Image processing
“Digital image modeling”

Digital image modeling

1. Representing digital image
2. Sampling and quantization
3. Color spaces
4. Relationships between pixels

Representing digital image (1)

Origin (0,0)
(top-left)

j/x

	0	1	2	3	4	5	M
0	0	0	0	0	0	0	
1	0	0	0	0	0	0	
2	0	0	1	2	2	0	
3	0	0	0	2	3	1	
4	0	0	0	1	1	0	
5	0	0	0	0	0	0	
6	0	0	0	0	0	0	
N							

i/y

a pixel

I(3,4) = 3

N = 6, M = 5
N×M = 30
q = 2 $0 \leq I(i, j) \leq 3$

The term image refers to a two dimensional array (i.e. raster) denoted by

N	is the number of line (or image height)
M	is the number of column (or image width)
N×M	is the size of the array (in elements/pixels)
i,j (e.g. y,x)	coordinates of a pixel with $i \in [0, N[$ and $j \in [0, M[$
2^q	is the intensity level numbers, with q the quantification parameter
$L = 2^{q-1}$	is the maximum intensity value
I(i,j)	a function to return the value of amplitude at spatial coordinates (i,j) with $0 \leq I(i, j) \leq L$

Representing digital image (2)

The histogram of a digital image is a representation of its intensity distribution such as

The image

I(i,j) = v	is a discrete function
i,j	the coordinates of a pixel
i ∈ [0, N[and j ∈ [0, M[	
v	is the pixel intensity value with $0 \le v \le L$
M×N	is the size of the array (in pixels)

The histogram

$h(k) = n_k$	is a discrete function
k	the intensity value
k∈ [0, L]	is the intensity level range
n_k	is the number of pixels in the image of intensity k

$$\sum_{k=0}^{L} h(k) = N \times M$$

e.g.

Raster with
N=3
M=4
N×M=12
q = 3
$0 \le I(i,j) \le 7$

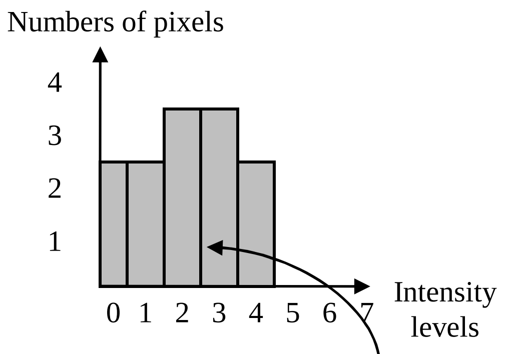
1	2	1	4
2	0	3	3
3	2	0	4

a pixel
I(1,2) = 3

→

Histogram with
k∈ [0, 7]

$$\sum_{k=0}^{7} h(k) = 12$$



a pixel distribution,
h(k=3) = 3
i.e. the number of “3”

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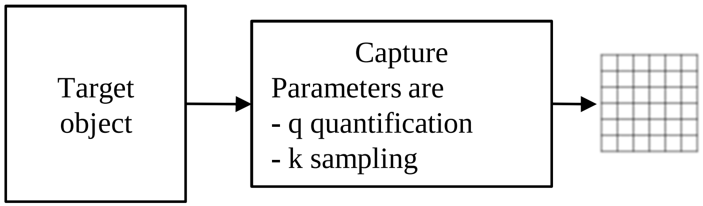
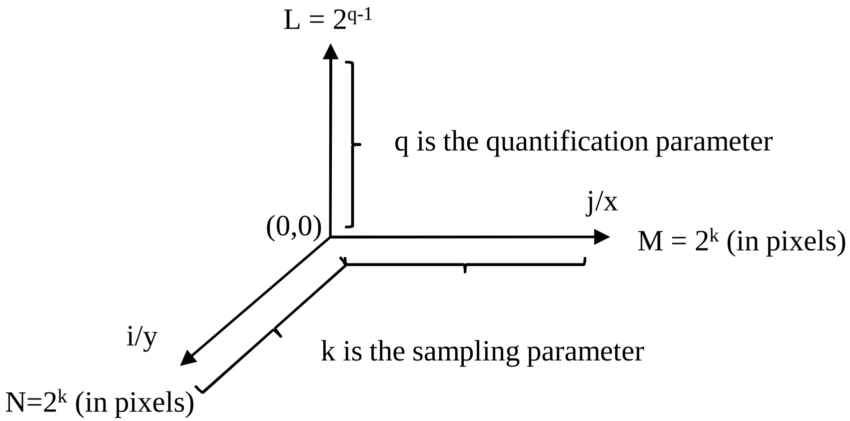
Sampling and quantization (1)

To be suitable for computer processing digital image are first captured (camera, digitized, screenshot …) both spatially and in amplitude

Specification of amplitude is called quantification

Specification of spatial coordinate is called sampling





$$b = 2^{2k} \times q$$

size of image (in bits)
$/2^3$ bytes
$/2^{13}$ Kbytes
$/2^{23}$ Mbytes
etc.

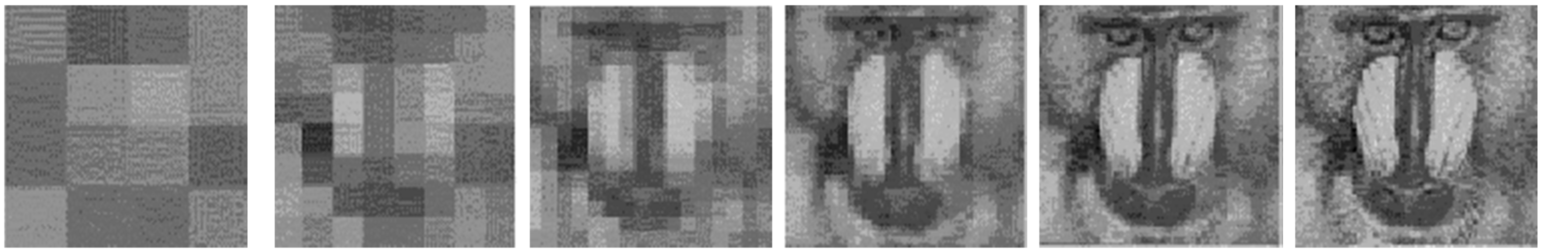
Sampling and quantization (2)

Quantification and sampling parameters impact the image quality, they must be set considering the image content.

quantification
$1 \leq q \leq 5$
k=6 (128×128)

q=1 2 levels | q=2 4 levels | q=3 8 levels | q=4 16 levels | q=5 32 levels

sampling
q=8 (256 values)
$2 \leq k \leq 7$



k=2 4×4 | k=3 8×8 | k=4 16×16 | k=5 32×32 | k=6 64×64 | k=7 128×128

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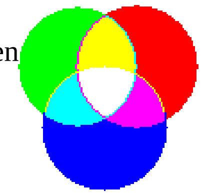
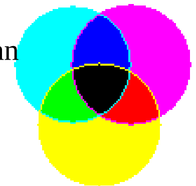
Color spaces (1)

Quantification specifies the maximum number of possible amplitude values, correspondence between these values and colors is ensured by a color space.

	q per channel	**channels**	**color codes**				
			black	**white**	**red**	**blue**	**green**
binary "miniswhite"	1	∅	1	0	∅	∅	∅
binary "minisblack"	1	∅	0	1	∅	∅	∅
gray level	8	∅	0	255	∅	∅	∅
RGB	8 (to 24)	R G B	0 0 0	255 255 255	255 0 0	0 255 0	0 0 255
CMY	8 (to 24)	C M Y	255 255 255	0 0 0	0 255 255	255 255 0	255 0 255





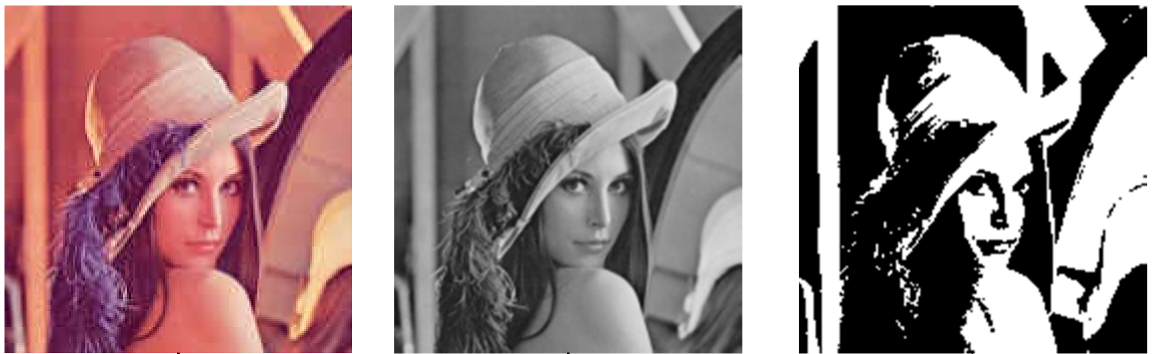
Others are at the corner: YIQ, HSV, etc.

Color spaces (2)

To convert color space is a tradeoff between preserving information and to gain storage capacity.

to gain storage capacity/ to loose information $\longrightarrow$

$(N\times M\times q)\times 3$	$(N\times M\times q)$	$(N\times M\times q)$
$q=8$	$q=8$	$q=1$



Conversion operation is “gray level conversion”

Conversion operation is “binarization”

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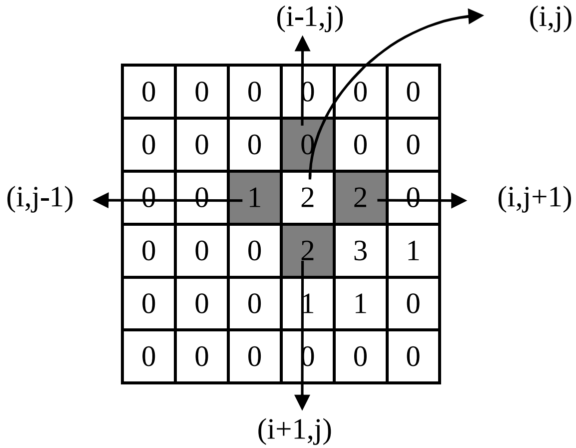
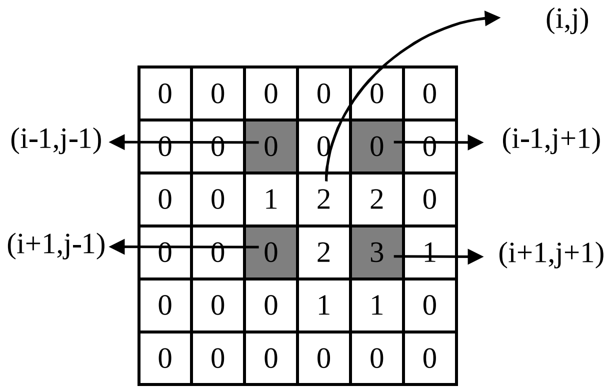
Relationships between pixels (1)

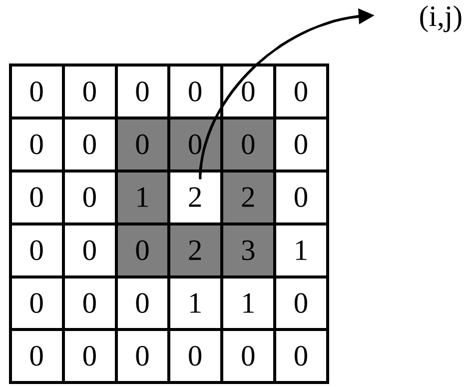
Considering a pixel p of coordinates (i,j), its 4 horizontal and vertical neighbors, denoted $\mathbf{N_4(p)}$, are



Considering a pixel p of coordinates (i,j), its 4 diagonal neighbors, denoted $\mathbf{N_D(p)}$, are



Considering a pixel p of coordinates (i,j), its 8 neighbors, denoted $\mathbf{N_8(p)}$, are a combination of $\mathbf{N_4(p)}$ and $\mathbf{N_D(p)}$



Considering two pixels, p of coordinates (i,j) and q of coordinates (u,v)

p and q are 4-adjacent if	$q \in N_4(p)$ and $p \in N_4(q)$
p and q are 8-adjacent if	$q \in N_8(p)$ and $p \in N_8(q)$

Relationships between pixels (2)

Considering the three pixels

- p of coordinates (i,j)
- q of coordinates (u,v)
- z of coordinates (x,y)

We define a relation D between p,q and z as distance if

(i)	$D(p,q) \geq 0$	non-negativity
(ii)	$D(p,q) = 0$ if $p = q$	reflexivity
(iii)	$D(p,q) = D(q,p)$	commutativity
(iv)	$D(p,z) \leq D(q,p) + D(q,z)$	triangle inequality

Definitions

Example

	0	1	2	3
0	0	0	0	0
1	0	7	4	0
2	0	7	3	2
3	0	5	1	0

p (2,1)

q (3,3)

The Euclidean distance is defined as

$$D_e(p,q) = \left[(i-u)^2 + (j-v)^2\right]^{\frac{1}{2}}$$

$$D_e(p,q) = \left[(2-3)^2 + (1-3)^2\right]^{\frac{1}{2}} = \sqrt{1+4} = \sqrt{5}$$

The city-block distance is defined as

$$D_4(p,q) = |i-u| + |j-v|$$

$$D_4(p,q) = |2-3| + |1-3| = 1 + 2 = 3$$

The chessboard distance is defined as

$$D_8(p,q) = \max(|i-u|, |j-v|)$$

$$D_8(p,q) = \max(|2-3|, |1-3|) = \max(1,2) = 2$$

Relationships between pixels (3)

Considering the three pixels

- p of coordinates (i,j)
- q of coordinates (u,v)
- z of coordinates (x,y)

We define a relation D between p,q and z as distance if

(i)	$D(p,q) \geq 0$	non-negativity
(ii)	$D(p,q) = 0$ if $p = q$	reflexivity
(iii)	$D(p,q) = D(q,p)$	commutativity
(iv)	$D(p,z) \leq D(q,p) + D(q,z)$	triangle inequality

Definitions

Distance maps of pixel p at coordinates (i,j)

The Euclidean distance is defined as

$$D_e(p,q) = \left[(i-u)^2 + (j-v)^2\right]^{\frac{1}{2}}$$

√8	√5	2	√5	√8
√5	√2	1	√2	√5
2	1	**p**	1	2
√5	√2	1	√2	√5
√8	√5	2	√5	√8

The city-block distance is defined as

$$D_4(p,q) = |i-u| + |j-v|$$

4	3	2	3	4
3	2	1	2	3
2	1	**p**	1	2
3	2	1	2	3
4	3	2	3	4

The chessboard distance is defined as

$$D_8(p,q) = \max(|i-u|, |j-v|)$$

2	2	2	2	2
2	1	1	1	2
2	1	**p**	1	2
2	1	1	1	2
2	2	2	2	2