

# Image processing

## “Histogram-based operators”

# Histogram-based operators

1. Fundamentals of histogram-based operators
2. Some histogram-based operators
  - 2.1. Image characterization
  - 2.2. Optimum thresholding
  - 2.3. Histogram equalization
  - 2.4. Histogram-based distances
3. Further investigations

# Fundamentals of histogram-based operators (1)

The histogram of a digital image is a representation of its intensity distribution such as

The image

$I(i,j) = v$  is a discrete function  
 $i, j$  the coordinates of a pixel  
 $i \in [0, N]$  and  $j \in [0, M]$   
 $v$  is the pixel intensity value with  
 $0 \leq v \leq L$   
 $M \times N$  is the size of the array (in pixels)

The histogram

$h(k) = n_k$  is a discrete function  
 $k$  the intensity value  
 $k \in [0, L]$  is the intensity level range  
 $n_k$  is the number of pixels in  
 $\sum_{k=0}^L h(k) = N \times M$

e.g.

Raster with

$N=3$   
 $M=4$   
 $N \times M = 12$   
 $q = 3$   
 $0 \leq I(i, j) \leq 7$

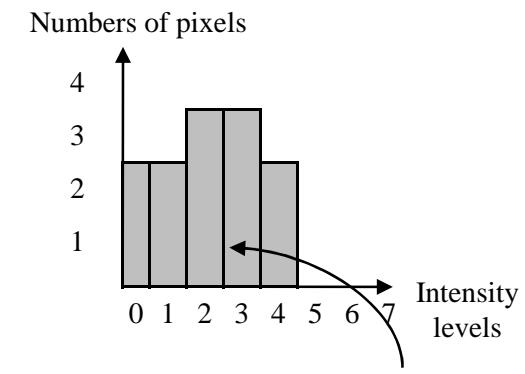
1	2	1	4
2	0	3	3
3	2	0	4

a pixel  
 $I(1,2) = 3$



Histogram with

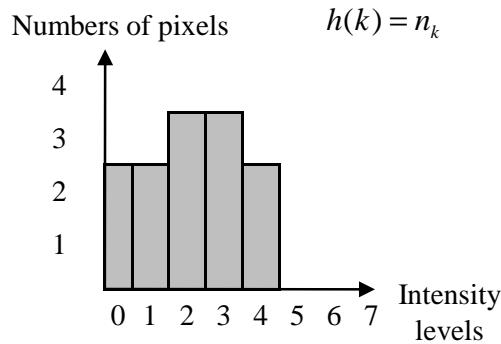
$k \in [0, 7]$   
 $\sum_{k=0}^7 h(k) = 12$



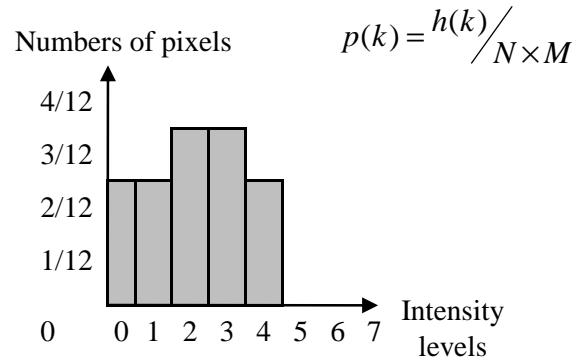
a pixel distribution,  
 $h(k=3) = 3$   
i.e. the number of "3"

# Fundamentals of histogram-based operators (2)

The basic image histogram is



The normalized image histogram is



$p(k)$  defines the probability to get the value  $k$  in the image  $I$

Raster with

$$N=3$$

$$M=4$$

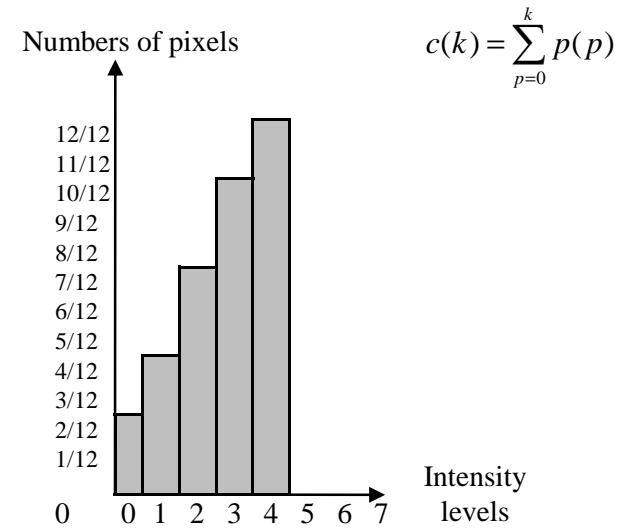
$$N \times M = 12$$

$$q = 3$$

1	2	1	4
2	0	3	3
3	2	0	4

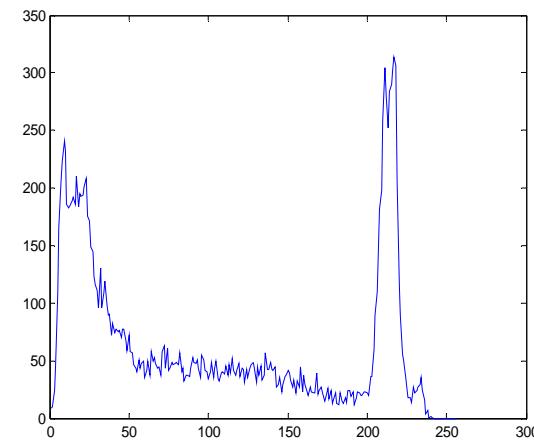
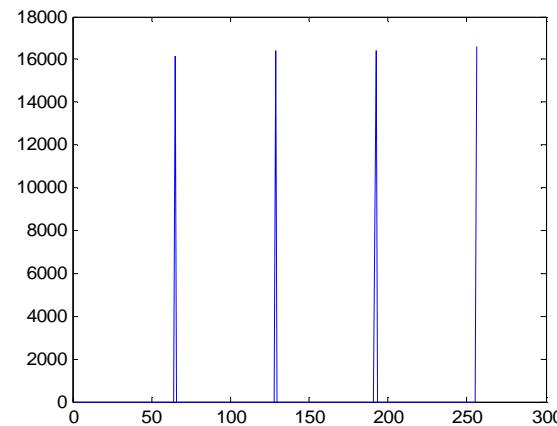
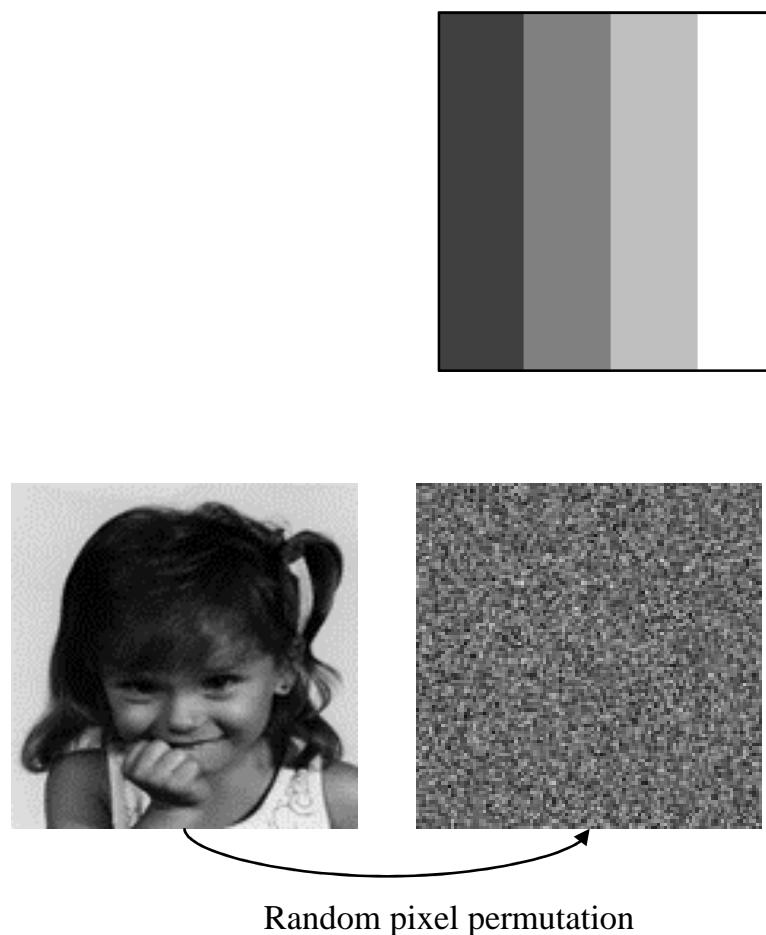
$$0 \leq I(i, j) \leq 7$$

The accumulated image histogram is



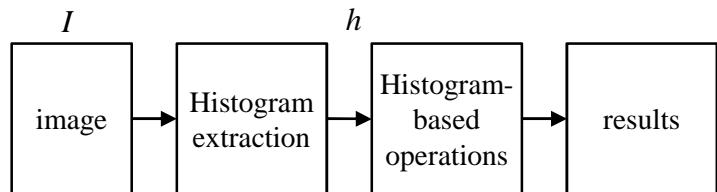
$c(k)$  defines the probability to get the value less or equal to  $k$  in the image  $I$

## Fundamentals of histogram-based operators (3)

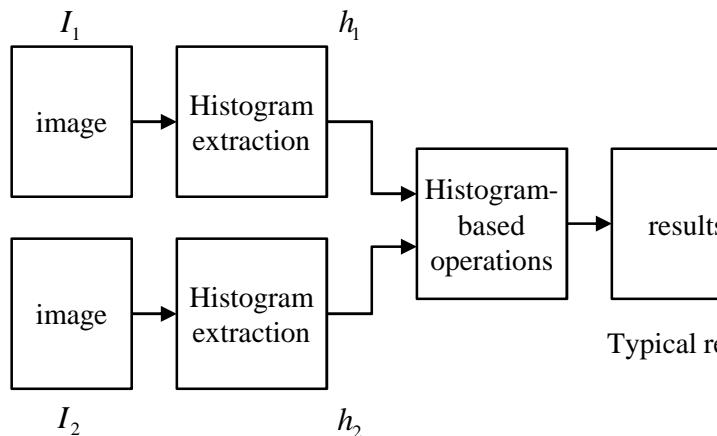


# Fundamentals of histogram-based operators (4)

Histogram-based operations include any statistical processes of intensity distribution. They could be based on single or multiple entries.

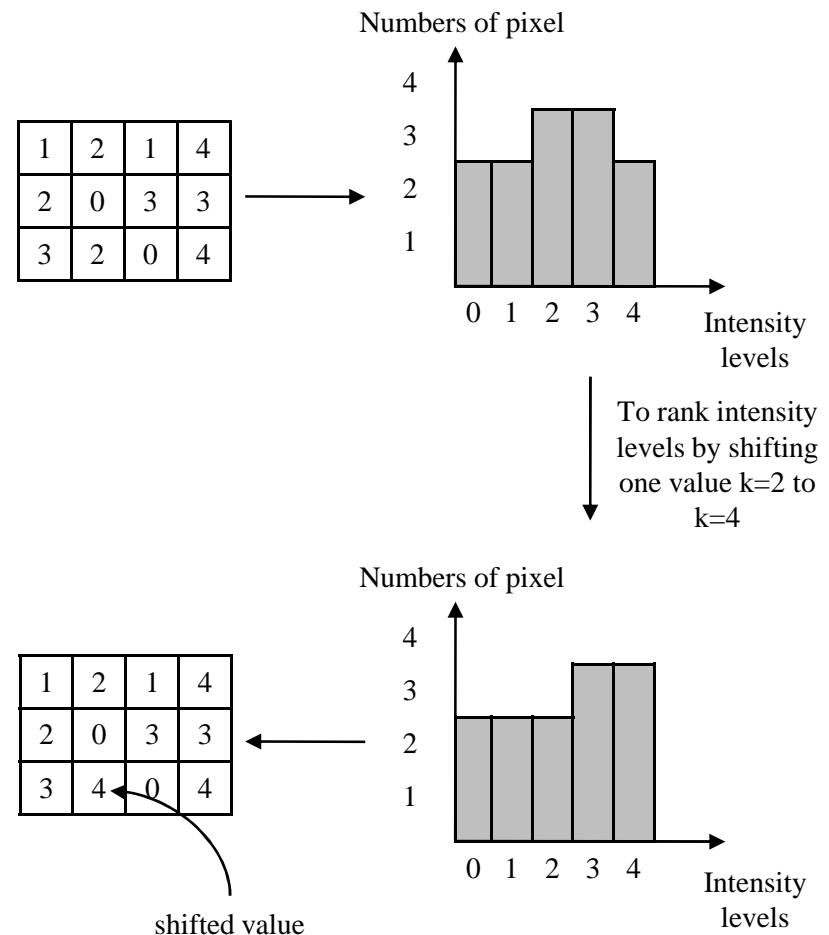


Typical results include  
 - an image  
 - some features



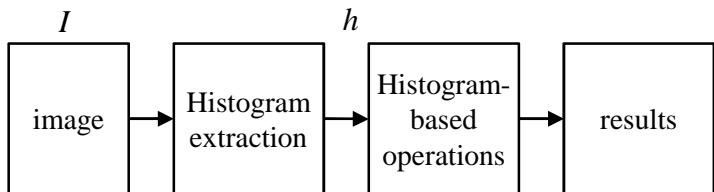
Typical results are distances

e.g.



# Fundamentals of histogram-based operators (5)

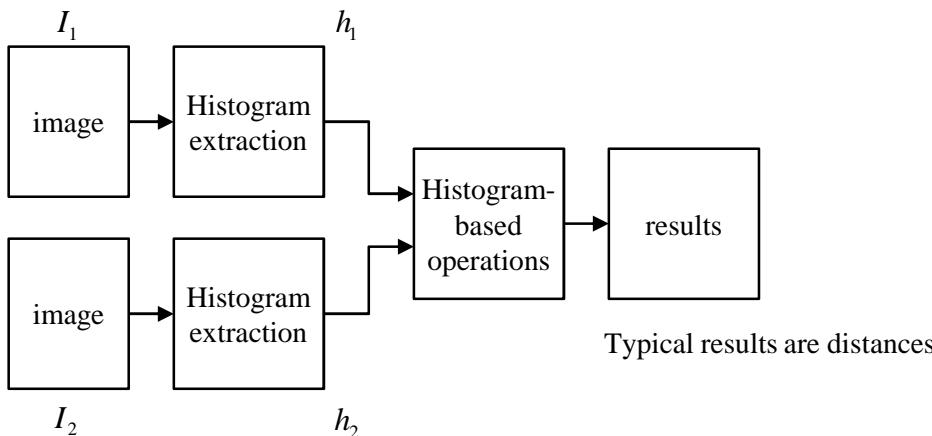
Histogram-based operations include any statistical processes of intensity distribution. They could be based on single or multiple entries.



Histogram-based operations are related to

- ✓ image characterization (or features extraction),
- ✓ automatic thresholding,
- ✓ image enhancement (histogram equalization),
- ✓ image matching (histogram-based distances),
- ✓ etc.

Typical results include  
- an image  
- some features



Typical results are distances

# Histogram-based operators

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  - 2.1. Image characterization
  - 2.2. Optimum thresholding
  - 2.3. Histogram equalization
  - 2.4. Histogram-based distances
3. Further investigations

Type	Methods	Application
Image characterization	Mean, standard deviation	Feature extraction
	Contrast	
	Moments	
	Entropy	
	Co-occurrence matrix	
	Uniformity, homogeneity	
	Correlation	
Thresholding	Otsu's method	Enhancement, segmentation
Histogram equalization	Histogram equalization	Enhancement
Histogram-based distance	Minkowski, $\chi^2$ , Kulback-Leibler and Jeffrey	Comparison, retrieval, spotting

# Image characterization

## “Mean and standard deviation”

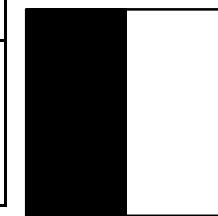
Characterization with mean and standard deviation

	From raster	From histogram	From normalized histogram
Mean (m)	$\frac{\sum_{i=0}^{N-1} \sum_{j=0}^{M-1} I(i, j)}{N \times M}$	$\frac{\sum_{k=0}^L k \times h(k)}{N \times M}$	$\sum_{k=0}^L k \times p(k)$
Standard deviation ( $\sigma$ )	$\sqrt{\frac{\sum_{i=0}^{N-1} \sum_{j=0}^{M-1} (I(i, j) - m)^2}{N \times M}}$	$\sqrt{\frac{\sum_{k=0}^L (k - m)^2 \times h(k)}{N \times M}}$	$\sqrt{\sum_{k=0}^L (k - m)^2 \times p(k)}$



$$m = \frac{L}{2}$$

$$\sigma = 0$$

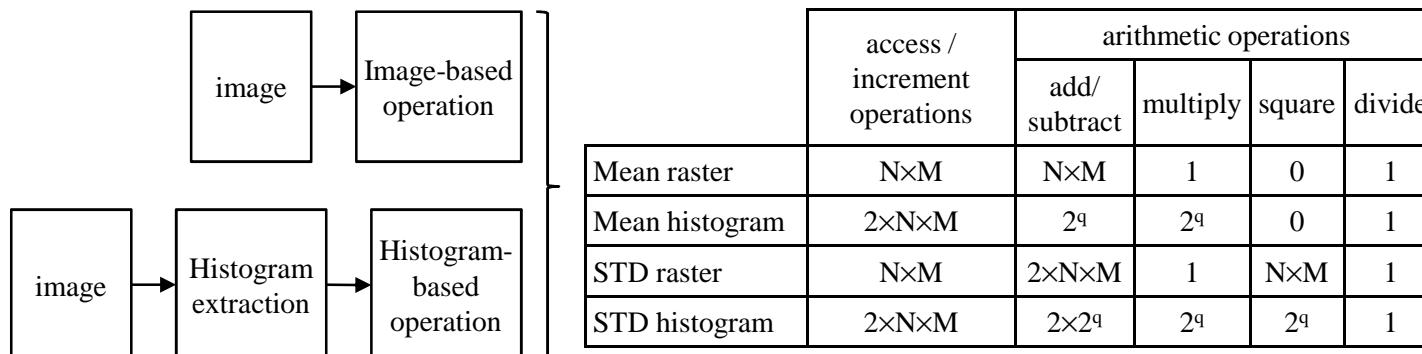


$$m = \frac{1}{2} \times 0 + \frac{1}{2} \times L = \frac{L}{2}$$

$$\sigma = \sqrt{2 \times \frac{1}{2} \times \left(\frac{L}{2}\right)^2} = \frac{L}{2}$$

Rq. Standard deviation is also defined as the squared root of the variance  $\sigma = \sqrt{v}$

Complexity comparison of raster vs. histogram-based operations



with  $N \times M \gg 2^q$   
and  $O(\text{access}) \ll O(\text{arithmetic})$   
the histogram-based operations  
are most efficient.

Type	Methods	Application
Image characterization	Mean, standard deviation	Feature extraction
	Contrast	
	Moments	
	Entropy	
	Co-occurrence matrix	
	Uniformity, homogeneity	
	Correlation	
Thresholding	Otsu's method	Enhancement, segmentation
Histogram equalization	Histogram equalization	Enhancement
Histogram-based distance	Minkowski, $\chi^2$ , Kulback-Leibler and Jeffrey	Comparison, retrieval, spotting

# Image characterization

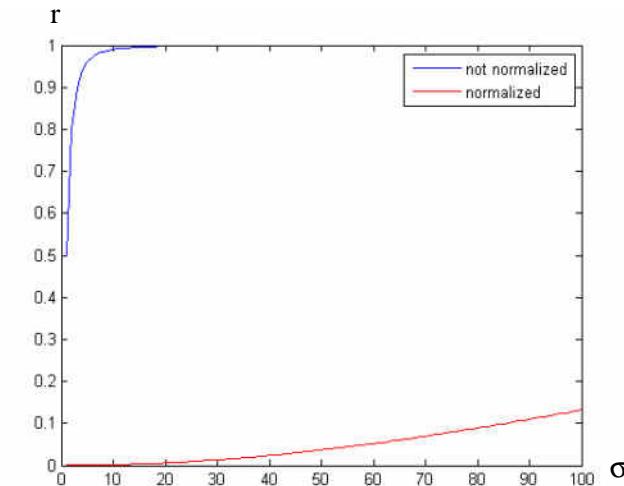
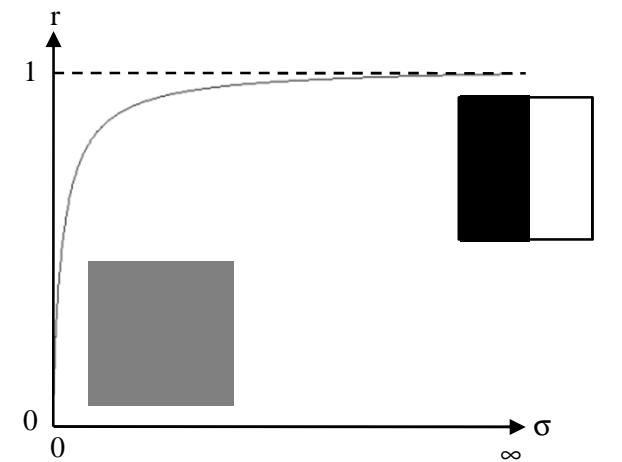
## “Contrast” (1)

Characterization with contrast

	From standard deviation	From standard deviation (normalized)
Contrast ( $r$ )	$r = 1 - \frac{1}{1 + \sigma^2}$	$r = 1 - \frac{1}{1 + \frac{\sigma^2}{L^2}}$

Contrast is the difference in visual properties that makes objects in an image distinguishable from other objects and the background. There are many possible definitions of contrast, the one above is the variance-based.

Due to the large values meet for variance, a normalization of variance with the number of square level intensity could improve the global curvature function.



# Image characterization

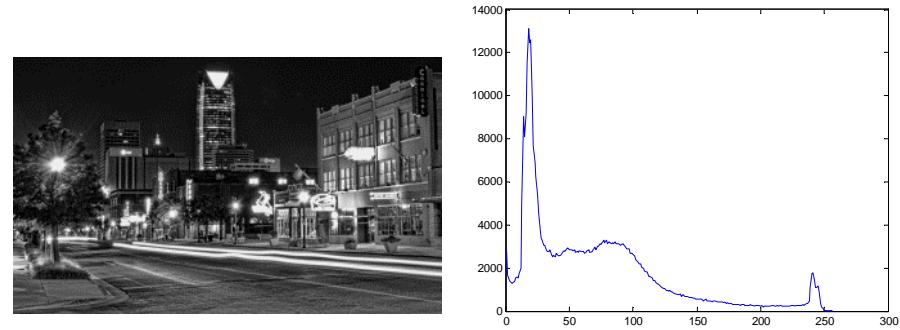
## “Contrast” (2)

Characterization with contrast

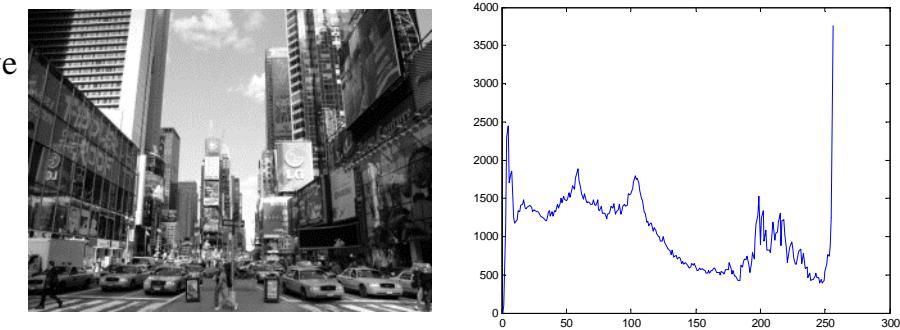
	From standard deviation	From standard deviation (normalized)
Contrast (r)	$r = 1 - \frac{1}{1 + \sigma^2}$	$r = 1 - \frac{1}{1 + \frac{\sigma^2}{L^2}}$

Contrast is the difference in visual properties that makes objects in an image distinguishable from other objects and the background. There are many possible definitions of contrast, the one above is the variance-based.

Due to the large values meet for variance, a normalization of variance with the number of square level intensity could improve the global curvature function.



Standard deviation $\sigma$	2 108
Contrast (r)	1
Normalized contrast (r)	0,98



Standard deviation $\sigma$	457
Contrast (r)	1
Normalized contrast (r)	0,76

Type	Methods	Application
Image characterization	Mean, standard deviation	Feature extraction
	Contrast	
	Moments	
	Entropy	
	Co-occurrence matrix	
	Uniformity, homogeneity	
	Correlation	
Thresholding	Otsu's method	Enhancement, segmentation
Histogram equalization	Histogram equalization	Enhancement
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# Image characterization

## “Statistical moments” (1)

Characterization with statistical moments

	From histogram	From normalized histogram
Moment ( $\mu_n$ )	$\mu_n = \sum_{k=0}^{2^q-1} \frac{(k-m)^n \times h(k)}{N \times M}$	$\mu_n = \sum_{k=0}^{2^q-1} (k-m)^n \times p(k)$

Statistical moments are the simplest approach to describe the image content, they are mainly applied to textures.

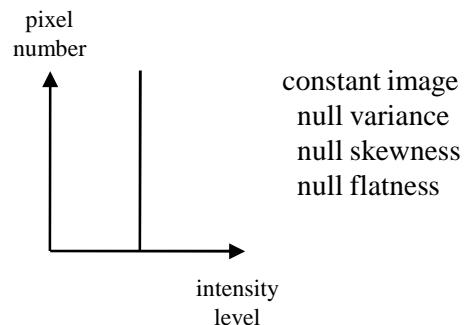
e.g.

n	Moments	Description
0	$\mu_0 = \sum_{k=0}^{2^q-1} (k-m)^0 \times p(k)$ $\mu_0 = \sum_{k=0}^{2^q-1} p(k) = 1$	The 0 <sup>th</sup> moment, is still equal to one
1	$\mu_1 = \sum_{k=0}^{2^q-1} (k-m) \times p(k)$ $\mu_1 = \sum_{k=0}^{2^q-1} 0 \times p(k) = 0$	The 1 <sup>st</sup> moment, is still equal to zero
2	$\mu_2 = \sum_{k=0}^{2^q-1} (k-m)^2 \times p(k)$	The 2 <sup>nd</sup> moment is the variance
3	$\mu_3 = \sum_{k=0}^{2^q-1} (k-m)^3 \times p(k)$	The 3 <sup>rd</sup> moment measure the skewness
4	$\mu_4 = \sum_{k=0}^{2^q-1} (k-m)^4 \times p(k)$	The 4 <sup>th</sup> moment measures the flatness

k	0	1	2	3	4	5	6	7
<b>h(k)</b>	0	0	0	80	0	0	0	0
<b>p(k)</b>	0	0	0	1	0	0	0	0
$(k-m)^0$	1	1	1	1	1	1	1	1
$(k-m)^1$	-3	-2	-1	0	1	2	3	4
$(k-m)^2$	9	4	1	0	1	4	9	4
$(k-m)^3$	-27	-8	-1	0	1	8	27	64
$(k-m)^4$	81	16	1	0	1	16	81	256

$$m = \sum_{k=0}^{2^q-1} k \times p(k) = 3$$

1	$\mu_0$
0	$\mu_1$
0	$\mu_2$
0	$\mu_3$
0	$\mu_4$



# Image characterization

## “Statistical moments” (2)

Characterization with statistical moments

	From histogram	From normalized histogram
Moment ( $\mu_n$ )	$\mu_n = \sum_{k=0}^{2^q-1} \frac{(k-m)^n \times h(k)}{N \times M}$	$\mu_n = \sum_{k=0}^{2^q-1} (k-m)^n \times p(k)$

Statistical moments are the simplest approach to describe the image content, they are mainly applied to textures.

e.g.

n	Moments	Description
0	$\mu_0 = \sum_{k=0}^{2^q-1} (k-m)^0 \times p(k)$ $\mu_0 = \sum_{k=0}^{2^q-1} p(k) = 1$	The 0 <sup>th</sup> moment, is still equal to one
1	$\mu_1 = \sum_{k=0}^{2^q-1} (k-m) \times p(k)$ $\mu_1 = \sum_{k=0}^{2^q-1} 0 \times p(k) = 0$	The 1 <sup>st</sup> moment, is still equal to zero
2	$\mu_2 = \sum_{k=0}^{2^q-1} (k-m)^2 \times p(k)$	The 2 <sup>nd</sup> moment is the variance
3	$\mu_3 = \sum_{k=0}^{2^q-1} (k-m)^3 \times p(k)$	The 3 <sup>rd</sup> moment measure the skewness
4	$\mu_4 = \sum_{k=0}^{2^q-1} (k-m)^4 \times p(k)$	The 4 <sup>th</sup> moment measures the flatness

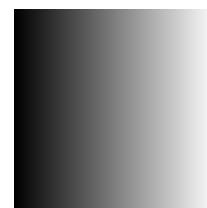
k	0	1	2	3	4	5	6	7
<b>h(k)</b>	10	10	10	10	10	10	10	10
<b>p(k)</b>	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8
$(k-m)^0$	1	1	1	1	1	1	1	1
$(k-m)^1$	-3,5	-2,5	-1,5	-0,5	0,5	1,5	2,5	3,5
$(k-m)^2$	12,5	6,25	2,25	0,25	0,25	2,25	6,25	12,25
$(k-m)^3$	-42,8	-15,6	-3,3	-0,12	0,12	3,3	15,6	42,8
$(k-m)^4$	150	39	5	0,06	0,06	5	39	150

$$m = \sum_{k=0}^{2^q-1} k \times p(k) = 3.5$$

1	$\mu_0$
0	$\mu_1$
5,25	$\mu_2$
0	$\mu_3$
48,6	$\mu_4$

pixel number  
↑  
intensity level  
→

gradient image  
strong variance  
null skewness  
strong flatness



# Image characterization

## “Statistical moments” (3)

Characterization with statistical moments

	From histogram	From normalized histogram
Moment ( $\mu_n$ )	$\mu_n = \sum_{k=0}^{2^q-1} \frac{(k-m)^n \times h(k)}{N \times M}$	$\mu_n = \sum_{k=0}^{2^q-1} (k-m)^n \times p(k)$

Statistical moments are the simplest approach to describe the image content, they are mainly applied to textures.

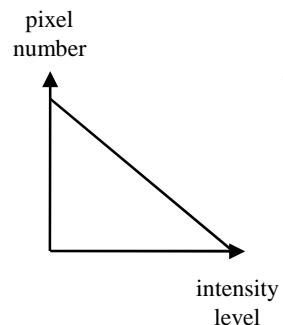
e.g.

n	Moments	Description
0	$\mu_0 = \sum_{k=0}^{2^q-1} (k-m)^0 \times p(k)$ $\mu_0 = \sum_{k=0}^{2^q-1} p(k) = 1$	The 0 <sup>th</sup> moment, is still equal to one
1	$\mu_1 = \sum_{k=0}^{2^q-1} (k-m) \times p(k)$ $\mu_1 = \sum_{k=0}^{2^q-1} 0 \times p(k) = 0$	The 1 <sup>st</sup> moment, is still equal to zero
2	$\mu_2 = \sum_{k=0}^{2^q-1} (k-m)^2 \times p(k)$	The 2 <sup>nd</sup> moment is the variance
3	$\mu_3 = \sum_{k=0}^{2^q-1} (k-m)^3 \times p(k)$	The 3 <sup>rd</sup> moment measure the skewness
4	$\mu_4 = \sum_{k=0}^{2^q-1} (k-m)^4 \times p(k)$	The 4 <sup>th</sup> moment measures the flatness

k	0	1	2	3	4	5	6	7
h(k)	70	60	50	40	30	20	10	00
p(k)	7/28	6/28	5/28	4/28	3/28	2/28	1/28	0
$(k-m)^0$	1	1	1	1	1	1	1	1
$(k-m)^1$	-2	-1	0	1	2	3	4	4
$(k-m)^2$	4	1	0	1	4	9	16	25
$(k-m)^3$	-8	-1	0	1	8	27	64	125
$(k-m)^4$	16	1	0	1	16	81	256	625

$$m = \sum_{k=0}^{2^q-1} k \times p(k) = 2$$

1	$\mu_0$
0	$\mu_1$
3	$\mu_2$
3	$\mu_3$
21	$\mu_4$



weighted gradient image  
medium variance  
strong skewness  
medium flatness



# Image characterization

## “Statistical moments” (4)

Characterization with statistical moments

	From histogram	From normalized histogram
Moment ( $\mu_n$ )	$\mu_n = \sum_{k=0}^{2^q-1} \frac{(k-m)^n \times h(k)}{N \times M}$	$\mu_n = \sum_{k=0}^{2^q-1} (k-m)^n \times p(k)$

Statistical moments are the simplest approach to describe the image content, they are mainly applied to textures.

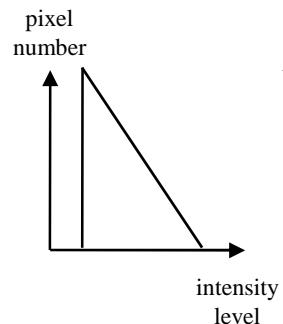
e.g.

n	Moments	Description
0	$\mu_0 = \sum_{k=0}^{2^q-1} (k-m)^0 \times p(k)$ $\mu_0 = \sum_{k=0}^{2^q-1} p(k) = 1$	The 0 <sup>th</sup> moment, is still equal to one
1	$\mu_1 = \sum_{k=0}^{2^q-1} (k-m) \times p(k)$ $\mu_1 = \sum_{k=0}^{2^q-1} 0 \times p(k) = 0$	The 1 <sup>st</sup> moment, is still equal to zero
2	$\mu_2 = \sum_{k=0}^{2^q-1} (k-m)^2 \times p(k)$	The 2 <sup>nd</sup> moment is the variance
3	$\mu_3 = \sum_{k=0}^{2^q-1} (k-m)^3 \times p(k)$	The 3 <sup>rd</sup> moment measure the skewness
4	$\mu_4 = \sum_{k=0}^{2^q-1} (k-m)^4 \times p(k)$	The 4 <sup>th</sup> moment measures the flatness

k	0	1	2	3	4	5	6	7
<b>h(k)</b>	0	0	0	40	30	20	10	00
<b>p(k)</b>	0	0	0	4/10	3/10	2/10	1/10	0
$(k-m)^0$	1	1	1	1	1	1	1	1
$(k-m)^1$	-4	-3	-2	-1	0	1	2	3
$(k-m)^2$	16	9	4	1	0	1	4	9
$(k-m)^3$	-64	-27	-8	-1	0	1	8	27
$(k-m)^4$	256	81	16	1	0	1	16	81

$$m = \sum_{k=0}^{2^q-1} k \times p(k) = 4$$

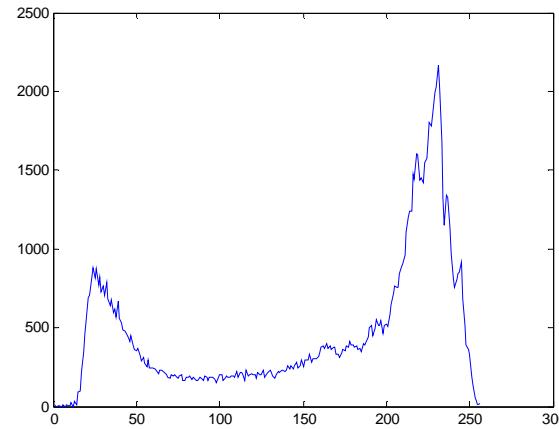
1	$\mu_0$
0	$\mu_1$
1	$\mu_2$
0.6	$\mu_3$
2.2	$\mu_4$



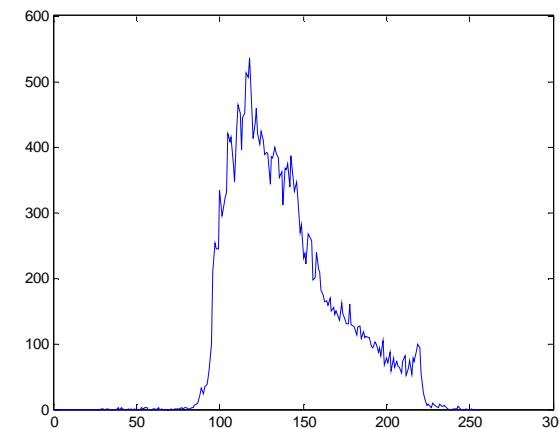
band weighted gradient image  
weak variance  
weak skewness  
weak flatness

# Image characterization

## “Statistical moments” (5)



n	value	scale
$\mu_0$	1	$10^0$
$\mu_1$	0	$10^0$
$\mu_2$	5.86	$10^3$
$\mu_3$	-32.07	$10^3$
$\mu_4$	6720.60	$10^3$



n	value	scale
$\mu_0$	1	$10^0$
$\mu_1$	0	$10^0$
$\mu_2$	0,96	$10^3$
$\mu_3$	23,05	$10^3$
$\mu_4$	2764.90	$10^3$

Type	Methods	Application
Image characterization	Mean, standard deviation	Feature extraction
	Contrast	
	Moments	
	Entropy	
	Co-occurrence matrix	
	Uniformity, homogeneity	
	Correlation	
Thresholding	Otsu's method	Enhancement, segmentation
Histogram equalization	Histogram equalization	Enhancement
Histogram-based distance	Minkowski, $\chi^2$ , Kulback-Leibler and Jeffrey	Comparison, retrieval, spotting

# Image characterization

## “Entropy” (1)

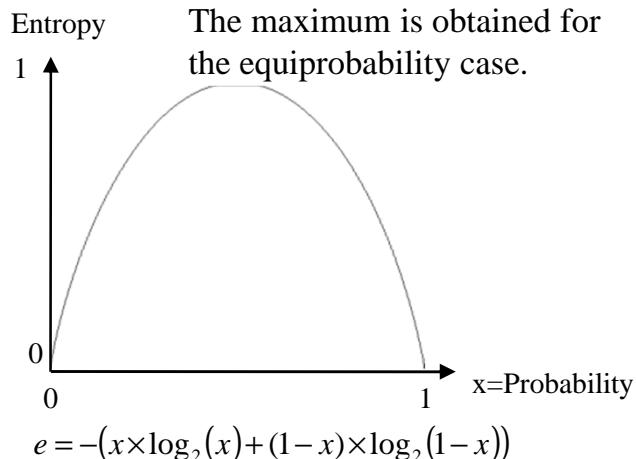
Characterization with Entropy

	From histogram	From normalized histogram
Entropy (e)	$-\sum_{k=0}^{2^q-1} \frac{h(k) \times \log_b \left( h(k) / (M \times N) \right)}{M \times N}$	$-\sum_{k=0}^{2^q-1} p(k) \times \log_b (p(k))$

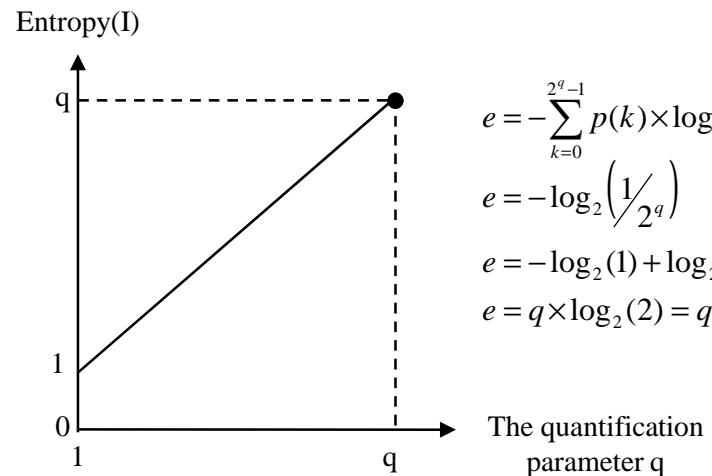
b is the base of the logarithm, common value is 2

Entropy measures the randomness of the image.

Entropy of two variables (x,y) with b=2 ( $\log_2$ )  
and  $x = p(0)$   $y = p(1) = 1 - x$



Max Entropy values (equiprobability) of n variables with b=2 ( $\log_2$ )



# Image characterization

## “Entropy” (2)

Characterization with Entropy

	From histogram	From normalized histogram
Entropy (e)	$-\sum_{k=0}^{2^q-1} h(k) \times \log_b \left( h(k) / (M \times N) \right)$	$-\sum_{k=0}^{2^q-1} p(k) \times \log_b (p(k))$

b is the base of the logarithm, common value is 2

Entropy measures the randomness of the image.

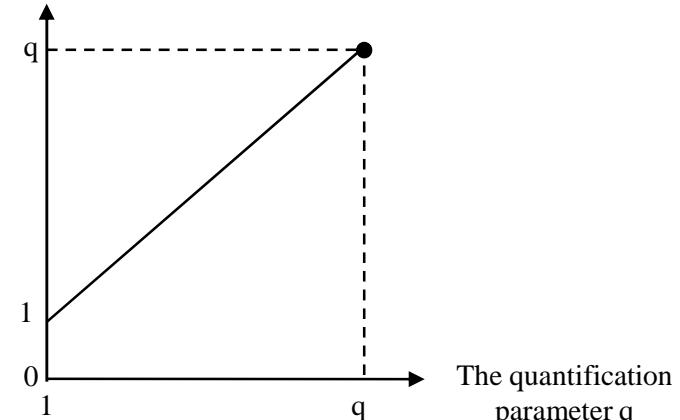
e.g.

k	0	1	2	3	4	5	6	7
h(k)	10	10	10	10	10	10	10	10
p(k)	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8
$\log_2(p(k))$	-3	-3	-3	-3	-3	-3	-3	-3

res	
80	pixel number
1	sum of probability
3	Entropy (e)

Max Entropy values (equiprobability)

Entropy(I)



The quantification parameter q

pixel  
number

gradient image  
max entropy



intensity  
level

# Image characterization

## “Entropy” (3)

Characterization with Entropy

	From histogram	From normalized histogram
Entropy (e)	$-\sum_{k=0}^{2^q-1} h(k) \times \log_b \left( h(k) / (M \times N) \right)$	$-\sum_{k=0}^{2^q-1} p(k) \times \log_b (p(k))$

b is the base of the logarithm, common value is 2

Entropy measures the randomness of the image.

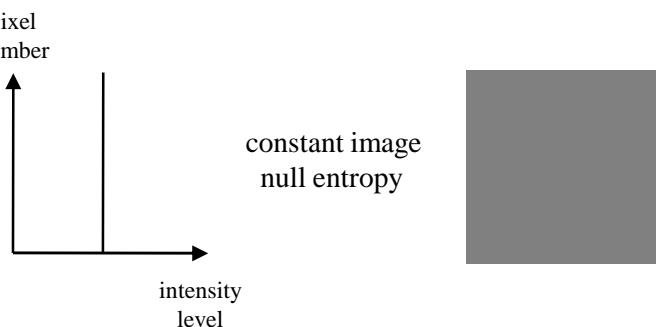
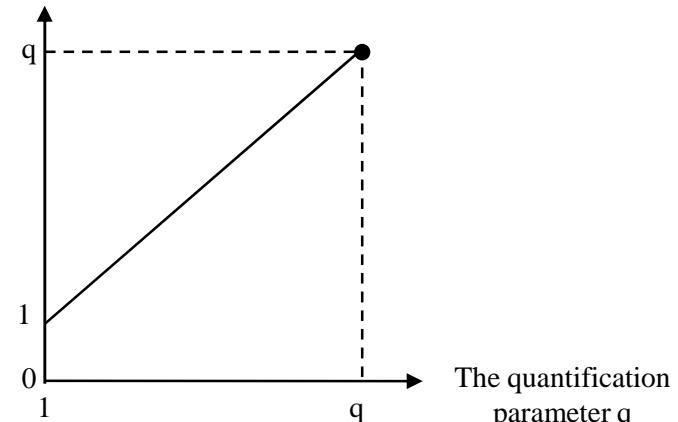
e.g.

k	0	1	2	3	4	5	6	7
h(k)				80				
p(k)	0	0	0	1	0	0	0	0
$\log_2(p(k))$	$-\infty$	$-\infty$	$-\infty$	0	$-\infty$	$-\infty$	$-\infty$	$-\infty$

res	
80	pixel number
1	sum of probability
0	Entropy (e)

Max Entropy values (equiprobability)

Entropy(I)



Type	Methods	Application
Image characterization	Mean, standard deviation	Feature extraction
	Contrast	
	Moments	
	Entropy	
	Co-occurrence matrix	
	Uniformity, homogeneity	
	Correlation	
Thresholding	Otsu's method	Enhancement, segmentation
Histogram equalization	Histogram equalization	Enhancement
Histogram-based distance	Minkowski, $\chi^2$ , Kulback-Leibler and Jeffrey	Comparison, retrieval, spotting

# Image characterization

## “Co-occurrence matrix”

A co-occurrence matrix is a distribution that is defined over an image to be the distribution of co-occurring values at a given offset.

an image I  
 $N \times M = 6 \times 6$   
 $q = 3$  (i.e.  $k \in [0-7]$ )

0	0	6	4	2	1
4	0	5	0	1	4
7	7	5	7	0	1
3	2	3	4	4	0
7	6	7	6	5	1
6	7	5	1	5	1

The corresponding co-occurrence matrix  $g$  using a [1 1] structuring element

k (intensity levels) - j							
0	1	2	3	4	5	6	7
0	1	2			1	1	
1				1	1		
2		1	1				
3			1	1			
4	2		1	1			
5	1	3					1
6				1	1		2
7	1				2	2	1

using a [1 1] structuring element, 5 is neighbor of 0,  
we increase the corresponding  $g(i,j)$  element in the co-occurrence matrix

equation	description
$n = \sum_{i=0}^{2^q-1} \sum_{j=0}^{2^q-1} g(i, j)$	Co-occurrence number
$p(i, j) = g(i, j) / n$ $\sum_{i=0}^{2^q-1} \sum_{j=0}^{2^q-1} p(i, j) = 1$	Probability estimation of $g$

The corresponding probability estimation  $p$ ,  $n$  equals 30

k (intensity levels) - j							
0	1	2	3	4	5	6	7
0	1/30	2/30			1/30	1/30	
1				1/30	1/30		
2		1/30	1/30				
3			1/30	1/30			
4	2/30		1/30	1/30			
5	1/30	3/30					1/30
6				1/30	1/30	2/30	
7	1/30			2/30	3/30	2/30	1/30

Type	Methods	Application
Image characterization	Mean, standard deviation	Feature extraction
	Contrast	
	Moments	
	Entropy	
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	Correlation	
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# Image characterization

## “Uniformity, homogeneity” (1)

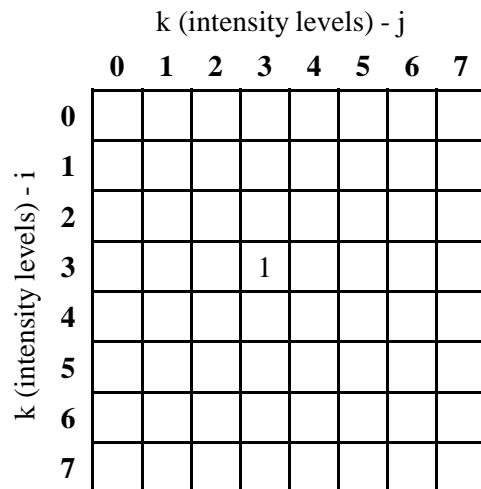
Characterization with uniformity and homogeneity

	<b>Equation</b>
<b>Uniformity</b>	$\sum_{i=0}^{2^q-1} \sum_{j=0}^{2^q-1} p(i, j)^2$
<b>Homogeneity</b>	$\sum_{i=0}^{2^q-1} \sum_{j=0}^{2^q-1} \frac{p(i, j)}{1 +  i - j }$

Uniformity (also called Energy) estimates image as a constant.

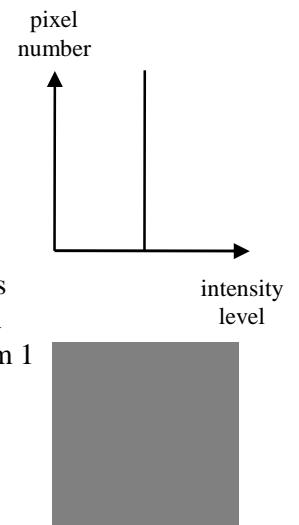
Homogeneity measures the spatial closeness of the element distribution in g.

e.g.



	<b>values</b>
<b>Uniformity</b>	$= 1^2 = 1$
<b>Homogeneity</b>	$\frac{1}{1 +  1 - 1 } = 1$

in a constant image,  
the uniformity equals  
the homogeneity and  
reaches the maximum 1



# Image characterization

## “Uniformity, homogeneity” (2)

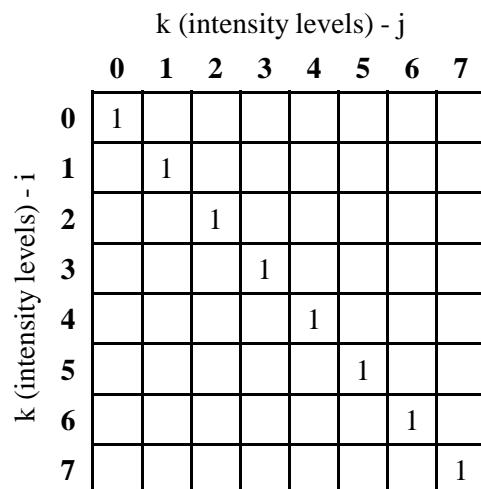
Characterization with uniformity and homogeneity

	<b>Equation</b>
<b>Uniformity</b>	$\sum_{i=0}^{2^q-1} \sum_{j=0}^{2^q-1} p(i, j)^2$
<b>Homogeneity</b>	$\sum_{i=0}^{2^q-1} \sum_{j=0}^{2^q-1} \frac{p(i, j)}{1 +  i - j }$

Uniformity (also called Energy) estimates image as a constant.

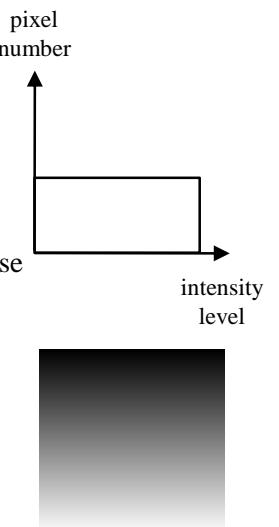
Homogeneity measures the spatial closeness of the element distribution in g.

e.g.



	<b>values</b>
<b>Uniformity</b>	$2^q \times \left(\frac{1}{2^q}\right)^2 = \frac{1}{2^q}$
<b>Homogeneity</b>	$2^q \times \frac{1}{2^q} = 1$

in a gradient image,  
uniformity is the inverse  
of the intensity level  
number, no impact is  
observed on  
homogeneity



# Image characterization

## “Uniformity, homogeneity” (3)

Characterization with uniformity and homogeneity

	Equation
Uniformity	$\sum_{i=0}^{2^q-1} \sum_{j=0}^{2^q-1} p(i, j)^2$
Homogeneity	$\sum_{i=0}^{2^q-1} \sum_{j=0}^{2^q-1} \frac{p(i, j)}{1 +  i - j }$

Uniformity (also called Energy) estimates image as a constant.

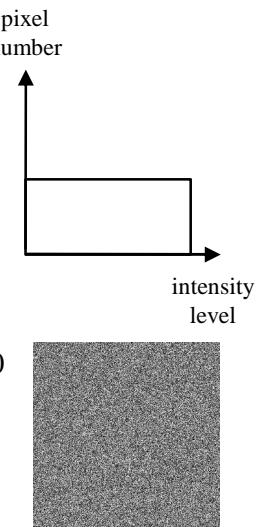
Homogeneity measures the spatial closeness of the element distribution in g.

e.g.

		k (intensity levels) - j									
		0 1 2 3 4 5 6 7									
		0	1	1	1	1	1	1	1	1	1
		1	1	1	1	1	1	1	1	1	1
		2	1	1	1	1	1	1	1	1	1
		3	1	1	1	1	1	1	1	1	1
		4	1	1	1	1	1	1	1	1	1
		5	1	1	1	1	1	1	1	1	1
		6	1	1	1	1	1	1	1	1	1
		7	1	1	1	1	1	1	1	1	1

	values
Uniformity	$(2^q)^2 \times \left( \frac{1}{(2^q)^2} \right)^2 = \frac{1}{(2^q)^2} \rightarrow 0$
Homogeneity	$\sum_{i=0}^{2^q-1} \sum_{j=0}^{2^q-1} \frac{p}{1 +  i - j } = \sum_{i=0}^{2^q-1} \sum_{j=0}^{2^q-1} \frac{1}{(2^q)^2} \times \frac{1}{1 +  i - j } \rightarrow 0$

in a random image,  
the uniformity equals  
the homogeneity and  
reaches the minimum 0



Type	Methods	Application
Image characterization	Mean, standard deviation	Feature extraction
	Contrast	
	Moments	
	Entropy	
	Co-occurrence matrix	
	Uniformity, homogeneity	
	Correlation	
Thresholding	Otsu's method	Enhancement, segmentation
Histogram equalization	Histogram equalization	Enhancement
Histogram-based distance	Minkowski, $\chi^2$ , Kulback-Leibler and Jeffrey	Comparison, retrieval, spotting

# Image characterization

## “Correlation” (1)

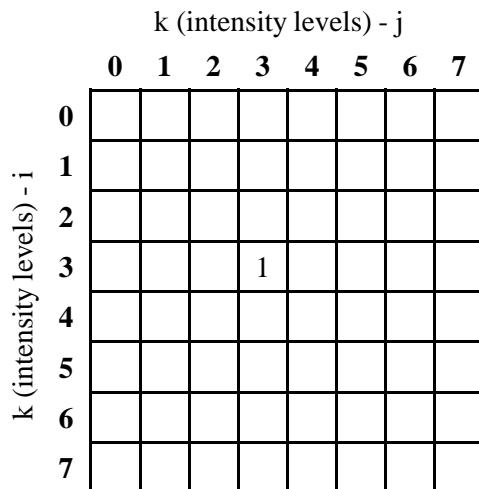
Characterization with correlation

	Equation
Correlation	$\sum_{i=0}^{2^q-1} \sum_{j=0}^{2^q-1} \frac{(i - m_r)(j - m_c)}{\sigma_r \sigma_c} p(i, j)$

	Equation
Row-mean and standard deviation	$m_r = \sum_{i=0}^{2^q-1} i \sum_{j=0}^{2^q-1} p(i, j) \quad \sigma_r^2 = \sum_{i=0}^{2^q-1} (i - m_r)^2 \sum_{j=0}^{2^q-1} p(i, j)$
Column-mean and standard deviation	$m_c = \sum_{j=0}^{2^q-1} j \sum_{i=0}^{2^q-1} p(i, j) \quad \sigma_c^2 = \sum_{j=0}^{2^q-1} (j - m_c)^2 \sum_{i=0}^{2^q-1} p(i, j)$

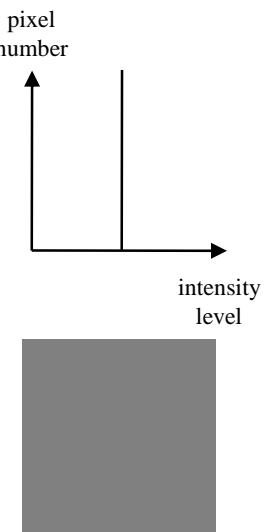
Correlation measures how correlated a pixel is to its neighbor over the entire image. Range of values is -1 to +1 corresponding to perfect correlation. This measure is not defined if the standard deviation is zero.

e.g.



	values
mean	$m_{r,c} = 3 \times 1 = 3$
standard deviation	$\sigma_{r,c} = (3-3)^2 \times 1 = 0$
Correlation	Not applicable

in a constant image,  
the correlation can't  
be defined if the  
standard deviation is  
zero



# Image characterization

## “Correlation” (2)

Characterization with correlation

	Equation
Correlation	$\sum_{i=0}^{2^q-1} \sum_{j=0}^{2^q-1} \frac{(i - m_r)(j - m_c)}{\sigma_r \sigma_c} p(i, j)$

	Equation
Row-mean and standard deviation	$m_r = \sum_{i=0}^{2^q-1} i \sum_{j=0}^{2^q-1} p(i, j) \quad \sigma_r^2 = \sum_{i=0}^{2^q-1} (i - m_r)^2 \sum_{j=0}^{2^q-1} p(i, j)$
Column-mean and standard deviation	$m_c = \sum_{j=0}^{2^q-1} j \sum_{i=0}^{2^q-1} p(i, j) \quad \sigma_c^2 = \sum_{j=0}^{2^q-1} (j - m_c)^2 \sum_{i=0}^{2^q-1} p(i, j)$

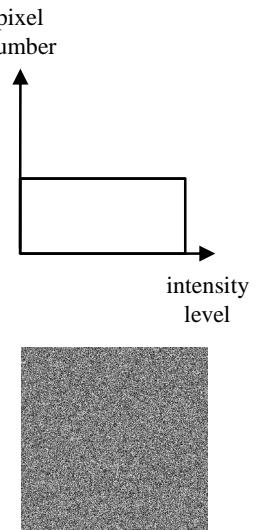
Correlation measures how correlated a pixel is to its neighbor over the entire image. Range of values is -1 to +1 corresponding to perfect correlation. This measure is not defined if the standard deviation is zero.

e.g.

		k (intensity levels) - j							
		0	1	2	3	4	5	6	7
k (intensity levels) - i	0	1	1	1	1	1	1	1	1
	1	1	1	1	1	1	1	1	1
	2	1	1	1	1	1	1	1	1
	3	1	1	1	1	1	1	1	1
	4	1	1	1	1	1	1	1	1
	5	1	1	1	1	1	1	1	1
	6	1	1	1	1	1	1	1	1
	7	1	1	1	1	1	1	1	1

	values
mean	$m_{r,c} \rightarrow \frac{2^q - 1}{2}$
standard deviation	$\sigma_{r,c} \rightarrow \infty$
Correlation	$\rightarrow 0$

in a random image,  
the correlation reaches  
the minimum 0



# Image characterization

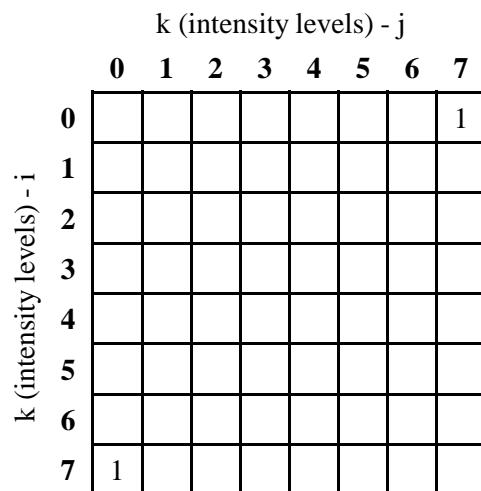
## “Correlation” (3)

Characterization with correlation

	Equation		Equation
Correlation	$\sum_{i=0}^{2^q-1} \sum_{j=0}^{2^q-1} \frac{(i - m_r)(j - m_c)}{\sigma_r \sigma_c} p(i, j)$	Row-mean and standard deviation	$m_r = \sum_{i=0}^{2^q-1} i \sum_{j=0}^{2^q-1} p(i, j) \quad \sigma_r^2 = \sum_{i=0}^{2^q-1} (i - m_r)^2 \sum_{j=0}^{2^q-1} p(i, j)$
		Column-mean and standard deviation	$m_c = \sum_{j=0}^{2^q-1} j \sum_{i=0}^{2^q-1} p(i, j) \quad \sigma_c^2 = \sum_{j=0}^{2^q-1} (j - m_c)^2 \sum_{i=0}^{2^q-1} p(i, j)$

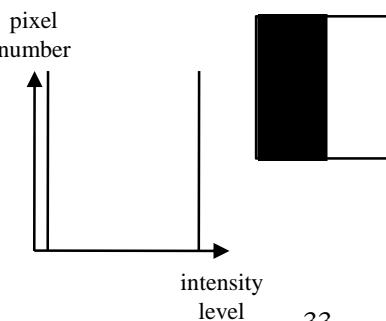
Correlation measures how correlated a pixel is to its neighbor over the entire image. Range of values is -1 to +1 corresponding to perfect correlation. This measure is not defined if the standard deviation is zero.

e.g.



	values
mean	$m_{r,c} = 0 \times \frac{1}{2} + 7 \times \frac{1}{2} = 3,5$
standard deviation	$\sigma_{r,c} = (0 - 3,5)^2 \times \frac{1}{2} + (7 - 3,5)^2 \times \frac{1}{2} = 3,5^2$
Correlation	$\frac{(0-3,5) \times (7-3,5) \times \frac{1}{2} + (7-3,5) \times (0-3,5) \times \frac{1}{2}}{\sqrt{3,5^2}^2} = -1$

in a band image with pixels that co-occur the correlation reaches the “maximum” -1



# Image characterization

## “Correlation” (4)

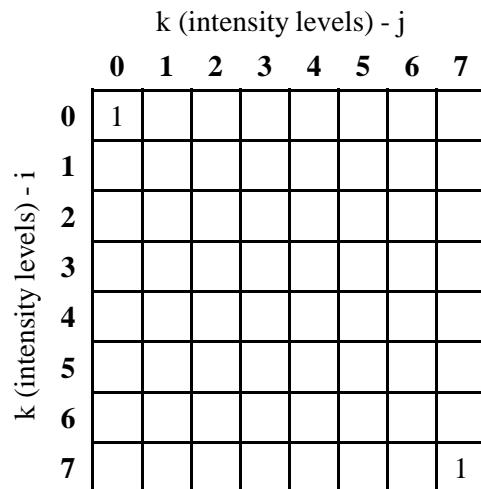
Characterization with correlation

	Equation
Correlation	$\sum_{i=0}^{2^q-1} \sum_{j=0}^{2^q-1} \frac{(i - m_r)(j - m_c)}{\sigma_r \sigma_c} p(i, j)$

	Equation
Row-mean and standard deviation	$m_r = \sum_{i=0}^{2^q-1} i \sum_{j=0}^{2^q-1} p(i, j) \quad \sigma_r^2 = \sum_{i=0}^{2^q-1} (i - m_r)^2 \sum_{j=0}^{2^q-1} p(i, j)$
Column-mean and standard deviation	$m_c = \sum_{j=0}^{2^q-1} j \sum_{i=0}^{2^q-1} p(i, j) \quad \sigma_c^2 = \sum_{j=0}^{2^q-1} (j - m_c)^2 \sum_{i=0}^{2^q-1} p(i, j)$

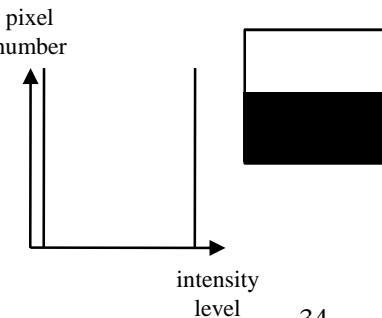
Correlation measures how correlated a pixel is to its neighbor over the entire image. Range of values is -1 to +1 corresponding to perfect correlation. This measure is not defined if the standard deviation is zero.

e.g.



	values
mean	$m_{r,c} = 0 \times \frac{1}{2} + 7 \times \frac{1}{2} = 3,5$
standard deviation	$\sigma_{r,c} = (0 - 3,5)^2 \times \frac{1}{2} + (7 - 3,5)^2 \times \frac{1}{2} = 3,5^2$
Correlation	$\frac{(0 - 3,5)^2}{(\sqrt{3,5^2})^2} \times \frac{1}{2} + \frac{(7 - 3,5)^2}{(\sqrt{3,5^2})^2} \times \frac{1}{2} = 1$

in a band image with pixels that don't co-occur the correlation reaches the “maximum” 1



# Histogram-based operators

1. Fundamentals of histogram-based operators
2. Some histogram-based operators
  - 2.1. Image characterization
  - 2.2. Optimum thresholding
  - 2.3. Histogram equalization
  - 2.4. Histogram-based distances
3. Further investigations

Type	Methods	Application
Image characterization	Mean, standard deviation	Feature extraction
	Contrast	
	Moments	
	Entropy	
	Co-occurrence matrix	
	Uniformity, homogeneity	
	Correlation	
Thresholding	Otsu's method	Enhancement, segmentation
Histogram equalization	Histogram equalization	Enhancement
Histogram-based distance	Minkowski, $\chi^2$ , Kulback-Leibler and Jeffrey	Comparison, retrieval, spotting

# Optimum thresholding “Otsu’s method” (1)

Thresholding may be viewed as a statistical-decision theory problem whose objective is to minimize the average error incurred in assigning pixels to two or more groups i.e. classes. The Otsu’s method is optimum in the sense that it maximizes the between-class variance.

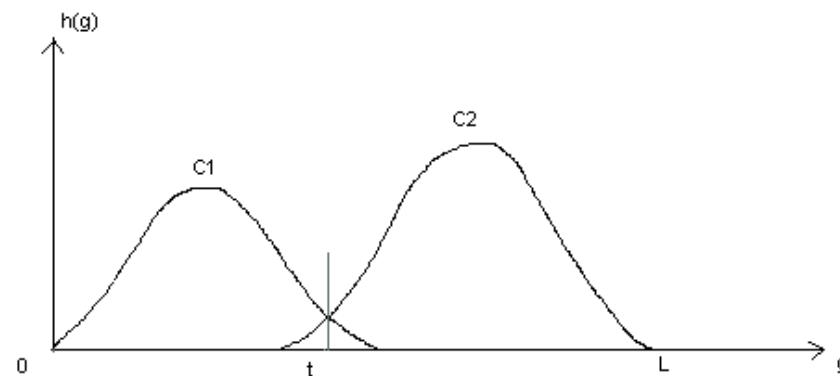


# Optimum thresholding

## “Otsu’s method” (2)

Thresholding may be viewed as a statistical-decision theory problem whose objective is to minimize the average error incurred in assigning pixels to two or more groups i.e. classes. The Otsu’s method is optimum in the sense that it maximizes the between-class variance.

	<b>Equation</b>		<b>Equation</b>
<b>Probability density function (i.e. normalized histogram)</b>	$\sum_{k=0}^{2^q-1} p(k) = 1$	<b>Probability <math>P_i</math> of a class <math>C_i</math> to have a pixel assigned to it</b>	$P_1 = \sum_{k=0}^t p(k) \quad P_2 = \sum_{k=t+1}^{2^q-1} p(k) \quad P_1 + P_2 = 1$
<b>Mean</b>	$m = \sum_{k=0}^{2^q-1} k \times p(k)$	<b>Mean intensity value of the pixel assigned to class <math>C_i</math></b>	$m_1 = \frac{1}{P_1} \sum_{k=0}^t k \times p(k) \quad m_2 = \frac{1}{P_2} \sum_{k=t+1}^{2^q-1} k \times p(k)$ $P_1 \times m_1 + P_2 \times m_2 = m$

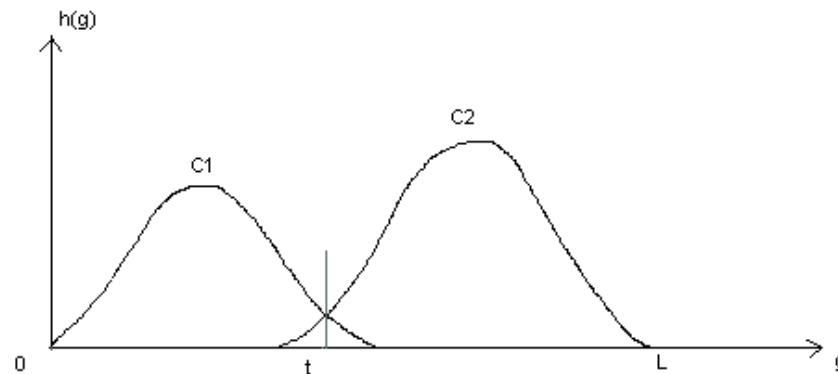


# Optimum thresholding

## “Otsu’s method” (3)

Thresholding may be viewed as a statistical-decision theory problem whose objective is to minimize the average error incurred in assigning pixels to two or more groups i.e. classes. The Otsu’s method is optimum in the sense that it maximizes the between-class variance.

	Equation		Equation
<b>Variance</b>	$\sigma^2 = \sum_{k=0}^{2^q-1} (k - m)^2 \times p(k)$	<b>The <math>C_i</math> variance</b>	$\sigma_i^2 = \frac{1}{P_1} \sum_{k=0}^t (k - m_1)^2 \times p(k) \quad \sigma_2^2 = \frac{1}{P_2} \sum_{k=t+1}^{2^q-1} (k - m_2)^2 \times p(k)$
<b>The between-class and intra-class variances</b>		$\sigma_B^2 = P_1(m_1 - m)^2 + P_2(m_2 - m)^2 \quad \sigma_I^2 = P_1\sigma_1^2 + P_2\sigma_2^2$ $\sigma^2 = \sigma_B^2 + \sigma_I^2$	
<b>Goodness of the threshold</b>		$\eta = \frac{\sigma_B^2}{\sigma^2}$	



# Optimum thresholding

## “Otsu’s method” (4)

Thresholding may be viewed as a statistical-decision theory problem whose objective is to minimize the average error incurred in assigning pixels to two or more groups i.e. classes. The Otsu’s method is optimum in the sense that it maximizes the between-class variance.

e.g.

<b>k</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>
<b>h(k)</b>	40	30	20	10	20	30	40	50
<b>p(k)</b>	4/24	3/24	2/24	1/24	2/24	3/24	4/24	5/24
<b>P<sub>1</sub>(k)</b>	4/24	7/24	9/24	10/24	12/24	15/24	19/24	<b>Na</b>
<b>P<sub>2</sub>(k)</b>	20/24	17/24	15/24	14/24	12/24	9/24	5/24	<b>Na</b>
<b>m<sub>1</sub>(k)</b>	0	0.42	0.77	1	1.5	2.2	3	<b>Na</b>
<b>m<sub>2</sub>(k)</b>	4.60	5.23	5.66	5.85	6.16	6.55	7	<b>Na</b>
<b>σ<sup>2</sup><sub>1</sub>(k)</b>	0	0.24	0.61	1	2.08	3.62	5.26	<b>Na</b>
<b>σ<sup>2</sup><sub>2</sub>(k)</b>	4.64	2.76	1.55	1.12	0.63	0.24	0	<b>Na</b>
<b>σ<sup>2</sup><sub>B</sub>(k)</b>	2.93	4.77	5.6	5.73	5.44	4.44	2.63	<b>Na</b>
<b>σ<sup>2</sup><sub>I</sub>(k)</b>	3.86	2.03	1.20	1.07	1.36	2.35	4.16	<b>Na</b>
<b>η(k)</b>	0.43	0.70	0.82	<b>0.84</b>	0.80	0.65	0.38	<b>Na</b>

$$m = \sum_{k=0}^{2^q-1} k \times p(k) = 4 \quad \sigma^2 = \sum_{k=0}^{2^q-1} (k - m)^2 \times p(k) = 6.8$$

$$P_1 = \sum_{k=0}^t p(k) \quad P_2 = \sum_{k=t+1}^{2^q-1} p(k) \quad P_1 + P_2 = 1$$

$$m_1 = \frac{1}{P_1} \sum_{k=0}^t k \times p(k) \quad m_2 = \frac{1}{P_2} \sum_{k=t+1}^{2^q-1} k \times p(k) \quad P_1 \times m_1 + P_2 \times m_2 = m$$

$$\sigma_1^2 = \frac{1}{P_1} \sum_{k=0}^t (k - m_1)^2 \times p(k) \quad \sigma_2^2 = \frac{1}{P_2} \sum_{k=t+1}^{2^q-1} (k - m_2)^2 \times p(k)$$

$$\sigma_B^2 = P_1(m_1 - m)^2 + P_2(m_2 - m)^2 \quad \sigma_I^2 = P_1\sigma_1^2 + P_2\sigma_2^2 \quad \sigma^2 = \sigma_B^2 + \sigma_I^2$$

$$\eta = \frac{\sigma_B^2}{\sigma^2}$$

# Optimum thresholding

## “Otsu’s method” (5)

Thresholding may be viewed as a statistical-decision theory problem whose objective is to minimize the average error incurred in assigning pixels to two or more groups i.e. classes. The Otsu’s method is optimum in the sense that it maximizes the between-class variance.

e.g. Implementation in Matlab of the Otsu algorithm

```
% probability density
k = [0 1 2 3 4 5 6 7];
h = [40 30 20 10 20 30 40 50];
p = h / sum(h); sum(p);

% probability
p1 = zeros(1,7); p2 = zeros(1,7);
for t=1:7
    p1(t) = sum(p(1:t));
    p2(t) = sum(p(t+1:8));
end
p1 + p2;

% global mean
m = sum(k.*p);

% mean
m1 = zeros(1,7); m2 = zeros(1,7);
for t=1:7
    m1(t) = sum(k(1:t).*p(1:t)) / p1(t);
    m2(t) = sum(k(t+1:8).*p(t+1:8)) / p2(t);
end
(p1.*m1 + p2.*m2)-m;

% global variance
v = sum( ((k-m).^2) .*p ) ;

% variance
v1 = zeros(1,7); v2 = zeros(1,7);
for t=1:7
    v1(t) = sum ( ((k(1:t)-m1(t)).^2).*p(1:t) ) / p1(t);
    v2(t) = sum ( ((k(t+1:8)-m2(t)).^2).*p(t+1:8) ) / p2(t)
end

% i and b variances
vb = zeros(1,7); vi = zeros(1,7);
for t=1:7
    vb(t) = p1(t)*(m1(t)-m)^2 + p2(t)*(m2(t)-m)^2;
    vi(t) = p1(t)*v1(t) + p2(t)*v2(t);
end
(vi+vb)-v;

% goodness
g = zeros(1,7); g = vb / v; [Y,I] = max(g);
```

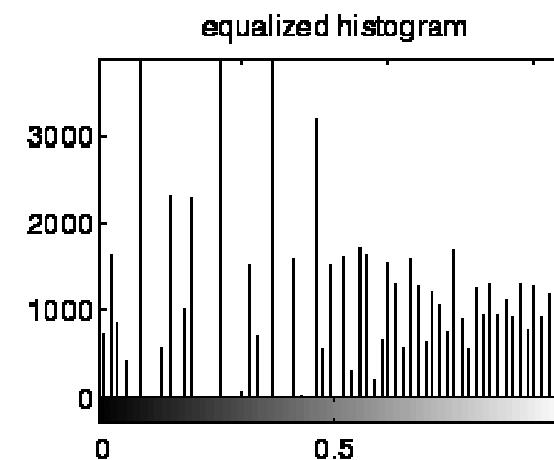
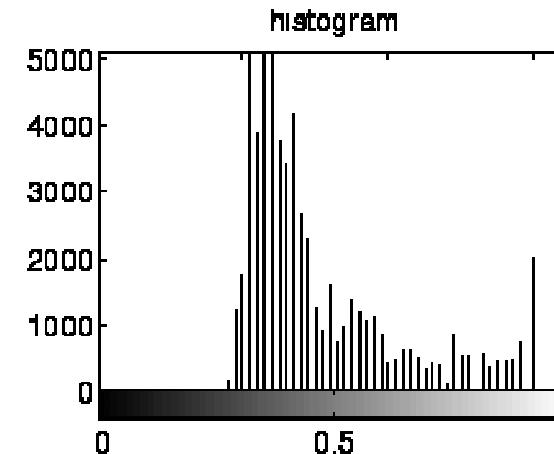
# Histogram-based operators

1. Fundamentals of histogram-based operators
2. Some histogram-based operators
  - 2.1. Image characterization
  - 2.2. Optimum thresholding
  - 2.3. Histogram equalization
  - 2.4. Histogram-based distances
3. Further investigations

Type	Methods	Application
Image characterization	Mean, standard deviation	Feature extraction
	Contrast	
	Moments	
	Entropy	
	Co-occurrence matrix	
	Uniformity, homogeneity	
	Correlation	
Thresholding	Otsu's method	Enhancement, segmentation
Histogram equalization	Histogram equalization	Enhancement
Histogram-based distance	Minkowski, $\chi^2$ , Kulback-Leibler and Jeffrey	Comparison, retrieval, spotting

# Histogram equalization (1)

Histogram equalization automatically determines a transformation function that seeks to produce an output image that has an uniform histogram. It aims to produce a picture with a flatter histogram to highlight image brightness.



# Histogram equalization (2)

Histogram equalization automatically determines a transformation function that seeks to produce an output image that has an uniform histogram. It aims to produce a picture with a flatter histogram to highlight image brightness.

	Equation	Equation	Comments
<b>The input histogram <math>r</math></b>	$\sum_{k=0}^{2^q-1} r(k)$	<b>Size equality</b> $\sum_{k=0}^{2^q-1} r(k) = \sum_{k=0}^{2^q-1} s(k) = NM$	Since the size is preserved in the input and output images, we have
<b>The output (equalized) histogram <math>s</math></b>	$\sum_{k=0}^{2^q-1} s(k)$	<b>Mapping rule for cumulative histograms <math>r(k \in [0,u]), s(k \in [0,v])</math></b> $\sum_{k=0}^u r(k) = \sum_{k=0}^v s(k)$	The cumulative input histogram $r$ up to level $u$ should be transformed to cover up to level $v$ in the output cumulative histogram $s$
	<b>Uniform output discrete function <math>s(k)</math></b>	$s(k) = \frac{NM}{2^q - 1} \quad \forall k$	Since the output histogram $s$ is uniformly flat, we have
	<b>Uniform cumulative histograms <math>r(k \in [0,u]), s(k \in [0,v])</math></b>	$\sum_{k=0}^v s(k) = v \times \frac{NM}{2^q - 1} = \sum_{k=0}^u r(k)$	The cumulative histograms $r,s$ of output and input images are then equal to
	<b>Mapping function</b>	$v = \frac{(2^q - 1)}{NM} \sum_{k=0}^u r(k) = (2^q - 1) \sum_{k=0}^u p_r(k)$ $\text{with } p_r(k) = \frac{r(k)}{NM}$ $v = T(u) = (2^q - 1) \sum_{k=0}^u p_r(k)$	The mapping function, for a pixel at level $v$ , from the input pixel at level $u$ , is then

# Histogram equalization (3)

Histogram equalization automatically determines a transformation function that seeks to produce an output image that has an uniform histogram. It aims to produce a picture with a flatter histogram to highlight image brightness.

	<b>Equation</b>
<b>The input histogram <math>r</math></b>	$\sum_{k=0}^{2^q-1} r(k)$
<b>Mapping function</b>	$p_r(k) = \frac{r(k)}{NM}$ $T(u) = (2^q - 1) \sum_{k=0}^u p_r(k)$
<b>The output (equalized) histogram <math>s</math></b>	$\sum_{k=0}^{2^q-1} s(k)$

# Histogram equalization (4)

Histogram equalization automatically determines a transformation function that seeks to produce an output image that has an uniform histogram. It aims to produce a picture with a flatter histogram to highlight image brightness.

	<b>Equation</b>
<b>The input histogram <math>r</math></b>	$\sum_{k=0}^{2^q-1} r(k)$
<b>Mapping function</b>	$p_r(k) = \frac{r(k)}{NM}$ $T(u) = (2^q - 1) \sum_{k=0}^u p_r(k)$
<b>The output (equalized) histogram <math>s</math></b>	$\sum_{k=0}^{2^q-1} s(k)$

e.g. to achieve the histogram equalization on the following  $r$  histogram  
(1) we compute  $p_r$  first

<b><math>k</math></b>	<b><math>r(k)</math></b>	<b><math>p_r(k)</math></b>
0	790	0,19
1	1023	0,25
2	850	0,21
3	656	0,16
4	329	0,08
5	245	0,06
6	122	0,03
7	81	0,02

$$\sum_{k=0}^{2^q-1} r(k) = 4096 = 64^2$$

# Histogram equalization (5)

Histogram equalization automatically determines a transformation function that seeks to produce an output image that has an uniform histogram. It aims to produce a picture with a flatter histogram to highlight image brightness.

	<b>Equation</b>
<b>The input histogram <math>r</math></b>	$\sum_{k=0}^{2^q-1} r(k)$
<b>Mapping function</b>	$p_r(k) = \frac{r(k)}{NM}$ $T(u) = (2^q - 1) \sum_{k=0}^u p_r(k)$
<b>The output (equalized) histogram <math>s</math></b>	$\sum_{k=0}^{2^q-1} s(k)$

e.g. to achieve the histogram equalization on the following  $r$  histogram  
(2) when  $p_r$  known, we compute  $T(u)$

<b><math>k</math></b>	<b><math>p_r(k)</math></b>
0	0,19
1	0,25
2	0,21
3	0,16
4	0,08
5	0,06
6	0,03
7	0,02

<b><math>u</math></b>	<b><math>T(u)</math></b>
0	1,33
1	3,08
2	4,55
3	5,67
4	6,23
5	6,65
6	6,86
7	7

$$= T(2) = 7 \sum_{k=0}^2 p_r(k) = 7 \times (0,19 + 0,25 + 0,21)$$

$$= T(5) = 7 \sum_{k=0}^5 p_r(k) \\ = 7 \times (0,19 + 0,25 + 0,21 + 0,16 + 0,08 + 0,06)$$

# Histogram equalization (6)

Histogram equalization automatically determines a transformation function that seeks to produce an output image that has an uniform histogram. It aims to produce a picture with a flatter histogram to highlight image brightness.

	<b>Equation</b>
<b>The input histogram <math>r</math></b>	$\sum_{k=0}^{2^q-1} r(k)$
<b>Mapping function</b>	$p_r(k) = \frac{r(k)}{NM}$ $T(u) = (2^q - 1) \sum_{k=0}^u p_r(k)$
<b>The output (equalized) histogram <math>s</math></b>	$\sum_{k=0}^{2^q-1} s(k)$

e.g. to achieve the histogram equalization on the following  $r$  histogram  
(3)  $T(u)$  have fractions because they are generated by summing probabilities,  
so  $T'(u)$  round them to the nearest integers

$k$	$p_r(k)$	$u$	$T(u)$	$T'(u)$
0	0,19	0	1,33	1
1	0,25	1	3,08	3
2	0,21	2	4,55	5
3	0,16	3	5,67	6
4	0,08	4	6,23	6
5	0,06	5	6,65	7
6	0,03	6	6,86	7
7	0,02	7	7	7

# Histogram equalization (7)

Histogram equalization automatically determines a transformation function that seeks to produce an output image that has an uniform histogram. It aims to produce a picture with a flatter histogram to highlight image brightness.

	Equation
The input histogram $r$	$\sum_{k=0}^{2^q-1} r(k)$
Mapping function	$p_r(k) = \frac{r(k)}{NM}$ $T(u) = (2^q - 1) \sum_{k=0}^u p_r(k)$
The output (equalized) histogram $s$	$\sum_{k=0}^{2^q-1} s(k)$

e.g. to achieve the histogram equalization on the following  $r$  histogram (4) when  $T'(u)$  known, we can obtain  $s$

$k$	$r(k)$	$u$	$T'(u)$	$k$	$s(k)$
0	790	0	1	0	0
1	1023	1	3	1	790
2	850	2	5	2	0
3	656	3	6	3	1023
4	329	4	6	4	0
5	245	5	7	5	850
6	122	6	7	6	985
7	81	7	7	7	448

$= r(0)$   
 $= r(1)$   
 $= r(2)$   
 $= r(3) + r(4) = 656 + 329$   
 $= r(5) + r(6) + r(7)$   
 $= 245 + 122 + 81$

# Histogram equalization (8)

Histogram equalization automatically determines a transformation function that seeks to produce an output image that has an uniform histogram. It aims to produce a picture with a flatter histogram to highlight image brightness.

	<b>Equation</b>
<b>The input histogram <math>r</math></b>	$\sum_{k=0}^{2^q-1} r(k)$
<b>Mapping function</b>	$p_r(k) = \frac{r(k)}{NM}$ $T(u) = (2^q - 1) \sum_{k=0}^u p_r(k)$
<b>The output (equalized) histogram <math>s</math></b>	$\sum_{k=0}^{2^q-1} s(k)$

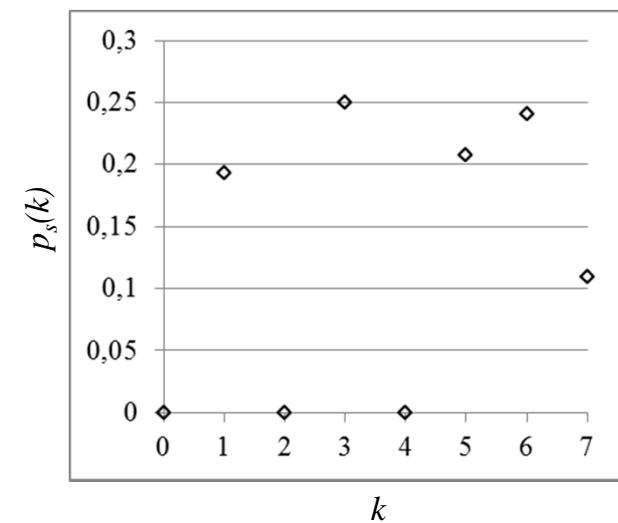
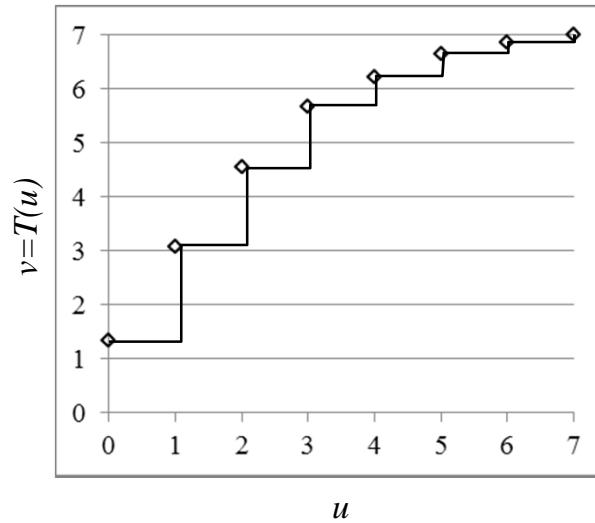
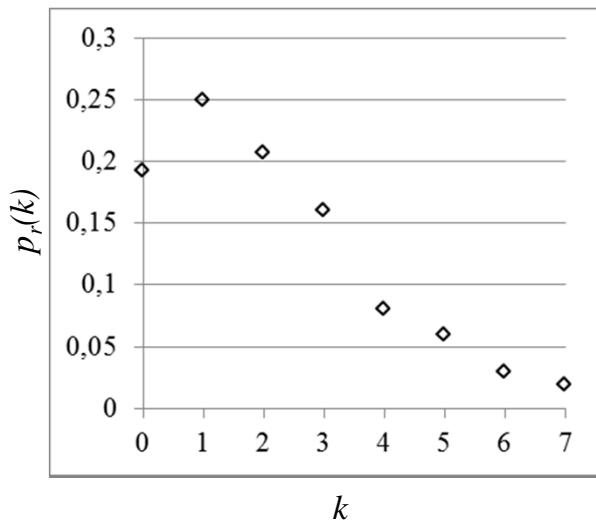
e.g. to achieve the histogram equalization on the following  $r$  histogram (5) and then  $p_s$

<b><math>k</math></b>	<b><math>r(k)</math></b>	<b><math>p_r(k)</math></b>	<b><math>u</math></b>	<b><math>T'(u)</math></b>	<b><math>k</math></b>	<b><math>s(k)</math></b>	<b><math>p_s(k)</math></b>
0	790	0,19	0	1	0	0	0
1	1023	0,25	1	3	1	790	0,19
2	850	0,21	2	5	2	0	0
3	656	0,16	3	6	3	1023	0,25
4	329	0,08	4	6	4	0	0
5	245	0,06	5	7	5	850	0,21
6	122	0,03	6	7	6	985	0,24
7	81	0,02	7	7	7	448	0,11

# Histogram equalization (9)

Histogram equalization automatically determines a transformation function that seeks to produce an output image that has an uniform histogram. It aims to produce a picture with a flatter histogram to highlight image brightness.

e.g. the final plot



# Histogram-based operators

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Histogram-based distance	Minkowski, $\chi^2$ , Kulback-Leibler and Jeffrey	Comparison, retrieval, spotting

# Histogram-based distances (1)

Histogram-based distances: given two (sub)images A and B whose (normalized) histograms are  $h_A$  and  $h_B$ , we define an histogram-based distance as  $D(h_A, h_B)$  between the histograms  $h_A$  and  $h_B$ .

Example Usage:

- Tracking
- Image retrieval
- Registration
- Detection
- Many more ...



*input*



*target*



*similarity*

# Histogram-based distances (2)

Histogram-based distances: given two (sub)images A and B whose (normalized) histograms are  $h_A$  and  $h_B$ , we define an histogram-based distance as  $D(h_A, h_B)$  between the histograms  $h_A$  and  $h_B$ .

e.g. within bin-by-bin distances, only pairs of bins in the two histograms that have the same index are matched.

**Minkowski distance:** is a metric on Euclidean space which can be considered as a generalization of both the Euclidean distance and the Manhattan distance.

$$D_p(A, B) = \left[ \sum_{k=0}^{2^q-1} |h_A(k) - h_B(k)|^p \right]^{1/p}$$

p	D(A,B)
$-\infty$	min distance
1	Manhattan distance
2	Euclidean distance
$+\infty$	max distance

**$\chi^2$  statistics:** measures how unlikely it is that one distribution was drawn from the population represented by the other.

$$D_{\chi^2}(A, B) = \sum_{k=0}^{2^q-1} \frac{(h_A(k) - m(k))^2}{m(k)}$$

$$m(k) = \frac{h_A(k) + h_B(k)}{2}$$

# Histogram-based distances (3)

Histogram-based distances: given two (sub)images A and B whose (normalized) histograms are  $h_A$  and  $h_B$ , we define an histogram-based distance as  $D(h_A, h_B)$  between the histograms  $h_A$  and  $h_B$ .

e.g. within bin-by-bin distances, only pairs of bins in the two histograms that have the same index are matched.

**Kullback-Leibler (KL) divergence** measures the amount of added information needed to encode image A based on the histogram of image B. The KL divergence is not symmetric  $D_{KL}(A,B) \neq D_{KL}(B,A)$ , and then not a distance

$$D_{KL}(A, B) = - \sum_{k=0}^{2^q-1} h_A(k) \log \frac{h_A(k)}{h_B(k)}$$

**Jeffrey distance:** is a modification of the K-L divergence that is numerically stable, symmetric (then a distance) and robust with respect to noise and the size of histogram bins

$$D_J(A, B) = \sum_{k=0}^{2^q-1} h_A(k) \log \frac{h_A(k)}{m(k)} + h_B(k) \log \frac{h_B(k)}{m(k)}$$

$$m(k) = \frac{h_A(k) + h_B(k)}{2}$$

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# Further investigations

- Other characterization methods / features (e.g. the Intensity Variation Number)
- Multiple thresholding (extension of the Otsu method)
- Histogram specification
- Comparison of histograms (cross-bin measures)
- Etc.