Assignment -- Sangram Keshari Patro

Q5) $f(x) = x^5 - 2x^3 + 2x^2 - 3x + 1 = f_0(x)$

$\Rightarrow f_1(x) = f_0'(x) = 5x^4 - 6x^2 + 4x - 3$

$f_2(x) = -\left(\text{remainder of } f(x)/f'(x)\right)$

$$\frac{f(x)}{f'(x)} = \left(\frac{x}{5} + \frac{-\frac{4x^3}{5} + \frac{6x^2}{5} - \frac{12x}{5} + 1}{5x^4 - 6x^2 + 4x - 3}\right) \rightarrow \text{remainder}$$

$\Rightarrow f_2(x) = 4x^3 - 6x^2 + 12x - 5$ (multiplying by 5 to make coefficients integers)

$f_3(x) = -\left(\text{remainder of } f_1(x)/f_2(x)\right)$

$$\frac{f_1(x)}{f_2(x)} = \left(\frac{5x}{4} + \frac{15}{8}\right) + \frac{-\frac{39}{4}x^2 - \frac{49x}{4} + \frac{51}{8}}{4x^3 - 6x^2 + 12x - 5} \rightarrow \text{remainder}$$

$\Rightarrow f_3(x) = 78x^2 + 98x - 51$ (multiplying by 8)

$f_4(x) = -\left(\text{remainder of } f_2(x)/f_3(x)\right)$

$$\Rightarrow \frac{f_2(x)}{f_3(x)} = \frac{4x^3 - 6x^2 + 12x - 5}{78x^2 + 98x - 51} = \left(\frac{2x}{39} - \frac{215}{1521}\right) + \frac{\frac{43300x}{1521} - \frac{6190}{507}}{78x^2 + 98x - 51}$$

$\Rightarrow f_4(x) = -43300x + 18570$ (multiplying by 1521 to make coefficients integer)

$$f_5(x) = -\left(\text{remainder of } \frac{78x^2 + 98x - 51}{-43300x + 18570}\right)$$

$$= -\frac{5039225}{9374450}$$

$f(x)$	-4	-2	-1	0	1	2	3
$f_0(x) = x^5 - 2x^3 + 2x^2 - 3x + 1$	$-$	$-$	$+$	$+$	$-$	$+$	$+$
$f_1(x) = 5x^4 - 6x^2 + 4x - 3$	$+$	$+$	$-$	$-$	$0(-)$	$+$	$+$
$f_2(x) = 4x^3 - 6x^2 + 12x - 5$	$-$	$-$	$-$	$-$	$+$	$+$	$+$
$f_3(x) = 78x^2 + 98x - 51$	$+$	$+$	$-$	$-$	$+$	$+$	$+$
$f_4(x) = -43300x + 18570$	$+$	$+$	$+$	$+$	$-$	$-$	$-$
$f_5(x) = -\frac{50392251}{9371150}$	$-$	$-$	$-$	$-$	$-$	$-$	$-$
No. of sign changes ($V(x)$)	4	4	3	3	2	1	1

From the above table it is clear that $f(x)$ has 3 real roots $\therefore$ 1 negative root in $(-2,-1)$ and 2 positive roots in $(0,1)$ & $(1,2)$. It also has 1 pair of complex root as total roots should be 5.

(Q2) $\boxed{e^z = z}$

$$\Rightarrow e^{x+iy} = x+iy$$

$$e^x(\cos y + i\sin y) = x + iy$$

$$e^x \cos y - x = 0 \quad , \quad e^x \sin y - y = 0$$

(i) $e^x = \dfrac{y}{\sin y} \Rightarrow x = \ln\left(\dfrac{y}{\sin y}\right)$

$$\Rightarrow e^{\ln\left(\frac{y}{\sin y}\right)} \cos y - \ln\left(\frac{y}{\sin y}\right) = 0$$

$$\Rightarrow y\cot y - \ln\left(\frac{y}{\sin y}\right) = 0$$

(ii) $\cos y = \dfrac{x}{e^x} \Rightarrow y = \cos^{-1}\left(\dfrac{x}{e^x}\right) + 2n\pi$

$$\Rightarrow e^x \sin\left(\cos^{-1}\left(\frac{x}{e^x}\right)\right) - \cos^{-1}\left(\frac{x}{e^x}\right) = 2n\pi$$

where $n = 0, 1, 2, 3, \ldots$. For each value of n there is one root.

But for $n = 0$ & 1 only the y is coming < 10.
(see the program attached.)