

# An Introduction to QCD

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## Introduction

QCD is a package that solves penalized quantile regression via exact coordinate descent method. The penalties considered are LASSO, SCAD, and MCP. Note that QCD algorithms for SCAD and MCP are experimental.

This vignette describes basic usage of functions related to  $\ell_1$  penalized quantile regression model in QCD package in R.

QCD mainly solves the following problem. Given data points  $(x_1, y_1), \dots, (x_n, y_n)$ , where  $y_i \in \mathbb{R}$  is a numerical response variable, and  $x_i \in \mathbb{R}^p$  is a  $p$ -dimensional covariate,

$$\arg \min_{\beta \in \mathbb{R}^p} \sum_{i=1}^n \rho_{\tau}(y_i - x_i^{\top} \beta) + \lambda \sum_{j=1}^p |\beta_j|$$

where  $\rho_{\tau}(u) := u(\tau - \mathbf{I}(u < 0))$  is the check loss function, and  $\lambda$  is a penalty parameter to be chosen in a data-driven fashion.  $\lambda$  could be a grid of values covering the entire range of possible solutions.

## Installation

You can download the QCD package by directly installing QCD\_0.0.0.9000.tar.gz file.

## Quick Start: QCD

The purpose of this section is to give users a general sense of the package regarding QCD. We will briefly go over the main functions, basic operations and outputs. QCD can be loaded using the `library` command:

```
library(QCD)
```

We create a data set for illustration:

```
set.seed(1)
data <- generate.data(n = 30, p = 30, tau = 0.5, signal = 1)
x <- data$X
y <- data$Y
```

This function generates simulation data based on Peng and Wang (2015). Note that in our work, we do not consider intercept and we can change the value of true beta through signal argument.

With single value of  $\lambda$ , we can fit the  $\ell_1$  penalized quantile regression using ‘qcd.lasso.fit’.

```
qr.lasso <- qcd.lasso.fit(x = x, y = y,
                        tau = 0.5, lambda = 0.01,
                        thresh = 1e-06, maxit = 100)

qr.lasso
#> $beta
#> [1] -0.092828056 -0.093605858 0.334601603 -0.139767059 0.307413743
#> [6] 0.895997273 -0.309618058 -0.066904169 0.181925297 -0.021908273
#> [11] -0.271969235 1.394531454 0.025730867 0.035676597 0.627937125
#> [16] -0.061635165 0.083219712 0.203526236 0.110415113 0.288514126
#> [21] 0.170748508 -0.018295819 0.083772780 0.051386300 0.077030655
#> [26] -0.095592290 -0.129518485 -0.090708140 -0.256023956 -0.009767324
#>
#> $df
#> [1] 30
#>
#> $lambda
#> [1] 0.01
#>
#> attr("class")
#> [1] "qcdlassofit"
```

We can fit the SCAD and MCP penalized quantile regression using ‘qcd.scad.fit’ and ‘qcd.mcp.fit’. These functions are experimental.

```
qr.scad <- qcd.scad.fit(x = x, y = y,
                      tau = 0.5, lambda = 0.01,
                      a = 2.2,
                      thresh = 1e-06, maxit = 100)

qr.scad
#> $beta
#> [1] 0.176752826 -0.121247322 0.594115599 -0.280112624 0.417956121
#> [6] 0.679449940 -0.035286819 0.268110897 -0.090038272 -0.002818645
#> [11] -0.416387713 1.126013449 -0.138645841 0.393340280 0.689371256
#> [16] -0.266273902 0.249587620 0.147824826 0.001737899 0.638888130
#> [21] 0.067816697 -0.077682990 -0.142755242 0.240249238 0.385999107
#> [26] -0.173536777 -0.065163988 0.410790718 -0.177674684 -0.039264227
#>
#> $df
#> [1] 30
#>
#> $lambda
#> [1] 0.01
#>
#> attr("class")
#> [1] "qcdscadfit"
```

```
qr.mcp <- qcd.mcp.fit(x = x, y = y,
                    tau = 0.5, lambda = 0.01,
                    a = 2.2,
                    thresh = 1e-06, maxit = 100)

qr.mcp
#> $beta
#> [1] 0.12718398 -0.12942573 0.54785270 -0.29481134 0.45576045 0.67201861
#> [7] -0.01550733 0.26119048 -0.06294352 0.15313129 -0.35603302 1.26391304
```

```

#> [13] -0.09530798  0.34376814  0.58087761 -0.25959496  0.24199422  0.15461116
#> [19] -0.05664122  0.62170089 -0.02336958 -0.06587700 -0.03259470  0.27173846
#> [25]  0.30868058 -0.15495523 -0.19792303  0.37649892 -0.12086170 -0.14789506
#>
#> $df
#> [1] 30
#>
#> $lambda
#> [1] 0.01
#>
#> attr("class")
#> [1] "qcdmcpfit"

```

Users are encouraged to utilize ‘qcd.path’ function to solve penalized quantile regression problems. A pathwise scheme ‘warm start’ is used as a default.

```

## Create lambda grid
upper <- 2; lower <- -6
lambda <- 2^seq(upper, lower, by = -0.2)

## warm start version
qr.lasso.warm = qcd.path(x = x, y = y, tau = 0.5,
                        funname = "LASSO", lambda = lambda,
                        nudge = FALSE,
                        thresh = 1e-06, maxit = 10000)

qr.lasso.warm
#> $beta
#>
#>      s0      s1      s2      s3      s4      s5
#> V1  0.000000 0.000000 0.000000 0.000000 0.000000 6.731544e-02
#> V2  0.000000 0.000000 0.000000 0.000000 0.000000 -8.636151e-03
#> V3  0.000000 0.000000 0.000000 0.000000 0.000000 0.000000e+00
#> V4  0.000000 0.000000 0.000000 0.000000 0.000000 -1.316437e-01
#> V5  0.3110855 0.3110855 0.3110855 0.3110855 0.5619431 2.073391e-01
#> V6  0.000000 0.000000 0.000000 0.000000 0.000000 7.819408e-01
#> V7  0.000000 0.000000 0.000000 0.000000 0.000000 0.000000e+00
#> V8  0.000000 0.000000 0.000000 0.000000 0.000000 0.000000e+00
#> V9  0.000000 0.000000 0.000000 0.000000 0.000000 -1.267696e-02
#> V10 0.000000 0.000000 0.000000 0.000000 0.000000 -5.740793e-05
#> V11 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000e+00
#> V12 0.000000 0.000000 0.000000 0.000000 0.000000 9.190977e-01
#> V13 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000e+00
#> V14 0.000000 0.000000 0.000000 0.000000 0.000000 -6.254023e-03
#> V15 0.000000 0.000000 0.000000 0.000000 0.000000 8.352232e-01
#> V16 0.000000 0.000000 0.000000 0.000000 0.000000 -7.015600e-03
#> V17 0.000000 0.000000 0.000000 0.000000 0.000000 6.700498e-03
#> V18 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000e+00
#> V19 0.000000 0.000000 0.000000 0.000000 0.000000 5.561056e-02
#> V20 0.000000 0.000000 0.000000 0.000000 0.000000 6.842728e-01
#> V21 0.000000 0.000000 0.000000 0.000000 0.000000 4.129693e-03
#> V22 0.000000 0.000000 0.000000 0.000000 0.000000 3.229291e-02
#> V23 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000e+00
#> V24 0.000000 0.000000 0.000000 0.000000 0.000000 -9.598278e-05
#> V25 0.000000 0.000000 0.000000 0.000000 0.000000 1.259680e-01

```

```

#> V26 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.000000e+00
#> V27 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 -6.011067e-02
#> V28 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.000000e+00
#> V29 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 -2.238831e-02
#> V30 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 -1.110061e-01
#>
#>          s6          s7          s8          s9          s10
#> V1  6.731544e-02 6.731540e-02 0.0642342837 6.423428e-02 6.394435e-02
#> V2 -8.636151e-03 -8.636128e-03 -0.0086265767 -8.626577e-03 -8.693383e-03
#> V3  0.000000e+00 0.000000e+00 0.0000000000 0.000000e+00 1.634601e-03
#> V4 -1.316436e-01 -1.316435e-01 -0.1267499973 -1.267500e-01 -1.262337e-01
#> V5  2.073390e-01 2.073388e-01 0.1952337742 1.952337e-01 1.928716e-01
#> V6  7.819409e-01 7.819413e-01 0.8078836532 8.078837e-01 8.110941e-01
#> V7  0.000000e+00 0.000000e+00 0.0000000000 0.000000e+00 3.799097e-04
#> V8  0.000000e+00 0.000000e+00 0.0000000000 0.000000e+00 2.741385e-04
#> V9 -1.267697e-02 -1.267697e-02 -0.0227591441 -2.275917e-02 -2.418438e-02
#> V10 -5.742005e-05 -5.742005e-05 0.0000000000 0.000000e+00 0.000000e+00
#> V11 0.000000e+00 0.000000e+00 0.0000000000 -2.790680e-08 -2.050330e-03
#> V12 9.190977e-01 9.190982e-01 0.9563652483 9.563653e-01 9.567205e-01
#> V13 0.000000e+00 0.000000e+00 0.0000000000 0.000000e+00 -1.677292e-08
#> V14 -6.253995e-03 -6.253995e-03 -0.0101456366 -1.014564e-02 -1.056192e-02
#> V15 8.352232e-01 8.352233e-01 0.8449265142 8.449265e-01 8.470345e-01
#> V16 -7.015602e-03 -7.015636e-03 -0.0093892217 -9.389261e-03 -1.014488e-02
#> V17 6.700498e-03 6.700658e-03 0.0255281328 2.552814e-02 2.623927e-02
#> V18 0.000000e+00 0.000000e+00 -0.0006850665 -6.851314e-04 -8.477501e-04
#> V19 5.561055e-02 5.561063e-02 0.0627296636 6.272968e-02 6.261725e-02
#> V20 6.842728e-01 6.842728e-01 0.6885162439 6.885162e-01 6.889572e-01
#> V21 4.129690e-03 4.129709e-03 0.0052342352 5.234247e-03 5.915774e-03
#> V22 3.229299e-02 3.229300e-02 0.0321664045 3.216640e-02 3.266759e-02
#> V23 0.000000e+00 0.000000e+00 0.0000000000 0.000000e+00 0.000000e+00
#> V24 -9.598278e-05 -9.598278e-05 -0.0032919629 -3.292008e-03 -4.709707e-03
#> V25 1.259679e-01 1.259679e-01 0.1207501640 1.207501e-01 1.204907e-01
#> V26 0.000000e+00 0.000000e+00 -0.0053654633 -5.365503e-03 -6.596449e-03
#> V27 -6.011074e-02 -6.011085e-02 -0.0738557550 -7.385581e-02 -7.440844e-02
#> V28 0.000000e+00 0.000000e+00 0.0000000000 0.000000e+00 -3.202633e-06
#> V29 -2.238831e-02 -2.238831e-02 -0.0198839353 -1.988394e-02 -1.969949e-02
#> V30 -1.110060e-01 -1.110059e-01 -0.1016507942 -1.016507e-01 -1.004403e-01
#>
#>          s11          s12          s13          s14          s15
#> V1  6.394435e-02 6.394437e-02 6.394437e-02 6.394447e-02 6.394447e-02
#> V2 -8.693382e-03 -8.693375e-03 -8.693383e-03 -8.693114e-03 -8.693114e-03
#> V3  1.634613e-03 1.634632e-03 1.634654e-03 1.634840e-03 1.634858e-03
#> V4 -1.262337e-01 -1.262337e-01 -1.262337e-01 -1.262337e-01 -1.262338e-01
#> V5  1.928715e-01 1.928715e-01 1.928714e-01 1.928709e-01 1.928709e-01
#> V6  8.110941e-01 8.110943e-01 8.110944e-01 8.110951e-01 8.110951e-01
#> V7  3.799256e-04 3.799883e-04 3.799944e-04 3.809648e-04 3.809648e-04
#> V8  2.741385e-04 2.741992e-04 2.742260e-04 2.745416e-04 2.745449e-04
#> V9 -2.418439e-02 -2.418455e-02 -2.418460e-02 -2.418540e-02 -2.418541e-02
#> V10 -5.353499e-09 0.000000e+00 0.000000e+00 -8.911968e-09 -5.903181e-09
#> V11 -2.050386e-03 -2.050432e-03 -2.050432e-03 -2.052684e-03 -2.052723e-03
#> V12 9.567205e-01 9.567206e-01 9.567206e-01 9.567207e-01 9.567207e-01
#> V13 -7.997758e-09 -7.997758e-09 -7.997758e-09 -6.493959e-09 -6.493959e-09
#> V14 -1.056194e-02 -1.056201e-02 -1.056206e-02 -1.056259e-02 -1.056261e-02
#> V15 8.470345e-01 8.470347e-01 8.470347e-01 8.470364e-01 8.470364e-01
#> V16 -1.014489e-02 -1.014496e-02 -1.014497e-02 -1.014539e-02 -1.014540e-02

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#> V17 2.623944e-02 2.623977e-02 2.623980e-02 2.624186e-02 2.624189e-02
#> V18 -8.478062e-04 -8.479306e-04 -8.479592e-04 -8.496434e-04 -8.496519e-04
#> V19 6.261730e-02 6.261739e-02 6.261739e-02 6.261848e-02 6.261849e-02
#> V20 6.889571e-01 6.889571e-01 6.889571e-01 6.889566e-01 6.889566e-01
#> V21 5.915774e-03 5.915792e-03 5.915805e-03 5.916046e-03 5.916046e-03
#> V22 3.266765e-02 3.266765e-02 3.266765e-02 3.266798e-02 3.266799e-02
#> V23 -5.398425e-18 0.000000e+00 0.000000e+00 0.000000e+00 -2.166870e-09
#> V24 -4.709806e-03 -4.709917e-03 -4.709917e-03 -4.710731e-03 -4.710731e-03
#> V25 1.204907e-01 1.204907e-01 1.204907e-01 1.204915e-01 1.204915e-01
#> V26 -6.596535e-03 -6.596664e-03 -6.596764e-03 -6.598198e-03 -6.598212e-03
#> V27 -7.440850e-02 -7.440854e-02 -7.440857e-02 -7.440961e-02 -7.440962e-02
#> V28 -3.155729e-06 -3.099711e-06 -3.074131e-06 -2.527785e-06 -2.527721e-06
#> V29 -1.969949e-02 -1.969951e-02 -1.969951e-02 -1.969961e-02 -1.969962e-02
#> V30 -1.004403e-01 -1.004403e-01 -1.004403e-01 -1.004398e-01 -1.004398e-01
#>          s16          s17          s18          s19          s20
#> V1  6.394447e-02 0.065969914 0.065969927 0.065969928 0.065969929
#> V2 -8.693113e-03 -0.006695020 -0.006695020 -0.006695022 -0.006695022
#> V3  1.634890e-03 0.004206090 0.004206132 0.004206137 0.004206140
#> V4 -1.262338e-01 -0.128006653 -0.128006667 -0.128006667 -0.128006668
#> V5  1.928709e-01 0.185466268 0.185466215 0.185466212 0.185466212
#> V6  8.110951e-01 0.821307776 0.821307838 0.821307843 0.821307844
#> V7  3.809659e-04 0.006124901 0.006124966 0.006124970 0.006124974
#> V8  2.745449e-04 0.006169617 0.006169639 0.006169647 0.006169665
#> V9 -2.418542e-02 -0.029294572 -0.029294578 -0.029294583 -0.029294593
#> V10 3.076850e-09 0.001062770 0.001062797 0.001062798 0.001062801
#> V11 -2.052760e-03 -0.016118994 -0.016118965 -0.016118964 -0.016118958
#> V12 9.567207e-01 0.957637101 0.957637101 0.957637101 0.957637104
#> V13 -6.493959e-09 0.001714065 0.001714055 0.001714062 0.001714071
#> V14 -1.056264e-02 -0.018906545 -0.018906545 -0.018906545 -0.018906546
#> V15 8.470365e-01 0.866482064 0.866482070 0.866482071 0.866482074
#> V16 -1.014543e-02 -0.015756588 -0.015756592 -0.015756592 -0.015756593
#> V17 2.624195e-02 0.044171829 0.044171829 0.044171829 0.044171827
#> V18 -8.496879e-04 -0.006888370 -0.006888370 -0.006888370 -0.006888370
#> V19 6.261853e-02 0.061808828 0.061808828 0.061808828 0.061808828
#> V20 6.889566e-01 0.698465853 0.698465853 0.698465853 0.698465853
#> V21 5.916063e-03 0.005812152 0.005812152 0.005812152 0.005812152
#> V22 3.266799e-02 0.034363112 0.034363128 0.034363129 0.034363130
#> V23 1.513975e-08 0.000000000 0.000000000 0.000000000 0.000000000
#> V24 -4.710712e-03 -0.002381540 -0.002381540 -0.002381539 -0.002381539
#> V25 1.204915e-01 0.119629286 0.119629294 0.119629293 0.119629292
#> V26 -6.598244e-03 -0.012039122 -0.012039127 -0.012039128 -0.012039132
#> V27 -7.440962e-02 -0.076090347 -0.076090351 -0.076090352 -0.076090348
#> V28 -2.516410e-06 0.003330336 0.003330356 0.003330359 0.003330358
#> V29 -1.969964e-02 -0.020795903 -0.020795900 -0.020795900 -0.020795900
#> V30 -1.004398e-01 -0.104124169 -0.104124171 -0.104124172 -0.104124172
#>          s21          s22          s23          s24          s25
#> V1  6.596993e-02 6.596994e-02 0.0673056781 0.0673056798 0.0673061239
#> V2 -6.695022e-03 -6.695025e-03 -0.0050745475 -0.0050745482 -0.0050747091
#> V3  4.206145e-03 4.206159e-03 0.0192132088 0.0192132938 0.0192150460
#> V4 -1.280067e-01 -1.280067e-01 -0.1302410032 -0.1302410605 -0.1302413175
#> V5  1.854662e-01 1.854662e-01 0.1761981961 0.1761981635 0.1761969784
#> V6  8.213078e-01 8.213079e-01 0.8300375255 0.8300375628 0.8300389654
#> V7  6.124981e-03 6.124996e-03 0.0140423330 0.0140423995 0.0140428957

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#> V8 6.169692e-03 6.169755e-03 0.0201134865 0.0201135854 0.0201142464
#> V9 -2.929461e-02 -2.929465e-02 -0.0405520508 -0.0405520969 -0.0405525116
#> V10 1.062806e-03 1.062816e-03 0.0073643594 0.0073643885 0.0073651057
#> V11 -1.611895e-02 -1.611893e-02 -0.0177597356 -0.0177597356 -0.0177600251
#> V12 9.576371e-01 9.576371e-01 0.9577454507 0.9577454705 0.9577458285
#> V13 1.714086e-03 1.714117e-03 0.0030775277 0.0030775521 0.0030775997
#> V14 -1.890655e-02 -1.890655e-02 -0.0213588178 -0.0213588390 -0.0213597408
#> V15 8.664821e-01 8.664821e-01 0.8678530457 0.8678530508 0.8678532148
#> V16 -1.575660e-02 -1.575661e-02 -0.0151074112 -0.0151073939 -0.0151071191
#> V17 4.417183e-02 4.417183e-02 0.0456516123 0.0456516123 0.0456528952
#> V18 -6.888369e-03 -6.888365e-03 -0.0071924315 -0.0071924315 -0.0071927724
#> V19 6.180883e-02 6.180883e-02 0.0723273990 0.0723274202 0.0723280298
#> V20 6.984659e-01 6.984659e-01 0.6955388735 0.6955388735 0.6955389023
#> V21 5.812152e-03 5.812151e-03 0.0008335431 0.0008335431 0.0008331342
#> V22 3.436313e-02 3.436314e-02 0.0360080164 0.0360080092 0.0360081196
#> V23 5.736675e-10 6.078580e-09 -0.0017649092 -0.0017649093 -0.0017648937
#> V24 -2.381539e-03 -2.381539e-03 -0.0023054163 -0.0023054005 -0.0023054069
#> V25 1.196293e-01 1.196293e-01 0.1195361892 0.1195361909 0.1195362064
#> V26 -1.203914e-02 -1.203916e-02 -0.0129099629 -0.0129099764 -0.0129101692
#> V27 -7.609035e-02 -7.609035e-02 -0.0763367643 -0.0763367643 -0.0763367821
#> V28 3.330358e-03 3.330360e-03 0.0038136592 0.0038136548 0.0038138667
#> V29 -2.079590e-02 -2.079590e-02 -0.0196838935 -0.0196838935 -0.0196837646
#> V30 -1.041242e-01 -1.041242e-01 -0.1039780050 -0.1039779985 -0.1039778593
#> s26 s27 s28 s29 s30
#> V1 0.067306149 0.0673061929 0.0673062068 0.0673062648 0.0673062743
#> V2 -0.005074720 -0.0050747405 -0.0050747469 -0.0050747793 -0.0050747844
#> V3 0.019215137 0.0192153411 0.0192154648 0.0192157857 0.0192159062
#> V4 -0.130241345 -0.1302413641 -0.1302414132 -0.1302414814 -0.1302414814
#> V5 0.176196923 0.1761967736 0.1761967100 0.1761965107 0.1761964364
#> V6 0.830039009 0.8300391837 0.8300392264 0.8300394736 0.8300395528
#> V7 0.014042932 0.0140429980 0.0140430724 0.0140431316 0.0140431436
#> V8 0.020114350 0.0201144949 0.0201146353 0.0201147588 0.0201148095
#> V9 -0.040552565 -0.0405526781 -0.0405527562 -0.0405528849 -0.0405529510
#> V10 0.007365124 0.0073651674 0.0073652193 0.0073652820 0.0073653272
#> V11 -0.017760025 -0.0177600215 -0.0177600215 -0.0177600593 -0.0177600649
#> V12 0.957745857 0.9577459418 0.9577459691 0.9577460013 0.9577460078
#> V13 0.003077634 0.0030776394 0.0030776730 0.0030776705 0.0030776650
#> V14 -0.021359783 -0.0213598603 -0.0213598941 -0.0213600990 -0.0213601626
#> V15 0.867853212 0.8678532035 0.8678532122 0.8678532191 0.8678532287
#> V16 -0.015107097 -0.0151070300 -0.0151070024 -0.0151069119 -0.0151068894
#> V17 0.045652931 0.0456530099 0.0456530293 0.0456531534 0.0456532070
#> V18 -0.007192799 -0.0071928955 -0.0071929088 -0.0071929431 -0.0071929663
#> V19 0.072328102 0.0723282297 0.0723282904 0.0723283228 0.0723283667
#> V20 0.695538902 0.6955389023 0.6955389023 0.6955389152 0.6955389152
#> V21 0.000833085 0.0008330199 0.0008330007 0.0008329638 0.0008329335
#> V22 0.036008123 0.0360081299 0.0360081262 0.0360081423 0.0360081538
#> V23 -0.001764894 -0.0017648961 -0.0017648961 -0.0017648961 -0.0017648961
#> V24 -0.002305400 -0.0023054184 -0.0023054073 -0.0023054138 -0.0023054138
#> V25 0.119536206 0.1195362061 0.1195362057 0.1195362057 0.1195362057
#> V26 -0.012910169 -0.0129101922 -0.0129101947 -0.0129102110 -0.0129102219
#> V27 -0.076336782 -0.0763367818 -0.0763367818 -0.0763367818 -0.0763367818
#> V28 0.003813868 0.0038138937 0.0038138995 0.0038139210 0.0038139405
#> V29 -0.019683762 -0.0196837459 -0.0196837459 -0.0196837097 -0.0196836988

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#> V30 -0.103977859 -0.1039778425 -0.1039778426 -0.1039778106 -0.1039777996
#>          s31          s32          s33          s34          s35
#> V1  0.0673062922 0.0673063092 0.067306330 0.0673063381 0.0673063545
#> V2 -0.0050747917 -0.0050747944 -0.005074801 -0.0050748046 -0.0050748127
#> V3  0.0192159799 0.0192160189 0.019216039 0.0192160771 0.0192160933
#> V4 -0.1302414814 -0.1302414817 -0.130241489 -0.1302415096 -0.1302415187
#> V5  0.1761963935 0.1761963663 0.176196358 0.1761963387 0.1761963328
#> V6  0.8300396117 0.8300396393 0.830039653 0.8300396815 0.8300396897
#> V7  0.0140431643 0.0140431673 0.014043178 0.0140431869 0.0140432003
#> V8  0.0201148573 0.0201148578 0.020114877 0.0201148996 0.0201149205
#> V9 -0.0405529935 -0.0405529943 -0.040553005 -0.0405530258 -0.0405530258
#> V10 0.0073653686 0.0073653690 0.007365383 0.0073653890 0.0073653921
#> V11 -0.0177600699 -0.0177600804 -0.017760083 -0.0177600837 -0.0177600843
#> V12 0.9577460078 0.9577460119 0.957746012 0.9577460119 0.9577460183
#> V13 0.0030776650 0.0030776661 0.003077655 0.0030776559 0.0030776535
#> V14 -0.0213601961 -0.0213602128 -0.021360224 -0.0213602408 -0.0213602408
#> V15 0.8678532298 0.8678532360 0.867853239 0.8678532485 0.8678532450
#> V16 -0.0151068741 -0.0151068741 -0.015106870 -0.0151068663 -0.0151068618
#> V17 0.0456532507 0.0456532906 0.045653302 0.0456533179 0.0456533345
#> V18 -0.0071929976 -0.0071929976 -0.007193003 -0.0071930033 -0.0071930196
#> V19 0.0723284297 0.0723284549 0.072328476 0.0723284802 0.0723285115
#> V20 0.6955389152 0.6955389152 0.695538915 0.6955389152 0.6955389152
#> V21 0.0008328914 0.0008328863 0.000832880 0.0008328777 0.0008328713
#> V22 0.0360081694 0.0360081694 0.036008169 0.0360081694 0.0360081694
#> V23 -0.0017648963 -0.0017648963 -0.001764898 -0.0017648975 -0.0017648988
#> V24 -0.0023054138 -0.0023054293 -0.002305429 -0.0023054310 -0.0023054310
#> V25 0.1195362057 0.1195362057 0.119536206 0.1195362062 0.1195362062
#> V26 -0.0129102367 -0.0129102498 -0.012910250 -0.0129102601 -0.0129102650
#> V27 -0.0763367818 -0.0763367815 -0.076336782 -0.0763367815 -0.0763367815
#> V28 0.0038139570 0.0038139712 0.003813977 0.0038139771 0.0038139886
#> V29 -0.0196836760 -0.0196836692 -0.019683668 -0.0196836675 -0.0196836631
#> V30 -0.1039778103 -0.1039778103 -0.103977804 -0.1039778110 -0.1039777998
#>          s36          s37          s38          s39          s40
#> V1  0.0673063648 0.0673063789 0.0673063838 0.0673063838 0.0673064134
#> V2 -0.0050748157 -0.0050748246 -0.0050748268 -0.0050748268 -0.0050748407
#> V3  0.0192161396 0.0192161988 0.0192162707 0.0192163833 0.0192165220
#> V4 -0.1302415439 -0.1302415849 -0.1302416345 -0.1302417070 -0.1302417920
#> V5  0.1761963149 0.1761962870 0.1761962579 0.1761962146 0.1761961681
#> V6  0.8300397207 0.8300397756 0.8300398328 0.8300399168 0.8300400083
#> V7  0.0140432210 0.0140432460 0.0140432811 0.0140433421 0.0140434137
#> V8  0.0201149927 0.0201150765 0.0201151933 0.0201153619 0.0201155311
#> V9 -0.0405530678 -0.0405530968 -0.0405531219 -0.0405531751 -0.0405532429
#> V10 0.0073654061 0.0073654141 0.0073654212 0.0073654428 0.0073654763
#> V11 -0.0177600843 -0.0177600852 -0.0177600852 -0.0177600852 -0.0177600852
#> V12 0.9577460393 0.9577460393 0.9577460421 0.9577460634 0.9577460929
#> V13 0.0030776793 0.0030776799 0.0030776834 0.0030777097 0.0030777461
#> V14 -0.0213602629 -0.0213602842 -0.0213602949 -0.0213603208 -0.0213603535
#> V15 0.8678532503 0.8678532630 0.8678532759 0.8678532894 0.8678533008
#> V16 -0.0151068549 -0.0151068481 -0.0151068476 -0.0151068379 -0.0151068322
#> V17 0.0456533467 0.0456533570 0.0456533639 0.0456533751 0.0456533975
#> V18 -0.0071930196 -0.0071930293 -0.0071930372 -0.0071930379 -0.0071930393
#> V19 0.0723285285 0.0723285459 0.0723285623 0.0723285706 0.0723285877
#> V20 0.6955389152 0.6955389210 0.6955389248 0.6955389248 0.6955389248

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#> V21 0.0008328634 0.0008328451 0.0008328306 0.0008328306 0.0008328199
#> V22 0.0360081656 0.0360081672 0.0360081604 0.0360081584 0.0360081511
#> V23 -0.0017648988 -0.0017648988 -0.0017649012 -0.0017648980 -0.0017648980
#> V24 -0.0023054279 -0.0023054278 -0.0023054274 -0.0023054274 -0.0023054221
#> V25 0.1195362062 0.1195362062 0.1195362087 0.1195362087 0.1195362087
#> V26 -0.0129102650 -0.0129102650 -0.0129102650 -0.0129102812 -0.0129102812
#> V27 -0.0763367815 -0.0763367815 -0.0763367815 -0.0763367815 -0.0763367815
#> V28 0.0038139888 0.0038139914 0.0038139916 0.0038139802 0.0038139805
#> V29 -0.0196836631 -0.0196836631 -0.0196836631 -0.0196836631 -0.0196836631
#> V30 -0.1039777998 -0.1039777998 -0.1039777998 -0.1039777998 -0.1039777998
#>
#> $df
#> s0 s1 s2 s3 s4 s5 s6 s7 s8 s9 s10 s11 s12 s13 s14 s15 s16 s17 s18 s19
#> 1 1 1 1 1 21 21 21 22 23 28 30 28 28 29 30 30 29 29 29
#> s20 s21 s22 s23 s24 s25 s26 s27 s28 s29 s30 s31 s32 s33 s34 s35 s36 s37 s38 s39
#> 29 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30
#> s40
#> 30
#>
#> $dim
#> [1] 30 41
#>
#> $lambda
#> [1] 4.00000000 3.48220225 3.03143313 2.63901582 2.29739671 2.00000000
#> [7] 1.74110113 1.51571657 1.31950791 1.14869835 1.00000000 0.87055056
#> [13] 0.75785828 0.65975396 0.57434918 0.50000000 0.43527528 0.37892914
#> [19] 0.32987698 0.28717459 0.25000000 0.21763764 0.18946457 0.16493849
#> [25] 0.14358729 0.12500000 0.10881882 0.09473229 0.08246924 0.07179365
#> [31] 0.06250000 0.05440941 0.04736614 0.04123462 0.03589682 0.03125000
#> [37] 0.02720471 0.02368307 0.02061731 0.01794841 0.01562500
#>
#> $nobs
#> [1] 30
#>
#> attr("class")
#> [1] "qcdpath"

## warm start and nudge version
set.seed(1)
qr.lasso.warm.nudge = qcd.path(x = x, y = y, tau = 0.5,
                               funname = "LASSO", lambda = lambda,
                               nudge = TRUE, nudgesd = 0.2,
                               thresh = 1e-06, maxit = 10000)

qr.lasso.warm.nudge
#> $beta
#> s0 s1 s2 s3 s4
#> V1 0.0000000 1.875189e-02 3.742754e-03 0.000000e+00 0.0009154288
#> V2 0.0000000 0.000000e+00 0.000000e+00 3.973581e-03 0.0000000000
#> V3 0.0000000 0.000000e+00 1.468481e-02 3.712587e-06 0.0033767145
#> V4 0.0000000 -1.445787e-07 -1.802037e-02 -7.676968e-02 -0.0001093258
#> V5 0.3110855 2.006387e-01 4.374324e-02 3.467623e-02 0.0068257787
#> V6 0.0000000 6.406430e-01 9.312066e-01 9.860243e-01 0.9242578086

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#> V7 0.0000000 0.000000e+00 0.000000e+00 0.000000e+00 -0.0018146505
#> V8 0.0000000 0.000000e+00 0.000000e+00 0.000000e+00 0.0000000000
#> V9 0.0000000 0.000000e+00 0.000000e+00 -3.469181e-03 -0.0105940102
#> V10 0.0000000 0.000000e+00 -1.228788e-03 0.000000e+00 0.0000000000
#> V11 0.0000000 0.000000e+00 -3.097514e-09 0.000000e+00 0.0000000000
#> V12 0.0000000 7.675892e-01 9.643367e-01 9.267420e-01 0.9870802005
#> V13 0.0000000 0.000000e+00 0.000000e+00 0.000000e+00 0.0000000000
#> V14 0.0000000 0.000000e+00 0.000000e+00 0.000000e+00 0.0000000000
#> V15 0.0000000 6.180611e-01 8.525992e-01 8.679650e-01 0.8679749077
#> V16 0.0000000 0.000000e+00 0.000000e+00 0.000000e+00 0.0000000000
#> V17 0.0000000 0.000000e+00 0.000000e+00 0.000000e+00 0.0000000000
#> V18 0.0000000 5.379706e-02 0.000000e+00 0.000000e+00 0.0101529191
#> V19 0.0000000 0.000000e+00 0.000000e+00 3.822299e-02 0.0000000000
#> V20 0.0000000 5.440848e-01 7.014941e-01 7.296368e-01 0.7660808483
#> V21 0.0000000 0.000000e+00 1.755036e-03 0.000000e+00 0.0000000000
#> V22 0.0000000 5.798751e-02 3.525397e-02 0.000000e+00 0.0325573504
#> V23 0.0000000 0.000000e+00 0.000000e+00 0.000000e+00 0.0000000000
#> V24 0.0000000 0.000000e+00 0.000000e+00 0.000000e+00 0.0742080614
#> V25 0.0000000 0.000000e+00 6.386184e-02 5.856033e-02 0.0001530013
#> V26 0.0000000 0.000000e+00 0.000000e+00 0.000000e+00 0.0000000000
#> V27 0.0000000 -6.265151e-02 -3.851578e-02 -5.393155e-02 -0.0147739391
#> V28 0.0000000 -4.561052e-03 -4.288593e-02 -7.643988e-02 -0.1138501352
#> V29 0.0000000 0.000000e+00 -3.728742e-05 0.000000e+00 0.0000000000
#> V30 0.0000000 0.000000e+00 -2.947437e-02 -1.315641e-02 -0.0469475366
#>
#> s5 s6 s7 s8 s9
#> V1 6.866099e-02 1.432427e-02 0.1137354789 5.752059e-02 0.000000e+00
#> V2 0.000000e+00 -6.661090e-04 -0.0008019823 8.908435e-02 0.000000e+00
#> V3 1.120390e-07 1.263760e-02 0.0352295716 1.050244e-02 6.784181e-02
#> V4 -3.068059e-02 -2.538759e-02 -0.0717971336 -8.161237e-03 -6.564822e-02
#> V5 2.613381e-03 3.773116e-02 0.0300860197 -2.413914e-02 2.039897e-01
#> V6 9.183016e-01 9.346685e-01 0.8701966680 1.100921e+00 7.893426e-01
#> V7 0.000000e+00 0.000000e+00 0.0000000000 -4.029473e-05 -2.253455e-02
#> V8 0.000000e+00 0.000000e+00 0.0000000000 1.289648e-03 0.000000e+00
#> V9 0.000000e+00 -3.174631e-02 -0.0337395618 -1.358959e-02 0.000000e+00
#> V10 -2.021466e-02 -2.752755e-05 0.0000000000 -2.910527e-02 -3.001992e-02
#> V11 -6.955499e-02 -1.486067e-04 -0.0709215001 0.000000e+00 -1.400910e-01
#> V12 8.728632e-01 1.004131e+00 0.9566326776 9.682461e-01 9.983066e-01
#> V13 -6.986891e-03 -2.527619e-05 -0.0226068412 -1.051597e-03 5.567097e-02
#> V14 0.000000e+00 0.000000e+00 -0.0003974008 -2.032148e-04 0.000000e+00
#> V15 9.909870e-01 8.510996e-01 1.0042410069 9.400168e-01 8.203664e-01
#> V16 -5.205196e-05 0.000000e+00 -0.0444233935 -4.638070e-02 1.965111e-02
#> V17 6.018537e-03 0.000000e+00 0.0905565652 -7.933212e-03 -7.789660e-03
#> V18 0.000000e+00 0.000000e+00 0.0000000000 3.176851e-02 0.000000e+00
#> V19 5.176167e-02 1.326135e-03 0.0092561623 6.917055e-09 4.897827e-02
#> V20 7.999706e-01 7.104500e-01 0.8011689743 6.491084e-01 5.064836e-01
#> V21 1.071534e-03 0.000000e+00 0.0000000000 1.883563e-01 2.029778e-01
#> V22 2.799580e-02 3.779420e-02 0.0476142081 -2.840382e-02 3.396929e-02
#> V23 0.000000e+00 0.000000e+00 0.0000000000 6.899908e-02 0.000000e+00
#> V24 0.000000e+00 2.242456e-02 0.0000000000 1.293924e-01 -1.139164e-05
#> V25 6.416887e-02 4.407741e-02 0.0000000000 8.250207e-03 1.968771e-01
#> V26 4.132600e-04 2.324643e-02 0.0000000000 -8.160813e-02 -3.728897e-02
#> V27 -4.372767e-02 -9.635428e-02 -0.0289019323 0.000000e+00 -1.262337e-03
#> V28 -2.024012e-02 -5.338405e-02 -0.1157301290 -1.234873e-01 -1.052318e-01

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#> V29 -3.828960e-03 0.000000e+00 -0.0204383794 0.000000e+00 -2.325809e-02
#> V30 -1.022323e-01 -6.595778e-02 -0.0455641140 -6.093525e-02 -7.663766e-02
#>          s10          s11          s12          s13          s14
#> V1  1.481614e-01 -9.528501e-03 8.989636e-02 6.149412e-02 0.0110210335
#> V2  3.099443e-02 -1.144637e-02 2.636851e-02 -6.658547e-05 -0.0211153347
#> V3  3.820054e-03 4.410886e-02 -2.650002e-02 4.013448e-02 0.1745791883
#> V4  -4.984232e-02 -2.156924e-02 -3.010869e-02 2.408958e-02 0.0857119160
#> V5  -8.647831e-03 -1.684309e-01 -7.928355e-02 -1.463673e-01 -0.0891125045
#> V6  9.995353e-01 1.078789e+00 8.951432e-01 1.009159e+00 0.8550845487
#> V7  -3.710606e-02 -3.455408e-01 -2.744064e-01 -2.550950e-01 -0.2525672119
#> V8  4.489058e-02 9.225576e-05 -9.671643e-06 -6.509985e-09 -0.0183776924
#> V9  0.000000e+00 2.719306e-02 2.438851e-02 -2.171213e-05 0.0354224188
#> V10 -2.390701e-01 -2.742821e-01 -5.000872e-01 -2.980319e-01 -0.1970935210
#> V11 -2.192056e-05 5.435465e-02 2.735198e-03 0.000000e+00 -0.3472705370
#> V12 9.481941e-01 1.210232e+00 1.052882e+00 1.192345e+00 1.3327464157
#> V13 -1.903550e-02 1.014144e-03 -7.679151e-03 -5.402122e-02 -0.0102523124
#> V14 5.817570e-03 -5.961981e-03 -2.037632e-03 1.460914e-01 0.1722554480
#> V15 8.948765e-01 7.571915e-01 7.613635e-01 6.081566e-01 0.6580456987
#> V16 -4.478414e-02 1.422500e-01 1.691986e-01 1.945530e-01 0.1192383640
#> V17 0.000000e+00 3.895763e-02 1.833364e-04 6.915021e-02 0.0221620077
#> V18 -2.722911e-03 -2.484297e-02 -1.538512e-01 -1.264993e-01 -0.2340619437
#> V19 6.165563e-02 2.126434e-03 4.086699e-01 9.282835e-02 0.2580116116
#> V20 5.792020e-01 4.127484e-01 1.025625e-01 1.986676e-01 0.2271625059
#> V21 8.409230e-02 -6.534032e-02 -2.584322e-02 5.772623e-03 -0.0583344376
#> V22 5.717557e-03 1.573872e-01 2.388237e-01 1.814598e-01 0.2193806706
#> V23 8.792974e-02 1.121050e-01 7.291859e-02 7.697040e-02 0.0271644528
#> V24 0.000000e+00 -1.109946e-01 -1.816245e-01 -2.305626e-01 -0.1487974154
#> V25 2.027193e-01 1.517607e-01 3.736138e-01 3.679282e-01 0.3073106743
#> V26 -2.203202e-02 6.798181e-02 2.566153e-02 -1.491300e-02 -0.0003873022
#> V27 -9.925602e-02 -1.837503e-01 -1.090097e-01 -2.269319e-01 -0.0788398833
#> V28 -2.903440e-02 -1.813126e-01 -1.179886e-01 -4.032997e-02 0.0168990233
#> V29 6.417416e-02 -3.498186e-02 -2.803242e-02 6.405674e-03 -0.2534720381
#> V30 -4.402105e-02 -1.999491e-02 -1.242001e-01 -7.183976e-02 -0.0007200669
#>          s15          s16          s17          s18          s19
#> V1  7.209176e-04 0.037032779 0.29871524 0.3352377529 0.37932176
#> V2  2.445719e-02 0.208410408 0.21815538 0.3955309369 0.19180558
#> V3  2.856160e-01 0.418800248 -0.12916297 -0.0289670449 -0.16947498
#> V4  4.888602e-02 0.099997548 0.07316914 0.1145883011 -0.12033871
#> V5  -8.355404e-02 -0.106098153 -0.09310139 -0.0009086178 0.11037841
#> V6  1.046188e+00 0.999043521 1.06383838 0.9419749633 0.78507462
#> V7  -1.614941e-01 0.281707947 0.24688603 0.7359691640 0.45915686
#> V8  -4.767335e-06 0.004850402 0.12725581 0.0578713279 0.14810058
#> V9  2.193337e-01 0.177202903 -0.02082269 0.1392637092 -0.11621559
#> V10 -2.245129e-02 -0.094272667 -0.04764387 -0.0620616560 -0.14613227
#> V11 -2.253146e-01 -0.376619085 -0.18472285 -0.4247778145 -0.18682510
#> V12 1.510858e+00 1.196570786 0.71695491 0.6951722590 0.34335067
#> V13 2.090458e-03 -0.191095581 -0.20297120 -0.2137613378 -0.16255418
#> V14 6.843923e-02 0.109358449 0.02871150 0.0456871439 -0.01985644
#> V15 5.818344e-01 0.897141079 1.11546762 1.1903585277 1.33031453
#> V16 1.808911e-01 -0.015308459 -0.12665448 -0.2699115924 -0.14953431
#> V17 2.494766e-01 0.001103745 -0.06228569 -0.2589127252 -0.37771835
#> V18 -1.953543e-01 -0.104286578 -0.06925207 0.0024062860 -0.05853942
#> V19 2.163023e-01 0.272791973 0.25366514 0.4121436918 0.50305483

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#> V20 2.511170e-01 0.686637611 0.74236935 0.8451410018 0.90514564
#> V21 -7.559755e-02 0.129699523 -0.01533035 -0.1899772652 -0.31937535
#> V22 3.280021e-01 0.091580135 0.15907304 0.2480009128 0.26023208
#> V23 -3.023344e-02 -0.093065772 0.04629731 0.0005824656 0.08610328
#> V24 -6.355937e-02 -0.013219557 0.10759213 0.0635686759 0.07350639
#> V25 3.208341e-01 0.378775844 0.20938159 0.4557802974 0.31916019
#> V26 -1.568961e-01 -0.227214245 -0.12255930 -0.2470378852 -0.03218987
#> V27 -1.068754e-01 0.028680141 0.03926748 -0.0214112255 -0.06885180
#> V28 -4.653896e-04 0.166190866 -0.04574659 0.1309586217 0.05587769
#> V29 -7.604536e-02 0.138778486 0.23676341 0.4260161757 0.26205966
#> V30 -1.847542e-01 -0.231338873 -0.11552136 -0.1343257246 -0.01323197
#>          s20          s21          s22          s23          s24          s25
#> V1 0.324725318 0.018220301 0.260672581 0.20798031 0.33381791 0.26659987
#> V2 -0.005518256 0.205098824 0.245708065 0.27187279 0.56761095 0.55783907
#> V3 -0.168509707 -0.122670307 0.115492033 0.19967754 0.16704239 0.29702376
#> V4 0.062691818 -0.076505758 -0.251399653 -0.10245166 -0.17454964 -0.09463670
#> V5 -0.043149532 -0.004023111 0.135757356 0.09086076 0.11194168 0.09163635
#> V6 0.922632346 1.289152281 0.971220014 1.11183386 1.23812112 1.17522766
#> V7 0.333692328 0.302203132 0.587140504 0.55540780 0.64196923 0.85223503
#> V8 0.113093108 0.104343766 0.154670877 0.02158058 0.06886481 0.20899756
#> V9 0.032406214 0.136105641 0.002262405 0.01864697 0.20987633 0.14009963
#> V10 0.007501317 0.106099696 0.034098363 0.27152639 -0.01534754 0.03983225
#> V11 -0.119192921 0.048680572 0.006791471 -0.08901803 -0.32924578 -0.31918887
#> V12 0.562118534 0.575784131 0.505013906 0.60437582 0.59681580 0.48541608
#> V13 -0.029617341 0.195535801 -0.085319680 -0.10984748 -0.03122743 -0.04626786
#> V14 -0.049332151 -0.268596442 -0.007364846 0.02537685 0.03877305 0.06792346
#> V15 1.211501728 1.337132232 1.246679536 1.20019076 1.29084600 1.48829855
#> V16 0.018031145 0.074143854 -0.278521111 -0.36061408 -0.55560817 -0.61632041
#> V17 -0.310181438 -0.297659431 -0.054935207 -0.17020839 -0.24014837 -0.26763764
#> V18 -0.303319199 -0.348679309 -0.063535028 0.19106344 0.12953888 0.14354382
#> V19 0.389926941 0.259926267 0.214479580 0.11773324 0.16649254 0.07233816
#> V20 1.204213537 1.231208066 1.057114796 1.02580512 0.99134195 1.29366893
#> V21 -0.486552962 -0.245298657 -0.151955631 0.11664599 0.15250049 0.13692391
#> V22 0.335749293 0.152125765 0.037324804 -0.13382246 0.08718711 -0.10866116
#> V23 0.203105384 0.401860025 0.073128404 0.03639073 -0.06130542 -0.11291091
#> V24 0.174688092 0.441354591 0.064623710 0.20823120 0.22476111 0.37595085
#> V25 0.021118810 0.006572632 0.336435680 0.10387854 0.42543511 0.46096906
#> V26 0.118709281 -0.163646664 -0.276962295 -0.40664355 -0.47690803 -0.53952393
#> V27 -0.175434387 0.075502606 -0.083827273 -0.04772232 0.02568188 0.17834130
#> V28 -0.034297684 -0.058277022 0.092358106 0.03845841 0.10916066 0.10770638
#> V29 0.290463755 0.237391317 0.358237545 0.37754677 0.45849637 0.45653060
#> V30 -0.283119877 -0.172119775 -0.182895980 -0.22302171 0.02133816 0.02754593
#>          s26          s27          s28          s29          s30          s31
#> V1 0.324645984 0.423030904 0.315305386 0.3817069 0.42214464 0.41184848
#> V2 0.580442808 0.375842817 0.301224200 0.0639892 0.07995192 0.34896017
#> V3 0.117657868 0.209066405 0.131373780 0.1854774 0.23120205 0.05286280
#> V4 -0.034847244 -0.070289640 0.125407076 0.3881653 0.33949675 0.20172343
#> V5 0.166943987 0.341747764 -0.047595381 -0.2663430 -0.24891351 -0.09475075
#> V6 1.104024458 0.821629517 0.987462787 0.8173839 0.88762234 0.85106607
#> V7 0.835711629 0.947563196 0.778079258 0.5794065 0.44291910 0.62088502
#> V8 0.121646209 0.234048953 0.142121698 0.2093922 0.33577668 0.31081840
#> V9 -0.074009935 -0.006308428 0.007138398 -0.1149506 0.04736791 0.11685961
#> V10 -0.010116195 0.094496714 0.009079590 -0.1450051 -0.10148593 -0.03793395

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#> V11 -0.254251047 -0.239396253 -0.351995421 -0.2981005 -0.52205472 -0.56180179
#> V12 0.542916470 0.723918969 0.608481044 0.7766108 0.90615577 0.74675881
#> V13 -0.214757943 -0.294895003 -0.302489401 -0.4583609 -0.55767611 -0.49593248
#> V14 0.009487362 0.221377174 0.311567220 0.5623305 0.34528263 0.32051923
#> V15 1.405331707 1.402833561 1.472410442 1.4711191 1.37436587 1.35194361
#> V16 -0.640540676 -0.825373159 -0.644482048 -0.4542555 -0.18604042 -0.27031658
#> V17 -0.071474615 0.003920361 -0.297689535 -0.2637778 -0.27556728 -0.21026828
#> V18 0.420884163 0.458770871 0.352043336 0.1187293 0.09519499 0.05212986
#> V19 -0.118891136 -0.291666802 0.011619641 -0.1416832 -0.01814562 -0.00484960
#> V20 1.138923311 1.341911837 1.155789513 1.2767029 1.07208524 1.08367474
#> V21 0.406263727 0.361288311 0.262705129 0.2211108 0.12840231 -0.13061036
#> V22 -0.243801726 -0.260794344 0.021818480 0.2014489 0.11792951 0.38706644
#> V23 -0.100382541 -0.078414190 0.030186955 -0.1437561 -0.20147639 -0.17887359
#> V24 0.371061474 0.327025870 0.285640670 0.1537993 0.01860915 -0.04844318
#> V25 0.328705389 0.258143292 0.223474160 0.2677324 0.30878826 0.51186561
#> V26 -0.597874013 -0.661145148 -0.476212474 -0.4315341 -0.27887855 -0.27581427
#> V27 0.242755771 0.358870785 0.450445483 0.5906107 0.31640491 0.17571740
#> V28 0.110194798 0.071226671 -0.162633994 -0.3406511 0.09858633 0.05444532
#> V29 0.475332570 0.612533676 0.602462186 0.5011863 0.22196074 0.48648885
#> V30 -0.018396856 -0.292409351 -0.288769463 -0.2404113 -0.26265551 -0.13713777
#>          s32          s33          s34          s35          s36          s37
#> V1 0.360840645 0.793100918 0.71864209 0.570439629 0.62386847 0.5872658
#> V2 0.340716335 0.677793979 0.79091597 0.752243042 0.74110344 0.8206948
#> V3 0.055553819 -0.497520102 -0.58273993 -0.394179995 -0.13501335 -0.2237616
#> V4 -0.086056375 0.012853265 -0.14199723 -0.318667337 -0.18580850 -0.4860619
#> V5 0.116868313 0.284108322 0.25883177 0.337497064 0.54244647 0.7043606
#> V6 0.912986319 0.889230299 0.74438501 0.777866869 0.78880381 0.7365661
#> V7 0.609448132 0.978683653 0.71455672 0.681011993 1.05240503 1.0376801
#> V8 0.328768699 0.416706588 0.60703437 0.700874119 0.61767615 0.5778754
#> V9 0.005739693 -0.231902892 -0.45700584 -0.419592697 -0.21987074 -0.4091759
#> V10 -0.136846676 -0.173949059 -0.12498624 -0.215528902 0.06058161 0.1868115
#> V11 -0.223263865 -0.346456484 -0.11537042 -0.108328107 -0.50861535 -0.2648159
#> V12 0.693705052 0.356528060 0.38075060 0.424310624 0.55488548 0.4256692
#> V13 -0.297808280 -0.245237004 -0.37361836 -0.366456475 -0.19886133 -0.2055218
#> V14 0.079818878 0.003691342 0.30245573 0.201306533 0.17031965 0.1320502
#> V15 1.202264530 1.265092758 1.31106704 1.346648490 1.28558354 1.2942562
#> V16 -0.281148671 -0.536185612 -0.63996851 -0.588714825 -0.69162344 -0.9334058
#> V17 -0.048913705 -0.055453993 0.30431934 0.286340945 0.22764170 0.4084306
#> V18 -0.036742914 0.134683670 -0.04407816 -0.090370963 0.17256597 0.3027074
#> V19 0.247081084 0.129953838 -0.01848087 -0.004203613 -0.12006906 -0.2498073
#> V20 1.037581088 1.021152740 1.27949195 1.227313514 1.21846049 1.1932828
#> V21 -0.058070246 -0.053269725 -0.23789335 -0.087643058 0.10535266 0.1634325
#> V22 0.099143438 0.073352068 0.14661992 0.143362974 0.10197847 -0.1317787
#> V23 -0.022628071 0.042981863 -0.19755570 -0.183145737 -0.33995862 -0.1556489
#> V24 0.110482324 0.374495682 0.20756853 0.147758627 0.30890185 0.2556723
#> V25 0.500928615 0.633655425 0.69142993 0.744701371 0.84139073 0.7329610
#> V26 -0.343478912 -0.406387333 -0.33014438 -0.368241163 -0.76678724 -0.7206605
#> V27 0.177218596 0.121540882 0.21094037 0.348744984 0.49679056 0.3252172
#> V28 0.115307005 0.094134866 -0.28533397 -0.180129090 -0.11993734 -0.1772209
#> V29 0.527029174 0.761886090 0.56073959 0.581382017 0.60640558 0.6046266
#> V30 -0.309992657 -0.059326259 0.21892319 0.086344166 0.11768206 0.2321597
#>          s38          s39          s40
#> V1 0.43501891 0.46365236 0.42355705

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#> V2 0.85146344 0.45072560 0.35888055
#> V3 -0.09225910 -0.18059312 -0.12051185
#> V4 -0.38420975 -0.33841429 -0.47593039
#> V5 0.49135303 0.51842057 0.40781947
#> V6 0.85772795 0.84460549 0.94855783
#> V7 0.74304114 0.63196177 0.42532434
#> V8 0.46128066 0.48786291 0.52151887
#> V9 -0.36010739 -0.17065489 -0.33414201
#> V10 0.02455605 -0.06363160 -0.11466073
#> V11 -0.18981188 -0.35009492 0.04707994
#> V12 0.53116565 0.56052614 0.34335527
#> V13 -0.27811068 -0.05463803 0.08336142
#> V14 -0.01941340 -0.04052352 -0.29144190
#> V15 1.29615510 1.33091121 1.35063989
#> V16 -0.78732195 -0.64016415 -0.50987959
#> V17 0.38046528 -0.02841434 0.21032762
#> V18 0.46761997 0.31206853 0.20466260
#> V19 -0.24151338 -0.04724723 0.07596618
#> V20 1.07290844 1.20002071 1.02203509
#> V21 0.39288536 0.30014508 0.24948710
#> V22 -0.25894816 -0.08085769 -0.18191258
#> V23 -0.13244642 -0.09602086 -0.05572430
#> V24 0.19521610 0.36104343 0.41541732
#> V25 0.60069920 0.35462665 0.30081246
#> V26 -0.58374358 -0.45029975 -0.33583391
#> V27 0.28099313 0.44437155 0.19350901
#> V28 -0.06357756 -0.17832506 0.07156053
#> V29 0.62346564 0.51200199 0.39270903
#> V30 0.09067412 0.03265217 -0.04430665
#>
#> $df
#> s0 s1 s2 s3 s4 s5 s6 s7 s8 s9 s10 s11 s12 s13 s14 s15 s16 s17 s18 s19
#> 1 11 17 14 17 22 21 21 27 23 27 30 30 29 30 30 30 30 30 30
#> s20 s21 s22 s23 s24 s25 s26 s27 s28 s29 s30 s31 s32 s33 s34 s35 s36 s37 s38 s39
#> 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30
#> s40
#> 30
#>
#> $dim
#> [1] 30 41
#>
#> $lambda
#> [1] 4.00000000 3.48220225 3.03143313 2.63901582 2.29739671 2.00000000
#> [7] 1.74110113 1.51571657 1.31950791 1.14869835 1.00000000 0.87055056
#> [13] 0.75785828 0.65975396 0.57434918 0.50000000 0.43527528 0.37892914
#> [19] 0.32987698 0.28717459 0.25000000 0.21763764 0.18946457 0.16493849
#> [25] 0.14358729 0.12500000 0.10881882 0.09473229 0.08246924 0.07179365
#> [31] 0.06250000 0.05440941 0.04736614 0.04123462 0.03589682 0.03125000
#> [37] 0.02720471 0.02368307 0.02061731 0.01794841 0.01562500
#>
#> $nobs
#> [1] 30
#>

```

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#> attr("class")  
#> [1] "qcdpath"
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