# WEEKLY JOURNAL-05


SUMMARY

*Detailed discussion on reliability modelling for series, parallel and complex systems. Explained various techniques such as conditional probability, cut set, connection matrix to model reliability of systems.*



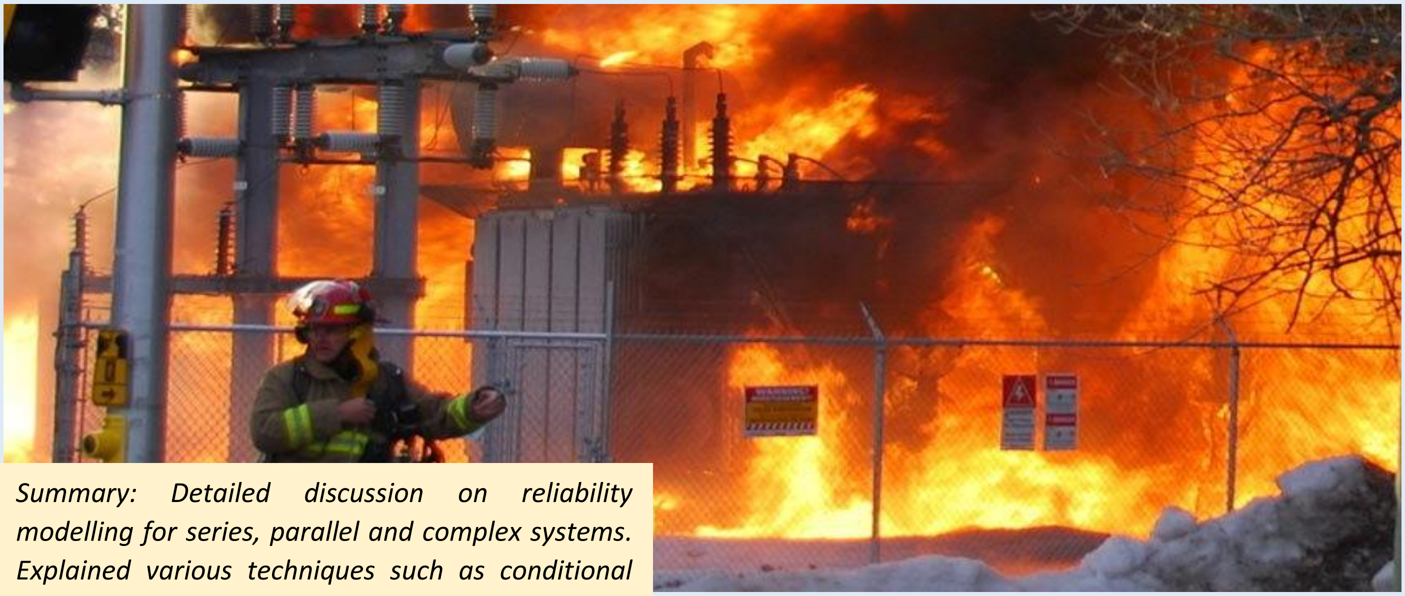
Shanthanam, Sangar

N359Z235

Group partner: Danielle Mouer

# Reliability Modelling

Week-04



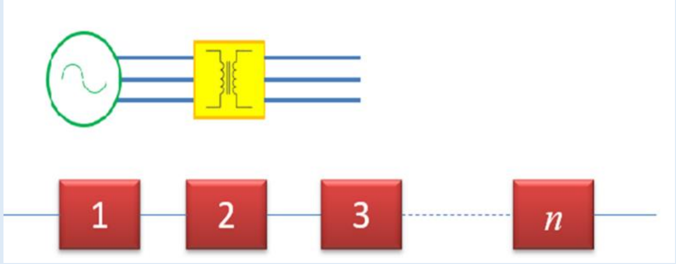
*Summary: Detailed discussion on reliability modelling for series, parallel and complex systems. Explained various techniques such as conditional probability, cut set, connection matrix to model reliability of systems.*

Reliability modelling of a bigger systems can be achieved by modelling the reliability of each component of that system.

These components are connected in series, parallel, meshed or combinations of these three. Components with series connection in electric network can be parallel in terms of reliability modelling.

In series connected reliability network, system can be affected with single component failure. Where as in parallel connected network failure of all components should fail.

*<u>Reliability modelling for series component only systems</u>*





Let $E_i$ is the event that component $i$ will be working, then

$$P(E_i) = R_i$$

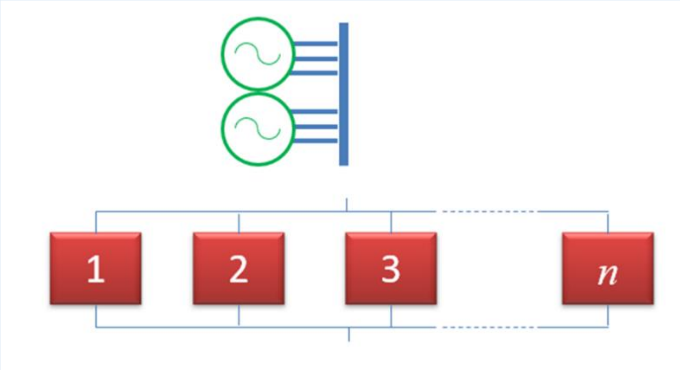
$$R_{sys} = P(E_1 \cap E_2 \cap E_3 \cap \ldots.)$$

But failure of individual components is independent. Therefore

$$R_{sys} = P(E_1 \cap E_2 \cap E_3 \cap \ldots.)$$
$$= P(E_1).P(E_2).P(E_3) \ldots.$$

$$R_{sys} = R_1.R_2.R_3 \ldots$$

*Reliability modelling for series component only systems*




Let $E_i^c$ is the event that the component $i$ will fail.

$$R_{sys} = 1 - P(E_1^c \cap E_2^c \cap E_3^c \cap \dots)$$

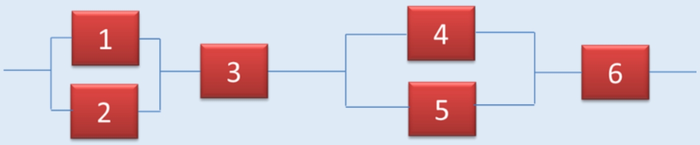
$$R_{sys} = 1 - P(E_1^c).P(E_2^c).P(E_3^c) \dots)$$

$$R_{sys} = 1 - (1 - R_1)(1 - R_2)(1 - R_3) \dots.$$

Most of the real-world systems contain both the series and parallel components.

Example:

Find the reliability of the system in the following in the figure:

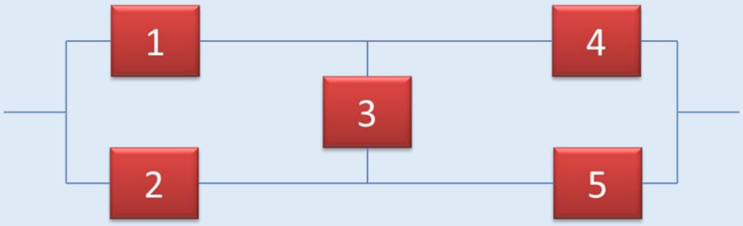



$$R_{sys} = \big(1 - (1 - R_1)(1 - R_2)\big).R_3(1 - R_4)(1 - R_5).R_6$$

*Solving complex systems*

There are some other systems which will have components that are neither series nor parallel. These systems can be solved by using

1. Conditional Probability method.
2. Cut set method
3. Connection matrix method




*Conditional probability method*

Use superposition method. Keep the specific component at working condition and find the reliability then keep the system to be in failure condition and find the reliability. System reliability will be sum of these two-event reliability.


System reliability = P(Sys working/ 3 works)+P(sys working/ 3 fails)

$$R_{sys} = \big(1 - (1 - R_1)(1 - R_2)\big)\big(1 - (1 - R_4)(1 - R_5)\big)R_3$$
$$+(1 - (1 - R_1R_4)(1 - R_2R_5))(1 - R_3)$$

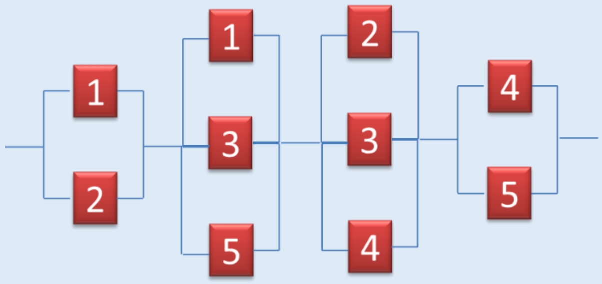
Components are removed until the system reliability can be simply found.

*Cut set method*

Cut set is defined as set of components which, when failed, causes system failure.

Minimal Cut Set is sub set of cut set, in which failure of the system happens only when all the components of the set fail. System does not fail when part of the set fails.

| Minimal cut set | Elements |
|---|---|
| A | 1,2 |
| B | 1,3,5 |
| C | 2,3,4 |
| D | 4,5 |




*Connection matrix technique*

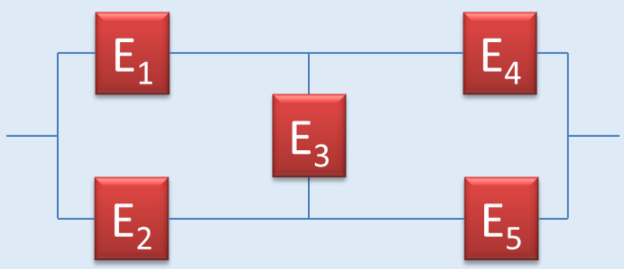
This technique is very useful to larger networks. Easy to program. The connection matrix is defined based on connectivity between nodes in a system.

0 is assigned if there is no connection between two nodes, 1 is assigned to show the connection between same nodes. And, the element is assigned when two nodes are connected by an element.

Then the matrix is simplified by removing all the nodes (except input and output nodes) using below formula. To remove node $k$, each element $N_{ij}$ ($i,j \neq k$) is replaced by

$$N_{ij}^{new} = N_{ij} + N_{ik}N_{kj}$$

Below steps shows how to apply connection matrix method for previously discussed method.




Step 1: develop the connection matrix based on previously discussed methods.

$$\begin{array}{c c} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 1 & E_1 & E_2 & 0 \\ 0 & 1 & E_3 & E_4 \\ 0 & E_3 & 1 & E_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Step 2: simplify the matrix by using node removal formula:

i. Removing node B

$$N_{AA}^{new} = N_{AA} + N_{AB}N_{BA}$$

$$N_{AA}^{new} = 1 + E_1 0 = 1$$

$$N_{AC}^{new} = N_{AC} + N_{AB}N_{BC}$$

$$N_{AA}^{new} = E_2 + E_1E_3$$

likewise other components will be found

$$\begin{array}{c c} & \begin{array}{ccc} A & C & D \end{array} \\ \begin{array}{c} A \\ C \\ D \end{array} & \begin{bmatrix} 1 & E_2 + E_1E_3 & E_1E_4 \\ 0 & 1 & E_5 + E_3E_4 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

ii. Now remove node C

$$N_{AD}^{new} = N_{AD} + N_{AC}N_{CD}$$

$$N_{AD}^{new} = E_1E_4 + (E_2 + E_1E_3)(E_5 + E_3E_4)$$

$$N_{DA}^{new} = N_{DA} + N_{DC}N_{CA} = 0 + 0 * 0 = 0$$

$$\begin{array}{c c} & \begin{array}{cc} A & D \end{array} \\ \begin{array}{c} A \\ D \end{array} & \begin{bmatrix} 1 & E_1E_4 + (E_2 + E_1E_3)(E_5 + E_3E_4) \\ 0 & 1 \end{bmatrix} \end{array}$$

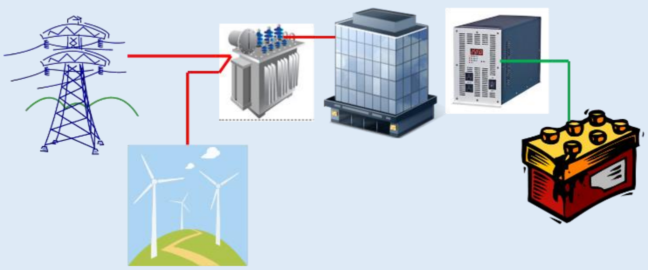
$$\begin{array}{c c} & \begin{array}{cc} A & D \end{array} \\ \begin{array}{c} A \\ D \end{array} & \begin{bmatrix} 1 & \boxed{E_1E_4 + E_2E_5 + E_1E_3E_5 + E_2E_3E_4} \\ 0 & 1 \end{bmatrix} \end{array}$$

Reliability for transition from A to D

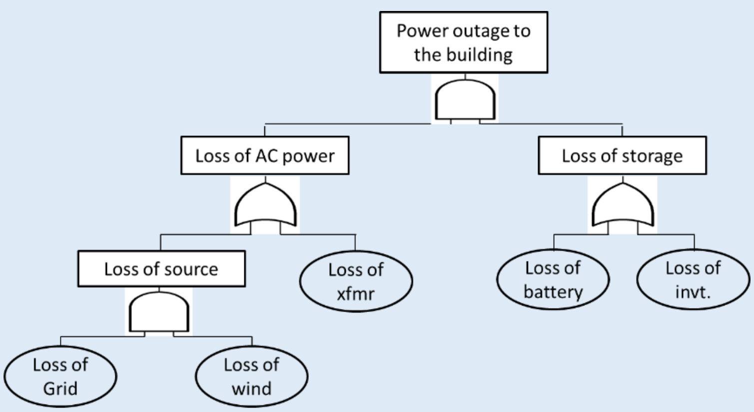
Fault tree method

Individual component failures together make the system failure. Therefore, system failure is modelled using Boolean algebra. Tree is based on the sequence of failures that could lead to the top event.

Apply fault tree method for following system.



Reliability of above system can be modelled as in below figure.




Failure of storage can be defined as

$$P(F_{ST}) = P(F_B + F_I) - P(F_B) + P(F_I) - P(F_B) \cdot P(F_I)$$

Loss of source can be modelled as

$$P(F_{So}) = P(F_G) \cdot P(F_W)$$

Loss of AC power is AND operation with grid and wind therefore

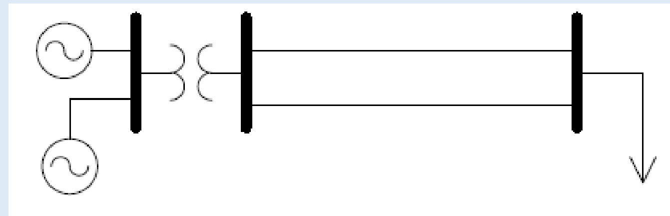
$$P(F_{AC}) = P(F_{Grid} + F_{Wind}) = P(F_{Grid}) + P(F_{Wind}) - P(F_{Grid})\, P(F_{Wind})$$

And the system reliability can be modelled as

$$P(F_{Sys}) = P(F_{AC}) \cdot P(F_{ST})$$

Example problem for the week

Consider the following power system. All these components have Weibull distribution with the following scale and shape parameters.



| | $\alpha$ | $\beta$ |
|---|---|---|
| Generator | 50 | 3 |
| Transformer | 65 | 2 |
| Lines | 30 | 2.5 |

a) Determine the general reliability function for system:

Let reliability of each generator be $R_G$, reliability of the transformer be $R_{Xf}$, and reliability of the lines to be $R_L$.

Cut set method can be used to estimate system reliability.

| | | |
|---|---|---|
| 1 | 2 | 3 |
| G1G2 | Transformer | L1L2 |

$$R_{sys} = \left(1 - (1 - R_{G1})(1 - R_{G2})\right) * R_{Xf} * (1 - (1 - R_{L1})(1 - R_{L2})\text{----------------------------------------------}(1)$$

Since each of these components follows Weibull distribution:

$$R(t) = \exp\left(-\left(\frac{t}{\alpha}\right)^{\beta}\right)$$

$$R_G = e^{-(\frac{t}{50})^3}$$

$$R_{Xf} = e^{-(\frac{t}{65})^2}$$

$$R_L = e^{-(\frac{t}{30})^{2.5}}$$

Therefore, system reliability can be calculated from eq 1.

$$R_{sys} = \left(1 - \left(1 - e^{-(\frac{t}{50})^3}\right)\left(1 - e^{-(\frac{t}{50})^3}\right)\right) * e^{-(\frac{t}{65})^2} * (1 - (1 - e^{-(\frac{t}{30})^{2.5}})(1 - e^{-(\frac{t}{30})^{2.5}})$$

## References:

[1] Power system reliability lecture notes.
[2] Problem set given for the class