Blind and Topological Interference Managements for Bistatic Integrated Sensing and Communication

Jiayu Liu, Kai Wan, *Member, IEEE,* Xinping Yi, *Member, IEEE,* Robert Caiming Qiu, *Fellow, IEEE*, and Giuseppe Caire, *Fellow, IEEE*

***Abstract*—Integrated sensing and communication (ISAC) systems provide significant enhancements in performance and resource efficiency compared to individual sensing and communication systems, primarily attributed to the collaborative use of wireless resources, radio waveforms, and hardware platforms. This paper focuses on the bistatic ISAC systems with dispersed multi-receivers and one sensor. Compared to a monostatic ISAC system, the main challenge in the bistatic setting is that the information messages are unknown to the sensor and therefore they are seen as interference, while the channel between the transmitters and the sensor is unknown to the transmitters. In order to mitigate the interference at the sensor while maximizing the communication degree of freedom, we introduce two strategies, namely, blind interference alignment and topological interference management. Although well-known in the context of Gaussian interference channels, these strategies are novel in the context of bistttic ISAC. For the bistatic ISAC models with heterogeneous coherence times or with heterogeneous connectivity, the achieved ISAC tradeoff points in terms of communication and sensing degrees of freedom are characterized. In particular, we show that the new tradeoff outperforms the time-sharing between the sensing-only and the communication-only schemes. Simulation results demonstrate that the proposed schemes significantly improve the channel estimation error for the sensing task compared to treating interference as noise at the sensor.**

***Index Terms*—Integrated Sensing and Communication, Degree of Freedom, Blind Interference Alignment, Topological Interference Management.**

I. Introduction

Driven by the continuous evolution of wireless communication technologies, future wireless networks are required to support both high-speed communication over wide-area coverage and large-scale sensing capabilities, thereby enabling high-precision modeling and real-time monitoring of the physical environment [1]. With the fast growth of communication terminals and radar devices, the coexistence of communication and sensing systems under limited spectrum resources has become increasingly prominent [2]. To address this challenge, Integrated Sensing and Communication (ISAC) has emerged, achieving dual-system synergy through innovative mechanisms such as hardware sharing and waveform fusion.

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J. Liu, K. Wan, and R. C. Qiu are with the School of Electronic Information and Communications, Huazhong University of Science and Technology, 430074 Wuhan, China, (e-mail: {jiayuliu,kai_wan,caiming}@hust.edu.cn).

X. Yi is with the Southeast University, Nanjing 210096, China, (e-mail:xyi@seu.edu.cn).

G. Caire is with the Electrical Engineering and Computer Science Department, Technische Universität Berlin, 10587 Berlin, Germany (e-mail: caire@tu-berlin.de).

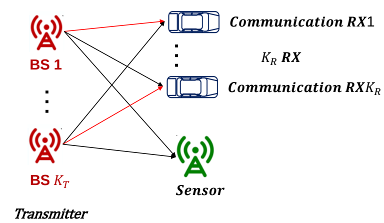


Fig. 1: The considered bistatic ISAC systems.

Despite the fundamental differences in information processing principles between sensing and communication, significant research efforts have been devoted to exploring the essence of “integration” gains in ISAC systems. These studies focus on the information-theoretic trade-offs between communication and sensing performance, aiming to establish a theoretical foundation for their co-design. The information-theoretic framework for ISAC was first introduced in [3], [4], where the authors investigated a monostatic ISAC system with state-dependent discrete memoryless channels and an independent and identically distributed (i.i.d.) state sequence. The capacity-distortion region was characterized by maximizing conditional mutual information under specific constraints. To explicitly derive the closed-form on the trade-off between communication and sensing performances, Xiong *et al.* [5] analyzed the MIMO Gaussian channel and explored the Cramér-Rao Bound (CRB)-rate region, which identified two extreme trade-off points: the communication-optimal point and the sensing-optimal point, which further revealed two critical aspects of ISAC trade-offs: subspace trade-offs and deterministic-random trade-offs. For scenarios with fixed sensing states, where the state remains constant over a transmission block, the optimal trade-off between communication rate and state detection error exponent was characterized in [6]. Monostatic ISAC over interference channels was considered in [7], [8], where two interfering ISAC transmitters communicate with their own users or a common user while sensing estimation through received echo signals, and an improved random coding than the superposition coding for interference channels was proposed. Readers can refer to the surveys in [9], [10] for more details on the information theoretic progress for ISAC.

The aforementioned researches focus on monostatic ISAC systems, where states are estimated by leveraging communication messages as side information. In contrast to monostatic sensing, bistatic sensing employs separated sensor and transmitter, which can avoid direct reflection or interference signals while covering larger areas. This approach exhibits advantages in the detection of low-altitude and ground targets, as well as in target detection utilizing multipath signals [11]. From an information-theoretic perspective[1], research on bistatic ISAC mainly concentrates on two scenarios depending on the relative positions of communication receivers and sensors.

- For bistatic ISAC models with colocated receiver and sensor, the work [12] characterizes the capacity-distortion region for i.i.d. state sequences, where the optimal strategy for the sensor is to first decode the communication message and then proceed with the estimation, referred to as unique decoding. The authors in [13] proposed an achievable region for the fixed state case by using composition codes and joint decoding-detection strategy, since in this case the above unique decoding is suboptimal. The authors studied the bistatic ISAC model for the joint multi-target localization and data communication task in [14], by designing a combination strategy of the pilot and data symbols. The bistatic ISAC model with colocated receiver and sensor over relay channels was considered in [15].
- For the bistatic ISAC model with dispersed receivers and sensor, a key challenge arises from the fact that communication messages are not necessary to be decoded by the sensor and thus act as its interference.[2] The authors in [17] developed three achievable decoding-and-estimation strategies for sensors, blind estimation (e.g., by treating interference as noise as in [18]), by partial-decoding-based estimation (e.g., by successive interference cancellation (SIC) as in [19]), and full-decoding-based estimation (i.e., fully decoding the communication message and then estimating). Physical secrecy issue was considered in [20], where the sensor cannot obtain any information about the communication message while estimating the state.

Main Contributions: This paper considers the bistatic ISAC model with multiple transmitters, dispersed multi-receivers, and one sensor. In order to characterize the fundamental tradeoff between communication and sensing performance, we consider "homogeneous" metrics, communication degree of freedom (cDoF) and sensing degree of freedom (sDoF), representing the average number of effective transmissions and observations in each time slot, respectively. Two new interference management strategies are first introduced into the bistatic ISAC model, in order to perfectly eliminate interference to the sensor.[3]

- One uses the blind interference alignment (BIA) strategy originally proposed in [23]. We assume that each transmitter is connected to all the receivers and sensor through wireless channels, and the channel between the transmitter and the sensor is slow-fading and unknown to the transmitters, while the channels between the transmitter and communication receivers are fast-fading and known to the transmitter and receivers.[4] We consider three types of wireless channels, including interference channel, multi-user MISO (MU-MISO) channel, and multi-user MIMO (MU-MIMO) channel.
- The other strategy is the topological interference management (TIM) originally proposed in [24]. We follow the topological interference network model in [24]–[26], assuming that each transmitter is connected to a subset of the sensor and receivers through slow-fading wireless channels, while the transmitters only know the prior information of the network topology, instead of the CSI. In this model, cooperative and non-cooperative transmissions among the transmitters are respectively considered, depending on whether the transmitters have all communication messages or their own messages.

By using the two interference management strategies, BIA and TIM, we propose new tradeoff points (in closed-form) between cDoF and sDoF, which are strictly better than the time-sharing between the two extreme sensing-optimal and communication-optimal points. The common ingredient of using the above two strategies into ISAC is to leverage the channel heterogeneity among the sensor and receivers to align interference at the sensor without knowing the CSI of the sensor channel, where the heterogeneity is on the coherence times for the first model, and on the connection topology for the second model. Finally, we perform simulations on practical ISAC system with QPSK modulation and additive white Gaussian noise. Considering the channel estimation error as the practical sensing metric, the proposed scheme improves the method by treating interference as noise at the sensor.

[1] The authors are fully aware of the fact that a whole host of problems related to the synchronization (time, frequency, phase) of transmitters and sensors play an important role in bistatic sensing. Nevertheless, these problems can be circumvented by appropriate system design. For example, we may imagine that base stations and sensor are part of the same network running a high precision synchronization protocol, which effectively eliminate the problem to the required degree of accuracy. In contrast, conveying the information bits sent to the users by multiple base stations without a suitable delay is a much harder task, since these messages depend on local (at the base station) scheduling decisions and the throughput of such traffic would be huge. Hence, it is reasonable to assume that the information messages are not know at the sensor, while the synchronization issues have been reduced to a degree that they can be neglected for the sake of analytical tractability.

[2] In this paper, we mainly focus on how to cancel the interference from the communication message to the sensor. Thus another existing bistatic ISAC model with dispersed receivers and sensor, where the sensor knows the communication message (such as in [16]), is out of scope.

[3] The interference alignment strategy was introduced into ISAC systems in [21], by using the original interference alignment scheme in [22] with the assumption that the channels from the communication transmitters and the (transformed) channels from the radar transmitter to the sensor are all known. In our paper, we assume that any channel to the sensor is unknown.

[4] The practical motivation is as follows: (i) Communication channels vary relatively quickly due to the mobility of communication users (with a coherence time of approximately 1-10 ms). Additionally, communication channels are periodically estimated via uplink/downlink (UL/DL) reciprocity and uplink pilots. Through these pilots, the transmitter obtains channel state information (CSI). For simplicity, we assume perfect CSI and do not consider pilot overhead or the actual channel estimation process. (ii) In contrast, sensing channels vary slowly because the sensors are stationary over few time slots. Furthermore, sensors and targets may be passive, meaning they do not transmit uplink pilots or provide channel state feedback. Therefore, even though the channel varies slowly, this CSI remains unknown to the transmitter. Note that the assumption that the CSI of communication channels is known and of sensing channels is unknown to the transmitters, has also been used in many information theoretic ISAC models such as [5], [17].

Notation Convention: We adopt boldface letters to refer to vectors and matrices. Sets are denoted using calligraphic symbols. Sans-serif symbols denote system parameters. For an arbitrary-size matrix $\mathbf{M}$, $\text{rank}(\mathbf{M})$, $\mathbf{M}^*$, $\mathbf{M}^T$, and $\mathbf{M}^H$ represent its rank, conjugate, transpose, and conjugate transpose, respectively. $\lceil \cdot \rceil$ represents the ceiling function, which denotes rounding up to the nearest integer. $\mathcal{A}^c$ is the complementary set of $\mathcal{A}$, and $|\mathcal{A}|$ is the cardinality of A. $\mathbf{A}_{ij}$ or $[\mathbf{A}]_{ij}$ presents the (i,j)-th entry of the matrix $\mathbf{A}$. If a is not divisible by b, $< a >_b$ denotes the least non-negative residue of a modulo b; otherwise, $< a >_b := b$.

II. System Model

A. Channel Models

Consider a bistatic ISAC system, where K_T transmitters are equipped with m_i antennas where $i \in \mathcal{T} = \{1, \ldots, \mathsf{K}_T\}$, and K_R communication receivers (simply called receivers), each receiver $k \in \mathcal{R} = \{1, \ldots, \mathsf{K}_R\}$ with n_k antennas. The sensor is equipped with a single antenna. The channel outputs at the K_R receivers and at the sensor at the t-th time slot are,

$$\mathbf{y}_c^{[k]}(t) = \sum_{i=1}^{\mathsf{K}_T} g_{ki}(\mathbf{H}^{[ki]}(t)\mathbf{x}^{[i]}(t) + \mathbf{z}^{[k]}(t)) \in \mathbb{C}^{\mathsf{n}_k \times 1}, \ k \in \mathcal{R},$$
$$y_s(t) = \sum_{i=1}^{\mathsf{K}_T} g_s(\mathbf{H}^{[(\mathsf{K}_R+1)i]}(t)\mathbf{x}^{[i]}(t) + z_s(t)),$$

where $\mathbf{H}^{[ki]}(t) \in \mathbb{C}^{\mathsf{n}_k \times \mathsf{m}_i}$ represents the channel from the i-th transmitter to the k-th receiver at time slot t, and $\mathbf{H}^{[(\mathsf{K}_R+1)i]}(t) \in \mathbb{C}^{1 \times \mathsf{m}_i}$ represents the channel from the i-th transmitter to the sensor at time slot t; $\mathbf{z}^{[k]}(t) \in \mathbb{C}^{\mathsf{n}_k \times 1}$ and $z_s(t) \in \mathbb{C}$ are the additive Gaussian white noise at the receivers and sensor, respectively, with each element i.i.d. as $\mathcal{CN}(0,1)$. $g_{ki} \in \{0,1\}$ and $g_s \in \{0,1\}$ indicate the connectivity parameters between the transmitter and the receivers/sensor. The network connectivity (i.e., topology) is known to all the transmitters, receivers, and sensor. Assume that each transmission block is composed of t_0 channel uses. The signal transmitted by each transmitter $i \in \mathcal{T}$ at time slot $t \in \{1, 2, \ldots, t_0\}$ is the sum of a communication signal and a dedicated sensing signal:

$$\mathbf{x}^{[i]}(t) = \mathbf{x}_c^{[i]}(t) + \mathbf{x}_s^{[i]}(t) \in \mathbb{C}^{\mathsf{m}_i \times 1}.$$

The transmitted signal $\mathbf{x}^{[i]}(t)$ satisfies the power constraint $\mathbb{E}[|\mathbf{x}^{[i]}(t)|^2] \leq P$. $\mathbf{x}_c^{[i]}(t) \in \mathbb{C}^{\mathsf{m}_i \times 1}$ represents the communication signal, generated by the product of some precoding matrix and some vector of message symbols,[5] and the dedicated sensing signal $\mathbf{x}_s^{[i]}(t) \in \mathbb{C}^{\mathsf{m} \times 1}$ is fixed and thus known to all the receivers and sensor.

Let the unbolded notations $y_c^{[k]}(t)$, $H^{[ki]}(t)$, $z^{[k]}(t)$, $x^{[i]}(t)$, $x_c^{[i]}(t)$, $x_s^{[i]}(t)$ represent $\mathbf{y}_c^{[k]}(t)$, $\mathbf{H}^{[ki]}(t)$, $\mathbf{z}^{[k]}(t)$, $\mathbf{x}^{[i]}(t)$, $\mathbf{x}_c^{[i]}(t)$, $\mathbf{x}_s^{[i]}(t)$ each only containing one element, respectively.

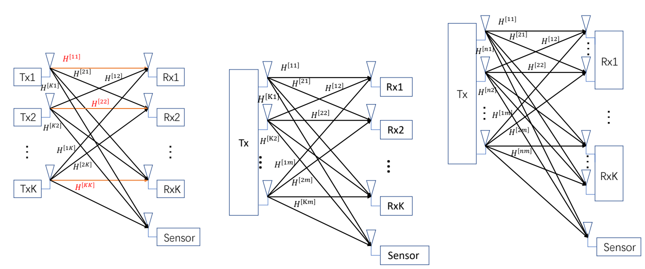
This paper aims to study interference mitigation based on the channel pattern among the receivers and the sensor. We consider two types of channel models: (i) heterogeneous coherence times, and (ii) heterogeneous connectivity.

1) Heterogeneous coherence times: Consider that the transmitters and receivers/sensor are fully connected. The communication channel follows a block fading model with a short coherence time, whereas the sensing channel follows a block fading model with a long coherence time. The channel state information of the communication channel is perfectly known by the transmitters and receivers.[6] The difference in coherence time arises because the receivers (such as vehicular terminals) experience significant Doppler shifts due to high mobility, leading to a short channel coherence time (on the order of milliseconds). In contrast, sensor (such as static sensors) experiences negligible Doppler effects due to their low-speed or stationary nature, resulting in a long channel coherence time (tens of milliseconds or more), which supports continuous signal integration to enhance sensing accuracy. Under this scenario, we assume that the communication channel varies in every time slot following a Rayleigh fading model, while the sensor channel remains approximately constant over the given time period t_0.

We investigate three different channels in this type:

- *Interference channel.* When $\mathsf{K}_T = \mathsf{K}_R = \mathsf{K}$, $\mathsf{m}_i = 1$ for each $i \in \mathcal{T}$, and $\mathsf{n}_j = 1$ for each $j \in \mathcal{R}$, we obtain the interference channel as shown in Fig. 2(a). Note that in the interference channel, the message symbols requested by the i-th receiver are only available at the i-th transmitter.
- *MU-MISO channel.* When $\mathsf{K}_T = 1$, $\mathsf{K}_R = \mathsf{K}$, $\mathsf{m}_1 = \mathsf{m}$, and $\mathsf{n}_j = 1$ for each $j \in \mathcal{R}$, we obtain the MU-MISO channel as shown in Fig. 2(a).
- *MU-MIMO channel.* When $\mathsf{K}_T = 1, \mathsf{K}_R = \mathsf{K}, \mathsf{m}_1 = \mathsf{m}$, we obtain the MU-MIMO channel as shown in Fig. 2(c).

For the MU-MISO and MU-MIMO channels, an additional assumption is that the CSI of receivers are known to the transmitter in a non-causal manner[7], implying that the transmitter has prior knowledge of the entire block's channel states.



(a) (b) (c)

Fig. 2: Heterogeneous coherence times model: (a) Interference channel, (b) MU-MISO channel, (c) MU-MIMO channel.

2) Heterogeneous connectivity: Now we consider the partial connectivity for network topology. Assume that the transmitters/sensor do not know the CSI of receivers and sensor, the

[5]Message symbols are encoded by using the Gaussian encoding with rate $\log P + o(\log P)$ (bit per message symbol). Thus each message symbol carries one DoF for large enough P.

[6]Through channel estimation and prediction, that is, the trajectories of the receivers movements are regular and predictable.

[7]There is a big body of literature in TDD/FDD massive MIMO systems [27], [28], where downlink channels can be efficiently estimated from uplink pilot signals by leveraging channel reciprocity in spatial and angular domains.

receivers have perfect CSIR about their connected links. Both the receivers and sensor have channels with long coherence time, such that we assume that the channels remain constant during a single transmission block (i.e., t_0 time slots). Thus we can exploit the inherent properties of the network structure in a stable network to enhance the network performance.

We investigate two different scenarios, depending on whether the transmitters in the network can cooperate or not.

- *Non-cooperative transmitters*. Each transmitter only has the communication messages desired by its corresponding receiver.
- *Cooperative transmitters*. Each transmitter has all the communication messages desired by the receivers.

For the network topology, we consider two exiting partial connection networks, the regular network in [26], [29]–[31] and the neighboring antidotes network in [25], [32].

Definition 1 ((K, d) regular network [26], [29]–[31])**.** *The* (K, d)*-regular network includes* K *single-antenna transmitters and* K *single-antenna users. Each user* j *receives signals from transmitter* j *as well as the successive* $d-1$ *ones, i.e., the set of connected transmitters by user* j *is* $\mathcal{R}_j = \{j, <j+1>_{\mathsf{K}}, \ldots, <j+d-1>_{\mathsf{K}}\}$. *This model with regular linear connectivity (cells are enumerated sequentially) is also known as Wyner-type model for cellular networks and has been widely investigated in information theory [33]–[35].*

Definition 2 ((K, U, D) neighboring antidotes network [25], [32])**.** *The* (K, U, D) *neighboring antidotes network with* $\mathsf{K} > U+D$ *and* $D \geq U$*, includes* K *single-antenna transmitters and* K *single-antenna users. Each user* j *is disconnected from the* U *preceding transmitters (indexed* $j-U, j-U+1, \ldots, j-1$*) and the* D *succeeding transmitters (indexed* $j+1, j+2, \ldots, j+D$*), where indices are interpreted modulo* K. *User* j *connects to all remaining* $\mathsf{K}-U-D$ *transmitters. Formally,* $\mathcal{R}_j = \{j, <j+D+1>_{\mathsf{K}}, \ldots, <j-U-1>_{\mathsf{K}}\}$. *In addition, exchanging the values of* U *and* D *results in an equivalent network.*

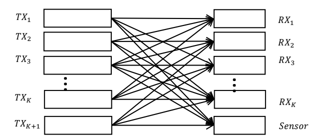
Based on the regular network in Definition 1, we consider two ways to add the sensor into the network:

- *Topology 1* (see Fig. 3(a)): In the $(\mathsf{K}+1, d)$ regular network or the $(\mathsf{K}+1, U, D)$ neighboring antidotes network, we set the first K users as the communication receivers and the last user as the sensor.
- *Topology 2* (see Fig. 3(b)): In the (K, d) regular network or the (K, U, D) neighboring antidotes network, with the first K users being communication receivers and one additional user as the sensor, where the set of connected transmitters of the sensor is a subset of one communication communication receiver.

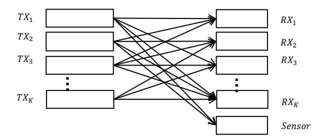
Define $\mathcal{R}_c^{[k]}$ and $\mathcal{R}_s$ as the set of connected transmitters by communication receiver k and by the sensor, respectively.

B. Performance Metrics

1) Communication Degree of Freedom: To evaluate communication performance, we define cDoF as the sum degree of freedom per time slot, averaged over t_0 time slots. Each independent message symbol contributes one unit of communication DoF. The cDoF is computed as the total number of message symbols successfully decoded by all communication receivers, normalized by t_0. This metric quantifies the efficiency of communication in terms of the number of recoverable symbols over a given transmission duration.



(a)



(b)

Fig. 3: Heterogeneous connectivity model: (a) Topology 1, (b) Topology 2.

2) Sensing Degree of Freedom: To facilitate the characterization of the fundamental tradeoff between communication and sensing, we adopt a dual metric to quantify both functionalities. Thus we consider the sensing degree of freedom based on the number of effective (i.e., independent) observations of the target during the estimation and detection process as in [5], [36]. More specifically, assuming the total number of transmitting antennas connected to the sensor is m', an effective observation sequence of length $N \geq \mathsf{m}'$ implies that the transmitter emits a set of orthogonal sensing signals $\mathbf{x}_1^{[0]}, \mathbf{x}_2^{[0]}, \ldots, \mathbf{x}_N^{[0]}$ (each with dimension $\mathsf{m}' \times 1$), satisfying $\left[\mathbf{x}_1^{[0]}, \mathbf{x}_2^{[0]}, \ldots, \mathbf{x}_N^{[0]}\right]\left[\mathbf{x}_1^{[0]}, \mathbf{x}_2^{[0]}, \ldots, \mathbf{x}_N^{[0]}\right]^H = I$. Accordingly, as in [5], [36] the total sensing degree of freedom (sDoF) is defined as the total number of the independent observations N. Assume that the sensor obtains these N independent observations by using T time slots. The average sDoF (or just simply called sDoF in the rest) is N/T.

Objective: The objective of this paper is to characterize the capacity region of all achievable tradeoff points (sDoF, cDoF), for large enough P and some finite t_0.

III. Main Results

The main technical contribution is to introduce the strategies of BIA and TIM into the bistatic ISAC systems to completely eliminate the interference from the communication messages at the sensor, under the constraint that the sensor channel is unknown to all. Since the dedicated sensing signals are known to the receivers while decoding the desired messages, they can eliminate the interference from the sensing signals.

- *Heterogeneous coherence times.* We can use the BIA strategy to eliminate the interference to the sensor. Meanwhile, to eliminate the interference from the undesired communication messages for each receiver, the zero forcing (ZF) strategy is used.
- *Heterogeneous connectivity.* Even though the CSI is unknown to the transmitters, we can use the TIM strategy to align interference for all the receivers and sensor by exploiting the symmetric network topology.

A. ISAC Model with Heterogeneous Coherence Times

Theorem 1 (Interference channel)**.** *For the bistatic ISAC systems with heterogeneous coherence times and interference* *channel including* K *single-antenna transmitters,* K *single-antenna communication receivers, and one sensor, the lower convex hull of the following tradeoff points is achievable:*

$$(sDoF, cDoF) = ((\mathsf{K}-1)/\mathsf{K}, 1), \tag{1}$$

$(sDoF, cDoF) = (1, 0)$*, and* $(sDoF, cDoF) = (0, 1)$*, where the latter two points are achieved by the existing sensing-only scheme and communication-only schemes.*[8]

Compared to the communication-only tradeoff point $(\mathrm{sDoF}, \mathrm{cDoF}) = (0, 1)$ achieved by ZF (the optimal one-shot linear coding), in (1) we can maintain the same cDoF while additionally obtaining a sDoF equal to $(\mathsf{K}-1)/\mathsf{K}$.

Theorem 2 (MU-MISO channel)**.** *For the bistatic ISAC systems with heterogeneous coherence times and the MU-MISO channel including one transmitter with* m *antennas,* K *single-antenna communication receivers, and one sensor, the lower convex hull of the following tradeoff points is achievable:*

$$(sDoF, cDoF) = \left(\frac{\lceil \frac{\mathsf{m}}{\mathsf{K}} \rceil - 1}{\lceil \frac{\mathsf{m}}{\mathsf{K}} \rceil}, \frac{\mathsf{m}}{\lceil \frac{\mathsf{m}}{\mathsf{K}} \rceil} \right), \tag{2}$$

$(sDoF, cDoF) = (1, 0)$*, and* $(sDoF, cDoF) = (0, \min\{\mathsf{m}, \mathsf{K}\})$*, where the latter two points are achieved by the existing sensing-only and communication-only schemes.*

Theorem 3 (MU-MIMO channel)**.** *For the bistatic ISAC systems with heterogeneous coherence times and the MU-MIMO channel including one transmitter with* m *antennas,* K *multi-antenna communication receivers (with the number of antennas* $\mathsf{n}_i, i = 1, \dots, \mathsf{K}$*), and one sensor, the lower convex hull of the following tradeoff points is achievable:*

$$(sDoF, cDoF) = \left(\frac{\left\lceil \frac{\mathsf{m}}{\sum_{k=1}^{\mathsf{K}} \mathsf{n}_k} \right\rceil - 1}{\left\lceil \frac{\mathsf{m}}{\sum_{k=1}^{\mathsf{K}} \mathsf{n}_k} \right\rceil}, \frac{\mathsf{m}}{\left\lceil \frac{\mathsf{m}}{\sum_{k=1}^{\mathsf{K}} \mathsf{n}_k} \right\rceil} \right), \tag{3}$$

$(sDoF, cDoF) = (1, 0)$*, and* $(sDoF, cDoF) = (0, \min\{\mathsf{m}, \sum_{k=1}^{\mathsf{K}} \mathsf{n}_k\})$*, where the latter two points are achieved by the existing sensing-only and communication-only schemes.*

B. ISAC Model with Heterogeneous Connectivity

For the bistatic ISAC model with heterogeneous connectivity described in Section II-A, Theorems 4 and Theorem 5 consider the non-cooperative and cooperative transmitters respectively, both for the Topology 1 and Topology 2.

Theorem 4 (Non-cooperative transmitters)**.** *For the bistatic ISAC systems with heterogeneous connectivity and non-cooperative transmitters,*

a. *for the Topology 1 based on the* $(\mathsf{K}+1, U, D)$ *neighboring antidotes network with* $\mathsf{K}+1 > U + D$ *and* $D \geq U$*, the lower convex hull of the following trade-off points is achievable:*

$$(sDoF, cDoF) = \left(\frac{1}{\mathsf{K} - D + U + 1}, \frac{\mathsf{K}(U+1)}{\mathsf{K} - D + U + 1} \right), \tag{4}$$

$(sDoF, cDoF) = (1, 0)$*, and* $(sDoF, cDoF) = \left(0, \frac{\mathsf{K}(U+1)}{\mathsf{K}-D+U+1}\right)$*, where the latter two points are achieved by the existing sensing-only scheme and the communication-only scheme [25], [32];*

b. *for the Topology 2 based on the* (K, U, D) *neighboring antidotes network with* $\mathsf{K} > U + D$ *and* $D \geq U$*, if there exists one receiver* $j \in \{1, \dots, \mathsf{K}\}$ *satisfying* $\mathcal{R}_s = \mathcal{R}_c^{[j]} \setminus \mathcal{A}$ *where* $j \in \mathcal{A}$*, the lower convex hull of the following trade-off points is achievable:*

$$(sDoF, cDoF) = \left(\frac{1}{\mathsf{K} - D + U}, \frac{\mathsf{K}(U+1)}{\mathsf{K} - D + U} \right), \tag{5}$$

$(sDoF, cDoF) = (1, 0)$*, and* $(sDoF, cDoF) = \left(0, \frac{\mathsf{K}(U+1)}{\mathsf{K}-D+U}\right)$*, where the latter two points are achieved by the existing sensing-only and communication-only schemes [25], [32].*

Theorem 5 (Cooperative transmitters)**.** *For the bistatic ISAC systems with heterogeneous connectivity and cooperative transmitters,*

a. *for the Topology 1 based on the* $(\mathsf{K}+1, d)$ *regular network, the lower convex hull of the following trade-off points are achievable:*

$$(sDoF, cDoF) = \left(\frac{1}{d+1}, \frac{2\mathsf{K}}{d+1} \right), \tag{6}$$

$(sDoF, cDoF) = (1, 0)$*, and* $(sDoF, cDoF) = \left(0, \frac{2\mathsf{K}}{d+1}\right)$*, where the latter two points are achieved by the existing sensing-only and communication-only schemes [26];*

b. *for the Topology 2 based on the* (K, d) *regular network, if there exists one receiver* $j \in \{1, \dots, \mathsf{K}\}$ *satisfying* $\mathcal{R}_s = \mathcal{R}_c^{[j]} \setminus \mathcal{A}$ *where* $j \in \mathcal{A}$*, the lower convex hull of the following trade-off points is achievable:*

$$(sDoF, cDoF) = \left(\frac{1}{d+1}, \frac{2K}{d+1} \right), \tag{7}$$

$(sDoF, cDoF) = (1, 0)$*, and* $(sDoF, cDoF) = \left(0, \frac{2K}{d+1}\right)$*, where the latter two points are achieved by the existing sensing-only and communication-only schemes [26].*

For the bistatic ISAC systems with heterogeneous connectivity, compared to the communication-only tradeoff points, in (4)-(7) we can maintain the same cDoFs while additionally obtaining non-zero sDoFs.

IV. Achievable schemes for the ISAC Model with Heterogeneous Coherence Times

This section provides the proposed ISAC schemes based on the BIA strategy for the bistatic ISAC system with heterogeneous coherence times, considering the interference and MU-MISO channels. The general description on the proposed

[8] Note that owing to the delay requirement of ISAC systems (especially for the sensing tasks), we consider zero forcing for communications within a limited number of coherence blocks, which excludes the vanilla interference alignment strategies that achieve precise interference alignment at the cost of an extremely large block length [37]. Moreover, even compared with the time-sharing between $(\mathrm{sDoF}, \mathrm{cDoF}) = (1, 0)$ and $(\mathrm{sDoF}, \mathrm{cDoF}) = (0, \mathsf{K}/2)$, when $\mathrm{sDoF} = (\mathsf{K}-1)/\mathsf{K}$ our scheme can achieve $\mathrm{cDoF} = 1$, while the above time-sharing can only achieve $\mathrm{cDoF} = 1/2$.

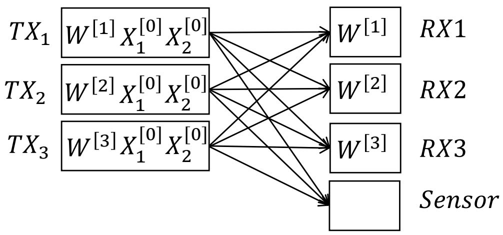


Fig. 4: 3×3 interference channel with a sensor for Example 1.

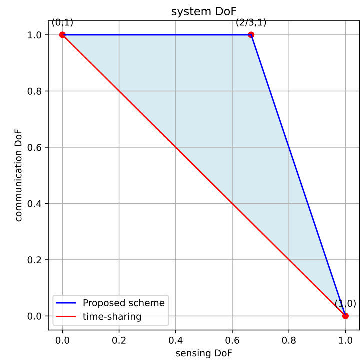


Fig. 5: Tradeoff between SDoF and CDoF for the 3×3 interference channel with a sensor.

scheme for the MU-MIMO channel (which extends the scheme for the MU-MISO channel) could be found in the Appendix B .

A. Proof of Theorem 1: Interference Channel

We first describe the main idea of the proposed scheme based on the BIA strategy through one example for the interference channel.

Example 1 (Interference channel with heterogeneous coherent times)**.** *Consider the bistatic ISAC system with heterogeneous coherence times and interference channel including 3 single-antenna transmitters, 3 single-antenna receivers, and one sensor, as illustrated in Fig. 4. For this system, the tradeoff points $(sDoF, cDoF) = (1,0)$, $(sDoF, cDoF) = (0,1)$ can be achieved by the sensing-only scheme and communication-only scheme, respectively. We then propose an ISAC scheme combining BIA and zero-forcing, achieving $(sDoF, cDoF) = (2/3, 1)$. Let $t_0 = 6$, and let the communication signals in the 6 time slots by each transmitter $k \in \{1,2,3\}$ be $x_c^{[k]}(1) = x_c^{[k]}(2) = x_c^{[k]}(3) = W_1^{[k]}$ and $x_c^{[k]}(4) = x_c^{[k]}(5) = x_c^{[k]}(6) = W_2^{[k]}$, where each $W_i^{[k]} \in \mathbb{C}$ represents a message symbol (as defined in Footnote 5) desired by the k-th receiver.*

Thus by removing the dedicated sensing signals (which are deterministic), the k-th communication receiver obtains from time slots $\{1,2,3\}$:

$$\hat{\mathbf{y}}_1^{[k]} = \begin{bmatrix} \hat{y}^{[k]}(1) \\ \hat{y}^{[k]}(2) \\ \hat{y}^{[k]}(3) \end{bmatrix} = \sum_{i=1}^{3} \begin{bmatrix} H^{[ki]}(1) x_c^{[k]}(1) \\ H^{[ki]}(2) x_c^{[k]}(2) \\ H^{[ki]}(3) x_c^{[k]}(3) \end{bmatrix} + \begin{bmatrix} z^{[k]}(1) \\ z^{[k]}(2) \\ z^{[k]}(3) \end{bmatrix}$$
$$= \sum_{i=1}^{3} \begin{bmatrix} H^{[ki]}(1) \\ H^{[ki]}(2) \\ H^{[ki]}(3) \end{bmatrix} W_1^{[i]} + \begin{bmatrix} z^{[k]}(1) \\ z^{[k]}(2) \\ z^{[k]}(3) \end{bmatrix}. \quad (8)$$

For the k-th receiver, since the channel coefficients $H^{[ki]}(t)$ where $t, i \in \{1,2,3\}$ are i.i.d. with circularly symmetric complex Gaussian distribution, we can apply the zero-forcing decoding to decode $W_1^{[k]}$ from $\hat{\mathbf{y}}_1^{[k]}$. Similarly, the k-th receiver can recover $W_2^{[k]}$ from time slots $\{4,5,6\}$. Thus the achieved communication DoF is 1.

The sensing DoF is $2/3$, by obtaining 4 effective observations can be obtained over 6 time slots. Next, we demonstrate the signal design using blind interference alignment. Let $\mathbf{x}_1^{[0]} = [x_{1,1}, x_{1,2}, x_{1,3}]^T \in \mathbb{C}^{3\times1}$, $\mathbf{x}_2^{[0]} = [x_{2,1}, x_{2,2}, x_{2,3}]^T \in \mathbb{C}^{3\times1}$, $\mathbf{x}_3^{[0]} = [x_{3,1}, x_{3,2}, x_{3,3}]^T \in \mathbb{C}^{3\times1}$, $\mathbf{x}_4^{[0]} = [x_{4,1}, x_{4,2}, x_{4,3}]^T \in \mathbb{C}^{3\times1}$, satisfying $[\mathbf{x}_1^{[0]}, \mathbf{x}_2^{[0]}, \mathbf{x}_3^{[0]}, \mathbf{x}_4^{[0]}][\mathbf{x}_1^{[0]}, \mathbf{x}_2^{[0]}, \mathbf{x}_3^{[0]}, \mathbf{x}_4^{[0]}]^H = I$. Our idea is to use the fact that the channel for the sensor remains constant over a certain period and that the communication signals also remain constant to cancel the communication signals, which are interference for the sensor. For the k-th transmitter where $k \in \{1,2,3\}$, the transmitted signals over time slots $\{1,2,3\}$ are: $x^{[k]}(1) = W^{[k]} + x_{1,k} + x_{2,k}$, $x^{[k]}(2) = W^{[k]} + x_{2,k}$, $x^{[k]}(3) = W^{[k]} + x_{1,k}$. The signals received by the sensor can be expressed as:

$$\mathbf{y}_{s,1}^{[4]} = \begin{bmatrix} y_s^{[4]}(1) \\ y_s^{[4]}(2) \\ y_s^{[4]}(3) \end{bmatrix}$$
$$= \sum_{i=1}^{3} \begin{bmatrix} H^{[4i]}(1)(W^{[i]} + x_{1,i} + x_{2,i}) \\ H^{[4i]}(1)(W^{[i]} + x_{2,i}) \\ H^{[4i]}(1)(W^{[i]} + x_{1,i}) \end{bmatrix} + \begin{bmatrix} z_s(1) \\ z_s(2) \\ z_s(3) \end{bmatrix}.$$

By subtracting the observations at the second and third time slots from the first time slot, we get:

$$\mathbf{y}_{s,1}^{[4]}(1) - \mathbf{y}_{s,1}^{[4]}(t) = \sum_{i=1}^{3} H^{[4i]}(1) x_{t-1,i} + z_s(1) - z_s(t)$$
$$= \mathbf{h} \begin{bmatrix} x_{t-1,1} \\ x_{t-1,2} \\ x_{t-1,3} \end{bmatrix} + z_s(1) - z_s(t),$$

where $\mathbf{h} = \left[H^{[41]}(1), H^{[42]}(1), H^{[43]}(1)\right]$ and $t = 2,3$. Therefore, the sensor obtains $\mathbf{h}\mathbf{x}_1^{[0]} + z_s(1) - z_s(2)$ and $\mathbf{h}\mathbf{x}_2^{[0]} + z_s(1) - z_s(3)$, meaning two effective observations through $\mathbf{x}_1^{[0]}, \mathbf{x}_2^{[0]}$ over the first three time slots. Similarly, the sensor obtains another two effective observations through $\mathbf{x}_3^{[0]}, \mathbf{x}_4^{[0]}$ over time slots $\{4,5,6\}$. Thus the SDoF is $2/3$, and the achieved tradeoff point by the proposed scheme is $(sDoF, cDoF) = (2/3, 1)$. Considering the three achieved tradeoff points, the lower convex hull is plotted by the blue line in Fig. 5, while the red line represents the time-sharing between the sensing-only and communication-only points. It can be seen that the achieved point by the proposed scheme significantly improves the one by time-sharing.

We are now ready to generalize the above example achieving the tradeoff point $(\text{sDoF}, \text{cDoF}) = ((\mathsf{K}-1)/\mathsf{K}, 1)$, for the $\mathsf{K} \times \mathsf{K}$ interference channel with an additional sensor.

Let $t_0 = 2\mathsf{K}$, and let the communication signals in the first K time slots by each transmitter $k \in \{1, \ldots, \mathsf{K}\}$ be $x_c^{[k]}(1) = \cdots = x_c^{[k]}(\mathsf{K}) = W_1^{[k]}$, and the communication signals in the second K time slots by each transmitter k be $x_c^{[k]}(\mathsf{K}+1) = \cdots = x_c^{[k]}(2\mathsf{K}) = W_2^{[k]}$, where each $W_i^{[k]} \in \mathbb{C}$ represents a message symbol desired by the k-th receiver.

At time slot t, the signal received by the k-th receiver is,

$$\begin{aligned}\mathbf{y}^{[k]}(t) = H^{[k1]}(t)(\mathbf{x}_c^{[1]}(t) + \mathbf{x}_s^{[1]}(t)) + H^{[k2]}(t)(\mathbf{x}_c^{[2]}(t) \\ + \mathbf{x}_s^{[2]}(t)) + \cdots + H^{[k\mathsf{K}]}(t)(\mathbf{x}_c^{[\mathsf{K}]}(t) + \mathbf{x}_s^{[\mathsf{K}]}(t)) + \mathbf{z}^{[k]}(t),\end{aligned}$$

for each $t \in \{1, \ldots, 2\mathsf{K}\}$. Since the CSI and the transmitted sensing signal $x_s^{[k]}(t)$ are known to the receivers, we can cancel this part of the signal at the receiver, resulting in an estimate $\hat{y}^{[k]}$ that only concerns the communication signals:

$$\begin{aligned}\hat{y}^{[k]}(t) &= H^{[k1]}(t)x_c^{[1]}(t) + \cdots + H^{[k\mathsf{K}]}(t)x^{[\mathsf{K}]}(t) + z^{[k]}(t) \\ &= H^{[k1]}(t)W_1^{[1]} + \cdots + H^{[k\mathsf{K}]}(t)W_1^{[\mathsf{K}]} + z^{[k]}(t),\end{aligned}$$

for each $t \in \{1, \ldots, \mathsf{K}\}$. Since the channel coefficients $H^{[ki]}(t)$ where $t, i \in \{1, \ldots, \mathsf{K}\}$ are i.i.d. with circularly symmetric complex Gaussian distribution, the k-th receiver can decode $W_1^{[k]}$ from the first K time slots with high probability by using ZF. Similarly, it can decode $W_2^{[k]}$ from the second K time slots. Thus the cDoF is 1.

Over $2\mathsf{K}$ time slots, the sensor can obtain $2\mathsf{K}-2$ effective observations, through $2\mathsf{K}-2$ sensing signals $\mathbf{x}_k^{[0]} = [x_{k,1}, \ldots, x_{k,\mathsf{K}}]^T \in \mathbb{C}^{\mathsf{K}\times 1}$, where $k = 1, 2, \ldots, 2\mathsf{K}-2$, satisfying $[\mathbf{x}_1^{[0]}, \mathbf{x}_2^{[0]}, \ldots, \mathbf{x}_{2\mathsf{K}-2}^{[0]}][\mathbf{x}_1^{[0]}, \mathbf{x}_2^{[0]}, \ldots, \mathbf{x}_{2\mathsf{K}-2}^{[0]}]^H = I$.

Let us first consider the first K time slots. We design the transmitted dedicated sensing signals $x_s^{[k]}(t)$ for $t \in \{1, \ldots, \mathsf{K}\}$ as follows,

$$\begin{bmatrix} x_s^{[k]}(1) \\ x_s^{[k]}(2) \\ \vdots \\ x_s^{[k]}(\mathsf{K}) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & \ldots & 1 \\ 0 & 1 & 1 & \ldots & 1 \\ 1 & 0 & 1 & \ldots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \ldots & 0 \end{bmatrix} \begin{bmatrix} x_{1,k} \\ x_{2,k} \\ x_{3,k} \\ \vdots \\ x_{\mathsf{K}-1,k} \end{bmatrix}. \tag{9}$$

Recall that the communication message transmitted by each transmitter during the first K time slots remains the same, as well as the channel from each transmitter to the sensor. Thus for each $t \in \{2, 3, \ldots, \mathsf{K}\}$, we have

$$\begin{aligned}&y_s^{[\mathsf{K}+1]}(1) - y_s^{[\mathsf{K}+1]}(t) = H^{[\mathsf{K}+1,1]}(1)x_{t-1,1} + \\ &H^{[\mathsf{K}+1,2]}(1)x_{t-1,2} + \cdots + H^{[\mathsf{K}+1,\mathsf{K}]}(1)x_{t-1,\mathsf{K}} + z_s(1) - z_s(t) \\ &= \left[H^{[\mathsf{K}+1,1]}(1), \ldots, H^{[\mathsf{K}+1,\mathsf{K}]}(1)\right] x_{t-1}^{[0]} + z_s(1) - z_s(t).\end{aligned}$$

Thus over the first K time slots, we obtain $\mathsf{K}-1$ effective observations through $\mathbf{x}_1^{[0]}, \ldots, \mathbf{x}_{\mathsf{K}-1}^{[0]}$. Similarly, over the second K time slots, we can obtain another $\mathsf{K}-1$ effective observations through $\mathbf{x}_{\mathsf{K}}^{[0]}, \ldots, \mathbf{x}_{2\mathsf{K}-2}^{[0]}$. Hence, the sensing degree of freedom (SDoF) is $(\mathsf{K}-1)/\mathsf{K}$.

B. Proof of Theorem 2: MU-MISO

Now we consider the bistatic ISAC systems with heterogeneous coherent times and the MU-MISO channel including one transmitter with m antennas, K single-antenna communication receivers, and one sensor. By Theorem 2, we propose a new BIA scheme to achieve $(\text{sDoF}, \text{cDoF}) = \left(\frac{\lceil \frac{\mathsf{m}}{\mathsf{K}} \rceil - 1}{\lceil \frac{\mathsf{m}}{\mathsf{K}} \rceil}, \frac{\mathsf{m}}{\lceil \frac{\mathsf{m}}{\mathsf{K}} \rceil}\right)$. Note that when $\mathsf{m} \le \mathsf{K}$, the above tradeoff point becomes $(0, \mathsf{m})$, which can be simply achieved by the communication-only scheme. Hence, in this proof we only consider the case $\mathsf{m} > \mathsf{K}$. For the ease of notation, we define that $\lceil \mathsf{m}/\mathsf{K} \rceil := \mathsf{a}$, thus $\mathsf{a} > 1$; define that $\mathsf{p} = < \mathsf{m} >_{\mathsf{K}}$.

Let $t_0 = \mathsf{a} \left\lceil \frac{\mathsf{m}}{\mathsf{a}-1} \right\rceil$. During these t_0 time slots, the sensor will obtain $\left\lceil \frac{\mathsf{m}}{\mathsf{a}-1} \right\rceil (\mathsf{a}-1)$ effective observations, through the sensing signals $\mathbf{x}_1^{[0]}, \ldots, \mathbf{x}_{\lceil \frac{\mathsf{m}}{\mathsf{a}-1} \rceil (\mathsf{a}-1)}^{[0]} \in \mathbb{C}^{\mathsf{m}\times 1}$, satisfying $\left[\mathbf{x}_1^{[0]}, \ldots, \mathbf{x}_{\lceil \frac{\mathsf{m}}{\mathsf{a}-1} \rceil (\mathsf{a}-1)}^{[0]}\right] \left[\mathbf{x}_1^{[0]}, \ldots, \mathbf{x}_{\lceil \frac{\mathsf{m}}{\mathsf{a}-1} \rceil (\mathsf{a}-1)}^{[0]}\right]^H = I$. In each period of a time slots, we let the receivers totally recover m communication messages, and let the sensor obtain $\mathsf{a}-1$ effective observation.

In the following, we illustrate our proposed scheme for the first a time slots. Each receiver $k \in \{1, \ldots, \mathsf{p}\}$ should decode the communication messages $W_j^{[i]} \in \mathbb{C}$ where $j \in \{1, 2, ..., \mathsf{a}\}$, and each receiver $k \in \{\mathsf{p}+1, \ldots, \mathsf{K}\}$ should decode the communication messages $W_j^{[i]} \in \mathbb{C}$ where $j \in \{1, 2, 3, ..., \mathsf{a}-1\}$.

For each time slot $t \in \{1, \ldots, \mathsf{a}\}$, we design the communication signals by the transmitter with m antennas as:

$$\mathbf{x}_c(1) = \cdots = \mathbf{x}_c(\mathsf{a}) = \sum_{\tau=1}^{\mathsf{a}-1} \sum_{k=1}^{\mathsf{K}} \mathbf{v}_\tau^{[k]} W_\tau^{[k]} + \sum_{j=1}^{\mathsf{p}} \mathbf{v}_{\mathsf{a}}^{[j]} W_{\mathsf{a}}^{[j]} \in \mathbb{C}^{\mathsf{m}\times 1},$$

where $\{\mathbf{v}_t^{[k]} : k \in \{1, \ldots, \mathsf{K}\}, t \in \{1, \ldots, \mathsf{a}-1\}\}$ and $\{\mathbf{v}_{\mathsf{a}}^{[j]} : j \in \{1, \ldots, \mathsf{p}\}\}$ are the precoding vectors to be determined later, each with dimension $\mathsf{m} \times 1$. At time slot $t \in \{1, \ldots, \mathsf{a}\}$, for each $k \in \{1, \ldots, \mathsf{K}\}$, the signal received by the k-th receiver is,

$$y^{[k]}(t) = [h^{[k1]}(t), \ldots, h^{[k\mathsf{m}]}(t)](\mathbf{x}_c(t) + \mathbf{x}_s(t)) + z^{[k]}(t),$$

where $h^{[ki]}(t)$ represents the channel coefficient from i-th antenna of the transmitter to k-th receiver at time slot t, for each $k \in \{1, \ldots, \mathsf{K}\}$ and $t \in \{1, \ldots, \mathsf{a}\}$. By removing the dedicated signals, the k-th receiver obtains,

$$\hat{y}^{[k]}(t) = [h^{[k1]}(t), \ldots, h^{[k\mathsf{m}]}(t)]\mathbf{x}_c(t) + z^{[k]}(t).$$

The precoding vectors are designed to ensure that

c1. for each $k \in \{1, \ldots, \mathsf{p}\}$ and $t \in \{1, \ldots, \mathsf{a}\}$, the k-th receiver can decode $W_t^{[k]}$ from $\hat{y}^{[k]}(t)$;
c2. for each $k \in \{\mathsf{p}+1, \ldots, \mathsf{K}\}$ and $t \in \{1, \ldots, \mathsf{a}-1\}$, the k-th receiver can decode $W_t^{[k]}$ from $\hat{y}^{[k]}(t)$.

To satisfy the above conditions, we propose the following design on the precoding vectors:

- Design on $\mathbf{v}_t^{[k]}$ for $k \in \{1, \ldots, \mathsf{p}\}, t \in \{1, \ldots, \mathsf{a}\}$. We let $\mathbf{v}_t^{[k]}$ be a right null vector of the following matrix with dimension $(\mathsf{m}-1) \times \mathsf{m}$:

$$\mathbf{H}_t^{[k]} = \begin{bmatrix} \mathbf{H}(1) \\ \mathbf{H}(2) \\ \vdots \\ \mathbf{H}(t) \backslash \mathbf{H}(t)[k,:] \\ \vdots \\ \mathbf{H}(\mathsf{a}) \end{bmatrix}, \quad (10)$$

where $\mathbf{H}(i)$ is the channel matrix from the transmitter to the K receivers at time slot $i \in \{1, 2, ..., \mathsf{a}-1\}$,

$$\mathbf{H}(i) = \begin{bmatrix} h^{[11]}(i) & h^{[12]}(i) & \cdots & h^{[1\mathsf{m}]}(i) \\ h^{[21]}(i) & h^{[22]}(i) & \cdots & h^{[2\mathsf{m}]}(i) \\ \vdots & \vdots & \ddots & \vdots \\ h^{[\mathsf{K}1]}(i) & h^{[\mathsf{K}2]}(i) & \cdots & h^{[\mathsf{Km}]}(i) \end{bmatrix} \in \mathbb{C}^{\mathsf{K} \times \mathsf{m}},$$

$\mathbf{H}(i)[j,:]$ represents the j-th row of $\mathbf{H}(i)$, $\mathbf{H}(i) \backslash \mathbf{H}(i)[j,:]$ represents the matrix $\mathbf{H}(i)$ with the j-th row removed,

$$\mathbf{H}(\mathsf{a}) = \begin{bmatrix} h^{[11]}(\mathsf{a}) & h^{[12]}(\mathsf{a}) & \cdots & h^{[1\mathsf{m}]}(\mathsf{a}) \\ h^{[21]}(\mathsf{a}) & h^{[22]}(\mathsf{a}) & \cdots & h^{[2\mathsf{m}]}(\mathsf{a}) \\ \vdots & \vdots & \ddots & \vdots \\ h^{[\mathsf{p}1]}(\mathsf{a}) & h^{[\mathsf{p}2]}(\mathsf{a}) & \cdots & h^{[\mathsf{pm}]}(\mathsf{a}) \end{bmatrix} \in \mathbb{C}^{\mathsf{p} \times \mathsf{m}}$$

is the channel matrix from the transmitter to the first p receivers at time slot a.
Note that the matrix $\mathbf{H}_t^{[k]}$ with dimension $(\mathsf{m}-1) \times \mathsf{m}$ has one non-zero right null vector with high probability.
- Design on $\mathbf{v}_t^{[k]}$ for $k \in \{\mathsf{p}+1, \ldots, \mathsf{a}\}, t \in \{1, \ldots, \mathsf{a}-1\}$. Let $\mathbf{v}_t^{[k]}$ be the right null vector of $\mathbf{H}_t^{[k]}$, with the same definition as (10).

By the above selection on the precoding vectors, one can check that the decodability conditions c1 and c2 are satisfied with high probability. For example, let us focus on receiver 1. Its received signal after removing the sensing signal at time slot $t \in \{1, \ldots, \mathsf{a}\}$ is

$$\begin{aligned} \hat{y}^{[1]}(t) &= \mathbf{H}(t)[1,:]\mathbf{x}_c(t) + z^{[1]}(t) \\ &= \mathbf{H}(t)[1,:](\sum_{\tau=1}^{\mathsf{a}-1} \sum_{k=1}^{\mathsf{K}} \mathbf{v}_\tau^{[k]} W_\tau^{[k]} + \sum_{j=1}^{\mathsf{p}} \mathbf{v}_{\mathsf{a}}^{[j]} W_{\mathsf{a}}^{[j]}) + z^{[1]}(t). \quad (11) \end{aligned}$$

It can be checked that the product of $\mathbf{H}(t)[1,:]$ and each precoding vector in (11) (except $\mathbf{v}_t^{[1]}$) is 0. Hence, receiver 1 can recover $W_t^{[1]}$ with high probability.

Thus the cDoF of the system is $\frac{\mathsf{pa}+(\mathsf{K}-\mathsf{p})(\mathsf{a}-1)}{\mathsf{a}} = \frac{m}{\mathsf{a}}$.

We then design the transmitted dedicated sensing signals $\mathbf{x}_s(t)$ for $t \in \{1, \ldots, \mathsf{a}\}$ as follows (recall that $\mathbf{x}_1^{[0]}, \ldots, \mathbf{x}_{\mathsf{a}}^{[0]}$ have been selected before)

$$\mathbf{x}_s(1) = \sum_{i=1}^{\mathsf{a}-1} \mathbf{x}_i^{[0]}, \quad \mathbf{x}_s(t) = \sum_{i=1, i \neq t-1}^{\mathsf{a}-1} \mathbf{x}_i^{[0]}, \ \forall t \in \{2, \ldots, \mathsf{a}\}.$$

At each time slot $t \in \{1, \ldots, \mathsf{a}\}$, the sensor receives

$$y_s^{[\mathsf{K}+1]}(t) = \mathbf{h}^{[\mathsf{K}+1]}(t)(\mathbf{x}_c(t) + \mathbf{x}_s(t)) + z_s(t),$$

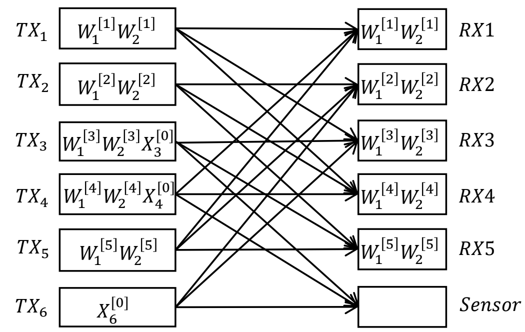


Fig. 6: TIM non-cooperative interference channel in Example 2.

where $\mathbf{h}^{[\mathsf{K}+1]}(t) = [h^{[\mathsf{K}+1,1]}(t), \ldots, h^{[\mathsf{K}+1,\mathsf{m}]}(t)]$, and $h^{[\mathsf{K}+1,i]}(t)$ represents the channel coefficient from i-th antenna of the transmitter to the sensor.

Since $\mathbf{h}^{[\mathsf{K}+1]}(1) = \cdots = \mathbf{h}^{[\mathsf{K}+1]}(\mathsf{a})$ and $\mathbf{x}_c(1) = \cdots = \mathbf{x}_c(\mathsf{a})$, the sensor can subtract the received signal at time slot $t \in \{2, \ldots, \mathsf{a}\}$ from the received signal at the first time slot:

$$\mathbf{y}_s^{[\mathsf{K}+1]}(1) - \mathbf{y}_s^{[\mathsf{K}+1]}(t) = \mathbf{h}^{[\mathsf{K}+1]}(1)\mathbf{x}_{t-1}^{[0]} + z_s(1) - z_s(t),$$

thereby canceling the interference caused by the communication signals and obtaining an effective observation of $\mathbf{x}_{t-1}^{[0]}$. Over a time slots, a total of $\mathsf{a}-1$ effective observations can be made, and the sensing degrees of freedom (SDoF) is $(\mathsf{a}-1)/\mathsf{a}$.

V. Achievable schemes for the ISAC Model with Heterogeneous Connectivity

This section provides the proposed ISAC schemes based on the TIM strategy for the bistatic ISAC system with heterogeneous connectivity.

A. Proof of Theorem 4-a: Topology 1 with Non-cooperative Transmitters

Considering the bistatic ISAC systems with heterogeneous connectivity and non-cooperative transmitters, we first describe the main idea of the proposed TIM scheme for the Topology 1 through the following example. In the bistatic ISAC systems with heterogeneous connectivity, all channels do not change during t_0 time slots; so for the ease of notation, we remove the time indices from channel coefficients.

Example 2 (Topology 1 with non-cooperative transmitters). *Consider the bistatic ISAC system with partially connected interference channel including* 6 *single-antenna transmitters,* 5 *single-antenna receivers and one sensor, as illustrated in Fig. 6. Each receiver or sensor is disconnected from the previous* $U = 1$ *and next* $D = 2$ *transmitters, and connected to other* 3 *transmitters; the topology belongs to Topology 1 from the* $(6, 1, 2)$*-neighboring antidotes network described in Definition 2. We have*

$$\mathcal{R}_c^{[1]} = \{1, 4, 5\}, \ \mathcal{R}_c^{[2]} = \{2, 5, 6\}, \ \mathcal{R}_c^{[3]} = \{1, 3, 6\},$$
$$\mathcal{R}_c^{[4]} = \{1, 2, 4\}, \ \mathcal{R}_c^{[5]} = \{2, 3, 5\}, \ \mathcal{R}_s = \{3, 4, 6\}.$$

For this system, the tradeoff points $(sDoF, cDoF) = (1, 0)$, $(sDoF, cDoF) = (0, 2)$ *can be achieved by the sensing-only* *and communication-only schemes, respectively. We then propose a TIM ISAC scheme, achieving* $(sDoF, cDoF) = (1/5, 2)$*.*

Let $t_0 = 15$*. During these* 15 *time slots, the sensor will obtain* 3 *effective observations, through the sensing signals* $\mathbf{x}_1^{[0]}, \mathbf{x}_2^{[0]}, \mathbf{x}_3^{[0]} \in \mathbb{C}^{3\times 1}$*, satisfying* $[\mathbf{x}_1^{[0]}, \mathbf{x}_2^{[0]}, \mathbf{x}_3^{[0]}][\mathbf{x}_1^{[0]}, \mathbf{x}_2^{[0]}, \mathbf{x}_3^{[0]}]^H = I$*. In each period of* 5 *time slots, we let each receiver recover* 2 *communication messages, and let the sensor obtain one effective observation. In the following, we illustrate our proposed scheme for the first* 5 *time slots, where each receiver* $j \in \{1, \ldots, 5\}$ *should recover* 2 *communication messages* $W_1^{[j]}, W_2^{[j]} \in \mathbb{C}$*.*

Denote the transmission signals of transmitter j *by* $\mathbf{x}^{[j]} = [x^{[j]}(1), \ldots, x^{[j]}(5)]^T \in \mathbb{C}^{5\times 1}$*. Then for the transmitters connected to the sensor, we design*

$$
\begin{aligned}
\mathbf{x}^{[3]} &= \mathbf{v}^{[3]} W_1^{[3]} + \mathbf{v}^{[4]} W_2^{[3]} + \mathbf{x}_s^{[3]}, \\
\mathbf{x}^{[4]} &= \mathbf{v}^{[4]} W_1^{[4]} + \mathbf{v}^{[5]} W_2^{[4]} + \mathbf{x}_s^{[4]}, \\
\mathbf{x}^{[6]} &= \mathbf{x}_s^{[6]};
\end{aligned}
$$

for the transmitters not connected to the sensor, we design

$$
\begin{aligned}
\mathbf{x}^{[1]} &= \mathbf{v}^{[1]} W_1^{[1]} + \mathbf{v}^{[2]} W_2^{[1]}, \\
\mathbf{x}^{[2]} &= \mathbf{v}^{[2]} W_1^{[2]} + \mathbf{v}^{[3]} W_2^{[2]}, \\
\mathbf{x}^{[5]} &= \mathbf{v}^{[5]} W_1^{[5]} + \mathbf{v}^{[6]} W_2^{[5]}.
\end{aligned}
$$

Note that $\mathbf{v}^{[1]}, \ldots, \mathbf{v}^{[6]} \in \mathbb{C}^{5\times 1}$ *represent* 6 *random i.i.d. vectors, where any* 5 *of them are linearly independent, and* $\mathbf{x}_s^{[3]}, \mathbf{x}_s^{[4]}, \mathbf{x}_s^{[6]} \in \mathbb{C}^{5\times 1}$ *are the transmitted sensing signals to be determined later.*

In this partially connected case, by exploiting the network topology, we align the interference at receivers 1*,* 4*, and* 5 *to a lower dimension. Let us consider receiver* 1 *as an example. The received signal after removing the dedicated sensing signals at time slot* $t \in \{1, \ldots, 5\}$ *is* $y^{[1]}(t) = \sum_{i\in\mathcal{R}_1=\{1,4,5\}} H^{[1i]} x^{[i]}(t) + z^{[1]}(t)$*. Hence, its received signals from the* 5 *time slots can be formulated as:*

$$
\begin{aligned}
\mathbf{y}^{[1]} = \begin{bmatrix} y^{[1]}(1) \\ \vdots \\ y^{[1]}(5) \end{bmatrix} &= H^{[11]}(\mathbf{v}^{[1]} W_1^{[1]} + \mathbf{v}^{[2]} W_2^{[1]}) \\
&+ H^{[14]}(\mathbf{v}^{[4]} W_1^{[4]} + \mathbf{v}^{[5]} W_2^{[4]}) \\
&+ H^{[15]}\left(\mathbf{v}^{[5]} W_1^{[5]} + \mathbf{v}^{[6]} W_2^{[5]}\right) + \mathbf{z}^{[1]} \\
&= \underbrace{\mathbf{v}^{[1]} H^{[11]} W_1^{[1]} + \mathbf{v}^{[2]} H^{[11]} W_2^{[1]}}_{\text{desired signals}} \\
&+ \mathbf{v}^{[4]} H^{[14]} W_1^{[4]} + \mathbf{v}^{[5]}\left(H^{[14]} W_2^{[4]} + H^{[15]} W_1^{[5]}\right) \\
&+ \mathbf{v}^{[6]} H^{[15]} W_2^{[5]} + \mathbf{z}^{[1]}.
\end{aligned} \tag{12}
$$

The dimension of the designed signals is 2*, and the dimension of the interference is aligned to* 3*. By the linearly independence of* $\mathbf{v}^{[1]}, \mathbf{v}^{[2]}, \mathbf{v}^{[4]}, \mathbf{v}^{[5]}, \mathbf{v}^{[6]}$*, receiver* 1 *can recover* $W_1^{[1]}, W_2^{[1]}$ *from the first* 5 *time slots. Similarly, each receiver can recover* 2 *communication messages from the first* 5 *time slots; thus the communication degree of freedom is* 2*.*

For the sensor, its received signal at time slot $t \in \{1, \ldots, 5\}$ *is* $y^{[6]}(t) = \sum_{i\in\mathcal{R}_6=\{3,4,6\}} H^{[6i]} \mathbf{x}^{[i]}(t) + z^{[6]}(t)$*. Hence, its received signals from the* 5 *time slots can be formulated as:*

$$
\begin{aligned}
\mathbf{y}^{[6]} = \begin{bmatrix} y^{[6]}(1) \\ \vdots \\ y^{[6]}(5) \end{bmatrix} &= H^{[63]}\left(\mathbf{v}^{[3]} W_1^{[3]} + \mathbf{v}^{[4]} W_2^{[3]} + \mathbf{x}_s^{[3]}\right) \\
&+ H^{[64]}\left(\mathbf{v}^{[4]} W_1^{[4]} + \mathbf{v}^{[5]} W_2^{[4]} + \mathbf{x}_s^{[4]}\right) + H^{[66]} \mathbf{x}_s^{[6]} + \mathbf{z}^{[6]} \\
&= \mathbf{v}^{[3]} H^{[63]} W_1^{[3]} + \mathbf{v}^{[4]}\left(H^{[63]} W_2^{[3]} + H^{[64]} W_1^{[4]}\right) \\
&+ \mathbf{v}^{[5]} H^{[64]} W_2^{[4]} + H^{[63]} \mathbf{x}_s^{[3]} + H^{[64]} \mathbf{x}_s^{[4]} + H^{[66]} \mathbf{x}_s^{[6]} + \mathbf{z}^{[6]}.
\end{aligned}
$$

Note that $[\mathbf{v}^{[2]}, \mathbf{v}^{[3]}, \mathbf{v}^{[6]}]$ *with dimension* 5×3 *has* 2 *linearly independent left null vectors with high probability; let* $\mathbf{v}_0 \in \mathbb{C}^{1\times 5}$ *be a non-zero left null vector. Thus we have*

$$
\begin{aligned}
\mathbf{v}_0 \mathbf{y}^{[6]} &= \mathbf{v}_0 \mathbf{v}^{[3]} H^{[63]} W_1^{[3]} + \mathbf{v}_0 \mathbf{v}^{[4]}\left(H^{[63]} W_2^{[3]} + H^{[64]} W_1^{[4]}\right) \\
&+ \mathbf{v}_0 \mathbf{v}^{[5]} H^{[64]} W_2^{[4]} + \mathbf{v}_0 H^{[63]} \mathbf{x}_s^{[3]} + \mathbf{v}_0 H^{[64]} \mathbf{x}_s^{[4]} \\
&+ \mathbf{v}_0 H^{[66]} \mathbf{x}_s^{[6]} + \mathbf{v}_0 \mathbf{z}^{[6]} \\
&= \begin{bmatrix} H^{[63]} & H^{[64]} & H^{[66]} \end{bmatrix} \begin{bmatrix} \mathbf{v}_0 \mathbf{x}_s^{[3]} \\ \mathbf{v}_0 \mathbf{x}_s^{[4]} \\ \mathbf{v}_0 \mathbf{x}_s^{[6]} \end{bmatrix} + z'^{[6]},
\end{aligned}
$$

where $z'^{[6]} = \mathbf{v}_0 \mathbf{z}^{[6]}$*. Hence, the sensor can eliminate the interference from communication messages, and obtain an effective observation from the first* 5 *time slots.*

We then fix the values of $\mathbf{x}_s^{[3]}, \mathbf{x}_s^{[4]}, \mathbf{x}_s^{[6]}$*. Recall that* $x_1^{[0]}, x_2^{[0]}, x_3^{[0]}$ *has been selected before, satisfying* $[\mathbf{x}_1^{[0]}, \mathbf{x}_2^{[0]}, \mathbf{x}_3^{[0]}][\mathbf{x}_1^{[0]}, \mathbf{x}_2^{[0]}, \mathbf{x}_3^{[0]}]^H = I$*. Now we let* $x_1^{[0]} = [\mathbf{v}_0 \mathbf{x}_s^{[3]}, \mathbf{v}_0 \mathbf{x}_s^{[4]}, \mathbf{v}_0 \mathbf{x}_s^{[6]}]$*; in other words, we solve* $\mathbf{x}_s^{[3]}$ *with* 5 *parameters satisfying* $\mathbf{v}_0 \mathbf{x}_s^{[3]} = x_{1,1}$*, where* $x_{1,1}$ *is the first element of* $\mathbf{x}_1^{[0]}$*, and similarly for* $\mathbf{x}_s^{[4]}$ *and* $\mathbf{x}_s^{[6]}$*. Therefore, the sensing degree of freedom is* $1/5$*.*

We next generalize Example 2 to achieve the tradeoff point $(\text{sDoF}, \text{cDoF}) = \left(\frac{1}{\mathsf{K}-D+U+1}, \frac{\mathsf{K}(U+1)]}{\mathsf{K}-D+U+1}\right)$ in (4). Each receiver or sensor is disconnected from the previous U and next $D \geq U$ transmitters, and connected to other $\mathsf{K}-U-D+1 > 0$ transmitters; thus the topology could be expressed as

$$
\begin{aligned}
&\mathcal{R}_c^{[k]} = \{k, <k+D+1>_{\mathsf{K}+1}, \ldots, <k-U-1>_{\mathsf{K}+1}\}, \\
&\mathcal{R}_s = \{\mathsf{K}+1, <D+1>_{\mathsf{K}+1}, <D+2>_{\mathsf{K}+1}, \ldots, <\mathsf{K}-U>_{\mathsf{K}+1}\},
\end{aligned}
$$

for each $k \in \{1, \ldots, \mathsf{K}\}$.

Let $t_0 = (\mathsf{K}-D-U+1)(\mathsf{K}-D+U+1)$, during these $(\mathsf{K}-D-U+1)(\mathsf{K}-D+U+1)$ time slots, the sensor will obtain $\mathsf{K}-D-U+1$ effective observations, through the sensing signals $\mathbf{x}_1^{[0]}, \mathbf{x}_2^{[0]}, \ldots, \mathbf{x}_{\mathsf{K}-D-U+1}^{[0]} \in \mathbb{C}^{(\mathsf{K}-D-U+1)\times 1}$, satisfying $[\mathbf{x}_1^{[0]}, \mathbf{x}_2^{[0]}, \ldots, \mathbf{x}_{\mathsf{K}-D-U+1}^{[0]}][\mathbf{x}_1^{[0]}, \mathbf{x}_2^{[0]}, \ldots, \mathbf{x}_{\mathsf{K}-D-U+1}^{[0]}]^H = I$. In each period of $(\mathsf{K}-D+U+1)$ time slots, we let each receiver decode $U+1$ communication messages, and let the sensor obtain one effective observation. In the next, we will illustrate our proposed scheme for the first $\mathsf{K}-D+U+1$ time slots, where each receiver $k \in \{1, 2, \ldots, \mathsf{K}\}$ should recover $U+1$ messages $W_1^{[k]}, \ldots, W_{U+1}^{[k]} \in \mathbb{C}$.

Denote the transmitted signals of transmitter $j \in \{1, \ldots, \mathsf{K}+1\}$ by $\mathbf{x}^{[j]} = [x^{[j]}(1), \ldots, x^{[j]}(\mathsf{K}-D+U+1)]^T \in$

$\mathbb{C}^{(\mathsf{K}-D+U+1)\times 1}$. For the transmitters connected to the sensor, we design based on the cyclic coding,

$$\mathbf{x}^{[i]}=\mathbf{v}^{[i]}W_1^{[i]}+\mathbf{v}^{[<i+1>_{\mathsf{K}+1}]}W_2^{[i]}+\ldots+\mathbf{v}^{[<i+U>_{\mathsf{K}+1}]}W_{U+1}^{[i]}+\mathbf{x}_s^{[i]},$$
$$\mathbf{x}^{[\mathsf{K}+1]}=\mathbf{x}_s^{[\mathsf{K}+1]},$$

for each $i\in\mathcal{R}_s\setminus\{\mathsf{K}+1\}$, where $\mathbf{x}_s^{[j]}, j\in\mathcal{R}_s$ are the sensing signals to be determined later. For the transmitters not connected to the sensor, design based on the cyclic coding,

$$\mathbf{x}^{[i]}=\mathbf{v}^{[i]}W_1^{[i]}+\mathbf{v}^{[<i+1>_{\mathsf{K}+1}]}W_2^{[i]}+\ldots+\mathbf{v}^{[<i+U>_{\mathsf{K}+1}]}W_{U+1}^{[i]},$$

for each $i\in\{1,\ldots,\mathsf{K}+1\}\setminus\mathcal{R}_s$. Note that $\mathbf{v}^{[1]},...,\mathbf{v}^{[U+\mathsf{K}]}\in\mathbb{C}^{\mathsf{K}-D+U+1}$ represent $\mathsf{K}+1$ random i.i.d vectors, where any $\mathsf{K}-D+U+1$ of them are linearly independent. For each $k\in\{1,\ldots,\mathsf{K}\}$, by removing the dedicated sensing signals from the received signals, the k-th receiver obtains

$$\begin{aligned}\hat{\mathbf{y}}^{[k]}&=H^{[kk]}(\mathbf{v}^{[k]}W_1^{[k]}+\cdots+\mathbf{v}^{[<k+U>_{\mathsf{K}+1}]}W_{U+1}^{[k]})\\&+\sum_{i\in\mathcal{R}_c^{[k]}\setminus\{k\}}H^{[ki]}(\mathbf{v}^{[i]}W_1^{[i]}+\ldots+\mathbf{v}^{[<i+U>_{\mathsf{K}+1}]}W_{U+1}^{[i]})+\mathbf{z}^{[k]}\\&=\underbrace{H^{[kk]}(\mathbf{v}^{[k]}W_1^{[k]}+\ldots+\mathbf{v}^{[<k+U>_{\mathsf{K}+1}]}W_{U+1}^{[k]})}_{\text{desire signals}}\\&+\underbrace{\sum_{i=k+D+1}^{k+\mathsf{K}}\mathbf{v}^{[<i>_{\mathsf{K}+1}]}\sum_{s=max(1,i-U)}^{min(\mathsf{K}-U-D,i)}H^{[ks]}W_{i-s+1}^{[s]}+\mathbf{z}^{[k]}}_{\text{interference signals}},\end{aligned}$$

where $\max(a,b)$ is the larger value (i.e., a if $a\geq b$, otherwise b) and $\min(a,b)$ is the smaller value (i.e., a if $a\leq b$, otherwise b). Therefore, the dimension of the interference signals is aligned to $\mathsf{K}-D$, by the linearly independece of $\mathsf{K}-D$ vectors $\mathbf{v}^{[<k+D+1>_{\mathsf{K}+1}]}$ $\mathbf{v}^{[<k+D+2>_{\mathsf{K}+1}]}$ $\ldots\mathbf{v}^{[<k+\mathsf{K}>_{\mathsf{K}+1}]}$, receiver k can recover $U+1$ messages from the first $\mathsf{K}-D+U+1$ time slots. Thus, the communication degree of freedom is $\frac{\mathsf{K}(U+1)}{\mathsf{K}-D+U+1}$.

For the sensor, the received signal is:

$$\begin{aligned}\mathbf{y}^{[\mathsf{K}+1]}&=H^{[\mathsf{K}+1,\mathsf{K}+1]}\mathbf{x}_s^{[\mathsf{K}+1]}+\sum_{i=D+1}^{\mathsf{K}-U}H^{[\mathsf{K}+1,i]}\mathbf{x}^{[i]}+\mathbf{z}^{[\mathsf{K}+1]}\\&=H^{[\mathsf{K}+1,\mathsf{K}+1]}\mathbf{x}_s^{[\mathsf{K}+1]}+\sum_{i=1}^{\mathsf{K}-D}\mathbf{v}^{[i]}\sum_{s=max(1,i-U)}^{min(\mathsf{K}-U-D,i)}H^{\mathsf{K}+1,s}W_{i-s+1}^{[s]}\\&+\sum_{i=D+1}^{\mathsf{K}-U}H^{[\mathsf{K}+1,i]}\mathbf{x}_s^{[i]}+\mathbf{z}^{[\mathsf{K}+1]}.\end{aligned}$$

The interference dimension from communication signals is the same as that for communication users, i.e., $\mathsf{K}-D$. Therefore, we can get a left null vector $\mathbf{v}_0$ in $\left[\mathbf{v}^{[D+1]}\mathbf{v}^{[D+2]}\ldots\mathbf{v}^{[\mathsf{K}]}\right]$ to remove the communication interference within $\mathsf{K}-D+U+1$ time slots, thus

$$\begin{aligned}&\mathbf{v}_0\mathbf{y}^{[\mathsf{K}+1]}\\&=H^{[\mathsf{K}+1,\mathsf{K}+1]}\mathbf{v}_0\mathbf{x}_s^{[\mathsf{K}+1]}+\sum_{i=D+1}^{\mathsf{K}-U}H^{[\mathsf{K}+1,i]}\mathbf{v}_0\mathbf{x}^{[i]}+\mathbf{v}_0\mathbf{z}^{[\mathsf{K}+1]}\\&=H^{[\mathsf{K}+1,\mathsf{K}+1]}\mathbf{v}_0\mathbf{x}_s^{[\mathsf{K}+1]}+\sum_{i=D+1}^{\mathsf{K}-U}H^{[\mathsf{K}+1,i]}\mathbf{v}_0\mathbf{x}_s^{[i]}+z'^{[\mathsf{K}+1]}.\end{aligned}$$

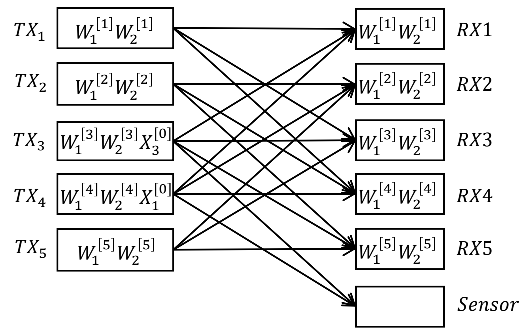


Fig. 7: Non-cooperative interference channel in Example 3.

Let $x_1^{[0]}=\left[\mathbf{v}_0\mathbf{x}_s^{[\mathsf{K}+1]}\quad\mathbf{v}_0\mathbf{x}_s^{[D+1]}\quad\cdots\quad\mathbf{v}_0\mathbf{x}_s^{[K-U]}\right]^H$, in other $(\mathsf{K}-D-U)$ transmission, we can use the same way to get $x_2^{[0]},x_3^{[0]},\cdots,x_{\mathsf{K}-D-U+1}^{[0]}$, satisfying $[\mathbf{x}_1^{[0]},\mathbf{x}_2^{[0]},\cdots,x_{\mathsf{K}-D-U+1}^{[0]}][\mathbf{x}_1^{[0]},\mathbf{x}_2^{[0]},\cdots,x_{\mathsf{K}-D-U+1}^{[0]}]^H=I$. So one effective observation of the channel can be obtained, achieving the sDoF equal to $\frac{1}{\mathsf{K}-D+U+1}$.

B. Proof of Theorem 4-b: Topology 2 with Non-cooperative Transmitters

Considering the bistatic ISAC systems with heterogeneous connectivity and non-cooperative transmitters, we describe the main idea of the proposed TIM scheme for the Topology 2 through the following example, while the general description can be found in the Appendix C and Appendix D.

Example 3 (Topology 2 with non-cooperative transmitters)**.** *Consider the bistatic ISAC system with partially connected interference channel including* 5 *single-antenna transmitters,* 5 *single-antenna receivers and one sensor, as illustrated in Fig. 7. Each receiver or sensor is disconnected from the previous* $U=1$ *and next* $D=1$ *transmitters, and connected to other* 3 *transmitters; the topology belongs to Topology 2 from the* $(5,1,1)$*-neighboring antidotes network described in Definition 2. We have*

$$\mathcal{R}_c^{[1]}=\{1,3,4\},\ \mathcal{R}_c^{[2]}=\{2,4,5\},\ \mathcal{R}_c^{[3]}=\{1,3,5\},$$
$$\mathcal{R}_c^{[4]}=\{1,2,4\},\ \mathcal{R}_c^{[5]}=\{2,3,5\},\ \mathcal{R}_s=\{3,4\}.$$

For this system, the tradeoff points $(\textit{sDoF},\textit{cDoF})=(1,0)$, $(\textit{sDoF},\textit{cDoF})=(0,2)$ *can be achieved by the sensing-only scheme and communication-only scheme, respectively. We then propose a TIM ISAC scheme with* $(\textit{sDoF},\textit{cDoF})=(1/5,2)$.

Let $t_0=10$*. During these* 10 *time slots, the sensor will obtain* 2 *effective observations, through the sensing signals* $\mathbf{x}_1^{[0]},\mathbf{x}_2^{[0]}\in\mathbb{C}^{3\times 1}$*, satisfying* $[\mathbf{x}_1^{[0]},\mathbf{x}_2^{[0]}][\mathbf{x}_1^{[0]},\mathbf{x}_2^{[0]}]^H=I$*. In each period of* 5 *time slots, we let each receiver recover* 2 *communication messages, and let the sensor obtain one effective observation. In the following, we illustrate our proposed scheme for the first* 5 *time slots, where each receiver* $j\in\{1,\ldots,5\}$ *should recover* 2 *communication messages* $W_1^{[j]},W_2^{[j]}\in\mathbb{C}$.

Denote the transmission signals of transmitter j *by* $\mathbf{x}^{[j]}=[\mathbf{x}^{[j]}(1),\ldots,\mathbf{x}^{[j]}(5)]^T\in\mathbb{C}^{5\times 1}$*. Then for the transmitters not connected to the sensor, we design*

$$\mathbf{x}^{[1]}=\mathbf{v}^{[1]}W_1^{[1]}+\mathbf{v}^{[2]}W_2^{[1]},$$

$$\mathbf{x}^{[2]} = \mathbf{v}^{[2]} W_1^{[2]} + \mathbf{v}^{[3]} W_2^{[2]},$$
$$\mathbf{x}^{[5]} = \mathbf{v}^{[5]} W_1^{[5]} + \mathbf{v}^{[1]} W_2^{[5]};$$

for the transmitters connected to the sensor, we design

$$\mathbf{x}^{[3]} = \mathbf{v}^{[3]} W_1^{[3]} + \mathbf{v}^{[4]} W_2^{[3]} + \mathbf{x}_s^{[3]},$$
$$\mathbf{x}^{[4]} = \mathbf{v}^{[4]} W_1^{[4]} + \mathbf{v}^{[5]} W_2^{[4]} + \mathbf{x}_s^{[4]}.$$

Note that $\mathbf{v}^{[1]}, \ldots, \mathbf{v}^{[5]} \in \mathbb{C}^{5\times 1}$ *represent* 5 *random i.i.d. vectors, which are linearly independent, and* $\mathbf{x}_s^{[3]}, \mathbf{x}_s^{[4]} \in \mathbb{C}^{5\times 1}$ *are the transmitted sensing signals to be determined later.*

In this partially connected case, by exploiting the network topology, we align the interference to each receiver. Let us consider receiver 1 *as an example. The received signal after removing the dedicated sensing signals at time slot* $t \in \{1, \ldots, 5\}$ *is* $\hat{y}^{[1]}(t) = \sum_{i\in\mathcal{R}_1=\{1,3,4\}} H^{[1i]}(t)\mathbf{x}^{[i]}(t) + z^{[1]}(t)$. *Hence, its received signals from the* 5 *time slots can be formulated as:*

$$\begin{aligned}\hat{\mathbf{y}}^{[1]} &= \begin{bmatrix} y^{[1]}(1) \\ \vdots \\ y^{[1]}(5) \end{bmatrix} = H^{[11]}\left(\mathbf{v}^{[1]} W_1^{[1]} + \mathbf{v}^{[2]} W_2^{[1]}\right) \\ &+ H^{[13]}\left(\mathbf{v}^{[3]} W_1^{[3]} + \mathbf{v}^{[4]} W_2^{[3]}\right) \\ &+ H^{[14]}\left(\mathbf{v}^{[4]} W_1^{[4]} + \mathbf{v}^{[5]} W_2^{[4]}\right) + \mathbf{z}^{[1]} \\ &= \underbrace{\mathbf{v}^{[1]} H^{[11]} W_1^{[1]} + \mathbf{v}^{[2]} H^{[11]} W_2^{[1]}}_{\textit{desired signals}} + \mathbf{v}^{[3]} H^{[13]} W_1^{[3]} \\ &+ \mathbf{v}^{[4]}\left(H^{[13]} W_2^{[3]} + H^{[14]} W_1^{[4]}\right) + \mathbf{v}^{[5]} H^{[14]} W_2^{[4]} + \mathbf{z}^{[1]}.\end{aligned}$$

Thus in $\hat{\mathbf{y}}^{[1]}$ *the dimension of the designed signals is* 2*, and the dimension of the interference is aligned to* 3*. By the linearly independence of* $\mathbf{v}^{[1]}, \mathbf{v}^{[2]}, \mathbf{v}^{[3]}, \mathbf{v}^{[4]}, \mathbf{v}^{[5]}$*, receiver* 1 *can recover* $W_1^{[1]}, W_2^{[1]}$ *from the first* 5 *time slots. Similarly, each receiver can recover* 2 *communication messages from the first* 5 *time slots; thus the cDoF is* 2*.*

For the sensor, its received signal at time slot $t \in \{1, \ldots, 5\}$ *is* $y^{[6]}(t) = \sum_{i\in\mathcal{R}_s=\{3,4\}} H^{[6i]}(t)\mathbf{x}^{[i]}(t) + z^{[6]}(t)$*. Hence, its received signals from the* 5 *time slots can be formulated as:*

$$\begin{aligned}\mathbf{y}^{[6]} &= \begin{bmatrix} y^{[6]}(1) \\ \vdots \\ y^{[6]}(5) \end{bmatrix} = H^{[63]}\left(\mathbf{v}^{[3]} W_1^{[3]} + \mathbf{v}^{[4]} W_2^{[3]} + \mathbf{x}_s^{[3]}\right) \\ &+ H^{[64]}\left(\mathbf{v}^{[4]} W_1^{[4]} + \mathbf{v}^{[5]} W_2^{[4]} + \mathbf{x}_s^{[4]}\right) + \mathbf{z}^{[6]} \\ &= \mathbf{v}^{[3]} H^{[63]} W_1^{[3]} + \mathbf{v}^{[4]}\left(H^{[53]} W_2^{[3]} + H^{[54]} W_1^{[4]}\right) \\ &+ \mathbf{v}^{[5]} H^{[64]} W_2^{[4]} + H^{[53]}\mathbf{x}_s^{[3]} + H^{[54]}\mathbf{x}_s^{[4]} + \mathbf{z}^{[6]}.\end{aligned}$$

Note that $[\mathbf{v}^{[3]}, \mathbf{v}^{4]}, \mathbf{v}^{[5]}]$ *with dimension* 5×3 *has* 2 *linearly independent left null vectors with high probability; let* $\mathbf{v}_0 \in \mathbb{C}^{1\times 5}$ *be a non-zero left null vector. Thus we have*

$$\begin{aligned}\mathbf{v}_0\mathbf{y}^{[6]} &= \mathbf{v}_0\mathbf{v}^{[3]} H^{[53]} W_1^{[3]} + \mathbf{v}_0\mathbf{v}^{[4]}\left(H^{[63]} W_2^{[3]} + H^{[64]} W_1^{[4]}\right) \\ &+ \mathbf{v}_0\mathbf{v}^{[5]} H^{[54]} W_2^{[4]} + \mathbf{v}_0 H^{[53]}\mathbf{x}_s^{[3]} + \mathbf{v}_0 H^{[54]}\mathbf{x}_s^{[4]} + \mathbf{v}_0\mathbf{z}^{[5]} \\ &= \begin{bmatrix} H^{[53]} & H^{[54]} \end{bmatrix} \begin{bmatrix} \mathbf{v}_0\mathbf{x}_s^{[3]} \\ \mathbf{v}_0\mathbf{x}_s^{[4]} \end{bmatrix} + \mathbf{z}'^{[5]},\end{aligned}$$

where $z'^{[5]} = \mathbf{v}_0\mathbf{z}^{[5]}$*. Hence, the sensor can eliminate the interference from communication messages, and obtain an effective observation from the first* 5 *time slots.*

We then fix the values of $\mathbf{x}_s^{[3]}, \mathbf{x}_s^{[4]}$*. Recall that* $\mathbf{x}_1^{[0]}, \mathbf{x}_2^{[0]}$ *has been selected before, satisfying* $[\mathbf{x}_1^{[0]}, \mathbf{x}_2^{[0]}][\mathbf{x}_1^{[0]}, \mathbf{x}_2^{[0]}]^H = I$*. Now we let* $x_1^{[0]} = [\mathbf{v}_0\mathbf{x}_s^{[3]}, \mathbf{v}_0\mathbf{x}_s^{[4]}]$*; in other words, we solve* $\mathbf{x}_s^{[3]}$ *with* 5 *parameters satisfying* $\mathbf{v}_0\mathbf{x}_s^{[3]} = x_{1,1}$*, where* $x_{1,1}$ *is the first element of* $\mathbf{x}_1^{[0]}$*, and similarly for* $\mathbf{x}_s^{[4]}$*. Therefore, the sensing degree of freedom is* $1/5$*.*

C. Proof of Theorem 5-a: Topology 1 with Cooperative Transmitters

We then consider the bistatic ISAC systems with heterogeneous connectivity and cooperative transmitters. For the Topology 1 based on the $(\mathsf{K}+1, d)$-regular network, we propose an ISAC scheme based on TIM achieving the tradeoff point $(\mathrm{sDoF}, \mathrm{cDoF}) = \left(\frac{1}{d+1}, \frac{2\mathsf{K}}{d+1}\right)$. Recall that the network topology could be expressed as

$$\mathcal{R}_c^{[k]} = \{k, <k+1>_{\mathsf{K}+1}, \ldots, <k+d-1>_{\mathsf{K}+1}\},$$
$$\mathcal{R}_s = \{1, 2, \ldots, d-1, \mathsf{K}+1\},$$

for each $k \in \{1, \ldots, \mathsf{K}\}$.

Let $t_0 = (d+1)(d+1)$. In $(d+1)(d+1)$ time slots, the sensor will obtain $d+1$ effective observations, through the sensing signals $\mathbf{x}_1^{[0]}, \mathbf{x}_2^{[0]}, ..., \mathbf{x}_{d+1}^{[0]} \in \mathbb{C}^{(d+1)\times 1}$, satisfying $[\mathbf{x}_1^{[0]}, \mathbf{x}_2^{[0]}, ..., \mathbf{x}_{d+1}^{[0]}][\mathbf{x}_1^{[0]}, \mathbf{x}_2^{[0]}, ..., \mathbf{x}_{d+1}^{[0]}]^H = I$. In each period of $(d+1)$ time slots, we let each receiver decode 2 communication messages, and let the sensor obtain one effective observation. In the next, we illustrate our proposed scheme for the first $d+1$ time slots, where each receiver $k \in \{1, 2, ..., \mathsf{K}\}$ should recover 2 messages $W_1^{[k]}, W_2^{[k]} \in \mathbb{C}$.

Denote the transmitted signals of transmitter $j \in \{1, \ldots, \mathsf{K}+1\}$ by $\mathbf{x}^{[j]} = [x^{[j]}(1), \ldots, x^{[j]}(d+1)]^T \in \mathbb{C}^{(d+1)\times 1}$. For the d transmitters connected to the sensor, let

$$\mathbf{x}^{[j]} = \mathbf{v}^{[j]} W_1^{[j]} + \mathbf{v}^{[j+1]} W_2^{[j-d+1]} + \mathbf{x}_s^{[j]}, \forall j \in \{1, 2, \ldots, d-2\},$$
$$\mathbf{x}^{[d-1]} = \mathbf{x}_s^{[d-1]} + \mathbf{v}^{[d-1]} W_1^{[d-1]},$$
$$\mathbf{x}^{[\mathsf{K}+1]} = \mathbf{x}_s^{[\mathsf{K}+1]} + \mathbf{v}^{[1]} W_2^{[\mathsf{K}-d+2]}.$$

For the transmitters in the set $\{d, d+1, \ldots, \mathsf{K}\}$, which are not connected to the sensor, let

$$\mathbf{x}^{[j]} = \mathbf{v}^{[j]} W_1^{[j]} + \mathbf{v}^{[j+1]} W_2^{[j-d+1]}, \forall j \in \{d, d+1, \ldots, \mathsf{K}\}.$$

The $k \in \{d, \ldots, \mathsf{K}-d+1\}$-th receiver only connected to the communication transmitter, the received signal is:

$$\begin{aligned}\mathbf{y}^{[k]} &= \sum_{j=k}^{k+d-1} H^{[kj]}\left(\mathbf{v}^{[j]} W_1^{[j]} + \mathbf{v}^{[j+1]} W_2^{[j-d+1]}\right) + \mathbf{z}^{[k]} \\ &= \underbrace{H^{[kk]}\mathbf{v}^{[k]} W_1^{[k]} + H^{[k,k+d-1]}\mathbf{v}^{[k+d]} W_2^{[k]}}_{\text{desired signal}} \\ &+ \sum_{j=k+1}^{k+d-1} \mathbf{v}^{[j]}\left(H^{[kj]} W_1^{[j]} + H^{[k,j-1]} W_2^{[j-d]}\right) + \mathbf{z}^{[k]}.\end{aligned}$$

The dimension of the desired signals is 2, and the dimension of the interference signals is $d-1$, we can achieve symmetric cDoF $= \frac{2}{d+1}$ for each receiver. Note that all interference at the receivers originates from communication signals, while sensing signals can be eliminated. The receiver that only receives communication signals experiences the strongest interference. For receivers that acquire both sensing and communication signals, the interference dimension does not exceed $d-1$. Given that the useful signal decoded by all receivers is 2, the communication degrees of freedom for all other receivers is $\frac{2}{d+1}$ as well. Thus, the cDoF is $\frac{2\mathsf{K}}{d+1}$.

For the sensor, the received signal $\mathbf{y}^{[s]}$ is:

$$\begin{aligned}\mathbf{y}^{[s]} &= \sum_{j=1}^{d-2} H^{[\mathsf{K}+1,j]}\left(\mathbf{v}^{[j]}W_1^{[j]} + \mathbf{v}^{[j+1]}W_2^{[j-d+1]} + \mathbf{x}_s^{[j]}\right) \\ &\quad + H^{[\mathsf{K}+1,\mathsf{K}+1]}\left(\mathbf{x}_s^{[\mathsf{K}+1]} + \mathbf{v}^{[1]}W_2^{[\mathsf{K}-d+2]}\right) \\ &\quad + H^{[\mathsf{K}+1,d-1]}\left(\mathbf{x}_s^{[d-1]} + \mathbf{v}^{[d-1]}W_1^{[d-1]}\right) \\ &= \sum_{j=1}^{d-1}\mathbf{v}^{[j]}(H^{[\mathsf{K}+1,j]}W_1^{[j]} + H^{[\mathsf{K}+1,j-1]}W_2^{[j-d]}) + \mathbf{z}^{[\mathsf{K}+1]}.\end{aligned}$$

Note that $[\mathbf{v}^{[1]}\mathbf{v}^{[2]}\cdots\mathbf{v}^{[d-1]}]$ with dimension $(d+1)\times(d-1)$ has 2 linearly independent left null vectors with high probability; let $\mathbf{v}_0\ in\mathbb{C}^{1\times d+1}$ be a non-zero left null vector. Thus

$$v_0\mathbf{y}^{[s]} = \sum_{j=1}^{d-1} H^{[K+1,j]}v_0\mathbf{x}_s^{[j]} + H^{[K+1,K+1]}v_0\mathbf{x}_s^{[K+1]} + \mathbf{v}_0\mathbf{z}^{[\mathsf{K}+1]}.$$

Let $x_1^{[0]} = \left[v_0\mathbf{x}_s^{[1]}, v_0\mathbf{x}_s^{[2]}, \cdots, v_0\mathbf{x}_s^{[d-1]}, v_0\mathbf{x}_s^{[\mathsf{K}+1]}\right]$, and this gives one valid observation. So one effective observation of the channel can be obtained, achieving a sDoF of $\frac{1}{d+1}$.

VI. Simulation Results

We simulate proposed schemes for the bistatic ISAC systems with heterogeneous coherent times and the interference channel in Example 2, and with heterogeneous connectivity and interference channel in Example 3, compared with the method by treating interference as noise (TIN) at the sensor.[9]

A. Heterogeneous Coherence Times

The simulation setting is as follows: the communication symbols are modulated by the QPSK modulation. The distance between the transmitter and the communication receiver is between $[50, 100]$ meters, and the distance between the transmitter and the sensor is between $[10, 30]$ meters. The fixed transmission power for both the communication signal and the sensing signal is set to 30 dBm, and the channel fading characteristics follow a Rayleigh distribution, with the SNR set to $[5, 35]$ dB. The transmitters have perfect CSI on communication channels but lack prior information about the sensing channel, and the dedicated sensing signal is a deterministic signal known by both the transmitting and receiving ends. The simulation flowchart is shown in Fig. 8. For the sensing task, we use the LS estimation method to estimate the channel, given by $\hat{\mathbf{H}} = (\mathbf{X}^{\mathbf{H}}\mathbf{X})^{-1}\mathbf{X}^{\mathbf{H}}\mathbf{Y}$, where $\hat{\mathbf{H}}$ represents the estimated channel, $\mathbf{X}$ represents the sensing signal, and $\mathbf{Y}$ represents the received signal. Since the LS method is very sensitive to noise, we observe the performance differences under SNR values ranging from [5, 35] dB. Analysis shows that at higher SNR conditions, the proposed method is less affected, and the channel estimation error (CEE) decreases with increasing SNR. The CEE is defined as CEE $= \|\mathbf{H}-\hat{\mathbf{H}}\|^2$, where $\mathbf{H}$ is the actual channel state, $\hat{\mathbf{H}}$ is the estimate channel state, $\|\cdot\|$ denotes the norm of the vector, commonly the Euclidean norm. In contrast, the reference method treats the communication signal as noise, so even with low noise power, the high transmission power of the communication signal results in poor estimation accuracy, with minimal changes in CEE as SNR increases. In comparison, our method provides a gain of [7, 75] dB on the estimation accuracy as shown in Fig. 9(a), while the communication performances (i.e., communication rate or bit error rate) of the two methods are the same.

B. Heterogeneous Connectivity

We modify the simulation in the above case to the interference channel with partial connection as in Example 3. Note that in this case, the transmitter only knows the network topology, without known the CSI of any receiver nor sensor. Our method provides a gain of [3.87, 18.892] dB on the estimation accuracy as shown in Fig. 9(b).

VII. Conclusion

This paper explored the interference management issue in bistatic ISAC systems, specifically how to effectively cancel the interference from the communication signals to the sensor, by applying the BIA and TIM strategies. The key is to leverage the heterogeneity (on the coherence times or on the connectivity) between the sensor and receivers. By using the communication and sensing degrees of freedom as the metric, the proposed schemes provide improved tradeoff points, compared to the time-sharing between the communication-only and sensing-only points. Simulations on the proposed schemes and the existing TIN scheme are provided, showing that the proposed schemes reduce the sensing CEE, while maintaining the same communication performance.

Appendix A
Sensing Degree Of Freedom

In the previous discussion, we introduced the definition of the sensing degree of freedom (sDoF) as the number of effective observational dimensions related to channel or target parameters. Our objective is to utilize this metric to characterize the upper bound of environmental information acquisition in an ISAC system under communication constraints.

In ISAC systems, the channel matrix $\mathbf{H}(\eta)$ not only describes the propagation characteristics of the communication link but also inherently encodes information about the sensing

[9]Since the sensor channel is unknown to the sensor, the successive interference cancellation (SIC) decoding strategy in [38] which first decodes the communication messages and then estimates the sensing state, cannot be used in our simulation.

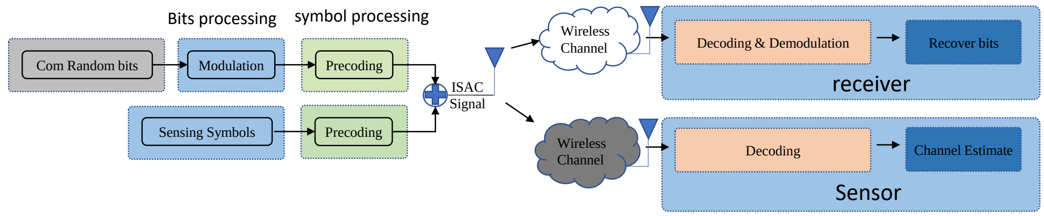
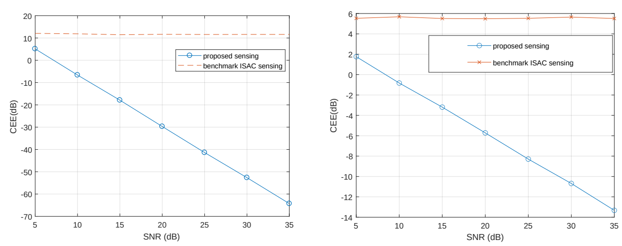


Fig. 8: Simulation flowchart.



(a) (b)

Fig. 9: Simulation results: (a) BIA (b) TIM.

targets, where η represents the sensing parameters. For multi-antenna systems (e.g., MIMO), $\mathbf{H}(\eta)$ can be expressed as:

$$\mathbf{H}(\eta) = \sum_{k=1}^{\mathsf{K}} \alpha_k \mathbf{a}_r(\theta_k)\mathbf{a}_t^H(\phi_k)e^{j2\pi\nu_k t}, \tag{13}$$

where:

K represents the number of propagation paths, including both direct and reflected paths, α_k is the complex gain of the k-th path, incorporating path loss and reflection coefficients, $\mathbf{a}_r(\theta_k)$ and $\mathbf{a}_t(\phi_k)$ are the receive and transmit array steering vectors, associated with the angle of arrival (AoA, θ_k) and angle of departure (AoD, ϕ_k) of the target, respectively, ν_k denotes the Doppler shift, which is related to the target's velocity.

From this formulation, it is evident that the channel matrix $\mathbf{H}(\eta)$ inherently contains information about the sensing targets, such as their location, velocity, and angular parameters. By analyzing the structure and properties of $\mathbf{H}(\eta)$, these sensing parameters can be extracted.

Assuming the transmitted signal is $\mathbf{X}$, the received signal can be expressed as:

$$\mathbf{Y} = \mathbf{H}(\eta)\mathbf{X} + \mathbf{N}, \tag{14}$$

where $\mathbf{N}$ represents noise. Given knowledge of $\mathbf{X}$ and the received signal $\mathbf{Y}$, we can estimate $\mathbf{H}(\eta)$ using techniques such as least squares (LS) or maximum likelihood (ML). For simplicity, we consider the LS estimation:

$$\mathbf{H}\hat{(\eta)} = \mathbf{Y}\mathbf{X}^{\dagger}, \tag{15}$$

where $\mathbf{X}^{\dagger}$ denotes the pseudo-inverse of $\mathbf{X}$. By analyzing $\mathbf{H}\hat{(\eta)}$, we can extract the following target parameters.

To further clarify this process, the channel matrix $\mathbf{H}(\eta)$ can be modeled as a function of the sensing parameters:

$$\mathbf{H}(\eta) = f(\boldsymbol{\theta}, \boldsymbol{\phi}, \boldsymbol{\tau}, \boldsymbol{\nu}), \tag{16}$$

where $\boldsymbol{\theta}$ is the vector of angles of arrival,$\boldsymbol{\phi}$ is the vector of angles of departure,$\boldsymbol{\tau}$ is the vector of path delays,$\boldsymbol{\nu}$ is the vector of Doppler shifts.

The sensing parameters can then be estimated by solving the following optimization problem:

$$\{\hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\phi}}, \hat{\boldsymbol{\tau}}, \hat{\boldsymbol{\nu}}\} = \arg \min_{\boldsymbol{\theta},\boldsymbol{\phi},\boldsymbol{\tau},\boldsymbol{\nu}} \|\mathbf{Y} - f(\boldsymbol{\theta}, \boldsymbol{\phi}, \boldsymbol{\tau}, \boldsymbol{\nu})\mathbf{X}\|_F^2, \tag{17}$$

where $\|\cdot\|_F$ denotes the Frobenius norm.It follows that the more data we acquire, the more accurate our parameter estimation becomes. Therefore, sensing performance can be directly linked to the sensing degree of freedom. The goal of this work is to optimize signal design to maximize SDoF, thereby improving sensing performance.

Appendix B
Proof of Theorem 3: MU-MIMO

Next, we consider the bistatic ISAC systems with heterogeneous coherence times and the MU-MIMO channel including one transmitter with m antennas, K multi-antenna receivers (receiver $k = 1, \ldots, \mathsf{K}$ with n_k antennas), and one single-antenna sensor. By Theorem 3, the proposed BIA scheme achieves the tradeoff point $(\mathrm{sDoF}, \mathrm{cDoF}) = \left(\frac{\left\lceil \frac{\mathsf{m}}{\sum_{k=1}^{\mathsf{K}} \mathsf{n}_k}\right\rceil - 1}{\left\lceil \frac{\mathsf{m}}{\sum_{k=1}^{\mathsf{K}} \mathsf{n}_k}\right\rceil}, \frac{\mathsf{m}}{\left\lceil \frac{\mathsf{m}}{\sum_{k=1}^{\mathsf{K}} \mathsf{n}_k}\right\rceil}\right)$. Note that when $\mathsf{m} \le \sum_{k=1}^{\mathsf{K}} \mathsf{n}_k$, the tradeoff point becomes $(0, \mathsf{m})$, which could be simply achieved by the communication-only scheme. Hence, we only consider the case $\mathsf{m} > \sum_{k=1}^{\mathsf{K}} \mathsf{n}_k$. For ease of notation, we define that $\left\lceil \frac{\mathsf{m}}{\sum_{k=1}^{\mathsf{K}} \mathsf{n}_k} \right\rceil := \mathsf{b}$, and $< \mathsf{m} >_{\sum_{k=1}^{\mathsf{K}} \mathsf{n}_k} := \mathsf{r}$. The number of receivers, denoted by S(which is determined by $\sum_{i=1}^{S} \mathsf{n}_i \le \mathsf{r} < \sum_{j=1}^{S+1} \mathsf{n}_i$), means that can perfectly exploit the multi-antenna diversity gain at the receiver during each transmission time slot (perfect meaning that the number of symbols decodable at each moment is equal to the number of antennas), $\mathsf{q} = \mathsf{r} - \sum_{i=1}^{\mathsf{S}} \mathsf{n}_i$.

Let $t_0 = (\mathsf{b}\left\lceil \frac{\mathsf{m}}{\mathsf{b}-1} \right\rceil)$, and let the receiver $k \in \{1, 2, ..., \mathsf{K}\}$ decode messages $W_{j,i,t}^{[k]}, i \in \{1, 2, ..., \mathsf{n}_k\}$ at time slot $t \in \{1, 2, ..., \mathsf{b} - 1\}$ in the j-th transmission, receiver $k \in \{1, 2, ..., \mathsf{S}\}$ decode messages $W_{j,i,\mathsf{b}}^{[k]}, i \in \{1, 2, ..., \mathsf{n}_k\}$ at time slot b in the j-th transmission in addition, and receiver $\mathsf{S}+1$ decode messages $W_{j,i,\mathsf{b}}^{[\mathsf{S}+1]}, i \in \{1, 2, ..., \mathsf{q}\}$ at time slot b in the j-th transmission in addition. Communication signals in the b time slots by transmitter be $\mathbf{x}_c(1) = \mathbf{x}_c(2) = \cdots = \mathbf{x}_c(\mathsf{b}) = \mathbf{x}_c \in \mathbb{C}^{m \times 1}$ (it's the same as previous section, b time slots represent the duration of one transmission in the system. We accumulate valid observation data for the sensing task during the $\left\lceil \frac{\mathsf{m}}{\mathsf{b}-1} \right\rceil$ transmissions. For simplicity and without loss of generality, we focus on a specific transmission process. For each time slot t, we design the communication signals by transmitter with m anttenas as :

$$\mathbf{x}_c(1) = \ldots = \mathbf{x}_c(b) = \sum_{t=1}^{\mathsf{b}-1} \sum_{i=1}^{\mathsf{n}_k} \sum_{k=1}^{\mathsf{K}} \mathbf{v}_{i,t}^{[k]} W_{i,t}^{[k]} + \sum_{i=1}^{\mathsf{n}_k} \sum_{k=1}^{\mathsf{S}} \mathbf{v}_{i,\mathsf{b}}^{[k]} W_{i,\mathsf{b}}^{[k]} + \sum_{i=1}^{\mathsf{q}} \mathbf{v}_{i,\mathsf{b}}^{[\mathsf{S}+1]} W_{i,\mathsf{b}}^{[\mathsf{S}+1]}$$

where $\mathbf{v}_{i,t}^{[k]} : k \in \{1,\ldots,\mathsf{K}\}, i \in \{1,\ldots,\mathsf{n}_k\}, t \in \{1,\ldots,\mathsf{b}-1\}$,$\mathbf{v}_{i,\mathsf{b}}^{[k]} : k \in \{1,\ldots,\mathsf{S}\}, i \in \{1,\ldots,\mathsf{n}_k\}$,$\mathbf{v}_{i,\mathsf{b}}^{[\mathsf{S}+1]} : i \in \{1,\ldots,\mathsf{q}\}$ are the precoding vectors to be determined later, each with dimension $\mathsf{m} \times 1$. At time slots $t \in \{1,\ldots,\mathsf{b}\}$, the signal received by the $k \in \{1,\ldots,\mathsf{K}\}$-th receiver is:

$$\mathbf{y}^{[k]} = \mathbf{H}^{[k1]}(t)(\mathbf{x}_c(t) + \mathbf{x}_s(t)) + \mathbf{z}^{[k]}(t),$$

we can remove the sensing signal from the received signal, resulting in an estimate siganl that only concerns the communictaion signals:

$$\hat{\mathbf{y}^{[k]}} = \begin{bmatrix} h^{[11]}(t) & h^{[12]}(t) & \cdots & h^{[1\mathsf{m}]}(t) \\ h^{[21]}(t) & h^{[22]}(t) & \cdots & h^{[2\mathsf{m}]}(t) \\ \vdots & \vdots & \ddots & \vdots \\ h^{[\mathsf{n}_k 1]}(t) & h^{[\mathsf{n}_k 2]}(t) & \cdots & h^{[\mathsf{n}_k \mathsf{m}]}(t) \end{bmatrix} \mathbf{x}_c(t) + \mathbf{z}^{[k]}(t),$$

where $h^{[ji]}(t) \in \mathbb{C}$ denotes the channel coefficient from the i-th antenna of the transmitter to the j-th antenna of user k at time slot t . The precoding vectors are designed to ensure that

M1. for each $k \in \{1,\ldots,\mathsf{S}\}$, $i \in \{1,\ldots,\mathsf{n}_k\}$and $t \in \{1,\ldots,\mathsf{b}\}$, the k-th receiver can decode $W_{i,t}^{[k]}, i \in \{1,\ldots,\mathsf{n}_k\}$ from $\hat{y}^{[k]}(t)$;

M2. for the $\mathsf{S}+1$-th receiver can decode $W_{i,t}^{[\mathsf{S}+1]}, i \in \{1,\ldots,\mathsf{n}_{\mathsf{S}+1}\}, t \in \{1,\ldots,\mathsf{b}-1\}$ and $W_{i,\mathsf{b}}^{[\mathsf{S}+1]}, i \in \{1,\ldots,\mathsf{q}\}$ from $\hat{y}^{[\mathsf{S}+1]}(t)$;

M3. for each $k \in \{\mathsf{S}+2,\ldots,\mathsf{K}\}$, $i \in \{1,\ldots,\mathsf{n}_k\}$and $t \in \{1,\ldots,\mathsf{b}-1\}$, the k-th receiver can decode $W_{i,t}^{[k]}, i \in \{1,\ldots,\mathsf{n}_k\}$ from $\hat{y}^{[k]}(t)$;

To satisfy the above conditions, we propose the following design on the precoding vectors: Concatenate the channel matrices of all users at the time slots when they can correctly decode, column by column, we can obtain:

$$\mathbf{H}_{all} = \begin{bmatrix} \mathbf{H}^{[11]}(1) \\ \vdots \\ \mathbf{H}^{[11]}(\mathsf{b}) \\ \vdots \\ \mathbf{H}^{[\mathsf{S}1]}(\mathsf{b}) \\ \mathbf{H}^{[\mathsf{S}+1,1]}(1) \\ \vdots \\ \mathbf{H}^{[\mathsf{S}+1,1]}(\mathsf{b})[1:\mathsf{q},:] \\ \mathbf{H}^{[\mathsf{S}+2,1]}(1) \\ \vdots \\ \mathbf{H}^{[\mathsf{S}+2,1]}(\mathsf{b}-1) \\ \vdots \\ \mathbf{H}^{[\mathsf{K},1]}(\mathsf{b}-1) \end{bmatrix} \in \mathbb{C}^{\mathsf{m}\times 1},$$

where $\mathbf{H}^{[\mathsf{S}+1,1]}(\mathsf{b})[1:\mathsf{q},:]$ represents taking only the first q rows of $\mathbf{H}^{[\mathsf{S}+1,1]}(\mathsf{b})$, which means that in time slot b, we are utilizing only the first q antennas of the $\mathsf{S}+1$ communication receivers.

- Design on $\mathbf{v}_{i,t}^{[k]}$ for $k \in \{1,\ldots,\mathsf{S}\}, i \in \{1,\ldots,\mathsf{n}_k\}, t \in \{1,\ldots,\mathsf{b}\}$. We let $\mathbf{v}_{i,t}^{[k]}, i \in \{1,\ldots,\mathsf{n}_k\}$ be a right null vector of the matrix $\mathbf{H}_{all} \backslash \mathbf{H}^{[k1]}(t)$ with dimension $(\mathsf{m}-\mathsf{n}_k) \times \mathsf{m}$, there are n_k right null vertors in its null space, which is the precoding vector $\mathbf{v}_{i,t}^{[k]}, i \in \{1,\ldots,\mathsf{n}_k\}$.
- Design on $\mathbf{v}_{i,t}^{[k]}$ for $k \in \{\mathsf{S}+1,\ldots,\mathsf{K}\}, i \in \{1,\ldots,\mathsf{n}_k\}, t \in \{1,\ldots,\mathsf{b}-1\}$. We let $\mathbf{v}_{i,t}^{[k]}, i \in \{1,\ldots,\mathsf{n}_k\}$ be a right null vector of the matrix $\mathbf{H}_{all} \backslash \mathbf{H}^{[k1]}(t)$ with dimension $(\mathsf{m}-\mathsf{n}_k) \times \mathsf{m}$, there are n_k right null vertors in its null space, which is the precoding vector $\mathbf{v}_{i,t}^{[k]}, i \in \{1,\ldots,\mathsf{n}_k\}$.
- Design on $\mathbf{v}_{i,\mathsf{b}}^{[\mathsf{S}+1]}$ for $i \in \{1,\ldots,\mathsf{q}\}$ for the $\mathsf{S}+1$-th receiver at time slot b. We let $\mathbf{v}_{i,\mathsf{b}}^{[\mathsf{S}+1]}$ be a right null vector of the matrix $\mathbf{H}_{all} \backslash \mathbf{H}^{[\mathsf{S}+1,1]}(\mathsf{b})[1:\mathsf{q},:]$ with dimension $\mathsf{m}-\mathsf{q}$, there are q right null vertors in its null space, which is the precoding vector $\mathbf{v}_{i,\mathsf{b}}^{[\mathsf{S}+1]}$.

By the above selection on the precoding vectors, one can check that the decodability conditions M1, M2 and M3 are satisfied with high probability. For example, let us focus on receiver 1. Its received signal after removing the sensing signal at time slot $t \in \{1,\ldots,\mathsf{b}\}$ is

$$\begin{aligned} \hat{y}^{[1]}(t) &= \mathbf{H}^{[11]}(t)\mathbf{x}_c(t) + z^{[1]}(t) \\ &= \mathbf{H}^{[11]}(t)(\sum_{t=1}^{\mathsf{b}-1} \sum_{i=1}^{\mathsf{n}_k} \sum_{k=1}^{\mathsf{K}} \mathbf{v}_{i,t}^{[k]} W_{i,t}^{[k]} \\ &+ \sum_{i=1}^{\mathsf{n}_k} \sum_{k=1}^{\mathsf{S}} \mathbf{v}_{i,\mathsf{b}}^{[k]} W_{i,\mathsf{b}}^{[k]} + \sum_{i=1}^{\mathsf{q}} \mathbf{v}_{i,\mathsf{b}}^{[\mathsf{S}+1]} W_{i,\mathsf{b}}^{[\mathsf{S}+1]}) + z^{[1]}(t). \end{aligned} \quad (18)$$

It can be checked that the product of $\mathbf{H}^{[11]}(t)$ and each precoding vector in (18) (except $\mathbf{v}_{i,t}^{[1]}, i \in \{1,\ldots,\mathsf{n}_1\}$) is 0. Hence, receiver 1 can recover $W_{i,t}^{[1]}, i \in \{1,\ldots,\mathsf{n}_1\}$ with high probability. Thus the cDoF $= \frac{\sum_{i=1}^{\mathsf{S}} \mathsf{b} n_i + \sum_{j=\mathsf{S}+1}^{\mathsf{K}} (\mathsf{b}-1) n_j + \mathsf{q}}{\mathsf{b}} = \frac{m}{\mathsf{b}}$

We then design the transmitted dedicated sensing signals $\mathbf{x}_s(t)$ for $t \in \{1,\dots,\mathsf{b}\}$ as follows (recall that $\mathbf{x}_1^{[0]},\dots,\mathbf{x}_\mathsf{b}^{[0]}$ have been selected before)

$$\mathbf{x}_s(1)=\sum_{i=1}^{\mathsf{b}-1}\mathbf{x}_i^{[0]},\quad \mathbf{x}_s(t)=\sum_{i=1,i\neq t-1}^{\mathsf{b}-1}\mathbf{x}_i^{[0]},\ \forall t\in\{2,\dots,\mathsf{b}\}.$$

At each time slot $t \in \{1,\dots,\mathsf{b}\}$, the sensor receives

$$y_s^{[\mathsf{K}+1]}(t)=\mathbf{h}^{[\mathsf{K}+1]}(t)(\mathbf{x}_c(t)+\mathbf{x}_s(t))+z_s(t),$$

where $\mathbf{h}^{[\mathsf{K}+1]}(t) = [h^{[\mathsf{K}+1,1]}(t),\dots,h^{[\mathsf{K}+1,\mathsf{m}]}(t)]$, and $h^{[\mathsf{K}+1,i]}(t)$ represents the channel coefficient from i-th antenna of the transmitter to the sensor.

Since $\mathbf{h}^{[\mathsf{K}+1]}(1)=\cdots=\mathbf{h}^{[\mathsf{K}+1]}(\mathsf{b})$ and $\mathbf{x}_c(1)=\cdots=\mathbf{x}_c(\mathsf{b})$, the sensor can subtract the received signal at time slot $t\in\{2,\dots,\mathsf{b}\}$ from the received signal at the first time slot:

$$\mathbf{y}_s^{[\mathsf{K}+1]}(1)-\mathbf{y}_s^{[\mathsf{K}+1]}(t)=\mathbf{h}^{[\mathsf{K}+1]}(1)\mathbf{x}_{t-1}^{[0]}+z_s(1)-z_s(t),$$

thereby canceling the interference caused by the communication signals and obtaining an effective observation of $\mathbf{x}_{t-1}^{[0]}$. Over b time slots, a total of $\mathsf{b}-1$ effective observations can be made, and the sensing degrees of freedom (SDoF) is $(\mathsf{b}-1)/\mathsf{b}$.

By the above selection we can select vectors from its null space to code the desired signals $W_{t,j}^{[k]}$. Through this zero-forcing precoding, we can achieve the placement of interference signals in the null space of the channel at the receiver, thereby enabling decoding. As a result, over b time slots, m symblos are totally decoded; thus the achieved communication DoF is $\frac{\mathsf{m}}{\mathsf{b}}=\frac{\mathsf{m}}{\left\lceil\frac{\mathsf{m}}{\sum_{k=1}^{\mathsf{K}}\mathsf{n}_k}\right\rceil}$. At the sensor, since the communication signal remain constant across a time slots, over $\mathsf{b}\left\lceil\frac{\mathsf{m}}{\mathsf{b}}\right\rceil$ time slots, the sensor can obtain $\mathsf{b}\left\lceil\frac{\mathsf{m}}{\mathsf{b}}\right\rceil-\mathsf{b}$ sensing signals where $k=1,2,...,t_0-\mathsf{b}$, satisfying $[\mathbf{x}_1^{[0]},\mathbf{x}_2^{[0]},\dots,\mathbf{x}_{\mathsf{b}\left\lceil\frac{\mathsf{m}}{\mathsf{b}-1}\right\rceil-\mathsf{b}}^{[0]}][\mathbf{x}_1^{[0]},\mathbf{x}_2^{[0]},\dots,\mathbf{x}_{\mathsf{b}\left\lceil\frac{\mathsf{m}}{\mathsf{b}-1}\right\rceil-\mathsf{b}}^{[0]}]^H=I$. And we can obtain the sensing signal $\mathbf{x}_s(t)$ to be transmitted by the k-th antenna at the transmitter as,

$$\begin{aligned}&\begin{bmatrix}x_{(1,k)}+x_{(2,k)}+x_{(3,k)}+\cdots+x_{(\mathsf{b}-1,k)}\\ x_{(2,k)}+x_{(3,k)}+\cdots+x_{(\mathsf{b}-1,k)}\\ x_{(1,k)}+x_{(3,k)}+\cdots+x_{(\mathsf{b}-1,k)}\\ \vdots\\ x_{(1,k)}+x_{(2,k)}+\cdots+x_{(\mathsf{b}-2,k)}\end{bmatrix}\\ &=\begin{bmatrix}1&1&1&\dots&1\\0&1&1&\dots&1\\1&0&1&\dots&1\\ \vdots&\vdots&\vdots&\ddots&\vdots\\1&1&1&\dots&0\end{bmatrix}\begin{bmatrix}x_{(1,k)}\\x_{(2,k)}\\x_{(3,k)}\\ \vdots\\x_{(\mathsf{b}-1,k)}\end{bmatrix}.\end{aligned}\tag{19}$$

For the sensor:

$$\mathbf{y}_s^{[\mathsf{K}+1]}(t)=\mathbf{h}^{[\mathsf{K}+1]}(t)\mathbf{x}(t)+z_s(t),$$

where $\mathbf{h}^{[\mathsf{K}+1]}(t)=\begin{bmatrix}H^{[\mathsf{K}+1,1]}(t)&\dots&H^{[\mathsf{K}+1,\mathsf{m}]}(t)\end{bmatrix}$.

Thus, Since the sensor experiences a slow fading process, the channel remains constant over several time slots, and the communication signal remains constant as well. The sensor can subtract the received signal at time slot t from the received signal at the first time slot:

$$\begin{aligned}\mathbf{y}_s^{[\mathsf{K}+1]}(1)-\mathbf{y}_s^{[\mathsf{K}+1]}(t)&=\mathbf{h}^{[\mathsf{K}+1]}(1)\mathbf{x}(1)-\mathbf{h}^{[\mathsf{K}+1]}(t)\mathbf{x}(t)\\&=\mathbf{h}^{[\mathsf{K}+1]}(t)\mathbf{x}_{t-1}^{[0]}+\mathbf{z}_s(1)-\mathbf{z}_s(t),\end{aligned}$$

thereby canceling the interference caused by the communication signals and obtaining an effective observation of $\mathbf{x}_{t-1}^{[0]}$. Over b time slots, a total of $\mathsf{b}-1$ effective observations can be made, and the sensing degrees of freedom (SDoF) is $(\mathsf{b}-1)/\mathsf{b}$.

The systems total sensing and communication degrees of freedom are $(\mathrm{SDoF},\mathrm{CDoF})=\left(\frac{\left\lceil\frac{\mathsf{m}}{\sum_{k=1}^{\mathsf{K}}\mathsf{n}_k}\right\rceil-1}{\left\lceil\frac{\mathsf{m}}{\sum_{k=1}^{\mathsf{K}}\mathsf{n}_k}\right\rceil},\frac{\mathsf{m}}{\left\lceil\frac{\mathsf{m}}{\sum_{k=1}^{\mathsf{K}}\mathsf{n}_k}\right\rceil}\right)$.

Appendix C
Proof of Theorem 4-b: Topology 2 with Non-cooperative Transmitters

This is the general description of the proposed TIM scheme for Theorem 4-b. For the Topology 2 based on the (K,U,D) neighboring antidotes network, we propose an ISAC scheme based on TIM achieving the tradeoff point $(\mathrm{sDoF},\mathrm{cDoF})=\left(\frac{1}{\mathsf{K}-D+U},\frac{\mathsf{K}(U+1)}{\mathsf{K}-D+U}\right)$ in (5). Each receiver is disconnected from the previous U and next $D\geq U$ transmitters, and connected to other $\mathsf{K}-U-D>0$ transmitters. Remind that the sensor is a subset of one communication that the network topology could be pressed as

$$\begin{aligned}&\mathcal{R}_c^{[k]}=\{k,<k+D+1>_\mathsf{K},\dots,<k-U-1>_\mathsf{K}\},\\&\mathcal{R}_s=\{<i+D+1>_\mathsf{K},\dots,<i-U-1>_\mathsf{K}\},\end{aligned}$$

for each $i,k\in\{1,\dots,\mathsf{K}\}$, To simplify notation without loss of generality, we set $i=1$, thus we can get $\mathcal{R}_s=\{2+D\ \dots,\mathsf{K}-U\}$. Let $t_0=(\mathsf{K}-D-U)(\mathsf{K}-D+U)$, during these $(\mathsf{K}-D-U)(\mathsf{K}-D+U)$ time slots, the sensor will obtain $\mathsf{K}-D-U$ effective observations, through the sensing signals $\mathbf{x}_1^{[0]},\mathbf{x}_2^{[0]},...,\mathbf{x}_{\mathsf{K}-D-U}^{[0]}\in\mathbb{C}^{(\mathsf{K}-D-U)\times1}$, satisfying $[\mathbf{x}_1^{[0]},\mathbf{x}_2^{[0]},...,\mathbf{x}_{\mathsf{K}-D-U}^{[0]}][\mathbf{x}_1^{[0]},\mathbf{x}_2^{[0]},...,\mathbf{x}_{\mathsf{K}-D-U}^{[0]}]^H=I$. In each period of $(\mathsf{K}-D+U)$ time slots, we let each receiver decode $U+1$ communication messages, and let the sensor obtain one effective observation. In the next, we will illustrate our proposed scheme for the first $K-D+U$ time slots, where each receiver $k\in\{1,2,...,\mathsf{K}\}$ should recover $U+1$ messages $W_1^{[k]},\dots,W_{U+1}^{[k]}\in\mathbb{C}$.

Denote the transmitted signals of transmitter $j\in\{1,\dots,\mathsf{K}\}$ by $\mathbf{x}^{[j]}=[x^{[j]}(1),\dots,x^{[j]}(\mathsf{K}-D+U)]^T\in\mathbb{C}^{(\mathsf{K}-D+U)\times1}$. For the transmitters connected to the sensor, we design based on the cyclic coding,

$$\mathbf{x}^{[i]}=\mathbf{v}^{[i]}W_1^{[i]}+\mathbf{v}^{[<i+1>_\mathsf{K}]}W_2^{[i]}+\dots+\mathbf{v}^{[<i+U>_\mathsf{K}]}W_{U+1}^{[i]}+\mathbf{x}_s^{[i]},$$

for each $i\in\mathcal{R}_s\}$, where $\mathbf{x}_s^{[j]},j\in\mathcal{R}_s$ are the sensing signals to be determined later. For the transmitters not connected to the sensor, design based on the cyclic coding,

$$\mathbf{x}^{[i]}=\mathbf{v}^{[i]}W_1^{[i]}+\mathbf{v}^{[<i+1>_\mathsf{K}]}W_2^{[i]}+\dots+\mathbf{v}^{[<i+U>_\mathsf{K}]>}W_{U+1}^{[i]},$$

for each $i\in\{1,\dots,\mathsf{K}\}\setminus\mathcal{R}_s$. Note that $\mathbf{v}^{[1]},...,\mathbf{v}^{[\mathsf{K}]}\in\mathbb{C}^{\mathsf{K}-D+U}$ represent K random i.i.d vectors, where any $\mathsf{K}-D+U$ of them are linearly independent.

For each $k \in \{1, \ldots, \mathsf{K}\}$, by removing the dedicated sensing signals from the received signals, the k-th receiver obtains

$$\hat{\mathbf{y}}^{[k]} = \sum_{i \in \mathcal{R}_c^{[k]} \setminus \{k\}} H^{[ki]}(\mathbf{v}^{[i]} W_1^{[i]} + \ldots + \mathbf{v}^{[<i+U>_\mathsf{K}]} W_{U+1}^{[i]}) + \mathbf{z}^{[k]}$$
$$= \underbrace{H^{[kk]}(\mathbf{v}^{[k]} W_1^{[k]} + \mathbf{v}^{[<k+1>_\mathsf{K}]} W_2^{[k]} + \ldots + \mathbf{v}^{[<k+U>_\mathsf{K}]} W_{U+1}^{[k]})}_{\text{desire signals}}$$
$$+ \underbrace{\sum_{i=k+D+1}^{k+\mathsf{K}-1} \mathbf{v}^{[<i>_\mathsf{K}]} \sum_{s=max(1,i-U)}^{min(\mathsf{K}-U-D,i)} H^{[ks]} W_{i-s+1}^{[s]}}_{\text{interference signals}} + \mathbf{z}^{[k]},$$

where $\max(a,b)$ is the larger value (i.e., a if $a \geq b$, otherwise b) and $\min(a,b)$ is the smaller value (i.e., a if $a \leq b$, otherwise b). Therefore, the dimension of the interference signals is aligned to $\mathsf{K}-D-1$, by the linearly independece of $\mathsf{K}-D-1$ vectors $\mathbf{v}^{[<k+D+1>_{\mathsf{K}+1}]}$ $\mathbf{v}^{[<k+D+2>_{\mathsf{K}+1}]}$ $\ldots \mathbf{v}^{[<k+\mathsf{K}>_{\mathsf{K}+1}]}$, receiver k can recover $U+1$ messages from the first $\mathsf{K}-D+U$ time slots. Thus, the communication degree of freedom is $\frac{\mathsf{K}(U+1)}{\mathsf{K}-D+U}$.

For the sensor, the received signal is:

$$\mathbf{y}^{[\mathsf{K}+1]} = \sum_{i=D+2}^{\mathsf{K}-U} H^{[\mathsf{K}+1,i]} \mathbf{x}^{[i]} + \mathbf{z}^{[\mathsf{K}+1]}$$
$$= \sum_{i=1}^{\mathsf{K}-D-1} \mathbf{v}^{[i]} \sum_{s=max(1,i-U)}^{min(\mathsf{K}-U-D,i)} H^{\mathsf{K}+1,s} W_{i-s+1}^{[s]}$$
$$+ \sum_{i=D+2}^{\mathsf{K}-U} H^{[\mathsf{K}+1,i]} \mathbf{x}_s^{[i]} + \mathbf{z}^{[\mathsf{K}+1]}.$$

The interference dimension from communication signals is the same as that for communication users, i.e., $\mathsf{K}-D-1$. Therefore, we can get a left null vector $\mathbf{v}_0$ in $\left[\mathbf{v}^{[D+2]} \ldots \mathbf{v}^{[\mathsf{K}]}\right]$ to remove the communication interference within $\mathsf{K}-D+U$ time slots, thus

$$\mathbf{v}_0 \mathbf{y}^{[\mathsf{K}+1]} = \sum_{i=D+2}^{\mathsf{K}-U} H^{[\mathsf{K}+1,i]} \mathbf{v}_0 \mathbf{x}^{[i]} + \mathbf{v}_0 \mathbf{z}^{[\mathsf{K}+1]}$$
$$= \sum_{i=D+2}^{\mathsf{K}-U} H^{[\mathsf{K}+1,i]} \mathbf{v}_0 \mathbf{x}_s^{[i]} + z'^{[\mathsf{K}+1]}.$$

Let $x_1^{[0]} = \left[\mathbf{v}_0 \mathbf{x}_s^{[D+2]} \ \cdots \ \mathbf{v}_0 \mathbf{x}_s^{[K-U]}\right]^H$, in other $(\mathsf{K}-D-U)$ transmission, we can use the same way to get $x_2^{[0]}, x_3^{[0]}, \cdots, x_{\mathsf{K}-D-U}^{[0]}$, satisfying $[\mathbf{x}_1^{[0]}, \mathbf{x}_2^{[0]}, \cdots, x_{\mathsf{K}-D-U}^{[0]}][\mathbf{x}_1^{[0]}, \mathbf{x}_2^{[0]}, \cdots, x_{\mathsf{K}-D-U}^{[0]}]^H = I$. So one effective observation of the channel can be obtained, achieving the sDoF equal to $\frac{1}{\mathsf{K}-D+U}$.

APPENDIX D
PROOF OF THEOREM 5-B: TOPOLOGY 2 WITH COOPERATIVE TRANSMITTERS

This is the general description of the proposed TIM scheme for Theorem 5-b. We then consider the bistatic ISAC systems with heterogeneous connectivity and cooperative transmitters. For the Topology 2 based on the (K, d)-regular network, we propose an ISAC scheme based on TIM achieving the tradeoff point $(\text{sDoF}, \text{cDoF}) = \left(\frac{1}{d+1}, \frac{2\mathsf{K}}{d+1}\right)$.

Recall that the set of connected transmitters of the sensor is a subset of one communication that the network topology could be expressed as

$$\mathcal{R}_c^{[k]} = \{k, <k+1>_{\mathsf{K}+1}, \ldots, <k+d-1>_{\mathsf{K}+1}\},$$
$$\mathcal{R}_s = \{<i+1>_{\mathsf{K}+1}, \ldots, <i+d-1>_{\mathsf{K}+1}\},$$

for each $i, k \in \{1, \ldots, \mathsf{K}\}$, To simplify notation without loss of generality, we set $i = 1$, thus we can get $\mathcal{R}_s = \{2 \ldots, d\}$.

Let $t_0 = (d+1)(d+1)$. In $(d+1)(d+1)$ time slots, the sensor will obtain $d+1$ effective observations, through the sensing signals $\mathbf{x}_1^{[0]}, \mathbf{x}_2^{[0]}, ..., \mathbf{x}_{d+1}^{[0]} \in \mathbb{C}^{(d+1)\times 1}$, satisfying $[\mathbf{x}_1^{[0]}, \mathbf{x}_2^{[0]}, ..., \mathbf{x}_{d+1}^{[0]}][\mathbf{x}_1^{[0]}, \mathbf{x}_2^{[0]}, ..., \mathbf{x}_{d+1}^{[0]}]^H = I$. In each period of $(d+1)$ time slots, we let each receiver decode 2 communication messages, and let the sensor obtain one effective observation. In the next, we illustrate our proposed scheme for the first $d+1$ time slots, where each receiver $k \in \{1, 2, ..., \mathsf{K}\}$ should recover 2 messages $W_1^{[k]}, W_2^{[k]} \in \mathbb{C}$.

Denote the transmitted signals of transmitter $j \in \{1, \ldots, \mathsf{K}\}$ by $\mathbf{x}^{[j]} = [x^{[j]}(1), \ldots, x^{[j]}(d+1)]^T \in \mathbb{C}^{(d+1)\times 1}$. For the d transmitters connected to the sensor, let

$$\mathbf{x}^{[j]} = \mathbf{v}^{[j]} W_1^{[j]} + \mathbf{v}^{[j+1]} W_2^{[j-d+1]} + \mathbf{x}_s^{[j]}, \forall j \in \{2, \ldots, d\}.$$

For the transmitters in the set $\{1, d+1, \ldots, \mathsf{K}\}$, which are not connected to the sensor, let

$$\mathbf{x}^{[j]} = \mathbf{v}^{[j]} W_1^{[j]} + \mathbf{v}^{[j+1]} W_2^{[j-d+1]}, \forall j \in \{1, d+1, \ldots, \mathsf{K}\}.$$

By removing the dedicate sensing signal, what the $k \in \{1, \ldots, \mathsf{K}\}$-th communication receiver received is:

$$\hat{\mathbf{y}}^{[k]} = \sum_{j=k}^{k+d-1} H^{[kj]} \left(\mathbf{v}^{[j]} W_1^{[j]} + \mathbf{v}^{[j+1]} W_2^{[j-d+1]}\right) + \mathbf{z}^{[k]}$$
$$= \underbrace{H^{[kk]} \mathbf{v}^{[k]} W_1^{[k]} + H^{[k,k+d-1]} \mathbf{v}^{[k+d]} W_2^{[k]}}_{\text{desired signal}}$$
$$+ \sum_{j=k+1}^{k+d-1} \mathbf{v}^{[j]} \left(H^{[kj]} W_1^{[j]} + H^{[k,j-1]} W_2^{[j-d]}\right) + \mathbf{z}^{[k]}.$$

The dimension of the desired signals is 2, and the dimension of the interference signals is $d-1$, we can achieve symmetric cDoF $= \frac{2}{d+1}$ for each receiver. Thus, the cDoF is $\frac{2\mathsf{K}}{d+1}$.

For the sensor, the received signal $\mathbf{y}^{[s]}$ is:

$$\mathbf{y}^{[s]} = \sum_{j=2}^{d} H^{[\mathsf{K}+1,j]} \left(\mathbf{v}^{[j]} W_1^{[j]} + \mathbf{v}^{[j+1]} W_2^{[j-d+1]} + \mathbf{x}_s^{[j]}\right) + \mathbf{z}^{[\mathsf{K}+1]}$$
$$= \sum_{j=2}^{d} \mathbf{v}^{[j]} (H^{[\mathsf{K}+1,j]} W_1^{[j]} + H^{[\mathsf{K}+1,j-1]} W_2^{[j-d]}) + \mathbf{z}^{[\mathsf{K}+1]}.$$

Note that $[\mathbf{v}^{[2]} \mathbf{v}^{[3]} \cdots \mathbf{v}^{[d+1]}]$ with dimension $(d+1) \times (d)$ has one linearly independent left null vectors with high probability; let $\mathbf{v}_0 \ in \mathbb{C}^{1 \times d+1}$ be the non-zero left null vector. Thus

$$v_0 \mathbf{y}^{[s]} = \sum_{j=2}^{d} H^{[\mathsf{K}+1,j]} v_0 \mathbf{x}_s^{[j]} + H^{[\mathsf{K}+1,\mathsf{K}+1]} v_0 \mathbf{x}_s^{[\mathsf{K}+1]} + \mathbf{v}_0 \mathbf{z}^{[\mathsf{K}+1]}.$$

Let $x_1^{[0]} = \left[v_0\mathbf{x}_s^{[2]}, v_0\mathbf{x}_s^{[3]}, \cdots, v_0\mathbf{x}_s^{[d]}, v_0\mathbf{x}_s^{[\mathsf{K}+1]}\right]$, and this gives one valid observation. So one effective observation of the channel can be obtained, achieving a sDoF of $\frac{1}{d+1}$.

References

[1] F. Liu, Y. Cui, C. Masouros, J. Xu, T. X. Han, Y. C. Eldar, and S. Buzzi, "Integrated sensing and communications: Toward dual-functional wireless networks for 6g and beyond," *IEEE journal on selected areas in communications*, vol. 40, no. 6, pp. 1728–1767, 2022.

[2] F. Liu, C. Masouros, A. Li, H. Sun, and L. Hanzo, "Mu-mimo communications with mimo radar: From co-existence to joint transmission," *IEEE Transactions on Wireless Communications*, vol. 17, no. 4, pp. 2755–2770, 2018.

[3] M. Kobayashi, G. Caire, and G. Kramer, "Joint state sensing and communication: Optimal tradeoff for a memoryless case," in *2018 IEEE International Symposium on Information Theory (ISIT)*, pp. 111–115.

[4] M. Ahmadipour, M. Kobayashi, M. Wigger, and G. Caire, "An information-theoretic approach to joint sensing and communication," *IEEE Trans. Information Theory*, vol. 70, no. 2, pp. 1124–1146, 2022.

[5] Y. Xiong, F. Liu, Y. Cui, W. Yuan, T. X. Han, and G. Caire, "On the fundamental tradeoff of integrated sensing and communications under gaussian channels," *IEEE Transactions on Information Theory*, vol. 69, no. 9, pp. 5723–5751, 2023.

[6] H. Joudeh and F. M. Willems, "Joint communication and binary state detection," *IEEE Journal on Selected Areas in Information Theory*, vol. 3, no. 1, pp. 113–124, 2022.

[7] Y. Liu, M. Li, Y. Han, and L. Ong, "Information-theoretic limits of integrated sensing and communication over interference channels," in *ICC 2024 - IEEE International Conference on Communications*, June 2024, pp. 3561–3566.

[8] Y. Liu, M. Li, A. Liu, L. Ong, and A. Yener, "Fundamental limits of multiple-access integrated sensing and communication systems," *arXiv preprint arXiv:2205.05328*, 2022.

[9] A. Liu, Z. Huang, M. Li, Y. Wan, W. Li, T. X. Han, C. Liu, R. Du, D. K. P. Tan, J. Lu *et al.*, "A survey on fundamental limits of integrated sensing and communication," *IEEE Communications Surveys & Tutorials*, vol. 24, no. 2, pp. 994–1034, 2022.

[10] Y. Xiong, F. Liu, K. Wan, W. Yuan, Y. Cui, and G. Caire, "From torch to projector: Fundamental tradeoff of integrated sensing and communications," *IEEE BITS Magazine*, pp. 1–13, 2024.

[11] N. J. Willis and H. D. Griffiths, *Advances in bistatic radar*. SciTech Publishing, 2007, vol. 2.

[12] W. Zhang, S. Vedantam, and U. Mitra, "Joint transmission and state estimation: A constrained channel coding approach," *IEEE Transactions on Information Theory*, vol. 57, no. 10, pp. 7084–7095, 2011.

[13] M.-C. Chang, S.-Y. Wang, T. Erdoan, and M. R. Bloch, "Rate and detection-error exponent tradeoff for joint communication and sensing of fixed channel states," *IEEE Journal on Selected Areas in Information Theory*, vol. 4, pp. 245–259, 2023.

[14] N. Zhao, Q. Chang, X. Shen, Y. Wang, and Y. Shen, "Joint target localization and data detection in bistatic isac networks," *IEEE Transactions on Communications*, Oct. 2024.

[15] Y. Liu, M. Li, L. Ong, and A. Yener, "Bistatic integrated sensing and communication over memoryless relay channels," in *2024 IEEE International Symposium on Information Theory (ISIT)*, pp. 2592–2597.

[16] H. Nikbakht, Y. C. Eldar, and H. V. Poor, "A MIMO ISAC system for ultra-reliable and low-latency communications," *arXiv preprint arXiv:2501.13025*, 2025.

[17] T. Jiao, Y. Geng, Z. Wei, K. Wan, Z. Yang, and G. Caire, "Information-theoretic limits of bistatic integrated sensing and communication," *arXiv e-prints*, pp. arXiv–2306, 2023.

[18] Y. Dong, F. Liu, and Y. Xiong, "Joint receiver design for integrated sensing and communications," *IEEE Communications Letters*, vol. 27, no. 7, pp. 1854–1858, 2023.

[19] C. Ouyang, Y. Liu, and H. Yang, "Revealing the impact of sic in noma-isac," *IEEE Wireless Communications Letters*, vol. 12, no. 10, pp. 1707–1711, 2023.

[20] Q. Gong, Y. Liu, M. Li, and L. Ong, "Secrecy rate- distortion tradeoff for integrated sensing and communication with an untrusted sensing node," in *2024 16th International Conference on Wireless Communications and Signal Processing (WCSP)*, Oct. 2024, pp. 109–114.

[21] Y. Cui, V. Koivunen, and X. Jing, "A perspective on degrees of freedom for radar in radar-communication interference channel," in *2018 52nd Asilomar Conference on Signals, Systems, and Computers*, pp. 403–408.

[22] V. R. Cadambe and S. A. Jafar, "Interference alignment and degrees of freedom of the k-user interference channel," *IEEE Transactions on Information Theory*, vol. 54, no. 8, pp. 3425–3441, 2008.

[23] S. A. Jafar, "Blind interference alignment," *IEEE Journal of Selected Topics in Signal Processing*, vol. 6, no. 3, pp. 216–227, 2012.

[24] ——, "Topological interference management through index coding," *IEEE Trans. Information Theory*, vol. 60, no. 1, pp. 529–568, 2013.

[25] H. Maleki, V. R. Cadambe, and S. A. Jafar, "Index codingan interference alignment perspective," *IEEE Transactions on Information Theory*, vol. 60, no. 9, pp. 5402–5432, 2014.

[26] X. Yi and D. Gesbert, "Topological interference management with transmitter cooperation," *IEEE Transactions on Information Theory*, vol. 61, no. 11, pp. 6107–6130, 2015.

[27] H. Xie, F. Gao, S. Zhang, and S. Jin, "A unified transmission strategy for TDD/FDD massive mimo systems with spatial basis expansion model," *IEEE Trans. Vehicular Technology*, vol. 66, no. 4, pp. 3170–3184, 2016.

[28] D. Fan, F. Gao, G. Wang, Z. Zhong, and A. Nallanathan, "Angle domain signal processing-aided channel estimation for indoor 60-ghz tdd/fdd massive mimo systems," *IEEE Journal on Selected Areas in Communications*, vol. 35, no. 9, pp. 1948–1961, 2017.

[29] J. A. Bondy, U. S. R. Murty *et al.*, *Graph theory with applications*. Macmillan London, 1976, vol. 290.

[30] P. Zhang and G. Chartrand, "Introduction to graph theory," *Tata McGraw-Hill*, vol. 2, pp. 2–1, 2006.

[31] E. R. Scheinerman and D. H. Ullman, *Fractional graph theory: a rational approach to the theory of graphs*. Courier Corporation, 2011.

[32] S. A. Jafar, "Elements of cellular blind interference alignment—aligned frequency reuse, wireless index coding and interference diversity," *arXiv preprint arXiv:1203.2384*, 2012.

[33] A. D. Wyner, "Shannon-theoretic approach to a gaussian cellular multiple-access channel," *IEEE Transactions on Information Theory*, vol. 40, no. 6, pp. 1713–1727, 2002.

[34] J. Xu, J. Zhang, and J. G. Andrews, "On the accuracy of the wyner model in cellular networks," *IEEE Transactions on Wireless Communications*, vol. 10, no. 9, pp. 3098–3109, 2011.

[35] O. Simeone, N. Levy, A. Sanderovich, O. Somekh, B. M. Zaidel, H. V. Poor, S. Shamai *et al.*, "Cooperative wireless cellular systems: An information-theoretic view," *Foundations and Trends® in Communications and Information Theory*, vol. 8, no. 1-2, pp. 1–177, 2012.

[36] L. Xie, F. Liu, J. Luo, and S. Song, "Sensing mutual information with random signals in gaussian channels: Bridging sensing and communication metrics," *arXiv preprint arXiv:2402.03919*, 2024.

[37] S. A. Jafar and S. Shamai, "Degrees of freedom region of the MIMO x channel," *IEEE Trans. Infor. Theory*, vol. 54, no. 1, pp. 151–170, 2008.

[38] H. Liu and E. Alsusa, "A novel ISAC approach for uplink NOMA system," *IEEE Communications Letters*, 2023.