Public Debt and Foreign Investment in Aging Developed Economies

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Abstract

This paper examines the government's foreign investment, funding higher-return foreign asset accumulation by issuing low-yield domestic bonds, as a strategy to manage public debt in aging developed economies. I develop a overlapping generations model with incomplete markets and a rich fiscal structure to analyze the strategy's implications for debt sustainability, macroeconomic outcomes, and welfare. Calibrating the model to Japan's post-1995 transition, I first find a critical intertemporal trade-off of the strategy. In the short run, the policy requires additional debt issuance, which crowds out capital, suppressing wages and growth. However, the long-run accumulation of high-yielding foreign assets improves the fiscal outlook, leading to a lower eventual debt-to-GDP ratio and higher capital stock. Second, the strategy was welfare-improving from a utilitarian perspective but created heterogeneous effects, making early generations worse off. The welfare improvement is robust to the risk of foreign returns. Third, I show that government foreign investment can weaken the standard $r < g$ condition for public debt sustainability by reducing the effective cost of borrowing. This study suggests that other aging economies might consider similar strategies.

Keywords: *Debt Sustainability, Demographic Transition, Public Investment*
JEL Classification: *E6, H6, J1.*

1 Introduction

Advanced economies increasingly face significant fiscal pressure from aging population. Longer life expectancies and lower fertility rates have resulted in higher dependency ratios. This demographic shift strains Pay-As-You-Go pension systems and balloons public debts. Figure 1 shows that the rise of public debt is ubiquitous among G7 economies as they experience population aging from 2000 to 2020. Given that these demographic trends are set to continue, a fundamental re-evaluation of fiscal and debt management strategies is essential for public debt sustainability in aging developed economies.

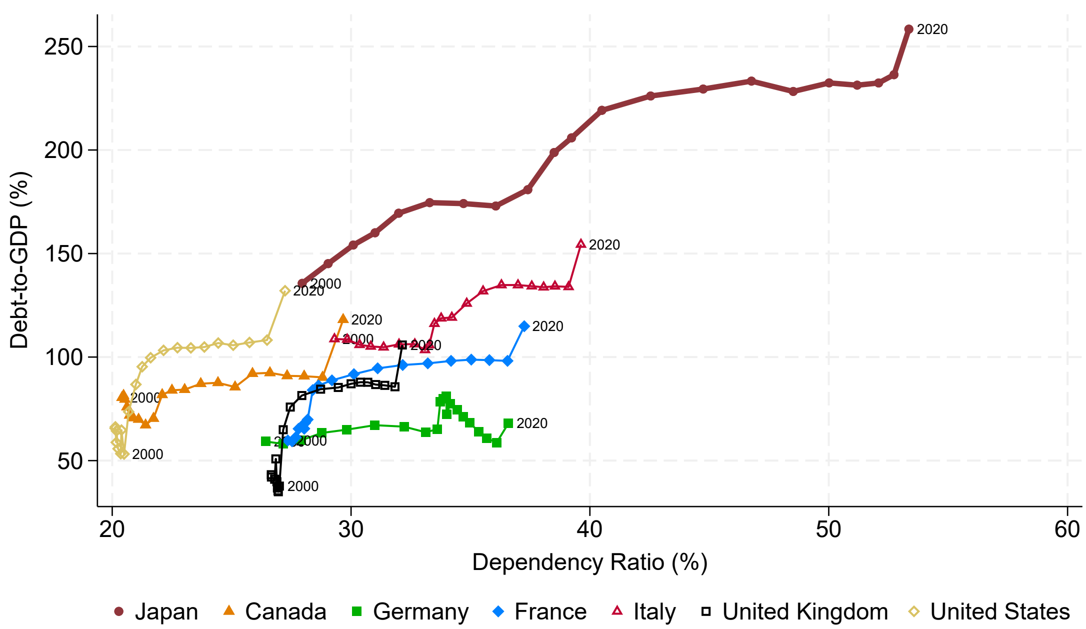


Figure 1: Debt-to-GDP and Dependency Ratio in G7 Economies

Note: The figure shows debt-to-GDP ratios of general government and dependency ratios (the ratio of population aged 65+ to population aged 20-64) for G7 economies from 2000 to 2020. Each observation represents a country-year pair. Data are sourced from IMF Global Debt Database and UN World Population Prospects.

Japan offers an ideal case study to investigate the demographic shift and government responses for public debt management. As the first among advanced economies to experience a sharp demographic transition as shown in Figure 1, Japan's dependency ratio more than doubled from 0.23 in 1995 to over 0.50 in 2020. While the population's composition shifted toward older age groups, the general government's debt-to-GDP ratio, which was 92.5% in 1995, grew rapidly and exceeded 250% in 2020. What makes Japan's case particularly interesting is the government's unconventional debt management strategy in response to these fiscal challenges, rebalancing its asset portfolio toward higher-return foreign assets. The Japanese government has substantially increased the share of foreign assets in its portfolio, from approximately 10% in 1995 to 37% in 2020 (see Figure 2), seeking for higher returns to alleviate fiscal burden from increasing pension liabilities.

This paper draws on this case of Japan to evaluate its foreign investment policies as a potential blueprint for other economies now experiencing rapid population aging. The policy may prove effective in aging economies due to the low domestic interest rate circumstances they tend to have. As households save more for a longer retirement period, they create a “savings glut" that drives down domestic interest rates (Auclert, Malmberg, Martenet, and Rognlie, 2025; Carvalho, Ferrero, Mazin, and Nechio, 2025). [1] If older economies tend to have lower interest rates, the government can find higher-return investment opportunities in younger economies so that it can exploit the interest rate differential by borrowing domestically at low rates and investing in higher-yielding foreign assets. However, the strategy may also backfire, as it requires additional debt issuance to fund foreign asset accumulation, which crowds out capital and suppresses domestic growth.

This paper studies the implication of this public foreign investment strategy for debt sustainability, macroeconomic outcomes, and welfare across cohorts. To do so, it develops a general equilibrium overlapping generations model with incomplete markets and a rich description of fiscal policy. The model is populated by households that are heterogeneous in age, wealth, and income. Households face stochastic death, retirement, and have a fertility rate. Households work and save during their younger years and live out of their savings and government pensions in retirement. On the production side, firms employ workers and rent capital subject to financial friction, facing competitive wages and rental rates. The government finances public spending, a pay-as-you-go pension system, and foreign asset accumulation through various taxes (on labor income, on capital income, and on consumption, i.e. a VAT) and by issuing debt. The government's ability to invest abroad leverages favorable domestic borrowing rates to issue debt domestically and invest the proceeds in higher-yielding foreign assets. This rich framework allows for a detailed analysis of the macroeconomic and welfare consequences of actively managing the public balance sheet, as well as a wide range of fiscal issues, including pension system reforms and changes in tax structure.

I calibrate the model to the Japanese economy's transition from 1995 to 2020, and consider a baseline scenario where I feed the model historical data on demographic changes,

[1] Low domestic rates can also result from financial repression policies in developed economies, since World War II, which incentivize financial institutions to hold government debt (Chari, Dovis, and Kehoe, 2020; Reinhart and Sbrancia, 2015).

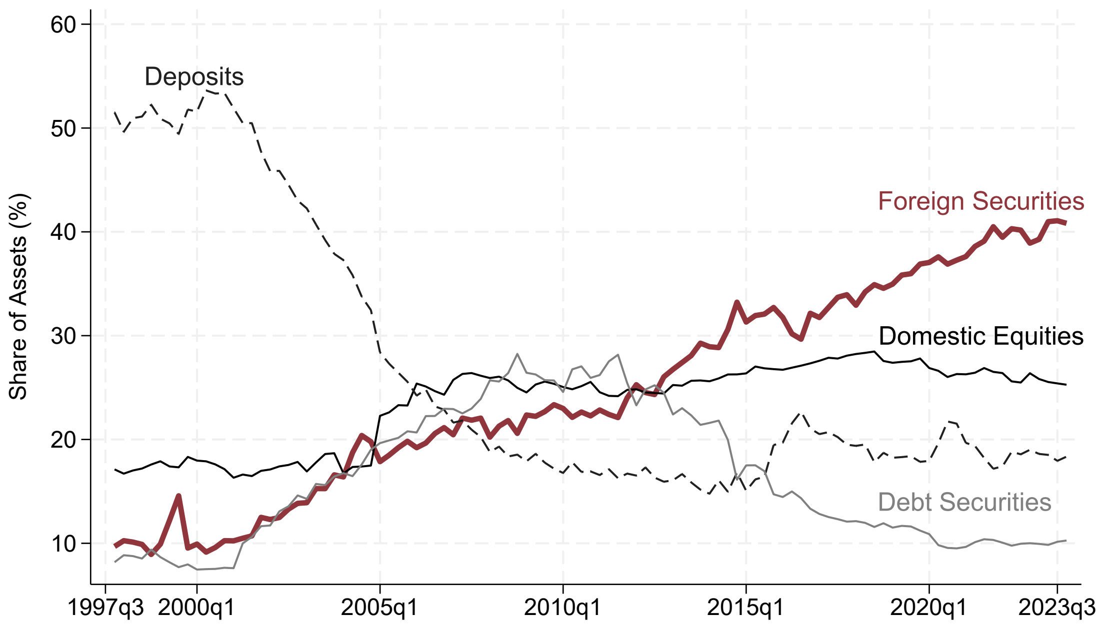


Figure 2: Asset Composition of General Government in Japan

Note: The figure shows the asset composition of Japan's general government, which comprises the central and local governments, the central bank, and the social security fund. Foreign securities include equities and bonds. Data are sourced from the Bank of Japan's Flow of Funds.

fiscal policy, and the government's balance sheet. The model's evolution under this baseline scenario shows a tight fit of the historical transition, explaining 78% of the increase in the debt-to-GDP ratio during this time period.

To understand the effect of the government's external investment strategy, I conduct a counterfactual experiment by simulating a scenario where the government's foreign investment share is held constant at its 1995 level. The results reveal a critical intertemporal trade-off. In the short run, increasing foreign asset holdings required additional debt issuance, which crowds out private capital, suppresses wages, and slows growth. Had the government kept its foreign asset share constant, the increase in the debt-to-GDP ratio would have been approximately 54% lower by 2020 compared to the baseline scenario, amounting to over a third of the observed rise in debt-to-GDP. In the long run, however, the external investment policy improves the fiscal outlook. The accumulation of high-yielding foreign assets generates significant investment income, reducing future borrowing needs. After the end of the accumulation phase, the debt-to-GDP ratio in the baseline scenario begins to fall, eventually dropping below the level of the counterfactual scenario. The eventual baseline steady state features a lower debt-to-GDP ratio, a higher capital stock, and higher wages compared to the scenario without aggressive foreign investment.

This stark contrast between short-run costs and long-run benefits suggests that the policy's welfare effects are heterogeneous across generations. I perform a welfare analysis to evaluate whether this ramp up in public investment is welfare-improving, in terms of a utilitarian social welfare function. The Japanese government's foreign investment was welfare-improving overall, but not Pareto improving. Individuals alive in 1995 were made worse off by the policy, except for the very wealthy and retirees. The young bear the short-run costs of lower wages and higher debt without fully enjoying the long-run benefits. In contrast, future generations, born after 1995, benefit from the higher wages and lower debt burden that the policy ultimately delivers. Even though the policy is desirable from a utilitarian point of view, it cannot be implemented in a Pareto-improving way, because the required transfers would be very large and not paid for themselves by present value..

A potential downside of public foreign investment is the risk associated with foreign asset returns. When the government's asset portfolio underperforms, it must either issue more debt, raise taxes, or cut spending to balance the budget. If this is the case, risk-averse households who anticipate the government's contingent fiscal response may not prefer the government to aggressively pursue a risky investment strategy. To better understand the impact of realization of low returns, I consider two additional analyses. First, I examine a worst-case scenario where unexpected negative shocks of foreign returns hits the economy during the initial phase of foreign asset accumulation, and find that the government needs to issue a significant amount of debt to cover the resulting fiscal shortfall. This leads to a much higher long-run debt-to-GDP ratio compared to the baseline scenario. Second, I extend the model to include aggregate uncertainty in foreign returns and simulate various steady states with different degrees of government foreign investment. I find that, even though households are risk-averse and the government's foreign investment introduces aggregate risks to the economy, the government's foreign investment can still improve the overall welfare of the economy. This result aligns with the findings of Kocherlakota (2023), who shows that there is more scope for the government's infinite debt rollover when the government's borrowing cost is stochastic.

Finally, as a theoretical contribution, I characterize conditions for the existence and the stability of a steady state featuring both primary deficits and a positive, finite debt level. These are generally weaker than the well-known $r < g$ condition (Blanchard, 2019; Reis, 2022). The government's foreign investment can help stabilize debt as long as the return on foreign assets is higher than the government's borrowing cost in the long run, so that a steady debt-to-GDP ratio can be maintained even without actively adjusting the primary deficit based on the debt level, unlike most existing models (Angeletos, Lian, and Wolf, 2024; Kaplan, Nikolakoudis, and Violante, 2023).

Beyond the analysis of the government's active balance sheet management, I also leverage the model to gain deeper insights into the driving forces of population aging and their implications for public debt. Specifically, I evaluate the effect of increasing longevity and lowering fertility rate on public debt. I consider two counterfactual scenarios of demographic changes: one where the mortality rates remain constant at 1995 levels, and another where the fertility rate remains constant at its 1995 level. The results show that change in mortality rate is substantially more impactful than declining fertility in driving up the debt-to-GDP ratio. If mortality rates had remained constant, the dependency ratio would have increased only to 0.25 by 2020, compared to 0.50 in the baseline scenario. However, if the fertility rate had remained constant, the dependency ratio would have still increased to 0.43 by 2020. This difference in the dependency ratio translates into a significant difference in the debt-to-GDP ratio. This result implies that, as opposed to popular belief, immigration policies to support the working-age population would have a limited effect on the fiscal outlook in aging economies.

Related Literature This paper extends the analysis of Chien, Cole, and Lustig (2025), who study the budgetary implications of the Japanese government's massive carry trade. They report that the Japanese public sector has expanded its fiscal space by borrowing domestically at floating rates and investing in longer-duration risky assets. According to their estimate, the government's carry trade yielded an annual return exceeding 6% of GDP for the public sector. This paper extends their analysis by examining the general equilibrium effects of the government's investment strategy in a tightly calibrated model. In particular, this paper clarifies the short-run and long-run effects of public external investment in a general equilibrium framework and provides a detailed welfare evaluation of the policy.

The model is built heavily upon the framework of Conesa and Krueger (1999); Krueger and Kubler (2006). Households in the model face a realistic life cycle, including aging, retirement, and stochastic death. The model is also similar to that of Kaplan et al. (2023), as both papers study the public debt accumulation within a heterogeneous agent quantitative macroeconomic model. This paper contributes to the literature by incorporating the role of government foreign investment. Furthermore, it provides a quantitative case study of Japan's fiscal experience since 1995.

This paper contributes to several more strands of the literature. It adds to the extensive body of work on public debt sustainability (Blanchard, 2019; Kocherlakota, 2023; Mehrotra and Sergeyev, 2021; Mian, Straub, and Sufi, 2025; Reis, 2022) by studying the role of government asset management and showing how foreign investment modifies the standard $r < g$ condition. This work also relates to studies that explore fiscal sustainability under conditions of $r > g$ (Angeletos et al., 2024). Second, this paper contributes to the literature on the pension system reform in aging economies. While much of this literature focuses on pension reform and its welfare effects (Conesa and Krueger, 1999; Gertler, 1999; Huggett and Ventura, 1999; Krueger and Kubler, 2006), this work explores the government's external investment as an alternative form of pension management. Third, the study also contributes to the literature on the link between population aging, capital flows, and interest rates (Attanasio, Bonfatti, Kitao, and Weber, 2016; Auclert et al., 2025; Carvalho et al., 2025). My quantitative model, which features realistic demographic dynamics, shows that an aging economy experiences capital deepening as individuals save more for longer retirements. The government in the model leverages the saving glut to invest in foreign assets. Finally, this paper examines the possibility of Pareto-improving fiscal policies in a heterogeneous agent model, contributing to the literature on Pareto improving fiscal and monetary policies (Aguiar, Amador, and Arellano, 2021, 2024).

Layout The remainder of the paper is structured as follows. Section 2 presents the data on Japan's demographic transition, fiscal pressure, and the government's foreign investment strategy. Section 3 lays out the overlapping generations model. Section 4 analyzes the theoretical conditions for debt sustainability. Section 5 describes the calibration strategy and the baseline scenario for the Japanese economy. Section 6 presents the results from counterfactual and welfare analyses. Section 7 studies the role of external returns. Section 8 concludes.

2 Demographic Shift and Fiscal Challenge in Japan

2.1 Demographic Shift

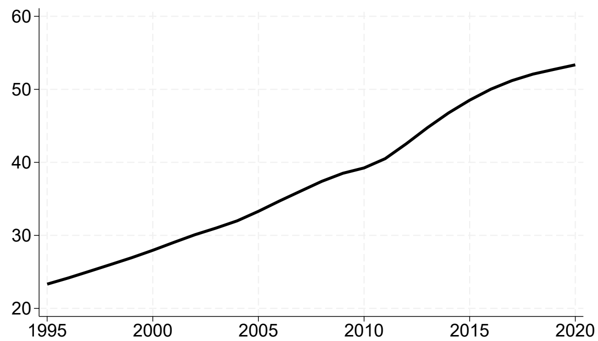
Japan had been experiencing population aging at unprecedented rates over the past few decades. The life expectancy at birth of Japanese individuals has increased significantly as shown in Figure 3 (a). The life expectancy at birth reached over 84 years in 2020, reflecting improvements in healthcare and living standards. In particular, the fact that the median length of life is greater than the life expectancy suggests a significant portion of the population lives well beyond the average. This adds to the burden on pension payments. As a result, the dependency ratio, which is computed as the ratio of the retired population to that of working-age population (age 20-64), has been steadily increasing since 1995. In 1995, one working-age individual supported approximately 0.23 retirees. By 2020, this figure had risen to over 0.50.

This aging trend can be attributed to two major factors. First, the decline in fertility rates. Figure 3 (c) shows the total fertility rate (TFR), the expected number of children born to a woman over her lifetime. Japan's TFR has steadily declined since 1950, remaining well below the replacement level of 2.1 since the 1970s. The decline halted in the 2000s, and the rate has since remained relatively constant. However, despite the significant funds governments allocate to increase birth rates, such as the Kishida administration's annual investment of 3.6 trillion yen, financial incentives alone appear insufficient. TFR is still below 1.5 in 2020s. The rising average age for first marriages and births, and the high proportion of single individuals, point to deeper social and cultural shifts.

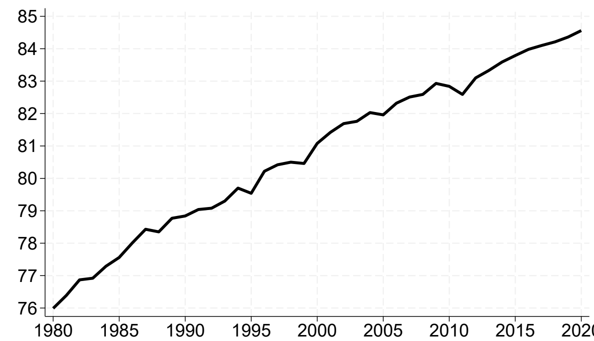
The second factor is the increase in survival probability. Figure 3 (d) illustrates the survival rates at different ages. Improvements in healthcare and living standards have led to higher survival rates, particularly for the elderly. This means that not only are people living longer, but they are also more likely to reach advanced ages. As a result, the proportion of the population that is elderly is increasing, further exacerbating the challenges posed by an aging society.

2.2 Fiscal Challenge and Reforms

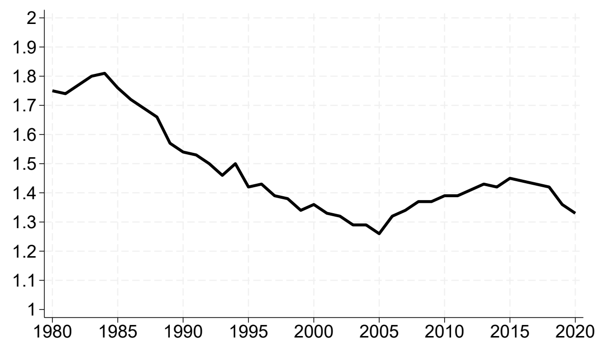
Fiscal Challenge Figure 4 shows the changes in the Japanese government's debt-to-GDP ratio and the capital-to-GDP ratio. As is well known, Figure 4 (a) shows that the debt ratio of the general government (central and local governments, central bank, and



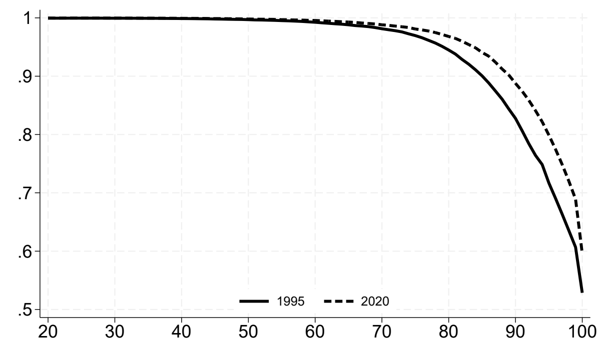
(a) Dependency Ratio (%)



(b) Life Expectancy



(c) Total Fertility Rate



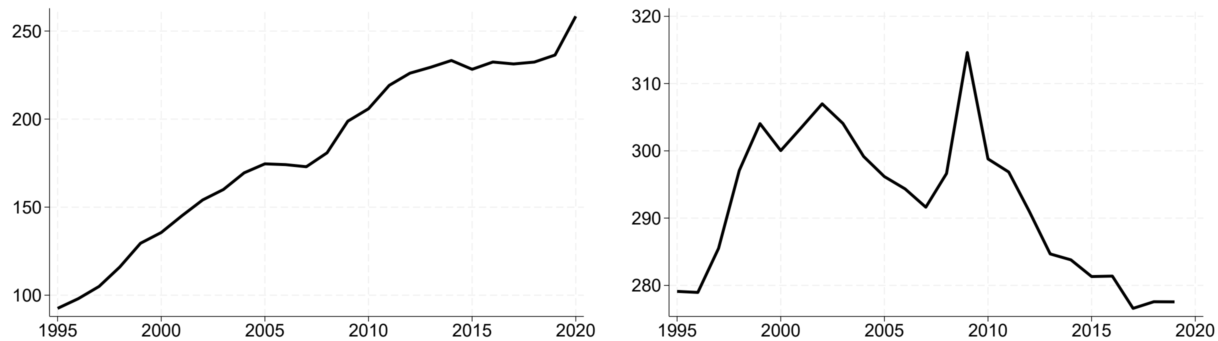
(d) Age-specific Survival Probability

Figure 3: Demographic Transition in Japan

Note: The dependency ratio is calculated as the ratio of the population aged 65 and over to the population aged 20–64 to measure the burden on the working-age population imposed by the pension system. Life expectancy is measured at birth. Age-specific survival probability is calculated ex-post using the age-specific population by year. All data are sourced from the UN World Population Prospects.

social security fund) surged from below 100% in the early 1990s to over 250% in 2020. This increase in debt leads to a capital crowd-out effect. As seen in Figure 4 (b), the capital-to-GDP ratio has shown a general downward trend since 2000. The decline in the capital-to-GDP ratio reduces household labor income, which in turn shrinks the government's labor income tax base, potentially worsening the debt problem.

Consumption Tax Reform Increasing value added tax has been often used as a fiscal response to increasing burden in social security budget in Japan. On April 1, 1989, Japan's consumption tax was first introduced at a rate of 3% under the Noboru Takeshita Cabinet. Despite being at the peak of the bubble economy at the time , this measure led to Prime Minister Takeshita's resignation and the ruling Liberal Democratic Party's (LDP) crushing defeat in the subsequent election, earning the tax the nickname "graveyard of adminis-



(a) Debt-to-GDP Ratio (%)

(b) Capital to Output Ratio (%)

Figure 4: Fiscal Pressure in Japan

Note: The debt-to-GDP ratio is the general government gross debt to nominal GDP, sourced from the IMF Global Debt Database. The general government comprises the central and local governments, the central bank, and the social security fund. The capital-to-GDP ratio is calculated by Perpetual Inventory Method and Solow accounting using Penn World Table data.

trations". Subsequently, on April 1, 1997, the Ryutaro Hashimoto Cabinet increased the consumption tax rate to 5%. This also resulted in Prime Minister Hashimoto's resignation and the LDP's defeat in the House of Councillors election. In the early 2000s, the Junichiro Koizumi Cabinet attempted institutional changes instead of direct rate hikes, such as lowering the tax-exempt threshold for consumption tax payers in 2003 and mandating tax-inclusive pricing in 2004. However, despite ongoing fiscal pressure, direct tax increases continued to face political hurdles. Discussions for a tax hike under the Taro Aso Cabinet in 2009 were halted when the LDP was replaced by the Democratic Party of Japan (DPJ), which had campaigned on a "no tax hike" platform. Even the ruling DPJ's Prime Minister Naoto Kan attempted to push for a consumption tax increase in 2010 due to worsening fiscal conditions, but this led to his party's significant defeat in the election and his resignation.

Despite these repeated failures, the need for fiscal consolidation intensified due to an aging population and rising national debt. In March 2012, the Yoshihiko Noda Cabinet submitted a bill to gradually increase the consumption tax rate to 8% in April 2014 and 10% in October 2015. This bill was passed through cross-party agreement with major opposition parties. As planned, the consumption tax was raised from 5% to 8% in April 2014. However, Prime Minister Shinzo Abe announced in November 2014 a postponement of the scheduled 10% increase (originally set for October 2015) by 18 months, citing concerns about an unexpected economic slowdown. Nevertheless, the Japanese government reached a fiscal limit where further postponement of the consumption tax increase was no longer feasible. Consequently, the final 10% consumption tax increase was implemented on October 1, 2019.

Public Foreign Investment and Asset Market Segmentation The Government Pension Investment Fund (GPIF) was established in 2001 to manage Japan's public pensions amid an aging population and to ensure fiscal stability. Its initial strategy was highly conservative, with a portfolio heavily concentrated in domestic bonds, specifically Japanese Government Bonds (JGBs), which accounted for 66–70% of its holdings. The allocation to equities and foreign assets remained very low. By 2012, the portfolio was composed of approximately 67% domestic bonds, 11% domestic equities, 8% foreign bonds, 9% foreign equities, and 5% short-term assets.

The Abenomics policy, introduced by Prime Minister Shinzo Abe in 2012, marked a significant pivot for the GPIF. Responding to the pressures of an aging population and a prolonged low-interest-rate environment, the fund shifted its focus from a domestic bond-centric approach to one that prioritized diversification and higher returns. The low yields on domestic bonds created a need for higher returns to meet long-term pension obligations. In 2013, The allocation to domestic bonds was reduced from 67% to 60%, while foreign equity and bond allocations were increased to 12% and 11%, respectively. In 2014, the combined bond allocation was reduced (domestic and foreign) from 75% to 50%, and the total equity allocation was increased from 24% to 50% (split equally between domestic and foreign equities).

Since April 2020, the GPIF has implemented a policy of equal asset allocation, targeting a 25% share for each of the four main asset classes: domestic bonds, foreign bonds, domestic equities, and foreign equities, with a permissible deviation of 7–11%. The fund has continued to increase its total foreign asset holdings, which now make up over 50% of the portfolio.

The government's active investment in high-yield assets results in a distinct divergence from household asset portfolio as shown in Figure 2 and 5.

Figure 5 shows the asset shares of households. A notable feature is that more than 50% of their assets are held as bank deposits. The second largest asset class is insurance and pension assets, which also offer relatively low returns. Domestic equities account for 10-20% of their assets, while foreign assets consistently represent less than 5% throughout the period.

In contrast, the general government actively restructured its asset portfolio between 1997 and 2023 (see Figure 2). It reduced its share of cash-like assets, which previously exceeded 50%, and increased its holdings of higher-yielding, riskier assets such as foreign securities and domestic equities. Notably, foreign securities grew from about 10% in 1997 to nearly 40% by the 2020s.

This is one of the unique characteristics of the Japanese economy. The government appears to act as a risk-taker on behalf of risk-averse households by investing in high-risk, high-return assets. This trend also suggests that households face domestic/foreign asset market segmentation, whereas the government is free from it. There are several possible reasons that the private sector does not replicate the government's foreign investment strategy. First, households may have home bias in their investment decisions, as modeled in Carvalho et al. (2025). Second, Chien et al. (2025) point out the specific regulation environment in Japan. In particular, Japanese government requires financial institutions to hedge the foreign exchange risk when they invest in foreign assets. Due to a high demand of hedge, the cost of hedging in Japan has been persistently and largely higher than the interest rate differential between domestic and foreign, which makes foreign investment less attractive for private sector. Last, risk averse households may not be willing to take additional risk if the government is already taking risk by investing in foreign assets. While studying the exact reason for this market segmentation is an interesting question, this paper does not try to rationalize it but simply takes it as a fact and reflects it in the model.

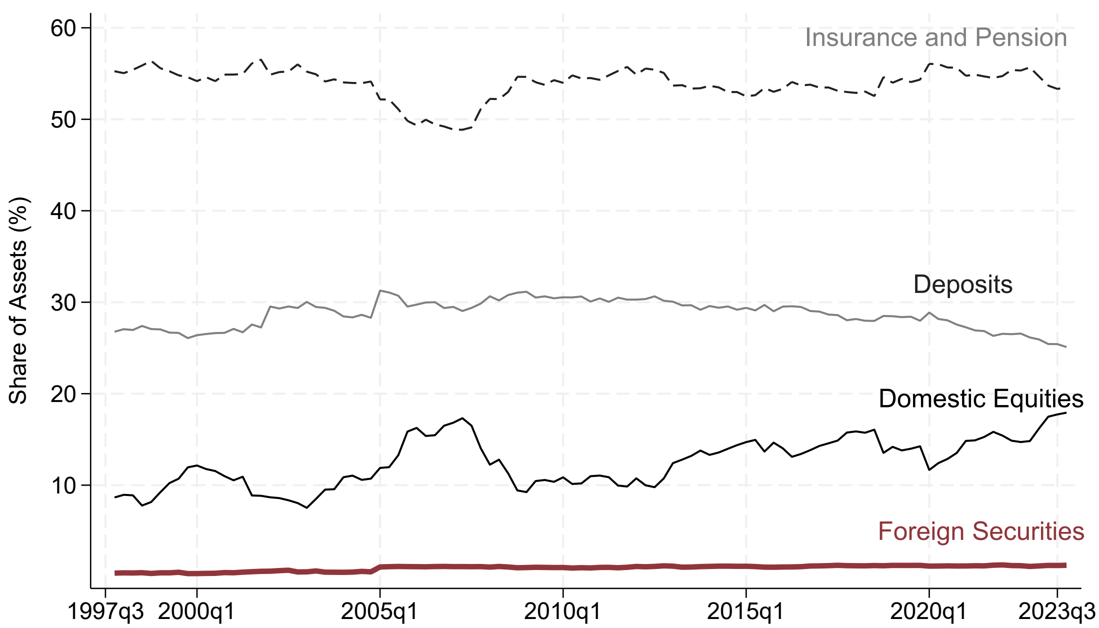


Figure 5: Asset Composition of Household in Japan

Note: The figure shows the asset composition of households in Japan. The data is sourced from the Flow of Funds Accounts by the Bank of Japan.

Immigration Policy Reform Historically, Japan has maintained a highly restrictive immigration policy, characterized by self-imposed isolation and strict controls on foreign entry and settlement. Unlike many advanced Western economies that relied on guest worker programs, Japan largely met its industrial manpower needs after 1945 by drawing on its internal rural labor force. Obtaining permanent residency was traditionally difficult, requiring 20 years of continuous residence, although this was later reduced to 10 years by 1998.

However, as the aging population began to strain labor markets, Japan introduced the Specified Skilled Worker (SSW) program in April 2019. This program is a cornerstone of its modern immigration strategy, designed to directly address severe labor shortages. It represents a significant shift away from the problematic Technical Intern Training Program (TITP), which is scheduled for abolition in 2024. The SSW program offers clearer pathways for foreign workers to contribute to Japan's economy. It targets 16 designated industrial sectors, with a projected need for 820,000 foreign workers over the five years from April 2024. The key sectors with the highest demand include manufacturing of industrial products (173,300), nursing care (135,000), manufacture of food and beverages (139,000), construction (80,000), and agriculture (78,000). This highly targeted approach highlights Japan's pragmatic focus on maintaining essential services and industrial output by addressing specific economic vulnerabilities.

3 Model

The model features growth due to the growth in both labor-augmenting technology and working-age population. I first present the non-detrended model, followed by the detrending of the model. Tilde notations are used for level variables, while non-tilde notations denote detrended variables. Lowercase letters represent individual variables, and uppercase letters are for aggregate variables.

3.1 Demographic

Households are indexed by $(\varepsilon, \tilde{a}, j)$, where $\varepsilon \in \mathcal{E}$ represents the endowed efficiency units of labor, $\tilde{a} \in \tilde{\mathcal{A}}$ denotes asset holdings, and $j \in \mathcal{J} = \{1, 2, \ldots, \bar{J}\}$ denotes age. The stochastic process of ε will be specified later. $\bar{J}$ is the maximum age that a household can reach, and all households die with probability 1 if they reach age $\bar{J}$. Let $d\widetilde{\Phi}_{jt}(\varepsilon, \tilde{a})$ denote the measure of households of type $(\varepsilon, \tilde{a})$ aged j at date t.

Households are born as age $j = 1$ workers and retire at age J_r. The measures of the working-age population and the retired population at date t are denoted by μ_t^w and μ_t^r, respectively.

$$\mu_t^w = \sum_{j=1}^{J_r-1} \widetilde{\Phi}_{jt}(\mathcal{E}, \tilde{\mathcal{A}}), \qquad \mu_t^r = \sum_{j=J_r}^{\bar{J}} \widetilde{\Phi}_{jt}(\mathcal{E}, \tilde{\mathcal{A}}).$$

Households face exogenous mortality risk. ψ_{jt} denotes the survival probability from date t to $t+1$ for a household of age j at date t. Working-age households give birth to a new household at rate ω_t. The measure of newborn households joining the economy at $t+1$ is $\omega_t \mu_t^w$. The law of motion for the working-age and retired populations are:

$$\mu_{t+1}^w = \omega_t \mu_t^w + \sum_{j<J_r-1} \psi_{jt} \widetilde{\Phi}_{jt}(\mathcal{E}, \tilde{\mathcal{A}}), \qquad \mu_{t+1}^r = \sum_{j \geq J_r-1} \psi_{jt} \widetilde{\Phi}_{jt}(\mathcal{E}, \tilde{\mathcal{A}}).$$

The growth rate of the working-age population, denoted by n_{t+1}, is defined as $n_{t+1} = \mu_{t+1}^w - \mu_t^w - 1$. The initial working-age population is normalized to 1. The dependency ratio (ζ_t) is calculated as the ratio of the retired population to the working-age population (μ_t^r / μ_t^w).

3.2 Timeline

See Appendix A for the timeline of the model.

3.3 Private Sector

Household Working-age households supply labor inelastically. The efficiency units of labor endowed to a household is denoted by ε, which follows AR(1) process with persistence ρ_ε and innovation σ_ε unless the household experiences an individual disaster. Irregardless of the current labor endowment, a household can experience a disaster with probability p_d in the following date. Household in disaster state will recover to one of normal endowed efficiency units of labor according to the ergodic distribution of the efficiency units of labor. Let $\pi_\varepsilon : \mathcal{E} \rightarrow \mathbb{R}$ denotes the ergodic distribution of endowed efficiency units of labor.

I assume asset market segmentation where households can only hold domestic assets.

This assumption is motivated by the data showing that Japanese households hold only a small share of foreign assets. Households can deposit their asset holdings into a bank at the deposit rate r_t.

Households are subject to capital income tax at rate τ_{kt}, labor income tax at rate τ_{lt}, and consumption tax at rate τ_{ct}. Working-age households supply labor inelastically and make labor income, $\tilde{w}_t$, per efficiency unit of labor. Retired households receive pension benefits. Household's cash in hand at date t is given by the sum of after-tax labor income ($(1-\tau_{lt})\tilde{w}_t\varepsilon_t$), pension benefits ($\widetilde{pb}_t$), the beggining of the date asset holdings ($\tilde{a}_t$), and the after-tax return on savings ($(1-\tau_{kt})r_t\tilde{a}_t$). Households allocate the cash in hand to consumption and savings for the next date under a borrowing limit ($\underline{\tilde{a}}$). A household's problem at date t is to maximize the expected discounted utility of consumption over the life, given by

$$
\begin{aligned}
\tilde{v}_{jt}(\varepsilon_t, \tilde{a}_t) = \max_{\tilde{c}_t, \tilde{a}_{t+1}} & \left[\tilde{c}_t^{\rho} + \tilde{\beta}\psi_{jt} \left(\mathbb{E}_{\varepsilon_{t+1}|\varepsilon_t} \tilde{v}_{j+1,t+1}(\varepsilon_{t+1}, \tilde{a}_{t+1})^{1-\sigma} \right)^{\frac{\rho}{1-\sigma}} \right]^{\frac{1}{\rho}} \\
\text{subject to} \quad & (1+\tau_{ct})\tilde{c}_t + \tilde{a}_{t+1} = \mathbf{1}_{j<J_r}(1-\tau_{lt})\tilde{w}_t\varepsilon_t + \mathbf{1}_{j \geq J_r}\widetilde{pb}_t + (1+(1-\tau_{kt})r_t)\,\tilde{a}_t \\
& \tilde{a}_{t+1} \geq \underline{\tilde{a}}, \quad \underline{\tilde{a}} \leq 0.
\end{aligned}
$$

The elasticity of intertemporal substitution (EIS) is $1 - 1/\rho$ and σ is the relative risk aversion. Note that capital income tax is imposed only for positive returns on savings. The pension scheme will be clarified in the government section.

With the households supplying labor inelastically, the aggregate labor supply in the efficiency units is given by

$$
\tilde{N}_t = \sum_j \int_{\varepsilon, \tilde{a}} \varepsilon \, d\widetilde{\Phi}_{jt}(\varepsilon, \tilde{a}).
$$

The average efficiency units of labor supplied by working-age household is denoted by L_t, which is defined as the ratio of the total amount of efficiency units of labor supplied to the measure of working-age households.

$$
L_t = \frac{\tilde{N}_t^s}{\mu_t^w} = \frac{\sum_{j=1}^{J_r-1} \int_{\varepsilon,\tilde{a}} \varepsilon \, d\widetilde{\Phi}_{jt}(\varepsilon, \tilde{\mathcal{A}})}{\sum_{j=1}^{J_r-1} \widetilde{\Phi}_{jt}(\mathcal{E}, \tilde{\mathcal{A}})}.
$$

Production and Financial Intermediary Firms produce output using capital and labor. The production function is identical across firms, given by

$$\tilde{Y}_t = \left(\tilde{K}_t\right)^{\alpha} \left(Z_t \tilde{N}_t\right)^{1-\alpha}$$

where Z_t represents the labor-augmenting technology. The growth rate of the labor-augmenting technology is denoted by $\gamma_t = Z_t / Z_{t-1} - 1$. $\tilde{K}_t$ and $\tilde{N}_t$ are the firm's demand for capital and labor, respectively.

Firms default on their debt with exogenous probability φ_t within the period. The representative bank originates funds from household deposits at the deposit rate r_t and lend money to firms and the government competitively. To offset the default risk, the bank charges a spread φ_t over the deposit rate for loans to firms. Contrarily, the government bond is not defaultable, allowing the government to issue bonds at the deposit rate r_t.

The equilibrium deposit rate r_t and the wage rate $\tilde{w}_t$ are determined based on the firm's marginal product of capital and labor:

$$r_t = \alpha \left(\frac{Z_t \tilde{N}_t}{\tilde{K}_t}\right)^{1-\alpha} - \delta - \varphi_t, \qquad \tilde{w}_t = (1-\alpha) Z_t \left(\frac{\tilde{K}_t}{Z_t \tilde{N}_t}\right)^{\alpha}$$

3.4 Government

Pension System The government pays out pension benefits to retired households regardless of their asset holdings and age. The generosity of the pension benefit is determined by the replacement ratio, θ_t, which is the ratio of the pension benefit to the equilibrium wage rate $\tilde{w}_t$.

$$\widetilde{pb}_t = \theta_t \tilde{w}_t.$$

Given that the median of the endowed efficiency units of labor is normalized to 1, the replacement ratio is the ratio of the retired household's pension benefit to the median labor income of working-age households.

Budget Constraint The government's final consumption is denoted by $\tilde{G}_t$. In addition to the general spending, the government pays pension benefits to the retired population, which is given by $\theta_t \tilde{w}_t \mu_t^r$. The government also services its debt by paying interest on the outstanding bonds, which is given by $(1 + r_t)\tilde{B}_t$, where $\tilde{B}_t$ is the total amount of government bonds issued at date t.

The government finances its spending, pension liabilities, and debt costs through tax revenue and issuing government bonds. The tax revenue consists of labor income tax ($\tau_{lt}\tilde{w}_t\tilde{N}_t$), capital income tax on positive returns to savings ($\tau_{kt}r_t\tilde{B}_t$), and consumption tax ($\tau_{ct}\tilde{C}_t$). Here, $\tilde{N}_t$ is the total amount of efficiency units of labor supplied by working-age households, and $\tilde{C}_t$ is the aggregate consumption at date t. The government also issues short-term and non-defaultable bonds at date t, denoted by $\tilde{B}_{t+1}$.

The government invests in foreign assets, which are assumed to yield a return of r_t^*. The total amount of foreign investment at date t is denoted by $\tilde{F}_t$. The government's budget constraint at date t is

$$\tau_{lt}\tilde{w}_t\tilde{N}_t + \tau_{ct}\tilde{C}_t + \tau_{kt}r_t(\tilde{B}_t + \tilde{K}_t) + \tilde{B}_{t+1} + (1 + r_t^*)\tilde{F}_t = \tilde{G}_t + \theta_t\tilde{w}_t\mu_t^r + (1 + r_t)\tilde{B}_t + \tilde{F}_{t+1}.$$

I define the government debt's utilization ratio as the ratio of the total amount of foreign investments to the total amount of debt ($\xi_t = \tilde{F}_t/\tilde{B}_t$). The budget constraint can be simplified with the utilization ratio as follows:

$$(1 - \xi_{t+1})\tilde{B}_{t+1} = \tilde{G}_t - \tau_{ct}\tilde{C}_t - \tau_{kt}r_t\tilde{K}_t + (\theta_t\mu_t^r - \tau_{lt}\tilde{N}_t)\tilde{w}_t + [1 + r_t(1 - \tau_{kt}) - \xi_t(1 + r_t^*)]\,\tilde{B}_t$$

The left-hand side represents the net amount of newly issued bonds available for financing primary deficits and servicing existing debt. The right-hand side comprises the primary deficit, which accounts for general spending and pension liabilities net of tax revenues, and the effective debt service cost, determined by the after-tax return on government bonds and the return on government investments in foreign assets.

3.5 Equilibrium

Market Clearing Conditions The market clearing conditions for capital market, labor market, and output market are

$$\tilde{K}_t + \tilde{B}_t = \sum_j \int_{\varepsilon,\tilde{a}} \tilde{a}\, d\widetilde{\Phi}_{jt}(\varepsilon,\tilde{a}),$$

$$\tilde{N}_t = \sum_{j=1}^{J_r-1} \int_{\varepsilon,\tilde{a}} \varepsilon\, d\widetilde{\Phi}_{jt}(\varepsilon,\tilde{a}),$$

$$\sum_j \int_{\varepsilon,\tilde{a}} \tilde{c}_t(\varepsilon,\tilde{a},j)\, d\widetilde{\Phi}_{jt}(\varepsilon,\tilde{a}) + \tilde{K}_{t+1} + \tilde{G}_t = \tilde{K}_t^\alpha \left(Z_t\tilde{N}_t\right)^{1-\alpha} + (1-\delta)\tilde{K}_t.$$

Law of Motion of Population Measure The law of motion of the population measure is summarized by the following equations: For any $E \subset \mathcal{E}, \tilde{A} \subset \tilde{\mathcal{A}}$ and $1 \leq j \leq \bar{J} - 1$,

$$\widetilde{\Phi}_{j+1,t+1}(E, \tilde{A}) = \int_{\tilde{a} \in \tilde{\mathcal{A}}} \tilde{P}_t(\varepsilon, \tilde{a}, j, E, \tilde{A}, j+1) \, d\widetilde{\Phi}_{jt}(\varepsilon, \tilde{a}),$$

where

$$\tilde{P}_t(\varepsilon, \tilde{a}, j, E, \tilde{A}, j+1) = \begin{cases} \Pr(\varepsilon_{t+1} \in E | \varepsilon_t = \varepsilon)\psi_{jt} & \text{if } \tilde{a}_{t+1} \in \tilde{A}, \\ 0 & \text{otherwise,} \end{cases}$$

New households are born with the equally distributed accidental bequests from deceased households. The total amount of accidental bequests collected by the government at the end of date t is

$$\text{Total Bequest}_t = \sum_j (1 - \psi_{jt}) \int_{\varepsilon, \tilde{a}} \tilde{a}'(\varepsilon, \tilde{a}, j) \, d\widetilde{\Phi}_{jt}(\varepsilon, \tilde{a})$$

The initial endowment for new born households at date $t+1$ is

$$\tilde{a}^i_{t+1} = \frac{\text{Total Bequest}_t}{\omega_t \mu^w_t}$$

A new-born household's initial endowment of efficiency units of labor is randomly drawn from the ergodic distribution of the labor productivity process. The measure of newborn households at date $t+1$ is given by

$$d\widetilde{\Phi}_{1,t+1}\left(\varepsilon, \{\tilde{a}^i_{t+1}\}\right) = \pi_\varepsilon \widetilde{\Phi}_{1,t+1}\left(\mathcal{E}, \{\tilde{a}^i_{t+1}\}\right) = \pi_\varepsilon \omega_t \sum_{j=1}^{J_r - 1} \widetilde{\Phi}_{jt}\left(\mathcal{E}, \tilde{\mathcal{A}}\right).$$

3.6 Model Detrending

The growth rate of the labor supply is

$$\frac{\Delta \tilde{N}_{t+1}}{\tilde{N}_t} = \frac{\Delta L_{t+1}}{L_t} + \frac{\Delta \mu^w_{t+1}}{\mu^w_t}.$$

Since the average efficiency unit of labor supplied by working-age households, L_t, is stationary, the growth rate of total labor supply equals the growth rate of the working-age population, n_{t+1}. The growth rate of total output is the sum of the growth rates of labor supply and productivity, $(1 + \gamma_{t+1})(1 + n_{t+1})$.

All aggregate variables are detrended by $\prod_{s=1}^{t}(1+\gamma_s)(1+n_s)$, and individual variables by $\prod_{s=1}^{t}(1+\gamma_s)$ to make the model stationary. This detrending formula is formalized as:

$$x_t = \frac{\tilde{x}_t}{\prod_{s=1}^{t}(1+\gamma_s)}, \quad X_t = \frac{\tilde{X}_t}{\prod_{s=1}^{t}(1+\gamma_s)(1+n_s)}.$$

The measure of population is detrended by the growth rate of working-age population. Given that the initial working-age population is normalized to 1 ($\mu_0^w = 1$), the detrended measure of population is:

$$\Phi_{jt}\,(E, A) = \Phi_{jt}\left(E, \frac{\tilde{A}}{\prod_{s=1}^{t}(1+\gamma_s)}\right) = \frac{\widetilde{\Phi}_{jt}(E, \tilde{A})}{\mu_t^w} = \frac{\widetilde{\Phi}_{jt}(E, \tilde{A})}{\prod_{s=1}^{t}(1+n_s)}.$$

See Appendix B for the detrended model.

4 Determinants of Debt Sustainability

This section examines how government's foreign investment affects fiscal sustainability. Section 4.1 studies the long-run impact by examining the possibility of a deficit-debt steady state. This is where the government can consistently run a primary deficit while keeping its debt finite and constant. I show that this is achievable even if the government's borrowing cost is greater than the growth rate ($r_b > g$), provided the government earns an excess return on its foreign investment. In Section 4.2 , I analyze how the government's foreign investment affects the dynamics of debt-to-GDP ratio. It will be shown that the outstanding debt can decay over time only when the rate of foreign asset accumulation is moderate. Furthermore, I show that, when the return on foreign assets surpasses the government's borrowing cost, the fiscal space can converge to a sustainable deficit steady state by appropriately managing the foreign asset position, even without actively adjusting spending and taxes.

4.1 Deficit Debt Steady State

I first define the sustainable deficit steady state. It is a steady state where the government runs a primary deficit and maintain a constant level of debt.

Definition 1 (Steady State)**.** Given a fiscal rule $\left\{\frac{G}{Y}, \theta, \tau_c, \tau_l, \tau_k, \xi\right\}$, demographic factors

$\{\omega, (\psi_j)_{j=1}^{\bar{J}}\}$, the production environment $\{\gamma, \varphi\}$, and the return from foreign asset r^*, the steady state of the economy is a set of (i) households' value functions $v_j(\varepsilon, a)$ and their optimal policy functions for saving and consumption $\{\mathbf{A}_j'(\varepsilon, a), \mathbf{C}_j(\varepsilon, a)\}$, (ii) aggregate variables $\{C, K, L, B, g\}$, (iii) prices $\{r, w\}$, (iv) the demographic structure $\{n, \zeta\}$, and (v) the measure of households $\Phi_j(\varepsilon, A)$ such that

1. the growth rate of working-age population (n) and the dependency ratio (ζ) satisfy

$$n = \omega + \sum_{j<J_r-1} \psi_j \Phi_j(\mathcal{E}, \mathcal{A}),$$
$$\zeta = \frac{\sum_{j \geq J_r} \Phi_j(\mathcal{E}, \mathcal{A})}{\sum_{j<J_r} \Phi_j(\mathcal{E}, \mathcal{A})}.$$

2. the growth rate of GDP (g) satisfies $g = (1+\gamma)(1+n) - 1$.

3. households maximize their values,

$$v_j(\varepsilon, a) = \max_{c, a'} \left[c^\rho + \beta \psi_j \left(\mathbb{E}_{\varepsilon'|\varepsilon} v_{j+1}(\varepsilon', a')^{1-\sigma} \right)^{\frac{\rho}{1-\sigma}} \right]^{\frac{1}{\rho}}$$
$$\text{subject to} \quad (1+\tau_c)c + (1+\gamma)a' = \mathbf{1}_{j<J_r}(1-\tau_l)w\varepsilon + \mathbf{1}_{j \geq J_r}\theta w + (1 + r(1-\tau_k))a$$
$$a' \geq \underline{a}$$

4. the aggregate consumption C and the aggreagate labor suppy L are given by

$$C = \sum_j \int_{\varepsilon, a} \mathbf{C}_j(\varepsilon, a) \, d\Phi_j(\varepsilon, a),$$
$$L = \frac{\sum_{j=1}^{J_r-1} \int_{\varepsilon, a} \varepsilon \, d\Phi_j(\varepsilon, a)}{\sum_{j=1}^{J_r-1} \int_{\varepsilon, a} d\Phi_j(\varepsilon, a)}.$$

5. the stationary distribution of the population $\Phi_j(\varepsilon, A)$ and the saving policy function $\mathbf{A}_j'(\varepsilon, a)$ satisfy, for $j \in [1, \bar{J} - 1]$,

$$\Phi_{j+1}(\varepsilon', A) = \int_{a \in A} P(\varepsilon, a, j, \varepsilon', A, j+1) \, d\Phi_j(\varepsilon, a)$$

where

$$\Phi_{j+1}(\varepsilon', A) = \begin{cases} \frac{\Pr(\varepsilon_{t+1} \in \varepsilon' | \varepsilon_t = \varepsilon)\psi_j}{1+n} & \text{if } \mathbf{A}'_j(\varepsilon, a) \in A \\ 0 & \text{otherwise} \end{cases}$$

The initial endowment for newborn households is

$$a^i = \frac{\sum_{j=1}^{J}(1-\psi_j)\int_{\varepsilon,a} \mathbf{A}'_j(\varepsilon, a)\, d\Phi_j(\varepsilon, a)}{\omega \sum_{j=1}^{J_r-1} \Phi_j(\mathcal{E}, \mathcal{A})}.$$

The measure of newborn households is

$$d\Phi_1(\varepsilon, \{a^i\}) = \pi_\varepsilon \Phi_1(\mathcal{E}, \{a^i\}) = \pi_\varepsilon \omega \sum_{j=1}^{J_r-1} \Phi_j(\mathcal{E}, \mathcal{A}).$$

6. the government's steady-state debt level B satisfies

$$B = -\frac{G - \tau_c C - \tau_k r K + (\theta\zeta - \tau_l L)w}{(r_b - g)(1-\xi) + (r_b - r^*)\xi}. \tag{1}$$

   where $r_b = (1-\tau_k)r$ is the government's borrowing cost.

7. prices $\{r, w\}$ are given by

$$r = \alpha \left(\frac{K}{L}\right)^{\alpha-1} - \delta - \varphi, \quad w = (1-\alpha)\left(\frac{K}{L}\right)^{\alpha}.$$

8. the market clearing conditions for capital is satisfied,

$$K + B = \sum_j \int_{\varepsilon,a} a\, d\Phi_j(\varepsilon, a).$$

■

Equation (1) describes the government's bond supply in the steady state. The numerator reflects the primary deficit, which increases due to (i) higher government expenditures, (ii) a more generous pension system relative to tax revenue, or (iii) an older demographic structure. The denominator captures the *effective debt service cost*. If the government invests a fraction ξ of its debt in foreign assets, the effective debt service cost decreases to the extent that the return on foreign investments exceeds the domestic borrowing cost. For the portion of debt not invested, the debt service cost can be reduced if the GDP growth rate surpasses the interest rate on government bonds. The average effective debt service cost is a weighted average of these two elements.

Deficit Debt Steady State A deficit debt steady state is a steady state featuring both a positive primary deficit and a finite level of debt. That is, it's a steady state that satisfies

$$G - \tau_c C - \tau_k r K + (\theta\zeta - \tau_l L) w \geq 0, \quad 0 < B < \infty.$$

This steady state can be represented as an intersection of the government's bond supply curve and the private sector's bond demand curve. If the steady state interest rate is r, the curves are given by

$$B^s(r) = -\frac{G - \tau_c C(r) - \tau_k r K(r) + (\theta\zeta - \tau_l L) w(r)}{(r_b - g)(1-\xi) + (r_b - r^*)\xi}, \qquad r_b = (1-\tau_k) r \tag{2}$$

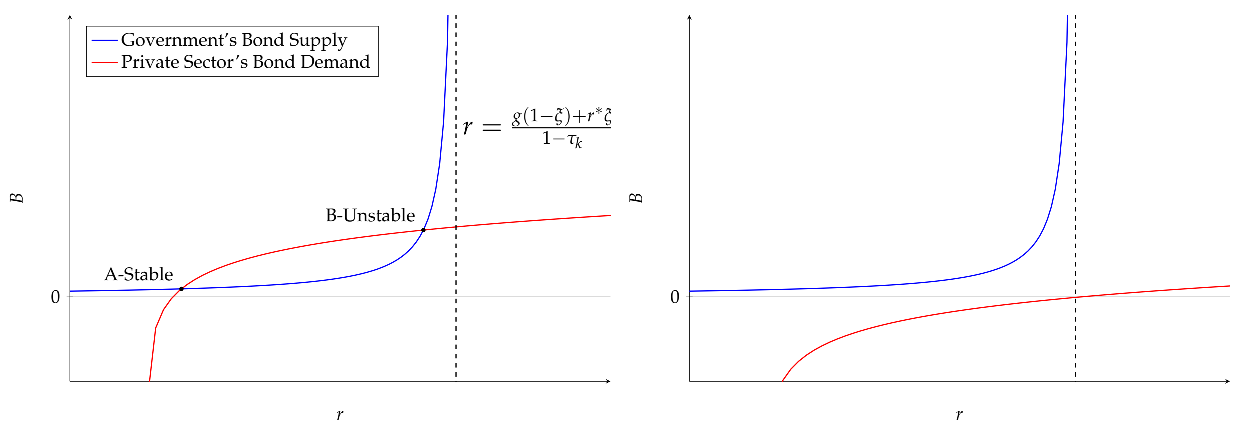
$$B^d(r) = \sum_j \int_{\varepsilon,a} a \, d\Phi_j(\varepsilon, a) - K(r). \tag{3}$$

The government's bond supply curve (2) reflects its financing needs, as discussed below. $K(r)$ in the private sector's bond demand (3) represents the capital stock required by the production function at interest rate r. The private sector's bond demand is households' total savings net of the capital stock.

When the primary deficit is positive and $r_b < r^*$, the government's bond supply increases with the steady-state interest rate. This occurs because the asset-to-debt ratio (ζ) is assumed to be fixed at a constant level. When r increases, the government's borrowing cost, $r_b = (1-\tau_k)r$, rises, the government requires more fiscal revenue to finance the primary deficit. Given that $r^* > r_b$, the only feasible strategy is to issue more government bonds and invest more in foreign assets, which yield a higher return. This mechanism ensures that the government can cover its financing needs while maintaining fiscal sustainability. Similarly, the private sector's net bond demand also increases with the interest rate. A higher interest rate incentivizes households to increase their savings, leading to a greater demand for government bonds.

Graphical Illustration of Sustainable Deficit Steady State A sustainable deficit steady state can be illustrated using a diagram as in Figure 6. The blue lines represent the government's bond supply curve against the steady state interest rate r. The red lines plot the private sector's bond demand curve. For a sustainable deficit steady state to exist, the two curves must intersect at a positive level of debt. The left panel of Figure 6 illustrates a case where the government can maintain a sustainable deficit steady state with a positive level of debt. The right panel shows a case where the government cannot sustain a positive level of debt, as the private sector's bond demand is insufficient to absorb the government's bond supply.

In the left panel of Figure 6, when a sustainable deficit steady state exists, there can be multiplicity of equilibrium. Point A represents a stable equilibrium, and Point B an unstable equilibrium. If the economy is at A and debt increases due to a government spending shock, the economy will return to A if the shock is not large enough to push the interest rate beyond that of B. However, if the shock is substantial, the government debt will diverge.



(a) Multiplicity of Steady State

(b) Non-existence of Steady State

Figure 6: Bond Supply and Demand in Steady State

Note: Blue and red lines represent the government's bond supply and the private sector's bond demand, given by (2) and (3), for a given steady state interest rate r. Dashed black line represents the asymptote of the government's bond supply curve, where the steady state debt level diverges. The left panel illustrates a case where a deficit debt steady state exists, while the right panel shows a case where it does not. The left panel also features the multiplicity of steady state, with point A being stable and point B unstable.

A Necessary Condition for the Existence of Sustainable Deficit Steady State Under a primary deficit, sustaining a positive level of debt requires the government's borrowing cost r_b to satisfy

$$r_b < g(1-\xi) + r^*\xi. \tag{4}$$

It is also obvious from Figure 6 given that the asymptote of the government's bond supply curve is $r = [g(1-\xi) + r^*\xi] / (1-\tau_k)$.

In the absence of foreign investment ($\xi = 0$), the condition reduces to the well-known condition $r_b < g$, implying that the debt-to-GDP ratio remains finite only if the growth rate of GDP is sufficiently high to offset the cost of servicing debt. However, if the government invests a portion of its debt in foreign assets ($\xi > 0$), the government can maintain fiscal sustainability even if the borrowing cost exceeds the GDP growth rate, as long as the return on foreign assets r^* is sufficiently high to cover the effective debt service cost.

The condition (4) is a necessary condition for the existence of a sustainable deficit steady state. If households are not willing to supply enough savings to absorb the government's bond issuance, the government may not be able to maintain a sustainable deficit steady state, as in the right panel of Figure 6. The government debt will diverge to infinity unless a fiscal adjustment is made.

The Role of ξ and r^* The government can influence the sustainability of its debt by adjusting its foreign investment strategy. Specifically, the government can reduce the effective debt service cost by increasing the proportion of debt invested in foreign assets (ξ) as far as the return on foreign assets (r^*) is greater than the government's borrowing cost. Alternatively, an increase in the return on foreign assets can also reduce the effective debt service cost under the same proportion of debt invested in foreign assets. This is also obvious from Figure 6, where the asymptote of the government's bond supply curve shifts to the right as ξ increases or r^* increases.

4.2 Stability of Debt Dynamics

Recall that the government's budget constraint is given by

$$(1+g_{t+1})(1-\xi_{t+1})B_{t+1} = G_t - \tau_{ct}C_t - \tau_{kt}r_tK_t + (\theta_t\zeta_t - \tau_{lt}L_t)w_t + \left[1 + r_t(1-\tau_{kt}) - \xi_t(1+r_t^*)\right]B_t.$$

To analyze the dynamics of the debt-to-GDP ratio, I normalize all relevant aggregate variables by GDP Y_t and use bar notation to denote GDP-normalized variables. For instance, $\bar{B}_t = B_t/Y_t$ denotes the debt-to-GDP ratio, $\bar{C}_t = C_t/Y_t$ denotes the consumption-to-GDP ratio, and so on. The normalized budget constraint is

$$\begin{aligned}(1-\xi_{t+1})\bar{B}_{t+1} &= \bar{G}_t - \tau_{ct}\bar{C}_t - \tau_{kt}r_t\bar{K}_t + (1-\alpha)\left(\theta_t\frac{\zeta_t}{L_t} - \tau_{lt}\right) + \left[1 + (1-\tau_{kt})r_t - (1+r_t^*)\xi_t\right]\bar{B}_t \\ &\equiv \bar{d}_t + \left[1 + (1-\tau_{kt})r_t - (1+r_t^*)\xi_t\right]\bar{B}_t. \qquad (5)\end{aligned}$$

where $\bar{d}_t = \bar{G}_t - \tau_{ct}\bar{C}_t - \tau_{kt} r_t \bar{K}_t + (1-\alpha)\left(\theta_t \frac{\zeta_t}{L_t} - \tau_{lt}\right)$ is the primary deficit-to-GDP ratio. (5) describes the recursive dynamics of the debt-to-GDP ratio over time.

Debt Decay and the Rate of Foreign Asset Accumulation For the outstanding debt at time t to decay between time t and $t+1$, the following condition must hold:

$$1 - \xi_{t+1} > [1 + (1-\tau_{kt})r_t - (1+r_t^*)\xi_t] \quad \Longleftrightarrow \quad (1+r_t^*)\xi_t > (1-\tau_{kt})r_t + \xi_{t+1}. \tag{6}$$

This condition implies that if the returns and principal from existing investment positions, $(1+r_t^*)\xi_t$, are sufficient to cover both the interest on outstanding debt ($(1-\tau_{kt})r_t$) and new investments (ξ_{t+1}), then the outstanding debt will decline.

The inequality highlights the appropriate rate of foreign asset accumulation necessary to prevent an uncontrolled increase in the debt-to-GDP ratio. If the government accumulates foreign asset position abruptly, it forces the government to issue more bonds to cover current spending and pension obligations, which can lead to a significant increase in the debt-to-GDP ratio. However, when the government has already accumulated a sufficient amount of foreign assets, it can manage the debt-to-GDP ratio more effectively. This suggests that governments should proactively accumulate foreign assets to prepare for future demographic challenges. Establishing a sufficiently high asset position in advance can help manage a significant increase in future pension obligations.

Convergence to a Sustainable Deficit Steady State Suppose a shock occurs at date t that increases the primary deficit ratio $\bar{d}_t$. The impact of 1 pp increase in $\bar{d}_t$ on the debt-to-GDP ratio at date $T > t$ is given by

$$\prod_{s=t}^{T} \frac{1 + (1-\tau_{ks})r_s - (1+r_s^*)\xi_s}{1 - \xi_{s+1}}.$$

The effect of the shock on debt-to-GDP ratio disappears asymptotically if

$$\lim_{T\to\infty} \prod_{s=t}^{T} \frac{1 + (1-\tau_{ks})r_s - (1+r_s^*)\xi_s}{1 - \xi_{s+1}} = 0.$$

This is a sufficient condition where an increase in primary deficit ratio at t has zero long-run cost.

Assuming the government maintains a constant foreign asset share and interest rates remain stable in the long run, the above condition simplifies to

$$\frac{1 + (1 - \tau_k)r - (1 + r^*)\xi}{1 - \xi} < 1 \quad \Longleftrightarrow \quad \xi > \frac{(1 - \tau_k)r}{r^*}. \tag{7}$$

Under this condition (7), if the primary deficit ratio converges to a long-run level ($\bar{d}_t \to \bar{d}_{\text{long-run}}$), the debt-to-GDP ratio will also converge to a long-run level ($\bar{B}_t \to \bar{B}_{\text{long-run}}$). Note that primary deficit ratio can converge to a long-run level simply by the government maintaining a constant fiscal policy stance on key variables like government spending to GDP ratio (G/Y), tax rates (τ_c, τ_k, τ_l), pension replacement ratio (θ), and foreign asset share (ξ). This presents a stark difference from the usual models in the literature, such as Kaplan et al. (2023) or Angeletos et al. (2024), which achieves long-run convergence of debt-to-GDP ratio by assuming active fiscal feedback rules that adjust government's spending based on target and current debt levels.

The above condition (7) also implies that if the foreign interest rate (r^*) exceeds the government's borrowing cost ($(1 - \tau_k)r$) in the long run, the government can stabilize its debt-to-GDP ratio by appropriately choosing its foreign investment share ξ. Specifically, this stable debt management is possible even when $r > 0$ if the government engages in foreign investment. In contrast, without foreign investment ($\xi = 0$), the impact of a one-time shock to the primary deficit on the debt-to-GDP ratio never disappears if $r > 0$. This difference is illustrated in Figure 7. With $r = 0.01, r^* = 0.04, \tau_k = 0.20$, the effect of a transitory shock in primary deficit on the debt-to-GDP disappears only when the government's foreign asset share is sufficiently high ($\xi = 0.25$). The effect persists when the government does not invest in foreign assets ($\xi = 0$).

5 Quantitative Analysis

5.1 Calibration

Calibration Strategy The model is calibrated to replicate the Japanese economy from 1995 to 2020. The model's initial steady state is set to match the Japanese economy in 1995. Subsequently, historical data on demographic factors, the pension system, fiscal policies, and the government's foreign investment strategy from 1995 to 2020 are fed into the model. The model's transition is computed under perfect foresight, based on a specific scenario assuming that the 2020 values of demographic factors, government policy

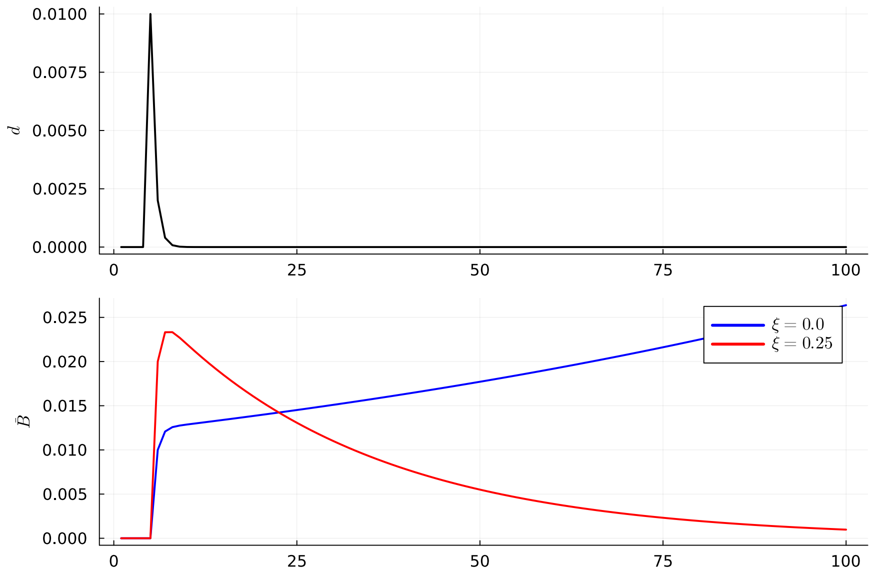


Figure 7: Convergence of Debt-to-GDP Ratio

stances, and foreign investment strategies remain constant post-2020. A fiscal reform in the form of spending cut is additionally assumed to be implemented after 2020 to ensure that the transition path converges to a sustainable deficit steady state. The final steady state in the model is not observed in the data, as it represents a long-term projection. The model's calibration is validated by comparing the model's transition path with historical data from 1995 to 2020.

Parameters The model parameterization is summarized in Table 1. The model frequency is annual, and households enter the model economy as age 20, retire at 65, and live up to age 100.

The model's initial age-specific survival probability profile is calibrated rather than taken directly from the data. The 1995 historical age-specific survival probability profile, combined with the 1995 historical fertility rate, does not generate the observed 1995 dependency ratio in the implied stationary distribution. To match the 1995 dependency ratio, I parameterize the logit of survival probability as a linear function of age. Specifically, I parameterize the survival probability of age-j household (ψ_j) by

$$\psi_j = \frac{\exp(\eta_0 + \eta_1 \times j)}{1 + \exp(\eta_0 + \eta_1 \times j)}$$

where η_0 and η_1 are calibrated to match the 1995 and 1996 dependency ratio using the historical 1995 and fertility rate. The calibrated age-specific survival probability generates

Table 1: Calibration of Initial Steady State

Variable		Value	Source / Target
Demographic			
J_{max}	Maximum age	81	Age 100 (Enter the model as 20)
J_r	Retirement age	46	Age 65
$\eta_0^\dagger$	Intercept of logit $\psi_{j,1995}$	-4.9616	Calibrated to match
$\eta_1^\dagger$	Slope of logit $\psi_{j,1995}$	0.0706	1995 dependency ratio
Preference			
$\beta^\dagger$	Discounting factor	0.99	Domestic rate in 1995
$\iota^\dagger$	EIS	0.10	Debt-to-GDP in 1995,
σ	Relative risk aversion	6.0	Guvenen (2009)
Income Process			
ρ_ε	Persistence of income shocks	0.95	Nakajima and Takahashi (2017)
σ_ε	Std. dev. of income shocks	0.15	$\mathrm{Var}(\log w) = 0.226$, Nakajima and Takahashi (2017)
p_d	Pr. of unemployment	0.02	Average long-term unemployment rate
Production			
α	Capital Share	0.36	National Accounts
δ	Depreciation rate	0.08	

Note: † indicates internally calibrated parameters.

a stationary distribution that closely replicates the historical initial dependency ratio.

The discounting factor β is calibrated to match the real interest rate of 1.01% in 1995, which is the average of the 1-year Japanese Government Bond (JGB) real yield in that year. The elasticity of the intertemporal substitution (EIS) is calibrated to match the government's historical debt-to-GDP ratio of 92.52% in 1995. A sufficiently low EIS is required to ensure that households are willing to hold government bonds when the government bond yield is lower than the GDP growth rate and their retirement income is ensured by the pension system. However, the chosen value is not far from those used in the literature, such as Guvenen (2009) or Gomes and Michaelides (2008). The relative risk aversion is externally fixed to 6.0. Again, this value aligns with choices in the literature (Gomes and Michaelides, 2008; Guvenen, 2009).

For the household's income process, I target the variance of raw log wage of 0.226 (Nakajima and Takahashi, 2017), with the persistence of idiosyncratic income shocks set to 0.95. The probability of unemployment is set to 2% that corresponds to the long-run average of unemployment rate of Japan.

The depreciation rate is externally chosen as 8% following the literature. The capital share is calibrated to match the capital-to-GDP ratio of 290.4% in 1995. The historical capital-to-GDP ratio is calculated using the depreciation rate that's consistent with the model.

Baseline Scenario The model's transition path is computed under the baseline scenario, details of which are summarized in Table 2. The baseline scenario incorporates historical data for key variables between 1995 and 2020, with post-2020 values assumed constant at their 2020 levels unless otherwise specified.

Table 2: Calibration of Baseline Scenario

Variable		**1995**	**2020-**	**1996-2019**	**Source / Target**
Demographic					
ω_t	Fertility rate	0.023	0.017	Data	UN Population
ψ_{jt}	Survival pr.			Data	UN Population
Production					
γ_t	Technology growth	0.011	0.011	Data	Penn World Table
φ_t	Financial friction	0.035	0.045	Linear	Capital-to-GDP in 1995 and 2020
Government					
τ_{kt}	Capital tax	0.210	0.210	Constant	Statutory values
τ_{ct}	Consumption tax	0.030	0.100	Data	Statutory values
τ_{lt}	Labor tax	0.241	0.327	Data	OECD Taxing Wages
θ_t	Pension replacement rate	0.315	0.315	Constant	OECD Pension Model
$(G/Y)_t$	Gov't spending to GDP	0.141	0.210	Data	National Accounts
ξ_t	Share of foreign assets	0.097	0.372	Data	Flow of Funds
r_t^*	Return on foreign assets	0.045	0.045	Data	GPIF foreign returns
Assumed Fiscal Reform after 2020		**2020**	**2040**	**Transition**	
$(G/Y)_{t>2020}$	Spending-to-GDP	0.210	0.180	Linear	
$(\tau_{ct})_{t>2020}$	Consumption tax rate	0.100	0.200	Linear	

Note: The table summarizes the calibration of the baseline scenario. The columns "1995" and "2020-" indicate the values of each variable in 1995 and from 2020 onwards, respectively. The column "1996-2019" describes how the variable evolves between 1996 and 2019. The last column provides the data source or target for each variable. See Appendix C for details of data processing and illustration of the baseline scenario.

Demographic factors are processed as described in Appendix C and fed into the model. The time series of fertility rate data is used as is, while age-specific survival probabilities in the initial steady state are calibrated to match the dependency ratio in 1995. Survival probability data between 1996 and 2020 are fed into the model as is, with one adjustment that working-age households are assumed to survive every year with certainty.

The technology growth rate data is computed from Penn World Table 10.01. For details of the data processing, see Appendix C. The technology growth rate has shown large fluctuations in the past three decades, with a significant decline in the 1990s and a recovery in the 2000s. Since the 1995 historical technology growth rate does not represent the long-run growth rate of technology, I used the average technology growth rate from 1985 to 2019 as the initial and final values in the baseline scenario. This average growth rate is 1.1%. For 1996-2019, historical data is fed into the model. The default probability is set to 3.5% in 1995 and increases linearly to 4.5% in 2020. This increasing trend in the default probability is necessary to match low capital-to-GDP ratio in 2020, given the low intereat rate circumstances.

Government's policy parameters are also informed by historical data. The capital income tax rate is held constant at 21% throughout the transition period. The consumption tax rate follows statutory values, increasing from 3% in 1995 to 10% in 2020. The labor income tax wedge is derived from OECD Taxing Wages data. As this data are available from 2000 to 2023, 2000 values are used for any year prior to 2000. The tax wedge rises from 24.1% in 1995 to 32.7% in 2020. The pension replacement ratio is sourced from the OECD Pension Model, using the ratio of pension benefits to average earnings. This rate is constant at 31.5%. The general government's spending-to-GDP ratio is from World Bank National Accounts data. The government expenditure-to-GDP ratio in 1995 was 15.1%, a figure higher than the 1975-1995 average. Indeed, until 1993, Japan's government expenditure-to-GDP ratio never exceeded 14%, and it remained below 15% in both 1993 and 1994. Based on this, I assume that 15.1% is not an appropriate value for the initial steady state and instead utilize the 1975-1995 average of 14.1% as the value for the initial steady state. The share of foreign assets in the government portfolio is from the Flow of Funds Japan data. The data ranges from 1997 to 2020, so the 1997 value is extended to 1995. The return on foreign assets also ranges from 2001 to 2020. For years prior to 2001, I use the average return on foreign assets from 2001 to 2020. The return rate is assumed to be constant at the average of 4.5% after 2020.

I assume the government engages in fiscal reform after 2020, which is necessary to ensure the economy converges to a sustainable deficit steady state. The government's spending-to-GDP ratio has risen from 14.1% in 1995 to 21.0% in 2020. The 2020 value is not sustainable in the long run, given demographic factors that remain constant at 2020 values and other fiscal policies, including tax rates and the pension replacement ratio, unchanged. To achieve a deficit debt steady state, the government should either (i) cut the spending-to-GDP ratio, (ii) increase tax rates, or (iii) reduce the pension replacement ratio. In the baseline scenario, I cut the government's spending and raises the consumption tax rate after 2020 as (i) the government's spending does not contribute to welfare in the model and (ii) actual Japanese government has been increasing the consumption tax rate to finance its spending. Other scenarios where the government increases the consumption tax rate or reduces the pension replacement ratio are examined in Appendix E. In the baseline scenario, the government's spending-to-GDP ratio is cut to 18% gradually within 20 years following 20200, and the consumption tax rate is increased to 20% over the same period.

5.2 Model Fit to Data

Table 3 summarizes how well the model's simulated values align with historical data for 1995 and 2020. In 1995, the model closely matches the dependency ratio (23.3% model vs. 23.3% data), debt-to-GDP ratio (92.3% model vs. 92.5% data), and capital-to-GDP ratio (288.8% model vs. 290.4% data). The interest rate shows a minor difference (0.96% model vs. 1.01% data).

Table 3: Model Fit to Data for 1995 and 2020

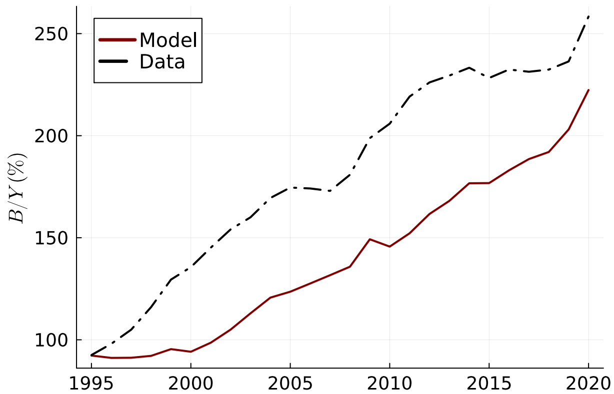
	1995		**2020**	
Variable	**Data**	**Model**	**Data**	**Model**
Dependency ratio (%)	23.3	23.3	53.4	51.9
Debt-to-GDP (%)	92.5	92.3	258.4	222.4
Capital-to-GDP (%)	290.4	288.8	294.2	281.1
Interest rate (%)	1.01	0.96	−0.10	0.03

Note: The "Model" column presents the model's simulated values for 1995 and 2020, with the initial steady state calibrated to match the 1995 data and the baseline scenario. For the details of calibration, see Table 1 and 2.

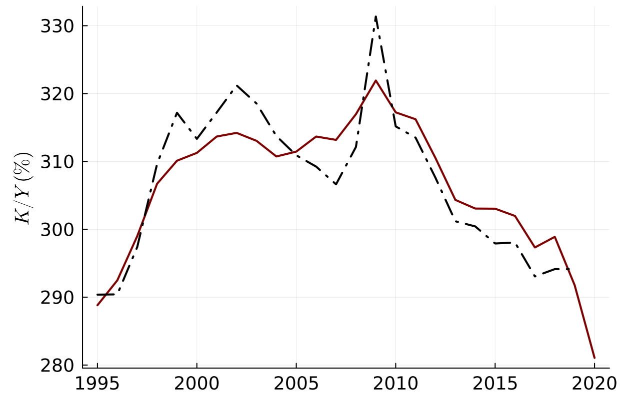
The model provides a reasonable fit for 2020. The dependency ratio is closely replicated (51.9% model vs. 53.4% data), and the capital-to-GDP ratio remains close to the data (281.1% model vs. 294.2% data). While the model's debt-to-GDP ratio is lower than the data, it still captures 78% of the increase observed (130.1%p model vs. 165.8%p data). The model's interest rate shows a positive value compared to the slightly negative historical rate (0.03% model vs. −0.10% data).

In addition to the summary in Table 3, the computed transition path during 1995 to 2020 of the model is presented in Figure 8. The model successfully replicates the demographic shifts observed in the data, particularly the rising dependency ratio and the short-term increase followed by a decline in the working-age population. Minor discrepancies between the model and data in demographic factors come from the model's simplification of ignoring the mortality rate for working-age households.

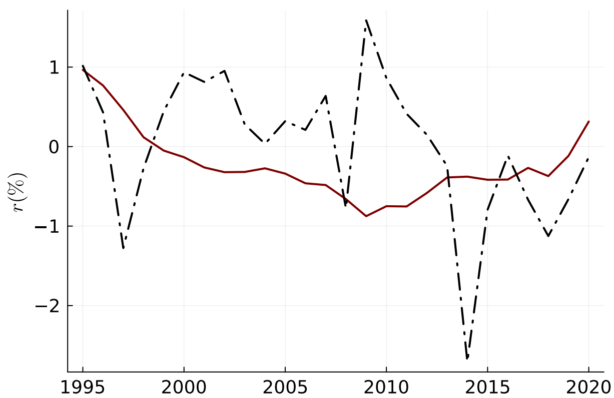
The model also shows a strong fit in the changes in the capital-to-GDP ratio. While the



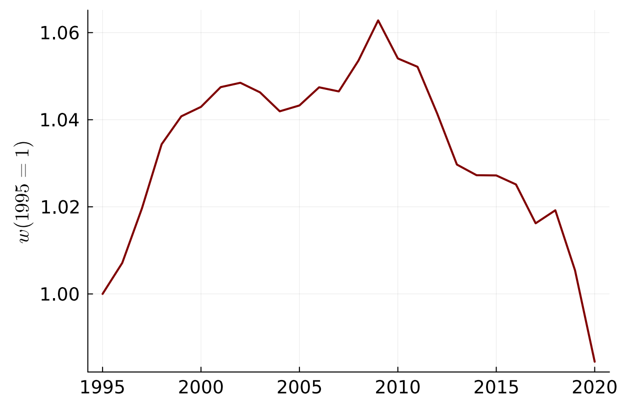
(a) Debt to GDP Ratio



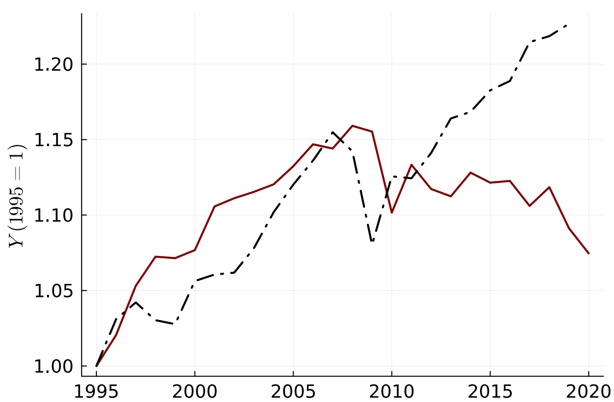
(b) Capital to GDP Ratio



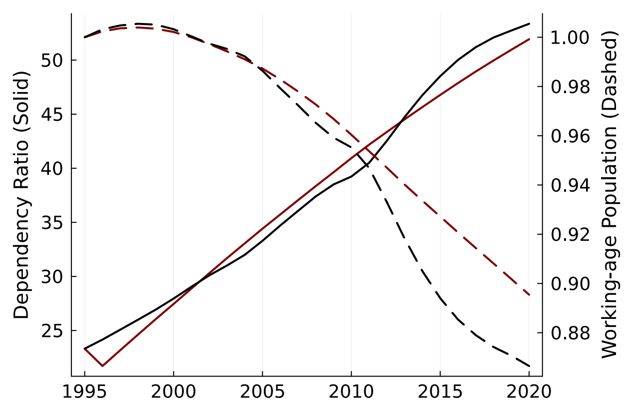
(c) Interest Rate



(d) Wage Rate



(e) GDP



(f) Demographics

Figure 8: Model Transition from 1995 to 2020

Note: The figure presents the model's transition path from 1995 to 2020 under the baseline scenario. The initial steady state of the model is calibrated to match 1995 data. See Table 1 for the initial steady state calibration. For details of the baseline scenario, see Table 2.

interest rate generally exhibits a good fit, the model cannot capture fluctuations due to the absence of financial frictions in its structure. Furthermore, the model predicts an increase in interest rates after 2010 due to rising government debt, which is not observed in the historical data. This divergence might be linked to the model's inability to fully explain GDP growth in the post-2010 period. A notable observation in the Japanese economy since the 1990s has been a clear decline in the labor share, often attributed to shifts in industrial structure (Higo, 2023). The increasing trend of capital share could help explain both the observed GDP growth and the persistently low interest rates after 2010, even with the decreasing capital-to-GDP ratio.

6 Counterfactuals and Welfare

Having calibrated the baseline model to the Japanese economy (see Section 5.1), I conduct counterfactual analyses to assess the impact of shifts in demographic factors and the government's foreign investment strategies on the increase in public debt. Additionally, I evaluate the welfare implications of the government pursuing foreign investments.

6.1 Driving Forces of Debt Accumulation among Demographic Factors

6.2 Counterfactual on Foreign Investment

I consider a counterfactual scenario where the share of foreign assets in the government's portfolio remains constant at its 1995 level of 9.7% throughout the transition period, instead of increasing to 37.2% by 2020, as it does in the baseline scenario. [2]

The results, summarized in Table 4 and Figure 9, highlight a clear difference between the two scenarios' short-term and long-term effects.

During the short-run phase (1995-2020), when the government is actively increasing its foreign asset share in the baseline scenario, the debt-to-GDP ratio rises more steeply than in the counterfactual scenario. The debt-to-GDP ratio in the baseline scenario increases by 121.11 percentage points by 2020, significantly more than the 54.71 percentage point increase in the counterfactual scenario. This is because a more aggressive invest-

[2] This approach can be used to decompose the effect of demographic transition on the debt accumulation into two factors: declining fertility rates and extending life expectancy. The decomposition results are presented together in Table 4 and details are presented in Appendix F.

ment strategy requires the government to issue additional debt to fund current spending and pension obligations.

However, the long-term prospects for 2120 reveal that a more aggressive accumulation of foreign assets ultimately leads to a lower debt-to-GDP ratio. As shown in Figure 9, the debt-to-GDP ratio in the baseline scenario falls below that of the counterfactual scenario within 10 years after the government completes its foreign asset accumulation. This shows that once the government stops increasing its foreign asset ratio, the previously accumulated foreign assets begin to lower borrowing costs and reduce the overall debt level.

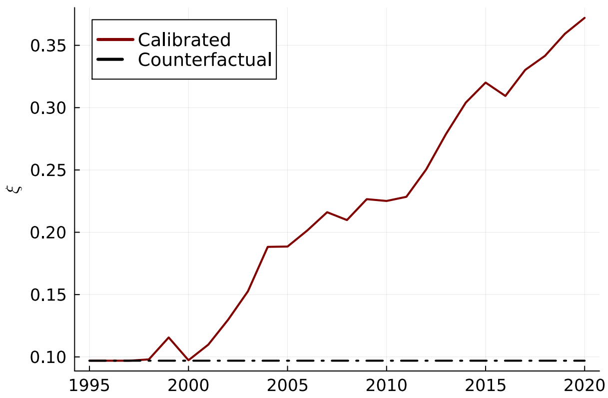
Other key variables also show consistent short-term and long-term dynamics. In the short term, the rising debt-to-GDP ratio in the baseline scenario leads to a capital crowding-out effect, keeping the capital-to-GDP ratio lower than in the counterfactual scenario. This lower rate of capital accumulation results in a lower GDP, higher interest rates, and lower wages in the baseline compared to the counterfactual. As a result, the short-term government accumulation of foreign assets negatively impacts household income and welfare.

In the long run, however, these effects are reversed. After the government finishes accumulating foreign assets, the baseline scenario projects a higher capital-to-GDP ratio. This leads to higher GDP, lower interest rates, and higher wage levels compared to the counterfactual.

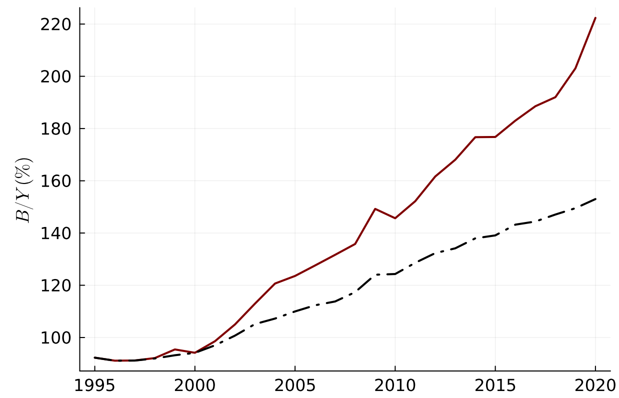
6.3 Optimal Investment and Welfare Evaluation

Optimal Path of Foreign Asset Accumulation As discussed in Section 6.2, the increase in the Japanese government's foreign asset share between 1995 and 2020 had contrasting effects. While it intensified capital crowd-out and lowered short-run wages, it led to higher long-run capital accumulation and wages. These conflicting outcomes create heterogeneous welfare effects across households, depending on age, asset holdings, and generation. The magnitude and rate of increase in the foreign asset share determine the severity of the capital crowding out effect in the short run and the long-run benefits from capital accumulation.

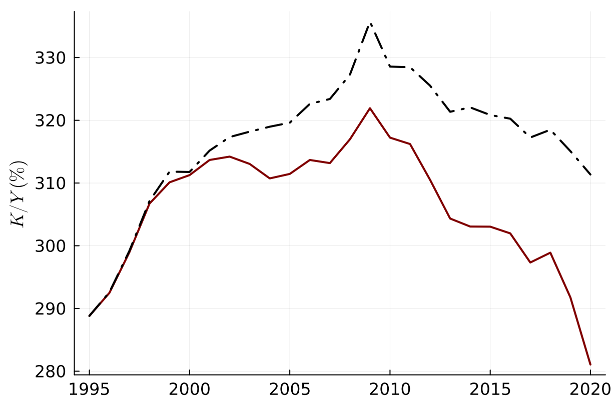
This observation raises the question of the optimal path of foreign asset share for maximizing social welfare. By finding the optimal path of external investment, I can evaluate whether the policy adopted by the Japanese government was welfare-improving compared to the optimum and to the counterfactual scenario where the government has not



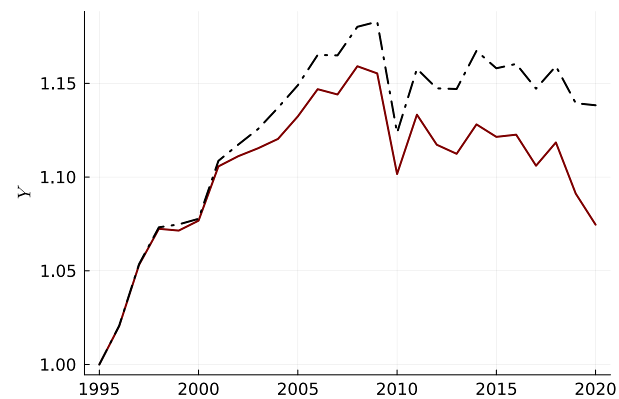
(a) Share of Foreign Assets



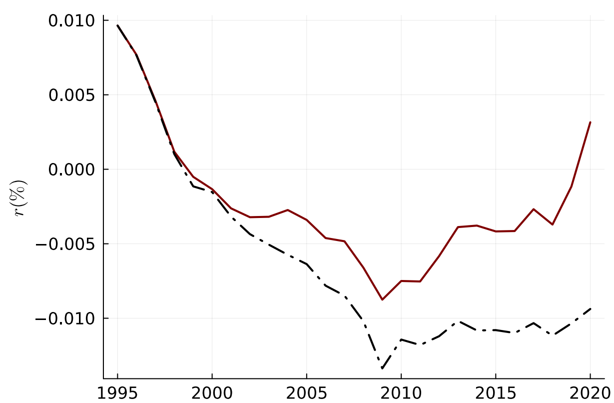
(b) Debt to GDP Ratio



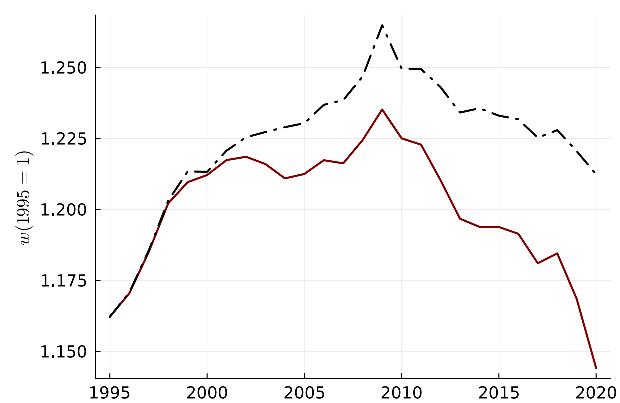
(c) Capital to GDP Ratio



(d) GDP



(e) Interest Rate



(f) Wage Rate

Figure 9: Counterfactual Analysis on Foreign Investment

Note: The label "Calibrated" indicates the baseline scenario calibrated in Section 5.1. The baseline scenario replicates the increasing share of foreign assets in the Japanese government's portfolio. The label "Counterfactual" indicates the counterfactual scenario where the share of foreign assets remains constant at the initial level.

Table 4: Counterfactual Analyses

Variable	1995 Baseline	2020 Baseline	2020 Const. F.I.	2020 Const. Fertility	2020 Const. Mortality
Dependency ratio (%)	23.3	51.9	51.9	43.8	25.7
Debt-to-GDP (%)	92.3	223.4	153.0	197.9	131.5
Capital-to-GDP (%)	288.8	281.1	311.4	258.3	304.1
Interest rate (%)	0.96	0.03	-0.94	0.92	-0.66

Variable	2120 Baseline	2120 Const. F.I.	2120 Const. Fertility	2120 Constant Mortality
Dependency ratio (%)	70.1	70.1	46.4	30.9
Debt-to-GDP (%)	28.8	146.3	-52.1	-100.3
Capital-to-GDP (%)	315.5	275.5	326.7	365.3
Interest rate (%)	-1.09	0.57	-1.48	-2.64

Note: The baseline scenario is calibrated in Section 5.1. Three counterfactual scenarios are compared to the baseline scenario. "Const. F.I." scenario assumes the share of foreign assets remains constant at the initial level ($\xi_t = \xi_{1995}$), "Const. Fertility" scenario assumes the fertility rate remains constant at the initial level ($\omega_t = \omega_{1995}$), and "Const. Mortality" scenario assumes the age-specific survival probability remains constant at the initial level ($\psi_{j,t} = \psi_{j,1995}$).

increased the foreign investment.

To compare welfare metrics across possible scenarios of foreign investment, I simplify the consideration set of foreign investment paths as follows. Let $\overrightarrow{\xi}(\xi_f) \in [0,1]^\infty$ denote the path of foreign asset shares defined by

$$\left(\overrightarrow{\xi}(\xi_f)\right)_t = \begin{cases} \xi_{1995} & \text{if } t = 1995, \\ \xi_{1995} + \frac{\xi_f - \xi_{1995}}{2020-1995} \times (t-1995) & \text{if } 1995 < t < 2020, \\ \xi_f & \text{if } t \geq 2020. \end{cases} \tag{8}$$

where ξ_{1995} is the historical foreign asset share in 1995. Considered paths of foreign investment shares are linear transitions from the historical foreign asset share in 1995 to a constant foreign asset share ξ_f after 2020. The consideration set of foreign investment paths is given by $\Xi = \{\overrightarrow{\xi}(\xi_f) \in [0,1]^\infty; \quad \xi_f \in [0,1]\}$.

For a given path $\overrightarrow{\xi}(\xi_f)$, I compute total social welfare by summing up the values of the current generation and the discounted values of all future generations.

$$
\begin{aligned}
\widetilde{W}\left(\overrightarrow{\xi}(\xi_f)\right) = &\sum_{j=1}^{\bar{J}} \int_{\varepsilon,\tilde{a}} \tilde{v}_{j,1995}(\varepsilon,\tilde{a})\, d\widetilde{\Phi}_{j,1995}(\varepsilon,\tilde{a}) \\
&+ \sum_{t=1996}^{\infty} \tilde{\beta}^{t-1995} \int_{\varepsilon,\tilde{a}} \tilde{v}_{1,t}(\varepsilon,\tilde{a})\, d\widetilde{\Phi}_{1,t}(\varepsilon,\tilde{a}).
\end{aligned} \tag{9}
$$

The first term on the right-hand side represents the total welfare of the generation alive in 1995, and the second term captures the discounted welfare of all future generations born after 1995.

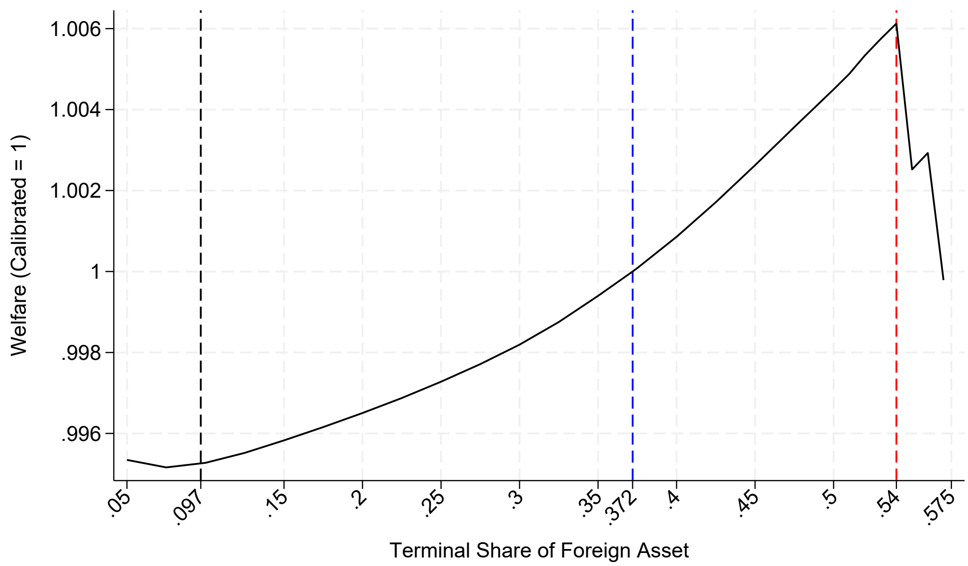


Figure 10: Optimal Foreign Investment Share

Note: The figure presents the total welfare computed by (9) for different values of ξ_f in (8). The welfare is normalized by the welfare under the calibrated baseline scenario. The black dashed line indicates the counterfactual scenario of no ramp-up of foreign investment. The red dashed line indicates the point maximizing welfare.

Figure 10 shows the computed welfare for different values of ξ_f. The figure shows that the overall welfare increases as the government's foreign investment share in the final steady state increases until the share reaches 54%. This suggests that up to 54%, the long-run benefits from capital accumulation dominates the short-run costs of capital crowding-out. However, if the government builds up the foreign assets rapidly between 1995 and 2020, the short-run capital crowding-out effect becomes severe, and the long-run benefits are not sufficient to offset this cost.

Considering that the Japanese government historically increased its share of foreign assets to 37.2% during this period, Figure 10 suggests that there was still room for welfare improvement by raising the share further to 54%. However, the historical policy was still more welfare-improving than the counterfactual scenario where the government had not increased its foreign asset share.

Distributional Consequences and Impossibility of Pareto-Improving Compensations It has been shown that the increase in the Japanese government's foreign investment share led to an improvement of utilitarian welfare compared to a counterfactual scenario in which the share of foreign investment had remained intact. Despite the improvement, the welfare effect under the active foreign investment can be heterogeneous across households depending on their cohort, asset, income, and age. Given this heterogeneity, it is natural to examine whether the government can use fiscal measures to address the distributional consequences. I investigate whether the government can make all households better off under the increasing foreign investment through its fiscal tools.

Following the methodology of Auerbach and Kotlikoff (1987) and Nishiyama and Smetters (2007), I examine if the government can make all households better off under the increasing foreign investment by providing lump-sum transfers based on their individual welfare changes. The transfer ensures that a household's lifetime value under the increasing investment share is equal to her value under the case of constant investment share. Specifically, let $\tilde{v}^{b}_{jt}(\varepsilon, \tilde{a})$ denote the household's value function in the baseline scenario (increasing share of foreign investment) and $\tilde{v}^{c}_{jt}(\varepsilon, \tilde{a})$ denote the household's value function in the counterfactual scenario (constant share of foreign investment). The required compensatory transfer $\tilde{T}_{jt}(\varepsilon, \tilde{a})$ is defined as follows:

$$\tilde{v}^{b}_{jt}(\varepsilon, \tilde{a} + \tilde{T}_{jt}(\varepsilon, \tilde{a})) = \tilde{v}^{c}_{jt}(\varepsilon, \tilde{a}). \tag{10}$$

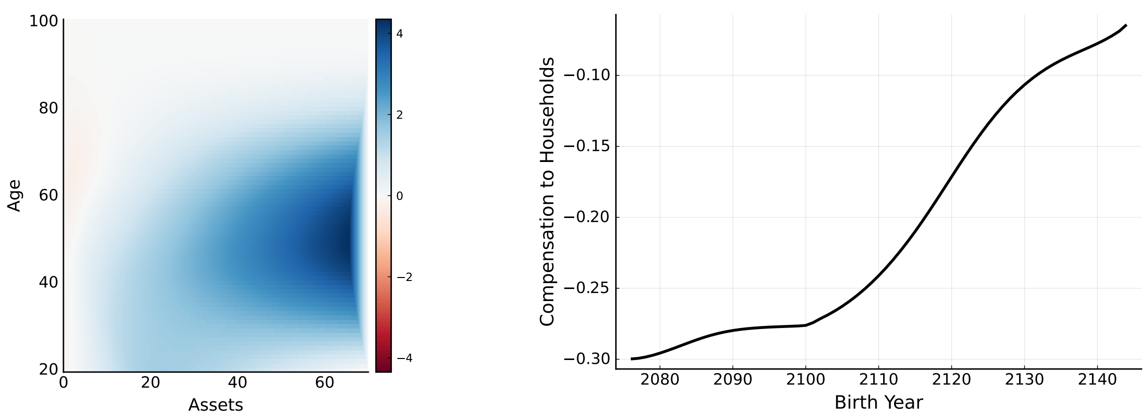
The compensatory transfer is provided to households as one-time lump-sum transfer. If the transfer is positive, it indicates that the household is worse off in the baseline scenario compared to the counterfactual scenario, and vice versa.

To assess the possibility of Pareto improvement through these transfers, I compute the net present value (NPV) of all compensatory transfers. This NPV sums up the transfers to all households living in the initial steady state and all future generations. The formula is as follows:

$$\text{NPV of Transfers} = \underbrace{\sum_j \int_{\varepsilon,\tilde{a}} \tilde{T}_{j,1995}(\varepsilon,\tilde{a})\, d\widetilde{\Phi}_{j,1995}(\varepsilon,\tilde{a})}_{\text{Transfers to the living households in 1995}}$$
$$+ \underbrace{\sum_{\tau=1996}^{\infty} \prod_{s=1995}^{\tau} \frac{1}{1+(1-\tau_{ks})r_s^b} \int_{\varepsilon,\tilde{a}} \tilde{T}_{1,\tau}(\varepsilon,\tilde{a})\, d\widetilde{\Phi}_{1,\tau}(\varepsilon,\tilde{a})}_{\text{Transfers to the future generations}}.$$

The first term represents the total transfers to all households alive in 1995. The second term is the NPV of transfers to all households born after 1995, discounted by the government's borrowing cost, since this reflects how the policy's effects are redistributed across generations through the government budget.

The sign of the NPV reveals the possibility of Pareto improvement. A negative NPV suggests that the government could compensate all individuals for any losses and still have a net gain, implying that the additional compensation plan can make the aggressive investment policy Pareto improving. Conversely, a positive NPV indicates that additional resources would be needed to compensate all households, suggesting that a Pareto improving compensation plan is not feasible.



(a) Transfer to Living Households

(b) Transfer to Future Generations

Figure 11: Welfare Effects of Foreign Investment

Note: Welfare effects are measured by the required compensations from the government to households to make them as well off as in the counterfactual scenario of low foreign investment (Equation (10)). Positive values indicate that the household is worse off under the increasing foreign investment, while negative values indicate that the household is better off under the increasing foreign investment. Panel (a) shows the transfers to households alive in 1995 with the median income shock. Panel (b) shows the transfers for future generations who born after all households alive in 1995 have died.

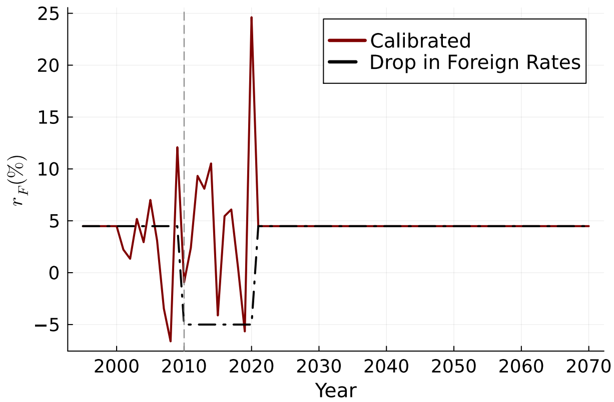
The required compensations are shown in Figure 11. Figure 11 (a) shows the required transfers for median-income households living in 1995. The results indicate that older or low-asset households benefit from the government's aggressive foreign investment policy, while high-asset or younger households experience losses. The primary reason for the losses among the initially living generation is the lower wage rate. The capital crowd-out effect caused by the government's increased foreign asset ratio between 1995 and 2020 means that this generation earns less in wages compared to the counterfactual scenario. Although the policy eventually raises wages in the long run, the initially living generation is already retired or nearing retirement by that time. Furthermore, the lower interest rates that follow retirement further reduce their lifetime welfare. Overall, 72% of households alive in 1995 experience welfare losses. In contrast, Figure 11 (b) shows that future generations generally benefit from the government's foreign investment. This is because they can enjoy higher wages as the government's strategy results in greater capital accumulation in the long run.

The net present value of transfers, calculated using the government's borrowing cost, is approximately 21% of the initial GDP. The positive indicates that the short-run cost borne by initially living households is not fully compensated by transfers funded by future generations. This finding highlights the impossibility of a self-financed, lump-sum transfer plan by the government that achieves a Pareto improvement with the increased foreign investment. Although the government's external investment can improve the utilitarian measure of social welfare, it cannot implement a transfer plan that makes all households better off under the policy change.

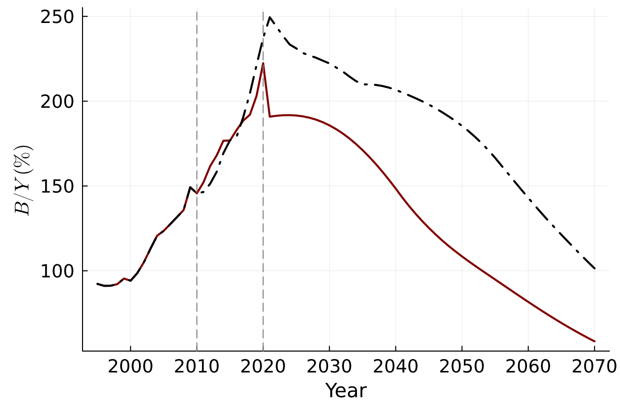
7 The Role of External Returns

7.1 An Unexpected Shock on External Returns

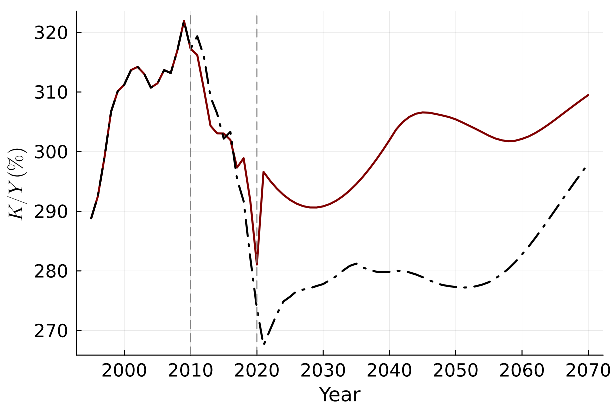
The main mechanism studied so far is the government's additional revenue from external investment, which reduces its need to issue bonds and thus mitigates the crowding-out effect on capital. However, external returns can move against the government, minimizing the extra revenue or even straining the fiscal space. To explore the effect of such unfavorable shocks, I simulate a counterfactual scenario where the government faces a series of unexpected negative shocks. Specifically, I assume that -5% external return is realized in 2010 and persists for 10 years. While the shock is unexpected, I assume that households form perfect foresight over the path of prices after the shock.



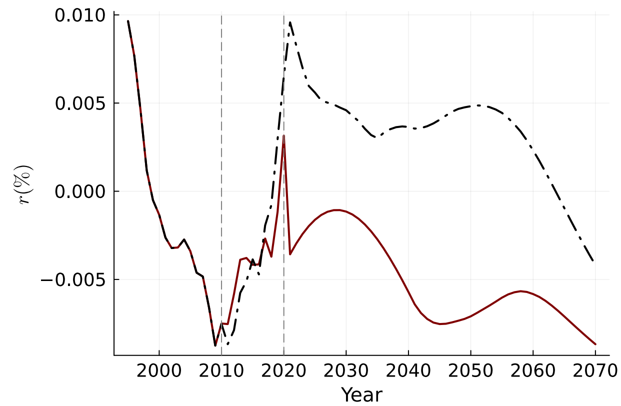
(a) External Return



(b) Debt to GDP Ratio



(c) Capital to GDP Ratio



(d) Interest Rate

Figure 12: The Effect of an Unexpected Negative Shock on External Returns

Note: The label "Calibrated" indicates the baseline scenario calibrated in Section 5.1. The label "Drop in Foreign Rates" indicates the counterfactual scenario where an unexpected negative shock of -5% on external returns is realized in 2010 and persists for 10 years. The shock is unexpected to households before 2010, but they form perfect foresight after the first realization of the shock.

The results are shown in Figure 12. Since the shock is unexpected, the transition path before the shock is the same as in the baseline case. After the negative shock to the government's balance sheet, the government increases its debt more sharply to finance its spending and losses from the foreign investment (Figure 12 (b)). That erodes the capital stock through the crowding-out effect, lowering the capital-to-GDP ratio (Figure 12 (c)). As a result, domestic rates rise sharply (Figure 12 (d)), and wage falls. Furthermore, the impacts of negative shock persist in the economy for a prolonged period, even after 60 years from the initial shock.

The results show that unexpected negative shocks are likely to have long-lasting adverse effects on the households' welfare. As the government's investment fails, more capital crowding-out occurs to balance the government's budget, leading to lower wages that shrinks households' income.

7.2 Uncertainty in External Returns

The perfect foresight of the external return r^* has been assumed so far. However, the external return is inherently stochastic, which can affect the risk-averse households' welfare. Specifically, as discussed in Section 7.1, the government needs to issue more debt to balance the budget upon a negative shock on the external return, which drives down the wages. Households forming rational expectations anticipate such fiscal responses to the stochastic foreign return, and expect more risk in their future income as the government pursue foreign investment strategy more aggressively. Risk-averse households may not favor the additional risk from the government's foreign investment strategy.

To see if the welfare improvement result in Section 6.3 is robust to the risky nature of the foreign investment, I explore a rational expectation equilibrium where households take into account the uncertainty of the government's investment performance. For simplicity, I assume that the external return is stochastically realized either high or low, $r^*_t \in \{r^*_H, r^*_L\}$. The stochastic realization of the current external return r^*_t affects the evolution of the government's debt from date t to $t+1$. Combined with the total savings made by all households at date t, the crowding out effect of the government debt at $t+1$ determines the capital stock in $t+1$ and the interest rate in $t+1$. Households take into account the expected future interest rate with rational expectations, which implies that household's value function includes the current aggregate variables (r^*_t, K_t) and the current distribution of households Φ_t as state variables. Specifically, the household's value function is given by

$$v_{jt}(\varepsilon_t, a_t; r^*_t, K_t, \Phi_t) = \max_{c_t, a_{t+1}} \left[c_t^{\rho} + \beta\psi_{jt} \left(\mathbb{E}_{(\varepsilon_{t+1}, r^*_{t+1})|(\varepsilon_t, r^*_t)} v_{j+1,t+1}(\varepsilon_{t+1}, a_{t+1}; r^*_{t+1}, K_{t+1}, \Phi_{t+1})^{1-\sigma} \right)^{\frac{\rho}{1-\sigma}} \right]^{\frac{1}{\rho}}$$

$$\begin{aligned} \text{subject to} \quad & (1+\tau_{ct})c_t + (1+\gamma_{t+1})a_{t+1} = \mathbf{1}_{j<J_r}(1-\tau_{lt})w_t\varepsilon_t + \mathbf{1}_{j\geq J_r}\theta_t w_t + (1+(1-\tau_{kt})r_t)\, a_t, \\ & a_{t+1} \geq \underline{a}, \\ & K_{t+1} = \Gamma^K_t(r^*_t, K_t, \Phi_t), \\ & \Phi_{t+1} = \Gamma^{\Phi}_t(r^*_t, K_t, \Phi_t). \end{aligned}$$

Γ^K_t and Γ^{Φ}_t denote the true law of motion of the aggregate capital and the aggregate distribution respectively. Notice that households are facing the uncertainty of the future external return, but they still perfectly foresight all the future policies and the economic environment, such as tax rates, pension replacement ratios, survival probabilities, and growth rates of labor-augmenting technology.

Following Krusell and Smith (1998), I approximate the distribution Φ_t by an aggregate moment A_t which denotes the total savings at the beginning of the time t, $\widehat{\Phi}_t = A_t$. The perceived laws of motion are parameterized as the following log-linear functions:

$$
\begin{aligned}
\log K_{t+1} =& \eta^H_{K1t} + \eta^H_{KKt} \log K_t + \eta^H_{KAt} \log A_t, \quad \text{if } r^*_t = r^*_H, \\
\log K_{t+1} =& \eta^L_{K1t} + \eta^L_{KKt} \log K_t + \eta^L_{KAt} \log A_t, \quad \text{if } r^*_t = r^*_L, \\
\log A_{t+1} =& \eta^H_{A1t} + \eta^H_{AKt} \log K_t + \eta^H_{AAt} \log A_t, \quad \text{if } r^*_t = r^*_H, \\
\log A_{t+1} =& \eta^L_{A1t} + \eta^L_{AKt} \log K_t + \eta^L_{AAt} \log A_t, \quad \text{if } r^*_t = r^*_L.
\end{aligned}
$$

To maintain consistency with the previous analysis, I should solve the transition dynamics using the Krusell-Smith algorithm. However, I focus only on the long-run equilibria realized under different values of the government's foreign investment policy (ξ) in this chapter. As the previous analysis suggested, the government's foreign investment accumulation incurs costs in the short run and benefits in the long run. Therefore, while the analysis presented here is limited, it is a reasonable comparison to show how the long-run benefit of foreign investment varies with government policy.

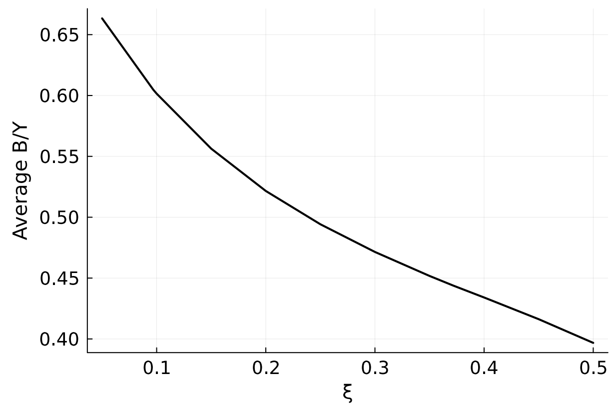
I calibrate all parameters except the government's foreign investment to debt ratio to 1995 historical values as in Section 5.1. To calibrate the stochastic process of the external return, I set $r^*_H > 0$ and $r^*_L < 0$. The value of r^*_H is calibrated to the average return of GPIF Japan conditional on being positive. Similarly, the value of r^*_L is calibrated to the average return conditional on being negative. The transition probability of the external return is computed from the historical data of GPIF's returns. The parameters for the stochastic process of foreign returns are presented in Table 5. The long-run average of the foreign return in simulation is positive (4.041%).

Table 5: Parameters for Krusell-Smith Economy

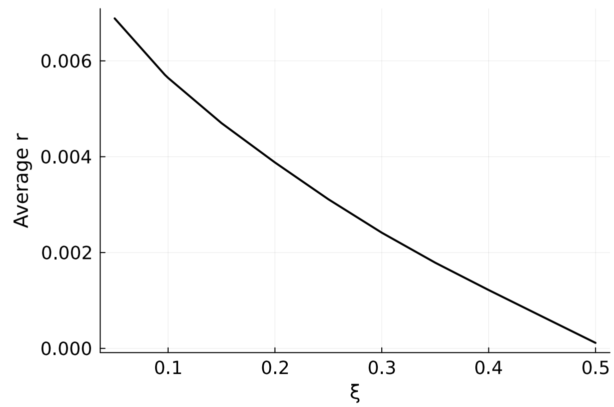
Parameter	Description	Value	Source
r^*_H	High external return	7.306%	GPIF historical returns conditional on being positive
r^*_L	Low external return	-3.506%	GPIF historical returns conditional on being negative
p_{HH}	$\Pr(r^*_{t+1} = r^*_H \mid r^*_t = r^*_H)$	0.6471	GPIF historical returns
p_{LL}	$\Pr(r^*_{t+1} = r^*_L \mid r^*_t = r^*_L)$	0.1667	GPIF historical returns
$\bar{r}^*$	Long-run average external return	4.041 %	Simulation results

Notes: All other parameters are calibrated to 1995 historical levels as in Section 5.1.

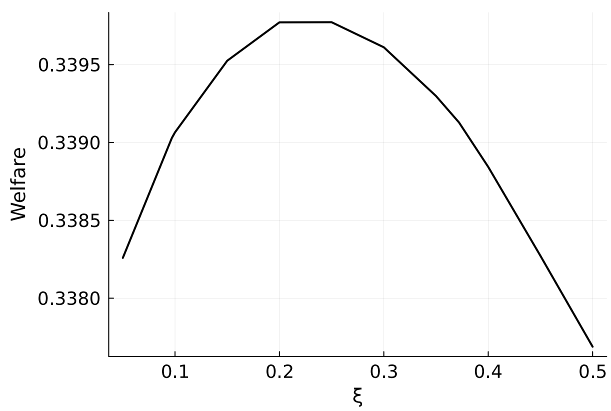
Figure 13 compares the long-run equilibria across different values of the government's foreign investment policy. As the foreign investment incurs positive returns on average in the long-run, a more intensive foreign investment policy leads to a lower average debt-to-GDP ratio. A lower debt-to-GDP ratio is associated with a lower capital crowding out effect and a higher capital-to-GDP ratio, implying a lower average domestic interest rate. Overall, the results suggest that even tough the foreign investment strategy introduces more risk to the economy, the ex-ante social welfare is still improving if the government engages in foreign investment thant not at all.



(a) Average Debt to GDP Ratio



(b) Average Domestic Interest Rate



(c) Social Welfare

Figure 13: Long-run Effect of Foreign Investment with Stochastic Returns

8 Conclusion

I developed a quantitative macroeconomics model to analyze the effects of government's foreign asset investment strategy.

The model shows that aggressive foreign investment or higher returns on external assets can contribute to long-term fiscal sustainability. In particular, the government can run permanent primary deficit while maintaining the debt level finite if it invests sufficient share of assets in high-yield foreign assets. Furthermore, if the return on foreign investment exceeds the government's borrowing cost, the debt-to-GDP ratio can be managed to converge to the initial level stably following a shock to the primary deficit.

Using a version of the model calibrated to the Japanese economy between 1995 and 2020, I conducted a counterfactual analysis. The analysis shows a stark inter-generational trade-off associated with the government's foreign asset investment strategy. While the strategy increased the capital stock and wages in the long run, it also crowded out private capital in the short run, leading to lower wages for households living during the transition period. However, the benefits of future generations outweighed the costs born by the initially living households, resulting in an overall improvement in utilitarian social welfare. The welfare improvement is robust even when considering the uncertainty in external returns.

The results suggest that government foreign asset investment strategies can play a crucial role in enhancing fiscal sustainability and inter-generational welfare, suggesting that such strategies can be considered seriously in aging societies facing fiscal challenges.

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A Model Timeline

Within date t, decisions are made in following order.

1. The government holds $\tilde{B}_t$ out of the beginning of the period total savings as predetermined debt. The remaining savings are supplied to the production sector as capital.

2. The production sector uses capital and labor to produce. Interests and wages are paid to households. The government also pays interest on its debt.

3. Households receive pension benefits, after-tax labor income and after-tax returns on savings. Households make saving decisions.

4. The governments borrows $\tilde{B}_{t+1}$ from households.

5. Mortality risk is realized. New households are born, and the accidental bequests of the deceased households are equally distributed to new households.

6. Moving on to date $t+1$.

B Detrended Model

Household The household's detrended value function $v_t = \tilde{v}_t / \prod_{s=1}^{t}(1+\gamma_s)$ satisfies the following Bellman equation:

$$
\begin{aligned}
v_{jt}(\varepsilon_t, a_t) = \max_{c_t, a_{t+1}} & \left[c_t^{\rho} + \beta_t \psi_{jt} \left(\mathbb{E}_{\varepsilon_{t+1}|\varepsilon_t} v_{j+1,t+1}(\varepsilon_{t+1}, a_{t+1})^{1-\sigma} \right)^{\frac{\rho}{1-\sigma}} \right]^{\frac{1}{\rho}} \\
\text{subject to} \quad & (1+\tau_{ct})c_t + (1+\gamma_{t+1})a_{t+1} = \mathbf{1}_{j<J_r}(1-\tau_{lt})w_t\varepsilon_t + \mathbf{1}_{j\geq J_r}\theta_t w_t + (1+r_t(1-\tau_{kt}))a_t, \\
& a_{t+1} \geq \underline{a}, \quad \underline{a} \leq 0.
\end{aligned}
$$

where $\beta_t = \tilde{\beta}(1+\gamma_{t+1})^{\rho}$.

Production Prices are determined by the capital-to-labor ratio,

$$
r_t = \alpha \left(\frac{K_t}{L_t} \right)^{\alpha-1} - \delta - \varphi_t, \qquad w_t = (1-\alpha) \left(\frac{K_t}{L_t} \right)^{\alpha}.
$$

Government The detrended government's budget constraint is

$$(1+g_{t+1})(1-\xi_{t+1})B_{t+1} = G_t - \tau_{ct}C_t - \tau_{kt}r_tK_t + (\theta_t\zeta_t - \tau_{lt}L_t)w_t + \left[1 + r_{bt} - \xi_t(1+r_t^*)\right] B_t$$

where $(1+g_{t+1}) = (1+\gamma_{t+1})(1+n_{t+1})$ is the sum of technology growth rate and population growth rate which equals to the growth rate of GDP in the steady state, and $r_{bt} = (1 - \tau_{kt} \cdot \mathbf{1}_{r_t \geq 0})r_t$ is the government's borrowing cost.

Market Clearing Conditions Detrended market clearing conditions are as follows.

$$\begin{aligned}
K_t + B_t &= \sum_j \int_{\varepsilon,a} a \; d\Phi_{jt}(\varepsilon, a) \\
L_t &= \frac{\sum_{j=1}^{J_r-1} \int_{\varepsilon,a} \varepsilon \; d\Phi_{jt}(\varepsilon, a)}{\sum_{j=1}^{J_r-1} \int_{\varepsilon,a} d\Phi_{jt}(\varepsilon, a)} \\
K_t^{\alpha} L_t^{1-\alpha} + (1-\delta)K_t + G_t &= \sum_j \int_{\varepsilon,a} c_t(\varepsilon_t, a_t, j) \; d\Phi_{jt}(\varepsilon, a) + (1+g_{t+1})K_{t+1}
\end{aligned}$$

Population Measure The law of motion of the detrended population measure is, for $E \subset \mathcal{E}, A \subset \mathcal{A}$ and $1 \leq j \leq \bar{J} - 1$,

$$\Phi_{j+1,t+1}(E, A) = \int_{\varepsilon,a} P_t(\varepsilon, a, j, E, A, j+1) \; d\Phi_{jt}(\varepsilon, a)$$

where

$$P_t(\varepsilon, a, j, E, A, j+1) = \begin{cases} \frac{\Pr(\varepsilon_{t+1} \in E | \varepsilon_t = \varepsilon)\psi_{jt}}{1+n_{t+1}} & \text{if } a \in A, \\ 0 & \text{otherwise.} \end{cases}$$

The initial endowment for newborn households at date $t+1$ is

$$a_{t+1}^i = \frac{\sum_{j=1}^{\bar{J}} (1-\psi_{jt}) \int_{\varepsilon,a} a'(\varepsilon, a, j) \; d\Phi_{jt}(\varepsilon, a)}{\omega_t \sum_{j=1}^{J_r-1} \Phi_{jt}(\mathcal{E}, \mathcal{A})}.$$

The newborn households at date $t+1$ is given by

$$d\Phi_{1,t+1}\left(\varepsilon, \{a_{t+1}^i\}\right) = \pi_\varepsilon \Phi_{1,t+1}\left(\mathcal{E}, \{a_{t+1}^i\}\right) = \pi_\varepsilon \omega_t \sum_{j=1}^{J_r-1} \Phi_{jt}\left(\mathcal{E}, \mathcal{A}\right).$$

C Data and Definitions

This appendix describes the data sources and the detailed assumptions made in data cleaning.

Fertility Rate Age-specific population data is sourced from the UN Population database, available from 1950 to 2024. The model's fertility rate is defined as the number of new working-age households joining in the following period per working-age household in the current period:

$$\omega_t = \frac{\text{Number of New Working-age Household}_{t+1}}{\text{Number of Working-age Household}_t}$$

Since working age is defined as between 20 and 64, the numerator is proxied by the population aged 20 in the next year. Similarly, the denominator is proxied by the population aged 20 to 64 in the current year. This provides the ex-post fertility rate from the data. While the model's fertility rate is intended to be the ex-ante fertility rate (the expected fertility rate at the beginning of the period), I assume that the ex-post fertility rate equals the ex-ante fertility rate.

The model's definition of the fertility rate differs from the standard definition of the Total Fertility Rate (TFR), which is the expected number of births a woman would have in her lifetime. It is important to note how to convert between the model's fertility rate and TFR. The model's fertility rate is the expected number of births a working-age household would have in a year. Given that the working age is 45 years in the model and that the model does not differentiate by sex, the following equation can be used to convert the model's fertility rate to TFR:

$$\text{Implied TFR} = \omega \times 45 \times 2.$$

If working-age households do not die, a TFR greater than 2 is required to maintain the working-age population. A TFR of 2 corresponds to the model's fertility rate of $1/45 \approx 0.0222$.

Survival Probability Age-specific mortality data is available from the UN Population database, ranging from 1950 to 2024. While detailed age-by-age survival probability data are available, I use a simpler parameterization. Specifically, it is assumed that all workers survive every year with a probability of 1, and households begin to die after retirement.

I use the data survival probability for each age for 1996 to 2020, but calibrate the age-specific survival probability in 1995 to replicate the initial steady state. The calibration is clarified in the next section.

Capital-to-GDP Ratio and Technology Growth Rate I calculate the capital-to-GDP ratio and the growth rate of labor-augmenting technology using Japan's national account data from Penn World Table 10.01, available from 1950 to 2019. The capital stock is calculated using the perpetual inventory method, with a depreciation rate consistent with that assumed in the calibrated model. The same capital share as in the calibrated model is used for Solow accounting.

Consumption Tax Rate The Japanese government first implemented a consumption tax in 1989 at a rate of 3%, which gradually increased to 10% by 2019. This tax became a significant source of government revenue, utilized to manage fiscal expansion driven by demographic shifts. I use the historical statutory consumption tax rate in the calibrated model.

Labor Income Tax Rate The labor income tax in the model represents the total wedge between the marginal product of labor and the household's disposable labor income. In this sense, the model's labor income tax should include not only the household's labor income tax but also the employer's social security contribution and the employee's payroll tax. I source the total tax wedge data from the OECD Taxing Wages database, specifically the sum of Employee Social Security Contributions and Employer Social Security Contributions for a single household, with no children, earning 100% of the average income. The data is available from 2000 to 2023.

Government-Spending-to-GDP Ratio General Government's Final Consumption Expenditure as a share of GDP is sourced from World Bank National Account data. The data covers the entire simulation range, from 1995 to 2020.

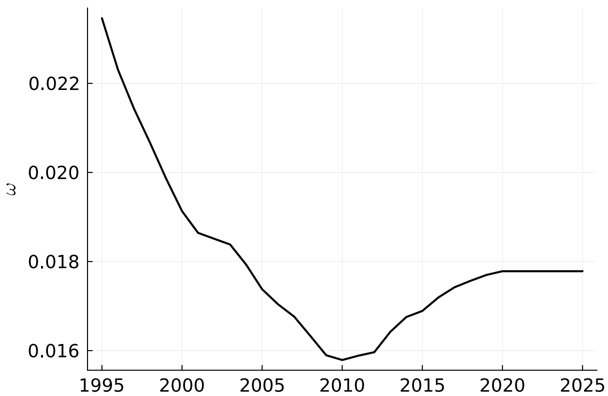
Government Debt-to-GDP Ratio I consider the general government debt, which includes the central government, local governments, the central bank, and social security funds.

1-Year Government Bond Yield I use the 1-year government bond real yield as a proxy for the interest rate in the model. The real yield is calculated using the CPI inflation rate from the Bank of Japan. One-week frequency data is used to calculate the annual average of the real yield.

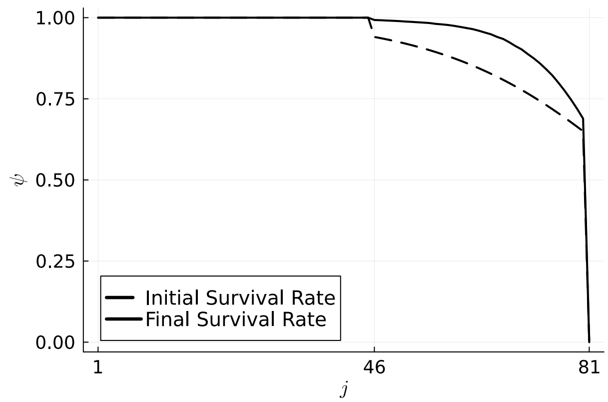
Share of Foreign Investment in Government Portfolio I identify the share of foreign investment in the government portfolio using the Flow of Funds data from the Bank of Japan. The data is available from 1997 to 2020. The share of foreign investment in the government portfolio is defined as the ratio of the total amount of Foreign Securities divided by the total amount of assets.

The total value of foreign assets can be affected by the exchange rate. The data manual for the Flow of Funds data states that for foreign currency-denominated deposits and loans, exchange gains and losses arising from conversions to yen should be recorded as reconciliation amounts. However, due to limitations in source data, it is difficult to identify all exchange gains and losses. Therefore, term-on-term differences in outstanding amounts after converting to yen are generally estimated as financial transaction flows.

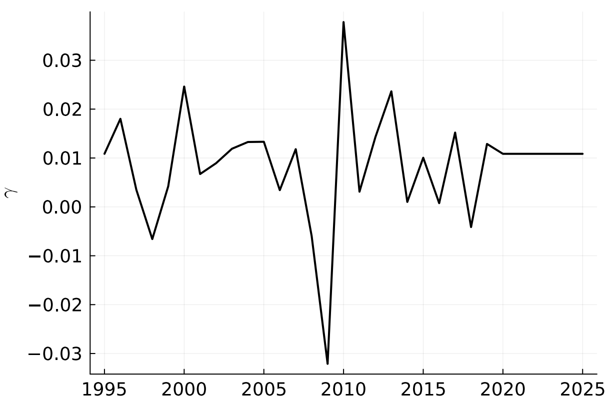
Return on Foreign Assets The Government Pension Investment Fund (GPIF) implements a diversified investment strategy, allocating its assets across domestic and foreign equities, domestic and foreign bonds, and alternative investments. GPIF reports the annual real return of its portfolio. I use the ex-post annual performance of GPIF's portfolio as the return on foreign assets in the model. The data is available from 2001 to 2020.



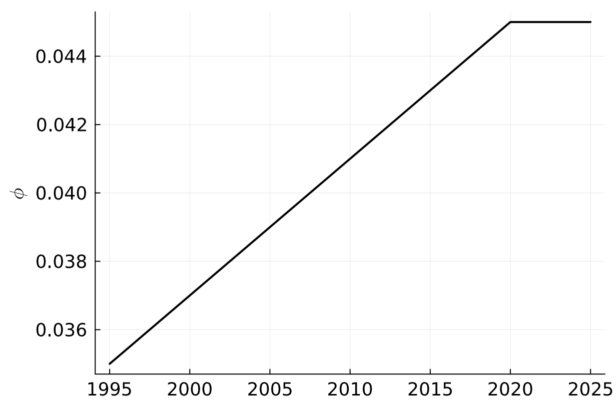
(a) Fertility Rate



(b) Age-specific Survival Probability

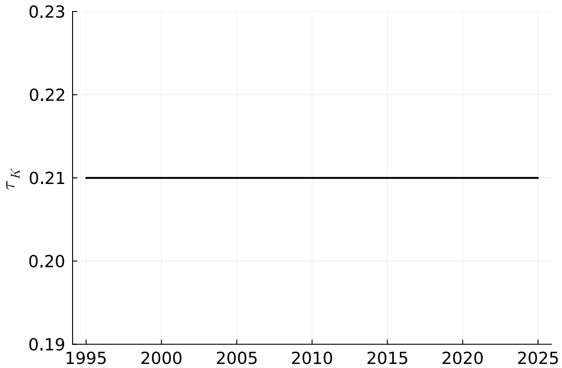


(c) TFP Growth Rate

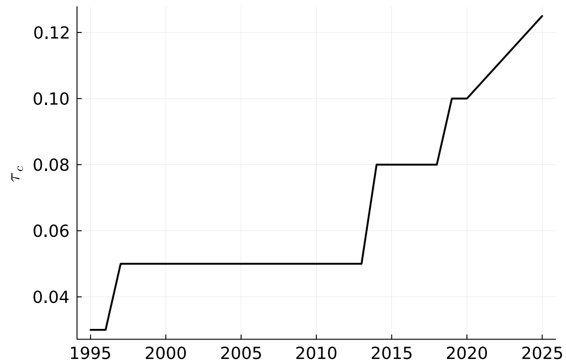


(d) Default Probability

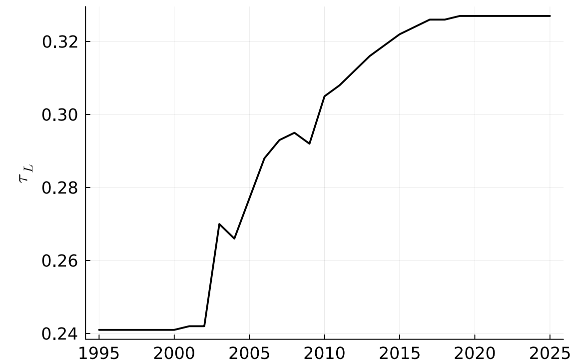
Figure 14: Demographic and Production Parameters in the Baseline Scenario



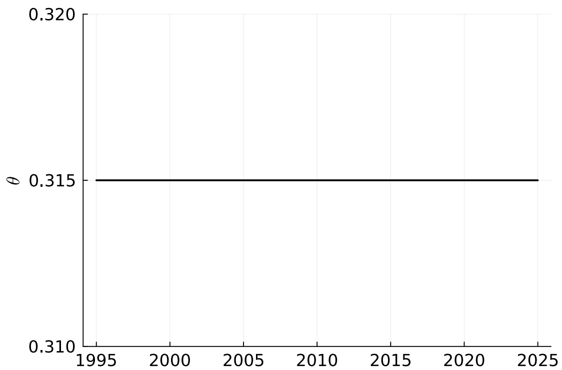
(a) Capital Income Tax Rate



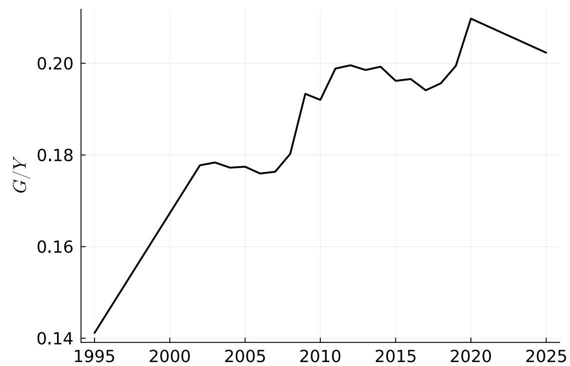
(b) Consumption Tax Rate



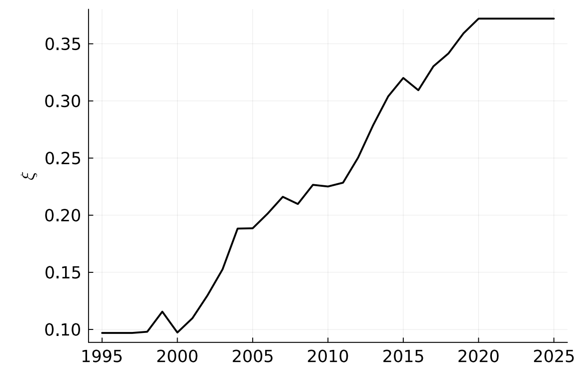
(c) Labor Income Tax Wedge



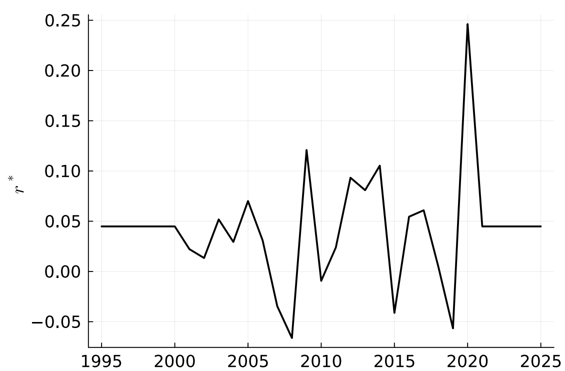
(d) Pension Replacement Ratio



(e) Government Spending to GDP Ratio



(f) Share of Foreign Assets



(g) Returns on Foreign Assets

Figure 15: Government Parameters in the Baseline Scenario

D Numerical Solution

The household's problem is solved using the endogenous grid method (EGM), as in Carroll (2006). This method leverages the first-order condition of the household's problem, which is presented as:

$$\frac{c_t^{\rho-1}}{1+\tau_{ct}} = \beta\psi_j \frac{1+r_{b,t+1}}{1+\gamma_{t+1}} \mathbb{E}_{\varepsilon'|\varepsilon}\left[\left(\frac{V_{t+1}}{\left(\mathbb{E}_t V_{t+1}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}} \right)^{1-\sigma-\rho} \frac{c_{t+1}^{\rho-1}}{1+\tau_{ct+1}} \right].$$

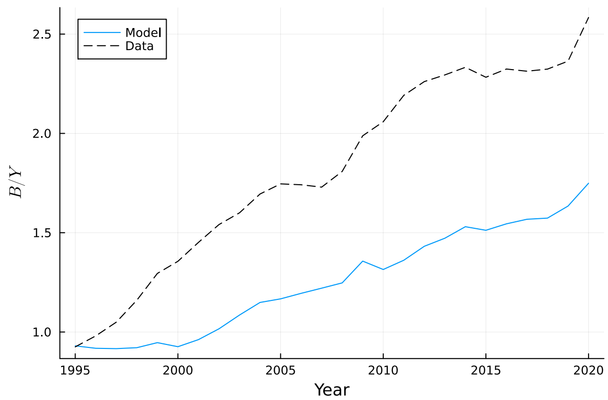
For all quantitative exercises, the same grids are used to discretize the household's state space. The state space is comprised of 12 points for ε, 2001 points of a, and 81 points of j. Given the high assumed value for relative risk aversion, $\sigma = 6$, it is crucial to use a sufficiently fine discretization for the asset holdings, a. A coarse grid for assets can introduce numerical errors, potentially overstating the variance of future cash-on-hand and consumption, which would lead to an exaggerated response in the household's precautionary savings.

E Alternative Fiscal Reforms after 2020

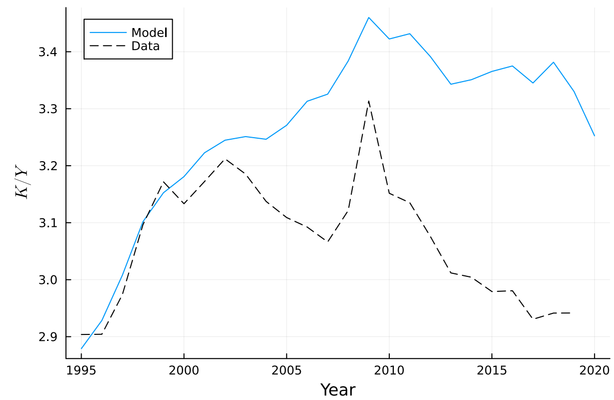
I additionally analyze the model's transition under alternative fiscal reform scenarios. Rather than cutting the government's spending-to-GDP ratio, the first alternative scenario reduces the pension replacement ratio from 32% to 23% in following 10 years after 2020. The reduced 23% replacement ratio is the maximum value that ensures the existence of a sustainable deficit steady state, given other parameters remain unchanged at the 2020 values. Results are shown in Figure 16. The second alternative scenario increases the consumption tax rate from 10% to 21% in following 10 years after 2020. Similarly, 21% consumption tax rate is the minimum value that ensures the existence of a sustainable deficit steady state. Results are shown in Figure 17.

F Decomposition of Demographic Effects

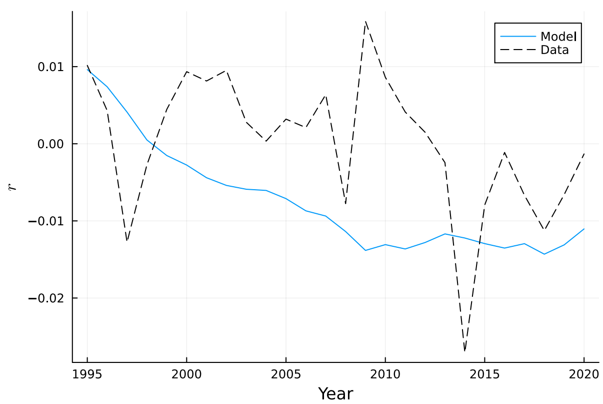
To assess the impact of population aging on debt accumulation, I differentiate between the two sources of population aging – extended life expectancy and decreased fertility rates –



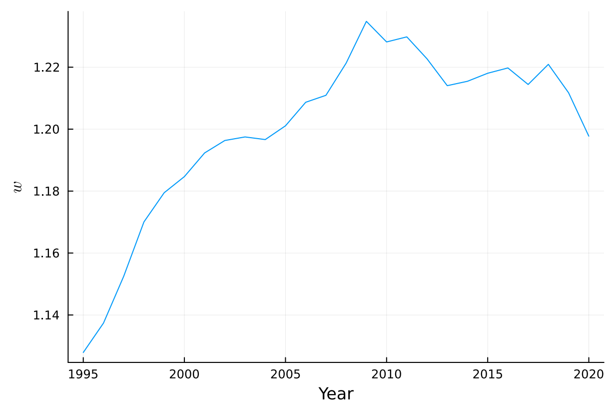
(a) Debt to GDP Ratio



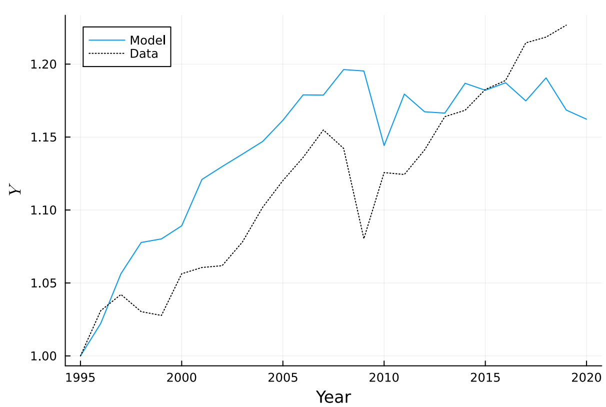
(b) Capital to GDP Ratio



(c) Interest Rate



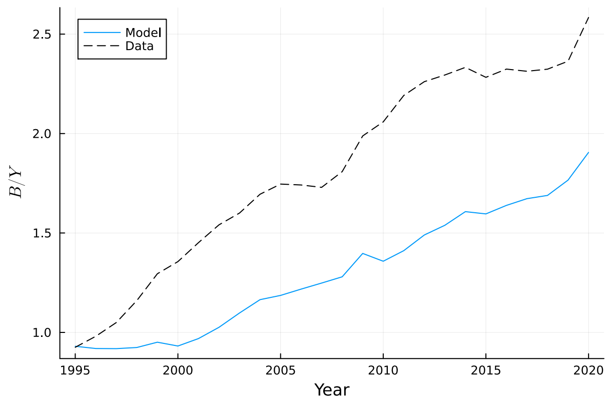
(d) Wage Rate



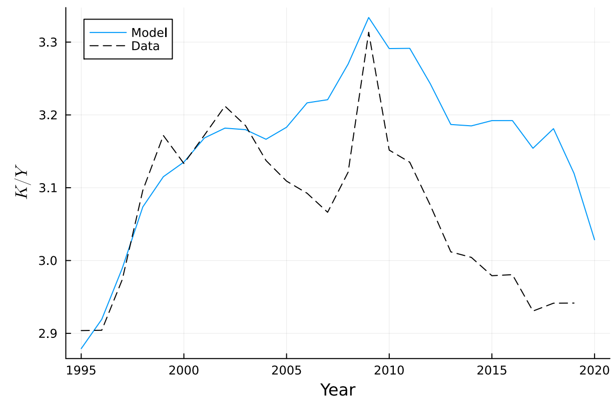
(e) GDP

Figure 16: Model Transition under Pension Reforms

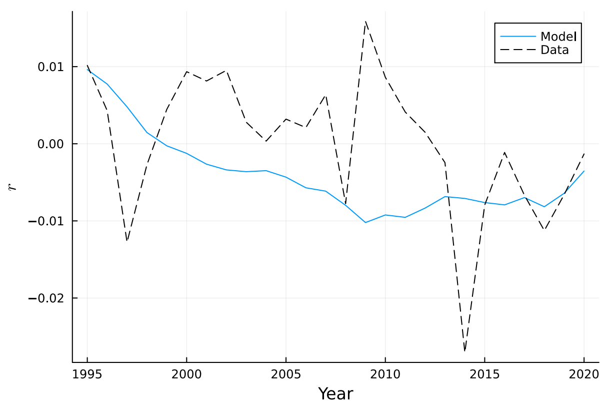
to identify which demographic factor contributes more significantly to government debt run up.



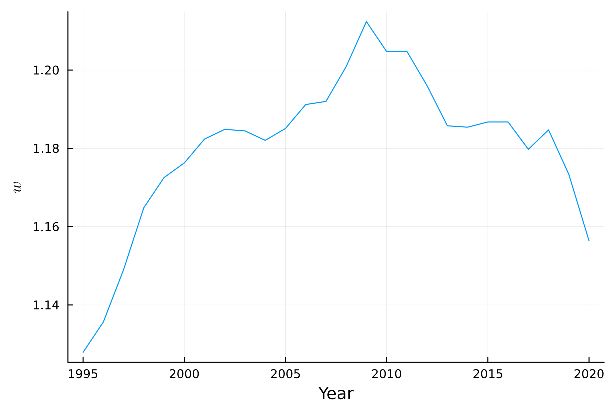
(a) Debt to GDP Ratio



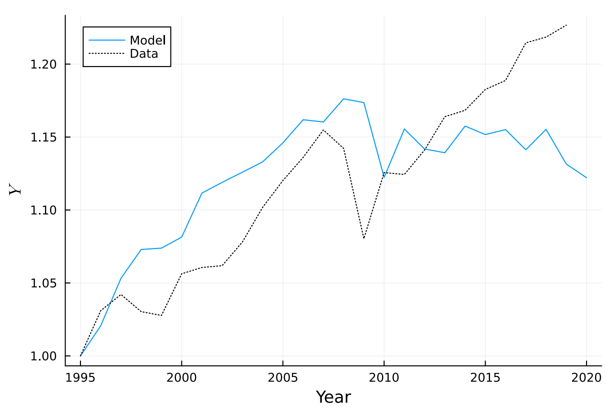
(b) Capital to GDP Ratio



(c) Interest Rate



(d) Wage Rate



(e) GDP

Figure 17: Model Transition under VAT Reforms

F.1 Effect of Lower Fertility Rate

As discussed in Section C, the threshold fertility rate that the working-age population does not decline is $1/45 \approx 0.0222$ ignoring the working-age population's mortality rate.

As shown in Figure 14 (a), the fertility rate in Japan used to be above this threshold level in 1995, but it had declined steeply to the lowest in 2010 which was below the threshold. In following 10 years Japan's fertility rate has been recovering, but it is still below the threshold level. This declining trend of fertility rate has been thought of as one of the main driver of population aging that pressures the government's fiscal sustainability. In the second counterfactual scenario, I assume that the fertility rate remains constant at its initial level ($\omega_t = \omega_{1995}$) throughout the transition period. This scenario allows for an analysis of the impact of lower fertility rates on fiscal space. The results are in Table 4 and Figure 18.

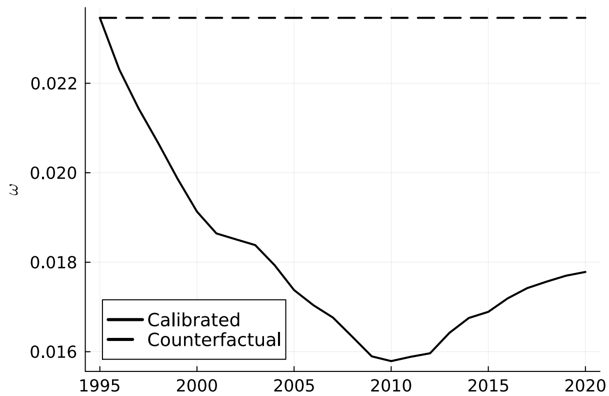
The effect of an undiminished fertility rate in the counterfactual scenario is immediately evident in the dependency ratio. In the baseline scenario, the dependency ratio surpasses 0.50 in 2020 and continues to rise above 0.70 thereafter. In contrast, the counterfactual scenario shows a dependency ratio of 0.4381 in 2020, converging to 0.4638 in the long run. This is underpinned by a continuously increasing working-age population in the counterfactual scenario, resulting from a fertility rate above the threshold fertility rate $1/45$.

The more abundant workforce in the counterfactual scenario leads to lower wage rates compared to the baseline scenario. Specifically, in the short term (1995-2020), the counterfactual scenario exhibits lower wage rates and higher interest rates. Due to these higher interest rates, the increase in the debt-to-GDP ratio does not significantly differ from the baseline scenario, even with a low dependency ratio that prevents a substantial rise in pension liability. As shown in Table 4, the baseline explains a 121.11%p increase in debt-to-GDP, whereas the counterfactual explains a 93.95%p increase. This suggests that the lower birth rate did not significantly influence the increase in the Japanese government's debt between 1995 and 2020.

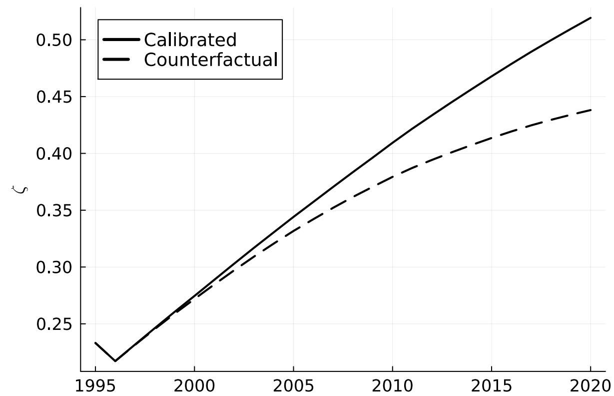
However, in the long term, a low birth rate is expected to significantly impact government debt. Projections for 2120 indicate that if the birth rate were to remain high, the government would be expected to have net assets rather than debt in the long run. Consequently, the Capital-to-GDP ratio and household wage income are also projected to be higher under the counterfactual scenario compared to the baseline.

F.2 Effect of Extended Life Expectancy

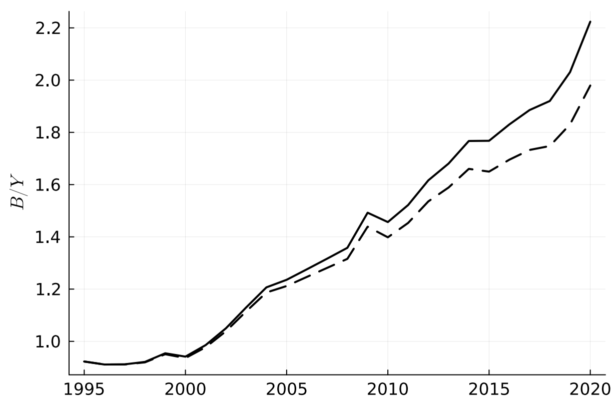
The third counterfactual scenario assumes that age-specific survival rates remain at 1995 levels. Between 1995 and 2020, Japan's life expectancy increased due to a continuous rise



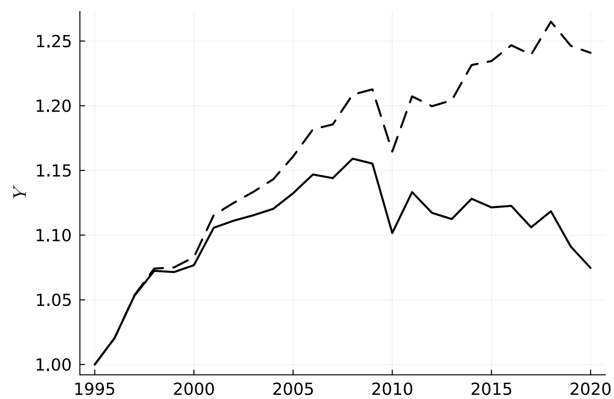
(a) Fertility Rate



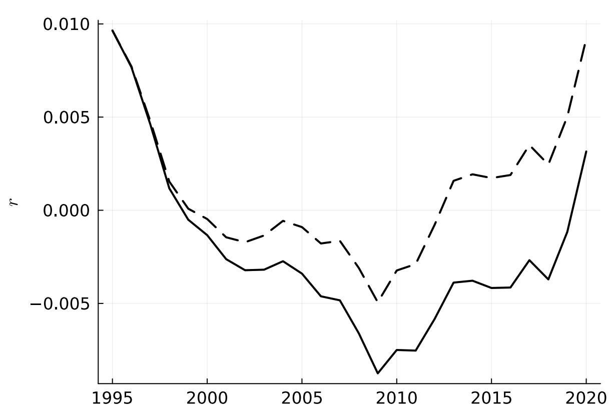
(b) Dependency Ratio



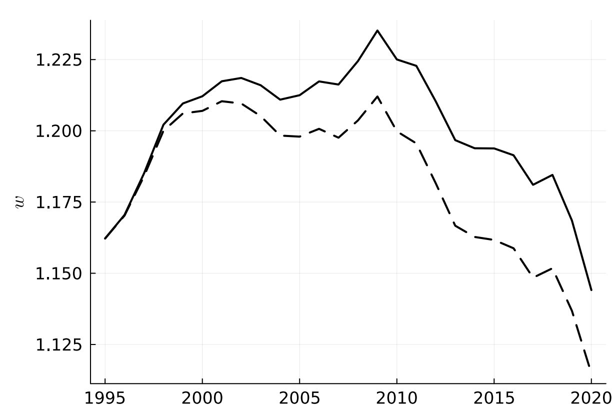
(c) Debt to GDP Ratio



(d) GDP



(e) Interest Rate



(f) Wage Rate

Figure 18: Counterfactual Scenario: Constant Fertility Rate

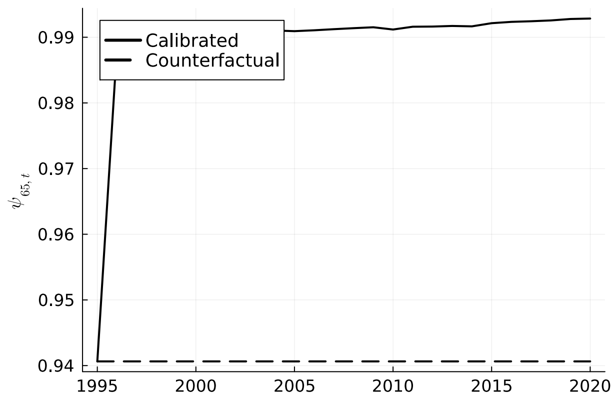
in age-specific survival rates. This is often thought of as a primary driver of population aging, with declining fertility rates. This counterfactual analysis illustrates the change in fiscal space resulting from increased life expectancy.

Similar to the scenario where the fertility rate remains high, the dependency ratio stays lower in the low-survival-rate scenario compared to the baseline scenario. A notable point is that the dependency ratio is even lower in the scenario without changes in survival rates compared to Section F.1. Table 4 shows that in 2020, the dependency ratio is 0.2574 in the low-survival-rate scenario, versus 0.4381 in the high-fertility-rate scenario. This suggests that the increase in survival rates played a greater role than the decline in fertility rates in population aging.

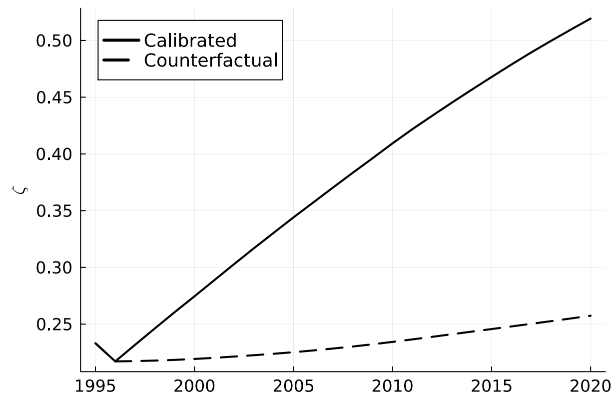
In the low-survival-rate scenario, a lower dependency ratio results in a smaller increase in government debt in the short run (1995-2020). It is calculated that the debt-to-GDP ratio would increase by only 23.98%p during 1995-2020 in the low-survival-rate scenario. This implies that a significant portion of the 121.11%p debt-to-GDP increase calculated in the baseline scenario originates from extended life expectancy.

Similar to the high-fertility-rate scenario, the relatively abundant labor force in the low-survival-rate scenario leads to lower wage rates and higher interest rates in the short term compared to the baseline. However, in the long term, as the government maintains a primary surplus and shifts from net debt to net asset position, it projects higher GDP and higher wage income.

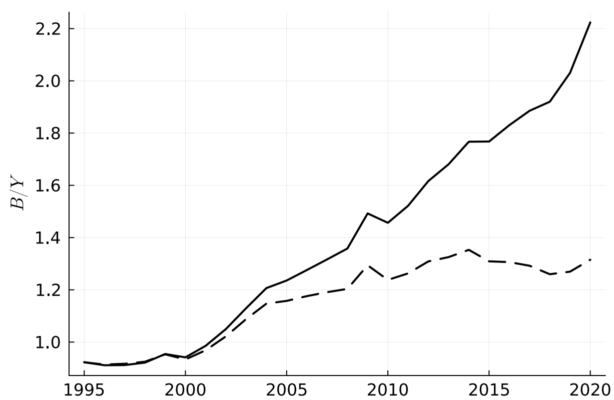
G Robustness



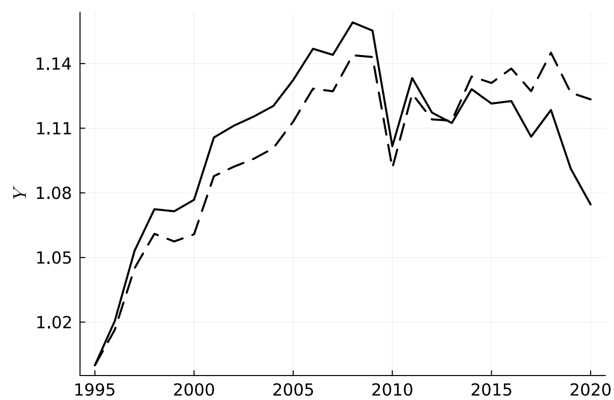
(a) Survival Probability of Age 65



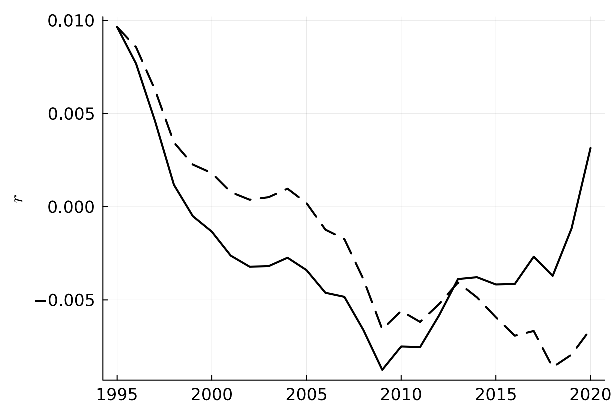
(b) Dependency Ratio



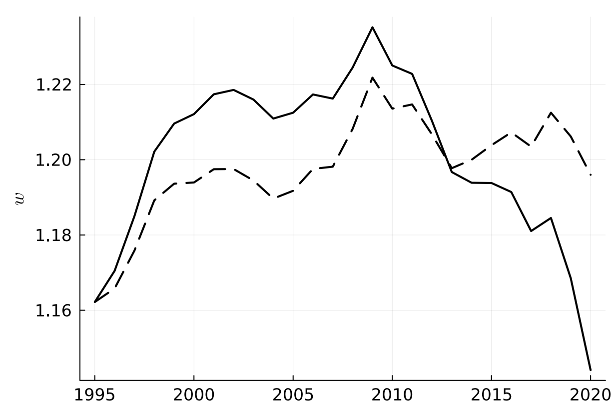
(c) Debt to GDP Ratio



(d) GDP



(e) Interest Rate



(f) Wage Rate

Figure 19: Counterfactual Scenario: Constant Survival Probability