WEEKLY JOURNAL-08

SUMMARY

Summary: continued the discussion on continuous Markov process. Various scenarios such as Load sharing, stand by systems were discussed.

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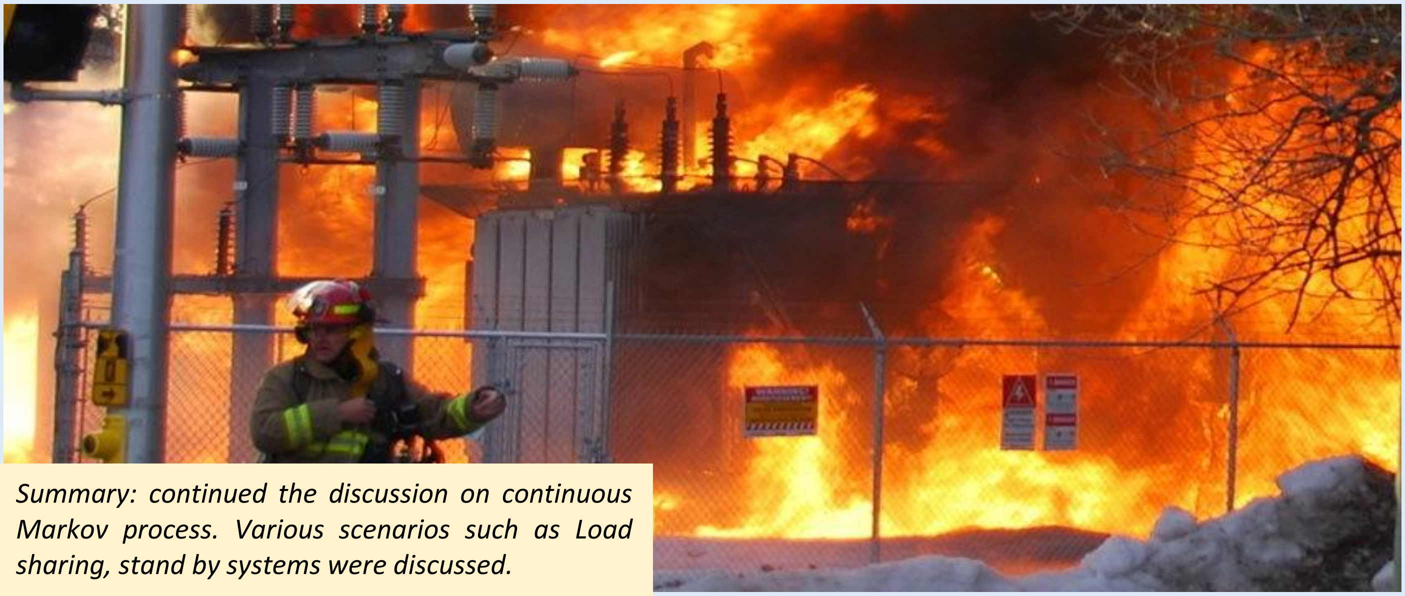
Markov Process

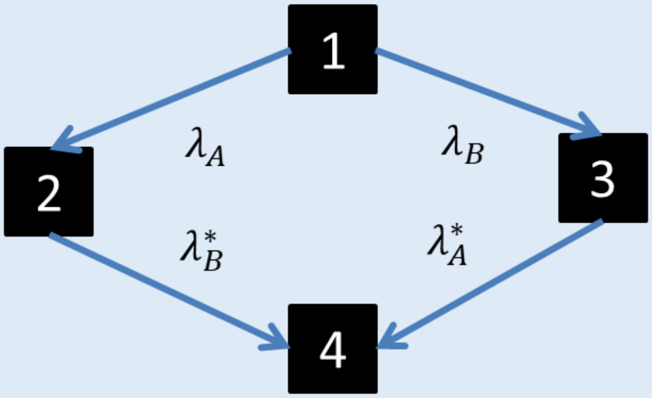
Week-08



Summary: continued the discussion on continuous Markov process. Various scenarios such as Load sharing, stand by systems were discussed.

Load sharing

A load can be supplied by two components in parallel. This is said to be load sharing. Failure of a component, in a network with 2 components in parallel, might affect the other component's failure rate. This can be represented as in below figure.



Failure rate between 2 and 4 becomes λ_B^*, since failure of component A will affect the failure rate of component B.

Now the probability relationships can be written as below.

$$\frac{dP_1(t)}{dt} = -(\lambda_A + \lambda_B)\, P_1(t)$$

$$\frac{dP_2(t)}{dt} = \lambda_A P_1(t) - \lambda_B^* P_2(t)$$

$$\frac{dP_3(t)}{dt} = \lambda_B P_1(t) - \lambda_A^* P_3(t)$$

$$P_3(t) = 1 - P_1(t) - P_2(t)$$

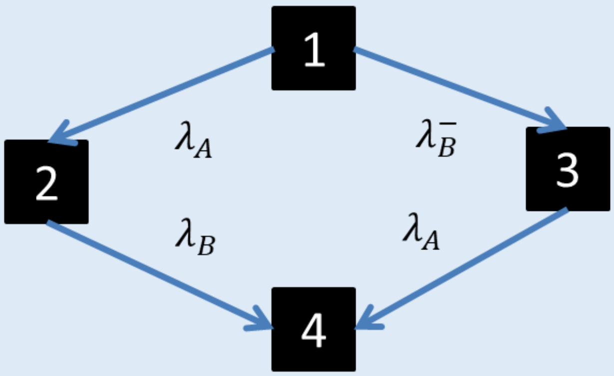
$P_2(t)$ *and* $P_2(t)$ can be found by using differential solutions method.

$$P_1(t) = \exp(-(\lambda_A + \lambda_B)t)$$

$$P_2(t) = \frac{\lambda_B}{\lambda_A + \lambda_B - \lambda_C}(\exp(-\lambda_C t) - \exp(-(\lambda_A + \lambda_B)t))$$

Standby systems

Standby systems are used in parallel and the stand by component stays does not operate until the other component fails. Failure rate of the standby component will become less than rated upon failure of the primary component.



Probability relationships can be written as

$$\frac{dP_1(t)}{dt} = -(\lambda_A + \lambda_{\bar{B}})\, P_1(t)$$

$$\frac{dP_2(t)}{dt} = \lambda_A P_1(t) - \lambda_B P_2(t)$$

$$\frac{dP_3(t)}{dt} = \lambda_{\bar{B}} P_1(t) - \lambda_A P_3(t)$$

$$P_1(t) = \exp(-(\lambda_A + \lambda_{\bar{B}})t)$$

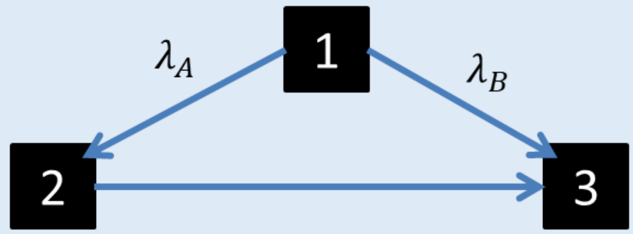
$$P_2(t) = \frac{\lambda_A}{\lambda_A + \lambda_{\bar{B}} - \lambda_B}(\exp(-\lambda_B t) - \exp(-(\lambda_A + \lambda_{\bar{B}})t))$$

$$P_3(t) = \exp(-\lambda_A t) - \exp(-(\lambda_A + \lambda_{\bar{B}})t)$$

One component operating in multiple states

So far, we discussed that a component can be in operation or failure state. However, in reality a component can take more than these two states. For example, a wind generator can generate electricity at 100% of it's rated value or 70% or 50%. The wind plant can operate anywhere between 0-100% of its rated value.

State diagram can be drawn for this scenario as in below:



Probability relationship for this can be written as

$$\frac{dP_1(t)}{dt} = -(\lambda_A + \lambda_B)\, P_1(t)$$

$$\frac{dP_2(t)}{dt} = \lambda_A P_1(t) - \lambda_C P_2(t)$$

$$P_3(t) = 1 - P_1(t) - P_2(t)$$

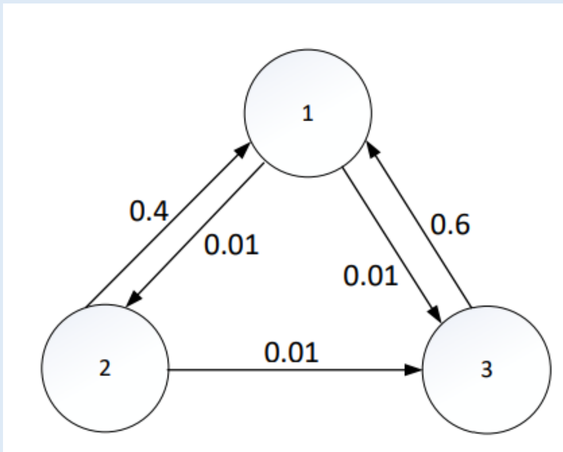
Then the solutions, using differential equation solving method, can be obtained as:

$$P_1(t) = \exp(-(\lambda_A + \lambda_B)t)$$

$$P_2(t) = \frac{\lambda_B}{\lambda_A + \lambda_B - \lambda_C}(\exp(-\lambda_C t) - \exp\,(-(\lambda_A + \lambda_B)t))$$

State space diagram and transition rates in f/hr of a continuous Markov process is shown in below Figure, Determine the following

1. Limiting probabilities of each state



Being in each state can be written as

$$\frac{dP_1(t)}{dt} = -0.02P_1(t) + 0.4 * P_2(t) + 0.6 * P_3(t)$$

$$\frac{dP_2(t)}{dt} = 0.01P_1(t) - 0.01 * P_2(t) + 0$$

$$\frac{dP_3(t)}{dt} = 0.01P_1(t) + 0.01 * P_2(t) - 0.6 * P_3(t)$$

Solution to these equations by using Laplace transformation

$$\frac{d}{dt}\begin{bmatrix} P_1(t) \\ P_2(t) \\ P_3(t) \end{bmatrix} = \begin{bmatrix} -0.02 & 0.4 & 0.6 \\ 0.01 & -0.01 & 0 \\ 0.01 & 0.01 & -0.6 \end{bmatrix} \begin{bmatrix} P_1(t) \\ P_2(t) \\ P_3(t) \end{bmatrix}$$

At limiting states $P^T p = \dot{p} = 0$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -0.02 & 0.4 & 0.6 \\ 0.01 & -0.01 & 0 \\ 0.01 & 0.01 & -0.6 \end{bmatrix} \begin{bmatrix} P_1(t) \\ P_2(t) \\ P_3(t) \end{bmatrix}$$

However, the 3rd row is dependent row. Therefore, we can use following property

$$P_1(t) + P_2(t) + P_3(t) = 1$$

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.02 & 0.4 & 0.6 \\ 0.01 & -0.01 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_1(t) \\ P_2(t) \\ P_3(t) \end{bmatrix}$$

Solution can be found by simplifying the above equation.