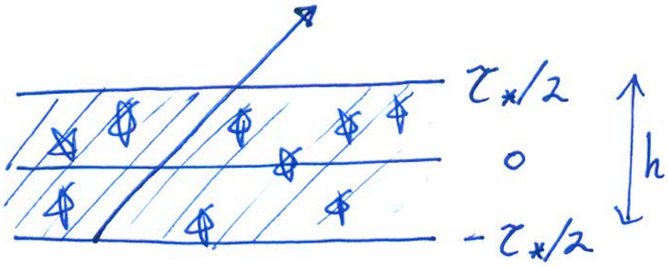
# $\boxed{190608}$. Derivation of $J_{FUV} = \Sigma_{FUV}\left(1 - E_2(\tau_*/2)\right)/(4\pi\tau_*)$ (OML10).




## Slab radiation transfer:

Source function $S = \frac{j}{\kappa}$ ; $\left\{ \begin{array}{l} j = \frac{L_*}{(Ah)(4\pi)} \quad \text{(assuming stars radiates isotropically)} \\ \kappa = \frac{\tau_*}{h} \quad \left( \frac{L_*}{A} \equiv \Sigma_{FUV} \right). \end{array} \right.$

$$S = \frac{\Sigma_{FUV}}{4\pi\tau_*}$$

Integrating from $-\tau_*/2$ to $\tau$ (upward rays) (inside source; $|\tau| < \tau_*/2$).

$$\int_{-\tau_*/2}^{\tau} \frac{d}{dt}\left(e^{t/\mu} I_+\right) dt = \int_{-\tau_*/2}^{\tau} \frac{S}{\mu} e^{t/\mu} dt$$

$$\cancel{e^{\tau/\mu} I_+} = \quad I_+ = \int_{-\tau_*/2}^{\tau} \frac{S}{\mu} e^{(t-\tau)/\mu} dt$$

Integrating from $\tau_*/2$ to $\tau$ (downward rays)

$$\int_{\tau_*/2}^{\tau} \frac{d}{dt}\left(e^{t/\mu} I_-\right) = \int_{\tau_*/2}^{\tau} \frac{S}{\mu} e^{t/\mu} d\mu$$

$$I_- = \int_{\tau_*/2}^{\tau} \frac{S}{\mu} e^{(t-\tau)/\mu} dt.$$

$$4\pi J_{FUV} = \int \cancel{I}\, d\Omega \rightarrow \cancel{2J = \int}$$

$$= \int_0^{\pi/2} I_+ \, d\Omega + \int_{\pi/2}^{\pi} I_- \, d\Omega$$

$$\rightarrow 2 J_{FUV} = \int_0^1 I_+ \, d\mu + \int_{-1}^0 I_- \, d\mu$$

$\frac{1}{\mu} \equiv X$

$\int_\infty^1 - X e \cdot dx \rightarrow {}_1^\infty$

$$\int_0^1 I_+ d\mu = \int_{-\tau_*/2}^{\tau} S \underbrace{\left( \int_0^1 \frac{e^{-(\tau-t)/\mu}}{\mu} d\mu \right)}_{\int_1^\infty \frac{e^{-(\tau-t)X}}{X} dX = E_1(\tau-t)} dt$$

$$= S \int_{-\tau_*/2}^{\tau} E_1(\tau - t) \, dt$$

$\tau - t \equiv X$

$$= S \int_{\tau+\tau_*/2}^{0} - E_1(X) \, dX$$

$E_1(X) = - \frac{dE_2}{dX}$

$E_2(0) = 1$.

(AS).

$$= S \left( 1 - E_2(\tau + \tau_*/2) \right)$$

$$\int I_- d\mu = \int_{\tau_*/2}^{\tau} S \underbrace{\left( \int_{-1}^0 \frac{e^{-(\tau-t)/\mu}}{\mu} d\mu \right)}_{-\int_1^\infty \frac{e^{-(t-\tau)X}}{X} dX = -E_1(t-\tau)} dt$$

$-\frac{1}{\mu} \equiv X$

$\frac{d\mu}{\mu^2} = dX$

$- X e^{-(\,)X} \frac{dX}{X^2}$

$$= - \int_{\tau_*/2}^{\tau} S \, E_1(t-\tau) \, dt$$

$t - \tau \equiv X$

$$= S \int_{\tau_*/2-\tau}^{0} (-E_1(X)) dX = S \left( 1 - E_2(\tau_*/2 - \tau) \right).$$

$$J_{FUV} = \frac{S}{2} \left( 2 - E_2(\tau_*/2 + \tau) - E_2(\tau_*/2 - \tau) \right)$$

$$\boxed{= \frac{\Sigma_{FUV}}{8\pi \tau_*} \left( 2 - E_2(\tau_*/2 + \tau) - E_2(\tau_*/2 - \tau) \right).}$$

$\cdot T2$

~~assume~~ Let $\tau = 0$ (at midplane)
and set $\tau_* = \tau_\perp$, we have

$$J_{FUV} = \frac{\Sigma_{FUV}}{8\pi\tau_\perp}\left(2 - 2E_2(\tau_\perp/2)\right)$$

$$= \Sigma_{FUV}\left(1 - E_2(\tau_\perp/2)\right)/(4\pi\tau_\perp)$$

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