WEEKLY JOURNAL-07

SUMMARY

Discussed about continuous Markov process. Advantages and conditions to apply Markov process were discussed. Second quiz was conducted in this week.

Shanthanam, Sangar

N359Z235

Group partner: Danielle Mouer

Markov Process Introducion

Week-07

Summary: Discussed about continuous Markov process. Advantages and conditions to apply Markov process were discussed. Second quiz was conducted in this week.

If a system has n number of states and its state transition depends only on current state then it is called as Markov process. Another important property of the Markov process is that a component can stay only at a single state at given time. Most reliability problems can be continuous time discrete state. If a component cannot be repaired then it is considered as nonrepairable case.

Following definitions are important in Markov Process:

Failure Rate $\lambda =$

$$\frac{\text{Number of Failures of a Component in the Given Time Period}}{\text{Total Period of Time the Component was Operating}}$$

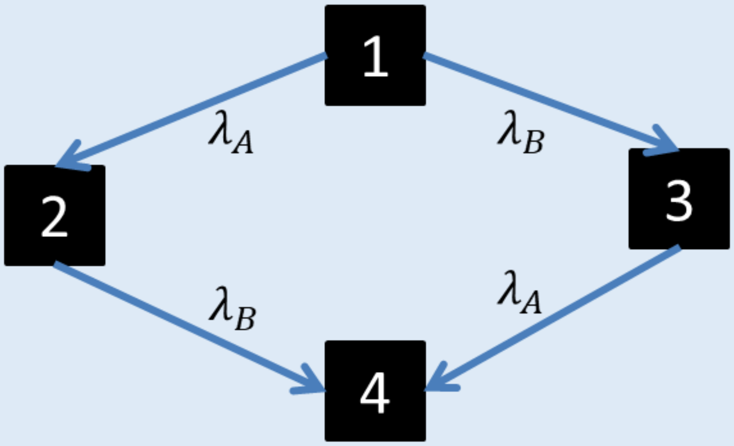
Repair Rate $\mu =$

$$\frac{\text{Number of Repairs of a Component in the Given Time Period}}{\text{Total Period of Time the Component was being Repaired}}$$

Transition Rate =

$$\frac{\text{Number of Times a Transition Occurs from a Given State}}{\text{Time Spent in that State}}$$

Below four state reliability of a system without repair is modelled using Markov process:



Here state 4 is an absorbing state. Arrows are pointed towards only one direction since repair is not considered.

State	Component A	Component B
1	Operating	Operating
2	Failed	Operating
3	Operating	Failed
4	Failed	Failed

If the system is at state 1, then Probability of being in state 1 at time $t+\Delta t$.

$$P_1(t+\Delta t) = P_1(t) - \lambda_A \Delta t P_1(t) - \lambda_B \Delta t\, P_1(t)$$

$$\frac{\left(P_1(t+\Delta t) - P_1(t)\right)}{\Delta t} = -\lambda_A P_1(t) - \lambda_B\, P_1(t)$$

$$\frac{dP_1(t)}{dt} = -(\lambda_A + \lambda_B)\, P_1(t)$$

$$\frac{dP_1(t)}{P_1(t)} = -(\lambda_A + \lambda_B) dt$$

$$\ln\left(P_1(t)\right) = -(\lambda_A + \lambda_B)t$$

$$P_1(t) = \exp(-(\lambda_A + \lambda_B)t)$$

When we do this calculation; the transition probability will be subtracted if the arrow goes out of the state. And probability will be added if the arrow comes in.

In the same way as described above, If the system is at state 2, then Probability of being in state 2 at time t+Δt can be written as:

$$P_2(t+\Delta t) = P_2(t) + \lambda_A \Delta t P_1(t) - \lambda_B \Delta t\, P_2(t)$$

$$\frac{dP_2(t)}{dt} = \lambda_A P_1(t) - \lambda_B\, P_2(t)$$
$$= \lambda_A \exp(-(\lambda_A + \lambda_B)t) - \lambda_B\, P_2(t)$$

The solution is:

$$P_2(t) = -\exp(-(\lambda_A + \lambda_B)t) + \exp(-\lambda_B t)$$

Ans we can find the Probability of being in state 3 at time t+Δt if the system is at state 3.

$$P_3(t+\Delta t) = P_3(t) + \lambda_B \Delta t P_1(t) - \lambda_A \Delta t\, P_2(t)$$

$$\frac{P_3(t+\Delta t) - P_3(t)}{\Delta t} = \lambda_B P_1(t) - \lambda_A P_2(t)$$

$$\frac{dP_3(t)}{dt} = \lambda_B P_1(t) - \lambda_A P_2(t)$$

And the solution can be written as:

$$P_3(t) = -\exp(-(\lambda_A + \lambda_B)t) + \exp(-\lambda_A t)$$

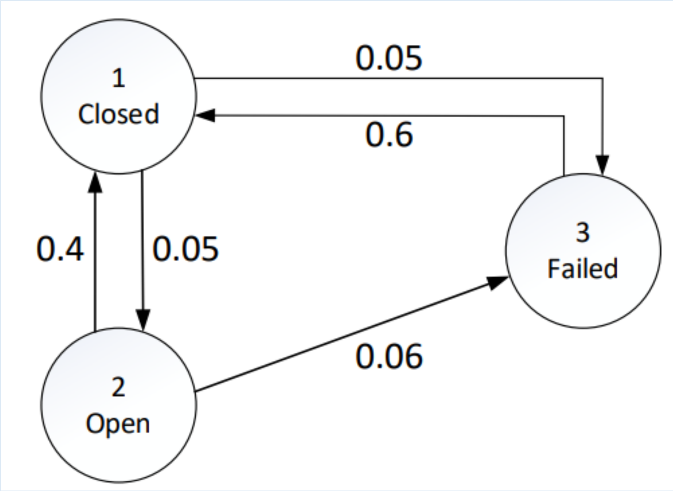
But we know that the total probability of the system is 1. Therefore

$$P_4(t) = 1 - P_1(t) - P_2(t) - P_3(t)$$

Problem of the week:

A circuit breaker can reside in one of the three mutually exclusive states as shown in Figure 1. The values shown are the probabilities of making the related transition at the end of each discrete time interval of 1 hour. System starts in State 1. Determine:

1. The probability of residing in each state after three-time interval.
2. Limiting state reliability of the system



Solution can be found using discrete Markov chain.

$$P(0) = [P_1^0 \quad P_2^0 \quad P_3^0]$$

The initial state is closed therefore:

$$P(0) = [1 \quad 0 \quad 0]$$

$$\boldsymbol{P} = \begin{bmatrix} P_{CC} & P_{CO} & P_{CF} \\ P_{OC} & P_{OC} & P_{OF} \\ P_{FC} & P_{FO} & P_{FF} \end{bmatrix} = \begin{bmatrix} 0.9 & 0.05 & 0.05 \\ 0.4 & 0.54 & 0.06 \\ 0.6 & 0 & 0.4 \end{bmatrix}$$

$$P(3) = P(0)\boldsymbol{P}^3$$

Answer can be found by simplifying this metrics.

State transition after convergence:

$$P^c(\boldsymbol{P}-\boldsymbol{I}) = [\boldsymbol{P_1^c} \quad \boldsymbol{P_2^c} \ \ \boldsymbol{P_3^c}]\left(\begin{bmatrix} 0.9 & 0.05 & 0.05 \\ 0.4 & 0.54 & 0.06 \\ 0.6 & 0 & 0.4 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right) = [\boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}]$$

$$P^c(\boldsymbol{P}-\boldsymbol{I}) = [\boldsymbol{P_1^c} \quad \boldsymbol{P_2^c} \ \ \boldsymbol{P_3^c}]\left(\begin{bmatrix} -0.1 & 0.05 & 0.05 \\ 0.4 & -0.46 & 0.06 \\ 0.6 & 0 & -0.6 \end{bmatrix}\right) = [\boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}]$$

Solution can be by simplifying this.

References:

[1] Power system reliability lecture notes.
[2] Problem set given for the class
[3] "stat.berkeley.edu," 2022. [Online]. Available: https://www.stat.berkeley.edu/~aldous/150/takis_exercises.pdf. [Accessed 14 10 2022].