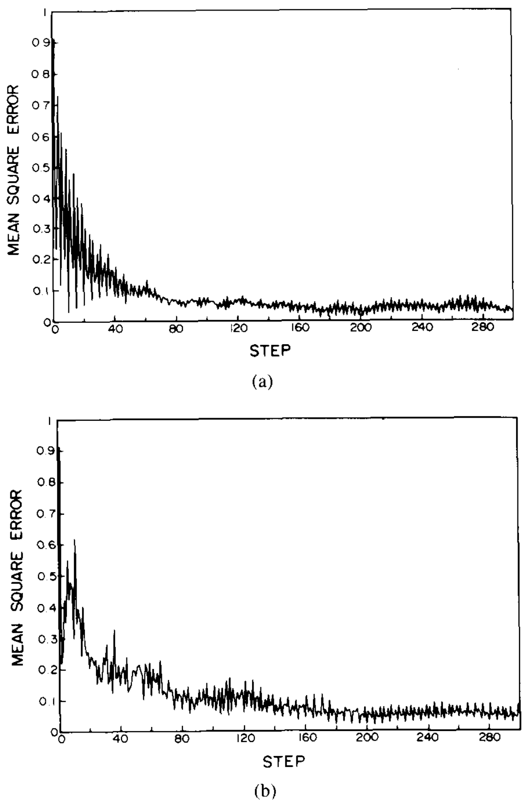

Fig. 4. Comparison of learning curves of method II and [6]. (a) Learning curve of method II. ($\alpha = 0.9025$, $\lambda = 0.90$.) (b) Learning curve of [6].

multiple sinusoids seems possible by cascading the second-order lattice filters and using the same type of algorithms proposed, but it also requires further study.

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Restoration of Randomly Blurred Images by the Wiener Filter

LING GUAN AND RABAB K. WARD

Abstract—**The restoration of images distorted by systems with noisy point spread functions and additive detection noise is considered. The criterion considered for the restoration is based on the Wiener technique. The proposed Wiener-based filter is iterative in nature. The overall computation of this modified Wiener filter can be carried out in the frequency domain using the FFT and circulant matrix approximation. Experimental results show that the modified Wiener filter outperforms its linear counterpart (based on neglecting the impulse-response noise). The modified Wiener filter also gives better restoration results than a recently proposed Backus-Gilbert technique. The Wiener-based filter is found to be computationally robust and inexpensive.**

I. Introduction

Restoration of images distorted by a system with a random blur function has been studied in recent years. The phenomenon of random blur describes an important class of imaging systems— those in which the impulse response function (or the transfer function) is random. The uncertainty of random blur may result from random amplitude and phase fluctuations of the pupil function of the optical system, due to such effects as optical propagation through turbulence or dust particles on the lens. Another class exhibiting the phenomenon of random blur is that of a scanning image-formation system (scanning microscope, microdensitometer, or laser printer) where the width of the beam has random noisy fluctuations. Random blur may also result from random vibrations of the imaging system relative to the object.

Previous work in this area has been limited. In 1967, Slepian [3] addressed this problem using the Wiener filter. Franks [4] used the same approach for random dispersive channels in communications. Heide [5] investigated this problem using several restoration schemes based on the assumption of Gaussian statistics for the random parameters in the blur function. Other methods applied to such restoration problems where the impulse response is unknown include the so-called blind deconvolution method [6]. Ward and Saleh [8] discussed the same problem via the Backus-Gilbert technique. But there, the randomness in the point spread function was confined in its width to that of the impulse response. In this work, the noise in the point spread function does not have to be confined to the width of the impulse response.

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Ward and Saleh [7] introduced restoration methods to the one-dimensional problem, based on modifications of the Wiener and the minimum variance unbiased (MVU) estimation methodologies. Iterative schemes were adopted. Estimates were not confined to the class of linear estimation, and the statistics of the unknown signal were not assumed to be completely known.

In this work, we extend the modified Wiener filter in [7] to the 2-D case. Since image restoration necessitates solving huge systems of linear equations whose computation in the space domain is prohibitively expensive, we shall have to resort to different methods of solution. We shall use circulant matrix approximation and the fast Fourier transform (FFT) whenever possible. The computation of the Wiener filter in the Fourier domain using circulant approximation is a straightforward matter. That is one of the main reasons for the popularity of that filter in image restoration. The main drawback of the Wiener filter is the assumption that the correlation matrix of the unknown object image (determined by averaging over the ensemble of the objects) is known. In most practical cases, this is unrealistic, and we shall use an iterative approach to estimate the correlation of the object image. Experimental results indicate that the proposed iterative Wiener filter is very worthwhile in that it outperforms the standard Wiener filter. It is also found that the iterative Wiener filter gives better results than those based on the Backus-Gilbert technique [8].

II. The 2-D Image Restoration

The original image F and the distorted image G are usually digitized and stored in N by N and M by M matrices, $M \geq N$, respectively. From F and G, we form the corresponding vectors f and g by stacking the matrix data in F and G column by column (or row by row). Then f is a vector of length N^2 and g is a vector of length M^2. The mathematical model is [1], [2]

$$g = \boldsymbol{H}f + n_2$$

where the noise term n_2 is a vector of length M^2 and $\boldsymbol{H}$ is a block Toeplitz matrix of dimension M^2 by N^2. Since here the blur function $\boldsymbol{H}$ is random, we split it into two parts, the deterministic part H and the stochastic part N_1, and rearrange the formula

$$g = Hf + n \tag{1a}$$

where

$$n = N_1 f + n_2. \tag{1b}$$

When N_1 and n_2 are stationary in space, the correlation matrix of the noise $\overline{R_n(f)}$ becomes block Toeplitz. Since $\overline{R_n(f)}$ is symmetric, we only need to store the first row. This row will be stored in a matrix of size M by M. We shall call this matrix $\overline{R}_n(f)$. Furthermore, if n_2 is also white, the entries in the matrix $\overline{R_n(f)}$ can be written explicitly. In order to make use of matrix circulant approximation and the FFT, we augment columns and rows of zeros to the image F to extend it to a matrix of size M by M. Thus,

$$f_{ij} = \begin{cases} \text{original value}, & i, j \leq N \\ 0, & \text{otherwise}. \end{cases} \tag{2}$$

After the modification, the vector f has length M^2, the matrix $\boldsymbol{H}$ has dimensions M^2 by M^2, and the correlation matrix R_f is block Teoplitz with dimensions M^2 by M^2. The first row of R_f will be stored in a matrix $\overline{R}_f$ of dimensions M by M.

A. The Wiener Filter

If the Wiener criterion is used to restore the 2-D image represented by (1a) and (1b), the estimator and statistics are represented by the following formulas:

$$\hat{f} = \hat{H}g \tag{3}$$

where

$$\hat{H} = R_f H' \left[H R_f H' + \overline{R_n(f)} \right]^{-1} \tag{3a}$$

and

$$\overline{R_n(f)} = E[nn'] = E\left[(N_1 f + n_2)(N_1 f + n_2)' \right]. \tag{3b}$$

If the noise sources N_1 and n_2 are white and uncorrelated with each other, the ijth entry in $\overline{R}_n(f)$ can be shown to be

$$\left[\overline{R}_n(f)_{ij} \right] = (M - i)(M - j)\, \sigma_1^2 [R_f]_{ij} + \sigma_2^2 \delta_{11}. \tag{4}$$

The estimate for R_f, the correlation matrix of the object image, is $R(\hat{f})$ where

$$\left[R(\hat{f}) \right]_{ij} = \frac{1}{M^2} \sum \sum \hat{f}_{lk} \hat{f}_{l+i-1, k+j-1}. \tag{5}$$

We solve (3a) iteratively using (4) and (5). Now consider (3a). Since every term on the right-hand side is block Toeplitz, we can approximate those matrices by circulant matrices [1], and the resulting system can be efficiently computed using the FFT. To be more general, the Wiener criterion used for the 2-D restoration is parametric. Thus, we need to add a parameter in the standard Wiener filter (3a) in order to optimize the restored images. To be explicit, we rewrite (3a) in its parametric form:

$$\hat{H} = R_f H' \left[H R_f H' + \gamma \overline{R_n(f)} \right]^{-1} \tag{6}$$

where γ is the optimum parameter determined experimentally. Taking the discrete Fourier transform on both sides of (6), we obtain

$$\hat{H}(\omega_k, \omega_l) = \frac{H^*(\omega_k, \omega_l)}{\left| H(\omega_k, \omega_l) \right|^2 + \gamma \dfrac{W_n(\omega_k, \omega_l)}{W_f(\omega_k, \omega_l)}} \tag{7}$$

where $W_n(\omega_k, \omega_l)$ is the power spectrum of n, $W_f(\omega_k, \omega_l)$ is the power spectrum of f, and * indicates complex conjugate. We also calculate the Fourier transform of the distorted image g, $G(\omega_k, \omega_l)$ and multiply (7) by $G(\omega_k, \omega_l)$. Finally, we perform the inverse Fourier transform to obtain the restored image $\hat{f}_{kl}$:

$$\hat{f}_{kl} = \frac{1}{M^2} \sum \sum \exp \left\{ \frac{2\pi i}{M} (k\omega_k - l\omega_l) \right\}^* \cdot \frac{H^*(\omega_k, \omega_l)\, G(\omega_k, \omega_l)}{\left| H(\omega_k, \omega_l) \right|^2 + \gamma \dfrac{W_n(\omega_k, \omega_l)}{W_f(\omega_k, \omega_l)}}. \tag{8}$$

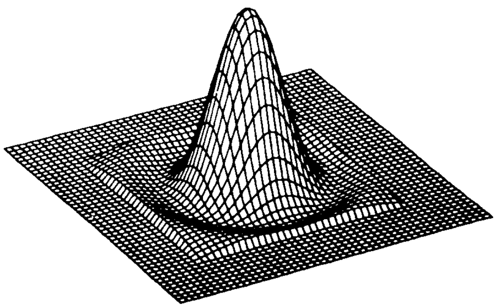
(For clarity, we dropped all the subscripts indicating iterations in the above discussion.)

B. Examples

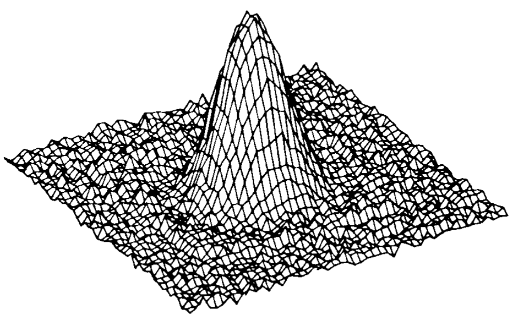
In order to test the 2-D Wiener filter, we use a 120 by 120 image, "Face," shown in Fig. 1. A random uncorrelated blur function is generated by superimposing white Gaussian noise N_1 on the deterministic sinc^2-shaped blur H. The sinc^2 blur function is known as the impulse response of an ideal diffraction-limited incoherent imaging system of a square aperture. The standard deviation of the noise N_1 is σ_1. The shape of the random blur is shown in Fig. 2 where (a) is the deterministic blur and its noisy counterpart is (b). The additive noise n_2 is (generated as) white and Gaussian, with standard deviation σ_2, and is uncorrelated with N_1, the randomness in the blur. The distorted image is given in Fig. 3(a) for which $\sigma_1 = 0.0005$, $\sigma_2 = 1.25$, and the root mean-squared error rms = 15.4896. Although σ_1^2, the strength of the noise N_1, is small, its contribution to the noise in g can be very large due to the relationship shown in (1b) and (4). The classical straightforward Wiener estimate, based on neglecting the effect of the impulse response noise N_1, is shown in Fig. 3(b) with rms = 36.8073. The magnification of noise by the linear Wiener filter is obvious. This estimate serves as an initial guess for the iterative approach of (6) in which both noise sources are included.

We first used the standard Wiener filter (3), that is, $\gamma = 1 \cdots$, and then the parametric Wiener filter (6) to perform the restoration. Experiments show that the restoration is optimized when

Fig. 1. The original (120 × 120) image, "Face."



(a)



(b)

Fig. 2. The sinc^2 impulse response function (a) without noise (9 × 9 pixels) and (b) with noise which extends to 120 × 120 pixels.

γ is equal to 2. See Fig. 3(c) and Fig. (d). The rms errors for the standard and parametric Wiener filters are 11.2911 and 10.3403, respectively. The parametric Wiener filter is seen to result in a smoother picture. The difference images between the original and those in Fig. 3 are shown in Fig. 4, from which we can see that the linear Wiener filter can eliminate the blur effect, but only at the expense of magnifying the image-dependent noise. On the other hand, the iterative filters give much better results. Comparison between the results of this modified Wiener filter and those given by using the Backus-Gilbert technique [8] are shown in Figs. 5 and 6. Fig. 5 is the random blur used for this comparison. According to the assumptions of the Backus-Gilbert technique, the noise is confined to the width of the blur function. The original image, Fig. 6(a), is the same as Fig. 1, but with a smaller scale. The blurred image is shown in Fig. 6(b) with the strengths of the noise components being $\sigma_1 = 0.006$ and $\sigma_2 = 3$, respectively. (Since the noise in the blur function is confined to the width of the blur in this case, larger σ_1 and σ_2 can be selected.) The rms of the blurred image is 11.6682. Fig. 6(c) and Fig. (d) are the results of the modified iterative Wiener filter (rms = 6.5391) and the technique in [8] (rms = 7.6349), respectively.

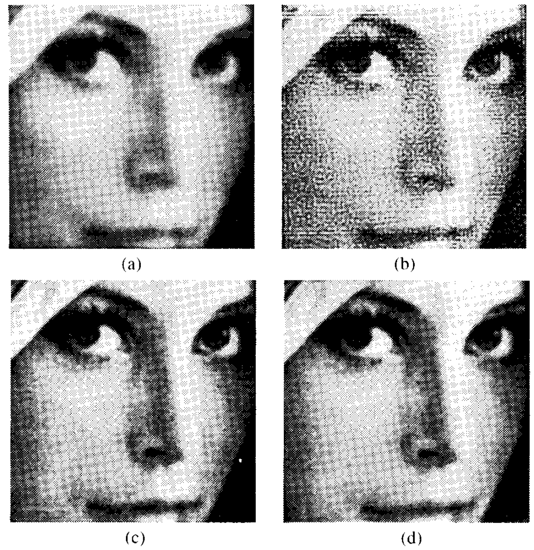


Fig. 3. (a) The blurred image of Fig. 1 with $\sigma_1 = 0.0005$, $\sigma_2 = 1.25$, and rms = 15.49. (b)–(d) The restored images of Fig. 3(a): the linear Wiener, rms = 36.81; the modified iterative Wiener, rms = 11.29; and the parametric modified iterative Wiener ($\gamma = 2$), rms = 10.34, respectively.

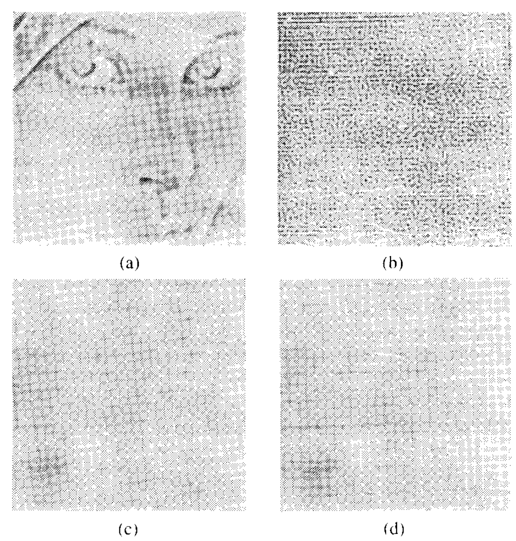


Fig. 4. (a)–(d) The difference images between the original and Fig. 3(a), (b), (c), and (d).

C. A Remark

In the process of restoring randomly distorted images, some technical details should be considered carefully. In order to better approximate the noise correlation matrix $\overline{R_n(f)}$ by a circulant matrix, we need to take into account the statistical nature of the noise. In the case that the noise is image independent, it possesses no predictable correlation beyond a distance d. That is,

$$R_{ij} = 0, \quad |i - j| \geq d.$$

Typically, d is about 20–30 pixels. But when the noise is image dependent, the distance d is related to the size of the image under

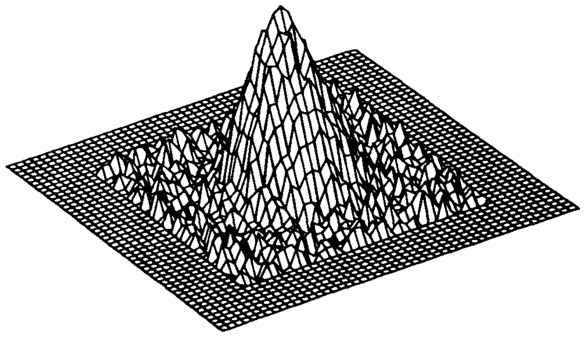


Fig. 5. The sinc^2 impulse response with noise confined to the width of the blur (9×9 pixels).

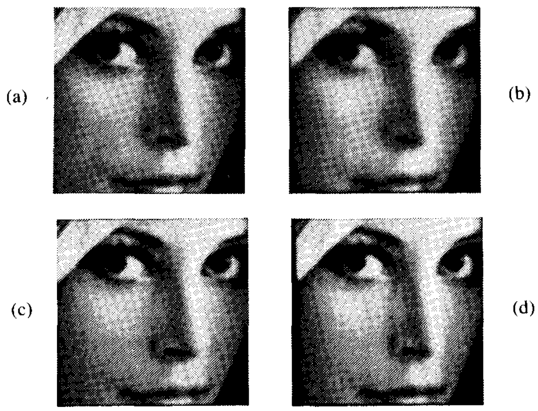


Fig. 6. (a) The original (120×120) image, same as in Fig. 1. (b) Fig. 6(a) blurred by the function shown in Fig. 5 with $\sigma_1 = 0.006$ and $\sigma_2 = 3$, respectively. rms = 11.668. (c), (d) The restoration of Fig. 6(b) by the iterative Wiener filter, rms = 6.539, and by the Backus-Gilbert technique, rms = 7.635, respectively.

consideration. This argument is supported by (4) and (5). If the distance d is not properly chosen, restoration may give incorrect results. If the distance chosen is too small, the approximated $\overline{R_n(f)}$ tends to become an identity matrix, and useful information is lost. On the other hand, if the distance chosen is too large, some artifacts are introduced, which may result in an unstable solution. Our experience is that the distance d is, approximately, linearly dependent on the image size. Thus, d satisfies

$$d \simeq (0.1 \rightarrow 0.2)M$$

for a matrix of size M by M.

III. Conclusions

For restoration of images distorted by systems having a noisy impulse response and additive noise, the Wiener filter is modified, extended, and applied to the 2-D case. The noise in the blur is not assumed to be necessarily confined to the width of the impulse response.

The simulation examples presented confirm that the modified Wiener filter works well for the restoration of random blur. The experimental results show that the newly derived filter always gives better results than its linear counterpart (based on ignoring the noise in the impulse noise). The modified parametric Wiener filter gives more pleasing (smoother) images than the standard modified Wiener filter. The latter also gives better results than those based on the Backus-Gilbert technique [8].

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Superresolution Reconstruction Through Object Modeling and Parameter Estimation

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***Abstract*—Fourier transform reconstruction with limited data is often encountered in tomographic imaging problems. Conventional techniques, such as FFT-based methods, the spatial-support-limited extrapolation method, and the maximum entropy method, have not been optimal in terms of both Gibbs ringing reduction and resolution enhancement. In this correspondence, a new method based on object modeling and parameter estimation is proposed to achieve superresolution reconstruction.**

I. Introduction

Many problems in physics and medicine involve imaging objects with high spatial frequency content in a limited amount of time. The limitation of available experimental data leads to the problem of diffraction-limited data which manifests itself by causing ringing in the image and is known as the Gibbs phenomenon. Due to the Gibbs phenomenon, the resolution of images reconstructed using the conventional Fourier transform method has been limited to $1/L$, with L being the data window size. Many methods have been proposed to recover information beyond this limit. A commonly used superresolution technique is the iterative algorithm of Gerchberg–Papoulis [1]. This algorithm uses the *a priori* knowledge that the object being imaged is of finite spatial support. It proceeds with an iterative scheme to perform Fourier transformation between the data and image space, imposing the anticipated object support and the consistency of the experimental data. It has been proved that in the ideal noiseless case, this procedure converges to the minimum norm solution [2]. However, when noise is present, the performance of this algorithm will be greatly limited. Other methods, including the maximum entropy method [3] and methods using the projection onto convex set (POCS) formalism [4], [5], attempt to employ stronger constraints to regularize the problem, but have not been optimal for Gibbs ringing reduction and resolution enhancement.

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