WEEKLY JOURNAL-04

SUMMARY

Reliability modelling by using probability theory was discussed.

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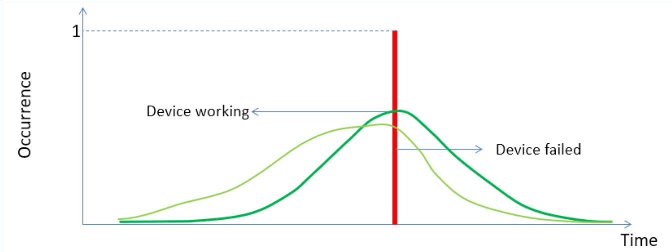
Reliability Modelling

Week-04

Summary: Overview of reliability modelling based on the probability theory. Different reliability indices were discussed. Modelling reliability using exponential and Weibull distribution was discussed.

Reliability of a system can be defined as the probability of that the system can perform its functions successfully under specified condition for a given time period.



Above figure shows a probability density function of a device failure rate. Device is in working condition until it reaches the red line. Then the device failed at time t (at red line point).

Reliability can also be defined as the probability that the device survived until time t. f(t) is the probability that the device will fail at time t.

Cumulative distribution function of failure distribution function can be written as:

$$F(t) = \int_0^t f(\tau)d(\tau)$$

From the above equation it can be written as:

$$f(t) = \frac{dF(t)}{dt}$$

Since the total probability of any function is 1. The reliability can be found by subtracting the CDF of failure from 1.

$$R(t) = 1 - \int_0^t f(\tau)d(\tau)$$

And

$$f(t) = -\frac{dR(t)}{dt}$$

MTTF-(Mean Time To Failure):- length of time that the device can work without failure.

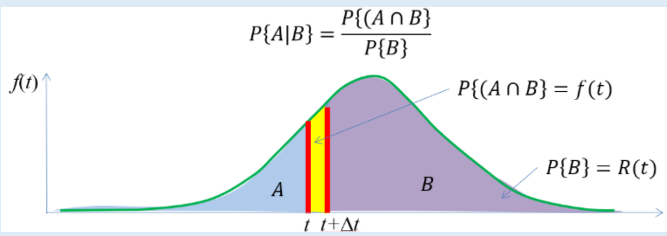
This can be written as:

$$MTTF = E[f(t)] = \int_0^{\infty} t - \frac{dR(t)}{dt} dt$$

$$= -tR(t)|_0^{\infty} + \int_0^{\infty} R(t)dt$$

$$MTTF = \int_0^{\infty} R(t)dt$$

Let A- be device in working condition and B-be the device in failed condition. Then, the device has been working up to this moment and failed in next moment can be modelled as conditional probability. As in below figure



$$P\{A/B\} = \frac{P\{(A \cap B\}}{P\{B\}} = \frac{f(t)}{R(t)}$$

This is an important index of reliability and named as hazard rate$\lambda(t)$. This will tell that if the device has survived up to this moment and at what rate it will fail in next moment.

$$\lambda(t) = P\{A/B\} = \frac{f(t)}{R(t)}$$

If the failure rate is constant then the reliability distribution will be exponential function.

$$\lambda = \frac{f(t)}{R(t)}$$

$$\lambda = -\frac{dR(t)}{dt}\frac{1}{R(t)}$$

$$\lambda dt = -\frac{1}{R(t)}dR(t)$$

$$\int_{R(0)=1}^{R(t)} \frac{1}{R(\tau)} dR(\tau) = -\int_0^t \lambda dt$$

$$\ln(R(t)) = -\lambda t$$

$$R(t) = \exp(-\lambda t)$$

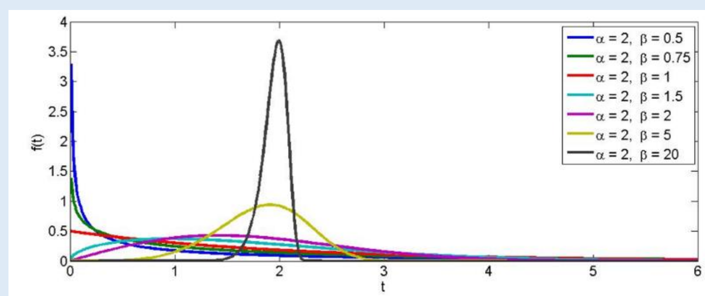
$$f(t) = -\frac{dR(t)}{dt} = -\frac{d\exp(-\lambda t)}{dt} = -(-\lambda)\exp(-\lambda t)$$

$$f(t) = \lambda\exp(-\lambda t)$$

Reliability of different systems have different distribution functions. Weibull distribution, is a very useful distribution, is used to model reliability of engineering system. advantage of Weibull distribution is that it can be used to model different shapes of distributions.

$$f(t) = \frac{\beta t^{\beta-1}}{\alpha^{\beta}} \exp\left(-\frac{t^{\beta}}{\alpha^{\beta}}\right)$$

Here α is known a scale factor and β is known as shape factor. Below figure shows the Weibull distribution plots for different α, β values.



Since we know the PDF, reliability function can be derived as follows:

$R(t) = 1 - \int_0^t f(\tau)d\tau$ or $R(t) = \int_t^{\infty} f(\tau)d\tau$

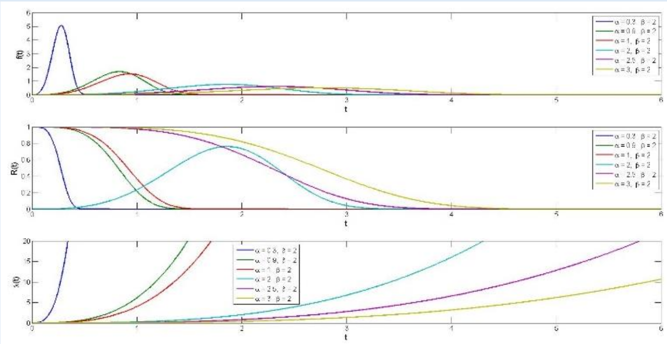
$$R(t) = \int_t^{\infty} \frac{\beta\tau^{\beta-1}}{\alpha^{\beta}} \exp\left(-\frac{\tau^{\beta}}{\alpha^{\beta}}\right) d\tau$$

$$R(t) = \exp\left(-\left(\frac{t}{\alpha}\right)^{\beta}\right)$$

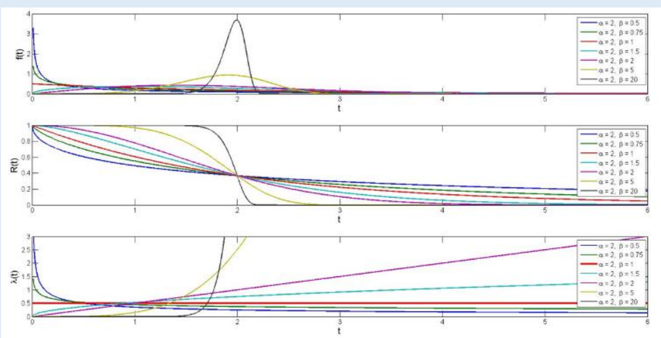
From this hazard rate can be calculated.

$$\lambda(t) = \frac{f(t)}{R(t)} = \frac{\frac{\beta t^{\beta-1}}{\alpha^{\beta}}\exp\left(-\left(\frac{t}{\alpha}\right)^{\beta}\right)}{\exp\left(-\left(\frac{t}{\alpha}\right)^{\beta}\right)} = \frac{\beta t^{\beta-1}}{\alpha^{\beta}}$$

Below figure shows variation of pdf for different α values.



Change of β value changes the shape of the pdf as in below figure.



It can be observed from above figures failure rate can increase, decrease or stay constant over time.

β < 1: Time decreasing failure rate; β = 1: Constant failure rate; β > 1: Time increasing failure rate.

The Weibull hazard rate can be linearized as in below:

$$\lambda(t) = \frac{\beta t^{\beta-1}}{\alpha^{\beta}}$$

Take natural log both side

$$\ln\big(\lambda(t)\big) = \ln\left(\frac{\beta t^{\beta-1}}{\alpha^{\beta}}\right) = \ln(\beta) + \ln\big(t^{\beta-1}\big) - \ln\big(\alpha^{\beta}\big)$$

$$\underbrace{\ln\big(\lambda(t)\big)}_{y} = \underbrace{(\beta-1)}_{m}\underbrace{\ln(t)}_{x} + \underbrace{\ln(\beta) - \ln\big(\alpha^{\beta}\big)}_{c}$$

Example:

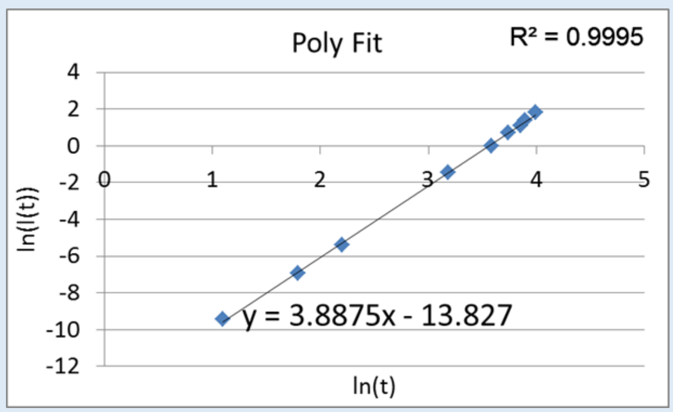
Model the following transformer failure rate:

Year	3	6	9	24	36	42	47	49	54
Failures / Year	0.00008	0.001	0.0045	0.23	1	2	3	4	6

The above information can be converted into reliability parameter table as in below table.

t	$\lambda(t)$	$\ln(t)$	$\ln(\lambda(t))$
3	0.00008	1.09861229	-9.4335
6	0.001	1.79175947	-6.9078
9	0.0045	2.19722458	-5.4037
24	0.23	3.17805383	-1.4697
36	1	3.58351894	0
42	2	3.73766962	0.69315
47	3	3.8501476	1.09861
49	4	3.8918203	1.38629
54	6	3.98898405	1.79176

This can be plotted as in below figure



β can be found from the slope of the line: -

$\beta - 1 = 3.8875 \Rightarrow \beta = 4.8875$

And α can be estimated as in below: -

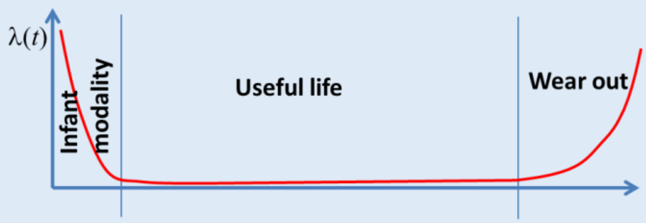
$$\ln(\beta) - \beta\ln(\alpha) = -13.827$$

$$\ln(\alpha) = 3.1536$$

$$\alpha = e^{3.1536} = 23.4225$$

Now this info can be used model the reliability using Weibull distribution.

Products may show higher failure rate at the start of life cycle due to manufacturing/ assembly defects. Then the failure rate may be small for some time. Again, the failure rate increases towards the end of the life cycle. This can be visually represented using bathtub curve as in below figure.



Problem of the day

A particular brand of identical relays has times to first failure that follow a Weibull distribution with parameters β = 0.5 and α = 10 years. What is the probability that a relay will survive (a) 1 year (b) 5 years and (c) 10 years without failure.

The failure rate function can be modelled using Weibull distribution as:

$$f(t) = \frac{\beta t^{\beta-1}}{\alpha^{\beta}} \exp\left(-\frac{t^{\beta}}{\alpha^{\beta}}\right)$$

$$f(t) = \frac{0.5t^{-0.5}}{10^{0.5}} \exp\left(-\frac{t^{0.5}}{\sqrt{10}}\right)$$

Probability that the device will survive for t years: -

$$R(t) = \int_{t}^{\infty} 0.5\sqrt{\frac{\tau}{10}} \exp\left(-\sqrt{\frac{\tau}{10}}\right) d\tau$$

$$\exp\left(-\sqrt{\frac{\tau}{10}}\right)$$

1) 1 year => -.73
2) 5 years => .49
3) 10 years => .37

References:

[1] Power system reliability lecture notes.