Elements of Machine Learning
Assigment 1 - Problem 3

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November 11, 2021

Gauss-Markov theorem

Part 1

Explain in your own words what the Gauss-Markov theorem is and why it is important.

Gauss-Markov theorem states that the least squares estimator is the best unbiased estimate in a linear regression model. There is practically no other linear unbiased estimator with smaller variance. In this the error expectation is null and are uncorrelated with zero variance.

The Gauss-Markov model is important because when applied in a linear regression model, we can find the best unbiased estimator if Markov assumptions apply.

Part 2

Explain in your own words and write down the formal math formulation of the three Gauss-Markov error term assumptions.

1. All error terms have a mean of exactly 0.

$$E[\epsilon_i] = 0; i = 0, 1, \ldots, n$$

2. Errors are not dependent on one another or are not correlated.

$$Cov(\epsilon_i, \epsilon_j) = 0; i, j = 0, 1, \ldots, n \text{ and } i \neq j$$

3. The variance of the error terms is constant and finite, regardless of how the variables look. So to speak "homoscedastic"

$$Var(\epsilon_i) = \sigma^2; i = 0, 1, \ldots, n$$

Part 3

Is it also the best (in terms of test error) linear unbiased estimate (argue with the bias-variance trade-off)?

Yes, Gauss-Markov model provides the best linear unbiased estimate for least-square fit. That is to say the estimates have the smallest variance among all unbiased estimates. For a least square model, it suffers from high variance and given that estimates are unbiased, they are mostly non-zero, making the model difficult to interpret. With Gauss Markov theorem, we get unbiased estimates, but with least variance.