



# The VIX, the variance premium and stock market volatility<sup>☆</sup>



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## ABSTRACT

We decompose the squared VIX index, derived from US S&P500 options prices, into the conditional variance of stock returns and the equity variance premium. We evaluate a plethora of state-of-the-art volatility forecasting models to produce an accurate measure of the conditional variance. We then examine the predictive power of the VIX and its two components for stock market returns, economic activity and financial instability. The variance premium predicts stock returns while the conditional stock market variance predicts economic activity and has a relatively higher predictive power for financial instability than does the variance premium.

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## 1. Introduction

The 2007–2009 crisis has intensified the need for indicators of the risk aversion of market participants. It has also become increasingly commonplace to assume that changes in risk appetites are an important determinant of asset prices. Not surprisingly, the behavioral finance literature (see e.g. Baker and Wurgler, 2007) has developed “sentiment indices”, and financial institutions have created a wide variety of “risk aversion” indicators (see Coudert and Gex, 2008, for a survey).

One simple candidate indicator is the equity variance premium, the difference between the squared VIX index and an estimate of the conditional variance of the stock market. The VIX index is the “risk-neutral” expected stock market variance for the US S&P500 contract and is computed from a panel of options prices.

Well-known as a “fear index” (Whaley, 2000) for asset markets, it reflects both stock market uncertainty (the “physical” expected volatility), and a variance risk premium, which is also the expected premium from selling stock market variance in a swap contract. Bollerslev et al. (2009) show that an estimate of this variance premium predicts stock returns; Bekaert et al. (2013) show that there are strong interactions between monetary policy and the variance premium, suggesting that monetary policy may actually affect risk aversion in the market place. The variance premium uses objective financial market information and naturally “cleanses” option-implied volatility from the effect of physical volatility dynamics and uncertainty, leaving a measure correlated with risk aversion.

How to measure the variance premium is not without controversy, however, because it relies on an estimate of the conditional variance of stock returns. For example, the measure proposed in Bollerslev et al. (2009), BTZ, henceforth, assumes that the conditional variance of stock market returns is a martingale, an assumption which is not supported by the data, leading to potentially biased variance premiums. In this paper, we tackle several measurement issues for the variance premium, assessing a plethora of state-of-the-art volatility models and making full use of overlapping daily data, rather than sparse end-of-month data, which is standard.

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The conditional variance measure is of interest in its own right. First, there is a long literature on the trade-off between risk, as measured by the conditional variance of stock market returns, and the aggregate risk premium on the market (see e.g. French et al., 1987, for a seminal contribution). This long line of research has mostly failed to uncover a strong positive relationship between risk and return (see Bali, 2008, for a summary). Second, stock market volatility can also be viewed as a market-based measure of economic uncertainty. For example, Bloom (2009) shows that heightened “economic uncertainty” decreases employment and output. Interestingly, he uses the VIX index to measure uncertainty, so that his results may actually be driven by the variance premium rather than uncertainty per se.

Using more plausible estimates of the variance premium and stock market volatility, we then assess whether they predict stock returns, economic activity, as well as financial instability, an economic outcome whose monitoring is of considerable policy interest. We find that the well-known results in BTZ exaggerate the predictive power of the variance premium for stock returns. However, the equity variance risk premium remains a reliable predictor of stock returns. Stock market volatility does not predict the stock market, but it is a much better predictor of economic activity than is the equity variance premium. It also predicts financial instability more strongly than does the variance premium, especially at longer horizons.

The remainder of the paper is organized as follows. Section 2 discusses the econometric framework that we use to forecast volatility, and lays out our model selection procedure. Section 3 reports the results of our specification analysis and forecasting performance comparison. Section 4 uses the preferred estimates of the variance premium and stock market volatility to predict stock returns, economic activity and financial instability. Section 5 concludes.

## 2. Econometric framework

We define the variance risk premium as:

$$VP_t = VIX_t^2 - E_t \left[ RV_{t+1}^{(22)} \right]. \quad (1)$$

Here the  $VIX$  is the implied option volatility of the S&P500 index for contracts with a maturity of one month, and  $RV_{t+1}^{(22)}$  is the S&P500 realized variance measured over the next month (22 trading days) using five-minute returns. Note that  $RV_{t+1}^{(22)} - VIX_t^2$  is the return to buying variance in a variance swap contract. Therefore, technically speaking, the variance risk premium refers to the negative of VP (see Carr and Wu, 2009). Since that number is mostly negative, we prefer to define it as we did in Eq. (1).

Economically, the squared  $VIX$  is the conditional return variance using a “risk-neutral” probability measure, whereas the conditional variance uses the actual “physical” probability measure. The risk-adjusted measure shifts probability mass to states with higher marginal utility (bad states) and this implies that in many realistic economic settings, the variance premium will be increasing in the economy’s risk aversion.

The unconditional mean of the variance premium is easy to compute by simply computing the average of  $VIX_t^2 - RV_{t+1}^{(22)}$ . However, we are interested in the conditional variance premium as described in Eq. (1), which relies on the physical conditional expected value of the future realized variance. The common approach to estimate this uses empirical projections of the realized variance on variables in the information set, and subtracts this estimated expected variance from the  $VIX^2$  to arrive at VP. Hence, the problem is reduced to one of variance forecasting.

Our data start on January 02, 1990 (the start of the model-free VIX series)<sup>1</sup> and covers the period until October 01, 2010. We have a total of 5208 daily, overlapping observations. The recent crisis period presents special challenges as stock market volatilities peaked at unprecedented levels, but at the same time the crisis represents an informative period during which uncertainty and risk aversion may have been particularly pronounced. Nevertheless, if we decompose the sample variance of the implied and realized volatility series in contributions by crisis and non-crisis observations, the crisis observations dominate despite representing a relatively small part of the sample. We deal with the crisis-induced challenges by considering both models that predict the level and the logarithm of realized variances, and by putting much emphasis on parameter stability in our model selection procedure. In addition, we focus on out-of-sample forecasting exercises where we conduct the in-sample estimations mostly on non-crisis observations, so that the influence of the crisis on the parameter estimates and model selection is mitigated.

### Variance forecasting

There is an extensive econometric literature on volatility forecasting. It is now generally accepted that models based on high frequency realized variances dominate standard models in the GARCH class (see e.g. Chen and Ghysels, 2012) and we therefore examine the state-of-the-art models in that class. These models stress the importance of persistence (using lagged realized variances as predictors), additional information content in the most recent return variances (Corsi, 2009), asymmetry between positive and negative return shocks (the classic volatility asymmetry, see e.g. Engle and Ng, 1993) and potentially differing predictive information present in jump versus continuous volatility components (Andersen et al., 2007). We accommodate all of these elements in our model.

In the finance literature, it has been pointed out as early as in Christensen and Prabhala (1998) that option prices as reflected in implied volatility should have information about future stock market volatility. This motivates using the  $VIX$  as a predictive variable. Recent articles using the  $VIX$  in similar forecasting exercises include Busch et al. (2011) who examine a number of variance forecasting models embedding option-implied volatility for bond, currency and stock markets, and Andersen and Bondarenko (2007) who mostly focus on measurement issues with the officially published  $VIX$  index. Of course, because the  $VIX$  also embeds a risk premium, it will not be an unbiased predictor of future realized volatility. Chernov (2007) argues that spot volatility is likely to have additional information about future volatility.

Finally, it is well-known that estimation noise hurts out-of-sample forecasting performance. Simple models such as the martingale model may therefore outperform more complex models. We therefore also consider a number of non-estimated models that are special cases of our general framework.

Our most general forecasting model can be represented as follows:

$$\begin{aligned} RV_t^{(22)} = & c + \alpha VIX_{t-22}^2 + \beta^m C_{t-22}^{(22)} + \beta^w C_{t-22}^{(5)} + \beta^d C_{t-22}^{(1)} \\ & + \gamma^m J_{t-22}^{(22)} + \gamma^w J_{t-22}^{(5)} + \gamma^d J_{t-22}^{(1)} + \delta^m r_{t-22}^{(22)-} \\ & + \delta^w r_{t-22}^{(5)-} + \delta^d r_{t-22}^{(1)-} + \varepsilon_t. \end{aligned} \quad (2)$$

<sup>1</sup> The CBOE changed the methodology for calculating the VIX, initially measuring implied volatility for the S&P100 index, to be measured in a model-free manner from a panel of option prices (see Bakshi et al., 2003, for details) only in September 2003. It then backdated the new model-free index to 1990 using historical option prices (see CBOE, 2004, for more details).

**Table 1**  
Models considered.

Variables	$VIX^2$	$C^{(22)}$	$C^{(5)}, C^{(1)}$	$J^{(22)}$	$J^{(5)}, J^{(1)}$	$r^{(22)-}$	$r^{(5)-}, r^{(1)-}$
Estimated models							
(Log) Model 1	X						
(Log) Model 2		X					
(Log) Model 3	X	X					
(Log) Model 4	X	X		X			
(Log) Model 5		X		X			
(Log) Model 6	X	X		X		X	
(Log) Model 7		X		X		X	
(Log) Model 8	X	X	X				
(Log) Model 9		X	X				
(Log) Model 10	X	X	X	X	X		
(Log) Model 11		X	X	X	X		
(Log) Model 12	X	X	X	X	X	X	X
(Log) Model 13		X	X	X	X	X	X
(Log) Model 14	Outcome of the general-to-specific (Gets) model selection — see <a href="#">Table 2</a>						
Non-estimated models							
Model 29	X						
Model 30		X					
Model 31	0.5 * X	0.5 * X					

Notes: Summary of variables included in estimated and non-estimated models. In models without jumps, the relevant realized variances ( $RV^{(22)}$ ,  $RV^{(5)}$  and  $RV^{(1)}$ ) are used instead of the continuous variations ( $C^{(22)}$ ,  $C^{(5)}$  and  $C^{(1)}$ ); i.e., in estimated (log) models 2, 3, 8, 9, and non-estimated models 30 and 31.

We want to forecast the monthly (22 trading days) S&P500 realized variance, denoted by  $RV_t^{(22)}$ , and defined as the sum of daily realized variances  $RV$  over the 22 days,  $RV_t^{(22)} = \sum_{j=1}^{22} RV_{t-j+1}$ . The daily realized variance sums squared five-minute intraday returns and the squared close-to-open return, with returns expressed in percentage form.<sup>2</sup> Our first independent variable is the  $VIX^2$  (expressed in monthly percentages squared, i.e.  $VIX^2/12$  where  $VIX$  is the quoted  $VIX$  index level in annualized percent), and we expect  $\alpha$  to be positive. The next six variables separate the realized variance into a continuous and a discontinuous (“jump”) component (at the monthly, weekly and daily frequencies), following Andersen et al. (2007). To isolate the jumps contribution to daily quadratic variation, we use threshold bipower variation proposed by Corsi et al. (2010). Corsi et al. (2010) show that this threshold measure substantially reduces the small-sample bias that the standard bipower variation (Barndorff-Nielsen and Shepherd, 2004) estimates exhibit.<sup>3</sup> The daily jump, denoted as  $J_t$ , is defined as:

$$J_t = \max[(RV_t - TBPV_t), 0] \quad (3)$$

where  $TBPV_t$  stands for threshold bipower variation defined in Corsi et al. (2010), equation 2.14. The continuous component of the daily quadratic variation is given by:

$$C_t = RV_t - J_t. \quad (4)$$

Weekly ( $h = 5$ ) and monthly ( $h = 22$ ) variables are averaged, and we express all variables in monthly units so that:  $J_t^{(h)} = \frac{22}{h} \sum_{j=1}^h C_{t-j+1}$ .

Finally, following Corsi and Renò (2012), we add negative returns over the past day, week and month, to incorporate a potential leverage effect (see Campbell and Hentschel, 1992; Bekaert

and Wu, 2000). To model the leverage effect at different frequencies, we define  $r_t^{(h)-} = \min[r_t^{(h)}, 0]$  where  $r_t^{(h)} = \frac{22}{h} \sum_{j=1}^h r_{t-j+1}$ .

In addition to forecasting realized variance in levels, we also consider models that predict the logarithm of the realized variance. The logarithmic counterpart of the model in (2) reads:

$$\begin{aligned} \ln RV_t^{(22)} = & c + \alpha \ln VIX_{t-22}^2 + \beta^m \ln C_{t-22}^{(22)} + \beta^w \ln C_{t-22}^{(5)} \\ & + \beta^d \ln C_{t-22}^{(1)} + \gamma^m \ln(1 + J_{t-22}^{(22)}) \\ & + \gamma^w \ln(1 + J_{t-22}^{(5)}) + \gamma^d \ln(1 + J_{t-22}^{(1)}) \\ & + \delta^m r_{t-22}^{(22)-} + \delta^w r_{t-22}^{(5)-} + \delta^d r_{t-22}^{(1)-} + \varepsilon_t. \end{aligned} \quad (5)$$

Because variances have right-skewed distributions, but logarithmic variances tend to have near Gaussian distributions, it may be easier to predict logarithmic variances with linear models. However, ultimately, we still need to identify the model that best forecasts the level of the realized variance. To this end, when we consider a logarithmic model, we assume log-normality to predict levels of monthly realized variances:

$$E_t[RV_{t+1}^{(22)}] = \exp\left(E_t[r_{t+1}^{(22)}] + \frac{1}{2} \text{var}[r_{t+1}^{(22)}]\right) \quad (6)$$

where  $r_{t+1}^{(22)} = \ln(RV_{t+1}^{(22)})$ . We use the logarithmic model to compute the conditional expectation of  $r_{t+1}^{(22)}$  and the sample variance of  $r_{t+1}^{(22)}$  to compute the variance term.

#### Model selection procedure

Our model selection procedure consists of comparing the out-of-sample forecasting performance of a set of 31 estimated and non-estimated models and examining their stability. The models are summarized in Table 1. For estimated models, we consider 14 variants of the encompassing model in levels (Eq. (2)) and in logarithms (Eq. (5)), respectively (i.e., we estimate 28 models in total). We estimate the models using OLS. Models 1 through 13 have fixed combinations of predictors. Models 14 for the level and the logarithmic specification, respectively, are models with a set of predictors selected by a general-to-specific (Gets) model selection procedure (using the full sample). While sequential Wald tests

<sup>2</sup> We use actual S&P500 returns. Other papers in the literature focused on S&P500 futures including, e.g., Andersen et al. (2007) and Corsi et al. (2010).

<sup>3</sup> The upward bias in bipower variation leads to a continuous variation that is too large and a jump component that is too small, and thus potentially also biases estimates of the jumps coefficients in models such as (2). To obtain threshold bipower variation, we use equation 2.14 in Corsi et al. (2010), with the threshold function as defined in their equation 2.15 and the scale-free constant set to 3. The construction of the estimator of the local variance is described in Appendix B of their paper.

**Table 2**  
General-to-specific (Gets) model selection.

Model	(1)	(2)
$VIX^2$		0.456*** [0.019]
$C^{(22)}$	−0.265 [0.267]	0.215*** [0.040]
$C^{(5)}$	0.282*** [0.094]	0.184*** [0.039]
$C^{(1)}$	0.126*** [0.047]	0.093*** [0.014]
$J^{(22)}$	2.052*** [0.635]	
$J^{(5)}$		
$J^{(1)}$		
$r^{(22)-}$	−0.955* [0.536]	
$r^{(5)-}$	−0.318** [0.138]	
$r^{(1)-}$	−0.211*** [0.072]	−0.002*** [0.0006]
# daily observations	5208	5208

Notes: Sample period January 02, 1990–October 01, 2010. Column 1 reports estimates for the level model while column 2 reports estimates for the logarithmic model. The standard errors reported in brackets are computed using 44 Newey–West lags.

\* Significance at the 0.10-level.

\*\* Significance at the 0.05-level.

\*\*\* Significance at the 0.01-level.

present problems in terms of selecting the size of tests (see e.g. Bhargava, 1987), we rely on the large body of work on model selection by David Hendry and co-authors, see e.g. Campos et al. (2003) and Hendry and Krolzig (2005). We use the recent implementation by Autometrics in the econometrics software package OxMetrics (see Doornik, 2009, for details). In terms of practical implementation, we use 5% as the target size for our tests, the default value in the software; we also do not use indicator saturation techniques, but test for parameter instability separately. We use PcGive, version 13 (see Doornik et al., 2009).

In Table 2, we discuss the level and logarithmic models chosen by the Gets model selection analysis. The standard errors below the parameters are computed using 44 Newey and West (1987) lags. The model selection yields models with monthly, weekly and daily continuous variation in both cases. In the level regression model, the Gets model selection procedure retains monthly jumps, in addition to negative returns at all three frequencies. In the logarithmic model, the squared  $VIX$  is chosen, along with the daily negative returns.

In addition, we consider 3 non-estimated models: the lagged squared  $VIX$  (model 29); the lagged realized variance (model 30; this is the model used in BTZ); and 0.5 times the lagged squared  $VIX$  plus 0.5 times the lagged realized variance (model 31).

We estimate the models using daily data between January 1, 1990 and July 15, 2005 (representing about 75% of the full sample) and use the rest of the sample (till October 01, 2010) to measure forecasting performance. The parameters are not updated.

We examine five different criteria. We compute the root-mean-squared error (RMSE), mean absolute error (MAE), and mean absolute percentage error (MAPE; the absolute error in percent of the actual realized variance). We evaluate whether the forecast error measures are significantly different among competing forecasting models through the Diebold and Mariano (1995) test (with standard errors computed using 44 Newey–West lags), using a 10% sig-

nificance level.<sup>4</sup> We also compute the  $R^2$  of Mincer and Zarnowitz (1969) forecasting regressions, that is, we compute the  $R^2$  in a regression of actual data on their forecasted values. The final criterion we examine is a simple joint Chow test for parameter stability over the last part of the sample versus the estimation part of the sample. Ericsson (1992) discusses formally how mean-squared-error minimization and parameter constancy are both necessary (but not sufficient) conditions to obtain a good forecasting model. We also produce the average correlation of the forecasts generated by a particular model with the forecasts generated by the winning model on each of the first 4 criteria. This gives a sense of how close different forecast models are economically.

### 3. Model selection results

Table 3 produces the statistics and the average ranking of our 31 considered models according to the criteria discussed above. Recall that for the logarithmic models, we are predicting the level of the realized variance as discussed in Section 2. The first column reports whether the model is stable according to the Chow test (using 10% significance level). Stable models are bolded. In the second column, we report the in-sample RMSE (denoted RMSE\*). In the next three columns, we report the out-of-sample RMSE, MAE and MAPE criteria.<sup>5</sup> We note that the out-of-sample RMSE is considerably larger than its in-sample counterpart. This is also true for the MAE criterion, but it is not true for the MAPE criterion, where out-of-sample errors are often smaller than in-sample errors (not reported). Because the realized variance became very large during the crisis, which constitutes a substantial part of the out sample, it is not surprising to see larger (absolute) errors out of sample. However, the results for the MAPE criterion suggest that, in relative terms, the errors did not increase. We test for each model whether it generates a statistic significantly different from the statistic generated by the best ranked model (i.e., model 14 for RMSE, model 8 for MAE, and model 4 for MAPE; all models in levels). When such a test fails to reject, the statistic is bolded. We view such tests as critical in model selection. A model may rank relatively low, but the criterion may have little power to distinguish different models and generate very similar forecast errors. For example, a quick glance at the table reveals that the RMSE criterion has little power to distinguish alternative models, while MAPE is the most distinguishing one. For the  $R^2$  criterion (in the 5th column), we view a difference of more than 5% with the winning model (model 14) as a significant difference in economic terms. Model statistics similar in  $R^2$  to the winning model are bolded. The 6th column produces an average correlation, averaging the correlation of each model's forecasts with the forecasts of the winning models in all four out-of-sample quantitative criteria (RMSE, MAE, MAPE, and  $R^2$ ). If a model were to be the top model on each criterion, it would get a correlation of 1. Finally, we rank models in each of the four categories (from 1, best, to 31, worst) and produce the average ranking score for each model in the last column of Table 3.

<sup>4</sup> We also computed two other statistics; the heteroskedasticity adjusted root-mean-squared error suggested in Bollerslev and Ghysels (1996) and the QLIKE loss function (see Patton, 2011). However, these statistics produce rankings very similar to the MAPE-criterion, so we do not discuss them further. Also note that the Diebold and Mariano test only uses the forecast errors and ignores the underlying model structure and estimation. While we could in principle use more complex statistics that take the model structure and estimation into account (see e.g. West, 2006), recent research suggests that the Diebold and Mariano test works well even in model-based out-of-sample forecasting comparisons (see Clark and McCracken, 2011; Diebold, 2013).

<sup>5</sup> Although the Diebold–Mariano test uses the mean-squared error (MSE) rather than RMSE, we report the RMSE so that RMSE and MAE criteria have a comparable scale (both are in the realized variance units, monthly percentages squared). MAPE is expressed in percent of the realized variance and is thus scale-free.



**Table 3**  
Model statistics and ranking.

Model	RMSE*	RMSE	MAE	MAPE	R <sup>2</sup>	Correlations	Average score
Estimated level models							
<b>Model 1</b>	11.462	<b>52.978</b>	<b>18.653</b>	<b>0.342</b>	0.411	0.954	15.50
<b>Model 2</b>	11.618	<b>49.610</b>	19.089	0.494	0.484	0.956	14.50
<b>Model 3</b>	10.909	<b>49.267</b>	<b>17.650</b>	0.354	0.491	0.974	9.50
Model 4	10.458	<b>55.336</b>	<b>19.047</b>	<b>0.318</b>	0.358	0.945	17.75
Model 5	11.485	<b>52.059</b>	19.190	0.498	0.432	0.951	19.75
Model 6	10.147	<b>53.171</b>	<b>18.785</b>	0.344	0.407	0.964	16.50
Model 7	10.600	<b>50.150</b>	18.798	0.468	0.472	0.967	14.50
<b>Model 8</b>	10.508	<b>46.077</b>	<b>16.856</b>	<b>0.347</b>	<b>0.555</b>	0.976	4.00
<b>Model 9</b>	10.822	<b>45.450</b>	17.523	0.445	<b>0.567</b>	0.969	4.75
Model 10	10.196	<b>50.635</b>	<b>17.603</b>	<b>0.324</b>	0.462	0.974	12.00
<b>Model 11</b>	10.658	<b>46.614</b>	<b>17.400</b>	0.446	<b>0.544</b>	0.975	6.50
Model 12	9.988	<b>50.261</b>	<b>17.863</b>	0.348	0.470	0.978	12.50
Model 13	10.237	<b>46.970</b>	<b>17.512</b>	0.435	<b>0.537</b>	0.981	6.75
Model 14	10.592	<b>45.147</b>	<b>17.537</b>	0.403	<b>0.572</b>	0.978	3.75
Estimated logarithmic models							
Log Model 1	13.137	<b>49.853</b>	19.059	0.406	0.479	0.957	13.50
<b>Log Model 2</b>	13.531	49.590	22.605	0.706	0.484	0.957	20.50
Log Model 3	12.985	<b>50.729</b>	21.762	0.562	0.460	0.974	22.00
Log Model 4	12.293	<b>50.794</b>	19.699	0.498	0.459	0.963	19.00
<b>Log Model 5</b>	13.225	49.066	22.066	0.730	0.495	0.957	19.25
Log Model 6	12.457	<b>50.590</b>	19.887	0.505	0.463	0.970	18.50
<b>Log Model 7</b>	13.226	<b>61.642</b>	25.194	0.723	0.203	0.932	30.25
Log Model 8	12.680	<b>48.004</b>	20.981	0.571	0.517	0.981	15.00
<b>Log Model 9</b>	12.852	<b>45.996</b>	21.072	0.662	<b>0.556</b>	0.977	13.50
Log Model 10	12.191	<b>48.031</b>	19.661	0.529	0.516	0.977	13.25
<b>Log Model 11</b>	12.485	<b>45.538</b>	21.077	0.694	<b>0.565</b>	0.977	13.50
Log Model 12	12.263	<b>49.271</b>	20.375	0.537	0.491	0.969	16.50
Log Model 13	12.792	<b>54.713</b>	23.452	0.695	0.372	0.942	27.50
Log Model 14	12.322	48.248	19.989	0.541	0.512	0.971	15.00
Non-estimated models							
Model 29	25.774	55.359	29.940	1.055	0.357	0.954	30.50
Model 30	12.548	<b>53.276</b>	22.254	0.500	0.405	0.956	24.25
Model 31	16.265	52.051	24.310	0.717	0.432	0.974	25.75

Notes: Selected model statistics based on the out-of-sample performance. Parameters are estimated using data between January 1, 1990 and July 15, 2005; the rest of the sample is used to assess forecasting performance. The 1st column reports the Chow stability test results (stable models bolded). The 2nd column shows the in-sample RMSE (RMSE\*). The next four columns show the out-of-sample RMSE, MAE, MAPE, and Mincer–Zarnowitz R<sup>2</sup> (see Section 3 for the “bolding” criteria). The 7th column produces the average correlation of each model with the best models in the RMSE, MAE, MAPE, R<sup>2</sup> categories. The 8th column produces the average ranking score of each model in the RMSE, MAE, MAPE, R<sup>2</sup> categories.

Using this information, we winnow down our set of models by requiring a good model to be bolded in at least 3 out of 4 quantitative criteria. This leaves us with only 7 models: level models 1, 4, 8, 10, 11, 13 and 14. Model 14 produces the best average ranking score (3.75) but is not stable. Indeed, only three of these seven models are stable: models 1, 8 and 11. Model 8 has the second best average score (4). It is also the only model which gets four bolds. Model 11 has the fourth best average score (6.5). Meanwhile, model 1 ranks only 16 in the average score. We therefore select models 8 and 11 as the winning models. Model 8 is Corsi's HAR model, supplemented with the squared VIX. Model 11 features continuous and jump variations at all three frequencies. More generally, the presence of realized variances (or their continuous components) at all three frequencies is important in delivering lower error statistics. In terms of R<sup>2</sup>, more complex models (level models 8, 9, 11, 13, 14 and logarithmic models 9 and 11) yield substantially higher values than the other models.

Over the full sample, the resulting coefficients for models 8 and 11 (with heteroskedasticity-robust standard errors in brackets) are:

$$\begin{aligned}
 RV_t^{(22)} = & 3.730 + 0.108 VIX_{t-22}^2 + 0.199 RV_{t-22}^{(22)} \\
 & (1.903) \quad (0.072) \quad (0.096) \\
 & + 0.330 RV_{t-22}^{(5)} + 0.107 RV_{t-22}^{(1)} \\
 & (0.117) \quad (0.026)
 \end{aligned} \quad (7)$$

$$\begin{aligned}
 RV_t^{(22)} = & 3.855 - 0.212 C_{t-22}^{(22)} + 0.237 C_{t-22}^{(5)} + 0.223 C_{t-22}^{(1)} \\
 & (1.164) \quad (0.237) \quad (0.064) \quad (0.064) \\
 & + 1.742 J_{t-22}^{(22)} + 0.327 J_{t-22}^{(5)} - 0.016 J_{t-22}^{(1)} \\
 & (0.702) \quad (0.262) \quad (0.056)
 \end{aligned} \quad (8)$$

Table 3 also shows statistics of some popular simple models used in the literature: the squared VIX – realized variance model used in Bekaert et al. (2013, our model 3), the martingale model of BTZ (model 30) and the AR(1) model of Londono (2011, model 2). Compared to the top models, the martingale and simple autoregressive models perform an order of magnitude worse but the squared VIX – realized variance model delivers quite similar performance. Of the simpler models, Model 3 is the best on all four criteria (only Model 1 does better on MAPE). It also has the best average score among the simpler models, and the sixth best average score overall.

#### Additional exercises

We performed two alternative exercises. First, we repeated our analysis with two alternative sample splits in the out-of-sample forecasting exercise. We estimated the models using data until June 16, 2003 (about 65% of the full sample) and until August 1, 2007 (about 85% of the sample), respectively. Results for the winning models 8 and 11 are remarkably robust. For the 65% split, out-of-sample forecasting performance is uniformly better, with lower

errors and higher  $R^2$ , with the best model attaining an  $R^2$  of just over 59%, i.e., 2% higher than in the 75% split. The winning models are the same as in the 75% split: model 4 on MAPE, model 8 on MAE, and model 14 on  $R^2$  and RMSE. Models that get 3 bolds out of 4 are models 1, 3, 4, 8, 10, and 14 (all in levels), with models 4, 10 and 14 being unstable (as in the 75% sample split). Model 8 is again the only model that gets 4 bolds. For the 85% split, where all out-of-sample predictions are made over the period that includes the recent financial crisis, all models produce higher errors and lower  $R^2$ . Model 8 is again the winning model on the MAE criterion, with level model 10 winning on MAPE and logarithmic model 11 on RMSE and  $R^2$ . Models that get 3 bolds out of 4 largely overlap with those in the 75% split: models 4, 8, 9, 10, 11, 13 and 14. Only models 8, 9 and 11 are stable, however. In sum, model 8 is consistently a top performer across all three sample splits. Model 11 does well in two of three sample splits, including the split that emphasizes performance during the financial crisis.<sup>6</sup>

Second, we re-consider our forecasting exercise with end-of-month data. In most of the existing articles (including BTZ, [Londono, 2011](#); [Busch et al., 2011](#)), end-of-month data are used to estimate conditional variance models. The use of daily data should lead to more efficient estimates, but the correlation between daily and monthly data induced by the overlapping data structure may make the increase in efficiency minor. We estimated all our models using end-of-the-month data till mid-2005, mimicking the 75% sample split used in our main forecasting exercise with daily data. We then use the obtained regression coefficients to construct daily out-of-sample realized variance forecasts for the remainder of the sample. Computing the usual criteria, we check whether we can accept the various models by looking for three bolds on our four quantitative criteria (MAPE, MAE, RMSE, and  $R^2$ ), and we verify the stability criterion.

With end-of-month data, the winning models in the four quantitative categories are the same as with the daily data (Model 4 on MAPE, Model 8 on MAE, and Model 14 on RMSE and  $R^2$ ). The winning models based on daily data beat those based on monthly data on all statistics with the exception of the MAPE criterion, where model 4 displays very comparable statistics (0.317 with end-of-month versus 0.318 with daily data). Models 1, 4 and 8 still get 3 bolds out of 4. However, model 4 is again unstable. Interestingly, model 8 based on monthly estimates does well relative to the best models based on the daily information. When estimated using the full sample, the monthly model puts less weight on the squared  $VIX$  and  $RV^{(22)}$ , and more weight on  $RV^{(5)}$  and  $RV^{(1)}$  than the daily model does. For the simpler models, model 3 (used in [Bekaert et al., 2013](#)) and model 1 are stable, and get 3 bolds out of 4. However, model 3 does better on the four quantitative criteria than model 1. The more complex models, like our previously winning model 11, which include jumps and/or asymmetric volatility, are, not surprisingly, more difficult to identify with monthly data. The use of monthly estimation samples should therefore best be restricted to relatively simple models, where the loss of efficiency is not very costly.

#### 4. Economics and predictability

##### *Risk and risk aversion*

In [Fig. 1](#), we plot the daily series for the variance risk premium (VP henceforth; displayed in Panel A), which may potentially

<sup>6</sup> We also verified that our winning models are overall stable. That is, using the monthly data set we conducted the standard unknown breakpoint test (Quandt–Andrews test; implemented in EViews 6), with 10% trimming, and found no evidence of instability. The test points to June 2008 as the most likely break for both models 8 and 11 but it is not statistically significant.

serve as a proxy for risk aversion, and the conditional (physical) variance of the stock market (CV henceforth; in Panel B), which may potentially serve as a measure of economic uncertainty. We show the two series obtained from the winning models 8 and 11 on one graph. The VP and CV series are positively correlated (correlation of 0.45 for model 8 and 0.27 for model 11) and display peaks at the expected times. The largest peaks for CV are observed during the Lehman aftermath in the recent crisis and at the time of the corporate scandals following the Enron debacle. Interestingly, the 1998 Russian crisis and the Gulf war did not generate much uncertainty, but these events do feature substantially elevated levels of VP. The Lehman event seems to have caused both massive uncertainty and massive risk aversion.

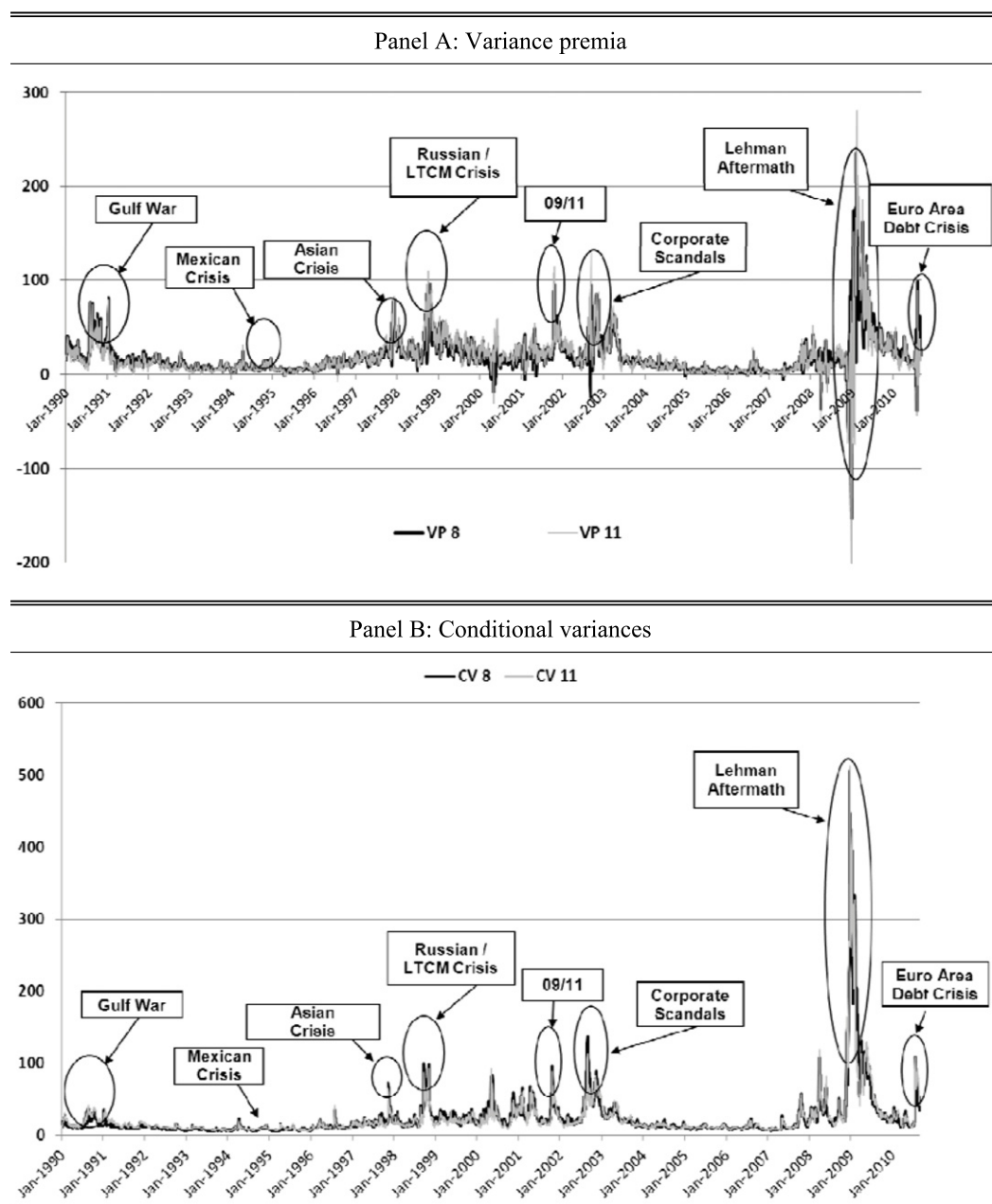
When realized variances show extreme peaks, the VP series can become negative, which happens more for model 11 than for model 8. This is a disadvantage of all these models. It is unlikely that during these periods of stress there was a sudden increase in risk appetite. The more mundane explanation is that realized variances likely have different components with different levels of mean reversion. In a massive crisis, some of the realized variance movements should probably be allowed to mean-revert more quickly and not affect the conditional variance as much as they do now. The models with jumps could theoretically capture this by having negative coefficients on the jump terms. However, model 11 puts a large positive coefficient on the monthly and weekly jump components, and a very small negative one on the daily jump component. Overall, it is likely that a non-linear model may be better equipped to capture the behavior of CV and VP in severe crises.

##### *Predicting stock market returns*

The two components of the squared  $VIX$  index have been considered as separate potential predictors of stock market returns. Starting with [French et al. \(1987\)](#), a large literature focuses on the relationship between aggregate stock market returns and their conditional variance. In a simple static CAPM model, the coefficient on the conditional stock market variance would be the wealth weighted risk aversion coefficient, but such a relationship need not hold perfectly in a dynamic model. In the literature on the risk–return relationship, estimates vary from positive to negative and the relationship is often insignificant. [Lundblad \(2007\)](#) suggests that the samples typically used are too short to uncover a relationship that is robustly and statistically significantly positive in the sample of over 150 years that he considers. Yet, the measurement of the conditional variance of stock returns may matter, too. The bulk of the extant literature has considered GARCH-in-mean models to measure the conditional stock market variance, which likely induces substantial measurement error in the regression. [Ghysels et al. \(2005\)](#) recover a positive risk–return trade-off measuring the conditional variance with a flexible function of past returns, applying MIDAS modeling. However, [Hedegaard and Hodrick \(2013\)](#) dispute these results, mostly finding insignificant coefficients with either GARCH or (adjusted) MIDAS models to measure the conditional variance.

BTZ recently showed that the variance risk premium has predictive power for future stock returns, which is logical since it harbors information about aggregate risk aversion. As shown above, their measure implicitly uses a volatility model that is strongly rejected by the data. We therefore re-consider the predictive power of both the equity variance risk premium (“risk”) and the conditional variance of the stock market (“uncertainty”), using our improved measures of the conditional variance of stock market returns.

We start with regressions using only the variance premium as a predictor of equity returns. Like BTZ, we rely on end-of-the-month observations but we consider various estimates of the variance



Notes: Daily series for the variance premium (VP) and the conditional variance (CV) from the winning models 8 and 11.

Fig. 1. Variance premium and conditional variance.

premium.<sup>7</sup> Table 4, Panel A contains the results. The left hand side variable is always excess stock returns (the S&P500 return in excess of the three-month T-bill rate; expressed in annualized percentages). We use three different horizons, monthly, quarterly and annual (denoted by 1, 3 and 12, respectively), averaging returns over a quarter/year. The overlap in the monthly data creates serial correlation in the error term that must be corrected for in creating standard errors. We use a relatively large number of Newey–West lags, namely  $\max\{3, 2 \times \text{horizon}\}$ , to do so, rather than create standard errors under the null of no predictability, as in Hodrick (1992). While

the Hodrick estimator has very good size properties, selecting a large number of lags may improve power (see Sun et al., 2008).

In the last specification in Panel A, we show that the squared VIX itself fails to predict stock returns. Just above, we repeat the BTZ specification that uses the past realized variance as the estimate of the conditional variance of stock market returns. The resulting variance premium proxy predicts stock market returns at all three horizons with the predictive power strongest at the quarterly horizon, both in terms of magnitude of the coefficient and the adjusted  $R^2$ . The  $R^2$  for the martingale model in the quarterly regression increases from 7% in the original BTZ sample to 13% in our sample. Thus, including the financial crisis actually strengthens BTZ's results. Compared to the predictability results when using the two best models – models 8 and 11 – to estimate the variance premium, BTZ's martingale model maximizes the predictive power of the variance premium for returns. For the best models, there is only

<sup>7</sup> We also performed all of our regressions using the BTZ sample, which ends in December 2007 and therefore conveniently excludes the crisis period. We note any interesting differences between results with and without the crisis period in the text.

**Table 4**  
Stock return regressions.

Horizon	1	3	12	1	3	12	1	3	12	1	3	12
Panel A: Monthly, quarterly and annual regressions with variance premium												
VP 8	0.311 [0.293]	0.426** [0.175]	0.241** [0.114]									
VP 11				0.304 [0.256]	0.289 [0.184]	0.169* [0.092]						
VP 30							0.527*** [0.121]	0.575*** [0.073]	0.168*** [0.055]			
VIX <sup>2</sup>										−0.030 [0.166]	0.019 [0.139]	0.061 [0.045]
Constant	−1.045 [5.285]	−3.069 [4.148]	0.841 [4.861]	−0.907 [5.434]	−0.574 [4.066]	2.148 [4.379]	−4.633 [3.439]	−5.408* [3.167]	2.273 [3.809]	5.835 [5.538]	3.954 [4.666]	2.829 [4.344]
Adj. R <sup>2</sup>	0.005	0.042	0.045	0.008	0.025	0.029	0.037	0.130	0.034	−0.004	−0.004	0.012
Panel B: Monthly, quarterly and annual regressions with variance premium and conditional variance												
VP 8	0.593* [0.330]	0.688*** [0.145]	0.287*** [0.092]									
CV 8	−0.358** [0.155]	−0.333** [0.061]	−0.058 [0.062]									
VP 11				0.421 [0.260]	0.362** [0.180]	0.171** [0.084]						
CV 11				−0.296* [0.177]	−0.183** [0.089]	−0.005 [0.051]						
VP 30							0.478*** [0.174]	0.564*** [0.109]	0.211*** [0.080]			
CV 30							−0.060 [0.078]	−0.013 [0.054]	0.052 [0.060]			
Constant	1.505 [4.760]	−0.696 [3.968]	1.258 [4.981]	3.324 [4.637]	2.045 [4.286]	2.214 [4.651]	−2.433 [4.434]	−4.918 [3.976]	0.383 [5.112]			
Adj. R <sup>2</sup>	0.024	0.095	0.046	0.024	0.043	0.025	0.035	0.127	0.042			
Horizon	1	3	12	1	3	12	1	3	12	1	3	12
Panel C: Monthly, quarterly and annual regressions with variance premium, conditional variance and other predictors												
3MTB	3.618 [3.146]	3.261 [3.344]	3.549 [3.070]	3.141 [3.091]	3.432 [3.765]	3.563 [3.147]	3.641 [3.365]	3.296 [3.440]	3.636 [3.109]			
Log(DY)	19.160 [14.411]	21.023 [12.904]	18.900* [10.510]	22.876 [14.267]	23.141* [13.490]	19.578* [10.502]	19.367 [14.468]	21.236 [12.951]	18.857* [10.510]			
CS	−15.022 [17.119]	−17.702 [13.153]	−6.335 [4.708]	−15.847 [18.529]	−17.936 [14.901]	−6.432 [5.151]	−10.290 [16.258]	−12.540 [12.602]	−5.120 [4.862]			
TS	1.944 [3.892]	2.095 [4.272]	3.983 [3.554]	1.649 [3.910]	2.255 [4.479]	4.004 [3.599]	1.806 [3.961]	1.953 [4.287]	4.008 [3.563]			
VP 8	0.674** [0.276]	0.796*** [0.146]	0.304*** [0.100]									
CV 8	−0.153 [0.211]	−0.110 [0.091]	0.054 [0.051]									
VP 11				0.560** [0.237]	0.515** [0.201]	0.234*** [0.075]						
CV 11				−0.110 [0.201]	0.025 [0.085]	0.087* [0.051]						
VP 30							0.533*** [0.174]	0.639*** [0.105]	0.238*** [0.075]			
CV 30							0.066 [0.124]	0.130 [0.096]	0.124** [0.052]			
Constant	−10.604 [16.083]	−12.335 [12.982]	−19.198 [12.433]	−10.258 [17.434]	−11.792 [14.004]	−19.057 [12.265]	−16.851 [15.829]	−19.132 [12.700]	−20.682 [12.530]			
Adj. R <sup>2</sup>	0.039	0.171	0.276	0.041	0.131	0.267	0.046	0.192	0.273			

Notes: Sample period January 1990–September 2010. All regressions are based on monthly observations. The standard errors reported in brackets are computed using max[3, 2\*horizon] Newey–West lags.

\* Significance at the 0.10-level.

\*\* Significance at the 0.05-level.

\*\*\* Significance at the 0.01-level.



statistically significant predictive power at the longer horizons and the  $R^2$  drops from 13% to somewhere between 2.5 and 4.5%.<sup>8</sup> In unreported results, we find that the realized variance predicts stock returns with a negative sign at the monthly and quarterly horizon (significant at the 10% and 5% level, respectively). However, the coefficients are an order of magnitude smaller than for the BTZ variance premium, and the  $R^2$  is just above 2% in the quarterly regression.

This generates somewhat of a puzzle regarding the origin of the strong predictive power of the BTZ-variance premium. If the realized variance is not a strong predictor of stock market returns and the  $VIX$  itself does not predict them at all, why does their difference provide strong predictive power? The coefficient on the variance premium can be decomposed as follows:

$$\beta_{VP} = \beta_{VIX^2} \frac{\text{var}[VIX^2]}{\text{var}[VP]} - \beta_{CV} \frac{\text{var}[RV]}{\text{var}[VP]}. \quad (9)$$

It turns out that the variance of the squared  $VIX$  is rather similar to the variance of  $RV$ , which is itself more than three times higher than the variance of the variance risk premium. Therefore, the variance premium coefficient at the quarterly frequency scales up the small positive coefficient on the  $VIX$  and the larger negative coefficient on stock market volatility by a factor of three.<sup>9</sup>

Economically, it does appear that the variance risk premium uncovers a component in the  $VIX$  index that is related to future stock market returns, but the statistical evidence is not very strong. Apart from the small sample, one possible reason for this is the well-known fact that equity risk premiums are likely driven by multiple state variables (see Ang and Bekaert, 2007; Menzly et al., 2004) so that the univariate regressions are necessarily mis-specified. In the consumption-based asset pricing model of Bekaert et al. (2009), risk aversion and uncertainty are the two state variables driving time-variation in the equity risk premium.<sup>10</sup>

We therefore investigate bivariate regressions using both the variance premium and the conditional variance as predictors. The results are in Panel B of Table 4. The VP is overall the stronger predictor over the quarterly and annual horizons. The CV coefficients are (with one exception) negative and sometimes significantly so for models 8 and 11.<sup>11</sup>

These results have implications for the consumption-based asset pricing literature, where there is a persistent debate about what economic mechanism generates a large equity premium, volatile stock market returns and long-horizon stock return predictability. In the Bansal and Yaron (2004) long-run risk model, time-variation in the equity premium comes from time-variation in economic uncertainty. Recent versions of the model (see e.g. Bansal et al., 2012) put more and more emphasis on the role of volatility and argue that substantial persistence in consumption volatility (which then generates high persistence in stock return volatility) is necessary to make the models fit the salient asset return features. However,

our empirical results cast doubt on this economic mechanism. The persistence of the conditional variance (at the monthly level) is modest, varying between 0.63 and 0.71 across models. Moreover, the time-varying risk premium component in equity returns comes predominantly from the variance risk premium, not from time-varying economic uncertainty. The effects of economic uncertainty on risk premiums we do document seem short-lived. This suggests that the alternative class of models (see Campbell and Cochrane, 1999), which relies on counter-cyclical changes in risk aversion to generate variation in risk premiums, has more chance of being the true economic mechanism explaining time-variation in equity risk premiums.<sup>12</sup>

In Panel C, we consider a multivariate regression including other well-known predictor-variables, namely the real 3-month rate (the three-month T-bill minus CPI inflation, denoted 3MTB), the logarithm of the dividend yield (denoted Log(DY)), the credit spread (the difference between Moody's BAA and AAA bond yield indices, denoted CS) and the term spread (the difference between the 10-year and the 3-month Treasury yields, denoted TS); all variables expressed in annualized percentages. The addition of the other variables strengthens the predictive power of the variance premium for equity returns, with the coefficients uniformly increasing. However, the uncertainty coefficients are now smaller and mostly insignificantly different from zero.

As to the other variables, the term structure variables are never significant. Both the real rate and the term spread have consistently positive coefficients. The credit spread obtains a negative coefficient that is not significantly different from zero, and the dividend yield is at best significant at the 10% level, mostly at the longer horizons. Here, the crisis adversely affected the predictive power of these variables. Excluding crisis data, the dividend yield and the credit spread are highly statistically significantly different from zero for all specifications (at all horizons in case of the dividend yield and at longer horizons in case of the credit spread), with the dividend yield having the expected positive coefficient, but the credit spread negatively affecting the equity premium.<sup>13</sup>

The adjusted  $R^2$ 's remain small at the one month horizon, but now become quite large at the quarterly (13%–19% range) and annual horizons (around 27%). It is likely that this high explanatory power may partially reflect statistical bias (see Boudoukh et al., 2007).

Given that our preferred VP and CV measures are based on the estimated models 8 and 11, we conduct a robustness check which accounts for the sampling error in the VP and CV in our regressions. Specifically, we draw 500 alternative VP and CV series from the distribution of VP and CV estimates. To do so, we retain the coefficients from the forecasting projection together with their asymptotic covariance matrix. Then, we draw 500 alternative parameter coefficients from the distribution of these estimates, generating alternative VP and CV estimates. We then feed these into our predictive regressions generating a distribution of coefficients, standard errors and t-statistics. As the first-stage projections are tightly estimated, the coefficients and standard errors are not materially affected, and our main inference regarding stock return predictability remains intact.

<sup>8</sup> One possibility is that because we pre-estimate the conditional variance and BTZ do not, measurement noise affects our estimates. However, our measurement provides proxies for the variance premium and the conditional stock market variance closer to the true economic concepts.

<sup>9</sup> In the BTZ sample, neither the squared  $VIX$  nor the realized variance predict stock returns, with the  $VIX$  getting a positive insignificant coefficient and the realized variance a zero coefficient in the quarterly regressions. In that sample, the variance of the squared  $VIX$  is more than twice as high as the variance of the BTZ variance risk premium so that the positive coefficient on the  $VIX$  gets scaled up by a factor of two.

<sup>10</sup> Anderson et al. (2009) also examine the impact of “risk” and “uncertainty”, but in their paper risk represents physical volatility and uncertainty disagreement among forecasters.

<sup>11</sup> In the BTZ sample, the CV coefficients are positive at the monthly, and negative at the quarterly and annual horizons, but mostly insignificant.

<sup>12</sup> Do note that the models we mention fail to generate realistic variance premiums in equilibrium (see Bekaert and Engstrom, 2013).

<sup>13</sup> There is no issue of multi-collinearity in the regression as the dividend yield–credit spread correlation is low (close to zero over the pre-crisis sample; and 0.2 over the full sample). While the negative credit spread coefficient may surprise some readers, BTZ also report negative coefficients for the credit spread in univariate excess return regressions. It is conceivable that the credit spread is a good indicator of economic prospects (for example, it is relatively highly correlated with economic uncertainty) and therefore helps cleanse the dividend yield from variation driven by cash flows, rather than risk premiums (see Goez, 2012, for a recent interesting attempt to cleanse the dividend yield of cash flow effects in a predictability regression).

**Table 5**  
Industrial production regressions.

Monthly, quarterly and annual regressions with variance premium and conditional variance												
Horizon	1	3	12	1	3	12	1	3	12	1	3	12
VP 8	−0.066 [0.052]	−0.028 [0.042]	0.025 [0.025]									
CV 8	−0.088*** [0.020]	−0.113*** [0.009]	−0.060*** [0.012]									
VP 11				−0.084 [0.054]	−0.046 [0.036]	0.007 [0.018]						
CV 11				−0.078*** [0.020]	−0.106*** [0.006]	−0.053*** [0.013]						
VP 30							−0.040 [0.034]	−0.027 [0.030]	0.014 [0.019]			
CV 30							−0.083*** [0.019]	−0.087*** [0.010]	−0.033*** [0.011]			
VIX <sup>2</sup>										−0.080*** [0.019]	−0.084*** [0.018]	−0.031*** [0.009]
Constant	5.009*** [0.884]	4.815*** [0.689]	2.758** [1.110]	5.132*** [0.892]	4.995*** [0.691]	2.935*** [1.047]	4.451*** [0.856]	4.287*** [0.699]	2.416** [1.172]	5.109*** [0.817]	5.204*** [0.675]	3.145*** [0.960]
Adj. R <sup>2</sup>	0.120	0.276	0.086	0.119	0.272	0.082	0.130	0.296	0.096	0.123	0.258	0.056

Notes: Sample period January 1990–September 2010. All regressions are based on monthly observations. The standard errors reported in brackets are computed using max[3, 2\*horizon] Newey–West lags.

\* Significance at the 0.10-level.

\*\* Significance at the 0.05-level.

\*\*\* Significance at the 0.01-level.

### Predicting the real economy and financial instability

In Table 5, we examine the predictive power of the variance risk premium and stock market volatility for economic activity as measured by industrial production growth (the log-difference of the total industrial production index expressed in annualized percentages; growth over a quarter/year is averaged). Bloom (2009) shows that uncertainty shocks lead to a rapid drop and rebound in aggregate output and employment. In a model with adjustment costs to labor and capital, this occurs because higher uncertainty causes firms to temporarily pause their investment and hiring. In some of his empirical work, Bloom actually uses the VIX to help measure uncertainty shocks. Here, we investigate whether the VIX and/or its two components predict economic activity in a simple regression framework.

The last specification shows that the squared VIX itself predicts economic activity with a negative sign at all horizons (significant at the 1% level). In terms of economic significance, a 1% (monthly) move in the VIX near the mean leads approximately to a 1% (annualized) drop in the industrial production growth over the next quarter. The bivariate regressions with its two components show that whatever predictive power the VIX has for future output, is coming from the uncertainty component. The coefficient on VP is negative at monthly and quarterly horizons, but it is always statistically insignificantly different from zero. The coefficient on CV is always negative and statistically significant at the 1% level for all three horizons. As with stock return regressions, we check robustness of our results to accounting for the sampling error in the VP and CV estimates. Our results are unaltered. We conclude that CV is a robust and significant predictor of economic activity.

Our results here add to a rapidly growing literature on predicting economic activity with economic uncertainty measures. For example, Stock and Watson (2012) argue that financial disruptions and heightened uncertainty helped produce the Great Recession. Allen et al. (2012) derive a measure of aggregate systemic risk using data on stock returns for banks and show that high levels of this measure predict future economic downturns. Bachmann et al. (2013) use survey expectations data to construct an empirical

proxy for time-varying business-level uncertainty. They show that surprise movements in the uncertainty proxy lead to significant reductions in production. Finally, Gilchrist et al. (2010) analyze how the combination of uncertainty shocks and financial market imperfections generate fluctuations in economic activity.

One economic outcome whose monitoring is of considerable policy interest is financial instability. In Table 6, we examine whether measures of the variance risk premium and stock market volatility have predictive power for financial instability. To measure financial instability, we use a financial stress indicator created by the European Central Bank (called CISS). The indicator is based on European Monetary Union (EMU) data, combining information from the money, equity, bond, and foreign exchange markets, and some financial intermediaries-related information. It mostly comprises realized volatilities for various return, currency or interest rate measures and it does not contain any implied volatility information (see Holló et al., 2012, for details). We regress the level of the CISS indicator 1, 3 and 12 months ahead on our VP and CV measures.

The VIX itself has a high predictive power for the one- and three-months ahead indicators (significant at the 1% level). The R<sup>2</sup> is over 40% at the monthly horizon and over 30% at the quarterly horizon. When both components of the VIX enter the regressions, the uncertainty component has a higher predictive power than the variance premium component. Uncertainty is significant at the 1% level at all three horizons, and the magnitude of its coefficients is uniformly higher than for the VP, particularly at the quarterly and annual horizons. The VP coefficient is significant at the monthly horizon (at the 5%–10% level) but not (with the exception of model 11) at the quarterly or annual frequency. These results are robust to accounting for the sampling error in the VP and CV estimates. Overall, a high predictive power of variables based on US data for a European financial stress indicator is noteworthy.<sup>14</sup>

<sup>14</sup> We also considered three alternative financial stress indicators: a (proprietary) CISS indicator based on US data, an indicator developed by the Kansas Fed and

**Table 6**  
Financial instability regressions.

Monthly, quarterly and annual regressions with variance premium and conditional variance												
Horizon	1	3	12	1	3	12	1	3	12	1	3	12
VP 8	0.222 <sup>*</sup> [0.121]	0.176 [0.137]	−0.054 [0.122]									
CV 8	0.301 <sup>***</sup> [0.048]	0.288 <sup>***</sup> [0.031]	0.200 <sup>***</sup> [0.044]									
VP 11				0.154 <sup>*</sup> [0.080]	0.191 <sup>*</sup> [0.110]	−0.015 [0.093]						
CV 11				0.345 <sup>***</sup> [0.055]	0.284 <sup>***</sup> [0.034]	0.187 <sup>***</sup> [0.036]						
VP 30							0.216 <sup>**</sup> [0.090]	0.134 [0.092]	−0.010 [0.087]			
CV 30							0.277 <sup>***</sup> [0.042]	0.256 <sup>***</sup> [0.036]	0.119 <sup>***</sup> [0.031]			
VIX <sup>2</sup>										0.274 <sup>***</sup> [0.040]	0.249 <sup>***</sup> [0.043]	0.112 <sup>***</sup> [0.037]
Constant	3.816 <sup>**</sup> [1.744]	5.071 <sup>**</sup> [2.052]	11.470 <sup>**</sup> [4.575]	4.121 <sup>**</sup> [1.682]	4.886 <sup>**</sup> [1.884]	11.021 <sup>**</sup> [4.347]	4.400 <sup>**</sup> [1.927]	6.443 <sup>***</sup> [2.116]	12.300 <sup>**</sup> [4.823]	3.455 <sup>**</sup> [1.401]	4.562 <sup>***</sup> [1.679]	10.317 <sup>***</sup> [3.969]
Adj. R <sup>2</sup>	0.422	0.349	0.090	0.446	0.350	0.093	0.426	0.369	0.092	0.422	0.346	0.067

Notes: Sample period January 1990–September 2010. All regressions are based on monthly observations. The CISS indicator was scaled by a factor of 100. The standard errors reported in brackets are computed using max[3, 2\*horizon] Newey–West lags.

<sup>\*</sup> Significance at the 0.10-level.

<sup>\*\*</sup> Significance at the 0.05-level.

<sup>\*\*\*</sup> Significance at the 0.01-level.

## 5. Conclusions

We decompose the squared VIX, the risk neutral expected stock market variance, into two components, the conditional (physical) variance of the stock market (CV) and the equity variance premium (VP), which is the difference between the two ( $VP = VIX^2 - CV$ ). Because this decomposition critically depends on the accuracy of the model for CV, we first conduct an extensive analysis of state-of-the-art variance forecasting models, where we make sure to also consider the squared VIX itself as a potential predictor. Indeed, one of our winning models includes the VIX.

We use these models to re-examine and expand the evidence on the predictive power of VP and CV for stock returns, economic activity (as measured by industrial production) and financial stress indicators (tracked by central banks). We find that the variance premium is a significant predictor of stock returns, but the conditional variance mostly is not. However, CV robustly and significantly predicts economic activity with a negative sign, whereas VP has no predictive power for future output growth. Lastly, CV has a relatively higher predictive power for financial instability than does the variance premium.

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an indicator developed by the St. Louis Fed. Results for the US CISS are very similar to those for the EMU CISS (the correlation between the two indicators is 0.8). For the Fed-developed indicators, results are similar with two qualifications. First, the coefficients on the VP component are now statistically significant at the 1% level at both the monthly and quarterly frequency. This is not surprising as both indicators include the VIX itself as one of the components (unlike the CISS indicators). Second, the R<sup>2</sup>'s are higher at the monthly and quarterly frequency compared to the regressions with the EMU CISS indicator. VP and CV have a higher predictive power for financial stress in the US compared to Europe.

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