Preference Learning in School Choice Problems*

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Abstract

We study a centralized school choice problem in which students acquire information about cardinal preferences at a cost before submitting rank-order lists to a given mechanism. Information acquisition is flexible in that a student chooses how much and what kinds of information to acquire. The allocation by the Deferred-Acceptance (DA) mechanism is inefficient relative to the Boston mechanism, and the inefficiency increases in the information cost. We find new informational sources of DA's inefficiency: greater homogeneity in rank-order reports and less information acquisition, mutually reinforcing each other. A higher information cost intensifies this reinforcing cycle, and so exacerbates the inefficiency.

1 Introduction

The market design literature on public school choices has proliferated since Abdulkadiroğlu and Sönmez (2003). A standard approach adopted by that literature takes Gale and Shapley (1962)'s two-sided matching model and views students and schools as agents with preferences over potential match partners. In practice, a district often determines schools' preferences based on multiple factors, including students' test scores, proximity to schools, and lotteries, and collects students' preference reports exclusively to assign students to schools. The two most prominent mechanisms are the Deferred–Acceptance mechanism (DA) and the Boston mechanism (or Immediate-Acceptance). Both mechanisms run multiple rounds and match students to their top

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choices in each round unless the schools have reached their capacities and must then reject some students. The DA mechanism defers the matches in that a student who is rejected by her preferred school can go down her rank-order list to another school and may displace a previously accepted student by that school. The Boston mechanism treats the matches as permanent and begins each new round with unmatched students and unfilled schools. Hence, if a student applies for a competitive school first, she must take the risk of not matching with her next preferred school when rejected by her top choice. Most studies support the use of the DA because of its strategyproofness and stability.[1]

These studies assume that students have complete information about their (ordinal) preferences. In reality, students have only limited knowledge about how well a particular school suits them due to the lack of personal experiences. Families seek guidance from sources such as review and rating websites (GreatSchools.org, niche.com, or SchoolDigger.com), parent forums, and school open houses. During that time-consuming process, students and families decide how much information to obtain and which aspects of the schools to investigate to serve their needs best while economizing on the learning cost. These decisions are made based on school qualities and acceptance odds (e.g., not learning about a school if the admission chance is nil), which ultimately depend on the mechanism used and other students' learning and reporting strategies.

We study a centralized school choice problem in which students acquire information about their cardinal preferences at a cost. We compare the Boston and DA mechanisms.[2] A mechanism defines a game in which students decide learning and reporting strategies. We find that the allocation of students to schools under the DA mechanism is less efficient than under the Boston mechanism. While the inefficiency of DA is well-known with complete-information models (Miralles (2009), Abdulkadiroğlu et al. (2011)), we find that costly information acquisition magnifies the DA's inefficiency and offer some explanations for why this occurs. This result is not so obvious. A simple idea may argue that the value of information is higher in the Boston mechanism because students who list competitive schools at the top must take the risk of matching badly.[3] Without such risks, students in the DA mechanism acquire less information, which

[1] In school choice, stability means no justified envy. A student has justified envy if a school she prefers to her match is assigned to another student who has a lower priority than her in that school.

[2] We omit Top Trading Cycle (TTC) mechanism because it is equivalent to the DA in our setting. See fn. 7 for an explanation.

[3] Persico (2000) establishes that the value of information is higher in a more risk-sensitive problem. As an application, he finds that the value of information is higher for a bidder in the first-price auction than in the second-price auction. Risk sensitivity differs from the risk that we associate with the Boston mechanism. It measures how sharply an agent's payoff decreases as her choice moves away from an optimal one.

contributes to the DA's inefficiency. However, this risk associated with the Boston mechanism is ill-defined when students do not know about their preferences. Even if the risk is well-defined, it can discourage students from learning about competitive schools as they tend not to apply for such schools.

We consider the model of Rational Inattention (RI) information acquisition, pioneered by Sims (1998) and Sims (2003). A key feature of RI information acquisition is its flexibility. In the current context, this flexibility means that students and families can decide not only how much information to acquire but also what kinds of information to acquire. We follow the RI paradigm because different types of information are valuable under different mechanisms, and students and families must focus on the information that matters most to economize the learning cost.

This RI information acquisition model is embedded into a simple school choice problem, which consists of a unit mass of infinitesimal students and three schools a (average), b (bad), and s (superior for average students). Suppose all schools have capacity 1/3, and students' match payoffs are $u_a = 1$, $u_b = 0$, and $u_s = v + \theta$, respectively. While $v > 1/2$ is fixed across students, θ is an idiosyncratic preference shock distributed independently across students. A given mechanism, DA or Boston, defines a Bayesian game in which each student first acquires information about her preference shock and then submits a rank-order list. Since $\min\{u_a, u_s\} > u_b$, students submit either sab or asb.

If information acquisition is costless (equivalently when θ is observable to students), a majority of students report sab rather than asb in either mechanism, because a majority of them prefer school s over a ($Pr[v + \theta > 1] = v > 1/2$). If the DA mechanism is given, then by strategyproofness, a student reports sab if and only if $v + \theta > 1$.[4] On the other hand, submitting sab is risky under the Boston mechanism. This rank-order report makes the student more likely to be rejected by a top choice school, and a failure to match in the first round of the mechanism is likely to trigger another rejection in the second round, resulting in a match with the worst school b. Hence, a student reports sab only if her preference type θ is significantly higher than $1 - v$ (Lemma 2). The DA mechanism does not deter near-indifferent students from reporting sab, so the rank-order submissions tend to be homogeneous. The mechanism thus has to rely more on random tie-breaking in assigning students to schools, and a larger number of near-indifferent students match with school s that is in short supply. Consequently, the allocation by the DA mechanism is inefficient (Corollary 1).

If information acquisition is costly, a student's rank-order submission is only par-

[4]For now, we assume no indifference, $v + \theta \neq 1$.

tially dependent on the (unobservable) preference type. In either mechanism, a student is more likely to get a signal that recommends reporting *sab* when her (unobservable) preference type is higher. But, the optimal signal structure and cost-justifying accuracy of the signals vary across a given mechanism and other students' strategies (Lemma 3). We find equilibrium learning and reporting strategies in the Boston and DA mechanisms (Proposition 1 and Proposition 2) and show that the allocation under the DA mechanism continues to be less efficient (Proposition 3 and Corollary 2).

Some sources of the DA's inefficiency are similar to what we observed from the costless information case. First, students under the DA mechanism are more likely to report *sab* and rank selective school s at the top because, when rejected by school s, the risk of matching with school b is lower with the DA mechanism than of the Boston mechanism (Proposition 2). Due to the homogeneous rank-order submissions, the DA mechanism resorts to a higher degree of random tie-breaking to assign students to schools. Second, costly information also contributes to the DA's inefficiency. Since the DA mechanism is strategyproof in the case of costless information acquisition, a student wants to submit *sab* when her (unobservable) type θ is above $1 - v$. Intuitively, an optimal information acquisition focuses around $1-v$ and signals if a student's preference type is above or below $1 - v$. Students with (unobservable) preference types higher than but close to $1 - v$ are likely to submit *sab* and match with school s. Under the Boston mechanism, students with such types are likely to submit *asb* because optimal information acquisition under that mechanism focuses around a cutoff point higher than $1 - v$ (Proposition 4).

An additional source of the DA's inefficiency is that greater use of tie-breaking discourages costly information acquisition. Students' rank-order reports are then more driven by ex-ante match utilities ($E[u_s] = v + \frac{1}{2} > u_a = 1$) and become even more homogeneous (Proposition 6). This interaction between homogeneous rank-order reports and less information acquisition is mutually reinforcing, becoming an intensifying cycle that increases the DA's inefficiency. This inefficiency is further exacerbated when the cost of information acquisition increases. Therefore, the DA mechanism becomes increasingly less efficient compared to the Boston as the marginal cost of information increases (Proposition 7).

1.1 Related Literature

Most studies in the market design literature take agents' preferences as given, with a few exceptions. The paper that is most related to our own may be Chen and He (2017), which similarly explores information acquisition in school choice and compares mechanisms. While Chen and He (2017) allows general preferences, they assume a restrictive form of learning. Students choose to learn ordinal preferences and then cardinal utilities. We take the opposite modeling approach: we allow information acquisition to be fully flexible at the cost of assuming a simple matching setup. This flexible learning framework offers comparative statics results and a clear intuition of how information costs result in the DA mechanism's increasing inefficiency.

A few other studies consider preference learning in matching. Immorlica et al. (2018) assumes that each student selects a school to investigate, one after another. They define stability over an outcome that consists of matching and agents' beliefs and study how to implement a stable outcome. Bade (2015) considers heterogeneous learning costs among students. Serial dictatorship lets agents with higher information costs choose early, incentivizing more information acquisition and improving efficiency. Harless and Manjunath (2018) considers a student's choice of which school to learn about, and Kloosterman and Troyan (2018) considers each student's learning about others' preferences.

The DA mechanism's inefficiency relative to the Boston mechanism is well-known with complete information models. Abdulkadiroğlu et al. (2011) show that DA is particularly inefficient when preferences are highly correlated, in which case the Boston mechanism screens students with strong preferences to allocate popular school seats. See also Miralles (2009), Pathak and Sönmez (2008), and Featherstone and Niederle (2011).[5] We find that such inefficiency of the DA is exacerbated if students need to learn about their preferences.

We study a mechanism design problem with RI agents, emphasizing how the mechanism being used affects the qualitative and quantitative natures of the information acquired by agents and vice versa. Other papers sharing this scheme include, but are not limited to: Yang (2019) and Li and Yang (forthcoming) on contract designs with an RI agent; and Bloedel and Segal (2018) on information design with an RI agent. We use mutual information to measure the cost of information acquisition. Shannon (1948), Cover and Thomas (2012) and Caplin and Dean (2015) provide information-theoretic, coding-theoretic, and revealed-preference foundations for this modeling choice, respectively. The growing literature on RI is recently surveyed by Caplin (2016) and Mackowiak et al. (2018).

[5] The inefficiency of the DA may not hold in some environments. Calsamiglia and Miralles (2020) considers a setting in which students are highly prioritized by their neighborhood schools. For example, consider a district with three neighborhoods, two with good but differentiated schools and one with a bad school. The risk of matching with the bad school may pressure students in good neighborhoods to pursue their neighborhood schools, leaving out welfare improving exchanges among the students that match the good and differentiated schools.

2 Model

A unit mass of students must be assigned to three schools s (superior), a (average), and b (bad). Each school j has a capacity $\lambda_i > 0$ and $\sum_{i=s,a,b} \lambda_i = 1$. Student preferences are represented by cardinal utilities $u_s = v + \theta$, $u_a = 1$, and $u_b = 0$. While $v \in (0,1)$ is constant, θ is a random variable that is uniformly distributed on $[0,1]$. Nature draws θ independently for each student, whose realization captures the student's idiosyncratic preference between schools s and a. Since a student with $\theta > 1 - v$ prefers school s to school a, v represents the fraction of students holding such a preference. School b is always the least preferred school, regardless of the realizations of preference shocks.

A matching mechanism solicits a rank-order list from each student that is either sab or asb. School b is always at the bottom of the rank-order list because it is always the least preferred. Given the populations of students who report sab and asb, the mechanism assigns students to schools such that no school exceeds its capacity. We will focus on two well-known mechanisms, the Boston and Deferred-Acceptance mechanisms.

Students do not observe their preference shocks but can acquire information about them at a cost before participating in the matching mechanism. We abstract away from the disclosure of information by schools (e.g., printing brochures, organizing internet forums and open houses), focusing instead on the aggregation of information into application decisions that consumes a great deal of energy and time in reality.

The information acquired by a student is captured by a signal structure $\Pi : [0,1] \to \Delta\mathcal{Z}$, where each $\Pi(\cdot \mid \theta)$, $\theta \in [0,1]$, specifies a probability distribution over a finite set $\mathcal{Z}$ of signal realizations conditional on her true preference being θ. For each signal realization $z \in \mathcal{Z}$, the student submits a rank-order list, based on which her matching outcome is determined. The cost of information acquisition is $\mu \cdot I(\Pi)$, where $\mu > 0$ is a marginal information cost, and $I(\Pi)$ is the mutual information between the preference shock and the signals generated by Π. That is,

$$I(\Pi) = H(\theta) - \mathbb{E}_{\Pi}[H(\theta \mid z)] \tag{1}$$

where $H(\cdot)$ denotes the Shannon entropy of a random variable.[6]

[6] See (2) for an expression of $I(\Pi)$ when the signals are binary.

2.1 Matching mechanisms

We consider two prominent mechanisms: Boston and Deferred-Acceptance (DA). For both mechanisms, we assume single tie-breaking, whereby student priorities are independently drawn from the uniform distribution $U[0,1]$ at the outset and are later used by all schools. A student with a higher priority realization is ranked higher. Students do not observe their priorities when submitting rank-order lists, and priorities are independent of student reports. Let $r \in [0,1]$ denote the population of students who submits *sab*.

Boston mechanism Students are assigned to schools in multiple rounds. In each round, the mechanism assigns unmatched students to their top choice schools. Schools keep accepting students based on their priorities until their capacities are reached.

To illustrate the mechanism, suppose a large fraction r of students report *sab* so that $r \geq \lambda_s$ and $1-r \leq \lambda_a$. In the first round, school s receives more applications than its capacity ($r \geq \lambda_s$). It accepts λ_s students with the highest priorities out of the r students who submitted *sab*. The remaining $r - \lambda_s$ students are rejected. Meanwhile, all students who submitted *asb* match with school a. By the end of this round, school s has reached its capacity, whereas school a still has opening $\lambda_a - (1-r)$. Students who submitted *sab* but were rejected by school s apply to school a in the second round. Only $\lambda_a - (1-r)$ with the highest priorities are accepted because school a has reached its capacity. The remaining students match with b in the third round. The final allocation of students to schools is reported in Table 1, which contains other cases whose derivations are omitted for brevity.

	s	a	b
sab	λ_s	$\lambda_a - (1-r)$	λ_b
asb	0	$1-r$	0

(a) if $r \geq 1 - \lambda_a$

	s	a	b
sab	λ_s	0	$r - \lambda_s$
asb	0	λ_a	$(1-r) - \lambda_a$

(b) if $\lambda_s \leq r \leq 1 - \lambda_a$

	s	a	b
sab	r	0	0
asb	$\lambda_s - r$	λ_a	λ_b

(c) if $r \leq \lambda_s$

Table 1: Boston mechanism. Each panel tabulates the populations of the students who submit the row rank-order lists and are matched with the column schools.

Deferred-Acceptance (DA) mechanism Similar to the Boston mechanism, the DA mechanism assigns students to schools in multiple rounds. However, the student assignment to schools is deferred. In each round, the mechanism assigns all students to their top choices among the schools that have not rejected them, up to the schools' full capacities. There exists an alternative characterization of the mechanism for the current setting (Azevedo and Leshno, 2016). The idea is to find market-clearing priority cutoffs $p_s, p_a, p_b \in [0,1]$. A student is assigned to a top-choice school among the schools that have cutoffs lower than the student's priority ranking. Unique market-clearing cutoffs exist so that the assignment of students to schools is feasible given school capacities.

To illustrate how to find market-clearing cutoffs, suppose a large fraction r of students report *sab* so that $r \geq \frac{\lambda_s}{\lambda_s+\lambda_a}$. Since school s receives more applications than school a, and every student is guaranteed to match at least with school b, we consider cutoffs such that $p_s > p_a > p_b = 0$. Among students who report *sab*, those with priorities higher than p_s match with school s. Other students with priority rankings either between p_a and p_s or below p_a match with school a or b, respectively. On the other hand, students who report *asb* match with either school a or b depending on their priorities to be above or below the cutoff p_a. The following table tabulates the cutoff-based assignment.

	s	a	b
sab	$(1-p_s)r$	$(p_s-p_a)r$	$p_a r$
asb	0	$(1-p_a)(1-r)$	$p_a(1-r)$

The market-clearing cutoffs satisfy $(1-p_s)r = \lambda_s$ and $(p_s-p_a)r+(1-p_a)(1-r) = \lambda_a$. The student assignment to schools by market-clearing cutoffs is given in Panel (a) of Table 2, which also contains the other case whose derivation is omitted for brevity.[7]

[7] The Top-Trading-Cycles (TTC) mechanism is equivalent to the DA mechanism in our setting. The TTC mechanism first distributes students to schools randomly so that the populations of the students according to their rank-order lists and the endowments are

	s	a	b
sab	$r\lambda_s$	$r\lambda_a$	$r\lambda_b$
asb	$(1-r)\lambda_s$	$(1-r)\lambda_a$	$(1-r)\lambda_b$

Then, the students who report *sab* but are endowed with school a trade with others who report *asb* but are endowed with school s. Students that are endowed with school b have no trading opportunities. The resulting population distribution is the same as in Table 2. See Leshno and Lo (2021) for a characterization of the TTC mechanism in a general environment with a continuum of students.

	s	a	b
sab	λ_s	$r\lambda_a - (1-r)\lambda_s$	$\lambda_b r$
asb	0	$(1-r)(\lambda_s+\lambda_a)$	$\lambda_b(1-r)$

(a) if $r \geq \frac{\lambda_s}{\lambda_s+\lambda_a}$

	s	a	b
sab	$r(\lambda_s+\lambda_a)$	0	$r\lambda_b$
asb	$(1-r)\lambda_s - r\lambda_a$	λ_a	$(1-r)\lambda_b$

(b) if $r < \frac{\lambda_s}{\lambda_s+\lambda_a}$

Table 2: DA mechanism. Each panel tabulates the populations of the students who submit the row rank-order lists and are matched with column schools.

2.2 Information Acquisition

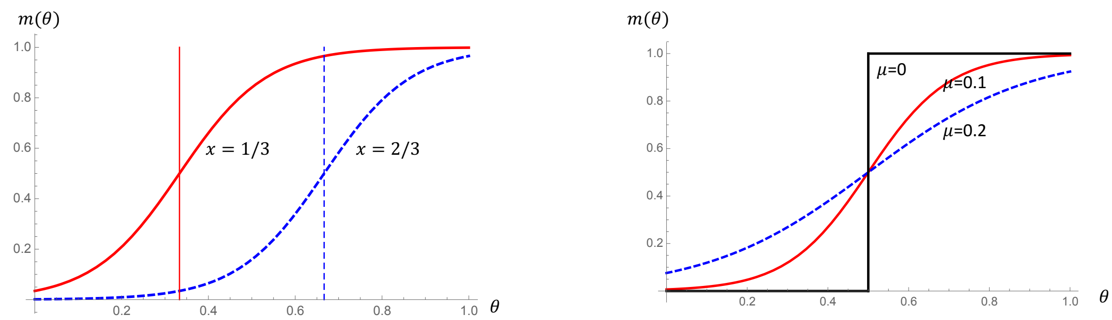
A student acquires any finite signal about preference shocks at a cost proportional to Shannon entropy reduction (Equation 1). The focus on finite signals isn't restrictive. According to Matějka and McKay (2015), it is without loss of generality to consider signal structures that recommend students to submit either *sab* or *asb*, and students strictly follow this recommendation. Since any signal realization induces a student to report either *sab* or *asb*, we can merge those realizations that induce *sab* into a single recommendation to submit *sab* and those that induce *asb* into another recommendation to submit *asb*. The resulting signal structure generates the same matching payoffs as the old one but is cheaper to acquire as it is less Blackwell informative. For the same reason, students must strictly prefer to follow the recommendations they receive. If a student is indifferent between the two recommendations, then merging them doesn't affect the consumption value of the signal but saves the information acquisition cost.

Hereafter, we shall represent the signal acquired by student i by an integrable function $m_i : [0,1] \to [0,1]$, where each $m_i(\theta)$ specifies the probability that the student is recommended to submit *sab* conditional on her true preference type being $\theta \in [0,1]$. Define $\overline{m}_i \equiv \int_0^1 m_i(\theta) d\theta$ as the average probability that the student is recommended to submit *sab*. The cost of acquiring the signal m_i is then $\mu \cdot I(m_i)$, where $\mu \geq 0$ and

$$\begin{aligned} I(m_i) = \int_0^1 & [m_i(\theta)\ln m_i(\theta) + (1-m_i(\theta))\ln(1-m_i(\theta))]d\theta \\ & - \overline{m}_i \ln \overline{m}_i - (1-\overline{m}_i)\ln(1-\overline{m}_i). \end{aligned} \tag{2}$$

To get an intuition of an optimal information-acquisition strategy, consider a binary-choice between $\{0,1\}$. An agent's payoff from each choice is either x or θ. While $x \in (0,1)$ is constant, θ is a random variable that is uniformly distributed on $[0,1]$. If the agent were fully informed, she would have chosen 1 if and only if her preference type θ is above x. The agent, however, needs to acquire information about θ. A signaling structure is represented by $m : [0,1] \to [0,1]$ such that the agent is recommended to choose 1 with probability $m(\theta)$ if her true preference type is θ.

Figure 1 illustrates an optimal information-acquisition strategy, either for various option values x (Panel (a)) or various marginal costs μ (Panel (b)). An optimal signal



(a) A choice of what to learn ($\mu = 0.1$). (b) A choice of how much to learn ($x = 1/2$).

Figure 1: An optimal flexible information acquisition when $\Delta(\theta) = \theta - x$.

structure $m(\theta)$ increases in θ because the payoff $\Delta(\theta)$ increases in θ. An agent wants to learn if her preference type θ is above or below a given option value x. Hence, the recommendation by $m(\theta)$ becomes dependent on θ being above or below x, as shown in Panel (a). Moreover, if the marginal cost of information μ decreases, the agent chooses to acquire more information, so the recommendation by $m(\theta)$ becomes dependent more precisely on θ, as shown in Panel (b).

2.3 The Students' Problem

A given mechanism $\Gamma \in \{B(oston), D(A)\}$ defines a game such that a student i's strategy is $m_i : [0,1] \to [0,1]$. We focus on symmetric equilibrium m^Γ. For any given fraction r of students that report sab, $U^\Gamma_{sab}(\theta; r)$ and $U^\Gamma_{asb}(\theta; r)$ denote the expected match utilities of a preference type θ who submits sab and asb, respectively. Let $\Delta^\Gamma(\theta; r) \equiv U^\Gamma_{sab}(\theta; r) - U^\Gamma_{asb}(\theta; r)$. Then, student i solves

$$\max_{m_i:[0,1]\to[0,1]} \int_0^1 m_i(\theta)\Delta^\Gamma(\theta; r^\Gamma)d\theta - \mu I(m_i), \tag{3}$$

taking other players' symmetric strategy m^Γ and $r^\Gamma \equiv \int m^\Gamma(\theta)d\theta$ as given. If a solution to this problem coincides with m^Γ almost everywhere, then m^Γ is a symmetric equilibrium.

3 Equilibrium Analysis

Throughout the analysis, we without loss of generality focus on parameter values such that school s is more selective than school a. In particular, for each μ, we assume that v is relatively high so that the equilibrium fraction of students that submit sab is larger than $\hat{r} \equiv \frac{\lambda_s}{\lambda_s+\lambda_a}$ in any given mechanism. We omit an analogous equilibrium analysis for the other parameter values.[8] The condition $r > \hat{r}$ holds if and only if $\frac{r}{\lambda_s} > \frac{1-r}{\lambda_a}$. If the given mechanism is Boston, school s reaches the capacity earlier than or in the same round as school a. If the given mechanism is DA, the priority cutoff for school s is (weakly) higher than the cutoff for school a.

3.1 Preliminary Result

We begin by computing the expected gain in match utilities $\Delta^\Gamma(\theta; r)$ for any mechanism $\Gamma \in \{B, D\}$, a fraction $r > \hat{r}$ of students that report sab, and an (unobservable) preference type θ.

First, consider the Boston mechanism. We distinguish between two cases: $r \geq 1-\lambda_a$ (Panel (a) of Table 1) and $\hat{r} < r < 1-\lambda_a$ (Panel (b) of Table 1). Suppose that $r \geq 1-\lambda_a$. If a student reports sab, she is assigned to schools s, a, and b, with probabilities $\frac{\lambda_s}{r}$, $\frac{\lambda_a-(1-r)}{r}$, and $\frac{\lambda_b}{r}$, respectively. If she reports asb, the match must be with school a. We repeat the exercise for the case of $\hat{r} < r < 1-\lambda_a$. Then,

$$\Delta^B(\theta; r) = \frac{\lambda_s(v+\theta)}{r} - \min\left\{\frac{\lambda_a}{1-r}, \frac{1-\lambda_a}{r}\right\}, \quad \forall r > \hat{r}. \tag{4}$$

Next, consider the DA mechanism (Panel (a) of Table 2). If student reports sab, she is assigned to schools s, a, and b, with probability $\frac{\lambda_s}{r}$, $\lambda_a - \frac{(1-r)\lambda_s}{r}$, and λ_b, respectively. If she reports asb instead, the probabilities become 0, $\lambda_s + \lambda_a$, and λ_b, respectively. Hence,

$$\begin{aligned}\Delta^D(\theta; r) &= \left(\frac{\lambda_s}{r}(v+\theta) + \lambda_a - \frac{1-r}{r}\lambda_s\right) - (\lambda_s + \lambda_a) \\ &= \frac{\lambda_s(v+\theta)}{r} - \frac{\lambda_s}{r} \qquad \forall r > \hat{r}. \end{aligned} \tag{5}$$

Lemma 1. *For any $r > \hat{r}$ and $\theta \in [0, 1]$, $\Delta^D(\theta; r) > \Delta^B(\theta; r)$.*

[8] To be more precise, consider an alternative normalization of the match utilities such that $\tilde{u}_b \equiv 0$, $\tilde{u}_s \equiv 1$, and, for each preference type θ, $\tilde{u}_a(\theta) \equiv u_a + (\tilde{u}_s - u_s(\theta)) = (1-v) + (1-\theta)$. With a change of variables, $\tilde{u}_a(\tilde{\theta}) = \tilde{v} + \tilde{\theta}$, where $\tilde{v} \equiv 1 - v$ and $\tilde{\theta} \equiv 1 - \theta \sim U[0, 1]$. Thus, the equilibrium analysis for the omitted cases – school a is more selective than school s due to high $\tilde{v}$ – follows immediately from the analysis that we present in this paper.

Proof. $\Delta^D - \Delta^B = \min\left\{\frac{\lambda_a}{1-r}, \frac{1-\lambda_a}{r}\right\} - \frac{\lambda_s}{r}$. Clearly, $\frac{1-\lambda_a}{r} = \frac{\lambda_b+\lambda_s}{r} > \frac{\lambda_s}{r}$. Moreover, $r > \hat{r}$ implies $\frac{\lambda_a}{1-r} > \frac{\lambda_s}{r}$. □

The intuition behind Lemma 1 is simple. When a large fraction of students apply for school s first ($r > \hat{r}$), submitting sab to the Boston mechanism is risky. It makes a student more likely to be rejected by a top choice school, and a failure to match with the school in the first round of the mechanism is likely to trigger another rejection in the second round, ultimately resulting in a final match with the worst school b. Such risk does not exist under the DA mechanism, under which schools either accept or reject students by the same priority cutoffs regardless of the students' rank-order reports. A student takes the same risk of matching with school b no matter which rank-order list is submitted.

3.2 Equilibrium with Costless Information Acquisition

We first analyze the equilibrium under costless information acquisition, $\mu = 0$. This case corresponds to the standard matching models in which a student knows her preferences and the distribution of others'. For any given mechanism, any symmetric equilibrium strategy must be a shifted unit step function:

$$m^\Gamma(\theta) = \begin{cases} 1 & \text{if } \theta > \theta^\Gamma, \\ 0 & \text{if } \theta < \theta^\Gamma. \end{cases}$$

The threshold θ^Γ depends on the given mechanism.[9]

It is well-known that the DA mechanism is strategyproof (when each student knows her preference type). It is a weakly dominant strategy for a student to report sab if and only if $\theta > 1 - v$. Thus, $\theta^D = 1 - v$ and the resulting fraction of students who report sab is $r^\Gamma = 1 - \theta^\Gamma = v$, which we assume to be larger than $\hat{r}$.

If the Boston mechanism is given, a student with the threshold preference type θ^B must be indifferent between sab and asb. The equilibrium cutoff θ^B satisfies $\Delta^B(\theta^B; r^B) = 0$, where $r^B = 1 - \theta^B$. It follows from (4) that

$$\begin{aligned} \text{either} \quad & \theta^B \leq \lambda_a \quad \text{and} \quad \lambda_s(v + \theta^B) = 1 - \lambda_a, \\ \text{or} \quad & \lambda_a \leq \theta^B \leq 1 - \lambda_s \quad \text{and} \quad \frac{\lambda_s(v + \theta^B)}{1 - \theta^B} = \frac{\lambda_a}{\theta^B}. \end{aligned} \tag{6}$$

Lemma 2. *If $v > \hat{r}$, then $\theta^B > \theta^D$.*

[9] We omit $m^\Gamma(\theta^\Gamma)$. There exists a zero measure of students with the threshold type θ^Γ and they are indifferent between sab and asb. Any tie-breaking $m^\Gamma(\theta^\Gamma) \in [0, 1]$ is consistent with our equilibrium analysis.

Lemma 2 shows that the Boston mechanism deters students that are near-indifferent between schools a and b from reporting sab. It would be better for near-indifferent students with $\theta \in (\theta^D, \theta^B)$ to submit asb instead and avoid the risk of getting multiple rejections and matching with school b. The DA mechanism does not discourage the near-indifferent students from submitting sab. As the rank-order submissions are more homogeneous ($r^D > r^B > \hat{r}$), the DA mechanism has to rely on random tie-breaking to a higher degree than the Boston mechanism. This yields allocation inefficiency.

For a formal result of allocation inefficiency, note that $u_a = 1$ and $u_b = 0$ for all students. Accordingly, the allocation efficiency depends only on the type distribution of students that match with school s. Let g and g' be two allocations of school s, defined as density functions such that $\int_0^1 g(\theta)d\theta = \int_0^1 g'(\theta)d\theta = \lambda_s$. We say that the allocation g is *more efficient than* g' if g first-order stochastically dominates g': $\int_0^{\overline{\theta}} g \, d\theta \leq \int_0^{\overline{\theta}} g' \, d\theta$, for every $\overline{\theta} \in (0,1)$, with some inequalities strict.

An equilibrium allocation of a mechanism $\Gamma \in \{B, D\}$ is

$$g^\Gamma(\theta) \equiv \begin{cases} \frac{\lambda_s}{1-\theta^\Gamma} & \text{if } \theta > \theta^\Gamma, \\ 0 & \text{if } \theta < \theta^\Gamma. \end{cases}$$

Lemma 2 ($\theta^B > \theta^D$) implies,

Corollary 1. *With costless information acquisition, the DA equilibrium allocation is less efficient than the Boston equilibrium allocation.*

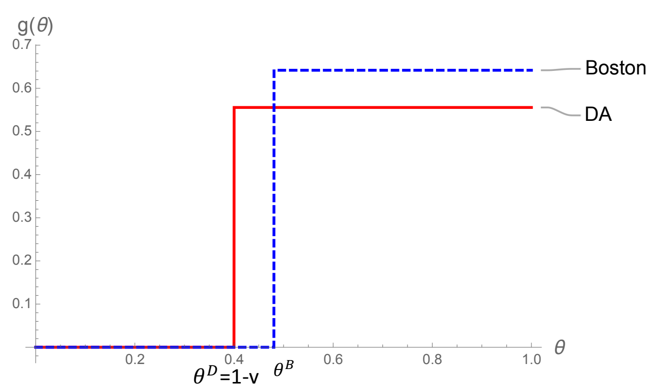


Figure 2: DA's inefficiency when $\mu = 0$, $v = 0.6$, and $\lambda_j = 1/3$ for $j \in \{s, a, b\}$. Each graph represents the probability that a student with preference type θ matches with school s.

3.3 Equilibrium with Costly Information Acquisition

Suppose that information acquisition is costly $\mu > 0$. We focus on parameter values – especially the value of v for any fixed μ – such that each mechanism $\Gamma \in \{B, D\}$ has an interior symmetric equilibrium such that $\hat{r} < r^\Gamma < 1$. We focus on interior equilibria because they are particularly interesting analytically. If v is too close to 1, then students in the DA mechanism would not acquire costly information, and the equilibrium allocation becomes independent of student (unobservable) preference types.

In an interior equilibrium of any given mechanism Γ, the problem faced by a student $i \in [0,1]$ (Equation 3) admits a solution $m_i : [0,1] \to [0,1]$ such that $\overline{m}_i \equiv \int m_i d\theta \in (0,1)$. An interior solution m_i exists if and only if the following first-order condition holds (Yang, 2015):

$$\Delta^\Gamma(\theta; r^\Gamma) = \mu \cdot \left[\ln\left(\frac{m_i(\theta)}{1 - m_i(\theta)}\right) - \ln\left(\frac{\overline{m}_i}{1 - \overline{m}_i}\right)\right], \quad \forall \theta \in [0,1]. \tag{7}$$

In a (symmetric) equilibrium, $\overline{m}_i = r^\Gamma$. By substituting $\overline{m}_i$ with r^Γ in (7), we obtain

$$m^\Gamma(\theta) = \left(1 + \frac{1 - r^\Gamma}{r^\Gamma} \exp\left(-\frac{\Delta^\Gamma(\theta; r^\Gamma)}{\mu}\right)\right)^{-1}, \quad \forall \theta \in [0,1], \tag{8}$$

and

$$r^\Gamma = \int_0^1 \left(1 + \frac{1 - r^\Gamma}{r^\Gamma} \exp\left(-\frac{\Delta^\Gamma(\theta; r^\Gamma)}{\mu}\right)\right)^{-1} d\theta. \tag{9}$$

Hence, we obtain an equilibrium condition for the fraction r^Γ.

Lemma 3. *A mechanism Γ has an interior equilibrium with a fraction $r^\Gamma \in (\hat{r}, 1)$ if and only if there exists a solution $r \in (\hat{r}, 1)$ of*

$$\exp\left(\frac{\lambda_s}{\mu}\right) = 1 + \frac{\exp\left(\frac{\lambda_s}{r\mu}\right) - 1}{\frac{1-r}{r}\exp\left(-\frac{\Delta^\Gamma(0;r)}{\mu}\right) + 1}. \tag{10}$$

A solution r^Γ of (10) *uniquely identifies an equilibrium strategy by* (8).

First, consider the DA mechanism. A careful inspection of (10), where $\Delta^D(0,r) = \frac{\lambda_s(v-1)}{r}$, reveals:

Proposition 1. *For any $\mu > 0$, there exist $\underline{v}, \overline{v} \in (0,1)$ with $\underline{v} < \overline{v}$ such that $v \in (\underline{v}, \overline{v})$ if and only if an interior equilibrium of the DA mechanism (uniquely) exists with $r^D \in (\hat{r}, 1)$.*

Figure 3 contains an intuition of the proof. The left-hand side (LHS) of (10) is constant at $\exp(\frac{\lambda_s}{\mu})$, and the right-hand side (RHS) under the DA mechanism varies in r (see a solid curve in Figure 3). The RHS (i) equals the LHS at $r = 1$ and (ii) increases indefinitely as r approaches 0. The RHS is shown to be a strictly single-dipped function in Appendix.[10] The intermediate value theorem implies that a unique solution of (10) exists in $(0, 1]$.

To ensure the solution to be in $(\hat{r}, 1)$, note that the RHS is strictly increasing in v for each fixed $r < 1$. The lower bound $\underline{v}$ is the value of v such that the RHS at $r = \hat{r}$ equals LHS (see the bottom dashed curve in Figure 3). We also observe that the derivative of the RHS at $r = 1$ is increasing in v. The upper bound $\overline{v}$ is attained when the derivative at $r = 1$ equals 0 (see the top dashed curve in Figure 3).[11]

The above exercise is intuitive. If v is very close to 1, then the (unobservable) match payoff $u_s = v + \theta$ is very likely to be higher than $u_a = 1$. A student submits a rank-order list *sab* without acquiring costly information. If v is sufficiently low, then school s is no longer more selective than school a in the DA mechanism.

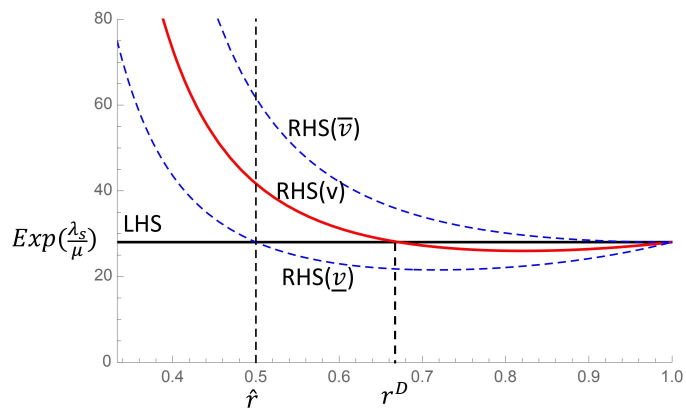


Figure 3: An intuition of the proof of Proposition 1.

We next consider the Boston mechanism. An interior equilibrium can be found by solving for a fraction r^B that satisfies (10), where $\Delta^B(0, r) = \frac{\lambda_s v}{r} - \min\left\{\frac{\lambda_a}{1-r}, \frac{1-\lambda_a}{r}\right\}$. Note that $\Delta^D(0, r) > \Delta^B(0, r)$ for every $r > \hat{r}$ (Lemma 1). Therefore, compared to

[10] A single-dipped function is the -1 multiple of a single-peaked function.

[11] In Appendix, we show that $\lim_{\mu\to 0} \underline{v} = \hat{r}$, $\lim_{\mu\to 0} \overline{v} = 1$, and $\lim_{\mu\to\infty} \underline{v} = \lim_{\mu\to\infty} \overline{v} = \frac{1}{2}$. If information acquisition is costless, $v \in (\hat{v}, 1)$ iff an equilibrium exists with $r^D = 1 - \theta^D = v \in (\hat{v}, 1)$. If information is very costly, and therefore students choose to acquire no information, then an interior equilibrium cannot exist unless $v = \frac{1}{2}$, and so $\mathbb{E}[u_s] = 1 = u_a$.

the RHS of (10) under the DA mechanism, the RHS under the Boston mechanism is smaller on every $r \in (\hat{r}, 1)$, and the two coincide at $r = \hat{r}$ and $r = 1$, as shown in Figure 4. Hence,

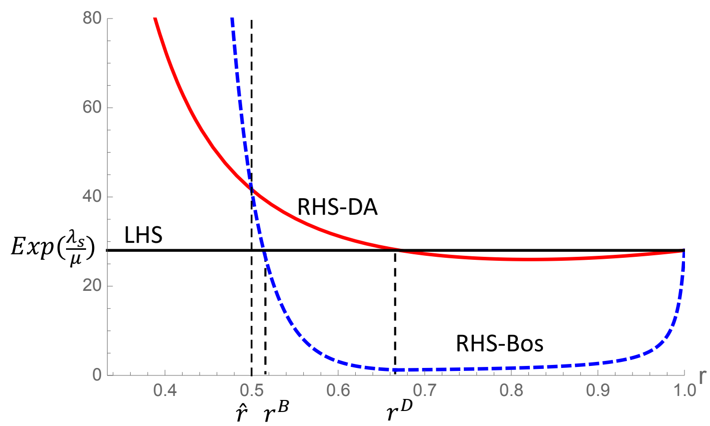


Figure 4: The equilibrium fraction of students that report *sab* under the DA or Boston mechanism.

Proposition 2. *If the DA mechanism has an interior equilibrium with $r^D \in (\hat{r}, 1)$, then the Boston mechanism has a (unique) interior equilibrium with $r^B \in (\hat{r}, r^D)$.*

The uniqueness of equilibrium under the Boston mechanism requires more proof that is relegated to the Appendix.

3.4 Allocation Inefficiency of the DA mechanism

We show that the DA mechanism continues to yield an inefficient allocation and find two sources of inefficiency. First, the rank-order submissions are more homogeneous under the DA mechanism than the Boston mechanism. Thus, the DA mechanism has to rely on random tie-breaking to a higher degree. This source of inefficiency is similar to what we have observed in the case of costless information acquisition. Second, the DA mechanism's increased reliance on tie-breaking discourages costly information acquisition. Students' rank-order reports become less dependent on their (unobserved) preference types and even more homogeneous. The consequences are an even higher-degree reliance on tie-breaking and further allocation inefficiencies. This intensifying loop of mutually reinforcing homogenous rank-order reports and less information acquisition is a newly found source of the DA's inefficiency.

As $u_a = 1$ and $u_b = 0$ are independent of preference type, we discuss an allocation efficiency by the type distribution of students that match with school s. Let g^Γ denote the density function of the equilibrium type distribution for each mechanism $\Gamma \in \{D, B\}$. That is, $g^\Gamma(\theta)$ is the probability that a student with (unobserved) preference type θ matches with school s in equilibrium. Note that $\int g^\Gamma(\theta)d\theta = \lambda_s$. We are focusing on equilibria in which school s is more selective than school a (r^Γ). Thus, students may match with school s only if they report sab to a given mechanism Γ. As such, $g^\Gamma(\theta) = m^\Gamma(\theta)\frac{\lambda_s}{r^\Gamma}$. We compare the equilibrium allocations g^B and g^D.[12]

Proposition 3. *Take any parameter values such that an equilibrium of the DA mechanism with $r^D \in (\hat{r}, 1)$ exists (so, an equilibrium of the Boston mechanism with $r^B \in (\hat{r}, r^D)$ also exists). Then, g^B is single crossing g^D from below.*[13]

Corollary 2. *The equilibrium allocation in DA is less efficient than in Boston.*

$$\int_0^{\overline{\theta}} g^D(\theta)d\theta > \int_0^{\overline{\theta}} g^B(\theta)d\theta, \quad \forall \overline{\theta} \in (0, 1).$$

Figure 5 illustrates the DA mechanism's inefficiency. The black horizontal line represents a constant function at λ_s, or a purely random allocation of school s. The blue dashed line is for the Boston mechanism equilibrium allocation g^B, and the red solid line is for the DA mechanism allocation g^D. The graph of g^B is single-crossing g^D from below, which shows the DA mechanism's inefficiency.

More homogeneous rank-order reports partly drive the inefficiency of the DA mechanism. Students that submit a rank-order list sab to the DA mechanism need not take a higher risk of matching with school b. These students have higher incentives to report sab and apply for a more selective school first, regardless of their (unobservable) preference types: $\forall\theta$ and $r > \hat{r}$, $\Delta^D(\theta; r) > \Delta^B(\theta; r)$. As a result, the mechanism has to rely on tie-breaking to a higher degree. This source of inefficiency – homogeneous rank-order submissions – is already observed with the case of costless information acquisition.

The costless information acquisition setup showed us that near-indifferent students match with school s more in the DA mechanism than the Boston mechanism, which results in the DA's inefficiency. Similarly, in the costly information acquisition setup, students with (unobservable) preference types only slightly above $1 - v$ tend to match

[12] It is without loss of generality to focus on the parameter values such that school s is more selective than school a ($r^\Gamma > \hat{r}$). For other parameter values, we can consider an alternative normalization of match utilities (see fn. 8) and still show the DA mechanism's inefficiency.

[13] That means, $\forall\theta' < \theta''$, $g^B(\theta') \geq g^D(\theta') \implies g^B(\theta'') > g^B(\theta'')$.

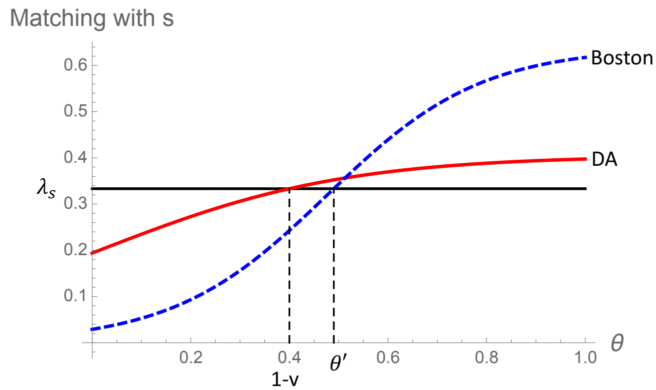


Figure 5: The inefficiency of the DA. Each graph shows the probability that a student with (unobservable) type θ matches with school s in equilibrium ($v = 0.6$, and $\lambda_j = 1/3$ for $j \in \{s, a, b\}$).

with school s more in the DA mechanism. Their acquired information under the DA mechanism focuses on whether preference types are above or below the cutoff $1 - v$. The relevant cutoff under the Boston mechanism is higher than $1 - v$ (it is θ' such that $\Delta^B(\theta', r^B) = 0$). Hence, students with near-indifferent preference types are likely to learn that their preference types are not significantly higher than $1 - v$. Such students are recommended to report asb and tend not to take seats in school s.

Formally, we use the derivative of an information acquisition strategy at θ, i.e., $\frac{dm(\theta)}{d\theta}$, to measure how much information is acquired to distinguish the preference types locally around θ.

Proposition 4. *Suppose that the DA and the Boston mechanisms have equilibrium with fractions r^D and r^B in $(\hat{r}, 1)$. Then, $\frac{dm^B(\theta)}{d\theta}$ is single crossing $\frac{dm^D(\theta)}{d\theta}$ from below.*

Another source of the DA mechanism's inefficiency is reinforcing homogeneous rank-order submissions and less information acquisition. The DA's higher reliance on tie-breaking discourages costly information acquisition. As the students' rank-order lists become less dependent on preference types, they become more homogeneous, and the mechanism has to rely on tie-breaking even more. To illustrate this reinforcement, we consider a hypothetical change of the mechanism from Boston to DA while holding the equilibrium fraction $r^B \in (\hat{r}, 1)$ fixed. The tatonnement best-response dynamics under the DA mechanism are illuminating.

Take any parameter values such that the DA and Boston mechanisms have equilibria with fractions in $(\hat{r}, 1)$. For any $r \in [r^B, r^D]$, let $m^H(\cdot; r, \mu)$ be students' interior best-response and denote $\overline{m}^H(\cdot; r, \mu) \equiv \int m^H(\theta; r, \mu) d\theta$. We measure the efficiency of the resulting allocation by $g^H(\theta; r, \mu) = m^H(\theta; r, \mu)\frac{\lambda_s}{\overline{m}^H(r,\mu)}$. First, we change the mechanism from the Boston to DA.

Proposition 5. *(i) $r^B < \overline{m}^H(r^B, \mu)$, and (ii) g^B is single crossing $g^H(\cdot; r^B, \mu)$ from below.*

The result is intuitive. A student benefits from reporting *sab* more under the DA mechanism then the Boston (Lemma 1). Hence, more than r^B fraction of students report *sab* under the DA mechanism. The resulting allocation is less efficient than the Boston mechanism equilibrium. The intuition is similar to the one for Proposition 3: a higher reliance on tie-breaking and near-indifferent students taking seats in school s.

We assume a relatively small marginal information cost μ to ensure that students in the DA mechanism have interior best responses for any $r \in (r^B, r^D)$.

Proposition 6. *Take any $\mu < \lambda_s(1 - v)$. For any $r \in (r^B, r^D)$, an interior best-response under the DA mechanism exists and $\overline{m}^H(r, \mu)$ is strictly increasing in r with a value in (r^B, r^D). If $r_1 < r_2$ and $r_1, r_2 \in (r^B, r^D)$, then the allocation $g^H(\cdot; r_1, \mu)$ is single crossing $g^H(\cdot; r_2, \mu)$ from below.*

Figure 6 depicts the tatonnement best-response dynamics. We start from the equilibrium allocation of the Boston mechanism (blue-dashed line). If the mechanism changes to the DA, while students best respond to r^B, they are more likely to submit *sab* than before, especially when their (unobservable) preference types are near $1 - v$. Hence, rank-order submissions become more homogeneous, and the mechanism relies more on tie-breaking, and the resulting allocation is less efficient. The following transitions, leading ultimately to the DA equilibrium (red-solid line), represent the reinforcement. More homogeneous rank-order submissions discourage costly information acquisition and lead to even more homogeneous rank-order reports. The resulting sequence of allocations is increasingly inefficient and less dependent on preference types.

3.5 The DA's Inefficiency Increases in μ

We study how the DA mechanism's inefficiency changes in μ.

If information costs are small so that $\mu < \lambda_s(1 - v)$, consider the reinforcing cycle between greater homogeneity in rank-order reports and less information acquisition. With a higher information cost, each step of the best-response dynamic increases the fraction of *sab* reporting students and the inefficiency of the resulting allocation more substantially. This result indicates an increased inefficiency of the DA mechanism relative to the Boston mechanism.

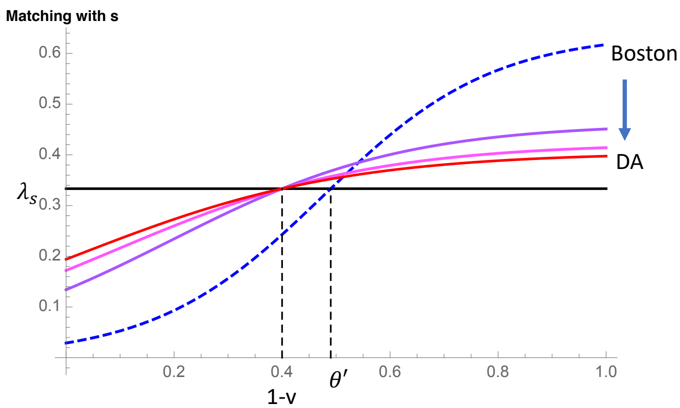


Figure 6: A change of mechanism from Boston to DA is followed by Tatonnement best-response dynamics ($v = 0.6$, and $\lambda_j = 1/3$ for $j \in \{s, a, b\}$).

Lemma 4. *Take $r > \hat{r}$ and $\mu_1 < \mu_2 < \lambda_s(1 - v)$ such that interior best-responses under the DA mechanism exist. Then, $\overline{m}^H(r, \mu_1) < \overline{m}^H(r, \mu_2)$, and $g^H(\cdot; r, \mu_1)$ is single crossing $g^H(\cdot; r, \mu_2)$ from below.*

To study the case of large information costs, we assume $v > 1/2$ and $\lambda_s \leq \lambda_a$. Then, school s is more selective than school a for every μ. Without this assumption, a quality school (e.g., school s when $E[u_s] = v + 1/2 > 1 = u_a$) can be less selective when it has a larger capacity than school a. Once we rule out such cases by assuming $\lambda_s < \lambda_a$, then school s becomes more selective than school a, whichever mechanism is used and however costly it is to acquire information. Focusing on such cases seems reasonable as quality schools are often more selective in practice.

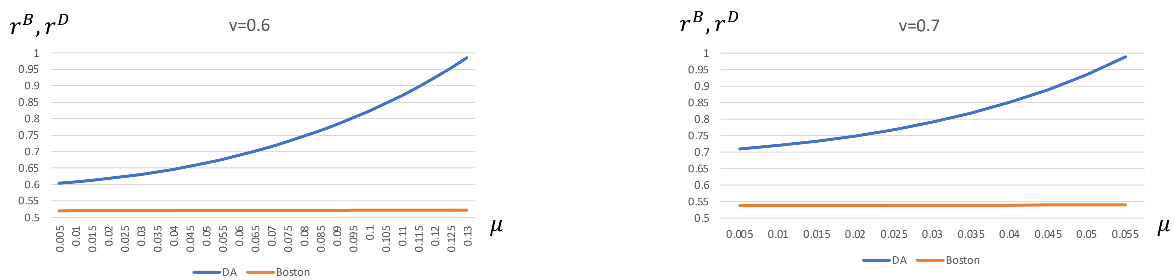
Intuitively, the equilibrium fractions r^B_μ and r^D_μ increase in μ. As μ increases, students acquire less information. Their rank-order reports then rely more on the expected utilities ($E[u_s] = v + 1/2 > 1$), and the report of *sab* is more likely to be recommended. We show that the fraction r^D_μ increases considerably faster than r^B_μ in μ. Unless v is close to 1, the equilibrium rank-order lists under the Boston mechanism never become too homogeneous.

Proposition 7. *Suppose $v > 1/2$ and $\lambda_s \leq \lambda_a$.*

1. *There exists $\overline{\mu} > 0$ such that, as μ increases from 0 to $\overline{\mu}$, the DA equilibrium fraction r^D_μ increases from v to 1.*
2. *If an equilibrium of the Boston mechanism exists for μ and μ' with $\mu < \mu'$ and $r^B_\mu > \frac{1}{2}(\geq \hat{r})$, then $r^B_\mu < r^B_{\mu'}$. However, if $v \leq \frac{1}{2} + \frac{\lambda_b}{\lambda_s}$, the equilibrium fraction r^B_μ*

is bounded above by $\max\{\frac{1}{2}, 1-\lambda_a\}$ *for every* μ*.*

An increasing difference between r^D_μ and r^B_μ suggests an increasingly greater efficiency loss in the DA mechanism, due to increasingly higher reliance on tie breaking and less incentive to acquire costly information. A numerical example can illustrate the comparison between r^B_μ and r^D_μ and the efficiencies of the equilibrium. Assume equal capacities ($\lambda_s = \lambda_a = \lambda_b = 1/3$) and $v \in \{0.6, 0.7\}$. For each v, we increase μ from 0 to $\overline{\mu}$ such that an interior equilibrium exists in the DA and Boston mechanisms. Figure 7 shows that the equilibrium fraction in the DA mechanism increases much faster than in the Boston mechanism.



(a) $v = 0.6$ (b) $v = 0.7$

Figure 7: As μ increases, students submit increasingly more homogeneous rank-order lists to the DA mechanism.

More homogeneous rank-order reports (i.e., $r^D > r^B > \hat{r}$) results in an inefficient allocation due to less information acquisition and more reliance on tie breaking. Hence, the increasing difference of $r^B - r^D$ in μ suggests that the DA mechanism can be increasingly more inefficient than the Boston mechanism. Let $g^\Gamma(\cdot)$ denote the type distribution of students that match with school s in a mechanism Γ. We define an average efficiency of each mechanism's equilibrium by $W^\Gamma \equiv \frac{\int_0^1 \theta g^\Gamma(\theta) d\theta}{\lambda_s}$.[14] The difference $W^B - W^D$ increases in μ (Figure 8).

4 Conclusion

We study a school choice problem in which students have to acquire information regarding their preferences before submitting rank-order lists to a given mechanism. We compare two prominent mechanisms: DA and Boston. The DA mechanism results in more homogeneous rank-order submissions, and consequently, it has to rely

[14]The division by λ_s is necessary because only λ_s mass of students match with s.

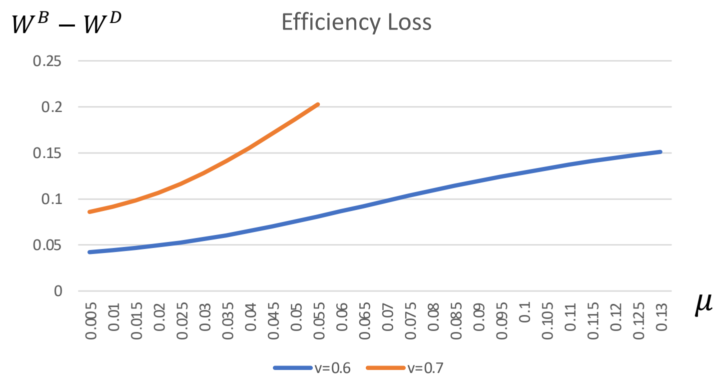


Figure 8: As μ increases, the DA mechanism becomes increasingly less efficiency than the Boston mechanism.

on tie-breaking more than the Boston mechanism. Thus, students have less incentive to acquire costly information, and their rank-order submissions become homogeneous furthermore. These properties lead to the inefficiency of the DA mechanism, which is exacerbated when information acquisition is more costly.

A Appendix

A.1 Proof of Lemma 2

We compare θ^B from (6) and $\theta^D = 1 - v$.

If $\theta^B \leq \lambda_a$, then $\lambda_s(v + \theta^B) = 1 - \lambda_a$, which implies

$$\theta^B = (\lambda_b/\lambda_s) + (1 - v) > 1 - v = \theta^D.$$

On the other hand, if $\lambda_a \leq \theta^B \leq 1 - \lambda_s$, then $\frac{\lambda_s(v+\theta^B)}{1-\theta^B} = \frac{\lambda_a}{\theta^B}$ implies

$$\theta^B = \frac{-x + \sqrt{x^2 + 4\lambda_s\lambda_a}}{2\lambda_s}, \quad \text{where} \quad x \equiv (\lambda_s + \lambda_a) - \lambda_s(1 - v).$$

Note that

$$\begin{aligned} \theta^B > 1 - v &\iff -x + \sqrt{x^2 + 4\lambda_s\lambda_a} > 2\lambda_s(1 - v) \\ &\iff \lambda_s\lambda_a > -x(\lambda_s + \lambda_a) + (\lambda_s + \lambda_a)^2 \\ &\iff v > \hat{r}. \end{aligned}$$

A.2 Proof of Lemma 3

We take (9) and find a solution r of

$$r = \int_0^1 \left(1 + \frac{1-r}{r} \exp\left(-\frac{\Delta^\Gamma(\theta; r)}{\mu}\right)\right)^{-1} d\theta.$$

For any mechanism $\Gamma \in \{B, D\}$, $\Delta^\Gamma(\theta; r) = \Delta^\Gamma(0; r) - \frac{\lambda_s \theta}{r}$ (Equations 4 and 5). Thus,

$$r = \int_0^1 \left(1 + \frac{1-r}{r} \exp\left(-\frac{\Delta^\Gamma(0; r)}{\mu}\right) \exp\left(\frac{\lambda_s \theta}{r\mu}\right)\right)^{-1} d\theta.$$

Observe that, for any $\delta > 0$,

$$\frac{d}{d\theta}\left[\frac{r\mu}{\lambda_s} \log\left(\delta + \exp\left(\frac{\lambda_s \theta}{r\mu}\right)\right)\right] = \left(1 + \delta \exp\left(-\frac{\lambda_s \theta}{r\mu}\right)\right)^{-1}.$$

Thus,

$$r = \frac{r\mu}{\lambda_s} \log\left[\frac{1-r}{r} \exp\left(-\frac{\Delta^\Gamma(0; r)}{\mu}\right) + \exp\left(\frac{\lambda_s \theta}{r\mu}\right)\right]\Bigg|_0^1.$$

We cancel out r from each side, take exponential of both sides, and obtain (10).

A.3 Proof of Proposition 1

Fix any μ, and let $z \equiv \frac{\lambda_s}{\mu}$ and $w \equiv \frac{\lambda_a}{\mu}$. We find the bounds $\overline{v}$ and $\underline{v}$ such that $v \in (\underline{v}, \overline{v})$ if and only if a solution r of the equilibrium condition (10) under the DA mechanism exists in $(\hat{r}, 1)$.

We let $x \equiv 1/r \in [1, \infty)$ and write (10) as

$$\begin{aligned} e^z &= 1 + \frac{e^{zx} - 1}{(x-1)e^{zx(1-v)} + 1} \\ \iff f(x, v) &\equiv \frac{e^{zx} - 1}{e^z - 1} - (x-1)e^{zx(1-v)} = 1. \end{aligned}$$

First, we find the upper bound $\overline{v}$. For any v, $f(1; v) = 1$ and $\lim_{x \to \infty} f(x; v) = \infty$. Moreover,

$$\begin{aligned} \frac{df(x; v)}{dx} < 0 &\iff \frac{ze^{zx}}{e^z - 1} - e^{zx(1-v)} - (x-1)e^{zx(1-v)} z(1-v) < 0 \\ &\iff g(x, v) \equiv \frac{ze^{zxv}}{e^z - 1} - 1 - (x-1)z(1-v) < 0. \end{aligned} \tag{11}$$

Let $\overline{v}$ be such that

$$g(1,\overline{v}) = 0 \iff ze^{z\overline{v}} = e^z - 1 \iff \overline{v} = \frac{1}{z}\log\left(\frac{e^z-1}{z}\right). \tag{12}$$

It can be verified that $\overline{v} \in (1/2, 1)$. Also, the last expression is increasing in z, a property that we will use later.

Suppose that $v < \overline{v}$. Note that $g(1;v) < 0$ because the function g is strictly increasing in v and $g(1;\overline{v}) = 0$. Moreover, $g(x;v)$ is strictly convex in x, and $\lim_{x\to\infty} g(x;v) = \infty$. Thus, as x increases from 1 to ∞, the function $g(x;v)$ changes the sign exactly once from strict negative to strict positive. This implies that $f(x;v)$ is a strictly single-dipped function, $f'(x;v) < 0$ at $x = 1$, and a solution $x^* > 1$ of $f(x;v) = 1$ (so, a solution $r^* = \frac{1}{x^*} < 1$ of (10)) exists uniquely.

Suppose that $v \geq \overline{v} \geq 1/2$. Then, $g(1;v) \geq 0$. Moreover, for any $x > 1$,

$$\frac{dg(x;v)}{dx} > \frac{dg(1;v)}{dx} \geq (z\overline{v})\frac{ze^{z\overline{v}}}{e^z-1} - z(1-\overline{v}) \geq z(2\overline{v}-1) \geq 0, \tag{13}$$

where the last two inequalities follow from (12) and $\overline{v} \geq 1/2$. Thus, $g(x;v) > 0$ for every $x > 1$, and there exists no solution $x^* > 1$ of $f(x;v) = 1$.

Second, we find the lower bound $\underline{v}$. We define $\hat{x} \equiv \frac{1}{\hat{r}} = \frac{\lambda_s+\lambda_a}{\lambda_s}$ and take $\underline{v}$ be such that

$$\begin{aligned} f(\hat{x};\underline{v}) = 1 \iff \frac{e^{z+w}-1}{e^z-1} - \frac{w}{z}e^{(z+w)(1-\underline{v})} = 1 \iff e^{(z+w)\underline{v}} = \frac{w}{1-e^{-w}}\frac{e^z-1}{z} \\ \iff \underline{v} = \frac{1}{z+w}\log\left(\frac{w}{1-e^{-w}}\frac{e^z-1}{z}\right). \end{aligned}$$

We can verify that $\overline{v} > \underline{v}$. Note that $f(1;\overline{v}) = 1$, and $\frac{df(x;v)}{dx} > 0$ for every $x > 1$ because of (11) and $g(x,\overline{v}) > 0$ for every $x > 1$. This implies $f(\hat{x},\overline{v}) > 1 = f(\hat{x},\underline{v})$, so $\overline{v} > \underline{v}$.

We showed that $v < \overline{v}$ if and only if a solution $x^* > 1$ of $f(x,v) = 1$ exists (uniquely). The function $f(x,v)$ is strictly and continuously increasing in v for every $x > 1$. Therefore, the unique solution $x^* \in (1,\hat{x})$ (i.e., $r^* \in (\hat{r},1)$) if and only if $v \in (\underline{v},\overline{v})$.

Last,

$$\lim_{\mu\to 0}\overline{v} = \lim_{z\to\infty}\frac{1}{z}\log\left(\frac{e^z-1}{z}\right) = 1, \quad \text{and} \quad \lim_{\mu\to\infty}\overline{v} = \lim_{z\to 0}\frac{1}{z}\log\left(\frac{e^z-1}{z}\right) = \frac{1}{2}.$$

Moreover,

$$\begin{aligned}
\underline{v} &= \frac{1}{z+w}\left[\log\left(\frac{e^z-1}{z}\right)+w-\log\left(\frac{e^w-1}{w}\right)\right] \\
&= \hat{r}\left[\frac{1}{z}\log\left(\frac{e^z-1}{z}\right)\right]+(1-\hat{r})\left[1-\frac{1}{w}\log\left(\frac{e^w-1}{w}\right)\right],
\end{aligned}$$

implies

$$\lim_{\mu\to 0}\underline{v} = \lim_{z,w\to\infty}\underline{v} = \hat{r}, \quad \text{and} \quad \lim_{\mu\to\infty}\underline{v} = \lim_{z,w\to 0}\underline{v} = \frac{1}{2}.$$

A.4 Proof of Proposition 2

Take any parameter values such that the DA mechanism has an interior equilibrium with $r^D \in (\hat{r}, 1)$. The proof of Proposition 1 showed r^D is a unique solution of (10) under the DA mechanism in $(0,1)$. Note that

$$\Delta^D(0;r) - \Delta^B(0;r) = \min\left\{\frac{\lambda_a}{1-r}, \frac{1-\lambda_a}{r}\right\} - \frac{\lambda_s}{r} > 0$$

if and only if $r > \hat{r}$. Either the opposite strictly inequality or an equality holds when $r < \hat{r}$ or $r = \hat{r}$, respectively. Thus, the right-hand side of (10) under the Boston mechanism is smaller than, larger than, or equal to the right-hand side under the DA mechanism when $r > \hat{r}$, $r < \hat{r}$, or $r = \hat{r}$, respectively. Then, an interior equilibrium exists with $r^B \in (\hat{r}, r^D)$ by the Intermediate Value Theorem.

To show the uniqueness of an interior equilibrium, let $z \equiv \frac{\lambda_s}{\mu}$, $w \equiv \frac{\lambda_a}{\mu}$, and $y \equiv \frac{\lambda_b}{\mu}$. Then, (10) under the Boston mechanism becomes

$$e^z = 1 + \frac{e^{\frac{z}{r}}-1}{\frac{1-r}{r}e^{\alpha(r)}e^{-\frac{zv}{r}}+1} \equiv h(r), \tag{14}$$

where $\alpha_1(r) \equiv \frac{w}{1-r}$, $\alpha_2(r) \equiv \frac{y+z}{r}$, and $\alpha(r) \equiv \min\{\alpha_1(r), \alpha_2(r)\}$.

Let $h_i(r) \equiv 1 + \frac{e^{\frac{z}{r}}-1}{\frac{1-r}{r}e^{\alpha_i(r)}e^{-\frac{zv}{r}}+1}$ so that $h(r) = \max\{h_1(r), h_2(r)\}$. If $r \leq 1-\lambda_a$, then $\alpha_1(r) \leq \alpha_2(r)$, so $h(r) = h_1(r)$. If $r \geq 1-\lambda_a$, then $\alpha_1(r) \geq \alpha_2(r)$, so $h(r) = h_2(r)$.

Claim 1. *$h_1(r)$ is single crossing e^z from above to below as r increases in $[\hat{r}, 1)$. Formally, if $e^z \geq h_1(r')$ for some $r' \in [\hat{r}, 1)$, then $e^z > h_1(r'')$ for every $r'' \in (r', 1)$.*

Proof. For any $r \in [\hat{r}, 1)$,

$$e^z \geq h_1(r) \iff \frac{1-r}{r} e^{\alpha_1(r)} e^{-\frac{zv}{r}} + 1 \geq \frac{e^{\frac{z}{r}} - 1}{e^z - 1}$$
$$\iff e^{\frac{w}{1-r}} e^{-\frac{zv}{r}} \geq \frac{e^z}{e^z - 1} \frac{(e^{z((1/r)-1)} - 1)}{(1/r) - 1},$$

where the weak inequalities can be replaced with strict ones. The left-hand side of the last inequality strictly increases in r. But, the right-hand side strictly decreases in r because

$$\left(\frac{e^{z(x-1)} - 1}{x - 1} \right)' > 0 \iff z(x-1)e^{z(x-1)} > e^{z(x-1)} - 1 \iff e^{-z(x-1)} > 1 - z(x-1),$$

which holds for every $x > 1$. Hence, Claim 1 follows. □

Claim 2. *$h_2(r)$ is single crossing e^z from above to below as r increases in $[\hat{r}, 1)$. Formally, if $e^z \geq h_2(r')$ for some $r' \in [\hat{r}, 1)$, then $e^z > h_2(r'')$ for every $r'' \in (r', 1)$.*

Proof. In the interval $[\hat{r}, 1)$,

$$e^z \geq h_2(r) \iff 1 \geq \frac{e^{\frac{z}{r}} - 1}{e^z - 1} - \frac{1-r}{r} e^{\alpha_2(r)} e^{-\frac{zv}{r}}$$

By a change of variable $x = \frac{1}{r} > 1$, we have $\alpha_2(x) \equiv (y+z)x$ and

$$e^z \geq h_2(r) \iff 1 \geq \frac{e^{zx} - 1}{e^z - 1} - (x-1)e^{\alpha_2(x)} e^{-zvx} \equiv \tilde{h}(x). \tag{15}$$

Note that

$$\tilde{h}'(x) < 0 \iff 0 > \frac{ze^{zx}}{e^z - 1} - e^{\alpha_2(x) - zvx} - (x-1)(\alpha_2' - zv)e^{\alpha_2(x) - zvx}$$
$$\iff 0 > \frac{ze^{z(1+v)x - \alpha_2(x)}}{e^z - 1} - 1 - (x-1)(\alpha_2' - zv) \equiv \tilde{g}(x, v),$$

where $\alpha_2' = y + z$ is independent of x.

If $zv > y$, then $\tilde{g}(x, v)$ is a strictly convex function of $x \in (1, 1/\hat{r}]$. So, as x increases from 1 to $\frac{1}{\hat{r}}$, either $\tilde{g}(x, v)$ remains to be strictly negative, or the sign of $\tilde{g}(x, v)$ changes exactly once from negative to 0 and then positive. On the other hand, if $zv \leq y$, then $\tilde{g}(x, z)$ is decreasing in x, so it remains to be strictly negative. In either case, $\tilde{h}(x)$ is a strictly single-dipped function of $x \in (1, 1/\hat{r}]$.

Finally, we began with parameter values such that an interior equilibrium of the DA mechanism exists with a fraction $r^D \in (\hat{r}, 1)$. Hence, $v \in (\underline{v}, \overline{v})$, and $g(x, v)$ of (11) in the proof of Proposition 1 satisfies $g(1, v) < 0$. Therefore, $\tilde{g}(1, v) < g(1; v) < 0$, which implies that $\tilde{h}'(1) < 0$. As $\tilde{h}(1) = 1$, $\tilde{h}'(1) < 0$, and $\tilde{h}(x)$ is a strictly single-dipped function of $x \in (1, 1/\hat{r}]$, the function $\tilde{h}(x)$ is single crossing from below as x increases in $(1, 1/\hat{r}]$. Last, Claim 2 follows from (15). $\square$

Finally, we prove the uniqueness of the interior equilibrium under the Boston mechanism. If $h(1-\lambda_a) \geq e^z$, then no solution of $h(r) = e^z$ exists in $(\hat{r}, 1-\lambda_a)$ by Claim 1, and at most one solution exists in $[1-\lambda_a, 1)$ by Claim 2. On the other hand, if $h(1-\lambda_a) < e^z$, then at most one solution of $h(r) = e^z$ exists in $(\hat{r}, 1-\lambda_a]$ by Claim 1, and no solution exists in $[1-\lambda_a, 1)$ by Claim 2.

A.5 Proof of Proposition 3

We prove that g^B is single crossing g^D from below:

$$(\forall \theta < \theta') \quad g^B(\theta) \geq g^D(\theta) \implies g^B(\theta') > g^D(\theta').$$

For each mechanism $\Gamma \in \{B, D\}$, we take the equilibrium conditions (8) and (10):

$$m^\Gamma(\theta) = \left(1 + \frac{1-r^\Gamma}{r^\Gamma} \exp\left(-\frac{\Delta^\Gamma(\theta; r^\Gamma)}{\mu}\right)\right)^{-1},$$

where $\Delta^\Gamma(\theta; r) = \Delta^\Gamma(0; r) - \frac{\lambda_s \theta}{r}$ (by (4) and (5)) and

$$\frac{1-r^\Gamma}{r^\Gamma} \exp\left(-\frac{\Delta^\Gamma(0; r^\Gamma)}{\mu}\right) = \frac{\exp\left(\frac{\lambda_s}{r^\Gamma \mu}\right) - 1}{\exp\left(\frac{\lambda_s}{\mu}\right) - 1} - 1.$$

Let $z \equiv \frac{\lambda_s}{\mu}$. Then,

$$m^\Gamma(\theta) = \left[1 + h(\theta, r^\Gamma)\right]^{-1}, \quad \text{where} \quad h(\theta, r) = \frac{\exp\left(-\frac{z\theta}{r}\right)\left(\exp\left(\frac{z}{r}\right) - \exp(z)\right)}{\exp(z) - 1}. \tag{16}$$

Note that $h(\theta, r)$ is independent of the mechanism Γ, and

$$\begin{aligned} \frac{dh(r;\theta)}{dr} &= \left[\frac{z\theta}{r^2} e^{-\frac{z\theta}{r}} (e^{\frac{z}{r}} - e^z) - \frac{z}{r^2} e^{-\frac{z\theta}{r}} e^{\frac{z}{r}}\right] \frac{1}{e^z - 1} \\ &= -\left[\frac{z}{r^2} e^{-\frac{z\theta}{r}} (e^{\frac{z}{r}}(1-\theta) + e^z \theta)\right] \frac{1}{e^z - 1} < 0. \end{aligned} \tag{17}$$

Finally, we take $g^\Gamma(\theta) = m^\Gamma(\theta)\frac{\lambda_s}{r^\Gamma}$ and obtain

$$\frac{d(\lambda_s/g^\Gamma(\theta))}{d\theta} = \frac{d(r^\Gamma(1+h(\theta,r^\Gamma))}{d\theta} = -z \cdot h(\theta, r^\Gamma).$$

Hence,

$$\begin{aligned} r^B < r^D & \implies (\forall \theta \in (0,1)) \quad \frac{d(\lambda_s/g^B(\theta))}{d\theta} < \frac{d(\lambda_s/g^D(\theta))}{d\theta} \\ & \implies (\forall \theta < \theta') \quad \frac{1}{g^B(\theta)} \leq \frac{1}{g^D(\theta)} \implies \frac{1}{g^B(\theta')} < \frac{1}{g^D(\theta')} \\ & \implies g^B(\theta) \text{ is single crossing } g^D(\theta) \text{ from below.} \end{aligned}$$

A.6 Proof of Proposition 4

We prove that

$$(\forall \theta' < \theta'') \quad \frac{dm^D(\theta')}{d\theta} \leq \frac{dm^B(\theta')}{d\theta} \implies \frac{dm^D(\theta'')}{d\theta} < \frac{dm^B(\theta'')}{d\theta}.$$

We write an equilibrium strategy of each mechanism $\Gamma \in \{D, B\}$ as in (16). Let $z \equiv \frac{\lambda_s}{\mu}$, and

$$m^\Gamma(\theta) = \left[1 + h(\theta, r^\Gamma)\right]^{-1} \quad \text{where} \quad h(\theta, r) = \frac{e^{-\frac{z\theta}{r}}(e^{\frac{z}{r}} - e^z)}{e^z - 1}.$$

Note that

$$\frac{dm(\theta,r)}{d\theta} = \frac{-\frac{dh(\theta,r)}{d\theta}}{(1+h(\theta,r))^2} = \frac{z}{r}\frac{h(\theta,r)}{(1+h(\theta,r))^2}.$$

Define

$$f(\theta, r) \equiv \left(\frac{dm(\theta,r)}{d\theta}\right)^{-1} = \frac{r}{z}\left(h(\theta,r) + \frac{1}{h(\theta,r)} + 2\right).$$

Then,

$$\frac{df(\theta,r)}{d\theta} = \frac{r}{z}\frac{dh}{d\theta}\left(1 - \frac{1}{h^2}\right) = -h\left(1 - \frac{1}{h^2}\right) = \frac{1}{h(\theta,r)} - h(\theta,r).$$

Note that $\frac{df(\theta,r)}{d\theta}$ increases in r because $h(\theta, r)$ is decreasing in r (see (17)). Thus,

$$(\forall \theta' < \theta'') \quad f(\theta'', r^B) - f(\theta', r^B) < f(\theta'', r^D) - f(\theta', r^D),$$

which implies

$$(\forall \theta' < \theta'') \quad f(\theta', r^B) \leq f(\theta', r^D) \implies f(\theta'', r^B) < f(\theta'', r^D),$$

or equivalently

$$(\forall \theta' < \theta'') \quad \frac{dm(\theta', r^D)}{d\theta} \leq \frac{dm(\theta', r^B)}{d\theta} \implies \frac{dm(\theta'', r^D)}{d\theta} < \frac{dm(\theta'', r^B)}{d\theta}.$$

A.7 Proof of Proposition 5

Suppose that the DA and the Boston mechanisms have equilibria with fractions such that $\hat{r} < r^B < r^D < 1$. We change the mechanism from Boston to DA while holding r fixed at r^B. Both $\Delta^D(\theta; r^B)$ and $\Delta^B(\theta; r^B)$ have the form of $\alpha(\theta + \beta)$ where $\alpha = \frac{\lambda_s}{r^B \mu}$ and β for $\Delta^D(\theta; r^B)$ is larger than one for $\Delta^B(\theta; r^B)$. Hence, in this proof, we write $\Delta^\Gamma(\theta; r^B)$ as $\alpha(\theta + \beta)$ and consider an increase of β.

If a student's unique optimal strategy $m : [0,1] \to [0,1]$ is an interior solution, the following first-order condition holds(Yang, 2015):

$$\alpha(\theta + \beta) = \ln\left(\frac{m(\theta)}{1 - m(\theta)}\right) - \ln\left(\frac{\overline{m}}{1 - \overline{m}}\right),$$

and the optimal strategy has the form $m(\theta) = \frac{Le^{\alpha(\theta+\beta)}}{Le^{\alpha(\theta+\beta)}+1}$, where $L = \frac{\overline{m}}{1-\overline{m}}$. We call L the likelihood ratio between the reports *sab* versus *asb*. The consistency $\overline{m} = \int m(\theta) d\theta$ requires that

$$\int_0^1 \frac{Le^{\alpha(\theta+\beta)}}{Le^{\alpha(\theta+\beta)} + 1} d\theta = \frac{L}{L+1} \iff \int_0^1 \frac{1}{e^{-\alpha(\theta+\beta)} + L} d\theta = \frac{1}{L+1}. \tag{18}$$

(Part 1) Let $L(\beta)$ be a unique solution of (18) for each β. By Implicit Function Theorem,

$$\int_0^1 \frac{-\alpha e^{-\alpha(\theta+\beta)} + L'(\beta)}{(e^{-\alpha(\theta+\beta)} + L(\beta))^2} d\theta = \frac{L'(\beta)}{(L(\beta)+1)^2},$$

or, equivalently

$$L'(\beta) \left[\int_0^1 \left(\frac{1}{e^{-\alpha(\theta+\beta)} + L(\beta)}\right)^2 d\theta - \left(\frac{1}{L(\beta)+1}\right)^2\right] = \int_0^1 \frac{\alpha e^{-\alpha(\theta+\beta)}}{(e^{-\alpha(\theta+\beta)} + L(\beta))^2} d\theta > 0.$$

Note that $\int_0^1 \left(\frac{1}{e^{-\alpha(\theta+\beta)}+L(\beta)}\right)^2 d\theta - \left(\frac{1}{L(\beta)+1}\right)^2 > 0$ because of (18).[15] Therefore, $L'(\beta) >$

[15]For a random variable X, $Var[X] = E[X^2] - (E[X])^2 > 0$. By (18), $\frac{1}{e^{-\alpha(\theta+\beta)}+L}$ is a random variable

0.

(Part 2) Define $g(\theta) \equiv m(\theta)\frac{\lambda_s}{\overline{m}}$. Then,

$$\frac{\lambda_s}{g(\theta)} = \frac{L + e^{-\alpha(\theta+\beta)}}{L+1}.$$

Take $\beta_1 < \beta_2$ and let L_1 and L_2 be the solutions of (18) for β_1 and β_2, respectively. Then, $L_1 < L_2$, and

$$\frac{\lambda_s}{g_2(\theta)} - \frac{\lambda_s}{g_1(\theta)} = \left(\frac{e^{-\alpha\beta_2}}{L_2+1} - \frac{e^{-\alpha\beta_1}}{L_1+1}\right) e^{-\alpha\theta} + \left(\frac{L_2}{L_2+1} - \frac{L_1}{L_1+1}\right),$$

which is strictly increasing in θ. Therefore, g_1 is single crossing g_2 from below.

A.8 Proof of Proposition 6 and Lemma 4

Take any μ and r such that $\mu < \lambda_s(1-v)$ and $r \in (r^B_\mu, r^D_\mu)$. Then, $\Delta^D(\theta; r) = \alpha(\theta + v - 1)$, where $\alpha = \frac{\lambda_s}{r\mu}$. A decrease of α corresponds to an increase of r when μ is fixed (for Proposition 6) and an increase of μ when r is fixed (for Lemma 4).

Given α and the DA mechanism, an optimal information acquisition strategy $m(\theta)$ satisfies (18).

$$\begin{aligned}
&\int_0^1 \frac{\overline{m}e^{\alpha(\theta+v-1)}}{\overline{m}e^{\alpha(\theta+v-1)} + 1 - \overline{m}} d\theta = \overline{m} \\
\iff& \log(\overline{m}e^{\alpha(\theta+v-1)} + 1 - \overline{m})\Big|_0^1 = \alpha\overline{m} \\
\iff& f(\overline{m}, \alpha) \equiv \overline{m}(e^{-\alpha(\overline{m}-v)} - e^{-\alpha(1-v)}) + (1-\overline{m})(e^{-\alpha\overline{m}} - 1) = 0. \qquad (19)
\end{aligned}$$

(Part 1) Note that $f(0, \alpha) = f(1, \alpha) = 0$, and

$$\frac{df(0,\alpha)}{d\overline{m}} = e^{\alpha v}(1 - e^{-\alpha}) - \alpha \geq e^{\alpha/2} - e^{-\alpha/2} - \alpha > 0.$$

Moreover, (12) implies that

$$\frac{df(1,\alpha)}{d\overline{m}} = e^{-\alpha}(e^{\alpha} - 1 - \alpha e^{\alpha v}) > 0,$$

because $v < \overline{v}$, $\alpha = \frac{\lambda_s}{r\mu} > \frac{\lambda_s}{\mu} \equiv z$, and $e^{\alpha} - 1 - \alpha e^{\alpha v}$ increases in α and decreases in v. As such, $\frac{df(\overline{m}^*,\alpha)}{d\overline{m}} < 0$ at a unique solution $\overline{m}^* \in (0,1)$ of $f(\overline{m}, \alpha) = 0$.

with the expected value $\frac{1}{L+1}$.

A sufficiently small $\mu < \lambda_s(1-v)$ implies $\frac{1}{\alpha} + v < \frac{\mu}{\lambda_s} + v < 1$. At $\overline{m} = \frac{1}{\alpha} + v (< 1)$,

$$f(\overline{m}, a) = \frac{1}{\alpha e}\Big((1+\alpha v)(1 - e^{1-\alpha(1-v)}) + (\alpha(1-v)-1)(e^{\alpha v} - e)\Big) > 0.$$

Hence, $\overline{m}^* > \frac{1}{\alpha} + v$. Then, as $1 < \alpha(\overline{m}^* - v) < \alpha(1-v)$,

$$\frac{d(e^{-\alpha(\overline{m}-v)} - e^{-\alpha(1-v)})}{d\alpha} = -(\overline{m}-v)e^{-\alpha(\overline{m}-v)} + (1-v)e^{-\alpha(1-v)} < 0,$$

where the last inequality holds because xe^{-x} decreases in $x > 1$. Thus, $\frac{df(\overline{m}^*,\alpha)}{d\alpha} < 0$. By Implicit Function Theorem, the solution $\overline{m}^*$ decreases in α (or, increases in r or μ).

Finally, for each $\mu < \lambda_s(1-v)$ and $r \in (r^B_\mu, r^D_\mu)$, let $\overline{m}^*(r,\mu)$ be the unique solution of (19). Then, $\overline{m}^*(r,\mu) \in (r^B_\mu, r^D_\mu)$, because $\overline{m}^*(r,\mu) > \overline{m}^*(r^B_\mu,\mu) > r^B_\mu$ (Proposition 5) and $\overline{m}^*(r,\mu) < \overline{m}^*(r^D_\mu,\mu) = r^D_\mu$ (r^D_μ is the DA equilibrium fraction).

(Part 2) Let $L(\alpha)$ be the unique solution of (19). Then, $m(\theta;\alpha) = \frac{L(\alpha)e^{\alpha(\theta+v-1)}}{L(\alpha)e^{\alpha(\theta+v-1)}+1}$ and

$$\frac{g(\theta;\alpha)}{\lambda_s} = \frac{L(\alpha)+1}{L(\alpha)+e^{-\alpha(\theta+v-1)}}.$$

In Step 1, we proved that $L(\alpha) = \frac{\overline{m}(\alpha)}{1-\overline{m}(\alpha)}$ decreases in α. Moreover,

$$\begin{aligned} &\frac{dg(\theta;\alpha)}{d\alpha} > 0 \\ \iff\ & L'(\alpha)(L(\alpha)+e^{-\alpha(\theta+v-1)}) > (L(\alpha)+1)(L'(\alpha) - (\theta+v-1)e^{-\alpha(\theta+v-1)}) \\ \iff\ & \theta + v - 1 > \frac{L'(\alpha)}{L(\alpha)+1}(e^{\alpha(\theta+v-1)} - 1). \end{aligned}$$

Thus, $\frac{dg(\theta;\alpha)}{d\alpha} > 0$ for every $\theta > 1-v$, and $\frac{dg(\theta;\alpha)}{d\alpha} < 0$ for every $\theta < 1-v$. Therefore, for $\alpha_1 > \alpha_2$, $g^H(\cdot;\alpha_1)$ is single crossing $g^H(\cdot;\alpha_2)$ from below.

A.9 Proof of Proposition 7

Suppose $v > 1/2$ and $\lambda_s \le \lambda_a$. Then, $v > \frac{1}{2} > \hat{r} = \frac{\lambda_s}{\lambda_s+\lambda_a}$.

DA mechanism For any μ, let $\overline{v}_\mu$ denote the upper bound of v such that an interior equilibrium with a fraction $r^D_\mu \in (\hat{r}, 1)$ exists. By (12), $\overline{v}_\mu = \frac{1}{z}\log\left(\frac{e^z-1}{z}\right)$, where $z \equiv \frac{\lambda_s}{\mu}$. It can be verified that $\overline{v}_\mu$ is strictly and continuously increasing in z from $\frac{1}{2}$ (as $z \to 0$) to 1 (as $z \to \infty$). For any $v > \frac{1}{2}$, we find $\overline{\mu}$ such that $v = \frac{1}{\underline{z}}\log\left(\frac{e^{\underline{z}}-1}{\underline{z}}\right)$, where $\underline{z} \equiv \frac{\lambda_s}{\overline{\mu}}$.

For the given v, an interior equilibrium under DA exists with fraction $r_\mu^D \in (\hat{r}, 1)$ only if $\mu < \overline{\mu}$ because

$$\begin{aligned} v &= (\overline{\mu}/\lambda_s) \log\left[(\overline{\mu}/\lambda_s)\left(\exp\left(\lambda_s/\overline{\mu}\right) - 1\right)\right] \\ &< (\mu/\lambda_s) \log\left[(\mu/\lambda_s)\left(\exp\left(\lambda_s/\mu\right) - 1\right)\right] \equiv \overline{v}_\mu \quad \text{(see (12))}. \end{aligned}$$

Moreover, $\lim_{\mu \to \overline{\mu}} \overline{v}_\mu = v$, and so $\lim_{\mu \to \overline{\mu}} r_\mu^D = 1$.

Next, we show that the equilibrium fraction r_μ^D strictly increases in μ.

We let $x \equiv \frac{1}{r}$ and $z \equiv \frac{\lambda_s}{\mu}$ and write the equilibrium condition (10) as

$$\begin{aligned} & e^z = 1 + \frac{e^{zx} - 1}{(x-1)e^{zx(1-v)} + 1} \\ \iff & e^{zx} - 1 - (e^z - 1)((x-1)e^{zx(1-v)} + 1) = 0 \\ \iff & (e^{zx} - e^z) - (x-1)(e^z - 1)e^{zx(1-v)} = 0 \\ \iff & h(x; z) \equiv e^{zxv}(1 - e^{z(1-x)}) - (x-1)(e^z - 1) = 0. \end{aligned}$$

A unique solution x^* of $h(x; z) = 0$ exists by Proposition 1; to ease expositions, we suppress the dependency of x^* on z. We make several claims to apply the Implicit Function Theorem.

Claim 3. *1. For any $y > 0$, $\frac{1}{y} - \frac{1}{e^y - 1} < \frac{1}{2}$, and*

2. $\log\left(\frac{y}{e^y - 1}\right) + y + \frac{y}{e^y - 1}$ is strictly increasing in y.

Proof. (Part 1) By L'Hopital's rule,

$$\lim_{y \to 0} \frac{1}{y} - \frac{1}{e^y - 1} = \lim_{y \to 0} \frac{e^y - 1 - y}{y(e^y - 1)} = \lim_{y \to 0} \frac{e^y - 1}{e^y - 1 + ye^y} = \lim_{y \to 0} \frac{e^y}{2e^y + ye^y} = \frac{1}{2}.$$

Next,

$$\begin{aligned} & \left(\frac{1}{y} - \frac{1}{e^y - 1}\right)' = -\frac{1}{y^2} + \frac{e^y}{(e^y - 1)^2} < 0 \iff y^2 e^y < (e^y - 1)^2 \\ \Longleftarrow\; & 2ye^y + y^2 e^y < 2(e^y - 1)e^y \iff 2y + y^2 < 2(e^y - 1) \\ \Longleftarrow\; & 2 + 2y < 2e^y. \end{aligned} \tag{20}$$

Each step above observes that two sides of an inequality converge as $y \to 0$ and compares their derivatives.

(Part 2) Define $f(y) \equiv \log\left(\frac{y}{e^y-1}\right) + y + \frac{y}{e^y-1}$. Then,

$$\begin{aligned}
f'(y) = 1 + \left(1 + \frac{e^y-1}{y}\right)\left(\frac{y}{e^y-1}\right)' &= 1 + \left(1 + \frac{e^y-1}{y}\right)\frac{e^y-1-ye^y}{(e^y-1)^2} > 0 \\
\iff \left(1 + \frac{e^y-1}{y}\right)\left(1 - \frac{ye^y}{e^y-1}\right) > 1 - e^y & \\
\iff \frac{e^y-1}{y} > \frac{ye^y}{e^y-1}. &
\end{aligned}$$

The last inequality follows from (20). □

Claim 4. $\frac{dh(x^*,z)}{dx} > 0.$

Proof. We want to show that, at $x = x^*$,

$$\frac{dh(x,z)}{dx} = e^{zxv}(zv)(1-e^{z(1-x)}) + e^{zxv}e^{z(1-x)}z - (e^z-1) > 0.$$

At $x = x^*$, since $h(x,z) = 0$, and so $e^{zxv}(1-e^{z(1-x)}) = (x-1)(e^z-1)$, which we divide from both sides of the above inequality.

$$\begin{aligned}
\frac{dh(x,z)}{dx} > 0 &\iff zv(x-1) + \frac{z(x-1)}{1-e^{z(1-x)}}e^{z(1-x)} > 1 \\
&\iff v > \frac{1}{z(x-1)} - \frac{1}{e^{z(x-1)}-1}.
\end{aligned}$$

The last inequality holds because of $v > 1/2$ and Part 1 of Claim 3. □

Claim 5. *If* $x^* < 2$, *then* $\frac{dh(x^*,z)}{dz} < 0.$

Proof. Note that

$$\frac{dh(x,z)}{dz} = e^{zxv}(xv)(1-e^{z(1-x)}) + e^{zxv}e^{z(1-x)}(x-1) - (x-1)e^z < 0.$$

At $x = x^*$, $h(x,z) = 0$, so $e^{zxv} = \frac{(x-1)(e^z-1)}{1-e^{z(1-x)}}$. Thus,

$$\begin{aligned}
\frac{dh(x,z)}{dz} < 0 &\iff (xv) + \frac{(x-1)e^{z(1-x)}}{1-e^{z(1-x)}} < \frac{e^z}{e^z-1} \\
&\iff (xv) + \frac{(x-1)}{e^{z(x-1)}-1} < 1 + \frac{1}{e^z-1}.
\end{aligned}$$

On the other hand, $h(x,z)=0$ implies that

$$zxv = \log\left(\frac{(x-1)(e^z-1)}{1-e^{z(1-x)}}\right) = \log\left(\frac{z(x-1)}{1-e^{z(1-x)}}\right) + \log\left(\frac{e^z-1}{z}\right)$$
$$= \log\left(\frac{z(x-1)}{e^{z(x-1)-1}}\right) + z(x-1) - \log\left(\frac{z}{e^z-1}\right).$$

Therefore, at $x=x^*$

$$\frac{dh(x,z)}{dz} < 0 \iff (zxv) + \frac{z(x-1)}{e^{z(x-1)}-1} < z + \frac{z}{e^z-1}$$
$$\iff \log\left(\frac{z(x-1)}{e^{z(x-1)-1}}\right) + z(x-1) + \frac{z(x-1)}{e^{z(x-1)}-1} < \log\left(\frac{z}{e^z-1}\right) + z + \frac{z}{e^z-1}.$$

The last inequality follows from the assumption $x^* < 2$ and Part 2 of Claim 3. □

By Implicit Function Theorem, Claim 4, and Claim 5, if $x^* < 2$, then

$$\frac{dx^*}{dz} = -\frac{dh(x^*,z)/dz}{dh(x^*,z)/dx} > 0.$$

If μ is sufficiently small (i.e., a large z), then r^D_μ is close to the complete information equilibrium fraction v, so $r^D_\mu > 1/2$, which implies that $x^* < 2$. Hence, μ increases (i.e., z decreases), $r^D > 1/2$ (i.e., $x^* < 2$) continues to hold. Therefore, r^D continues to increase in μ.

Boston mechanism We take v and μ such that an equilibrium with fraction r^B_μ exists in $(1/2, 1)$. We use a change of variables so that $x \equiv \frac{1}{r}$, $z \equiv \frac{\lambda_s}{\mu}$, $w \equiv \frac{\lambda_a}{\mu}$, $y \equiv \frac{\lambda_b}{\mu}$, and the equilibrium condition (14) as

$$e^z = 1 + \frac{e^{zx}-1}{(x-1)e^{\alpha(x)}e^{-zvx}+1}, \tag{21}$$

where $\alpha_1(x) \equiv \frac{wx}{x-1}$, $\alpha_2(x) \equiv (y+z)x$, and $\alpha(x) \equiv \min\{\alpha_1(x), \alpha_2(x)\}$.

Define

$$(\forall i = 1, 2,) \quad h_i(x,\mu) \equiv e^{zvx}(1-e^{z(1-x)}) - (x-1)(e^z-1)e^{\alpha_i(x)-zx},$$

so that, if $x^B_\mu \geq \frac{1}{1-\lambda_a}$, then $h_1(x^B_\mu, \mu) = 0$, and if $x^B_\mu \leq \frac{1}{1-\lambda_a}$, then $h_2(x^B_\mu, \mu) = 0$.

For each $i = 1, 2$, we prove that if a solution x^*_i of $h_i(x;\mu) = 0$ exists, then $\frac{dx^*_i}{d\mu} < 0$. Similar to the proof of Proposition 7, we apply the Implicit Function Theorem.

Claim 6. *For any $x \in (1,2)$ and $y > 0$, $\log\left(\frac{(x-1)(e^z-1)}{1-e^{z(1-x)}}\right) > \frac{zx}{2}$.*

Proof. We prove that, for any $x \in (1,2)$, $(x-1)(e^y - 1) > e^{xy/2} - e^{y(1-(x/2))}$. Both sides of the inequality are equal to 0 at $x = 1$ and to $e^y - 1$ at $x = 2$. The left-hand side is a linear function of x. The right-hand side is a strictly convex function of $x \in (1,2)$ because $(e^{xy/2} - e^{y(1-(x/2))})'' = (y/2)(e^{xy/2} + e^{y(1-(x/2))})' = (y/2)^2(e^{xy/2} - e^{y(1-(x/2))}) > 0$. □

Claim 7. *For $i = 1, 2$, if $x_i^* \in (1,2)$, then* $\frac{dh_i(x_i^*, \mu)}{dx} > 0$.

Proof. For each $i = 1, 2$,

$$\begin{aligned}\frac{dh_i(x,\mu)}{dx} =& e^{zvx} zv(1 - e^{z(1-x)}) + e^{zvx} e^{z(1-x)} z \\ & - (e^z - 1)e^{\alpha_i - zx} - (x-1)(e^z - 1)e^{\alpha_i - zx}(\alpha_i' - z),\end{aligned}$$

where $\alpha_1' \equiv \frac{d\alpha_1}{dx} = -\frac{w}{(x-1)^2}$ and $\alpha_2' \equiv \frac{d\alpha_2}{dx} = y + z$.

As $h_i(x_i^*, \mu) = 0$, $e^{zvx_i^*}(1 - e^{z(1-x_i^*)}) = (x_i^* - 1)(e^z - 1)e^{\alpha_i - zx_i^*}$, by which and z we divide the above derivative.

$$\begin{aligned}\frac{dh_i(x_i^*,\mu)}{dx} > 0 &\iff v + \frac{1}{1 - e^{z(1-x_i^*)}} e^{z(1-x_i^*)} > \frac{1}{z(x_i^* - 1)} + \left(\frac{\alpha_i'}{z} - 1\right) \\ &\iff v - \left(\frac{\alpha_i'}{z} - 1\right) > \frac{1}{z(x_i^* - 1)} - \frac{1}{e^{z(x_i^* - 1)} - 1}. \end{aligned} \tag{22}$$

The left-hand side of (22) is greater than $\frac{1}{2}$ because,

- if $i = 1$, then $v - \left(\frac{\alpha_1'}{z} - 1\right) > v + 1 > \frac{1}{2}$, and
- if $i = 2$, then $h_2(x_2^*, \mu) = 0$ implies $zvx_2^* = \log\left(\frac{(x_2^*-1)(e^z-1)}{1-e^{z(1-x_2^*)}}\right) + (\alpha_2 - zx_2^*)$, and $\alpha_2(x_2^*) = \alpha_2' x_2^*$. Thus

  $$v - \left(\frac{\alpha_2'}{z} - 1\right) = v + 1 - \frac{\alpha_2}{zx_2^*} > \frac{1}{2} \iff \log\left(\frac{(x_2^* - 1)(e^z - 1)}{1 - e^{z(1-x_2^*)}}\right) > \frac{zx_2^*}{2},$$

  and the last inequality holds by Claim 6.

On the other hand, the right-hand side of (22) is less than $\frac{1}{2}$ (Part 1 of Claim 3). Thus, $\frac{dh_i(x_i^*,\mu)}{dx} > 0$. □

Claim 8. *For $i = 1, 2$, if $x_i^* \in (1,2)$, then* $\frac{dh_i(x_i^*, \mu)}{d\mu} > 0$.

Proof. For each $i = 1, 2$,

$$\begin{aligned}\frac{dh_i(x,\mu)}{d\mu} =& z'\left(e^{zvx} vx(1 - e^{z(1-x)}) - e^{zvx} e^{z(1-x)}(1-x) - (x-1)e^z e^{\alpha_i - zx}\right) \\ & - (x-1)(e^z - 1)e^{\alpha_i - zx}(\alpha_i' - z'x) > 0,\end{aligned}$$

where $z' = -\frac{\lambda_s}{\mu^2}$, $w' = -\frac{\lambda_a}{\mu^2}$, $y' = -\frac{\lambda_b}{\mu^2}$, $\alpha_1' = \frac{d\alpha_1}{d\mu} = \frac{w'x}{x-1} < 0$, and $\alpha_2' = \frac{d\alpha_2}{d\mu} = (y'+z')x < 0$.

As $h_i(x_i^*, \mu) = 0$, $e^{zvx_i^*}(1 - e^{z(1-x_i^*)}) = (x_i^* - 1)(e^z - 1)e^{\alpha_i - zx_i^*}$, by which and z' we divide the above derivative.

$$\begin{aligned}\frac{dh_i(x_i^*, \mu)}{d\mu} < 0 &\iff vx_i^* - \frac{e^{z(1-x_i^*)}(1-x_i^*)}{1-e^{z(1-x_i^*)}} < \frac{e^z}{e^z - 1} + \left(\frac{\alpha_i'}{z'} - x_i^*\right) \\ &\iff zvx_i^* + \frac{z(x_i^* - 1)}{e^{z(x_i^*-1)} - 1} < z + \frac{z}{e^z - 1} + (\alpha_i - zx_i^*),\end{aligned}$$

where we find the last inequality from $z' = -\frac{z}{\mu}$ and $\alpha_i' = -\frac{\alpha_i}{\mu}$ for $i = 1, 2$.

On the other hand, $h_i(x_i^*, \mu) = 0$ implies that

$$\begin{aligned}zvx_i^* &= \log\left(\frac{(x_i^* - 1)(e^z - 1)e^{\alpha_i - zx_i^*}}{1 - e^{z(1-x_i^*)}}\right) \\ &= \log\left(\frac{z(x_i^* - 1)}{e^{z(x_i^*-1)} - 1}\right) + z(x_i^* - 1) - \log\left(\frac{z}{e^z - 1}\right) + (\alpha_i - zx_i^*).\end{aligned}$$

Therefore,

$$\begin{aligned}\frac{dh_i(x_i^*, \mu)}{d\mu} < 0 \iff \log\left(\frac{z(x_i^* - 1)}{e^{z(x_i^*-1)} - 1}\right) + z(x_i^* - 1) + \frac{z(x_i^* - 1)}{e^{z(x_i^*-1)} - 1} \\ < \log\left(\frac{z}{e^z - 1}\right) + z + \frac{z}{e^z - 1}.\end{aligned}$$

The last inequality follows from $x_i^* < 2$ and Part 2 of Claim 3. $\square$

By Claim 7 and Claim 8, $\frac{dx_i^*}{d\mu} = -\frac{dh_i(x_i^*,\mu)/d\mu}{dh_i(x_i^*,\mu)/d\mu} < 0$. Then, $x_\mu^B \in (1, 2)$ decreases in μ because $x_\mu^B = x_i^*$ for either $i = 1$ or $i = 2$, and the solution x_i^* of $h_i(x, \mu) = 0$ decreases in μ.

Finally, we prove that $v \leq \frac{1}{2} + \frac{\lambda_b}{\lambda_s}$ implies $r_\mu^B \leq \max\{\frac{1}{2}, 1 - \lambda_a\}$.

Suppose that $r_\mu^B > 1 - \lambda_a$. Then, by the equilibrium condition (14), r_μ^B is a unique solution of

$$e^z = 1 + \frac{e^{\frac{z}{r}} - 1}{\frac{1-r}{r}e^{\frac{y+z}{r}}e^{-\frac{zv}{r}} + 1} \equiv h_2(r).$$

At $r = \frac{1}{2}$,

$$\begin{aligned}e^z \geq h_2(1/2) &\iff e^{2(y+z)}e^{-2zv} + 1 \geq \frac{e^{2z} - 1}{e^z - 1} = e^z + 1 \\ &\iff 2(y + z - zv) > z \iff v \leq \frac{1}{2} + \frac{y}{z} = \frac{1}{2} + \frac{\lambda_b}{\lambda_s}.\end{aligned}$$

Claim 2 proved that $h_2(r)$ is single-crossing e^z from above to below as r increases in $[\hat{r}, 1)$. Hence, $r_\mu^B \leq 1/2$.

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