UNIVERSIDAD PANAMERICANA

Mecatrónica y Control de Sistemas

Image Processing in the Frequency Domain

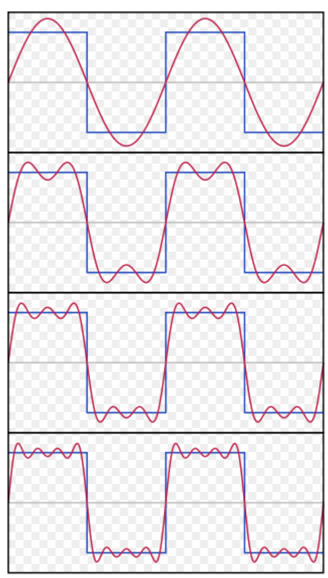
Introduction

Fourier Series:

Any periodic function can be approximated by a series of sines and cosines of different frequencies and amplitudes

$$f(t) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos \frac{2n\pi}{T} t + b_n \sin \frac{2n\pi}{T} t \right]$$

For those functions that are not periodic we can turn them into periodic with the **Fourier Transform**



Fourier Transform

Continuous:

1D

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux}\, dx$$

2D

$$F(u,v) = \iint_{-\infty}^{\infty} f(x,y) e^{-j2\pi(ux+vy)} dxdy$$

Discrete:

1D

$$F(u) = \frac{1}{M} \sum_{x=0}^{M-1} f(x) e^{-j2\pi ux/M}$$

u=0,1,2,…M-1

2D

$$F(u,v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi(\frac{ux}{M}+\frac{vy}{N})}$$

u=0,1,2,…M-1

v=0,1,2,…N-1

Frequency concept: $e^{j\theta} = cos\theta + jsen\theta$

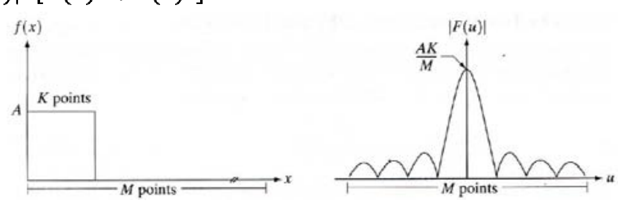
$$F(u) = \frac{1}{M} \sum_{x=0}^{M-1} f(x) \left[cos\left(\frac{2\pi ux}{M}\right) - jsen\left(\frac{2\pi ux}{M}\right) \right]$$

u=0,1,2,…M-1

Fourier Transform

Spectrum

In Signals: $|F(u)| = [\mathbb{R}(u)^2 + \mathbb{C}(u)^2]^{\frac{1}{2}}$



In Images: $|F(u,v)| = [\mathbb{R}(u,v)^2 + \mathbb{C}(u,v)^2]^{\frac{1}{2}}$

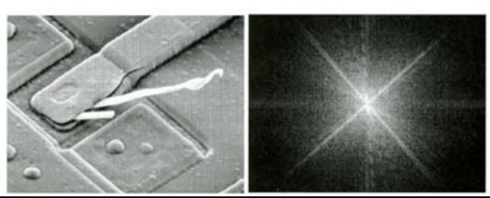
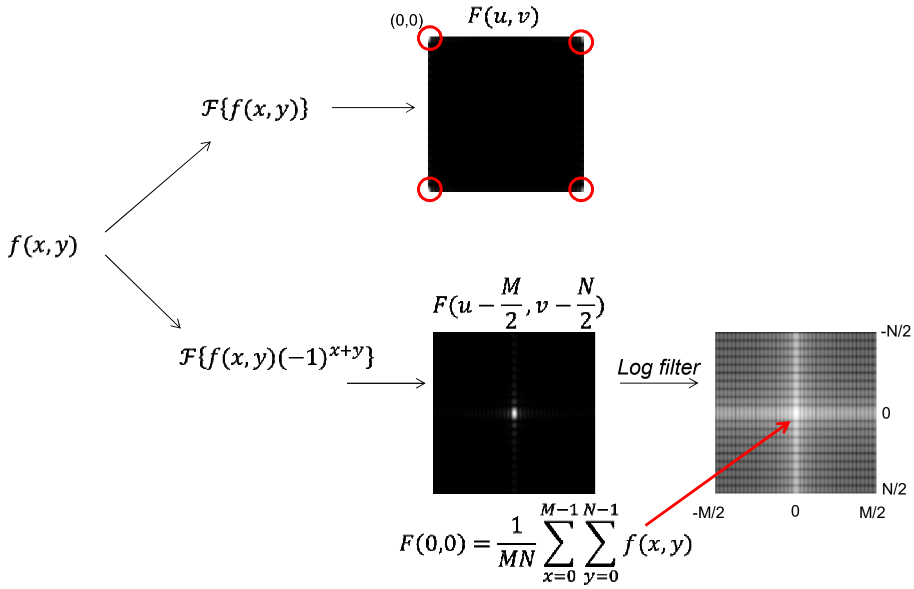
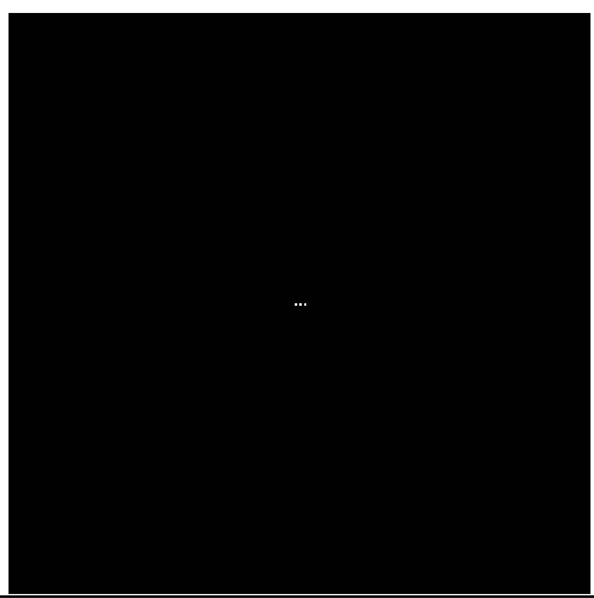


Image Spectrum



How to interpret the Spectrum of an image



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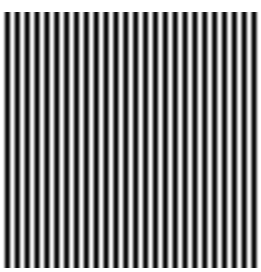

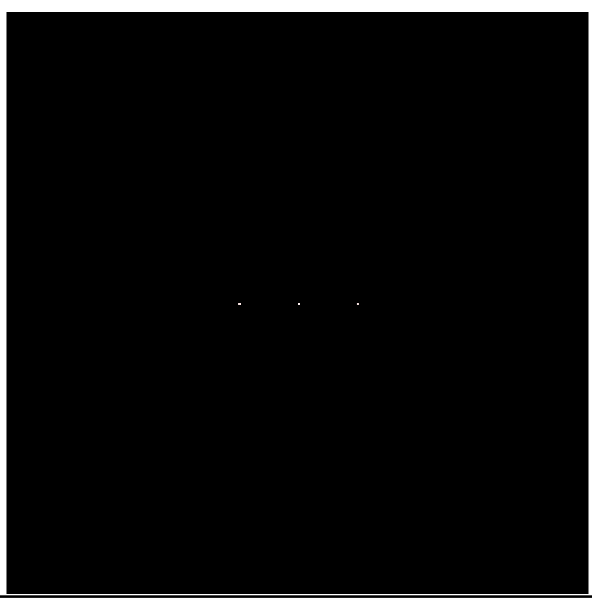

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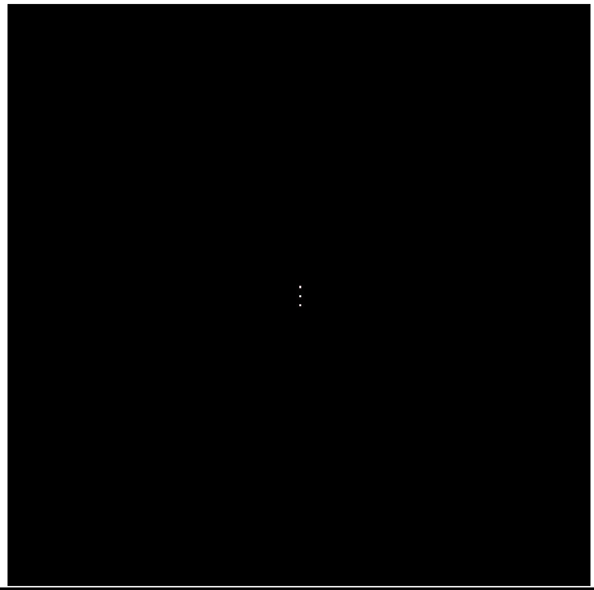

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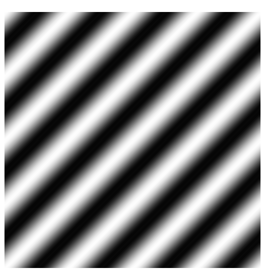

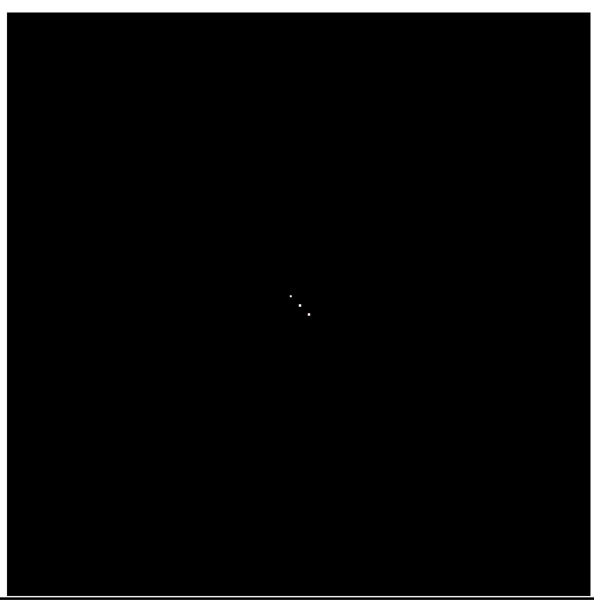

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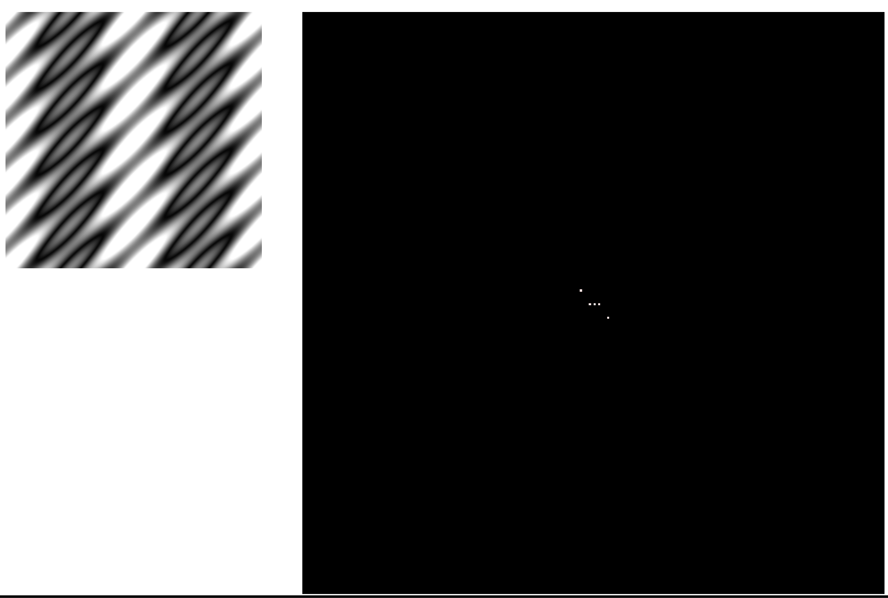

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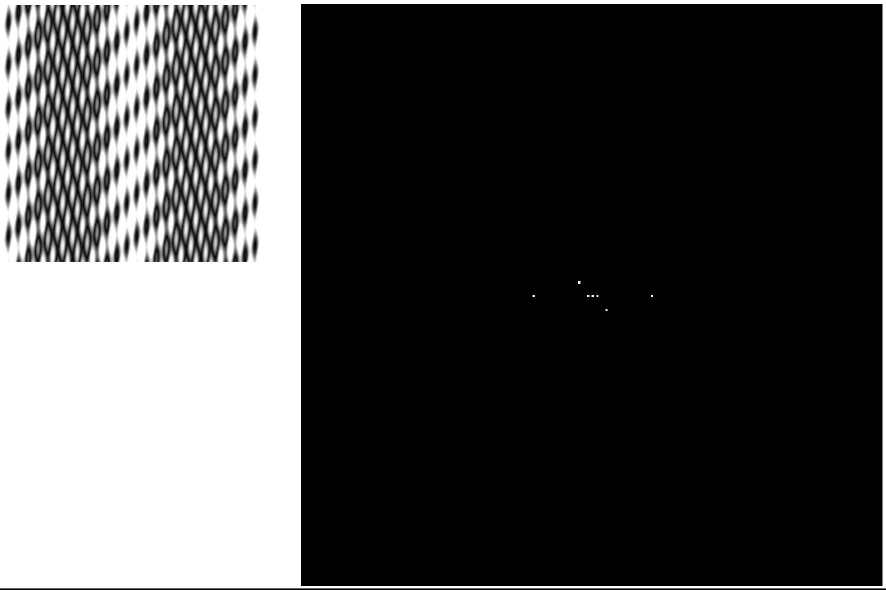

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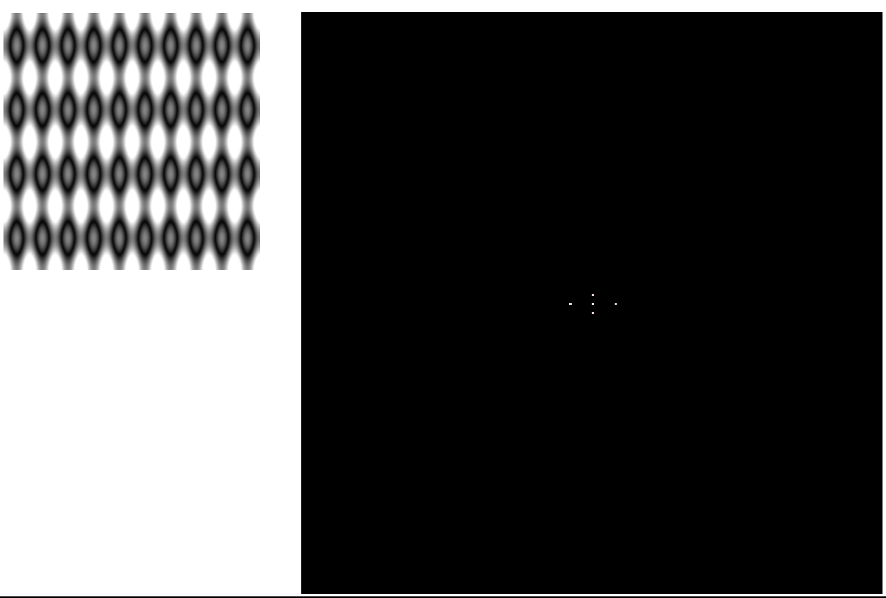

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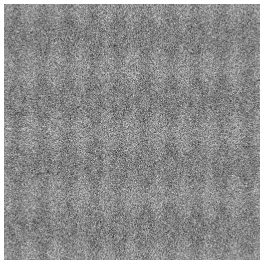

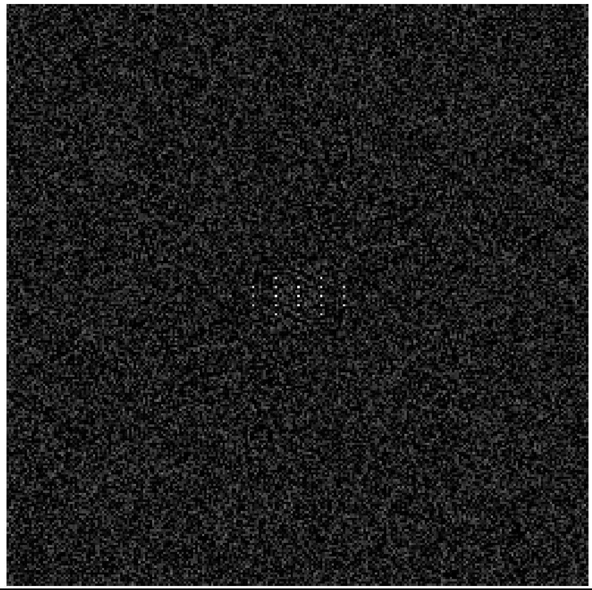

Filtering in Frequency Domain