B. Tech (E&C) 3rd Semester

ECS-303: Circuit Analysis and Synthesis

2nd Sessional Test (November 2023)

Time: 1hr | **MM:16** | **Course Outcome**

Questions *1. Attempt all questions* *2. Make suitable assumption if required*

Q.1 | **8M** | **CO1**

(i) Find the Norton equivalent with respect to terminals *a-b* in the circuit shown in **Fig. 1** below. **(2M)**

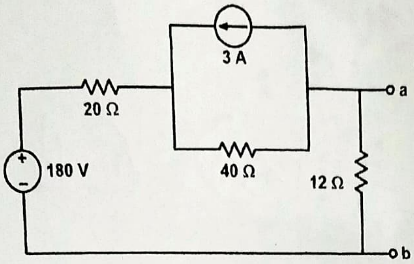


Fig. 1

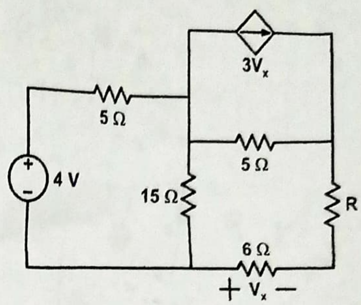
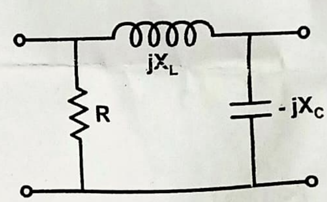


Fig. 2

(ii) For the circuit shown in **Fig. 2** above, determine the maximum power delivered to the variable resistor R. **(3M)**

(iii) Determine the z-parameters of the network shown below. **(3M)**



Q.2 | **8M** | **CO2**

(i) Develop the state equations for the following differential equation given below. **(2M)**

$$\frac{d^2y(t)}{dt^2}+\frac{4dy(t)}{dt}+3y(t)=z(t)$$

(ii) Develop the state equations for the circuit shown in **Fig. 1** below. **(3M)**

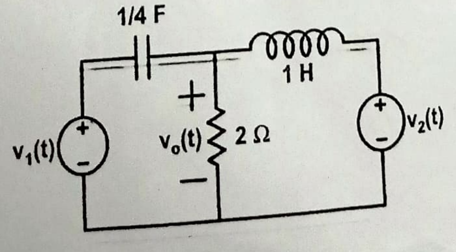


Fig. 1

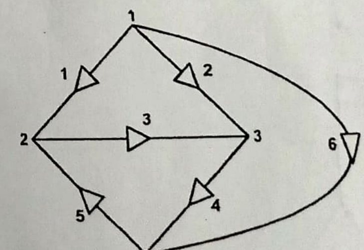


Fig. 2

(iii) Find the incidence matrix and cut-set matrix for the given graph in **Fig. 2** above. **(3M)**

15

B. Tech (E&C) 3rd Semester

ECS-303: Circuit Analysis and Synthesis

1st Sessional Test (September 2023)

Time: 1hr **MM:16** **Course Outcome**

Questions *1. Attempt all questions* *2. Make suitable assumption if required*

Q.1 8M CO1

(i) Calculate v and i_x in the circuit of Fig.1 shown below. (2M)

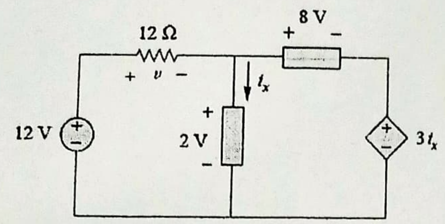


Fig. 1

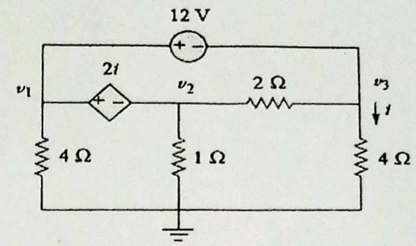
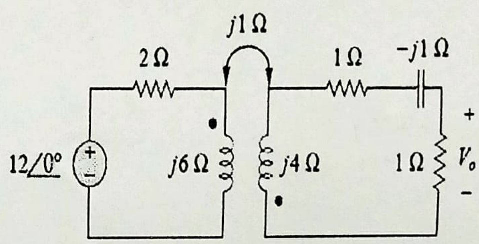


Fig. 2

(ii) For the circuit shown in Fig. 2 above, find $v1$, $v2$, and $v3$ using nodal analysis. (3M)

(iii) For the circuit shown in Fig. below, find V_o (3M)



Q.2 8M CO2

(i) A system has the transfer function

$$H(s) = \frac{s}{(s+1)(s+2)}$$

(a) Find the impulse response of the system. (1M)

(b) Determine the output y(t), given that the input is x(t)=u(t). (1M)

(ii) The switch in Fig. 1 shown below has been in position a for a long time. At t=0, it moves to position b. Calculate i(t) for all t>0. (3M)

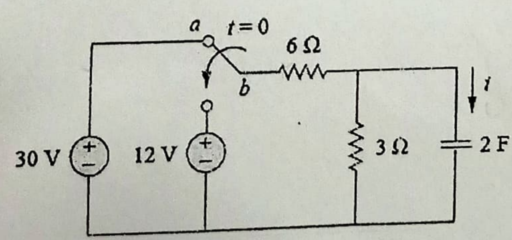


Fig. 1

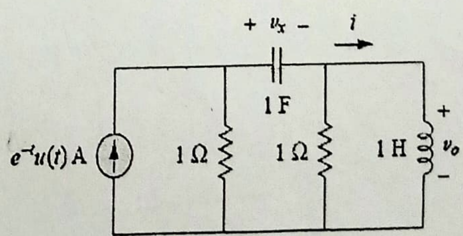


Fig. 2

(iii) Find $v_o(t)$ in the circuit of Fig. 2 shown above, if $v_x(0)$=2 V and i(0)=1 A. (3M)

Basic

• charge (Q) → $i(t) = \frac{dQ}{dt}$

$$i(t) = \frac{dq}{dt}$$

$1e^- = -1.6 \times 10^{-19} C$

$$\int i(t)\,dt = dq$$

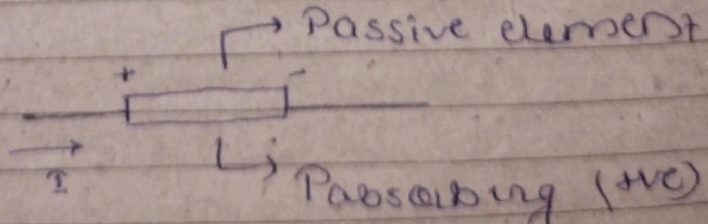
$$\left(q = \int_{-\infty}^{t} i(t)\,dt \right)$$

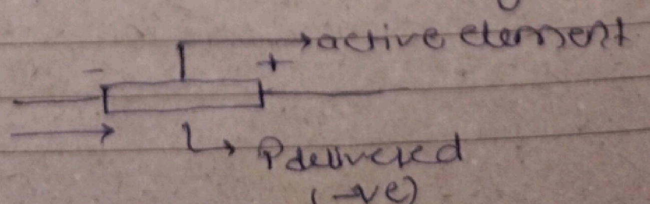
• V (potential) $= \frac{dE}{dq}$

• $\left(P(t) = \frac{dE}{dt} \right)$

$$= \frac{dE}{dq} \cdot \frac{dq}{dt}$$

$$\left(P(t) = V(t) \cdot I(t) \right)$$





Basic

• charge (Q) → $i(t) = \frac{dQ}{dt}$

$i(t) = \frac{dq}{dt}$

$1e^- = -1.6 \times 10^{-19} C$

$$\int i(t)\,dt = dq$$

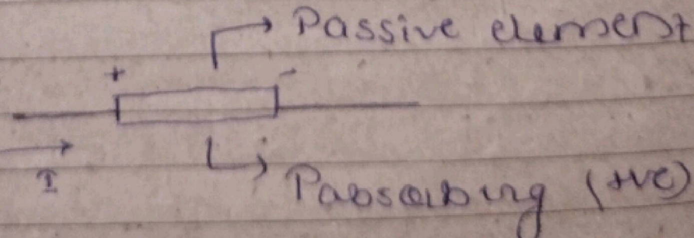
$$\left(q = \int_{-\infty}^{t} i(t)\,dt \right)$$

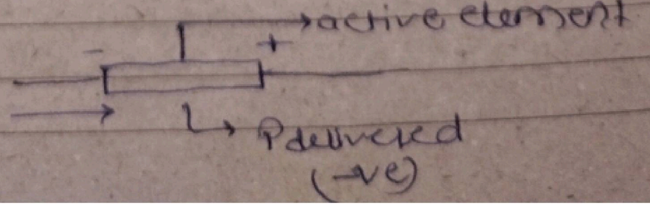
• V (potential) $= \frac{dE}{dq}$

• $\left(P(t) = \frac{dE}{dt} \right)$

$$= \frac{dE}{dq} \cdot \frac{dq}{dt}$$

$$\left(P(t) = V(t) \cdot I(t) \right)$$





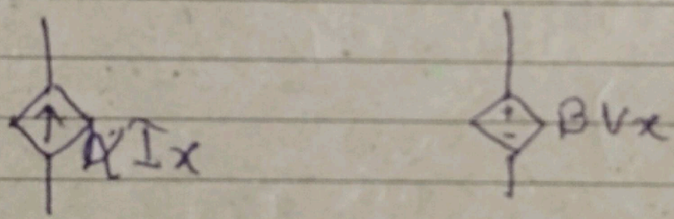
- Those element which alway absorb power then it is passive element

- Those element which alway deliver power then it is active element

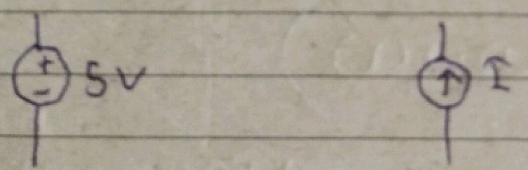
* Dependent source & independent source :-

→ It may be current, voltage source

→ Dependent source depend on any parameter and this connect with any element of the circuit.



→ Independent source ~~depend~~ does not depend on any parameter.



Q:- $q = 5t \cdot \sin(4\pi t)$ mC

$i = ?$ $t = 0.5s$

$$i = \frac{dq}{dt}$$

$i = 5\sin(4\pi t) + 20\pi t\cos(4\pi t)$

$i = 5\cancel{(2t)}^{0} + 20\pi t\cos(2\pi)$

$\left(i \Big|_{t=0.5} = 10\pi \text{ mA} \right)$

Q:- $i = (3t^2 - t)A$, $q = ?$

$t = 1s \text{ to } 2s$

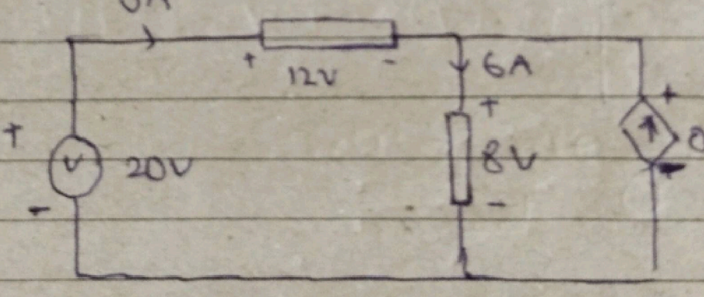
$\int dq = \int i\,dt$

$Q = \int_1^2 (3t^2 - t)\,dt$

$Q = t^3 - \frac{t^2}{2} = \left[(8-1) - \frac{(4-1)}{2} \right]$

$7 - \frac{3}{2} \Rightarrow \underline{\underline{5.5\,C}}$

Q:-



→ 8V as it is in parallel

$P_{absorb} = 12 \times 5 + 6 \times 8 + 8 \times 1$

$60 + 48$

P_{absorb} 108 W

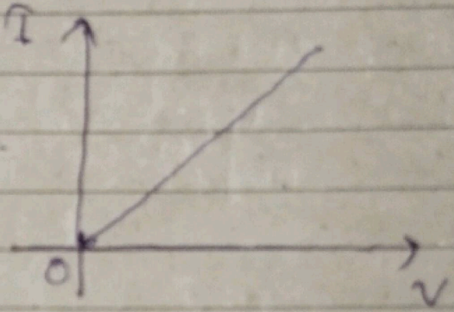
$P_{deliver} = -20 \times 5 - 8 \times 1 \Rightarrow -108\,W$

To check

$$P_{delivered} + P_{absorbed} = 0$$

$$-108 + 108 = 0$$

* <u>Ohm's law</u> :

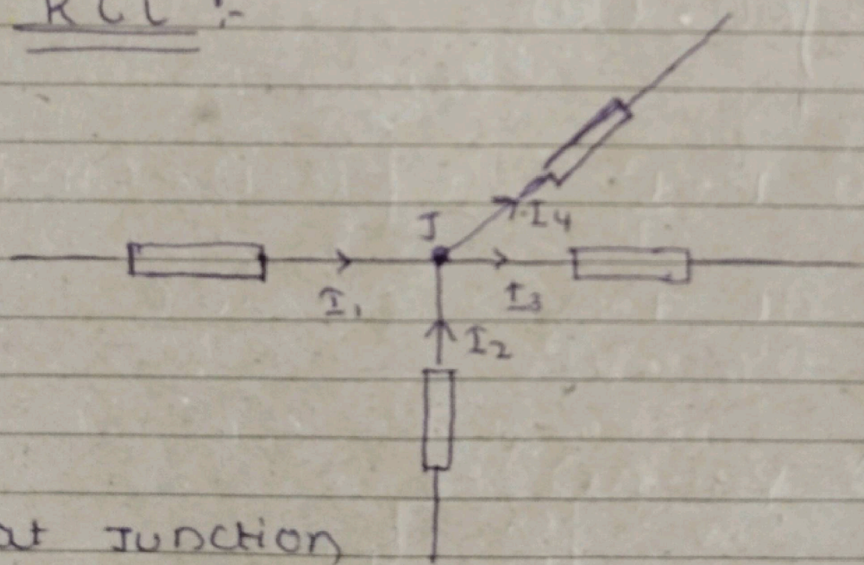


$V \propto I$

$(V = IR)$

$\left(R = \frac{\rho l}{A}\right)$

* <u>KCL</u> :-



at junction

$$\sum_{n=1}^{n} I_n = 0$$

$$I_1 + I_2 - I_3 - I_4 = 0$$

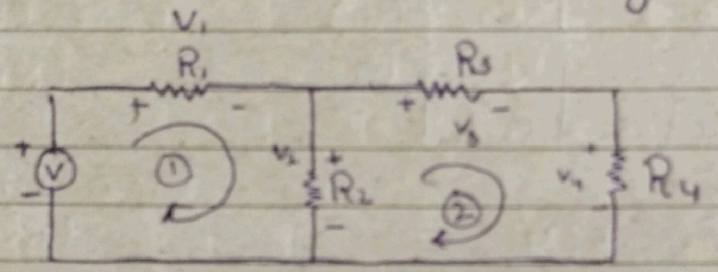
$$(I_1 + I_2 = I_3 + I_4)$$

Delta

Incoming current = outgoing current

→ based on conservation of charge
↳ neither be created nor destroyed

* KVL (Kirchoff's voltage law) :-



In loop
$$\sum_{N=1}^{M} V_N = 0$$

In loop ①

$$V - V_1 - V_2 = 0$$

$$(V = V_1 + V_2)$$

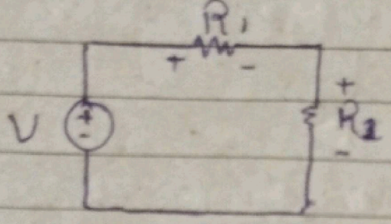
In loop ②

$$-V_3 - V_4 + V_2 = 0$$

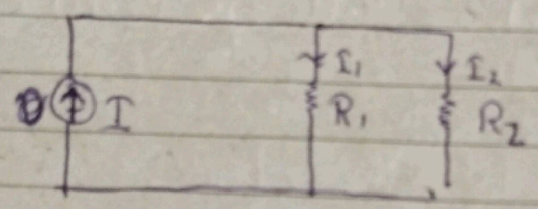
$$(V_2 = V_3 + V_4)$$

$\rightarrow V_{R_1} = \dfrac{V R_1}{R_1 + R_2}$

*



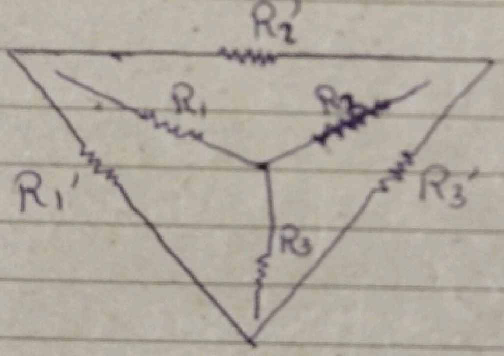
$\rightarrow V_{R_2} = \dfrac{V R_2}{R_1 + R_2}$



$I_2 = \dfrac{I R_1}{(R_1 + R_2)}$

$I_1 = \dfrac{I R_2}{(R_1 + R_2)}$

* Star-Delta Transformation:-



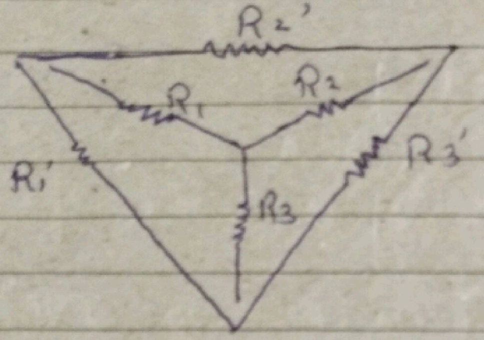
$$R_1' = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_2}$$

$$R_2' = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_3}$$

$$R_3' = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_3}$$

Delta

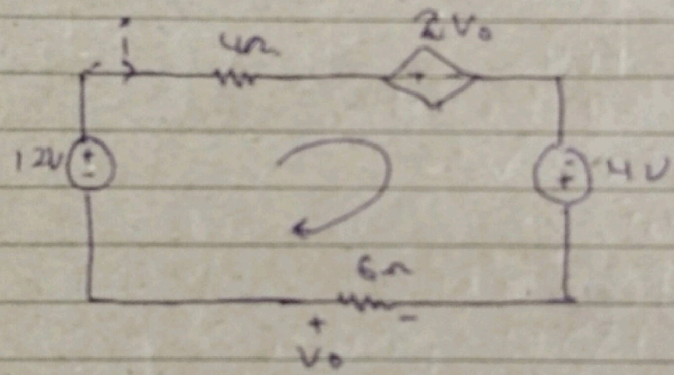
Delta – star transformation



$$R_1 = \frac{R_1' R_2'}{R_1' + R_2' + R_3'}$$

$$R_2 = \frac{R_1' R_3'}{R_1' + R_2' + R_3'}$$

Q:- Find i, V_0?



$\xrightarrow{I} \overset{-\ \ +}{V}$ I +ve

$I \rightarrow \overset{+\ \ -}{V}$ I -ve

$V_0 = -6i$

In this loop

$$+12V - 4i - 2V_0 + 4V + V_0 = 0$$

$$12V - 4i + 12i + 4V - 6i = 0$$

$$16V + 2i = 0$$

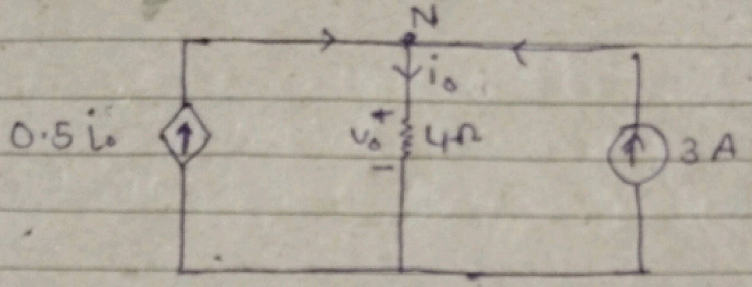
$$(i = -8A)$$

as,

$$V_0 = -6i$$

$$(V_0 = 48V)$$

Q:- Calculate i_o & v_o



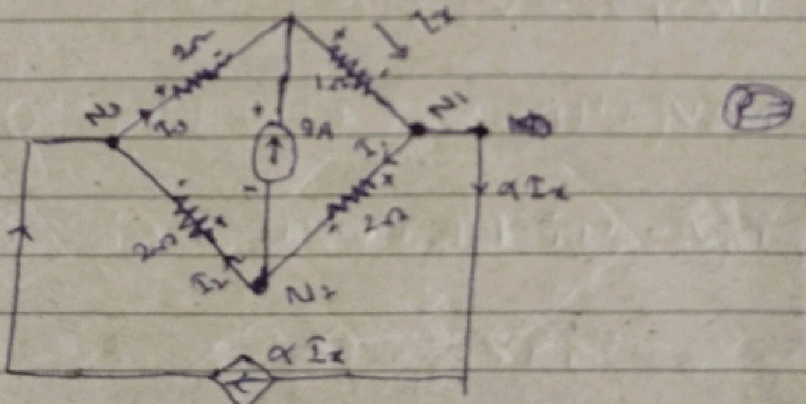
$$0.5 i_o + 3 - i_o = 0$$

$$(i_o = 6A)$$

$$v_o = +i_o 4$$

$$v_o = +24V$$

Q:- Find α for which 9A current source supplies 180 W power



Sol. $P = VI$

$$180 = V(9)$$

$$(V = 20V)$$

Delta

at N_1, (Apply KCL)

$$I_x = I_1 + \alpha I_x$$

$$\left(I_1 = (1-\alpha) I_x \right)$$

at N_2

$$I_1 = 9 + I_2$$

$$I_2 = I_1 - 9$$

$$\left(I_2 = (1-\alpha) I_x - 9 \right)$$

at N_3:

$$I_3 = I_2 + \alpha I_x$$

$$= (I_x - \alpha I_x - 9 + \alpha I_x)$$

$$\left(I_3 = I_x - 9 \right)$$

→ Apply KVL in left loop

$$-2(I_x - 9) - 20 - 2[(1-\alpha) I_x - 9] = 0$$

$$-2I_x + 18 - 20 - 2[I_x - \alpha I_x - 9] = 0$$

$$-2I_x - 2 - 2I_x + 2\alpha I_x + 18 = 0$$

$$-4I_x + 2\alpha I_x + 16 = 0 \quad \text{— } (1)$$

Date
Page No.

In right loop

$$-I_x - 2I_1 + 20 = 0$$

$$-I_x - 2I_x + 2\alpha I_x + 20 = 0$$

$$-3I_x + 2\alpha I_x + 20 = 0 \quad \text{(ii)}$$

from eq (i) & (2) subtract both

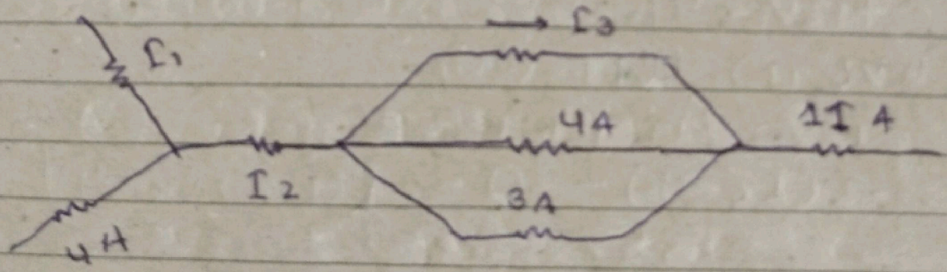
$$-I_x + 4 = 0$$

$$(I_x = -4)$$

Now,

$$(\alpha = 4)$$

Q:- Find $(I_1, I_2 \; \& \; I_3)$



$$4A + 3A + I_3 = 11A$$

$$11 - 7A = I_3$$

$$(I_3 = 4A)$$

$I_2 = 4+4+3$

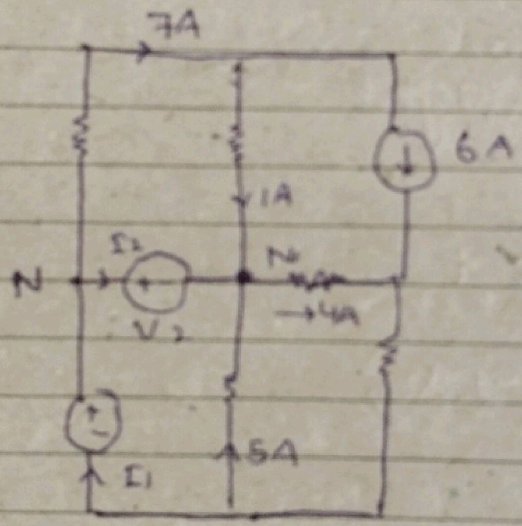
$I_2 = 11\,A$

~~$I_1 = $~~ $I_1 + 4 = 11$

$(I_1 = 7\,A)$

→ (7A, 11A, 4A)

Q:- Find I_1 & I_2



at N_1

$I_2 + 5 + 1 = 4$

$(I_2 = -2\,A)$

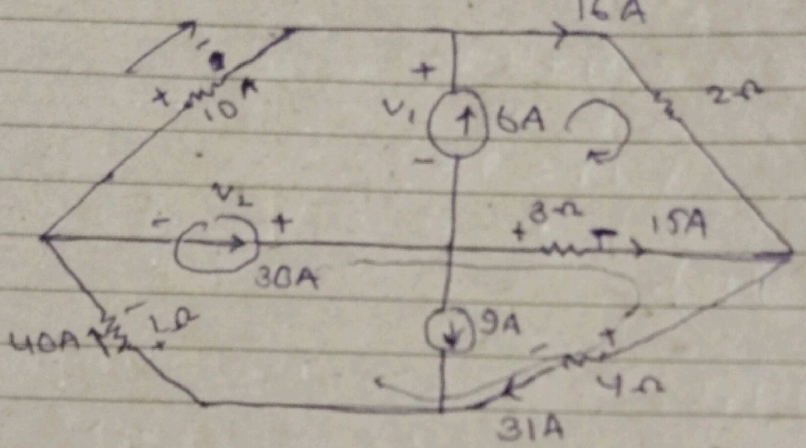
at N

$I_1 = I_2 + 7$

$\boxed{I_1 = 5\,A}$

Date
Page No.

Q:- find V_1 & V_2?



In loop top right

$+V_1 - 32V + 45$

$(V_1 = -13V)$

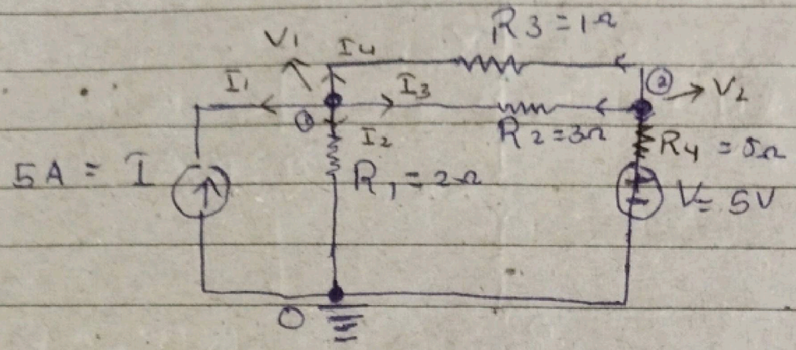
In bottom loop

$V_2 - 45 - 124 - 40$

$(V_2 = 209V)$.

Delta

Nodal Analysis :-



at Node ①

$$I_1 + I_2 + I_3 + I_4 = 0$$

$$-5 + \frac{V_1 - 0}{2} + \frac{V_1 - V_2}{3} + \frac{V_1 - V_2}{1} = 0$$

$$-30 + 3V_1 + 2V_1 - 2V_2 + 6V_1 - 6V_2 = 0$$

$$11V_1 - 8V_2 - 30 = 0 \quad \text{—— (1)}$$

at Node ②

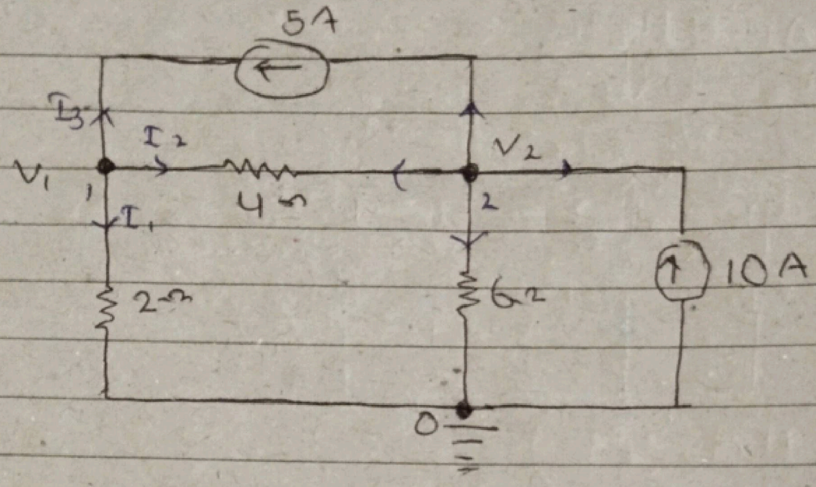
$$\frac{V_2 - V_1}{1} + \frac{V_2 - V_1}{3} + \frac{V_2 - 5 - 0}{5} = 0$$

$$15V_2 - 15V_1 + 5V_2 - 5V_1 + 3V_2 - 15 = 0$$

$$-20V_1 + 23V_2 - 15 = 0 \quad \text{—— (11)}$$

Date________
Page No.______

(11)



at Node - (1)

$$I_1 + I_2 + I_3 = 0$$

$$\frac{V_1 - 0}{2} + \frac{V_1 - V_2}{4} - 5 = 0$$

$$V_1 + 2V_1 - 2V_2 - 20 = 0$$

$$3V_1 - V_2 = 20 \quad \text{—— (1)};$$

at Node (2)

$$I_1 + I_2 + I_3 + I_4 = 0$$

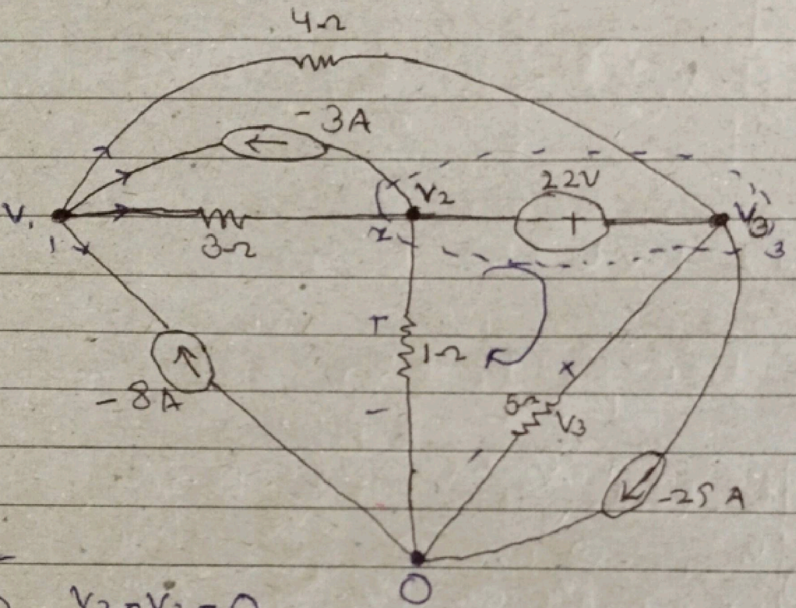
$$= \frac{V_2 - 0}{6} + \frac{V_2 - V_1}{4} + 5 - 10 = 0$$

$$2V_2 + 3V_2 - 3V_1 - 60 = 0$$

$$-3V_1 + 5V_2 - 60 = 0 \quad \text{—— (11)}$$

Note :-
Har node se outgoing current as a convention le lenge if convention current is in ← and given current direction → 10A current will become -ve

(11)



Apply KVL
in loop $V_2, V_3, 0$

$$22V - V_3 + V_2 = 0$$

$$V_2 - V_3 = 22 \quad \text{—— (1)}$$

apply KCL at (1)

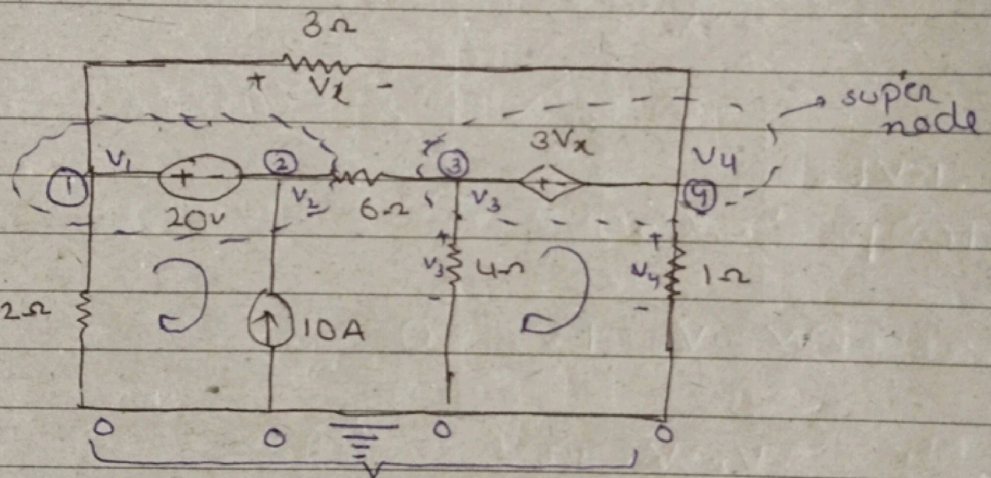
$$I_1 + I_2 + I_3 + I_4 = 0$$

$$+8 + \frac{V_1 - V_2}{3} + \cancel{\ldots} + 3 + \frac{V_1 - V_3}{4} = 0$$

$$132 + 7V_1 - 4V_2 - 3V_3 = 0 \quad \text{—— (11)}$$

We assume node ② & ③ as super node, when ever there is a voltage source in btw 2 nodes it will become super node (we let it 1 node).

(IV)



⑤ grounded

Apply KVL

$-3V_x - V_4 + V_3 = 0$

$V_3 - V_4 = 3V_x$ — (i)

at node ① & ② as super node means of node ① & ② eqn be written in single eqn

$$\frac{V_1 - 0}{2} + \frac{V_1 - V_4}{3} + \frac{V_2 - V_3}{6} - 10 = 0 \quad \text{— (ii)}$$

$$\Rightarrow \quad 5V_1 + V_2 - V_3 - 2V_4 = 60 \quad \text{—— (ii)}$$

similarly

Node (3) & (4) are super node

$$\frac{V_3}{4} + \frac{V_3 - V_2}{6} + \frac{V_4 - V_1}{3} + \frac{V_4}{1} = 0$$

$$-4V_1 - 2V_2 + 5V_3 + 16V_4 = 0 \quad \text{—— (iii)}$$

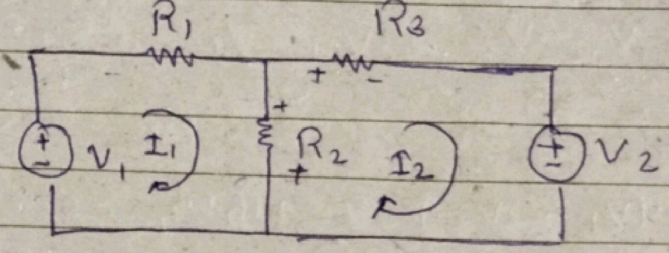
Apply KVL in loop $V_1, V_2, 0$

$$-20 - V_2 + V_1$$

$$V_1 - V_2 = 20 \quad \text{—— (iv)}$$

Mesh Analysis :-

→ A mesh is a closed path during a circuit with no other path inside it



⇒ Apply KVL in both loop

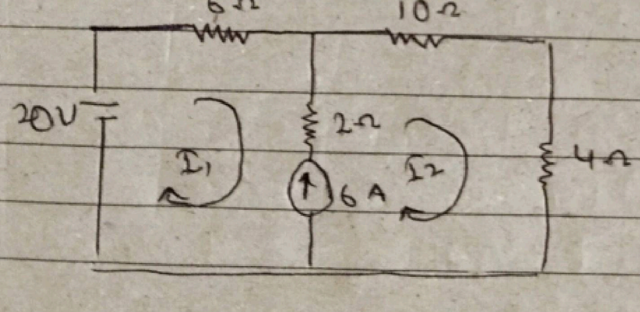
$$(R_1 + R_2) I_1 + V_1 - I_2 R_2 = 0 \quad \text{— (I)}$$

Apply KVL in other loop

$$-I_2 (R_2 + R_3) - V_2 - I_1 R_2 = 0 \quad \text{— (II)}$$

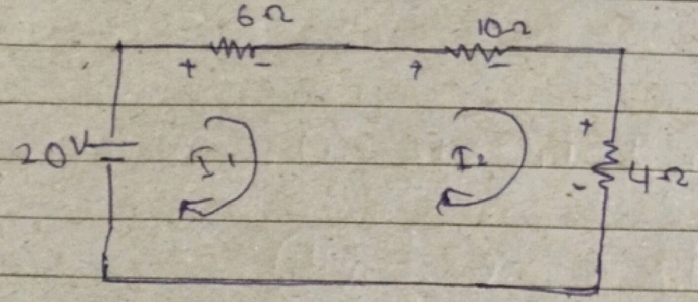
solve both eqn to get answer

(ii)



→ It is a super mesh type question mean whenever there is a current source in btw 2 mesh it will become super mesh & to solve the question we remove that branch.

Apply super mesh concept



Apply KVL in that loop

$$-6I_1 - 10I_2 - 14I_2 + 20 = 0 \quad \text{--- (1)}$$

for eqn (2) we know in super mesh branch

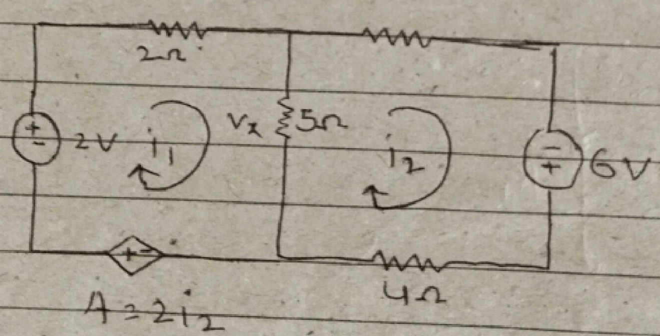
$$I_1 - I_2 = -6 \quad \text{--- (11)}$$

After solving both eqn

$I_1 = -3.2$

$I_2 = 2.8$

(111)



if $A = 2i_2$

$i_1 = ?$

Apply KVL in loop (1)

$$+2 - 2i_1 - 5i_1 + 5i_2 + 2i_2 = 0$$

$$-7i_1 + 7i_2 = -2 \quad \text{--- (1)}$$

Date
Page No.

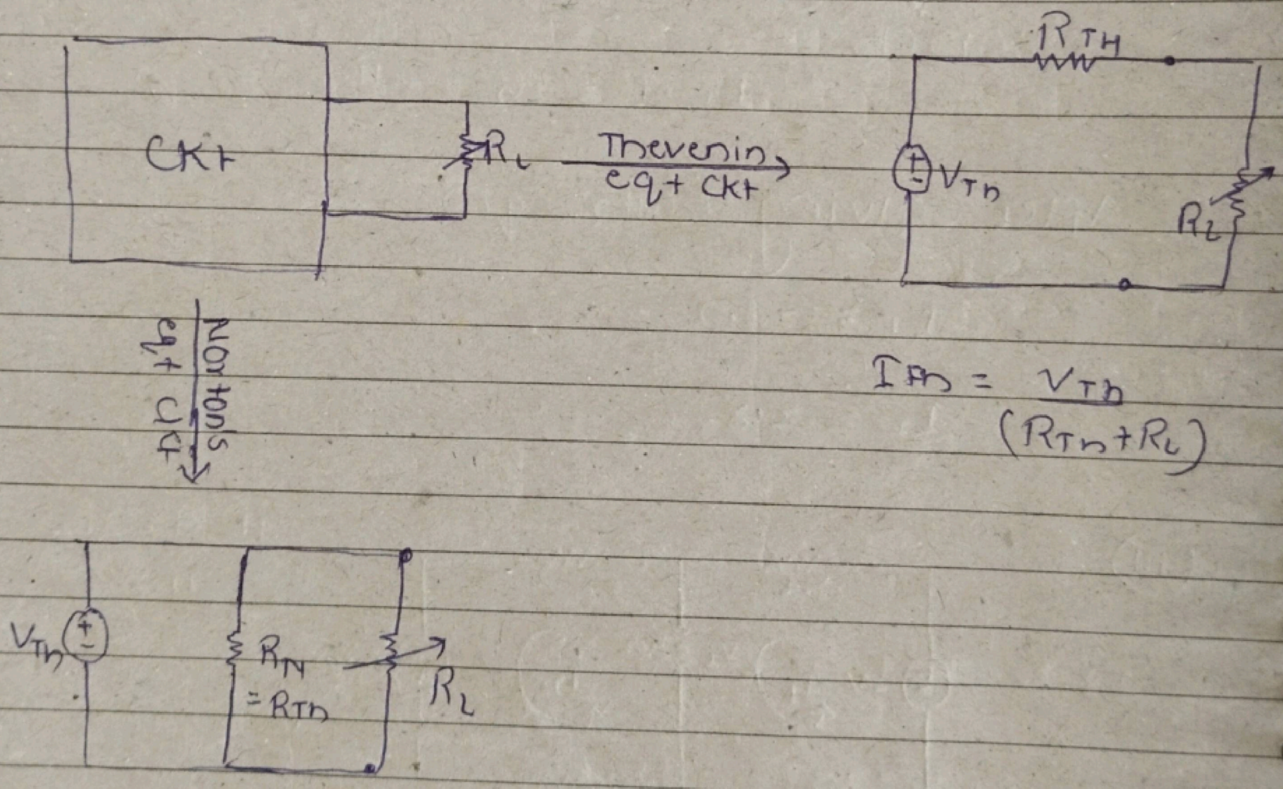
In loop ②

$$-3i_2+6-4i_2+5i_1-5i_2=0$$

$$5i_1-12i_2=-6 \quad \text{—(ii)}$$

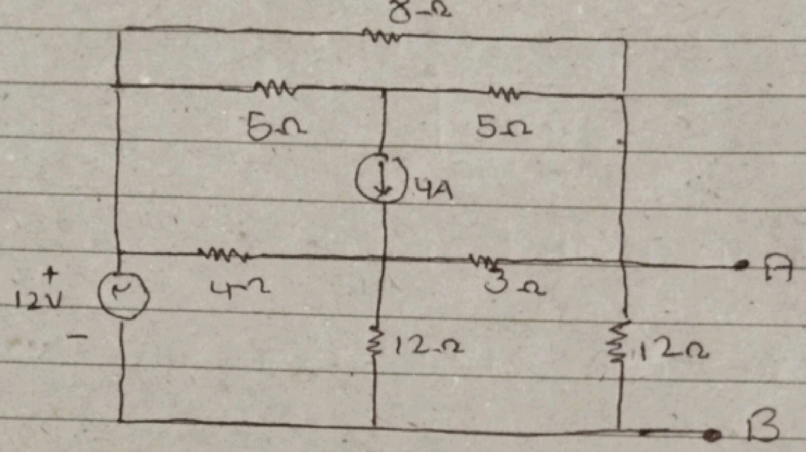
solve both eqn to get i_1 & i_2 & then find A

• Thevenin's & Norton's Theorem :-

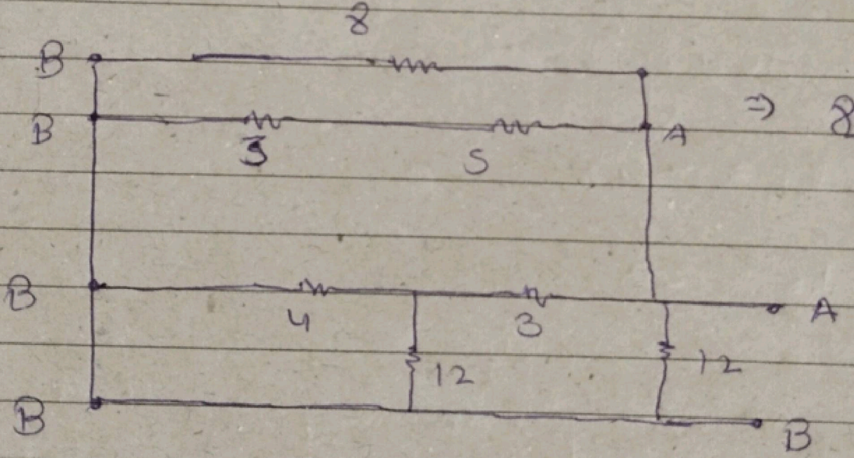


$$I_{Th} = \frac{V_{Th}}{(R_{Th}+R_L)}$$

Q:- Find Thevini's Eqn across A-B ?

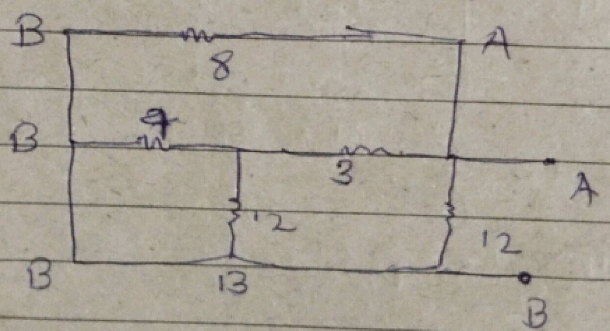


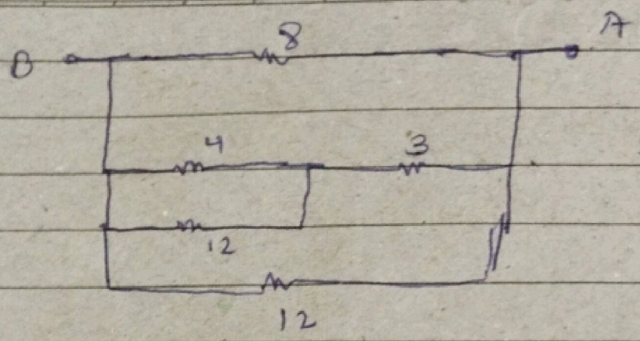
Sol To Find R_{Th} short voltage source & open current source (independent)



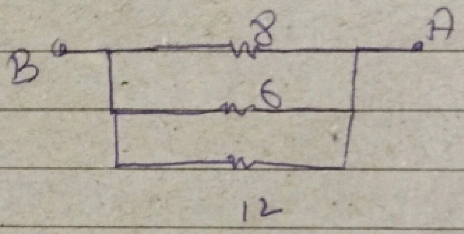
$\Rightarrow \frac{8 \times 8}{8+8} = \frac{64}{16}$

$= 4$



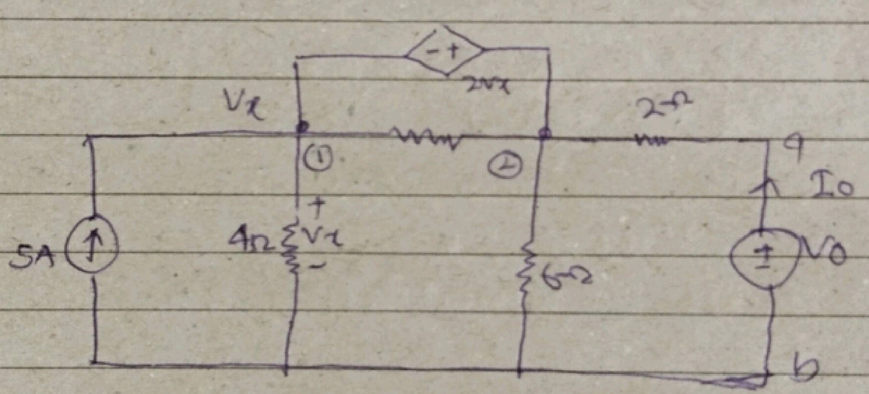


⇒



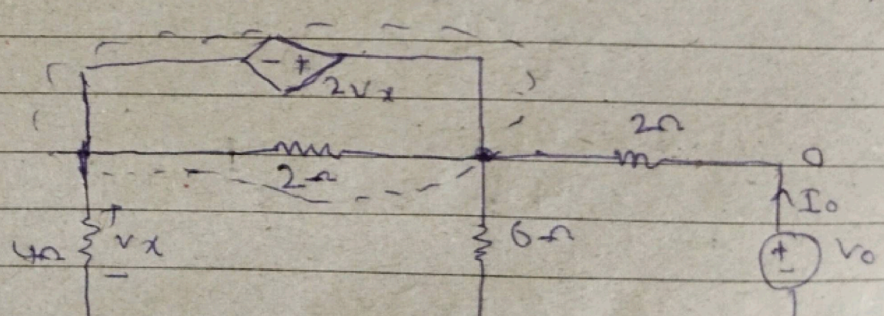
⇒ 2Ω $= R_{th}$

Q:- Find Thevenin's Eqt across A-B



⇒ let V_o across a-b & open independent current source & node ① & ② are supernode

⇒

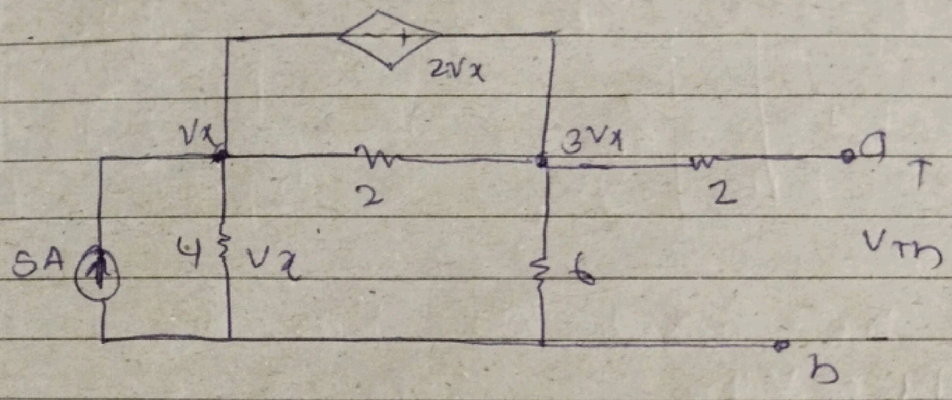


To find $R_{th} = \frac{V_o}{I_o}$ ratio of V_o/I_o

Let V_y across node (2)

at node (1)

To find V_{Th}



$V_{Th} = 3V_x$ from previous finding

as node ① & ② are supernode

$$\frac{V_x}{4} + \frac{V_x - V_{Th}}{2} - 5 + \frac{V_{Th} - V_x}{2} + \frac{V_{Th}}{6} = 0$$

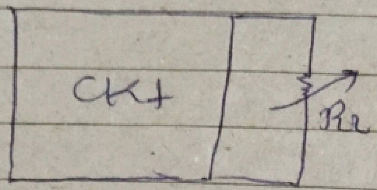
$$\frac{V_{th}}{3\times4} + \frac{V_{th}}{3\times2} - \frac{V_{th}}{2} + \frac{V_{th}}{2} - \frac{V_{th}}{3\times2} + \frac{V_{th}}{6} = 5$$

$$\Rightarrow \frac{V_{th}}{12} + \frac{2V_{th}}{6\times2} = 5$$

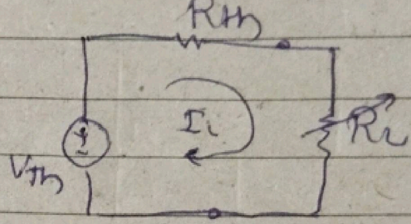
$$3V_{th} = 12\times5$$

$$(V_{th} = 20V)$$

• Maximum power transfer theorem :-



$\Rightarrow$



$$I_L = \frac{V_{th}}{R_{th} + R_L}$$

$$P_L = I_L^2 \cdot R_L$$

$$P_L = \left[\frac{V_{th}}{R_{th} + R_L}\right]^2 \cdot R_L$$

$$= V_{th}^2 \cdot \frac{R_L}{(R_{th} + R_L)^2}$$

For maximum power $(R_L = R_{th})$

$$P_L = V_{th}^2 \cdot \frac{R_L}{(2R_L)^2} = \frac{V_{th}^2}{4} \frac{\cancel{R_L}}{R_L^{\cancel{2}}}$$

$$P_L = \frac{V_{th}^2}{4R_L^2}$$

$$P_L = V_{th}^2 / 4R_L^2$$

To max. power $dP_L/dR_L = 0$

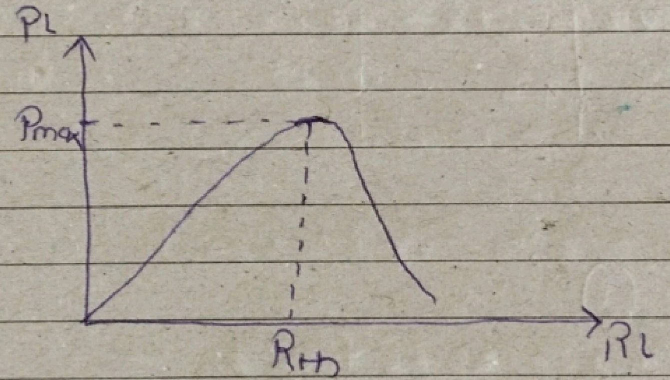
$$\frac{d}{dR_L}\left[V_{th}^2 \cdot \frac{R_L}{(R_{th}+R_L)^2}\right]$$

$$= V_{th}^2\left[(R_L+R_{th})^2 \cdot \frac{dR_L}{dR_L} - R_L \cdot \frac{d}{dR_L}(R_{th}+R_L)\right]$$

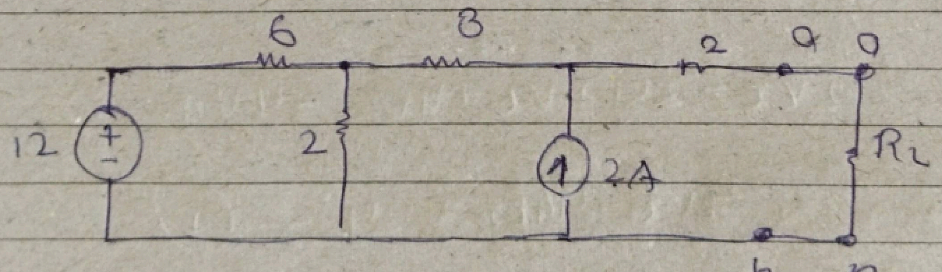
on solving it we get

$(R_{th} = R_L)$ hence proved

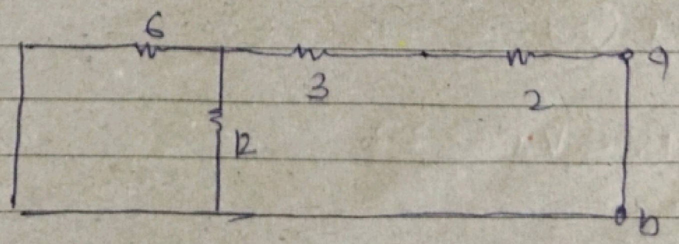
P_L vs R_L



Q :-

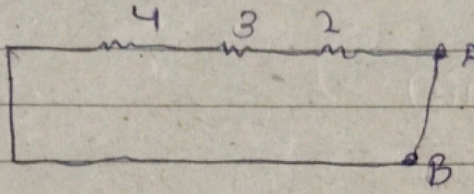


Sol :-



$\Rightarrow \frac{6 \times 12}{6+2} = \frac{72}{8}$

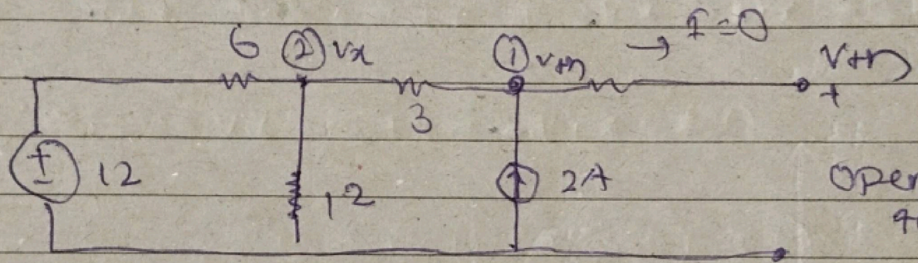
$\Rightarrow 4$



$(R_{th} = 9\Omega)$

(thus $R_L = 9\Omega$) as $R_{th} = R_L$

To find P_{max} we have to caculate V_{th}



Open karenge to find Vth

at node ①

$$\frac{Vx - 12}{6} + \frac{Vx}{12} + \frac{Vx - V_{th}}{3} = 0$$

$$2Vx - 24 + Vx + 4Vx - 4V_{th} = 0$$

$$7Vx - 4V_{th} = 24 \quad \text{①}$$

at node ②

$$\frac{V_{th} - Vx}{3} = 2$$

Date
Page No.

$V_{th} - V_x = 6$

$V_{th} - 6 = V_x$ —— (11)

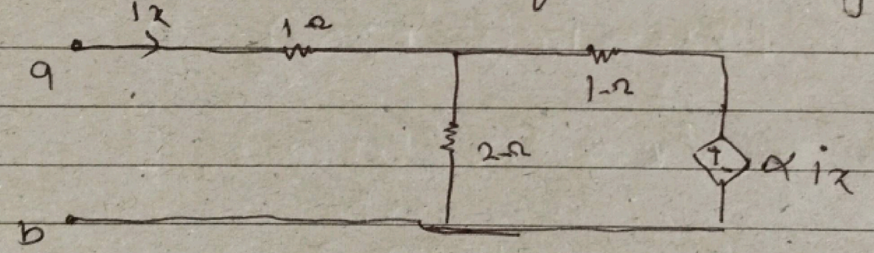
from eq (1) & (2)

$(V_{th} = 22V)$.

to find $P_{max} = \frac{V_{th}^2}{4R_L}$

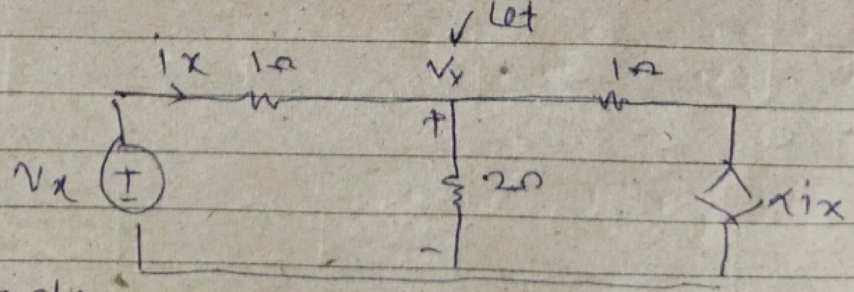
$\Rightarrow \frac{22 \times 22}{4 \times 9} = \underline{\underline{13.44\ W}}$

Q:- find thevenin's eqt ckt as a func of α?



<u>Note</u>
If there is no independent source in the ckt thus V_{th} is alway is zero

<u>sol</u> to find R_{th} let V_x across a-b



at node

$\frac{V_y - V_x}{1} + \frac{V_y}{2} + \frac{V_y - \alpha i_x}{1} = 0$ —— (1)

Page No.
Date

also Apply KVL in the loop

$$+V_x - 1 \times i_x - V_y = 0$$

$$V_y = V_x - i_x \quad \text{— (ii)}$$

put (ii) in (i)

$$\cancel{V_x} - i_x - \cancel{V_x} + \frac{V_x - i_x}{2} + \frac{V_x - i_x - \alpha i_x}{1} = 0$$

$$-2i_x + V_x - i_x + 2V_x - 2i_x - 2\alpha i_x = 0$$

$$3V_x - 5i_x - 2\alpha i_x = 0$$

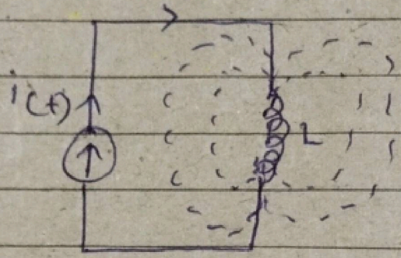
$$3V_x - i_x(5 + 2\alpha) = 0$$

$$3V_x = i_x(5 + 2\alpha)$$

$$R_{th} = \frac{V_x}{i_x} = \frac{(5 + 2\alpha)}{3}$$

Date ______
Page No. ______

• Magnetically coupled ckt :-



$$V \propto \frac{d\phi}{dt}$$

$$V \propto N$$

$$V = N\frac{d\phi}{dt}$$

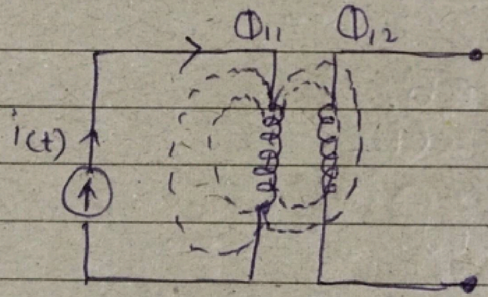
$$V = N \cdot \frac{d\phi}{di} \cdot \frac{di}{dt}$$

Self inductance

$$\boxed{V = L\frac{di}{dt}}$$

L; inductance

Mutual inductance

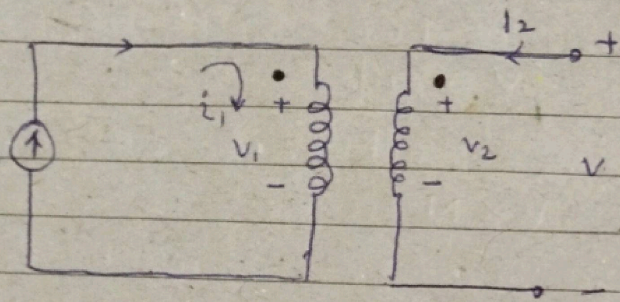


$$\left(V_1 = N_1 \frac{d\phi_{11}}{di} \cdot \frac{di}{dt} \right) \qquad \left(V_2 = N_2 \frac{d\phi_{12}}{di} \cdot \frac{di}{dt} \right)$$

$$\left(V_2 = M_{12}\frac{di}{dt} \right)$$

M_{12} : mutual inductance

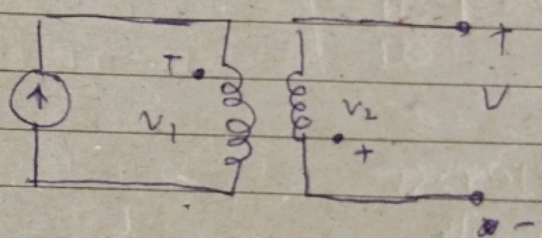
Dot convention for coil :-



i_1 & i_2 are time varying

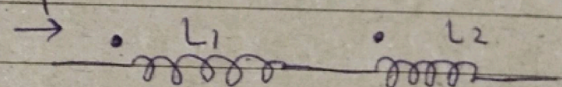
$(M = M_{12} = M_{21})$

$$v_1 = L\frac{di_1}{dt} + M\cdot\frac{di_2}{dt}$$

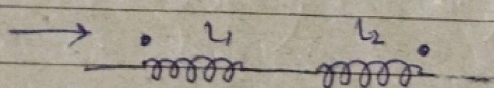


$$v_1 = L\frac{di_1}{dt} - M\frac{di_2}{dt}$$

<u>concept</u> :



$(L_{eq} = L_1 + L_2 + 2M)$



$(L_{eq} = L_1 + L_2 - 2M)$

- concept

$$L = sL$$

$$C = 1/sC$$

$$s = \sigma + j\omega$$

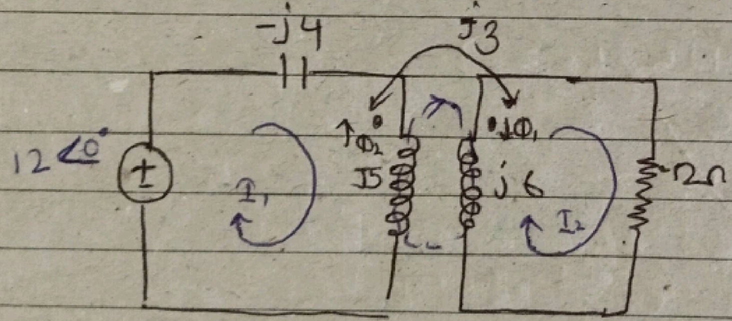
$$\sigma = 0$$

$$s = j\omega$$

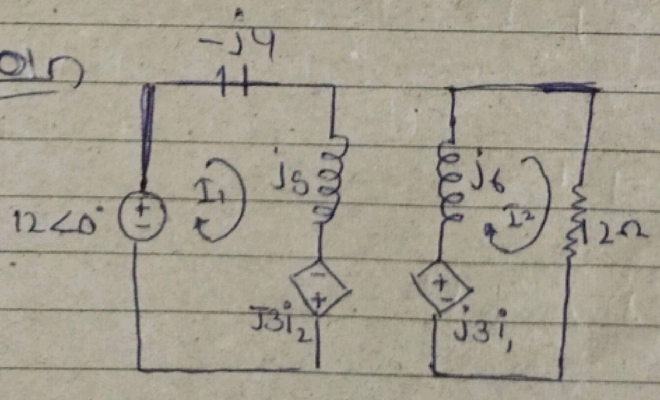
$$X_L = j\omega L \qquad X_M = j\omega M$$

$$X_C = 1/j\omega C = \frac{-j}{\omega C}$$

Q:- solve the ckt to find I_1 & I_2



soln



⇒ simplified ckt

Apply KVL in loop 1

$$12 - (-j4i_1) - j5I_1 + j3I_2 = 0$$

$$12 - jI_1 + j3I_2 = 0 \quad \text{— (1)}$$

in loop 2

$$-j6I_2 - 12I_2 + j3I_1 = 0 \quad \text{— (II)}$$

On solving eq (1)

$$jI_1 - j3i_2 = 12$$

$$I_1 = \frac{(12+j6)I_2}{j3} \quad \text{— (A)}$$

put (A) in (IIa)

$$\cancel{+j3I_1 - (12+j6)I_2 = 0}$$

$$\cancel{j3\left[\frac{12+j6}{j3}\right]I_2 -}$$

$$j\left(\frac{12+j6}{j3}\right)I_2 - j3I_2 = 12$$

$$(4+j2)I_2 - j3I_2 = 12$$

$$I_2(4-j) = 12$$

$$|x - iy| = \left(\sqrt{x^2+y^2}\right)$$

$$\theta = \tan^{-1}(y/x)$$

Date

Page No.

$$I_2 = \frac{12 + j \cdot 0}{4 - j} = \frac{\sqrt{12^2+0^2}}{\sqrt{4^2+(-1)^2}}$$

$$I_2 = \frac{12}{\sqrt{17}}$$

For angle,

$$\frac{\tan^{-1}(0)}{\tan^{-1}\left(\frac{1}{4}\right)} = \angle 14.04$$

$$I_2 \Rightarrow \frac{12}{\sqrt{17}} \angle 14.04$$

put I_2 in (A)

$$I_1 = \left(\frac{12 + j6}{j3}\right)\left(\frac{12}{\sqrt{17}}\right)$$

Page No.
Date

Laplace Transformation

$$F(t) \underset{\text{inverse laplace}}{\overset{\text{laplace}}{\longleftrightarrow}} F(\omega) \text{ (frequency)}$$

$s = \cancel{\sigma}^{\to 0} + j\omega$

$s = j\omega$

$X_C = \frac{1}{sC}$ $\quad X_L = sL$

$X_C = \frac{1}{j\omega C}$ $\quad X_L = j\omega L$

$$F(s) = \int_{-\infty}^{\infty} F(t) e^{-st} \cdot dt$$

$$\left(F(s) = \int_{0^-}^{\infty} f(t) e^{-st} dt \right) \rightarrow \text{for unilateral ckt}$$

$$F(s) = \int_{0^-}^{\infty} F(t) e^{-(\sigma + j\omega)t} \cdot dt)$$

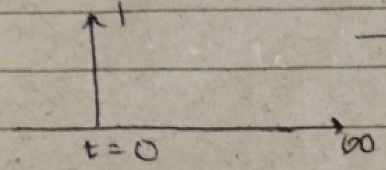
$$f(s) = \int_{0^-} f(t) e^{-\sigma t} e^{-j\omega t} dt \quad \rightarrow |e^{-j\omega t}| = 1$$

$$\boxed{F(s) = \left| \int f(t) e^{-\sigma t} dt \right|} < \infty$$

↳ absolutely integrable

- Impulse signal :-

→ ~~...~~ at an instant eg:- lightening



→ unit impulse

$f(t) = \delta(t)$

$$1 \cdot \delta(t) \xleftrightarrow{L.T} 1$$

$$A\,\delta(t) \xleftrightarrow{LT} A$$

since,

$\therefore \; F(s) = \int_{0^-}^{\infty} f(t) e^{-st} dt$

$$\delta(t) \xleftrightarrow{L.T} \int_{0^-}^{\infty} \delta(t) e^{-st} dt$$

e^{-st} , $t=0$

$e^{-0 \times s} = 1$

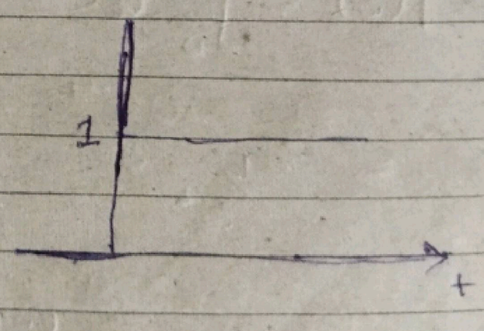
$e^{0} = 1$

$$\left(\delta(t) \longleftrightarrow \int_{0^-}^{\infty} \delta(t) e^{-0 \times s} dt = 1 \right)$$

- unit step signal

$f(t) = u(t)$

$$f(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$



$\because\ F(s) = \int_{0^-}^{\infty} f(t)\, e^{-st}\, dt$

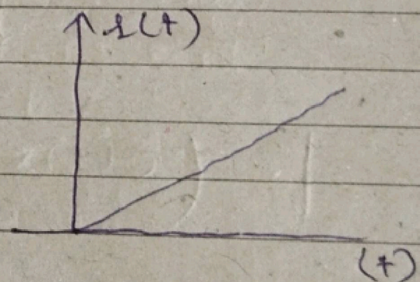
$= \int_{0}^{\infty} 1 \cdot e^{-st}\, dt$

$= \left. \frac{e^{-st}}{s} \right|_{0}^{\infty} \Rightarrow \frac{e^{-s\times\infty}}{-s} + \frac{e^{-s\times 0}}{s}$

$\Rightarrow 1/s$

$$\boxed{u(t) \xleftrightarrow{LT} 1/s}$$

- <u>ramp ~~sing~~ signal :-</u>



$r(t) = t$

$\because\ f(s) = \int_{0}^{\infty} F(t)\, e^{-st}\, dt$

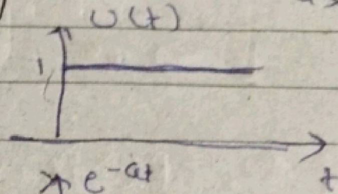
ILATE

$f(s) = \int_{0}^{\infty} t\, e^{-st}\, dt$ $\left[\int uv\, dx = u \int v\, dx - \int \frac{du}{dx} \int v\, dx \right.$

$t = 1/s^2$

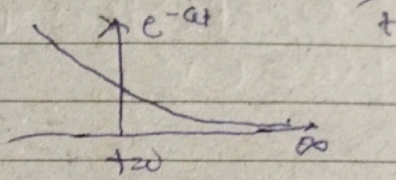
$$\boxed{r(t) \xleftrightarrow{L.T} 1/s^2}$$

Q:- Find laplace transform of $f(t) = e^{-at}u(t)$, $a>0$

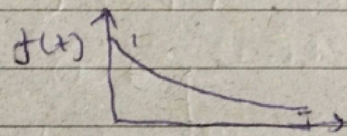


Sol $F(s) = \int_0^\infty f(t) e^{-st} dt$

$\int_0^\infty e^{-at}\, u(t)\, e^{-st} dt$ $\quad (u(t) = 1)$



$= \int_0^\infty e^{-at} e^{-st}\, dt$



$\int_0^\infty e^{-(a+s)t} dt$

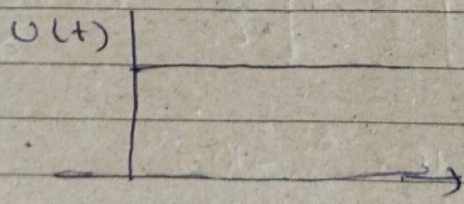
$\left. \frac{e^{-(a+s)t}}{-(s+a)} \right|_0^\infty = \left(\frac{1}{s+a} \right)$

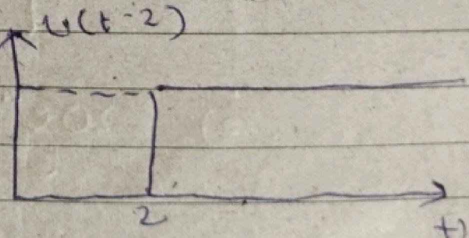
Q:- Find L.T?

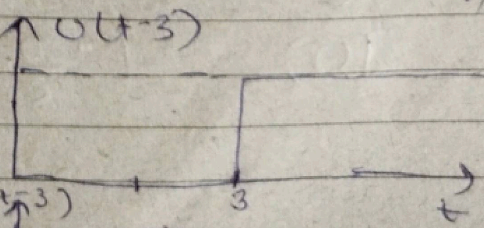
$F(t) = 10\left[u(t-2) - u(t-3)\right]$

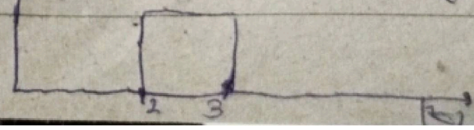
Sol

$u(t-2) \Rightarrow t-2 = 0 \quad t = 2$







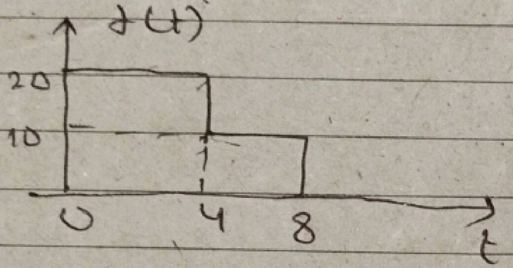


Page No.
Date

$$F(s) = 10\left[\frac{e^{-2s}}{s} - \frac{e^{-3s}}{s}\right]$$

$$F(s) = \frac{10}{s}\left[e^{-2s} - e^{-3s}\right]$$

Q:- Find L.T



Sol)
$$f(s) = \int_0^4 f(t)e^{-st}\,dt + \int_4^8 f(t)e^{-st}\,dt$$

$$20\int_0^4 e^{-st}\,dt + 10\int_4^8 e^{-st}\,dt$$

$$20\left[\frac{e^{-4s}}{s} - \frac{1}{s}\right] + 10\left[\frac{e^{-8s}}{s} - \frac{e^{-4s}}{s}\right]$$

$$\Rightarrow \frac{20e^{-4s}}{s} - \frac{20}{s} + \frac{10e^{-8s}}{s} - \frac{10e^{-4s}}{s}$$

$$f(s) = \left(-\frac{20}{s} + \frac{10e^{-4s}}{s} + \frac{10e^{-8s}}{s}\right)$$

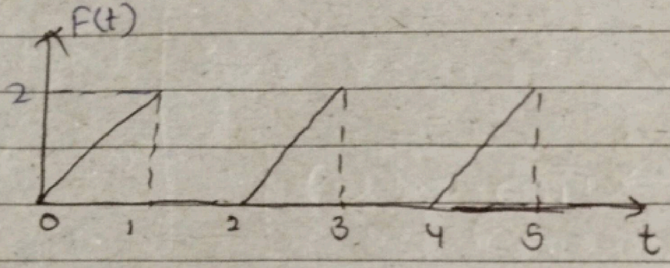
Note

$$\left[\frac{1}{1-e^{-Ts}} = \frac{1}{1-e^{-2s}} \right]$$

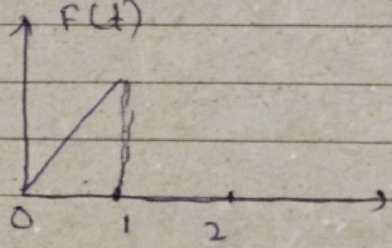
Date ______
Page No. ______

Q:- Find L.T

$(T = 2s)$

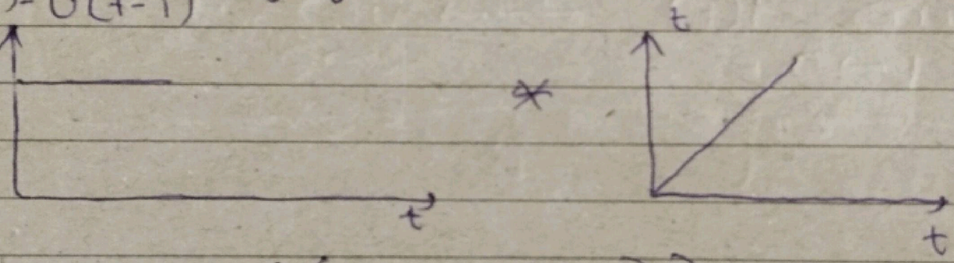


Sol =>



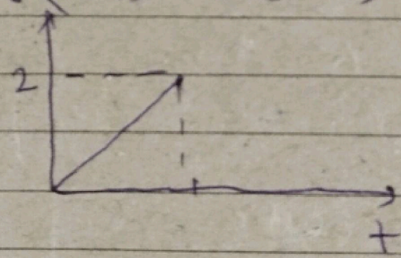
simulation of graph

$\Rightarrow u(t) - u(t-1)$



$t\,(u(t) - u(t-1))$

=



$$F(t) = t\,u(t) - t\,u(t-1)$$

$$2\left[t\,u(t) - (t-1+1)\,u(t-1)\right]$$

$$2\left[t(u(t)) - (t-1)u(t-1) + u(t-1)\right]$$

Page No. ______
Date ______

- Final value theorem :-

$t \to \infty$

$$\boxed{\lim_{t \to \infty} f(t) \xleftrightarrow{L.T} \lim_{s \to 0} s \cdot F(s)}$$

$t \leftrightarrow \frac{1}{s}$

$\infty \leftrightarrow \frac{1}{\infty}$

$s = 0$

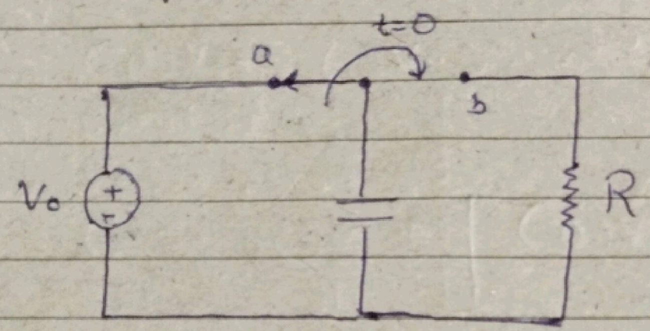
$(s=0)$

- Initial value theorem :-

$t \longrightarrow 0$

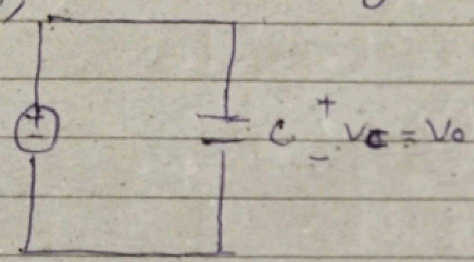
$$\boxed{\lim_{t \to 0} f(t) \xleftrightarrow{L.T} \lim_{s \to 0} s \cdot F(s)}$$

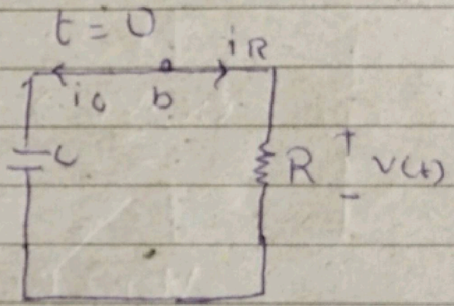
• Source free R.C circuit :-



Capacitor initially charged to V_0

$t<0$,





we know

$$i = dq/dt$$

$$q = CV \Rightarrow \left(i_C = C \cdot \frac{dv}{dt} \right)$$

also

$$i_R = \frac{v(t)}{R}$$

Apply KCL at b

$$i_C + i_R = 0$$

$$C \cdot \frac{dv(t)}{dt} + \frac{v(t)}{R} = 0$$

across the capacitor

$$\frac{d\,V(t)}{dt} \xleftrightarrow{L.T} S \cdot V(s) - V(0^-) \quad \text{initial cond}^n$$

from (1)

$$C\left[S \cdot V(s) - V(0^-)\right] + \frac{V(s)}{R} = 0$$

$$C\left[S \cdot V(s) - V_0\right] + \frac{V(s)}{R} = 0$$

$$CS \cdot V(s) - CV_0 + \frac{V(s)}{R} = 0$$

$$V(s)\left\{C \cdot S + \frac{1}{R}\right) = CV_0$$

$$V(s) = \frac{CV_0}{\left(CS + 1/R\right)}$$

$$\boxed{V(s) = \frac{V_0}{S + 1/RC}}$$

We have to find I.L.T of $V(s)$

we know

$$L\{e^{-at}\} = \frac{1}{s+a}$$

$$L^{-1}\left(\frac{1}{(s+a)}\right) = e^{-at}$$

Date
Page No.

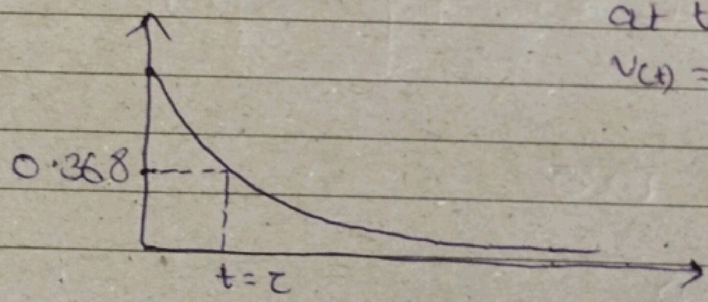
$$L^{-1}\{V(s)\} = L^{-1}\left\{\frac{V_0}{s+1/RC}\right\}$$

$$V(t) = V_0 e^{-\frac{1}{RC} t}$$

$$\tau = 1/RC$$

$$\boxed{V(t) = V_0 e^{-t/\tau}}$$

V(t) vs t



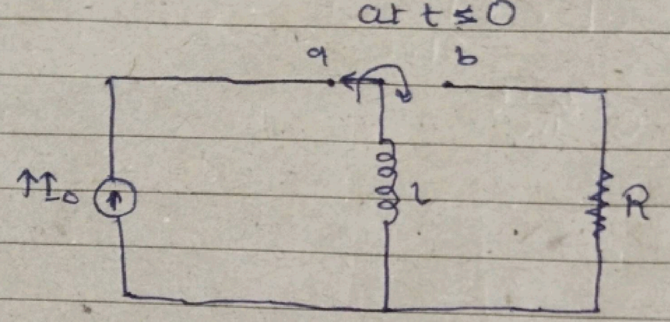
at $t = \tau$

$V(t) = V_0 e^{-1}$

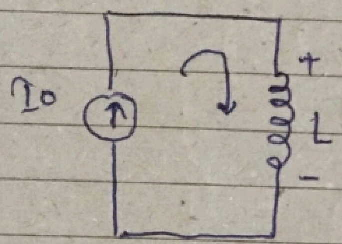
$V(t) = 0.36 V_0$

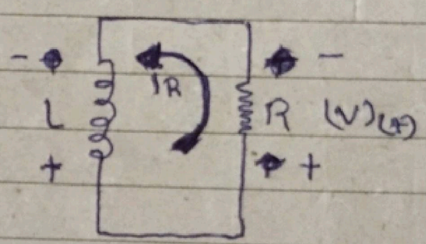
Page No.
Date

Source free R-L ckt



let
Initially inductor has I_0 current





Apply KVL when $t \geq 0$

$$+V_L(t) + v(t) = 0$$

$$\Rightarrow L\frac{di(t)}{dt} + i_{(t)}R = 0 \quad \text{--- (1)}$$

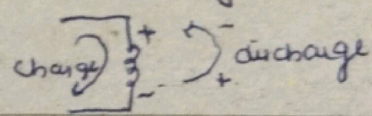
Applying Laplace transformation on (1)

$$\mathcal{L}\left\{L\frac{di(t)}{dt} + i(t)R\right\} = 0$$

$$L\{sI(s) - \overset{\text{initial}}{I(0^-)}\} + I(s)R$$

$$sLI(s) - LI_0 + I(s)\cdot R = 0$$

when inductor discharge its polarity will changed



$$I(s) = \frac{L I_0}{(sL+R)}$$

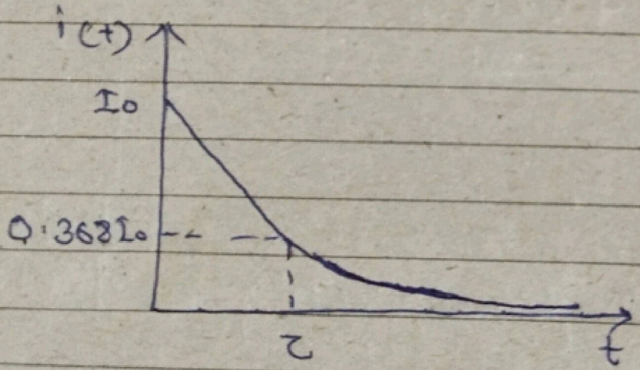
$$I(s) = \frac{I_0}{(s+R/L)} \quad \text{——— (11)}$$

taking inverse laplace transform of (11)

$$i(t) = I_0 e^{-R/L\,t}$$

$$i(t) = I_0 e^{-t/L/R} \qquad (\tau = L/R)$$

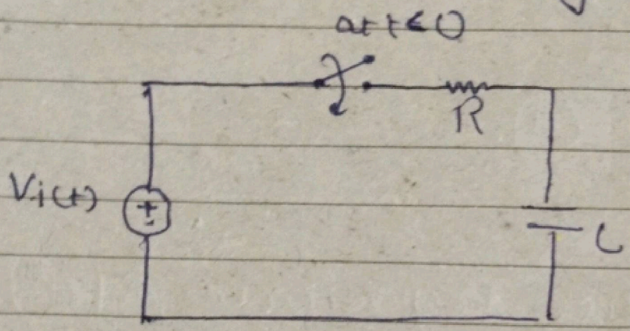
$$\boxed{i(t) = I_0 e^{-t/\tau}}$$

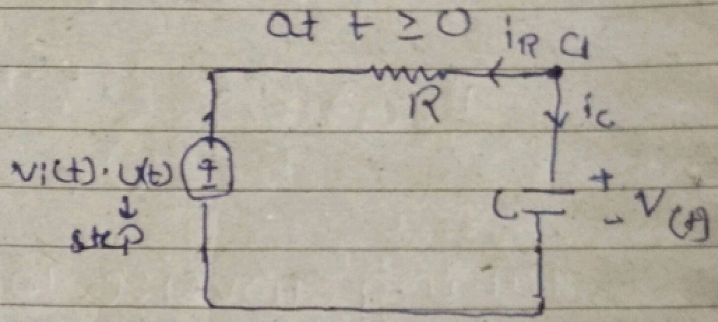


at $t = \tau$

$i(t) = I_0 e^{-1}$

• Step Response of RC ckt :-





at node a apply KCL

$$i_R + i_C = 0$$

$$\frac{V(t) - V_i(t)U(t)}{R} + i_C = 0$$

$$\frac{V(t) - V_i(t)\cdot U(t)}{R} + C\cdot\frac{dV(t)}{dt} = 0$$

$i = \frac{dq}{dt}$

$q = Cv$

$i = \frac{dCv}{dt}$

$i = C\cdot\frac{dv}{dt}$

$i_C = C\frac{dV(t)}{dt}$

$V_i \to s$

$V(t) \to S$

Apply Laplace

$$\frac{V(S) - \frac{V_i(S)}{S}}{R} + C\left\{SV(S) - V(0^-)\right\} = 0$$

$$V(S) - \frac{V_i(S)}{S} + RC\left\{SV(S) - V_0\right\} = 0$$

$$V(S) - V_i(S) + RCSV(S) - RCV_0 = 0$$

$$V(S)\left\{1 + RCS\right\} = \frac{V_i(S)}{S} + RCV_0$$

Date

Page No.

$$V(S) = \frac{Vi(s)}{S(1+RCS)} + \frac{RCV_0}{(1+RCS)}$$

$\longrightarrow$ apply partial fraction as S(is in multiply)

$$V(S) = \frac{RCV_0}{(1+RCS)} + \frac{Vi(S)}{S(1+RCS)}$$

Partial fraction for,

$$\frac{Vi(s)}{S(1+RCS)} = \frac{A}{S} + \frac{B}{(1+RCS)}$$

$$A\Big|_{S=0} = \frac{Vi(s)}{1} = Vi(s)$$

$$B\Big|_{S=-1/RC} = \frac{Vi}{-1/RC} = -RCVi(s)$$

$$\cancel{V(S) = \frac{RCV_0}{(1+RCS)}}$$

$$V(S) = \frac{V_0}{(S+1/RC)} + \frac{Vi(s)}{S} - \frac{RCVi(s)}{(1+RCS)}$$

$$V(S) = \frac{V_0}{(S+1/RC)} + \frac{Vi(S)}{S} - \frac{Vi(S)}{(S+1/RC)} \quad \text{—— } (11)$$

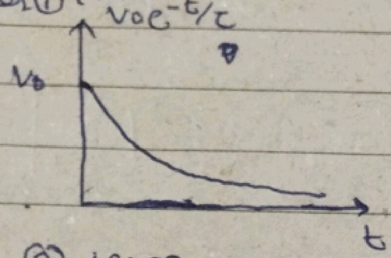
apply Laplace inverse on (11)

$$V(t) = V_0e^{-t/\tau} + Vi(t)U(t) - Vi(t)e^{-t/\tau}\cdot U(t)$$

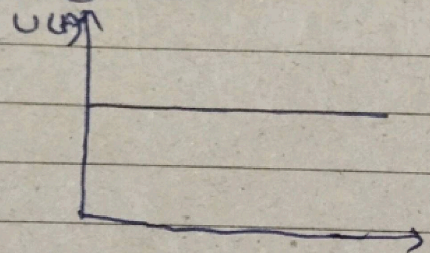
$$\boxed{V(t) = V_0e^{-t/\tau} + Vi(t)\left[1-e^{-t/\tau}\right]\cdot U(t)} \quad \tau = RC$$

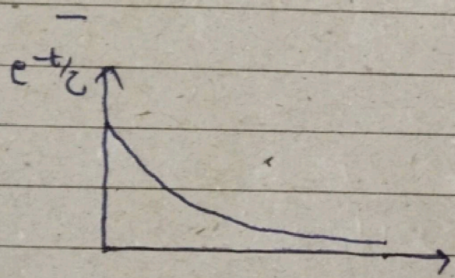
Graph for final expression

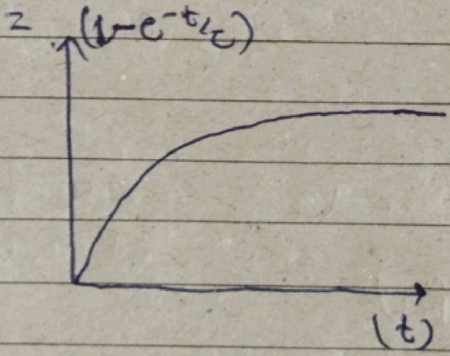
For ① term



For ② term

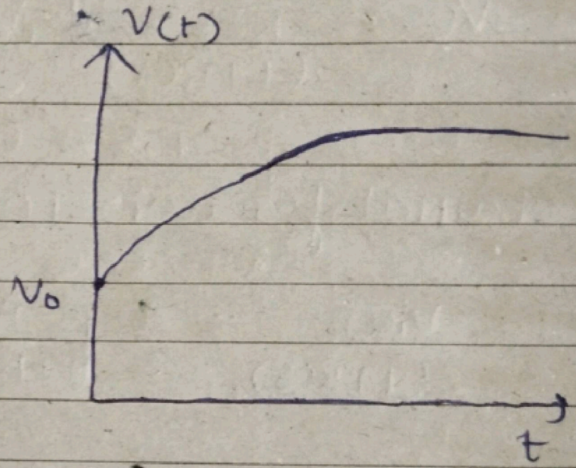






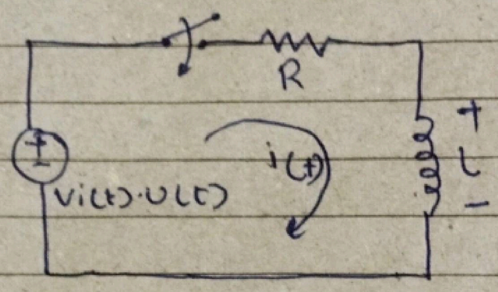
$\Rightarrow$

Final

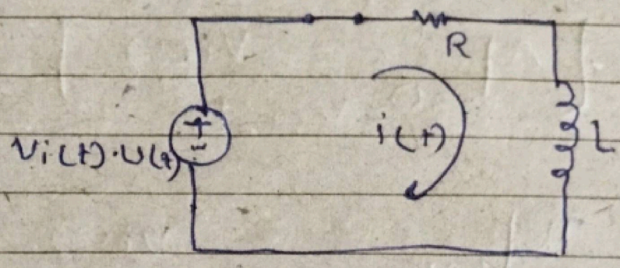


- Step Responce of RL ckt :-

at t=0



at t>0,



Apply KVL,

$$V_i(t)\cdot U(t) - R\,i(t) - V_L(t) = 0$$

$$V_i(t)\cdot U(t) - R\,i(t) - L\frac{di}{dt} = 0$$

<u>L.T</u>

$$\frac{V_i(s)}{S} - R\,I(S) - L\left\{ S\,I(S) - I(0^-)\right\} = 0$$

$$\frac{V_i(s)}{S} - R\,I(S) - SL\,I(s) + L I_0 = 0$$

$$\frac{V_i(s)}{S} + L I_0 = R\,I(S) + SL\,I(S)$$

$$\frac{V_i(s)}{s} + L I_0 = I(S)\,(R+SL)$$

$$I(S) = \frac{V_i(s)}{S(R+SL)} + \frac{L I_0}{R+SL}$$

↳ Apply partial fraction

P.F

$$\frac{V_i(s)}{s(R+sL)} = \frac{A}{s} + \frac{B}{R+sL}$$

$$\left[A\Big|_{s=0} = \frac{V_i(s)}{R} \right]$$

$$B\Big|_{s=R/L} = \frac{V_i(s)}{-R/L}$$

$$\left[B = -\frac{L}{R} V_i(s) \right]$$

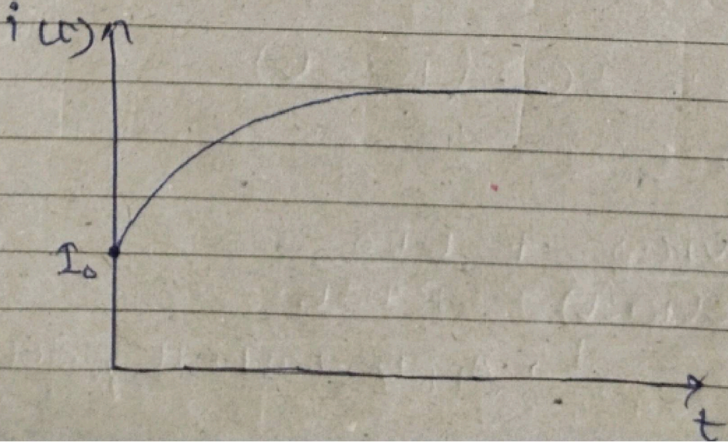
$$I(s) = \frac{I_0}{(s+R/L)} + \frac{V_i(s)}{R}\frac{1}{s} - \frac{L}{R} V_i(s) \frac{1}{(R/L+s)}$$

Apply ILT

$$i(t) = I_0 e^{-R/L\,t} + \frac{V_i(t)}{R} \cdot u(t) - \frac{V_i(t)}{R} e^{-R/L\,t} \cdot u(t)$$

$$i(t) = I_0 e^{-R/L\,t} + \frac{V_i(t)}{R} \left[1 - e^{-R/L\,t} \right] u(t)$$

i(t) vs t



- **Note**

$t = \infty \quad , \quad t = 0$

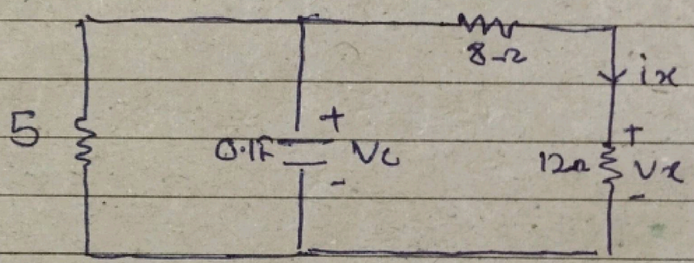
RC

$$v(t) = v(\infty) - \left[v(\infty) - v_0\right] e^{-t/\tau}$$

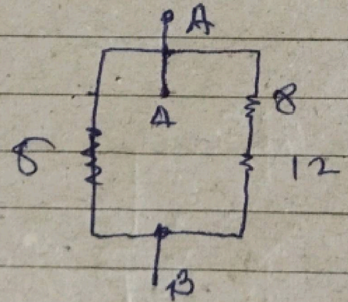
RL

$$i(t) = i(\infty) - \left[i(\infty) - i(0)\right] e^{-t/\tau}$$

Q :- Given $V_c(0) = 15V$, Find V_c, V_x, i_x for $t > 0$



Sol: To find τ we have to find R_{net}, to find R_{net} we open capacitor & find ~~Rnet~~ R_{net} across it.



$\Rightarrow 8 + 12 = 20$

$$R_{net} = \frac{20 \times 5}{20 + 5} = 4\,\Omega$$

$\tau = RC$

$\tau = 4 \times 0.1$

$\tau = 0.4\,s$

$\because$ $V(t) = V_0 e^{-t/\tau}$ as $\tau = 0.4$

$V(t) = 15 e^{-t/0.4}$

$V_c(t) = V(t) = 15 e^{-2.5t}$

For,

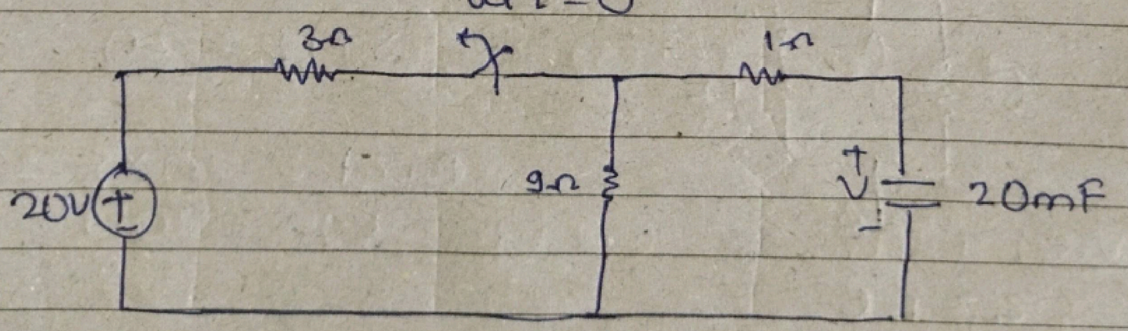
$V_x = V_c(t) \cdot \frac{12}{8+12} = \frac{12}{20} V_c$

$V_x = 9 e^{-2.5t}$

$i_x = \frac{V_c(t) - V(x)t}{8} = \frac{15 e^{-2.5t} - 9 e^{-2.5t}}{8}$

$i_x = \frac{3}{4} e^{-2.5t}$

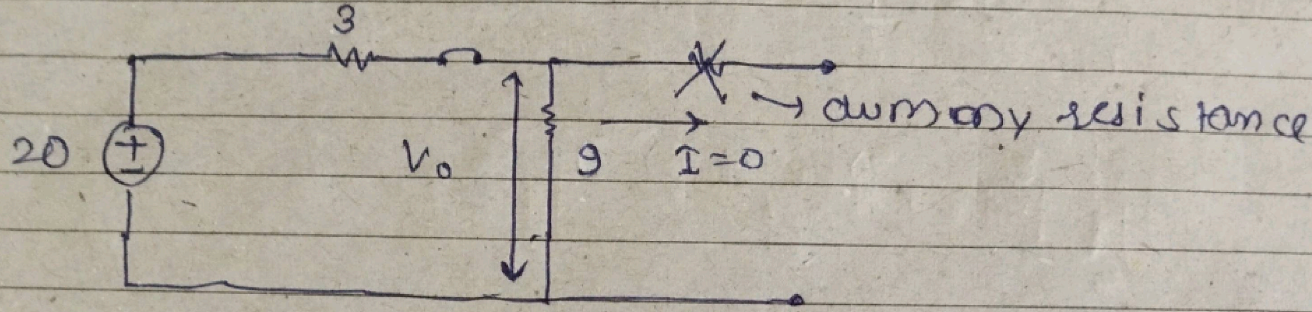
Q:- $V(t)$, initial energy stored in capacitor at $t>0$



Date ____
Page No. ____

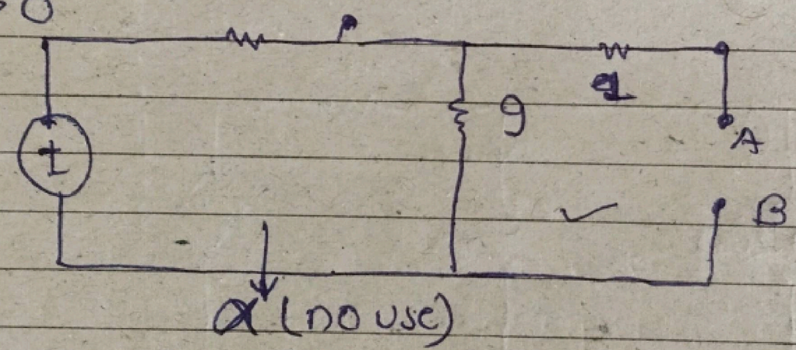
Sol at $t<0$

capacitor connected with DC thus it behave as open ckt as $X_C = \infty$



$$V_0 = 20 \times \frac{9}{(9+3)} \Rightarrow 15\,V$$

at $t>0$



$R_{th} = 9+1$

$R_{th} = 10\,\Omega$

$\tau = R_{th} C = 10 \times 20 \times 10^{-3}$

$\tau = 0.2\,s$

$V(t) = V_0 e^{-t/\tau}$

$V(t) = 15 e^{-t/0.2} \Rightarrow \left(V(t) = 15 e^{-5t} \right)$

Use KVL during inductor

Page No.
Date

Energy,

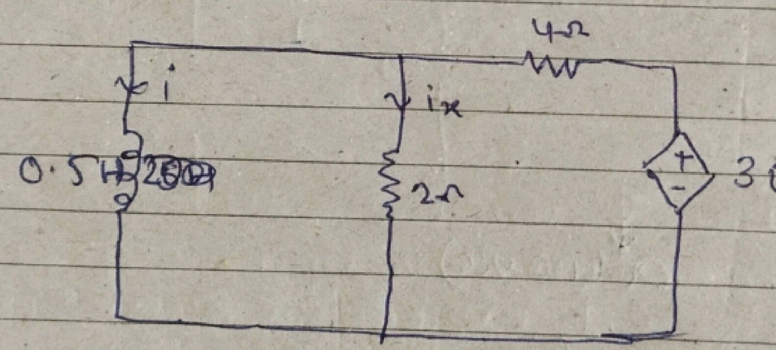
$$= \frac{1}{2} C V_0^2$$

$$= \frac{1}{\cancel{2}} \times \overset{10}{\cancel{20}} \times 10^{-3} \times 15 \times 15$$

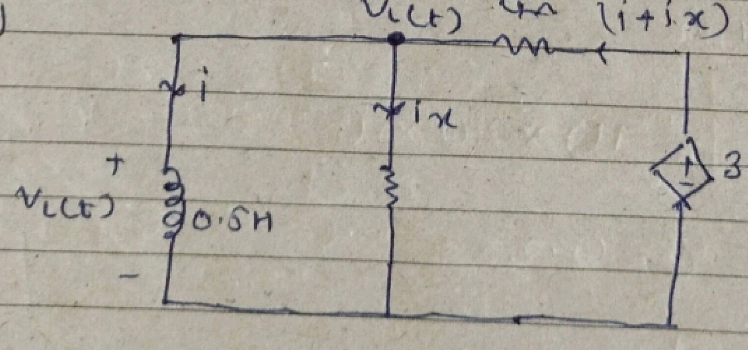
$$\left(E \Rightarrow 2.25\,J \right)$$

H.A

Q: Assuming $i(0) = 10A$ find $i(t)$ & $i_x(t)$ in the ckt



Sol)



Apply KVL in outer loop

$$+V_L(t) + 4(i + i_x) - 3i = 0$$

$$V_L(t) = L\frac{di}{dt} = L\{SI(s) - i(0^-)\}$$

$$L\frac{di}{dt} + 4i + 4ix - 3i = 0$$

$$L\frac{di}{dt} + i + 4ix = 0$$

$$L\{SI(s) - i(0^-)\} + I(s) + 4Ix(s) = 0 \quad \text{--- } (1)$$

Also,

$$ix(t) = \frac{V_L(t)}{2}$$

$$V_L(t) = L\frac{di}{dt}$$

$$Ix(t) = \frac{L\,di/dt}{2} = \frac{L}{2}\{SI(s) - i(0^-)\}$$

$$= \frac{L}{2}SI(s) - \frac{L}{2}(10)$$

$$Ix(s) = \frac{SL}{2}I(s) - 5L \quad \text{put in } (1)$$

$$SLI(s) - Li(0^-) + I(s) + 4\{S\frac{L}{2}I(s) - 5L\} = 0$$

$$SLI(s) - 10L + I(s) + 2SLI(s) - 20L = 0$$

$$3SLI(s) - 30L + I(s) = 0$$

$$I(s)\{3SL + 1\} = 30L$$

$$I(s) = \frac{30L}{3SL+1} = \frac{30 \times 0.5}{(3 \times 0.5s + 1)}$$

Page No.
Date

$$I(s) = \frac{15}{\frac{3}{2}s + 1}$$

$$I(s) = \frac{15 \times 2/3}{s + 2/3} = \frac{10}{s + 2/3} \quad \text{—— (11)}$$

inverse Laplace transform of (11)

$$i(t) = 10e^{-2/3 t}$$

Now we have

$$I_x(s) = \frac{sL}{2} I(s) - 5L$$

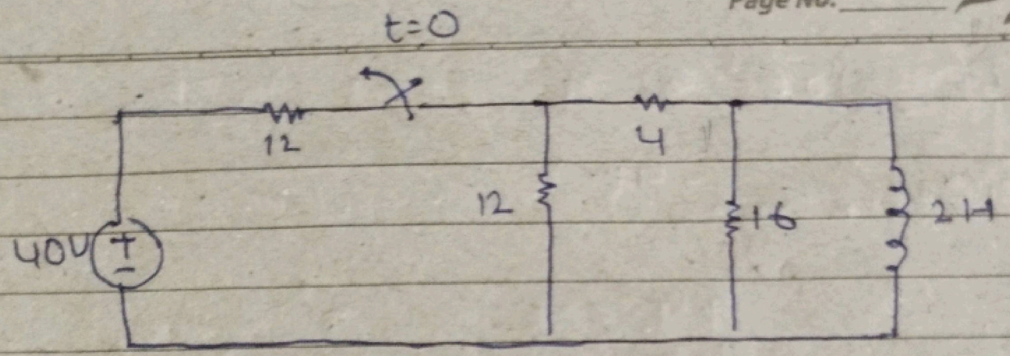
$$I_x(s) = \frac{1}{4} s I(s) - .25$$

Apply I.L.T

$$I_x(t) = \frac{1}{4}$$

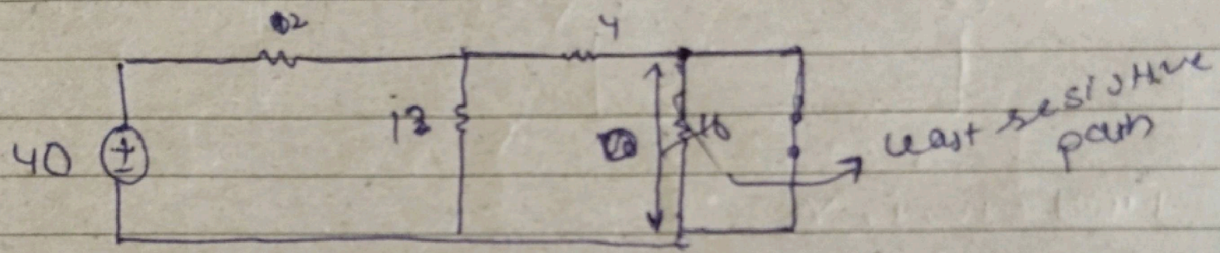
Date
Page No.

Q:-



Find $i(t)|_{t>0} = ?$

inductor behave like close ckt



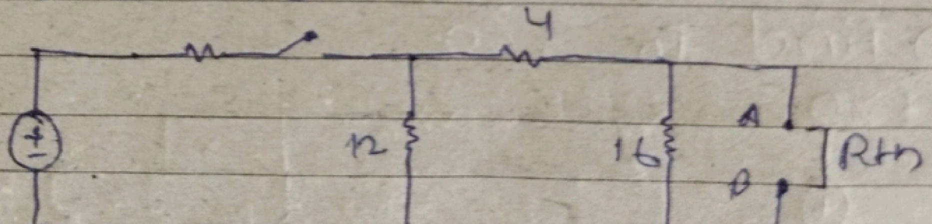
$\frac{12 \times 4}{16} = 3\Omega$

$R_{net} = 3+2 = 5\Omega$

$I = \frac{40}{5} = 8\,A$

$\left(I' = \frac{8 \times 12}{16} = 6A \right)$

To find τ at $t>0$



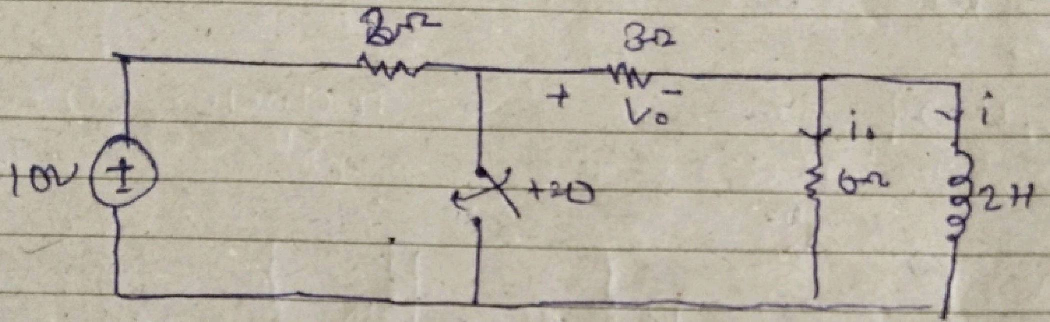
$R_{th} = \frac{16}{2} = 8\Omega$

$\tau = \frac{L}{R} = \frac{1}{4}$ NOW,

$i(t) = i_0 e^{-t/\tau}$

$\left(i(t) = 6e^{-4t} \right)$

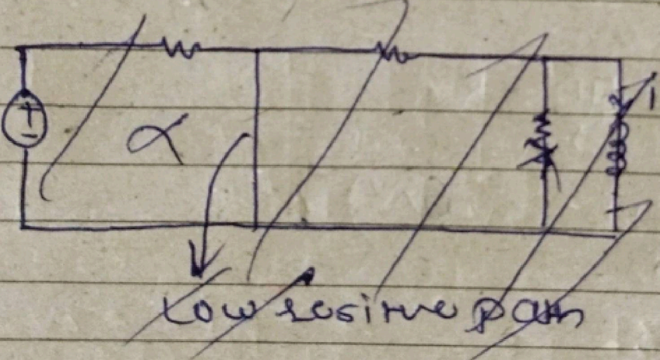
Q:



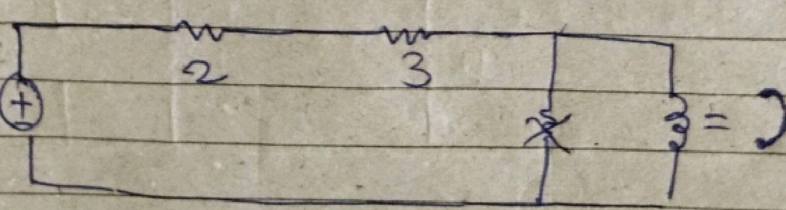
Find i_o, i, v_o

at $t \geq 0$ close

$i_o = 0$ because 2H get shorted and current follow low resistive path



TO find I_o $t<0$

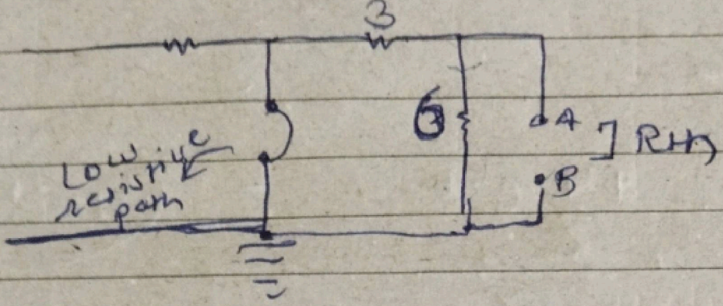


$R_{eq} = 5\,\Omega$

$I_0 = \frac{10}{5} = 2A$

Date ______
Page No. ______

To find τ



$R_{th} = \frac{3 \times 6}{9}$

$R_{th} = 2\Omega$

$\tau = L/R = \frac{2}{2} = 1$

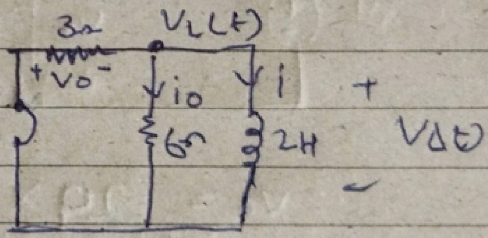
$i(t) = I_0 e^{-t/\tau}$

$$\boxed{i(t) = 2e^{-t}}$$

$(V_0 = 2 \times 3 = 6V)$

Now find

$\left.\begin{array}{l} i_0 = ? \\ V_0 = ? \end{array}\right\} t > 0$

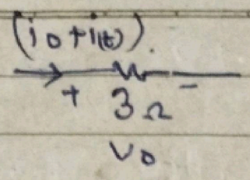


$i(t) = 2e^{-t}$

$i_0 = \frac{V_L(t)}{6} = i_0 = \frac{L\,di/dt}{6}$

$i_0 = \frac{2 \cdot d(2e^{-t})/dt)}{6}$

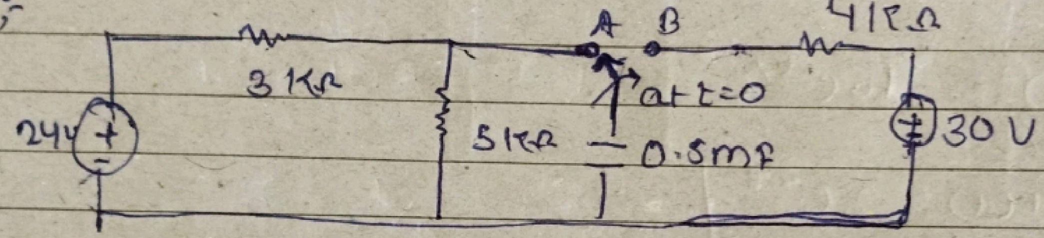
$i_0 = -\frac{4(e^{-t})}{6} = \left(\frac{-2}{3}e^{-t}\right)$



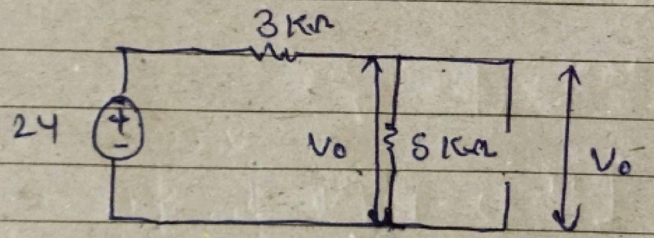
$$\Rightarrow \frac{\left(-\frac{2}{3}+2\right)e^{-t}}{3}$$

$$V_0 = \frac{+\frac{4}{\cancel{3}}e^{-t}}{\cancel{3}} = 4e^{-t}$$

Q:-

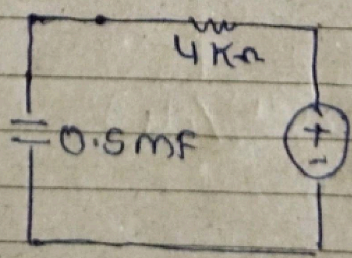


at $t<0$



$$V_0 = 24 \times \frac{5}{5+3} \Rightarrow \frac{120}{8} = 15V$$

at $t \geq 0$



~~at node~~ c)

We know as it is not source free RC ckt

$$V(t) = V(\infty) - [V_\infty - V_0]e^{-t/\tau}$$

$$V(t) = 30 - [30-15]e^{-t/\tau}$$

$$V(t) = \cancel{30-15} \quad 30 - 15e^{-t/2}$$

$\tau = RC$

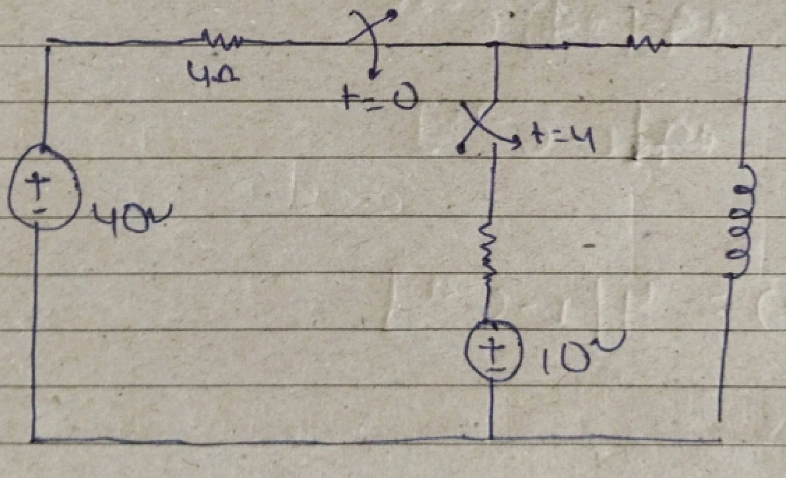
$\tau = 4k\Omega \cdot 0.5m$

$\tau = 2$

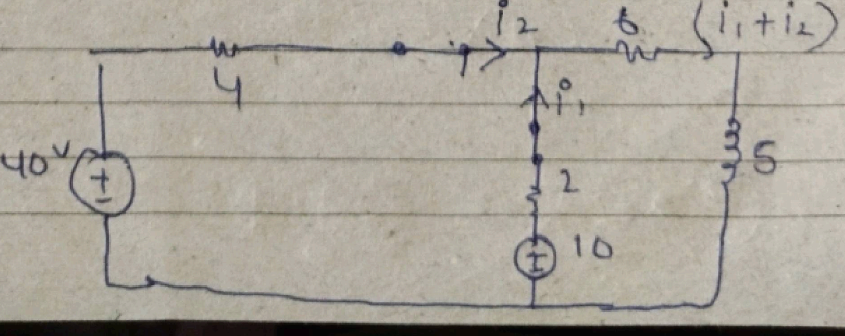
$$V(t)\big|_{1s} = 30 - 15e^{-1/2}$$

$$V(t)\big|_{4s} = 30 - 15e^{-2}$$

Q:-



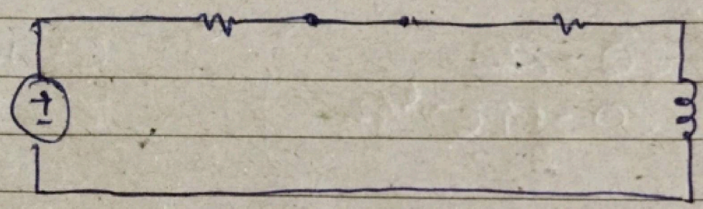
at $t > 0$



Apply KVL in loop (1)

40 − 10i −

4 > t ≥ 0



$i_\infty = \frac{40}{10} A = 4A$

$\tau = L/R = \frac{5}{10} = \frac{1}{2}$

$i(t)_{4>t\geq 0} = i_\infty - (i_\infty - i_0)\, e^{-t/\tau}$

$4 - (4)\, e^{-2t}$

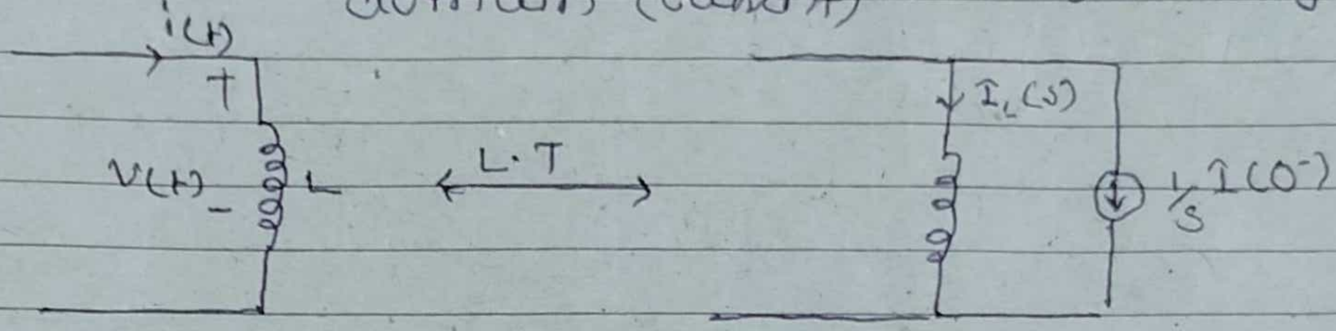
$4[1 - e^{-2t}]$

$i(t) = 4[1 - e^{-2t}]$

inductor L ko SL se replace kar denge

Date ____ Page No. ____

* ~~Capa~~ charging time domain ~~and~~ to frequency domain (current)



$V(t) = L\frac{di}{dt}$

Applying L.T

$V(s) = L\{sI(s) - I(0^-)\}$

$V(s) = sLI_s - L\cdot I(0^-)$

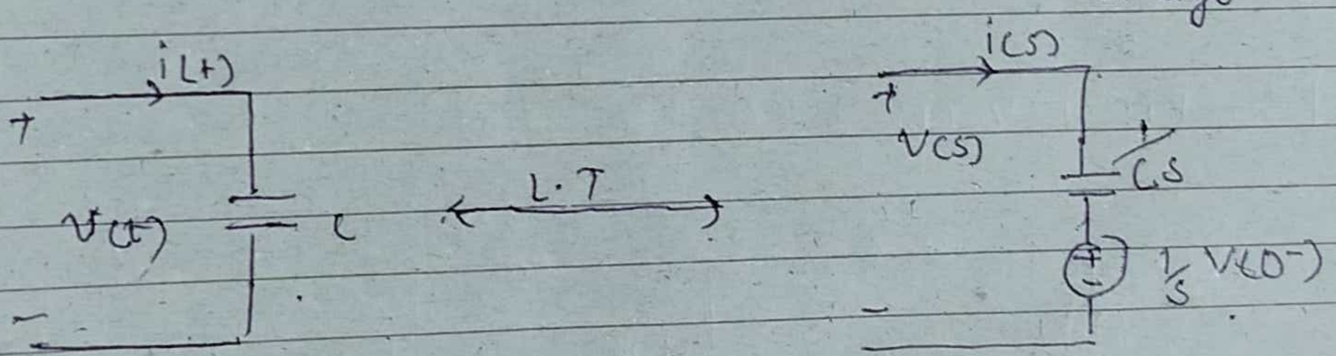
$I_s \cdot sL = V(s) + LI(0^-)$

$I(s) = \frac{1}{sL}V_s + \frac{I(0^-)}{s}$

$I(s) = I_L(s) + \frac{1}{s}I(0^-)$

* In case of capacitor :-

voltage domain



$i = \frac{dq}{dt}$ $\quad q = CV$

$i(t) = C\frac{dV(t)}{dt}$

$$I(s) = C\{sV(s) - V(0^-)\}$$

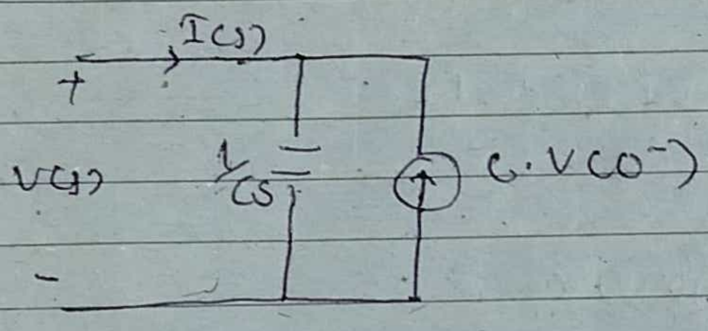
$$I(s) = CsV(s) - CV(0^-)$$

$$V_s = \frac{I(s)}{Cs} + \frac{V(0^-)}{s}$$

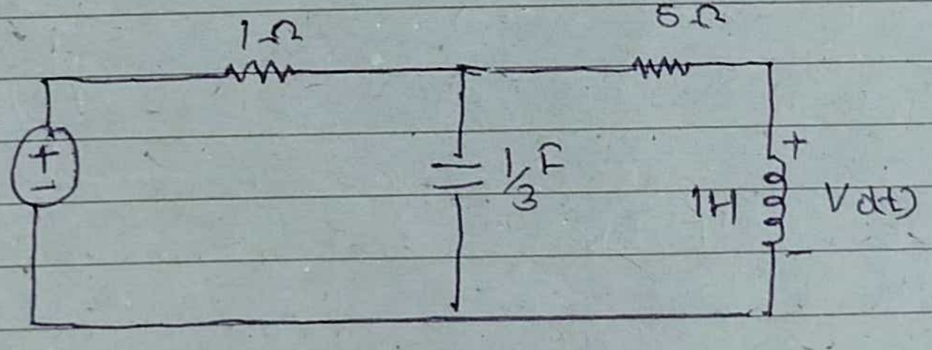
$$V(s) = \frac{1}{Cs} I(s) + \frac{1}{s} V(0^-)$$

$$I(s) = \frac{V(s)}{1/Cs} - C \cdot V(0^-)$$

current domain



Q:- Find $V_o(t)$?



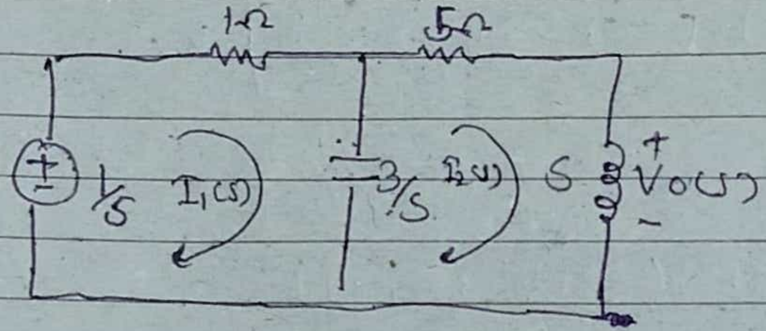
Date ______
Page No. ______

Sol $u(t) \xleftrightarrow{LT} 1/s$

$1, 5\,\Omega \xleftrightarrow{LT} 1, 5\,\Omega$

$\frac{1}{3}F \xleftrightarrow{LT} \frac{1}{Cs} = \frac{1}{\frac{1}{3}s} = 3s$

$1H \xleftrightarrow{LT} sL = s$



as intial condn is not given then intial value is zero

Apply KVL

$$\frac{1}{s} - I_1(s) - \frac{3}{s}\left(I_1(s) - I_2(s)\right) = 0 \quad \text{--- (1)}$$

$$-5I_2(s) - I_2(s)(s) - \frac{3}{s}\left(I_2(s) - I_1(s)\right)$$

$$-5I_2(s) - I_2(s)(s) - \frac{3}{s}I_2(s) + \frac{3}{s}I_1(s) = 0$$

$$\frac{3}{s} \cdot I_1(s) = \left(5 + s + \frac{3}{s}\right) I_2(s)$$

—— (ii)

$$I_1(s) = \left(5 + s + \frac{3}{s}\right) s/3 \; I_2(s)$$ put in (1)

Page No.
Date

$$\frac{1}{s} = \left(\frac{1}{\cancel{8}} + \frac{3}{s}\right)\frac{s}{3}\left(5 + s + \frac{3}{s}\right) I_2(s) - \frac{3}{s} I_2(s)$$

$$\frac{1}{\cancel{s}} = \frac{(s+3)}{\cancel{s}} \cdot \frac{s}{3} \left(\frac{s^2+5s+3}{s}\right) I_2(s) - \frac{3}{\cancel{s}} I_2(s)$$

$$1 = (s+3)\frac{1}{3}(s^2+5s+3) I_2(s) - 3 I_2(s)$$

$$1 = \left[\frac{(s+3)(s^2+5s+3) - 9}{3}\right] I_2(s)$$

$$I_2(s) = \frac{3}{[(s+3)(s^2+5s+3) - 9]}$$

$$I_2(s) = \frac{3}{s^3 + 5s^2 + 3s + 3s^2 + 15s + \cancel{9} - \cancel{9}}$$

$$I_2(s) = \frac{3}{s^3 + 8s^2 + 18s}$$

$$I_2(s) = \frac{3}{s(s^2 + 8s + 18)}$$

$$V_0(s) = s I_2(s) = \frac{3 \cancel{s}}{\cancel{s}(s^2 + 8s + 18)}$$

Date
Page No.

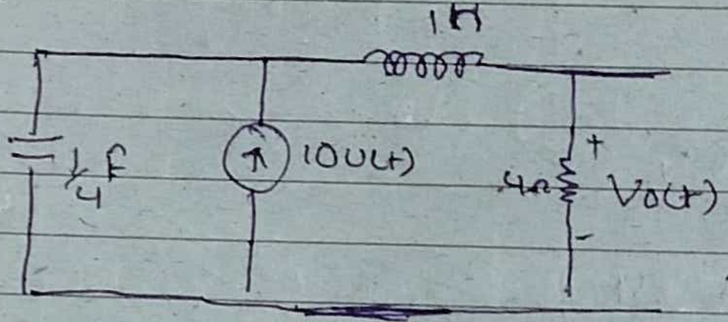
$$V(s) = \frac{3}{s^2+8s+16+2}$$

$$V(s) = \frac{1}{\sqrt{2}} \frac{3\times\sqrt{2}}{(s+4)^2+(\sqrt{2})^2}$$

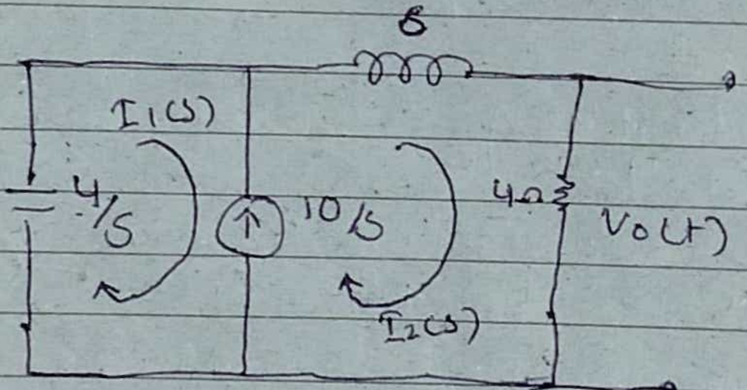
I.L.T

$$V_0(t) = \frac{3}{\sqrt{2}} e^{-4t}\cdot \sin\sqrt{2}t \cdot u(t)$$

Q:- Find $v_0(t)$



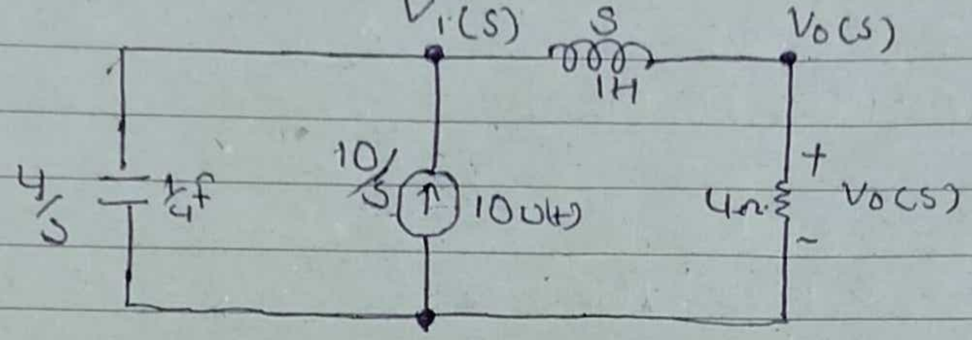
Sol



$$I_2 - I_1 = 10/s$$

$$-\frac{4}{s} I_1(s) -$$

Not Apply supermesh it become complicated



Apply nodal analysis :-

$$\frac{V_1}{4/s} - \frac{10}{s} + \frac{V_1 - V_0}{s} = 0$$

$$\frac{sV_1}{4} - \frac{10}{s} + \frac{V_1}{s} - \frac{V_0}{s} = 0$$

$$V_1\left(\frac{s}{4} + \frac{1}{s}\right) - \frac{V_0}{s} = \frac{10}{s}$$

$$V_1\left(\frac{s^2+4}{4s}\right) - \frac{V_0}{s} = \frac{10}{s}$$

$$V_1\left(\frac{(s^2+4s)}{4\cancel{s}}\right) - \frac{V_0}{\cancel{s}} = \frac{10}{\cancel{s}}$$

$$V_1(s^2+4) - 4V_0 = 40 \quad \text{— (1)}$$

$$\frac{V_0 - V_1}{s} + \frac{V_0}{4} = 0$$

$$\frac{V_0}{s} - \frac{V_1}{s} + \frac{V_0}{4} = 0$$

Date
Page No.

$$v_0\left(\frac{1}{s}+\frac{1}{4}\right)=\frac{v_1}{s}$$

$$v_0\left(\frac{4+s}{4\cancel{s}}\right)=\frac{v_1}{\cancel{s}} \quad \text{——— (ii)}$$

(ii) − (i)

$$v_0\left(\frac{s+4}{4}\right)\cdot(s^2+4)-4v_0=40$$

$$v_0\left(s^2+4s+4s^2+\cancel{16}-\cancel{16}\right)=160$$

$$v_0=\frac{160}{s(s^2+4s+4)}$$

$$v_0=\frac{160}{s(s+2)^2}$$

Partial fraction

$$\frac{160}{s(s+2)^2}=\frac{A}{s}+\frac{B}{(s+2)}+\frac{C}{(s+2)^2}$$

$$160=A(s+2)^2+B(s+2)s+Cs$$

$160=-2C$

$(C=-80)$

$160=A4$

$(A=40)$

$160=360+3B-80$

$B=-40$

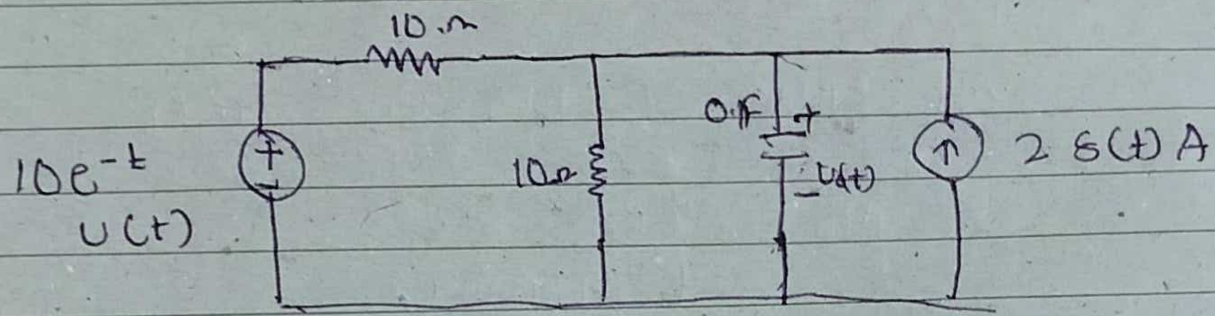
Page No.
Date

$$V_o(s) = \frac{40}{s} - \frac{40}{(s+2)} - \frac{80}{(s+2)^2}$$

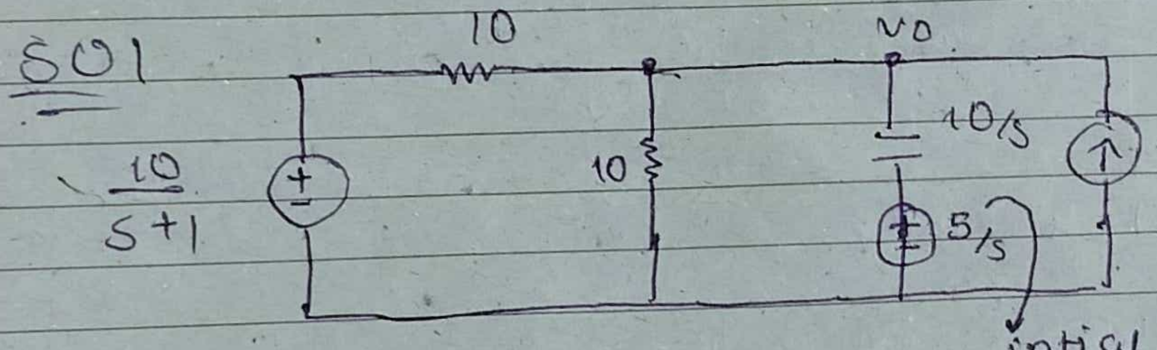
ILT

$$v_o(t) = 40\left(1 - e^{-2t} - 2e^{-2t}\right)$$

Q:- Find $v_o(t)$, Given $v_o(0^-) = 5V$



Sol



$$\frac{v_o - \frac{10}{s+1}}{10} + \frac{v_o}{10} + \frac{v_o - 5/s}{10/s} - 2 = 0$$

$$\frac{v_o}{10} - \frac{1}{s+1} + \frac{v_o}{10} + \frac{s v_o}{10} - \frac{1}{2} - 2 = 0$$

$$v_o\left(\frac{1}{5} + \frac{s}{10}\right) = \frac{3}{2} + \frac{1}{s+1}$$

Date
Page No.

$$V_o(s) = \frac{10}{(s+1)(s+2)} + \frac{s}{2}\frac{(10)s}{s+2}$$

$$V_o(s) = \frac{10}{(s+1)(s+2)} + \frac{2s}{(s+2)}$$

$$\frac{10}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

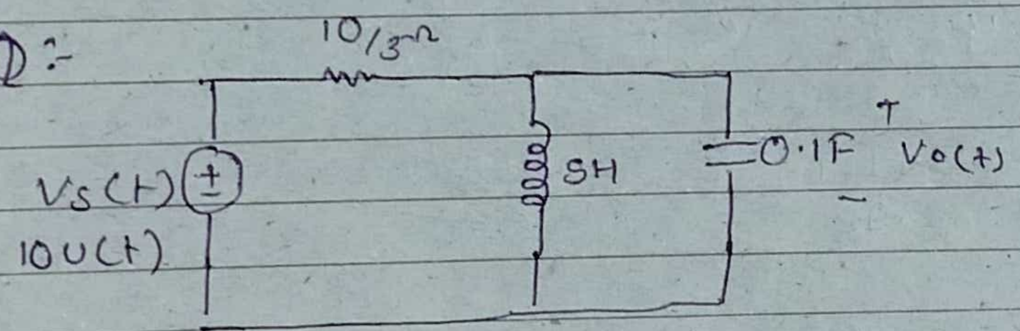
$$10 = A(s+2) + B(s+1)$$

$$(A = 10) \quad , \quad 10 = -B$$
$$(B = -10)$$

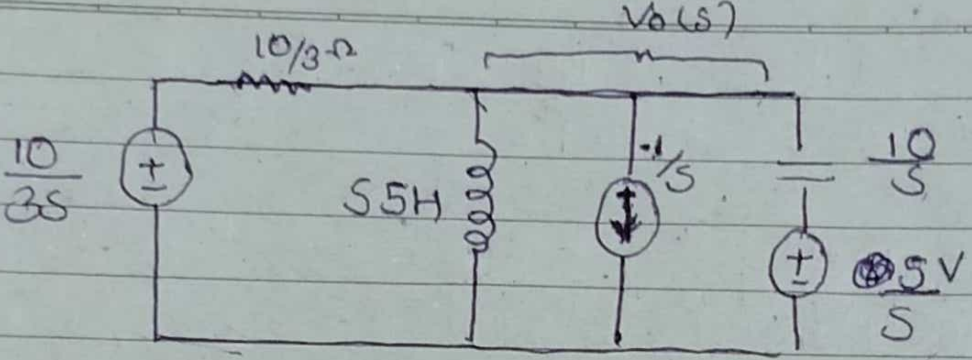
$$V_o(s) = \frac{10}{s+1} - \frac{10}{(s+2)} + \frac{2s}{s+2}$$

$$= \{10e^{-t} + 15e^{-2t}\} u(t)$$

Q :-



assume that at t=0, −1A flow through inductor & +5V is across the capacitor



Applying nodal analysis

$$\frac{V_o(s) - 10/3s}{10/3} + \frac{V_o(s)}{5s} + \left(-\frac{1}{s}\right) + \frac{V_o(s) - 5/s}{10/s} = 0$$

$$\frac{3V_o(s)}{10} - \frac{\cancel{10}/s}{\cancel{10}/3} + \frac{V_o(s)}{5s} - \frac{1}{s} + \frac{s V_o(s)}{10} - \frac{\cancel{5}}{\cancel{10}}\frac{1}{2} = 0$$

$$\frac{3}{10}V_o(s) - \frac{3}{s} + \frac{V_o(s)}{5s} - \frac{1}{s} + \frac{s}{10}V_o(s) + \frac{1}{2} = 0$$

$$\frac{3}{10}V_o(s) + \frac{1}{5s}V_o(s) + \frac{s}{10}V_o(s) = \frac{4}{s} + \frac{1}{2}$$

$$V_o(s)\left\{\frac{3}{10} + \frac{1}{5s} + \frac{s}{10}\right\} = \frac{8+s}{2s}$$

$$V_o(s)\left\{\frac{3s+1+s^2}{5\,\cancel{10s}}\right\} = \frac{s+8}{\cancel{2s}}$$

$$V_o(s) = \frac{5(s+8)}{s^2+3s+2}$$

$$\frac{5(s+8)}{(s+1)(s+2)} = \frac{A}{(s+1)} + \frac{B}{(s+2)}$$

Date
Page No.

$A\big|_{S=-1} = 5 \times 7 = 35 \quad , \quad B = -30$

$$V_0(s) = \frac{35}{(s+1)} - \frac{30}{s+2}$$

I.L.T

$$\left[35e^{-t} - 30e^{-2t}\right]u(t)$$

- Transfer function

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

input funct. — O/P func.

$$y(t) = x(t) * h(t)$$

$$Y(s) = X(s) \cdot H(s)$$

$$X(s) \longrightarrow \boxed{H(s)} \longrightarrow Y(s)$$

$$\boxed{H(s) = \frac{Y(s)}{X(s)}} \longrightarrow \text{" Transfer function}$$

Q:- If input / O/P func is $y(t) = 10e^{-t}\cos 4t\, u(t)$
$x(t) = e^{-t} \cdot u(t)$
Find transfer func ? & impulse response?

$$H(s) = \frac{Y(s)}{X(s)} \longrightarrow (1)$$

$$Y(s) = 10 \cdot \frac{(s+1)}{(s+1)^2 + 4^2} = \frac{10(s+1)}{(s+1)^2 + 4^2}$$

$$X(s) = \frac{1}{(s+1)}$$

Put in eq (1)

Date

Page No.

$$H(s) = \frac{\frac{10(s+1)}{(s+1)^2+4^2}}{\frac{1}{(s+1)}}$$

$$H(s) = 10\left\{\frac{(s+1)^2+16-16}{(s+1)^2+4^2}\right\}$$

$$H(s) = 10\left\{\frac{(s+1)^2+16}{(s+1)^2+4^2} - \frac{16}{(s+1)^2+4^2}\right\}$$

$$H(s) = 10\left\{1 - \frac{16}{(s+1)^2+4^2}\right\} = \left\{1 - \frac{4\times4}{(s+1)^2+4^2}\right.$$

I.L.T

$$h(t) = 10\{\delta(t) - 4e^{-t}\sin 4t\}$$

$$h(t) = 10\delta(t) - 40e^{-t}\sin 4t \cdot u(t)$$

⇒ impulse response means $x(t) = \delta(t)$ & $X(s) = 1$ thus $Y(s) = H(s)$, $\underbrace{y(t) = h(t)}_{\text{impulse respons}}$

thus,

$$y(t) = 10\delta(t) - 40e^{-t}\sin 4t\, u(t)$$

Page No.
Date

Q:- $H(s) = \frac{2s}{(s+6)}$ Find $y(t)$, when "

$x_{(t)} = 10e^{-3t}u(t)$ & $y(t)$

Sol $H(s) = \frac{2s}{s+6}$

I.L.T

$h(t) = 2e^{-6t}$

$x(t) = 1$

thus,

$y(t) = h(t)$

$y(t) = 2e^{-6t}$

$Y(s) = X(s) \cdot H(s)$

$\frac{10}{s+3} \cdot \frac{2s}{s+6} = \frac{20s}{(s+3)(s+6)}$

$Y(s) = \frac{-20}{s+3} + \frac{40}{s+6}$

$y(t) = (-20e^{-3t} + 40e^{-6t})u(t)$

when impulse response

$Y(s) = H(s)$

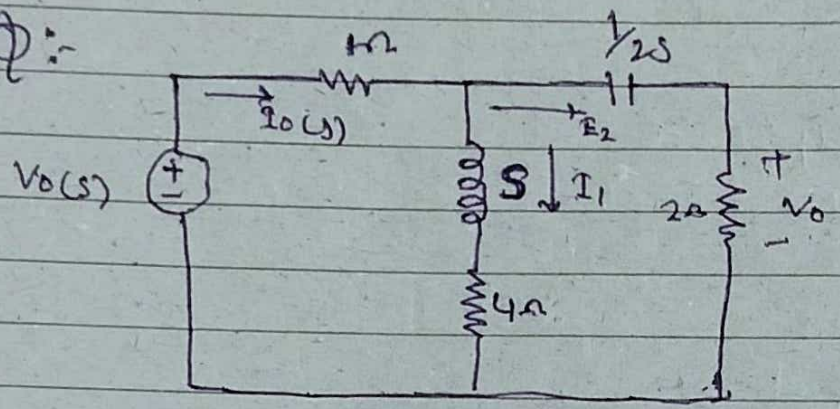
$Y(s) = \frac{2s}{s+6}$

$$= \frac{2S}{S+6}$$

$$\Rightarrow 2\left(\frac{S+6-6}{S+6}\right) \Rightarrow 2\left(1-\frac{6}{S+6}\right)$$

$$\Rightarrow 2\delta(t) - 12e^{-6t}u(t)$$

Q:-



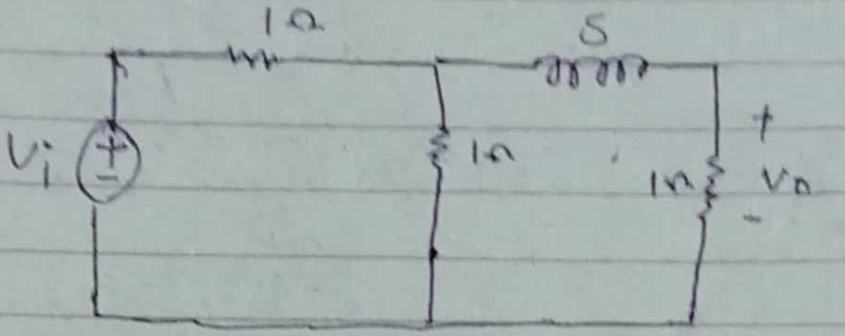
⇒ find Gain $H(s) = \frac{V_o(s)}{I_o(s)}$

Sol $I_2 = \frac{I_o(s)\cdot(S+4)}{\left(S+4+\frac{1}{2s}+2\right)}$ ⟶ current divider rule

$$V_o(s) = I_2 \cdot 2 = 2\,\frac{I_o(s)\,(s+4)}{\left(S+4+\frac{1}{2s}+2\right)}$$

$$\left(H(s) = \frac{V_o(s)}{I_o(s)} = \frac{2(S+4)}{\left(S+4+\frac{1}{2s}+2\right)} \right)$$

Q:



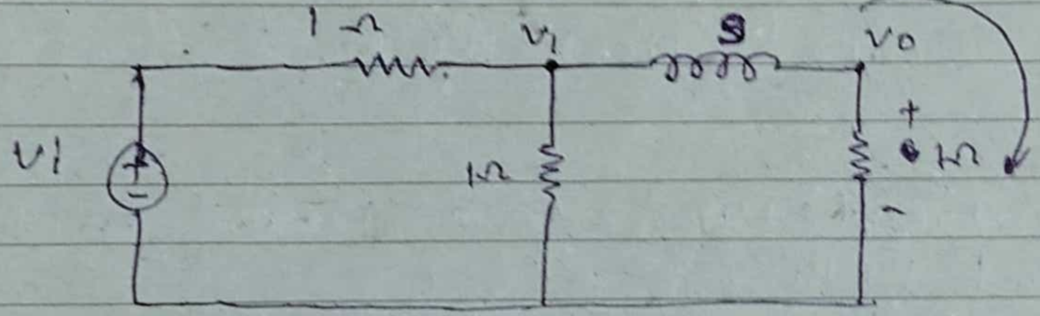
Find (a) $H(s) = \frac{V_o}{V_i} = ?$

(b) impulse response

(c) $v_i(t) = u(t)$

(d) $v_i(t) = 8\cos 2t$.

Sol



Apply Nodal at V_1

$$\frac{V_1 - V_i}{1} + \frac{V_1}{1} + \frac{V_1 - 0}{(S+1)} = 0$$

$$V_1 - V_i + V_1 + \frac{V_1}{S+1}$$

$$2V_1 - V_i + \frac{V_1}{S+1} = 0$$

$$2V_1 + \frac{V_1}{S+1} = V_i$$

Date
Page No.

(i) $\left\{\frac{q\Omega(s+1)+V_1}{s+1}\right\} = Vi$

$$V_1\left\{\frac{2s+2+1}{(s+1)}\right\} = V_i$$

$$V_1\left[\frac{2s+3}{(s+1)}\right] = Vi$$

$$V_1 = \frac{(s+1)\,Vi}{(2s+3)}$$

Apply voltage divider rule

$$Vo = V_1 \cdot \frac{1}{s+1}$$

forward R

net R

$$Vo = \frac{(s+1)\,Vi}{(2s+3)} \cdot \frac{1}{(s+1)}$$

$$\frac{Vo}{Vi} = \frac{1}{(2s+3)} = \frac{1}{2}\left[\frac{1}{s+3/2}\right]$$

(a) $H(s) = \left(\frac{Vo}{Vi} = \frac{1}{2}\left[\frac{1}{s+3/2}\right]\right)$

Apply ILT

$$h(t) = \frac{1}{2} e^{-3/2 t} u(t)$$

(b) $x(t) = 1$

$$y(t) = \frac{1}{2} e^{-3/2 t} u(t)$$

Page No.
Date

(c) $= v_i(t) = u(t)$

$$v_i(s) = 1/s$$

$$Y(s) = H(s) \cdot X(s)$$

$$Y(s) = \frac{1}{2s+3} \cdot \frac{1}{s} = \frac{1}{2s(s+3/2)}$$

apply P.F then ILaplace transform,

$$y(t) = \frac{1}{3}\left[1 - e^{-3t/2}\right] u(t)$$

(d) $v_i(t) = 8\cos 2t$

$$Y(s) = \frac{8s}{s^2+4} + \frac{1}{2(s+3/2)}$$

$$Y(s) = \frac{4s}{(s+3/2)(s^2+4)}$$

Applying P.F

$$\frac{4s}{(s+3/2)(s^2+4)} = \frac{A}{(s+3/2)} + \frac{Bs+C}{(s^2+4)} \quad , -A = B = \frac{24}{25}$$

~~MSR~~

Date
Page No.

- ## Network Stability

$$e^{-at} \longleftrightarrow \frac{1}{s+a}$$

how we find signal is stable not

$\frac{1}{(s+a)}$ → to find poles ~~But~~ Equate Denomenator $=0$

$s+a=0$,

$\boxed{s=-a}$ pole is -ve

if poles of the system is -ve then system is -ve & converging graph & vice versa

$$H(s) = \frac{N(s)}{D(s)} = \frac{(s+a_1)(s+a_2)(s+a_3)----(s+a_n)}{(s+b_1)(s+b_2)(s+b_3)----(s+b_m)}$$

$s = -a_1, -a_2, -a_3, -a_4 --- -a_n$

↑

zero's of the system & it does not give any idea about the stability of system

$s = -b_1, -b_2, -b_3, -b_4 ----, -b_m$

↓

Pole's of the system it gives idea of the stability of the system

25-2-24

Page No.

Date

- Two port Network :-

Z - parameter

Y - " "

h - " "

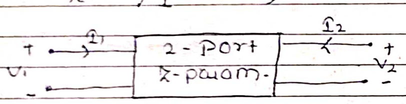
g - " "

ABCD - " "

① Z-parameter :-

↳ impedence parameter

$Z = V/I \Rightarrow V = Z \cdot I$



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$V_1 = Z_{11} I_1 + Z_{12} I_2$ —— ①

$V_2 = Z_{21} I_2 + Z_{22} I_2$ —— ②

Now from eqn ①

$$Z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2 = 0}$$

Z_{11} : input driving impedence when O/P port is open

Date

Page No.

ⓑ $Z_{12} = \left. \dfrac{V_1}{I_2} \right|_{I_1 = 0}$

→ Reverse transfer impedence when I/P is open

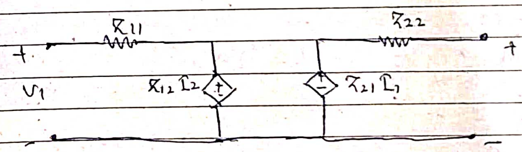
ⓒ $Z_{21} = \left. \dfrac{V_2}{I_1} \right|_{I_2 = 0}$

→ Forward transfer impedence when O/P is open

d) $Z_{22} = \left. \dfrac{V_2}{I_2} \right|_{I_1 = 0}$

→ O/P driving impedence when I/P is ope

Circuit for the same -



Page No.
Date

(ii) Y parameter :
↳ Admittance parameter

$$Y = \frac{1}{Z} = \frac{I}{V} \Rightarrow (I = Y \cdot V)$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{—— (I)}$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \text{—— (II)}$$

from the eqn

a) $Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2 = 0}$

→ input driving admittance, when O/P port is short ckt

b) $Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2 = 0}$

→ forward transfer admittance, when O/P is short ckt

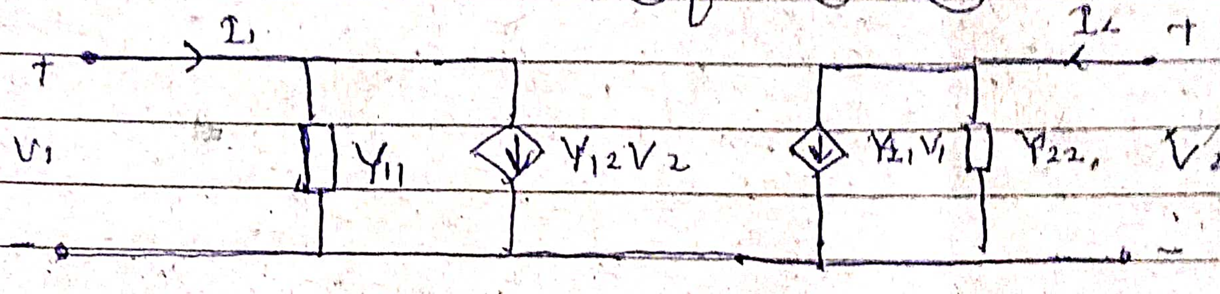
c) $Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1 = 0}$

Reverse transfer admittance, when I/P short ckt

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

→ O/p driving admittance when I/P short ckt

* Model for the eqn (i) & (ii)



(iii) h-parameter :-
↳ hybrid parameter

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{—— (i)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{—— (ii)}$$

a) $h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$

→ I/p driving impedance, when o/p is short ckt.

b) $h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$

→ reverse voltage gain when I/P is open ckt

Page No.
Date

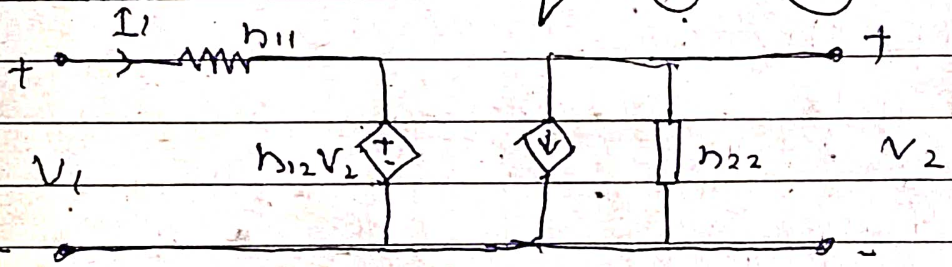
c) $h_{21} = \left.\dfrac{I_2}{I_1}\right|_{V_2=0}$

→ forward current gain when O/P shorted

d) $h_{22} = \left.\dfrac{I_2}{V_2}\right|_{I_1=0}$

→ O/P admittance when I/P is open ckt

* Model for the eqn (I) & (II)



(IV) g-parameters (inverse h-parameter)

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

$$I_1 = g_{11}V_1 + g_{12}I_2 \quad \text{— (I)}$$

$$V_2 = g_{21}V_1 + g_{22}I_2 \quad \text{— (II)}$$

a) $g_{11} = \left.\dfrac{I_1}{V_1}\right|_{I_2=0}$

→

Date
Page No.

b) $g_{12} = \left.\dfrac{I_1}{I_2}\right|_{V_1=0}$

$\rightarrow$

c) $g_{21} = \left.\dfrac{V_2}{V_1}\right|_{I_2=0}$

$\rightarrow$

d) $g_{22} = \left.\dfrac{V_2}{I_2}\right|_{V_1=0}$

-

(V) <u>ABCD parameter</u> :-

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$V_1 = AV_2 + B(-I_2) \quad \text{—— (I)}$$

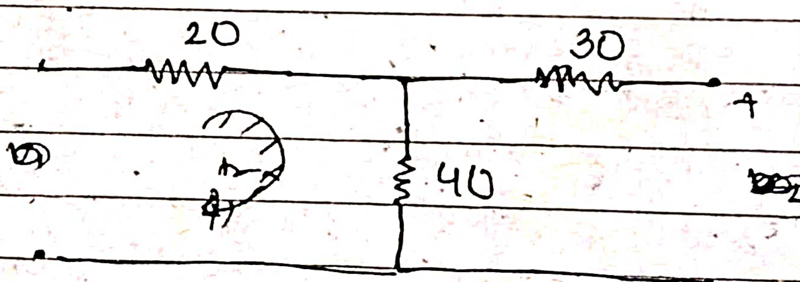
$$I_1 = CV_2 + D(-I_2) \quad \text{—— (II)}$$

Date ______ Page No. ______

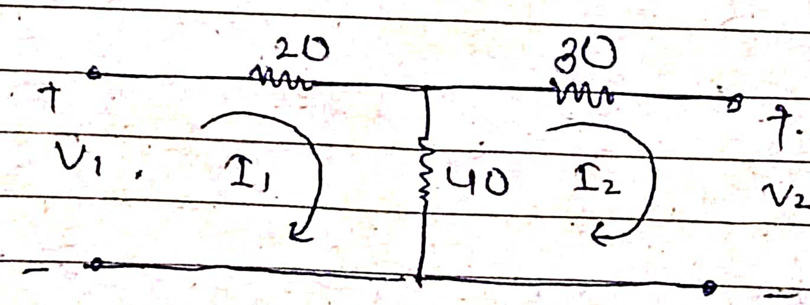
symmetrical reciprocal

	Symmetrical Ntw	Reciprocal
$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$	$Z_{11} = Z_{22}$	$Z_{21} = Z_{12}$
$\begin{bmatrix} Y_{11} & Y_{22} \\ Y_{21} & Y_{22} \end{bmatrix}$	$Y_{11} = Y_{22}$	$Y_{21} = Y_{12}$
$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$	$h_{11}h_{22} - h_{21}h_{12} = 1$	$h_{21} = -h_{12}$
$\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$	$g_{11}g_{22} - g_{12}g_{21} = 1$	$g_{21} = -g_{12}$
$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$	$A = D$	$AD - BC = 1$

Q: Find Z-parameter



Sol



$$V_1 - 20I_1 - 40(I_1 + I_2) = 0$$

$$V_1 = 60I_1 + 40I_2 \quad \text{——— (1)}$$

Page No.
Date

$V_2 = 30I_2 - 40(I_1+I_2) = 0$

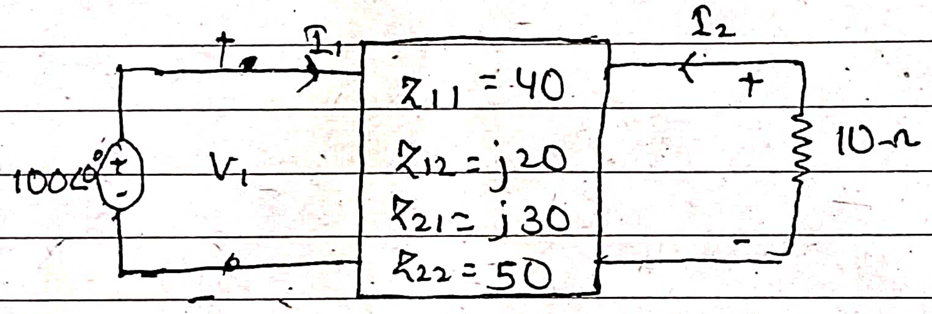
$V_2 = 40I_1 + 70I_2$ —— (ii)

now,

$Z_{11} = 60$, $Z_{12} = 40$
$Z_{21} = 40$, $Z_{22} = 70$

As $Z_{12} = Z_{21}$ it is reciprocal ckt

Q:- Find I_1 & I_2



Sol $V_1 = Z_{11}I_1 + Z_{12}I_2$
$V_2 = Z_{21}I_1 + Z_{22}I_2$

$\therefore V_2 = -I_2 \times 10$
$= -10I_2$

$100 = 40I_1 + j20I_2$ —— (i)

$-60I_2 = j30I_1$
$I_1 = 2jI_2$

$V_2 = j30I_1 + 50I_2$

$-10I_2 = j30I_1 + 50I_2$ —— (ii)

$60I_2 + j30I_1 = 0$
$40I_1 + j20I_2 = 100$

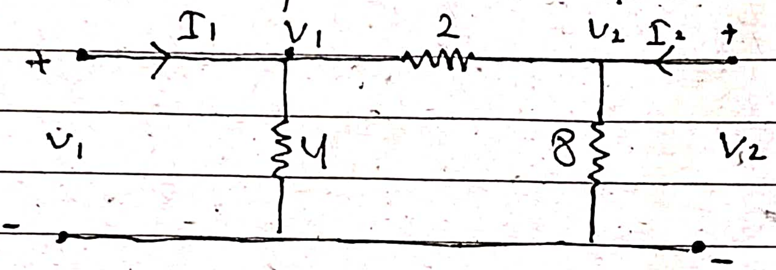
from this

$I_1 = 2j I_2$

$I_2 = -j \Rightarrow I_2 = 1\angle \tan(-\infty) = 1\angle 90'$

$I_1 = -2j^2 = 2A$

Q:- Find Y parameter?



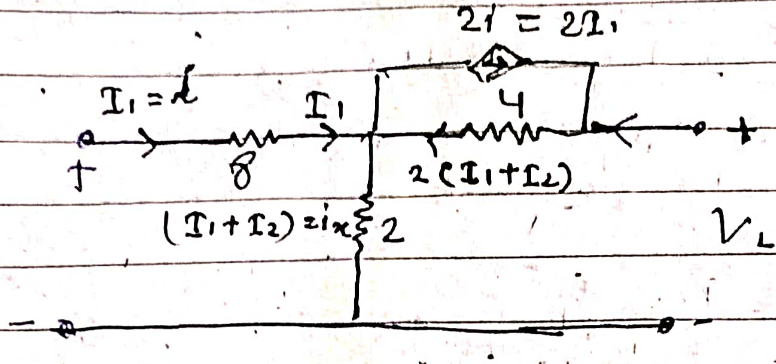
$$\frac{V_1}{4} + \frac{V_1 - V_2}{2} - I_1 = 0$$

$$\frac{V_1}{4} + \frac{2V_1}{2\times 2} - \frac{V_2}{2} = I_1$$

$$\left(\frac{3V_1}{4} - \frac{1}{2}V_2 = I_1\right)$$

Page No.
Date

Q: Find Y parameter



Apply KCL

$$I_1 + \cancel{2I_1} + I_2 = i_x + \cancel{2I_1}$$

$$i_x = (I_1 + I_2) \quad \text{——— (I)}$$

$$V_1 - 8I_1 - 2(I_1 + I_2) = 0$$

$$V_1 = 10I_1 + 2I_2 \quad \text{——— (II)}$$

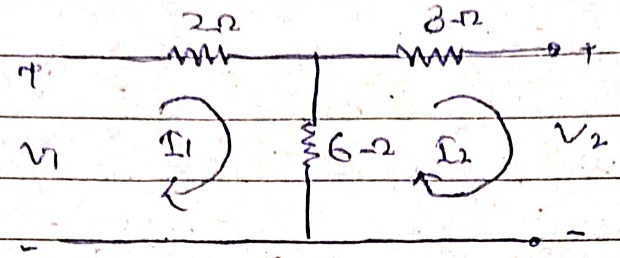
$$V_2 = 4(2I_1 + I_2) - 2(I_1 + I_2) = 0$$

$$V_2 = 8I_1 + 4I_2 + 2I_1 + 2I_2$$

$$V_2 = 10I_1 + 6I_2 \quad \text{——— (III)}$$

Date ______
Page No. ______

Q: Find h-parameter = ?



<u>Sol</u> Apply KVL

$$V_1 - 2I_1 - 6(I_1 + I_2) = 0$$

$$V_1 = 8I_1 + 6I_2 \quad \text{—— (1)}$$

$(h_{11} = 8)$ or $h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$

$$V_2 - 3I_2 - 6I_2 - 6I_1 = 0$$

$$V_2 - 9I_2 - 6I_1 = 0$$

$$I_2 = \frac{V_2}{9} - \frac{6I_1}{9}$$

$$I_2 = \frac{1}{9}V_2 - \frac{2}{3}I_1$$

$(h_{21}) = -2/3$, $\left(h_{22} = \frac{1}{9}\right)$

• Interconnection of 2 port N/w :-

→ series-series connection → Z-parameter

→ parallel parallel " → Y "

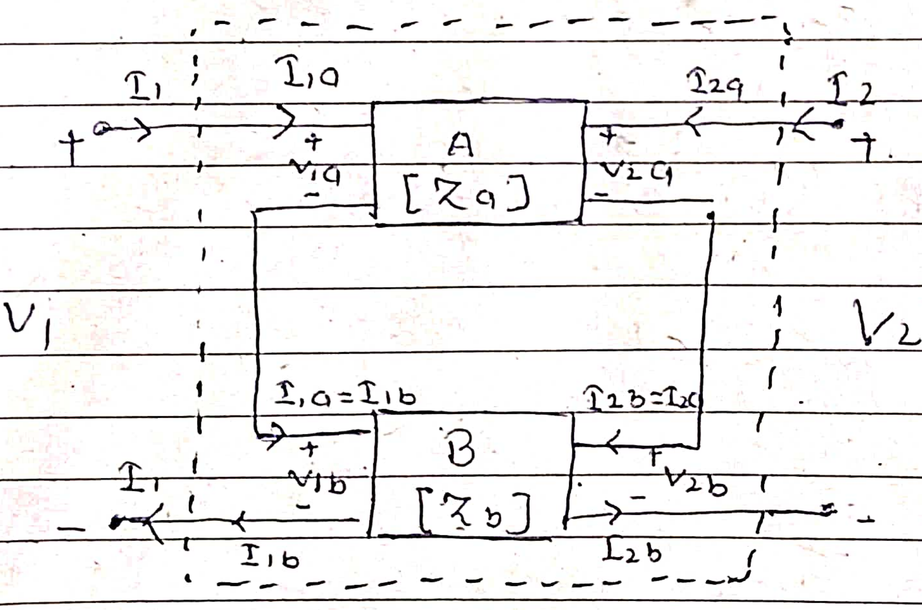
→ series - " " → I/p - Z , O/p - Y

→ parallel series " → I/p Y , O/P - Z

→ cascade " " → ABCD parameter

Note
We connect Z in series & Y in parallel

①



Now Apply KVL

$$V_1 - V_{1a} - V_{1b} = 0$$

$$(V_1 = V_{1a} + V_{1b})$$

$(V_2 = V_{2a} + V_{2b} = 0)$

$(I_1 = I_{1a} = I_{1b})$

$(I_2 = I_{2a} = I_{2b})$

Thus we can write ~~its~~ eqn for z parameter as it is 2, 2port network

$V_{1a} = z_{11a} I_{1a} + z_{12a} \cdot I_{2a}$ — (1)

$V_{2a} = z_{21a} I_{1a} + z_{22a} \cdot I_{2a}$ — (2)

$V_{1b} = z_{11b} I_{1b} + z_{12b} I_{2b}$ — (3)

$V_{2b} = z_{21b} I_{1b} + z_{22b} I_{2b}$ — (4)

add eqn (1) & (3)

$V_{1a} + V_{1b} = z_{11a} \cdot I_{1a} + z_{11b} I_{1b} + z_{12a} I_{2a} + z_{12b} I$

as $V_{1a} + V_{1b} = V_1$

$$\boxed{V_1 = (z_{11a} + z_{11b}) I_1 + (z_{12a} + z_{12b}) I_2}$$

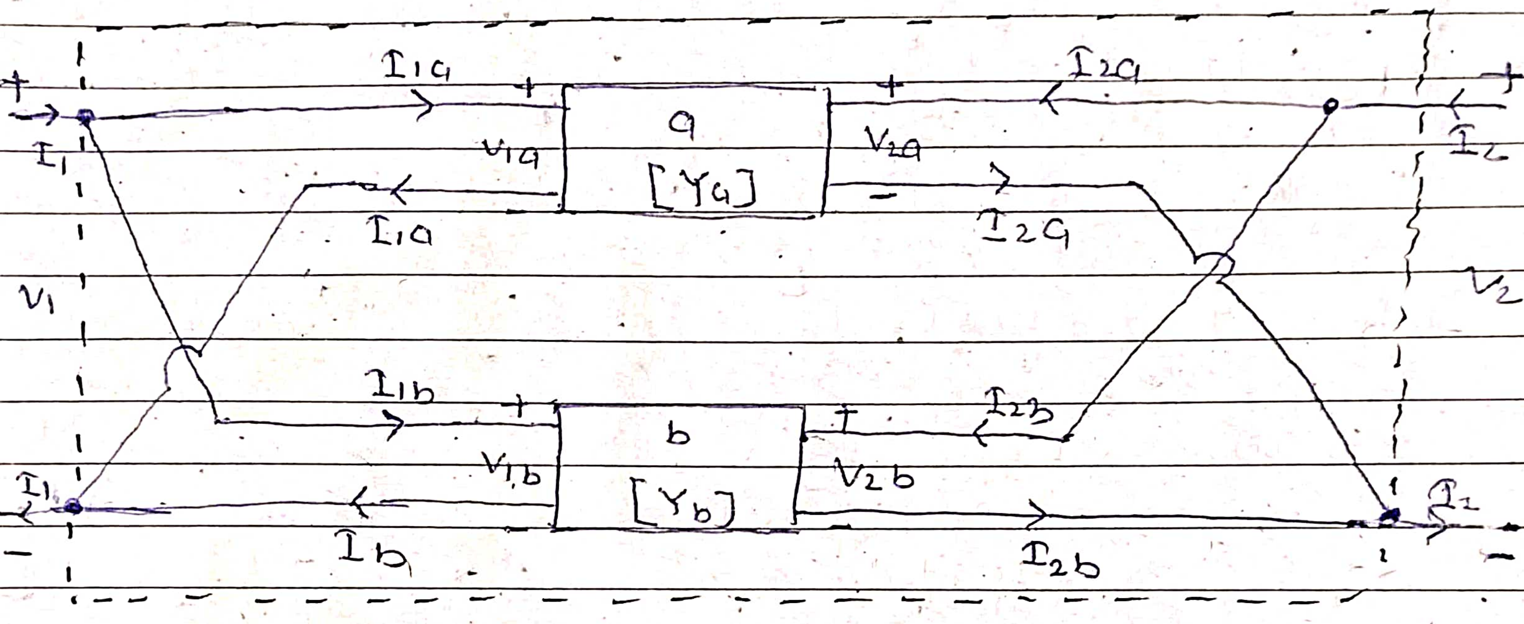
Now similarly for eq 2 & (4)

$V_{2a} + V_{2b} = z_{21a} I_{1a} + z_{21b} \cdot I_{1b} + z_{22a} I_{2a} + z_{22b} I_{2b}$

$V_2 = (z_{21a} + z_{21b}) I_1 + (z_{22a} + z_{22b}) I_2$

$$[Z] = \begin{bmatrix} (Z_{11a} + Z_{11b}) & (Z_{12a} + Z_{12b}) \\ (Z_{21a} + Z_{21b}) & (Z_{22a} + Z_{22b}) \end{bmatrix}$$

(11) parallel parallel connection -



Now on analysis

$V_1 = V_{1a} = V_{1b}$

$V_2 = V_{2a} = V_{2b}$

$I_1 = I_{1a} + I_{1b}$

$I_2 = I_{2a} + I_{2b}$

Date
Page No.

Now we can write eqn for Z-parameter

$$I_{1a} = Y_{11a} \cdot V_{1a} + Y_{12a} \cdot V_{2a} \quad \text{--- (1)}$$

$$I_{2a} = Y_{21} V_{1a} + Y_{22a} \cdot V_{2a} \quad \text{--- (2)}$$

$$I_{1b} = Y_{11b} \cdot V_{1b} + Y_{12b} V_{2b} \quad \text{--- (3)}$$

$$I_{2b} = Y_{21b} V_{1b} + Y_{22b} V_{2b} \quad \text{--- (4)}$$

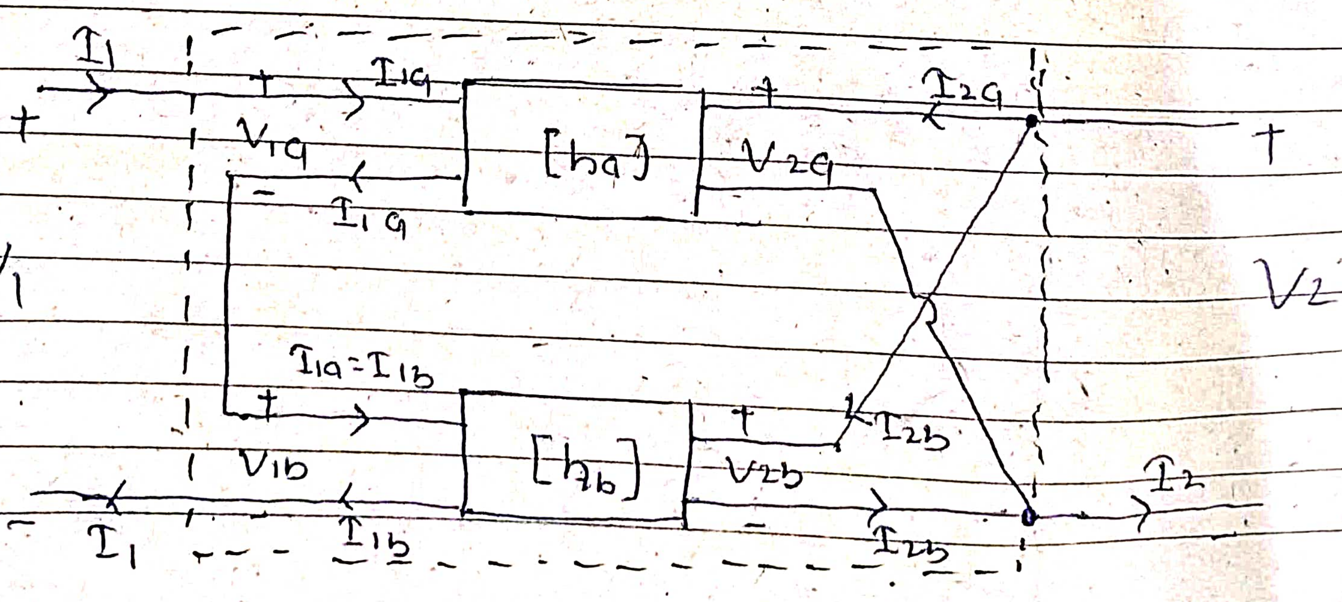
Now Add eqn (1) & (1)

$$I_1 = (I_{1a} + I_{1b}) = (Y_{11a} + Y_{11b}) V_1 + (Y_{12a} + Y_{12b}) V_2$$

$$I_2 = (I_{2a} + I_{2b}) = (Y_{21} + Y_{21b}) V_1 + (Y_{22a} + Y_{22b}) V_2$$

$$[Y] = \begin{bmatrix} (Y_{11a} + Y_{11b}) & (Y_{12a} + Y_{12b}) \\ (Y_{21a} + Y_{21b}) & (Y_{22a} + Y_{22b}) \end{bmatrix}$$

(3) Series - Parallel



Page No.
Date

On analysis,

$I_1 = I_a = I_{1b}$

$V_1 - V_{1a} - V_{1b} = 0$

$V_1 = (V_{1a} + V_{1b})$

$I_2 = I_{2a} + I_{2b}$, $V_2 = V_{2a} = V_{2b}$

Now,

$V_{1a} = h_{11a} I_{1a} + h_{12a} V_{2a}$

$V_{1b} = h_{11b} I_{1b} + h_{12b} V_{2b}$

$I_{2a} = h_{21a} I_{1a} + h_{22a} V_{2a}$

$I_{2b} = h_{21b} I_{1b} + h_{22b} \cdot V_{2b}$

Add

(I) & (II)

$V_{1a} + V_{1b} = V_1 = (h_{11a} + h_{11b}) I_1 + (h_{12a} + h_{12b}) V_2$

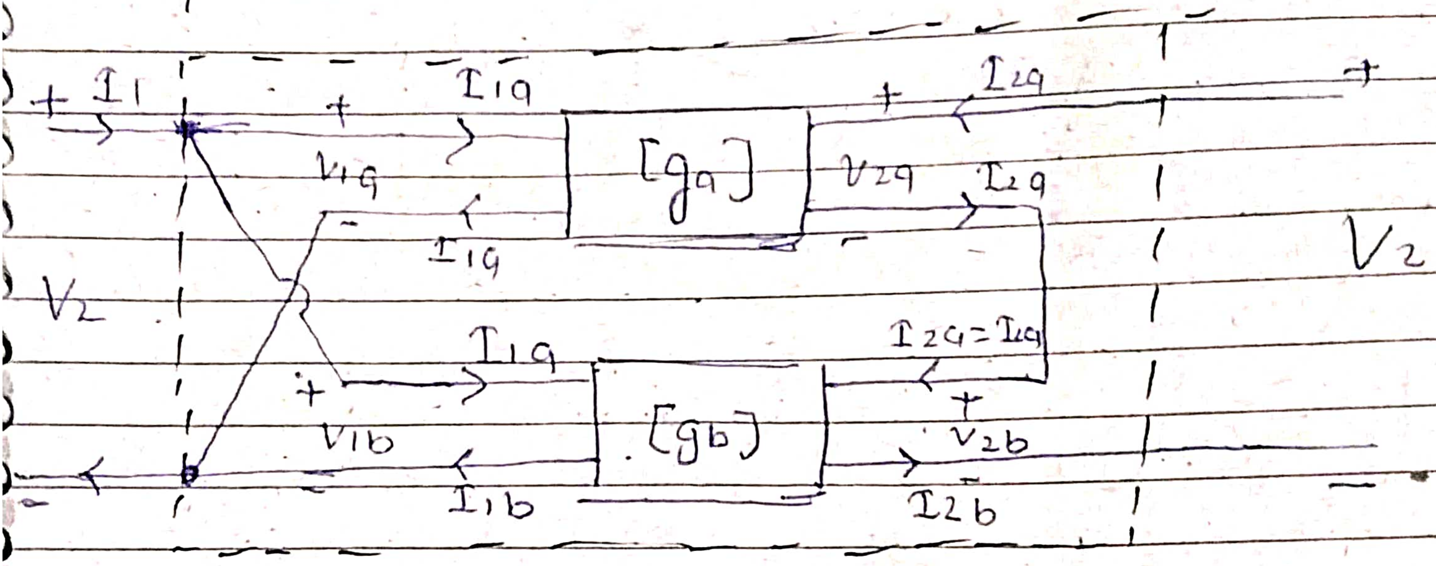
(III) & (IV)

$I_{2a} + I_{2b} = I_2 = (h_{21a} + h_{21b}) I_1 + (h_{22a} + h_{22b}) V_2$

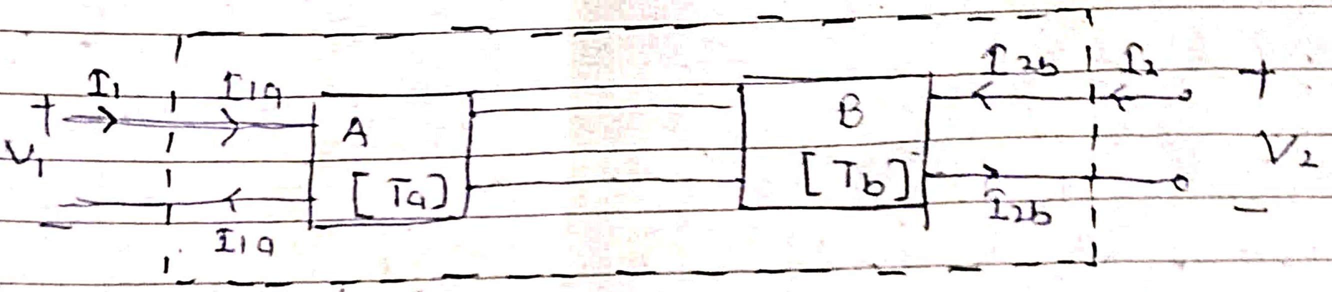
$$[h] = \begin{bmatrix} (h_{11a} + h_{11b}) & (h_{12a} + h_{12b}) \\ (h_{21a} + h_{21b}) & (h_{22a} + h_{22b}) \end{bmatrix}$$

Date ____________
Page No. ____________

→ parallel - series connection :-



⑤ Cascade



$$[T] = [T_a][T_b]$$

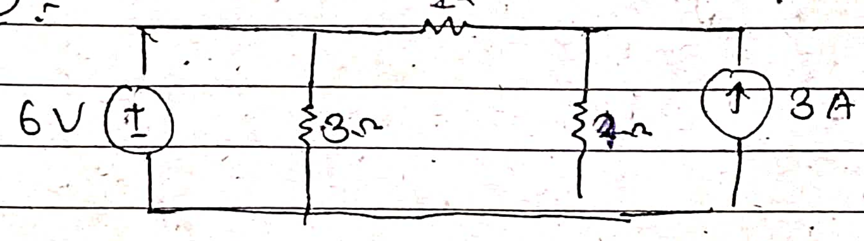
$$[T] = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix}$$

UNIT – 3

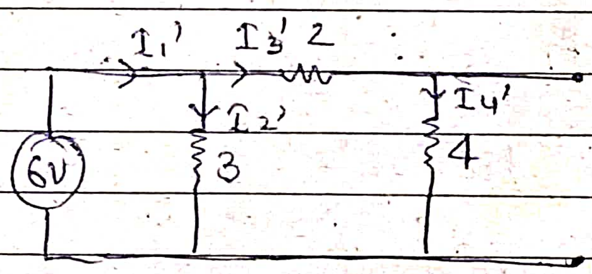
- **Superposition theorem** -

Condition to apply superpos. theorem
→ linear Ntw
→ Independent source

Q:-



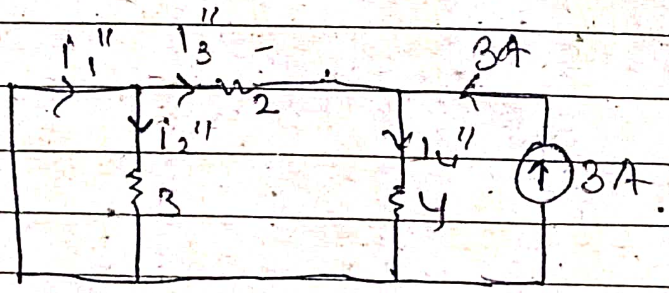
Sol



$\left[i_2' = \frac{6}{3} = 2A \right)$, $\left(i_3' = i_4' = \frac{6}{6} = 1A \right)$

$\left(i_1' = 3A \right)$

Case-2



$i_2'' = 0$, $i_4'' = 1A$

$i_1'' = i_3'' = -2A$

$i_1 = i_1' + i_1''$

$= 3 + (-2)$

$i_1 = 1A$

$i_2 = i_2' + i_2''$

$i_2 = 2 + 0 = 2A$

$i_2 = 2A$

$i_4 = i_4' + i_4''$

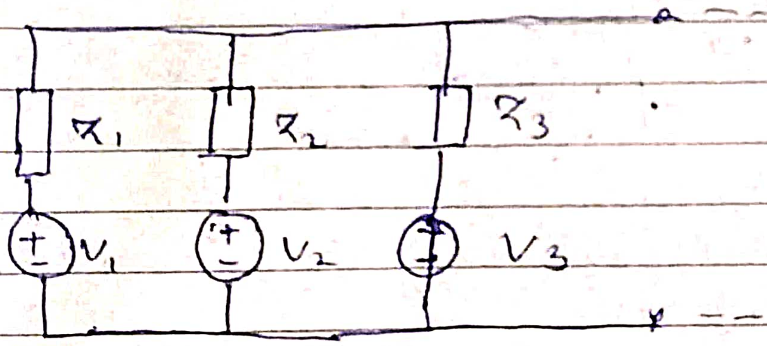
$1 + 1$

$i_4 = 2A$

$i_3 = i_3' + i_3'' = 1 + (-2)$

$i_3 = -1A$

- Millman's theorem :-



According to this theorem we can this multiple combination of z & V in ~~sig~~ single voltage & single impedence.

$Y_1 = \frac{1}{z_1}$, $Y_2 = \frac{1}{z_2}$, $Y_3 = \frac{1}{z_3}$

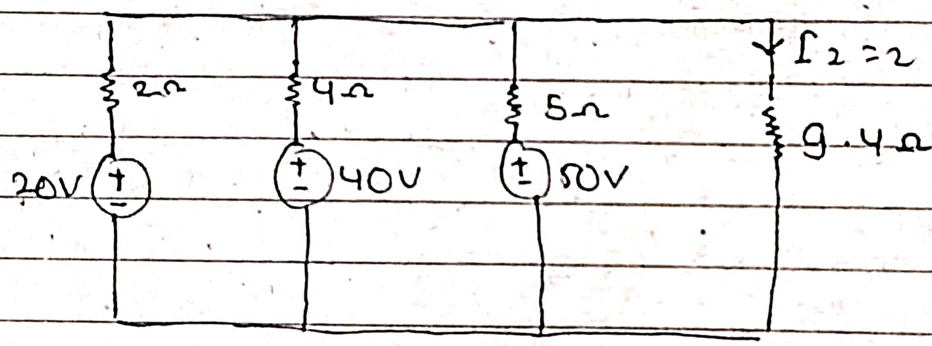
$$Y = Y_1 + Y_2 + Y_3$$

$$Y = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$$

$$\mathcal{E} = \frac{V_1 Y_1 + V_2 Y_2 + V_3 Y_3}{Y}$$

$$\mathcal{E} = \frac{V_1 Y_1 + V_2 Y_2 + V_3 Y_3}{(Y_1 + Y_2 + Y_3}$$

Q:-



Sol

$$Y_1 = \frac{1}{Z_1} = \frac{1}{2} = 0.5$$

$$Y_2 = \frac{1}{Z_2} = \frac{1}{4} = 0.25$$

$$Y_2 = \frac{1}{Z_3} = \frac{1}{5} = 0.2$$

$$Y = 0.5 + 0.25 + 0.2$$

$$Y = 0.95 \quad , \quad Z = \frac{1}{Y} = 1.0052$$

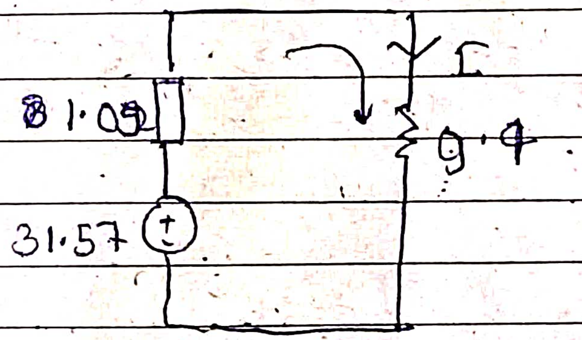
Page No.
Date

$$V = \frac{V_1 Y_1 + V_2 Y_2 + V_3 Y_3}{(Y_1 + Y_2 + Y_3)}$$

$$V = \frac{20 \times 0.5 + 40 \times 0.25 + 50 \times 0.2}{0.95}$$

$$V = \frac{30}{0.95} = 31.57$$

Equivalent CKt

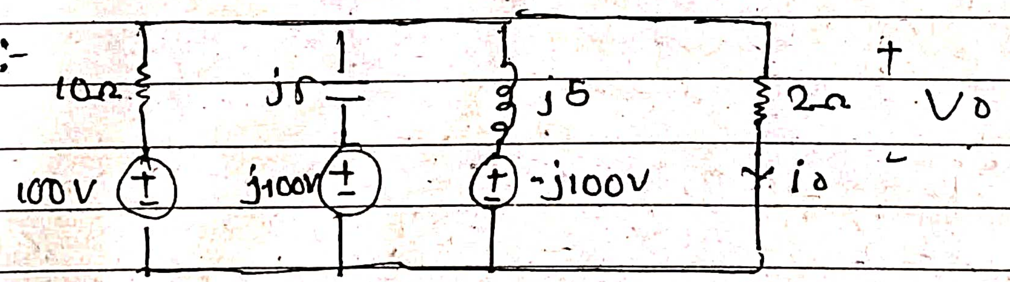


$$I_2 = \frac{31.57}{9.4 + 1.052}$$

$$\boxed{I_2 = 3.02A}$$

HA

Q :-

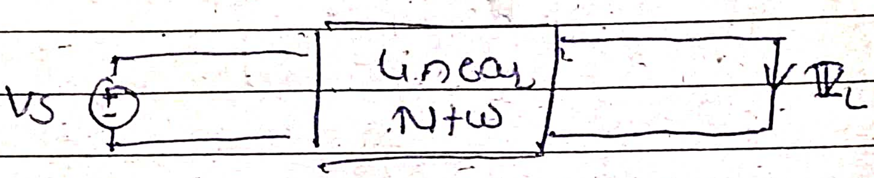


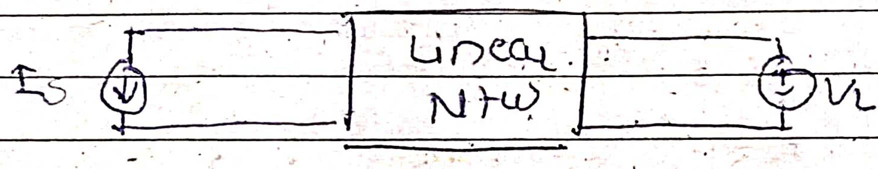
Find V_o, I_o = ?

* ~~Respons~~

* Reciprocity theorem -

→ $\dfrac{\text{Response}}{\text{Excitation}} = \text{constant}$





$$\boxed{\frac{I_L}{V_S} = \frac{I_S}{V_L}}$$

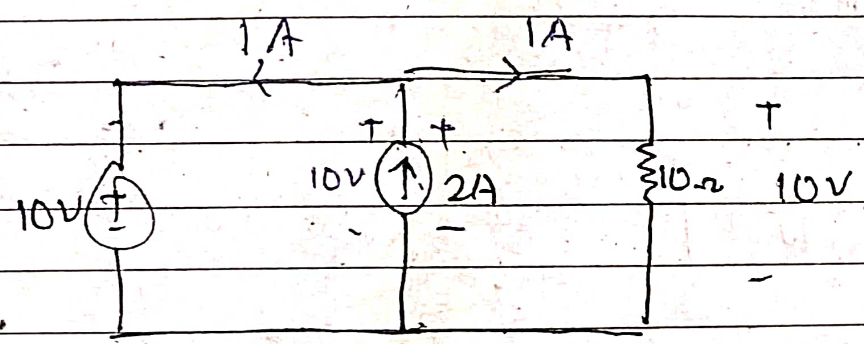
Q :-

Page No.
Date

• Tellegan's theorem :-

→ (Power absorb = Power delivered)

Note
(+) sign me current enter kar raha hai P_{absorb} & (+) sign se current nikal raha hai $P_{deliver}$



$P_{10V} = 10 \times 1 = 10W$ → $P_{absorbed}$

$P_{2A} = 10 \times 2 = 20W$ → $P_{delivered}$

$P_{10\Omega} = 10 \times 1 = 10W$ $P_{absorbed}$

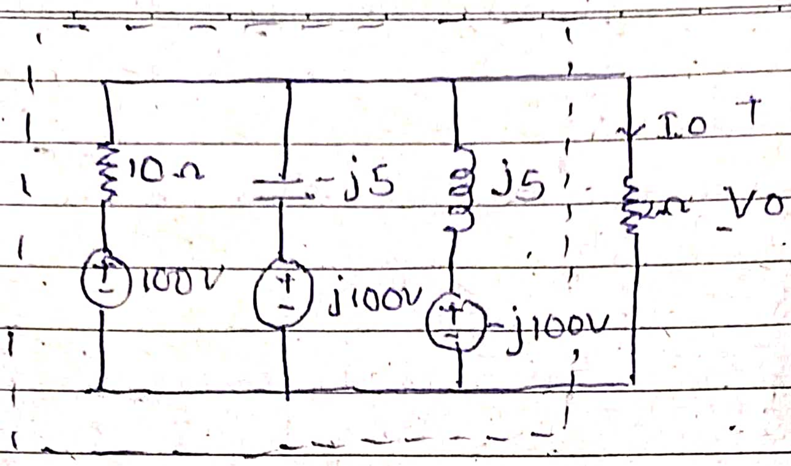
$P_{absorbed} = P_{delivered}$

$20W = 10W + 10W$

$(20W = 20W)$

Date
Page No.

Q:-



$$V = \frac{V_1Y_1 + V_2Y_2 + V_3Y_3}{(Y_1 + Y_2 + Y_3)} \qquad Y = \frac{1}{Z}, \quad Z = 1/Y$$

$$Y_1 = \frac{1}{Z_1} = \frac{1}{10} = 0.1\ \Omega$$

$$Y_2 = 1/Z_2 = \frac{1}{-j5} = j0.2$$

$$Y_3 = 1/Z_3 = \frac{1}{j5} = -j0.2$$

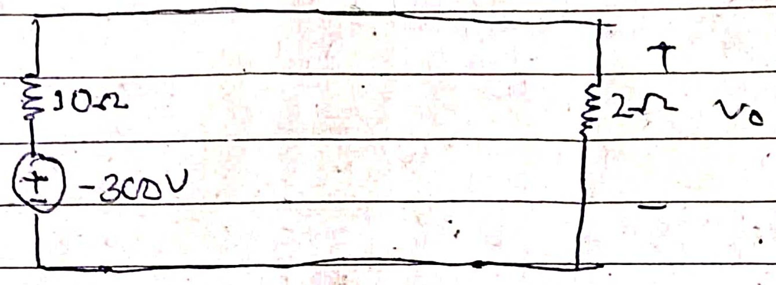
$$Y = 0.1 + \cancel{j0.2} - \cancel{j0.2} = 0.1$$

$$Z = \frac{1}{Y} = \frac{1}{0.1} = 10$$

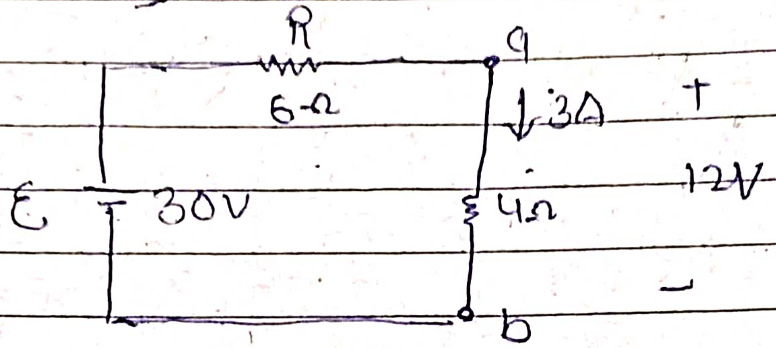
$$V = \frac{(100 \times 0.1 + j100 \times j0.2 + (-j100) \times (-j0.2))}{0.1}$$

$$V = \frac{10 - 20 - 20}{0.1} = \frac{-30}{0.1} = -300V$$

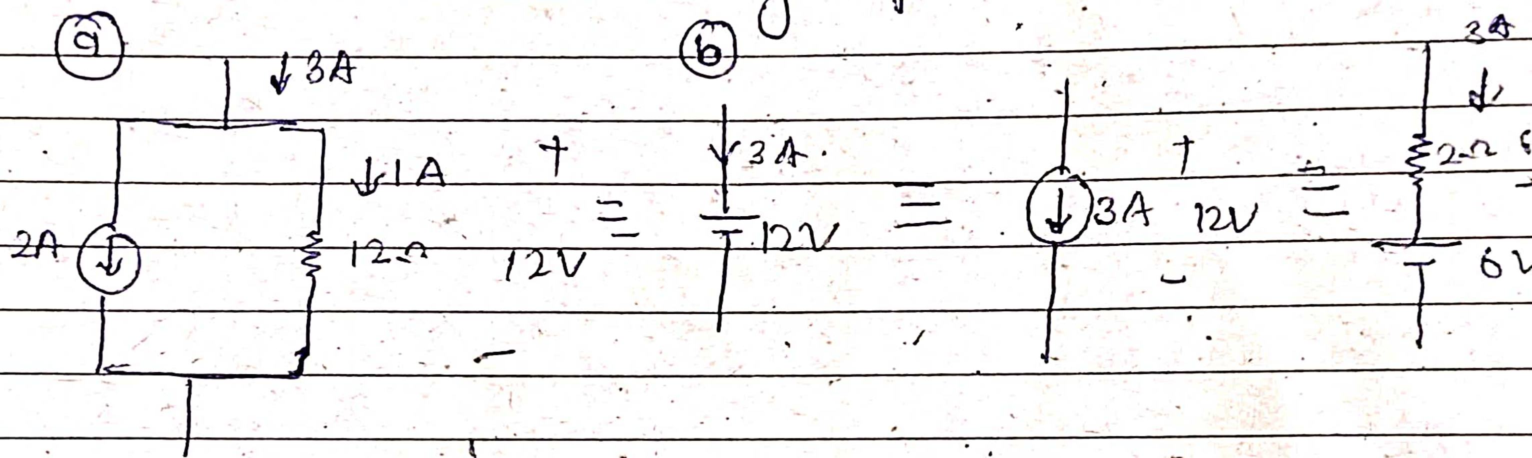
Equivalent Ckt,



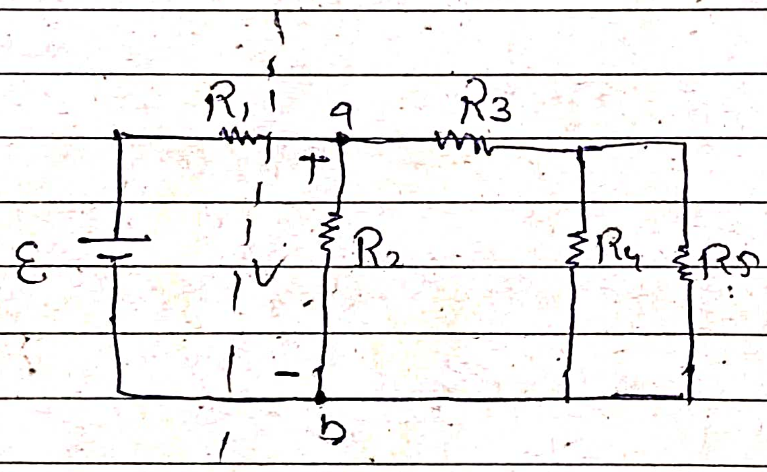
Substitution theorem



a & b section may represent as



Ex:-



Substitution theorem

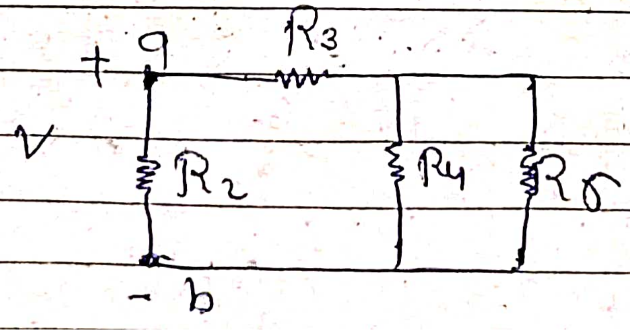
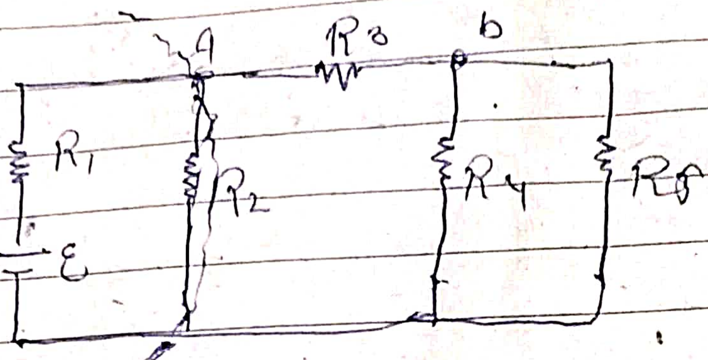
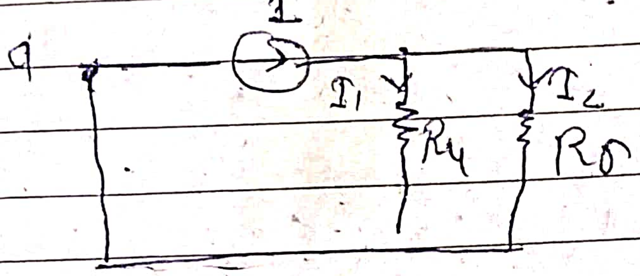


Illustration 2



⇓ subst. theorem

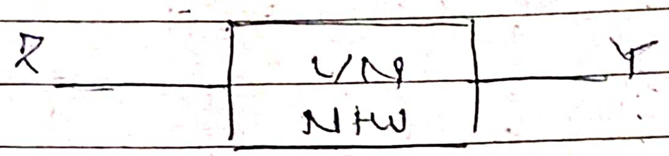


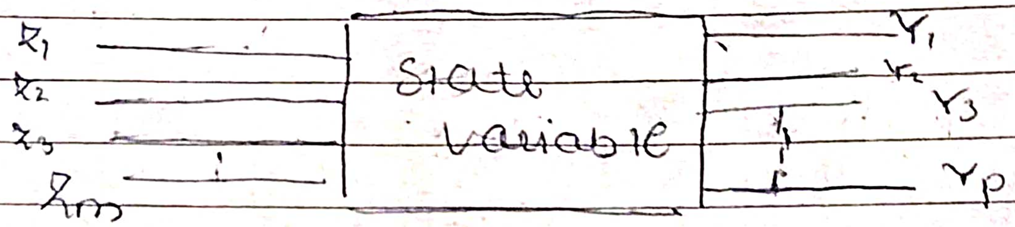
$$I_1 = \frac{I \times R_5}{R_5 + R_4}$$

$$I_2 = \frac{I \times R_4}{R_5 + R_4}$$

Date ______
Page No. ______

State variable :-





derivative

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ \vdots \\ x_n(t) \end{bmatrix}, \quad \dot{x}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \vdots \\ \dot{x}_n(t) \end{bmatrix}$$

$$\left(\dot{x}(t) = \frac{dx(t)}{dt} \right), \quad r(t) = \begin{bmatrix} r_1(t) \\ r_2(t) \\ r_3(t) \\ \vdots \\ r_m(t) \end{bmatrix}$$

$$Y(t) = \begin{bmatrix} Y_1(t) \\ Y_2(t) \\ Y_3(t) \\ \vdots \\ Y_p(t) \end{bmatrix}$$

$$\boxed{\begin{aligned} \dot{x} &= Ax + Br \\ y &= Cx + Dr \end{aligned}}$$

A = system matrix
B = input coupling matrix
C = O/P matrix
D = feed forward matrix

$[A]_{n \times n}$, $[B]_{n \times m}$
$[C]_{p \times n}$, $[D]_{p \times m}$

Page No.
Date

$\dot{x} = AX + BZ$

apply L.T

$S \cdot X(s) = AX(s) + BZ(s)$

$S(X(s)) - AX(s) = BZ(s)$

identity matrix

$X(s)\left[S \cdot I - A\right] = B\,Z(s)$

$\frac{Y(s)}{Z(s)} = X(s)$

$Y(s) = X(s)\,Z(s)$

$X(s) = (S \cdot I - A)^{-1} \cdot B \cdot Z(s)$

$$X(s) = (S \cdot I - A)^{-1} \cdot B \cdot Z(s)$$

$Y(s) = C \cdot X(s) + \cancel{D}^{\,0} \cdot Z(s)$

$D \to 0$, maximium case

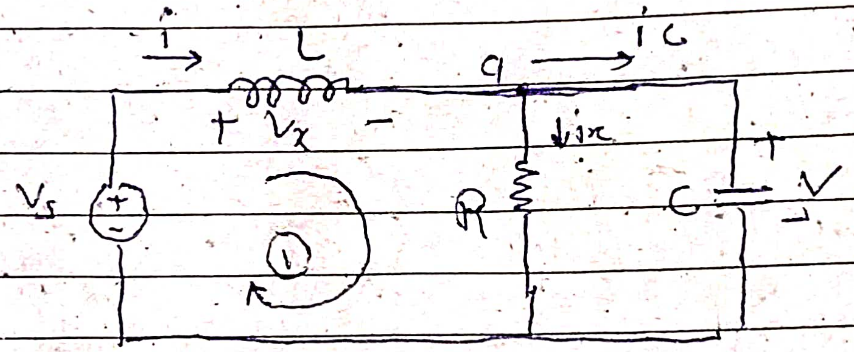
$Y(s) = C \cdot X(s)$

$Y(s) = C\,(SI - A)^{-1} \cdot B \cdot Z(s)$

Date
Page No.

Note
storing elements are capacitor & inductor

Q:



sol

$L \rightarrow i \longrightarrow V_L = L\frac{di}{dt} = L \cdot \dot{i}(t)$

$C \rightarrow V \rightarrow i = C\frac{dv}{dt} = C\dot{V}(t)$

KCL at a,

$i = i_x + i_c$

$i = \frac{V}{R} + C \cdot \frac{dV(t)}{dt}$

$i = \frac{V}{R} + C\,\dot{V}(t)$

$C \cdot \dot{V}(t) = i - V/R$

$$\Rightarrow \left(\dot{V}(t) = \frac{-V}{RC} + i/C \right) \longrightarrow 1$$

KVL at loop 1

$$V_s - V_L - V = 0$$

$$V_s - V = V_L$$

$$\because V_L = L\frac{di}{dt}$$

$$V_L = L\dot{i}_{(t)}$$

$$L\dot{i}(t) = -V + V_s$$

$$\dot{i}'(t) = -\frac{V}{L} + \frac{V_s}{L} \quad \text{—— (2)}$$

NOW,

$$V' = \frac{-1}{RC}V + \frac{1}{C} \cdot i$$

$$i' = \frac{-1}{L}V + \frac{1}{V}V_s$$

We can this in form

$$\dot{x} = AX + B?$$

$$\begin{bmatrix} v' \\ t' \end{bmatrix} = \begin{bmatrix} -1/RC & 1/C \\ -1/L & 0 \end{bmatrix} \begin{bmatrix} v \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L \end{bmatrix} V_s$$

Date

Page No.

$$i_x = \frac{V}{R} = \frac{1}{R}V + 0 \cdot i$$

$$i_x = \begin{bmatrix} 1/R & 0 \end{bmatrix} \begin{bmatrix} V \\ i \end{bmatrix}$$

Let $R = 1\Omega$, $C = 0.25F$, $L = 0.5H$

$$A = \begin{bmatrix} -4 & 4 \\ -2 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$C = [1, 0]$$

$$Y_S = C(SI - A)^{-1} \cdot B \cdot Z(s)$$

$$H(s) = \frac{Y(s)}{Z(s)}$$

$$H(s) = C(SI - A)^{-1}$$

$$s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} s & 0 \\ 0 & 3 \end{bmatrix}$$

$$SI - A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -4 & 4 \\ -2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} s & 4 \\ -2 & s+2 \end{bmatrix}$$

$$|SI - A| = s^2 + 4s + 8$$

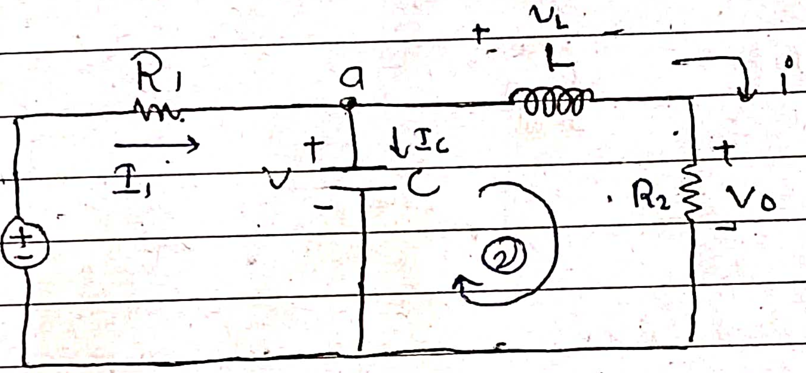
Page No.
Date

$$(SI-A)^{-1} = \frac{adj(SI-A)}{\det(SI-A)} = \frac{\begin{bmatrix} s & 4 \\ -2 & s+4 \end{bmatrix}^T}{s^2+4s+8}$$

$$\frac{\begin{bmatrix} 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 8 \\ 0 & 2s+8 \end{bmatrix}}{s^2+4s+8}$$

$$H(s) = \frac{8}{s^2+4s+8}$$

Q:-



Find H(s) ? $R_1 = 1$, $R_2 = 2$ $C = 0.5$, $L = 0.2$

Sol Apply KCL at a,

$$I_1 = I_c + i$$

$$I_1 = C \cdot \frac{dv}{dt} + i \qquad \Bigg| \qquad I_1 = \frac{V_s - v}{R_1}$$

$$\frac{V_s - v}{R_1} = C \cdot \dot{v}(t) + i$$

$$C\dot{v}(t) = \frac{-v}{R_1} - i + \frac{V_s}{R_1}$$

Date
Page No.

$$\boxed{\dot{v}(t) = -\frac{v}{R_1C} - \frac{i}{C} + \frac{V_s}{R_1C}} \quad \text{—— (1)}$$

Apply KVL at loop-2,

$$+v - V_L - V_0 = 0$$

$$V_L = v - V_0$$

$$L\frac{di}{dt} = v - V_0$$

$$\dot{i}(t) = \frac{v}{L} - \frac{V_0}{L}$$

$$V_0 = iR_2 \quad \text{—— (3)}$$

$$\boxed{\dot{i}(t) = \frac{v}{L} - \frac{iR_2}{L}} \quad \text{—— (2)}$$

We know $\dot{x} = A\cancel{X} + BZ$

$$\begin{bmatrix} \dot{v}(t) \\ \dot{i}(t) \end{bmatrix} = \begin{bmatrix} -1/R_1C & -1/C \\ 1/L & -R_2/L \end{bmatrix} \begin{bmatrix} v \\ i \end{bmatrix} + \begin{bmatrix} 1/R_1C \\ 0 \end{bmatrix} V_s$$

$\because Y = CX$

$$V_0 = \begin{bmatrix} 0 & R_2 \end{bmatrix} \begin{bmatrix} v \\ i \end{bmatrix}$$

$$A = \begin{bmatrix} -2 & -2 \\ 5 & -10 \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 2 \end{bmatrix}$$

Page No. ____
Date ____

We know,

$$H(s) = C \cdot (sI - A)^{-1} \cdot B$$

$$(sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -2 & -2 \\ 5 & -10 \end{bmatrix} \Rightarrow \begin{bmatrix} s+2 & 2 \\ -5 & s+10 \end{bmatrix}$$

$$(sI - A)^{-1} = \frac{\operatorname{adj}(sI - A)}{|sI - A|} = \frac{\begin{bmatrix} (s+10) & -2 \\ 5 & (s+2) \end{bmatrix}}{s^2 + 12s + 30}$$

$$H(s) = \frac{\begin{bmatrix} 0 & 2 \end{bmatrix} \begin{bmatrix} (s+10) & -2 \\ 5 & (s+2) \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix}}{s^2 + 12s + 30}$$

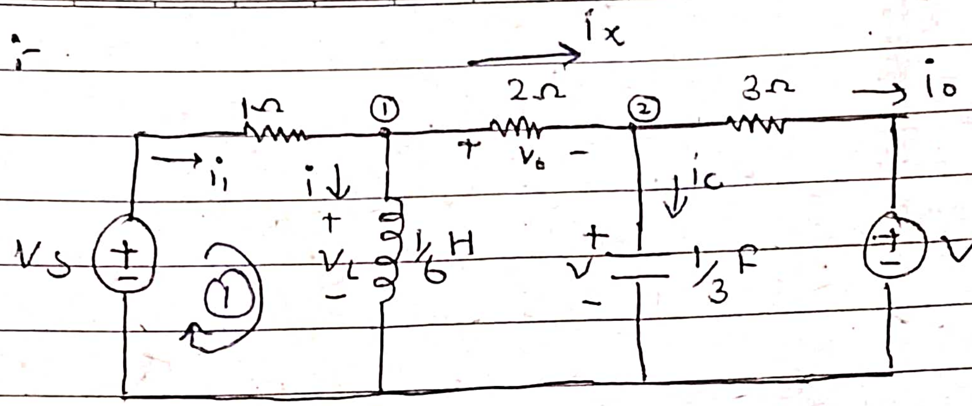
$$H(s) = \frac{\begin{bmatrix} 0 \cdot 2 \end{bmatrix} \begin{bmatrix} 2(s+10) \\ 10 \end{bmatrix}}{s^2 + 12s + 30}$$

$$H(s) = \frac{20}{(s^2 + 12s + 30)}$$

Date ____________

Page No. ____________

Q :-



Apply KVL at ~~1~~ loop 1

$$V_s - i_1 \times 1 - V_L = 0$$

$$V_s - i_1 = L\frac{di}{dt}$$

$$\left(V_s - i_1 = L\dot{i}(t) \right) \quad \text{—— } (1)$$

KVL at loop 2

$$V_s - i_1 - V_s - V = 0$$

$$V_s - V - V_o = i_1 \quad \text{—— } (2)$$

We know,

$$i_x = i - i_1 \quad , \quad V_o = (i_1 - i)2 \quad \text{put in (ii)}$$

$$i_1 = V_s - V - V_o$$

$$i_1 = V_s - V - (i_1 - i)2$$

$$3i_1 = V_s - V + 2i$$

$$\left[i_1 = \frac{V_s - V + 2i}{3} \right] \quad \text{—— put in (1)}$$

Page No. ______
Date ______

$$L\,\dot{i}(t) = V_s - \left[\frac{(V_s - V + 2i)}{3}\right]$$

$$= V_s - \frac{V_s}{3} + \frac{V}{3} - \frac{2}{3}i$$

$$\dot{i}(t) = \frac{V_s}{L} - \frac{V_s}{3} + \frac{V}{3L} - \frac{2i}{3L}$$

$$\dot{i}(t) = \frac{3V_s - LV_s}{3L} + \frac{V}{3L} - \frac{2i}{3L}$$

as $L = \frac{1}{6}H$ put in above eqn

$$\left(\dot{i}(t) = 2V - 4i + 4V_s\right) \quad \text{——— } A$$

Now apply KVL at (2)

$$(i_1 - i) = I_c + i_0 \qquad \Bigg| \; i_0 = \frac{V - V_i}{3}$$

$$i_1 - i = C\frac{dv}{dt} + i_0$$

$$\left(\frac{V_s - V + 2i}{3} - i\right) = C\,\dot{V}(t) + \frac{V - V_i}{3}$$

$$\frac{V_s - V + 2i - 3i}{3} - \frac{V}{3} + \frac{V_i}{3} = C\,\dot{V}(t)$$

$$\frac{V_s}{3} - \frac{V}{3} + \frac{2i}{3} - \frac{V}{3} + \frac{V_i}{3} = C\,\dot{V}(t)$$

$$C\,\dot{V}(t) = \frac{V_s}{3} - \frac{2V}{3} + \frac{V_i}{3} - \frac{i}{3}$$

Date ______
Page No. ______

as $C = \frac{1}{3}F$.

$$\frac{1}{\cancel{3}}\dot{V}(t) = \frac{V_S}{\cancel{3}} - \frac{2V}{\cancel{3}} + \frac{V_i}{\cancel{3}} - \frac{i}{\cancel{3}}$$

$$\left(\dot{V}(t) = V_S - 2V + V_i - i\right) \text{ —— } (B).$$

Now for V_0,

$$V_0 = 2i_1 - 2i$$

$$= 2\left[\frac{V_S - V + 2i}{3}\right] - 2i$$

$$= \frac{2}{3}V_S - \frac{2}{3}V + \frac{4}{3}i - 2i$$

$$V_0 = \frac{2}{3}V_S - \frac{2}{3}V - \frac{2}{3}i \text{ —— } (C)$$

$$\begin{bmatrix} \dot{V}(t) \\ \dot{i}(t) \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ 2 & -4 \end{bmatrix} \begin{bmatrix} V \\ i \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} V_S \\ V_i \end{bmatrix}$$

$$\begin{bmatrix} V_0 \\ i_0 \end{bmatrix} = \begin{bmatrix} -2/3 & -2/3 \\ 1/3 & 0 \end{bmatrix} \begin{bmatrix} V \\ i \end{bmatrix} + \begin{bmatrix} 2/3 & 0 \\ 0 & -1/3 \end{bmatrix} \begin{bmatrix} V_S \\ V_i \end{bmatrix}$$

Q :- $\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = 5z(t)$

Sol

Let $x_1 = y(t)$

$x_2 = \dot{x}_1 = \dot{y}(t)$

$\dot{x}_2 = \ddot{x}_1 = \ddot{y}(t)$

Now from given eqn,

$$\dot{x}_2 + 3x_2 + 2x_1 = 5z(t)$$

$$\dot{x}_2 = -3x_2 - 2x_1 + 5z(t)$$

$$\dot{x}_1 = x_2 + 0x_1 + 0z(t)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 5 \end{bmatrix} z(t)$$

$$Y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$H(s) = C\left[(sI-A)^{-1}\right] \cdot B$$

$$(sI-A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}$$

$$(sI-A)^{-1} = \frac{\begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix}}{s^2+3s+2}$$

Date
Page No.

$$H(s) = \frac{\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix} \begin{bmatrix} 0 \\ 5 \end{bmatrix}}{s^2+3s+2}$$

$$\frac{\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 5s \end{bmatrix}}{s^2+3s+2}$$

$$H(s) = \frac{5}{s^2+3s+2}$$

Q:- $\frac{d^3 y(t)}{dt^3} + 18\frac{d^2 y}{dt^2} + 20\frac{dy}{dt} + 5y = z(t)$ Find, [A], [B] & [C]

Sol

$$x_1 = y_1(t)$$

$$x_2 = \dot{x}_1 = \dot{y}(t)$$

$$x_3 = \dot{x}_2 = \ddot{x}_1 = \ddot{y}(t)$$

$$\dot{x}_3 = \ddot{x}_2 = \dddot{x}_1 = \dddot{y}(t)$$

$$\dot{x}_3 + 18x_3 + 20x_2 + 5x_1 = z(t)$$

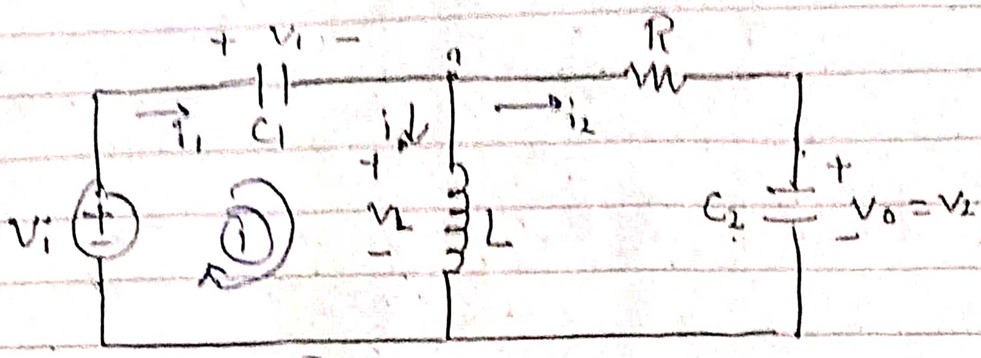
$$\dot{x}_3 = z(t) - 18x_3 - 20x_2 - 5x_1$$

$$\begin{bmatrix} \dot{x}_3 \\ \dot{x}_2 \\ \dot{x}_1 \end{bmatrix} = \underbrace{\begin{bmatrix} -18 & -20 & -5 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}}_{[A]} \begin{bmatrix} x_3 \\ x_2 \\ x_1 \end{bmatrix} + \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}_{[B]} z(t)$$

$$y = \underbrace{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}}_{[C]} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Page No.
Date

Q:- Find $H(s)$



sol

KVL in loop 1

$$V_i - V_1 - V_L = 0$$

$$V_i - V_1 = V_L = L\frac{di}{dt}$$

$$V_i - V_1 = L\,di/dt$$

$$i(t) = -\frac{V_1}{L} + \frac{V_i}{L} \quad \text{— (A)}$$

at node a

$$i_1 = i + i_2$$

$$i_1 = C\frac{dV_1}{dt} \quad \text{— (1)}$$

Apply KVL in outer loop

$$V_i - V_1 - I_2 R - V_2 = 0$$

$$V_i - V_1 - V_2 = i_2 R$$

$$i_2 = \frac{V_i}{R} - \frac{V_1}{R} - \frac{V_2}{R}$$

also,

$$i_2 = C_2 \frac{dV_2}{dt}$$

$$\frac{i_2}{C_2} = \dot{V}_2(t)$$

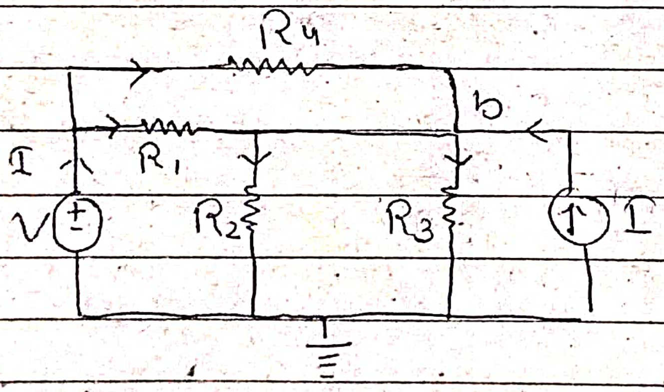
$$\dot{V}_2(t) = \frac{V_i}{RC_2} - \frac{V_1}{RC_2} - \frac{V_2}{RC_2} \qquad \text{—— (B)}$$

from (1)

$$\dot{V}_1(t) = \frac{i_1}{C_1} + \frac{V_i}{RC_1} - \frac{V_1}{RC_1} - \frac{V_2}{RC_1} \qquad \text{—— (C)}$$

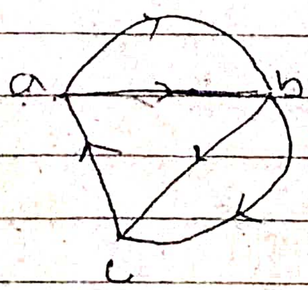
$$\begin{bmatrix} \dot{V}_1 \\ \dot{V}_2 \\ \dot{i} \end{bmatrix} = \begin{bmatrix} -1/RC_1 & -1/RC_1 & 1/C_1 \\ -1/RC_2 & -1/R_2 & 0 \\ -1/L & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} + \begin{bmatrix} 1/RC_1 \\ 1/RC_2 \\ 1/L \end{bmatrix} V_i$$

- <u>Graph theory</u> :



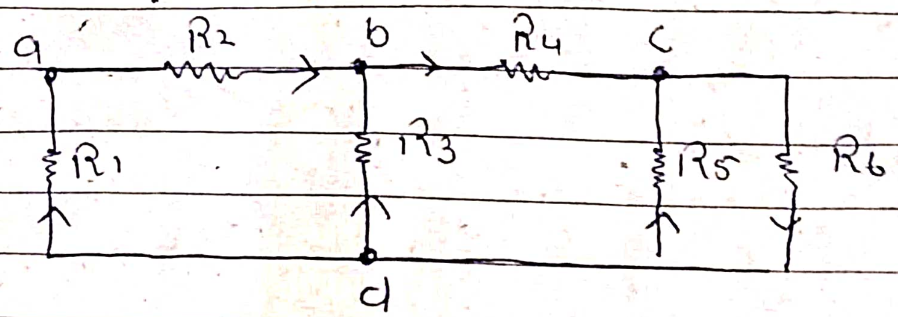
$I \longrightarrow O.C$

$V \longrightarrow S.C$

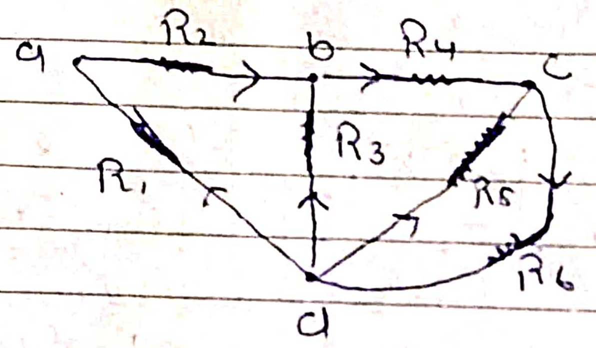


directed graph

• Incidence Matrix



⇒ directed graph ↴



4-node
6-~~Br~~ branches

↱ incidence matrix

branch ⟶
node ↓

$$[A] = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline a & -1 & 1 & 0 & 0 & 0 & 0 \\ b & 0 & -1 & -1 & 1 & 0 & 0 \\ c & 0 & 0 & 0 & -1 & -1 & 1 \\ \hdashline d & 1 & 0 & 1 & 0 & 1 & -1 \end{array}$$

Note:-
Jis node se current nikal raha hai uss ko (+1) lenge & Jis me enter kar raha hai uss ko (-1)

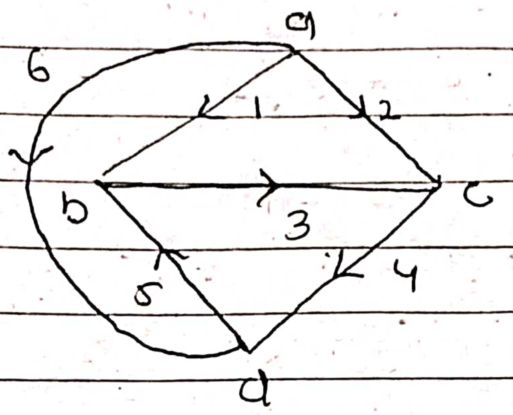
→ If we eleminate the refrence node element then this matrix ~~bec~~ becomes reduce incidence matrix

$$\begin{bmatrix} -1 & +1 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 \end{bmatrix} \rightarrow \text{Reduce incidence matrix}$$

Date

Page No.

Q:- form its incidence matrix



$$[A] = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline a & 1 & 1 & 0 & 0 & 0 & 1 \\ b & -1 & 0 & 1 & 0 & -1 & 0 \\ c & 0 & -1 & -1 & 1 & 0 & 0 \\ d & 0 & 0 & 0 & -1 & 1 & -1 \end{array}$$

→ incidence matrix

Now apply KCL at node a

$$i_1 + i_2 + i_6 = 0$$

KCL at node b

$$-i_1 - i_5 + i_3 = 0$$

KCL at node c

$$-i_2 + i_4 - i_3 = 0$$

KCL at node d

$$-i_6 - i_4 + i_5 = 0$$

$[A][i_b] = 0$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 & -1 & 0 \\ 0 & -1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = 0$$

~~The~~ After solving this we get all KCL eqn.

KVL in all branches :-

$b_1 \rightarrow$ branch 1

$V_{b1} = V_a - V_b$
$V_{b2} = V_a - V_c$
$V_{b3} = V_b - V_c$
$V_{b4} = V_c - V_d$
$V_{b5} = V_d - V_b$
$V_{b6} = V_a - V_d$

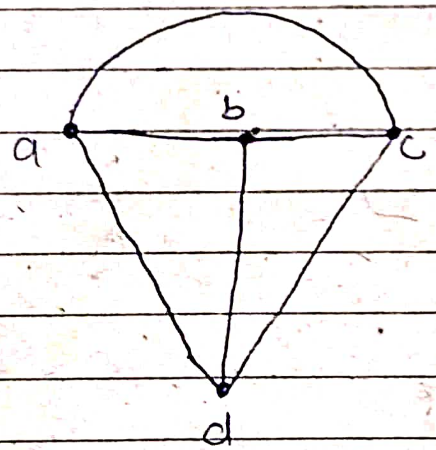
$$[A] = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 & -1 & 0 \\ 0 & -1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & -1 \end{bmatrix}$$

$$[A]^T [V_{node}] = [V_{branch}]$$

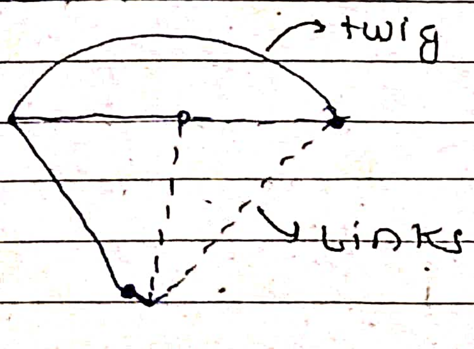
$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \end{bmatrix} = \begin{bmatrix} V_{b1} \\ V_{b2} \\ V_{b3} \\ V_{b4} \\ V_{b5} \\ V_{b6} \end{bmatrix}$$

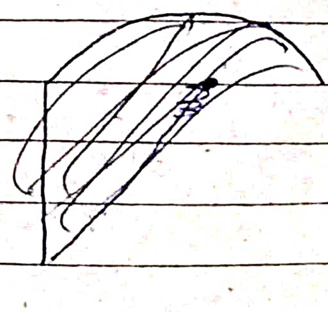
After solving this we get all KVL eqn's.

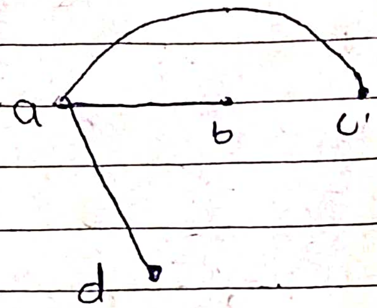
* Tree and Co-Tree :-



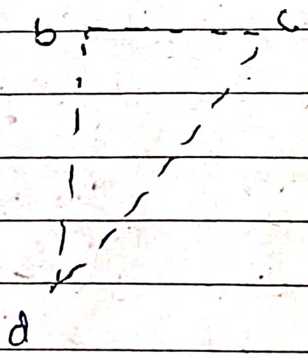
→ While making tree we have to remember that loop should not be made.





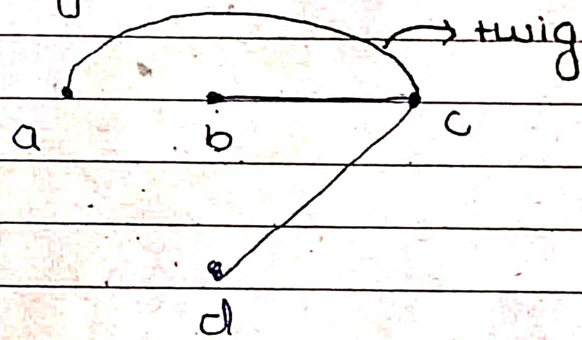


=> Tree

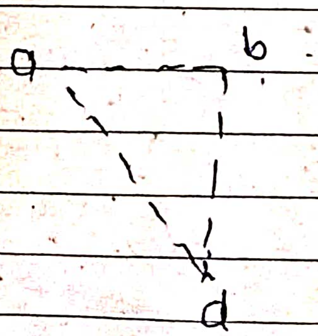


=> co tree

Another ~~possible~~ possible combination of tree & co-tree are



=> tree



=> co-tree

no. of possible tree $= \det\{[A] * [A]^T\}$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 & -1 & 0 \\ 0 & -1 & -1 & 1 & 0 & 0 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{-1} & \cancel{1} & \cancel{-1} \end{bmatrix}_{4\times 6} \begin{bmatrix} 1 & -1 & 0 & \cancel{0} \\ 1 & 0 & -1 & \cancel{0} \\ 0 & 1 & -1 & \cancel{0} \\ 0 & 0 & 1 & \cancel{-1} \\ 0 & -1 & 0 & \cancel{1} \\ 1 & 0 & 0 & \cancel{-1} \end{bmatrix}_{6\times 4}$$

$$\Rightarrow \begin{bmatrix} 3 & -1 & -1 & \cancel{-1} \\ -1 & 3 & -1 & \cancel{-1} \\ -1 & -1 & 3 & \cancel{-1} \\ \cancel{-1} & \cancel{-1} & \cancel{-1} & \cancel{3} \end{bmatrix}$$

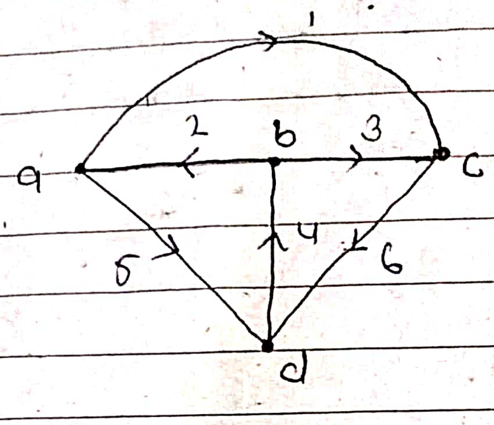
After considering reduce incidence matrix we get 3x3 matrix

$$\begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix}$$

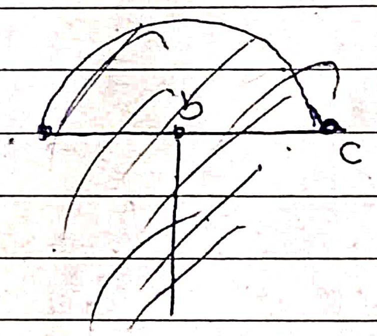
$\Rightarrow 3(9-1)+1(-3-1)-1(1+3)$

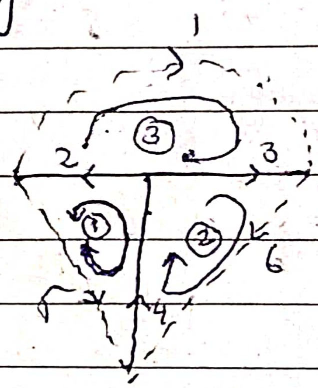
$\Rightarrow 16 \longrightarrow$ ~~tree~~ possible tree will form

• Tie-set Matrix



Law :-
→ We have to consider only those loop that has 1 link & 2 twig





branch →
loop ↓

loop	1	2	3	4	5	6
1	0	1	0	1	1	0
2	0	0	1	1	0	1
3	1	1	-1	0	0	0

→ Tie set matrix

→ Jis loop ko dekh rahe hai then us loop ke branch ko consider karege baaki sab zero

→ If current ka dirn same as the loop dirn the take 1 otherwise 0

Date
Page No.

* KVL in tieset matrix

in loop 1

$$V_4 + V_2 + V_5 = 0$$

in loop 2

$$V_3 + V_6 + V_4 = 0$$

in loop 3

$$V_1 - V_3 + V_2 = 0$$

then $[A_T][V_b] = 0$

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & -1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = 0$$

after solving this we get all KVL eqns

* KCL in Tie set

i_b, i_L

$i_1 = i_{L2}$, $i_2 = i_{L1} + i_{L3}$
$i_3 = i_{L2} - i_{L3}$, $i_4 = i_{L4} + i_{L2}$
$i_5 = i_{L1}$, $i_{L6} = i_{L2}$

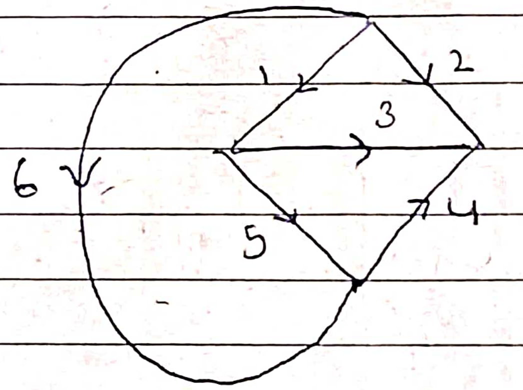
then,

$$[i_b] = [A_T]^T [i_L]$$

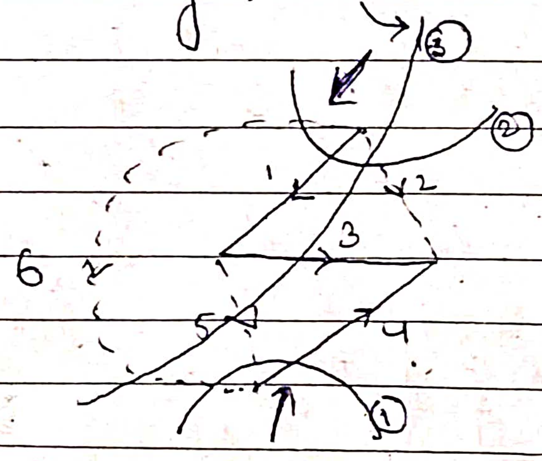
$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_{L1} \\ i_{L2} \\ i_{L3} \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix}$$

After solving this we get all KCL eqns

- Cut set Matrix :-



→ cut those branch that include only one twig and atleast 2 link



→ Cut set Jitni branches ko cut kar rahe hai sirf unhi branches pe value assign hogi baaki sab zero

→ cut dirn ki dirn twig ke dirn ko dekh ke assign karenge

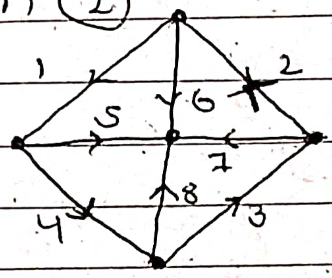
→ cut set ke dirn me current hoga to (+1) assign karenge otherwise (-1)

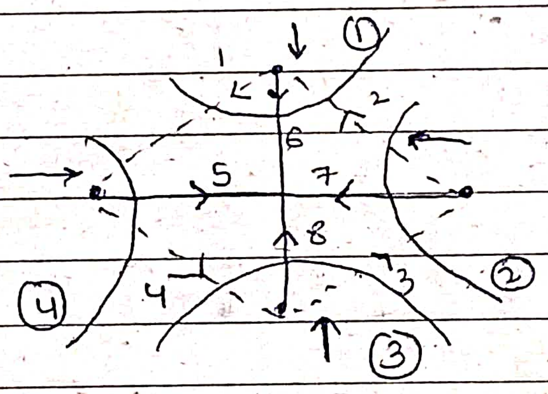
branch →
cutset ↓

	1	2	3	4	5	6
1	0	0	0	1	-1	-1
2	1	1	0	0	0	1
3	0	1	1	0	1	1

← cut set matrix

Illustration 2



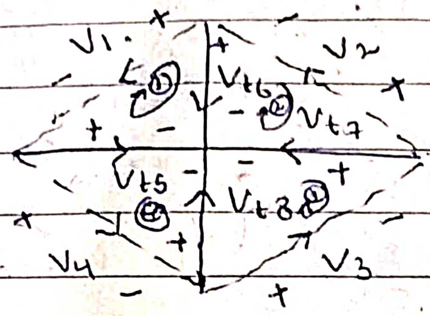


~~1 2 3 4 5 6~~
~~1~~
~~2~~
~~3~~

	1	2	3	4	5	6	7	8
1	1	-1	0	0	0	1	0	0
2	0	1	-1	0	0	0	1	0
3	0	0	1	-1	0	0	0	1
4	-1	0	0	1	1	0	0	0

Page No.
Date

KVL in cut set matrix



In loop ①

$$-V_1 - V_{t5} + V_{t6} = 0$$

$$V_1 = -V_{t5} + V_{t6} \quad \text{—— ①}$$

In loop ②

$$V_2 = -V_{t6} + V_{t7} \quad \text{—— ②}$$

In loop 3

$$V_3 = -V_{t7} + V_{t8} \quad \text{—— ③}$$

In loop 4

$$V_4 = -V_{t8} + V_{t5} \quad \text{—— ④}$$

In general

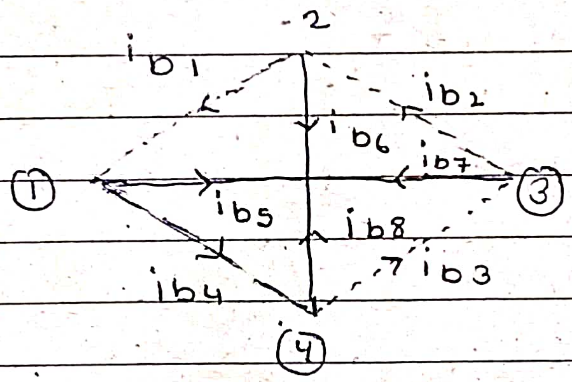
$$V_b = [Q_t]^T [V_t]$$

t: twig links
b: branch

$$\begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_{t5} \\ V_{t6} \\ V_{t7} \\ V_{t8} \end{bmatrix} = V_b$$

After solving this we get all KVL eqns

KCL in cut set matrix :-



at node 1

$i_{b1} = i_{b4} + i_{b5}$

at node 2

$i_{b2} = i_{b1} + i_{b6}$

at node 3

$i_{b3} = i_{b7} + i_{b8}$

at node 4

$i_{b4} = i_{b8} + i_{b3}$

In General

$$[Q][i_b] = 0$$

$$\begin{bmatrix} -1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_{b1} \\ i_{b2} \\ i_{b3} \\ i_{b4} \\ i_{b5} \\ i_{b6} \\ i_{b7} \\ i_{b8} \end{bmatrix} = 0$$

Date
Page No.

* Network synthesis :-
→ There is no unique soln
→ For passive elements only

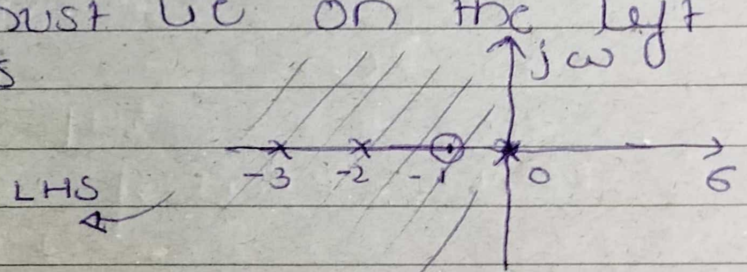
• Hurwitz polynomial :-

$$H(s) = \frac{N(s)}{D(s)} = \frac{s+1}{s(s+2)(s+3)}$$

when H(s) become zero then that value of s gives zeros here only one zero exist ($s=-1$)

when H(s) become infinite then that value of s gives poles here 3 poles exist. $s=0$, $s=-2$, $s=-3$

→ For a system to become stable then all system must lie on the left side of the y axis



→ For a complicated expression of H(s) Hurwitz gives the idea of that system is stable or not.

Q:- check whether polynomial is hurwitz?

(a) $P(s) = s^3 + 2s^2 + 4s + 2$

Sol even part of P(s) $m(s) = 2s^2 + 2$

odd " " " $n(s) = s^3 + 4s$

$$Q(s) = \frac{m(s)}{n(s)} \text{ or } \frac{n(s)}{m(s)}$$

→ we take that part of P(s) as numerator which have highest degree

$$Q(s) = \frac{s^3 + 4s}{2s^2 + 2}$$

By continued fraction expansion

$$2s^2+2 \,\big)\, \cancel{s^3} + 4s \,\big(\, \frac{s}{2}$$
$$\cancel{s^3} + s$$
$$- \quad -$$
$$3s \,\big)\, 2\cancel{s^2} + 2 \,\big(\, \frac{2s}{3}$$
$$\cancel{2s^2}$$
$$- \quad -$$
$$2 \,\big)\, 3s \,\big(\, \frac{3}{2}s$$
$$3s$$
$$0$$

all quotient terms are +ve so P(s) is hurwitz

(b) $P(s) = s^4 + s^3 + 4s^2 + 2s + 3$

Sol $m(s) = s^4 + 4s^2 + 3$

$n(s) = s^3 + 2s$

Date ____
Page No. ____

$$Q(s) = \frac{m(s)}{n(s)} = \frac{s^4+4s^2+3}{s^3+2s}$$

$s^3+2s \,\big)\, s^4+4s^2+3 \,\big(\, s$

$\quad s^4 + 2s^2$

$\quad - \quad -$

$\quad 2s^2+3 \,\big)\, s^3+2s \,\big(\, s/2$

$\qquad s^3 + \frac{3s}{2}$

$\qquad - \quad -$

$\qquad \frac{1}{2}s \,\big)\, 2s^2+3 \,\big(\, 4s$

$\qquad\quad 2s^2$

$\qquad\quad 3 \,\big)\, \frac{1}{2}s \,\big(\, \frac{1}{6}s$

$\qquad\qquad \frac{1}{2}s$

$\qquad\qquad 0$

all quotiets are +ve then
it is hurwitz polynomial

(c) $P(s) = s^5 + 2s^3 + 3$

sol As all even part are missing thus
it satisfy hurwitz polynomial property

$$P(s) = s^5 + 2s^3 + 3s = N(s)$$

$$\frac{dP(s)}{d(s)} = 5s^4 + 6s^2 + 3 = M(s)$$

$$Q(s) = \frac{N(s)}{M(s)} = \frac{s^5+2s^3+3s}{5s^4+6s^2+3}$$

Page No.
Date

- Positive Real Function (PRF) :-

$$F(s) = Z(s) = \frac{P(s)}{Q(s)} = \frac{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}{b_m s^m + b_{m-1} s^{m-1} + \cdots b_1 s + b_0}$$

$$\frac{P(s)}{Q(s)}$$

m_1, n_1 ← → m_2, n_2

if, $\left(m_1 m_2 - n_1 n_2 \Big|_{s = j\omega} \geq 0 \right)$

if this condition satify then it is PRF.

Q:- Check whether $F(s) = \frac{s+2}{s+4}$ is +ve real function?

Sol Given $F(s) = \frac{s+2}{s+4}$

$$\therefore m_1 m_2 - n_1 n_2 \Big|_{s=j\omega} \geq 0$$

$m_1 = 2$, $m_2 = 4$
$n_1 = s$, $n_2 = s$

$$8 - s^2 \Big|_{s = j\omega} \geq 0$$

$$= 8 - (j\omega)^2 \Rightarrow (8 + \omega^2 > 0)$$

thus it is PRF

Date
Page No.

(b) $F(s) = \dfrac{s^2+7s+12}{s^3+9s^2+20s+12}$

Sol

$m_1 = s^2+12$, $m_2 = 9s^2+12$

$n_1 = 7s$, $n_2 = s^3+20s$

$\therefore m_1 m_2 - n_1 n_2 \Big|_{s=j\omega} \geq 0$

$(s^2+12)(9s^2+12) - 7s(s^3+20s)$

$= (9s^4+12s^2+108s^2+144) - 7s^4 - 140s^2$

$2s^4 + 120s^2 - 140s + 144$

$2s^4 - 20s^2 + 144$, put $s=j\omega$

$2(j\omega)^4 - 20(j\omega)^2 + 144$

$\left(2\omega^4 + 20\omega^2 + 144 \geq 0\right)$

as it is greater than zero thus it is PRF

(c) $F(s) = \dfrac{s^2+9}{s^3+s}$

$s^2+9=0$

$s^2=-9$

$s = \pm j3$

$F(s) = \dfrac{(s+j3)(s-j3)}{s(s^2+1)}$

$= \dfrac{(s+j3)(s-j3)}{s(s+j)(s-j)}$ Find residue

thus By Partial fraction

$\dfrac{(s+j3)(s-j3)}{s(s+j)(s-j)} = \dfrac{A}{s} + \dfrac{B}{s+j} + \dfrac{B^*}{s-j}$

$$A\Big|_{s=0} = \frac{(s+j3)(s-j3)}{(s+j)(s-j)} = \frac{3j \times -3j}{j \times -j} = 9$$

$$B\Big|_{s=-j} = \frac{(s+j3)(s-j3)}{s(s-j)} = \frac{(-j+j3)(-j-j3)}{-j} = -4$$

$$B^*\Big|_{s=j} = \frac{(s+j3)(s-j3)}{s \times (s+j)} = \frac{(j+j3)(j-j3)}{j(j+j)} = -4$$

The residue B & B^* are real, but not positive so the F(s) is not PRF

- Network synthesis

Q:- synthesize the impedance function

(a) $Z(s) = \frac{2s^3+7s}{s^3+3}$

Sol

$$s^2+3 \overline{\big) 2s^3+7s \big(} 2s$$
$$\underline{2s^3+6s}$$
$$s$$

$$\Rightarrow 2s + \frac{s}{s^3+3}$$

$$Z(s) = Z_1(s) + Z_2(s)$$

$$Z_1(s) = 2s \quad , \quad Z_2(s) = \frac{s}{s^3+3} = Y_2(s) = \frac{1}{Z_2(s)} = \frac{s^3+3}{s}$$

$$Y_2(s) = s + \frac{3}{s} \Rightarrow Y_2(s) = Y_3(s) + Y_4(s)$$

$Y_3(s) \to s$, $Y_4(s) \to 3/s$

$$\Rightarrow Z(s) = 2s + \frac{1}{Y_2(s)}$$

$$= 2s + \frac{1}{Y_3(s) + Y_4(s)}$$

$$= 2s + \frac{1}{\left(s + 3/s\right)}$$

series as R | Y → parallel

$C \to \frac{1}{SC}$ | $\to SC$

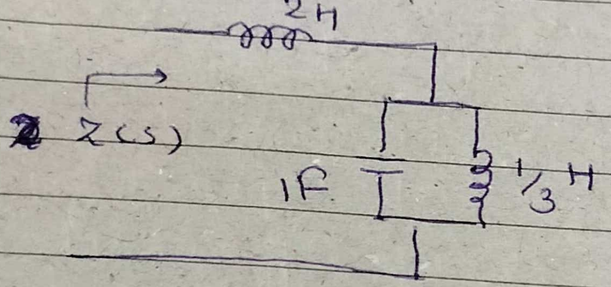
$L \to SL$ | $1/SL$

$Y_3(s) = S = SC \Rightarrow C = 1F$

$Y_4(s) = \frac{3}{S} = \frac{1}{SL} \Rightarrow L = \frac{1}{3} H$

$Z_1(s) = 2S \to L = 2H$

Ckt look like.



(b) $Z(s) = \dfrac{8s^3 + 4s^2 + 4s + 1}{8s^3 + 4s}$

Sol.

$$\begin{array}{r|l} 8s^3+4s & \cancel{8s^3} + 4s^2 + \cancel{4s} + 1 \quad (1 \\ & \underline{-8s^3 \qquad - 4s} \\ & 4s^2+1 \end{array}$$

$1 + \dfrac{4s^2+1}{8s^3+4s}$

$Z_1(s)$ $\quad$ $Z_2(s)$

$Z(s) = Z_1(s) + Z_2(s)$

$Y_2(s) = \dfrac{1}{Z_2(s)} = \dfrac{8s^3+4s}{4s^2+1}$

$$\begin{array}{r|l} 4s^2+1 & \cancel{8s^3}+4s \quad (2s \\ & \underline{8s^3+2s} \\ & 2s \end{array}$$

$Y_2(s) = 2s + \dfrac{2s}{4s^2+1}$

$= 2s + \dfrac{2}{(4s + 1/s)}$

Date
Page No.

$$Z(s) = Z_1(s) + \frac{1}{Y_2(s)} = Z_1(s) + \frac{1}{Y_3(s) + Y_4(s)}$$

$$\Rightarrow 1 + \frac{1}{2s + \frac{2}{4s + 1/s}} \Rightarrow Y(4) = \frac{2}{4s + 1/s}$$

$$Z_4(s) = \frac{4s + 1/s}{2}$$

$$Z(4)s = 2s + \frac{1}{2s} = \frac{1}{Y_4(s)}$$

$$Z_0(s) = Z_1(s) + \frac{1}{Y_3(s) + \frac{1}{Z_4(s)}}$$

$Z_1(s) = 1$

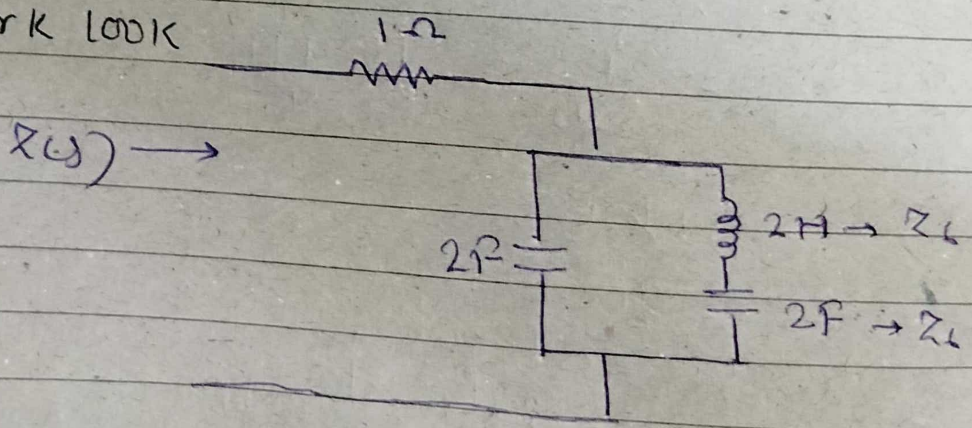
$(R = 1\Omega)$

$Y_3(s) = 2s = Cs \quad (C = 2F)$

$Z_5 = 2s, \quad (L = 2H)$

$Z_6 = \frac{1}{2s}, \quad (C = 2F)$

Network look like,



(C) $Z(s) = \frac{s^2 + 3s + 12}{s(s+4)}$

Page No.
Date

Sol $s^2+4s \overline{) s^2+3s+12 } \; (+1$

s^2+4s

$-s+12$ → as it become -ve thus we stop here & do

$s^2+4s \overline{) s^2+3s+12 } \; (3/s$

$0 \; +3s+12$

$- \quad -$

s^2

$\Rightarrow \frac{3}{s} + \underbrace{\frac{s^2}{s^2+4s}}_{Z_2(s)}$

$Z_0(s) = Z_1(s) + Z_2(s)$

$Z(s) = \frac{3}{s} + \frac{s}{s+4}$

$Y_2(s) = Z_2(s) = \frac{s+4}{s}$

$\underbrace{1}_{y_3} + \underbrace{4/s}_{y_4}$

$Z(s) = Z_1(s) + \frac{1}{Y_3(s)+Y_4(s)}$

NOW,

$Z_1(s) = \frac{3}{s}$ $\quad C = \frac{1}{3}F$

$Y_3(s) = 1$, $R = 1\Omega$

$Y_4(s) = \frac{4}{s} = \frac{1}{sL} = L = \frac{1}{4}H$