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# Curricular Flows: Trajectories, Turning Points, and Assignment Criteria in High School Math Careers

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What do course trajectories look like? Do career paths intersect? Do courses assume distinct roles in educational careers? What factors lead students to adopt various curricular moves? This article addresses these questions by examining the structure and dynamics of participant flows across courses in two high school math curricula. Drawing on Markov models and network analytic methods, it examines the length, rate of progression, volume, and direction of multiple trajectories in math and identifies certain courses as important career turning points. The findings suggest that schools can design a variety of mobility systems by altering course offerings, prerequisites, and grading policies. Analyses of individual curricular moves show that nonsanctioned moves, such as leaving school, are guided by students' background characteristics, while sanctioned moves, such as course transitions, are guided primarily by structural constraints and adaptations to signals about classroom performance.

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**M**ore than two decades ago, Sorenson (1984:41) wrote that discussions of curricular differentiation should attend to the *patterned flow of students across courses*:

[The] organizational differentiation of students defines a structure of flows in an educational system. Even when research focuses on the causes and consequences of grouping, it tends to focus on single assignments to, for example, college tracks or ability groups. That such assignments are part of sequences of assignments producing educational careers is often a neglected fact.

However, research on curricular differentiation has not focused directly on the patterned flow of students across courses, and, as a result, important questions about educational careers have not been addressed. For example, are their multiple career trajectories

in curricula? What are they, and how do they vary? Do distinct career paths intersect or overlap? Do certain courses play a special role in students' educational careers? Do some course locations afford greater attainment opportunities than do others? What factors lead students to adopt various kinds of curricular moves?

This article addresses these questions by analyzing the *structure* and *dynamics* of participant flows across math courses in two high schools. A focus on flow relations turns attention from abstract career strata (i.e., tracks and track mobility) to career trajectories that link actual curricular units over time. This shift in focus identifies various structural features of career systems that have been overlooked in prior accounts, such as the composition, shape, length, and rate of progression of different career trajectories; the regions of track overlap that constitute con-

fluences and turning points in educational careers; and the variety of mobility patterns that arise in school curricula. Instead of finding a general form of curricular differentiation, the analyses presented here reveal how schools can have vastly different career systems.

A focus on math curricula in two distinct schools enables a comprehensive, comparative analysis of career systems and how students move within them. Comparative case studies have been frequently used to depict social structures in sociological subfields, such as social networks (Chase 1991; White 1970; White, Boorman, and Breiger 1976) and formal organizations (March and Simon 1958; Stewman 1975, 1986; Stewman and Konda 1983), but they have been underutilized in the sociology of education's research on course-taking structures (Heck, Price, and Thomas 2004). The case approach used here complements and informs national studies of transcripts. National studies identify central, generalizable tendencies in course taking across the nation's schools and thereby inform federal policy makers whose decisions arguably result in some far-reaching changes in schooling. In contrast, case studies afford a detailed inspection of the internal variation of schools (where arguably much of the variance in schooling resides). Cases take key concepts of national studies, give them concrete representation, and illustrate both their nuances and limitations. Hence, the case-study approach affords a local, detailed perspective on curricular differentiation that is largely absent in current research and informs national studies.

Math is a focus of analysis because the completion of math courses remains key to college admissions and has long-term educational and occupational consequences. In addition, math is the most rigidly defined subject matter in high schools with the most readily identifiable tracks (Oakes, Gamoran, and Page 1992), course sequences (Stevenson, Schiller, and Schneider 1994), and mobility structure (e.g., sponsored; see Turner 1960). One would therefore expect math to be a subject that clearly depicts the central concepts of prior research (e.g., discrete tracks, staged course taking, and "tournament"

mobility patterns). However, the depiction of curricular flows in math provides a mixed story. In many instances, there are clearly identifiable career paths that align with certain ability levels. But in others, multiple career paths overlap and span ability levels. Moreover, the aggregate pattern of course taking reveals hybridized forms of educational mobility and suggests that upward and downward mobility are partly a matter of curricular design.

This article has two aims. The first is to describe the *structure* of educational careers by mapping the pattern of students' movement, or curricular flows, across math courses in two schools. The pattern of students' movement reveals that schools can have widely different systems of mobility whose internal variations reflects how the experience of each career path can constantly change. Because of course offerings and institutional rules, curricula can provide students with different course-taking opportunities and constraints (Hallinan and Sorenson 1986; Sorenson 1970, 1984). Some promote upward movement into more demanding math careers, while others pressure students either to move on or to end their math careers (Hallinan 1996b; Lucas 1999; Rosenbaum 1978). Within these mobility systems, there are multiple career paths or course trajectories of different lengths (number of stages), volumes (of students), and rates of progression (e.g., whether students repeat courses or take more than one). In many instances, these career paths interpenetrate, and certain courses become way stations for populations of students with different abilities, ages, and motives (Friedkin and Thomas 1997; Hallinan 1991; Heck et al. 2004). Some courses even act as career turning points, either by giving students a wide assortment of choices (starting forks) or by pressuring them into another career trajectory (dead ends). Hence, the structure of educational careers entails a great deal of internal variation that characterizes distinct career paths and mobility across them.

The second aim is to describe the *dynamics* of educational careers by identifying assignment processes that bring about the types of individual curricular moves (e.g.,

stopping math, leaving school, or moving upstream or downstream) composing career structures (Baron, Davis-Blake, and Bielby 1986; Hallinan 1996b; Lucas 1999; Sorenson 1984; Spilerman 1972). Although a wide assortment of explanatory factors is possible, the results indicate that most sanctioned curricular moves are guided by the opportunities and constraints that various course locations afford (structural controls) and the messages about competence and the appropriateness of course membership that achieved grades communicate. Students' background plays a secondary role in most transitions and best explains extreme moves, such as dropping out of school altogether.

## CONCEPTIONS OF COURSE-TAKING PATTERNS

This article builds on and refines prior conceptions of course-taking structures. In addition, it identifies where the prior conceptions fail to hold and discusses the variety of roles that courses can play in students' educational careers. Perhaps the most recognized conceptions of course-taking structures stem from research on *tracking* and *track mobility*. Research on tracking describes course-taking structures in terms of discrete career strata, or curricular programs that are composed of sets of courses (Oakes et al. 1992:574–75). Some of the most convincing work has used course schedules and designations of ability level to identify career strata (Garet and DeLany 1988; Hallinan 1994; Oakes et al. 1992). The general finding of this work has been that affiliations with high-status tracks have positive effects on students' outcomes, whereas affiliations with low-status tracks are detrimental. More recent work has applied network analytic methods to data from students' transcripts and has identified sets of structurally equivalent courses that are reflective of various educational programs (Friedkin and Thomas 1997; Heck et al. 2004). Friedkin and Thomas identified upward of eight career strata that predict educational outcomes better than did prior tripartite tracking schemes. In addition, they identified new curricular programs that do not fit traditional nominal

classifications of vocational, general, and college tracks (or remedial, regular, and honors) and appear to call them into question (e.g., business and applied math). Hence, research on tracking has consistently conceived of course-taking structures as career strata and has refined the representation of these strata and shown their substantial influence on a variety of student outcomes.

Whereas research on tracking has highlighted curricular programs as course sets, research on *track mobility* has highlighted the pattern of student flow across these sets (Garet and DeLany 1988; Hallinan 1994, 1996b; Lucas 1999; Rhoades 1987; Rosenbaum 1978; Turner 1960). Research on track mobility describes course-taking structures by the pattern of moves across tracks and has argued that the educational system in the United States has a distinct pattern of mobility (e.g., contest and, more recently, tournament) from that observed in Europe (e.g., sponsored). At least four ideal types of movement are discussed in the literature: (1) *contest mobility*, in which upward and downward movement across ability levels occurs from achievement contests (Turner 1960); (2) *sponsored mobility*, in which the initial differentiation of ability is followed by path-dependent moves onward in each ability stream without switching (Turner 1960); (3) *tournament mobility*, in which achievement contests relegate losers to a lower ability level and allow winners to continue in the same ability level (Lucas 1999; Rosenbaum 1978); and (4) *progressive mobility*, in which achievement contests position winners in higher ability levels and losers in the same ability level (Hallinan 1996b).

A third line of research is more fine-grained and perhaps more directly related to the approach presented in this article. It concerns *course sequences* in curricular subjects. Researchers on course sequences have argued that courses are staged or given a modal order. For example, students who want to take geometry must first take some form of algebra. Hence, algebra classes are antecedent to geometry classes, and this ordering extends to other courses in the subject, such that all math course taking is arranged in a sequence of course stages.

Because of this staging of prerequisites, course taking in subjects like math can be regarded as an opportunity sequence (Schneider, Swanson, and Rieggle-Crumb 1997; Stevenson et al. 1994). When course taking is rigidly staged with prerequisites (e.g., math), future course taking and achievement levels are more path dependent than when course taking is less rigidly sequenced (e.g., history).

In theory, the study of curricular flows should identify whether these different conceptualizations of course-taking structures persist, in what form, and where they fail to hold. In practice, it affords a concrete means of comparing and contrasting key concepts of curricular differentiation.<sup>1</sup> Hence, patterns of student flows across courses illustrate whether tracks or distinct programs exist, what internal structure these programs afford students who participate in them, and what kinds of mobility potentials are likely from any particular course location. In the cases presented in this article, there emerge course trajectories with some semblance to tracks, and their structural properties (e.g., flow volume, career progression, and length) help explain changes in students' experience. In addition, patterns of student flows help explain overlaps in tracks in more concrete terms. The structure of participant flows identifies courses in which career paths intersect and expose different age and ability-level populations to one another.

The mapping of student flows across courses also brings greater clarity and specificity to conceptions of track mobility by revealing where upward and downward moves across career paths occur and thereby defines educational opportunities that are timed and course specific. Hence, curricular flows reveal career turning points or locations where the greatest educational opportunities occur. By focusing treatments at these career junctures, educators have the greatest potential to have a positive influence on students' careers.

The study of curricular flows reveals the limitations of tracking and track mobility conceptions because course trajectories do not align cleanly with ability-level distinctions, and many trajectories (or tracks) intersect and

overlap. Hence, the study of curricular flows changes the conception of stratified careers, revealing that the internal order of career trajectories differs and that multiple trajectories can interrelate. Moreover, in the aggregate, flow patterns render pure forms of mobility problematic and show that different forms of mobility apply to different career paths. In shorter, lower-ability career paths, tournament mobility explains how career paths interrelate, while in longer, higher-ability career paths, contest mobility explains how career paths interrelate. Thus, different stories of educational opportunity arise depending on where a student is in the curriculum.

The study of curricular flows also informs the core concept of course sequences and reveals its' limitations. Curricular flow patterns in math entail career trajectories that partially align with ability levels and have stages akin to course sequences. However, not all courses constitute equivalent stages in these sequences. Supposedly, Algebra 1 courses occupy the same sequence stage regardless of ability level. However, Algebra A and B (a two-year course sequence) are equivalent to a single course of Algebra 1, indicating that one-to-one stage comparisons may not exist in many curricula. In addition, there are points at which prerequisites are unclear and sequences break down so that large proportions of students take career "shortcuts."

In sum, a detailed comparative analysis of math course-taking flows interrelates prior conceptions of course-taking structures, presents them in a concrete light, and illustrates their limitations. Moreover, such nuanced depictions come closer to revealing how educational career structures vary and where policies can be better focused and offers a new way of conceiving of course-taking structures: one that rests on flow patterns and has the potential to reveal the variable roles that courses play in students' educational careers (e.g., starting points, end points, turning points, and points of convergence). In short, through the structural analysis of curricular flows, a new and fruitful extension of tracking, track mobility, and course-sequence research emerges.

## MOBILITY DYNAMICS

Career structures are composed of individual curricular moves, and these curricular moves are driven by a finite set of factors that characterize the mobility dynamics of educational careers (Hallinan 1996b; Lucas 1999; Sorenson 1984; Spilerman 1972). It is possible that career structures differ across schools merely because schools establish course offerings and rules that guide students to adopt certain kinds of moves. It is also possible that students' characteristics and behaviors guide the adoption of curricular moves. To determine whether student populations or school rules matter, a different approach is needed, one that focuses on the types of individual curricular moves and factors that drive their adoption.

Within the sociology of education, research has focused on a narrow range of curricular moves (e.g., same track, up track, down track, or out of the subject altogether; see Hallinan 1996b; Lucas 1999). This research has also been characterized by debates over which factors guide students' decisions to adopt these moves. Hallinan (1996b) found that the structural constraints of course locations are highly associated with the likelihood of experiencing each type of move. In contrast, Lucas (1999) showed that background characteristics are highly associated with the likelihood of experiencing each type of move. It is therefore unclear whether structural or compositional factors most determine the mobility dynamics that shape course-taking structures.

This article contributes to research on curricular moves in two ways. First, it expands the scope of study to focus on a wider array of individual curricular moves that students make: (1) dropping out, (2) stopping math, (3) repeating a course, (4) moving downstream, (5) staying in the same stream, (6) moving upstream, or (7) graduating. These are mutually exclusive sets of moves that are interdependent and, in the aggregate, form the larger career structure. Hence, readers are given a more developed understanding of how career dynamics relate to the larger career structures that they compose. Second, the analyses attempt to build upon Lucas's

(1999) and Hallinan's (1996b) depictions of background and structural constraints on mobility dynamics, and to reveal how individuals react to the immediate classroom environment and the achievement messages that teachers convey. In particular, statistical models are developed to test whether *background characteristics* (race, gender, class, and family contexts), *structural locations* (opportunities and constraints defined by organizational rules), and/or *achievement selection* (adaptation and selection processes) influence each type of curricular move (1–7).

2x2 2x3 2x4 2x5

## DATA AND METHODS

Two objectives guide the empirical analyses. The first is to describe the structure of participant flows across math courses in two high schools (Chase 1991; Sorenson 1970, 1984). The second is to identify explanatory factors that are associated with various types of curricular moves that students adopt in their educational careers (Hallinan 1996b; Lucas 1999).

### Data

The data analyzed include records and field observations at two schools. Records at each school consist of students' course schedules and grade transcripts of each semester for two years (the 1996–97 and 1997–98 school years). From the school records, it is possible to match the students' addresses with 1989 census block tracts to acquire median household incomes. Guardian listings provide information on the types of households, and yearbooks provide information on the students' gender and race (see Appendix Table A).<sup>2</sup>

3x3 3x2 → 3x2

### The Structure of Course Taking

To study the structure of students' math careers, two pieces of information are needed: (1) a map of the entire math career structure that identifies the overall pattern of participant flows across course work and (2) a sensible rendering of individual career paths and their structural characteristics so that interrelations can be described and some



understanding of career experiences can be gained. To represent participant flows across courses, students' schedules are used from two consecutive years (1997 and 1998) to develop large affiliation matrices (Friedkin and Thomas 1997; Sorenson 1984). In these affiliation matrices, rows are students, columns are courses, and cell values of 1 and 0 indicate membership in a particular year. Schedules are acquired at the end of each school year, so many of the scheduling issues and midsemester transfers are not an issue for coding (Hallinan 2003). Given that math courses in these schools span the academic year, movement between courses is kept at a minimum. In the few cases in which it occurs, the later affiliation is used. If a student fails or drops out midyear, his or her original course membership is used.

After the affiliation networks are constructed (the enumeration of all students' memberships each year =  $A_{97}$  and  $A_{98}$ ), matrix algebra is used to calculate participant flows across courses (Wasserman and Faust 1994).<sup>3</sup> To construct a mobility matrix,  $A_{97}$  and  $A_{98}$  need to have the same number and order of students (rows) so that the transposed affiliation network of 1997 ( $A_{97}^T$ ) can be multiplied by the affiliation network of 1998 ( $A_{98}$ ). Through such matrix multiplication, a mobility matrix of origins by destinations is created,  $M = A_{97}^T \times A_{98}$ . The cell values in this matrix indicate the number of students who moved from 1997 math courses to 1998 math courses. When presented in table form, the results are considered transition frequencies from origin states to destination states (see Chase 1991).<sup>4</sup>

Transition frequencies tell us about the volume of flows but do not test the likelihood that students will move from an origin state to a destination. Such a test can be constructed from the transition frequency tables by dividing the values in each row by the total number of students who are in the course during 1997 (Chase 1991; Sorenson 1984). The new cell values represent maximum likelihood tests or transition probabilities that indicate the proportion of students from each course who move to the next position. These probabilities are presented in Tables 1 and 2. The rows and columns of

each matrix are permuted in an effort to indicate where distinct ability labels are used.

High probabilities of transition with a unidirectional flow identify *career paths*. Career paths can have different *lengths*, where some trajectories entail the completion of two courses and others entail the completion of five. The length of a career path is recognized as the number of moves that students make upon entry into a system until the point of exit or terminal move. Predicted lengths of vacancy chain serve as proximate measures of the length of various students' careers (Abbott 1990; Chase 1991; Smith and Abbott 1983; Sorenson 1984; White 1970).<sup>5</sup> Predicted chain lengths are used because full transcript data extending across the students' high school tenure are not available. Predicted chain lengths require only two years of course schedules and reveal the general transition patterns from which a population's primary career paths can be identified. Of course, complete career data on students' course taking is preferable and afford more detailed information on individual trajectories and the likelihoods of taking one path over another. Future research could advance career modeling by using sequence analysis to predict individual career paths and the likelihood of switching among them at various stages of students' careers (Abbott 1995).

The way to read predicted chain lengths (i.e., values of  $n$ ) is as an opportunity sequence: the higher the predicted chain length for a course, the more course-taking opportunities are available from that location. By mapping the career paths of students across math courses in conjunction with these predicted chain lengths, it is possible to identify where careers fan out and where multiple chains meet in bottlenecks (Stewman and Konda 1983). In essence, the structure of mobility in terms of career paths and their interrelations can be described.

## MOBILITY DYNAMICS

The general pattern of curricular flows captures the structure of educational careers but does not explain how course transitions arise. To study career dynamics, one must identify

the assignment criteria that are associated with types of individual curricular moves (Lucas 1999; Sorenson 1984). Identifying which curricular moves to analyze can be difficult because the array of *token* curricular moves is enormous. Hence, prior research (Hallinan 1996b; Lucas 1999) has analyzed types of curricular moves. This article also looks at types of curricular moves, but improves on prior research by examining a mutually exclusive set. All students must adopt one of these moves every round of course assignment: (1) *leave school*, (2) *stop math*, (3) *repeat a course*, (4) *move onward and downstream*, (5) *move onward in the same stream*, (6) *move onward and upstream*, or (7) *graduate* (see Appendix Table A for the construction of the variables and descriptive statistics). By studying a mutually exclusive set of moves, one can constrain and compare the results so as to overcome prior methodological criticisms (see Lucas 1999:chap. 6).

The argument of this article is that students are agents in course-taking decisions, but that their decisions are structurally constrained and enabled. To illustrate this argument, three clusters of independent variables are used to predict each type of curricular move (see Appendix Table A): *students' background*, *achievement selection*, and *organizational locations*. Considerable research has argued that *students' background* influences individual course-taking decisions by shaping individual experiences and their store of resources. In such a fashion, certain moves and career paths seem more aligned with prior experiences and resemble contexts in which students feel they can succeed in the setting's activities (Bourdieu and Wacquant 1992; Lareau and Horvat 1999). Dummy variables are used in coding *female gender*, *disadvantaged minority*, and *alternative family structure*.<sup>6</sup> *Median household income* is used as a proxy for socioeconomic status. The literature suggests that females (Catsambis 1994), disadvantaged minorities (e.g., Native Americans, nonwhite Hispanics, and African Americans; see Hallinan 1996a; Lucas 1999), youths from broken homes (Lee 1993; Useem 1991), and families with lower incomes (Lucas 1999; Oakes and Guiton 1995) will likely adopt downward moves over upward

ones because more advanced and difficult course work entails activities and experiences that are misaligned with their prior experiences and reserve of social resources (capital).

In addition to background proclivities, students are adaptive agents who respond to more immediate signals that are communicated in the school setting. For example, performance evaluations, such as grades, clearly influence students' decisions about course taking (Garet and DeLany 1988; Oakes et al. 1992; Rosenbaum 1978; Sorenson 1970). Grades communicate to the students their relative performance and academic status (Bidwell and Friedkin 1988; Cohen and Lotan 1997; Sorenson 1987), and therefore indicate how well they fit certain career paths over others. Students who repeatedly acquire low grades will feel they are out of place in their current career path and will be inclined to move downward to another, while those with higher grades may be inclined to move upward. Achieved grades thus act as a selection factor or socialization message that is aimed at encouraging a student to learn and adapt. Students' grades are measured by *past math grades* and by whether or not the students receive *nonletter grades* or alternative forms of grading.

However, even if actors have background proclivities and react to status signals in the immediate field of schooling, they are still beholden to the structural constraints of *organizational locations* and the course-taking rules that are affixed to them (DeLany 1991; March and Simon 1958). Being in certain *ability-level courses* and in certain *grade levels* greatly influences the range of curricular choices that are available to students. If career paths are discrete and stratified, then one may expect ability levels to have the same effects as the tracking literature suggests: greater path-dependent moves within ability levels, more advantageous moves for students in high-ability courses, and more disadvantaged moves for lower-strata students (Lucas 1999). If all careers are staged in a similar fashion by graduation requirements and minimum grade-level prerequisites, then grade levels should influence students' movement in the curriculum. These rules make timed exit and progression common in many subjects (Hallinan 1996b).

In this study, a third type of organizational location is identified, referred to as “*turning points*.” Turning points are locations at which students are encouraged to redirect their careers because path-dependent moves are no longer the primary option. Some turning points are *dead ends* that force students either to exit the system or to change tracks because no further courses are offered in the particular career path. For example, a student may arrive in graduate school with low math skills, and to meet the statistics requirements, must take a remedial sequence (e.g., Introduction to Data or Introduction to Statistics) that merely prepares him or her for the required sequence, which the student must eventually take. Other turning-point courses are *starting forks*, which afford students the option to move into a newly established career path. These starting forks represent career bottlenecks or gatekeeping courses that constrain participant flows. For example, at many universities, students must complete a particular introductory course before they can take more specialized and advanced courses that are of greater interest (e.g., Introduction to Sociology as a prerequisite to Sociology of Education or Social Networks).

Logistic regression (Allison 1999) is used to identify factors that are most associated with each type of curricular move in comparison to all others—for example, leaving school (1) compared to all other moves (2–6), or moving uptrack (6) in comparison to all other moves (1–5). Although 12th graders and graduation moves are included in structural maps, they are omitted from the logit models. Simply put, being a 12th grader is collinear with graduating, and therefore logit models predicting graduation moves produce biased parameter estimates of no statistical value. Therefore, all the logit models analyze the sample of students from Grades 9 to 11 (or 8 to 11 for one of the schools) and their mutually exclusive set of moves (1–6). It is important to note that there is some debate about which method is the most appropriate for models of curricular moves (see Lucas 1999:chap. 6).<sup>7</sup> For a variety of reasons, the most familiar and practical method is used to predict the mutually exclusive set of dependent variables described earlier.

## PROFILES OF TWO SCHOOLS

### *Rural High*

Before the results are presented, it is useful to introduce the two high schools and to describe the course-taking rules that are described in their curriculum guides. The first high school, Rural High, is located in a small town and serves communities within a 15-mile radius. Rural High is a traditional rural school composed of approximately 1,600 students (in Grades 9–12) from predominantly middle- to lower-income households that are engaged in white-collar, agricultural, and blue-collar employment. Rural High is ethnically homogeneous (97% white) but heterogeneous in terms of students’ abilities. Some students test several grade levels below their peers, while others perform well above the state mean.

In an effort to facilitate instruction, Rural High places incoming students in different ability-level math classes according to their test scores, teachers’ recommendations, prior grades (from middle school), and prior track placement in middle school.<sup>8</sup> Its curriculum handbook explicitly lists a set of courses under each ability level. The set labeled “accelerated” is comprised of five separate courses (through Calculus), and the set labeled “regular” entails four courses (through Precalculus). Two final sets, labeled “basic” and “special education,” entail only three and two courses, respectively. The curriculum handbook, then, suggests that there will be four discrete career paths (reflecting ability tracks) with staged course work (reflecting sequences), in which basic and special education careers are shorter than those in regular and accelerated course work.

The curriculum handbook also describes the requirements for graduation and the prerequisites for college entry. Rural High students must complete two years of math (at any level) to graduate, but students take more, depending on the status of the college they want to attend. To attend junior college, students must complete the equivalent of Algebra 1 (Basic Math and Special Education Math will not suffice). To attend a less competitive four-year local college, students must



complete at least two years of math through Geometry. To attend most state colleges, students need to finish at least three years of math through Algebra 2 and at least finish Precalculus or Calculus to attend the premier state university. These graduation and college requirements suggest that Rural High will have no career paths of fewer than two steps and that, depending on the track, career paths will pertain to different post-high school locations. However, most Rural High students aspire to attend college, and the lower tracks fail to meet these eligibility requirements, so it is likely that some students will span career paths in an effort to meet such demands. In sum, Rural High's course offerings, rules, and population characteristics suggest there will be distinct career paths of various lengths that align with tracks but that may intersect as students try to acquire eligibility for college.

### **Magnet High**

The second high school, Magnet High, is an elite arts and science magnet school in an inner-city neighborhood of a large midwestern metropolitan area about 500 miles from Rural High. Magnet is an integrated high school composed of around 900 high-ability students (Grades 8–12) from predominantly lower-income households. Approximately 35 percent of the school is African American, 6 percent is Latino, 2 percent is Asian, and 57 percent is white. The salient identity distinction within Magnet High's population separates the urban minority students (Latino and African American) from the more suburban white and Asian populations.

Although Magnet High is heterogeneous in ethnic background, it is homogeneous in terms of students' ability and motivation. Students must apply to attend Magnet and then are admitted only if they take an entrance examination and receive a high score. Consequently, all the students are motivated and high achieving. To serve these students, Magnet High has a college preparatory curriculum whose classes are at least equivalent to the honors-level course work offered at other high schools in the district. The high school's "magnet" is its internation-

al baccalaureate program (IB) whose curriculum is more demanding than honors or Advanced Placement course work.

Magnet High's curriculum handbook lists math courses by grade level and ability label. The math curriculum begins in the eighth grade with a single course and then differentiates offerings into higher ability-level tracks at later grade levels. In order of increasing difficulty, the ability levels are honors, IB-general, IB-standard, and IB-honors. The delayed differentiation of offerings at Magnet High suggests that the career structure may branch up and outward like a tree. The curriculum handbook states that a student must take at least four years of math to graduate with a "certificate." However, in 1997, Magnet increased the requirement to five years of math, resulting in more students taking math in 1998 than in 1997. To acquire the more prestigious "diploma," a student must take the more demanding IB classes (five years of math) and attain a passing score on the IB examination. Doing so gives the student enough college credits to be a sophomore or junior at the state university. Magnet High's curriculum guide does not list college requirements because the graduation requirements surpass even those at the elite level.

## **RESULTS**

### **Mapping Math Career Structures**

The first set of results presents transition probabilities that make it possible to discern the structure of math careers at both schools. Table 1 presents curricular flows for Rural High math and represents the courses in descending order with borders around the ability levels that counselors and students commonly recognize. The values in the matrix indicate the proportion of students from each origin row who move to the destination column. Listed at the bottom of the matrix are multiple points of entry from outside the math curriculum.

By following the flows of students from their points of entry through their courses and out to exit states, one can identify various career paths through math courses (Sorenson

1984). Courses that are linked by a series of high probability flows form identifiable *trajectories or career paths*. Figure 1 presents a graphic rendering of the transition probability matrix and identifies five career paths: special education, lower-basic, upper-basic, regular, and accelerated.

The special education trajectory entails the fewest students and produces the shortest career path. The expected path length that is due to transition probabilities (2.09 course steps)<sup>9</sup> suggests that most special education students stop at the second course, although some move upstream into the first course of the lower-basic trajectory. The lower-basic math career starts in Basic Math and extends through Algebra B. The expected career length suggests that many students in this trajectory make it to Algebra B (2.89 steps). The upper-basic trajectory begins in Algebra A and extends upstream into Geometry (2.73 steps). It overlaps the lower-basic trajectory at multiple courses and extends into the regular sequence at Geometry. This trajectory entails a turning-point course (or dead end), where students are forced to move up into regular ability-level course work at the end of their careers so as to meet college entry requirements. The regular-level career path starts in Algebra 1 and extends into Algebra 2. The percentages of students retaking (see circles) and opting out (see off-shooting downward arrows) of basic and regular math make the expected career lengths somewhat shorter and the progression slower. Consequently, the regular trajectory stops short of Precalculus (expected chain length of 2.78), where only 17 percent of Algebra 2 students move onward to Precalculus. This seemingly premature end to the regular-ability level trajectory may seem odd, but a cursory investigation indicates that many of these students never had to retake a course, and some began in the honors career path and drifted downstream. The final career path of accelerated course work starts in Accelerated Algebra 1 and extends up to Calculus. The expected career length in honors course work is the longest in Rural High's math curriculum (3.91). Because most accelerated students take two math courses in the freshman or sophomore year, the predicted chain length

is underestimated (since students take two courses in the same year) and reflects the progression rate of the trajectory (i.e., five courses in four years).

The structural maps of career paths in Table 1 and Figure 1 reveal a variety of internal qualities of educational careers that have been overlooked in prior research. Perhaps the most obvious is how career paths vary in *length*, as was discussed earlier. But careers also vary in other regards, such as *volume*, *direction*, and *rate of progression*. The *volume* of each trajectory can be gauged by the size of each square in Figure 1 and by the final column of Table 1. The regular trajectory entails the majority of students in math, and only a small fraction of students are involved in the special education career path. The volume of students in lower ability-level sequences diminishes from attrition, while the regular sequences diminish when a large number of students fulfill graduation and/or college entry requirements and decide to stop taking math in their senior year.

Not all the career trajectories move in the same *direction*. Several career paths begin and end in the same ability-level designation (e.g., lower basic, regular, and accelerated), while others "dead-end" and force students into a higher ability-level class late in their careers (e.g., special education and upper basic). Last, some career paths move at different *rates*. The basic and regular trajectories entail a significant percentage of students who retake the same course and slow career progression. In contrast, the accelerated trajectory entails students who take two courses in the same year, thereby accelerating the progression of courses and extending their math careers.

Another key finding is that *career paths can interpenetrate*. Three forms of career interpenetration are identified: *drift*, *overlap*, and *intersection*. *Drift* arises as relatively minor transition probabilities across trajectories and is most readily observed between the regular and accelerated sequences. Probabilities indicate that downstream flows are more likely, with 15 percent of Accelerated Algebra students going to regular Geometry and 16 percent of Accelerated Geometry students going to Algebra 2. In contrast, the likelihood of

Table 1. Rural High Mathematics: Transition Probability Matrix for Student Moves

Destination States													
Origin State	Transient States										Points of Exit		
	Acc Calculus	Acc Precalculus	Acc Alg/Trig	Acc Geometry	Acc Algebra 1	Precalculus	Algebra 2	Geometry	Algebra 1	Algebra B	Algebra A	Basic Math	Special Ed Math 2
	Special Ed Math 1	Special Ed Math 2	Graduate	Outside Math	Leave School	# Students 1997							
Acc Calculus	.40	.	.	.	.	.	.	.	.	.	.	1.00	.
Acc Precalculus	.77	.01	.01	.	.	.01	.	.	.	.	.06	.51	.
Acc Algebra II/Trig	.11	.01	.63	.02	.	.06	.	.02	.	.	.06	.03	.
Acc Geometry <sup>a</sup>	.	.78	.18	.15	.	.	.	.02	.	.	.02	.09	.
Acc Algebra 1 <sup>a</sup>	.	.	.	.	.	.	.	.	.	.	.02	.02	.
Precalculus	.07	.	.	.	.	.	.	.	.	.20	.03	.70	.
Algebra 2	.	.01	.	.	.	.17	.11	.	.	.38	.03	.31	.
Geometry	.	.	.09	.09	.	.	.47	.09	.	.22	.06	.08	.
Algebra 1	.	.	.	.72	.05	.	.	.72	.01	.02	.12	.01	.
Algebra B	.	.	.	.38	.	.	.	.12	.12	.28	.10	.12	.
Algebra A	.	.	.	.01	.	.	.	.64	.15	.09	.12	.09	.
Basic Math	.	.	.	.	.05	.	.	.46	.17	.10	.22	.41	.
Special Ed Math 2	.	.	.	.	.	.	.	.07	.05	.43	.14	.21	.
Special Ed Math 1	.	.	.	.	.	.	.	.	.	.05	.34	.	.
Points of Entry													
8th Grade <sup>a</sup>	.10	.02	.02	.08	.12	.	.17	.05	.43	.	.19	.06	.06
Transfer	.	.	.00	.	.	.	.01	.24	.14	.	.02	.05	.02
Outside Math	.	.	.	.	.	.00	.01	.02	.	.	.44	.03	.01
Est. Chain Length	1.00	1.43	2.21	2.83	3.91	1.07	1.34	2.01	2.78	2.00	2.73	2.89	2.09

Note: All values greater than .10 are in bold face.  
a Row sums exceed 1, since some students from this location take more than one course at the next stage.

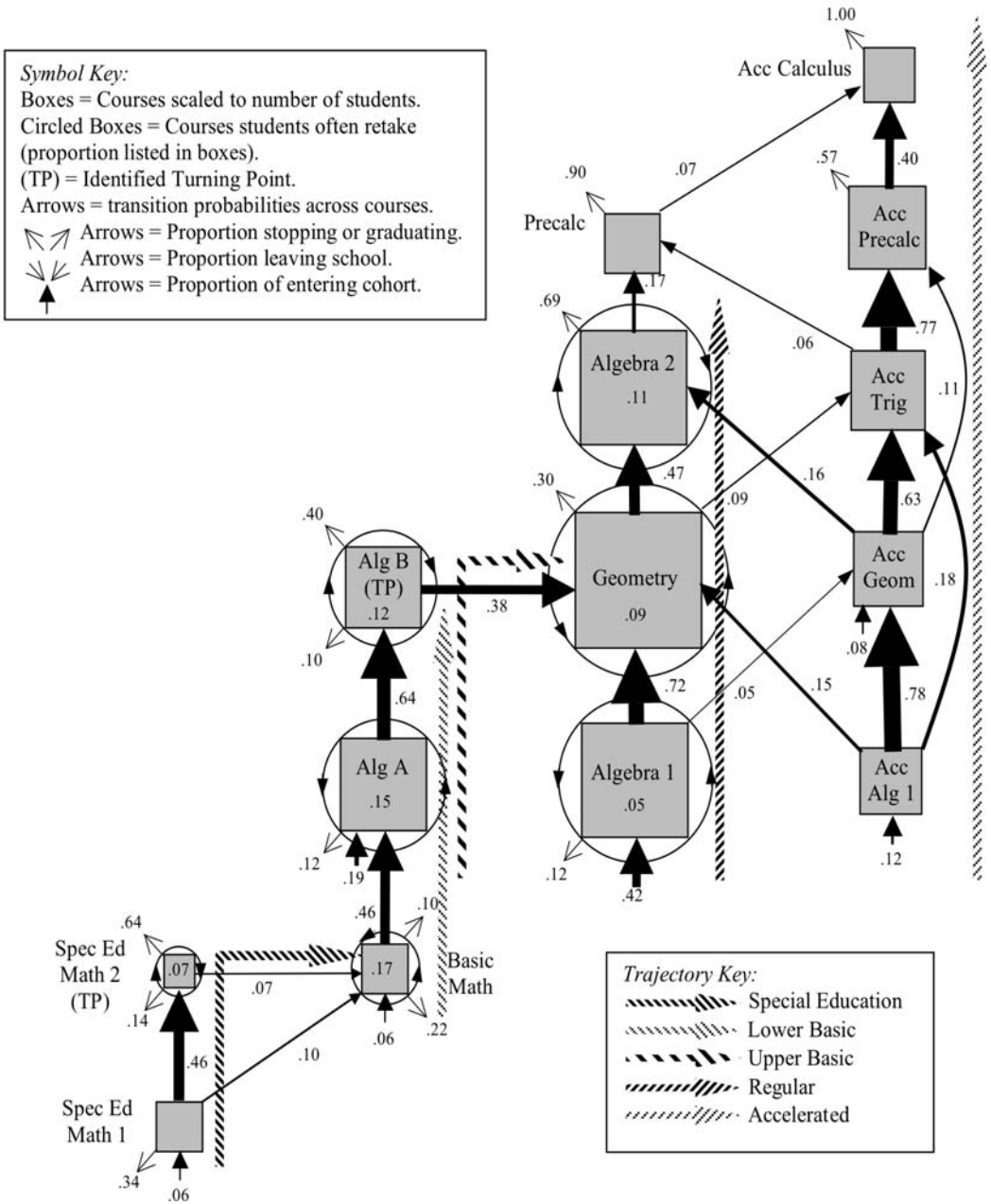


Figure 1 Rural High Mathematics Transition Probabilities

upward placement is between 5 percent and 9 percent. However, it is important to keep in mind that the general track is much larger, so the smaller proportion moving up sustains the volume of students in the accelerated sequence.<sup>10</sup>

*Career overlaps* arise across multiple courses and reveal settings in which heterogeneous populations of students meet. Such overlap can be found across the lower- and upper-basic careers. Since Rural High makes initial student allocations on the basis of test scores, older students in the “overlap courses” are frequently those who took lower ability-level course work. By contrast, *career intersections* are interpenetrations that are focused on particular course junctures. For example, a highly salient intersection occurs when the upper-basic sequence intersects with the regular sequence at Geometry. As a result, Geometry is composed of an age-heterogeneous population consisting of regular 10th graders, upper-basic 11th graders, and 12th graders who are retaking the course to be eligible at the local college (see Figure 1). One cause for this track intersection is found in the preceding course of Algebra B that represents a “dead end.” As such, Algebra B is a *turning point* where students must change career paths or exit math.

A final interesting characteristic of Rural High’s career structure concerns the *staging* of course sequences. There is some evidence that Special Education Math 1 *short-circuits* Special Education Math 2 by allowing 10 percent of its students to go on to Basic Math. Here, rules about prerequisites are more loosely applied. In addition, the one-to-one staging of courses is problematic in two instances. First, Algebra A and B are a two-year version of Algebra 1, so there is no course equivalent to Algebra A. Thus, lateral moves are difficult from this location. Second, accelerated students take two math classes in the same year, calling into question sequential ordering. Since a student can take Accelerated Algebra 1 and Accelerated Geometry in the same year (or Accelerated Geometry and Accelerated Trigonometry), it appears as if algebra is not a prerequisite for geometry, and geometry is not a prerequisite of trigonometry. None of these features

undermines the fact that courses are sequenced, but the features do complicate how the ordering is done and suggest that there may be substantial error in the sequence coding of math courses in national data sets.

With regard to track mobility, the results suggest that a significant number of students move into higher ability-level course work, but it would be a mistake to interpret this as a rosy picture for students in lower ability-level courses. Path-dependent movement in the two basic trajectories and the special education trajectory is weaker than that observed in the regular or honors trajectory. In addition, the points of exit in the basic and special education sequences reveal how downward mobility arises: It is less a move downstream than a move out of the system altogether. Consequently, the flow pattern for basic and special education students is either onward and up into regular course work or out of math and the high school altogether.<sup>11</sup> In regular and accelerated courses, a different context of mobility exists. Accelerated courses show a substantial degree of downward flow to regular courses that replenishes the population in advanced regular courses like Precalculus. However, this relation is somewhat symbiotic in that the larger regular curriculum replenishes the number of students who leave honors course work. The downflows from regular course work are another matter. There is little to no downward mobility from regular to basic. Instead, students either retake their regular-level courses or stop taking math. For both the regular and the two basic-level sequences, upward mobility patterns are eclipsed by exits from the system. Hence, Rural High has a hybrid mobility system in which contest mobility arises across the “higher” paths and tournament mobility arises across the “lower” paths.

The structure of Magnet High’s math curriculum is displayed in Table 2 and Figure 2. Magnet High’s math career structure is different from Rural High’s and begins with a single course, Honors Algebra 1. By following the transition probabilities, one sees that students in Honors Algebra 1 are differentiated into two ability levels consisting of Honors Geometry and Geometry Pre-IB (a course for



aspiring IB students). This distinction continues into the 11th grade when students are divided into four ability levels, three of which become two-year IB sequences that end with an IB examination (general—IB, standard—IBS, and honors—IBH). Regardless of the course, 12th-grade students invariably exit as graduates. Stopping math is really an action adopted only by 12th graders in the lowest trajectory. And leaving Magnet is an action that is adopted primarily in the lowest trajectory during Grades 8–10, before sunk-costs accrue.

Figure 2 depicts the structure of participant flows across Magnet's math courses and reveals the expected treelike structure composed of four career paths (see the trajectory key). Most students go on to the IB programs early in their careers, rather than remain in the lower-status honors sequence. Students appear eager for this type of course work in the 9th grade when Pre-IB Geometry has twice the number of students as Honors Geometry (i.e., 115 to 55, see the last column of Table 2). After students enter the 9th grade, there is a good deal of path dependence within ability levels. Movement across ability levels is less common than is movement within the same level. At the end of the 10th grade, students have the option of entering one of three baccalaureate sequences. These last two years of course work have the most stable movement, partly because students pay a fee (\$75) to take the specialized examinations and partly because the school greatly values students' IB test performance.

Within Magnet's curriculum, two courses act as *turning points*—Algebra 1 Honors and Algebra II/Trigonometry Pre-IB. Both courses represent *starting forks*, or locations after which a wider array of course offerings is made available. As a result of such opportunity, there is a great deal of dispersion up- and downstream from these locations. From Algebra I Honors, nearly half the students go up into more difficult course work. From Trigonometry, nearly 34 percent move up into demanding IB sequences, while 21 percent decide to opt out of such course work after having had a preliminary taste of it. In each location, the tendency is to depart from

the past course trajectory for new ones that are suddenly made available.

Chain lengths tell something about the likelihood of how far students will go in their math careers from any particular course. Looking at the bottom row of the transition probability matrix in Table 2, one sees the predicted chain lengths. Students who enter math often complete 3.54 courses. This is less than the required four years of math and reveals that some students exit the school during Grades 8–10. Each year, about 12 percent of the school's student body leaves, and most of them are in Grades 8–10. This attrition rate means that an 8th-grade class will be nearly half its size by the time of graduation. Writing in the predicted chain lengths on Figure 2's courses, one sees that with each step onward, a student is more likely to finish his or her math career and graduate. In addition, courses in higher ability levels are likely to have students with long careers. Therefore, while a number of students exit math early, those who stay in the curriculum appear to compete for the higher-status slots that become available and stay on to graduation.

Magnet High's curriculum resembles a treelike structure in which students are increasingly differentiated. Such differentiation is induced by increases in course offerings at timed stages of students' careers (e.g., the 8th and 10th grades). In certain regards, this flow pattern resembles a system of progressive mobility implied by Hallinan (1996b). Upon entry to the school, students are placed in a single course, and those who succeed are given the opportunity to move to a higher ability level, while those who do not may continue in the same career path. High rates of exit in the lowest career path show that the bottom of the system moves onward, upward, or out.

In conclusion, maps of participant flows reveal multiple career paths in each curriculum and illustrate that the two schools have distinct career systems. Rural High has multiple career paths of different lengths, volumes, and speeds that intersect at particular courses and sometimes overlap across several. In contrast, Magnet High has multiple overlapping careers of the same length that are increasingly differentiated from one another. The

Table 2. Magnet High Mathematics: Transition Probability Matrix of Student Moves<sup>a</sup>

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Transient States																																																																																																																																						
Points of Exit																																																																																																																																						
Origin State	Adv Calc IB-h	Precalc & Trig IB-h	Calculus IB-s	Precalculus IB-s	Precalc & Disc Math IB	Functions, Stat, Trig IB	Alg II & Trig Pre-IB	Geometry Pre-IB	Discrete Math	Functions	Algebra II Hon	Geometry Hon	Algebra I Hon																																																																																																																									
18 27 16 34 23 44 105 115 NA 83 94 55 177	Graduate Outside Math Leave School	1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00	.04 .15 .14 .10 new course 1998 .18 .04 .27 .04 .20 .18	.07 .09 .10 new course 1998 .04 .04 .27 .20 .18	.03 .09 .09 new course 1998 .52 .63 .58 .39 .02	.03 .09 .09 new course 1998 .52 .63 .58 .39 .02	.03 .09 .09 new course 1998 .52 .63 .58 .39 .02	.03 .09 .09 new course 1998 .52 .63 .58 .39 .02	.03 .09 .09 new course 1998 .52 .63 .58 .39 .02	.03 .09 .09 new course 1998 .52 .63 .58 .39 .02	.03 .09 .09 new course 1998 .52 .63 .58 .39 .02	.03 .09 .09 new course 1998 .52 .63 .58 .39 .02	.03 .09 .09 new course 1998 .52 .63 .58 .39 .02																																																																																																																									
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<sup>a</sup> All values greater than .10 are in bold face.

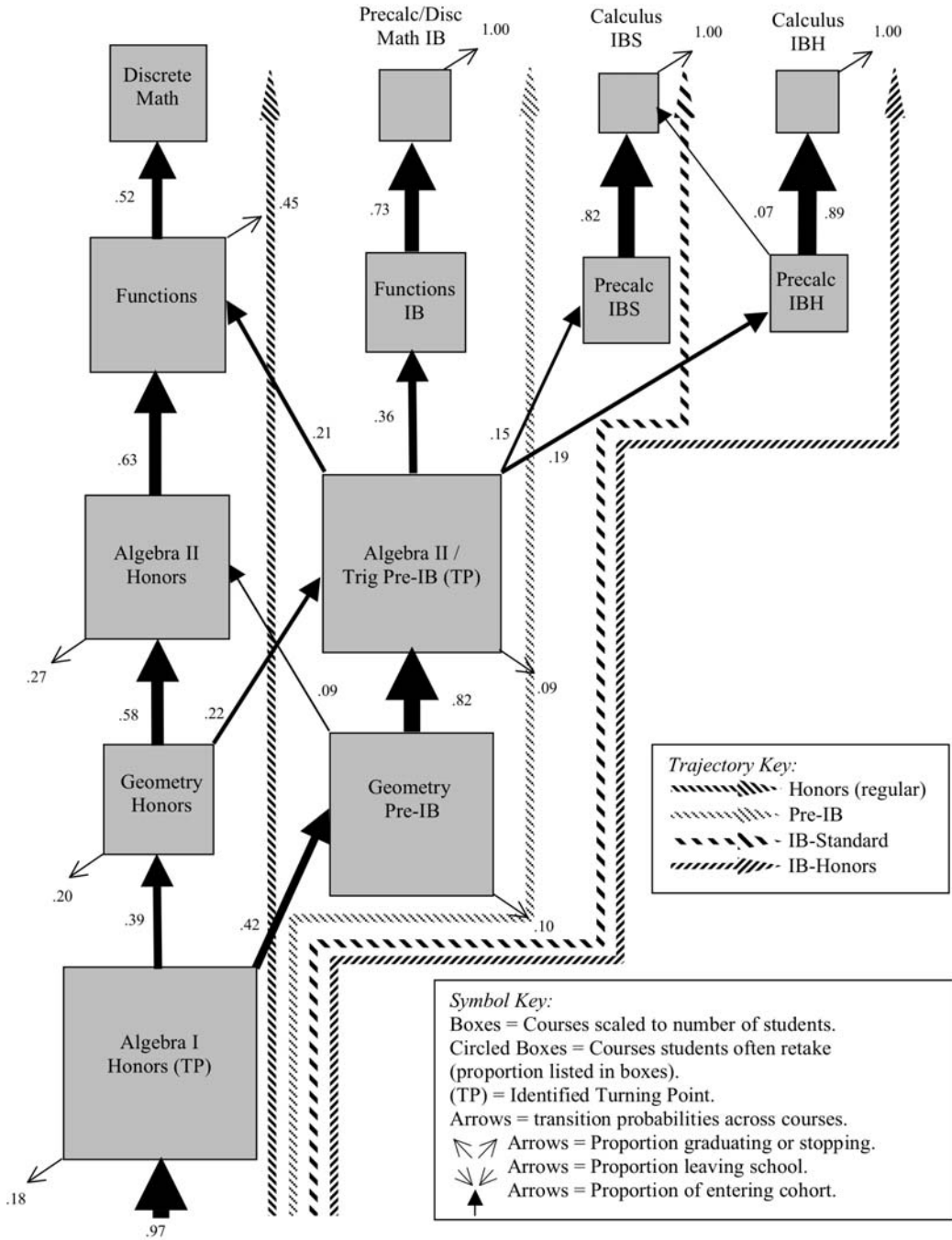


Figure 2 Magnet High Mathematics Transition Probabilities

structural characteristics of each course trajectory reveal how educational careers vary and shape students' experience.

Last, in comparing math at the two schools, the results indicate that the same subject can use different systems of mobility that do not cleanly fit the modal types that have been identified in research on track mobility. Magnet High's mobility structure resembles a triangle resting on its point (▼) and closely resembles a hybrid system of contest and progressive mobility. This is the case because differentiation is delayed and upward flows are observed across different ability levels as higher ability-level courses become available. Rural High has the inverse mobility structure to Magnet. Its mobility structure resembles a triangle resting on its base (▲) and is a hybrid system of sponsored, contest, and tournament mobility. Rural High's structure resembles sponsored mobility because students are differentiated on their entrance to the system and retain a degree of path dependence in different ability sequences. Across the higher ability-level careers, the movement resembles contest mobility because upward and downward selection persists. Last, across the lower ability-level careers, the movement resembles tournament mobility because students move either onward or out of the system altogether.

### **Model of Mobility Dynamics**

Earlier, math career structures were described as having multiple, interrelated course trajectories of different structural properties. Here, the focus shifts to the compositional moves of these trajectories so as to identify assignment criteria that generate such career structures. Logistic regressions are used to predict six mutually exclusive moves at each school: leave school, stop math, repeat, move on but downtrack, move on in the same track, and move on but uptrack. Three types of assignment criteria are hypothesized to generate these moves: students' background, achievement selection, and curricular locations. The aim of this section is to identify processes that are associated with curricular moves in Grades 9–11 at Rural High and Grades 8–11 at Magnet High.

Table 3 reveals how different sets of variables fit the data and explain each curricular move at each school.<sup>12</sup> The point in presenting these results is to show not only which sets of variables best explain each kind of curricular move, but to determine if these relations hold across different career systems. Even though each school has a different career structure and is composed of different frequencies of each curricular move, the pattern of results is consistent across the schools.<sup>13</sup> The most extreme and unsanctioned move of leaving school is best predicted by students' background characteristics and achievement selection. There is a slight difference between the schools, however, since achievement appears to have a slight edge in explaining dropping out of Rural High, while background clearly predominates at Magnet High. One reason for this slight difference is that most students who leave Magnet High are transferring to another school, while students at Rural High are dropping out. Another extreme type of move is repeating a course. Although this move is modeled only for Rural High, it shows that achievement selection is the primary factor that drives such decisions. More sanctioned moves like stopping math and moving downtrack, in the same track, and uptrack are all best predicted by structural locations. Achieved grades play a secondary role in predicting these sanctioned moves, and students' background plays a tertiary one or has no effect. Extreme moves thus appear to be guided more by students' background factors and the signals that grades communicate, while sanctioned moves appear to be highly guided by the opportunities and constraints that course locations afford and the achievement signals that lead youths to more "appropriate" or fitting assignments. This makes some sense, since one may expect sanctioned moves to be guided by rules and performance adjustments and unsanctioned moves to be guided by external concerns and instances of failure.

The most consistent finding in Table 3 is that the full model, which includes all the sets of variables, fits the data best and does the best job of explaining why students adopt each move (see the change in -2 log-likeli-

Table 3. Model Fitness and Predictive Power, by Theoretical Sets of Variables

Models	Leave	Stop	Repeat	Down	Same	Up
<i>Rural High School (N = 1,137)</i>						
Model Fitness (Change -2LL, chi-square test)						
Only student background ( <i>df</i> = 6)	42***	5NS	14*	11NS	21**	3NS
Only achievement selection ( <i>df</i> = 2)	51***	48***	273***	6NS	179***	58***
Only organizational locations ( <i>df</i> = 3)	23***	295***	17***	45***	255***	95***
Full model ( <i>df</i> = 11)	96***	316***	278***	94***	378***	173***
Predictive Power (maximum rescaled <i>R</i> <sup>2</sup> )						
Only student background ( <i>df</i> = 6)	.08	.01	.03	.05	.02	.01
Only achievement selection ( <i>df</i> = 2)	.10	.07	.50	.02	.19	.11
Only organizational locations ( <i>df</i> = 3)	.05	.38	.03	.17	.27	.17
Full model ( <i>df</i> = 11)	.18	.40	.51	.36	.38	.30
Number of Observations	99	203	91	31	603	110
% of Observations = <i>n</i> / <i>N</i>	9	18	8	3	53	10
<i>Magnet High School (N = 707)</i>						
Model Fitness (change -2LL, chi-square test)						
Only student background ( <i>df</i> = 6)	172***	9NS	NA <sup>a</sup>	14*	41***	23**
Only achievement selection ( <i>df</i> = 2)	116***	8*		20***	42***	69***
Only organizational locations ( <i>df</i> = 3)	31***	84***		41***	102***	158***
Full model ( <i>df</i> = 11)	245***	97***		80***	166***	274***
Predictive Power (maximum rescaled <i>R</i> <sup>2</sup> )						
Only student background ( <i>df</i> = 6)	.41	.04		.06	.08	.05
Only achievement selection ( <i>df</i> = 2)	.29	.04		.09	.08	.15
Only organizational locations ( <i>df</i> = 3)	.08	.37		.17	.18	.33
Full model ( <i>df</i> = 11)	.55	.42		.32	.28	.53
Number of Observations	89	31	3	36	419	127
% of Observations = <i>n</i> / <i>N</i>	13	4	0	5	59	18

Source: McFarland (1999).  
\* *p* < .05, \*\* *p* < .01, \*\*\* *p* < .001.  
Note: Missing HHMI is omitted from the results table.  
<sup>a</sup> There are not enough observations for a logit model (Magnet repeaters = 3).



hood scores and the maximum rescaled  $R$ -square). The specific coefficients for these results are presented in Table 4. The results in these models have highly variable magnitudes because of the small cell sizes. In some cases, coefficients translate into high (e.g., 145) and low (e.g., .07) odds ratios (odds ratios =  $\exp(\text{coefficient})$ ). Collapsing the school samples together provides results that are consistent with those presented and produces a greater number of fit models with less extreme coefficients. However, a central goal of the analysis is to perform a comprehensive comparison, so the results are presented independently for each school. In spite of the effect magnitudes, the pattern of coefficients is relatively consistent across the schools.

Looking at the results for *leaving school*, one sees that being from a nonnuclear family and acquiring low grades (especially a non-letter grade) are strong indicators that a student will decide to leave the school. There is also some evidence that students in lower grade levels are more likely to exit than are those who have greater investments in their educational careers. Supplementary analyses (not shown) indicate that ability levels may have an indirect effect on leaving school by being associated with lower grades.

The results for *stopping math* show that students in later grade levels and lower tracks are most likely to stop taking math. In addition, low grades are associated with decisions to stop math. Such results reveal that sanctioned exits are primarily timed decisions by students who have fulfilled graduation requirements, struggle with math, and aspire to less prestigious post-high school destinations.

The results for *course repetition* indicate that Rural High students retake a class when they acquire low grades (F) or a nonletter grade (Incomplete) in their math classes. Hence, it is almost entirely a move determined by failure. Supplementary analyses (not shown) indicate that being from a broken family and lower tracks may have an indirect effect on these decisions by lowering students' achievement.

*Downward moves* to lower ability-level course work are driven primarily by the opportunities and constraints that course

locations afford. Thus, students in higher ability-level course work have greater odds of moving down because there is nowhere up to go. The odds of moving down also increase when students acquire low grades. Magnet High differs from Rural High in that the starting-fork courses (turning points), such as Geometry Pre-IB and Trigonometry Pre-IB, increase the likelihood of downward movement. Simply put, some students decide that the taxing IB work is not for them. Notably, this decision has nothing to do with race, since minority students (net of household income and family structure) are less likely to adopt such a disadvantageous move.

*Same-stream moves* are also driven primarily by the opportunities and constraints that course locations afford. At Rural High, same-track moves are likely to be adopted by students in higher ability-level courses, students in lower grade levels, students with higher grades, and students who are not getting alternative kinds of grades (e.g., pass or incomplete). At Magnet High, same-track moves are likely to be adopted by students from traditional families, students in higher ability-level courses, students who avoid non-letter grades (I = Incomplete, P = Pass), and students who are not located in a turning-point course that promotes career changes. As such, stable career paths primarily emerge in higher ability-level course work, in earlier grade levels, and because students have familial support and good grades.

The final results in Table 4 concern *upward movement*. Again, structural conditions and achievement selection play the most salient roles in the student allocation process. Students in lower grade levels, in lower ability levels, and with high grades are most likely to move upstream. Higher ability locations do not encourage upward mobility because there is no higher stream for students to advance to; only the same and lower ability trajectories are available (Hallinan 1996b; Lucas 1999). Hence, lower ability levels afford greater opportunities for upward movement. High grades communicate who should take these available opportunities. High letter grades and the avoidance of alternative forms of grading thus result in greater chances of moving upward. Upward movement is also

Table 4. Full Model Results for Moves in Grades 9–11

Variables	Leave	Stop	Repeat	Down	Same	Up
<i>Rural High School (N = 1,137)</i>						
Student Background						
Minority	.24	-.18	.10	-12.93	.25	-.88
Female	.11	.00	-.37	-.73	.04	.20
Household median income	-.02	.02	-.02	-.03	.01	.02
Nontraditional family	1.06***	-.37	-.14	-.96	-.20	.06
Achievement Selection						
Math grade 1996–97	-.18	-.29***	-2.72***	-1.24***	.50***	.80***
Nonletter grade 1996–97	2.17***	.17	4.16***	-.03	-2.54***	-13.55
Organizational Locations						
Grade level	-.32*	2.09***	-.18	-.29	-.65***	-.56**
Ability level	-.24	-.90***	.04	3.62***	.40***	-.65***
Turning point (dead end)	-.04	.13	.58	-7.89	-15.97	2.12***
Constant	1.06	-19.89***	1.86	-10.00**	4.69***	2.78
<i>Magnet High School (N = 707)</i>						
Student Background						
Minority	-.43	-1.05*	NA <sup>a</sup>	-1.01*	.39*	.42
Female	.06	.28		-.21	.09	-.50
Household median income	.09	-.02		.01	-.07*	.09
Nontraditional family	4.98***	-.50		-.11	-.95***	-.37
Achievement Selection						
Math grade 1996–97	-.94***	-.37		-1.06***	.02	1.53***
Nonletter grade 1996–97	3.06***	-12.74		-11.71	-2.33***	-.99
Organizational Locations						
Grade level	-.28	3.52***		-.24	-.17	-.14
Ability level	-.28	-.40*		1.41***	.27*	-.92***
Turning point (starting fork)	-.25	-10.64		1.58**	-1.88***	3.28***
Constant	.30	-36.97**		-4.94*	1.77	-2.53

Source: McFarland (1999).

Note: Missing HHMI is omitted from the results table.

<sup>a</sup> There are not enough observations for a logit model (Magnet repeaters = 3).

greatly facilitated by turning-point courses. At Rural High, career dead ends put pressure on students to change career paths, and upstream trajectories are all that are available. At Magnet High, starting forks or bottleneck courses pressure students into either up- or downstream moves. Starting forks clearly pressure students to change career paths, but the sizes of the coefficients across the models suggest that they promote upward movement more than downward movement.

In sum, curricular moves across math courses and trajectories—or sanctioned moves—are primarily explained by the constraints of curricular locations (via rules and opportunity sets; see DeLany 1991; Hallinan 1996b) and the process of achievement selection (Garet and DeLany 1988; Sorenson 1987). Even decisions to stop taking math are constrained by timing rules and the fulfillment of graduation requirements. Since some sanctioned moves allow for career changes, achievement selection becomes an important process in communicating the appropriateness of a student's course selections. Successful students move onward and upward, while unsuccessful ones move downward or stop taking math as soon as their requirements for graduation are fulfilled.

The more "extreme" and unsanctioned curricular moves like dropping out or repeating a course follow a different allocation process. Family issues and failure better explain students' decisions to leave school and to retake a course than do the constraints of various curricular positions. The decision to leave school arises when youths perform so poorly that they consistently fail courses and see no reward for schooling. It also arises because parents get divorced and move (Swanson and Schneider 1999) or because single parents have only so much time to monitor and redirect their children when difficulties arise. The decision to repeat a course is almost purely an issue of achievement selection. When students at Rural High fail a math class that is required to graduate, they are allowed to retake it. When students at Magnet High fail a math class, their failure usually brings them below the minimum grade-point-average standards and forces them to leave the school. Overall, these two

schools' math curricula are composed of mobility dynamics that follow the same general processes. However, in the aggregate, these moves form different career structures.

These results suggest that students' careers are structured primarily by organizational rules and structural opportunities; secondarily by achievement selection processes; and last, by the background composition of students. The vast majority of students and their moves are decided by structural opportunities and constraints, and their sanctioned movement within and across such opportunity sets is guided by an achievement selection process that communicates a better or worse fit within the system. Only more uncommon and unsanctioned moves, such as leaving school and retaking courses, are predicted by factors of students' background and failure to achieve. This finding suggests that more ambitious career structures could be designed for most schools and feasibly accomplished as long as unsanctioned movement (dropping out and repeating) is minimized through appropriate intervention (e.g., tutoring or encouraging participation in extracurricular activities).

## CONCLUSION

The structural analysis of flow relations across courses furthers prior research on tracking, track mobility, and course sequences by illustrating how their core concepts apply to concrete cases, what nuances emerge, and how each conceptualization has certain limitations. Using fully enumerated math-course schedules from two schools over two years, this article has presented a structural representation of career trajectories that enriches our understanding of how career systems vary across schools. In addition, statistical analyses of curricular moves have identified processes that guide sanctioned (curricular locations and achievement selection) and unsanctioned (background conditions and achievement selection) moves that give these career structures their form.<sup>14</sup>

Several findings have been presented about the structure and dynamics of intraorganizational careers in high school math cur-

ricula. First, *participant flows exhibit certain structural characteristics of math careers* that help explain students' differential career experiences: (1) participants can enter a career system at a single point or at several (*starting configurations* differ); (2) careers can proceed in *multiple directions*, most often within the same ability level, but sometimes across them (either up or down an ability level); (3) flows form *career paths* when courses are connected with high probabilities of transition and unidirectional flow patterns; (4) flows can proceed at *different rates* so that progress is slower when students repeat the same course and faster when students take two courses on the same career path in a single year (e.g., Rural High's accelerated courses); (5) *different volumes* of students can flow through each career path, so that some are "trickles" with only a small number of individuals, while others are "floods" composed of a vast number of youths; (6) career paths can even vary in *length*, depending on the number of consecutive courses through which high probabilities of transition occur so that some math careers consist of only one or two completed courses before exit, while others can entail up to five; and (7) *multiple career paths* can arise in the same curriculum, where some intersect at particular junctures (turning points) and others extend across several courses (areas of overlap).

Second, *the general pattern of course taking in the two cases presented illustrates that schools can have different mobility structures for the same subject*. Rural High's math is characterized by five course trajectories, and there is a decrease in the number of course offerings for lower ability-level courses over time, such that students move either upstream or out of the system altogether. In contrast, Magnet High's math begins with a single ability level and then branches off into four course trajectories by offering more challenging course work in later grade levels.

Third, *certain conclusions can be drawn about the dynamics of math careers*, or the compositional moves of each career structure. Even though the aggregate pattern of curricular moves can reveal different career structures, essentially the same processes explain each type of move. In short, career

paths are shaped by background dispositions, curricular opportunities that various course locations afford (i.e., organizational rules), and messages about competence and the appropriateness of course membership that achieved grades communicate. In particular, sanctioned curricular movement entails following rules and making choices that are based on achievement messages about appropriate placement, while unsanctioned moves occur because family situations and achievement messages imply that a break from organizational norms is needed.

The implications of this research are wide ranging. For example, mobility systems are more complex and differentiated than the literature's description of modal types conveys (Lucas 1999; Rosenbaum 1978; Turner 1960). Thus, the math career structure at one school may be different from that of another school, and within the same school, the math career structure may be different from the science career structure (McFarland 1999).<sup>15</sup> Research that examines the variety of curricular structures that are present within today's schools and subjects should inductively arrive at a set of modal types that will likely differ from our current abstract forms of contest, sponsored, tournament, and progressive mobility. By comparing career structures and their effect on students' learning and achievement, educational researchers may better understand how to redesign curricula so as to improve educational experiences and maximize positive student outcomes.

A structural analysis of student flows across course work also reveals overlaps and turning points in students' educational careers. The literature on tracking has long discussed overlaps but has given little thought to when and how track boundaries are obscured. A structural analysis of participant flows makes it possible to address this issue. Certain courses act as structural *locales*, where students of different career trajectories and stages meet and experience the same curricular contents (Lofland 1991; McFarland 1999). These locales can be a single course through which several trajectories intersect (such as turning points) or a series of courses in which two trajectories have overlapping participation (e.g., where one trajectory lags a step behind

another like “slow” and “fast lanes” on the same road). This article has identified and described such locales and, in particular, has found that turning-point courses alter path-dependent curricular flows, often sending students to higher ability-level trajectories. Future work may want to focus on the mechanisms that encourage youths to take advantage of these opportune moments because they may be where the leverage is the greatest for changing the trajectory of individual educational careers.

A third benefit of this type of analysis is that it shows that a wider array of moves is possible than that described in the literature on track mobility (Hallinan 1996b; Lucas 1999). As such, additional dynamics have been identified that are salient to educational research (e.g., repeating the same course, as well as entry and exit points that have been previously overlooked). If future research finds that the same variables induce the same curricular moves across schools and modal types of mobility, then the case for altering the structure to optimize such mechanisms will be stronger. Future work may turn its focus to *designing* career structures that take better advantage of the causal factors that are common to most American high schools (Coleman 1993).<sup>16</sup>

Finally, this type of structural analysis is useful for educational *assessment* because it describes methods of analyzing the organizational context that relies on information that is gleaned from school records (consisting of course schedules, grades, attendance, and the like). Thus, principals could use the various techniques described in this article to identify the career structure of each subject individually or of the entire school curriculum as a whole (Friedkin and Thomas 1997; Heck et al. 2004; McFarland 1999). With a map of these career structures, educators can assess whether there are certain “problem spots” where students drop out or stop course work and, through simple statistical methods, identify variables that are associated with problematic moves. These methods can enable schools and organizations to assess themselves and thereby implement solutions that they deem useful in relation to their goals.

## NOTES

1. Prior research has used comemberships as the basis for identifying sets of structurally equivalent courses that are representative of tracks (see Friedkin and Thomas 1997; Heck et al. 2004). In theory, tracks should be identified as cohesive chains in participant flows (structures in which courses with high probabilities of transition form trajectories), and course sequences should be identified as structurally equivalent sets of courses in these flows (courses with the same sets of student-flow patterns, such as stages in course sequences and broker-turning point types of courses; see McFarland 1999; Wasserman and Faust 1994). However, the theory assumes that the types are pure, when in all likelihood less ideal representations exist that complicate matters. Nevertheless, a focus on the structure of curricular flows affords perhaps the first view of courses as assuming different roles in educational careers. Since curricular flows are asymmetric ties between courses, block-model analysis should reveal a variety of interesting structural roles for courses. In contrast, prior network analysis on comemberships has assumed symmetric relations between courses and affords a far more limited and arguably less interesting and useful structural representation.

2. Inferences that are drawn from school records and yearbooks regarding household income, family structure, gender, and race have nearly perfect reliability when compared to a sample of student surveys that are used at each site (a sample of 15 percent at each school). When distinctions were difficult, the researcher fell back on information gleaned from a year of field observations conducted at both schools. For example, in cases in which Hispanic ethnicity was uncertain, counselors or the student were questioned. Given that neither school had a sizable population of Hispanics, such inquiries were readily managed.

3. This affiliation matrix may be described as  $\mathbf{A} = [a_{ij}]$ , with  $N$  students and  $K$  math courses, where  $a_{ij}$  indicates the membership affiliation of student  $i$  in math course  $j$ .

$$\mathbf{A} = \begin{matrix} & a_{11} & a_{12} & \dots & a_{1K} \\ \begin{matrix} a_{N1} \\ a_{N2} \end{matrix} & \dots & \dots & \dots & \dots \end{matrix}$$



The value of  $a_{ij}$  is binary and indicates the presence or absence of the student's participation in the course.

An affiliation network was created for each separate school year (e.g.,  $A_{97}$  and  $A_{98}$ ). However, additional affiliations besides courses were needed in these networks because the roles of math courses are defined by the patterns of flows into and out of the math curriculum, not just between courses within it. Hence, additional columns were added to  $A_{97}$  and  $A_{98}$  so as to indicate points of entry and exit. Additional columns were constructed in  $A_{97}$  to identify various *entry states* outside the math curriculum. One column pertains to students attending middle school during the 1996–97 school year. A second column identifies transfer students, or those who entered the school from upper grade levels. A third column identifies students attending the high school in 1996–97 but who were not taking math (“outsiders”). Similarly, several columns were constructed in  $A_{98}$  to identify *exit states*. One column pertains to students who left the high school before graduation. A second column identifies students who attended school in 1997–98 but who did not take math (“outsiders”). A third column identifies students who graduated in 1997–98. All membership states are readily inferred from the records.

4. In some instances, students took more than one math course. In the affiliation networks, this fact was recorded as two memberships. If the students went from one to two memberships, they essentially became two people in the transition matrix. If they went from two to one membership, then the two memberships represent separate moves to the next course in the transition matrix. This discrepancy is not an issue for calculations of the probabilities of transition because they reflect actual flow patterns. However, it is an issue for statistical models that predict types of moves; in that case, only the highest membership was included in the analyses.

5. A predicted vacancy-chain length is often described by its multiplier effect (White 1970). The predicted distribution of vacancy chain lengths is given by what is called the multiplier matrix in the Markov formulation (Chase 1991:141; McFarland 1970; Sorenson 1984:31):

$$N = (I - Q)^{-1} \quad (1)$$

where  $Q$  is an  $m \times m$  submatrix of the transition probability matrix including only the probabilities of transitions among the various transient states (where entry and exit states are omitted), and  $I$  is an  $m \times m$  identity matrix (where the diagonal has values of 1). Each element  $n_{ij}$  of the multiplier matrix  $N$  equals the expected number of times a vacancy starting out in state  $i$  will be in resource units in state  $j$  before exiting the system. The row sums of  $N$  give the expected multiplier effects for chains starting in each of the transient states (White 1970). In matrix notation,

$$n = N1 \quad (2)$$

where  $n$  is a column vector giving the predicted multiplier effects and  $1$  is a column vector of 1's (this equation simply sums the rows of matrix  $N$ ). These row sums capture the expected length of curricular paths extending from each course (Smith and Abbott 1983:1153). Courses from which most students graduate have a one-step chain, while those that represent starting points in students' careers can entail upward of four steps.

6. Alternative family structures consist of a single parent, parents with surnames that are different from the child's, the father with a different surname from the child, and so forth. When the two parents are male-female combinations with the same surname as their child, they are considered a “traditional” family. This coding of school records was compared with survey data in which students reported on their parents' or guardians' marital status. The coding of traditional versus alternative types of families was found to be accurate (a 98 percent match). More specific designations, such as whether listed guardians' names reflected foster parents or single parents who were divorced, were found to be inaccurate. The fact that the coding of traditional, biological parents versus other types of families was accurately identified at these two sites does not mean that it will necessarily work for other high schools (although school records have improved and the marital status of parents is often listed

explicitly). The two schools in this study are from conservative states, so the distinction applies well.

7. Several sophisticated methods have been used to predict curricular moves and the factors that are associated with them: logistic regression models associating origins with the likelihood of adopting various types of moves (Hallinan 1996b; Spilerman 1972), ordered probit models associating origins with destinations (Lucas 1999), and multinomial logit models (Breen and Jonsson 2000). The use of logistic regression has been criticized as inaccurate (Lucas 1999), and while ordered probit models provided an elegant solution for Lucas, they are ill suited for the current analyses. An ordered probit model is applicable when the slopes of variables are the same across levels of the dependent variable. To test whether this was an acceptable method, a preliminary test statistic was used in cumulative logit models to ascertain whether the slopes were parallel (Allison 1999). Regardless of the ordering used—including the ordinal variable used by Lucas—statistically significant parallel slopes that permitted the accurate use of probit models could not be found. Usually, the lack of parallel slopes leads researchers to adopt multinomial logit models that render the moves into unordered or categorical variables. While technically valid in this case, the use of multinomial logit models remains impractical because it would require the comparison of six outcomes with each other (i.e., leave school x stop math, stop math x down track—a total of 15 regressions for each school). In the end, it was decided that each move should be predicted in comparison to all others in simple logit models (akin to Hallinan 1996b). Although the biases in likelihood were well explained by Lucas, it is believed that analyzing an array of mutually exclusive moves and their changing coefficients prevents any “rosy” misinterpretations that could arise (as Lucas suggested was the case in Hallinan’s work).

8. In most cases, aggressive parents (regardless of background) can alter their children’s track placement. In fact, Rural High’s counselors told me that they documented when a student was placed against their recommendation, so as to be prepared for litigation if the student failed.

9. The way to interpret estimated chain lengths is to regard the first move as entering the origin course. So 2.09 suggests that the first step is from the eighth grade to Special Ed 1; the second from Special Ed 1 to 2; and beyond that, to Basic Math. These observed chains are comparable to the predicted measures of chain length for each course using Equation 2 described in note 5. The bottom row of the matrix presents predicted chain-length values (of *n*) indicating the number of steps or the number of courses a student is likely to take from the course he or she is currently in. Predicted chain lengths offer a summary understanding of the opportunity sequences that each course affords (Chase 1991; Sorenson 1984; White 1970). Of special interest are courses with longer chain lengths because they are early stages of each math sequence. If one looks at the predicted chain lengths for each course in a trajectory, it is clear that students increase their chances of completing a trajectory with each step forward that they move in it.

10. It is likely that with full transcript data one could determine whether downward-moving students are also more likely to move upward the next year (see Lucas 1999). Markov models of transition probabilities afford only year-by-year movement likelihoods and may not offer the best means for studying individual career trajectories; rather, they may fit the structure of population flows in a curricular subject more generally.

11. In certain respects, this flow pattern presents another take on what the head counselor described in an interview: “Students placed in basic generally have two reactions: (a) “I can do better” or (b) “I’d rather take basic because it is easier.” The former generally have college aspirations and know that many schools will not accept basic-level credits (i.e., Basic Math). The latter are simply looking to “get by.” What the counselor neglected to mention (or took for granted) was that the students who like the “easier” courses are on their way to dropping out of school.

12. The maximum rescaled *R*-square reflects the explained variance, but it rescales the generalized *R*-square measure so that the upper bound equals 1 (see Allison

1999:56–57 for a detailed description). The change in the  $-2$  log-likelihood (LL) statistic is used to perform a chi-square test (according to a change in  $df$ ) of model improvement (see Allison 1999:20), making it possible to identify better models according to their predictive power and fitness.

13. A simple correlation of changes in  $-2LL$  and maximum rescaled  $R$ -squares across schools runs near .98. The results for leaving school and same-track moves have a .90 correlation, and no comparison can be made for repeating a course.

14. The data available for this study make it difficult to test alternative explanations for why students move the way they do. Future research can improve on this work by collecting fully enumerated course schedules that span students' careers from a larger sample of schools, by capturing changes in scheduling over the course of a year (Hallinan 2003), and by using attitudinal surveys. Through larger and more detailed study, scholars can test a wider array of explanations.

15. These findings are potentially generalizable to other schools and subjects. The findings most readily apply to math curricula at other schools because math curricula are usually organized in a hierarchical, rule-based fashion, making structural maps easy to generate and compare. While the general patterns of math at Rural High and Magnet High

are different, their compositional moves follow similar logics, regardless of whether models are collapsed or run separately by school. The findings on math course flows extend to English and science curricula at both schools, although as timing rules diminish and the range of course options increases (i.e., electives and semester courses), the identification of distinct career trajectories becomes more difficult. For these reasons, the results on math careers are least applicable to the social science curricula. In the social science curricula, the structure of course taking entails required courses taken at particular times (e.g., 9th-grade World History and 11th-grade U.S. History), interspersed with clusters of electives taken in a random order (e.g., sociology, psychology, economics, and so forth). The structure of course taking in these types of curricula represents more of a network or branching tree structure that repeatedly converges and diverges in form.

16. Past structural models, such as vacancy chains and venturi tubes (Chase 1991; Sorenson 1984; Stewman 1975, 1986; Stewman and Konda 1983), are platforms for future designs. Those models predict how structural changes would alter career progression. Other platforms could stem from operations research (Sterman 2000) and its effort to model organizational structures and decision processes in firms.

Appendix Table A. Construction of Variables for Logistic Regressions (Grades 9–11 at Rural High School and Grades 8–11 at Magnet High School)

Variable	Rural		Magnet	
	Mean	SD	Mean	SD
<i>Student Background</i>				
Minority race	.04	.20	.40	.49
Female gender	.50	.50	.58	.49
Household median income	.05	5.48	-.05	2.93
Missing HHMI	.26	.44	.08	.27
Alternative family	.31	.46	.40	.49
<i>Achievement Selection</i>				
Math letter grade (1997)	2.14	1.20	2.81	.96
0 = F, 1 = D, 2 = C, 3 = B, 4 = A. Other grades of P (pass), I (incomplete), or E (satisfactory) = 2 or C at Magnet and = 1 or D at Rural.				
No math letter grade	.06	.24	.05	.21
1 = Grades of P, I, or E; 0 = other.				
Rural: 1 = Special Ed, 2 = Basic, 3 = Regular, 4 = Honors. Magnet: 1 = Honors, 2 = IB (International Baccalaureate), 3 = IBS (Standard), 4 = IBH (Honors).				
Grade level (or age proxy)	2.94	.77	1.45	.91
Turning-point course	9.93	.80	9.53	1.12
9th through 12th grades at Rural and 8th through 12th at Magnet.				
Rural: "Special Ed Math 2" or "Algebra B" Magnet: "Algebra I Hon" or "Algebra II/ Trig IB"				
.07 .26 .40 .49				
<i>Dependent Variables (Curricular Moves)</i>				
Leave school	.09	.28	.13	.33
Stop taking math	.18	.38	.04	.20
Repeat course	.08	.27	.00	.07
Move downstream	.03	.16	.05	.22
Move same stream	.53	.50	.59	.49
Move upstream	.10	.30	.18	.38
Student leaves school (drops out) in 1998.				
Student exits math in 1998 but stays in school.				
Student repeats the same course in 1998.				
Student goes on in math but moves downstream in 1998.				
Student goes on in math and stays in the same stream in 1998.				
Student goes on in math and moves upstream in 1998.				

Note: Rural N = 1,137, Magnet N = 707 (all logit models omit 12th-grade students).

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