CI2611 - Algoritmos y estructuras I

Parcial 1

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Tarea 4 2012

Problema 1-d Probar la correctitud.

$\{N > 0\}$

$pal, k := true, 0$

$\{inv : pal \equiv (\forall i | 0 \leq i < k : S[i] = S[N - i - 1]) \wedge 0 \leq k \leq N\}\{cota : N - k\}$

$do\ (k \neq N) \wedge pal \rightarrow$

$\quad pal, k := S[k] = S[N - k - 1], k + 1$

od

$\{pal \equiv (\forall i | 0 \leq i < N : S[i] = S[N - i - 1])\}$

Demostración

Prueba 1.a: $\{P \wedge B0\}S0\{P\}$

$\{pal \equiv (\forall i|0 \leq i < k : S[i] = S[N-i-1]) \wedge 0 \leq k \leq N \wedge (k \neq N \wedge pal)\}$

$pal, k := (S[k] = S[n-k-1]), k+1$

$\{pal \equiv (\forall i|0 \leq i < k : S[i] = S[N-i-1])\} \wedge 0 \leq k \leq N$

Por regla de la asignación:

$pal \equiv (\forall i|0 \leq i < k : S[i] = S[N-i-1]) \wedge 0 \leq k \leq N \wedge (k \neq N \wedge pal) \Rightarrow$

$(pal \equiv (\forall i|0 \leq i < k : S[i] = S[N-i-1]) \wedge 0 \leq k \leq N)(pal, k := (S[k] = S[n-k-1]), k+1)$

Suposicion del antecedente, empezando con el consecuente para llegar a true:

$(pal \equiv (\forall i|0 \leq i < k : S[i] = S[N-i-1]) \wedge 0 \leq k \leq N)(pal, k := (S[k] = S[n-k-1]), k+1)$

$\equiv \quad \langle Sustitucion\ textual \rangle$

$S[k] = S[n-k-1] \equiv (\forall i|0 \leq i < k+1 : S[i] = S[N-i-1]) \wedge 0 \leq k+1 \leq N$

$\equiv \quad \langle Extrayendo\ ultimo\ elemento\ cuantificador \rangle$

$S[k] = S[n-k-1] \equiv (\forall i|0 \leq i < k : S[i] = S[N-i-1]) \wedge S[k] = S[N-k-1] \wedge 0 \leq k+1 \leq N$

$\equiv \quad \langle p \wedge (p \equiv q) \equiv p \wedge q \rangle$

$S[k] = S[n-k-1] \wedge (\forall i|0 \leq i < k : S[i] = S[N-i-1]) \wedge S[k] = S[N-k-1] \wedge 0 \leq k+1 \leq N$

$\Rightarrow \quad \langle p \wedge q \Rightarrow p \rangle$

$(\forall i|0 \leq i < k : S[i] = S[N-i-1])$

$\equiv \quad \langle Hipotesis:\ pal \equiv (\forall i|0 \leq i < k : S[i] = S[N-i-1]) \equiv true \rangle$

$true$

Prueba 2: $P \wedge \neg B0 \Rightarrow Q$

$pal \equiv (\forall i | 0 \leq i < k : S[i] = S[N-i-1]) \wedge 0 \leq k \leq N \wedge (k = N \vee \neg pal) \Rightarrow pal \equiv (\forall i | 0 \leq i < N : S[i] = S[N-i-1])$

Suposición del antecedente:

$H0 : pal \equiv (\forall i | 0 \leq i < k : S[i] = S[N-i-1]) \equiv true$

$H1 : 0 \leq k \leq N \equiv true$

$H2 : (k = N \vee \neg pal) \equiv true$

$true$

$\equiv \quad \langle H2 \rangle$

$k = N \vee \neg pal$

Por casos:

$k = N$

$\equiv \quad \langle H0 \rangle$

$k = N \wedge pal \equiv (\forall i | 0 \leq i < k : S[i] = S[N-i-1])$

$\equiv \quad \langle Sustitucion\ k = N \rangle$

$k = N \wedge pal \equiv (\forall i | 0 \leq i < N : S[i] = S[N-i-1])$

$\Rightarrow \quad \langle p \wedge q \Rightarrow p \rangle$

$pal \equiv (\forall i | 0 \leq i < N : S[i] = S[N-i-1])$

$\therefore\ H0 \wedge H1 \wedge H2 \Rightarrow pal \equiv (\forall i | 0 \leq i < N : S[i] = S[N-i-1])$

Por Sup. Antecedente + Caso: $k = N$

$\neg pal$

$\equiv \quad \langle H0 \rangle$

$\neg true$

$\equiv \quad \langle \neg true \equiv false \rangle$

$false$

$\equiv \quad \langle false \Rightarrow p \rangle$

$pal \equiv (\forall i | 0 \leq i < N : S[i] = S[N-i-1])$

$\therefore\ H0 \wedge H1 \wedge H2 \Rightarrow pal \equiv (\forall i | 0 \leq i < N : S[i] = S[N-i-1])$

Por Sup. Antecedente + Caso: $\neg pal$

$\therefore\ H0 \wedge H1 \wedge H2 \Rightarrow pal \equiv (\forall i | 0 \leq i < N : S[i] = S[N-i-1])$

Por Sup. Antecedente

Tarea 5 2012

Problema 3-d Calcular el índice académico de un trimestre. Suponga que en un arreglo de enteros de tamaño N se almacena la nota obtenida en cada materia y en otro arreglo se almacena el número de créditos correspondientes a esas materia. N representa el número de materias inscritas en el trimestre.

[

$const\ n, c : seq\ of\ int;$

$const\ N : int := |n|;$

$const\ M : int := |c|;$

$var\ i, sc : int;$

$var\ ia : float;$

$\{N > 0 \wedge M > 0 \wedge N = M\}$

$sc, i, ia := 0, 0, 0$

$\{inv : sc = (\sum k|0 \leq k < i : c[k]) \wedge ia = (\sum k|0 \leq k < i : n[k] * c[k]) \wedge 0 \leq i \leq M \wedge 0 \leq i \leq N \wedge N = M\}\{Cota : N - i\}$

$do\ i < N \rightarrow$

$sc, i, ia := c[i] + sc, i + 1, ia + n[i] * c[i]$

od

$\{wp.(ia := ia/sc).Q\}$

$ia := ia/sc$

$\{sc = (\sum k|0 \leq k < M : c[k]) \wedge ia = (\sum k|0 \leq k < N : n[k] * c[k])/sc)\}$

]

Hallar la precondición más débil de la asignación $wp.(ia := ia/sc).Q$.

$(sc = (\sum k|0 \leq k < M : c[k]) \wedge ia = (\sum k|0 \leq k < N : n[k] * c[k])/sc)(ia := ia/sc)$

$\equiv \quad \langle Sustitucion\ textual \rangle$

$sc = (\sum k|0 \leq k < M : c[k]) \wedge ia/sc = (\sum k|0 \leq k < N : n[k] * c[k])/sc$

$\equiv \quad \langle Aritemetica \rangle$

$sc = (\sum k|0 \leq k < M : c[k]) \wedge ia = (\sum k|0 \leq k < N : n[k] * c[k])$

Dado que la precondición más débil asegura correctitud, ya se tiene probada esta parte.

Prueba 0

$\{N > 0 \wedge M > 0 \wedge N = M\}$

$sc, i, ia := 0, 0, 0$

$\{inv : sc = (\sum k|0 \leq k < i : c[k]) \wedge ia = (\sum k|0 \leq k < i : n[k] * c[k])/sc) \wedge 0 \leq i \leq M \wedge 0 \leq i \leq N \wedge N = M\}$

Por regla de la asignación:

$N > 0 \wedge M > 0 \wedge N = M \Rightarrow$

$(sc = (\sum k|0 \leq k < i : c[k]) \wedge ia = (\sum k|0 \leq k < i : n[k] * c[k])/sc \wedge 0 \leq i \leq M \wedge 0 \leq i \leq N \wedge N = M)(sc, i, ia := 0, 0, 0)$

Método por fortalecimiento.

$(sc = (\sum k|0 \leq k < i : c[k]) \wedge ia = (\sum k|0 \leq k < i : n[k] * c[k])/sc \wedge 0 \leq i \leq M \wedge 0 \leq i \leq N \wedge N = M)(sc, i, ia := 0, 0, 0)$

$\equiv \quad \langle Sustitucion\ Textual \rangle$

$0 = (\sum k|0 \leq k < 0 : c[k]) \wedge 0 = (\sum k|0 \leq k < 0 : n[k] * c[k])/sc \wedge 0 \leq 0 \leq M \wedge 0 \leq 0 \leq N \wedge N = M$

$\equiv \quad \langle Rango\ vacio :\ 0 \leq k < 0 \equiv false; (\sum k|false : P) = 0 \rangle$

$0 = 0 \wedge 0 = 0/sc \wedge 0 \leq 0 \leq M \wedge 0 \leq 0 \leq N \wedge N = M$

$\equiv \quad \langle 0 = 0 \equiv true; a \leq b < c \equiv a \leq b \wedge b < c \rangle$

$0 \leq 0 \wedge 0 \leq M \wedge 0 \leq 0 \wedge 0 \leq N \wedge N = M$

$\equiv \quad \langle 0 \leq 0 \equiv true;\ a \leq b \equiv a < b \vee a = b \rangle$

$(0 < N \vee 0 = N) \wedge (0 < M \vee 0 = M) \wedge N = M$

$\Leftarrow \quad \langle p \Rightarrow p \vee q \rangle$

$0 < N \wedge 0 < M \wedge N = M$

$\equiv \quad \langle a < b \equiv b > a \rangle$

$N > 0 \wedge M > 0 \wedge N = M$

Prueba 1.a: $\{P \wedge B0\}S0\{P\}$

$\{sc = (\sum k|0 \leq k < i : c[k]) \wedge ia = (\sum k|0 \leq k < i : n[k] * c[k]) \wedge 0 \leq i \leq M \wedge 0 \leq i \leq N \wedge N = M \wedge i < N\}$

$sc, i, ia := c[i] + sc, i + 1, ia + n[i] * c[i]$

$\{sc = (\sum k|0 \leq k < i : c[k]) \wedge ia = (\sum k|0 \leq k < i : n[k] * c[k]) \wedge 0 \leq i \leq M \wedge 0 \leq i \leq N \wedge N = M\}$

Por regla de la asignación.

$sc = (\sum k|0 \leq k < i : c[k]) \wedge ia = (\sum k|0 \leq k < i : n[k] * c[k]) \wedge 0 \leq i \leq M \wedge 0 \leq i \leq N \wedge N = M \wedge i < N \Rightarrow (sc = (\sum k|0 \leq k < i : c[k]) \wedge ia = (\sum k|0 \leq k < i : n[k] * c[k]) \wedge 0 \leq i \leq M \wedge 0 \leq i \leq N \wedge N = M)(sc, i, ia := c[i] + sc, i + 1, ia + n[i] * c[i])$

Fortalecimiento.

$(sc = (\sum k|0 \leq k < i : c[k]) \wedge ia = (\sum k|0 \leq k < i : n[k] * c[k]) \wedge 0 \leq i \leq M \wedge 0 \leq i \leq N \wedge N = M)(sc, i, ia := c[i] + sc, i + 1, ia + n[i] * c[i])$

$\equiv \quad \langle Sustitucion\ Textual \rangle$

$sc + c[i] = (\sum k|0 \leq k < i+1 : c[k]) \wedge ia + n[i] * c[i] = (\sum k|0 \leq k < i+1 : n[k] * c[k]) \wedge 0 \leq i+1 \leq M \wedge 0 \leq i+1 \leq N \wedge N = M$

$\equiv \quad \langle Sacando\ ultimo\ termino \rangle$

$sc + c[i] = (\sum k|0 \leq k < i : c[k]) + c[i] \wedge ia + n[i] * c[i] = (\sum k|0 \leq k < i : n[k] * c[k]) + n[i] * c[i] \wedge 0 \leq i+1 \leq M \wedge 0 \leq i+1 \leq N \wedge N = M$

$\equiv \quad \langle Aritmetica \rangle$

$sc = (\sum k|0 \leq k < i : c[k]) \wedge ia = (\sum k|0 \leq k < i : n[k] * c[k]) \wedge 0 \leq i+1 \leq M \wedge 0 \leq i+1 \leq N \wedge N = M$

$\equiv \quad \langle a \leq b < c \equiv a \leq b \wedge b < c \rangle$

$sc = (\sum k|0 \leq k < i : c[k]) \wedge ia = (\sum k|0 \leq k < i : n[k] * c[k]) \wedge 0 \leq i+1 \wedge i+1 \leq M \wedge 0 \leq i+1 \wedge i+1 \leq N \wedge N = M$

$\equiv \quad \langle Aritmetica \rangle$

$sc = (\sum k|0 \leq k < i : c[k]) \wedge ia = (\sum k|0 \leq k < i : n[k] * c[k]) \wedge -1 \leq i \wedge i \leq M - 1 \wedge -1 \leq i \wedge i \leq N - 1 \wedge N = M$

$\Leftarrow \quad \langle a + 1 \leq b \Rightarrow a \leq b \rangle$

$sc = (\sum k|0 \leq k < i : c[k]) \wedge ia = (\sum k|0 \leq k < i : n[k] * c[k]) \wedge 0 \leq i \wedge i \leq M - 1 \wedge 0 \leq i \wedge i \leq N - 1 \wedge N = M$

$\Leftarrow \quad \langle a \leq b \wedge a \neq b \Rightarrow a \leq b - 1 \rangle$

$sc = (\sum k|0 \leq k < i : c[k]) \wedge ia = (\sum k|0 \leq k < i : n[k] * c[k]) \wedge 0 \leq i \wedge i \leq M \wedge i \neq M \wedge 0 \leq i \wedge i \leq N \wedge i \neq N \wedge N = M$

$\equiv \quad \langle Sustitucion\ N = M \rangle$

$sc = (\sum k|0 \leq k < i : c[k]) \wedge ia = (\sum k|0 \leq k < i : n[k] * c[k]) \wedge 0 \leq i \wedge i \leq N \wedge i \neq N \wedge N = M$

$\equiv \quad \langle a \leq b \wedge a \neq b \equiv a \leq b \wedge a < b \rangle$

$sc = (\sum k|0 \leq k < i : c[k]) \wedge ia = (\sum k|0 \leq k < i : n[k] * c[k]) \wedge 0 \leq i \wedge i \leq N \wedge i < N \wedge N = M$

$\Leftarrow \quad \langle p \wedge q \Rightarrow p \rangle$

$sc = (\sum k|0 \leq k < i : c[k]) \wedge ia = (\sum k|0 \leq k < i : n[k] * c[k]) \wedge 0 \leq i \wedge i < M \wedge 0 \leq i \wedge i < N \wedge i < N \wedge N = M$

$\equiv \quad \langle a \leq b < c \equiv a \leq b \wedge b < c \rangle$

$sc = (\sum k|0 \leq k < i : c[k]) \wedge ia = (\sum k|0 \leq k < i : n[k] * c[k]) \wedge 0 \leq i \leq M \wedge 0 \leq i \leq N \wedge N = M \wedge i < N$

Queda demostrado!

Prueba 2: $[P \wedge \neg B0 \Rightarrow Q]$

$sc = (\sum k | 0 \leq k < i : c[k]) \wedge ia = (\sum k | 0 \leq k < i : n[k] * c[k]) \wedge 0 \leq i \leq M \wedge 0 \leq i \leq N \wedge N = M \wedge i \geq N \Rightarrow$
$sc = (\sum k | 0 \leq k < M : c[k]) \wedge ia = (\sum k | 0 \leq k < N : n[k] * c[k])$

Debilitamiento.

$sc = (\sum k | 0 \leq k < i : c[k]) \wedge ia = (\sum k | 0 \leq k < i : n[k] * c[k]) \wedge 0 \leq i \leq M \wedge 0 \leq i \leq N \wedge N = M \wedge i \geq N$

$\Rightarrow \quad \langle i \leq N \wedge i \geq N \Rightarrow i = N; Sustituvion\ N = M \rangle$

$sc = (\sum k | 0 \leq k < M : c[k]) \wedge ia = (\sum k | 0 \leq k < N : n[k] * c[k]) \wedge 0 \leq i \leq M \wedge 0 \leq i \leq N \wedge N = M \wedge i \geq N$

$\Rightarrow \quad \langle p \wedge q \Rightarrow p \rangle$

$sc = (\sum k | 0 \leq k < M : c[k]) \wedge ia = (\sum k | 0 \leq k < N : n[k] * c[k])$

QDP

Prueba 3.1: $[P \wedge B0 \Rightarrow t \geq 0]$

$sc = (\sum k|0 \leq k < i : c[k]) \wedge ia = (\sum k|0 \leq k < i : n[k] * c[k]) \wedge 0 \leq i \leq M \wedge 0 \leq i \leq N \wedge N = M \wedge i < N \Rightarrow N - i \geq 0$

Debilitamiento.

$sc = (\sum k|0 \leq k < i : c[k]) \wedge ia = (\sum k|0 \leq k < i : n[k] * c[k]) \wedge 0 \leq i \leq M \wedge 0 \leq i \leq N \wedge N = M \wedge i < N$

$\equiv \quad \langle Sustitucion\ N = M; a \leq b \leq c \equiv a \leq b \wedge b \leq c \rangle$

$sc = (\sum k|0 \leq k < i : c[k]) \wedge ia = (\sum k|0 \leq k < i : n[k] * c[k]) \wedge 0 \leq i \wedge i \leq N \wedge N = M \wedge i < N$

$\Rightarrow \quad \langle p \wedge q \Rightarrow p \rangle$

$i \leq N$

$\equiv \quad \langle Aritmetica \rangle$

$N - i \geq 0$

QDP

Prueba 3.2.a: $\{P \wedge B0 \wedge t = C\}S0\{t < C\}$

$\{sc = (\sum k|0 \leq k < i : c[k]) \wedge ia = (\sum k|0 \leq k < i : n[k] * c[k]) \wedge 0 \leq i \leq M \wedge 0 \leq i \leq N \wedge N = M \wedge i < N \wedge N - i = C\}$

$sc, i, ia := c[i] + sc, i + 1, ia + n[i] * c[i]$

$\{N - i < C\}$

Por regla de la asignación.

$sc = (\sum k|0 \leq k < i : c[k]) \wedge ia = (\sum k|0 \leq k < i : n[k] * c[k]) \wedge 0 \leq i \leq M \wedge 0 \leq i \leq N \wedge N = M \wedge i < N \wedge N - i = C \Rightarrow$
$(N - i < C)(sc, i, ia := c[i] + sc, i + 1, ia + n[i] * c[i])$

Suposición del antecedente y empezando con el consecuente.

$(N - i < C)(sc, i, ia := c[i] + sc, i + 1, ia + n[i] * c[i])$

$\equiv$ $\langle Sustitucion\ textual \rangle$

$N - (i + 1) < C$

$\equiv$ $\langle Aritmetica \rangle$

$N - i < C + 1$

$\equiv$ $\langle Hipotesis:\ N - i = C \rangle$

$C < C + 1$

$\equiv$ $\langle Aritmetica \rangle$

$true$

QDP