

# An Introduction to QCD

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## Introduction

QCD is a package that solves penalized quantile regression via exact coordinate descent method. The penalties considered are LASSO, SCAD, and MCP. Note that QCD algorithms for SCAD and MCP are experimental.

This vignette describes basic usage of functions related to  $\ell_1$  penalized quantile regression model in QCD package in R.

QCD mainly solves the following problem. Given data points  $(x_1, y_1), \dots, (x_n, y_n)$ , where  $y_i \in \mathbb{R}$  is a numerical response variable, and  $x_i \in \mathbb{R}^p$  is a  $p$ -dimensional covariate,

$$\arg \min_{\beta \in \mathbb{R}^p} \sum_{i=1}^n \rho_{\tau}(y_i - x_i^{\top} \beta) + \lambda \sum_{j=1}^p |\beta_j|$$

where  $\rho_{\tau}(u) := u(\tau - \mathbf{I}(u < 0))$  is the check loss function, and  $\lambda$  is a penalty parameter to be chosen in a data-driven fashion.  $\lambda$  could be a grid of values covering the entire range of possible solutions.

## Installation

You can download the QCD package from Github.

```
library(devtools)
install_github("sangheekim96/QCD")
```

## Quick Start: QCD

The purpose of this section is to give users a general sense of the package regarding QCD. We will briefly go over the main functions, basic operations and outputs. QCD can be loaded using the `library` command:

```
library(QCD)
```

We create a data set for illustration:

```
set.seed(1)
data <- generate.data(n = 30, p = 30, tau = 0.5,
                     signal = 1, autocorrelation = 0.5)
x <- data$X
y <- data$Y
```

This function generates simulation data based on Peng and Wang (2015). User can freely adjust signal-to-noise ratio and autocorrelation rate in the covariance function. Note that in our work, we do not consider intercept and we can change the value of true beta through signal argument.

With single value of  $\lambda$ , we can fit the  $\ell_1$  penalized quantile regression using 'qcd.lasso.fit'.

```
qr.lasso <- qcd.lasso.fit(x = x, y = y,
                        tau = 0.5, lambda = 0.01,
                        thresh = 1e-06, maxit = 100)

qr.lasso
#> $beta
#> [1] -0.09256468 -0.09480112 0.33449157 -0.14254155 0.30993666 0.89490069
#> [7] -0.30986422 -0.06745045 0.18230786 -0.01938970 -0.27150800 1.39848894
#> [13] 0.02520689 0.04075439 0.62510606 -0.06056419 0.08319574 0.20341591
#> [19] 0.10956987 0.28977844 0.17342721 -0.01824379 0.08093966 0.05482822
#> [25] 0.07697402 -0.09648412 -0.12904320 -0.09072932 -0.25575207 -0.00937032
#>
#> $df
#> [1] 30
#>
#> $lambda
#> [1] 0.01
#>
#> attr("class")
#> [1] "qcdlassofit"
```

We can fit the SCAD and MCP penalized quantile regression using 'qcd.scad.fit' and 'qcd.mcp.fit'. These functions are experimental.

```
qr.scad <- qcd.scad.fit(x = x, y = y,
                      tau = 0.5, lambda = 0.01,
                      a = 2.2,
                      thresh = 1e-06, maxit = 100)

qr.scad
#> $beta
#> [1] 0.1743577376 -0.1263904817 0.5959691385 -0.2879099910 0.4067507253
#> [6] 0.7118756713 -0.0420400208 0.2808683538 -0.0939448263 -0.0007431164
#> [11] -0.4131171829 1.1094042124 -0.1328959994 0.3990382303 0.6941466915
#> [16] -0.2774672989 0.2552531120 0.1318407565 0.0013966464 0.6517688366
#> [21] 0.0741647926 -0.0919081998 -0.1645784771 0.2459348425 0.3791098823
#> [26] -0.1673704894 -0.0491277138 0.4202162073 -0.1796384008 -0.0357884071
#>
#> $df
#> [1] 30
#>
#> $lambda
#> [1] 0.01
#>
#> attr("class")
#> [1] "qcdscadfit"
```

```
qr.mcp <- qcd.mcp.fit(x = x, y = y,
                    tau = 0.5, lambda = 0.01,
                    a = 2.2,
                    thresh = 1e-06, maxit = 100)

qr.mcp
```

```

#> $beta
#> [1] 0.12132477 -0.13073813 0.55451853 -0.29607662 0.45747159 0.66863174
#> [7] -0.02304193 0.26160263 -0.06633177 0.15825593 -0.36543470 1.25995534
#> [13] -0.09591821 0.34699148 0.57907060 -0.24909971 0.23707390 0.14943369
#> [19] -0.04626106 0.61662189 -0.01243126 -0.06666772 -0.03325702 0.27326112
#> [25] 0.31362824 -0.15559160 -0.18926676 0.37399161 -0.11994415 -0.15041834
#>
#> $df
#> [1] 30
#>
#> $lambda
#> [1] 0.01
#>
#> attr("class")
#> [1] "qcdmcpfit"

```

Users are encouraged to utilize ‘qcd.path’ function to solve penalized quantile regression problems. A pathwise scheme ‘warm start’ is used as a default.

```

## Create lambda grid
upper <- 2; lower <- -6
lambda <- 2^seq(upper, lower, by = -0.2)

## warm start version
qr.lasso.warm = qcd.path(x = x, y = y, tau = 0.5,
                        funname = "LASSO", lambda = lambda,
                        nudge = FALSE,
                        thresh = 1e-06, maxit = 10000)
#> 2 1.8 1.6 1.4 1.2 1 0.8 0.6 0.4 0.2 0 -0.2 -0.4 -0.6 -0.8 -1 -1.2 -1.4 -1.6 -1.8

qr.lasso.warm
#> $beta
#>
#>      s0      s1      s2      s3      s4      s5
#> V1 0.000000 0.000000 0.000000 0.000000 0.000000 1.601275e-02
#> V2 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000e+00
#> V3 0.000000 0.000000 0.000000 0.000000 0.000000 3.266672e-02
#> V4 0.000000 0.000000 0.000000 0.000000 0.000000 -9.415331e-02
#> V5 0.3110855 0.3110855 0.3110855 0.3110855 0.5619431 7.480393e-02
#> V6 0.000000 0.000000 0.000000 0.000000 0.000000 9.666279e-01
#> V7 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000e+00
#> V8 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000e+00
#> V9 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000e+00
#> V10 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000e+00
#> V11 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000e+00
#> V12 0.000000 0.000000 0.000000 0.000000 0.000000 9.334824e-01
#> V13 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000e+00
#> V14 0.000000 0.000000 0.000000 0.000000 0.000000 -4.031492e-03
#> V15 0.000000 0.000000 0.000000 0.000000 0.000000 8.886618e-01
#> V16 0.000000 0.000000 0.000000 0.000000 0.000000 -2.856327e-03
#> V17 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000e+00
#> V18 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000e+00
#> V19 0.000000 0.000000 0.000000 0.000000 0.000000 8.158906e-02
#> V20 0.000000 0.000000 0.000000 0.000000 0.000000 7.323217e-01
#> V21 0.000000 0.000000 0.000000 0.000000 0.000000 4.321074e-05

```

```

#> V22 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.000000e+00
#> V23 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.000000e+00
#> V24 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.000000e+00
#> V25 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 5.113813e-02
#> V26 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 3.567738e-04
#> V27 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 -8.245844e-02
#> V28 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 -1.683173e-03
#> V29 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 -2.840739e-02
#> V30 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 -3.989660e-02
#>
#>          s6          s7          s8          s9          s10
#> V1  1.601271e-02 1.303944e-02 1.300074e-02 1.300074e-02 1.300075e-02
#> V2  0.000000e+00 0.000000e+00 0.000000e+00 -3.237784e-08 -3.560477e-08
#> V3  3.266679e-02 3.390683e-02 3.397635e-02 3.397644e-02 3.397653e-02
#> V4 -9.415331e-02 -9.295255e-02 -9.296690e-02 -9.296686e-02 -9.296683e-02
#> V5  7.480390e-02 6.746412e-02 6.732290e-02 6.732279e-02 6.732269e-02
#> V6  9.666279e-01 9.688902e-01 9.690151e-01 9.690152e-01 9.690152e-01
#> V7  0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
#> V8  0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
#> V9  0.000000e+00 -1.575662e-04 -2.606079e-04 -2.606248e-04 -2.606284e-04
#> V10 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 -1.479542e-08
#> V11 0.000000e+00 -5.491667e-09 -2.083176e-05 -2.087139e-05 -2.087958e-05
#> V12 9.334824e-01 9.384394e-01 9.386045e-01 9.386046e-01 9.386046e-01
#> V13 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
#> V14 -4.031483e-03 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
#> V15 8.886618e-01 8.946183e-01 8.947236e-01 8.947237e-01 8.947237e-01
#> V16 -2.856328e-03 -4.462177e-03 -4.579275e-03 -4.579294e-03 -4.579304e-03
#> V17 0.000000e+00 0.000000e+00 1.596380e-04 1.596570e-04 1.596665e-04
#> V18 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
#> V19 8.158906e-02 7.785849e-02 7.782812e-02 7.782811e-02 7.782811e-02
#> V20 7.323217e-01 7.301918e-01 7.301797e-01 7.301797e-01 7.301796e-01
#> V21 4.321074e-05 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
#> V22 3.801628e-10 1.413159e-02 1.415462e-02 1.415465e-02 1.415465e-02
#> V23 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
#> V24 0.000000e+00 9.669476e-04 9.870555e-04 9.870804e-04 9.870863e-04
#> V25 5.113813e-02 5.461153e-02 5.476626e-02 5.476629e-02 5.476632e-02
#> V26 3.567855e-04 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
#> V27 -8.245845e-02 -8.965198e-02 -8.959844e-02 -8.959844e-02 -8.959844e-02
#> V28 -1.683173e-03 -3.739611e-03 -3.759084e-03 -3.759084e-03 -3.759142e-03
#> V29 -2.840739e-02 -2.482804e-02 -2.475661e-02 -2.475661e-02 -2.475659e-02
#> V30 -3.989660e-02 -4.060227e-02 -4.063916e-02 -4.063917e-02 -4.063917e-02
#>
#>          s11          s12          s13          s14          s15
#> V1  1.300074e-02 1.300074e-02 1.300074e-02 0.0154043412 1.540436e-02
#> V2 -3.187958e-08 -3.187958e-08 -3.263081e-08 0.0006440341 6.440341e-04
#> V3  3.397660e-02 3.397664e-02 3.397665e-02 0.0712750326 7.127504e-02
#> V4 -9.296683e-02 -9.296683e-02 -9.296683e-02 -0.0894231256 -8.942313e-02
#> V5  6.732264e-02 6.732258e-02 6.732255e-02 0.0404204866 4.042049e-02
#> V6  9.690152e-01 9.690152e-01 9.690152e-01 0.9647603850 9.647604e-01
#> V7  0.000000e+00 0.000000e+00 0.000000e+00 0.0119310328 1.193103e-02
#> V8  1.490687e-08 5.906909e-08 5.547274e-08 0.0050273796 5.027372e-03
#> V9 -2.606284e-04 -2.606621e-04 -2.606729e-04 -0.0019010512 -1.901047e-03
#> V10 -1.793765e-08 -1.793765e-08 -5.904329e-09 -0.0024699874 -2.469987e-03
#> V11 -2.088441e-05 -2.087080e-05 -2.087387e-05 -0.0004816322 -4.816320e-04
#> V12 9.386045e-01 9.386045e-01 9.386045e-01 0.9498030557 9.498031e-01

```

```

#> V13 0.000000e+00 -1.646725e-08 -2.030072e-08 -0.0000249630 -2.495874e-05
#> V14 0.000000e+00 0.000000e+00 0.000000e+00 -0.0006219995 -6.220013e-04
#> V15 8.947237e-01 8.947237e-01 8.947237e-01 0.9120088686 9.120089e-01
#> V16 -4.579310e-03 -4.579347e-03 -4.579354e-03 -0.0164948251 -1.649483e-02
#> V17 1.596679e-04 1.596989e-04 1.596989e-04 0.0054447772 5.444777e-03
#> V18 0.000000e+00 0.000000e+00 0.000000e+00 -0.0013322810 -1.332286e-03
#> V19 7.782811e-02 7.782811e-02 7.782811e-02 0.0727699148 7.276991e-02
#> V20 7.301796e-01 7.301796e-01 7.301796e-01 0.7232582023 7.232582e-01
#> V21 0.000000e+00 0.000000e+00 0.000000e+00 0.0000000000 0.000000e+00
#> V22 1.415465e-02 1.415469e-02 1.415471e-02 0.0220486511 2.204865e-02
#> V23 1.333307e-10 9.835267e-09 2.302720e-08 0.0078905831 7.890582e-03
#> V24 9.870871e-04 9.871045e-04 9.871218e-04 0.0052387320 5.238731e-03
#> V25 5.476636e-02 5.476636e-02 5.476636e-02 0.0759199702 7.591997e-02
#> V26 0.000000e+00 0.000000e+00 0.000000e+00 0.0008587776 8.587778e-04
#> V27 -8.959844e-02 -8.959842e-02 -8.959842e-02 -0.0947227261 -9.472273e-02
#> V28 -3.759226e-03 -3.759333e-03 -3.759361e-03 -0.0090083182 -9.008318e-03
#> V29 -2.475655e-02 -2.475651e-02 -2.475650e-02 -0.0224258817 -2.242588e-02
#> V30 -4.063917e-02 -4.063917e-02 -4.063917e-02 -0.0484842801 -4.848428e-02
#>          s16          s17          s18          s19          s20
#> V1  1.540436e-02 1.540436e-02 1.540436e-02 1.540436e-02 1.540436e-02
#> V2  6.440349e-04 6.440354e-04 6.440355e-04 6.440360e-04 6.440365e-04
#> V3  7.127504e-02 7.127504e-02 7.127504e-02 7.127504e-02 7.127504e-02
#> V4 -8.942313e-02 -8.942313e-02 -8.942313e-02 -8.942313e-02 -8.942313e-02
#> V5  4.042049e-02 4.042049e-02 4.042049e-02 4.042049e-02 4.042049e-02
#> V6  9.647604e-01 9.647604e-01 9.647604e-01 9.647604e-01 9.647604e-01
#> V7  1.193103e-02 1.193103e-02 1.193103e-02 1.193103e-02 1.193103e-02
#> V8  5.027371e-03 5.027371e-03 5.027371e-03 5.027371e-03 5.027370e-03
#> V9 -1.901046e-03 -1.901046e-03 -1.901046e-03 -1.901046e-03 -1.901046e-03
#> V10 -2.469987e-03 -2.469987e-03 -2.469987e-03 -2.469987e-03 -2.469987e-03
#> V11 -4.816320e-04 -4.816320e-04 -4.816325e-04 -4.816331e-04 -4.816340e-04
#> V12 9.498031e-01 9.498031e-01 9.498031e-01 9.498031e-01 9.498031e-01
#> V13 -2.495841e-05 -2.495839e-05 -2.495826e-05 -2.495808e-05 -2.495782e-05
#> V14 -6.220015e-04 -6.220018e-04 -6.220022e-04 -6.220025e-04 -6.220031e-04
#> V15 9.120089e-01 9.120089e-01 9.120089e-01 9.120089e-01 9.120089e-01
#> V16 -1.649483e-02 -1.649483e-02 -1.649483e-02 -1.649482e-02 -1.649482e-02
#> V17 5.444777e-03 5.444777e-03 5.444777e-03 5.444777e-03 5.444777e-03
#> V18 -1.332286e-03 -1.332286e-03 -1.332286e-03 -1.332287e-03 -1.332287e-03
#> V19 7.276991e-02 7.276991e-02 7.276991e-02 7.276991e-02 7.276991e-02
#> V20 7.232582e-01 7.232582e-01 7.232582e-01 7.232582e-01 7.232582e-01
#> V21 0.000000e+00 0.000000e+00 1.056360e-16 1.056360e-16 1.056360e-16
#> V22 2.204865e-02 2.204865e-02 2.204865e-02 2.204865e-02 2.204865e-02
#> V23 7.890581e-03 7.890581e-03 7.890581e-03 7.890581e-03 7.890581e-03
#> V24 5.238731e-03 5.238730e-03 5.238730e-03 5.238730e-03 5.238730e-03
#> V25 7.591997e-02 7.591997e-02 7.591997e-02 7.591997e-02 7.591997e-02
#> V26 8.587782e-04 8.587784e-04 8.587788e-04 8.587794e-04 8.587802e-04
#> V27 -9.472273e-02 -9.472273e-02 -9.472273e-02 -9.472273e-02 -9.472273e-02
#> V28 -9.008318e-03 -9.008318e-03 -9.008318e-03 -9.008318e-03 -9.008318e-03
#> V29 -2.242588e-02 -2.242588e-02 -2.242588e-02 -2.242588e-02 -2.242588e-02
#> V30 -4.848428e-02 -4.848428e-02 -4.848428e-02 -4.848428e-02 -4.848428e-02
#>          s21          s22          s23          s24          s25
#> V1  1.543433e-02 1.543433e-02 1.543435e-02 1.543435e-02 1.543436e-02
#> V2  8.168889e-04 8.169298e-04 8.169494e-04 8.169541e-04 8.169601e-04
#> V3  7.134876e-02 7.134874e-02 7.134875e-02 7.134875e-02 7.134877e-02

```

```

#> V4 -8.956326e-02 -8.956330e-02 -8.956330e-02 -8.956330e-02 -8.956331e-02
#> V5 4.034921e-02 4.034920e-02 4.034916e-02 4.034916e-02 4.034916e-02
#> V6 9.647032e-01 9.647032e-01 9.647033e-01 9.647033e-01 9.647033e-01
#> V7 1.218763e-02 1.218765e-02 1.218765e-02 1.218765e-02 1.218764e-02
#> V8 5.083756e-03 5.083802e-03 5.083831e-03 5.083831e-03 5.083829e-03
#> V9 -1.922797e-03 -1.922797e-03 -1.922797e-03 -1.922799e-03 -1.922799e-03
#> V10 -2.559163e-03 -2.559173e-03 -2.559224e-03 -2.559224e-03 -2.559236e-03
#> V11 -5.154649e-04 -5.154606e-04 -5.154576e-04 -5.154642e-04 -5.154793e-04
#> V12 9.498073e-01 9.498073e-01 9.498073e-01 9.498073e-01 9.498073e-01
#> V13 -2.543802e-05 -2.543802e-05 -2.543802e-05 -2.543735e-05 -2.543735e-05
#> V14 -6.694259e-04 -6.694267e-04 -6.694319e-04 -6.694395e-04 -6.694558e-04
#> V15 9.120223e-01 9.120223e-01 9.120223e-01 9.120223e-01 9.120223e-01
#> V16 -1.643299e-02 -1.643298e-02 -1.643297e-02 -1.643297e-02 -1.643297e-02
#> V17 5.334810e-03 5.334791e-03 5.334789e-03 5.334780e-03 5.334769e-03
#> V18 -1.340954e-03 -1.340964e-03 -1.340966e-03 -1.340966e-03 -1.340966e-03
#> V19 7.278656e-02 7.278657e-02 7.278658e-02 7.278658e-02 7.278658e-02
#> V20 7.232964e-01 7.232964e-01 7.232964e-01 7.232964e-01 7.232964e-01
#> V21 2.106536e-05 2.109034e-05 2.108494e-05 2.108463e-05 2.108401e-05
#> V22 2.206907e-02 2.206909e-02 2.206909e-02 2.206910e-02 2.206910e-02
#> V23 7.909302e-03 7.909306e-03 7.909312e-03 7.909328e-03 7.909365e-03
#> V24 5.251300e-03 5.251329e-03 5.251330e-03 5.251341e-03 5.251365e-03
#> V25 7.593974e-02 7.593975e-02 7.593975e-02 7.593975e-02 7.593976e-02
#> V26 8.692436e-04 8.692285e-04 8.692282e-04 8.692282e-04 8.692280e-04
#> V27 -9.473195e-02 -9.473195e-02 -9.473195e-02 -9.473195e-02 -9.473195e-02
#> V28 -9.672597e-03 -9.672605e-03 -9.672605e-03 -9.672615e-03 -9.672649e-03
#> V29 -2.213353e-02 -2.213353e-02 -2.213353e-02 -2.213353e-02 -2.213353e-02
#> V30 -4.878326e-02 -4.878328e-02 -4.878328e-02 -4.878329e-02 -4.878331e-02
#> s26 s27 s28 s29 s30
#> V1 1.543437e-02 1.543437e-02 1.543437e-02 1.543437e-02 1.543438e-02
#> V2 8.169671e-04 8.169718e-04 8.169764e-04 8.169772e-04 8.169872e-04
#> V3 7.134877e-02 7.134877e-02 7.134877e-02 7.134878e-02 7.134878e-02
#> V4 -8.956332e-02 -8.956332e-02 -8.956332e-02 -8.956333e-02 -8.956333e-02
#> V5 4.034916e-02 4.034916e-02 4.034916e-02 4.034916e-02 4.034916e-02
#> V6 9.647033e-01 9.647033e-01 9.647033e-01 9.647032e-01 9.647033e-01
#> V7 1.218765e-02 1.218765e-02 1.218765e-02 1.218765e-02 1.218765e-02
#> V8 5.083835e-03 5.083834e-03 5.083833e-03 5.083838e-03 5.083838e-03
#> V9 -1.922801e-03 -1.922801e-03 -1.922801e-03 -1.922804e-03 -1.922804e-03
#> V10 -2.559241e-03 -2.559243e-03 -2.559245e-03 -2.559261e-03 -2.559262e-03
#> V11 -5.154818e-04 -5.154854e-04 -5.154917e-04 -5.154888e-04 -5.154952e-04
#> V12 9.498073e-01 9.498073e-01 9.498073e-01 9.498073e-01 9.498073e-01
#> V13 -2.543735e-05 -2.543688e-05 -2.543638e-05 -2.543819e-05 -2.543819e-05
#> V14 -6.694599e-04 -6.694633e-04 -6.694701e-04 -6.694725e-04 -6.694778e-04
#> V15 9.120223e-01 9.120223e-01 9.120223e-01 9.120223e-01 9.120224e-01
#> V16 -1.643297e-02 -1.643297e-02 -1.643297e-02 -1.643297e-02 -1.643297e-02
#> V17 5.334766e-03 5.334764e-03 5.334755e-03 5.334754e-03 5.334750e-03
#> V18 -1.340966e-03 -1.340966e-03 -1.340966e-03 -1.340967e-03 -1.340967e-03
#> V19 7.278658e-02 7.278658e-02 7.278658e-02 7.278658e-02 7.278657e-02
#> V20 7.232964e-01 7.232964e-01 7.232964e-01 7.232964e-01 7.232964e-01
#> V21 2.108296e-05 2.108279e-05 2.108272e-05 2.108230e-05 2.107838e-05
#> V22 2.206910e-02 2.206910e-02 2.206910e-02 2.206910e-02 2.206910e-02
#> V23 7.909373e-03 7.909381e-03 7.909387e-03 7.909387e-03 7.909394e-03
#> V24 5.251370e-03 5.251375e-03 5.251379e-03 5.251380e-03 5.251384e-03
#> V25 7.593977e-02 7.593977e-02 7.593977e-02 7.593977e-02 7.593977e-02

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#> V26 8.692275e-04 8.692274e-04 8.692274e-04 8.692276e-04 8.692269e-04
#> V27 -9.473195e-02 -9.473195e-02 -9.473195e-02 -9.473195e-02 -9.473195e-02
#> V28 -9.672654e-03 -9.672659e-03 -9.672667e-03 -9.672667e-03 -9.672673e-03
#> V29 -2.213353e-02 -2.213353e-02 -2.213353e-02 -2.213353e-02 -2.213352e-02
#> V30 -4.878331e-02 -4.878331e-02 -4.878332e-02 -4.878332e-02 -4.878332e-02
#>          s31          s32          s33          s34          s35
#> V1  1.543438e-02 1.543438e-02 1.543438e-02 1.543438e-02 1.543437e-02
#> V2  8.169902e-04 8.169902e-04 8.169918e-04 8.169926e-04 8.169935e-04
#> V3  7.134878e-02 7.134878e-02 7.134878e-02 7.134878e-02 7.134879e-02
#> V4 -8.956334e-02 -8.956334e-02 -8.956334e-02 -8.956334e-02 -8.956334e-02
#> V5  4.034915e-02 4.034915e-02 4.034915e-02 4.034915e-02 4.034915e-02
#> V6  9.647033e-01 9.647032e-01 9.647032e-01 9.647032e-01 9.647032e-01
#> V7  1.218765e-02 1.218765e-02 1.218765e-02 1.218765e-02 1.218766e-02
#> V8  5.083842e-03 5.083842e-03 5.083842e-03 5.083843e-03 5.083843e-03
#> V9 -1.922809e-03 -1.922809e-03 -1.922809e-03 -1.922809e-03 -1.922810e-03
#> V10 -2.559267e-03 -2.559272e-03 -2.559272e-03 -2.559273e-03 -2.559274e-03
#> V11 -5.154943e-04 -5.154933e-04 -5.154943e-04 -5.154953e-04 -5.154959e-04
#> V12  9.498073e-01 9.498073e-01 9.498073e-01 9.498073e-01 9.498073e-01
#> V13 -2.544160e-05 -2.544160e-05 -2.544164e-05 -2.544201e-05 -2.544240e-05
#> V14 -6.694788e-04 -6.694787e-04 -6.694806e-04 -6.694807e-04 -6.694807e-04
#> V15  9.120224e-01 9.120224e-01 9.120224e-01 9.120224e-01 9.120224e-01
#> V16 -1.643297e-02 -1.643297e-02 -1.643296e-02 -1.643296e-02 -1.643296e-02
#> V17  5.334744e-03 5.334743e-03 5.334742e-03 5.334741e-03 5.334741e-03
#> V18 -1.340969e-03 -1.340969e-03 -1.340969e-03 -1.340970e-03 -1.340970e-03
#> V19  7.278657e-02 7.278658e-02 7.278657e-02 7.278658e-02 7.278657e-02
#> V20  7.232964e-01 7.232964e-01 7.232964e-01 7.232964e-01 7.232964e-01
#> V21  2.107964e-05 2.107964e-05 2.107865e-05 2.107865e-05 2.107873e-05
#> V22  2.206910e-02 2.206910e-02 2.206910e-02 2.206910e-02 2.206910e-02
#> V23  7.909394e-03 7.909394e-03 7.909395e-03 7.909395e-03 7.909395e-03
#> V24  5.251387e-03 5.251387e-03 5.251388e-03 5.251388e-03 5.251389e-03
#> V25  7.593977e-02 7.593978e-02 7.593978e-02 7.593978e-02 7.593978e-02
#> V26 8.692269e-04 8.692273e-04 8.692267e-04 8.692274e-04 8.692281e-04
#> V27 -9.473195e-02 -9.473195e-02 -9.473195e-02 -9.473195e-02 -9.473195e-02
#> V28 -9.672680e-03 -9.672680e-03 -9.672681e-03 -9.672682e-03 -9.672683e-03
#> V29 -2.213352e-02 -2.213352e-02 -2.213352e-02 -2.213352e-02 -2.213352e-02
#> V30 -4.878332e-02 -4.878332e-02 -4.878332e-02 -4.878332e-02 -4.878333e-02
#>          s36          s37          s38          s39          s40
#> V1  1.543437e-02 1.543437e-02 1.543437e-02 1.543437e-02 1.543437e-02
#> V2  8.169941e-04 8.169946e-04 8.169956e-04 8.169961e-04 8.169961e-04
#> V3  7.134879e-02 7.134879e-02 7.134879e-02 7.134879e-02 7.134879e-02
#> V4 -8.956334e-02 -8.956334e-02 -8.956334e-02 -8.956334e-02 -8.956334e-02
#> V5  4.034915e-02 4.034915e-02 4.034915e-02 4.034915e-02 4.034915e-02
#> V6  9.647032e-01 9.647032e-01 9.647032e-01 9.647032e-01 9.647032e-01
#> V7  1.218766e-02 1.218766e-02 1.218766e-02 1.218766e-02 1.218766e-02
#> V8  5.083843e-03 5.083843e-03 5.083843e-03 5.083844e-03 5.083844e-03
#> V9 -1.922810e-03 -1.922810e-03 -1.922810e-03 -1.922811e-03 -1.922811e-03
#> V10 -2.559274e-03 -2.559274e-03 -2.559275e-03 -2.559275e-03 -2.559275e-03
#> V11 -5.154961e-04 -5.154957e-04 -5.154964e-04 -5.154967e-04 -5.154967e-04
#> V12  9.498073e-01 9.498073e-01 9.498073e-01 9.498073e-01 9.498073e-01
#> V13 -2.544240e-05 -2.544242e-05 -2.544288e-05 -2.544286e-05 -2.544296e-05
#> V14 -6.694808e-04 -6.694809e-04 -6.694813e-04 -6.694816e-04 -6.694816e-04
#> V15  9.120224e-01 9.120224e-01 9.120224e-01 9.120224e-01 9.120224e-01
#> V16 -1.643296e-02 -1.643296e-02 -1.643296e-02 -1.643296e-02 -1.643296e-02

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#> V17 5.334741e-03 5.334741e-03 5.334740e-03 5.334740e-03 5.334740e-03
#> V18 -1.340970e-03 -1.340970e-03 -1.340970e-03 -1.340970e-03 -1.340970e-03
#> V19 7.278658e-02 7.278658e-02 7.278658e-02 7.278658e-02 7.278658e-02
#> V20 7.232964e-01 7.232964e-01 7.232964e-01 7.232964e-01 7.232964e-01
#> V21 2.107928e-05 2.107924e-05 2.107920e-05 2.107917e-05 2.107917e-05
#> V22 2.206910e-02 2.206910e-02 2.206910e-02 2.206910e-02 2.206910e-02
#> V23 7.909396e-03 7.909396e-03 7.909397e-03 7.909397e-03 7.909397e-03
#> V24 5.251389e-03 5.251389e-03 5.251389e-03 5.251390e-03 5.251390e-03
#> V25 7.593978e-02 7.593978e-02 7.593978e-02 7.593978e-02 7.593978e-02
#> V26 8.692284e-04 8.692284e-04 8.692283e-04 8.692283e-04 8.692284e-04
#> V27 -9.473195e-02 -9.473195e-02 -9.473195e-02 -9.473195e-02 -9.473195e-02
#> V28 -9.672685e-03 -9.672685e-03 -9.672686e-03 -9.672686e-03 -9.672686e-03
#> V29 -2.213352e-02 -2.213352e-02 -2.213352e-02 -2.213352e-02 -2.213352e-02
#> V30 -4.878333e-02 -4.878333e-02 -4.878333e-02 -4.878333e-02 -4.878333e-02
#>
#> $df
#> s0 s1 s2 s3 s4 s5 s6 s7 s8 s9 s10 s11 s12 s13 s14 s15 s16 s17 s18 s19
#> 1 1 1 1 1 18 19 19 20 21 22 24 25 25 29 29 29 29 30 30
#> s20 s21 s22 s23 s24 s25 s26 s27 s28 s29 s30 s31 s32 s33 s34 s35 s36 s37 s38 s39
#> 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30
#> s40
#> 30
#>
#> $dim
#> [1] 30 41
#>
#> $lambda
#> [1] 4.00000000 3.48220225 3.03143313 2.63901582 2.29739671 2.00000000
#> [7] 1.74110113 1.51571657 1.31950791 1.14869835 1.00000000 0.87055056
#> [13] 0.75785828 0.65975396 0.57434918 0.50000000 0.43527528 0.37892914
#> [19] 0.32987698 0.28717459 0.25000000 0.21763764 0.18946457 0.16493849
#> [25] 0.14358729 0.12500000 0.10881882 0.09473229 0.08246924 0.07179365
#> [31] 0.06250000 0.05440941 0.04736614 0.04123462 0.03589682 0.03125000
#> [37] 0.02720471 0.02368307 0.02061731 0.01794841 0.01562500
#>
#> $nobs
#> [1] 30
#>
#> attr("class")
#> [1] "qcdpath"

## warm start and nudge version
set.seed(1)
qr.lasso.warm.nudge = qcd.path(x = x, y = y, tau = 0.5,
                               funname = "LASSO", lambda = lambda,
                               nudge = TRUE, nudgesd = 0.2,
                               thresh = 1e-06, maxit = 10000)
#> 2 1.8 1.6 1.4 1.2 1 0.8 0.6 0.4 0.2 0 -0.2 -0.4 -0.6 -0.8 -1 -1.2 -1.4 -1.6 -1.8

qr.lasso.warm.nudge
#> $beta
#> s0 s1 s2 s3 s4
#> V1 0.0000000 0.0000000 0.0000000000 4.033334e-02 0.000000e+00

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#> V2 0.0000000 0.0000000 0.0000000000 0.000000e+00 0.000000e+00
#> V3 0.0000000 0.0000000 0.0000000000 2.037843e-03 3.442298e-02
#> V4 0.0000000 0.0000000 -0.0039091680 -7.054952e-03 -2.573166e-02
#> V5 0.3110855 0.1994542 0.1621813175 1.623507e-06 1.218325e-06
#> V6 0.0000000 0.7659266 0.8548982670 8.798166e-01 9.356097e-01
#> V7 0.0000000 0.0000000 0.0000000000 0.000000e+00 -3.247124e-04
#> V8 0.0000000 0.0000000 0.0000000000 -5.113389e-03 0.000000e+00
#> V9 0.0000000 0.0000000 -0.0079134224 -8.704439e-04 0.000000e+00
#> V10 0.0000000 0.0000000 0.0000000000 -2.892864e-02 0.000000e+00
#> V11 0.0000000 0.0000000 0.0000000000 0.000000e+00 -3.925389e-04
#> V12 0.0000000 0.8977639 0.9232098360 9.162608e-01 9.770787e-01
#> V13 0.0000000 0.0000000 0.0000000000 0.000000e+00 0.000000e+00
#> V14 0.0000000 0.0000000 0.0000000000 0.000000e+00 0.000000e+00
#> V15 0.0000000 0.4312670 0.6034515268 8.201827e-01 8.841701e-01
#> V16 0.0000000 0.0000000 0.0000000000 0.000000e+00 0.000000e+00
#> V17 0.0000000 0.0000000 0.0000000000 0.000000e+00 0.000000e+00
#> V18 0.0000000 0.2483612 0.1509390008 3.788861e-02 0.000000e+00
#> V19 0.0000000 0.0000000 0.0001932628 9.698421e-02 0.000000e+00
#> V20 0.0000000 0.0000000 0.3935668040 6.650706e-01 7.694439e-01
#> V21 0.0000000 0.2001262 0.1663891588 0.000000e+00 0.000000e+00
#> V22 0.0000000 0.0000000 0.0000000000 7.568037e-03 3.199633e-02
#> V23 0.0000000 0.0000000 0.0000000000 0.000000e+00 0.000000e+00
#> V24 0.0000000 0.0000000 0.0000000000 1.207072e-02 5.741677e-03
#> V25 0.0000000 0.0000000 0.0000000000 4.117569e-02 4.932135e-02
#> V26 0.0000000 0.0000000 0.0000000000 0.000000e+00 0.000000e+00
#> V27 0.0000000 -0.1090953 -0.0257236254 -3.042928e-02 -3.178814e-02
#> V28 0.0000000 0.0000000 -0.0051965211 -3.802625e-02 -9.464803e-02
#> V29 0.0000000 0.0000000 0.0000000000 0.000000e+00 0.000000e+00
#> V30 0.0000000 0.0000000 -0.0109959671 -7.601508e-02 -5.154106e-02
#>
#> s5 s6 s7 s8 s9
#> V1 2.970618e-02 4.150789e-02 6.507620e-02 0.056836936 2.974398e-03
#> V2 0.000000e+00 2.309580e-18 0.000000e+00 0.0000000000 -1.163488e-02
#> V3 7.477197e-02 1.964593e-03 2.186719e-02 0.0000000000 4.855558e-02
#> V4 -1.869649e-02 -4.507786e-02 -5.010031e-02 -0.039876073 0.000000e+00
#> V5 0.000000e+00 7.087140e-02 0.000000e+00 -0.060632692 1.416290e-01
#> V6 9.189129e-01 9.253844e-01 9.578681e-01 1.104557457 8.755191e-01
#> V7 0.000000e+00 9.071075e-03 -7.241260e-03 -0.002558294 -1.480770e-02
#> V8 0.000000e+00 0.000000e+00 0.000000e+00 0.0000000000 0.000000e+00
#> V9 0.000000e+00 -4.088077e-02 -2.499540e-03 0.0000000000 -5.079708e-03
#> V10 0.000000e+00 0.000000e+00 3.724614e-03 -0.017259243 0.000000e+00
#> V11 -9.324288e-02 0.000000e+00 -1.294918e-02 -0.003086534 -1.266671e-01
#> V12 9.340488e-01 9.615770e-01 9.785288e-01 0.938791254 1.014170e+00
#> V13 -3.273153e-02 -1.264366e-03 0.000000e+00 0.0000000000 2.412655e-02
#> V14 0.000000e+00 0.000000e+00 -2.160188e-04 0.0000000000 0.000000e+00
#> V15 9.547054e-01 8.358403e-01 9.712439e-01 0.969689570 7.893006e-01
#> V16 -1.473193e-03 5.506654e-09 -3.263819e-03 -0.001757951 6.680953e-02
#> V17 0.000000e+00 0.000000e+00 2.027096e-02 -0.026756292 -2.316653e-02
#> V18 -6.253797e-03 3.109316e-02 -4.096007e-03 -0.002380800 0.000000e+00
#> V19 5.986221e-02 0.000000e+00 6.053449e-05 0.0000000000 5.670689e-02
#> V20 8.092052e-01 6.604917e-01 8.073417e-01 0.763064766 4.988570e-01
#> V21 1.023011e-16 0.000000e+00 0.000000e+00 0.034832707 1.382876e-01
#> V22 3.814538e-02 6.917994e-02 4.304192e-02 0.060371873 9.777148e-02
#> V23 0.000000e+00 0.000000e+00 0.000000e+00 0.079153078 -1.347451e-02

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#> V24 9.847945e-03 0.000000e+00 3.992708e-02 0.090567013 -5.871438e-08
#> V25 5.542607e-02 7.771457e-02 2.153751e-03 0.000000000 9.910378e-02
#> V26 0.000000e+00 9.446343e-04 3.592291e-06 -0.055002762 -4.286187e-02
#> V27 -2.782506e-02 -1.272370e-01 -5.829217e-02 0.000000000 -4.062163e-02
#> V28 -3.708341e-02 -3.276377e-02 -8.218221e-02 -0.151739018 -7.520943e-02
#> V29 -1.965745e-03 0.000000e+00 -6.791322e-02 0.010156320 -4.217791e-08
#> V30 -9.652522e-02 -3.680618e-02 -4.491553e-02 -0.064949901 -1.327169e-01
#>          s10          s11          s12          s13          s14
#> V1  1.133902e-01 -0.013181621 7.646816e-02 0.0364032225 0.022719793
#> V2  1.560458e-06 0.000000000 3.425886e-02 0.0306093133 -0.045421764
#> V3  1.184365e-02 0.003726523 -1.604931e-02 0.0343370025 0.087047077
#> V4  -2.992812e-02 -0.027286143 -3.327833e-02 -0.0001568070 0.014503132
#> V5  -1.085667e-04 -0.151206999 7.637255e-04 -0.1215548792 -0.053868121
#> V6  9.327722e-01 1.018445927 7.980291e-01 0.9854730097 0.844330662
#> V7  0.000000e+00 -0.308165122 -2.755348e-01 -0.3045685827 -0.222790619
#> V8  6.942842e-02 0.011719569 4.777069e-05 0.0264915952 -0.007582458
#> V9  -1.265953e-02 0.002555133 1.292023e-03 -0.0004169259 0.078934571
#> V10 -1.210196e-01 -0.204747658 -4.458434e-01 -0.3027327367 -0.241792572
#> V11 -9.680452e-02 -0.006873685 -3.000967e-03 -0.0204963158 -0.241110950
#> V12 9.827734e-01 1.208985495 1.145446e+00 1.2089738849 1.196616303
#> V13 -1.149298e-02 0.041406185 -1.506091e-03 -0.0165586645 0.084781473
#> V14 -4.512040e-05 -0.046197830 -6.037527e-03 0.0163492575 -0.005454431
#> V15 8.881379e-01 0.754996749 7.240510e-01 0.6745226187 0.777613703
#> V16 1.471036e-02 0.222078976 1.672131e-01 0.2110065041 0.127036827
#> V17 0.000000e+00 0.053940079 3.387036e-02 0.0821425571 0.006622134
#> V18 -5.318716e-02 -0.062951367 -1.322141e-01 -0.1208095327 -0.166497869
#> V19 1.047993e-01 -0.032453230 1.933242e-01 0.0392412847 0.181041784
#> V20 6.229715e-01 0.419982013 1.743743e-01 0.2429014505 0.431063529
#> V21 -6.406432e-04 -0.071728959 0.000000e+00 -0.0748329501 -0.157006179
#> V22 7.489645e-02 0.257613581 1.847871e-01 0.2586012392 0.270075256
#> V23 7.745112e-02 0.032905506 5.791152e-02 0.0165063211 0.046658951
#> V24 -7.719465e-05 -0.117709325 -1.389094e-01 -0.1197714470 -0.059805608
#> V25 2.061925e-01 0.194788534 3.851171e-01 0.3128012108 0.207488074
#> V26 -2.851613e-03 0.067050836 0.000000e+00 0.0053504512 0.115372153
#> V27 -8.354274e-02 -0.155518381 -1.342004e-01 -0.2353854137 -0.144472102
#> V28 -4.599979e-02 -0.162297692 -7.217123e-02 -0.0138234379 -0.077749202
#> V29 4.592282e-02 -0.006458690 -2.441140e-02 -0.0065743163 -0.084325622
#> V30 -6.113191e-02 -0.007568013 -1.234283e-01 -0.0049413750 -0.039938738
#>          s15          s16          s17          s18          s19
#> V1  6.494470e-07 0.083370677 0.0908991961 2.840423e-01 0.329598679
#> V2  1.416801e-03 0.054299019 0.2091185143 2.703217e-01 0.053873709
#> V3  2.185976e-01 0.229862796 -0.0803236426 -1.327487e-01 -0.262246035
#> V4  1.291944e-01 0.159008407 0.1760556485 9.678892e-02 -0.050093032
#> V5  -8.998111e-02 -0.145513453 -0.1248515743 -2.668963e-05 0.044110608
#> V6  1.040123e+00 1.077456138 0.9733710424 9.559332e-01 0.852941154
#> V7  3.509222e-02 0.297620143 0.1638415781 5.273410e-01 0.294088110
#> V8  -6.797962e-02 0.048095299 0.0250140403 1.035908e-01 0.231220586
#> V9  2.581393e-01 0.128717200 0.0320720674 -6.320063e-04 -0.173313404
#> V10 -3.670832e-04 -0.049876433 -0.0133677365 1.625556e-01 0.006139055
#> V11 -2.352473e-01 -0.249312570 -0.2509477009 -3.418275e-01 -0.069520019
#> V12 1.366148e+00 1.002861737 0.9151947256 7.527646e-01 0.628446415
#> V13 3.980875e-02 -0.057242917 -0.2046043083 -1.932395e-01 -0.106438447
#> V14 -1.205817e-02 0.028775674 -0.0606865999 7.169393e-03 -0.000132512

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#> V15 7.236202e-01 1.033476141 1.0868491730 1.201061e+00 1.186801237
#> V16 2.075660e-01 0.007933420 0.0007295853 -2.096956e-01 -0.023422700
#> V17 1.997975e-03 -0.258698390 -0.0974510828 -1.432014e-01 -0.190449625
#> V18 -1.734343e-01 -0.000863073 -0.0100275692 -6.070447e-02 -0.132624258
#> V19 2.180323e-01 0.072239603 0.0722597123 2.326729e-01 0.049918915
#> V20 4.688454e-01 0.835558288 0.8782026768 1.038423e+00 1.225242625
#> V21 -1.830872e-01 0.032415072 0.0179801138 -2.210589e-01 -0.412189916
#> V22 3.712390e-01 0.140979398 0.1756845540 2.430886e-01 0.270841574
#> V23 5.326585e-02 -0.095665557 0.1247972283 4.506848e-02 0.105158846
#> V24 7.349795e-03 0.101424168 0.0133524323 1.948354e-01 0.163125677
#> V25 1.977189e-01 0.230298467 0.0523340224 1.883646e-01 0.055763722
#> V26 -1.257929e-01 -0.248870729 -0.0121274014 -1.115023e-01 0.050731673
#> V27 -1.727486e-01 0.048240353 -0.0010501497 -3.684669e-02 -0.096736126
#> V28 -1.373135e-02 -0.048095798 -0.1965550362 -2.423517e-02 -0.132543557
#> V29 5.498637e-02 0.246119585 0.3761851581 4.355958e-01 0.255982051
#> V30 -2.058750e-01 -0.157264530 -0.2334014392 -2.287365e-01 -0.066965064
#>          s20          s21          s22          s23          s24
#> V1  0.197780932 4.807243e-02 0.13740384 0.0695859492 0.24614230
#> V2 -0.055394342 3.205313e-02 0.12880005 0.2216371585 0.27496617
#> V3 -0.128394491 2.579426e-04 0.19214031 0.2754439355 0.13903411
#> V4 -0.138878024 -1.918113e-01 -0.06834576 0.0894933264 -0.11295884
#> V5 -0.012350747 1.872729e-02 0.03549186 0.0718639484 0.22999863
#> V6  1.038502675 1.115040e+00 0.99990572 1.0629549687 0.98485290
#> V7 -0.001991733 8.216798e-02 0.33747590 0.4270890913 0.42401731
#> V8  0.359679779 2.362167e-01 0.13052717 -0.0309987929 0.15017987
#> V9 -0.146539604 -9.810734e-02 -0.06884297 -0.0351023373 0.12826201
#> V10 0.154038363 6.300638e-02 0.25331623 0.3848188348 -0.14731050
#> V11 0.148356669 4.425258e-02 -0.25055480 -0.3321081355 -0.39783329
#> V12 0.778313503 8.485573e-01 0.80253937 0.8826669156 0.85592856
#> V13 -0.135858873 3.427462e-02 -0.02842343 -0.1381073689 -0.04079770
#> V14 0.077230650 -1.125198e-01 0.09418510 0.0116304489 0.01107757
#> V15 1.049220384 1.070305e+00 1.12401972 1.1004961214 1.12374866
#> V16 0.120350137 -3.102861e-02 -0.20962404 -0.2536620601 -0.39680334
#> V17 -0.187711298 -1.454182e-03 -0.03145240 -0.1041467076 -0.15208958
#> V18 -0.225528980 -1.596235e-01 0.06951156 0.3615390173 0.28397540
#> V19 0.129228645 7.529796e-02 -0.03100211 -0.0417768608 0.05235878
#> V20 1.202278657 1.104779e+00 1.03063392 0.9870294310 0.83803859
#> V21 -0.491703491 -7.265438e-02 -0.01213981 0.2507013184 0.30890326
#> V22 0.275246790 1.187541e-02 -0.02331700 -0.2212286520 -0.04301617
#> V23 0.149540748 1.758353e-01 -0.06679738 -0.0482051617 -0.15617992
#> V24 0.202698304 2.505016e-01 0.11711876 0.2897797342 0.14850574
#> V25 -0.029073014 -4.057837e-05 0.20652257 0.0008244825 0.34350222
#> V26 0.079897560 -5.370607e-02 -0.29889037 -0.3462479168 -0.31656362
#> V27 -0.110058995 1.004723e-02 0.01126818 -0.0428163604 0.09358983
#> V28 -0.227341896 -1.828136e-01 -0.07154965 0.0638463094 0.04914681
#> V29 0.162991616 1.796049e-02 0.08701409 0.1885463035 0.27367329
#> V30 -0.180281911 -1.255019e-01 -0.02578006 -0.1134555081 0.14673545
#>          s25          s26          s27          s28          s29
#> V1  0.185607857 0.294487597 0.43983531 0.3892844894 0.396165561
#> V2  0.383962826 0.523183040 0.40117495 0.3998724890 0.136149432
#> V3  0.239436919 0.234276456 0.18470399 0.0428325532 0.043324345
#> V4 -0.169090126 -0.072371586 -0.09206754 -0.1727632632 0.117802492
#> V5  0.172465013 0.438113233 0.69165540 0.3892097719 0.202394255

```

```

#> V6 1.083509171 0.854630338 0.63473026 0.8620850219 0.758699665
#> V7 0.667884341 0.900308574 1.05892184 0.8792264882 0.603762930
#> V8 0.131749445 0.070572620 0.19047345 0.1569751188 0.336773062
#> V9 0.079845861 -0.101185733 -0.09730418 0.0054379977 -0.064135286
#> V10 0.266938828 0.368304350 0.45971401 0.2245618515 0.110093623
#> V11 -0.464239165 -0.548259354 -0.61453525 -0.6909571845 -0.695615174
#> V12 0.788503599 0.678105942 0.76880970 0.6196474571 0.828174035
#> V13 0.026093306 -0.162787655 -0.15868728 -0.1798014678 -0.103571661
#> V14 -0.039299934 -0.009063756 0.10381090 -0.0620810842 -0.004450225
#> V15 1.259977063 1.281290968 1.20465591 1.3872146171 1.228641443
#> V16 -0.500582537 -0.644904740 -0.76114284 -0.5157346377 -0.285296684
#> V17 -0.024210233 -0.010972957 0.02397736 -0.1676093013 -0.192085463
#> V18 0.297733754 0.530021392 0.55019784 0.3928262389 0.092535396
#> V19 -0.124786456 -0.052312302 -0.41587002 -0.0947520566 -0.143683643
#> V20 1.142976593 1.176389179 1.49254157 1.3958983568 1.237150539
#> V21 0.205665706 0.345307145 0.19246964 0.0955215996 0.017633180
#> V22 -0.073679043 -0.274055316 -0.33475352 -0.0570738012 0.228036686
#> V23 -0.147467854 -0.200378147 -0.12021093 -0.0420659299 -0.209661719
#> V24 0.381030493 0.391255013 0.50732833 0.2481760497 0.290683059
#> V25 0.198269816 0.187918013 0.14834943 0.2600737485 0.272888636
#> V26 -0.397266646 -0.517913425 -0.50117471 -0.3574611288 -0.249462814
#> V27 0.156258899 0.203749312 0.16435869 0.3121575411 0.318155205
#> V28 0.002578848 0.145952579 0.20026367 0.0679188235 -0.218410132
#> V29 0.280348045 0.355004298 0.48812890 0.3762164890 0.418367788
#> V30 0.031910730 0.005919696 -0.07093163 -0.0006755611 0.002654263
#> s30 s31 s32 s33 s34 s35
#> V1 0.450617166 0.464889274 0.258085108 0.71784094 0.620491871 0.46643352
#> V2 0.069410145 0.332313136 0.267168486 0.64301358 0.773370520 0.68766645
#> V3 0.005269373 0.057502936 0.269899668 -0.20930690 -0.428532299 -0.32527439
#> V4 0.218089007 0.027406641 -0.114051833 0.13460473 0.090895311 -0.08110622
#> V5 0.066388769 0.141457996 0.224600610 0.41945041 0.258711189 0.30159354
#> V6 0.836800970 0.696362792 0.831525589 0.78689552 0.721857704 0.73210079
#> V7 0.513021380 0.626326252 0.570071801 1.18911250 0.944237860 0.69755314
#> V8 0.291751613 0.185335785 0.228253769 0.01202886 0.193648101 0.38013549
#> V9 0.082785210 0.194446397 0.210702117 0.25192212 -0.144226385 -0.19796739
#> V10 0.149503434 -0.038675238 0.003122057 0.03041195 -0.003698947 -0.03031893
#> V11 -0.742808642 -0.738875793 -0.497427158 -0.83323739 -0.664705930 -0.48286201
#> V12 0.872602860 0.838873500 0.838414060 0.62905604 0.713781506 0.63501403
#> V13 -0.252356682 -0.290054154 -0.138479109 -0.02481011 -0.156550094 -0.07348976
#> V14 0.007742281 -0.074601733 -0.054745983 -0.17853460 0.017146233 -0.07973606
#> V15 1.302176463 1.334140873 1.173509876 1.27699750 1.155702478 1.20326299
#> V16 -0.039282620 -0.112285113 -0.223419185 -0.47184524 -0.452080894 -0.35451117
#> V17 -0.226578390 -0.159702073 0.049039714 -0.16854089 0.096245185 0.15668945
#> V18 0.148681287 0.101613997 -0.003222007 0.20920873 0.148585381 0.07400578
#> V19 -0.064444697 -0.066060218 0.229361953 0.18304306 -0.146238397 -0.15429161
#> V20 1.210840888 1.125910003 0.918455479 0.88445058 0.953321410 0.96156064
#> V21 -0.094297509 -0.246765651 -0.008533084 -0.08209606 -0.199459408 -0.15774303
#> V22 0.133727972 0.410530569 0.199706026 0.34598385 0.384746383 0.39348267
#> V23 -0.104375523 -0.323729450 -0.117174676 -0.07826221 -0.162035730 -0.25022579
#> V24 0.145293650 -0.051194271 0.080844298 0.14393330 0.082231625 0.15054097
#> V25 0.181507821 0.460799609 0.462556067 0.58338592 0.778813781 0.62811084
#> V26 -0.141713566 -0.169026093 -0.290664419 -0.41701711 -0.380272040 -0.33039963
#> V27 0.145925508 -0.009294823 0.102912180 -0.05861444 -0.039365834 0.01512082

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#> V28 0.162272704 0.317487380 0.296220156 0.28357780 -0.066397097 -0.08393364
#> V29 0.120408916 0.305971131 0.387342838 0.63089194 0.520856299 0.57620682
#> V30 -0.089814621 0.036716420 -0.305645927 0.02169080 0.356935355 0.24523864
#>          s36          s37          s38          s39          s40
#> V1 0.526360617 0.68469309 0.6285136745 0.63582850 0.51398528
#> V2 0.686861355 0.77577224 0.8370980344 0.51033218 0.71670795
#> V3 -0.157328213 -0.36438907 -0.3375287131 -0.44323315 -0.62621960
#> V4 0.036252361 -0.18063289 0.0254581139 0.31246020 0.10841752
#> V5 0.465230926 0.73216428 0.2795204278 0.37369466 0.30797198
#> V6 0.760888954 0.74612423 0.8652945987 0.82391979 1.17280334
#> V7 1.033882051 1.12501124 1.0622565077 0.95733253 0.81265556
#> V8 0.263109109 0.21246467 0.2537996017 0.04517819 0.20772433
#> V9 0.046771634 -0.04654960 -0.2526949687 -0.07441255 -0.25237030
#> V10 -0.011018196 0.09802186 -0.0691533420 0.25042464 0.28186081
#> V11 -0.827602007 -0.64601318 -0.4949296266 -0.82188272 -0.35663520
#> V12 0.717869006 0.50066142 0.5540439946 0.65398456 0.39052540
#> V13 0.034386644 0.21090652 -0.0004104799 0.24604283 0.19964145
#> V14 -0.290146233 -0.36178107 -0.2826668342 -0.31775960 -0.53656452
#> V15 1.201680526 1.22285831 1.3069062063 1.20209307 1.22510002
#> V16 -0.579165070 -0.88357476 -0.7974223948 -0.55678019 -0.25700191
#> V17 -0.005541145 0.16993545 0.0407885011 -0.05811565 -0.07462794
#> V18 0.411168018 0.62707583 0.5676183452 0.55687854 0.25678006
#> V19 -0.179928717 -0.19876690 -0.2450395375 -0.23509825 0.02688760
#> V20 0.975826912 0.92515694 0.9524291925 0.93742349 0.79560454
#> V21 0.094315476 0.02917494 0.2362533992 0.12926062 0.15210160
#> V22 0.296035680 0.07409975 0.0713259361 0.24645131 0.26346871
#> V23 -0.277785243 -0.06419001 -0.1337356845 -0.07240882 -0.06023844
#> V24 0.439710601 0.45681748 0.5146549395 0.68493966 0.70461692
#> V25 0.561099732 0.55926310 0.5408270179 0.19066240 0.08278580
#> V26 -0.417326451 -0.46709158 -0.4270648676 -0.32028438 -0.39484964
#> V27 0.064722652 -0.02679802 -0.0435648314 0.07190615 -0.07201552
#> V28 0.014449881 -0.06415209 -0.0112351195 -0.19174420 -0.17255834
#> V29 0.780822411 0.76463615 0.9018094996 0.76044274 0.59538457
#> V30 0.129670607 0.25075417 0.1676467615 0.17114168 0.16323809
#>
#> $df
#> s0 s1 s2 s3 s4 s5 s6 s7 s8 s9 s10 s11 s12 s13 s14 s15 s16 s17 s18 s19
#> 1 7 13 19 15 20 20 24 21 25 28 29 28 30 30 30 30 30 30 30
#> s20 s21 s22 s23 s24 s25 s26 s27 s28 s29 s30 s31 s32 s33 s34 s35 s36 s37 s38 s39
#> 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30
#> s40
#> 30
#>
#> $dim
#> [1] 30 41
#>
#> $lambda
#> [1] 4.00000000 3.48220225 3.03143313 2.63901582 2.29739671 2.00000000
#> [7] 1.74110113 1.51571657 1.31950791 1.14869835 1.00000000 0.87055056
#> [13] 0.75785828 0.65975396 0.57434918 0.50000000 0.43527528 0.37892914
#> [19] 0.32987698 0.28717459 0.25000000 0.21763764 0.18946457 0.16493849
#> [25] 0.14358729 0.12500000 0.10881882 0.09473229 0.08246924 0.07179365
#> [31] 0.06250000 0.05440941 0.04736614 0.04123462 0.03589682 0.03125000

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#> [37] 0.02720471 0.02368307 0.02061731 0.01794841 0.01562500  
#>  
#> $nobs  
#> [1] 30  
#>  
#> attr("class")  
#> [1] "qcdpath"
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