UNIVERSIDAD PANAMERICANA

Mecatrónica y Control de Sistemas

Image Restoration

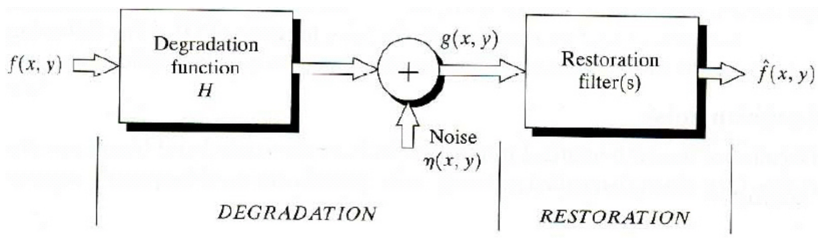
Introduction

Enhancement vs Restoration

Enhancement: Image manipulation to obtain the most of the psychophysics aspects of human vision (subjective)

Restoration: Recovery or reconstruction of an image that has been degraded by some phenomenon (objective).

General Model: Degradation/Restoration



Spatial Domain

$$g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$$

Frequency Domain

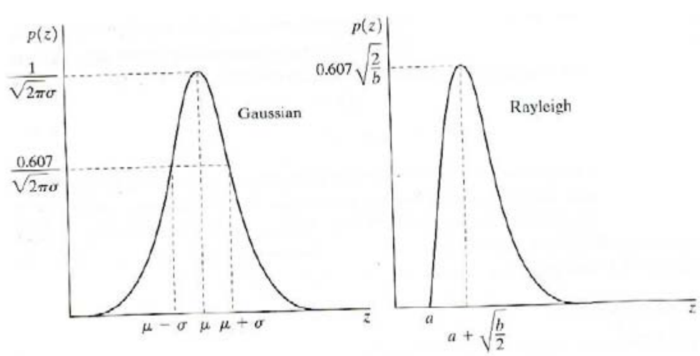
$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

Most common types of noise:
- Acquisition (digitalization)
- Transmission

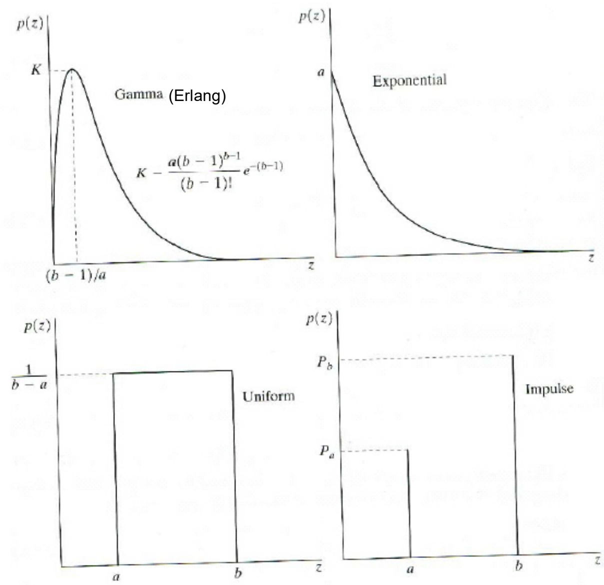
Noise Models

Noise in images:
Random variables characterized by a probability density function (PDF) which describes the statistical behavior of the gray-level values in the noise component.

Some PDF:



Noise Models



Periodic Noise

It comes from electric or electromechanic interference during acquisition. It is a sinusoidal noise of several frequencies:

$$r(x, y) = A \sin[2\pi u_0(x + B_x)/M + 2\pi v_0(y + B_y)/N]$$

A: Amplitude
u_0: Sinusoidal frequency in x
v_0: Sinusoidal frequency in y
B_x, B_y: Phase displacement

Matlab:
[n,R,S]=imnoise3(M,N,C)

n: The noise's spatial pattern MxN
R: Fourier transform
S: Spectrum
C: k pair of frequency coordinates *(u,v)* that locate the impulses in S

Noise Estimation

How to identify the type of noise in an image?

Periodic noise: With the Fourier Spectrum. Points can be detected visually.
Spatial noise: Identification of the PDF.

Restoration with noise

Image with noise:

$$g(x, y) = f(x, y) + \eta(x, y)$$

$$G(u, v) = F(u, v) + N(u, v)$$

So?

Image without noise

$$f(x, y) = g(x, y) - \eta(x, y)$$

How to know the type of spatial noise?

Spatial filters

Mean filters:

- -Arithmetic
- -Geometric

Gaussian, uniform

- -Harmonic
- -Contraharmonic

Salt & Pepper (not both)

Order-statistics filters:

- -Median — Salt & Pepper (both)
- -Max — Pepper
- -Min — Salt
- -Midpoint — Gaussian, uniform
- -Alpha-trimmed mean — Gaussian, uniform + Salt & Pepper

Adaptive Filters

Adaptive filters change their behavior based on the statistical characteristics of the image inside a rectangular region S_{xy} of dimensions m x n. Their adaptive characteristics allow them to eliminate practically any type of spatial noise.

Two adapting parameters:

Mean: The average gray-level in a region

Variance: The average contrast in a region

The adaptive filter is based on 4 parameters:

(a) g(x,y) the noisy image

(b) σ^2_n: the noise variance that corrupts *f(x,y)* and transforms it into *g(x,y)*

(c) m_L: the local mean of the pixels inside S_{xy}

(d) σ^2_L: the local variance of the pixels inside S_{xy}

Adaptive Filters

Algorithm:

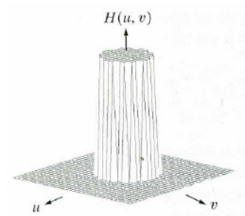
1) If σ^2_n=0, g(x,y)=f(x,y) (there is no noise)
2) If $\sigma^2_L >> \sigma^2_n$, the filter returns a value close to g(x,y). A high local variance σ^2_L means borders, which must be preserved.
3) If $\sigma^2_L = \sigma^2_n$, the filter returns the arithmetic average of the pixels in $S_{xy.}$ This condition occurs when the local area has the same properties of the total image and local noise can be reduced by simple averaging.

$$\hat{f}(x, y) = g(x, y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left[g(x, y) - m_L\right].$$

Important: Noise reduction performance might the same as the one of other filters but generally, images tend to have more contrast.

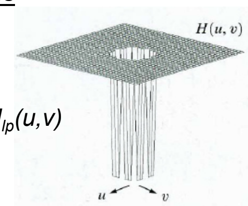
Periodic noise reduction with frequency filters

Lowpass

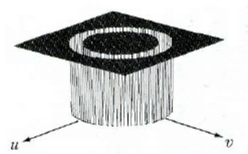


Highpass

$H_{hp}(u,v)=1-H_{lp}(u,v)$



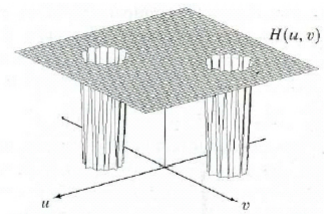
Band-reject



Band-pass

$H_{bp}(u,v)=1-H_{br}(u,v)$

Notch



Degradation Phenomenon

Most common degradation phenomenon:
Camera motion

Inverse Filtering - Weiner filter

Minimum square method

$$e^2 = E\{(f - \hat{f})^2\}$$

Expected value

Frequency domain:

$$\hat{F}(u,v) = \overbrace{\left[\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2 + \underbrace{S_\eta(u,v)/S_f(u,v)}_{\text{NSR: noise to signal ratio}}}\right]}^{\text{Minimum square filter method}} G(u,v)$$

where

$H(u,v)$ = the degradation function
$|H(u,v)|^2 = H^*(u,v)H(u,v)$
$H^*(u,v)$ = the complex conjugate of $H(u,v)$
$S_\eta(u,v) = |N(u,v)|^2$ = the power spectrum of the noise
$S_f(u,v) = |F(u,v)|^2$ = the power spectrum of the undegraded image

Implementation in Matlab

DECONVWNR: Image restoration using Wiener filter.

Case 1: $N(u,v)=0 \longrightarrow NSR=0 \longrightarrow \hat{F}(u,v) = \dfrac{G(u,v)}{H(u,v)}$

Inverse filter

fe=deconvwnr(g,H)

fe: estimated image
g: degraded image
H: Degradation function

Case: $N(u,v)\neq 0 \longrightarrow NSR\neq 0$

Autocorrelation Wiener Filter

fe=deconvwnr(g,H, NCORR,FCORR)

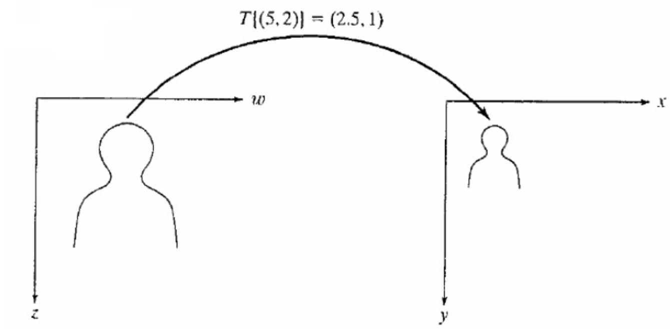
$NCORR=\Im\left[Sn(u,v)\right]$

Needs previous knowledge of the original image

Geometric Transformations

$(x, y) = T\{(w, z)\}$ An image *f* defined in *(w,z)* distorts geometrically to produce image *g* defined in *(x,y)*

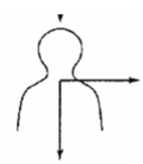
Ex: $(x, y) = T\{(w, v)\} = (w/2, z/2)$



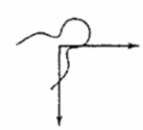
Affine transform matrix **T**: $[x \quad y \quad 1] = [w \quad z \quad 1]\,\mathbf{T} = [w \quad z \quad 1]\begin{bmatrix} t_{11} & t_{12} & 0 \\ t_{21} & t_{22} & 0 \\ t_{31} & t_{32} & 1 \end{bmatrix}$

Geometric Transformations

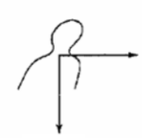
Scaling $\begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $x = s_x w$, $y = s_y z$



Rotation $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $x = w\cos\theta - z\sin\theta$, $y = w\sin\theta + z\cos\theta$



Shear (horizontal) $\begin{bmatrix} 1 & 0 & 0 \\ \alpha & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $x = w + \alpha z$, $y = z$



Shear (vertical) $\begin{bmatrix} 1 & \beta & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $x = w$, $y = \beta w + z$