In[1]:=
```
p1 = (1 - x / h) (1 - y / h)
p2 = (1 - x / h) (y / h)
p3 = (x / h) (1 - y / h)
p4 = (x / h) (y / h)
```

Out[1]= $\left(1-\frac{x}{h}\right)\left(1-\frac{y}{h}\right)$

Out[2]= $\frac{\left(1-\frac{x}{h}\right) y}{h}$

Out[3]= $\frac{x\left(1-\frac{y}{h}\right)}{h}$

Out[4]= $\frac{x y}{h^2}$

In[6]:= $\text{Integrate}\left[\nabla_{\{x,y\}}\text{p1}.\nabla_{\{x,y\}}\text{p1}+\text{p1}*\text{p1},\ \{x,\ 0,\ h\},\ \{y,\ 0,\ h\}\right]$

Out[6]= $\frac{1}{9}\left(6+h^2\right)$

In[7]:= $\text{Integrate}\left[\nabla_{\{x,y\}}\text{p1}.\nabla_{\{x,y\}}\text{p2}+\text{p1}*\text{p2},\ \{x,\ 0,\ h\},\ \{y,\ 0,\ h\}\right]$

Out[7]= $\frac{1}{18}\left(-3+h^2\right)$

In[8]:= $\text{Integrate}\left[\nabla_{\{x,y\}}\text{p1}.\nabla_{\{x,y\}}\text{p3}+\text{p1}*\text{p3},\ \{x,\ 0,\ h\},\ \{y,\ 0,\ h\}\right]$

Out[8]= $\frac{1}{18}\left(-3+h^2\right)$

In[9]:= $\text{Integrate}\left[\nabla_{\{x,y\}}\text{p1}.\nabla_{\{x,y\}}\text{p4}+\text{p1}*\text{p4},\ \{x,\ 0,\ h\},\ \{y,\ 0,\ h\}\right]$

Out[9]= $\frac{1}{36}\left(-12+h^2\right)$

In[10]:= $\text{Integrate}\left[\nabla_{\{x,y\}}\text{p2}.\nabla_{\{x,y\}}\text{p2}+\text{p2}*\text{p2},\ \{x,\ 0,\ h\},\ \{y,\ 0,\ h\}\right]$

Out[10]= $\frac{2}{3}+\frac{h^2}{9}$

In[11]:= $\text{Integrate}\left[\nabla_{\{x,y\}}\text{p2}.\nabla_{\{x,y\}}\text{p3}+\text{p2}*\text{p3},\ \{x,\ 0,\ h\},\ \{y,\ 0,\ h\}\right]$

Out[11]= $\frac{1}{36}\left(-12+h^2\right)$

In[12]:= $\text{Integrate}\left[\nabla_{\{x,y\}}\text{p2}.\nabla_{\{x,y\}}\text{p4}+\text{p2}*\text{p4},\ \{x,\ 0,\ h\},\ \{y,\ 0,\ h\}\right]$

Out[12]= $-\frac{1}{6}+\frac{h^2}{18}$

In[13]:= `Integrate[∇{x,y} p3.∇{x,y} p3 + p3 * p3, {x, 0, h}, {y, 0, h}]`

Out[13]= $\frac{1}{9}\left(6+h^2\right)$

In[14]:= `Integrate[∇{x,y} p3.∇{x,y} p4 + p3 * p4, {x, 0, h}, {y, 0, h}]`

Out[14]= $\frac{1}{18}\left(-3+h^2\right)$

In[15]:= `Integrate[∇{x,y} p4.∇{x,y} p4 + p4 * p4, {x, 0, h}, {y, 0, h}]`

Out[15]= $\frac{1}{3}+\frac{1}{3}\left(\frac{1}{h^3}+\frac{1}{3\,h}\right)h^3$

In[16]:= `f = Cos[π (x + xi)] - Cos[π (y + yi)]`

Out[16]= Cos[π (x + xi)] - Cos[π (y + yi)]

In[17]:= `Integrate[p1 * f, {x, 0, h}, {y, 0, h}]`

Out[17]= $\frac{\mathrm{Cos}[\pi\,\mathrm{xi}]-\mathrm{Cos}[\pi\,(h+\mathrm{xi})]-\mathrm{Cos}[\pi\,\mathrm{yi}]+\mathrm{Cos}[\pi\,(h+\mathrm{yi})]-h\,\pi\,\mathrm{Sin}[\pi\,\mathrm{xi}]+h\,\pi\,\mathrm{Sin}[\pi\,\mathrm{yi}]}{2\,\pi^2}$

In[19]:= `Integrate[p2 * f, {x, 0, h}, {y, 0, h}]`

Out[19]= $-\frac{-\mathrm{Cos}[\pi\,\mathrm{xi}]+\mathrm{Cos}[\pi\,(h+\mathrm{xi})]-\mathrm{Cos}[\pi\,\mathrm{yi}]+\mathrm{Cos}[\pi\,(h+\mathrm{yi})]+h\,\pi\,\mathrm{Sin}[\pi\,\mathrm{xi}]+h\,\pi\,\mathrm{Sin}[\pi\,(h+\mathrm{yi})]}{2\,\pi^2}$

In[20]:= `Integrate[p3 * f, {x, 0, h}, {y, 0, h}]`

Out[20]= $\frac{-\mathrm{Cos}[\pi\,\mathrm{xi}]+\mathrm{Cos}[\pi\,(h+\mathrm{xi})]-\mathrm{Cos}[\pi\,\mathrm{yi}]+\mathrm{Cos}[\pi\,(h+\mathrm{yi})]+h\,\pi\,\mathrm{Sin}[\pi\,(h+\mathrm{xi})]+h\,\pi\,\mathrm{Sin}[\pi\,\mathrm{yi}]}{2\,\pi^2}$

In[21]:= `Integrate[p4 * f, {x, 0, h}, {y, 0, h}]`

Out[21]= $\frac{1}{2\,\pi^2}\left(-\mathrm{Cos}[\pi\,\mathrm{xi}]+\mathrm{Cos}[\pi\,(h+\mathrm{xi})]+\mathrm{Cos}[\pi\,\mathrm{yi}]-\right.$
$\left.\mathrm{Cos}[\pi\,(h+\mathrm{yi})]+h\,\pi\,\mathrm{Sin}[\pi\,(h+\mathrm{xi})]-h\,\pi\,\mathrm{Sin}[\pi\,(h+\mathrm{yi})]\right)$