Computer Vision 2016 Spring HW#3 theory

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Q1.

(a) Known part is like below.

$$\text{for world point X, camera 1 has } \mathrm{X}_{c1}, \text{camera 2 has } X_{c2}.$$

$$\text{for } \mathrm{M}_1 = A_1[R_1|t_1] \text{ and } M_2 = A_2[R_2|t_2], X_{c1} = R_1X + t_1 \ (1), X_{c2} = R_2X + t_2 \ (2).$$

$$\text{for length constant } \mathrm{z}_1 \text{ and } z_2, p = \frac{1}{z_1}X_{c1} \ (3), q = \frac{1}{z_2}X_{c2} \ (4).$$

Let's think about a projection of reference camera center to plane π. If the distance between the plane π and the center of camera 2 is d_2, we can consider a normal vectors n_2 from camera 2 center to plane π, which's length is $\frac{1}{\mathrm{d}_2}$.

Then, we can know that

$$n_2^T X_{c2} = 1 \ (5).$$

Now, let's think about $\frac{z_1}{z_1}p$.

$\frac{z_1}{z_2}p = \frac{1}{z_2}(R_1X + t_1) = \frac{1}{z_2}(\mathrm{R}_1R_2^{-1}(X_{c2} - t_2) + t_1)$ (by (3) and (1))

(R_1 must have inverse because it's rotation)

$$= \frac{1}{z_2}(R_1R_2^{-1}(z_2q - t_2) + t_1) = R_1R_2^{-1}q - \frac{1}{z_2}R_1R_2^{-1}t_2 + \frac{1}{z_2}t_1 \ \text{(by (4))}$$

$$= (\mathrm{R}_1R_2^{-1} - R_1R_2^{-1}t_2 \cdot n_2^T + t_1 \cdot n_2^T) \ q \ \text{(by (5) and (4))}$$

Then, we can get the homography matrix $\mathrm{H} = (\mathrm{R}_1R_2^{-1} - R_1R_2^{-1}t_2 \cdot n_2^T + t_1 \cdot n_2^T)$.

This H satisfy the equation $\frac{z_1}{z_2}p = Hq$, which means it is a projective transformation of the plane.

Therefore, this H exists for any point X.

(b) Homography matrix is 3X3, therefore we have 9 unknowns. However, because it shows homogeneous equality, we can re-scale it. Therefore, only 8 unknowns left.

$$\begin{bmatrix} wx_i' \\ wy_i' \\ w \end{bmatrix} = \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

Because of above matrix equation, one pair of point makes two equations like below.

$$\mathrm{x}_\mathrm{i}'(h_{20}x_i + h_{21}y_i + h_{22}) = h_{00}x_i + h_{01}y_i + h_{02}$$

$$y_i'(h_{20}x_i + h_{21}y_i + h_{22}) = h_{10}x_i + h_{11}y_i + h_{12}$$

Therefore, to reveal 8 unknowns, 4 pairs of points are needed.

(c) From above equation, we can construct next equation using n pairs of points.

$$\mathbf{Ah} = \mathbf{0} \text{ when } \boldsymbol{A} = \begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1'x_1 & -x_1'y_1 & -x_1' \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -y_1'x_1 & -y_1'y_1 & -y_1' \\ x_2 & y_2 & 1 & 0 & 0 & 0 & -x_2'x_2 & -x_2'y_2 & -x_2' \\ 0 & 0 & 0 & x_2 & y_2 & 1 & -y_2'x_2 & -y_2'y_2 & -y_2' \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_n & y_n & 1 & 0 & 0 & 0 & -x_n'x_n & -x_n'y_n & -x_n' \\ 0 & 0 & 0 & x_n & y_n & 1 & -y_n'x_n & -y_n'y_n & -y_n' \end{bmatrix}, \quad \mathbf{h} = \begin{bmatrix} h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \\ h_{22} \end{bmatrix}, \quad \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix},$$

Then, a least squares could be performed to $\left|\left|\mathbf{Ah} - \mathbf{0}\right|\right|^2$.

For $\hat{h}$ which is unit vector and $h \equiv \hat{h}$, solution of least squares problem $\left|\left|\mathbf{A}\hat{\boldsymbol{h}} - \mathbf{0}\right|\right|^2$ is

$$\hat{h} = eigenvector \text{ of } A^T A \text{ with smallest } eigenvalue.$$