

# Supplemental Appendix of “Asset Reallocation in Markets with Intermediaries Under Selling Pressure”

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## Abstract

In this supplemental appendix, we perform a calibration exercise to evaluate the quantitative significance of our theoretical results applied to the mid-size US corporate acquisition market. A typical transaction involves a sale of assets – either all the corporation assets or a subdivision – by corporate investors or by private equity (PE) buyout funds.

## S.1 Calibration

The mid-size corporate acquisition market lends itself naturally to a search framework. Corporate sellers take about a year to find an appropriate buyer and close a transaction (Boone and Mulherin (2011)). A typical PE buyout fund acquires a small number of portfolio firms, holds the firms as inventory and adds operational value through better management, and exits by selling their portfolio firms to provide liquidity to fund investors.<sup>1</sup> PE funds divest the acquired firms to corporate investors through primary buyouts (PBOs), or to other PE funds through secondary buyouts (SBOs), each of which accounted for 52% and 42% of PE exits in 2017. PE buyout funds have become pivotal players in the US corporate acquisition

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<sup>1</sup>We distinguish buyout funds from other PE funds such as venture capital funds, which usually invest in fractional equity stakes of start-ups and early-stage firms.

market. In 2017, their investment represents \$538 billion out of the \$2.1 trillion (or 4,053 out of 10,769 deals) with a compounded annual growth rate of 7.5% since 2011.<sup>2</sup>

It is worth noting that private equity funds in practice have some unique features that our model omits. The funds can hold multiple assets, directly improve the fundamental value of the holding assets, and anticipate selling pressure to arrive at the end of their life (Kaplan and Stromberg (2009)). Nonetheless, the corporate acquisition market is an interesting new application of the OTC literature, and the calibration exercise of our model can potentially uncover new empirical insights.

We address various quantitative questions such as: (i) Is there an over or undersupply of tradable assets relative to the market demand (i.e., the number of high-type investors and PE funds)? (ii) How are fund valuations affected by liquidity provision through SBOs and fund managers' operational improvement? (iii) What is the impact of PE entry on fund valuation and transaction prices? (iv) What are the welfare losses associated with search frictions?

Our model has 14 exogenous parameters:  $n = (n_v, n_f, n_a)$ ,  $u = (u_l, u_h, u_f, u_e)$ ,  $\rho = (\rho_u, \rho_d, \rho_e)$ ,  $\lambda = (\lambda_d, \lambda_f, \lambda_s)$  and  $r$ . Some parameters  $(n, u)$  are either directly observed or obtained from other empirical studies, and the remaining seven parameters  $(\rho, \lambda, r)$  are estimated to fit the model statistics in Table S.1 to the data. The calibrated parameters are reported in Table S.2.

### S.1.1 Data and Methodology

We focus on acquisitions by mid-size US companies, defined as companies with annual revenues between \$20 million and \$1,000 million. We exclude small companies, for which there is no reliable data on acquisition activities, and large companies to maintain an overall homogeneity in our sample set. According to the latest available U.S. Economic Census Data (2012), there are approximately 102,626 mid-size companies. We normalize  $n_v = 1$  and apply the same rescaling to the number of PE funds, assets, trading volumes, etc. dividing by the number of companies.

The primary data is from the 2018 US PE middle market report by Pitchbook Data Inc. The data includes corporate acquisition deals with transaction values between \$25 million and \$1,000 million. There are 1,893 PE funds targeting the middle market (thus  $n_f \simeq 0.02$

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<sup>2</sup>The data on PE activities in the corporate acquisition market from PitchBook Data, Inc., can be found at <https://pitchbook.com/news/reports/2q-2018-ma-report>.

Description	Data	Model Statistic (for normalized values)
Corporate (Direct) acquisitions	6387	$\eta_{lo-hn} = \lambda_d \mu_{lo} \mu_{hn}$
Primary Buyouts (PBOs)	1440	$\eta_{lo-fn} = \lambda_f \mu_{lo} \mu_{fn}$
Secondary Buyouts (SBOs)	359	$\eta_{fn-fe} = \lambda_s \mu_{fn} \mu_{fe}$
Avg time to sell for corporate investors	1.25 years	$E[\tau_{sv}]$ (eq. (4))
Avg time to sell for PE funds	0.91 years	$E[\tau_{sf}]$ (eq. (5))
Fund performance (PME)	1.01	PME (eq. (S.1))
Price multiple (EV/EBITDA)	9.0	$P_{lo-hn}/u_l$

Table S.1: Key Statistics on Corporate Acquisitions and Private Equity

with normalization).<sup>3</sup> The dataset also provides the transaction volumes for various kinds of trading, summarized in Table S.1, along with other model statistics. The number of direct transactions among companies is 9,626 per-year from 2007 to 2017. In the same period, PE funds acquired an average of 1,799 firms per year, of which 359 ( $\approx 20\%$ ) are through SBOs and 1,440 are through PBOs. The steady state of our model implies that the number of PE buyouts must be equal to the number of exits (i.e,  $\eta_{fe-hn} + \eta_{fo-hn} = \eta_{lo-fn}$ ). Then, the total number of deals equals the sum  $\eta_{lo-hn} + 2\eta_{lo-fn} + \eta_{fe-fn}$ , which gives us an estimate of 6,387 direct transactions ( $\eta_{lo-hn}$ ) per year. The trade volumes allow us to identify the search intensities ( $\lambda_d, \lambda_f, \lambda_s$ ).

For the average time to sell by corporate investors and PE funds ( $E[\tau_{sv}]$  and  $E[\tau_{sf}]$  in Lemma 4), we resort to various reports available online and take their average. According to reports prepared by selling agents such as business brokers or investment bankers (see Section S.3 for our references), selling a firm takes an average of 11 months (0.91 years) for PE funds and 15 months (1.25 years) for corporate investors.<sup>4</sup>

A common performance measure of PE funds is the Public Market Equivalent (PME), introduced by Kaplan and Schoar (2005) (see also Sorensen and Jagannathan (2015)) and defined as

$$PME = \frac{\text{Present value of distributions to fund investors}}{\text{Present value of capital calls made by fund investors}}. \quad (\text{S.1})$$

<sup>3</sup>Pitchbook reports the number of PE funds raised in each year targeting the middle market. We cumulate these numbers for 2007-2018, as the average lifespan of PE funds is 12 years (Metrick and Yasuda (2010)).

<sup>4</sup>We include both time taken in the preparation process and the listing-to-sale process. The preparation process for PE funds takes only an average of 2 months – much shorter than an average of 6 months for corporate investors. Portfolio firms of PE funds are usually in a better state of readiness to approach the market due to high-quality governance, accounting, and information systems. The listing-to-sale process takes an average of 9 months for selling agents.

	Parameters	Variable	Value
(Observed)	No. of corporate investors	$n_v$	1.0
	No. of PE funds	$n_f$	0.02
	No. of assets	$n_a$	0.5
	Flow Payoff low type	$u_l$	1
	Flow payoff high type	$u_h$	1.4
	Flow payoff PE (harvesting)	$u_f$	1.3
	Flow payoff PE (exiting)	$u_e$	1.1
(Estimated)	Low valuation shock	$\rho_d$	0.23
	High valuation shock	$\rho_u$	0.16
	Liquidity shock	$\rho_e$	0.38
	Match intensity (direct trading)	$\lambda_d$	46.1
	Match intensity (PBO)	$\lambda_f$	61.7
	Match intensity (SBO)	$\lambda_s$	699
	Discount rate	$r$	11.8%

Table S.2: Fitted Parameters of Calibration

PME is the ratio of cash outflows over cash contributions, both discounted at the public market total return (e.g., S&P 500 index) after subtracting management fees paid to the fund managers – see Section S.2 for a derivation of the closed-form expression.<sup>5</sup> A PME of one is indicative of a fund’s performance in line with the public market, and a lower PME indicates underperformance. We take the average PME of 1.01 from various estimates.<sup>6</sup>

The most widely used metric for the relative acquisition prices is the EV/EBITDA multiple. This transaction multiple, which is matched to our model’s  $P_{lo-hn}/u_l$ , is on average 9.0 from 2005 to 2017.<sup>7</sup>

We now turn to the payoff parameters. Low-type corporate investors’ flow payoff is normalized as  $u_l = 1$ . For high-type corporate investors’ flow payoff, we use Muscarella and Vetsuypens (1990), Opler (1992) and Andrade and Kaplan (1998). Each of these papers estimate the increase in operating profits (or cash flows) for firms after fund buyouts as 23.5%, 16.5%, and 52.9%, respectively. We take the average and set  $u_f = 1.3$  (i.e., 30%

<sup>5</sup>According to Metrick and Yasuda (2010), the management fees are usually 2% of the committed capital and paid from the inception of the fund until its liquidation.

<sup>6</sup>Kaplan and Schoar (2005) estimate an average PME of 0.93 for PE funds in the period 1980-1994, while Phalippou and Gottschalg (2008), using similar dataset but different methodology, report an average PME of 0.88. Harris, Jenkinson, and Kaplan (2014), on the other hand, report significantly better performance with an average PME of 1.22 for the period 1984-2008. The estimates of PME by PitchBook Data, Inc. yields an average of 1.00 for the period 2006-15.

<sup>7</sup>See a recent report on EV/EBITDA by FactSet Research Systems Inc. at [https://www.factset.com/hubfs/mergerstat\\_em/monthly/US-Flashwire-Monthly.pdf](https://www.factset.com/hubfs/mergerstat_em/monthly/US-Flashwire-Monthly.pdf).

increase from  $u_l$ ), which is also close to the average 28.5% of the takeover premium paid by PE acquirers (Bargeron, Schlingemann, Stulz, and Zutter, 2008). For the flow payoff net of liquidity cost  $u_e$ , Nadauld, Sensoy, Vorkink, and Weisbach (2016) find that fund investors under liquidity shocks sell their PE ownership to other fund investors at a 13.8% discount. This observation motivates our choice of  $u_e = (1 - 0.138) \times u_f \simeq 1.1$ . Last, it is difficult to observe the payoff improvement from acquisitions ( $u_h - u_l$ ) because the target's and acquirer's operations often blend together. As such, we seek an indirect evidence from the premium paid by acquirers. Betton, Eckbo, and Thorburn (2008) reports an average 43% takeover premium over 4,880 acquisitions during 1980-2002, and Bargeron, Schlingemann, Stulz, and Zutter (2008) find that the takeover premium paid by a private acquirer is 40.9%. From these estimates, we choose  $u_h = 1.4$ .

We do not directly observe the number of assets. In our benchmark analysis, we set  $n_a = 0.5$ . The estimates of the parameter values in Table S.2 with  $n_a = 0.25$  and  $n_a = 0.75$  change insignificantly, except for  $\rho_d$  and  $\rho_u$ .

We estimate the remaining parameters  $\beta \equiv (\rho, \lambda, r)$  that best explain the key statistics in Table S.1. Each choice of the remaining parameters' values, together with the directly observed parameters  $(n, u)$ , defines a market  $\theta = (n, r, u, \rho, \lambda)$ . We compute the statistics  $Y_i(\beta; n, u)$  for each row  $i = 1, \dots, 7$  in Table S.1 and compare them with the observed data  $Y_i^{obs}$ . The estimate of  $\beta$  minimizes the sum of squared residuals (SSR), subject to positive trade gains in the unique steady-state solution of the market  $(\beta, n, u)$ :

$$\min_{\beta} SSR(\beta; n, u) \equiv \sum_{i=1}^7 \left( \frac{Y_i(\beta; n, u) - Y_i^{obs}}{Y_i^{obs}} \right)^2$$

subject to  $g_m(\beta; n, u) \geq 0$ , for each  $m \in \mathcal{M}$ .

The lower section of Table S.2 summarizes the parameter estimates. Our model fits the observed data with a high degree of accuracy: the minimum SSR is approximately  $2.9 \times 10^{-5}$ .

The parameter estimates are of reasonable magnitudes. The estimated type transition rates  $\rho_u$  and  $\rho_d$  suggest that the type transitions (from high to low, and vice versa) happen within 6.3 to 4.3 years on average. The meeting rates  $\lambda = (\lambda_d, \lambda_f, \lambda_s)$  are of a much higher order, due to normalization, as discussed in Section 5. The meeting rate for funds is higher than corporate investors ( $\lambda_f > \lambda_d$ ). The calibrated inter-PE meeting rate  $\lambda_s$  is much higher than the other two, reflecting the high volume of SBOs relative to the small fraction of PE funds. Lastly, the estimated discount rate  $r = 11.8\%$ , although high, seems reasonable given

	Description	Symbol	Value
(i)	Oversupply of assets	$n_a - (n_h + n_f)$	0.08
(ii)	Impact of SBOs on fund valuation (changing of $\lambda_s$ from 0 to 699)	$\frac{\Delta v_{fn}}{v_{fn}}$	26.2%
	Sensitivity of fund value to operations	$\frac{\partial \log v_{fn}}{\partial \log u_f}$	4.22
	Sensitivity of fund value to liquidity shocks	$\frac{\partial \log v_{fn}}{\partial \log u_e}$	0.52
(iii)	Impact of PE entry on fund valuation (changing the number of funds from $n_f$ to $2n_f$ )	$\frac{\Delta v_{fn}}{v_{fn}}$	0.7%
	Impact of PE entry on transaction prices (changing the number of funds from $n_f$ to $2n_f$ )	$\frac{\Delta P_{lo-hn}}{P_{lo-hn}}$	-0.9%
(iv)	Welfare gain by asset reallocations (relative to the welfare with no asset reallocations)	$\frac{W - \bar{W}}{\bar{W}}$	13.6%
	Welfare gain by asset reallocations (relative to the maximum welfare gain)	$\frac{W - \bar{W}}{\bar{W} - \bar{W}}$	92.1%
	corporate investors' share of the welfare gain	$\frac{W_v - \bar{W}}{\bar{W} - \bar{W}}$	73.3%

Table S.3: Calibration Results

that assets represent stakes in mid-size private firms.

**Results** The calibration results in Table S.3 answer the quantitative questions (i)-(iv) at the beginning of this section.

First, there is an oversupply of assets. The calibrated  $\rho_u$  and  $\rho_d$  imply  $n_h \equiv \frac{\rho_u}{\rho_u + \rho_d} = 0.40$  and an excess supply of assets  $n_a > n_h + n_f$ . The result, together with large meeting rates, suggests that the US corporate acquisition market is close to Case C of the fast-search market (Proposition 7).<sup>8</sup>

Second, SBOs significantly improve fund valuations, as explained with Figure 2. PE fund values with SBOs are 26.2% higher than without SBOs, highlighting the complementarities among PE funds. While we acknowledge the criticism against SBOs, maybe it is because of (and not in spite of) SBOs that PE funds generate high returns. An improvement in firms' operation  $u_f$  by 1% leads to a significant 4.22% increase in fund value  $v_{fn}$ . However, a similar improvement of  $u_e$  is attenuated by a vibrant SBO market and has a negligible influence on fund value: sensitivity is 0.52.

Third, when the number of PE funds increases, the percentage of SBOs relative to the total fund exits would increase (Figure 3). This pattern is indeed observed in the data. The

<sup>8</sup>The calibration result on the oversupply of assets is not sensitive to our choice of  $n_a = 0.5$ . While a choice of  $n_a = 0.25$  or  $0.75$  results in different estimates of  $\rho_u$  and  $\rho_d$ , the oversupply remains about the same ( $n_a - (n_h + n_f) \approx 0.08$ ).

share of firms sold by PE funds through SBOs has increased from 13% in the 1980s, 19% in 2009, to 42% in 2017. Doubling the number of PE funds would increase the average fund value by 0.7%, suggesting that the complementarities among funds plays a stronger role than the competitive effect. Doubling the number of funds leads to only a 0.9% decrease in direct transaction prices. This is because more purchases by funds empower selling investors and more sales by funds increase buying investors' bargaining position.

Finally, the welfare gain by asset reallocations is 13.6%, relative to the autarkic situation welfare  $\underline{W}$  (see page 20). This welfare gain ( $W - \underline{W}$ ) attains 92.1% of the best possible gain ( $\overline{W} - \underline{W}$ ). This fraction is lower than the gain in OTC markets for municipal bonds as described in Hugonnier, Lester, and Weill (2020), likely due to higher search frictions in the corporate acquisition market. The corporate investors' percentage share of this welfare gain is 73.3%, which leaves 26.7% to PE funds. The PE funds' welfare share is very large relative to their small number  $n_f = 0.02$ .

## S.2 Public Market Equivalent (PME)

We provide closed-form expressions of PME. Consider a fund that does not hold an asset in a steady-state equilibrium. The fund takes  $\tau_b$  period of time until purchasing an asset at a price of  $P_b$  and takes  $\tau_s$  period of time (after purchasing) until selling the asset at a price  $P_s$ . Let  $u(t) \in \{u_f, u_e\}$  denote the payoff flow while holding the asset at  $t \in [0, \tau_s]$ .

We modify Sorensen and Jagannathan (2015)'s definition of PME in discrete time with a stochastic discount. For our case of continuous time and deterministic discount, we define PME as

$$PME \equiv \frac{\text{Present value of distributions to fund investors}}{\text{Present value of capital calls made by fund investors}} = \frac{PV_{\text{dist}}}{PV_{\text{calls}}},$$

where

$$PV_{\text{dist}} \equiv E \left[ e^{-r\tau_b} \int_0^{\tau_s} e^{-rt} u(t) dt + e^{-r\tau_s} P_s \right],$$

$$PV_{\text{calls}} \equiv PV_{\text{purchasing price}} + PV_{\text{management fees}} = E [P_b e^{-r\tau_b}] + E \left[ (fP_b) \int_0^{\tau_b + \tau_s} e^{-rt} dt \right].$$

The management fees are paid retrospectively, as if the flow of fees which equals a fraction of the fund size (i.e.,  $fP_b$ ) is paid throughout the fund's lifetime. For calibration, we set  $f \simeq 2\%$  based on Metrick and Yasuda (2010), which finds that management fees are usually

2% of committed capital and paid from the inception of a fund until its liquidation. For discount rate  $r$ , Kaplan and Schoar (2005) use the return on the S&P 500, whereas we use our estimate of the same.

First, we obtain the closed-form expression of  $PV_{\text{dist}}$ . Since the time to purchase,  $\tau_b$ , is independent of the time to sell  $\tau_s$  (post-purchase) and the selling price  $P_s$ ,

$$PV_{\text{dist}} = E[e^{-r\tau_b}] E\left[\int_0^{\tau_s} e^{-rt} u(t) dt + e^{-r\tau_s} P_s\right].$$

A purchase of an asset occurs on meeting a corporate investor or a fund at the exit phase, whichever happens first ( $\tau_b \equiv \min\{\tau_{lo-fn}, \tau_{fe-fn}\}$ ).  $\tau_b$  follows an exponential distribution with parameter  $\lambda_f \mu_{lo} + \lambda_s \mu_{fe}$ . As such,

$$E[e^{-r\tau_b}] = \frac{\lambda_f \mu_{lo} + \lambda_s \mu_{fe}}{\lambda_f \mu_{lo} + \lambda_s \mu_{fe} + r}.^9$$

The fund can sell either (i) before receiving a liquidity shock to a corporate investor, or (ii) after receiving a liquidity shock to either a corporate investor or a fund buyer. The expected continuation payoff, upon receiving a liquidity shock before selling an asset, is

$$V_e \equiv E\left[u_e \left(\int_0^{\tau_e} e^{-rt} dt\right) + e^{-r\tau_e} P_e\right],$$

where  $\tau_e$  denotes the time that the fund remains as type  $fe$ , and  $P_e$  denotes the selling price. Note that  $\tau_e \equiv \min\{\tau_{fe-hn}, \tau_{fe-fn}\}$  follows an exponential distribution with parameter  $\lambda_f \mu_{hn} + \lambda_s \mu_{fn}$ . The probability of selling to a corporate investor  $\frac{\lambda_f \mu_{hn}}{\lambda_f \mu_{hn} + \lambda_s \mu_{fn}}$  is independent of the selling time  $\tau_e$ . Thus

$$\begin{aligned} V_e &= \frac{u_e}{\lambda_f \mu_{hn} + \lambda_s \mu_{fn} + r} + \frac{\lambda_f \mu_{hn} + \lambda_s \mu_{fn}}{\lambda_f \mu_{hn} + \lambda_s \mu_{fn} + r} \frac{\lambda_f \mu_{hn} P_{fe-hn} + \lambda_s \mu_{fn} P_{fe-fn}}{\lambda_f \mu_{hn} + \lambda_s \mu_{fn}} \\ &= \frac{u_e + \lambda_f \mu_{hn} P_{fe-hn} + \lambda_s \mu_{fn} P_{fe-fn}}{\lambda_f \mu_{hn} + \lambda_s \mu_{fn} + r}. \end{aligned}$$

Similarly, an  $fo$  type fund receives a payoff flow  $u_f$  during a lifetime spanning  $\tau_{fo} \equiv \min\{\tau_{fo-hn}, \tau_e\}$ . Eventually, the fund either sells its asset to a buying investor at price

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<sup>9</sup>We use (i)  $\int_0^{\bar{t}} e^{-rt} dt = -\frac{e^{-rt}}{r} \Big|_0^{\bar{t}} = \frac{1-e^{-r\bar{t}}}{r}$ , (ii) for  $x \sim \exp(\alpha)$ ,  $E[e^{-rx}] = \int_0^\infty e^{-rx} \alpha e^{-\alpha x} dx = \frac{\alpha}{\alpha+r}$ , and (iii) for  $x \sim \exp(\alpha)$ ,  $\int_0^x e^{-rt} dt = E\left[\frac{1-e^{-rx}}{r}\right] = \frac{1}{\alpha+r}$ .



$P_{fo-hn}$  or receives a liquidity shock and a continuation payoff  $V_e$ . Thus,

$$E \left[ \int_0^{\tau_s} e^{-rt} u(t) dt + e^{-r\tau_s} P_s \right] = \frac{u_f + \lambda_f \mu_{hn} P_{fo-hn} + \rho_e V_e}{\lambda_f \mu_{hn} + \rho_e + r}.$$

It follows that

$$PV_{\text{dist}} = \left( \frac{\lambda_f \mu_{lo} + \lambda_s \mu_{fe}}{\lambda_f \mu_{lo} + \lambda_s \mu_{fe} + r} \right) \left( \frac{u_f + \lambda_f \mu_{hn} P_{fo-hn} + \rho_e \left( \frac{u_e + \lambda_f \mu_{hn} P_{fe-hn} + \lambda_s \mu_{fn} P_{fe-fn}}{\lambda_f \mu_{hn} + \lambda_s \mu_{fn} + r} \right)}{\lambda_f \mu_{hn} + \rho_e + r} \right).$$

Second, we find the closed-form expression of  $PV_{\text{calls}}$ . The time taken to buy  $\tau_b$ , the time taken to sell  $\tau_s$ , and the event of purchasing from a low-type investor, rather than an exiting fund, are all independent from each other. Thus,

$$PV_{\text{calls}} = E[P_b] E[e^{-r\tau_b}] + E[(fP_b)] E \left[ \int_0^{\tau_b + \tau_s} e^{-rt} dt \right],$$

where

$$\begin{aligned} E \left[ \int_0^{\tau_b + \tau_s} e^{-rt} dt \right] &= E \left[ \int_0^{\tau_b} e^{-rt} dt \right] + E \left[ \int_{\tau_b}^{\tau_b + \tau_s} e^{-rt} dt \right] \\ &= E \left[ \int_0^{\tau_b} e^{-rt} dt \right] + E[e^{-r\tau_b}] E \left[ \int_0^{\tau_s} e^{-rt} dt \right]. \end{aligned}$$

Note that

$$\begin{aligned} E[P_b] &= \frac{\lambda_f \mu_{lo} P_{lo-fn} + \lambda_s \mu_{fe} P_{fe-fn}}{\lambda_f \mu_{lo} + \lambda_s \mu_{fe}}, \\ E[e^{-r\tau_b}] &= \frac{\lambda_f \mu_{lo} + \lambda_s \mu_{fe}}{\lambda_f \mu_{lo} + \lambda_s \mu_{fe} + r}, \quad \text{and} \\ E \left[ \int_0^{\tau_b} e^{-rt} dt \right] &= \frac{1}{\lambda_f \mu_{lo} + \lambda_s \mu_{fe} + r}. \end{aligned}$$

Last, recall that a fund's type remains  $fo$  or  $fe$  for the time period  $\tau_{fo} \equiv \min\{\tau_{fo-hn}, \tau_e\}$  or  $\tau_e \equiv \min\{\tau_{fe-hn}, \tau_{fe-fn}\}$ , respectively. Then,

$$\begin{aligned}
E \left[ \int_0^{\tau_s} e^{-rt} dt \right] &= E \left[ \int_0^{\tau_{fo}} e^{-rt} dt \right] + E \left[ \mathbf{1}_{\tau_{fo}=\tau_e} \right] E \left[ e^{-r\tau_{fo}} \right] E \left[ \int_0^{\tau_e} e^{-rt} dt \right] \\
&= \frac{1}{\lambda_f \mu_{hn} + \rho_e + r} + \frac{\rho_e}{\lambda_f \mu_{hn} + \rho_e} \frac{\lambda_f \mu_{hn} + \rho_e}{\lambda_f \mu_{hn} + \rho_e + r} \frac{1}{\lambda_f \mu_{hn} + \lambda_s \mu_{fn} + r}.
\end{aligned}$$

It follows that

$$PV_{\text{calls}} = \frac{\lambda_f \mu_{lo} P_{lo-fn} + \lambda_s \mu_{fe} P_{fe-fn}}{\lambda_f \mu_{lo} + \lambda_s \mu_{fe} + r} \left( 1 + f \left( \frac{1}{\lambda_f \mu_{lo} + \lambda_s \mu_{fe}} + \frac{1 + \rho_e \left( \frac{1}{\lambda_f \mu_{hn} + \lambda_s \mu_{fn} + r} \right)}{\lambda_f \mu_{hn} + \rho_e + r} \right) \right).$$

### S.3 Various Estimates of the Time to Sell

A sale of a private firm consists of two major processes: the preparation and the listing-to-sale process. The preparation takes less time if a firm already has high-quality accounting and information systems, which is the case of PE-backed firms (Kaplan and Stromberg (2009)). The preparation for PE-backed firms takes an average of 2 months, while other firms need an average of 6 months (see the upper part of Table S.4). The listing-to-sale process takes about 9 months for various selling agents (see the lower part of Table S.4). We set the total time for selling a firm as 11 months for PE funds and 15 months for corporate investors.

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Ave. Time Taken	Source
<b>For preparations</b>	
1-6 months	<a href="https://www.highrockpartners.com/how-long-does-it-take-to-sell-a-company/">https://www.highrockpartners.com/how-long-does-it-take-to-sell-a-company/</a>
12 months	<a href="https://www.businessinsider.com/11-stages-of-selling-a-company-2011-4">https://www.businessinsider.com/11-stages-of-selling-a-company-2011-4</a>
<b>From listing to sale</b>	
6-9 months	<a href="https://www.mabusinessbrokers.com/blog/how-long-does-it-take-to-sell-a-business">https://www.mabusinessbrokers.com/blog/how-long-does-it-take-to-sell-a-business</a>
9 months	<a href="https://www.exitadviser.com/seller-status.aspx?id=long-does-take-sell">https://www.exitadviser.com/seller-status.aspx?id=long-does-take-sell</a>
9 months	<a href="https://www.allbusiness.com/how-long-does-it-take-to-sell-a-business-2-6592268-1.html">https://www.allbusiness.com/how-long-does-it-take-to-sell-a-business-2-6592268-1.html</a>
12 months	<a href="https://www.businessinsider.com/11-stages-of-selling-a-company-2011-4">https://www.businessinsider.com/11-stages-of-selling-a-company-2011-4</a>
9 months	<a href="https://www.moorestephens.co.uk/msuk/moore-stephens-south/news/july-2017-(1)/how-long-does-it-take-to-sell-a-small-business">https://www.moorestephens.co.uk/msuk/moore-stephens-south/news/july-2017-(1)/how-long-does-it-take-to-sell-a-small-business</a>
9 months	<a href="https://www.tvba.co.uk/article/how-long-does-it-take-to-sell-a-company">https://www.tvba.co.uk/article/how-long-does-it-take-to-sell-a-company</a>
6-9 months	<a href="https://www.simonscottcmc.co.uk/blog/long-take-sell-business/">https://www.simonscottcmc.co.uk/blog/long-take-sell-business/</a>
10 months	<a href="https://www.ibgbusiness.com/tips-sell-business-long-take-sell-business/">https://www.ibgbusiness.com/tips-sell-business-long-take-sell-business/</a>
10 months	<a href="https://www.highrockpartners.com/how-long-does-it-take-to-sell-a-company/">https://www.highrockpartners.com/how-long-does-it-take-to-sell-a-company/</a>

Table S.4: Estimated time to sell a firm

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