Business Report - 2

PG Program in Data Science and Business Analytics

submitted by

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1 Problem 1

A physiotherapist with a male football team is interested in studying the relationship between foot injuries and the positions at which the players play from the data collected:

Position	**Players Injured**	**Players Not Injured**	**Total**
Striker	45	32	77
Forward	56	38	94
Attacking Midfielder	24	11	35
Winger	20	9	29
Total	**145**	**90**	**235**

Table 1: Data on Players and Injuries

1.1 What is the probability that a randomly chosen player would suffer an injury?

The probability that a randomly chosen player would suffer an injury is calculated as follows:

$$P(\text{Injured}) = \frac{\text{Total Injured Players}}{\text{Total Players}} = \frac{145}{235}$$

Simplifying this gives:

$$P(\text{Injured}) = 0.617$$

Thus, the probability that a randomly chosen player would suffer an injury is **0.617** or 61.7%.

1.2 What is the probability that a player is a forward or a winger?

The probability that a randomly chosen player is either a forward or a winger is:

$$P(\text{Forward or Winger}) = \frac{\text{Total Forwards} + \text{Total Wingers}}{\text{Total Players}} = \frac{94 + 29}{235}$$

Simplifying this gives:

$$P(\text{Forward or Winger}) = \frac{123}{235} = 0.523$$

Thus, the probability that a randomly chosen player is a forward or a winger is **0.523** or 52.3%.

1.3 What is the probability that a randomly chosen player plays in a striker position and has a foot injury?

The probability that a randomly chosen player plays in a striker position and has a foot injury is:

$$P(\text{Striker and Injured}) = \frac{\text{Injured Strikers}}{\text{Total Players}} = \frac{45}{235}$$

Simplifying this gives:

$$P(\text{Striker and Injured}) = 0.191$$

Thus, the probability that a randomly chosen player plays in a striker position and has a foot injury is **0.191** or 19.1%.

1.4 What is the probability that a randomly chosen injured player is a striker?

The probability that a randomly chosen injured player is a striker is:

$$P(\text{Striker} \mid \text{Injured}) = \frac{\text{Injured Strikers}}{\text{Total Injured Players}} = \frac{45}{145}$$

Simplifying this gives:

$$P(\text{Striker} \mid \text{Injured}) = 0.310$$

Thus, the probability that a randomly chosen injured player is a striker is **0.310** or 31.0%.

2 Problem 2

2.1 What proportion of the gunny bags have a breaking strength of less than 3.17 kg per sq cm?

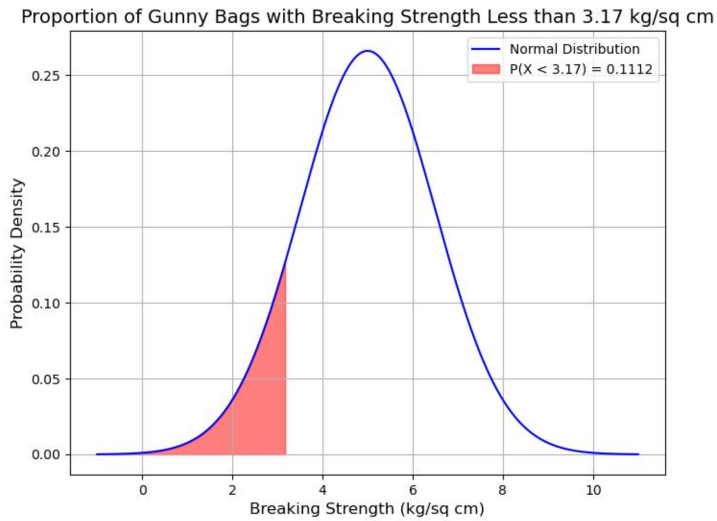


Figure 1: Proportion of Gunny Bags with Breaking Strength Less than 3.17 kg/sq cm

Here is the visualization that shows the normal distribution of breaking strength for the gunny bags. The shaded red area represents the proportion of gunny bags with a breaking strength of less than 3.17 kg per sq cm, which is approximately 11.12%.

2.2 What proportion of the gunny bags have a breaking strength of at least 3.6 kg per sq cm.?

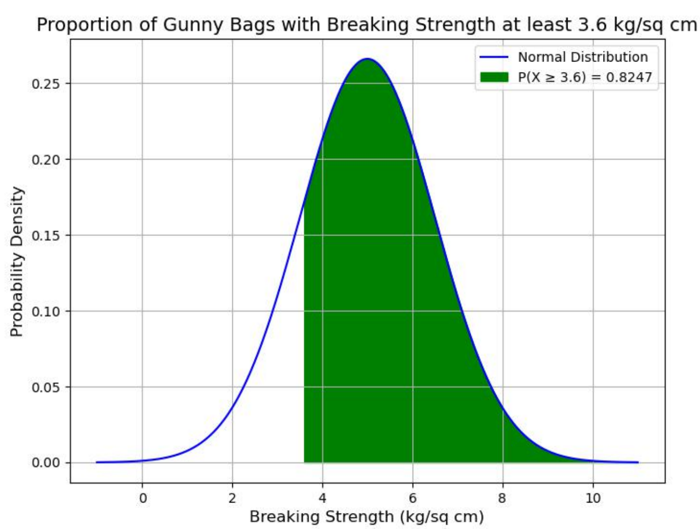


Figure 2: Proportion of Gunny Bags with Breaking Strength of at least 3.6 kg per sq cm.

Here is the visualization that shows the normal distribution of breaking strength for the gunny bags. The shaded green region shows the proportion of bags whose strength is 3.6 kg/sq cm or more, representing about 82.47%.

2.3 What proportion of the gunny bags have a breaking strength between 5 and 5.5 kg per sq cm.?

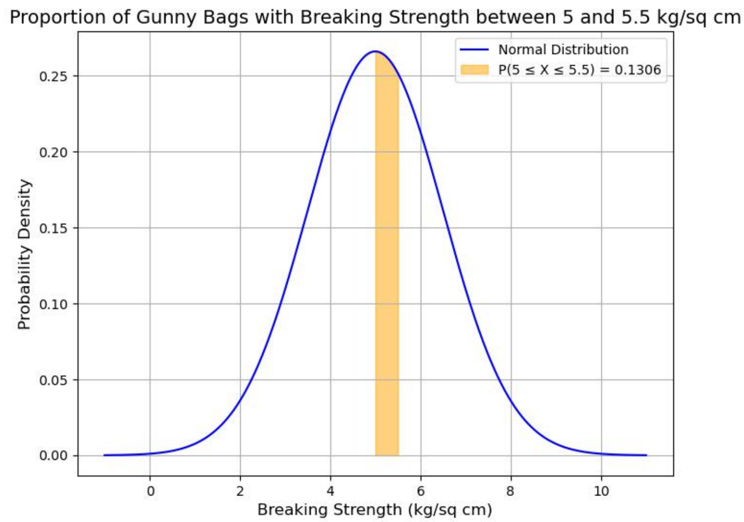


Figure 3: Proportion of Gunny Bags with Breaking Strength between 5 and 5.5 kg per sq cm.

Here is the visualization that shows the normal distribution of breaking strength for the gunny bags. The shaded orange region shows the proportion of bags within this range, approximately 13.06%.

2.4 What proportion of the gunny bags have a breaking strength NOT between 3 and 7.5 kg per sq cm.?

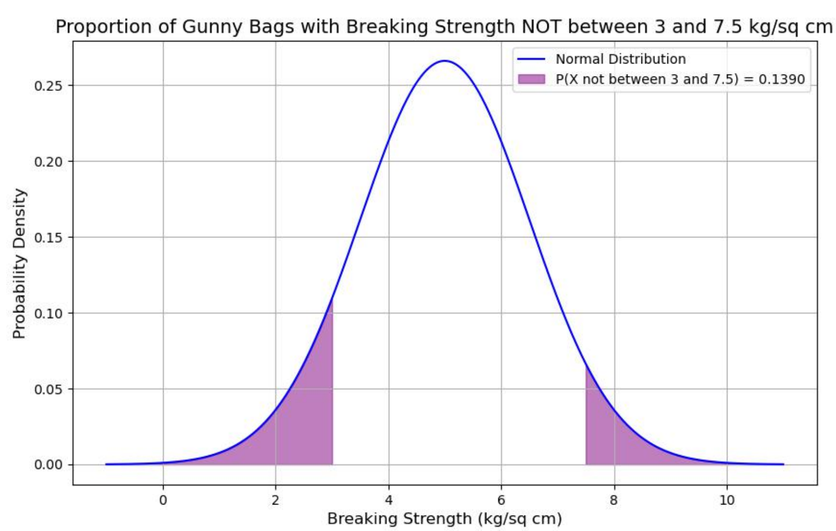


Figure 4: Proportion of Gunny Bags with Breaking Strength NOT between 3 and 7.5 kg per sq cm.

Here is the visualization that shows the normal distribution of breaking strength for the gunny bags. The shaded purple regions show the proportion of bags whose strength falls outside the 3 to 7.5 kg/sq cm range, about 13.9%.

3 Problem 3

3.1 Zingaro has reason to believe that the unpolished stones may not be suitable for printing. Do you think Zingaro is justified in thinking so?

Null Hypothesis (H_0): The mean Brinell hardness index of unpolished stones is greater than or equal to 150 (i.e., the stones are suitable for printing).

$$H_0 : \mu \geq 150 \quad \textbf{(Unpolished stones are suitable for printing)}$$

Alternative Hypothesis (H_A): The mean Brinell hardness index of unpolished stones is less than 150 (i.e., the stones may not be suitable for printing).

$$H_A : \mu < 150 \quad \textbf{(Unpolished stones are not suitable for printing)}$$

Significance Level: We set the significance level at $\alpha = 0.05$.

Conducting the t-test: We use a one-sample t-test to compare the sample mean hardness of unpolished stones to the population mean of 150. The t-test provides a t-statistic and p-value to be -4.16 and 4.17e-05 respectively.

Decision Rule: - The p-value is less than $\alpha = 0.05$ hence we reject the null hypothesis i.e., the stones may not be suitable for printing

3.2 Is the mean hardness of the polished and unpolished stones the same?

Hypotheses: Null Hypothesis (H_0): The mean Brinell hardness index of polished stones is equal to the mean Brinell hardness index of unpolished stones.

$$H_0 : \mu_{polished} = \mu_{unpolished}$$

Alternative Hypothesis (H_A): The mean Brinell hardness index of polished stones is different from the mean Brinell hardness index of unpolished stones.

$$H_A : \mu_{polished} \neq \mu_{unpolished}$$

Significance Level: We set the significance level at $\alpha = 0.05$.

Test Statistic: We use a two-sample t-test to compare the means of the two groups. The t-statistic is calculated as:

$$t = \frac{\bar{x}_{polished} - \bar{x}_{unpolished}}{\sqrt{\frac{s^2_{polished}}{n_{polished}} + \frac{s^2_{unpolished}}{n_{unpolished}}}}$$

Decision Rule: - The t-test provides a t-statistic and p-value to be -3.24 and 0.0016 respectively. The p-value is less than $\alpha = 0.05$ hence we reject the null hypothesis i.e.,The mean Brinell hardness index of polished stones is different from the mean Brinell hardness index of unpolished stones.

4 Problem 4

4.1 How does the hardness of implants vary depending on dentists?

In this analysis, we aim to determine how the hardness of dental implants varies depending on the dentists. The response variable of interest is the hardness of dental implants, and we will analyze this separately for Alloy 1 and Alloy 2.

Significance Level: We set the significance level at $\alpha = 0.05$. For both types of alloys, we consider the following hypotheses:

4.1.1 Null Hypothesis (H_0)

The hardness of implants does not differ between dentists:

$$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$$

4.1.2 Alternate Hypothesis (H_A)

The hardness of implants differs between at least one pair of dentists:

$$H_A : \textbf{At least one } \mu_i \textbf{ differs from the others.}$$

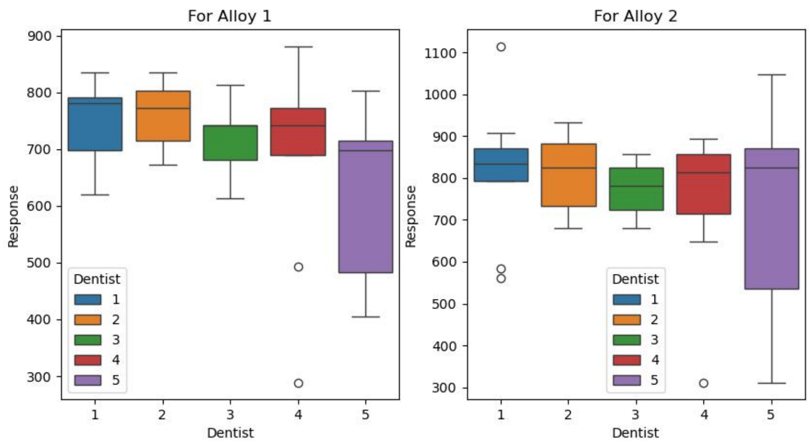


Figure 5: Hardness of implants vary depending on dentists for two different alloys

4.1.3 Assumptions of the One-Way ANOVA

Before conducting the ANOVA, the following assumptions must be checked:

- **Normality**: The response variable (hardness) is normally distributed for each group (dentist).
- **Homogeneity of variance**: The variances of hardness between dentists are equal.
- **Independence**: Observations are independent.

Note: The p-values of Shapiro's test indicate the distribution is normal for both alloys (except one case for Alloy 2 with Dentist 4) and the population variances are equal for both alloys between dentists. Even if the assumptions are not fully met, we will still proceed with the test as instructed.

4.1.4 Analysis and Results

The analysis is conducted separately for Alloy 1 and Alloy 2.

- **Alloy 1**: The results of the ANOVA test indicate whether we can reject the null hypothesis. The p-value is greater than 0.05, hence we accept the null hypothesis and conclude that the implant hardness doesn't differ between dentists for alloy 1.

Source	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	106683.688889	26670.922222	1.977112	0.116567
Residual	40.0	539593.555556	13489.838889	NaN	NaN

Table 2: ANOVA Results for Alloy-1

It doesn't differ so we don't require to perform Tukey HSD Test.

- **Alloy 2**: The results of the ANOVA test indicate whether we can reject the null hypothesis. The p-value is greater than 0.05, hence we accept the null hypothesis and conclude that the implant hardness doesn't differs between dentists for alloy 2.

Source	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	5.679791e+04	14199.477778	0.524835	0.718031
Residual	40.0	1.082205e+06	27055.122222	NaN	NaN

Table 3: ANOVA Results for Alloy-2

It doesn't differ so we don't require to perform Tukey HSD Test.

4.1.5 Conclusion

The analysis separately for both Alloy 1 and Alloy 2 provides insights into whether the hardness of dental implants varies based on the dentist. The results of the ANOVA test indicate whether there are no significant differences in hardness across dentists, and the same is confirmed by Tukey HSD test.

4.2 How does the hardness of implants vary depending on methods?

In this analysis, we aim to determine how the hardness of dental implants varies depending on the methods used. The variable of interest is the hardness of dental implants, and we will analyze this separately for Alloy 1 and Alloy 2.

Null Hypothesis (H_0)

The hardness of implants does not differ between methods:

$$H_0 : \mu_1 = \mu_2 = \mu_3$$

Alternate Hypothesis (H_A)

The hardness of implants differs between at least one pair of methods:

$$H_A : \textbf{At least one } \mu_i \textbf{ differ from the others.}$$

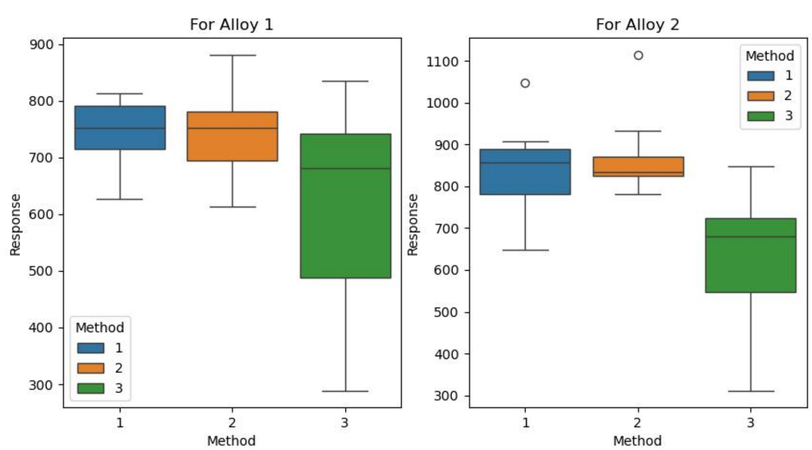


Figure 6: Hardness of implants vary depending on methods for two different alloys

4.2.1 Assumptions of the One-Way ANOVA

Before conducting the ANOVA, the following assumptions must be checked:

- **Normality**: The response variable (hardness) is normally distributed for each group (method).
- **Homogeneity of variance**: The variances of hardness between methods are equal.
- **Independence**: Observations are independent.

Note: The p-values of Shapiro's test indicate the distribution is normal for both alloys (except one case for Alloy 2 with Method 2) and the population variances are not equal for both alloys between methods. Even if the assumptions are not fully met, we will still proceed with the test as instructed.

The analysis is conducted separately for Alloy 1 and Alloy 2.

- **Alloy 1**: The results of the ANOVA test for Alloy 1 are shown in Table 4. Since the p-value is less than 0.05, we reject the null hypothesis for Alloy 1. This indicates that the hardness of implants differs between methods.

Source	df	sum_sq	mean_sq	F	PR(>F)
C(Method)	2.0	148472.177778	74236.088889	6.263327	0.004163
Residual	42.0	497805.066667	11852.501587	NaN	NaN

Table 4: ANOVA Results for Alloy-1

4.2.2 Tukey HSD Test for Alloy 1

The Tukey HSD test helps identify which pairs of methods differ significantly in terms of hardness. For Alloy 1, significant differences in hardness are found between Method 1 and Method 3, as well as

Group 1	Group 2	Mean Difference	p-adj	Lower Bound	Upper Bound	Reject
1	2	-6.1333	0.987	-102.714	90.4473	False
1	3	-124.8000	0.0085	-221.3807	-28.2193	True
2	3	-118.6667	0.0128	-215.2473	-22.0860	True

Table 5: Tukey HSD Test Results for Alloy 1

between Method 2 and Method 3. However, the difference between Method 1 and Method 2 is not statistically significant.

- **Alloy 2**:

4.2.3 ANOVA Results for Alloy 2

The results of the ANOVA test for Alloy 2 are as follows:

Source	df	sum_sq	mean_sq	F	PR(>F)
C(Method)	2.0	499640.4	249820.200000	16.4108	0.000005
Residual	42.0	639362.4	15222.914286	NaN	NaN

Table 6: ANOVA Results for Alloy 2

Since the p-value is much less than 0.05, we reject the null hypothesis for Alloy 2. This indicates that the hardness of implants differs significantly between methods.

Group 1	Group 2	Mean Difference	p-adj	Lower Bound	Upper Bound	Reject
1	2	27.0000	0.8212	-82.4546	136.4546	False
1	3	-208.8000	0.0001	-318.2546	-99.3454	True
2	3	-235.8000	0.0000	-345.2546	-126.3454	True

Table 7: Tukey HSD Test Results for Alloy 2

4.2.4 Tukey HSD Test for Alloy 2

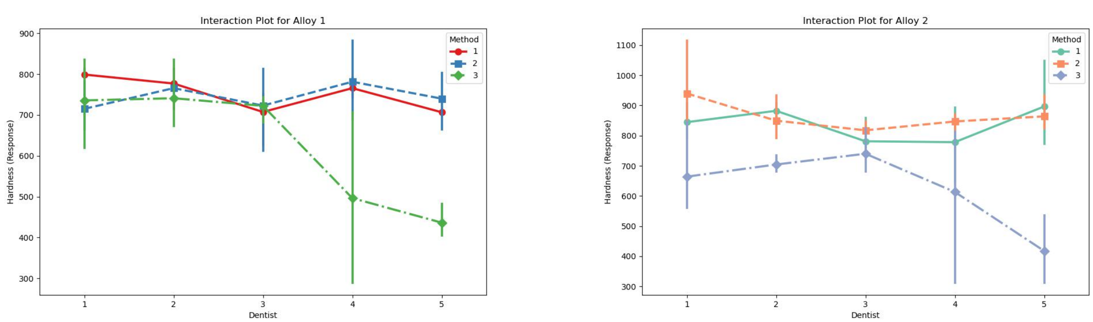
The Tukey HSD test helps identify which pairs of methods differ significantly in terms of hardness.

For Alloy 2, significant differences in hardness are found between Method 1 and Method 3, as well as between Method 2 and Method 3. However, the difference between Method 1 and Method 2 is not statistically significant.

4.2.5 Conclusion

The analysis separately for both Alloy 1 and Alloy 2 provides insights into whether the hardness of dental implants varies based on the methods. For **Alloy 1**, the ANOVA test reveals significant differences in hardness across the methods ($p = 0.004$). This indicates that at least one method differs from the others in terms of hardness. The Tukey HSD test confirms that Methods 1 and 3, as well as Methods 2 and 3, have statistically significant differences in hardness, while no significant difference is found between Methods 1 and 2. For **Alloy 2**, the ANOVA test also shows significant differences in hardness across the methods ($p = 0.000005$). Similarly, the Tukey HSD test indicates significant differences between Methods 1 and 3, and Methods 2 and 3, while no significant difference is observed between Methods 1 and 2. In both alloys, the ANOVA tests highlight that hardness varies significantly among the methods, and the Tukey HSD test pinpoints the specific pairs of methods where these differences occur.

4.3 What is the interaction effect between the dentist and method on the hardness of dental implants for each type of alloy?



(a) Alloy 1 (b) Alloy 2

Figure 7: Interaction Plots of how Hardness of implants vary depending on methods & dentists for two different alloys

Alloy 1:

- **Method 1**: Consistent hardness (~ 800) across dentists, slight drop for dentists 3 and 5.
- **Method 2**: Starts low (~ 700), slight decline across dentists, with some variation.
- **Method 3**: Sharp drop in hardness for dentist 4, but large variability.

Alloy 2:

- **Method 1**: Hardness remains stable (~ 800) across all dentists.

- **Method 2**: Starts high (~ 950), gradually decreases as dentists change, showing large variation.
- **Method 3**: Lowest hardness (~ 700 to ~ 600), declines steadily across dentists, with large error bars.

In this analysis, we aim to determine how the hardness of dental implants is affected by the interaction between the dentist and method. The response variable is the hardness of dental implants, and we will analyze this separately for Alloy 1 and Alloy 2.

Null Hypothesis (H_0)

There is no interaction effect between dentist and method on the hardness of dental implants:

$$H_0 : \textbf{There is no significant interaction between dentist and method.}$$

Alternate Hypothesis (H_A)

There is an interaction effect between dentist and method on the hardness of dental implants:

$$H_A : \textbf{There is a significant interaction between dentist and method.}$$

4.3.1 Assumptions of the Two-Way ANOVA

Before conducting the ANOVA, the following assumptions must be checked:

- **Normality**: The response variable (hardness) is normally distributed for each combination of dentist and method.
- **Homogeneity of variance**: The variances of hardness between groups (dentist-method combinations) are equal.
- **Independence**: Observations are independent.

Note: We have already verified these conditions separately for both methods and alloys, so there is no need to check them explicitly for their combinations. Regardless of whether the assumptions are fully met, we will proceed with the test as directed. The interaction effects are analyzed separately for Alloy 1 and Alloy 2.

Alloy 1

The results of the two-way ANOVA for Alloy 1 are shown in Table 8.

Source	**sum_sq**	**df**	**F**	**PR(>F)**
C(Dentist)	106683.69	4.0	3.90	0.011484
C(Method)	148472.18	2.0	10.85	0.000284
C(Dentist):C(Method)	185941.38	8.0	3.40	0.006793
Residual	205180.00	30.0	NaN	NaN

Table 8: ANOVA Results for Interaction Effect

Since the p-value for the interaction effect is 0.007, we conclude that there is significant interaction effect between dentist and method on the hardness of implants for Alloy 1.

Alloy 2

For Alloy 2, the results of the two-way ANOVA are shown in Table 9.

Since the p-value for the interaction effect is 0.09, we conclude that there is no significant interaction effect between dentist and method on the hardness of implants for Alloy 2.

Table 9: ANOVA Table

Source	Sum of Squares (sum_sq)	Degrees of Freedom (df)	F	$PR(> F)$
C(Dentist)	56797.911111	4.0	1.106152	0.371833
C(Method)	499640.400000	2.0	19.461218	0.000004
C(Dentist):C(Method)	197459.822222	8.0	1.922787	0.093234
Residual	385104.666667	30.0	NaN	NaN

4.3.2 Conclusion

The analysis for both Alloy 1 and Alloy 2 provides insights into whether the interaction between dentist and method affects the hardness of dental implants.

- For **Alloy 1**,
  - **Effect of Dentist**: A p-value of 0.011484 indicates a statistically significant difference in implant hardness among dentists, suggesting the choice of dentist significantly impacts hardness.
  - **Effect of Method**: A p-value of 0.000284 shows a significant effect of the method on implant hardness, with different methods leading to notably different outcomes.
  - **Interaction Effect**: The p-value for the interaction term $C(Dentist) : C(Method)$ is 0.006793, indicating a significant interaction effect on implant hardness, implying that the effect of the method varies by dentist.
- For **Alloy 2**,
  - **Effect of Dentist**: A p-value of 0.3718 suggests no significant difference in implant hardness among dentists.
  - **Effect of Method**: The p-value of 0.000004 indicates a significant effect of the method on implant hardness, with different methods yielding significantly different outcomes.
  - **Interaction Effect**: The interaction term $C(Dentist) : C(Method)$ has a p-value of 0.0932, indicating a potential interaction effect; however, it does not reach significance at the 0.05 level, suggesting no significant influence on hardness for Alloy 2.

4.4 How does the hardness of implants vary depending on dentists and methods together?

In this analysis, we aim to determine how the hardness of dental implants is affected by taking contributions of dentists and methods together, separately for Alloy 1 and Alloy 2. The response variable is the hardness of dental implants.

Null Hypotheses (H_0):

- There is no significant effect of **Dentist** on the **Response**:

$$H_0 : \mu_{Dentist1} = \mu_{Dentist2} = \ldots = \mu_{Dentistn}$$

  This means that the mean responses for all dentists are the same.

- There is no significant effect of **Method** on the **Response**:

$$H_0 : \mu_{Method1} = \mu_{Method2} = \ldots = \mu_{Methodn}$$

  This means that the mean responses for all methods are the same.

Alternate Hypotheses (H_A):

- There is a significant effect of **Dentist** on the **Response**:

$$H_A : \mu_{Dentist1} \neq \mu_{Dentist2} \neq \cdots \neq \mu_{Dentistn}$$

  This means that at least one dentist's mean response differs from the others.

- There is a significant effect of **Method** on the **Response**:

$$H_A : \mu_{Method1} \neq \mu_{Method2} \neq \cdots \neq \mu_{Methodn}$$

  This means that at least one method's mean response differs from the others.

Note: We have already verified these conditions separately for both methods and alloys, so there is no need to check them explicitly for their combinations. Regardless of whether the assumptions are fully met, we will proceed with the test as directed. The analysis is conducted separately for Alloy 1 and Alloy 2.

Alloy 1:

4.4.1 ANOVA Results for Alloy 1

The results of the two-way ANOVA test for Alloy 1 are shown in Table 10.

- For **Method**: The p-value is 0.002211, which is less than 0.05. Therefore, we **reject the null hypothesis** and conclude that there is a significant effect of the method on the response.
- For **Dentist**: The p-value is 0.051875, which is slightly greater than 0.05. Therefore, we **fail to reject the null hypothesis**, suggesting that dentist differences are not statistically significant at the 0.05 level.

Source	**sum_sq**	**df**	**F**	**PR(>F)**
C(Dentist)	106683.688889	4.0	2.591255	0.051875
C(Method)	148472.177778	2.0	7.212522	0.002211
Residual	391121.377778	38.0	NaN	NaN

Table 10: ANOVA Results for Dentist and Method for alloy 1

4.4.2 Tukey HSD Test for Alloy 1

The Tukey HSD test helps identify which combinations of dentists and methods differ significantly in terms of hardness. Look into the support file for the table.

The Tukey HSD test further reveals the significant pairwise comparisons: For the combinations with reject = True signifies that at least one dentist's mean response differs from the others.

Alloy 2:

4.4.3 ANOVA Results for Alloy 2

The results of the two-way ANOVA test for Alloy 2 are as follows:

Source	**sum_sq**	**df**	**F**	**PR(>F)**
C(Dentist)	56797.911111	4.0	0.926215	0.458933
C(Method)	499640.400000	2.0	16.295479	0.000008
Residual	582564.488889	38.0	NaN	NaN

Table 11: ANOVA Results for Dentist and Method for alloy 2

The results of the two-way ANOVA test for Alloy 1 are shown in Table 11.

- For **Method**: The p-value is 0.000008, which is less than 0.05. Therefore, we **reject the null hypothesis** and conclude that there is a significant effect of the method on the response.
- For **Dentist**: The p-value is 0.458933, which is slightly greater than 0.05. Therefore, we **fail to reject the null hypothesis**, suggesting that dentist differences are not statistically significant at the 0.05 level.

4.4.4 Tukey HSD Test for Alloy 2

The Tukey HSD test helps identify which combinations of dentists and methods differ significantly in terms of hardness. Look into the support file for the table. The Tukey HSD test further reveals the significant pairwise comparisons: For the combinations with reject = True signifies that at least one dentist's mean response differs from the others.