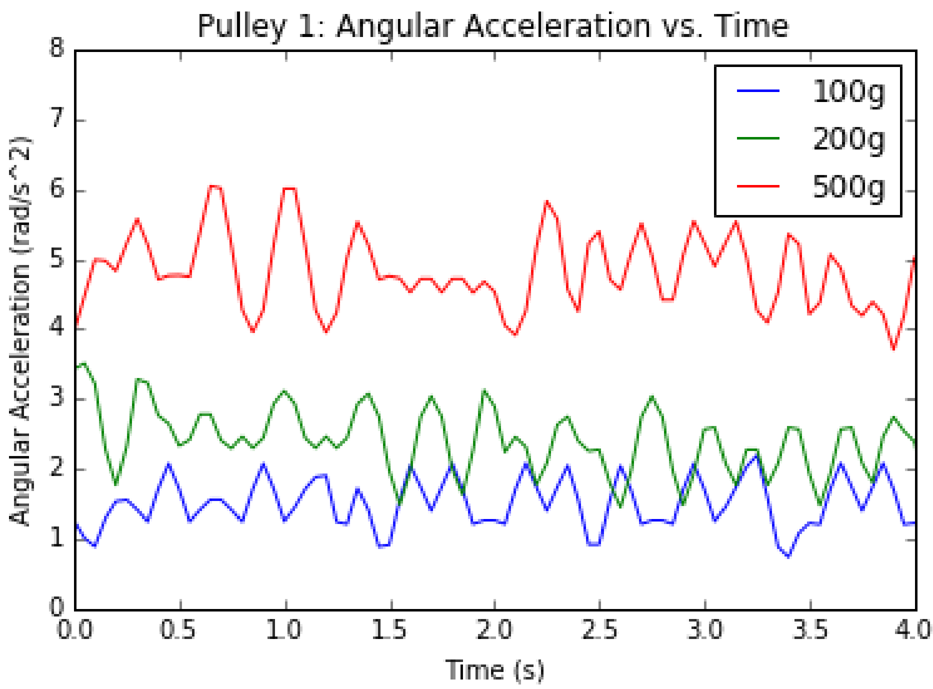

FIG. 5: Depicted above is an acceleration vs. time plot for three different masses on pulley 1. The graph shows variation, however, it displays a constant slope over time.

Since there is some variation in our angular acceleration over time, we calculate the mean and standard error in the mean to use in our analysis. To calculate the mean and standard deviation in the angular acceleration we use the built in functions in the numpy Python package. These values are shown in Table 2 of the Data section.

The next step in this lab is calculating the torque for each mass-pulley system. We calculate torque for each mass-pulley system by using the formula:

$$\tau = mgr$$

where m is the combined mass of the hanging mass and slotted masses, g is the acceleration due to gravity, and r is the radius of the pulley. A sample calculation is shown below. These values and their corresponding uncertainties are shown in Table 3 in the Data section above.

$$\tau = mrg = (0.15kg)(0.005m)(9.8m/s^2) = 0.0074Nm$$

To calculate the uncertainty in our torque values, we add in quadrature using the following formula:

$$d\tau = \tau\sqrt{\left(\frac{\delta r}{r}\right)^2 + \left(\frac{\delta m}{m}\right)^2}$$

τ represents our torque value, r is the radius of the pulley, δr is the uncertainty in our radius, m is the mass of the hanging mass and slotted masses, and δm is the uncertainty in that mass. A sample calculation for our first torque value is detailed below.

$$d\tau = 0.00735Nm\sqrt{\left(\frac{0.001m}{0.005m}\right)^2 + \left(\frac{0.001kg}{0.15kg}\right)^2} = 0.0015Nm$$

Therefore, we report our torque value as 0.0074 ± 0.0015 Nm.

Once we obtain all values of angular acceleration, torque, and the uncertainties in each, we begin our analysis of the linear relationship between τ and α using logarithms. To do this, we take the logarithm of all torque and angular acceleration values in addition to their uncertainties.

We know that

$$\tau = I\alpha$$

. By taking the log of both sides, we can show that:

$$log(\tau) = log(I\alpha)$$

$$log(\tau) = log(I) + log(\alpha)$$

$$log(\alpha) = -log(I) + log(\tau)$$

We recognize this equation to be in y=mx+b form where $log(\alpha)$ is y, 1 is m, $log(\tau)$ is x, and $-log(I)$ is b.

To obtain the logs of torque and angular acceleration, we do np.log10 of the numpy array. The uncertainty calculations are more involved. We compute the uncertainty in the log of torque and the log of angular acceleration as shown below. All of these values are found in Table 2 of the Data section.

$$\delta log(\tau) = \frac{\delta\tau}{ln(10)}$$

$$\delta log(\alpha) = \frac{\delta\alpha}{ln(10)}$$

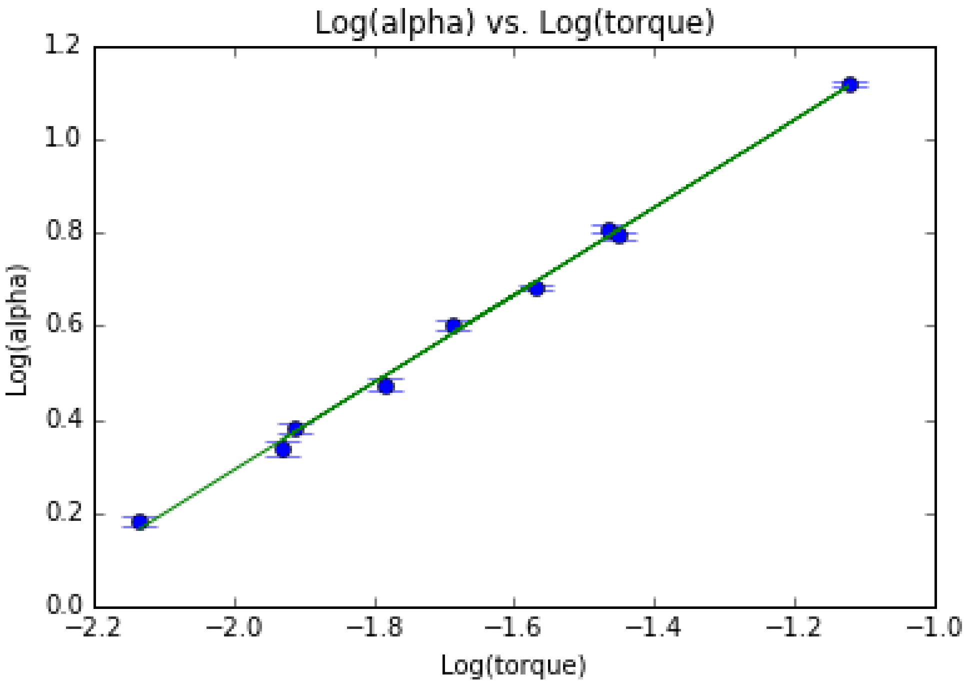

FIG. 6: Depicted above is the log(alpha) vs. log(torque) plot. This shows our calculated torque values with error bars taking into account uncertainty. The best-fitting slope is 0.935 and the y-intercept is 2.162. We calculate these values using polyfit.

With these values, we generate a $log(\alpha)$ vs. $log(\tau)$ plot by using polyfit shown above in Figure 6. We create a scatter plot of all torque and angular acceleration values. Additionally, we calculate the error for each point and plot them as error bars on the graph. We choose polyfit as the linear least-squares optimization function in order to determine the best-fitting slope and y-intercept of our data. We find the slope to be 0.935 and the y-intercept to be 2.162.

To calculate the uncertainty in the slope and y-intercept, we take the diagonal of the covariant matrix in Python. The uncertainty in the slope is 0.018 while the uncertainty in the y-intercept is 0.028. Therefore, our slope is equal to 0.935 $\pm$ 0.018 and our y-intercept is equal to 2.162 $\pm$ 0.028.

From these results, we can obtain the moment of inertia. We know that:

$$-log(I) = b$$

We also know that there is an implicit base of 10. Therefore, we can say:

$$10^{-b} = I$$

$$10^{-2.162} = 0.0069 kg/m^2$$

We calculate the uncertainty in the moment of inertia in Python as follows:

$$\delta I = log(10) * I * np.mean(\delta log(I))$$

We find δI to be 0.0001. Therefore, our experimental value of moment of inertia is 0.0069 $\pm$ 0.0001 kg/m^2.

Next, we calculate the theoretical or accepted value of moment inertia by using the formula below:

$$I_{true} = I_{rod} + 2I_m$$

where I_{rod} is the moment of inertia of the rod and $2I_m$ is the moment of inertia of the two masses attached at both ends of the rod. The formula above can be simplified to:

$$I_{true} = \frac{1}{12}ML^2 + 2mx^2$$

where M is the mass of the rod, L is the length of the rod, m is the mass of the screw-on mass, and x is the distance the screw-on mass is from the center of the rod.

Plugging in values, we calculate the moment of inertia as follows:

$$I_{true} = \frac{1}{12}(0.042kg)(0.382m)^2 + 2(0.08kg)(0.184m)^2 = 0.0059kg/m^2$$

Our accepted value of moment of inertia is 0.0059 kg/m^2 while our experimental value is 0.0069 ± 0.0001 kg/m^2.

V. DISCUSSION

Our experimental and theoretical values of moment of inertia are relatively consistent within uncertainties. Although our experimental value of 0.0069 ± 0.0001 kg/m^2 does not include the theoretical value, 0.0059 kg/m^2, the numbers are still close to one another and are likely off due to human error. It is possible that the combined mass fell at a non-constant acceleration due to the string getting caught on the pulley. It's also possible that the combined mass swung back and forth while falling, which could have affected the read-out measurements of angular acceleration. Our results could be off due to an underestimate in the error of radius measurements. It is possible that we could have been off by more than 1 mm.

In the Analysis section above, we derived our value of the slope from the $log(\alpha)$ vs. $log(\tau)$ graph to be 0.935 ± 0.018. The accepted value of the slope is 1. Within uncertainty, I would argue that the value of the slope is consistent because it is very close to 1. Similar to the value of the moment of inertia, the experimental value does not include the theoretical value. However, it is possible that the same human error and underestimates in uncertainties led to the small uncertainty that we have.

In this lab, we assumed that the angle of the falling mass was always 90 degrees relative to the rod. It's likely that each trial was not exactly 90 degrees each time. However, this is negligible in the calculation of the torque. We are also neglecting air resistance of the falling mass and the spinning rod. This is because air resistance does not affect our results significantly enough to report.

For future labs, I would recommend including a picture of the experimental setup in the procedure. At first, I was looking for three separate pulleys rather than the 3-pulley system. There may have been a diagram in the materials box, but it would have been helpful to have the diagram in the lab report as well so that I could see what each material looked like. I would also recommend to stray away from heavier slinging masses because it gives you less data to work with in your analysis.

VI. CONCLUSIONS

Overall, this experiment illustrates that the linear relationship between torque and angular acceleration can be used to determine moments of inertia. Through the use of LoggerPro, I was able to understand how the angle, angular velocity, and angular acceleration changed with time. This lab taught me more about how we can use logarithmic functions to interpret linear relationships. I gained experience in uncertainty calculations as well as the interpretation of my results. Additionally, I learned more about linear least-square optimization functions like polyfit, which are used to obtain linear fits to data. Although our moment of inertia and slope values are not within range, I argue that they can be considered consistent since they are very close to the accepted values. All in all, we successfully determined the angular acceleration for each dataset and experimentally determined an accurate moment of inertia for the simple pulley system.

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[1] https://www.britannica.com/science/moment-of-inertia