ECE212H1F – Circuit Analysis

Lecture #15

Second-Order Circuits

Natural Response – Part 1

Second-Order Circuits: *General Layout*

R G B O

- We have discussed the transient response of first-order circuits, which may include sources, resistors, and storage elements which can be reduced to a single storage element:
- Second-order circuits include sources, resistors, and at least two unique storage elements:

R G B O

Source-Free Series *RLC* Circuit: *Natural Response*

- Recall, the ***natural response*** of a circuit with transients relates to the release of the ***initial stored energy*** within the storage elements.
- Consider the ***series RLC*** circuit shown below. How can we find the expression for the current in the circuit for $t > 0$?
  - We can use the general *differential equation approach*.
- If we do this (using KVL) we find that the current can be determined by solving the differential equation given by:

R G B O

Clicker

If R = 0 in the series RLC circuit shown below, what will the i(t) waveform look like?

A. Constant voltage

B. Decaying exponential

C. Growing exponential

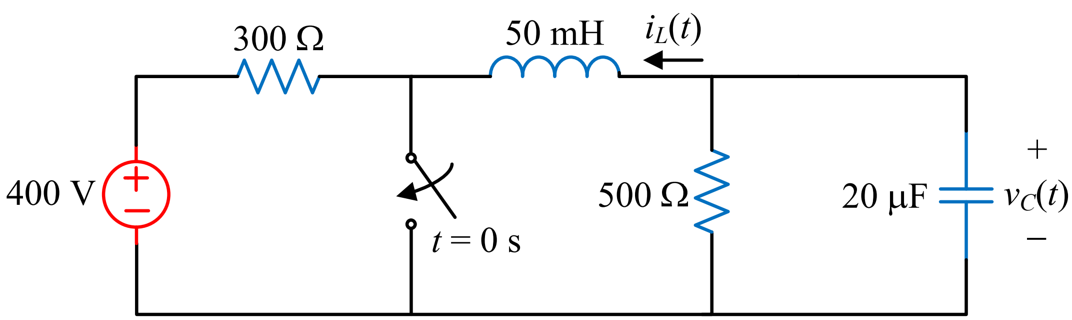
D. Sinusoid

E. Damped sinusoid

R G B O

Example:
Parallel RLC Circuit – Natural Response

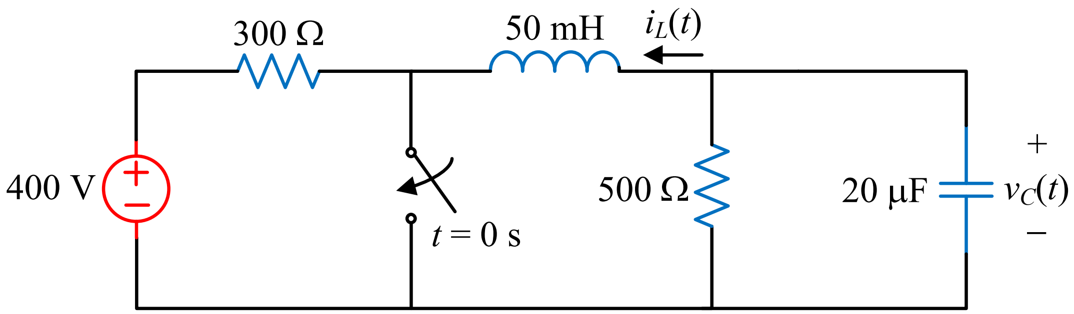
- How does the situation change if *R*, *L*, and *C* are connected in parallel?
- Consider the circuit shown below. Find $v_C(t)$ and $i_L(t)$ for $t > 0$.



R G B Ω

Clicker

For the circuit given below what is the initial value of the current through the inductor (as indicated in the circuit)?



A. $i_L(0+) = 2.66$ A

B. $i_L(0+) = 1.33$ A

C. $i_L(0+) = 0.5$ A

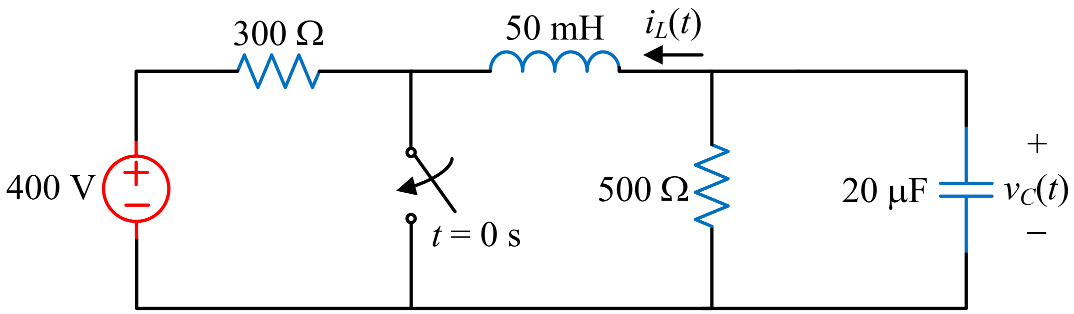
D. $i_L(0+) = 0$ A

E. $i_L(0+) = -0.5$ A

R G B Ω

Clicker

For the circuit given below what is the final value of the voltage across the capacitor?



A. $v_C(\infty) = 0$ V

B. $v_C(\infty) = 40$ V

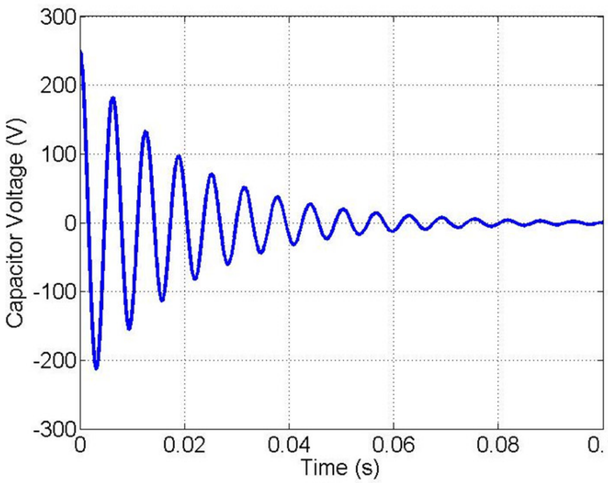
C. $v_C(\infty) = 25$ V

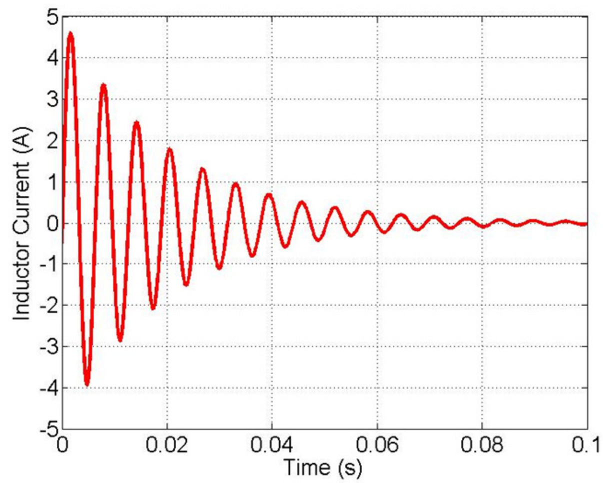
D. $v_C(\infty) = 20$ V

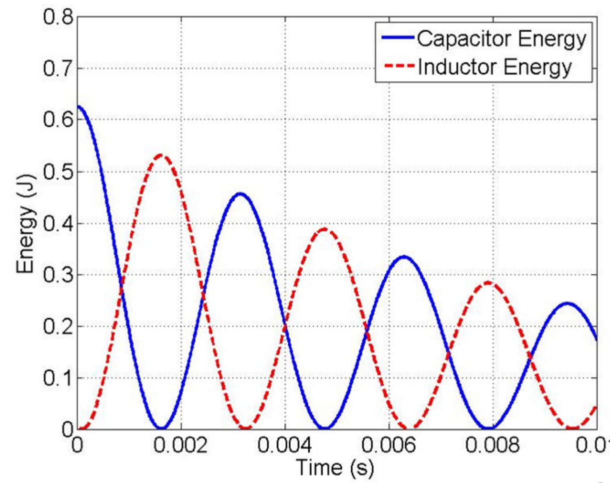
E. $v_C(\infty) = -40$ V

Example:
Parallel RLC Circuit – Natural Response (cont'd)

Example:
Parallel RLC Circuit – Natural Response (cont’d)







Second-Order Circuits: *Definitions*

R G B O

- There are some important quantities which govern the response for *all* second-order circuits. These are:

- Using these we can write the general form of the differential equation which governs the ***natural response of all second-order circuits***:

- This results in the characteristic equation (auxiliary equation) of:

R G B O

Key Points

1. The ***natural response*** of a circuit relates to the situation where there is no independent source connected to the circuit after the switch moves (e.g., for $t > 0$)
   - This response is caused by the release of the initial energy stored in the storage elements (i.e., *L*'s and *C*'s)

2. The final value of the natural response will always be zero due to the damping of the resistor in the circuit.
   - Meaning that all of the initial stored energy is eventually released as heat

3. The differential equation which characterizes the ***natural response*** of ***any RLC* circuit** will be homogeneous, and will have a form that looks like:

$$\frac{d^2x}{dt^2} + 2\zeta\omega_0\frac{dx}{dt} + \omega_0^2 x = 0 \qquad \text{or} \qquad \frac{d^2x}{dt^2} + 2\alpha\frac{dx}{dt} + \omega_0^2 x = 0$$

4. The **solution to this differential equation** can be found by finding the roots of the characteristic equation, i.e., $m_1, m_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$

5. The basic quantities are: ω_0 = undamped natural frequency,

   α = damping factor,

   ζ = damping ratio, and

   $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$ damped natural frequency.

R G B Q

Key Points

6. For general *RLC* circuits, the way in which these basic quantities relate to the circuit elements *R's*, *L's*, and *C's*, depend on the how that circuit is connected.

7. If the circuit has a ***series RLC*** connection circuits the roots are $m_1, m_2 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$ and the basic quantities are

   Undamped natural frequency = $\omega_0 = \frac{1}{\sqrt{LC}}$

   Damping factor = $\alpha = \frac{R}{2L}$

   Damping ratio = $\zeta = \frac{\alpha}{\omega_0}$

8. If the circuit has a ***parallel RLC*** circuits the are $m_1, m_2 = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$, and the basic quantities are

   Undamped natural frequency = $\omega_0 = \frac{1}{\sqrt{LC}}$

   Damping factor = $\alpha = \frac{1}{2RC}$

   Damping ratio = $\zeta = \frac{\alpha}{\omega_0}$

9. In the expressions for the series and parallel *RLC* circuits, the **R is the *total resistance* that the storage elements "see"** (for example, this is the total resistance that the *L* and *C* are in series with, or the total resistance that the *L* and the *C* are in parallel with)