

## SOME USEFUL INFORMATIONS

### Arithmetic Sequence Size

$$1, 2, 3, 4, 5, 6, \dots, N \leq O(N)$$

$$1, 3, 5, 7, \dots, N \leq N/2 \text{ i.e. } O(N)$$

$$1, 4, 7, 10, \dots, N \leq N/3 \text{ i.e. } O(N)$$

$$1, 1+k, 1+2k, 1+3k, 1+4k, 1+5k, \dots, N \leq N/k \text{ i.e. } O(N) \text{ if } k \text{ is a constant}$$

Arithmetic Series Applications of  $1+2+3+4+\dots+N = \frac{N(N+1)}{2}$

$$1+2+3+4+5+6+\dots+N-3+N-2+N-1+N \leq O(N^2)$$

$$1+2+3+4+5+6+\dots+(N/2-3)+(N/2-2)+(N/2-1)+N/2 \leq O(N^2)$$

$$1+2+3+4+5+6+\dots+(N/3-3)+(N/3-2)+(N/3-1)+N/3 \leq O(N^2)$$

$$1+2+3+4+5+6+\dots+\sqrt{N} \leq O((\sqrt{N})^2) \leq O(N)$$

$$1+2+3+4+5+6+\dots+N^2 \leq O(N^4)$$

$$1+2+3+4+5+6+\dots+N^3 \leq O(N^6)$$

$$1+2+3+4+5+6+\dots+N^k \leq O(N^{k+1})$$

$$1+2^2+3^2+4^2+5^2+6^2+\dots+N^2 \leq O(N^3)$$

$$1+2^3+3^3+4^3+5^3+6^3+\dots+N^3 \leq O(N^4)$$

$$1^k+2^k+3^k+4^k+5^k+6^k+\dots+N^k \leq O(N^{k+1})$$

### Geometric Sequence Size

$$N, N/2, N/4, N/8, N/2^4, N/2^5, \dots, 8, 4, 2, 1 \leq \log_2 N$$

$$N, N/3, N/9, N/27, N/3^4, N/3^5, \dots, 3^3, 9, 3, 1 \leq \log_3 N$$

$$N, N/5, N/25, N/125, N/5^4, N/5^5, \dots, 5^3, 5^2, 5, 1 \leq \log_5 N$$

$$N, N/k, N/k^2, N/k^3, N/k^4, N/k^5, N/k^6, \dots, k^3, k^2, k, 1 \leq \log_k N$$

$O(\log N)$

### Application

$$\sqrt{N} * \sqrt{N} = N$$

**for(int i=1; i\*i<=N; i++)** Complexity of this loop is  $O(\sqrt{N})$   
**Sum++;**

$$N \times N = N^2$$

**for(int i=1; i\*i<=N\*N; i++)** Complexity of this loop is  $O(N)$   
**Sum++;**

### GEOMETRIC SERIES

$$N < 1+2+4+8+16+32+\dots+N/4+N/2+N < 2N$$

$$N < 1+3+9+3^3+3^4+3^5+\dots+N/3^2+N/3+N < 2N$$

$$N < 1+5+2^5+5^3+5^4+5^5+\dots+N/5^2+N/5+N < 2N$$

**For any constant - ratio(multiplication factor greater than 2 the above inequality is valid).**

$O(N)$