Public Debt and Investment in Aging Developed Economies

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Abstract

This paper investigates government foreign investment as a strategy to manage public debt in aging developed economies. Using a quantitative overlapping generations model with incomplete markets, I show that it is possible for a government to sustain permanent fiscal deficits and maintain a finite amount of debt even when the domestic interest rate exceeds the economic growth rate. This is achievable if the government invests a sufficiently large share of its debt in foreign assets or if the return on foreign investment is high enough to offset the unfavorable domestic interest rate and growth rate differential. It is also found that government foreign investment can help stabilize the debt-to-GDP ratio in the long run as long as the return on foreign assets is higher than the government's borrowing cost, without relying on an active fiscal rule targeting a debt-to-GDP ratio. Calibrated to the Japanese economy from 1995 to 2020, the model explains 72.22% of the increase in Japan's debt-to-GDP ratio during this period, with the government's foreign investment strategy accounting for 55% of the debt accumulation. A counterfactual analysis indicates that while this external investment initially increases debt and crowds out domestic capital in the short run, it ultimately leads to a lower debt-to-GDP ratio, a larger capital stock, and higher wages in the long run. A welfare analysis follows and shows that Japanese government's foreign investment strategy is welfare-improving in terms of utilitarian social welfare but not Pareto-improving, and a self-financed lump-sum compensation plan that makes Pareto-improving redistribution is not feasible. This study provides an ideal laboratory for studying fiscal issues in aging economies and valuable insights for other advanced economies facing similar demographic and fiscal challenges.

Keywords: *Debt Sustainability, Demographic Transition, Public Investment*
JEL Classification: *E6, H6, J1.*

1 Introduction

Many advanced economies face significant fiscal pressure from population aging, which strains social security systems and inflates public debt. Addressing high public debt in aging economies is a long-standing challenge in macroeconomics. Literature has extensively explored conventional solutions, such as pension reforms that adjust contribution or benefit levels and policies that encourage later retirement to expand the labor force (Auerbach & Kotlikoff, 1987; Huggett & Ventura, 1999; Krueger & Kubler, 2006).

This paper investigates an alternative, less-explored strategy for alleviating the aging economy's public debt burden: generating extra returns on government debt through foreign asset accumulation. As documented by Chien, Cole, and Lustig (2023), the Japanese government, which faces both an aging population and a high debt-to-GDP ratio, has substantially increased the share of foreign assets in its portfolio throughout its period of rapid aging. They estimate that this strategy has expanded Japan's fiscal space by as much as 4% of GDP.

This strategy appears increasingly feasible in aging economies. As households accumulate savings for a longer retirement period, they can drive down domestic interest rates, creating a "savings glut" (Auclert, Malmberg, Martenet, & Rognlie, 2021; Carvalho, Ferrero, Mazin, & Nechio, 2025). This environment allows the government to borrow cheaply at home and invest in higher-yielding foreign assets. Yet, while potentially beneficial for fiscal accounts, the broader macroeconomic and welfare implications of this strategy remain unclear. The expansion of the government's balance sheet to finance foreign investments may crowd out domestic capital and create complex distributional effects across generations. This paper studies whether this foreign investment strategy is ultimately welfare-improving.

To this end, I develop a quantitative overlapping generations (OLG) model populated by households that are heterogeneous in age, wealth, and income shock. Households follow a realistic life cycle: they work and save during their younger years and live on their savings and government pensions in retirement. On the production side, competitive firms determine market wages and the return on domestic investment. A feature of the model's financial market is that private firms face a default risk premium, which allows the government to borrow at a lower, risk-free interest rate. The government finances public spending and a pay-as-you-go pension system through taxes and by issuing non-defaultable short-term debt. The model's crucial feature is the government's ability to invest abroad. It leverages its favorable borrowing position to issue debt domestically at the low risk-free rate and invest the proceeds in higher-yielding foreign assets. This paper analyzes the macroeconomic and welfare consequences of this active management of the public balance sheet. Beyond analyzing this specific policy, the rich framework serves as an ideal laboratory to study a wide range of fiscal issues, including pension system reforms, changes in tax structure, and adjustments to the retirement age.

The theoretical analysis of the model yields a key insight into debt sustainability. I first examine the possibility of a "sustainable deficit" steady state, where the government permanently runs fiscal deficits while maintaining a finite and constant debt. I show that such a steady state can exist even when the domestic interest rate exceeds the economy's growth rate, a condition that violates the well-known $r < g$ condition for debt sustainability. This is possible if the government invests a sufficiently large share of its debt in foreign assets, or if the return on foreign investment is high enough to offset the unfavorable domestic interest rate and growth rate differential. I also show that the government's foreign investment can help stabilize the debt-to-GDP ratio as long as the return on foreign assets is higher than the government's cost in the long run. This implies that a stable debt-to-GDP ratio can be maintained even without actively adjusting the primary deficit based on the current debt level, unlike existing models (Kaplan, Nikolakoudis, & Violante, 2023).

The model is calibrated to the Japanese economy, which provides a compelling case study. As the world's earliest and most rapidly aging advanced economy, Japan has faced significant fiscal pressures, leading to a surge in government debt that exceeds 250% of its GDP in 2020. The Japanese government has also rapidly increased the share of foreign assets in its portfolio, from approximately 10% in 1995 to 37% in 2020. To conduct a quantitative analysis, I construct a baseline scenario where I feed the model with historical data on Japan's demographic changes, fiscal policy, and the government's foreign asset portfolio. A fiscal reform after 2020 is assumed in the baseline scenario as it appears to be necessary to ensure a sustainable deficit final steady state. I calibrate the model under the baseline scenario to replicate Japan's historical transition from 1995 to 2020. The model shows a strong fit with the historical data, explaining 72.22% of the increase in the debt-to-GDP ratio during this period.

To understand the effect of the government's external investment strategy, I conduct a counterfactual experiment by simulating an alternative scenario where the government's foreign investment share is held constant at its 1995 level. The results of the counterfactual experiment reveal a critical intertemporal trade-off. In the short run, increasing foreign asset holdings required additional debt issuance, which crowded out private capital, suppressed wages, and slowed GDP growth. The simulation shows that had the government kept its foreign asset share constant, the rise in the debt-to-GDP ratio would have been approximately 55% lower by 2020 compared to the calibrated model.

Despite these adverse short-run effects, the external investment policy is projected to improve fiscal outcomes in the long run. The accumulation of high-yielding foreign assets generates significant investment income, reducing future borrowing needs. As a result, after the accumulation phase ends, the debt-to-GDP ratio in the baseline scenario begins to fall, eventually dropping below the level of the counterfactual scenario. The new steady state features a lower debt-to-GDP ratio, a higher capital stock, and higher wages compared to the scenario without aggressive foreign investment.

This stark contrast between short-run costs and long-run benefits suggests that the policy's welfare effects are heterogeneous across generations. Consequently, I perform a welfare analysis to evaluate if the public investment is welfare-improving. The analysis shows that Japanese government's foreign investment was welfare-improving in terms of utilitarian social welfare, but it was not Pareto improving. The analysis shows that generations living in the initial state are made worse off by the policy. They bear the short-run costs of lower wages and higher debt without fully enjoying the long-run benefits. In contrast, future generations born in the future benefit from the higher wages and lower debt burden that the policy ultimately delivers.

I also investigate whether the government can implement a lump-sum transfer plan to mitigate the policy's distributional effects. This involves calculating the transfers needed for each household type to ensure they are as well off in the high-investment scenario as they would have been in the counterfactual scenario. The analysis shows that the net present value of compensations required for current households in the steady state exceeds the net present value of the willingness-to-pay from future generations. This indicates that a Pareto-improving redistribution of the public investment policy's effects is not feasible.

The main mechanism behind the welfare effect is the extra fiscal revenue generated by the government's foreign investment. The size of this additional revenue determines the extent of capital crowding out and the overall welfare impact of the investment strategy. Consequently, if households and the government face an uncertainty in foreign asset returns unlike the baseline model, the economy's transition dynamics can be altered. Section 7 extends the model to include the uncertainty in foreign asset returns and studies the effect of foreign asset returns.

(This section is still in progress. I describe how I study the topic.)

The remainder of the paper is structured as follows. Section 2 presents the data on Japan's demographic transition, fiscal pressure, and the government's foreign investment strategy. Section 3 lays out the overlapping generations model. Section 4 analyzes the theoretical conditions for debt sustainability. Section 5 describes the calibration strategy and the baseline scenario for the Japanese economy. Section 6 presents the results from the counterfactual and welfare analysis. Section 7 studies the role of external returns. Section 8 concludes.

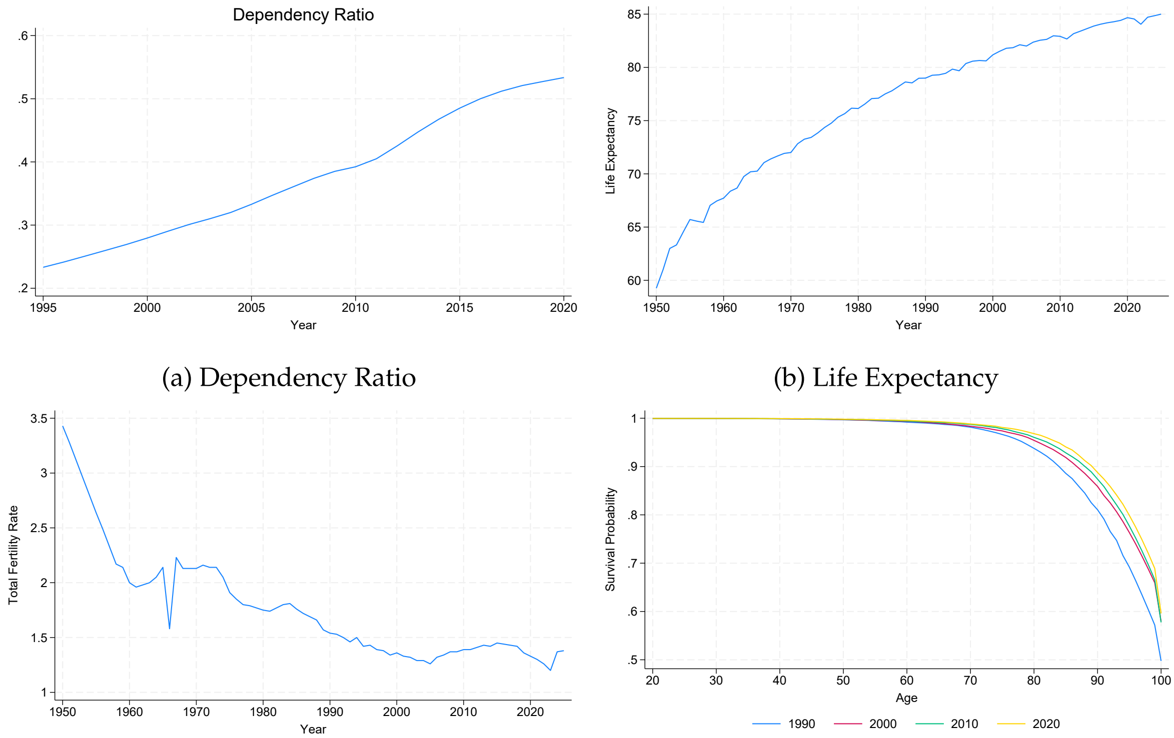
Related Literature This paper contributes to several strands of the literature. It adds to the extensive body of work on public debt sustainability (Blanchard, 2019; Mehrotra & Sergeyev, 2021; Reis, 2022) by formally incorporating the role of government asset management and showing how foreign investment modifies the standard $r < g$ condition. This work also relates to studies that explore fiscal sustainability under conditions of $r > g$ (Angeletos, Lian, & Wolf, 2024). Second, this paper contributes to the literature on the pension system reform in aging economies. While much of this literature focuses on pension reform and its welfare effects (Conesa & Krueger, 1999; Gertler, 1999; Huggett & Ventura, 1999; Krueger & Kubler, 2006), this work explores the government's external investment as an alternative form of pension management. Third, the study also contributes to the literature on the link between population aging, capital flows, and interest rates (Attanasio, Bonfatti, Kitao, & Weber, 2016; Auclert et al., 2021; Carvalho et al., 2025). My quantitative model, which features realistic demographic dynamics, shows that an aging economy experiences capital deepening as individuals save more for longer retirements. The government in the model leverages the saving glut to invest in foreign assets. Finally, this paper extends the analysis of Chien et al. (2023) by investigating the effects of the government's external investment with a quantitative macro model.

2 Data: Demographic Transition and Fiscal Challenge in Japan

This section describes population aging in Japan and its implications for fiscal issues using data.

2.1 Demographic Transition

Japan had been experiencing population aging at unprecedented rates over the past few decades. The life expectancy at birth of Japanese individuals has increased significantly as shown in Figure 1 (a). The life expectancy at birth reached over 84 years in 2020, reflecting improvements in healthcare and living standards. In particular, the fact that the median length of life is greater than the life expectancy suggests a significant portion of the population lives well beyond the average. This adds to the burden on pension payments. As a result, the dependency ratio, which is computed as the ratio of the retired population to that of working-age population (age 20-64), has been steadily increasing since 1995. In 1995, one working-age individual supported approximately 0.23 retirees. By 2020, this figure had risen to over 0.50.



(a) Dependency Ratio

(b) Life Expectancy

(c) Total Fertility Rate

(d) Age-specific Survival Probability

Figure 1: Demographic Transition in Japan

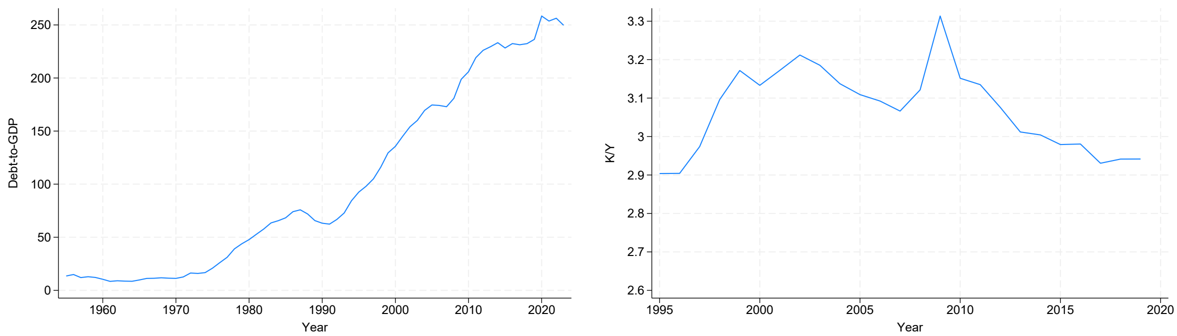
This aging trend can be attributed to two major factors. First, the decline in fertility rates. Figure 1 (c) shows the total fertility rate (TFR), the expected number of children born to a woman over her lifetime. Japan's TFR has steadily declined since 1950, remaining well below the replacement level of 2.1 since the 1970s. The decline halted in the 2000s, and the rate has since remained relatively constant. However, despite the significant funds governments allocate to increase birth rates, such as the Kishida administration's annual investment of 3.6

trillion yen, financial incentives alone appear insufficient. TFR is still below 1.5 in 2020s. The rising average age for first marriages and births, and the high proportion of single individuals, point to deeper social and cultural shifts.

The second factor is the increase in survival probability. Figure 1 (d) illustrates the survival rates at different ages. Improvements in healthcare and living standards have led to higher survival rates, particularly for the elderly. This means that not only are people living longer, but they are also more likely to reach advanced ages. As a result, the proportion of the population that is elderly is increasing, further exacerbating the challenges posed by an aging society.

2.2 Fiscal Challenge and Reforms

Fiscal Challenge Figure 2 shows the changes in the Japanese government's debt-to-GDP ratio and the capital-to-GDP ratio. As is well known, Figure 2 (a) shows that the debt ratio of the general government (central and local governments, central bank, and social security fund) surged from below 100% in the early 1990s to over 250% in 2020. This increase in debt leads to a capital crowd-out effect. As seen in Figure 2 (b), the capital-to-GDP ratio has shown a general downward trend since 2000. The decline in the capital-to-GDP ratio reduces household labor income, which in turn shrinks the government's labor income tax base, potentially worsening the debt problem.



(a) Debt-to-GDP Ratio

(b) Capital to Output Ratio

Figure 2: Fiscal Pressure in Japan

Consumption Tax Reform Increasing value added tax has been often used as a fiscal response to increasing burden in social security budget in Japan. On April 1, 1989, Japan's consumption tax was first introduced at a rate of 3% under the Noboru Takeshita Cabinet. Despite being at the peak of the bubble economy at the time , this measure led to Prime Minister Takeshita's resignation and the ruling Liberal Democratic Party's (LDP) crushing defeat in the subsequent election, earning the tax the nickname "graveyard of administrations". Subsequently, on April 1, 1997, the Ryutaro Hashimoto Cabinet increased the consumption tax rate to 5%. This also resulted in Prime Minister Hashimoto's resignation and the LDP's defeat in the House of Councillors election. In the early 2000s, the Junichiro Koizumi Cabinet attempted institutional changes instead of direct rate hikes, such as lowering the tax-exempt threshold for consumption tax payers in 2003 and mandating tax-inclusive pricing in 2004. However, despite ongoing fiscal pressure, direct tax increases continued to face political hurdles. Discussions for a tax hike under the Taro Aso Cabinet in 2009 were halted when the LDP was replaced by the Democratic Party of Japan (DPJ), which had campaigned on a "no tax hike" platform. Even the ruling DPJ's Prime Minister Naoto Kan attempted to push for a consumption tax increase in 2010 due to worsening fiscal conditions, but this led to his party's significant defeat in the election and his resignation.

Despite these repeated failures, the need for fiscal consolidation intensified due to an aging population and rising national debt. In March 2012, the Yoshihiko Noda Cabinet submitted a bill to gradually increase the consumption tax rate to 8% in April 2014 and 10% in October 2015. This bill was passed through cross-party agreement with major opposition parties. As planned, the consumption tax was raised from 5% to 8% in April 2014. However, Prime Minister Shinzo Abe announced in November 2014 a postponement of the scheduled 10% increase (originally set for October 2015) by 18 months, citing concerns about an unexpected economic slowdown. Nevertheless, the Japanese government reached a fiscal limit where further postponement of the consumption tax increase was no longer feasible. Consequently, the final 10% consumption tax increase was implemented on October 1, 2019.

Table 1: Consumption Tax Reform in Japan

Date	Introduced VAT Rate	Administration	Notes
April 1, 1989	3%	Noboru Takeshita	First introduction of consumption tax in Japan.
April 1, 1997	5%	Ryutaro Hashimoto	Included 1% local consumption tax.
April 1, 2014	8%	Yoshihiko Noda	Implemented as planned by Noda Cabinet's bill.
October 1, 2019	10%	Shinzo Abe	Increased consumption tax

Public Foreign Investment and Asset Market Segmentation The Government Pension Investment Fund (GPIF) was established in 2001 to manage Japan's public pensions amid an aging population and to ensure fiscal stability. Its initial strategy was highly conservative, with a portfolio heavily concentrated in domestic bonds, specifically Japanese Government Bonds (JGBs), which accounted for 66–70% of its holdings. The allocation to equities and foreign assets remained very low. By 2012, the portfolio was composed of approximately 67% domestic bonds, 11% domestic equities, 8% foreign bonds, 9% foreign equities, and 5% short-term assets.

The Abenomics policy, introduced by Prime Minister Shinzo Abe in 2012, marked a significant pivot for the GPIF. Responding to the pressures of an aging population and a prolonged low-interest-rate environment, the fund shifted its focus from a domestic bond-centric approach to one that prioritized diversification and higher returns. The low yields on domestic bonds created a need for higher returns to meet long-term pension obligations. In 2013, The allocation to domestic bonds was reduced from 67% to 60%, while foreign equity and bond allocations were increased to 12% and 11%, respectively. In 2014, the combined bond allocation was reduced (domestic and foreign) from 75% to 50%, and the total equity allocation was increased from 24% to 50% (split equally between domestic and foreign equities).

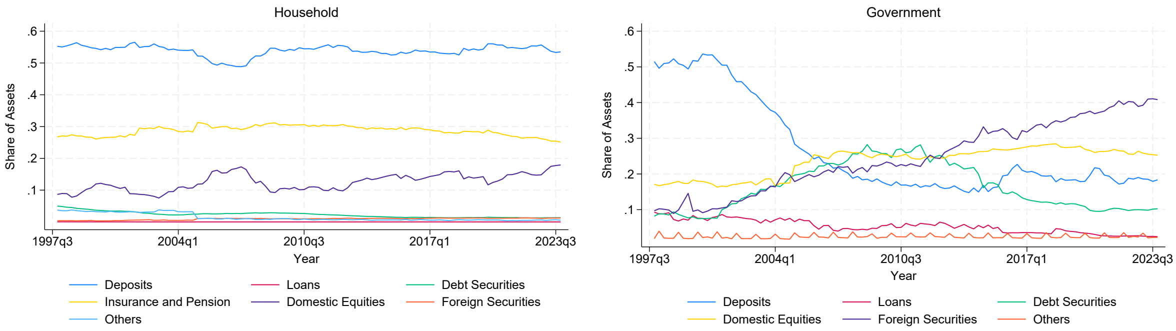
Since April 2020, the GPIF has implemented a policy of equal asset allocation, targeting a 25% share for each of the four main asset classes: domestic bonds, foreign bonds, domestic equities, and foreign equities, with a permissible deviation of 7–11%. The fund has continued to increase its total foreign asset holdings, which now make up over 50% of the portfolio.

The government's active investment in high-yield assets results in a distinct divergence from household asset portfolio as shown in Figure 3.

Figure 3 (a) shows the asset shares of households. A notable feature is that more than 50% of their assets are held as bank deposits. The second largest asset class is insurance and pension assets, which also offer relatively low returns. Domestic equities account for 10-20% of their assets, while foreign assets consistently represent less than 5% throughout the period.

In contrast, the general government actively restructured its asset portfolio between 1997 and 2023. It reduced its share of cash-like assets, which previously exceeded 50%, and increased its holdings of higher-yielding, riskier assets such as foreign securities and domestic equities. Notably, foreign securities grew from about 10% in 1997 to nearly 40% by the 2020s.

This is one of the unique characteristics of the Japanese economy. The government appears to act as a risk-taker on behalf of risk-averse households by investing in high-risk, high-return assets. This trend also suggests that households face domestic/foreign asset market segmentation, whereas the government is free from it. While there could be various reasons for households facing this market segmentation, and exploring those reasons would be an interesting avenue for research, this study assumes that households are constrained by it.



(a) Household

(b) General Government

Figure 3: Asset Share by Asset Type

Immigration Policy Reform Historically, Japan has maintained a highly restrictive immigration policy, characterized by self-imposed isolation and strict controls on foreign entry and settlement. Unlike many advanced Western economies that relied on guest worker programs, Japan largely met its industrial manpower needs after 1945 by drawing on its internal rural labor force. Obtaining permanent residency was traditionally difficult, requiring 20 years of continuous residence, although this was later reduced to 10 years by 1998.

However, as the aging population began to strain labor markets, Japan introduced the Specified Skilled Worker (SSW) program in April 2019. This program is a cornerstone of its modern immigration strategy, designed to directly address severe labor shortages. It represents a significant shift away from the problematic Technical Intern Training Program (TITP), which is scheduled for abolition in 2024. The SSW program offers clearer pathways for foreign workers to contribute to Japan's economy. It targets 16 designated industrial sectors, with a projected need for 820,000 foreign workers over the five years from April 2024. The key sectors with the highest demand include manufacturing of industrial products (173,300), nursing care (135,000), manufacture of food and beverages (139,000), construction (80,000), and agriculture (78,000). This highly targeted approach highlights Japan's pragmatic focus on maintaining essential services and industrial output by addressing specific economic vulnerabilities.

3 Model

This section outlays a model of the economy featuring overlapping generations of households, identical firms, a bank, and the government. The model incorporates a growth process driven by advancements in labor-augmenting technology and a change in working-age population. I first present the non-detrended model, followed by the detrending of the model. Tilde notation is used for level variables, while non-tilde notation denotes detrended variables. Lowercase letters represent individual variables, and uppercase letters represent aggregate variables.

3.1 Demographic

Households are indexed by $(\varepsilon, \tilde{a}, j)$, where $\varepsilon \in \mathcal{E}$ represents the endowed efficiency units of labor, $\tilde{a} \in \tilde{\mathcal{A}}$ denotes asset holdings, and $j \in \mathcal{J} = \{1, 2, \ldots, \bar{J}\}$ denotes age. The stochastic process of ε will be specified later. $\bar{J}$ is the maximum age that a household can reach, and all households die with probability 1 if they reach age $\bar{J}$. Let $d\widetilde{\Phi}_{jt}(\varepsilon, \tilde{a})$ denote the measure of households of type $(\varepsilon, \tilde{a})$ aged j at date t.

Households are born as age $j = 1$ workers and retire at age J_r. The measures of the working-age population and the retired population at date t are denoted by μ_t^w and μ_t^r, respectively.

$$\mu_t^w = \sum_{j=1}^{J_r-1} \widetilde{\Phi}_{jt}(\mathcal{E}, \tilde{\mathcal{A}}), \quad \mu_t^r = \sum_{j=J_r}^{\bar{J}} \widetilde{\Phi}_{jt}(\mathcal{E}, \tilde{\mathcal{A}}).$$

Households face exogenous mortality risk. ψ_{jt} denotes the survival probability from date t to $t+1$ for a household of age j at date t. Working-age households give birth to a new household at rate ω_t. The measure of newborn households joining the economy at $t+1$ is $\omega_t \mu_t^w$. The law of motion for the working-age and retired populations are:

$$\mu_{t+1}^w = \omega_t \mu_t^w + \sum_{j<J_r-1} \psi_{jt} \widetilde{\Phi}_{jt}(\mathcal{E}, \tilde{\mathcal{A}}),$$
$$\mu_{t+1}^r = \sum_{j \geq J_r-1} \psi_{jt} \widetilde{\Phi}_{jt}(\mathcal{E}, \tilde{\mathcal{A}}).$$

The growth rate of the working-age population, denoted by n_{t+1}, is defined as:

$$1 + n_{t+1} = \frac{\mu_{t+1}^w}{\mu_t^w} = \omega_t + \frac{\sum_{j<J_r-1} \psi_{jt} \widetilde{\Phi}_{jt}(\mathcal{E}, \tilde{\mathcal{A}})}{\mu_t^w}.$$

The initial working-age population is normalized to 1. The dependency ratio (ζ_t) is calculated as the ratio of the retired population to the working-age population (μ_t^r / μ_t^w).

3.2 Timeline

Within date t, decisions are made in following order.

1. The government holds $\tilde{B}_t$ out of the beginning of the period total savings as predetermined debt. The remaining savings are supplied to the production sector as capital.
2. The production sector uses capital and labor to produce. Interests and wages are paid to households. The government also pays interest on its debt.
3. Households receive pension benefits, after-tax labor income and after-tax returns on savings. Households make saving decisions.
4. The governments borrows $\tilde{B}_{t+1}$ from households.
5. Mortality risk is realized. New households are born, and the accidental bequests of the deceased households are equally distributed to new households.
6. Moving on to date $t+1$.

3.3 Private Sector

Household Working-age households supply labor inelastically. The efficiency units of labor endowed to a household is denoted by ε, which follows AR(1) process with persistence ρ_ε and innovation σ_ε unless the household experiences an individual disaster. Irregardless of the current labor endowment, a household can experience a disaster with probability p_d in the following date. Household in disaster state will recover to one of normal endowed efficiency units of labor according to the ergodic distribution of the efficiency units of labor. Let $\pi_\varepsilon : \mathcal{E} \to \mathbb{R}$ denotes the ergodic distribution of endowed efficiency units of labor.

I assume a asset market segmentation where households can only hold domestic assets. This assumption is motivated by the data showing that Japanese households hold only a small share of foreign assets. Households can deposit their asset holdings into a bank at the deposit rate r_t.

Households are subject to capital income tax at rate τ_{kt}, labor income tax at rate τ_{lt}, and consumption tax at rate τ_{ct}. Working-age households supply labor inelastically and make labor income, $\tilde{w}_t$, per efficiency unit of labor. Retired households receive pension benefits. Household's cash in hand at date t is given by the sum of after-tax labor income ($(1-\tau_{lt})\tilde{w}_t\varepsilon_t$), pension benefits ($\widetilde{pb}_t$), the beggining of the date asset holdings ($\tilde{a}_t$), and the after-tax return on savings ($(1-\tau_{kt})r_t\tilde{a}_t$). Households allocate the cash in hand to consumption and savings for the next date under a borrowing limit ($\underline{\tilde{a}}$). A household's problem at date t is to maximize the expected discounted utility of consumption over the life, given by

$$
\begin{aligned}
\tilde{v}_{jt}(\varepsilon_t,\tilde{a}_t) &= \max_{\tilde{c}_t,\tilde{a}_{t+1}} \left[\tilde{c}_t^{\rho} + \tilde{\beta}\psi_{jt}\left(\mathbb{E}_{\varepsilon_{t+1}|\varepsilon_t}\tilde{v}_{j+1,t+1}(\varepsilon_{t+1},\tilde{a}_{t+1})^{1-\sigma}\right)^{\frac{\rho}{1-\sigma}}\right]^{\frac{1}{\rho}} \\
\text{s.t.}\quad & (1+\tau_{ct})\tilde{c}_t + \tilde{a}_{t+1} = \mathbf{1}_{j<J_r}(1-\tau_{lt})\tilde{w}_t\varepsilon_t + \mathbf{1}_{j\geq J_r}\widetilde{pb}_t + (1+(1-\tau_{kt})r_t)\,\tilde{a}_t \\
& \tilde{a}_{t+1} \geq \underline{\tilde{a}}, \quad \underline{\tilde{a}} \leq 0.
\end{aligned}
$$

The elasticity of intertemporal subsitution (EIS) is $1-1/\rho$ and σ is the relatie risk aversion. Note that capital income tax is imposed only for positive returns on savings. The pension scheme will be clarified in the government section.

With the households supplying labor inelastically, the aggregate labor supply in the efficiency units is given by

$$
\tilde{N}_t = \sum_j \int_{\varepsilon,\tilde{a}} \varepsilon \, d\widetilde{\Phi}_{jt}(\varepsilon,\tilde{a}).
$$

The average efficiency units of labor supplied by working-age household is denoted by L_t, which is defined as the ratio of the total amount of efficiency units of labor supplied to the measure of working-age households.

$$
L_t = \frac{\tilde{N}_t^s}{\mu_t^w} = \frac{\sum_{j=1}^{J_r-1}\int_{\varepsilon,\tilde{a}} \varepsilon \, d\widetilde{\Phi}_{jt}(\varepsilon,\tilde{\mathcal{A}})}{\sum_{j=1}^{J_r-1}\widetilde{\Phi}_{jt}(\mathcal{E},\tilde{\mathcal{A}})}.
$$

Production and Financial Intermediary Firms produce output using capital and labor. The production function is identical across firms, given by

$$
\tilde{Y}_{it} = \left(\tilde{K}_{it}\right)^{\alpha}\left(Z_t\tilde{N}_{it}\right)^{1-\alpha}
$$

where Z_t represents the labor-augmenting technology. The growth rate of the labor-augmenting technology is denoted by $\gamma_t = Z_t/Z_{t-1} - 1$. $\tilde{K}_{it}$ and $\tilde{N}_{it}$ are the firm's demand for capital and labor, respectively.

Firms default on their debt with exogenous probability φ_t within the period. The representative bank originates funds from household deposits at the deposit rate r_t and lend money to firms and the goverment competitively. To offset the default risk, the bank charges a spread φ_t over the deposit rate for loans to firms. Contrarily, the goverment bond is not defaultable, allowing the govermnet to issue bonds at the deposit rate r_t.

The equilibrium deposit rate r_t and the wage rate $\tilde{w}_t$ are determined based on the firm's marginal product of capital and labor:

$$
\begin{aligned}
r_k &= \alpha \left(\frac{Z_t \tilde{N}_t}{\tilde{K}_t} \right)^{1-\alpha} - \delta - \varphi_t, \\
\tilde{w}_t &= (1-\alpha) Z_t \left(\frac{\tilde{K}_t}{Z_t \tilde{N}_t} \right)^{\alpha}
\end{aligned}
$$

3.4 Government

Pension System The government pays out pension benefits to retired households regardless of their asset holdings and age. The generosity of the pension benefit is determined by the replacement ratio, θ_t, which is the ratio of the pension benefit to the equilibrium wage rate $\tilde{w}_t$.

$$
\widetilde{pb}_t = \theta_t \tilde{w}_t.
$$

Given that the median of the endowed efficiency unitos of labor is normalized to 1, the replacement ratio is the ratio of the retired household's pension benefit to the median labor income of working-age households.

Budget Constraint The government's final consumption is denoted by $\tilde{G}_t$. In addition to the general spendings, the government pays pension benefits to the retired population, which is given by $\theta_t \tilde{w}_t \mu_t^r$. The government also services its debt by paying interest on the outstanding bonds, which is given by $(1 + r_t)\tilde{B}_t$, where $\tilde{B}_t$ is the total amount of government bonds issued at date t.

The government finances its spendings, pension liabilities, and debt costs through tax revenue and issuing government bonds. The tax revenue consists of labor income tax ($\tau_{lt} \tilde{w}_t \tilde{N}_t$), capital income tax on positive returns to savings ($\tau_{kt} r_t \tilde{B}_t$), and consumption tax ($\tau_{ct} \tilde{C}_t$). Here, $\tilde{N}_t$ is the total amount of efficiency units of labor supplied by working-age households, and

$\tilde{C}_t$ is the aggregate consumption at date t. The government also issues short-term and non-defaultable bonds at date t, denoted by $\tilde{B}_{t+1}$.

The government invests in foreign assets, which are assumed to yield a return of r_t^*. The total amount of foreign investment at date t is denoted by $\tilde{F}_t$. The government's budget constraint at date t is

$$\tau_{lt}\tilde{w}_t\tilde{N}_t + \tau_{ct}\tilde{C}_t + \tau_{kt}r_t\tilde{B}_t + \tilde{B}_{t+1} + (1+r_t^*)\tilde{F}_t = \tilde{G}_t + \theta_t\tilde{w}_t\mu_t^r + (1+r_t)\tilde{B}_t + \tilde{F}_{t+1}.$$

I define the government debt's utilization ratio as the ratio of the total amount of foreign investments to the total amount of debt ($\xi_t = \tilde{F}_t/\tilde{B}_t$). The budget constraint can be simplified with the utilization ratio as follows:

$$(1-\xi_{t+1})\tilde{B}_{t+1} = \tilde{G}_t - \tau_{ct}\tilde{C}_t + (\theta_t\mu_t^r - \tau_{lt}\tilde{N}_t)\tilde{w}_t + \left[1 + r_t(1-\tau_{kt}) - \xi_t(1+r_t^*)\right]\tilde{B}_t$$

The left-hand side represents the net amount of newly issued bonds available for financing primary deficits and servicing existing debt. The right-hand side comprises the primary deficit, which accounts for general spending and pension liabilities net of tax revenues, and the effective debt service cost, determined by the after-tax return on government bonds and the return on government investments in foreign assets.

3.5 Equilibrium

Market Clearing Conditions The market clearing conditions for capital market, labor market, and output market are

$$\begin{aligned}
\tilde{K}_t + \tilde{B}_t &= \sum_j \int_{\varepsilon,\tilde{a}} \tilde{a}\, d\widetilde{\Phi}_{jt}(\varepsilon,\tilde{a}), \\
\tilde{N}_t &= \sum_{j=1}^{J_r-1} \int_{\varepsilon,\tilde{a}} \varepsilon\, d\widetilde{\Phi}_{jt}(\varepsilon,\tilde{a}), \\
\sum_j \int_{\varepsilon,\tilde{a}} \tilde{c}_t(\varepsilon,\tilde{a},j)\, d\widetilde{\Phi}_{jt}(\varepsilon,\tilde{a}) + \tilde{K}_{t+1} + \tilde{G}_t &= \tilde{K}_t^{\alpha}\left(Z_t\tilde{N}_t\right)^{1-\alpha} + (1-\delta)\tilde{K}_t.
\end{aligned}$$

Law of Motion of Population Measure The law of motion of the population measure is summarized by the following equations: For any $E \subset \mathcal{E}, \tilde{A} \subset \tilde{\mathcal{A}}$ and $1 \le j \le \bar{J} - 1$,

$$\widetilde{\Phi}_{j+1,t+1}(E, \tilde{A}) = \int_{\tilde{a} \in \tilde{\mathcal{A}}} \tilde{P}_t(\varepsilon, \tilde{a}, j, E, \tilde{A}, j+1) \, d\widetilde{\Phi}_{jt}(\varepsilon, \tilde{a}),$$

where

$$\tilde{P}_t(\varepsilon, \tilde{a}, j, E, \tilde{A}, j+1) = \begin{cases} \Pr(\varepsilon_{t+1} \in E | \varepsilon_t = \varepsilon)\psi_{jt} & \text{if } \tilde{a}_{t+1} \in \tilde{A}, \\ 0 & \text{otherwise,} \end{cases}$$

New households are born with the equally distributed accidental bequests from deceased households. The total amount of accidental bequests collected by the government at the end of date t is

$$\text{Total Bequest}_t = \sum_j (1 - \psi_{jt}) \int_{\varepsilon, \tilde{a}} \tilde{a}'(\varepsilon, \tilde{a}, j) \, d\widetilde{\Phi}_{jt}(\varepsilon, \tilde{a})$$

The initial endowment for new born households at date $t+1$ is

$$\tilde{a}^i_{t+1} = \frac{\text{Total Bequest}_t}{\omega_t \mu^w_t}$$

A new-born household's initial endowment of effiency units of labor is randomly drawn from the ergodic distribution of the labor productivity process. The measure of newborn households at date $t+1$ is given by

$$d\widetilde{\Phi}_{1,t+1}\left(\varepsilon, \{\tilde{a}^i_{t+1}\}\right) = \pi_\varepsilon \widetilde{\Phi}_{1,t+1}\left(\mathcal{E}, \{\tilde{a}^i_{t+1}\}\right) = \pi_\varepsilon \omega_t \sum_{j=1}^{J_r - 1} \widetilde{\Phi}_{jt}\left(\mathcal{E}, \tilde{\mathcal{A}}\right).$$

3.6 Model Detrending

The growth rate of the labor supply is

$$\frac{\Delta \tilde{N}_{t+1}}{\tilde{N}_t} = \frac{\Delta L_{t+1}}{L_t} + \frac{\Delta \mu^w_{t+1}}{\mu^w_t}.$$

Since the average efficiency unit of labor supplied by working-age households, L_t, is stationary, the growth rate of total labor supply equals the growth rate of the working-age population, n_{t+1}. The growth rate of total output is the sum of the growth rates of labor supply and productivity, $(1+\gamma_{t+1})(1+n_{t+1})$.

All aggregate variables are detrended by $\prod_{s=1}^{t}(1+\gamma_s)(1+n_s)$, and individual variables by $\prod_{s=1}^{t}(1+\gamma_s)$ to make the model stationary. This detrending formula is formalized as:

$$x_t = \frac{\tilde{x}_t}{\prod_{s=1}^{t}(1+\gamma_s)}, \quad X_t = \frac{\tilde{X}_t}{\prod_{s=1}^{t}(1+\gamma_s)(1+n_s)}.$$

The measure of population is detrended by the growth rate of working-age population. Given that the initial working-age population is normalized to 1 ($\mu_0^w = 1$), the detrended measure of population is:

$$\Phi_{jt}(E, A) = \Phi_{jt}\left(E, \frac{\tilde{A}}{\prod_{s=1}^{t}(1+\gamma_s)}\right) = \frac{\widetilde{\Phi}_{jt}(E, \tilde{A})}{\mu_t^w} = \frac{\widetilde{\Phi}_{jt}(E, \tilde{A})}{\prod_{s=1}^{t}(1+n_s)}.$$

Household The household's detrended value function $v_t = \tilde{v}_t / \prod_{s=1}^{t}(1+\gamma_s)$ satisfies the following Bellman equation:

$$v_{jt}(\varepsilon_t, a_t) = \max_{c_t, a_{t+1}} \left[c_t^{\rho} + \beta_t \psi_{jt} \left(\mathbb{E}_{\varepsilon_{t+1}|\varepsilon_t} v_{j+1,t+1}(\varepsilon_{t+1}, a_{t+1})^{1-\sigma} \right)^{\frac{\rho}{1-\sigma}} \right]^{\frac{1}{\rho}}$$

$$\text{s.t.} \quad (1+\tau_{ct})c_t + (1+\gamma_{t+1})a_{t+1} = \mathbf{1}_{j<J_r}(1-\tau_{lt})w_t\varepsilon_t + \mathbf{1}_{j\geq J_r}\theta_t w_t + (1 + r_t(1-\tau_{kt}))a_t,$$

$$a_{t+1} \geq \underline{a}, \quad \underline{a} \leq 0.$$

where $\beta_t = \tilde{\beta}(1+\gamma_{t+1})^{\rho}$.

Production Prices are determined by the capital-to-labor ratio,

$$r_t = \alpha \left(\frac{K_t}{L_t}\right)^{\alpha-1} - \delta - \varphi_t,$$
$$w_t = (1-\alpha)\left(\frac{K_t}{L_t}\right)^{1-\alpha}.$$

Government The detrended government's budget constraint is

$$(1+g_{t+1})(1-\xi_{t+1})B_{t+1} = G_t - \tau_{ct}C_t + (\theta_t\zeta_t - \tau_{lt}L_t)w_t + \left[1 + r_{bt} - \xi_t(1+r_t^*)\right]B_t$$

where $(1+g_{t+1}) = (1+\gamma_{t+1})(1+n_{t+1})$ is the growth rate of GDP and $r_{bt} = (1-\tau_{kt})r_t$ is the government's borrowing cost.

Market Clearing Conditions Detrended market clearing conditions are as follows.

$$
\begin{aligned}
K_t + B_t &= \sum_j \int_{\varepsilon,a} a \, d\Phi_{jt}(\varepsilon, a) \\
L_t &= \frac{\sum_{j=1}^{J_r-1} \int_{\varepsilon,a} \varepsilon \, d\Phi_{jt}(\varepsilon, a)}{\sum_{j=1}^{J_r-1} \int_{\varepsilon,a} d\Phi_{jt}(\varepsilon, a)} \\
K_t^{\alpha} L_t^{1-\alpha} + (1-\delta)K_t + G_t &= \sum_j \int_{\varepsilon,a} c_t(\varepsilon_t, a_t, j) \, d\Phi_{jt}(\varepsilon, a) + (1 + g_{t+1})K_{t+1}
\end{aligned}
$$

Population Measure The law of motion of the detrended population measure is, for $E \subset \mathcal{E}, A \subset \mathcal{A}$ and $1 \leq j \leq \bar{J} - 1$,

$$
\Phi_{j+1,t+1}(E, A) = \int_{\varepsilon,a} P_t(\varepsilon, a, j, E, A, j+1) \, d\Phi_{jt}(\varepsilon, a)
$$

where

$$
P_t(\varepsilon, a, j, E, A, j+1) = \begin{cases} \frac{\Pr(\varepsilon_{t+1} \in E | \varepsilon_t = \varepsilon)\psi_{jt}}{1+n_{t+1}} & \text{if } a \in A, \\ 0 & \text{otherwise.} \end{cases}
$$

The initial endowment for newborn households at date $t+1$ is

$$
a_{t+1}^{i} = \frac{\sum_{j=1}^{\bar{J}} (1 - \psi_{jt}) \int_{\varepsilon,a} a'(\varepsilon, a, j) \, d\Phi_{jt}(\varepsilon, a)}{\omega_t \sum_{j=1}^{J_r-1} \Phi_{jt}(\mathcal{E}, \mathcal{A})}.
$$

The newborn households at date $t+1$ is given by

$$
d\Phi_{1,t+1}\left(\varepsilon, \{a_{t+1}^{i}\}\right) = \pi_{\varepsilon}\Phi_{1,t+1}\left(\mathcal{E}, \{a_{t+1}^{i}\}\right) = \pi_{\varepsilon}\omega_t \sum_{j=1}^{J_r-1} \Phi_{jt}\left(\mathcal{E}, \mathcal{A}\right).
$$

4 Determinants of Debt Sustainability

This section examines how government's foreign investment affects fiscal sustainability. Section 4.1 studies the long-run impact by examining the possibility of a "sustainable deficit" steady state. This is where the government can consistently run a primary deficit while keeping its debt finite and constant. I show that this is achievable even if the government's borrowing cost is greater than the growth rate ($r_b > g$), provided the government earns an excess return on its foreign investment. In Section 4.2 , I analyze how the government's foreign investment affects the dynamics of debt-to-GDP ratio. It will be shown that the outstanding debt can decay over time only when the rate of foreign asset accumulation is moderate. Furthermore, I show that, when the return on foreign assets surpasses the government's borrowing cost, the fiscal space can converge to a sustainable deficit steady state by appropriately managing the foreign asset position, even without actively adjusting spendings and taxes.

4.1 Sustainable Deficit Steady State

I first define the sustainable deficit steady state. It is a steady state where the government runs a primary deficit and maintain a constant level of debt.

Definition 1 (Steady State)**.** Given a fiscal rule $\{G, \theta, \tau_c, \tau_l, \tau_k, \xi\}$, demographic factors $\{\omega, (\psi_j)_{j=1}^{J}\}$, the production environment $\{\gamma, \varphi\}$, and the return from foreign asset r^*, the steady state of the economy is a set of (i) households' value functions $v_j(\varepsilon, a)$ and their optimal policy functions for saving and consumption $\{\mathbf{A}'_j(\varepsilon, a), \mathbf{C}_j(\varepsilon, a)\}$, (ii) aggregate variables $\{C, K, L, B, g\}$, (iii) prices $\{r, w\}$, (iv) the demographic structure $\{n, \zeta\}$, and (v) the measure of households $\Phi_j(\varepsilon, A)$ such that

1. the growth rate of working-age population (n) and the dependency ratio (ζ) satisfy

$$n = \omega + \sum_{j<J_r-1} \psi_j \Phi_j(\mathcal{E}, \mathcal{A}),$$
$$\zeta = \frac{\sum_{j\geq J_r} \Phi_j(\mathcal{E}, \mathcal{A})}{\sum_{j<J_r} \Phi_j(\mathcal{E}, \mathcal{A})}.$$

2. the growth rate of GDP (g) satisfies

$$g = (1+\gamma)(1+n) - 1.$$

3. households maximize their values,

$$v_j(\varepsilon, a) = \max_{c,a'} \left[c^\rho + \beta\psi_j \left(\mathbb{E}_{\varepsilon'|\varepsilon} v_{j+1}(\varepsilon', a')^{1-\sigma} \right)^{\frac{\rho}{1-\sigma}} \right]^{\frac{1}{\rho}}$$
$$\text{s.t.} \quad (1+\tau_c)c + (1+\gamma)a' = \mathbf{1}_{j<J_r}(1-\tau_l)w\varepsilon + \mathbf{1}_{j\geq J_r}\theta w + (1 + r(1-\tau_k))a$$
$$a' \geq \underline{a}$$

4. the aggregate consumption C and the aggreagate labor suppy L are given by

$$C = \sum_j \int_{\varepsilon,a} \mathbf{C}_j(\varepsilon,a)\, d\Phi_j(\varepsilon,a),$$
$$L = \frac{\sum_{j=1}^{J_r-1} \int_{\varepsilon,a} \varepsilon\, d\Phi_j(\varepsilon,a)}{\sum_{j=1}^{J_r-1} \int_{\varepsilon,a} d\Phi_j(\varepsilon,a)}.$$

5. the stationary distribution of the population $\Phi_j(\varepsilon, A)$ and the saving policy function $\mathbf{A}'_j(\varepsilon,a)$ satisfy, for $j \in [1, \bar{J}-1]$,

$$\Phi_{j+1}(\varepsilon', A) = \int_{a \in A} P(\varepsilon, a, j, \varepsilon', A, j+1)\, d\Phi_j(\varepsilon,a)$$

where

$$\Phi_{j+1}(\varepsilon', A) = \begin{cases} \frac{\Pr(\varepsilon_{t+1} \in \varepsilon' | \varepsilon_t = \varepsilon)\psi_j}{1+n} & \text{if } \mathbf{A}'_j(\varepsilon,a) \in A \\ 0 & \text{otherwise} \end{cases}$$

The initial endowment for newborn households is

$$a^i = \frac{\sum_{j=1}^{\bar{J}} (1-\psi_j) \int_{\varepsilon,a} \mathbf{A}'_j(\varepsilon,a)\, d\Phi_j(\varepsilon,a)}{\omega \sum_{j=1}^{J_r-1} \Phi_j(\mathcal{E}, \mathcal{A})}.$$

The measure of newborn households is

$$d\Phi_1(\varepsilon, \{a^i\}) = \pi_\varepsilon \Phi_1(\mathcal{E}, \{a^i\}) = \pi_\varepsilon \omega \sum_{j=1}^{J_r-1} \Phi_j(\mathcal{E}, \mathcal{A}).$$

6. the government's steady-state debt level B satisfies

$$B = -\frac{G - \tau_c C + (\theta\zeta - \tau_l L)w}{(r_b - g)(1-\xi) + (r_b - r^*)\xi}. \tag{1}$$

where $r_b = (1-\tau_k)r$ is the government's borrowing cost.

7. prices $\{r, w\}$ are given by

$$r = \alpha \left(\frac{K}{L}\right)^{\alpha-1} - \delta - \varphi, \quad w = (1-\alpha)\left(\frac{K}{L}\right)^{1-\alpha}.$$

8. the market clearing conditions for capital is satisfied,

$$K + B = \sum_j \int_{\varepsilon,a} a \, d\Phi_j(\varepsilon, a).$$

■

Equation (1) describes the government's bond supply in the steady state. The numerator reflects the primary deficit, which increases due to (i) higher government expenditures, (ii) a more generous pension system relative to tax revenue, or (iii) an older demographic structure. The denominator captures the *effective debt service cost*. If the government invests a fraction ξ of its debt in foreign assets, the effective debt service cost decreases to the extent that the return on foreign investments exceeds the domestic borrowing cost. For the portion of debt not invested, the debt service cost can be reduced if the GDP growth rate surpasses the interest rate on government bonds. The average effective debt service cost is a weighted average of these two elements.

I now define the sustainable deficit steady state.

Definition 2 (Sustainable Deficit Steady State)**.** A sustainable deficit steady state is a steady state in which the government maintains a permanent primary deficit while the government carries a finite and constant amount of debt. This requires:

$$G - \tau_c C + (\theta\zeta - \tau_l L)w \geq 0, \quad 0 < B < \infty.$$

■

A fiscally sustainable steady state can be represented as an intersection of the government's bond supply curve and the private sector's bond demand curve. If the steady state interest rate is r, the curves are given by

$$B^s(r) = -\frac{G - \tau_c C + (\theta\zeta - \tau_l L)w(r)}{(r_b - g)(1-\xi) + (r_b - r^*)\xi}, \qquad r_b = (1-\tau_k)r \tag{2}$$

$$B^d(r) = \sum_j \int_{\varepsilon,a} a \, d\Phi_j(\varepsilon, a) - K(r). \tag{3}$$

The government's bond supply curve (2) reflects its financing needs, as discussed below. $K(r)$ in the private sector's bond demand (3) represents the capital stock required by the production function at interest rate r. The private sector's bond demand is households' total savings net of the capital stock.

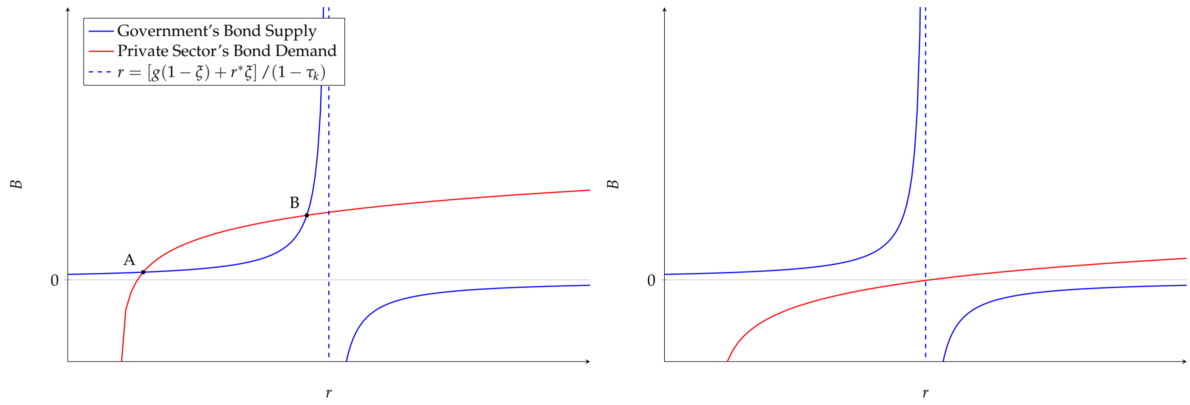
When the primary deficit is positive and $r_b < r^*$, the government's bond supply increases with the steady-state interest rate. This occurs because the asset-to-debt ratio (ζ) is assumed to be fixed at a constant level. When r increases, the government's borrowing cost, $r_b = (1 - \tau_k)r$, rises, the government requires more fiscal revenue to finance the primary deficit. Given that $r^* > r_b$, the only feasible strategy is to issue more government bonds and invest more in foreign assets, which yield a higher return. This mechanism ensures that the government can cover its financing needs while maintaining fiscal sustainability. Similarly, the private sector's net bond demand also increases with the interest rate. A higher interest rate incentivizes households to increase their savings, leading to a greater demand for government bonds.

Graphical Illustration of Sustainable Deficit Steady State A sustainable deficit steady state can be illustrated using a diagram as in Figure 4. The blue lines represent the government's bond supply curve against the steady state interest rate r. The red lines plot the private sector's bond demand curve. For a sustainable deficit steady state to exist, the two curves must intersect at a positive level of debt. The left panel of Figure 4 illustrates a case where the government can maintain a sustainable deficit steady state with a positive level of debt. The right panel shows a case where the government cannot sustain a positive level of debt, as the private sector's bond demand is insufficient to absorb the government's bond supply.

In the left panel of Figure 4, when a sustainable deficit steady state exists, there can be multiplicity of equilibrium. Point A represents a stable equilibrium, and Point B an unstable equilibrium. If the economy is at A and debt increases due to a government spending shock, the economy will return to A if the shock is not large enough to push the interest rate beyond that of B. However, if the shock is substantial, the government debt will diverge.

A Necessary Condition for the Existence of Sustainable Deficit Steady State Under a primary deficit, sustaining a positive level of debt requires the government's borrowing cost r_b to satisfy

$$r_b < g(1 - \xi) + r^* \xi. \tag{4}$$



(a) Multiplicity of Steady State

(b) Non-existence of Steady State

Figure 4: Bond Supply and Demand Diagram

It is also obvious from Figure 4 given that the asymptote of the government's bond supply curve is $r = [g(1-\xi) + r^*\xi] / (1-\tau_k)$.

In the absence of foreign investment ($\xi = 0$), the condition reduces to the well-known condition $r_b < g$, implying that the debt-to-GDP ratio remains finite only if the growth rate of GDP is sufficiently high to offset the cost of servicing debt. However, if the government invests a portion of its debt in foreign assets ($\xi > 0$), the government can maintain fiscal sustainability even if the borrowing cost exceeds the GDP growth rate, as long as the return on foreign assets r^* is sufficiently high to cover the effective debt service cost.

The condition (4) is a necessary condition for the existence of a sustainable deficit steady state. If households are not willing to supply enough savings to absorb the government's bond issuance, the government may not be able to maintain a sustainable deficit steady state, as in the right panel of Figure 4. The government debt will diverge to infinity unless a fiscal adjustment is made.

The Role of ξ and r^* The government can influence the sustainability of its debt by adjusting its foreign investment strategy. Specifically, the government can reduce the effective debt service cost by increasing the proportion of debt invested in foreign assets (ξ) as far as the return on foreign assets (r^*) is greater than the government's borrowing cost. Alternatively, an increase in the return on foreign assets can also reduce the effective debt service cost under the same proportion of debt invested in foreign assets. This is also obvious from Figure 4, where the asymptote of the government's bond supply curve shifts to the right as ξ increases or r^* increases.

4.2 Stability of Debt Dynamics

Recall that the government's budget constraint is given by

$$(1+g_{t+1})(1-\xi_{t+1})B_{t+1} = G_t - \tau_{ct}C_t + (\theta_t\zeta_t - \tau_{lt}L_t)w_t + \left[1 + r_t(1-\tau_{kt}) - \xi_t(1+r_t^*)\right]B_t.$$

To analyze the dynamics of the debt-to-GDP ratio, I introduce new notations for the debt-to-GDP ratio $\bar{B}_t = B_t/Y_t$ and the government spending-to-GDP ratio $\bar{G}_t = G_t/Y_t$. With these, the budget constraint can be rewritten as

$$\begin{aligned}(1-\xi_{t+1})\bar{B}_{t+1} &= \bar{G}_t - \tau_{ct}\bar{C}_t + (1-\alpha)\left(\theta_t\frac{\zeta_t}{L_t} - \tau_{lt}\right) + \left[1 + (1-\tau_{kt})r_t - (1+r_t^*)\xi_t\right]\bar{B}_t \\ &\equiv d_t + \left[1 + (1-\tau_{kt})r_t - (1+r_t^*)\xi_t\right]\bar{B}_t. \qquad (5)\end{aligned}$$

where $\bar{C}_t$ is the aggregate consumption-to-GDP ratio, and $d_t = \bar{G}_t - \tau_{ct}\bar{C}_t + (1-\alpha)\left(\theta_t\frac{\zeta_t}{L_t} - \tau_{lt}\right)$ is the primary deficit-to-GDP ratio. (5) describes the recursive dynamics of the debt-to-GDP ratio over time.

Debt Decay and the Rate of Foreign Asset Accumulation For the outstanding debt at time t to decay between time t and $t+1$, the following condition must hold:

$$1 - \xi_{t+1} > \left[1 + (1-\tau_{kt})r_t - (1+r_t^*)\xi_t\right] \iff (1+r_t^*)\xi_t > (1-\tau_{kt})r_t + \xi_{t+1}. \qquad (6)$$

This condition implies that if the returns and principal from existing investment positions, $(1+r_t^*)\xi_t$, are sufficient to cover both the interest on outstanding debt ($(1-\tau_{kt})r_t$) and new investments (ξ_{t+1}), then the outstanding debt will decline.

The inequality highlights the appropriate rate of foreign asset accumulation necessary to prevent an uncontrolled increase in the debt-to-GDP ratio. If the government accumulates foreign asset position abruptly, it forces the government to issue more bonds to cover current spending and pension obligations, which can lead to a significant increase in the debt-to-GDP ratio. However, when the government has already accumulated a sufficient amount of foreign assets, it can manage the debt-to-GDP ratio more effectively. This suggests that governments should proactively accumulate foreign assets to prepare for future demographic challenges. Establishing a sufficiently high asset position in advance can help manage a significant increase in future pension obligations.

Convergence to a Sustainable Deficit Steady State If the primary deficit ratio d_t is subject to a random shock at date t, its impact on the future debt-to-GDP ratio diminishes asymptotically if

$$\lim_{T\to\infty}\prod_{s=t}^{T}\frac{1+(1-\tau_{ks})r_s-(1+r_s^*)\xi_s}{1-\xi_{s+1}}=0.$$

Assuming the government maintains a constant proportion of foreign investment and interest rates remain stable in the long run, this condition simplifies to

$$\frac{1+(1-\tau_k)r-(1+r^*)\xi}{1-\xi}<1 \quad\Longleftrightarrow\quad \xi>\frac{(1-\tau_k)r}{r^*}. \tag{7}$$

This result indicates that if the foreign interest rate (r^*) exceeds the government's borrowing cost ($(1-\tau_k)r$), the government can achieve a sustainable deficit steady state by appropriately managing its foreign investment ratio ξ. Furthermore, upon a one-time shock to the primary deficit, the debt-to-GDP ratio converges to the original level if the government maintains a constant foreign investment ratio ξ that satisfies (7). This can occur even without active fiscal adjustments to spending or taxes.

5 Quantitative Analysis

5.1 Calibration

Calibration Strategy The model is calibrated to replicate the Japanese economy from 1995 to 2020. The model's initial steady state is set to match the Japanese economy in 1995. Subsequently, historical data on demographic factors, the pension system, fiscal policies, and the government's foreign investment strategy from 1995 to 2020 are fed into the model. The model's transition is computed under perfect foresight, based on a specific scenario assuming that the 2020 values of demographic factors, government policy stances, and foreign investment strategies remain constant post-2020. A fiscal reform in the form of spending cut is additionally assumed to be implemented after 2020 to ensure that the transition path converges to a sustainable deficit steady state. The final steady state in the model is not observed in the data, as it represents a long-term projection. The model's calibration is validated by comparing the model's transition path with historical data from 1995 to 2020.

Parameters The model parameterization is summarized in Table 2. The model frequency is annual, and households are born at age 20, live up tp age 100, and retire at age 65.

Table 2: Summary of Model Parameters

Variable		Value	Source / Target
Demographic			
J_{max}	Maximum age	81	Age 100 (Born as 20)
J_r	Retirement age	46	Age 65
$\psi^{\dagger}_{j,1995}$	Age-specific survival probability in 1995		$\zeta_{1995} = 0.2332$
Preference			
$\beta^{\dagger}$	Discounting factor	0.99	$r_{1995} = 0.0101$
$\iota^{\dagger}$	Elasticity of intertemporal substitution	0.108	$\left(\frac{B}{Y}\right)_{1995} = 0.9253$
σ	Relative risk aversion	6.0	Guvenen (2009)
Income Process			
ρ_{ε}	Persistence of idiosyncratic income shocks	0.95	
σ_{ε}	Std. dev. of idiosyncratic income shocks	$\sqrt{0.226(1-\rho_{\varepsilon}^2)}$	$\text{Var}(\log w) = 0.226$
p_d	Probability of disaster state	0.02	Long-term unemployment rate
ε_d	Value of ε at disaster state	0.0	
$\underline{a}$	Borrowing constraint	0.0	
Production			
$\alpha^{\dagger}$	Capital Share	0.330	$\left(\frac{B}{Y}\right)_{1995} = 2.9038$
δ	Depreciation rate	0.075	

Note: † indicates internally calibrated parameters.

The model's initial age-specific survival probability profile is calibrated rather than taken directly from the data. The 1995 historical age-specific survival probability profile, combined with the 1995 historical fertility rate, does not generate the observed 1995 dependency ratio in the implied stationary distribution. To match the 1995 dependency ratio, I parameterize the logit of survival probability as a linear function of age. Specifically, I parameterize the survival probability of age-j household (p_j) by

$$p_j = \frac{e^{\beta_0+\beta_1\times j}}{1+e^{\beta_0+\beta_1\times j}}$$

where β_0 and β_1 are calibrated to match the 1995 dependency ratio using the historical 1995 fertility rate. The calibrated age-specific survival probability generates a stationary distribution that closely replicates the historical dependency ratio of 1995.

The discounting factor β is calibrated to match the real interest rate of 1.01% in 1995, which is the average of the 1-year Japanes Government Bond (JGB) real yield in that year. The elasticity of the intertemporal substitution (EIS) is calibrated to match the government's historical debt-to-GDP ratio of 92.52% in 1995. A sufficiently low EIS is required to ensure that households are willing to hold government bonds when the government bond yield is lower than the GDP growth rate and their retirement income is ensured by the pension system. However, this value is not far from those used in the literature, such as 0.1 in Guvenen (2009) or 0.3 in Gomes and Michaelides (2008). The relative risk aversion is externally fixed to 6.0. Again, this value aligns with the literature, with 6.0 in Guvenen (2009) and 5.0 in Gomes and Michaelides (2008).

For the household's income process, I target the variance of raw log wage of 0.226 (Nakajima & Takahashi, 2017), with the persistence of idiosyncratic income shocks set to 0.95. The probability of disaster state is set to 2% that corresponds to the long-term unemployment rate of Japan. The value of ε under the disaster is set to 0.

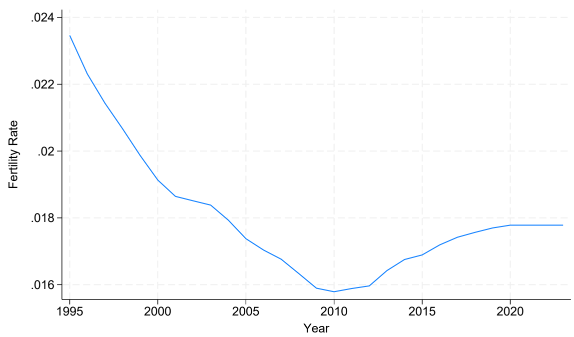
The depreciation rate is externally chosen as 7.5%. The capital share is calibrated to match the capital-to-GDP ratio of 2.9038 in 1995. The historical capital-to-GDP ratio is calculated using the depreciation rate of 7.5%.

Baseline Scenario The model's transition path is computed under the baseline scenario, details of which are summarized in Table 3 and illustrated in Figures 5 - 6. The baseline scenario incorporates historical data for key variables between 1995 and 2020, with post-2020 values assumed constant at their 2020 levels unless otherwise specified.

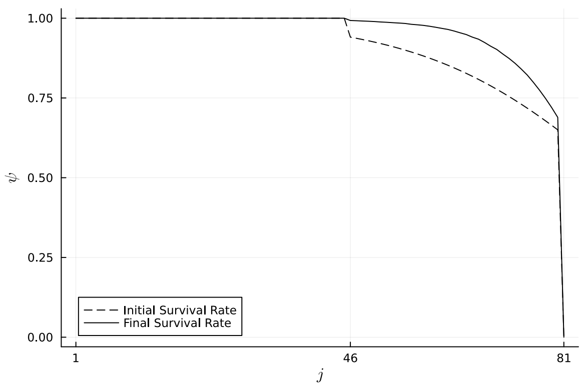
Table 3: Summary of the Baseline Scenario

Variable		Initial	Terminal	1995 - 2020	Source / Target
Demographic					
ω_t	fertility rate	0.023	0.017	Data	UN Population
ψ_{jt}	Age-specific survival probability			Data	UN Population
Production					
γ_t	Technology growth rate	0.011	0.011	Data	Penn World Table 10.01
φ_t	Default probability	0.030	0.0375	Linear	Low interest rates post 2010
Government					
τ_{kt}	Capital income tax rate	0.220	0.220	Constant	
τ_{ct}	Consumption tax rate	0.030	0.100	Data	Statutory values
τ_{lt}	Tax wedge on labor income	0.241	0.327	Data	OECD Taxing Wages
θ_t	Pension replacement ratio	0.320	0.320	Constant	OECD Pension Model
$\left(\frac{G}{Y}\right)_t$	Gov't spending to GDP	0.141	0.210	Data	World Bank National Accounts
ξ_t	Share of foreign assets	0.097	0.372	Data	Flow of Funds Japan
r_t^*	Return on foreign assets	0.045	0.045	Data	GPIF real returns
Fiscal Reform after 2020			**Assumed Value**		
$\left(\frac{G}{Y}\right)_{t>2020}$	Spending-to-GDP after 2020		0.140		Spending cut for fiscal sustainability

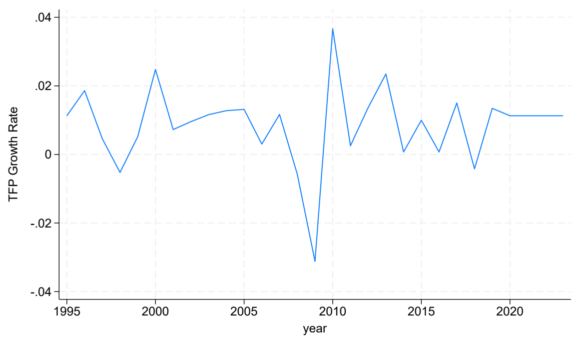
Demographic factors are processed as described in Appendix A and fed into the model. The time series of fertility rate data is used as is, while age-specific survival probabilities in the initial steady state are calibrated to match the dependency ratio in 1995. Survival probability data between 1996 and 2020 are fed into the model as is, with one adjustment that working-age



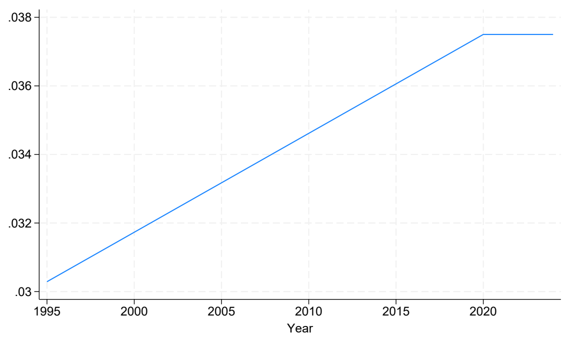
(a) Fertility Rate



(b) Age-specific Survival Probability



(c) TFP Growth Rate



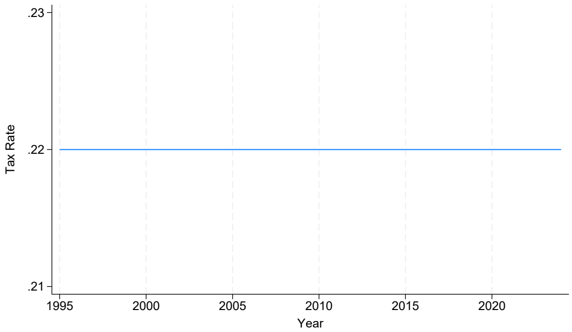
(d) Default Probability

Figure 5: Demographic and Production Parameters in the Baseline Scenario

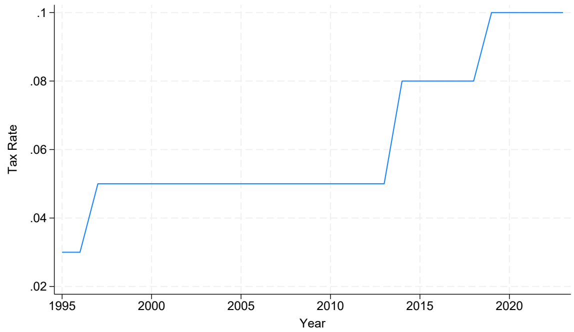
households are assumed to survive every year with certainty.

The technology growth rate data is computed from Penn World Table 10.01. For details of the data processing, see Appendix A. The technology growth rate has shown large fluctuations in the past three decades, with a significant decline in the 1990s and a recovery in the 2000s. Since the 1995 historical technology growth rate does not represent the long-run growth rate of technology, I used the average technology growth rate from 1985 to 2019 as the initial and final values in the baseline scenario. This average growth rate is 1.1%. For 1996-2019, historical data is fed into the model. The default probability is set to 3% in 1995 and increases linearly to 3.75% in 2020. This increasing trend in the default probability is necessary to match the low interest rates in the post-2010 period.

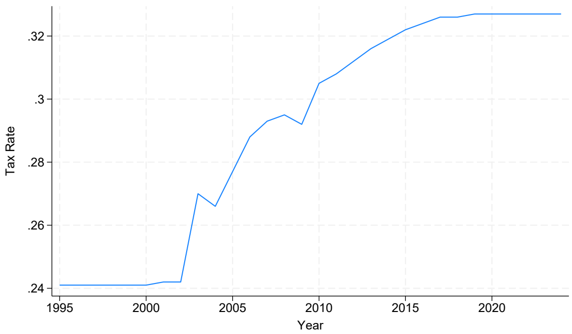
Government's policy parameters are also informed by historical data. The capital income tax rate is held constant at 22% throughout the transition period. The consumption tax rate follows statutory values, increasing from 3% in the initial steady state to 10% in 2020. The labor income tax wedge is derived from OECD Taxing Wages data. As this data ranges from 2000 to 2023, 2000 values are used for any year prior to 2000. The tax wedge rises from 24.1%



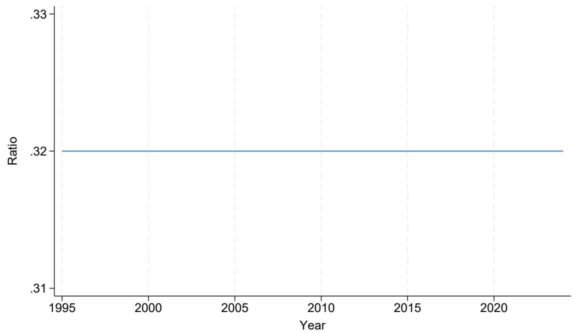
(a) Capital Income Tax Rate



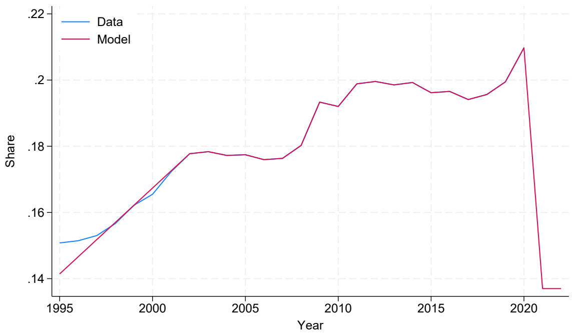
(b) Consumption Tax Rate



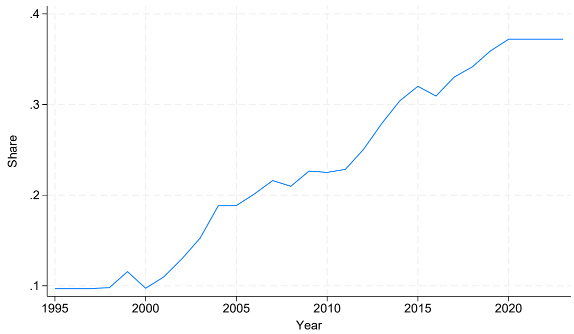
(c) Labor Income Tax Wedge



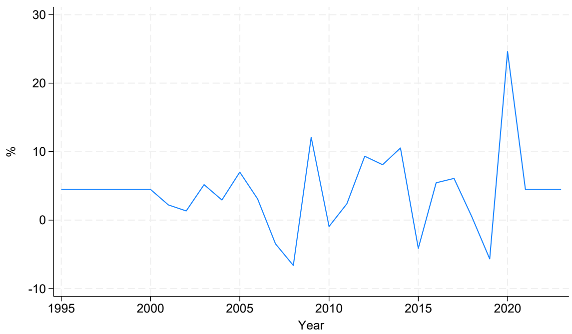
(d) Pension Replacement Ratio



(e) Government Spending to GDP Ratio



(f) Share of Foreign Assets



(g) Returns on Foreign Assets

Figure 6: Government Parameters in the Baseline Scenario

in 1995 to 32.7% in 2020. The pension replacement ratio is sourced from the OECD Pension Model, using the ratio of pension benefits to average earnings. This rate is constant at 32%. The general government's spending-to-GDP ratio is from World Bank National Accounts data. The government expenditure-to-GDP ratio in 1995 was 15.1%, a figure higher than the 1975-1995 average. Indeed, until 1993, Japan's government expenditure-to-GDP ratio never exceeded 14%, and it remained below 15% in both 1993 and 1994. Based on this, I assume that 15.1% is not an appropriate value for the initial steady state and instead utilize the 1975-1995 average of 14.1% as the value for the initial steady state. The share of foreign assets in the government portfolio is from the Flow of Funds Japan data. The data ranges from 1997 to 2020, so the 1997 value is extended to 1995. The return on foreign assets also ranges from 2001 to 2020. For years prior to 2001, I use the average return on foreign assets from 2001 to 2020. The return rate is assumed to be constant at the average of 4.5% after 2020.

I assume the government engages in fiscal reform after 2020, which is necessary to ensure the economy converges to a sustainable deficit steady state. The government's spending-to-GDP ratio has risen from 14.1% in 1995 to 21.0% in 2020. The 2020 value is not sustainable in the long run, given demographic factors that remain constant at 2020 values and other fiscal policies, including tax rates and the pension replacement ratio, unchanged. To achieve a sustainable deficit steady state, the government should either (i) cut the spending-to-GDP ratio, (ii) increase tax rates, or (iii) reduce the pension replacement ratio. In the baseline scenario, I cut government spending since spending does not contribute to welfare in the model. Other scenarios where the government increases the consumption tax rate or reduces the pension replacement ratio are examined in Appendix C. In the baseline scenario, the government's spending-to-GDP ratio is cut to 14.0% immediately after 2020 and remains constant thereafter. The value of 14.0% is the maximum value that ensures the existence of a fiscally sustainable steady state, given the demographic factors and other fiscal parameters in the model.

5.2 Model Fit to Data

Table 4 summarizes how well the model's simulated values align with historical data for 1995 and 2020. In 1995, the model closely matches the dependency ratio (0.2331 model vs. 0.2332 data), debt-to-GDP ratio (0.9119 model vs. 0.9253 data), and capital-to-GDP ratio (2.8790 model vs. 2.9038 data). The interest rate shows a minor difference (0.83% model vs. 1.01% data).

The model provides a reasonable fit for 2020. The dependency ratio is closely replicated (0.5192 model vs. 0.5335 data), and the capital-to-GDP ratio remains close to the data (2.7559 model vs. 2.9416 data). While the model's debt-to-GDP ratio is lower than the data, it still

Table 4: Model Fit to Data for 1995 and 2020

	1995		2020	
Variable	**Data**	**Model**	**Data**	**Model**
Dependency ratio	0.2332	0.2331	0.5335	0.5192
Debt-to-GDP	0.9253	0.9119	2.5837	2.1230
Capital-to-GDP	2.9038	2.8790	2.9416	2.7559
Interest rate	0.0101	0.0083	-0.0010	0.0073

captures the significant increase observed (119.77%p model vs. 165.84%p data). The model's interest rate shows a positive value compared to the slightly negative historical rate (0.73% model vs. -0.1% data).

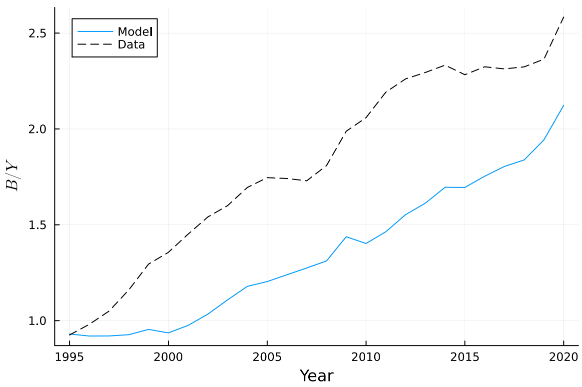
In addition to the summary in Table 4, the computed transition path during 1995 to 2020 of the model is presented in Figure 7. The model successfully replicates the demographic shifts observed in the data, particularly the rising dependency ratio and the short-term increase followed by a decline in the working-age population. Minor discrepancies between the model and data in demographic factors come from the model's simplification of ignoring the mortality rate for working-age households.

The model also shows a strong fit in the changes in the capital-to-GDP ratio. While the interest rate generally exhibits a good fit, the model cannot capture fluctuations due to the absence of financial frictions in its structure. Furthermore, the model predicts an increase in interest rates after 2010 due to rising government debt, which is not observed in the historical data. This divergence might be linked to the model's inability to fully explain GDP growth in the post-2010 period. A notable observation in the Japanese economy since the 1990s has been a clear decline in the labor share, often attributed to shifts in industrial structure (Higo, 2023). The increasing trend of capital share could help explain both the observed GDP growth and the persistently low interest rates after 2010, even with the decreasing capital-to-GDP ratio.

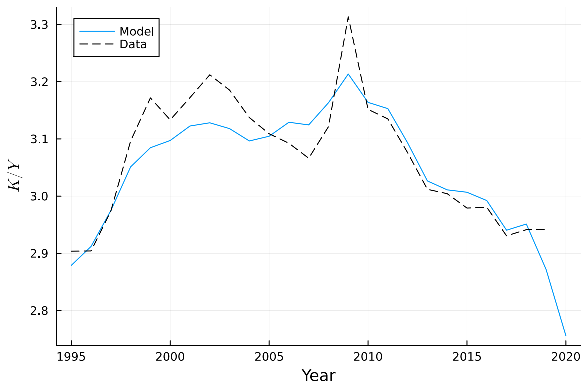
6 Policy Counterfactuals and Welfare

6.1 A Counterfactual Scenario on External Investment

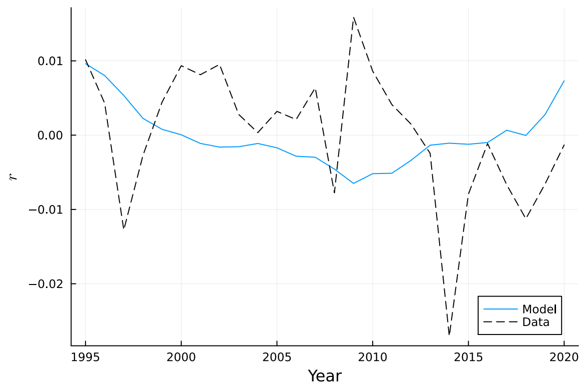
Having calibrated the baseline model to the Japanese economy (see Section 5.1), I conduct a counterfactual analysis to assess the impact of the government's foreign asset investment strategy on the increase in government debt between 1995 and 2020. The counterfactual scenario



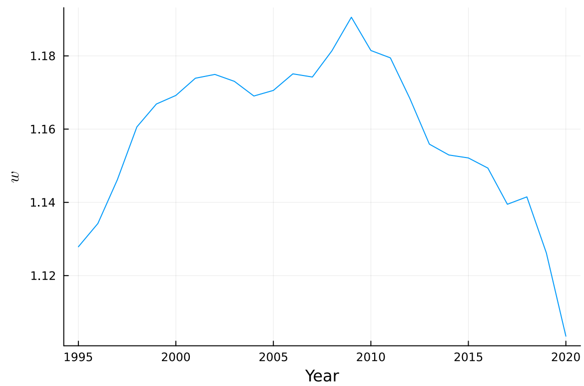
(a) Debt to GDP Ratio



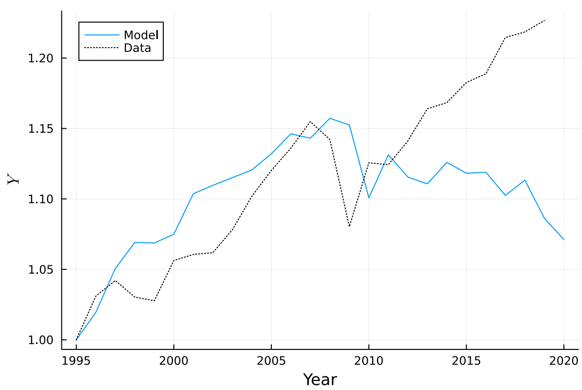
(b) Capital to GDP Ratio



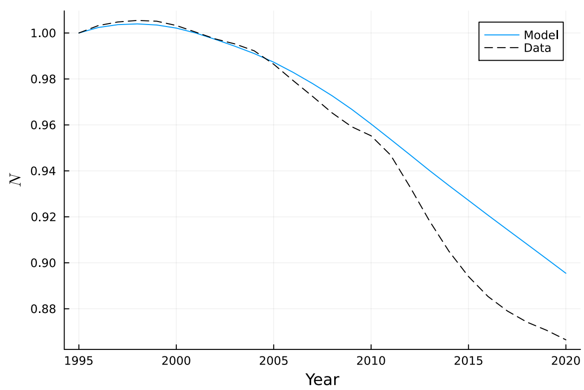
(c) Interest Rate



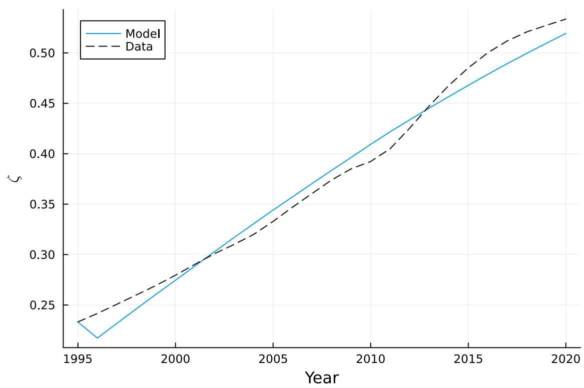
(d) Wage Rate



(e) GDP



(f) Working-Age Population (1995=1)



(g) Dependency Ratio

Figure 7: Model Transition from 1995 to 2020

assumes that the share of foreign assets in the government's portfolio remains constant at its 1995 level of 9.7% throughout the transition period, instead of increasing to 37.2% by 2020, as it does in the baseline scenario. [1]

The results, summarized in Table 5 and Figure 8, highlight a clear difference between the two scenarios' short-term and long-term effects.

During the short-run phase (1995-2020), when the government is actively increasing its foreign asset share in the baseline scenario, the debt-to-GDP ratio rises more steeply than in the counterfactual scenario. The debt-to-GDP ratio in the baseline scenario increases by 121.11 percentage points by 2020, significantly more than the 54.71 percentage point increase in the counterfactual scenario. This is because a more aggressive investment strategy requires the government to issue additional debt to fund current spending and pension obligations.

However, the long-term prospects for 2120 reveal that a more aggressive accumulation of foreign assets ultimately leads to a lower debt-to-GDP ratio. As shown in Figure 8, the debt-to-GDP ratio in the baseline scenario falls below that of the counterfactual scenario within 10 years after the government completes its foreign asset accumulation. This shows that once the government stops increasing its foreign asset ratio, the previously accumulated foreign assets begin to lower borrowing costs and reduce the overall debt level.

Other key variables also show consistent short-term and long-term dynamics. In the short term, the rising debt-to-GDP ratio in the baseline scenario leads to a capital crowding-out effect, keeping the capital-to-GDP ratio lower than in the counterfactual scenario. This lower rate of capital accumulation results in a lower GDP, higher interest rates, and lower wages in the baseline compared to the counterfactual. As a result, the short-term government accumulation of foreign assets negatively impacts household income and welfare.

In the long run, however, these effects are reversed. After the government finishes accumulating foreign assets, the baseline scenario projects a higher capital-to-GDP ratio. This leads to higher GDP, lower interest rates, and higher wage levels compared to the counterfactual.

6.2 Optimal Investment and Welfare Evaluation

Optimal Path of Foreign Asset Accumulation As discussed in Section 6.1, the increase in the Japanese government's foreign asset share between 1995 and 2020 had contrasting effects.

[1] This approach can be used to decompose the effect of demographic transition on the debt accumulation into two factors: declining fertility rates and extending life expectancy. The decomposition results are presented together in Table 5 and details are presented in Appendix D.

Table 5: Counterfactual Scenarios

	1995	2020			
Variable	**Baseline**	**Baseline**	**Constant ξ**	**Constant ω**	**Constant ψ_j**
Dependency ratio	0.2331	0.5192		0.4381	0.2574
Debt-to-GDP	0.9119	2.1230	1.4590	1.8514	1.1517
Capital-to-GDP	2.8790	2.7559	3.0727	2.6356	3.0189
Interest rate	0.0083	0.0073	-0.0051	0.0128	-0.0282

	2120			
Variable	**Baseline**	**Constant ξ**	**Constant ω**	**Constant ψ_j**
Dependency ratio	0.7008		0.4638	0.3090
Debt-to-GDP	0.0819	0.3319	-0.7978	-1.3283
Capital-to-GDP	3.2688	3.1874	3.4784	3.9163
Interest rate	-0.0121	-0.0089	-0.0176	-0.0282

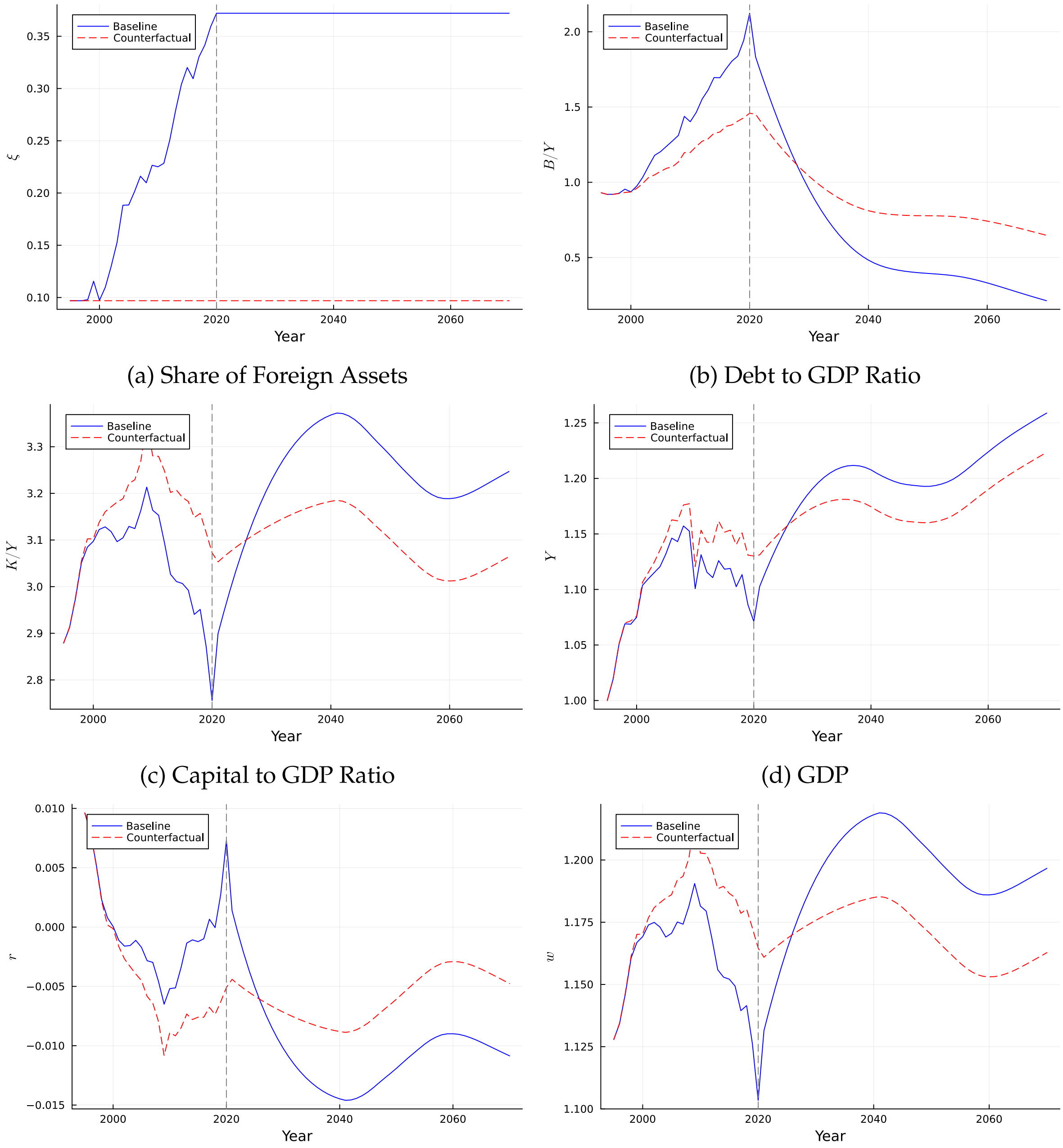
Note: The baseline scenario is calibrated in Section 5.1. Three counterfactual scenarios are compared to the baseline scenario. "Constant ξ" scenario assumes the share of foreign assets remains constant at the initial level ($\xi_t = \xi_{1995}$), "Constant ω" scenario assumes the fertility rate remains constant at the initial level ($\omega_t = \omega_{1995}$), and "Constant ψ_j" scenario assumes the age-specific survival probability remains constant at the initial level ($\psi_{j,t} = \psi_{j,1995}$).

While it intensified capital crowd-out and lowered short-run wages, it led to higher long-run capital accumulation and wages. These conflicting outcomes create heterogeneous welfare effects across households, depending on age, asset holdings, and generation. The magnitude and rate of increase in the foreign asset share determine the severity of the capital crowding out effect in the short run and the long-run benefits from capital accumulation.

This observation raises the question of the optimal path of foreign asset share for maximizing social welfare. By finding the optimal path of external investment, I can evaluate whether the policy adopted by the Japanese government was welfare-improving compared to the optimum and to the counterfactual scenario where the government has not increased the foreign investment.

To compare welfare metrics across possible scenarios of foreign investment, I simplify the feasibility set of foreign investment paths as follows. Let $\overrightarrow{\xi}(\xi_f) \in [0,1]^\infty$ denote the path of foreign asset shares defined by

$$\left(\overrightarrow{\xi}(\xi_f)\right)_t = \begin{cases} \xi_{1995} & \text{if } t = 1995, \\ \xi_{1995} + \frac{\xi_f - \xi_{1995}}{2020-1995} \times (t - 1995) & \text{if } 1995 < t < 2020, \\ \xi_f & \text{if } t \geq 2020. \end{cases}$$



(a) Share of Foreign Assets

(b) Debt to GDP Ratio

(c) Capital to GDP Ratio

(d) GDP

(e) Interest Rate

(f) Wage Rate

Figure 8: Counterfactual Scenario: Constant Share of Foreign Assets

where ξ_{1995} is the historical foreign asset share in 1995. Feasible paths of foreign investment shares are linear transitions from the historical foreign asset share in 1995 to a constant foreign asset share ξ_f after 2020. The feasibility set of foreign investment paths is given by $\Xi = \{ \overrightarrow{\xi}(\xi_f) \in [0,1]^\infty; \quad \xi_f \in [0,1]\}$.

For a given path $\overrightarrow{\xi}(\xi_f)$, I compute total social welfare by summing up the values of the current generation and the discounted values of all future generations.

$$\widetilde{W}\left(\overrightarrow{\xi}(\xi_f)\right) = \sum_{j=1}^{J} \int_{\varepsilon,\tilde{a}} \tilde{v}_{j,1995}(\varepsilon,\tilde{a})\, d\widetilde{\Phi}_{j,1995}(\varepsilon,\tilde{a}) + \sum_{t=1996}^{\infty} \tilde{\beta}^{t-1995} \int_{\varepsilon,\tilde{a}} \tilde{v}_{1,t}(\varepsilon,\tilde{a})\, d\widetilde{\Phi}_{1,t}(\varepsilon,\tilde{a}).$$

The first term on the right-hand side represents the total welfare of the generation alive in 1995, and the second term captures the discounted welfare of all future generations born after 1995.

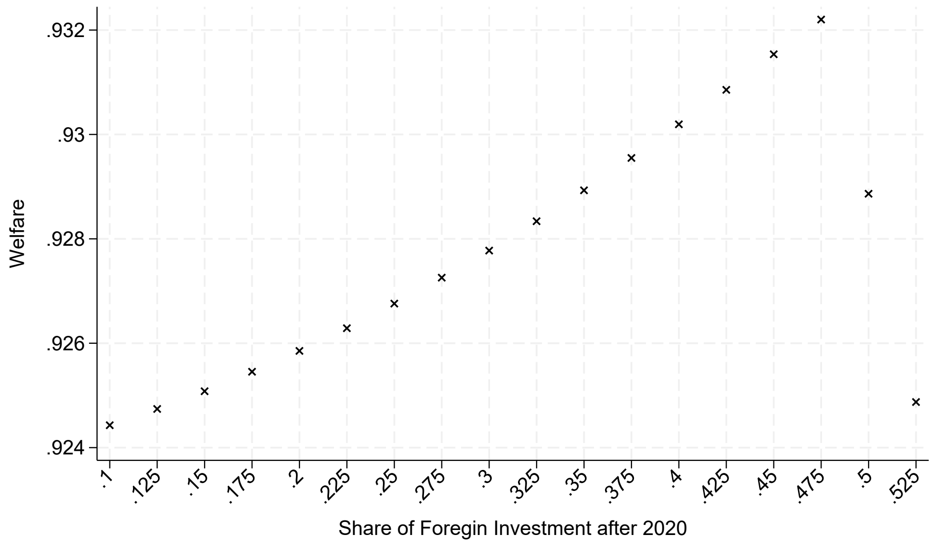


Figure 9: Optimal Foreign Investment Share

Figure 9 shows the computed welfare for different values of ξ_f, which are chosen between 10% and 52.5% at 2.5% increments. The figure shows that the overall welfare increases as the government's foreign investment share in the final steady state increases until the share reaches 50%. This suggests that up to 50%, the long-run benefits from capital accumulation dominates the short-run costs of capital crowding-out. However, if the government builds up the foreign assets rapidly between 1995 and 2020, the short-run capital crowding-out effect becomes severe, and the long-run benefits are not sufficient to offset this cost.

Considering that the Japanese government historically increased its share of foreign assets to 37.2% during this period, Figure 9 suggests that there was still room for welfare improvement by raising the share further to 47.5%. However, the historical policy was still more welfare-improving than the counterfactual scenario where the government had not increased its foreign asset share. Specifically, the total welfare under the baseline scenario, which replicates the historical path, is computed to be xxx, while the welfare under the counterfactual scenario is computed to be xxx.

Distributional Consequences and Impossibility of Pareto-Improving Compensations It has been shown that the increase in the Japanese government's foreign investment share led to an improvement of utilitarian welfare compared to a counterfactual scenario in which the share of foreign investment had remained intact. Despite the improvement, the welfare effect under the active foreign investment can be heterogeneous across households depending on their cohort, asset, income, and age. Given this heterogeneity, it is natural to examine whether the government can use fiscal measures to address the distributional consequences. I investigate whether the government can make all households better off under the increasing foreign investment through its fiscal tools.

Following the methodology of Auerbach and Kotlikoff (1987) and Nishiyama and Smetters (2007), I examine if the government can make all households better off under the increasing foreign investment by providing lump-sum transfers based on their individual welfare changes. The transfer ensures that a household's lifetime value under the increasing investment share is equal to her value under the case of constant investment share. Specifically, let $\tilde{v}^{b}_{jt}(\varepsilon, \tilde{a})$ denote the household's value function in the baseline scenario (increasing share of foreign investment) and $\tilde{v}^{c}_{jt}(\varepsilon, \tilde{a})$ denote the household's value function in the counterfactual scenario (constant share of foreign investment). The required compensatory transfer $\tilde{T}_{jt}(\varepsilon, \tilde{a})$ is defined as follows:

$$\tilde{v}^{b}_{jt}(\varepsilon, \tilde{a} + \tilde{T}_{jt}(\varepsilon, \tilde{a})) = \tilde{v}^{c}_{jt}(\varepsilon, \tilde{a}), \quad \forall j, t, \varepsilon, \tilde{a}.$$

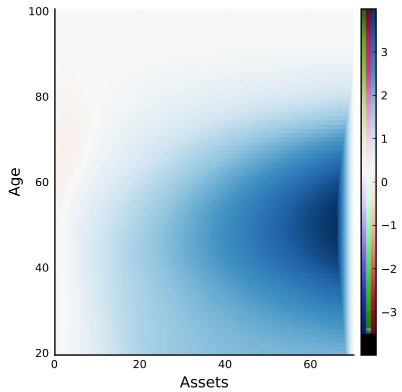
The compensatory transfer is provided to households as one-time lump-sum transfer. If the transfer is positive, it indicates that the household is worse off in the baseline scenario compared to the counterfactual scenario, and vice versa.

To assess the possibility of Pareto improvement through these transfers, I compute the net present value (NPV) of all compensatory transfers. This NPV sums up the transfers to all households living in the initial steady state and all future generations. The formula is as follows:

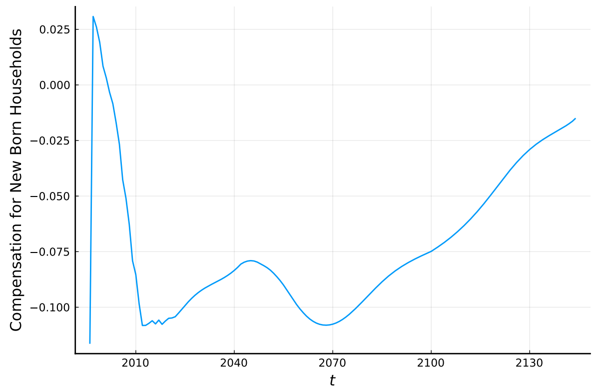
$$\text{NPV of Transfers} = \underbrace{\sum_{j} \int_{\varepsilon, \tilde{a}} \tilde{T}_{j,1995}(\varepsilon, \tilde{a})\, d\widetilde{\Phi}_{j,1995}(\varepsilon, \tilde{a})}_{\text{Transfers to the living households in 1995}} + \underbrace{\sum_{\tau=1996}^{\infty} \prod_{s=1995}^{\tau} \frac{1}{1 + (1 - \tau_{ks}) r^{b}_{s}} \int_{\varepsilon, \tilde{a}} \tilde{T}_{1,\tau}(\varepsilon, \tilde{a})\, d\widetilde{\Phi}_{1,\tau}(\varepsilon, \tilde{a})}_{\text{Transfers to the future generations}}.$$

The first term represents the total transfers to all households alive in 1995. The second term is the NPV of transfers to all households born after 1995, discounted by the government's borrowing cost, since this reflects how the policy's effects are redistributed across generations through the government budget.

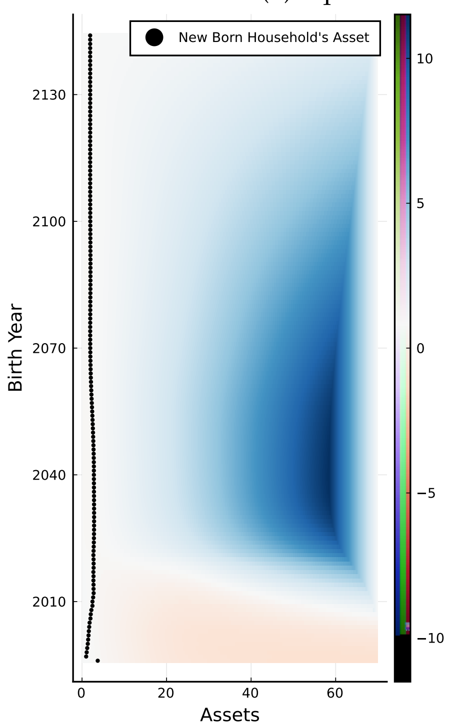
The sign of the NPV reveals the possibility of Pareto improvement. A negative NPV suggests that the government could compensate all individuals for any losses and still have a net gain, implying that the additional compensation plan can make the aggressive investment policy Pareto improving. Conversely, a positive NPV indicates that additional resources would be needed to compensate all households, suggesting that a Pareto improving compensation plan is not feasible.



(a) Transfer to Living Households



(b) Equilibrium Transfer to Future Generations



(c) Transfer to Future Generations

Figure 10: Welfare Effects of Foreign Investment

The required compensations are shown in Figure 10. Figure 10 (a) shows the required transfers for median-income households living in 1995. The results indicate that older or low-asset households benefit from the government's aggressive foreign investment policy, while high-asset or younger households experience losses. The primary reason for the losses among the initially living generation is the lower wage rate. The capital crowd-out effect caused by the government's increased foreign asset ratio between 1995 and 2020 means that this generation earns less in wages compared to the counterfactual scenario. Although the policy eventually raises wages in the long run, the initially living generation is already retired or nearing retirement by that time. Furthermore, the lower interest rates that follow retirement further reduce their lifetime welfare.

Figures 10 (b) and (c) show the transfers for future generations. Households born in the future generally experience an increase in lifetime value due to the government's foreign investment strategy. Specifically, generations born between 2010 and 2070 are found to benefit most from the Japanese government's increase in foreign investment from 1995 to 2020.

The net present value of transfers, calculated using the government's borrowing cost, is approximately 0.2415. The positive indicates that the short-run cost borne by initially living households is not fully compensated by transfers funded by future generations.

This finding highlights the impossibility of a self-financed, lump-sum transfer plan by the government that achieves a Pareto improvement with the increased foreign investment. Although the government's external investment can improve the utilitarian measure of social welfare, it cannot implement a transfer plan that makes all households better off under the policy change.

7 The Role of External Returns

7.1 An Unexpected Shock on External Returns

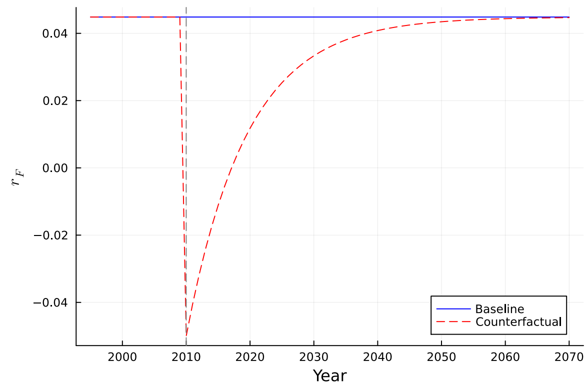
The main mechanism studied so far is the government's additional revenue from external investment, which reduces its need to issue bonds and thus mitigates the capital crowding-out effect. However, external returns can move against the government, minimizing the extra revenue or even aggravating the fiscal space. To explore the effect of such unfavorable shocks, I compare two scenarios again. The first scenario features constant external returns at its average level throughout the transition period, while the second scenario shows an unexpected negative shock to external returns during the time the government is accumulating foreign assets. Specifically, I assume that a -5% external return is realized in 2010. This shock is persistent with an autocorrelation of 0.7. While the shock itself is unexpected, I assume that households have perfect foresight over the path of prices after the shock.

The results are shown in Figure 11. Since the shock is unexpected, the transition path before the shock is the same as in the baseline case. After the negative shock to the government's balance sheet, households adjust their savings to save more, anticipating a future spike in interest rates (Figure 11 (b)). With the abundance of savings, the capital stock remains high in the short term following the shock (Figure 11 (c)). As a result, GDP remains high for about a decade (Figure 11 (d)), and the interest rate remains low (Figure 11 (g)). Due to the low short-run interest rate, the debt-to-GDP ratio paradoxically remains low in the short run (Figure 11 (e)). However, as the shock persists and the negative external return pressures government debt, the debt-to-GDP ratio begins to rise rapidly after 2020. Consequently, the capital-to-GDP ratio falls sharply (Figure 11 (f)), and the interest rate rises significantly (Figure 11 (g)). It shows that the negative shock on external returns leaves a prolonged effect on the labor income for more than 50 years, as the debt-to-GDP ratio remains high for a long time.

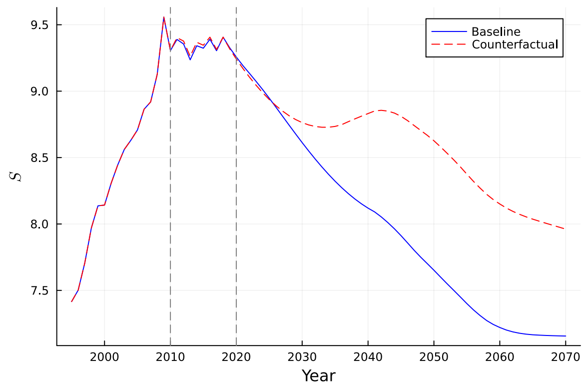
7.2 Uncertainty in External Returns

The perfect foresight of the external return r^* has been assumed so far. However, the external return is inherently stochastic, which intertwines and the government's debt dynamics and the household's saving decisions. In particular, the stochastic realization of the external return r^* affects the future government's debt and the future interest rate, which in turn affects the household's saving motive in the current period. In this section, I explore a rational expectation equilibrium where households take into account the effect of the stochastic external return on the future interest rate.

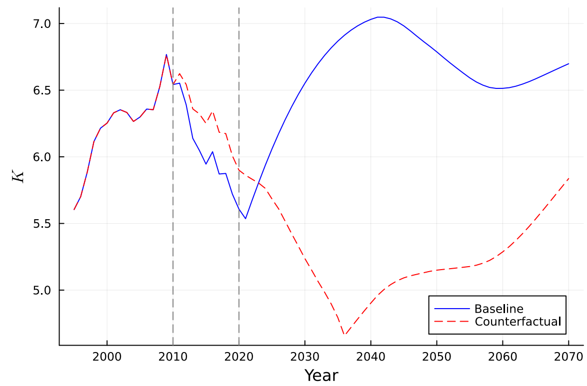
For simplicity, I assume that the external return is stochastically realized either high or low, $r_t^* \in \{r_H^*, r_L^*\}$. The stochastic realization of the current external return r_t^* affects the evolution of the government's debt from date t to $t+1$. Combined with the total savings made by all households at date t, the crowding out effect of the government debt at $t+1$ determines the capital stock in $t+1$ and the interest rate in $t+1$. Households take into account the expected future interest rate with rational expectations, which implies that household's value function



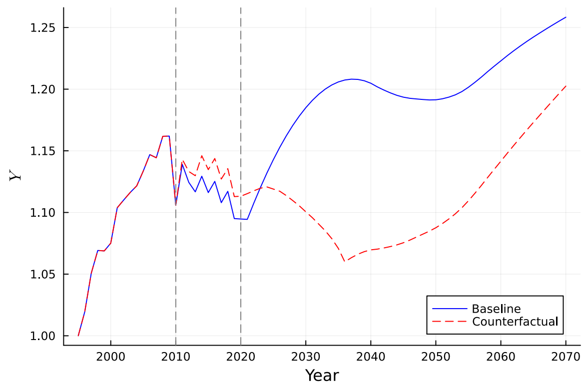
(a) External Return



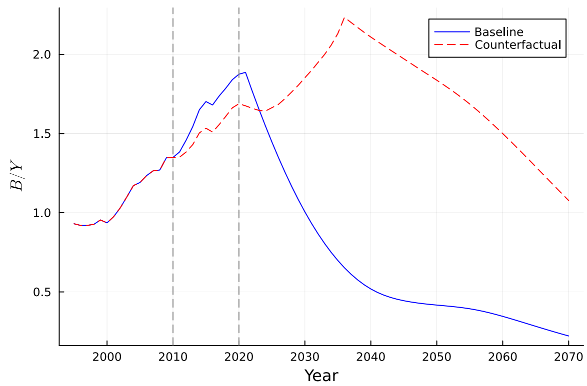
(b) Total Savings



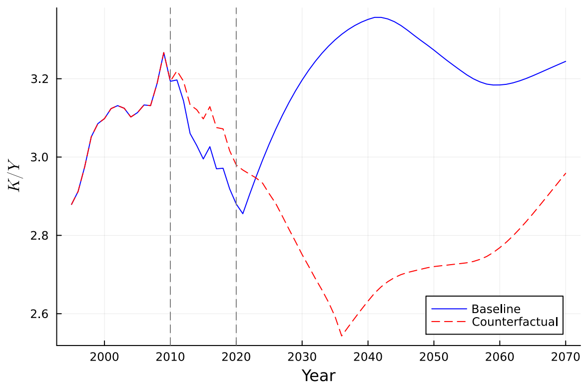
(c) Capital Stock



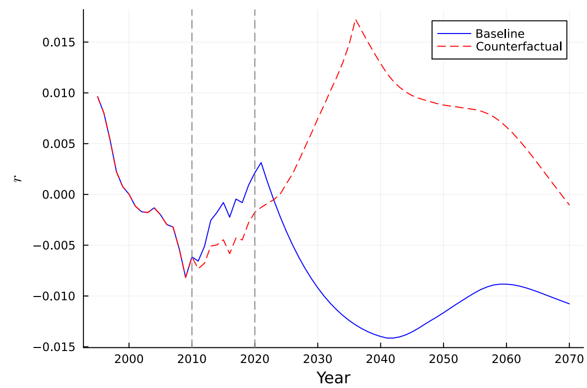
(d) GDP



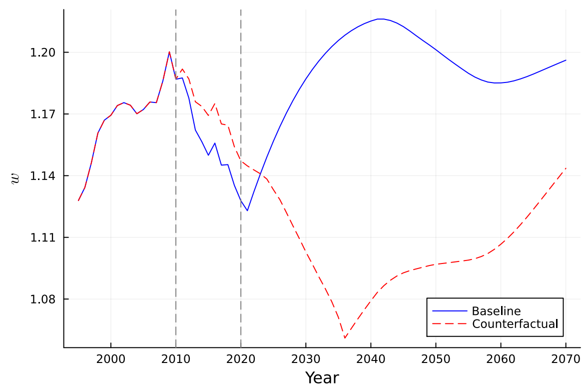
(e) Debt to GDP Ratio



(f) Capital to GDP Ratio



(g) Interest Rate



(h) Wage Rate

Figure 11: The Effect of an Unexpected Negative Shock on External Returns

includes the current external return r_t^* and the current distribution of households Φ_t as state variables. The household's value function is given by

$$v_{jt}(\varepsilon_t, a_t; r_t^*, \Phi_t) = \max_{c_t, a_{t+1}} \left[c_t^{\rho} + \beta\psi_{jt} \left(\mathbb{E}_{(\varepsilon_{t+1}, r_{t+1}^*)|(\varepsilon_t, r_t^*)} v_{j+1,t+1}(\varepsilon_{t+1}, a_{t+1}; r_{t+1}^*, \Phi_{t+1})^{1-\sigma} \right)^{\frac{\rho}{1-\sigma}} \right]^{\frac{1}{\rho}}$$

$$\text{s.t.} \quad (1+\tau_{ct})c_t + (1+\gamma_{t+1})a_{t+1} = \mathbf{1}_{j<J_r}(1-\tau_{lt})w_t\varepsilon_t + \mathbf{1}_{j\geq J_r}\theta_t w_t + (1+(1-\tau_{kt})r_t)\, a_t,$$

$$a_{t+1} \geq \underline{a},$$

$$\Phi_{t+1} = \Gamma_t(\Phi_t, r_t^*).$$

Notice that households are facing the uncertainty of the future external return, but they still perfectly foresight all the future policies and the economic environment, such as tax rates, pension replacement ratios, survival probabilities, and growth rates of labor-augmenting technology.

Following Krusell and Smith (1998), I approximate the distribution Φ_t by two first aggregate moments $\widehat{\Phi}_t = (K_t, B_t)$ and approximate the perceived law of motion $\widehat{\Gamma}_t(\widehat{\Phi}_t, r_t^*)$ by the following log-linear function.

$$\begin{aligned}
\log K_{t+1} =& \eta_{0t}^H + \eta_{KKt}^H \log K_t + \eta_{KBt}^H \log B_t, \quad \text{if } r_t^* = r_H^*, \\
\log K_{t+1} =& \eta_{0t}^L + \eta_{KKt}^L \log K_t + \eta_{KBt}^L \log B_t, \quad \text{if } r_t^* = r_L^*, \\
\log B_{t+1} =& \eta_{0t}^H + \eta_{BKt}^H \log K_t + \eta_{BBt}^H \log B_t, \quad \text{if } r_t^* = r_H^*, \\
\log B_{t+1} =& \eta_{0t}^L + \eta_{BKt}^L \log K_t + \eta_{BBt}^L \log B_t, \quad \text{if } r_t^* = r_L^*.
\end{aligned}$$

To calibrate the stochastic process of the external return, I set $r_H^* > 0$ and $r_L^* < 0$. The value of r_H^* is calibrated to the average return of Government Pension Invest Fund (GPIF) in Japan conditional on being positive. Similarly, the value of r_L^* is calibrated to the average return conditional on being negative. The transition probability of the external return is computed from the historical data of GPIF's returns. The stochastic process is specified in Table 6.

Table 6: Transition Matrix of External Returns

		To	
		r_H^*	r_L^*
From	r_H^*	.6471	.3529
	r_L^*	.1667	.8333

Note: $r_H^* = 7.306\%$ and $r_L^* = -3.506\%$ are calibrated to the average returns of GPIF in Japan conditional on being positive and negative, respectively.

8 Conclusion

I developed a quantitative overlapping generations model with incomplete markets to analyze the effects of government's foreign asset investment strategy.

The model shows that aggressive foreign investment or higher returns on external assets can contribute to long-term fiscal sustainability. In particular, the government can run permanent primary deficit while maintaining the debt level finite if it invests sufficient share of assets in high-yield foreign assets. Futhermore, if the return on foreign investment exceeds the government's borrowing cost, the debt-to-GDP ratio can be managed to converge to the initial level stably following a shock to the primary deficit.

Using a version of the model calibrated to the Japanese economy between 1995 and 2020, I conducted a counterfactual analysis. The results indicates that roughly 55% of the increase in Japan's government debt during this period can be attributed to its foreign asset investment strategy. While this policy initially increased debt in the short term, long-term projections for 2120 shows that it ultimately leads to a lower government debt level.

A subsequent welfare analysis revealed that the government's foreign asset investment policy had a negative impact on households living during the initial period but a positive impact on future generations. The overall welfare change is evaluated to be positive according to utilitarian social welfare. However, it is found that Japanese government couldn't redistribute the welfare consequence across generations to make all households better off. To compensate the short-run costs borne by living households in the initial steady state, the government should raise extra resources in addition to the willingness to pay from future generations. This provides a comprehensive understanding of the trade-offs involved in government foreign asset investment strategies and their implications for fiscal sustainability and household welfare.

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A Data and Definitions

This appendix describes the data sources and the detailed assumptions made in data cleaning.

Fertility Rate Age-specific population data is sourced from the UN Population database, available from 1950 to 2024. The model's fertility rate is defined as the number of new working-age households joining in the following period per working-age household in the current period:

$$\omega_t = \frac{\text{Number of New Working-age Household}_{t+1}}{\text{Number of Working-age Household}_t}$$

Since working age is defined as between 20 and 64, the numerator is proxied by the population aged 20 in the next year. Similarly, the denominator is proxied by the population aged 20 to 64 in the current year. This provides the ex-post fertility rate from the data. While the model's fertility rate is intended to be the ex-ante fertility rate (the expected fertility rate at the beginning of the period), I assume that the ex-post fertility rate equals the ex-ante fertility rate.

The model's definition of the fertility rate differs from the standard definition of the Total Fertility Rate (TFR), which is the expected number of births a woman would have in her lifetime. It is important to note how to convert between the model's fertility rate and TFR. The model's fertility rate is the expected number of births a working-age household would have in a year. Given that the working age is 45 years in the model and that the model does not differentiate by sex, the following equation can be used to convert the model's fertility rate to TFR:

$$\text{Implied TFR} = \omega \times 45 \times 2.$$

If working-age households do not die, a TFR greater than 2 is required to maintain the working-age population. A TFR of 2 corresponds to the model's fertility rate of $1/45 \approx 0.0222$.

Survival Probability Age-specific mortality data is available from the UN Population database, ranging from 1950 to 2024. While detailed age-by-age survival probability data are available, I use a simpler parameterization. Specifically, it is assumed that all workers survive every year with a probability of 1, and households begin to die after retirement. I use the data survival probability for each age for 1996 to 2020, but calibrate the age-specific survival probability in

1995 to replicate the initial steady state. The calibration is clarified in the next section.

Capital-to-GDP Ratio and Technology Growth Rate I calculate the capital-to-GDP ratio and the growth rate of labor-augmenting technology using Japan's national account data from Penn World Table 10.01, available from 1950 to 2019. The capital stock is calculated using the perpetual inventory method, with a depreciation rate consistent with that assumed in the calibrated model. The same capital share as in the calibrated model is used for Solow accounting.

Consumption Tax Rate The Japanese government first implemented a consumption tax in 1989 at a rate of 3%, which gradually increased to 10% by 2019. This tax became a significant source of government revenue, utilized to manage fiscal expansion driven by demographic shifts. I use the historical statutory consumption tax rate in the calibrated model.

Labor Income Tax Rate The labor income tax in the model represents the total wedge between the marginal product of labor and the household's disposable labor income. In this sense, the model's labor income tax should include not only the household's labor income tax but also the employer's social security contribution and the employee's payroll tax. I source the total tax wedge data from the OECD Taxing Wages database, specifically the sum of Employee Social Security Contributions and Employer Social Security Contributions for a single household, with no children, earning 100% of the average income. The data is available from 2000 to 2023.

Government-Spending-to-GDP Ratio General Government's Final Consumption Expenditure as a share of GDP is sourced from World Bank National Account data. The data covers the entire simulation range, from 1995 to 2020.

Government Debt-to-GDP Ratio I consider the general government debt, which includes the central government, local governments, the central bank, and social security funds.

1-Year Government Bond Yield I use the 1-year government bond real yield as a proxy for the interest rate in the model. The real yield is calculated using the CPI inflation rate from the Bank of Japan. One-week frequency data is used to calculate the annual average of the real yield.

Share of Foreign Investment in Government Portfolio I identify the share of foreign investment in the government portfolio using the Flow of Funds data from the Bank of Japan. The data is available from 1997 to 2020. The share of foreign investment in the government portfolio is defined as the ratio of the total amount of Foreign Securities divided by the total amount of assets.

The total value of foreign assets can be affected by the exchange rate. The data manual for the Flow of Funds data states that for foreign currency-denominated deposits and loans, exchange gains and losses arising from conversions to yen should be recorded as reconciliation amounts. However, due to limitations in source data, it is difficult to identify all exchange gains and losses. Therefore, term-on-term differences in outstanding amounts after converting to yen are generally estimated as financial transaction flows.

Return on Foreign Assets The Government Pension Investment Fund (GPIF) implements a diversified investment strategy, allocating its assets across domestic and foreign equities, domestic and foreign bonds, and alternative investments. GPIF reports the annual real return of its portfolio. I use the ex-post annual performance of GPIF's portfolio as the return on foreign assets in the model. The data is available from 2001 to 2020.

B Numerical Solution

The household's problem is solved using the endogenous grid method (EGM), as in Carroll (2006). This method leverages the first-order condition of the household's problem, which is presented as:

$$\frac{c_t^{\rho-1}}{1+\tau_{ct}} = \beta\psi_j \frac{1+r_{b,t+1}}{1+\gamma_{t+1}} \mathbb{E}_{\varepsilon'|\varepsilon}\left[\left(\frac{V_{t+1}}{\left(\mathbb{E}_t V_{t+1}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}}\right)^{1-\sigma-\rho} \frac{c_{t+1}^{\rho-1}}{1+\tau_{ct+1}}\right].$$

For all quantitative exercises, the same grids are used to discretize the household's state space. The state space is comprised of 12 points for ε, 2001 points of a, and 81 points of j. Given the high assumed value for relative risk aversion, $\sigma = 6$, it is crucial to use a sufficiently fine discretization for the asset holdings, a. A coarse grid for assets can introduce numerical errors, potentially overstating the variance of future cash-on-hand and consumption, which would lead to an exaggerated response in the household's precautionary savings.

C Alternative Fiscal Reforms after 2020

I additionally analyze the model's transition under alternative fiscal reform scenarios. Rather than cutting the government's spending-to-GDP ratio, the first alternative scenario reduces the pension replacement ratio from 32% to 23% in following 10 years after 2020. The reduced 23% replacement ratio is the maximum value that ensures the existence of a sustainable deficit steady state, given other parameters remain unchanged at the 2020 values. Results are shown in Figure 12. The second alternative scenario increases the consumption tax rate from 10% to 21% in following 10 years after 2020. Similarly, 21% consumption tax rate is the minimum value that ensures the existence of a sustainable deficit steady state. Results are shown in Figure 13.

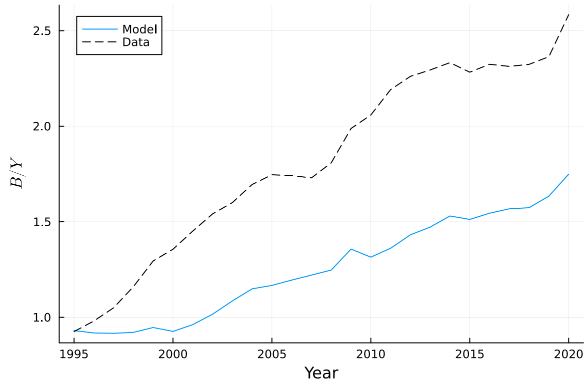
D Decomposition of Demographic Effects

To assess the impact of population aging on debt accumulation, I differentiate between the two sources of population aging – extended life expectancy and decreased fertility rates – to identify which demographic factor contributes more significantly to government debt run up.

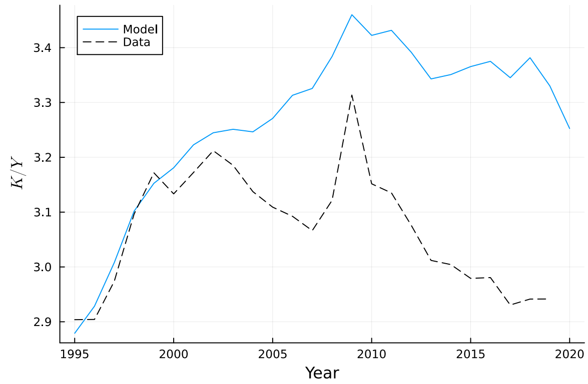
D.1 Effect of Lower Fertility Rate

As discussed in Section A, the threshold fertility rate that the working-age population does not decline is $1/45 \approx 0.0222$ ignoring the working-age population's mortality rate. As shown in Figure 5 (a), the fertility rate in Japan used to be above this threshold level in 1995, but it had declined steeply to the lowest in 2010 which was below the threshold. In following 10 years Japan's fertility rate has been recovering, but it is still below the threshold level. This declining trend of fertility rate has been thought of as one of the main driver of population aging that pressures the government's fiscal sustainability. In the second counterfactual scenario, I assume that the fertility rate remains constant at its initial level ($\omega_t = \omega_{1995}$) throughout the transition period. This scenario allows for an analysis of the impact of lower fertility rates on fiscal space. The results are in Table 5 and Figure 14.

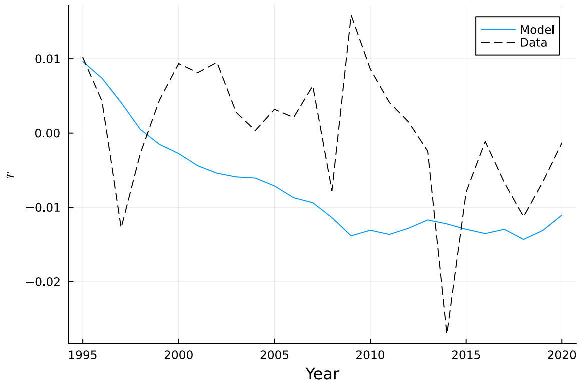
The effect of an undiminished fertility rate in the counterfactual scenario is immediately evident in the dependency ratio. In the baseline scenario, the dependency ratio surpasses 0.50 in 2020 and continues to rise above 0.70 thereafter. In contrast, the counterfactual scenario shows



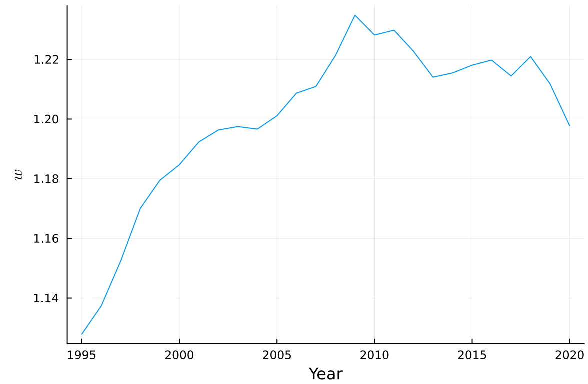
(a) Debt to GDP Ratio



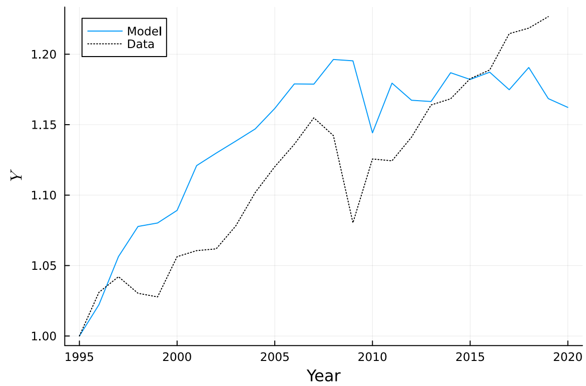
(b) Capital to GDP Ratio



(c) Interest Rate



(d) Wage Rate

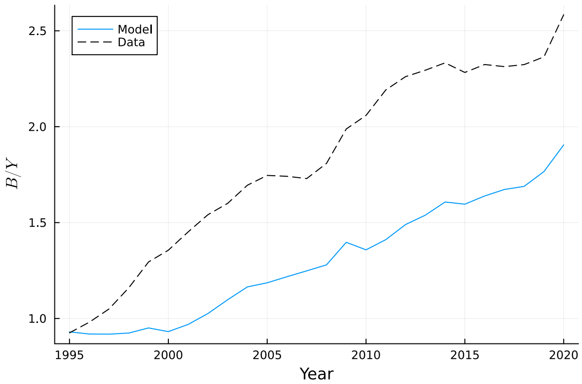


(e) GDP

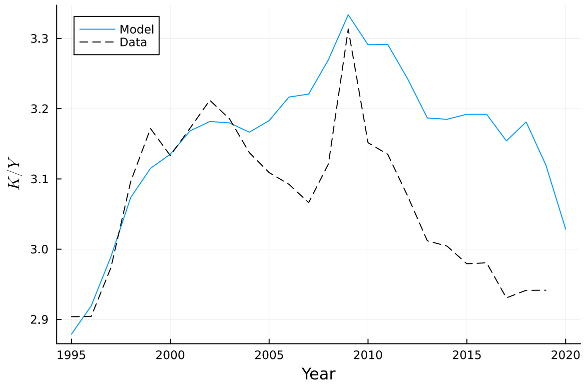
Figure 12: Model Transition under Pension Reforms

a dependency ratio of 0.4381 in 2020, converging to 0.4638 in the long run. This is underpinned by a continuously increasing working-age population in the counterfactual scenario, resulting from a fertility rate above the threshold fertility rate $1/45$.

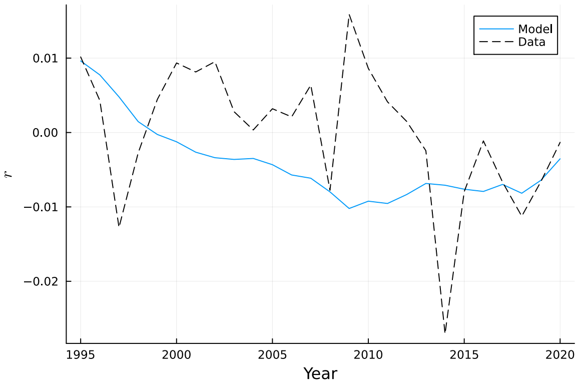
The more abundant workforce in the counterfactual scenario leads to lower wage rates compared to the baseline scenario. Specifically, in the short term (1995-2020), the counterfactual scenario exhibits lower wage rates and higher interest rates. Due to these higher interest rates, the increase in the debt-to-GDP ratio does not significantly differ from the baseline sce-



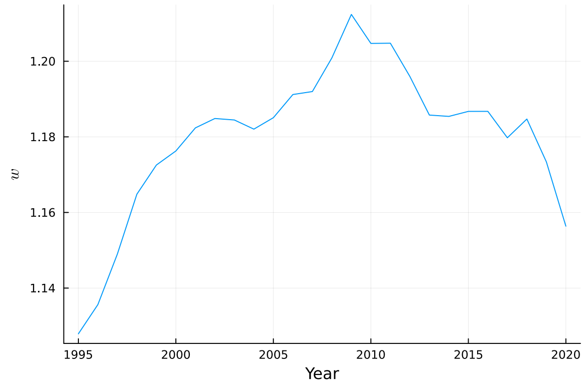
(a) Debt to GDP Ratio



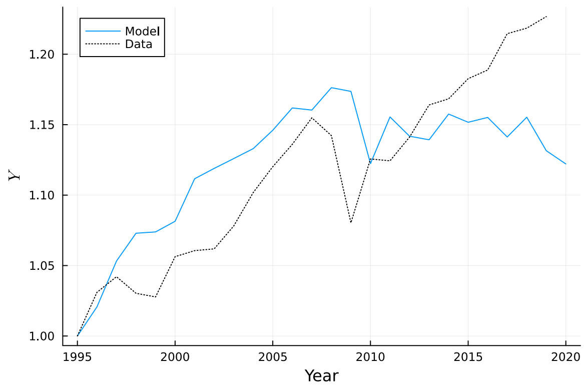
(b) Capital to GDP Ratio



(c) Interest Rate



(d) Wage Rate



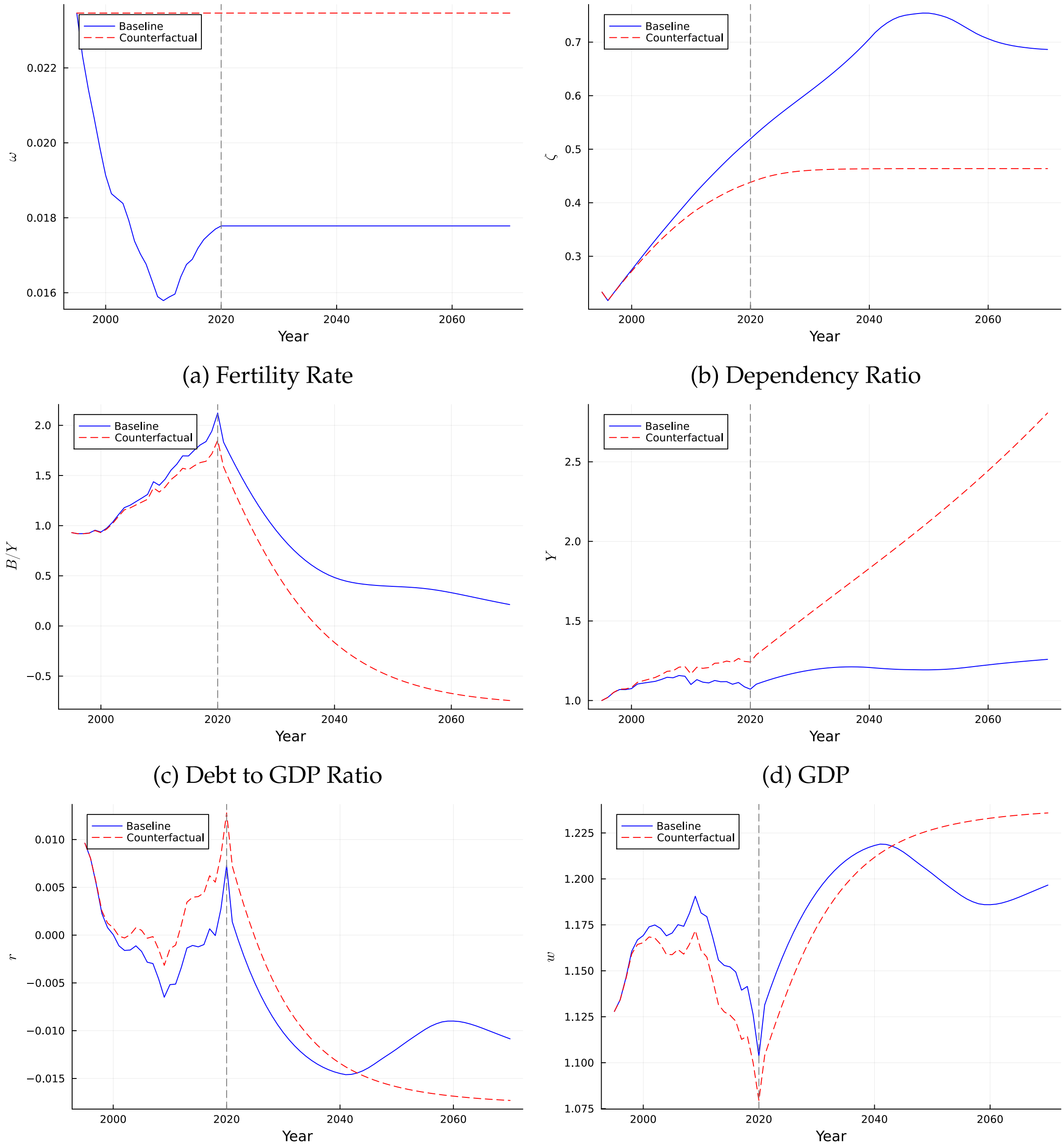
(e) GDP

Figure 13: Model Transition under VAT Reforms

nario, even with a low dependency ratio that prevents a substantial rise in pension liability. As shown in Table 5, the baseline explains a 121.11%p increase in debt-to-GDP, whereas the counterfactual explains a 93.95%p increase. This suggests that the lower birth rate did not significantly influence the increase in the Japanese government's debt between 1995 and 2020.

However, in the long term, a low birth rate is expected to significantly impact government debt. Projections for 2120 indicate that if the birth rate were to remain high, the government would be expected to have net assets rather than debt in the long run. Consequently, the

Capital-to-GDP ratio and household wage income are also projected to be higher under the counterfactual scenario compared to the baseline.



(a) Fertility Rate

(b) Dependency Ratio

(c) Debt to GDP Ratio

(d) GDP

(e) Interest Rate

(f) Wage Rate

Figure 14: Counterfactual Scenario: Constant Fertility Rate

D.2 Effect of Extended Life Expectancy

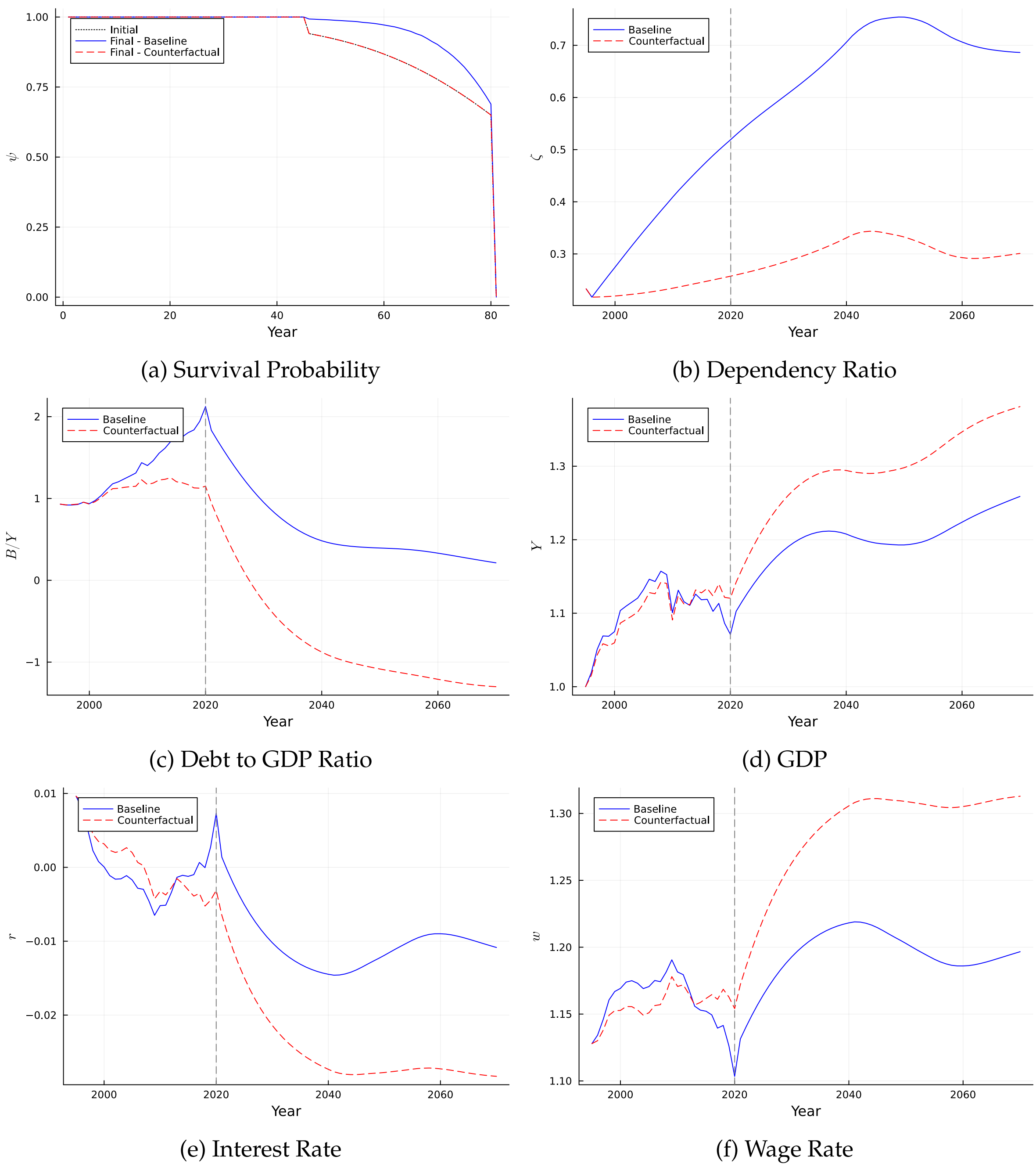
The third counterfactual scenario assumes that age-specific survival rates remain at 1995 levels. Between 1995 and 2020, Japan's life expectancy increased due to a continuous rise in age-specific survival rates. This is often thought of as a primary driver of population aging, with declining fertility rates. This counterfactual analysis illustrates the change in fiscal space resulting from increased life expectancy.

Similar to the scenario where the fertility rate remains high, the dependency ratio stays lower in the low-survival-rate scenario compared to the baseline scenario. A notable point is that the dependency ratio is even lower in the scenario without changes in survival rates compared to Section D.1. Table 5 shows that in 2020, the dependency ratio is 0.2574 in the low-survival-rate scenario, versus 0.4381 in the high-fertility-rate scenario. This suggests that the increase in survival rates played a greater role than the decline in fertility rates in population aging.

In the low-survival-rate scenario, a lower dependency ratio results in a smaller increase in government debt in the short run (1995-2020). It is calculated that the debt-to-GDP ratio would increase by only 23.98%p during 1995-2020 in the low-survival-rate scenario. This implies that a significant portion of the 121.11%p debt-to-GDP increase calculated in the baseline scenario originates from extended life expectancy.

Similar to the high-fertility-rate scenario, the relatively abundant labor force in the low-survival-rate scenario leads to lower wage rates and higher interest rates in the short term compared to the baseline. However, in the long term, as the government maintains a primary surplus and shifts from net debt to net asset position, it projects higher GDP and higher wage income.

E Robustness



(a) Survival Probability (b) Dependency Ratio

(c) Debt to GDP Ratio (d) GDP

(e) Interest Rate (f) Wage Rate

Figure 15: Counterfactual Scenario: Constant Survival Probability