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The Sullying Effect of Recessions

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Previous work has established that recessions involve a "cleansing" effect, so that in downturns, only high productivity jobs remain. But empirical evidence suggests job quality is procyclical: jobs created in recessions are likely to be low-paying and temporary. This paper modifies previous models by adding on-the-job search, which leads to an additional "sullying" effect. Calibration of the model suggests this offsetting sullying effect is likely to be much larger than the cleansing effect, and can account for the procyclical match quality we observe in the data.

1. INTRODUCTION

Since as far back as Schumpeter (1939), economists have been interested in how business cycles affect the allocation of resources. Schumpeter originally advanced the argument that recessions promote a more efficient allocation of resources by driving out bad investments and freeing up resources for more productive uses. This view has been revived recently in a series of theoretical papers inspired by the work of Davis and Haltiwanger (1992), who demonstrated that recessions are associated with increased job reallocation in the manufacturing sector. In trying to explain these cyclical patterns in reallocation, authors such as Hall (1991, 2000); Mortensen and Pissarides (1994); Caballero and Hammour (1994, 1996); and Gomes, Greenwood and Rebelo (1999) have devised models that formalize the notion that recessions promote a more efficient allocation of resources by "cleansing" out less efficient production arrangements and redirecting resources into relatively more productive uses. The intuition behind these models is as follows. Suppose there is some friction that prevents resources from moving instantaneously to their most efficient uses, creating slack in the economy because of an inefficient allocation of resources. Since recessions drive down the profitability of all production arrangements, the least efficient uses of resources will cease to be viable and shut down, and the owners of those resources will have incentive to search for other, more productive arrangements. Hence, recessions reduce slack in production by limiting the scope for allocative inefficiency and shifting resources towards their more productive uses.[1]

Although the cleansing view rests on sound theoretical foundations, some of its predictions have not held up well to subsequent empirical work. For example, studies such as Griliches and Regev (1995) and Bailey, Bartelsman and Haltiwanger (1998) that

1. It is important to emphasize that this need not imply recessions lead to higher welfare. For example, the shutting down of less efficient production arrangements might be caused by a negative shock such as a fall in aggregate productivity, which on its own makes agents worse off. Moreover, to the extent that frictions impede the immediate allocation of resources into more productive uses, part of the gains will not be realized as long as resources remain unemployed while seeking out better uses. The papers cited above are quite careful in making this distinction, although their normative implications have often been misinterpreted.

examine panel datasets of manufacturing plants where establishments can be ranked according to their productivity find only weak evidence that slumps are associated with a shift of resources into more productive plants. Others have gone even further in arguing that jobs created during recessions appear to be of lower average quality: Bowlus (1993) and Davis, Haltiwanger and Schuh (1996) both find evidence that jobs created during recessions are more likely to be destroyed than jobs created during booms, suggesting there is little incentive to keep such production arrangements in operation for long periods. Bowlus (1993) also finds that jobs created during recessions come from the lower part of the wage distribution, which accords with work by Bils (1985) and Shin (1994). If wages are increasing in the inherent productivity of a job, this too would suggest recessions encourage the creation of relatively less productive matches. Another problem for the cleansing hypothesis is evidence that worker reallocation, in contrast with job reallocation, is procyclical. This problem was pointed out early on by Summers in his discussion of Hall (1991). Summers argues that since workers are less likely to quit during recessions, downturns must impede the transition of workers into their most productive uses rather than facilitate it. This view is confirmed by worker surveys that suggest employed workers are more likely to report being mismatched and underutilized during recessions.[2] The question, then, is whether these observations are consistent with the cleansing hypothesis: should we view recessions as alleviating the misallocation of resources, as the cleansing view would hold, or should we view them as exacerbating it?

This paper argues that the empirical evidence above can be potentially reconciled with the theoretical literature on the cleansing effect of recessions. More specifically, it claims that while recessions hasten the destruction of less efficient matches, they also stifle the creation of the most efficient matches. Thus, the fact that recessions kill off marginal production arrangements does not necessarily imply that they serve to improve allocative efficiency and reduce slack in the economy. The explanation I offer involves a conceptually simple modification of existing theoretical models, namely that I allow workers to search on-the-job as well as through unemployment. This feature, which has been neglected for convenience in the theoretical literature cited above, gives rise to a countervailing "sullying" effect that works against the cleansing effect. To understand the role of on-the-job search, consider Figure 1. This figure depicts the distribution of jobs across levels of surplus. A negative productivity shock implies all jobs offer less surplus than before, inducing an affine shift of the distribution across levels of surplus, as illustrated in the first two panels. Jobs towards the bottom of the distribution that yield positive surplus when economic conditions are favourable fail to generate positive surplus when conditions are unfavourable, and are destroyed. This reflects the cleansing effect, and can explain why job reallocation appears to be countercyclical. But with on-the-job search, we also need to take into account how resources are allocated across those matches that survive cleansing. In what follows, I show that entrepreneurs create relatively fewer vacancies during recessions, so workers have a more difficult time moving into those jobs which they are best suited for. Thus, workers reallocate into their most productive uses more slowly during recessions, and the mass of the distribution shifts towards matches that yield less surplus. This shift is illustrated in the last panel of Figure 1, and is what I refer to as the sullying effect: even as the economy cleanses out its most inferior matches, more workers are stuck in mediocre matches, and fewer high

2. See, for example, Akerlof, Rose and Yellen (1988). Further evidence comes from Acemoglu (1999), who reports workers were more likely to classify themselves as being overeducated during the mid 1970s, when unemployment was high, than during the mid 1980s.

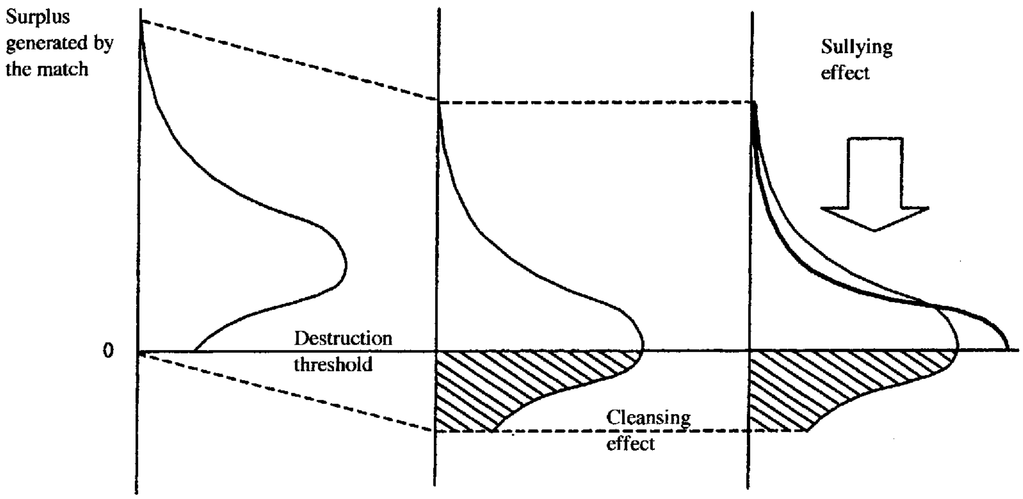

FIGURE 1
The cleansing and sullying effects

quality matches are created. The overall effect of recessions on allocative efficiency thus depends on the relative magnitude of the two effects. To address this issue, I calibrate the model in the second half of the paper. Of the two forces, it is the sullying effect that appears to be the more dominant. As an illustrative example, a simulation of the model suggests that cumulative reallocation of workers across job sites during the three recessions between 1970–1983 contributed to an almost 1% *decline* in productivity from its original 1970 level. This is a far cry from the Schumpeterian notion that reallocation during recessions serves only to purge the economy of mismatch and inefficient production arrangements.

While several recent authors have taken on the original cleansing hypothesis, what distinguishes this paper from the existing literature is its potential to explain why aggregate downturns discourage the creation of more productive matches while fostering the existence of relatively less efficient one. Previous work, such as Ramey and Watson (1997) and Caballero and Hammour (1999), critique aspects of the cleansing hypothesis, but they are motivated by the distinct question of whether reallocation during recessions is socially efficient. They demonstrate that under certain conditions, agents could be made better off if some of the relatively less productive matches that are destroyed during recessions were kept intact. These results are similar in spirit to those of the present model, since they also imply that reallocation during recessions need not lead to more efficient outcomes. But these models do not predict that recessions stifle the creation of more efficient matches or worsen match quality among surviving production arrangements. For example, Ramey and Watson introduce moral hazard into the production decisions of agents, and show that the destruction threshold in Figure 1 will generally not be efficient. But the cyclical implications of their model are identical to those of conventional models, *i.e.* it is still true in their model that only the least productive jobs are destroyed during recessions, and the average quality of surviving matches rises. Caballero and Hammour consider moral hazard in credit markets rather than in production. In their model, entrepreneurs with insufficient funds are forced to destroy jobs which are then re-created by entrepreneurs who are not as severely constrained financially. This implies that some of the jobs above the optimal destruction threshold will nevertheless be destroyed during recessions. This type of reallocation represents a pure drain, since resources are wasted as labour is needlessly reallocated to uses in which they are no more productive than they were previously. But since only jobs above the destruction threshold will be created during recessions and the threshold still rises during recessions, their model again implies average match quality rises in recessions and allocative inefficiency declines.[3]

The paper is organized as follows. Section 2 lays out the model and defines a symmetric steady-state equilibrium. Section 3 uses comparative statics around the symmetric steady state to motivate the cleansing and sullying effects of recessions. In Section 4, I numerically solve a version of the model in which aggregate productivity is allowed to fluctuate over time, and consider its quantitative implications. Section 5 concludes.

2. THE MODEL

This section develops a matching model in which agents are allowed to search on-the-job as well as when unemployed. The model builds on the work of Mortensen and Pissarides (1994), who generate a cleansing effect in a matching model with only off-the-job search. In the very same symposium issue, Pissarides (1994) discusses how this matching framework can be modified to allow for on-the-job search. While the present model shares certain aspects with his model, it differs in several important respects. First, allocative inefficiency in my model arises when workers occupy jobs in which they do not enjoy a comparative advantage. By contrast, allocative inefficiency in the Pissarides model arises because bargaining distortions cause firms to create too few jobs of a particular type relative to the socially efficient level. This source of slack makes it more difficult to quantify the degree of allocative inefficiency over the cycle. Second, his framework confounds the direct effects of aggregate productivity shocks with indirect effects that arise because shocks impact different sectors asymmetrically. By focusing on a symmetric framework, the model developed here makes the role of aggregate shocks more transparent. Finally, his framework does not easily generalize to more than two job categories, which is essential for studying cleansing and sullying simultaneously.[4]

Consider an economy with N different types of workers, indexed by $i \in \{1, \ldots, N\}$, where each type accounts for $1/N$-th of the total population. A worker can be employed in any of N different types of jobs that entrepreneurs can create, where jobs are indexed by $j \in \{1, \ldots, N\}$. Entrepreneurs can create any number of any type of job they want. Each job requires only one worker. Once an entrepreneur creates a job, he encounters workers who can fill that job through a random matching process that will be described in more detail below. When an entrepreneur finds a worker he wishes to hire and the worker agrees

3. In fact, the financial constraints in Caballero and Hammour magnify the countercyclical nature of match quality, since jobs that are vulnerable to financial constraints are less productive on average. As they observe in a footnote, "the 'selectivity' of spurious destruction makes the difference with Schumpeterian destruction less stark than may appear at first glance." Barlevy (2000*a*) considers a different model in which the most productive jobs are also more vulnerable to financial constraints. His model can also generate procyclical match quality, but for different reasons than the ones considered here.

4. In a separate paper, Mortensen (1994) also modifies his model with Pissarides to allow for on-the-job search. However, his formulation assumes new matches always generate the highest level of surplus, and so cannot produce fluctuations in the quality of new jobs over the cycle.

to form a match, the entrepreneur will retain that worker and stop searching until either the original worker he hired leaves him or the two somehow become separated.[5]

The production technology governing the various pairings of workers and job types is as follows. On his own, any worker can produce b units of output per unit of time (the model is set in continuous time). When a worker of type i is matched with a job of type j, he can produce y_{ij} units of output. The output y_{ij} is assumed to be the product of an aggregate component y and a match-specific component ε_{ij}:

$$y_{ij} = y(1 + \varepsilon_{ij}).$$

Although I will ultimately want to study the effects of fluctuations in aggregate productivity y, it will be helpful for now to treat the level of aggregate productivity y as a constant.

I next impose structure on the idiosyncratic component ε_{ij}. Specifically, let

$$n = N - [(N + j - i) \bmod N],$$

denote an inverse measure of the distance between i and j, *i.e.* n is a measure which assumes its highest value N when i and j are equal, and its lowest value 1 when i and j are furthest apart, modulo N. I assume $\varepsilon_{ij} = \varepsilon_n$, *i.e.* the productivity of worker i in job j only depends on the difference between i and j. This imposes a convenient symmetry in production technology. That is, each of the ε_{ij} can only assume one of N different values, *i.e.* $\varepsilon_{ij} \in \{\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_N\}$. Without loss of generality, I arrange jobs so that $\varepsilon_1 < \varepsilon_2 < \cdots < \varepsilon_N$, which implies individuals are most productive on the job that bears their name and then become increasingly less productive as the index of the job number rises, modulo N. It follows that n can be interpreted as a measure of match quality, since a higher n connotes a more productive match. This pattern can be illustrated in the following matrix, which denotes the match quality n associated with each possible pair i and j:

$$\begin{array}{cc} & j \\ i & \begin{bmatrix} N & N-1 & & 2 & 1 \\ 1 & N & & 3 & 2 \\ & & \ddots & & \\ N-2 & N-3 & & N & N-1 \\ N-1 & N-2 & & 1 & N \end{bmatrix} \end{array}.$$

This specification for the production technology is virtually identical to the one used by Marimon and Zilibotti (1999). However, they rule out on-the-job search in their framework, which lies at the heart of the analysis here. Moscarini (1999) also considers a closely related production technology. In contrast with Marimon and Zilibotti, he does allow for on-the-job search in his framework, although using a different specification than the one considered here. His paper focuses on the question of how selective workers are regarding which jobs they apply for, while on-the-job search plays only a minimal role in his analysis. Here, the emphasis is reversed; the matching technology is such that workers

5. Thus, entrepreneurs cannot engage in on-the-job search for better workers. However, given free entry into the market for vacancies, this assumption is irrelevant: entrants drive the expected profits of posting a vacancy to zero, so entrepreneurs will never find it profitable to search for a better worker than the one they currently employ. Put another way, under the assumption of constant returns to scale implicit in the model, entrepreneurs do not need to get rid of their current workers to make room for a new one, and so if they encounter a new worker they will simply create a new position.

are selective in a trivial way, at least in the steady-state version of the model, but the role of on-the-job search is explored in more detail.[6]

In a frictionless environment, the efficient allocation of resources would assign each worker to the job that bears his name; this is the job in which the worker enjoys a comparative advantage and, as I discuss below, all jobs are equally costly to create. However, in a decentralized market with search frictions, individuals are allocated randomly across jobs. They arrive at good matches only through ongoing search. There will therefore be an endogenous amount of resource misallocation, depending on how agents in this economy respond to the underlying search frictions. I now describe the assumptions characterizing the search technology available in this economy. These assumptions are adapted from the matching framework of Mortensen and Pissarides (1994):

- Let v_j denote the total number of vacancies posted by entrepreneurs for jobs of type j at a given point in time. There is an infinite number of potential entrepreneurs, so $v_j \in [0, \infty)$. The total number of vacancies is denoted by

$$v = \sum_{j=1}^{N} v_j.$$

- The names of all workers who are not employed in their most productive match are submitted to a centralized matching algorithm which periodically assigns them to one of the posted vacancies chosen at random. If u denotes the number of unemployed workers and e_n denotes the fraction of workers employed in a match of quality n, then the total number of job applications is given by

$$z = u + \sum_{n=1}^{N-1} e_n.$$

- The number of contacts suggested by the matching algorithm per unit of time depends on the total number of vacancies and applications. Specifically, it is given by a constant returns to scale function $m(z, v)$, where $m(\cdot, \cdot)$ is increasing in both arguments, $m(x, 0) = m(0, x) = 0$ for any $x \geqq 0$, and $\lim_{x\to\infty} m(1, x) = \lim_{x\to\infty} m(x, 1) = \infty$. Contacts are distributed uniformly across all possible worker-job pairs.
- A contact turns into a match if and only if both parties agree to a partnership.
- Matches break up exogenously at a rate s.

Although the matching technology is standard, it should be pointed out that in contrast with some of the previously mentioned models that allow for on-the-job search, the rate at which offers arrive in the above specification is the same regardless of whether the worker is employed or not. This implies that the reservation productivity for a worker in steady state is equal to b, a fact confirmed in a lemma in the Appendix. Intuitively, since accepting a job does not require the worker to sacrifice a high rate of encountering better offers, there is no reason not to accept a job that offers a higher flow payment than unemployment, and no reason to accept a job that offers a lower flow payment.

6. An analogous structure has also been used in search models of money, such as Kiyotaki and Wright (1991). There, the assumption that different workers have a comparative advantage in the production of different goods is replaced with one where different agents have an acquired taste for the consumption of different goods. In these models agents either consume a good or keep it to trade for goods they do consume, and there is no sense in which agents are better off searching while holding a good that is closer to the one they most prefer than one that is further away. Hence, the analogue to on-the-job search in these models—trading goods several times before finding a good an agent ultimately wants to consume—does not play the same role as it does in the environment described here, where the distance between the current match and the desired match plays an important role.

Given the matching technology, one can readily compute the rate at which workers of type i encounter jobs of type j. In what follows, I consider only symmetric equilibria where neither the distribution of workers across match quality nor the number of vacancies posted depends on i and j. Under symmetry, the rate at which a worker on a match of quality m encounters a job of quality n is given by

$$p_{mn} = \frac{v_n}{v} \cdot \frac{m(z, v)}{z} = \frac{1}{N} \cdot m\left(1, \frac{v}{z}\right),$$

where v_n denotes the number of vacancies for the job which corresponds with a match of quality n for a given worker type i. As long as $n > m$, the worker will agree to join in a match with the new entrepreneur. For unemployed workers, the rate at which they encounter a match of quality n is likewise given by

$$p_n = \frac{1}{N} \cdot m\left(1, \frac{v}{z}\right).$$

Analogously, the rate at which an entrepreneur encounters a worker with whom he forms a match of quality n is given by $1/N \cdot m(z/v, 1)$. However, not all of the workers an employer encounters will agree to enter a match, since some of these workers might already be employed in a job with match quality $m \geqq n$. If a worker only switches to jobs with strictly higher match quality, the rate at which an entrepreneur encounters a worker of match quality n who agrees to form a match is given by

$$q_n = \frac{1}{N}\left[\frac{u + \sum_{m=1}^{n-1} e_m}{u + \sum_{m=1}^{N-1} e_m}\right] \cdot m\left(\frac{z}{v}, 1\right).$$

To determine how many jobs will be created by entrepreneurs, I need to specify the incentives for entrepreneurs to post vacancies. For this, I need to impose assumptions on the preferences of entrepreneurs as well as the share of the output they expect to receive once a match is formed. I assume both entrepreneurs and workers are risk neutral over earnings, and that both discount the future at the same rate r. I further assume entrepreneurs incur a fixed flow cost k per unit of time from posting a vacancy. In specifying the share of the output entrepreneurs expect to receive from a match, I follow previous literature in assuming that the two parties split the surplus formed by the match, *i.e.* wages are set to insure that the surplus for a worker from being employed in a job of quality n represents a fraction $a \in (0, 1)$ of the total surplus the two parties receive over their outside option of separating and searching for different partners. Formally, if we let Π_n denote the value for an entrepreneur of presently employing a worker in a match of quality n, W_n the value for a worker of being presently employed in match of quality n, U the value of being presently unemployed, and V the value of a presently posted vacancy, then wages are set so that the worker receives a fraction a of the total surplus from remaining in the match:

$$W_n - U = a(\Pi_n + W_n - U - V) \equiv aS_n.$$

Following Binmore, Rubinstein, and Wolinsky (1985), one can motivate this wage-setting rule as the equilibrium outcome of standard bargaining games in which the two parties take turns at making offers how to split the surplus. Characterizing the steady state equilibrium turns out to be a non-trivial exercise under this rule, since the value of a match of a given quality depends on what happens to the worker and entrepreneur in all future contingencies, including the possibility that the worker moves on to a better match. Thus, the value of a match of a given quality depends directly on the values associated with matches of different quality levels, in contrast with models with only off-the-job search where the value of a given match depends on the expected value associated with matches of different quality levels only indirectly, through the value of unemployment. To analyse the steady-state, it turns out that we require analytical expressions for the surplus associated with each quality. Fortunately, the framework described above yields a manageable system of equations for steady-state surplus that can still be solved in closed-form. That said, when I later move beyond steady-states to simulate a stochastic version of the model, I will need to assume agents split output rather than surplus in order to minimize on the computational complexity.[7]

To summarize, given the wage setting rule outlined above, entrepreneurs in this economy choose how many vacancies to post, what type of vacancies to post, and whether or not to form a match when a worker comes along. Workers choose only whether to form a match when an entrepreneur comes along. Given these decisions, the following constitutes a symmetric equilibrium for this economy:

Definition. *A symmetric equilibrium is a set of time paths* $\{v, u, e_n\}_{n=1}^N$ *that satisfies the following conditions:*

1. *Free entry*: given the paths for $\{v, u, e_n\}_{n=1}^N$, the expected value of posting a vacancy V is equal to 0 for any job j.
2. *Individual rationality*: at each point in time, an entrepreneur and a worker remain in a match of quality n if and only if they receive more from the match than from leaving to search for a new partner, *i.e.* if and only if the surplus associated with the match is positive:

$$S_n \geqq 0. \tag{1.1}$$

3. *Laws of motion*: the time paths of u and e_n are given by an initial condition $\{u, e_n\}_{n=1}^N$ at $t = 0$ and the laws of motion

$$\dot{u} = s(1-u) - u\textstyle\sum_{i=1}^N p_n + \sum_{i=1}^N \delta_n, \tag{1.2}$$

$$\dot{e}_n = p_n u + \textstyle\sum_{m<n} p_{mn} e_m - \left(\sum_{m>n} p_{nm} + s\right) e_n - \delta_n, \tag{1.3}$$

where δ_n denotes a Dirac delta function that captures discrete jumps from match quality n into unemployment due to endogenous job destruction.[8]

I begin by solving for a steady-state equilibrium in which, in addition to the above requirements, all endogenous variables are constant over time, *i.e.* where the initial values of $\{u, e_n\}_{n=1}^N$ persist indefinitely, and consequently so will the number of vacancies entrepreneurs post. The next proposition establishes there is a unique symmetric steady-state equilibrium. The proof of Proposition 1, as well as other propositions in the paper, is presented in an Appendix.

7. It is quite common to resort to such alternative wage rules in models with on-the-job search. For example, while Pissarides (1994) initially assumes wages are set according to a surplus-sharing rule, he subsequently assumes in a footnote that wages do not depend on any of the additional surplus generated by on-the-job search to maintain mathematical tractability. Burdett, Imai and Wright (1999) and Shimer (2000) assume agents split output rather than surplus, again to simplify the algebra. Mortensen (1994) considers splitting the surplus, but simplifies the problem by assuming all new matches start with the same productivity level.

8. When y is constant, this jump component will be equal to zero for any date $t > 0$. It will be positive for $t > 0$, however, when the productivity of a match fluctuates over time.

Proposition 1. *Suppose y is constant over time. Then there exists a unique set of time-invariant paths $\{u, e_n, v_n\}_{n=1}^N$ which satisfies free entry, individual rationality, and the laws of motion above.*

Two remarks are in order about the steady-state equilibrium above. First, as long as there is more than one n for which $y_n > b$, the steady state exhibits some endogenous mismatch, *i.e.* the steady-state distribution across match quality will not correspond to a mass point at $n = N$ in which all employed workers are in the job where they are most productive. The reason labour resources are misallocated in equilibrium is that there is a positive cost k to posting a vacancy. Even though mismatched workers could be employed more productively elsewhere, entrepreneurs are deterred from creating vacancies to seek out such workers because it is costly to do so. Second, the focus on steady-states is often justified in models with only off-the-job search since those models exhibit global asymptotic stability, *i.e.* from any initial conditions, the economy will eventually converge to the unique steady-state. With on-the-job search, it can be shown that the distribution of workers across match quality levels $\{e_n\}_{n=1}^N$ converges to a unique value for a given ratio v/z. However, the evolution of this ratio depends on the distribution $\{e_n\}_{n=1}^N$ in a complicated non-monotonic way, making it difficult to determine whether this ratio settles down starting from an arbitrary initial condition. Thus, it is not obvious that this matching model continues to exhibit global asymptotic stability. Still, for the numerical simulations of a discrete-time version of the model reported below, the endogenous variables always converged to the steady state, suggesting the system remains globally stable even in the presence of on-the-job search.

3. CLEANSING AND SULLYING

The previous section established the existence of a unique symmetric steady-state equilibrium. But in order to characterize the allocation of resources over the business cycle, we need to allow for deviations from steady-state. Mortensen and Pissarides (1994) do precisely this in their model with only off-the-job search by allowing aggregate productivity y to fluctuate over time. They are able to characterize the equilibrium of the stochastic model analytically. However, the presence of on-the-job search introduces the distribution $\{e_n\}_{n=1}^N$ as an additional state variable, making it impossible to characterize the transitional dynamics associated with deviations from steady-state. Still, judging by the results in Mortensen and Pissarides (1994), the effects of temporary changes in aggregate productivity are likely to be qualitatively similar to the effects of permanent changes in the steady-state level of aggregate productivity. I therefore begin with comparative static exercises on the steady-state equilibrium described in Proposition 1. These capture an essential part of how the level of aggregate productivity can affect the allocation of resources, and thus provides a more rigorous intuition for the cleansing and sullying effects described in the Introduction. Of course, comparative static exercises are limited, since they abstract from intertemporal considerations and provide no insight on the transitional dynamics associated with regime changes. For this reason, I subsequently turn to a numerical analysis of a stochastic version of the model, which is taken up in the next section. It confirms that the effects of permanent changes in the steady-state level of aggregate productivity are qualitatively similar to the effects of temporary changes in the level of aggregate productivity in a stochastic version of the model.

I first illustrate that across steady-states, the model delivers the conventional cleansing effect discussed in previous literature. Intuitively, at lower levels of aggregate productivity, each job generates less output and thus less surplus given the flow value of unemployment remains constant. Hence, the least productive jobs that yield marginally positive surplus at high levels of aggregate productivity will be unprofitable at lower values of aggregate productivity. This can be formalized in the following proposition:

Proposition 2. *In a steady-state equilibrium, there exists a cutoff match quality $n^*(y)$ such that $S_n > 0$ if and only if $n \geq n^*(y)$, where $n^*(\cdot)$ is weakly decreasing in the steady-state level of aggregate productivity y.*

Proposition 2 is straightforward. Since the productivity of each match remains constant over time, being matched yields positive surplus if and only if $y_n > b$. Given $y_n = y(1+\varepsilon_n)$, the threshold $n^* = \min\{n : y_n > b\}$ decreases with aggregate productivity y. However, the conclusion of Proposition 2 would follow even if the productivity of a match were allowed to change over time. To see why, suppose y fluctuates between two levels, $y_0 < y_1$. Consider a match of quality n such that $y_1(1+\varepsilon_n) > b > y_0(1+\varepsilon_n)$, *i.e.* which yields more output than if the worker were to produce on his own when $y = y_1$ but less output when $y = y_0$. At the higher level of productivity, working on a match of quality n and searching for an even better match strictly dominates unemployment, so $n > n^*(y_1)$. When $y = y_0$, remaining on a match of quality n may or may not dominate unemployment: remaining in the match is valuable since the parties can produce a high level of output immediately the instant y recovers, but by remaining in the match the parties collectively give up $b - y_0(1+\varepsilon_n)$ units of output per unit time as long as aggregate conditions prevail. Taking the limit as $y_1(1+\varepsilon_n) \to b$, the benefit of the option to retain the match goes to 0, while the cost remains bounded away from zero. Hence, we can find a job that will be suspended when aggregate productivity is low but not when it is high, *i.e.* there exists an n such that $n^*(y_0) > n > n^*(y_1)$, implying the cutoff n^* must be decreasing in y.

In the absence of on-the-job search, the average match quality depends only on the cutoff level n^*, and Proposition 2 would imply that average match quality rises during recessions. This is why previous authors have concluded that recessions alleviate allocative inefficiency. But once we allow for on-the-job search, we also need to take into account the reallocation of resources among surviving matches with quality $n \geq n^*$; that is, allocative efficiency depends not just on how the destruction threshold evolves over the cycle, but also on how resources are allocated across matches with a quality level *above* the threshold. The next proposition establishes how the level of aggregate productivity affects the distribution of workers across matches of quality $n \geq n^*$ in steady state:

Proposition 3. *Consider two values $y_1 > y_0$, and let $n^* \equiv n^*(y_0) > n^*(y_1)$. Then the steady-state ratio v/z is higher when $y = y_1$, and the steady-state distribution $\{e_n/[\sum_{n^*}^N e_n]\}_{n=n^*}^N$ when $y = y_1$ first-order stochastically dominates the corresponding steady-state distribution at $y = y_0$.*

Proposition 3 can be understood as follows. The zero profit condition requires that the expected value of a vacancy is equal to its cost, *i.e.*

$$\sum_{n=n^*}^N q_n \Pi_n = k,$$

or alternatively

$$\frac{1}{N}\sum_{n=n^*}^N m\left(\frac{z}{v}, 1\right)\left[\frac{u + \sum_{m=1}^{n-1} e_m}{u + \sum_{m=1}^{N-1} e_m}\right](1-a)S_n = k.$$

As noted above, at lower levels of aggregate productivity, matches will be associated with less surplus. This reduces the value to an employer of a filled vacancy $\Pi_n = (1-a)S_n$.

To insure entrepreneurs still earn zero expected profits on each vacancy posted, the endogenous variable v/z will have to adjust until the expected value of posting a vacancy is once again equal to its cost k. As can be seen from the free entry condition above, the value of a vacancy depends on the ratio v/z in three distinct ways. First, the lower this ratio, the more applicants there are for each vacancy, so entrepreneurs face a higher rate $m(z/v, 1)$ of encountering workers when they post a vacancy. This raises the expected value from posting a vacancy. Second, as v/z falls, the surplus associated with each match S_n increases. Intuitively, workers are less likely to move on to better matches at lower values of v/z, so the output produced on a given match will accumulate over a longer expected duration. Finally, as implied by the second part of Proposition 3, a lower value of v/z will induce a first-order stochastic shift of the distribution across match qualities towards lower quality matches. But with fewer well-matched workers, there is a higher probability of finding a worker who is willing to leave his previous match. This too raises the expected value of posting a vacancy. Thus, by all considerations, employers are better off when there are fewer vacancies per applicant, or conversely when there are more applicants per vacancy. When aggregate productivity declines, then, the ratio v/z would have to fall in order to keep the expected value from posting a vacancy unchanged.

This last observation is the key to the shift in the distribution described in the proposition. At lower values of aggregate productivity, labour market conditions must tilt in favour of entrepreneurs at the expense of workers. Hence, workers are less likely to bump into a better match when aggregate productivity is low, while the separation rate s remains unchanged by assumption. This reduces the odds for a worker of making it to a high quality match before an exogenous separation throws him back into the unemployment pool, and hence fewer workers will be employed in high quality matches in steady state. I refer to this shift of the steady-state distribution as the sullying effect of recessions. In contrast with the cleansing effect, the sullying effect implies recessions exacerbate allocative inefficiency by steering fewer workers towards jobs where they are more productive. This can potentially explain why a greater fraction of employed workers report being mismatched during recessions, despite the fact that less efficient matches are destroyed; even though no workers occupy the least productive matches when aggregate conditions are unfavourable, there are also fewer workers who occupy the most productive matches. In addition, the sullying effect could potentially explain why *new* matches that form during recessions are of lower average quality than the matches that form during booms. The reason is that with more workers employed in lower quality matches, workers will agree to enter matches of lower average quality given that they accept any match exceeding the quality of their current match.

When we move beyond steady states to allow for cyclical fluctuations, part of the intuition behind the sullying effect still carries over. Because of discounting, a decline in aggregate productivity will lower the surplus associated with each match, other things equal. Hence, the endogenous variables will have to adjust in order to maintain the zero profit condition at the lower productivity level. In contrast with the steady state analysis, where it is a single ratio v/z that has to adjust, now the entire path of v/z over time can adjust to maintain the zero profit condition. The same logic as before allows us to rule out the scenario in which a decline in aggregate productivity causes the entire path of v/z to attain higher values than would have prevailed under the original level of aggregate productivity. Thus, a decline in aggregate productivity must lead to a decline in the number of vacancies per searcher at some point relative to what this ratio would have been had aggregate productivity remained at its higher level. As such, recessions will still stifle the reallocation of labour into its most productive uses to some degree. But precisely how the sullying effect manifests itself in this environment requires a more rigorous treatment of the transitional dynamics that comparative static exercises cannot capture.

Before I turn to the numerical analysis to characterize the transitional dynamics, I close with some remarks regarding endogenous search intensity. As the model is formulated, the amount of mismatch depends only on the actions of entrepreneurs who choose how many vacancies to post. More generally, though, workers also make decisions that can affect the extent of mismatch. For example, suppose workers choose the number of applications $\theta \in [0, \infty)$ to submit to the centralized matching algorithm described in Section 2 at a cost of $c(\theta)$, where $c(\cdot)$ is increasing and convex. Let θ_n denote the number of applications submitted by workers on a match of quality n, and θ_u denote the number of applications submitted by unemployed workers. Then the total number of applications is equal to

$$z = \theta_u u + \sum_{n=1}^{N-1} \theta_n e_n,$$

and the rate at which a worker in a match of quality m encounters a match of quality n is proportional to the number of applications he submits,

$$p_{mn} = \frac{\theta_m}{N} \cdot m\left(1, \frac{v}{z}\right),$$

while the rate at which entrepreneurs are matched with workers of quality n becomes

$$q_n = \frac{1}{N}\left[\frac{\theta_u u + \sum_{m=1}^{n-1} \theta_m e_m}{\theta_u u + \sum_{m=1}^{N-1} \theta_m e_m}\right] \cdot m\left(\frac{z}{v}, 1\right).$$

Allowing workers to choose search intensity has ambiguous implications for the sullying effect. On the one hand, since low levels of productivity are associated with fewer vacancies per application, workers will be discouraged from searching for jobs that are more elusive to find. This would amplify the sullying effect as workers move into better matches at an even slower rate than if search intensity were held fixed. However, since the costs and benefits of search might change along with the level of productivity, endogenous search intensity could undo the sullying effect if agents have more incentive to search for better matches at low levels of aggregate productivity. For example, if search requires workers to devote time that they could alternatively use in production, the opportunity cost of search will be lower when aggregate productivity is low, encouraging agents to search. Various authors have invoked this type of intertemporal substitution mechanism to argue that recessions are ideal times to invest in looking for a better match or in growth-enhancing activities more generally.[9] However, at least in the context of search, this argument abstracts from the fact that search frictions for workers become more severe during recessions, making them less ideal times to seek out better matches. Ultimately, the direction of search over the cycle depends on which of the various forces that influence the incentives of agents to search is more dominant. But this does not take away from the main point of this paper, which is that recessions tend to slow down the rate at which workers encounter matches, generating a greater degree of resource misallocation than would otherwise arise.[10]

9. For example, Hall (1991, 2000) and Greenwood, Gomes and Rebelo (1999) have made such an argument in models of search; Cooper and Haltiwanger (1993) and Aghion and Saint Paul (1998) apply this logic to examine the cyclicality of technological innovation; and Dellas and Sakellaris (1997) and Barlevy and Tsiddon (2000) discuss countercyclical investment in human capital.

10. In addition to the two forces described here, the search decisions of agents will also be influenced by the fact that differences in productivity across matches are proportional to aggregate productivity, so that the gains from moving to a better match will be higher during booms for a fixed cost of searching. This effect is particularly exploited in Pissarides (1994) in arguing that search is likely to be procyclical.

4. QUANTITATIVE IMPLICATIONS

The fact that the distribution of workers across match qualities $\{e_n\}_{n=1}^N$ enters as a state variable in the model makes the transitional dynamics difficult to characterize analytically. This section therefore turns to numerical techniques to analyse and simulate a stochastic version of the model in which productivity is allowed to fluctuate over time. The section serves two purposes. First, I confirm that the basic insights from the comparative static exercises above carry over when aggregate productivity is allowed to fluctuate over time. Second, by choosing parameter values judiciously, I can examine whether the sullying effect is likely to be empirically relevant. Specifically, I calibrate the model so that the equilibrium unemployment rate implied by the model mimics key features of the unemployment series in the data. As can be seen from the laws of motion for e_n and u above, the same factors which determine the evolution of the unemployment rate, namely the separation rate s and the rate workers encounter new matches $m(1, v/z)$, also determine the evolution of the distribution of matches across different match qualities. The model therefore allows us to use unemployment, for which we have reliable measures, to make inference on the distribution of match quality for which we have only indirect measures.

To keep the calculations tractable, I introduce two modifications to the basic framework above. First, I move to a discrete-time formulation, which is necessary in order to simulate the model. The discrete-time model is briefly sketched out in an Appendix. Second, I assume the parties split the output y_n rather than the surplus S_n associated with the match, where the worker receives a fraction a of the output and the entrepreneur receiving a fraction $1 - a$. A match forms only if both parties agree to the match, *i.e.* if $\Pi_n \geqq V$ and $W_n \geqq U$. Since output is increasing in n, there will again be some cutoff n^* below which at least one of the parties refuses to engage in production, and a similar argument as before can be used to establish that n^* is decreasing in y. As noted in an earlier footnote, previous authors have also assumed that agents split the output from a match instead of the surplus to simplify the equilibrium conditions. This assumption proves to be convenient, since the value of a match of quality n to the entrepreneur Π_n can be evaluated separately for each n rather than collectively for all n. Solving for the functions Π_n, which map the distribution $\{e_n\}_{n=1}^N$ into the set of real numbers, is done using a collocation method in which each Π_n is approximated with a polynomial function $\tilde{\Pi}_n(e_1, \ldots, e_N)$, with the coefficients for each polynomial chosen so that the approximate Bellman equations where $\tilde{\Pi}_n$ are substituted for Π_n hold exactly for particular values of $(e_1, \ldots, e_N)$. The technical details behind the collocation procedure are again relegated to an Appendix.

Solving the model numerically requires choosing particular functional forms and parameter values. I begin with the choices regarding the dimensionality of the model. First, I assume aggregate productivity y_t can take on only two values, y_0 and y_1, where the latter is normalized to equal 1. Aggregate productivity is assumed to switch between these two values with a constant probability of 0·05 per period, where a period is taken to be a quarter, so that a given level of productivity will persist for 5 years on average, consistent with business cycle frequencies. Next, I need to choose the number N of potential match qualities. To avoid the curse of dimensionality, N cannot be set to arbitrarily large values. However, N should also not be too small; at a minimum, modelling both the destruction of less efficient matches during recessions together with shifts in the distribution of resources across matches above the destruction threshold requires at least three quality levels. I settle on $N = 6$, which allows me to keep a relatively large number of matches above the destruction threshold. This choice is motivated in part by the work of Topel and Ward (1992), who find that the typical labour market entrant moves through 6 to 7 jobs during his first 10 years. Assuming these are mostly voluntary would suggest 6 as a lower bound on the number of match qualities workers can be employed in. Still larger values of N require keeping track of increasingly more coefficients for each polynomial function, as well as a bigger number of polynomials, which slows down the iterative collocation procedure. For $N = 6$, we have $2 \times 6 = 12$ value functions $\Pi_n^i : X \to R$ to keep track of, one for each pair of aggregate productivity i and match quality n, where the domain of each function is given by $X = \{(e_1, \ldots, e_6) : \sum e_j \leqq 1, e_j \geqq 0\}$.

Next, I need to specify a functional form for the matching function $m(z, v)$. In the continuous-time version of the model, this function represents the rate at which matches are formed, and can take on any value in $[0, \infty)$. But in the discrete-time version, this function denotes the actual number of contacts in a period, and is therefore bounded by $\min(z, v)$. Following den Haan, Ramey and Watson (2000), I choose the function

$$m(z, v) = \frac{zv}{v + z},$$

which exhibits constant returns to scale and satisfies the required boundedness condition.

Finally, I turn to parameter values. For lack of a better choice, I assume workers and entrepreneurs split the output y_n equally, *i.e.* $a = 0{\cdot}5$; however, this is really just a normalization, since the share of output $1 - a$ that goes to entrepreneurs matters only relative to the cost of posting a vacancy k, which itself is a free parameter. In choosing the discount rate, I allow a period to represent one quarter, and set the quarterly discount rate r equal to 0·01 as is standard in real business cycle models. In choosing values governing the productivity of the different matches, I proceed as follows. First, in choosing b, I simply assume that given all of the other parameter values, b is such that jobs with $n = 1$ would be destroyed when aggregate productivity is low, while all match qualities operate when aggregate productivity is high. Next, in setting the values for ε_n, I make use of previous results in Topel and Ward (1992) that suggest workers who change jobs experience a wage gain of about 10% on average. Barlevy (2000*b*) estimates a model based on the one developed in this paper and finds that the gains from search further exhibit a memoryless property, *i.e.* workers appear to earn an additional 10% from moving to a new job regardless of how many matches they have already moved through.[11] Since wages are proportional to productivity, setting

$$\varepsilon_n = 0{\cdot}1 \times \textstyle\sum_{N+1-n}^{N} \frac{1}{j}$$

uniquely insures that regardless of his current match quality n, a worker expects to gain 10% on average when he switches to a better match. Thus, we have

$$\varepsilon_1 = 0{\cdot}0167, \qquad \varepsilon_4 = 0{\cdot}0950$$

$$\varepsilon_2 = 0{\cdot}0367, \qquad \varepsilon_5 = 0{\cdot}1450$$

$$\varepsilon_3 = 0{\cdot}0617, \qquad \varepsilon_6 = 0{\cdot}2450.$$

11. This would suggest the distribution of match quality ε_n is approximately exponential.

As for the values of aggregate productivity y, recall that aggregate productivity in one of the states, y_1, is normalized to 1. The level of aggregate productivity in the other state, y_0, is chosen, together with the separation rate s and the cost parameter k, to generate a time series for unemployment that replicates the same series in the data. These three parameters are calibrated in the following manner. As noted above, numerical simulations suggest that the dynamical system eventually settles down along any path where aggregate productivity remains constant. From the laws of motion for u, the unemployment rate at this fixed point is given by

$$u = \frac{s(1-m')}{1+m'+s(1-m')},$$

where m' denotes the probability a worker finds a viable match. The parameters were chosen so that this expression fluctuates between 3% and 9%, corresponding to the historical range for the national unemployment in the U.S. over the post-War era. This imposes two moment conditions for the three parameters of y_0, k, and s. The third condition involves the transitional dynamics for the unemployment rate. Specifically, I want to rule out the case in which the destruction of low quality matches during recessions leads to an unrealistically large jump in the unemployment rate. Thus, I effectively limit the fraction of workers who can ever be employed in the lowest quality match with $n = 1$ at a point in time. Judging by seasonally adjusted unemployment data from the Bureau of Labor Statistics (BLS), the largest jump in the national unemployment between two consecutive quarters in the post-War period (1948–1999) is 1·67 percentage points, back in the first quarter of 1975. I therefore rule out cases in which the unemployment rate jumps upwards by more than 2 percentage points between any two consecutive quarters. To summarize, y_0, k, and s are chosen so that (1) the long run unemployment ranges between 3 and 9%, and (2) from its long-run value of 3%, unemployment jumps to 5% in the quarter aggregate productivity declines and low quality matches are destroyed.

Using a search algorithm over different values of y_0, k, and s, I solve for the combination of parameters that satisfies these three moment conditions: $y_0 = 0{\cdot}280$, $k = 1{\cdot}892$, and $s = 0{\cdot}018$. The implied separation rate s is identical to what Hall (1995) reports for the quarterly rate of permanent separations excluding quits. However, it is smaller than the 5% quarterly rate he reports for leaving a job into unemployment. The cost of maintaining a vacancy k is a little less than the total amount of output that can be generated by a match over a period of six months, which seems plausible. By contrast, the required change in aggregate productivity is implausibly large. This is not very surprising; it is well known that reasonable productivity shocks cannot on their own account for fluctuations in the unemployment rate in search models. However, as illustrated by den Haan, Ramey and Watson (2000), more reasonable productivity shocks can lead to large changes in unemployment as long as they induce even small changes in the equilibrium interest rate r. While this suggests the interest rate should be endogenized, such a modification is not essential for gauging the magnitude of the sullying effect. The reason is that both unemployment and the sullying effect depend on the same underlying endogenous variable, namely the probability $m(v/z, 1)$ of finding a job. Since any source of fluctuations would have to be calibrated in such a way that $m(v/z, 1)$ fluctuates in a manner consistent with observed movements in the unemployment series, it would lead to the same predictions regarding the implied sullying effect.

With all of the parameter values assigned, I can simulate the model and investigate the behaviour of its key variables. It is natural to begin with the evolution of the ratio v/z, which lies at the heart of the sullying effect. The previous section illustrated that across steady states, this ratio is positively related to aggregate productivity, *i.e.* lower levels of productivity are associated with fewer vacancies per searcher. To determine how this ratio is related to aggregate productivity when the latter is allowed to fluctuate over time, I trace out the equilibrium value of $m(v/z, 1)$ along a particular realization of aggregate productivity. This is done by approximating the value of a match to the entrepreneur $\Pi_n(e_1, \ldots, e_N)$ for different match levels and using these to solve for the ratio v/z which satisfies the zero profit condition at a given level of aggregate productivity and a given distribution $\{e_n\}_{n=1}^N$. Given this ratio, the laws of motion for e_n can be used to compute the distribution $\{e_n\}_{n=1}^N$ at the beginning of next period, and thus the path of v/z can be computed recursively for a sequence of realizations for aggregate productivity and an initial condition for $\{e_n\}_{n=1}^N$. To assess the effect of a negative productivity shock, I simulated the model with aggregate productivity equal to y_1 for the first 100 periods. For all of the initial conditions I experimented with, the probability $m(v/z, 1)$ settled down to 0·36, suggesting the dynamical system implied by the model is globally stable. Once this probability converged to this value, I let aggregate productivity assume its lower value y_0 and simulated the effects of a negative productivity shock as it persisted for the next 20 quarters. The path of $m(v/z, 1)$ is illustrated in Figure 2, with the quarter labelled 0 denoting the period in which the negative productivity shock first strikes. As can be seen from the figure, the probability for a worker of finding a job falls from 0·36 to 0·13 on impact. Thus, the immediate effect of a negative productivity shock is to lower the number of vacancies per searcher. From the analysis of the previous section, we can surmise that this will shift the mass of the distribution of workers towards lower quality matches, to a degree that will be quantified further below. But for now, it is enough to note that this shift implies entrepreneurs encounter more dissatisfied workers employed in low-quality matches in the period right after the shock. Since dissatisfied workers are more likely to accept a job offer that comes their way, the expected value of a vacancy will rise, implying the ratio v/z must increase in order to maintain the zero profit condition. Eventually, the probability of finding a match converges to 0·18, still below the value this probability assumes when aggregate productivity is high. Hence, the basic insight from the comparative static exercises above carries over to the case in which productivity fluctuates over

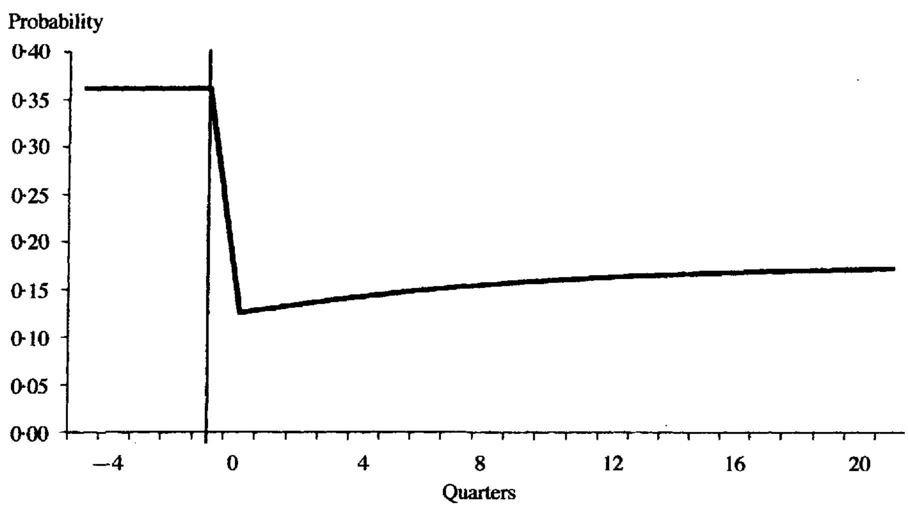

FIGURE 2

Effect of a negative productivity shock on the probability of finding a job time: the number of vacancies per application is uniformly lower when aggregate productivity falls, implying workers will move more slowly to higher quality matches.

The magnitude of the fluctuations in the probability $m(v/z, 1)$ illustrated in Figure 2 ultimately determines the size of the sullying effect. For this reason, it is important to verify that fluctuations as suggested in Figure 2 are in fact plausible. One way to confirm this is to use the behaviour of quits over the cycle. After all, quit rates depend on the same probability $m(v/z, 1)$, and are directly related to movement between match qualities that govern the increase in mismatch inherent in the sullying effect. To construct an empirical series on quit rates, I turn to the Panel Survey on Income Dynamics (PSID), which tracks down a representative sample of household heads in the U.S. that originate from a core sample of households starting in 1967.[12] Although the PSID begins in 1967, a consistent series on quits can be constructed starting only from 1974; prior to that year, the PSID failed to distinguish between quits from one job to another and internal promotions. The quit series was assembled by taking only those workers who were classified as employed for the year in which they were interviewed, and report having quit or resigned their main job in the year they were being queried about.[13] The number of workers who report a quit is then divided by the number of workers who are classified as employed, to arrive at an annual quit rate per employed worker. For the period covering 1974–1991, quits ranged between 5·9% and 10·0% per year, with an average of 8·1% per year. These estimates are comparable to estimates of annual quit rates computed in Akerlof, Rose, and Yellen (1988) based on BLS data for the manufacturing sector covering 1948–1982. The top panel of Figure 3 plots the quit series for the years available, along with the unemployment rate for the years covered by the PSID, measured on a different scale. As can be seen in the figure, the quit rate is clearly procyclical.

To allow for a meaningful comparison between the model and the data, I evaluated quit rates in the model along a path of realized values for aggregate productivity designed to mimic the unemployment rate during this same period. I then aggregated over quarters to produce annual quit rates and unemployment rates. With only two levels of aggregate productivity, the replication is necessarily crude; for example, the simulated unemployment series rises and falls too quickly relative to what we observe in the data. Still, the simulated unemployment series provides a reasonable approximation of the actual unemployment series, as can be seen in Figure 3. The average annual quit rate implied by the model is equal to 7·5%, roughly on par with actual quit rates. Moreover, the quit rate in the model is procyclical, as it is in the data. However, quits are significantly more volatile in the model than in the data, with quits ranging between 2·8 and 13·5%, well outside the scope of anything observed in the data. The greater volatility exhibited by the model is probably due to the inability of the simulation to capture the slow evolution of stocks observed in the data. Since many quits come from workers in the bottom of the distribution, the rapid destruction and reconstitution of matches of quality $n = 1$ that occurs as aggregate productivity switches between y_0 and y_1 is likely to generate spuriously large fluctuations in aggregate quits. Indeed, the large swings in the quit rate appear to be largely a transient phenomenon in the model. In the long run, the quit rate converges to 6·3% for the lower level of aggregate productivity, and to 8·5% for the higher level of aggregate productivity—both well within the range observed in the data. This is reassuring, since it is the long-run quit rate that determines how changes in productivity

12. As users of the PSID are well aware of, the data includes both a random sample and a group of low income households. All figures computed for the PSID in this paper are based on the random sample.

13. Following Polsky (1999), the data was adjusted for the survey years 1984–1987 in which changes in the wording of the questions contribute to an overstatement of actual turnover.

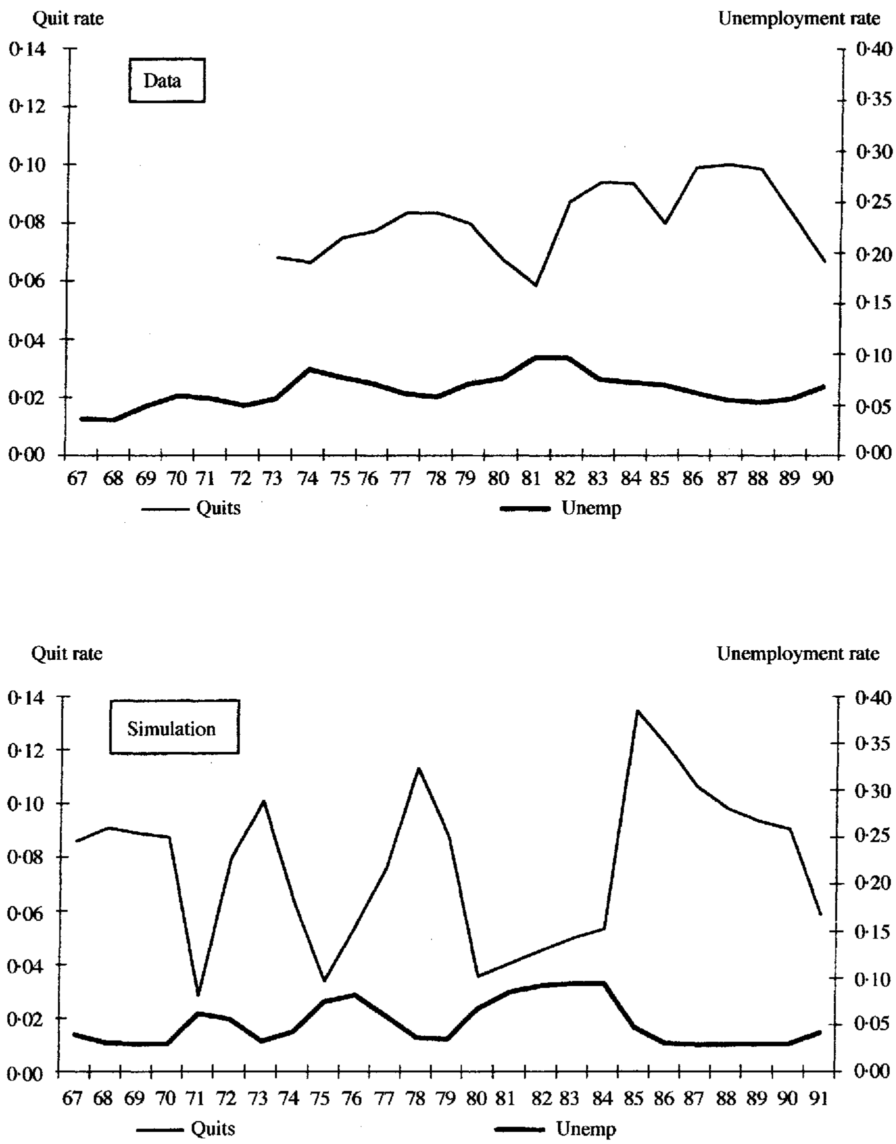

FIGURE 3

Quit rates over the cycle

ultimately affect the allocation of resources across matches. It also implies that the predictions of the model involving the cumulative effects of productivity shocks over longer periods are likely to be more reliable than those involving very high frequencies.

Since the implied volatility of $m(v/z, 1)$ appears to be of a sensible order of magnitude, at least for longer horizons, I next turn to the quantitative implications of the model regarding match quality. As a first pass, I examine the implications of the model regarding the fraction of employed workers who are mismatched, *i.e.* the fraction of all workers employed in a job of quality $n < N$. This provides a measure of the degree of slack in production. Moreover, it is something that we can potentially compare with job dissatisfaction measures that are available in the data. In particular, the PSID surveys workers as to whether they wish to change out of their current job. If we asked workers in the model such a question, all who are not employed in their most preferred job would provide an affirmative answer to this question. Hence, the fraction of workers who express a desire to change jobs should provide an analogue to the fraction of workers in the model who are employed in a match with $n < N$. In practice, though, the wording of the question in the PSID—asking workers whether they have been thinking about getting a new job or whether will they keep the job they have now—invites subjective interpretations. As such, the comparison between the model and the corresponding series in the PSID is at best suggestive. The top panel of Figure 4 reports the fraction of workers classified as employed who report they have thought about changing a job for those years in which the question was asked. This question was asked between 1967–1974, then again (with the exact same wording) between 1978–1987. The series ranges from a low of 9·6% towards the beginning of the sample, when unemployment rates were still quite low, to 17·8% in 1984. The fraction of mismatched workers appears to be countercyclical, with a lag of about a year or two relative to unemployment.

The bottom panel of Figure 4 reports the fraction of employed workers in jobs where $n < N$, averaged over four quarters to yield annual rates, for the path of aggregate productivity that simulates unemployment over this period. The fraction of workers who are mismatched ranges from a low of 18·8% towards the beginning of the sample to 27·0% at around the same period as implied by the data. Thus, the model implies a greater degree of mismatch than is observed in the survey data. One possible explanation for this is that workers who are close to their optimal match, *e.g.* workers employed on a match of quality $N-1$, rationalize that since there is only a small chance they will encounter a match they would be willing to change into, they should tell the interviewer that they are likely to keep the job they have now. In this case, the data would underestimate the true extent of mismatch. Of course, this is but one way in which individuals can interpret the question; workers who are already on the highest possible match might equally report they have thought about changing a job to convey how great it would be if a better match existed even when no such match exists in practice, in which case the data would overstate the true extent of mismatch (although the wording of the question presumably discourages this interpretation). Abstracting for a moment from discrepancies in the level of mismatch between the model and the data, the model accords quite well with the survey data along other dimensions. The fraction of workers who are mismatched exhibits the same countercyclical pattern in both the model and the data, and both estimate that after the three recessions that hit the U.S. economy between 1970–1983, an additional 8% of workers would have shifted into jobs where they consider themselves mismatched. By this standard, the model again seems to generate fairly reasonable changes in the composition of match quality over the cycle. In fact, if the discrepancy in levels between the model and the data reflects a tendency of the data to underestimate mismatch because of misreporting, the 8% increase in mismatch in the data is likely to understate the real increase in mismatch over this period. This is because the bias induced by misclassifying mismatched workers as well-matched is more severe when mismatch is higher. As an illustrative example, suppose we misidentified all workers with $n = 5$ as well matched. The fraction of workers who are mismatched in the model would have grown from 9·1% to a high of only 13·3% over the same simulated path for aggregate productivity, capturing only half of the increase in actual mismatch.

Although the model suggests a fairly significant increase in the number of workers who are underemployed in jobs where they are not being utilized to their full productive potential, this figure could represent just the tip of the iceberg when it comes to the effects

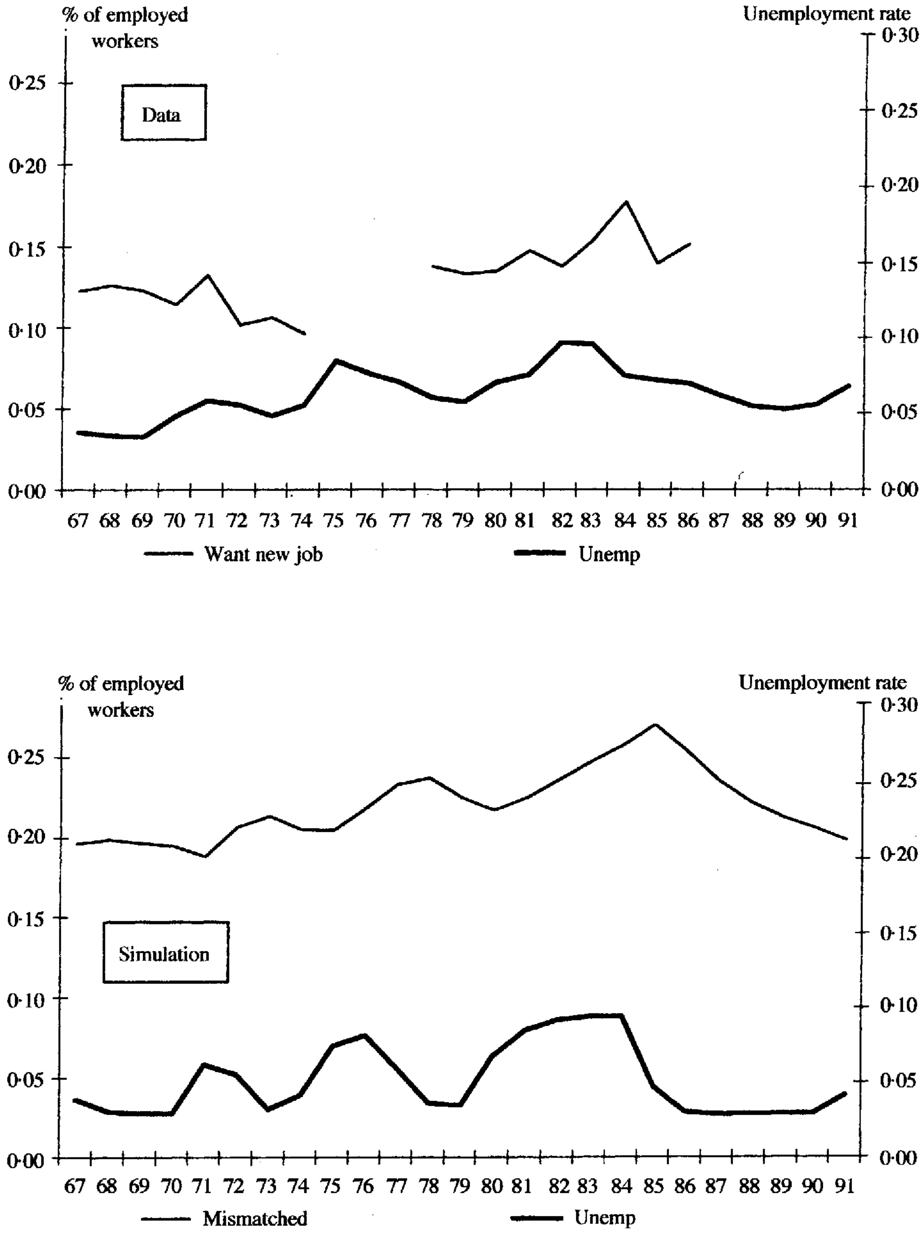

FIGURE 4
Mismatch over the cycle

of aggregate fluctuations on the allocation of resources. After all, not only do fewer workers move into the matches where $n = 6$, but also into matches where $n = 5$, 4, and 3. To gauge these effects, I turn to the evolution of average match productivity, *i.e.* the average of $(1 + \varepsilon_n)$ across all existing matches. In evaluating this measure, recall that there are two effects pulling at average productivity. On the one hand, the cleansing effect purges the least productive matches, causing average match quality to rise. On the other hand, the sullying effect shifts resources away from very high quality matches, causing average match quality to fall. To compare the relative magnitude of these two effects, the top panel of Figure 5 traces the evolution of average match productivity in response to a negative productivity shock. At the higher level of aggregate productivity, average match quality converges to 1·219, *i.e.* one unit of labour produces on average 1·219y units of output. At the onset of the recession, average match productivity rises because of the cleansing of less productive matches, so that the average employed worker produces 1·221y units of output. This is a relatively modest increase, in line with the estimates reported in a footnote in Caballero and Hammour (1994), as well as empirical estimates in Bailey, Bartelsman and Haltiwanger (1998). But after 7 quarters, aggregate productivity is below its original level,

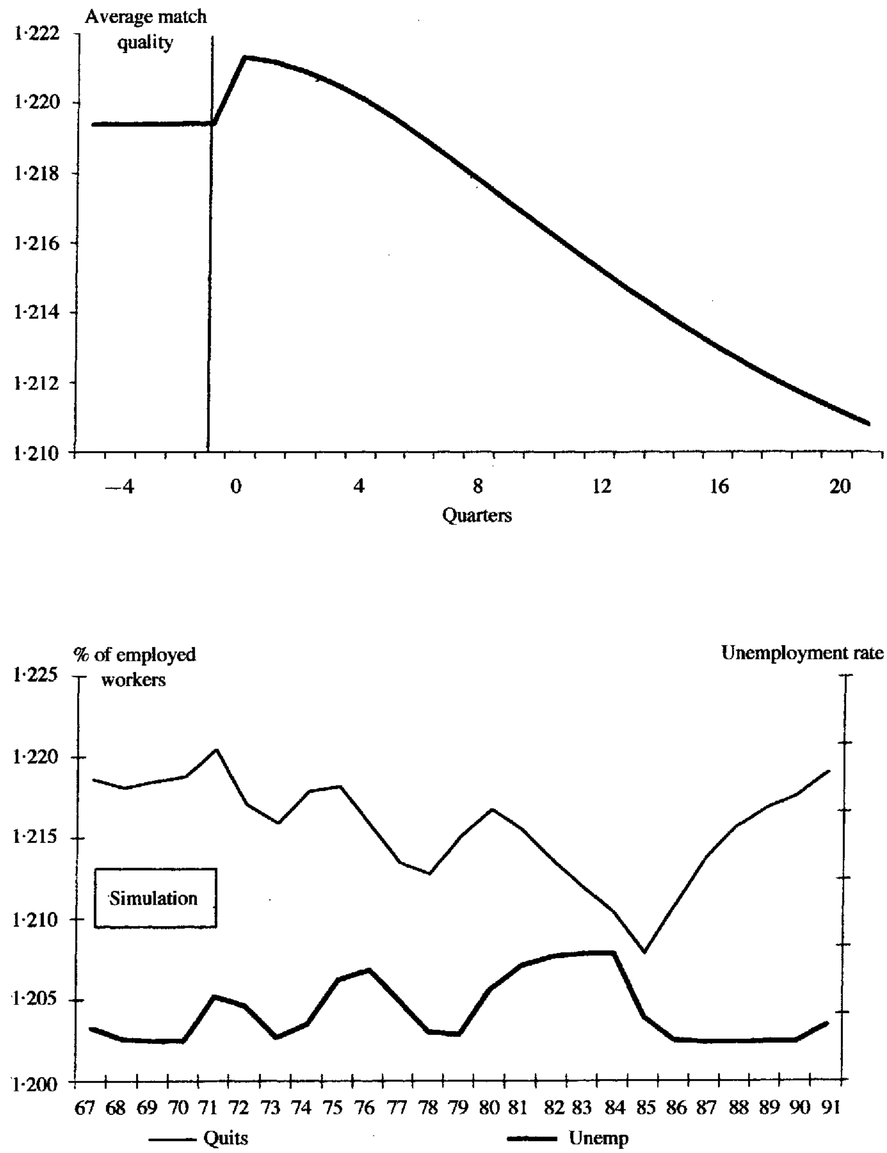

FIGURE 5
Average match productivity

and after 5 years is down to $1{\cdot}211y$, a decline of almost 1%. In the long run, average match productivity converges to $1{\cdot}206$, a decline that eclipses any of the changes in average productivity that arise because of the cleansing of less productive matches. However, convergence to this long run is quite slow. To gauge whether the sullying effect is likely to be relevant at business cycle frequencies, the bottom panel of Figure 5 reports average match productivity along the path of aggregate productivity that replicates fluctuations in the U.S. unemployment rate over the relevant time period. Despite jumps in productivity associated with the purging of inefficient matches at the onset of recessions, the model predicts a cumulative decline in productivity from $1{\cdot}219y$ in 1970 to $1{\cdot}210y$ at the peak unemployment rate over the period, so a decline of almost 1% in aggregate productivity. Thus, rather than inducing a reallocation of resources towards their most productive uses as the Schumpeterian view would argue, the model suggests that the sequence of recessions that struck the U.S. in the 1970s ultimately contributed to greater underemployment and allocative inefficiency than if good times had prevailed.

The quantitative exercise above helps to reconcile why despite the destruction of less productive matches, recessions can appear to be periods of mismatch and underutilization. The failure to move workers into better matches appears to have more dramatic consequences for the allocation of resources, especially over longer horizons, than the destruction of the least efficient matches. However, this is based on the stock of all existing matches, whereas some of the evidence cited in the Introduction concerned the flow of new matches. For example, Bowlus (1993) and Davis, Haltiwanger and Schuh (1996) find evidence that matches created during recession tend to be of lower quality than the average new job that is created during booms, at least when measured in terms of the expected lifetime of the job. In principle, the model can account for this pattern as well; as the distribution of workers across workers shifts towards lower quality matches, workers are more willing to enter into new matches even if they are not very productive. Hence, the average new match might be less productive during a recession. To see whether this effect is quantitatively significant, I gauge the effects of a negative productivity shock on the average match quality across new matches, *i.e.* the average of $(1+\varepsilon_n)$ across newly created matches.[14] This is illustrated in Figure 6. Several features about this series are noteworthy. First, the average productivity of a new match is approximately $1{\cdot}15y$, compared with about $1{\cdot}22y$ across all matches. This is due to selection induced by on-the-job search; since higher quality matches tend to attract more workers over time, we will observe relatively more workers employed in higher quality matches when we average productivity across all matches than when we look only at newly created jobs. Second, the productivity of new matches is much more volatile than the average productivity across all matches. This just reflects the fact that flows are more volatile than stocks. Finally, the model can generate a decline in the productivity of new matches, although a fairly transient one. On impact, the fact that matches of quality $n = 1$ are no longer created leads to an upward spike in the average productivity of all new matches. The cleansing effect is much more pronounced for new matches than across all matches as a whole. But after 5 quarters, the average quality of a new match also falls below its original value. It remains below the original level for the next 4 years, at which point it exceeds its original level. To gauge the implications of this for actual data, I evaluated the productivity of new matches along the simulated path that mimics fluctuations in unemployment in the U.S. during the 1970s and 1980s. The correlation between the productivity of new matches and unemployment is

14. As in the data, I include any match that is newly created, regardless of whether it originates from an unemployed worker or directly from a quit.

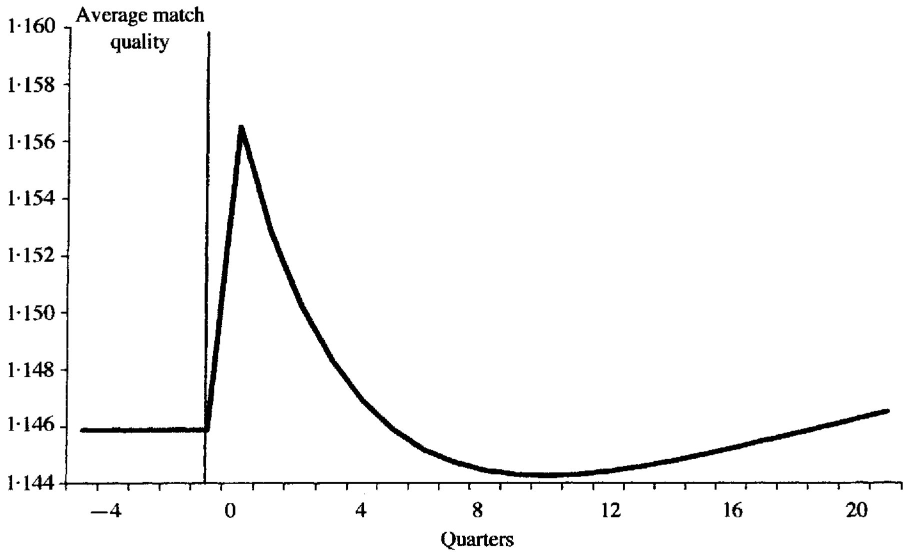

FIGURE 6
Effect of a negative productivity shock on the quality of new matches

$-0\cdot14$, reflecting that fact that at cyclical frequencies of about 5 years, the negative relationship apparent in Figure 6 is likely to dominate fluctuations in match quality over the cycle. Hence, the model can replicate the finding that the average match created during recessions is less productive than the average match created during booms. However, since flows are more sensitive to the somewhat arbitrary choice of N, and since the model does not perform well at very high frequencies, it is difficult to seriously compare this correlation with the specific findings reported in the Introduction.

5. CONCLUSION

This paper demonstrates that allowing workers to engage in on-the-job search has important consequences for the question of how the allocation of resources changes over the cycle. Previous work has argued that recessions should mitigate those forces that prevent resources from being allocated to their most productive uses. The basic argument is that recessions are associated with less profitable production, which makes them an opportune time to forgo production and invest in improving match quality. Thus, workers will be more inclined to quit less productive matches and search for new jobs during recessions. However, this paper illustrates that at the same time, recessions can exacerbate search frictions, since firms post fewer vacancies per application during recessions. If workers search off-the-job, this fact has no consequences for the observed allocation of resources, since it only means that workers have to remain unemployed for longer periods before they find a match. But with on-the-job search, less productive matches will survive for longer periods, and the allocation of resources shifts towards mediocre matches.

The presence of the sullying effect should help in revising the view of recessions as periods of healthy reallocation and reorganization; although recessions could serve this role by discouraging some of the least productive matches from forming, they also work against moving resources towards their best uses. The last section of the paper suggests that fairly plausible changes in quit rates over the cycle can have a fairly significant effect on average productivity and allocative efficiency. By laying out the various effects, the model can hopefully serve as a basis to better understand the nature of reallocation over the cycle. For example, in a companion piece to this paper, Barlevy (2000*b*) estimates the model developed here (and in turn, its estimates are used to calibrate the model in this paper) in order to gain a better understanding of the cleansing effect of recessions. That is, by separating cleansing and sullying, it tries to better understand the nature of job destruction during the cycle and changes in the threshold level at which jobs are either created or destroyed. Moving to a richer framework such as the one developed here would appear to be a positive direction towards understanding the nature of job reallocation over the business cycle and its ultimate consequences and implications.

APPENDIX A. PROOFS

Before turning to the propositions, I begin with some preliminary results. The asset equations for the values of vacancies, jobs, and unemployment are given by

$$rV = -k + \frac{1}{N} m\left(\frac{z}{v}, 1\right) \Sigma_{n=1}^{N} \left[\frac{u + \sum_{j=1}^{n-1} e_j}{u + \sum_{j=1}^{N-1} e_j}\right] \max(\Pi_n, V) + \dot{V}, \tag{A.1}$$

$$rU = b + \frac{1}{N} m\left(1, \frac{v}{z}\right) \Sigma_{j=1}^{N} \max[W_j - U, 0] + \dot{U}, \tag{A.2}$$

$$r\Pi_n = y_n - w_n - \left[s + \frac{N-n}{N} m\left(1, \frac{v}{z}\right)\right] \Pi_n + \dot{\Pi}_n, \tag{A.3}$$

$$rW_n = w_n + s(U - W_n) + \frac{1}{N} m\left(1, \frac{v}{z}\right) \Sigma_{j=1}^{N} \max[W_j - W_n, 0] + \dot{W}_n. \tag{A.4}$$

Substituting in the equilibrium condition $V = 0$ $(\Rightarrow \dot{V} = 0)$, we can combine these equations to get a single equation for the surplus $S_n = \Pi_n + W_n - U$:

$$\begin{aligned}
(r+s)S_n - \dot{S}_n &= y_n - b - \left[\frac{N-n}{N} m\right] \Pi_n \\
&\quad + \frac{m}{N}\left(\Sigma_{j=1}^{N} \max[W_j - W_n, 0] - \Sigma_{j=1}^{N} \max[W_j - U, 0]\right) \\
&= y_n - b - \left[\frac{N-n}{N} m\right] (1-a) \max(S_n, 0) \\
&\quad + \frac{am}{N}\left(\Sigma_{j=1}^{N} \max[S_j - S_n, 0] - \Sigma_{j=1}^{N} \max[S_j, 0]\right)
\end{aligned}$$

In steady state, $\dot{S}_n = 0$, and the above equations can be rewritten as

$$\begin{aligned}
(r+s)S_n &= y_n - b - \left[\frac{N-n}{N} m\right](1-a)S_n + \frac{a}{N} m\left(\Sigma_{j=1}^{N} (\max(S_j, S_n) - S_n - \max(S_j, 0))\right) \\
&= y_n - b - \left[\frac{N-n}{N} m\right](1-a)S_n + \frac{a}{N} m\left(\Sigma_{j=1}^{N} (\max(S_j, S_n) - \max(S_j, 0)) - NS_n\right) \\
&= y_n - b - \left[\frac{N-n}{N} m\right](1-a)S_n + \frac{a}{N} m\left(\Sigma_{j=1}^{n} (S_n - \max(S_j, 0)) - NS_n\right) \\
&= y_n - b - \left[\frac{N-n}{N} m\right](1-a)S_n + \frac{a}{N} m\left((n-N)S_n - \Sigma_{j=1}^{n} \max(S_j, 0)\right) \\
&= y_n - b - \left[\frac{N-n}{N} m\right] S_n - \frac{a}{N} m \Sigma_{j=1}^{n} \max(S_j, 0).
\end{aligned}$$

The above equations use the fact that S_n is increasing in n, which can be shown using an argument similar to that in Lemma 1 below. Finally, since it is always possible not to engage in production, $S_n \geqq 0$, and the above equation should be rewritten as

$$(r+s)S_n = \max\left\{y_n - b - \left[\frac{N-n}{N}m\right]S_n - \frac{a}{N}m\sum_{j=1}^{n}\max(S_j, 0), 0\right\}.$$

I now present two lemmas. The first lemma establishes that there exists a cutoff n^* below which $S_n = 0$ and above which it is positive, so that we can focus on solving for S_n only for $n = n^*, n^*+1, \ldots, N$. The second one establishes a useful result in matrix algebra that will be useful in the proof of Proposition 1.

Lemma 1. *Define $n^* = \min\{n : y_n > b\}$. Then $S_n = 0$ for all $n < n^*$ and $S_n > 0$ for all $n \geqq n^*$, with S_n strictly increasing in n.*

Proof. Since y_n is increasing in n, we can start at $n = 1$ and proceed inductively. From the asset equation,

$$S_1 = \frac{\max(y_1 - b, 0)}{r+s+((N-1+a)/N)m},$$

so that $S_1 > 0$ iff $y_1 > b$. Next, by induction, suppose $y_j < b$ and $S_j = 0$ for all $j = 1, 2, \ldots, n$. Then

$$S_{n+1} = \frac{\max(y_{n+1} - b, 0)}{r+s+((N-n+a)/N)m},$$

which is positive iff $y_{n+1} > b$. Hence, $S_n = 0$ for all $n < n^*$, and $S_{n^*} > 0$. We next induct from n^*. Suppose $y_j > b$ and $S_j > 0$ for $j = n^*, \ldots, n$. Then the asset equations imply

$$(r+s)S_{n+1} = \max\left\{y_{n+1} - b - \left[\frac{N-n-1}{N}m\right]S_{n+1} - \frac{a}{N}m\sum_{j=n^*}^{n+1}S_j, 0\right\}.$$

However, from the asset equation for $S_n > 0$, we have

$$\frac{a}{N}m\sum_{j=n^*}^{n}S_j = y_n - b - \left[\frac{N-n}{N}m\right]S_n - (r+s)S_n,$$

so

$$\begin{aligned}(r+s)S_{n+1} &= \max\left\{y_{n+1} - y_n - \left[\frac{N-n-1}{N}m\right]S_{n+1} + \left[\frac{N-n}{N}m\right]S_n + (r+s)S_n + \frac{a}{N}mS_{n+1}, 0\right\}\\ &= \max\left\{y_{n+1} - y_n + \left[\frac{N-n}{N}m\right](S_n - S_{n+1}) + (r+s)S_n + \frac{1-a}{N}mS_{n+1}, 0\right\}.\end{aligned}$$

To show that $S_{n+1} > 0$, suppose $S_{n+1} = 0$. Then it follows that

$$y_{n+1} - y_n + \left[\frac{N-n}{N}m\right]S_n + (r+s)S_n \leqq 0,$$

which is a contradiction given $y_{n+1} \geqq y_n$ and $S_n > 0$ by assumption. Hence, $S_{n+1} > 0$. Moreover, since $S_{n+1} > 0$, we know that we can rewrite the above equation as an equality without the max operator, *i.e.*

$$\left(r+s+\left[\frac{N-n}{N}m\right]\right)(S_{n+1} - S_n) = y_{n+1} - y_n + \frac{1-a}{N}mS_{n+1} > 0,$$

so that $S_{n+1} > S_n$. ||

Lemma 2. *Results in matrix algebra:*

(a) *Consider an $n \times n$ matrix M_n which has the form*

$$M_n = \begin{bmatrix} q & q+p_{n-1} & 0 & & 0 \\ q & q & q+p_{n-2} & & \\ q & q & q & \ddots & 0 \\ q & q & q & & q+p_1 \\ q & q & q & & q \end{bmatrix}.$$

Then $\det M_n = (-1)^{n+1} q \prod_{j=1}^{n-1} p_j$.

(b) *Consider an* $n \times n$ *matrix* M_n *which has the form*

$$M_n = \begin{bmatrix} q+p_{n-1} & & 0 & 1 \\ & \ddots & & 1 \\ q & & q+p_1 & 1 \\ q & & q & 1 \end{bmatrix}.$$

Then $\det M_n = \prod_{j=1}^{n-1} p_j$.

Proof. The proof of both statements is by induction.

(a) *For* $n = 1$, $\det M_n = q$ *which satisfies the criterion. Next, suppose the statement holds for all integers up to n. Then*

$$\det M_{n+1} = \det \begin{bmatrix} q & q+p_n & \cdots & 0 \\ q & & & \\ \vdots & & M_n & \\ q & & & \end{bmatrix}$$

$$= q \det M_n - (q+p_{n-1}) \det M_n$$

$$= -p_{n-1} \det M_n$$

$$= (-1)^{n+2} q \prod_{j=1}^{n} p_j.$$

(b) For $n = 1$, $\det M_n = 1$ which satisfies the criterion. Next, suppose the statement holds for all integers up to n. Then

$$\det M_{n+1} = \det \begin{bmatrix} q+p_n & 0 & \cdots & 1 \\ q & & & \\ \vdots & & M_n & \\ q & & & \end{bmatrix}.$$

Expanding along the top row:

$$\det M_{n+1} = (q+p_n)\det M_n + (-1)^{n+1} \det \begin{bmatrix} q & q+p_{n-1} & 0 & 0 \\ q & q & 0 & 0 \\ \vdots & & & \\ q & q & q & q+p_1 \\ q & q & q & q \end{bmatrix}$$

$$= (q+p_n) \prod_{j=1}^{n-1} p_j + (-1)^{n+1}(-1)^{n+2} q \prod_{j=1}^{n-1} p_j$$

$$= (q+p_n) \prod_{j=1}^{n-1} p_j - q \prod_{j=1}^{n-1} p_j$$

$$= \prod_{j=1}^{n} p_j. \quad \|$$

Proof of Proposition 1. Using Lemma 1, we can abstract from all $n < n^*$ and abstract from the max operator for $n \geq n^*$. Hence, $\{S_n\}_{n=n^*}^{N}$ are characterized by a linear system of equations, which in matrix notation can be written compactly as

$$A \begin{bmatrix} S_{n^*} \\ \vdots \\ S_N \end{bmatrix} = \begin{bmatrix} y_{n^*} - b \\ \vdots \\ y_N - b \end{bmatrix},$$

where

$$A = \begin{bmatrix} r+s+\dfrac{N-n^*+a}{N} m & 0 & 0 \\ \dfrac{a}{N} m & r+s+\dfrac{N-n^*-1+a}{N} m & 0 \\ & & \ddots \\ \dfrac{a}{N} m & \dfrac{a}{N} m & r+s+\dfrac{a}{N} m \end{bmatrix}.$$

Since A is a lower triangular matrix with non-zero diagonals, A is invertible, and so there exists a solution $\{S_n\}_{n=n^*}^N$ given the a value for m and thus v/z. We can therefore express S_n as a function of m. To prove there exists a unique equilibrium value of m, we will need the monotonicity result that $dS_n/dm < 0$. However, since

$$S_n = \frac{y_n - b - am/N \sum_{j=n^*}^{n-1} S_j}{r + s + ((N-n+a)/N)m},$$

the fact that S_j is decreasing in m for all $j \in \{n^*, \dots, n-1\}$ implies that S_n could either increase and decrease in m. Intuitively, since the value of unemployment is increasing in m, a higher value of m could act to either increase or decrease the surplus from an existing match. Thus, it is necessary to compute S_n explicitly to establish monotonicity. Using Cremer's rule, we have

$$S_n = \frac{\det A_n}{\det A} = \frac{\det A_n}{\prod_{j=n^*}^{n}(r+s+((N-j+a)/N)m)}$$

where A_n is the matrix A with the n-th column replaced with the vector $[y_n - b]_{n=n^*}^N$, *i.e.*

$$A_n = \begin{bmatrix} r+s+\frac{N-n^*+a}{N}m & 0 & y_{n^*}-b & 0 \\ \frac{a}{N}m & r+s+\frac{N-n^*-1+a}{N}m & y_{n^*+1}-b & 0 \\ & & \ddots & \\ & & y_n - b & \\ & & & \ddots \\ \frac{a}{N}m & \frac{a}{N}m & y_N - b & r+s+\frac{a}{N}m \end{bmatrix}$$

$$\equiv \begin{bmatrix} A_{n1} & 0 \\ A_{n2} & A_{n3} \end{bmatrix},$$

where A_{n1} is an $(n+1-n^*) \times (n+1-n^*)$ matrix, A_{n2} is an $(N-n) \times (n+1-n^*)$ matrix, and A_{n2} is an $(N-n) \times (N-n)$ matrix that is lower triangular. It follows that

$$\det A_n = \det A_{n1} \det A_{n3} = \det A_{n1} \cdot \left[\prod_{j=n+1}^{N}\left(r+s+\frac{N-j+a}{N}m\right)\right].$$

Next, I claim that

$$\det A_{n1} = (y_n - b)\prod_{j=k+1}^{n-1}\left(r+s+\frac{N-j}{N}m\right)$$
$$+\sum_{k=1}^{n-1}\frac{am}{N}\left[\prod_{j=1}^{k-1}\left(r+s+\frac{N-j}{N}m\right)\right]\left[\prod_{j=k+1}^{n-1}\left(r+s+\frac{N-j+a}{N}m\right)\right](y_n - y_k).$$

To see this, evaluate the determinant of A_{n1} using cofactor expansion along the final column. This involves multiplying $(y_k - b)(-1)^{2(n+1-n^*)-k+1}$ by the determinant of a matrix containing two blocks, one of which is lower triangular and the other of which has the same form as in Lemma 2:

$$\det A_{n1} = \textstyle\sum_{k=n^*}^{n}(y_k - b)(-1)^{n+k+2-2n^*}$$

$$\times \det\begin{bmatrix} q+p_{n^*} & & 0 & 0 & 0 & & 0 & 0 \\ & \ddots & & & & & 0 & 0 \\ q & & q+p_{k-1} & 0 & 0 & & & \\ q & & q & q & q+p_{k+1} & & & \\ & & q & q & q & & & \\ & & & & & \ddots & & \\ q & q & & & & & q & q+p_{n-1} \\ q & q & & & & & q & q \end{bmatrix}$$

$$= \textstyle\sum_{k=n^*}^{n}(y_k - b)(-1)^{n+k+2-2n^*}\left[\prod_{j=n^*}^{k-1}(q+p_j)\right]\det\begin{bmatrix} q & q+p_{k+1} & & 0 \\ & & \ddots & \\ q & q & & q+p_{n-1} \\ q & q & & q \end{bmatrix}$$

$$= \textstyle\sum_{k=n^*}^{n}(y_k - b)(-1)^{n+k+2-2n^*}\left((-1)^{n-k+1}q\prod_{j=k+1}^{n-1}p_j\right)$$

$$= (y_n - b)(-1)^{2(n+1-n^*)}(q+p_j) + \textstyle\sum_{k=n^*}^{n-1}(y_k - b)(-1)^{2n+1-n^*+1}\left(q\prod_{j=k+1}^{n-1}p_j\right)$$

$$= (y_n - b)\textstyle\prod_{j=n^*}^{n-1}(q+p_j) - \sum_{k=n^*}^{n-1}q(y_k - b)\left[\prod_{j=k+1}^{n-1}p_j\right]\left[\prod_{j=n^*}^{k-1}(q+p_j)\right],$$

where $q = am/N$ and $p_j = r + s + ((N-j)/N)m$. Finally, using part (b) of Lemma 2, we know that $\det A_{1n}$ when the last column is replaced with a vector of 1 instead of $[y_j - b]_{j=n^*}^{n}$, is equal to

$$\textstyle\prod_{j=n^*}^{n-1}(q+p_j) - \sum_{k=n^*}^{n-1}q\left[\prod_{j=k+1}^{n-1}p_j\right]\left[\prod_{j=n^*}^{k-1}(q+p_j)\right] = \prod_{j=n^*}^{n-1}p_j. \tag{A.5}$$

Given this restriction on the coefficients, we can rewrite

$$(y_n - b)\textstyle\prod_{j=n^*}^{n-1}(q+p_j) = (y_n - b)\prod_{j=n^*}^{n-1}p_j + (y_n - b)\sum_{k=n^*}^{n-1}q\left[\prod_{j=k+1}^{n-1}p_j\right]\left[\prod_{j=n^*}^{k-1}(q+p_j)\right]$$

and distributing terms out allows us to write express this determinant as

$$(y_n - b)\textstyle\prod_{j=n^*}^{n-1}p_j + \sum_{k=n^*}^{n-1}q(y_n - y_k)\left[\prod_{j=k+1}^{n-1}p_j\right]\left[\prod_{j=n^*}^{k-1}(q+p_j)\right].$$

Substituting for q and p_j, we have

$$S_n = \frac{\prod_{j=n^*}^{n-1}(r+s+((N-j)m)]/N)}{\prod_{j=n^*}^{n}(r+s+((N-j+a)m)/N)}(y_n - b) + \sum_{k=n^*}^{n}\frac{am}{N}\frac{\prod_{j=n^*}^{k-1}(r+s+((N-j)m)/N)}{\prod_{j=n^*}^{k}(r+s+((N-j+a)m)/N)}(y_n - y_k)$$

as desired. Finally, I show that $(am/N)S_n$ is increasing in m. To do this, note that

$$\frac{am}{N}\frac{\prod_{j=n^*}^{k-1}((r+s)N+(N-j)m)}{\prod_{j=n^*}^{k}((r+s)N+(N-j+a)m)}$$

$$= \frac{am}{\big((r+s)N+(N-n^*+a)m\big)N}\prod_{j=n^*}^{k-1}\frac{(r+s)N+(N-j)m}{(r+s)N+(N-j-1+a)m},$$

which is a product of strictly positive functions all of which are increasing in m. Using this result and multiplying S_n by am/N establishes that this expression is increasing in m. Since

$$S_n = \frac{y_n - b - am/N\sum_{j=n^*}^{n-1}S_j}{r+s+((N-n+a)/N)m}$$

then it follows that S_n is decreasing in m, as desired.

To solve for the equilibrium level of entry, we use the free entry condition $V = 0$:

$$\frac{1}{N} m\left(\frac{z}{v}, 1\right) \sum_{n=n^*}^{N} (1-a) \frac{F_{n-1}}{F_{N-1}} S_n = k. \tag{A.6}$$

where

$$F_n = u + \sum_{m \leq n} e_m.$$

Since the vector $\{F_n\}_{n=n^*-1}^{N-1}$ is isomorphic to $\{e_n\}_{n=n^*}^{N}$, we can solve for the steady-state distribution by looking at the steady state properties of F_n. Using the laws of motion (1.2) and (1.3) implies that in steady state,

$$\dot{F}_n = s(1 - F_n) - \frac{N-n}{N} m\left(1, \frac{v}{z}\right) F_n = 0,$$

so that

$$F_n = \frac{s}{s + ((N-n)/N) m(1, v/z)},$$

i.e. the distribution $\{e_n\}_{n=n^*}^{N}$ is uniquely determined for a given value of $m(1, v/z)$. Finally, since

$$\frac{F_n}{F_{N-1}} = \frac{s + (1/N) m(1, v/z)}{s + ((N-n)/N) m(1, v/z)},$$

is decreasing in v/z for $n < N$, it follows that the L.H.S. of (A.6) is monotonically decreasing in v/z; all three terms involving v/z are decreasing in this term. Thus, there exists at most one value v/z which can satisfy the free entry condition. Taking limits, we have that the L.H.S. of (A.6) converges to ∞ as $v/z \to 0$, and to 0 as $v/z \to \infty$, which insures existence for any $k \in (0, \infty)$. Hence, there is a unique ratio v/z that is consistent with steady state. The steady-state distribution $\{e_n\}_{n=n^*}^{N}$ can be backed out from the steady-state values of F_n, while the values of surplus can be computed using the formula above. ||

Proof of Proposition 3. The first part of the proposition follows from (A.6) and the fact that the L.H.S. is increasing in y and decreasing in v/z (the former can be easily established from the formula for S_n). For the second part of the proposition, note that a first order stochastic shift is equivalent to the statement that for all $n^* \leqq n < N$, the ratio

$$\frac{\sum_{m=n^*}^{n} e_m}{\sum_{m=n^*}^{N} e_m}$$

is decreasing in v/z. But we know

$$\frac{\sum_{m=n^*}^{n} e_m}{\sum_{m=n^*}^{N} e_m} = \frac{F_{n-1} - F_{n^*-1}}{F_{N-1} - F_{n^*-1}} = \frac{n - n^*}{N - n^*} \cdot \frac{s + m/N}{s + (N - n + 1)m/N}$$

which is in fact decreasing in v/z. ||

APPENDIX B. THE DISCRETE-TIME MODEL AND COLLOCATION METHOD

This appendix describes the discrete-time version of the model that is used for calibration, as well as the collocation method used for approximating the value functions Π_n^i, where i denotes the level of aggregate productivity and n denotes the quality of the match. In contrast with the continuous time model presented in the text, the model here assumes the parties split output rather than surplus when match quality exceeds some level n^*.

B.1. *Timing within a period*

1. Let $(e_1, \ldots, e_N)$ denote the distribution at the beginning of period t. At this stage, aggregate productivity $y_t \in \{y_0, y_1\}$ is determined, where y_t follows a Markov chain with probability μ of transition.
2. Exogenous and endogenous separations occur at the beginning of the period, before production takes place. Hence,

$$e_n'' = (1 - s)e_n,$$

for $n \geqq n^*$ and

$$e_n'' = 0,$$

for $n < n^*$.

3. After $(e''_1, \ldots, e''_N)$ is observed, and vacancies are posted by firms so as to insure zero profits.
4. Search takes place and workers change jobs:

$$e'_n = \left(1 - \frac{N-n}{N} m(x)\right) e''_n + \frac{1}{N} m(x)\left(1 - \sum_{j=n}^{N} e''_j\right),$$

where $m(x) = m(1, v/z)$.

5. Period $t+1$ starts with a distribution (e'_1, e'_2, e'_3).

6.2. *Computing the value of a match* Π^i_n

Without loss of generality, I set $n^* = 1$ during a boom. I further assume that during recessions, only the least productive match quality, $n = 1$, is destroyed. This leaves $2N - 1$ functions that affect the decision to post a vacancy: $\{\Pi^1_n\}^N_{n=1}$ and $\{\Pi^0_n\}^N_{n=2}$. Of these functions, Π^1_N and Π^0_N, do not depend on $(e_1, \ldots, e_3)$ and can be solved directly using Bellman equation. Given the timing within a period above, these Bellman equations are given by

$$\Pi^i_N = (1-a)y_i(1+\varepsilon_N) + \beta(1-s)\left[(1-\mu)\Pi^i_N + \mu\Pi^{-i}_N\right],$$

which yields

$$\Pi^i_N = \frac{(1-\beta(1-s)(1-\mu))y_i + \beta(1-s)\mu y_{-i}}{1-\beta(1-s)(1-2\mu)} \frac{(1-a)(1+\varepsilon_N)}{1-\beta(1-s)}.$$

I approximate the remaining $2N - 3$ functions with second order polynomials. That is,

$$\tilde{\Pi}(e_1, \ldots, e_N) = a_0 + \sum_{k=1}^{N} a_k e_k + \sum_{j=k}^{N} \sum_{k=1}^{N} a_{jk} e_j e_k.$$

Following den Haan, Ramey and Watson (2000), the number of contacts out of the population of 1 that form within a period is given by

$$m(z, v) = \frac{zv}{z+v}.$$

The probability of filling a vacancy can be rewritten as

$$m\left(\frac{z}{v}, 1\right) = \frac{z/v}{z/v+1} = \frac{1}{1+x},$$

where $x = v/z$.

The free entry condition implies that in the good state of the world,

$$\frac{1}{N}\frac{1}{1+x} \sum_{n=1}^{N} \left(\frac{1 - \sum_{m=1}^{n} e''_j}{1 - e''_N}\right) \Pi^1_n = k,$$

and in the bad state,

$$\frac{1}{N}\frac{1}{1+x} \sum_{n=2}^{N} \left(\frac{1 - \sum_{m=2}^{n} e''_j}{1 - e''_N}\right) \Pi^0_n = k,$$

which implies $x = x^i(e''_1, \ldots, e''_N)$.

Finally, for jobs of quality n that survive both levels of aggregate productivity, we have that the value functions satisfy the following Bellman equation:

$$\Pi^i_n(e_1, \ldots, e_N) = (1-a)y_i(1+\varepsilon_n) + \beta(1-s) \times \left[\begin{array}{l} (1-\mu)\left(1 - \frac{N-n}{N} m(x_i(e''_1, \ldots, e''_N))\right)\Pi^i_n(e'_1, \ldots, e'_N) \\ \quad + \mu\left(1 - \frac{N-n}{N} m(x_{-i}(e''_1, \ldots, e''_N))\right)\Pi^{-i}_n(e'_1, \ldots, e'_N) \end{array}\right],$$

where $m(x) = x/(1+x)$ is the probability that the worker encounters a match. If a job is destroyed when aggregate productivity is low, then the same equation still holds for $i = 1$, but $\Pi^0_n = 0$.

The above equations can be written more compactly as

$$\Pi^i_n(e_1, \ldots, e_N) = f^i_n(e_1, \ldots, e_N).$$

Now, suppose we approximate Π^i_n everywhere with polynomial functions $\tilde{\Pi}^i_n$, where a^i_n is the vector of coefficients associated with this polynomial. This yields the system of approximate Bellman equations

$$\tilde{\Pi}^i_n(a^i_n, e_1, \ldots, e_N) = f^i_n(a^i_n, e_1, \ldots, e_N).$$

The method of collocation involves choosing points S distinct points $(e_{1s}, \ldots, e_{Ns})$, where $S = \dim(a)$, and imposing that the above equations are exactly satisfied at those points. In other words, we choose one point for each coefficient we need to estimate. For each i and n, this is just a linear system of equations

$$Xa = f(a),$$

where

$$f(a) = \left[f_n^i(a, e_{1s}, \ldots, e_{Ns})\right]_{s=1}^{S},$$

and

$$X = \left[1 \quad e_{1s} \quad \ldots \quad e_{Ns} \quad e_{1s}^2 \quad \ldots \quad e_{1s}e_{Ns} \quad e_{2s}^2 \quad \ldots \quad e_{2s}e_{Ns}\right]_{s=1}^{S},$$

The R.H.S. of this system of equations is linear in a, but the L.H.S. is not. To solve this system, I use an iterative procedure where I use a_t to construct $f(a_t)$, and then construct a_{t+1} by solving for the vector a_{t+1} such that $Xa_{t+1} = f(a_t)$, *i.e.*

$$a_{t+1} = (X'X)^{-1}X'f(a_t).$$

I start with an initial guess a_0 and then iterate until a_t converges.

For the set of points $(e_{1s}, \ldots, e_{Ns})$, I allowed the computer to choose points at random such that $\Sigma e_{1n} \leqq 1$. Experimenting with several such points suggested that the values of $\widetilde{\Pi}_n^i$ did not differ by more than 0·01 in the relevant range where the model ultimately settled down to across such specifications, and by no more than 0·5 in some of the extreme points. I therefore chose one particular realization and used it to generate all of the sequences reported in the paper.

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