

Image Processing in the Spatial Domain

Introduction

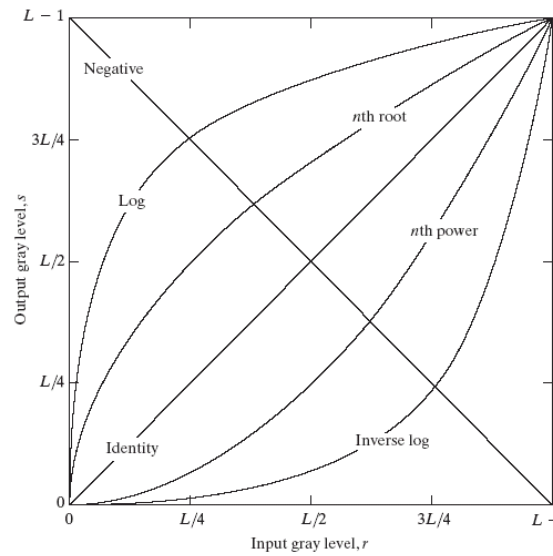
Objective: To process an image so that the result is more suitable than the original image for a specific application.

Types of processing:

Spatial Domain: A direct processing on the image; it is based on direct manipulation of pixels

Frequency Domain: Techniques based on modification of the Fourier Spectrum.

Intensity Transformation

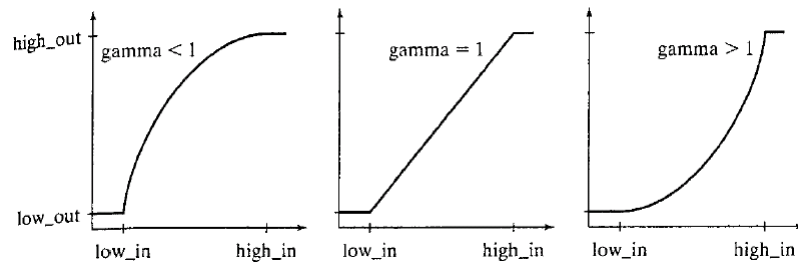


Negative - Matlab

$$s = 1 - r$$

$J = \text{imadjust}(f, [\text{low_in}; \text{high_in}], [\text{low_out}; \text{high_out}], \text{gamma})$

Normalized
[0 ; 1]



Log and Power Transformations

Logs

$$s = c \log(1 + r)$$

Expands the dark values in the image while compressing the higher-level values.
Opposite case for the inverse log transformation:

$$s = ce^{(r-1)}$$

Powers

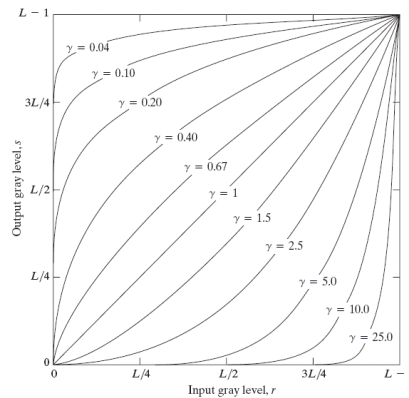
$$s = cr^\gamma \leftarrow \text{Gamma}$$

$\gamma > 1$

Compresses the dark values
Expands the highest values

$\gamma < 1$

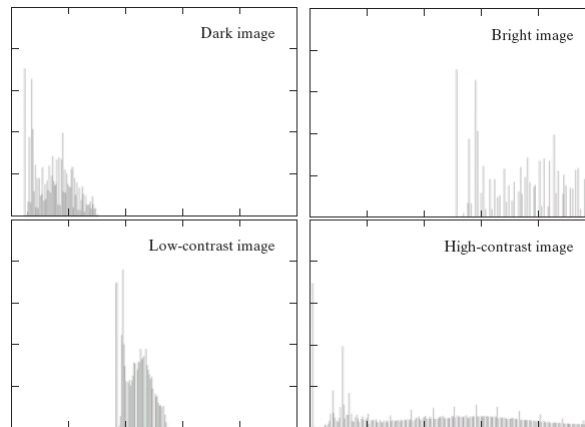
Expands the dark values
Compresses the highest values



Histogram

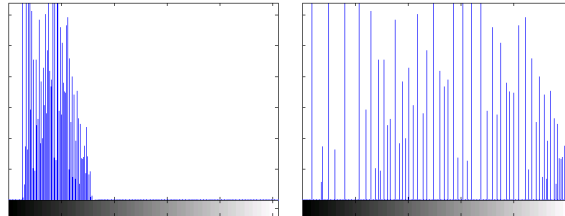
The histogram of a digital image with gray levels in the range **[0, L-1]** is a discrete function $h(r_k) = n_k$ where r_k is the k th gray level and n_k is the number of pixels in the image having the gray level r_k .

Besides of being a statistics tool for image analysis, the histogram is also helpful for performing intensity transformations.

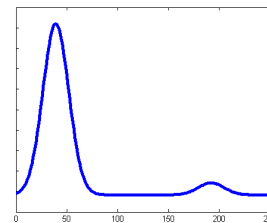


Histogram Equalization

An histogram equalization creates an image with similar gray levels along the range [0 x] → High contrast images



Gaussian Equalization: tailored histogram



Pixel by pixel processing – Arithmetic Operations

Sum of 2 images:

- Overlaps objects (attention with saturation: $A+B > 255$)
- Brightens an image (not really used)

Subtraction of 2 images:

- Finds differences between images (attention with saturation: $A-B < 0$)
- Darkens an image (not really used)

Multiplication of 2 images :

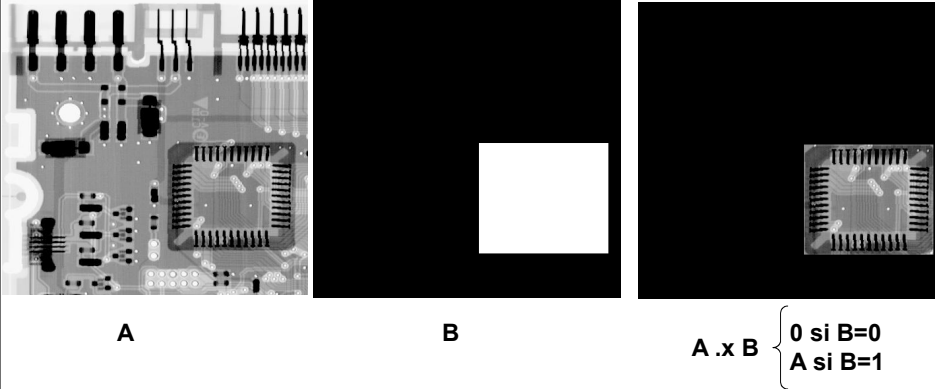
- Isolates an area for further processing

Division of 2 images:

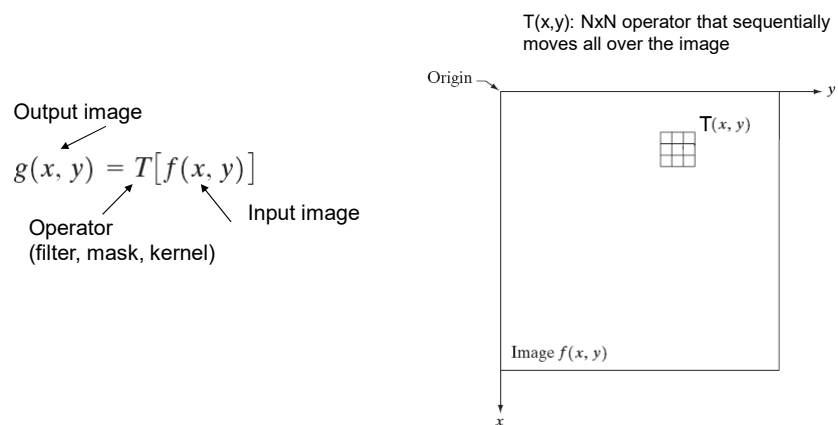
- Not used at all

Pixel by pixel processing – Arithmetic Operations

Hadamard product: Element-wise multiplication



Filters



Filters

Original image
 $f(x,y)$

Mask

$w(-1,-1)$	$w(-1,0)$	$w(-1,1)$
$w(0,-1)$	$w(0,0)$	$w(0,1)$
$w(1,-1)$	$w(1,0)$	$w(1,1)$

Mask coefficients, showing coordinate arrangement

Pixels of image section under mask

Element-wise multiplication:

$$R = w(-1,-1)f(x-1,y-1) + w(-1,0)f(x-1,y) + \dots + w(0,0)f(x,y) + \dots + w(1,0)f(x+1,y) + w(1,1)f(x+1,y+1)$$

In general:

$$g(x,y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s,t)f(x+s,y+t)$$

Filters: Correlation and Convolution

Correlation: The aforementioned procedure (**$w \times f$**)

Convolution: Same, just... **w** is rotated 180° before being applied to **f**

$G = \text{imfilter}(f, w, \text{filtering mode}, \text{boundary options}, \text{size options})$

Options	Description
Filtering Mode	
'corr'	Filtering is done using correlation. This is the default.
'conv'	Filtering is done using convolution.
Boundary Options	
P	The boundaries of the input image are extended by padding with a value, P (written without quotes). This is the default, with value 0.
'replicate'	The size of the image is extended by replicating the values in its outer border.
'symmetric'	The size of the image is extended by mirror-reflecting it across its border.
'circular'	The size of the image is extended by treating the image as one period a 2-D periodic function.
Size Options	
'full'	The output is of the same size as the extended (padded) image
'same'	The output is of the same size as the input. This is achieved by limiting the excursions of the center of the filter mask to points contained in the original image. This is the default.

Linear filters

Averaging filters

The average of the pixels contained in the mask → smooths transitions between gray levels. Reduces contrast.

Ex 1

$$\frac{1}{9} \times$$

1	1	1
1	1	1
1	1	1

Ex 2

$$\frac{1}{16} \times$$

1	2	1
2	4	2
1	2	1

Linear filters (Cont...)

Relief filter

Produces reliefs in an image with protuberances and holes. The light areas are raised while the dark areas are sculpted.

-2	-1	0
-1	1	1
0	1	2

Line drawing filter

Produces a drawing effect to an image

-1	-1	-1	-1	-1
-1	-2	-2	-2	-1
-1	-2	34	-2	-1
-1	-2	-2	-2	-1
-1	-1	-1	-1	-1

Non Linear or Statistic Filters

Laplacian

Continuous:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \quad \left\{ \begin{array}{l} \text{In x: } \frac{\partial^2 f}{\partial^2 x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y) \\ \text{In y: } \frac{\partial^2 f}{\partial^2 y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y) \end{array} \right.$$

Then:

$$\nabla^2 f = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)] - 4f(x, y)$$

Laplacian Filter (Cont...)

Implementation:

0	1	0	1	1	1
1	-4	1	1	-8	1
0	1	0	1	1	1
0	-1	0	-1	-1	-1
-1	4	-1	-1	8	-1
0	-1	0	-1	-1	-1

$$g(x, y) = \begin{cases} f(x, y) - \nabla^2 f(x, y) & \text{When the central coefficient of the Laplacian mask is negative.} \\ f(x, y) + \nabla^2 f(x, y) & \text{When the central coefficient of the Laplacian mask is positive.} \end{cases}$$