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fractalRegression: An R package for multiscale regression and fractal analyses

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Software

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Summary

Time series data from scientific fields as diverse as astrophysics, economics, human movement science, and neuroscience all exhibit fractal properties. That is, these time series often exhibit self-similarity and long-range correlations. This `fractalRegression` package implements a number of univariate and bivariate time series tools appropriate for analyzing noisy data exhibiting these properties. These methods, especially the bivariate tools (Kristoufek 2015; Likens et al. 2019) have yet to be implemented in a complete package for the R Statistical Software environment. As both practitioners and developers of these methods, we expect these tools will be of interest to a wide audience of R users, especially those from fields such as the human movement, cognitive, and other behavioral sciences. The algorithms have been developed in C++ using the popular Rcpp (Eddelbuettel and Francois 2011) and RcppArmadillo (Eddelbuettel and Sanderson 2014) packages. The result is a collection of efficient functions that perform well even on long time series (e.g., 10,000 data points).

Statement of need

Fractal analysis, in its many forms, has become an important framework in virtually every area of science, often serving as an indicator of system health (Goldberger et al. 2002), adaptability (Bak, Tang, and Wiesenfeld 1987), control (Likens et al. 2015), cognitive function (Euler et al. 2016), and multi-scale interactions (Kelty-Stephen 2017). In particular, various methods related to Detrended Fluctuation Analysis (DFA) (Peng et al. 1994) have rose to prominence due to their ease of understanding and broad applicability to stationary and nonstationary time series, alike. The basic DFA algorithm has been implemented in numerous packages and software programs. However, advanced methods such as Multifractal Detrended Fluctuation Analysis (MFDFA) (Kantelhardt et al. 2002), Detrended Cross Correlation (DCCA) (Zebende 2011; Podobnik et al. 2011), and, in particular, fractal regression techniques such as Multiscale Regression Analysis (MRA) (Kristoufek 2015; Likens et al. 2019) have not yet been implemented in a comprehensive CRAN Package for the R Statistical Software Environment. Thus, there is a clear need for a package that incorporates this functionality in order to advance theoretical research focused on understanding the time varying properties of natural phenomena and applied research that uses those insights in important areas such as healthcare (Cavanaugh, Kelty-Stephen, and Stergiou 2017) and education(Snow et al. 2016).

*Custom footnotes for e.g. denoting who the corresponding author is can be included like this.

Comparison to other Packages

Some foundational efforts in fractal analyses, which partially overlap with the functionality of this package, have been implemented elsewhere. For example, a number of univariate fractal and multifractal analyses have been implemented in the 'fracLab' library for MATLAB (Legrand and Véhel 2003) and other toolboxes that are mainly targeted at multifractal analysis (Ihlen and Vereijken 2010; Ihlen et al. 2012). In terms of open access packages, there are other packages that implement some, but not all of the same functions such as the `fathon` package (Bianchi 2020) that has been implemented in Python as well as the R packages: deprecated `fractal` package, `nonlinearTseries` (Garcia 2020), and `MFDFA` (Laib et al. 2018). However, none of the above packages incorporate monofractal and multifractal DFA with DCCA and MRA and run on a C++ architecture. Our `fractalRegression` package is unique in this combination of analyses, efficiency. For instance, we are not aware of any other packages that feature MRA and Multiscale Lagged Regression (MLRA).

Example

In this example, we provide a brief demonstration of multiscale regression analysis using the 'handmovement' data provided in the `crqa` package (Coco and Dale 2014) for R. In the study cited in that package (Wallot et al. 2016), participants performed a collaborative task wherein they built model cars while their hand movements were measured via accelerometers attached to the wrist. Presented below, and also in the vignette provided with the package, are the results of applying the `mra` function to the bivariate time series formed from the movements of the collaborators dominant hands. We arbitrarily choose person 1 (P1) from that dataset to serve as the predictor series and person 2 (P2) to serve as the criterion in the multiscale regression. The two time series are depicted in .

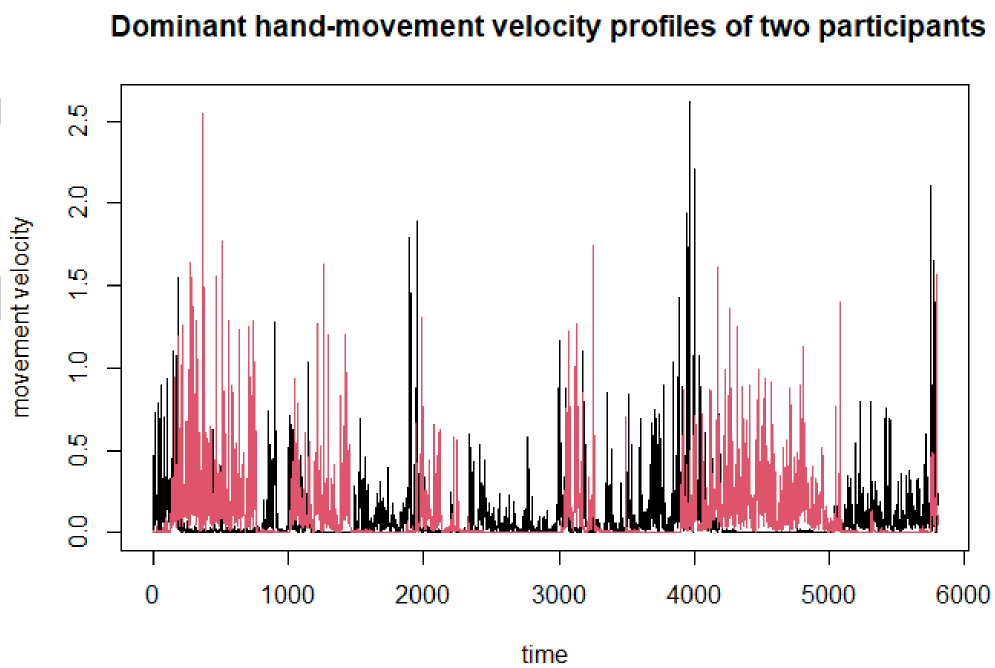

Figure 1: Handmovement time series.

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Visual inspection of these series suggests that the two collaborators tended to take turns during the experiment, alternating between periods of inactivity and activity. This is visually evident from the bursts of red and black lines that almost perfectly alternate in activity level. One might expect, based on these two series that correlation between the collaborator's movements would not be correlated at shorter time scales, but would become negatively correlated over longer time scales. The results from Multiscale Regression Analysis (MRA) are given in Figure 2 and support this expectation.

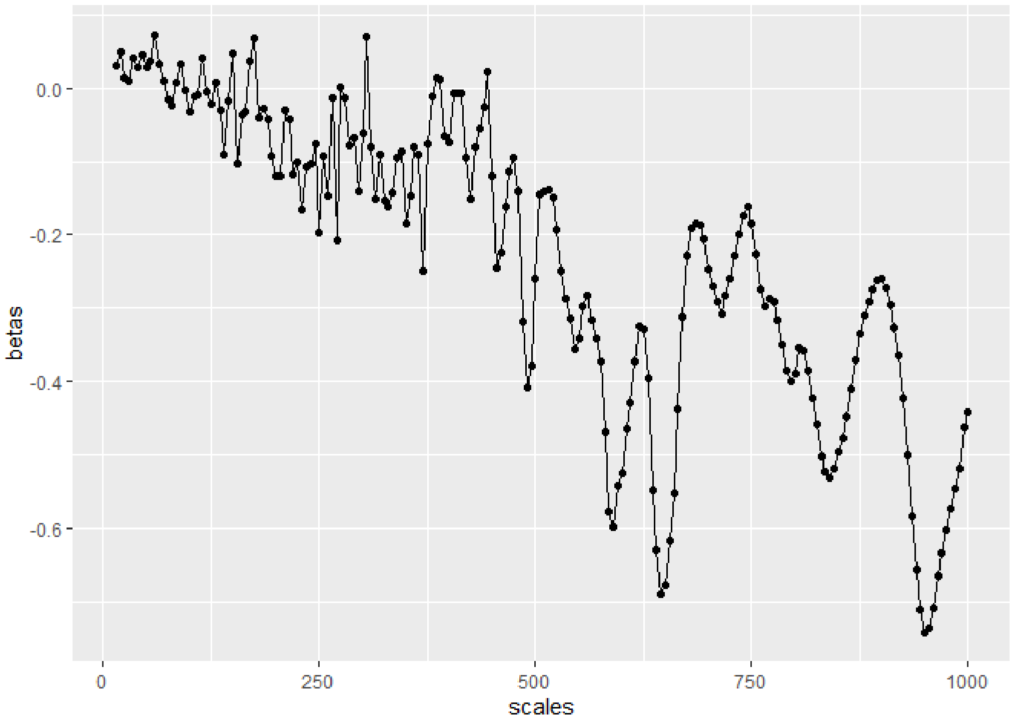

Figure 2: MRA plot of handmovement time series.

In particular, note that $\beta(s)$ are very close to zero at small scales, but becomes increasingly negative at larger time scales, as anticipated from visual inspection of the time series. And, one would expect DCCA to produce similar results (see vignette).

In summary, this `fractalRegression` package collects older univariate and bivariate techniques into a single, efficient package that can be installed directly from the CRAN network or github. We anticipate this software will be of benefit to scientists across many disciplines.

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