Granular Search in Monopsonistic Labor Market

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Modeling Monopsony Power in Labor Market

Modern monopsony approach

- Robinson (1933); Burdett and Mortensen (1998); Manning (2011)
- Finite elasticity of labor supply to individual firm

Granular search approach

- Jarosch, Nimczik and Sorkin (2024)
- DMP + finite number of firms $\longrightarrow$ possibility of future re-encounter
- "Outside option that's really outside"
- Market power based on the relative vacancy size
- Such size-based market power depresses wages by about 2.6% (1,500 euros annually)

Modeling Monopsony Power in Labor Market

Limitation of granular search approach

- Taking the firm size distribution as given, determine the wages.
- Cannot capture firms' decision to post vacancies
  - Specifically, how firms respond to competitors' actions
  - The market structure determines each firm's optimal strategy
- It limits the scope to study policy counterfactuals
  - Policies that accompany entry and exit of firms
  - Policy impact through the strategic interaction of firms cannot be studied

What We Do

1. Simple static model
   - Clarifies the incentive of vacancy postings in the presence of granular search
   - Endogenous firm size distribution
   - Mechanism of size-based wage suppression
2. Dynamic model with heterogeneous firms
   - Embed the granular search into firm decision problem, à la Cahuc, Marque and Wasmer (2008)
   - Hetero monopsonists with different productivity + the fringe firm
   - Define the market equilibrium
3. Counterfactual analyses (only roadmap today)
   - Minimum wage policy: the effect of the size of fringe firm
   - Competition policy (e.g., merger policy): # of firms / productivity distribution
   - Atomistic limit: measuring the effect of size-based market power

Simple Static Model

Simple Static Model

Setup

- Static, pseudo-dynamic model
- Two firms with productivity levels $\{z_i\}_{i=1,2}$ and a continuum of workers
- Workers search for a job
  - will be matched to a firm with exogenous probability f
  - the prob. of match with firm i is $f \times s_i$
  - receive wage w_i if matched to firm i
  - can remain unemployed, receive unemployment benefit b
  - claim his outside option as if he could continue searching in the future (pseudo-dynamic feature)

Simple Static Model

Setup

- Firms post vacancies and produce using labor
  - with CRS production function $y_i = z_i n_i$
  - with convex vacancy posting cost $\frac{c}{1+\gamma} v_i^{1+\gamma}$
  - each vacancy is matched with a worker with exogenous prob. q
- Firm i's profit:
  given competing firm's wage and vacancy postings $\{w_{-i}, v_{-i}\}$,

$$\pi_i = \max_{v_i} \; z_i n_i - \frac{c}{1+\gamma} v_i^{1+\gamma} - w_i(v_i) \, n_i$$

where $n_i = q \; v_i$.

Simple Static Model

Worker's Outside Option

- U_i: worker's value in case of trade breakdown with firm i
  - worker claims the value as if he could continue searching in the next period
  - (i) can be matched to firm $-i$ or (ii) remain unemployed

$$U_i = f s_{-i}\ \hat{w}_{-i} + (1 - f s_{-i})\ b$$

- E_i: worker's value when working for firm i

$$E_i = w_i$$

- Worker's surplus from match with firm i:

$$E_i - U_i = f\ s_{-i}\,(w_i - \hat{w}_{-i}) + \left(1 - f\ s_{-i}\right)(w_i - b)$$

Simple Static Model

Wage Function

- Firm i's surplus from the match with worker:

$$\Pi_i = z_i - w_i$$

- Nash bargaining with worker's share η:

$$w_i - b = \eta\Big(z_i - b\Big) + (1-\eta) f s_{-i}\Big(\hat{w}_{-i} - b\Big).$$

  - $z_i - b$: total surplus from the match
  - $f s_{-i}(\hat{w}_{-i} - b)$: worker's expected surplus from the competing firm in case of trade breakdown
  - s_{-i} is under firm i's choice in firm i's profit maximization problem

$$s_{-i} = \frac{\hat{v}_{-i}}{v_i + \hat{v}_{-i}}$$

Simple Static Model

Firm i's Best Response

Given the competing firm's strategy $\{\hat{w}_{-i}, \hat{v}_{-i}\}$, the foc of v_i is

$$q\left(z_i - w_i\right) - n_i \ \partial_{v_i} w_i = c v_i^{\gamma}$$

- $q\left(z_i - w_i\right)$: expected firm's surplus from additional match
- $-n_i \ \partial_{v_i} w_i$: savings on the total wage cost through wage suppression
- $c v_i^{\gamma}$: marginal cost of posting a vacancy

Simple Static Model

Firm i's Best Response

The benefit from wage suppression is

$$-n_i \ \partial_{v_i} w_i = (1-\eta)\left(\frac{n_i}{v}\right)\left(f s_{-i}(\hat{w}_{-i} - b)\right) > 0$$

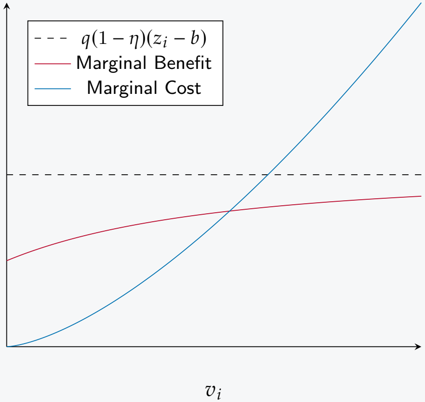
- $1-\eta$: firm's bargaining power
- n_i/v: firm i's size relative to the total vacancy size
  extensive margin of wage suppression
- $f s_{-i}(\hat{w}_{-i} - b)$: worker's expected surplus in case of trade breakdown
  intensive margin of wage suppression

Simple Static Model

Firm i's Best Response

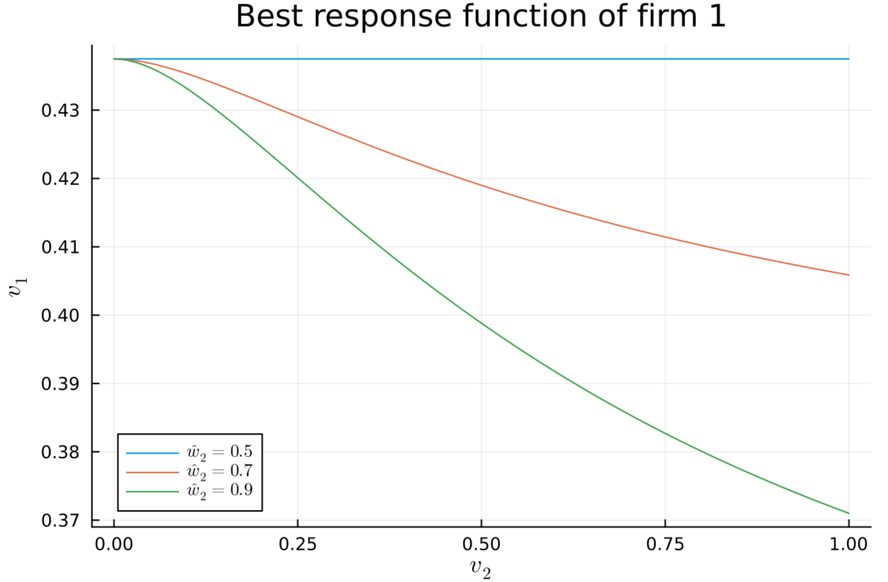
After substituting the wage term and the wage suppression term, the foc of v_i becomes

$$q(1-\eta)\left(z_i - b - f\left(\frac{\hat{v}_{-i}}{v_i + \hat{v}_{-i}}\right)^2 (\hat{w}_{-i} - b)\right) = c v_i^{\gamma}.$$



Simple Static Model

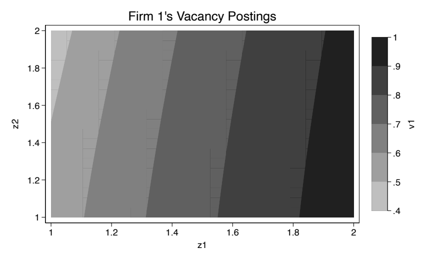
Firm i's Best Response

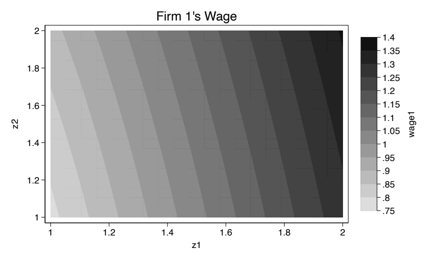


Simple Static Model

Equilibrium

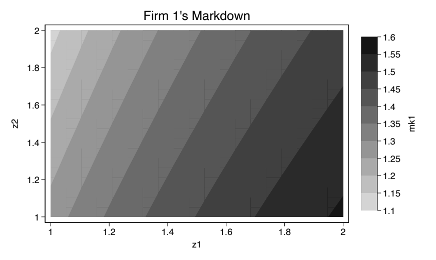
parameters	b	c	γ	η	q	f
values	0.5	0.4	2.0	0.5	0.5	0.5





Simple Static Model

Equilibrium



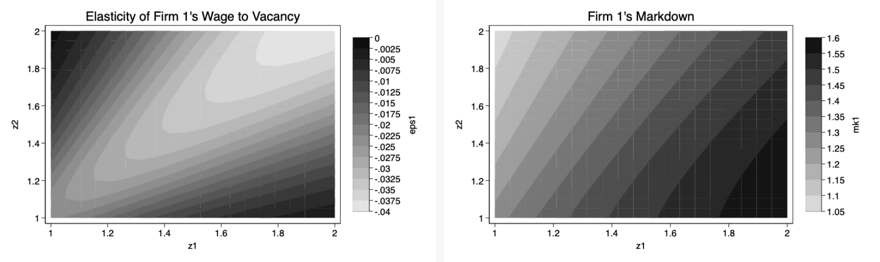
$$\varepsilon_{w,v} = \frac{z_i - \frac{cv_i^{\gamma}}{q}}{w_i} - 1$$

$$\mu_i = \frac{z_i}{w_i}$$

Simple Static Model

Equilibrium

With $\gamma = 0.5$



For $v_i < 1$, v_i^γ varies more in v_i when γ is smaller.

Dynamic Model with Heterogeneous Firms

Dynamic Model with Heterogeneous Firms

Environment

- The model is continuous in time
- N_m monopsonists with $\{z_i\}_{i=1,\cdots,N_m}$ and one fringe firm with $z_{N_m+1} = 1$
- Monopsonists can send threat of exclusion, but the fringe firm cannot
- Exogenous separation rate δ
- Market tightness $\theta \longrightarrow$ matching rate $q(\theta)$ and job-finding rate $f(\theta)$
- Worker meets firm i at rate of $f(\theta)s_i$ where $s_i = v_i/\sum_j v_j$

Dynamic Model with Heterogeneous Firms

Worker's Values

- The flow value of unemployed worker:

$$\rho U = b + f(\theta) \sum_{i=1}^{N_m+1} s_i (E_i - U)$$

- The flow value of worker working for firm i:

$$\rho E_i = w_i + \delta(U - E_i)$$

- The flow value of unemployed worker in case of trade breakdown with firm i:

$$\rho U_i = b + f(\theta) \sum_{j \neq i} s_j (E_j - U_i)$$

Dynamic Model with Heterogeneous Firms

Worker's Values

- The worker's instantaneous surplus from match with
  - a monopsonist $1 \le i \le N_m$ is $\rho(E_i - U_i)$
  - the fringe firm $i = N_m + 1$ is $\rho(E_i - U)$
- When worker is matched to a monopsonist,

$$\rho(E_i - U_i) = \rho \frac{\rho + f(\theta)}{\rho + f(\theta)(1 - s_i)} \frac{w_i - \rho U}{\rho + \delta}$$

- When worker is matched to the fringe firm,

$$\rho(E_i - U) = \rho \frac{w_i - \rho U}{\rho + \delta}$$

Dynamic Model with Heterogeneous Firms

Worker's Values

- Useful expression of the unemployed worker's flow value:

$$\rho U = \frac{(\rho + \delta)b + f(\theta) \sum_i s_i w_i}{\rho + \delta + f(\theta)}$$

Dynamic Model with Heterogeneous Firms

Fringe Firm

Given $\{w_{it}\}_{i=1}^{N_m}$ and $\{v_{it}\}_{i=1}^{N_m}$, the fringe firm does not internalize the effect of its vacancy postings on the wage.

$$V_f(n_{ft}) = \max_{v_{ft}} \frac{1}{1+\rho\Delta t} \left[\left\{ z_f n_{ft} - w_{ft} n_{ft} - C_f(v_{ft}) \right\} \Delta t + V_f(n_{ft+\Delta t}) \right]$$
$$\text{s.t.} \quad n_{ft+\Delta t} = (1-\delta\Delta t) n_{ft} + q(\theta_t) v_{ft} \Delta t$$

- CRS production function + convex vacancy posting cost
- Not internalizing: the fringe firm is non-strategic

Dynamic Model with Heterogeneous Firms

Fringe Firm

The fringe firm's steady state job creation condition is

$$q(\theta)\left(z_f - w_f\right) = (\rho + \delta)\ \partial_v C_f(v_f)$$

The fringe firm's surplus from a match is

$$\rho \partial_n V_f(n_f) = \rho \frac{z_f - w_f}{\rho + \delta}$$

Dynamic Model with Heterogeneous Firms

Fringe Firm's Wage

Nash bargaining rule:

$$\eta \ \rho \partial_n V_f(n_f) = (1-\eta) \ \rho (E_f - U)$$

Nash bargaining result:

$$\begin{aligned} w_f &= \eta z_f + (1-\eta) \ \rho U \\ &= \eta z_f + (1-\eta) \frac{(\rho+\delta) b + f(\theta) \sum_j s_j w_j}{\rho + \delta + f(\theta)} \end{aligned}$$

We write $w_f = w_f(\mathcal{S}, \mathcal{W}, \theta)$

$\mathcal{S}$ and $\mathcal{W}$ are vectors of shares and wages.

Dynamic Model with Heterogeneous Firms

Monopsonist

Given $\{w_{jt}, v_{jt}\}_{j \neq i}$, a monopsonist i solves

$$V_m(n_{it}) = \max_{v_{it}} \frac{1}{1 + \rho \Delta t} \left[\{ z_i F(n_{it}) - w_{it} n_{it} - C_m(v_{it}) \} \Delta t + V_m(n_{it+\Delta t}) \right]$$
$$\text{s.t.} \quad n_{it+\Delta t} = (1 - \delta \Delta t) n_{it} + q(\theta_t) v_{it} \Delta t$$

- DRS production function + convex vacancy posting cost
- Monopsonists internalize the effect of vacancy postings on the wage

Dynamic Model with Heterogeneous Firms

Monopsonist

Job creation condition in steady state:

$$q(\theta)\left(z_i\partial_n F(n_i) - w_i - n_i\partial_n w_i\right) = (\rho+\delta)\left(\partial_v C_m(v_i) + n_i\partial_v w_i\right)$$

The surplus from a match to a monopsonist is

$$\rho\partial_n V_m(n_i) = \rho\frac{z_i\partial_n F(n_i) - w_i - n_i\partial_n w_i}{\rho+\delta}$$

Dynamic Model with Heterogeneous Firms

Monopsonist's Wage

The same Nash bargaining rule, but with worker's surplus $\rho(E_i - U_i)$.

We get the following expression for the monopsonist's wage:

$$w^i = \Xi_0(s_i;\theta)\,\rho U + \Big(1 - \Xi_0(s_i;\theta)\Big)\Big(z_i \partial_n F(n_i) - (\partial_n w_i) n_i\Big), \qquad i = 1, 2, \cdots, N_m$$

with

$$\Xi_0(s_i;\theta) = \frac{\Big(1-\eta\Big)\Big(\rho + f(\theta)\Big)}{\Big(1-\eta\Big)\Big(\rho + f(\theta)\Big) + \eta\Big(\rho + f(\theta)(1 - s_i)\Big)} \geq 1 - \eta$$

$\partial_s \Xi_0(s_i;\theta) > 0.$

Dynamic Model with Heterogeneous Firms

Monopsonist's Wage

In the firm i's wage bargaining,

- ρU is not fixed
- the vector of shares $\mathcal{S} = (s_1, \cdots, s_{N_m+1})$ is not fixed

Collecting the terms, we have the following PDE for w_i.

$$w_i = \Xi_1(s_i;\theta)\frac{(\rho+\delta)b + f(\theta)\sum_{j\neq i} s_j w_j}{\rho+\delta+f(\theta)} + \Xi_2(s_i;\theta)\left(z_i\partial_n F(n_i) - (\partial_n w_i)n_i\right)$$

where

$$\Xi_1(s_i;\theta) = \frac{\Xi_0(s_i;\theta)}{\Omega(s_i;\theta)}, \quad \Xi_2(s_i;\theta) = \frac{1-\Xi_0(s_i;\theta)}{\Omega(s_i;\theta)}, \quad \Omega(s_i;\theta) = \frac{\rho+\delta+f(\theta)(1-s^i\Xi_0)}{\rho+\delta+f(\theta)} < 1$$

Dynamic Model with Heterogeneous Firms

Monopsonist's Wage

Assuming $F(n) = n^\alpha$ and imposing $\lim_{n_i \to 0} n_i w_i = 0$, we have closed-form wage function

$$w_i(n_i, v_i; \theta, \{w_j, v_j\}_{j \neq i}) = \Xi_1(s_i, \theta) \frac{(\rho + \delta) b + f(\theta) \sum_{j \neq i} s_j w_j}{\rho + \delta + f(\theta)} + \frac{\Xi_2(s_i, \theta)}{1 - (1 - \alpha) \Xi_2(s_i, \theta)} \alpha z_i (n_i)^{\alpha - 1}$$

or write

$$w_i(\mathcal{N}, \mathcal{S}, \mathcal{W}, \theta) = \Xi_1^i(\mathcal{S}, \theta) \frac{(\rho + \delta) b + f(\theta) \sum_{j \neq i} s_j w_j}{\rho + \delta + f(\theta)} + \frac{\Xi_2^i(\mathcal{S}, \theta)}{1 - (1 - \alpha) \Xi_2^i(\mathcal{S}, \theta)} \alpha z_i (n_i)^{\alpha - 1}$$

Dynamic Model with Heterogeneous Firms

Monopsonist's Wage

Firm i's additional vacancy posting shifts the distribution of shares $\mathcal{S}$ into a certain direction.

$$\partial_{v_i} s_j = \begin{cases} \frac{1-s_j}{v} & \text{if } j = i, \\ -\frac{s_j}{v} & \text{if } j \neq i. \end{cases}$$

If firm i posts $\Delta \approx 0$ additional vacancies, the new vacancy share vector $\mathcal{S}'$ is

$$\mathcal{S}' = \mathcal{S} + \frac{\Delta}{v}(e_i - \mathcal{S})$$

The first derivative of wage is

$$\partial_{v_i} w_i(\mathcal{N}, \mathcal{S}, \mathcal{W}, \theta) = \lim_{\Delta \to 0} \frac{w_i(\mathcal{N}, \mathcal{S}', \mathcal{W}, \theta) - w_i(\mathcal{N}, \mathcal{S}, \mathcal{W}, \theta)}{\Delta}$$

Dynamic Model with Heterogeneous Firms

Labor Market Equilibrium

In the steady state equilibrium,

- all firms' job-creation condition should be satisfied
- all firms' employment should be in steady state
- the bathtub condition for the unemployment rate should be satisfied

Dynamic Model with Heterogeneous Firms

Labor Market Equilibrium

The equilibrium is a vector $(\mathcal{S}, \mathcal{N}, \mathcal{W}, \theta, u)$ that satisfies

$$\frac{z_i \alpha (n_i)^{\alpha-1} - w_i(\mathcal{N}, \mathcal{S}, \mathcal{W}, \theta) - n_i \; \partial_n w_i(\mathcal{N}, \mathcal{S}, \mathcal{W}, \theta)}{\rho + \delta} = \frac{\partial_v C_m(v_i) + n_i \; \partial_v w_i(\mathcal{N}, \mathcal{S}, \mathcal{W}, \theta)}{q(\theta)},$$

$$\frac{z_f - w_f(\mathcal{S}, \mathcal{W}, \theta)}{\rho + \delta} = \frac{\partial_v C_f(v_f)}{q(\theta)},$$

$$\delta n^i = q(\theta) v^i,$$

$$\delta(1-u) = f(\theta) u,$$

$$\sum_{i=1}^{N} s_i = 1.$$

Computation-wise, we can solve the system of equations by finding the fixed point $(\mathcal{S}, \mathcal{W}, \theta)$ given $\mathcal{Z}$ and parameters.

Dynamic Model with Heterogeneous Firms

Computation Issues

Cannot find a vector $(\mathcal{S}, \mathcal{W}, \theta)$ that satisfies all firms' job-creation conditions.

$$q(\theta) \; (\mathrm{mpl}_m - w_m - n_m \partial_{n_m} w_m) > (\rho + \delta) \; (\partial_{v_m} C_m + n_m \partial_{v_m} w_m)$$
$$q(\theta) \; \left(\mathrm{mpl}_f - w_f\right) < (\rho + \delta) \; \partial_{v_f} C_f$$

Potential reasons

- The fringe firm's productivity is too low
  - If fix $z_f = 1.0$, mpl_m is about 30% higher than mpl_f
  - Set $z_f > 1.3$??
  - The lowest productivity that makes the fringe firm post vacancy depends on N_m.
- Two types of firms share the same matching technology
  - If $MB < MC$, $\downarrow v_f \rightarrow \uparrow q(\theta) \rightarrow \uparrow MB$
  - However, $\uparrow q(\theta)$ increases the monopsonists' benefit as well

Counterfactual Analyses

Counterfactual Analyses

Minimum Wage Policy

Suppose the fringe firm's vacancy posting cost is linear.

When a binding minimum wage $\underline{w} \geq w_f^0$ is implemented, if θ doesn't change, the fringe firm will not post any vacancies.

$$\frac{z_f - \underline{w}}{\rho + \delta} < \frac{z_f - w_f^0}{\rho + \delta} = \frac{c}{q(\theta_0)}$$

If the fringe firm does not post vacancies, θ will decrease.

The maximum market tightness θ' where the fringe firm posts vacancies is

$$\frac{z_f - \underline{w}}{\rho + \delta} = \frac{c}{q(\theta')}$$

$\theta_0 \searrow \theta'$.

Counterfactual Analyses

Competition Policy

When a merger deal is closed, the number of firms N_m and the productivity vector $\mathcal{Z}$ will change.

Our model can easily simulate the effect of the changes in labor market equilibrium.

We can address the following issues:

- Competing firms' response and the change in their employments
- How does the fringe firm adjust its vacancy postings?
- Impact on competing firms' endogenous bargaining powers (markdowns)
- Change in wage distribution
- Is pre-merger HHI informative about the post-merger wage distribution?

Counterfactual Analyses

Atomistic Limit

As in Jarosch, Nimczik and Sorkin (2024). Eliminating the monopsonists' size-based market power.

Mechanically, this is to impose

$$\Xi_0(s_i;\theta) = 1 - \eta$$

and

$$\partial_{v_i} w_i(\mathcal{N}, \mathcal{S}, \mathcal{W}, \theta) = 0.$$

Compared to the atomistic limit, do monopsonists post more vacancies?