On the Application of Blind Interference Alignment for Bistatic Integrated Communication and Sensing Integration Systems

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***Abstract*—Integrated sensing and communication (ISAC) systems provide significant enhancements in performance and resource efficiency compared to individual sensing and communication systems, primarily attributed to the collaborative use of wireless resources, radio waveforms, and hardware platforms. The performance limits of a system are crucial for guiding its design; however, the performance limits of integrated sensing and communication (ISAC) systems remain an open question. This paper focuses on the bistatic ISAC systems with dispersed multi-receivers and one sensor. Compared to the monostatic ISAC systems, the main challenge is that that the communication messages are unknown to the sensor and thus become its interference, while the channel information between the transmitters and the sensor is unknown to the transmitters. In order to mitigate the interference at the sensor while maximizing the communication degree of freedom, we propose an interference management method for various bistatic ISAC channels based on blind interference alignment, including interference channels, MU-MISO channels, and MU-MIMO channels. Under each of such system, the achieved ISAC tradeoff points by the proposed schemes in terms of communication and sensing degrees of freedom are characterized. Simulation results also demonstrate that the proposed schemes significantly improvement on the ISAC performance compared to the state-of-the-art.**

***Index Terms*—Integrated Sensing and Communication, Blind Interference Alignment, Degree of Freedom.**

I. Introduction

With the continuous advancement of wireless communication systems, next-generation wireless networks are required to provide high-quality wireless connections over broad spatial domains and also to deliver high-precision estimation and detection services to complete sensing tasks [1]–[4]. The number of wireless communication and radar devices will rapidly increase, making it a challenging problem to meet the demands of communication and radar systems without expanding spectrum resources [5], [6]. To address this issue, Integrated Sensing and Communication (ISAC) technology has emerged, which allows for the sharing of equipments and time-frequency resources between sensing and communication tasks to improve resource utilization, achieving a mutually beneficial effect through the synergy of communication and sensing technologies [7].

However, sensing and communication operate on distinct information processing principles. To delve into the essence of the “integration” gain in ISAC systems, substantial research

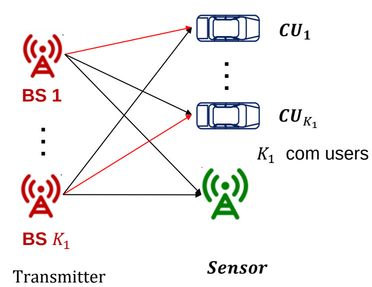


Fig. 1: The ISAC scenarios

has been conducted on the information theoretic tradeoffs between communication and sensing performance based on these metrics. The information theoretic ISAC work was originally proposed in [8], [9] for the monostatic ISAC system with state-dependent discrete memoryless channels and i.i.d. state sequence to be estimated. The capacity-distortion region was characterized by a conditional mutual information maximized over some conditional probability satisfying some constraints. In order to explicitly characterize the close-form of the fundamental tradeoff between the communication and sensing performances, Xiong *et al.* considered the MIMO Gaussian channel in [10] and aimed to search the Cramer-Rao Bound (CRB)-rate region. Two extreme tradeoff points were characterized in [10], a.k.a, the communication-optimal point and the sensing-optimal point, which also reveal two fundamental aspects of tradeoffs in ISAC systems: subspace tradeoffs and deterministic-random tradeoffs. For the fixed sensing state where the state to be sensed remains fixed throughout one transmission block, the optimal tradeoff between the communication rate and state detection error exponent was characterized in [11]. Although research on communication-sensing tradeoffs has made some progress, it remains an open question [12], [13].

The aforementioned works mainly studied the monostatic ISAC systems, where the state is estimated by leveraging the communication message as the side information. Compared to monostatic radar, bistatic radar, by using separate sensor and transmitter, can avoid direct reflections or interference signals and cover larger areas, providing advantages in the detection of low-altitude and ground targets and also in the utilization of multipath signals for target detection [14], [15]. Information theoretic ISAC works on the bistatic model basically focused on two cases, depending on the location of the communication receiver and sensor. For the bistatic ISAC model with co-located receiver and sensor, the capacity-distortion region for the i.i.d. state sequence was characterized in [16] and achievable region for the fixed state was proposed in [17].

For the bistatic ISAC model with dispersed receiver(s) and sensor, whose fundamental tradeoff between communication and sensing performances is generally open. A key challenge is that communication messages act as interference for the sensor. The authors in [18] introduced three achievable decoding-and-estimation strategies for the sensor which does not necessarily recover the communication message, i.e., blind estimation (e.g., the strategy of treating interference as noise as in [19]), partial-decoding-based estimation (e.g., the strategy of successive interference cancellation (SIC) as in [20]), and full-decoding-based estimation (i.e., first fully decoding the communication message and then estimating). Considering the general distortion function as the sensing metric, the achievable the rate-distortion regions by these three strategies in [18] are in the form of single-letter regions with auxiliary random variables.

Main Contributions: This paper considers the bistatic ISAC model with dispersed multi-receivers and one sensor. In order to characterize the fundamental tradeoff between the communication and sensing performances, we consider 'homogeneous' metrics, say communication degree of freedom (cDoF) and sensing degree of freedom (sDoF), which represent the average numbers of effective transmissions and observations in each time slot, respectively. As a new interference management strategy for perfectly eliminating the interference to the sensor in the bistatic ISAC model, we use the blind interference alignment (BIA) strategy originally proposed in [21], with the assumption that the channel between the transmitters and the sensor is in slow-fading unknown to the transmitters and the channel between the transmitters and the communication receivers is in fast-fading known to the transmitters. We consider three types of wireless channels, interference channels, multi-user MISO (MU-MISO) channels, and multi-user MIMO (MU-MIMO) channels. By using BIA, we propose new tradeoff points between cDoF and sDoF, which are strictly better than the time-sharing between the two extreme sensing-optimal and communication-optimal points. Starting from the perspective of degrees of freedom, we design the transmission signal to provide the system with more degree of freedom. A simulation comparison with the method in [22] that precoding the transmission signal from the viewpoint of degrees of freedom can yield a gain of approximately [7, 75] dB.

Notation Convention: Sets are denoted using calligraphic symbols. Vectors and matrices are represented in bold. For an arbitrary-size matrix $\mathbf{M}$, rank($\mathbf{M}$), $\mathbf{M}^*$, $\mathbf{M}^T$, and $\mathbf{M}^H$ represent its rank, conjugate, transpose, and conjugate transpose, respectively. $\lceil\cdot\rceil$ represents the ceiling function, which denotes rounding up to the nearest integer.

II. System Model and Performance Metrics

A. System Model

Let us consider a typical bistatic ISAC system, where K_T ISAC transmitters are equipped with m_i antennas each ($i = 1, 2, \ldots, \mathsf{K}_T$), and K_R communication receivers are equipped with n_j antennas each ($j = 1, 2, \ldots, \mathsf{K}_R$). The sensing receiver is equipped with n_s antennas (in this paper, we first consider the case where $n_s = 1$). We consider a multiple-shot decoding scenario, that is, in the time slots $K_T \frac{\sum_{i=1}^{K_T} m_i}{\sum_{j=1}^{K_R} n_j}$, the receiver aims to decode the desired signal $W^{[k]}$ (where $k = 1, 2, \ldots, K_R$) over the corresponding time period by jointly observing across multiple time instances. The channel output at the receiver in the t-th time slot follows the input-output relationship as follows:

$$\mathbf{Y}_c^{[k]}(t) = \sum_{i=1}^{\mathsf{K}_T} \mathbf{H}_c^{[ki]}(t)\mathbf{X}^{[i]}(t) + \mathbf{Z}^{[k]}(t) \tag{1}$$

$$\mathbf{Y}_s(t) = \sum_{i=1}^{\mathsf{K}_T} \mathbf{H}_s^{[(\mathsf{K}_R+1)i]}(t)\mathbf{X}^{[i]}(t) + \mathbf{Z}_s(t) \tag{2}$$

where $k = \{1, 2, \ldots, \mathsf{K}_R\}$ represents the user index, and $t \in \mathbb{N}$ is the time slot index. The term $\mathbf{H}_c^{[ki]}(t) \in \mathbb{C}^{n_k \times m_i}$ denotes the channel experienced by the signal sent from the i-th transmitter to the k-th communication user at time slot t. Similarly, $\mathbf{H}_s^{[(\mathsf{K}_R+1)i]}(t) \in \mathbb{C}^{n_s \times m_i}$ represents the channel experienced by the signal sent from the i-th transmitter to the sensing user at time slot t.

The received signal by the k-th communication user at time slot t is given by $\mathbf{Y}_c^{[k]}(t) \in \mathbb{C}^{n_k \times 1}$, and the received signal by the sensing receiver at time slot t is denoted by $\mathbf{Y}_s(t) \in \mathbb{C}^{n_s \times 1}$. The terms $\mathbf{Z}^{[k]}(t) \in \mathbb{C}^{n_k \times 1}$ and $\mathbf{Z}_s(t) \in \mathbb{C}^{n_s \times 1}$ represent the additive Gaussian white noise at the communication and sensing receivers, respectively, both distributed as $\mathcal{CN}(0, 1)$.

For the communication task, the channel state information (CSI) is known to both the transmitter and the communication receiver. Moreover, in the case where the receiver has multiple antennas, the CSI is causal, meaning the transmitter knows the channel state at all times. For the sensing task, however, the CSI is unknown to both the transmitter and the receiver.

In order to achieve the sensing task, the transmitted signal $\mathbf{X}_i^{[0]} \in \mathbb{C}^{m \times 1}$ is known to all users. The signal transmitted by each transmitter can be written as:

$$\mathbf{X} = \mathbf{X}^{[i]} + \mathbf{X}^{[0]} \tag{3}$$

where $\mathbf{X}^{[i]} = \mathbf{V}^{[i]} W^{[i]}$, with $\mathbf{V}^{[i]} \in \mathbb{C}^{m \times 1}$ being the matrix used to encode the message $W^{[i]} \in \mathbb{C}^K$.

The transmitted signal $\mathbf{X}^{[i]}(t)$ satisfies the power constraint $\mathbb{E}[|\mathbf{X}^{[i]}(t)|^2] \leq P$. The total power across all transmitters is assumed to be equal to ρ. The size of the message set is denoted by $|\mathbf{W}^{[i]}(\rho)|$.

For codewords spanning t_0 channel uses, the rates $R_i(\rho) = \frac{\log|\mathbf{W}^{[i]}(\rho)|}{t_0}$ are achievable if the probability of error for all messages can be simultaneously made arbitrarily small by choosing an appropriately large t_0 (where $t_0 = K_T \frac{\sum_{i=1}^{K_T} m_i}{\sum_{j=1}^{K_R} n_j}$, which also implies that the larger the ratio of transmit/receive antennas, the better). During this transmission time, the receiver can correctly decode $W^{[k]}$ (where $k = 1, 2, \ldots, K_R$) from interference through multiple observations. Therefore, we can define the system's average degrees of freedom as:

$$D = \frac{K_R}{t_0} \tag{4}$$

which represents the average amount of independent information that can be successfully transmitted at each time instance. Since this is a multi-shot process, transmission can only be completed after accumulating sufficient information. By observing each time slot individually, no symbols can be decoded.

In this process, the movement of communication users is assumed to be regular within a limited spatial range. The rapid changes in their motion state result in a short coherence time of the channel, leading to time-selective fading. The channel varies for each transmission in every time slot, and we assume it follows Rayleigh fading, where both the real and imaginary parts of the channel coefficients are independent and identically distributed Gaussian variables with zero mean and variance of 1 (normal distribution), i.e.:

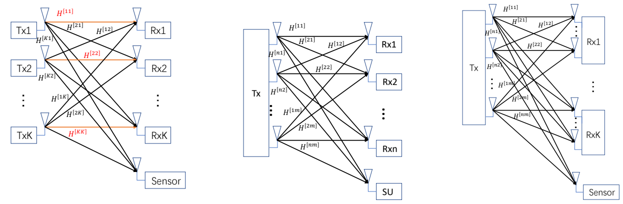
$$h = h_I + jh_Q \tag{5}$$

where $h_I \sim N(0,1)$, $h_Q \sim N(0,1)$. Conversely, the motion of sensors is characterized by irregularity, leading to slower changes in their motion state and consequently a longer coherence time. This indicates that the channel encountered by sensors undergoes frequency-selective fading. Specifically, the channel experienced by communication users evolves with the transmission of the signal, exhibiting time-varying characteristics. In contrast, the channel for sensors remains approximately constant over a given time period, indicating that the channel variations are relatively stable during the transmission of the signal.

By varying the number of antennas at the transmitter and receiver, we can derive three different system models. When $\mathsf{K}_T = \mathsf{K}_R = \mathsf{K}, m_i = 1 (i = 1, 2, \ldots, \mathsf{K}_T), n_j = 1 (j = 1, 2, \ldots, \mathsf{K}_R)$, we obtain a typical channel model in wireless communication known as the Multi-User Interference Channel as shown in figure 2(a). When $\mathsf{K}_T = 1, \mathsf{K}_R = n, m_1 = m, n_j = 1 (j = 1, 2, \ldots, n)$, the system evolves into a MU-MISO channel as shown in figure 2(a). When $\mathsf{K}_T = 1, \mathsf{K}_R = \mathsf{K}, m_1 = m$, the system evolves into a MU-MIMO channel as shown in 2(c).

B. Performance Matrics

1) Communication Degree of Freedom: In the study of wireless communications, channel capacity is commonly used



(a) (b) (c)

Fig. 2: System Model. (a) Interference channels, (b) MU-MISO channels, (c) MU-MIMO channels

to represent the transmission capacity and maximum achievable rate of a system:

$$C = B \log_2 (1 + \mathrm{SNR}), \tag{6}$$

where C denotes the channel capacity of the system, B represents the system bandwidth, and SNR stands for the signal-to-noise ratio.

However, in practical scenarios, the analysis of channel capacity can become highly complex in multi-user and multi-antenna systems due to the influence of noise during the communication process. Consequently, the academic community has proposed an approximate capacity under high SNR conditions, which we refer to as the Degrees of Freedom (DoF). Also known as multiplexing gain, DoF represents the maximum number of independent data streams that a communication system can support. It is defined as follows:

$$D = \lim_{\mathrm{SNR}\to\infty} \frac{C(\mathrm{SNR})}{\log_2(\mathrm{SNR})}. \tag{7}$$

2) Sensing Degree of Freedom: The concept of DoF in wireless communication is difficult to directly apply to radar signal processing because channel capacity has no corresponding physical significance in radar. Therefore, we define the sensing degree of freedom based on the number of effective observations of the target during the estimation and detection process [23].

In sensing tasks, the transmitted signals are denoted as $X_i^{[0]}$ ($i = 1, 2, \ldots, N$), and the receiver completes the sensing task based on the echoes of the transmitted signals. The transmitted signals satisfy:

$$\left[X_1^{[0]}, X_2^{[0]}, \ldots, X_N^{[0]}\right] \left[X_1^{[0]}, X_2^{[0]}, \ldots, X_N^{[0]}\right]^H = I, \tag{8}$$

where $\mathbf{H}$ represents the channel matrix. The term "effective" means that at the receiver end, the product of $X_i^{[0]}$ and $\mathbf{H}$ can be obtained independently. Once we obtain the product with the channel, we can complete the task of channel estimation.

Theorem 1([24]). *If for a system, the rate pairs (R_{11}, R_{21}) and (R_{12}, R_{22}) are achievable, then $(R_1, R_2) = (\alpha R_{11} + (1-\alpha) R_{12}, \alpha R_{21} + (1-\alpha) R_{22})$ is achievable for any $\alpha \in [0, 1]$.*

Theorem 2([25]). *In a point-to-point MIMO system with N_T transmit antennas and N_R receive antennas, the maximum* *achievable degrees of freedom at high SNR is given by* $\min\{N_T, N_R\}$.

Objective: We aim to ensure that the system's communication degrees of freedom can achieve the constraints of Theorem 2 and that our sensing degrees of freedom and communication degrees of freedom (SDoF, CDoF) outperform the effect obtained by applying Theorem 1 to separate systems.

III. Main Result

In this section, we will demonstrate how to apply blind interference alignment techniques to system design and analyze the communication degrees of freedom (CDoF) and sensing degrees of freedom (SDoF) to obtain the trade-off curve of degrees of freedom.

A. Multi-user Interference Channel

We begin by introducing the multi-user interference channel scenario. In this setup, we have K single-antenna ISAC transmitters and K+1 single-antenna receivers. The first K receivers are tasked with decoding communication symbols, while the $(\mathsf{K}+1)$-th receiver is responsible for estimating or detecting the target based on the transmitted signals.In this system, by employing the zero-forcing method combined with blind interference alignment, we can achieve $(\text{SDoF}, \text{CDoF}) = \left(\frac{\mathsf{K}-1}{\mathsf{K}}, 1\right)$.

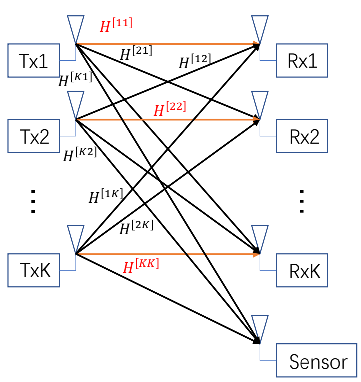


Fig. 3: Multi-user interference channel with a sensing user.

To better analyze the system's degrees of freedom, we start by examining a 3×3 communication interference channel with an additional sensing user, and then extend it to a K-user interference channel. We propose a transmission scheme combining blind interference alignment and zero-forcing, achieving (SDoF, CDoF) = (2/3, 1). First, we analyze the communication users. The received signal at time t for the k-th communication user is:

$$\begin{aligned} Y^{[k]}(t) = & H^{[k1]}(t)(X^{[1]}(t) + X^{[0]}(t)) \\ & + H^{[k2]}(t)(X^{[2]}(t) + X^{[0]}(t)) \\ & + H^{[k3]}(t)(X^{[3]}(t) + X^{[0]}(t)) + Z(t) \end{aligned} \tag{9}$$

The CSI and the sensing signal $X^{[0]}(t)$ are known to the communication users, allowing this part of the signal to be canceled at the receiver. The estimated signal $\hat{Y}^{[k]}$ at the receiver, only concerning the communication signals, is:

$$\begin{aligned} \hat{Y}^{[k]}(t) = & H^{[k1]}(t)X^{[1]}(t) + H^{[k2]}(t)X^{[2]}(t) \\ & + H^{[k3]}(t)X^{[3]}(t) + Z(t) \end{aligned} \tag{10}$$

Where $\hat{Y}^{[k]}(t)$ represents the observed value obtained for the estimate excluding the sensing channel.For any receiver, we know that the dimensionality of the useful information is 1, while the dimensionality of interference is 2. Therefore, we need data from three time slots to decode the required information. Equation (10) can be rewritten (ignoring noise due to high SNR in zero-forcing,k=1,2,3):

$$\hat{\mathbf{Y}}^{[k]} = \begin{bmatrix} \hat{Y}^{[k]}(1) \\ \hat{Y}^{[k]}(2) \\ \hat{Y}^{[k]}(3) \end{bmatrix} = \sum_{i=1}^{3} \begin{bmatrix} H^{[ki]}(1) \\ H^{[ki]}(2) \\ H^{[ki]}(3) \end{bmatrix} X^{[i]} \tag{11}$$

For any receiver, we can use the values of its undesired channels to apply zero-forcing on the interference signals, ultimately recovering the desired signal. Consequently, the system's communication degrees of freedom can be determined to be 1. Assuming $k = 1$ (the channel variation process for the user is the same, so choosing any user does not affect the result).Selecting the channel parameters of the undesired signals to form the matrix $\mathbf{H}_{\text{cob}} = \begin{bmatrix} H^{[12]}(1) & H^{[13]}(1) \\ H^{[12]}(2) & H^{[13]}(2) \\ H^{[12]}(3) & H^{[13]}(3) \end{bmatrix}$, it is easy to see that $\text{rank}(\mathbf{H}_{\text{cob}}) = 2$, and the dimension of its row space is 3. Hence, the dimension of the left null space of $\mathbf{H}_{\text{cob}}$ should be $3 - 2 = 1$. Let $\mathbf{v}_1 \in \mathbb{C}^{1\times 3}$ be a vector in the left null space. Then, $\mathbf{v}_1 \begin{bmatrix} H^{[12]}(1) \\ H^{[12]}(2) \\ H^{[12]}(3) \end{bmatrix} = \mathbf{v}_1 \begin{bmatrix} H^{[13]}(1) \\ H^{[13]}(2) \\ H^{[13]}(3) \end{bmatrix} = 0$. Therefore, we obtain:

$$\mathbf{v}_1 \hat{\mathbf{Y}}^{[1]} = \mathbf{v}_1 \begin{bmatrix} H^{[11]}(1) \\ H^{[11]}(2) \\ H^{[11]}(3) \end{bmatrix} X^{[1]} \tag{12}$$

Thus, user 1 can obtain the desired signal $X^{[1]}$, and three symbols can be solved over three time slots, giving a communication DoF (CDoF) of 3/3 = 1.

The sensing DoF (SDoF) is 2/3, meaning two effective observations of $X_1^{[0]}$ and $X_2^{[0]}$ can be obtained over three time slots. Next, we demonstrate the signal design using blind interference alignment. Let $X_1^{[0]} \in \mathbb{Z}^{3\times 1}$ and $X_2^{[0]} \in \mathbb{Z}^{3\times 1}$, satisfying $[X_1^{[0]}, X_2^{[0]}][X_1^{[0]}, X_2^{[0]}]^H = I$. Our idea is to use the fact that the channel for the sensing user remains constant over a certain period and that the communication signals also remain constant to cancel the communication signals, which are interference for the sensing user.

Let $X_1^{[0]} = [x_{1,1}, x_{1,2}, x_{1,3}]^T$ and $X_2^{[0]} = [x_{2,1}, x_{2,2}, x_{2,3}]^T$. For the i-th transmitter, the transmitted signals over three time slots are: $X_i + x_{1,i} + x_{2,i}$, $X_i + x_{2,i}$, $X_i + x_{1,i}$. The messages received by the sensing user can be expressed as:

$$\mathbf{Y}^{[4]} = \sum_{i=1}^{3} \begin{bmatrix} H^{[4i]}(1)(X^{[i]} + x_{1,i} + x_{2,i}) \\ H^{[4i]}(1)(X^{[i]} + x_{2,i}) \\ H^{[4i]}(1)(X^{[i]} + x_{1,i}) \end{bmatrix} \tag{13}$$

By subtracting the observations at the second and third time slots from the first time slot, we get:

$$\begin{aligned} \mathbf{Y}^{[4]}(1) - \mathbf{Y}^{[4]}(t) &= \sum_{i=1}^{3} H^{[4i]}(1) x_{t-1,i} \\ &= \mathbf{H} \begin{bmatrix} x_{t-1,1} \\ x_{t-1,2} \\ x_{t-1,3} \end{bmatrix} \end{aligned} \tag{14}$$

where $\mathbf{H} = \left[H^{[41]}(1), H^{[42]}(1), H^{[43]}(1)\right]$, $t = 2, 3$. Therefore, we obtain $\mathbf{H}X_1^{[0]}$ and $\mathbf{H}X_2^{[0]}$, meaning two effective observations over three time slots. Thus, the SDoF is 2/3, and the overall system degrees of freedom for sensing and communication are (SDoF, CDoF) = (2/3, 1).

By decomposing this system, if we only have the 3×3 communication interference channel, we can see that the degrees of freedom for sensing and communication are (SDoF, CDoF) = (0, 1). If there is only one sensing user, the degrees of freedom are (SDoF, CDoF) = (1, 0).By the time-sharing method, we can plot the trade-off curve of degrees of freedom obtained by combining two discrete systems, as shown by the red line in the figure. Furthermore, using our proposed method, a new inflection point (SDoF, CDoF) = (2/3, 1) can be obtained, resulting in a new curve, as illustrated by the blue line in the figure 4.

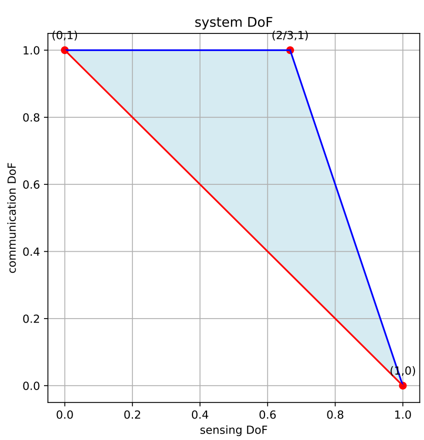


Fig. 4: system DoF vs timesharing DoF.

Next, we will demonstrate how the system achieves

$$(\mathrm{SDoF}, \mathrm{CDoF}) = \left(\frac{K-1}{K}, 1\right) \tag{15}$$

in the case of a $K \times K$ interference channel with an additional sensing user. First, let's analyze the communication users. The signal received by the k-th communication user is:

$$\begin{aligned} Y^{[k]}(t) &= H^{[k1]}(t)(X^{[1]}(t) + X^{[0]}(t)) + H^{[k2]}(t)(X^{[2]}(t) \\ &+ X^{[0]}(t)) + \cdots + H^{[kK]}(t)(X^{[K]}(t) + X^{[0]}(t)) + Z(t) \end{aligned} \tag{16}$$

Similarly, since the CSI and the transmitted sensing signal $X^{[0]}(t)$ are known to the communication users, we can cancel this part of the signal at the receiver, resulting in an estimate $\hat{Y}^{[k]}$ that only concerns the communication signals:

$$\begin{aligned} \hat{Y}^{[k]}(t) = H^{[k1]}(t)X^{[1]}(t) + H^{[k2]}(t)X^{[2]}(t) + \cdots + \\ H^{[kK]}(t)X^{[K]}(t) + Z(t) \end{aligned} \tag{17}$$

Using the same method as in the previous section, we combine the channel matrices experienced by all the undesired signals to obtain H_{cob}, and it is easy to find that $\mathrm{rank}(H_{\mathrm{cob}}) = K-1$. The dimension of its row space is K, so the dimension of its left null space should be $K - (K-1) = 1$. Let this vector be $v_k \in \mathbb{C}^{1\times K}$, and following the same steps as before, we can obtain $X^{[k]}(t)$. Thus, we achieve a total of K degrees of freedom over K time slots, and the communication degrees of freedom (CDoF) of the system is $K/K = 1$.

For the sensing user, we can obtain $K-1$ sensing signals over K time slots, with $X_k^{[0]} \in \mathbb{Z}^{K\times 1}$ (for $k = 1, 2, \ldots, K-1$). The transmitted sensing signals need to satisfy our constraint $[X_1^{[0]}, X_2^{[0]}, \ldots, X_{K-1}^{[0]}][X_1^{[0]}, X_2^{[0]}, \ldots, X_{K-1}^{[0]}]^H = I$. Besides transmitting the fixed signal $X^{[k]}(t)$ at each time slot, the k-th transmitter also needs to transmit the sensing signal, which we present in matrix form. In our previous toy example, the transmitted signal was already given by the equation, and we can see that the encoding for the transmitted sensing signal is the same; we only need to ensure that after subtracting the signals at two different time slots, the corresponding coefficient's parameter is 1, as follows:

$$\begin{bmatrix} x_{(1,k)} + x_{(2,k)} \\ x_{(2,k)} \\ x_{(1,k)} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_{(1,k)} \\ x_{(2,k)} \end{bmatrix} \tag{18}$$

The encoding matrix for the sensing signal to be transmitted is not full-rank. From the perspective of solving the equation, any encoding method can be used to solve for the desired sensing signal, as long as the encoding matrix's columns are full-rank. However, since the actual sensing system does not know the encoding method used by the transmitter, we must ensure that the desired signal can be obtained by the simplest operation at the receiver. We can write out the encoding method for K users:

$$
\begin{bmatrix}
x_{(1,k)} + x_{(2,k)} + x_{(3,k)} + \cdots + x_{(K-1,k)} \\
x_{(2,k)} + x_{(3,k)} + \cdots + x_{(K-1,k)} \\
x_{(1,k)} + x_{(3,k)} + \cdots + x_{(K-1,k)} \\
\vdots \\
x_{(1,k)} + x_{(2,k)} + \cdots + x_{(K-2,k)}
\end{bmatrix}
= \begin{bmatrix}
1 & 1 & 1 & \dots & 1 \\
0 & 1 & 1 & \dots & 1 \\
1 & 0 & 1 & \dots & 1 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & 1 & 1 & \dots & 0
\end{bmatrix}
\begin{bmatrix}
x_{(1,k)} \\ x_{(2,k)} \\ x_{(3,k)} \\ \vdots \\ x_{(K-1,k)}
\end{bmatrix} \tag{19}
$$

In this encoding scheme, since the interference channel requires transmitting the same signal over K time slots to decode the corresponding communication signal, we only need to subtract the received values at other time slots from the received value at the first time slot to obtain the result at time t:

$$
Y^{[K+1]}(1) - Y^{[K+1]}(t) = H^{[K+1,1]}(1)x_{(t-1,1)} + H^{[K+1,2]}(1)x_{(t-1,2)} + \cdots + H^{[K+1,K]}(1)x_{(t-1,K)} \tag{20}
$$

In matrix multiplication form:

$$
\left[H^{[K+1,1]}(1), H^{[K+1,2]}(1), \dots, H^{[K+1,K]}(1)\right]
\begin{bmatrix}
x_{(t-1,1)} \\ x_{(t-1,2)} \\ x_{(t-1,3)} \\ \vdots \\ x_{(t-1,K)}
\end{bmatrix} \tag{21}
$$

Thus, at each time slot after the first, we can obtain an effective observation of the channel. Over K time slots, we obtain $K-1$ effective observations, and the sensing degrees of freedom (SDoF) is $\frac{K-1}{K}$.

B. MISO Channel

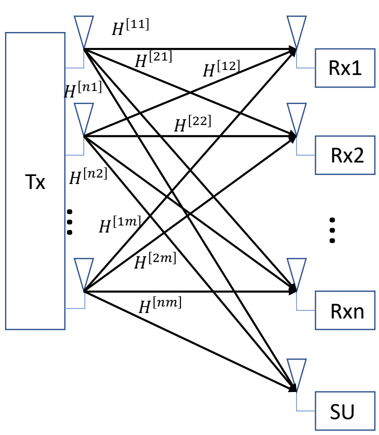


Fig. 5: MISO channel with a sensing user.

We set up the system with a transmitter equipped with m antennas, a communication receiver with n single-antenna receivers, and a sensing receiver with a single antenna. Next, we will demonstrate how to extend the system model to scenarios with a larger number of antennas at the transmitter. We denote the channel matrix from the transmitter to the receiver at time t as:

$$
\mathbf{H}(t) = \begin{bmatrix}
H^{[11]}(t) & H^{[12]}(t) & \dots & H^{[1m]}(t) \\
H^{[21]}(t) & H^{[22]}(t) & \dots & H^{[2m]}(t) \\
\vdots & \vdots & \ddots & \vdots \\
H^{[n1]}(t) & H^{[n2]}(t) & \dots & H^{[nm]}(t)
\end{bmatrix} \tag{22}
$$

Here, $H^{[ij]}(t)$ represents the channel from the j-th transmit antenna to the i-th receive antenna at time t. Our main idea is to maximize the sensing degree of freedom while maintaining the original communication degrees of freedom of the system. To obtain the demand $W_t^{[i]}$ for user i, we demonstrate the pre-coding matrix obtained using the zero forcing method. The matrix $\mathbf{H}^i_{unexp}(t)$, composed of all non-expected channel elements at non-expected times, is given by:

$$
\mathbf{H}^i_{unexp}(t) = \begin{bmatrix}
\mathbf{H}(1) \\ \mathbf{H}(2) \\ \vdots \\ \mathbf{H}(t) \backslash \mathbf{H}(t)[i,:] \\ \vdots \\ \mathbf{H}\left(\lceil \frac{m}{n} \rceil\right)
\end{bmatrix} \tag{23}
$$

where $H(t)[i,:]$ denotes the i-th row of $H(t)$, and $H(t)/H(t)[i,:]$ represents the matrix $H(t)$ with the i-th row removed, $\lceil m/n \rceil$ denotes the ceiling of m/n. $H(\lceil m/n \rceil)$ is given by:

$$
\begin{bmatrix}
H^{[11]}\left(\lceil \frac{m}{n} \rceil\right) & H^{[12]}\left(\lceil \frac{m}{n} \rceil\right) & \cdots & H^{[1m]}\left(\lceil \frac{m}{n} \rceil\right) \\
H^{[21]}\left(\lceil \frac{m}{n} \rceil\right) & H^{[22]}\left(\lceil \frac{m}{n} \rceil\right) & \cdots & H^{[2m]}\left(\lceil \frac{m}{n} \rceil\right) \\
\vdots & \vdots & \ddots & \vdots \\
H^{[p1]}\left(\lceil \frac{m}{n} \rceil\right) & H^{[p2]}\left(\lceil \frac{m}{n} \rceil\right) & \cdots & H^{[pm]}\left(\lceil \frac{m}{n} \rceil\right)
\end{bmatrix} \tag{24}
$$

where $p = m \mod n$. By analyzing the dimensions of the matrix, we know that the null space of $H^i_{\text{unexp}}(t)$ is non-empty. Therefore, we can use the vectors in its null space to encode the transmitted signals such that they are nulled out for other users or at undesired times. Next, we provide a detailed example of our method. Assume the transmitter has 3 antennas, and there are two single-antenna communication users and one single-antenna sensing user at the receiver. For the first communication user (CU1), we want it to decode two communication symbols $W_1^{[1]}$ and $W_2^{[1]}$ over two time slots; for the second communication user (CU2), we want it to decode one communication symbol $W_1^{[2]}$ over two time slots.

For $W_1^{[1]}$, the matrix $H^1_{\text{unexp}}(1)$ is:

$$
H^1_{\text{unexp}}(1) = \begin{bmatrix}
H^{[21]}(1) & H^{[22]}(1) & H^{[23]}(1) \\
H^{[11]}(2) & H^{[12]}(2) & H^{[13]}(2)
\end{bmatrix} \tag{25}
$$

The null space of $H^1_{\text{unexp}}(1)$ contains a vector denoted by $v_1^{[1]}$. We use $v_1^{[1]}$ to encode $W_1^{[1]}$, and similarly, we obtain $v_2^{[1]}$ to encode $W_2^{[1]}$.

For $W_1^{[2]}$, the matrix $H^2_{\text{unexp}}(1)$ is:

$$H^2_{\text{unexp}}(1) = \begin{bmatrix} H^{[11]}(1) & H^{[12]}(1) & H^{[13]}(1) \\ H^{[11]}(2) & H^{[12]}(2) & H^{[13]}(2) \end{bmatrix} \tag{26}$$

We obtain $v_1^{[2]}$ to encode $W_1^{[2]}$. Ultimately, we obtain our transmitted communication signals:

$$\mathbf{X} = \sum_{j=1}^{2} v_1^j W_1^j + v_2^1 W_2^1 \tag{27}$$

In this setup, all signals that are either not intended for user i or not required at time t are nullified because the pre-coding vector v_t^i is a vector in the null space of the channel matrix that the signal experiences. After multiplication, this results in zero, and the desired signal W_t^i can be decoded from the received signal based on the channel parameter polynomial. Thus, we can continuously transmit the same signal $\mathbf{X}$ across 2 time slots, with each time slot revealing a different communication signal, ensuring the system's communication degree of freedom. With 2 time slots, 3 symbols are decoded, so the system's communication degree of freedom is $\frac{3}{2}$.

Similar to the previous analysis, since the communication signals remain constant across 2 time slots, we can obtain effective observations for 1 signals, namely $\mathbf{X}_1^{[0]} \in \mathbb{R}^{m\times 1}$, and satisfies the required orthogonality condition. Let $X_1^{[0]} = [x_{1,1}, x_{1,2}, x_{1,3}]^T$, the transmitted signals over two time slots are: $\mathbf{X} + X_1^{[0]}$, $\mathbf{X}$. The messages received by the sensing user can be expressed as:

$$\mathbf{Y}^{[3]} = \begin{bmatrix} \mathbf{H}^{[3]}(1)(\mathbf{X} + X_1^{[0]}) \\ \mathbf{H}^{[3]}(1)(\mathbf{X}) \end{bmatrix} \tag{28}$$

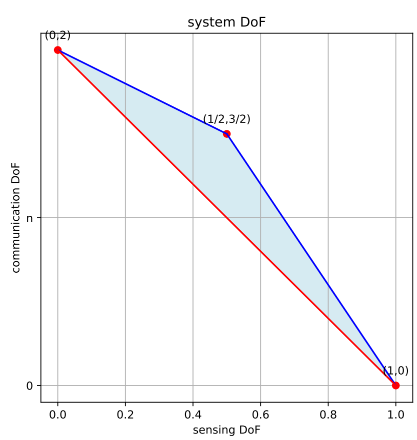


Fig. 6: system DoF vs timesharing DoF.

By subtracting the observations at the second time slot from the first time slot, we get:

$$\mathbf{Y}^{[3]}(1) - \mathbf{Y}^{[3]}(2) = \mathbf{H}^{[3]}(1) X_1^{[0]} \tag{29}$$

where $\mathbf{H}^{[3]}(1) = \left[H^{[31]}(1), H^{[32]}(1), H^{[33]}(1)\right]$. Therefore, we obtain $\mathbf{H}^{[3]}(1) X_1^{[0]}$, meaning one effective observations over three time slots. Thus, the SDoF is 1/2, and the overall system degrees of freedom for sensing and communication are (SDoF, CDoF) = (1/2, 3/2).

By decomposing this system, if we only have the 3×2 communication MISO channel, we can see that the degrees of freedom for sensing and communication are (SDoF, CDoF) = (0, 2). If there is only one sensing user, the degrees of freedom are (SDoF, CDoF) = (1, 0).By the time-sharing method, we can plot the trade-off curve of degrees of freedom obtained by combining two discrete systems, as shown by the red line in the figure. Furthermore, using our proposed method, a new inflection point (SDoF, CDoF) = (1/2, 3/2) can be obtained, resulting in a new curve, as illustrated by the blue line in the figure 6.

In this way, we can continuously transmit X over p time slots, and the communication signals decoded at each time slot are different, ensuring the communication degrees of freedom (DoF) in the system. On this basis, blind interference alignment can also be utilized to achieve higher sensing degrees of freedom. For communication user i, the received signal at time t is given by:

$$\begin{aligned} Y^{[i]}(t) &= \begin{bmatrix} H^{[i1]}(t) & H^{[i2]}(t) & H^{[i3]}(t) & \dots & H^{[im]}(t) \end{bmatrix} X \\ &= \begin{bmatrix} H^{[i1]}(t) & H^{[i2]}(t) & H^{[i3]}(t) & \dots & H^{[im]}(t) \end{bmatrix} v_t^{[i]} W_t^{[i]} \end{aligned} \tag{30}$$

where all the non-user i signals or signals not required at time t are pre-coded by $v_t^{[i]}$, which is a vector in the null space of the experienced channel matrix. Therefore, after multiplying, these signals become zero. Based on the channel polynomial and the received signal, we can decode $W_t^{[i]}$. With n single-antenna users, we can achieve n communication degrees of freedom at one time slot. For the sensing user:

$$Y^{[n+1]}(t) = \begin{bmatrix} H^{[n+1,1]}(t) & H^{[n+1,2]}(t) & \dots & H^{[n+1,m]}(t) \end{bmatrix} X \tag{31}$$

Similar to the previous analysis, since the communication signal remains constant over p time slots, we can obtain $p-1$ effective observations of $X_1^{[0]}, X_2^{[0]}, \dots, X_{p-1}^{[0]}$. Here, $X_j^{[0]} \in \mathbb{R}^{m\times 1}$, and they satisfy the orthogonality condition we require. Let $X_j^{[0]} = \left[x_{(j,1)}, x_{(j,2)}, x_{(j,3)}, \dots, x_{(j,m)}\right]^T$, and we can obtain the sensing signal to be transmitted by the k-th antenna at the transmitter as:

$$
\begin{bmatrix}
x_{(1,k)}+x_{(2,k)}+x_{(3,k)}+\cdots+x_{(p-1,k)}\\
x_{(2,k)}+x_{(3,k)}+\cdots+x_{(p-1,k)}\\
x_{(1,k)}+x_{(3,k)}+\cdots+x_{(p-1,k)}\\
\vdots\\
x_{(1,k)}+x_{(2,k)}+\cdots+x_{(p-2,k)}
\end{bmatrix}
=\begin{bmatrix}
1 & 1 & 1 & \dots & 1\\
0 & 1 & 1 & \dots & 1\\
1 & 0 & 1 & \dots & 1\\
\vdots & \vdots & \vdots & \ddots & \vdots\\
1 & 1 & 1 & \dots & 0
\end{bmatrix}
\begin{bmatrix}
x_{(1,k)}\\
x_{(2,k)}\\
x_{(3,k)}\\
\vdots\\
x_{(p-1,k)}
\end{bmatrix}
\tag{32}
$$

Thus, the sensing user can subtract the received signal at time t from the received signal at the first time slot, thereby canceling the interference caused by the communication signals and obtaining an effective observation of $X^{[0]}_{t-1}$. Over p time slots, a total of $p-1$ effective observations can be made, and the sensing degrees of freedom (SDoF) is $p-1/p$.

The system's total sensing and communication degrees of freedom are $(\mathrm{SDoF},\mathrm{CDoF}) = \left(\frac{p-1}{p}, n\right)$. For an $m \times n$ system, we only need to ensure that the number of antennas at the transmitter is an integer multiple of the number of antennas at the receiver. In this way, we can achieve sensing and communication degrees of freedom $\left(\frac{p-1}{p}, n\right)$. Although increasing the number of antennas generally increases the communication degrees of freedom in the system, due to the limitation on the number of antennas at the receiver, the maximum number of linearly independent data streams that can be transmitted at one time is restricted. Thus, the originally available degrees of freedom can be repurposed as sensing degrees of freedom.

C. MU-MIMO

To generalize the model, we further extend it to the case where the number of antennas at the receiver can be arbitrary. In the setup from the previous section, we modify the number of antennas at the k-th receiver to n_k. The channel matrix from the transmitter to the k-th receiver at time t is denoted as $\mathbf{H}^{[k]}(t) \in \mathbb{C}^{n_k \times m}$.

$$
\mathbf{H}^{[k]}(t) = \begin{bmatrix}
H^{[11]}(t) & H^{[12]}(t) & \cdots & H^{[1m]}(t)\\
H^{[21]}(t) & H^{[22]}(t) & \cdots & H^{[2m]}(t)\\
\vdots & \vdots & \ddots & \vdots\\
H^{[n_k 1]}(t) & H^{[n_k 2]}(t) & \cdots & H^{[n_k m]}(t)
\end{bmatrix}
\tag{33}
$$

where $H^{[ji]}(t)$ denotes the channel coefficient from the i-th antenna of the transmitter to the j-th antenna of user k. Similar to previous discussions, we first consider the communication task in isolation. At time t, the signal that user k expects to receive is denoted as $W^{[k]}_i(t)$ (where $i = 1, 2, \ldots, n_k$). Using the same approach, we aim to encode the signal such that it is zeroed out at non-desired targets and non-desired times.

For user k, the matrix of non-desired times and targets that the signal $W^{[k]}_i(t)$ experiences at time t can be represented as:

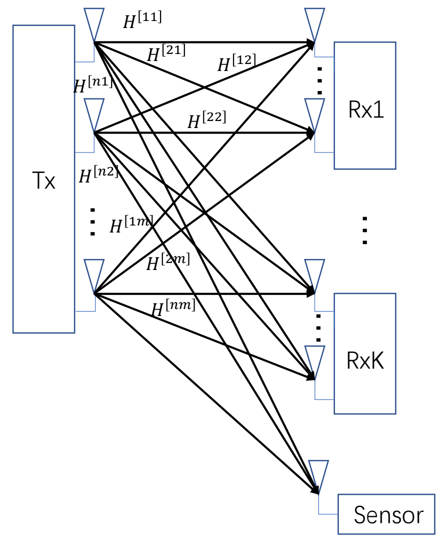


Fig. 7: MU-MIMO senior.

$$
\mathbf{H}^{[k]}_{(t-\mathrm{unexp})} = \begin{bmatrix}
H^{[1]}(1)\\
\vdots\\
H^{[1]}(p)\\
\vdots\\
H^{[k]}(1)\\
\vdots\\
H^{[k]}(t-1)\\
H^{[k]}(t+1)\\
\vdots\\
H^{[k]}\left(\left\lceil \frac{m}{\sum_{k=1}^{K} n_k} \right\rceil\right)
\end{bmatrix}
\tag{34}
$$

where $H^{[k]}\left(\left\lceil \frac{m}{\sum_{k=1}^{K} n_k} \right\rceil\right)$ denotes the channel matrix at the last time slot that can be sent to the maximum number of receivers.

where $p = m \mod \sum_{k=1}^{K} n_k$, let the last k satisfying the first condition be k_1, and $q = p - \sum_{i=1}^{k_1} n_i$. To better understand, we take an example where the transmitter has 6 antennas, the receiver has two communication users each with two antennas, and one sensing user with a single antenna. We hope the first communication user (CU1) can decode 4 symbols in two time slots, and the second communication user (CU2) can decode 2 symbols in two time slots. Similar to the previous section, the undesired matrix for CU1 at the first time slot can be written as:

$$
H^{[1]}_{(1-\mathrm{unexp})} = \begin{bmatrix} H^{[1]}(2)\\ H^{[2]}(1) \end{bmatrix}
\tag{36}
$$

We can see that this matrix is a 4×6 matrix. Therefore, we can use its null space to find two zero vectors $v^{[1]}_1(1)$ and $v^{[1]}_2(1)$ to encode the symbols to be transmitted. Similarly, we

$$H_{(t-\text{unexp})}^{[k]} = \begin{cases} \begin{bmatrix} H^{[11]}\left(\frac{m}{\sum_{k=1}^{K} n_k}\right) & H^{[12]}\left(\frac{m}{\sum_{k=1}^{K} n_k}\right) & \cdots & H^{[1m]}\left(\frac{m}{\sum_{k=1}^{K} n_k}\right) \\ H^{[21]}\left(\frac{m}{\sum_{k=1}^{K} n_k}\right) & H^{[22]}\left(\frac{m}{\sum_{k=1}^{K} n_k}\right) & \cdots & H^{[2m]}\left(\frac{m}{\sum_{k=1}^{K} n_k}\right) \\ \vdots & \vdots & \ddots & \vdots \\ H^{[n_k 1]}\left(\frac{m}{\sum_{k=1}^{K} n_k}\right) & H^{[n_k 2]}\left(\frac{m}{\sum_{k=1}^{K} n_k}\right) & \cdots & H^{[n_k m]}\left(\frac{m}{\sum_{k=1}^{K} n_k}\right) \end{bmatrix}, & \sum_{i=1}^{k} n_i < p \\ \begin{bmatrix} H^{[11]}\left(\frac{m}{\sum_{k=1}^{K} n_k}\right) & H^{[12]}\left(\frac{m}{\sum_{k=1}^{K} n_k}\right) & \cdots & H^{[1m]}\left(\frac{m}{\sum_{k=1}^{K} n_k}\right) \\ H^{[21]}\left(\frac{m}{\sum_{k=1}^{K} n_k}\right) & H^{[22]}\left(\frac{m}{\sum_{k=1}^{K} n_k}\right) & \cdots & H^{[2m]}\left(\frac{m}{\sum_{k=1}^{K} n_k}\right) \\ \vdots & \vdots & \ddots & \vdots \\ H^{[q1]}\left(\frac{m}{\sum_{k=1}^{K} n_k}\right) & H^{[q2]}\left(\frac{m}{\sum_{k=1}^{K} n_k}\right) & \cdots & H^{[qm]}\left(\frac{m}{\sum_{k=1}^{K} n_k}\right) \end{bmatrix}, & p < \sum_{i=1}^{k} n_i < p + n_k \\ \emptyset, & \text{else} \end{cases} \tag{35}$$

can obtain the encoding vectors for the other two symbols. Finally, we get the transmitted signal:

$$X = (v_1^{[1]}(1)W_1^{[1]}(1)) + \cdots + (v_1^{[2]}(1)W_1^{[2]}(1)) + (v_2^{[2]}(1)W_2^{[2]}(1)) \tag{37}$$

Thus, except for the channel experienced by user k at time t, all other channels are zeroed out. In this way, the receiver can decode the desired signal. Therefore, we can decode the received signal using the channel parameters polynomial and the pre-coding matrix. The communication degrees of freedom obtained in 2 time slots are $6/2 = 3$. For the sensing user, the received signal is:

$$Y^{[3]}(t) = \begin{bmatrix} H^{[3,1]}(t) & H^{[3,2]}(t) & \cdots & H^{[3,6]}(t) \end{bmatrix} (X + X_0) \tag{38}$$

Similar to the analysis of the MISO system, since the communication signal remains unchanged in 2 time slots, we can use the signal $X_1^{[0]}$ to obtain effective observations of the target. Let $X_1^{[0]} \in \mathbb{R}^{6\times 1}$, satisfying our orthogonality requirements. Let $X_1^{[0]} = [x_{1,1}, x_{1,2}, x_{1,3}, \ldots, x_{1,m}]^T$, then the transmitted signal is:

$$\begin{bmatrix} X + X_1^{[0]} \\ X \end{bmatrix} \tag{39}$$

In this way, the sensing user only needs to subtract the received signal at the 2nd time slot from the received signal at the 1st time slot to eliminate the interference caused by the communication signal.

$$\mathbf{Y}^{[3]}(1) - \mathbf{Y}^{[3]}(2) = \mathbf{H}^{[3]}(1)X_1^{[0]} \tag{40}$$

where

$$\mathbf{H}^{[3]}(1) = \left[H^{[31]}(1), ..., H^{[36]}(1)\right] \tag{41}$$

Therefore, we obtain $\mathbf{H}^{[3]}(1)X_1^{[0]}$, meaning one effective observations over three time slots. Thus, the SDoF is 1/2, and the overall system degrees of freedom for sensing and communication are (SDoF, CDoF) = (1/2, 3). By decomposing this system, if we only have the 6×4 communication MU-MIMO channel, we can see that the degrees of freedom for sensing and communication are (SDoF, CDoF) = (0, 4). If there is only one sensing user, the degrees of freedom are (SDoF, CDoF) = (1, 0).By the time-sharing method, we can plot the trade-off curve of degrees of freedom obtained by combining two discrete systems, as shown by the red line in the figure. Furthermore, using our proposed method, a new inflection point (SDoF, CDoF) = (1/2, 3) can be obtained, resulting in a new curve, as illustrated by the blue line in the figure 8.

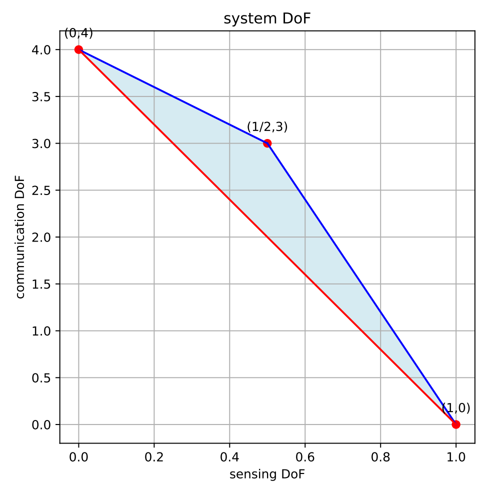


Fig. 8: system DoF vs timesharing DoF

IV. Achievale Scheme

In this section, we introduce the zero-forcing + BIA scheme mentioned earlier. As discussed in the previous section, we categorize based on the number of receive antennas per user and demonstrate the corresponding schemes in the following order: interference channel and MU-MISO channel.

A. Interference Channel

To better analyze the system's degrees of freedom, we start by examining a 3×3 communication interference channel with an additional sensor, and then extend it to a K-user interference channel. We propose a transmission scheme combining BIA and zero-forcing, achieving (SDoF, CDoF) = (2/3, 1). First, we analyze the communication users. The received signal at time t for the k-th communication user is:

$$\hat{\mathbf{Y}}^{[k]} = \begin{bmatrix} \hat{Y}^{[k]}(1) \\ \hat{Y}^{[k]}(2) \\ \hat{Y}^{[k]}(3) \end{bmatrix} = \sum_{i=1}^{3} \begin{bmatrix} H^{[ki]}(1) \\ H^{[ki]}(2) \\ H^{[ki]}(3) \end{bmatrix} X^{[i]}, \tag{42}$$

Here, $H^{[ij]}(t)$ represents the channel from the j-th transmit antenna to the i-th receive antenna at time t, $\hat{Y}^{[k]}(t)$ represents the observed value obtained for the estimate excluding the sensing channel. For any receiver, we can use the values of its undesired channels to apply zero-forcing on the interference signals, ultimately recovering the desired signal. Consequently, the system's communication degrees of freedom can be determined to be 1 [26].

The sensing DoF (SDoF) is 2/3, meaning two effective observations of $X_1^{[0]}$ and $X_2^{[0]}$ can be obtained over three time slots. Next, we demonstrate the signal design using blind interference alignment. Let $X_1^{[0]} \in \mathbb{Z}^{3\times 1}$ and $X_2^{[0]} \in \mathbb{Z}^{3\times 1}$, satisfying $[X_1^{[0]}, X_2^{[0]}][X_1^{[0]}, X_2^{[0]}]^H = I$. Our idea is to use the fact that the channel for the sensor remains constant over a certain period and that the communication signals also remain constant to cancel the communication signals, which are interference for the sensor.

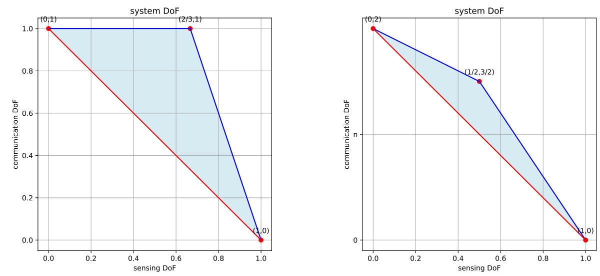
Let $X_1^{[0]} = [x_{1,1}, x_{1,2}, x_{1,3}]^T$ and $X_2^{[0]} = [x_{2,1}, x_{2,2}, x_{2,3}]^T$. For the i-th transmitter, the transmitted signals over three time slots are: $X_i + x_{1,i} + x_{2,i}$, $X_i + x_{2,i}$, $X_i + x_{1,i}$. The messages received by the sensor can be expressed as:

$$\mathbf{Y}^{[4]} = \sum_{i=1}^{3} \begin{bmatrix} H^{[4i]}(1)(X^{[i]} + x_{1,i} + x_{2,i}) \\ H^{[4i]}(1)(X^{[i]} + x_{2,i}) \\ H^{[4i]}(1)(X^{[i]} + x_{1,i}) \end{bmatrix}. \tag{43}$$

By subtracting the observations at the second and third time slots from the first time slot, we get:

$$\begin{aligned} \mathbf{Y}^{[4]}(1) - \mathbf{Y}^{[4]}(t) &= \sum_{i=1}^{3} H^{[4i]}(1) x_{t-1,i} \\ &= \mathbf{H} \begin{bmatrix} x_{t-1,1} \\ x_{t-1,2} \\ x_{t-1,3} \end{bmatrix} \end{aligned}, \tag{44}$$

where $\mathbf{H} = \left[H^{[41]}(1), H^{[42]}(1), H^{[43]}(1)\right]$, $t = 2, 3$. Therefore, we obtain $\mathbf{H}X_1^{[0]}$ and $\mathbf{H}X_2^{[0]}$, meaning two effective observations over three time slots. Thus, the SDoF is 2/3, and the overall system degrees of freedom for sensing and communication are (SDoF, CDoF) = (2/3, 1). By decomposing this system, if we only have the 3×3 communication interference channel, we can see that the degrees of freedom for sensing and communication are (SDoF, CDoF) = (0, 1). If there is only one sensor, the degrees of freedom are (SDoF, CDoF) = (1, 0).By the time-sharing method, we can plot the tradeoff curve of degrees of freedom obtained by combining two discrete



(a) (b)

Fig. 9: system DoF vs timesharing DoF.. (a) Interference channels, (b) MU-MISO channels

systems, as shown by the red line in the figure. Furthermore, using our proposed method, a new inflection point (SDoF, CDoF) = (2/3, 1) can be obtained, resulting in a new curve, as illustrated by the blue line in the figure 9(a).

B. MU-MISO Channel

Next, we provide a detailed example of our method. Assume the transmitter has 3 antennas, and there are two single-antenna communication users and one single-antenna sensor at the receiver. For the first communication user (CU1), we want it to decode two communication symbols $W_1^{[1]}$ and $W_2^{[1]}$ over two time slots; for the second communication user (CU2), we want it to decode one communication symbol $W_1^{[2]}$ over two time slots.

For $W_1^{[1]}$, the matrix $\mathbf{H}_{\text{unexp}}^{1}(1)$ (composed of all non-expected channel elements at non-expected times) is:

$$\mathbf{H}_{\text{unexp}}^{1}(1) = \begin{bmatrix} H^{[21]}(1) & H^{[22]}(1) & H^{[23]}(1) \\ H^{[11]}(2) & H^{[12]}(2) & H^{[13]}(2) \end{bmatrix}, \tag{45}$$

The null space of $\mathbf{H}_{\text{unexp}}^{1}(1)$ contains a vector denoted by $\mathbf{v}_1^{[1]}$. We use $\mathbf{v}_1^{[1]}$ to encode $W_1^{[1]}$, and similarly, we obtain $\mathbf{v}_2^{[1]}$ to encode $W_2^{[1]}$.

For $W_1^{[2]}$, the matrix $\mathbf{H}_{\text{unexp}}^{2}(1)$ is:

$$\mathbf{H}_{\text{unexp}}^{2}(1) = \begin{bmatrix} H^{[11]}(1) & H^{[12]}(1) & H^{[13]}(1) \\ H^{[11]}(2) & H^{[12]}(2) & H^{[13]}(2) \end{bmatrix}, \tag{46}$$

We obtain $\mathbf{v}_1^{[2]}$ to encode $W_1^{[2]}$. Ultimately, we obtain our transmitted communication signals:

$$\mathbf{X} = \sum_{j=1}^{2} \mathbf{v}_1^{j} W_1^{j} + \mathbf{v}_2^{1} W_2^{1}. \tag{47}$$

In this setup, all signals that are either not intended for user i or not required at time t are nullified because the pre-coding vector $\mathbf{v}_t^{i}$ is a vector in the null space of the channel matrix that the signal experiences. After multiplication, this results in zero, and the desired signal W_t^{i} can be decoded from the received signal based on the channel parameter polynomial. Thus, we can continuously transmit the same signal $\mathbf{X}$ across

2 time slots, with each time slot revealing a different communication signal, ensuring the system's communication degree of freedom. With 2 time slots, 3 symbols are decoded, so the system's communication degree of freedom is $\frac{3}{2}$.

Similar to the previous analysis, since the communication signals remain constant across 2 time slots, we can obtain effective observations for 1 signals, namely $\mathbf{X}_1^{[0]} \in \mathbb{R}^{m\times 1}$, and satisfies the required orthogonality condition. Let $X_1^{[0]} = [x_{1,1}, x_{1,2}, x_{1,3}]^T$, the transmitted signals over two time slots are: $\mathbf{X} + X_1^{[0]}$, $\mathbf{X}$. The messages received by the sensor can be expressed as:

$$\mathbf{Y}^{[3]} = \begin{bmatrix} \mathbf{H}^{[3]}(1)(\mathbf{X} + X_1^{[0]}) \\ \mathbf{H}^{[3]}(1)(\mathbf{X}) \end{bmatrix}, \tag{48}$$

By subtracting the observations at the second time slot from the first time slot, we get:

$$\mathbf{Y}^{[3]}(1) - \mathbf{Y}^{[3]}(2) = \mathbf{H}^{[3]}(1)X_1^{[0]}, \tag{49}$$

where $\mathbf{H}^{[3]}(1) = \left[H^{[31]}(1), H^{[32]}(1), H^{[33]}(1)\right]$. Therefore, we obtain $\mathbf{H}^{[3]}(1)X_1^{[0]}$, meaning one effective observations over three time slots. Thus, the SDoF is 1/2, and the overall system degrees of freedom for sensing and communication are (SDoF, CDoF) = (1/2, 3/2).

By decomposing this system, if we only have the 3×2 communication MISO channel, we can see that the degrees of freedom for sensing and communication are (SDoF, CDoF) = (0, 2). If there is only one sensor, the degrees of freedom are (SDoF, CDoF) = (1, 0).By the time-sharing method, we can plot the tradeoff curve of degrees of freedom obtained by combining two discrete systems, as shown by the red line in the figure. Furthermore, using our proposed method, a new inflection point (SDoF, CDoF) = (1/2, 3/2) can be obtained, resulting in a new curve, as illustrated by the blue line in the figure 9(b).

V. Simulation

In this section, we simulate the previously discussed 3x4 interference channel example and compare it with the methods mentioned in the [22]. The specific simulation parameters are set as follows: the carrier frequency is 5.8 GHz with a bandwidth of 10 MHz. To ensure the stability of communication transmission, the communication symbols to be transmitted are modulated by QPSK and then transformed into time-domain signals via IFFT for transmission. The distance between the transmitter and the communication receiver is between [50, 100] meters, and between the transmitter and the sensor is between [10, 30] meters. The fixed transmission power for both the communication signal and the sensing signal is set to 30 dBm, and the channel fading characteristics follow a Rayleigh distribution.

For the task settings, we remain consistent with this paper: the transmitter is assumed to know the communication channel but not the sensing channel. In the comparison method under the same assumptions, the sensing signal is transmitted with

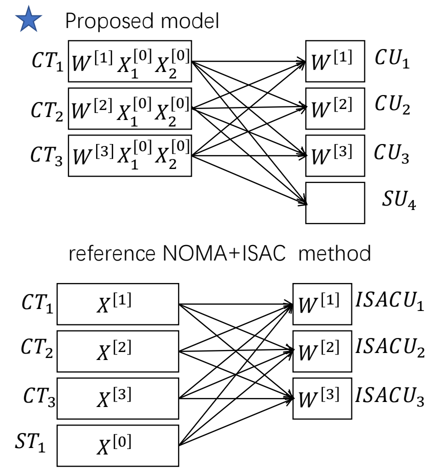


Fig. 10: simulation senior

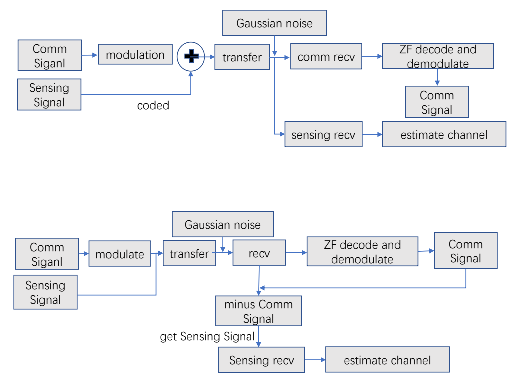


Fig. 11: simulation flowchart

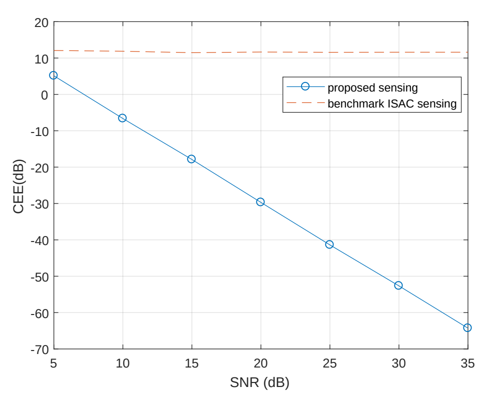


Fig. 12: simulation result

opposite polarity at two different time slots to cancel out the interference at the receiver. However, due to the unknown channel, decoding is impossible, and the mutual interference between communication and sensing signals is treated as noise. In contrast, our method designs the sensing signals using the blind interference alignment technique, which effectively reduces the interference dimension caused by the communication signals at the receiver.The flowchart is shown in Figure10 and Figure11.

For the sensing task, we use the LS estimation method to estimate the channel, given by $\hat{\mathbf{H}} = (\mathbf{X^H X})^{-1}\mathbf{X^H Y}$, where $\hat{\mathbf{H}}$ represents the estimated channel, $\mathbf{X}$ represents the sensing signal, and $\mathbf{Y}$ represents the received signal. Since the LS method is very sensitive to noise, we observe the performance differences under SNR values ranging from [5, 35] dB. Analysis shows that at higher SNR conditions, the proposed method is less affected, and the estimation error decreases with increasing SNR. In contrast, the reference method treats the communication signal as noise, so even with low noise power, the high transmission power of the communication signal results in poor estimation accuracy, with minimal changes in estimation error as SNR increases.In comparison, our method provides a gain of [7, 75] dB in estimation accuracy.

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