STRATEGIC DEMANDS FOR INVENTORS AND AGGREGATE GROWTH

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ABSTRACT. This paper develops a simple Schumpeterian growth model where firm-level strategic demand for inventors can be described. All firms should produce the latest invention to be the monopolist in output market using inventors as the only input of innovation. Given the necessity of inventor as input of innovation, frontier innovating firms can exert market power in inventor market to strategically deter their competitors' innovation. Using a tractable model featured by Stackelberg competition in inventor market, I theoretically characterize the impact of misallocation of inventors among firms on aggregate growth. I also propose an empirical strategy to measure the effect of a change in the inventor market structure on aggregate growth.

KEYWORDS: Innovation, Human Capital, Misallocation, Growth

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1. INTRODUCTION

Since Romer (1990) and Lucas (1988), human resource allocation has been always at the center of endogenous growth theory. In particular, the allocation of skilled workforces (inventors or scientists) across firms has been thought of as the main determinant of the aggregate growth by Schumpeterian growth theories (Klette and Kortum, 2004; Acemoglu, Aghion, and Zilibotti, 2006; Acemoglu, Akcigit, Alp, Bloom, and Kerr, 2018; Akcigit and Ates, 2021). In the Schumpeterian growth perspective, firms should first come up with an innovation to collect revenues in the output market. Inventor is the only input of this innovation. Firms undertake R&D by hiring inventors, employment level of inventors determines firm-level innovation rate, and firm-level innovation rates pin down the aggregate growth of economy.

However, firm-level strategic interaction in the inventor market has been understudied despite the importance of skilled labor allocation. Since hiring enough number of inventors is essential to innovation, and since the total supply of inventor is limited, firms can wither out their competitors' inventor employment by strategically determine their own demands for inventors. The strategic behavior can undermine the competitors' total innovation capacity so that it can improve the firm's value by reducing potential technological threats from competitors.

This paper develops a Schumpeterian growth model where I can take into account such firm-level strategic behavior in inventor labor market. The model can be much tractable when I assume Stackelberg competition in skilled labor market. In particular, frontier firms are assumed to move as the first mover in each industry's inventor market. Since frontier firms have better prestige or recruiting power in each industry, it is reasonable to assume that frontier firms can move as the first mover. And to justify this, I also assume the "industry-specific skill sets" of inventors. When the limited mobility of inventors is assumed, strategic interactions between frontier firms from different industries can be ignored which make the model more tractable without special coordination mechanism among frontier firms. The novel incorporation of separate labor market enables me to study the intra-industry competition over skilled workers without a complicated coordination mechanism.

Under the hypothesis that the U.S. inventor market structure has shifted from competitive to Stackelberg, I theoretically study the impact of such input market change on the aggregate growth rate. When the inventor market structure changes, frontier firms buy up more inventors than they do in competitive inventor market to deter laggard firms' innovation. This new allocation can either improve or disimprove the aggregate growth rate. I show that the change in aggregate growth rate can be conceptually decomposed into two factors - misallocation effect and distribution effect - and show that the change can be entirely accounted for by the misallocation effect.

Whether the aggregate growth rate is improved by the new allocation depends on the answer of which effect is more dominant between the effect of decreasing returns to scale of innovation and the effect of frontier firms' higher innovation capacity. Unless frontier firms' innovation capacity is relatively high enough so that frontier firms' better utilization of inventors can fully offset the inefficiency of inventor concentration, frontier firms' market power exercise in inventor market should cause a suboptimal allocation of inventors.

I also propose an empirical strategy to study the impact of change in inventor market structure on aggregate growth. By calibrating my model to the 1990 U.S. economy and switching the inventor market structure from competitive to Stackelberg, I can measure how many fractions of changes in U.S. economic growth rate can be accounted for by the misallocation of inventors. I provide a simple baseline Method of Simulated Moments framework that can conduct such an analysis.

This article is organized as follows. Section 2 outlays the model setup. Section 3 lists the equilibrium characterization of each competitive inventor market model and non-competitive inventor market model. Section 5 presents the computational algorithm. Section 4 provides a decomposition of change in aggregate growth rate along the transition of inventor market structure. Section 6 describes the empirical strategy to calibrate the model to the U.S. economy. Section 7 concludes.

2. Model

2.1. **Preference.** I assume the economy is in continuous time and is populated with a representative household having the following constant relative risk aversion preference.

$$U_0 = \int_0^\infty e^{-\rho t} \frac{C(t)^{1-\sigma} - 1}{1-\sigma} dt.$$

Here, ρ is the discount factor, and $C(t)$ is the aggregate consumption level. I assume the economy is closed the consumption good is perishable. The aggregate consumption level $C(t)$ should be equal to the aggregate output level $Y(t)$ in equilibrium.

The representative household is endowed with two types of labor, skilled and unskilled. Unskilled labor can be used only for the production of intermediate goods. Skilled labor is supplied in labor market in the form of inventor, and once hired by firms, they can contribute to firm-level R&D. Skilled labor is assumed to have specially oriented innate talents which can be only used in a certain industry; an inventor who is oriented to industry i cannot be hired by any firm in industry j. The total supply of inventor in each industry has the same measure L^s across industries. With the assumptions above, the representative household's budget constraint is

$$\dot{A}(t) + C(t) \leq r(t)A(t) + w^u(t)L^u + \int_0^1 w_j^s(t)L^s dj,$$

where $A(t)$ is the asset position, $r(t)$ is the equilibrium interest rate on assets, w^u is the unskilled labor wage, and w_j^s is the skilled wage in industry j. L^u and L^s are measure of unskilled and skilled labor respectively. Notice that as I have assumed skilled labor cannot be transferred to another industry rather than it is originally oriented, each industry has its own skilled wage.

With the market clearance condition in the output market, $Y(t) = C(t)$, the following standard Euler equation is derived

$$g \equiv \frac{\dot{Y}}{Y} = \frac{r-\rho}{\sigma},$$

which pins down the real interest rate.

2.2. **Production Technologies.** All firms are single-industry firms. In each industry, there are one frontier firm and N_j laggard firms. The number of laggards N_j is determined endogenously in the equilibrium. I define frontier firm as firm with the best production productivity in the industry where it is active. All other firms are defined as laggard firms. In each industry, firms compete in Bertrand pricing. As the result of the competition, only the frontier firm in each industry can produce output goods.

The frontier firm in industry j produces output goods using the following linear production function.

$$y_j = q_j l_j,$$

where l_j is employment of unskilled labor by the frontier firm, q_j is the best production productivity in industry j held by the frontier, and y_j is quantity produced.

Suppose firm f has the highest productivity level q_j in industry j. This firm f produces all the output from the industry j,

$$y_j = q_j l_j,$$

where l_j is the unskilled labor demand by the frontier firm f.

The aggregate output level $Y(t)$ is determined by the following constant elasticity of substitution aggregator,

$$Y(t) = \left(\int_0^1 y_j^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where $\varepsilon > 1$ is the elasticity of substitution between products. It is easy to show that

2.3. **Innovation.** Innovation is defined as a technological breakthrough that gives the innovating firm the leading-edge technology in the industry where the firm is innovating. All frontier and laggard firms hire inventors and fight for the next innovation. The arrival of innovation is stochastic by nature and follows Poisson process with the following intensity.

$$x_f = \theta_f^{\gamma} h_f^{\gamma}, \quad f \in \{F, L\}, \gamma \in (0,1).$$

Here, $f \in \{F, L\}$ is type of firm (frontier or laggard), h is inventor employment of firm f, θ is innovation capacity, and x is Poisson arrival intensity of innovation. I assume $\theta_F > \theta_L$ which means that frontier firms can utilize given inventors more efficiently than laggard firms can.

One point is noteworthy in this assumption; γ is assumed to less than 1 so that the innovation function has decreasing returns to scale. Because of the DRS property, the best allocation of given amount of inventor supply can be accomplished when equal measure of inventors are hired by as many firms as possible. Therefore, when inventors are more concentrated to the frontier firm in each industry, each industry are less innovative which leads to a slowdown of the aggregate growth.

Once a firm produces innovation, the innovating firm pushes the frontier production productivity forward and becomes the next frontier firm in the industry. When the existing frontier firm innovates earlier than any other laggard firms, it can stay as the frontier firm until the next innovation. Let $q_j(t)$ be the best production productivity in industry j at time t. The production productivity is held by the existing frontier firm at time t. Any firm that makes the first next innovation within a short time interval $(t, \Delta t)$will hold the best production productivity at time $t + \Delta t$ which is

$$q_j(t + \Delta t) = q_j(t) + \lambda \bar{q}, \quad \lambda > 0,$$

where $\bar{q}$ is the average of best productivity from each industry in the economy. In words, firms can learn a certain fraction of the economy-wide average productivity once they innovate.

I also assume that any firms investing in R&D should hire ϕ measure of skilled workers as fixed costs of R&D. It can be thought of as hiring managerial staffs of R&D projects. Because of the existence of fixed cost of R&D, all firms have a discrete choice over whether or not to invest in R&D. When a firm decides to invest in R&D, the total cost of R&D in units of consumption goods is

$$C(x, \theta) = w^s \left(\frac{x_f^{1/\gamma}}{\theta} + \phi \right),$$

where w^s is the skilled wage in industry j.

2.4. **Inventor Market.** Each skilled labor market in industry j has the total skilled labor supply L^s. Since the skilled workers have an innate talent that is oriented to only one industry, skilled workers located in one industry cannot be transferred to another industry. The skilled labor market in each industry has its own wages w_j^s due to the limited mobility of skilled labor.

In order to analyze the effect of the frontier firm's strategic employment decision, I define two different skilled labor market structures as follows.

2.4.1. *Competitive Inventor Market.* All firms active in industry j take the skilled wage w_j^s as given. Then, one frontier firm and N_j laggard firms decide their optimal demands for inventors simultaneously. In the simultaneous decision, each firm does not consider the other firms' possible best responses against the decision of itself. Taking the laggard firms' inventor demands as given, the frontier firm decides its own demand by comparing the employment cost and the expected benefit of R&D.

2.4.2. *Non-competitive Inventor Market.* In the non-competitive skilled labor market, the frontier firm moves as the first-mover. Since the frontier firm's optimal demand decision for inventors are followed by laggard firms' optimal demand decisions, the frontier firm can take into account laggard firms' best response and the market clearing condition of inventor market. Market clearing condition of inventor market is

$$L^s = (h_F + \phi) + N \times (h_L + \phi).$$

When one laggard firm's optimal demand for inventors (h_L) is not sensitive to frontier firm's decision (h_F), the frontier firm can indirectly choose the measure of laggard firms (N) by choosing its own demand for inventors. This way, the frontier firm can reduce the total innovation rate from laggard firms ($N \times x_L$) and improve its firm value.

I assume that the frontier firm and laggards in the same industry pay the same skilled wage. This setup can be justified by the homogeneity of inventors. No differential in inventors' productivity is assumed so that frontier firms do not need to offer a high wage to inventors for screening purposes. On the other hand, the frontier firm cannot pay a lower wage to inventors since, if so, inventors do not choose to work for the frontier. The first-mover nature of the frontier firm is plugged into this model as an exogenous model design, not as an endogenous result of other model mechanisms.

2.5. **Free Entry and Exit.** I assume free entry and exit of laggard firms in skilled labor markets. When free entry is allowed, a laggard firm's value in any industry should be zero.

$$V^L(\hat{q}_j) = 0 \quad \text{for all } \hat{q}_j. \tag{1}$$

Even with this condition, laggards firms can suffer from a negative profit in a given point of time. The condition just imposes the zero value in the expected value of firms. Some economic assumptions like the completeness of the financial market can guarantee this condition. Along with the market clear condition, the above free entry-exit condition determines the skilled wages and the number of laggard firms.

2.6. **Value Functions.** The purpose of deriving the firm's HJB equation of each type is to describe each type's optimal decision rule in the stationary equilibrium. To this end, I use the hat notation for the normalized value of the corresponding variable. For instance, $\hat{w}^s$ denotes the normalized skilled wage (w^s/w^u).

2.6.1. *Laggard Firms.* Laggard firms do not produce output at all as a result of pricing competition, which means that laggards hire none of unskilled workers. On the other hand, they still can hire skilled workers to conduct R&D and aim to be the next frontier firm in the industry.

A laggard firm in industry $\hat{q}$ solves the following maximization problem.

$$r\hat{V}^L(\hat{q}) = \xi(\hat{q}) \cdot \left(V^L(\hat{q} + \lambda\bar{\hat{q}}) - V^L(\hat{q}) \right) + \frac{\partial V^L(\hat{q})}{\partial \hat{q}} \frac{\partial \hat{q}}{\partial w^u} \frac{\partial w^u}{\partial t}$$
$$+ \max \left\{ 0, \max_{h_L > 0} \left[x(h_L) \cdot \left(V^F(\hat{q} + \lambda\bar{\hat{q}}) - V^L(\hat{q}) \right) - w^s(\hat{q}) \cdot (h_L + \phi) \right] \right\}$$

Given discounting at the rate r, the left hand side is the flow value of a laggard firm in a industry where the best production productivity of which is $\hat{q}$. The right hand side shows the components that compose this flow value. A laggard firm is facing a binary decision - whether or not to conduct R&D - since the fixed cost of R&D is assumed in the model. With intensity $\xi(\hat{q})$, other firms can produce innovation. In that case, the laggard firm still remains as a laggard firm, but the best production productivity of the industry improves to $\hat{q} + \lambda\bar{\hat{q}}$. The second term represents the effect of aggregate growth. The third term is from the discrete choice over whether or not to invest in R&D due to the existence of fixed cost of R&D. When the firm decides not to invest in R&D, there is no change in the flow value. When the firm decides to invest in R&D, it should optimally choose the demand for inventors by comparing expected benefit of R&D investment to the total costs of R&D.

The equation can be simplified from the following observations. First, from the definition of the normalized productivity($\hat{q} = q/w^u$) and the aggregate growth rate ($g = \dot{w^u}/w^u$), the change in firm value on the balanced growth path can be written as

$$\frac{\partial V^L(\hat{q})}{\partial \hat{q}} \frac{\partial \hat{q}}{\partial w^u} \frac{\partial w^u}{\partial t} = -g\hat{q} \frac{\partial V^L(\hat{q})}{\partial \hat{q}}.$$

In addition to this, I impose free entry and exit of laggard firms, (1). The HJB equation reduces to

$$\max_{h_L \geq 0} \left[x_L(h_L) \cdot V^F(\hat{q}_j + \lambda\bar{\hat{q}}) - \hat{w}^s(\hat{q}_j)(h_L + \phi) \right] = 0. \tag{2}$$

This equation and the market clearance condition of skilled labor market (see (4)) jointly determine the number of laggard firms and the skilled wage.

2.6.2. *Frontier Firms.* Frontier firms behave differently in the competitive and the non-competitive skilled labor market. In the competitive skilled labor market, since the frontier firm and laggard firms decide their own optimal skilled labor demand simultaneously, the frontier firm cannot control the number of laggards when it decides its own demands. In contrary to that, in the non-competitive labor market, the frontier firm takes into account the laggard firms' best responses and the market clearance condition.

2.6.3. *Competitive Inventor Market.* The frontier firm in industry $\hat{q}$ solves the following functional equation under given $w^s(\hat{q}), N(\hat{q}), \bar{\hat{q}}$ and g.

$$
\begin{aligned}
rV^F(\hat{q}) &= \pi(\hat{q}) - g\hat{q}\frac{\partial V^F(\hat{q})}{\partial \hat{q}} + \max\{\mathcal{V}_N, \mathcal{V}_{RD}\}, \\
\mathcal{V}_N &= N_0 \cdot x_L(\hat{q}) \cdot \Big(0 - V^F(\hat{q})\Big), \\
\mathcal{V}_{RD} &= \max_{h_F \geq 0} \Big[- w^s(\hat{q}) \cdot (h_F + \phi) \\
&\quad + N(\hat{q}) \cdot x_L(\hat{q}) \cdot \Big(0 - V^F(\hat{q})\Big) + x_F(h_F) \cdot \Big[V^F(\hat{q} + \lambda\bar{\hat{q}}) - V^F(\hat{q})\Big] \Big], \\
N_0 &= \frac{L^s}{h_L(\hat{q}) + \phi}.
\end{aligned} \tag{3}
$$

The frontier firm collects instantaneous profit $\pi(\hat{q})$ from output goods market and is exposed to the aggregate growth effect. When it does not invest in R&D, the measure of laggard firms is given by N_0 and the intensity of at least one laggard firm innovates is $N_0 \times x_L(\hat{q})$. When at least one laggard firm makes innovation, the frontier firm loses its technology leadership and becomes a laggard. When the frontier firm invests in R&D, the flow value due to the R&D investment is $\mathcal{V}_{RD}$. However, because the market structure is competitive, frontier firm does not take into account the measure of laggard firm in the optimal decision. The frontier firm only compares the total costs of R&D ($-w^s(\hat{q}) \cdot (h_F + \phi)$) to the expected benefit of it ($x_F(h_F) \cdot \left[V^F(\hat{q} + \lambda\bar{\hat{q}}) - V^F(\hat{q})\right]$).

Notice that, once the optimal skilled demand for each frontier $h_F^*(\hat{q})$ and laggard $h_L^*(\hat{q})$ are determined, the number of laggard firms consistent with the optimal demands is fixed by the market clearance condition of the skilled labor market,

$$
(h_F^*(\hat{q}) + \phi) + N(\hat{q}) \cdot (h_L^*(\hat{q}) + \phi) = L^s, \quad \text{for all } \hat{q}. \tag{4}
$$

2.6.4. *Non-competitive Inventor Market.* When the frontier firm moves as the first-mover in the skilled labor market, it can anticipate the laggard firms' responses. Then, it is obvious from (4) that the frontier firms can indirectly decide the number of laggard firms by stifling the remainder of inventors. More precisely, the frontier firm in industry $\hat{q}$ solves the following HJB equation.

$$
\begin{aligned}
rV^F(\hat{q}) =& \pi(\hat{q}) - g\hat{q}\frac{\partial V^F(\hat{q})}{\partial \hat{q}} + \max\{\mathcal{V}_N, \mathcal{V}_{RD}\}, \\
\mathcal{V}_N =& N_0 \cdot x_L(\hat{q}) \cdot \Big(0 - V^F(\hat{q})\Big), \\
\mathcal{V}_{RD} =& \max_{h_F \geq 0} \Bigg[- w^s(\hat{q}) \cdot (h_F + \phi) \\
& + N\left(h_F(\hat{q})\right) \cdot x_L(\hat{q}) \cdot \Big(0 - V^F(\hat{q})\Big) + x_F(h_F) \cdot \Big[V^F(\hat{q} + \lambda\bar{\hat{q}}) - V^F(\hat{q})\Big] \Bigg], \\
N_0 =& \frac{L^s}{h_L(\hat{q}) + \phi}, \quad N\left(h_F(\hat{q})\right) = \frac{L^s - h_F(\hat{q}) - \phi}{h_L(\hat{q}) + \phi}
\end{aligned}
\tag{5}
$$

The only difference between competitive and non-competitive skilled labor market is that in the non-competitive market the frontier firm can minimize the threat from the laggard competitors by buying up skilled workers.

2.7. **Market Clearing.** There are three types of markets in the economy. (i) output goods market in each industry j, (ii) one unskilled labor market in the economy, and (iii) skilled labor market in each industry j.

For the output market, the output from industry $j(y_j)$ constitutes the overall output level Y, and the overall output is consumed by the representative household.

There is the unique unskilled labor market. Since only the frontier firm in each industry has non-zero demand for unskilled labor, the market clearance condition for unskilled labor is expressed as follows.

$$\int_0^\infty l_F^*(\hat{q}) \cdot dF(\hat{q}) = L^u,$$

where $l_F^*(\hat{q})$ is the optimal unskilled labor demand of the frontier firm in industry $\hat{q}$ and $F(\hat{q})$ is the distribution of the industry-leading technology $\hat{q}$.

Unlike the unskilled labor demand, all frontier laggard firms can hire positive amounts of skilled labor. Since skilled labor markets are segmented in industry level, market clearance condition is

$$(h_F^*(\hat{q}) + \phi) + N(\hat{q}) \cdot (h_L^*(\hat{q}) + \phi) = L^s,$$

where $h^*(\hat{q})$ is the skilled labor demand by each type, $N(\hat{q})$ is the number of laggard firms in industry $\hat{q}$, and ϕ is the fixed cost of R&D in units of skilled labor.

3. EQUILIBRIUM

3.1. **Static Equilibrium.** The inverse demand function for the monopolist in industry j is

$$p_j = Y^{\frac{1}{\varepsilon}} y_j^{-\frac{1}{\varepsilon}}.$$

Then, the frontier firm in each industry collects the following monopolistic profit.

$$\pi(q_j) = \max_{y_j \geq 0} \left(Y^{\frac{1}{\varepsilon}} y_j^{-\frac{1}{\varepsilon}} - \frac{w^u}{q_j} \right) y_j = \frac{1}{\varepsilon - 1} \left(\frac{\varepsilon - 1}{\varepsilon} \right)^{\varepsilon} Y \left(\frac{q_j}{w^u} \right)^{\varepsilon - 1}.$$

From the output aggregator, the unskilled wage is given as follow.

$$w^u = \frac{\varepsilon - 1}{\varepsilon} \left[\int_0^1 q_j^{\varepsilon - 1} dj \right]^{\frac{1}{\varepsilon - 1}}.$$

Thus, w^u increases as the overall technology level matures. To obtain the stationary equilibrium of the models, I normalize all growing variables by w^u. The detrended instantaneous monopolistic profit is

$$\pi(\hat{q}) = \frac{1}{\varepsilon - 1} \left(\frac{\varepsilon - 1}{\varepsilon} \right)^{\varepsilon} \hat{q}^{\varepsilon - 1}.$$

3.2. **Optimal Decision of Laggard Firms.** A laggard firm's maximization problem (2) can be solved under given value function $V^F(\hat{q})$ and skilled wages $w^s(\hat{q})$. From (2) and the functional form of the innovation rate, $x_L(h_L) = \theta_L^{\gamma} h_L^{\gamma}$, a laggard firm's optimal skilled labor demand and the optimal innovation rate are given. The value of maximand with the optimal decisions should be zero. This pins down the skilled wage.

$$\begin{aligned} h_L^*(\hat{q}) &= \left[\frac{\gamma \theta_L^{\gamma} \Delta(\hat{q})}{w^s(\hat{q})} \right]^{\frac{1}{1-\gamma}}, \\ x_L^*(\hat{q}) &= \left[\frac{\gamma \theta_L \Delta(\hat{q})}{w^s(\hat{q})} \right]^{\frac{\gamma}{1-\gamma}}, \\ w^s(\hat{q}) &= \left[\frac{\gamma^{\frac{\gamma}{1-\gamma}} - \gamma^{\frac{1}{1-\gamma}}}{\phi} \right]^{1-\gamma} \theta_L^{\gamma} \Delta(\hat{q}), \end{aligned} \tag{6}$$

where $\Delta(\hat{q}) = V^F(\hat{q} + \lambda \bar{q}) - V^L(\hat{q})$ is the incentive for a laggard to innovate. Since $V^L(\hat{q}) = 0$ in equilibrium, $\Delta(\hat{q}) = V^F(\hat{q} + \lambda \bar{q})$ in equilibrium.

The size of a laggard firm can be rephrased as

$$h^*_L(\hat{q}) = \left[\frac{\gamma}{\gamma^{\frac{\gamma}{1-\gamma}} - \gamma^{\frac{1}{1-\gamma}}} \right]^{1-\gamma} \phi.$$

The optimal size of inventor employment is proportional to the fixed costs of R&D. This equilibrium results is intuitive given the zero value constraint for laggard firms' firm value: high fixed costs of R&D should be offset with high expected benefit of R&D by hiring more inventors.

Furthermore, the skilled wage profile function $w^s(\hat{q})$ is an increasing function as long as $V^F(\hat{q})$ increases in $\hat{q}$. To be precise, the skilled wage in industry $\hat{q}$ is proportional to $V^F(\hat{q} + \lambda\bar{\hat{q}})$ which is the improvement in laggard firm's firm value when it innovates.

$$w^s(\hat{q}) = \left[\frac{\gamma^{\frac{\gamma}{1-\gamma}} - \gamma^{\frac{1}{1-\gamma}}}{\phi} \right]^{1-\gamma} \theta_L^{\gamma} V^F(\hat{q} + \lambda\bar{\hat{q}}) \equiv \gamma_{ws} V^F(\hat{q} + \lambda\bar{\hat{q}}).$$

As a result of free-entry-exit condition, the skilled wage can be flexibly adjusted until the potential benefit for laggard firms to produce innovation is totally absorbed by the skilled wage. In turn, the laggard firm's size profile($h_L(\hat{q})$) does not change over $\hat{q}$.

3.3. **Optimal Decision of Frontier Firms.**

3.3.1. *Competitive Inventor Market.* Taking the laggard firm's optimal decision (6), it can be easily shown from (3) that , when $\gamma = 1/2$, the frontier firm's optimal demands for inventors conditional on investing in R&D is

$$h_F(\hat{q}) = \frac{\theta_F \phi}{\theta_L} \left[\frac{V^F(\hat{q} + \lambda\bar{\hat{q}}) - V^F(\hat{q})}{V^F(\hat{q} + \lambda\bar{\hat{q}})} \right]^2.$$

Frontier firms compare the potential benefit of innovation ($V^F(\hat{q} + \lambda\bar{\hat{q}}) - V^F(\hat{q})$) to the costs of hiring R&D staffs ($\gamma_{ws} V^F(\hat{q} + \lambda\bar{\hat{q}})$). Frontier firms hire more inventors when the potential benefits more outweigh the costs of R&D.

Given the optimal demands for inventors, it is also easy to show that frontier firms engage in R&D if and only if

$$\left[\frac{V^F(\hat{q} + \lambda\bar{\hat{q}}) - V^F(\hat{q})}{V^F(\hat{q} + \lambda\bar{\hat{q}})} \right]^2 \geq \frac{\theta_L}{\theta_F}.$$

If $V^F(\hat{q})$ is convex, then there exists the unique threshold $\hat{q}_{thr}$ above which frontier firms do not invest in R&D and below which frontier firms invest in R&D. This is again a result of frontier firm's cost-benefit analysis. The cost of hiring inventors ($w^s(\hat{q}) = \gamma_{ws} V^F(\hat{q} + \lambda\bar{\hat{q}})$) increases when $\hat{q}$ is high. On the other hand, the improvement of production productivity $\lambda\bar{\hat{q}}$ is relatively small when $\hat{q}$ is high. Since the benefit of R&D decreases in $\hat{q}$ and the cost increases in $\hat{q}$, high-productivity frontier firms do not invest in R&D.

3.3.2. *Non-competitive Inventor Market.* Similarly, when $\gamma = 1/2$, it is easy to show that optimal demands for inventors of frontier firm in industry $\hat{q}$ is

$$h_F(\hat{q}) = \frac{\theta_L}{\theta_F}\frac{1}{\phi}.$$

Two factors affect the frontier firm's inventor hiring decision. One is the relative innovation capacity of itself θ_F/θ_L. When the frontier firms know it is relatively more innovative than laggard firms, it does not need to pay much to hire inventors in order to elongate their technology leadership. The other factor is the fixed cost of R&D. When the R&D investment incurs costlier fixed costs, a laggard firm's size should be large enough to offset the fixed cost. Large size of laggard firm decreases the potential reduction in the number of laggard firms when the frontier firm buys up more inventors. Because the impact of extra employment on laggard firms' total innovation rate is small, the frontier does not hire much inventors when ϕ is high.

It is also easy to show that all frontier firms invest in R&D regardless of the level of $\hat{q}$. The necessary-sufficient condition for frontier firm to invest in R&D is

$$\frac{V^F(\hat{q} + \lambda\bar{\hat{q}})}{V^F(\hat{q})} \geq 1.$$

This condition is satisfied for all $\hat{q}$ as long as $V^F(\hat{q})$ increases in $\hat{q}$. In Stackelberg inventor market, frontier firms additionally benefit by stifling laggard firms' innovations. The result above implies that this benefit is enough to make all high-productivity firm invest in R&D unlike how frontiers are in competitive inventor market.

3.4. **Stationary Equilibrium Distribution of $\hat{q}$.** The distribution of the industry-leading relative productivity($\hat{q}$) may change according to the firm-level optimal decisions on R&D investment. To be more precise, let $F_t(\hat{q})$ be the cumulative measure of leading-edge productivity $\hat{q}$ at time t. This represents the measure of industries whose leading firm's normalized productivity is less than $\hat{q}$. For notational simplicity, I write the total innovation rate in industry $\hat{q}$ as $\tau(\hat{q}) = x_F(\hat{q}) + N(\hat{q}) \cdot x_L(\hat{q})$. This is the Poisson arrival intensity of event where at least one firm in industry $\hat{q}$ innovates.

The Kolmogorov forward equation for $F_t(\hat{q})$ is

$$F_{t+\Delta t}(\hat{q}) = F_t(\hat{q}(1+g\Delta t)) - \int_{\hat{q}-\lambda\bar{\hat{q}}}^{\hat{q}} \tau(\hat{q}_j)\Delta t \cdot dF_t(\hat{q}_j). \tag{7}$$

This equation can be interpreted as follows. Recall that $\hat{q} = q/w^u$. The unskilled wage grows over time at rate of g. Without innovation, an industry-leading $\hat{q}(1+g\Delta t)$ will have become obsolete to $\hat{q}$ at time $t+\Delta t$. Therefore, the measure of industries whose leading relative productivity is less than $\hat{q}(1+g\Delta t)$ accounts for a part of the measure of industries having best relative productivity less than $\hat{q}$ at time $t+\Delta t$. Meanwhile, industries which are in $[0, \hat{q}(1+g\Delta t)]$ at time t can innovate and improve their productivity over $\hat{q}$ at $t+\Delta t$. Given the innovation step-size $\lambda\bar{\hat{q}}$, only industries with best productivity in between $[\hat{q}-\lambda\bar{\hat{q}}, \hat{q}]$ can improve over $\hat{q}$ at $t+\Delta t$. The second term of the right hand side means the measure of industries which are placed at $[\hat{q}-\lambda\bar{\hat{q}}, \hat{q}]$ at time t but innovate over $\hat{q}$ during Δt.

It is clear that, in the stationary equilibrium, the distribution $F_t(\hat{q})$ should be time invariant, i.e. $F_t(\hat{q}) = F_{t+\Delta t}(\hat{q})$. Letting $\Delta t \to 0$ in (7), we arrive at

$$g\hat{q}f(\hat{q}) = \int_{\hat{q}-\lambda\bar{\hat{q}}}^{\hat{q}} \tau(\hat{q}_j) \cdot dF(\hat{q}_j). \tag{8}$$

To get a computable version of this equation, I differentiate both sides with respect to $\hat{q}$.

$$g\hat{q}f'(\hat{q}) = (\tau(\hat{q}) - g) \cdot f(\hat{q}) - \tau(\hat{q}-\lambda\bar{\hat{q}}) \cdot f(\hat{q}-\lambda\bar{\hat{q}}). \tag{9}$$

This is a delay differential equation with a constant lag $\lambda\bar{\hat{q}}$. With a history function $f(\hat{q}) = 0$ for all $\hat{q} < \hat{q}_{min}$, this equation can be solved easily.

One more concern related to the computation is that the probability density function $f(\hat{q})$ should satisfy

$$\int_{-\infty}^{\infty} f(\hat{q}) \cdot d\hat{q} = 1. \tag{10}$$

Let $\tilde{f}(\hat{q})$ be the solution of (9) which does not satisfy the constraint (10). Since (9) is linear, I can attain the solution of (9) satisfying (10) just by normalizing $\tilde{f}(\hat{q})$ by $\int_{-\infty}^{\infty} \tilde{f}(\hat{q}) d\hat{q}$.

Once the stationary distribution is solved, the average productivity $\bar{\hat{q}}$ can be computed easily by

$$\bar{\hat{q}} = \mathbb{E}(\hat{q}) = \int_0^\infty \hat{q} \cdot dF(\hat{q}) \tag{11}$$

By integrating both sides of (8), I get

$$g \int_0^\infty \hat{q} f(\hat{q}) \cdot d\hat{q} = \int_0^\infty \left[\int_{\hat{q}-\lambda-\bar{\hat{q}}}^{\hat{q}} \tau(\hat{q}_j) \cdot dF(\hat{q}_j) \right] d\hat{q} = \lambda \bar{\hat{q}} \int_0^\infty \tau(\hat{q}) \cdot dF(\hat{q}).$$

Using (11), the last line amounts to

$$g = \lambda \int_0^\infty \tau(\hat{q}) \cdot dF(\hat{q}). \tag{12}$$

4. Misallocation of Inventors and the Aggregate Growth Rate

The change in the aggregate growth rate when the structure of inventor markets transits from competitive to non-competitive can be decomposed into two effects - misallocation effect and distribution effect - using (12).

(13)

$$g_N - g_C = \underbrace{\lambda \left[\int_0^\infty (\tau_N(\hat{q}) - \tau_C(\hat{q})) \cdot dF_C(\hat{q}) \right]}_{\text{misallocation effect}} + \underbrace{\lambda \left[\int_0^\infty \tau_N(\hat{q}) \cdot (dF_N(\hat{q}) - dF_C(\hat{q})) \right]}_{\text{distribution effect}=0}.$$

The first term captures the effect of change in industry-level innovation intensity, which I call misallocation effect, and the second term is the effect of change in the distribution of $\hat{q}$.

The distribution effect should be 0. As shown in Section 3, the optimal demands for inventors of frontier and laggard firm are both constant over $\hat{q}$ when inventor markets are non-competitive. Therefore, $\tau_N(\hat{q})$ is also constant over $\hat{q}$ and the distribution effect should be 0. The change in the aggregate growth rate can be entirely accounted for by the first term - misallocation effect.

In the non-competitive inventor market, more inventors are concentrated to frontier firms than they are in the competitive model. A new allocation of inventors in each industry may either improve or disimprove industry-level innovation rate. Due to decreasing returns to scale of innovation function ($\gamma < 1$), the concentration of inventors deteriorates the allocative efficiency of inventors. On the other hand, frontier firms can more efficiently utilize inventors because they are assumed to have better innovation capacity than laggards ($\theta_F > \theta_L$).

An additional factor that can change the aggregate growth rate is the measure of inventors. When the household provides more inventors than ever, without any change in the share of frontier firm's inventor demand and each type firm's innovation capacity, the industry-level innovation intensity will be improved.

5. Computational Algorithm

5.1. The Competitive Model. The stationary equilibrium of the competitive model can be solved computationally by finding a fixed point of $\{\bar{\hat{q}}, g\} \in \mathbb{R}^2_+$. Specifically, using an initial guess on the vector, $\{\bar{\hat{q}}_i, g_i\}$,

(1) I solve the frontier value firms' value $V^F(\hat{q})$ by the following algorithm.
   (a) fix an initial guess on the value function, $V_i^F(\hat{q})$.
   (b) compute $h_L(\hat{q}), x_L(\hat{q})$, and $w^s(\hat{q})$ by (6).
   (c) compute $h_F(\hat{q})$ and $x_F(\hat{q})$ by solving the maximization problem of (3).
   (d) compare the value of investing in R&D to the value of not investing in R&D. Save the optimal discrete decision in $RND(\hat{q})$ vector.
   (e) With the optimal choices($h_L(\hat{q}), h_F(\hat{q}), x_L(\hat{q}), x_F(\hat{q}), RND(\hat{q})$) and the skilled wage profile($w^s(\hat{q})$), update the guess on firm value to $V_f^F(\hat{q})$ by solving

$$\begin{aligned} rV^F(\hat{q}) =& \pi(\hat{q}) - g\hat{q}\frac{\partial V^F(\hat{q})}{\partial \hat{q}} + N(h_F(\hat{q})) \cdot x_L(\hat{q}) \cdot \Big(0 - V^F(\hat{q})\Big) \\ &+ RND(\hat{q}) \cdot \Big[x_F(h_F(\hat{q})) \cdot \Big[V^F(\hat{q} + \lambda\bar{\hat{q}}) - V^F(\hat{q})\Big] - w^s(\hat{q}) \cdot (h_F(\hat{q}) + \phi)\Big] \end{aligned}$$

   using Steepest Descent algorithm.
   (f) Define cost function as $||V_f^F - V_i^F||_2$. Find $V_i^F(\hat{q})$ that minimizes the criterion using Steepest Descent algorithm again.

(2) With the converged $h_L(\hat{q}), h_F(\hat{q}), x_L(\hat{q})$, and $x_F(\hat{q})$,I compute the stationary productivity distribution $F(\hat{q})$ by solving (9).
(3) Withe the computed productivity distribution, I compute $\bar{\hat{q}}_f$ and g_f by (11) and (12).
(4) Define the criterion function by

$$\frac{|g_f - g_i|}{\frac{1}{2}(|g_f| + |g_i|)} + \frac{|\bar{\hat{q}}_i - \bar{\hat{q}}_i|}{\frac{1}{2}(|\bar{\hat{q}}_f| + |\bar{\hat{q}}_i|)}.$$

   Find $\{\bar{\hat{q}}_i, g_i\}$ that minimizes the criterion.

5.2. The Non-competitive Model. Computation of non-competitive model is easier since I know that all frontier firms should invest in R&D regardless of the level of $\hat{q}$. Except for not comparing the value of R&D to that of non-R&D, computation algorithm is the same as in the competitive inventor market model.

6. Calibration Plan

The purpose of the main simulation is to measure the impact of change in inventor market structure on the aggregate growth rate. To this end, I first calibrate the competitive inventor market model to 1990 U.S. economy. Then, I only switch the inventor market structure to the non-competitive one. By comparing the model predicted change in the aggregate growth rate to the change in empirical data, I can measure how many fractions of the secular change in the aggregate growth rate can be accounted for by frontier firms' strategic decision.

I set the elasticity of substitution across goods to 3, which gives a convex instantaneous profit profile. I choose $L^s = 0.17018$ to match the share of bachelor or higher degree work forces among persons age 25 and over in 1990. Following Blundell, Griffith, and Windmeijer (2002) and Acemoglu, Akcigit, Alp, Bloom, and Kerr (2018), the elasticity of innovation rate to inventor input is set to 0.5. I set the discount rate $\rho = 2$, which roughly corresponds to an annual discount factor of 0.97, and Constant Relative Risk Aversion to 2.

TABLE 1. Parameter Choice

Parameter	Description	Value
ε	Elasticity of substitution	3.00000
L^s	Share of bachelor or higher degree among persons age 25 and over in 1990	0.17018
γ	Innovation elasticity	0.50000
σ	CRRA	2.00000
ρ	Discount rate	0.02000

Other parameters $(\phi, \theta_F, \theta_L, \lambda)$ are estimated using Method of Simulated Moments with the following criterion.

$$\sum_{i=1}^{3} \frac{|\text{model}(i) - \text{data}(i)|}{\frac{1}{2}|\text{model}(i)| + \frac{1}{2}|\text{data}(i)|}.$$

In the baseline estimation, I target three empirical moments listed in Table 2. Using the U.S data between 1960 and 2021, I observe the declining trend of the annual growth rate of real GDP per capital during the period. I target the trend-fitted value of real GDP per capital growth rate in 1990, which is 2.033%.

Since the misallocation of inventors is the main source of the change in the aggregate growth rate in my model, I also match the average size of frontier and laggard to that of U.S. 1990 empirical value. I cannot directly observe inventor employment size in firm-level from Compustat data. Therefore, following Bloom, Jones, Van Reenen,

TABLE 2. Data Moments

Moments	Data
Aggregate growth in 1990	0.02033
Average R&D share of frontier in 1990	0.38419
Average R&D share of laggard in 1990	0.07569

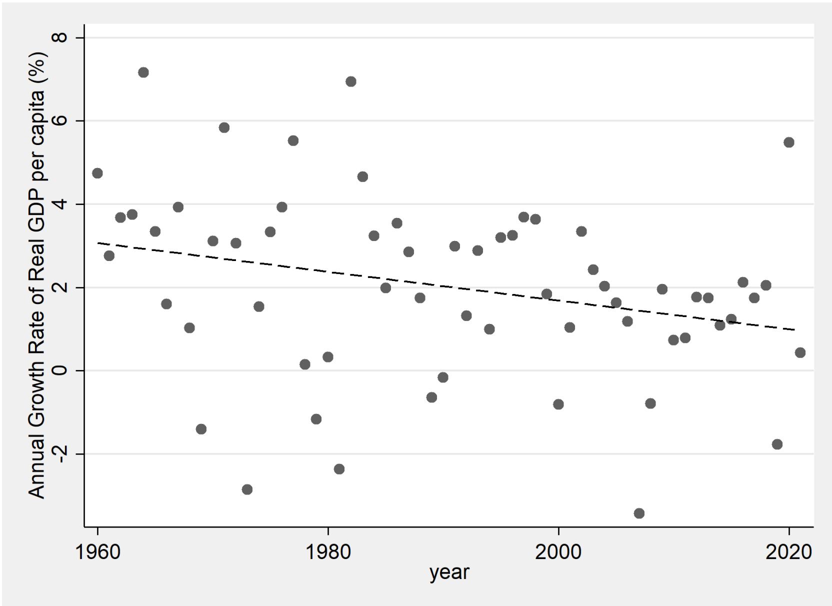

FIGURE 1. Annual Growth Rate of the U.S. Real GDP per capita

and Webb (2020), I proxy inventor employment by R&D expenditure under the assumption that most of R&D expenditures are spent hiring inventors. To rule out the effect of skilled wage and filter out the effective number of inventors, I assume that each industry share the same skilled wage and compute the fraction of each type firm's R&D expenditure out of the industry total R&D expenditure. I use 4-digit SIC as the definition of industry. In each industry, I identify frontier firms as top-revenue firms those jointly account for 50% of total industry revenue. The rest of firms are identified as laggard firms. The trend-fitted value of average share of frontier firms' inventor demand share is 38.419% in 1990. For laggard firms, the average share is 7.569%.

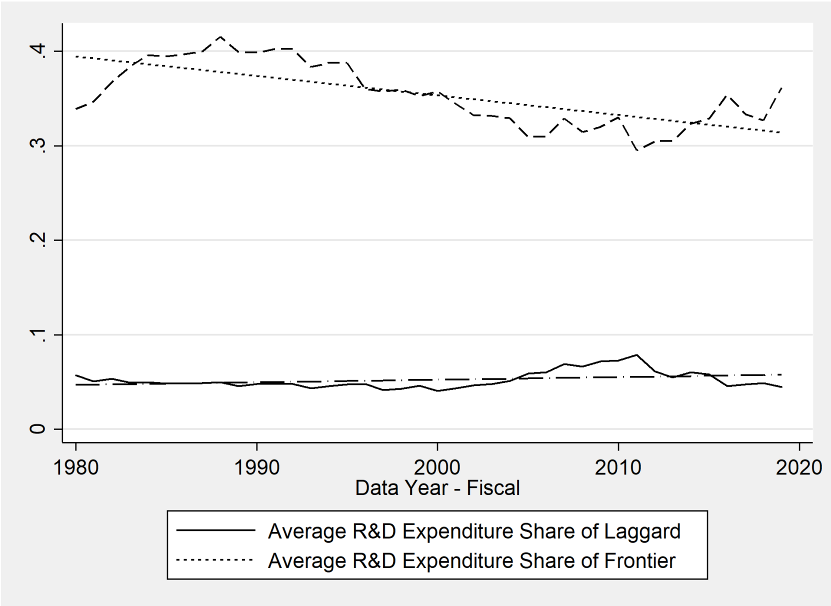

FIGURE 2. R&D Expenditure Share of Frontier and Laggard
Data: Compustat
Notes: 4-digit SIC is used as the definition of industry. Frontier firms in each industry are identified as top revenue firms that jointly account for 50% of total industry revenue. The rest of the firms are identified as laggard firms. The average R&D expenditure is computed using as weight the industry-level total revenue.

7. CONCLUSION

I proposed a Schumpeterian growth model with non-competitive inventor market structure and provided the equilibrium characterization of the model. In the competitive inventor market model, high-productivity frontier firms do not invest in R&D because the cost of hiring inventors is too high in high-productivity industries. Contrary to that, in the non-competitive inventor market model, frontier firms can reduce the total technology threats from laggard firms by hiring more inventors. The additional gain of deterring laggard firms' innovations is substantial enough to make all frontier firms invest in R&D. As a result, inventor allocation is more concentrated to frontier firms in the non-competitive market model.

I also characterized that this concentration of inventors may improve or disimprove the aggregate growth rate. Unless frontier firms' relative innovation capacity (θ_F/θ_L) is high enough to fully offset the effect of decreasing returns to scale of innovation, the aggregate growth rate should be lowered along the transition of inventor market.

Whether frontier firms' relative innovation capacity is high enough should be checked by careful calibration of the competitive inventor market to the U.S. economy. For this purpose, I provided an empirical strategy to estimate the relative innovation capacity. Since the main channel of change in the aggregate growth rate is from the change in allocative efficiency of inventors, the average size of frontier and laggard measured by their inventor employment should be calibrated. By using R&D expenditure as a proxy of inventor employment, I can calibrate the average size of each type of firms to real data.

The calibration is a computationally burdensome task. Solving the value function once requires solving the Steepest Descent minimization twice in nested order, and the value function computation should be done hundreds of times to compute the general equilibrium under a given set of parameters. I leave the computation as future research.

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