ASSIGNMENT - 11

→ Sangram Keshari Patro

③ $g(t) = 2\sin(2\pi f_1 t) + \cos(2\pi f_2 t)$

$N = 128$, $f_1 = 20$, $f_2 = 80$, $t_K = \frac{K}{N}$ for $K = 0, 1, \ldots N-1$

So, $f_c = \frac{1}{2\Delta} = \frac{128}{2} = 64$ Hz

f_2 is aliased to $80 - 2f_c = 80 - 128 = -48$ Hz

So, The frequencies which will be seen in DFT are.

$(-f_c, f_c) \rightarrow \{-48, -20, 20, 48\}$ Hz

$(0, 2f_c) \rightarrow (20, 48, 80, 108\}$ Hz

rest all frequencies are 0.

and ~~$G_{20} \ldots G_{108}$~~ $G_{48} = G_{-80}$ & $G_{108} = G_{-20}$

$$G_j = \sum_{K=0}^{N-1} g_K W^{jK}, \quad g_K = \frac{1}{N}\sum_{j=0}^{N-1} G_j W^{-jK}$$

(where $W = e^{\frac{2\pi i}{N}}$)

$$g_K = \frac{1}{N}\sum_{j=0}^{N-1} G_j W^{-jK} = \frac{1}{N}\left(G_{20} \underbrace{e^{\frac{2\pi i}{N}}}_{W}{}^{\times(-20K)} + G_{48} W^{(-48K)} + G_{80} W^{(-80K)} + G_{108} W^{(-108K)}\right)$$

$$g_K = \frac{1}{N}\left(G_{20} W^{-20K} + G_{-80} W^{80K} + G_{80} W^{-80K} + G_{-20} W^{20K}\right) \rightarrow ①$$

and $g_K = 2\sin\left(2\pi f_1 \times \frac{K}{N}\right) + \cos\left(2\pi f_2 \frac{K}{N}\right)$

$$= \cancel{2}\left(\frac{e^{i(2\pi f_1 \frac{K}{N})} - e^{-i(2\pi f_1 \frac{K}{N})}}{\cancel{2}i}\right) + \left(\frac{e^{i(2\pi f_2 \frac{K}{N})} + e^{-i(2\pi f_2 \frac{K}{N})}}{2}\right)$$

$$= \frac{1}{2}\left(W^{80K} + W^{-80K}\right) + i\left(W^{-20K} - W^{20K}\right) \rightarrow ②$$

by comparing → ① & → ② we get,

$G_{-80} = \frac{N}{2}$, $G_{80} = \frac{N}{2}$, $G_{20} = Ni$ & $G_{-20} = -Ni$

If we divide by N to all of G_j we get

$G_{20} = i$, $G_{48} = \frac{1}{2}$, $G_{80} = \frac{1}{2}$ and $G_{108} = -i$

rest all G_js are 0.

Q4) $H_j = \sum_{k=0}^{N-1} h_k w^{jk}$

$$= \sum_{k=0}^{N-1} (f_k + i g_k) w^{jk} \longrightarrow ①$$

$$H_{N-j} = \sum_{k=0}^{N-1} (f_k + i g_k) w^{-jk}$$

$$H_{N-j}^* = \sum_{k=0}^{N-1} (f_k^* - i g_k^*) w^{jk}$$

$$H_j = H_{N-j}^* = \sum_{k=0}^{N-1} f_k w^{jk} + i \sum_{k=0}^{N-1} g_k w^{jk} \longrightarrow ②$$

(as h_k is real)

$$\Rightarrow H_j + H_{N-j}^* = 2 \sum_{k=0}^{N-1} f_k w^{jk} = 2F_j \quad (\text{from} \to ① \ \& \ ②)$$

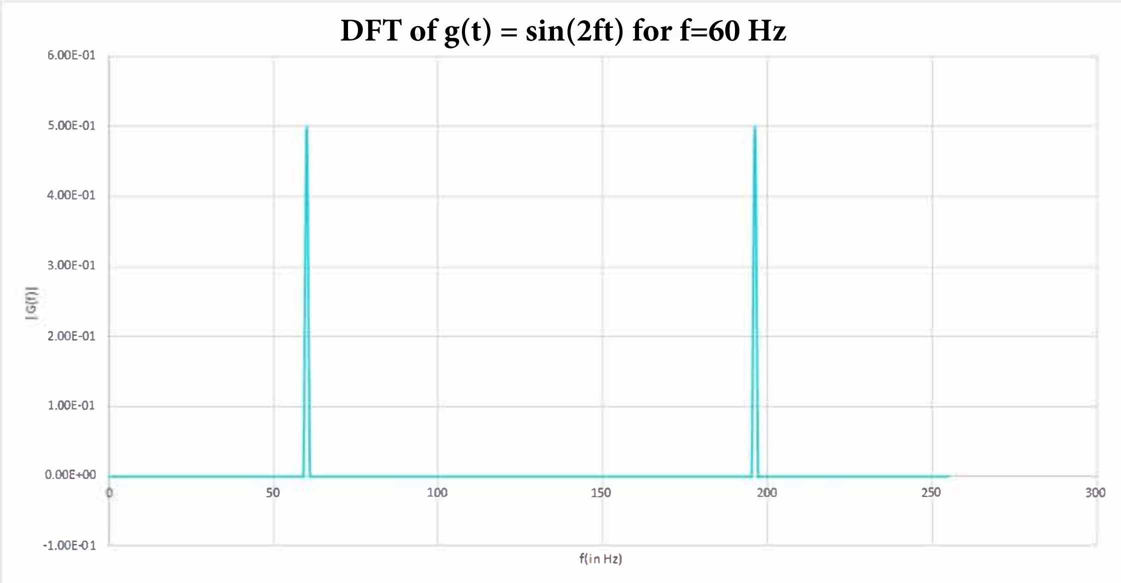
$$\Rightarrow \boxed{F_k = \frac{1}{2}(H_k + H_{N-k}^*)}$$

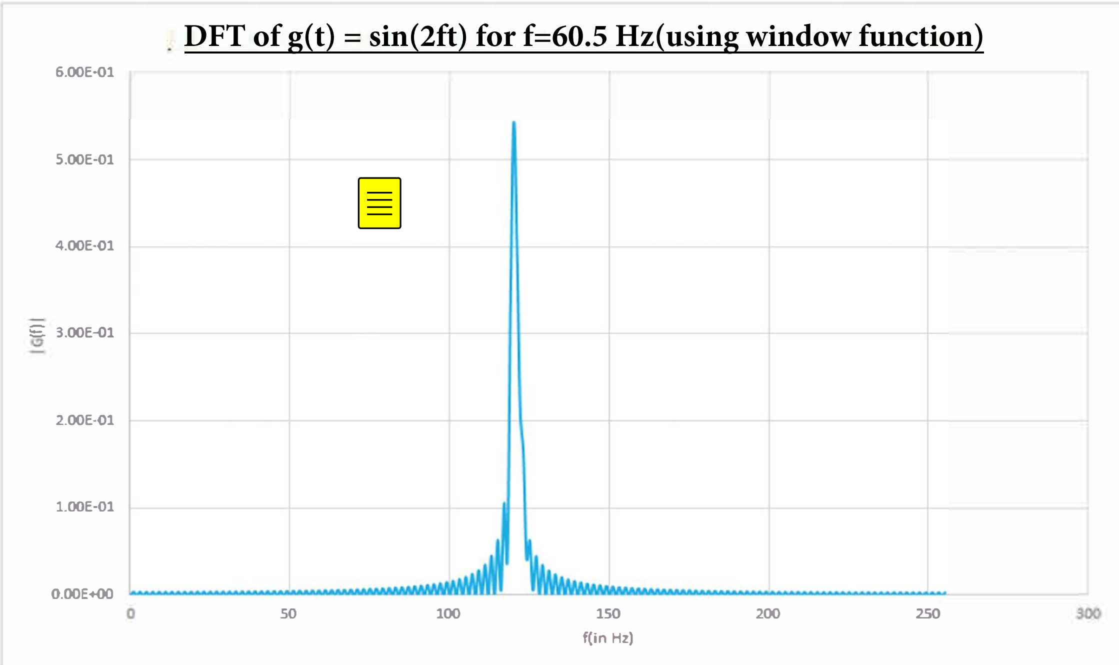
$$H_j - H_{N-j}^* = 2i \times \sum_{k=0}^{N-1} g_k w^{jk} \quad (\to ① - \to ②)$$

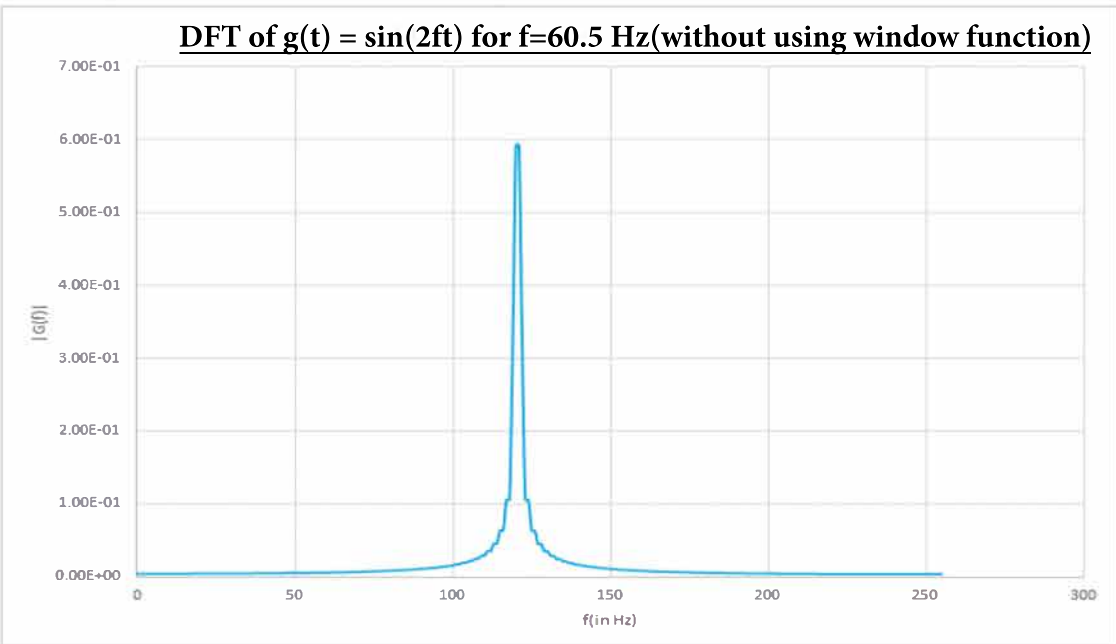
$$= 2i G_k$$

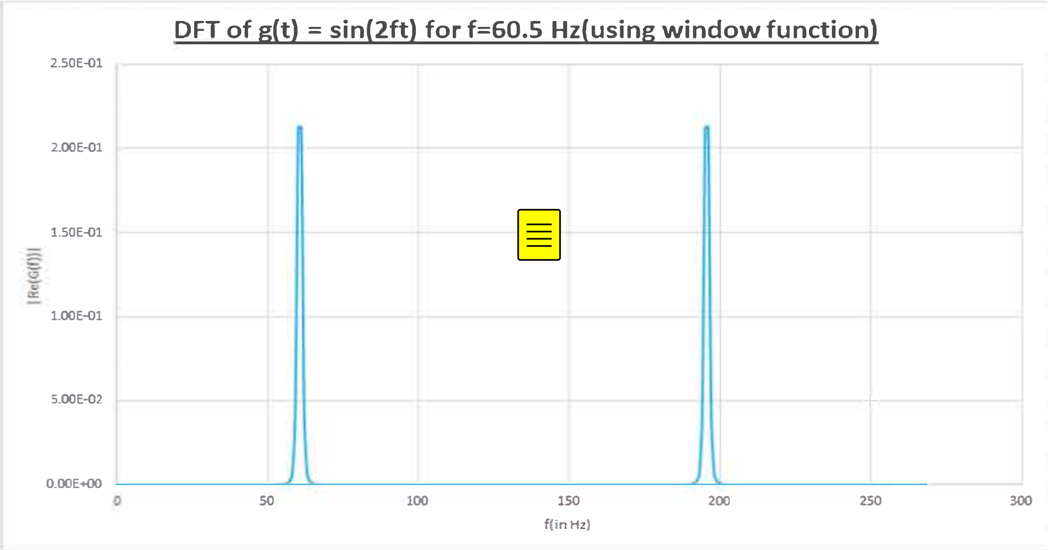
$$\Rightarrow G_k = \frac{1}{2i}(H_k - H_{N-k}^*)$$

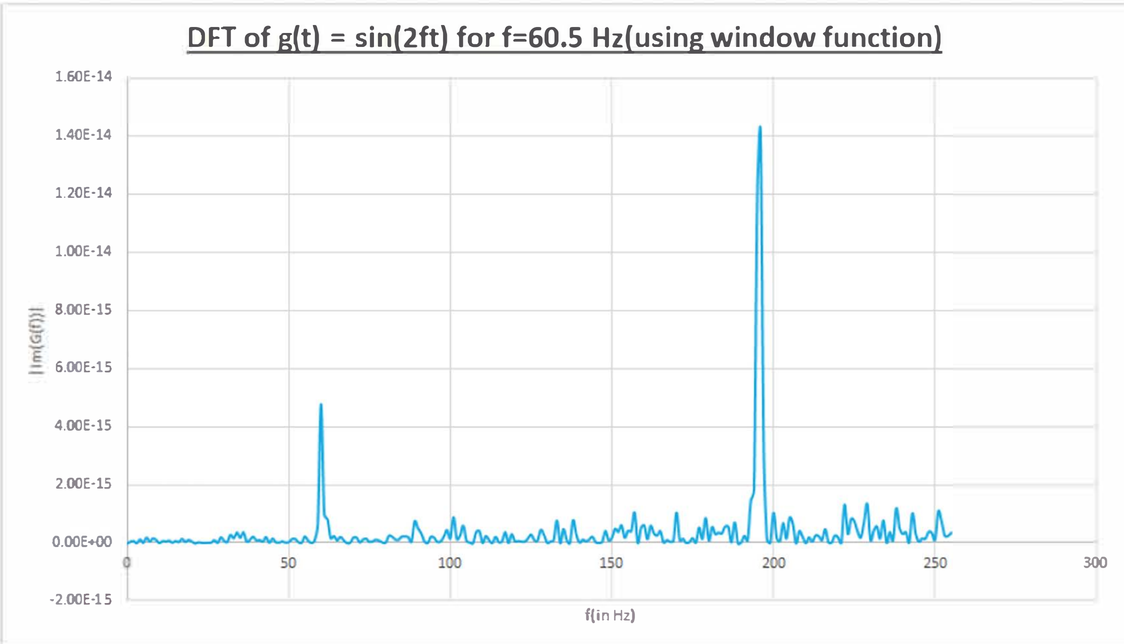
Q1) **DFT of g(t) = sin(2ft) for f=60 and 60.5 Hz**

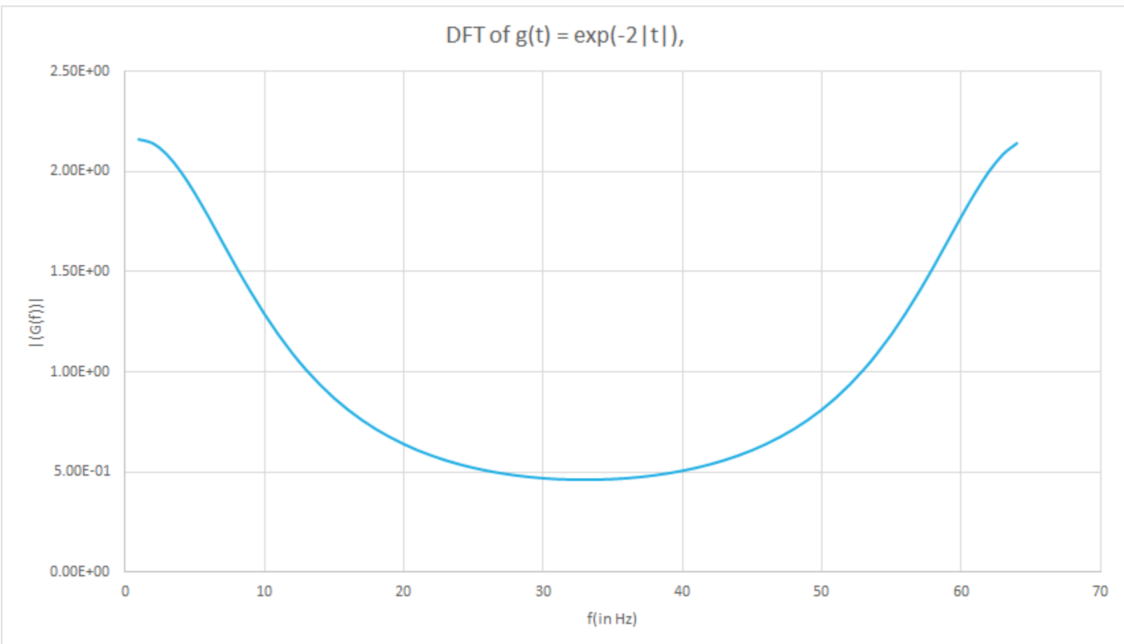







DFT of g(t) = sin(2ft) for f=60.5 Hz(using window function)
2.50E-01
2.00E-01
1.50E-01
1.00E-01
5.00E-02
0.00E+00
|Re(G(f))|
0
50
100
150
200
250
300
f(in Hz)


DFT of g(t) = sin(2ft) for f=60.5 Hz(using window function)
1.60E-14
1.40E-14
1.20E-14
1.00E-14
8.00E-15
6.00E-15
4.00E-15
2.00E-15
0.00E+00
-2.00E-15
|Im(G(f))|
0
50
100
150
200
250
300
f(in Hz)


DFT of g(t) = exp(-2|t|),
2.50E+00
2.00E+00
1.50E+00
1.00E+00
5.00E-01
0.00E+00
|(G(f))|
0
10
20
30
40
50
60
70
f(in Hz)