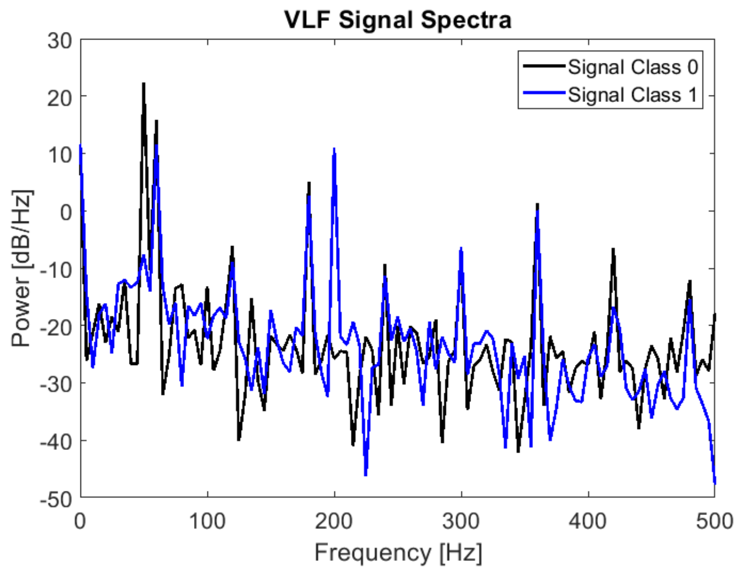
1. **Linear Regression with Intercept:** Consider the problem of a single input variable x and output variable y. Assume we want to fit a simple linear model of the form $\hat{y} = \theta_1 x + \theta_0$, and accordingly find the best value of θ_1 and θ_0. Assume we have N measurements of the input and output where the n-th measurement is given by x_n and y_n with corresponding estimate $\hat{y}_n = \theta_1 x_n + \theta_0$. To find the best model, we want to minimize the mean square error (MSE), $MSE = \frac{1}{N}\sum_{n=1}^{N}(\hat{y}_n - y_n)^2$.

   Find the optimum values of θ_1 and θ_0 that minimizes the MSE. Make sure to show all your work.

2. **VLF Signal Detection:** Very Low Frequency (VLF) signals are often used for underwater and underground communications. For this problem, two classes of signal spectra from a low frequency loop transmitter have been collected between 0 and 500 Hz. The figure below shows a picture of the transmitter along with sample cases of the two different signal classes.



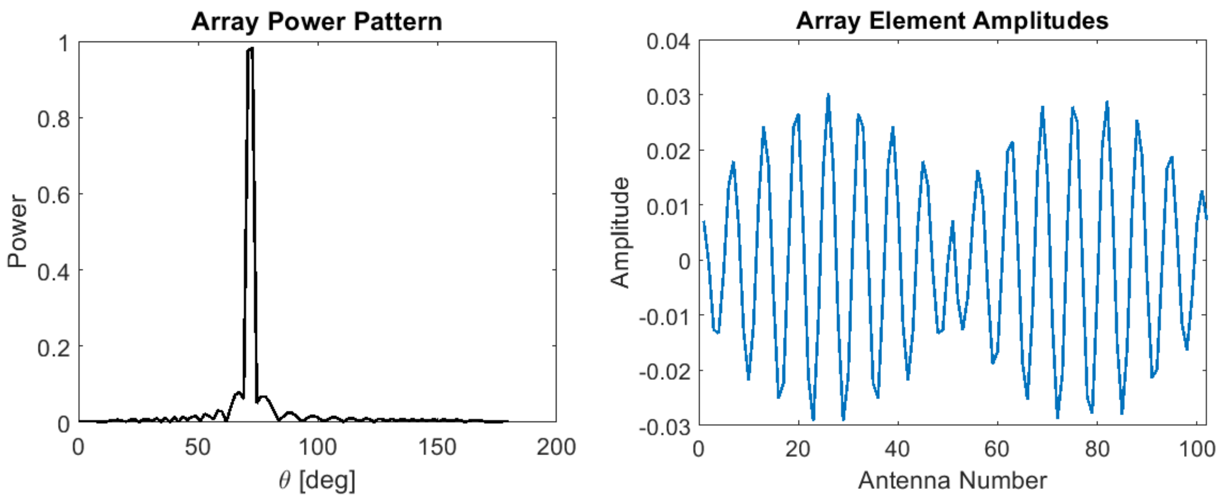
The input data consists of 1000 signal collections with 101 frequency bins (input features). The two signal classes are labeled '0' and '1' respectively. The input file is 'Signal_input_data.csv' and corresponding class labels are in 'Signal_class_data.csv'.

Split the data using an 80-20 split of training to test data. Using the training data, build a classifier model using:

a) Perceptron
b) Logistic Regression
c) Support Vector Machine with the RBF Kernel
d) Multi-layer Perceptron Neural Network with 2 hidden layers with 10 neurons each (ReLu activation function).

Make sure to note any regularization parameters you use in your models. Run of each of the above models on the testing data and show a confusion matrix. *Which model is the best option for this dataset?*

3. **Antenna Array Design:** Antenna arrays are commonly used for creating directive beams for radio communications. Arrays consist of several antenna elements where each one is driven by a different signal, this allows a beam pattern to be engineered mathematically. The input dataset 'Antenna_beam_data.csv' contains 10,000 beam patterns (power as a function of angle) for a specific antenna array where each beam pattern has been collected for 100 angles (so there are 10,000 rows and 100 columns). The output dataset 'Array_data.csv' consists of the corresponding antenna amplitudes (for 102 antenna elements). The figure below shows an example of the beam power pattern (left) and the corresponding amplitudes of each of the 102 antenna elements (right). For this problem you will design models that can determine the antenna element amplitudes that can generate a beam pattern.



a) Split the data using an 80-20 split of training to test data. Using the training data, build a regression model that can predict the array element amplitudes for a desired beam pattern using:
    i. Linear Regression
    ii. Multi-layer Perceptron Neural Network with 2 hidden layers with 100 neurons each (ReLu activation function)
b) Plot a comparison of one testing example output alongside the predicted output for both models.
c) For the models from (a) make a scatterplot of the predicted output vs the actual output for all the testing data and calculate the RMSE of both models.
d) Which model is the better option?

Hint Make sure to scale both the input and output before fitting the models.*