Universität des Saarlandes
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Lehrstuhl für Signalverarbeitung
NNIA Winter Term 2019/2020

# Exercise Sheet 2

Linear Algebra ctd.

**Deadline: 24.11.2020, 23:59**

## Exercise 2.1 - Eigenvalue decomposition & SVD (4 * 0.5 + 1 + 1 = 4 points)

The exercise is based on slides and chapters 2.7 *Eigendecomposition* and 2.8 *Singular value decomposition* from Deep Learning book.

Let $\boldsymbol{A}$ be a matrix with eigenvalues $a = -2$ and $b = 0$ and corresponding eigenvectors $\boldsymbol{v} = \begin{bmatrix} 0.6 \\ 0.8 \end{bmatrix}$ and $\boldsymbol{w} = \begin{bmatrix} 0.8 \\ -0.6 \end{bmatrix}$.

a) Draw vectors $\boldsymbol{v}$ and $\boldsymbol{w}$ in one color and vectors $\boldsymbol{Av}$ and $\boldsymbol{Aw}$ on the same graph in another color. Explain the transformation of the vectors.

b) Are there any other eigenvectors that have $a$ as their eigenvalue? If yes, provide an example; if no, give the reason why.

c) Compute matrix $\boldsymbol{A}$ and show how you did it.

d) What is the inverse of matrix $\begin{bmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{bmatrix}$? Explain how you found the inverse.

e) Provide definitions of **singular**, **positive definite**, **positive semi-definite**, **negative definite** and **negative semi-definite** matrices. Does matrix $\boldsymbol{A}$ belong to any of these types of matrices? Argue why / why not.

f) In which case do we perform singular value decomposition? Is there a case when we want to perform both eigenvalue and singular value decomposition? If yes, provide an example; if no, give the reason why.

## Exercise 2.2 - Principal Component Analysis (4 points)

Principal component analysis (PCA) is an example of applying eigendecomposition when working with data. Here is a great post by Zakaria Jaadi explaining how PCA works.

In Figure 1 you can see a dataset of birds, where each bird is described by its head length and skull size.

a) Draw the centered dataset (no need to be very exact) and mark all the principle components. Why do we need to center the dataset?

b) Represent the data in one-dimensional space after applying the encoding matrix. How do we compute the encoding matrix?

c) Represent the data in the original space after applying the decoding matrix. How do we compute the decoding matrix?

d) Give an example of a two-dimensional dataset with two classes for which PCA performs badly (i.e. the first principal component does not give us information needed to separate the classes). Support your answer with a drawing of the original dataset and the dataset reduced to one dimension. Provide a short explanation (ca. 2 sentences).

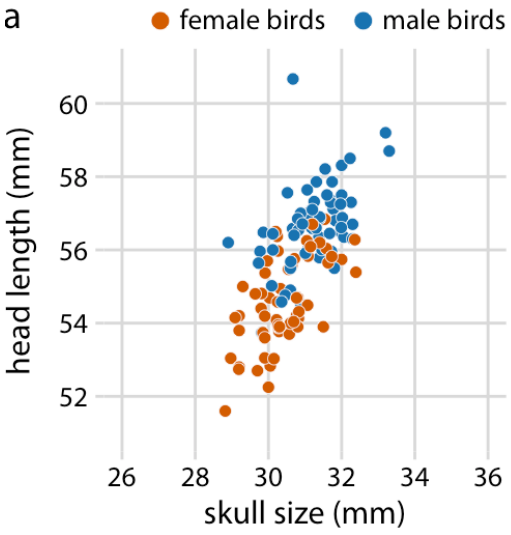


Figure 1: Bird dataset.

**Exercise 2.3 - Applications of eigendecomposition in machine learning** (2 points)

Describe (shortly, without mathematical details) at least two other machine learning methods that use eigendecomposition. Provide the source where you found the information.

# Submission instructions

The following instructions are mandatory. If you are not following them, tutors can decide to not correct your exercise.

- You have to submit the solutions of this assignment sheet as a team of 2-3 students.
- Hand in a **single** PDF file with your solutions.
- Therefore Make sure to write the student ID and the name of each member of your team on your submission.
- Your assignment solution must be uploaded by only **one** of your team members to the course website.
- If you have any trouble with the submission, contact your tutor **before** the deadline.