Estimating dynamic treatment effects of events: An imputation approach to Interrupted Time Series design

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Motivation

- The **Interrupted Time Series (ITS)** is an appealing ID strategy to assess the effect of events when we have (1) no temporal variation in treatment timing and (2) no suitable control unit
- **DID**, **Synthetic Control** are impossible; **RDiT** targets only at a cutoff
- **Classical ITS** with static regression (1) lacks causal assumptions, (2) requires assumed structure of effect (e.g., level shift), and (3) often misses time-series nature of the data (e.g., auto-correlation)

Overview

- **Imputation Approach**: A model is fit to pre-treatment period, then used to impute $y_{it}(0)$ over post-treatment periods
- **Models for Estimation**: (1) ARIMA (Autoregressive Integrated Moving Average) (2) GP (Gaussian Process)
- **Applications**: Effect of treatment every units experience: (1) Supreme Court ruling (DC v. Heller 2008), (2) War (Russian invasion of Ukraine)

Proposed Method

- **Estimands**: Individual Treatment Effect on Treated $\delta_{is} = Y_{is}(1) - Y_{is}(0)$, where s denotes post-treatment periods; then aggregate across units ($ATT_s = \frac{1}{N}\sum_i \delta_{is}$) or periods ($ATT_i = \frac{1}{|S|}\sum_s \delta_{is}$)
- **Key Assumption**: The predicted $Y_{is}(0)$ values well represent what would have happened absent treatment, implying no "other event" relevant to the outcome; Longer post-treatment period is more likely to violate it

ARIMA Approach

$$Y_t(0) = \underbrace{\frac{1}{(1-L)^d(1-L^s)^D}}_{\text{differencing}} \frac{\Phi_Q(L^s)\overbrace{\theta_q(L)}^{\text{lag (MA)}}}{\Phi_P(L^s)\underbrace{\phi_p(L)}_{\text{lag (AR)}}}\epsilon_t,$$

where L is lag operator ($L^k Y_t = Y_{t-k}$) and upper cases are seasonal parts

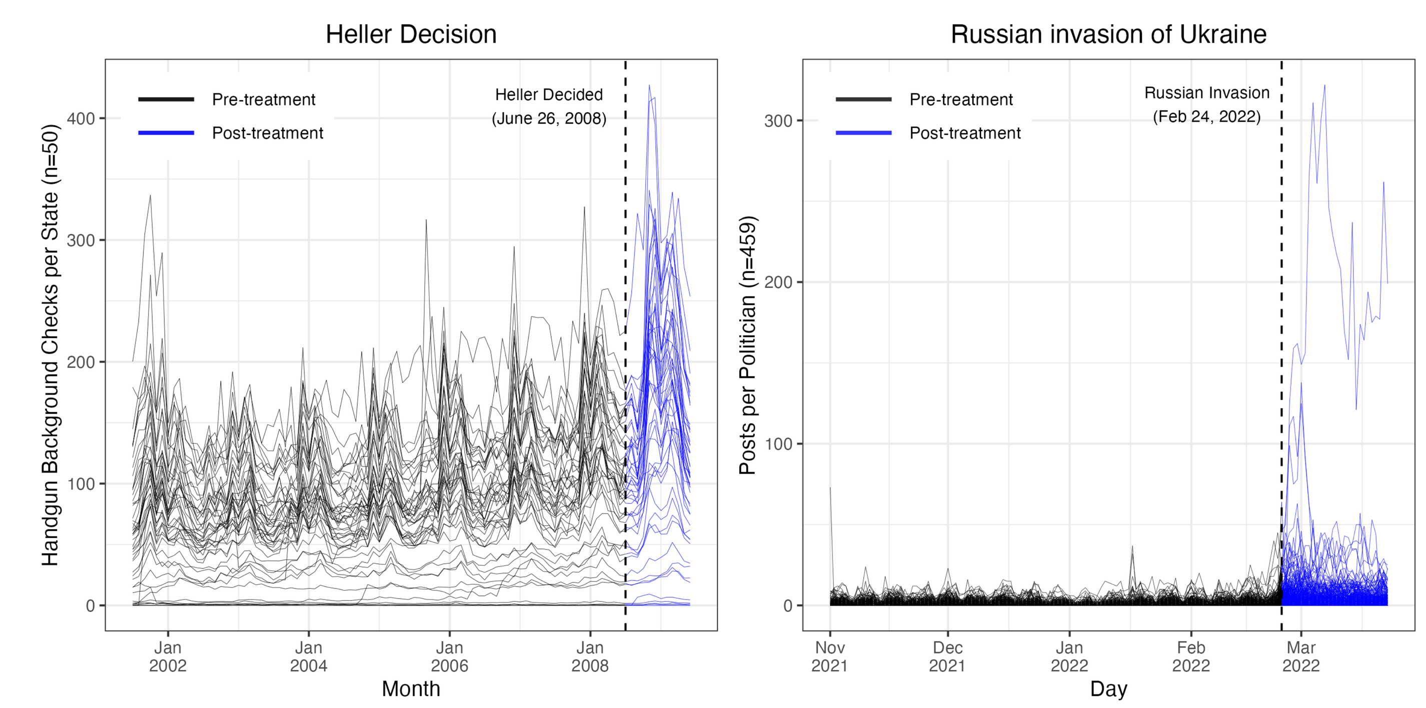
GP Approach

$$Y_t(0)|X \sim \mathcal{N}(0, \mathbf{K} + \sigma_y^2)$$

$$k(x,x') = \underbrace{\sigma_{f1}^2 xx'}_{\text{linear}} + \underbrace{\sigma_{f2}^2 exp(-\frac{(x-x')^2}{2l_{Gau}^2})}_{\text{Gaussian}} + \underbrace{\sigma_{f3}^2 exp(-\frac{2sin^2(\pi(x-x')/p}{l_{per}^2})}_{\text{periodic}}$$

- **Combined Kernel for GP**: A combined kernel (**linear + Gaussian + periodic**) is employed to identify temporal patterns in data, i.e., a long-term linear trend with some obvious periodicity and local deviations
- **Inference** by residual bootstrap (ARIMA) or predictive distribution (GP).

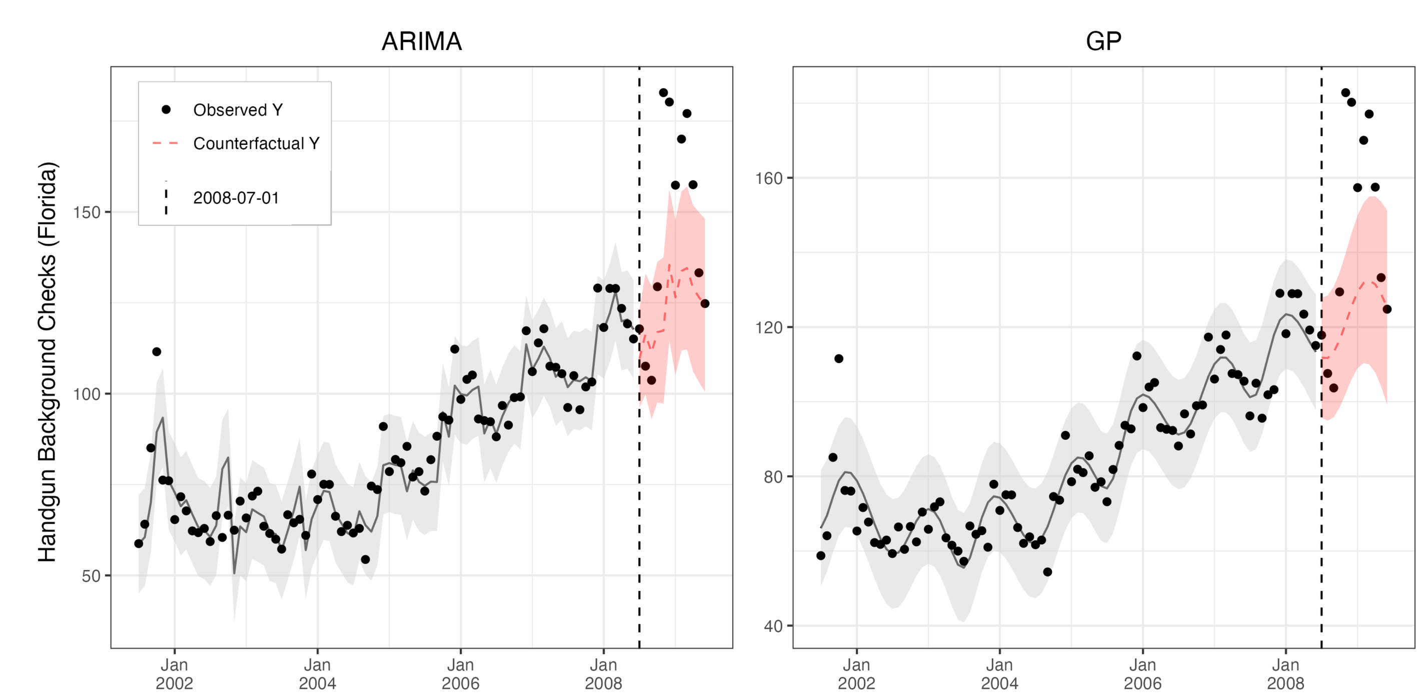
Applications: Raw Data

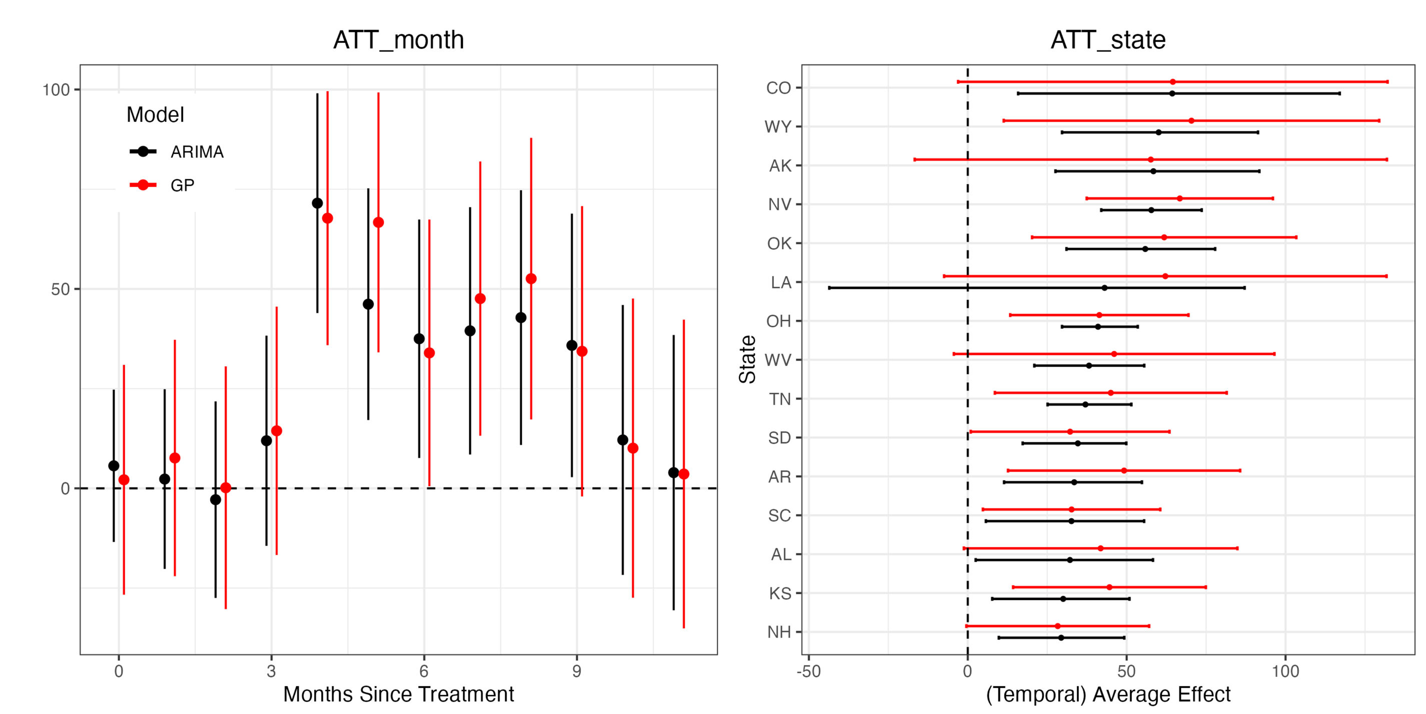


- Models that well address complexities in time series are required

Application 1: Heller Effect

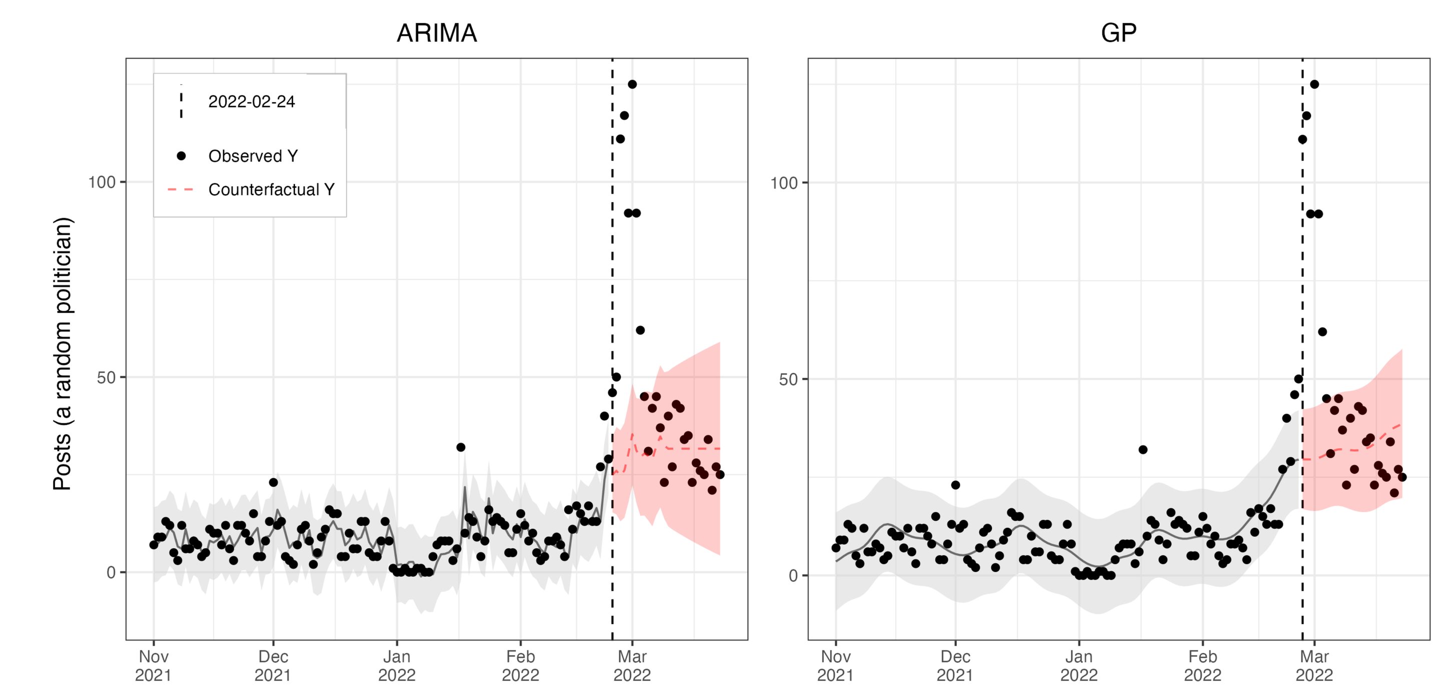
- Treatment event: *District of Columbia v. Heller* ruling (June 2008)
- Periods: 7 years before, 1 year after
- Outcome: Handgun background checks per 100k (proxy for purchase)
- Heller led to increase in handgun sales following the ruling, with lagged/decaying (ATT_s, left) and heterogeneous effects (ATT_i, right)

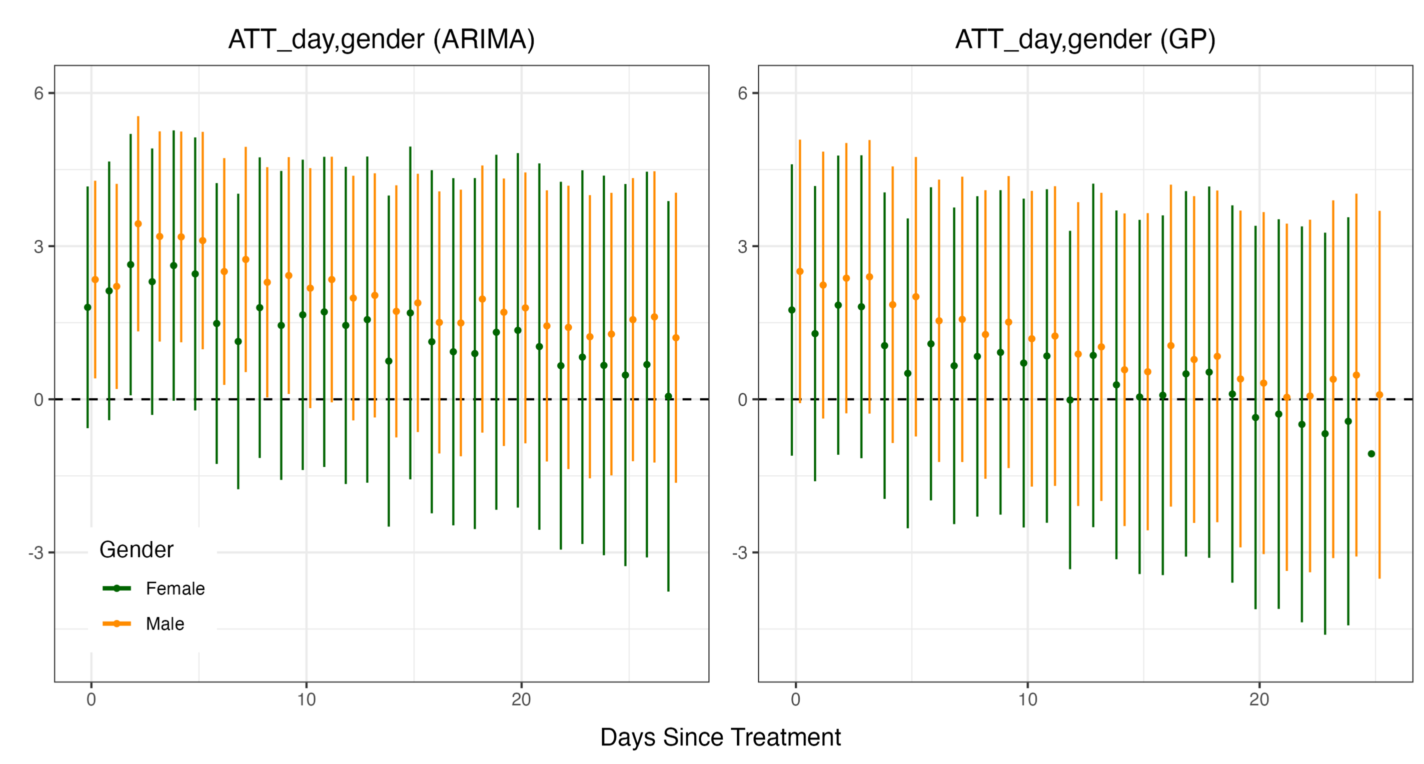




Application 2: Politicians' Responses to War

- Replication study (modified) of Damann et al. (2024)
- Treatment event: Russian invasion of Ukraine (Feb 24, 2022)
- Periods: 115 days before, 28 days after
- Outcome: Number of Facebook posts by 469 Ukrainian politicians
- Russian invasion immediately caused increase in politicians' posts, but gender gap in engagement with the public seems weak (ATT_s) when we account for individual trends and forecast uncertainty





References

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