NM - ASSIGNMENT

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$y = mx + c$

2)

$$\chi^2 = \sum_{i=1}^{n} \left(\frac{y_i - (mx_i + c)}{\sigma_i} \right)^2$$

$$\frac{\partial \chi^2}{\partial c} = 2 \times \sum_{i=1}^{n} \left(\frac{y_i - (mx_i + c)}{\sigma_i^2} \right) \times -1 = 0 \rightarrow (1)$$

$$\Rightarrow \sum \frac{y_i}{\sigma_i^2} - m \sum \frac{x_i}{\sigma_i^2} - c \sum \frac{1}{\sigma_i^2} = 0$$

$$\Rightarrow m S_1 + c S_0 = t_0 \rightarrow (2)$$

$$\frac{\partial \chi^2}{\partial m} = 2 \times \sum_{i=1}^{n} \left(\frac{y_i - (mx_i + c)}{\sigma_i^2} \right) \times (-x_i) = 0 \rightarrow (3)$$

$$\Rightarrow \sum_{i=1}^{n} \frac{x_i y_i}{\sigma_i^2} - m \sum_{i=1}^{n} \frac{x_i^2}{\sigma_i^2} - c \sum_{i=1}^{n} \frac{x_i}{\sigma_i^2} = 0$$

$$\Rightarrow m S_2 + c S_1 = t_1 \rightarrow (4)$$

from (2) & (4)

$$(2) \times S_2 - (4) \times S_1 \Rightarrow c(S_0 S_2 - S_1^2) = t_0 S_2 - t_1 S_1$$

$$\Rightarrow c = \frac{S_2 t_0 - S_1 t_1}{S_0 S_2 - S_1^2}$$

$$\text{and } m = \frac{t_0 - c S_0}{S_1} = \frac{t_0 - \dfrac{S_0 t_0 S_2 + S_0 S_1 t_1}{S_0 S_2 - S_1^2}}{S_1} = \frac{S_0 t_1 - S_1 t_0}{S_0 S_2 - S_1^2}$$

Consider the form,

$$y = m(x - \bar{x}) + c, \text{ where } \bar{x} = \frac{S_1}{S_0},$$

$$\text{so, } \chi^2 = \sum_{i=1}^{n} \left(\frac{y_i - \left[m\left(x_i - \frac{S_1}{S_0}\right) + c \right]}{\sigma_i} \right)^2$$

$$\Rightarrow \frac{\partial \chi^2}{\partial c} = 2 \times \sum \left(\frac{y_i - \left[m\left(x_i - \frac{S_1}{S_0}\right) + c \right]}{\sigma_i^2} \right) \times (-1) = 0$$

$$\Rightarrow \sum \frac{y_i}{\sigma_i^2} - m \sum \frac{x_i}{\sigma_i^2} + m \frac{S_1}{S_0} \times \sum \frac{1}{\sigma_i^2} + c \sum \frac{1}{\sigma_i^2} = 0$$

$$t_0 - m S_1 + m \frac{S_1}{\cancel{S_0}} \times \cancel{S_0} + c S_0 = 0 \Rightarrow \cancel{m\left(S_2 - \frac{S_1^2}{S_0}\right) + cS_1 = t_1}$$

$$c = \frac{t_0}{S_0} \rightarrow (1)$$

$$\Rightarrow \frac{\partial \chi^2}{\partial m} = 2 \sum \frac{y_i - \left[m\left(x_i - \frac{S_1}{S_0}\right) + c \right]}{\sigma_i^2} \times (-x_i) = 0$$

$$\Rightarrow -\underbrace{\sum \frac{x_i y_i}{\sigma_i^2}}_{t_1} + m \underbrace{\sum \frac{x_i^2}{\sigma_i^2}}_{S_2} + c \underbrace{\sum \frac{x_i}{\sigma_i^2}}_{S_1} - \frac{m S_1}{S_0} \underbrace{\sum \frac{x_i}{\sigma_i^2}}_{S_1} = 0$$

$$\Rightarrow m\left(S_2 - \frac{S_1^2}{S_0}\right) + cS_1 = t_1 \quad \text{— II.}$$

using $c = \frac{t_0}{S_0}$ in eq→ II,

$$m = t_1 - \frac{S_1 \times t_0}{S_0} \Big/ \left(S_2 - \frac{S_1^2}{S_0}\right) = \frac{S_0 t_1 - S_1 t_0}{S_2 S_0 - S_1^2}$$

(3) $$F(a,x) = \frac{a_0}{2} + \sum_{j=1}^{m} a_j \cos(jx)$$

$$\chi^2 = \int_0^{\pi} w(x)\left[f(x) - \left(\frac{a_0}{2} + \sum_{j=1}^{m} a_j \cos(jx)\right)\right]^2 dx$$

take $w(x) = 1$

$$\frac{\partial \chi^2}{\partial a_0} = 2\int_0^{\pi} \left[f(x) - \left(\frac{a_0}{2} + \sum_{j=1}^{m} a_j \cos(jx)\right)\right] \times -\frac{1}{2}\, dx = 0$$

$$\Rightarrow \int_0^{\pi} f(x)\, dx = \frac{a_0}{2}\int_0^{\pi} dx + \cancelto{0}{\int_0^{\pi} \sum_{j=1}^{m} a_j \cos(jx)\, dx}$$

$$\Rightarrow a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\, dx \rightarrow (1)$$

$$\frac{\partial \chi^2}{\partial a_j} = \int_0^{\pi} 2\left[f(x) - \left(\frac{a_0}{2} + \sum_{j=1}^{m} a_j \cos(jx)\right)\right] \cos jx\, dx = 0$$

$$\Rightarrow \int_0^{\pi} f(x)\cos(jx)\, dx = \cancelto{0}{\int_0^{\pi} \frac{a_0}{2}\cos(jx)} + \int_0^{\pi} \left[\sum_{j=1}^{m} a_j \cos(jx)\right] \cos(jx)\, dx$$

$$\Rightarrow \int_0^{\pi} f(x)\cos(jx)\, dx = a_j \int_0^{\pi} \cos^2(jx)\, dx = a_j \times \int_0^{\pi} \left(\frac{1+\cos(2jx)}{2}\right) dx$$

$$= a_j \times \frac{\pi}{2}$$

$$\Rightarrow a_j = \frac{2}{\pi}\int_0^{\pi} f(x)\cos(jx)\, dx \rightarrow (2)$$

so, from → (1) & (2)

$$a_j = \frac{2}{\pi}\int_0^{\pi} f(x)\cos(jx) \rightarrow [\text{for } j = 0, 1, \ldots n]$$