Computer Vision 2016 Spring HW # 1 Theory questions

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Q1. (a)

$$\mathrm{g(x)} = f(x) * g(x)$$

When $\mathrm{F(u)}$ *is Fourier transformed form of* $f(x)$ *and* $G(u)$ *is also Fourier transformed form of* $g(x)$,

$$\mathrm{G(u)} = F(u)G(u). Therefore, F(u) = 1$$

$$\mathrm{F(u)} = \int_{-\infty}^{\infty} f(x)e^{-i2\pi ux}dx = 1. \quad \mathrm{f(x)} = \delta(x).$$

Q1. (b)

Let the edge orientation is $\tan^{-1}\frac{I_y}{I_x} = \theta$, and $\mathrm{G(u,\sigma)} = \frac{1}{\sigma\sqrt{(2\pi)}}e^{-\frac{u^2}{2\sigma^2}}$

Then, we can apply rotational transform by angle θ to create new plane with axis X and Y.

$$\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Also, we can represent x and y with X and Y.

$$\mathrm{x = Xcos\theta + Ysin\theta}, \qquad \mathrm{y = -Xsin\theta + Ycos\theta}$$

Now, we can represent our smoothing function with X and Y like this.

$$\mathrm{I}'(Xcos\theta + Ysin\theta, \mathrm{-Xsin\theta + Ycos\theta}) = \mathrm{I}(Xcos\theta + Ysin\theta, \mathrm{-Xsin\theta + Ycos\theta}) * \mathrm{G(X,\sigma_1)} * \mathrm{G(Y,\sigma_2)}$$

In other words,

$$\mathrm{I'(x,y)} = \mathrm{I(x,y)} * G(xcos - ysin\theta, \sigma_1) * G(-xsin\theta + ycos\theta, \sigma_2)$$

Q2. (a)

Let each evidence $(\mathrm{x_i, y_i})$. Circle equation is $(x_i - a)^2 + (y_i - b)^2 = r^2$.

Then, hough space (parameter space) would have a, b, r axis.

For certain r, there would be circle per each evidence which has itself as the center and r as it's radius.

If we construct accumulator array A(a, b, r) with these circles and get local maximum, we would find a point which has a, b, r that we have to know.

If we know radius r already, this problem become much easier.

Hough space would have a, b axis only, and there would be circles in that plane.

If we construct accumulator array $A(a, b)$ with these circles and get local maximum, we would find a point which has a, b that we have to know.