Computer Vision 2016 Spring HW#2 theory

2013-11415 Sanha Lee

Q1.

(1) Let the albedo of this Lambertian surface is ρ_d, each light source's intensity is equal as I, and surface normal is $\overrightarrow{n_1}$.

Then, because of properties of Lambertian surface, for all viewing directions, radiance of surface L_1 from first source and radiance of surface L_2 from second source are like this.

$$L_1 = \frac{\rho_d}{\pi} I \vec{n} \cdot \overrightarrow{s_1}$$

$$L_2 = \frac{\rho_d}{\pi} I \vec{n} \cdot \overrightarrow{s_2}$$

Now, let's think about radiance of surface L_3 from single "effective" source.

$$L_3 = L_1 + L_2$$

$$= \frac{\rho_d}{\pi} I \overrightarrow{n_1} \cdot (\overrightarrow{s_1} + \overrightarrow{s_2})$$

By above equality, we can regard the "effective" source as a light source having intensity of I and direction $(\overrightarrow{s_1} + \overrightarrow{s_2})$.

Therefore, the "effective" direction s_3 is $\overrightarrow{s_1} + \overrightarrow{s_2}$.

(2) With different intensity I_1 *and* I_2, radiances are changed like below.

$$L_1 = \frac{\rho_d}{\pi} I_1 \vec{n} \cdot \overrightarrow{s_1}$$

$$L_2 = \frac{\rho_d}{\pi} I_2 \vec{n} \cdot \overrightarrow{s_2}$$

Now, let's think about "effective" source's direction s_3 and intensity I_3.

Like (1), we can get this equality.

$$I_3 \overrightarrow{s_3} = I_1 \overrightarrow{s_1} + I_2 \overrightarrow{s_2}$$

Therefore, "effective" direction s_3 is $\frac{I_1}{I_3} \overrightarrow{s_1} + \frac{I_2}{I_3} \overrightarrow{s_2}$.

(There are two unknowns (I_3 *and* s_3) but there is just one equation, so I couldn't get both unknowns respectively.)

Q2.

Let's think about 3D coordinate system with origin at the sphere's center and with x and y axes oriented with the image axes.

At a point (u, v, z) on this sphere's surface, $\mathrm{R}^2 = u^2 + v^2 + z^2$ because of the definition of sphere and Pythagorean theorem.

Therefore, the normal vector formula for (u,v) is $(\frac{\mathrm{u}}{\mathrm{R}}, \frac{v}{R}, \frac{\sqrt{R^2-u^2-v^2}}{R})$.