## Supplementary Material: Models of Wind Turbine and Photovoltaic, Detailed Controllers for the Primal and Dual Variables, Parameters, and Additional System States

A. Wind Speed and Active Power Generation of Distributed Wind Generator Modeling

The hourly wind speed is modeled by the following Weibull probability distribution function (PDF) [1]

$$PDF(v) = \frac{k}{c} \left(\frac{v}{c}\right)^{k-1} e^{\left(-\frac{v}{c}\right)^k} \quad (k > 0, v > 0, c > 1) \quad (1)$$

where v, k, and c are the wind speed (m/s), the shape coefficient, and the scale coefficient, respectively.

The conversion from the wind speed to active power generation of a WT at node i can be calculated by [1]

$$P_i^{WT} = \begin{cases} 0 & v < V_{cutin} \\ \frac{1}{2}\rho v^3 A C_P & V_{cutin} \le v \le V_{rate} \\ P_i^{rate} & V_{rate} < v \le V_{cutout} \\ 0 & v > V_{cutout} \end{cases}$$
(2)

where  $P_i^{WT}$ ,  $\rho$ , A, and  $C_P$  are the output power of the WT at node i with wind speed v, air density (kg/m³), total area of the WT swept surface (m²) and the power coefficient, respectively.  $V_{cutin}$ ,  $V_{rate}$ ,  $V_{cutout}$  and  $P_i^{rate}$  are the cut-in speed, rated speed, cut-out speed, and rated power of the WT at node i, respectively.

B. Solar Irradiance and Active Power Generation of Distributed Solar Generator Modeling

Hourly solar irradiance is modeled by the following Beta PDF [2]

$$PDF(s) = \begin{cases} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)+\Gamma(\beta)} \times \\ s^{\alpha-1} \times (1-s)^{\beta-1} & if \ 0 \le s \le 1, 0 \le \alpha, \beta \\ 0 & \text{else} \end{cases}$$
(3)

where s is the solar irradiance (kW/m<sup>2</sup>).  $\alpha$  and  $\beta$  are the parameters of Beta PDF, which can be calculated via the following equations ( $\mu$  and  $\sigma$  are the mean and standard deviation of s):

$$\beta = (\frac{\mu(1+\mu)}{\sigma^2} - 1) \times (1-\mu), \quad \alpha = \frac{\mu\beta}{1-\mu}.$$
 (4)

The active power generation of the solar DG at bus i is given by [2]

$$P_i^{PV} = \eta \times A \times s \tag{5}$$

where  $P_i^{PV}$  is the output power of the PV at node i for solar irradiance s.  $\eta$  and A are the efficiency (%) of the power conversion and total area (m<sup>2</sup>) of the PV panel, respectively.

C. Distributed Controllers

The controllers for the primal variables are:

$$\begin{split} \dot{H}_{i}^{L} &= -k_{H_{i}^{L}} \left( \frac{\partial L}{\partial H_{i}^{L}} \right) = -k_{H_{i}^{L}} \left( \frac{\omega_{i}^{H}}{\lambda_{i}} (H_{i}^{L} - H_{i}^{L,Set}) \right) \\ &- \lambda_{i}^{H} - \psi_{i}^{-} + \psi_{i}^{+} \right) & (6a) \\ \dot{H}_{i}^{CHP} &= -k_{H_{i}^{CHP}} \left( \frac{\partial L}{\partial H_{i}^{CHP}} \right) = -k_{H_{i}^{CHP}} \left( \lambda_{i}^{HG} \right) \\ &- v_{i}^{-} + v_{i}^{+} + \phi_{i}^{-} c_{i}^{m} + \phi_{i}^{+} c_{i}^{v} \right) & (6b) \\ \dot{H}_{i}^{GB} &= -k_{H_{i}^{GB}} \left( \frac{\partial L}{\partial H_{i}^{GB}} \right) = -k_{H_{i}^{GB}} \left( \frac{\lambda_{i}^{GB}}{\eta_{i}^{GB}} \right) \\ &+ \lambda_{i}^{HG} - \kappa_{i}^{-} + \kappa_{i}^{+} \right) & (6c) \\ \dot{H}_{i}^{EB} &= -k_{H_{i}^{EB}} \left( \frac{\partial L}{\partial H_{i}^{EB}} \right) = -k_{H_{i}^{EB}} \left( \frac{\lambda_{i}^{EB}}{\eta_{i}^{EB}} \right) \\ &+ \lambda_{i}^{HG} - \iota_{i}^{-} + \iota_{i}^{+} \right) & (6d) \\ \dot{H}_{i}^{G} &= -k_{H_{i}^{G}} \left( \frac{\partial L}{\partial H_{i}^{G}} \right) = -k_{H_{i}^{G}} \left( -\lambda_{i}^{HG} + \lambda_{i}^{H} \right) & (6e) \\ \dot{G}_{i}^{CHP} &= -k_{G_{i}^{GHP}} \left( \frac{\partial L}{\partial G_{i}^{CHP}} \right) = -k_{G_{i}^{GHP}} \left( \lambda_{i}^{Gas} - \lambda_{i}^{GHP} \right) \\ \dot{G}_{i}^{GB} &= -k_{G_{i}^{GB}} \left( \frac{\partial L}{\partial G_{i}^{GB}} \right) = -k_{G_{i}^{GB}} \left( \lambda_{i}^{Gas} - \lambda_{i}^{GB} \right) & (6f) \\ \dot{G}_{i}^{GB} &= -k_{H_{i}} \left( \frac{\partial L}{\partial H_{i}} \right) = -k_{H_{i}} \left( \omega_{i}^{HP} (H_{i} - H_{i}^{Set}) + \sum_{ji \in \mathcal{E}_{Pump}(H)} \lambda_{ji}^{HP} - \sum_{ji \in \mathcal{E}_{Pump}(H)} \lambda_{ji}^{HP} - \sum_{ji \in \mathcal{E}_{Ioad}(H)} \lambda_{ji}^{HP} - \sigma_{i}^{-} + \sigma_{i}^{+} \right) \\ \dot{H}_{i}^{Pump} &= -k_{H_{i}^{Pump}} \left( \frac{\partial L}{\partial H_{i}^{Pump}} \right) = -k_{H_{i}^{Pump}} \left( -\frac{\rho g}{\eta_{i}} M_{ji} \lambda_{i}^{P} - \sum_{ji \in \mathcal{E}_{Nump}(H)} \lambda_{ji}^{Pump} - \rho_{i}^{-} + \rho_{i}^{+} \right) \\ &- \sum_{ji \in \mathcal{E}_{Nump}(H)} \lambda_{ji}^{Pump} - \rho_{i}^{-} + \rho_{i}^{+} \right) \\ &- \sum_{ji \in \mathcal{E}_{Nump}(H)} \lambda_{ji}^{Pump} - \rho_{i}^{-} + \rho_{i}^{+} \right) \end{aligned}$$

 $\dot{T}_i = -k_{T_i} \left( \frac{\partial L}{\partial T_i} \right) = -k_{T_i} \left( -\sum_{i \in T_i} \lambda_i^H c M_{ik} + \frac{\partial L}{\partial T_i} \right)$ 

$$\sum_{k \in \mathcal{N}_{out}(i)} \lambda_{ik}^{T} \exp\left(-\frac{\nu L_{ik}}{c M_{ik}}\right) + \nu_{i}^{+} - \nu_{i}^{-}\right) \qquad (6j)$$

$$\dot{T}_{ji} = -k_{T_{ji}} \left(\frac{\partial L}{\partial T_{ji}}\right) = -k_{T_{ji}} \left(\lambda_{i}^{H} c M_{ji} - \lambda_{ji}^{T} + \xi_{ji}^{+}\right)$$

$$-\xi_{ji}^{-}\right) \qquad (6k)$$

$$\dot{P}_{i}^{CHP} = -k_{P_{i}^{CHP}} \left(\frac{\partial L}{\partial P_{i}^{CHP}}\right) = -k_{P_{i}^{CHP}} \left(\lambda_{i}^{PG} + \frac{\lambda_{i}^{CHP}}{\eta_{i}^{CHP}}\right)$$

$$-\zeta_{i}^{-} + \zeta_{i}^{+} - \phi_{i}^{-} + \phi_{i}^{+}\right) \qquad (6l)$$

$$\dot{P}_{i}^{pump} = -k_{P_{i}^{pump}} \left(\frac{\partial L}{\partial P_{i}^{Pump}}\right) = -k_{P_{i}^{pump}} \left(-\lambda_{i}^{PG} + \lambda_{i}^{P}\right) \qquad (6m)$$

$$\dot{P}_{i}^{EB} = -k_{P_{i}^{EB}} \left(\frac{\partial L}{\partial P_{i}^{PE}}\right) = -k_{P_{i}^{EB}} \left(-\lambda_{i}^{PG} - \lambda_{i}^{EB}\right) \qquad (6n)$$

$$\dot{P}_{1}^{Grid} = -k_{P_{1}^{Grid}} \left(\frac{\partial L}{\partial P_{i}^{Grid}}\right) = -k_{P_{1}^{Grid}} \left(\lambda_{i}^{Grid} \left(P_{1}^{Grid} - P_{1}^{Grid} + \lambda_{i}^{PG}\right)\right) \qquad (6p)$$

$$\dot{Q}^{Grid} = -k_{Q_{i}^{Grid}} \left(\frac{\partial L}{\partial Q_{i}^{Grid}}\right) = -k_{Q_{i}^{Grid}} \left(\lambda_{i}^{QG}\right) \qquad (6p)$$

$$\dot{P}_{i} = -k_{P_{i}} \left(\frac{\partial L}{\partial P_{i}}\right) = -k_{P_{i}} \left(-\lambda_{i}^{PG} + \lambda_{i}^{PF}\right) \qquad (6q)$$

$$\dot{Q}_{i} = -k_{Q_{i}} \left(\frac{\partial L}{\partial Q_{i}^{G}}\right) = -k_{Q_{i}} \left(-\lambda_{i}^{QG} + \lambda_{i}^{QF}\right) \qquad (6r)$$

$$\dot{Q}_{i}^{G} = -k_{Q_{i}^{G}} \left(\frac{\partial L}{\partial Q_{i}^{G}}\right) = -k_{Q_{i}^{G}} \left(\lambda_{i}^{QG} - \chi_{i}^{-} + \chi_{i}^{+}\right) \qquad (6t)$$

$$\dot{P}_{ij} = -k_{P_{ij}} \left(\frac{\partial L}{\partial P_{ij}}\right) = -k_{P_{ij}} \left(\lambda_{j}^{PF} - \lambda_{i}^{PF} - 2\lambda_{i}^{V} r_{ij} - \vartheta_{ij}^{-} + \vartheta_{ij}^{+}\right) \qquad (6t)$$

$$\dot{Q}_{ij} = -k_{Q_{ij}} \left(\frac{\partial L}{\partial Q_{ij}}\right) = -k_{Q_{ij}} \left(\lambda_{j}^{QF} - \lambda_{i}^{QF} - 2\lambda_{i}^{V} r_{ij} - \vartheta_{ij}^{-} + \vartheta_{ij}^{+}\right) \qquad (6t)$$

$$\dot{V}_{i} = -k_{V_{i}} \left(\frac{\partial L}{\partial V_{i}}\right) = -k_{V_{i}} \left(\sum_{ij \in \mathcal{E}(\mathcal{P})} \lambda_{i}^{V} - \sum_{mi \in \mathcal{E}(\mathcal{P})} \lambda_{m}^{V} - \vartheta_{i}^{-} + \vartheta_{i}^{+}\right). \qquad (6v)$$

The controllers for the dual variables related to the equality constraints are:

$$\begin{split} \dot{\lambda}_{i}^{GB} = & k_{\lambda_{i}^{GB}} \left( \frac{\partial L}{\partial \lambda_{i}^{GB}} \right) = k_{\lambda_{i}^{GB}} \left( \frac{H_{i}^{GB}}{\eta_{i}^{GB}} - G_{i}^{GB} \right) \end{split} \tag{7a}$$
 
$$\dot{\lambda}_{i}^{EB} = & k_{\lambda_{i}^{EB}} \left( \frac{\partial L}{\partial \lambda_{i}^{EB}} \right) = k_{\lambda_{i}^{EB}} \left( \frac{H_{i}^{EB}}{\eta_{i}^{EB}} - P_{i}^{EB} \right) \end{split} \tag{7b}$$
 
$$\dot{\lambda}_{i}^{CHP} = & k_{\lambda_{i}^{CHP}} \left( \frac{\partial L}{\partial \lambda_{i}^{CHP}} \right) = k_{\lambda_{i}^{CHP}} \left( \frac{P_{i}^{CHP}}{\eta_{i}^{CHP}} - G_{i}^{CHP} \right)$$
 
$$\dot{\lambda}_{i}^{PG} = & k_{\lambda_{i}^{PG}} \left( \frac{\partial L}{\partial \lambda_{i}^{PG}} \right) = k_{\lambda_{i}^{PG}} \left( P_{i}^{CHP} + P_{i}^{WT} \right)$$

$$\begin{split} &+P_{i}^{PV}+P_{i}^{Grid}-P_{i}^{L}-P_{i}^{pump}-P_{i}^{EB}-P_{i})\\ \dot{\lambda}_{i}^{QG}=k_{\lambda_{i}^{QG}}\left(\frac{\partial L}{\partial \lambda_{i}^{QG}}\right)=k_{\lambda_{i}^{QG}}\left(Q_{i}^{G}+Q_{i}^{Grid}-Q_{i}^{L}-Q_{i}\right)\\ \dot{\lambda}_{j}^{PF}=k_{\lambda_{j}^{PF}}\left(\frac{\partial L}{\partial \lambda_{j}^{PF}}\right)=k_{\lambda_{j}^{PF}}\left(\sum_{i\in\Omega_{i\rightarrow j}}P_{ij}+P_{j}\right)\\ &-\sum_{k\in\Omega_{j\rightarrow k}}P_{jk}\right)\\ \dot{\lambda}_{j}^{QF}=k_{\lambda_{j}^{QF}}\left(\frac{\partial L}{\partial \lambda_{i}^{QF}}\right)=k_{\lambda_{j}^{QF}}\left(\sum_{i\in\Omega_{i\rightarrow j}}Q_{ij}+Q_{j}\right)\\ &-\sum_{k\in\Omega_{j\rightarrow k}}Q_{jk}\right)\\ \dot{\lambda}_{i}^{V}=k_{\lambda_{i}^{V}}\left(\frac{\partial L}{\partial \lambda_{i}^{V}}\right)=k_{\lambda_{i}^{V}}\left(V_{i}-V_{j}\right)\\ &-2(r_{ij}P_{ij}+x_{ij}Q_{ij})\right),\ ij\in\mathcal{E}(\mathcal{P})\\ &-k_{i}^{HG}=k_{\lambda_{i}^{HG}}\left(\frac{\partial L}{\partial \lambda_{i}^{HG}}\right)=k_{\lambda_{i}^{HG}}\left(H_{i}^{GB}+H_{i}^{EB}+H_{i}^{CHP}-H_{i}^{CHP}\right)\\ &-H_{i}^{G}\right)\\ \dot{\lambda}_{i}^{H}=k_{\lambda_{i}^{H}}\left(\frac{\partial L}{\partial \lambda_{i}^{H}}\right)=k_{\lambda_{i}^{H}}\left(H_{i}^{G}-H_{i}^{L}-c\sum_{k\in\mathcal{N}_{out}(i)}M_{ik}T_{i}\right)\\ &+c\sum_{j\in\mathcal{N}_{in}(i)}M_{ji}T_{ji}\right)\\ \dot{\lambda}_{ji}^{T}=k_{\lambda_{ji}^{T}}\left(\frac{\partial L}{\partial \lambda_{ji}^{T}}\right)=k_{\lambda_{ji}^{T}}\left(-T_{ji}\right)\\ &+\left(T_{j}-T_{a}\right)\exp\left(-\frac{\nu L_{ji}}{cM_{ji}}\right)+T_{a}\right)\\ \dot{\lambda}_{ji}^{HP}=k_{\lambda_{ji}^{HP}}\left(\frac{\partial L}{\partial \lambda_{ji}^{HP}}\right)=k_{\lambda_{ji}^{HP}}\left(H_{j}-H_{i}-H_{j}-H_{i}^{pump}\right)\\ \dot{\lambda}_{ji}^{HP}=k_{\lambda_{ji}^{P}}\left(\frac{\partial L}{\partial \lambda_{ji}^{HP}}\right)=k_{\lambda_{ji}^{P}}\left(H_{j}-H_{i}-F_{ji}M_{ji}^{2}\right). \end{aligned} (7m)$$

The controllers for the dual variables related to the inequality constraints are:

$$\dot{\lambda}_{ji}^{HD} = k_{\lambda_{ji}^{HD}} \left( \frac{\partial L}{\partial \lambda_{ji}^{HD}} \right) = k_{\lambda_{ji}^{HD}} \left( -H_j + H_i + H_{i,\min}^c \right)_{\lambda_{ji}^{HD}}^+ \tag{8a}$$

$$\dot{\kappa}_i^+ = k_{\kappa_i^+} \left( \frac{\partial L}{\partial \kappa_i^+} \right) = k_{\kappa_i^+} \left( H_i^{GB} - H_{i, \max}^{GB} \right)_{\kappa_i^+}^+$$

$$\dot{\kappa}_i^- = k_{\kappa_i^-} \left( \frac{\partial L}{\partial \kappa_i^-} \right) = k_{\kappa_i^-} \left( H_{i, \min}^{GB} - H_i^{GB} \right)_{\kappa_i^-}^+$$

$$i_i^+ = k_{\iota_i^+} \left(\frac{\partial L}{\partial \iota_i^+}\right) = k_{\iota_i^+} \left(H_i^{EB} - H_{i,\max}^{EB}\right)_{\iota_i^+}^+$$

$$i_i^- = k_{\iota_i^-} \left( \frac{\partial L}{\partial \iota_i^-} \right) = k_{\iota_i^-} \left( H_{i, \min}^{EB} - H_i^{EB} \right)_{\iota_i^-}^+$$

$$\dot{\theta}_i^+ = k_{\theta_i^+} \left( \frac{\partial L}{\partial \theta_i^+} \right) = k_{\theta_i^+} \left( V_i - V_{i, \max} \right)_{\theta_i^+}^+$$

$$\dot{\theta}_i^- = k_{\theta_i^-} \left( \frac{\partial L}{\partial \theta_i^-} \right) = k_{\theta_i^-} \left( V_{i, \min} - V_i \right)_{\theta_i^-}^+$$

$$\dot{\vartheta}_{ij}^{+} = k_{\vartheta_{ij}^{+}} \left( \frac{\partial L}{\partial \vartheta_{ij}^{+}} \right) = k_{\vartheta_{ij}^{+}} \left( P_{ij} - P_{ij,\max} \right)_{\vartheta_{ij}^{+}}^{+}$$

$$\dot{\vartheta}_{ij}^{-} = k_{\vartheta_{ij}^{-}} \left( \frac{\partial L}{\partial \vartheta_{ij}^{-}} \right) = k_{\vartheta_{ij}^{-}} \left( P_{ij, \min} - P_{ij} \right)_{\vartheta_{ij}^{-}}^{+}$$

$$\dot{\epsilon}_{ij}^{+} = k_{\epsilon_{ij}^{+}} \left( \frac{\partial L}{\partial \epsilon_{ij}^{+}} \right) = k_{\epsilon_{ij}^{+}} \left( Q_{ij} - Q_{ij, \max} \right)_{\epsilon_{ij}^{+}}^{+}$$

$$\dot{\epsilon}_{ij}^{-} = k_{\epsilon_{ij}^{-}} \left( \frac{\partial L}{\partial \epsilon_{ij}^{-}} \right) = k_{\epsilon_{ij}^{-}} \left( Q_{ij, \min} - Q_{ij} \right)_{\epsilon_{ij}^{-}}^{+}$$

$$\dot{\chi}_i^- = k_{\chi_i^-} \left( \frac{\partial L}{\partial \chi_i^-} \right) = k_{\chi_i^-} \left( Q_{i, \min}^G - Q_i^G \right)_{\chi_i^-}^+$$

$$\dot{\chi}_i^+ = k_{\chi_i^+} \Big(\frac{\partial \dot{L}}{\partial \chi_i^+}\Big) = k_{\chi_i^+} \Big(Q_i^G - Q_{i,\max}^G\Big)_{\chi_i^+}^+.$$

$$\dot{\sigma}_i^+ = k_{\sigma_i^+} \left( \frac{\partial L}{\partial \sigma_i^+} \right) = k_{\sigma_i^+} \left( H_i - H_{i, \max} \right)_{\sigma_i^+}^+$$

$$\dot{\sigma}_{i}^{-} = k_{\sigma_{i}^{-}} \left( \frac{\partial \dot{L}}{\partial \sigma_{i}^{-}} \right) = k_{\sigma_{i}^{-}} \left( H_{i, \min} - H_{i} \right)_{\sigma_{i}^{-}}^{+}$$

$$\dot{\psi}_i^+ = k_{\psi_i^+} \left(\frac{\partial L}{\partial \psi_i^+}\right) = k_{\psi_i^+} \left(H_i^L - H_{i,\max}^L\right)_{\psi_i^+}^+$$

$$\dot{\psi}_i^- = k_{\psi_i^-} \Big(\frac{\partial L}{\partial \psi_i^-}\Big) = k_{\psi_i^-} \Big(H_{i,\min}^L - H_i^L\Big)_{\psi_i^-}^+$$

$$\dot{\nu}_{i}^{+} = k_{\nu_{i}^{+}} \left( \frac{\partial L}{\partial \nu_{i}^{+}} \right) = k_{\nu_{i}^{+}} \left( T_{i} - T_{i, \max} \right)_{\nu_{i}^{+}}^{+}$$

$$\dot{\nu}_i^- = k_{\nu_i^-} \left( \frac{\partial \dot{L}}{\partial \nu_i^-} \right) = k_{\nu_i^-} \left( T_{i,\min} - T_i \right)_{\nu_i^-}^+$$

$$\dot{\xi}_{ji}^{+} = k_{\xi_{ji}^{+}} \left( \frac{\partial L}{\partial \xi_{ii}^{+}} \right) = k_{\xi_{ji}^{+}} \left( T_{ji} - T_{ji, \max} \right)_{\xi_{ii}^{+}}^{+}$$

$$\dot{\xi}_{ji}^- = k_{\xi_{ji}^-} \left( \frac{\partial L}{\partial \xi_{ji}^-} \right) = k_{\xi_{ji}^-} \left( T_{ji, \min} - T_{ji} \right)_{\xi_{ji}^-}^+$$

$$\dot{\rho}_i^+ = k_{\rho_i^+} \left( \frac{\partial L}{\partial \rho_i^+} \right) = k_{\rho_i^+} \left( H_i^{pump} - H_{i,max}^{pump} \right)_{\rho_i^+}^+$$

$$\dot{\rho}_{i}^{-}=k_{\rho_{i}^{-}}\left(\frac{\partial L}{\partial \rho_{i}^{-}}\right)=k_{\rho_{i}^{-}}\left(H_{i,min}^{pump}-H_{i}^{pump}\right)_{\rho_{i}^{-}}^{+}$$

$$\dot{\varsigma}_{i}^{-}=k_{\varsigma_{i}^{-}}\Big(\frac{\partial L}{\partial \varsigma_{i}^{-}}\Big)=k_{\varsigma_{i}^{-}}\Big(P_{i,\min}^{CHP}-P_{i}^{CHP}\Big)_{\varsigma_{i}^{-}}^{+}$$

$$\dot{\varsigma}_{i}^{+} = k_{\varsigma_{i}^{+}} \left( \frac{\partial L}{\partial \varsigma_{i}^{+}} \right) = k_{\varsigma_{i}^{+}} \left( P_{i}^{CHP} - P_{i, \max}^{CHP} \right)_{\varsigma_{i}^{+}}^{+}$$

$$\dot{v}_i^- = k_{v_i^-} \Big(\frac{\partial L}{\partial v_i^-}\Big) = k_{v_i^-} \Big(H_{i,\min}^{CHP} - H_i^{CHP}\Big)_{v_i^-}^+$$

$$= k_{\theta^{-}} \left( \frac{\partial L}{\partial \theta_{i}^{+}} \right) = k_{\theta^{-}} \left( V_{i \min} - V_{i} \right)^{+}$$

$$= k_{\theta^{-}} \left( \frac{\partial L}{\partial \theta_{i}^{-}} \right) = k_{\theta^{-}} \left( V_{i \min} - V_{i} \right)^{+}$$

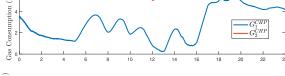
$$(8g)$$

$$\dot{\theta}_{ij}^{+} = k_{artheta_{ij}^{+}} \left( rac{\partial L}{\partial artheta_{ij}^{+}} 
ight) = k_{artheta_{ij}^{+}} \left( P_{ij} - P_{ij, ext{max}} 
ight)_{artheta_{ij}^{+}}^{+}$$

$$\dot{\epsilon}_{ij}^{+} = k_{\epsilon_{ij}^{+}} \left( \frac{\partial L}{\partial \epsilon_{i,i}^{+}} \right) = k_{\epsilon_{ij}^{+}} \left( Q_{ij} - Q_{ij,\max} \right)_{\epsilon_{i,i}^{+}}^{+}$$

$$\dot{\epsilon}_{ij}^{-} = k_{\epsilon_{ij}^{-}} \left( \frac{\partial L}{\partial \epsilon_{ij}^{-}} \right) = k_{\epsilon_{ij}^{-}} \left( Q_{ij, \min} - Q_{ij} \right)_{\epsilon_{i}^{-}}^{+}$$

$$\dot{\chi}_i^- = k_{\chi_i^-} \left( \frac{\partial L}{\partial \chi_i^-} \right) = k_{\chi_i^-} \left( Q_{i, \min}^G - Q_i^G \right)_{\chi_i^-}^+$$



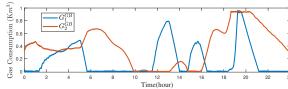


Fig. 1: Gas Consumption of  $H_i^{CHP}$  and  $H_i^{GB}$ .

(9e)

(8i) 
$$\dot{v}_{i}^{+} = k_{v_{i}^{+}} \left(\frac{\partial L}{\partial v_{i}^{+}}\right) = k_{v_{i}^{+}} \left(H_{i}^{CHP} - H_{i,\max}^{CHP}\right)_{v_{i}^{+}}^{+}$$
 (9n)

(8j) 
$$\dot{\phi}_{i}^{-} = k_{\phi_{i}^{-}} \left( \frac{\partial L}{\partial \phi_{i}^{-}} \right) = k_{\phi_{i}^{-}} \left( c_{i}^{m} H_{i}^{CHP} + P_{i,\min}^{CHP} - P_{i}^{CHP} \right)_{\phi_{i}^{-}}^{+}$$
(9o)

(8k) 
$$\dot{\phi}_{i}^{+} = k_{\phi_{i}^{+}} \left( \frac{\partial L}{\partial \phi_{i}^{+}} \right) = k_{\phi_{i}^{+}} \left( P_{i}^{CHP} - P_{i,\text{max}}^{CHP} + c_{i}^{v} H_{i}^{CHP} \right)_{\phi_{i}^{+}}^{+}$$
(8l) (9p)

## (8m)D. Parameters and Additional Results

TABLE I: Parameters of Part I

(9a)	Parameter	Value	Parameter	Value
(9b)	$ \begin{bmatrix} H_{i,\min}^{GB}, H_{i,\max}^{GB} \end{bmatrix} $ $ \eta_i^{GB} $	[0, 0.8]	$\begin{bmatrix} [H_{i,\min}^{EB}, H_{i,\max}^{EB}] \\ \eta_i^{EB} \end{bmatrix}$	[0, 0.8]
(9c)	$\begin{bmatrix} P_{i,\min}^{CHP}, P_{i,\max}^{CHP} \end{bmatrix} \\ \eta_i^{CHP}$	[0.1, 2.4]	$[H_{i,\min}^{CHP}, H_{i,\max}^{CHP}]$ $c \text{ kJ/(kg} \cdot ^{\circ}\text{C})$	[0, 2] 4.2
(60)	$c_i^m$	0.75	$c_i^v$	0.25

TABLE II: Parameters of Part II

Paramete	r   Value	Parameter	Value
$\overline{\rho}$	1000 kg/m <sup>3</sup>	g	$9.8m/s^2$
$A_i$	9.60E-05	$B_i$	3.38E-05
$C_i$	-1.09E-04	$F_{ji}$	$0.00010941 \ s^2m^{-5}$
$\omega_{i,\mathrm{min}}$	100 rpm	$\omega_{i,\max}$	1525 rpm
$T_a$	-10 °C	$\mid \eta_i \mid$	0.8075
$\nu$	0.33 W/(m⋅°C)	$\lambda_i$	1
$egin{aligned} \omega_i^H \ \lambda^{Gas} \end{aligned}$	1	$\omega_i^{HP}$	0.5
$\lambda^{Gas}$	0.02	$\lambda^{Grid}$	5

- Tables I-II list the parameters of Part I-II. Figure 1 demon-(9k) strates the gas consumption of the GBs and the CHPs at nodes 1 and 8, respectively. Figure 2 demonstrates the total
- (91) heat generation profiles of nodes 1 and 8. Figures 3 show the voltage profile of each bus in the microgrid. Figure 4 shows (9m)the outlet temperature of the nodes and pipes in the DHS.

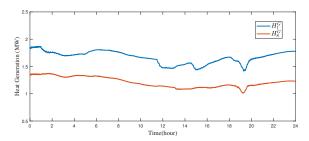


Fig. 2: Total heat generation profiles of nodes 1 and 8.

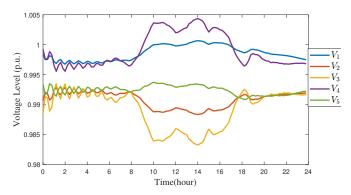


Fig. 3: Voltage Profile of  $V_i$ .

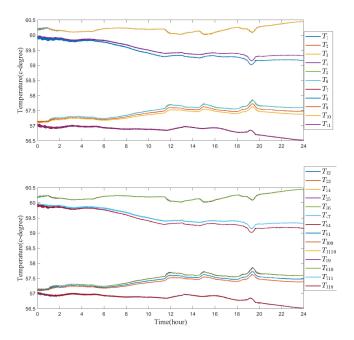


Fig. 4: Top: Temperature of  $T_i$ ; Bottom: Temperature of  $T_{ji}$ .

## REFERENCES

- P. Siano and G. Mokryani, "Probabilistic assessment of the impact of wind energy integration into distribution networks," *IEEE Transactions* on *Power Systems*, vol. 28, no. 4, pp. 4209–4217, Nov 2013.
   G. Mokryani, A. Majumdar, and B. C. Pal, "Probabilistic method for the
- [2] G. Mokryani, A. Majumdar, and B. C. Pal, "Probabilistic method for the operation of three-phase unbalanced active distribution networks," *IET Renewable Power Generation*, vol. 10, no. 7, pp. 944–954, 2016.