Econometrica, Vol. 77, No. 3 (May, 2009), 623–685

THE IMPACT OF UNCERTAINTY SHOCKS

BY NICHOLAS BLOOM[1]

Uncertainty appears to jump up after major shocks like the Cuban Missile crisis, the assassination of JFK, the OPEC I oil-price shock, and the 9/11 terrorist attacks. This paper offers a structural framework to analyze the impact of these uncertainty shocks. I build a model with a time-varying second moment, which is numerically solved and estimated using firm-level data. The parameterized model is then used to simulate a macro uncertainty shock, which produces a rapid drop and rebound in aggregate output and employment. This occurs because higher uncertainty causes firms to temporarily pause their investment and hiring. Productivity growth also falls because this pause in activity freezes reallocation across units. In the medium term the increased volatility from the shock induces an overshoot in output, employment, and productivity. Thus, uncertainty shocks generate short sharp recessions and recoveries. This simulated impact of an uncertainty shock is compared to vector autoregression estimations on actual data, showing a good match in both magnitude and timing. The paper also jointly estimates labor and capital adjustment costs (both convex and nonconvex). Ignoring capital adjustment costs is shown to lead to substantial bias, while ignoring labor adjustment costs does not.

KEYWORDS: Adjustment costs, uncertainty, real options, labor and investment.

1. INTRODUCTION

UNCERTAINTY APPEARS TO dramatically increase after major economic and political shocks like the Cuban missile crisis, the assassination of JFK, the OPEC I oil-price shock, and the 9/11 terrorist attacks. Figure 1 plots stock-market volatility—one proxy for uncertainty—which displays large bursts of uncertainty after major shocks, which temporarily double (implied) volatility on average.[2] These volatility shocks are strongly correlated with other measures of uncertainty, like the cross-sectional spread of firm- and industry-level earnings and productivity growth. Vector autoregression (VAR) estimations suggest that they also have a large real impact, generating a substantial drop and rebound in output and employment over the following 6 months.

[1]This article was the main chapter of my Ph.D. thesis, previously called "The Impact of Uncertainty Shocks: A Firm-Level Estimation and a 9/11 Simulation." I would like to thank my advisors Richard Blundell and John Van Reenen; the co-editor and the referees; my formal discussants Susantu Basu, Russell Cooper, Janice Eberly, Eduardo Engel, John Haltiwanger, Valerie Ramey, and Chris Sims; Max Floetotto; and many seminar audiences. Financial support of the ESRC and the Sloan Foundation is gratefully acknowledged.

[2]In financial markets implied share-returns volatility is the canonical measure for uncertainty. Bloom, Bond, and Van Reenen (2007) showed that firm-level share-returns volatility is significantly correlated with a range of alternative uncertainty proxies, including real sales growth volatility and the cross-sectional distribution of financial analysts' forecasts. While Shiller (1981) has argued that the *level* of stock-price volatility is excessively high, Figure 1 suggests that *changes* in stock-price volatility are nevertheless linked with real and financial shocks.

 DOI: 10.3982/ECTA6248

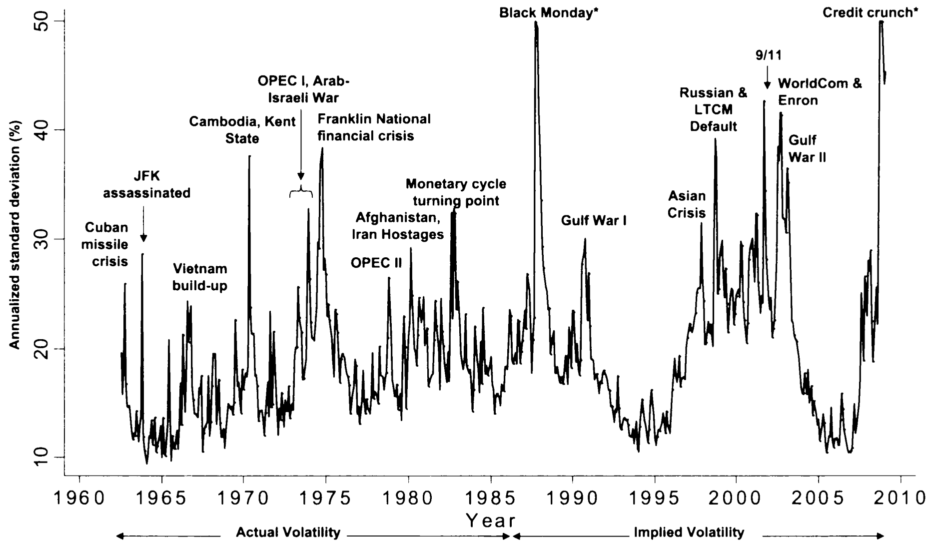

FIGURE 1.—Monthly U.S. stock market volatility. *Notes*: Chicago Board of Options Exchange VXO index of percentage implied volatility, on a hypothetical at the money S&P100 option 30 days to expiration, from 1986 onward. Pre-1986 the VXO index is unavailable, so actual monthly returns volatilities are calculated as the monthly standard deviation of the daily S&P500 index normalized to the same mean and variance as the VXO index when they overlap from 1986 onward. Actual and VXO are correlated at 0.874 over this period. A brief description of the nature and exact timing of every shock is contained in Appendix A. The asterisks indicate that for scaling purposes the monthly VXO was capped at 50. Uncapped values for the Black Monday peak are 58.2 and for the credit crunch peak are 64.4. LTCM is Long Term Capital Management.

Uncertainty is also a ubiquitous concern of policymakers. For example, after 9/11 the Federal Open Market Committee (FOMC), worried about exactly the type of real-options effects analyzed in this paper, stated in October 2001 that "the events of September 11 produced a marked increase in uncertainty [...] depressing investment by fostering an increasingly widespread wait-and-see attitude." Similarly, during the credit crunch the FOMC noted that "Several [survey] participants reported that uncertainty about the economic outlook was leading firms to defer spending projects until prospects for economic activity became clearer."

Despite the size and regularity of these second-moment (uncertainty) shocks, there is no model that analyzes their effects. This is surprising given the extensive literature on the impact of first-moment (levels) shocks. This leaves open a wide variety of questions on the impact of major macroeconomic shocks, since these typically have both a first- and a second-moment component.

The primary contribution of this paper is to structurally analyze these types of uncertainty shocks. This is achieved by extending a standard firm-level model with a time-varying second moment of the driving process and a mix of labor and capital adjustment costs. The model yields a central region of inaction in hiring and investment space due to nonconvex adjustment costs. Firms only hire and invest when business conditions are sufficiently good, and only fire and disinvest when they are sufficiently bad. When uncertainty is higher, this region of inaction expands—firms become more cautious in responding to business conditions.

I use this model to simulate the impact of a large temporary uncertainty shock and find that it generates a rapid drop, rebound, and overshoot in employment, output, and productivity growth. Hiring and investment rates fall dramatically in the 4 months after the shock because higher uncertainty increases the real-option value to waiting, so firms scale back their plans. Once uncertainty has subsided, activity quickly bounces back as firms address their pent-up demand for labor and capital. Aggregate productivity growth also falls dramatically after the shock because the drop in hiring and investment reduces the rate of reallocation from low to high productivity firms, which drives the majority of productivity growth in the model as in the real economy.[3] Again productivity growth rapidly bounces back as pent-up reallocation occurs.

In the medium term the increased volatility arising from the uncertainty shock generates a "volatility overshoot." The reason is that most firms are located near their hiring and investment thresholds, above which they hire/invest and below which they have a zone of inaction. So small positive shocks generate a hiring and investment response while small negative shocks generate no response. Hence, hiring and investment are locally convex in business conditions (demand and productivity). The increased volatility of business conditions growth after a second-moment shock therefore leads to a medium-term rise in labor and capital.

In sum, these second-moment effects generate a rapid slowdown and bounce-back in economic activity, entirely consistent with the empirical evidence. This is very different from the much more persistent slowdown that typically occurs in response to the type of first-moment productivity and/or demand shock that is usually modelled in the literature.[4] This highlights the importance to policymakers of distinguishing between the persistent first-moment effects and the temporary second-moment effects of major shocks.

I then evaluate the robustness of these predictions to general equilibrium effects, which for computational reasons are not included in my baseline model. To investigate this I build the falls in interest rates, prices, and wages that occur after actual uncertainty shocks into the simulation. This has little short-run effect on the simulations, suggesting that the results are robust to general equilibrium effects. The reason is that the rise in uncertainty following a second-moment shock not only generates a slowdown in activity, but it also makes firms

[3]See Foster, Haltiwanger, and Krizan (2000, 2006).

[4]See, for example, Christiano, Eichenbaum, and Evans (2005) and the references therein.

temporarily extremely insensitive to price changes. This raises a second policy implication that the economy will be particularly unresponsive to monetary or fiscal policy immediately after an uncertainty shock, suggesting additional caution when thinking about the policy response to these types of events.

The secondary contribution of this paper is to analyze the importance of jointly modelling labor and capital adjustment costs. For analytical tractability and aggregation constraints the empirical literature has estimated either labor or capital adjustment costs individually, assuming the other factor is flexible, or estimated them jointly, assuming only convex adjustment costs.[5] I jointly estimate a mix of labor and capital adjustment costs (both convex and nonconvex) by exploiting the properties of homogeneous functions to reduce the state space. The estimation uses simulated method of moments on firm-level data to overcome the identification problem associated with the limited sample size of macro data. I find moderate nonconvex labor adjustment costs and substantial nonconvex capital adjustment costs. I also find that assuming capital adjustment costs only—as is standard in the investment literature—generates an acceptable overall fit, while assuming labor adjustment costs only—as is standard in the labor demand literature—produces a poor fit.

The analysis of uncertainty shocks links with the earlier work of Bernanke (1983) and Hassler (1996) who highlighted the importance of variations in uncertainty.[6] In this paper I quantify and substantially extend their predictions through two major advances: first, by introducing uncertainty as a stochastic process which is critical for evaluating the high-frequency impact of major shocks and, second, by considering a joint mix of labor and capital adjustment costs which is critical for understanding the dynamics of employment, investment, and productivity.

This framework also suggests a range of future research. Looking at individual events, it could be used, for example, to analyze the uncertainty impact of trade reforms, major deregulations, tax changes, or political elections. It also suggests there is a trade-off between policy "correctness" and "decisiveness"—it may be better to act decisively (but occasionally incorrectly) than to deliber-

[5]See, for example: on capital, Cooper and Haltiwanger (1993), Caballero, Engel, and Haltiwanger (1995), Cooper, Haltiwanger, and Power (1999), and Cooper and Haltiwanger (2006); on labor, Hammermesh (1989), Bertola and Bentolila (1990), Davis and Haltiwanger (1992), Caballero and Engel (1993), Caballero, Engel, and Haltiwanger (1997), and Cooper, Haltiwanger, and Willis (2004); on joint estimation with convex adjustment costs, Shapiro (1986), Hall (2004), and Merz and Yashiv (2007); see Bond and Van Reenen (2007) for a full survey of the literature.

[6]Bernanke developed an example of uncertainty in an oil cartel for capital investment, while Hassler solved a model with time-varying uncertainty and fixed adjustment costs. There are of course many other linked recent strands of literature, including work on growth and volatility such as Ramey and Ramey (1995) and Aghion, Angeletos, Banerjee, and Manova (2005), on investment and uncertainty such as Leahy and Whited (1996) and Bloom, Bond, and Van Reenen (2007), on the business-cycle and uncertainty such as Barlevy (2004) and Gilchrist and Williams (2005), on policy uncertainty such as Adda and Cooper (2000), and on income and consumption uncertainty such as Meghir and Pistaferri (2004).

ate on policy, generating policy-induced uncertainty. For example, when the Federal Open Markets Committee discussed the negative impact of uncertainty after 9/11 it noted that "A key uncertainty in the outlook for investment spending was the outcome of the ongoing Congressional debate relating to tax incentives for investment in equipment and software" (November 6th, 2001). Hence, in this case Congress's attempt to revive the economy with tax incentives may have been counterproductive due to the increased uncertainty the lengthy policy process induced.

More generally, the framework in this paper also provides one response to the "where are the negative productivity shocks?" critique of real business cycle theories.[7] In particular, since second-moment shocks generate large falls in output, employment, and productivity growth, it provides an alternative mechanism to first-moment shocks for generating recessions. Recessions could simply be periods of high uncertainty without negative productivity shocks. Encouragingly, recessions do indeed appear in periods of significantly higher uncertainty, suggesting an uncertainty approach to modelling business cycles (see Bloom, Floetotto, and Jaimovich (2007)). Taking a longer-run perspective this paper also links to the volatility and growth literature, given the large negative impact of uncertainty on output and productivity growth.

The rest of the paper is organized as follows: in Section 2, I empirically investigate the importance of jumps in stock-market volatility; in Section 3, I set up and solve my model of the firm; in Section 4, I characterize the solution of the model and present the main simulation results; in Section 5, I outline my simulated method of moments estimation approach and report the parameter estimates using U.S. firm data; and in Section 6, I run some robustness test on the simulation results. Finally, Section 7 offers some concluding remarks. Data and programs are provided in an online supplement (Bloom (2009)).

2. DO JUMPS IN STOCK-MARKET VOLATILITY MATTER?

Two key questions to address before introducing any models of uncertainty shocks are (i) do jumps[8] in the volatility index in Figure 1 correlate with other measures of uncertainty and (ii) do these have any impact on real economic outcomes? In Section 2.1, I address the first question by presenting evidence showing that stock-market volatility is strongly linked to other measures of productivity and demand uncertainty. In Section 2.2, I address the second question by presenting vector autoregression (VAR) estimations showing that volatility shocks generate a short-run drop in industrial production of 1%, lasting about 6 months, and a longer-run overshoot. First-moment shocks to the interest rates and stock-market levels generate a much more gradual drop and

[7]See the extensive discussion in King and Rebelo (1999).

[8]I tested for jumps in the volatility series using the bipower variation test of Barndorff-Nielsen and Shephard (2006) and found statistically significant evidence for jumps. See Appendix A.1.

rebound in activity lasting 2 to 3 years. A full data description for both sections is contained in Appendix A.[9]

2.1. *Empirical Evidence on the Links Between Stock-Market Volatility and Uncertainty*

The evidence presented in Table I shows that a number of *cross-sectional* measures of uncertainty are highly correlated with *time-series* stock-market volatility. Stock-market volatility has also been previously used as a proxy for uncertainty at the firm level (e.g., Leahy and Whited (1996) and Bloom, Bond, and Van Reenen (2007)).

Columns 1–3 of Table I use the cross-sectional standard deviation of firms' pretax profit growth, taken from the quarterly accounts of public companies. As can be seen from column 1 stock-market time-series volatility is strongly correlated with the cross-sectional spread of firm-level profit growth. All variables in Table I have been normalized by their standard deviations (SD). The coefficient implies that the 2.47 SD rise in stock-market time-series volatility that occurred on average after the shocks highlighted in Figure 1 would be associated with a 1.31 SD ($1.31 = 2.47 \times 0.532$) rise in the cross-sectional spread of the growth rate of profits, a large increase. Column 2 reestimates this including a full set of quarterly dummies and a time trend, finding very similar results.[10] Column 3 also includes quarterly standard industrial criterion (SIC) three-digit industry controls and again finds similar results,[11] suggesting that idiosyncratic firm-level shocks are driving the time-series variations in volatility.

Columns 4–6 use a monthly cross-sectional stock-return measure and show that this is also strongly correlated with the stock-return volatility index. Columns 7 and 8 report the results from using the standard deviation of annual five-factor Total Factor Productivity (TFP) growth within the National Bureau of Economic Research (NBER) manufacturing industry data base. There is also a large and significant correlation of the cross-sectional spread of industry productivity growth and stock-market volatility. Finally, columns 9 and 10 use a measure of the dispersion across macro forecasters over their predictions for future gross domestic product (GDP), calculated from the Livingstone half-yearly survey of professional forecasters. Once again, periods of high stock-market volatility are significantly correlated with cross-sectional dispersion, in this case in terms of disagreement across macro forecasters.

[9]All data and program files are also available at http://www.stanford.edu/~nbloom/.

[10]This helps to control for any secular changes in volatility (see Davis, Haltiwanger, Jarmin, and Miranda (2006)).

[11]This addresses the type of concerns that Abraham and Katz (1986) raised about Lillien's (1982) work on unemployment, where time-series variations in cross-sectional unemployment appeared to be driven by heterogeneous responses to common macro shocks.

TABLE I

THE STOCK-MARKET VOLATILITY INDEX REGRESSED ON CROSS-SECTIONAL MEASURES OF UNCERTAINTY[a]

Explanatory Variable Is Period by Period Cross-Sectional Standard Deviation of	Dependent Variable Is *Stock-Market Volatility*[b] 1	2	3	4	5	6	7	8	9	10
Firm profit growth,[c] Compustat quarterly	0.532 *(0.064)*	0.526 *(0.092)*	0.469 *(0.115)*							
Firm stock returns,[d] CRSP monthly				0.543 *(0.037)*	0.544 *(0.038)*	0.570 *(0.037)*				
Industry TFP growth,[e] SIC 4-digit yearly							0.429 *(0.119)*	0.419 *(0.125)*		
GDP forecasts,[f] Livingstone half-yearly									0.614 *(0.111)*	0.579 *(0.121)*
Time trend	No	Yes	Yes	No	Yes	Yes	No	Yes	No	Yes
Month/quarter/half-year dummies[g]	No	Yes	Yes	No	Yes	Yes	n/a	n/a	No	Yes
Controls for SIC 3-digit industry[h]	No	No	Yes	No	No	Yes	n/a	n/a	n/a	n/a
R^2	0.287	0.301	0.238	0.287	0.339	0.373	0.282	0.284	0.332	0.381
Time span		62Q3–05Q1			62M7–06M12		1962–1996		62H2–98H2	
Average units in cross section[i]		327			355		425		57.4	
Observations in regression		171			534		35		63	

[a]Each column reports the coefficient from regressing the time series of stock-market volatility on the within period cross-sectional standard deviation (SD) of the explanatory variable calculated from an underlying panel. All variables normalized to a SD of 1. Standard errors are given in italics in parentheses below. So, for example, column 1 reports that the stock-market volatility index is on average 0.532 SD higher in a quarter when the cross-sectional spread of firms' profit growth is 1 SD higher.

[b]The stock-market volatility index measures monthly volatility on the U.S. stock market and is plotted in Figure 1. The quarterly, half-yearly, and annual values are calculated by averaging across the months within the period.

[c]The standard deviation of firm profit growth measures the within-quarter cross-sectional spread of profit growth rates normalized by average sales, defined as $(\text{profits}_t - \text{profits}_{t-1})/(0.5 \times \text{sales}_t + 0.5 \times \text{sales}_{t-1})$ and uses firms with 150+ quarters of data in Compustat quarterly accounts.

[d]The standard deviation of firm stock returns measures the within month cross-sectional standard deviation of firm-level stock returns for firm with 500+ months of data in the Center for Research in Securities Prices (CRSP) stock-returns file.

[e]The standard deviation of industry TFP growth measures the within-year cross-industry spread of SIC 4-digit manufacturing TFP growth rates, calculated using the five-factor TFP growth figures from the NBER data base.

[f]The standard deviation of GDP forecasts comes from the Philadelphia Federal Reserve Bank's biannual Livingstone survey, calculated as the (standard deviation/mean) of forecasts of nominal GDP 1 year ahead, using half-years with 50+ forecasts, linearly detrended to remove a long-run downward drift.

[g]Month/quarter/half-year dummies refers to quarter, month, and half-year controls for period effects.

[h]Controls for SIC 3-digit industry denotes that the cross-sectional spread is calculated with SIC 3-digit by period dummies so the profit growth and stock returns are measured relative to the industry period average.

[i]Average units in cross section refers to the *average* number of units (firms, industries, or forecasters) used to measure the cross-sectional spread.

2.2. *VAR Estimates on the Impact of Stock-Market Volatility Shocks*

To evaluate the impact of uncertainty shocks on real economic outcomes I estimate a range of VARs on monthly data from June 1962 to June 2008.[12] The variables in the estimation order are log(S&P500 stock market index), a stock-market volatility indicator (described below), Federal Funds Rate, log(average hourly earnings), log(consumer price index), hours, log(employment), and log(industrial production). This ordering is based on the assumptions that shocks instantaneously influence the stock market (levels and volatility), then prices (wages, the consumer price index (CPI), and interest rates), and finally quantities (hours, employment, and output). Including the stock-market levels as the first variable in the VAR ensures the impact of stock-market levels is already controlled for when looking at the impact of volatility shocks. All variables are Hodrick–Prescott (HP) detrended ($\lambda = 129{,}600$) in the baseline estimations.

The main stock-market volatility indicator is constructed to take a value 1 for each of the shocks labelled in Figure 1 and a 0 otherwise. These 17 shocks were explicitly chosen as those events when the peak of HP detrended volatility level rose significantly above the mean.[13] This indicator function is used to ensure that identification comes only from these large, and arguably exogenous, volatility shocks rather than from the smaller ongoing fluctuations.

Figure 2 plots the impulse response function of industrial production (the solid line with plus symbols) to a volatility shock. Industrial production displays a rapid fall of around 1% within 4 months, with a subsequent recovery and rebound from 7 months after the shock. The 1 standard-error bands (dashed lines) are plotted around this, highlighting that this drop and rebound is statistically significant at the 5% level. For comparison to a first-moment shock, the response to a 1% impulse to the Federal funds rate (FFR) is also plotted (solid line with circular symbols), displaying a much more persistent drop and recovery of up to 0.7% over the subsequent 2 years.[14] Figure 3 repeats the same exercise for employment, displaying a similar drop and recovery in activity. Figures A1, A2, and A3 in the Appendix confirm the robustness of these VAR results to a range of alternative approaches over variable ordering, variable inclusion, shock definitions, shock timing, and detrending. In particular, these results are robust to identification from uncertainty shocks defined by the 10 exogenous shocks arising from wars, OPEC shocks, and terror events.

[12]Note that this period excludes most of the Credit Crunch, which is too recent to have full VAR data available. I would like to thank Valerie Ramey and Chris Sims (my discussants) for their initial VAR estimations and subsequent discussions.

[13]The threshold was 1.65 standard deviations above the mean, selected as the 5% one-tailed significance level treating each month as an independent observation. The VAR estimation also uses the full volatility series (which does not require defining shocks) and finds very similar results, as shown in Figure A1.

[14]The response to a 5% fall the stock-market levels (not plotted) is very similar in size and magnitude to the response to a 1% rise in the FFR.

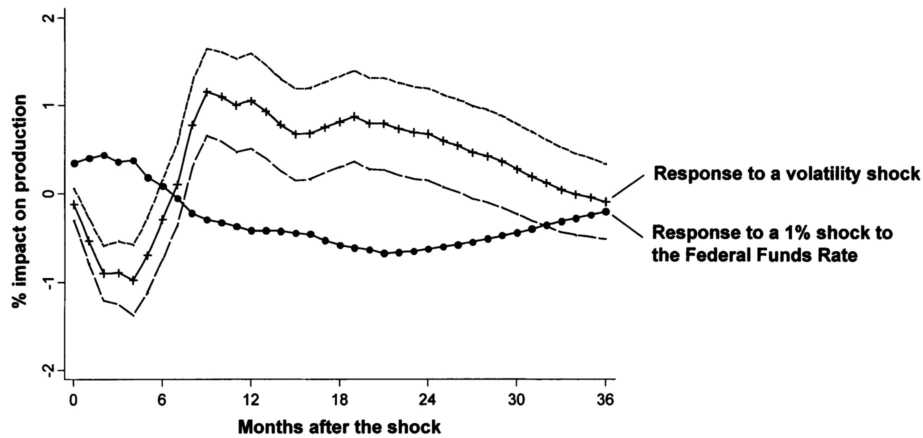

FIGURE 2.—VAR estimation of the impact of a volatility shock on industrial production. *Notes*: Dashed lines are 1 standard-error bands around the response to a volatility shock.

3. MODELLING THE IMPACT OF AN UNCERTAINTY SHOCK

In this section I model the impact of an uncertainty shock. I take a standard model of the firm[15] and extend it in two ways. First, I introduce uncertainty as a stochastic process to evaluate the impact of the uncertainty shocks shown in Figure 1. Second, I allow a joint mix of convex and nonconvex adjustment costs for both labor and capital. The time-varying uncertainty interacts with

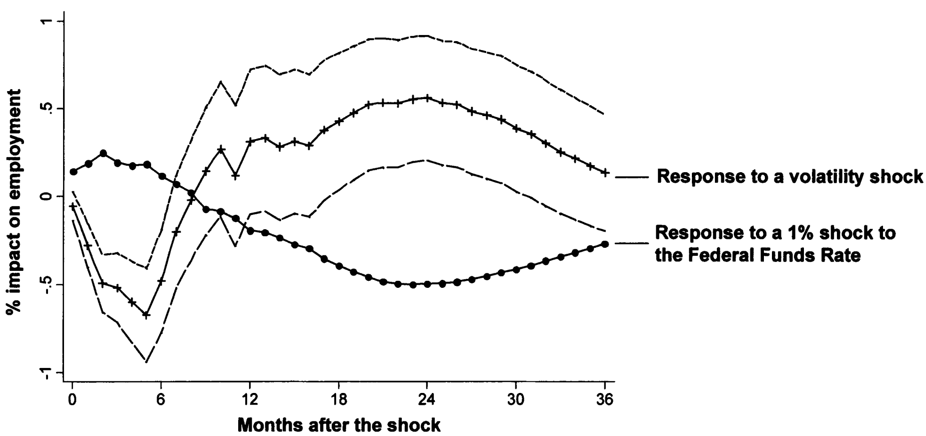

FIGURE 3.—VAR estimation of the impact of a volatility shock on employment. *Notes*: Dashed lines are 1 standard-error bands around the response to a volatility shock.

[15]See, for example, Bertola and Caballero (1994), Abel and Eberly (1996), or Caballero and Engel (1999).

the nonconvex adjustment costs to generate time-varying real-option effects, which drive fluctuations in hiring and investment. I also build in temporal and cross-sectional aggregation by assuming firms own large numbers of production units, which allows me to estimate the model's parameters on firm-level data.

3.1. *The Production and Revenue Function*

Each production unit has a Cobb–Douglas[16] production function

$$F(\widetilde{A}, K, L, H) = \widetilde{A}K^{\alpha}(LH)^{1-\alpha} \tag{3.1}$$

in productivity ($\widetilde{A}$), capital (K), labor (L), and hours (H). The firm faces an isoelastic demand curve with elasticity (ε),

$$Q = BP^{-\varepsilon},$$

where B is a (potentially stochastic) demand shifter. These can be combined into a revenue function $R(\widetilde{A}, B, K, L, H) = \widetilde{A}^{1-1/\varepsilon}B^{1/\varepsilon}K^{\alpha(1-1/\varepsilon)}(LH)^{(1-\alpha)(1-1/\varepsilon)}$. For analytical tractability I define $a = \alpha(1-1/\varepsilon)$ and $b = (1-\alpha)(1-1/\varepsilon)$, and substitute $A^{1-a-b} = \widetilde{A}^{1-1/\varepsilon}B^{1/\varepsilon}$, where A combines the unit-level productivity and demand terms into one index, which for expositional simplicity I will refer to as *business conditions*. With these redefinitions we have[17]

$$S(A, K, L, H) = A^{1-a-b}K^{a}(LH)^{b}.$$

Wages are determined by undertime and overtime hours around the standard working week of 40 hours. Following the approach in Caballero and Engel (1993), this is parameterized as $w(H) = w_1(1 + w_2H^{\gamma})$, where w_1, w_2, and γ are parameters of the wage equation to be determined empirically.

3.2. *The Stochastic Process for Demand and Productivity*

I assume business conditions evolve as an augmented geometric random walk. Uncertainty shocks are modelled as time variations in the standard deviation of the driving process, consistent with the stochastic volatility measure of uncertainty in Figure 1.

[16]While I assume a Cobb–Douglas production function, other supermodular homogeneous unit revenue functions could be used. For example, when replacing (3.1) with a constant elasticity of substitution aggregator over capital and labor, where $F(\widetilde{A}, K, L, H) = \widetilde{A}(\alpha_1K^{\sigma} + \alpha_2(LH)^{\sigma})^{1/\sigma}$, I obtained similar simulation results.

[17]This reformulation to A as the stochastic variable to yield a jointly homogeneous revenue function avoids long-run effects of uncertainty reducing or increasing output because of convexity or concavity in the production function. See Abel and Eberly (1996) for a discussion.

Business conditions are in fact modelled as a multiplicative composite of three separate random walks[18]: a macro-level component (A_t^M), a firm-level component ($A_{i,t}^F$), and a unit-level component ($A_{i,j,t}^U$), where $A_{i,j,t} = A_t^M A_{i,t}^F A_{i,j,t}^U$ and i indexes firms, j indexes units, and t indexes time. The macro-level component is modelled as

$$A_t^M = A_{t-1}^M(1 + \sigma_{t-1} W_t^M), \quad W_t^M \sim N(0,1), \tag{3.2}$$

where σ_t is the standard deviation of business conditions and W_t^M is a macro-level independent and identically distributed (i.i.d.) normal shock. The firm-level component is modelled as

$$A_{i,t}^F = A_{i,t-1}^F(1 + \mu_{i,t} + \sigma_{t-1} W_{i,t}^F), \quad W_{i,t}^F \sim N(0,1), \tag{3.3}$$

where $\mu_{i,t}$ is a firm-level drift in business conditions and $W_{i,t}^F$ is a firm-level i.i.d. normal shock. The unit-level component is modelled as

$$A_{i,j,t}^U = A_{i,j,t-1}^U(1 + \sigma_{t-1} W_{i,j,t}^U), \quad W_{i,j,t}^U \sim N(0,1), \tag{3.4}$$

where $W_{i,j,t}^U$ is a unit-level i.i.d. normal shock. I assume W_t^M, $W_{i,t}^F$, and $W_{i,j,t}^U$ are all independent of each other.

While this demand structure may seem complex, it is formulated to ensure that (i) units within the same firm have linked investment behavior due to common firm-level business conditions, and (ii) they display some independent behavior due to the idiosyncratic unit-level shocks, which is essential for smoothing under aggregation. This demand structure also assumes that macro-, firm-, and unit-level uncertainty are the same.[19] This is broadly consistent with the results from Table I for firm and macro uncertainty, which show these are highly

[18]A random-walk driving process is assumed for analytical tractability, in that it helps to deliver a homogenous value function (details in the next section). It is also consistent with Gibrat's law. An equally plausible alternative assumption would be a persistent AR(1) process, such as the following based on Cooper and Haltiwanger (2006): $\log(A_t) = \alpha + \rho \log(A_{t-1}) + v_t$, where $v_t \sim N(0, \sigma_{t-1})$, $\rho = 0.885$. To investigate this alternative I programmed another monthly simulation with autoregressive business conditions and no labor adjustment costs (so I could drop the labor state) and all other modelling assumptions the same. I found in this setup there were still large real-options effects of uncertainty shocks on output, as plotted in Figure S1 in the supplemental material (Bloom (2009)).

[19]This formulation also generates business-conditions shocks at the unit level (firm level) that have three (two) times more variance than at the macro level. This appears to be inconsistent with actual data, since establishment data on things like output and employment are many times more volatile than the macro equivalent. However, it is worth noting two points. First, micro data also typically have much more measurement error than macro data so this could be causing the much greater variance of micro data. In stock-returns data, one of the few micro and macro indicators with almost no measurement error, firm stock returns have twice the variance of aggregate returns consistent with the modelling assumption. Second, because of the nonlinearities in the investment and hiring response functions (due to nonconvex adjustment costs), output and input growth is much more volatile at the *unit* level than at the macro level, which is smoothed by aggregation. So

correlated. For unit-level uncertainty there is no direct evidence on this, but to the extent that this assumption does not hold the quantitative impact of macro uncertainty, shocks will be reduced (since total uncertainty will fluctuate less than one for one with macro uncertainty), while the qualitative findings will remain. I also evaluate this assumption in Section 6.3 by simulating an uncertainty shock which only changes the variance of W_t^M (rather than changing the variance of W_t^M, $W_{i,t}^F$, and $W_{i,j,t}^U$), with broadly similar results.

The firm-level business conditions drift ($\mu_{i,t}$) is also assumed to be stochastic to allow autocorrelated changes over time within firms. This is important for empirically identifying adjustment costs from persistent differences in growth rates across firms, as Section 5 discusses in more detail.

The stochastic volatility process (σ_t^2) and the demand conditions drift ($\mu_{i,t}$) are both assumed for simplicity to follow two-point Markov chains

$$\sigma_t \in \{\sigma_L, \sigma_H\}, \quad \text{where} \quad \Pr(\sigma_{t+1} = \sigma_j | \sigma_t = \sigma_k) = \pi_{k,j}^{\sigma}, \tag{3.5}$$

$$\mu_{i,t} \in \{\mu_L, \mu_H\}, \quad \text{where} \quad \Pr(\mu_{i,t+1} = \mu_j | \mu_{i,t} = \mu_k) = \pi_{k,j}^{\mu}. \tag{3.6}$$

3.3. *Adjustment Costs*

The third piece of technology that determines the firms' activities is the adjustment costs. There is a large literature on investment and employment adjustment costs which typically focuses on three terms, all of which I include in my specification:

Partial Irreversibilities: Labor partial irreversibility, labelled C_L^P, derives from *per capita* hiring, training, and firing costs, and is denominated as a fraction of annual wages (at the standard working week). For simplicity I assume these costs apply equally to gross hiring and gross firing of workers.[20] Capital partial irreversibilities arise from resale losses due to transactions costs, the market for lemons phenomenon, and the physical costs of resale. The resale loss of capital is labelled C_K^P and is denominated as a fraction of the relative purchase price of capital.

Fixed Disruption Costs: When new workers are added into the production process and new capital is installed, there may be a fixed loss of output. For example, adding workers may require fixed costs of advertising, interviewing, and training, or the factory may need to close for a few days while a capital refit is occurring. I model these fixed costs as C_L^F and C_K^F for hiring/firing and investment, respectively, both denominated as fractions of annual sales.

even if the unit-, firm-, and macro-level business conditions processes all have the same variance, the unit- and firm-level employment, capital, and sales growth outcomes will be more volatile due to more lumpy hiring and investment.

[20]Micro data evidence, for example, Davis and Haltiwanger (1992), suggests both *gross* and *net* hiring/firing costs may be present. For analytical simplicity I have restricted the model to *gross* costs, noting that *net* costs could also be introduced and estimated in future research through the addition of two *net* firing cost parameters.

Quadratic Adjustment Costs: The costs of hiring/firing and investment may also be related to the rate of adjustment due to higher costs for more rapid changes, where $C_L^Q L(\frac{E}{L})^2$ are the quadratic hiring/firing costs and E denotes gross hiring/firing, and $C_K^Q K(\frac{I}{K})^2$ are the quadratic investment costs and I denotes gross investment.

The combination of all adjustment costs is given by the adjustment cost function

$$\begin{aligned} &C(A, K, L, H, I, E, p_t^K) \\ &\quad = 52w(40)C_L^P(E^+ + E^-) + (I^+ - (1 - C_K^P)I^-) \\ &\qquad + \left(C_L^F 1_{\{E\neq 0\}} + C_K^F 1_{\{I\neq 0\}}\right)S(A, K, L, H) \\ &\qquad + C_L^Q L\left(\frac{E}{L}\right)^2 + C_K^Q K\left(\frac{I}{K}\right)^2, \end{aligned}$$

where E^+ (I^+) and E^- (I^-) are the absolute values of positive and negative hiring (investment), respectively, and $1_{\{E\neq 0\}}$ and $1_{\{I\neq 0\}}$ are indicator functions which equal 1 if true and 0 otherwise. New labor and capital take one period to enter production due to time to build. This assumption is made to allow me to pre-optimize hours (explained in Section 3.5 below), but is unlikely to play a major role in the simulations given the monthly periodicity. At the end of each period there is labor attrition and capital depreciation proportionate to δ_L and δ_K, respectively.

3.4. *Dealing With Cross-Sectional and Time Aggregation*

Gross hiring and investment is typically lumpy with frequent zeros in single-plant establishment-level data, but much smoother and continuous in multiplant establishment and firm-level data. This appears to be because of extensive aggregation across two dimensions: cross-sectional aggregation across types of capital and production plants; and temporal aggregation across higher-frequency periods within each year. I build this aggregation into the model by explicitly assuming that firms own a large number of production *units* and that these operate at a higher frequency than yearly. The units can be thought of as different production plants, different geographic or product markets, or different divisions within the same firm.

To solve this model I need to define the relationship between production units within the firm. This requires several simplifying assumptions to ensure analytical tractability. These are not attractive, but are necessary to enable me to derive numerical results and incorporate aggregation into the model. In doing this I follow the general stochastic aggregation approach of Bertola and Caballero (1994) and Caballero and Engel (1999) in modelling macro and industry investment, respectively, and most specifically Abel and Eberly (2002) in modelling firm-level investment.

Production units are assumed to independently optimize to determine investment and employment. Thus, all linkages across units within the same firm are modelled by the common shocks to demand, uncertainty, or the price of capital. So, to the extent that units are linked over and above these common shocks, the implicit assumption is that they independently optimize due to bounded rationality and/or localized incentive mechanisms (i.e., managers being assessed only on their own unit's profit and loss account).[21]

In the simulation the number of units per firm is set at 250. This number was obtained from two pieces of analysis. First, I estimated the number of production units in my Compustat firms. To do this I started with the work of Davis et al. (2006), showing that Compustat firms with 500+ employees (in the Census Bureau data) have on average 185 establishments each *in the United States*.[22] Their sample is similar to mine, which has Compustat firms with 500+ employees and \$10m+ of sales (details in Section 5.4). I then used the results of Bloom, Schankerman, and Van Reenen (2007), who used Bureau Van Dijk data to show that for a sample of 715 Compustat firms, 61.5% of their subsidiaries are located overseas. Again, their sample is similar to mine, having a median of 3839 employees compared to 3450 for my sample. Combining these facts suggests that—if the number of establishments per subsidiary is approximately the same overseas as in the United States—the Compustat firms in my sample should have around 480 establishments: about 185 in the United States and about 295 overseas ($295 = 185 \times 61.5/(100 - 61.5)$).

Second, the simulation results are insensitive to the number of units once firms have 250 or more units. The reason is that with 250 units, the firm is effectively smoothed across independent unit-level shocks, so that more units do not materially change the simulation moments. Since running simulations with large numbers of units is computationally intensive, I used 250 units as a good approximation to the 480 units my firms approximately have on average.

Of course this assumption on 250 units per firm will have a direct effect on the estimated adjustment costs (since aggregation and adjustment costs are both sources of smoothing) and thereby have an indirect effect on the simulation. Hence, in Section 5 I reestimate the adjustment costs, assuming instead the firm has 1 and 25 units to investigate this further.

The model also assumes no entry or exit for analytical tractability. This seems acceptable in the monthly time frame (entry/exit accounts for around 2% of employment on an annual basis), but is an important assumption to explore in future research. My intuition is that relaxing this assumption should increase

[21]The semiindependent operation of plants may be theoretically optimal for incentive reasons (to motivate local managers) and technical reasons (the complexity of centralized information gathering and processing). The empirical evidence on decentralization in U.S. firms suggests that plant managers have substantial hiring and investment discretion (see, for example, Bloom and Van Reenen (2007) and Bloom, Sadun, and Van Reenen (2008)).

[22]I wish to thank Javier Miranda for helping with these figures.

the effect of uncertainty shocks, since entry and exit decisions are extremely nonconvex, although this may have some offsetting effects through the estimation of slightly "smoother" adjustment costs.

There is also the issue of time-series aggregation. Shocks and decisions in a typical business unit are likely to occur at a much higher frequency than annually, so annual data will be temporally aggregated, and I need to explicitly model this. There is little information on the frequency of decision making in firms. The anecdotal evidence suggests monthly frequencies are typical, due to the need for senior managers to schedule regular meetings, which I assume in my main results.[23] Section 5.7 undertakes some robustness tests on this assumption and finds that time aggregation is actually quite important. This highlights the importance of obtaining better data on decision making frequency for future research.

3.5. *Optimal Investment and Employment*

The optimization problem is to maximize the present discounted flow of revenues less the wage bill and adjustment costs. As noted above, each unit within the firm is assumed to optimize independently. Units are also assumed to be risk neutral to focus on the real-options effects of uncertainty.

Analytical methods suggest that a unique solution to the unit's optimization problem exists that is continuous and strictly increasing in (A, K, L) with an almost everywhere unique policy function.[24] The model is too complex, however, to be fully solved using analytical methods, so I use numerical methods, knowing that this solution is convergent with the unique analytical solution.

Given current computing power, however, I have too many state and control variables to solve the problem as stated, but the optimization problem can be substantially simplified in two steps. First, hours are a flexible factor of production and depend only on the variables (A, K, L), which are predetermined in period t given the time to build assumption. Therefore, hours can be optimized out in a prior step, which reduces the *control* space by one dimension. Second, the revenue function, adjustment cost function, depreciation schedules, and demand processes are all jointly homogenous of degree 1 in (A, K, L), allowing the whole problem to be normalized by one state variable, reducing the *state* space by one dimension.[25] I normalize by capital to operate on $\frac{A}{K}$ and $\frac{L}{K}$. These two steps dramatically speed up the numerical simulation, which is run

[23]Note that even if shocks continuously hit the firm, if decision making only happens monthly, then there is no loss of generality from assuming a monthly shock process.

[24]The application of Stokey and Lucas (1989) for the continuous, concave, and almost surely bounded normalized returns and cost function in (3.7) for quadratic adjustment costs and partial irreversibilities, and Caballero and Leahy (1996) for the extension to fixed costs.

[25]The key to this homogeneity result is the random-walk assumption on the demand process. Adjustment costs and depreciation are naturally scaled by unit size, since otherwise units would "outgrow" adjustment costs and depreciation. The demand function is homogeneous through the trivial renormalization $A^{1-a-b} = \widetilde{A}^{1-1/\varepsilon}B^{1/\varepsilon}$.

on a state space of $(\frac{A}{K}, \frac{L}{K}, \sigma, \mu)$, making numerical estimation feasible. Appendix B contains a description of the numerical solution method.

The Bellman equation of the optimization problem before simplification (dropping the unit subscripts) can be stated as

$$\begin{aligned} &V(A_t, K_t, L_t, \sigma_t, \mu_t) \\ &= \max_{I_t, E_t, H_t} \Big\{ S(A_t, K_t, L_t, H_t) - C(A_t, K_t, L_t, H_t, I_t, E_t) - w(H_t)L_t \\ &\quad + \frac{1}{1+r} E\big[V(A_{t+1}, K_t(1-\delta_K) + I_t, L_t(1-\delta_L) \\ &\quad + E_t, \sigma_{t+1}, \mu_{t+1})\big]\Big\}, \end{aligned}$$

where r is the discount rate and $E[\cdot]$ is the expectation operator. Optimizing over hours and exploiting the homogeneity in (A, K, L) to take out a factor of K_t enables this to be rewritten as

$$\begin{aligned} Q(a_t, l_t, \sigma_t, \mu_t) = \max_{i_t, e_t} \Big\{ & S^*(a_t, l_t) - C^*(a_t, l_t, i_t, l_t e_t) \\ & + \frac{1 - \delta_K + i_t}{1+r} E[Q(a_{t+1}, l_{t+1}, \sigma_{t+1}, \mu_{t+1})]\Big\}, \end{aligned} \tag{3.7}$$

where the normalized variables are $l_t = L_t/K_t$, $a_t = A_t/K_t$, $i_t = I_t/K_t$, and $e_t = E_t/L_t$, $S^*(a_t, l_t)$ and $C^*(a_t, l_t, i_t, l_t e_t)$ are sales and costs after optimization over hours, and $Q(a_t, l_t, \sigma_t, \mu_t) = V(a_t, 1, l_t, \sigma_t, \mu_t)$, which is Tobin's Q.

4. THE MODEL'S SOLUTION AND SIMULATING AN UNCERTAINTY SHOCK

In this section I present the main results of the model and the uncertainty simulations. I do this before detailing the parameter values to enable readers to get to the main results more quickly. I list all the parameter values in Tables II and III and discuss how I obtained them in Section 5. Simulation parameter robustness tests can be found in Section 6.

4.1. *The Model's Solution*

The model yields a central region of inaction in $(\frac{A}{K}, \frac{A}{L})$ space, due to the nonconvex costs of adjustment. Units only hire and invest when business conditions are sufficiently good, and only fire and disinvest when they are sufficiently bad. When uncertainty is higher, these thresholds move out: units become more cautious in responding to business conditions.

To provide some graphical intuition, Figure 4 plots in $(\frac{A}{K}, \frac{A}{L})$ space the values of the fire and hire thresholds (left and right lines) and the sell and buy

TABLE II

PREDEFINED PARAMETERS IN THE MODEL

Parameter	Value	Rationale (Also See the Text)
α	1/3	Capital share in output is one-third, labor share is two-thirds.
ε	4	33% markup. With constant returns to scale yields $a+b=0.75$. I also try a 20% markup to yield $a+b=0.833$.
w_1	0.8	Hourly wages minimized at a 40 hour week.
w_2	2.4e–9	Arbitrary scaling parameter. Set so the wage bill equals unity at 40 hours.
σ_H	$2\times\sigma_L$	Uncertainty shocks 2× baseline uncertainty (Figure 1 data). σ_L estimated. I also try 1.5× and 3× baseline shocks.
$\pi^{\sigma}_{L,H}$	1/36	Uncertainty shocks expected every 3 years (17 shocks in 46 years in Figure 1).
$\pi^{\sigma}_{H,H}$	0.71	Average 2-month half-life of an uncertainty shock (Figure 1 data). I also try 1 and 6 month half-lives.
$(\mu_H+\mu_L)/2$	0.02	Average real growth rate equals 2% per year. The spread $\mu_H-\mu_L$ is estimated.
$\pi^{\mu}_{L,H}$	$\pi^{\mu}_{H,L}$	Firm-level demand growth transition matrix assumed symmetric. The parameter $\pi^{\mu}_{H,L}$ estimated.
δ_K	0.1	Capital depreciation rate assumed 10% per year.
δ_L	0.1	Labor attrition assumed 10% for numerical speed (since $\delta_L=\delta_K$). I also try $\delta_L=0.2$.
r	6.5%	Long-run average value for U.S. firm-level discount rate (King and Rebello (1999)).
N	250	Firms operate 250 units, chosen to match data on establishments per firm. I also try $N=25$ and $N=1$.

capital thresholds (top and bottom lines) for low uncertainty (σ_L) and the preferred parameter estimates in Table III column All. The inner region is the region of inaction ($i=0$ and $e=0$), where the real-option value of waiting is worth more than the returns to investment and/or hiring. Outside the region of inaction, investment and hiring will be taking place according to the optimal values of i and e. This diagram is a two-dimensional (two-factor) version of the one-dimensional investment models of Abel and Eberly (1996) and Caballero and Leahy (1996). The gap between the investment/disinvestment thresholds is higher than between the hire/fire thresholds due to the higher adjustment costs of capital.

Figure 5 displays the same lines for both low uncertainty (the inner box of lines) and high uncertainty (the outer box of lines). It can be seen that the comparative static intuition that higher uncertainty increases real options is confirmed here, suggesting that large changes in σ_t can have an important impact on investment and hiring behavior.

To quantify the impact of these real-option values I ran the thought experiment of calculating what *temporary* fall in wages and interest rates would be required to keep units hiring and investment thresholds unchanged when un-

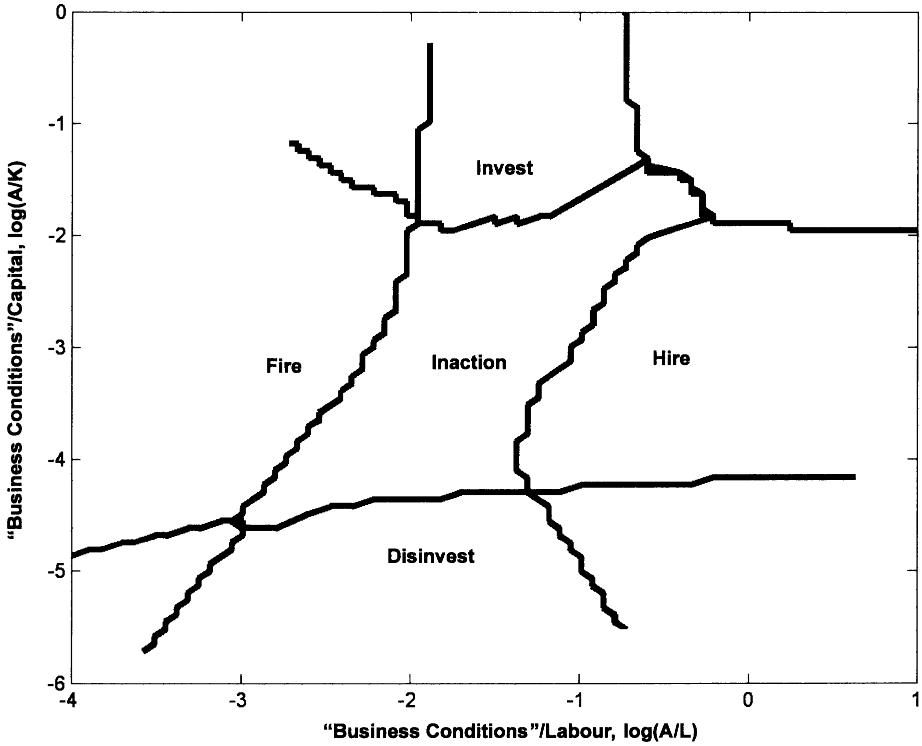

FIGURE 4.—Hiring/firing and investment/disinvestment thresholds. Simulated thresholds using the adjustment cost estimates from the column All in Table III. Although the optimal policies are of the (s, S) type, it cannot be proven that this is always the case.

certainty temporarily rises from σ_L to σ_H. The required wage and interest rate falls turn out to be quantitatively large: units would need a 25% reduction in wages in periods of high uncertainty to leave their marginal hiring decisions unchanged and a 7% (700 basis point) reduction in the interest rates in periods of high uncertainty to leave their marginal investment decisions unchanged.[26] The reason this uncertainty effect is so large is that labor and capital adjustment costs lead units to be cautious about hiring and investing. It is expensive to hire and then rapidly fire a worker or to buy a piece of equipment and then quickly resell it. So when uncertainty is high, units optimally postpone hiring and investment decisions for a few months until business conditions become clearer.

[26]This can be graphically seen in supplemental material Figure S2, which plots the low and high uncertainty thresholds, but with the change that when $\sigma_t = \sigma_H$, interest rates are 7 percentage points lower and wage rates are 25% lower than when $\sigma_t = \sigma_L$.

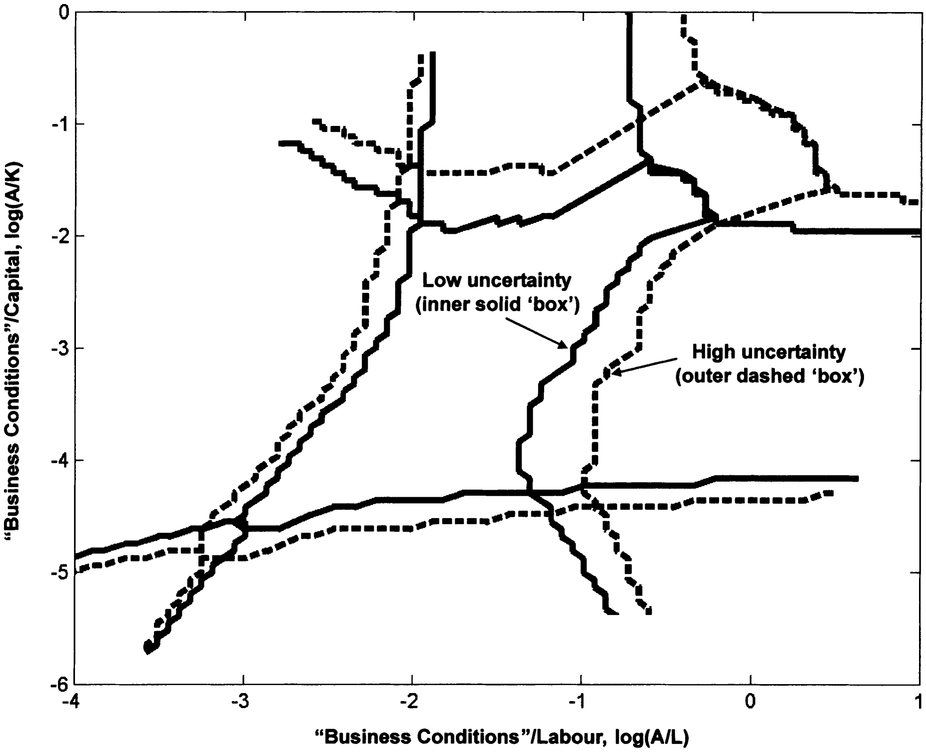

FIGURE 5.—Thresholds at low and high uncertainty. Simulated thresholds using the adjustment cost estimates from the column All in Table III. High uncertainty is twice the value of low uncertainty ($\sigma_H = 2 \times \sigma_L$).

Interestingly, recomputing these thresholds with permanent (time invariant) differences in uncertainty results in an even stronger impact on the investment and employment thresholds. So the standard comparative static result on changes in uncertainty will tend to overpredict the expected impact of time changing uncertainty. The reason is that units evaluate the uncertainty of their discounted value of marginal returns over the lifetime of an investment or hire, so high current uncertainty only matters to the extent that it drives up long-run uncertainty. When uncertainty is mean reverting, high current values have a lower impact on expected long-run values than if uncertainty were constant.

Figure 6 shows a one-dimensional cut of Figure 4 (using the same x-axis), with the level of hiring/firing (solid line, left y-axis) and cross-sectional density of units (dashed line, right y-axis) plotted. These are drawn for one illustrative set of parameters: baseline uncertainty (σ_L), high demand growth (μ_H), and

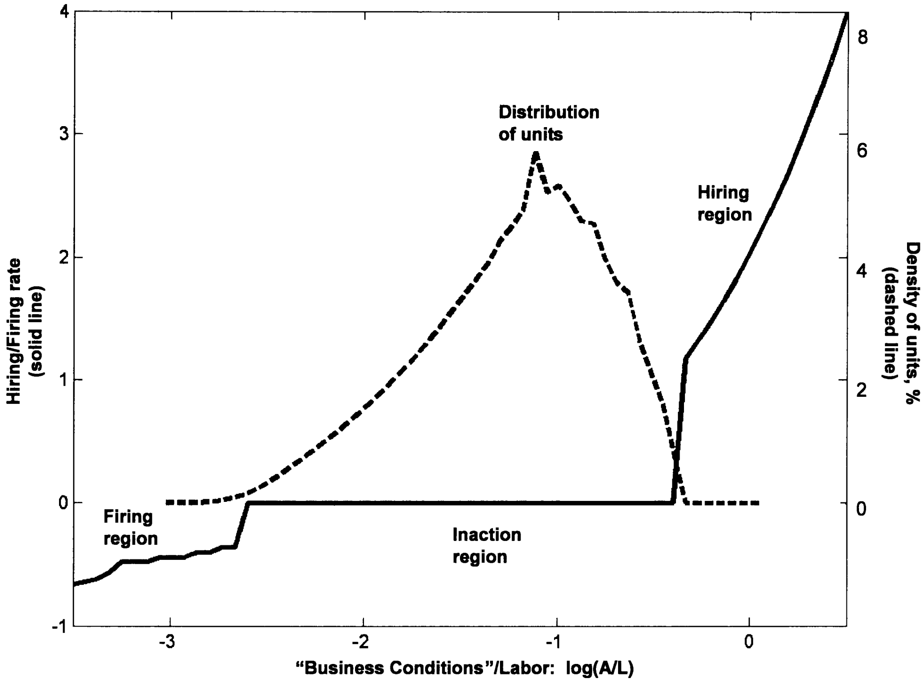

FIGURE 6.—The distribution of units between the hiring and firing thresholds. The hiring response (solid line) and unit-level density (dashed line) for low uncertainty (σ_L), high drift (μ_H), and the most common capital/labor (K/L) ratio. The distribution of units in (A/L) space is skewed to the right because productivity growth generates an upward drift in A and attrition generates a downward drift in L. The density peaks internally because of lumpy hiring due to fixed costs.

the modal value of capital/labor.[27] Three things stand out: first, the distribution is skewed to the right due to positive demand growth and labor attrition; second, the density just below the hiring threshold is low because whenever the unit hits the hiring threshold, it undertakes a burst of activity (due to hiring fixed costs) that moves it to the interior of the space; and third, the density peaks at the interior which reflects the level of hiring that is optimally undertaken at the hiring threshold.

4.2. *The Simulation Outline*

The simulation models the impact of a large, but temporary, rise in the variance of business-conditions (productivity and demand) growth. This second-moment shock generates a rapid drop in hiring, investment, and productivity

[27]Figure 6 is actually a 45° cut across Figure 4. The reason is Figure 6 holds K/L constant while allowing A to vary.

growth as units become much more cautious due to the rise in uncertainty. Once the uncertainty shock passes, however, activity bounces back as units clear their pent-up demand for labor and capital. This also leads to a drop and rebound in productivity growth, since the temporary pause in activity slows down the reallocation of labor and capital from low to high productivity units. In the medium term this burst of volatility generates an overshoot in activity due to the convexity of hiring and investment in business conditions.

Of course this is a stylized simulation, since other factors also typically change around major shocks. Some of these factors can and will be added to the simulation, for example allowing for a simultaneous negative shock to the first moment. I start by focusing on a second-moment shock only, however, to isolate the pure uncertainty effects and demonstrate that these alone are capable of generating large short-run fluctuations. I then discuss the robustness of this analysis to price changes from general equilibrium effects and a combined first- and second-moment shock. In Section 6 I also show robustness to a range of different parameter values, including adjustment costs and the stochastic process for uncertainty.

4.3. *The Baseline Simulation*

I simulate an economy of 1000 units (four firms) for 15 years at a monthly frequency. This simulation is then repeated 25,000 times, with the values for labor, capital, output, and productivity averaged over all these runs. In each simulation the model is hit with an uncertainty shock in month 1 of year 11, defined as $\sigma_t = \sigma_H$ in equation (3.5). All other micro and macro shocks are randomly drawn as per Section 3. This generates the average impact of an uncertainty shock, where the average is taken over the distribution of micro and macro shocks. There are both fixed cost and partial irreversibility adjustment costs for labor and capital, which are estimated from Compustat data as explained in Section 5 (in particular see the All column in Table III).

Before presenting the simulation results, it is worth first showing the precise impulse that will drive the results. Figure 7a reports the average value of σ_t normalized to unity before the shock. It is plotted on a monthly basis, with the month normalized to zero on the date of the shock. Three things are clear from Figure 7a: first, the uncertainty shock generates a sharp spike in the average σ_t across the 25,000 simulations; second, this dies off rapidly with a half-life of 2 months; and third, the shock almost doubles average σ_t (the rise is less than 100% because some of the 25,000 simulations already had $\sigma_t = \sigma_H$ when the shock occurred). In Figure 7b I show the average time path of business conditions ($A_{j,t}$), showing that the uncertainty shock has no first-moment effect.

In Figure 8, I plot aggregate detrended labor, again normalized to 1 at the month before the shock. This displays a substantial fall in the 6 months immediately after the uncertainty shock and then overshoots from month 8 onward, eventually returning to level by around 3 years.

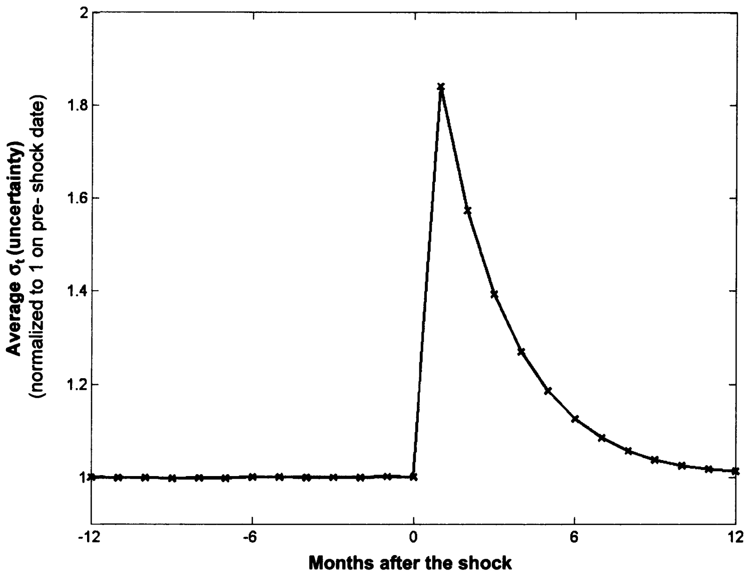

FIGURE 7A.—The simulation has a large second-moment shock.

The initial drop occurs because the rise in uncertainty increases the real-option value of inaction, leading the majority of units to temporarily freeze

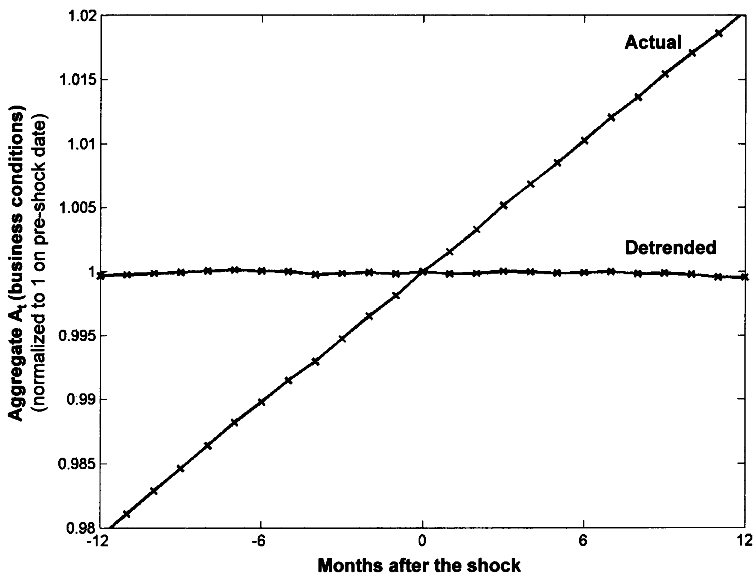

FIGURE 7B.—The simulation has no first moment shock.

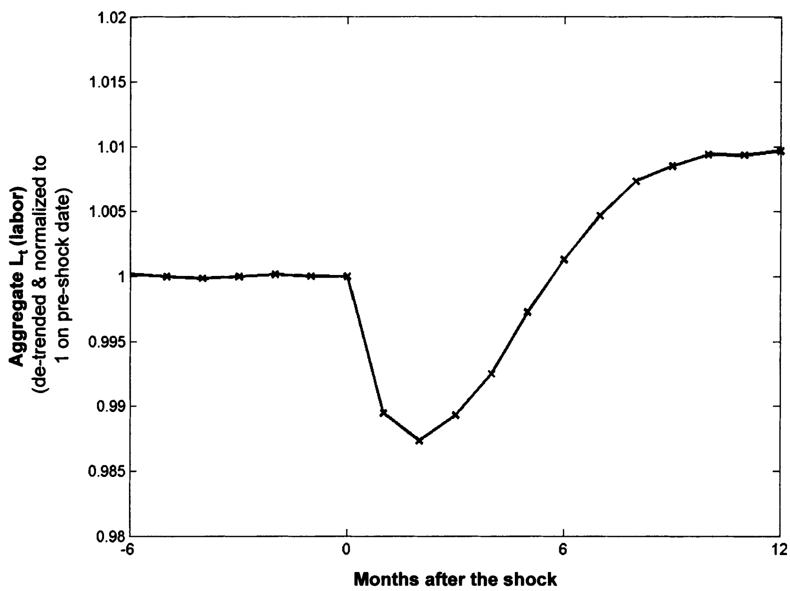

FIGURE 8.—Aggregate (detrended) labor drops, rebounds, and overshoots. The aggregate figures for L_t are calculated by summing across all units within the simulation.

hiring. Because of the ongoing exogenous attrition of workers, this generates a fall in net employment. Endogenizing quits would of course reduce the impact of these shocks, since the quit rate would presumably fall after a shock. In the model, to offset this I have conservatively assumed a 10% annual quit rate—well below the 15% to 25% quit rate observed over the business cycle in recent Job Openings and Labor Turnover Survey (JOLTS) data (see Davis, Faberman, and Haltiwanger (2006)). This low fixed quit rate could be thought of as the exogenous component due to retirement, maternity, sickness, family relocation, and so forth.

The rebound from month 4 onward occurs because of the combination of falling uncertainty (since the shock is only temporary) and rising pent-up demand for hiring (because units paused hiring over the previous 3 months). To make up the shortfall in labor, units begin to hire at a faster pace than usual so the labor force heads back toward its trend level. This generates the rapid rebound in the total labor from month 3 until about month 6.

4.4. *The Volatility Overshoot*

One seemingly puzzling phenomenon, however, is the overshoot from month 7 onward. Pure real-options effects of uncertainty should generate a drop and overshoot in the *growth rate* of labor (that is the hiring rate), but only a drop and convergence back to trend in the *level* of the labor force. So the question is what is causing this medium-term overshoot in the level of the labor force?

This medium-term overshoot arises because the increased volatility of business conditions leads more units to hit both the hiring and firing thresholds. Since more units are clustered around the hiring threshold than the firing threshold, due to labor attrition and business-conditions growth (see Figure 6), this leads to a medium-term burst of net hiring. In effect hiring is convex in productivity just below the hiring threshold—units that receive a small positive shock hire and units that receive a small negative shock do not respond. So total hiring rises in the medium term with the increased volatility of productivity growth. Of course once units have undertaken a burst of hiring, they jump to the interior of the region of inaction and so do not hire again for some time. So in the long-run this results in labor falling back to its long-run trend path. I label this phenomenon the *volatility overshoot*, since this medium-term hiring boom is induced by the higher unit-level volatility of business-conditions.[28]

Thus, the effect of a rise in σ_t is twofold. First, the real-options effect from increased uncertainty over future business conditions causes an initial drop in activity as units pause investment and hiring. This happens rapidly since expectations change upon impact of the uncertainty shock, so that hiring and investment instantly freeze. Second, the effect from increased volatility of realized business conditions causes a medium-term hiring boom. This takes more time to occur because it is driven by the rise in the realized volatility of productivity growth. This rise in volatility accrues over several months. Thus, the uncertainty drop always precedes the volatility overshoot.

These distinct uncertainty and volatility effects are shown in Figure 9. This splits out the expectational effects of higher σ_t from the realized volatility effects of higher σ_t. These simulations are shown for 36 months after the shock to highlight the long-run differences between these effects.[29] The uncertainty effect is simulated by allowing unit expectations over σ_t to change after the shock (as in the baseline) but holds the variance of the actual draw of shocks constant.

[28]This initially appears similar to the type of "echo effect" that appears in demand for products like cars in response to demand shocks, but these echo effects are actually quite distinct from a volatility overshoot. In the echo effect case, what arises is a lumpy response to a first-moment shock. The fixed costs of adjustment lead to a burst of investment, with subsequent future bursts (echo effects). This can arise in models with one representative agent and perfect certainty, but requires lump-sum adjustment costs. The volatility overshoot in this paper arises from time variation is the cross-sectional distribution, leading to an initial overshoot and then a gradual return to trend. This can arise in a model with no lump-sum adjustment costs (for example, partial irreversibility is sufficient), but it does require a cross section of agents and time variation in the second moment.

[29]In general, I plot response for the first 12 months due to the partial equilibrium nature of the analysis, unless longer-run plots are expositionally helpful.

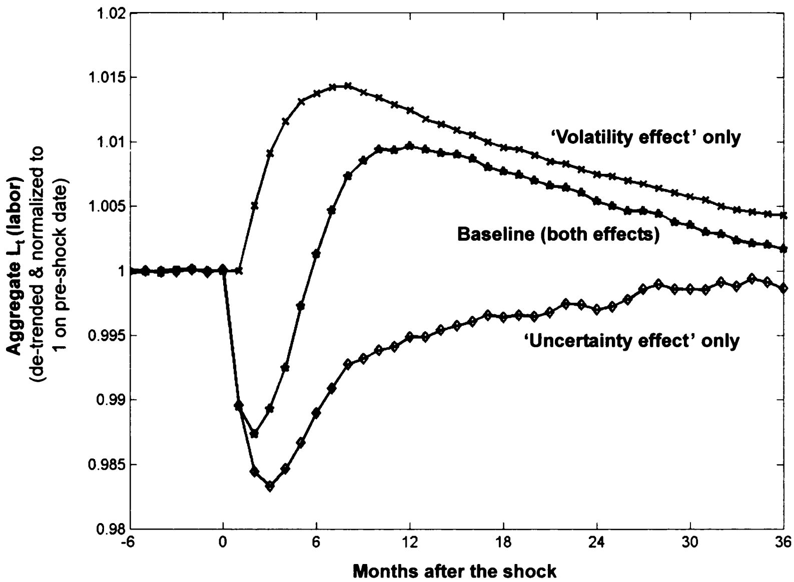

FIGURE 9.—Separating out the uncertainty and volatility effects. The baseline plot is the same as in Figure 8 but extended out for 36 months. For the volatility effect only plot, firms have expectations set to $\sigma_t = \sigma_L$ in all periods (i.e., uncertainty effects are turned off), while in the uncertainty effect only, they have the actual shocks drawn from a distribution $\sigma_t = \sigma_L$ in all periods (i.e., the volatility effects are turned off).

This generates a drop and rebound back to levels, but no volatility overshoot. The volatility effect is simulated by holding unit expectations over σ_t constant but allowing the realized volatility of the business conditions to change after the shock (as in the baseline). This generates a volatility overshoot, but no initial drop in activity from a pause in hiring.[30] The baseline figure in the graph is simply the aggregate detrended labor (as in Figure 8). This suggests that uncertainty and volatility have very different effects on economic activity, despite often being driven by the same underlying phenomena.

The response of aggregate capital to the uncertainty shock is similar to labor. There is a short-run drop in capital as units postpone investing, followed by a rebound as they address their pent-up demand for investment, and a subsequent volatility driven overshoot (see supplementary material Figure S3).

[30]In the figure, the volatility effects also take 1 extra month to begin. This is simply because of the standard finance timing assumption in (3.2) that σ_{t-1} drives the volatility of $A_{j,t}$. Allowing volatility to be driven by σ_t delivers similar results because in the short run the uncertainty effect of moving out the hiring and investment thresholds dominates.

4.5. *Why Uncertainty Reduces Productivity Growth*

Figure 10a plots the time series for the growth of aggregate productivity, defined as $\sum_j A_{j,t}L_{j,t}$, where the sum is taken over all j production units in the economy in month t. In this calculation the growth of business conditions ($A_{j,t}$) can be used as a proxy for the growth of productivity under the assumption that shocks to demand are small in comparison to productivity (or that the shocks are independent). Following Baily, Hulten, and Campbell (1992), I define three indices as follows[31]:

$$\underbrace{\frac{\sum A_{j,t}L_{j,t} - \sum A_{j,t-1}L_{j,t-1}}{\sum A_{j,t-1}L_{j,t-1}}}_{\text{aggregate productivity growth}}$$

$$= \underbrace{\frac{\sum (A_{j,t} - A_{j,t-1})L_{j,t-1}}{\sum A_{j,t-1}L_{j,t-1}}}_{\text{within productivity growth}} + \underbrace{\frac{\sum A_{j,t}(L_{j,t} - L_{j,t-1})}{\sum A_{j,t-1}L_{j,t-1}}}_{\text{reallocation productivity growth}}.$$

The first term, aggregate productivity growth, is the increase in productivity weighted by employment across units. This can be broken down into two subterms: within productivity growth, which measures the productivity increase within each production unit (holding the employment of each unit constant), and reallocation productivity growth, which measures the reallocation of employment from low to high productivity units (holding the productivity of each unit constant).

In Figure 10a aggregate productivity growth shows a large fall after the uncertainty shock. The reason is that uncertainty reduces the shrinkage of low productivity units and the expansion of high productivity units, reducing the reallocation of resources toward more productive units.[32] This reallocation from low to high productivity units drives the majority of productivity growth in the model so that higher uncertainty has a first-order effect on productivity growth. This is clear from the decomposition which shows that the fall in total is entirely driven by the fall in the reallocation term. The within term is constant since, by assumption, the first moment of the demand conditions shocks is unchanged.[33]

[31]Strictly speaking, Bailey, Hulten, and Campbell (1992) defined four terms, but for simplicity I have combined the between and cross terms into a reallocation term.

[32]Formally there is no reallocation in the model because it is partial equilibrium. However, with the large distribution of contracting and expanding units all experiencing independent shocks, gross changes in unit factor demand are far larger than net changes, with the difference equivalent to reallocation.

[33]These plots are not completely smooth because the terms are summations of functions which are approximately squared in $A_{j,t}$. For example $A_{j,t}L_{j,t} \approx \lambda A_{j,t}^2$ for some scalar λ since $L_{i,t}$ is ap-

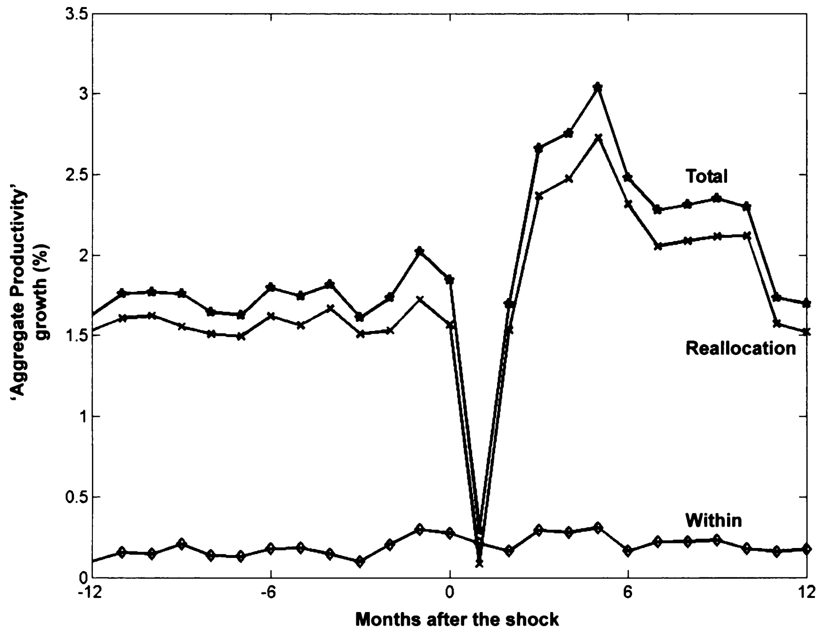

FIGURE 10A.—Aggregate productivity growth falls and rebounds after the shock.

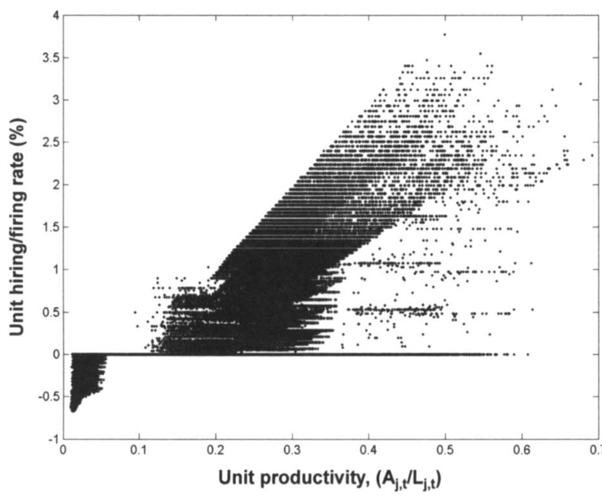

FIGURE 10B.—Unit level productivity and hiring for the period before the uncertainty shock.

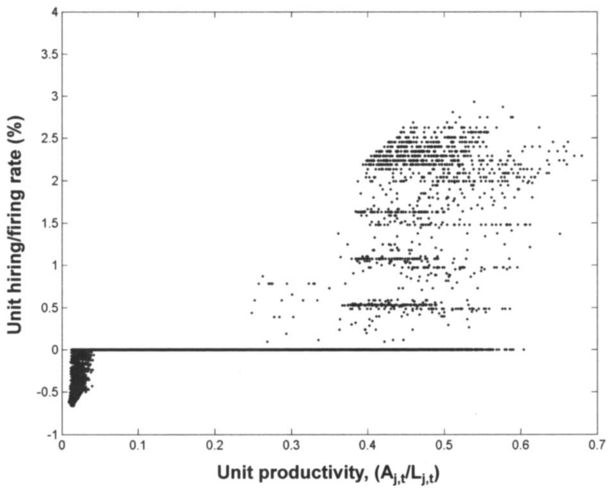

FIGURE 10C.—Unit level productivity and hiring in the period after the uncertainty shock.

In the bottom two panels this reallocative effect is illustrated by two unit-level scatter plots of gross hiring against log productivity in the month before the shock (Figure 10b) and the month after the shock (Figure 10c). It can be seen that after the shock much less reallocative activity takes place with a substantially lower fraction of expanding productive units and shrinking unproductive units. Since actual U.S. aggregate productivity growth appears to be 70% to 80% driven by reallocation,[34] these uncertainty effects should play an important role in the real impact of large uncertainty shocks.

Figure 11 plots the *level* of an alternative productivity measure—Solow productivity. This is defined as aggregate output divided by factor share weighted aggregate inputs:

$$\text{Solow aggregate productivity} = \frac{\sum_j A_{j,t}^{1/(\varepsilon-1)} K_{j,t}^{\alpha} (L_{j,t} \times H_{j,t})^{1-\alpha}}{\alpha \sum_j K_{j,t} + (1-\alpha) \sum_j L_{j,t} \times H_{j,t}}.$$

proximately linear in $A_{j,t}$. Combined with the random-walk nature of the driving process (which means some individual units grow very large), this results in lumpy aggregate productivity growth even in very large samples of units.

[34]Foster, Haltiwanger, and Krizan (2000, 2006) reported that reallocation, broadly defined to include entry and exit, accounts for around 50% of manufacturing and 90% of retail productivity growth. These figures will in fact underestimate the full contribution of reallocation since they miss the within establishment reallocation, which Bernard, Redding, and Schott's (2006) results on product switching suggests could be substantial.

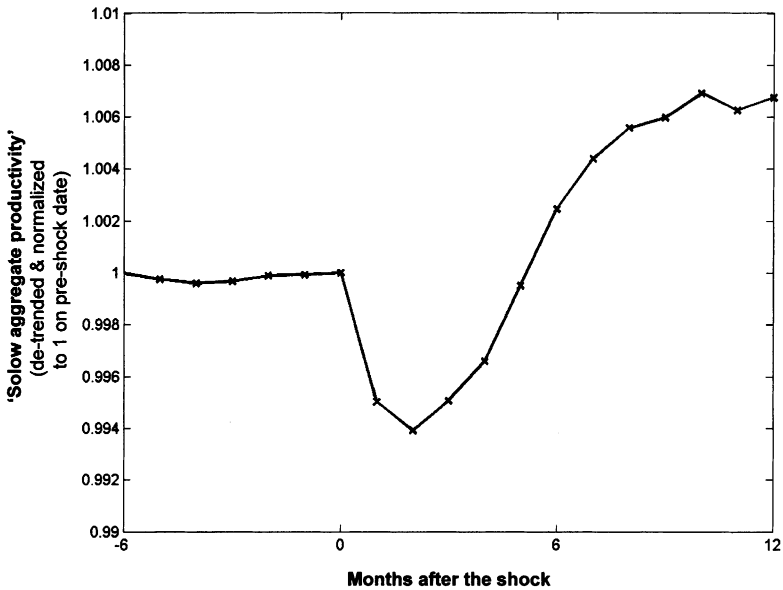

FIGURE 11.—Solow aggregate productivity (detrended) drops, rebounds, and overshoots. Solow productivity is defined as aggregate output divided by the factor share weighted aggregate inputs.

I report this series because macro productivity measures are typically calculated in this way using only macro data (note that the previous aggregate productivity measure would require micro data to calculate). As can be seen in Figure 11, the detrended Solow productivity series also falls and rebounds after the uncertainty shock. Again, this initial drop and rebound is because of the initial pause and subsequent catch-up in the level of reallocation across units immediately after the uncertainty shock. The medium-run overshoot is again due to the increased level of cross-sectional volatility, which increases the potential for reallocation, leading to higher aggregate medium-term productivity growth.

Finally, Figure 12 plots the effects of an uncertainty shock on output. This shows a clear drop, rebound, and overshoot, very similar to the behavior of the labor, capital, and productivity. What is striking about Figure 12 is the similarity of the size, duration, and time profile of the simulated response to an uncertainty shock compared to the VAR results on actual data shown in Figure 2. In particular, both the simulated and actual data show a drop of detrended activity of around 1% to 2% after about 3 months, a return to trend at around 6 months, and a longer-run gradual overshoot.

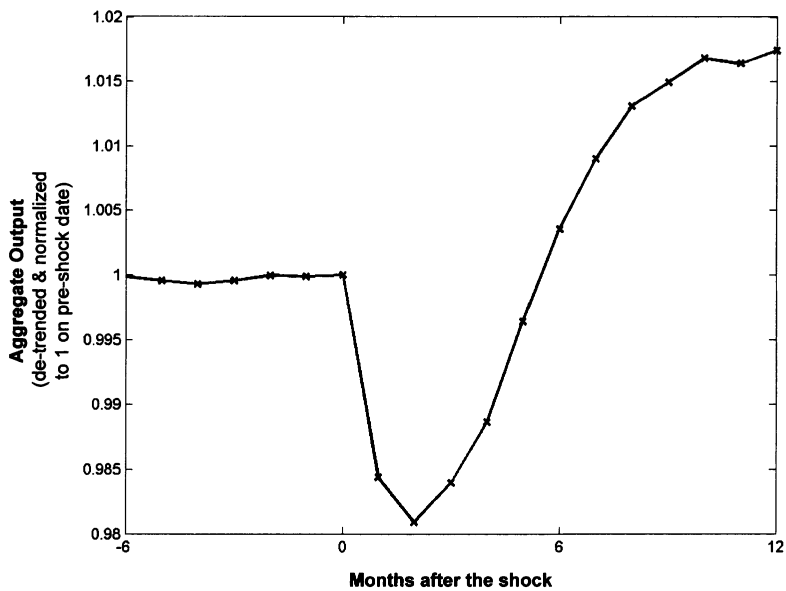

FIGURE 12.—Aggregate (detrended) output drops, rebounds, and overshoots.

4.6. *Investigating Robustness to General Equilibrium*

Ideally I would set up my model within a general equilibrium (GE) framework, allowing prices to endogenously change. This could be done, for example, by assuming agents approximate the cross-sectional distribution of units within the economy using a finite set of moments and then using these moments in a representative consumer framework to compute a recursive competitive equilibrium (see, for example, Krusell and Smith (1998), Khan and Thomas (2003), and Bachman, Caballero, and Engel (2008)). However, this would involve another loop in the routine to match the labor, capital, and output markets between units and the consumer, making the program too slow to then loop in the simulated method of moments estimation routine. Hence, there is a trade-off between two options: (i) a GE model with flexible prices but assumed adjustment costs[35] and (ii) estimated adjustment costs but in a fixed price model. Since the effects of uncertainty are sensitive to the nature of adjustment costs, I chose to take the second option and leave GE analysis to future work.

[35]Unfortunately there are no "off the shelf" adjustment cost estimates that can be used, since no paper has previously jointly estimated convex and nonconvex labor and capital adjustment costs.

This means the results in this model could be compromised by GE effects if factor prices changed sufficiently to counteract factor demand changes.[36] One way to investigate this is to estimate the actual changes in wages, prices, and interest rates that arise after a stock-market volatility shock and feed them into the model in an expectations consistent way. If these empirically plausible changes in factor prices radically changed these results, this would suggest they are not robust to GE, while if they have only a small impact, it is more reassuring on GE robustness.

To do this I use the estimated changes in factor prices from the VAR (see Section 2.2), which are plotted in Figure 13. An uncertainty shock leads to a short-run drop and rebound of interest rates of up to 1.1% points (110 basis point), of prices of up to 0.5%, and of wages of up to 0.3%. I take these numbers and structurally build them into the model so that when $\sigma_t = \sigma_H$, interest

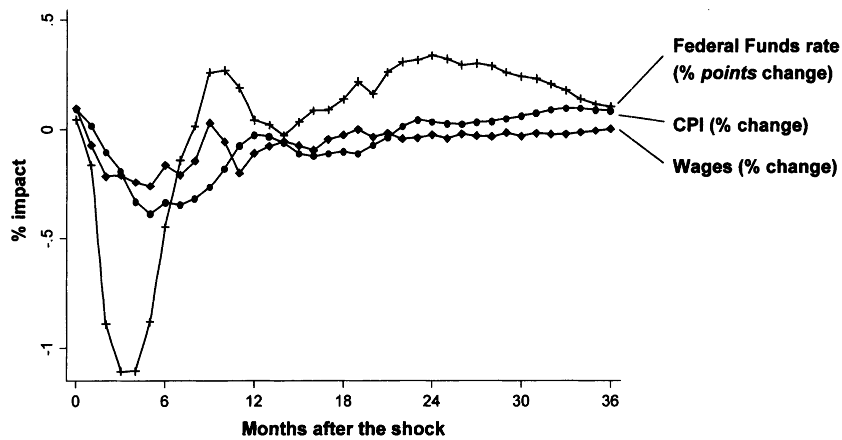

FIGURE 13.—VAR estimation of the impact of a volatility shock on prices. *Notes*: VAR Cholesky orthogonalized impulse response functions to a volatility shock. Estimated monthly from June 1962 to June 2008. Impact on the Federal Funds rate is plotted as a percentage point change so the shock reduces rates by up to 110 basis points. The impact on the CPI and wages is plotted as percentage change.

[36]Khan and Thomas (2008) found in their micro to macro investment model that with GE, the response of the economy to productivity shocks is not influenced by the presence of nonconvex adjustment costs. With a slight abuse of notation this can be characterized as $(\partial(\partial K_t/\partial A_t))/\partial NC \approx 0$, where K_t is aggregate capital, A_t is aggregate productivity, and NC are nonconvex adjustment costs. The focus of my paper on the direct impact of uncertainty on aggregate variables is different and can be characterized instead as $\partial K_t/\partial\sigma_t$. Thus, their results are not necessarily inconsistent with mine. More recent work by Bachman, Caballero, and Engel (2008), found their results depend on the choice of parameter values. Sim (2006) built a GE model with capital adjustment costs and time-varying uncertainty and found that the impact of temporary increases in uncertainty on investment is robust to GE effects.

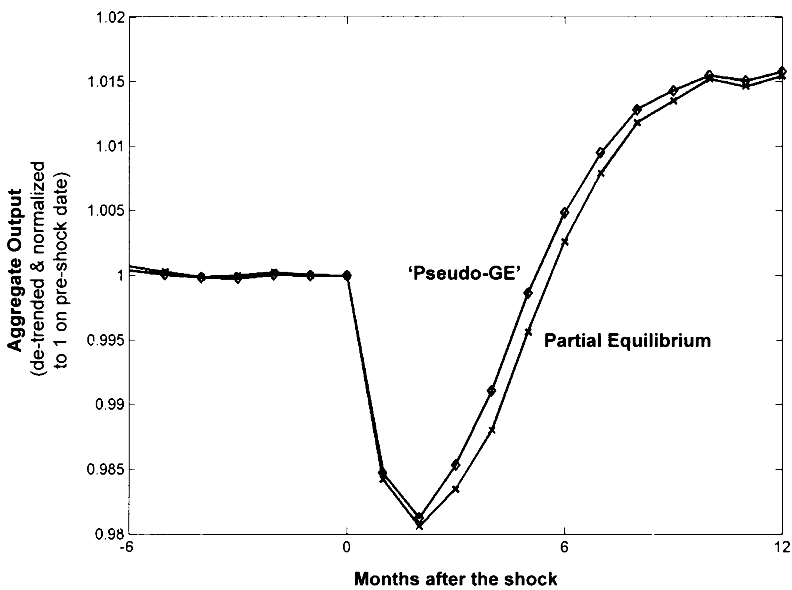

FIGURE 14.—Aggregate (detrended) output: partial equilibrium and pseudo GE. Pseudo-GE allows interest rates, wages, and prices to be 1.1% points, 0.5%, and 0.3%, respectively, lower during periods of high uncertainty.

rates are 1.1% lower, prices (of output and capital) are 0.5% lower, and wages are 0.3% lower. Units expect this to occur, so expectations are rational.

In Figure 14, I plot the level of output after an uncertainty shock with and without these "pseudo-GE" prices changes. This reveals two surprising outcomes: first, the effects of these empirically reasonable changes in interest rates, prices, and wages have very little impact on output in the *immediate* aftermath of an uncertainty shock; and second, the limited pseudo-GE effects that do occur are greatest at around 3–5 months, when the level of uncertainty (and so the level of the interest rate, price, and wage reductions) is much smaller. To highlight the surprising nature of these two findings, Figure S4 (supplemental material) plots the impact of the pseudo-GE price effects on capital, labor, and output in a simulation *without* adjustment costs. In the absence of any adjustment costs, these interest rate, prices, and wages changes do have an extremely large effect. So the introduction of adjustment costs both dampens and delays the response of the economy to the pseudo-GE price changes.

The reason for this limited impact of pseudo-GE price changes is that after an uncertainty shock occurs, the hiring/firing and investment/disinvestment thresholds jump out, as shown in Figure 5. As a result there are no units near any of the response thresholds. This makes the economy insensitive to changes in interest rates, prices, or wages. The only way to get an impact would be to shift the thresholds back to the original low uncertainty position where the majority of units are located, but as noted in Section 4.1 the quantitative impact of these uncertainty shocks is equivalent to something like a 7% (700 basis point) higher interest rate and a 25% higher wage rate, so these pseudo-GE price reductions of 1.1% in interest rates, 0.5% in prices, and 0.3% in wages are not sufficient to do this.

Of course once the level of uncertainty starts to fall back again, the hiring/firing and investment/disinvestment thresholds begin to move back toward their low uncertainty values. This means they start to move back toward the region in (A/K) and (A/L) space where the units are located, so the economy becomes more sensitive to changes in interest rates, prices, and wages. Thus, these pseudo-GE price effects start to play a role, but only with a lag.

In summary, the rise in uncertainty not only reduces levels of labor, capital, productivity, and output, but it also makes the economy temporarily extremely insensitive to changes in factor prices. This is the macro equivalent to the "cautionary effects" of uncertainty demonstrated on firm-level panel data by Bloom, Bond, and Van Reenen (2007).

For policymakers this is important since it suggests a monetary or fiscal response to an uncertainty shock is likely to have almost no impact in the immediate aftermath of a shock. But as uncertainty falls back down and the economy rebounds, it will become more responsive, so any response to policy will occur with a lag. Hence, a policymaker trying, for example, to cut interest rates to counteract the fall in output after an uncertainty shock would find no immediate response, but rather a delayed response when the economy was already starting to recover. This cautions against using first-moment policy levers to respond to the second-moment component of shocks; policies aimed directly at reducing the underlying increase in uncertainty are likely to be far more effective.

4.7. *A Combined First- and Second-Moment Shock*

All the large macro shocks highlighted in Figure 1 comprise both a first- and a second-moment element, suggesting a more realistic simulation would analyze these together. This is undertaken in Figure 15, where the output response to a pure second-moment shock (from Figure 12) is plotted alongside the output response to the same second-moment shock with an additional first-moment shock of -2% to business conditions.[37] Adding an additional first-moment shock leaves the main character of the second-moment shock unchanged—a large drop and rebound.

Interestingly, a first-moment shock on its own shows the type of slow response dynamics that the real data display (see, for example, the response to a

[37] I choose -2% because this is equivalent to 1 year of business-conditions growth in the model. Larger or smaller shocks yield a proportionally larger or smaller impact.

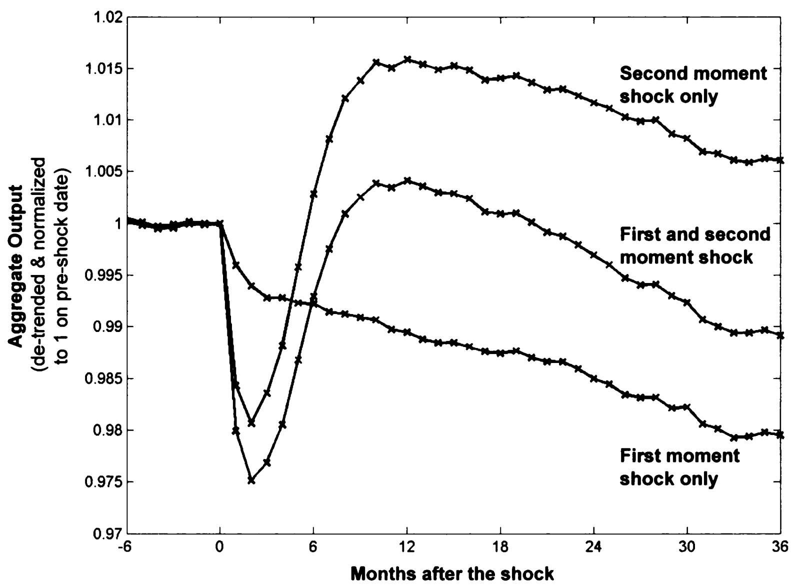

FIGURE 15.—Combined first- and second-moment shocks. The second-moment shock only has σ_t set to σ_H. The first- and second-moment shock has σ_t set to σ_H and also a -2% macro business-conditions shock. The first-moment shock only just has a -2% macro business-conditions shock.

monetary shock in Figure 3). This is because the cross-sectional distribution of units generates a dynamic response to shocks.[38]

This rapid drop and rebound in response to a second-moment shock is clearly very different from the persistent drop over several quarters in response to a more traditional first-moment shock. Thus, to the extent a large shock is more a second-moment phenomenon—for example, 9/11—the response is likely to involve a rapid drop and rebound, while to the extent it is more a first-moment phenomenon—for example, OPEC II—it is likely to generate a persistent slowdown. However, in the immediate aftermath of these shocks, distinguishing them will be difficult, as both the first- and second-moment components will generate an immediate drop in employment, investment, and productivity. The analysis in Section 2.1 suggests, however, there are empirical proxies for uncertainty that are available in real time to aid policymakers, such as the VXO series for implied volatility (see notes to Figure 1), the cross-sectional spread of stock-market returns, and the cross-sectional spread of professional forecasters.

Of course these first- and second-moment shocks differ both in terms of the moments they impact and in terms of their duration: permanent and tempo-

[38]See the earlier work on this by, for example, Caballero and Engel (1993) and Bertola and Caballero (1994).

rary, respectively. The reason is that the second-moment component of shocks is almost always temporary while the first-moment component tends to be persistent. For completeness a persistent second-moment shock would generate a similar effect on investment and employment as a persistent first-moment shock, but would generate a slowdown in productivity *growth* through the reallocation term rather than a one-time reduction in productivity *levels* through the within term. Thus, the temporary/permanent distinction is important for the predicted time profile of the impact of the shocks on hiring and investment, and the first-/second-moment distinction is important for the route through which these shocks impact productivity.

The only historical example of a persistent second-moment shock was the Great Depression, when uncertainty—as measured by share-returns volatility—rose to an incredible 130% of 9/11 levels on average for the 4 years of 1929 to 1932. While this type of event is unsuitable for analysis using my model given the lack of general equilibrium effects and the range of other factors at work, the broad predictions do seem to match up with the evidence. Romer (1990) argued that uncertainty played an important real-options role in reducing output during the *onset* of the Great Depression, while Ohanian (2001) and Bresnahan and Raff (1991) reported "inexplicably" low levels of productivity growth with an "odd" lack of output *reallocation* over this period.

5. ESTIMATING THE MODEL PARAMETERS

This section explains how the individual parameter values used to solve the model and to simulate uncertainty shocks in the previous section were obtained. Readers who are focused on the simulation may want to skip to Section 6.

The full set of parameters is the vector θ that characterizes the firm's revenue function, stochastic processes, adjustment costs, and discount rate. The econometric problem consists of estimating this parameter vector θ. Since the model has no analytical closed form solution, this vector cannot be estimated using standard regression techniques. Instead estimation of the parameters is achieved by simulated method of moments (SMM), which minimizes a distance criterion between key moments from actual data (a panel of publicly traded firms from Compustat) and simulated data. Because SMM is computationally intensive, only 10 parameters can be estimated; the remaining 13 are predefined.

5.1. *Simulated Method of Moments*

SMM proceeds as follows: a set of actual data moments Ψ^A is selected for the model to match.[39] For an arbitrary value of θ the dynamic program is solved

[39]See McFadden (1989) and Pakes and Pollard (1989) for the statistical properties of the SMM estimator.

and the policy functions are generated. These policy functions are used to create a simulated data panel of size $(\kappa N, T+10)$, where κ is a strictly positive integer, N is the number of firms in the actual data, and T is the time dimension of the actual data. The first 10 years are discarded so as to start from the ergodic distribution. The simulated moments $\Psi^S(\theta)$ are then calculated on the remaining simulated data panel, along with an associated criterion function $\Gamma(\theta)$, where $\Gamma(\theta) = [\Psi^A - \Psi^S(\theta)]' W [\Psi^A - \Psi^S(\theta)]$, which is a weighted distance between the simulated moments $\Psi^S(\theta)$ and the actual moments Ψ^A.

The parameter estimate $\widehat{\theta}$ is then derived by searching over the parameter space to find the parameter vector which minimizes the criterion function:

$$\widehat{\theta} = \arg\min_{\theta \in \Theta} [\Psi^A - \Psi^S(\theta)]' W [\Psi^A - \Psi^S(\theta)]. \tag{5.1}$$

Given the potential for discontinuities in the model and the discretization of the state space, I use an annealing algorithm for the parameter search (see Appendix B). Different initial values of θ are selected to ensure the solution converges to the global minimum. I also run robustness tests in Section 5.7 with different initial distributions.

The efficient choice for W is the inverse of the variance–covariance matrix of $[\Psi^A - \Psi^S(\theta)]$. Defining Ω to be the variance–covariance matrix of the data moments, Lee and Ingram (1991) showed that under the estimating null, the variance–covariance of the simulated moments is equal to $\frac{1}{\kappa}\Omega$. Since Ψ^A and $\Psi^S(\theta)$ are independent by construction, $W = [(1+\frac{1}{\kappa})\Omega]^{-1}$, where the first term in the inner brackets represents the randomness in the actual data and the second term represents the randomness in the simulated data. Ω is calculated by block bootstrap with replacement on the actual data. The asymptotic variance of the efficient estimator $\widehat{\theta}$ is proportional to $(1+\frac{1}{\kappa})$. I use $\kappa = 25$, with each of these 25 firm panels having independent draws of macro shocks. This implies the standard error of $\widehat{\theta}$ is increased by 4% by using simulation estimation.

5.2. *Predefined Parameters*

In principle every parameter could be estimated, but in practice the size of the estimated parameter space is limited by computational constraints. I therefore focus on the parameters about which there are probably the weakest priors—the six adjustment cost parameters, the wage/hours trade-off slope, the baseline level of uncertainty, and the two key parameters that determine the firm-level demand drift, $\Theta = (PR_L, FC_L, QC_L, PR_K, FC_K, QC_K, \gamma, \sigma_L, \pi^{\mu}_{H,H}, \mu_L)$. The other 13 parameters are based on values in the data and the literature, and are displayed in Table II.

The predefined parameters outlined in Table II are mostly self-explanatory, although a few require further discussion. One of these is ε, which is the elasticity of demand. In a constant returns to scale (CRS) production function setup this translates directly into the returns to scale parameter on the revenue function, $a + b$. There is a wide range of estimates of the revenue returns to scale, recent examples being 0.905 in Khan and Thomas (2003), 0.82 in Bachman, Caballero, and Engel (2008), and 0.592 in Cooper and Haltiwanger (2006). I chose a parameter value of $\varepsilon = 4$, which (under CRS) yields $a + b = 0.75$, which is (i) roughly in the midpoint of this literature and (ii) optimal for the speed of the numerical simulation since $a = 0.25$ and $b = 0.5$ so that capital and labor have integer fraction exponentials which compute much faster.[40] This implies a markup of 33%, which is toward the upper end of the range estimates for price–cost markups. I also check the robustness of my results to a parameter value of $a + b = 0.83$ (given by $\varepsilon = 6$ with CRS), which is consistent with a 20% markup.

The uncertainty process parameters are primarily taken from the macro volatility process in Figure 1, with the baseline level of uncertainty estimated in the simulation. The labor attrition rate is chosen at 10% per annum. This low figure is selected for two reasons: (i) to be conservative in the simulations of an uncertainty shock, since attrition drives the fall in employment levels, and (ii) for numerical speed, as this matches the capital depreciation rate, so that the (L/K) dimension can be ignored if no investment and hiring/firing occur. I also report a robustness test for using an annualized labor attrition rate of 20%, which more closely matches the figures for annualized manufacturing quits in Davis, Faberman, and Haltiwanger (2006).

5.3. *Estimation*

Under the null, a full-rank set of moments (Ψ^A) will *consistently* estimate the parameter of the adjustment costs (Θ).[41] The choice of moments is also important for the *efficiency* of the estimator. This suggests that moments which are "informative" about the underlying structural parameters should be included. The basic insights of plant- and firm-level data on labor and capital is the presence of highly skewed cross-sectional growth rates and rich time-series dynamics, suggesting some combination of cross-sectional and time-series moments. Two additional issues help to guide the exact choice of moments.

5.3.1. *Distinguishing the Driving Process From Adjustment Costs*

A key challenge in estimating adjustment costs for factor inputs is distinguishing between the dynamics of the driving process and factor adjust-

[40] Integer fractional exponentials are more easily approximated in binary calculations (see Judd (1998, Chapter 2) for details). This is quantitatively important due to the intensity of exponential calculations in the simulation, for example, moving from $a + b = 0.75$ to $a + b = 0.76$ slows down the simulation by around 15%. Choosing a lower value of $a + b$ also induces more curvature in the value function so that less grid points are required to map the relevant space.

[41] Note that even with a full-rank set of moments the parameters are only identified pointwise.

ment costs. Concentrating on the moments from only one factor—for example, capital—makes it very hard to do this. To illustrate this, first consider a very smooth driving process without adjustment costs, which would produce a smooth investment series. Alternatively consider a volatile driving process with convex capital adjustment costs, which would also produce a smooth investment series. Hence, without some additional moments (or assumptions), it would be very hard to estimate adjustment costs using just the investment series data.

So I focus on the joint (cross-sectional and dynamic) moments of the investment, employment, and sales growth series. The difference in responses across the three series (investment, employment, and sales growth) identifies the two sets of adjustment costs (for capital and labor).[42]

5.3.2. *Distinguishing Persistent Differences From Adjustment Costs*

A stylized fact from the estimation of firm- and plant-level investment and labor demand equations is the empirical importance of "fixed effects," that is, persistent differences across firms and plants in their levels of investment, employment, and output growth rates. Without controls for these persistent differences, the estimates of the adjustment costs could be biased. For example, persistent between-firm differences in investment, employment, and sales growth rates due to different growth rates of demand would (in the absence of controls for this) lead to the estimation of large quadratic adjustment costs, which are necessary to induce the required firm-level autocorrelation.

To control for differential firm-level growth rates, the estimator includes two parameters: the spread of firm-level business-conditions growth, $\mu_H - \mu_L$, which determines the degree of firm-level heterogeneity in the average growth rates of business conditions as defined in (3.3); and the persistence of firm-level business-conditions growth, $\pi^{\mu}_{H,H}$, as defined in (3.6). When $\mu_H - \mu_L$ is large there will be large differences in the growth rates of labor, capital, and output across firms, and when $\pi^{\mu}_{H,H}$ is close to unity these will be highly persistent.[43] To identify these parameters separately from adjustment costs requires information on the time path of autocorrelation across the investment, employment, and sales growth series. For example, persistent correlations between investment, sales, and employment growth rates going back over many years would help to identify fixed differences in the growth rates of the driving process, while decaying correlations in the investment series *only* would suggest convex capital adjustment costs.

[42]An alternative is a two-step estimation process in which the driving process is estimated first and then the adjustment costs are estimated given this driving process (see, for example, Cooper and Haltiwanger (2006)).

[43]Note that with $\pi^{\mu}_{H,H} = 1$ these will be truly fixed effect differences.

So I include moments for the second-order and fourth-order correlations of the investment, employment growth, and sales growth series.[44] The second-order autocorrelation is chosen to avoid a negative bias in these moments from underlying level measurement errors which would arise in a first-order autocorrelation measure, while the fourth-order autocorrelation is chosen to allow a sufficiently large additional time period to pass (2 years) to identify the decay in the autocorrelation series. Shorter and longer lags, like the third-order, fifth-order, and sixth-order autocorrelations could also be used, but in experimentations did not make much difference.[45]

5.4. *Firm-Level Data*

There are too little data at the macroeconomic level to provide sufficient identification for the model. I therefore identify my parameters using a panel of firm-level data from U.S. Compustat. I select the 20 years of data that cover 1981 to 2000.

The data were cleaned to remove major mergers and acquisitions by dropping the top and bottom 0.5% of employment growth, sales growth, and investment rates. Firms with an average of at least 500 employees and \$10m sales (in 2000 prices) were kept to focus on firms which are more size homogeneous. This generated a sample of 2548 firms and 22,950 observations with mean (median) employees of 13,540 (3450) and mean (median) sales of \$2247m (\$495m) in 2000 prices. In selecting all Compustat firms I am conflating the parameter estimates across a range of different industries, and a strong argument can be made for running this estimation on an industry by industry basis. However, in the interests of obtaining the "average" parameters for a macro simulation and to ensure a reasonable sample size, I keep the full panel, leaving industry-specific estimation to future work.

Capital stocks for firm i in industry m in year t are constructed by the perpetual inventory method[46]: labor figures come from company accounts, while sales figures come from accounts after deflation using the CPI. The investment rate is calculated as $(I/K)_{i,t} = I_{i,t}/(0.5 * (K_{i,t} + K_{i,t-1}))$, the employment growth rate as $(\Delta L/L)_{i,t} = (L_{i,t} - L_{i,t-1})/(0.5 * (L_{i,t} + L_{i,t-1}))$, and the sales growth as $(\Delta S/S)_{i,t} = (S_{i,t} - S_{i,t-1})/(0.5 * (S_{i,t} + S_{i,t-1}))$.[47]

[44]Note: A kth-order correlation for series $x_{i,t}$ and $y_{i,t}$ is defined as $\mathrm{Corr}(x_{it}, y_{it-k})$.

[45]Note that because the optimal weighting matrix takes into account the covariance across moments, adding extra moments that are highly correlated to included moments has very little impact on the parameters estimates.

[46]$K_{i,t} = (1 - \delta_K)K_{i,t-1}(P_{m,t}/P_{m,t-1}) + I_{i,t}$, initialized using the net book value of capital, where $I_{i,t}$ is net capital expenditure on plant, property, and equipment, and $P_{m,t}$ is the industry-level capital goods deflator from Bartelsman, Becker, and Grey (2000).

[47]*Gross* investment rates and *net* employment growth rates are used since these are directly observed in the data. Under the null that the model is correctly specified, the choice of net versus gross is not important for the consistency of parameter estimates as long as the *same* actual and simulated moments are matched.

The simulated data are constructed in exactly the same way as company accounts are built. First, firm value is created by adding up across the N units in each firm. It is then converted into annual figures using standard accounting techniques: simulated data for flow figures from the accounting profit & loss and cash-flow statements (such as sales and capital expenditure) are added up across the 12 months of the year; simulated data for stock figures from the accounting balance sheet statement (such as capital stock and labor force) are taken from the year end values.

By constructing my simulation data in the same manner as company accounts I can estimate adjustment costs using firm-level data sets like Compustat. This has some advantages versus using census data sets like the Logitudinal Research Dataset (LRD) because firm-level data are (i) easily available to all researchers in a range of different countries; (ii) matched into firm level financial and cash-flow data; and (iii) available as a yearly panel stretching back several decades (for example to the 1950s in the United States). Thus, this technique of explicitly building aggregation into estimators to match against aggregated quoted firm-level data should have a broader use in other applications. The obvious disadvantage of using Compustat is it represents only about one-third of employment in the United States (Davis et al. (2006)).

5.5. *Measurement Errors*

Employment figures are often poorly measured in company accounts, typically including all parttime, seasonal, and temporary workers in the total employment figures without any adjustment for hours, usually after heavy rounding. This problem is then made much worse by the differencing to generate growth rates.

As a first step toward reducing the sensitivity toward these measurement errors, the autocorrelations of growth rates are taken over longer periods (as noted above). As a second step, I explicitly introduce employment measurement error into the simulated moments to try to mimic the bias these impute into the actual data moments. To estimate the size of the measurement error, I assume that firm wages (W_{it}) can be decomposed into $W_{it} = \eta_t \lambda_{j,t} \phi_i L_{it}$, where η_t is the absolute price level, $\lambda_{j,t}$ is the relative industry wage rate, ϕ_i is a firm-specific salary rate (or skill/seniority mix), and L_{it} is the average annual firm labor force (hours adjusted). I then regress $\log W_{it}$ on a full set of year dummies, a log of the 4-digit SIC industry average wage from Bartelsman, Becker, and Gray (2000), a full set of firm-specific fixed effects, and $\log L_{it}$. Under my null on the decomposition of W_{it} the coefficient on $\log L_{it}$ will be approximately $\sigma_L^2/(\sigma_L^2 + \sigma_{ME}^2)$, where σ_L^2 is the variation in log employment and σ_{ME}^2 is the measurement error in log employment. I find a coefficient (standard error (s.e.)) on $\log L_{it}$ of 0.882 (0.007), implying a measurement error of 13%

in the logged labor force numbers.[48] This is reassuringly similar to the 8% estimate for measurement error in Compustat manufacturing firms' labor figures that Hall (1987) calculated comparing ordinary least squares and instrumental variable estimates. I take the average of these figures and incorporate this into the simulation estimation by multiplying the aggregated annual firm labor force by $me_{i,t}$, where $me_{i,t} \sim$ i.i.d. $LN(0, 0.105)$ before calculating simulated moments.

The other variable which is also potentially affected by measurement error is the capital stock.[49] Actual depreciation rates are not observed, so the perpetual inventory capital stock is potentially mismeasured. To investigate the importance of this in Section 5.7 I reestimate the model assuming capital stocks also have a 10% log-normal measurement error.

5.6. *Baseline Adjustment-Cost Estimates*

In this section I present the estimates of the units' capital and labor adjustment costs. Starting with Table III, the column labelled Data in the bottom panel reports the actual moments from Compustat. These demonstrate that investment rates have a low spread but a heavy right skew, due to the lack of disinvestment, and strong dynamic correlations. Labor growth rates are relatively variable but unskewed, with weaker dynamic correlations. Sales growth rates have similar moments to those of labor, although slightly lower spread and higher degree of dynamics correlations.

The column in Table III labelled All presents the results from estimating the preferred specification, allowing for all types of adjustment costs. The estimated adjustment costs for capital imply a large resale loss of around 34% on capital, fixed investment costs of 1.5% of annual sales (about 4 working days), and no quadratic adjustment costs. The estimated labor adjustment costs imply limited hiring and firing costs of about 1.8% of annual wages (about 5 working days), and a high fixed cost of around 2.1% of annual revenue (about 6 working days), with no quadratic adjustment costs. The standard errors suggest all of these point estimates are statistically significant except for the fixed cost of capital adjustment (C_K^F).

One question is how do these estimates compare to those previously estimated in the literature? Table IV presents a comparison for some other estimates from the literature. Three factors stand out: first, there is tremendous variation is estimated adjustment costs, reflecting the variety of data, techniques, and assumptions used in the different papers; second, my estimates

[48]Adding firm- or industry-specific wage trends reduces the coefficient on log W_{it}, implying an even higher degree of measurement error. Running the reverse regression of log labor on log wages plus the same controls generates a coefficient (s.e.) of 0.990 (0.008), indicating that the proportional measurement error in wages (a better recorded financial variable) is many times smaller than that of employment. The regressions are run using 2468 observations on 219 firms.

[49]Sales and capital expenditure values are usually easier to audit and so much better measured.

TABLE III

ADJUSTMENT COST ESTIMATES (TOP PANEL)[a]

Estimated Parameters	All	Capital	Labor	Quad	None
	Adjustment Costs Specification				
C_K^P: investment resale loss (%)	33.9	42.7			
	(6.8)	*(14.2)*			
C_K^F: investment fixed cost (% annual sales)	1.5	1.1			
	(1.5)	*(0.2)*			
C_K^Q: capital quadratic adjustment cost (parameter)	0	0.996		4.844	
	(0.009)	*(0.044)*		*(454.15)*	
C_L^P: per capita hiring/firing cost (% annual wages)	1.8		16.7		
	(0.8)		*(0.1)*		
C_L^F: fixed hiring/firing costs (% annual sales)	2.1		1.1		
	(0.9)		*(0.1)*		
C_L^Q: labor quadratic adjustment cost (parameter)	0		1.010	0	
	(0.037)		*(0.017)*	*(0.002)*	
σ_L: baseline level of uncertainty	0.443	0.413	0.216	0.171	0.100
	(0.009)	*(0.012)*	*(0.005)*	*(0.005)*	*(0.005)*
$\mu_H - \mu_L$: spread of firm business conditions growth	0.121	0.122	0.258	0.082	0.158
	(0.002)	*(0.002)*	*(0.001)*	*(0.001)*	*(0.001)*
$\pi_{H,L}^{\mu}$: transition of firm business conditions growth	0	0	0.016	0	0.011
	(0.001)	*(0.001)*	*(0.001)*	*(0.001)*	*(0.001)*
γ: curvature of the hours/wages function	2.093	2.221	3.421	2.000	2.013
	(0.272)	*(0.146)*	*(0.052)*	*(0.009)*	*(14.71)*

(Continues)

of positive capital and labor adjustment costs appear broadly consistent with other papers which jointly estimate these; and third, studies which estimate nonconvex adjustment costs report positive and typically very substantial values.

For interpretation, in Table III I also display results for four illustrative restricted models. First, a model with capital adjustment costs only, assuming labor is fully flexible, as is typical in the investment literature. In the Capital columns we see that the fit of the model is worse, as shown by the significant rise in the criterion function from 404 to 625.[50] This reduction in fit is primarily due to the worse fit of the labor moments, suggesting ignoring labor adjustment costs is a reasonable approximation for modelling investment. Second, a model with labor adjustment costs only—as is typical in the dynamic labor demand literature—is estimated in the column Labor: the fit is substantially worse. This suggests that ignoring capital adjustment costs is problematic. Third, a model

[50]The χ^2 value for 3 degrees of freedom is 7.82, so the column Capital can easily be rejected against the null of All given the difference in criterion values of 221. It is also true, however, that the preferred All specification can also be rejected as the true representation of the data given that the χ^2 value for 10 degrees of freedom is 18.31.

TABLE III

(BOTTOM PANEL)

Moments	Data	Data Moments – Simulated Moments				
Correlation $(I/K)_{i,t}$ with $(I/K)_{i,t-2}$	*0.328*	0.060	−0.015	0.049	−0.043	0.148
Correlation $(I/K)_{i,t}$ with $(I/K)_{i,t-4}$	*0.258*	0.037	0.004	0.088	0.031	0.162
Correlation $(I/K)_{i,t}$ with $(\Delta L/L)_{i,t-2}$	*0.208*	0.003	−0.025	0.004	−0.056	0.078
Correlation $(I/K)_{i,t}$ with $(\Delta L/L)_{i,t-4}$	*0.158*	−0.015	−0.009	0.036	0.008	0.091
Correlation $(I/K)_{i,t}$ with $(\Delta S/S)_{i,t-2}$	*0.260*	−0.023	−0.062	−0.044	−0.102	0.024
Correlation $(I/K)_{i,t}$ with $(\Delta S/S)_{i,t-4}$	*0.201*	−0.010	−0.024	0.018	−0.036	0.087
Standard deviation $(I/K)_{i,t}$	*0.139*	−0.010	0.010	−0.012	0.038	0.006
Coefficient of skewness $(I/K)_{i,t}$	*1.789*	0.004	0.092	1.195	1.311	1.916
Correlation $(\Delta L/L)_{i,t}$ with $(I/K)_{i,t-2}$	*0.188*	−0.007	0.052	−0.075	0.055	0.053
Correlation $(\Delta L/L)_{i,t}$ with $(I/K)_{i,t-4}$	*0.133*	−0.021	0.024	−0.061	0.038	0.062
Correlation $(\Delta L/L)_{i,t}$ with $(\Delta L/L)_{i,t-2}$	*0.160*	0.011	0.083	−0.033	0.071	0.068
Correlation $(\Delta L/L)_{i,t}$ with $(\Delta L/L)_{i,t-4}$	*0.108*	−0.013	0.054	−0.026	0.045	0.060
Correlation $(\Delta L/L)_{i,t}$ with $(\Delta S/S)_{i,t-2}$	*0.193*	−0.019	0.063	−0.091	0.064	0.023
Correlation $(\Delta L/L)_{i,t}$ with $(\Delta S/S)_{i,t-4}$	*0.152*	0.003	0.056	−0.051	0.059	0.063
Standard deviation $(\Delta L/L)_{i,t}$	*0.189*	−0.022	−0.039	0.001	−0.001	0.005
Coefficient of skewness $(\Delta L/L)_{i,t}$	*0.445*	−0.136	0.294	−0.013	0.395	0.470
Correlation $(\Delta S/S)_{i,t}$ with $(I/K)_{i,t-2}$	*0.203*	−0.016	−0.015	−0.164	−0.063	−0.068
Correlation $(\Delta S/S)_{i,t}$ with $(I/K)_{i,t-4}$	*0.142*	−0.008	−0.010	−0.081	−0.030	−0.027
Correlation $(\Delta S/S)_{i,t}$ with $(\Delta L/L)_{i,t-2}$	*0.161*	−0.005	0.032	−0.105	−0.024	−0.037
Correlation $(\Delta S/S)_{i,t}$ with $(\Delta L/L)_{i,t-4}$	*0.103*	−0.015	0.011	−0.054	−0.005	−0.020
Correlation $(\Delta S/S)_{i,t}$ with $(\Delta S/S)_{i,t-2}$	*0.207*	−0.033	0.002	−0.188	−0.040	−0.158
Correlation $(\Delta S/S)_{i,t}$ with $(\Delta S/S)_{i,t-4}$	*0.156*	0.002	0.032	−0.071	−0.021	−0.027
Standard deviation $(\Delta S/S)_{i,t}$	*0.165*	0.004	0.003	0.033	0.051	0.062
Coefficient of skewness $(\Delta S/S)_{i,t}$	*0.342*	−0.407	−0.075	−0.365	0.178	0.370
Criterion, $\Gamma(\theta)$		404	625	3618	2798	6922

[a]The Data column (bottom panel only) contains the moments from 22,950 observations on investment (I), capital (K), labor (L) and sales (S) for 2548 firms. The other columns contain the adjustment costs estimates (top panel) and data moments minus the simulated moments (bottom panel) for all adjustment costs (All), just capital adjustment costs (Capital), just labor adjustment costs (Labor), just quadratic adjustment costs with 1 unit per firm (Quad), and no adjustment costs (None). So, for example, the number 0.328 at the top of the Data column reports that the second-lag of the autocorrelation of investment in the data is 0.328, and the number 0.060 to the right reports that in the All specification the simulated moment is 0.060 less than the data moment (so is 0.268 in total). In the top panel standard-errors are given in italics in parentheses below the point estimates. Parameters were estimated using simulated method of moments, and standard were errors calculated using numerical derivatives.

with quadratic costs only and no cross-sectional aggregation—as is typical in convex adjustment-costs models—is estimated in the Quad column, leading to a moderate reduction in fit generated by excessive intertemporal correlation and an inadequate investment skew. Interestingly, industry and aggregate data are much more autocorrelated and less skewed due to extensive aggregation, suggesting quadratic adjustments costs could be a reasonable approximation at this level. Finally, a model with no adjustment costs is estimated in column None. Omitting adjustment costs clearly reduces the model fit. It also biases the estimation of the business-conditions process to have much larger firm-

TABLE IV

A COMPARISON WITH OTHER CAPITAL AND LABOR ADJUSTMENT-COST ESTIMATES

	Capital			Labor		
Source	PI (%)	Fixed (%)	Quad	PI (%)	Fixed (%)	Quad
Column All in Table III (this paper)	33.9	1.5	0	1.8	2.1	0
Ramey and Shapiro (2001)	40–80					
Caballero and Engel (1999)		16.5				
Hayashi (1982)			20			
Cooper and Haltiwanger (2006)	2.5	20.4	0.049			
Shapiro (1986)			3			1.33
Merz and Yashiv (2007)			14.2			4.2
Hall (2004)			0			0
Nickell (1986)				8–25		
Cooper, Haltiwanger, and Willis (2004)					1.7	0

[a]PI denotes partial irreversibilities, Fixed denotes fixed costs, and Quad denotes quadratic adjustment costs. Missing values indicate no parameter estimated in the main specification. Zeros indicate the parameter was not significantly different from zero. Nickell's (1986) lower [higher] value is for unskilled [skilled] workers. Shapiro's (1986) value is a weighted average of $(2/3) \times 0$ for production workers and $(1/3) \times 4$ for nonproduction workers. Merz and Yashiv's (2007) values are the approximated quadratic adjustment costs at the sample mean. Comparability subject to variation in data sample, estimation technique, and maintained assumptions.

level growth fixed effects and lower variance of the idiosyncratic shocks. This helps to highlight the importance of jointly estimating adjustment costs and the driving process.

In Table III there are also some estimates of the driving process parameters σ_L, $\mu_H - \mu_L$, and $\pi^{\mu}_{H,H}$, as well as the wage–hours curve parameter γ. What is clear is that changes in the adjustment-cost parameters leads to changes in these parameters. For example, the lack of adjustment costs in the column Quad generates an estimated uncertainty parameter of around one-third of the baseline All value and a spread in firm-level fixed costs of about two-thirds of the baseline All value. This provides support for the selection of moments that can separately identify the driving process and adjustment-cost parameters.

5.7. *Robustness Tests on Adjustment-Cost Estimates*

In Table V I run some robustness tests on the modelling assumptions. The column All repeats the baseline results from Table III for ease of comparison.

The column $\delta_L = 0.2$ reports the results from reestimating the model with a 20% (rather than 10%) annual attrition rate for labor. This higher rate of attrition leads to higher estimates of quadratic adjustment costs for labor and capital, and lower fixed costs for labor. This is because with higher labor attrition rates, hiring and firing become more sensitive to current demand shocks (since higher attrition reduces the sensitivity to past shocks). To compensate, the estimated quadratic adjustment-cost estimates are higher and fixed costs

TABLE V

ADJUSTMENT COST ROBUSTNESS TESTS[a]

Estimated Parameters	All	$\delta_L = 20\%$	$a+b=0.83$	$N=25$	$N=1$	Pre −5%	Pre +5%	Cap ME	Yearly	Weekly
	Adjustment Costs Specification									
C_K^P: investment resale loss (%)	33.9	28.6	29.8	30.3	47.0	33.9	36.1	35.3	45.3	50.0
	(6.8)	(4.8)	(4.8)	(8.7)	(9.1)	(6.8)	(4.9)	(5.1)	(5.2)	(30.8)
C_K^F: investment fixed cost (% annual sales)	1.5	2.1	2.1	0.9	1.3	1.5	1.6	0.9	2.1	1.6
	(1.0)	(0.9)	(0.5)	(0.4)	(0.2)	(1.0)	(0.5)	(0.4)	(0.3)	(0.4)
C_K^Q: capital quadratic adjustment cost	0	0.461	0	0.616	2.056	0	0.058	0.525	0.025	1.488
	(0.009)	(0.054)	(0.007)	(0.154)	(0.284)	(0.009)	(0.037)	(0.078)	(0.015)	(0.729)
C_L^P: per capita hiring/firing cost (% wages)	1.8	1.0	0	0	0	1.8	0	0	2.0	0
	(0.8)	(0.1)	(0.0)	(0.1)	(0.1)	(0.8)	(0.04)	(0.7)	(0.9)	(6.1)
C_L^F: fixed hiring/firing costs (% sales)	2.1	0.3	1.7	1.3	0	2.1	1.6	0.9	2.0	1.3
	(0.9)	(0.1)	(0.6)	(0.8)	(0.0)	(0.9)	(0.7)	(0.4)	(0.5)	(0.3)
C_L^Q: labor quadratic adjustment cost	0	0.360	0	0.199	0.070	0	0	0	1.039	0.808
	(0.037)	(0.087)	(0.021)	(0.062)	(0.031)	(0.037)	(0.018)	(0.017)	(0.165)	(0.612)
σ_L: baseline level of uncertainty	0.443	0.490	0.498	0.393	0.248	0.443	0.469	0.515	0.339	0.600
	(0.009)	(0.019)	(0.012)	(0.013)	(0.008)	(0.009)	(0.011)	(0.017)	(0.011)	(0.035)
$\mu_H - \mu_L$: spread of business conditions growth	0.121	0.137	0.123	0.163	0.126	0.121	0.132	0.152	0.228	0
	(0.002)	(0.002)	(0.001)	(0.002)	(0.002)	(0.002)	(0.002)	(0.003)	(0.005)	(0.018)
$\pi^{\mu}_{H,L}$: transition business conditions growth	0	0	0	0	0	0	0	0	0.016	n/a
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	n/a
γ: curvature of hours/wages function	2.093	2.129	2.000	2.148	2.108	2.093	2.056	2.000	2.000	2.129
	(0.272)	(0.222)	(0.353)	(0.266)	(0.147)	(0.272)	(0.246)	(0.211)	(0.166)	(0.254)
Criterion, $\Gamma(\theta)$	404	496	379	556	593	403	380	394	656	52

[a] Adjustment costs estimates using the same methodology as in Table III for the baseline model (All), except 20% annualized labor attrition ($\delta_L = 20\%$), 20% markup ($a+b=0.83$), only 25 units per firm ($N=25$), only 1 unit per firm ($N=1$), a 5% negative business-conditions shock 6 months prior to the start of the simulation period (pre −5%), a 5% positive business-conditions shock 6 months prior to the simulation period (pre +5%), a 10% log-normal capital measurement error (cap ME), and simulation run at a yearly frequency (Yearly). The final column (Weekly) is different. This evaluates temporal aggregation bias. It reports parameters estimated using moments from simulated data created by taking the parameters values from the All column, simulating at a weekly frequency, and then aggregating up to yearly data. This is done to test the bias from estimating the model assuming an underlying monthly process when in fact the moments are generated from an underlying weekly process. Standard errors are given in Italics below the point estimates.

are lower. The column $a + b = 0.83$ reports the results for a specification with a 20% markup, in which the estimated adjustment costs look very similar to the baseline results.

In columns $N = 25$ and $N = 1$, the results are reported for simulations assuming the firm operates 25 units and 1 unit, respectively.[51] These assumptions also lead to higher estimates for the quadratic adjustment costs and lower estimates for the nonconvex adjustment costs to compensate for the reduction in smoothing by aggregation.

In columns Pre -5% and Pre $+5\%$, the results are reported for simulations assuming there is a -5% and $+5\%$ shock, respectively, to business conditions in the 6 months before the simulation begins. This simulates running the simulation during a recession and boom, respectively, to investigate how the initial conditions influence the results. As shown in Table V the results are numerically identical for Pre -5% and similar for Pre $+5\%$ to those for All, suggesting the results are robust to different initial conditions. The reason is that the long time period of the simulation (20 years) and the limited persistence of macro shocks means that the impact of initial conditions dies quickly.

In column Cap ME, the parameters are estimated including a log-normal 10% measurement error for capital, with broadly similar results. Finally, the last two columns investigate the impact of time aggregation. First, the column Yearly reports the results for running the simulation at a yearly frequency without any time aggregation. Dropping time aggregation leads to higher estimated quadratic adjustment costs to compensate for the loss of smoothing by aggregation. Second, the column Weekly reports the results from (i) taking the baseline All parameter estimates and using them to run a weekly frequency simulation, (ii) aggregating these data to a yearly frequency, and (iii) using this to estimate the model assuming a monthly underlying frequency. Thus, this seeks to understand the bias from assuming the model is monthly if the underlying generating process was in fact weekly. Comparing the All parameter values used to generate the data with the Weekly values estimated from (incorrectly) assuming a monthly underlying frequency reveals a number of differences. This highlights that modelling time aggregation correctly appears to matter for correctly estimating adjustment costs. Given the lack of prior empirical or simulation results on temporal aggregation, this suggests an area in which further research would be particularly valuable.

6. SIMULATION ROBUSTNESS

In this section, I undertake a set of simulation robustness tests for different values of adjustment costs, the predetermined parameters, and the uncertainty process.

[51]The specification with $N = 1$ is included to provide guidance on the impact of simulated aggregation rather than for empirical realism. The evidence from the annual reports of almost all large companies suggests aggregation is pervasive.

6.1. *Adjustment Costs*

To evaluate the effects of different types of adjustment, I ran three simulations: the first with fixed costs only, the second with partial irreversibilities only, and the third with quadratic adjustment costs only.[52] The output from these three simulations is shown in Figure 16. As can be seen, the two specifications with nonconvex adjustment costs generate a distinct drop and rebound in economic activity. The rebound with fixed costs is faster than with partial irreversibilities because of the bunching in hiring and investment, but otherwise they are remarkably similar in size, duration, and profile. The quadratic adjustment-cost specification appears to generate no response to an uncertainty shock. The reason is that there is no kink in adjustment costs around zero, so there is no option value associated with doing nothing.

In summary, this suggests the predictions are very sensitive to the inclusion of some degree of nonconvex adjustment costs, but are much less sensitive to the type (or indeed level) of these nonconvex adjustment costs. This highlights

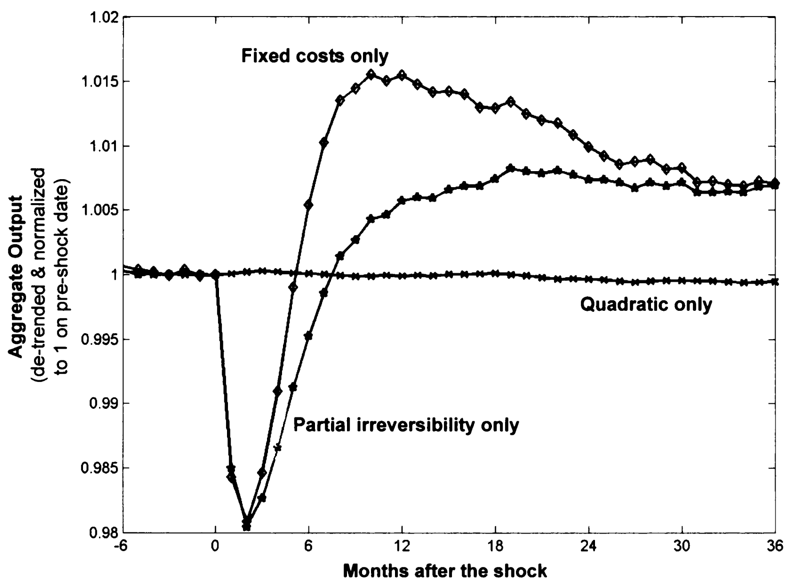

FIGURE 16.—Different adjustment costs. Adjustment costs in the fixed costs specification have only the C_K^F and C_L^F adjustment costs from the All estimation in Table III the partial irreversibility has only the C_K^P and C_L^P from the baseline All estimation in Table III, and the quadratic has the adjustment costs from the Quad column in Table III.

[52]For fixed costs and partial irreversibilities the adjustment costs are the fixed cost and partial irreversibility components of the parameter values from the All column in Table III. For quadratic adjustment costs, the values are from the Quad column in Table III.

the importance of the prior step of estimating the size and nature of the underlying labor and capital adjustment costs.

6.2. *Predefined Parameters*

To investigate the robustness of the simulation results to the assumptions over the predefined parameters, I reran the simulations using the different parameters from Table V. The results, shown in Figure 17, highlight that the *qualitative* result of a drop and rebound in activity is robust to the different assumptions over the predetermined parameters. This is because of the presence of some nonconvex component in all the sets of estimated adjustment costs in Table V.

The size of this drop and rebound did vary across specifications, however. Running the simulation with the $N = 1$ parameter estimates from Table V leads to a drop of only 1%, about half the baseline drop of about 1.8%. This smaller drop was due to the very high levels of estimated quadratic adjustment costs that were required to smooth the investment and employment series in the absence of cross-sectional aggregation. Of course, the assumption of no cross-sectional aggregation ($N = 1$) is inconsistent with the pervasive aggregation in the typical large firm. This simulation is presented simply to highlight the importance of building aggregation into estimation routines when it is also present in the data.

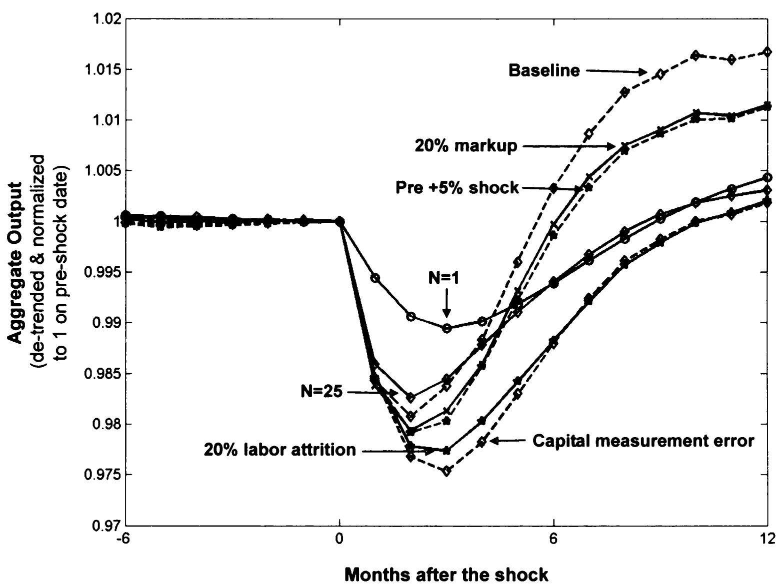

FIGURE 17.—Simulation robustness to different parameter assumptions.

In the $\delta_L = 0.2$ specification the drop was around 2.25%, about 30% above the baseline drop, due to the greater labor attrition after the shock. Hence, this more realistic assumption on 20% annual labor attrition (rather than 10% in the baseline) generates a larger drop and rebound in activity. The results for assuming partial cross-sectional aggregation ($N = 25$), a 20% markup ($a + b = 0.83$), a preestimation boom (Pre +5%), and capital measurement errors (Cap ME) are all pretty similar to the baseline simulation (which has full cross-sectional aggregation and a 33% markup).

6.3. *Durations and Sizes of Uncertainty Shocks*

Finally, I also evaluate the effects of robustness of the simulation predictions to different durations and sizes of uncertainty shocks. In Figure 18, I plot the output response to a shorter-lived shock (a 1-month half-life) and a longer-lived shock (a 6-month half-life). Also plotted is the baseline (a 2-month half-life). It is clear that longer-lived shocks generate larger and more persistent falls in output. The reason is that the pause in hiring and investment lasts for longer if the rise in uncertainty is more persistent. Of course, because the rise in uncertainty is more persistent, the cumulative increase in volatility is also larger so that the medium-term "volatility overshoot" is also greater. Hence, more persistent uncertainty shocks generate a larger and more persistent drop, rebound, and overshoot in activity. This is interesting in the context of the Great Depression, a period in which uncertainty rose to 260% of the baseline

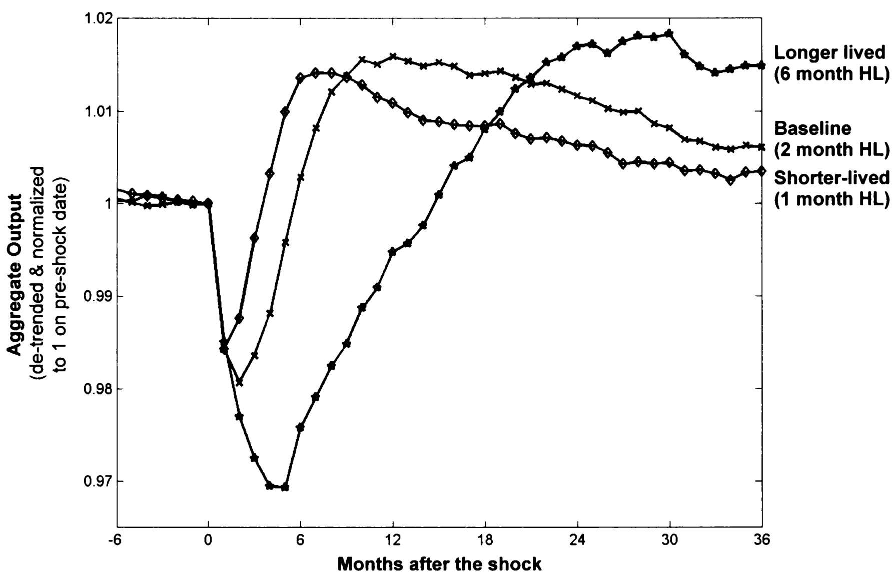

FIGURE 18.—Uncertainty shocks with half-lives (HL) of 1 month, 2 months, and 6 months.

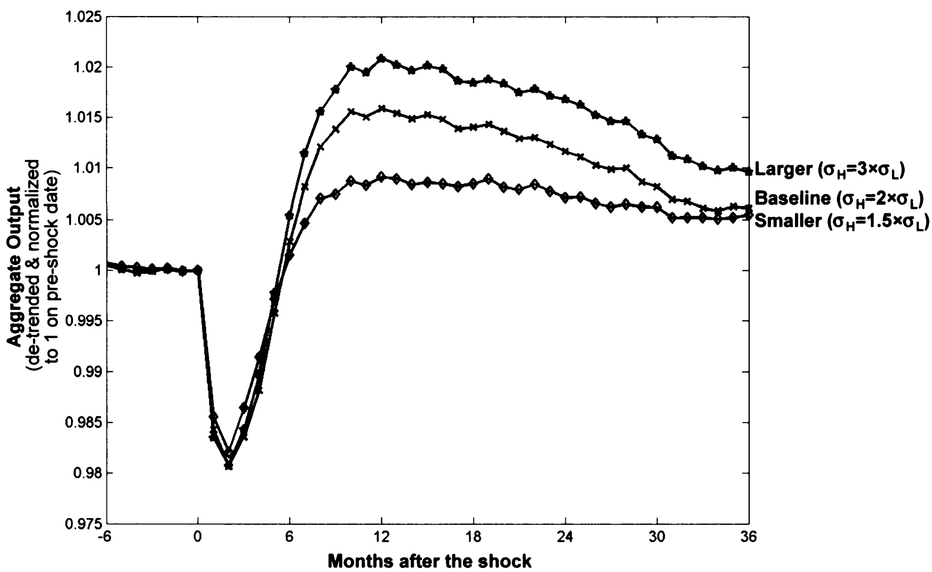

FIGURE 19.—Different sizes of uncertainty shocks. The larger and smaller uncertainty shocks have values of σ_H equal to 150% and 300% of the σ_L level (baseline is 200%).

level for over 4 years, which in my (partial equilibrium) model would generate an extremely large and persistent drop in output and employment.

In Figure 19, I plot the output response to a smaller uncertainty shock ($\sigma_H = 1.5 \times \sigma_L$), a larger uncertainty shock ($\sigma_H = 3 \times \sigma_L$), and the baseline uncertainty shock ($\sigma_H = 2 \times \sigma_L$). Surprisingly, the three different sizes of uncertainty shock lead to similar sized drops in activity. The reason is that real-option values are increasing, but concave, in the level of uncertainty,[53] so the impact of a 50% rise in uncertainty on the hiring and investment thresholds is about two-thirds of the size of the baseline 100% rise in uncertainty. Since the baseline impact on the hiring and investment thresholds is so large, even two-thirds of this pauses almost all hiring and investment. What is different across the different sizes of shocks, however, is that larger uncertainty shocks generate a larger medium-term volatility overshoot because the cumulative increase in volatility is greater.

Finally, in Figure 20, I evaluate the effects of an uncertainty shock which only changes the variance of macro shocks, and not the variance of firm- or unit-level shocks. This changes two things in the simulation. First, overall uncertainty only rises by 33% after a shock, since while macro uncertainty doubles, firm and micro uncertainty are unchanged. Despite this the initial drop is similar to the baseline simulation for a 100% increase in overall uncertainty.

[53]See Dixit (1993) and Abel and Eberly (1996) for an analytical derivation and discussion.

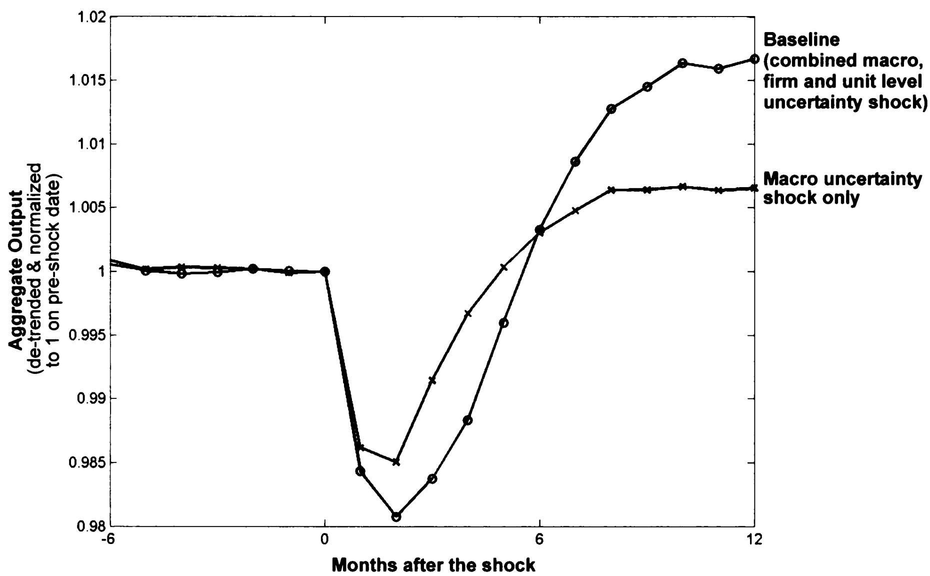

FIGURE 20.—Macro uncertainty shock only. At month 0 the simulation plot Macro uncertainty shock only has macro shocks only set to σ_H at month 0. Firm and unit level shocks are σ_L at all times.

This confirms the results from Figure 19 that even moderately sized uncertainty shocks are sufficient to pause activity. Second, there is no cross-sectional increase in firm- and unit-level variance, substantially reducing the volatility overshoot.[54]

7. CONCLUSIONS

Uncertainty appears to dramatically increase after major economic and political shocks like the Cuban missile crisis, the assassination of JFK, the OPEC I oil-price shock, and the 9/11 terrorist attacks. If firms have nonconvex adjustment costs, these uncertainty shocks will generate powerful real-option effects, driving the dynamics of investment and hiring behavior. These shocks appear to have large real effects: the uncertainty component alone generates a 1% drop and rebound in employment and output over the following 6 months, and a milder long-run overshoot.

This paper offers a structural framework to analyze these types of uncertainty shocks, building a model with a time-varying second moment of the

[54]There is still some volatility overshoot due to the averaging across macro shocks in the 25,000 macro draws. The reason for this is that the cross-sectional distribution is right-skewed on average (as shown in Figure 6) so that investment responds more to positive than negative macro shocks.

driving process, and a mix of labor and capital adjustment costs. The model is numerically solved and estimated on firm-level data using simulated method of moments. The parameterized model is then used to simulate a large macro *uncertainty* shock, which produces a rapid drop and rebound in output, employment, and productivity growth. This is due to the effect of higher uncertainty making firms temporarily pause their hiring and investment behavior. In the medium term, the increased volatility arising from the uncertainty shock generates a volatility overshoot as firms respond to the increased variance of productivity shocks, which drives a medium-term overshoot and longer-run return to trend. Hence, the simulated response to uncertainty shocks generates a drop, rebound, and longer-run overshoot, much the same as their actual empirical impact.

This temporary impact of a second-moment shock is different from the typically persistent impact of a first moment shock. While the second-moment effect has its biggest drop by month 3 and has rebounded by about month 6, persistent first-moment shocks generate falls in activity that last several quarters. Thus, for a policymaker in the immediate aftermath of a shock it is critical to distinguish the relative contributions of the first- and second-moment components of shocks for predicting the future evolution of output.

The uncertainty shock also induces a strong insensitivity to other economic stimuli. At high levels of uncertainty the real-option value of inaction is very large, which makes firms extremely cautious. As a result, the effects of empirically realistic general equilibrium type interest rate, wage, and price falls have a very limited short-run effect on reducing the drop and rebound in activity. This raises a second policy implication, that in the immediate aftermath of an uncertainty shock, monetary or fiscal policy is likely to be particularly ineffective.

This framework also enables a range of future research. Looking at individual events it could be used, for example, to analyze the uncertainty impact of major deregulations, tax changes, trade reforms, or political elections. It also suggests there is a trade-off between policy correctness and decisiveness—it may be better to act decisively (but occasionally incorrectly) than to deliberate on policy, generating policy-induced uncertainty.

More generally these second-moment effects contribute to the "where are the negative productivity shocks?" debate in the business cycle literature. It appears that second-moment shocks can generate short sharp drops and rebounds in output, employment, investment, and productivity growth without the need for a first-moment productivity shock. Thus, recessions could potentially be driven by increases in uncertainty. Encouragingly, recessions do indeed appear in periods of significantly higher uncertainty, suggesting an uncertainty approach to modelling business cycles (see Bloom, Floetotto, and Jaimovich (2007)). Taking a longer-run perspective, this model also links to the volatility and growth literature given the evidence for the primary role of reallocation in productivity growth.

The paper also jointly estimates nonconvex and convex labor and capital adjustment costs. I find substantial fixed costs of hiring/firing and investment, a large loss from capital resale, and some moderate per-worker hiring/firing costs. I find no evidence for quadratic investment or hiring/firing adjustment costs. I also find that assuming capital adjustment costs only—as is standard in the investment literature—generates an acceptable overall fit, while assuming labor adjustment costs only—as is standard in the labor demand literature—produces a poor fit.

APPENDIX A: DATA

All data and Stata do files used to create the empirical Figures 1, 2, and 3 and Table I are available at http://www.stanford.edu/~nbloom/. In this appendix I describe the contents and construction of these data sets.

A.1. *Stock-Market Volatility Data*

A.1.1. *Testing for Jumps in Stock-Market Volatility*

To test for jumps in stock-market volatility, I use the nonparametric bipower variation test of Barndorff-Nielsen and Shephard (2006). The test works for a time series $\{x_t, t = 1, 2, \dots, N\}$ by comparing the squared variation, $\mathrm{SV} = \sum_{t=3}^{N}(x_t - x_{t-1})^2$ with the bipower variation, $\mathrm{BPV} = \sum_{t=3}^{N}(x_t - x_{t-1})(x_{t-1} - x_{t-2})$. In the limit as $dt \to 0$, if there are no jumps in the data, then $E[\mathrm{SV}] \to E[\mathrm{BPV}]$ since the variation is driven by a continuous process. If there are jumps, however, then $E[\mathrm{SV}] > E[\mathrm{BPV}]$ since jumps have a squared impact on SV but only a linear impact on BPV. Barndorff-Nielsen and Shephard (2006) suggested two different test statistics—the *linear-jump* and the *ratio-jump* test—which have the same asymptotic distribution but different finite-sample properties. Using the monthly data from Figure 1, I reject the null of no jumps at the 2.2% and 1.6% level using the linear and ratio tests, respectively. Using the daily VXO data underlying Figure 1 (available from January 1986 onward), I reject the null of no jumps using both tests at the 0.0% level.

A.1.2. *Defining Stock-Market Volatility Shocks*

Given the evidence for the existence of stock-market volatility jumps, I need to define what they are. The main measure is an indicator that takes a value of 1 for each of the 17 events labelled in Figure 1, and 0 otherwise. These 17 events are chosen as those with stock-market volatility more than 1.65 standard deviations above the Hodrick–Prescott detrended ($\lambda = 129{,}600$) mean of the stock-market volatility series (the raw undetrended series is plotted in Figure 1). While some of these shocks occur in 1 month only, others span multiple months so there was a choice over the exact allocation of their timing. I tried

TABLE A.1

MAJOR STOCK-MARKET VOLATILITY SHOCKS

Event	Max Volatility	First Volatility	Type
Cuban missile crisis	October 1962	October 1962	Terror
Assassination of JFK	November 1963	November 1963	Terror
Vietnam buildup	August 1966	August 1966	War
Cambodia and Kent State	May 1970	May 1970	War
OPEC I, Arab–Israeli War	December 1973	December 1973	Oil
Franklin National	October 1974	September 1974	Economic
OPEC II	November 1978	November 1978	Oil
Afghanistan, Iran hostages	March 1980	March 1980	War
Monetary cycle turning point	October 1982	August 1982	Economic
Black Monday	November 1987	October 1987	Economic
Gulf War I	October 1990	September 1990	War
Asian Crisis	November 1997	November 1997	Economic
Russian, LTCM default	September 1998	September 1998	Economic
9/11 terrorist attack	September 2001	September 2001	Terror
Worldcom and Enron	September 2002	July 2002	Economic
Gulf War II	February 2003	February 2003	War
Credit crunch	October 2008	August 2007	Economic

two different approaches: the primary approach is to allocate each event to the month with the largest volatility spike for that event; an alternative approach is to allocate each event to the first month in which volatility went more than 2 standard deviations above the HP detrended mean. The events can also be categorized in terms of terror, war, oil, or economic shocks. So a third volatility indicator uses only the arguably most exogenous terms of terror, war, and oil shocks.

The volatility shock events, their dates under each timing scheme, and their classification are shown in Table A.1. It is noticeable from Table A.1 that almost all the shocks are bad events. So one question for empirical identification is how distinct are stock-market volatility shocks from stock-market levels shocks? Fortunately, it turns out that these series do move reasonably independently because some events—like the Cuban missile crisis—raise volatility without impacting stock-market levels, while others—like hurricane Katrina—generate falls in the stock market without raising volatility. So, for example, the log detrended stock-market level has a correlation of −0.192 with the main 1/0 volatility shock indicator, a correlation of −0.136 with the 1/0 oil, terror, and war shock indicator, and a −0.340 correlation with the log detrended volatility index itself. Thus, the impact of stock-market volatility can be separately identified from stock-market levels. In the working paper version of this paper (Bloom (2008)), I briefly described each of the 17 volatility shocks shown on Figure 1 to highlight the fact that these are typically linked to real shocks.

A.2. *Cross-Sectional Uncertainty Measures*

There are four key cross-sectional uncertainty measures:

Standard Deviation of Firm-Level Profits Growth: This is measured on a quarterly basis using Compustat quarterly accounts. It is the cross-sectional standard deviation of the growth rates of pretax profits (data item 23). Profit growth has a close fit to productivity and demand growth in homogeneous revenue functions, and is one of the few variables to have been continuously reported in quarterly accounts since the 1960s. This is normalized by the firms' average sales (data item 2) and is defined as $(\text{profits}_t - \text{profits}_{t-1})/(0.5 \times \text{sales}_t + 0.5 \times \text{sales}_{t-1})$. Only firms with 150 or more quarters of accounts with sales and pretax profit figures are used to minimize the effects of sample composition changes.[55] The growth rates are windsorized at the top and bottom 0.05% growth rates to prevent the series from being driven by extreme outliers.

Standard Deviation of Firm-Level Stock Returns: This is measured on a monthly basis using the CRSP data file. It is the cross-sectional standard deviation of the monthly stock returns. The sample is all firms with 500 or more months of stock-returns data. The returns are windsorized at the top and bottom 0.5% growth rates to prevent the series from being driven by extreme outliers.

Standard Deviation of Industry-Level TFP Growth: This is measured on an annual basis using the NBER industry data base (Bartelsman, Becker, and Gray (2000)). The cross-sectional spread is defined as the standard deviation of the five-factor TFP growth rates, taken across all SIC 4-digit manufacturing industries. The complete sample is a balanced panel for 422 of the 425 industries (results are robust to dropping these three industries).

Standard Deviation of GDP Forecasts. This is measured on a half-yearly basis using the Philadelphia Federal Reserve Bank's Livingstone survey of professional forecasters. It is defined as the cross-sectional standard deviation of the 1-year-ahead GDP forecasts normalized by the mean of the 1-year-ahead GDP forecasts. Only half-years with 50+ forecasts are used to ensure sufficient sample size for the calculations. This series is linearly detrended across the sample (1950–2006) to remove a long-run downward drift of forecaster variance.

A.3. *VAR Data*

The VAR estimations are run using monthly data from June 1962 through June 2008. The full set of VAR variables in the estimation are log industrial production in manufacturing (Federal Reserve Board of Governors, seasonally adjusted), employment in manufacturing (BLS, seasonally adjusted), average hours in manufacturing (BLS, seasonally adjusted), log consumer price

[55]Limiting compositional change helps to address some of the issues raised by Davis, Faberman, and Haltiwanger (2006), who found rising sales volatility of publicly quoted firms but flat volatility of privately held firms. I also include a time trend in column 2 of Table I to directly control for this and focus on short-run movements.

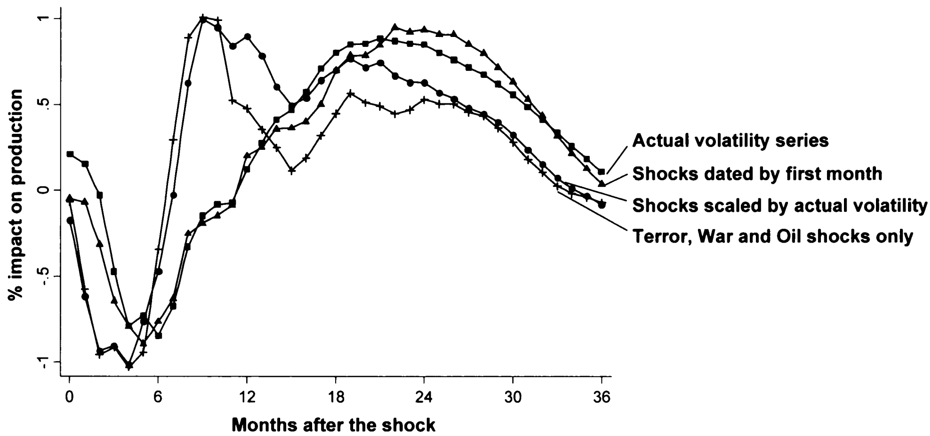

FIGURE A1.—VAR robustness to different shock definitions.

index (all urban consumers, seasonally adjusted), log average hourly earnings for production workers (manufacturing), Federal Funds rate (effective rate, Federal Reserve Board of Governors), a monthly stock-market volatility indicator (described below), and the log of the S&P500 stock-market index. All variables are HP detrended using a filter value of $\lambda = 129{,}600$.

In Figure A1, the industrial production impulse response function is shown for four different measures of volatility: the actual series in Figure 1 after HP detrending (square symbols), the 1/0 volatility indicator with the shocks scaled by the HP detrended series (dot symbols), an alternative volatility indicator which dates shocks by their first month (rather than their highest month) (triangle symbols), and a series which only uses the shocks linked to terror, war, and oil (plus symbols). As can be seen, each one of these shock measures generates a rapid drop and rebound in the predicted industrial production. In Figure A2, the VAR results are also shown to be robust to a variety of alternative variable sets and orderings. The VAR is reestimated using a simple trivariate VAR (the volatility indicator, log employment, and industrial production only) (square symbols) also displays a drop and rebound. The "quadvariate" VAR (the volatility indicator, log stock-market levels, log employment, and industrial production) also displays a similar drop and rebound (cross symbols), as does the quadvariate VAR with the variable ordering reversed (circular symbols). Hence the response of industrial production to a volatility shock appears robust to both the basic selection and the ordering of variables. In Figure A3, I plot the results using different HP detrending filter values: the linear detrended series ($\lambda = \infty$) is plotted (square symbols) alongside the baseline detrending ($\lambda = 129{,}600$) (cross symbols) and the "flexible" detrending ($\lambda = 1296$). As can be seen, the results again appear robust. I also conducted a range of other experiments, such as adding controls for the oil price (spot price of West Texas), and found the results to be similar.

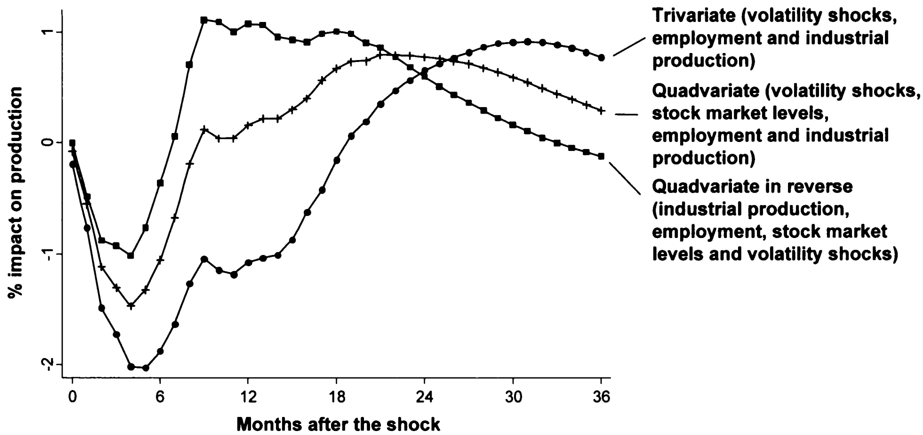

FIGURE A2.—VAR robustness to different variable sets and ordering.

APPENDIX B: NUMERICAL SOLUTION METHOD

This appendix describes some of the key steps in the numerical techniques used to solve the firm's maximization problem. The full program, which runs on Matlab 64-bit, is at http://www.stanford.edu/~nbloom/.

B.1. *Value Function Iteration*

The objective is to solve the value function (3.7). This value function solution procedure is used in two parts of the paper. The first is in the simulated method of moments estimation of the unknown adjustment cost parameters,

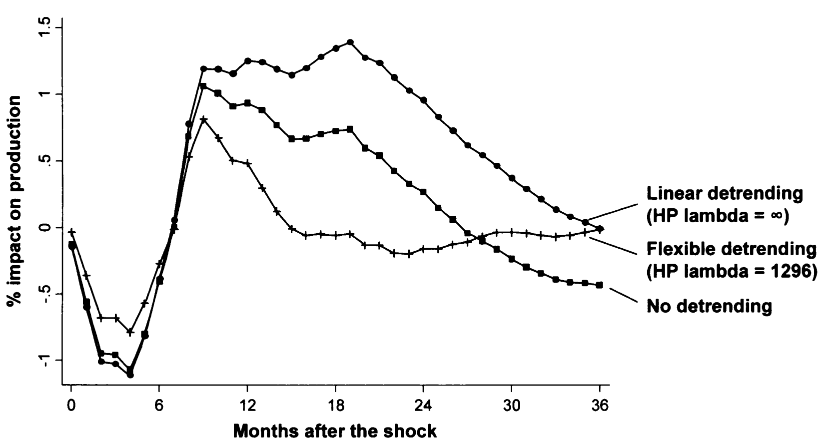

FIGURE A3.—VAR robustness to different variable detrending assumptions.

whereby the value function is repeatedly solved for a variety of different parameters, the data are simulated, and these moments are used in the parameter search algorithm. The second is in the simulation where the value function is solved just once—using the estimated parameters choices—and then used to simulate a panel of 1000 units, repeated 25,000 times. The numerical contraction mapping procedure used to solve the value function in both cases is the same. This proceeds following three steps:

STEP 1: Choose a grid of points in (a, l, σ, μ) space. Given the log-linear structure of the demand process, I use a grid of points in $(\log(a), \log(l), \sigma, \mu)$ space. In the $\log(a)$ and $\log(l)$ dimensions this is equidistantly spaced; in the σ and μ spacing this is determined by the estimated parameters. The normalization by capital in a and l—noting that $a = A/K$ and $l = L/K$—also requires that the grid spacing in the $\log(a)$ and $\log(l)$ dimensions be the same (i.e., $a_{i+1}/a_i = l_{j+1}/l_j$, where $i, j = 1, 2, \dots, N$ index grid points) so that the set of investment rates $\{a_i/a_1, a_i/a_2, \dots, a_i/a_N\}$ maintains the state space on the grid.[56] This equivalency between the grid spaces in the $\log(a)$ and $\log(l)$ dimensions means that the solution is substantially simplified if the values of δ_K and δ_L are equal, so that depreciation leaves the $\log(l)$ dimension unchanged. When δ_K and δ_L are unequal, the difference between them needs to be an integer of the grid spacing. For the $\log(a)$ dimension depreciation is added to the drift in the stochastic process, so there is no constraint on δ_K. Given the conversion to logs, I need to apply the standard Jensen correction to the driving process (3.2), (3.3), and (3.4); for example, for (3.2), $\log(A_t^M) = \log(A_{t-1}^M) - (\sigma_{t-1}^2 - \sigma_L^2)/2 + \sigma_{t-1} W_t^M$. The uncertainty effect on the drift rate is second order compared to the real-options effect, so the simulations are virtually unchanged if this correction is omitted.

I used a grid of 40,000 points ($100 \times 100 \times 2 \times 2$). I also experimented with finer and coarser partitions and found that there were some changes in the value functions and policy choices as the partition changed, but the characteristics of the solution (i.e., a threshold response space as depicted in Figure 3) were unchanged as long as about 60 or more grid points were used in the $\log(a)$ and $\log(l)$ dimensions. Hence, the qualitative nature of the simulation results was robust to moderate changes in the number of points in the state space partition.

STEP 2: Define the value function on the grid of points. This is straightforward for most of the grid, but toward the edge of the grid, due to the random-walk nature of the demand process, this requires taking expectations of the value function off the edge of the state space. To address this, an extrapolation procedure is used to approximate the value function off the edge of the

[56]Note that some extreme choices of the investment rate will move the state off the l grid which induces an offsetting choice of employment growth rates e to ensure this does not occur.

state space. Under partial irreversibilities and/or fixed costs, the value function is log-linear outside the zone of inaction, so that as long as the state space is defined to include the region of inaction, this approximation is exact. Under quadratic adjustment costs the value function, however, is concave so a log-linear approach is only approximately correct. With a sufficiently large state space, however, the probability of being at a point off the edge of the state space is very low, so any approximation error will have little impact.

STEP 3: The value function iteration process. First, select a starting value for the value function in the first loop. I used the solution for the value function without any adjustment costs, which can be easily derived. In the SMM estimation routine I initially tried using the last solution in the next iteration, but found this could generate instability in the estimations loop. So instead I always used the same initial value function. The speed of value function iteration depends on the modulus of contraction, which with a monthly frequency and a 6.5% annual discount rate is relatively slow. The number of loops was fixed at 250, which was chosen to ensure convergence in the *policy* functions. In practice, value functions typically converge more slowly than the policy function rules associated with them. Thus, it is generally more efficient to stop the iterations when the policy functions have converged, even if the value function has not yet fully converged.

B.2. *Simulated Method of Moments Estimation*

To generate the simulated data for the SMM estimation (used to create $\Psi^S(\theta)$ in Equation (5.1)), I simulate an economy with 1000 firms, with 250 units each. This is run for 30 years, with the first 10 years discarded to eliminate the effects of any assumptions on initial conditions. Each firm is randomly assigned an initial drift parameter μ_L or μ_H. I run this simulations 25 times to try to average out over the impact of any individual macro shocks. The same seed is always used in every simulation iteration. I also assume firms are initially distributed equally across μ_L and μ_H given the symmetry of the transition matrix for $\mu_{i,t}$. To ensure that first-moment draws have a constant aggregate drift rate, I numerically set

$$\sum_{i,j} A_{i,j,t} = \exp^{(\mu_L+\mu_H)/2t} \sum_{i,j} A_{i,j,0},$$

consistent with (3.6) as $N \to \infty$, which in smaller samples stops extreme draws for individual units from driving macro averages.

I use a simulated annealing algorithm for minimizing the criterion function in the estimation step in Equation (5.1). This starts with a predefined first and second guess. For the third guess onward it takes the best prior guess and randomizes from this to generate a new set of parameter guesses. That is, it takes the best-fit parameters and randomly "jumps off" from this point for its next guess. Over time the algorithm "cools," so that the variance of the parameter jumps falls, allowing the estimator to fine tune its parameter estimates around the global best fit. I restart the program with different initial conditions to ensure the estimator converges to the global minimum. The simulated annealing algorithm is extremely slow, which is an issue since it restricts the size of the parameter space which can be estimated. Nevertheless, I use this because it is robust to the presence of local minima and discontinuities in the criterion function across the parameter space.

To generate the standard errors for the parameter point estimates, I generate numerical derivatives of the simulation moments with respect to the parameters and weight them using the optimal weighting matrix. One practical issue with this is that the value of the numerical derivative, defined as $f'(x) = \frac{f(x+\varepsilon)-f(x)}{\varepsilon}$, is sensitive to the exact value of ε chosen. This is a common problem with calculating numerical derivatives using simulated data with underlying discontinuities, arising, for example, from grid-point-defined value functions. To address this, I calculate four values of the numerical derivative for an ε of $+1\%$, $+2.5\%$, $+5\%$, and -1% of the midpoint of the parameter space[57] and then take the median value of these numerical derivatives. This helps to ensure that the numerical derivative is robust to outliers arising from any discontinuities in the criterion function.

REFERENCES

ABEL, A., AND J. EBERLY (1996): "Optimal Investment With Costly Reversibility," *Review of Economic Studies*, 63, 581–593. [631,632,639,672]

——— (2002): "Investment and q With Fixed Costs: An Empirical Analysis," Mimeo, Wharton School, University of Pennsylvania. [635]

ABRAHAM, K., AND L. KATZ (1986): "Cyclical Unemployment: Sectoral Shifts or Aggregate Disturbances?" *Journal of Political Economy*, 94, 507–522. [628]

ADDA, J., AND R. COOPER (2000): "Balladurette and Juppette: A Discrete Analysis of Scrapping Subsidies," *Journal of Political Economy*, 108, 778–806. [626]

AGHION, P., G. ANGELETOS, A. BANERJEE, AND K. MANOVA (2005): "Volatility and Growth: Credit Constraints and Productivity-Enhancing Investment," Working Paper 11349, NBER. [626]

BACHMAN, R., R. CABALLERO, AND E. ENGEL (2008): "Lumpy Investment in Dynamic General Equilibrium," Mimeo, Yale. [652,653,659]

BAILY, M., C. HULTEN, AND D. CAMPBELL (1992): "Productivity Dynamics in Manufacturing Plants," *Brookings Papers on Economic Activity*, 1992, 187–249. [648]

BARLEVY, G. (2004): "The Cost of Business Cycles Under Endogenous Growth," *American Economic Review*, 94, 964–990. [626]

BARNDORFF-NIELSEN, O., AND N. SHEPHARD (2006): "Econometrics of Testing for Jumps in Financial Economics Using Bipower Variation," *Journal of Financial Econometrics*, 4, 1–30. [627,675]

[57]For example, the midpoint of the parameter space for C_K^F is taken as 0.01, so that ε is defined as 0.0001, 0.00025, 0.0005, and -0.0001.

BARTELSMAN, E., R. BECKER, AND W. GRAY (2000): "NBER Productivity Database," available at www.nber.org. [661,662,677]

BERNANKE, B. (1983): "Irreversibility, Uncertainty and Cyclical Investment," *Quarterly Journal of Economics*, 98, 85–106. [626]

BERNARD, A., S. REDDING, AND P. SCHOTT (2006): "Multi-Product Firms and Product Switching," Working Paper 12293, NBER. [650]

BERTOLA, G., AND S. BENTOLILA (1990): "Firing Costs and Labor Demand: How Bad Is Eurosclerosis," *Review of Economic Studies*, 54, 318–402. [626]

BERTOLA, G., AND R. CABALLERO (1994): "Irreversibility and Aggregate Investment," *Review of Economic Studies*, 61, 223–246. [631,635,656]

BLOOM, N. (2008): "The Impact of Uncertainty Shocks," Working Paper 13385, NBER. [676]

——— (2009): "Supplement to 'The Impact of Uncertainty Shocks'," *Econometrica Supplemental Material*, 77, http://econometricsociety.org/ecta/Supmat/6248_data and programs.zip. [627,633]

BLOOM, N., S. BOND, AND J. VAN REENEN (2007): "Uncertainty and Investment Dynamics," *Review of Economic Studies*, 74, 391–415. [623,626,628,655]

BLOOM, N., M. FLOETOTTO, AND N. JAIMOVICH (2007): "Really Uncertain Business Cycles," Mimeo, Stanford. [627,674]

BLOOM, N., R. SADUN, AND J. VAN REENEN (2008): "The Organization of Firms Across Countries," Mimeo, LSE. [636]

BLOOM, N., M. SCHANKERMAN, AND J. VAN REENEN (2007): "Identifying Technology Spillovers and Product Market Rivalry," Working Paper 13060, NBER. [636]

BLOOM, N., AND J. VAN REENEN (2007): "Measuring and Explaining Management Practices Across Firms and Countries," *Quarterly Journal of Economics*, 122, 1351–1408. [636]

BOND, S., AND J. VAN REENEN (2007): "Microeconometric Models of Investment and Employment," in *Handbook of Econometrics* (forthcoming). [626]

BRESNAHAN, T., AND D. RAFF (1991): "Intra-Industry Heterogeneity and the Great Depression: The American Motor Vehicles Industry, 1929–1935," *Journal of Economic History*, 51, 317–331. [657]

CABALLERO, R., AND E. ENGEL (1993): "Microeconomic Adjustment Hazards and Aggregate Dynamics," *Quarterly Journal of Economics*, 108, 359–383. [626,632,656]

——— (1999): "Explaining Investment Dynamics in U.S. Manufacturing: A Generalized (S, s) Approach," *Econometrica*, 67, 783–826. [631,635,666]

CABALLERO, R., E. ENGEL, AND J. HALTIWANGER (1995): "Plant-Level Adjustment and Aggregate Investment Dynamics," *Brookings Papers on Economic Activity*, 2, 1–54. [626]

——— (1997): "Aggregate Employment Dynamics: Building From Microeconomics Evidence," *American Economic Review*, 87, 115–137. [626]

CABALLERO, R., AND J. LEAHY (1996): "Fixed Costs: The Demise of Marginal Q," Working Paper 5508, NBER. [637,639]

CHRISTIANO, L., M. EICHENBAUM, AND C. EVANS (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 113, 1–45. [625]

COOPER, R., AND J. HALTIWANGER (1993): "The Aggregate Implications of Machine Replacement: Theory and Evidence," *American Economic Review*, 83, 360–382. [626]

——— (2006): "On the Nature of the Capital Adjustment Process," *Review of Economic Studies*, 73, 611–633. [626,633,659,660,666]

COOPER, R., J. HALTIWANGER, AND J. WILLIS (2004): "Dynamics of Labor Demand: Evidence From Plant-Level Observations and Aggregate Implications," Working Paper 10297, NBER. [626,666]

COOPER, R., J. HALTIWANGER, AND L. POWER (1999): "Machine Replacement and the Business Cycle: Lumps and Bumps," *American Economic Review*, 89, 921–946. [626]

DAVIS, S., R. FABERMAN, AND J. HALTIWANGER (2006): "The Flow Approach to Labor Markets: New Data Sources and Micro–Macro Links," *Journal of Economic Perspectives*, 20, 3–26. [645, 659,677]

DAVIS, S., AND J. HALTIWANGER (1992): "Gross Job Creation, Gross Job Destruction, and Employment Reallocation," *Quarterly Journal of Economics*, 107, 819–863. [626,634]

DAVIS, S., J. HALTIWANGER, R. JARMIN, AND J. MIRANDA (2006): "Volatility and Dispersion in Business Growth Rates: Publicly Traded versus Privately Held Firms," Working Paper 12354, NBER. [628,636,662]

DIXIT, A. (1993): *The Art of Smooth Pasting, Fundamentals of Pure and Applied Economics*. Reading, U.K.: Harwood Academic. [672]

DIXIT, A., AND R. PINDYCK (1994): *Investment Under Uncertainty*. Princeton, NJ: Princeton University Press.

FOSTER, L., J. HALTIWANGER, AND C. KRIZAN (2000): "Aggregate Productivity Growth: Lessons From Microeconomic Evidence," in *New Developments in Productivity Analysis*. Chicago, IL: University of Chicago Press. [625,650]

——— (2006): "Market Selection, Reallocation and Restructuring in the U.S. Retail Trade Sector in the 1990s," *Review of Economics and Statistics*, 88, 748–758. [625,650]

GILCHRIST, S., AND J. WILLIAMS (2005): "Investment, Capacity and Uncertainty: A Putty–Clay Approach," *Review of Economic Dynamics*, 8, 1–27. [626]

HALL, B. (1987): "The Relationship Between Firm Size and Firm Growth in the U.S. Manufacturing Sector," *Journal of Industrial Economics*, 35, 583–606. [663]

HALL, R. (2004): "Measuring Factor Adjustment Costs," *Quarterly Journal of Economics*, 119, 899–927. [626,666]

HAMERMESH, D. (1989): "Labor Demand and the Structure of Adjustment Costs," *American Economic Review*, 79, 674–89. [626]

HASSLER, J. (1996): "Variations in Risk and Fluctuations in Demand—A Theoretical Model," *Journal of Economic Dynamics and Control*, 20, 1115–1143. [626]

HAYASHI, F. (1982): "Tobin's Marginal Q and Average Q: A Neoclassical Interpretation," *Econometrica*, 50, 213–224. [666]

JUDD, K. (1998): *Numerical Methods in Economics*. Cambridge, MA: MIT Press. [659]

KHAN, A., AND J. THOMAS (2003): "Nonconvex Factor Adjustments in Equilibrium Business Cycle Models: Do Nonlinearities Matter?" *Journal of Monetary Economics*, 50, 331–360. [652, 659]

——— (2008): "Idiosyncratic Shocks and the Role of Nonconvexities in Plant and Aggregate Investment Dynamics," *Econometrica*, 76, 395–436. [653]

KING, R., AND S. REBELO (1999): "Resuscitating Real Business Cycles," in *Handbook of Macroeconomics*, ed. by J. Taylor and M. Woodford. Amsterdam: Elsevier. [627,639]

KRUSELL, P., AND A. SMITH (1998): "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, 106, 867–896. [652]

LEAHY, J., AND T. WHITED (1996): "The Effects of Uncertainty on Investment: Some Stylized Facts," *Journal of Money Credit and Banking*, 28, 64–83. [626,628]

LEE, B., AND B. INGRAM (1991): "Simulation Estimation of Time Series Models," *Journal of Econometrics*, 47, 197–205. [658]

LILLIEN, E. (1982): "Sectoral Shifts and Cyclical Unemployment," *Journal of Political Economy*, 90, 777–793. [628]

MCFADDEN, D. (1989): "A Method of Simulated Moments for Estimation of Discrete Response Models Without Numerical Integration," *Econometrica*, 57, 995–1026. [657]

MEGHIR, C., AND L. PISTAFERRI (2004): "Income Variance Dynamics and Heterogeneity," *Econometrica*, 72, 1–32. [626]

MERZ, M., AND E. YASHIV (2007): "Labor and the Market Value of the Firm," *American Economic Review*, 97, 1419–1431. [626,666]

NICKELL, S. (1986): "Dynamic Models of Labor Demand," in *Handbook of Labor Economics*, Vol. 1, ed. by O. Ashenfelter and R. Layard. Amsterdam: North-Holland. [666]

OHANIAN, L. (2001): "Why Did Productivity Fall So Much During The Great Depression?" *American Economic Review*, 91, 34–38. [657]

PAKES, A., AND D. POLLARD (1989): "Simulation and the Asymptotics of Optimization Estimators," *Econometrica*, 57, 1027–1057. [657]

RAMEY, G., AND V. RAMEY (1995): "Cross-Country Evidence on the Link Between Volatility and Growth," *American Economic Review*, 85, 1138–1151. [626]

RAMEY, V., AND M. SHAPIRO (2001): "Displaced Capital A Study of Aerospace Plant Closings," *Journal of Political Economy*, 109, 958–992. [666]

ROMER, C. (1990): "The Great Crash and the Onset of the Great Depression," *Quarterly Journal of Economics*, 105, 597–624. [657]

SHAPIRO, M. (1986): "The Dynamic Demand for Labor and Capital," *Quarterly Journal of Economics*, 101, 513–542. [626,666]

SHILLER, R. (1981): "Do Stock Prices Move Too Much to Be Justified by Subsequent Changes in Dividends?" *American Economic Review*, 71, 421–436. [623]

SIM, J. (2006): "Uncertainty, Irreversible Investment and General Equilibrium," Mimeo, Boston University. [653]

STOKEY, N., AND R. LUCAS (1989): *Recursive Methods in Economic Dynamics, With E. Prescott*. Cambridge, MA: Harvard University Press. [637]

Dept. of Economics, Stanford University, 579 Serra Mall, Stanford, CA 94305, U.S.A. and the Centre for Economic Performance, London School of Economics and Political Science, London, U.K. and NBER; nbloom@stanford.edu.

Manuscript received January, 2006; final revision received December, 2008.