Machine Translation: Summer Term 2021

A Bluffer's Guide to NNs

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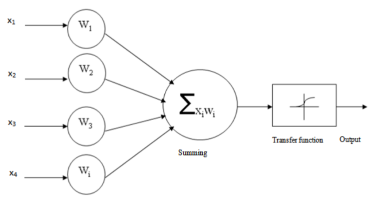
1. **In your own words, please describe supervised machine learning. Think about data, descriptive features, labels to be predicted, finding a function that describes the data, models, model bias, parameters, error functions, what is adjusted by learning, how learning is driven by the error, how learning tries to minimize error, overfitting/underfitting the data, generalizing to new unseen data, training test, and development data, etc. Use an example, if you find this useful.**
   In supervised learning, learning happens with the help of labeled data and a function is learned that describes the given data well. The function uses descriptive features underlying the labeled data to do the learning process. We call this learned function a trained model. During the training process, the model initializes some random parameters weights and biases and after each iteration, the model computes an error for each prediction made with respect to the actual output. The model further minimizes this error recursively by updating the weights and biases. Once the error is well below a tolerance level, we call the model trained and ready to predict labels of the unlabelled examples.

2. **Given the following AN:**



   where $x_1 = 0.9, x_2 = -0.3, x_3 = 1.2, x_4 = 0.1$ **and** $w_1 = 0.5, w_2 = 0.8, w_3 = -1.1, w_2 = 0.2$ **please compute**

$$z = \sum_{i=1}^{4} x_i \cdot w_i$$
$$a = sigmoid(z)$$

$$z = \sum_{i=1}^{4} x_i \cdot w_i$$
$$z = (0.9 \cdot 0.5) + (-0.3 \cdot 0.8) + (1.2 \cdot -1.1) + (0.1 \cdot 0.2)$$
$$a = sigmoid(-1.09)$$
$$a = \frac{1}{1 + e^{-(-1.09)}}$$
$$a = 0.748$$

3. **Please write the input in (2) above as a column vector $\boldsymbol{x}$. What is the transpose $\boldsymbol{x^T}$ of $\boldsymbol{x}$? Please write the weights in (2) above as a column vector $\boldsymbol{w}$. What is the transpose $\boldsymbol{w^T}$ of $\boldsymbol{w}$?**

$$\boldsymbol{x} = \begin{bmatrix} 0.9 \\ -0.3 \\ 1.2 \\ 0.1 \end{bmatrix} \qquad \boldsymbol{x^T} = \begin{bmatrix} 0.9 & -0.3 & 1.2 & 0.1 \end{bmatrix}$$

$$\boldsymbol{w} = \begin{bmatrix} 0.5 \\ 0.8 \\ -1.1 \\ 0.2 \end{bmatrix} \qquad \boldsymbol{w^T} = \begin{bmatrix} 0.5 & 0.8 & -1.2 & 0.2 \end{bmatrix}$$

4. **Vectorise the AN in (2) above: please express the same AN as in (2) above in terms of an input column vector $\boldsymbol{x}$ and a weight column vector $\boldsymbol{w}$ (with the same values as in (2) above) in terms of an inner product between the vectors and an element-wise application of the *sigmoid* to the result of the inner product. Please do this once with w followed by x, and once the other way around. (Make sure you remember how to place the transpose to make this work).**

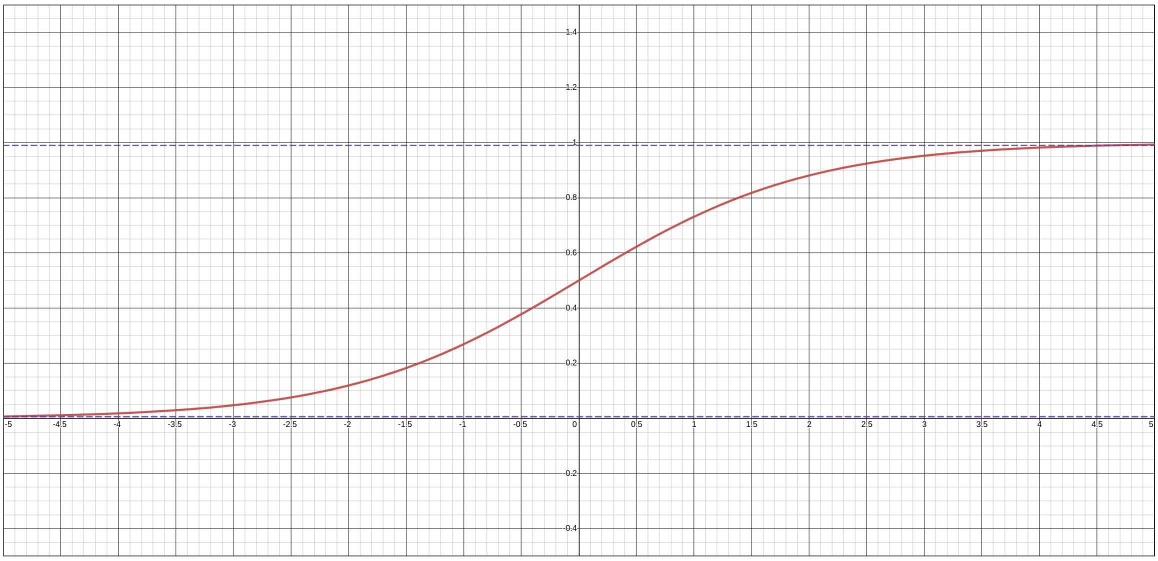
$$\begin{aligned}
\boldsymbol{O} &= sigmoid(\boldsymbol{w^T} \cdot \boldsymbol{x}) \\
&= sigmoid(\begin{bmatrix} 0.5 & 0.8 & -1.2 & 0.2 \end{bmatrix} \cdot \begin{bmatrix} 0.9 \\ -0.3 \\ 1.2 \\ 0.1 \end{bmatrix}) \\
&= sigmoid(-1.09) \\
&= 0.748 \\
\boldsymbol{O} &= sigmoid(\boldsymbol{x^T} \cdot \boldsymbol{w}) \\
&= sigmoid(\begin{bmatrix} 0.9 & -0.3 & 1.2 & 0.1 \end{bmatrix} \cdot \begin{bmatrix} 0.5 \\ 0.8 \\ -1.1 \\ 0.2 \end{bmatrix}) \\
&= sigmoid(-1.09) \\
&= 0.748
\end{aligned}$$

5. **Please complete the definition of the sigmoid below**

$$sigmoid(z) = \cdots$$

and draw the sigmoid function in a coordinate system where $z \in [-5, 5]$ and $sigmoid(z) \in [-0.5, 1.5]$. What are $sigmoid(0)$, $sigmoid(-4)$ and $sigmoid(4)$?

$$\begin{aligned}
sigmoid(z) &= \frac{1}{1 + e^{-x}} \\
sigmoid(0) &= \frac{1}{1 + e^{-0}} = 0.5 \\
sigmoid(4) &= \frac{1}{1 + e^{-4}} = 0.98 \\
sigmoid(-4) &= \frac{1}{1 + e^{-(-4)}} = 0.017
\end{aligned}$$

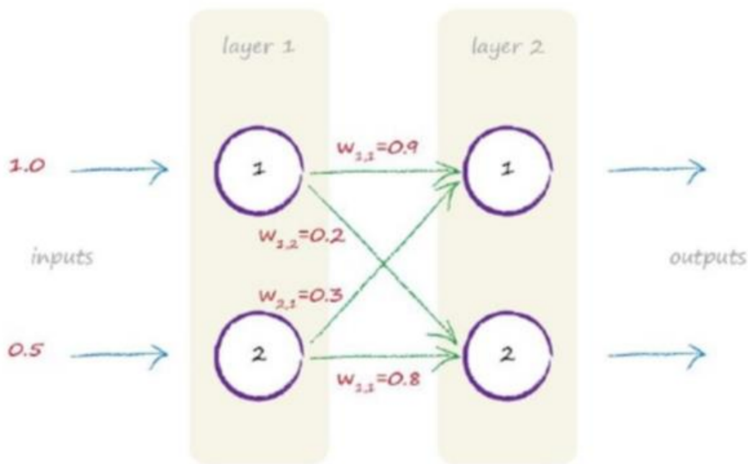


6. **Please work out the inner product between the following two matrices:**

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 18 & 22 \\ 42 & 50 \end{bmatrix}$$

7. **Given the following simple FFNN, please compute the activations (the outputs) of the two ANs in layer 2. Please assume that layer 1 ANs are just input neurons ("sensors") without activation functions, and that layer 2 ANs have sigmoid activation functions.**



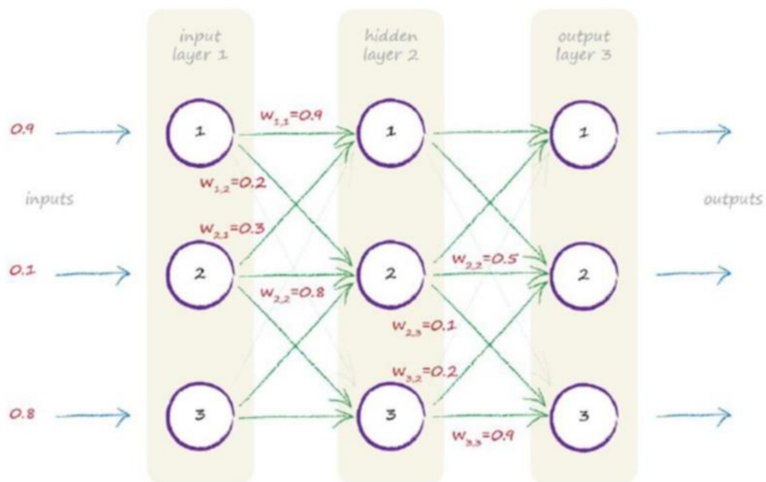
$$
\begin{aligned}
\boldsymbol{W}_{\texttt{input hidden}} &= \begin{bmatrix} w_{11} & w_{21} \\ w_{12} & w_{22} \end{bmatrix} \\
&= \begin{bmatrix} 0.9 & 0.3 \\ 0.2 & 0.8 \end{bmatrix} \\
\boldsymbol{X}_{\texttt{hidden}} &= \boldsymbol{W} \cdot \boldsymbol{X} \\
&= \begin{bmatrix} 0.9 & 0.3 \\ 0.2 & 0.8 \end{bmatrix} \cdot \begin{bmatrix} 0.1 \\ 0.5 \end{bmatrix} \\
&= \begin{bmatrix} 0.24 \\ 0.42 \end{bmatrix} \\
\boldsymbol{O}_{\texttt{hidden}} &= \sigma \boldsymbol{X}_{\texttt{hidden}} \\
&= \sigma \begin{bmatrix} 0.24 \\ 0.42 \end{bmatrix} \\
&= \begin{bmatrix} 0.55 \\ 0.60 \end{bmatrix}
\end{aligned}
$$

8. **Please vectorise the simple FFNN in (7) above: define an input column vector $\boldsymbol{I}$ and a weight matrix $\boldsymbol{W}$ such that output column vector $O = sigmoid(W \bullet I)$, where "•" is the inner product, and $\boldsymbol{I}$ and $\boldsymbol{O}$ are column vectors.**

$$
\begin{aligned}
\boldsymbol{I} &= \begin{bmatrix} 0.1 \\ 0.5 \end{bmatrix} \\
\boldsymbol{W} &= \begin{bmatrix} 0.9 & 0.3 \\ 0.2 & 0.8 \end{bmatrix} \\
\boldsymbol{O}_{\texttt{hidden}} &= sigmoid(\boldsymbol{W} \cdot \boldsymbol{I}) \\
&= sigmoid(\begin{bmatrix} 0.9 & 0.3 \\ 0.2 & 0.8 \end{bmatrix} \cdot \begin{bmatrix} 0.1 \\ 0.5 \end{bmatrix}) \\
&= sigmoid(\begin{bmatrix} 0.25 \\ 0.42 \end{bmatrix}) \\
&= \begin{bmatrix} 0.55 \\ 0.60 \end{bmatrix}
\end{aligned}
$$

9. **Why do we vectorize ANs and ANNs?**
   Vectorization helps perform complex calculations in neural networks easily and flexibly. It is efficient and easy to interpret.

10. **Given the following FFNN**



with

$$\boldsymbol{I} = \begin{bmatrix} 0.9 \\ 0.1 \\ 0.8 \end{bmatrix}$$

$$\boldsymbol{W}_{\texttt{input hidden}} = \begin{bmatrix} 0.9 & 0.3 & 0.4 \\ 0.2 & 0.8 & 0.2 \\ 0.1 & 0.5 & 0.6 \end{bmatrix}$$

$$\boldsymbol{W}_{\texttt{hidden output}} = \begin{bmatrix} 0.3 & 0.7 & 0.5 \\ 0.6 & 0.5 & 0.2 \\ 0.8 & 0.1 & 0.9 \end{bmatrix}$$

please work out :

$$\boldsymbol{X}_{\texttt{hidden}} = \boldsymbol{W}_{\texttt{input hidden}} \cdot \boldsymbol{I}$$
$$\boldsymbol{O}_{\texttt{hidden}} = sigmoid(\boldsymbol{X}_{\texttt{hidden}})$$
$$\boldsymbol{X}_{\texttt{output}} = \boldsymbol{W}_{\texttt{hidden output}} \cdot \boldsymbol{O}_{\texttt{hidden}}$$

and

$$\boldsymbol{O}_{\texttt{output}} = sigmoid(\boldsymbol{X}_{\texttt{output}})$$

$$\begin{aligned}
\boldsymbol{X}_{\texttt{hidden}} &= \boldsymbol{W}_{\texttt{input hidden}} \cdot \boldsymbol{I} \\
&= \begin{bmatrix} 0.9 & 0.3 & 0.4 \\ 0.2 & 0.8 & 0.2 \\ 0.1 & 0.5 & 0.6 \end{bmatrix} \cdot \begin{bmatrix} 0.9 \\ 0.1 \\ 0.8 \end{bmatrix} \\
&= \begin{bmatrix} 1.16 \\ 0.42 \\ 0.62 \end{bmatrix} \\
\boldsymbol{O}_{\texttt{hidden}} &= sigmoid(\boldsymbol{X}_{\texttt{hidden}}) \\
&= sigmoid(\begin{bmatrix} 1.16 \\ 0.42 \\ 0.62 \end{bmatrix}) \\
&= \begin{bmatrix} 0.76 \\ 0.60 \\ 0.65 \end{bmatrix} \\
\boldsymbol{X}_{\texttt{output}} &= \boldsymbol{W}_{\texttt{hidden output}} \cdot \boldsymbol{O}_{\texttt{hidden}} \\
&= \begin{bmatrix} 0.3 & 0.7 & 0.5 \\ 0.6 & 0.5 & 0.2 \\ 0.8 & 0.1 & 0.9 \end{bmatrix} \cdot \begin{bmatrix} 0.76 \\ 0.60 \\ 0.65 \end{bmatrix} \\
&= \begin{bmatrix} 0.973 \\ 0.886 \\ 0.773 \end{bmatrix} \\
\boldsymbol{O}_{\texttt{output}} &= sigmoid(\boldsymbol{X}_{\texttt{output}}) \\
&= sigmoid(\begin{bmatrix} 0.973 \\ 0.886 \\ 0.773 \end{bmatrix}) \\
&= \begin{bmatrix} 0.725 \\ 0.708 \\ 0.675 \end{bmatrix}
\end{aligned}$$

NOTE:

1. Check alternative activation functions (plot and equation)
2. Check XOR

References