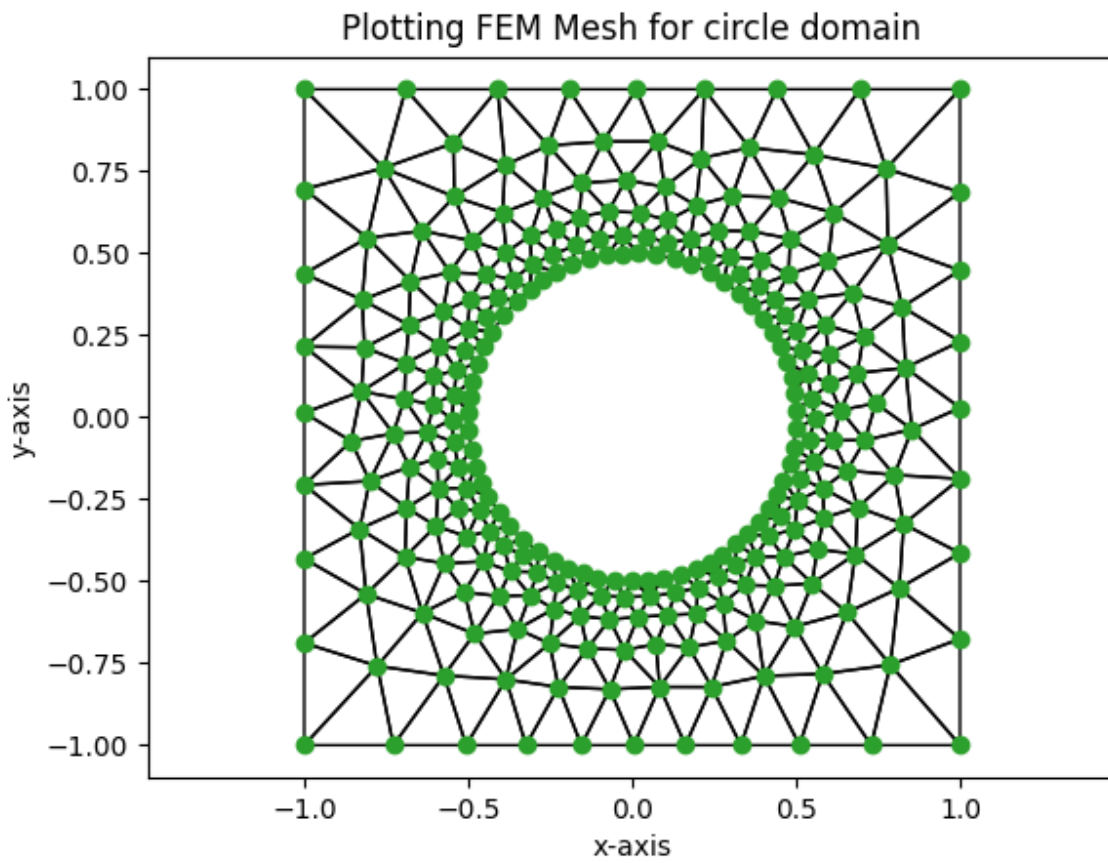
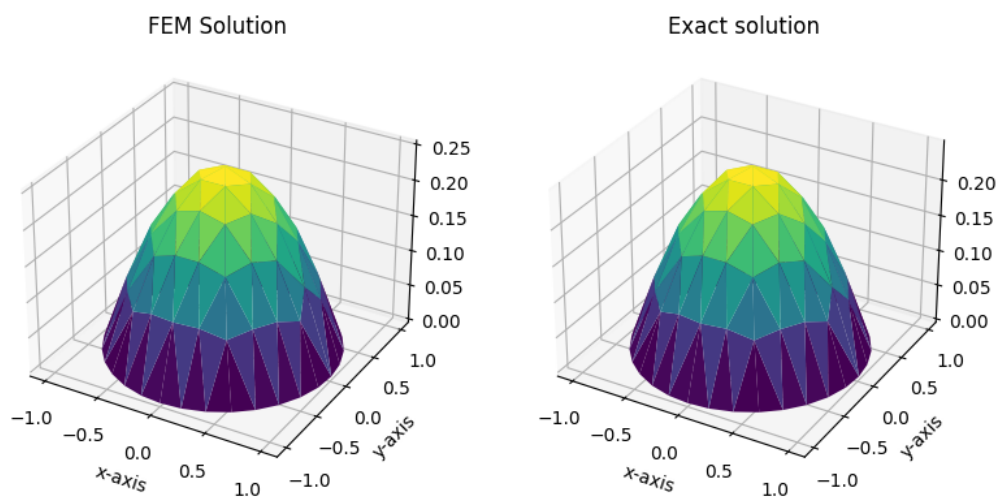


Question 2a

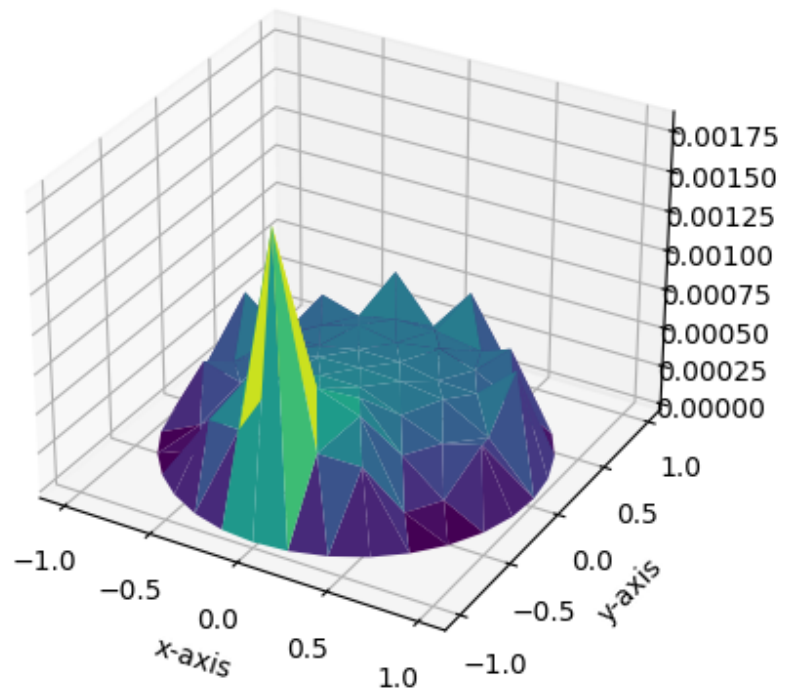


Question 2c



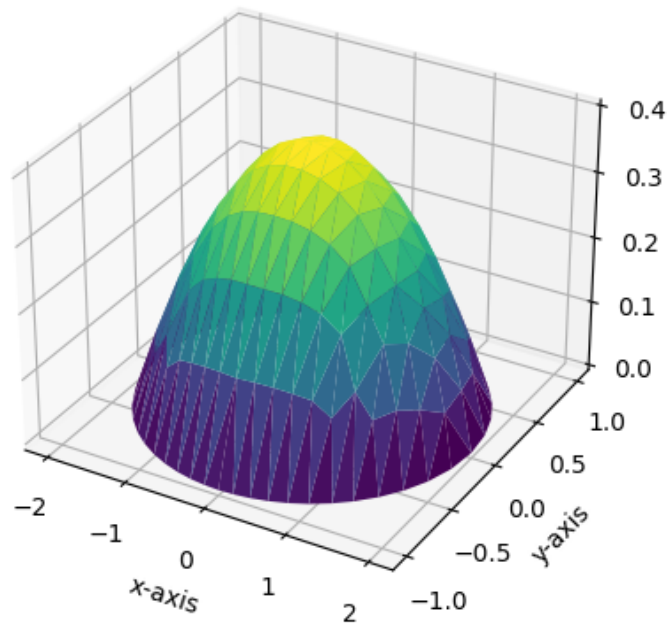
Error plot:

Plotting error against exact solution

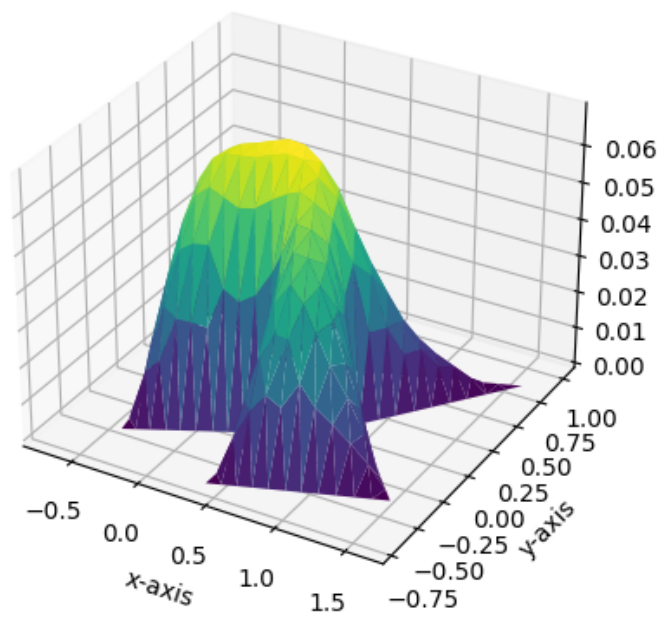


Question 2d

Plotting FEM Solution on Ellipse

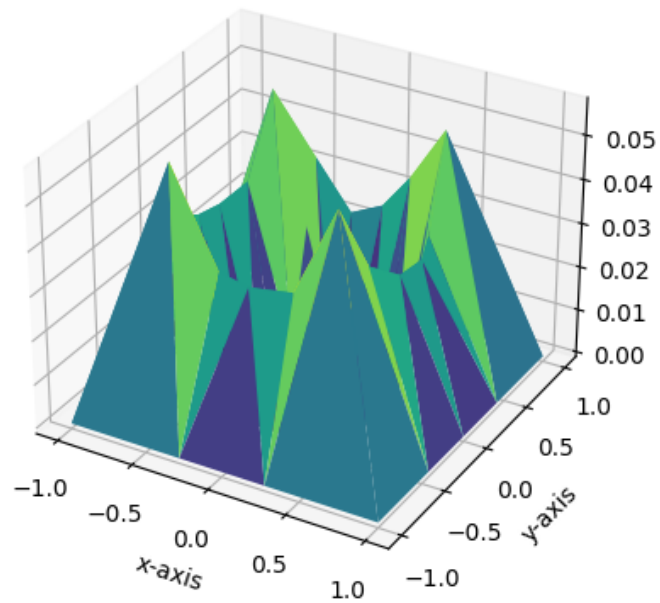


Plotting FEM Solution on Polygon

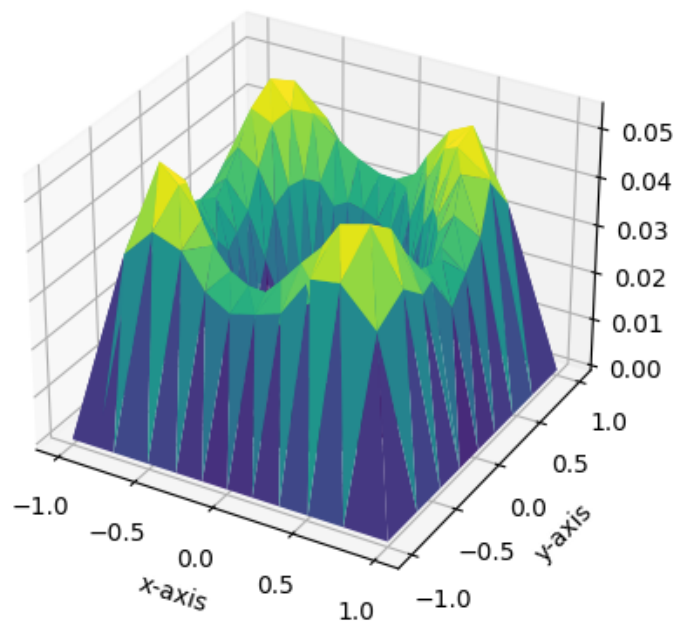


Question 2e

Plotting FEM Solution on Concave region, $h=0.1$



Plotting FEM Solution on Concave region, $h=0.05$



Resolution Study:

Consider the concave region:

For $h = 0.01$, the L2 norm of the solution = 0.18239867646815075

For $h = 0.05$, the L2 norm of the solution = 0.45766497250426924

Now, consider the circular region:

For $h=0.1$, the L2 norm of $u = 2.610957571554937$

For $h=0.5$, the L2 norm of $u = 0.4458913952006164$

We can say that in the concave region, as h reduces or the mesh becomes more fine, the L2 norm of the solution increases.

However, for the same factor of decrease in h , the L2 norm of u reduces.

Since the exact solution for the concave case is not known, this study is inconclusive for the behavior of the solution u on a concave region as h is refined.

The right way to do it would be to compute the exact solution for the concave region and compare the FEM solution with this exact solution by refining the mesh.

Question 5c

Num Elements = 16 $h=0.25$ Error = $|u - u_{\text{exact}}| = 0.0008465312128414781$

Num Elements = 64 $h=0.125$ Error = $|u - u_{\text{exact}}| = 0.00021610678305636344$

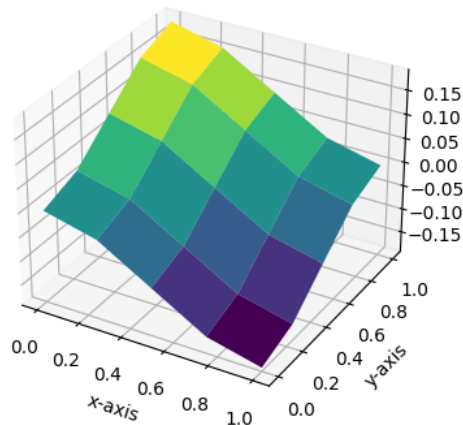
Num Elements = 256 $h=0.0625$ Error = $|u - u_{\text{exact}}| = 5.429626130537546e-05$

$\text{Error}(h=0.25)/\text{Error}(h=0.125) = 3.9171894600859978$

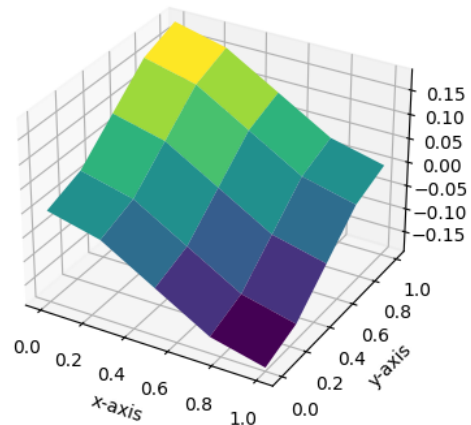
$\text{Error}(h=0.125)/\text{Error}(h=0.0625) = 3.9171894600859978$

Thus, the expected convergence of error, which is quadratic, is satisfied. When h is reduced by a factor of 2, the error reduces by a factor of 4.

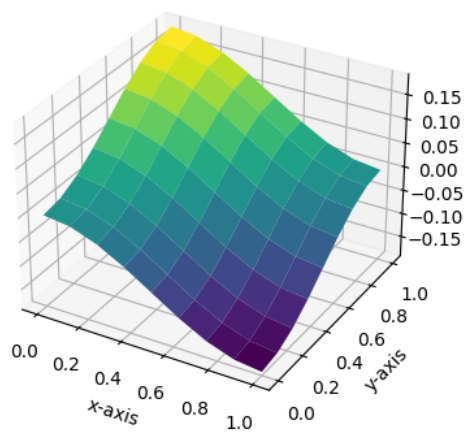
FEM Solution for Q5, num_elements=16



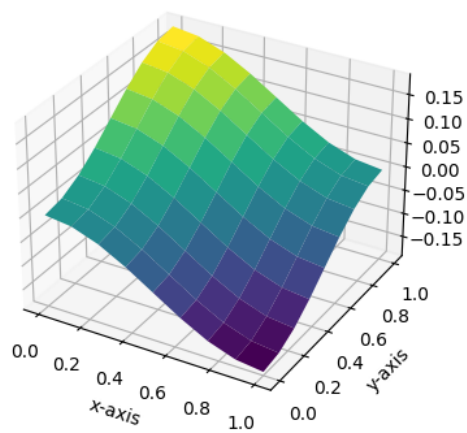
Exact Solution for Q5, num_elements=16



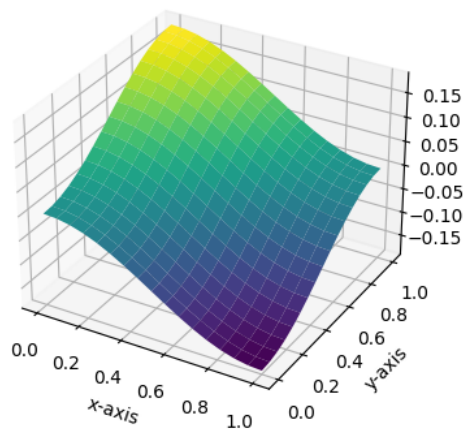
FEM Solution for Q5, num_elements=64



Exact Solution for Q5, num_elements=64



FEM Solution for Q5, num_elements=256



Exact Solution for Q5, num_elements=256

