ORIGINAL ARTICLE

Understanding the Estimation of Circumference of the Earth by of Eratosthenes based on the History of Science, For Earth Science Education

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ABSTRACT

The first accurate estimate of the Earth's circumference was made by the Hellenism scientist Eratosthenes (276-195 B.C.) in about 240 B.C. The simplicity and elegance of Eratosthenes' measurement of the circumference of the Earth by mathematics abstraction strategies were an excellent example of ancient Greek ingenuity. Eratosthenes's success was a triumph of logic and the scientific method, the method required that he assume that Sun was so far away that its light reached Earth along parallel lines. That assumption, however, should be supported by another set of measurements made by the ancient Hellenism, Aristarchus, namely, a rough measurement of the relative diameters and distances of the Sun and Moon. Eratosthenes formulated the simple proportional formula, by mathematic abstraction strategies based on perfect sphere and a simple mathematical rule as well as in the geometry in this world. The Earth must be a sphere by a logical and empirical argument of Aristotle, based on the Greek word symmetry including harmony and beauty of form. We discuss the justification of these three bold assumptions for mathematical abstraction of Eratosthenes's experiment for calculating the circumference of the Earth, and justifying all three assumptions from historical perspective for mathematics and science education. Also it is important that the simplicity about the measurement of the earth's circumstance at the history of science.

Key words : Eratosthenes, estimate of the Earth's circumference, mathematics abstraction, Symmetry, Mathematics & science education, simplicity

Ⅰ. INTRODUCTION

During the Hellenism period, Greek became the international language and Greek culture was seen as the ideal. One of the most important Hellenism cities was Alexandria in Egypt. It was where Euclid, the Greek mathematician, taught geometry and founded a school of mathematics. Plato actually thought that abstraction had a greater reality (symmetry) than the properties of objects in the physical world (Nisbett, 2003, pp. 156- 157).

The simplicity and elegance of Eratosthenes' measurement of the circumference of the Earth by mathematics abstraction were an excellent example of ancient Greek ingenuity. Thus, our research should be consider Eratosthenes' activities at that time(Greek philosophy, and Alexandra mathematics),

as both mathematics abstraction involving idealization, and symmetry based on Plato's thinking, geometry based on Euclid, and Aristarchus's estimate between the Sun and the Earth and early Atomism.

In scientific activity, mathematics abstraction allows the scientists to focus on an object's particular properties which our mathematics theory deals, while deliberately removing (subtract) any other properties present in the concrete circumstances, by Bold assumptions. Idealization is the consideration of properties that the object definitely does not possess in actual system, by Idealized assumptions, the notion of abstraction, by contrast, deliberately ignores certain features the object possesses in concrete circumstances (Oh, 2016)

It would be wrong to conclude that it was a divine hand that created the Earth as a perfect sphere with a religious perfect sphere belief. Symmetry is a powerful tool, and a source of hypotheses that we can make to understand the physical world (Lederman, and Hill, 2004, pp. 18-20).

An acute awareness of the beauty inherent in scientific ideas and scientific discovery necessarily draws us in to its study.

We remind that "The main recommendations for improving teaching and learning in astronomy and physics is that teachers should be encouraged to use thought-experiments based on historical examples (Eratosthenes' measurement of the circumference of the Earth), which appeared in all textbooks of South Korea, to stretch students' imaginations and enhance their reasoning.

Research questions of this study are: 1) to assume specially the Earth's perfect sphere logically and empirically, and parallel sun rays at that time, for mathematics abstraction based on Greek Philosophy and Alexandria Mathematics, 2) to justify all three bold assumptions from Greek history, and 3) to search weakness of Eratosthenes's work (Estimation of Circumference of the Earth) in modern view.
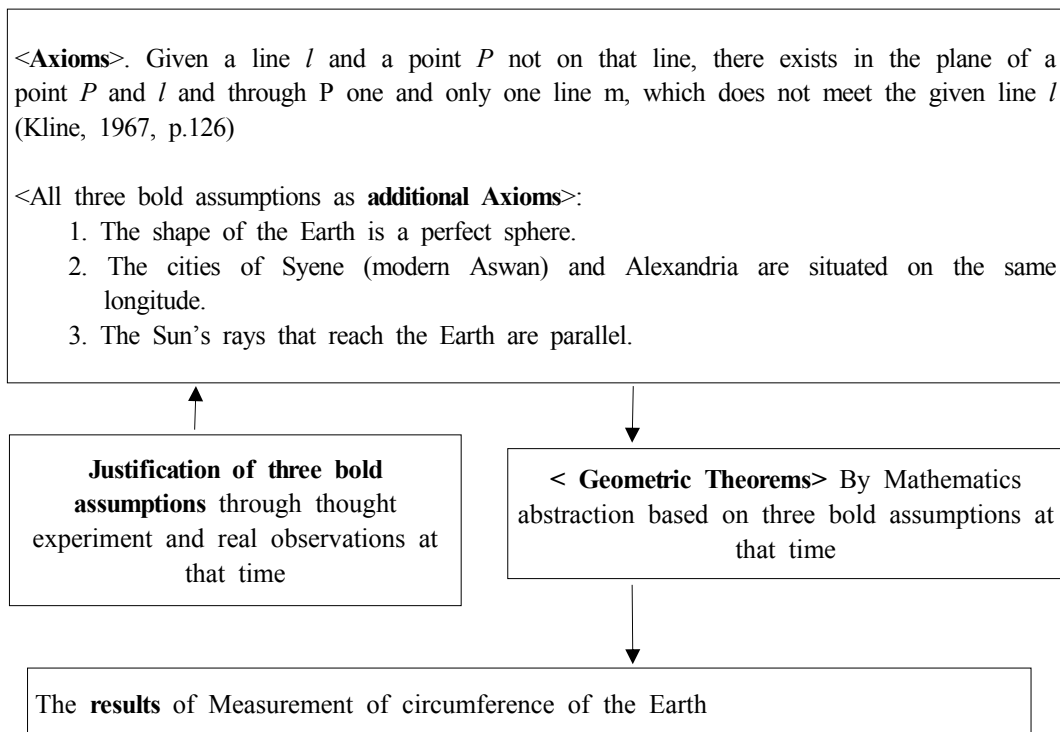
<Axioms>. Given a line *l* and a point *P* not on that line, there exists in the plane of a point *P* and *l* and through P one and only one line m, which does not meet the given line *l* (Kline, 1967, p.126)

<All three bold assumptions as **additional Axioms**>:
1. The shape of the Earth is a perfect sphere.
2. The cities of Syene (modern Aswan) and Alexandria are situated on the same longitude.
3. The Sun's rays that reach the Earth are parallel.

Justification of three bold assumptions through thought experiment and real observations at that time

< **Geometric Theorems**> By Mathematics abstraction based on three bold assumptions at that time

The **results** of Measurement of circumference of the Earth

Fig 1. Eratosthenes' Mathematics abstraction strategies based on Euclidian
Geometry for the Estimation of Circumference of the Earth

II. Measurement of circumference of the Earth by Eratosthenes three bold assumptions

Ancients' Chinese historical elements are chosen and organized according to specific educational and conceptual constraints that include the construction of the quasi-parallelism of solar rays reaching Earths' surface, and the spontaneous modeling of the propagation of Sunlight leaning on divergent rays.
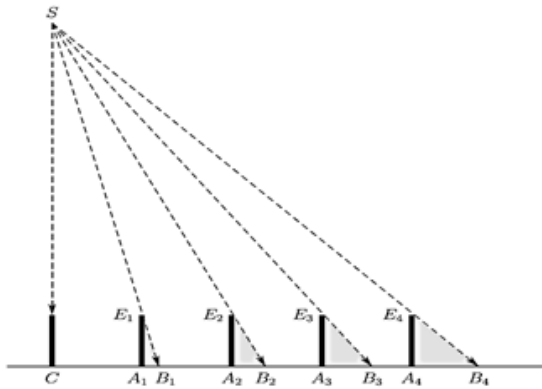

Fig 2. According to the Zhou cosmology, every displacement on the Earth surface produces a length change in the shadow cast by the gnomon (Hosson & Decamp, 2014)

The Chinese text presented hereafter is taken from the Chin Shu, a book written around 635 A.D. The astronomical part of this book has been written by Li Shun-feˆng. The proposed excerpt refers to the astronomical knowledge under the Zhou dynasty that began about a thousand year B.C. According to Eratosthenes, however, the measurement of the size of the Earth considers two assumptions: the flat surface by the Chinese and the rays of light parallel to the spherical Earth, which in the age of Hellenism, was an audacious assumption, when compared to the Sun that emits lights radially. Therefore, this study aims to investigate, from the literature of Greek philosophers and scientists in Alexandria rather than that of China, what the origin of the assumption that the Earth is round was and how it was possible to consider that the Sun is far away.
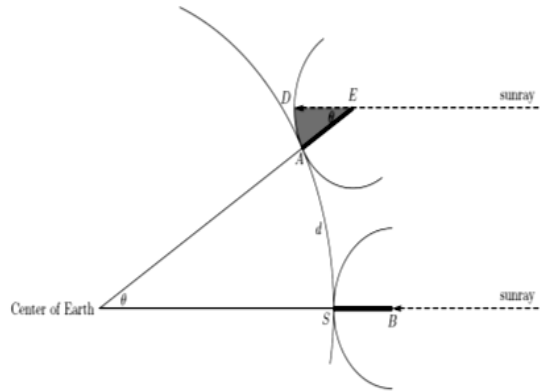

Fig 3. Illustration of Eratosthenes' procedure as described by Diomedes, That was the circular motion of the celestial bodies.

Greek philosophy began to spread along with the conquests of Alexander the Great (356-323 B.C.). Alexander the Great entertains great interest in the sciences. This is partially because his private teacher had been Aristotle. Alexander erected a city called Alexandria in Egypt, and the Great Library of Alexandria was established in 300 B.C. This continued as the site of globally leading center for research for the next 700 years. Eratosthenes (310- 230? B.C.) was one of several brilliant astronomers to emerge from the so-called Alexandrian school, which by his time already had distinguished tradition. Based on the abstract logic that a perfect sphere and a simple mathematical rule exist in reality as in geometry, Eratosthenes abstracted a simple proportion. The Earth's roundness was already logically and empirically proven by Aristotle (Crease, 2003, p.4).

Eratosthenes's success was a triumph of logic and the scientific technique, but it suffered from one weakness. The method required that he assume that Sun was so far away that its light reached Earth along parallel lines. That assumption, however, was supported by another set of measurements made by the ancient Greek, namely, a rough measurement of the relative diameters and distances of the Sun and Moon. As the chief librarian at Alexandria's city library, you can imagine that he read many books. One of those spoke of a well in Syene, Egypt (near Aswan) – now

known to lie close to the tropic of cancer. He read that every year on the 21st of June at noon, the sun was positioned so that light rays went straight down the well, the sun being directly overhead. This was a property that never happened in his hometown Alexandria. This gave him a brilliant idea to calculate the circumference of the earth.

At noon he placed a stick perpendicular to the ground, using its shadow to calculate the circumference. His method was as follows:

The value of the Earth's circumference was calculated as early as the 4th century BC by Aristotle and 3rd century BC by Archimedes (Evans 1998). But a more precise value, close to the modern one, was obtained by Eratosthenes in the 3rd century BC (Layser 1984). He calculated the circumference of the Earth through mathematics abstraction based on following bold assumptions by an application of Euclidean geometry (Kline, 1967, p.136)
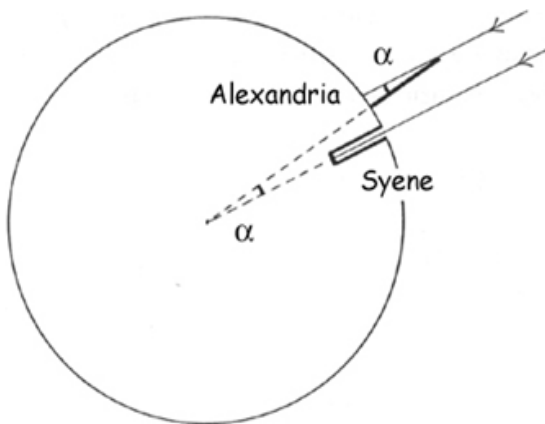

Fig 4. Geometry of Eratosthenes's experiment for calculating the circumference of the Earth (3rd century BC)

<All three bold assumptions>:
1. The shape of the Earth is a perfect sphere.
2. The cities of Syene (modern Aswan) and Alexandria are situated on the same longitude.
3. The Sun's rays that reach the Earth are parallel.

<Axiom based on Euclidian Geometry >. Given a line l and a point P not on that line, there exists in the plane of a point P and l and through P one and only one line m, which does not meet the given line l (Kline, 1967, p.126)

He stuck a stick in the ground at Alexandria, measuring the angle between the sun's rays and the stick and it measured to be about 7.2° (α). As the two triangles – well, center, stick and stick, ray, shadow – are similar triangles, the angle at the center must also be about 7.2° (see Figure 1), as deduced from the parallel line-equal angle theorem. The reason is that the angles formed where a single line crosses two parallel lines are equal **<a Geometric theorem>**.

Assuming all three bold assumptions as additional Axioms, he stuck a stick in the ground at Alexandria, measuring the angle between the sun's rays and the stick and it measured to be about 7° (α), and the bwell thus represent 7/360, or about 1/50, of the distance around the Earth. He then calculated the distance between Syene and Alexandria, which was 5,000 stadia, and now **the mathematics was simple**: 50×5,000 = 250,000 stadia (**The results** of Measurement of circumference of the Earth)

The following section discuss the justification of these three bold assumptions for mathematics abstraction of Eratosthenes's experiment for calculating the circumference of the Earth, and justifying All three assumptions both from modern and historical perspective (see Figure 1).

Why can Eratosthenes consider the Shape of the Earth as a perfect sphere?

Pythagoras (570?-500 B.C.), born in the island of Samos near the Aegean Sea, asserted that based on geometry and trigonometry, all natural phenomena could be described in numbers. t\The Pythagoreans hand taught that the Earth was round and all celestial bodies moved in circular orbits. This was generally accepted. This way of thinking began at this time and dominated the ancient times. Anaxagoras (500?- 428

B.C.), a Pythagorean, asserted that the moon emitted light as it reflected the light of the sun. From this observation, he realized that the solar and lunar eclipses occurred due to the shadows of the moon and the Earth, respectively.

In Greece, Plato (428‑347) asserted that all motions occurring in the universe are perfectly circular motions, and all celestial bodies are round in shape. He also accepted Pythagoras' assertion that the sun, the moon, and the planets move in circular motions around the Earth. His disciple, Aristotle, adopted the first comprehensive theory of physics, and asserted that the reason why the universe appears as it does currently was due to this theory. As a postscript to Aristotle, let us consider one example (Toulmin & Goodfield, 1961):

Namely, the passage in which he argue for the view that the view that the Earth is a sphere. He begins with theoretical considerations: all earthy matter tends to converge into a comment center, so if the Earth came into existence by the congregation of the matter formulating it, a spherical form was the result to be expected. (This argument is equally sound on the more modern view, which the planets condensed out of the hot gaseous material from the Sun) (p.110)

Aristotle adds evidence based on direct observation, from astronomical observations (Toulmin & Goodfield, 1961):

From the shape of the Earth's shadow, as observed on the Moon's face in lunar eclipse, and from the way in which stars differ in elevation according to the latitude of the place from which you observed them (p. 111).

Among the quadrangles of the same circumferences, a square has the largest area, and among the triangles, it is the equilateral triangle that has the largest area. Both are beautiful geometrical figures with symmetry. In this regard, a circle is the one that exhibits the most ideal beauty. Moreover, a sphere reveals a circle from anywhere it is cut. It is thus natural to consider that

the Greeks who had been influenced by the traditional Euclid geometry, centred on the geometry of circles and lines, may have posited these spheres to be mysterious, and thus thought of the Earth and the celestial bodies also as spheres. The Pythagoreans had taught that the Earth was a spherical sphere very early in the Greek philosophy. And by 400 B.C. this was generally accepted (Nisbett, 2003).

According to his some empirical evidences for the earth's round shape, a ship a long way away seems mysteriously below sea-level, since this happens equally in all directions, the earth is a sphere. Strictly, only the part of the earth which had been explored was seen to be spherical (Crease, 2003, p.4), and there are evidence from the shape of the Earth's shadow, as observed on the Moon's face in a lunar eclipse, from the way in which stars differ in elevation according to the latitude of the place from which you observe them.

A physicist as beholder, say that beauty means symmetry. Given theories, he think that the more symmetrical, beautiful and generally theory (Zee, 2007, p.13). Scientist has become imbued with the faith that Nature has an underlying design of symmetry and simplicity (Zee, 2007, p.8).

As Poincar´e (1946) describes,
"The scientist does not study nature because it is useful; he studies it because he delights in it, and he delights in it because it is beautiful. . . Intellectual beauty is what makes intelligence sure and strong" (pp. 366‑367).

It would be wrong to conclude that it was a divine hand that created the Earth as a perfect sphere with a religious perfect sphere belief. Symmetry is a powerful tool, and a source of hypotheses that we can make to understand the physical world. An acute awareness of the beauty inherent in scientific ideas and scientific discovery necessarily draws us in to its study.

Why can Eratosthenes Consider Sunrays as Parallel Lines?

In the year 260 BC, Aristarchus of Samos had estimated the relative size of the Earth, Moon, and Sun, and the relative distances to the Moon and Sun., by assuming that the Sun and the stars are motionless, that the Earth revolves around the Sun, and that the fixed stars are so far away, that the distances of the

Earth and of the Sun to the stars are practically the same (Hoyle 1972; Evans 1998; Heath 1913).

When the Moon is in its first or last quarter, it forms, together with the Sun and the Earth, a right triangle with angle α = 90°. Determining the angle β, Aristarchus estimated for the first time the distance between the Earth and the Sun as related to distance L to the Moon. Since his measurements of this angle were not very accurate, the distance to the Sun was calculated as 20L. The actual value is closer to 400L, which is 20 times as large as Aristarchus's value.

However, despite the results of such calculations by Aristarchus, it is very bold to assume that the sun's rays hit the Earth's surface in an approximately parallel manner. Therefore, to justify that such a proportional expression holds, an idealization strategy that logically proves the parallel incidence of the sun's rays is required. Therefore, their studies were not immediately accepted.
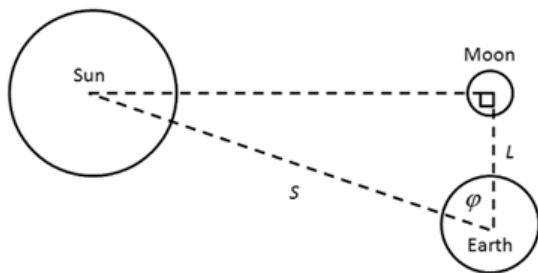

Fig 5. Earth-Sun-Moon relation at half-moon

First, there are difficulties to assume the sun to be a faraway **point source of light,** when the sun actually has volume.

Second, as the time zones were not discovered then and Aristarchus' heliocentric theory was not accepted widely; therefore, the geocentric celestial sphere could

not be so large. In the Geocentric system, the universe is finite. Because all celestial bodies, including the fixed stars, would need to rotate around the Earth once a day, size of the universe should be finite. If stars were infinitely distant from the Earth, they would not have been able to rotate the Earth once a day at a finite speed. However, stars in the Heliocentric universe would not need to rotate, and therefore, there is no limitation in the distance from the Earth. As shown in Figure 7, for a light source with a volume to be far away enough to become a point, the Sun must be far removed from the Earth as much as possible (further Sun C in figure 5 rather than nearer Sun C).
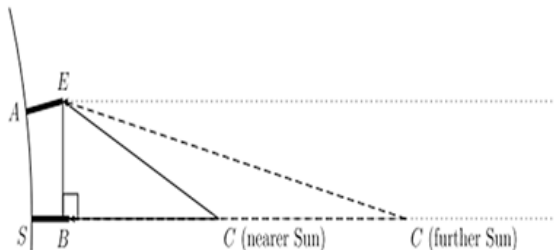

Fig 6. Further the Sun is away, smaller is the angle ∠ C, and more are the lines

If ∠E + ∠B + ∠C = 180°, and ∠E = 90°- ∠C, Thus ∠E ≒ 90°, since ∠C is small (≒ 0.02°)

According to Aristarchus calculation, Distances to the Sun is equal to about 180 Earth Diameters (Heath, 1913, pp. 350-352), and Earth diameter is equal to 250,000/3.14 stadias, because the distance between Syene and Alexandra was equal to 5,000 stadias, Tan (∠C) = 5000(stadia)/180* 250,000/3.14(stadia), thus ∠C is equal to about 0.02°; the angle a is then almost zero. With this idea in mind, Eratosthenes is justified in making the assumption that sunrays striking the Earth are parallel.

Aristarchus calculated the distance between the Earth and the Sun, compared the size of the Earth's orbit to a grain of sand, and explained that the distance of the location of the star could not be observed well due to the miniscule size of the Earth's orbit compared

to the distance to the star. This is an excellent answer to the reason why the parallax of the star could not be observed (Copernicus also provided the same answer much later). As such, this argument was subject to the objection that the size of the cosmos needed to be expanded beyond belief (with the beliefs of the time). The scientific issues faced by the heliocentric hypothesis were mighty, and ancient astronomers objected to the heliocentric hypothesis with definite evidence. There were religious objections in placing the lowly and shifting Earth on the holy and incorruptible heavens. As such, he was blamed to be blasphemous (McClean Ⅲ, & Dorn, 1999, p.83).

Furthermore, to assume the expanded universe, the ancient Greek atomism, the birth of modern chemistry, posed that matter was thought to consist of small particles moving in a completely empty space, that the world of atoms was infinite, and that there would be infinite voids between these particles (Frank, 2011, pp. 82-83.)

Alexandria and Syene lie on the same meridian?

Eratosthenes' first assumption is based on a map which he constructed using the wealth of knowledge available at the library of Alexandria.

Eratosthenes' map (Siebold, 1998) of the world appeared in his work entitled Geography, which was long regarded as the highest authority on geography in the ancient world (Dick, 1971, p.389). However, since the current map location is not located on the same meridian, it differs from the actual real value and the size of the exaggerated earth is calculated.

Ⅲ. Clearly can We Consider the value (15% off) obtained as Modern real value?

A stadium was a standard distance over which races were held. An Olympic stadium was 185m, in which case the circumference would be 46,250km, and an Egyptian stadium is 157m so the calculation would be 39,259km.

We do not actually know which stadium Eratosthenes was working with, but both are pretty close to the actual value of 40,100km. Using the Olympic stadium his value is 15% off, but with the Egyptian stadium, only 1% off! Therefore, given the uncertainty of the various Greek stadia, utilized as units of distance by Eratosthenes (Fraknoi et al., 1998), this gives rise to a logical fallacy from the modern
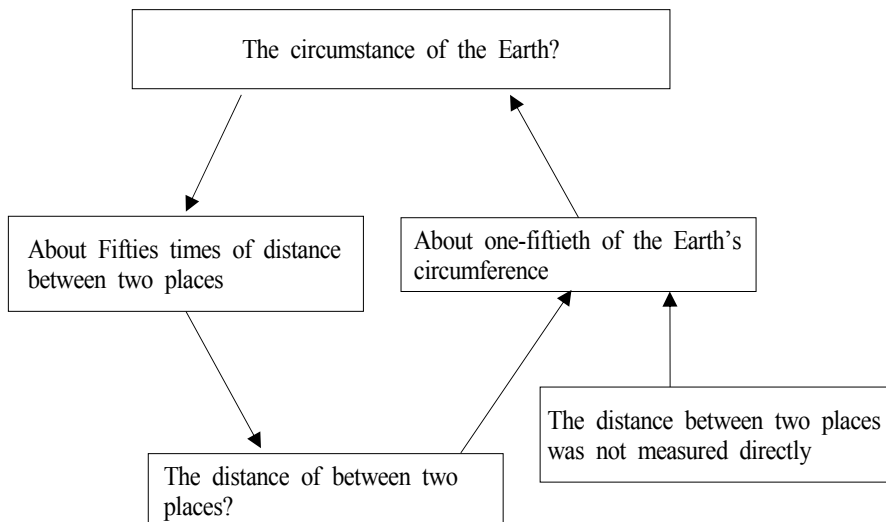

Fig 7. Circular Argument, Pointiness in Modern Terms

perspective given the following circular argument (see Figure 2), Unfortunately, we do not know exactly how many stadias there were to modern unit of distance: suggested values range from seven and half to ten. So we cannot compare Eratosthenes' estimate of 250,000 stadias with the modern measurement. Yet clearly the value obtained cannot have been far from the truth (Toulmin & Goodfield, 1961).

Wesenberg refers to them as the "Attic" (from Asia Minor and southern Italy), the "Doric" (from Greece and Sicily), and the "Ionic" (used throughout the Greek civilization), and, Pliny (29-79 A.D.) gives an estimate equivalent to 516.73 ft (157.50m). Based on this value, Eratosthenes'' value for the circumference of the earth turn out to be 21,466 mile (3.93 x 10³ km) (Cushing, 1998, p.978). We refer to this the most frequently accepted stadia (+16.2%) as the "Italian" stadia (Dicks, 1960, pp.42-44).

Therefore, we suggests that the value of 1 stadia is not clear from the viewpoint of modern, however it is highly effective methodologically rather than a accuracy.

Ⅳ.Mathematical abstraction & justification

Using Aristotle's principle that nature always moves in simple and economical rules and Plato's simple mathematical claim (Oh, 2016), Eratosthenes' results were made possible owing to the logic that all natural phenomena are simple and move according to economic feasibility, Aristarchus' calculation results, as well as the idea according to his calculations that the universe could be much larger than it was thought to be according to his calculations, though they were not accepted then.

Plato would like suppose that the concepts of mathematics (more particularly of plane geometry), such as a circle, line and triangle, exist only in the abstract and that knowledge of them is not gained through sense experience. This was Plato's answer to the quest for the immutable amidst the change of the sensible world (Cushing, 1998, p.4). It was an outstanding victory of observation and reasoning that Eratosthenes calculated the circumference of the earth exceptionally bright and precisely. Unfortunately, these great thoughts - that the earth is round, atomic, and solar center - have disappeared for a while in history.

According to a recently study, Oh (2016), the study was essentially introduced a mathematical abstraction and idealization of his causal idealization in thought experiments. In this study, just as in Oh (2016) recent research, the mathematical abstraction and causal idealization from Galileo's thought experiment strategy were fundamentally adopted. Why he adopted a mathematical abstraction based on Greek geometry.

According to a **Euclid geometric theorem**, the reason is that the angles formed where a single line crosses two parallel lines are equal. Now imagine drawing a straight line from the center of the Earth outward so that it passes vertically through the Earth's surface in Alexandria. The angle A, call it, between line the Sun's rays in Alexandria is the same as the angle between that line and the line from the center of the Earth up through the well in southern Egypt (see Figure 1).

The distance between Alexandria and Cyrene, and the difference between the meridians of altitudes of Sun were measured between two places at noon at summer solstice. In other words, while the length of a shadow was measured at Alexandria, but as the sun shone right to the bottom of the well at Cyrene, there were no shadows. What relationship do these two measurements have?

If, **in a geometric theorem**, the central angle of a circle and the length of the arc on the central angle are proportional, and if a part of the Earth's surface is the same was a part of the geometric circle, what is the relationship, and with what does the relationship occur, for the difference between the meridian of altitudes of the Sun of the two places belonging to the same parts of the Earth's surface with the same longitude on the same day? Then, it is proportional to the distance between the two points.

Therefore, there are sufficient reasons to assert that the difference between the meridian transit altitudes of the sunrays on the same day on two places on the Earth's round surface is proportional to the distance between two places.

One is asked to imagine what happens in an idealized world if a certain cause is assumed, by reconstructing a phenomenon that can be observed in an experiential world so that an abstract proportional expression is possible. The causal idealization strategy is utilized which then directs to the idealized world from an experiential world.

First, it is already known that According to Aristarchus calculation, Distances to the Sun is equal to about 180 Earth Diameters (Heath, 1913, pp. 350-352), moreover, it is necessary to assume that parallel rays are required for the condition of alternate angles to be met in geometry.

If, a source of light with volume near the surface of Earth is put further away from the surface, it can be imagined that the angle between the rays grow smaller (about 0.02°, see Figure 4). From the source of light, all rays of light move in a straight line, and the sun is infinitely far away and becomes a point source of light, then what would happen to such rays?

In effect, the sun with volume will become a point source of light and will shine on the surface of the Earth in an approximately parallel fashion.

Therefore, the fact that Distances to the Sun is equal to about 180 Earth Diameters farther away from the moon provides the rationale for the sunrays to shine on the surface of the Earth in an approximately parallel fashion. Therefore, the difference between the meridians of altitudes of the Sun of the two points becomes the central angle of the two points.

The causal idealization of this study is functional in its destruction of the size of the small cosmos of the time, and formulated a size of the cosmos that was difficult to be accepted at the time. From a functional standpoint, this study formulated that the difference between the meridians of altitudes of the Sun of two

points formed the central angle between the two points through the fact that the Sun was very far away and by destroying the existing small cosmos of the time that the Sun was close. Therefore, Brown (2011) asserted that there were a small number of thought experiments which were both destructive and constructive, and termed such experiments as Plato's thought experiments.

Unfortunately, Eratosthenes' great thoughts - accepted the earth's roundness, ancient atomism, and heliocentric hypothesis at that time - have been forgotten in history for many centuries. Such great thoughts turned out in Copernicus, Galileo, and Kepler, after the Renaissance

Ⅴ. Discussion and Conclusion

When we turn to The Greeks' cosmology, we find there an equally strong commitment to symmetry and simplicity in modern science.

First, Eratosthenes' estimate of circumference of the Earth by abstraction based on the belief that the theorems of geometry, is used in the territorial world as well as in heavens An assumption that Sun's light reached Earth along parallel line, suffered from one weakness at that time (small cosmos), so it is justified using Aristarchus's distance between the Sun and the Earth, Atomism's concept for space at that time.

Second, the question of Eratosthenes's accuracy is pointless in modern terms, because the distance between two places was not measured directly (Circular Argument). However, the beauty of his experiment is its breathtaking simplicity and elegance. There are many more interesting experiments done by Eratosthenes and many other philosophers, from this point onwards. But, in my eyes, this was really the fundamental building block or milestone in cosmology. Really, the mathematics he used was so simple, taught to us at the age of 12, yet it is not the learning of things that makes one intelligent or stand out, it is the

manner in which one applies it.

Third, it can be observed that the assumption that the sun's rays must be parallel was heavily influenced by the heliocentric theory of Aristarchus which bred scientific and religious debates. In other words, the simple measurement of the size of the Earth was founded on a bold assumption that could not be accepted at that time. Such traditional thought experiments are both destructive and constructive at the same time, and such experiments can be called Plato's thought experiments (Brown, 2011).

Fourth, Eratosthenes had utilized a quantitative methodology, rather than the qualitative methodology commonly utilized at the time. "He was not only an eminent Alexandrian mathematician of the third century B.C., but was ranked in antiquity among the rare group of man who were equally proficient in all fields of knowledge."(Gulbekian, 1987). Such methods were passed in Galileo and Kepler.

Fifth, this experiment fits well with the conditions of Heidegger's Aesthetics. In other words, the mundane to the holy, the limited to the unlimited, and this experiment displays that things of all dimensions are ultimately connected including the small, temporary things. This experiment has changed the quality of experience of humankind towards the world (Crease, 2003, pp. 3-14).

Sixth, the most significant achievement of the early Greek and Hellenism sciences is in astronomy. Astronomy is the only field of science whereby mathematical methods were applied before the end of fourth century B.C. and saw great success (Lloyd, 1970). Eratosthenes' measurement of the circumference of the Earth is a scientific method that connected the earth and the heavens through mathematics based on symmetry including simplicity. The Greeks introduced true scientific methods. Their scientific method was mainly based on reasoning. The great leap of mankind that replaced mysticism was the ancient Greek civilization.

We fully endorse Collins's suggestion (Collins, 1998) that mathematics and science involving philosophy are propelled forward in new and surprising directions by the production of technology. Eratosthenes' approximation of the Earth's circumference is a beautiful mathematical argument, regardless of the accuracy of its result. Eratosthenes helped to lay the foundation for science based on Alexandria mathematics and empirical observation as well as Greek philosophical speculations. Most importantly, he demonstrated the astonishing power of mathematics as a tool to model our world.

According to Chae (2012), the teachers mentioned that the experiments for measuring the size of the earth in the current curriculum gave rise to difficulties in measuring precisely the angles between the string and the post. Also, there has been a scientific contradiction that solar altitudes were increased in a high latitude region, instead of decreased. For this reason, an alternative method has been developed to measure the earth's size using the distance and the solar altitude difference of two places. The teachers all agreed that by using the updated measuring experiment, they can acquire more precise measurements and it is easier, faster and consequently more effective than the existing methods. I agree that he suggest that the newly developed experiment by the researchers can overhaul the problems of the current experiments and it can be an effective alternative to the current experiment.

However, according to (Crease, 2003), The beauty of the Eratosthenes' experiment was that it was in its simplicity that it was possible to find Earth's circumstance related to the scale of the universe at that time, just by performing short shadows. In other words, if sunlight comes in at a right angle to Siena, the length of the shadow of Alexandria's sundial and its central angle are measured at 1/50 of the total circle. Therefore, the distance between the two places was 1/50 of the circumference of the total earth. It was possible to calculate the circumference of the earth just by measuring the shadow of one side rather than two

places (refer to Figure 3).

The important characteristic of this experiment is that it is so simple that his experiment is still beautiful. Therefore, we should not overlook the scientific historical characteristics of Eratosthenes' experiment.

국문요약

지구크기에 대한 최초의 정확한 측정은 기원전 230년, 헬레니즘의 과학자인, 에라토스테네스 (276-195 B.C.)에 의하여 이루어졌다. 역사적으로 수학적 추상화는 유럽의 고대 그리스인의 천재성을 보여주는 좋은 예이다. 그 당시에는 상상하기 어려운 태양이 멀리 떨어져 있기에 태양광선이 평행하게 지구에 입사한다는 전제를 요구하는, 논리적이고 과학적인 기법의 에라토스테네스의 과학적 방법의 성공이었다. 중요한 것은 간단한 수학적 비례식을 성립하기 위해서는 지구가 둥글고, 광선이 지구에 나란하게 들어온다는 가정이 필요하였다. 즉 천상으로부터 지상으로 유클리드 기하학이 연결된다는 내용이다. 그것은 최초로 태양중심을 주장한 아리스타쿠스의 제안을 받아들여야 했고, 아리스토텔레스의 자연관인 우주는 아름답고 우아하다는 사상에 기반을 두어 지구는 구처럼 대칭적이라는 것이다. 우리는 이러한 가정들을 현대가 아닌 그 당시 어떻게 정당화 했는지를 탐색하는 것이다. 또한 실험의 미적 관점에서 에라토스테네스의 지구측정의 중요한 특징은 단순성에 있다는 것을 강조하는 것이다.

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