An Introducion to QCD

Sanghee Kim Sumanta Basu

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Contents

Introduction

`QCD` is a package that solves penalized quantile regression via exact coordinate descent method. The penalties considered are LASSO, SCAD, and MCP. Note that QCD algorithms for SCAD and MCP are experimental.

This vignette describes basic usage of functions related to ℓ_1 penalized quantile regression model in `QCD` package in R.

`QCD` mainly solves the following problem. Given data points $(x_1, y_1), \ldots, (x_n, y_n)$, where $y_i \in \mathbb{R}$ is a numerical response variable, and $x_i \in \mathbb{R}^p$ is a p-dimensional covariate,

$$\arg\min_{\beta \in \mathbb{R}^p} \sum_{i=1}^{n} \rho_\tau \left(y_i - x_i^\top \beta\right) + \lambda \sum_{j=1}^{p} |\beta_j|$$

where $\rho_\tau(u) := u\left(\tau - \mathbf{I}(u < 0)\right)$ is the check loss function, and λ is a penalty parameter to be chosen in a data-driven fashion. λ could be a grid of values covering the entire range of possible solutions.

Installation

You can download the `QCD` package from Github.

```
library(devtools)
install_github("sangheekim96/QCD")
```

Quick Start: QCD

The purpose of this section is to give users a general sense of the package regarding QCD. We will briefly go over the main functions, basic operations and outputs. `QCD` can be loaded using the `library` command:

```
library(QCD)
```

We create a data set for illustration:

```
set.seed(1)
data <- generate.data(n = 30, p = 30, tau = 0.5, signal = 1)
x <- data$X
y <- data$Y
```

This function generates simulation data based on Peng and Wang (2015). Note that in our work, we do not consider intercept and we can change the value of true beta through signal argument.

With single value of λ, we can fit the ℓ_1 penalized quantile regression using 'qcd.lasso.fit'.

```
qr.lasso <- qcd.lasso.fit(x = x, y = y,
                          tau = 0.5, lambda = 0.01,
                          thresh = 1e-06, maxit = 100)
qr.lasso
#> $beta
#>  [1] -0.092828056 -0.093605858  0.334601603 -0.139767059  0.307413743
#>  [6]  0.895997273 -0.309618058 -0.066904169  0.181925297 -0.021908273
#> [11] -0.271969235  1.394531454  0.025730867  0.035676597  0.627937125
#> [16] -0.061635165  0.083219712  0.203526236  0.110415113  0.288514126
#> [21]  0.170748508 -0.018295819  0.083772780  0.051386300  0.077030655
#> [26] -0.095592290 -0.129518485 -0.090708140 -0.256023956 -0.009767324
#>
#> $df
#> [1] 30
#>
#> $lambda
#> [1] 0.01
#>
#> attr(,"class")
#> [1] "qcdlassofit"
```

We can fit the SCAD and MCP penalized quantile regression using 'qcd.scad.fit' and 'qcd.mcp.fit'. These functions are experimental.

```
qr.scad <- qcd.scad.fit(x = x, y = y,
                        tau = 0.5, lambda = 0.01,
                        a = 2.2,
                        thresh = 1e-06, maxit = 100)
qr.scad
#> $beta
#>  [1]  0.176752826 -0.121247322  0.594115599 -0.280112624  0.417956121
#>  [6]  0.679449940 -0.035286819  0.268110897 -0.090038272 -0.002818645
#> [11] -0.416387713  1.126013449 -0.138645841  0.393340280  0.689371256
#> [16] -0.266273902  0.249587620  0.147824826  0.001737899  0.638888130
#> [21]  0.067816697 -0.077682990 -0.142755242  0.240249238  0.385999107
#> [26] -0.173536777 -0.065163988  0.410790718 -0.177674684 -0.039264227
#>
#> $df
#> [1] 30
#>
#> $lambda
#> [1] 0.01
#>
#> attr(,"class")
#> [1] "qcdscadfit"
```

```
qr.mcp <- qcd.mcp.fit(x = x, y = y,
                      tau = 0.5, lambda = 0.01,
                      a = 2.2,
                      thresh = 1e-06, maxit = 100)
qr.mcp
```

```
#> $beta
#>  [1]  0.12718398 -0.12942573  0.54785270 -0.29481134  0.45576045  0.67201861
#>  [7] -0.01550733  0.26119048 -0.06294352  0.15313129 -0.35603302  1.26391304
#> [13] -0.09530798  0.34376814  0.58087761 -0.25959496  0.24199422  0.15461116
#> [19] -0.05664122  0.62170089 -0.02336958 -0.06587700 -0.03259470  0.27173846
#> [25]  0.30868058 -0.15495523 -0.19792303  0.37649892 -0.12086170 -0.14789506
#>
#> $df
#> [1] 30
#>
#> $lambda
#> [1] 0.01
#>
#> attr(,"class")
#> [1] "qcdmcpfit"
```

Users are encouraged to utilize 'qcd.path' function to solve penalized quantile regression problems. A pathwise scheme 'warm start' is used as a default.

```
## Create lambda grid
upper <- 2; lower <- -6
lambda <-  2^seq(upper, lower, by = -0.2)

## warm start version
qr.lasso.warm = qcd.path(x = x, y = y, tau = 0.5,
                         funname = "LASSO", lambda = lambda,
                         nudge = FALSE,
                         thresh = 1e-06, maxit = 10000)

qr.lasso.warm
#> $beta
#>            s0        s1        s2        s3        s4            s5
#> V1  0.0000000 0.0000000 0.0000000 0.0000000 0.0000000  6.731544e-02
#> V2  0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 -8.636151e-03
#> V3  0.0000000 0.0000000 0.0000000 0.0000000 0.0000000  0.000000e+00
#> V4  0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 -1.316437e-01
#> V5  0.3110855 0.3110855 0.3110855 0.3110855 0.5619431  2.073391e-01
#> V6  0.0000000 0.0000000 0.0000000 0.0000000 0.0000000  7.819408e-01
#> V7  0.0000000 0.0000000 0.0000000 0.0000000 0.0000000  0.000000e+00
#> V8  0.0000000 0.0000000 0.0000000 0.0000000 0.0000000  0.000000e+00
#> V9  0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 -1.267696e-02
#> V10 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 -5.740793e-05
#> V11 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000  0.000000e+00
#> V12 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000  9.190977e-01
#> V13 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000  0.000000e+00
#> V14 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 -6.254023e-03
#> V15 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000  8.352232e-01
#> V16 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 -7.015600e-03
#> V17 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000  6.700498e-03
#> V18 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000  0.000000e+00
#> V19 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000  5.561056e-02
#> V20 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000  6.842728e-01
#> V21 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000  4.129693e-03
#> V22 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000  3.229291e-02
```

```
#> V23 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000  0.000000e+00
#> V24 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 -9.598278e-05
#> V25 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000  1.259680e-01
#> V26 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000  0.000000e+00
#> V27 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 -6.011067e-02
#> V28 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000  0.000000e+00
#> V29 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 -2.238831e-02
#> V30 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 -1.110061e-01
#>                s6           s7           s8           s9           s10
#> V1   6.731544e-02  6.731540e-02  0.0642342837  6.423428e-02  6.394435e-02
#> V2  -8.636151e-03 -8.636128e-03 -0.0086265767 -8.626577e-03 -8.693383e-03
#> V3   0.000000e+00  0.000000e+00  0.0000000000  0.000000e+00  1.634601e-03
#> V4  -1.316436e-01 -1.316435e-01 -0.1267499973 -1.267500e-01 -1.262337e-01
#> V5   2.073390e-01  2.073388e-01  0.1952337742  1.952337e-01  1.928716e-01
#> V6   7.819409e-01  7.819413e-01  0.8078836532  8.078837e-01  8.110941e-01
#> V7   0.000000e+00  0.000000e+00  0.0000000000  0.000000e+00  3.799097e-04
#> V8   0.000000e+00  0.000000e+00  0.0000000000  0.000000e+00  2.741385e-04
#> V9  -1.267697e-02 -1.267697e-02 -0.0227591441 -2.275917e-02 -2.418438e-02
#> V10 -5.742005e-05 -5.742005e-05  0.0000000000  0.000000e+00  0.000000e+00
#> V11  0.000000e+00  0.000000e+00  0.0000000000 -2.790680e-08 -2.050330e-03
#> V12  9.190977e-01  9.190982e-01  0.9563652483  9.563653e-01  9.567205e-01
#> V13  0.000000e+00  0.000000e+00  0.0000000000  0.000000e+00 -1.677292e-08
#> V14 -6.253995e-03 -6.253995e-03 -0.0101456366 -1.014564e-02 -1.056192e-02
#> V15  8.352232e-01  8.352233e-01  0.8449265142  8.449265e-01  8.470345e-01
#> V16 -7.015602e-03 -7.015636e-03 -0.0093892217 -9.389261e-03 -1.014488e-02
#> V17  6.700498e-03  6.700658e-03  0.0255281328  2.552814e-02  2.623927e-02
#> V18  0.000000e+00  0.000000e+00 -0.0006850665 -6.851314e-04 -8.477501e-04
#> V19  5.561055e-02  5.561063e-02  0.0627296636  6.272968e-02  6.261725e-02
#> V20  6.842728e-01  6.842728e-01  0.6885162439  6.885162e-01  6.889572e-01
#> V21  4.129690e-03  4.129709e-03  0.0052342352  5.234247e-03  5.915774e-03
#> V22  3.229299e-02  3.229300e-02  0.0321664045  3.216640e-02  3.266759e-02
#> V23  0.000000e+00  0.000000e+00  0.0000000000  0.000000e+00  0.000000e+00
#> V24 -9.598278e-05 -9.598278e-05 -0.0032919629 -3.292008e-03 -4.709707e-03
#> V25  1.259679e-01  1.259679e-01  0.1207501640  1.207501e-01  1.204907e-01
#> V26  0.000000e+00  0.000000e+00 -0.0053654633 -5.365503e-03 -6.596449e-03
#> V27 -6.011074e-02 -6.011085e-02 -0.0738557550 -7.385581e-02 -7.440844e-02
#> V28  0.000000e+00  0.000000e+00  0.0000000000  0.000000e+00 -3.202633e-06
#> V29 -2.238831e-02 -2.238831e-02 -0.0198839353 -1.988394e-02 -1.969949e-02
#> V30 -1.110060e-01 -1.110059e-01 -0.1016507942 -1.016507e-01 -1.004403e-01
#>               s11          s12          s13          s14          s15
#> V1   6.394435e-02  6.394437e-02  6.394437e-02  6.394447e-02  6.394447e-02
#> V2  -8.693382e-03 -8.693375e-03 -8.693383e-03 -8.693114e-03 -8.693114e-03
#> V3   1.634613e-03  1.634632e-03  1.634654e-03  1.634840e-03  1.634858e-03
#> V4  -1.262337e-01 -1.262337e-01 -1.262337e-01 -1.262337e-01 -1.262338e-01
#> V5   1.928715e-01  1.928715e-01  1.928714e-01  1.928709e-01  1.928709e-01
#> V6   8.110941e-01  8.110943e-01  8.110944e-01  8.110951e-01  8.110951e-01
#> V7   3.799256e-04  3.799883e-04  3.799944e-04  3.809648e-04  3.809648e-04
#> V8   2.741385e-04  2.741992e-04  2.742260e-04  2.745416e-04  2.745449e-04
#> V9  -2.418439e-02 -2.418455e-02 -2.418460e-02 -2.418540e-02 -2.418541e-02
#> V10 -5.353499e-09  0.000000e+00  0.000000e+00 -8.911968e-09 -5.903181e-09
#> V11 -2.050386e-03 -2.050432e-03 -2.050432e-03 -2.052684e-03 -2.052723e-03
#> V12  9.567205e-01  9.567206e-01  9.567206e-01  9.567207e-01  9.567207e-01
#> V13 -7.997758e-09 -7.997758e-09 -7.997758e-09 -6.493959e-09 -6.493959e-09
```

```
#> V14 -1.056194e-02 -1.056201e-02 -1.056206e-02 -1.056259e-02 -1.056261e-02
#> V15  8.470345e-01  8.470347e-01  8.470347e-01  8.470364e-01  8.470364e-01
#> V16 -1.014489e-02 -1.014496e-02 -1.014497e-02 -1.014539e-02 -1.014540e-02
#> V17  2.623944e-02  2.623977e-02  2.623980e-02  2.624186e-02  2.624189e-02
#> V18 -8.478062e-04 -8.479306e-04 -8.479592e-04 -8.496434e-04 -8.496519e-04
#> V19  6.261730e-02  6.261739e-02  6.261739e-02  6.261848e-02  6.261849e-02
#> V20  6.889571e-01  6.889571e-01  6.889571e-01  6.889566e-01  6.889566e-01
#> V21  5.915774e-03  5.915792e-03  5.915805e-03  5.916046e-03  5.916046e-03
#> V22  3.266765e-02  3.266765e-02  3.266765e-02  3.266798e-02  3.266799e-02
#> V23 -5.398425e-18  0.000000e+00  0.000000e+00  0.000000e+00 -2.166870e-09
#> V24 -4.709806e-03 -4.709917e-03 -4.709917e-03 -4.710731e-03 -4.710731e-03
#> V25  1.204907e-01  1.204907e-01  1.204907e-01  1.204915e-01  1.204915e-01
#> V26 -6.596535e-03 -6.596664e-03 -6.596764e-03 -6.598198e-03 -6.598212e-03
#> V27 -7.440850e-02 -7.440854e-02 -7.440857e-02 -7.440961e-02 -7.440962e-02
#> V28 -3.155729e-06 -3.099711e-06 -3.074131e-06 -2.527785e-06 -2.527721e-06
#> V29 -1.969949e-02 -1.969951e-02 -1.969951e-02 -1.969961e-02 -1.969962e-02
#> V30 -1.004403e-01 -1.004403e-01 -1.004403e-01 -1.004398e-01 -1.004398e-01
#>               s16          s17          s18          s19          s20
#> V1   6.394447e-02  0.065969914  0.065969927  0.065969928  0.065969929
#> V2  -8.693113e-03 -0.006695020 -0.006695020 -0.006695022 -0.006695022
#> V3   1.634890e-03  0.004206090  0.004206132  0.004206137  0.004206140
#> V4  -1.262338e-01 -0.128006653 -0.128006667 -0.128006667 -0.128006668
#> V5   1.928709e-01  0.185466268  0.185466215  0.185466212  0.185466212
#> V6   8.110951e-01  0.821307776  0.821307838  0.821307843  0.821307844
#> V7   3.809659e-04  0.006124901  0.006124966  0.006124970  0.006124974
#> V8   2.745449e-04  0.006169617  0.006169639  0.006169647  0.006169665
#> V9  -2.418542e-02 -0.029294572 -0.029294578 -0.029294583 -0.029294593
#> V10  3.076850e-09  0.001062770  0.001062797  0.001062798  0.001062801
#> V11 -2.052760e-03 -0.016118994 -0.016118965 -0.016118964 -0.016118958
#> V12  9.567207e-01  0.957637101  0.957637101  0.957637101  0.957637104
#> V13 -6.493959e-09  0.001714065  0.001714055  0.001714062  0.001714071
#> V14 -1.056264e-02 -0.018906545 -0.018906545 -0.018906545 -0.018906546
#> V15  8.470365e-01  0.866482064  0.866482070  0.866482071  0.866482074
#> V16 -1.014543e-02 -0.015756588 -0.015756592 -0.015756592 -0.015756593
#> V17  2.624195e-02  0.044171829  0.044171829  0.044171829  0.044171827
#> V18 -8.496879e-04 -0.006888370 -0.006888370 -0.006888370 -0.006888370
#> V19  6.261853e-02  0.061808828  0.061808828  0.061808828  0.061808828
#> V20  6.889566e-01  0.698465853  0.698465853  0.698465853  0.698465853
#> V21  5.916063e-03  0.005812152  0.005812152  0.005812152  0.005812152
#> V22  3.266799e-02  0.034363112  0.034363128  0.034363129  0.034363130
#> V23  1.513975e-08  0.000000000  0.000000000  0.000000000  0.000000000
#> V24 -4.710712e-03 -0.002381540 -0.002381540 -0.002381539 -0.002381539
#> V25  1.204915e-01  0.119629286  0.119629294  0.119629293  0.119629292
#> V26 -6.598244e-03 -0.012039122 -0.012039127 -0.012039128 -0.012039132
#> V27 -7.440962e-02 -0.076090347 -0.076090351 -0.076090352 -0.076090348
#> V28 -2.516410e-06  0.003330336  0.003330356  0.003330359  0.003330358
#> V29 -1.969964e-02 -0.020795903 -0.020795900 -0.020795900 -0.020795900
#> V30 -1.004398e-01 -0.104124169 -0.104124171 -0.104124172 -0.104124172
#>               s21          s22           s23           s24           s25
#> V1   6.596993e-02  6.596994e-02  0.0673056781  0.0673056798  0.0673061239
#> V2  -6.695022e-03 -6.695025e-03 -0.0050745475 -0.0050745482 -0.0050747091
#> V3   4.206145e-03  4.206159e-03  0.0192132088  0.0192132938  0.0192150460
#> V4  -1.280067e-01 -1.280067e-01 -0.1302410032 -0.1302410605 -0.1302413175
```

```
#> V5   1.854662e-01  1.854662e-01  0.1761981961  0.1761981635  0.1761969784
#> V6   8.213078e-01  8.213079e-01  0.8300375255  0.8300375628  0.8300389654
#> V7   6.124981e-03  6.124996e-03  0.0140423330  0.0140423995  0.0140428957
#> V8   6.169692e-03  6.169755e-03  0.0201134865  0.0201135854  0.0201142464
#> V9  -2.929461e-02 -2.929465e-02 -0.0405520508 -0.0405520969 -0.0405525116
#> V10  1.062806e-03  1.062816e-03  0.0073643594  0.0073643885  0.0073651057
#> V11 -1.611895e-02 -1.611893e-02 -0.0177597356 -0.0177597356 -0.0177600251
#> V12  9.576371e-01  9.576371e-01  0.9577454507  0.9577454705  0.9577458285
#> V13  1.714086e-03  1.714117e-03  0.0030775277  0.0030775521  0.0030775997
#> V14 -1.890655e-02 -1.890655e-02 -0.0213588178 -0.0213588390 -0.0213597408
#> V15  8.664821e-01  8.664821e-01  0.8678530457  0.8678530508  0.8678532148
#> V16 -1.575660e-02 -1.575661e-02 -0.0151074112 -0.0151073939 -0.0151071191
#> V17  4.417183e-02  4.417183e-02  0.0456516123  0.0456516123  0.0456528952
#> V18 -6.888369e-03 -6.888365e-03 -0.0071924315 -0.0071924315 -0.0071927724
#> V19  6.180883e-02  6.180883e-02  0.0723273990  0.0723274202  0.0723280298
#> V20  6.984659e-01  6.984659e-01  0.6955388735  0.6955388735  0.6955389023
#> V21  5.812152e-03  5.812151e-03  0.0008335431  0.0008335431  0.0008331342
#> V22  3.436313e-02  3.436314e-02  0.0360080164  0.0360080092  0.0360081196
#> V23  5.736675e-10  6.078580e-09 -0.0017649092 -0.0017649093 -0.0017648937
#> V24 -2.381539e-03 -2.381539e-03 -0.0023054163 -0.0023054005 -0.0023054069
#> V25  1.196293e-01  1.196293e-01  0.1195361892  0.1195361909  0.1195362064
#> V26 -1.203914e-02 -1.203916e-02 -0.0129099629 -0.0129099764 -0.0129101692
#> V27 -7.609035e-02 -7.609035e-02 -0.0763367643 -0.0763367643 -0.0763367821
#> V28  3.330358e-03  3.330360e-03  0.0038136592  0.0038136548  0.0038138667
#> V29 -2.079590e-02 -2.079590e-02 -0.0196838935 -0.0196838935 -0.0196837646
#> V30 -1.041242e-01 -1.041242e-01 -0.1039780050 -0.1039779985 -0.1039778593
#>              s26           s27           s28           s29           s30
#> V1   0.067306149  0.0673061929  0.0673062068  0.0673062648  0.0673062743
#> V2  -0.005074720 -0.0050747405 -0.0050747469 -0.0050747793 -0.0050747844
#> V3   0.019215137  0.0192153411  0.0192154648  0.0192157857  0.0192159062
#> V4  -0.130241345 -0.1302413641 -0.1302414132 -0.1302414814 -0.1302414814
#> V5   0.176196923  0.1761967736  0.1761967100  0.1761965107  0.1761964364
#> V6   0.830039009  0.8300391837  0.8300392264  0.8300394736  0.8300395528
#> V7   0.014042932  0.0140429980  0.0140430724  0.0140431316  0.0140431436
#> V8   0.020114350  0.0201144949  0.0201146353  0.0201147588  0.0201148095
#> V9  -0.040552565 -0.0405526781 -0.0405527562 -0.0405528849 -0.0405529510
#> V10  0.007365124  0.0073651674  0.0073652193  0.0073652820  0.0073653272
#> V11 -0.017760025 -0.0177600215 -0.0177600215 -0.0177600593 -0.0177600649
#> V12  0.957745857  0.9577459418  0.9577459691  0.9577460013  0.9577460078
#> V13  0.003077634  0.0030776394  0.0030776730  0.0030776705  0.0030776650
#> V14 -0.021359783 -0.0213598603 -0.0213598941 -0.0213600990 -0.0213601626
#> V15  0.867853212  0.8678532035  0.8678532122  0.8678532191  0.8678532287
#> V16 -0.015107097 -0.0151070300 -0.0151070024 -0.0151069119 -0.0151068894
#> V17  0.045652931  0.0456530099  0.0456530293  0.0456531534  0.0456532070
#> V18 -0.007192799 -0.0071928955 -0.0071929088 -0.0071929431 -0.0071929663
#> V19  0.072328102  0.0723282297  0.0723282904  0.0723283228  0.0723283667
#> V20  0.695538902  0.6955389023  0.6955389023  0.6955389152  0.6955389152
#> V21  0.000833085  0.0008330199  0.0008330007  0.0008329638  0.0008329335
#> V22  0.036008123  0.0360081299  0.0360081262  0.0360081423  0.0360081538
#> V23 -0.001764894 -0.0017648961 -0.0017648961 -0.0017648961 -0.0017648961
#> V24 -0.002305400 -0.0023054184 -0.0023054073 -0.0023054138 -0.0023054138
#> V25  0.119536206  0.1195362061  0.1195362057  0.1195362057  0.1195362057
#> V26 -0.012910169 -0.0129101922 -0.0129101947 -0.0129102110 -0.0129102219
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#> V27 -0.076336782 -0.0763367818 -0.0763367818 -0.0763367818 -0.0763367818
#> V28  0.003813868  0.0038138937  0.0038138995  0.0038139210  0.0038139405
#> V29 -0.019683762 -0.0196837459 -0.0196837459 -0.0196837097 -0.0196836988
#> V30 -0.103977859 -0.1039778425 -0.1039778426 -0.1039778106 -0.1039777996
#>              s31           s32          s33           s34           s35
#> V1   0.0673062922  0.0673063092  0.067306330  0.0673063381  0.0673063545
#> V2  -0.0050747917 -0.0050747944 -0.005074801 -0.0050748046 -0.0050748127
#> V3   0.0192159799  0.0192160189  0.019216039  0.0192160771  0.0192160933
#> V4  -0.1302414814 -0.1302414817 -0.130241489 -0.1302415096 -0.1302415187
#> V5   0.1761963935  0.1761963663  0.176196358  0.1761963387  0.1761963328
#> V6   0.8300396117  0.8300396393  0.830039653  0.8300396815  0.8300396897
#> V7   0.0140431643  0.0140431673  0.014043178  0.0140431869  0.0140432003
#> V8   0.0201148573  0.0201148578  0.020114877  0.0201148996  0.0201149205
#> V9  -0.0405529935 -0.0405529943 -0.040553005 -0.0405530258 -0.0405530258
#> V10  0.0073653686  0.0073653690  0.007365383  0.0073653890  0.0073653921
#> V11 -0.0177600699 -0.0177600804 -0.017760083 -0.0177600837 -0.0177600843
#> V12  0.9577460078  0.9577460119  0.957746012  0.9577460119  0.9577460183
#> V13  0.0030776650  0.0030776661  0.003077655  0.0030776559  0.0030776535
#> V14 -0.0213601961 -0.0213602128 -0.021360224 -0.0213602408 -0.0213602408
#> V15  0.8678532298  0.8678532360  0.867853239  0.8678532485  0.8678532450
#> V16 -0.0151068741 -0.0151068741 -0.015106870 -0.0151068663 -0.0151068618
#> V17  0.0456532507  0.0456532906  0.045653302  0.0456533179  0.0456533345
#> V18 -0.0071929976 -0.0071929976 -0.007193003 -0.0071930033 -0.0071930196
#> V19  0.0723284297  0.0723284549  0.072328476  0.0723284802  0.0723285115
#> V20  0.6955389152  0.6955389152  0.695538915  0.6955389152  0.6955389152
#> V21  0.0008328914  0.0008328863  0.000832880  0.0008328777  0.0008328713
#> V22  0.0360081694  0.0360081694  0.036008169  0.0360081694  0.0360081694
#> V23 -0.0017648963 -0.0017648963 -0.001764898 -0.0017648975 -0.0017648988
#> V24 -0.0023054138 -0.0023054293 -0.002305429 -0.0023054310 -0.0023054310
#> V25  0.1195362057  0.1195362057  0.119536206  0.1195362062  0.1195362062
#> V26 -0.0129102367 -0.0129102498 -0.012910250 -0.0129102601 -0.0129102650
#> V27 -0.0763367818 -0.0763367815 -0.076336782 -0.0763367815 -0.0763367815
#> V28  0.0038139570  0.0038139712  0.003813977  0.0038139771  0.0038139886
#> V29 -0.0196836760 -0.0196836692 -0.019683668 -0.0196836675 -0.0196836631
#> V30 -0.1039778103 -0.1039778103 -0.103977804 -0.1039778110 -0.1039777998
#>              s36           s37          s38           s39           s40
#> V1   0.0673063648  0.0673063789  0.0673063838  0.0673063838  0.0673064134
#> V2  -0.0050748157 -0.0050748246 -0.0050748268 -0.0050748268 -0.0050748407
#> V3   0.0192161396  0.0192161988  0.0192162707  0.0192163833  0.0192165220
#> V4  -0.1302415439 -0.1302415849 -0.1302416345 -0.1302417070 -0.1302417920
#> V5   0.1761963149  0.1761962870  0.1761962579  0.1761962146  0.1761961681
#> V6   0.8300397207  0.8300397756  0.8300398328  0.8300399168  0.8300400083
#> V7   0.0140432210  0.0140432460  0.0140432811  0.0140433421  0.0140434137
#> V8   0.0201149927  0.0201150765  0.0201151933  0.0201153619  0.0201155311
#> V9  -0.0405530678 -0.0405530968 -0.0405531219 -0.0405531751 -0.0405532429
#> V10  0.0073654061  0.0073654141  0.0073654212  0.0073654428  0.0073654763
#> V11 -0.0177600843 -0.0177600852 -0.0177600852 -0.0177600852 -0.0177600852
#> V12  0.9577460393  0.9577460393  0.9577460421  0.9577460634  0.9577460929
#> V13  0.0030776793  0.0030776799  0.0030776834  0.0030777097  0.0030777461
#> V14 -0.0213602629 -0.0213602842 -0.0213602949 -0.0213603208 -0.0213603535
#> V15  0.8678532503  0.8678532630  0.8678532759  0.8678532894  0.8678533008
#> V16 -0.0151068549 -0.0151068481 -0.0151068476 -0.0151068379 -0.0151068322
#> V17  0.0456533467  0.0456533570  0.0456533639  0.0456533751  0.0456533975
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#> V18 -0.0071930196 -0.0071930293 -0.0071930372 -0.0071930379 -0.0071930393
#> V19  0.0723285285  0.0723285459  0.0723285623  0.0723285706  0.0723285877
#> V20  0.6955389152  0.6955389210  0.6955389248  0.6955389248  0.6955389248
#> V21  0.0008328634  0.0008328451  0.0008328306  0.0008328306  0.0008328199
#> V22  0.0360081656  0.0360081672  0.0360081604  0.0360081584  0.0360081511
#> V23 -0.0017648988 -0.0017648988 -0.0017649012 -0.0017648980 -0.0017648980
#> V24 -0.0023054279 -0.0023054278 -0.0023054274 -0.0023054274 -0.0023054221
#> V25  0.1195362062  0.1195362062  0.1195362087  0.1195362087  0.1195362087
#> V26 -0.0129102650 -0.0129102650 -0.0129102650 -0.0129102812 -0.0129102812
#> V27 -0.0763367815 -0.0763367815 -0.0763367815 -0.0763367815 -0.0763367815
#> V28  0.0038139888  0.0038139914  0.0038139916  0.0038139802  0.0038139805
#> V29 -0.0196836631 -0.0196836631 -0.0196836631 -0.0196836631 -0.0196836631
#> V30 -0.1039777998 -0.1039777998 -0.1039777998 -0.1039777998 -0.1039777998
#>
#> $df
#>  s0  s1  s2  s3  s4  s5  s6  s7  s8  s9 s10 s11 s12 s13 s14 s15 s16 s17 s18 s19
#>   1   1   1   1   1  21  21  21  22  23  28  30  28  28  29  30  30  29  29  29
#> s20 s21 s22 s23 s24 s25 s26 s27 s28 s29 s30 s31 s32 s33 s34 s35 s36 s37 s38 s39
#>  29  30  30  30  30  30  30  30  30  30  30  30  30  30  30  30  30  30  30  30
#> s40
#>  30
#>
#> $dim
#> [1] 30 41
#>
#> $lambda
#>  [1] 4.00000000 3.48220225 3.03143313 2.63901582 2.29739671 2.00000000
#>  [7] 1.74110113 1.51571657 1.31950791 1.14869835 1.00000000 0.87055056
#> [13] 0.75785828 0.65975396 0.57434918 0.50000000 0.43527528 0.37892914
#> [19] 0.32987698 0.28717459 0.25000000 0.21763764 0.18946457 0.16493849
#> [25] 0.14358729 0.12500000 0.10881882 0.09473229 0.08246924 0.07179365
#> [31] 0.06250000 0.05440941 0.04736614 0.04123462 0.03589682 0.03125000
#> [37] 0.02720471 0.02368307 0.02061731 0.01794841 0.01562500
#>
#> $nobs
#> [1] 30
#>
#> attr(,"class")
#> [1] "qcdpath"

## warm start and nudge version
set.seed(1)
qr.lasso.warm.nudge = qcd.path(x = x, y = y, tau = 0.5,
                               funname = "LASSO", lambda = lambda,
                               nudge = TRUE, nudgesd = 0.2,
                               thresh = 1e-06, maxit = 10000)

qr.lasso.warm.nudge
#> $beta
#>            s0            s1            s2            s3            s4
#> V1  0.0000000  1.875189e-02  3.742754e-03  0.000000e+00  0.0009154288
#> V2  0.0000000  0.000000e+00  0.000000e+00  3.973581e-03  0.0000000000
#> V3  0.0000000  0.000000e+00  1.468481e-02  3.712587e-06  0.0033767145
```

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#> V4  0.0000000 -1.445787e-07 -1.802037e-02 -7.676968e-02 -0.0001093258
#> V5  0.3110855  2.006387e-01  4.374324e-02  3.467623e-02  0.0068257787
#> V6  0.0000000  6.406430e-01  9.312066e-01  9.860243e-01  0.9242578086
#> V7  0.0000000  0.000000e+00  0.000000e+00  0.000000e+00 -0.0018146505
#> V8  0.0000000  0.000000e+00  0.000000e+00  0.000000e+00  0.0000000000
#> V9  0.0000000  0.000000e+00  0.000000e+00 -3.469181e-03 -0.0105940102
#> V10 0.0000000  0.000000e+00 -1.228788e-03  0.000000e+00  0.0000000000
#> V11 0.0000000  0.000000e+00 -3.097514e-09  0.000000e+00  0.0000000000
#> V12 0.0000000  7.675892e-01  9.643367e-01  9.267420e-01  0.9870802005
#> V13 0.0000000  0.000000e+00  0.000000e+00  0.000000e+00  0.0000000000
#> V14 0.0000000  0.000000e+00  0.000000e+00  0.000000e+00  0.0000000000
#> V15 0.0000000  6.180611e-01  8.525992e-01  8.679650e-01  0.8679749077
#> V16 0.0000000  0.000000e+00  0.000000e+00  0.000000e+00  0.0000000000
#> V17 0.0000000  0.000000e+00  0.000000e+00  0.000000e+00  0.0000000000
#> V18 0.0000000  5.379706e-02  0.000000e+00  0.000000e+00  0.0101529191
#> V19 0.0000000  0.000000e+00  0.000000e+00  3.822299e-02  0.0000000000
#> V20 0.0000000  5.440848e-01  7.014941e-01  7.296368e-01  0.7660808483
#> V21 0.0000000  0.000000e+00  1.755036e-03  0.000000e+00  0.0000000000
#> V22 0.0000000  5.798751e-02  3.525397e-02  0.000000e+00  0.0325573504
#> V23 0.0000000  0.000000e+00  0.000000e+00  0.000000e+00  0.0000000000
#> V24 0.0000000  0.000000e+00  0.000000e+00  0.000000e+00  0.0742080614
#> V25 0.0000000  0.000000e+00  6.386184e-02  5.856033e-02  0.0001530013
#> V26 0.0000000  0.000000e+00  0.000000e+00  0.000000e+00  0.0000000000
#> V27 0.0000000 -6.265151e-02 -3.851578e-02 -5.393155e-02 -0.0147739391
#> V28 0.0000000 -4.561052e-03 -4.288593e-02 -7.643988e-02 -0.1138501352
#> V29 0.0000000  0.000000e+00 -3.728742e-05  0.000000e+00  0.0000000000
#> V30 0.0000000  0.000000e+00 -2.947437e-02 -1.315641e-02 -0.0469475366
#>                s5           s6           s7           s8           s9
#> V1   6.866099e-02  1.432427e-02  0.1137354789  5.752059e-02  0.000000e+00
#> V2   0.000000e+00 -6.661090e-04 -0.0008019823  8.908435e-02  0.000000e+00
#> V3   1.120390e-07  1.263760e-02  0.0352295716  1.050244e-02  6.784181e-02
#> V4  -3.068059e-02 -2.538759e-02 -0.0717971336 -8.161237e-03 -6.564822e-02
#> V5   2.613381e-03  3.773116e-02  0.0300860197 -2.413914e-02  2.039897e-01
#> V6   9.183016e-01  9.346685e-01  0.8701966680  1.100921e+00  7.893426e-01
#> V7   0.000000e+00  0.000000e+00  0.0000000000 -4.029473e-05 -2.253455e-02
#> V8   0.000000e+00  0.000000e+00  0.0000000000  1.289648e-03  0.000000e+00
#> V9   0.000000e+00 -3.174631e-02 -0.0337395618 -1.358959e-02  0.000000e+00
#> V10 -2.021466e-02 -2.752755e-05  0.0000000000 -2.910527e-02 -3.001992e-02
#> V11 -6.955499e-02 -1.486067e-04 -0.0709215001  0.000000e+00 -1.400910e-01
#> V12  8.728632e-01  1.004131e+00  0.95663267776  9.682461e-01  9.983066e-01
#> V13 -6.986891e-03 -2.527619e-05 -0.0226068412 -1.051597e-03  5.567097e-02
#> V14  0.000000e+00  0.000000e+00 -0.0003974008 -2.032148e-04  0.000000e+00
#> V15  9.909870e-01  8.510996e-01  1.0042410069  9.400168e-01  8.203664e-01
#> V16 -5.205196e-05  0.000000e+00 -0.0444233935 -4.638070e-02  1.965111e-02
#> V17  6.018537e-03  0.000000e+00  0.0905565652 -7.933212e-03 -7.789660e-03
#> V18  0.000000e+00  0.000000e+00  0.0000000000  3.176851e-02  0.000000e+00
#> V19  5.176167e-02  1.326135e-03  0.0092561623  6.917055e-09  4.897827e-02
#> V20  7.999706e-01  7.104500e-01  0.8011689743  6.491084e-01  5.064836e-01
#> V21  1.071534e-03  0.000000e+00  0.0000000000  1.883563e-01  2.029778e-01
#> V22  2.799580e-02  3.779420e-02  0.0476142081 -2.840382e-02  3.396929e-02
#> V23  0.000000e+00  0.000000e+00  0.0000000000  6.899908e-02  0.000000e+00
#> V24  0.000000e+00  2.242456e-02  0.0000000000  1.293924e-01 -1.139164e-05
#> V25  6.416887e-02  4.407741e-02  0.0000000000  8.250207e-03  1.968771e-01
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#> V26  4.132600e-04  2.324643e-02  0.0000000000 -8.160813e-02 -3.728897e-02
#> V27 -4.372767e-02 -9.635428e-02 -0.0289019323  0.000000e+00 -1.262337e-03
#> V28 -2.024012e-02 -5.338405e-02 -0.1157301290 -1.234873e-01 -1.052318e-01
#> V29 -3.828960e-03  0.000000e+00 -0.0204383794  0.000000e+00 -2.325809e-02
#> V30 -1.022323e-01 -6.595778e-02 -0.0455641140 -6.093525e-02 -7.663766e-02
#>               s10           s11           s12           s13           s14
#> V1   1.481614e-01 -9.528501e-03  8.989636e-02  6.149412e-02  0.0110210335
#> V2   3.099443e-02 -1.144637e-02  2.636851e-02 -6.658547e-05 -0.0211153347
#> V3   3.820054e-03  4.410886e-02 -2.650002e-02  4.013448e-02  0.1745791883
#> V4  -4.984232e-02 -2.156924e-02 -3.010869e-02  2.408958e-02  0.0857119160
#> V5  -8.647831e-03 -1.684309e-01 -7.928355e-02 -1.463673e-01 -0.0891125045
#> V6   9.995353e-01  1.078789e+00  8.951432e-01  1.009159e+00  0.8550845487
#> V7  -3.710606e-02 -3.455408e-01 -2.744064e-01 -2.550950e-01 -0.2525672119
#> V8   4.489058e-02  9.225576e-05 -9.671643e-06 -6.509985e-09 -0.0183776924
#> V9   0.000000e+00  2.719306e-02  2.438851e-02 -2.171213e-05  0.0354224188
#> V10 -2.390701e-01 -2.742821e-01 -5.000872e-01 -2.980319e-01 -0.1970935210
#> V11 -2.192056e-05  5.435465e-02  2.735198e-03  0.000000e+00 -0.3472705370
#> V12  9.481941e-01  1.210232e+00  1.052882e+00  1.192345e+00  1.3327464157
#> V13 -1.903550e-02  1.014144e-03 -7.679151e-03 -5.402122e-02 -0.0102523124
#> V14  5.817570e-03 -5.961981e-03 -2.037632e-03  1.460914e-01  0.1722554480
#> V15  8.948765e-01  7.571915e-01  7.613635e-01  6.081566e-01  0.6580456987
#> V16 -4.478414e-02  1.422500e-01  1.691986e-01  1.945530e-01  0.1192383640
#> V17  0.000000e+00  3.895763e-02  1.833364e-04  6.915021e-02  0.0221620077
#> V18 -2.722911e-03 -2.484297e-02 -1.538512e-01 -1.264993e-01 -0.2340619437
#> V19  6.165563e-02  2.126434e-03  4.086699e-01  9.282835e-02  0.2580116116
#> V20  5.792020e-01  4.127484e-01  1.025625e-01  1.986676e-01  0.2271625059
#> V21  8.409230e-02 -6.534032e-02 -2.584322e-02  5.772623e-03 -0.0583344376
#> V22  5.717557e-03  1.573872e-01  2.388237e-01  1.814598e-01  0.2193806706
#> V23  8.792974e-02  1.121050e-01  7.291859e-02  7.697040e-02  0.0271644528
#> V24  0.000000e+00 -1.109946e-01 -1.816245e-01 -2.305626e-01 -0.1487974154
#> V25  2.027193e-01  1.517607e-01  3.736138e-01  3.679282e-01  0.3073106743
#> V26 -2.203202e-02  6.798181e-02  2.566153e-02 -1.491300e-02 -0.0003873022
#> V27 -9.925602e-02 -1.837503e-01 -1.090097e-01 -2.269319e-01 -0.0788398833
#> V28 -2.903440e-02 -1.813126e-01 -1.179886e-01 -4.032997e-02  0.0168990233
#> V29  6.417416e-02 -3.498186e-02 -2.803242e-02  6.405674e-03 -0.2534720381
#> V30 -4.402105e-02 -1.999491e-02 -1.242001e-01 -7.183976e-02 -0.0007200669
#>               s15           s16           s17           s18           s19
#> V1   7.209176e-04  0.0370327791  0.29871524  0.3352377529  0.37932176
#> V2   2.445719e-02  0.2084104080  0.21815538  0.3955309369  0.19180558
#> V3   2.856160e-01  0.4188002480 -0.12916297 -0.0289670449 -0.16947498
#> V4   4.888602e-02  0.0999975480  0.07316914  0.1145883011 -0.12033871
#> V5  -8.355404e-02 -0.1060981530 -0.09310139 -0.0009086178  0.11037841
#> V6   1.046188e+00  0.9990435210  1.06383838  0.9419749633  0.78507462
#> V7  -1.614941e-01  0.2817079470  0.24688603  0.7359691640  0.45915686
#> V8  -4.767335e-06  0.0048504020  0.12725581  0.0578713279  0.14810058
#> V9   2.193337e-01  0.1772029030 -0.02082269  0.1392637092 -0.11621559
#> V10 -2.245129e-02 -0.0942726670 -0.04764387 -0.0620616560 -0.14613227
#> V11 -2.253146e-01 -0.3766190850 -0.18472285 -0.4247778145 -0.18682510
#> V12  1.510858e+00  1.1965707860  0.71695491  0.6951722590  0.34335067
#> V13  2.090458e-03 -0.1910955810 -0.20297120 -0.2137613378 -0.16255418
#> V14  6.843923e-02  0.1093584490  0.02871150  0.0456871439 -0.01985644
#> V15  5.818344e-01  0.8971410790  1.11546762  1.1903585277  1.33031453
#> V16  1.808911e-01 -0.0153084590 -0.12665448 -0.2699115924 -0.14953431
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#> V17  2.494766e-01  0.001103745 -0.06228569 -0.2589127252 -0.37771835
#> V18 -1.953543e-01 -0.104286578 -0.06925207  0.0024062860 -0.05853942
#> V19  2.163023e-01  0.272791973  0.25366514  0.4121436918  0.50305483
#> V20  2.511170e-01  0.686637611  0.74236935  0.8451410018  0.90514564
#> V21 -7.559755e-02  0.129699523 -0.01533035 -0.1899772652 -0.31937535
#> V22  3.280021e-01  0.091580135  0.15907304  0.2480009128  0.26023208
#> V23 -3.023344e-02 -0.093065772  0.04629731  0.0005824656  0.08610328
#> V24 -6.355937e-02 -0.013219557  0.10759213  0.0635686759  0.07350639
#> V25  3.208341e-01  0.378775844  0.20938159  0.4557802974  0.31916019
#> V26 -1.568961e-01 -0.227214245 -0.12255930 -0.2470378852 -0.03218987
#> V27 -1.068754e-01  0.028680141  0.03926748 -0.0214112255 -0.06885180
#> V28 -4.653896e-04  0.166190866 -0.04574659  0.1309586217  0.05587769
#> V29 -7.604536e-02  0.138778486  0.23676341  0.4260161757  0.26205966
#> V30 -1.847542e-01 -0.231338873 -0.11552136 -0.1343257246 -0.01323197
#>              s20          s21          s22         s23         s24         s25
#> V1   0.324725318  0.018220301  0.260672581  0.20798031  0.33381791  0.26659987
#> V2  -0.005518256  0.205098824  0.245708065  0.27187279  0.56761095  0.55783907
#> V3  -0.168509707 -0.122670307  0.115492033  0.19967754  0.16704239  0.29702376
#> V4   0.062691818 -0.076505758 -0.251399653 -0.10245166 -0.17454964 -0.09463670
#> V5  -0.043149532 -0.004023111  0.135757356  0.09086076  0.11194168  0.09163635
#> V6   0.922632346  1.289152281  0.971220014  1.11183386  1.23812112  1.17522766
#> V7   0.333692328  0.302203132  0.587140504  0.55540780  0.64196923  0.85223503
#> V8   0.113093108  0.104343766  0.154670877  0.02158058  0.06886481  0.20899756
#> V9   0.032406214  0.136105641  0.002262405  0.01864697  0.20987633  0.14009963
#> V10  0.007501317  0.106099696  0.034098363  0.27152639 -0.01534754  0.03983225
#> V11 -0.119192921  0.048680572  0.006791471 -0.08901803 -0.32924578 -0.31918887
#> V12  0.562118534  0.575784131  0.505013906  0.60437582  0.59681580  0.48541608
#> V13 -0.029617341  0.195535801 -0.085319680 -0.10984748 -0.03122743 -0.04626786
#> V14 -0.049332151 -0.268596442 -0.007364846  0.02537685  0.03877305  0.06792346
#> V15  1.211501728  1.337132232  1.246679536  1.20019076  1.29084600  1.48829855
#> V16  0.018031145  0.074143854 -0.278521111 -0.36061408 -0.55560817 -0.61632041
#> V17 -0.310181438 -0.297659431 -0.054935207 -0.17020839 -0.24014837 -0.26763764
#> V18 -0.303319199 -0.348679309 -0.063535028  0.19106344  0.12953888  0.14354382
#> V19  0.389926941  0.259926267  0.214479580  0.11773324  0.16649254  0.07233816
#> V20  1.204213537  1.231208066  1.057114796  1.02580512  0.99134195  1.29366893
#> V21 -0.486552962 -0.245298657 -0.151955631  0.11664599  0.15250049  0.13692391
#> V22  0.335749293  0.152125765  0.037324804 -0.13382246  0.08718711 -0.10866116
#> V23  0.203105384  0.401860025  0.073128404  0.03639073 -0.06130542 -0.11291091
#> V24  0.174688092  0.441354591  0.064623710  0.20823120  0.22476111  0.37595085
#> V25  0.021118810  0.006572632  0.336435680  0.10387854  0.42543511  0.46096906
#> V26  0.118709281 -0.163646664 -0.276962295 -0.40664355 -0.47690803 -0.53952393
#> V27 -0.175434387  0.075502606 -0.083827273 -0.04772232  0.02568188  0.17834130
#> V28 -0.034297684 -0.058277022  0.092358106  0.03845841  0.10916066  0.10770638
#> V29  0.290463755  0.237391317  0.358237545  0.37754677  0.45849637  0.45653060
#> V30 -0.283119877 -0.172119775 -0.182895980 -0.22302171  0.02133816  0.02754593
#>              s26          s27          s28         s29         s30         s31
#> V1   0.324645984  0.423030904  0.315305386  0.3817069  0.42214464  0.41184848
#> V2   0.580442808  0.375842817  0.301224200  0.0639892  0.07995192  0.34896017
#> V3   0.117657868  0.209066405  0.131373780  0.1854774  0.23120205  0.05286280
#> V4  -0.034847244 -0.070289640  0.125407076  0.3881653  0.33949675  0.20172343
#> V5   0.166943987  0.341747764 -0.047595381 -0.2663430 -0.24891351 -0.09475075
#> V6   1.104024458  0.821629517  0.987462787  0.8173839  0.88762234  0.85106607
#> V7   0.835711629  0.947563196  0.778079258  0.5794065  0.44291910  0.62088502
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```
#> V8   0.121646209  0.234048953  0.142121698  0.2093922  0.33577668  0.31081840
#> V9  -0.074009935 -0.006308428  0.007138398 -0.1149506  0.04736791  0.11685961
#> V10 -0.010116195  0.094496714  0.009079590 -0.1450051 -0.10148593 -0.03793395
#> V11 -0.254251047 -0.239396253 -0.351995421 -0.2981005 -0.52205472 -0.56180179
#> V12  0.542916470  0.723918969  0.608481044  0.7766108  0.90615577  0.74675881
#> V13 -0.214757943 -0.294895003 -0.302489401 -0.4583609 -0.55767611 -0.49593248
#> V14  0.009487362  0.221377174  0.311567220  0.5623305  0.34528263  0.32051923
#> V15  1.405331707  1.402833561  1.472410442  1.4711191  1.37436587  1.35194361
#> V16 -0.640540676 -0.825373159 -0.644482048 -0.4542555 -0.18604042 -0.27031658
#> V17 -0.071474615  0.003920361 -0.297689535 -0.2637778 -0.27556728 -0.21026828
#> V18  0.420884163  0.458770871  0.352043336  0.1187293  0.09519499  0.05212986
#> V19 -0.118891136 -0.291666802  0.011619641 -0.1416832 -0.01814562 -0.00484960
#> V20  1.138923311  1.341911837  1.155789513  1.2767029  1.07208524  1.08367474
#> V21  0.406263727  0.361288311  0.262705129  0.2211108  0.12840231 -0.13061036
#> V22 -0.243801726 -0.260794344  0.021818480  0.2014489  0.11792951  0.38706644
#> V23 -0.100382541 -0.078414190  0.030186955 -0.1437561 -0.20147639 -0.17887359
#> V24  0.371061474  0.327025870  0.285640670  0.1537993  0.01860915 -0.04844318
#> V25  0.328705389  0.258143292  0.223474160  0.2677324  0.30878826  0.51186561
#> V26 -0.597874013 -0.661145148 -0.476212474 -0.4315341 -0.27887855 -0.27581427
#> V27  0.242755771  0.358870785  0.450445483  0.5906107  0.31640491  0.17571740
#> V28  0.110194798  0.071226671 -0.162633994 -0.3406511  0.09858633  0.05444532
#> V29  0.475332570  0.612533676  0.602462186  0.5011863  0.22196074  0.48648885
#> V30 -0.018396856 -0.292409351 -0.288769463 -0.2404113 -0.26265551 -0.13713777
#>              s32          s33         s34          s35         s36         s37
#> V1   0.360840645  0.793100918  0.71864209  0.570439629  0.62386847  0.5872658
#> V2   0.340716335  0.677793979  0.79091597  0.752243042  0.74110344  0.8206948
#> V3   0.055553819 -0.497520102 -0.58273993 -0.394179995 -0.13501335 -0.2237616
#> V4  -0.086056375  0.012853265 -0.14199723 -0.318667337 -0.18580850 -0.4860619
#> V5   0.116868313  0.284108322  0.25883177  0.337497064  0.54244647  0.7043606
#> V6   0.912986319  0.889230299  0.74438501  0.777866869  0.78880381  0.7365661
#> V7   0.609448132  0.978683653  0.71455672  0.681011993  1.05240503  1.0376801
#> V8   0.328768699  0.416706588  0.60703437  0.700874119  0.61767615  0.5778754
#> V9   0.005739693 -0.231902892 -0.45700584 -0.419592697 -0.21987074 -0.4091759
#> V10 -0.136846676 -0.173949059 -0.12498624 -0.215528902  0.06058161  0.1868115
#> V11 -0.223263865 -0.346456484 -0.11537042 -0.108328107 -0.50861535 -0.2648159
#> V12  0.693705052  0.356528060  0.38075060  0.424310624  0.55488548  0.4256692
#> V13 -0.297808280 -0.245237004 -0.37361836 -0.366456475 -0.19886133 -0.2055218
#> V14  0.079818878  0.003691342  0.30245573  0.201306533  0.17031965  0.1320502
#> V15  1.202264530  1.265092758  1.31106704  1.346648490  1.28558354  1.2942562
#> V16 -0.281148671 -0.536185612 -0.63996851 -0.588714825 -0.69162344 -0.9334058
#> V17 -0.048913705 -0.055453993  0.30431934  0.286340945  0.22764170  0.4084306
#> V18 -0.036742914  0.134683670 -0.04407816 -0.090370963  0.17256597  0.3027074
#> V19  0.247081084  0.129953838 -0.01848087 -0.004203613 -0.12006906 -0.2498073
#> V20  1.037581088  1.021152740  1.27949195  1.227313514  1.21846049  1.1932828
#> V21 -0.058070246 -0.053269725 -0.23789335 -0.087643058  0.10535266  0.1634325
#> V22  0.099143438  0.073352068  0.14661992  0.143362974  0.10197847 -0.1317787
#> V23 -0.022628071  0.042981863 -0.19755570 -0.183145737 -0.33995862 -0.1556489
#> V24  0.110482324  0.374495682  0.20756853  0.147758627  0.30890185  0.2556723
#> V25  0.500928615  0.633655425  0.69142993  0.744701371  0.84139073  0.7329610
#> V26 -0.343478912 -0.406387333 -0.33014438 -0.368241163 -0.76678724 -0.7206605
#> V27  0.177218596  0.121540882  0.21094037  0.348744984  0.49679056  0.3252172
#> V28  0.115307005  0.094134866 -0.28533397 -0.180129090 -0.11993734 -0.1772209
#> V29  0.527029174  0.761886090  0.56073959  0.581382017  0.60640558  0.6046266
```

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#> V30 -0.309992657 -0.059326259  0.21892319  0.086344166  0.11768206  0.2321597
#>             s38         s39         s40
#> V1   0.43501891  0.46365236  0.42355705
#> V2   0.85146344  0.45072560  0.35888055
#> V3  -0.09225910 -0.18059312 -0.12051185
#> V4  -0.38420975 -0.33841429 -0.47593039
#> V5   0.49135303  0.51842057  0.40781947
#> V6   0.85772795  0.84460549  0.94855783
#> V7   0.74304114  0.63196177  0.42532434
#> V8   0.46128066  0.48786291  0.52151887
#> V9  -0.36010739 -0.17065489 -0.33414201
#> V10  0.02455605 -0.06363160 -0.11466073
#> V11 -0.18981188 -0.35009492  0.04707994
#> V12  0.53116565  0.56052614  0.34335527
#> V13 -0.27811068 -0.05463803  0.08336142
#> V14 -0.01941340 -0.04052352 -0.29144190
#> V15  1.29615510  1.33091121  1.35063989
#> V16 -0.78732195 -0.64016415 -0.50987959
#> V17  0.38046528 -0.02841434  0.21032762
#> V18  0.46761997  0.31206853  0.20466260
#> V19 -0.24151338 -0.04724723  0.07596618
#> V20  1.07290844  1.20002071  1.02203509
#> V21  0.39288536  0.30014508  0.24948710
#> V22 -0.25894816 -0.08085769 -0.18191258
#> V23 -0.13244642 -0.09602086 -0.05572430
#> V24  0.19521610  0.36104343  0.41541732
#> V25  0.60069920  0.35462665  0.30081246
#> V26 -0.58374358 -0.45029975 -0.33583391
#> V27  0.28099313  0.44437155  0.19350901
#> V28 -0.06357756 -0.17832506  0.07156053
#> V29  0.62346564  0.51200199  0.39270903
#> V30  0.09067412  0.03265217 -0.04430665
#>
#> $df
#>  s0  s1  s2  s3  s4  s5  s6  s7  s8  s9 s10 s11 s12 s13 s14 s15 s16 s17 s18 s19
#>   1  11  17  14  17  22  21  21  27  23  27  30  30  29  30  30  30  30  30  30
#> s20 s21 s22 s23 s24 s25 s26 s27 s28 s29 s30 s31 s32 s33 s34 s35 s36 s37 s38 s39
#>  30  30  30  30  30  30  30  30  30  30  30  30  30  30  30  30  30  30  30  30
#> s40
#>  30
#>
#> $dim
#> [1] 30 41
#>
#> $lambda
#>  [1] 4.00000000 3.48220225 3.03143313 2.63901582 2.29739671 2.00000000
#>  [7] 1.74110113 1.51571657 1.31950791 1.14869835 1.00000000 0.87055056
#> [13] 0.75785828 0.65975396 0.57434918 0.50000000 0.43527528 0.37892914
#> [19] 0.32987698 0.28717459 0.25000000 0.21763764 0.18946457 0.16493849
#> [25] 0.14358729 0.12500000 0.10881882 0.09473229 0.08246924 0.07179365
#> [31] 0.06250000 0.05440941 0.04736614 0.04123462 0.03589682 0.03125000
#> [37] 0.02720471 0.02368307 0.02061731 0.01794841 0.01562500
#>
```

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#> $nobs
#> [1] 30
#>
#> attr(,"class")
#> [1] "qcdpath"
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