# Assignment 3

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March 14, 2022

All the source codes are available here.

## 1 MMSE estimation

For the noise model:

$$Y = X + Z \tag{1}$$

The MMSE estimate is given by

$$\begin{aligned}
\hat{X} &= E(X|Y) \\
&= \frac{\int X * P_{y|x}(Y|X)P_x(X)dX}{\int P_{y|x}(Y|X)P_x(X)dX} == \frac{\int X * P_z(Y-X)P_x(X)dX}{\int P_z(Y-X)P_x(X)dX} \\
&= \frac{\frac{1}{2\sqrt{2\pi\sigma_x\sigma_z}} \int x \exp - \frac{(y-x)^2}{2\sigma_z^2} \exp - \frac{|x|}{\sigma_x} dx}{\frac{1}{2\sqrt{2\pi\sigma_x\sigma_z}} \int \exp - \frac{(y-x)^2}{2\sigma_z^2} \exp - \frac{|x|}{\sigma_x} dx} == \frac{\int x \exp - \frac{(y-x)^2}{2\sigma_z^2} \exp - \frac{|x|}{\sigma_x} dx}{\int \exp - \frac{(y-x)^2}{2\sigma_z^2} \exp - \frac{|x|}{\sigma_x} dx}
\end{aligned} \tag{2}$$

where $X \sim L(0, \sigma_x)$ and $Z \sim N(0, \sigma_z)$ and $\sigma_z = \sqrt{0.1}$

- The expectation value has no closed form solution, so I have integrated it numerically
- I have used quad function of scipy for numerical integration
- For X given Y, as the value of X is in neighbourhood of Y, we are integrating from -Y to Y and the Y is varied from -4*Vx to 4*Vx.
- The estimate $\hat{x}$ as a function of y is shown in Fig. 1.

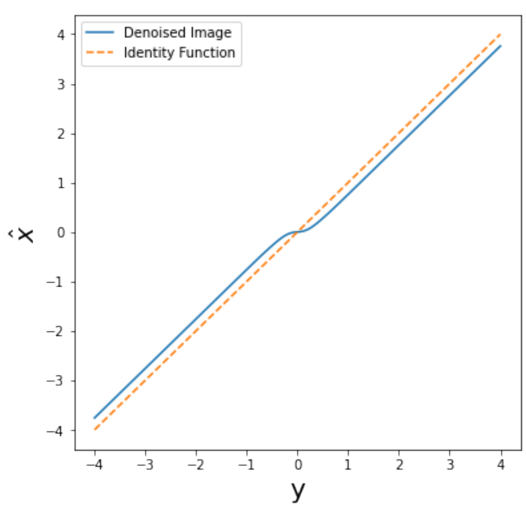



Figure 1: Value of $\hat{x}$

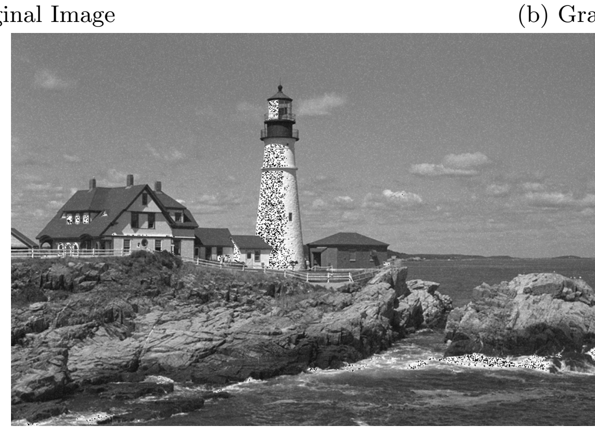
# 2 Question 2

## 2.1 Part 1

- I have read the "lighthouse2.bmp" image and converted it into Gray scale and saved it as "gray-image.png"
- A gaussian noise with $\sigma_z^2 = 100$ is added to the Gray scale image. I have generated this noise with randn function of cv2
- The noisy image is clipped between 0 and 255
- Original, Grayscale, Noisy images are shown in Fig. 2a, 2b, 2c

(a) Original Image

(b) Gray Image



(c) Noisy Image

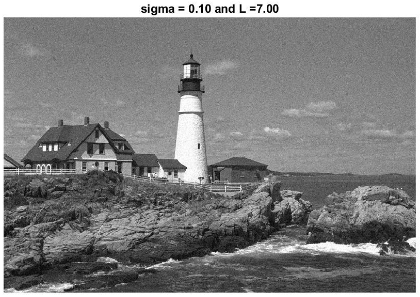
The noisy image is filtered with different low pass filters of kernel size $L = [3, 7, 11]$ and $\sigma = [0.1, 1, 2, 4, 8]$. The mean square error for different combination of $\sigma$ and L are shown in table 1

Table 1: Comparison of MSE for different kernel size and $\sigma$

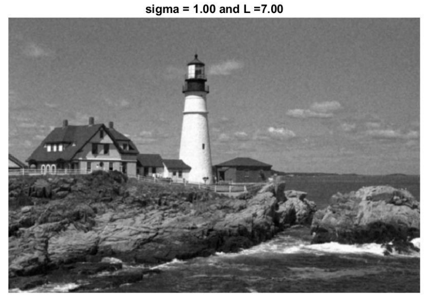
| MSE(L,$\sigma$) | 0.1 | 1 | 2 | 4 | 8 |
|---|---|---|---|---|---|
| 3 | 30.3924 | 43.6770 | 45.7789 | 46.3646 | 46.4587 |
| 7 | 30.3924 | 46.1494 | 52.3061 | 54.3864 | 54.8935 |
| 11 | 30.3924 | 46.1379 | 53.4426 | 57.2564 | 58.3677 |

We can see that the least MSE is with $\sigma = 0.1$ and $L = 3$. It is noticed that with increase in the size for filter keeping $\sigma$ fixed or vice-versa or both, the sharpness of the image increase and the image becomes more and more blurry. Here I am showing the comparison for $L = 7$ and vary $\sigma$. As the value of $\sigma$ increases the image gets more blurred losses details. As increase in $\sigma$ decreases the bandwidth of LPF in frequency domain.

sigma = 0.10 and L =7.00



sigma = 1.00 and L =7.00



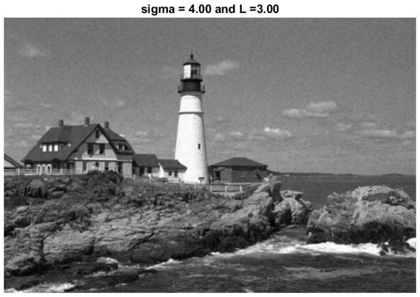
sigma = 2.00 and L =7.00

sigma = 4.00 and L =7.00

sigma = 8.00 and L =7.00

Here I am showing the comparison for fixed $\sigma = 4$ and varying L. It can be seen that with varying L as well the image gets more and more blurry

sigma = 4.00 and L =3.00



sigma = 4.00 and L =7.00

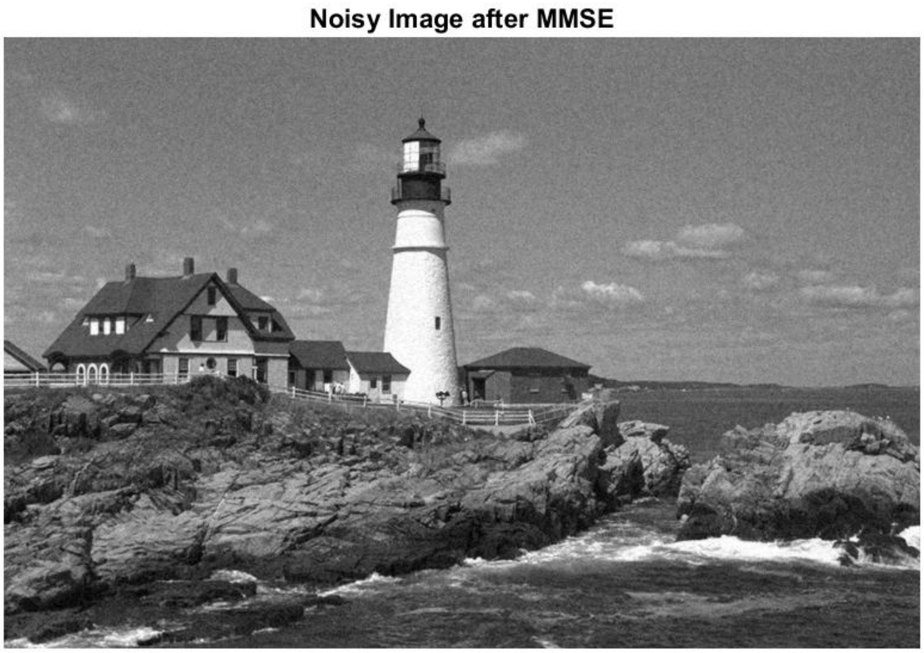
### 2.2 Part 2: MMSE filter

We split the noisy image Y into $Y_1$ and $\mu_Y$

$$Y_1 = Y - \mu_Y \tag{3}$$

where $Y_1$ is the high pass image, $\mu_Y$ is the low pass image, MMSE estimate of X i.e $\hat{X}$ is given by:

$$\begin{aligned}
\hat{X} &= \hat{X}_1 + \mu_Y \\
\hat{X}_1 &= \frac{\sigma_{X1}^2}{\sigma_{Y1}^2} Y_1 \\
\sigma_{X1}^2 &= \sigma_{Y1}^2 - \sigma_{Z1}^2 \\
\sigma_{Z1}^2 &= \sigma_Z^2 (1 + \sum_{i=-1}^{1} \sum_{j=-1}^{1} ) h[i,j]^2 - 2h[0,0])
\end{aligned} \tag{4}$$

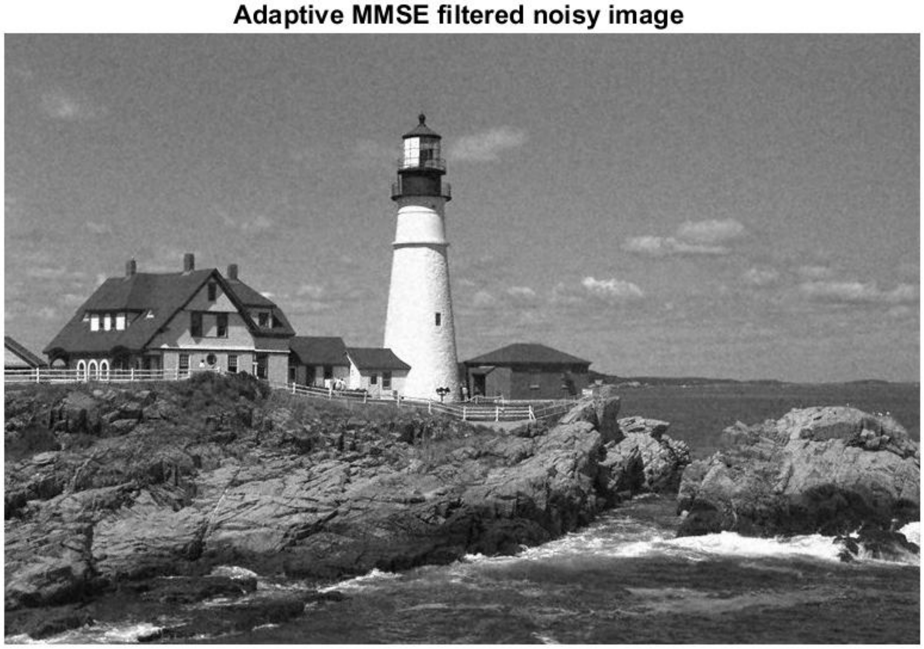



### 2.3 Part 3: Adaptive MMSE filter

In this part we use the MMSE filter on small patches of images of size 11X11 and shift the patches by 5 pixels. The pixels which belong to multiple patches will be averaged out to get the final value. **** The technique used is similar to the ones used in question above with slight modification.

- If in any patch $\sigma_{Y1}^2 < \sigma_{Z1}^2$, then $\sigma_{Y1}^2 = \sigma_{Z1}^2$
- In above case $\sigma_{X1}^2 = 0$

Adaptive MMSE filtered noisy image



# 3 Ques 3

## 3.1 Part 1:

The output of the low pass gaussian filter as shown in Fig. ?? is sharpened using the high pass filter. MSE is calculated for different values of gain and the minimum $MSE = 69.3721$ is achieved for $gain = 1$. The output for $gain = 1$ is shown in Fig. 3

Figure 3: Low pass filtered image

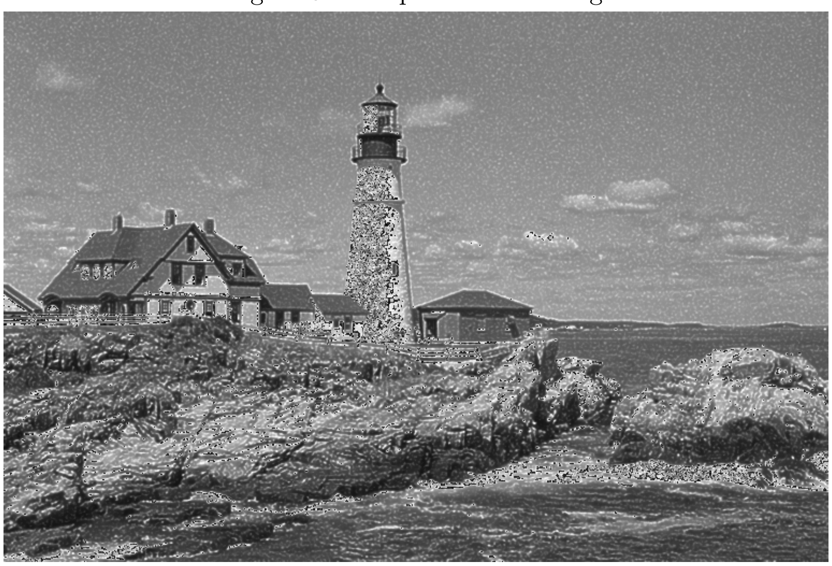


Figure 4: Sharpened image

## 3.2 Part 2:

In the sharpened image the smoother region has become rough. It doesn't look good in terms of the perception of quality. Here I am showing the images with a larger gain for a better comparison.

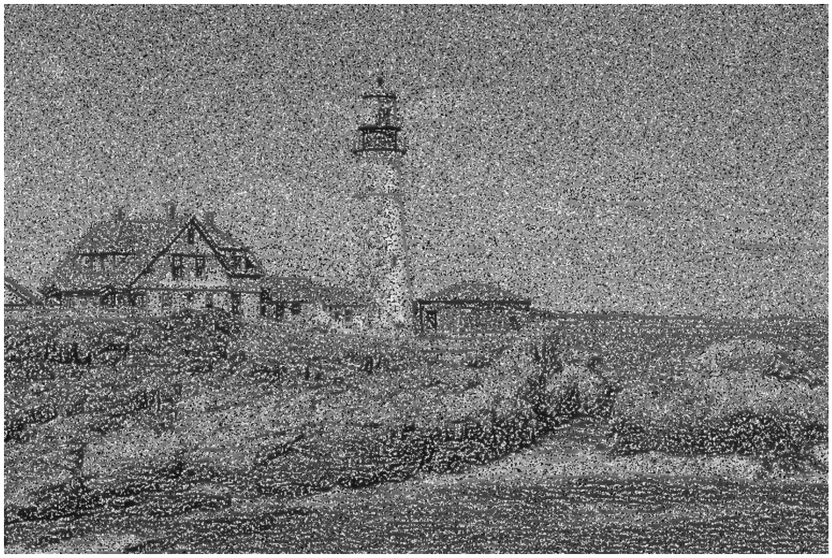


Figure 5: Sharpened Image with a larger gain = 50

As we increase the gain, the image is becoming more noisy and doesn't look good.