Rotation-induced nucleon pair breaking in nuclei with quadrupole and octupole deformations

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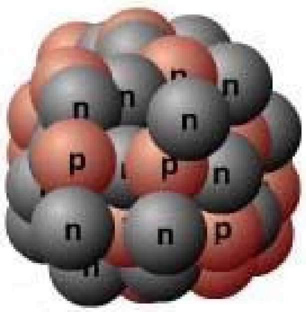
Nuclear models

- *A* tremendous amount of progress in the study of atomic energy levels has been possible because the mathematical form of coulomb's law is known. *W*hereas, the mathematical form of the nuclear force is not yet known.
- *A* simple form (such as a potential function dependent only on the separation of two nucleons) is not a possible solution to the problem. So we require nuclear models.
- *N*uclear models can be divided into two categories.



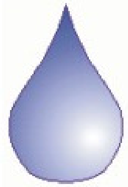
1. Models with strong interaction between the nucleons:

- Liquid drop model
- α-cluster model
- Shell model

2. Models of non-interacting nucleons:

- Fermi gas model
- Optical model

The liquid drop model



The liquid drop model is a model in nuclear physics which treats the nucleus as a drop of incompressible nuclear fluid

- first proposed by George Gamow and developed by Niels Bohr and John Archibald Wheeler

- The fluid is made of nucleons (protons and neutrons), which are held together by the strong nuclear force.

This is a crude model that does not explain all the properties of the nucleus, but (!)

- does explain the spherical shape of most nuclei.
- It also helps to predict the binding energy of the nucleus.

George Gamow
(1930-1968)

Niels Henrik David Bohr
(1885-1962)

John Archibald Wheeler
(1911-2008)

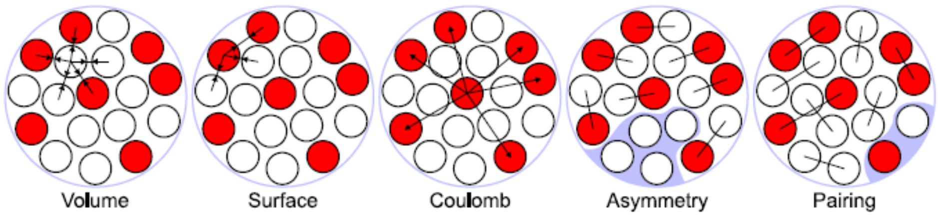
The liquid drop model

The parametrisation of nuclear masses as a function of *A* and *Z*, which is known as the Weizsäcker formula or the semi-empirical mass formula, was first introduced in 1935 by German physicist Carl Friedrich von Weizsäcker:

$$M(A,Z) = Zm_p + Nm_n - E_B$$

E_B is the binding energy of the nucleus :

$$E_B = \underbrace{a_V \cdot A}_{\text{Volum term}} - \underbrace{a_S \cdot A^{\frac{2}{3}}}_{\text{Surface term}} - \underbrace{a_C \cdot \frac{Z^2}{A^{\frac{1}{3}}}}_{\text{Coulomb term}} - \underbrace{a_{Sym} \cdot \frac{(N-Z)^2}{A}}_{\text{Assymetry term}} - \underbrace{\frac{\delta}{A^{1/2}}}_{\text{Pairing term}}$$



Empirical parameters:

$$a_V \approx 16MeV$$
$$a_S \approx 20MeV$$
$$a_C \approx 0,75MeV$$
$$a_{Sym} \approx 21MeV$$

$$\delta = \begin{cases} -11.2\ \mathrm{MeV}/c^2 & \text{for even } Z \text{ and } N \text{ (even-even nuclei)} \\ 0\ \mathrm{MeV}/c^2 & \text{for odd } A \text{ (odd-even nuclei)} \\ +11.2\ \mathrm{MeV}/c^2 & \text{for odd } Z \text{ and } N \text{ (odd-odd nuclei).} \end{cases}$$

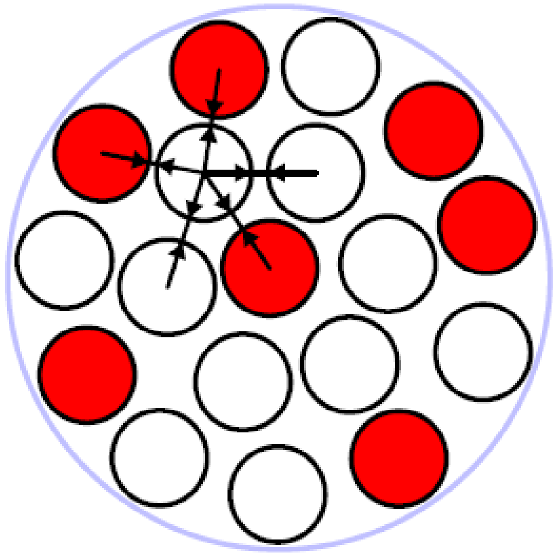
Binding energy of the nucleus

Volume energy (dominant term):

$$E_V = a_V A$$

A - mass number

Coefficient $a_V \approx 16 \ \ MeV$

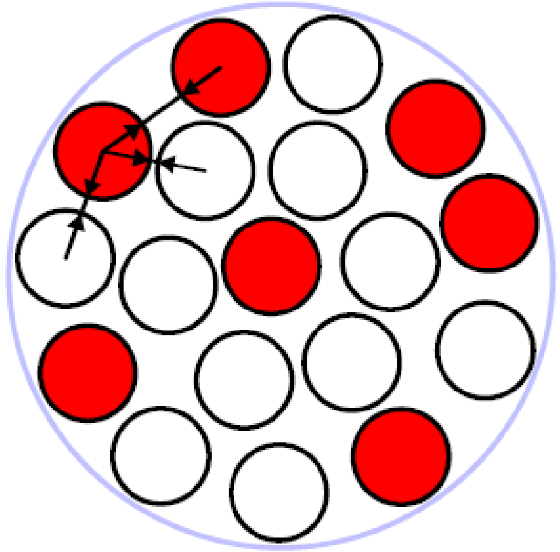


- The basis for this term is the strong nuclear force.
- The strong force affects both protons and neutrons ➔ this term is independent of Z.
- Because the number of pairs that can be taken from A particles is $A(A-1)/2$, one might expect a term proportional to A^2 . However, the strong force has a very limited range, and a given nucleon may only interact strongly with its nearest neighbors and next nearest neighbors. Therefore, the number of pairs of particles that actually interact is roughly proportional to A.

Binding energy of the nucleus

Surface energy: $$\boxed{E_S = -a_S A^{2/3}}$$

Coefficient $a_S \approx 20 \ \ MeV$



- **This term, also based on the strong force, is a correction to the volume term.**

- **A nucleon at the surface of a nucleus interacts with less number of nucleons than one in the interior of the nucleus, so its binding energy is less. This can also be thought of as a surface tension term, and indeed a similar mechanism creates surface tension in liquids.**

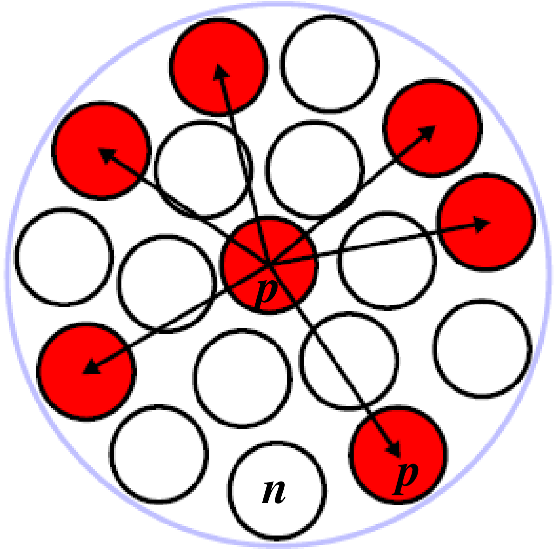
- **The surface energy term is therefore negative and is proportional to the surface area :**
if the volume of the nucleus is proportional to A $(V=4/3\pi R^3)$**,**
then the radius should be proportional to $A^{1/3}$ $(R\sim A^{1/3})$ **and the surface area to** $A^{2/3}$ $(S=\pi R^2=\pi A^{2/3})$**.**

Binding energy of the nucleus

Coulomb (or electric) energy:

$$\boxed{E_C = -a_C \frac{Z^2}{A^{1/3}}}$$

Coefficient $a_C \approx 0.75 \quad MeV$



Coulomb

- **The basis for this term is the electrostatic repulsion between protons.**
- **The electric repulsion between each pair of protons in a nucleus contributes toward decreasing its binding energy:**

from QED - interaction energy for the charges q_1, q_2 inside the ball

$$E_{\text{int}} \sim \frac{q_1 q_2}{R}$$

Here R - empirical nuclear radius: $R \sim A^{1/3}$

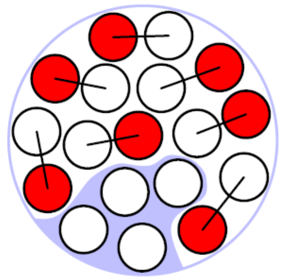
Thus, $$E_C \sim \frac{Z^2}{A^{1/3}}$$

Binding energy of the nucleus

Asymmetry energy (also called Pauli Energy):

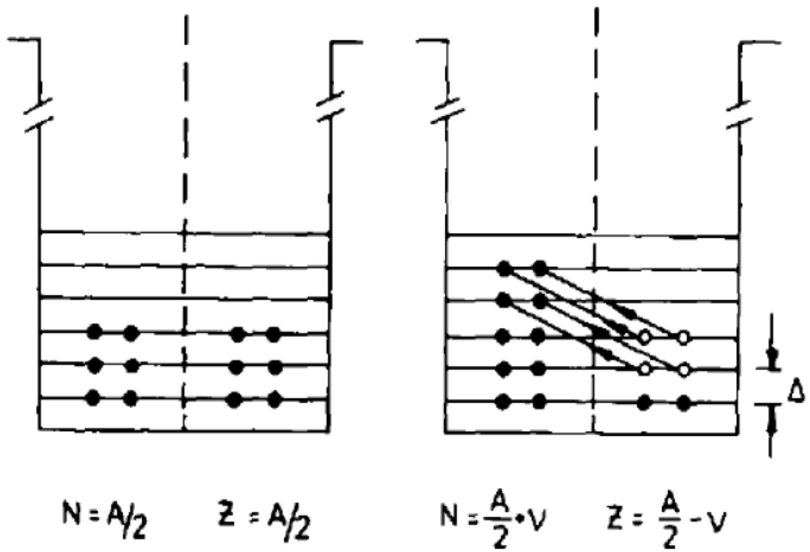
Coefficient $a_{sym} \approx 21\ MeV$

$$E_{asym} = -a_{Sym}\frac{(N-Z)^2}{A}$$



$$N = A/2 + \upsilon,\ Z = A/2 - \upsilon \Longrightarrow \upsilon = \frac{N-Z}{2} = \frac{A-2Z}{2}$$

If the average energy separation between adjacent orbital is taken to be Δ, replacing υ nucleons will cost an energy loss of $\Delta BE = \upsilon(\Delta\upsilon/2)$ (See the figure). The spacing between the highest empty energy level and highest occupied energy level is $\frac{\upsilon}{2}\times$ spacing between adjacent orbital(i.e. Δ) and No. of nucleons are υ So, net change in BE $=\frac{\upsilon}{2}\times\Delta\times\upsilon = \frac{\upsilon^2}{2}\Delta = \frac{(A-2Z)^2}{8}\Delta$. The potential depth describing the nuclear well does not vary much with changing mean number So, energy spacing Δ should vary inversely to fit in more levels in that same potential well. i.e. $\Delta \propto \frac{1}{A}$. So, Assymmetry term becomes $\sim \frac{(A-2Z)^2}{A}$.



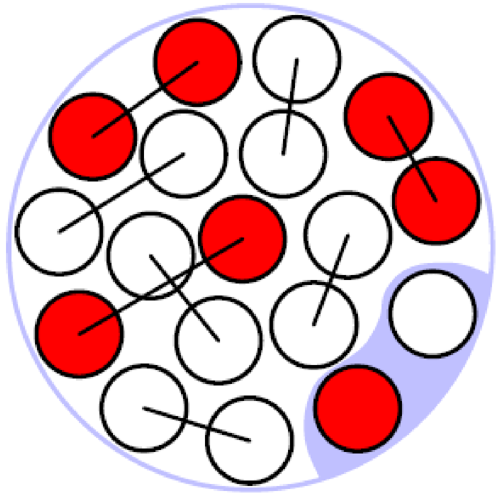
Schematic single-particle model description, two different distributions of A nucleons over the proton and neutron orbital with two-fold (spin) degeneracy. For the drawn figure $\upsilon = 4$ So, the $\Delta BE = \frac{4}{2}\times\Delta\times 4 = 8\Delta$.

Binding energy of the nucleus

Pairing energy:

$$\boxed{E_{pair} = -\frac{\delta}{A^{1/3}}}$$

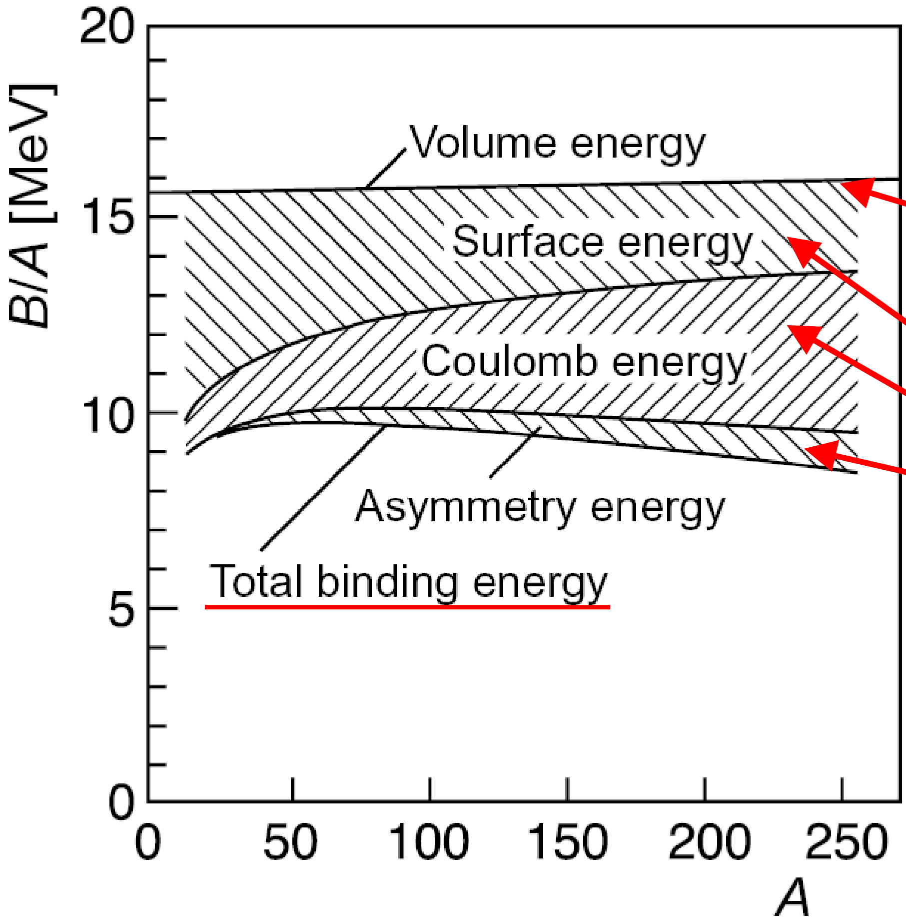
$$\delta = \begin{cases} -11.2\ \mathrm{MeV}/c^2 \text{ for even } Z \text{ and } N \text{ (even-even nuclei)} \\ 0\ \mathrm{MeV}/c^2 \text{ for odd } A \text{ (odd-even nuclei)} \\ +11.2\ \mathrm{MeV}/c^2 \text{ for odd } Z \text{ and } N \text{ (odd-odd nuclei).} \end{cases}$$



Pairing

- **An energy which is a correction term that arises from the effect o spin-coupling.**
- **Due to the Pauli exclusion principle the nucleus would have a lower energy if the number of protons with spin up will be equal to the number of protons with spin down. This is also true for neutrons.**
- **Only if both *Z* and *N* are even, both protons and neutrons can have equal numbers of spin up and spin down particles.**
- **An even number of particles is more stable (δ<0 for even-even nuclei) than an odd number (δ>0).**

The liquid drop model



The different contributions to the binding energy per nucleon versus mass number *A*:

The horizontal line at ≈ 16 MeV represents the contribution of the volume energy.

This is reduced by the surface energy, the asymmetry energy and the Coulomb energy to the effective binding energy of ≈ 8 MeV(*lower line*).

The contributions of the asymmetry and Coulomb terms increase rapidly with *A*, while the contribution of the surface term decreases.

$$E_B = \underbrace{a_V \cdot A}_{\text{Volum term}} - \underbrace{a_S \cdot A^{\frac{2}{3}}}_{\text{Surface term}} - \underbrace{a_C \cdot \frac{Z^2}{A^{\frac{1}{3}}}}_{\text{Coulomb term}} - \underbrace{a_{Sym} \cdot \frac{(N-Z)^2}{A}}_{\text{Assymetry term}} - \underbrace{\frac{\delta}{A^{1/2}}}_{\text{Pairing term}}$$

The basic concept of the Fermi-gas model

The theoretical concept of a Fermi-gas may be applied for systems of weakly interacting fermions, i.e. particles obeying Fermi-Dirac statistics leading to the Pauli exclusion principle ➔

• Simple picture of the nucleus:

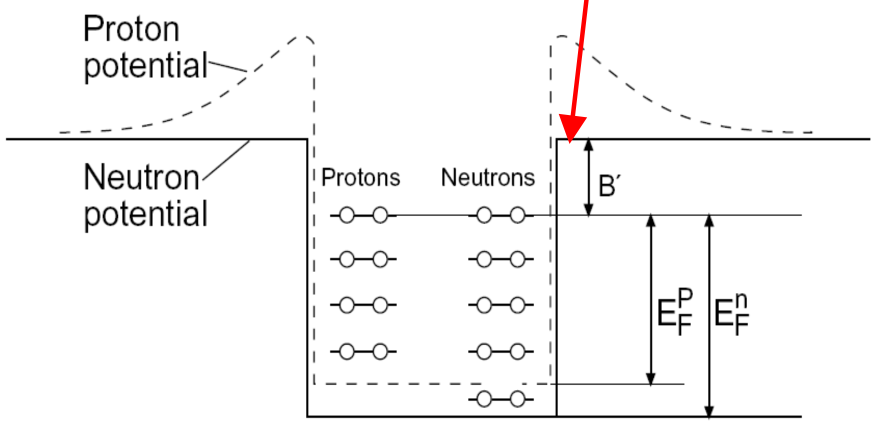
— Protons and neutrons are considered as moving freely within the nuclear volume. The binding potential is generated by all nucleons

— In a first approximation, these nuclear potential wells are considered as rectangular: it is constant inside the nucleus and stops sharply at its end

— Neutrons and protons are distinguishable fermions and are therefore situated in two separate potential wells

— Each energy state can be ocupied by two nucleons with different spin projections

— All available energy states are filled by the pairs of nucleons ➔ no free states , no transitions between the states



— The energy of the highest occupied state is the Fermi energy E_F

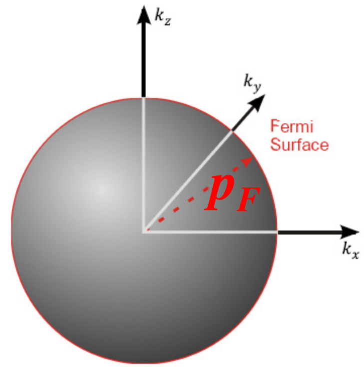
— The difference *B'* between the top of the well and the Fermi level is constant for most nuclei and is just the average binding energy per nucleon *B'/A* = 7–8 MeV.

Number of nucleon states

Heisenberg Uncertainty Principle: $\Delta x\, \Delta p \geq \frac{1}{2}\hbar$

The volume of one particle in phase space: $2\pi\,\hbar$

The number of nucleon states in a volume V:



$$\tilde{n} = \frac{\int\int d^3r\, d^3p}{(2\pi\hbar)^3} = \frac{V \cdot 4\pi \int_0^{p_{max}} p^2 dp}{(2\pi\hbar)^3} \qquad (1)$$

$(V = \int d^3r)$

At temperature T = 0, i.e. for the nucleus in its ground state, the lowest states will be filled up to a maximum momentum, called the Fermi momentum p_F. The number of these states follows from integrating eq.(1) from 0 to $p_{max}=p_F$:

$$\tilde{n} = \frac{V \cdot 4\pi \int_0^{p_F} p^2 dp}{(2\pi\hbar)^3} = \frac{V \cdot 4\pi\; p_F^3}{(2\pi\hbar)^3 \cdot 3} \quad \Rightarrow \quad \boxed{\tilde{n} = \frac{V \cdot p_F^3}{6\pi^2\hbar^3}} \qquad (2)$$

Since an energy state can contain two fermions of the same species, we can have

Neutrons: $N = \dfrac{V \cdot (p_F^n)^3}{3\pi^2\hbar^3}$ Protons: $Z = \dfrac{V \cdot (p_F^p)^3}{3\pi^2\hbar^3}$

p_F^n is the fermi momentum for neutrons, p_F^p – for protons

Fermi momentum

Let's estimate Fermi momentum p_F: Use $R = R_0 \cdot A^{1/3}\, fm$, $V = \frac{4\pi}{3} R^3 = \frac{4\pi}{3} R_0^{\,3} A$

The density of nucleons in a nucleus = number of nucleons in a volume V (cf. Eq.(2)):

$$n = 2 \cdot \tilde{n} = 2 \cdot \frac{V \cdot p_F^3}{6\pi^2 \hbar^3} = 2 \cdot \frac{4\pi}{3} R_0^{\,3} A \cdot \frac{p_F^3}{6\pi^2 \hbar^3} = \frac{4A}{9\pi} \frac{R_0^{\,3} p_F^3}{\hbar^3} \qquad (3)$$

two spin states !

Fermi momentum p_F:

$$p_F = \left(\frac{6\pi^2 \hbar^3 n}{2V} \right)^{1/3} = \left(\frac{9\pi\hbar^3}{4A} \frac{n}{R_0^{\,3}} \right)^{1/3} = \left(\frac{9\pi \cdot n}{4A} \right)^{1/3} \cdot \frac{\hbar}{R_0} \qquad (4)$$

After assuming that the proton and neutron potential wells have the same radius, we find for a nucleus with $n=Z=N=A/2$ the Fermi momentum p_F:

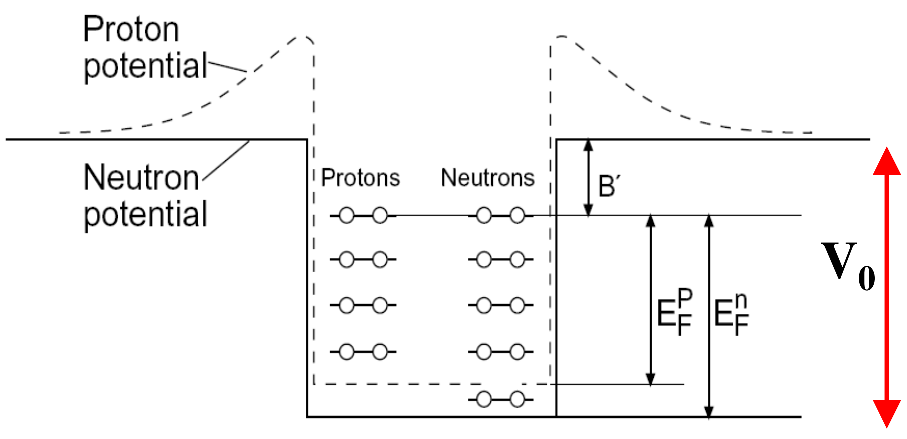
$$p_F = p_F^n = p_F^p = \left(\frac{9\pi}{8} \right)^{1/3} \cdot \frac{\hbar}{R_0} \approx \underline{250\ MeV/c}$$

The nucleons move freely inside the nucleus with large momentum!

Fermi energy: $$E_F = \frac{p_F^2}{2M} \approx \underline{33\ \ MeV}$$

M =938 MeV- the mass of nucleon

Nucleon potential



❑ The difference B' between the top of the well and the Fermi level is constant for most nuclei and is just the average binding energy per nucleon $B/A = 7\text{–}8\ MeV$.

➔ The depth of the potential V_0 and the Fermi energy are independent of the mass number A:

$$\boxed{V_0 = E_F + B' \approx 40\ MeV}$$

❑ Heavy nuclei have a surplus of neutrons. Since the Fermi level of the protons and neutrons in a stable nucleus have to be equal (otherwise the nucleus would enter a more energetically favourable state through β-decay) this implies that the depth of the potential well as it is experienced by the neutron gas has to be larger than of the proton gas (cf. Fig.), i.e. $E_F^n > E_F^p$

➔ protons are therefore on average less strongly bound in nuclei than neutrons. This may be understood as a consequence of the Coulomb repulsion of the charged protons and leads to an extra term in the potential:

$$V_C = (Z-1)\frac{\alpha \cdot \hbar c}{R}$$

Kinetic energy

The dependence of the binding energy on the surplus of neutrons may be calculated within the Fermi gas model.

- **First we find the average kinetic energy per nucleon:**

$$\langle E \rangle = \frac{\int_0^{E_F} E \cdot \frac{dn}{dE} dE}{\int_0^{E_F} \frac{dn}{dE} dE} = \frac{\int_0^{p_F} E \cdot \frac{dn}{dp} dp}{\int_0^{p_F} \frac{dn}{dp} dp}$$

where $\frac{dn}{dp} = Const \cdot p^2$ **and** $dE = \frac{p}{E} dp$

distribution function of the nucleons – from (1)

$$\langle E_{kin} \rangle = \frac{\int_0^{p_F} E_{kin}(p) p^2 dp}{\int_0^{p_F} p^2 dp} = \frac{\int_0^{p_F} \frac{p^2}{2M} p^2 dp}{\int_0^{p_F} p^2 dp} = \frac{3}{5} \frac{p_F^2}{2M} \approx 20 MeV$$

Non-relativistic:

$$E_{kin}(p) = \frac{p^2}{2M}$$

- **The total kinetic energy of the nucleus is therefore**

$$E_{\mathrm{kin}}(N,Z) = N\langle E_{\mathrm{n}} \rangle + Z\langle E_{\mathrm{p}} \rangle = \frac{3}{10M}\left(N \cdot (p_{\mathrm{F}}^{\mathrm{n}})^2 + Z \cdot (p_{\mathrm{F}}^{\mathrm{p}})^2\right)$$

$$E_{\mathrm{kin}}(N,Z) = \frac{3}{10M}\frac{\hbar^2}{R_0^2}\left(\frac{9\pi}{4}\right)^{2/3}\frac{N^{5/3}+Z^{5/3}}{A^{2/3}} \qquad (5)$$

where the radii of the proton and the neutron potential well have again been taken the same.

Binding energy

This average kinetic energy has a minimum at $N = Z$ for fixed mass number A (but varying N or, equivalently, Z). Hence the binding energy gets maximal for $N = Z$.

Expanding this result around the symmetric case with N = Z = A/2.
Taking Z − N = ϵ and, Z + N = A we have:

$$Z = \frac{A}{2}\left(1 + \frac{\epsilon}{A}\right) \text{ and } N = \frac{A}{2}\left(1 - \frac{\epsilon}{A}\right) \text{ with } \frac{\epsilon}{A} << 1$$

and inserting expansion of $(1+x)^n = 1+nx+{}^nC_2x^2+\ldots$ we get:

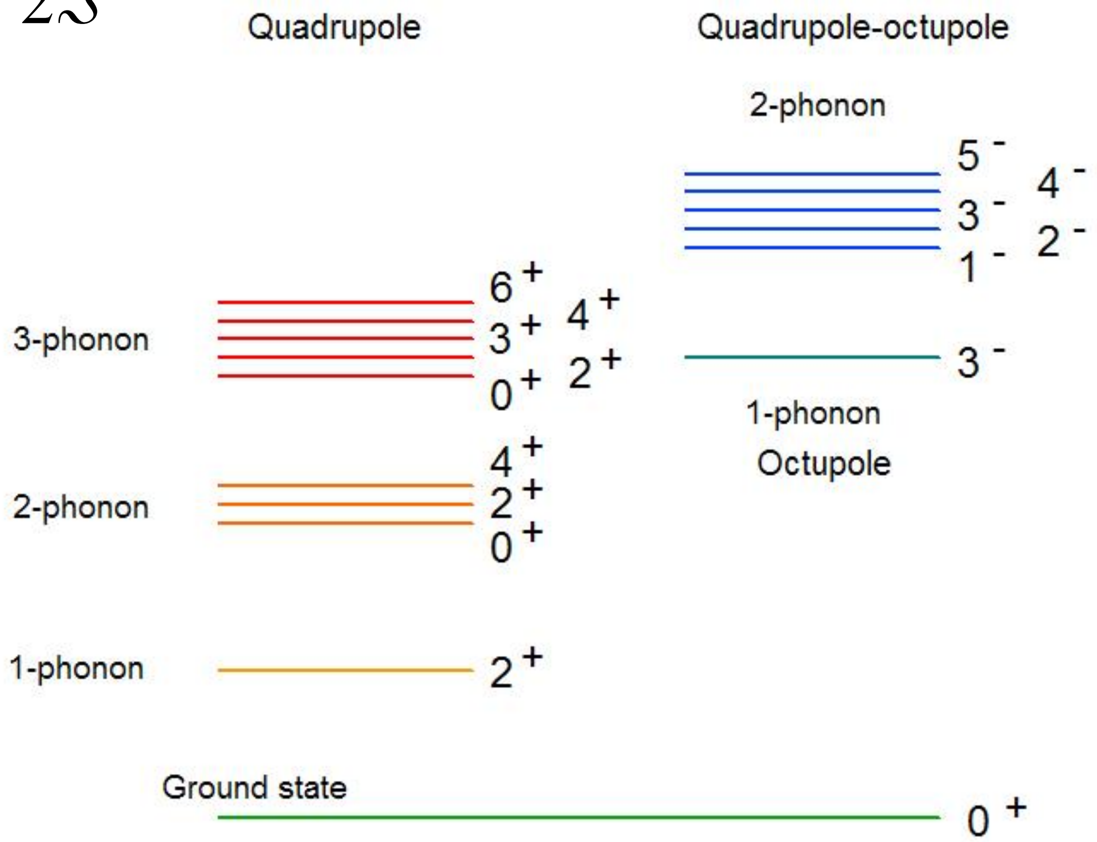
$$E_{\mathrm{kin}}(N,Z) = \frac{3}{10M}\frac{\hbar^2}{R_0^2}\left(\frac{9\pi}{8}\right)^{2/3}\left(A + \frac{5}{9}\frac{(N-Z)^2}{A} + \cdots\right)$$

The first term corresponds to the volume energy in the Weizsäcker mass formula, the second one to the asymmetry energy. The asymmetry energy grows with the neutron (or proton) surplus, thereby reducing the binding energy

Collective Nuclear Rotation

- Deformed nuclei exhibit collective rotational bands. The rotational motion involve contributions of many nucleons so, collective term is used.
- The excitation energy E, and the angular momentum,I are related as:

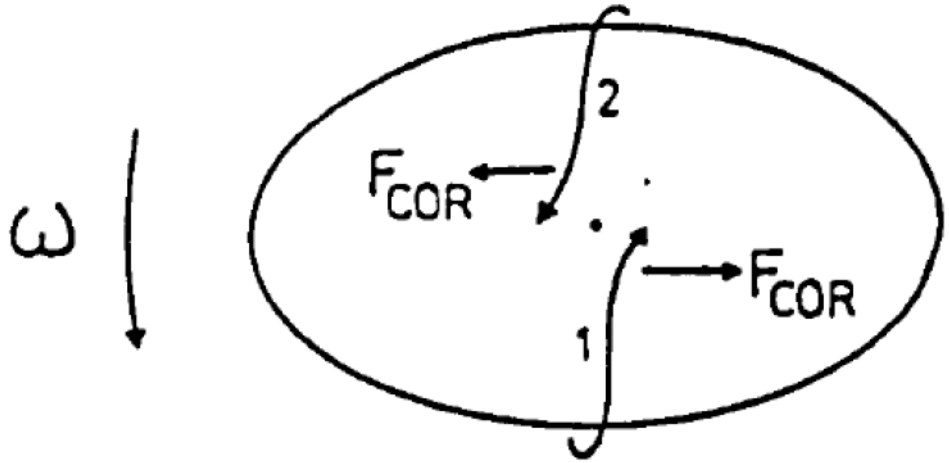
$$E_{\text{rot}}\left(I\right)=\frac{\hbar^{2}}{2\Im}I\left(I+1\right) \qquad \equiv A\,I\left(I+1\right), \quad I^{\pi}=0^{+},2^{+},\ldots$$



The series of states with consecutively increasing angular momentum is known as the 'rotational band'. The lowest state of the band is referred to as the 'bandhead'. The state of lowest energy at a given angular momentum is called the 'yrast state'.

Collective Nuclear Rotation

- **In an even-even nucleus at low rotational frequency the yrast band will be based on the ground state configuration, i.e. all the nucleons are in pairwise occupation of time-reversed orbits with the lowest excitation energy possible.**
- **As the rotational frequency increases, the excitation energy of this fir st excited band may be lowered with respect to that of the ground state band until at some critical frequency,ω_{crit} . This is known as the fir st band crossing. It can be explained by the effect of Coriolis force on two nucleons as shown in the figure.**



The effect of the Coriolis force, $F_{cor} = -2m(\overrightarrow{\omega} \times \overrightarrow{v})$ for two nucleons moving in time-reversed orbits. The force acts in opposite directions for each nucleon as the velocity of two nucleons are opposite to each other, tending to pull them apart and eventually breaking the pairing.

Collective Nuclear Rotation

❖ Moments of Inertia

- **It is found empirically that the static moment of inertia for a rotating nucleus is typically 50-80% of the rigid body value. This is due in particular to the presence of pairing correlations.**
- **Two important quantities known as the nuclear moments of inertia can be introduced,**
  - **'Kinematic' moment of inertia** $\mathscr{I}^{(1)} = \hbar \frac{I_x}{\omega}$
  - **'Dynamic' moment of inertia** $\mathscr{I}^{(2)} = \hbar \frac{dI_x}{d\omega}$

Here, I_X, represents the component of the total angular momentum on the rotation axis.

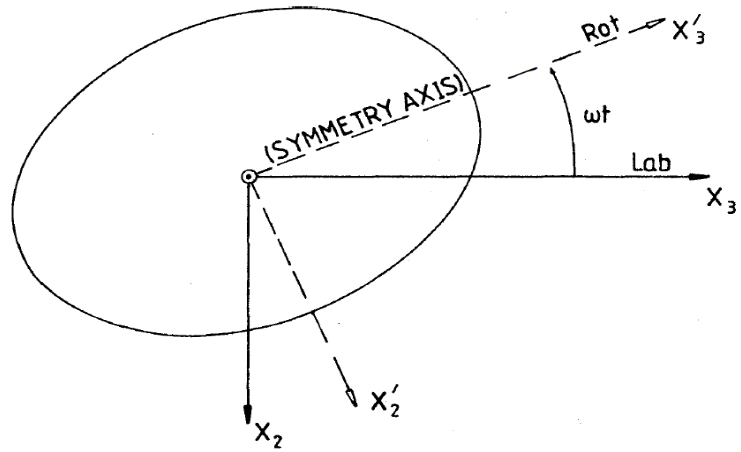
The dynamic moment of inertia, $\mathscr{I}^{(2)}$, is very sensitive to alignment effects, whereas the kinematic moment of inertia, $\mathscr{I}^{(1)}$, describes the nuclear rotation at a given angular momentum, I, and rotational frequency,

- **The total angular momentum, I, of a rotating nucleus can be decomposed into two parts. $I = R + J$.**
- **R is the angular momentum generated by the collective rotation of the inert even-even core.**
- **J is generated by the intrinsic motion of the valence nucleons.**

Cranking model

- The cranking model is a model describing independent-particle motion in such a nucleus with both static deformed shape and pair fields and which is rotating.
- A very direct way of investigating the properties of a rotating nucleus is to force it to rotate with some fixed frequency, ω, or to "crank" the nucleus.
- The mathematical formulation of a rotating single-particle potential was first given by Inglis (1954). With the coordinates in the laboratory system given by x, y and z and those in the rotating system by x_1, x_2 and x_3, we get, for constant angular velocity, ω, around the x_1-axis,

$$x'_1 = x_1$$
$$x'_2 = x_2 \cos \omega t + x_3 \sin \omega t$$
$$x'_3 = -x_2 \sin \omega t + x_3 \cos \omega t$$



- The kinetic energy, T, of a particle with mass, m, at a position (x_1, x_2, x_3) is given by:

$$T = \frac{m}{2}\left(\dot{x}_1^2 + \dot{x}_2^2 + \dot{x}_3^2\right) = \frac{m}{2}\left\{\dot{x}_1'^2 + \dot{x}_2'^2 + \dot{x}_3'^2 - 2\omega\left(\dot{x}_2' x_3' - \dot{x}_3' x_2'\right) + \omega^2\left(x_2'^2 + x_3'^2\right)\right\}$$

- So, the lagrangian becomes $L = T - V(x'_1, x'_2, x'_3)$ and using the Lagrange equations we get:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}'_i} - \frac{\partial L}{\partial x'_i} = 0$$

Cranking model

$$m\ddot{x}'_1 = -\frac{\partial \mathrm{V}}{\partial \mathrm{x}'_1}$$

$$m\ddot{x}'_2 = -\frac{\partial V}{\partial x'_2} + 2m\omega\dot{x}'_3 + m\omega x_2^2$$

$$m\ddot{x}'_3 = -\frac{\partial V}{\partial x_3}, -2m\omega\dot{x}'_2 + m\omega^2 x'_3$$

- **By introducing the rotational vector $\overline{\omega} = \omega\hat{x}'_1$,where x_1 is the unit vector along the x_1-axis these equations can be written in the condensed form:**

$$m\ddot{\bar{r}}' = -\nabla V - 2m\bar{\bar{\omega}} \times \dot{\mathrm{r}}' - m\bar{\omega} \times (\bar{\bar{\omega}} \times \bar{r}')$$

Coriolis Centrifugal

As we know the canonical momenta is $p'_i = \frac{\partial \mathrm{L}}{\partial \dot{\mathrm{x}}'_i}$ **So the equations become:**

$$p_1'^2 = m^2 \dot{x}_1'^2$$

$$p_2'^2 = m^2 \dot{x}_2'^2 - 2m^2\omega \dot{x}'_2 x'_3 + m^2\omega^2 x_3'^2$$

$$\mathrm{p}_3'^2 = \mathrm{m}^2 \dot{x}_3'^2 + 2\,\mathrm{m}^2\omega \dot{x}'_3 \mathrm{x}'_2 + \mathrm{m}^2\omega_2^2 x_2'^2$$

So, the hamiltonian in rotating frame becomes:

$$\mathrm{H}_\omega(x'_i, p'_i) = \sum_i \dot{x}'_i p' - L = \sum_i \dot{x}'_i \frac{\partial L}{\partial \dot{x}'_i} - L$$

$$= \sum_{\mathrm{i}} \frac{1}{2m} p_i'^2 + v(x'_i) - \omega(x'_2 p'_3 - x'_3 p'_2)$$

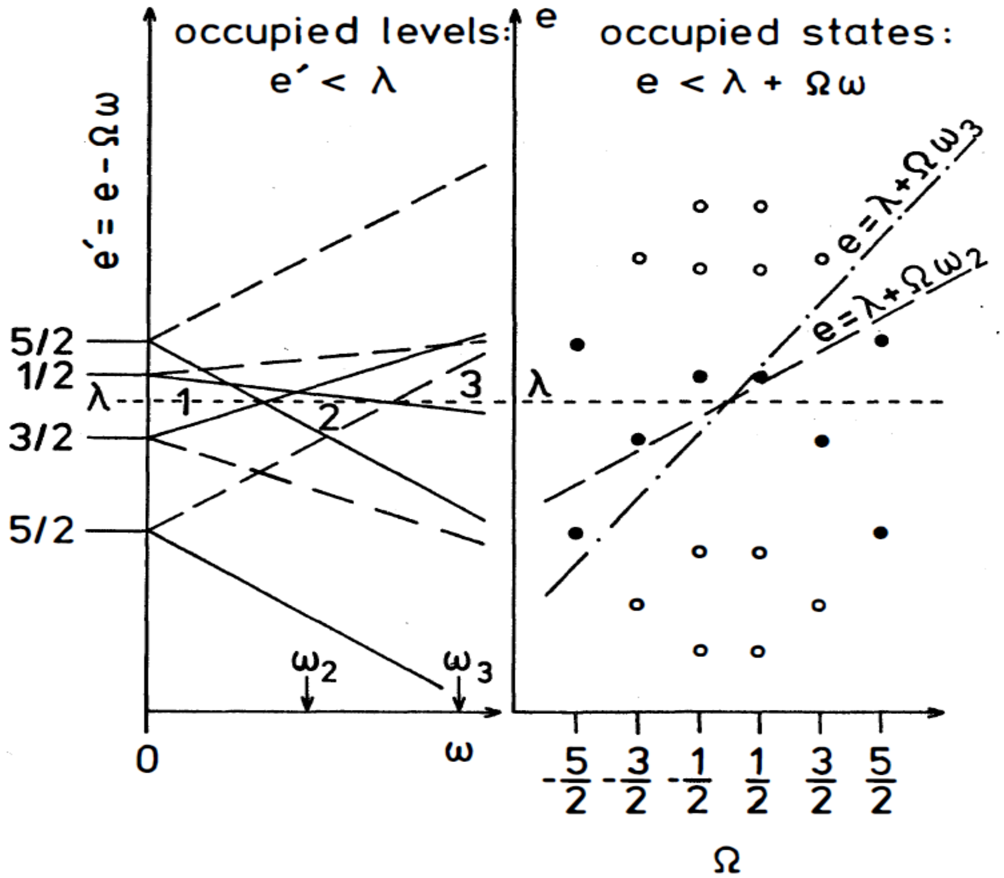
$$= H_0(x'_i, p'_i) - \omega L'_1 (\text{As } \bar{L}' = \bar{r}' \times \bar{p}')$$

Cranking model

- where H_0 is the hamiltonian of the system when $\omega = 0$ or the normal non-rotating Hamiltonian. We must also take into account of the spin s of the nucleus, which means we must replace L'_1 with $J'_1 = L'_1 + S'_1$ giving the equation:

$$H_\omega(x'_i, p'_i) = H_0(x'_i, p'_i) - \omega J'_1$$

- In a rotation-symmetric potential, the wavefunctions of the nonrotating single-particle potential have a well-defined projection quantum number, Ω, of the angular momentum on the symmetry axis, which we shall define as the z-axis. So, the cranking hamiltonian becomes $H_\omega = H_0 - \omega J_z$ and the eigenvalues become $e'_i = e_i - \omega\Omega_i$.



Cranking model

- In the rotating system, the total energy is sum of the energies of all the levels below the fermi energy, i.e. $E' = \sum_{i=1}^{N} e'_i$ where N, is the no. of nucleons.
- Energy in the lab frame is:

$$E = \sum_{\mu=1}^{N} \langle \mu | H_0 | \mu \rangle = \sum_{\mu=1}^{N} \langle \mu | H_\omega | \mu \rangle + \omega \langle \mu | J_x | \mu \rangle = E' + \omega I_x \qquad \textbf{(1)}$$

- which is independent of the rotational frequency, ω.
- The eigenvalues of H_ω can be written as:

$$e'_\mu = \langle \mu | H_\omega | \mu \rangle = \langle \mu | H_0 | \mu \rangle - \omega \langle \mu | J_x | \mu \rangle$$

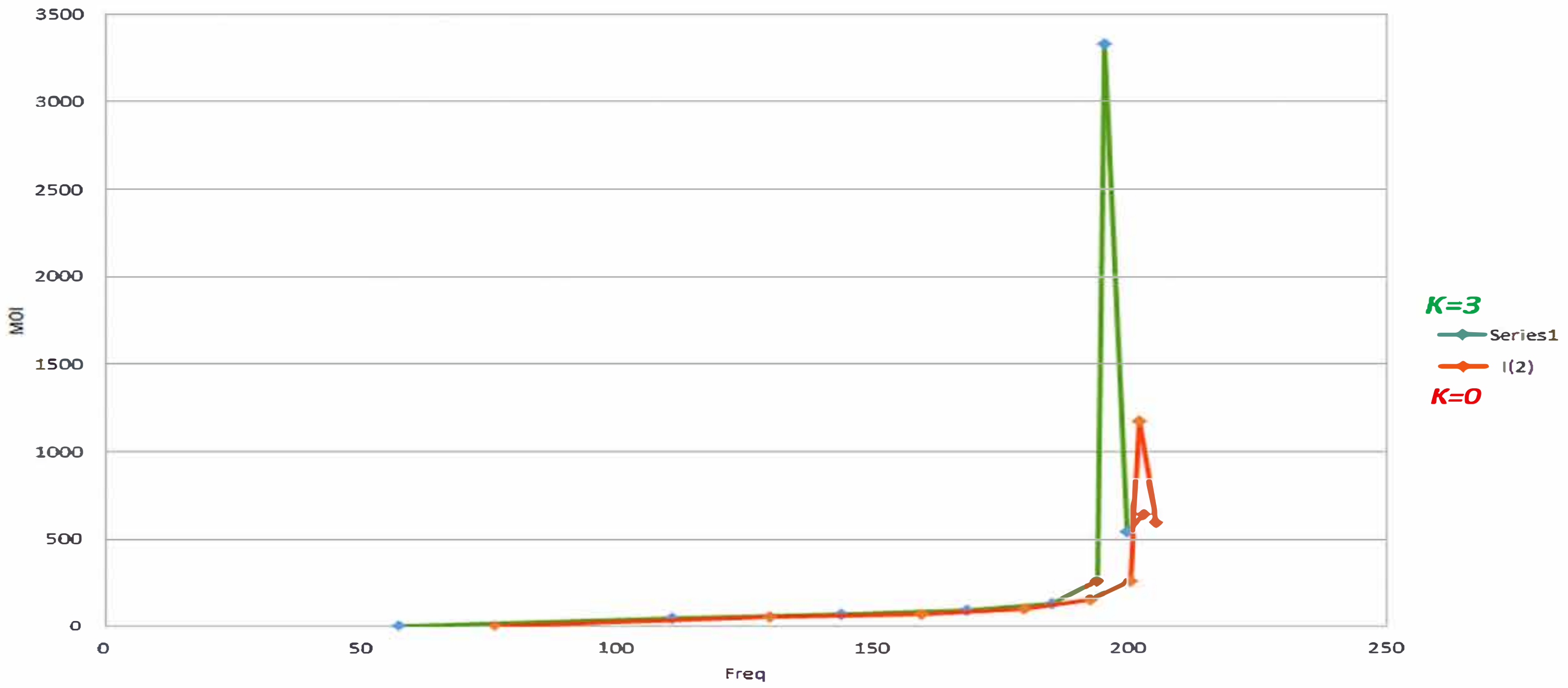
which implies that $\frac{de'_\mu}{d\omega} = -\langle \mu | J_x | \mu \rangle = -I_x$

- For a system containing N nucleons the total angular momentum along the rotational axis is calculated as $I_x = \sum_{\mu=1}^{N} \langle \mu | J_x | \mu \rangle$
- Differentiating eq(1) wrt I_x *we have:* $\omega = \frac{dE(I)}{dI_x}$
- Approximating the differential quotient by a quotient of finite differences (by restricting ourselves to se-quences with ,△I = 2, when defining a rotational band.) we get, $\omega(I) = \frac{E(I+1) - E(I-1)}{I_x(I+1) - I_x(I-1)}$, where $I_x(I) = \sqrt{(I + \frac{1}{2})^2 - K^2}$

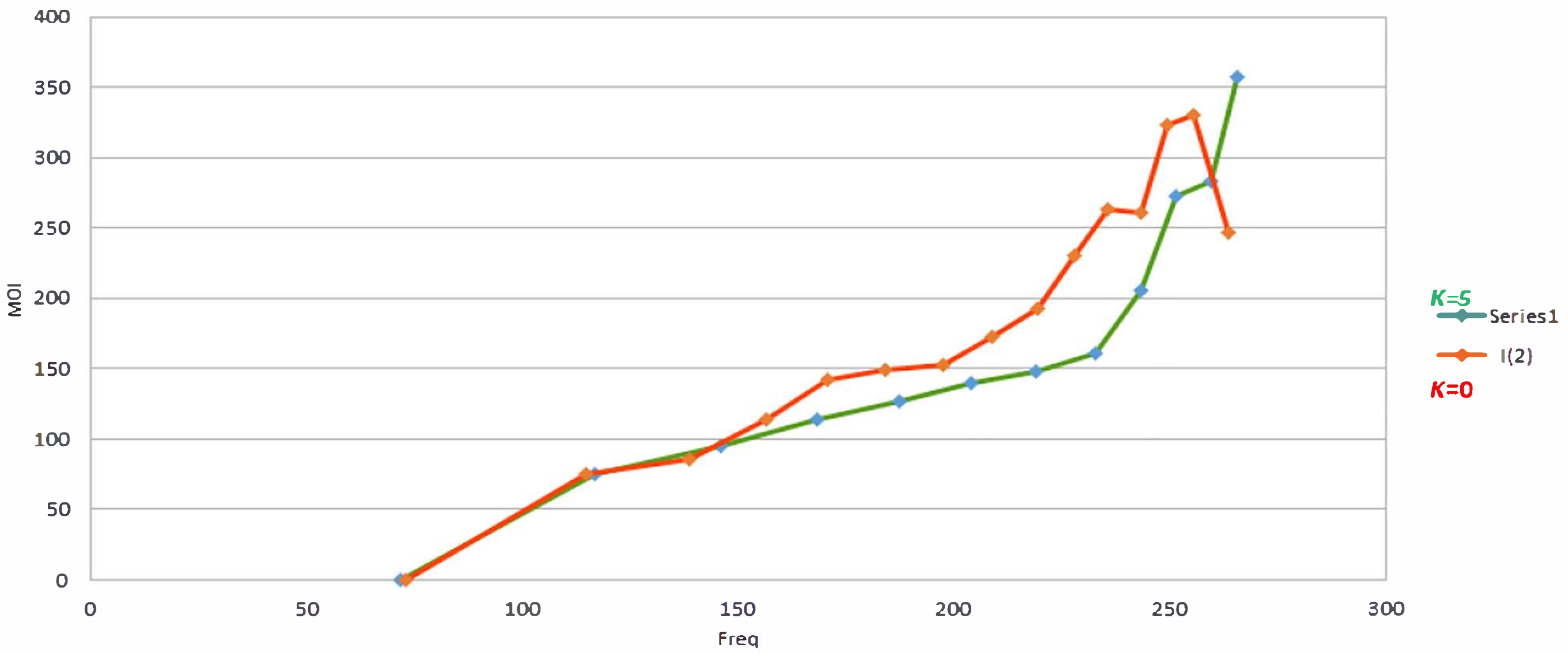
where K is the projection of the angular momentum onto the symmetry axis(The value of the angular momentum of the band head)

  - **'Kinematic' moment of inertia** $\mathscr{I}^{(1)} = \hbar \frac{I_x}{\omega}$
  - **'Dynamic' moment of inertia** $\mathscr{I}^{(2)} = \hbar \frac{dI_x}{d\omega}$

222Rn I(2) vs Frequency graph

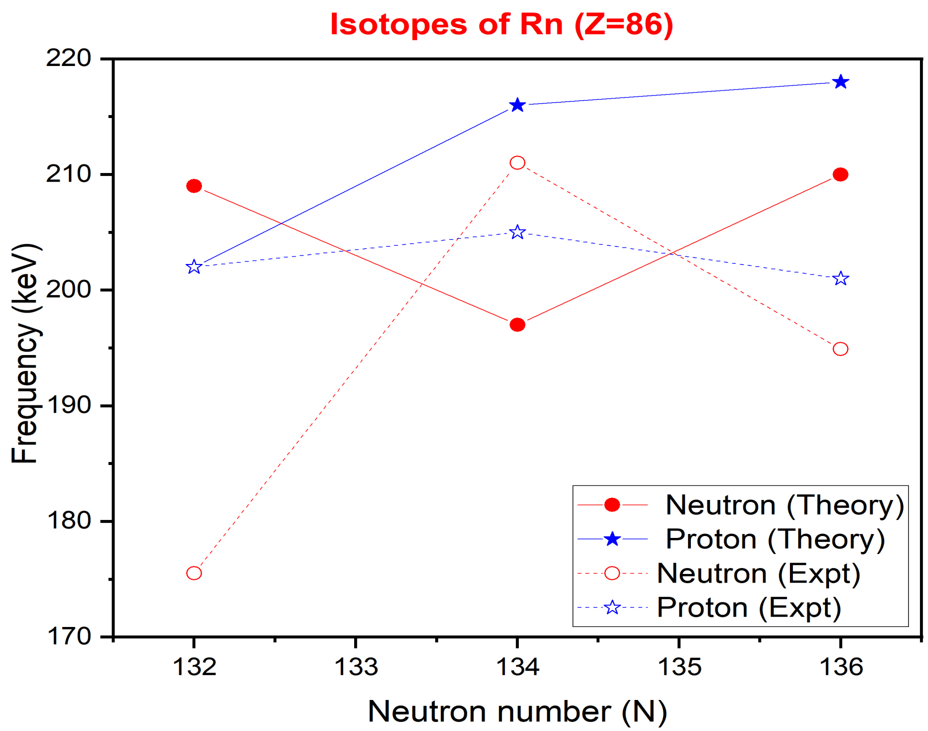


220Ra I(2) vs Frequency graph



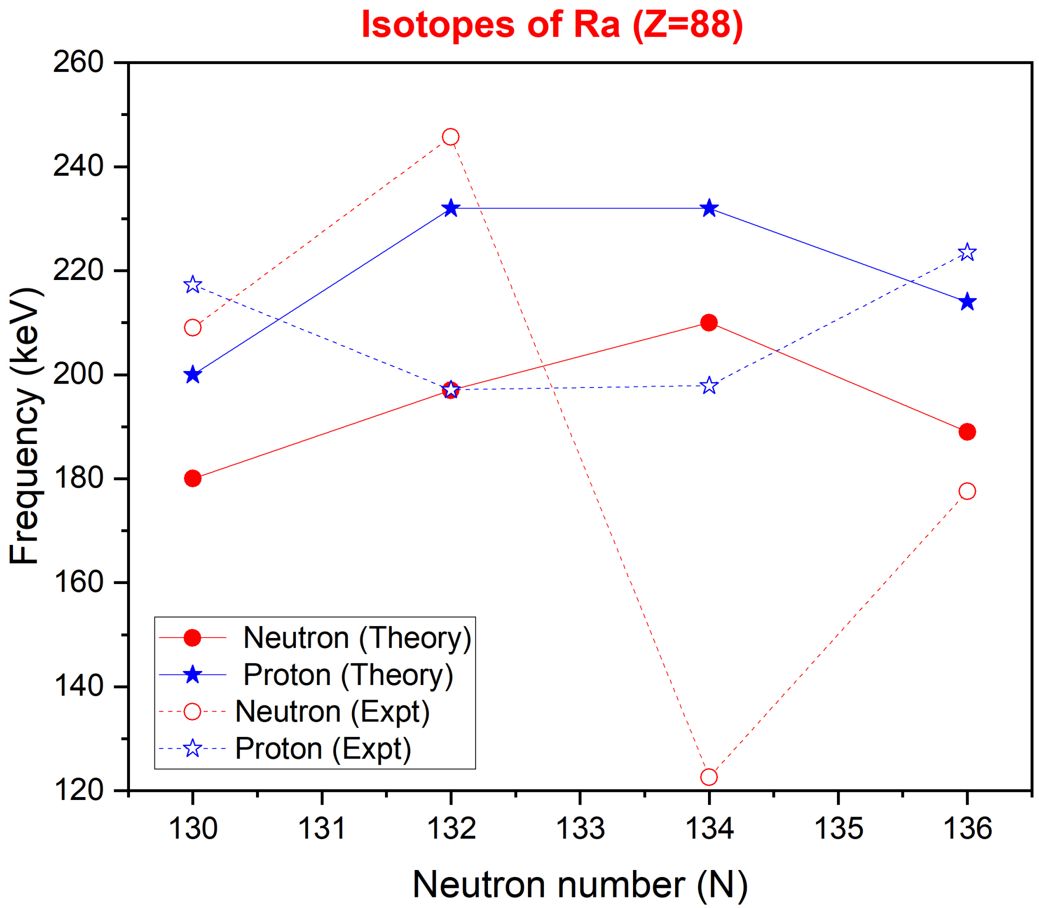
Nucleon pair breaking frequency

- The plots of $\mathscr{J}^{(2)}$ vs breaking frequency is attached below for all even-even and even-odd nuclei. Cranked shell-model calculations have been performed for a number of even-mass isotopes of Rn (Z=86) up to U (Z=92).
- The universal parameterization of the Woods-Saxon potential, incorporating quadrupole, octupole and higher-order degrees of deformation has been used.
- The values of the frequencies at which a pair of nucleons (either protons or neutrons) is broken due to the Coriolis force is obtained from the calculations. These values are compared with those from experiment, where data are available. In other cases, the calculations constitute predictions in regions which cannot presently be accessed by experiment.



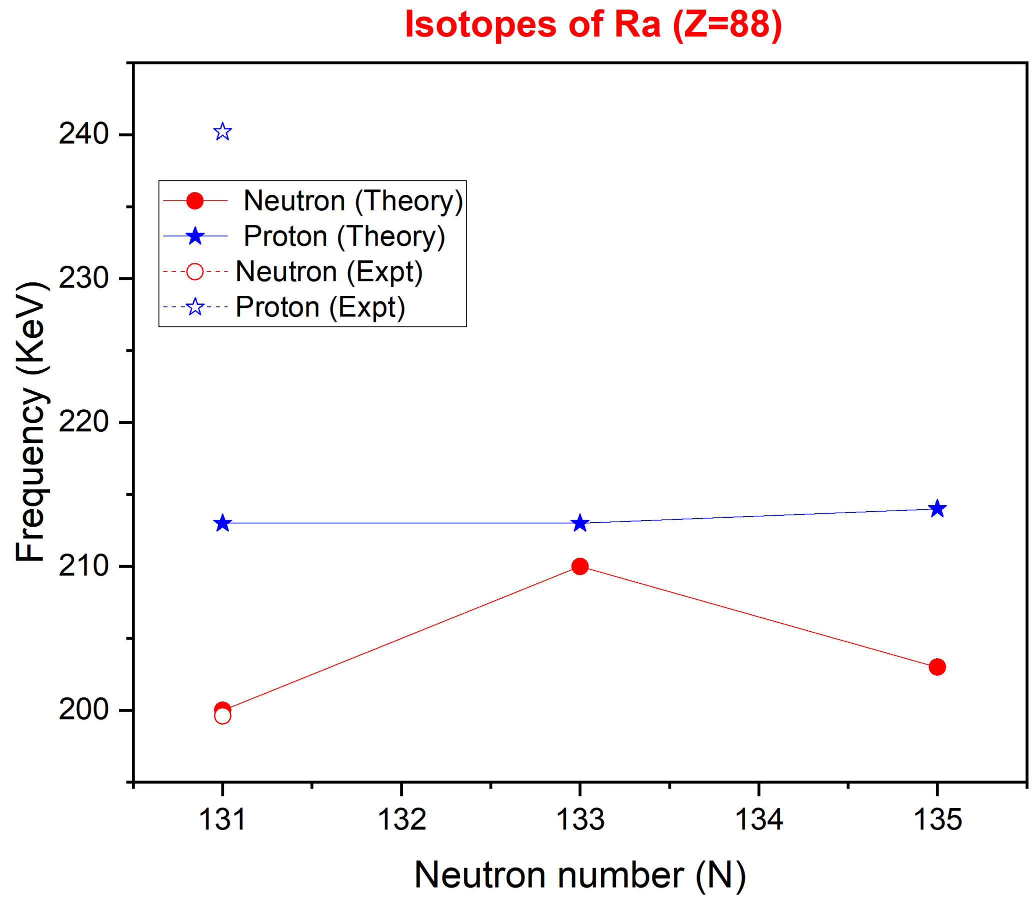
Nucleon pair breaking frequency

- ***Even-even mass isotopes of Ra (Z=88)***



Rotational frequency for breaking a pair of nucleons in Ra isotopes from theoretical calculations performed in this work and their comparison with experimental values.

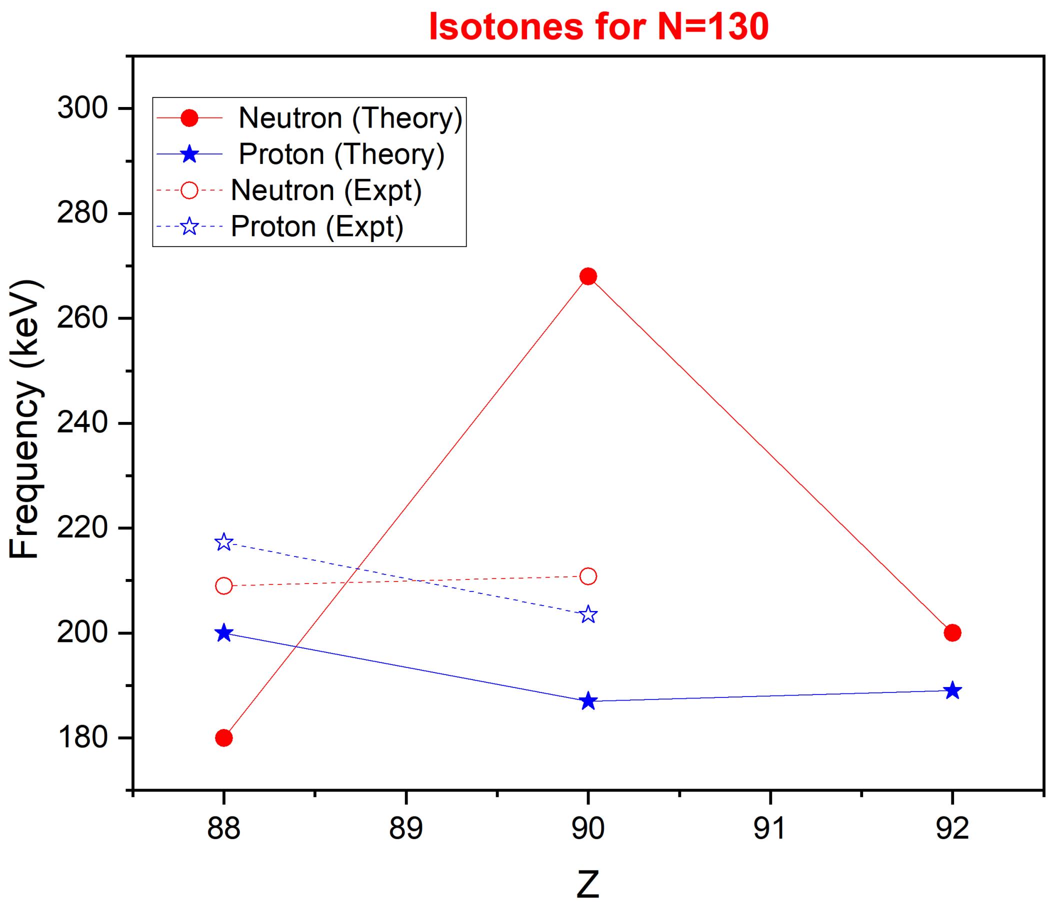
- ***Even-odd mass isotopes of Ra (Z=88)***



Rotational frequency for breaking a pair of nucleons in Ra isotopes from theoretical calculations performed in this work and their comparison with experimental values.

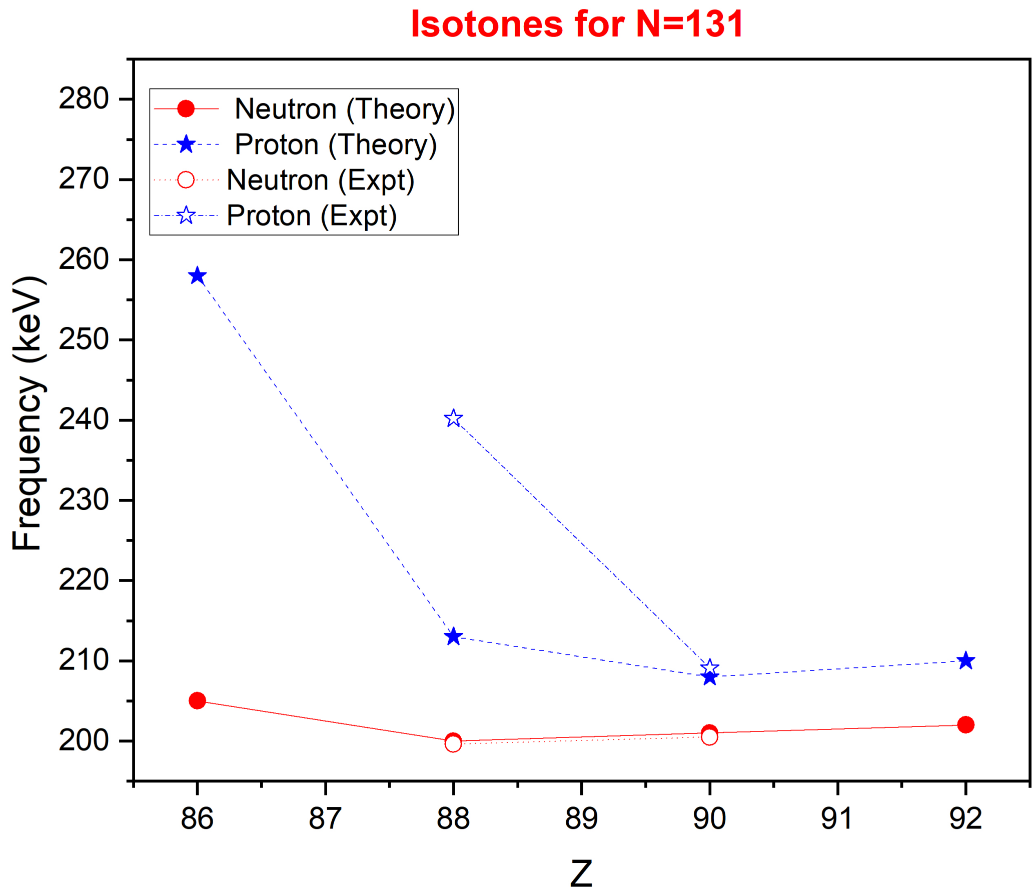
Nucleon pair breaking frequency

- ***Even-even mass isotones***



Rotational frequency for breaking a pair of nucleons in isotones for N=130 from theoretical calculations performed in this work and their comparison with experimental values.

- ***Even-odd mass isotones***



Rotational frequency for breaking a pair of nucleons in isotones for N=131 from theoretical calculations performed in this work and their comparison with experimental values.

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THANK YOU