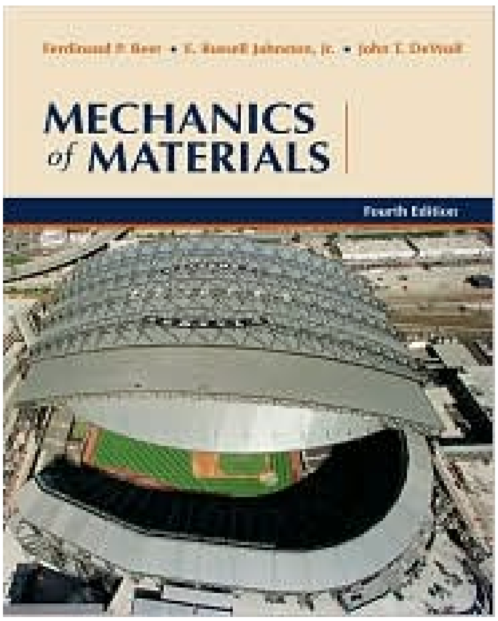
Instructor's and Solutions Manual

to accompany

Mechanics of Materials

Fourth Edition

Volume I, Chapters 1-6



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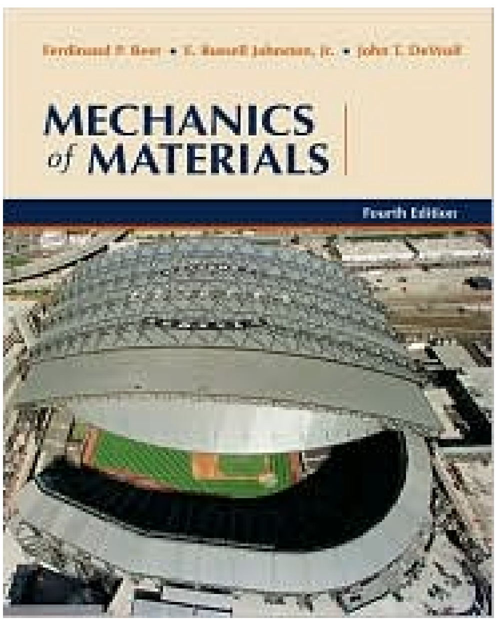
Volume II, Chapters 7-11



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Ferdinand P. Beer, E. Russell Johnston, Jr., and John T. DeWolf

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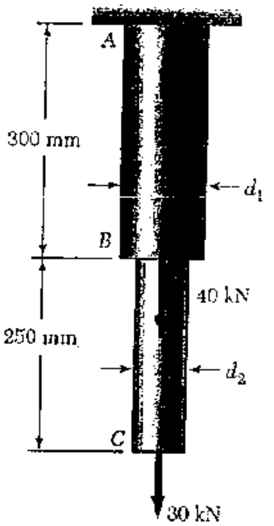
Problem Solutions

by

Dean Updike

Chapter 1

Problem 1.1



1.1 Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Knowing that $d_1 = 50$ mm and $d_2 = 30$ mm, find average normal stress at the midsection of (*a*) rod *AB*, (*b*) rod *BC*.

(a) Rod AB

$$P = 40 + 30 = 70 \text{ kN} = 70 \times 10^3 \text{ N}$$

$$A = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}(50)^2 = 1.9635 \times 10^3 \text{ mm}^2 = 1.9635 \times 10^{-3} \text{ m}^2$$

$$\sigma_{AB} = \frac{P}{A} = \frac{70 \times 10^3}{1.9635 \times 10^{-3}} = 35.7 \times 10^6 \text{ Pa}$$

$$\sigma_{AB} = 35.7 \text{ MPa}$$

(b) Rod BC

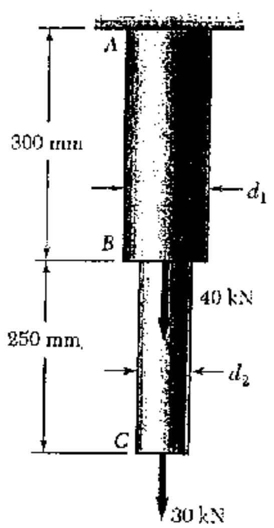
$$P = 30 \text{ kN} = 30 \times 10^3 \text{ N}$$

$$A = \frac{\pi}{4} d_2^2 = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86 \times 10^{-6} \text{ m}^2$$

$$\sigma_{BC} = \frac{P}{A} = \frac{30 \times 10^3}{706.86 \times 10^{-6}} = 42.4 \times 10^6 \text{ Pa}$$

$$\sigma_{BC} = 42.4 \text{ MPa}$$

Problem 1.2



1.2 Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Knowing that the average normal stress must not exceed 140 MPa in either rod, determine the smallest allowable values of d_1 and d_2.

Rod AB.

$$P = 40 + 30 = 70 \text{ kN} = 70 \times 10^3 \text{ N}$$

$$\sigma_{AB} = \frac{P}{A_{AB}} = \frac{P}{\frac{\pi}{4} d_1^2} = \frac{4P}{\pi d_1^2}$$

$$d_1 = \sqrt{\frac{4P}{\pi \sigma_{AB}}} = \sqrt{\frac{(4)(70 \times 10^3)}{\pi (140 \times 10^6)}} = 25.2 \times 10^{-3} \text{ m}$$

$$d_1 = 25.2 \text{ mm}$$

Rod BC

$$P = 30 \text{ kN} = 30 \times 10^3 \text{ N}$$

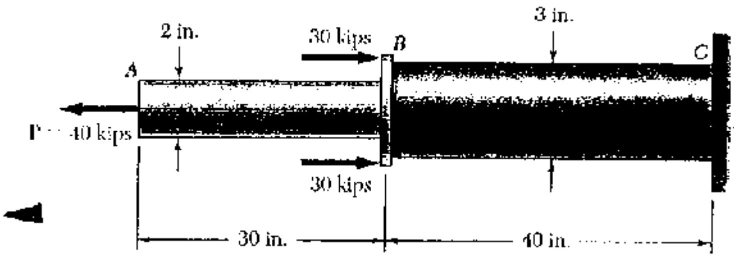
$$\sigma_{BC} = \frac{P}{A_{BC}} = \frac{P}{\frac{\pi}{4} d_2^2} = \frac{4P}{\pi d_2^2}$$

$$d_2 = \sqrt{\frac{4P}{\pi \sigma_{BC}}} = \sqrt{\frac{(4)(30 \times 10^3)}{\pi (140 \times 10^6)}} = 16.52 \times 10^{-3} \text{ m}$$

$$d_2 = 16.52 \text{ mm}$$

Problem 1.3

1.3 Two solid cylindrical rods AB and BC are welded together at B and loaded as shown. Determine the average normal stress at the midsection of (*a*) rod AB, (*b*) rod BC.



(a) Rod AB.

$P = 40$ kips (tension)

$A_{AB} = \dfrac{\pi d_{AB}^2}{4} = \dfrac{\pi (2)^2}{4} = 3.1416 \text{ in}^2$

$\sigma_{AB} = \dfrac{P}{A_{AB}} = \dfrac{40}{3.1416} = 12.73$ ksi ◄

(b) Rod BC.

$F = 40 - (2)(30) = -20$ kips. i.e. 20 kips compression.

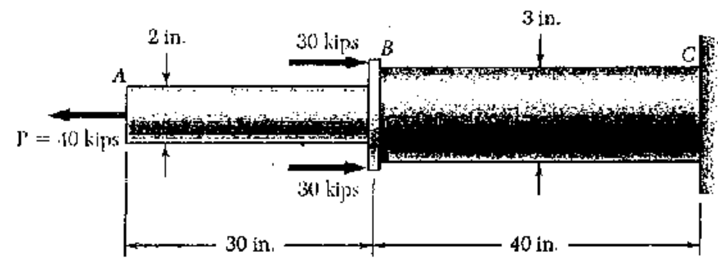
$A_{BC} = \dfrac{\pi d_{BC}^2}{4} = \dfrac{\pi (3)^2}{4} = 7.0686 \text{ in}^2$

$\sigma_{BC} = \dfrac{F}{A_{BC}} = \dfrac{-20}{7.0686} = -2.83$ ksi ◄

Problem 1.4

1.4 In Prob. 1.3, determine the magnitude of the force **P** for which the tensile stress in rod AB has the same magnitude as the compressive stress in rod BC.

1.3 Two solid cylindrical rods AB and BC are welded together at B and loaded as shown. Determine the average normal stress at the midsection of (*a*) rod AB, (*b*) rod BC.



$A_{AB} = \frac{\pi}{4}(2)^2 = 3.1416 \text{ in}^2$

$\sigma_{AB} = \dfrac{P}{A_{AB}} = \dfrac{P}{3.1416}$

$= 0.31831 \, P$

$A_{BC} = \frac{\pi}{4}(3)^2 = 7.0686 \text{ in}^2$

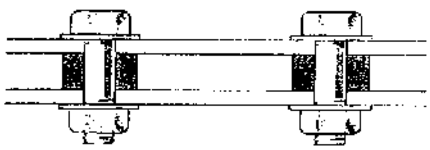
$\sigma_{BC} = \dfrac{(2)(30) - P}{A_{AB}}$

$= \dfrac{60 - P}{7.0686} = 8.4883 - 0.14147 \, P$

Equating σ_{AB} to σ_{BC}

$0.31831 \, P = 8.4883 - 0.14147 \, P$ $\qquad P = 18.46$ kips ◄

Problem 1.5



1.5 Two steel plates are to be held together by means of 16-mm-diameter high-strength steel bolts fitting snugly inside cylindrical brass spacers. Knowing that the average normal stress must not exceed 200 MPa in the bolts and 130 MPa in the spacers, determine the outer diameter of the spacers that yields the most economical and safe design.

At each bolt location the upper plate is pulled down by the tensile force P_b of the bolt. At the same time the spacer pushes that plate upward with a compressive force P_s. In order to maintain equilibrium.

$$P_b = P_s$$

For the bolt $\quad \sigma_b = \frac{P_b}{A_b} = \frac{4P_b}{\pi d_b^2} \quad$ or $\quad P_b = \frac{\pi}{4}\sigma_b d_b^2$

For the spacer $\quad \sigma_s = \frac{P_s}{A_s} = \frac{4P_s}{\pi(d_s^2 - d_b^2)} \quad$ or $\quad P_s = \frac{\pi}{4}\sigma_s(d_s^2 - d_b^2)$

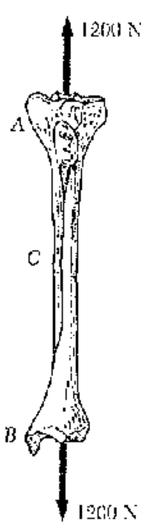
Equating P_b and P_s

$$\frac{\pi}{4}\sigma_b d_b^2 = \frac{\pi}{4}\sigma_s(d_s^2 - d_b^2)$$

$$d_s = \sqrt{1 + \frac{\sigma_b}{\sigma_s}}\, d_b \quad \sqrt{1 + \frac{200}{130}}\,(16) \qquad d_s = 25.5 \text{ mm}$$

Problem 1.6

1.6 A strain gage located at C on the surface of bone AB indicates that the average normal stress in the bone is 3.80 MPa when the bone is subjected to two 1200-N forces as shown. Assuming the cross section of the bone at C to be annular and knowing that its outer diameter is 25 mm, determine the inner diameter of the bone's cross section at C.



$$\sigma = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma}$$

Geometry: $\quad A = \frac{\pi}{4}(d_1^2 - d_2^2)$

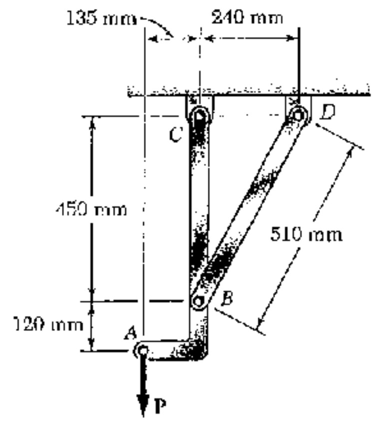
$$d_2^2 = d_1^2 - \frac{4A}{\pi} = d_1^2 - \frac{4P}{\pi\sigma}$$

$$d_2^2 = (25\times10^{-3})^2 - \frac{(4)(1200)}{\pi(3.80\times10^6}$$

$$= 222.9\times10^{-6} \text{ m}^2$$

$$d_2 = 14.93\times10^{-3} \text{ m} \qquad d_2 = 14.93 \text{ mm}$$

Problem 1.7



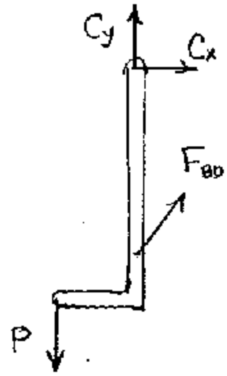
1.7 Knowing that the central portion of the link BD has a uniform cross-sectional area of 800 mm^2, determine the magnitude of the load **P** for which the normal stress in that portion of BD is 50 MPa.

$\sigma_{BD} = 50\ \text{MPa} = 50 \times 10^6\ \text{Pa}$

$A_{BD} = 800\ \text{mm}^2 = 800 \times 10^{-6}\ \text{m}^2$

$F_{BD} = \sigma_{BD} A_{BD} = (50 \times 10^6)(800 \times 10^{-6}) = 40 \times 10^3\ \text{N}$

Draw free body diagram of body ABD.



$+\circlearrowleft M_C = 0$

$(0.450)\left(\frac{240}{510} F_{BD}\right) - 0.135\,P = 0$

$P = 1.5686\,F_{BD}$

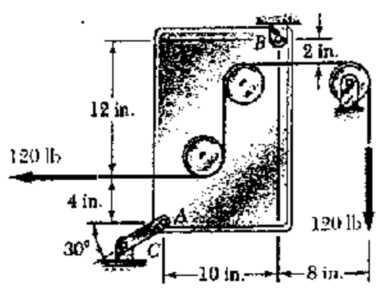
$P = (1.5686)(40 \times 10^3)$

$= 62.7 \times 10^3\ \text{N}$

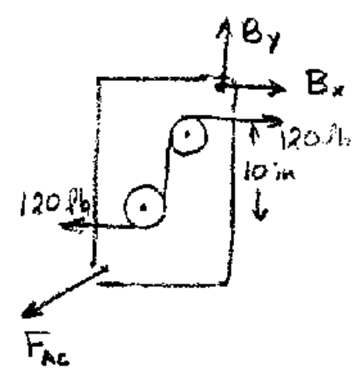
62.7 kN ◄

Problem 1.8

1.8 Link AC has a uniform rectangular cross section $\frac{1}{8}$ in. thick and 1 in. wide Determine the normal stress in the central portion of the link.



Use the plate together with two pulleys as a free body. Note that the cable tension causes at 1200 lb-in clockwise couple to act on the body.



$\circlearrowleft \Sigma M_B = 0$

$-(12+4)(F_{AC} \cos 30°) + (10)(F_{AC} \sin 30°) - 1200 = 0$

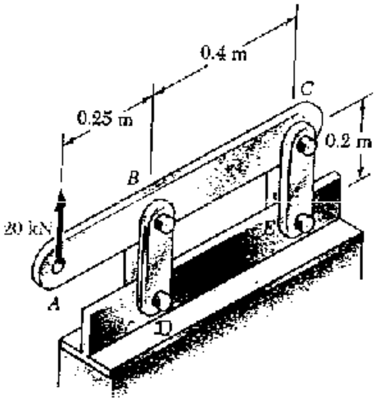
$F_{AC} = -\dfrac{1200}{16\cos 30° - 10 \sin 30°} = -135.50\ \text{lb.}$

Area of link AC: $A = 1\ \text{in.} \times \frac{1}{8}\ \text{in} = 0.125\ \text{in}^2$

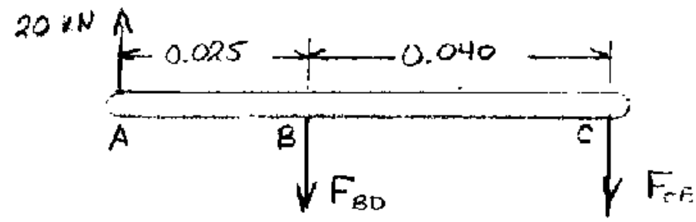
Stress in link AC: $\sigma_{AC} = \dfrac{F_{AC}}{A} = -\dfrac{135.50}{0.125} = 1084\ \text{psi} = 1.084\ \text{ksi}$ ◄

Problem 1.9

1.9 Each of the four vertical links has an 8 × 36- mm uniform rectangular cross section and each of the four pins has a 16- mm diameter. Determine the maximum value of the average normal stress in the links connecting (*a*) points *B* and *D*, (*b*) points *C* and *E*.



Use bar ABC as a free body.



$\Sigma M_C = 0 \qquad (0.040)F_{BD} - (0.025+0.040)(20\times10^3) = 0$

$F_{BD} = 32.5\times10^3$ N $\qquad$ Link BD is in tension

$\Sigma M_B = 0 \qquad -(0.040)F_{CE} - (0.025)(20\times10^3) = 0$

$F_{CE} = -12.5\times10^3$ N $\qquad$ Link CE is in compression

Net area of one link for tension = $(0.008)(0.036 - 0.016)$

$= 160\times10^{-6}$ m^2. For two parallel links $A_{net} = 320\times10^{-6}$ m^2

Tensile stress in link BD

(a) $\quad \sigma_{BD} = \dfrac{F_{BD}}{A_{net}} = \dfrac{32.5\times10^3}{320\times10^{-6}} = 101.56\times10^6$ or 101.6 MPa ◄

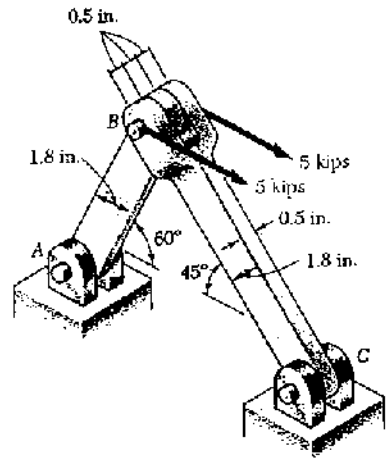
Area for one link in compression = $(0.008)(0.036)$

$= 288\times10^{-6}$ m^2. For two parallel links $A = 576\times10^{-6}$ m^2

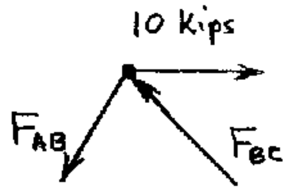
(b) $\quad \sigma_{CE} = \dfrac{F_{CE}}{A} = \dfrac{-12.5\times10^3}{576\times10^{-6}} = -21.70\times10^6$ or -21.7 MPa ◄

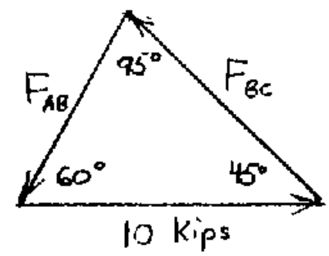
Problem 1.10

1.10 Two horizontal 5-kip forces are applied to pin B of the assembly shown. Knowing that a pin of 0.8-in. diameter is used at each connection, determine the maximum value of the average normal stress (a) in link AB, (b) in link BC.



Use joint B as free body





Force triangle

Law of Sines

$$\frac{F_{AB}}{\sin 45^\circ} = \frac{F_{BC}}{\sin 60^\circ} = \frac{10}{\sin 95^\circ}$$

$F_{AB} = 7.3205$ kips $F_{BC} = 8.9658$ kips.

Link AB is a tension member

Minimum section at pin $A_{net} = (1.8 - 0.8)(0.5) = 0.5 \text{ in}^2$

(a) Stress in AB $\sigma_{AB} = \frac{F_{AB}}{A_{net}} = \frac{7.3205}{0.5} = 14.64$ ksi

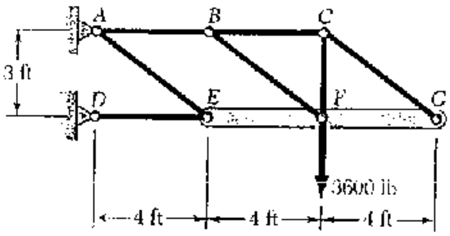
Link BC is a compression member

Cross sectional area is $A = (1.8)(0.5) = 0.9 \text{ in}^2$

(b) Stress in BC $\sigma_{BC} = \frac{-F_{BC}}{A} = \frac{-8.9658}{0.9} = -9.96$ ksi

Problem 1.11

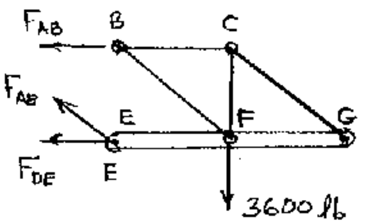
1.11 The rigid bar *EFG* is supported by the truss system shown. Knowing that the member *CG* is a solid circular rod of 0.75-in. diameter, determine the normal stress in *CG*.



Using portion EFGCB as a free body

$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5}F_{AB} - 3600 = 0$

$F_{AE} = 6000$ lb.



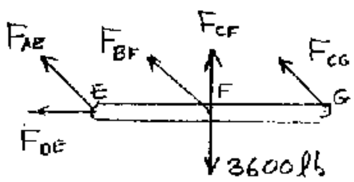
Using beam EFG as a free body

$\circlearrowleft M_F = 0: \quad -(4)\frac{3}{5}F_{AE} + (4)(\frac{3}{5}F_{CG}) = 0$

$F_{CG} = F_{AE} = 6000$ lb.

Cross sectional area of member CG

$A_{CG} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.75)^2 = 0.44179 \text{ in}^2$



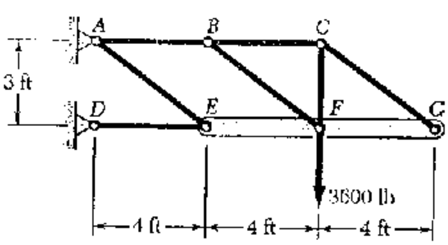
Normal stress in CG.

$\sigma_{CG} = \frac{F_{CG}}{A_{CG}} = \frac{6000}{0.44179} = 13\,580$ psi

13.58 ksi

Problem 1.12

1.12 The rigid bar *EFG* is supported by the truss system shown. Determine the cross-sectional area of member *AE* for which the normal stress in the member is 15 ksi.



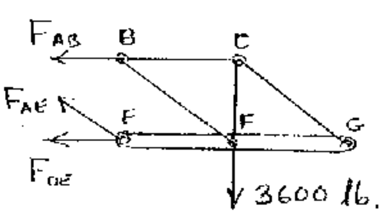
Using portion EFGCB as a free body

$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5}F_{AE} - 3600 = 0$

$F_{AE} = 6000$ lb. $= 6.00$ kips

Stress in member AE $\qquad \sigma_{AE} = 15$ ksi

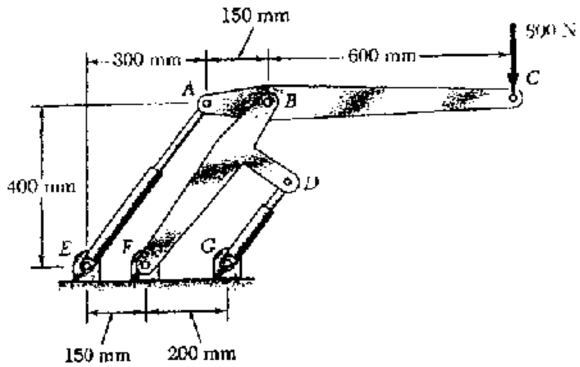
$\sigma_{AE} = \frac{F_{AE}}{A_{AE}}$



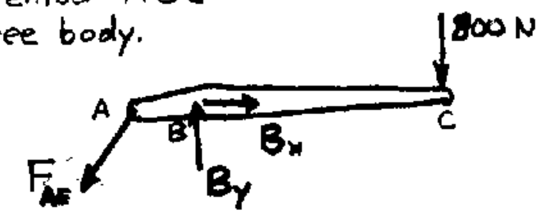
$A_{AE} = \frac{F_{AE}}{\sigma_{AE}} = \frac{6.00}{15} = 0.400 \text{ in}^2$

Problem 1.13

1.13 Two hydraulic cylinders are used to control the position of the robotic arm *ABC*. Knowing that the control rods attached at *A* and *D* each have a 20-mm diameter and happen to be parallel in the position shown, determine the average normal stress in (*a*) member *AE*, (*b*) member *DG*.



Use member ABC as free body.

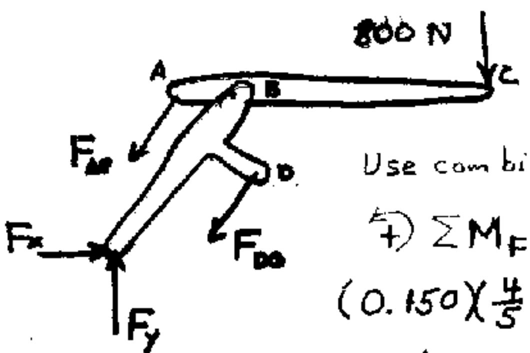


$\circlearrowleft\!\!+\ \Sigma M_B = 0 \qquad (0.150)\frac{4}{5}F_{AE} - (0.600)(800) = 0 \qquad F_{AE} = 4\times10^3\ \text{N}$

Area of rod in member AE is $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(20\times10^{-3})^2 = 314.16\times10^{-6}\ \text{m}^2$

Stress in rod AE: $\sigma_{AE} = \dfrac{F_{AE}}{A} = \dfrac{4\times10^3}{314.16\times10^{-6}} = 12.73\times10^6\ \text{Pa}$

(a) $\sigma_{AE} = 12.73\ \text{MPa}$ ◄



Use combined members ABC and BFD as free body.

$\circlearrowleft\!\!+\ \Sigma M_F = 0$

$(0.150)(\frac{4}{5}F_{AE}) - (0.200)(\frac{4}{5}F_{DG})$

$-(1.050 - 0.350)(800) = 0 \qquad F_{DG} = -1500\ \text{N}$

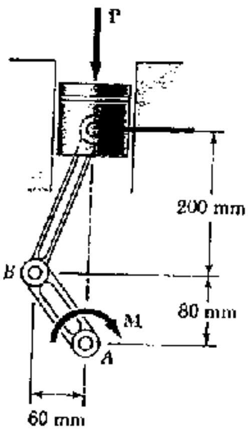
Area in rod DG is $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(20\times10^{-3})^2 = 314.16\times10^{-6}\ \text{m}^2$

Stress in rod DG: $\sigma_{DG} = \dfrac{F_{DG}}{A} = \dfrac{-1500}{3.1416\times10^{-6}} = -4.77\times10^6\ \text{Pa}$

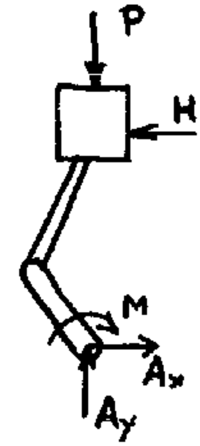
(b) $\sigma_{DG} = -4.77\ \text{MPa}$ ◄

Problem 1.14

1.14 A couple **M** of magnitude 1500 N · m is applied to the crank of an engine. For the position shown, determine (*a*) the force **P** required to hold the engine system in equilibrium, (*b*) the average normal stress in the connecting rod *BC*, which has a 450-mm² uniform cross section.



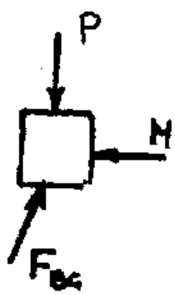
Use piston, rod, and crank together as free body. Add wall reaction H and bearing reactions A_x and A_y.

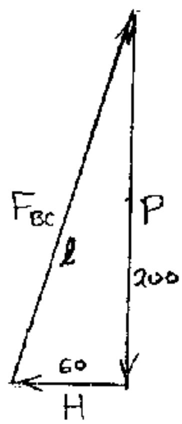


$\circlearrowleft \Sigma M_A = 0$

$(0.280 \text{ m})H - 1500 \text{ N·m} = 0$

$H = 5.3571 \times 10^3 \text{ N}$





Use piston alone as free body. Note that rod is a two-force member; hence the direction of force F_{BC} is known. Draw the force triangle and solve for P and F_{BC} by proportions.

$\ell = \sqrt{200^2 + 60^2} = 208.81 \text{ mm}$

$\frac{P}{H} = \frac{200}{60} \quad \therefore \quad P = 17.86 \times 10^3 \text{ N}$

(a) $P = 17.86 \text{ kN}$ ◄

$\frac{F_{BC}}{H} = \frac{208.81}{60} \quad \therefore \quad F_{BC} = 18.643 \times 10^3 \text{ N}$

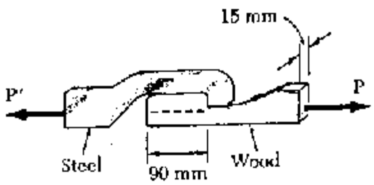
Rod BC is a compression member. Its area is $450 \text{ mm}^2 = 450 \times 10^{-6} \text{ m}^2$

Stress $\quad \sigma_{BC} = \frac{-F_{BC}}{A} = \frac{-18.643 \times 10^3}{450 \times 10^{-6}} = -41.4 \times 10^6 \text{ Pa}$

(b) $\sigma_{BC} = -41.4 \text{ MPa}$ ◄

Problem 1.15

1.15 When the force **P** reached 8 kN, the wooden specimen shown failed in shear along the surface indicated by the dashed line. Determine the average shearing stress along that surface at the time of failure.



Area being sheared

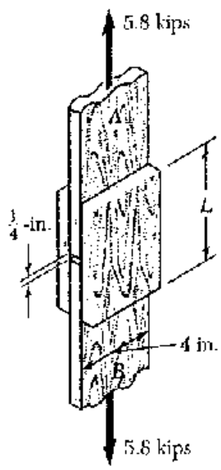
$A = 90\text{ mm} \times 15\text{ mm} = 1350\text{ mm}^2 = 1350 \times 10^{-6}\text{ m}^2$

Force $P = 8 \times 10^3$ N

Shearing stress $\tau = \dfrac{P}{A} = \dfrac{8 \times 10^3}{1350 \times 10^{-6}} = 5.93 \times 10^3\text{ Pa} = 5.93\text{ MPa}$ ◄

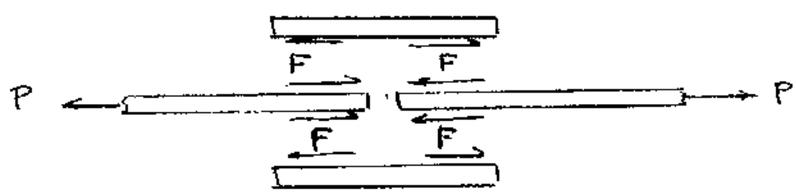
Problem 1.16

1.16 The wooden members *A* and *B* are to be joined by plywood splice plates which will be fully glued on the surfaces in contact. As part of the design of the joint, and knowing that the clearance between the ends of the members is to be $\frac{1}{4}$ in., determine the smallest allowable length *L* if the average shearing stress in the glue is not to exceed 120 psi.



There are four separate areas that are glued. Each of these areas transmits one half the the 5.8 kip force. Thus

$F = \frac{1}{2}P = \frac{1}{2}(5.8) = 2.9\text{ kips} = 2900\text{ lb.}$



Let l = length of one glued area and $w = 4$ in. be its width.

For each glued area $A = lw$

Average shearing stress $\tau = \dfrac{F}{A} = \dfrac{F}{lw}$

The allowable shearing stress is $\tau = 120$ psi

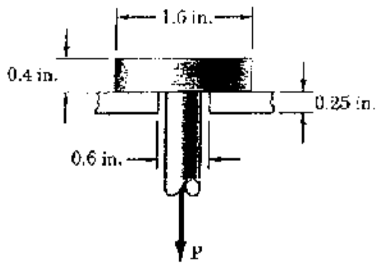
Solving for l $\quad l = \dfrac{F}{\tau w} = \dfrac{2900}{(120)(4)} = 6.0417$ in.

Total length L $\quad L = l + (\text{gap}) + l = 6.0417 + \frac{1}{4} + 6.0417$

$= 12.33$ in. ◄

Problem 1.17

1.17 A load **P** is applied to a steel rod supported as shown by an aluminum plate into which a 0.6-in.-diameter hole has been drilled. Knowing that the shearing stress must not exceed 18 ksi in the steel rod and 10 ksi in the aluminum plate, determine the largest load **P** that may be applied to the rod.



For steel $A_1 = \pi d t = \pi(0.6)(0.4)$

$= 0.7540 \text{ in}^2$

$\tau_1 = \dfrac{P}{A_1} \quad \therefore \quad P = A_1 \tau_1 = (0.7540)(18)$

$= 13.57 \text{ kips}$

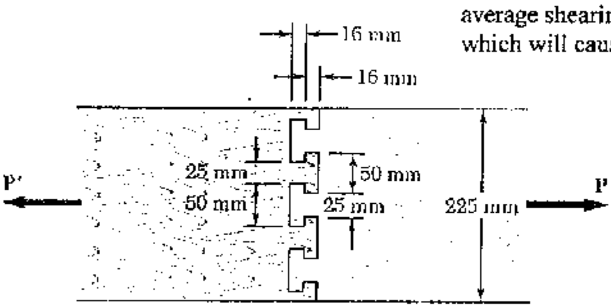
For aluminum $A_2 = \pi d t = \pi(1.6)(0.25) = 1.2566 \text{ in}^2$

$\tau_2 = \dfrac{P}{A_2} \quad \therefore \quad P = A_2 \tau_2 = (1.2566)(10) = 12.57 \text{ kips}$

Limiting value of P is the smaller value $\therefore$ $P = 12.57$ kips

Problem 1.18

1.18 Two wooden planks, each 12 mm thick and 225 mm wide, are joined by the dry mortise joint shown. Knowing that the wood used shears off along its grain when the average shearing stress reaches 8 MPa, determine the magnitude P of the axial load which will cause the joint to fail.



Six areas must be sheared off when the joint fails. Each of these areas has dimensions 16 mm × 12 mm, its area being

$A = (16)(12) = 192 \text{ mm}^2 = 192 \times 10^{-6} \text{ m}^2$

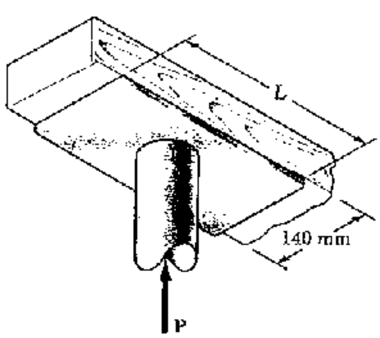
At failure the force F carried by each of areas is

$F = \tau A = (8 \times 10^6)(192 \times 10^{-6}) = 1536 \text{ N} = 1.536 \text{ kN}$

Since there are six failure areas $P = 6F = (6)(1.536) = 9.22 \text{ kN}$

Problem 1.19

1.19 The axial force in the column supporting the timber beam shown is $P = 75$ kN. Determine the smallest allowable length L of the bearing plate if the bearing stress in the timber is not to exceed 3.0 MPa.



SOLUTION

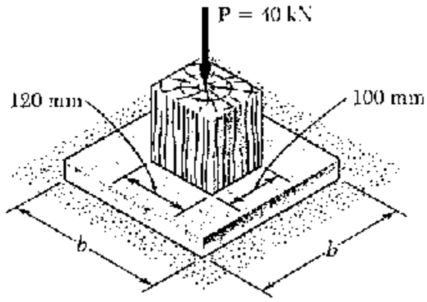
$\sigma_b = \dfrac{P}{A} = \dfrac{P}{Lw}$

Solving for L: $L = \dfrac{P}{\sigma_b w} = \dfrac{75 \times 10^3}{(3.0 \times 10^6)(0.140)}$

178.6×10^{-3} m

$L = 178.6$ mm

Problem 1.20



1.20 A 40-kN axial load is applied to a short wooden post that is supported by a concrete footing resting on undisturbed soil. Determine (*a*) the maximum bearing stress on the concrete footing, (*b*) the size of the footing for which the average bearing stress in the soil is 145 kPa.

(a) Bearing stress on concrete footing.

$$P = 40 \text{ kN} = 40 \times 10^3 \text{ N}$$

$$A = (100)(120) = 12 \times 10^3 \text{ mm}^2 = 12 \times 10^{-3} \text{ m}^2$$

$$\sigma = \frac{P}{A} = \frac{40 \times 10^3}{12 \times 10^{-3}} = 3.33 \times 10^6 \text{ Pa}$$

$$3.33 \text{ MPa}$$

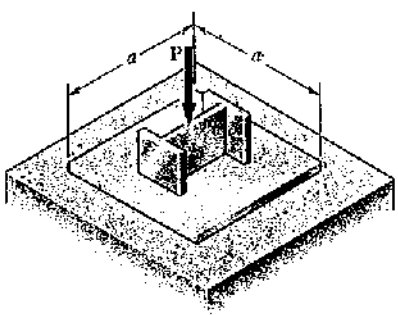
(b) Footing area. $P = 40 \times 10^3$ N $\quad \sigma = 145$ kPa $= 45 \times 10^3$ Pa

$$\sigma = \frac{P}{A} \qquad A = \frac{P}{\sigma} = \frac{40 \times 10^3}{145 \times 10^3} = 0.27586 \text{ m}^2$$

Since the area is square $\quad A = b^2$

$$b = \sqrt{A} = \sqrt{0.27586} = 0.525 \text{ m} \qquad b = 525 \text{ mm}$$

Problem 1.21



1.21 An axial load **P** is supported by a short W8 × 40 column of cross-sectional area $A = 11.7$ in.^{2} and is distributed to a concrete foundation by a square plate as shown. Knowing that the average normal stress in the column must not exceed 30 ksi and that the bearing stress on the concrete foundation must not exceed 3.0 ksi, determine the side *a* of the plate that will provide the most economical and safe design.

For the column $\quad \sigma = \dfrac{P}{A}$

or $\quad P = \sigma A = (30)(11.7) = 351$ kips

For the $a \times a$ plate, $\quad \sigma = 3.0$ ksi

$$A = \frac{P}{\sigma} = \frac{351}{3.0} = 117 \text{ in}^2$$

Since the plate is square $\quad A = a^2$

$$a = \sqrt{A} = \sqrt{117} = 10.82 \text{ in.}$$

Problem 1.22

1.22 Three wooden planks are fastened together by a series of bolts to form a column. The diameter of each bolt is $\frac{1}{2}$ in. and the inner diameter of each washer is $\frac{5}{8}$ in., which is slightly larger than the diameter of the holes in the planks. Determine the smallest allowable outer diameter d of the washers, knowing that the average normal stress in the bolts is 5 ksi and that the bearing stress between the washers and the planks must not exceed 1.2 ksi.

Bolt: $A_{bolt} = \frac{\pi}{4} d_b^2 = \frac{\pi}{4}\left(\frac{1}{2}\right)^2 = 0.19635 \text{ in}^2$

$\sigma_b = \frac{P}{A}$ ∴ Tensile force in bolt $P = \sigma_b A = (5)(0.19635) = 0.98175$ kips

Washer: inside diameter $= d_i = \frac{5}{8}$ in, outside diameter $= d_o$

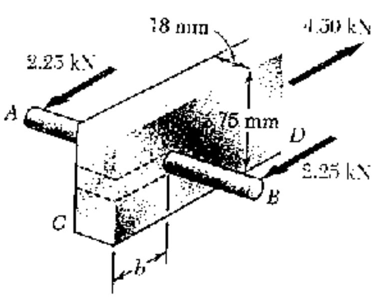
Bearing area $A_w = \frac{\pi}{4}(d_o^2 - d_i^2)$ and $A_w = \frac{P}{\sigma_b}$.

Equating $\frac{\pi}{4}(d_o^2 - d_i^2) = \frac{P}{\sigma_b}$

$$d_o^2 = d_i^2 + \frac{4P}{\pi\sigma_b} = \left(\frac{5}{8}\right)^2 + \frac{(4)(0.98175)}{\pi(12.5)} = 1.4323 \text{ in}^2$$

$$d_o = 1.197 \text{ in.}$$ ◄

Problem 1.23



1.23 A 0.12-mm-diameter steel rod AB is fitted to a round hole near end C of the wooden member CD. For the loading shown, determine (*a*) the maximum average normal stress in the wood, (*b*) the distance b for which the average shearing stress is 620 kPa on the surfaces indicated by the dashed lines, (*c*) the average bearing stress on the wood.

(a) Maximum average normal stress in the wood.

$A_{net} = (75 - 12)(18) = 1.134 \times 10^3 \text{ mm}^2 = 1.134 \times 10^{-3} \text{ m}^2$

$P = 4.50 \text{ kN} = 4.50 \times 10^3 \text{ N}$

$\sigma = \frac{P}{A_{net}} = \frac{4.50 \times 10^3}{1.134 \times 10^{-3}} = 3.97 \times 10^6 \text{ Pa}$ 3.97 MPa ◄

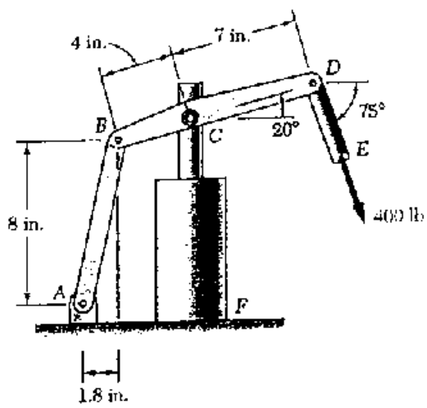
(b) $\tau = \frac{P}{A} = \frac{P}{2bt}$ $b = \frac{P}{2t\tau} = \frac{4.50 \times 10^3}{(2)(18 \times 10^{-3})(620 \times 10^3)} = 202 \times 10^{-3} \text{ m}$

$b = 202 \text{ mm}$ ◄

(c) $\sigma_b = \frac{P}{dt} = \frac{4.50 \times 10^3}{(12 \times 10^{-3})(18 \times 10^{-3})} = 20.8 \times 10^6 \text{ Pa}$ 20.8 MPa ◄

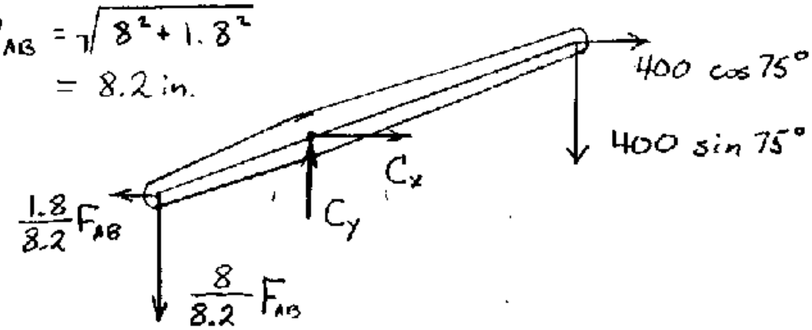
Problem 1.24

1.24 The hydraulic cylinder *CF*, which partially controls the position of rod *DE*, has been locked in the position shown. Member *BD* is $\frac{5}{8}$ in. thick and is connected to the vertical rod by a $\frac{3}{8}$-in.-diameter bolt. Determine (*a*) the average shearing stress in the bolt, (*b*) the bearing stress at *C* in member *BD*.



Use member BCD as a free body, and note that AB is a two force member.

$$\ell_{AB} = \sqrt{8^2 + 1.8^2} = 8.2 \text{ in.}$$



$\circlearrowleft \Sigma M_C = 0 \qquad (4\cos 20°)(\frac{8}{8.2} F_{AB}) - (4\sin 20°)(\frac{1.8}{8.2} F_{AB})$

$\qquad - (7\cos 20°)(400 \sin 75°) - (7 \sin 20°)(400 \cos 75°) = 0$

$3.36678\ F_{AB} - 2789.35 = 0 \qquad \therefore F_{AB} = 828.49 \text{ lb.}$

$\xrightarrow{+} \Sigma F_x = 0 \qquad -\frac{1.8}{8.2} F_{AB} + C_x + 400 \cos 75° = 0$

$C_x = \frac{(1.8)(828.49)}{8.2} - 400 \cos 75° = 78.34 \text{ lb.}$

$+\uparrow \Sigma F_y = 0 \qquad -\frac{8}{8.2} F_{AB} + C_y - 400 \sin 75° = 0$

$C_y = \frac{(8)(828.49)}{8.2} + 400 \sin 75° = 1194.65 \text{ lb.}$

$C = \sqrt{C_x^2 + C_y^2} = 1197.2 \text{ lb.}$

(a) Shearing stress in the bolt: $\qquad P = 1197.2 \text{ lb.}$

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}\left(\frac{3}{8}\right)^2 = 0.11045 \text{ in}^2$

$\tau = \frac{P}{A} = \frac{1197.2}{0.11045} = 10.84 \times 10^3 \text{ psi} = 10.84 \text{ ksi}$ ◄

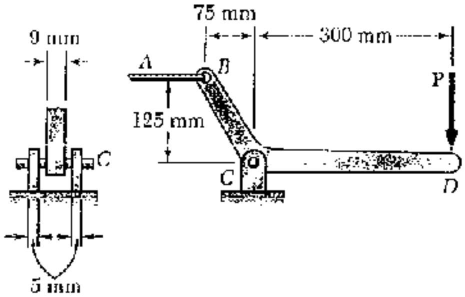
(b) Bearing stress at C in member BCD: $\qquad P = 1197.2 \text{ lb.}$

$A_b = dt = \left(\frac{3}{8}\right)\left(\frac{5}{8}\right) = 0.234375 \text{ in}^2$

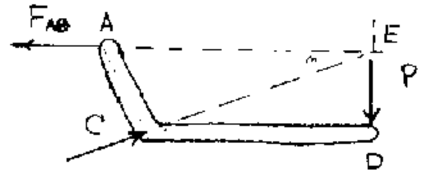
$\sigma_b = \frac{P}{A_b} = \frac{1197.2}{0.234375} = 5.11 \times 10^3 \text{ psi} = 5.11 \text{ ksi}$ ◄

Problem 1.25

1.25 A 6-mm-diameter pin is used at connection C of the pedal shown. Knowing that $P = 500$ N, determine (*a*) the average shearing stress in the pin, (*b*) the nominal bearing stress in the pedal at C, (*c*) the nominal bearing stress in each support bracket at C.



Draw free body diagram of ACD.
Since ACD is a 3-force member, the reaction at C is directed toward point E, the intersection of the lines of action of the other two forces.



From geometry, $CE = \sqrt{300^2 + 125^2} = 325$ mm.

$+\uparrow \Sigma F_y = 0: \quad \frac{125}{325} C - P = 0 \qquad C = 2.6P = (2.6)(500) = 1300$ N

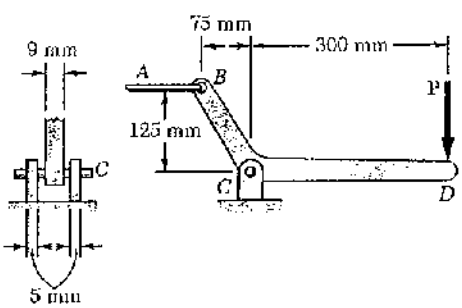
(a) $\tau_{pin} = \dfrac{\frac{1}{2}C}{A_{pin}} = \dfrac{\frac{1}{2}C}{\frac{\pi}{4}d^2} = \dfrac{2C}{\pi d^2} = \dfrac{(2)(1300)}{\pi(6\times10^{-3})^2} = 23.0\times10^6$ Pa $\qquad$ **23.0 MPa** ◄

(b) $\sigma_b = \dfrac{C}{A_b} = \dfrac{C}{dt} = \dfrac{1300}{(6\times10^{-3})(9\times10^{-3})} = 24.1\times10^6$ Pa $\qquad$ **24.1 MPa** ◄

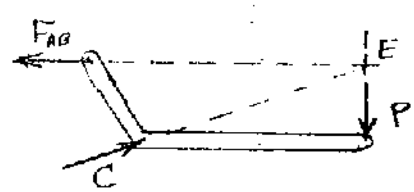
(c) $\sigma_b = \dfrac{\frac{1}{2}C}{A_b} = \dfrac{C}{2dt} = \dfrac{1300}{(2)(6\times10^{-3})(5\times10^{-3})} = 21.7\times10^6$ Pa $\qquad$ **21.7 MPa** ◄

Problem 1.26

1.26 Knowing that a force **P** of magnitude 750 N is applied to the pedal shown, determine (*a*) the diameter of the pin at C for which the average shearing stress in the pin is 40 MPa, (*b*) the corresponding bearing stress in the pedal at C, (*c*) the corresponding bearing stress in the each support bracket at C.



Draw free body diagram of ACD.
Since ACD is a 3-force member, the reaction at C is directed toward point E, the intersection of the lines of action of the other two forces.



From geometry, $CE = \sqrt{300^2 + 125^2} = 325$ mm

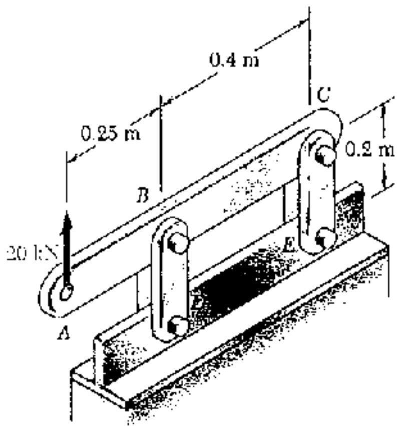
$+\uparrow \Sigma F_y = 0: \quad \frac{125}{325} C - P = 0 \qquad C = 2.6P = (2.6)(750) = 1950$ N

(a) $\tau_{pin} = \dfrac{\frac{1}{2}C}{A_{pin}} = \dfrac{\frac{1}{2}C}{\frac{\pi}{4}d^2} \qquad d = \sqrt{\dfrac{2C}{\pi \tau_{pin}}} = \sqrt{\dfrac{(2)(1950)}{\pi(40\times10^6)}} = 5.57\times10^{-3}$ m $\qquad$ **5.57 mm** ◄

(b) $\sigma_b = \dfrac{C}{A_b} = \dfrac{C}{dt} = \dfrac{1950}{(5.57\times10^{-3})(9\times10^{-3})} = 38.9\times10^6$ Pa $\qquad$ **38.9 MPa** ◄

(c) $\sigma_b = \dfrac{\frac{1}{2}C}{A_b} = \dfrac{C}{2dt} = \dfrac{1950}{(2)(5.57\times10^{-3})(5\times10^{-3})} = 35.0\times10^6$ Pa $\qquad$ **35.0 MPa** ◄

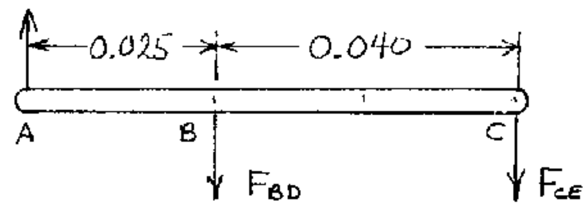
Problem 1.27



1.27 For the assembly and loading of Prob. 1.9, determine (*a*) the average shearing stress in the pin at *B*, (*b*) the average bearing stress at *B* in member *BD*, (*c*) the average bearing stress at *B* in member *ABC*, knowing that this member has a 10 × 50-mm uniform rectangular cross section.

1.9 Each of the four vertical links has an 8 × 36-mm uniform rectangular cross section and each of the four pins has a 16-mm diameter. Determine the maximum value of the average normal stress in the links connecting (*a*) points *B* and *D*, (*b*) points *C* and *E*.

Use bar ABC as a free body



$$\Sigma M_C = 0 \qquad (0.040)F_{BD} - (0.025 + 0.040)(20 \times 10^3) = 0$$

$$F_{BD} = 32.5 \times 10^3 \text{ N}$$

(a) Shear pin at B $\qquad \tau = \dfrac{F_{BD}}{2A}$ for double shear

where $A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.016)^2 = 201.06 \times 10^{-6} \text{ m}^2$

$$\tau = \frac{32.5 \times 10^3}{(2)(201.06 \times 10^{-6})} = 80.8 \times 10^6 \qquad 80.8 \text{ MPa}$$

(b) Bearing: link BD $\qquad A = dt = (0.016)(0.008) = 128 \times 10^{-6} \text{ m}^2$

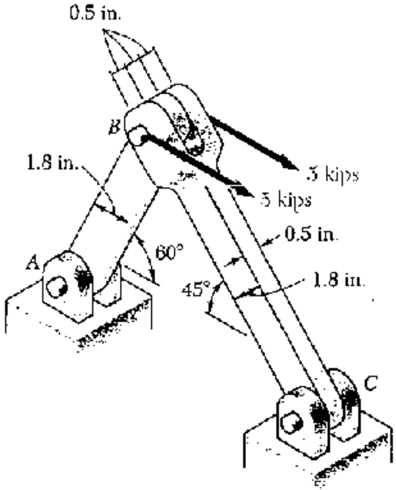
$$\sigma_b = \frac{\frac{1}{2} F_{BD}}{A} = \frac{(0.5)(32.5 \times 10^3)}{128 \times 10^{-6}} = 126.95 \times 10^6 \qquad 127.0 \text{ MPa}$$

(c) Bearing in ABC at B

$$A = dt = (0.016)(0.010) = 160 \times 10^{-6} \text{ m}^2$$

$$\sigma_b = \frac{F_{BD}}{A} = \frac{32.5 \times 10^3}{160 \times 10^{-6}} = 203 \times 10^6 \qquad 203 \text{ MPa}$$

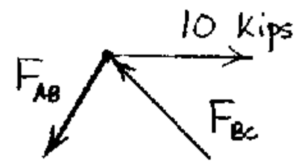
Problem 1.28

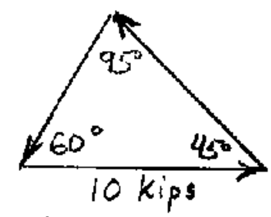


1.28 For the assembly and loading of Prob. 1.10, determine (*a*) the average shearing stress in the pin at *C*, (*b*) the average bearing stress at *C* in member *BC*, (*c*) the average bearing stress at *B* in member *BC*.

1.10 Two horizontal 5-kip forces are applied to pin *B* of the assembly shown. Knowing that a pin of 0.8-in. diameter is used at each connection, determine the maximum value of the average normal stress (*a*) in link *AB*, (*b*) in link *BC*.

Use joint B as free body





Force triangle

Law of Sines

$$\frac{F_{AB}}{\sin 45°} = \frac{F_{BC}}{\sin 60°} = \frac{10}{\sin 95°} \qquad F_{BC} = 8.9658 \text{ kips}$$

(a) Shearing stress in pin at C $\qquad \tau = \frac{F_{BC}}{2A_P}$

$$A_P = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.8)^2 = 0.5026 \text{ in}^2$$

$$\tau = \frac{8.9658}{(2)(0.5026)} = 8.92 \qquad 8.92 \text{ ksi}$$

(b) Bearing stress at C in member BC $\qquad \sigma_b = \frac{F_{BC}}{A}$

$$A = t\,d = (0.5)(0.8) = 0.4 \text{ in}^2$$

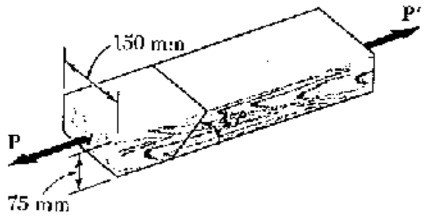
$$\sigma_b = \frac{8.9658}{0.4} = 22.4 \qquad 22.4 \text{ ksi}$$

(c) Bearing stress at B in member BC $\qquad \sigma_b = \frac{F_{BC}}{A}$

$$A = 2t\,d = 2(0.5)(0.8) = 0.8 \text{ in}^2$$

$$\sigma_b = \frac{8.9658}{0.8} = 11.21 \qquad 11.21 \text{ ksi}$$

Problem 1.29



1.29 Two wooden members of uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $P = 11$ kN, determine the normal and shearing stresses in the glued splice.

$$\theta = 90° - 45° = 45°$$

$$P = 11 \text{ kN} = 11 \times 10^3 \text{ N}$$

$$A_o = (150)(75) = 11.25 \times 10^3 \text{ mm}^2 = 11.25 \times 10^{-3} \text{ m}^2$$

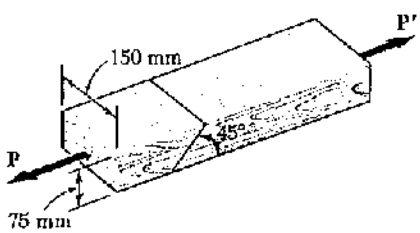
$$\sigma = \frac{P \cos^2\theta}{A_o} = \frac{(11 \times 10^3)\cos^2 45°}{11.25 \times 10^{-3}}$$

$$= 489 \times 10^3 \text{ Pa} \qquad \sigma = 489 \text{ kPA}$$

$$\tau = \frac{P \sin 2\theta}{2A_o} = \frac{(11 \times 10^3)(\sin 90°)}{(2)(11.25 \times 10^{-3})}$$

$$= 4.89 \times 10^3 \text{ Pa} \qquad \tau = 489 \text{ kPA}$$

Problem 1.30



1.30 Two wooden members of uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that the maximum allowable shearing stress in the glued splice is 620 kPa, determine (*a*) the largest load **P** that can be safely applied, (*b*) the corresponding tensile stress in the splice.

$$\theta = 90° - 45° = 45°$$

$$A_o = (150)(75) = 11.25 \times 10^3 \text{ mm}^2 = 11.25 \times 10^{-3} \text{ m}^2$$

$$\tau = 620 \text{ kPa} = 620 \times 10^3 \text{ Pa}$$

$$\tau = \frac{P \sin 2\theta}{2A_o}$$

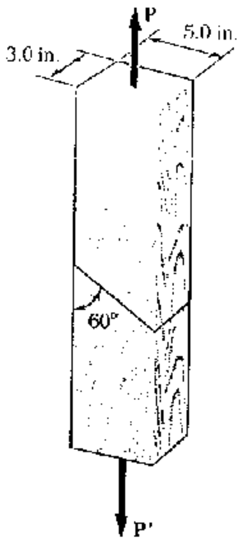
(a) $$P = \frac{2A_o\tau}{\sin 2\theta} = \frac{(2)(11.25 \times 10^{-3})(620 \times 10^3)}{\sin 90°}$$

$$= 13.95 \times 10^3 \text{ N} \qquad 13.95 \text{ kN}$$

(b) $$\sigma = \frac{P\cos^2\theta}{A_o} = \frac{(13.95 \times 10^3)(\cos 45°)^2}{11.25 \times 10^{-3}}$$

$$= 620 \times 10^3 \text{ Pa} \qquad 620 \text{ kPa}$$

Problem 1.31



1.31 The 1.4 kip load **P** is supported by two wooden members of uniform cross section that are joined by the simple glued scarf splice shown. Determine the normal and shearing stresses in the glued splice.

$P = 1400 \text{ lb} \qquad \theta = 90° - 60° = 30°$

$A_o = (5.0)(3.0) = 15 \text{ in}^2$

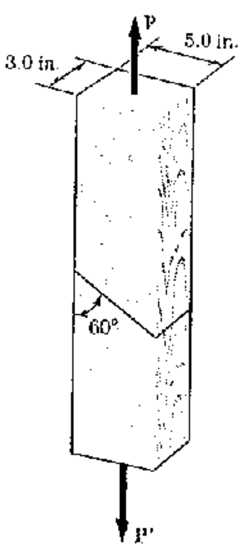
$$\sigma = \frac{P\cos^2\theta}{A_o} = \frac{(1400)(\cos 30°)^2}{15}$$

$\sigma = 70.0 \text{ psi}$ ◄

$$\tau = \frac{P\sin 2\theta}{2A_o} = \frac{(1400)\sin 60°}{(2)(15)}$$

$\tau = 40.4 \text{ psi}$ ◄

Problem 1.32



1.32 Two wooden members of uniform cross section are joined by the simple scarf splice shown. Knowing that the maximum allowable tensile stress in the glued splice is 75 psi, determine (*a*) the largest load **P** that can be safely supported, (*b*) the corresponding stress in the splice.

$A_o = (5.0)(3.0) = 15 \text{ in}^2$

$\theta = 90° - 60° = 30°$

$$\sigma = \frac{P\cos^2\theta}{A_o}$$

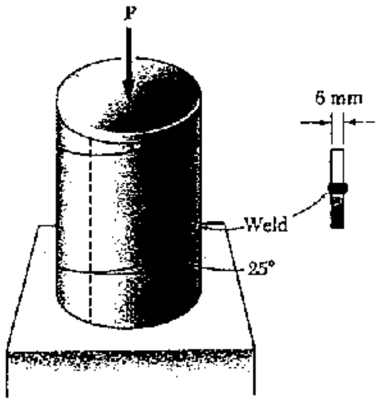
(a) $$P = \frac{\sigma A_o}{\cos^2\theta} = \frac{(75)(15)}{\cos^2 30°} = 1500 \text{ lb}$$

$P = 1.500 \text{ kips}$ ◄

(b) $$\tau = \frac{P\sin 2\theta}{2A_o} = \frac{(1500)\sin 60°}{(2)(15)}$$

$\tau = 43.3 \text{ psi}$ ◄

Problem 1.33



1.33 A steel pipe of 300-mm outer diameter is fabricated from 6-mm-thick plate by welding along a helix that forms an angle of 25° with a plane perpendicular to the axis of the pipe. Knowing that a 250-kN axial force **P** is applied to the pipe, determine the normal and shearing stresses in directions respectively normal and tangential to the weld.

$d_o = 0.300\ m \qquad r_o = \frac{1}{2}d_o = 0.150\ m$

$r_i = r_o - t = 0.150 - 0.006 = 0.144\ m$

$A_o = \pi(r_o^2 - r_i^2) = \pi(0.150^2 - 0.144^2)$

$= 5.54 \times 10^{-3}\ m^2$

$\theta = 25°$

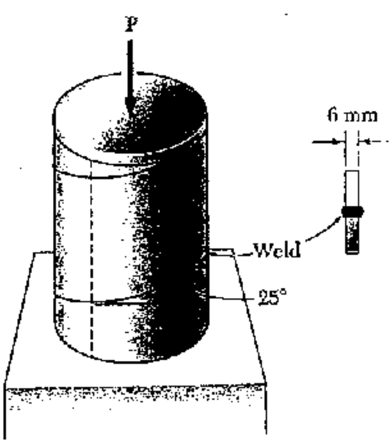
$\sigma = \frac{P}{A_o}\cos^2\theta = \frac{-250 \times 10^3 \cos^2 25°}{5.54 \times 10^{-3}}$

$= -37.1 \times 10^6 \qquad \sigma = -37.1\ MPa$

$\tau = \frac{P}{2A_o}\sin 2\theta = \frac{-250 \times 10^3 \sin 50°}{(2)(5.54 \times 10^{-3})}$

$= -17.28 \times 10^6 \qquad \tau = 17.28\ MPa$

Problem 1.34



1.34 A steel pipe of 300-mm outer diameter is fabricated from 6-mm-thick plate by welding along a helix that forms an angle of 25° with a plane perpendicular to the axis of the pipe. Knowing that the maximum allowable normal and shearing stresses in the directions respectively normal and tangential to the weld are $\sigma = 50$ MPa and $\tau = 30$ MPa, determine the magnitude P of the largest axial force that can be applied to the pipe.

$d_o = 0.300\ m \qquad r_o = \frac{1}{2}d_o = 0.150\ m$

$r_i = r_o - t = 0.150 - 0.006 = 0.144\ m$

$A_o = \pi(r_o^2 - r_i^2) = \pi(0.150^2 - 0.144^2)$

$= 5.54 \times 10^{-3}\ m^2$

$\theta = 25°$

Based on $|\sigma| = 50$ MPa: $\quad \sigma = \frac{P}{A_o}\cos^2\theta$

$P = \frac{A_o\sigma}{\cos^2\theta} = \frac{(5.54 \times 10^{-3})(50 \times 10^6)}{\cos^2 25°} = 337 \times 10^3$

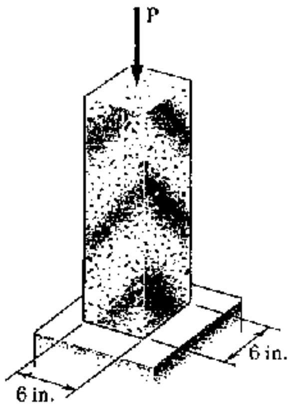
Based on $|\tau| = 30$ MPa $\quad \tau = \frac{P}{2A_o}\sin 2\theta$

$P = \frac{2A_o\tau}{\sin 2\theta} = \frac{(2)(5.54 \times 10^{-3})(30 \times 10^6)}{\sin 50°} = 434 \times 10^3$

Smaller value is the allowable value of P $\therefore \quad P = 337$ kN

Problem 1.35

1.35 A 240-kip load **P** is applied to the granite block shown. Determine the resulting maximum value of (*a*) the normal stress, (*b*) the shearing stress. Specify the orientation of the plane on which each of these maximum values occurs.



$A_o = (6)(6) = 36 \text{ in}^2$

$\sigma = \dfrac{P}{A_o}\cos^2\theta = \dfrac{-240}{36}\cos^2\theta = -6.67\cos^2\theta$

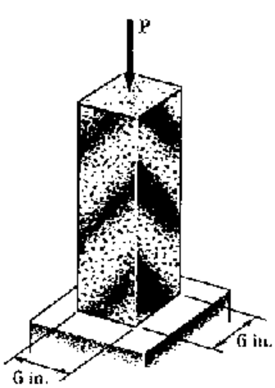
(a) max. tensile stress $= 0$ at $\theta = 90°$

max. compressive stress $= 6.67$ ksi at $\theta = 0°$

(b) $\tau_{max} = \dfrac{P}{2A_o} = \dfrac{240}{(2)(36)} = 3.33$ ksi at $\theta = 45°$

Problem 1.36

1.36 A centric load **P** is applied to the granite block shown. Knowing that the resulting maximum value of the shearing stress in the block is 2.5 ksi, determine (*a*) the magnitude of **P**, (*b*) the orientation of the surface on which the maximum shearing stress occurs, (*c*) the normal stress exerted on the surface, (*d*) the maximum value of the normal stress in the block.



$A_o = (6)(6) = 36 \text{ in}^2 \qquad \tau_{max} = 2.5$ ksi

$\theta = 45°$ for plane of τ_{max}

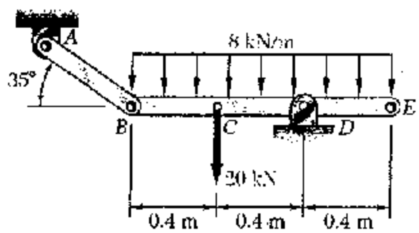
(a) $\tau_{max} = \dfrac{|P|}{2A_o} \therefore |P| = 2A_o\tau_{max} = (2)(36)(2.5) = 180$ Kips

(b) $\sin 2\theta = 1 \quad 2\theta = 90° \quad \theta = 45°$

(c) $\sigma_{45} = \dfrac{P}{A_o}\cos^2 45° = \dfrac{P}{2A_o} = \dfrac{-180}{(2)(36)} = -2.5$ ksi

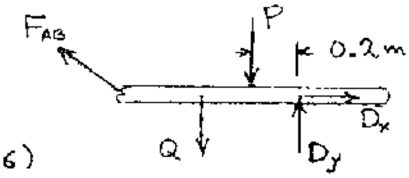
(d) $\sigma_{max} = \dfrac{P}{A_o} = \dfrac{-180}{36} = -5$ ksi

Problem 1.37



1.37 Link *AB* is to be made of a steel for which the ultimate normal stress is 450 MPa. Determine the cross-sectional area for *AB* for which the factor of safety will be 3.50. Assume that the link will be adequately reinforced around the pins at *A* and *B*.

$P = (1.2)(8) = 9.6 \text{ kN}$



$+\circlearrowleft \Sigma M_D = 0$

$-(0.8)(F_{AB} \sin 35°) + (0.2)(9.6) + (0.4)(20) = 0$

$F_{AB} = 21.619 \text{ kN} = 21.619 \times 10^3 \text{ N}$

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{\sigma_{ult}}{F.S.}$$

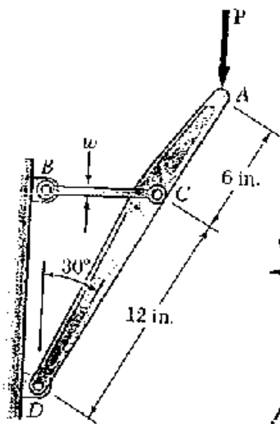
$$A_{AB} = \frac{(F.S.) F_{AB}}{\sigma_{ult}} = \frac{(3.50)(21.619 \times 10^3)}{450 \times 10^6}$$

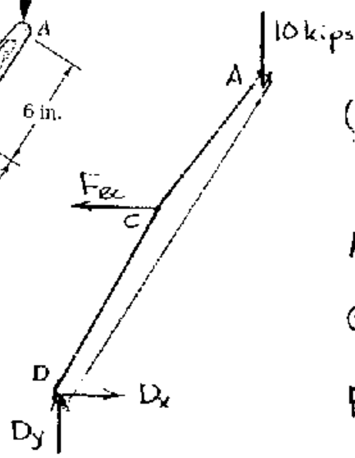
$= 168.1 \times 10^{-6} \text{ m}^2$

$A_{AB} = 168.1 \text{ mm}^2$

Problem 1.38

1.38 The horizontal link BC is $\frac{1}{4}$ in. thick, has a width $w = 1.25$ in., and is made of a steel with a 65-ksi ultimate strength in tension. What is the factor of safety if the structure shown is designed to support a load of $P = 10$ kips?





$+\circlearrowleft \Sigma M_C = 0$

$(12 \cos 30°) F_{AB} - (18 \sin 30°)(10) = 0$

$F_{BC} = 8.6603$ kips

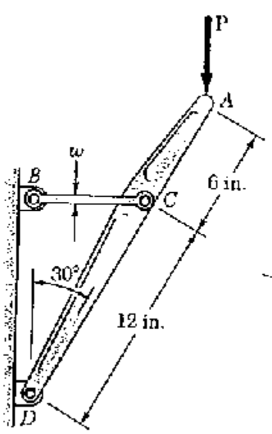
$A_{BC} = (\frac{1}{4})(1.25) = 0.3125 \text{ in}^2$

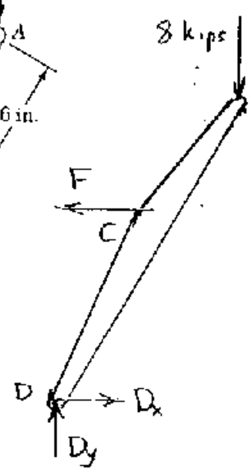
$\sigma_{BC} = \dfrac{F_{BC}}{A_{BC}} = \dfrac{\sigma_{ult}}{F.S}$

$F.S = \dfrac{A_{BC}\, \sigma_{ult}}{F_{BC}} = \dfrac{(0.3125)(65)}{8.6603} = 2.35$

Problem 1.39

1.39 The horizontal link BC is $\frac{1}{4}$ in. thick and is made of a steel with a 65-ksi ultimate strength in tension. What should be the width w of the link if the structure shown is to be designed to support a load $P = 8$ kips with a factor of safety equal to 3?





$+\circlearrowleft \Sigma M_C = 0$

$(12 \cos 30°) F_{AB} - (18 \sin 30°)(8) = 0$

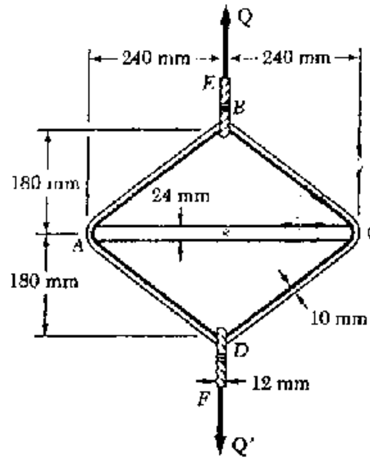
$F_{AB} = 6.9282$ kips

$\sigma_{AB} = \dfrac{F_{AB}}{A_{AB}} = \dfrac{F_{AB}}{t w} = \dfrac{\sigma_{ult}}{F.S.}$

$w = \dfrac{(F.S.)\, F_{AB}}{t\, \sigma_{ult}} = \dfrac{(3)(6.9282)}{(\frac{1}{4})(65)}$

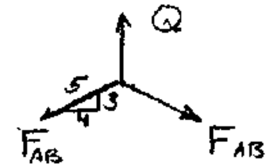
$w = 1.279$ in.

Problem 1.40



1.40 A steel loop *ABCD* of length 1.2 m and of 10-mm diameter is placed as shown around a 24-mm-diameter aluminum rod *AC*. Cables *BE* and *DF*, each of 12-mm diameter, are used to apply the load **Q**. Knowing that the ultimate strength of the steel used for the loop and the cables is 480 MPa, determine the largest load **Q** that can be applied if an overall factor of safety of 3 is desired.

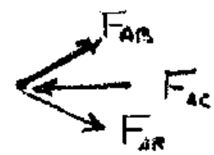
Using joint B as free body and considering symmetry



$$2 \cdot \frac{3}{5} F_{AB} - Q = 0$$

$$Q = \frac{6}{5} F_{AB}$$

Using joint A as a free body and considering symmetry



$$2 \cdot \frac{4}{5} F_{AB} - F_{AC} = 0$$

$$\frac{8}{5} \cdot \frac{5}{6} Q - F_{AC} = 0 \quad \therefore \quad Q = \frac{3}{4} F_{AC}$$

Based on strength of cable BF

$$Q_U = \sigma_U A = \sigma_U \frac{\pi}{4} d^2 = (480 \times 10^6) \frac{\pi}{4} (0.012)^2 = 54.29 \times 10^3 \text{ N}$$

Based on strength of steel loop

$$Q_U = \frac{6}{5} F_{AB,U} = \frac{6}{5} \sigma_U A = \frac{6}{5} \sigma_U \frac{\pi}{4} d^2$$

$$= \frac{6}{5} (480 \times 10^6) \frac{\pi}{4} (0.010)^2 = 45.24 \times 10^3 \text{ N}$$

Based on strength of rod AC

$$Q_U = \frac{3}{4} F_{AC,U} = \frac{3}{4} \sigma_U A = \frac{3}{4} \sigma_U \frac{\pi}{4} d^2$$

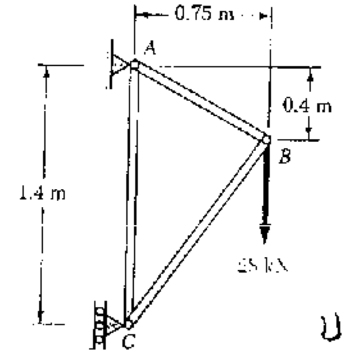
$$= \frac{3}{4} (260 \times 10^6) \frac{\pi}{4} (0.024)^2 = 88.22 \times 10^3 \text{ N}$$

Actual ultimate load Q_U is the smallest $\therefore \quad Q_U = 45.24 \times 10^3$ N

Allowable load $\quad Q = \frac{Q_U}{F.S.} = \frac{45.24 \times 10^3}{3} = 15.08 \times 10^3$ N

$= 15.08$ kN

Problem 1.41

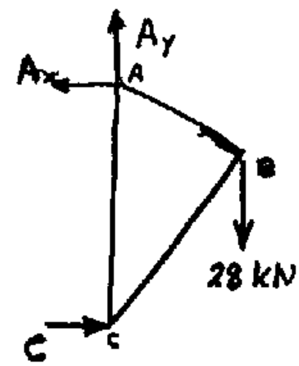


1.41 Members *AB* and *BC* of the truss shown are made of the same alloy. It is known that a 20-mm-square bar of the same alloy was tested to failure and that an ultimate load of 120 kN was recorded. If a factor of safety of 3.2 is to be achieved for both bars, determine the required cross- sectional area of (*a*) bar *AB*, (*b*) bar *AC*.

Length of member AB

$$l_{AB} = \sqrt{0.75^2 + 0.4^2} = 0.85 \text{ m}$$

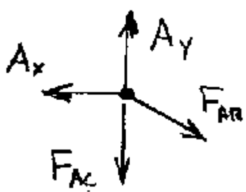
Use entire truss as a free body



$+\circlearrowleft \Sigma M_c = 0 \qquad 1.4 A_x - (0.75)(28) = 0 \qquad A_x = 15 \text{ kN}$

$+\uparrow \Sigma F_y = 0 \qquad A_y - 28 = 0 \qquad A_y = 28 \text{ kN}$

Use joint A as free body



$\xrightarrow{+} \Sigma F_x = 0 \qquad \frac{0.75}{0.85} F_{AB} - A_x = 0$

$$F_{AB} = \frac{(0.85)(15)}{0.75} = 17 \text{ kN}$$

$+\uparrow \Sigma F_y = 0 \qquad A_y - F_{AC} - \frac{0.4}{0.85} F_{AB} = 0$

$$F_{AC} = 28 - \frac{(0.4)(17)}{0.85} = 20 \text{ kN}$$

For the test bar $\quad A = (0.020)^2 = 400 \times 10^{-6} \text{ m}^2 \qquad P_u = 120 \times 10^3 \text{ N}$

For the material $\quad \sigma_u = \frac{P_u}{A} = \frac{120 \times 10^3}{400 \times 10^{-6}} = 300 \times 10^6 \text{ Pa}$

(a) For member AB $\quad \text{F.S.} = \frac{P_u}{F_{AB}} = \frac{\sigma_u A_{AB}}{F_{AB}}$

$$A_{AB} = \frac{(\text{F.S.}) F_{AB}}{\sigma_u} = \frac{(3.2)(17 \times 10^3)}{300 \times 10^6} = 181.33 \times 10^{-6} \text{ m}^2$$

$$A_{AB} = 181.3 \text{ mm}^2 \blacktriangleleft$$

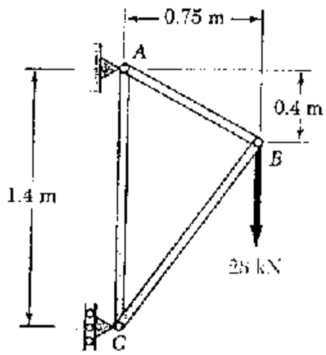
(b) For member AC $\quad \text{F.S.} = \frac{P_u}{F_{AC}} = \frac{\sigma_u A_{AC}}{F_{AC}}$

$$A_{AC} = \frac{(\text{F.S.}) F_{AC}}{\sigma_u} = \frac{(3.2)(20 \times 10^3)}{300 \times 10^6} = 213.33 \times 10^{-6} \text{ m}^2$$

$$A_{AC} = 213 \text{ mm}^2 \blacktriangleleft$$

Problem 1.42

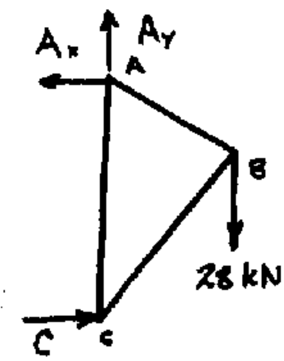
1.42 Members AB and BC of the truss shown are made of the same alloy. It is known that a 20-mm-square bar of the same alloy was tested to failure and that an ultimate load of 120 kN was recorded. If bar AB has a cross-sectional area of 225 mm^2, determine (*a*) the factor of safety for bar AB, (*b*) the cross-sectional area of bar AC if it is to have the same factor of safety as bar AB.



Length of member AB

$$\ell_{AB} = \sqrt{0.75^2 + 0.4^2} = 0.85 \text{ m}$$

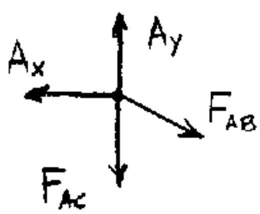
Use entire truss as a free body



$\circlearrowleft \Sigma M_C = 0 \qquad 1.4 A_x - (0.75)(28) = 0 \qquad A_x = 15 \text{ kN}$

$+\uparrow \Sigma F_y = 0 \qquad A_y - 28 = 0 \qquad A_y = 28 \text{ kN}$

Use joint A as free body



$\xrightarrow{+} \Sigma F_x = 0 \qquad \frac{0.75}{0.85} F_{AB} - A_x = 0$

$$F_{AB} = \frac{(0.85)(15)}{0.75} = 17 \text{ kN}$$

$+\uparrow \Sigma F_y = 0 \qquad A_y - F_{AC} - \frac{0.4}{0.85} F_{AB} = 0$

$$F_{AC} = 28 - \frac{(0.4)(17)}{0.85} = 20 \text{ kN}$$

For the test bar $\quad A = (0.020)^2 = 400 \times 10^{-6} \text{ m}^2 \qquad P_U = 120 \times 10^3 \text{ N}$

For the material $\quad \sigma_U = \frac{P_U}{A} = \frac{120 \times 10^3}{400 \times 10^{-6}} = 300 \times 10^6 \text{ Pa}$

(a) For bar AB $\quad \text{F.S.} = \frac{F_U}{F_{AB}} = \frac{\sigma_U A_{AB}}{F_{AB}} = \frac{(300 \times 10^6)(225 \times 10^{-6})}{17 \times 10^3}$

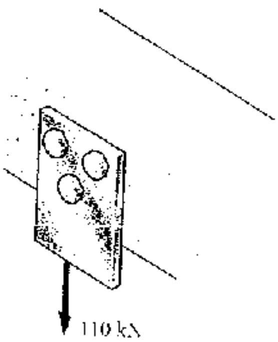
F.S. = 3.97 ◄

(b) For bar AC $\quad \text{F.S.} = \frac{F_U}{F_{AC}} = \frac{\sigma_U A_{AC}}{F_{AC}}$

$$A_{AC} = \frac{(\text{F.S.}) F_{AC}}{\sigma_U} = \frac{(3.97)(20 \times 10^3)}{300 \times 10^6} = 264.7 \times 10^{-6} \text{ m}^2$$

$A_{AC} = 265 \text{ mm}^2$ ◄

Problem 1.43



1.43 Three steel bolts are to be used to attach the steel plate shown to a wooden beam. Knowing that the plate will support a 110 kN load, that the ultimate shearing stress for the steel used is 360 MPa, and that a factor of safety of 3.35 is desired, determine the required diameter of the bolts.

For each bolt $P = \frac{110}{3} = 36.667$ kN

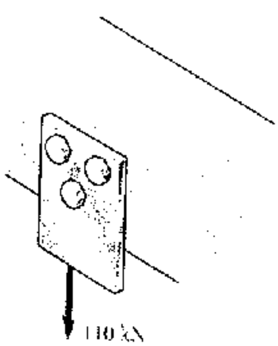
Required $P_U = (F.S.)P = (3.35)(36.667) = 122.83$ kN

$$\tau_U = \frac{P_U}{A} = \frac{P_U}{\frac{\pi}{4}d^2} = \frac{4P_U}{\pi d^2}$$

$$d = \sqrt{\frac{4P_U}{\pi \tau_U}} = \sqrt{\frac{(4)(122.83 \times 10^3)}{\pi(360 \times 10^6)}} = 20.8 \times 10^{-3} \text{ m}$$

$d = 20.8$ mm

Problem 1.44



1.44 Three 18-mm-diameter steel bolts are to be used to attach the steel plate shown to a wooden beam. Knowing that the plate will support a 110-kN load and that the ultimate shearing stress for the steel used is 360 MPa, determine the factor of safety for this design.

For each bolt $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(18)^2 = 254.47 \text{ mm}^2$

$= 254.47 \times 10^{-6} \text{ m}^2$

$P_U = A\tau_U = (254.47 \times 10^{-6})(360 \times 10^6)$

$= 91.609 \times 10^3$ N

For the three bolts $P_U = (3)(91.609 \times 10^3)$

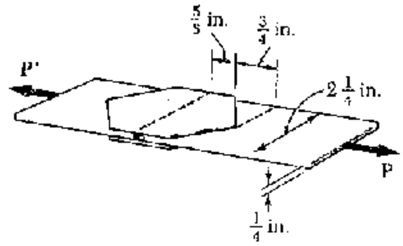
$= 274.83 \times 10^3$ N

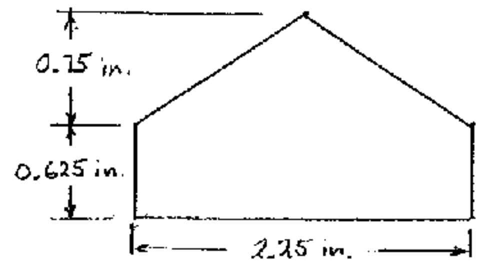
Factor of safety

$$F.S. = \frac{P_U}{P} = \frac{274 \times 10^3}{110 \times 10^3} = 2.50$$

Problem 1.45

1.45 Two plates, each $\frac{1}{8}$ in. thick, are used to splice a plastic strip as shown. Knowing that the ultimate shearing stress of the bonding between the surfaces is 130 psi, determine the factor of safety with respect to shear when $P = 325$ lb.





Bond area: (See figure)

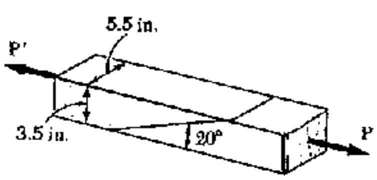
$$A = \frac{1}{2}(2.25)(0.75) + (2.25)(0.625) = 2.25 \text{ in}^2$$

$$P_U = 2 A \tau_U = (2)(2.25)(130) = 585 \text{ lb.}$$

$$F.S. = \frac{P_U}{P} = \frac{585}{325} = 1.800$$

Problem 1.46

1.46 Two wooden members of 3.5 × 5.5-in. uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that the maximum allowable shearing stress in the glued splice is 75 psi, determine the largest axial load **P** that can be safely applied.



$$A_0 = (3.5)(5.5) = 19.25 \text{ in}^2$$

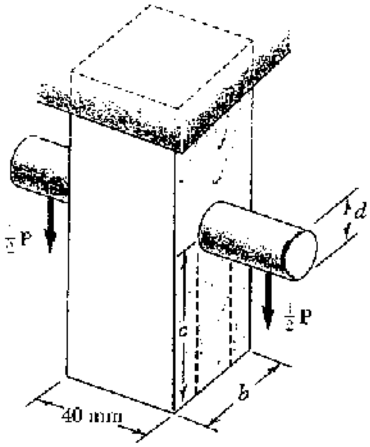
$$\theta = 90° - 20° = 70°$$

$$\tau = \frac{P}{A_0} \sin\theta\cos\theta = \frac{P}{2A_0} \sin 2\theta$$

$$P = \frac{2A_0\tau}{\sin 2\theta} = \frac{(2)(19.25)(75)}{\sin 140°} = 4492 \text{ lb} = 4.49 \text{ kips}$$

Problem 1.47

1.47 A load **P** is supported as shown by a steel pin that has been inserted in a short wooden member hanging from the ceiling. The ultimate strength of the wood used is 60 MPa in tension and 7.5 MPa in shear, while the ultimate strength of the steel is 145 MPa in shear. Knowing that $b = 40$ mm, $c = 55$ mm, and $d = 12$ mm, determine the load **P** if an overall factor of safety of 3.2 is desired.



Based on double shear in pin

$$P_U = 2A\tau_U = 2\frac{\pi}{4}d^2\tau_U$$
$$= \frac{\pi}{4}(2)(0.012)^2(145\times10^6) = 32.80\times10^3 \text{ N}$$

Based on tension in wood

$$P_U = A\sigma_U = w(b-d)\sigma_U$$
$$= (0.040)(0.040-0.012)(60\times10^6)$$
$$= 67.2\times10^3 \text{ N}$$

Based on double shear in the wood

$$P_U = 2A\tau_U = 2wc\tau_U = (2)(0.040)(0.055)(7.5\times10^6)$$
$$= 33.0\times10^3 \text{ N}$$

Use smallest $\quad P_U = 32.8\times10^3$ N

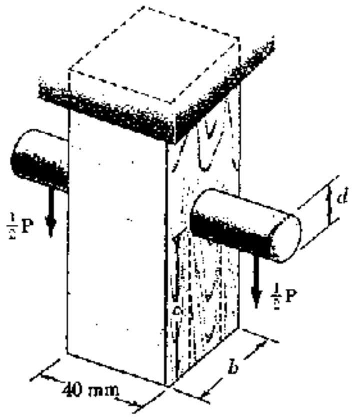
Allowable $\quad P = \frac{P_U}{F.S.} = \frac{32.8\times10^3}{3.2} = 10.25\times10^3$ N

10.25 kN

Problem 1.48

1.48 For the support of Prob. 1.47, knowing that the diameter of the pin is d = 16 mm and that the magnitude of the load is P = 20 kN, determine (*a*) the factor of safety for the pin, (*b*) the required values of *b* and *c* if the factor of safety for the wooden members is the same as that found in part *a* for the pin.

1.47 A load **P** is supported as shown by a steel pin that has been inserted in a short wooden member hanging from the ceiling. The ultimate strength of the wood used is 60 MPa in tension and 7.5 MPa in shear, while the ultimate strength of the steel is 145 MPa in shear. Knowing that b = 40 mm, c = 55 mm, and d = 12 mm, determine the load **P** if an overall factor of safety of 3.2 is desired.



$P = 20 \text{ kN} = 20 \times 10^3 \text{ N}$

(a) Pin: $A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.016)^2 = 201.06 \times 10^{-6} \text{ m}^2$

Double shear $\quad \tau = \frac{P}{2A} \qquad \tau_U = \frac{P_U}{2A}$

$$P_U = 2A\tau_U = (2)(201.16\times10^{-6})(145\times10^6)$$

$$= 58.336 \times 10^3 \text{ N}$$

$$F.S. = \frac{P_U}{P} = \frac{58.336\times10^3}{20\times10^3} = 2.92$$

(b) Tension in wood $\quad P_U = 58.336\times10^3$ N for same F.S.

$$\sigma_U = \frac{P_U}{A} = \frac{P_U}{w(b-d)} \qquad \text{where } w = 40 \text{ mm} = 0.040 \text{ m}$$

$$b = d + \frac{P_U}{w\sigma_U} = 0.016 + \frac{58.336\times10^3}{(0.040)(60\times10^6)} = 40.3\times10^3 \text{ m}$$

$$b = 40.3 \text{ mm}$$

Shear in wood $\quad P_U = 58.336\times10^3$ N for same F.S.

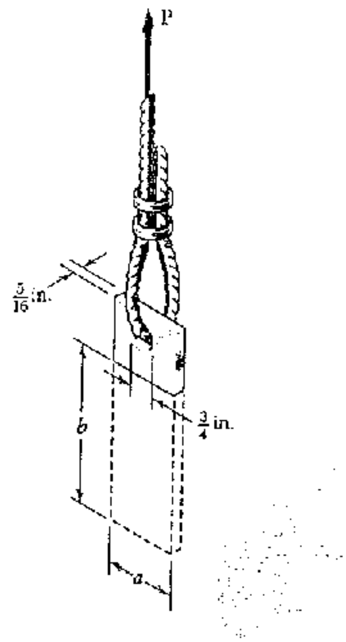
Double shear; each area is $A = wc$

$$\tau_U = \frac{P_U}{2A} = \frac{P_U}{2wc}$$

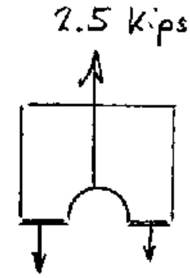
$$c = \frac{P_U}{2w\tau_U} = \frac{58.336\times10^3}{(2)(0.040)(7.5\times10^6)} = 97.2\times10^{-3} \text{ m}$$

$$c = 97.2 \text{ mm}$$

Problem 1.49



1.49 A steel plate $\frac{5}{16}$ in. thick is embedded in a horizontal concrete slab and is used to anchor a high-strength vertical cable as shown. The diameter of the hole in the plate is $\frac{3}{4}$ in., the ultimate strength of the steel used is 36 ksi, and the ultimate bonding stress between plate and concrete is 300 psi. Knowing that a factor of safety of 3.60 is desired when $P = 2.5$ kips, determine (*a*) the required width a of the plate, (*b*) the minimum depth b to which a plate of that width should be embedded in the concrete slab. (Neglect the normal stresses between the concrete and the lower end of the plate.)



Based on tension in plate

$$A = (a - d)\,t$$

$$P_U = \sigma_U A$$

$$F.S. = \frac{P_U}{P} = \frac{\sigma_U (a-d)t}{P}$$

Solving for a

$$a = d + \frac{(F.S.)P}{\sigma_U t} = \frac{3}{4} + \frac{(3.60)(2.5)}{(36)(\frac{5}{16})}$$

(a) $\quad a = 1.550$ in.

Based on shear between plate and concrete slab

$$A = \text{perimeter} \times \text{depth} = 2(a+t)\,b \qquad \tau_U = 0.300 \text{ ksi}$$

$$P_U = \tau_U A = 2\tau_U (a+t)\,b \qquad F.S. = \frac{P_U}{P}$$

Solving for b

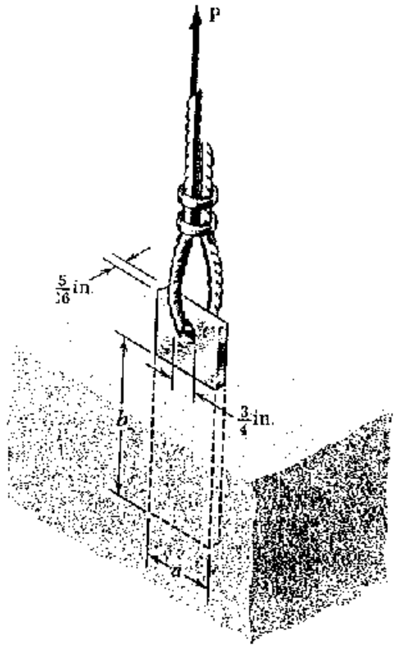
$$b = \frac{(F.S.)P}{2(a+t)\tau_U} = \frac{(3.6)(2.5)}{(2)(1.550 + \frac{5}{16})(0.300)}$$

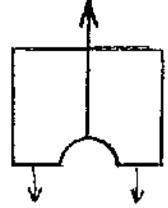
(b) $\quad b = 8.05$ in.

Problem 1.50

1.50 Determine the factor of safety for the cable anchor in Prob. 1.49 when $P = 3$ kips, knowing that $a = 2$ in. and $b = 7.5$ in.

1.49 A steel plate $\frac{5}{16}$ in. thick is embedded in a horizontal concrete slab and is used to anchor a high-strength vertical cable as shown. The diameter of the hole in the plate is $\frac{3}{4}$ in., the ultimate strength of the steel used is 36 ksi, and the ultimate bonding stress between plate and concrete is 300 psi. Knowing that a factor of safety of 3.60 is desired when $P = 2.5$ kips, determine (*a*) the required width *a* of the plate, (*b*) the minimum depth *b* to which a plate of that width should be embedded in the concrete slab. (Neglect the normal stresses between the concrete and the lower end of the plate.)





Based on tension in plate

$A = (a - d)t$

$= (2 - \frac{3}{4})(\frac{5}{16}) = 0.3906 \text{ in}^2$

$P_U = \sigma_U A$

$= (36)(0.3906) = 14.06 \text{ kips}$

$F.S. = \frac{P_U}{P} = \frac{14.06}{3} = 4.69$

Based on shear between plate and concrete slab

$A = \text{perimeter} \times \text{depth} = 2(a + t)b = 2(2 + \frac{5}{16})(7.5)$

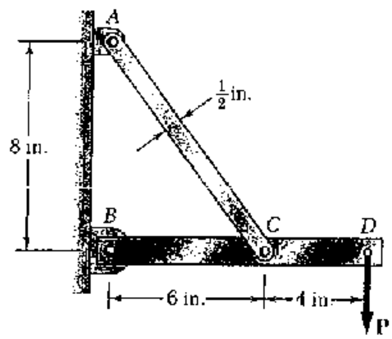
$A = 34.69 \text{ in}^2 \qquad \tau_U = 0.300 \text{ ksi}$

$P_U = \tau_U A = (0.300)(34.69) = 10.41 \text{ kips}$

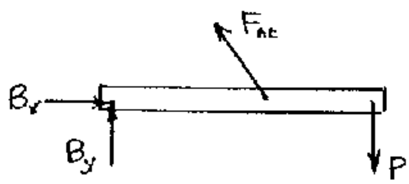
$F.S. = \frac{P_U}{P} = \frac{10.41}{3} = 3.47$

Actual factor of safety is the smaller value $\quad F.S. = 3.47$ ◄

Problem 1.51



1.51 Link AC is made of a steel with a 65-ksi ultimate normal stress and has a $\frac{1}{4} \times \frac{1}{2}$-in. uniform rectangular cross section. It is connected to a support at A and to member BCD at C by $\frac{3}{8}$-in.-diameter pins, while member BCD is connected to its support at B by a $\frac{5}{16}$-in.-diameter pin; all of the pins are made of a steel with a 25-ksi ultimate shearing stress and are in single shear. Knowing that a factor of safety of 3.25 is desired, determine the largest load **P** that can be applied at D. Note that link AC is not reinforced around the pin holes



Use free body BCD.

$$+\circlearrowleft M_B = 0: \ (6)\left(\tfrac{8}{10}F_{AC}\right) - 10\,P = 0$$

$$P = 0.48\ F_{AC} \qquad (1)$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0: \quad B_x - \tfrac{6}{10}F_{AC} = 0$$

$$B_x = \tfrac{6}{10}F_{AC} = 1.25\,P \ \rightarrow$$

$$+\circlearrowleft M_C = 0: \ -6B_y - 4P = 0$$

$$B_y = -\tfrac{2}{3}P \qquad \text{i.e. } B_y = \tfrac{2}{3}P \downarrow$$

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{1.25^2 + \left(\tfrac{2}{3}\right)^2}\ P = 1.41667\,P \qquad P = 0.70588\ B \qquad (2)$$

Shear in pins at A and C.

$$F_{AC} = \tau A_{pin} = \frac{\tau_U}{F.S.}\frac{\pi}{4}d^2 = \left(\frac{25}{3.25}\right)\left(\frac{\pi}{4}\right)\left(\frac{3}{8}\right)^2 = 0.84959 \text{ kips}$$

Tension on net section at A and C.

$$F_{AC} = \sigma A_{net} = \frac{\sigma_U}{F.S.}A_{net} = \left(\frac{65}{3.25}\right)\left(\frac{1}{4}\right)\left(\frac{1}{2} - \frac{3}{8}\right) = 0.625 \text{ kips}$$

Smaller value of F_{AC} is 0.625 kips.

From (1) $\quad P = (0.48)(0.625) = 0.300$ kips

Shear in pin at B.

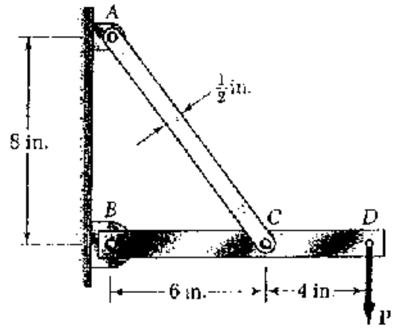
$$B = \tau A_{pin} = \frac{\tau_U}{F.S.}\frac{\pi}{4}d^2 = \left(\frac{25}{3.25}\right)\left(\frac{\pi}{4}\right)\left(\frac{5}{16}\right)^2 = 0.58999 \text{ kips}$$

From (2) $\quad P = (0.70588)(0.58999) = 0.416$ kips

Allowable value of P is the smaller value. $\qquad P = 0.300$ kips

or $P = 300$ lb

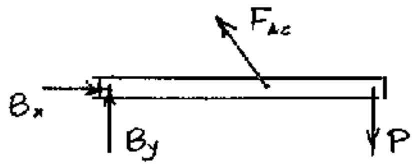
Problem 1.52



1.52 Solve Prob. 1.51, assuming that the structure has been redesigned to use $\frac{5}{16}$-in.-diameter pins at A and C as well as at B and that no other change has been made.

1.51 Link AC is made of a steel with a 65-ksi ultimate normal stress and has a $\frac{1}{4} \times \frac{1}{2}$ in. uniform rectangular cross section. It is connected to a support at A and to member BCD at C by $\frac{3}{8}$-in.-diameter pins, while member BCD is connected to its support at B by a $\frac{5}{16}$-in.-diameter pin; all of the pins are made of a steel with a 25-ksi ultimate shearing stress and are in single shear. Knowing that a factor of safety of 3.25 is desired, determine the largest load $\mathbf{P}$ that may be applied at D. Note that link AC is not reinforced around the pin holes.

Use free body BCD



$$+\circlearrowleft \Sigma M_B = 0: \quad (6)(\tfrac{8}{10}F_{AC}) - 10P = 0$$

$$P = 0.48\, F_{AC} \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad B_x - \tfrac{6}{10}F_{AC} = 0$$

$$B_x = \tfrac{6}{10}F_{AC} = 1.25\,P \rightarrow$$

$$+\circlearrowleft \Sigma M_C = 0: \quad -6B_y - 4P = 0 \qquad B_y = -\tfrac{2}{3}P \quad \text{i.e.} \quad B_y = \tfrac{2}{3}P \downarrow$$

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{1.25^2 + (\tfrac{2}{3})^2}\; P = 1.41667\,P \qquad P = 0.70583\,B \qquad (2)$$

Shear in pins at A and C.

$$F_{AC} = \tau A_{pin} = \frac{\tau_U}{F.S.}\frac{\pi}{4}d^2 = \left(\frac{25}{3.25}\right)\left(\frac{\pi}{4}\right)\left(\frac{5}{16}\right)^2 = 0.58999 \text{ kips}$$

Tension on net section at A and C.

$$F_{AC} = \sigma A_{net} = \frac{\sigma_U}{F.S.}A_{net} = \left(\frac{65}{3.25}\right)\left(\frac{1}{4}\right)\left(\frac{1}{2} - \frac{5}{16}\right) = 0.9375 \text{ kips.}$$

Smaller value of F_{AC} is 0.58999 kips

From (1) $\qquad P = (0.48)(0.58999) = 0.283$ kips

Shear in pin at B.

$$B = \tau A_{pin} = \frac{\tau_U}{F.S.}\frac{\pi}{4}d^2 = \left(\frac{25}{3.25}\right)\left(\frac{\pi}{4}\right)\left(\frac{5}{16}\right)^2 = 0.58999 \text{ kips}$$

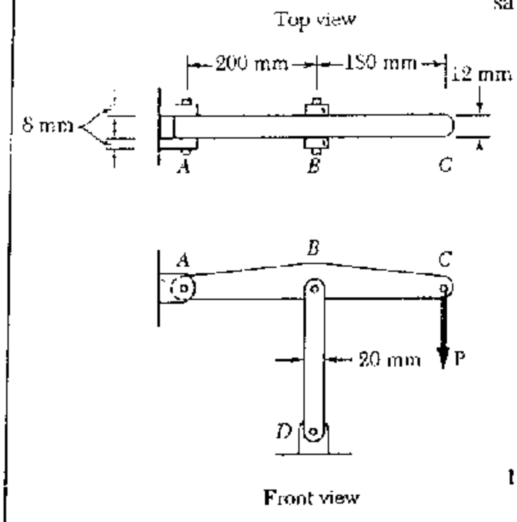
From (2) $\qquad P = (0.70588)(0.58999) = 0.416$ kips

Allowable value of P is the smaller value. $\qquad P = 0.283$ kips

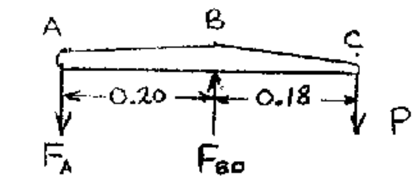
$$P = 283 \text{ lb}$$

Problem 1.53

1.53 In the structure shown, an 8-mm-diameter pin is used at A, and 12-mm-diameter pins are used at B and D. Knowing that the ultimate shearing stress is 100 MPa at all connections and that the ultimate normal stress is 250 MPa in each of the two links joining B and D, determine the allowable load $\mathbf{P}$ if an overall factor of safety of 3.0 is desired.



Statics: Use ABC as free body.



$\Sigma M_B = 0 \qquad 0.20\,F_A - 0.18\,P = 0$

$P = \frac{10}{9} F_A$

$\Sigma M_A = 0 \qquad 0.20\,F_{BD} - 0.38\,P = 0$

$P = \frac{10}{19} F_{BD}$

Based on double shear in pin A

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.008)^2 = 50.266 \times 10^{-6}\ \text{m}^2$

$F_A = \frac{2\tau_U A}{\text{F.S.}} = \frac{(2)(100\times10^6)(50.266\times10^{-6})}{3.0} = 3.351 \times 10^3\ \text{N}$

$P = \frac{10}{9} F_A = 3.72 \times 10^3\ \text{N}$

Based on double shear in pins at B and D

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.012)^2 = 113.10 \times 10^{-6}\ \text{m}^2$

$F_{BD} = \frac{2\tau_U A}{\text{F.S.}} = \frac{(2)(100\times10^6)(113.10\times10^{-6})}{3.0} = 7.54 \times 10^3\ \text{N}$

$P = \frac{10}{19} F_{BD} = 3.97 \times 10^3\ \text{N}$

Based on compression in links BD

For one link $\quad A = (0.020)(0.008) = 160 \times 10^{-6}\ \text{m}^2$

$F_{BD} = \frac{2\sigma_U A}{\text{F.S.}} = \frac{(2)(250\times10^6)(160\times10^{-6})}{3.0} = 26.7 \times 10^3\ \text{N}$

$P = \frac{10}{19} F_{BD} = 14.04 \times 10^3\ \text{N}$

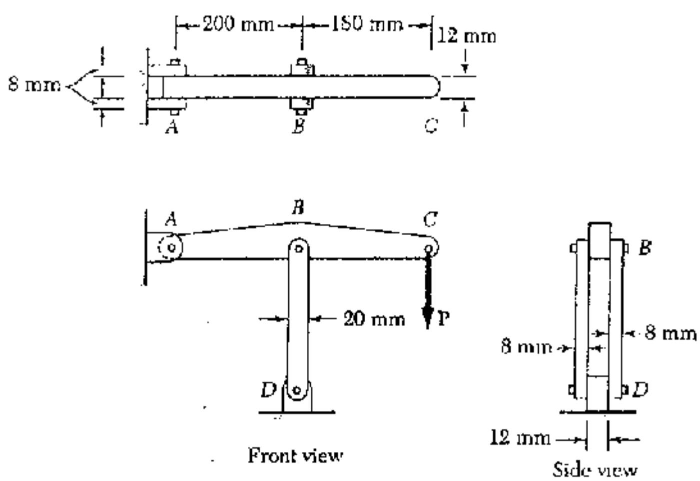
Allowable value of P is smallest $\therefore \quad P = 3.72 \times 10^3\ \text{N}$

3.72 kN

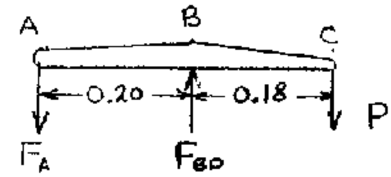
Problem 1.54

1.54 In an alternative design for the structure of Prob. 1.53, a pin of 10-mm-diameter is to be used at *A*. Assuming that all other specifications remain unchanged, determine the allowable load **P** if an overall factor of safety of 3.0 is desired.

1.53 In the structure shown, an 8-mm-diameter pin is used at *A*, and 12-mm-diameter pins are used at *B* and *D*. Knowing that the ultimate shearing stress is 100 MPa at all connections and that the ultimate normal stress is 250 MPa in each of the two links joining *B* and *D*, determine the allowable load **P** if an overall factor of safety of 3.0 is desired.



Statics : Use ABC as free body.



$\Sigma M_B = 0 \qquad 0.20\, F_A - 0.18\, P = 0$

$P = \frac{10}{9} F_A$

$\Sigma M_A = 0 \qquad 0.20\, F_{BD} - 0.38\, P = 0$

$P = \frac{10}{19} F_{BD}$

Based on double shear in pin A

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.010)^2 = 78.54 \times 10^{-6}\ m^2$

$F_A = \frac{2\tau_u A}{F.S.} = \frac{(2)(100\times10^6)(78.54\times10^{-6})}{3.0} = 5.236\times10^3\ N$

$P = \frac{10}{9} F_A = 5.82\times10^3\ N$

Based on double shear in pins at B and D

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.012)^2 = 113.10\times10^{-6}\ m^2$

$F_{BD} = \frac{2\tau_u A}{F.S.} = \frac{(2)(100\times10^6)(113.10\times10^{-6})}{3.0} = 7.54\times10^3\ N$

$P = \frac{10}{19} F_{BD} = 3.97\times10^3\ N$

Based on compression in links BD

For one link $\quad A = (0.020)(0.008) = 160\times10^{-6}\ m^2$

$F_{BD} = \frac{2\sigma_u A}{F.S.} = \frac{(2)(250\times10^6)(160\times10^{-6})}{3.0} = 26.7\times10^3\ N$

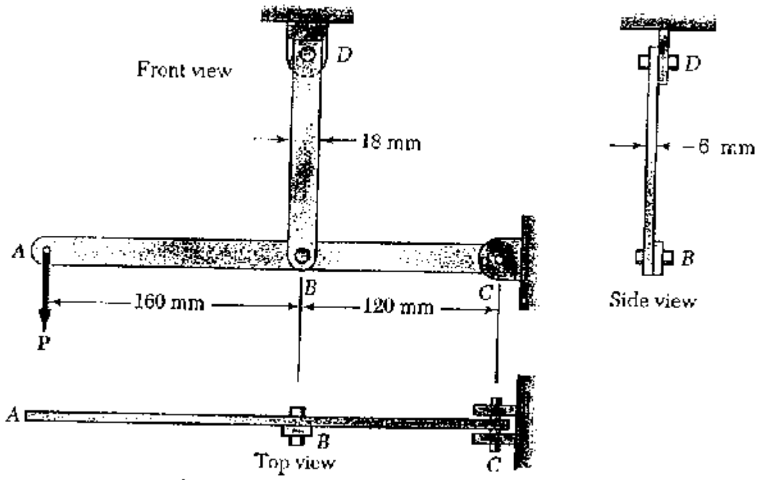
$P = \frac{10}{19} F_{BD} = 14.04\times10^3\ N$

Allowable value of P is smallest $\therefore \quad P = 3.97\times10^3\ N$

3.97 kN ◄

Problem 1.55

1.55 In the steel structure shown, a 6-mm-diameter pin is used at C and 10-mm-diameter pins are used at B and D. The ultimate shearing stress is 150 MPa at all connections, and the ultimate normal stress is 400 MPa in link BD. Knowing that a factor of safety of 3 is desired, determine the largest load **P** that can be applied at A. Note that link BD is not reinforced around the pin holes.



Use free body ABC.

$+\circlearrowleft \Sigma M_C = 0:$

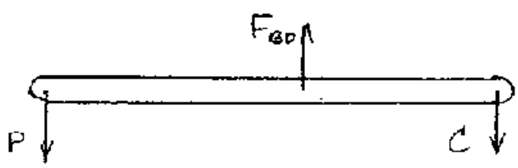
$0.280\,P - 0.120\,F_{BD} = 0$

$P = \frac{3}{7} F_{BD} \qquad (1)$

$+\circlearrowleft \Sigma M_B = 0:$

$0.160\,P - 0.120\,C = 0$

$P = \frac{3}{4} C \qquad (2)$



Tension on net section of link BD.

$F_{BD} = \sigma A_{net} = \frac{\sigma_U}{F.S.} A_{net}$

$= \left(\frac{400\times10^6}{3}\right)(6\times10^{-3})(18-10)(10^{-3})$

$= 6.40\times10^3\ N$

Shear in pins at B and D.

$F_{BD} = \tau A_{pin} = \frac{\tau_U}{F.S.}\frac{\pi}{4}d^2 = \left(\frac{150\times10^6}{3}\right)\left(\frac{\pi}{4}\right)(10\times10^{-3})^2 = 3.9270\times10^3\ N$

Smaller value of F_{BD} is 3.9270×10^3 N.

From (1) $\quad P = \left(\frac{3}{7}\right)(3.9270\times10^3) = 1.683\times10^3\ N$

Shear in pin at C

$C = 2\tau A_{pin} = 2\frac{\tau_U}{F.S.}\frac{\pi}{4}d^2 = (2)\left(\frac{150\times10^6}{3}\right)\left(\frac{\pi}{4}\right)(6\times10^{-3})^2 = 2.8274\times10^3\ N$

From (2) $\quad P = \left(\frac{3}{4}\right)(2.8274\times10^3) = 2.12\times10^3\ N$

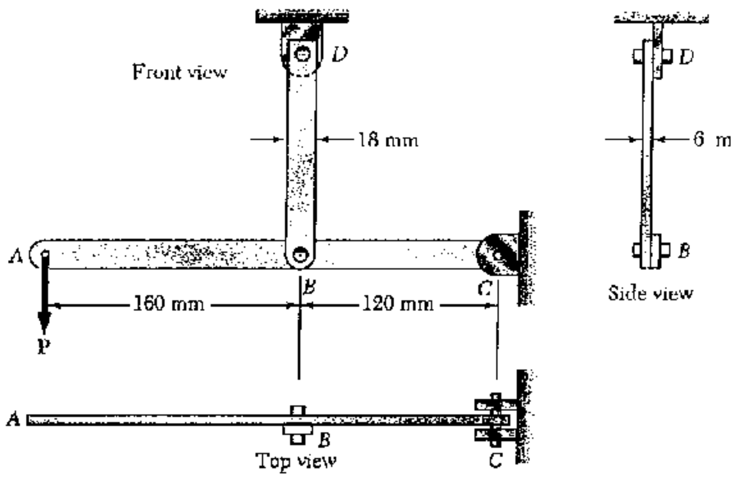
Smaller value of P is allowable value. $\qquad P = 1.683\times10^3\ N$

$P = 1.683\ kN$

Problem 1.56

1.56 Solve Prob. 1.55, assuming that the structure has been redesigned to use 12-mm-diameter pins at *B* and *D* and no other change has been made.

1.55 In the steel structure shown, a 6-mm-diameter pin is used at *C* and 10-mm-diameter pins are used at *B* and *D*. The ultimate shearing stress is 150 MPa at all connections, and the ultimate normal stress is 400 MPa in link *BD*. Knowng that a factor of safety of 3 is desired, determine the largest load **P** that many be applied at *A*. Note that link *BD* is not reinforced around the pin holes.



Use free body ABC.

$+\circlearrowleft \Sigma M_C = 0:$

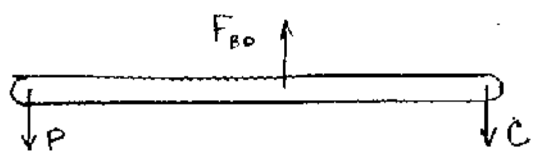
$0.280 P - 0.120 F_{BD} = 0$

$P = \frac{3}{7} F_{BD}$ (1)

$+\circlearrowleft \Sigma M_B = 0:$

$0.160 P - 0.120 C = 0$

$P = \frac{3}{4} C$ (2)



Tension on net section of link BD.

$$F_{BD} = \sigma A_{net} = \frac{\sigma_U}{F.S.} A_{net}$$
$$= \left(\frac{400\times10^6}{3}\right)(6\times10^{-3})(18-12)(10^{-3})$$
$$= 4.80\times10^3 \text{ N}$$

Shear in pins at B and D.

$$F_{BD} = \tau A_{pin} = \frac{\tau_U}{F.S.}\frac{\pi}{4}d^2 = \left(\frac{150\times10^6}{3}\right)\left(\frac{\pi}{4}\right)(12\times10^{-3})^2 = 5.6549\times10^3 \text{ N}$$

Smaller value of F_{BD} is 4.80×10^3 N.

From (1) $\quad P = \left(\frac{3}{7}\right)(4.80\times10^3) = 2.06\times10^3$ N

Shear in pin at C

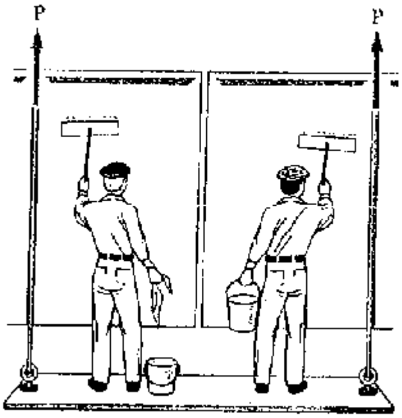
$$C = 2\tau A_{pin} = 2\frac{\tau_U}{F.S.}\frac{\pi}{4}d^2 = (2)\left(\frac{150\times10^6}{3}\right)\left(\frac{\pi}{4}\right)(6\times10^{-3})^2 = 2.8274\times10^3 \text{ N}$$

From (2) $\quad P = \left(\frac{3}{4}\right)(2.8274\times10^3) = 2.12\times10^3$ N

Smaller value of P is the allowable value. $\quad P = 2.06\times10^3$ N

$P = 2.06$ kN

Problem 1.57



*1.57 The Load and Resistance Factor Design method is to be used to select the two cables that will raise and lower a platform supporting two window washers. The platform weights 160 lb and each of the window washers is assumed to weight 195 lb with equipment. Since these workers are free to move on the platform, 75% of their total weight and the weight of their equipment will be used as the design live load of each cable. (*a*) Assuming a resistance factor $\phi = 0.85$ and load factors γ_D 1.2 and $\gamma_L = 1.5$, determine the required minimum ultimate load of one cable. (*b*) What is the conventional factor of safety for the selected cables?

$$\gamma_D P_D + \gamma_L P_L = \varphi P_U$$

$$P_U = \frac{\gamma_D P_D + \gamma_L P_L}{\varphi}$$

$$= \frac{(1.2)(\frac{1}{2} \times 160) + (1.5)(\frac{3}{4} \times 2 \times 195)}{0.85}$$

$$= 629 \text{ lb.}$$

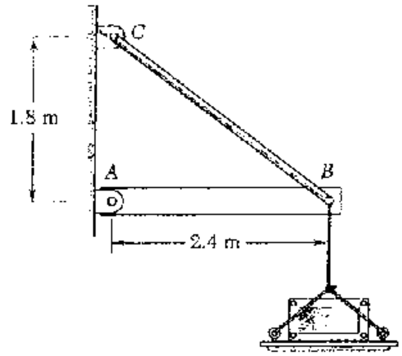
Conventional factor of safety

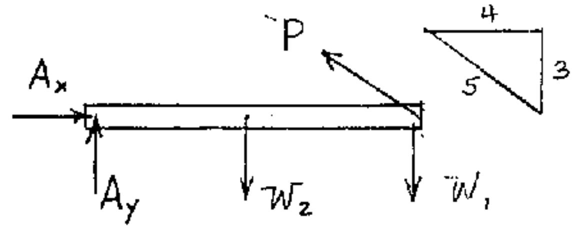
$$P = P_D + P_L = \frac{1}{2} \times 80 + 0.75 \times 2 \times 195 = 372.5 \text{ lb}$$

$$F.S. = \frac{P_U}{P} = \frac{629}{372.5} = 1.689$$

Problem 1.58

*1.58 A 40-kg platform is attached to the end B of a 50-kg wooden beam AB, which is supported as shown by a pin at A and by a slender steel rod BC with a 12-kN ultimate load. (a) Using the Load and Resistance Factor Design method with a resistance factor $\phi = 0.90$ and load factors $\gamma_D = 1.25$ and $\gamma_L = 1.6$, determine the largest load that can be safely placed on the platform. (b) What is the corresponding conventional factor of safety for rod BC?





$\circlearrowleft \Sigma M_A = 0 \qquad (2.4)\frac{3}{5}P - 2.4\,W_1 - 1.2\,W_2 \quad \therefore \; P = \frac{5}{3}W_1 + \frac{5}{6}W_2$

For dead loading $\quad W_1 = (40)(9.81) = 392.4 \text{ N}$

$W_2 = (50)(9.81) = 490.5 \text{ N}$

$P_D = (\frac{5}{3})(392.4) + (\frac{5}{6})(490.5) = 1.0628 \times 10^3 \text{ N}$

For live loading $\quad W_1 = mg \quad W_2 = 0$

$P_L = \frac{5}{3}mg \quad$ from which $\quad m = \frac{3}{5}\frac{P_L}{g}$

Design criterion

$\gamma_D P_D + \gamma_L P_L = \phi P_U$

$P_L = \dfrac{\phi P_U - \gamma_D P_D}{\gamma_L} = \dfrac{(0.90)(12\times10^3) - (1.25)(1.0628\times10^{-3})}{1.6}$

$= 5.920\times10^3 \text{ N}$

Allowable load $\quad m = \frac{3}{5}\,\dfrac{5.92\times10^3}{9.81} = 362 \text{ kg}$ ◀

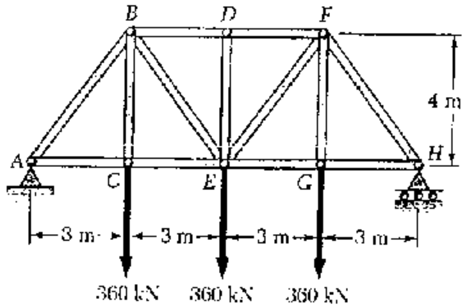
Conventional factor safety

$P = P_D + P_L = 1.0628\times10^3 + 5.920\times10^3 = 6.983\times10^3 \text{ N}$

$\text{F.S.} = \dfrac{P_U}{P} = \dfrac{12\times10^3}{6.983\times10^3} = 1.718$ ◀

Problem 1.59

1.59 For the Pratt bridge truss and loading shown, determine the average stress in member BE, knowing that the cross-sectional area of that member is 3750 mm^2.

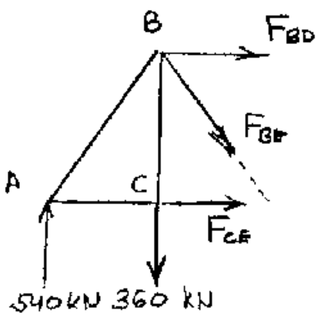


Use entire truss as free body

$+\circlearrowleft \Sigma M_H = 0:$

$(3)(360) + (6)(360) + (9)(360) - 12 A_y = 0$

$A_y = 540 \text{ kN}$



Use portion of truss to the left of a section cutting members BD, BE, and CE.

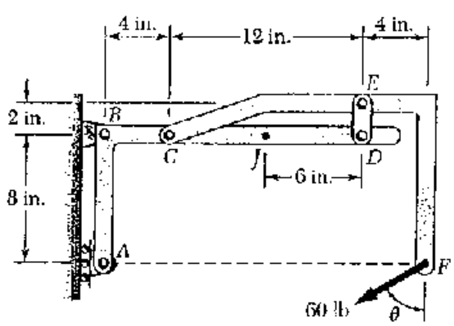
$+\uparrow \Sigma F_y = 0$

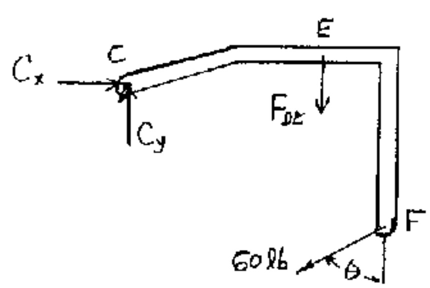
$540 - 360 - \frac{4}{5} F_{BE} = 0 \qquad F_{BE} = 225 \text{ kN}$

$\sigma_{BE} = \frac{F_{BE}}{A_{BE}} = \frac{225 \times 10^3}{3750 \times 10^{-6}} = 60 \times 10^6 \text{ Pa} \qquad 60.0 \text{ MPa}$

Problem 1.60

1.60 Knowing that link DE is 1 in. wide and $\frac{1}{8}$ in. thick, determine the normal stress in the central portion of that link when (*a*) $\theta = 0$, (*b*) $\theta = 90°$.





Use member CEF as a free body

$+\circlearrowleft \Sigma M_C = 0$

$-12 F_{DE} - (8)(60 \sin\theta) - (16)(60 \cos\theta) = 0$

$F_{DE} = -40 \sin\theta - 80 \cos\theta \quad \text{lb}$

$A_{DE} = (1)(\frac{1}{8}) = 0.125 \text{ in}^2$

$\sigma_{DE} = \frac{F_{DE}}{A_{DE}}$

(a) $\theta = 0: \quad F_{DE} = -80 \text{ lb.}$

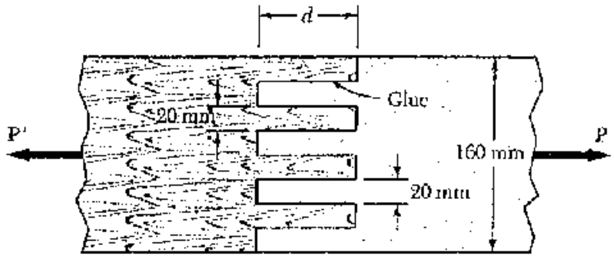
$\sigma_{DE} = \frac{-80}{0.125} = -640 \text{ psi}$

(b) $\theta = 90°: \quad F_{DE} = -40 \text{ lb.}$

$\sigma_{DE} = \frac{-40}{0.125} = -320 \text{ psi}$

Problem 1.61

1.61 Two wooden planks, each 22 mm thick and 160 mm wide, are joined by the glued mortise joint shown. Knowing that the joint will fail when the average shearing stress in the glue reaches 820 kPa, determine the smallest allowable length d of the cuts if the joint is to withstand an axial load of magnitude $P = 7.6$ kN.



Seven surfaces carry the total load $P = 7.6\text{ kN} = 7.6 \times 10^3$

Let $t = 22$ mm

Each glue area is $A = dt$

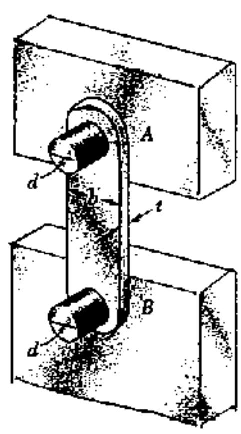
$$\tau = \frac{P}{7A} \qquad A = \frac{P}{7\tau} = \frac{7.6 \times 10^3}{(7)(820 \times 10^3)} = 1.32404 \times 10^{-3}\ \text{m}^2$$

$$= 1.32404 \times 10^3\ \text{mm}^2$$

$$d = \frac{A}{t} = \frac{1.32404 \times 10^3}{22} = 60.2 \qquad d = 60.2\ \text{mm}$$

Problem 1.62

1.62 Link AB, of width $b = 2$ in. and thickness $t = \frac{1}{4}$ in., is used to support the end of a horizontal beam. Knowing that the average normal stress in the link is -20 ksi, and that the average shearing stress in each of the two pins is 12 ksi, determine (a) the diameter d of the pins, (b) the average bearing stress in the link.



Rod AB is in compression.

$A = bt$ where $b = 2$ in and $t = \frac{1}{4}$ in

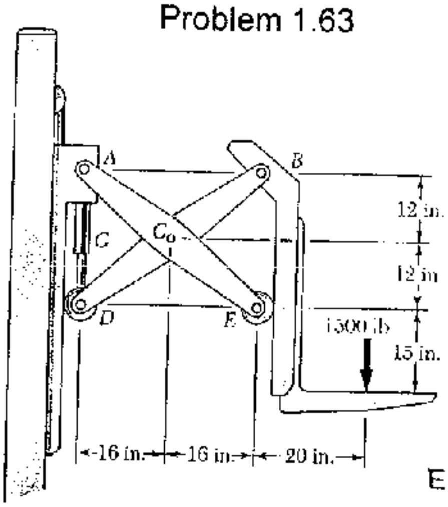
$P = -\sigma A = -(-20)(2)(\frac{1}{4}) = 10$ kips

Pin: $\tau_P = \dfrac{P}{A_P}$ and $A_P = \dfrac{\pi}{4} d^2$

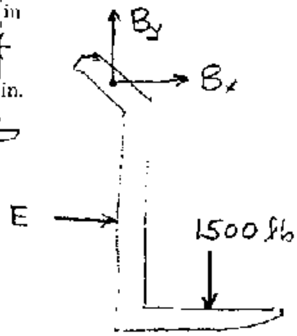
(a) $d = \sqrt{\dfrac{4A_P}{\pi}} = \sqrt{\dfrac{4P}{\pi \tau_P}} = \sqrt{\dfrac{(4)(10)}{\pi(12)}} = 1.030$ in.

(b) $\sigma_b = \dfrac{P}{dt} = \dfrac{10}{(1.030)(0.25)} = 38.8$ ksi

Problem 1.63



1.63 Two identical linkage-and-hydraulic-cylinder systems control the position of the forks of a fork-lift truck. The load supported by the one system shown is 1500 lb. Knowing that the thickness of member BD is $\frac{5}{8}$ in., determine (*a*) the average shearing stress in the $\frac{1}{2}$-in.-diameter pin at B, (*b*) the bearing stress at B in member BD.



Use one fork as a free body.

$+\circlearrowleft \Sigma M_B = 0:$

$24E - (20)(1500) = 0$

$E = 1250 \text{ lb.} \rightarrow$

$\xrightarrow{+} \Sigma F_x = 0$

$E + B_x = 0 \qquad B_x = -E$

$B_x = 1250 \text{ lb} \leftarrow$

$+\uparrow \Sigma F_y = 0: \quad B_y - 1500 = 0 \qquad B_y = 1500 \text{ lb}$

$B = \sqrt{B_x^2 + B_y^2} = \sqrt{1250^2 + 1500^2} = 1952.56 \text{ lb.}$

(a) Shearing stress in pin at B.

$A_{pin} = \frac{\pi}{4} d_{pin}^2 = \frac{\pi}{4}\left(\frac{1}{2}\right)^2 = 0.19635 \text{ in}^2$

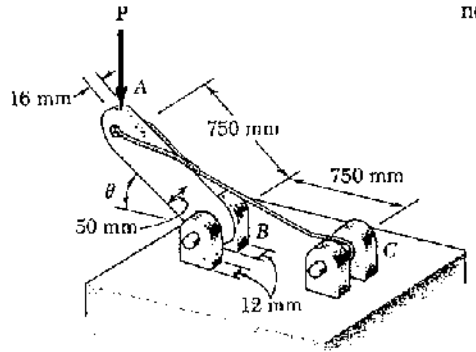
$\tau = \frac{B}{A_{pin}} = \frac{1952.56}{0.19635} = 9.94 \times 10^3 \text{ psi}$ 9.94 ksi ◄

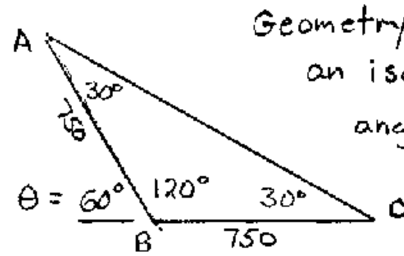
(b) Bearing stress at B.

$\sigma = \frac{B}{dt} = \frac{1952.56}{\left(\frac{1}{2}\right)\left(\frac{5}{8}\right)} = 6.25 \times 10^3 \text{ psi}$ 6.25 ksi ◄

Problem 1.64

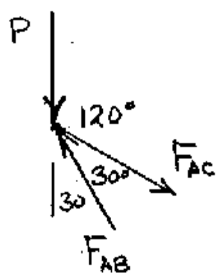
1.64 Determine the largest load **P** that can be applied at A when $\theta = 60°$, knowing that the average shearing stress in the 10-mm-diameter pin at B must not exceed 120 MPa and that the average bearing stress in member AB and in the bracket at B must not exceed 90 MPa.

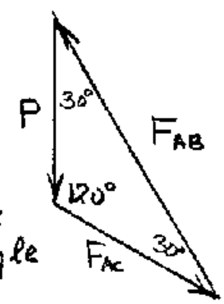




Geometry: Triangle ABC is an isosceles triangle with angles shown here

Use joint A as free body





Force triangle

Law of sines applied to force triangle

$$\frac{P}{\sin 30°} = \frac{F_{AB}}{\sin 120°} = \frac{F_{AC}}{\sin 30°}$$

$$P = \frac{F_{AB}\sin 30°}{\sin 120°} = 0.57735\, F_{AB}$$

$$P = \frac{F_{AC}\sin 30°}{\sin 30°} = F_{AC}$$

If shearing stress in pin at B is critical

$$A_P = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.010)^2 = 78.54\times10^{-6}\ \text{m}^2$$

$$F_{AB} = 2A\tau = (2)(78.54\times10^{-6})(120\times10^{6}) = 18.850\times10^{3}\ \text{N}$$

If bearing stress in member AB at bracket at A is critical

$$A_b = td = (0.016)(0.010) = 160\times10^{-6}\ \text{m}^2$$

$$F_{AB} = A_b\sigma_b = (160\times10^{-6})(90\times10^{6}) = 14.40\times10^{3}\ \text{N}$$

If bearing stress in the bracket at B is critical

$$A_b = 2td = (2)(0.012)(0.010) = 240\times10^{-6}\ \text{m}^2$$

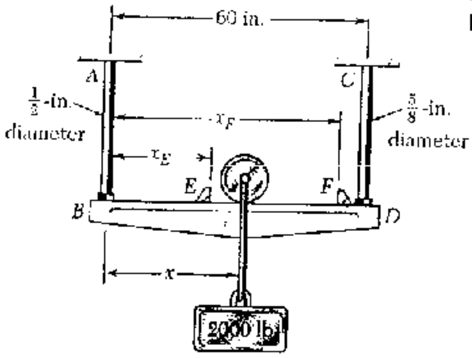
$$F_{AB} = A_b\sigma_b = (240\times10^{-6})(90\times10^{6}) = 21.6\times10^{3}\ \text{N}$$

Allowable F_{AB} is the smallest, i.e. 14.40×10^{3} N

Then, from Statics $\quad P_{allow} = (0.57735)(14.40\times10^{3})$

$$= 8.31\times10^{3}\ \text{N} \qquad 8.31\ \text{kN}$$

Problem 1.65



1.65 The 2000-lb load may be moved along the beam BD to any position between stops at E and F. Knowing that $\sigma_{all} = 6$ ksi for the steel used in rods AB and CD, determine where the stops should be placed if the permitted motion of the load is to be as large as possible.

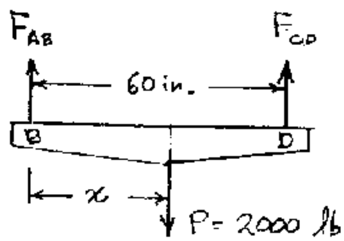
Permitted member forces:

AB: $(F_{AB})_{max} = \sigma_{all} A_{AB} = (6)(\frac{\pi}{4})(\frac{1}{2})^2$

$= 1.17810$ kips

CD: $(F_{CD})_{max} = \sigma_{all} A_{CD} = (6)(\frac{\pi}{4})(\frac{5}{8})^2$

$= 1.84078$ kips



Use member BEFD as a free body.

$P = 2000$ lb $= 2.000$ kips

$+\circlearrowleft \Sigma M_D = 0$

$-(60)F_{AB} + (60 - x_E)P = 0$

$60 - x_E = \dfrac{60 F_{AB}}{P} = \dfrac{(60)(1.17810)}{2.000}$

$= 35.343$

$x_E = 24.7$ in. ◄

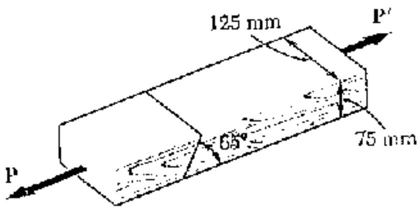
$+\circlearrowleft \Sigma M_B = 0$

$60 F_{CD} - xP = 0$

$x_F = \dfrac{60 F_{CD}}{P} = \dfrac{(60)(1.84078)}{2.000}$

$x_F = 55.2$ in. ◄

Problem 1.66



1.66 Two wooden members of 75 × 125-mm uniform rectangular cross section are joined by the simple glued joint shown. Knowing that P = 3.6 kN and that the ultimate strength of the glue is 1.1 MPa in tension and 1.4 MPa in shear, determine the factor of safety.

$$\theta = 90° - 65° = 25°$$

$$P = 3.6 \text{ kN} = 3.6 \times 10^3 \text{ N}$$

$$A_0 = (75)(125) = 9.375 \times 10^3 \text{ mm}^2 = 9.375 \times 10^{-3} \text{ m}^2$$

Tension on glue area. $\sigma = \dfrac{P\cos^2\theta}{A_0} = \dfrac{(3.6 \times 10^3)(\cos 25°)^2}{9.375 \times 10^{-3}}$

$$= 315.415 \times 10^3 \text{ Pa}$$

$$F.S. = \frac{\sigma_U}{\sigma} = \frac{1.1 \times 10^6}{315.415 \times 10^3} = 3.49$$

Shear on glue area. $\tau = \dfrac{P\sin 2\theta}{2A_0} = \dfrac{(3.6 \times 10^3)(\sin 50°)}{(2)(9.375 \times 10^{-3})}$

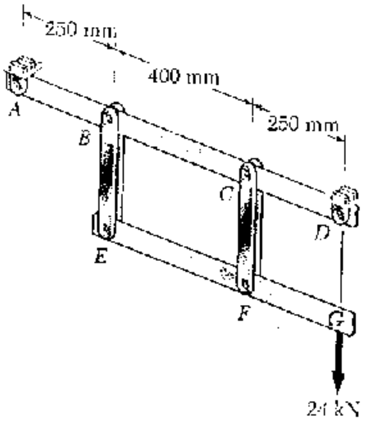
$$= 147.081 \times 10^3 \text{ Pa}$$

$$F.S. = \frac{\tau_U}{\tau} = \frac{1.4 \times 10^6}{147.081 \times 10^3} = 9.52$$

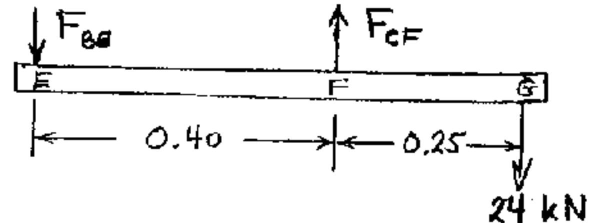
The smaller factor of safety governs $\qquad F.S. = 3.49$ ◄

Problem 1.67

1.67 Each of the two vertical links CF connecting the two horizontal members AD and EG has a 10×40-mm uniform rectangular cross section and is made of a steel with an ultimate strength in tension of 400 MPa, while each of the pins at C and F has a 20-mm diameter and is made of a steel with an ultimate strength in shear of 150 MPa. Determine the overall factor of safety for the links CF and the pins connecting them to the horizontal members.



Use member EFG as free body.



$\circlearrowleft \Sigma M_E = 0$

$0.40\, F_{CF} - (0.65)(24 \times 10^3) = 0$

$F_{CF} = 39 \times 10^3 \text{ N}$

Based on tension in links CF

$A = (b - d)t = (0.040 - 0.02)(0.010) = 200 \times 10^{-6} \text{ m}^2$ (one link)

$F_U = 2\sigma_U A = (2)(400 \times 10^6)(200 \times 10^{-6}) = 160.0 \times 10^3 \text{ N}$

Based on double shear in pins

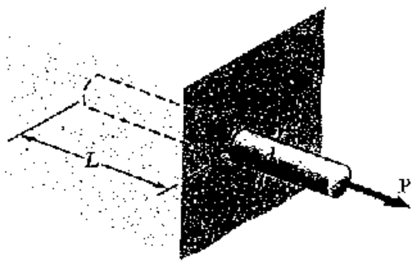
$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.020)^2 = 314.16 \times 10^{-6} \text{ m}^2$

$F_U = 2\tau_U A = (2)(150 \times 10^6)(314.16 \times 10^{-6}) = 94.248 \times 10^3 \text{ N}$

Actual F_U is smaller value, i.e. $F_U = 94.248 \times 10^3$ N

Factor of safety $F.S. = \dfrac{F_U}{F_{CF}} = \dfrac{94.248 \times 10^3}{39 \times 10^3} = 2.42$ ◄

Problem 1.68



1.68 A force **P** is applied as shown to a steel reinforcing bar that has been embedded in a block of concrete. Determine the smallest length L for which the full allowable normal stress in the bar can be developed. Express the result in terms of the diameter d of the bar, the allowable normal stress σ_{all} in the steel, and the average allowable bond stress τ_{all} between the concrete and the cylindrical surface of the bar. (Neglect the normal stresses between the concrete and the end of the bar.)

For shear, $A = \pi d L$

$P = \tau_{all} A = \tau_{all} \pi d L$

For tension, $A = \frac{\pi}{4} d^2$

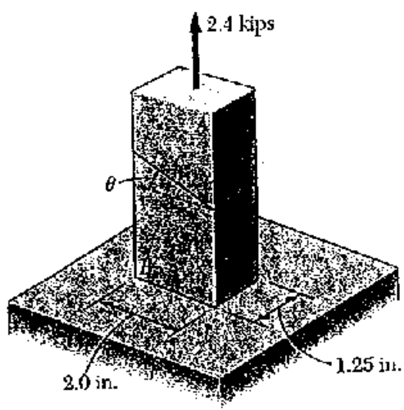
$P = \sigma_{all} A = \sigma_{all}(\frac{\pi}{4} d^2)$

Equating $\quad \tau_{all} \pi d L = \sigma_{all} \frac{\pi}{4} d^2$

Solving for L

$L_{min} = \sigma_{all} d / 4 \tau_{all}$ ◄

Problem 1.69



1.69 The two portions of member AB are glued together along a plane forming an angle θ with the horizontal. Knowing that the ultimate stress for the glued joint is 2.5 ksi in tension and 1.3 ksi in shear, determine (*a*) the value of θ for which the factor of safety of the member is maximum, (*b*) the corresponding value of the factor of safety. (*Hint*: Equate the expressions obtained for the factors of safety with respect to normal stress and shear.)

$A_o = (2.0)(1.25) = 2.50 \text{ in}^2$

At the optimum angle $\quad (F.S.)_\sigma = (F.S.)_\tau$

Normal stress: $\sigma = \frac{P}{A_o}\cos^2\theta \quad \therefore \quad P_{U,\sigma} = \frac{\sigma_U A_o}{\cos^2\theta}$

$(F.S.)_\sigma = \frac{P_{U,\sigma}}{P} = \frac{\sigma_U A_o}{P\cos^2\theta}$

Shearing stress: $\tau = \frac{P}{A_o}\sin\theta\cos\theta \quad \therefore \quad P_{U,\tau} = \frac{\tau_U A_o}{\sin\theta\cos\theta}$

$(F.S.)_\tau = \frac{P_{U,\tau}}{P} = \frac{\tau_U A_o}{P\sin\theta\cos\theta}$

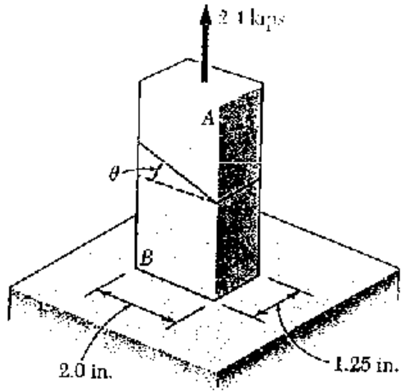
Equating: $\frac{\sigma_U A_o}{P\cos^2\theta} = \frac{\tau_U A_o}{P\sin\theta\cos\theta}$

Solving: $\frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{\tau_U}{\sigma_U} = \frac{1.3}{2.5} = 0.520 \qquad$ (a) $\theta_{opt} = 27.5°$ ◄

(b) $\quad P_U = \frac{\sigma_U A_o}{\cos^2\theta} = \frac{(2.5)(2.50)}{\cos^2 27.5°} = 7.94$ kips

$F.S. = \frac{P_U}{P} = \frac{7.94}{2.4} = 3.31$ ◄

Problem 1.70



1.70 The two portions of member *AB* are glued together along a plane forming an angle θ with the horizontal. Knowing that the ultimate stress for the glued joint is 2.5 ksi in tension and 1.3 ksi in shear, determine the range of values of θ for which the factor of safety of the members is at least 3.0.

$A_o = (2.0)(1.25) = 2.50 \text{ in}^2$

$P = 2.4 \text{ kips} \qquad P_U = (F.S)P = 7.2 \text{ kips}$

Based on tensile stress

$$\sigma_U = \frac{P_U}{A_o}\cos^2\theta$$

$$\cos^2\theta = \frac{\sigma_U A_o}{P_U} = \frac{(2.5)(2.50)}{7.2} = 0.86806$$

$\cos\theta = 0.93169 \qquad \theta = 21.3° \qquad \theta \geq 21.3°$

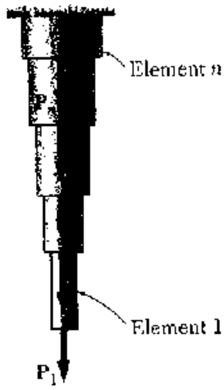
Based on shearing stress $\quad \tau_U = \frac{P_U}{A_o}\sin\theta\cos\theta = \frac{P_U}{2A_o}\sin 2\theta$

$$\sin 2\theta = \frac{2A_o\tau_U}{P_U} = \frac{(2)(2.50)(1.3)}{7.2} = 0.90278$$

$2\theta = 64.52° \qquad \theta = 32.3° \qquad \theta \leq 32.3°$

Hence $\quad 21.3° \leq \theta \leq 32.3°$

PROBLEM 1.C1



1.C1 A solid steel rod consisting of n cylindrical elements welded together is subjected to the loading shown. The diameter of element i is denoted by d_i and the load applied to its lower end by $\mathbf{P}_i$ with the magnitude P_i of this load being assumed positive if $\mathbf{P}_i$ is directed downward as shown and negative otherwise. (*a*) Write a computer program that can be used with either SI or U.S. customary units to determine the average stress in each element of the rod. (*b*) Use this program to solve Probs. 1.1 and 1.3.

SOLUTION

FORCE IN ELEMENT i:

It is the sum of the forces applied to that element and all lower ones:

$$F_i = \sum_{k=1}^{i} P_k$$

AVERAGE STRESS IN ELEMENT i:

Area $= A_i = \frac{1}{4}\pi d_i^2$ Ave stress $= \dfrac{F_i}{A_i}$

PROGRAM OUTPUTS

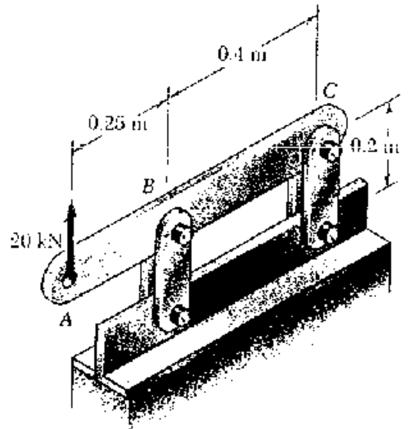
Problem 1.1

Element	Stress (MPa)
1	42.441
2	35.651

Problem 1.3

Element	Stress (ksi)
1	12.732
2	-2.829

PROBLEM 1.C2

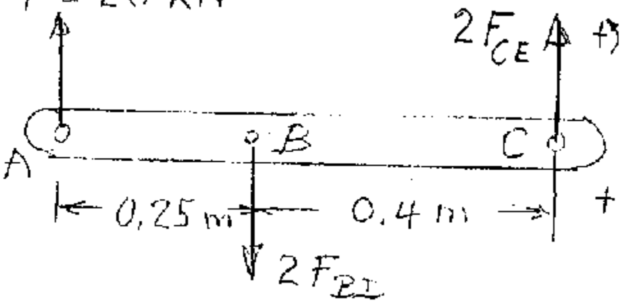


1.C2 A 20-kN load is applied as shown to the horizontal member *ABC*. Member *ABC* has a 10 × 50-mm uniform rectangular cross section and is supported by four vertical links, each of 8 × 36-mm uniform rectangular cross section. Each of the four pins at *A*, *B*, *C*, and *D* has the same diameter *d* and is in double shear. (*a*) Write a computer program to calculate for values of *d* from 10 to 30 mm, using 1-mm increments, (1) the maximum value of the average normal stress in the links connecting pins *B* and *D*, (2) the average normal stress in the links connecting pins *C* and *E*, (3) the average shearing stress in pin *B*, (4) the average shearing stress in pin *C*, (5) the average bearing stress at *B* in member *ABC*, (6) the average bearing stress at *C* in member *ABC*. (*b*) Check your program by comparing the values obtained for *d* = 16 mm with the answers given for Probs. 1.9 and 1.27. (*c*) Use this program to find the permissible values of the diameter *d* of the pins, knowing that the allowable values of the normal, shearing, and bearing stresses for the steel used are, respectively 150 MPa, 90 MPa, and 230 MPa. (*d*) Solve part *c*, assuming that the thickness of member *ABC* has been reduced from 10 to 8 mm.

SOLUTION

FORCES IN LINKS

$P = 20$ kN



F.B. DIAGRAM OF ABC:

$+\circlearrowleft \Sigma M_C = 0:\ 2F_{BD}(BC) - P(AC) = 0$

$F_{BD} = P(AC)/2(BC)$ (TENSION)

$+\circlearrowleft \Sigma M_B = 0:\ 2F_{CE}(BC) - P(AB) = 0$

$F_{CE} = P(AB)/2(BC)$ (COMP.)

(1) LINK BD

Thickness $= t_L$

$A_{BD} = t_L(w_L - d)$

$\sigma_{BD} = +F_{BD}/A_{BD}$

(2) LINK CE

Thickness $= t_L$

$A_{CE} = t_L w_L$

$\sigma_{CE} = -F_{CE}/A_{CE}$

(3) PIN B

$\tau_B = F_{BD}/(\pi d^2/4)$

(4) PIN C

$\tau_C = F_{CE}/(\pi d^2/4)$

(5) BEARING STRESS AT B

Thickness of member $AC = t_{AC}$

Sig Bear B $= F_{BD}/(d\, t_{AC})$

(6) BEARING STRESS AT C

Sig Bear C $= F_{CE}/(d\, t_{AC})$

SHEARING STRESS IN ABC UNDER PIN B

$F_B = \tau_{AC}\, t_{AC}\,(w_{AC}/2)$

$\Sigma F_y = 0:\ 2F_B = 2F_{BD}$

$\tau_{AC} = \dfrac{2F_{BD}}{t_{AC}\, w_{AC}}$

(CONTINUED)

PROBLEM 1.C2 CONTINUED **PROGRAM OUTPUTS**

INPUT DATA FOR PARTS (a), (b), (c): P = 20 kN, AB = 0.25 m, BC = 0.40 m, AC = 0.65 m, TL = 8 mm, WL = 36 mm, TAC = 10 mm, WAC = 50 mm

d	Sigma BD	Sigma CE	Tau B	Tau C	SigBear B	SigBear C
10.00	78.13	-21.70	206.90	79.58	325.00	125.00
11.00	81.25	-21.70	170.99	65.77	295.45	113.64
12.00	84.64	-21.70	143.68	55.26	270.83	104.17
13.00	88.32	-21.70	122.43	47.09	250.00	96.15
14.00	92.33	-21.70	105.56	40.60	232.14	89.29
15.00	96.73	-21.70	91.96	35.37	216.67	83.33
16.00	101.56	-21.70	80.82	31.08	203.12	78.13 ← (b)
17.00	106.91	-21.70	71.59	27.54	191.18	73.53
18.00	112.85	-21.70	63.86	24.56	180.56	69.44
19.00	119.49	-21.70	57.31	22.04	171.05	65.79
20.00	126.95	-21.70	51.73	19.89	162.50	62.50
21.00	135.42	-21.70	46.92	18.04	154.76	59.52
22.00	145.09	-21.70	42.75	16.44	147.73	56.82
23.00	156.25	-21.70	39.11	15.04	141.30	54.35
24.00	169.27	-21.70	35.92	13.82	135.42	52.08
25.00	184.66	-21.70	33.10	12.73	130.00	50.00
26.00	203.13	-21.70	30.61	11.77	125.00	48.08
27.00	225.69	-21.70	28.38	10.92	120.37	46.30
28.00	253.91	-21.70	26.39	10.15	116.07	44.64
29.00	290.18	-21.70	24.60	9.46	112.07	43.10
30.00	338.54	-21.70	22.99	8.84	108.33	41.67

(c) ANSWER : 16 mm ≤ d ≤ 22 mm ◄ (c)

CHECK: For d = 22 mm, Tau AC = 65 MPa < 90 MPa O.K.

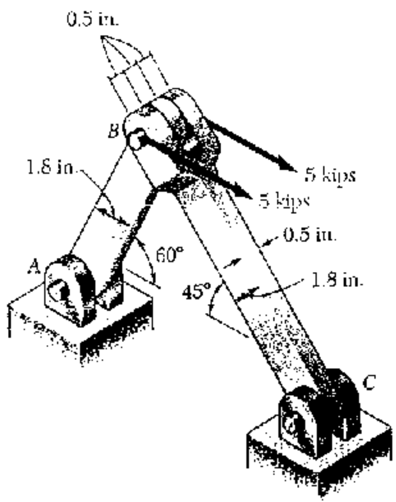
INPUT DATA FOR PART (d): P = 20 kN, AB = 0.25 m, BC = 0.40 m, AC = 0.65 m, TL = 8 mm, WL = 36 mm, TAC = 8 mm, WAC = 50 mm

d	Sigma BD	Sigma CE	Tau B	Tau C	SigBear B	SigBear C
10.00	78.13	-21.70	206.90	79.58	406.25	156.25
11.00	81.25	-21.70	170.99	65.77	369.32	142.05
12.00	84.64	-21.70	143.68	55.26	338.54	130.21
13.00	88.32	-21.70	122.43	47.09	312.50	120.19
14.00	92.33	-21.70	105.56	40.60	290.18	111.61
15.00	96.73	-21.70	91.96	35.37	270.83	104.17
16.00	101.56	-21.70	80.82	31.08	253.91	97.66
17.00	106.91	-21.70	71.59	27.54	238.97	91.91
18.00	112.85	-21.70	63.86	24.56	225.69	86.81
19.00	119.49	-21.70	57.31	22.04	213.82	82.24
20.00	126.95	-21.70	51.73	19.89	203.12	78.13
21.00	135.42	-21.70	46.92	18.04	193.45	74.40
22.00	145.09	-21.70	42.75	16.44	184.66	71.02
23.00	156.25	-21.70	39.11	15.04	176.63	67.93
24.00	169.27	-21.70	35.92	13.82	169.27	65.10
25.00	184.66	-21.70	33.10	12.73	162.50	62.50
26.00	203.13	-21.70	30.61	11.77	156.25	60.10
27.00	225.69	-21.70	28.38	10.92	150.46	57.87
28.00	253.91	-21.70	26.39	10.15	145.09	55.80
29.00	290.18	-21.70	24.60	9.46	140.09	53.88
30.00	338.54	-21.70	22.99	8.84	135.42	52.08

(d) ANSWER : 18 mm ≤ d ≤ 22 mm ◄ (d)

CHECK: For d = 22 mm, Tau AC = 81.25 MPa < 90 MPa O.K.

PROBLEM 1.C3

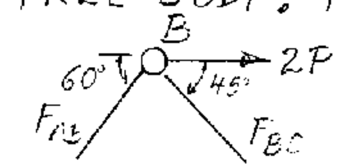


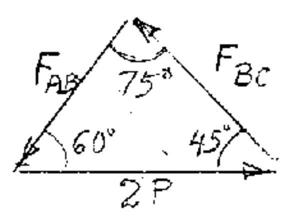
1.C3 Two horizontal 5-kip forces are applied to pin B of the assembly shown. Each of the three pins at A, B, and C has the same diameter d and is double shear. (a) Write a computer program to calculate for values of d from 0.50 to 1.50 in., using 0.05-in. increments, (1) the maximum value of the average normal stress in member AB, (2) the average normal stress in member BC, (3) the average shearing stress in pin A, (4) the average shearing stress in pin C, (5) the average bearing stress at A in member AB, (6) the average bearing stress at C in member BC, (7) the average bearing stress at B in member BC. (b) Check your program by comparing the values obtained for $d = 0.8$ in. with the answers given for Probs. 1.10 and 1.28. (c) Use this program to find the permissible values of the diameter d of the pins, knowing that the allowable values of the normal, shearing, and bearing stresses for the steel used, are respectively, 22 ksi, 13 ksi, and 36 ksi. (d) Solve part c, assuming that a new design is being investigated, in which the thickness and width of the two members are changed, respectively, form 0.5 to 0.3 in. and from 1.8 to 2.4 in.

SOLUTION

FORCES IN MEMBERS AB AND BC

FREE BODY: PIN B





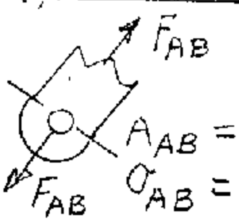
FROM FORCE TRIANGLE:

$$\frac{F_{AB}}{\sin 45°} = \frac{F_{BC}}{\sin 60°} = \frac{2P}{\sin 75°}$$

$$F_{AB} = 2P(\sin 45°/\sin 75°)$$

$$F_{BC} = 2P(\sin 60°/\sin 75°)$$

(1) MAX. AVE. STRESS IN AB



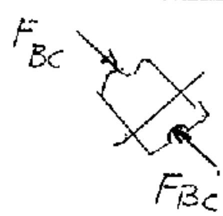
Width $= w$

Thickness $= t$

$$A_{AB} = (w - d)t$$

$$\sigma_{AB} = F_{AB}/A_{AB}$$

(2) AVE. STRESS IN BC



$$A_{BC} = wt$$

$$\sigma_{BC} = F_{BC}/A_{BC}$$

(3) PIN A

$$\tau_A = (F_{AB}/2)/(\pi d^2/4)$$

(4) PIN C

$$\tau_C = (F_{BC}/2)/(\pi d^2/4)$$

(5) BEARING STRESS AT A

Sig Bear A = F_{AB}/dt

(6) BEARING STRESS AT C

Sig Bear C = F_{BC}/dt

(7) BEARING STRESS AT B IN MEMBER BC

Sig Bear B = $F_{BC}/2dt$

(CONTINUED)

PROBLEM 1.C3 CONTINUED

PROGRAM OUTPUTS

INPUT DATA FOR PARTS (a), (b), (c): P = 5 kips, w = 1.8 in., t = 0.5 in.

D in.	SIGAB ksi	SIGBC ksi	TAUA ksi	TAUC ksi	SIGBRGA ksi	SIGBRGC ksi	SIGBRGB ksi
0.500	11.262	-9.962	18.642	22.831	29.282	35.863	17.932
0.550	11.713	-9.962	15.406	18.869	26.620	32.603	16.301
0.600	12.201	-9.962	12.945	15.855	24.402	29.886	14.943
0.650	12.731	-9.962	11.030	13.510	22.525	27.587	13.793
0.700	13.310	-9.962	9.511	11.649	20.916	25.616	12.808
0.750	13.944	-9.962	8.285	10.147	19.521	23.909	11.954
0.800	14.641	-9.962	7.282	8.918	18.301	22.414	11.207 ← (b)
0.850	15.412	-9.962	6.450	7.900	17.225	21.096	10.548
0.900	16.268	-9.962	5.754	7.047	16.268	19.924	9.962
0.950	17.225	-9.962	5.164	6.324	15.412	18.875	9.438
1.000	18.301	-9.962	4.660	5.708	14.641	17.932	8.966
1.050	19.521	-9.962	4.227	5.177	13.944	17.078	8.539
1.100	20.916	-9.962	3.852	4.717	13.310	16.301	8.151
1.150	22.525	-9.962	3.524	4.316	12.731	15.593	7.796
1.200	24.402	-9.962	3.236	3.964	12.201	14.943	7.471
1.250	26.620	-9.962	2.983	3.653	11.713	14.345	7.173
1.300	29.282	-9.962	2.758	3.377	11.262	13.793	6.897
1.350	32.536	-9.962	2.557	3.132	10.845	13.283	6.641
1.400	36.603	-9.962	2.378	2.912	10.458	12.808	6.404
1.450	41.831	-9.962	2.217	2.715	10.097	12.367	6.183
1.500	48.803	-9.962	2.071	2.537	9.761	11.954	5.977

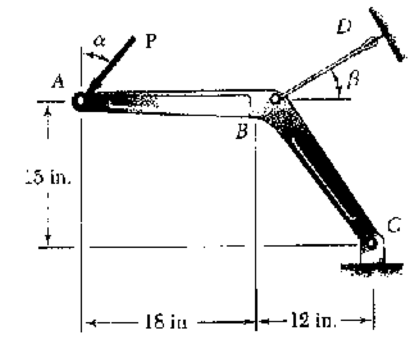
(c) ANSWER: 0.70 in. ≤ d ≤ 1.10 in. (c)

INPUT DATA FOR PART (d): P = 5 kips, w = 2.4 in., t = 0.3 in.

D in.	SIGAB ksi	SIGBC ksi	TAUA ksi	TAUC ksi	SIGBRGA ksi	SIGBRGC ksi	SIGBRGB ksi
0.500	12.843	-12.452	18.642	22.831	48.803	59.772	29.886
0.550	13.190	-12.452	15.406	18.869	44.367	54.338	27.169
0.600	13.556	-12.452	12.945	15.855	40.669	49.810	24.905
0.650	13.944	-12.452	11.030	13.510	37.541	45.978	22.989
0.700	14.354	-12.452	9.511	11.649	34.860	42.694	21.347
0.750	14.789	-12.452	8.285	10.147	32.536	39.848	19.924
0.800	15.251	-12.452	7.282	8.918	30.502	37.357	18.679
0.850	15.743	-12.452	6.450	7.900	28.708	35.160	17.580
0.900	16.268	-12.452	5.754	7.047	27.113	33.206	16.603
0.950	16.829	-12.452	5.164	6.324	25.686	31.459	15.729
1.000	17.430	-12.452	4.660	5.708	24.402	29.886	14.943
1.050	18.075	-12.452	4.227	5.177	23.240	28.463	14.231
1.100	18.771	-12.452	3.852	4.717	22.183	27.169	13.584
1.150	19.521	-12.452	3.524	4.316	21.219	25.988	12.994
1.200	20.335	-12.452	3.236	3.964	20.335	24.905	12.452
1.250	21.219	-12.452	2.983	3.653	19.521	23.909	11.954
1.300	22.183	-12.452	2.758	3.377	18.771	22.989	11.495
1.350	23.240	-12.452	2.557	3.132	18.075	22.138	11.069
1.400	24.402	-12.452	2.378	2.912	17.430	21.347	10.674
1.450	25.686	-12.452	2.217	2.715	16.829	20.611	10.305
1.500	27.113	-12.452	2.071	2.537	16.268	19.924	9.962

(d) ANSWER: 0.85 in. ≤ d ≤ 1.25 in. (d)

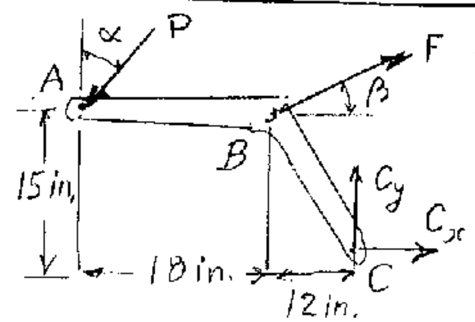
PROBLEM 1.C4



1.C4 A 4-kip force **P** forming an angle α with the vertical is applied as shown to member ABC, which is supported by a pin and bracket at C and by a cable BD forming an angle β with the horizontal. (*a*) Knowing that the ultimate load of the cable is 25 kips, write a computer program to construct a table of the values of the factor of safety of the cable for values of α and β from 0 to 45°, using increments in α and β corresponding to 0.1 increments in $\tan \alpha$ and $\tan \beta$. (*b*) Check that for any given value of α the maximum value of the factor of safety is obtained for $\beta = 38.66°$ and explain why. (*c*) Determine the smallest possible value of the factor of safety for $\beta = 38.66°$, as well as the corresponding value of α, and explain the result obtained.

SOLUTION

(a) DRAW F.B. DIAGRAM OF ABC:



$$+\circlearrowleft \sum M_C = 0: (P\sin\alpha)(15\text{ in.}) + (P\cos\alpha)(30\text{ in.}) - (F\cos\beta)(15\text{ in.}) - (F\sin\beta)(12\text{ in.}) = 0$$

$$F = P\,\frac{15\sin\alpha + 30\cos\alpha}{15\cos\beta + 12\sin\beta}$$

$$F.S. = F_{ult}/F$$

OUTPUT FOR $P = 4$ kips AND $F_{ult} = 20$ kips

VALUES OF FS

BETA

ALPHA	0	5.71	11.31	16.70	21.80	26.56	30.96	34.99	38.66	41.99	45.00
0.000	3.125	3.358	3.555	3.712	3.830	3.913	3.966	3.994	4.002	3.995	3.977
5.711	2.991	3.214	3.402	3.552	3.666	3.745	3.796	3.823	3.830	3.824	3.807
11.310	2.897	3.113	3.295	3.441	3.551	3.628	3.677	3.703	3.710	3.704	3.687
16.699	2.837	3.049	3.227	3.370	3.477	3.553	3.600	3.626	3.633	3.627	3.611
21.801	2.805	3.014	3.190	3.331	3.438	3.512	3.560	3.585	3.592	3.586	3.570
26.565	2.795	3.004	3.179	3.320	3.426	3.500	3.547	3.572	3.579	3.573	3.558
30.964	2.803	3.013	3.189	3.330	3.436	3.510	3.558	3.583	3.590	3.584	3.568
34.992	2.826	3.036	3.214	3.356	3.463	3.538	3.586	3.611	3.619	3.612	3.596
38.660	2.859	3.072	3.252	3.395	3.503	3.579	3.628	3.653	3.661	3.655	3.638
41.987	2.899	3.116	3.298	3.444	3.554	3.631	3.680	3.706	3.713	3.707	3.690
45.000	2.946	3.166	3.351	3.499	3.611	3.689	3.739	3.765	3.773	3.767	3.750

↑(b)

(b) When $\beta = 38.66°$, $\tan\beta = 0.8$ and cable BD is perpendicular to the lever arm BC.

(c) $F.S. = 3.579$ for $\alpha = 26.6°$; P is perpendicular to the lever arm AC

NOTE:

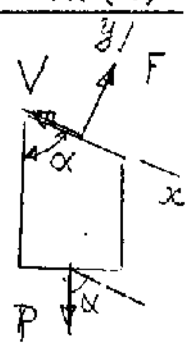
The value $F.S. = 3.579$ is the smallest of the values of F.S. corresponding to $\beta = 38.66°$ and the largest of those corresponding to $\alpha = 26.6°$. The point $\alpha = 26.6°$, $\beta = 38.66°$ is a "saddle point", or "minimax" of the function $F.S.(\alpha, \beta)$.

PROBLEM 1.C5

1.C5 A load **P** is supported as shown by two wooden members of uniform rectangular cross section that are joined by a simple glued scarf splice. (*a*) Denoting by σ_U and τ_U, respectively, the ultimate strength of the joint in tension and in shear, write a computer program which, for given values of a, b, P, σ_U and τ_U, expressed in either SI or U.S. customary units, and for values of α from 5 to 85° at 5° intervals, can be used to calculate (1) the normal stress in the joint, (2) the shearing stress in the joint, (3) the factor of safety relative to failure in tension, (4) the factor of safety relative to failure in shear, (5) the overall factor of safety for the glued joint. (*b*) Apply this program, using the dimensions and loading of the members of Probs. 1.29 and 1.31, knowing that σ_U = 1.26 MP and τ_U = 1.50 MPa for the glue used in Prob. 1.29, and that σ_U = 150 psi and τ_U = 214 psi for the glue used in Prob. 1.31. (*c*) Verify in each of these two cases that the shearing stress is maximum for α = 45°.

SOLUTION

(1) and (2) Draw the F.B. diagram of lower member:



$\overset{+}{\searrow}\sum F_x = 0: \quad -V + P\cos\alpha = 0 \qquad V = P\cos\alpha$

$+\nearrow\sum F_y = 0: \quad F - P\sin\alpha = 0 \qquad F = P\sin\alpha$

Area $= ab/\sin\alpha$

Normal stress:

$$\sigma = \frac{F}{\text{Area}} = (P/ab)\sin^2\alpha$$

Shearing stress; $\tau = \dfrac{V}{\text{Area}} = (P/ab)\sin\alpha\cos\alpha$

(3) F.S. for tension (normal stresses)

$$FSN = \sigma_U/\sigma$$

(4) F.S. for shear:

$$FSS = \tau_U/\tau$$

(5) OVERALL F.S.:

FS = The <u>smaller</u> of FSN and FSS.

(CONTINUED)

PROGRAM OUTPUTS

```
Problem 1.29
    a =    150  mm
    b =    75   mm
    P =    11   kN
    SIGU =    1.26 MPa
    TAUU =    1.50 MPa
```

ALPHA	SIG (MPa)	TAU (MPa)	FSN	FSS	FS	
5	.007	.085	169.644	17.669	17.669	
10	.029	.167	42.736	8.971	8.971	
15	.065	.244	19.237	6.136	6.136	
20	.114	.314	11.016	4.773	4.773	
25	.175	.375	7.215	4.005	4.005	
30	.244	.423	5.155	3.543	3.543	
35	.322	.459	3.917	3.265	3.265	
40	.404	.481	3.119	3.116	3.116	
45	.489	.489	2.577	3.068	2.577	◄ (b), (c)
50	.574	.481	2.196	3.116	2.196	
55	.656	.459	1.920	3.265	1.920	
60	.733	.423	1.718	3.543	1.718	
65	.803	.375	1.569	4.005	1.569	
70	.863	.314	1.459	4.773	1.459	
75	.912	.244	1.381	6.136	1.381	
80	.948	.167	1.329	8.971	1.329	
85	.970	.085	1.298	17.669	1.298	

```
Problem 1.31
    a =    5 in.
    b =    3 in.
    P =    1400 lb
    SIGU =    150  psi
    TAUU =    214  psi
```

ALPHA	SIG (psi)	TAU (psi)	FSN	FSS	FS	
5	.709	8.104	211.574	26.408	26.408	
10	2.814	15.961	53.298	13.408	13.408	
15	6.252	23.333	23.992	9.171	9.171	
20	10.918	29.997	13.739	7.134	7.134	
25	16.670	35.749	8.998	5.986	5.986	
30	23.333	40.415	6.429	5.295	5.295	
35	30.706	43.852	4.885	4.880	4.880	
40	38.563	45.958	3.890	4.656	3.890	
45	46.667	46.667	3.214	4.586	3.214	◄ (c)
50	54.770	45.958	2.739	4.656	2.739	
55	62.628	43.852	2.395	4.880	2.395	
60	70.000	40.415	2.143	5.295	2.143	◄ (b)
65	76.663	35.749	1.957	5.986	1.957	
70	82.415	29.997	1.820	7.134	1.820	
75	87.081	23.333	1.723	9.171	1.723	
80	90.519	15.961	1.657	13.408	1.657	
85	92.624	8.104	1.619	26.408	1.619	

Chapter 2

Problem 2.1

2.1 A steel rod that is 5.5 ft long stretches 0.04 in. when a 2-kip tensile load is applied to it. Knowing that $E = 29 \times 10^6$ psi, determine (*a*) the smallest diameter rod that should be used, (*b*) the corresponding normal stress caused by the load.

$L = 5.5 \text{ ft} = 66 \text{ in.}, \quad \delta = 0.04 \text{ in.}, \quad P = 2000 \text{ lb.} \qquad \delta = \dfrac{PL}{AE}$

$$A = \frac{PL}{E\delta} = \frac{(2000)(66)}{(29\times10^6)(0.04)} = 0.113793 \text{ in}^2$$

(a) $A = \frac{\pi}{4}d^2 \qquad d = \sqrt{\dfrac{4A}{\pi}} = \sqrt{\dfrac{(4)(0.113793)}{\pi}} = 0.381 \text{ in.}$

(b) $\sigma = \dfrac{P}{A} = \dfrac{2000}{0.113793} = 17576 \text{ psi} \qquad 17.56 \text{ ksi}$

Problem 2.2

2.2 A 60-m-long steel wire is subjected to 6 kN tensile force. Knowing that $E = 200$ GPa and that the length of the rod increases by 48 mm, determine (*a*) the smallest diameter that may be selected for the wire, (*b*) the corresponding normal stress.

$P = 6\times10^3 \text{ N}, \quad \delta = 48\times10^{-3} \text{ m}, \quad E = 200\times10^9 \text{ Pa} \qquad \delta = \dfrac{PL}{AE}$

$$A = \frac{PL}{E\delta} = \frac{(6\times10^3)(60)}{(200\times10^9)(48\times10^{-3})} = 37.5\times10^{-6} \text{ m}^2$$

(a) $A = \frac{\pi}{4}d^2 \qquad d = \sqrt{\dfrac{4A}{\pi}} = \sqrt{\dfrac{(4)(37.5\times10^{-6})}{\pi}} = 6.91\times10^{-3} \text{ m} \qquad 6.91 \text{ mm}$

(b) $\sigma = \dfrac{P}{A} = \dfrac{6\times10^3}{37.5\times10^{-6}} = 160\times10^6 \text{ Pa} \qquad 160.0 \text{ MPa}$

Problem 2.3

2.3 A control rod made of yellow brass must not stretch more than $\frac{1}{8}$ in. when the tension in the wire is 800 lb. Knowing that $E = 15 \times 10^6$ psi and that the maximum allowable normal stress is 32 ksi, determine (*a*) the smallest diameter that can be selected for the rod, (*b*) the corresponding maximum length of the rod.

$\delta = \frac{1}{8} \text{ in} = 0.125 \text{ in.}$

$\sigma = 32 \text{ ksi} = 32\times10^3 \text{ psi}$

$\sigma = \dfrac{P}{A} \quad \text{or} \quad A = \dfrac{P}{\sigma} = \dfrac{800}{32\times10^3} = 25\times10^{-3} \text{ in}^2$

(a) $A = \frac{\pi}{4}d^2 \qquad d = \sqrt{\dfrac{4A}{\pi}} = \sqrt{\dfrac{(4)(25\times10^{-3})}{\pi}} = 0.1784 \text{ in.}$

(b) $\delta = \dfrac{PL}{AE} \qquad L = \dfrac{AE\delta}{P} = \dfrac{(25\times10^{-3})(15\times10^6)(0.125)}{800} = 58.6 \text{ in.}$

Problem 2.4

2.4 Two gage marks are placed exactly 250 mm apart on a 12-mm-diameter aluminum rod with $E = 73$ GPa and an ultimate strength of 140 MPa. Knowing that the distance between the gage marks is 250.28 mm after a load is applied, determine (*a*) the stress in the rod, (*b*) the factor of safety.

$E = 73\times10^9 \text{ Pa}$

$\delta = 250.28 - 250.00 = 0.28 \text{ mm} = 0.28\times10^{-3} \text{ m}$

$L = 250 \text{ mm} = 250\times10^{-3} \text{ m}$

(a) $\sigma = E\varepsilon = \dfrac{E\delta}{L} = \dfrac{(73\times10^9)(0.28\times10^{-3})}{250\times10^{-3}} = 81.76\times10^6 \text{ Pa} \qquad 81.8 \text{ MPa}$

(b) $\text{F.S.} = \dfrac{\sigma_U}{\sigma} = \dfrac{140\times10^6}{81.76\times10^6} = 1.712$

Problem 2.5

2.5 A nylon thread is subjected to a 2-lb tension force. Knowing that $E = 0.7 \times 10^6$ psi. and that the length of the thread increases by 1.1 %, determine (*a*) the diameter of the thread, (*b*) the stress in the thread.

$$\varepsilon = \frac{\delta}{L} = \frac{0.011L}{L} = 0.011$$

$$\delta = \frac{PL}{AE} \qquad A = \frac{PL}{E\delta} = \frac{P}{E\varepsilon} = \frac{2}{(0.7\times10^6)(0.011)} = 259.74\times10^{-6}\ \text{in}^2$$

(a) $A = \frac{\pi}{4}d^2 \qquad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(259.74\times10^{-6})}{\pi}} = 0.01819$ in.

(b) $\sigma = \frac{P}{A} = \frac{2}{259.74\times10^{-6}} = 7.70\times10^3$ psi $\qquad$ 7.70 ksi

Problem 2.6

2.6 A cast-iron tube is used to support a compressive load. Knowing that $E = 69$ GPa and that the maximum allowable change in length is 0.025 %, determine (*a*) the maximum normal stress in the tube, (*b*) the minimum wall thickness for a load of 7.2 kN if the outside diameter of the tube is 50 mm.

$$E = 69\ \text{GPa} = 69\times10^9\ \text{Pa}$$

$$\varepsilon = \frac{\delta}{L} = \frac{0.00025L}{L} = 0.00025$$

(a) $\sigma = E\varepsilon = (69\times10^9)(0.00025) = 17.25\times10^6$ Pa $\qquad$ 17.25 MPa

(b) $\sigma = \frac{P}{A} \qquad A = \frac{P}{\sigma} = \frac{7.2\times10^3}{17.25\times10^6} = 417.39\times10^{-6}\ \text{m}^2 = 417.39\ \text{mm}^2$

$$A = \frac{\pi}{4}(d_o^2 - d_i^2) \qquad d_i^2 = d_o^2 - \frac{4A}{\pi} = 50^2 - \frac{(4)(417.39)}{\pi} = 1968.56\ \text{mm}^2$$

$$d_i = 44.368\ \text{mm} \qquad t = \tfrac{1}{2}(d_o - d_i) = \tfrac{1}{2}(50 - 44.368) = 2.82\ \text{mm}$$

Problem 2.7

2.7 A 28-ft length of 0.25-in.-diameter steel wire is to be used in a hanger. It is noted that the wire stretches 0.45 in. when a tensile force **P** is applied. Knowing that $E = 29 \times 10^6$ psi , determine (*a*) the magnitude of the force **P**, (*b*) the corresponding normal stress in the wire

$$L = 28\ \text{ft} = 336\ \text{in.}$$

$$A = \frac{\pi}{4}d^2 = \left(\frac{\pi}{4}\right)(0.25)^2 = 49.087\times10^{-3}\ \text{in}^2$$

(a) $\delta = \frac{PL}{AE} \qquad P = \frac{AE\delta}{L} = \frac{(49.087\times10^{-3})(29\times10^6)(0.45)}{336} = 1.907\times10^3$ lb

1.907 kips

(b) $\sigma = E\varepsilon = \frac{E\delta}{L} = \frac{(29\times10^6)(0.45)}{336} = 38.8\times10^3$ psi $\qquad$ 38.8 ksi

Problem 2.8

2.8 A square aluminum bar should not stretch more than 1.4 mm when it is subjected to a tensile load. Knowing that $E = 70$ GPa and that the allowable tensile strength is 120 MPa, determine (*a*) the maximum allowable length of the pipe, (*b*) the required dimensions of the cross-section if the tensile load is 28 kN.

$\sigma = 120 \times 10^6$ Pa

$E = 70 \times 10^9$ Pa $\qquad \delta = 1.4 \times 10^{-3}$ m

(a) $\delta = \dfrac{PL}{AE} = \dfrac{\sigma L}{E} \qquad L = \dfrac{E\delta}{\sigma} = \dfrac{(70\times10^9)(1.4\times10^{-3})}{120\times10^6} = 0.817$ m

$L = 817$ mm ◄

(b) $\sigma = \dfrac{P}{A} \qquad A = \dfrac{P}{\sigma} = \dfrac{28\times10^3}{120\times10^6} = 233.333\times10^{-6}\ m^2 = 233.333\ mm^2$

$A = a^2 \qquad a = \sqrt{A} = \sqrt{233.333} = 15.28$ mm ◄

Problem 2.9

2.9 A 9-kN tensile load will be applied to a 50-m length of steel wire with $E = 200$ GPa. Determine the smallest diameter wire which can be used, knowing that the normal stress must not exceed 150 MPa and that the increase in the length of the wire should be at most 25 mm.

Considering allowable stress $\quad \sigma = 150 \times 10^6$ Pa

$\sigma = \dfrac{P}{A} \quad \therefore \quad A = \dfrac{P}{\sigma} = \dfrac{9\times10^3}{150\times10^6} = 60\times10^{-6}\ m^2$

Considering allowable elongation $\quad \delta = 25\times10^{-3}$ m

$\delta = \dfrac{PL}{AE} \quad \therefore \quad A = \dfrac{PL}{E\delta} = \dfrac{(9\times10^3)(50)}{(200\times10^9)(25\times10^{-3})} = 90\times10^{-6}\ m^2$

Larger area governs $\qquad A = 90\times10^{-6}\ m^2$

$A = \dfrac{\pi}{4} d^2 \qquad d = \sqrt{\dfrac{4A}{\pi}} = \sqrt{\dfrac{(4)(90\times10^{-6})}{\pi}} = 10.70\times10^{-3}$ m

$= 10.70$ mm ◄

Problem 2.10

2.10 A block of 250-mm length and 50×40 mm cross section is to support a centric compressive load P. The material to be used is a bronze for which $E = 95$ GPa. Determine the largest load which can be applied, knowing that the normal stress must not exceed 80 MPa and that the decrease in length of the block should be at most 0.12% of its original length.

$A = (50)(40) = 2000\ mm^2 = 2\times10^{-3}\ m^2$

$\sigma_{all} = 80$ MPa $= 80\times10^6$ Pa, $\qquad E = 95\times10^9$ Pa

Considering allowable stress

$\sigma = \dfrac{P}{A} \qquad P = A\sigma = (2\times10^{-3})(80\times10^6) = 160\times10^3$ N

Considering allowable deformation

$\delta = \dfrac{PL}{EA} \qquad P = EA\left(\dfrac{\delta}{L}\right) = (95\times10^9)(2\times10^{-3})(0.0012) = 228\times10^3$ N

Smaller value of P governs $\qquad P = 160\times10^3$ N $= 160.0$ kN ◄

Problem 2.11

2.11 A 1.5-m-long aluminum rod must not stretch more than 1 mm and the normal stress must not exceed 40 MPa when the rod is subjected to a 3-kN axial load. Knowing that E = 70 GPa, determine the required diameter of the rod.

$L = 1.5$ m

$\delta = 1 \times 10^{-3}$ m, $\quad \sigma = 40 \times 10^6$ Pa, $\quad E = 70 \times 10^9$ Pa, $\quad P = 3 \times 10^3$ N

Stress: $\sigma = \dfrac{P}{A} \qquad A = \dfrac{P}{\sigma} = \dfrac{3 \times 10^3}{40 \times 10^6} = 75 \times 10^{-6}\ \text{m}^2 = 75\ \text{mm}^2$

Deformation: $\delta = \dfrac{PL}{AE}$

$$A = \frac{PL}{E\delta} = \frac{(3 \times 10^3)(1.5)}{(70 \times 10^9)(1 \times 10^{-3})} = 64.29 \times 10^{-6}\ \text{m}^2 = 64.29\ \text{mm}^2$$

Larger value of A governs. $\qquad A = 75\ \text{mm}^2$

$A = \dfrac{\pi}{4} d^2 \qquad d = \sqrt{\dfrac{4A}{\pi}} = \sqrt{\dfrac{(4)(75)}{\pi}} = 9.77$ mm ◄

Problem 2.12

2.12 A nylon thread is to be subjected to a 2.5-lb tensile load. Knowing that $E = 0.5 \times 10^6$ psi, that the maximum allowable normal stress is 6 ksi, and that the length of the thread must not increase by more than 1%, determine the required diameter of the thread.

$\sigma = 6$ ksi $= 6 \times 10^3$ psi

Stress: $\sigma = \dfrac{P}{A} \qquad A = \dfrac{P}{\sigma} = \dfrac{2.5}{6 \times 10^3} = 416.667 \times 10^{-6}\ \text{in}^2$

Deformation: $\delta = \dfrac{PL}{AE}$

$$A = \frac{P}{E}\left(\frac{L}{\delta}\right) = \left(\frac{2.5}{0.5 \times 10^6}\right)(100) = 500 \times 10^{-6}\ \text{in}^2$$

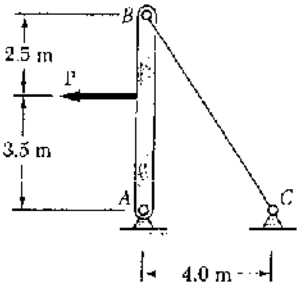
Larger value of A governs. $\qquad A = 500 \times 10^{-6}\ \text{in}^2$

$A = \dfrac{\pi}{4} d^2 \qquad d = \sqrt{\dfrac{4A}{\pi}} = \sqrt{\dfrac{(4)(500 \times 10^{-6})}{\pi}} = 25.2 \times 10^{-3}$ in.

0.0252 in. ◄

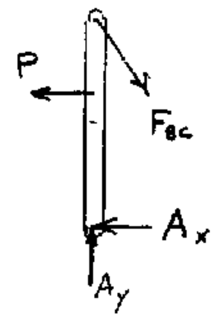
Problem 2.13

2.13 The 4-mm-diameter cable BC is made of a steel with $E = 200$ GPa. Knowing that the maximum stress in the cable must not exceed 190 MPa and that the elongation of the cable must not exceed 6 mm, find the maximum load **P** that can be applied as shown.



$L_{BC} = \sqrt{6^2 + 4^2} = 7.2111$ m

Use bar AB as a free body



$\circlearrowleft \Sigma M_A = 0 \qquad 3.5P - (6)\left(\frac{4}{7.2111} F_{BC}\right) = 0$

$P = 0.9509\ F_{BC}$

Considering allowable stress $\sigma = 190 \times 10^6$ Pa

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.004)^2 = 12.566 \times 10^{-6}\ m^2$

$\sigma = \frac{F_{BC}}{A} \therefore \quad F_{BC} = \sigma A = (190 \times 10^6)(12.566 \times 10^{-6}) = 2.388 \times 10^3$ N

Considering allowable elongation $\delta = 6 \times 10^{-3}$ m

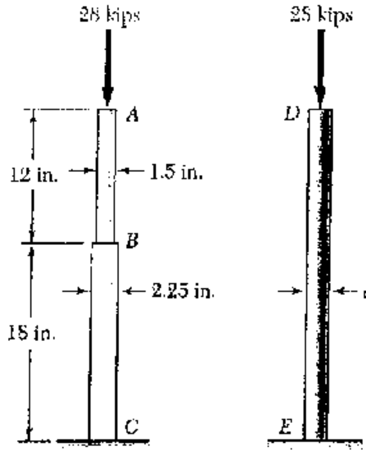
$\delta = \frac{F_{BC} L_{BC}}{AE} \therefore F_{BC} = \frac{AE\delta}{L_{BC}} = \frac{(12.566 \times 10^{-6})(200 \times 10^9)(6 \times 10^{-3})}{7.2111} = 2.091 \times 10^3$ N

Smaller value governs $\qquad F_{BC} = 2.091 \times 10^3$ N

$P = 0.9509\ F_{BC} = (0.9509)(2.091 \times 10^3) = 1.988 \times 10^3\ N = 1.988$ kN ◄

Problem 2.14

2.14 The aluminum rod ABC ($E = 10.1 \times 10^6$ psi), which consists of two cylindrical portions AB and BC, is to be replaced with a cylindrical steel rod DE ($E = 29 \times 10^6$ psi) of the same overall length. Determine the minimum required diameter d of the steel rod if its vertical deformation is not to exceed the deformation of the aluminum rod under the same load and if the allowable stress in the steel rod is not to exceed 24 ksi.



Deformation of aluminum rod

$$\delta_A = \frac{PL_{AB}}{A_{AB}E} + \frac{PL_{BC}}{A_{BC}E} = \frac{P}{E}\left(\frac{L_{AB}}{A_{AB}} + \frac{L_{BC}}{A_{BC}}\right)$$

$$= \frac{28\times10^3}{10.1\times10^6}\left(\frac{12}{\frac{\pi}{4}(1.5)^2} + \frac{18}{\frac{\pi}{4}(2.25)^2}\right)^2 = 0.031376 \text{ in}$$

Steel rod $\qquad \delta = 0.031376$ in

$$\delta = \frac{PL}{EA} \quad \therefore \quad A = \frac{PL}{E\delta} = \frac{(28\times10^3)(30)}{(29\times10^6)(0.031376)} = 0.92317 \text{ in}^2$$

$$\sigma = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma} = \frac{28\times10^3}{24\times10^3} = 1.1667 \text{ in}^2$$

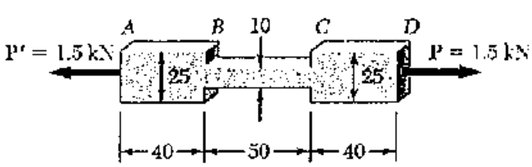
Required area is the larger value $\qquad A = 1.1667 \text{ in}^2$

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(1.6667)}{\pi}} = 1.219 \text{ in.}$$

Problem 2.15

2.15 The specimen shown has been cut from a 5-mm-thick sheet of vinyl ($E = 3.10$ GPa) and is subjected to a 1.5-kN tensile load. Determine (a) the total deformation of the specimen, (b) the deformation of its central portion BC.

Dimensions in mm



$$\delta_{AB} = \frac{PL_{AB}}{EA_{AB}} = \frac{(1.5\times10^3)(40\times10^{-3})}{(3.10\times10^9)(25\times10^{-3})(5\times10^{-3})}$$

$$= 154.84\times10^{-6} \text{ m}$$

$$\delta_{BC} = \frac{PL_{BC}}{EA_{BC}} = \frac{(1.5\times10^3)(50\times10^{-3})}{(3.10\times10^9)(10\times10^{-3})(5\times10^{-3})}$$

$$= 483.87\times10^{-6} \text{ m}$$

$$\delta_{CD} = \frac{PL_{CD}}{EA_{CD}} = \delta_{AB} = 154.84\times10^{-6} \text{ m}$$

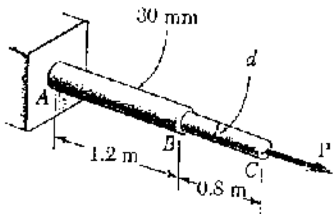
(a) Total deformation: $\delta = \delta_{AB} + \delta_{BC} + \delta_{CD} = 793.52\times10^{-6}$ m

0.794 mm

(b) Over BC $\qquad \delta_{BC} = 483.87\times10^{-6}$ $\qquad$ 0.484 mm

Problem 2.16

2.16 A single axial load of magnitude $P = 58$ kN is applied at end C of the brass rod ABC. Knowing that $E = 105$ GPa, determine the diameter d of portion BC for which the deflection of point C will be 3 mm.



$$\delta_C = \sum \frac{P_i L_i}{A_i E} = \frac{P}{E}\left\{\frac{L_{AB}}{A_{AB}} + \frac{L_{BC}}{A_{BC}}\right\}$$

$$\frac{L_{BC}}{A_{BC}} = \frac{E\delta_C}{P} - \frac{L_{AB}}{A_{AB}} = \frac{(105\times10^9)(3\times10^{-3})}{58\times10^3} - \frac{1.2}{\frac{\pi}{4}(0.030)^2} = 3.7334\times10^3 \ m^{-1}$$

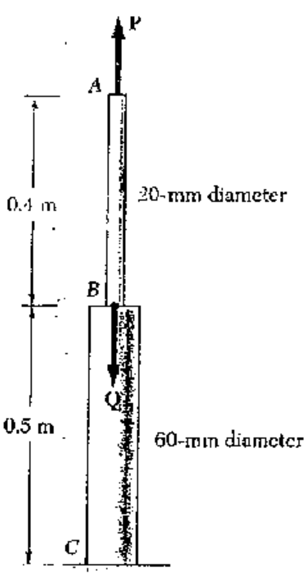
$$A_{BC} = \frac{L_{BC}}{3.7334\times10^3} = \frac{0.8}{3.7334\times10^3} = 214.28\times10^{-6}\ m^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 \quad \therefore \quad d_{BC} = \sqrt{\frac{4A_{BC}}{\pi}} = \sqrt{\frac{(4)(214.28\times10^{-6})}{\pi}} = 16.52\times10^{-3}\ m$$

$$= 16.52\ mm$$

Problem 2.17

2.17 Both portions of the rod ABC are made of an aluminum for which $E = 70$ GPa. Knowing that the magnitude of **P** is 4 kN, determine (*a*) the value of **Q** so that the deflection at A is zero, (*b*) the corresponding deflection of B.



(a) $A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(0.020)^2 = 314.16\times10^{-6}\ m^2$

$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(0.060)^2 = 2.8274\times10^{-3}\ m^2$

Force in member AB is P tension

Elongation $\delta_{AB} = \frac{P L_{AB}}{E A_{AB}} = \frac{(4\times10^3)(0.4)}{(70\times10^9)(314.16\times10^{-6})}$

$= 72.756\times10^{-6}\ m$

Force in member BC is Q − P compression

Shortening $\delta_{BC} = \frac{(Q-P)L_{BC}}{E A_{BC}} = \frac{(Q-P)(0.5)}{(70\times10^9)(2.8274\times10^{-3})}$

$= 2.5263\times10^{-9}(Q-P)$

For zero deflection at A $\quad \delta_{BC} = \delta_{AB}$

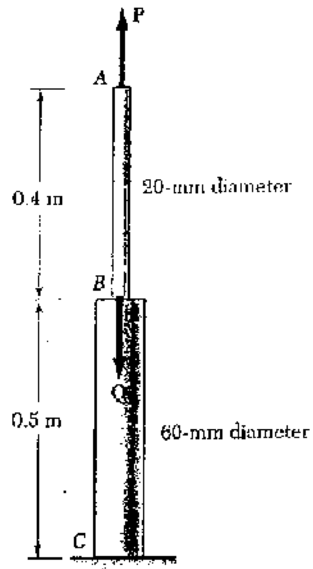
$2.5263\times10^{-9}(Q-P) = 72.756\times10^{-6} \quad \therefore \quad Q - P = 28.8\times10^3\ N$

$Q = 28.3\times10^3 + 4\times10^3 = 32.8\times10^3\ N = 32.8\ kN$

(b) $\delta_{AB} = \delta_{BC} = \delta_B = 72.756\times10^{-6}\ m = 0.0728\ mm \downarrow$

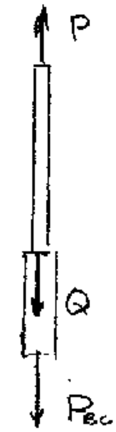
Problem 2.18

2.18 The rod *ABC* is made of an aluminum for which $E = 70$ GPa. Knowing that $P = 6$ kN and $Q = 42$ kN, determine the deflection of (*a*) point *A*, (*b*) point *B*.



(a) $A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(0.020)^2 = 314.16 \times 10^{-6}\ m^2$

$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(0.060)^2 = 2.8274 \times 10^{-3}\ m^2$



$P_{AB} = P = 6 \times 10^3$ N

$P_{BC} = P - Q = 6 \times 10^3 - 42 \times 10^3 = -36 \times 10^3$ N

$L_{AB} = 0.4$ m $\qquad L_{BC} = 0.5$ m

$\delta_{AB} = \frac{P_{AB} L_{AB}}{A_{AB} E_A} = \frac{(6 \times 10^3)(0.4)}{(314.16 \times 10^{-6})(70 \times 10^9)}$

$= 109.135 \times 10^{-6}$ m

$\delta_{BC} = \frac{P_{BC} L_{BC}}{A_{BC} E} = \frac{(-36 \times 10^3)(0.5)}{(2.8274 \times 10^{-3})(70 \times 10^9)}$

$= -90.947 \times 10^{-6}$ m

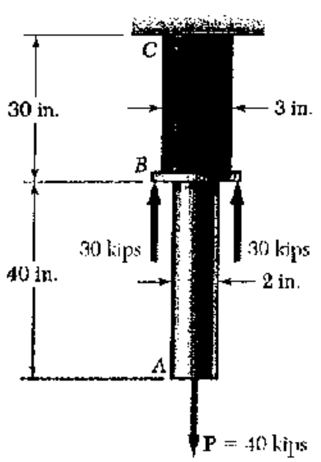
$\delta_A = \delta_{AB} + \delta_{BC} = 109.135 \times 10^{-6} - 90.947 \times 10^{-6}\ m = 18.19 \times 10^{-6}$ m

$= 0.01819$ mm ↑

(b) $\delta_B = \delta_{BC} = -90.9 \times 10^{-6}\ m = -0.0909$ mm or 0.0919 mm ↓

Problem 2.19

2.19 Two solid cylindrical rods are joined at *B* and loaded as shown. Rod *AB* is made of steel ($E = 29 \times 10^6$ psi), and rod *BC* of brass ($E = 15 \times 10^6$ psi). Determine (*a*) the total deformation of the composite rod *ABC*, (*b*) the deflection of point *B*.



Portion AB: $P_{AB} = 40 \times 10^3$ lb, $L_{AB} = 40$ in., $d = 2$ in.,

$A_{AB} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(2)^2 = 3.1416\ in^2$, $E_{AB} = 29 \times 10^6$ psi

$\delta_{AB} = \frac{P_{AB} L_{AB}}{E_{AB} A_{AB}} = \frac{(40 \times 10^3)(40)}{(29 \times 10^6)(3.1416)} = 17.5619 \times 10^{-3}$ in.

Portion BC: $P_{BC} = -20 \times 10^3$ lb, $L_{BC} = 30$ in., $d = 3$ in.,

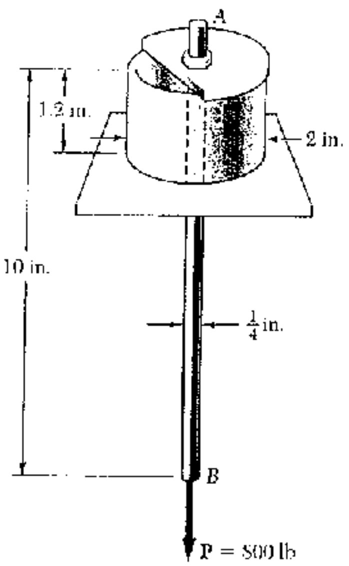
$A_{BC} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(3)^2 = 7.0686\ in^2$, $E_{BC} = 15 \times 10^6$ psi

$\delta_{BC} = \frac{P_{BC} L_{BC}}{E_{BC} A_{BC}} = \frac{(-20 \times 10^3)(30)}{(15 \times 10^6)(7.0686)} = -5.6588 \times 10^{-3}$ in.

(a) $\delta = \delta_{AB} + \delta_{BC} = 17.5619 \times 10^{-3} - 5.6588 \times 10^{-3}$ $\qquad \delta = 11.90 \times 10^{-3}$ in. ↓

(b) $\delta_B = -\delta_{BC}$ $\qquad \delta_B = 5.66 \times 10^{-3}$ in. ↑

Problem 2.20



2.20 A $\frac{1}{8}$-in.-thick hollow polystyrene cylinder ($E = 0.45 \times 10^6$ psi) and a rigid circular plate (only part of which is shown) are used to support a 10-in.-long steel rod AB ($E = 29 \times 10^6$ psi) of $\frac{1}{4}$-in. diameter. If an 800-lb load **P** is applied at B, determine (*a*) the elongation of rod AB, (*b*) the deflection of point B, (*c*) the average normal stress in rod AB.

Rod AB: $P_{AB} = 800$ lb, $L_{AB} = 10$ in. $d = \frac{1}{4}$ in

$A_{AB} = \frac{\pi}{4}d^2 = \frac{\pi}{4}\left(\frac{1}{4}\right)^2 = 49.087 \times 10^{-3}$ in

(a) $\delta_{AB} = \dfrac{P_{AB}L_{AB}}{E_{AB}A_{AB}} = \dfrac{(800)(10)}{(29\times10^6)(49.087\times10^{-6})} = 5.62 \times 10^{-3}$ in.

Hollow cylinder: $d_o = 2$ in. $d_i = 2 - (2)\left(\frac{1}{8}\right) = 1.75$ in.

$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(2^2 - 1.75^2) = 736.31 \times 10^{-3}$ in^2

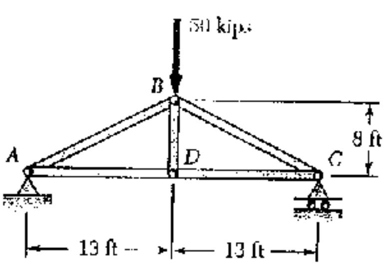
$L = 1.2$ in., $P = 800$ lb.

$\delta_{cyl} = \dfrac{PL}{EA} = \dfrac{(800)(1.2)}{(0.45\times10^6)(736.31\times10^{-3})} = 2.90\times10^{-3}$ in.

(b) Deflection of point B $\quad \delta_B = \delta_{AB} + \delta_{cyl} = \quad 8.52 \times 10^{-3}$ in. ↓

(c) Stress in rod AB $\quad \sigma = \dfrac{P}{A_{AB}} = \dfrac{800}{49.087\times10^{-3}} = 16.30 \times 10^3$ psi $= 16.30$ ksi

Problem 2.21



2.21 For the steel truss ($E = 29 \times 10^6$ psi) and loading shown, determine the deformations of members AB and AD, knowing that their cross-sectional areas are 4.0 in^2 and 2.8 in^2, respectively.

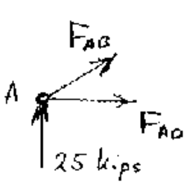
Statics: Reactions at A and C. 25 kips ↑

Member BD is a zero force member.

$L_{AB} = \sqrt{13^2 + 8^2} = 15.2643$ ft $= 183.172$ in.

$L_{AD} = 13$ ft $= 156$ in.

Use joint A as free body



$+\uparrow \Sigma F_y = 0: \quad 25 + \dfrac{8}{15.2643} F_{AB} = 0 \qquad F_{AB} = -47.701$ kips

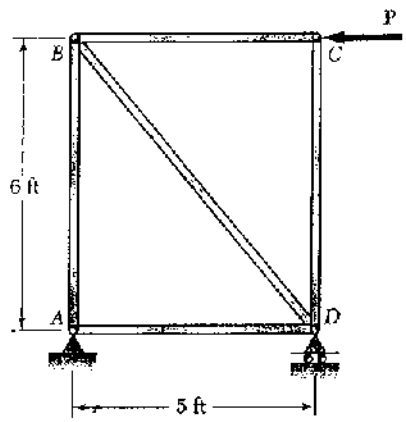
$\pm \Sigma F_x = 0: \quad F_{AD} + \dfrac{13}{15.2643} F_{AB} = 0 \qquad F_{AD} = 40.625$ kips

$\delta_{AB} = \dfrac{F_{AB}L_{AB}}{EA_{AB}} = \dfrac{(-47.701\times10^3)(183.172)}{(29\times10^6)(4.0)} = -0.0753$ in.

$\delta_{AD} = \dfrac{F_{AD}L_{AD}}{EA_{AD}} = \dfrac{(40.625\times10^3)(156)}{(29\times10^6)(2.8)} = 0.0780$ in.

Problem 2.22

2.22 The steel frame ($E = 29 \times 10^6$ psi) shown has a diagonal brace BD with an area of 3.2 in^2. Determine the largest allowable load **P** if the change in length of member BD is not to exceed $\frac{1}{16}$ in.



$\delta_{BD} = \frac{1}{16}$ in. $= 0.0625$ in.

$L_{BD} = \sqrt{5^2 + 6^2} = 7.8102$ ft $= 93.723$ in.

$\delta_{BD} = \dfrac{F_{BD} L_{BD}}{E A_{BD}} \qquad F_{BD} = \dfrac{E A_{BD} \delta_{BD}}{L_{BD}}$

$F_{BD} = \dfrac{(29 \times 10^6)(3.2)(0.0625)}{93.723} = 61.884 \times 10^3$ lb.

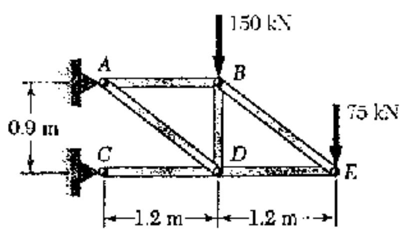
Use joint B as a free body.

$\overset{+}{\leftarrow} \Sigma F_x = 0: \quad -P + \frac{5}{7.8102} F_{BD} = 0$

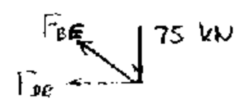
$P = \dfrac{(5)(61.884 \times 10^3)}{7.8102} = 39.6 \times 10^3$ lb $= 39.6$ kips ◄

Problem 2.23

2.23 Members AB and BE of the truss shown consist of 25-mm-diameter steel rods ($E = 200$ GPa). For the loading shown, determine the elongation of (*a*) rod AB, (*b*) rod BE.



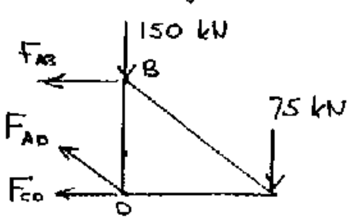
$L_{BE} = \sqrt{1.2^2 + 0.9^2} = 1.5$ m



Use joint E as a free body.

$+\uparrow \Sigma F_y = 0: \quad \frac{0.9}{1.5} F_{BE} - 75 = 0 \qquad F_{BE} = 125$ kN

Use triangle BDE as a free body: $\circlearrowleft \Sigma M_D = 0$



$0.9 F_{AB} - (1.2)(75) = 0 \qquad F_{AB} = 100$ kN

$A_{AB} = A_{BE} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(25)^2 = 490.87$ mm^2

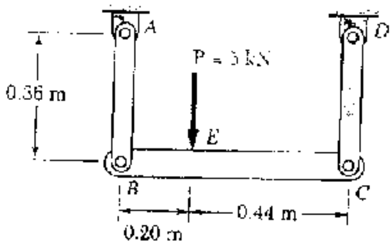
$= 490.87 \times 10^{-6}$ m^2

$E = 200 \times 10^9$ Pa

(a) $\delta_{AB} = \dfrac{F_{AB} L_{AB}}{E A_{AB}} = \dfrac{(100 \times 10^3)(1.2)}{(200 \times 10^9)(490.87 \times 10^{-6})} = 1.222 \times 10^{-3}$ m $\qquad$ 1.222 mm ◄

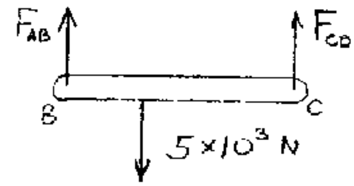
(b) $\delta_{BE} = \dfrac{F_{BE} L_{BE}}{E A_{BE}} = \dfrac{(125 \times 10^3)(1.5)}{(200 \times 10^9)(490.87 \times 10^{-6})} = 1.910 \times 10^{-3}$ m $\qquad$ 1.910 mm ◄

Problem 2.24



2.24 Each of the links *AB* and *CD* is made of aluminum (E = 75 GPa) and has a cross-sectional area of 125 mm^2. Knowing that they support the rigid member *BC*, determine the deflection of point *E*.

SOLUTION



Use member BC as a free body

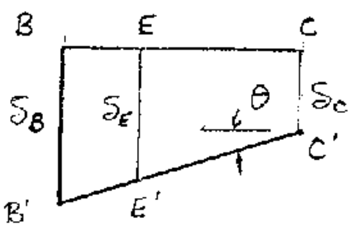
$\circlearrowleft \Sigma M_C = 0 \quad -(0.64)F_{AB} + (0.44)(5\times10^3) = 0 \qquad F_{AB} = 3.4375\times10^3\ N$

$\circlearrowleft \Sigma M_B = 0 \quad (0.64)F_{CD} - (0.20)(5\times10^3) = 0 \qquad F_{CD} = 1.5625\times10^3\ N$

For links AB and CD $\quad A = 125\ mm^2 = 125\times10^{-6}\ m^2$

$$\delta_{AB} = \frac{F_{AB}L_{AB}}{EA} = \frac{(3.4375\times10^3)(0.36)}{(75\times10^9)(125\times10^{-6})} = 132.00\times10^{-6}\ m = \delta_B$$

$$\delta_{CD} = \frac{F_{CD}L_{CD}}{EA} = \frac{(1.5625\times10^3)(0.36)}{(75\times10^9)(125\times10^{-6})} = 60.00\times10^{-6}\ m = \delta_C$$



Deformation diagram

$$\text{Slope } \theta = \frac{\delta_B - \delta_C}{l_{BC}} = \frac{72.00\times10^{-6}}{0.64}$$

$$= 112.5\times10^{-6}\ rad$$

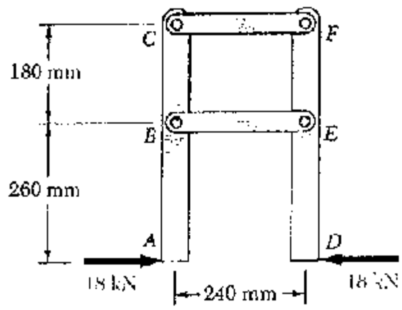
$$\delta_E = \delta_C + l_{EC}\theta$$

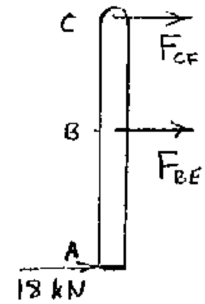
$$= 60.00\times10^{-6} + (0.44)(112.5\times10^{-6})$$

$$= 109.5\times10^{-6}\ m = 0.1095\ mm \downarrow$$

Problem 2.25

2.25 Members *ABC* and *DEF* are joined with steel links ($E = 200$ GPa). Each of the links is made of a pair of 25×35-mm plates. Determine the change in length of (*a*) member *BE*, (*b*) member *CF*.





Use member ABC as a free body

$\circlearrowleft \Sigma M_B = 0$

$$(0.260)(18\times10^3) - (0.180)F_{CF} = 0$$

$$F_{CF} = \frac{(0.260)(18\times10^3)}{0.180} = 26\times10^3\ \text{N}$$

$\circlearrowright \Sigma M_C = 0 \qquad (0.440)(18\times10^3) + (0.180)F_{BE} = 0$

$$F_{BE} = -\frac{(0.440)(18\times10^3)}{0.180} = -44\times10^3\ \text{N}$$

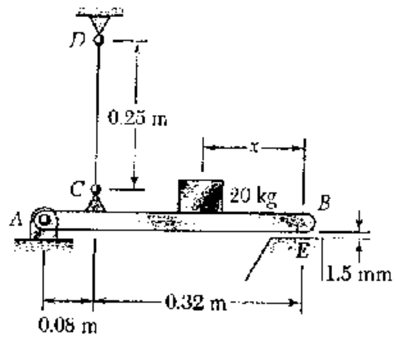
Area for link made of two plates

$$A = (2)(0.025)(0.035) = 1.75\times10^{-3}\ \text{m}^2$$

(a) $$\delta_{BE} = \frac{F_{BE}L_{BE}}{EA} = \frac{(-44\times10^3)(0.240)}{(200\times10^9)(1.75\times10^{-3})} = -30.2\times10^{-6}\ \text{m} = -0.0302\ \text{mm}$$

(b) $$\delta_{CF} = \frac{F_{CF}L_{CF}}{EA} = \frac{(26\times10^3)(0.240)}{(200\times10^9)(1.75\times10^{-3})} = 17.83\times10^{-6}\ \text{m} = 0.01783\ \text{mm}$$

Problem 2.26



2.26 The length of the 2-mm-diameter steel wire CD has been adjusted so that with no load applied, a gap of 1.5 mm exists between the end B of the rigid beam ACB and a contact point E. Knowing that $E = 200$ GPa, determine where a 20-kg block should be placed on the beam in order to cause contact between B and E.

Rigid rod ACE rotates through angle θ to close gap.

$$\theta = \frac{1.5\times10^{-3}}{0.40} = 3.75\times10^{-3} \text{ rad}$$

Point C moves downward.

$$\delta_C = 0.08\,\theta = (0.08)(3.75\times10^{-3}) = 300\times10^{-6} \text{ m}$$

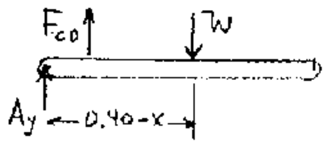
$$\delta_{CD} = \delta_C = 300\times10^{-6} \text{ m}$$

$$A_{CD} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(2)^2 = 3.1416 \text{ mm}^2 = 3.1416\times10^{-6} \text{ m}^2$$

$$E = 200\times10^9 \text{ Pa} \qquad \delta_{CD} = \frac{F_{CD}L_{CD}}{EA_{CD}}$$

$$F_{CD} = \frac{EA_{CD}\delta_{CD}}{L_{CD}} = \frac{(200\times10^9)(3.1416\times10^{-6})(300\times10^{-6})}{0.25} = 753.98 \text{ N}$$

Use beam ACB as a free body. $\quad W = mg = (20)(9.81) = 196.2$ N

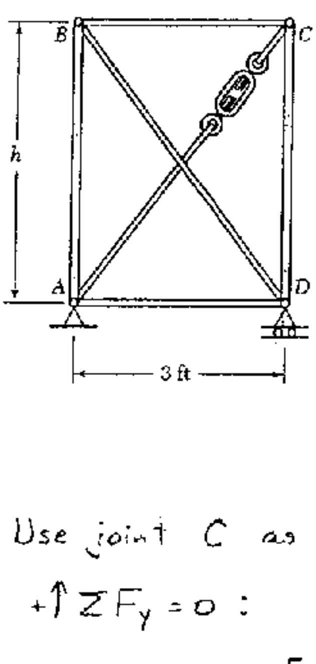


$$+\circlearrowleft \Sigma M_A = 0: \quad 0.08\,F_{CD} - (0.40 - x)W = 0$$

$$0.40 - x = \frac{(0.08)(753.98)}{196.2} = 0.30743 \text{ m}$$

$$x = 0.0926 \text{ m} = 92.6 \text{ mm}$$

Problem 2.27



2.27 Members AB and CD are $1\frac{1}{8}$-in.-diameter steel rods, and members BC and AD are $\frac{7}{8}$-in.-diameter steel rods. When the turnbuckle is tightened, the diagonal member AC is put in tension. Knowing that $E = 29\times10^6$ psi, determine the largest allowable tension in AC so that the deformations in members AB and CD do not exceed 0.04 in.

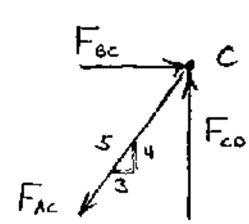
$$\delta_{AB} = \delta_{CD} = 0.04 \text{ in} \qquad h = 4 \text{ ft} = 48 \text{ in.} = L_{CD}$$

$$A_{CD} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(1.125)^2 = 0.99402 \text{ in}^2$$

$$\delta_{CD} = \frac{F_{CD}L_{CD}}{EA_{CD}}$$

$$F_{CD} = \frac{EA_{CD}\delta_{CD}}{L_{CD}} = \frac{(29\times10^6)(0.99402)(0.04)}{48} = 24.022\times10^3 \text{ lb.}$$

Use joint C as a free body

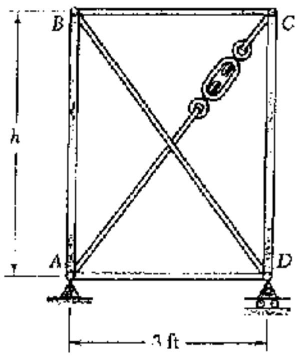


$$+\uparrow \Sigma F_y = 0: \quad F_{CD} - \frac{4}{5}F_{AC} = 0 \quad \therefore F_{AC} = \frac{5}{4}F_{CD}$$

$$F_{AC} = \frac{5}{4}(24.022\times10^3) = 30.0\times10^3 \text{ lb.}$$

$$= 30.0 \text{ kips}$$

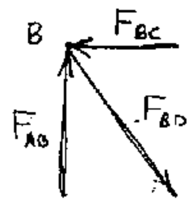
Problem 2.28

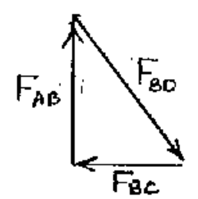


2.28 For the structure in Prob. of 2.27, determine (*a*) the distance h so that the deformations in members AB, BC, CD and AD are equal, (*b*) the corresponding tension in member AC.

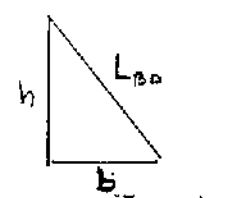
2.27 Members AB and CD are $1\frac{1}{8}$-in.-diameter steel rods, and members BC and AD are $\frac{7}{8}$-in.-diameter steel rods. When the turnbuckle is tightened, the diagonal member AC is put in tension. Knowing that $E = 29 \times 10^6$ psi and $h = 4$ ft, determine the largest allowable tension in AC so that the deformations in members AB and CD do not exceed 0.04 in.

(a) Statics: Use joint B as a free body





Force Triangle



Geometry

From similar triangles

$$\frac{F_{AB}}{h} = \frac{F_{BC}}{b} = \frac{F_{BD}}{L_{BC}}$$

$$F_{AB} = \frac{h}{b} F_{BC}$$

For equal deformations

$$\delta_{AB} = \delta_{BC} \quad \frac{F_{AB}\, h}{E A_{AB}} = \frac{F_{BC}\, b}{E A_{BC}} \quad \therefore \quad F_{AB} = \frac{b}{h} \cdot \frac{A_{AB}}{A_{BC}} F_{BC}$$

Equating expressions for F_{AB}

$$\frac{h}{b} F_{BC} = \frac{b}{h} \frac{A_{AB}}{A_{BC}} F_{BC} \qquad \frac{h^2}{b^2} = \frac{A_{AB}}{A_{BC}} = \frac{\frac{\pi}{4} d_{AB}^2}{\frac{\pi}{4} d_{BC}^2} = \frac{d_{AB}^2}{d_{BC}^2}$$

$$\frac{h}{b} = \frac{d_{AB}}{d_{BC}} = \frac{9/8}{7/8} = \frac{9}{7} \qquad b = 3 \text{ ft} = 36 \text{ in.}$$

$$h = \frac{9}{7} b = \frac{9}{7}(3) = 3.86 \text{ ft} = 46.3 \text{ in.}$$

(b) Setting $\delta_{AB} = \delta_{BC} = 0.04$ in.

$$\delta_{BC} = \frac{F_{BC}\, b}{E A_{BC}} \quad \therefore \quad F_{BC} = \frac{E A_{BC} \delta_{BC}}{b} = \frac{(29 \times 10^6) \frac{\pi}{4} \left(\frac{7}{8}\right)^2 (0.04)}{36}$$

$$= 19.376 \times 10^3 \text{ lb.}$$

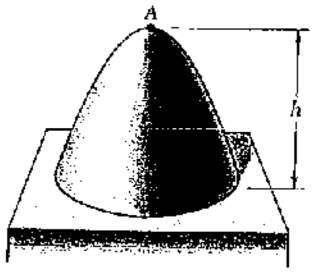
$$F_{AB} = \frac{h}{b} F_{BC} = \frac{9}{7}(19.376 \times 10^3) = 24.912 \times 10^3 \text{ lb}$$

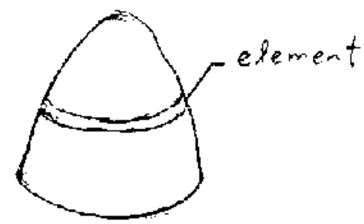
From the force triangle

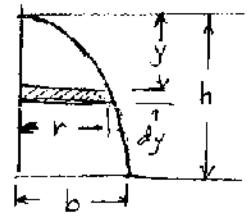
$$F_{BD} = F_{AC} = \sqrt{F_{BC}^2 + F_{AB}^2} = 31.6 \times 10^3 \text{ lb.} = 31.6 \text{ kips}$$

Problem 2.29

2.29 Determine the deflection of the apex A of a homogeneous paraboloid of revolution of height h, density ρ, and modulus of elasticity E, due to its own weight.







Let b = radius of the base and r = radius at section with coordinate y.

For the element $r = b\left(\frac{y}{h}\right)^{1/2}$

$$A = \pi r^2 = \pi b^2 \frac{y}{h}$$

$$dP = \rho g A\, dy = \pi b^2 \rho g \frac{y}{h}$$

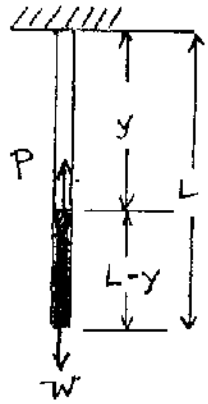
$$P = \int_0^y dP = \frac{\pi b^2 \rho g}{h}\int_0^y y\, dy = \frac{\pi b^2 \rho g y^2}{2h}$$

$$\delta = \Sigma \frac{P\,\Delta y}{EA} = \int_0^h \frac{P\,dy}{EA(y)} = \frac{\pi b^2 \rho g}{2Eh}\cdot\frac{h}{\pi b^2}\int_0^h y\,dy = \frac{\rho g h^2}{4E}$$ ◄

Problem 2.30

2.30 A homogeneous cable of length L and uniform cross section is suspended from one end. (*a*) Denoting by ρ the density (mass per unit volume) of the cable and by E its modulus of elasticity, determine the elongation of the cable due to its own weight. (*b*) Show that the same elongation would be obtained if the cable were horizontal and if a force equal to half of its weight were applied at each end.

SOLUTION



(a) For element at point identified by coordinate y

P = weight of portion below the point

$$= \rho g A(L-y)$$

$$d\delta = \frac{P\,dy}{EA} = \frac{\rho g A(L-y)\,dy}{EA} = \frac{\rho g(L-y)}{E}\,dy$$

$$\delta = \int_0^L \frac{\rho g(L-y)}{E}\,dy = \frac{\rho g}{E}\left(Ly - \tfrac{1}{2}y^2\right)\Big|_0^L$$

$$= \frac{\rho g}{E}\left(L^2 - \frac{L^2}{2}\right) = \frac{1}{2}\frac{\rho g L^2}{E}$$ ◄

(b) Total weight $\quad W = \rho g A L$

$$F = \frac{EA\delta}{L} = \frac{EA}{L}\cdot\frac{1}{2}\frac{\rho g L^2}{E} = \frac{1}{2}\rho g A L = \frac{1}{2}W$$ ◄

Problem 2.31

2.31 Denoting by ε the "engineering strain" in a tensile specimen, show that the true strain is $\epsilon_t = \ln(1 + \epsilon)$.

SOLUTION

$$\varepsilon_t = \ln \frac{L}{L_o} = \ln \frac{L_o + \delta}{L_o} = \ln\left(1 + \frac{\delta}{L_o}\right) = \ln(1+\varepsilon)$$

Thus $\qquad \varepsilon_t = \ln(1+\varepsilon)$

Problem 2.32

2.32 The volume of a tensile specimen is essentially constant while plastic deformation occurs. If the initial diameter of the specimen is d_1, show that when the diameter is d, the true strain is $\epsilon_t = 2\ln(d_1/d)$.

SOLUTION

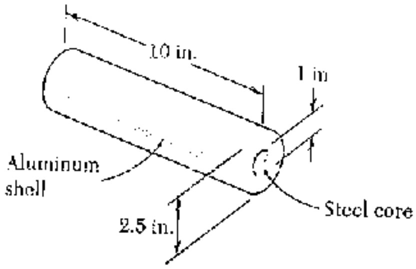
If the volume is constant $\qquad \frac{\pi}{4} d^2 L = \frac{\pi}{4} d_1^2 L_o$

$$\frac{L}{L_o} = \frac{d_1^2}{d^2} = \left(\frac{d_1}{d}\right)^2$$

$$\varepsilon_t = \ln \frac{L}{L_o} = \ln\left(\frac{d_1}{d}\right)^2 = 2 \ln \frac{d_1}{d}$$

Problem 2.33

2.33 Compressive centric forces of 40 kips are applied at both ends of the assembly shown by means of rigid plates. Knowing that $E_s = 29 \times 10^6$ psi and E_a 10.1 × 10^6 psi, determine (*a*) the normal stresses in the steel core and the aluminum shell, (*b*) the deformation of the assembly.



Let P_a = portion of axial force carried by shell.

P_s = portion of axial force carried by core.

$$\delta = \frac{P_a L}{E_a A_a} \qquad P_a = \frac{E_a A_a}{L}\delta$$

$$\delta = \frac{P_s L}{E_s A_s} \qquad P_s = \frac{E_s A_s}{L}\delta$$

Total force $\quad P = P_a + P_s = (E_a A_a + E_s A_s)\frac{\delta}{L}$

$$\frac{\delta}{L} = \varepsilon = \frac{P}{E_a A_a + E_s A_s}$$

Data: $P = 40 \times 10^3$ lb

$$A_a = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(2.5^2 - 1.0)^2 = 4.1233 \text{ in}^2$$

$$A_s = \frac{\pi}{4}d^2 = \frac{\pi}{4}(1)^2 = 0.7854 \text{ in}^2$$

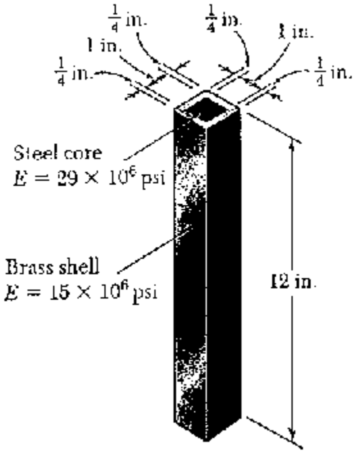
$$\varepsilon = \frac{-40 \times 10^3}{(10.1 \times 10^6)(4.1233) + (29 \times 10^6)(0.7854)} = -620.91 \times 10^{-6}$$

(a) $\sigma_s = E_s\varepsilon = (29 \times 10^6)(620.91 \times 10^{-6}) = -18.01 \times 10^3$ psi $\qquad -18.01$ ksi

$\sigma_a = E_a\varepsilon = (10.1 \times 10^6)(620.91 \times 10^{-6}) = -6.27 \times 10^3$ psi $\qquad -6.27$ ksi

(b) $\delta = L\varepsilon = (10)(620.91 \times 10^{-6}) = -6.21 \times 10^{-3}$ in.

Problem 2.34



2.34 The length of the assembly decreases by 0.006 in. when an axial force is applied by means of rigid end plates. Determine (*a*) the magnitude of the applied force, (b) the corresponding stress in the steel core.

Let P_b = portion of axial force carried by brass shell.

P_s = portion of axial force carried by steel core.

$$\delta = \frac{P_b L}{A_b E_b} \qquad P_b = \frac{E_b A_b \delta}{L}$$

$$\delta = \frac{P_s L}{A_s E_s} \qquad P_s = \frac{E_s A_s \delta}{L}$$

$$P = P_b + P_s = (E_b A_b + E_s A_s)\frac{\delta}{L}$$

$$A_s = (1)(1) = 1.00 \text{ in}^2$$

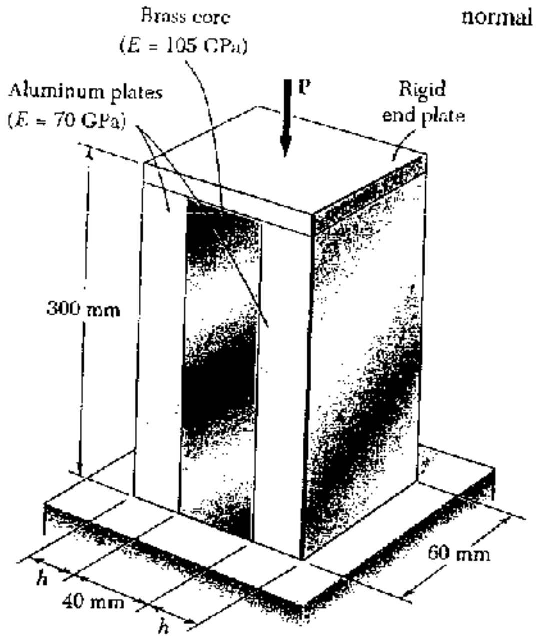
$$A_b = (1.5)(1.5) - (1)(1) = 1.25 \text{ in}^2$$

(a) $P = [(15\times10^6)(1.25) + (29\times10^6)(1.00)]\dfrac{0.006}{12} = 23.9\times10^3 \text{ lb}$ 23.9 kips ◄

(b) $\sigma_s = E_s \varepsilon = E_s \dfrac{\delta}{L} = (29\times10^6)\dfrac{0.006}{12} = 14.50\times10^3 \text{ psi}$ 14.50 ksi ◄

Problem 2.35

2.35 An axial centric force of magnitude $P = 450$ kN is applied to the composite block shown by means of a rigid end plate. Knowing that $h = 10$ mm, determine the normal stress in (*a*) the brass core, (*b*) the aluminum plates.



Let P_b = portion of axial force carried by brass core

P_a = portion carried by two aluminum plates

$$\delta = \frac{P_b L}{E_b A_b} \qquad P_b = \frac{E_b A_b \delta}{L}$$

$$\delta = \frac{P_a L}{E_a A_a} \qquad P_a = \frac{E_a A_a \delta}{L}$$

$$P = P_b + P_a = (E_b A_b + E_a A_a)\frac{\delta}{L}$$

$$\varepsilon = \frac{\delta}{L} = \frac{P}{E_b A_b + E_a A_a}$$

$$A_b = (60)(40) = 2400 \text{ mm}^2 = 2400 \times 10^{-6} \text{ m}^2$$

$$A_a = (2)(60)(10) = 1200 \text{ mm}^2 = 1200 \times 10^{-6} \text{ m}^2$$

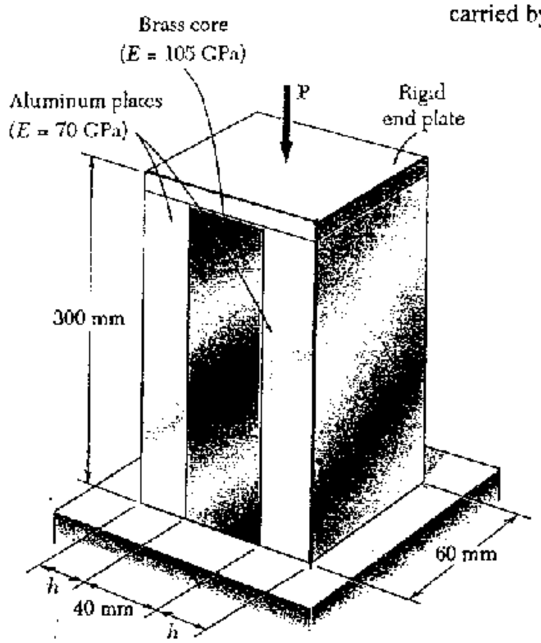
$$\varepsilon = \frac{450 \times 10^3}{(105 \times 10^9)(2400 \times 10^{-6}) + (70 \times 10^9)(1200 \times 10^{-6})} = 1.3393 \times 10^{-3}$$

(a) $\sigma_b = E_b \varepsilon = (105 \times 10^9)(1.3393 \times 10^{-3}) = 140.6 \times 10^6 \text{ Pa} = 140.6 \text{ MPa}$ ◄

(b) $\sigma_a = E_a \varepsilon = (70 \times 10^9)(1.3393 \times 10^{-3}) = 93.75 \times 10^6 \text{ Pa} = 93.75 \text{ MPa}$ ◄

Problem 2.36

2.36 For the composite block shown in Prob. 2.35, determine (*a*) the value of *h* if the portion of the load carried by the aluminum plates is half the portion of the load carried by the brass core, (*b*) the total load if the stress in the brass is 80 MPa.



Let P_b = portion of axial force carried by brass core

P_a = portion carried by the two aluminum plates

$$\delta = \frac{P_b L}{E_b A_b} \qquad P_b = \frac{E_b A_b \delta}{L}$$

$$\delta = \frac{P_a L}{E_a A_a} \qquad P_a = \frac{E_a A_a \delta}{L}$$

(a) Given $P_a = \frac{1}{2} P_b$

$$\frac{E_a A_a \delta}{L} = \frac{1}{2} \frac{E_b A_b \delta}{L}$$

$$A_a = \frac{1}{2} \frac{E_b}{E_a} A_b$$

$$A_b = (40)(60) = 2400 \text{ mm}^2 = 2400 \times 10^{-6} \text{ m}^2$$

$$A_a = \frac{1}{2} \frac{105 \times 10^9}{70 \times 10^9} 2400 = 1800 \text{ mm}^2 = (2)(60)h$$

$$h = \frac{1800}{(2)(60)} = 15 \text{ mm}$$

(b) $\sigma_b = \frac{P_b}{A_b}$

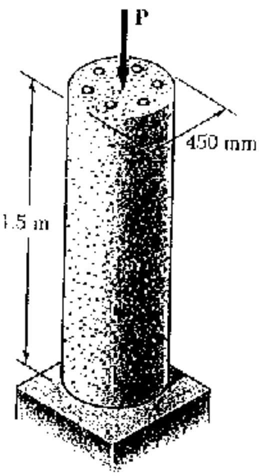
$$P_b = A_b \sigma_b = (2400 \times 10^{-6})(80 \times 10^6) = 192 \times 10^3 \text{ N}$$

$$P_a = \frac{1}{2} P_b = 96 \times 10^3 \text{ N}$$

$$P = P_b + P_a = 288 \times 10^3 \text{ N} = 288 \text{ kN}$$

Problem 2.37

2.37 The 1.5-m concrete post is reinforced with six steel bars, each with a 28-mm diameter. Knowing that $E_s = 200$ GPa and $E_c = 25$ GPa, determine the normal stresses in the steel and in the concrete when a 1550 kN axial centric force **P** is applied to the post.



Let P_c = portion of axial force carried by concrete

P_s = portion carried by the six steel rods

$$\delta = \frac{P_c L}{E_c A_c} \qquad P_c = \frac{E_c A_c \delta}{L}$$

$$\delta = \frac{P_s L}{E_s A_s} \qquad P_s = \frac{E_s A_s \delta}{L}$$

$$P = P_c + P_s = (E_c A_c + E_s A_s)\frac{\delta}{L}$$

$$\varepsilon = \frac{\delta}{L} = \frac{P}{E_c A_c + E_s A_s}$$

$$A_s = 6\,\frac{\pi}{4} d_s^2 = \frac{6\pi}{4}(28)^2 = 3.6945\times10^3\ mm^2 = 3.6945\times10^{-3}\ m^2$$

$$A_c = \frac{\pi}{4} d_c^2 - A_s = \frac{\pi}{4}(450)^2 - 3.6945\times10^3 = 155.349\times10^3\ mm^2$$

$$= 153.349\times10^{-3}\ m^2$$

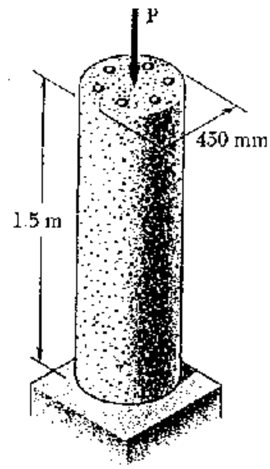
$$L = 1.5\ m$$

$$\varepsilon = \frac{1550\times10^3}{(25\times10^9)(155.349\times10^{-3}) + (200\times10^9)(3.6945\times10^{-3})} = 335.31\times10^{-6}$$

$$\sigma_s = E_s\varepsilon = (200\times10^9)(335.31\times10^{-6}) = 67.1\times10^6\ Pa \qquad 67.1\ MPa$$

$$\sigma_c = E_c\varepsilon = (25\times10^9)(335.31\times10^{-6}) = 8.38\times10^6\ Pa \qquad 8.38\ MPa$$

Problem 2.38



2.38 For the post of Prob. 2.37, determine the maximum centric force which may be applied if the allowable normal stress is 160 MPa in the steel and 18 MPa in the concrete.

2.37 The 1.5-m concrete post is reinforced with six steel bars, each with a 28-mm diameter. Knowing that $E_s = 200$ GPa and $E_c = 25$ GPa, determine the normal stresses in the steel and in the concrete when a 1550 kN axial centric force **P** is applied to the post.

Determine allowable strain in each material

Steel: $\varepsilon_s = \dfrac{\sigma_s}{E_s} = \dfrac{160\times10^6}{200\times10^9} = 800\times10^{-6}$

Concrete: $\varepsilon_c = \dfrac{\sigma_c}{E_c} = \dfrac{18\times10^6}{25\times10^9} = 720\times10^{-6}$

Smaller value governs $\quad \varepsilon = \dfrac{\delta}{L} = 720\times10^{-6}$

Let P_c = portion of load carried by concrete

P_s = portion carried by six steel rods

$\delta = \dfrac{P_c L}{E_c A_c}, \quad P_c = E_c A_c \dfrac{\delta}{L} = E_c A_c \varepsilon$

$\delta = \dfrac{P_s L}{E_s A_s}, \quad P_s = E_s A_s \dfrac{\delta}{L} = E_s A_s \varepsilon$

$P = P_c + P_s = (E_c A_c + E_s A_s)\varepsilon$

$A_s = 6\frac{\pi}{4} d_s^2 = \frac{6\pi}{4}(28)^2 = 3.6945\times10^3\ \text{mm}^2 = 3.6945\times10^{-3}\ \text{m}^2$

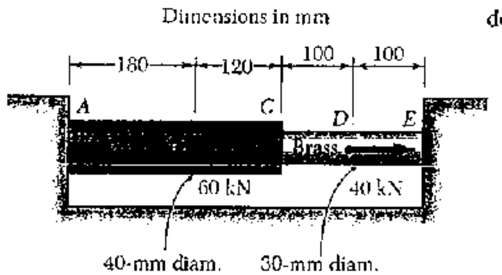
$A_c = \frac{\pi}{4}d_c^2 - A_s = \frac{\pi}{4}(450)^2 - 3.6945\times10^3 = 155.349\times10^3\ \text{mm}^2$
$= 155.349\times10^{-3}\ \text{m}^2$

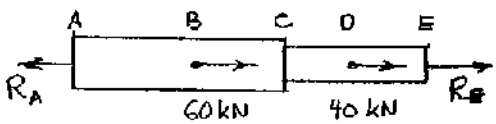
$P = [(25\times10^9)(155.349\times10^{-3}) + (200\times10^9)(3.6945\times10^{-3})](720\times10^{-6})$

$= 3.33\times10^6$ N $\qquad$ 3330 kN

Problem 2.39

2.39 Two cylindrical rods, one of steel and the other of brass, are joined at C and restrained by rigid supports at A and E. For the loading shown and knowing that $E_s = 200$ GPa and $E_b = 105$ GPa, determine (*a*) the reactions at A and E, (*b*) the deflection of point C.





A to C: $E = 200 \times 10^9$ Pa

$A = \frac{\pi}{4}(40)^2 = 1.25664 \times 10^3 \text{ mm}^2 = 1.25664 \times 10^{-3} \text{ m}^2$

$EA = 251.327 \times 10^6$ N

C to E: $E = 105 \times 10^9$ Pa

$A = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86 \times 10^{-6} \text{ m}^2$

$EA = 74.220 \times 10^6$ N

A to B: $P = R_A$ $\qquad L = 180 \text{ mm} = 0.180 \text{ m}$

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0.180)}{251.327 \times 10^6} = 716.20 \times 10^{-12} R_A$$

B to C: $P = R_A - 60 \times 10^3$ $\qquad L = 120 \text{ mm} = 0.120 \text{ m}$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60 \times 10^3)(0.120)}{251.327 \times 10^6} = 447.47 \times 10^{-12} R_A - 26.848 \times 10^{-6}$$

C to D: $P = R_A - 60 \times 10^3$ $\qquad L = 100 \text{ mm} = 0.100 \text{ m}$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60 \times 10^3)(0.100)}{74.220 \times 10^6} = 1.34735 \times 10^{-9} R_A - 80.841 \times 10^{-6}$$

D to E: $P = R_A - 100 \times 10^3$ $\qquad L = 100 \text{ mm} = 0.100 \text{ m}$

$$\delta_{DE} = \frac{PL}{EA} = \frac{(R_A - 100 \times 10^3)(0.100)}{74.220 \times 10^6} = 1.34735 \times 10^{-9} R_A - 134.735 \times 10^{-6}$$

A to E: $\delta_{AE} = \delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE} = 3.85837 \times 10^{-9} R_A - 242.424 \times 10^{-6}$

Since point E cannot move relative to A, $\delta_{AE} = 0$

(a) $3.85837 \times 10^{-9} R_A - 242.424 \times 10^{-6} = 0 \qquad R_A = 62.831 \times 10^3$ N $\qquad$ 62.8 kN ←

$R_E = R_A - 100 \times 10^3 = 62.8 \times 10^3 - 100 \times 10^3 = -37.2 \times 10^3$ N $\qquad$ 37.2 kN ←

(b) $\delta_C = \delta_{AB} + \delta_{BC} = 1.16367 \times 10^{-9} R_A - 26.848 \times 10^{-6}$

$= (1.16369 \times 10^{-9})(62.831 \times 10^3) - 26.848 \times 10^{-6}$

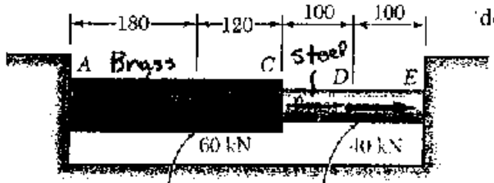
$= 46.3 \times 10^{-6}$ m $\qquad$ 46.3 μm →

Problem 2.40

2.40 Solve Prob. 2.39, assuming that rod AC is made of brass and rod CE is made of steel.

2.39 Two cylindrical rods, one of steel and the other of brass, are joined at C and restrained by rigid supports at A and E. For the loading shown and knowing that $E_s = 200$ GPa and $E_b = 105$ GPa, determine (*a*) the reactions at A and E, (*b*) the deflection of point C.

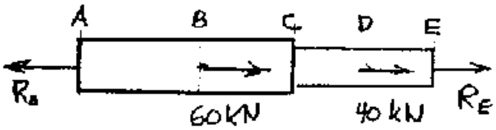
Dimensions in mm



A to C: $E = 105 \times 10^9$ Pa

$A = \frac{\pi}{4}(40)^2 = 1.25664 \times 10^3\ \text{mm}^2 = 1.25664 \times 10^{-3}\ \text{m}^2$

$EA = 131.947 \times 10^6$ N



C to E: $E = 200 \times 10^9$ Pa

$A = \frac{\pi}{4}(30)^2 = 706.86\ \text{mm}^2 = 706.86 \times 10^{-6}\ \text{m}^2$

$EA = 141.372 \times 10^6$ N

A to B: $P = R_A$ $\quad L = 180\ \text{mm} = 0.180\ \text{m}$

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0.180)}{131.947 \times 10^6} = 1.36418 \times 10^{-9} R_A$$

B to C: $P = R_A - 60 \times 10^3$ $\quad L = 120\ \text{mm} = 0.120\ \text{m}$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60 \times 10^3)(0.120)}{131.947 \times 10^6} = 909.456 \times 10^{-12} R_A - 54.567 \times 10^{-6}$$

C to D: $P = R_A - 60 \times 10^3$ $\quad L = 100\ \text{mm} = 0.100\ \text{m}$

$$\delta_{CD} = \frac{PL}{EA} = \frac{(R_A - 60 \times 10^3)(0.100)}{141.372 \times 10^6} = 707.354 \times 10^{-12} R_A - 42.441 \times 10^{-6}$$

D to E: $P = R_A - 100 \times 10^3$ $\quad L = 100\ \text{mm} = 0.100\ \text{m}$

$$\delta_{DE} = \frac{PL}{EA} = \frac{(R_A - 100 \times 10^3)(0.100)}{141.372 \times 10^6} = 707.354 \times 10^{-12} R_A - 70.735 \times 10^{-6}$$

A to E: $\delta_{AE} = \delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE} = 3.68834 \times 10^{-9} R_A - 167.743 \times 10^{-6}$

Since point E cannot move relative to A, $\delta_{AE} = 0$

(a) $3.68834 \times 10^{-9} R_A - 167.743 \times 10^{-6} = 0 \qquad R_A = 45.479 \times 10^3$ N $\qquad$ 45.5 kN ←

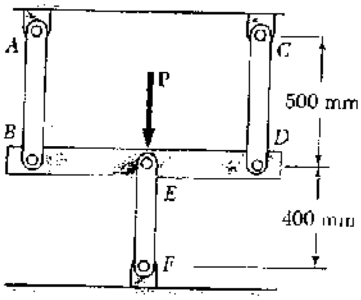
$R_E = R_A - 100 \times 10^3 = 45.479 \times 10^3 - 100 \times 10^3 = -54.521 \times 10^3 \qquad$ 54.5 kN ←

(b) $\delta_C = \delta_{AB} + \delta_{BC} = 2.27364 \times 10^{-9} R_A - 54.567 \times 10^{-6}$

$= (2.27364 \times 10^{-9})(45.479 \times 10^3) - 54.567 \times 10^{-6}$

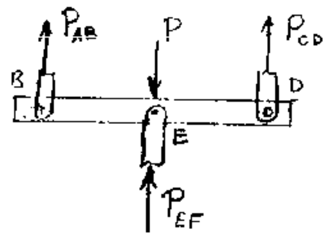
$= 48.8 \times 10^{-6}$ m $\qquad$ 48.8 μm →

Problem 2.41



2.41 Three steel rods ($E = 200$ GPa) support a 36-kN load **P**. Each of the rods AB and CD has a 200-mm^2 cross-sectional area and rod EF has a 625- mm^2 cross-sectional area. Determine the (a) the change in length of rod EF, (b) the stress in each rod.

Use member BED as a free body



By symmetry, or by $\Sigma M_E = 0$

$$P_{CD} = P_{AB}$$

$$\Sigma F_y = 0$$

$$P_{AB} + P_{CD} + P_{EF} - P = 0$$

$$P = 2P_{AB} + P_{EF}$$

$$\delta_{AB} = \frac{P_{AB}L_{AB}}{EA_{AB}}, \quad \delta_{CD} = \frac{P_{CD}L_{CD}}{EA_{CD}}, \quad \delta_{EF} = \frac{P_{EF}L_{EF}}{EA_{EF}}$$

Since $L_{AB} = L_{CD}$ and $A_{AB} = A_{CD}$, $\delta_{AB} = \delta_{CD}$

Since points A, C, and E are fixed $\delta_B = \delta_{AB}$, $\delta_D = \delta_{CD}$, $\delta_E = \delta_{EF}$

Since member BED is rigid $\delta_E = \delta_B = \delta_C$

$$\frac{P_{AB}L_{AB}}{EA_{AB}} = \frac{P_{EF}L_{EF}}{EA_{EF}} \quad \therefore \quad P_{AB} = \frac{A_{AB}}{A_{EF}} \cdot \frac{L_{EF}}{L_{AB}} P_{EF} = \frac{200}{625} \cdot \frac{400}{500} P_{EF}$$

$$= 0.256 \, P_{EF}$$

$$P = 2P_{AB} + P_{EF} = (2)(0.256)P_{EF} + P_{EF} = 1.512 \, P_{EF}$$

$$P_{EF} = \frac{P}{1.512} = \frac{36 \times 10^3}{1.512} = 23.810 \times 10^3 \text{ N}$$

$$P_{AB} = P_{CD} = (0.256)(23.810 \times 10^3) = 6.095 \times 10^3 \text{ N}$$

(a) $$\delta = \delta_{EF} = \frac{(23.810 \times 10^3)(400 \times 10^{-3})}{(200 \times 10^9)(625 \times 10^{-6})} = 76.2 \times 10^{-6} \text{ m}$$

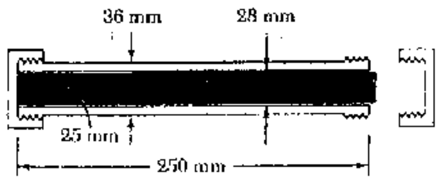
$$= 0.0762 \text{ mm}$$

or $$\delta = \delta_{AB} = \frac{(6.095 \times 10^3)(500 \times 10^{-3})}{(200 \times 10^9)(200 \times 10^{-6})} = 76.2 \times 10^{-6} \text{ m}$$

(b) $$\sigma_{AB} = \sigma_{CD} = \frac{P_{AB}}{A_{AB}} = \frac{6.095 \times 10^3}{200 \times 10^{-6}} = 30.5 \times 10^6 \text{ Pa} = 30.5 \text{ MPa}$$

$$\sigma_{EF} = -\frac{P_{EF}}{A_{EF}} = -\frac{23.810 \times 10^3}{625 \times 10^{-6}} = -38.1 \times 10^6 \text{ Pa} = 38.1 \text{ MPa}$$

Problem 2.42



2.42 A 250-mm-long aluminum tube ($E = 70$ GPa) of 36-mm outer diameter and 28-mm inner diameter may be closed at both ends by means of single-threaded screw-on covers of 1.5-mm pitch. With one cover screwed on tight, a solid brass rod ($E = 105$ GPa) of 25-mm diameter is placed inside the tube and the second cover is screwed on. Since the rod is slightly longer than the tube, it is observed that the cover must be forced against the rod by rotating it one-quarter of a turn before it can be tightly closed. Determine (*a*) the average normal stress in the tube and in the rod, (*b*) the deformations of the tube and of the rod.

$$A_{tube} = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(36^2 - 28^2) = 402.12 \text{ mm}^2 = 402.12 \times 10^{-6} \text{ m}^2$$

$$A_{rod} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(25)^2 = 490.87 \text{ mm}^2 = 490.87 \times 10^{-6} \text{ m}^2$$

$$\delta_{tube} = \frac{PL}{E_{tube} A_{tube}} = \frac{P(0.250)}{(70 \times 10^9)(402.12 \times 10^{-6})} = 8.8815 \times 10^{-9} P$$

$$\delta_{rod} = -\frac{PL}{E_{rod} A_{rod}} = \frac{P(0.250)}{(105 \times 10^9)(490.87 \times 10^{-6})} = -4.8505 \times 10^{-9} P$$

$$\delta^* = \frac{1}{4} \text{ turn} \times 1.5 \text{ mm} = 0.375 \text{ mm} = 375 \times 10^{-6} \text{ m}$$

$$\delta_{tube} = \delta^* + \delta_{rod} \quad \text{or} \quad \delta_{tube} - \delta_{rod} = \delta^*$$

$$8.8815 \times 10^{-9} P + 4.8505 \times 10^{-9} P = 375 \times 10^{-6}$$

$$P = \frac{0.375 \times 10^{-3}}{(8.8815 + 4.8505)(10^{-9})} = 27.308 \times 10^3 \text{ N}$$

(a) $$\sigma_{tube} = \frac{P}{A_{tube}} = \frac{27.308 \cdot 10^3}{402.12 \times 10^{-6}} = 67.9 \times 10^6 \text{ Pa} = 67.9 \text{ MPa} \blacktriangleleft$$

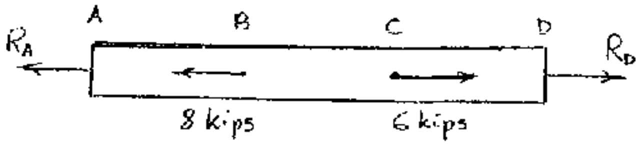
$$\sigma_{rod} = -\frac{P}{A_{rod}} = -\frac{27.308 \times 10^3}{490.87 \times 10^{-6}} = -55.6 \times 10^6 \text{ Pa} = -55.6 \text{ MPa} \blacktriangleleft$$

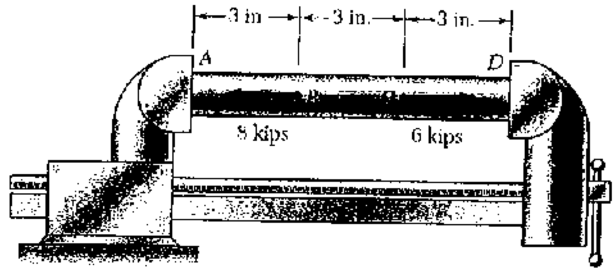
(b) $$\delta_{tube} = (8.8815 \times 10^{-9})(27.308 \times 10^3) = 242.5 \times 10^{-6} \text{ m} = 0.2425 \text{ mm} \blacktriangleleft$$

$$\delta_{rod} = -(4.8505 \times 10^{-9})(27.308 \times 10^3) = -132.5 \times 10^{-6} \text{ m} = -0.1325 \text{ mm} \blacktriangleleft$$

Problem 2.43

2.43 A steel tube ($E = 29 \times 10^6$ psi) with a $1\frac{1}{4}$ in.-outer diameter and a $\frac{1}{8}$ in.-thickness is placed in a vise that is adjusted so that its jaws just touch the ends of the tube without exerting any pressure on them. The two forces shown are then applied to the tube. After these forces are applied, the vise is adjusted to decrease the distance between its jaws by 0.008 in. Determine (*a*) the forces exerted by the vise on the tube at *A* and *D*, (*b*) the change in length of the portion *BC* of the tube.





For the tube $\quad d_o = 1\frac{1}{4}$ in. $= 1.25$ in.

$$d_i = d_o - 2t = 1.25 - (2)(\tfrac{1}{8}) = 1.00 \text{ in.}$$

$$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(1.25^2 - 1.00^2) = 0.44179 \text{ in}^2$$

A to B: $\quad P = R_A$ lb $\qquad L = 3$ in.

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(3)}{(29 \times 10^6)(0.44179)} = 234.157 \times 10^{-9} R_A \text{ in.}$$

B to C: $\quad P = R_A + 8000$ lb, $\quad L = 3$ in.

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A + 8000)(3)}{(29 \times 10^6)(0.44179)} = 234.157 \times 10^{-9} R_A + 1.87326 \times 10^{-3} \text{ in.}$$

C to D: $\quad P = R_A + 2000$ lb, $\quad L = 3$ in.

$$\delta_{CD} = \frac{PL}{EA} = \frac{(R_A + 2000)(3)}{(29 \times 10^6)(0.44179)} = 234.157 \times 10^{-9} R_A + 0.46831 \, 10^{-3} \text{ in.}$$

A to D: $\quad \delta_{AD} = \delta_{AB} + \delta_{BC} + \delta_{CD} = 702.471 \times 10^{-9} R_A + 2.34157 \times 10^{-3}$ in.

Given jaw movement $\qquad \delta_{AD} = -0.008$ in.

(a) $\quad 702.471 \times 10^{-9} R_A + 2.34157 \times 10^{-3} = -0.008$

$$R_A = -14.7217 \times 10^3 \text{ lb} \qquad\qquad R_A = 14.72 \text{ kips} \rightarrow$$

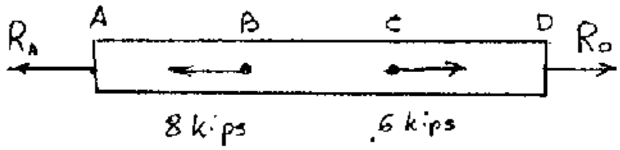
$$R_D = R_A + 2000 = -12.7217 \times 10^3 \text{ lb} \qquad\qquad R_D = 12.72 \text{ kips} \leftarrow$$

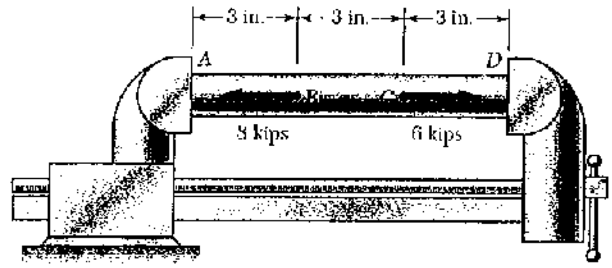
(b) $\quad \delta_{BC} = (234.157 \times 10^{-9})(-14.7217 \times 10^3) + 1.87326 \times 10^{-3} = -1.574 \times 10^{-3}$ in.

Problem 2.44

2.44 Solve Prob. 2.43, assuming that after the forces have been applied, the vise is adjusted to increase the distance between its jaws by 0.004 in.

2.43 A steel tube ($E = 29 \times 10^6$ psi) with a $1\frac{1}{4}$ in.-outer diameter and a $\frac{1}{8}$ in.-thickness is placed in a vise that is adjusted so that its jaws just touch the ends of the tube without exerting any pressure on them. The two forces shown are then applied to the tube. After these forces are applied, the vise is adjusted to decrease the distance between its jaws by 0.008 in. Determine (*a*) the forces exerted by the vise on the tube at *A* and *D*, (*b*) the change in length of the portion *BC* of the tube.





For the tube $\quad d_o = 1\frac{1}{4}\text{ in.} = 1.25\text{ in.}$

$$d_i = d_o - 2t = 1.25 - (2)(\tfrac{1}{8}) = 1.00\text{ in.}$$

$$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(1.25^2 - 1.00^2) = 0.44179\text{ in}^2$$

A to B: $\quad P = R_A \qquad L = 3\text{ in.}$

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(3)}{(29\times10^6)(0.44179)} = 234.157\times10^{-9}R_A\text{ in.}$$

B to C: $\quad P = R_A + 8000\text{ lb} \qquad L = 3\text{ in.}$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A+8000)(3)}{(29\times10^6)(0.44179)} = 234.157\times10^{-9}R_A + 1.87326\times10^{-3}\text{ in.}$$

C to D: $\quad P = R_A + 2000\text{ lb} \qquad L = 3\text{ in.}$

$$\delta_{CD} = \frac{PL}{EA} = \frac{(R_A+2000)(3)}{(29\times10^6)(0.44179)} = 234.157\times10^{-9}R_A + 0.46831\times10^{-3}\text{ in.}$$

A to D: $\quad \delta_{AD} = \delta_{AB} + \delta_{BC} + \delta_{CD} = 702.471\times10^{-9}R_A + 2.34157\times10^{-3}\text{ in.}$

Given jaw movement $\quad \delta_{AD} = -0.004\text{ in.}$

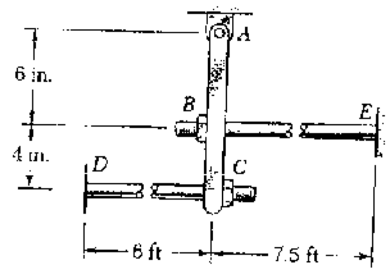
(a) $\quad 702.471\times10^{-9}R_A + 2.34157\times10^{-3} = -0.004$

$$R_A = -9.0275\times10^3\text{ lb} \qquad R_A = 9.03\text{ kips} \rightarrow$$

$$R_D = R_A + 2000 = -7.0275\times10^3\text{ lb} \qquad R_D = 7.03\text{ kips} \leftarrow$$

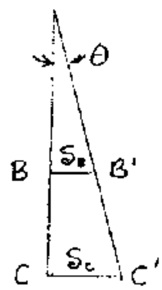
(b) $\quad \delta_{BC} = (234.157\times10^{-9})(-9.0275\times10^3) + 1.87326\times10^{-3} = -241\times10^{-6}\text{ in.}$

Problem 2.45



2.45 The steel rods BE and CD each have a diameter of $\frac{5}{8}$ in. ($E = 29 \times 10^6$ psi). The ends are threaded with a pitch of 0.1 in. Knowing that after being snugly fit, the nut at B is tightened one full turn, determine (*a*) the tension in rod CD, (*b*) the deflection of point C of the rigid member ABC.

Let θ be the rotation of bar ABC as shown



Then, $\delta_B = 6\theta$ and $\delta_C = 10\,\theta$

But $\delta_B = \delta_{turn} - \dfrac{P_{BE} L_{BE}}{E_{BE} A_{BE}}$

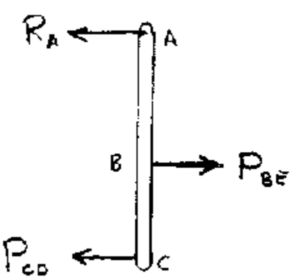
$$P_{BE} = (E_{BE} A_{BE})(\delta_{turn} - \delta_B)/L_{BE}$$

$L_{BE} = 7.5 \text{ ft} = 90 \text{ in}, \quad \delta_{turn} = 0.1 \text{ in}$

$A_{BE} = \frac{\pi}{4} d^2 = \frac{\pi}{4}\left(\frac{5}{8}\right)^2 = 0.3068 \text{ in}^2$

$$P_{BE} = \frac{(29 \times 10^6)(0.3068)(0.1 - 6\theta)}{90}$$

$$= 9.886 \times 10^3 - 593.15 \times 10^3\,\theta$$



$\delta_C = \dfrac{P_{CD} L_{CD}}{E A_{CD}} \quad \therefore \quad P_{CD} = \dfrac{E A \delta_C}{L_{CD}}$

$L_{CD} = 6 \text{ ft} = 72 \text{ in}, \quad A_{CD} = 0.3068 \text{ in}^2$

$$P_{CD} = \frac{(29 \times 10^6)(0.3068)(10\,\theta)}{72}$$

$$= 1.23572 \times 10^6\ \theta$$

$\circlearrowleft \Sigma M_A = 0 \qquad 6 P_{BE} - 10 P_{CD} = 0$

$$(6)(9.886 \times 10^3 - 593.16 \times 10^3\,\theta) - (10)(1.23572 \times 10^6)\,\theta = 0$$

$$59.316 \times 10^3 - 15.916 \times 10^6\,\theta = 0 \qquad \theta = 3.7268 \times 10^{-3} \text{ rad}$$

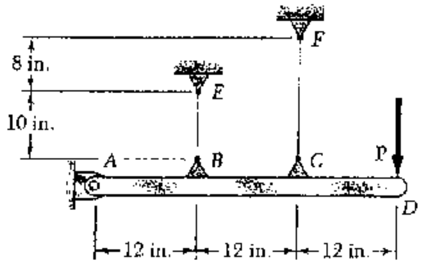
(a) $P_{CD} = (1.23572 \times 10^6)(3.7268 \times 10^{-3}) = 4.61 \times 10^3$

$= 4.61$ kips ◀

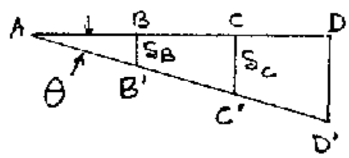
(b) $\delta_C = 10\,\theta = (10)(3.7268 \times 10^{-3}) = 37.3 \times 10^{-3}$ in

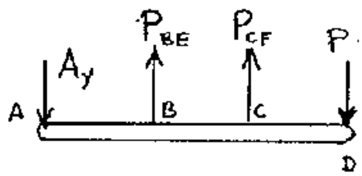
$= 0.0373$ in. → ◀

Problem 2.46



2.46 The rigid bar AD is supported by two steel wires of $\frac{1}{16}$-in. diameter ($E = 29 \times 10^6$ psi) and a pin and bracket at D. Knowing that the wires were initially taught, determine (*a*) the additional tension in each wire when a 220-lb load **P** is applied at D, (*b*) the corresponding deflection of point D.





Let θ be the rotation of bar ABCD

Then $\quad \delta_B = 12\,\theta$

$$\delta_C = 24\,\theta$$

$$\delta_B = \frac{P_{BE} L_{BE}}{AE}$$

$$P_{BE} = \frac{EA\,\delta_{BE}}{L_{BE}} = \frac{(29\times10^6)\frac{\pi}{4}\left(\frac{1}{16}\right)^2(12\,\theta)}{10}$$

$$= 106.77\times10^3\,\theta$$

$$\delta_C = \frac{P_{CF} L_{CF}}{EA}$$

$$P_{CF} = \frac{EA\,\delta_{CF}}{L_{CF}} = \frac{(29\times10^6)\frac{\pi}{4}\left(\frac{1}{16}\right)^2(24\,\theta)}{18}$$

$$= 118.63\times10^3\,\theta$$

Using free body ABCD

$\circlearrowleft \Sigma M_A = 0 \qquad 12\,P_{BE} + 24\,P_{CF} - 36\,P = 0$

$$(12)(106.77\times10^3\,\theta) + (24)(118.63\times10^6\,\theta) - (36)(220) = 0$$

$$4.1283\times10^6\,\theta = (36)(220)$$

$$\theta = 1.9185\times10^{-3}\ \text{rad}$$

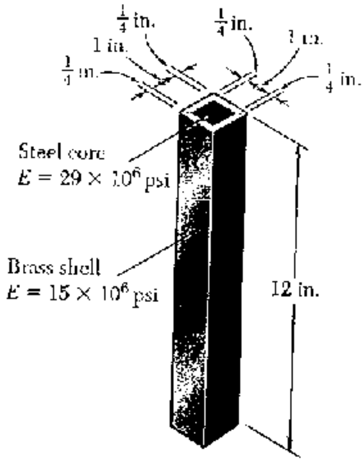
(a) $\quad P_{BE} = (106.77\times10^3)(1.9185\times10^{-3}) = 204.8\ \text{lb}$

$\quad P_{CF} = (118.63\times10^3)(1.9185\times10^{-3}) = 227.6\ \text{lb}$

(b) $\quad \delta_D = 36\,\theta = (36)(1.9185\times10^{-3}) = 69.1\times10^{-3}\ \text{in}$

$\quad = 0.0691\ \text{in.}$

Problem 2.47



2.47 The brass shell ($\alpha_b = 11.6 \times 10^{-6}/°F$) is fully bonded to the steel core ($\alpha_s = 6.5 \times 10^{-6}/°F$). Determine the largest allowable increase in temperature if the stress in the steel core is not to exceed 8 ksi.

Let P_s = axial force developed in the steel core

For equilibrium with zero total force, the compressive force in the brass shell is P_s.

Strains $\qquad \varepsilon_s = \dfrac{P_s}{E_s A_s} + \alpha_s(\Delta T)$

$$\varepsilon_b = -\frac{P_s}{E_b A_b} + \alpha_b(\Delta T)$$

Matching $\quad \varepsilon_s = \varepsilon_b$

$$\frac{P_s}{E_s A_s} + \alpha_s(\Delta T) = -\frac{P_s}{E_b A_b} + \alpha_b(\Delta T)$$

$$\left(\frac{1}{E_s A_s} + \frac{1}{E_b A_b}\right) P_s = (\alpha_b - \alpha_s)(\Delta T) \qquad (1)$$

$A_b = (1.5)(1.5) - (1.0)(1.0) = 1.25 \text{ in}^2$

$A_s = (1.0)(1.0) = 1.0 \text{ in}^2$

$\alpha_b - \alpha_s = 5.1 \times 10^{-6}\ /°F$

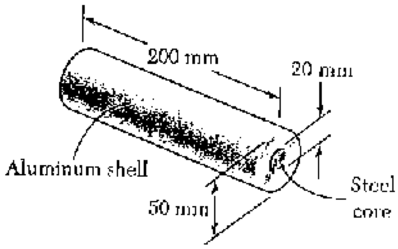
$P_s = \sigma_s A_s = (8 \times 10^3)(1.0) = 8 \times 10^3 \text{ lb}$

$$\frac{1}{E_s A_s} + \frac{1}{E_b A_b} = \frac{1}{(29 \times 10^6)(1.0)} + \frac{1}{(15 \times 10^6)(1.25)} = 87.816 \times 10^{-9} \text{ lb}^{-1}$$

From (1) $\quad (87.816 \times 10^{-9})(8 \times 10^3) = (5.1 \times 10^{-6})(\Delta T)$

$$\Delta T = 137.8\ °F$$

Problem 2.48



2.48 The assembly shown consists of an aluminum shell ($E_a = 70$ GPa, $\alpha_a = 23.6 \times 10^{-6}/°C$) fully bonded to a steel core ($E_s = 200$ GPa, $\alpha_s = 11.7 \times 10^{-6}/°C$) and is unstressed at a temperature of 20 °C. Considering only axial deformations, determine the stress in the aluminum shell when the temperature reaches 180 °C.

$$A_s = \frac{\pi}{4}(20)^2 = 314.159 \text{ mm}^2 = 314.159 \times 10^{-6} \text{ m}^2$$

$$A_a = \frac{\pi}{4}(50^2 - 20^2) = 1.64934 \times 10^3 \text{ mm}^2 = 1.64934 \times 10^{-3} \text{ m}^2$$

Let P_s be the axial force carried by the steel core and P_a that carried by the aluminum shell.

Total axial force $\quad P = P_a + P_s = 0$

$$P_s = -P_a \qquad (1)$$

Deformation.

$$\delta = \frac{P_a L}{E_a A_a} + L\alpha_a(\Delta T) = \frac{P_s L}{E_s A_s} + L\alpha_s(\Delta T)$$

$$\frac{P_a}{E_a A_a} - \frac{P_s}{E_s A_s} = (\alpha_s - \alpha_a)(\Delta T)$$

Using (1) $\quad \left(\frac{1}{E_a A_a} + \frac{1}{E_s A_s}\right) P_a = (\alpha_s - \alpha_a)(\Delta T)$

$$\left(\frac{1}{(70\times 10^9)(1.64934\times 10^{-3})} + \frac{1}{(200\times 10^9)(314.159\times 10^{-6})}\right) P_a = (11.7\times 10^{-6} - 23.6\times 10^{-6})(180 - 20)$$

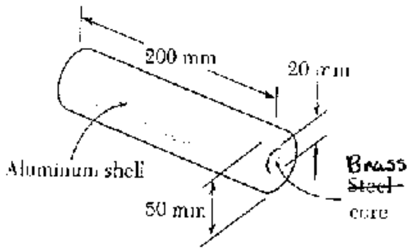
$$(24.577 \times 10^{-9}) P_a = -1.904 \times 10^{-3}$$

$$P_a = -77.471 \times 10^3 \text{ N}$$

Stress in aluminum shell $\quad \sigma_a = \frac{P_a}{A_a} = \frac{-77.471 \times 10^3}{1.64934 \times 10^{-3}} = -47.0 \times 10^6$ Pa

-47.0 MPa ◄

Problem 2.49



2.49 Solve Prob. 2.48, assuming that the core is made of brass (E_b = 105 GPa, α_b = 20.9 × 10^{-6}/°C).

2.48 The assembly shown consists of an aluminum shell (E_a = 70 GPa, α_a = 23.6 × 10^{-6}/°C) fully bonded to a steel core (E_s = 200 GPa, α_s = 11.7 × 10^{-6}/°C) and is unstressed at a temperature of 20 °C. Considering only axial deformations, determine the stress in the aluminum shell when the temperature reaches 180 °C.

$$A_b = \frac{\pi}{4}(20)^2 = 314.159 \text{ mm}^2 = 314.159 \times 10^{-6} \text{ m}^2$$

$$A_a = \frac{\pi}{4}(50^2 - 20^2) = 1.64934 \times 10^3 \text{ mm}^2 = 1.64934 \times 10^{-3} \text{ m}^2$$

Let P_b be the axial force carried by the brass core and P_a that carried by the aluminum shell.

Total axial force $\quad P = P_a + P_b = 0$

$$P_b = -P_a \qquad (1)$$

Deformation.

$$\delta = \frac{P_a L}{E_a A_a} + L\alpha_a(\Delta T) = \frac{P_b L}{E_b A_b} + L\alpha_b(\Delta T)$$

$$\frac{P_a}{E_a A_a} - \frac{P_b}{E_b A_b} = (\alpha_b - \alpha_a)(\Delta T)$$

Using (1), $\left(\frac{1}{E_a A_a} + \frac{1}{E_b A_b}\right) P_a = (\alpha_b - \alpha_a)(\Delta T)$

$$\left(\frac{1}{(70\times 10^9)(1.64934\times 10^{-3})} + \frac{1}{(105\times 10^9)(314.159\times 10^{-6})}\right) P_a$$
$$= (20.9\times 10^{-6} - 23.6\times 10^{-6})(180 - 20)$$

$$(38.977\times 10^{-9}) P_a = -432\times 10^{-6}$$

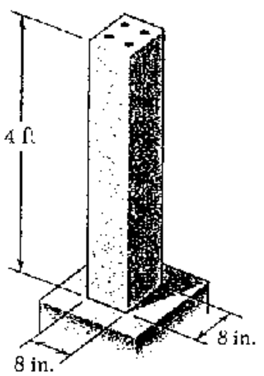
$$P_a = -11.0835 \times 10^3 \text{ N}$$

Stress in aluminum shell $\quad \sigma_a = \frac{P_a}{A_a} = \frac{-11.0835\times 10^3}{1.64934\times 10^{-3}} = -6.72\times 10^6 \text{ Pa}$

-6.72 MPa

Problem 2.50

2.50 A 4-ft concrete post is reinforced by four steel bars, each of $\frac{3}{4}$ in. diameter. Knowing that $E_s = 29 \times 10^6$ psi, $\alpha_s = 6.5 \times 10^{-6}/°\text{F}$ and $E_c = 3.6 \times 10^6$ psi and $\alpha_c = 5.5 \times 10^{-6}/°\text{F}$, determine the normal stresses induced in the steel and in the concrete by a temperature rise of 80°F.



$$A_s = 4\,\frac{\pi}{4} d^2 = (4)\left(\frac{\pi}{4}\right)\left(\frac{3}{4}\right)^2 = 1.76715 \text{ in}^2$$

$$A_c = A - A_s = 8^2 - 1.76715 = 62.233 \text{ in}^2$$

Let P_c be the tensile in the concrete. For equilibrium with zero total force, the compressive force in the four steel rods is $-P_c$.

Strains:

$$\varepsilon_s = \frac{P_s}{E_s A_s} + \alpha_s(\Delta T) = -\frac{P_c}{E_s A_s} + \alpha_s(\Delta T)$$

$$\varepsilon_c = \frac{P_c}{E_c A_c} + \alpha_c(\Delta T)$$

Matching:

$$\varepsilon_c = \varepsilon_s \qquad \frac{P_c}{E_c A_c} + \alpha_c(\Delta T) = -\frac{P_c}{E_s A_s} + \alpha_s(\Delta T)$$

$$\left(\frac{1}{E_c A_c} + \frac{1}{E_s A_s}\right) P_c = (\alpha_s - \alpha_c)(\Delta T)$$

$$\left(\frac{1}{(3.6\times10^6)(62.233)} + \frac{1}{(29\times10^6)(1.76715)}\right) P_c$$

$$= (6.5\times10^{-6} - 5.5\times10^{-6})(80)$$

$$P_c = 3.3366\times10^3 \text{ lb} \qquad P_s = -3.3366\times10^3 \text{ lb}$$

Stress in steel $\quad \sigma_s = \dfrac{P_s}{A_s} = \dfrac{-3.3366\times10^3}{1.76715} = -1.888\times10^3$ psi ◄

Stress in concrete $\quad \sigma_c = \dfrac{P_c}{A_c} = \dfrac{3.3366\times10^3}{62.233} = 53.6$ psi ◄

Problem 2.51

2.51 A steel railroad track ($E_s = 29 \times 10^6$ psi, $\alpha_s = 6.5 \times 10^{-6}/°\text{F}$) was laid out at a temperature of 30°F. Determine the normal stress in the rails when the temperature reaches 125 °F, assuming that the rails (*a*) are welded to form a continuous track, (*b*) are 39 ft long with $\frac{1}{4}$-in. gaps between them.

$L = 39$ ft $= 468$ in.

$$\delta_T = L\alpha(\Delta T) = (468)(6.5\times10^{-6})(125-30) = 288.99\times10^{-3} \text{ in.}$$

$$\delta_P = \frac{PL}{EA} = \frac{L}{E}\sigma = \frac{468}{29\times10^6}\sigma$$

(a) $\delta_P + \delta_T = 0 \qquad \dfrac{468}{29\times10^6}\sigma + 288.99\times10^{-3} = 0 \qquad \sigma = -17.91\times10^3$ psi

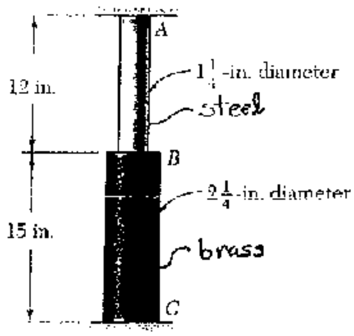
-17.91 ksi ◄

(b) $\delta_P + \delta_T = \frac{1}{4} \qquad \dfrac{468}{29\times10^6}\sigma + 288.99\times10^{-3} = \dfrac{1}{4} \qquad \sigma = -2.42\times10^3$ psi

-2.42 ksi ◄

Problem 2.52

2.52 A rod consisting of two cylindrical portions AB and BC is restrained at both ends. Portion AB is made of steel ($E_s = 29 \times 10^6$ psi, $\alpha_s = 6.5 \times 10^{-6}/°F$) and portion BC is made of brass ($E_b = 17 \times 10^6$ psi, $\alpha_b = 10.4 \times 10^{-6}/°F$). Knowing that the rod is initially unstressed, determine (*a*) the normal stresses induced in portions AB and BC by a temperature rise of 65°F, (*b*) the corresponding deflection of point B.



$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(1.25)^2 = 1.2272 \text{ in}^2$$

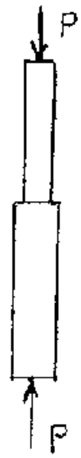
$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(2.25)^2 = 3.9761 \text{ in}^2$$

Free thermal expansion

$$\delta_T = L_{AB}\alpha_s(\Delta T) + L_{BC}\alpha_b(\Delta T)$$

$$= (12)(6.5\times10^{-6})(65) + (15)(10.4\times10^{-6})(65)$$

$$= 15.21\times10^{-3} \text{ in.}$$



Shortening due to induced compressive force P

$$\delta_P = \frac{PL_{AB}}{E_s A_{AB}} + \frac{PL_{BC}}{E_b A_{BC}}$$

$$= \frac{12\ P}{(29\times10^6)(1.2272)} + \frac{15\ P}{(17\times10^6)(3.9761)} = 559.10\times10^{-9}\ P$$

For zero net deflection $\delta_P = \delta_T$

$$(559.10\times10^{-9})P = 15.21\times10^{-3}$$

$$P = 27.204\times10^3 \text{ lb.}$$

(a) $$\sigma_{AB} = -\frac{P}{A_{AB}} = -\frac{27.204\times10^3}{1.2272} = -22.2\times10^3 \text{ psi} = -22.2 \text{ ksi}$$

$$\sigma_{BC} = -\frac{P}{A_{BC}} = -\frac{27.204\times10^3}{3.9761} = -6.84\times10^3 \text{ psi} = -6.84 \text{ ksi}$$

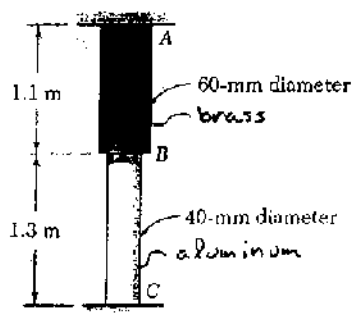
(b) $$\delta_B = +\frac{PL_{AB}}{E_s A_{AB}} - L_{AB}\alpha_s(\Delta T)$$

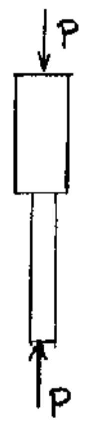
$$= +\frac{(27.204\times10^3)(12)}{(29\times10^6)(1.2272)} - (12)(6.5\times10^{-6})(65) = +4.10\times10^{-3} \text{ in}$$

i.e. 4.10×10^{-3} in. ↑

Problem 2.53

2.53 A rod consisting of two cylindrical portions AB and BC is restrained at both ends. Portion AB is made of brass ($E_b = 105$ GPa, $\alpha_b = 20.9 \times 10^{-6}/°C$) and portion BC is made of aluminum ($E_a = 72$ GPa, $\alpha_a = 23.9 \times 10^{-6}/°C$). Knowing that the rod is initially unstressed, determine (*a*) the normal stresses induced in portions AB and BC by a temperature rise of 42°C, (*b*) the corresponding deflection of point B.





$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(60)^2 = 2.8274 \times 10^3 \text{ mm}^2 = 2.8274 \times 10^{-5} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(40)^2 = 1.2566 \times 10^3 \text{ mm}^2 = 1.2566 \times 10^{-3} \text{ m}^2$$

Free thermal expansion

$$\delta_T = L_{AB}\alpha_b(\Delta T) + L_{BC}\alpha_a(\Delta T)$$

$$= (1.1)(20.9 \times 10^{-6})(42) + (1.3)(23.9 \times 10^{-6})(42)$$

$$= 2.2705 \times 10^{-3} \text{ m}$$

Shortening due to induced compressive force

$$\delta_P = \frac{P L_{AB}}{E_b A_{AB}} + \frac{P L_{BC}}{E_a A_{BC}}$$

$$= \frac{1.1\, P}{(105 \times 10^9)(2.8274 \times 10^{-3})} + \frac{1.3\, P}{(72 \times 10^9)(1.2566 \times 10^{-3})}$$

$$= 18.074 \times 10^{-9}\, P$$

For zero net deflection $\quad \delta_P = \delta_T$

$$18.074 \times 10^{-9}\, P = 2.2705 \times 10^{-3}$$

$$P = 125.62 \times 10^3 \text{ N}$$

(a) $$\sigma_{AB} = -\frac{P}{A_{AB}} = -\frac{125.62 \times 10^3}{2.8274 \times 10^{-3}} = -44.4 \times 10^6 \text{ Pa} = -44.4 \text{ MPa}$$

$$\sigma_{BC} = -\frac{P}{A_{BC}} = -\frac{125.62 \times 10^3}{1.2566 \times 10^{-3}} = -100.0 \times 10^6 \text{ Pa} = -100.0 \text{ MPa}$$

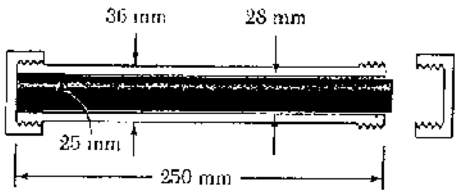
(b) $$\delta_B = +\frac{P L_{AB}}{E_b A_{AB}} - L_{AB}\alpha_b(\Delta T)$$

$$= \frac{(125.62 \times 10^3)(1.1)}{(105 \times 10^9)(2.8274 \times 10^{-3})} - (1.1)(20.9 \times 10^{-6})(42)$$

$$= -500 \times 10^{-6} \text{ m} = -0.500 \text{ mm}$$

i.e. 0.500 mm $\downarrow$

Problem 2.54



2.54 In Prob. 2.42, determine the average normal stress in the tube and the rod, assuming that the temperature was 15° C when the nuts were snugly fitted and that the final temperature is 55° C. (For aluminum, $\alpha_a = 23.6 \times 10^{-6}/°C$; for brass, $\alpha_b = 20.9 \times 10^{-6}/°C$).

2.42 A 250-mm-long aluminum tube (E = 70 GPa) of 36-mm outer diameter and 28-mm inner diameter may be closed at both ends by means of single-threaded screw-on covers of 1.5-mm pitch. With one cover screwed on tight, a solid brass rod (E = 105 GPa) of 25-mm diameter is placed inside the tube and the second cover is screwed on. Since the rod is slightly longer than the tube, it is observed that the cover must be forced against the rod by rotating it one-quarter of a turn before it can be tightly closed. Determine (a) the average normal stress in the tube and in the rod, (b) the deformations of the tube and of the rod.

$$A_{tube} = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(36^2 - 28^2) = 402.12 \text{ mm}^2 = 402.12 \times 10^{-6} \text{ m}^2$$

$$A_{rod} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(25)^2 = 490.87 \text{ mm}^2 = 490.87 \times 10^{-6} \text{ m}^2$$

$$\Delta T = 55 - 15 = 40 \text{ °C}$$

$$\delta_{tube} = \frac{PL}{E_{tube} A_{tube}} + L\alpha_{tube}(\Delta T) = \frac{P(0.250)}{(70 \times 10^9)(402.12 \times 10^{-6})} + (0.250)(23.6 \times 10^{-6})(40)$$

$$= 8.8815 \times 10^{-9} P + 236 \times 10^{-6}$$

$$\delta_{rod} = -\frac{PL}{E_{rod} A_{rod}} + L\alpha_{rod}(\Delta T) = -\frac{P(0.250)}{(105 \times 10^9)(490.87 \times 10^{-6})} + (0.250)(20.9 \times 10^{-6})(40)$$

$$= -4.8505 \times 10^{-9} P + 209 \times 10^{-6}$$

$$\delta^* = \frac{1}{4} \text{ turn} \times 1.5 \text{ mm} = 0.375 \text{ mm} = 375 \times 10^{-6} \text{ m}$$

$$\delta_{tube} = \delta_{rod} + \delta^*$$

$$8.8815 \times 10^{-9} P + 236 \times 10^{-6} = -4.8505 \times 10^{-9} P + 209 \times 10^{-6} + 375 \times 10^{-6}$$

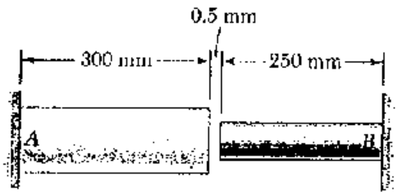
$$13.732 \times 10^{-9} P = 348 \times 10^{-6} \qquad P = 25.342 \times 10^3 \text{ N}$$

$$\sigma_{tube} = \frac{P}{A_{tube}} = \frac{25.342 \times 10^3}{402.12 \times 10^{-6}} = 63.0 \times 10^6 \text{ Pa} = 63.0 \text{ MPa}$$ ◄

$$\sigma_{rod} = -\frac{P}{A_{rod}} = -\frac{25.342 \times 10^3}{490.87 \times 10^{-6}} = -51.6 \times 10^6 \text{ Pa} = -51.6 \text{ MPa}$$ ◄

Problem 2.55

2.55 At room temperature (20°C) a 0.5-mm gap exists between the ends of the rods shown. At a later time when the temperature has reached 140°C, determine (*a*) the normal stress in the aluminum rod, (*b*) the change in length of the aluminum rod.



Aluminum
$A = 2000\ \text{mm}^2$
$E = 75$ GPa
$\alpha = 23 \times 10^{-6}/°\text{C}$

Stainless steel
$A = 800\ \text{mm}^2$
$E = 190$ GPa
$\alpha = 17.3 \times 10^{-6}/°\text{C}$

$\Delta T = 140 - 20 = 120\ °\text{C}$

Free thermal expansion

$$\delta_T = L_a \alpha_a (\Delta T) + L_s \alpha_s (\Delta T)$$
$$= (0.300)(23\times10^{-6})(120) + (0.250)(17.3\times10^{-6})(120)$$
$$= 1.347 \times 10^{-3}\ \text{m}$$

Shortening due to P to meet constraint.

$$\delta_P = 1.347\times10^{-3} - 0.5\times10^{-3} = 0.847\times10^{-3}\ \text{m}$$

$$\delta_P = \frac{PL_a}{E_a A_a} + \frac{PL_s}{E_s A_s} = \left(\frac{L_a}{E_a A_a} + \frac{L_s}{E_s A_s}\right)P$$
$$= \left(\frac{0.300}{(75\times10^9)(2000\times10^{-6})} + \frac{0.250}{(190\times10^9)(800\times10^{-6})}\right)P = 3.6447\times10^{-9}\ P$$

Equating: $3.6447\times10^{-9}\ P = 0.847\times10^{-3}$ $\qquad P = 232.39\times10^3\ \text{N}$

(a) $\sigma_a = -\dfrac{P}{A_a} = -\dfrac{232.39\times10^3}{2000\times10^{-6}} = -116.2\times10^6\ \text{Pa}$ $\qquad -116.2\ \text{MPa}$ ◄

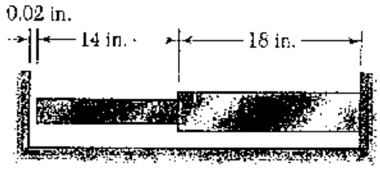
(b) $\delta_a = L_a \alpha_a (\Delta T) - \dfrac{PL_a}{E_a A_a}$

$$= (0.300)(23\times10^{-6})(120) - \frac{(232.39\times10^3)(0.300)}{(75\times10^9)(2000\times10^{-6})} = 363\times10^{-6}\ \text{m}$$

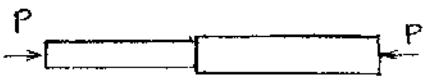
0.363 mm ◄

Problem 2.56

2.56 Knowing that a 0.02-in. gap exists when the temperature is 75 °F, determine (*a*) the temperature at which the normal stress in the aluminum bar will be equal to -11 ksi, (*b*) the corresponding exact length of the aluminum bar.



Bronze	Aluminum
$A = 2.4\ \text{in.}^2$	$A = 2.8\ \text{in.}^2$
$E = 15 \times 10^6$ psi	$E = 10.6 \times 10^6$ psi
$\alpha = 12 \times 10^{-6}/°\text{F}$	$\alpha = 12.9 \times 10^{-6}/°\text{F}$



$\sigma_a = -11\ \text{ksi} = -11 \times 10^3\ \text{psi}$

$P = -\sigma_a A_a = (11 \times 10^3)(2.8) = 30.8 \times 10^3\ \text{lb.}$

Shortening due to P

$$\delta_P = \frac{PL_b}{E_b A_b} + \frac{PL_a}{E_a A_a}$$

$$= \frac{(30.8 \times 10^3)(14)}{(15 \times 10^6)(2.4)} + \frac{(30.8 \times 10^3)(18)}{(10.6 \times 10^6)(2.8)}$$

$$= 30.657 \times 10^{-3}\ \text{in.}$$

Available elongation for thermal expansion

$$\delta_T = 0.02 + 30.657 \times 10^{-3} = 50.657 \times 10^{-3}\ \text{in.}$$

But $\quad \delta_T = L_b \alpha_b (\Delta T) + L_a \alpha_a (\Delta T)$

$$= (14)(12 \times 10^{-6})(\Delta T) + (18)(12.9 \times 10^{-6})(\Delta T) = (400.2 \times 10^{-6})\Delta T$$

Equating $\quad (400.2 \times 10^{-6})\Delta T = 50.657 \times 10^{-3} \qquad \Delta T = 126.6\ °\text{F}$

(a) $\qquad T_{hot} = T_{cold} + \Delta T = 75 + 126.6 = 201.6\ °\text{F}$

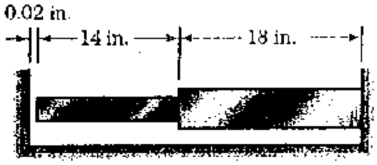
(b) $\quad \delta_a = L_a \alpha_a (\Delta T) - \dfrac{PL_a}{E_a A_a}$

$$= (18)(12.9 \times 10^{-6})(126.6) - \frac{(30.8 \times 10^3)(18)}{(10.6 \times 10^6)(2.8)} = 10.712 \times 10^{-3}\ \text{in}$$

$$L_{exact} = 18 + 10.712 \times 10^{-3} = 18.0107\ \text{in.}$$

Problem 2.57

2.57 Determine (*a*) the compressive force in the bars shown after a temperature rise of 180°F, (*b*) the corresponding change in length of the bronze bar.



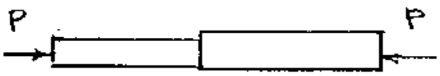
Bronze	Aluminum
$A = 2.4\text{ in.}^2$	$A = 2.8\text{ in.}^2$
$E = 15 \times 10^6$ psi	$E = 10.6 \times 10^6$ psi
$\alpha = 12 \times 10^{-6}/°\text{F}$	$\alpha = 12.9 \times 10^{-6}/°\text{F}$

Thermal expansion if free of constraint

$$\delta_T = L_b \alpha_b (\Delta T) + L_a \alpha_a (\Delta T)$$

$$= (14)(12\times10^{-6})(180) + (18)(12.9\times10^{-6})(180)$$

$$= 72.036 \times 10^{-3} \text{ in.}$$



Constrained expansion. $\delta = 0.02$ in

Shortening due to induced compressive force P

$$\delta_P = 72.036\times10^{-3} - 0.02 = 52.036\times10^{-3} \text{ in.}$$

But $\delta_P = \frac{PL_b}{E_b A_b} + \frac{PL_a}{E_a A_a} = \left(\frac{L_b}{E_b A_b} + \frac{L_a}{E_a A_a}\right) P$

$$= \left(\frac{14}{(15\times10^6)(2.4)} + \frac{18}{(10.6\times10^6)(2.8)}\right) P = 995.36\times10^{-9}\, P$$

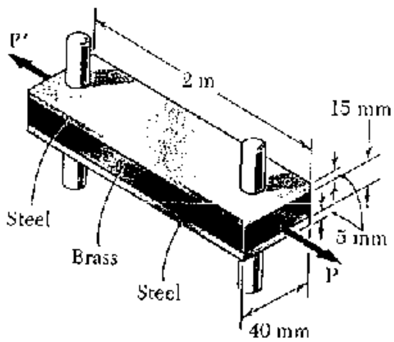
(a) Equating $995.36\times10^{-9}\, P = 52.036\times10^{-3}$ $\quad P = 52.279\times10^3$ lb

52.3 kips ◄

(b) $\delta_b = L_b \alpha_b (\Delta T) - \frac{PL_b}{E_b A_b}$

$$= (14)(12\times10^{-6})(180) - \frac{(52.279\times10^3)(14)}{(15\times10^6)(2.4)} = 9.91\times10^{-3} \text{ in.}$$ ◄

Problem 2.58



2.58 Two steel bars (E_s = 200 GPa and $\alpha_s = 11.7 \times 10^{-6}/°C$) are used to reinforce a brass bar (E_b = 105 GPa, $\alpha_b = 20.9 \times 10^{-6}/°C$) which is subjected to a load P = 25 kN. When the steel bars were fabricated, the distance between the centers of the holes which were to fit on the pins was made 0.5 mm smaller than the 2 m needed. The steel bars were then placed in an oven to increase their length so that they would just fit on the pins. Following fabrication, the temperature in the steel bars dropped back to room temperature. Determine (*a*) the increase in temperature that was required to fit the steel bars on the pins, (*b*) the stress in the brass bar after the load is applied to it.

(a) Required temperature change for fabrication

$\delta_T = 0.5$ mm $= 0.5 \times 10^{-3}$ m

(a) Temperature change required to expand steel bar by this amount

$\delta_T = L\alpha_s \Delta T, \quad 0.5 \times 10^{-3} = (2.00)(11.7 \times 10^{-6})(\Delta T), \quad \Delta T =$

$$0.5 \times 10^{-3} = (2)(11.7 \times 10^{-6})(\Delta T)$$

$$\Delta T = 21.368 \text{ °C} \qquad 21.4 \text{ °C}$$

(b) Once assembled, a tensile force P^* develops in the steel and a compressive force P^* develops in the brass, in order to elongate the steel and contract the brass.

Elongation of steel: $A_s = (2)(5)(40) = 400 \text{ mm}^2 = 400 \times 10^{-6} \text{ m}^2$

$$(\delta_P)_s = \frac{F^*L}{A_s E_s} = \frac{P^*(2.00)}{(400 \times 10^{-6})(200 \times 10^{9})} = 25 \times 10^{-9} P^*$$

Contraction of brass: $A_b = (40)(15) = 600 \text{ mm}^2 = 600 \times 10^{-6} \text{ m}^2$

$$(\delta_P)_b = \frac{P^*L}{A_b E_b} = \frac{P^*(2.00)}{(600 \times 10^{-6})(105 \times 10^{9})} = 31.746 \times 10^{-9} P^*$$

But $(\delta_P)_s + (\delta_P)_b$ is equal to the initial amount of misfit

$$(\delta_P)_s + (\delta_P)_b = 0.5 \times 10^{-3}, \quad 56.746 \times 10^{-9} P^* = 0.5 \times 10^{-3}$$

$$P^* = 8.811 \times 10^{3} \text{ N}$$

Stresses due to fabrication

Steel: $\sigma_s^* = \dfrac{P^*}{A_s} = \dfrac{8.811 \times 10^{3}}{400 \times 10^{-6}} = 22.03 \times 10^{6} \text{ Pa} = 22.03 \text{ MPa}$

Brass: $\sigma_b^* = -\dfrac{P^*}{A_b} = -\dfrac{8.811 \times 10^{3}}{600 \times 10^{-6}} = -14.68 \times 10^{6} \text{ Pa} = -14.68 \text{ MPa}$

To these stresses must be added the stresses due to the 25 kN load.

continued

Problem 2.58 continued

For the added load, the additional deformation is the same for both the steel and the brass. Let δ' be the additional displacement. Also, let P_s and P_b be the additional forces developed in the steel and brass, respectively.

$$\delta' = \frac{P_s L}{A_s E_s} = \frac{P_b L}{A_b E_b}$$

$$P_s = \frac{A_s E_s}{L}\delta' = \frac{(400\times10^{-6})(200\times10^{9})}{2.00}\delta' = 40\times10^{6}\,\delta'$$

$$P_b = \frac{A_b E_b}{L}\delta' = \frac{(600\times10^{-6})(105\times10^{9})}{2.00}\delta' = 31.5\times10^{6}\,\delta'$$

Total $\quad P = P_s + P_b = 25\times10^{3}$ N

$$40\times10^{6}\,\delta' + 31.5\times10^{6}\,\delta' = 25\times10^{3} \qquad \delta' = 349.65\times10^{-6}\ \text{m}$$

$$P_s = (40\times10^{6})(349.65\times10^{-6}) = 13.986\times10^{3}\ \text{N}$$

$$P_b = (31.5\times10^{6})(349.65\times10^{-6}) = 11.140\times10^{3}\ \text{N}$$

$$\sigma_s = \frac{P_s}{A_s} = \frac{13.986\times10^{3}}{400\times10^{-6}} = 34.97\times10^{6}\ \text{Pa}$$

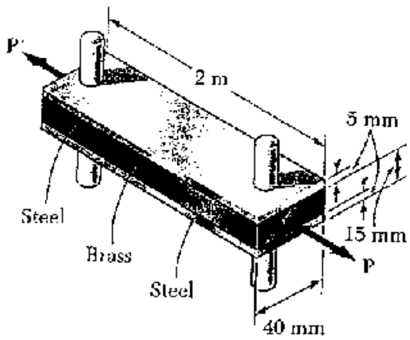
$$\sigma_b = \frac{P_b}{A_b} = \frac{11.140\times10^{3}}{600\times10^{-6}} = 18.36\times10^{6}\ \text{Pa}$$

Add stress due to fabrication

$$\sigma_s = 34.97\times10^{6} + 22.03\times10^{6} = 57.0\times10^{6}\ \text{Pa} = 57.0\ \text{MPa}$$

$$\sigma_b = 18.36\times10^{6} - 14.68\times10^{6} = 3.68\times10^{6}\ \text{Pa} = 3.68\ \text{MPa}$$

Problem 2.59



2.59 Determine the maximum load P that may be applied to the brass bar of Prob. 2.58 if the allowable stress in the steel bars is 30 MPa and the allowable stress in the brass bar is 25 MPa.

2.58 Two steel bars ($E_s = 200$ GPa and $\alpha_s = 11.7 \times 10^{-6}/°C$) are used to reinforce a brass bar ($E_b = 105$ GPa, $\alpha_b = 20.9 \times 10^{-6}/°C$) which is subjected to a load $P = 25$ kN. When the steel bars were fabricated, the distance between the centers of the holes which were to fit on the pins was made 0.5 mm smaller than the 2 m needed. The steel bars were then placed in an oven to increase their length so that they would just fit on the pins. Following fabrication, the temperature in the steel bars dropped back to room temperature. Determine (*a*) the increase in temperature that was required to fit the steel bars on the pins, (*b*) the stress in the brass bar after the load is applied to it.

See solution to Problem 2.58 to obtain the fabrication stresses

$\sigma_s^* = 22.03$ MPa $\qquad \sigma_b^* = -14.68$ MPa

Allowable stresses: $\sigma_{s,all} = 30$ MPa , $\sigma_{b,all} = 25$ MPa

Available stress increase from load

$$\sigma_s = 30 - 22.03 = 7.97 \text{ MPa}$$
$$\sigma_b = 25 + 14.68 = 39.68 \text{ MPa}$$

Corresponding available strains

$$\varepsilon_s = \frac{\sigma_s}{E_s} = \frac{7.97\times10^6}{200\times10^9} = 39.85\times10^{-6}$$
$$\varepsilon_b = \frac{\sigma_b}{E_b} = \frac{39.68\times10^6}{105\times10^9} = 377.9\times10^{-6}$$

Smaller value governs $\therefore \quad \varepsilon = 39.85\times10^{-6}$

Areas: $A_s = (2)(5)(40) = 400 \text{ mm}^2 = 400\times10^{-6} \text{ m}^2$
$A_b = (15)(40) = 600 \text{ mm}^2 = 600\times10^{-6} \text{ m}^2$

$$P_s = E_s A_s \varepsilon = (200\times10^9)(400\times10^{-6})(39.85\times10^{-6}) = 3.188\times10^3 \text{ N}$$

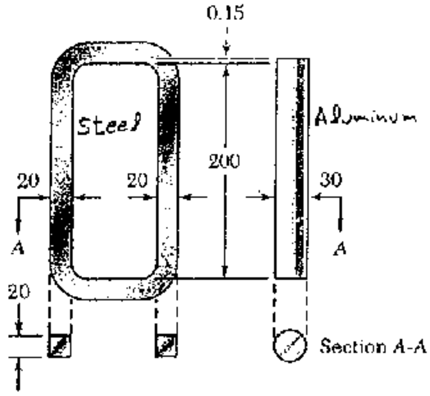
$$P_b = E_b A_b \varepsilon = (105\times10^9)(600\times10^{-6})(39.85\times10^{-6}) = 2.511\times10^3 \text{ N}$$

Total allowable additional force

$$P = P_s + P_b = 3.188\times10^3 + 2.511\times10^3 = 5.70\times10^3 \text{ N} = 5.70 \text{ kN}$$

Problem 2.60

Dimensions in mm



2.60 An aluminum rod (E_a = 70 GPa, α_a = 23.6 × 10^{-6}/°C) and a steel link (E_s = 200 GPa, α_s = 11.7 × 10^{-6}/°C) have the dimensions shown at a temperature of 20°C. The steel link is heated until the aluminum rod can be fitted freely into in. The temperature of the whole assembly is then raised to 150°C. Determine (*a*) the final stress in the rod, (*b*) in the link.

$$\Delta T = T_{final} - T_{initial} = 150°C - 20°C = 130°C$$

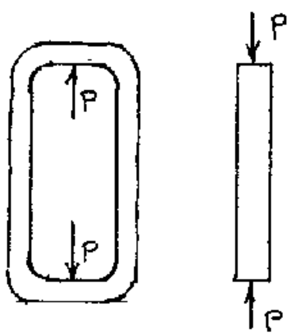
Free thermal expansion of each part

Aluminum rod: $(\delta_T)_a = L\alpha_a(\Delta T)$

$$(\delta_T)_a = (0.200)(23.6\times10^{-6})(130) = 613.6\times10^{-6}\ m$$

Steel link: $(\delta_T)_s = L\alpha_s(\Delta T)$

$$(\delta_T)_s = (0.200)(11.7\times10^{-6})(130) = 304.2\times10^{-6}\ m$$



Let P be the compressive force developed in the aluminum rod. It is also the tensile force in the steel link.

Aluminum rod: $A_a = \frac{\pi}{4}d^2 = \frac{\pi}{4}(30)^2 = 706.86\ mm^2$
$= 706.86\times10^{-6}\ m^2$

$$(\delta_P)_a = \frac{PL}{E_aA_a} = \frac{(P)(0.200)}{(70\times10^9)(706.86\times10^{-6})} = 4.04203\times10^{-9}\ P$$

Steel link: $A_s = (2)(20)^2 = 800\ mm^2 = 800\times10^{-6}\ m^2$

$$(\delta_P)_s = \frac{PL}{E_sA_s} = \frac{(P)(0.200)}{(200\times10^9)(800\times10^{-6})} = 1.25\times10^{-9}\ P$$

Matching the deformed lengths

$$\cancel{0.200} + (\delta_T)_s + (\delta_P)_s = \cancel{0.200} + 0.15\times10^{-3} + (\delta_T)_a - (\delta_P)_a$$

$$(\delta_P)_s + (\delta_P)_a = 0.15\times10^{-3} + (\delta_T)_a - (\delta_T)_s$$

$$1.25\times10^{-9}\ P + 4.04203\times10^{-9}\ P = 0.15\times10^{-3} + 613.6\times10^{-6} - 304.2\times10^{-6}$$

$$P = 86.810\times10^3\ N$$

(a) Stress in rod $\quad \sigma_a = -\frac{P}{A_a} = -\frac{86.810\times10^3}{706.86\times10^{-6}} = -122.8\times10^6\ Pa$

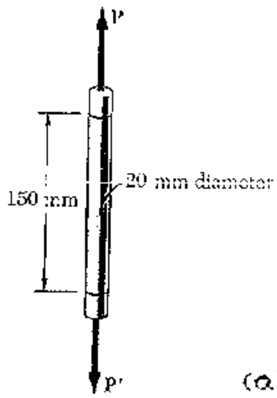
−122.8 MPa ◄

(b) Stress in link $\quad \sigma_s = \frac{P}{A_s} = \frac{86.810\times10^3}{800\times10^{-6}} = 108.5\times10^6$

108.5 MPa ◄

Problem 2.61

2.61 In a standard tensile test an aluminum rod of 20-mm diameter is subjected to a tension force of $P = 30$ kN. Knowing that $\nu = 0.35$ and $E = 70$ GPa, determine (*a*) the elongation of the rod in an 150-mm gage length, (*b*) the change in diameter of the rod.



$$P = 30\times10^{3}\ \text{N}$$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(20)^2 = 314.159\ \text{mm}^2 = 314.159\times10^{-6}\ \text{m}^2$$

$$\sigma_y = \frac{P}{A} = \frac{30\times10^3}{314.159\times10^{-6}} = 95.493\times10^6\ \text{Pa}$$

$$\varepsilon_y = \frac{\sigma_y}{E} = \frac{95.493\times10^6}{70\times10^9} = 1.36418\times10^{-3}$$

(a) $\delta_y = L\varepsilon_y = (150\times10^{-3})(1.36418\times10^{-3}) = 205\times10^{-6}$ m

0.205 mm ◄

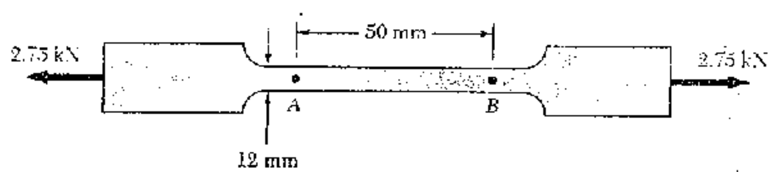
$$\varepsilon_x = -\nu\varepsilon_y = -(0.35)(1.36418\times10^{-3}) = -477.46\times10^{-6}$$

(b) $\delta_x = d\,\varepsilon_x = (20\times10^{-3})(-477.46\times10^{-6}) = -9.55\times10^{-6}$ m

-0.00955 mm ◄

Problem 2.62

2.62 A 2.75 kN tensile load is applied to a test coupon made from 1.6-mm flat steel plate ($E = 200$ GPa, $\nu = 0.30$). Determine the resulting change (*a*) in the 50-mm gage length, (*b*) in the width of portion *AB* of the test coupon, (*c*) in the thickness of portion *AB*, (*d*) in the cross-sectional area of portion *AB*.



$$A = (1.6)(12) = 19.2\ \text{mm}^2 = 19.2\times10^{-6}\ \text{m}^2$$

$$P = 2.75\times10^3\ \text{N}$$

$$\sigma_x = \frac{P}{A} = \frac{2.75\times10^3}{19.2\times10^{-6}} = 143.229\times10^6\ \text{Pa}$$

$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{143.229\times10^6}{200\times10^9} = 716.15\times10^{-6}$$

$$\varepsilon_y = \varepsilon_z = -\nu\varepsilon_x = -(0.30)(716.15\times10^{-6}) = -214.84\times10^{-6}$$

(a) $L = 0.050$ m $\quad \delta_x = L\varepsilon_x = (0.050)(716.15\times10^{-6}) = 35.81\times10^{-6}$ m

0.0358 mm ◄

(b) $w = 0.012$ m $\quad \delta_y = w\varepsilon_y = (0.012)(-214.84\times10^{-6}) = -2.578\times10^{-6}$ m

-0.00258 mm ◄

(c) $t = 0.0016$ m $\quad \delta_z = t\varepsilon_z = (0.0016)(-214.84\times10^{-6}) = -343.7\times10^{-9}$ m

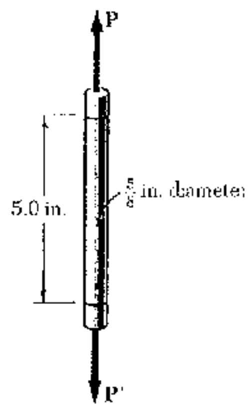
-0.0003437 mm ◄

(d) $A = w_0(1+\varepsilon_y)t_0(1+\varepsilon_z) = w_0t_0(1+\varepsilon_y+\varepsilon_z+\varepsilon_y\varepsilon_z) \qquad A_0 = w_0t_0$

$\Delta A = A - A_0 = w_0t_0(\varepsilon_y+\varepsilon_z+\text{negligible term}) = 2w_0t_0\varepsilon_y$

$= (2)(0.012)(0.0016)(-214.84\times10^{-6}) = -8.25\times10^{-9}\ \text{m}^2 = -0.00825\ \text{mm}^2$ ◄

Problem 2.63



2.63 A standard tension test is used to determine the properties of an experimental plastic. The test specimen is a $\frac{5}{8}$-in.-diameter rod and it is subjected to a 800 lb tensile force. Knowing that an elongation of 0.45 in. and a decrease in diameter of 0.025 in. are observed in a 5-in. gage length, determine the modulus of elasticity, the modulus of rigidity, and Poisson's ratio of the material.

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}\left(\frac{5}{8}\right)^2 = 0.306796 \text{ in}^2$$

$$P = 800 \text{ lb}$$

$$\sigma_y = \frac{P}{A} = \frac{800}{0.306796} = 2.6076 \times 10^3 \text{ psi}$$

$$\varepsilon_y = \frac{\delta_y}{L} = \frac{0.45}{5.0} = 0.090$$

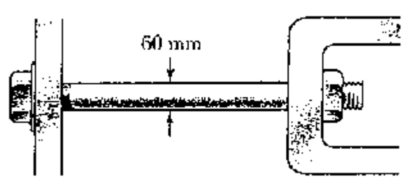
$$\varepsilon_x = \frac{\delta_x}{d} = \frac{-0.025}{0.625} = -0.040$$

$$E = \frac{\sigma_y}{\varepsilon_y} = \frac{2.6076 \times 10^3}{0.090} = 28.973 \times 10^3 \text{ psi} \qquad E = 29.0 \times 10^3 \text{ psi}$$

$$\nu = -\frac{\varepsilon_x}{\varepsilon_y} = -\frac{-0.040}{0.090} = 0.44444 \qquad \nu = 0.444$$

$$G = \frac{E}{2(1+\nu)} = \frac{28.973 \times 10^3}{(2)(1+0.4444)} = 10.029 \times 10^3 \text{ psi} \qquad G = 10.03 \times 10^3 \text{ psi}$$

Problem 2.64



2.64 The change in diameter of a large steel bolt is carefully measured as the nut is tightened. Knowing that $E = 200$ GPa and $\nu = 0.29$, determine the internal force in the bolt, if the diameter is observed to decrease by 13 μm.

$$\delta_y = -13 \times 10^{-6} \text{ m} \qquad d = 60 \times 10^{-3} \text{ m}$$

$$\varepsilon_y = \frac{\delta_y}{d} = -\frac{13 \times 10^{-6}}{60 \times 10^{-3}} = -216.67 \times 10^{-6}$$

$$\nu = -\frac{\varepsilon_y}{\varepsilon_x} \quad \therefore \quad \varepsilon_x = -\frac{\varepsilon_y}{\nu} = \frac{216.67 \times 10^{-6}}{0.29} = 747.13 \times 10^{-6}$$

$$\sigma_x = E\varepsilon_x = (200 \times 10^9)(747.13 \times 10^{-6}) = 149.43 \times 10^6 \text{ Pa}$$

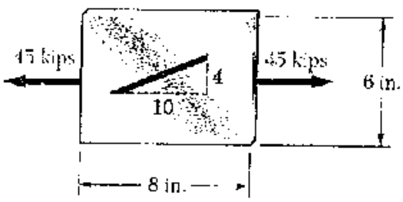
$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(60)^2 = 2.827 \times 10^3 \text{ mm}^2 = 2.827 \times 10^{-3} \text{ m}^2$$

$$F = \sigma_x A = (149.43 \times 10^6)(2.827 \times 10^{-3}) = 422 \times 10^3 \text{ N}$$

$$= 422 \text{ kN}$$

Problem 2.65

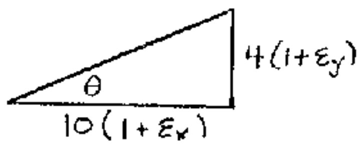
2.65 A line of slope 4:10 has been scribed on a cold-rolled yellow-brass plate, 6. in. wide and $\frac{1}{4}$ in. thick. Using the data available in Appendix B, determine the slope of the line when the plate is subjected as shown to a 45-kip centric axial load.



From Appendix B, $E = 15 \times 10^6$ psi, $G = 5.6 \times 10^6$ psi

$$G = \frac{E}{2(1+\nu)} \qquad \nu = \frac{E}{2G} - 1 = \frac{15}{(2)(5.6)} - 1 = 0.3393$$

$$A = (6)(\tfrac{1}{4}) = 1.50 \text{ in}^2$$



$$\sigma_x = \frac{P}{A} = \frac{45 \times 10^3}{1.50} = 30 \times 10^3 \text{ psi}$$

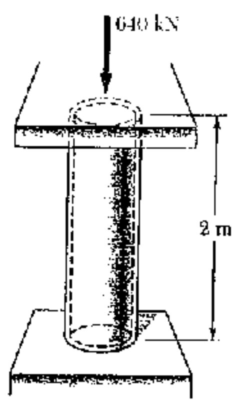
$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{30 \times 10^3}{15 \times 10^6} = 0.00200$$

$$\varepsilon_y = -\nu \varepsilon_x = -(0.3393)(0.00200) = -0.0006786$$

$$\tan\theta = \frac{4(1+\varepsilon_y)}{10(1+\varepsilon_x)} = \frac{(4)(1 - 0.0006786)}{(10)(1 + 0.00200)} = 0.39893$$

Problem 2.66

2.66 A 2-m length of an aluminum pipe of 240-mm outer diameter and 10-mm wall thickness is used as a short column and carries a centric axial load of 640 kN. Knowing that $E = 73$ GPa and $\nu = 0.33$, determine (*a*) the change in length of the pipe, (*b*) the change in its outer diameter, (*c*) the change in its wall thickness.



$$d_o = 240 \text{ mm} \quad t = 10 \text{ mm} \quad d_i = d_o - 2t = 220 \text{ mm}$$

$$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(240^2 - 220^2) = 7.2257 \times 10^3 \text{ mm}^2$$
$$= 7.2257 \times 10^{-3} \text{ m}^2$$

$$P = 640 \times 10^3 \text{ N}$$

$$\text{(a)} \quad \delta = -\frac{PL}{AE} = -\frac{(640 \times 10^3)(2.00)}{(7.2257 \times 10^{-3})(73 \times 10^9)} = -2.427 \times 10^{-3} \text{ m}$$
$$= -2.43 \text{ mm}$$

$$\varepsilon = \frac{\delta}{L} = -\frac{2.427 \times 10^{-3}}{2.00} = -1.2133 \times 10^{-3}$$

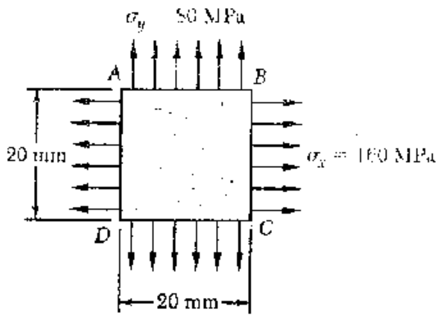
$$\varepsilon_{lat} = -\nu\varepsilon = -(0.33)(-1.2133 \times 10^{-3}) = 400.4 \times 10^{-6}$$

$$\text{(b)} \quad \Delta d_o = d_o \varepsilon_{lat} = (240)(400.4 \times 10^{-6}) = 0.0961 \text{ mm}$$

$$\text{(c)} \quad \Delta t = t \varepsilon_{lat} = (10)(400.4 \times 10^{-6}) = 0.00400 \text{ mm}$$

Problem 2.67

2.67 A 20-mm square has been scribed on the side of a large steel pressure vessel. After pressurization, the biaxial stress condition of the square is as shown. Using the data available in Appendix B, for structural steel, determine the percent change in the slope of diagonal DB due to the pressurization of the vessel.



For structural steel Appendix B gives

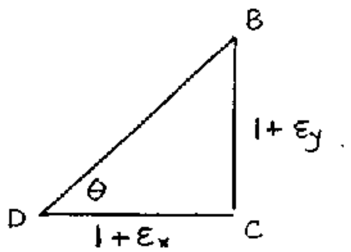
$E = 200 \text{ GPa}, \quad G = 77.2 \text{ GPa}$

$G = \dfrac{E}{2(1+\nu)}$

$\nu = \dfrac{E}{2G} - 1 = \dfrac{200}{(2)(77.2)} - 1 = 0.2953$

$\sigma_x = 160 \times 10^6 \text{ Pa} \qquad \sigma_y = 80 \times 10^6 \text{ Pa}$

$\sigma_z \approx 0$



$\varepsilon_x = \dfrac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z)$

$= \dfrac{1}{200 \times 10^9}\left[160 \times 10^6 - (0.2953)(80 \times 10^6)\right]$

$= 0.00068187$

$\varepsilon_y = \dfrac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z)$

$= \dfrac{1}{200 \times 10^9}\left[-(0.2953)(160 \times 10^6) + 80 \times 10^6\right]$

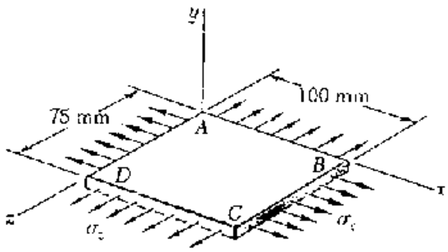
$= 0.00016373$

Slope of DB $\quad \tan\theta = \dfrac{1+\varepsilon_y}{1+\varepsilon_x} = \dfrac{1 + 0.00016373}{1 + 0.00068187} = 0.9994822$

Change in slope $\quad 0.999482 - 1 = -0.000518$

$\dfrac{\Delta \tan\theta}{\tan 45^\circ} \times 100\% = -0.0518\ \%$

Problem 2.68



2.68 A fabric used in air-inflated structures is subjected to a biaxial loading that results in normal stresses $\sigma_x = 120$ MPa and $\sigma_y = 160$ MPa. Knowing that the properties of the fabric can be approximated as $E = 87$ GPa and $\nu = 0.34$, determine the change in length of (*a*) side *AB*, (*b*) side *BC*, (*c*) diagonal *AC*.

$$\sigma_x = 120 \times 10^6 \text{ Pa}, \quad \sigma_y = 0, \quad \sigma_z = 160 \times 10^6 \text{ Pa}$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z)$$

$$= \frac{1}{87 \times 10^9}\left[120 \times 10^6 - (0.34)(160 \times 10^6)\right]$$

$$= 754.02 \times 10^{-6}$$

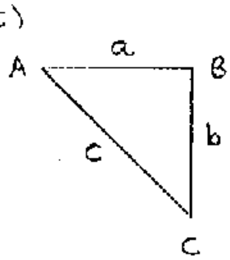
$$\varepsilon_z = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y + \sigma_z) = \frac{1}{87 \times 10^9}\left[-(0.34)(120 \times 10^6) + 160 \times 10^6\right]$$

$$= 1.3701 \times 10^{-3}$$

(a) $\delta_{AB} = (\overline{AB})\varepsilon_x = (100 \text{ mm})(754.02 \times 10^{-6}) = 0.0754 \text{ mm}$ ◄

(b) $\delta_{BC} = (\overline{BC})\varepsilon_z = (75 \text{ mm})(1.3701 \times 10^{-6}) = 0.1028 \text{ mm}$ ◄

(c)



Label sides of right triangle ABC as a, b, and c.

$$c^2 = a^2 + b^2$$

Obtain differentials by calculus.

$$2c\,dc = 2a\,da + 2b\,db$$

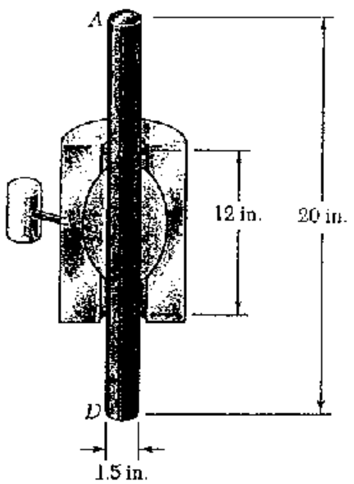
$$dc = \frac{a}{c}\,da + \frac{b}{c}\,db$$

But, $a = 100$ mm, $b = 100$ mm, $c = \sqrt{100^2 + 75^2)} = 125$ mm

$da = \delta_{AB} = 0.0754$ mm $\qquad db = \delta_{BC} = 0.1370$ mm

$$\delta_{AC} = dc = \frac{100}{125}(0.0754) + \frac{75}{125}(0.1028) = 0.1220 \text{ mm}$$ ◄

Problem 2.69



2.69 The aluminum rod AD is fitted with a jacket that is used to apply a hydrostatic pressure of 6000 psi to the 12-in. portion BC of the rod. Knowing that $E = 10.1 \times 10^6$ psi and $\nu = 0.36$, determine (*a*) the change in the total length AD, (*b*) the change in diameter at the middle of the rod.

$$\sigma_x = \sigma_z = -p = -6000 \text{ psi} \qquad \sigma_y = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z)$$

$$= \frac{1}{10.1\times10^6}\left[-6000 - (0.36)(0) - (0.36)(-6000)\right]$$

$$= -380.198\times10^{-6}$$

$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z)$$

$$= \frac{1}{10.1\times10^6}\left[-(0.36)(-6000) + 0 - (0.36)(-6000)\right]$$

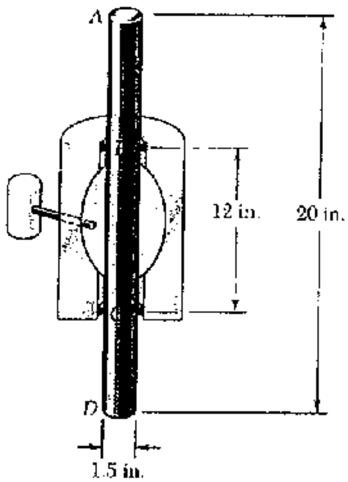
$$= 427.72\times10^{-6}$$

Length subject to strain ε_x: $\quad L = 12$ in.

(a) $\delta_y = L\varepsilon_y = (12)(427.72\times10^{-6}) = 5.13\times10^{-3}$ in. ◄

(b) $\delta_x = d\varepsilon_x = (1.5)(-380.198\times10^{-6}) = -0.570\times10^{-3}$ in. ◄

Problem 2.70



2.70 For the rod of Prob. 2.69, determine the forces that should be applied to the ends A and D of the rod (a) if the axial strain in portion BC of the rod is to remain zero as the hydrostatic pressure is applied, (b) if the total length AD of the rod is to remain unchanged.

2.69 The aluminum rod AD is fitted with a jacket that is used to apply a hydrostatic pressure of 6000 psi to the 12-in. portion BC of the rod. Knowing that $E = 10.1 \times 10^6$ psi and $\nu = 0.36$, determine (a) the change in the total length AD, (b) the change in diameter at the middle of the rod.

Over the pressurized portion BC

$$\sigma_x = \sigma_z = -p \qquad \sigma_y = \sigma_y$$

$$(\varepsilon_y)_{BC} = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z)$$

$$= \frac{1}{E}(2\nu p + \sigma_y)$$

(a) $(\varepsilon_y)_{BC} = 0 \qquad 2\nu p + \sigma_y = 0$

$$\sigma_y = -2\nu p = -(2)(0.36)(6000)$$

$$= -4320 \text{ psi}$$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(1.5)^2 = 1.76715 \text{ in}^2$$

$$F = A\sigma_y = (1.76715)(-4320) = -7630 \text{ lb.}$$

i.e. 7630 lb compression

(b) Over unpressurized portions AB and CD $\qquad \sigma_x = \sigma_z = 0$

$$(\varepsilon_y)_{AB} = (\varepsilon_y)_{CD} = \frac{\sigma_y}{E}$$

For no change in length

$$\delta = L_{AB}(\varepsilon_y)_{AB} + L_{BC}(\varepsilon_y)_{BC} + L_{CD}(\varepsilon_y)_{CD} = 0$$

$$(L_{AB} + L_{CD})(\varepsilon_y)_{AB} + L_{BC}(\varepsilon_y)_{BC} = 0$$

$$(20-12)\frac{\sigma_y}{E} + \frac{12}{E}(2\nu p + \sigma_y) = 0$$

$$\sigma_y = -\frac{24\nu p}{20} = -\frac{(24)(0.36)(6000)}{20} = -2592 \text{ psi}$$

$$P = A\sigma_y = (1.76715)(-2592) = -4580 \text{ lb.}$$

i.e. 4580 lb compression

Problem 2.71

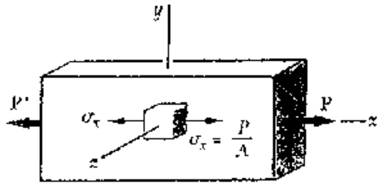


Fig 1.40 (a)

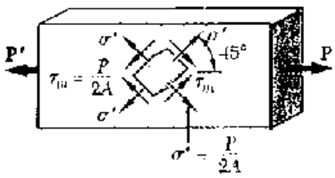
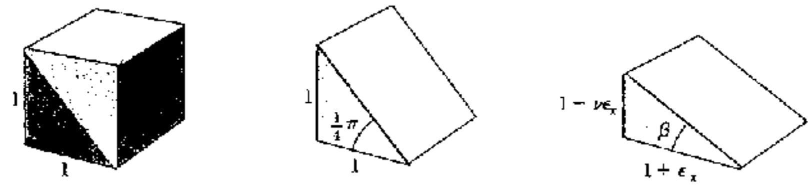


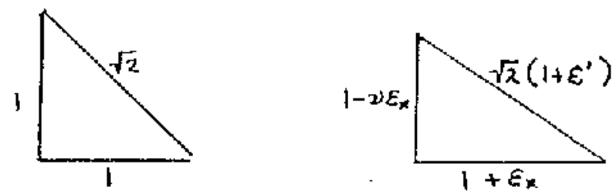
Fig 1.40 (b)

2.71 For a member under axial loading, express the normal strain ε' in a direction forming an angle of 45° with the axis of the load in terms of the axial strain ε_x by (*a*) comparing the hypothenuses of the triangles shown in Fig. 2.54, which represent respectively an element before and after deformation, (*b*) using the values of the corresponding stresses σ' and σ_x shown in Fig. 1.40, and the generalized Hooke's law.



(*a*) (*b*) (*c*)

Fig 2.54



Before deformation After deformation

(a)

$$\left[\sqrt{2}(1+\varepsilon')\right]^2 = (1+\varepsilon_x)^2 + (1-\nu\varepsilon_x)^2$$

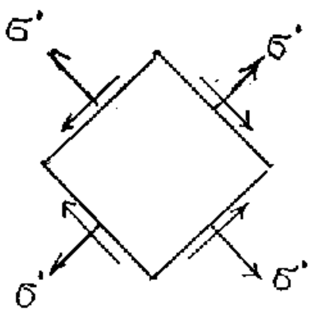
$$2(1+2\varepsilon'+\varepsilon'^2) = 1+2\varepsilon_x+\varepsilon_x^2+1-2\nu\varepsilon_x+\nu^2\varepsilon_x^2$$

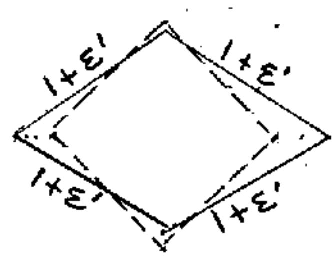
$$4\varepsilon'+2\varepsilon'^2 = 2\varepsilon_x+\varepsilon_x^2-2\nu\varepsilon_x+\nu^2\varepsilon_x^2$$

Neglect squares as small $\quad 4\varepsilon' = 2\varepsilon_x - 2\nu\varepsilon_x$

$$\varepsilon' = \frac{1-\nu}{2}\varepsilon_x$$

(b)





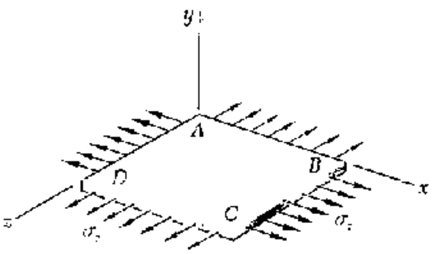
$$\varepsilon' = \frac{\sigma'}{E} - \frac{\nu\sigma'}{E}$$

$$= \frac{1-\nu}{E}\cdot\frac{P}{2A}$$

$$= \frac{1-\nu}{2E}\sigma_x$$

$$= \frac{1-\nu}{2}\varepsilon_x$$

Problem 2.72



2.72 The homogeneous plate *ABCD* is subjected to a biaxial loading as shown. It is known that $\sigma_z = \sigma_0$ and that the change in length of the plate in the x direction must be zero, that is, $\varepsilon_x = 0$. Denoting by E the modulus of elasticity and by ν Poisson's ratio, determine (*a*) the required magnitude of σ_x, (*b*) the ratio σ_0 / ε_z.

$$\sigma_z = \sigma_0, \quad \sigma_y = 0, \quad \varepsilon_x = 0$$

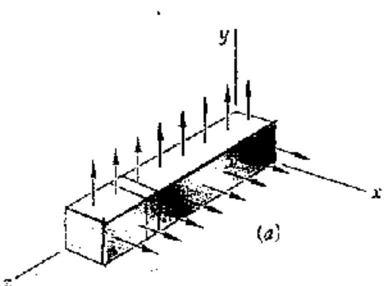
$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \frac{1}{E}(\sigma_x - \nu\sigma_0)$$

(a) $\sigma_x = \nu\sigma_0$ ◄

(b) $\varepsilon_z = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y + \sigma_z) = \frac{1}{E}(-\nu^2\sigma_0 - 0 + \sigma_0) = \frac{1-\nu^2}{E}\sigma_0$

$$\frac{\sigma_0}{\varepsilon_z} = \frac{E}{1-\nu^2}$$ ◄

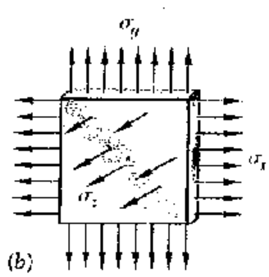
Problem 2.73



2.73 In many situations physical constraints prevent strain from occurring in a given direction; for example $\varepsilon_z = 0$ in the case shown, where longitudinal movement of the long prism is prevented at every point. Plane sections perpendicular to the longitudinal axis remain plane and the same distance apart. Show that for the this situation, which is known as *plane strain*, we can express σ_z, ε_x and ε_y as follows:

$$\sigma_z = \nu(\sigma_x + \sigma_y)$$

$$\varepsilon_x = \frac{1}{E}[(1-\nu^2)\sigma_x - \nu(1+\nu)\sigma_y] \qquad \varepsilon_y = \frac{1}{E}[(1-\nu^2)\sigma_y - \nu(1+\nu)\sigma_x]$$

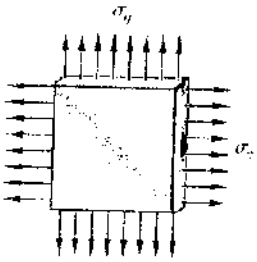


$\varepsilon_z = 0 = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y + \sigma_z)$ or $\sigma_z = \nu(\sigma_x + \sigma_y)$ ◄

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \frac{1}{E}[\sigma_x - \nu\sigma_y - \nu^2(\sigma_x + \sigma_y)]$$
$$= \frac{1}{E}[(1-\nu^2)\sigma_x - \nu(1+\nu)\sigma_y]$$ ◄

$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = \frac{1}{E}[-\nu\sigma_x + \sigma_y - \nu^2(\sigma_x + \sigma_y)]$$
$$= \frac{1}{E}[(1-\nu^2)\sigma_y - \nu(1+\nu)\sigma_x]$$ ◄

Problem 2.74



2.74 In many situations it is known that the normal stress in a given direction is zero, for example $\sigma_z = 0$ in the case of the thin plate shown. For this case, which is known as *plane stress*, show that if the strains ε_x and ε_y have been determined experimentally, we can express σ_x, σ_y and ε_z as follows:

$$\sigma_x = E\frac{\varepsilon_x + \nu\varepsilon_y}{1-\nu^2} \qquad \sigma_y = E\frac{\varepsilon_y + \nu\varepsilon_x}{1-\nu^2} \qquad \varepsilon_z = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y)$$

$$\sigma_z = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) \quad (1) \qquad \varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y) \quad (2)$$

Multiplying (2) by ν and adding to (1)

$$\varepsilon_x + \nu\varepsilon_y = \frac{1-\nu^2}{E}\sigma_x \quad \text{or} \quad \sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y)$$

Multiplying (1) by ν and adding to (2)

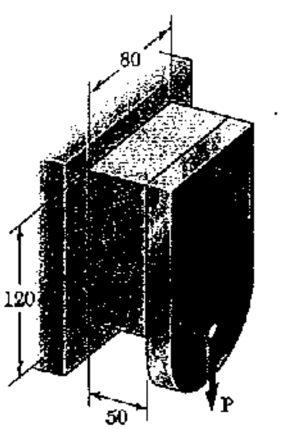
$$\varepsilon_y + \nu\varepsilon_x = \frac{1-\nu^2}{E}\sigma_y \quad \text{or} \quad \sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x)$$

$$\varepsilon_z = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y) = -\frac{\nu}{\cancel{E}}\cdot\frac{\cancel{E}}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y + \varepsilon_y + \nu\varepsilon_x)$$

$$= -\frac{\nu(1+\nu)}{1-\nu^2}(\varepsilon_x + \varepsilon_y) = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y)$$

Problem 2.75

2.75 The plastic block shown is bonded to a rigid support and to a vertical plate to which a 240-kN load P is applied. Knowing that for the plastic used $G = 1050$ MPa, determine the deflection of the plate.



Dimensions in mm

$$A = (80)(120) = 9.6\times10^3\ \text{mm}^2 = 9.6\times10^{-3}\ \text{m}^2$$

$$P = 240\times10^3\ \text{N}$$

$$\tau = \frac{P}{A} = \frac{240\times10^3}{9.6\times10^{-3}} = 25\times10^6\ \text{Pa}$$

$$G = 1050\times10^6\ \text{Pa}$$

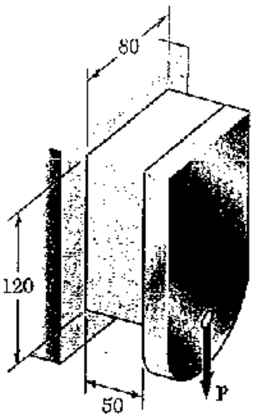
$$\gamma = \frac{\tau}{G} = \frac{25\times10^6}{1050\times10^6} = 23.810\times10^{-3}$$

$$h = 50\ \text{mm} = 0.050\ \text{m}$$

$$\delta = h\gamma = (0.050)(23.810\times10^{-3})$$

$$= 1.190\times10^{-3}\ \text{m} \qquad 1.091\ \text{mm}\downarrow$$

Problem 2.76



Dimensions in mm

2.76 What load **P** should be applied to the plate of Prob. 2.75 to produce a 1.5-mm deflection?

2.75 The plastic block shown is bonded to a rigid support and to a vertical plate to which a 240-kN load **P** is applied. Knowing that for the plastic used $G = 1050$ MPa, determine the deflection of the plate.

$$\delta = 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{ m}$$

$$h = 50 \text{ mm} = 50 \times 10^{-3} \text{ m}$$

$$\gamma = \frac{\delta}{h} = \frac{1.5 \times 10^{-3}}{50 \times 10^{-3}} = 30 \times 10^{-3}$$

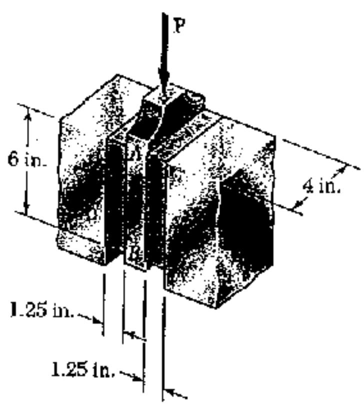
$$G = 1050 \times 10^{6} \text{ Pa}$$

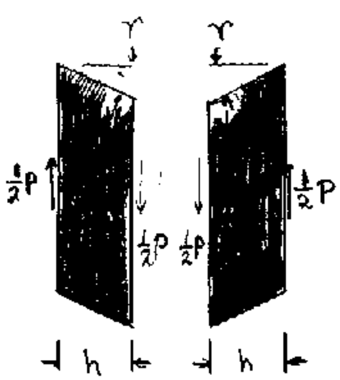
$$\tau = G\gamma = (1050 \times 10^{6})(30 \times 10^{-3}) = 31.5 \times 10^{6} \text{ Pa}$$

$$A = (120)(80) = 9.6 \times 10^{3} \text{ mm}^2 = 9.6 \times 10^{-3} \text{ m}^2$$

$$P = \tau A = (31.5 \times 10^{6})(9.6 \times 10^{-3}) = 302 \times 10^{3} \text{ N} \qquad 302 \text{ kN}$$

Problem 2.77





2.77 A vibration isolation unit consists of two blocks of hard rubber bonded a plate AB and to rigid supports as shown. Knowing that a force of magnitude $P = 6$ kips causes a deflection $\delta = \frac{1}{16}$ in. of plate AB, determine the modulus of rigidity of the rubber used.

$$F = \frac{1}{2}P = \frac{1}{2}(6000) = 3000 \text{ lb}$$

$$A = (6)(4) = 24 \text{ in}^2$$

$$\tau = \frac{F}{A} = \frac{3000}{24} = 125 \text{ psi}$$

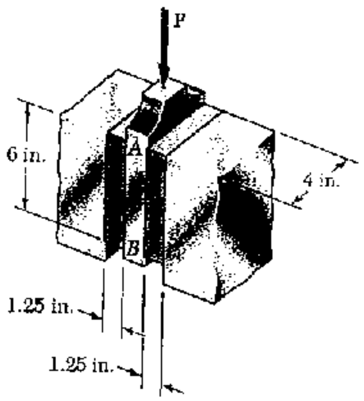
$$\delta = \frac{1}{16} \text{ in.} = 0.0625 \text{ in}$$

$$h = 1.25 \text{ in.}$$

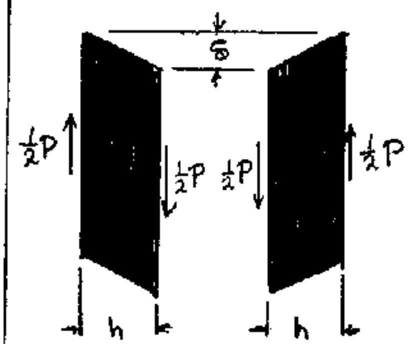
$$\gamma = \frac{\delta}{h} = \frac{0.0625}{1.25} = 50 \times 10^{-3} \text{ in.}$$

$$G = \frac{\tau}{\gamma} = \frac{125}{50 \times 10^{-3}} = 2.50 \times 10^{3} \text{ psi}$$

Problem 2.78



2.78 A vibration isolation unit consists of two blocks of hard rubber with a modulus of rigidity $G = 2.75$ ksi bonded to a plate AB and to rigid supports as shown. Denoting by P the magnitude of the force applied to the plate and by δ the corresponding deflection, determine the effective spring constant, $k = P/\delta$, of the system.



Shearing strain $\quad \gamma = \dfrac{\delta}{h}$

Shearing stress $\quad \tau = G\gamma = \dfrac{G\delta}{h}$

Force $\quad \frac{1}{2}P = A\tau = \dfrac{GA\delta}{h}$

$$P = \frac{2GA\delta}{h}$$

Effective spring constant

$$k = \frac{P}{\delta} = \frac{2GA}{h}$$

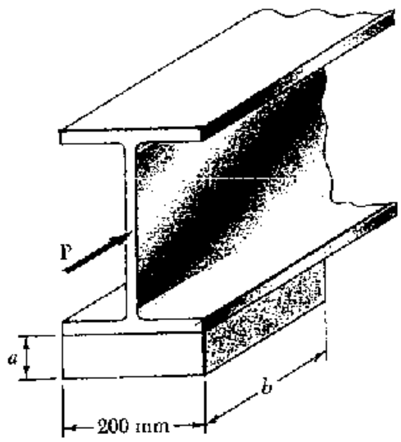
Data: $\quad G = 2.75 \times 10^3$ psi

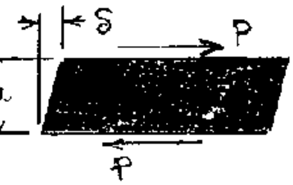
$A = (6)(4) = 24 \text{ in}^2$

$h = 1.25$ in.

$$k = \frac{(2)(2.75\times10^3)(24)}{1.25} = 105.6\times10^3 \text{ lb/in} \blacktriangleleft$$

Problem 2.79





2.79 An elastomeric bearing ($G = 0.9$ MPa) is used to support a bridge girder as shown to provide flexibility during earthquakes. The beam must not displace more than 10 mm when a 22 kN lateral load is applied as shown. Knowing that the maximum allowable shearing stress is 420 kPa, determine *(a)* the smallest allowable dimension b, *(b)* the smallest required thickness a.

Shearing force $P = 22 \times 10^3$ N

Shearing stress $\tau = 420 \times 10^3$ Pa

$$\tau = \frac{P}{A} \therefore A = \frac{P}{\tau} = \frac{22 \times 10^3}{420 \times 10^3} = 52.381 \times 10^{-3}\ \text{m}^2$$

$$= 52.381 \times 10^3\ \text{mm}^2$$

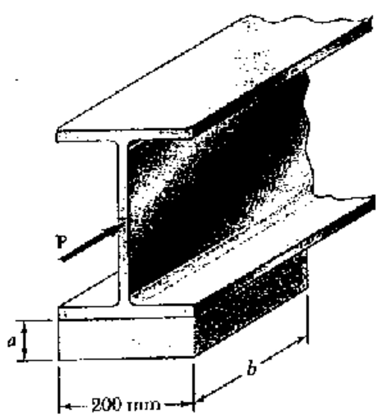
$$A = (200\ \text{mm})(b)$$

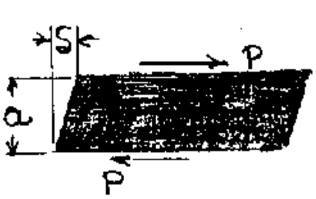
$$b = \frac{A}{200} = \frac{52.381 \times 10^3}{200} = 262\ \text{mm}$$

$$\gamma = \frac{\tau}{G} = \frac{420 \times 10^3}{0.9 \times 10^6} = 466.67 \times 10^{-3}$$

But $\gamma = \frac{\delta}{a} \therefore \quad a = \frac{\delta}{\gamma} = \frac{10\ \text{mm}}{466.67 \times 10^{-3}} = 21.4\ \text{mm}$

Problem 2.80





2.80 For the elastomeric bearing in Prob. 2.79 with $b = 220$ mm and $a = 30$ mm, determine the shearing modulus G and the shear stress τ for a maximum lateral load $P = 19$ kN and a maximum displacement $\delta = 12$ mm.

Shearing force $P = 19 \times 10^3$ N

Area $A = (200\ \text{mm})(220\ \text{mm}) = 44 \times 10^3\ \text{mm}^2$
$= 44 \times 10^{-3}\ \text{m}^2$

$$\tau = \frac{P}{A} = \frac{19 \times 10^3}{44 \times 10^{-3}} = 431.81 \times 10^8\ \text{Pa}$$

$$= 431\ \text{kPa}$$

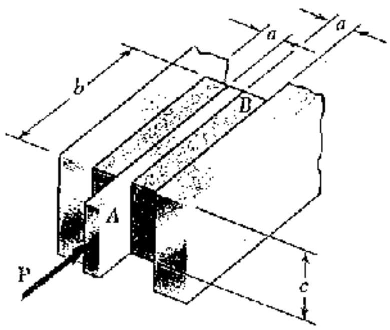
Shearing strain $\gamma = \frac{\delta}{a} = \frac{12\ \text{mm}}{30\ \text{mm}} = 0.400$

Shearing modulus

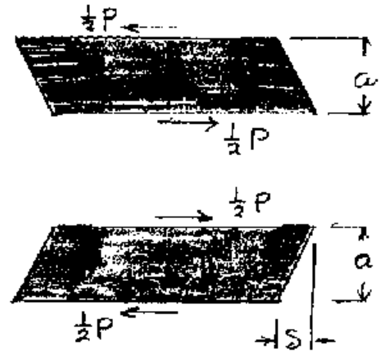
$$G = \frac{\tau}{\gamma} = \frac{431.81 \times 10^3}{0.4} = 1.080 \times 10^6\ \text{Pa}$$

$$= 1.080\ \text{MPa}$$

Problem 2.81



2.81 Two blocks of rubber with a modulus of rigidity $G = 1.75$ ksi are bonded to rigid supports and to a plate AB. Knowing that $c = 4$ in. and $P = 10$ kips, determine the smallest allowable dimensions a and b of the blocks if the shearing stress in the rubber is not to exceed 200 psi and the deflection of the plate is to be at least $\frac{3}{16}$ in.



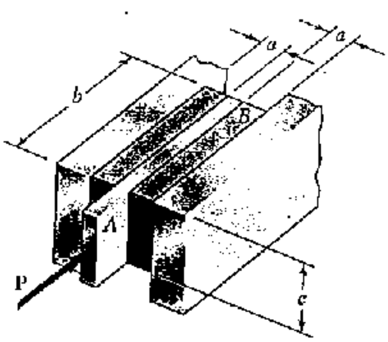
Shearing strain $\quad \gamma = \frac{\delta}{a} = \frac{\tau}{G}$

$$a = \frac{G\delta}{\tau} = \frac{(1750)(\frac{3}{16})}{200} = 1.641 \text{ in.}$$

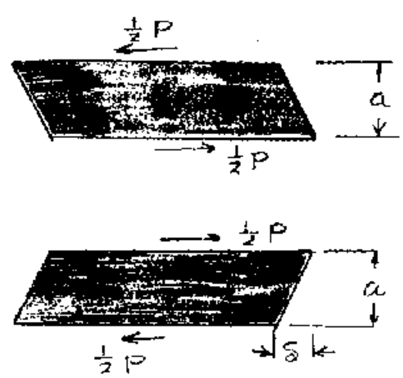
Shearing stress $\quad \tau = \frac{\frac{1}{2}P}{A} = \frac{P}{2bc}$

$$b = \frac{P}{2c\tau} = \frac{10 \times 10^3}{(2)(4)(200)} = 6.25 \text{ in.}$$

Problem 2.82



2.82 Two blocks of rubber with a modulus of rigidity $G = 1.50$ ksi are bonded to rigid supports and to a plate AB. Knowing that $b = 8$ in. and $c = 5$ in., determine the largest allowable load P and the smallest allowable thickness a of the blocks if the shearing stress in the rubber is not to exceed 210 psis and the deflection of the plate is to be at least $\frac{1}{4}$ in.



Shearing stress $\quad \tau = \frac{\frac{1}{2}P}{A} = \frac{P}{2bc}$

$$P = 2bc\tau = (2)(8)(5)(210) = 16.80 \times 10^3 \text{ lb} = 16.80 \text{ kips}$$

Shearing strain $\quad \gamma = \frac{\delta}{a} = \frac{\tau}{G}$

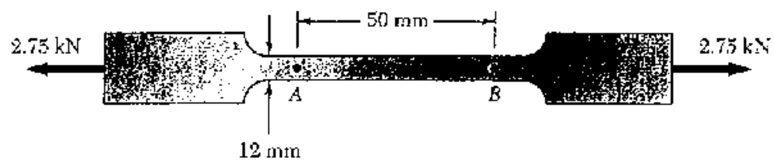
$$a = \frac{G\delta}{\tau} = \frac{(1500)(\frac{1}{4})}{210} = 1.786 \text{ in.}$$

Problem 2.83

***2.83** Determine the change in volume of the 50-mm gage length segment AB in Prob. 2.62 (*a*) by computing the dilatation of the material, (*b*) by subtracting the original volume of portion AB from its final volume.

2.62 A 2.75 kN tensile load is applied to a test coupon made from 1.6-mm flat steel plate ($E = 200$ GPa, $\nu = 0.30$). Determine the resulting change (*a*) in the 50-mm gage length, (*b*) in the width of portion AB of the test coupon, (*c*) in the thickness of portion AB, (*d*) in the cross-sectional area of portion AB.

All calculations here are in mm



(a) $A_o = (12)(1.6) = 19.2 \text{ mm}^2 = 19.2 \times 10^{-6} \text{ m}^2$

Volume $\quad \mathcal{V}_o = L_o A_o = (50)(19.2) = 960 \text{ mm}^3$

$\sigma_x = \dfrac{P}{A_o} = \dfrac{2.75 \times 10^3}{19.2 \times 10^{-6}} = 143.229 \times 10^6 \text{ Pa} \qquad \sigma_y = \sigma_z = 0$

$\varepsilon_x = \dfrac{1}{E}\left(\sigma_x - \nu\sigma_y - \nu\sigma_z\right) = \dfrac{\sigma_x}{E} = \dfrac{143.229 \times 10^6}{200 \times 10^9} = 716.15 \times 10^{-6}$

$\varepsilon_y = \varepsilon_z = -\nu\varepsilon_x = -(0.30)(716.15 \times 10^{-3}) = -214.84 \times 10^{-6}$

$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = 286.46 \times 10^{-6}$

$\Delta\mathcal{V} = \mathcal{V}_o e = (960)(286.46 \times 10^{-6}) = 0.275 \text{ mm}^3$

(b) From the solution to Problem 2.62

$\delta_x = 0.03581 \text{ mm} \qquad \delta_y = -0.002578 \text{ mm} \qquad \delta_z = -0.0003437 \text{ mm}$

The dimensions when under the 2.75 kN load are:

length $\quad L = L_o + \delta_x = 50 + 0.03581 \quad 50.03581 \text{ mm}$

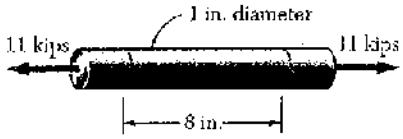
width $\quad w = w_o + \delta_y = 12 - 0.002578 = 11.997422 \text{ mm}$

thickness $\quad t = t_o + \delta_z = 1.6 - 0.0003437 = 1.5996563 \text{ mm}$

volume $\quad \mathcal{V} = Lwt = (50.03581)(11.997422)(1.5996563) = 960.275 \text{ mm}^3$

$\Delta\mathcal{V} = \mathcal{V} - \mathcal{V}_o = 960.275 - 960 = 0.275 \text{ mm}^3$

Problem 2.84



*2.84 Determine the dilatation e and the change in volume of the 8-in. length of the rod shown if (a) the rod is made of steel with $E = 29 \times 10^6$ psi and $\nu = 0.30$, (b) the rod is made of aluminum with $E = 10.6 \times 10^6$ psi and $\nu = 0.35$.

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(1)^2 = 0.785398 \text{ in}^2$$

$$P = 11 \times 10^3 \text{ lb}$$

Stresses: $\sigma_x = \dfrac{P}{A} = \dfrac{11 \times 10^3}{0.785398} = 14.0056 \times 10^3$ psi

$$\sigma_y = \sigma_z = 0$$

(a) Steel. $E = 29 \times 10^6$ psi $\quad \nu = 0.30$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \frac{\sigma_x}{E} = \frac{14.0056 \times 10^3}{29 \times 10^6} = 482.95 \times 10^{-6}$$

$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = -\frac{\nu\sigma_x}{E} = -\nu\varepsilon_x = -(0.30)(482.95 \times 10^{-6})$$
$$= -144.885 \times 10^{-6}$$

$$\varepsilon_z = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y + \sigma_z) = -\frac{\nu\sigma_x}{E} = \varepsilon_y = -144.885 \times 10^{-6}$$

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = 193.2 \times 10^{-6}$$ ◄

$$\Delta V = Ve = ALe = (0.785398)(8)(193.2 \times 10^{-6}) = 1.214 \times 10^{-3} \text{ in}^3$$ ◄

(b) Aluminum. $E = 10.6 \times 10^6$ psi $\quad \nu = 0.35$

$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{14.0056 \times 10^3}{10.6 \times 10^6} = 1.32128 \times 10^{-3}$$

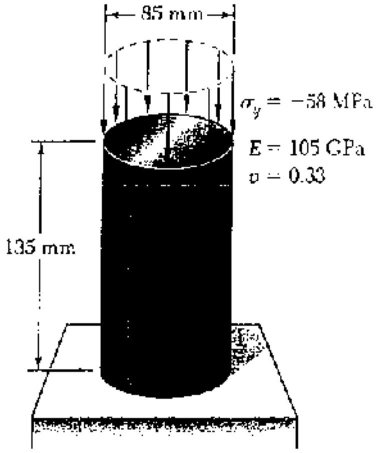
$$\varepsilon_y = -\nu\varepsilon_x = -(0.35)(1.32128 \times 10^{-3}) = -462.45 \times 10^{-6}$$

$$\varepsilon_z = \varepsilon_y = -462.45 \times 10^{-6}$$

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = 396 \times 10^{-6}$$ ◄

$$\Delta V = Ve = ALe = (0.785398)(8)(396 \times 10^{-6}) = 2.49 \times 10^{-3} \text{ in}^3$$ ◄

Problem 2.85



*2.85 (*a*) For the axial loading shown, determine the change in height and the change in volume of the brass cylinder shown. (*b*) Solve part *a*, assuming that the loading is hydrostatic with $\sigma_x = \sigma_y = \sigma_z = -70$ MPa.

$$h_o = 135 \text{ mm} = 0.135 \text{ m}$$

$$A_o = \frac{\pi}{4} d_o^2 = \frac{\pi}{4}(85)^2 = 5.6745 \times 10^3 \text{ mm}^2 = 5.6745 \times 10^{-3} \text{ m}^2$$

$$V_o = A_o h_o = 766.06 \times 10^3 \text{ mm}^3 = 766.06 \times 10^{-6} \text{ m}^3$$

(a) $\sigma_x = 0$, $\sigma_y = -58 \times 10^6$ Pa, $\sigma_z = 0$

$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = \frac{\sigma_y}{E}$$

$$= -\frac{58 \times 10^6}{105 \times 10^9} = -552.38 \times 10^{-6}$$

$$\Delta h = h_o \varepsilon_y = (135 \text{ mm})(-552.38 \times 10^{-6}) = -0.0746 \text{ mm}$$

$$e = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z) = \frac{(1-2\nu)\sigma_y}{E} = \frac{(0.34)(-58 \times 10^6)}{105 \times 10^9}$$

$$= -187.81 \times 10^{-6}$$

$$\Delta V = V_o e = (766.06 \times 10^3 \text{ mm}^3)(-187.81 \times 10^{-6}) = -143.9 \text{ mm}^3$$

(b) $\sigma_x = \sigma_y = \sigma_z = -70 \times 10^6$ Pa $\qquad \sigma_x + \sigma_y + \sigma_z = -210 \times 10^6$ Pa

$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = \frac{1-2\nu}{E}\sigma_y$$

$$= \frac{(0.34)(-70 \times 10^6)}{105 \times 10^9} = -226.67 \times 10^{-6}$$

$$\Delta h = h_o \varepsilon_y = (135 \text{ mm})(-226.67 \times 10^{-6}) = -0.0306 \text{ mm}$$

$$e = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z) = \frac{(0.34)(-210 \times 10^6)}{105 \times 10^9} = -680 \times 10^{-6}$$

$$\Delta V = V_o e = (766.06 \times 10^3 \text{ mm}^3)(-680 \times 10^{-6}) = -521 \text{ mm}^3$$

Problem 2.86

*2.86 A 6-in. diameter solid steel sphere is lowered into the ocean to a point where the pressure is 7.1 ksi (about 3 miles below the surface). Knowing that $E = 29 \times 10^6$ psi and $\nu = 0.30$, determine (*a*) the decrease in diameter of the sphere, (*b*) the decrease in volume of the sphere, (*c*) the percent increase in the density of the sphere.

For a solid sphere $\quad \mathcal{V}_0 = \frac{\pi}{6} d_0^3 = \frac{\pi}{6}(6.00)^3 = 113.097 \text{ in}^3$

$$\sigma_x = \sigma_y = \sigma_z = -p = -7.1 \times 10^3 \text{ psi}$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = -\frac{(1-2\nu)p}{E} = -\frac{(0.4)(7.1\times10^3)}{29\times10^6} = -97.93\times10^{-6}$$

Likewise $\quad \varepsilon_y = \varepsilon_z = -97.93\times10^{-6}$

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = -293.79\times10^{-6}$$

(a) $-\Delta d = -d_0\varepsilon_x = -(6.00)(97.93\times10^{-6}) = 588\times10^{-6}$ in. ◄

(b) $-\Delta\mathcal{V} = -\mathcal{V}_0 e = -(113.097)(-293.79\times10^{-6}) = 33.2\times10^{-3} \text{ in}^3$ ◄

(c) Let m = mass of sphere. $\qquad m$ = constant.

$$m = \rho_0\mathcal{V}_0 = \rho\mathcal{V} = \rho\mathcal{V}_0(1+e)$$

$$\frac{\rho-\rho_0}{\rho_0} = \frac{\rho}{\rho_0} - 1 = \frac{m}{\mathcal{V}_0(1+e)}\cdot\frac{\mathcal{V}_0}{m} - 1 = \frac{1}{1+e} - 1$$

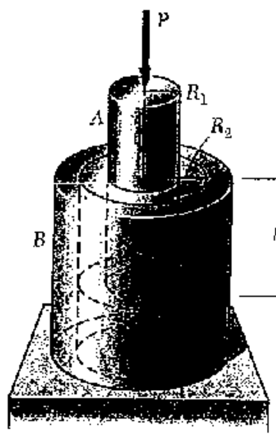
$$= (1 - e + e^2 - e^3 + \dots) - 1 = -e + e^2 - e^3 + \dots$$

$$\approx -e = 293.79\times10^{-6}$$

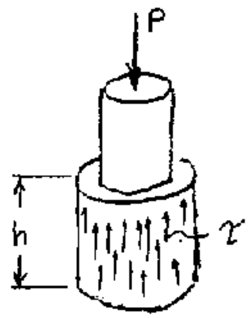
$$\frac{\rho-\rho_0}{\rho_0}\times100\% = (293.79\times10^{-6})(100\%) = 0.0294\%$$ ◄

Problem 2.87

*2.87 A vibration isolation support consists of a rod A of radius R_1 and a tube B of inner radius R_2 bonded to a 80-mm-long hollow rubber cylinder with a modulus of rigidity $G = 10.93$ MPa. Determine the required value of the ratio R_2/R_1 if if a 10-kN force $\mathbf{P}$ is to cause a 2-mm deflection of rod A.



Let r be a radial coordinate. Over the hollow rubber cylinder $R_1 \le r \le R_2$

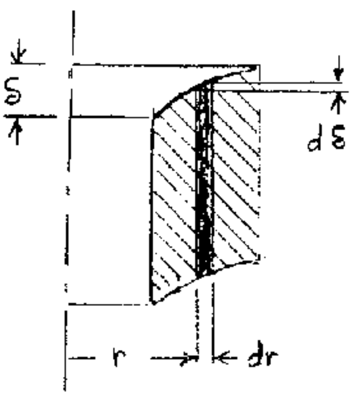


Shearing stress τ acting on a cylindrical surface of radius r is

$$\tau = \frac{P}{A} = \frac{P}{2\pi r h}$$

The shearing strain is

$$\gamma = \frac{\tau}{G} = \frac{P}{2\pi G h r}$$



Shearing deformation over radial length dr

$$\frac{d\delta}{dr} = \gamma$$

$$d\delta = \gamma\, dr = \frac{P}{2\pi G h}\,\frac{dr}{r}$$

Total deformation

$$\delta = \int_{R_1}^{R_2} d\delta = \frac{P}{2\pi G h}\int_{R_1}^{R_2}\frac{dr}{r}$$

$$= \frac{P}{2\pi G h}\ln r\Big|_{R_1}^{R_2} = \frac{P}{2\pi G h}(\ln R_2 - \ln R_1)$$

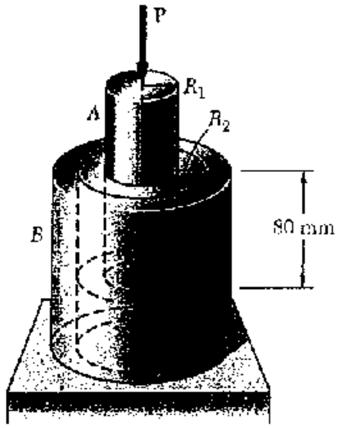
$$= \frac{P}{2\pi G h}\ln\frac{R_2}{R_1}$$

$$\ln\frac{R_2}{R_1} = \frac{2\pi G h \delta}{P} = \frac{(2\pi)(10.93\times10^6)(0.080)(0.002)}{10\times10^3} = 1.0988$$

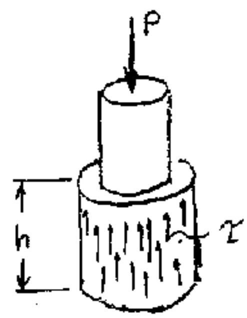
$$\frac{R_2}{R_1} = \exp(1.0988) = 3.00$$

Problem 2.88

*2.88 A vibration isolation support consists of a rod A of radius $R_1 = 10$ mm and a tube B of inner radius $R_2 = 25$ mm bonded to an 80-mm-long hollow rubber cylinder with a modulus of rigidity $G = 12$ MPa. Determine the largest allowable force **P** which may be applied to rod A if its deflection is not to exceed 2.50 mm.



Let r be a radial coordinate. Over the hollow rubber cylinder $R_1 \le r \le R_2$

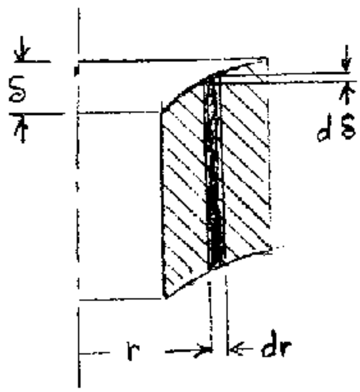


Shearing stress τ acting on a cylindrical surface of radius r is

$$\tau = \frac{P}{A} = \frac{P}{2\pi r h}$$

The shearing strain is

$$\gamma = \frac{\tau}{G} = \frac{P}{2\pi G h r}$$



Shearing deformation over radial length dr

$$\frac{d\delta}{dr} = \gamma$$

$$d\delta = \gamma\, dr = \frac{P}{2\pi G h}\frac{dr}{r}$$

Total deformation

$$\delta = \int_{R_1}^{R_2} d\delta = \frac{P}{2\pi G h}\int_{R_1}^{R_2}\frac{dr}{r}$$

$$= \frac{P}{2\pi G h}\ln r \Big|_{R_1}^{R_2} = \frac{P}{2\pi G h}(\ln R_2 - \ln R_1)$$

$$= \frac{P}{2\pi G h}\ln\frac{R_2}{R_1} \quad \text{or} \quad P = \frac{2\pi G h \delta}{\ln(R_2/R_1)}$$

Data: $R_1 = 10 \text{ mm} = 0.010 \text{ m}$, $R_2 = 25 \text{ mm} = 0.025 \text{ m}$, $h = 80 \text{ mm} = 0.080 \text{ m}$

$G = 12 \times 10^6$ Pa $\qquad \delta = 2.50 \times 10^{-3}$ m

$$P = \frac{(2\pi)(12\times10^6)(0.080)(2.50\times10^{-3})}{\ln(0.025/0.010)} = 16.46 \times 10^3 \text{ N} \qquad 16.46 \text{ kN}$$

Problem 2.89

***2.89** The material constants E, G, k, and ν are related by Eqs. (2.33) and (2.43). Show that any one of these constants may be expressed in terms of any other two constants. For example, show that (*a*) $k = GE/(9G - 3E)$ and (*b*) $\nu = (3k - 2G)/(6k + 2G)$.

$$k = \frac{E}{3(1-2\nu)} \quad \text{and} \quad G = \frac{E}{2(1+\nu)}$$

(a) $1 + \nu = \frac{E}{2G}$ or $\nu = \frac{E}{2G} - 1$

$$k = \frac{E}{3\left[1 - 2\left(\frac{E}{2G} - 1\right)\right]} = \frac{2EG}{3[2G - 2E + 4G]} = \frac{2EG}{18G - 6E}$$

$$= \frac{EG}{9G - 6E}$$

(b) $\frac{k}{G} = \frac{2(1+\nu)}{3(1-2\nu)}$

$$3k - 6k\nu = 2G + 2G\nu$$

$$3k - 2G = 2G + 6k$$

$$\nu = \frac{3k - 2G}{6k + 2G}$$

Problem 2.90

***2.90** Show that for any given material, the ratio G/E of the modulus of rigidity over the modulus of elasticity is always less than $\frac{1}{2}$ but more than $\frac{1}{3}$ [*Hint*: Refer to Eq. (2.43) and to Sec. 2.13.]

$$G = \frac{E}{2(1+\nu)} \quad \text{or} \quad \frac{E}{G} = 2(1+\nu)$$

Assume $\nu \geq 0$ for almost all materials and $\nu < \frac{1}{2}$ for a positive bulk modulus

Applying the bounds $\quad 2 \leq \frac{E}{G} < 2(1+\frac{1}{2}) = 3$

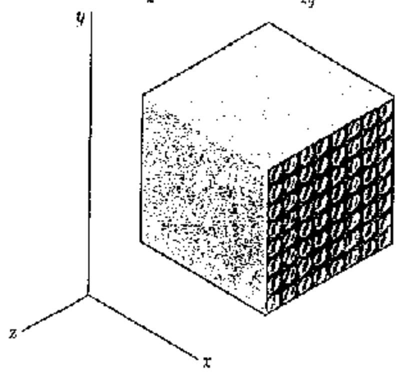
Taking the reciprocals $\quad \frac{1}{2} \geq \frac{G}{E} > \frac{1}{3}$

or $\quad \frac{1}{3} \leq \frac{G}{E} \leq \frac{1}{2}$

Problem 2.91

*2.91 A composite cube with 40-mm sides and the properties shown is made with glass polymer fibers aligned in the x direction. The cube is constrained against deformations in the y and z directions and is subjected to a tensile load of 65 kN in the x direction. Determine (a) the change in the length of the cube in the x direction, (b) the stresses σ_x, σ_y, and σ_z.

$E_x = 50$ GPa $\quad \nu_{xz} = 0.254$
$E_y = 15.2$ GPa $\quad \nu_{xy} = 0.254$
$E_z = 15.2$ GPa $\quad \nu_{zy} = 0.428$



Stress-to-strain equations are

$$\varepsilon_x = \frac{\sigma_x}{E_x} - \frac{\nu_{yx}\sigma_y}{E_y} - \frac{\nu_{zx}\sigma_z}{E_z} \quad (1) \qquad \frac{\nu_{xy}}{E_x} = \frac{\nu_{yx}}{E_y} \quad (4)$$

$$\varepsilon_y = -\frac{\nu_{xy}\sigma_x}{E_x} + \frac{\sigma_y}{E_y} - \frac{\nu_{zy}\sigma_z}{E_z} \quad (2) \qquad \frac{\nu_{yz}}{E_y} = \frac{\nu_{zy}}{E_z} \quad (5)$$

$$\varepsilon_z = -\frac{\nu_{xz}\sigma_x}{E_x} - \frac{\nu_{yz}\sigma_y}{E_y} + \frac{\sigma_z}{E_z} \quad (3) \qquad \frac{\nu_{zx}}{E_z} = \frac{\nu_{xz}}{E_x} \quad (6)$$

The constraint conditions are $\varepsilon_y = 0$ and $\varepsilon_z = 0$.

Using (2) and (3) with the constraint conditions gives

$$\frac{1}{E_y}\sigma_y - \frac{\nu_{zy}}{E_z}\sigma_z = \frac{\nu_{xy}}{E_x}\sigma_x \quad (7)$$

$$-\frac{\nu_{yz}}{E_y}\sigma_y + \frac{1}{E_z}\sigma_z = \frac{\nu_{xz}}{E_x}\sigma_x \quad (8)$$

$$\frac{1}{15.2}\sigma_y - \frac{0.428}{15.2}\sigma_z = \frac{0.254}{50}\sigma_x \quad \text{or} \quad \sigma_y - 0.428\sigma_z = 0.077216\,\sigma_x$$

$$-\frac{0.428}{15.2}\sigma_y + \frac{1}{15.2}\sigma_z = \frac{0.254}{50}\sigma_x \quad \text{or} \quad -0.428\sigma_y + \sigma_z = 0.077216\,\sigma_x$$

Solving simultaneously $\quad \sigma_y = \sigma_z = 0.134993\,\sigma_x$

Using (4) and (5) in (1) $\quad \varepsilon_x = \frac{1}{E_x}\sigma_x - \frac{\nu_{xy}}{E_x}\sigma_y - \frac{\nu_{xz}}{E}\sigma_z$

$$\varepsilon_x = \frac{1}{E_x}\left[1 - (0.254)(0.134993) - (0.254)(0.134993)\right]\sigma_x$$

$$= \frac{0.93142\,\sigma_x}{E_x}$$

$$A = (40)(40) = 1600 \text{ mm}^2 = 1600 \times 10^{-6} \text{ m}^2$$

$$\sigma_x = \frac{P}{A} = \frac{65 \times 10^3}{1600 \times 10^{-6}} = 40.625 \times 10^6 \text{ Pa}$$

continued

Problem 2.91 continued

$$\varepsilon_x = \frac{(0.93142)(40.625\times10^3)}{50\times10^9} = 756.78\times10^{-6}$$

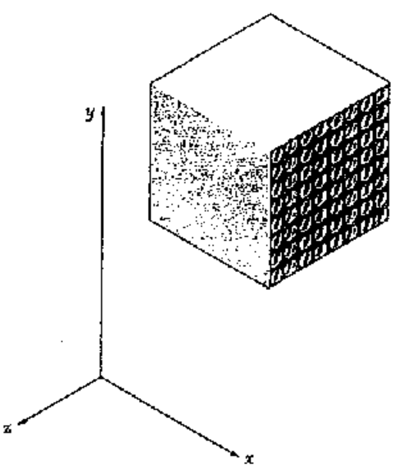
(a) $\delta_x = L_x\varepsilon_x = (40\text{ mm})(756.78\times10^{-6}) = 0.0303\text{ mm}$ ◄

(b) $\sigma_x = 40.625\times10^6\text{ Pa} = 40.6\text{ MPa}$ ◄

$\sigma_y = \sigma_z = (0.134993)(40.625\times10^6) = 5.48\times10^6\text{ Pa}$
$= 5.48\text{ MPa}$ ◄

Problem 2.92

***2.92** The composite cube of Prob. 2.91 is constrained against deformation in the z direction and elongated in the x direction by 0.035 mm due to a tensile load in the x direction. Determine (*a*) the stresses σ_x, σ_y, and σ_z, (*b*) the change in the dimension in the y direction.



$E_x = 50$ GPa $\quad \nu_{xz} = 0.254$
$E_y = 15.2$ GPa $\quad \nu_{xy} = 0.254$
$E_z = 15.2$ GPa $\quad \nu_{zy} = 0.428$

$$\varepsilon_x = \frac{\sigma_x}{E_x} - \frac{\nu_{yx}\sigma_y}{E_y} - \frac{\nu_{zx}\sigma_z}{E_z} \quad (1) \qquad \frac{\nu_{xy}}{E_x} = \frac{\nu_{yx}}{E_y} \quad (4)$$

$$\varepsilon_y = -\frac{\nu_{xy}\sigma_x}{E_x} + \frac{\sigma_y}{E_y} - \frac{\nu_{zy}\sigma_z}{E_z} \quad (2) \qquad \frac{\nu_{yz}}{E_y} = \frac{\nu_{zy}}{E_z} \quad (5)$$

$$\varepsilon_z = -\frac{\nu_{xz}\sigma_x}{E_x} - \frac{\nu_{yz}\sigma_y}{E_y} + \frac{\sigma_z}{E_z} \quad (3) \qquad \frac{\nu_{zx}}{E_z} = \frac{\nu_{xz}}{E_x} \quad (6)$$

Constraint condition $\quad \varepsilon_z = 0$

Load condition $\quad \sigma_y = 0$

From equation (3) $\quad 0 = -\frac{\nu_{xz}}{E_x}\sigma_x + \frac{1}{E_z}\sigma_z$

$$\sigma_z = \frac{\nu_{xz}E_z}{E_x}\sigma_x = \frac{(0.254)(15.2)}{50} = 0.077216\,\sigma_x$$

continued

Problem 2.92 continued

From equation (1) with $\sigma_y = 0$

$$\varepsilon_x = \frac{1}{E_x}\sigma_x - \frac{\nu_{zx}}{E_z}\sigma_z = \frac{1}{E_x}\sigma_x - \frac{\nu_{xz}}{E_x}\sigma_z$$

$$= \frac{1}{E_x}\left[\sigma_x - 0.254\,\sigma_z\right] = \frac{1}{E_x}\left[1 - (0.254)(0.077216)\right]\sigma_x$$

$$= \frac{0.98039}{E_x}\sigma_x$$

$$\sigma_x = \frac{E_x\,\varepsilon_x}{0.98039}$$

But $\varepsilon_x = \frac{\delta_x}{L_x} = \frac{0.035 \text{ mm}}{40 \text{ mm}} = 875 \times 10^{-6}$

(a) $\sigma_x = \frac{(50\times 10^9)(875\times 10^{-6})}{0.98039} = 44.625\times 10^3 \text{ Pa} = 44.6 \text{ MPa}$ ◄

$\sigma_y = 0$ ◄

$\sigma_z = (0.077216)(44.625\times 10^6) = 3.446\times 10^6 \text{ Pa} = 3.45 \text{ MPa}$ ◄

From (2) $\quad \varepsilon_y = -\frac{\nu_{xy}}{E_x}\sigma_x + \frac{1}{E_y}\sigma_y - \frac{\nu_{zy}}{E_z}\sigma_z$

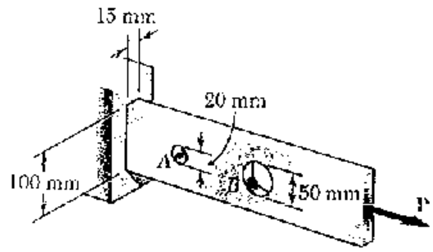
$$= -\frac{(0.254)(44.625\times 10^6)}{50\times 10^9} + 0 - \frac{(0.428)(3.446\times 10^6)}{15.2\times 10^9}$$

$$= -323.73\times 10^{-6}$$

$\delta_y = L_y\,\varepsilon_y = (40 \text{ mm})(-323.73\times 10^{-6}) = -0.0129 \text{ mm}$ ◄

Problem 2.93

2.93 Knowing that $\sigma_{all} = 120$ MPa, determine the maximum allowable value of the centric axial load **P**.



At hole A $\quad r = \frac{1}{2}(20) = 10$ mm

$d = D - 2r = 100 - 20 = 80$ mm

$A_{net} = dt = (80)(15) = 1200\ \text{mm}^2 = 1.20 \times 10^{-3}\ \text{m}^2$

$\frac{r}{d} = \frac{10}{80} = 0.125$

From Fig. 2.64a, $K = 2.65$

$$\sigma_{max} = \frac{KP}{A_{net}} \qquad P = \frac{A_{net}\sigma_{max}}{K} = \frac{(1.20 \times 10^{-3})(120 \times 10^{6})}{2.65} = 54.3 \times 10^3\ N$$

At hole B $\quad r = \frac{1}{2}(50) = 25$ mm $\qquad d = 100 - 50 = 50$ mm

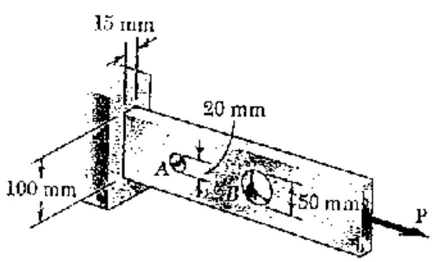
$A_{net} = (50)(15) = 750\ \text{mm}^2 = 750 \times 10^{-6}\ \text{m}^2, \qquad \frac{r}{d} = \frac{25}{50} = 0.50, \quad K = 2.16$

$$P = \frac{(750 \times 10^{-6})(120 \times 10^{6})}{2.16} = 41.7 \times 10^3\ N$$

Allowable value of P is the smaller $\qquad P = 41.7 \times 10^3\ N = 41.7\ kN$ ◄

Problem 2.94

2.94 Two holes have been drilled through a long steel bar that is subjected to a centric axial load as shown. For $P = 32$ kN, determine the maximum value of the stress (*a*) at *A*, (*b*) at *B*.



(a) At hole A $\quad r = \frac{1}{2}(20) = 10$ mm

$d = D - 2r = 100 - 20 = 80$ mm

$A_{net} = dt = (80)(15) = 1200\ \text{mm}^2 = 1.20 \times 10^{-3}\ \text{m}^2$

$\frac{r}{d} = \frac{10}{80} = 0.125$

From Fig. 2.64a, $K = 2.65$

$$\sigma_{max} = \frac{KP}{A_{net}} = \frac{(2.65)(32 \times 10^3)}{1.20 \times 10^{-3}} = 70.7 \times 10^6\ Pa$$

70.7 MPa ◄

(b) At hole B $\quad r = \frac{1}{2}(50) = 25$ mm, $\quad d = 100 - 50 = 50$ mm

$A_{net} = (50)(15) = 750\ \text{mm}^2 = 750 \times 10^{-6}\ \text{m}^2$

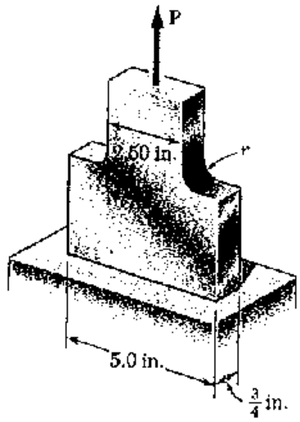
$\frac{r}{d} = \frac{25}{50} = 0.50 \qquad K = 2.16$

$$\sigma_{max} = \frac{KP}{A_{net}} = \frac{(2.16)(32 \times 10^3)}{750 \times 10^{-6}} = 92.2 \times 10^6\ Pa$$

92.2 MPa ◄

Problem 2.95

2.95 Knowing that $P = 10$ kips, determine the maximum stress when (*a*) $r = 0.50$ in., (*b*) $r = 0.625$ in.



$P = 10 \times 10^3$ lb $\quad D = 5.0$ in. $\quad d = 2.50$ in.

$$\frac{D}{d} = \frac{5.0}{2.50} = 2.00$$

$$A_{min} = dt = (2.50)(\tfrac{3}{4}) = 1.875 \text{ in}^2$$

(a) $r = 0.50$ in. $\quad \dfrac{r}{d} = \dfrac{0.50}{2.50} = 0.20$

From Fig. 2.64 b $\quad K = 1.94$

$$\sigma_{max} = \frac{KP}{A_{min}} = \frac{(1.94)(10 \times 10^3)}{1.875} = 10.35 \times 10^3 \text{ psi}$$

10.35 ksi ◄

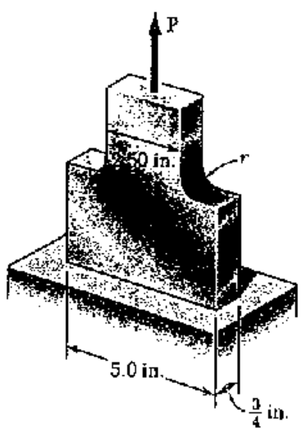
(b) $r = 0.625 \quad \dfrac{r}{d} = \dfrac{0.625}{2.50} = 0.25 \quad K = 1.82$

$$\sigma_{max} = \frac{KP}{A_{min}} = \frac{(1.82)(10 \times 10^3)}{1.875} = 9.71 \times 10^3 \text{ psi}$$

9.71 ksi ◄

Problem 2.96

2.96 Knowing that, for the plate shown, the allowable stress is 16 ksi, determine the maximum allowable value of P when (*a*) $r = \frac{3}{8}$ in., (*b*) $r = \frac{3}{4}$ in.



$\sigma_{max} = 16 \times 10^3$ psi

$D = 5.0$ in. $\quad d = 2.50$ in. $\quad \dfrac{D}{d} = 2.00$

$$A_{min} = dt = (2.50)(\tfrac{3}{4}) = 1.875 \text{ in}^2$$

(a) $r = \frac{3}{8}$ in. $= 0.375$ in. $\quad \dfrac{r}{d} = \dfrac{0.375}{2.50} = 0.15$

From Fig. 2.64 b $\quad K = 2.14$

$$\sigma_{max} = \frac{KP}{A_{min}}$$

$$P = \frac{A_{min}\sigma_{max}}{K} = \frac{(1.875)(16 \times 10^3)}{2.14} = 14.02 \times 10^3 \text{ lb}$$

14.02 kips ◄

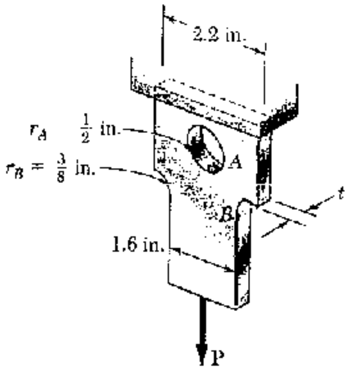
(b) $r = \frac{3}{4}$ in. $= 0.75$ in. $\quad \dfrac{r}{d} = \dfrac{0.75}{2.50} = 0.30 \quad K = 1.75$

$$P = \frac{A_{min}\sigma_{max}}{K} = \frac{(1.875)(16 \times 10^3)}{1.75} = 17.14 \times 10^3 \text{ lb}$$

17.14 kips ◄

Problem 2.97

2.97 For $P = 8.5$ kips, determine the minimum plate thickness t required if the allowable stress is 18 ksi.



At the hole: $r_A = \frac{1}{2}$ in $\quad d_A = 2.2 - 1.0 = 1.2$ in

$$\frac{r_A}{d_A} = \frac{1/2}{1.2} = 0.417$$

From Fig 2.64 a $\quad K = 2.22$

$$\sigma_{max} = \frac{KP}{A_{net}} = \frac{KP}{d_A t} \quad \therefore \quad t = \frac{KP}{d_A \sigma_{max}}$$

$$t = \frac{(2.22)(8.5)}{(1.2)(18)} = 0.874 \text{ in.}$$

At the fillet $\quad D = 2.2$ in, $\quad d_B = 1.6$ in $\quad \frac{D}{d_B} = \frac{2.2}{1.6} = 1.375$

$$r_B = \frac{3}{8} = 0.375 \text{ in} \qquad \frac{r_B}{d_B} = \frac{0.375}{1.6} = 0.2344$$

From Fig 2.64 b $\quad K = 1.70 \qquad \sigma_{max} = \frac{KP}{A_{min}} = \frac{KP}{d_B t}$

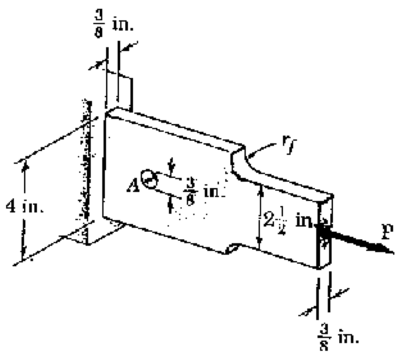
$$t = \frac{KP}{d_B \sigma_{max}} = \frac{(1.70)(8.5)}{(1.6)(18)} = 0.502 \text{ in.}$$

The larger value is the required minimum plate thickness

$t = 0.874$ in.

Problem 2.98

2.98 Knowing that the hole has a diameter of $\frac{3}{8}$-in., determine (*a*) the radius r_f of the fillets for which the same maximum stress occurs at the hole *A* and at the fillets, (*b*) the corresponding maximum allowable load **P** if the allowable stress is 15 ksi.



For the circular hole $\quad r = (\frac{1}{2})(\frac{3}{8}) = 0.1875$ in

$d = 4 - \frac{3}{8} = 3.625$ in $\qquad \frac{r}{d} = \frac{0.1875}{3.625} = 0.0517$

$A_{net} = d\,t = (3.625)(\frac{3}{8}) = 1.3594 \text{ in}^2$

From Fig 2.64 a $\quad K_{hole} = 2.82$

$$\sigma_{max} = \frac{K_{hole} P}{A_{net}}$$

(b) $\quad P = \frac{A_{net}\sigma_{max}}{K_{hole}} = \frac{(1.3594)(15)}{2.82} = 7.23$ kips

(a) For fillet $\quad D = 4$ in, $\quad d = 2.5$ in $\qquad \frac{D}{d} = \frac{4.0}{2.5} = 1.60$

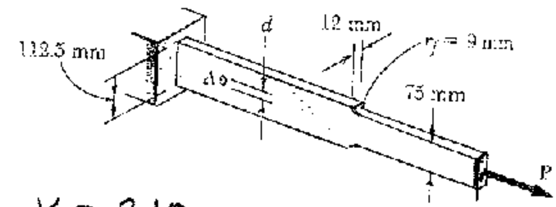
$A_{min} = d\,t = (2.5)(\frac{3}{8}) = 0.9375 \text{ in}^2$

$$\sigma_{max} = \frac{K_{fillet} P}{A_{min}} \quad \therefore \quad K_{fillet} = \frac{A_{min}\sigma_{max}}{P} = \frac{(0.9375)(15)}{7.23} = 1.945$$

From Fig 2.64 b $\quad \frac{r_f}{d} \approx 0.17 \quad \therefore \quad r_f \approx 0.17d = (0.17)(2.5) = 0.425$ in

Problem 2.99

2.99 A hole is to be drilled in the plate at A. The diameters of the bits available to drill the hole range from 12 to 24 mm in 3-mm increments. (a) Determine the diameter d of the largest bit that can be used if the allowable load at the hole is to exceed that at the fillets. (b) If the allowable stress in the plate is 145 MPa, what is the corresponding allowable load **P**?



At the fillets, $r = 9$ mm $d = 75$ mm

$D = 112.5$ mm $\frac{D}{d} = \frac{112.5}{75} = 1.5$

$\frac{r}{d} = \frac{9}{75} = 0.12$ From Fig 2.64 b $K = 2.10$

$A_{min} = (75)(12) = 900 \text{ mm}^2 = 900 \text{ mm}^2 = 900 \times 10^{-6} \text{ m}^2$

$$\sigma_{max} = K\frac{P_{all}}{A_{min}} = \sigma_{all} \therefore P_{all} = \frac{A_{min}\sigma_{all}}{K} = \frac{(900\times10^{-6})(145\times10^{6})}{2.10}$$

$$= 62.1 \times 10^3 \text{ N} = 62.1 \text{ kN}$$

At the hole: $A_{net} = (D - 2r)t$, $\frac{r}{d} = \frac{r}{D-2r}$

where $D = 112.5$ mm r = radius of circle $t = 12$ mm

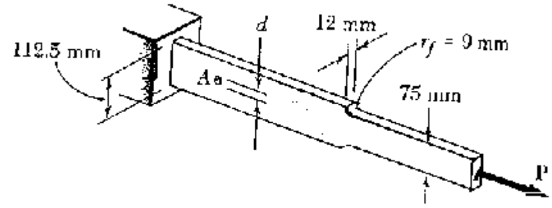
K is taken from Fig 2.64 a

$$\sigma_{max} = K\frac{P}{A_{net}} = \sigma_{all} \therefore P_{all} = \frac{A_{net}\sigma_{all}}{K}$$

Hole diam	r	$d = D-2r$	r/d	K:	A_{net} (m^2)	P_{all}, (N)
12 mm	6.0 mm	100.5	0.060	2.80	1206×10^{-6}	62.5×10^3
15 mm	7.5 mm	97.5 mm	0.077	2.75	1170×10^{-6}	61.7×10^3
18 mm	9.0 mm	94.5 mm	0.095	2.71	1134×10^{-6}	60.7×10^3
21 mm	10.5 mm	91.5 mm	0.115	2.67	1098×10^{-6}	59.6×10^3
24 mm	12.0 mm	88.5 mm	0.136	2.62	1062×10^{-6}	58.8×10^3

Problem 2.100

2.100 (*a*) For $P = 58$ kN and $d = 12$ mm, determine the maximum stress in the plate shown. (*b*) Solve part *a*, assuming that the hole at *A* is not drilled.



Maximum stress at hole.

Use Fig. 2.64 a for values of K

$$\frac{r}{d} = \frac{6}{112.5 - 12} = 0.0597, \qquad K = 2.80$$

$$A_{net} = (12)(112.5 - 12) = 1206 \text{ mm}^2 = 1206 \times 10^{-6} \text{ m}^2$$

$$\sigma_{max} = K\frac{P}{A_{net}} = \frac{(2.80)(58 \times 10^3)}{1206 \times 10^{-6}} = 134.7 \times 10^6 \text{ Pa}$$

Maximum stress at fillets

Use Fig. 2.64 b

$$\frac{r}{d} = \frac{9}{75} = 0.12, \qquad \frac{D}{d} = \frac{112.5}{75} = 1.50, \qquad K = 2.10$$

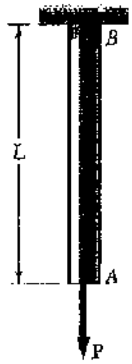
$$A_{min} = (12)(75) = 900 \text{ mm}^2 = 900 \times 10^{-6} \text{ m}^2$$

$$\sigma_{max} = K\frac{P}{A_{min}} = \frac{(2.10)(58 \times 10^3)}{900 \times 10^{-6}} = 135.3 \times 10^6 \text{ Pa}$$

(a) With hole and fillets $\qquad \sigma_{max} = 134.7$ MPa ◀

(b) Without hole $\qquad \sigma_{max} = 135.3$ MPa ◀

Problem 2.101



2.101 The cylindrical rod AB has a length $L = 5$ ft and a 0.75-in. diameter; it is ; it is made of mild steel that is assumed to be elastoplastic with $E = 29 \times 10^6$ psi and $\sigma_Y = 36$ ksi. A force **P** is applied to the bar and then removed to give it a permanent set δ_p. Determine the maximum value of the force **P** and the maximum amount δ_m by which the bar should be stretched if the desired value of δ_p is (*a*) 0.1 in., (*b*) 0.2 in.

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.75)^2 = 0.44179 \text{ in}^2 \qquad L = 5 \text{ ft} = 60 \text{ in.}$$

$$\delta_Y = L\varepsilon_Y = \frac{L\sigma_Y}{E} = \frac{(60)(36\times10^3)}{29\times10^3} = 0.074483 \text{ in.}$$

When δ_m exceeds δ_Y, thus causing permanent stretch δ_p, the maximum force is

$$P_m = A\sigma_Y = (0.44179)(36\times10^3) = 15.9043\times10^3 \text{ lb.}$$

15.90 kips

$$\delta_p = \delta_m - \delta' = \delta_m - \delta_Y \quad \text{so that} \quad \delta_m = \delta_p + \delta_Y$$

(a) $\delta_p = 0.1$ in. $\qquad \delta_m = 0.1 + 0.074483 = 0.1745$ in.

(b) $\delta_p = 0.2$ in. $\qquad \delta_m = 0.2 + 0.074483 = 0.274$ in.

Problem 2.102

2.102 The cylindrical rod AB has a length $L = 6$ ft and a 1.25-in. diameter; it is made of a mild steel that is assumed to be elastoplastic with $E = 29 \times 10^6$ psi and $\sigma_Y = 36$ ksi. A force **P** is applied to the bar until end A has moved down by an amount δ_m. Determine the maximum value of the force **P** and the permanent set of the bar after the force has been removed, knowing that (*a*) $\delta_m = 0.125$ in., (*b*) $\delta_m = 0.250$ in.

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(1.25)^2 = 1.22718 \text{ in.}^2 \qquad L = 6 \text{ ft} = 72 \text{ in.}$$

$$\delta_Y = L\varepsilon_Y = \frac{L\sigma_Y}{E} = \frac{(72)(36\times10^3)}{29\times10^6} = 0.089379 \text{ in.}$$

If $\delta_m \geq \delta_Y$, $\quad P_m = A\sigma_Y = (1.22718)(36\times10^3)$

$$= 44.179\times10^3 \text{ lb.} = 44.2 \text{ kips}$$

(a) $\delta_m = 0.125$ in. $> \delta_Y$ so that $P_m = 44.2$ kips

$$\delta' = \frac{P_m L}{AE} = \frac{\sigma_Y L}{E} = \delta_Y = 0.089379$$

$$\delta_p = \delta_m - \delta' = 0.125 - 0.089379 = 0.0356 \text{ in.}$$

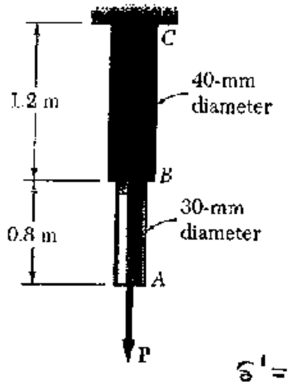
(b) $\delta_m = 0.250$ in. $> \delta_Y$ so that $P_m = 44.2$ kips

$$\delta' = \delta_Y$$

$$\delta_p = \delta_m - \delta' = 0.250 - 0.089379 = 0.1606 \text{ in.}$$

Problem 2.103

2.103 Rod ABC consists of two cylindrical portions AB and BC; it is made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. A force **P** is applied to the rod and then removed to give it a permanent set $\delta_p = 2$ mm. Determine the maximum value of the force **P** and the maximum amount δ_m by which the rod should be stretched to give it the desired permanent set.



$$A_{AB} = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4}(40)^2 = 1.25664 \times 10^3 \text{ mm}^2 = 1.25664 \times 10^{-3} \text{ m}^2$$

$$P_{max} = A_{min}\sigma_Y = (706.86 \times 10^{-6})(250 \times 10^6) = 176.715 \times 10^3 \text{ N}$$

176.7 kN ◄

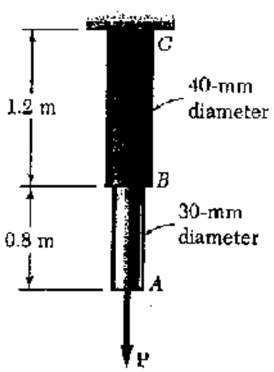
$$\delta' = \frac{P'L_{AB}}{EA_{AB}} + \frac{P'L_{BC}}{EA_{BC}} = \frac{(176.715 \times 10^3)(0.8)}{(200 \times 10^9)(706.86 \times 10^{-6})} + \frac{(176.715 \times 10^3)(1.2)}{(200 \times 10^9)(1.25664 \times 10^{-3})}$$

$$= 1.84375 \times 10^{-3} \text{ m} = 1.84375 \text{ mm}$$

$$\delta_p = \delta_m - \delta' \quad \text{or} \quad \delta_m = \delta_p + \delta' = 2 + 1.84375 = 3.84 \text{ mm}$$ ◄

Problem 2.104

2.104 Rod ABC consists of two cylindrical portions AB and BC; it is made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. A force **P** is applied to the rod until its end A has moved down by an amount $\delta_m = 5$ mm. Determine the maximum value of the force **P** and the permanent set of the rod after the force has been removed.



$$A_{AB} = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4}(40)^2 = 1.25664 \times 10^3 \text{ mm}^2 = 1.25644 \times 10^{-3} \text{ m}^2$$

$$P_{max} = A_{min}\sigma_Y = (706.86 \times 10^{-6})(250 \times 10^6) = 176.715 \times 10^3 \text{ N}$$

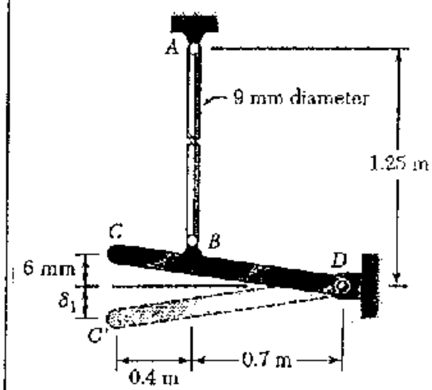
176.7 kN ◄

$$\delta' = \frac{P'L_{AB}}{EA_{AB}} + \frac{P'L_{BC}}{EA_{BC}} = \frac{(176.715 \times 10^3)(0.8)}{(200 \times 10^9)(706.68 \times 10^{-6})} + \frac{(176.715 \times 10^3)(1.2)}{(200 \times 10^9)(1.25664 \times 10^{-3})}$$

$$= 1.84375 \times 10^{-3} \text{ m} = 1.84375 \text{ mm}$$

$$\delta_p = \delta_m - \delta' = 5 - 1.84375 = 3.16 \text{ mm}$$ ◄

Problem 2.105



2.105 Rod *AB* is made of a mild steel that is assumed to be elastoplastic with E = 200 GPa and σ_Y = 345 MPa. After the rod has been attached to the rigid lever *CD*, it is found that end *C* is 6 mm too high. A vertical force **Q** is then applied at *C* until this point has moved to position *C'*. Determine the required magnitude of **Q** and the deflection δ_1 if the lever is to *snap* back to a horizontal position after **Q** is removed.

$A_{AB} = \frac{\pi}{4}(9)^2 = 63.617\ mm^2 = 63.617 \times 10^{-6}\ m^2$

Since rod AB is to be stretched permanently,

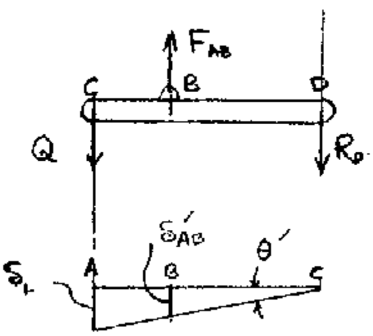
$(F_{AB})_{max} = A_{AB}\sigma_Y = (63.617\times10^{-6})(345\times10^6)$

$= 21.948\times10^3\ N$

$+\circlearrowleft \Sigma M_D = 0: \quad 1.1\,Q - 0.7\,F_{AB} = 0$

$Q_{max} = \frac{0.7}{1.1}(21.948\times10^3) = 13.967\times10^3\ N$

13.97 kN

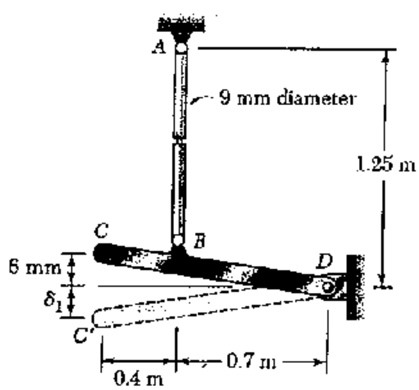


$\delta'_{AB} = \frac{(F_{AB})_{max} L_{AB}}{E A_{AB}} = \frac{(21.948\times10^3)(1.25)}{(200\times10^9)(63.617\times10^{-6})} = 2.15625\times10^{-3}\ m$

$\theta' = \frac{\delta'_{AB}}{0.7} = 3.0804\times10^{-3}\ rad$

$\delta_1 = 1.1\,\theta' = 3.39\times10^{-3}\ m$ 3.39 mm

Problem 2.106



2.106 Solve Prob. 2.105, assuming that the yield point of the mild steel is 250 MPa.

2.105 Rod *AB* is made of a mild steel that is assumed to be elastoplastic with E = 200 GPa and σ_Y = 345 MPa. After the rod has been attached to the rigid lever *CD*, it is found that end *C* is 6 mm too high. A vertical force **Q** is then applied at *C* until this point has moved to position *C'*. Determine the required magnitude of **Q** and the deflection δ_1 if the lever is to *snap* back to a horizontal position after **Q** is removed.

$A_{AB} = \frac{\pi}{4}(9)^2 = 63.617\ mm^2 = 63.617\times10^{-6}\ m^2$

Since rod AB is to be stretched permanently,

$(F_{AB})_{max} = A_{AB}\sigma_Y = (63.617\times10^{-6})(250\times10^6)$

$= 15.9043\times10^3\ N$

$+\circlearrowleft \Sigma M_D = 0 \quad 1.1\,Q - 0.7\,F_{AB} = 0$

$Q_{max} = \frac{0.7}{1.1}(15.9043\times10^3) = 10.12\times10^3\ N$

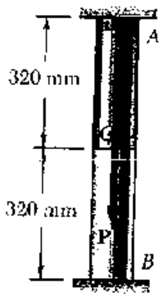
10.12 kN

$\delta'_{AB} = \frac{(F_{AB})_{max} L_{AB}}{E A_{AB}} = \frac{(15.9043\times10^3)(1.25)}{(200\times10^9)(63.617\times10^{-6})} = 1.5625\times10^{-3}\ m$

$\theta' = \frac{\delta'_{AB}}{0.7} = 2.2321\times10^{-3}\ rad$

$\delta_1 = 1.1\,\theta' = 2.46\times10^{-3}\ m$ 2.46 mm

Problem 2.107



2.107 Rod AB consists of two cylindrical portions AC and BC, each with a cross-sectional area of 2950 mm^2. Portion AC is made of a mild steel with $E = 200$ GPa and $\sigma_Y = 250$ MPa, and portion CB is made of a high-strength steel with $E = 200$ GPa and $\sigma_Y = 345$ MPa. A load P is applied at C as shown. Assuming both steels to be elastoplastic, determine (*a*) the maximum deflection of C if P is gradually increased from zero to 1625 kN and then reduced back to zero, (*b*) the maximum stress in each portion of the rod, (*c*) the permanent deflection of C.

Displacement at C to cause yielding of AC

$$\delta_{C,Y} = L_{AC}\varepsilon_{Y,AC} = \frac{L_{AC}\sigma_{Y,AC}}{E} = \frac{(0.320)(250\times10^6)}{200\times10^9} = 0.400\times10^{-3}\ m$$

Corresponding force $\quad F_{AC} = A\sigma_{Y,AC} = (2950\times10^{-6})(250\times10^6) = 737.5\times10^3\ N$

$$F_{CB} = -\frac{EA\delta_C}{L_{CB}} = -\frac{(200\times10^9)(2950\times10^{-6})(0.400\times10^{-3})}{0.320} = -737.5\times10^3\ N$$

F_{AC}

C

$F_{CB}+P$

For equilibrium of element at C,

$$F_{AC} - (F_{CB} + P_Y) = 0 \qquad P_Y = F_{AC} - F_{CB} = 1475\times10^3\ N$$

Since applied load $P = 1625\times10^3\ N > 1475\times10^3\ N$, portion AC yields.

$$F_{CB} = F_{AC} - P = 737.5\times10^3 - 1625\times10^3\ N = -887.5\times10^3\ N$$

(a) $$\delta_C = -\frac{F_{CB}L_{CB}}{EA} = \frac{(887.5\times10^3)(0.320)}{(200\times10^9)(2950\times10^{-6})} = 0.48136\times10^{-3}\ m = 0.481\ mm$$

(b) Maximum stresses. $\quad \sigma_{AC} = \sigma_{Y,AC} = 250\ MPa$

$$\sigma_{BC} = \frac{F_{BC}}{A} = -\frac{887.5\times10^3}{2950\times10^{-6}} = -300.81\times10^6\ Pa = -301\ MPa$$

(c) Deflection and forces for unloading.

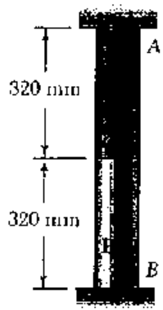
$$\delta' = \frac{P'_{AC}L_{AC}}{EA} = -\frac{P'_{CB}L_{CB}}{EA} \quad \therefore\ P'_{CB} = -P'_{AC}\frac{L_{AC}}{L_{CB}} = -P'_{AC}$$

$$P' = 1625\times10^3 = P'_{AC} - P'_{CB} = 2P'_{AC} \qquad P'_{AC} = 812.5\times10^3\ N$$

$$\delta' = \frac{(812.5\times10^3)(0.320)}{(200\times10^9)(2950\times10^{-6})} = 0.44068\times10^{-3}\ m$$

$$\delta_p = \delta_m - \delta' = 0.48136\times10^{-3} - 0.44068\times10^{-3} = 0.04068\times10^{-3}\ m = 0.0407\ mm\ \downarrow$$

Problem 2.108



2.108 For the composite rod of Prob. 2.107, if P is gradually increased from zero until the deflection of point C reaches a maximum value of $\delta_m = 0.5$ mm and then decreased back to zero, determine, (*a*) the maximum value of P, (*b*) the maximum stress in each.

2.107 Rod AB consists of two cylindrical portions AC and BC, each with a cross-sectional area of 2950 mm^2. Portion AC is made of a mild steel with E = 200 GPa and σ_Y = 250 MPa, and portion CB is made of a high-strength steel with E = 200 GPa and σ_Y = 345 MPa. A load **P** is applied at C as shown. Assuming both steels to be elastoplastic, determine (*a*) the maximum deflection of C if P is gradually increased from zero to 1625 kN and then reduced back to zero, (*b*) the maximum stress in each portion of the rod, (*c*) the permanent deflection of C.

Displacement at C is $\delta_m = 0.30$ mm. The corresponding strains are

$$\varepsilon_{AC} = \frac{\delta_m}{L_{AC}} = \frac{0.50 \text{ mm}}{320 \text{ mm}} = 1.5625 \times 10^{-3}$$

$$\varepsilon_{CB} = -\frac{\delta_m}{L_{CB}} = -\frac{0.50 \text{ mm}}{320 \text{ mm}} = -1.5625 \times 10^{-3}$$

Strains at initial yielding

$$\varepsilon_{Y,AC} = \frac{\sigma_{Y,AC}}{E} = \frac{250 \times 10^6}{200 \times 10^9} = 1.25 \times 10^{-3} \qquad \text{(yielding)}$$

$$\varepsilon_{Y,CB} = -\frac{\sigma_{Y,BC}}{E} = -\frac{345 \times 10^6}{200 \times 10^9} = -1.725 \times 10^{-3} \qquad \text{(elastic)}$$

(a) Forces: $F_{AC} = A\sigma_Y = (2950 \times 10^{-6})(250 \times 10^6) = 737.5 \times 10^{-3}$ N

$$F_{CB} = EA\varepsilon_{CB} = (200 \times 10^9)(2950 \times 10^{-6})(-1.5625 \times 10^{-3}) = -921.875 \times 10^3 \text{ N}$$

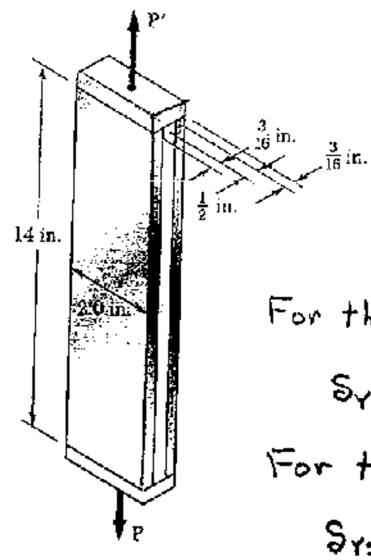
For equilibrium of element at C $\quad F_{AC} - F_{CB} - P = 0$

$$P = F_{AC} - F_{CB} = 737.5 \times 10^3 + 921.875 \times 10^3 = 1.659375 \times 10^6 \text{ N} = 1659 \text{ kN}$$

(b) Stresses: AC $\quad \sigma_{AC} = \sigma_{Y,AC} = 250$ MPa

CB $\quad \sigma_{CB} = \frac{F_{CB}}{A} = -\frac{921.875 \times 10^3}{2950 \times 10^{-6}} = -312.5 \times 10^6 \text{ Pa} = -312.5 \text{ MPa}$

Problem 2.109



2.109 Two tempered-steel bars, each $\frac{3}{16}$-in. thick, are bonded to a $\frac{1}{2}$-in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 29 \times 10^6$ psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel. The load P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 0.04$ in. and then decreased back to zero. Determine (*a*) the maximum value of P, (*b*) the maximum stress in the tempered-steel bars, (*c*) the permanent set after the load is removed.

For the mild steel $\quad A_1 = (\frac{1}{2})(2) = 1.00 \text{ in}^2$

$$\delta_{Y1} = \frac{L\sigma_{Y1}}{E} = \frac{(14)(50\times10^3)}{29\times10^6} = 0.024138 \text{ in.}$$

For the tempered steel $\quad A_2 = 2(\frac{3}{16})(2) = 0.75 \text{ in}^2$

$$\delta_{Y2} = \frac{L\sigma_{Y2}}{E} = \frac{(14)(100\times10^3)}{29\times10^3} = 0.048276 \text{ in.}$$

Total area: $\quad A = A_1 + A_2 = 1.75 \text{ in}^2$

$\delta_{Y1} < \delta_m < \delta_{Y2}$ The mild steel yields. Tempered steel is elastic.

(a) Forces $\quad P_1 = A_1\sigma_{Y1} = (1.00)(50\times10^3) = 50\times10^3$ lb.

$$P_2 = \frac{EA_2\delta_m}{L} = \frac{(29\times10^3)(0.75)(0.04)}{14} = 62.14\times10^3 \text{ lb.}$$

$$P = P_1 + P_2 = 112.14\times10^3 \text{ lb} = 112.1 \text{ kips}$$

(b) Stresses $\quad \sigma_1 = \frac{P_1}{A_1} = \sigma_{Y1} = 50\times10^3 \text{ psi} = 50 \text{ ksi}$

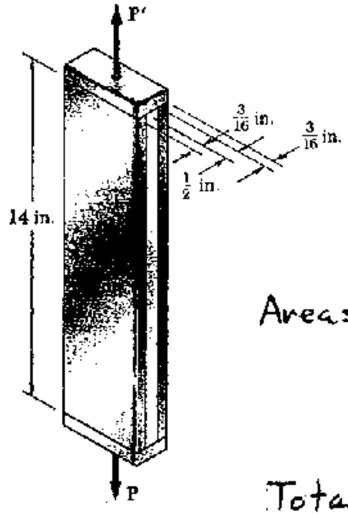
$$\sigma_2 = \frac{P_2}{A_2} = \frac{62.14\times10^3}{0.75} = 82.86\times10^3 \text{ psi} = 82.86 \text{ ksi}$$

Unloading $\quad \delta' = \frac{PL}{EA} = \frac{(112.14\times10^3)(14)}{(29\times10^6)(1.75)} = 0.03094$ in.

(c) Permanent set $\quad \delta_p = \delta_m - \delta' = 0.04 - 0.03094$

$$= 0.00906 \text{ in.}$$

Problem 2.110



2.110 For the composite bar of Prob. 2.109, if P is gradually increased from zero to 98 kips and then decreased back to zero, determine (*a*) the maximum deformation of the bar, (*b*) the maximum stress in the tempered-steel bars, (*c*) the permanent set after the load is removed.

2.109 Two tempered-steel bars, each $\frac{3}{16}$-in. thick, are bonded to a $\frac{1}{2}$-in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 29 \times 10^6$ psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel.

Areas: Mild steel $A_1 = (\frac{1}{2})(2) = 1.00 \text{ in}^2$

Tempered steel $A_2 = 2(\frac{3}{16})(2) = 0.75 \text{ in}^2$

Total: $A = A_1 + A_2 = 1.75 \text{ in}^2$

Total force to yield the mild steel

$$\sigma_{Y1} = \frac{P_Y}{A} \quad \therefore \quad P_Y = A\sigma_{Y1} = (1.75)(50 \times 10^3) = 87.50 \times 10^3 \text{ lb.}$$

$P > P_Y$, therefore mild steel yields.

Let P_1 = force carried by mild steel
P_2 = force carried by tempered steel

$$P_1 = A_1\sigma_1 = (1.00)(50 \times 10^3) = 50 \times 10^3 \text{ lb.}$$

$$P_1 + P_2 = P, \quad P_2 = P - P_1 = 98 \times 10^3 - 50 \times 10^3 = 48 \times 10^3 \text{ lb.}$$

(a) $$\delta_m = \frac{P_2 L}{E A_2} = \frac{(48 \times 10^3)(14)}{(29 \times 10^6)(0.75)} = 0.03090 \text{ in.}$$

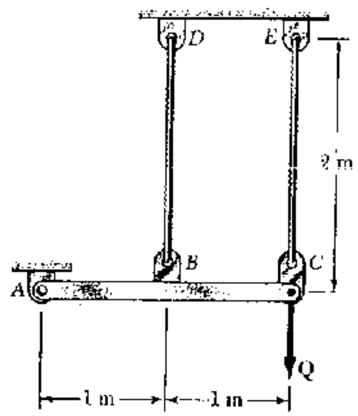
(b) $$\sigma_2 = \frac{P_2}{A_2} = \frac{48 \times 10^3}{0.75} = 64 \times 10^3 \text{ psi} = 64 \text{ ksi}$$

Unloading $$\delta' = \frac{PL}{EA} = \frac{(98 \times 10^3)(14)}{(29 \times 10^6)(1.75)} = 0.02703 \text{ in}$$

(c) $$\delta_p = \delta_m - \delta' = 0.03090 - 0.02703 = 0.00387 \text{ in.}$$

Problem 2.111

2.111 Each cable has a cross-sectional area of 100 mm^2 and is made of an elastoplastic material for which $\sigma_Y = 345$ MPa and $E = 200$ GPa. A force **Q** is applied at C to the rigid bar ABC and is gradually increased from 0 to 50 Kn and then reduced to zero. Knowing that the cables were initially taut, determine (*a*) the maximum stress that occurs in cable BD, (*b*) the maximum deflection of point C, (*c*) the final displacement of point C. (*Hint:* In part *c*, cable CE is not taut.)



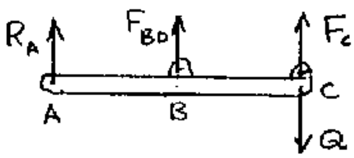
Elongation constraints for taut cables.

Let θ = rotation angle of rigid bar ABC

$$\theta = \frac{\delta_{BD}}{L_{AB}} = \frac{\delta_{CE}}{L_{AC}}$$

$$\delta_{BD} = \frac{L_{AB}}{L_{AC}}\delta_{CE} = \frac{1}{2}\delta_{CE} \qquad (1)$$

Equilibrium of bar ABC.



$$+\circlearrowleft M_A = 0: \quad L_{AB}F_{BD} + L_{AC}F_{CE} - L_{AC}Q = 0$$

$$Q = F_{CE} + \frac{L_{AB}}{L_{AC}}F_{BD} = F_{CE} + \frac{1}{2}F_{BD} \qquad (2)$$

Assume cable CE is yielded. $F_{CE} = A\sigma_Y = (100\times10^{-6})(345\times10^6) = 34.5\times10^3$ N

From (2) $F_{BD} = 2(Q - F_{CE}) = (2)(50\times10^3 - 34.5\times10^3) = 31.0\times10^3$ N

Since $F_{BD} \le A\sigma_Y = 34.5\times10^3$ N, cable BD is elastic when $Q = 50$ kN.

(a) Maximum stresses. $\sigma_{CE} = \sigma_Y = 345$ MPa

$$\sigma_{BD} = \frac{F_{BD}}{A} = \frac{31.0\times10^3}{100\times10^{-6}} = 310\times10^6 \text{ Pa} \qquad \sigma_{BD} = 310 \text{ MPa}$$

(b) Maximum of deflection of point C

$$\delta_{BD} = \frac{F_{BD}L_{BD}}{EA} = \frac{(31.0\times10^3)(2)}{(200\times10^9)(100\times10^{-6})} = 3.1\times10^{-3} \text{ m}$$

From (1) $\delta_C = \delta_{CE} = 2\delta_{BD} = 6.2\times10^{-3}$ m $\qquad$ 6.20 mm $\downarrow$

Permanent elongation of cable CE: $(\delta_{CE})_p = (\delta_{CE}) - \dfrac{\sigma_Y L_{CE}}{E}$

$$(\delta_{CE})_p = (\delta_{CE})_{max} - \frac{F_{CE}L_{CE}}{EA} = (\delta_{CE})_{max} - \frac{\sigma_Y L_{CE}}{E} =$$

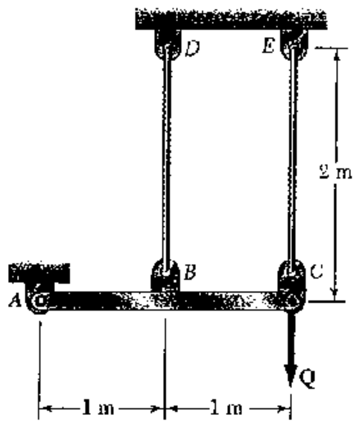
$$= 6.20\times10^{-3} - \frac{(345\times10^6)(2)}{200\times10^9} = 2.75\times10^{-3} \text{ m}$$

(c) Unloading. Cable CE is slack ($F_{CE} = 0$) at $Q = 0$.

From (2) $F_{BD} = 2(Q - F_{CE}) = 2(0 - 0) = 0$

Since cable BD remained elastic $\delta_{BD} = \dfrac{F_{BD}L_{BD}}{EA} = 0$

Problem 2.112



2.112 Solve Prob. 2.111, assuming that the cables are replaced by rods of the same cross-sectional area and material. Further assume that the rods are braced so that they can carry compressive forces.

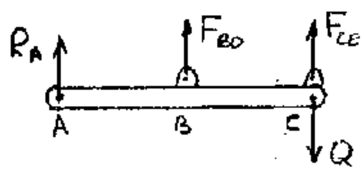
2.111 Each cable has a cross-sectional area of 100 mm² and is made of an elastoplastic material for which $\sigma_Y = 345$ MPa and $E = 200$ GPa. A force **Q** is applied at C to the rigid bar ABC and is gradually increased from 0 to 50 Kn and then reduced to zero. Knowing that the cables were initially taut, determine (*a*) the maximum stress that occurs in cable BD, (*b*) the maximum deflection of point C, (*c*) the final displacement of point C. (*Hint:* In part *c*, cable CE is not taut.)

Elongation constraints.

Let θ = rotation angle of rigid bar ABC.

$$\theta = \frac{\delta_{BC}}{L_{AB}} = \frac{\delta_{CE}}{L_{AC}}$$

$$\delta_{BD} = \frac{L_{AB}}{L_{AC}}\,\delta_{CE} = \tfrac{1}{2}\,\delta_{CE} \qquad (1)$$



Equilibrium of bar ABC

$$+\circlearrowleft \Sigma M_A = 0: \quad L_{AB} F_{BD} + L_{AC} F_{CE} - L_{AC} Q = 0$$

$$Q = F_{CE} + \frac{L_{AB}}{L_{AC}} F_{BD} = F_{CE} + \tfrac{1}{2} F_{BD} \qquad (2)$$

Assume cable CE is yielded. $F_{CE} = A\sigma_Y = (100\times10^{-6})(345\times10^{6}) = 34.5\times10^{3}$ N

From (2) $F_{BD} = 2(Q - F_{CE}) = (2)(50\times10^{3} - 34.5\times10^{3}) = 31.0\times10^{3}$ N

Since $F_{BD} < A\sigma_Y = 34.5\times10^{3}$ N, cable BD is elastic when $Q = 50$ kN.

(a) Maximum stresses $\sigma_{CE} = \sigma_Y = 345$ MPa

$$\sigma_{BD} = \frac{F_{BD}}{A} = \frac{31.0\times10^{3}}{100\times10^{-6}} = 310\times10^{6}\ \text{Pa} \qquad \sigma_{BD} = 310\ \text{MPa}$$

(b) Maximum deflection of point C

$$\delta_{BD} = \frac{F_{BD} L_{BD}}{EA} = \frac{(31.0\times10^{3})(2)}{(200\times10^{9})(100\times10^{-6})} = 3.1\times10^{-3}\ \text{m}$$

From (1) $(\delta_C)_m = \delta_{CE} = 2\delta_{BD} = 6.2\times10^{-3}$ m 6.20 mm ↓

Unloading. $Q' = 50\times10^{3}$ N, $\delta_{CE}' = \delta_C'$ From (1) $\delta_{BD}' = \tfrac{1}{2}\delta_C'$

Elastic

$$F_{BD}' = \frac{EA\delta_{BD}'}{L_{BD}} = \frac{(200\times10^{9})(100\times10^{-6})(\frac{1}{2}\delta_C')}{2} = 5\times10^{6}\ \delta_C'$$

$$F_{CE}' = \frac{EA\delta_{CE}'}{L_{CE}} = \frac{(200\times10^{9})(100\times10^{-6})(\delta_C')}{2} = 10\times10^{6}\ \delta_C'$$

From (2) $Q' = F_{CE}' + \tfrac{1}{2}F_{BD}' = 12.5\times10^{6}\ \delta_C'$

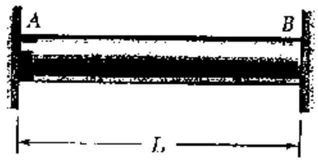
Equating expressions for Q' $12.5\times10^{6}\delta_C' = 50\times10^{3}$

$$\delta_C' = 4\times10^{-3}\ \text{m}$$

(c) Final displacement. $\delta_C = (\delta_C)_m - \delta_C' = 6.2\times10^{-3} - 4\times10^{-3} = 2.2\times10^{-3}$ m

2.20 mm ↓

Problem 2.113



2.113 A uniform steel rod of cross-sectional area A is attached to rigid supports and is unstressed at a temperature of 45°F. The steel is assumed to be elastoplastic with $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi. Knowing that $\alpha = 6.5 \times 10^{-6}/°F$, determine the stress in the bar (*a*) when the temperature is raised to 320°F, (*b*) after the temperature has returned to 45°F.

Let P be the compressive force in the rod.

Determine temperature change to cause yielding.

$$\delta = -\frac{PL}{AE} + L\alpha(\Delta T) = -\frac{\sigma_Y L}{E} + L\alpha(\Delta T)_Y = 0$$

$$(\Delta T)_Y = \frac{\sigma_Y}{E\alpha} = \frac{36 \times 10^3}{(29 \times 10^6)(6.5 \times 10^{-6})} = 190.98\ °F$$

But $\Delta T = 320 - 45 = 275°F > (\Delta T_Y)$

(a) Yielding occurs $\qquad \sigma = -\sigma_Y = -36$ ksi ◄

Cooling: $\quad (\Delta T)' = 275°F$

$$\delta' = \delta_P' + \delta_T' = -\frac{P'L}{AE} + L\alpha(\Delta T)' = 0$$

$$\sigma' = \frac{P'}{A} = -E\alpha(\Delta T)' =$$

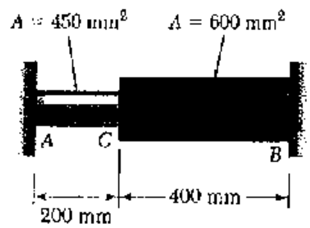
$$= -(29 \times 10^6)(6.5 \times 10^{-6})(275) = -51.8375 \times 10^3 \text{ psi}$$

(b) Residual stress

$$\sigma_{res} = -\sigma_Y - \sigma' = -36 \times 10^3 + 51.8375 \times 10^3 = 15.84 \times 10^3 \text{ psi}$$

15.84 ksi ◄

Problem 2.114

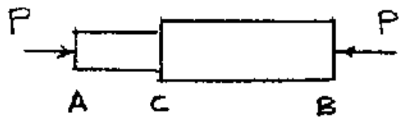


2.114 The steel rod ABC is attached to rigid supports and is unstressed at a temperature of 20°C. The steel is assumed elastoplastic, with $\sigma_Y = 250$ MPa and $E = 200$ GPa. The temperature of both portions of the rod is then raised to 120 °C. Knowing that $\alpha = 11.7 \times 10^{-6}/°\text{C}$, determine (*a*) the stress in portion AC, (*b*) the deflection of point C.

$$A_{AC} = 450 \times 10^{-6}\ \text{m}^2 \qquad L_{AC} = 0.200\ \text{m}$$

$$A_{CB} = 600 \times 10^{-6}\ \text{m}^2 \qquad L_{CB} = 0.400\ \text{m}$$

$$(\delta_{B/A})_P + (\delta_{B/A})_T = 0 \qquad \text{constraint}$$



Determine ΔT to cause yielding in AC.

$$-\frac{PL_{AC}}{A_{AB}E} - \frac{PL_{CB}}{A_{CB}E} + L_{AB}\,\alpha(\Delta T)_Y = 0$$

$$\Delta T = \frac{P}{L_{AB}E\alpha}\left(\frac{L_{AB}}{A_{AB}} + \frac{L_{CB}}{A_{CB}}\right)$$

At yielding $\quad P = A_{AC}\sigma_Y$

$$(\Delta T)_Y = \frac{A_{AC}\sigma_Y}{L_{AB}E\alpha}\left(\frac{L_{AB}}{A_{AB}} + \frac{L_{BC}}{A_{BC}}\right) =$$

$$= \frac{(450\times10^{-6})(250\times10^{6})}{(0.600)(200\times10^{9})(11.7\times10^{-6})}\left(\frac{0.200}{450\times10^{-6}} + \frac{0.400}{600\times10^{-6}}\right) = 89.03\ °\text{C}$$

Actual $\Delta T = 120 - 20 = 100°\ \text{C} > (\Delta T)_Y$. $\qquad$ Yielding occurs.

(a) $\sigma_{AC} = -\sigma_Y = -250$ MPa ◄

$$P = A_{AC}\sigma_Y = (450\times10^{-6})(250\times10^{6}) = 112.5\times10^{3}\ \text{N}$$

$$\delta_C = -\delta_{C/B} = \frac{PL_{CB}}{EA_{CB}} - L_{CB}\,\alpha\,(\Delta T)$$

$$= \frac{(112.5\times10^{3})(0.400)}{(200\times10^{9})(600\times10^{-6})} - (0.400)(11.7\times10^{-6})(100)$$

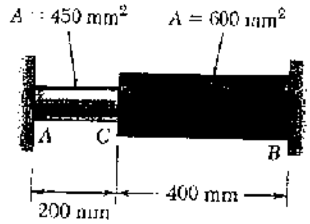
$$= 375\times10^{-6} - 468\times10^{-6} = -93.0\times10^{-6}\ \text{m} = -0.0930\ \text{mm}$$

$$\delta_C = 0.0930\ \text{mm} \leftarrow$$ ◄

Problem 2.115

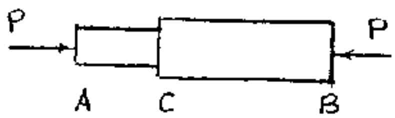
*2.115 Solve Prob. 2.114, assuming that the temperature of the rod is raised to 120 °C and then returned to 20 °C.

2.114 The steel rod ABC is attached to rigid supports and is unstressed at a temperature of 20°C. The steel is assumed elastoplastic, with $\sigma_Y = 250$ MPa and E = 200 GPa. The temperature of both portions of the rod is then raised to 120 °C. Knowing that $\alpha = 11.7 \times 10^{-6}/°C$, determine (*a*) the stress in portion AC, (*b*) the deflection of point C.



$A_{AC} = 450 \times 10^{-6}\ m^2 \qquad L_{AC} = 0.200\ m$

$A_{CB} = 600 \times 10^{6}\ m^2 \qquad L_{CB} = 0.400\ m$



$(\delta_{B/A})_P + (\delta_{B/A})_T = 0 \qquad$ constraint

Determine ΔT to cause yielding in AC.

$$-\frac{PL_{AC}}{A_{AC}E} - \frac{PL_{CB}}{A_{CB}E} + L_{AB}\alpha(\Delta T) = 0$$

$$\Delta T = \frac{P}{L_{AB}E\alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right) \qquad \text{At yielding} \quad P_Y = A_{AC}\sigma_Y$$

$$(\Delta T)_Y = \frac{A_{AC}\sigma_Y}{L_{AB}E\alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right) = \frac{(450\times10^{-6})(250\times10^{6})}{(0.600)(200\times10^{9})(11.7\times10^{-6})}\left(\frac{0.200}{450\times10^{-6}} + \frac{0.400}{600\times10^{-6}}\right)$$

$$= 89.03\ °C$$

Actual $(\Delta T) = 120 - 20 = 100°C > (\Delta T)_Y$. Yielding occurs.

At 120°C $\quad \sigma_{AC} = -\sigma_Y \quad P = A_{AC}\sigma_Y = (450\times10^{-6})(250\times10^{6}) = 112.5\times10^{3}$ N

Cooling $\Delta T' = 100°$ C

$$P' = \frac{L_{AB}E\alpha(\Delta T)'}{\left(\frac{L_{AB}}{A_{AB}} + \frac{L_{CB}}{A_{CB}}\right)} = \frac{(0.600)(200\times10^{9})(11.7\times10^{-6})(100)}{\left(\frac{0.200}{450\times10^{-6}} + \frac{0.400}{600\times10^{-6}}\right)} = 126.36\times10^{3}\ N$$

Residual force $\quad P = P - P' = 112.5\times10^{3} - 126.36\times10^{3} = -13.86\times10^{3}$ N

(a) Residual stress $\quad \sigma_{AC} = -\dfrac{P}{A_{AC}} = \dfrac{13.86\times10^{3}}{450\times10^{-6}} = 30.8\times10^{6}$ Pa

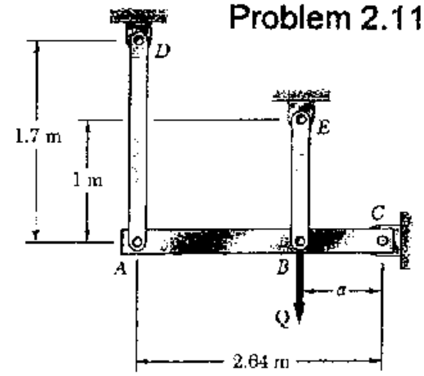
$\sigma_{AC} = 30.8$ MPa ◄

(b) $\delta_C = -\delta_{C/B} = \dfrac{PL_{CB}}{EA_{CB}} = \dfrac{(-13.86\times10^{3})(0.400)}{(200\times10^{9})(600\times10^{-6})} = -46.2\times10^{-6}$ m

$= -0.0462$ mm

$\delta_C = 0.0462$ mm ← ◄

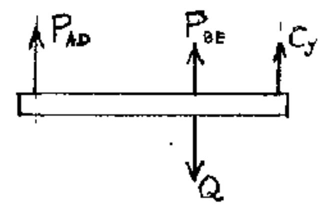
Problem 2.116



2.116 The rigid bar *ABC* is supported by two links, *AD* and *BE*, of uniform 37.5×6-mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The magnitude of the force **Q** applied at *B* is gradually increased from zero to 260 kN. Knowing that $a = 0.640$ m, determine (*a*) the value of the normal stress in each link, (*b*) the maximum deflection of point *B*.

Statics: $\Sigma M_C = 0 \quad 0.640(Q - P_{BE}) - 2.64 P_{AD} = 0$

Deformation: $\delta_A = 2.64\,\theta, \quad \delta_B = a\theta = 0.640\,\theta$

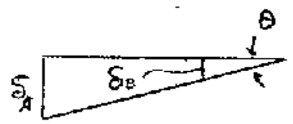


Elastic Analysis:

$$A = (37.5)(6) = 225 \text{ mm}^2 = 225\times10^{-6} \text{ m}^2$$

$$P_{AD} = \frac{EA}{L_{AD}}\delta_A = \frac{(200\times10^9)(225\times10^{-6})}{1.7}\delta_A = 26.47\times10^6\,\delta_A$$

$$= (26.47\times10^6)(2.64\,\theta) = 69.88\times10^6\,\theta$$



$$\sigma_{AD} = \frac{P_{AD}}{A} = 310.6\times10^9\,\theta$$

$$P_{BE} = \frac{EA}{L_{BE}}\delta_B = \frac{(200\times10^9)(225\times10^{-6})}{1.0}\delta_B = 45\times10^6\,\delta_B$$

$$= (45\times10^6)(0.640\,\theta) = 28.80\times10^6\,\theta$$

$$\sigma_{BE} = \frac{P_{BE}}{A} = 128\times10^9\,\theta$$

From Statics $\quad Q = P_{BE} + \frac{2.64}{0.640}P_{AD} = P_{BE} + 4.125\,P_{AD}$

$$= [28.80\times10^6 + (4.125)(69.88\times10^6)]\theta = 317.06\times10^6\,\theta$$

θ_Y at yielding of link AD $\quad \sigma_{AD} = \sigma_Y = 250\times10^6 = 310.6\times10^9\,\theta$

$\theta_Y = 804.89\times10^{-6}$

$Q_Y = (317.06\times10^6)(804.89\times10^{-6}) = 255.2\times10^3$ N

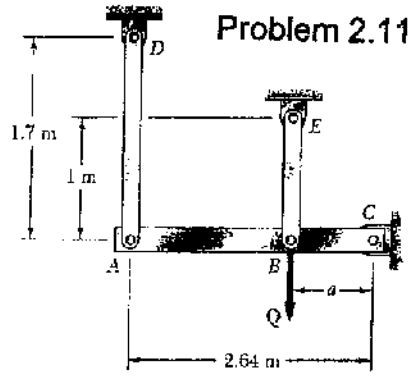
Since $Q = 260\times10^3 > Q_Y$, link AD yields. $\quad \sigma_{AD} = 250$ MPa ◄

$P_{AD} = A\,\sigma_Y = (225\times10^{-6})(250\times10^{-6}) = 56.25\times10^3$ N

From Statics $\quad P_{BE} = Q - 4.125\,P_{AD} = 260\times10^3 - (4.125)(56.25\times10^3)$

$P_{BE} = 27.97\times10^3$ N $\qquad \sigma_{BE} = \frac{P_{BE}}{A} = \frac{27.97\times10^3}{225\times10^{-6}} = 124.3\times10^6$ Pa $= 124.3$ MPa ◄

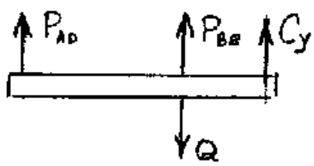
$$\delta_B = \frac{P_{BE}L_{BE}}{EA} = \frac{(27.97\times10^3)(1.0)}{(200\times10^9)(225\times10^{-6})} = 621.53\times10^{-6} \text{ m} = 0.622 \text{ mm} \downarrow$$ ◄



Problem 2.117

2.117 Solve Prob. 2.116, knowing that $a = 1.76$ m and that the magnitude of the force **Q** applied at B is gradually increased from zero to 135 kN.

2.116 The rigid bar ABC is supported by two links, AD and BE, of uniform 37.5×6-mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The magnitude of the force **Q** applied at B is gradually increased from zero to 260 kN. Knowing that $a = 0.640$ m, determine (*a*) the value of the normal stress in each link, (*b*) the maximum deflection of point B.



Statics: $\Sigma M_C = 0 \quad 1.76(Q - P_{BE}) - 2.64 P_{AD} = 0$

Deformation: $\delta_A = 2.64\,\theta, \quad \delta_B = 1.76\,\theta$

Elastic Analysis

$A = (37.5)(6) = 225\ \text{mm}^2 = 225\times10^{-6}\ \text{m}^2$

$P_{AD} = \dfrac{EA}{L_{AD}}\delta_A = \dfrac{(200\times10^9)(225\times10^{-6})}{1.7}\delta_A = 26.47\times10^6\,\delta_A$

$= (26.47\times10^6)(2.64\,\theta) = 69.88\times10^6\,\theta$

$\sigma_{AD} = \dfrac{P_{AD}}{A} = 310.6\times10^9\,\theta$

$P_{BE} = \dfrac{EA}{L_{BE}}\delta_B = \dfrac{(200\times10^9)(225\times10^{-6})}{1.0}\delta_B = 45\times10^6\,\delta_B = (45\times10^6)(1.76\,\theta)$

$= 79.2\times10^6\,\theta \qquad \sigma_{BE} = \dfrac{P_{BE}}{A} = 352\times10^9\,\theta$

From Statics $\quad Q = P_{BE} + \dfrac{2.64}{1.76}P_{AD} = P_{BE} + 1.500\,P_{AD}$

$= [73.8\times10^6 + (1.500)(69.88\times10^6)]\,\theta = 178.62\times10^6\,\theta$

θ_Y at yielding of link BE $\qquad \sigma_{BE} = \sigma_Y = 250\times10^6 = 352\times10^9\,\theta_Y$

$\theta_Y = 710.23\times10^{-6}$

$Q_Y = (178.62\times10^6)(710.23\times10^{-6}) = 126.86\times10^3\ \text{N}$

Since $Q = 135\times10^3\,\text{N} > Q_Y$, link BE yields $\quad \sigma_{BE} = \sigma_Y = 250\ \text{MPa}$ ◄

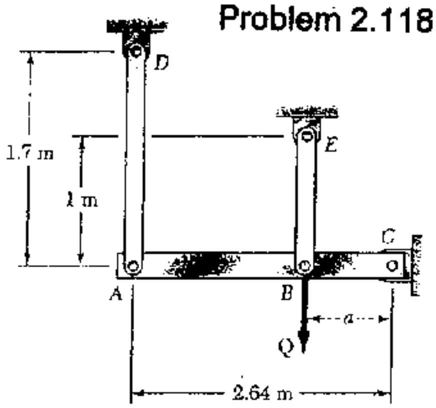
$P_{BE} = A\sigma_Y = (225\times10^{-6})(250\times10^6) = 56.25\times10^3\ \text{N}$

From Statics $\qquad P_{AD} = \dfrac{1}{1.500}(Q - P_{BE}) = 52.5\times10^3\ \text{N}$

$\sigma_{AD} = \dfrac{P_{AD}}{A} = \dfrac{52.5\times10^3}{225\times10^{-6}} = 233.3\times10^6 = 233\ \text{MPa}$ ◄

From elastic analysis of AD $\qquad \theta = \dfrac{P_{AD}}{69.88\times10^6} = 751.29\times10^{-6}\ \text{rad}$

$\delta_B = 1.76\,\theta = 1.322\times10^{-3}\ \text{m} = 1.322\ \text{mm}$ ◄



Problem 2.118

***2.118** Solve Prob. 2.116, assuming that the magnitude of the force **Q** applied at *B* is gradually increased from zero to 260 kN and then decreased back to zero. Knowing that $a = 0.640$ m, determine (*a*) the residual stress in each link, (*b*) the final deflection of point *B*. Assume that the links are braced so that they can carry compressive forces without buckling.

2.116 The rigid bar *ABC* is supported by two links, *AD* and *BE*, of uniform 37.5×6-mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The magnitude of the force **Q** applied at *B* is gradually increased from zero to 260 kN. Knowing that $a = 0.640$ m, determine (*a*) the value of the normal stress in each link, (*b*) the maximum deflection of point *B*.

See solution to Problem 2.116 for the normal stresses in each link and the deflection of point B after loading

$$\sigma_{AD} = 250 \times 10^6 \text{ Pa} \qquad \sigma_{BE} = 124.3 \times 10^6 \text{ Pa}$$

$$\delta_B = 621.53 \times 10^{-6} \text{ m}$$

The elastic analysis given in the solution to PROBLEM 2.121 applies to the unloading

$$Q = 317.06 \times 10^6 \, \theta'$$

$$\theta' = \frac{Q}{317.06 \times 10^6} = \frac{260 \times 10^3}{317.06 \times 10^6} = 820.03 \times 10^{-6}$$

$$\sigma'_{AD} = 310.6 \times 10^9 \, \theta = (310.6 \times 10^9)(820.03 \times 10^{-6}) = 254.70 \times 10^6 \text{ Pa}$$

$$\sigma'_{BE} = 128 \times 10^9 \, \theta = (128 \times 10^9)(820.03 \times 10^{-6}) = 104.96 \times 10^6 \text{ Pa}$$

$$\delta'_B = 0.640 \, \theta' = 524.82 \times 10^{-6} \text{ m}$$

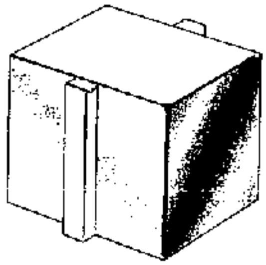
(a) Residual stresses

$$\sigma_{AD,res} = \sigma_{AD} - \sigma'_{AD} = 250 \times 10^6 - 254.70 \times 10^6 = -4.70 \times 10^6 \text{ Pa} = -4.70 \text{ MPa}$$

$$\sigma_{BE,res} = \sigma_{BE} - \sigma'_{BE} = 124.3 \times 10^6 - 104.96 \times 10^6 = 19.34 \times 10^6 \text{ Pa} = 19.34 \text{ MPa}$$

(b) $$\delta_{B,P} = \delta_B - \delta'_B = 621.53 \times 10^{-6} - 524.82 \times 10^{-6} = 96.71 \times 10^{-6} \text{ m} = 0.0967 \text{ mm} \downarrow$$

Problem 2.119



*2.119 A narrow bar of aluminum is bonded to the side of a thick steel plate as shown. Initially, at $T_1 = 70°F$, all stresses are zero. Knowing that the temperature will be slowly raised to T_2 and then reduced to T_1, determine (*a*) the highest temperature T_2 that does *not* result in residual stresses, (*b*) the temperature T_2 that will result in a residual stress in the aluminum equal to 58 ksi. Assume $\alpha_a = 12.8 \times 10^{-6}/°F$ for the aluminum and $\alpha_s = 6.5 \times 10^{-6}/°F$ for the steel. Further assume that the aluminum is elastoplastic, with $E = 10.9 \times 10^6$ psi and $\sigma_Y = 58$ ksi. (*Hint*: Neglect the small stresses in the plate.)

Determine temperature change to cause yielding

$$\delta = \frac{PL}{EA} + L\alpha_a(\Delta T)_Y = L\alpha_s(\Delta T)_Y$$

$$\frac{P}{A} = \sigma = -E(\alpha_a - \alpha_s)(\Delta T)_Y = -\sigma_Y$$

$$(\Delta T_Y) = \frac{\sigma_Y}{E(\alpha_a - \alpha_s)} = \frac{58 \times 10^3}{(10.9\times10^6)(12.8 - 6.5)(10^{-6})} = 844.62\ °F$$

(a) $T_{2Y} = T_1 + (\Delta T)_Y = 70 + 844.62 = 915°F$

After yielding

$$\delta = \frac{\sigma_Y L}{E} + L\alpha_a(\Delta T) = L\alpha_s(\Delta T)$$

Cooling

$$\delta' = \frac{P'L}{AE} + L\alpha_a(\Delta T)' = L\alpha_s(\Delta T)'$$

The residual stress is

$$\sigma_{res} = \sigma_Y - \frac{P'}{A} = \sigma_Y - E(\alpha_a - \alpha_s)(\Delta T)$$

Set $\sigma_{res} = -\sigma_Y$

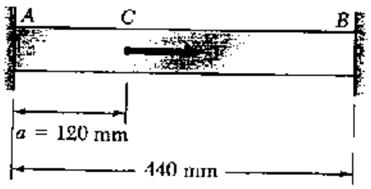
$$-\sigma_Y = \sigma_Y - E(\alpha_a - \alpha_s)(\Delta T)$$

$$\Delta T = \frac{2\sigma_Y}{E(\alpha_a - \alpha_s)} = \frac{(2)(58\times10^3)}{(10.9\times10^6)(12.8 - 6.5)(10^{-6})} = 1689°F$$

(b) $T_2 = T_1 + \Delta T = 70 + 1689 = 1759°F$

If $T_2 > 1759°F$, the aluminum bar will most likely yield in compression.

Problem 2.120

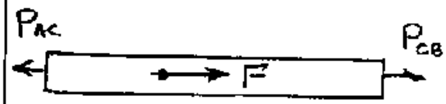


2.120 Bar AB has a cross-sectional area of 1200 mm^2 and is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. Knowing that the force **F** increases from 0 to 520 kN and then decreases to zero, determine (*a*) the permanent deflection of point C, (*b*) the residual stress in the bar.

$$A = 1200 \text{ mm}^2 = 1200\times10^{-6} \text{ m}^2$$

Force to yield portion AC : $P_{AC} = A\sigma_Y = (1200\times10^{-6})(250\times10^{6})$

$$= 300\times10^{3} \text{ N}$$



For equilibrium $F + P_{CB} - P_{AC} = 0$

$$P_{CB} = P_{AC} - F = 300\times10^{3} - 520\times10^{3}$$

$$= -220\times10^{3} \text{ N}$$

$$\delta_C = -\frac{P_{CB}L_{CB}}{EA} = \frac{(220\times10^{3})(0.440-0.120)}{(200\times10^{9})(1200\times10^{-6})} = 0.293333\times10^{-3} \text{ m}$$

$$\sigma_{CB} = \frac{P_{CB}}{A} = \frac{220\times10^{3}}{1200\times10^{-6}} = -183.333\times10^{6} \text{ Pa}$$

Unloading

$$\delta_C' = \frac{P_{AC}'L_{AC}}{EA} = -\frac{P_{CB}'L_{CB}}{EA} = \frac{(F-P_{AC}')L_{CB}}{EA}$$

$$P_{AC}'\left(\frac{L_{AC}}{EA} + \frac{L_{BC}}{EA}\right) = \frac{F L_{CB}}{EA}$$

$$P_{AC}' = \frac{F L_{CB}}{L_{AC}+L_{CB}} = \frac{(520\times10^{3})(0.440-0.120)}{0.440} = 378.182\times10^{3} \text{ N}$$

$$P_{CB}' = P_{AC}' - F = 378.182\times10^{3} - 520\times10^{3} = -141.818\times10^{3} \text{ N}$$

$$\sigma_{AC}' = \frac{P_{AC}'}{A} = \frac{378.182\times10^{3}}{1200\times10^{-6}} = 315.152\times10^{6} \text{ Pa}$$

$$\sigma_{BC}' = \frac{P_{BC}'}{A} = -\frac{141.818\times10^{3}}{1200\times10^{-6}} = -118.182\times10^{6} \text{ Pa}$$

$$\delta_C' = \frac{(378.182)(0.120)}{(200\times10^{9})(1200\times10^{6})} = 0.189091\times10^{-3} \text{ m}$$

(a) $\delta_{C_P} = \delta_C - \delta_C' = 0.293333\times10^{-3} - 0.189091\times10^{-3} = 0.1042\times10^{-3}$ m

$= 0.1042$ mm

(b) $\sigma_{AC,res} = \sigma_Y - \sigma_{AC}' = 250\times10^{6} - 315.152\times10^{6} = -65.2\times10^{6}$ Pa

$= -65.2$ MPa

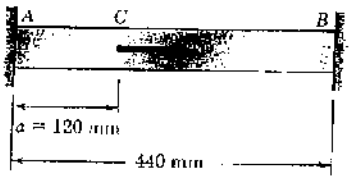
$\sigma_{CB,res} = \sigma_{CB} - \sigma_{CB}' = -183.333\times10^{6} + 118.182\times10^{6} = -65.2\times10^{6}$ Pa

$= -65.2$ MPa

Problem 2.121

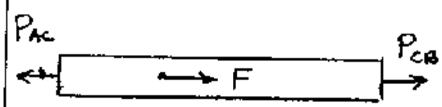
2.121 Solve Prob. 2.120, assuming that $a = 180$ mm.

2.120 Bar AB has a cross-sectional area of 1200 mm^2 and is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. Knowing that the force **F** increases from 0 to 520 kN and then decreases to zero, determine (*a*) the permanent deflection of point C, (*b*) the residual stress in the bar.



$$A = 1200 \text{ mm}^2 = 1200 \times 10^{-6} \text{ m}^2$$

Force to yield portion AC: $P_{AC} = A\sigma_Y = (1200\times10^{-6})(250\times10^{6})$
$= 300\times10^{3}$ N



For equilibrium $\quad F + P_{CB} - P_{AC} = 0$

$$P_{CB} = P_{AC} - F = 300\times10^{3} - 520\times10^{3} = -220\times10^{3} \text{ N}$$

$$\delta_C = -\frac{P_{CB}L_{CB}}{EA} = \frac{(220\times10^{3})(0.440-0.180)}{(200\times10^{9})(1200\times10^{-6})} = 0.238333\times10^{-3} \text{ m}$$

$$\sigma_{CB} = \frac{P_{CB}}{A} = -\frac{220\times10^{3}}{1200\times10^{-6}} = -183.333\times10^{6} \text{ Pa}$$

Unloading

$$\delta_C' = \frac{P_{AC}'L_{AC}}{EA} = -\frac{P_{CB}'L_{CB}}{EA} = \frac{(F-P_{AC}')L_{CB}}{EA} \therefore \quad P_{AC}'\left(\frac{L_{AC}}{EA} + \frac{L_{CB}}{EA}\right) = \frac{FL_{CB}}{EA}$$

$$P_{AC}' = \frac{FL_{CB}}{L_{AC}+L_{CB}} = \frac{(520\times10^{3})(0.440-0.180)}{0.440} = 307.273\times10^{3} \text{ N}$$

$$P_{CB}' = P_{AC}' - F = 307.273\times10^{3} - 520\times10^{3} = -212.727\times10^{3} \text{ N}$$

$$\delta_C' = \frac{(307.273\times10^{3})(0.180)}{(200\times10^{9})(1200\times10^{-6})} = 0.230455\times10^{-3} \text{ m}$$

$$\sigma_{AC}' = \frac{P_{AC}'}{A} = \frac{307.273\times10^{3}}{1200\times10^{-6}} = 256.061\times10^{6} \text{ Pa}$$

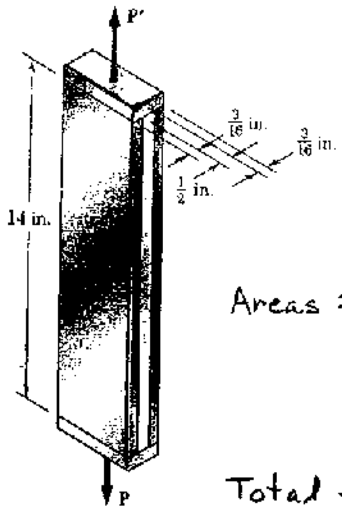
$$\sigma_{CB}' = \frac{P_{CB}'}{A} = \frac{-212.727\times10^{3}}{1200\times10^{-6}} = -177.273\times10^{6} \text{ P}$$

(a) $\quad \delta_{C,p} = \delta_C - \delta_C' = 0.238333\times10^{-3} - 0.230455\times10^{-3} = 0.00788\times10^{-3}$ m
$= 0.00788$ mm ◄

(b) $\quad \sigma_{AC,res} = \sigma_{AC} - \sigma_{AC}' = 250\times10^{6} - 256.061\times10^{6} = -6.06\times10^{6}$ Pa
$= -6.06$ MPa ◄

$\sigma_{CB,res} = \sigma_{CB} - \sigma_{CB}' = -183.333\times10^{6} + 177.273\times10^{6} = -6.06\times10^{6}$ Pa
$= -6.06$ MPa ◄

Problem 2.122



***2.122** For the composite bar of Prob. 2.109, determine the residual stresses in the tempered-steel bars if P is gradually increased from zero to 98 kips and then decreased back to zero.

2.109 Two tempered-steel bars, each $\frac{3}{16}$-in. thick, are bonded to a $\frac{1}{2}$-in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 29 \times 10^6$ psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel. The load P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 0.04$ in. and then decreased back to zero.

Areas: Mild steel $A_1 = (\frac{1}{2})(2) = 1.00 \text{ in}^2$

Tempered steel $A_2 = (2)(\frac{3}{16})(2) = 0.75 \text{ in}^2$

Total: $A = A_1 + A_2 = 1.75 \text{ in}^2$

Total force to yield the mild steel

$$\sigma_{Y1} = \frac{P_Y}{A} \quad \therefore \quad P_Y = A\sigma_{Y1} = (1.75)(50\times10^3) = 87.50\times10^3 \text{ lb.}$$

$P > P_Y$; therefore mild steel yields

Let P_1 = force carried by mild steel
P_2 = force carried by tempered steel

$$P_1 = A_1\sigma_{Y1} = (1.00)(50\times10^3) = 50\times10^3 \text{ lb.}$$

$$P_1 + P_2 = P, \qquad P_2 = P - P_1 = 98\times10^3 - 50\times10^3 = 48\times10^3 \text{ lb.}$$

$$\sigma_2 = \frac{P_2}{A_2} = \frac{48\times10^3}{0.75} = 64\times10^3 \text{ psi}$$

Unloading $\quad \sigma' = \frac{P}{A} = \frac{98\times10^3}{1.75} = 56\times10^3 \text{ psi}$

Residual stresses

mild steel $\quad \sigma_{1,res} = \sigma_1 - \sigma' = 50\times10^3 - 56\times10^3 = -6\times10^{-3} \text{ psi}$

$= -6$ ksi

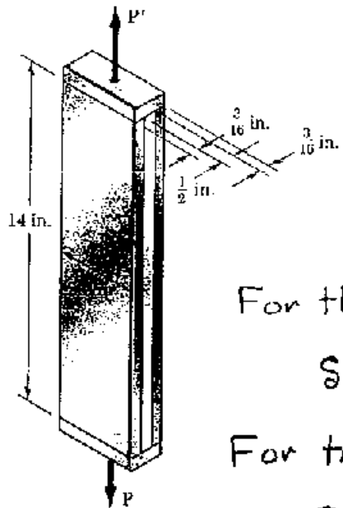
tempered steel $\quad \sigma_{2,res} = \sigma_2 - \sigma_1 = 64\times10^3 - 56\times10^3$

$= 8\times10^3 \text{ psi} = 8.00 \text{ ksi}$ ◄

Problem 2.123

*2.123 For the composite bar in Prob. 2.109, determine the residual stresses in the tempered-steel bars if P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 0.4$ in. and is then decreased back to zero.

2.109 Two tempered-steel bars, each $\frac{3}{16}$-in. thick, are bonded to a $\frac{1}{2}$-in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 29 \times 10^6$ psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel. The load P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 0.04$ in. and then decreased back to zero.



For the mild steel $\quad A_1 = (\frac{1}{2})(2) = 1.00 \text{ in}^2$

$$\delta_{Y1} = \frac{L\sigma_{Y1}}{E} = \frac{(14)(50\times10^3)}{29\times10^6} = 0.024138 \text{ in}$$

For the tempered steel $\quad A_2 = 2(\frac{3}{16})(2) = 0.75 \text{ in}^2$

$$\delta_{Y2} = \frac{L\sigma_{Y2}}{E} = \frac{(14)(100\times10^3)}{29\times10^6} = 0.048276 \text{ in.}$$

Total area: $\quad A = A_1 + A_2 = 1.75 \text{ in}^2$

$\delta_{Y1} < \delta_m < \delta_{Y2}$ The mild steel yields. Tempered steel is elastic.

Forces $\quad P_1 = A_1\sigma_{Y1} = (1.00)(50\times10^3) = 50\times10^3$ lb

$$P_2 = \frac{EA_2\delta_m}{L} = \frac{(29\times10^6)(0.75)(0.04)}{14} = 62.14\times10^3 \text{ lb.}$$

Stresses $\quad \sigma_1 = \frac{P_1}{A_1} = \sigma_{Y1} = 50\times10^3$ psi

$$\sigma_2 = \frac{P_2}{A_2} = \frac{62.14\times10^3}{0.75} = 82.86\times10^3 \text{ psi}$$

Unloading $\quad \sigma' = \frac{P}{A} = \frac{112.14}{1.75} = 64.08\times10^5$ psi

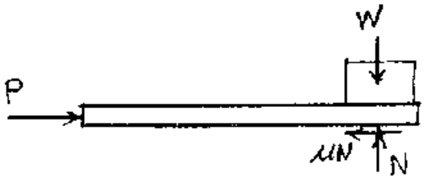
Residual stresses

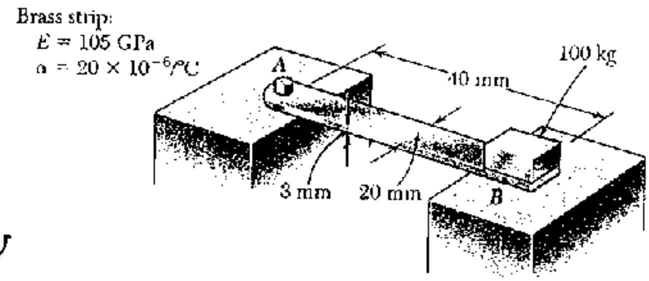
$$\sigma_{1,res} = \sigma_1 - \sigma' = 50\times10^3 - 64.08\times10^3 = -14.08\times10^3 \text{ psi} = -14.08 \text{ ksi}$$

$$\sigma_{2,res} = \sigma_2 - \sigma' = 82.86\times10^3 - 64.08\times10^3 = 18.78\times10^3 \text{ psi} = 18.78 \text{ ksi}$$

Problem 2.124

2.124 The brass strip AB has been attached to a fixed support at A and rests on a rough support at B. Knowing that the coefficient of friction is 0.60 between the strip and the support at B, determine the decrease in temperature for which slipping will impend.





$+\uparrow \Sigma F_y = 0: \quad N - W = 0 \qquad N = W$

$\overset{+}{\rightarrow} \Sigma F_x = 0 \quad P - \mu N = 0 \qquad P = \mu W = \mu m g$

$\delta = -\dfrac{PL}{EA} + L\alpha(\Delta T) = 0 \qquad \Delta T = \dfrac{P}{EA\alpha} = \dfrac{\mu m g}{EA\alpha}$

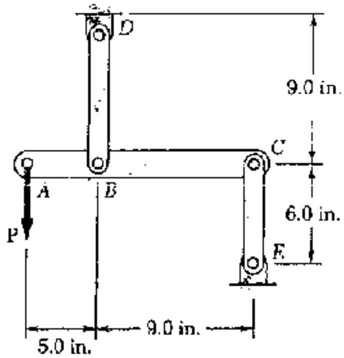
Data: $\mu = 0.60 \qquad A = (20)(3) = 60\ \text{mm}^2 = 60 \times 10^{-6}\ \text{m}^2$

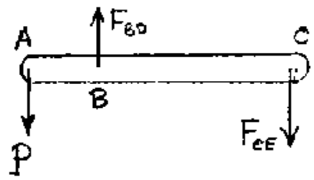
$m = 100\ \text{kg} \qquad g = 9.81\ \text{m/s}^2 \qquad E = 105 \times 10^9\ \text{Pa}$

$$\Delta T = \frac{(0.60)(100)(9.81)}{(105\times10^9)(60\times10^{-6})(20\times10^{-6})} = 4.67^\circ\text{C}$$

Problem 2.125

2.125 Link *BD* is made of brass ($E = 15 \times 10^6$ psi) and has a cross-sectional area of 0.40 in^2. Link *CE* is made of aluminum ($E = 10.4 \times 10^6$ psi) and has a cross-sectional area of 0.50 in^2. Determine the maximum force **P** that can be applied vertically at point *A* if the deflection of *A* is not to exceed 0.014 in.





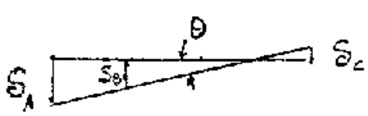
Use member ABC as a free body

$\circlearrowleft \Sigma M_C = 0, \quad 14P - 9F_{BD} = 0, \qquad F_{BD} = 1.5556\ P$

$\circlearrowleft \Sigma M_B = 0 \quad 5P - 9F_{CE} = 0, \qquad F_{CE} = 0.5556\ P$

$$\delta_B = \delta_{BD} = \frac{F_{BD}L_{BD}}{E_{BD}A_{BD}} = \frac{(1.5556\ P)(9.0)}{(15\times10^6)(0.40)} = 2.3333\times10^{-6}\ P \downarrow$$

$$\delta_C = \delta_{CE} = \frac{F_{CE}L_{CE}}{E_{CE}A_{CE}} = \frac{(0.5556\ P)(6.0)}{(10.4\times10^6)(0.50)} = 0.6410\times10^{-6}\ P \uparrow$$



Deformation Diagram

From the deformation diagram

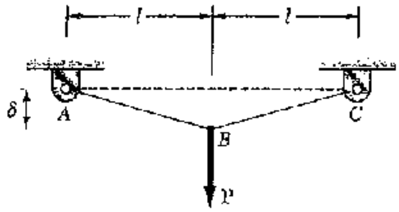
$$\text{Slope } \theta = \frac{\delta_B + \delta_C}{l_{BC}} = \frac{2.9743\times10^{-6}\ P}{9} = 0.3305\times10^{-6}\ P$$

$$\delta_A = \delta_B + l_{AB}\,\theta$$
$$= 2.3333\times10^{-6}P + (5)(0.3305\times10^{-6})P$$
$$= 3.9858\times10^{-6}\ P$$

Apply displacement limit $\quad \delta_A = 0.014 \text{ in} = 3.9858\times10^{-6}\ P$

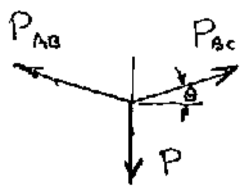
$$P = \frac{0.014}{3.9858\times10^{-6}} = 3.51\times10^3 \text{ lb} = 3.51 \text{ kips}$$

Problem 2.126



2.126 The uniform wire ABC, of unstretched length $2l$, is attached to the supports shown and a vertical load **P** is applied at the midpoint B. Denoting by A the cross-sectional area of the wire and by E the modulus of elasticity, show that, for $\delta << l$, the deflection at the midpoint B is

$$\delta = l\sqrt[3]{\frac{P}{AE}}$$



Use approximation

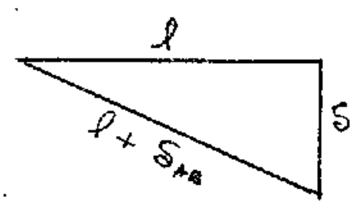
$$\sin\theta \approx \tan\theta \approx \frac{\delta}{l}$$

Statics $\quad \Sigma F_y = 0 \qquad 2P_{AB}\sin\theta - P = 0$

$$P_{AB} = \frac{P}{2\sin\theta} \approx \frac{Pl}{2\delta}$$

Elongation $\quad \delta_{AB} = \frac{P_{AB}\,l}{AE} = \frac{Pl^2}{2AE\delta}$

Deflection



From the right triangle

$$(l + \delta_{AB})^2 = l^2 + \delta^2$$

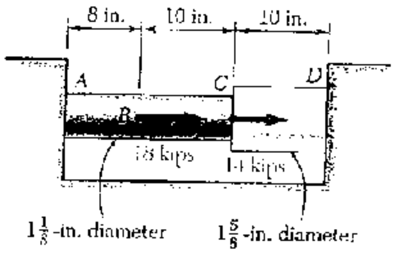
$$\delta^2 = \cancel{l^2} + 2l\,\delta_{AB} + \delta_{AB}^2 - \cancel{l^2}$$

$$= 2l\,\delta_{AB}\left(1 + \frac{1}{2}\frac{\delta_{AB}}{l}\right) \approx 2l\,\delta_{AB}$$

$$\approx \frac{Pl^3}{AE\delta}$$

$$\delta^3 \approx \frac{Pl^3}{AE} \quad \therefore \quad \delta \approx l\sqrt[3]{\frac{P}{AE}}$$

Problem 2.127



2.127 Two cylindrical rods, CD made of steel ($E = 29 \times 10^6$ psi) and AC made of aluminum ($E = 10.4 \times 10^6$ psi), are joined at C and restrained by rigid supports at A and D. Determine (*a*) the reactions at A and D, (*b*) the deflection of point C.

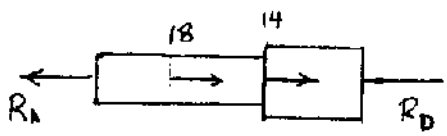
AB: $P = R_A, \quad L_{AB} = 8 \text{ in}$

$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(1.125)^2 = 0.99402 \text{ in}^2$

$\delta_{AB} = \frac{PL}{EA}$

$= \frac{R_A(8)}{(10.4\times10^6)(0.99402)}$

$= 0.77386 \times 10^{-6}\, R_A$



BC: $P = R_A - 18\times10^3, \quad L = 10 \text{ in}, \quad A = 0.99402 \text{ in}^2$

$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 18\times10^3)(10)}{(10.4\times10^6)(0.99402)} = 0.96732\times10^{-6} R_A - 17.412\times10^{-3}$

CD: $P = R_A - 18\times10^3 - 14\times10^3 = R_A - 32\times10^3$

$L = 10 \text{ in} \qquad A = \frac{\pi}{4} d_{CD}^2 = \frac{\pi}{4}(1.625)^2 = 2.0739 \text{ in}^2$

$\delta_{CD} = \frac{PL}{EA} = \frac{(R_A - 32\times10^3)(10)}{(29\times10^6)(2.0739)} = 0.16627\times10^{-6} R_A - 5.321\times10^{-3}$

$\delta_{AD} = \delta_{AB} + \delta_{BC} + \delta_{CD} = 1.9075\times10^{-6} R_A - 22.733\times10^{-3}$

Since point D cannot move relative to A $\quad \delta_{AD} = 0$

(a) $1.9075\times10^{-6} R_A - 22.733\times10^{-3} = 0 \qquad R_A = 11.92\times10^3 \text{ lb.} \leftarrow$

$R_D = 32\times10^3 - R_A = 20.08\times10^3 \text{ lb} \leftarrow$

(b) $\delta_C = \delta_{AB} + \delta_{CD}$

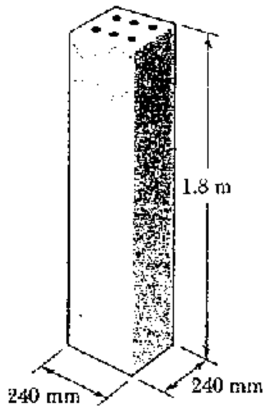
$= 1.7412\times10^{-6} R_A - 17.412\times10^{-3}$

$= (1.7412\times10^{-6})(11.92\times10^3) - 17.412\times10^{-3} = 3.34\times10^{-3} \text{ in}$

or $\delta_C = \frac{R_D L_{CD}}{E_{CD} A_{CD}} = \frac{(20.08\times10^3)(10)}{(29\times10^6)(2.0739)} = 3.34\times10^{-3} \text{ in}$

Problem 2.128

2.128 The concrete post (E_c = 25 GPa and α_c = 9.9 × 10⁻⁶/°C) is reinforced with six steel bars, each of 22-mm diameter (E_s = 200 GPa and α_s = 11.7 × 10⁻⁶/°C). Determine the normal stresses induced in the steel and in the concrete by a temperature rise of 35°C.



$$A_s = 6\frac{\pi}{4}d^2 = 6\cdot\frac{\pi}{4}(22)^2 = 2.2808\times10^3\ \text{mm}^2 = 2.2808\times10^{-3}\ \text{m}^2$$

$$A_c = 240^2 - A_s = 240^2 - 2.2808\times10^3 = 55.32\times10^3\ \text{mm}^2 = 55.32\times10^{-3}\ \text{m}^2$$

Let P_c = tensile force developed in the concrete

For equilibrium with zero total force, the compressive force in the six steel rods is P_c

Strains: $\varepsilon_s = -\dfrac{P_c}{E_sA_s} + \alpha_s(\Delta T)$, $\quad \varepsilon_c = \dfrac{P_c}{E_cA_c} + \alpha_c(\Delta T)$

Matching: $\varepsilon_c = \varepsilon_s \qquad \dfrac{P_c}{E_cA_c} + \alpha_c(\Delta T) = -\dfrac{P_c}{E_sA_s} + \alpha_s(\Delta T)$

$$\left(\frac{1}{E_cA_c} + \frac{1}{E_sA_s}\right)P_c = (\alpha_s - \alpha_c)(\Delta T)$$

$$\left[\frac{1}{(25\times10^9)(55.32\times10^{-3})} + \frac{1}{(200\times10^9)(2.2808\times10^{-3})}\right]P_c = (1.8\times10^{-6})(35)$$

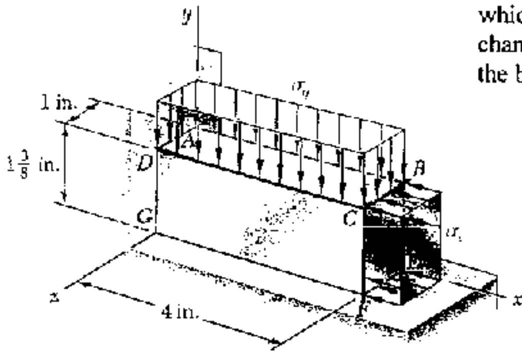
$$P_c = 21.61\times10^3\ \text{N}$$

$$\sigma_c = \frac{P_c}{A_c} = \frac{21.61\times10^3}{55.32\times10^{-3}} = 0.391\times10^6\ \text{Pa} = 0.391\ \text{MPa} \blacktriangleleft$$

$$\sigma_s = -\frac{P_c}{A_c} = \frac{21.61\times10^3}{2.2808\times10^{-3}} = -9.47\times10^6\ \text{Pa} = -9.47\ \text{MPa} \blacktriangleleft$$

Problem 2.129

2.129 The block shown is made of a magnesium alloy for which $E = 6.5 \times 10^6$ psi and $\nu = 0.35$. Knowing that $\sigma_x = -20$ ksi, determine (*a*) the magnitude of σ_y for which the change in the height of the block will be zero, (*b*) the corresponding change in the area of the face *ABCD*, (*c*) the corresponding change in the volume of the block.



$\delta_y = 0 \qquad \varepsilon_y = 0$

$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = 0$

(a) $\sigma_y = \nu\sigma_x = (0.35)(-20 \times 10^3)$

$= -7 \times 10^3 \text{ psi} = -7 \text{ ksi}$

(b) $\varepsilon_z = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y) = -\frac{\nu(\sigma_x + \sigma_y)}{E}$

$= \frac{(0.35)(-20 \times 10^3 - 7 \times 10^3)}{6.5 \times 10^6} = 1.4538 \times 10^{-3}$

$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{-20 \times 10^3 - (0.35)(-7 \times 10^3)}{6.5 \times 10^6}$

$= -2.7 \times 10^{-3}$

$A_o + \Delta A = L_x(1 + \varepsilon_x)L_z(1 + \varepsilon_z) = L_x L_z(1 + \varepsilon_x + \varepsilon_z + \varepsilon_x\varepsilon_z)$

But $A_o = L_x L_z$

$\Delta A = L_x L_z(\varepsilon_x + \varepsilon_z + \varepsilon_x\varepsilon_z)$

$= (4.0)(1.0)(1.4538 \times 10^{-3} - 2.7 \times 10^{-3} + \text{small term})$

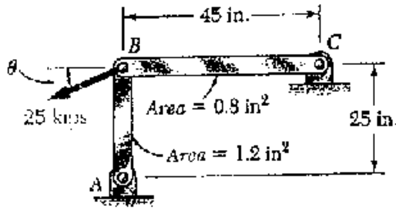
$= -4.98 \times 10^{-3} \text{ in}^2 = -0.00498 \text{ in}^2$

(c) Since L_y is constant

$\Delta V = L_y(\Delta A) = (1.375)(-4.98 \times 10^{-3}) = -6.85 \times 10^{-3} \text{ in}^3$

$= -0.00685 \text{ in}^3$

Problem 2.130

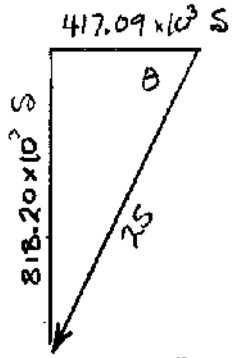


2.130 Knowing that $E = 29 \times 10^6$ psi, determine (*a*) the value of θ for which the deflection of point B is down and to the left along a line forming an angle of 36° with the horizontal, (*b*) the corresponding magnitude of the deflection of B.

$$\delta_{BC} = \delta \cos 36^\circ$$

$$P_{BC} = \frac{E A_{BC} \delta_{BC}}{L_{BC}} = \frac{(29 \times 10^6)(0.8)\,\delta \cos 36^\circ}{45}$$

$$= 417.09 \times 10^3\ \delta$$



$$\delta_{AB} = \delta \sin 36^\circ$$

$$P_{AC} = \frac{E A_{AC} \delta_{AC}}{L_{AC}} = \frac{(29 \times 10^6)(1.2)\,\delta \sin 36^\circ}{25}$$

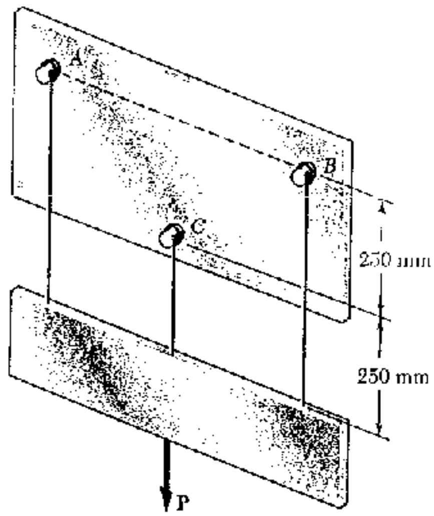
$$= 818.20 \times 10^3\ \delta$$

$$\tan \theta = \frac{818.20 \times 10^3\ \delta}{417.09 \times 10^3\ \delta} = 1.9617 \qquad \theta = 63.0^\circ$$

$$P = 25 \times 10^3 = \sqrt{(417.09 \times 10^3\ \delta)^2 + (818.20 \times 10^{-3}\ \delta)^2} = 918.38 \times 10^3\ \delta$$

$$\delta = \frac{25 \times 10^3}{918.38 \times 10^3} = 0.0272 \text{ in.}$$

Problem 2.131



2.131 Steel wires of 3.25-mm diameter are used at A and B while an aluminum wire of 2-mm diameter is used at C. Knowing that each wire is initially taut, determine the additional tension in each wire when a 900-N force **P** is applied to the midpoint of the lower edge of the plate. Use $E_s = 200$ GPa for steel and $E_a = 70$ GPa for aluminum.

Let P_A, P_B, and P_C be the wire tensions.

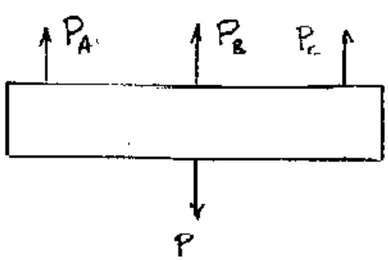
By symmetry $P_A = P_B$, $\delta_A = \delta_B$

Also $\delta_C = \delta_A = \delta_B = \delta$

$$A_A = \frac{\pi}{4}(3.25)^2 = 8.2958 \text{ mm}^2 = 8.2958 \times 10^{-6} \text{ m}^2$$

$$A_B = A_A$$

$$L_A = L_B = 500 \text{ mm} = 0.500 \text{ m}$$



$$P_A = P_B = \frac{E_A A_A \delta}{L_A} = \frac{(200\times10^9)(8.2958\times10^{-6})}{0.500}\delta$$

$$= 3.3183 \times 10^6 \ \delta$$

$$A_C = \frac{\pi}{4}(2)^2 = 3.1416 \text{ mm}^2 = 3.1416 \times 10^{-6} \text{ m}^2$$

$$L_C = 250 \text{ mm} = 0.250 \text{ m}$$

$$P_C = \frac{E_C A_C \delta}{L_C} = \frac{(70\times10^9)(3.1416\times10^{-6})}{0.250}$$

$$= 0.87965 \times 10^{-6} \ \delta$$

$+\uparrow \Sigma F_y = 0 \qquad P_A + P_B + P_C - P = 0$

$$(3.3183\times10^6)\delta + (3.3183\times10^6)\delta + (0.87965\times10^6)\delta - 900 = 0$$

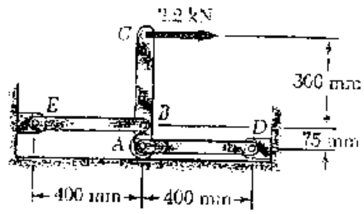
$$\delta = 119.74 \times 10^{-6} \text{ m}$$

$$P_A = P_B = (3.3183\times10^6)(119.74\times10^{-6}) = 397 \text{ N}$$ ◄

$$P_C = (0.87965\times10^{-6})(119.74\times10^{-6}) = 105.3 \text{ N}$$ ◄

Problem 2.132

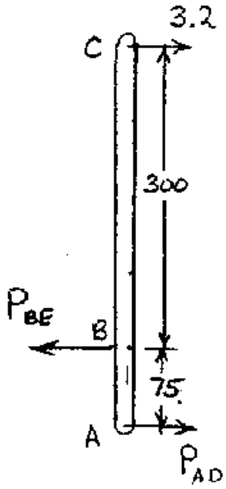
2.132 The steel bars BE and AD each have a 6×18-mm cross section. Knowing that E = 200 GPa, determine the deflections of points A, B, and C of the rigid bar ABC.



Use rigid bar ABC as a free body

$$\circlearrowleft + \; \Sigma M_B = 0 \qquad (75)P_{AD} - (300)(3.2) = 0$$

$$P_{AD} = 12.8 \text{ kN}$$



$$\xrightarrow{+} \Sigma F_x = 0 \qquad -P_{BE} + 3.2 + P_{AD} = 0$$

$$P_{BE} = 16 \text{ kN}$$

Deformations

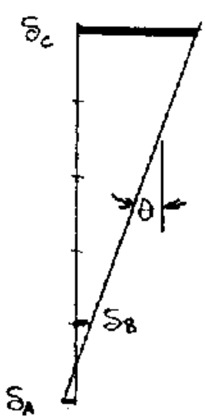
$$A = (6)(18) = 108 \text{ mm}^2 = 108 \times 10^{-6} \text{ m}^2$$

$$\leftarrow \; \delta_A = \delta_{AD} = \frac{P_{AD} L_{AD}}{EA} = \frac{(12.8 \times 10^3)(400 \times 10^{-3})}{(200 \times 10^9)(108 \times 10^{-6})}$$

$$= 237.04 \times 10^{-6} \text{ m} = 0.237 \text{ mm} \leftarrow$$

$$\leftarrow \; \delta_B = \delta_{BE} = \frac{P_{BE} L_{BE}}{EA} = \frac{(16 \times 10^3)(400 \times 10^{-3})}{(200 \times 10^9)(108 \times 10^{-6})}$$

$$= 296.30 \times 10^{-6} \text{ m} = 0.296 \text{ mm} \rightarrow$$



$$\theta = \frac{\delta_A + \delta_B}{L_{AB}} = \frac{(237.04 + 296.30)(10^{-6})}{75 \times 10^{-3}}$$

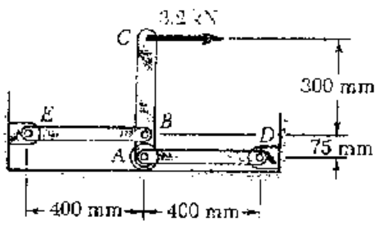
$$= 7.1112 \times 10^{-3}$$

$$\delta_C = \delta_B + L_{BC}\,\theta$$

$$= 296.30 \times 10^{-6} + (300 \times 10^{-3})(7.1112 \times 10^{-3})$$

$$= 2.4297 \times 10^{-3} \text{ m} = 2.43 \text{ mm} \rightarrow$$

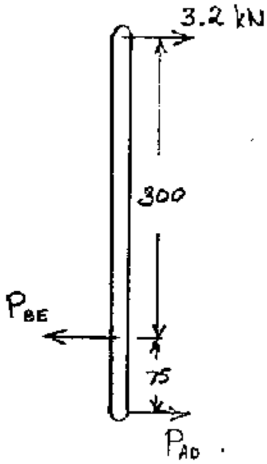
Problem 2.133



2.133 In Prob. 2.132, the 3.2-kN force caused point C to deflect to the right. Using $\alpha = 11.7 \times 10^{6}/°C$, determine the ($a$) the overall change in temperature that causes point C to return to its original position, (b) the corresponding total deflection of points A and B.

2.132 The steel bars BE and AD each have a 6×18-mm cross section. Knowing that $E = 200$ GPa, determine the deflections of points A, B, and C of the rigid bar ABC.

Use rigid ABC as a free body



$\circlearrowleft \Sigma M_B = 0 \qquad 75\, P_{AD} - (300)(3.2) = 0$

$P_{AD} = 12.8 \text{ kN}$

$\xrightarrow{+} \Sigma F_x = 0 \qquad -P_{BE} + 3.2 + P_{AD} = 0$

$P_{BE} = 16 \text{ kN}$

Deformations:

$$\leftarrow \delta_A = \delta_{AD} = \frac{P_{AD} L_{AD}}{EA} + L_{AD}\alpha(\Delta T)$$

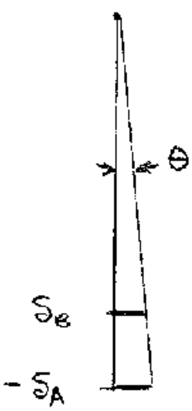
$$= \frac{(12.8\times10^3)(400\times10^{-3})}{(200\times10^9)(108\times10^{-6})} + (400\times10^{-3})(11.7\times10^{-6})(\Delta T)$$

$$= 237.04\times10^{-6} + 4.68\times10^{-6}(\Delta T)$$

$$\rightarrow \delta_B = \delta_{BE} = \frac{P_{BE} L_{BE}}{EA} + L_{BE}\alpha(\Delta T)$$

$$= \frac{(16\times10^3)(400\times10^{-3})}{(200\times10^9)(108\times10^{-6})} + (400\times10^{-3})(11.7\times10^{-6})(\Delta T)$$

$$= 296.30\times10^{-6} + 4.68\times10^{-6}(\Delta T)$$



$\delta_C = 0 \qquad \delta_B = 0.300\,\theta \qquad -\delta_A = 0.375\,\theta$

$$-\delta_A = \frac{0.375}{0.300}\delta_A = 1.25\,\delta_A$$

$$-(237.04\times10^{-6} + 4.68\times10^{-6}(\Delta T)) = (1.25)[296.30\times10^{-6} + 4.68\times10^{-6}(\Delta T)]$$

$$-10.53\times10^{-6}(\Delta T) = 607.415\times10^{-6} \quad \therefore \quad \Delta T = -57.684 \text{ C}$$

$$= -57.7\ °C$$

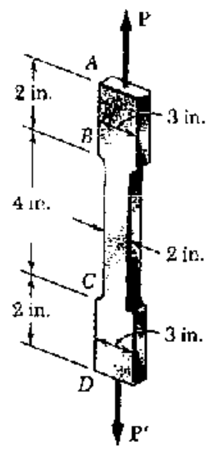
$$\delta_A = 237.04\times10^{-6} - (4.68\times10^{-6})(57.684) = -32.92\times10^{-6} \text{ m}$$

$$\delta_A = 0.0329 \text{ mm} \rightarrow$$

$$\delta_B = 296.30\times10^{-6} - (4.68\times10^{-6})(57.684) = +26.34\times10^{-6} \text{ m}$$

$$\delta_B = 0.0263 \text{ mm} \rightarrow$$

Problem 2.134



2.134 The steel tensile specimen $ABCD$ ($E = 29 \times 10^6$ psi and $\sigma_Y = 50$ ksi) is loaded in tension until the maximum strain is $\epsilon = 0.0025$. (*a*) Neglecting the effect of the fillets on the change in length of the specimen, determine the resulting overall length AD of the specimen after the load is removed. (*b*) Following the removal of the load in part *a*, a compressive load is applied until the maximum compressive strain is $\epsilon = 0.0020$. Determine the resulting change in length AD after the compressive load is removed.

(a) $\varepsilon_Y = \dfrac{\sigma_Y}{E} = \dfrac{50 \times 10^3}{29 \times 10^6} = 0.001724$

$\varepsilon_{max} = 0.0025 > \varepsilon_Y$ Yielding occurs in portion BC

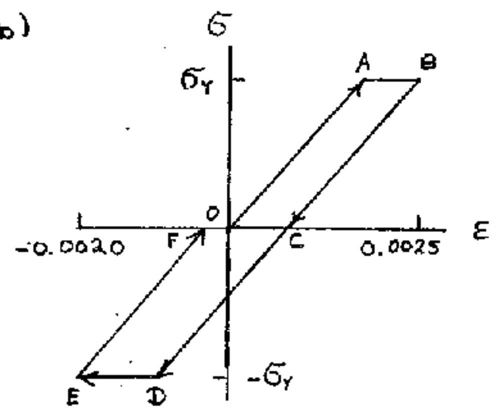
$\sigma_{BC} = \sigma_Y = 50 \times 10^3$ psi

Permanent strain in BC

$\varepsilon_{BC} = \varepsilon_{max} - \varepsilon_Y = 0.0025 - 0.001724 = 0.000776$

$\delta_{BC} = L_{BC}\,\varepsilon_B = (4)(0.000776) = 0.00310$ in. ◄

(b)



In reversed loading, at point E on stress-strain plot

$\varepsilon = -0.0020$

as given. During removal of the reversed load, the change in strain is $\sigma_Y / E = 0.001724$.

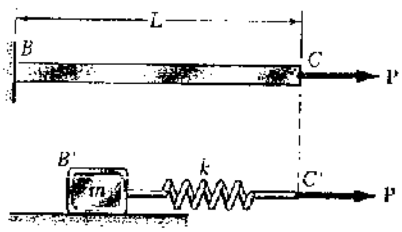
The permanent strain in BC is

$\varepsilon_{BC} = -0.0020 + 0.001724 = 0.000276$

$\delta_{BC} = L\,\varepsilon_{BC} = (4)(-0.000276) = -0.001104$ in. ◄

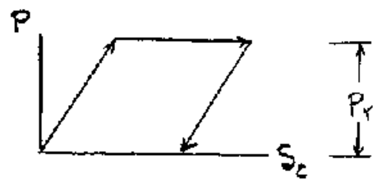
Note that portions AB and CD are always elastic, thus their deformations during loading and unloading do not contribute to any permanent deformation. Hence $\delta_{AD} = \delta_{BC}$ whenever the load P is zero.

Problem 2.135



2.135 The uniform rod BC has a cross-sectional area A and is made of a mild steel which can be assumed to be elastoplastic with a modulus of elasticity E and a yield strength σ_y. Using the block-and-spring system shown, it is desired to simulate the deflection of end C of the rod as the axial force **P** is gradually applied and removed, that is, the deflection of points C and C' should be the same for all values of P. Denoting by μ the coefficient of friction between the block and the horizontal surface, derive and expression for (a) the required mass m of the block, (b) the required constant k of the spring.

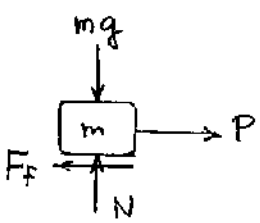
Force-deflection diagram for point C of rod BC.



For $P < P_Y = A\sigma_Y$

$$\delta_C = \frac{PL}{EA} \qquad P = \frac{EA}{L}\delta_C$$

$$P_{max} = P_Y = A\sigma_Y$$



Force-deflection diagram for point C' of block and spring system.

$+\uparrow \Sigma F_y = 0: \quad N - mg = 0 \qquad N = mg$

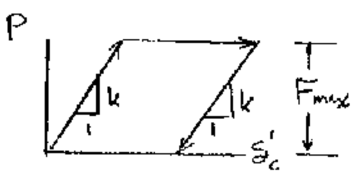
$\xrightarrow{+} \Sigma F_x = 0: \quad P - F_f = 0 \qquad P = F_f$

If block does not move, i.e $F_f < \mu N = \mu mg$ or $P < \mu mg$,

then $\delta_c' = \dfrac{P}{k}$ or $P = k\delta_c'$

If $P = \mu mg$, then slip at $P = F_m = \mu mg$ occurs.

If the force P is the removed, the spring returns to its initial length.



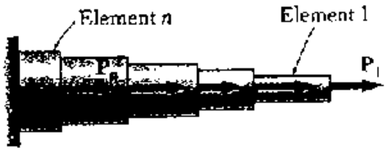
(a) Equating P_Y and F_{max}

$$A\sigma_Y = \mu mg \qquad m = \frac{A\sigma_Y}{\mu g}$$

(b) Equating slopes:

$$k = \frac{EA}{L}$$

PROBLEM 2.C1



2.C1 A rod consisting of n elements, each of which is homogeneous and of uniform cross section, is subjected to the loading shown. The length of element i is denoted by L_i, its cross-sectional area by A_i, modulus of elasticity by E_i, and the load applied to its right end by $\mathbf{P}_i$, the magnitude P_i of this load being assumed to be positive if $\mathbf{P}_i$ is directed to the right and negative otherwise. (*a*) Write a computer program that can be used to determine the average normal stress in each element, the deformation of each element, and the total deformation of the rod. (*b*) Use this program to solve Probs. 2.18 and 2.19.

SOLUTION

FOR EACH ELEMENT, ENTER

L_i, A_i, E_i

COMPUTE DEFORMATION

UPDATE AXIAL LOAD $P = P + P_i$

COMPUTE FOR EACH ELEMENT

$\sigma_i = P/A_i$

$\delta_i = PL_i/A_iE_i$

TOTAL DEFORMATION:

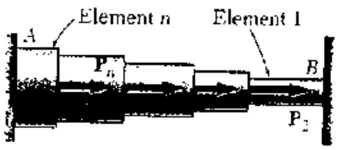
UPDATE THROUGH n ELEMENTS

$\delta = \delta + \delta_i$

PROGRAM OUTPUT

```
Problem  2.18
  Element  Stress (MPa)  Deformation (mm)

     1       19.0986        .1091
     2      -12.7324       -.0909
  Total Deformation =     .0182 mm

Problem  2.19
  Element  Stress (ksi)  Deformation (in.)

     1        12.7324         .0176
     2        -2.8294        -.0057
  Total Deformation =   .01190 in.
```

PROBLEM 2.C2



2.C2 Rod AB is horizontal with both ends fixed; it consists of n elements, each of which is homogeneous and of uniform cross section, and is subjected to the loading shown. The length of element i is denoted by L_i, its cross-sectional area by A_i, its modulus of elasticity by E_i, and the load applied to its right end by $\mathbf{P}_i$, the magnitude P_i of this load being assumed to be positive if $\mathbf{P}_i$ is directed to the right and negative otherwise. (Note that $P_1 = 0$.) (*a*) Write a computer program which can be used to determine the reactions at A and B, the average normal stress in each element, and the deformation of each element. (*b*) Use this program to solve Probs. 2.39 and 2.40.

SOLUTION

WE CONSIDER THE REACTION AT B REDUNDANT AND RELEASE THE ROD AT B

COMPUTE δ_B WITH $R_B = 0$

FOR EACH ELEMENT, ENTER

$L_i,\ A_i,\ E_i$

UPDATE AXIAL LOAD

$P = P + P_i$

COMPUTE FOR EACH ELEMENT

$\sigma_i = P/A_i$

$\delta_i = PL_i/A_iE_i$

UPDATE TOTAL DEFORMATION

$\delta_B = \delta_B + \delta_i$

COMPUTE δ_B DUE TO UNIT LOAD AT B

UNIT $\sigma_i = 1/A_i$

UNIT $\delta_i = L_i/A_iE_i$

UPDATE TOTAL UNIT DEFORMATION

UNIT δ_B = UNIT δ_B + UNIT δ_i

SUPERPOSITION

FOR TOTAL DISPLACEMENT AT B = ZERO

$\delta_B + R_B$ UNIT $\delta_B = 0$

SOLVING:

$R_B = -\delta_B/$UNIT δ_B

THEN:

$R_A = \Sigma P_i + R_B$

CONTINUED

FOR EACH ELEMENT

$$\sigma = \sigma_i + R_B \text{ UNIT } \sigma_i$$

$$\delta = \delta_i + R_B \text{ UNIT } \delta_i$$

PROGRAM OUTPUT

```
Problem  2.39
     RA =  -62.809 kN
     RB =  -37.191 kN
   Element  Stress (MPa)  Deformation (mm)

     1       -52.615           -.05011
     2         3.974            .00378
     3         2.235            .00134
     4        49.982            .04498

Problem  2.40
     RA =  -45.479 kN
     RB =  -54.521 kN
   Element  Stress (MPa)  Deformation (mm)

     1       -77.131           -.03857
     2       -20.542           -.01027
     3       -11.555           -.01321
     4        36.191            .06204
```

PROBLEM 2.C3

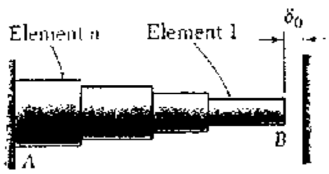


Fig. P2.C3

2.C3 Rod AB consists of n elements, each of which is homogeneous and of uniform cross section. End A is fixed, while initially there is a gap δ_0 between end B and the fixed vertical surface on the right. The length of element i is denoted by L_i, its cross-sectional area by A_i, its modulus of elasticity by E_i, and its coefficient of thermal expansion by α_i. After the temperature of the rod has been increased by ΔT, the gap at B is closed and the vertical surfaces exert equal and opposite forces on the rod. (a) Write a computer program which can be used to determine the magnitude of the reactions at A and B, the normal stress in each element, and the deformation of each element. (b) Use this program to solve Probs. 2.52, 2.53, 2.55, and 2.57.

SOLUTION

WE COMPUTE THE DISPLACEMENTS AT B ASSUMING THERE IS NO SUPPORT AT B:

ENTER L_i, A_i, E_i, α_i

ENTER TEMPERATURE CHANGE T

COMPUTE FOR EACH ELEMENT

$$\delta_i = \alpha_i L_i T$$

UPDATE TOTAL DEFORMATION

$$\delta_B = \delta_B + \delta_i$$

<u>COMPUTE δ_B DUE TO UNIT LOAD AT B</u>

$$\text{UNIT } \delta_i = L_i / A_i E_i$$

UPDATE TOTAL UNIT DEFORMATION

$$\text{UNIT } \delta_B = \text{UNIT } \delta_B + \text{UNIT } \delta_i$$

<u>COMPUTE REACTIONS</u>

FROM SUPERPOSITION

$$R_B = (\delta_B - \delta_0) / \text{UNIT } \delta_B$$

THEN

$$R_A = -R_B$$

<u>FOR EACH ELEMENT</u>

$$\sigma_i = -R_B / A_i$$

$$\delta_i = \alpha_i L_i T + R_B L_i / A_i E_i$$

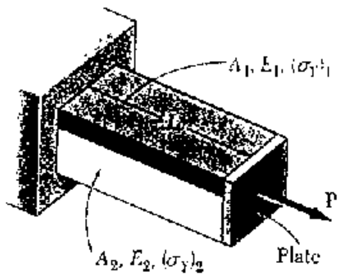
CONTINUED

PROBLEM 2.C3 CONTINUED

PROGRAM OUTPUT

```
Problem  2.52
 R =     27.204 kips

Element    Stress (ksi)   Deform.(10*-3 in.)

    1        -22.168         -4.103
    2         -6.842          4.103

Problem  2.53
 R =    125.628 kN

 Element    Stress (MPa)   Deform.(microm)

    1        -44.432           .500
    2        -99.972          -.500

Problem  2.55
 R =    232.390 kN

 Element    Stress (MPa)   Deform.(microm)

    1       -116.195        363.220
    2       -290.487        136.780

Problem  2.57
 R =     52.279 kips

 Element    Stress (ksi)   Deform.(10*-3 in.)

    1        -21.783          9.909
    2        -18.671         10.091
```

PROBLEM 2.C4



2.C4 Bar AB has a length L and is made of two different materials of given cross-sectional area, modulus of elasticity, and yield strength. The bar is subjected as shown to a load **P** which is gradually increased from zero until the deformation of the bar has reached a maximum value δ_m and then decreased back to zero. (*a*) Write a computer program which, for each of 25 values of δ_m equally spaced over a range extending from 0 to a value equal to 120% of the deformation causing both materials to yield, can be used to determine the maximum value P_m of the load, the maximum normal stress in each material, the permanent deformation δ_p of the bar, and the residual stress in each material. (*b*) Use this program to solve Probs. 2.109, and 2.110.

SOLUTION

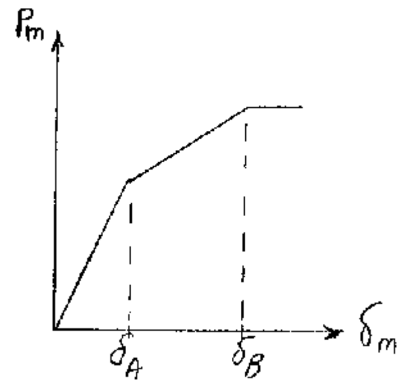
NOTE: THE FOLLOWING ASSUMES $(\sigma_Y)_1 < (\sigma_Y)_2$

DISPLACEMENT INCREMENT

$$\delta_m = 0.05\,(\sigma_Y)_2\,L/E_2$$

DISPLACEMENTS AT YIELDING

$$\delta_A = (\sigma_Y)_1\,L/E_1 \qquad \delta_B = (\sigma_Y)_2\,L/E_2$$



FOR EACH DISPLACEMENT

IF $\delta_m < \delta_A$:

$$\sigma_1 = \delta_m E_1/L$$
$$\sigma_2 = \delta_m E_2/L$$
$$P_m = (\delta_m/L)(A_1E_1 + A_2E_2)$$

IF $\delta_A < \delta_m < \delta_B$:

$$\sigma_1 = (\sigma_Y)_1$$
$$\sigma_2 = \delta_m E_2/L$$
$$P_m = A_1\sigma_1 + (\delta_m/L)A_2E_2$$

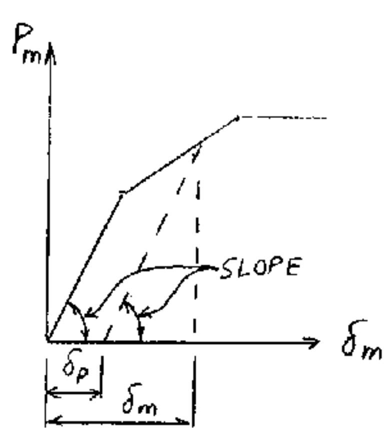
IF $\delta_m > \delta_B$:

$$\sigma_1 = (\sigma_Y)_1 \qquad \sigma_2 = (\sigma_Y)_2$$
$$P_m = A_1\sigma_1 + A_2\sigma_2$$

PERMANENT DEFORMATIONS, RESIDUAL STRESSES

SLOPE OF FIRST (ELASTIC) SEGMENT

$$\text{SLOPE} = (A_1E_1 + A_2E_2)/L$$
$$\delta_p = \delta_m - (P_m/\text{SLOPE})$$
$$(\sigma_1)_{res} = \sigma_1 - (E_1P_m/(L\ \text{SLOPE}))$$
$$(\sigma_2)_{res} = \sigma_2 - (E_2P_m/(L\ \text{SLOPE}))$$



CONTINUED

PROBLEM 2.C4 CONTINUED

PROGRAM OUTPUT

Problems 2.109 and 2.110

DM 10**-3 in.	PM kips	SIGM(1) ksi	SIGM(2) ksi	DP 10**-3 in.	SIGR(1) ksi	SIG(2) ksi	
.000	.000	.000	.000	.000	.000	.000	
2.414	8.750	5.000	5.000	.000	.000	.000	
4.828	17.500	10.000	10.000	.000	.000	.000	
7.241	26.250	15.000	15.000	.000	.000	.000	
9.655	35.000	20.000	20.000	.000	.000	.000	
12.069	43.750	25.000	25.000	.000	.000	.000	
14.483	52.500	30.000	30.000	.000	.000	.000	
16.897	61.250	35.000	35.000	.000	.000	.000	
19.310	70.000	40.000	40.000	.000	.000	.000	
21.724	78.750	45.000	45.000	.000	.000	.000	
24.138	87.500	50.000	50.000	.000	.000	.000	
26.552	91.250	50.000	55.000	1.379	-2.143	2.857	
28.966	95.000	50.000	60.000	2.759	-4.286	5.714	◄ 2.110
31.379	98.750	50.000	65.000	4.138	-6.429	8.571	
33.793	102.500	50.000	70.000	5.517	-8.571	11.429	
36.207	106.250	50.000	75.000	6.897	-10.714	14.286	
38.621	110.000	50.000	80.000	8.276	-12.857	17.143	◄ 2.109
41.034	113.750	50.000	85.000	9.655	-15.000	20.000	
43.448	117.500	50.000	90.000	11.034	-17.143	22.857	
45.862	121.250	50.000	95.000	12.414	-19.286	25.714	
48.276	125.000	50.000	100.000	13.793	-21.429	28.571	
50.690	125.000	50.000	100.000	16.207	-21.429	28.571	
53.103	125.000	50.000	100.000	18.621	-21.429	28.571	
55.517	125.000	50.000	100.000	21.034	-21.429	28.571	
57.931	125.000	50.000	100.000	23.448	-21.429	28.571	

PROBLEM 2.C5

2.C5 The plate has a hole centered across the width. The stress concentration factor for a flat bar under axial loading with a centric hole is:

$$K = 3.00 - 3.13\left(\frac{2r}{D}\right) + 3.66\left(\frac{2r}{D}\right)^2 - 1.53\left(\frac{2r}{D}\right)^3$$

where r is the radius of the hole and D is the width of the bar. Write a computer program to determine the allowable load **P** for the given values of r, D, the thickness t of the bar, and the allowable stress σ_{all} of the material. Knowing that $t = \frac{1}{4}$ in., $D = 3.0$ in., and $\sigma_{all} = 16$ ksi., determine the allowable load **P** for values of r from 0.125 in. to 0.75 in., using 0.125 in. increments.

SOLUTION

ENTER

r, D, t, σ_{all}

COMPUTE K

$RD = 2.0\ r/D$

$K = 3.00 - 3.13\ RD + 3.66\ RD^2 - 1.53\ RD^3$

COMPUTE AVERAGE STRESS

$\sigma_{ave} = \sigma_{all}/K$

ALLOWABLE LOAD

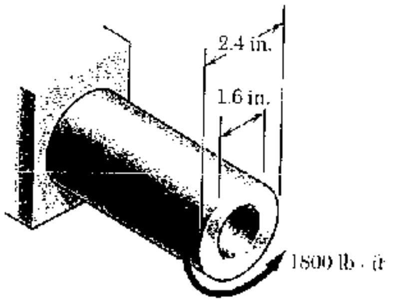
$P_{all} = \sigma_{ave}\ (D - 2.0\,r)\,t$

PROGRAM OUTPUT

Radius (in.)	Allowable Load (kips)
.1250	3.9802
.2500	3.8866
.3750	3.7154
.5000	3.4682
.6250	3.1523
.7500	2.7794

Chapter 3

Problem 3.1



3.1 (*a*) For the hollow shaft and loading shown, determine the maximum shearing stress. (*b*) Determine the diameter of a solid shaft for which the maximum shearing stress in the loading shown is the same as in part a.

$$c_1 = \tfrac{1}{2} d_1 = (\tfrac{1}{2})(1.6) = 0.8 \text{ in}$$

$$c_2 = \tfrac{1}{2} d_2 = (\tfrac{1}{2})(2.4) = 1.2 \text{ in.}$$

$$J = \tfrac{\pi}{2}(c_2^4 - c_1^4) = \tfrac{\pi}{2}(1.2^4 - 0.8^4) = 2.6138 \text{ in}^4$$

$$T = 1800 \text{ lb·ft} = 21600 \text{ lb·in}$$

(a) $$\tau_{max} = \frac{Tc_2}{J} = \frac{(21600)(1.2)}{2.6138} = 9917 \text{ psi}$$

$$\tau_{max} = 9.92 \text{ ksi}$$

(b) $\tau = \dfrac{Tc}{J}$, $\quad J = \dfrac{\pi}{2}c^4 \qquad \tau = \dfrac{2T}{\pi c^3}$

$$c^3 = \frac{2T}{\pi\tau} = \frac{(2)(21600)}{\pi(9917)} = 1.38666 \text{ in}^3$$

$c = 1.115$ in. $\qquad d = 2c = 2.23$ in. $\qquad$ $d = 2.23$ in.

Problem 3.2

3.2 (*a*) Determine the torque that can be applied to a solid shaft of 3.6-in. outer diameter without exceeding an allowable shearing stress of 10 ksi. (*b*) Solve part *a*, assuming that the solid shaft is replaced by a hollow shaft of the same mass and of 3.6- in. inner diameter.

(a) For the solid shaft $\quad c = \tfrac{1}{2} d = (\tfrac{1}{2})(3.6) = 1.8$ in.

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\pi}{2}(1.8)^3 = 9.1609 \text{ in}^3$$

$$\tau_{max} = \frac{Tc}{J} \quad \text{or} \quad T = \frac{\tau_{max} J}{c} = (10)(9.1609) = 91.609 \text{ kip·in}$$

$$T = 7.63 \text{ kip·ft}$$

(b) Hollow shaft: $\quad c_1 = \tfrac{1}{2} d_1 = (\tfrac{1}{2})(3.6) = 1.8$ in.

For equal masses the cross sectional areas must be equal.

$$A = \pi c^2 = \pi(c_2^2 - c_1^2) \quad \text{or} \quad c_2 = \sqrt{c_1^2 + c^2}$$

$$c_2 = \sqrt{(1.8)^2 + (1.8)^2} = 2.5456 \text{ in.}$$

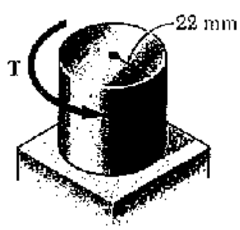
$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = 49.469 \text{ in}^4$$

$$T = \frac{\tau_{max} J}{c_2} = \frac{(10)(49.469)}{2.5456} = 194.33 \text{ kip·in}$$

$$T = 16.19 \text{ kip·ft}$$

Problem 3.3

3.3 Determine the torque T that causes a maximum shearing stress of 80 MPa in the steel cylindrical shaft shown.



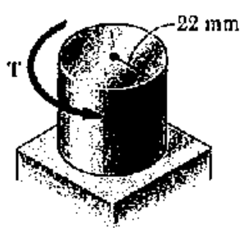
$$\tau_{max} = \frac{Tc}{J} \qquad J = \frac{\pi}{2} c^4$$

$$T = \frac{\pi}{2} c^3 \tau_{max} = \frac{\pi}{2}(0.022)^3(80 \times 10^6)$$

$$= 1338.1 \text{ N}\cdot\text{m} \qquad T = 133.8 \text{ kN}\cdot\text{m} \blacktriangleleft$$

Problem 3.4

3.4 For the cylindrical shaft shown, determine the maximum shearing stress caused by a torque of magnitude $T = 1.5$ kN · m.



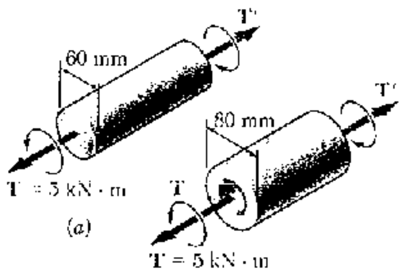
$$\tau_{max} = \frac{Tc}{J} \qquad J = \frac{\pi}{2} c^4$$

$$\tau_{max} = \frac{2T}{\pi c^3} = \frac{(2)(1500)}{\pi (0.022)^3} = 89.682 \times 10^6$$

$$\tau_{max} = 89.7 \text{ MPa} \blacktriangleleft$$

Problem 3.5

3.5 (*a*) For the 60-mm-diameter solid cylinder and loading shown, determine the maximum shearing stress. (*b*) Determine the inner diameter of the hollow cylinder, of 80-mm outer diameter, for which the maximum stress is the same as in part a.



(a) Solid shaft: $c = \frac{1}{2}d = \frac{1}{2}(0.060) = 0.030\ m$

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(5000)}{\pi(0.030)^3}$$

$$= 117.893 \times 10^6 \qquad \tau_{max} = 117.9\ MPa$$

(b) Hollow shaft: $c_2 = \frac{1}{2}d = \frac{1}{2}(0.080) = 0.040\ m$

$$\frac{J}{c_2} = \frac{\frac{\pi}{2}(c_2^4 - c_1^4)}{c_2} = \frac{T}{\tau_{max}}$$

$$c_1^4 = c_2^4 - \frac{2Tc_2}{\pi \tau_{max}} = (0.040)^4 - \frac{(2)(5000)(0.040)}{\pi(117.893 \times 10^6)} = 1.48 \times 10^{-6}\ m^4$$

$$c_1 = 0.032237\ m = 34.879\ mm \qquad d_1 = 2c_1 = 69.8\ mm$$

Problem 3.6

3.6 (*a*) Determine the torque that can be applied to a solid shaft of 20-mm diameter without exceeding an allowable shearing stress of 80 MPa. (*b*) Solve part *a*, assuming that the solid shaft has been replaced by a hollow shaft of the same cross-sectional area and with an inner diameter equal to half of its own outer diameter .

(a) Solid shaft: $c = \frac{1}{2}d = \frac{1}{2}(0.020) = 0.010\ m$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.10)^4 = 15.7080 \times 10^{-9}\ m^4$$

$$T = \frac{J\tau_{max}}{c} = \frac{(15.7080 \times 10^{-9})(80 \times 10^6)}{0.010} = 125.664 \qquad T = 125.7\ N\cdot m$$

(b) Hollow shaft: Same area as solid shaft.

$$A = \pi(c_2^2 - c_1^2) = \pi\left[c_2^2 - (\tfrac{1}{2}c_2)^2\right] = \frac{3}{4}\pi c_2^2 = \pi c^2$$

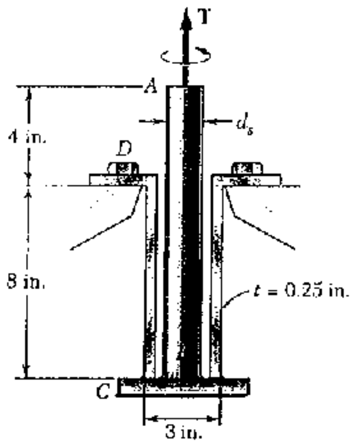
$$c_2 = \frac{2}{\sqrt{3}}c = \frac{2}{\sqrt{3}}(0.010) = 0.0115470\ m$$

$$c_1 = \frac{1}{2}c_2 = 0.0057735\ m$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.0115470^4 - 0.0057735^4) = 26.180 \times 10^{-9}\ m^4$$

$$T = \frac{\tau_{max} J}{c_2} = \frac{(80 \times 10^6)(26.180 \times 10^{-9})}{0.0115470} = 181.38 \qquad T = 181.4\ N\cdot m$$

Problem 3.7



3.7 The solid spindle AB has a diameter $d_s = 1.5$ in. and is made of a steel with an allowable shearing stress of 12 ksi, while sleeve CD is made of a brass with an allowable shearing stress of 7 ksi. Determine the largest torque **T** that can be applied at A.

Solid spindle AB: $\quad c = \frac{1}{2}d_s = \frac{1}{2}(1.5) = 0.75$ in.

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.75)^4 = 0.49701 \text{ in}^4$$

$$\tau_{max} = \frac{Tc}{J}$$

$$T_{AB} = \frac{J\tau_{al}}{c} = \frac{(0.49701)(12)}{0.75} = 7.952 \text{ kip}\cdot\text{in}$$

Sleeve CD: $\quad c_2 = \frac{1}{2}d_2 = \frac{1}{2}(3.0) = 1.5$ in.

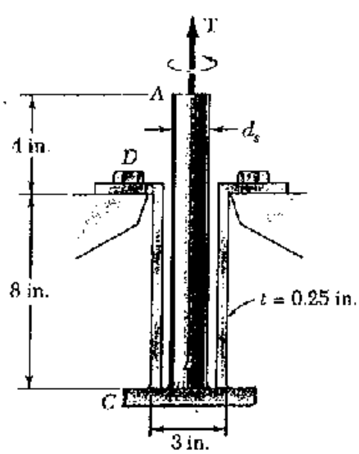
$$c_1 = c_2 - t = 1.5 - 0.25 = 1.25 \text{ in.}$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(1.5^4 - 1.25^4) = 4.1172 \text{ in}^4$$

$$T_{CD} = \frac{J\tau_{al}}{c_2} = \frac{(4.1172)(7)}{1.5} = 19.213 \text{ kip}\cdot\text{in}$$

Allowable value of torque T is the smaller. $\qquad T = 7.95$ kip·in ◄

Problem 3.8



3.8 The solid spindle AB is made of a steel with an allowable shearing stress of 12 ksi, and sleeve CD is made of a brass with an allowable shearing stress of 7 ksi. Determine (*a*) the largest torque **T** that can be applied at A if the allowable shearing stress is not to be exceeded in sleeve CD, (*b*) the corresponding required value of the diameter d_s of spindle AB.

(a) Sleeve CD: $\quad c_2 = \frac{1}{2}d_2 = \frac{1}{2}(3.0) = 1.5$ in.

$$c_1 = c_2 - t = 1.5 - 0.25 = 1.25 \text{ in.}$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(1.5^4 - 1.25^4) = 4.1172 \text{ in}^4$$

$$\tau_{max} = \frac{Tc_2}{J}$$

$$T_{CD} = \frac{J\tau_{CD}}{c_2} = \frac{(4.1172)(7)}{1.5} = 19.213 \text{ kip}\cdot\text{in}$$

For equilibrium $\qquad T = 19.21$ kip·in ◄

(b) Solid spindle AB: $\qquad T = 19.213$ kip·in

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

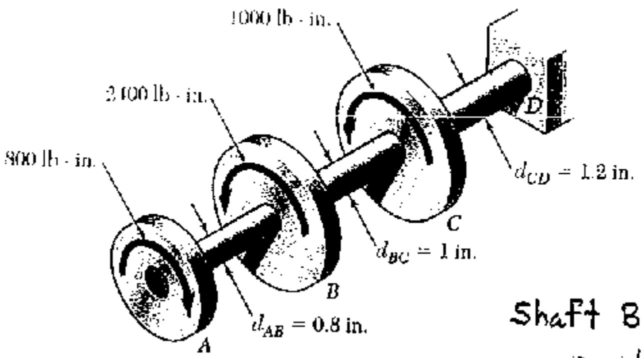
$$c = \sqrt[3]{\frac{2T}{\pi\tau}} = \sqrt[3]{\frac{(2)(19.213)}{\pi(12)}} = 1.0064 \text{ in.}$$

$$d_s = 2c = (2)(1.0064)$$

$d_s = 2.01$ in. ◄

Problem 3.9

3.9 Knowing that each of the shafts AB, BC, and CD consist of solid circular rods, determine (*a*) the shaft in which the maximum shearing stress occurs, (*b*) the magnitude of that stress.



Shaft AB: $T = 800$ lb·in

$c = \frac{1}{2}d = 0.4$ in.

$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$\tau_{max} = \frac{(2)(800)}{\pi(0.4)^3} = 7957$ psi

Shaft BC: $T = -800 + 2400 = 1600$ lb·in

$c = \frac{1}{2}d = 0.5$ in.

$\tau_{max} = \frac{(2)(1600)}{\pi(0.5)^3} = 8149$ psi (largest)

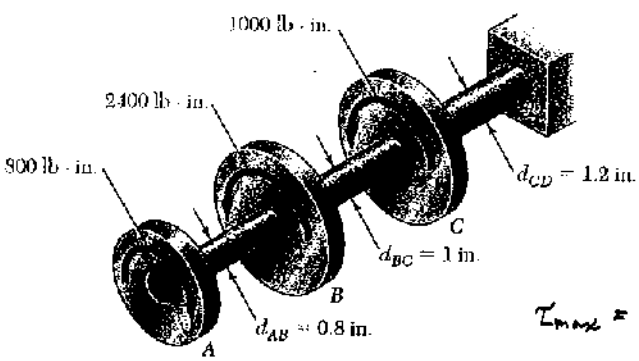
Shaft CD: $T = -800 + 2400 + 1000 = 2600$ lb·in $\quad c = \frac{1}{2}d = 0.6$ in.

$\tau_{max} = \frac{(2)(2600)}{\pi(0.6)^3} = 7663$ psi

Answers: (a) Shaft BC $\qquad$ (b) 8.15 ksi

Problem 3.10

3.10 Knowing that a 0.40-in.-diameter hole has been drilled through each of the shafts AB, BC, and CD, determine (*a*) the shaft in which the maximum shearing stress occurs, (*b*) the magnitude of that stress.



Hole: $c_1 = \frac{1}{2}d_1 = \frac{1}{2}(0.40) = 0.20$ in.

Shaft AB: $T = 800$ lb·in

$c_2 = \frac{1}{2}d_2 = 0.4$ in.

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.4^4 - 0.20^4)$

$= 0.037699$ in^4

$\tau_{max} = \frac{Tc_2}{J} = \frac{(800)(0.4)}{0.037699} = 8488$ psi (largest)

Shaft BC: $T = -800 + 2400 = 1600$ lb·in $\quad c_2 = \frac{1}{2}d_2 = 0.5$ in.

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.5^4 - 0.20^4) = 0.095661$ in^4

$\tau_{max} = \frac{Tc_2}{J} = \frac{(1600)(0.5)}{0.095661} = 8363$ psi

Shaft CD: $T = -800 + 2400 + 1000 = 2600$ lb·in $\quad c_2 = \frac{1}{2}d_2 = 0.6$ in.

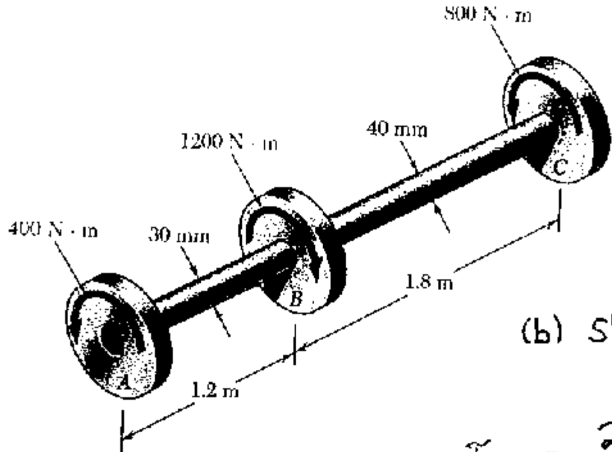
$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.6^4 - 0.20^4) = 0.20106$ in^4

$\tau_{max} = \frac{Tc_2}{J} = \frac{(2600)(0.6)}{0.20106} = 7759$ psi

Answers: (a) Shaft AB $\qquad$ (b) 8.49 ksi

Problem 3.11

3.11 The torques shown are exerted on pulleys A, B, and C. Knowing that both shafts are solid, determine the maximum shearing stress in (*a*) shaft AB, (*b*) shaft BC.



(a) Shaft AB: $T = 400$ N·m

$c = \frac{1}{2}d = \frac{1}{2}(0.030) = 0.015$ m

$J = \frac{\pi}{2}c^4 \qquad \tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$\tau_{max} = \frac{(2)(400)}{\pi(0.015)^3} = 75.5 \times 10^6$ Pa

$\tau_{max} = 75.5$ MPa ◄

(b) Shaft BC: $T = 800$ N·m

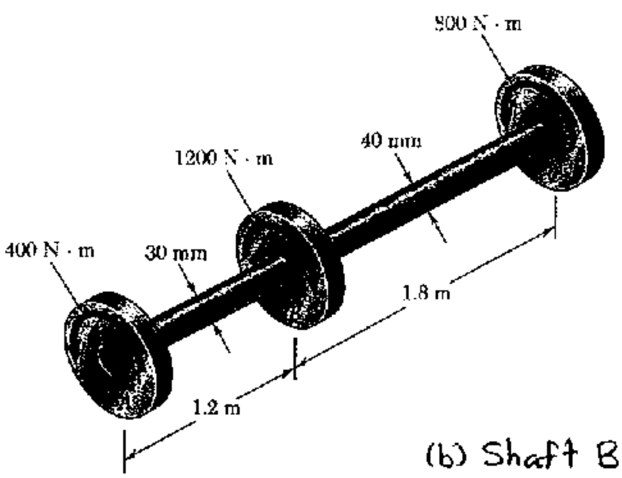
$c = \frac{1}{2}d = 0.020$ m

$\tau_{max} = \frac{2T}{\pi c^3} = \frac{(2)(800)}{\pi(0.020)^3} = 63.7 \times 10^6$ Pa

$\tau_{max} = 63.7$ MPa ◄

Problem 3.12

3.12 The shafts of the pulley assembly shown are to be redesigned. Knowing that the allowable shearing stress in each shaft is 60 MPa, determine the smallest allowable diameter of (*a*) shaft AB, (*b*) shaft BC.



(a) Shaft AB: $T = 400$ N·m

$\tau_{max} = 60$ MPa $= 60 \times 10^6$ Pa

$J = \frac{\pi}{2}c^4 \qquad \tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$c = \sqrt[3]{\frac{2T}{\pi \tau_{max}}} = \sqrt[3]{\frac{(2)(400)}{\pi(60 \times 10^6)}}$

$= 16.19 \times 10^{-3}$ m $= 16.19$ mm

$d_{AB} = 2c = 32.4$ mm ◄

(b) Shaft BC: $T = 800$ N·m

$\tau_{max} = 60$ MPa $= 60 \times 10^6$ Pa

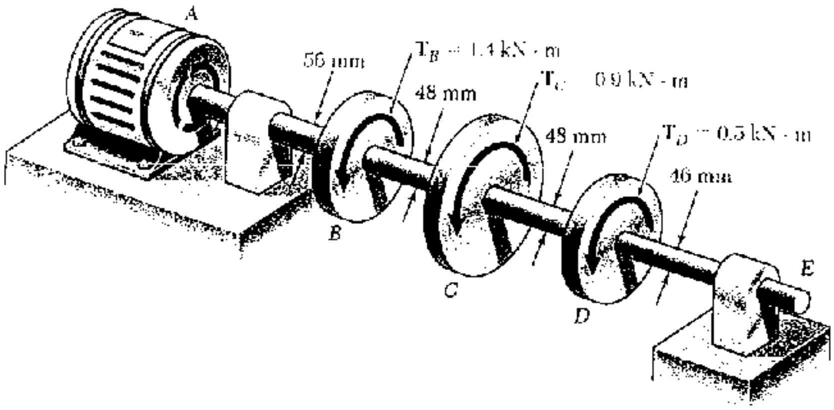
$c = \sqrt[3]{\frac{2T}{\pi c^3}} = \sqrt[3]{\frac{(2)(800)}{\pi(60 \times 10^6)}}$

$= 20.40 \times 10^{-3}$ m $= 20.40$ mm

$d_{BC} = 2c = 40.8$ mm ◄

Problem 3.13

3.13 Under normal operating conditions, the electric motor exerts a torque of 2.8 kN · m on shaft *AB*. Knowing that each shaft is solid, determine the maximum shearing stress in (*a*) shaft *AB*, (*b*) shaft *BC*, (*c*) shaft *CD*.



(a) Shaft AB: $T_{AB} = 2.8 \text{ kN}\cdot\text{m} = 2.8\times10^3 \text{ N}\cdot\text{m}$, $c = \frac{1}{2}d = 28 \text{ mm} = 0.028 \text{ m}$

$$\tau_{AB} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(2.8\times10^3)}{\pi(0.028)^3} = 81.20\times10^6 \text{ Pa} \qquad \tau_{AB} = 81.2 \text{ MPa}$$

(b) Shaft BC: $T_{BC} = 1.4 \text{ kN}\cdot\text{m} = 1.4\times10^3 \text{ N}\cdot\text{m}$, $c = \frac{1}{2}d = 24 \text{ mm} = 0.024 \text{ m}$

$$\tau_{BC} = \frac{2T}{\pi c^3} = \frac{(2)(1.4\times10^3)}{\pi(0.024)^3} = 64.47\times10^6 \text{ Pa} \qquad \tau_{BC} = 64.5 \text{ MPa}$$

(c) Shaft CD: $T_{CD} = 0.5 \text{ kN}\cdot\text{m} = 0.5\times10^3 \text{ N}\cdot\text{m}$, $c = \frac{1}{2}d = 24 \text{ mm} = 0.024 \text{ m}$

$$\tau_{CD} = \frac{2T}{\pi c^3} = \frac{(2)(0.5\times10^3)}{\pi(0.024)^3} = 23.03\times10^6 \text{ Pa} \qquad \tau_{CD} = 23.0 \text{ MPa}$$

Problem 3.14

3.14 In order to reduce the total mass of the assembly of Prob 3.13, a new design is being considered in which the diameter of shaft *BC* will be smaller. Determine the smallest diameter of shaft *BC* for which the maximum value of the shearing stress in the assembly will not be increased.

See the solution to Problem 3.13 for figure and for maximum shearing stresses in shafts AB, BC, and CD. The largest shearing occurs in shaft AB. Its magnitude is 81.2 MPa. Adjust the diameter of shaft BC so that its maximum shearing stress is 81.2 MPa.

$\tau_{BC} = 81.2\times10^6$ Pa $\qquad T_{BC} = 1.4\times10^3$ N·m

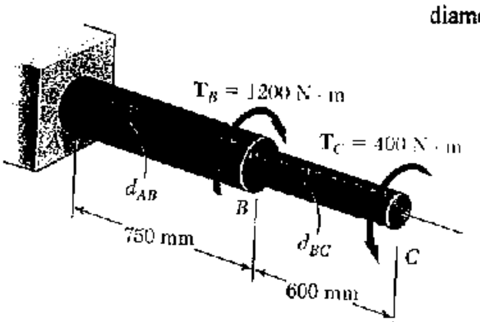
$$\tau_{BC} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c = \sqrt[3]{\frac{2T_{BC}}{\pi\tau_{BC}}} = \sqrt[3]{\frac{(2)(1.4\times10^3)}{\pi(81.2\times10^6)}} = 22.22\times10^{-3} \text{ m} = 22.22 \text{ mm}$$

$$d = 2c = 44.4 \text{ mm}$$

Problem 3.15

3.15 The solid shaft shown is formed of a brass for which the allowable shearing stress 55 MPa. Neglecting the effect of stress concentrations, determine smallest diameters d_{AB} and d_{BC} for which the allowable shearing stress is not exceeded..



$$\tau_{max} = 55\ \text{MPa} = 55 \times 10^6\ \text{Pa}$$

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c = \sqrt[3]{\frac{2T}{\pi \tau_{max}}}$$

Shaft AB: $T_{AB} = 1200 - 400 = 800$ N·m

$$c = \sqrt[3]{\frac{(2)(800)}{\pi(55\times10^6)}} = 21.00\times10^{-3}\ \text{m} = 21.0\ \text{m}$$

minimum $d_{AB} = 2c = 42.0$ mm ◄

Shaft BC: $T_{BC} = 400$ N·m

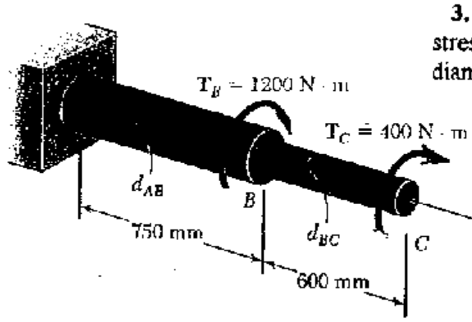
$$c = \sqrt[3]{\frac{(2)(400)}{\pi(55\times10^6)}} = 16.667\times10^{-3}\ \text{m} = 16.67\ \text{mm}$$

minimum $d_{BC} = 2c = 33.3$ mm ◄

Problem 3.16

3.16 Solve Prob. 3.15 assuming that the direction of $\mathbf{T}_C$ is reversed.

3.15 The solid shaft shown is formed of a brass for which the allowable shearing stress 55 MPa. Neglecting the effect of stress concentrations, determine smallest diameters d_{AB} and d_{BC} for which the allowable shearing stress is not exceeded..



Note that the direction of T_C has been reversed in the figure to the left.

$$\tau_{max} = 55\ \text{MPa} = 55\times10^6\ \text{Pa}$$

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c = \sqrt[3]{\frac{2T}{\pi \tau_{max}}}$$

Shaft AB: $T_{AB} = 1200 + 400 = 1600$ N·m

$$c = \sqrt[3]{\frac{(2)(1600)}{\pi(55\times10^6)}} = 26.46\times10^{-3}\ \text{m} = 26.46\ \text{mm}$$

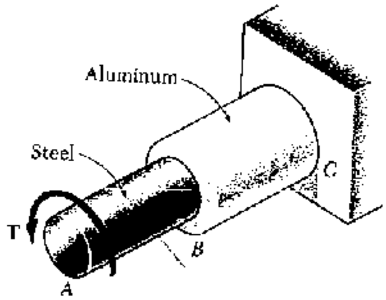
minimum $d_{AB} = 2c = 52.9$ mm ◄

Shaft BC: $T_{BC} = 400$ N·m

$$c = \sqrt[3]{\frac{(2)(400)}{\pi(55\times10^6)}} = 16.667\times10^{-3}\ \text{m} = 16.67\ \text{mm}$$

minimum $d_{BC} = 2c = 33.3$ mm ◄

Problem 3.17



3.17 Shaft *AB* is made of a steel with an allowable shearing stress of 90 MPa and shaft *BC* is made of an aluminum with an allowable shearing stress of 60 MPa. Knowing that the diameter of shaft *BC* is 50 mm and neglecting the effect of stress concentrations, determine (*a*) the largest torque T that can be applied at A if the allowable stress is not to be exceeded in shaft *BC*, (*b*) the corresponding required diameter of shaft *AB*.

(a) Shaft BC: $\tau_{max} = 60 \text{ MPa} = 90 \times 10^6 \text{ Pa}$

$d_{BC} = 50 \text{ mm}$ $\quad c = \frac{1}{2}d = 25 \text{ mm} = 0.025 \text{ m}$

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad T = \frac{\pi}{2}\tau_{max} c^3$$

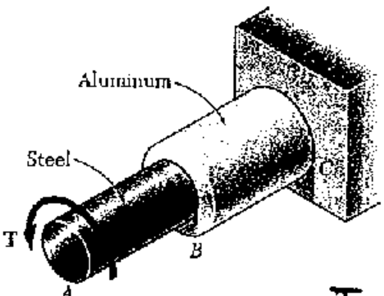
$$T = \frac{\pi}{2}(60 \times 10^6)(0.025)^3 = 1.47262 \times 10^3 \text{ N}\cdot\text{m} \qquad T = 1.473 \text{ kN}\cdot\text{m}$$

(b) Shaft AB: $\tau_{max} = 90 \text{ MPa} = 90 \times 10^6 \text{ Pa}$ $\qquad T = 1.47262 \times 10^3 \text{ N-m}$

$$c = \sqrt[3]{\frac{2T}{\pi \tau_{max}}} = \sqrt[3]{\frac{(2)(1.47262 \times 10^3)}{\pi (90 \times 10^6)}} = 21.84 \times 10^{-3} \text{ m} = 21.84 \text{ mm}$$

$$d_{AB} = 2c = 43.7 \text{ mm}$$

Problem 3.18



3.18 Shaft *AB* has a 30-mm diameter and is made of a steel with an allowable shearing stress of 90 MPa, while shaft *BC* has a 50-mm diameter and is made of an aluminum alloy with an allowable shearing stress of 60 MPa. Neglecting the effect of stress concentrations, determine the largest torque *T* that can be applied at *A*.

Shaft AB: $c = \frac{1}{2}d_{AB} = \frac{1}{2}(30) = 15 \text{ mm} = 0.015 \text{ m}$

$\tau_{max} = 90 \text{ MPa} = 90 \times 10^6 \text{ Pa}$

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad T = \frac{\pi}{2}\tau_{max} c^3$$

$$T_{AB} = \frac{\pi}{2}(90 \times 10^6)(0.015)^3 = 477 \text{ N}\cdot\text{m}$$

Shaft BC: $c = \frac{1}{2}d_{BC} = \frac{1}{2}(50) = 25 \text{ mm} = 0.025 \text{ m}$

$\tau_{max} = 60 \text{ MPa} = 60 \times 10^6 \text{ Pa}$

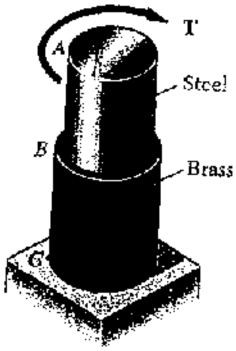
$$T_{BC} = \frac{\pi}{2}\tau_{max} c^3 = \frac{\pi}{2}(60 \times 10^6)(0.025)^3 = 1473 \text{ N}\cdot\text{m}$$

The allowable torque for the entire shaft is the smaller of T_{AB} and T_{BC}

$$T = T_{AB} = 477 \text{ N}\cdot\text{m}$$

Problem 3.19

3.19 The allowable shearing stress is 100 MPa in the 36-mm-diameter steel rod *AB* and 60 MPa in the 40-mm-diameter rod *BC*. Neglecting the effect of stress concentrations, determine the largest torque that can be applied at *A*.



$$\tau_{max} = \frac{Tc}{J}, \quad J = \frac{\pi}{2}c^4, \quad T = \frac{\pi}{2}\tau_{max}c^3$$

Shaft AB: $\tau_{max} = 100 \text{ MPa} = 100 \times 10^6 \text{ Pa}$

$$c = \tfrac{1}{2}d_{AB} = \tfrac{1}{2}(36) = 18 \text{ mm} = 0.018 \text{ m}$$

$$T_{AB} = \frac{\pi}{2}(100\times10^6)(0.018)^3 = 916 \text{ N}\cdot\text{m}$$

Shaft BC: $\tau_{max} = 60 \text{ MPa} = 60\times10^6 \text{ Pa}$

$$c = \tfrac{1}{2}d_{BC} = \tfrac{1}{2}(40) = 20 \text{ mm} = 0.020 \text{ m}$$

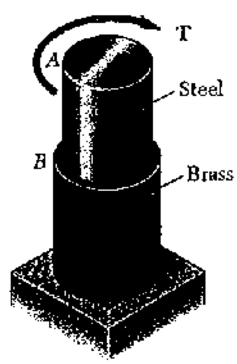
$$T_{BC} = \frac{\pi}{2}(60\times10^6)(0.020)^3 = 754 \text{ N}\cdot\text{m}$$

The allowable torque is the smaller of T_{AB} and T_{BC}.

$$T = 754 \text{ N}\cdot\text{m}$$

Problem 3.20

3.20 The allowable shearing stress is 100 MPa in the steel rod *AB* and 60 MPa in the brass rod *BC*. Knowing that a torque of magnitude $T = 900$ N · m is applied at *A* and neglecting the effect of stress concentrations, determine the required diameter of (*a*) rod AB, (*b*) rod. *BC*.



$$\tau_{max} = \frac{Tc}{J}, \quad J = \frac{\pi}{2}c^4, \quad c = \sqrt[3]{\frac{2T}{\pi\tau_{max}}}$$

Shaft AB: $\tau_{max} = 100 \text{ MPa} = 100\times10^6 \text{ Pa}$

$$c = \sqrt[3]{\frac{(2)(900)}{\pi(100\times10^6)}} = 17.894\times10^{-3} \text{ m} = 17.894 \text{ mm}$$

$$d_{AB} = 2c = 35.8 \text{ mm}$$

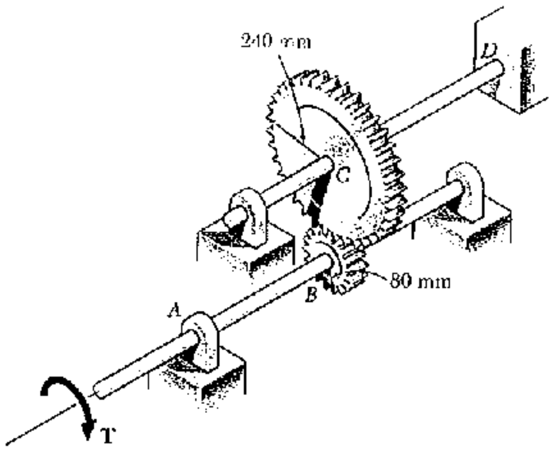
Shaft BC: $\tau_{max} = 60 \text{ MPa} = 60\times10^6 \text{ Pa}$

$$c = \sqrt[3]{\frac{(2)(900)}{\pi(60\times10^6)}} = 21.216\times10^{-3} \text{ m} = 21.216 \text{ mm}$$

$$d_{BC} = 2c = 42.4 \text{ mm}$$

Problem 3.21

3.21 Two solid steel shafts are connected by the gears shown. A torque of magnitude $T = 900$ N · m is applied to shaft AB. knowing that the allowable shearing stress is 50 MPa and considering only stresses due to twisting, determine the required diameter of (*a*) shaft AB, (*b*) shaft CD.



$$T_{AB} = T = 900 \text{ N·m}$$

$$T_{CD} = \frac{r_C}{r_B} T_{AB} = \frac{240}{80}(900) = 2700 \text{ N·m}$$

(a) Shaft AB: $\tau_{max} = 50 \text{ MPa} = 50 \times 10^6 \text{ Pa}$

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}, \qquad c = \sqrt[3]{\frac{2T}{\pi \tau_{max}}}$$

$$c = \sqrt[3]{\frac{(2)(900)}{\pi(50 \times 10^6)}} = 22.55 \times 10^{-3} \text{ m} = 22.55 \text{ mm}$$

$$d_{AB} = 2c = 45.1 \text{ mm}$$

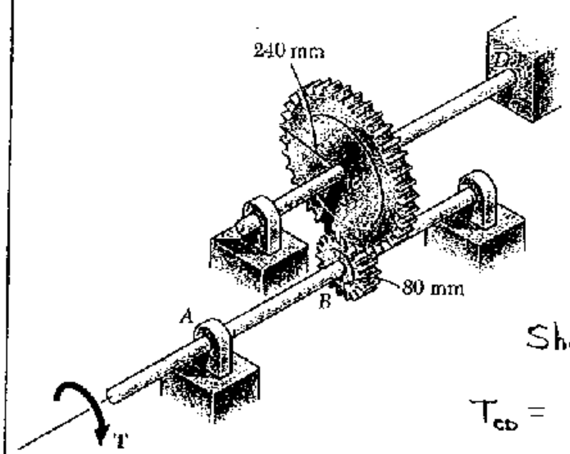
(b) Shaft CD: $\tau_{max} = 50$ MPa

$$c = \sqrt[3]{\frac{(2)(2700)}{\pi(50 \times 10^6)}} = 32.52 \times 10^{-3} \text{ m} = 32.52 \text{ mm}$$

$$d_{CD} = 2c = 65.0 \text{ mm}$$

Problem 3.22

3.22 Shaft CD is made from a 66-mm-diameter rod and is connected to the 48-mm-diameter shaft AB as shown. Considering only stresses due to twisting and knowing that the allowable shearing stress is 60 MPa for each shaft, determine the largest torque **T** that can be applied.



$$\tau_{max} = 60 \text{ MPa} = 60 \times 10^6 \text{ Pa}$$

Shaft AB: $c = \frac{1}{2} d_{AB} = 24 \text{ mm} = 0.024 \text{ m}$

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad T = \frac{\pi}{2} \tau_{max} c^3$$

$$T = \frac{\pi}{2}(60 \times 10^6)(0.024)^3 = 1303 \text{ N·m}$$

Shaft CD: $c = \frac{1}{2} d_{CD} = 33 \text{ mm} = 0.033 \text{ m}$

$$T_{CD} = \frac{\pi}{2}(60 \times 10^6)(0.033)^3 = 3387 \text{ N·m}$$

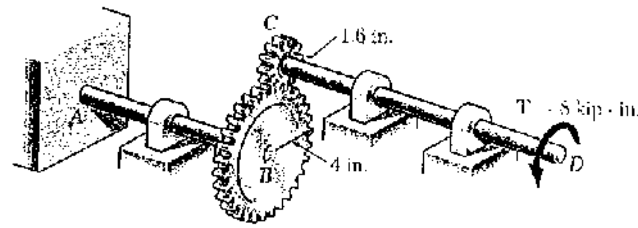
$$T = \frac{r_B}{r_C} T_{CD} = \frac{80}{240}(3387) = 1129 \text{ N·m}$$

The allowable torque is the smaller of the two calculated values.

$$T = 1129 \text{ N·m} = 1.129 \text{ kN·m}$$

Problem 3.23

3.23 A torque of magnitude $T = 8$ kip · in. is applied at D as shown. Knowing that the diameter of shaft of shaft AB is 2.25 in. and that the diameter of shaft CD is 1.75 in., determine the maximum shearing stress in (*a*) shaft AB, (*b*) shaft CD.



$T_{CD} = 8 \text{ kip}\cdot\text{in}$

$T_{AB} = \frac{r_B}{r_C} T_{CD} = \frac{4}{1.6}(8) = 20 \text{ kip}\cdot\text{in}$

(a) Shaft AB: $c = \frac{1}{2} d_{AB} = 1.125 \text{ in.}$

$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$\tau_{max} = \frac{(2)(20)}{\pi (1.125)^3} = 8.94 \text{ ksi}$

(a) $\tau_{max} = 8.94 \text{ ksi}$ ◄

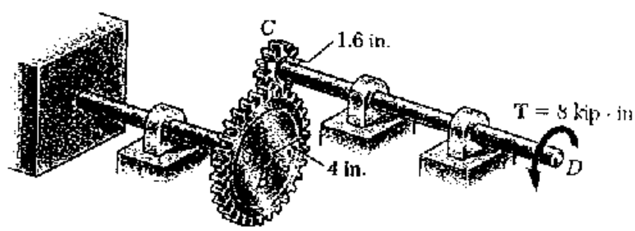
(b) Shaft CD: $c = \frac{1}{2} d_{CD} = 0.875 \text{ in.}$

$\tau_{max} = \frac{2T}{\pi c^3} = \frac{(2)(8)}{\pi (0.875)^3} = 7.60 \text{ ksi}$

(b) $\tau_{max} = 7.60 \text{ ksi}$ ◄

Problem 3.24

3.24 A torque of magnitude $T = 8$ kip · in. is applied at D as shown. Knowing that the allowable shearing stress is 7.5 ksi in each shaft, determine the required diameter of (*a*) shaft AB, (*b*) shaft CD.



$T_{CD} = 8 \text{ kip}\cdot\text{in}$

$T_{AB} = \frac{r_B}{r_C} T_{AB} = \frac{4}{1.6}(8) = 20 \text{ kip}\cdot\text{in}$

$\tau_{max} = 7.5 \text{ ksi}$

$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$c = \sqrt[3]{\frac{2T}{\pi \tau_{max}}}$

(a) Shaft AB: $c = \sqrt[3]{\frac{(2)(20)}{\pi(7.5)}} = 1.1929 \text{ in.}$

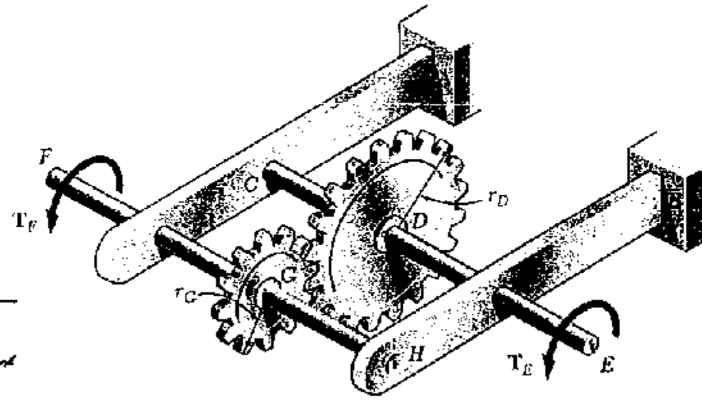
$d_{AB} = 2c = 2.39 \text{ in.}$ ◄

(b) Shaft CD: $c = \sqrt[3]{\frac{(2)(8)}{\pi(7.5)}} = 0.8789 \text{ in.}$

$d_{CD} = 2c = 1.758 \text{ in.}$ ◄

Problem 3.25

3.25 Under normal operating conditions a motor exerts a torque of magnitude T_F = 150 N · m at F. The shafts are made of a steel for which the allowable shearing stress is 75 MPa. Knowing that for the gears r_D = 200 mm and r_G = 75 mm, determine the required diameter of (*a*) shaft *CDE*, (*b*) shaft *FGH*.



$T_{FG} = T_F = 150$ N·m

$T_{DE} = \dfrac{r_D}{r_G} T_{FG} = \dfrac{200}{75}(150)$

$= 400$ N·m

$\tau_{max} = 75$ MPa $= 75 \times 10^6$ Pa

$\tau_{max} = \dfrac{Tc}{J} = \dfrac{2T}{\pi c^3}, \quad c = \sqrt[3]{\dfrac{2T}{\pi \tau_{max}}}$

(a) Shaft CDE:

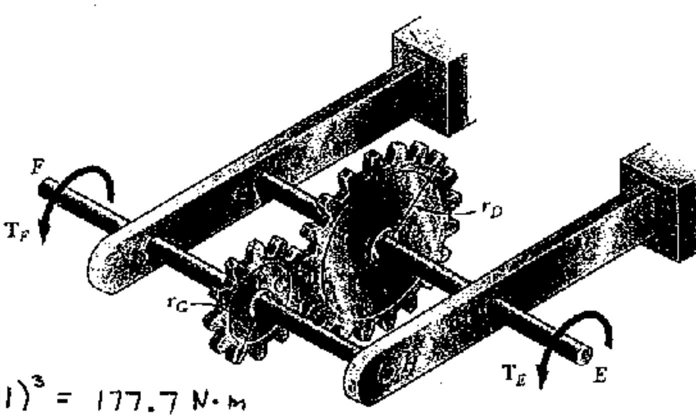
$c = \sqrt[3]{\dfrac{(2)(400)}{\pi(75 \times 10^6)}} = 15.03 \times 10^{-3}$ m $= 15.03$ mm $d_{CDE} = 2c = 30.1$ mm ◄

(b) Shaft FGH:

$c = \sqrt[3]{\dfrac{(2)(150)}{\pi(75 \times 10^6)}} = 10.84 \times 10^{-3}$ m $= 10.84$ mm $d_{FGH} = 2c = 21.7$ mm ◄

Problem 3.26

3.26 Under normal operating conditions a motor exerts a torque of magnitude T_F at F. The shafts are made of a steel for which the allowable shearing stress is 85 MPa and have diameters d_{CDE} = 22 mm and d_{FGH} = 20 mm. Knowing that r_D = 150 mm and r_G = 100 mm, determine the largest allowable value of T_F.



$\tau_{max} = 85$ MPa $= 85 \times 10^6$ Pa

Shaft FG: $c = \frac{1}{2}d = 10$ mm $= 0.010$ m

$T_F = \dfrac{J \tau_{max}}{c} = \dfrac{\pi}{2} \tau_{max} c^3$

$= \frac{\pi}{2}(85 \times 10^6)(0.010)^3 = 133.5$ N·m

Shaft DE: $c = \frac{1}{2}d = 11$ mm $= 0.011$ m

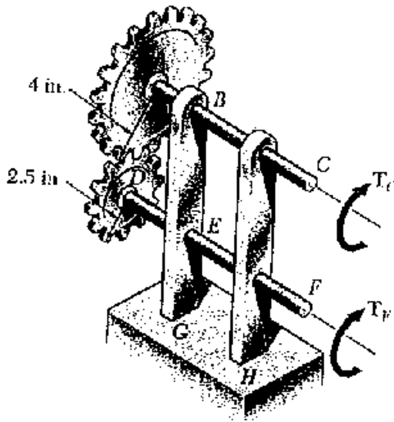
$T_E = \frac{\pi}{2} \tau_{max} c^3 = \frac{\pi}{2}(85 \times 10^6)(0.011)^3 = 177.7$ N·m

By statics $T_F = \dfrac{r_G}{r_D} T_E = \dfrac{100}{150}(177.7) = 118.5$ N·m

The largest allowable value of T_F is the smaller of the two calculated values.

$T_F = 118.5$ N·m ◄

Problem 3.27



3.27 The two solid shafts are connected by gears as shown and are made of a steel for which the allowable shearing stress is 8500 psi. Knowing that a torque of magnitude $T_C = 5$ kip · in. is applied at C and that the assembly is in equilibrium, determine the required diameter of (*a*) shaft *BC*, (*b*) shaft *EF*.

$\tau_{max} = 8500 \text{ psi} = 8.5 \text{ ksi}$

(a) Shaft BC: $T_c = 5$ kip·in

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c = \sqrt[3]{\frac{2T}{\pi \tau_{max}}}$$

$$c = \sqrt[3]{\frac{(2)(5)}{\pi (8.5)}} = 0.7208 \text{ in.}$$

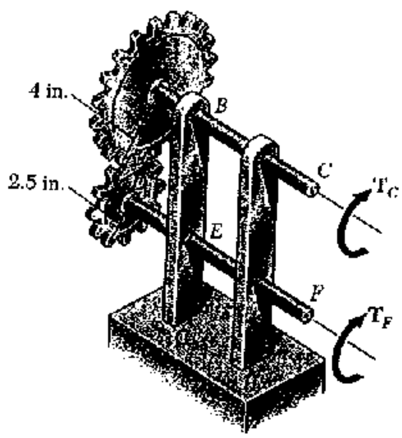
$$d_{BC} = 2c = 1.442 \text{ in.}$$

Shaft EF: $T_F = \frac{r_D}{r_A} T_C = \frac{2.5}{4}(5) = 3.125$ kip·in

$$c = \sqrt[3]{\frac{2T}{\pi \tau_{max}}} = \sqrt[3]{\frac{(2)(3.125)}{\pi (8.5)}} = 0.6163 \text{ in.}$$

$$d_{EF} = 2c = 1.233 \text{ in.}$$

Problem 3.28



3.28 The two solid shafts are connected by gears as shown and are made of a steel for which the allowable shearing stress is 7000 psi. Knowing the diameters of the two shafts are, respectively, $d_{BC} = 1.6$ in. and $d_{EF} = 1.25$ in. determine the largest torque $\mathbf{T}_C$ that can be applied at C.

$\tau_{max} = 7000 \text{ psi} = 7.0 \text{ ksi}$

Shaft BC: $d_{BC} = 1.6$ in. $\qquad c = \frac{1}{2}d = 0.8$ in.

$$T_c = \frac{J\tau_{max}}{c} = \frac{\pi}{2}\tau_{max} c^3$$
$$= \frac{\pi}{2}(7.0)(0.8)^3 = 5.63 \text{ kip·in}$$

Shaft EF: $d_{EF} = 1.25$ in. $\qquad c = \frac{1}{2}d = 0.625$ in.

$$T_F = \frac{J\tau_{max}}{c} = \frac{\pi}{2}\tau_{max} c^3$$
$$= \frac{\pi}{2}(7.0)(0.625)^3 = 2.684 \text{ kip·in}$$

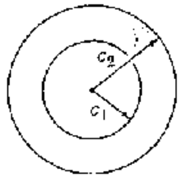
By statics $\quad T_c = \frac{r_A}{r_D} T_F = \frac{4}{2.5}(2.684) = 4.30$ kip·in

Allowable value of T_c is the smaller.

$$T_c = 4.30 \text{ kip·in}$$

Problem 3.29

3.29 (*a*) For a given allowable stress, determine the ratio T/w of the maximum allowable torque T and the weight per unit length w for the hollow shaft shown. (*b*) Denoting by $(T/w)_0$ the value of this ratio computed for a solid shaft of the same radius c_2, express the ratio T/w for the hollow shaft in terms of $(T/w)_0$ and c_1/c_2.



w = weight per unit length , ρg = specific weight

W = total weight , L = length

$$w = \frac{W}{L} = \frac{\rho g L A}{L} = \rho g A = \rho g \pi \left(c_2^2 - c_1^2\right)$$

$$T = \frac{J \tau_{al}}{c_2} = \frac{\pi}{2}\,\frac{c_2^4 - c_1^4}{c_2}\,\tau_{al} = \frac{\pi}{2}\,\frac{(c_2^2+c_1^2)(c_2^2-c_1^2)}{c_2}\,\tau_{al}$$

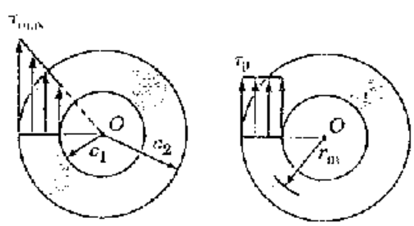
$$\left(\frac{T}{w}\right)_h = \frac{(c_2^2+c_1^2)\,\tau_{al}}{2\rho g c_2} \qquad \text{(hollow shaft)}$$

$c_1 = 0$ for solid shaft $\quad \left(\dfrac{T}{w}\right)_0 = \dfrac{c_2 \tau_{al}}{2\gamma} \qquad$ (solid shaft)

$$\frac{(T/w)_h}{(T/w)_0} = 1 + \frac{c_1^2}{c_2^2} \qquad \left(\frac{T}{w}\right)_h = \left(\frac{T}{w}\right)_0 \left(1 + \frac{c_1^2}{c_2^2}\right)$$

Problem 3.30

3.30 While the exact distribution of the shearing stresses in a hollow cylinder shaft is a shown in Fig. (1), an approximate value may be obtained for τ_{max} by assuming the stresses to be uniformly distributed over the area A of the cross section, as shown in Fig. (2), and then further assuming that all the elementary shearing forces act a distance from O equal to the mean radius $r_m = \frac{1}{2}(c_1 + c_2)$ of the cross section. This approximate value is $\tau_0 = T/Ar_m$, where T is the applied torque. Determine the ratio τ_{max}/τ_0 of the true value of the maximum shearing stress and its approximate value τ_0 for values of c_1/c_2 respectively equal to 1.00, 0.95, 0.75, 0.50, and 0.



(*a*) (*b*)

For a hollow shaft: $\displaystyle \tau_{max} = \frac{Tc_2}{J} = \frac{2Tc_2}{\pi(c_2^4 - c_1^4)} = \frac{2Tc_2}{\pi(c_2^2 - c_1^2)(c_2^2 + c_1^2)}$

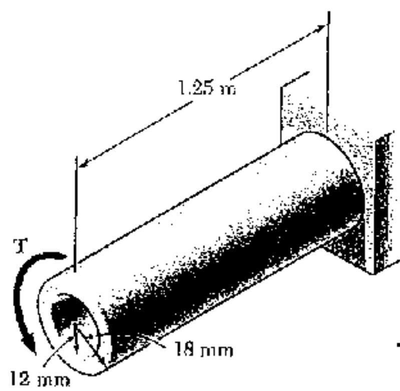
$$= \frac{2Tc_2}{A(c_2^2 + c_1^2)}$$

By definition $\displaystyle \tau_0 = \frac{T}{A r_m} = \frac{2T}{A(c_2 + c_1)}$

Dividing $\displaystyle \frac{\tau_{max}}{\tau_0} = \frac{c_2(c_2 + c_1)}{c_2^2 + c_1^2} = \frac{1 + (c_1/c_2)}{1 + (c_1/c_2)^2}$

c_1/c_2	1.0	0.95	0.75	0.5	0.0
τ_{max}/τ_0	1.0	1.025	1.120	1.200	1.0

Problem 3.31



3.31 For the aluminum shaft shown ($G = 27$ GPa), determine (*a*) the torque **T** what causes an angle of twist of 4°, (*b*) the angle of twist caused by the same torque **T** in a solid cylindrical shaft of the same length and cross sectional area.

(a) $\varphi = \dfrac{TL}{GJ}$, $\quad T = \dfrac{GJ\varphi}{L}$

$\varphi = 4° = 69.813 \times 10^{-3}$ rad, $\quad L = 1.25$ m

$G = 27$ GPa $= 27 \times 10^9$ Pa

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.018^4 - 0.012^4) = 132.324 \times 10^{-9}$ m^4

$$T = \frac{(27 \times 10^9)(132.324 \times 10^{-9})(69.813 \times 10^{-3})}{1.25}$$

$= 199.539$ N·m $\qquad T = 199.5$ N·m ◄

(b) Matching areas: $A = \pi c^2 = \pi(c_2^2 - c_1^2)$

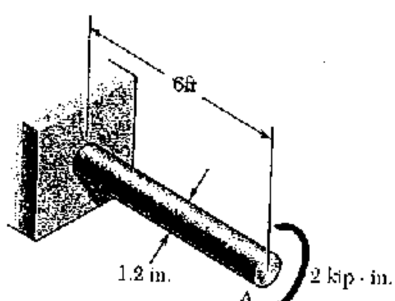
$c = \sqrt{c_2^2 - c_1^2} = \sqrt{0.018^2 - 0.012^2} = 0.013416$ m

$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.013416)^4 = 50.894 \times 10^{-9}$ m^4

$$\varphi = \frac{TL}{GJ} = \frac{(195.539)(1.25)}{(27 \times 10^9)(50.894 \times 10^{-9})} = 181.514 \times 10^{-3} \text{ rad}$$

$\varphi = 10.40°$ ◄

Problem 3.32



3.32(*a*) For the solid steel shaft shown ($G = 11.2 \times 10^6$ psi), determine the angle of twist at A. (*b*) Solve part (*a*), assuming that the steel shaft is hollow with a 1.2-in.-outer diameter and a 0.8-in.-inner diameter.

(a) $c = \frac{1}{2}d = 0.6$ in. $\quad J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.6)^4 = 0.203575$ in^4

$L = 6$ ft $= 72$ in.

$T = 2$ kip·in $= 2000$ lb·in

$$\varphi = \frac{(2000)(72)}{(11.2 \times 10^6)(0.203575)} = 63.157 \times 10^{-3} \text{ rad}$$

$\varphi = 3.62°$ ◄

(b) $c_2 = \frac{1}{2}d_2 = 0.6$ in. $\qquad c_1 = \frac{1}{2}d_1 = 0.4$ in.

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.6^4 - 0.4^4) = 0.163363$ in^4

$$\varphi = \frac{TL}{GJ} = \frac{(2000)(72)}{(11.2 \times 10^6)(0.163363)} = 78.703 \text{ rad}$$

$\varphi = 4.51°$ ◄

Problem 3.33

3.33 Determine the largest allowable diameter of a 10-ft-long steel rod ($G = 11.2 \times 10^6$ psi) if the rod is to be twisted through 30° without exceeding a shearing stress of 12 ksi.

$L = 10 \text{ ft} = 120 \text{ in.} \qquad \varphi = 30° = \frac{30\pi}{180} = 0.52360 \text{ rad}$

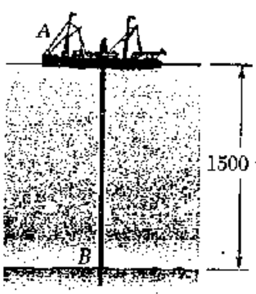
$\tau = 12 \text{ ksi} = 12 \times 10^3 \text{ psi}$

$\varphi = \frac{TL}{GJ}, \quad T = \frac{GJ\varphi}{L}, \quad \tau = \frac{Tc}{J} = \frac{GJ\varphi c}{JL} = \frac{G\varphi c}{L}, \qquad c = \frac{\tau L}{G\varphi}$

$c = \frac{(12 \times 10^3)(120)}{(11.2 \times 10^6)(0.52360)} = 0.24555 \text{ in.} \qquad d = 2c = 0.491 \text{ in.}$ ◄

Problem 3.34

3.34 The ship at *A* has just started to drill for oil on the ocean floor at a depth of 1500 m. Knowing that the top of the 200-mm-diameter steel drill pipe ($G = 77.2$ GPa) rotates through two complete revolutions before the drill bit at *B* starts to operate, determine the maximum shearing stress caused in the pipe by torsion.



$\varphi = \frac{TL}{GJ} \qquad T = \frac{GJ\varphi}{L}$

$\tau = \frac{Tc}{J} = \frac{GJ\varphi c}{JL} = \frac{G\varphi c}{L}$

$\varphi = 2 \text{ rev} = 2(2\pi) \text{ rad} = 12.566 \text{ rad}$

$c = \frac{1}{2}d = 100 \text{ mm} = 0.100 \text{ m}$

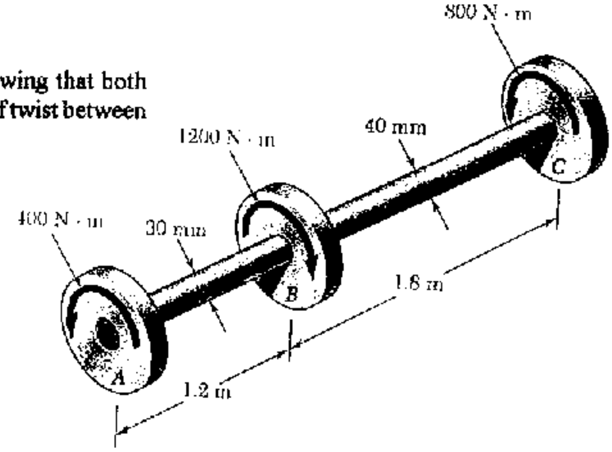
$G = 77.2 \text{ GPa} = 77.2 \times 10^9 \text{ Pa}$

$\tau = \frac{(77.2 \times 10^9)(12.566)(0.100)}{1500} = 64.7 \times 10^6 \text{ Pa}$

$\tau = 64.7 \text{ MPa}$ ◄

Problem 3.35

3.35 The torques shown are exerted on pulleys *A*, *B*, and *C*. Knowing that both shafts are solid and made of brass ($G = 39$ GPa), determine the angle of twist between (*a*) *A* and *B*, (*b*) *A* and *C*.



(a) Angle of twist between A and B

$T_{AB} = 400\ \text{N}\cdot\text{m}, \quad L_{AB} = 1.2\ \text{m}$

$c = \frac{1}{2}d = 0.015\ \text{m} \qquad G = 39\times10^9\ \text{Pa}$

$J_{AB} = \frac{\pi}{2}c^4 = 79.52\times10^{-9}\ \text{m}^4$

$$\varphi_{A/B} = \frac{TL}{GJ} = \frac{(400)(1.2)}{(39\times10^9)(79.52\times10^{-9})}$$

$= 0.154772$ rad ↺

$\varphi_{A/B} = 8.87°$ ↺ ◄

(b) Angle of twist between A and C

$T_{BC} = 800\ \text{N}\cdot\text{m}, \quad L_{BC} = 1.8\ \text{m}, \quad c = \frac{1}{2}d = 0.020\ \text{m}, \quad G = 39\times10^6\ \text{Pa}$

$J_{BC} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.020)^4 = 251.327\times10^{-9}\ \text{m}^4$

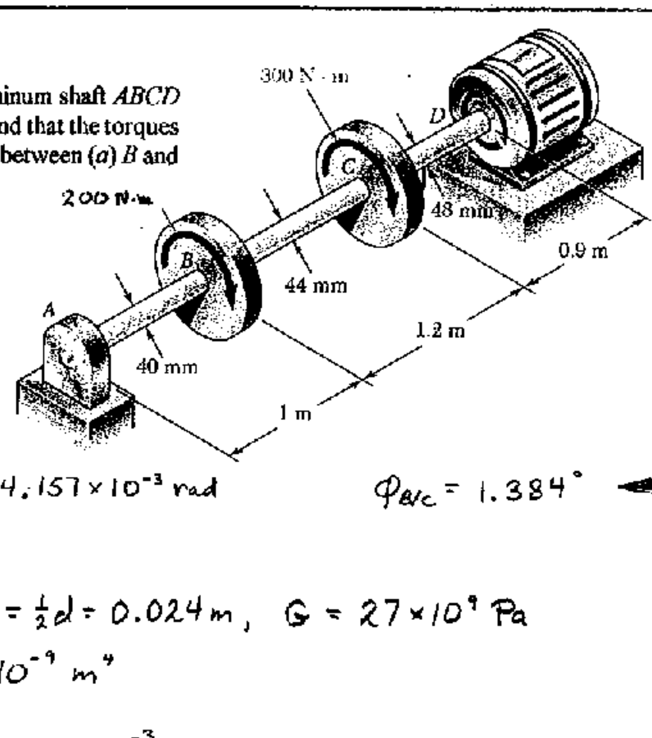
$$\varphi_{B/C} = \frac{(800)(1.8)}{(39\times10^9)(251.327\times10^{-9})} = 0.146912\ \text{rad}$$

$\varphi_{A/C} = \varphi_{A/B} + \varphi_{B/C} = 0.154772 - 0.146912 = 0.007860$ rad ↺

$\varphi_{C/A} = 0.450°$ ↺ ◄

Problem 3.36

3.36 The electric motor exerts a 500 N · m torque on the aluminum shaft *ABCD* when it is rotating at a constant speed. Knowing that $G = 27$ GPa and that the torques exerted on pulleys *B* and *C* are as shown, determine the angle twist between (*a*) *B* and *C*, (*b*) *B* and *D*.



(a) Angle of twist between B and C

$T_{BC} = 200\ \text{N}\cdot\text{m}, \quad L_{BC} = 1.2\ \text{m}$

$c = \frac{1}{2}d = 0.022\ \text{m}, \quad G = 27\times10^9\ \text{Pa}$

$J_{BC} = \frac{\pi}{2}c^4 = 367.97\times10^{-9}\ \text{m}$

$$\varphi_{B/C} = \frac{TL}{GJ} = \frac{(200)(1.2)}{(27\times10^9)(367.97\times10^{-9})} = 24.157\times10^{-3}\ \text{rad}$$

$\varphi_{B/C} = 1.384°$ ◄

(b) Angle of twist between B and D.

$T_{CD} = 500\ \text{N}\cdot\text{m}, \quad L_{CD} = 0.9\ \text{m}, \quad c = \frac{1}{2}d = 0.024\ \text{m}, \quad G = 27\times10^9\ \text{Pa}$

$J_{CD} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.024)^4 = 521.153\times10^{-9}\ \text{m}^4$

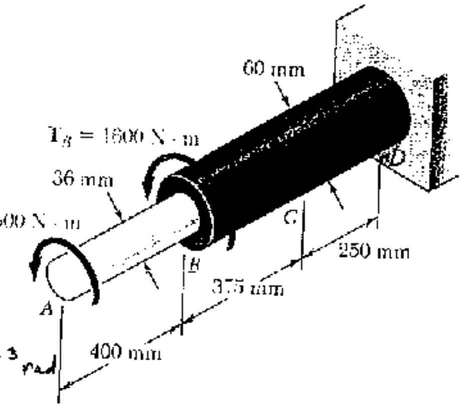
$$\varphi_{C/D} = \frac{(500)(0.9)}{(27\times10^9)(521.153\times10^{9})} = 31.980\times10^{-3}\ \text{rad}$$

$\varphi_{B/D} = \varphi_{B/C} + \varphi_{C/D} = 24.157\times10^{-3} + 31.980\times10^{-3} = 56.137\times10^{-3}$ rad

$\varphi_{B/D} = 3.22°$ ◄

Problem 3.37

3.37 The aluminum rod AB (G = 27 GPa) is bonded to the brass rod BD (G = 39 GPa). Knowing that portion CD of the brass rod is hollow and has an inner diameter of 40 mm determine the angle of twist at A.



Rod AB: $G = 27\times10^9$ Pa, $L = 0.400$ m

$T = 800$ N·m $\quad c = \frac{1}{2}d = 0.018$ m

$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.018)^4 = 164.896\times10^{-9}$ m

$\varphi_{A/B} = \frac{TL}{GJ} = \frac{(800)(0.400)}{(27\times10^9)(164.896\times10^{-9})} = 71.875\times10^{-3}$ rad

Part BC: $G = 39\times10^9$ Pa $\quad L = 0.375$ m, $\quad c = \frac{1}{2}d = 0.030$ m

$T = 800 + 1600 = 2400$ N·m, $\quad J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.030)^4 = 1.27234\times10^{-6}$ m^4

$\varphi_{B/C} = \frac{TL}{GJ} = \frac{(2400)(0.375)}{(39\times10^9)(1.27234\times10^{-6})} = 18.137\times10^{-3}$ rad

Part CD: $c_1 = \frac{1}{2}d_1 = 0.020$ m $\quad c_2 = \frac{1}{2}d_2 = 0.030$ m, $\quad L = 0.250$ m

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.030^4 - 0.020^4) = 1.02102\times10^{-6}$ m^4

$\varphi_{C/D} = \frac{TL}{GJ} = \frac{(2400)(0.250)}{(39\times10^9)(1.02102\times10^{-6})} = 15.068\times10^{-3}$ rad

Angle of twist at A $\quad \varphi_A = \varphi_{A/B} + \varphi_{B/C} + \varphi_{C/D}$

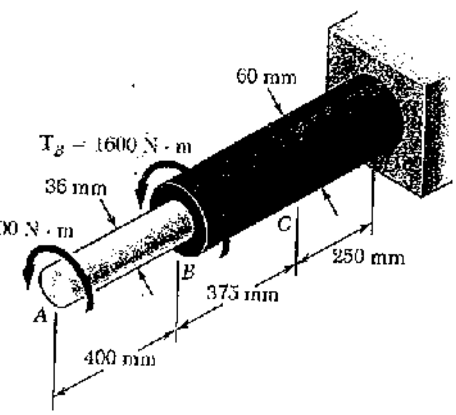
$= 105.080\times10^{-3}$ rad

$\varphi_A = 6.02°$ ◄

Problem 3.38

3.38 Solve Prob. 3.37, assuming that portion BD is a solid 60-mm-diameter rod of length 625 mm.

3.37 The aluminum rod AB (G = 27 GPa) is bonded to the brass rod BD (G = 39 GPa). Knowing that portion CD of the brass rod is hollow and has an inner diameter of 40 mm determine the angle of twist at A.



Rod AB: $G = 27\times10^9$ Pa, $L = 0.400$ m

$T = 800$ N·m $\quad c = \frac{1}{2}d = 0.18$ m

$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.018)^4 = 164.896\times10^{-9}$ m^4

$\varphi_{A/B} = \frac{TL}{GJ} = \frac{(800)(0.400)}{(27\times10^6)(164.896\times10^{-9})} = 71.875\times10^{-3}$ rad

Rod BCD: $G = 39\times10^9$ Pa, $L = 0.375 + 0.250 = 0.625$ m, $c = \frac{1}{2}d = 0.030$ m

$T = 800 + 1600 = 2400$ N·m $\quad J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.030)^4 = 1.27234\times10^{-6}$ m^4

$\varphi_{B/D} = \frac{TL}{GJ} = \frac{(2400)(0.625)}{(39\times10^6)(1.27234\times10^{-6})} = 30.229\times10^{-3}$ rad

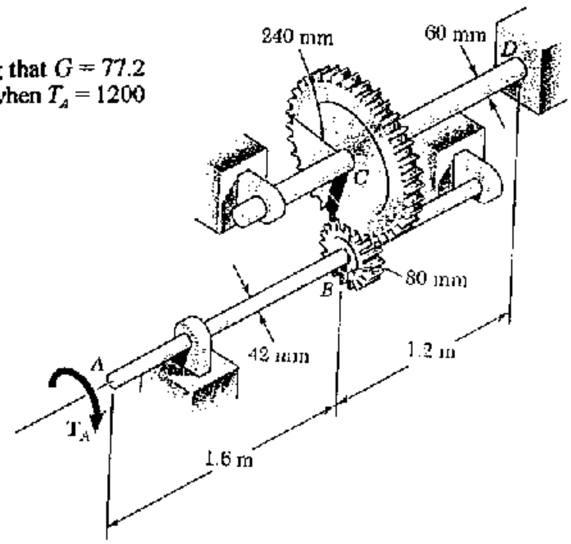
Angle of twist at A $\quad \varphi_A = \varphi_{A/B} + \varphi_{B/D}$

$= 102.104\times10^{-3}$ rad

$\varphi_A = 5.85°$ ◄

Problem 3.39

3.39 Two solid shafts are connected by gears as shown. Knowing that $G = 77.2$ GPa for each shaft, determine the angle through which end A rotates when $T_A = 1200$ N · m.



Calculation of torques.

Circumferential contact force between gears B and C

$$F = \frac{T_{AB}}{r_B} = \frac{T_{CD}}{r_C} \qquad T_{CD} = \frac{r_C}{r_B} T_{AB}$$

$$T_{AB} = 1200 \text{ N-m}$$

$$T_{CD} = \frac{240}{80}(1200) = 3600 \text{ N·m}$$

Twist in shaft CD: $c = \frac{1}{2}d = 0.030\text{ m}$, $L = 1.2\text{ m}$, $G = 77.2 \times 10^9$ Pa

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.030)^4 = 1.27234 \times 10^{-6} \text{ m}^4$$

$$\varphi_{C/D} = \frac{TL}{GJ} = \frac{(3600)(1.2)}{(77.2 \times 10^9)(1.27234 \times 10^{-9})} = 43.981 \times 10^{-3} \text{ rad}$$

Rotation angle at C $\qquad \varphi_C = \varphi_{C/D} = 43.981 \times 10^{-3}$ rad

Circumferential displacement at contact points of gears B an C

$$\delta = r_C \varphi_C = r_B \varphi_B$$

Rotation angle at B $\qquad \varphi_B = \frac{r_C}{r_B}\varphi_C = \frac{240}{80}(43.981 \times 10^{-3}) = 131.942 \times 10^{-3}$ rad

Twist in shaft AB: $c = \frac{1}{2}d = 0.021\text{ m}$, $L = 1.6\text{ m}$, $G = 77.2 \times 10^9$ Pa

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.021)^4 = 305.49 \times 10^{-9} \text{ m}^4$$

$$\varphi_{A/B} = \frac{TL}{GJ} = \frac{(1200)(1.6)}{(77.2 \times 10^9)(305.49 \times 10^{-9})} = 81.412 \times 10^{-3} \text{ rad}$$

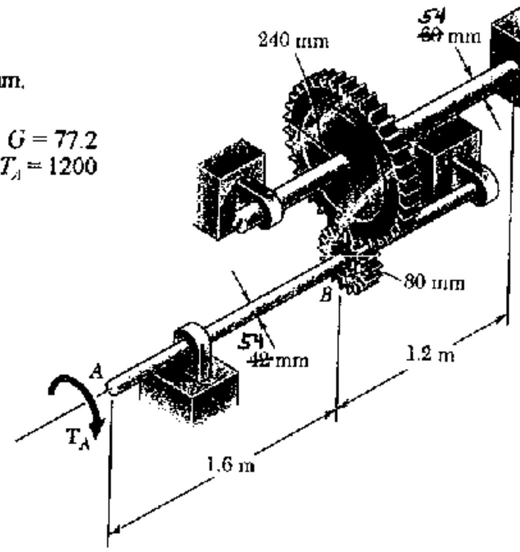
Rotation angle at A $\qquad \varphi_A = \varphi_B + \varphi_{A/B} = 213.354 \times 10^{-3}$ rad

$$\varphi_A = 12.22^\circ$$

Problem 3.40

3.40 Solve Prob. 3.39, assuming that the diameter of each shaft is 54 mm.

3.39 Two solid shafts are connected by gears as shown. Knowing that $G = 77.2$ GPa for each shaft, determine the angle through which end A rotates when $T_A = 1200$ N · m.



Calculation of torques

Circumferential contact force between gears B and C

$$F = \frac{T_{AB}}{r_B} = \frac{T_{CD}}{r_C} \qquad T_{CD} = \frac{r_C}{r_B} T_{AB}$$

$$T_{AB} = 1200 \text{ N-m}$$

$$T_{CD} = \frac{240}{80}(1200) = 3600 \text{ N}\cdot\text{m}$$

Twist in shaft CD: $c = \frac{1}{2}d = 0.027$ m, $L = 1.2$ m, $G = 77.2 \times 10^9$ Pa

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.027)^4 = 834.79 \times 10^{-9} \text{ m}^4$$

$$\varphi_{C/D} = \frac{TL}{GJ} = \frac{(3600)(1.2)}{(77.2\times10^9)(834.78\times10^{-9})} = 67.033 \times 10^{-3} \text{ rad}$$

Rotation angle at C $\qquad \varphi_C = \varphi_{C/D} = 67.033 \times 10^{-3}$ rad

Circumferential displacement at contact points of gears B and C

$$\delta = r_C \varphi_C = r_B \varphi_B$$

Rotation angle at B $\qquad \varphi_B = \frac{r_C}{r_B}\varphi_C = \frac{240}{3}(67.033\times10^{-3}) = 201.10\times10^{-3}$ rad

Twist in shaft AB: $c = \frac{1}{2}d = 0.027$ m, $L = 1.6$ m, $G = 77.2 \times 10^7$ Pa

$$J = 834.79 \times 10^{-9} \text{ m}$$

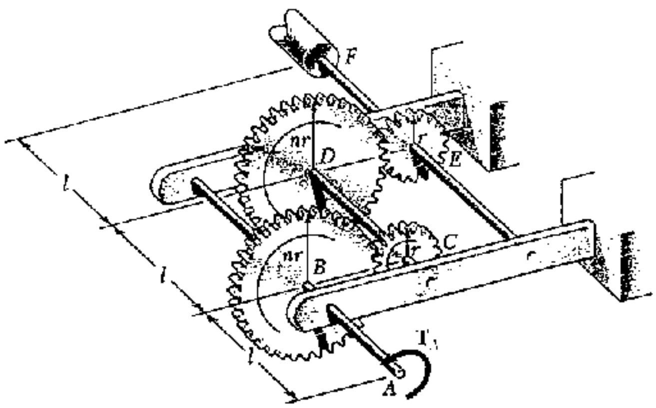
$$\varphi_{A/B} = \frac{TL}{GJ} = \frac{(1200)(1.6)}{(77.2\times10^9)(834.79\times10^{-9})} = 29.792\times10^{-3} \text{ rad}$$

Rotation angle at A $\qquad \varphi_A = \varphi_B + \varphi_{A/B} = 230.89 \times 10^{-3}$ rad

$$\varphi_A = 13.23°$$

Problem 3.41

3.41 A coder F, used to record in digital form the rotation of shaft A, is connected to the shaft by means of the gear train shown, which consists of four gears and three solid steel shafts each of diameter d. Two of the gears have a radius r and the other two a radius nr. If the rotation of the coder F is prevented, determine in terms of T, l, G, J, and n the angle through which end A rotates.



$$T_{AB} = T_A$$

$$T_{CD} = \frac{r_C}{r_B} T_{AB} = \frac{T_{AB}}{n} = \frac{T_A}{n}$$

$$T_{EF} = \frac{r_E}{r_D} T_{CD} = \frac{T_{CD}}{n} = \frac{T_A}{n^2}$$

$$\varphi_E = \varphi_{EF} = \frac{T_{EF} l_{EF}}{GJ} = \frac{T_A l}{n^2 GJ}$$

$$\varphi_D = \frac{r_E}{r_D} \varphi_E = \frac{\varphi_E}{n} = \frac{T_A l}{n^3 GJ}$$

$$\varphi_{CD} = \frac{T_{CD} l_{CD}}{GJ} = \frac{T_A l}{nGJ}$$

$$\varphi_C = \varphi_D + \varphi_{CD} = \frac{T_A l}{n^3 GJ} + \frac{T_A l}{nGJ} = \frac{T_A l}{GJ}\left(\frac{1}{n^3} + \frac{1}{n}\right)$$

$$\varphi_B = \frac{r_C}{r_B} \varphi_C = \frac{\varphi_C}{n} = \frac{Tl}{GJ}\left(\frac{1}{n^4} + \frac{1}{n^2}\right)$$

$$\varphi_{AB} = \frac{T_{AB} l_{AB}}{GJ} = \frac{T_A l}{GJ}$$

$$\varphi_A = \varphi_B + \varphi_{AB} = \frac{T_A l}{GJ}\left(\frac{1}{n^4} + \frac{1}{n^2} + 1\right)$$

Problem 3.42

3.42 For the gear train described in Prob. 3.41, determine the angle through which end A rotates when $T = 5$ lb · in., $l = 2.4$ in., $d = 1/16$ in., $G = 11.2 \times 10^6$ psi, and $n = 2$.

3.41 A coder F, used to record in digital form the rotation of shaft A, is connected to the shaft by means of the gear train shown, which consists of four gears and three solid steel shafts each of diameter d. Two of the gears have a radius r and the other two a radius nr. If the rotation of the coder F is prevented, determine in terms of T, l, G, J, and n the angle through which end A rotates.

See solution to PROBLEM 3.41 for development of equation for φ_A

$$\varphi_A = \frac{Tl}{GJ}\left(1 + \frac{1}{n^2} + \frac{1}{n^4}\right)$$

Data: $T = 5$ lb·in $\quad l = 2.4$ in., $\quad c = \frac{1}{2}d = \frac{1}{32}$ in., $\quad G = 11.2\times10^6$ psi

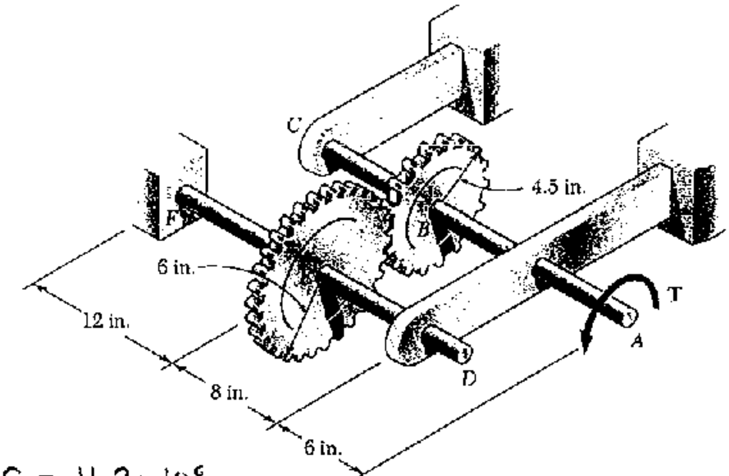
$n = 2$, $\quad J = \frac{\pi}{2}c^4 = \frac{\pi}{2}\left(\frac{1}{32}\right)^4 = 1.49803 \times 10^{-6}$ in^4

$$\varphi_A = \frac{(5)(2.4)}{(11.2\times10^6)(1.49803\times10^{-6})}\left(1 + \frac{1}{4} + \frac{1}{16}\right) = 938.73 \times 10^{-3} \text{ rad}$$

$$\varphi_A = 53.8°$$

Problem 3.43

3.43 Two shafts, each of 7/8 in. diameter are connected by the gears shown. Knowing that $G = 11.2 \times 10^6$ psi and that the shaft at F is fixed, determine the angle through which end A rotates when a 1.2 kip · in. torque is applied at A.



Calculation of torques.

Circumferential contact force between gears B and E.

$$F = \frac{T_{AB}}{r_B} = \frac{T_{EF}}{r_E} \qquad T_{EF} = \frac{r_E}{r_B} T_{AB}$$

$$T_{AB} = 1.2 \text{ kip·in} = 1200 \text{ lb·in}$$

$$T_{EF} = \frac{6}{4.5}(1200) = 1600 \text{ lb·in}$$

Twist in shaft FE.

$L = 12$ in., $\quad c = \frac{1}{2}d = \frac{7}{16}$ in., $\quad G = 11.2 \times 10^6$ psi

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}\left(\frac{7}{16}\right)^4 = 57.548 \times 10^{-3} \text{ in}^4$$

$$\varphi_{E/F} = \frac{TL}{GJ} = \frac{(1600)(12)}{(11.2 \times 10^6)(57.548 \times 10^{-3})} = 29.789 \times 10^{-3} \text{ rad}$$

Rotation at E. $\quad \varphi_E = \varphi_{E/F} = 29.789 \times 10^{-3}$ rad

Tangential displacement at gear circle $\quad s = r_E \varphi_E = r_B \varphi_B$

Rotation at B $\quad \varphi_B = \frac{r_E}{r_B}\varphi_E = \frac{6}{4.5}(29.789 \times 10^{-3}) = 39.718 \times 10^{-3}$ rad

Twist in shaft BA. $\quad L = 8 + 6 = 14$ in. $\quad J = 57.548 \times 10^{-3}$ in^4

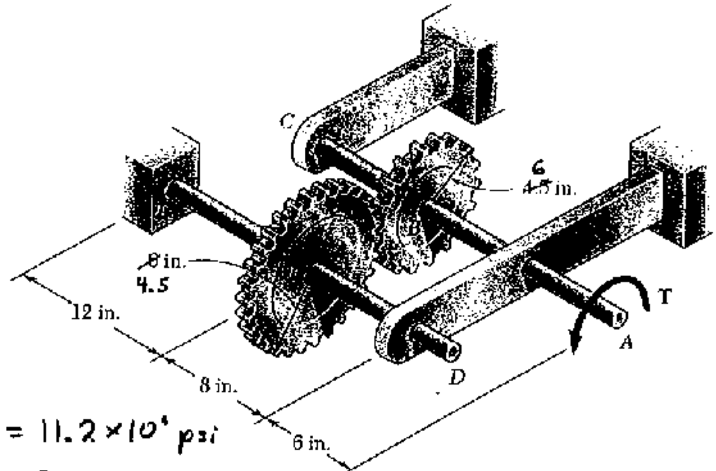
$$\varphi_{A/B} = \frac{TL}{GJ} = \frac{(1200)(14)}{(11.2 \times 10^6)(57.548 \times 10^{-3})} = 26.065 \times 10^{-3} \text{ rad}$$

Rotation at A. $\quad \varphi_A = \varphi_B + \varphi_{A/B} = 65.783 \times 10^{-3}$ rad $\qquad \varphi_A = 3.77°$ ◄

Problem 3.44

3.44 Solve Prob. 3.43, assuming that after a design change the radius of gear B is 6 in. and the radius of gear E is 4.5 in.

3.43 Two shafts, each of 7/8 in. diameter are connected by the gears shown. Knowing that $G = 11.2 \times 10^6$ psi and that the shaft at F is fixed, determine the angle through which end A rotates when a 1.2 kip · in. torque is applied at A.



Calculation of torques.

Circumferential contact force between gears B and E,

$$F = \frac{T_{AB}}{r_B} = \frac{T_{EF}}{r_E} \qquad T_{EF} = \frac{r_E}{r_B} T_{AB}$$

$$T_{AB} = 1.2 \text{ kip-in} = 1200 \text{ lb-in}$$

$$T_{EF} = \frac{4.5}{6}(1200) = 900 \text{ lb-in}$$

Twist in shaft FE

$L = 12$ in. $\quad c = \frac{1}{2}d = \frac{7}{16}$ in., $\quad G = 11.2 \times 10^6$ psi

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}\left(\frac{7}{16}\right)^4 = 57.548 \times 10^{-3} \text{ in}^4$$

$$\varphi_{E/F} = \frac{TL}{GJ} = \frac{(900)(12)}{(11.2 \times 10^6)(57.548 \times 10^{-3})} = 16.756 \times 10^{-3} \text{ rad}$$

Rotation at E. $\quad \varphi_E = \varphi_{E/F} = 16.756 \times 10^{-3}$ rad

Tangential displacement at gear circle. $\quad \delta = r_E \varphi_E = r_B \varphi_B$

Rotation at B. $\quad \varphi_B = \frac{r_E}{r_B}\varphi_E = \frac{4.5}{6}(16.756 \times 10^{-3}) = 12.567 \times 10^{-3}$ rad

Twist in shaft BA. $\quad L = 8 + 6 = 14$ in. $\quad J = 57.548 \times 10^{-3}$ in^4

$$\varphi_{A/B} = \frac{TL}{GJ} = \frac{(1200)(14)}{(11.2 \times 10^6)(57.548 \times 10^{-3})} = 26.065 \times 10^{-3} \text{ rad}$$

Rotation at A. $\quad \varphi_A = \varphi_B + \varphi_{B/A} = 38.632 \times 10^{-3}$ rad $\qquad \varphi_A = 2.21°$ ◄

Problem 3.45

3.45 The design specifications of a 6-ft-long solid circular transmission shaft require that the angle of twist of the shaft not exceed 0.5° when a torque of 60 kip · in. is applied. Determine the required diameter of the shaft, knowing that the shaft is made of a steel with an allowable shearing stress of 12.5 ksi and a modulus of rigidity of 11.2 × 10^6 psi.

$T = 60$ kip·in $= 60 \times 10^3$ lb·in $\qquad \varphi = 0.5° = 8.7266 \times 10^{-3}$ rad $\qquad L = 6$ ft $= 72$ in.

$\tau = 12.5$ ksi $= 12.5 \times 10^3$ psi $\qquad G = 11.2 \times 10^6$ psi $\qquad J = \frac{\pi}{2} c^4$

Based on angle of twist. $\quad \varphi = \frac{TL}{GJ} = \frac{2TL}{\pi G c^4}$

$$c = \sqrt[4]{\frac{2TL}{\pi G \varphi}} = \sqrt[4]{\frac{(2)(60 \times 10^3)(72)}{\pi (11.2 \times 10^6)(8.7266 \times 10^{-3})}} = 2.303 \text{ in.}$$

Based on shearing stress. $\quad \tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$$c = \sqrt[3]{\frac{2T}{\pi \tau}} = \sqrt[3]{\frac{(2)(60 \times 10^3)}{\pi (12.5 \times 10^3)}} = 1.451 \text{ in.}$$

Use larger value. $\quad c = 2.303$ in. $\qquad d = 2c = 4.61$ in. ◄

Problem 3.46

3.46 The design specifications of a 1.2-m-long solid transmission shaft require that the angle of twist of the shaft not exceed 4° when a torque of 750 N · m is applied. Determine the required diameter of the shaft, knowing that the shaft is made of a steel with an allowable shearing stress of 90 MPa and a modulus of rigidity of 77.2 GPa.

$T = 750$ N·m, $\quad \varphi = 4° = 69.813 \times 10^{-3}$ rad, $\quad L = 1.2$ m, $\quad J = \frac{\pi}{2} c^4$

$\tau = 90$ MPa $= 90 \times 10^6$ Pa $\qquad G = 77.2$ GPa $= 77.2 \times 10^9$ Pa

Based on angle of twist. $\quad \varphi = \frac{TL}{GJ} = \frac{2TL}{\pi G c^4}$

$$c = \sqrt[4]{\frac{2TL}{\pi G \varphi}} = \sqrt[4]{\frac{(2)(750)(1.2)}{\pi (77.2 \times 10^9)(69.813 \times 10^{-3})}} = 18.06 \times 10^{-3} \text{ m}$$

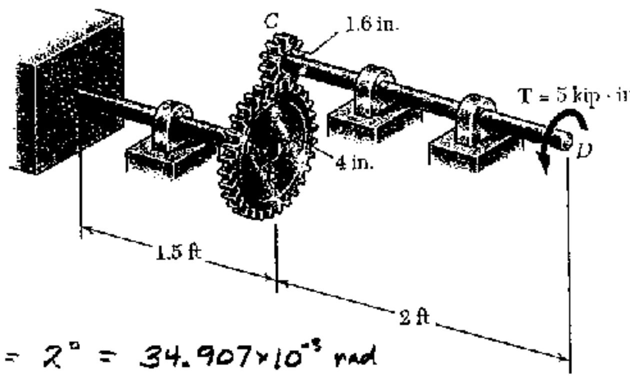
Based on shearing stress $\quad \tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$$c = \sqrt[3]{\frac{2T}{\pi \tau}} = \sqrt[3]{\frac{(2)(750)}{\pi (90 \times 10^6)}} = 17.44 \times 10^{-3} \text{ m}$$

Use larger value $\quad c = 18.06 \times 10^{-3}$ m $= 18.06$ mm $\qquad d = 2c = 36.1$ mm ◄

Problem 3.47

3.47 The design of the gear-and-shaft system shown requires that steel shafts of the same diameter be used for both AB and CD. It is further required that $\tau_{max} \le 9$ ksi and that the angle ϕ_D through which end D of shaft CD rotates not exceed 2°. Knowing that $G = 11.2 \times 10^6$ psi, determine the required diameter of the shafts.



$T_{CD} = 5 \text{ kip}\cdot\text{in} = 5\times10^3 \text{ lb}\cdot\text{in}$

$T_{AB} = \frac{r_B}{r_A} T_{CD} = \frac{4}{1.6}(5\times10^3) = 12.5\times10^3 \text{ lb}\cdot\text{in}$

For design based on stress use the larger value of torque. $T = 12.5\times10^3$ lb·in

$\tau = 9 \text{ ksi} = 9\times10^3 \text{ psi}$

$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c = \sqrt[3]{\frac{2T}{\pi\tau}}$

$c = \sqrt[3]{\frac{(2)(12.5\times10^3)}{\pi(9\times10^3)}} = 0.8842 \text{ in.}$

Design based on rotation angle $\varphi_D = 2° = 34.907\times10^{-3}$ rad

Shaft AB: $\varphi_B = \varphi_{B/A} = \frac{T_{AB} L_{AB}}{GJ} \qquad L_{AB} = 1.5 \text{ ft} = 18 \text{ in.}$

Gears: $\delta = r_B \varphi_B = r_C \varphi_C \qquad \varphi_C = \frac{r_B}{r_C}\varphi_B$

$\varphi_C = \frac{r_B}{r_C}\frac{T_{AB} L_{AB}}{GJ} = \frac{(4)(12.5\times10^3)(18)}{1.6\ GJ} = \frac{562.5\times10^3}{GJ}$

Shaft CD $\quad \varphi_{D/C} = \frac{T_{CD} L_{CD}}{GJ} \qquad L_{CD} = 2 \text{ ft} = 24 \text{ in.}$

$\varphi_{D/C} = \frac{(5\times10^3)(24)}{GJ} = \frac{120\times10^3}{GJ}$

Rotation at D $\quad \varphi_D = \varphi_C + \varphi_{D/C} = \frac{682.5\times10^3}{GJ} = 34.907\times10^{-3}$

$GJ = \frac{\pi}{2} G c^4 = \frac{682.5\times10^3}{34.907\times10^{-3}} = 19.5520\times10^6 \text{ lb}\cdot\text{in}$

$GJ = \frac{\pi}{2} G c^4$

$c = \sqrt[4]{\frac{2GJ}{\pi G}} = \sqrt[4]{\frac{(2)(19.5520\times10^6)}{\pi(11.2\times10^6)}} = 1.0267 \text{ in.}$

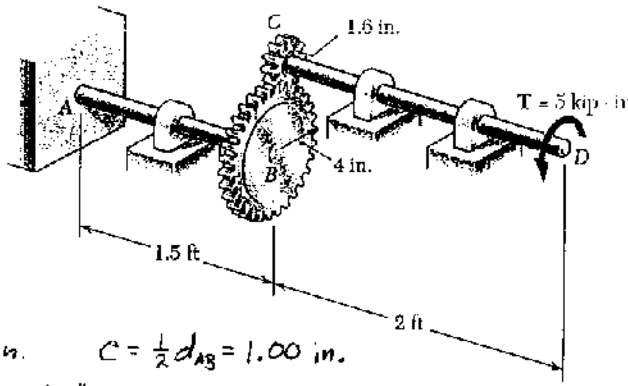
Use the larger value $\quad c = 1.0267$ in. $\qquad d = 2c = 2.05$ in. ◄

Problem 3.48

3.48 In the gear-and-shaft system shown the diameter of shafts are $d_{AB} = 2$ in. and $d_{CD} = 1.5$ in. Knowing that $G = 11.2 \times 10^6$ psi, determine the angle through which end D of shaft CD rotates.

Calculation of torques

Circumferential contact force between gears B and C



$F = \frac{T_{AB}}{r_B} = \frac{T_{CD}}{r_C}$ $T_{AB} = \frac{r_B}{r_C} T_{CD}$

$T_{CD} = 5$ kip·in $= 5 \times 10^3$ lb·in

$T_{AB} = \frac{4}{1.6}(5 \times 10^3) = 12.5 \times 10^3$ kip·in

Twist in shaft AB: $L = 1.5$ ft $= 18$ in. $c = \frac{1}{2} d_{AB} = 1.00$ in.

$J = \frac{\pi}{2} c^4 = \frac{\pi}{2}(1.00)^4 = 1.5708$ in^4

$\varphi_{B/A} = \frac{TL}{GJ} = \frac{(12.5 \times 10^3)(18)}{(11.2 \times 10^6)(1.5708)} = 12.789 \times 10^{-3}$ rad

Rotation at B. $\varphi_B = \varphi_{B/A} = 12.789 \times 10^{-3}$ rad

Tangential displacement at gear circle $\delta = r_B \varphi_B = r_C \varphi_C$

Rotation at C $\varphi_B = \frac{r_B}{r_C} \varphi_C = \frac{4}{1.6}(12.789 \times 10^{-3}) = 31.973 \times 10^{-3}$ rad

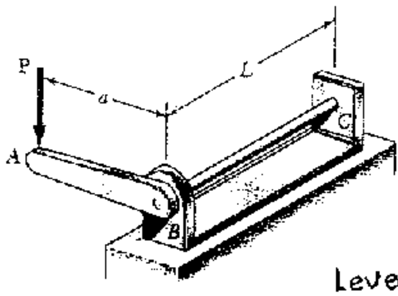
Twist in shaft CD: $L = 2$ ft $= 24$ in. $c = \frac{1}{2} d_{CD} = 0.75$ in.

$J = \frac{\pi}{2} c^4 = \frac{\pi}{2}(0.75)^4 = 0.49701$ in^4

$\varphi_{D/C} = \frac{TL}{GJ} = \frac{(5 \times 10^3)(24)}{(11.2 \times 10^6)(0.49701)} = 21.557 \times 10^{-3}$ rad

Rotation at D. $\varphi_D = \varphi_C + \varphi_{D/C} = 53.530 \times 10^{-3}$ rad $\varphi_D = 3.07°$

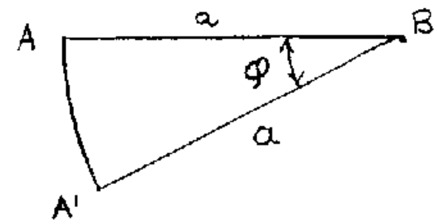
Problem 3.49



3.49 The solid cylindrical rod BC is attached to the rigid lever AB and to the fixed support at C. The vertical force **P** applied at A causes a small displacement Δ at point A. Show that the corresponding maximum shearing stress in the rod is

$$\tau = \frac{Gd}{2La}\Delta$$

where d is the diameter of the rod and G its modulus of rigidity.



Lever AB turns through angle φ to position A'B as shown in the auxiliary figure.

Vertical displacement is $\Delta = a \sin\varphi$

from which $\varphi = \arcsin \frac{\Delta}{a}$

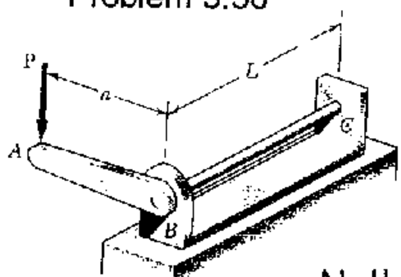
The maximum shearing stress in rod BC is

$$\tau_{max} = G\gamma_{max} = G\frac{c\varphi}{L} = G\frac{d\varphi}{2L} = \frac{Gd}{2L}\arcsin\frac{\Delta}{a}$$

For small $\frac{\Delta}{a}$, $\arcsin\frac{\Delta}{a} \approx \frac{\Delta}{a}$

$$\tau_{max} = \frac{Gd\Delta}{2La}$$

Problem 3.50



3.50* and *3.51 The solid cylindrical rod BC of length $L = 24$ in. is attached to the rigid lever AB of length $a = 16$ in. and to the support at C. Design specifications require that the displacement of A not exceed 1 in. when a 100-lb force **P** is applied at A. For the material indicated determine the required diameter of the rod.

3.50 *Steel:* $\tau_{max} = 12$ ksi, $G = 11.2 \times 10^6$ psi.

At the allowable twist angle $\quad \sin\varphi = \frac{\Delta}{a} = \frac{1}{16} = 0.0625$

$\varphi = 3.5833° = 0.062541$ rad.

$T = Pa\cos\varphi = (100)(16)\cos 3.5833° = 1596.9$ lb·in

Based on twist $\quad \varphi = \frac{TL}{GJ} = \frac{2TL}{\pi G c^4} \quad \therefore \quad c^4 = \frac{2TL}{\pi G \varphi}$

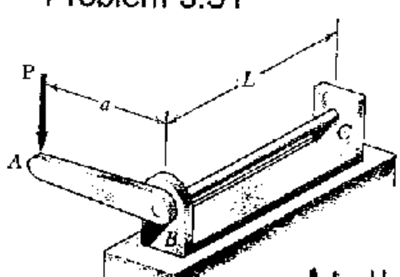
$c^4 = \frac{(2)(1596.9)(24)}{\pi(11.2\times10^6)(0.062541)} = 34.833\times10^{-3}\ \text{in}^4 \qquad c = 0.4320$ in.

Based on stress $\quad \tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad \therefore \quad c^3 = \frac{2T}{\pi\tau} \qquad (\tau = 12\,000\ \text{psi})$

$c^3 = \frac{2(1596.9)}{\pi(12\,000)} = 84.718\times10^{-3}\ \text{in}^3 \qquad c = 0.4392$ in

Use larger value for design $\quad c = 0.4392$ in. $\quad d = 2c = 0.878$ in. ◄

Problem 3.51



3.50* and *3.51 The solid cylindrical rod BC of length $L = 24$ in. is attached to the rigid lever AB of length $a = 16$ in. and to the support at C. Design specifications require that the displacement of A not exceed 1 in. when a 100-lb force **P** is applied at A. For the material indicated determine the required diameter of the rod.

3.51 *Aluminum:* $\tau_{max} = 9$ ksi, $G = 3.9 \times 10^6$ psi.

At the allowable twist angle $\quad \sin\varphi = \frac{\Delta}{a} = \frac{1}{16} = 0.0625$

$\varphi = 3.5833° = 0.062541$ rad.

$T = Pa\cos\varphi = (100)(15)\cos 3.5833° = 1596.9$ lb·in

Based on twist $\quad \varphi = \frac{TL}{GJ} = \frac{2TL}{\pi G c^4} \quad \therefore \quad c^4 = \frac{2TL}{\pi G \varphi}$

$c^4 = \frac{(2)(1596.9)(24)}{\pi(3.9\times10^6)(0.062541)} = 100.033\times10^{-3}\ \text{in}^4 \qquad c = 0.5624$ in.

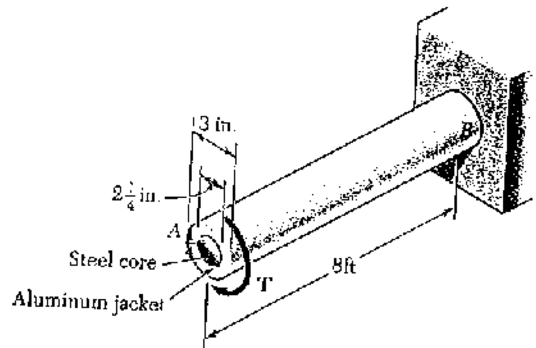
Based on stress $\quad \tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad \therefore \quad c^3 = \frac{2T}{\pi\tau} \qquad (\tau = 9\,000\ \text{psi})$

$c^3 = \frac{(2)(1596.9)}{\pi(9000)} = 112.958\times10^{-3}\ \text{in}^3 \qquad c = 0.4834$ in.

Use larger value for design $\quad c = 0.5624$ in. $\quad d = 2c = 1.125$ in. ◄

Problem 3.52

3.52 A torque of magnitude $T = 35$ kip · in. is applied at end A of the composite shaft shown. Knowing that the modulus of rigidity is 11.2×10^6 psi for the steel and 3.9×10^6 psi for the aluminum, determine (*a*) the maximum shearing stress in the steel core, (*b*) the maximum shearing stress in the aluminum jacket, (*c*) the angle of twist at A.



$L = 8\text{ ft} = 96\text{ in.} \qquad T = 35 \times 10^3\ \text{lb}\cdot\text{in}$

Steel core: $c = \frac{1}{2}d = 1\frac{1}{8}\text{ in.} = 1.125\text{ in.}$

$J_1 = \frac{\pi}{2}c^4 = 2.5161\ \text{in}^4$

$G_1 J_1 = (11.2\times10^6)(2.5161) = 28.180\times10^6\ \text{lb}\cdot\text{in}$

Torque carried by steel core $\quad T_1 = \dfrac{G_1 J_1 \varphi}{L}$

Aluminum jacket: $c_1 = \frac{1}{2}d_1 = 1.125\text{ in},\ c_2 = \frac{1}{2}d_2 = 1.5\text{ in.}$

$J_2 = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(1.5^4 - 1.125^4) = 5.4360\ \text{in}^4$

$G_2 J_2 = (3.9\times10^6)(5.4360) = 21.201\times10^6\ \text{lb}\cdot\text{in}$

Torque carried by aluminum jacket $\quad T_2 = \dfrac{G_2 J_2 \varphi}{L}$

Total torque $\quad T = T_1 + T_2 = (G_1 J_1 + G_2 J_2)\dfrac{\varphi}{L}$

$$\frac{\varphi}{L} = \frac{T}{G_1 J_1 + G_2 J_2} = \frac{35\times10^3}{28.180\times10^6 + 21.201\times10^6} = 708.78\times10^{-6}\ \text{rad/in}$$

(a) Maximum stress in steel core

$\tau = G_1\gamma = G_1 c \dfrac{\varphi}{L} = (11.2\times10^6)(1.125)(708.78\times10^{-6})$

$= 8.93\times10^3$ psi $\qquad \tau_{st} = 8.93$ ksi ◄

(b) Maximum stress in aluminum jacket

$\tau = G_2\gamma = G_2 c_2 \dfrac{\varphi}{L} = (3.9\times10^6)(1.5)(708.78\times10^{-6})$

$= 4.15\times10^3$ psi $\qquad \tau_{al} = 4.14$ ksi ◄

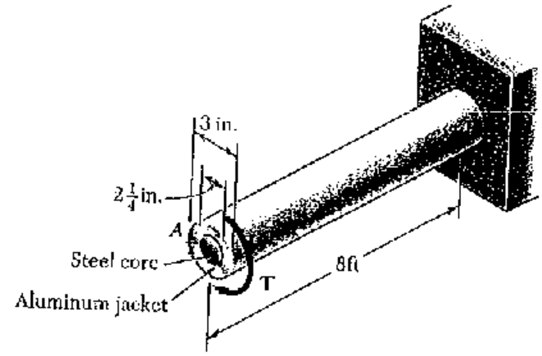
(c) Angle of twist

$\varphi = L\dfrac{\varphi}{L} = (96)(708.78\times10^{-6}) = 68.043\times10^{-3}$ rad

$\varphi = 3.90°$ ◄

Problem 3.53

3.53 The composite shaft shown is to be twisted by applying a torque **T** at end A. Knowing that the modulus of rigidity is 11.2×10^6 psi for the steel and 3.9×10^6 psi for the aluminum, determine the largest angle through which end A can be rotated if the following allowable stresses are not be exceeded $\tau_{steel} = 8500$ psi and $\tau_{aluminum} = 6500$psi.



$L = 8 ft = 96$ in.

$$\tau_{max} = G\gamma_{max} = G c_{max} \frac{\varphi}{L}$$

$$\frac{\varphi_{all}}{L} = \frac{\tau_{all}}{G c_{max}} \quad \text{for each material}$$

Steel core: $\tau_{all} = 8500$ psi

$c_{max} = \frac{1}{2}d = 1\frac{1}{8}$ in. $= 1.125$ in.

$$\frac{\varphi_{all}}{L} = \frac{8500}{(11.2\times10^6)(1.125)} = 674.60\times10^{-6} \text{ rad/in}$$

Aluminum jacket: $\tau_{all} = 6500$ psi $\qquad c_{max} = \frac{1}{2}d_2 = 1.5$ in.

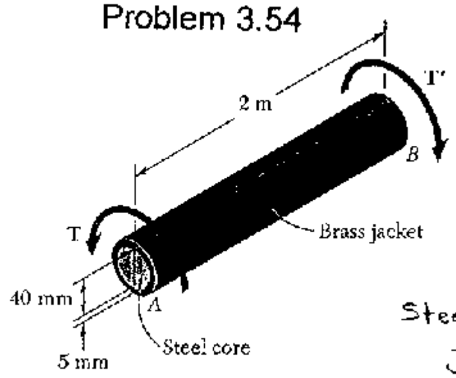
$$\frac{\varphi_{all}}{L} = \frac{6500}{(3.9\times10^6)(1.5)} = 1.1111\times10^{-6} \text{ rad/in}$$

Smaller value of $\frac{\varphi_{all}}{L}$ governs $\qquad \frac{\varphi_{all}}{L} = 674.60\times10^{-6}$ rad/in

Allowable angle of twist: $\varphi_{all} = L\frac{\varphi_{all}}{L} = (96)(674.60\times10^{-6})$

$= 64.76\times10^{-3}$ rad $\qquad \varphi_{all} = 3.71°$

Problem 3.54



3.54 The composite shaft shown consists of a 5-mm-thick brass jacket ($G_{brass} = 39$ GPa) bonded to a 40-mm-diameter steel core ($G_{steel} = 77.2$ GPa). Knowing that the shaft is subjected to a 600 N · m torque, determine (*a*) the maximum shearing stress in the brass jacket, (*b*) the maximum shearing stress in the steel core, (*c*) the angle of twist of *B* relative to *A*.

$L = 2\text{ m}$

Steel core: $c = \frac{1}{2}d = 20\text{ mm} = 0.020\text{ m}$

$J_1 = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.020)^4 = 251.33\times10^{-9}\text{ m}^4$

$G_1J_1 = (77.2\times10^9)(251.33\times10^{-9}) = 19.4025\times10^3\text{ N-m}^2$

Torque carried by steel core $T_1 = G_1J_1\frac{\varphi}{L}$

Brass jacket: $c_1 = \frac{1}{2}d_1 = 20\text{ mm} = 0.020\text{ m}$ $c_2 = 20+5 = 25\text{ mm} = 0.025\text{ m}$

$J_2 = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.025^4 - 0.020^4) = 362.265\times10^{-9}\text{ m}^4$

$G_2J_2 = (39\times10^9)(362.265\times10^{-9}) = 14.1283\times10^3\text{ N}\cdot\text{m}^2$

Torque carried by brass jacket $T_2 = G_2J_2\frac{\varphi}{L}$

Total torque $T = T_1 + T_2 = (G_1J_1 + G_2J_2)\frac{\varphi}{L}$

$$\frac{\varphi}{L} = \frac{T}{G_1J_1 + G_2J_2} = \frac{600}{19.4025\times10^3 + 14.1283\times10^3} = 17.894\times10^{-3}\text{ rad/m}$$

(a) Maximum shearing stress in brass jacket

$\tau_{max} = G\gamma_{max} = G_2c_2\frac{\varphi}{L} = (39\times10^9)(0.025)(17.894\times10^{-3})$

$= 17.45\times10^6\text{ Pa}$ $\tau_{brass} = 17.45\text{ MPa}$ ◄

(b) Maximum shearing stress in steel core

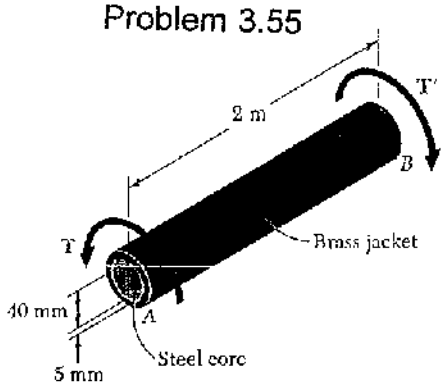
$\tau_{max} = G\gamma_{max} = G_1c\frac{\varphi}{L} = (77.2\times10^9)(0.020)(17.894\times10^{-3})$

$= 27.6\times10^6\text{ Pa}$ $\tau_{steel} = 27.6\text{ MPa}$ ◄

(c) Angle of twist

$\varphi = L\frac{\varphi}{L} = (2)(17.894\times10^{-3}) = 35.788\times10^{-3}\text{ rad}$ $\varphi = 2.05°$ ◄

Problem 3.55



3.55 For the composite shaft of Prob. 3.54, the allowable shearing stress in the brass jacket is 20 MPa and 45 MPa in the steel core. Determine (*a*) the largest torque which can be applied to the shaft, (*b*) the corresponding angle of twist of *B* relative to *A*.

3.54 The composite shaft shown consists of a 5-mm-thick brass jacket (G_{brass} = 39 GPa) bonded to a 40-mm-diameter steel core (G_{steel} = 77.2 GPa). Knowing that the shaft is subjected to a 600 N · m torque, determine (*a*) the maximum shearing stress in the brass jacket, (*b*) the maximum shearing stress in the steel core, (*c*) the angle of twist of *B* relative to *A*.

$$\tau_{max} = G\gamma_{max} = G c_{max} \frac{\varphi}{L}$$

$$\frac{\varphi_{all}}{L} = \frac{\tau_{all}}{G c_{max}} \quad \text{for each material}$$

$$T_i = G_i J_i \frac{\varphi}{L} \quad \text{for each material}$$

Brass jacket: $\tau_{all} = 20\times10^6$ Pa, $c_1 = 20$ mm $= 0.020$ m, $c_2 = 20+5 = 25$ mm $= 0.025$ m

$$\frac{\varphi_{all}}{L} = \frac{20\times10^6}{(39\times10^9)(0.025)} = 20.513\times10^{-3} \text{ rad/m}$$

$$J_2 = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.025^4 - 0.020^4) = 362.265\times10^{-9} \text{ m}^4$$

Steel core: $\tau_{all} = 45\times10^6$ Pa, $c = 0.020$ m

$$\frac{\varphi_{all}}{L} = \frac{45\times10^6}{(77.2\times10^9)(0.020)} = 29.145\times10^{-3} \text{ rad/m}$$

$$J_1 = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.020)^4 = 251.33\times10^{-9} \text{ m}^4$$

Smaller value of $\frac{\varphi_{all}}{L}$ governs $\quad \frac{\varphi_{all}}{L} = 20.513\times10^{-3}$ rad/m

Torque carried by brass sleeve

$$T_2 = G_2 J_2 \frac{\varphi_{all}}{L} = (39\times10^9)(362.265\times10^{-9})(20.513\times10^{-3}) = 289.8 \text{ N}\cdot\text{m}$$

Torque carried by steel core

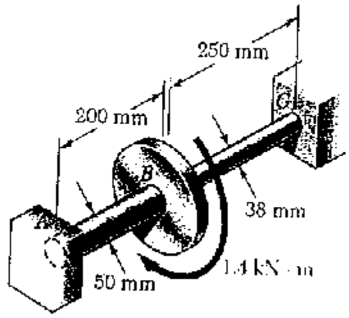
$$T_1 = G_1 J_1 \frac{\varphi_{all}}{L} = (77.2\times10^9)(251.33\times10^{-9})(20.513\times10^{-3}) = 398.0 \text{ N}\cdot\text{m}$$

(a) Allowable torque $\quad T = T_1 + T_2 = 687.8$ N·m $\qquad T_{all} = 688$ N·m

(b) Allowable twist angle

$$\varphi_{all} = L\frac{\varphi_{all}}{L} = (2)(20.513\times10^{-3}) = 41.026\times10^{-3} \text{ rad} \qquad \varphi_{all} = 2.35°$$

Problem 3.56



3.56 Two solid steel shafts ($G = 77.2$ GPa) are connected to a coupling disk B and to fixed supports at A and C. For the loading shown, determine (*a*) the reaction at each support, (*b*) the maximum shearing stress in shaft AB, (*c*) the maximum shearing stress in shaft BC.

Shaft AB

$T = T_{AB}$, $L_{AB} = 0.200\ m$, $c = \frac{1}{2}d = 25\ mm = 0.025\ m$

$J_{AB} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.025)^4 = 613.59 \times 10^{-9}\ m^4$

$\varphi_B = \frac{T_{AB}L_{AB}}{GJ_{AB}}$

$T_{AB} = \frac{GJ_{AB}}{L_{AB}}\varphi_B = \frac{(77.2\times10^9)(613.59\times10^{-9})}{0.200}\varphi_B = 236.847\times10^3\ \varphi_B$

Shaft BC

$T = T_{BC}$, $L_{BC} = 0.250\ m$, $c = \frac{1}{2}d = 19\ mm = 0.019\ m$

$J_{BC} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.019)^4 = 204.71\times10^{-9}\ m^4$ $\qquad \varphi_B = \frac{T_{BC}L_{BC}}{GJ_{BC}}$

$T_{BC} = \frac{GJ_{BC}}{L_{BC}}\varphi_B = \frac{(77.2\times10^9)(204.71\times10^{-9})}{0.250} = 63.214\times10^3\ \varphi_B$

Equilibrium of coupling disk. $\quad T = T_{AB} + T_{BC}$

$1.4\times10^3 = 236.847\times10^3\ \varphi_B + 63.214\times10^3\ \varphi_B$

$\varphi_B = 4.6657\times10^{-3}$ rad.

$T_{AB} = (236.847\times10^3)(4.6657\times10^{-3}) = 1.10506\times10^3\ N\cdot m$

$T_{BC} = (63.214\times10^3)(4.6657\times10^{-3}) = 294.94\ N\cdot m$

(a) Reactions at supports $\qquad T_A = T_{AB} = 1105\ N\cdot m$ ◄

$T_C = T_{BC} = 295\ N\cdot m$ ◄

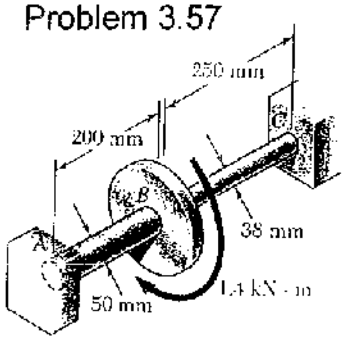
(b) Maximum shearing stress in AB

$\tau_{AB} = \frac{T_{AB}c}{J_{AB}} = \frac{(1.10506\times10^3)(0.025)}{613.59\times10^{-9}} = 45.0\times10^6\ Pa$ $\qquad \tau_{AB} = 45.0\ MPa$ ◄

(c) Maximum shearing stress in BC

$\tau_{BC} = \frac{T_{BC}c}{J_{BC}} = \frac{(294.94)(0.019)}{204.71\times10^{-9}} = 27.4\times10^9\ Pa$ $\qquad \tau_{BC} = 27.4\ MPa$ ◄

Problem 3.57



3.57 Solve Prob. 3.56, assuming that shaft AB is replaced by a hollow shaft of the same outer diameter and of 25-mm inner diameter.

3.56 Two solid steel shafts ($G = 77.2$ GPa) are connected to a coupling disk B and to fixed supports at A and C. For the loading shown, determine (*a*) the reaction at each support, (*b*) the maximum shearing stress in shaft AB, (*c*) the maximum shearing stress in shaft BC.

Shaft AB

$T = T_{AB}$, $L_{AB} = 0.200\text{ m}$, $c_2 = 25\text{ mm} = 0.025\text{ m}$

$c_1 = 12.5\text{ mm} = 0.0125\text{ m}$

$$J_{AB} = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.025^4 - 0.0125^4) = 575.243\times10^{-9}\text{ m}^4$$

$$T_{AB} = \frac{GJ_{AB}}{L_{AB}}\varphi_B = \frac{(77.2\times10^9)(575.243\times10^{-9})}{0.200}\varphi_B = 222.044\times10^3\,\varphi_B$$

Shaft BC

$T = T_{BC}$, $L_{BC} = 0.250\text{ m}$, $c = \frac{1}{2}d = 19\text{ mm} = 0.019\text{ m}$

$$J_{BC} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.019)^4 = 204.71\times10^{-9}\text{ m}$$

$$T_{BC} = \frac{GJ_{BC}}{L_{BC}}\varphi_B = \frac{(77.2\times10^9)(204.71\times10^{-9})}{0.250}\varphi_B = 63.214\times10^3\,\varphi_B$$

Equilibrium of coupling disk $\qquad T = T_{AB} + T_{BC}$

$$1.4\times10^3 = 222.044\times10^3\,\varphi_B + 63.214\times10^3\,\varphi_B$$

$$\varphi_B = 4.9078\times10^{-3}\text{ rad}$$

$$T_{AB} = (222.044\times10^3)(4.9078\times10^{-3}) = 1.0897\times10^3\text{ N}\cdot\text{m}$$

$$T_{BC} = (63.214\times10^3)(4.9078\times10^{-3}) = 310.24\text{ N}\cdot\text{m}$$

(a) Reactions at supports $\qquad T_A = T_{AB} = 1090\text{ N}\cdot\text{m}$ ◄

$T_C = T_{BC} = 310\text{ N}\cdot\text{m}$ ◄

(b) Maximum shearing stress in AB

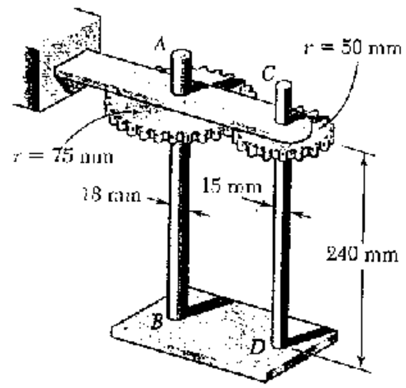
$$\tau_{AB} = \frac{T_{AB}c_2}{J_{AB}} = \frac{(1.0897\times10^3)(0.025)}{575.243\times10^{-9}} = 47.4\times10^6\text{ Pa} \qquad \tau_{AB} = 47.4\text{ MPa}$$ ◄

(c) Maximum shearing stress in BC

$$\tau_{BC} = \frac{T_{BC}c}{J_{BC}} = \frac{(310.24)(0.019)}{204.71\times10^{-9}} = 28.8\times10^6\text{ Pa} \qquad \tau_{BC} = 28.8\text{ MPa}$$ ◄

Problem 3.58

3.58 At a time when rotation is prevented at the lower end of each shaft, a 80-N · m torque is applied to end A of shaft AB. Knowing that $G = 77.2$ GPa for both shafts, determine (*a*) the maximum shearing stress in shaft CD, (*b*) the angle of rotation at A.



Let T_A = torque applied at A = 50 N·m

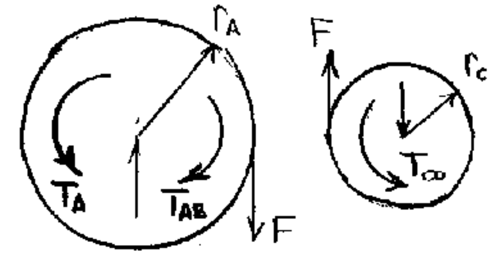
T_{AB} = torque in shaft AB

T_{CD} = torque in shaft CD

Statics

$$T_A - T_{AB} - F r_A = 0$$

$$T_{CD} - F r_c = 0$$



Gear A　　Gear C

$$T_{CD} = \frac{r_c}{r_A}(T_A - T_{AB}) = \frac{2}{3}(T_A - T_{AB})$$

Kinematics: $r_A \varphi_A = r_c \varphi_c$　　$\varphi_A = \frac{r_c}{r_A}\varphi_c = \frac{2}{3}\varphi_c$

Angles of twist　$\varphi_A = \frac{T_{AB}L}{GJ_{AB}}$　　$\varphi_C = \frac{T_{CD}L}{GJ_{CD}} = \frac{2}{3}\frac{(T_A - T_{AB})L}{GJ_{CD}}$

$$\frac{T_{AB}L}{GJ_{AB}} = \frac{2}{3}\cdot\frac{2}{3}\frac{(T_A - T_{AB})L}{GJ_{CD}}$$

$$\left(\frac{4}{9} + \frac{J_{CD}}{J_{AB}}\right)T_{AB} = \left(\frac{4}{9} + \left(\frac{15}{18}\right)^4\right)T_{AB} = \frac{4}{9}T_A$$

$$T_{AB} = 0. \qquad T_A = (0.47960)(80) = 38.368 \text{ N·m}$$

$$T_{CD} = \frac{2}{3}(80 - 38.368) = 27.755 \text{ N·m}$$

(a) Maximum shearing stress in shaft CD

$$\tau_{CD} = \frac{T_{CD}c}{J_{CD}} = \frac{2T_{CD}}{\pi c^3} = \frac{(2)(27.755)}{\pi(0.0075)^3} = 41.9\times10^6 \text{ Pa} \qquad \tau_{CD} = 41.9 \text{ MPa}$$

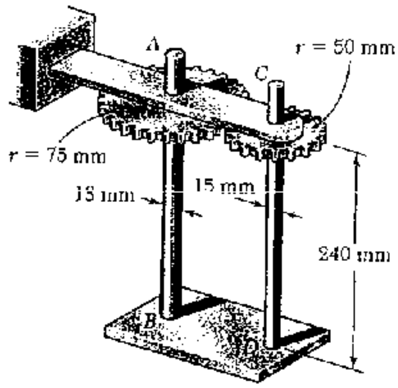
(b) Angle of rotation at A

$$\varphi_A = \frac{T_{AB}L}{GJ_A} = \frac{2T_{AB}L}{\pi G c^4} = \frac{(2)(38.368)(0.240)}{\pi(77.2\times10^9)(0.009)^4} = 11.57\times10^{-3} \text{ rad}$$

$$\varphi_A = 0.663^\circ$$

Problem 3.59

3.59 Solve Prob. 3.58, assuming that the 80-N · m torque is applied to end *C* of shaft *CD*.



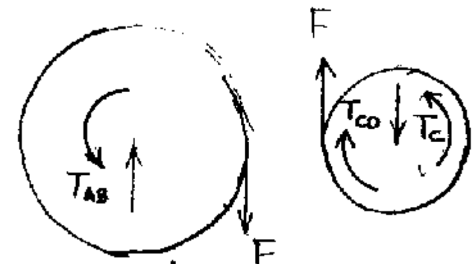
3.58 At a time when rotation is prevented at the lower end of each shaft, a 80-N · m torque is applied to end *A* of shaft *AB*. Knowing that $G = 77.2$ GPa for both shafts, determine (*a*) the maximum shearing stress in shaft *CD*, (*b*) the angle of rotation at *A*.

Let T_C = torque applied at C = 50 N·m

T_{CD} = torque in shaft CD

T_{AB} = torque in shaft AB

Statics.



Gear A   Gear C

$$T_{AB} - r_A F = 0$$

$$T_C - T_{CD} - r_C F = 0$$

$$T_{AB} = \frac{r_A}{r_C}(T_C - T_{CD}) = \frac{3}{2}(T_C - T_{CD})$$

Kinematics: $r_A \varphi_A = r_C \varphi_C$ $\quad \varphi_C = \frac{r_A}{r_C}\varphi_A = \frac{3}{2}\varphi_A$

Angles of twist $\quad \varphi_C = \frac{T_{CD}L}{GJ_{CD}} \qquad \varphi_A = \frac{T_{AB}L}{GJ_{AB}} = \frac{3}{2}\frac{(T_C - T_{CD})L}{GJ_{AB}}$

$$\frac{T_{CD}L}{GJ_{CD}} = \frac{3}{2}\cdot\frac{3}{2}\;\frac{T_C - T_{CD}}{GJ_{AB}}$$

$$\left(\frac{J_{AB}}{J_{CD}} + \frac{9}{4}\right)T_{CD} = \left(\left(\frac{18}{15}\right)^4 + \frac{9}{4}\right)T_{CD} = \frac{9}{4}T_C$$

$$T_{CD} = 0.4796\,T_C = (0.52040)(80) = 41.632 \text{ N·m}$$

$$T_{AB} = \frac{3}{2}(80 - 41.632) = 57.552 \text{ N·m}$$

(a) Maximum shearing stress in CD

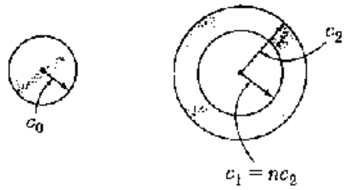
$$\tau_{CD} = \frac{T_{CD}c}{J_{CD}} = \frac{2T_{CD}}{\pi c^3} = \frac{(2)(41.632)}{\pi(0.0075)^3} = 62.8\times10^6 \text{ Pa} \qquad \tau_{CD} = 62.8 \text{ MPa}$$

(b) Angle of rotation at A

$$\varphi_A = \frac{T_{AB}L}{GJ_{AB}} = \frac{2T_{AB}L}{\pi G c^4} = \frac{(2)(57.552)(0.240)}{\pi(77.2\times10^9)(0.009)^4} = 17.36\times10^{-3} \text{ rad}$$

$$\varphi_A = 0.995^\circ$$

Problem 3.60



3.60 A solid shaft and a hollow shaft are made of the same material and are of the same weight and length. Denoting by n the ratio c_1/c_2, show that the ratio T_s/T_h of the torque T_s in the solid shaft to the torque T_h in the hollow shaft is (*a*) $\sqrt{1 - n^2}/(1 + n^2)$ if the maximum shearing stress is the same in each shaft, (*b*) $(1 - n)/(1 + n^2)$ if the angle of twist is the same for each shaft.

For equal weight and length, the areas are equal

$$\pi c_0^2 = \pi(c_2^2 - c_1^2) = \pi c_2^2(1-n^2) \quad \therefore \quad c_0 = c_2(1-n^2)^{1/2}$$

$$J_s = \frac{\pi}{2}c_0^4 = \frac{\pi}{2}c_2^4(1-n^2)^2 \qquad J_h = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}c_2^4(1-n^4)$$

(a) For equal stresses

$$\tau = \frac{T_s c_0}{J_s} = \frac{T_h c_2}{J_h}$$

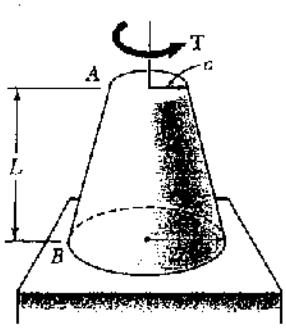
$$\frac{T_s}{T_h} = \frac{J_s c_2}{J_h c_0} = \frac{\frac{\pi}{2}c_2^4(1-n^2)^2 c_2}{\frac{\pi}{2}c_2^4(1-n^4)c_2(1-n^2)^{1/2}} = \frac{1-n^2}{(1+n^2)(1-n^2)^{1/2}} = \frac{(1-n^2)^{1/2}}{1+n^2}$$

(b) For equal angles of twist

$$\varphi = \frac{T_s L}{G J_s} = \frac{T_h L}{G J_h}$$

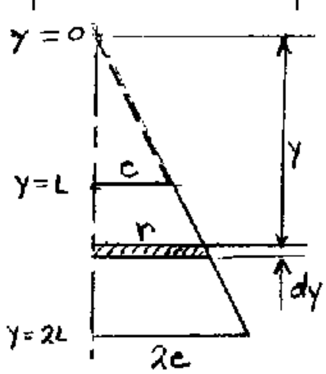
$$\frac{T_s}{T_h} = \frac{J_s}{J_h} = \frac{\frac{\pi}{2}c_2^4(1-n^2)^2}{\frac{\pi}{2}c_2^4(1-n^4)} = \frac{(1-n^2)^2}{1-n^4} = \frac{1-n^2}{1+n^2}$$

Problem 3.61



3.61 A torque **T** is applied as shown to a solid tapered shaft AB. Show by integration that the angle of twist at A is

$$\phi = \frac{7TL}{12\pi G c^4}$$



Introduce coordinate y as shown.

$$r = \frac{cy}{L}$$

Twist in length dy

$$d\varphi = \frac{T\,dy}{GJ} = \frac{T\,dy}{G\frac{\pi}{2}r^4} = \frac{2TL^4\,dy}{\pi G c^4 y^4}$$

$$\varphi = \int_L^{2L} \frac{2TL^4}{\pi G c^4}\frac{dy}{y^4} = \frac{2TL^4}{\pi G c^4}\int_L^{2L}\frac{dy}{y^4}$$

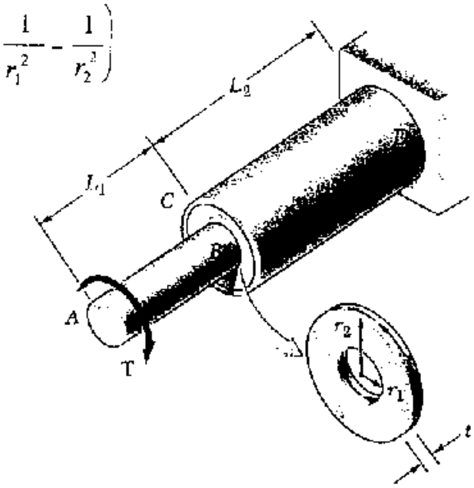
$$= \frac{2TL^4}{\pi G c^4}\left\{-\frac{1}{3y^3}\right\}_L^{2L} = \frac{2TL^4}{\pi G c^4}\left\{-\frac{1}{24L^3} + \frac{1}{3L^3}\right\}$$

$$= \frac{2TL^4}{\pi G c^4}\left\{\frac{7}{24L^3}\right\} = \frac{7TL}{12\pi G c^4}$$

Problem 3.62

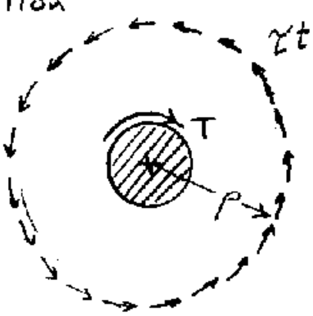
3.62 An annular plate of thickness t and modulus of rigidity G is used to connect shaft AB of radius r_1 to tube CD of inner radius r_2. Knowing that a torque **T** is applied to end A of shaft AB and that end D of tube CD is fixed, (a) determine the magnitude and location of the maximum shearing stress in the annular plate, (b) show that the angle through which end B of the shaft rotates with respect to end C of the tube is

$$\phi_{B/C} = \frac{T}{4\pi G t}\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right)$$



Use a free body consisting of shaft AB and an inner portion of the plate BC, the outer radius of this portion being ρ

The force per unit length of circumference is τt.



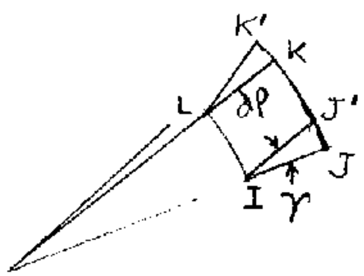
$\Sigma M = 0$

$$\tau t(2\pi\rho)\rho - T = 0$$

$$\tau = \frac{T}{2\pi t \rho^2}$$

(a) Maximum shearing stress occurs at $\rho = r_1$ $\qquad \tau_{max} = \dfrac{T}{2\pi t r_1^2}$ (1)

Shearing strain $\quad \gamma = \dfrac{\tau}{G} = \dfrac{T}{2\pi G t \rho^2}$



The relative circumferential displacement in radial length $d\rho$ is

$$dS = \gamma\, d\rho = \rho\, d\varphi$$

$$d\varphi = \gamma \frac{d\rho}{\rho}$$

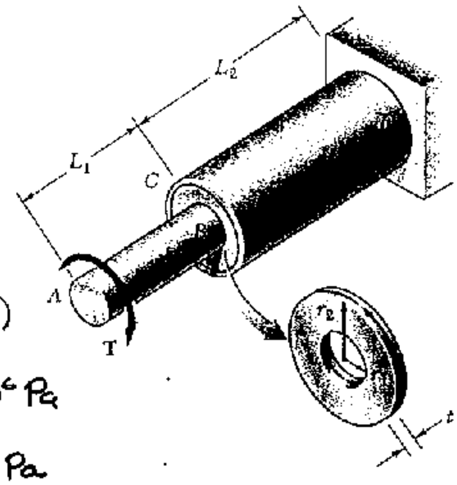
$$d\varphi = \frac{T}{2\pi G t \rho^2}\frac{d\rho}{\rho} = \frac{T}{2\pi G t}\frac{d\rho}{\rho^3}$$

$$\varphi_{B/C} = \int_{r_1}^{r_2} \frac{T}{2\pi G t}\frac{d\rho}{\rho^3} = \frac{T}{2\pi G t}\int_{r_1}^{r_2}\frac{d\rho}{\rho^3} = \frac{T}{2\pi G t}\left\{-\frac{1}{2\rho^2}\right\}\Bigg|_{r_1}^{r_2}$$

$$= \frac{T}{2\pi G t}\left\{-\frac{1}{2r_2^2} + \frac{1}{2r_1^2}\right\} = \frac{T}{4\pi G t}\left\{\frac{1}{r_1^2} - \frac{1}{r_2^2}\right\}$$

Problem 3.63

3.63 An annular brass plate (G = 39 GPa), of thickness t = 6 mm, is used to connect the brass shaft AB, of length L_1 = 50 mm and radius r_1 = 30 mm, to the brass tube CD, of length L_2 = 125 mm, inner radius r_2 = 75 mm, and thickness t = 3 mm. Knowing that a 2.8 kN · m torque **T** is applied to end A of shaft AB and that end D of tube CD is fixed, determine (*a*) the maximum shearing stress in the shaft-plate-tube system, (*b*) the angle through which end A rotates. (*Hint:* Use the formula derived in Prob. 3.62 to solve part *b*.)



Shaft AB

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2T}{\pi r_1^3}$$

$$= \frac{(2)(2800)}{\pi(0.030)^3} = 66.0\times 10^6 \text{ Pa}$$

$$= 66.0 \text{ MPa}$$

Plate BC (See PROBLEM 3.62 for derivation)

$$\tau = \frac{T}{2\pi t r_1^2} = \frac{2800}{2\pi(0.006)(0.030)^2} = 82.5\times10^6 \text{ Pa}$$

$$= 82.5 \text{ MPa}$$

Shaft CD $\quad c_1 = r_2 = 0.075 \text{ m}, \quad c_2 = r_2 + t = 0.075 + 0.003 = 0.078 \text{ m}$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.078^4 - 0.075^4) = 8.4421\times10^{-6} \text{ m}^4$$

$$\tau = \frac{Tc_2}{J} = \frac{(2800)(0.078)}{8.4421\times10^{-6}} = 25.9\times10^6 \text{ Pa} = 25.9 \text{ MPa}$$

(a) Largest stress $\qquad \tau_{max} = 82.5 \text{ MPa}$ ◄

Shaft AB $\quad \varphi_{AB} = \frac{TL_1}{GJ} = \frac{2TL_1}{\pi G c^4} = \frac{(2)(2800)(0.050)}{\pi(39\times10^9)(0.030)^4}$

$$= 2.821\times10^{-3} \text{ rad}$$

Plate BC (See Problem 3.62 for derivation)

$$\varphi_{BC} = \frac{T}{4\pi G t}\left\{\frac{1}{r_1^2} - \frac{1}{r_2^2}\right\} = \frac{2800}{4\pi(39\times10^9)(0.006)}\left\{\frac{1}{0.030^2} - \frac{1}{0.075^2}\right\}$$

$$= 0.889\times10^{-3} \text{ rad}$$

Shaft CD $\quad \varphi_{CD} = \frac{TL_2}{GJ} = \frac{(2800)(0.125)}{(39\times10^9)(8.4421\times10^{-6})} = 1.063\times10^{-3} \text{ rad}$

(b) Total rotation angle $\quad \varphi_A = \varphi_{AB} + \varphi_{BC} + \varphi_{CD} = 4.773\times10^{-3} \text{ rad}$

$$\varphi_A = 0.273^\circ$$ ◄

Problem 3.64

3.64 Using an allowable shearing stress of 4.5 ksi, design a solid steel shaft to transmit 12 hp at speed of (*a*) 1200 rpm, (*b*) 2400 rpm.

$\tau_{all} = 4.5 \text{ ksi} = 4500 \text{ psi}$ $\qquad P = 12 \text{ hp} = (12)(6600) = 79.2 \times 10^3 \text{ lb}\cdot\text{in/s}$

(a) $f = \dfrac{1200}{60} = 20 \text{ Hz}$

$T = \dfrac{P}{2\pi f} = \dfrac{79.2 \times 10^3}{2\pi(20)} = 630.25 \text{ lb}\cdot\text{in}$

$\tau = \dfrac{Tc}{J} = \dfrac{2T}{\pi c^3}$

$c = \sqrt[3]{\dfrac{2T}{\pi\tau}} = \sqrt{\dfrac{(2)(630.25)}{\pi(4500)}} = 0.44675 \text{ in.}$ $\qquad d = 2c = 0.893 \text{ in.}$

(b) $f = \dfrac{2400}{60} = 40 \text{ Hz}$

$T = \dfrac{P}{2\pi f} = \dfrac{79.2 \times 10^3}{2\pi(40)} = 315.127 \text{ lb}\cdot\text{in}$

$c = \sqrt[3]{\dfrac{(2)(315.127)}{\pi(4500)}} = 0.35458 \text{ in.}$ $\qquad d = 2c = 0.709 \text{ in.}$

Problem 3.65

3.65 Using an allowable shearing stress of 50 MPa, design a solid steel shaft to transmit 15 kW at a frequency of (*a*) 30 Hz, (*b*) 60 Hz.

$\tau_{all} = 50 \text{ MPa} = 50 \times 10^6 \text{ Pa}$ $\qquad P = 15 \text{ kW} = 15 \times 10^3 \text{ W}$

(a) $f = 30 \text{ Hz}$

$T = \dfrac{P}{2\pi f} = \dfrac{15 \times 10^3}{2\pi(30)} = 79.577 \text{ N}\cdot\text{m}$

$\tau = \dfrac{Tc}{J} = \dfrac{2T}{\pi c^3}$

$c = \sqrt[3]{\dfrac{2T}{\pi\tau}} = \sqrt[3]{\dfrac{(2)(79.577)}{\pi(50 \times 10^6)}} = 10.044 \times 10^{-3} \text{ m} = 10.044 \text{ mm}$

$d = 2c = 20.1 \text{ mm}$

(b) $f = 60 \text{ Hz}$

$T = \dfrac{P}{2\pi f} = \dfrac{15 \times 10^3}{2\pi(60)} = 39.789 \text{ N}\cdot\text{m}$

$c = \sqrt[3]{\dfrac{(2)(39.789)}{\pi(50 \times 10^6)}} = 7.972 \times 10^{-3} \text{ m} = 7.972 \text{ mm}$

$d = 2c = 15.94 \text{ mm}$

Problem 3.66

3.66 Determine the maximum shearing stress in a solid shaft of 12-mm diameter as it transmits 2.5 kW at a frequency of (*a*) 25 Hz, (*b*) 50 Hz.

$c = \frac{1}{2}d = 6\text{ mm} = 0.006\text{ m} \qquad P = 2.5\text{ kW} = 2500\text{ W}$

(a) $f = 25\text{ Hz} \qquad T = \frac{P}{2\pi f} = \frac{2500}{2\pi(25)} = 15.9155\text{ N}\cdot\text{m}$

$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2(15.9155)}{\pi(0.006)^3} = 46.9\times10^6\text{ Pa} \qquad \tau = 46.9\text{ MPa}$

(b) $f = 50\text{ Hz} \qquad T = \frac{2500}{2\pi(50)} = 7.9577\text{ N}\cdot\text{m}$

$\tau = \frac{2(7.9577)}{\pi(0.006)^3} = 23.5\times10^6\text{ Pa} \qquad \tau = 23.5\text{ MPa}$

Problem 3.67

3.67 Determine the maximum shearing stress in a solid shaft of 1.5-in. diameter as it transmits 75 hp at a speed of (*a*) 750 rpm, (*b*) 1500 rpm.

$c = \frac{1}{2}d = 0.75\text{ in.} \qquad P = 75\text{ hp} = (75)(6600) = 495\times10^3\text{ lb}\cdot\text{in/s}$

(a) $f = \frac{750}{60} = 12.5\text{ Hz}$

$T = \frac{P}{2\pi f} = \frac{495\times10^3}{2\pi(12.5)} = 6.3025\times10^3\text{ lb}\cdot\text{in}$

$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(6.3025\times10^3)}{\pi(0.75)^3} = 9.51\times10^3\text{ psi} \qquad \tau = 9.51\text{ ksi}$

(b) $f = \frac{1500}{60} = 25\text{ Hz}$

$T = \frac{495\times10^3}{2\pi(25)} = 3.1513\times10^3\text{ lb}\cdot\text{in}$

$\tau = \frac{(2)(3.1513\times10^3)}{\pi(0.75)^3} = 4.76\times10^3\text{ psi} \qquad \tau = 4.76\text{ ksi}$

Problem 3.68

3.68 A steel drive shaft is 6 ft long and its outer and inner diameters are respectively equal to 2.25 in. and 1.75 in. Knowing that the shaft transmits 240 hp while rotating at 1800 rpm, determine (*a*) the maximum shearing stress, (*b*) the angle of twist of the shaft ($G = 11.2 \times 10^6$ psi).

$L = 6\text{ ft} = 72\text{ in.} \qquad c_2 = \frac{1}{2}d_o = 1.125\text{ in.} \qquad c_1 = \frac{1}{2}d_i = 0.875\text{ in.}$

$P = 240\text{ hp} = (240)(6600) = 1.584 \times 10^6\text{ lb}\cdot\text{in/s}$

$f = \frac{1800}{60} = 30\text{ Hz} \qquad T = \frac{P}{2\pi f} = \frac{1.584 \times 10^6}{2\pi(30)} = 8.4034 \times 10^3\text{ lb}\cdot\text{in}$

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(1.125^4 - 0.875^4) = 1.59534\text{ in}^4$

(a) Maximum shearing stress. $\tau = \frac{Tc_2}{J}$

$\tau = \frac{(8.4034 \times 10^3)(1.125)}{1.59534} = 5.9259 \times 10^3\text{ psi} \qquad \tau = 5.93\text{ ksi}$ ◄

(b) Angle of twist. $\varphi = \frac{TL}{GJ}$

$\varphi = \frac{(8.4034 \times 10^3)(72)}{(11.2 \times 10^6)(1.59534)} = 33.86 \times 10^{-3}\text{ rad} \qquad \varphi = 1.940^\circ$ ◄

Problem 3.69

3.69 One of two hollow drive shafts of a cruise ship is 40 m long, and its outer and inner diameters are 400 mm and 200 mm, respectively. The shaft is made of a steel for which $\tau_{all} = 60$ MPa and $G = 77.2$ GPa. Knowing that the maximum speed of rotation of the shaft is 160 rpm, determine (*a*) the maximum power that can be transmitted by one shaft to its propeller, (*b*) the corresponding angle of twist of the shaft.

$L = 40\text{ m}$

$c_2 = \frac{1}{2}d_o = 200\text{ mm} = 0.200\text{ m} \qquad c_1 = \frac{1}{2}d_i = 100\text{ mm} = 0.100\text{ m}$

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.200^4 - 0.100^4) = 2.3562 \times 10^{-3}\text{ m}^4$

$\tau = \frac{Tc_{max}}{J} \qquad T = \frac{J\tau}{c_{max}} = \frac{(2.3562 \times 10^{-3})(60 \times 10^6)}{0.200} = 706.86 \times 10^3\text{ N}\cdot\text{m}$

$f = \frac{160}{60} = 2.66667\text{ Hz}$

(a) Maximum power $P = 2\pi f T$

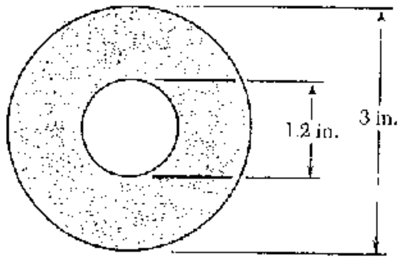
$P = 2\pi(2.6667)(706.86 \times 10^3) = 11.844 \times 10^6\text{ W} \qquad P = 11.84\text{ MW}$ ◄

(b) Angle of twist $\varphi = \frac{TL}{GJ}$

$\varphi = \frac{(706.86 \times 10^3)(40)}{(77.2 \times 10^9)(2.3562 \times 10^{-3})} = 155.44 \times 10^{-3}\text{ rad} \qquad \varphi = 8.91^\circ$ ◄

Problem 3.70

3.70 While a steel shaft of the cross section shown rotates at 120 rpm, a stroboscopic measurement indicates that the angle of twist is 2° in a 12 ft length. Using $G = 11.2 \times 10^6$ psi, determine the power being transmitted.



$\varphi = 2° = 34.907 \times 10^{-3}$ rad $\qquad L = 12$ ft $= 144$ in.

$c_2 = \frac{1}{2} d_o = 1.5$ in. $\qquad c_1 = \frac{1}{2} d_i = 0.6$ in

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(1.5^4 - 0.6^4) = 7.7486$ in^4

$f = \frac{120}{60} = 2$ Hz

$$T = \frac{GJ\varphi}{L} = \frac{(11.2 \times 10^6)(7.7486)(34.907 \times 10^{-3})}{144} = 21.037 \times 10^3 \text{ lb} \cdot \text{in}$$

$$P = 2\pi f T = 2\pi(2)(21.037 \times 10^3) = 264.36 \times 10^3 \text{ lb} \cdot \text{in/s}$$

$$\frac{264.36}{6600} = 40.1 \qquad\qquad 40.1 \text{ hp}$$

Problem 3.71

3.71 Determine the required thickness of the 50-mm tubular shaft of Example 3.07, if it is to transmit the same power while rotating at a frequency of 30 Hz.

From Example 3.07 $\qquad P = 100$ kW $= 100 \times 10^3$ W

$\tau_{all} = 60$ MPa $= 60 \times 10^6$ Pa $\qquad c_2 = \frac{1}{2} d = 25$ mm $= 0.025$ m

Given $\qquad f = 30$ Hz

$$T = \frac{P}{2\pi f} = 530.52 \text{ N} \cdot \text{m}$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) \qquad \tau = \frac{Tc_2}{J} = \frac{2Tc_2}{\pi(c_2^4 - c_1^4)}$$

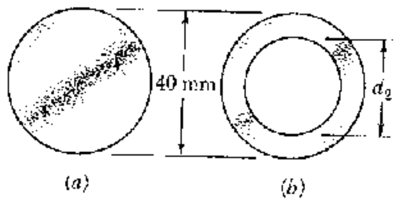
$$c_1^4 = c_2^4 - \frac{2Tc_2}{\pi \tau} = 0.025^4 - \frac{(2)(530.52)(0.025)}{\pi(60 \times 10^6)} = 249.90 \times 10^{-9} \text{ m}^4$$

$$c_1 = 22.358 \times 10^{-3} \text{ m} = 22.358 \text{ mm}$$

$$t = c_2 - c_1 = 25 \text{ mm} - 22.358 \text{ mm} = 2.642 \text{ mm} \qquad\qquad t = 2.64 \text{ mm}$$

Problem 3.72

3.72 The design of a machine element calls for a 40 mm-outer-diameter shaft to transmit 45 kW. (*a*) If the speed of rotation is 720 rpm, determine the maximum shearing stress in shaft *a*. (*b*) If the shaft of rotation can be increased 50% to 1080 rpm, determine the largest inner diameter of shaft *b* for which the maximum shearing stress will be the same in each shaft.



(a) $f = \frac{720}{60} = 12 \text{ Hz} \qquad P = 45 \text{ kW} = 45\times10^3 \text{ W}$

$$T = \frac{P}{2\pi f} = \frac{45\times10^3}{2\pi(12)} = 596.83 \text{ N}\cdot\text{m}$$

$$c = \tfrac{1}{2}d = 20 \text{ mm} = 0.020 \text{ m}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(596.83)}{\pi(0.020)^3} = 47.494\times10^6 \text{ Pa} \qquad \tau_{max} = 47.5 \text{ MPa}$$

(b) $f = \frac{1080}{60} = 18 \text{ Hz} \qquad T = \frac{45\times10^3}{2\pi(18)} = 397.89 \text{ N}\cdot\text{m}$

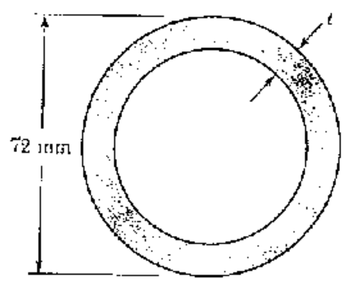
$$\tau = \frac{Tc_2}{J} = \frac{2Tc_2}{\pi(c_2^4 - c_1^4)} \qquad c_1^4 = c_2^4 - \frac{2Tc_2}{\pi\tau}$$

$$c_1^4 = 0.020^4 - \frac{(2)(397.89)(0.020)}{\pi(47.494\times10^6)} = 53.333\times10^{-9} \qquad c_1 = 15.20\times10^{-3} \text{ m}$$

$$c_1 = 15.20\times10^{-3} \text{ m} = 15.20 \text{ mm} \qquad d_2 = 2c_1 = 30.4 \text{ mm}$$

Problem 3.73

3.73 A steel pipe of 72-mm outer diameter is to be used to transmit a torque of 2500 N · m without exceeding an allowable shearing stress of 55 MPa. A series of 72-mm-outer-diameter pipes is available for use. Knowing that the wall thickness of the available pipes varies from 4 mm to 10 mm in 2-mm increments, choose the lightest pipe that can be used.



$$c_2 = \tfrac{1}{2}d_o = 36 \text{ mm} = 0.036 \text{ m}$$

$$\tau = \frac{Tc_2}{J} = \frac{2Tc_2}{\pi(c_2^4 - c_1^4)}$$

$$c_1^4 = c_2^4 - \frac{2Tc_2}{\pi\tau} = 0.036^4 - \frac{(2)(2500)(0.036)}{\pi(55\times10^6)} = 637.875\times10^{-9}$$

$$c_1 = 28.26\times10^3 \text{ m} = 28.26 \text{ mm}$$

$$t = c_2 - c_1 = 36 \text{ mm} - 28.26 \text{ mm} = 7.74 \text{ mm} \qquad \text{Use } t = 8 \text{ mm}$$

Problem 3.74

3.74 A 2.5-m-long solid steel shaft is to transmit 10 kW at a frequency of 25 Hz. Determine the required diameter of the shaft, knowing that $G = 77.2$ GPa, that the allowable shearing stress is 30 MPa, and that the angle of twist must not exceed 4°.

$P = 10\ kW = 10\times10^3\ W \qquad f = 25\ Hz \qquad \varphi = 4° = 69.813\times10^{-3}\ rad$

$$T = \frac{P}{2\pi f} = \frac{10\times10^3}{2\pi(25)} = 63.662\ N\cdot m$$

Stress requirement $\qquad \tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$$c = \sqrt[3]{\frac{2T}{\pi\tau}} = \sqrt[3]{\frac{(2)(63.662)}{\pi(30\times10^6)}} = 11.055\times10^{-3}\ m = 11.055\ mm$$

Twist angle requirement $\qquad \varphi = \frac{TL}{GJ} = \frac{2TL}{\pi G c^4}$

$$c = \sqrt[4]{\frac{2TL}{\pi G\varphi}} = \sqrt[4]{\frac{(2)(63.662)(2.5)}{\pi(77.2\times10^9)(69.813\times10^{-3})}} = 11.709\ mm$$

Use the larger value $\qquad c = 11.709\ mm \qquad\qquad d = 2c = 23.4\ mm$ ◄

Problem 3.75

3.75 A 1.5-m-long solid steel shaft of 22-mm diameter is to transmit 12 kW. Determine the minimum frequency at which the shaft can rotate, knowing that $G = 77.2$ GPa, that the allowable shearing stress is 30 MPa, and that the angle of twist must not exceed 3.5°.

$L = 1.5\ m \qquad c = \frac{1}{2}d = 11\ mm = 0.011\ m \qquad P = 12\ kW = 12\times10^3\ W$

$\varphi = 3.5° = 61.087\times10^{-3}\ rad \qquad G = 77.2\times10^9\ Pa \qquad \tau = 30\times10^6\ Pa$

Stress requirement $\qquad \tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$$T = \frac{\pi}{2}\tau c^3 = \frac{\pi}{2}(30\times10^6)(0.011)^3 = 62.722\ N\cdot m$$

Twist angle requirement $\qquad \varphi = \frac{TL}{GJ} = \frac{2TL}{\pi G c^4}$

$$T = \frac{\pi}{2}\frac{Gc^4\varphi}{L} = \frac{\pi(77.2\times10^9)(0.011)^4(61.087\times10^{-3})}{(2)(1.5)} = 72.305\ N\cdot m$$

Maximum allowable torque is the smaller value $\qquad T = 62.722\ N\cdot m$

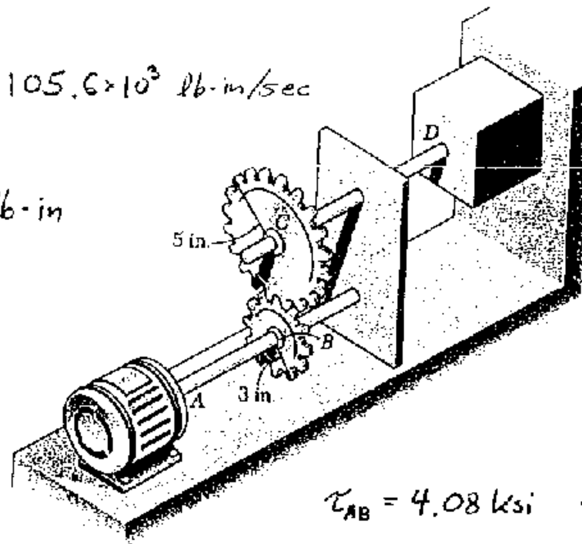
Minimum frequency to transmit a power of 12 kW.

$$P = 2\pi f T$$

$$f = \frac{P}{2\pi T} = \frac{12\times10^3}{2\pi(62.722)} = 30.4\ Hz \qquad\qquad f = 30.4\ Hz ◄$$

Problem 3.76

3.76 The two solid shafts and gears shown are used to transmit 16 hp from the motor at A operating at a speed of 1260 rpm, to a machine tool at D. Knowing that each shaft has a diameter of 1 in., determine the maximum shearing stress (a) in shaft AB, (b) in shaft CD.



(a) Shaft AB: $P = 16 \text{ hp} = (16)(6600) = 105.6 \times 10^3 \text{ lb}\cdot\text{in/sec}$

$f = \frac{1260}{60} = 21 \text{ Hz}$

$T_{AB} = \frac{P}{2\pi f} = \frac{105.6\times 10^3}{2\pi(21)} = 800.32 \text{ lb}\cdot\text{in}$

$c = \frac{1}{2}d = 0.5 \text{ in.}$

$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$= \frac{(2)(800.32)}{\pi (0.5)^3} = 4.08 \times 10^3 \text{ psi}$

$\tau_{AB} = 4.08 \text{ ksi}$ ◄

(b) Shaft CD

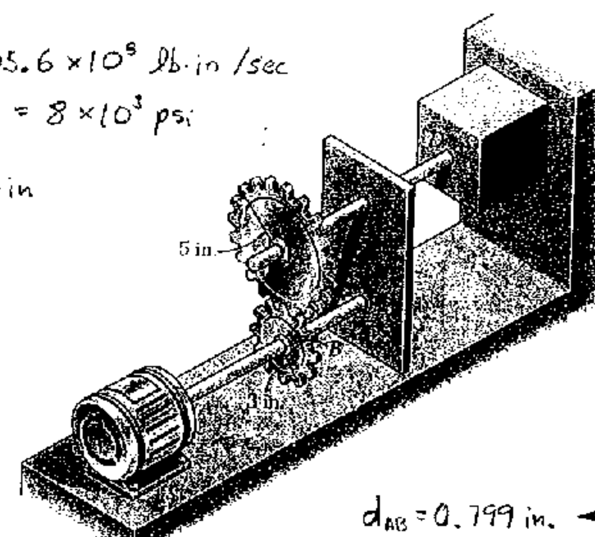
$T_{CD} = \frac{r_C}{r_B} T_{AB} = \frac{5}{3}(800.32) = 1.33387 \times 10^3 \text{ lb}\cdot\text{in}$

$\tau = \frac{2T}{\pi c^3} = \frac{(2)(1.33387\times 10^3)}{\pi (0.5)^3} = 6.79 \times 10^3 \text{ psi}$

$\tau_{CD} = 6.79 \text{ ksi}$ ◄

Problem 3.77

3.77 The two solid shafts and gears shown are used to transmit 16 hp from the motor at A operating at a speed of 1260 rpm, to a machine tool at D. Knowing that the maximum allowable shearing stress is 8 ksi, determine the required diameter (a) of shaft AB, (b) of shaft CD.



(a) Shaft AB: $P = 16 \text{ hp} = (16)(6600) = 105.6 \times 10^3 \text{ lb}\cdot\text{in/sec}$

$f = \frac{1260}{60} = 21 \text{ Hz}$ $\qquad \tau = 8 \text{ ksi} = 8 \times 10^3 \text{ psi}$

$T_{AB} = \frac{P}{2\pi f} = \frac{105.6\times 10^3}{2\pi(21)} = 800.32 \text{ lb}\cdot\text{in}$

$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c = \sqrt[3]{\frac{2T}{\pi \tau}}$

$c = \sqrt[3]{\frac{(2)(800.32)}{\pi (8\times 10^3)}} = 0.399 \text{ in}$

$d_{AB} = 2c = 0.799 \text{ in.}$

$d_{AB} = 0.799 \text{ in.}$ ◄

(b) Shaft CD

$T_{CD} = \frac{r_C}{r_B} T_{AB} = \frac{5}{3}(800.32) = 1.33387 \times 10^3 \text{ lb}\cdot\text{in}$

$c = \sqrt[3]{\frac{2T}{\pi\tau}} = \sqrt[3]{\frac{(2)(1.33387\times 10^3)}{\pi (8 \times 10^3)}} = 0.473 \text{ in.}$

$d_{CD} = 2c = 0.947 \text{ in.}$

$d_{CD} = 0.947 \text{ in.}$ ◄

Problem 3.78

3.78 A steel shaft must transmit 150 kW at speed of 360 rpm. Knowing that G = 77.2 GPa, design a solid shaft so that the maximum stress will not exceed 50 MPa and the angle of twist in a 2.5-m length must not exceed 3°.

$P = 150 \text{ kW} = 150 \times 10^3 \text{ W} \qquad f = \frac{360}{60} = 6 \text{ Hz}$

$T = \frac{P}{2\pi f} = \frac{150 \times 10^3}{2\pi(6)} = 3.9789 \times 10^3 \text{ N}\cdot\text{m}$

Stress requirement $\tau = 50 \times 10^6 \text{ Pa} \qquad \tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$c = \sqrt[3]{\frac{2T}{\pi\tau}} = \sqrt[3]{\frac{(2)(3.9789 \times 10^3)}{\pi(50 \times 10^6)}} = 37.00 \times 10^{-3} \text{ m} = 37.00 \text{ mm}$

Angle of twist requirement: $\varphi = 3° = 52.36 \times 10^{-3} \text{ rad}$

$\varphi = \frac{TL}{GJ} = \frac{2TL}{\pi G c^4} \qquad G = 77.2 \times 10^9 \text{ Pa}, \quad L = 2.5 \text{ m}$

$c = \sqrt[4]{\frac{2TL}{\pi G \varphi}} = \sqrt[4]{\frac{(2)(3.9789 \times 10^3)(2.5)}{\pi(77.2 \times 10^9)(52.36 \times 10^{-3})}} = 35.38 \times 10^{-3} \text{ m} = 35.38 \text{ mm}$

Use larger value. $c = 37.00 \text{ mm} \qquad d = 2c = 74.0 \text{ mm}$ ◄

Problem 3.79

3.79 A 2.5-m-long steel shaft of 30-mm diameter rotates at a frequency of 30 Hz. Determine the maximum power that the shaft can transmit, knowing that G = 77.2 GPa, that the allowable shearing stress is 50 MPa, and that the angle of twist must not exceed 7.5°.

$c = \frac{1}{2}d = 15 \text{ mm} = 0.015 \text{ m} \qquad L = 2.5 \text{ m}$

Stress requirement $\tau = 50 \times 10^6 \text{ Pa} \qquad \tau = \frac{Tc}{J}$

$T = \frac{\tau J}{c} = \frac{\pi}{2}\tau c^3 = \frac{\pi}{2}(50 \times 10^6)(0.015)^3 = 265.07 \text{ N}\cdot\text{m}$

Twist angle requirement $\varphi = 7.5° = 130.90 \times 10^{-3} \text{ rad} \qquad G = 77.2 \times 10^9 \text{ Pa}$

$\varphi = \frac{TL}{GJ} = \frac{2TL}{\pi G c^4}$

$T = \frac{\pi}{2} G c^4 \varphi = \frac{\pi}{2}(77.2 \times 10^9)(0.015)^4(130.90 \times 10^{-3}) = 803.60 \text{ N}\cdot\text{m}$

Smaller value of T is the maximum allowable torque. $T = 265.07 \text{ N}\cdot\text{m}$

Power transmitted at $f = 30$ Hz

$P = 2\pi f T = 2\pi(30)(265.07) = 49.96 \times 10^3 \text{ W}$

$P = 50.0 \text{ kW}$ ◄

Problem 3.80

3.80 A 1.5-in.-diameter steel shaft of length 4 ft will be used to transmit 60 hp between a motor and a pump. Knowing that $G = 11.2 \times 10^6$ psi, determine the lowest speed of rotation at which the stress does not exceed 8500 psi and the angle of twist does not exceed 2°.

$c = \frac{1}{2}d = 0.75$ in. $\quad L = 4$ ft $= 48$ in. $\quad P = 60$ hp $= 396 \times 10^3$ lb·in/s

Stress requirement $\quad \tau = 8500$ psi $\quad \tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$$T = \frac{\pi}{2}\tau c^3 = \frac{\pi}{2}(8500)(0.75)^3 = 5.6328 \times 10^3 \text{ lb·in}$$

Twist angle requirement $\quad \varphi = 2° = 34.907 \times 10^{-3}$ rad

$$\varphi = \frac{TL}{GJ} = \frac{2TL}{\pi G c^4}$$

$$T = \frac{\pi G c^4 \varphi}{2L} = \frac{\pi(11.2\times10^6)(0.75)^4(34.907\times10^{-3})}{(2)(48)} = 4.0481\times10^3 \text{ lb·in}$$

Maximum allowable torque is the smaller value. $\quad T = 4.0481 \times 10^3$ lb·in

$$P = 2\pi f T$$

$$f = \frac{P}{2\pi T} = \frac{396\times10^3}{2\pi(4.0481\times10^3)} = 15.569 \text{ Hz} \qquad f = 734 \text{ rpm}$$

Problem 3.81

3.81 A 5-ft-long solid steel shaft of 0.875-in. diameter is to transmit 18 hp. Determine the minimum speed at which the shaft can rotate, knowing that $G = 11.2 \times 10^6$ psi, that the allowable shearing stress is 4.5 ksi, and that the angle of twist must not exceed 3.5°.

$L = 5$ ft $= 60$ in. $\quad c = \frac{1}{2}d = 0.4375$ in. $\quad P = 18$ hp $= 118.8 \times 10^3$ lb·in/s

Stress requirement $\quad \tau = 4.5$ ksi $= 4.5 \times 10^3$ psi $\quad \tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$$T = \frac{\pi}{2}\tau c^3 = \frac{\pi}{2}(4.5\times10^3)(0.4375)^3 = 591.92 \text{ lb·in}$$

Twist angle requirement $\quad \varphi = 3.5° = 61.087 \times 10^{-3}$ rad $\quad \varphi = \frac{TL}{GJ}$

$$T = \frac{GJ\varphi}{L} = \frac{\pi G c^4 \varphi}{2L} = \frac{\pi(11.2\times10^6)(0.4375)^4(61.087\times10^{-3})}{(2)(60)}$$

$$= 656.21 \text{ lb·in}$$

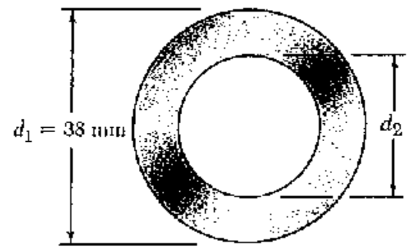
Maximum allowable torque is the smaller value $\quad T = 591.92$ lb·in

$$P = 2\pi f T$$

$$f = \frac{P}{2\pi T} = \frac{118.8\times10^3}{2\pi(591.92)} = 31.943 \text{ Hz} \qquad f = 1917 \text{ rpm}$$

Problem 3.82

3.82 A 1.5-m-long tubular steel shaft of 38-mm outer diameter d_1 is to be made of a steel for which $\tau_{all} = 65$ MPa and $G = 77.2$ GPa. Knowing that the angle of twist must not exceed 4° when the shaft is subjected to a torque of 600 N · m, determine the largest inner diameter d_2 which can be specified in the design.



$L = 1.5\ m \qquad c_2 = \frac{1}{2} d_o = 19\ mm = 0.019\ m$

$\tau = 65 \times 10^6\ Pa \qquad \varphi = 4° = 69.813 \times 10^{-3}\ rad$

$J = \frac{\pi}{2}(c_2^4 - c_1^4)$

Stress requirement $\qquad \tau = \frac{Tc_2}{J} = \frac{2Tc_2}{\pi(c_2^4 - c_1^4)}$

$c_1 = \sqrt[4]{c_2^4 - \frac{2Tc_2}{\pi\tau}} = \sqrt[4]{0.019^4 - \frac{(2)(600)(0.019)}{\pi(65\times 10^6)}} = 11.689 \times 10^{-3}\ m = 11.689\ mm$

Twist angle requirement $\qquad \varphi = \frac{TL}{GJ} = \frac{2TL}{\pi G(c_2^4 - c_1^4)}$

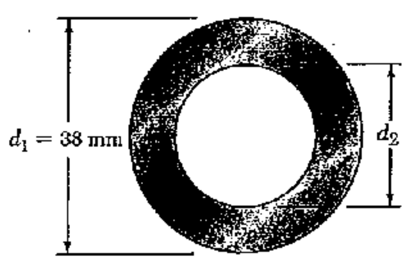
$c_1 = \sqrt[4]{c_2^4 - \frac{2TL}{\pi G \varphi}} = \sqrt[4]{0.019^4 - \frac{(2)(600)(1.5)}{\pi(77.2\times 10^9)(69.813\times 10^{-3})}}$

$c_1 = 12.448 \times 10^{-3}\ m = 12.448\ mm$

Use smaller value of $c_1 \qquad c_1 = 11.689\ mm \qquad d_i = 2c_1 = 23.4\ mm$ ◄

Problem 3.83

3.83 A 1.5-m-long tubular steel shaft of 38-mm outer diameter d_1 and 30-mm inner diameter d_2 is to transmit 100 kW between a turbine and a generator. Determine the minimum frequency at which the shaft can rotate, knowing that $G = 77.2$ GPa, that the allowable shearing stress is 60 MPa, and that the angle of twist must not exceed 3°.



$L = 1.5\ m \qquad \varphi = 3° = 52.360 \times 10^{-3}\ rad$

$c_2 = \frac{1}{2} d_o = 19\ mm = 0.019\ m, \quad c_1 = \frac{1}{2} d_i = 15\ mm = 0.015\ m$

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.019^4 - 0.015^4) = 125.186 \times 10^{-9}\ m^4$

Stress requirement $\qquad \tau = 60 \times 10^6\ Pa \qquad \tau = \frac{Tc_2}{J}$

$T = \frac{J\tau}{c_2} = \frac{(125.186\times 10^{-9})(60\times 10^6)}{0.019} = 395.32\ N\cdot m$

Twist angle requirement $\qquad \varphi = \frac{TL}{GJ}$

$T = \frac{GJ\varphi}{L} = \frac{(77.2\times 10^9)(125.186\times 10^{-9})(52.360\times 10^{-3})}{1.5} = 337.35\ N\cdot m$

Maximum allowable torque is the smaller value $\qquad T = 337.35\ N\cdot m$

Power transmitted $\qquad P = 100\ kW = 100 \times 10^3\ W \qquad P = 2\pi f T$

Frequency $\qquad f = \frac{P}{2\pi T} = \frac{100 \times 10^3}{2\pi(337.35)} = 47.2\ Hz \qquad f = 47.2\ Hz$ ◄

Problem 3.84

3.84 The stepped shaft shown rotates at 450 rpm. Knowing that $r = 0.5$ in., determine that maximum power that can be transmitted without exceeding an allowable shearing stress of 7500 psi.

$d = 5$ in. $\quad D = 6$ in. $\quad r = 0.5$ in.

$\frac{D}{d} = \frac{6}{5} = 1.20 \qquad \frac{r}{d} = \frac{0.5}{5} = 0.10$ From Fig 3.32 $K = 1.33$

For the smaller side $\quad c = \frac{1}{2}d = 2.5$ in. $\qquad \tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$

$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi(2.5)^3(7500)}{(2)(1.33)} = 138.404 \times 10^3 \text{ lb}\cdot\text{in.}$$

$f = 450$ rpm $= 7.5$ Hz

Power $\quad P = 2\pi f T = 2\pi(7.5)(138.404\times10^3) = 6.52\times10^6$ lb·in/s

$P = 988$ hp ◄

Problem 3.85

3.85 The stepped shaft shown rotates at 450 rpm. Knowing that $r = 0.2$ in., determine that maximum power that can be transmitted without exceeding an allowable shearing stress of 7500 psi.

$d = 5$ in. $\quad D = 6$ in. $\quad r = 0.2$ in.

$\frac{D}{d} = \frac{6}{5} = 1.20 \qquad \frac{r}{d} = \frac{0.2}{5} = 0.04$ From Fig. 3.32 $K = 1.55$

For smaller side $\quad c = \frac{1}{2}d = 2.5$ in. $\qquad \tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$

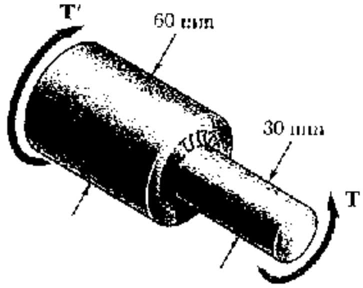
$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi(2.5)^3(7500)}{(2)(1.55)} = 118.76\times10^3 \text{ lb}\cdot\text{in}$$

$f = 450$ rpm $= 7.5$ Hz

Power $\quad P = 2\pi f T = 2\pi(7.5)(118.76\times10^3) = 5.60\times10^6$ lb·in/s

$P = 848$ hp ◄

Problem 3.86



3.86 Knowing that the stepped shaft shown must transmit 45 kW speed of 2100 rpm, determine the minimum radius r of the fillet if an allowable shearing stress of 50 MPa is not to be exceeded.

$$f = \frac{2100}{60} = 35 \text{ Hz}$$

$$P = 45 \times 10^3 \text{ W}$$

$$T = \frac{P}{2\pi f} = \frac{45 \times 10^3}{2\pi(35)} = 204.63 \text{ N}\cdot\text{m}$$

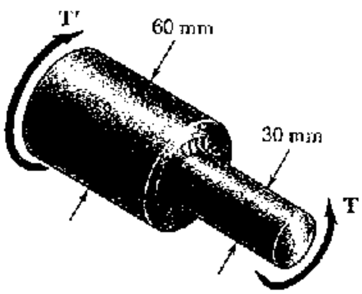
For smaller side $\quad c = \frac{1}{2}d = 15 \text{ mm} = 0.015 \text{ m} \qquad \tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$

$$K = \frac{\pi \tau c^3}{2T} = \frac{\pi(50 \times 10^6)(0.015)^3}{(2)(204.63)} = 1.295$$

$\frac{D}{d} = \frac{60}{30} = 2 \qquad$ From Fig. 3.32 $\quad \frac{r}{d} = 0.17$

$r = 0.17\, d = (0.17)(30) = 5.1 \text{ mm} \qquad\qquad r = 5.1 \text{ mm}$ ◄

Problem 3.87



3.87 The stepped shaft shown must transmit 45 kW. Knowing that the allowable shearing stress in the shaft is 40 MPa and that the radius of the fillet is $r = 6$ mm, determine the smallest permissible speed of the shaft.

$\frac{r}{d} = \frac{6}{30} = 0.2 \qquad \frac{D}{d} = \frac{60}{30} = 2$

From Fig. 3.32 $\quad K = 1.26$

For smaller side $\quad c = \frac{1}{2}d = 15 \text{ mm} = 0.015 \text{ m}$

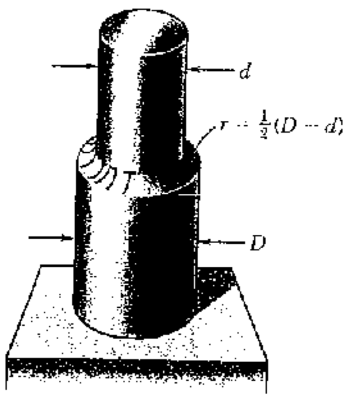
$$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$$

$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi(0.015)^3(40 \times 10^6)}{(2)(1.26)} = 168.30 \times 10^3 \text{ N}\cdot\text{m}$$

$P = 45 \text{ kW} = 45 \times 10^3 \qquad\qquad P = 2\pi f T$

$f = \frac{P}{2\pi T} = \frac{45 \times 10^3}{2\pi(168.30 \times 10^3)} = 42.6 \text{ Hz} \qquad\qquad f = 42.6 \text{ Hz}$ ◄

Problem 3.88



Full quarter-circular fillet extends to edge of larger shaft

3.88 A torque of magnitude $T = 200$ lb · in. is applied to the stepped shaft shown, which has a full quarter-circular fillet. Knowing that $D = 1$ in., determine the maximum shearing stress in the shaft when (*a*) $d = 0.8$ in., (*b*) $d = 0.9$ in.

(a) $\frac{D}{d} = \frac{1.0}{0.8} = 1.25 \qquad r = \frac{1}{2}(D - d) = 0.1 \text{ in.}$

$\frac{r}{d} = \frac{0.1}{0.8} = 0.125$

From Fig. 3.32 $\quad K = 1.31$

For smaller side $\quad c = \frac{1}{2}d = 0.4$ in.

$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3} = \frac{(2)(1.31)(200)}{\pi(0.4)^3} = 2.61 \times 10^3 \text{ psi}$

$\tau = 2.61 \text{ ksi}$ ◄

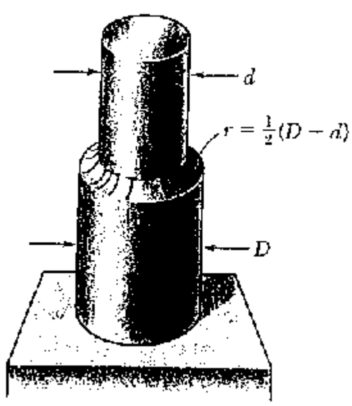
(b) $\frac{D}{d} = \frac{1.0}{0.9} = 1.111 \qquad r = \frac{1}{2}(D - d) = 0.05$

$\frac{r}{d} = \frac{0.05}{1.0} = 0.05 \qquad$ From Fig. 3.32 $\quad K = 1.44$

For smaller side $\quad c = \frac{1}{2}d = 0.45$ in

$\tau = \frac{2KT}{\pi c^3} = \frac{(2)(1.44)(200)}{\pi(0.45)^3} = 2.01 \times 10^3 \text{ psi} \qquad \tau = 2.01 \text{ ksi}$ ◄

Problem 3.89



Full quarter-circular fillet extends to edge of larger shaft

3.89 In the stepped shaft shown, which has a full quarter-circular fillet, $D = 1.25$ in. and $d = 1$ in. Knowing that the speed of the shaft is 2400 rpm and that the allowable shearing stress is 7500 psi, determine the maximum power that can be transmitted by the shaft.

$\frac{D}{d} = \frac{1.25}{1.0} = 1.25 \qquad r = \frac{1}{2}(D - d) = 0.15 \text{ in.}$

$\frac{r}{d} = \frac{0.15}{1.0} = 0.15$

From Fig. 3.32 $\quad K = 1.31$

For smaller side $\quad c = \frac{1}{2}d = 0.5$ in.

$\tau = \frac{KTc}{J} \qquad T = \frac{J\tau}{Kc} = \frac{\pi c^3 \tau}{2K}$

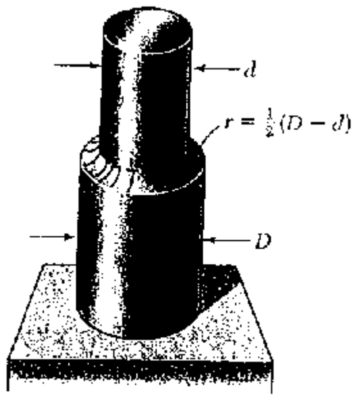
$T = \frac{\pi(0.5)^3(7500)}{(2)(1.31)} = 1.1241 \times 10^3 \text{ lb} \cdot \text{in}$

$f = 2400 \text{ rpm} = 40 \text{ Hz}$

$P = 2\pi f T = 2\pi(40)(1.1241 \times 10^3) = 282.5 \times 10^3 \text{ lb} \cdot \text{in/s}$

$P = 42.8 \text{ hp}$ ◄

Problem 3.90



Full quarter-circular fillet extends to edge of larger shaft

3.90 In the stepped shaft shown, which has a full quarter-circular fillet, the allowable shearing stress is 80 MPa. Knowing that $D = 30$ mm, determine the largest allowable torque that can be applied to the shaft is (*a*) $d = 26$ mm, (*b*) $d = 24$ mm.

$\tau = 80 \times 10^6$ Pa

(a) $\frac{D}{d} = \frac{30}{26} = 1.154$ $\quad r = \frac{1}{2}(D-d) = 2$ mm

$\frac{r}{d} = \frac{2}{26} = 0.0768$

From Fig. 3.32 $\quad K = 1.38$

Smaller side $\quad c = \frac{1}{2}d = 13$ mm $= 0.013$ m

$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$

$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi(0.013)^3(80\times10^6)}{(2)(1.38)}$

$= 203$ N-m $\qquad T = 203$ N-m ◄

(b) $\frac{D}{d} = \frac{30}{24} = 1.25$ $\quad r = \frac{1}{2}(D-d) = 3$ mm $\quad \frac{r}{d} = \frac{3}{24} = 0.125$

From Fig. 3.32 $\quad K = 1.31$ $\qquad c = \frac{1}{2}d = 12$ mm $= 0.012$ m

$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi(0.012)^3(80\times10^6)}{(2)(1.31)} = 165.8$ N·m $\qquad T = 165.8$ N·m ◄

Problem 3.91

3.91 A 2-in.-diameter solid shaft is made of a mild steel that is assumed to be elastoplastic with $\tau_Y = 20$ ksi. Determine the maximum shearing stress and the radius of the elastic core caused by the application of a torque of magnitude (*a*) 30 kip · in., (*b*) 40 kip · in.

$c = \frac{1}{2}d = 1.0$ in. $\qquad \tau_Y = 20$ ksi

Compute T_Y $\qquad T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(1.0)^3(20) = 31.416$ kip·in

(a) $T = 30$ kip·in $< T_Y$ $\qquad$ elastic $\qquad \rho = c = 1.0$ in.

$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(30)}{\pi(1.0)^3} = 19.10$ ksi $\qquad \tau_{max} = 19.10$ ksi

(b) $T = 40$ kip·in $> T_Y$ $\qquad$ plastic region with elastic core

The maximum shearing stress is $\qquad \tau_{max} = \tau_Y = 20$ ksi

$T = \frac{4}{3}T_Y\left(1 - \frac{\rho_Y^3}{c^3}\right)$

$\frac{\rho_Y^3}{c^3} = 4 - \frac{3T}{T_Y} = 4 - \frac{(3)(40)}{31.416} = 0.18028 \qquad \frac{\rho_Y}{c} = 0.56492$

$\rho_Y = 0.56492c = (0.56492)(1.0) \qquad \rho_Y = 0.565$ in.

Problem 3.92

3.92 A 38-mm-diameter solid shaft is made of a mild steel that is assumed to be elastoplastic with $\tau_Y = 145$ MPa. Determine the maximum shearing stress and the radius of the elastic core caused by the application of a torque of magnitude (*a*) 1.2 kN · m, (*b*) 1.8 kN · m.

$c = \frac{1}{2}d = 0.019$ m $\qquad \tau_Y = 150 \times 10^6$ Pa

Compute T_Y $\qquad T_Y = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(0.019)^3(145\times10^6) = 1.56224\times10^3$ N·m

(a) $T = 1.2\times10^3$ N·m $< T_Y$ $\qquad$ elastic $\qquad \rho = c = 19$ mm

$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(1.2\times10^3)}{\pi(0.019)^3} = 111.378\times10^6$ Pa $\qquad 111.4$ MPa

(b) $T = 1.6\times10^3$ N·m $> T_Y$ $\qquad$ plastic region with elastic core.

The maximum shearing stress is $\qquad \tau_{max} = \tau_Y = 145$ MPa

$T = \frac{4}{3}T_Y\left(1 - \frac{\rho_Y^3}{c^3}\right)$

$\frac{\rho_Y^3}{c^3} = 4 - \frac{3T}{T_Y} = 4 - \frac{(3)(1.8\times10^3)}{1.56224\times10^3} = 0.54342 \qquad \frac{\rho_Y}{c} = 0.81604$

$\rho_Y = 0.81604c = (0.81604)(0.019) = 0.01550$ m $\qquad \rho_Y = 15.50$ mm

Problem 3.93

3.93 It is observed that a straightened paper clip can be twisted through several revolutions by the application of a torque of approximately 60 mN · m. Knowing that the diameter of the wire in the paper clip is 0.9 mm, determine the approximate value of the yield stress of the steel.

$c = \frac{1}{2}d = 0.45\,\text{mm} = 0.45\times10^{-3}\,\text{m}$ $\qquad T_P = 60\,\text{mN}\cdot\text{m} = 60\times10^{-3}\,\text{N}\cdot\text{m}$

$$T_P = \frac{4}{3}T_Y = \frac{4}{3}\frac{J\tau_Y}{c} = \frac{4}{3}\cdot\frac{\pi}{2}c^3\tau_Y = \frac{2\pi}{3}c^3\tau_Y$$

$$\tau_Y = \frac{3T_P}{2\pi c^3} = \frac{(3)(60\times10^{-3})}{2\pi(0.45\times10^{-3})^3} = 314\times10^6\,\text{Pa}$$

$\tau_Y = 314\,\text{MPa}$ ◄

Problem 3.94

3.94 A 1.25-in.-diameter solid rod is made of an elastoplastic material with τ_Y = 5 ksi. Knowing that the elastic core of the rod is of diameter 1 in., determine the magnitude of the torque applied to the rod.

$c = \frac{1}{2}d = 0.625\,\text{in.}$ $\qquad \tau_Y = 5\times10^3\,\text{psi}$ $\qquad \rho_Y = \frac{1}{2}d_Y = 0.5\,\text{in.}$

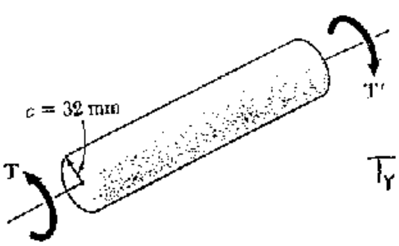
$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(0.625)^3(5\times10^3) = 1.91747\times10^3\,\text{lb}\cdot\text{in}$$

$$T = \frac{4}{3}T_Y\left(1-\frac{1}{4}\frac{\rho_Y^3}{c^3}\right) = \frac{4}{3}(1.91747\times10^3)\left(1-\frac{1}{4}\frac{0.5^3}{0.625^3}\right) = 2.23\times10^3\,\text{lb}\cdot\text{in}$$

$T = 2230\,\text{lb}\cdot\text{in}$ ◄

Problem 3.95

3.95 The solid circular shaft shown is made of a steel that is assumed to be elastoplastic with τ_Y = 145 MPa. Determine that magnitude T of the applied torque when the plastic zone is (*a*) 16 mm deep, (*b*) 24 mm deep.



$c = 32\,\text{mm} = 0.32\,\text{m}$ $\qquad \tau_Y = 145\times10^6\,\text{Pa}$

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(0.032)^3(145\times10^6) = 7.4634\times10^3\,\text{N}\cdot\text{m}$$

(a) $t_p = 16\,\text{mm} = 0.016\,\text{m}$ $\qquad \rho_Y = c - t_p = 0.032 - 0.016 = 0.016\,\text{m}$

$$T = \frac{4}{3}T_Y\left(1-\frac{1}{4}\frac{\rho_Y^3}{c^3}\right) = \frac{4}{3}(7.4634\times10^3)\left(1-\frac{1}{4}\frac{0.016^3}{0.032^3}\right) = 9.6402\times10^3\,\text{N}\cdot\text{m}$$

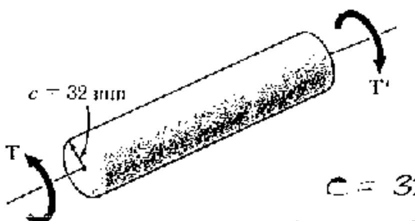
$T = 9.64\,\text{kN}\cdot\text{m}$ ◄

(b) $t_p = 24\,\text{mm} = 0.024\,\text{m}$ $\qquad \rho_Y = c - t_p = 0.032 - 0.024 = 0.008\,\text{m}$

$$T = \frac{4}{3}T_Y\left(1-\frac{1}{4}\frac{\rho_Y^3}{c^3}\right) = \frac{4}{3}(7.4634\times10^3)\left(1-\frac{1}{4}\frac{0.008^3}{0.032^3}\right) = 9.9123\times10^3\,\text{N}\cdot\text{m}$$

$T = 9.91\,\text{kN}\cdot\text{m}$ ◄

Problem 3.96



3.96 For the shaft and loading of problem 3.95, assuming that $G = 77.2$ GPa determine the angle of twist in a 1.5-m length of the shaft.

3.95 The solid circular shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145$ MPa. Determine that magnitude T of the applied torque when the plastic zone is (*a*) 16 mm deep, (*b*) 24 mm deep.

$c = 32\text{ mm} = 0.032\text{ m} \qquad \tau_Y = 145\times10^6\text{ Pa} \qquad G = 77.2\times10^9\text{ Pa}$

$L = 1.5\text{ m}$

$\gamma = \dfrac{\rho\varphi}{L} \qquad \gamma_Y = \dfrac{c\,\varphi_Y}{L} \qquad \varphi_Y = \dfrac{L\gamma_Y}{c} = \dfrac{L\tau_Y}{cG} = \dfrac{(1.5)(145\times10^6)}{(0.032)(77.2\times10^9)} = 88.042\times10^{-3}\text{ rad}$

$\dfrac{\rho_Y}{c} = \dfrac{\varphi_Y}{\varphi} \qquad \varphi = \dfrac{c}{\rho_Y}\varphi_Y$

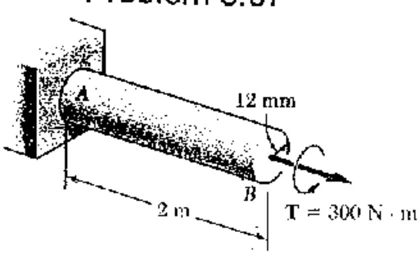
(a) $t_p = 16\text{ mm} = 0.016\text{ m} \qquad \rho_Y = c - t_p = 0.016\text{ m} \qquad \dfrac{\rho_Y}{c} = 0.500$

$\varphi = \dfrac{\varphi_Y}{\rho_Y/c} = \dfrac{88\times042\times10^{-3}}{0.500} = 176.08\times10^{-3}\text{ rad} \qquad \varphi = 10.09^\circ$ ◄

(b) $t_p = 24\text{ mm} = 0.024\text{ m} \qquad \rho_Y = c - t_p = 0.008\text{ m} \qquad \dfrac{\rho_Y}{c} = 0.250$

$\varphi = \dfrac{\varphi_Y}{\rho_Y/c} = \dfrac{88.042\times10^{-3}}{0.250} = 352.168\times10^{-3}\text{ rad} \qquad \varphi = 20.2^\circ$ ◄

Problem 3.97



3.97 The shaft *AB* is made of a material that is elastoplastic with $\tau_Y = 90$ MPa and $G = 30$ GPa. For the loading shown, determine (*a*) the radius of the elastic core of the shaft, (*b*) the angle of twist at end *B*.

(a) $c = 12\text{ mm} = 0.012\text{ m} \qquad \tau_Y = 90\times10^6\text{ Pa}$

$T_Y = \dfrac{J\tau_Y}{c} = \dfrac{\pi}{2}c^3\tau_Y$

$= \dfrac{\pi}{2}(0.012)^3(90\times10^6) = 244.29\text{ N}\cdot\text{m}$

$T = 300\text{ N}\cdot\text{m} > T_Y$ plastic region with elastic core

$T = \dfrac{4}{3}T_Y\left(1 - \dfrac{1}{4}\dfrac{\rho_Y^3}{c^3}\right) \qquad \dfrac{\rho_Y^3}{c^3} = 1 - \dfrac{3T}{T_Y} = 1 - \dfrac{(3)(300)}{244.29} = 0.31585$

$\dfrac{\rho_Y}{c} = 0.68102 \qquad \rho_Y = (0.68102)(0.012) = 8.17\times10^{-3}\text{ m} \qquad \rho_Y = 8.17\text{ mm}$ ◄

(b) $L = 2\text{ m} \qquad G = 30\times10^9\text{ Pa}$

$\varphi_Y = \dfrac{T_Y L}{JG} = \dfrac{2T_Y L}{\pi c^4 G} = \dfrac{(2)(244.29)(2)}{\pi(0.012)^4(30\times10^9)} = 0.5000\text{ rad}$

$\dfrac{\varphi_Y}{\varphi} = \dfrac{\rho_Y}{c} \qquad \varphi = \dfrac{\varphi_Y}{\rho_Y/c} = \dfrac{0.5000}{0.68120} = 0.734\text{ rad} \qquad \varphi = 42.1^\circ$ ◄

Problem 3.98

3.98 A 0.75-in.-diameter solid circular shaft is made of material that is assumed to be elastoplastic with $\tau_Y = 20$ ksi and $G = 11.2 \times 10^6$ psi. For a 4-ft length of the shaft, determine the maximum shearing stress and the angle of twist caused by a 1800 lb · in. torque.

$\tau_Y = 20$ ksi $= 20 \times 10^3$ psi, $c = \frac{1}{2} = 0.375$ in., $L = 4$ ft $= 48$ in., $T = 1800$ lb·in

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2} c^3 \tau_Y = \frac{\pi}{2}(0.375)^3(20 \times 10^3) = 1656.70 \text{ lb}\cdot\text{in.}$$

$T > T_Y$ Plastic region with elastic core $\quad \tau_{max} = \tau_Y = 20$ ksi ◄

$$\phi_Y = \frac{T_Y L}{GJ} = \frac{2T_Y L}{\pi c^4 G} = \frac{(2)(1656.70)(48)}{\pi(0.375)^4(11.2 \times 10^6)} = 0.22857 \text{ rad}$$

$$T = \frac{4}{3} T_Y \left(1 - \frac{1}{4}\frac{\phi_Y^3}{\phi^3}\right)$$

$$\left(\frac{\phi_Y}{\phi}\right)^3 = 4 - \frac{3T}{T_Y} = 4 - \frac{(3)(1800)}{1656.70} = 0.7451 \qquad \frac{\phi_Y}{\phi} = 0.90471$$

$$\phi = \frac{\phi_Y}{0.90471} = \frac{0.22857}{0.9047} = 0.25264 \text{ rad} \qquad \phi_Y = 14.48^\circ$$ ◄

Problem 3.99

3.99 A solid circular rod is made of a material that is assumed to be elastoplastic. Denoting by τ_Y and ϕ_Y, respectively, the torque and the angle of twist at the onset of yield, determine the angle of twist is the torque is increased to (*a*) $T = 1.1T$, (*b*) $T = 1.25T$, (*c*) $T = 1.3T$.

$$T = \frac{4}{3} T_Y \left(1 - \frac{1}{4}\frac{\phi_Y^3}{\phi^3}\right)$$

$$\frac{\phi_Y}{\phi} = \sqrt[3]{4 - \frac{3T}{T_Y}} \quad \text{or} \quad \frac{\phi}{\phi_Y} = \frac{1}{\sqrt[3]{4 - \frac{3T}{T_Y}}}$$

(a) $\frac{T}{T_Y} = 1.10 \qquad \frac{\phi}{\phi_Y} = \frac{1}{\sqrt[3]{4 - (3)(1.10)}} = 1.126 \qquad \phi = 1.126\,\phi_Y$ ◄

(b) $\frac{T}{T_Y} = 1.25 \qquad \frac{\phi}{\phi_Y} = \frac{1}{\sqrt[3]{4 - (3)(1.25)}} = 1.587 \qquad \phi = 1.587\,\phi_Y$ ◄

(c) $\frac{T}{T_Y} = 1.3 \qquad \frac{\phi}{\phi_Y} = \frac{1}{\sqrt[3]{4 - (3)(1.3)}} = 2.15 \qquad \phi = 2.15\,\phi_Y$ ◄

Problem 3.100

3.100 A 1.25-in. diameter solid circular shaft is made of a material that is assumed to be elastoplastic with $\tau_Y = 18$ ksi and $G = 11.2 \times 10^6$ psi. For an 8-ft length of the shaft, determine the maximum shearing stress and the angle of twist caused by a 7.5 kip · in. torque.

$c = \frac{1}{2}d = 0.625$ in., $G = 11.2 \times 10^6$ psi, $\tau_Y = 18$ ksi $= 18000$ psi

$L = 8$ ft. $= 96$ in. $T = 7.5$ kip-in $= 7.5 \times 10^3$ lb-in

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(0.625)^3(18000) = 6.9029 \times 10^3 \text{ lb-in}$$

$T > T_Y$ plastic region with elastic core $\therefore$ $\tau_{max} = \tau_Y = 18$ ksi ◄

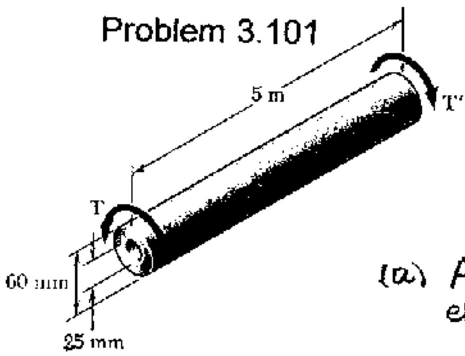
$$\gamma_Y = \frac{c\varphi_Y}{L} \quad \therefore \varphi_Y = \frac{L\gamma_Y}{c} = \frac{L\tau_Y}{cG} = \frac{(96)(18000)}{(0.625)(11.2 \times 10^6)} = 246.86 \times 10^{-3} \text{ rad}$$

$$T = T_Y\left(1 - \frac{1}{4}\frac{\varphi_Y^3}{\varphi^3}\right)$$

$$\frac{\varphi}{\varphi_Y} = \frac{1}{\sqrt[3]{4 - \frac{3T}{T_Y}}} = \frac{1}{\sqrt[3]{4 - \frac{(3)(7.5\times10^3)}{6.9029\times10^3}}} = 1.10533$$

$\varphi = 1.10533\,\varphi_Y = (1.10533)(246.86 \times 10^{-3}) = 272.86 \times 10^{-3}$ rad $\qquad \varphi = 15.63°$ ◄

Problem 3.101



3.101 The hollow shaft shown is made of steel which is assumed to be elastoplastic with $\tau_Y = 145$ MPa and $G = 77.2$ GPa. Determine the magnitude T of the torque and the corresponding angle of twist (*a*) at the onset of yield, (*b*) when the plastic zone is 10 mm deep.

(a) At the onset of yield, the stress distribution is the elastic distribution with $\tau_{max} = \tau_Y$

$$c_2 = \frac{1}{2} d_2 = 0.030 \text{ m}, \quad c_1 = \frac{1}{2} d_1 = 0.0125 \text{ m}$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.030^4 - 0.0125^4) = 1.2340 \times 10^{-6} \text{ m}$$

$$\tau_{max} = \tau_Y = \frac{T_Y c_2}{J} \quad \therefore \quad T_Y = \frac{J\tau_Y}{c_2} = \frac{(1.2340\times10^{-6})(145\times10^6)}{0.030} = 5.9648\times10^3 \text{ N}\cdot\text{m}$$

$$T_Y = 5.96 \text{ kN}\cdot\text{m}$$

$$\varphi_Y = \frac{T_Y L}{GJ} = \frac{(5.9643\times10^3)(5)}{(77.2\times10^9)(1.234010^{-6})} = 313.04\times10^{-3} \text{ rad} \qquad \varphi_Y = 17.94°$$

(b) $t = 0.010$ m $\qquad \rho_Y = c_2 - t = 0.030 - 0.010 = 0.020$ m

$$\gamma = \frac{\rho\varphi}{L} = \frac{\rho_Y\varphi}{L} = \gamma_Y = \frac{\tau_Y}{G}$$

$$\varphi = \frac{\tau_Y L}{G\rho_Y} = \frac{(145\times10^6)(5)}{(77.2\times10^9)(0.020)} = 469.56\times10^{-3} \text{ rad} \qquad \varphi = 26.9°$$

Torque T_1 carried by elastic portion $c_1 \le \rho \le \rho_Y$

$\tau = \tau_Y$ at $\rho = \rho_Y$. $\quad \tau_Y = \frac{T_1 \rho_Y}{J_1}$ $\quad$ where $J_1 = \frac{\pi}{2}(\rho_Y^4 - c_1^4)$

$$J_1 = \frac{\pi}{2}(0.020^4 - 0.0125^4) = 212.978\times10^{-9} \text{ m}^4$$

$$T_1 = \frac{J_1 \tau_Y}{\rho_Y} = \frac{(212.978\times10^{-9})(145\times10^6)}{0.020} = 1.5441\times10^3 \text{ N}\cdot\text{m}$$

Torque T_2 carried by plastic portion

$$T_2 = 2\pi\int_{\rho_Y}^{c_2} \tau_Y \rho^2 \, d\rho = 2\pi\tau_Y \frac{\rho^3}{3}\Big|_{\rho_Y}^{c_2} = \frac{2\pi}{3}\tau_Y(c_2^3 - \rho_Y^3)$$

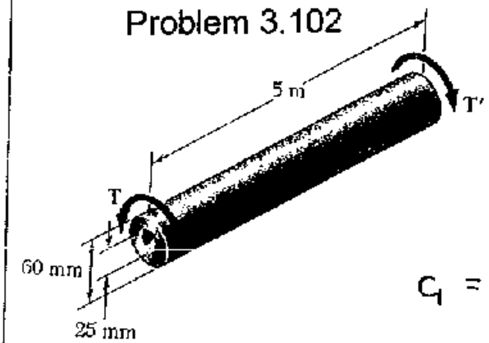
$$= \frac{2\pi}{3}(145\times10^6)(0.030^3 - 0.020^3) = 5.7701\times10^3 \text{ N}\cdot\text{m}$$

Total torque

$$T = T_1 + T_2 = 1.5541\times10^3 + 5.7701\times10^3 = 7.3142\times10^3 \text{ N}\cdot\text{m}$$

$$T = 7.31 \text{ kN}\cdot\text{m}$$

Problem 3.102



3.102 For the shaft of Prob. 3.101, determine (*a*) the angle of twist at which the shaft first becomes fully plastic, (*b*) the corresponding magnitude T of the torque. Sketch the T-ϕ curve for the shaft.

3.101 The hollow shaft shown is made of steel which is assumed to be elastoplastic with $\tau_Y = 145$ MPa and $G = 77.2$ GPa. Determine the magnitude T of the torque and the corresponding angle of twist (*a*) at the onset of yield, (*b*) when the plastic zone is 10 mm deep.

$$c_1 = \tfrac{1}{2} d_1 = 0.0125 \text{ m} \qquad c_2 = \tfrac{1}{2} d_2 = 0.030 \text{ m}$$

(a) For onset of fully plastic yielding, $\rho_Y = c_1$

$$\tau = \tau_Y \quad \therefore \quad \gamma = \frac{\tau_Y}{G} = \frac{\rho_Y \varphi}{L} = \frac{c_1 \varphi}{L}$$

$$\varphi_P = \frac{L \tau_Y}{c_1 G} = \frac{(5)(145 \times 10^6)}{(0.0125)(77.2 \times 10^9)} = 751.295 \times 10^{-3} \text{ rad} \qquad \varphi_P = 43.0^\circ$$

$$\text{(b)} \quad T_P = 2\pi \int_{c_1}^{c_2} \tau_Y \rho^2 \, d\rho = 2\pi \tau_Y \frac{\rho^3}{3}\Big|_{c_1}^{c_2} = \frac{2\pi}{3} \tau_Y (c_2^3 - c_1^3)$$

$$= \frac{2\pi}{3} (145 \times 10^6)(0.030^3 - 0.0125^3) = 7.606 \times 10^3 \text{ N}\cdot\text{m}$$

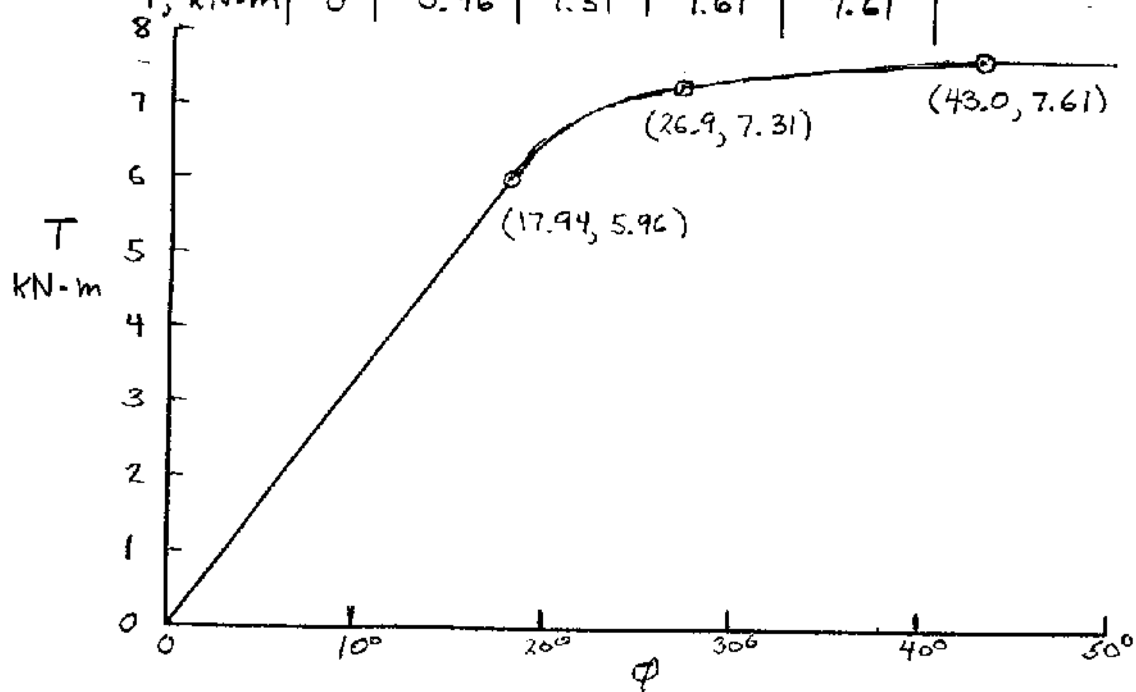
$$T_P = 7.61 \text{ kN}\cdot\text{m}$$

From Problem 3.101 $\qquad \varphi_Y = 17.94^\circ \qquad T_Y = 5.96$ kN·m

Also from Problem 3.101 $\qquad \varphi = 26.9^\circ \qquad T = 7.31$ kN·m

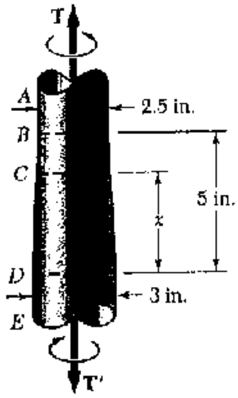
Plot T vs φ using the following data.

φ, deg	0	17.94	26.9	43.0	> 43.0
T, kN·m	0	5.96	7.31	7.61	7.61



Problem 3.103

3.103 A steel rod is machined to the shape shown to form a tapered solid shaft to which torques of magnitude $T = 75$ kip · in. are applied. Assuming the steel to be elastoplastic with $\tau_Y = 21$ ksi and $G = 11.2 \times 10^6$ psi, determine (*a*) the radius of the elastic core in portion *AB* of the shaft, (*b*) the length of portion *CD* which remains fully elastic.



(a) In portion AB $\quad c = \frac{1}{2}d = 1.25$ in.

$$T_Y = \frac{J_{AB}\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(1.25)^3(21\times10^3) = 64.427\times10^3 \text{ lb}\cdot\text{in}$$

$$T = \frac{4}{3}T_Y\left(1 - \frac{\rho_Y^3}{c^3}\right)$$

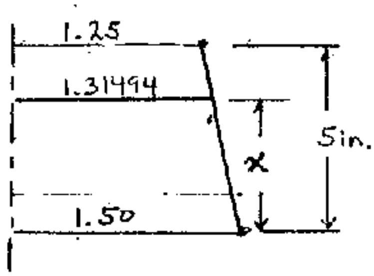
$$\frac{\rho_Y}{c} = \sqrt[3]{4 - \frac{3T}{T_Y}} = \sqrt[3]{4 - \frac{(3)(75\times10^3)}{64.427\times10^3}} = 0.79775$$

$$\rho_Y = 0.79775c = (0.79775)(1.25) = 0.99718 \text{ in.}$$

$\rho_Y = 0.997$ in.

(b) For yielding at point C $\quad \tau = \tau_Y, \; c = c_x, \; T = 75\times10^3$ lb·in

$$T = \frac{J_c\tau_Y}{c_x} = \frac{\pi}{2}c_x^3\tau_Y \qquad c_x = \sqrt[3]{\frac{2T}{\pi\tau_Y}} = \sqrt[3]{\frac{(2)(75\times10^3)}{\pi(21\times10^3)}} = 1.31494 \text{ in.}$$

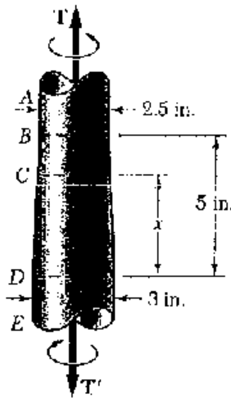


Using proportions from the sketch

$$\frac{1.50 - 1.31494}{1.50 - 1.25} = \frac{x}{5}$$

$x = 3.70$ in.

Problem 3.104



3.104 If the torques applied to the tapered shaft of Prob. 3.103 are slowly increased, determine (*a*) the magnitude T of the largest torques which can be applied to the shaft, (*b*) the length of the portion CD which remains fully elastic.

3.103 A steel rod is machined to the shape shown to form a tapered solid shaft to which torques of magnitude $T = 75$ kip · in. are applied. Assuming the steel to be elastoplastic with $\tau_Y = 21$ ksi and $G = 11.2 \times 10^6$ psi, determine (*a*) the radius of the elastic core in portion AB of the shaft, (*b*) the length of portion CD which remains fully elastic.

(a) The largest torque that may be applied is that which makes portion AB fully plastic.

In portion AB $\quad c = \frac{1}{2}d = 1.25$ in.

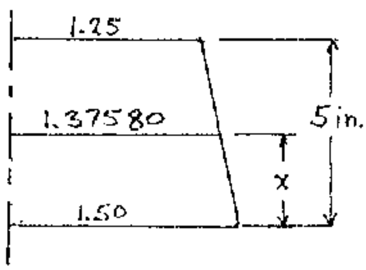
$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(1.25)^3(21\times10^3) = 64.427\times10^3 \text{ lb}\cdot\text{in}$$

For fully plastic shaft $\quad \rho_Y = 0 \qquad T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right) = \frac{4}{3}T_Y$

$$T = \frac{4}{3}(64.427\times10^3) = 85.903\times10^3 \text{ lb}\cdot\text{in} \qquad T = 85.9 \text{ kip}\cdot\text{in}$$

(b) For yielding at point C, $\tau = \tau_Y$, $c = c_x$, $T = 85.903\times10^3$ lb·in

$$\tau_Y = \frac{Tc_x}{J_x} = \frac{2T}{\pi c_x^3} \qquad c_x = \sqrt[3]{\frac{2T}{\pi\tau_Y}} = \sqrt[3]{\frac{(2)(85.903\times10^3)}{\pi(21\times10^3)}} = 1.37580 \text{ in.}$$

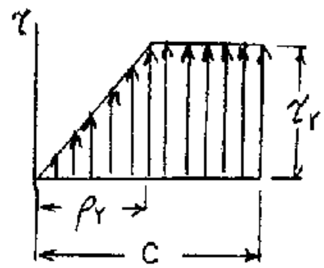


Using proportions from the sketch

$$\frac{1.50 - 1.37580}{1.50 - 1.25} = \frac{x}{5} \qquad x = 2.48 \text{ in.}$$

Problem 3.105

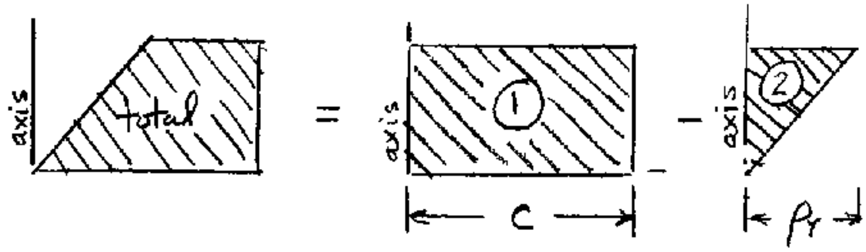
3.105 Considering the partially plastic shaft of Fig. 3.38*a*, derive Eq.(3.32) by recalling that the integral in Eq. (3.26) representing the second moment about the τ axis of the area under the τ-ρ curve.



The stress is that shown on the left.

$$T = 2\pi \int_0^c \rho^2 \tau \, d\rho = 2\pi \int \rho^2 \, dA = 2\pi I$$

where $dA = \tau \, d\rho$ and I = 2nd moment about the τ-axis.



$$I = I_1 - I_2$$

$$= \frac{1}{3}\tau_Y c^3 - \left\{ \frac{1}{36}\tau_Y \rho_Y^3 + \frac{1}{2}\tau_Y \rho_Y \left(\frac{1}{3}\rho_Y\right)^2 \right\}$$

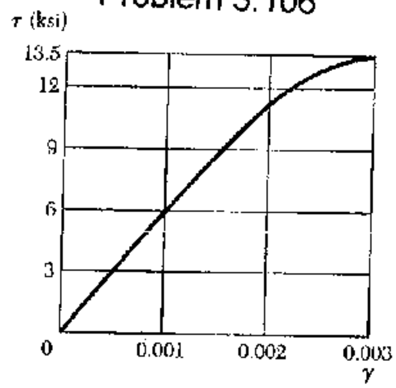
$$= \frac{1}{3}\tau_Y c^3 - \frac{1}{12}\tau_Y \rho_Y^3 = \frac{1}{3}\tau_Y c^3 \left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right)$$

$$T = 2\pi I = \frac{2\pi}{3}\tau_Y c^3 \left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right)$$

Recall that $T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2} c^3 \tau_Y \quad \therefore \quad \frac{2\pi}{3}\tau_Y c^3 = \frac{4}{3} T_Y$

Hence, $$T = \frac{4}{3} T_Y \left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right)$$

Problem 3.106



3.106 A solid brass rod of 1.2-in. diameter is subjected to a torque which causes a maximum shearing stress of 13.5 ksi in the rod. Using the τ-γ diagram shown for the brass used, determine (*a*) the magnitude of the torque, (*b*) the angle of twist in a 24-in. length of the rod.

(a) $\tau_{max} = 13.5$ ksi $\qquad c = \frac{1}{2}d = 0.600$ in

From the stress-strain diagram $\gamma_{max} = 0.003$

Let $z = \dfrac{\gamma}{\gamma_{max}} = \dfrac{\rho}{c}$

$$T = 2\pi \int_0^c \rho^2 \tau \, d\rho = 2\pi c^3 \int_0^1 z^2 \tau \, dz = 2\pi c^3 I$$

where the integral I is given by $\quad I = \int_0^1 z^2 \tau \, dz$

Evaluate I using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$I = \frac{\Delta z}{3} \sum w z^2 \tau$$

where w is a weighting factor. Using $\Delta z = 0.25$ we get the values given in the table below:

z	γ	τ, ksi	$z^2\tau$, ksi	w	$wz^2\tau$, ksi
0	0	0	0.000	1	0
0.25	0.00075	4.5	0.281	4	1.125
0.5	0.0015	8.6	2.15	2	4.30
0.75	0.00225	12.2	6.86	4	27.45
1.0	0.003	13.5	13.5	1	13.5
					46.375 ← $\sum wz^2\tau$

$$I = \frac{(0.25)(46.375)}{3} = 3.865 \times 10^{-3} \text{ ksi}$$

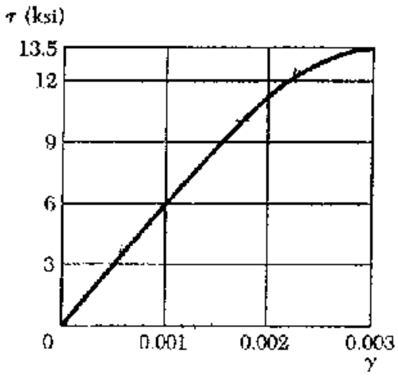
(a) $T = 2\pi c^3 I = 2\pi (0.600)^3 (3.865) = 5.24$ kip·in $\qquad T = 5.24$ kip·in ◄

(b) $\gamma_{max} = \dfrac{c\varphi}{L}$

$$\varphi = \frac{L\gamma_m}{c} = \frac{(24)(0.003)}{0.600} = 120 \times 10^{-3} \text{ rad} \qquad \varphi = 6.88^\circ \text{ ◄}$$

Note: Answer may differ slightly due to reading of graph and choice of numerical integration formula.

Problem 3.107



3.107 A solid brass rod of 0.8-in. diameter and 30-in. length is twisted through an angle of 10°. Using the τ-γ diagram shown for the brass used, determine (*a*) the magnitude of the torque applied to the rod, (*b*) the maximum shearing stress in the rod.

(a) $\varphi = 10° = 174.53 \times 10^{-3}$ rad

$c = \frac{1}{2} d = 0.400$ in. $\quad L = 30$ in.

$$\gamma_{max} = \frac{c\varphi}{L} = \frac{(0.4)(174.53\times10^{-3})}{30}$$

$$= 0.00233$$

Let $z = \frac{\gamma}{\gamma_{max}} = \frac{\rho}{c}$

$$T = 2\pi \int_0^c \rho^2 \tau \, d\rho = 2\pi c_2^3 \int_0^1 z^2 \tau \, dz = 2\pi c_2^3 I$$

where the integral I is given by $\quad I = \int_{1/3}^1 z^2 \tau \, dz$

Evaluate I using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$I = \frac{\Delta z}{3} \sum w z^2 \tau$$

where w is a weighting factor. Using $\Delta z = \frac{1}{6}$ we get the values given in the table below.

z	γ	τ, ksi	$z^2\tau$, ksi	w	$wz^2\tau$, ksi
0	0	0	0.	1	0
0.25	0.000583	3.5	0.219	4	0.88
0.5	0.001165	7.0	1.75	2	3.50
0.75	0.001748	10.0	5.625	4	22.50
1	0.00233	12.2	12.6	1	12.60
					39.48 ← $\sum w z^2 \tau$

$$I = \frac{(0.25)(39.48)}{3} = 3.29 \text{ ksi}$$

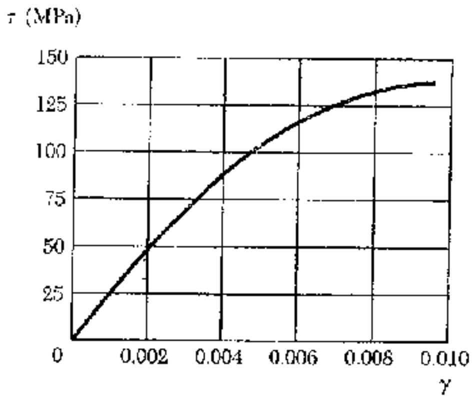
$$T = 2\pi c_2^3 I = 2\pi (0.400)^3 (3.29)$$

$T = 1.322$ kip-in ◄

Note: Answer may differ slightly due to differences of opinion in reading the stress-strain curve.

(b) From the graph, $\quad \tau_{max} = 12.6$ ksi ◄

Problem 3.108



3.108 A solid aluminum rod of 40-mm diameter is subjected to a torque which produces in the rod a maximum shearing strain of 0.008. Using the τ-γ diagram shown for the aluminum alloy used, determine (*a*) the magnitude of the torque applied to the rod, (*b*) the angle of twist in a 750-mm length of the rod.

$\gamma_{max} = 0.008 \qquad c = \frac{1}{2}d = 0.020 \text{ m}$

$L = 750 \text{ mm} = 0.750 \text{ m}$

(a) Let $z = \dfrac{\gamma}{\gamma_{max}} = \dfrac{\rho}{c}$

$$T = 2\pi \int_0^c \rho^2 \tau \, d\rho = 2\pi c^3 \int_0^1 z^2 \tau \, dz = 2\pi c^3 I$$

where the integral I is given by $\qquad I = \int_0^1 z^2 \tau \, dz$

Evaluate I using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$I = \frac{\Delta z}{3} \sum w z^2 \tau$$

where w is a weighting factor. Using $\Delta z = 0.25$ we get the values given in the table below.

z	γ	τ, MPa	$z^2\tau$, MPa	w	$wz^2\tau$, MPa
0	0	0	0	1	0
0.25	0.002	48	3.0	4	12.0
0.5	0.004	88	22.0	2	44.0
0.75	0.006	115	64.7	4	258.8
1	0.008	133	133.0	1	133.0
					447.8 ← $\sum wz^2\tau$

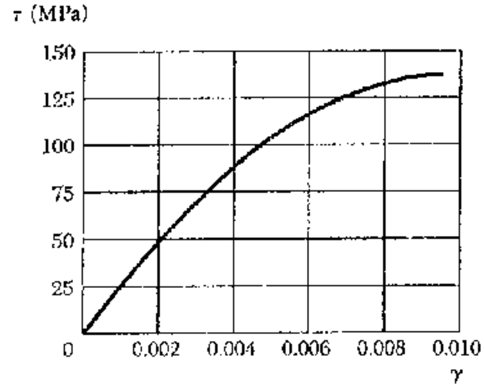
$$I = \frac{(0.25)(447.8)}{3} = 37.3 \text{ MPa} = 37.3 \times 10^6 \text{ Pa}$$

$$T = 2\pi (0.020)^3 (37.3 \times 10^6) = 1.876 \times 10^3 \text{ N·m} \qquad T = 1.876 \text{ kN·m}$$

(b) $\gamma_{max} = \dfrac{c\varphi}{L}$

$$\varphi = \frac{L\gamma_{max}}{c} = \frac{(0.750)(0.008)}{0.020} = 300 \times 10^{-3} \text{ rad} \qquad \varphi = 17.19°$$

Problem 3.109



3.109 The curve shown in Fig. P3.108 can be approximated by the relation

$$\tau = 27.8 \times 10^9 \gamma - 1.390 \times 10^{12} \gamma^2$$

Using this relation and Eqs. (3.2) and (3.26), solve problem 3.108.

3.108 A solid aluminum rod of 40-mm diameter is subjected to a torque which produces in the rod a maximum shearing strain of 0.008. Using the τ-γ diagram shown for the aluminum alloy used, determine (*a*) the magnitude of the torque applied to the rod, (*b*) the angle of twist in a 750-mm length of the rod.

$\gamma_{max} = 0.008 \qquad c = \frac{1}{2}d = 0.020 \text{ m}$

$L = 750 \text{ mm} = 0.750 \text{ m}$

(a) Let $z = \frac{\gamma}{\gamma_{max}} = \frac{\rho}{c}$

$$T = 2\pi \int_0^c \rho^2 \tau \, d\rho = 2\pi c^3 \int_0^1 z^2 \tau \, dz$$

The given stress-strain curve is $\tau = B\gamma + C\gamma^2 = B\gamma_{max} z + C\gamma_{max}^2 z^2$

where $B = 27.8 \times 10^9$ and $C = -1.390 \times 10^{12}$

$$T = 2\pi c^3 \int_0^1 z^2 (B\gamma_{max} z + C\gamma_{max}^2 z^2) dz$$

$$= 2\pi c^3 \left\{ B\gamma_{max} \int_0^1 z^3 dz + C\gamma_{max}^2 \int_0^4 z^4 dz \right\}$$

$$= 2\pi c^3 \left\{ \frac{1}{4} B\gamma_{max} + \frac{1}{5} C\gamma_{max}^2 \right\}$$

$$= 2\pi (0.020)^3 \left\{ \frac{1}{4}(27.8 \times 10^9)(0.008) + \frac{1}{5}(-1.390 \times 10^{12})(0.008)^2 \right\}$$

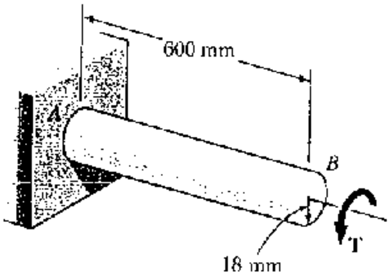
$$= 1.900 \times 10^3 \text{ N-m} \qquad T = 1.900 \text{ kN-m}$$

(b) $\gamma_{max} = \frac{c\varphi}{L}$

$$\varphi = \frac{L\gamma_{max}}{c} = \frac{(0.750)(0.008)}{0.020} = 300 \times 10^{-3} \text{ rad} \qquad \varphi = 17.19°$$

Problem 3.110

3.110 The solid circular shaft AB is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145$ MPa and $G = 77.2$ GPa. The torque **T** is increased until the radius of the elastic core is 6 mm. Determine the maximum residual shearing stress in the shaft after the torque **T** is removed.



$c = 0.018$ m $\quad \rho_Y = 0.006$ m $\quad \frac{\rho_Y}{c} = \frac{0.006}{0.018} = \frac{1}{3}$

$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.018)^4 = 164.896 \times 10^{-9}$ m^4

$T_Y = \frac{J\tau_Y}{c} = \frac{(164.896\times10^{-9})(145\times10^6)}{0.018} = 1.32833\times10^3$ N·m

At end of loading: $T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right) = \frac{4}{3}(1.32833\times10^3)\left[1 - \frac{1}{4}\left(\frac{1}{3}\right)^3\right]$

$= 1.7547\times10^3$ N·m

The stresses are $\tau_{load} = 0$ at $\rho = 0$
$\tau_{load} = \tau_Y = 145$ MPa at $\rho = 0.006$ m
$\tau_{load} = \tau_Y = 145$ MPa at $\rho = 0.018$ m

Torque change during unloading $\quad T = -1.7547\times10^3$ N·m

Stress changes during unloading $\quad \tau' = \frac{T\rho}{J}$ (elastic)

At $\rho = 0$ $\quad \tau' = 0$

At $\rho = 0.006$ m $\quad \tau' = -\frac{(1.7547\times10^3)(0.006)}{164.896\times10^{-9}} = 63.8$ MPa

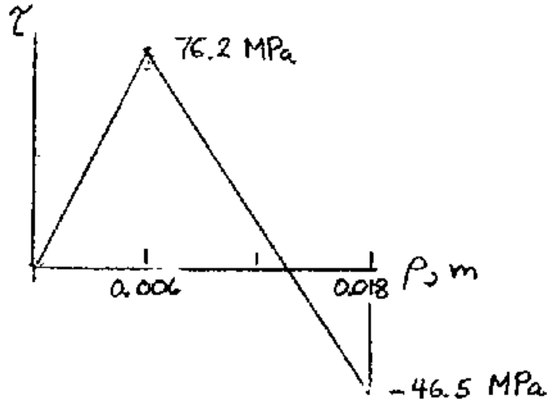
At $\rho = 0.018$ m $\quad \tau' = -\frac{(1.7547\times10^3)(0.018)}{164.896\times10^{-9}} = 191.5$ MPa

Residual stresses are found by adding $\quad \tau_{res} = \tau_{load} - \tau'$

At $\rho = 0$ $\quad \tau_{res} = 0$

At $\rho = 0.006$ m $\quad \tau_{res} = 145 - 63.8 = 81.2$ MPa

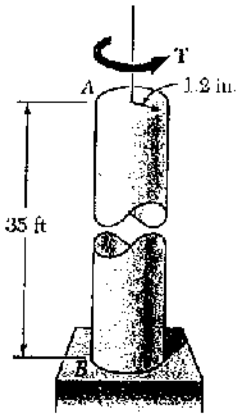
At $\rho = 0.018$ m $\quad \tau_{res} = 145 - 191.5 = -46.5$ MPa



maximum $\tau_{res} = 81.2$ MPa

Problem 3.111

3.111 The solid circular drill rod *AB* is made of a steel that is assumed to be elastoplastic with $\tau_Y = 22$ ksi and $G = 11.2 \times 10^6$ psi. Knowing that a torque $T = 75$ kip in. is applied to the rod and then removed, determine the maximum residual shearing stress in the rod.



SOLUTION

$c = 1.2$ in. $\qquad L = 35$ ft $= 420$ in.

$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(1.2)^4 = 3.2572$ in^4

$T_Y = \frac{J\tau_Y}{c} = \frac{(3.2572)(22)}{1.2} = 59.715$ kip·in

Loading: $T = 75$ kip·in

$T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right)$

$\frac{\rho_Y^3}{c^3} = 4 - \frac{3T}{T_Y} = 4 - \frac{(3)(75)}{59.715} = 0.23213$

$\frac{\rho_Y}{c} = 0.61458, \qquad \rho_Y = 0.61458c = 0.73749$ in.

Unloading: $\quad \tau' = \frac{T\rho}{J}$ where $T = 75$ kip·in

At $\rho = c$ $\quad \tau' = \frac{(75)(1.2)}{3.2572} = 27.63$ ksi

At $\rho = \rho_Y$ $\quad \tau' = \frac{(75)(0.73749)}{3.2572} = 16.98$ ksi

Residual: $\quad \tau_{res} = \tau_{load} - \tau'$

At $\rho = c$ $\quad \tau_{res} = 22 - 27.63 = -5.63$ ksi

At $\rho = \rho_Y$ $\quad \tau_{res} = 22 - 16.98 = 5.02$ ksi

maximum $\tau_{res} = 5.63$ ksi

Problem 3.112

3.112 In Prob. 3.111, determine the permanent angle of twist of the rod.

3.111 The solid circular drill rod AB is made of a steel that is assumed to be elastoplastic with $\tau_Y = 22$ ksi and $G = 11.2 \times 10^6$ psi. Knowing that a torque $T = 75$ kip in. is applied to the rod and then removed, determine the maximum residual shearing stress in the rod.

From the solution to PROBLEM 3.111

$c = 1.2$ in. $\quad J = 3.2572\ \text{in}^4 \qquad \frac{\rho_Y}{c} = 0.61458, \quad \rho_Y = 0.73749$ in.

After loading $\quad \gamma = \frac{\rho\varphi}{L} \therefore \quad \varphi = \frac{L\gamma}{\rho} = \frac{L\gamma_Y}{\rho_Y} = \frac{L\tau_Y}{\rho_Y G} \qquad L = 35\ \text{ft} = 420$ in.

$$\varphi_{load} = \frac{(420)(22\times 10^3)}{(0.73749)(11.2\times 10^6)} = 1.11865\ \text{rad} = 64.09°$$

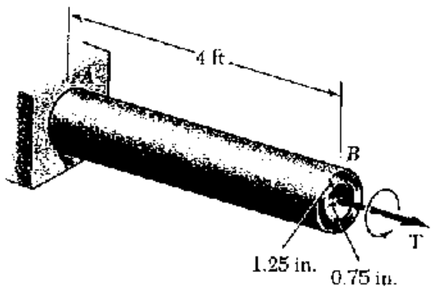
During unloading $\quad \varphi' = \frac{TL}{GJ} \quad$ (elastic) $\qquad T = 5\times 10^3$ N·m

$$\varphi' = \frac{(75\times 10^3)(420)}{(11.2\times 10^6)(3.2572)} = 0.86347\ \text{rad} = 49.47°$$

Permanent twist angle

$$\varphi_{perm} = \varphi_{load} - \varphi' = 1.11865 - 0.86347 = 0.25518 \qquad \varphi = 14.62°$$

Problem 3.113



3.113 The hollow shaft AB is made of a mild steel that is assumed to be elastoplastic with $\tau_Y = 18$ ksi and $G = 11.2 \times 10^6$ psi. The magnitude T of the torque is slowly increased until the plastic zone first reaches the inner surface; the torque is then removed. Determine (*a*) the maximum residual shearing stress, (*b*) the permanent angle of twist.

$c_1 = 0.75$ in. $\quad c_2 = 1.25$ in. $\quad L = 4$ ft $= 48$ in.

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(1.25^4 - 0.75^4) = 3.3379$ in^4

$\tau_Y = 18$ ksi $= 18 \times 10^3$ psi

Loading: When ρ_Y reaches the inner surface, then $\tau = \tau_Y$ everywhere.

$$T_{load} = 2\pi \int_{c_1}^{c_2} \rho^2 \tau_Y \, d\rho = 2\pi \tau_Y \frac{\rho^3}{3}\Big|_{c_1}^{c_2} = \frac{2\pi}{3}\tau_Y (c_2^3 - c_1^3)$$

$$= \frac{2\pi}{3}(18 \times 10^3)(1.25^3 - 0.75^3) = 57.727 \times 10^3 \text{ lb}\cdot\text{in}$$

$$\gamma = \frac{\tau_Y}{G} \text{ at } \rho = c_1 \quad \text{and} \quad \gamma = \frac{\rho\varphi}{L} \text{ or } \varphi = \frac{L\gamma}{\rho} = \frac{L\tau_Y}{c_1 G}$$

$$\varphi_{load} = \frac{(48)(18 \times 10^3)}{(0.75)(11.2 \times 10^6)} = 0.102857 \text{ rad} = 5.8933^\circ$$

Unloading: $T = 57.727 \times 10^3$ lb·in $\qquad$ Elastic $\tau' = \frac{T\rho}{J}$

At $\rho = c_2 = 1.25$ in. $\quad \tau' = \frac{(57.727 \times 10^3)(1.25)}{3.3379} = 21.62 \times 10^3$ psi

At $\rho = c_1 = 0.75$ in. $\quad \tau' = \frac{(57.727 \times 10^3)(0.75)}{3.3379} = 12.97 \times 10^3$ psi

$$\varphi' = \frac{TL}{GJ} = \frac{(57.727 \times 10^3)(48)}{(11.2 \times 10^6)(3.3379)} = 0.074119 \text{ rad} = 4.2467^\circ$$

Residual: $\quad \tau_{res} = \tau_{load} - \tau' \qquad \varphi_{perm} = \varphi_{load} - \varphi'$

(a) At $\rho = c_2 \quad \tau_{res} = 18 \times 10^3 - 21.62 \times 10^3 = -3.62 \times 10^3$ psi

$\tau_{res} = -3.62$ ksi

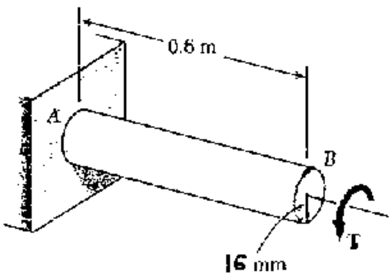
At $\rho = c_1 \quad \tau_{res} = 18 \times 10^3 - 12.97 \times 10^3 = 5.03 \times 10^3$ psi

maximum $\tau_{res} = 5.03$ ksi ◄

(b) $\varphi_{perm} = 0.102857 - 0.074119 = 0.028738$ rad $\qquad \varphi_{perm} = 1.647^\circ$ ◄

Problem 3.114

3.114 The solid shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145$ MPa and $G = 77.2$ GPa. The torque is increased in magnitude until the shaft has been twisted through 6°; the torque is then removed. Determine (*a*) the magnitude and location of the maximum residual shearing stress, (*b*) the permanent angle of twist.



$c = 0.016$ m $\qquad \varphi = 6° = 104.72 \times 10^{-3}$ rad

$$\gamma_{max} = \frac{c\varphi}{L} = \frac{(0.016)(104.72\times10^{-3})}{0.6} = 0.0027925$$

$$\gamma_Y = \frac{\tau_Y}{G} = \frac{145\times10^6}{77.2\times10^9} = 0.0018782$$

$$\frac{\rho_Y}{c} = \frac{\gamma_Y}{\gamma_{max}} = \frac{0.0018}{0.0027925} = 0.67260$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.016)^4 = 102.944\times10^{-9}\ \text{m}^4$$

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(0.016)^3(145\times10^6) = 932.93\ \text{N}\cdot\text{m}$$

At end of loading $\quad T_{load} = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right) = \frac{4}{3}(932.93)\left[1-\frac{1}{4}(0.67433)^3\right]$

$$= 1.14855\times10^3\ \text{N}\cdot\text{m}$$

Unloading: $\quad$ elastic $\qquad T' = -1.14855\times10^3$ N·m

At $\rho = c \qquad \tau' = \frac{T'c}{J} = \frac{(-1.14855\times10^3)(0.016)}{102.944\times10^{-9}} = -178.52\times10^6$ Pa

At $\rho = \rho_Y \qquad \tau' = \frac{T'c}{J}\frac{\rho_Y}{c} = (-178.52\times10^6)(0.67433) = 120.38\times10^6$ Pa

$$\varphi' = \frac{T'L}{GJ} = \frac{(-1.14855\times10^3)(0.6)}{(77.2\times10^9)(102.944\times10^{-9})} = -86.71\times10^{-3}\ \text{rad} = 4.97°$$

Residual: $\qquad \tau_{res} = \tau_{load} - \tau' \qquad\qquad \varphi_{perm} = \varphi_{load} - \varphi$

(a) At $\rho = c \qquad \tau_{res} = 145\times10^6 - 178.52\times10^6 = -33.52\times10^6$ Pa
$= -33.5$ MPa

At $\rho = \rho_Y \qquad \tau_{res} = 145\times10^6 - 120.38\times10^6 = 24.62\times10^6$ Pa
24.6 MPa

Maximum residual stress: 33.5 MPa at $\rho = 16$ mm ◀

(b) $\quad \varphi_{perm} = 104.72\times10^{-3} - 86.71\times10^{-3} = 17.78\times10^{-3}$ rad $\qquad\qquad \varphi_{perm} = 1.032°$ ◀

Problem 3.115

3.115 In Prob. 3.110, determine the permanent angle of twist of the shaft.

3.110 The solid circular shaft *AB* is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145$ MPa and $G = 77.2$ GPa. The torque **T** is increased until the radius of the elastic core is 6 mm. Determine the maximum residual shearing stress in the shaft after the torque **T** is removed.

From the solution to PROBLEM 3.110 $\quad c = 0.018\text{ m}, \; J = 164.896 \times 10^{-9}\text{ m}^4$

After loading $\quad T = 1.7547 \times 10^3\text{ N}\cdot\text{m}, \quad \rho_Y = 0.006\text{ m}$

$$\gamma = \frac{\rho\varphi}{L} \quad \text{or} \quad \varphi = \frac{L\gamma}{\rho} = \frac{L\tau_Y}{\rho_Y G}$$

where $L = 600\text{ mm} = 0.600\text{ m}, \; \tau_Y = 145 \times 10^6\text{ Pa}, \; G = 77.2 \times 10^9\text{ Pa}$

$$\varphi_{load} = \frac{(0.600)(145 \times 10^6)}{(0.006)(77.2 \times 10^9)} = 0.18782\text{ rad} = 10.76^\circ$$

Unloading $\quad T = 1.7547 \times 10^3\text{ N}\cdot\text{m}$

$$\varphi' = \frac{TL}{GJ} = \frac{(1.7547 \times 10^3)(0.600)}{(77.2 \times 10^9)(164.896 \times 10^{-9})} = 0.08270\text{ rad} = 4.74^\circ$$

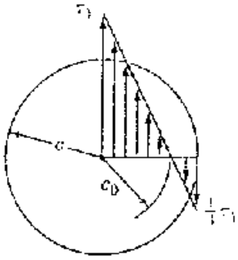
Permanent angle of twist $\quad \varphi_{perm} = \varphi_{load} - \varphi'$

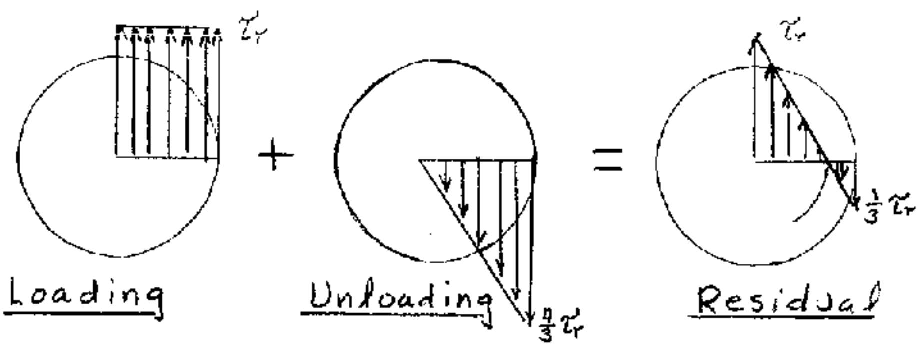
$$\varphi_{perm} = 0.18782 - 0.08270 = 0.10512\text{ rad}$$

$\varphi_{perm} = 6.02^\circ$ ◀

Problem 3.116

3.116 A torque **T** applied to a solid rod made of an elastoplastic material is increased until the rod is fully plastic and then removed. (*a*) Show that the distribution of residual shearing stresses is a represented in the figure. (*b*) Determine the magnitude of the torque due to the stresses acting on the portion of the rod located within a circle of radius c_0.





(a)

After loading $\rho_Y = 0$, $T_{load} = \frac{4}{3}T_Y = \frac{4}{3}\frac{\pi}{2}c^3\tau_Y = \frac{2\pi}{3}c^3\tau_Y$

Unloading $\tau' = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2(T_{load})}{\pi c^3} = \frac{4}{3}\tau_Y$ at $\rho = c$

$\tau' = \frac{4}{3}\tau_Y\frac{\rho}{c}$

Residual $\tau_{res} = \tau_Y - \frac{4}{3}\tau_Y\frac{\rho}{c} = \tau_Y\left(1 - \frac{4\rho}{3c}\right)$

To find c_0 set $\tau_{res} = 0$ and $\rho = c_0$

$0 = 1 - \frac{4c_0}{3c} \quad \therefore \quad c_0 = \frac{3}{4}c$

(b) $T = 2\pi\int_0^{c_0}\rho^2\tau\,d\rho = 2\pi\int_0^{\frac{3}{4}c}\rho^2\tau_Y\left(1 - \frac{4}{3}\frac{\rho}{c}\right)d\rho$

$= 2\pi\tau_Y\left(\frac{\rho^3}{3} - \frac{4}{3}\frac{\rho^4}{4c}\right)\Big|_0^{\frac{3}{4}c} = 2\pi\tau_Y c^3\left\{\frac{1}{3}\left(\frac{3}{4}\right)^3 - \left(\frac{4}{3}\right)\frac{1}{4}\left(\frac{3}{4}\right)^4\right\}$

$= 2\pi\tau_Y c^3\left\{\frac{9}{64} - \frac{27}{256}\right\} = \frac{9\pi}{128}\tau_Y c^3 = 0.2209\,\tau_Y c^3$

Problem 3.117

3.117 After the hollow shaft of Prob. 3.113 has been loaded and unloaded as described in that problem, a torque $\mathbf{T}_1$ of sense opposite to the original torque $\mathbf{T}$ is applied to the shaft. Assuming no change in the value of τ_Y, determine the magnitude T_1 of the torque $\mathbf{T}_1$ required to initiate yield in this second loading and compare it with the magnitude T_Y of the torque $\mathbf{T}$ that caused the shaft to yield in the original loading.

3.113 The hollow shaft AB is made of a mild steel that is assumed to be elastoplastic with $\tau_Y = 18$ ksi and $G = 11.2 \times 10^6$ psi. The magnitude T of the torque is slowly increased until the plastic zone first reaches the inner surface; the torque is then removed. Determine (*a*) the maximum residual shearing stress, (*b*) the permanent angle of twist.

From the solution of PROBLEM 3.113 $\quad c_1 = 0.75$ in., $c_2 = 1.25$ in.

$\tau_Y = 18$ ksi $= 18 \times 10^3$ psi $\qquad J = 3.3379$ in^4

At $\rho = c_2 \qquad \tau_{res} = -3.62$ ksi $= -3.62 \times 10^3$ psi

Allowable reversed stress for reversed load

$$\tau_{all} = \tau_Y + \tau_{res} = 18 \times 10^3 - 3.62 \times 10^3 = 14.38 \times 10^3 \text{ psi}$$

For elastic behavior $\qquad \tau = \dfrac{Tc_2}{J}$

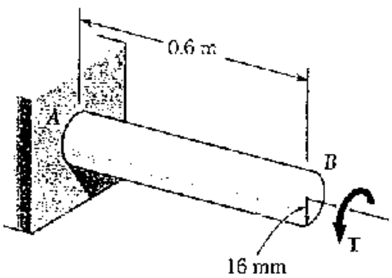
$$T_1 = \frac{J\tau_{all}}{c_2} = \frac{(3.3379)(14.38 \times 10^3)}{1.25} = 38.4 \times 10^3 \text{ lb}\cdot\text{in.}$$

$T_1 = 38.4$ kip·in ◄

$$T_Y = \frac{J\tau_Y}{c_2} = \frac{(3.3379)(18 \times 10^3)}{1.25} = 48.1 \times 10^3 \text{ lb}\cdot\text{in}$$

$T_Y = 48.1$ kip·in ◄

Problem 3.118



3.118 After the solid shaft of Prob. 3.114 has been loaded and unloaded as described in that problem, a torque $\mathbf{T}_1$ of sense opposite to the original torque $\mathbf{T}$ is applied to the shaft. Assuming no change in the value of ϕ_Y, determine the angle of twist ϕ_1 for which yield is initiated in this second loading and compare it with the angle ϕ_Y for which the shaft started to yield in the original loading.

3.114 The solid shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145$ MPa and $G = 77.2$ GPa. The torque is increased in magnitude until the shaft has been twisted through 6°; the torque is then removed. Determine (*a*) the magnitude and location of the maximum residual shearing stress, (*b*) the permanent angle of twist.

From the solution to PROBLEM 3.114 $\quad c = 0.016\text{ m}, \; L = 0.6\text{ m}$

$\tau_Y = 145 \times 10^6 \text{ Pa}, \quad J = 102.944 \times 10^{-9} \text{ m}^4$

The residual stress at $\rho = c$ is $\quad \tau_{res} = 33.5 \text{ MPa}$

For loading in the opposite sense, the change in stress to produce reversed yielding is

$$\tau_1 = \tau_Y - \tau_{res} = 145 \times 10^6 - 33.5 \times 10^6 = 111.5 \times 10^6 \text{ Pa}$$

$$\tau_1 = \frac{T_1 c}{J} \quad \therefore \quad T_1 = \frac{J\tau_1}{c} = \frac{(102.944 \times 10^{-9})(111.5 \times 10^6)}{0.016}$$

$$= 717 \text{ N}\cdot\text{m}$$

Angle of twist at yielding under reversed torque.

$$\phi_1' = \frac{T_1 L}{GJ} = \frac{(717 \times 10^3)(0.6)}{(77.2 \times 10^9)(102.944 \times 10^{-9})} = 54.16 \times 10^{-3} \text{ rad}$$

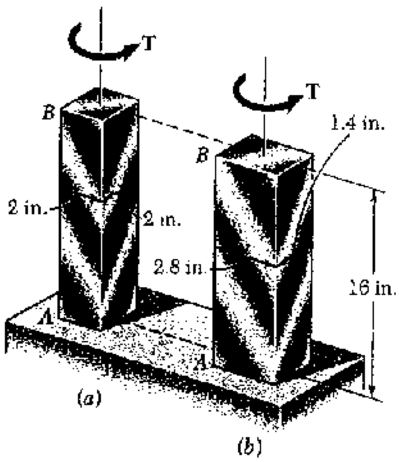
$\phi_1 = 3.10°$ ◄

Angle of twist for yielding in original loading.

$$\gamma = \frac{\tau_Y}{G} = \frac{c\phi_Y}{L}$$

$$\phi_Y = \frac{L\tau_Y}{cG} = \frac{(0.6)(145 \times 10^6)}{(0.016)(77.2 \times 10^9)} = 70.434 \times 10^{-3} \text{ rad} \qquad \phi_Y = 4.04°$$ ◄

Problem 3.119



3.119 Knowing that $T = 7$ kip · in. and that $G = 5.6 \times 10^6$ psi, determine for each of the cold-rolled yellow brass bars shown the maximum shearing stress and the angle of twist of end B.

$T = 7 \times 10^3$ lb·in $\qquad L = 16$ in.

(a) $\quad a = 2$ in. $\quad b = 2$ in. $\quad \frac{a}{b} = 1.0$

From Table 3.1, $c_1 = 0.208$, $c_2 = 0.1406$

$$\tau_{max} = \frac{T}{c_1 a b^2} = \frac{7 \times 10^3}{(0.208)(2)(2)^2} = 4.21 \times 10^3 \text{ psi}$$

$\tau_{max} = 4.21$ ksi

$$\varphi = \frac{TL}{c_2 a b^3 G} = \frac{(7 \times 10^3)(16)}{(0.1406)(2)(2)^3(5.6 \times 10^6)}$$

$= 8.89 \times 10^{-3}$ rad $\qquad \varphi = 0.509°$

(b) $\quad a = 2.8$ in. $\quad b = 1.4$ in. $\quad \frac{a}{b} = 2.0 \quad$ From Table 3.1 $\quad c_1 = 0.246$, $c_2 = 0.229$

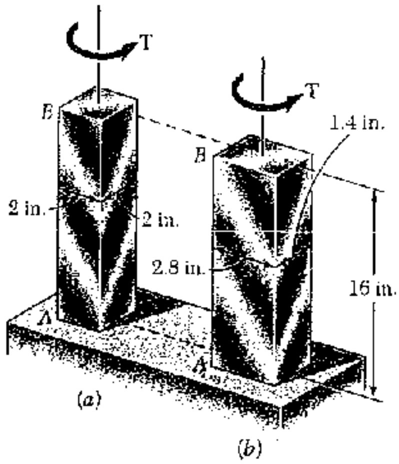
$$\tau_{max} = \frac{T}{c_1 a b^2} = \frac{7 \times 10^3}{(0.246)(2.8)(1.4)^2} = 5.19 \times 10^3 \text{ psi}$$

$\tau_{max} = 5.19$ ksi

$$\varphi = \frac{TL}{c_2 a b^3 G} = \frac{(7 \times 10^3)(16)}{(0.229)(2.8)(1.4)^3(5.6 \times 10^6)} = 11.37 \times 10^{-3} \text{ rad}$$

$\varphi = 0.651°$

Problem 3.120



3.120 Using $\tau_{all} = 7.5$ ksi and knowing that $G = 5.6 \times 10^6$ psi, determine for each of the cold-rolled yellow brass bars shown the largest torque T that can be applied and the corresponding angle of twist at end B.

$\tau_{al} = 7.5 \times 10^3$ psi $\quad L = 16$ in.

(a) $a = 2$ in. $\quad b = 2$ in. $\quad \frac{a}{b} = 1.0$

From Table 3.1 $\quad c_1 = 0.208, \quad c_2 = 0.1406$

$\tau_{max} = \frac{T}{c_1 a b^2} \qquad T = c_1 a b^2 \tau_{max}$

$T = (0.208)(2)(2)^2(7.5 \times 10^3) = 12.48 \times 10^3$ lb·in

$T = 12.48$ kip·in

$$\varphi = \frac{TL}{c_2 a b^3 G} = \frac{(12.48 \times 10^3)(16)}{(0.1406)(2)(2)^3(5.6 \times 10^6)}$$

$= 15.85 \times 10^{-3}$ rad $\qquad \varphi = 0.908°$

(b) $a = 2.8$ in. $\quad b = 1.4$ in. $\quad \frac{a}{b} = 2.0 \quad$ From Table 3.1 $\quad c_1 = 0.246, \ c_2 = 0.229$

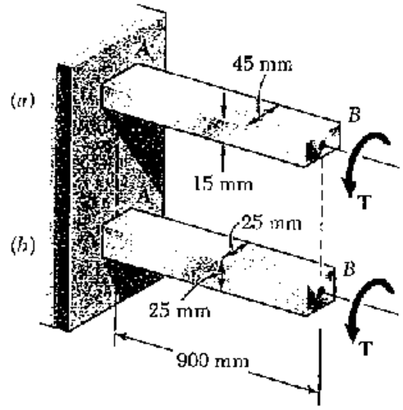
$\tau_{max} = \frac{T}{c_1 a b^2} \qquad T = c_1 a b^2 \tau_{max} = (0.246)(2.8)(1.4)^2(7.5 \times 10^3) = 10.13 \times 10^3$ lb·in

$T = 10.31$ kip·in

$$\varphi = \frac{TL}{c_2 a b^3 G} = \frac{(10.13 \times 10^3)(16)}{(0.229)(2.8)(1.4)^3(5.6 \times 10^6)} = 16.44 \times 10^{-3} \text{ rad} \qquad \varphi = 0.942°$$

Problem 3.121

3.121 Knowing that the magnitude of the torque **T** is 200 N · m and that $G = 27$ GPa, determine for each of the aluminum bars shown that maximum shearing stress and the angle of twist at end B.



$T = 200\ \text{N}\cdot\text{m} \quad L = 0.900\ \text{m} \quad G = 27\times10^{9}\ \text{Pa}$

(a) $a = 45\ \text{mm} \quad b = 15\ \text{mm} \quad \frac{a}{b} = 3.0$

From Table 3.1 $\quad c_1 = 0.267 \quad c_2 = 0.263$

$$\tau_{max} = \frac{T}{c_1 a b^2} = \frac{200}{(0.267)(0.045)(0.015)^2}$$

$= 74.0\times10^{6}\ \text{Pa} \qquad \tau_{max} = 74.0\ \text{MPa}$ ◄

$$\varphi = \frac{TL}{c_2 a b^3 G} = \frac{(200)(0.900)}{(0.263)(0.045)(0.015)^3(27\times10^{9})}$$

$= 166.9\times10^{-3}\ \text{rad} \qquad \varphi = 9.56°$ ◄

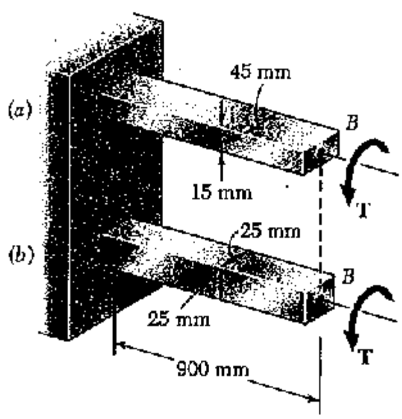
(b) $a = 25\ \text{mm} \quad b = 25\ \text{mm} \quad \frac{a}{b} = 1.0$ From Table 3.1 $\quad c_1 = 0.208,\ c_2 = 0.1406$

$$\tau_{max} = \frac{T}{c_1 a b^2} = \frac{200}{(0.208)(0.025)(0.025)^2} = 61.5\times10^{6}\ \text{Pa} \qquad \tau_{max} = 61.5\ \text{MPa} ◄$$

$$\varphi = \frac{TL}{c_2 a b^3 G} = \frac{(200)(0.900)}{(0.1406)(0.025)(0.025)^3(27\times10^{9})} = 121.4\times10^{-3}\ \text{rad} \qquad \varphi = 6.95° ◄$$

Problem 3.122

3.122 Using $\tau_{all} = 70$ MPa and $G = 27$ GPa, determine for each of the aluminum bars shown the largest torque **T** that can be applied and the corresponding angle of twist at end B.



$\tau_{all} = 70\times10^{6}\ \text{Pa} \quad G = 27\times10^{9}\ \text{Pa} \quad L = 0.900\ \text{m}$

(a) $a = 45\ \text{mm} \quad b = 15\ \text{mm} \quad \frac{a}{b} = 3.0$

From Table 3.1 $\quad c_1 = 0.267,\ c_2 = 0.263$

$$\tau_{max} = \frac{T}{c_1 a b^2} \qquad T = c_1 a b^2 \tau_{max}$$

$T = (0.267)(0.045)(0.015)^2(70\times10^{6}) = 189.2\ \text{N}\cdot\text{m}$

$T = 189.2\ \text{N}\cdot\text{m}$ ◄

$$\varphi = \frac{TL}{c_2 a b^3 G} = \frac{(189.2)(0.900)}{(0.263)(0.045)(0.015)^3(27\times10^{9})} = 157.9\times10^{-3}\ \text{rad} \qquad \varphi = 9.05° ◄$$

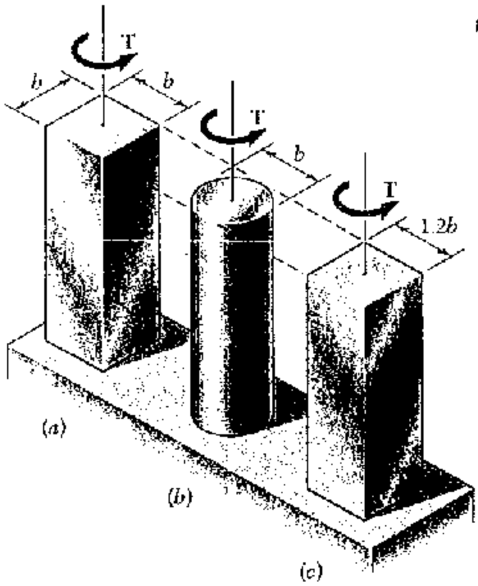
(b) $a = 25\ \text{mm} \quad b = 25\ \text{mm} \quad \frac{a}{b} = 1.0$ From Table 3.1 $\quad c_1 = 0.208,\ c_2 = 0.1406$

$\tau_{max} = \frac{T}{ab^2} \quad T = c_1 a b^2 \tau_{max} = (0.208)(0.025)(0.025)^2(70\times10^{6})$

$= 227.5\ \text{N}\cdot\text{m} \qquad T = 228\ \text{N}\cdot\text{m}$ ◄

$$\varphi = \frac{TL}{c_2 a b^3 G} = \frac{(227.5)(0.900)}{(0.1406)(0.025)(0.025)^3(27\times10^{9})} = 138.1\times10^{-3}\ \text{rad} \quad \varphi = 7.91° ◄$$

Problem 3.123



3.123 **Each of the three steel bars shown is subjected to a torque of magnitude $T = 275\ \mathrm{N \cdot m}$. Knowing that the allowable shearing stress is 50 MPa, determine the required dimension b for each bar.**

$T = 275\ \mathrm{N \cdot m} \qquad \tau_{max} = 50 \times 10^6\ \mathrm{Pa}$

(a) Square: $a = b \qquad \frac{a}{b} = 1.0$

From Table 3.1 $\quad c_1 = 0.208$

$$\tau_{max} = \frac{T}{c_1 a b^2} = \frac{T}{c_1 b^3}$$

$$b = \sqrt[3]{\frac{T}{c_1 \tau_{max}}} = \sqrt[3]{\frac{275}{(0.208)(50\times10^6)}}$$

$= 29.8 \times 10^{-3}\ \mathrm{m} \qquad\qquad b = 29.8\ \mathrm{mm}$ ◄

(b) Circle $\quad c = \frac{1}{2} b \qquad \tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

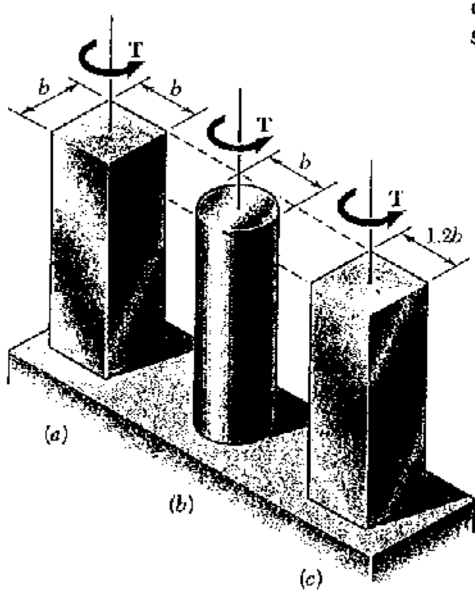
$$c = \sqrt[3]{\frac{2T}{\pi \tau_{max}}} = \sqrt[3]{\frac{(2)(275)}{\pi(50\times10^6)}} = 15.18 \times 10^{-3}\ \mathrm{m}$$

$c = 15.18\ \mathrm{m} \qquad\qquad b = 2c = 30.4\ \mathrm{mm}$ ◄

(c) Rectangle: $a = 1.2b \quad$ From Table 3.1 $\quad c_1 = 0.219 \quad \tau_{max} = \frac{T}{c_1 a b^2} = \frac{T}{1.2 c_1 b^3}$

$$b = \sqrt[3]{\frac{T}{1.2 c_1 \tau_{max}}} = \sqrt[3]{\frac{275}{(1.2)(0.219)(50\times10^6)}} = 27.6 \times 10^{-3}\ \mathrm{m} \qquad b = 27.6\ \mathrm{mm}$$ ◄

Problem 3.124



3.124 Each of the three aluminum bars shown is to be twisted through an angle of 2°. Knowing that $b = 30$ mm, $\tau_{all} = 50$ MPa, and $G = 27$ GPa, determine the shortest allowable length of each bar.

$\varphi = 2° = 34.907 \times 10^{-3}$ rad $\qquad \tau = 50 \times 10^{6}$ Pa

$G = 27 \times 10^{9}$ Pa $\qquad b = 30$ mm $= 0.030$ m

For square and rectangle

$$\tau = \frac{T}{c_1 a b^2} \qquad \varphi = \frac{TL}{c_2 a b^3 G}$$

Divide to eliminate T; then solve for L

$$\frac{\varphi}{\tau} = \frac{c_1 a b^2 L}{c_2 a b^3 G} \qquad L = \frac{c_2 b G \varphi}{c_1 \tau}$$

(a) Square: $\frac{a}{b} = 1.0$

From Table 3.1 $\quad c_1 = 0.208, \ c_2 = 0.1406$

$$L = \frac{(0.1406)(0.030)(27 \times 10^{9})(34.907 \times 10^{-3})}{(0.208)(50 \times 10^{6})} = 382 \times 10^{-3} \text{ m} \qquad L = 382 \text{ mm}$$

(b) Circle $\quad c = \frac{1}{2} b = 0.015$ m $\qquad \tau = \frac{Tc}{J} \qquad \varphi = \frac{TL}{GJ}$

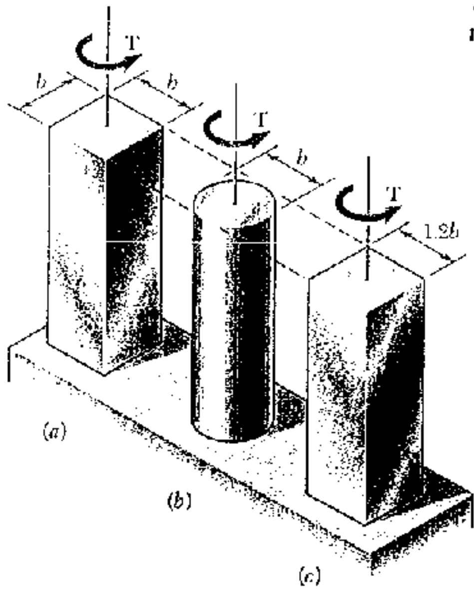
Divide to eliminate T; then solve for L $\qquad \frac{\varphi}{\tau} = \frac{JL}{cGJ} = \frac{L}{cG}$

$$L = \frac{cG\varphi}{\tau} = \frac{(0.015)(27 \times 10^{9})(34.907 \times 10^{-3})}{50 \times 10^{6}} = 283 \times 10^{-3} \text{ m} \qquad L = 283 \text{ mm}$$

(c) Rectangle: $a = 1.2b \qquad \frac{a}{b} = 1.2 \qquad$ From Table 3.1 $\quad c_1 = 0.219, \ c_2 = 0.1661$

$$L = \frac{(0.1661)(0.030)(27 \times 10^{9})(34.907 \times 10^{-3})}{(0.219)(50 \times 10^{6})} = 429 \times 10^{-3} \text{ m} \qquad L = 429 \text{ mm}$$

Problem 3.125



3.125 Each of the three steel bars shown is subjected to a torque of magnitude $T = 5$ kip · in. Knowing that the allowable shearing stress is 8 ksi, determine the required dimension b for each bar.

$T = 5\times10^3$ lb·in $\qquad \tau = 8\times10^3$ psi

(a) Square: $a = b \qquad \frac{a}{b} = 1.0$

From Table 3.1 $\qquad c_1 = 0.208$

$$\tau = \frac{T}{c_1 a b^2} = \frac{T}{c_1 b^3}$$

$$b = \sqrt[3]{\frac{T}{c_1 \tau}} = \sqrt[3]{\frac{5\times10^3}{(0.208)(8\times10^3)}} \qquad b = 1.443 \text{ in.}$$

(b) Circle: $\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

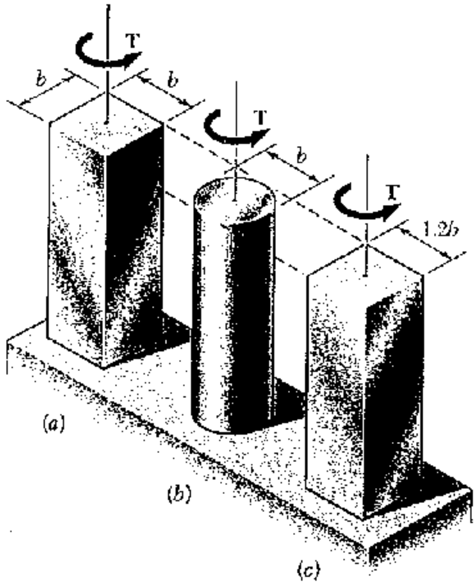
$$c = \sqrt[3]{\frac{2T}{\pi\tau}} = \sqrt[3]{\frac{(2)(5\times10^3)}{\pi(8\times10^3)}} = 0.7355 \text{ in.}$$

$$b = 2c = 1.471 \text{ in.}$$

(c) Rectangle: $a = 1.2\,b \qquad \frac{a}{b} = 1.2 \qquad$ From Table 3.1 $\qquad c_1 = 0.219$

$$\tau = \frac{T}{c_1 a b^2} = \frac{T}{1.2 c_1 b^3} \qquad b = \sqrt[3]{\frac{T}{1.2 c_1 \tau}} = \sqrt[3]{\frac{5\times10^3}{(1.2)(0.219)(8\times10^3)}} \qquad b = 1.335 \text{ in.}$$

Problem 3.126



3.126 Each of the three aluminum bars shown is to be twisted through an angle of 1.25°. Knowing that $b = 1.5$ in., $\tau_{all} = 7.5$ ksi, and $G = 3.9 \times 10^6$ psi, determine the shortest allowable length of each bar.

$\varphi = 1.25° = 21.817 \times 10^{-3}$ rad $\qquad \tau = 7.5 \times 10^3$ psi

For square and rectangle

$$\tau = \frac{T}{c_1 a b^2} \qquad \varphi = \frac{TL}{c_2 a b^3 G}$$

Divide to eliminate T; then solve for L

$$\frac{\varphi}{\tau} = \frac{c_1 a b^2 L}{c_2 a b^3 G} \qquad L = \frac{c_2 b G \varphi}{c_1 \tau}$$

(a) Square $\frac{a}{b} = 1.0$

From Table 3.1 $\quad c_1 = 0.208, \; c_2 = 0.1406$

$$L = \frac{(0.1406)(1.5)(3.9 \times 10^6)(21.817 \times 10^{-3})}{(0.208)(7.5 \times 10^3)} \qquad L = 11.50 \text{ in.}$$

(b) Circle: $c = \frac{1}{2} b = 0.75$ in. $\qquad \tau = \frac{Tc}{J} \qquad \varphi = \frac{TL}{GJ}$

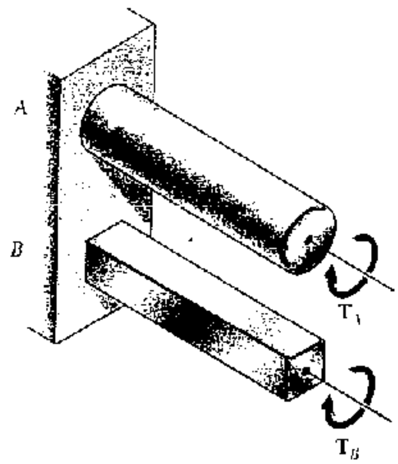
Divide to eliminate T; then solve for L $\qquad \frac{\varphi}{\tau} = \frac{JL}{cGJ} = \frac{L}{cG}$

$$L = \frac{cG\varphi}{\tau} = \frac{(0.75)(3.9 \times 10^6)(21.817 \times 10^{-3})}{7.5 \times 10^3} \qquad L = 8.51 \text{ in.}$$

(c) Rectangle: $a = 1.2b \qquad \frac{a}{b} = 1.2$ $\quad$ From Table 3.1 $\quad c_1 = 0.219, \; c_2 = 0.1661$

$$L = \frac{(0.1661)(1.5)(3.9 \times 10^6)(21.817 \times 10^{-3})}{(0.219)(7.5 \times 10^3)} \qquad L = 12.91 \text{ in.}$$

Problem 3.127



3.127 Shafts A and B are made of the same material and have the same cross-sectional area, but A has a circular cross section and B has a square cross section. Determine the ratio of the maximum torques T_A and T_B that can be safely applied to A and B, respectively.

SOLUTION

Let c = radius of circular section A and b = side of square section B.

For equal areas $\pi c^2 = b^2$

$$c = \frac{b}{\sqrt{\pi}}$$

Circle: $\tau_A = \frac{Tc}{J} = \frac{2T_A}{\pi c^3} \therefore T_A = \frac{\pi}{2} c^3 \tau_A$

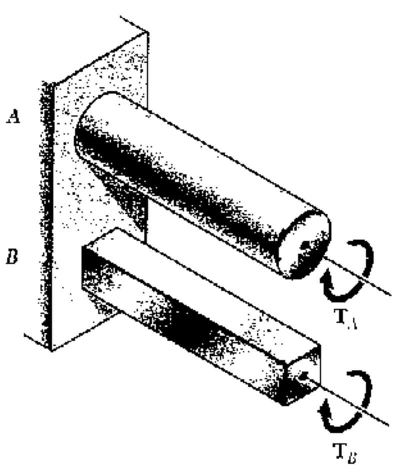
Square: $c_1 = 0.208$ from Table 3.1

$$\tau_B = \frac{T_A}{c_1 a b^2} = \frac{T_B}{c_1 b^3} \therefore T_B = c_1 b^3 \tau_B$$

Ratio: $\frac{T_A}{T_B} = \frac{\frac{\pi}{2} c^3 \tau_B}{c_1 b^3 \tau_B} = \frac{\frac{\pi}{2} \cdot \frac{b^3}{\pi^{3/2}} \tau_A}{c_1 b^3 \tau_B} = \frac{1}{2c_1\sqrt{\pi}} \frac{\tau_A}{\tau_B}$

For the same stresses $\tau_B = \tau_A$ $\therefore$ $\frac{T_A}{T_B} = \frac{1}{(2)(0.208)\sqrt{\pi}} = 1.356$ ◄

Problem 3.128



3.128 Shafts A and B are made of the same material and have the same length and cross-sectional area, but A has a circular cross section and B has a square cross section. Determine the ratio of the maximum values of the angles ϕ_A and ϕ_B through which shafts A and B, respectively, can be twisted.

SOLUTION

Let c = radius of circular section A and b = side of square section B.

For equal areas $\pi c^2 = b^2 \therefore \quad b = \sqrt{\pi}\, c$

Circle: $\gamma_{max} = \frac{\tau_A}{G} = \frac{c\varphi_A}{L} \therefore \varphi_A = \frac{L\tau_A}{cG}$

Square: Table 3.1 $c_1 = 0.208$, $c_2 = 0.1406$

$$\tau_B = \frac{T_B}{c_1 a b^2} = \frac{T_B}{0.208\, b^3} \therefore \quad T_B = 0.208\, b^3 \tau_B$$

$$\varphi_B = \frac{T_B L}{c_2 a b^3 G} = \frac{0.208\, b^3 \tau_B L}{0.1406\, b^4 G} = \frac{1.4794\, L \tau_B}{bG}$$

Ratio $\frac{\varphi_A}{\varphi_B} = \frac{L\tau_A}{cG} \cdot \frac{bG}{1.4794 L \tau_B} = 0.676 \frac{b\tau_A}{c\tau_B} = 0.676\sqrt{\pi} \frac{\tau_A}{\tau_B}$

For equal stresses $\tau_A = \tau_B$ $\qquad \frac{\varphi_B}{\varphi_A} = 0.676\sqrt{\pi} = 1.198$ ◄

Problem 3.129

3.129 Determine the largest allowable square cross section of a steel shaft of length 6 m if the maximum shearing stress is not to exceed 120 MPa when the shaft is twisted through one complete revolution. Use $G = 77.2$ GPa.

$L = 6\text{ m}, \quad \tau = 120\times10^6\text{ Pa}, \quad G = 77.2\times10^9\text{ Pa} \qquad \varphi = 1\text{ rev} = 2\pi\text{ rad}$

$\tau = \dfrac{T}{c_1 a b^2} \qquad \varphi = \dfrac{TL}{c_2 a b^3 G}$

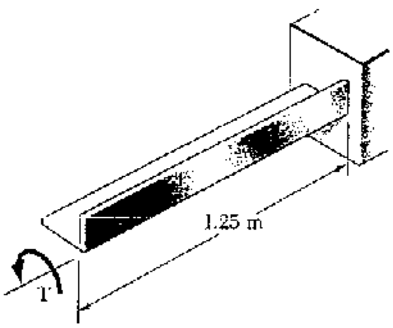
Divide to eliminate T; then solve for b. $\dfrac{\varphi}{\tau} = \dfrac{c_1 a b^2 L}{c_2 a b^3 G} = \dfrac{c_1 L}{c_2 b G}$

$b = \dfrac{c_1 L \tau}{c_2 G \varphi}$ For a square cross section $\dfrac{a}{b} = 1.0$

From Table 3.1 $\quad c_1 = 0.208, \ c_2 = 0.1406$

$$b = \frac{(0.208)(6)(120\times10^6)}{(0.1406)(77.2\times10^9)(2\pi)} = 2.20\times10^{-3}\text{ m} \qquad b = 2.20\text{ mm}$$

Problem 3.130



3.130 A 1.25-m-long steel angle has an L127 × 76 × 6.4 cross section. From Appendix C we find that the thickness of the section is 6.4 mm and that its area is 1252 mm^2. Knowing that $\tau_{all} = 60$ MPa and that $G = 77.2$ GPa, and ignoring the effect of stress concentration, determine (*a*) the largest torque T that can be applied, (*b*) the corresponding angle of twist.

$$A = 1252 \text{ mm}^2 \qquad b = 6.4 \text{ mm} = 0.0064 \text{ m}$$

$$a = \frac{A}{b} = \frac{1252}{6.4} = 195.6 \text{ mm} = 0.1956 \text{ m}$$

$$\frac{a}{b} = \frac{195.6}{6.4} = 30.56$$

$$c_1 = c_2 = \frac{1}{3}\left(1 - 0.630\frac{b}{a}\right) = 0.3265$$

$$\tau_{max} = \frac{T}{c_1 a b^2} \quad \therefore \quad T = c_1 a b^2 \tau_{max}$$

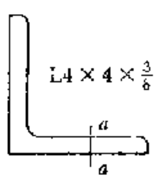
(a) $T = (0.3265)(0.1956)(0.0064)^2(60\times10^6) = 156.95\times10^3$ N·m

$T = 157.0$ kN·m ◄

(b) $\varphi = \dfrac{TL}{c_2 a b^3 G} = \dfrac{c_1 a b^2 \tau_{max} L}{c_2 a b^3 G} = \dfrac{c_1 \tau_{max} L}{c_2 b G} = \dfrac{\tau_{max} L}{bG}$

$\varphi = \dfrac{(60\times10^6)(1.25)}{(0.0064)(77.2\times10^9)} = 151.80\times10^{-3}$ rad $\qquad \varphi = 8.70°$ ◄

Problem 3.131



3.131 A 3000 lb · in. torque is applied to a 6-ft-long steel angle with an L× 4 × 4 × $\frac{3}{8}$ cross section. From Appendix C we find that the thickness of the section is $\frac{3}{8}$ in. and that its area is 2.86 in^2. Knowing that $G = 11.2 \times 10^6$ psi, determine (*a*) the maximum shearing stress along line *a-a*, (*b*) the angle of twist.

$$A = 2.86 \text{ in}^2, \quad b = \frac{3}{8} \text{ in} = 0.375 \text{ in}, \quad a = \frac{A}{a} = \frac{2.86}{0.375} = 7.627 \text{ in}$$

$$\frac{a}{b} = \frac{7.627}{0.375} = 20.34 \qquad c_1 = c_2 = \frac{1}{3}\left(1 - 0.630\frac{b}{a}\right) = 0.3230$$

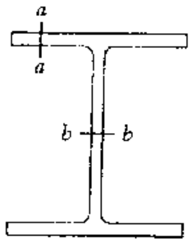
(a) $\tau_{max} = \dfrac{T}{c_1 a b^2} = \dfrac{3000}{(0.3230)(7.627)(0.375)^2} = 8.66\times10^3$ psi $= 8.66$ ksi ◄

(b) $\varphi = \dfrac{TL}{c_2 a b^3 G} = \dfrac{(3000)(72)}{(0.3230)(7.627)(0.375)^3(11.2\times10^6)} = 148.45\times10^{-3}$ rad

$\varphi = 8.51°$ ◄

Note: $L = 6$ ft. $= 72$ in

Problem 3.132



W8 × 31

3.132 An 8-ft-long steel member with a W8 × 31 cross section is subjected to a 5-kip · in. torque. The properties of the rolled-steel section are given in Appendix C. Knowing that $G = 11.2 \times 10^6$ psi, determine (*a*) the maximum shearing stress along line *a-a*, (*b*) the maximum shearing stress along line *b-b*, (*c*) the angle of twist. (*Hint*: consider the web and flanges separately and obtain a relation between the torques exerted on the web and a flange, respectively, by expressing that the resulting angles of twist are equal.)

SOLUTION

<u>Flange:</u> $a = 7.995$ in, $b = 0.435$, $\frac{a}{b} = \frac{7.995}{0.435} = 18.38$

$c_1 = c_2 = \frac{1}{3}\left(1 - 0.630\frac{b}{a}\right) = 0.3219 \qquad \varphi_F = \frac{T_F L}{c_2 a b^3 G}$

$T_F = c_2 a b^3 \frac{G\varphi_F}{L} = K_F \frac{G\varphi}{L} \qquad$ where $K_F = c_2 a b^3$

$K_F = (0.3219)(7.995)(0.435)^3 = 0.2138 \text{ in}^4$

<u>Web:</u> $a = 8.0 - (2)(0.435) = 7.13$ in, $b = 0.285$ in, $\frac{a}{b} = \frac{7.13}{0.285} = 25.02$

$c_1 = c_2 = \frac{1}{3}\left(1 - 0.630\frac{b}{a}\right) = 0.3249 \qquad \varphi_w = \frac{T_w L}{c_2 a b^3 G}$

$T_w = c_2 a b^3 \frac{G\varphi_w}{L} = K_w \frac{G\varphi}{L} \qquad$ where $K_w = c_2 a b^3$

$K_w = (0.3249)(7.13)(0.285)^3 = 0.0563 \text{ in}^4$

For matching twist angles $\quad \varphi_F = \varphi_w = \varphi$

Total torque $\quad T = 2T_F + T_w = (2K_F + K_w)\frac{G\varphi}{L}$

$\frac{G\varphi}{L} = \frac{T}{2K_F + K_w}, \quad T_F = \frac{K_F T}{2K_F + K_w}, \quad T_w = \frac{K_w T}{2K_w + K_w}$

$T_F = \frac{(0.2138)(5000)}{(2)(0.2138) + 0.0563} = 2221 \text{ lb·in}; \quad T_w = \frac{(0.0563)(5000)}{(2)(0.2138) + 0.0563} = 557 \text{ lb·in}$

(a) $\tau_F = \frac{T_F}{c_1 a b^2} = \frac{2221}{(0.3219)(7.995)(0.435)^2} = 4570 \text{ psi} = 4.57 \text{ ksi}$ ◄

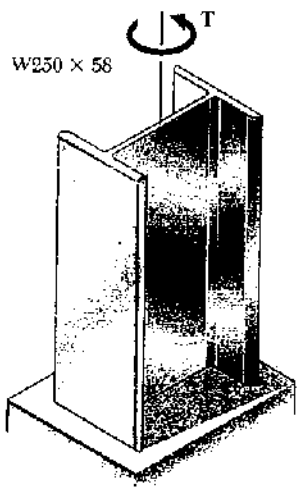
(b) $\tau_w = \frac{T_w}{c_1 a b^2} = \frac{557}{(0.3249)(7.13)(0.285)^2} = 2960 \text{ psi} = 2.96 \text{ ksi}$ ◄

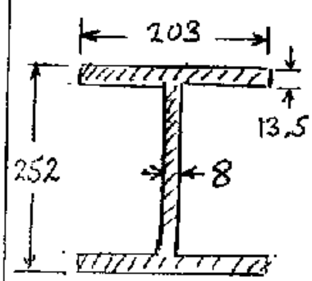
(c) $\frac{G\varphi}{L} = \frac{T}{2K_F + K_w} \quad \therefore \varphi = \frac{TL}{G(2K_F + K_w)} \quad$ where $L = 8 \text{ ft} = 96$ in.

$\varphi = \frac{(5000)(96)}{(11.2 \times 10^6)[(2)(0.2138) + 0.563]} = 88.6 \times 10^{-3} \text{ rad} = 5.08°$ ◄

Problem 3.133

3.133 A 3-m-long steel member has a W250 × 58 cross section. Knowing that $G = 77.2$ GPa and that the allowable shearing stress is 35 MPa, determine (*a*) the largest torque **T** that can be applied, (*b*) the corresponding angle of twist. Refer to Appendix C for the dimensions of the cross section and neglect the effect of stress concentrations. (See hint of Prob. 3.132.)





SOLUTION

Flange: $a = 203$ mm, $b = 13.5$ mm, $\frac{a}{b} = 15.04$

$$C_1 = C_2 = \frac{1}{3}\left(1 - 0.630\frac{b}{a}\right) = 0.3194$$

$$\varphi_F = \frac{T_F L}{C_2 a b^3 G} \quad \therefore \quad T_F = C_2 a b^3 \frac{G\varphi}{L} = K_F \frac{G\varphi}{L}$$

$$K_F = (0.3194)(0.203)(0.0135)^3 = 159.53 \times 10^{-9} \text{ m}^4$$

Web: $a = 252 - (2)(13.5) = 225$ mm, $b = 8$ mm

$$\frac{a}{b} = 28.13, \quad C_1 = C_2 = \frac{1}{3}\left(1 - 0.63\frac{b}{a}\right) = 0.3259$$

$$\varphi_W = \frac{T_W L}{C_2 a b^3 G} \quad \therefore \quad T_W = C_2 a b^3 \frac{G\varphi}{L} = K_W \frac{G\varphi}{L}$$

$$K_W = (0.3259)(0.225)(0.008)^3 = 37.54 \times 10^{-9} \text{ m}^4$$

For matching twist angles $\varphi_F = \varphi_W = \varphi$

Total torque: $T = 2T_F + T_W = (2K_F + K_W)\frac{G\varphi}{L}$

$$\frac{G\varphi}{L} = \frac{T}{2K_F + K_W}, \qquad T_F = \frac{K_F T}{2K_F + K_W} \quad \therefore \quad T = \frac{2K_F + K_W}{K_F} T_F$$

$$T_W = \frac{K_W T}{2K_F + K_W} \quad \therefore \quad T = \frac{2K_F + K_W}{K_W} T_W$$

Allowable value for T based on allowable value for T_F

$$T_F = c_1 a b^2 \tau = (0.3194)(0.203)(0.0135)^2(35 \times 10^6) = 413.6 \text{ N·m}$$

$$T = \frac{(2)(159.53) + (37.54)}{159.53}(413.6) = 924.5 \text{ N·m}$$

Allowable value for T based on allowable value for T_W

$$T_W = c_1 a b^2 \tau = (0.3259)(0.225)(0.008)^2(35 \times 10^6) = 164.25 \text{ N·m}$$

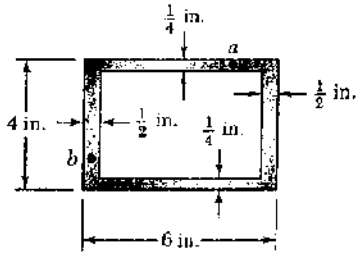
$$T = \frac{(2)(159.53) + 37.54}{37.54}(164.25) = 1560 \text{ N·m}$$

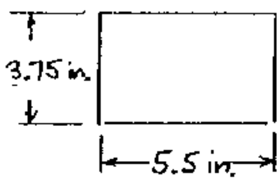
Choose smaller value $\qquad T = 925$ N·m ◄

$$\varphi = \frac{TL}{(2K_F + K_W)G} = \frac{(924.5)(3.00)}{(356.6 \times 10^{-9})(77.2 \times 10^9)} = 100.7 \times 10^{-3} \text{ rad} \qquad \varphi = 5.77° \blacktriangleleft$$

Problem 3.134

3.134 A 5 kip · ft torque is applied to a hollow aluminum shaft having the cross section shown. Neglecting the effect of stress concentrations, determine the shearing stress at points a and b.





$T = (5)(10^3)(12) = 60 \times 10^3$ lb · in

Area bounded by center line

$a = bh = (5.5)(3.75) = 20.625 \text{ in}^2$

At point a $t = 0.25$ in.

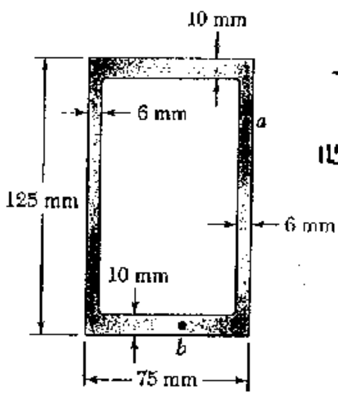
$\tau = \frac{T}{2ta} = \frac{60 \times 10^3}{(2)(0.25)(20.625)} = 5.82 \times 10^3 \text{ psi}$ $\tau = 5.82$ ksi ◄

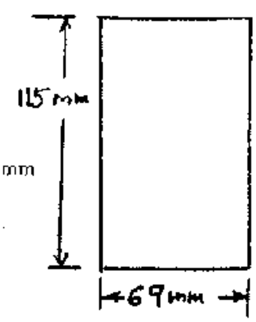
At point b $t = 0.50$ in.

$\tau = \frac{T}{2ta} = \frac{60 \times 10^3}{(2)(0.50)(20.625)} = 2.91 \times 10^3 \text{ psi}$ $\tau = 2.91$ ksi ◄

Problem 3.135

3.135 A 5 kN · m torque is applied to a hollow shaft having the cross section shown. Neglecting the effect of stress concentrations, determine the shearing stress at points a and b.





$T = 5 \times 10^3$ N · m

Area bounded by center line

$a = bh = (69)(115) = 7.935 \times 10^3 \text{ mm}^2$

$= 7.935 \times 10^{-3} \text{ m}^2$

At point a $t = 6 \text{ mm} = 0.006$ m

$\tau = \frac{T}{2ta} = \frac{5 \times 10^3}{(2)(0.006)(7.935 \times 10^{-3})}$

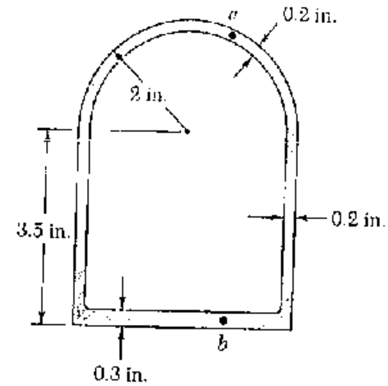
$= 52.5 \times 10^6$ Pa $\tau = 52.5$ MPa ◄

At point b $t = 10 \text{ mm} = 0.010$ m

$\tau = \frac{T}{2ta} = \frac{5 \times 10^3}{(2)(0.010)(7.935 \times 10^{-3})} = 31.5 \times 10^6$ Pa $\tau = 31.5$ MPa ◄

Problem 3.136

3.136 A 50 kip · in. torque is applied to a hollow shaft having the cross section shown. Neglecting the effect of stress concentrations, determine the shearing stress at points a and b.



Area bounded by centerline.

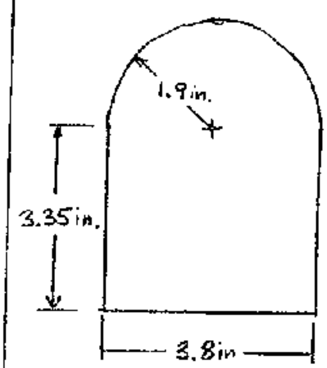
$$\mathcal{a} = (3.8)(3.35) + \frac{\pi}{2}(1.9)^2 = 18.4006 \text{ in}^2$$

At point a. $t = 0.2$ in

$$\tau = \frac{T}{2\mathcal{a}t} = \frac{50}{(2)(18.4006)(0.2)} = 6.79 \text{ ksi}$$

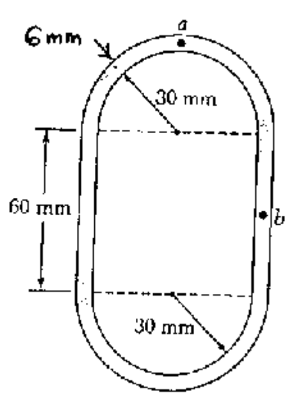
At point b. $t = 0.3$ in.

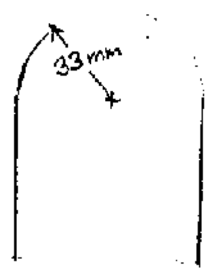
$$\tau = \frac{T}{2\mathcal{a}t} = \frac{50}{(2)(18.0046)(0.3)} = 4.53 \text{ ksi}$$



Problem 3.137

3.137 A 750 N · m torque is applied to a hollow shaft having the cross section shown and a uniform 6-mm wall thickness. Neglecting the effect of stress concentrations, determine the shearing stress at points a and b.





Area bounded by center line

$$\mathcal{a} = 2\,\frac{\pi}{2}(33)^2 + (60)(66) = 7381 \text{ mm}^2$$

$$= 7381 \times 10^{-6} \text{ m}^2$$

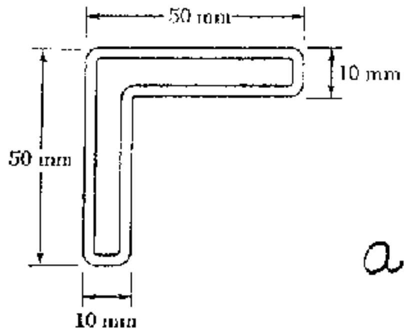
$t = 0.006$ m at both a and b.

Then at points a and b

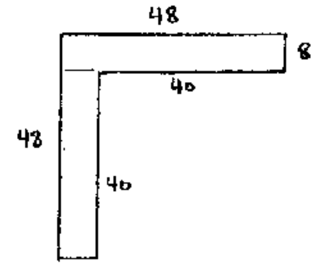
$$\tau = \frac{T}{2t\mathcal{a}} = \frac{750}{(2)(0.006)(7381\times10^{-6})} = 8.47\times10^6 \text{ Pa} = 8.47 \text{ MPa}$$

Problem 3.138

3.138 A hollow member having the cross section shown is formed from sheet metal of 2-mm thickness. Knowing that the shearing stress must not exceed 3 MPa, determine the largest torque that can be applied to the member.



Area bounded by centerline



$$a = (48)(8) + (40)(8)$$

$$= 704\ mm^2 = 704 \times 10^{-6}\ m^2$$

$$t = 0.002\ m$$

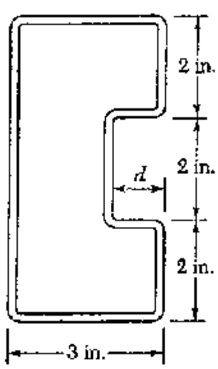
$$\tau = \frac{T}{2ta} \quad \therefore \quad T = 2ta\tau = (2)(0.002)(704\times10^{-6})(3\times10^{6})$$

$$= 8.45\ N\cdot m$$

$T = 8.45\ N\cdot m$

Problem 3.139

3.139 and 3.140 A hollow member having the cross section shown is to be formed from sheet metal of 0.06-in. thickness. Knowing that a 1250 lb · in. torque will be applied to the member, determine the smallest dimension d that can be used if the shearing stress is not to exceed 750 psi.

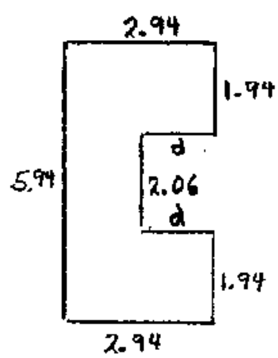


Area bounded by centerline

$$a = (5.94)(2.94) - 2.06d = 17.4636 - 2.06d$$

$$t = 0.06\ in, \quad \tau = 750\ psi, \quad T = 1250\ lb\cdot in$$

$$\tau = \frac{T}{2ta}$$



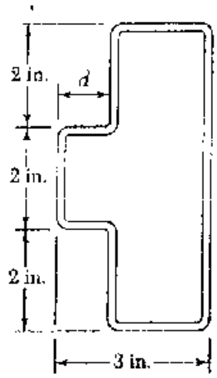
$$a = \frac{T}{2t\tau}$$

$$17.4636 - 2.06d = \frac{1250}{(2)(0.06)(750)} = 13.8889$$

$$d = \frac{3.5747}{2.06} = 1.735\ in.$$

$d = 1.735\ in.$

Problem 3.140



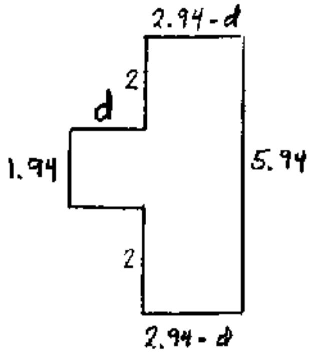
3.139 and 3.140 A hollow member having the cross section shown is to be formed from sheet metal of 0.06-in. thickness. Knowing that a 1250 lb · in. torque will be applied to the member, determine the smallest dimension d that can be used if the shearing stress is not to exceed 750 psi.

Area bounded by center

$$a = (5.94)(2.94 - d) + 1.94\,d = 17.4636 - 4.00\,d$$

$t = 0.06$ in, $\tau = 750$ psi, $T = 1250$ lb·in

$$\tau = \frac{T}{2ta}$$



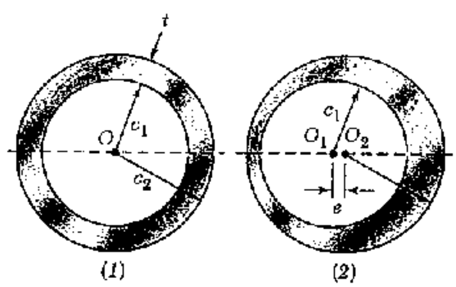
$$a = \frac{T}{2t\tau}$$

$$17.4636 - 4.00\,d = \frac{1250}{(2)(0.06)(750)} = 13.8889$$

$$d = \frac{3.5747}{4.00} = 0.894 \text{ in.}$$

$d = 0.894$ in.

Problem 3.141



3.141 A hollow cylindrical shaft was designed with the cross section shown in Fig. (*1*) to withstand a maximum torque T_0. Defective fabrication, however, resulted in a slight eccentricity e between the inner and outer cylindrical surfaces of the shaft as shown in Fig. (*2*). (*a*) Express the maximum torque T which can be safely applied to the defective shaft in terms of T_0, e, and t. (*b*) Calculate the percent decrease in the allowable torque for values of the ratio e/t equal to 0.1, 0.5, and 0.9.

(a) For both configurations $a = \pi\left(\frac{c_1 + c_2}{2}\right)^2$

Let $t = c_2 - c_1$ be the thickness of (1)

The minimum thickness of (2) is $t - e$.

For (1) $\tau_{al} = \frac{T_0}{2ta}$ For (2) $\tau_{al} = \frac{T}{2(t-e)a}$

Forming the ratio $\frac{\tau_{al}}{\tau_{al}} = 1 = \frac{T}{2(t-e)a} \cdot \frac{2ta}{T_0} = \frac{Tt}{T_0(t-e)}$

$$T = T_0\left(1 - \frac{e}{t}\right)$$

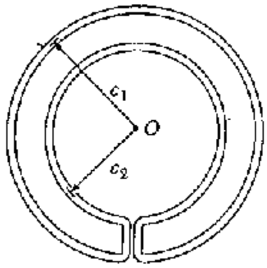
(b) Reduction in torque $\frac{T_0 - T}{T_0} = \frac{e}{t}$

Expressed as a % $\quad$ % reduction $= \frac{e}{t} \times 100\%$

$\frac{e}{t}$	% red
0.1	10%
0.5	50%
0.9	90%

Problem 3.142

3.142 A cooling tube having the cross section shown is formed from a sheet of stainless steel of 3-mm thickness. The radii $c_1 = 150$ mm and $c_2 = 100$ mm are measured to the center line of the sheet metal. Knowing that a torque of magnitude $T = 3$ kN · m is applied to the tube, determine (*a*) the maximum shearing stress in the tube, (*b*) the magnitude of the torque carried by the outer circular shell. Neglect the dimension of the small opening where the outer and inner shells are connected.



Area bounded by centerline

$$a = \pi(c_1^2 - c_2^2) = \pi(150^2 - 100^2) = 39.27 \times 10^3 \text{ mm}^2$$

$$= 39.27 \times 10^{-3} \text{ m}^2$$

$$t = 0.003 \text{ m}$$

(a) $$\tau = \frac{T}{2ta} = \frac{3 \times 10^3}{(2)(0.003)(39.27 \times 10^{-3})} = 12.73 \times 10^6 \text{ Pa} \qquad \tau = 12.76 \text{ MPa}$$

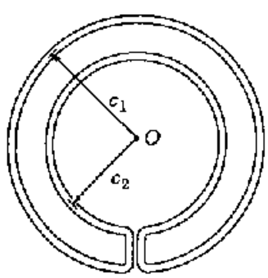
(b) $$T_1 = (2\pi c_1 t \tau_1 c_1) = 2\pi c_1^2 t \tau$$

$$= 2\pi (0.150)^2 (0.003)(12.73 \times 10^6) = 5.40 \times 10^3 \text{ N·m}$$

$$T_1 = 5.40 \text{ kN·m}$$

Problem 3.143

3.143 A cooling tube having the cross section shown is formed from a sheet of stainless steel of thickness t. The radii c_1 and c_2 are measured to the center line of the sheet metal. Knowing that a torque **T** is applied to the tube, determine in terms of T, c_1, c_2, and t the maximum shearing stress in the tube.



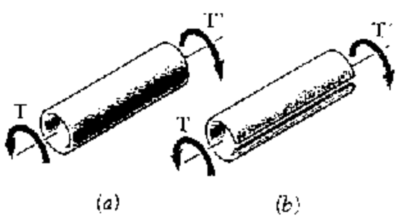
Area bounded by centerline

$$a = \pi(c_1^2 - c_2^2)$$

Shearing stress

$$\tau = \frac{T}{2ta} = \frac{T}{2\pi t(c_1^2 - c_2^2)}$$

Problem 3.144



3.144 Equal torques are applied to thin-walled tubes of the same length L, same thickness t, and same radius c. One of the tubes has been slit lengthwise as shown. Determine (a) the ratio τ_b/τ_a of the maximum shearing stresses in the tubes, (b) the ratio ϕ_b/ϕ_a of the angles of twist of the shafts.

Without slit

Area bounded by centerline: $\mathcal{a} = \pi c^2$

$$\tau_a = \frac{T}{2t\mathcal{a}} = \frac{T}{2\pi c^2 t}$$

$$J \approx 2\pi c^3 t \qquad \varphi_a = \frac{TL}{GJ} = \frac{TL}{2\pi c^3 t G}$$

With slit: $\quad a = 2\pi c, \quad b = t, \quad \frac{a}{b} = \frac{2\pi c}{t} >> 1$

$$c_1 = c_2 = \frac{1}{3}$$

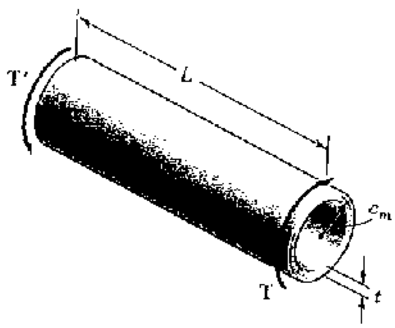
$$\tau_b = \frac{T}{c_1 a b^2} = \frac{3T}{2\pi c t^2}$$

$$\varphi_b = \frac{T}{c_2 a b^3 G} = \frac{3TL}{2\pi c t^3 G}$$

(a) Stress ratio: $\quad \frac{\tau_b}{\tau_a} = \frac{3T}{2\pi c t^2} \cdot \frac{2\pi c^2 t}{T} = \frac{3c}{t} \qquad \frac{\tau_b}{\tau_a} = \frac{3c}{t}$ ◄

(b) Twist ratio: $\quad \frac{\varphi_b}{\varphi_a} = \frac{3TL}{2\pi c t^3 G} \cdot \frac{2\pi c^3 t G}{TL} = \frac{3c^2}{t^2} \qquad \frac{\varphi_b}{\varphi_a} = \frac{3c^2}{t^2}$ ◄

Problem 3.145



3.145 A hollow cylindrical shaft of length L, mean radius c_m, and uniform thickness t is subjected to torques of magnitude T. Consider, on the one hand, the values of the average shearing stress τ_{ave} and the angle of twist ϕ obtained from the elastic torsion formulas developed in Secs. 3.4 and 3.5 and, on the other hand, the corresponding values obtained from the formulas developed in Sec. 3.13 for thin-walled hollow shafts. (*a*) Show that the relative error introduced by using the thin-wall-shaft formulas rather than the elastic torsion formulas is the same for τ_{ave} and ϕ and that the relative error is positive and proportional to the square of the ratio t/c_m. (*b*) Compare the percent error corresponding to values of the ratio t/c_m equal 0.1, 0.2 and 0.4.

Let c_2 = outer radius $= c_m + \frac{1}{2}t$ and c_1 = inner radius $= c_m - \frac{1}{2}t$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(c_2^2 + c_1^2)(c_2 + c_1)(c_2 - c_1)$$

$$= \frac{\pi}{2}(c_m^2 + c_m t + \tfrac{1}{4}t^2 + c_m^2 - c_m t + \tfrac{1}{4}t^2)(2c_m)\,t$$

$$= 2\pi(c_m^2 + \tfrac{1}{4}t^2)\,c_m t$$

$$\tau_m = \frac{Tc_m}{J} = \frac{T}{2\pi(c_m^2 + \frac{1}{4}t^2)\,t}$$

$$\varphi_1 = \frac{TL}{JG} = \frac{TL}{2\pi(c_m^2 + \frac{1}{4}t^2)\,c_m t\, G}$$

Area bounded by centerline $\quad \mathcal{a} = \pi c_m^2$

$$\tau_{ave} = \frac{T}{2t\mathcal{a}} = \frac{T}{2\pi c_m^2 t}$$

$$\varphi_2 = \frac{TL}{4\mathcal{a}^2 G}\oint\frac{ds}{t} = \frac{TL(2\pi c_m/t)}{4(\pi c_m^2)^2 G} = \frac{TL}{2\pi c_m^3 t G}$$

Ratios:

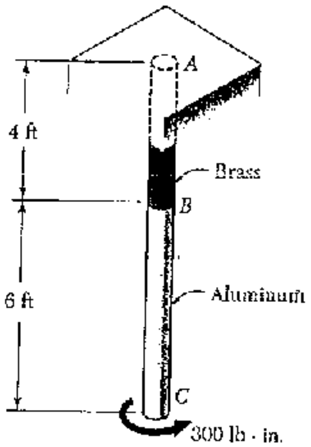
$$\frac{\tau_{ave}}{\tau_m} = \frac{T}{2\pi c_m^2 t}\cdot\frac{2\pi(c_m^2 + \frac{1}{4}t^2)\,t}{T} = 1 + \frac{1}{4}\frac{t^2}{c_m^2}$$

$$\frac{\varphi_2}{\varphi_1} = \frac{TL}{2\pi c_m^3 t G}\cdot\frac{2\pi(c_m^2 + \frac{1}{4}t^2)\,c_m t G}{TL} = 1 + \frac{1}{4}\frac{t^2}{c_m}$$

$$\frac{\tau_{ave}}{\tau_m} - 1 = \frac{\varphi_2}{\varphi_1} - 1 = \frac{1}{4}\frac{t^2}{c_m^2}$$

$\frac{t}{c_m}$	0.1	0.2	0.4
$\frac{1}{4}\frac{t^2}{c_m^2}$	0.0025	0.01	0.04
	0.25%	1%	4%

Problem 3.146



3.146 The aluminum rod BC ($G = 3.9 \times 10^6$ psi) is bonded to the brass for AB ($G = 5.6 \times 10^6$ psi). Knowing that each rod is solid and has a diameter of 0.5 in., determine the angle of twist (*a*) at B, (*b*) at C.

Both portions $c = \frac{1}{2}d = 0.25$ in

$J = \frac{\pi}{2}c^4 = 6.1359 \times 10^{-3}$ in^4 $\qquad T = 300$ lb·in

Shaft AB: $G_{AB} = 5.6 \times 10^6$ psi $\quad L_{AB} = 4$ ft $= 48$ in

$$\varphi_B = \varphi_{AB} = \frac{T L_{AB}}{G_{AB} J} = \frac{(300)(48)}{(5.6\times10^6)(6.1359\times10^{-3})}$$

$= 0.419$ rad $\qquad \varphi_B$ $24.0°$

Shaft BC: $G = 3.9 \times 10^6$ psi $\quad L_{BC} = 6$ ft $= 72$ in.

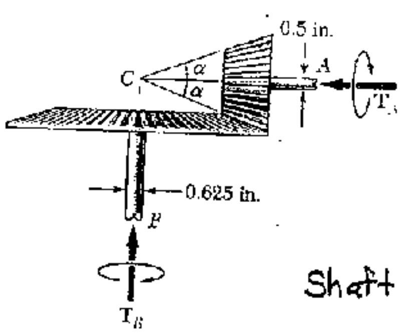
$$\varphi_{BC} = \frac{T L_{BC}}{G_{BC} J} = \frac{(300)(72)}{(3.9\times10^6)(6.1359\times10^{-3})}$$

$= 0.903$ rad $= 51.7°$

$\varphi_C = \varphi_B + \varphi_{BC}$

$= 0.417 + 0.903 = 1.320$ rad $\qquad \varphi_C = 76.6°$

Problem 3.147



3.147 In the bevel-gear system shown, $\alpha = 18.43°$. Knowing that the allowable shearing stress is 8 ksi in each shaft and that the system is in equilibrium, determine the largest torque $\mathbf{T}_A$ that can be applied at A.

Shaft A: $\tau = 8$ ksi $\quad c = \frac{1}{2}d = 0.25$ in

$$T_A = \frac{J\tau}{c} = \frac{\pi}{2}c^3\tau = \frac{\pi}{2}(0.25)^3(8) = 0.19635 \text{ kip-in}$$

Shaft B: $\tau = 8$ ksi $\quad c = \frac{1}{2}d = 0.3125$ in

$$T_B = \frac{J\tau}{c} = \frac{\pi}{2}c^3\tau = \frac{\pi}{2}(0.3125)^3(8) = 0.3835 \text{ kip·in}$$

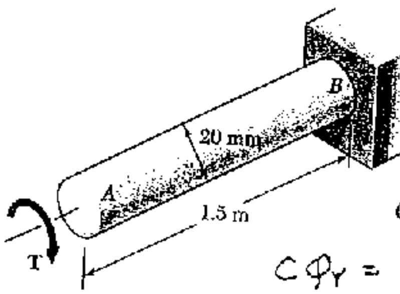
From Statics: $T_A = \frac{r_A}{r_B} T_B = (\tan\alpha) T_B = (\tan 18.43°)(0.3885)$

$= 0.12779$ kip·in

Allowable value of T_A is the smaller

$T_A = 0.1278$ kip·in $= \qquad T_A = 127.8$ Kip-in

Problem 3.148



3.148 A torque **T** is applied to the 20-mm-diameter steel rod AB. Assuming that the steel is elastoplastic with $G = 77.2$ GPa and $\tau_Y = 145$ MPa, determine (*a*) the torque **T** when the angle of twist at A is 25°, (*b*) the corresponding diameter of the elastic core of the shaft.

$c = \frac{1}{2}d = 0.010$ m, $L = 1.5$ m, $G = 77 \times 10^9$ Pa

$c\varphi_Y = L\gamma_Y = \frac{L\tau_Y}{G}$ $\qquad \varphi_Y = \frac{L\tau_Y}{Gc}$

$$\varphi_Y = \frac{(1.5)(145 \times 10^6)}{(77.2 \times 10^9)(0.010)} = 281.74 \times 10^{-3} \text{ rad}$$

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(0.010)^3(145 \times 10^6) = 227.77 \text{ N·m}$$

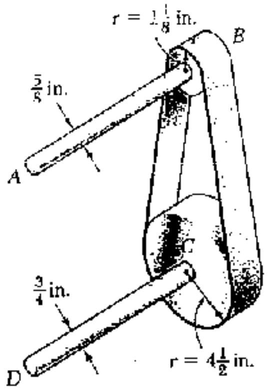
$$\varphi = 25° = 436.33 \times 10^{-3} \text{ rad} > \varphi_Y \qquad \frac{\varphi_Y}{\varphi} = \frac{281.74 \times 10^{-3}}{436.33 \times 10^{-3}} = 0.64569$$

(a) $T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\varphi_Y^3}{\varphi^3}\right) = \frac{4}{3}(227.77)\left[1 - \frac{1}{4}(0.64569)^3\right]$ $\qquad T = 283$ N·m ◄

(b) $\frac{\rho_Y}{c} = \frac{\varphi_Y}{\varphi} = 0.64569$ $\qquad \rho_Y = 0.64737\, c = (0.64737)(0.010)$

$\rho_Y = 6.4569 \times 10^{-3}$ m $= 6.4569$ mm $\qquad d_Y = 2\rho_Y = 12.91$ mm ◄

Problem 3.149



3.149 The shaft-disk-belt arrangement shown is used to transmit 3 hp from point A to point D. (a) Using an allowable shearing stress of 9500 psi, determine the required speed of shaft AB. (b) Solve part a, assuming that the diameters of shafts AB and CD are respectively 0.75 in. and 0.625 in.

SOLUTION

$\tau = 9500$ psi, $\quad P = 3\text{ hp} = (3)(6600) = 19800$ lb·in/s

$\tau = \dfrac{Tc}{J} = \dfrac{2T}{\pi c^3} \qquad T = \dfrac{\pi}{2} c^3 \tau$

Allowable torques

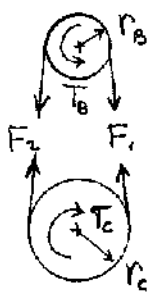
$\frac{5}{8}$ in. diameter shaft

$c = \frac{5}{16}$ in, $\quad T_{all} = \frac{\pi}{2}\left(\frac{5}{16}\right)^3 (9500) = 455.4$ lb·in

$\frac{3}{4}$ in diameter shaft

$c = \frac{3}{8}$ in, $\quad T_{all} = \frac{\pi}{2}\left(\frac{3}{8}\right)^3 (9500) = 786.9$ lb·in

Statics:



$T_B = r_B (F_1 - F_2) \qquad T_C = r_C (F_1 - F_2)$

$T_B = \dfrac{r_B}{r_C} T_C = \dfrac{1.125}{4.5} T_C = 0.25\, T_C$

(a) Allowable torques $\quad T_{B,all} = 455.4$ lb.in, $\quad T_{C,all} = 786.9$ lb·in

Assume $T_C = 786.9$ lb·in. Then $T_B = (0.25)(786.9) = 196.73$ lb.in < 455.4 lb in (okay)

$P = 2\pi f T \qquad f_{AB} = \dfrac{P}{2\pi T_B} = \dfrac{19800}{2\pi(196.73)} \qquad f_{AB} = 16.02$ Hz

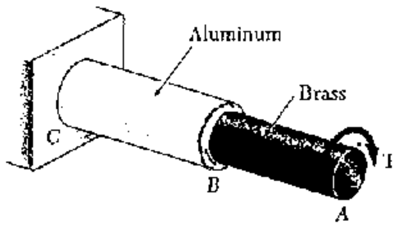
(b) Allowable torques $\quad T_{B,all} = 786.9$ lb·in, $\quad T_{C,all} = 455.4$ lb·in.

Assume $T_C = 455.4$ lb·in $\quad$ Then $T_B = (0.25)(455.4) = 113.85$ lb·in < 786.9 lb·in

$P = 2\pi f T \qquad f_{AB} = \dfrac{P}{2\pi T_B} = \dfrac{19800}{2\pi(113.85)} \qquad f_{AB} = 27.2$ Hz

Problem 3.150

3.150 The allowable stress is 50 MPa in the brass rod AB and 25 MPa in the aluminum rod BC. Knowing that a torque of magnitude $T = 1250$ N · m is applied at A, determine the required diameter (a) of rod AB, (b) of rod BC.



$$\tau_{max} = \frac{Tc}{J} \qquad J = \frac{\pi}{2}c^4 \qquad c^3 = \frac{2T}{\pi \tau_{max}}$$

Rod AB: $c^3 = \frac{(2)(1250)}{\pi(50 \times 10^6)} = 15.915 \times 10^{-6}\ m^3$

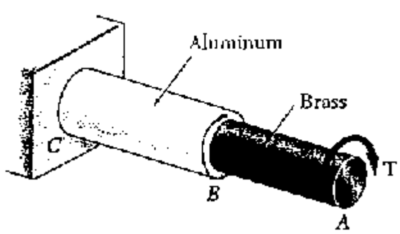
$c = 25.15 \times 10^{-3}$ m $= 25.15$ mm $\qquad d_{AB} = 2c = 50.3$ mm ◄

Rod BC: $c^3 = \frac{(2)(1250)}{\pi(25 \times 10^6)} = 31.831 \times 10^{-6}\ m^3$

$c = 31.69 \times 10^{-3}$ m $= 31.69$ mm $\qquad d_{BC} = 2c = 63.4$ mm ◄

Problem 3.151

3.151 The *solid* rod BC has a diameter of 30 mm and is made of an aluminum for which the allowable shearing stress is 25 MPa. Rod AB is *hollow* and has an outer diameter of 25 mm; it is made of a brass for which the allowable shearing stress is 50 MPa. Determine (a) the largest inner diameter of rod AB for which the factor of safety is the same for each rod, (b) the largest torque that can be applied at A.



Solid rod BC: $\tau = \frac{Tc}{J} \qquad J = \frac{\pi}{2}c^4$

$\tau_{all} = 25 \times 10^6$ Pa $\qquad c = \frac{1}{2}d = 0.015$ m

$T_{all} = \frac{\pi}{2}c^3\tau_{all} = \frac{\pi}{2}(0.015)^3(25 \times 10^6) = 132.536$ N·m

Hollow rod AB: $\tau_{all} = 50 \times 10^6$ Pa $\qquad T_{all} = 132.536$ N·m

$c_2 = \frac{1}{2}d_2 = \frac{1}{2}(0.025) = 0.0125$ m $\qquad c_1 = ?$

$$T_{all} = \frac{J\tau_{all}}{c_2} = \frac{\pi}{2}(c_2^4 - c_1^4)\frac{\tau_{all}}{c_2}$$

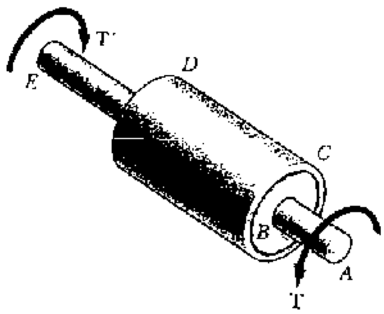
$$c_1^4 = c_2^4 - \frac{2T_{all}c_2}{\pi\tau_{all}} = 0.0125^4 - \frac{(2)(132.536)(0.0125)}{\pi(50 \times 10^6)} = 3.3203 \times 10^{-9}\ m^4$$

(a) $c_1 = 7.59 \times 10^{-3}$ m $= 7.59$ mm $\qquad d_1 = 2c_1 = 15.18$ mm ◄

(b) Allowable torque $\qquad T_{all} = 132.5$ N·m ◄

Problem 3.152

3.152 The steel jacket CD has been attached to the 40-mm-diameter steel shaft AE by means of *rigid* flanges welded to the jacket and to the rod. The outer diameter of the jacket is 80 mm and its wall thickness is 4 mm. If 500 N · m torques are applied as shown, determine the maximum shearing stress in the jacket.



SOLUTION

Solid shaft: $c = \frac{1}{2}d = 0.020 \text{ m}$

$$J_S = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.020)^4 = 251.33 \times 10^{-9} \text{ m}^4$$

Jacket: $c_2 = \frac{1}{2}d = 0.040 \text{ m}$

$$c_1 = c_2 - t = 0.040 - 0.004 = 0.036 \text{ m}$$

$$J_J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.040^4 - 0.036^4) = 1.3829 \times 10^{-6} \text{ m}^4$$

Torque carried by shaft $\quad T_S = GJ_S\varphi/L$

Torque carried by jacket $\quad T_J = GJ_J\varphi/L$

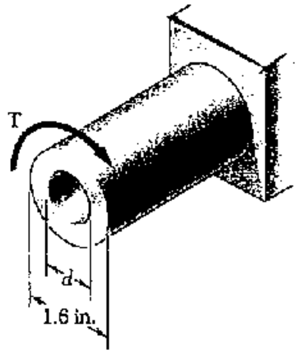
Total torque $\quad T = T_S + T_J = (J_S + J_J)G\varphi/L \quad \therefore \quad \frac{G\varphi}{L} = \frac{T}{J_S + J_J}$

$$T_J = \frac{J_J}{J_S + J_J}T = \frac{(1.3829\times10^{-6})(500)}{1.3829\times10^{-6} + 251.33\times10^{-6}} = 423.1 \text{ N}\cdot\text{m}$$

Maximum shearing stress in jacket

$$\tau = \frac{T_J c}{J_c} = \frac{(423.1)(0.040)}{1.3829\times10^{-6}} = 12.24\times10^{6} \text{ Pa} \qquad 12.24 \text{ MPa}$$

Problem 3.153



3.153 Knowing that the internal diameter of the hollow shaft shown is $d = 0.9$ in., determine the maximum shearing stress caused by a torque of magnitude $T = 9$ kip · in.

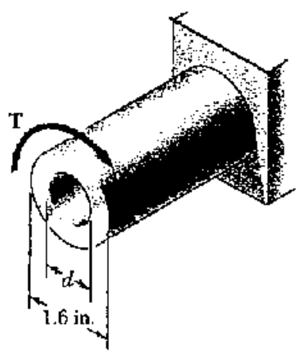
$c_2 = \frac{1}{2} d_2 = (\frac{1}{2})(1.6) = 0.8$ in. $\qquad c = 0.8$ in.

$c_1 = \frac{1}{2} d_1 = (\frac{1}{2})(0.9) = 0.45$ in

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.8^4 - 0.45^4) = 0.5790 \text{ in}^4$

$\tau_{max} = \frac{Tc}{J} = \frac{(9)(0.8)}{0.5790} = 12.44$ ksi ◄

Problem 3.154



3.154 Knowing that $d = 1.2$ in., determine the torque **T** that causes a maximum shearing stress of 7.5 ksi in the hollow shaft shown.

$c_2 = \frac{1}{2} d_2 = (\frac{1}{2})(1.6) = 0.8$ in $\qquad c = 0.8$ in

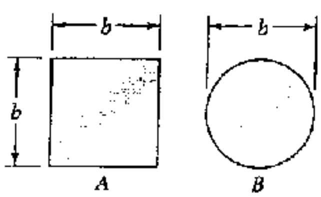
$c_1 = \frac{1}{2} d_1 = (\frac{1}{2})(1.2) = 0.6$ in

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.8^4 - 0.6^4) = 0.4398 \text{ in}^4$

$\tau_{max} = \frac{Tc}{J}$

$T = \frac{J\tau_{max}}{c} = \frac{(0.4398)(7.5)}{0.8} = 4.12$ kip·in ◄

Problem 3.155



3.155 Two shafts are made of the same material. The cross section of shaft *A* is a square of side *b* and that of shaft *B* is circle of diameter *b*. Knowing that the shafts re subjected to the same torque, determine the ratio τ_A / τ_B of the maximum shearing stresses occurring in the shafts.

A. square $\quad \frac{a}{b} = 1, \quad c_1 = 0.208$ (Table 3.1)

$\tau_A = \frac{T}{c_1 a b^2} = \frac{T}{0.208\, b^3}$

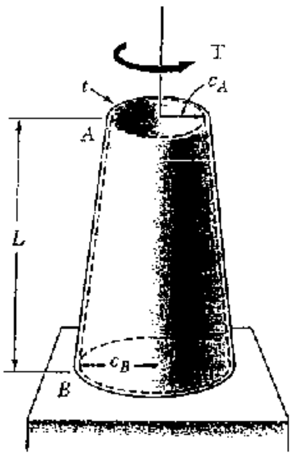
B. circle $\quad c = \frac{1}{2} b \qquad \tau_B = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{16\, T}{\pi b^3}$

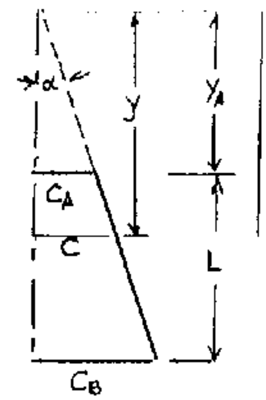
Ratio $\quad \frac{\tau_A}{\tau_B} = \frac{1}{0.208} \cdot \frac{\pi}{16} = 0.3005\,\pi \qquad \frac{\tau_A}{\tau_B} = 0.944$ ◄

Problem 3.156

3.156 The long, hollow, tapered shaft AB has a uniform thickness t. Denoting by G the modulus of rigidity, show that the angle of twist at end A is

$$\phi_A = \frac{TL}{4\pi G t}\frac{c_A + c_B}{c_A^2 c_B^2}$$





From geometry

$$\tan\alpha = \frac{c_B - c_A}{L}$$

$$c = y\tan\alpha = \frac{c_B - c_A}{L}y$$

$$y_A = \frac{c_A}{\tan\alpha} = \frac{c_A L}{c_B - c_A}$$

$$y_B = \frac{c_B}{\tan\alpha} = \frac{c_B L}{c_B - c_A}$$

$$J = 2\pi c^3 t = 2\pi\frac{(c_B - c_A)^3}{L^3}y^3 t$$

$$\varphi = \int_{y_A}^{y_B}\frac{T\,dy}{GJ} = \frac{TL^3}{2\pi(c_B-c_A)^3 tG}\int_{y_A}^{y_B}\frac{dy}{y^3} = \frac{TL^3}{2\pi(c_B-c_A)^3 tG}\left(-\frac{1}{2y^2}\Big|_{y_A}^{y_B}\right)$$

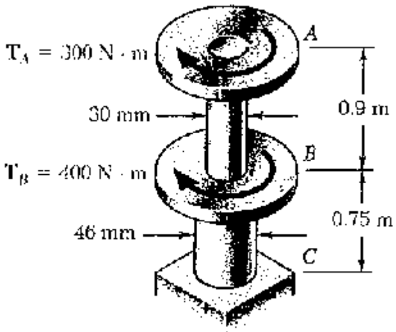
$$= \frac{TL^3}{4\pi(c_B-c_A)^3 tG}\left\{\frac{1}{y_A^2} - \frac{1}{y_B^2}\right\} = \frac{TL^3}{4\pi(c_B-c_A)^3 tG}\cdot\left\{\frac{(c_B-c_A)^2}{L^2c_A^2} - \frac{(c_B-c_A)^2}{L^2c_B^2}\right\}$$

$$= \frac{TL}{4\pi(c_B-c_A)tG}\left\{\frac{1}{c_A^2} - \frac{1}{c_B^2}\right\} = \frac{TL\,(c_B^2 - c_A^2)}{4\pi(c_B-c_A)tGc_A^2c_B^2}$$

$$= \frac{TL\,(c_B + c_A)}{4\pi G t\, c_A^2 c_B^2}$$ ◄

Problem 3.157

3.157 The torques shown are exerted on pulleys A and B. Knowing that the steel shafts are soli' and that $G = 77.2$ GPa, determine the angle of twist between (a) A and B, (b) A and C.



(a) $T_{AB} = 300$ N·m, $L_{AB} = 0.9$ m, $c_{AB} = \frac{1}{2}d = 0.015$ m

$$J_{AB} = \frac{\pi}{2}(0.015)^4 = 79.522 \times 10^{-9}\ \text{m}^4$$

$$\varphi_{AB} = \frac{T_{AB}L_{AB}}{GJ_{AB}} = \frac{(300)(0.9)}{(77.2\times10^9)(79.522\times10^{-9})}$$

$$= 43.98\times10^{-3}\ \text{rad}$$

$\varphi_{AB} = 2.52°$ ◄

(b) $T_{BC} = 300 + 400 = 700$ N·m, $L_{BC} = 0.75$ m, $c_{BC} = \frac{1}{2}d = 0.023$ m

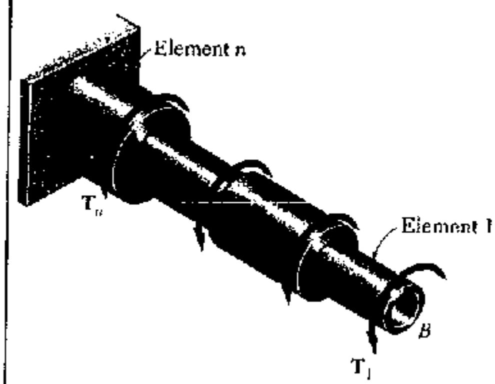
$$J_{BC} = \frac{\pi}{2}(0.023)^4 = 439.573\times10^{-9}\ \text{m}^4$$

$$\varphi_{BC} = \frac{T_{BC}L_{BC}}{GJ_{BC}} = \frac{(700)(0.75)}{(77.2\times10^9)(439.573\times10^{-9})} = 15.47\times10^{-3}\ \text{rad}$$

$$\varphi_{AC} = \varphi_{AB} + \varphi_{BC} = 59.45\times10^{-3}\ \text{rad}$$

$\varphi_{AC} = 3.41°$ ◄

PROBLEM 3.C1



3.C1 Shaft AB consists of n homogeneous cylindrical elements, which can be solid or hollow. Its end A is fixed, while its end B is free, and it is subjected to the loading shown. The length of element i is denoted by L_i, its outer diameter by OD_i, its inner diameter by ID_i, its modulus of rigidity by G_i, and the torque applied to its right end by $\mathbf{T}_i$, the magnitude T_i of this torque being assumed to be positive if $\mathbf{T}_i$ is observed as counterclockwise from end B and negative otherwise. (Note that $ID_i = 0$ if the element is solid.) (*a*) Write a computer program that can be used to determine the maximum shearing stress in each element, the angle of twist of each element, and the angle of twist of the entire shaft. (*b*) Use this program to solve Probs. 3.36, 3.37, and 3.157.
3.151.

SOLUTION

FOR EACH CYLINDRICAL ELEMENT, ENTER

$L_i\ ,\ OD_i\ ,\ ID_i\ ,\ G_i\ ,\ T_i$

AND COMPUTE

$$J_i = (\pi/32)(OD_i^4 - ID_i^4)$$

OUTLINE OF PROGRAM

UPDATE TORQUE $\quad T = T + T_i$

AND COMPUTE

$$TAU_i = T\,(OD_i/2)/J_i$$

$$PHI_i = T L_i / G_i J_i$$

ANGLE OF TWIST OF ENTIRE SHAFT, STARTING WITH $\Theta = 0$, UPDATE THROUGH n^{th} ELEMENT

$$\Theta = \Theta + PHI_i$$

PROGRAM OUTPUT

Problem 3.36

Element	Maximum Stress (MPa)	Angle of Twist (degrees)
1.0000	11.9575	1.3841
2.0000	23.0259	1.8323

Angle of twist for entire shaft = 3.2164 °

Problem 3.37

Element	Maximum Stress (MPa)	Angle of Twist (degrees)
1.0000	87.3278	4.1181
2.0000	56.5884	1.0392
3.0000	70.5179	0.8633

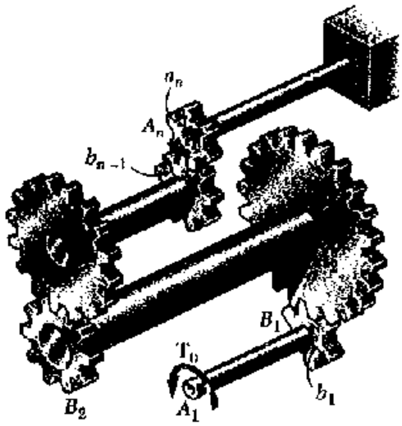
Angle of twist for entire snaft = 6.0206 °

Problem 3.157

Element	Maximum Stress (MPa)	Angle of Twist (degrees)
1.0000	56.5884	2.5199
2.0000	36.6264	0.8864

Angle of twist for entire shaft = 3.4063 °

PROBLEM 3.C2



3.C2 The assembly shown consists of n cylindrical shafts, which can be solid or hollow, connected by gears and supported by brackets (not shown). End A_1 of the first shaft is free and is subjected to a torque $\mathbf{T}_0$, while end B_n of the last shaft is fixed. The length of shaft A_iB_i is denoted by L_i, its outer diameter by OD_i, its inner diameter by ID_i, and its modulus of rigidity by G_i. (Note that $ID_i = 0$ if the element is solid.) The radius of gear A_i is denoted by a_i, and the radius of gear B_i by b_i. (a) Write a computer program that can be used to determine the maximum shearing stress in each shaft, the angle of twist of each shaft, and the angle through which end A_i rotates. (*b*) Use this program to solve Probs. 3.42, 3.43, and 3.44.

SOLUTION

TORQUE IN SHAFTS. ENTER $T_i = T_0$

$$T_{i+1} = T_i\,(A_{i+1}/B_i)$$

FOR EACH SHAFT, ENTER

$L_i \quad OD_i \quad ID_i \quad G_i$

COMPUTE: $J_i = (\pi/32)(OD_i^4 - ID_i^4)$

$$TAU_i = T_i\,(OD_i/2)/J_i$$

$$PHI_i = T_i\,L_i/G_i\,J_i$$

ANGLE OF ROTATION AT END A_1

COMPUTE ROTATION AT THE "A" END OF EACH SHAFT

START WITH ANGLE = PHI_n AND UPDATE

FROM n TO 1, AND ADD PHI_i

$$ANGLE = ANGLE\,(A_i)/B_{i-1} + PHI_{i-1}$$

PROGRAM OUTPUT

Problem 3.42

Shaft No.	Max Stress) (ksi)	Angle of Twist (degrees)
1	104.31	40.979
2	52.15	20.490
3	26.08	10.245

Angle through which A1 rotates = 53.785°

Problem 3.43

Shaft No.	Max Stress (ksi)	Angle of Twist (degrees)
1	9.29	1.493
2	12.16	1.707

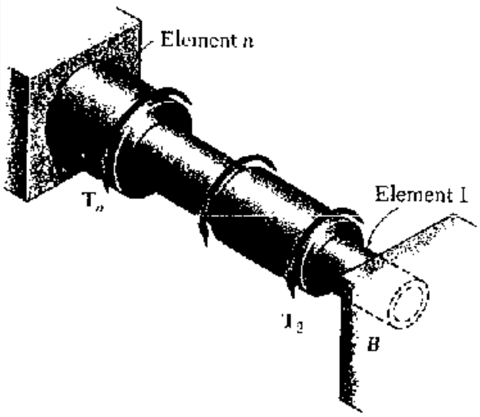
Angle through which A1 rotates = 3.769°

Problem 3.44

Shaft No.	Max Stress) (ksi	Angle of Twist (degrees)
1	9.12	1.493
2	6.84	0.960

Angle through which A1 rotates = 2.213°

PROBLEM 3.C3



3.C3 Shaft *AB* consists of *n* homogeneous cylindrical elements, which can be solid or hollow. Both of its ends are fixed, and it is subjected to the loading shown. The length of element *i* is denoted by L_i, its outer diameter by OD_i, its inner diameter by ID_i, its modulus of rigidity by G_i, and the torque applied to its right end by $\mathbf{T}_i$, the magnitude T_i of this torque being assumed to be positive if $\mathbf{T}_i$ is observed as counterclockwise from end *B* and negative otherwise. Note that $ID_1 = 0$ if the element is solid and also that $T_1 = 0$. Write a computer program that can be used to determine the reactions at *A* and *B*, the maximum shearing stress in each element, and the angle of twist of each element. Use this program (*a*) to solve Prob. 3.56, (*b*) to determine the maximum shearing stress in the shaft of Example 3.05.

SOLUTION WE CONSIDER THE REACTION AT B AS REDUNDANT AND RELEASE THE SHAFT AT B.

COMPUTE Θ_B WITH $T_B = 0$:

FOR EACH ELEMENT, ENTER

L_i, OD_i, ID_i, G_i, T_i (NOTE $T_1 = T_B = 0$)

COMPUTE

$$J_i = (\pi/32)(OD_i^4 - ID_i^4)$$

UPDATE TORQUE

$$T = T + T_i$$

AND COMPUTE FOR EACH ELEMENT

$$TAU_i = T(OD_i/2)/J_i$$

$$PHI_i = T L_i / G_i J_i$$

COMPUTE Θ_B: STARTING WITH $\Theta = 0$ AND UPDATING THROUGH n ELEMENTS

$$\Theta_i = \Theta_i + PHI_i \qquad ; \qquad \Theta_B = \Theta_n$$

COMPUTE Θ_B DUE TO UNIT TORQUE AT B

$$UNIT\ TAU_i = OD_i/2J_i$$

$$UNIT\ PHI_i = L_i/G_i J_i$$

FOR n ELEMENTS:

$$UNIT\Theta_B(i) = UNIT\Theta_B(i) + UNIT\ PHI_i$$

SUPERPOSITION:

FOR TOTAL ANGLE AT B TO BE ZERO. $\Theta_B + T_B(UNIT\Theta_B(n)) = 0$

$$T_B = -\Theta_B / (UNIT\Theta_B(n))$$

THEN $T_A = \sum T(i) + T_B$

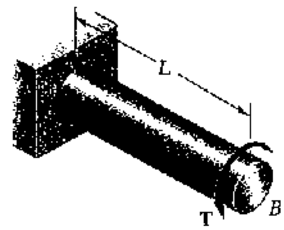
FOR EACH ELEMENT: MAX STRESS: $TOTAL\ TAU_i = TAU_i + T_B(UNIT\ TAU_i)$

ANGLE OF TWIST: $TOTAL\ PHI_i = PHI_i + T_B(UNIT\ PHI_i)$

PROGRAM OUTPUT

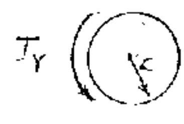
```
Problem 3.56            TA =    -0.295 kN*m
                        TB =    -1.105 kN*m

Element      tau max         Angle of Twist
              (MPa)            (degrees)
   1         -45.024            -0.267
   2          27.375             -.267

Problem 3.05         TA =  -51.733 lb*ft
                     TB =  -38.267 lb*ft
```

PROBLEM 3.C4



3.C4 The homogeneous, solid cylindrical shaft AB has a length L, a diameter d, a modulus of rigidity G, and a yield strength τ_Y. It is subjected to a torque $\mathbf{T}$ that is gradually increased from zero until the angle of twist of the shaft has reached a maximum value ϕ_m and then decreased back to zero. (*a*) Write a computer program that, for each of 16 values of ϕ_m equally spaced over a range extending from 0 to a value 3 times as large as the angle of twist at the onset of yield, can be used to determine the maximum value T_m of the torque, the radius of the elastic core, the maximum shearing stress, the permanent twist, and the residual shearing stress both at the surface of the shaft and at the interface of the elastic core and the plastic region. (*b*) Use this program to obtain approximate answers to Probs. 3.111, 3.112, 3.114.

SOLUTION

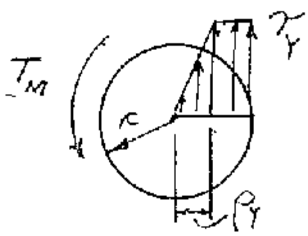


AT ONSET OF YIELD

$$T_Y = \tau_Y \frac{J}{c} = \frac{\pi}{2}\tau_Y c^3$$

$$\phi_Y = \frac{T_Y L}{GJ} = \left(\frac{\tau_Y J}{c}\right)\frac{L}{GJ} = \frac{\tau_Y L}{cG}$$

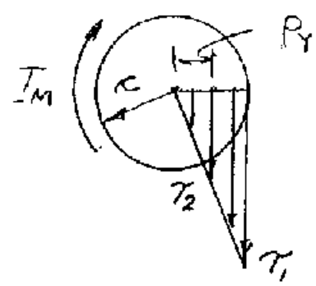
LOADING: $T_M > T_Y$



$$T_M = \frac{4}{3} T_M\left[1 - \frac{1}{4}\left(\frac{\phi_Y}{\phi_M}\right)^3\right] \qquad \text{EQ. (1)}$$

$$\rho_Y = c\,\frac{\phi_Y}{\phi_M} \qquad \text{EQ. (2)}$$

UNLOADING (ELASTIC)



$$\phi_U = \frac{T_M L}{GJ} \qquad \phi_U = \text{ ANGLE OF TWIST FOR UNLOADING}$$

$$\tau_1 = T_M \frac{c}{J} \qquad \tau_1 = \text{ TAU AT } \rho = c$$

$$\tau_2 = \tau_1 \frac{\rho_Y}{c} \qquad \tau_2 = \text{ TAU AT } \rho = \rho_Y$$

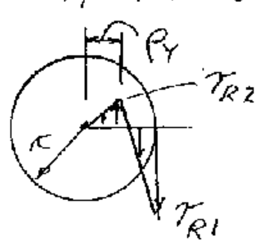
SUPERPOSE LOADING AND UNLOADING

FOR $\phi = 0$ TO $\phi = 3\phi_Y$ USING $0.2\phi_Y$ INCREMENTS

WHEN $\phi < \phi_Y$: $\quad T_M = T_Y \dfrac{\phi}{\phi_Y} \quad \rho_Y = \frac{1}{2}d \quad \phi_M = \phi_Y \dfrac{\phi}{\phi_Y}$

WHEN $\phi > \phi_Y$: $\quad T_M$ USE EQ(1) $\quad \rho_Y$ USE EQ(2)

RESIDUAL: $\quad \phi_p = \phi_M - \phi_U \quad \tau_{R1} = \tau_1 - \tau_Y \quad \tau_{R2} = \tau_2 - \tau_Y$



CONTINUED

PROBLEM 3.C4 - CONTINUED

Interpolate between values at the values of T_{max} or ϕ_{max} indicated

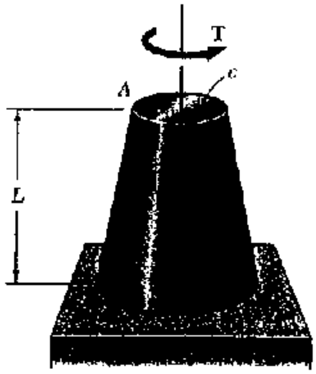
Problem 3.111 and 3.112

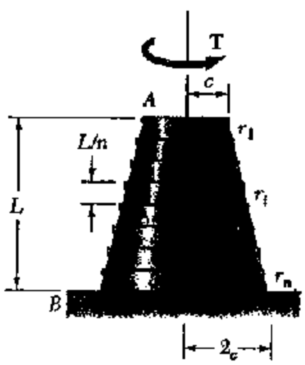
PHIM deg	TM kip*in.	RY in.	TAUM ksi	PHIP deg	TAUR1 ksi	TAUR2 ksi	
0.000	0.000	1.200	0.000	0.000	0.000	0.000	
7.878	11.943	1.200	4.400	0.000	0.000	0.000	
15.756	23.886	1.200	8.800	0.000	0.000	0.000	
23.635	35.829	1.200	13.200	0.000	0.000	0.000	
31.513	47.772	1.200	17.600	0.000	0.000	0.000	
39.391	59.715	1.200	22.000	0.000	0.000	0.000	
47.269	68.101	1.000	22.000	2.346	1.092	0.000	
55.147	72.366	0.857	22.000	7.411	2.957	-3.090	
63.025	74.761	0.750	22.000	13.710	4.786	-4.661	
70.904	76.207	0.667	22.000	20.634	6.402	-5.543	
78.782	77.132	0.600	22.000	27.902	7.792	-6.076	← $T_{MAX} = 75$ kip·in.
86.660	77.751	0.545	22.000	35.372	8.980	-6.417	
94.538	78.181	0.500	22.000	42.967	9.999	-6.645	
102.416	78.488	0.462	22.000	50.642	10.878	-6.803	
110.294	78.714	0.429	22.000	58.371	11.643	-6.916	
118.173	78.883	0.400	22.000	66.138	12.313	-6.999	
						-7.062	

Problem 3.114

PHIM deg	TM kN*m	RY mm	TAUM MPa	PHIP deg	TAUR1 MPa	TAUR2 MPa	
0.000	0.000	16.000	0.000	0.000	0.000	0.000	
0.807	0.187	16.000	29.000	0.000	0.000	0.000	
1.614	0.373	16.000	58.000	0.000	0.000	0.000	
2.421	0.560	16.000	87.000	0.000	0.000	0.000	
3.228	0.746	16.000	116.000	0.000	0.000	0.000	
4.036	0.933	16.000	145.000	0.000	0.000	0.000	
4.843	1.064	13.333	145.000	0.000	0.000	0.000	
5.650	1.131	11.429	145.000	0.240	7.198	-20.363	
6.457	1.168	10.000	145.000	0.759	19.486	-30.719	← $\phi_{MAX} = 6°$
7.264	1.191	8.889	145.000	1.405	31.542	-36.533	
8.071	1.205	8.000	145.000	2.114	42.197	-40.046	
8.878	1.215	7.273	145.000	2.859	51.354	-42.292	
9.685	1.221	6.667	145.000	3.624	59.184	-43.794	
10.492	1.226	6.154	145.000	4.402	65.901	-44.837	
11.300	1.230	5.714	145.000	5.188	71.699	-45.583	
12.107	1.232	5.333	145.000	5.980	76.739	-46.132	
				6.776	81.152	-46.543	

PROBLEM 3.C5





3.C5 The exact expression is given in Prob. 3.64 for the angle of twist of the solid tapered shaft AB when a torque $\mathbf{T}$ is applied as shown. Derive an approximate expression for the angle of twist by replacing the tapered shaft by n cylindrical shafts of equal length and of radius $r_i = (n + i - \frac{1}{2})(c/n)$, where $i = 1, 2, \ldots, n$. Using for T, L, G, and c values of your choice, determine the percentage error in the approximate expression when (a) $n = 4$, (b) $n = 8$, (c) $n = 20$, (d) $n = 100$.

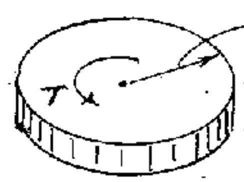
SOLUTION

FROM PROB. 3.64 EXACT EXPRESSION:

$$\phi = \frac{7TL}{12\pi G c^4}$$

$$\text{OR,} \quad \phi = \left(\frac{7}{12\pi}\right)\frac{TL}{Gc^4} = 0.18568\,\frac{TL}{Gc^4}$$

CONSIDER TYPICAL ith SHAFT



$$J_i = \frac{\pi}{2}(r_i)^4$$

$$\Delta\phi = \frac{T(L/n)}{G J_i}$$

ENTER UNIT VALUES OF T, L, G, AND c.
(NOTE: SPECIFIC VALUES CAN BE ENTERED)

ENTER INITIAL VALUE OF ZERO FOR ϕ
ENTER n = NUMBER CYLINDRICAL SHAFTS

FOR $i = 1$ TO n, UPDATE ϕ

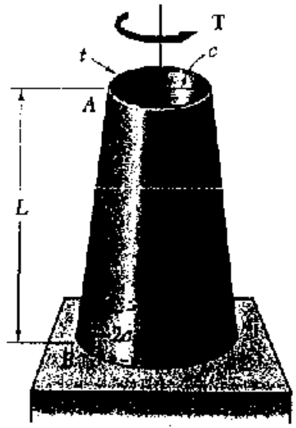
$$\phi = \phi + \Delta\phi$$

OUTPUT OF PROGRAM

```
              COEFFICIENT of TL/Gc^4

    Exact coefficient from Prob. 3.64 is 0.18568
           Number of elemental disks = n
```

n	approximate	exact	percent error
4	0.17959	0.18568	-3.28185
8	0.18410	0.18568	-0.85311
20	0.18542	0.18568	-0.13810
100	0.18567	0.18568	-0.00554

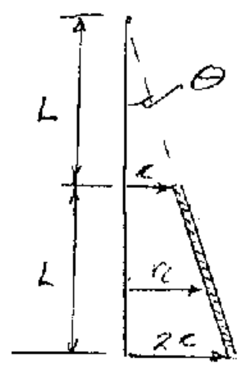
PROBLEM 3.C6



3.C6 A torque **T** is applied as shown to the long, hollow, tapered shaft AB of uniform thickness t. The exact expression for the angle of twist of the shaft can be obtained from the expression given in Prob. 3.153. Derive an approximate expression for the angle of twist by replacing the tapered shaft by n cylindrical rings of equal length and of radius $r_i = (n + i - \frac{1}{2})(c/n)$, where $i = 1, 2, \ldots, n$. Using for T, L, G, c and t values of your choice, determine the percentage error in the approximate expression when (a) $n = 4$, (b) $n = 8$, (c) $n = 20$, (d) $n = 100$.

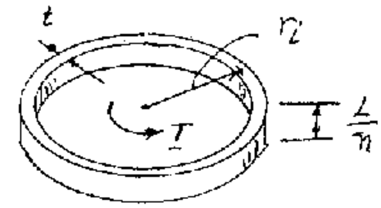
SOLUTION

SINCE THE SHAFT IS LONG $c \ll L$, THE ANGLE θ IS SMALL AND WE CAN USE t AS THE THICKNESS OF THE n CYLINDRICAL RINGS.



FOR $c \ll L$

$$\theta \approx \tan\theta = \frac{2c - c}{L} = \frac{c}{L}$$



$$r_i = (n + i - \tfrac{1}{2})(\tfrac{c}{n})$$

$$J_i \approx (\text{AREA})\, r_i^2 = (2\pi r_i t) r_i^2 = 2\pi t r_i^3$$

$$\Delta\phi = \frac{T(L/n)}{G J_i}$$

ENTER UNIT VALUES FOR T, L, G, t, AND c

(NOTE: SPECIFIC VALUES CAN BE ENTERED IF DESIRED)

ENTER INITIAL VALUE OF ZERO FOR ϕ

ENTER n = NUMBER OF CYLINDRICAL RINGS

FOR $i = 1$ TO n, UPDATE ϕ

$$\phi = \phi + \Delta\phi$$

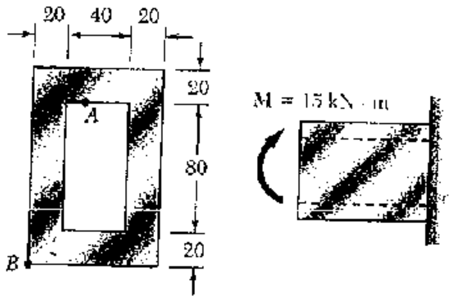
OUTPUT OF PROGRAM

```
                COEFFICIENT of TL/Gtc^3

     Exact coefficient from Prob. 3.153 is 0.05968
             Number of elemental disks = n

      n      approximate       exact        percent error

      4       0.058559       0.059683        -1.883078
      8       0.059394       0.059683        -0.483688
     20       0.059637       0.059683        -0.078022
    100       0.059681       0.059683        -0.003127
```

Chapter 4

Problem 4.1

4.1 and 4.2 Knowing that the couple shown acts in a vertical plane, determine the stress at (*a*) point *A*, (*b*) point *B*.



Dimensions in mm

For rectangle $I = \frac{1}{12} b h^3$

Outside rectangle: $I_1 = \frac{1}{12}(80)(120)^3$

$I_1 = 11.52 \times 10^6 \text{ mm}^4 = 11.52 \times 10^{-6} \text{ m}^4$

Cutout: $I_2 = \frac{1}{12}(40)(80)^3$

$I_2 = 1.70667 \times 10^6 \text{ mm}^4 = 1.70667 \times 10^{-6} \text{ m}^4$

Section: $I = I_1 - I_2 = 9.81333 \times 10^{-6} \text{ m}^4$

(a) $y_A = 40 \text{ mm} = 0.040 \text{ m}$ $\sigma_A = -\frac{M y_A}{I} = -\frac{(15 \times 10^3)(0.040)}{9.81333 \times 10^{-6}} = -61.6 \times 10^6 \text{ Pa}$

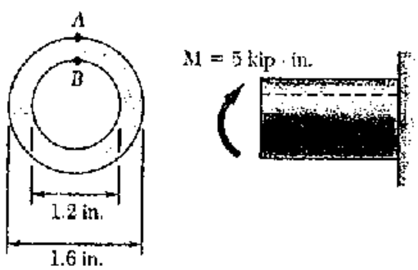
$\sigma_A = -61.6 \text{ MPa}$ ◄

(b) $y_B = -60 \text{ mm} = -0.060 \text{ m}$ $\sigma_B = -\frac{M y_B}{I} = -\frac{(15 \times 10^3)(-0.060)}{9.81333 \times 10^{-6}} = 91.7 \times 10^6 \text{ Pa}$

$\sigma_B = 91.7 \text{ MPa}$ ◄

Problem 4.2

4.1 and 4.2 Knowing that the couple shown acts in a vertical plane, determine the stress at (*a*) point *A*, (*b*) point *B*.



$r_o = \frac{1}{2}(1.6) = 0.8 \text{ in.}$ $r_i = \frac{1}{2}(1.2) = 0.6 \text{ in.}$

$I = \frac{\pi}{4}(r_o^4 - r_i^4) = \frac{\pi}{4}(0.8^4 - 0.6^4)$

$= 0.21991 \text{ in}^4$

(a) $y_A = 0.8 \text{ in.}$ $\sigma_A = -\frac{M y_A}{I} = -\frac{(5 \times 10^3)(0.8)}{0.21991} = -18.19 \times 10^3 \text{ psi}$

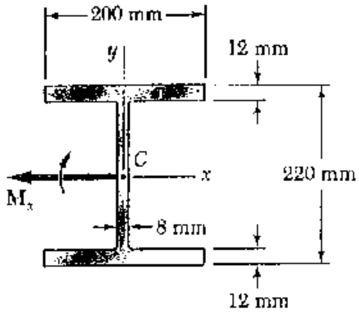
$\sigma_A \; -18.19 \text{ ksi}$ ◄

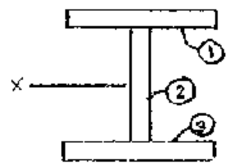
(b) $y_B = 0.6 \text{ in.}$ $\sigma_B = -\frac{M y_B}{I} = -\frac{(5 \times 10^3)(0.6)}{0.21991} = -13.64 \times 10^3 \text{ psi}$

$\sigma_B = -13.64 \text{ ksi}$ ◄

Problem 4.3

4.3 Using an allowable stress of 155 MPa, determine the largest bending moment M that can be applied to the wide flange beam shown. Neglect the effect of fillets.





Moment of inertia about x-axis

$I_1 = \frac{1}{12}(200)(12)^3 + (200)(12)(104)^2$
$= 25.9872 \times 10^6 \text{ mm}^4$

$I_2 = \frac{1}{12}(8)(196)^3 = 5.0197 \times 10^6 \text{ mm}^4$

$I_3 = I_1 = 25.9872 \times 10^6 \text{ mm}^4$

$I = I_1 + I_2 + I_3 = 56.944 \times 10^6 \text{ mm}^4 = 56.944 \times 10^{-6} \text{ m}^4$

$\sigma = \frac{Mc}{I}$ with $c = \frac{1}{2}(220) = 110 \text{ mm} = 0.110 \text{ m}$

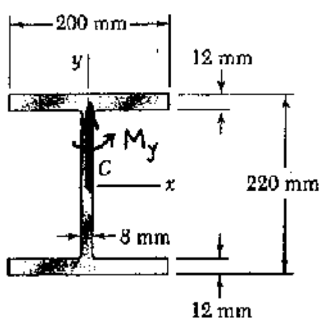
$M = \frac{I\sigma}{c}$ with $\sigma = 155 \times 10^6 \text{ Pa}$

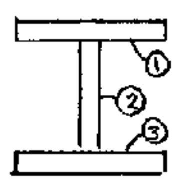
$M_x = \frac{(56.944 \times 10^{-6})(155 \times 10^6)}{0.110} = 80.2 \times 10^3 \text{ N-m}$ $M_x = 80.2 \text{ kN}\cdot\text{m}$

Problem 4.4

4.4 Solve Prob. 4.3, assuming that the wide-flange beam is bent about the y axis by a couple of moment M_y.

4.3 Using an allowable stress of 155 MPa, determine the largest bending moment M that can be applied to the wide flange beam shown. Neglect the effect of fillets.





Moment of inertia about y-axis

$I_1 = \frac{1}{12}(12)(200)^3 = 8 \times 10^6 \text{ mm}^4$

$I_2 = \frac{1}{12}(196)(8)^3 = 8.36 \times 10^6 \text{ mm}^4$

$I_3 = I_1 = 8 \times 10^6 \text{ mm}^4$

$I = I_1 + I_2 + I_3 = 16.00836 \times 10^6 \text{ mm}^4 = 16.00836 \times 10^{-6} \text{ m}^4$

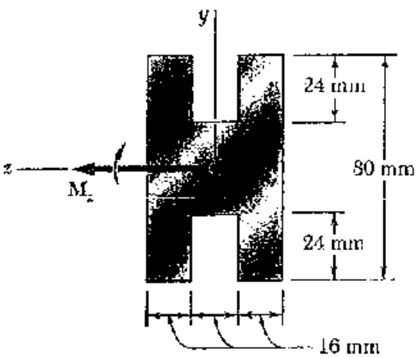
$\sigma = \frac{Mc}{I}$ with $c = \frac{1}{2}(200) = 100 \text{ mm} = 0.100 \text{ m}$

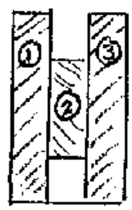
$M_y = \frac{I\sigma}{c}$ with $\sigma = 155 \times 10^6 \text{ Pa}$

$M_y = \frac{(16.00836 \times 10^{-6})(155 \times 10^6)}{0.100} = 24.8 \times 10^3 \text{ N}\cdot\text{m}$ $M_y = 24.8 \text{ kN}\cdot\text{m}$

Problem 4.5

4.5 A beam of the cross section shown is extruded from an aluminum alloy for which $\sigma_Y = 250$ MPa and $\sigma_U = 450$ MPa. Using a factor of safety of 3.00, determine the largest couple that can be applied to the beam when it is bent about the z axis.





Allowable stress $= \dfrac{\sigma_U}{F.S.} = \dfrac{450}{3} = 150$ MPa

$= 150 \times 10^6$ Pa

Moment of inertia about z axis

$I_1 = \frac{1}{12}(16)(80)^3 = 682.67 \times 10^3 \ mm^4$

$I_2 = \frac{1}{12}(16)(32)^3 = 43.69 \times 10^3 \ mm^4$

$I_3 = I_1 = 682.67 \times 10^3 \ mm^4$

$I = I_1 + I_2 + I_3 = 1.40902 \times 10^6 \ mm^4 = 1.40902 \times 10^{-6} \ m^4$

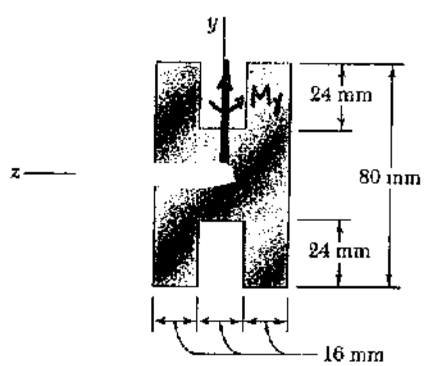
$\sigma = \dfrac{Mc}{I}$ with $c = \frac{1}{2}(80) = 40 \ mm = 0.040 \ m$

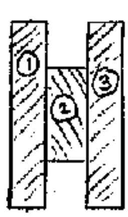
$$M = \frac{I\sigma}{c} = \frac{(1.40902 \times 10^{-6})(150 \times 10^6)}{0.040} = 5.28 \times 10^3 \ N \cdot m \qquad M = 5.28 \ kN \cdot m$$

Problem 4.6

4.6 Solve Prob. 4.5, assuming that the beam is bent about the y axis.

4.5 A beam of the cross section shown is extruded from an aluminum alloy for which $\sigma_Y = 250$ MPa and $\sigma_U = 450$ MPa. Using a factor of safety of 3.00, determine the largest couple that can be applied to the beam when it is bent about the z axis.





Allowable stress $= \dfrac{\sigma_U}{F.S.} = \dfrac{450}{3.00} = 150$ MPa

$= 150 \times 10^6$ Pa

Moment of inertia about y-axis

$I_1 = \frac{1}{12}(80)(16)^3 + (80)(16)(16)^2 = 354.987 \times 10^3 \ mm^4$

$I_2 = \frac{1}{12}(32)(16)^3 = 10.923 \times 10^3 \ mm^4$

$I_3 = I_1 = 354.987 \times 10^3 \ mm^4$

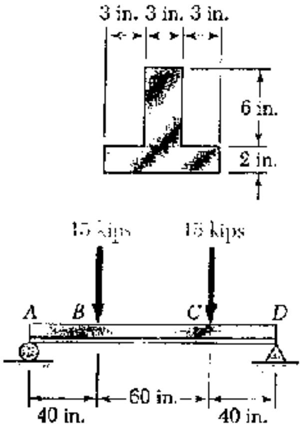
$I = I_1 + I_2 + I_3 = 720.9 \times 10^3 \ mm^4 = 720.9 \times 10^{-9} \ m^4$

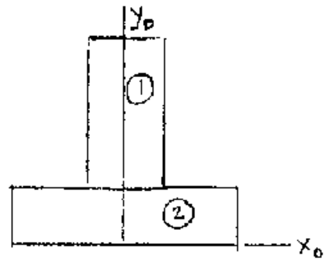
$\sigma = \dfrac{Mc}{I}$ with $c = \frac{1}{2}(48) = 24 \ mm = 0.024 \ m$

$$M = \frac{I\sigma}{c} = \frac{(720.9 \times 10^{-9})(150 \times 10^6)}{0.024} = 4.51 \times 10^3 \ N \cdot m \qquad M = 4.51 \ kN \cdot m$$

Problem 4.7

4.7 through 4.9 Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion BC of the beam.

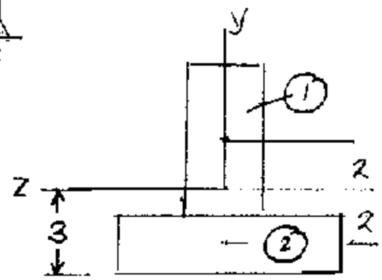




	A	$\bar{y}_o$	$A\bar{y}_o$
①	18	5	90
②	18	1	18
Σ	36		108

$$\bar{Y}_o = \frac{108}{36} = 3 \text{ in}$$

Neutral axis lies 3 in. above the base.

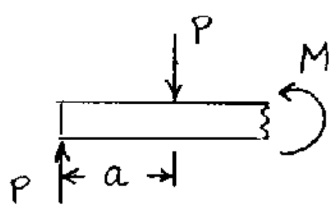


$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(3)(6)^3 + (18)(2)^2 = 126 \text{ in}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(9)(2)^3 + (18)(2)^2 = 78 \text{ in}^4$$

$$I = I_1 + I_2 = 126 + 78 = 204 \text{ in}^4$$

$$y_{top} = 5 \text{ in.} \qquad y_{bot} = -3 \text{ in.}$$



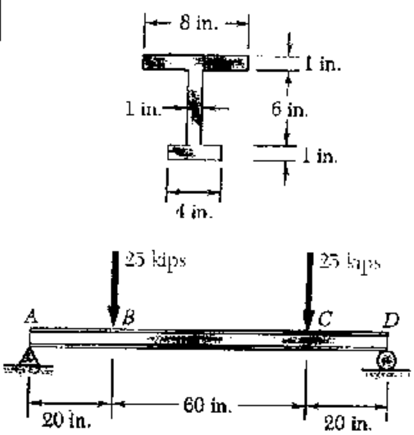
$$M - Pa = 0$$

$$M = Pa = (15)(40) = 600 \text{ kip}\cdot\text{in.}$$

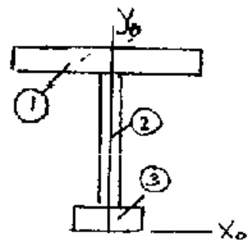
$$\sigma_{top} = -\frac{M y_{top}}{I} = -\frac{(600)(5)}{204} = -14.71 \text{ ksi} \quad \text{(compression)}$$

$$\sigma_{bot} = -\frac{M y_{bot}}{I} = -\frac{(600)(-3)}{204} = 8.82 \text{ ksi} \quad \text{(tension)}$$

Problem 4.8



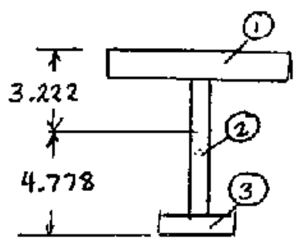
4.7 through 4.9 Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.



	A	$\bar{y}_o$	$A\bar{y}_o$
①	8	7.5	60
②	6	4	24
③	4	0.5	2
Σ	18		86

$\bar{Y}_o = \frac{86}{18} = 4.778$ in

Neutral axis lies 4.778 in above the base.



$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(8)(1)^3 + (8)(2.722)^2$

$= 59.94 \text{ in}^4$

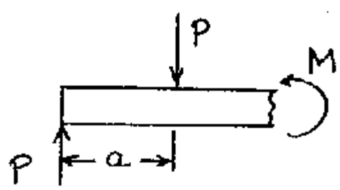
$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(1)(6)^3 + (6)(0.778)^2$

$= 21.63 \text{ in}^4$

$I_3 = \frac{1}{12} b_3 h_3^3 + A_3 d_3^2 = \frac{1}{12}(4)(1)^3 + (4)(4.278)^2 = 73.54 \text{ in}^4$

$I = I_1 + I_2 + I_3 = 59.94 + 21.63 + 73.54 = 155.16 \text{ in}^4$

$y_{top} = 3.222$ in $\qquad y_{bot} = -4.778$ in



$M - Pa = 0$

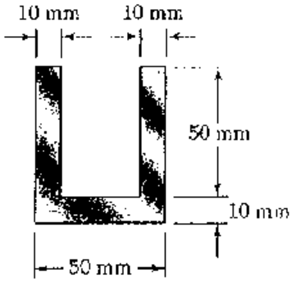
$M = Pa = (25)(20) = 500$ kip·in.

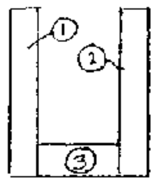
$\sigma_{top} = -\frac{M y_{top}}{I} = -\frac{(500)(3.222)}{155.16} = -10.38$ ksi (compression) ◀

$\sigma_{bot} = -\frac{M y_{bot}}{I} = -\frac{(500)(-4.778)}{155.16} = 15.40$ ksi (tension) ◀

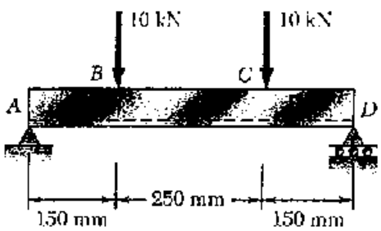
Problem 4.9

4.7 through 4.9 Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.

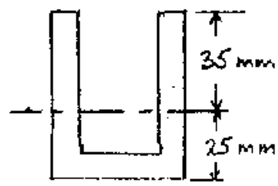




	A, mm^2	$\bar{y}_o$, mm	$A\bar{y}_o$, mm^3
1	600	30	18×10^3
2	600	30	18×10^3
3	300	5	1.5×10^3
	1500		37.5×10^3



$$\bar{Y}_o = \frac{37.5\times10^3}{1500} = 25 \text{ mm}$$



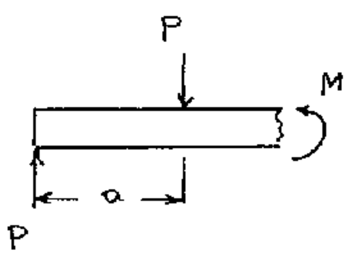
Neutral axis lies 25 mm above the base.

$$I_1 = \frac{1}{12}(10)(60)^3 + (600)(5)^2 = 195\times10^3 \text{ mm}^4 \qquad I_2 = I_1 = 195 \text{ mm}^4$$

$$I_3 = \frac{1}{12}(30)(10)^3 + (300)(20)^2 = 122.5\times10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 512.5\times10^3 \text{ mm}^4 = 512.5\times10^{-9} \text{ m}^4$$

$$y_{top} = 35 \text{ mm} = 0.035 \text{ m} \qquad y_{bot} = -25 \text{ mm} = -0.025 \text{ m}$$



$$a = 150 \text{ mm} = 0.150 \text{ m} \qquad P = 10\times10^3 \text{ N}$$

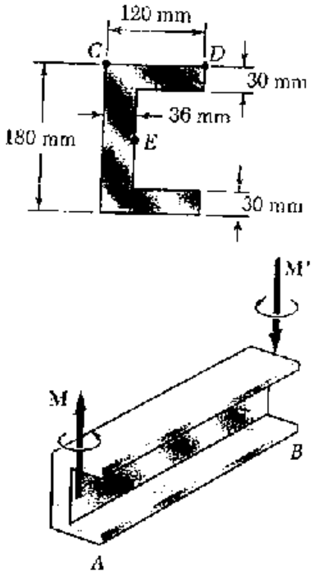
$$M = Pa = (10\times10^3)(0.150) = 1.5\times10^3 \text{ N}\cdot\text{m}$$

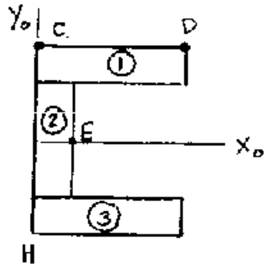
$$\sigma_{top} = -\frac{My_{top}}{I} = -\frac{(1.5\times10^3)(0.035)}{512.5\times10^{-9}} = -102.4\times10^6 \text{ Pa} \qquad \sigma_{top} = -102.4 \text{ MPa (compression)}$$

$$\sigma_{bot} = -\frac{My_{bot}}{I} = -\frac{(1.5\times10^3)(-0.025)}{512.5\times10^{-9}} = 73.2\times10^6 \text{ Pa} \qquad \sigma_{bot} = 73.2 \text{ MPa (tension)}$$

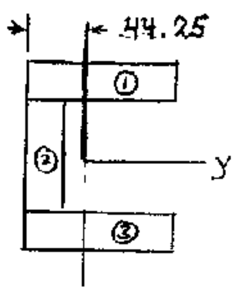
Problem 4.10

4.10 Two equal and opposite couples of magnitude $M = 25$ kN · m are applied to the channel-shaped beam AB. Observing that the couples cause the beam to bend in a horizontal plane, determine the stress at (*a*) point C, (*b*) point D, (*c*) point E.





	A, mm^2	$\bar{X}_0$, mm	$A\bar{X}_0$, mm^3
①	3600	60	216×10^3
②	4320	18	77.76×10^3
③	3600	60	216×10^3
Σ	11520		509.76×10^3



$$\bar{X} = \frac{509.76 \times 10^3}{11520} = 44.25 \text{ mm}$$

$$y_C = -44.25 \text{ mm} = -0.04425 \text{ m}$$

$$y_D = 120 - 44.25 = 75.75 \text{ mm} = 0.07575 \text{ m}$$

$$y_E = 36 - 44.25 = -8.25 \text{ mm} = -0.00825 \text{ m}$$

$$d_1 = 60 - 44.25 = 15.75 \text{ mm}$$
$$d_2 = 44.25 - 18 = 26.26 \text{ mm}$$
$$d_3 = d_1$$

$$I_1 = I_3 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(30)(120)^3 + (3600)(15.75)^2 = 5.2130 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(120)(36)^3 + (4320)(26.25)^2 = 3.4433 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 6.5187 \times 10^6 \text{ mm}^4 = 13.8694 \times 10^{-6} \text{ m}^4$$

$$M = 15 \times 10^3 \text{ N·m}$$

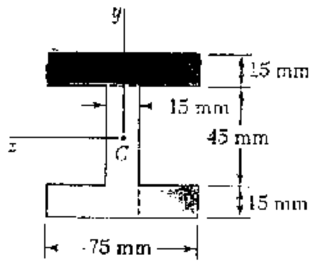
(a) Point C: $$\sigma_C = -\frac{My_C}{I} = -\frac{(25 \times 10^3)(-0.04425)}{13.8699 \times 10^{-6}} = 79.8 \times 10^6 \text{ Pa} = 79.8 \text{ MPa}$$

(b) Point D: $$\sigma_D = -\frac{My_D}{I} = -\frac{(25 \times 10^3)(0.07575)}{13.8694 \times 10^{-6}} = -136.5 \times 10^6 \text{ Pa} = -136.5 \text{ MPa}$$

(c) Point E: $$\sigma_E = -\frac{My_E}{I} = -\frac{(25 \times 10^3)(-0.00825)}{13.8694 \times 10^{-6}} = 14.87 \times 10^6 \text{ Pa} = 14.87 \text{ MPa}$$

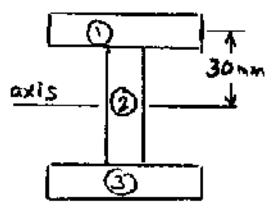
Problem 4.11

4.11 Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 8 kN · m, determine the total force acting on the top flange.



The stress distribution over the entire cross section is given by the bending stress formula

$$\sigma_x = -\frac{My}{I}$$



where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded is calculated from this stress distribution. Over an area element dA the force is

$$dF = \sigma_x \, dA = -\frac{My}{I} dA$$

The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I} dA = -\frac{M}{I}\int y\, dA = -\frac{M}{I}\bar{y}^* A^*$$

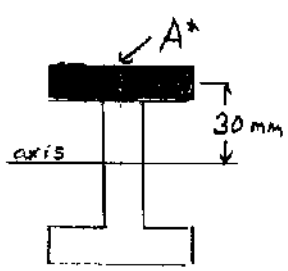
where $\bar{y}^*$ is the centroidal coordinate of the shaded portion and A^* is its area.

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(75)(15)^3 + (75)(15)(30)^2 = 1.0336 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(15)(45)^3 = 0.1139 \times 10^6 \text{ mm}^4$$

$$I_3 = I_1 = 1.0336 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 2.1811 \times 10^6 \text{ mm}^4 = 2.1811 \times 10^{-6} \text{ m}^4$$



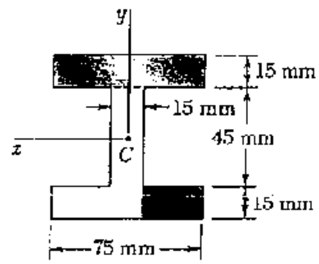
$$A^* = (75)(15) = 1125 \text{ mm}^2 = 1125 \times 10^{-6} \text{ m}^2$$

$$\bar{y}^* = 30 \text{ mm} = 0.030 \text{ m}$$

$$F = -\frac{M\bar{y}^* A}{I} = -\frac{(8\times10^3)(0.030)(1125\times10^{-6})}{2.1811\times10^{-6}}$$

$$= -123.8 \times 10^3 \text{ N} = -123.8 \text{ kN}$$

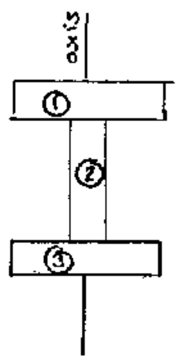
Problem 4.12



4.12 Knowing that a beam of the cross section shown is bent about a vertical axis and that the bending moment is 4 kN · m, determine the total force acting on the shaded portion of the lower flange.

The stress distribution over the entire cross section is given by the bending stress formula

$$\sigma_x = -\frac{My}{I}$$



where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded is calculated from this stress distribution. Over an area element dA the force is

$$dF = \sigma_x\, dA = -\frac{My}{I}\, dA$$

The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I}\, dA = -\frac{M}{I}\int y\, dA = -\frac{M}{I}\bar{y}^* A^*$$

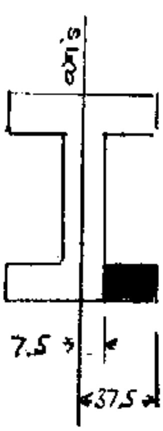
where $\bar{y}^*$ is the centroidal coordinate of the shaded portion and A^* is its area.

$$I_1 = \tfrac{1}{12} b_1 h_1^3 = \tfrac{1}{12}(15)(75)^3 = 0.52734\times10^{6}\ \text{mm}^4$$

$$I_2 = \tfrac{1}{12} b_2 h_2^3 = \tfrac{1}{12}(45)(15)^3 = 0.01256\times10^{6}\ \text{mm}^4$$

$$I_3 = I_1 = 0.5273\times10^{-6}$$

$$I = I_1 + I_2 + I_3 = 1.0672\times10^{6}\ \text{mm}^4 = 1.0672\times10^{-6}\ \text{m}^4$$



$$A^* = (37.5-7.5)(15) = 450\ \text{mm}^2 = 450\times10^{-6}\ \text{m}^2$$

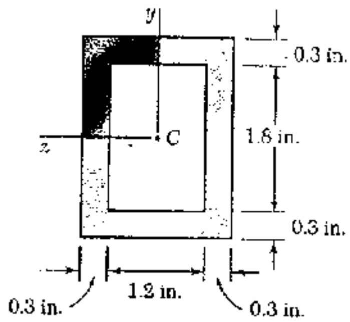
$$\bar{y}^* = \tfrac{1}{2}(37.5+7.5) = 22.5\ \text{mm} = 0.0225\ \text{m}$$

$$F = \frac{M\bar{y}^*A^*}{I} = \frac{(4\times10^3)(0.0225)(450\times10^{6})}{1.0672\times10^{-6}}$$

$$= 37.9\times10^3\ \text{N} = 37.9\ \text{kN}$$

Problem 4.13

4.13 Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 8 kip · in., determine the total force acting on the shaded portion of the beam.



The stress distribution over the entire cross section is given by the bending stress formula

$$\sigma_x = -\frac{My}{I}$$

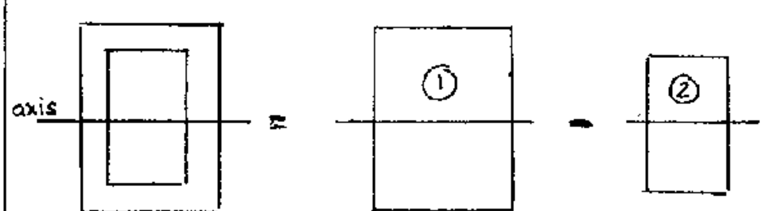
where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded is calculated from this stress distribution. Over an area element dA the force is

$$dF = \sigma_x \, dA = -\frac{My}{I} dA$$

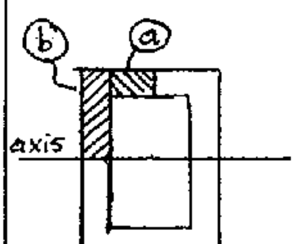
The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I} dA = -\frac{M}{I}\int y\, dA = -\frac{M}{I}\bar{y}^* A^*$$

where $\bar{y}^*$ is the centroidal coordinate of the shaded portion and A^* is its area.



$$I = I_1 - I_2$$
$$= \frac{1}{12} b_1 h_1^3 - \frac{1}{12} b_2 h_2^3$$
$$= \frac{1}{12}(1.8)(2.4)^3 - \frac{1}{12}(1.2)(1.8)^3$$
$$= 1.4904 \text{ in}^4$$



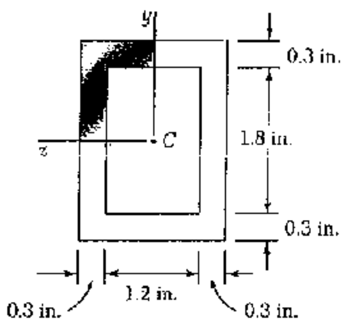
$$\bar{y}^* A^* = \bar{y}_a A_a + \bar{y}_b A_b$$
$$= (1.05)(0.9)(0.3) + (0.45)(0.3)(0.9) = 0.405 \text{ in}^3$$

$$F = \frac{M\bar{y}^* A^*}{I} = \frac{(8)(0.405)}{1.4904} = 2.17 \text{ kips}$$

Problem 4.14

4.14 Solve Prob. 4.13, assuming that the beam is bent about a vertical axis and that the bending moment is 8 kip · in.

4.13 Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 8 kip · in., determine the total force acting on the shaded portion of the beam.



The stress distribution over the entire cross section is given by the bending stress formula

$$\sigma_x = -\frac{My}{I}$$

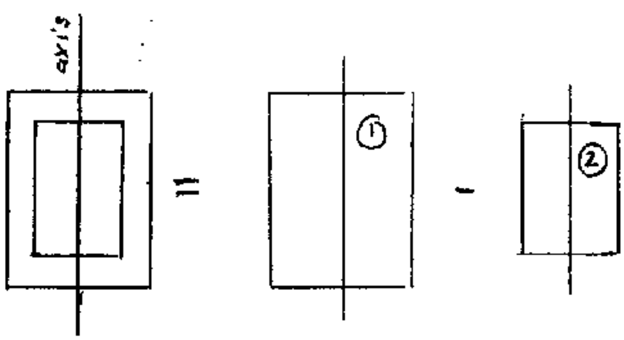
where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded is calculated from this stress distribution. Over an area element dA the force is

$$dF = \sigma_x \, dA = -\frac{My}{I} dA$$

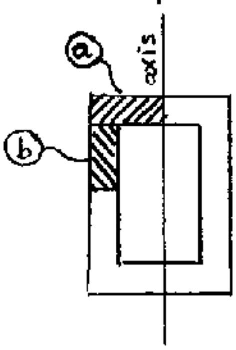
The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I} dA = -\frac{M}{I}\int y \, dA = -\frac{M}{I}\bar{y}^* A^*$$

where $\bar{y}^*$ is the centroidal coordinate of the shaded portion and A^* is its area.



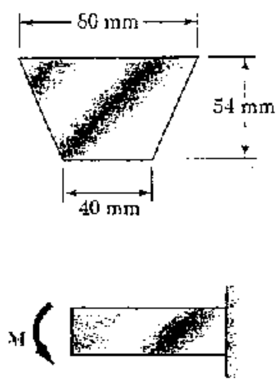
$$I = I_1 - I_2$$
$$= \frac{1}{12} b_1 h_1^3 - \frac{1}{12} b_2 h_2^3$$
$$= \frac{1}{12}(2.4)(1.8)^3 - \frac{1}{12}(1.8)(1.2)^3$$
$$= 0.9072 \text{ in}^4$$



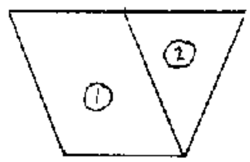
$$\bar{y}^* A^* = \bar{y}_a A_a + \bar{y}_b A_b$$
$$= (0.45)(0.3)(0.9) + (0.75)(0.9)(0.3)$$
$$= 0.324 \text{ in}^3$$

$$F = \frac{M \bar{y}^* A^*}{I} = \frac{(8)(0.324)}{0.9072} = 2.89 \text{ kips.}$$

Problem 4.15



4.15 Knowing that for the extruded beam shown the allowable stress is 120 MPa in tension and 150 MPa in compression, determine the largest couple **M** that can be applied.



	A, mm^2	$\bar{y}_o$, mm	$A\bar{y}_o$, mm^3
①	2160	27	58320
②	1080	36	38880
Σ	3240		97200

$$\bar{Y} = \frac{97200}{3240} = 30 \text{ mm}$$

The neutral axis lies 30 mm above the bottom.

$y_{top} = 54 - 30 = 24 \text{ mm} = 0.024 \text{ m}$

$y_{bot} = -30 \text{ mm} = -0.030 \text{ m}$

$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(40)(54)^3 + (40)(54)(3)^2 = 544.32 \times 10^3 \text{ mm}^4$

$I_2 = b_2h_2^2 + A_2d_2^2 = \frac{1}{36}(40)(54)^3 + \frac{1}{2}(40)(54)(6)^2 = 213.84 \times 10^3 \text{ mm}^4$

$I = I_1 + I_2 = 758.16 \times 10^3 \text{ mm}^4 = 758.16 \times 10^{-9} \text{ m}^4$

$|\sigma| = \left|\frac{My}{I}\right| \qquad |M| = \left|\frac{\sigma I}{y}\right|$

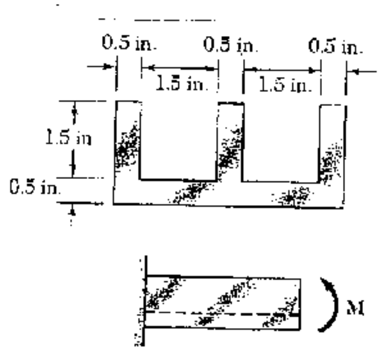
top: tension side $\quad M = \frac{(120 \times 10^6)(758.16 \times 10^{-9})}{0.024} = 3.7908 \times 10^3 \text{ N}\cdot\text{m}$

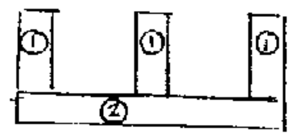
bottom: compression $\quad M = \frac{(150 \times 10^6)(758.16 \times 10^{-9})}{0.030} = 3.7908 \times 10^3 \text{ N}\cdot\text{m}$

Choose the smaller as M_{all} $\qquad M_{all} = 3.7908 \times 10^3 \text{ N}\cdot\text{m} = 3.79 \text{ kN}\cdot\text{m}$ ◄

Problem 4.16

4.16 Knowing that for the extruded beam shown the allowable stress is 12 ksi in tension and 16 ksi in compression, determine the largest couple **M** that can be applied.





	A	$\bar{y}_o$	$A\bar{y}_o$
①	2.25	1.25	2.8125
②	2.25	0.25	0.5625
	4.50		3.375

$$\bar{Y} = \frac{3.375}{4.50} = 0.75 \text{ in}$$

The neutral axis lies 0.75 in. above bottom.

$y_{top} = 2.0 - 0.75 = 1.25$ in, $\quad y_{bot} = -0.75$ in.

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(1.5)(1.5)^3 + (2.25)(0.5)^2 = 0.984375 \text{ in}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(4.5)(0.5)^3 + (2.25)(0.5)^2 = 0.609375 \text{ in}^4$$

$$I = I_1 + I_2 = 1.59375 \text{ in}^4$$

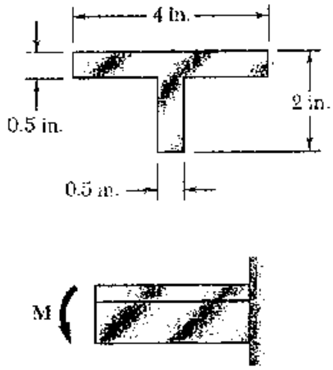
$$|\sigma| = \left|\frac{My}{I}\right| \qquad M = \left|\frac{\sigma I}{y}\right|$$

Top: compression $\quad M = \frac{(16)(1.59375)}{1.25} = 20.4$ kip·in

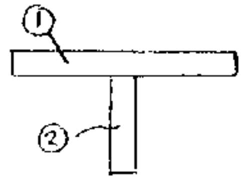
Bottom: tension $\quad M = \frac{(12)(1.59375)}{0.75} = 25.5$ kip·in

Choose the smaller as M_{all} $\quad M_{all} = 20.4$ kip·in.

Problem 4.17



4.17 For the casting shown, determine the largest couple **M** that can be applied without exceeding either of the following allowable stresses: $\sigma_{all} = +6$ ksi, $\sigma_{all} = -15$ ksi.



	A, in^2	$\bar{y}_o$, in.	$A\bar{y}_o$, in^3
①	2.00	1.75	3.50
②	0.75	0.75	0.5625
Σ	2.75		4.0625

$$\bar{Y}_o = \frac{4.0625}{2.75} = 1.4773 \text{ in.}$$

The neutral axis lies 1.4773 in. above the bottom.

$y_{top} = 2 - 1.4773 = 0.5227$ in. $\qquad y_{bot} = -1.4773$ in.

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(4)(0.5)^3 + (2.00)(0.2727)^2 = 0.19040 \text{ in}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(0.5)(1.5)^3 + (0.75)(0.7273)^2 = 0.53735 \text{ in}^4$$

$$I = I_1 + I_2 = 0.72775 \text{ in}^4$$

$$\sigma = \frac{My}{I} \qquad M = \frac{\sigma I}{y}$$

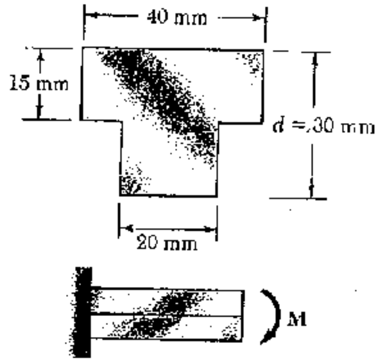
Top: tension side $\qquad M = \dfrac{(6 \times 10^3)(0.72775)}{0.5227} = 8.35 \times 10^3$ lb·in

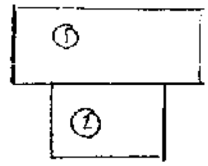
Bottom: compression $\qquad M = \dfrac{(-15 \times 10^3)(0.72775)}{-1.4773} = 7.39 \times 10^3$ lb·in

Choose the smaller value $\qquad M = 7.39 \times 10^3$ lb·in $\qquad M = 7.39$ kip·in ◀

Problem 4.18

4.18 The beam shown is made of a nylon for which the allowable stress is 24 MPa in tension and 30 MPa in compression. Determine the largest couple **M** that can be applied to the beam.





	A, mm^2	$\bar{y}_o$, mm	$A\bar{y}_o$, mm^3
①	600	22.5	13.5×10^3
②	300	7.5	2.25×10^3
Σ	900		15.75×10^3

$$\bar{Y}_o = \frac{15.5 \times 10^3}{900} = 17.5 \text{ mm}$$

The neutral axis lies 17.5mm above the bottom.

$y_{top} = 30 - 17.5 = 12.5 \text{ mm} = 0.0125 \text{ m}, \quad y_{bot} = -17.5 \text{ mm} = -0.0175 \text{ m}$

$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(40)(15)^3 + (600)(5)^2 = 26.25 \times 10^3 \text{ mm}^4$

$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(20)(15)^3 + (300)(10)^2 = 35.625 \times 10^3 \text{ mm}^4$

$I = I_1 + I_2 = 61.875 \times 10^3 \text{ mm}^4 = 61.875 \times 10^{-9} \text{ m}^4$

$|\sigma| = \left|\frac{My}{I}\right| \qquad M = \left|\frac{\sigma I}{y}\right|$

Top: tension side $\quad M = \frac{(24 \times 10^6)(61.875 \times 10^{-9})}{0.0125} = 118.8 \text{ N·m}$

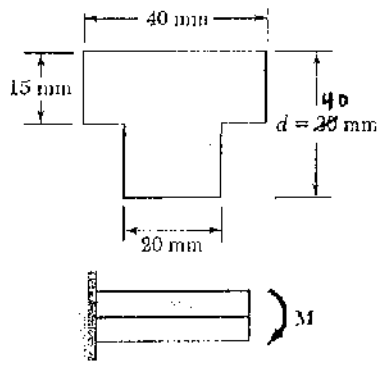
Bottom: compression $\quad M = \frac{(30 \times 10^6)(61.875 \times 10^{-9})}{0.0175} = 106.1 \text{ N·m}$

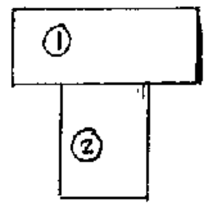
Choose smaller value $\qquad M = 106.1 \text{ N·m}$

Problem 4.19

4.19 Solve Prob. 4.18, assuming that $d = 40$ mm.

4.18 The beam shown is made of a nylon for which the allowable stress is 24 MPa in tension and 30 MPa in compression. Determine the largest couple **M** that can be applied to the beam.





	A, mm²	$\bar{y}_o$, mm	$A\bar{y}_o$, mm³
①	600	32.5	19.5×10^3
②	500	12.5	6.25×10^3
Σ	1100		25.75×10^3

$$\bar{Y}_o = \frac{25.75\times10^3}{1100} = 23.41 \text{ mm}$$

The neutral axis lies 23.41 mm above the bottom.

$y_{top} = 40 - 23.41 = 16.59 \text{ mm} = 0.01659 \text{ m}$

$y_{bot} = -23.41 \text{ mm} = -0.02341 \text{ m}$

$$I_1 = \tfrac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \tfrac{1}{12}(40)(15)^3 + (600)(9.09)^2 = 60.827\times10^3 \text{ mm}^4$$

$$I_2 = \tfrac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \tfrac{1}{12}(20)(25)^3 + (500)(10.91)^2 = 85.556\times10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 146.383\times10^3 \text{ mm}^4 = 146.383\times10^{-9} \text{ m}^4$$

$$|\sigma| = \left|\frac{My}{I}\right| \qquad M = \left|\frac{\sigma I}{y}\right|$$

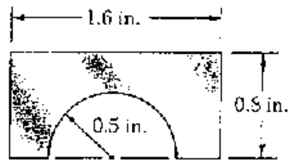
Top: tension side $\quad M = \dfrac{(24\times10^6)(146.383\times10^{-9})}{0.01659} = 212 \text{ N}\cdot\text{m}$

Bottom: compression $\quad M = \dfrac{(30\times10^6)(146.383\times10^{-9})}{0.02341} = 187.6 \text{ N}\cdot\text{m}$

Choose smaller value $\qquad M = 187.6 \text{ N}\cdot\text{m}$

Problem 4.20

4.20 Knowing that for the beam shown the allowable stress is 12 ksi in tension and 16 ksi in compression, determine the largest couple **M** that can be applied.



① = rectangle ② = semi-circular cutout

$A_1 = (1.6)(0.8) = 1.28 \text{ in}^2$

$A_2 = \frac{\pi}{2}(0.5)^2 = 0.3927 \text{ in}^2$

$A = 1.28 - 0.3927 = 0.8873 \text{ in}^2$

$\bar{y}_1 = 0.4 \text{ in.} \qquad \bar{y}_2 = \frac{4r}{3\pi} = \frac{(4)(0.5)}{3\pi} = 0.2122 \text{ in}$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{(1.28)(0.4) - (0.3927)(0.2122)}{0.8873} = 0.4831 \text{ in.}$$

Neutral axis lies 0.4831 in above the bottom

Moment of inertia about the base

$$I_b = \frac{1}{3}bh^3 - \frac{\pi}{8}r^4 = \frac{1}{3}(1.6)(0.8)^3 - \frac{\pi}{8}(0.5)^4 = 0.24852 \text{ in}^4$$

Centroidal moment of inertia

$$\bar{I} = I_b - A\bar{Y}^2 = 0.24852 - (0.8873)(0.4831)^2$$

$$= 0.04144 \text{ in}^4$$

$y_{top} = 0.8 - 0.4831 = 0.3169 \text{ in}, \qquad y_{bot} = -0.4831 \text{ in}^4$

$|\sigma| = \left|\frac{My}{I}\right| \qquad M = \left|\frac{\sigma I}{y}\right|$

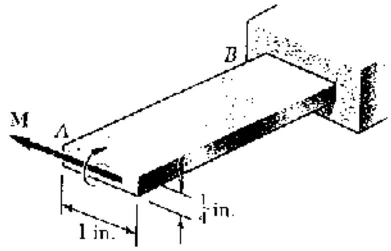
Top: tension side $\quad M = \frac{(12)(0.04144)}{0.3169} = 1.569 \text{ kip}\cdot\text{in}$

Bottom: compression $\quad M = \frac{(16)(0.04144)}{0.4831} = 1.372 \text{ kip}\cdot\text{in}$

Choose the smaller value $\qquad M = 1.372 \text{ kip}\cdot\text{in.}$ ◄

Problem 4.21

4.21 Knowing that $\sigma_{all} = 24$ ksi for the steel strip *AB*, determine (*a*) the largest couple **M** that can be applied, (*b*) the corresponding radius of curvature. Use $E = 29 \times 10^6$ psi.



$I = \frac{1}{12} b h^3 = (\frac{1}{12})(1)(\frac{1}{4})^3 = 1.30208 \times 10^{-3}$ in^4

$\sigma = \frac{Mc}{I}$ $\qquad c = (\frac{1}{2})(\frac{1}{4}) = 0.125$ in.

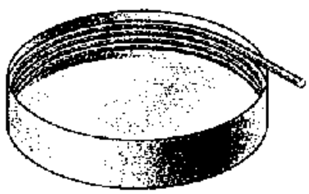
(a) $M = \frac{\sigma I}{c} = \frac{(24\times10^3)(1.30208\times10^{-3})}{0.125}$

$M = 250$ lb·in ◄

(b) $\frac{c}{\rho} = \frac{\sigma_{max}}{E}$ $\qquad \rho = \frac{Ec}{\sigma_{max}} = \frac{(29\times10^6)(0.125)}{24\times10^3}$ $\qquad \rho = 151.0$ in. ◄

Problem 4.22

4.22 Straight rods of 6-mm diameter and 30-m length are stored by coiling the rods inside a drum of 1.25-m inside diameter. Assuming that the yield strength is not exceeded, determine (*a*) the maximum stress in a coiled rod, (*b*) the corresponding bending moment in the rod. Use $E = 200$ GPa.



Let D = inside diameter of the drum
d = diameter of rod, $c = \frac{1}{2} d$,
ρ = radius of curvature of center line of rods when bent.

$\rho = \frac{1}{2} D - \frac{1}{2} d = \frac{1}{2}(1.25) - \frac{1}{2}(6\times10^{-3}) = 0.622$ m

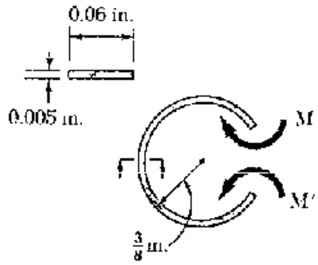
$I = \frac{\pi}{4} c^4 = \frac{\pi}{4}(0.003)^4 = 63.617\times10^{-12}$ m^4

(a) $\sigma_{max} = \frac{Ec}{\rho} = \frac{(200\times10^9)(0.003)}{0.622} = 965\times10^6$ Pa $\qquad \sigma = 965$ MPa ◄

(b) $M = \frac{EI}{\rho} = \frac{(200\times10^9)(63.617\times10^{-12})}{0.622} = 20.5$ N·m $\qquad M = 20.5$ N·m ◄

Problem 4.23

4.23 It is observed that a thin steel strip of 0.06-in. width can be bent into a circle of $\frac{3}{4}$-in. diameter without any resulting permanent deformation. Knowing that $E = 29 \times 10^6$ psi, determine (*a*) the maximum stress in the bent strip, (*b*) the magnitude of the couples required to bend the strip.



$$I = \frac{1}{12} b h^3 = \frac{1}{12}(0.06)(0.005)^3 = 625 \times 10^{-12} \text{ in}^4$$

$$\rho = \frac{1}{2} D = \frac{1}{2}\left(\frac{3}{4}\right) = 0.375 \text{ in}$$

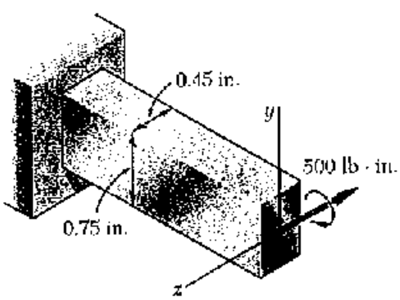
$$c = \frac{1}{2} h = 0.0025 \text{ in}$$

(a) $$\sigma_{max} = \frac{Ec}{\rho} = \frac{(29 \times 10^6)(0.0025)}{0.375} = 193.3 \times 10^3 \text{ psi} = 193.3 \text{ ksi}$$

(b) $$M = \frac{EI}{\rho} = \frac{(29 \times 10^6)(625 \times 10^{-12})}{0.375} = 0.0483 \text{ lb} \cdot \text{in}$$

Problem 4.24

4.24 A 500 lb · in. couple is applied to the steel bar shown. (*a*) Assuming that the couple is applied about the *z* axis as shown, determine the maximum stress and the radius of curvature of the bar. (*b*) Solve part *a*, assuming that the couple is applied about the *y* axis. Use $E = 29 \times 10^6$ psi.



(a) Bending about z-axis.

$$I = \frac{1}{12} b h^3 = \frac{1}{12}(0.45)(0.75)^3 = 15.8203 \times 10^{-3} \text{ in}^4$$

$$c = \frac{1}{2} h = \frac{1}{2}(0.75) = 0.375 \text{ in.}$$

$$\sigma = \frac{Mc}{I} = \frac{(500)(0.375)}{15.8203 \times 10^{-3}} = 11.85 \times 10^3 \text{ psi}$$

$$\sigma = 11.85 \text{ ksi}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{500}{(29 \times 10^6)(15.8203 \times 10^{-3})} = 1.0898 \times 10^{-3} \text{ in}^{-1}$$

$$\rho = 918 \text{ in} \qquad \rho = 76.5 \text{ ft}$$

(b) Bending about y-axis

$$I = \frac{1}{12} b h^3 = \frac{1}{12}(0.75)(0.45)^3 = 5.6953 \times 10^{-3} \text{ in}^4$$

$$c = \frac{1}{2} h = \frac{1}{2}(0.45) = 0.225 \text{ in.}$$

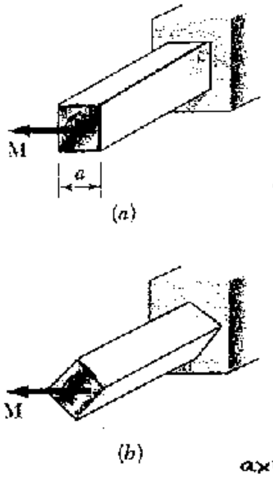
$$\sigma = \frac{Mc}{I} = \frac{(500)(0.225)}{5.6953 \times 10^{-3}} = 19.75 \times 10^3 \text{ psi} \qquad \sigma = 19.75 \text{ ksi}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{500}{(29 \times 10^6)(5.6953 \times 10^{-3})} = 3.0273 \times 10^{-3} \text{ in}^{-1}$$

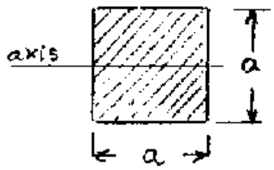
$$\rho = 330 \text{ in.} \qquad \rho = 27.5 \text{ ft}$$

Problem 4.25

4.25 A couple of magnitude M is applied to a square bar of side a. For each of the orientations shown, determine the maximum stress and the curvature of the bar.



(a)



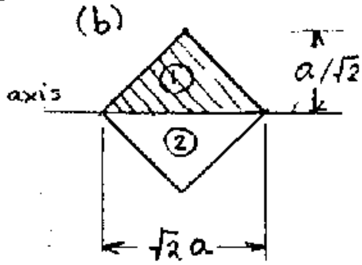
$$I = \frac{1}{12}bh^3 = \frac{1}{12}a\,a^3 = \frac{a^4}{12}$$

$$c = \frac{a}{2}$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{M\frac{a}{2}}{\frac{a^4}{12}} = \frac{6M}{a^3}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{M}{E\frac{a^4}{12}} = \frac{12M}{Ea^4}$$

(b)



For one triangle, the moment of inertia about its base is

$$I_1 = \frac{1}{12}bh^3 = \frac{1}{12}(\sqrt{2}a)\left(\frac{a}{\sqrt{2}}\right)^3 = \frac{a^4}{24}$$

$$I_2 = I_1 = \frac{a^4}{24}$$

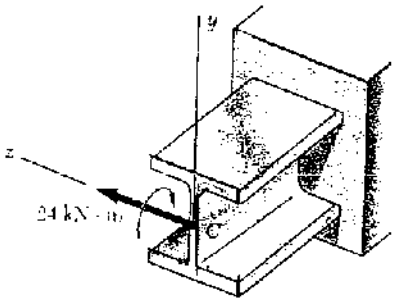
$$I = I_1 + I_2 = \frac{a^4}{12}$$

$$c = \frac{a}{\sqrt{2}} \qquad \sigma_{max} = \frac{Mc}{I} = \frac{M\,a/\sqrt{2}}{a^4/12} = \frac{6\sqrt{2}\,M}{a^3}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{M}{E\frac{a^4}{12}} = \frac{12M}{Ea^4}$$

Problem 4.26

4.26 A 24 kN · m couple is applied to the W200 × 46.1 rolled-steel beam shown. (*a*) Assuming that the couple is applied about the *z* axis as shown, determine the maximum stress and the radius of curvature of the beam. (*b*) Solve part *a*, assuming that the couple is applied about the *y* axis. Use $E = 200$ GPa.



For W 200 × 46.1 rolled steel section

$I_x = 45.5 \times 10^6 \text{ mm}^4 = 45.5 \times 10^{-6} \text{ m}^4$

$S_x = 448 \times 10^3 \text{ mm}^3 = 448 \times 10^{-6} \text{ m}^3$

$I_y = 15.3 \times 10^6 \text{ mm}^4 = 15.3 \times 10^{-6} \text{ m}^4$

$S_y = 151 \times 10^3 \text{ mm}^3 = 151 \times 10^{-6} \text{ m}^3$

(a) $M_z = 24 \text{ kN}\cdot\text{m} = 24 \times 10^3 \text{ N}\cdot\text{m}$

$$\sigma = \frac{M}{S} = \frac{24 \times 10^3}{448 \times 10^{-6}} = 53.6 \times 10^6 \text{ Pa} = 53.6 \text{ MPa}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{24 \times 10^3}{(200 \times 10^9)(45.5 \times 10^{-6})} = 2.637 \times 10^{-3} \text{ m}^{-1}$$

$$\rho = 379 \text{ m}$$

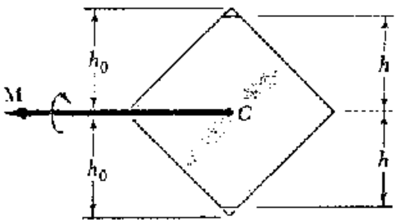
(b) $M_y = 24 \text{ kN}\cdot\text{m} = 24 \times 10^3 \text{ N}\cdot\text{m}$

$$\sigma = \frac{M}{S} = \frac{24 \times 10^3}{151 \times 10^{-6}} = 158.9 \times 10^6 \text{ Pa} = 158.9 \text{ MPa}$$

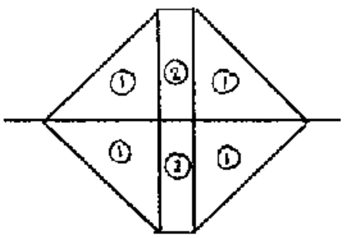
$$\frac{1}{\rho} = \frac{M}{EI} = \frac{24 \times 10^3}{(200 \times 10^9)(15.3 \times 10^{-6})} = 7.84 \times 10^{-3} \text{ m}^{-1}$$

$$\rho = 127.5 \text{ m}$$

Problem 4.27



4.27 A portion of a square bar is removed by milling, so that its cross section is as shown. The bar is then bent about its horizontal axis by a couple **M**. Considering the case where $h = 0.9h_0$, express the maximum stress in the bar in the form $\sigma_m = k\sigma_0$ where σ_0 is the maximum stress that would have occurred if the original square bar had been bent by the same couple **M**, and determine the value of k.



$$I = 4I_1 + 2I_2$$

$$= (4)\left(\tfrac{1}{12}\right)h\,h^3 + (2)\left(\tfrac{1}{3}\right)(2h_0 - 2h)(h^3)$$

$$= \tfrac{1}{3}h^4 + \tfrac{4}{3}h_0h^3 - \tfrac{4}{3}h\,h^3 = \tfrac{4}{3}h_0h^3 - h^4$$

$$c = h$$

$$\sigma_1 = \frac{Mc}{I} = \frac{Mh}{\frac{4}{3}h_0h^3 - h^4} = \frac{3M}{(4h_0 - 3h)h^2}$$

For the original square $h = h_0$, $c = h_0$

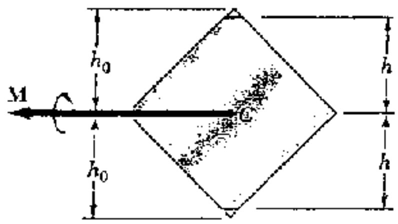
$$\sigma_0 = \frac{3M}{(4h_0 - 3h_0)h_0^2} = \frac{3M}{h_0^3}$$

$$\frac{\sigma}{\sigma_0} = \frac{h_0^3}{(4h_0 - 3h)h^2} = \frac{h_0^3}{(4h_0 - (3)(0.9)h_0)(0.9h_0^2)} = 0.950$$

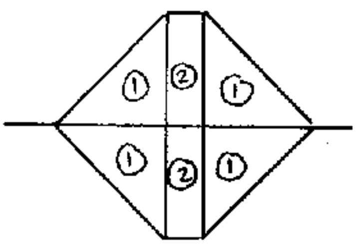
$$\sigma = 0.950\,\sigma_0 \qquad k = 0.950$$

Problem 4.28

4.28 In Prob. 4.27, determine (*a*) the value of h for which the maximum stress σ_m is as small as possible, (*b*) the corresponding value of k.



4.27 A portion of a square bar is removed by milling, so that its cross section is as shown. The bar is then bent about its horizontal axis by a couple **M**. Considering the case where $h = 0.9h_0$, express the maximum stress in the bar in the form $\sigma_m = k\sigma_0$ where σ_0 is the maximum stress that would have occurred if the original square bar had been bent by the same couple **M**, and determine the value of k.



$$I = 4I_1 + 2I_2$$

$$= (4)(\tfrac{1}{12})hh^3 + (2)(\tfrac{1}{3})(2h_0 - 2h)h^3$$

$$= \tfrac{1}{3}h^4 - \tfrac{4}{3}h_0h^3 - \tfrac{4}{3}h^3 \quad = \quad \tfrac{4}{3}h_0h^3 - h^4$$

$$c = h \qquad \frac{I}{c} = \tfrac{4}{3}h_0h^2 - h^3$$

$\frac{I}{c}$ is maximum at $\frac{d}{dh}\left[\tfrac{4}{3}h_0h^2 - h^3\right] = 0$

$$\tfrac{8}{3}h_0h - 3h^2 = 0 \qquad h = \tfrac{8}{9}h_0$$

$$\frac{I}{c} = \tfrac{4}{3}h_0(\tfrac{8}{9}h_0)^2 - (\tfrac{8}{9}h_0)^3 = \frac{256}{729}h_0^3 \qquad \sigma = \frac{Mc}{I} = \frac{729}{256}\frac{M}{h_0^3}$$

For the original square $\quad h = h_0 \quad c = h_0 \quad \frac{I_0}{c_0} = \frac{1}{3}h_0^3$

$$\sigma_0 = \frac{Mc_0}{I_0} = \frac{3M}{h_0^3}$$

$$\frac{\sigma}{\sigma_0} = \frac{729}{256}\cdot\frac{1}{3} = \frac{729}{768} = 0.949 \qquad k = 0.949$$

Problem 4.29

4.29 For the bar and loading of Example 4.01, determine (a) the radius of curvature ρ, (b) the radius of curvature ρ' of a transverse cross section, (c) the angle between the sides of the bar that were originally vertical. Use $E = 29 \times 10^6$ psi and $\nu = 0.29$.

From Example 4.01 $\quad M = 30$ kip·in, $\quad I = 1.042$ in^4

(a) $\dfrac{1}{\rho} = \dfrac{M}{EI} = \dfrac{(30 \times 10^3)}{(29 \times 10^6)(1.042)} = 993 \times 10^{-6}$ in^{-1} $\qquad \rho = 1007$ in.

(b) $\varepsilon' = \nu \varepsilon = \dfrac{\nu c}{\rho} = \nu \dfrac{c}{\rho'}$

$\dfrac{1}{\rho'} = \nu \dfrac{1}{\rho} = (0.29)(993 \times 10^{-6})$ in^{-1} $= 288$ in^{-1} $\qquad \rho' = 3470$ in.

(c) $\theta = \dfrac{\text{length of arc}}{\text{radius}} = \dfrac{b}{\rho'} = \dfrac{0.8}{3470} = 230 \times 10^{-6}$ rad $= 0.01320°$

Problem 4.30

4.30 For the aluminum bar and loading of Sample Problem 4.1, determine (*a*) the radius of curvature ρ' of a transverse cross section, (*b*) the angle between the sides of the bar that were originally vertical. Use $E = 10.6 \times 10^6$ psi and $\nu = 0.33$.

From Sample Problem 4.1 $\quad I = 12.97$ in^4 $\quad M = 103.8$ kip·in

$\dfrac{1}{\rho} = \dfrac{M}{EI} = \dfrac{103.8 \times 10^3}{(10.6 \times 10^6)(12.97)} = 755 \times 10^{-6}$ in^{-1}

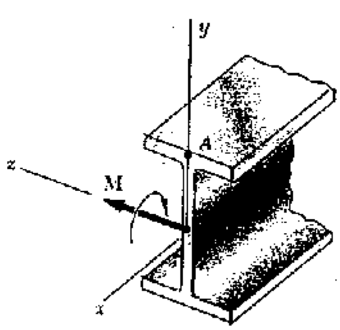
(a) $\dfrac{1}{\rho'} = \nu \dfrac{1}{\rho} = (0.33)(755 \times 10^{-6}) = 249 \times 10^{-6}$ in^{-1}

$\rho' = 4010$ in. $= 334$ ft.

(b) $\theta = \dfrac{\text{length of arc}}{\text{radius}} = \dfrac{b}{\rho'} = \dfrac{3.25}{4010} = 810 \times 10^{-6}$ rad $= 0.0464°$

Problem 4.31

4.31 A W200 × 31.3 rolled-steel beam is subjected to a couple **M** of moment 45 kN · m. Knowing that $E = 200$ GPa and $\nu = 0.29$, determine (*a*) the radius of curvature ρ, (*b*) the radius of curvature ρ' of a transverse cross section.



For W 200 × 31.3 rolled steel section

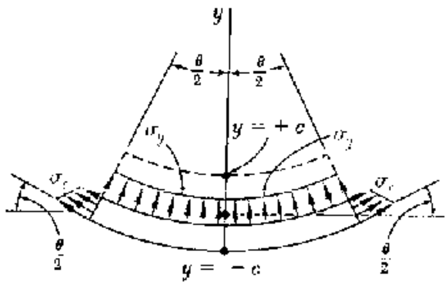
$I = 31.4 \times 10^6$ mm^4 $= 31.4 \times 10^{-6}$ m^4

(a) $\dfrac{1}{\rho} = \dfrac{M}{EI} = \dfrac{45 \times 10^3}{(200 \times 10^9)(31.4 \times 10^{-6})} = 7.17 \times 10^{-3}$ m^{-1}

$\rho = 139.6$ m

(b) $\dfrac{1}{\rho'} = \nu \dfrac{1}{\rho} = (0.29)(7.17 \times 10^{-3}) = 2.07 \times 10^{-3}$ m^{-1} $\qquad \rho' = 481$ m

Problem 4.32



4.32 It was assumed in Sec. 4.3 that the normal stresses σ_y in a member in pure bending are negligible. For an initially straight elastic member of rectangular cross section, (*a*) derive an approximate expression for σ_y as a function of y, (*b*) show that $(\sigma_y)_{max} = -(c/2\rho)(\sigma_x)_{max}$ and, thus, that σ_y can be neglected in all practical situations. (*Hint*: Consider the free-body diagram of the portion of beam located below the surface of ordinate y and assume that the distribution of the stress σ_x is still linear.)

Denote the width of the beam by b and the length by L.

$$\theta = \frac{L}{\rho}$$

Using the free body diagram above, with $\cos\frac{\theta}{2} \approx 1$

$$\Sigma F_y = 0 \qquad \sigma_y b L + 2\int_{-c}^{y} \sigma_x b\,dy \sin\frac{\theta}{2} = 0$$

$$\sigma_y = -\frac{2}{L}\sin\frac{\theta}{2}\int_{-c}^{y}\sigma_x\,dy \approx -\frac{\theta}{L}\int_{-c}^{y}\sigma_x\,dy = -\frac{1}{\rho}\int_{-c}^{y}\sigma_x\,dy$$

But $\quad \sigma_x = -(\sigma_x)_{max}\dfrac{y}{c}$

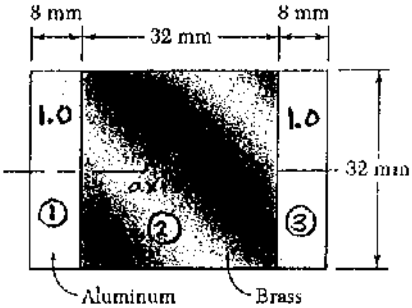
(a) $$\sigma_y = \frac{(\sigma_x)_{max}}{\rho c}\int_{-c}^{y} y\,dy = \frac{(\sigma_x)_{max}}{\rho c}\left.\frac{y^2}{2}\right|_{-c}^{y} = \frac{(\sigma_x)_{max}}{2\rho c}(y^2 - c^2)$$

The maximum value σ_y occurs at $y = 0$

(b) $$(\sigma_y)_{max} = -\frac{(\sigma_x)_{max}c^2}{2\rho c} = -\frac{(\sigma_x)_{max}c}{2\rho}$$

Problem 4.33

4.33 and 4.34 A bar having the cross section shown has been formed by securely bonding brass and aluminum stock. Using the data given below, determine the largest permissible bending moment when the composite bar is bent about a horizontal axis.



	Aluminum	Brass
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

Use aluminum as the reference material.

For aluminum $n = 1.0$

For brass $n = E_b/E_a = 105/70 = 1.5$

Values of n are shown on the figure.

For the transformed section

$$I_1 = \frac{n_1}{12} b_1 h_1^3 = \frac{1.0}{12}(8)(32)^3 = 21.8453 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 = \frac{1.5}{12}(32)(32)^3 = 131.072 \times 10^3 \text{ mm}^4$$

$$I_3 = I_1 = 21.8453 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 174.7626 \times 10^3 \text{ mm}^4 = 174.7626 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad M = \left|\frac{\sigma I}{ny}\right|$$

Aluminum: $n = 1.0$, $|y| = 16 \text{ mm} = 0.016 \text{ m}$, $\sigma = 100 \times 10^6$ Pa

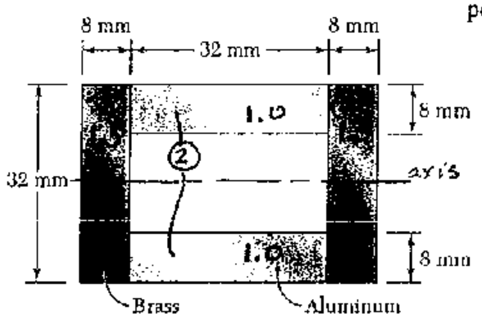
$$M = \frac{(100 \times 10^6)(174.7626 \times 10^{-9})}{(1.0)(0.016)} = 1.0923 \times 10^3 \text{ N}\cdot\text{m}$$

Brass: $n = 1.5$, $|y| = 16 \text{ mm} = 0.016 \text{ m}$, $\sigma = 160 \times 10^6$ Pa

$$M = \frac{(160 \times 10^6)(174.7626 \times 10^{-9})}{(1.5)(0.016)} = 1.1651 \times 10^3 \text{ N}\cdot\text{m}$$

Choose the smaller value. $M = 1.092 \times 10^3$ N-m $\qquad$ $M = 1.092$ kN-m

Problem 4.34



4.33 and 4.34 A bar having the cross section shown has been formed by securely bonding brass and aluminum stock. Using the data given below, determine the largest permissible bending moment when the composite bar is bent about a horizontal axis.

	Aluminum	Brass
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

Use aluminum as the reference material

For aluminum $n = 1.0$

For brass $n = E_b/E_a = 105/70 = 1.5$

Values of n are shown on the sketch.

For the tranformed section

$I_1 = \frac{n_1}{12} b_1 h_1^3 = \frac{1.5}{12}(8)(32)^3 = 32.768 \times 10^3 \text{ mm}^4$

$I_2 = \frac{n_2}{12} b_2 (H_2^3 - h_2^3) = \frac{1.0}{12}(32)(32^3 - 16^3) = 76.459 \times 10^3 \text{ mm}^4$

$I_3 = I_1 = 32.768 \times 10^3 \text{ mm}^4$

$I = I_1 + I_2 + I_3 = 141.995 \times 10^3 \text{ mm}^4 = 141.995 \times 10^{-9} \text{ m}^4$

$|\sigma| = \left|\frac{nMy}{I}\right| \qquad M = \left|\frac{\sigma I}{ny}\right|$

Aluminum: $n = 1.0$, $|y| = 16 \text{ mm} = 0.016 \text{ m}$, $\sigma = 100 \times 10^6$ Pa

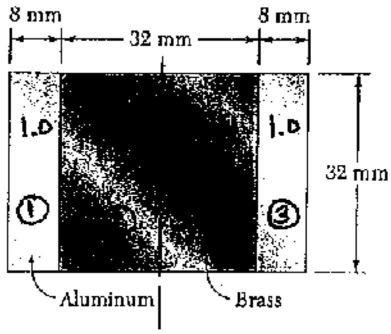
$$M = \frac{(100 \times 10^6)(141.995 \times 10^{-9})}{(1.0)(0.016)} = 887.47 \text{ N}\cdot\text{m}$$

Brass: $n = 1.5$, $|y| = 16 \text{ mm} = 0.016 \text{ m}$, $\sigma = 160 \times 10^6$ Pa

$$M = \frac{(160 \times 10^6)(141.995 \times 10^{-9})}{(1.5)(0.016)} = 946.63 \text{ N}\cdot\text{m}$$

Choose the smaller value. $M = 887 \text{ N}\cdot\text{m}$

Problem 4.35



4.35* and *4.36 **For the composite bar indicated, determine the largest permissible bending moment when the bar is bent about a <u>vertical</u> axis.**

4.35 Bar of Prob. 4.33.

	Aluminum	Brass
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

Use aluminum as the reference material

For aluminum, $n = 1.0$

For brass, $n = E_b/E_a = 105/70 = 1.5$

Values of n are shown on the figure.

For the transformed section

$$I_1 = \frac{n_1}{12} h_1 b_1^3 + n_1 A_1 d_1^2 = \frac{1.0}{12}(32)(8)^3 + (1.0)[(32)(8)](20)^2 = 103.7653 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} h_2 b_2^3 = \frac{1.5}{12}(32)(32)^3 = 131.072 \times 10^3 \text{ mm}^4$$

$$I_3 = I_1 = 103.7653 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 338.58 \times 10^3 \text{ mm}^4 = 338.58 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \frac{nMy}{I} \qquad M = \left|\frac{\sigma I}{ny}\right|$$

Aluminum: $n = 1.0$, $|y| = 24 \text{ mm} = 0.024 \text{ m}$, $\sigma = 100 \times 10^6$ Pa

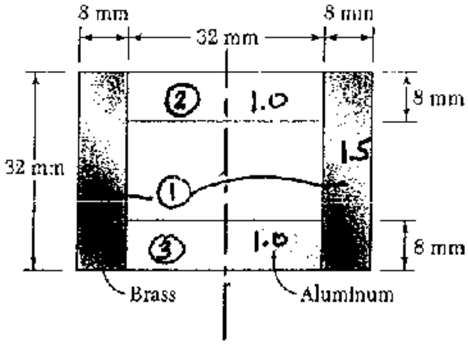
$$M = \frac{(100 \times 10^6)(338.58 \times 10^{-9})}{(1.0)(0.024)} = 1.411 \times 10^3 \text{ N}\cdot\text{m}$$

Brass: $n = 1.5$, $|y| = 16 \text{ mm} = 0.016 \text{ m}$, $\sigma = 160 \times 10^6$ Pa

$$M = \frac{(160 \times 10^6)(338.58 \times 10^{-9})}{(1.5)(0.016)} = 2.257 \times 10^3 \text{ N}\cdot\text{m}$$

Choose the smaller value. $M = 1.4 \quad \times 10^3$ N·m $\qquad M = 1.411$ kN·m ◄

Problem 4.36



4.35* and *4.36 For the composite bar indicated, determine the largest permissible bending moment when the bar is bent about a vertical axis.

4.36 Bar of Prob. 4.34

	Aluminum	Brass
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

Use aluminum as the reference material.

For aluminum $n = 1.0$

For brass $n = E_b/E_a = 105/70 = 1.5$

Values of n and shown on the sketch.

For the transformed section

$$I_1 = \frac{n_1}{12} h_1 (B_1^3 - b_1^3) = \frac{1.5}{12}(32)(48^3 - 32^3) = 311.296 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} h_2 b_2^3 = \frac{1.0}{12}(8)(32)^3 = 21.8453 \times 10^3 \text{ mm}^4$$

$$I_3 = I_2 = 21.8453 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 354.99 \times 10^3 \text{ mm}^4 = 354.99 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad M = \left|\frac{\sigma I}{ny}\right|$$

Aluminum: $n = 1.0$, $|y| = 16 \text{ mm} = 0.016 \text{ m}$ $\quad \sigma = 100 \times 10^6 \text{ Pa}$

$$M = \frac{(100 \times 10^6)(354.99 \times 10^{-9})}{(1.0)(0.016)} = 2.2187 \times 10^3 \text{ N}\cdot\text{m}$$

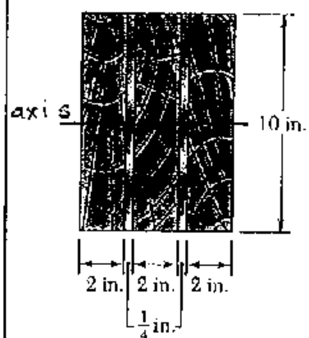
Brass: $n = 1.5$ $\quad |y| = 24 \text{ mm} = 0.024 \text{ m}$ $\quad \sigma = 160 \times 10^6 \text{ Pa}$

$$M = \frac{(160 \times 10^6)(354.99 \times 10^{-9})}{(1.5)(0.024)} = 1.57773 \times 10^3 \text{ N}\cdot\text{m}$$

Choose the smaller value. $\quad M = 1.57773 \times 10^3 \text{ N}\cdot\text{m}$ $\qquad M = 1.578 \text{ kN}\cdot\text{m}$ ◄

Problem 4.37

4.37 Three wooden beams and two steel plates are securely bolted together to form the composite member shown. Using the data given below, determine the largest permissible bending moment when the member is bent about a horizontal axis.



	Wood	Steel
Modulus of elasticity	2×10^6 psi	30×10^6 psi
Allowable stress	2000 psi	22,000 psi

Use wood as the reference material

For wood $n_w = 1$

For steel $n = E_s/E_w = 30/2 = 15$

Properties of the geometric section.

Steel: $I_s = \frac{1}{12}(\frac{1}{4}+\frac{1}{4})(10)^3 = 41.6667 \text{ in}^4$

Wood: $I_w = \frac{1}{12}(2+2+2)(10^3) = 500 \text{ in}^4$

Transformed section

$I_{trans} = n_s I_s + n_w I_w = (15)(41.6667) + (1)(500) = 1125 \text{ in}^4$

$|\sigma| = \left|\frac{nMy}{I}\right| \qquad M = \left|\frac{\sigma I}{ny}\right|$

Wood: $n = 1 \quad |y| = 5 \text{ in.} \quad \sigma = 2000 \text{ psi}$

$M = \frac{(2000)(1125)}{(1)(5)} = 450 \times 10^3 \text{ lb·in}$

Steel: $n = 15 \quad |y| = 5 \text{ in} \quad \sigma = 22000 \text{ psi}$

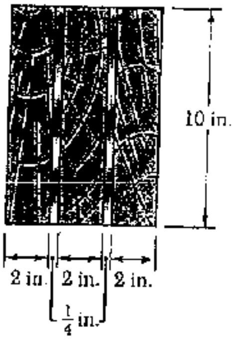
$M = \frac{(22000)(1125)}{(15)(5)} = 330 \times 10^3 \text{ lb·in}$

Choose the smaller value $M = 330 \times 10^3 \text{ lb·in}$

$M = 330 \text{ kip·in}$ ◄

Problem 4.38

4.38 For the composite member of Prob. 4.37, determine the largest permissible bending moment when the member is bent about a vertical axis.



4.37 Three wooden beams and two steel plates are securely bolted together to form the composite member shown. Using the data given below, determine the largest permissible bending moment when the member is bent about a horizontal axis.

	Wood	Steel
Modulus of elasticity	2×10^6 psi	30×10^6 psi
Allowable stress	2000 psi	22,000 psi

Use wood as the reference material

For wood $\quad n_w = 1$

For steel $\quad n_s = E_s/E_w = 30/2 = 15$

Properties of the geometric section

Total: $\quad I_t = \frac{1}{12} h b^3 = \frac{1}{12}(10)(6.5)^3 = 228.854 \text{ in}^4$

Steel: $\quad I_s = \frac{1}{12} h(B^3 - b^3) = \frac{1}{12}(10)(2.5^3 - 2^3) = 6.3542 \text{ in}^4$

Wood: $\quad I_w = I_t - I_s = 222.5 \text{ in}^4$

Transformed section

$$I_{trans} = n_s I_s + n_w I_w = (15)(6.3542) + (1)(222.5) = 317.81 \text{ in}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad M = \left|\frac{\sigma I}{ny}\right|$$

Wood: $\quad n = 1 \quad |y| = 3.25 \text{ in.} \quad \sigma = 2000 \text{ psi}$

$$M = \frac{(2000)(317.81)}{(1)(3.25)} = 195.6 \times 10^3 \text{ lb}\cdot\text{in}$$

Steel: $\quad n = 15 \quad |y| = 1.25 \text{ in.} \quad \sigma = 22000 \text{ psi}$

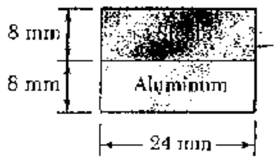
$$M = \frac{(22000)(317.81)}{(15)(1.25)} = 372.90 \times 10^3 \text{ lb}\cdot\text{in}$$

Choose the smaller value. $\quad M = 195.6 \times 10^3 \text{ lb}\cdot\text{in}$

$M = 195.6 \text{ kip}\cdot\text{in}$ ◄

Problem 4.39

4.39 and 4.40 A steel bar ($E_s = 210$ GPa) and an aluminum bar ($E_a = 70$ GPa) are bonded together to form the composite bar shown. Determine the maximum stress in (*a*) the aluminum, (b) the steel, when the bar is bent about a horizontal axis with $M = 60$ N · m.

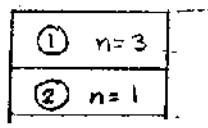


Use aluminum as the reference material.

For aluminum $n = 1$

For steel $n = E_s/E_a = 210/70 = 3$

Transformed section



	A, mm^2	nA, mm^2	$\bar{y}_o, mm$	$nA\bar{y}_o, mm^3$
①	192	576	12	6912
②	192	192	4	768
Σ		768		7680

$$\bar{Y}_o = \frac{7680}{768} = 10 \text{ mm}$$

The neutral axis lies 10 mm above the bottom.

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{3}{12}(24)(8)^3 + (576)(2)^2 = 5.376 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1}{12}(24)(8)^3 + (192)(6)^3 = 7.936 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 13.312 \times 10^3 \text{ mm}^4 = 13.312 \times 10^{-9} \text{ m}^4$$

$$M = 60 \text{ N·m}$$

$$\sigma = -\frac{nMy}{I}$$

(a) Aluminum $n = 1$, $y = -10 \text{ mm} = -0.010 \text{ m}$

$$\sigma_a = -\frac{(1)(60)(-0.010)}{13.312 \times 10^{-9}} = 45.072 \times 10^6 \text{ Pa}$$

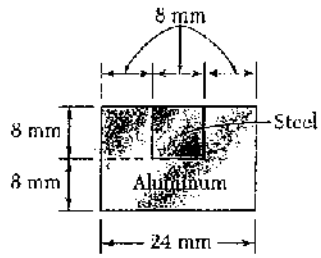
$\sigma_a = 45.1$ MPa ◄

(b) Steel $n = 3$, $y = 6 \text{ mm} = 0.006 \text{ m}$

$$\sigma_s = -\frac{(3)(60)(0.006)}{13.312 \times 10^{-9}} = 81.130 \times 10^6 \text{ Pa}$$

$\sigma_s = -81.1$ MPa ◄

Problem 4.40



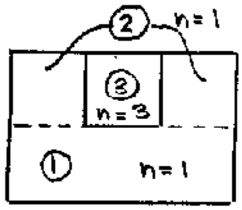
4.39 and 4.40 A steel bar (E_s = 210 GPa) and an aluminum bar (E_a = 70 GPa) are bonded together to form the composite bar shown. Determine the maximum stress in (*a*) the aluminum, (b) the steel, when the bar is bent about a horizontal axis with M = 60 N · m.

Use aluminum as the reference material.

For aluminum $n = 1$

For steel $n = E_s/E_a = 210/70 = 3$

Transformed section



	A, mm^2	nA, mm^2	$\bar{y}_o$, mm	$nA\bar{y}_o$, mm^3
1	192	192	4	768
2	128	128	12	1536
3	64	192	12	2304
		512		4608

$\bar{Y}_o = \frac{4608}{512} = 9$ mm   The neutral axis lies 9 mm above the bottom.

$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1}{12}(24)(8)^3 + (192)(5)^2 = 5.824 \times 10^3$ mm^4

$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1}{12}(16)(8)^3 + (128)(3)^2 = 1.83467 \times 10^3$ mm^4

$I_3 = \frac{n_3}{12} b_3 h_3^3 + n_3 A_3 d_3^2 = \frac{3}{12}(8)(8)^3 + (192)(3)^2 = 2.752 \times 10^3$ mm^4

$I = I_1 + I_2 + I_3 = 10.41067 \times 10^3$ mm^4 $= 10.4106 \times 10^{-9}$ m^4

$M = 60$ N·m

$$\sigma = -\frac{nMy}{I}$$

(a) Aluminum $n = 1$, $y = -9$ mm $= -0.009$ m

$$\sigma_a = -\frac{(1)(60)(-0.009)}{10.41067 \times 10^{-9}} = 51.870 \times 10^6 \text{ Pa}$$

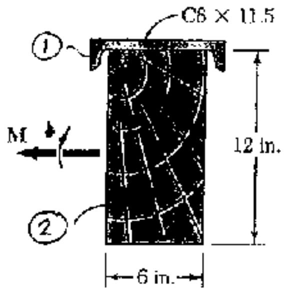
$\sigma_a = 51.9$ MPa ◄

(b) Steel $n = 3$, $y = 7$ mm $= 0.007$ m

$$\sigma_s = -\frac{(3)(60)(0.007)}{10.41067 \times 10^{-9}} = -121.03 \times 10^6 \text{ Pa}$$

$\sigma_s = -121.0$ MPa ◄

Problem 4.41

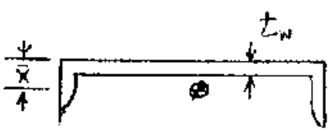


4.41 and 4.42 The 6 × 20-in. timber bean has been strengthened by bolting to it the steel reinforcement shown. The modulus of elasticity for wood is 1.8×10^6 psi and for steel 29×10^6 psi. knowing that the beam is bent about a horizontal axis by a couple of moment $M = 450$ kip · in., determine the maximum stress in (*a*) the wood, (*b*) the steel.

Use wood as the reference material.

For wood $\quad n = 1$

For steel $\quad n = E_s/E_w = 29/1.8 = 16.1111$



For C8×11.5 channel section, $A = 3.38\ in^2$

$t_w = 0.220$ in., $\bar{x} = 0.571$ in., $I_y = 1.32\ in^4$

For the composite section the centroid of the channel lies $12 + 0.220 - 0.571 = 11.649$ in. above the base. $\bar{y}_o = 11.649$ in. for channel.

Transformed section

$$\bar{Y}_o = \frac{1066.353}{126.456} = 8.433 \text{ in.}$$

The neutral axis lies 8.433 in. above the bottom.

	A, in^2	nA, in^2	$\bar{y}_o$, in.	$nA\bar{y}_o$, in^3
①	3.38	54.456	11.649	634.353
②	72	72	6	432
		126.456		1066.353

$$I_1 = n_1 \bar{I}_x + n_1 A_1 d_1^2 = (16.1111)(1.32) + (54.456)(11.649 - 8.433)^2 = 584.49 \text{ in}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1}{12}(6)(12)^3 + (72)(8.433 - 6)^2 = 1290.20 \text{ in}^4$$

$$I = I_1 + I_2 = 1874.69 \text{ in}^4$$

$M = 450$ kip · in $\qquad \sigma = -\frac{nMy}{I}$

(a) Wood: $\quad n = 1, \quad y = -8.433$ in.

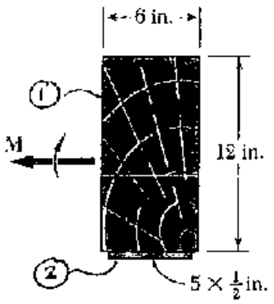
$$\sigma_w = -\frac{(450)(-8.433)}{1874.69} = 2.02 \text{ ksi} \qquad \sigma_w = 2.02 \text{ ksi}$$

(b) Steel: $\quad n = 16.1111 \qquad y = 12 + 0.220 - 8.433 = 3.787$ in.

$$\sigma_s = -\frac{(16.1111)(450)(3.787)}{1874.69} = -14.65 \text{ ksi} \qquad \sigma_s = -14.65 \text{ ksi}$$

Problem 4.42

4.41 and 4.42 The 6 × 20-in. timber bean has been strengthened by bolting to it the steel reinforcement shown. The modulus of elasticity for wood is 1.8×10^6 psi and for steel 29×10^6 psi. knowing that the beam is bent about a horizontal axis by a couple of moment $M = 450$ kip · in., determine the maximum stress in (*a*) the wood, (*b*) the steel.



Use wood as the reference material.

For wood $n = 1$

For steel $n = E_s/E_w = 29/1.8 = 16.1111$

Transformed section

	A, in^2	nA, in^2	$\bar{y}_o$	$nA\bar{y}_o$, in^3
1	72	72	6	432
2	2.5	40.278	−0.25	−10.069
		112.278		421.931

$\bar{Y}_o = \dfrac{421.931}{112.278} = 3.758$ in.

The neutral axis lies 3.758 in. above the wood-steel interface.

$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1}{12}(6)(12)^3 + (72)(6 - 3.758)^2 = 1225.91$ in^4

$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{16.1111}{12}(5)(0.5)^3 + (40.278)(3.758 + 0.25)^2$
$= 647.87$ in^4

$I = I_1 + I_2 = 1873.77$ in^4

$M = 450$ kip · in $\qquad \sigma = -\dfrac{nMy}{I}$

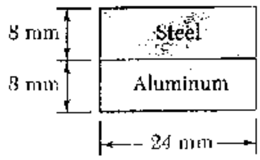
(a) Wood: $n = 1 \qquad y = 12 - 3.758 = 8.242$ in.

$\sigma_w = -\dfrac{(1)(450)(8.242)}{1873.77} = -1.979$ ksi $\qquad \sigma_w = -1.979$ ksi ◄

(b) Steel: $n = 16.1111 \qquad y = -3.758 - 0.5 = -4.258$ in.

$\sigma_s = -\dfrac{(16.1111)(450)(-4.258)}{1873.77} = 16.48$ ksi $\qquad \sigma_s = 16.48$ ksi ◄

Problem 4.43



4.43 and 4.44 For the composite bar indicated, determine the radius of curvature caused by the couple of moment 60 N · m.

4.43 Bar of Prob. 4.39

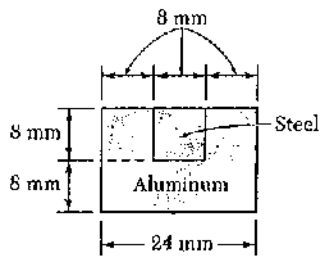
See solution to Problem 4.39 for calculation of I.

$I = 13.312 \times 10^{-9}\ m^4 \qquad E_a = 70 \times 10^9\ Pa$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{60}{(70 \times 10^9)(13.312 \times 10^{-9})} = 64.39 \times 10^{-3}\ m^{-1}$$

$\rho = 15.53\ m$

Problem 4.44



4.43 and 4.44 For the composite bar indicated, determine the radius of curvature caused by the couple of moment 60 N · m.

4.44 Bar of Prob. 4.40

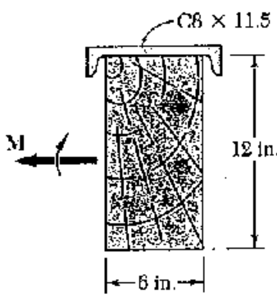
See solution to Problem 4.40 for calculation of I.

$I = 10.4106 \times 10^{-9}\ m^4 \qquad E_a = 70 \times 10^9\ Pa$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{60}{(70 \times 10^9)(10.4106 \times 10^{-9})} = 82.33\ m^{-1}$$

$\rho = 12.15\ m$

Problem 4.45



4.45* and *4.46 For the composite beam indicated, determine the radius of curvature caused by the couple of moment 450 kip · in.

4.45 Bar of Prob. 4.41

See solution to Problem 4.41 for calculation of I.

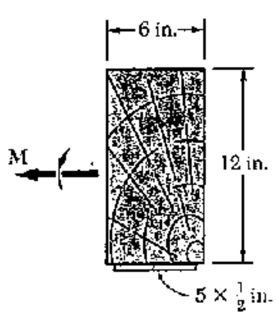
$I = 1874.69\ in^4 \qquad E_w = 1.8 \times 10^6\ psi$

$M = 450\ kip \cdot in = 450 \times 10^3\ lb \cdot in$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{450 \times 10^3}{(1.8 \times 10^6)(1874.69)} = 133.36 \times 10^{-6}\ in^{-1}$$

$\rho = 7499\ in. = 625\ ft$

Problem 4.46



4.45* and *4.46 For the composite beam indicated, determine the radius of curvature caused by the couple of moment 450 kip · in.

4.46 Bar of Prob. 4.42

See solution to Problem 4.42 for calculation of I.

$I = 1873.77\ in^4 \qquad E_w = 1.8 \times 10^6\ psi$

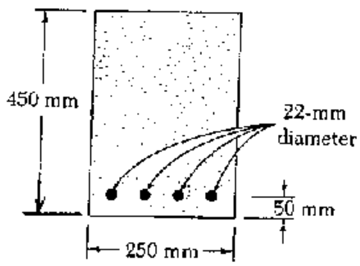
$M = 450\ kip \cdot in = 450 \times 10^3\ lb \cdot in$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{450 \times 10^3}{(1.8 \times 10^6)(1873.77)} = 133.42 \times 10^{-6}\ in^{-1}$$

$\rho = 7495\ in. = 625\ ft$

Problem 4.47

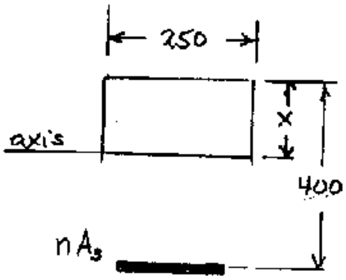
4.47 The reinforced concrete beam shown is subjected to a positive bending moment of 175 kN · m. knowing that the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel, determine (*a*) the stress in the steel, (*b*) the maximum stress in the concrete.



$$n = \frac{E_s}{E_c} = \frac{200\ \text{GPa}}{25\ \text{GPa}} = 8.0$$

$$A_s = 4 \cdot \frac{\pi}{4} d^2 = (4)\left(\frac{\pi}{4}\right)(22)^2 = 1.5205 \times 10^3\ \text{mm}^2$$

$$nA_s = 12.164 \times 10^3\ \text{mm}^2$$



Locate the neutral axis

$$250\, x \frac{x}{2} - (12.164 \times 10^3)(400 - x) = 0$$

$$125 x^2 + 12.164 \times 10^3 x - 4.8657 \times 10^6 = 0$$

Solving for x $\quad x = \dfrac{-12.164 \times 10^3 + \sqrt{(12.164 \times 10^3)^2 + (4)(125)(4.8657 \times 10^6)}}{(2)(125)}$

$$x = 154.55\ \text{mm}, \qquad 400 - x = 245.45\ \text{mm}$$

$$I = \frac{1}{3}\, 250\, x^3 + (12.164 \times 10^3)(400 - x)^2$$

$$= \frac{1}{3}(250)(154.55)^3 + (12.164 \times 10^3)(245.45)^2$$

$$= 1.0404 \times 10^9\ \text{mm}^4 = 1.0404 \times 10^{-3}\ \text{m}^4$$

$$\sigma = -\frac{nMy}{I}$$

(a) Steel: $\quad y = -245.45\ \text{mm} = -0.24545\ \text{m}$

$$\sigma = -\frac{(8.0)(175 \times 10^3)(-0.24545)}{1.0404 \times 10^{-3}} = 330 \times 10^6\ \text{Pa} = 330\ \text{MPa} \quad \blacktriangleleft$$

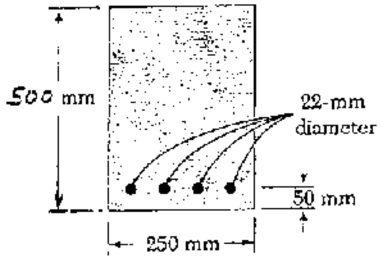
(b) Concrete: $\quad y = 154.55\ \text{mm} = 0.15455\ \text{m}$

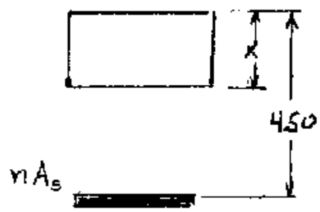
$$\sigma = -\frac{(1.0)(175 \times 10^3)(0.15455)}{1.0404 \times 10^{-3}} = -26.0 \times 10^6\ \text{Pa} = -26.0\ \text{MPa} \quad \blacktriangleleft$$

Problem 4.48

4.48 Solve Prob. 4.47, assuming that the 450-mm depth is increased to 500 mm.

4.47 The reinforced concrete beam shown is subjected to a positive bending moment of 175 kN · m. knowing that the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel, determine (*a*) the stress in the steel, (*b*) the maximum stress in the concrete.





$$n = \frac{E_s}{E_c} = \frac{200\text{ GPa}}{25\text{ GPa}} = 8.0$$

$$A_s = 4\,\frac{\pi}{4}d^2 = (4)\left(\frac{\pi}{4}\right)(22)^2 = 1.5205\times10^3\text{ mm}^2$$

$$nA_s = 12.164\times10^3\text{ mm}^2$$

Locate the neutral axis

$$250\,x\,\frac{x}{2} - (12.164\times10^3)(450-x) = 0$$

$$125x^2 + 12.164\times10^3 x - 5.4738\times10^6 = 0$$

Solving for x

$$x = \frac{-12.164\times10^3 + \sqrt{(12.164\times10^3)^2 + (4)(125)(5.4738\times10^6)}}{(2)(125)}$$

$$x = 166.19\text{ mm}, \qquad 450 - x = 283.81\text{ mm}$$

$$I = \frac{1}{3}(250)x^3 + (12.164\times10^3)(450-x)^2$$

$$= \frac{1}{3}(250)(166.19)^3 + (12.164\times10^3)(283.81)^2$$

$$= 1.3623\times10^9\text{ mm}^4 = 1.3623\times10^{-3}\text{ m}^4$$

$$\sigma = -\frac{nMy}{I}$$

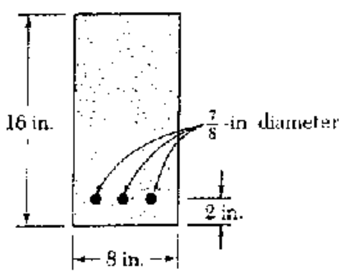
(a) Steel: $y = -283.81\text{ mm} = -0.28381\text{ m}$

$$\sigma = -\frac{(8.0)(175\times10^3)(-0.28381)}{1.3623\times10^{-3}} = 292\times10^6\text{ Pa} = 292\text{ MPa}$$

(b) Concrete: $y = 166.19\text{ mm} = 0.16619\text{ m}$

$$\sigma = -\frac{(1.0)(175\times10^3)(0.16619)}{1.3623\times10^{-3}} = -21.3\times10^6\text{ Pa} = -21.3\text{ MPa}$$

Problem 4.49

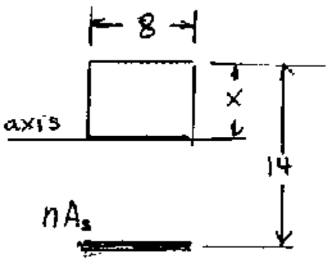


4.49 A concrete beam is reinforced by three steel rods placed as shown. The modulus of elasticity is 3×10^6 psi for the concrete and 30×10^6 psi for the steel. Using an allowable stress os 1350 psi for the concrete and 20 ksi for the steel, determine the largest allowable positive bending moment in the beam.

$$n = \frac{E_s}{E_c} = \frac{30\times10^6}{3\times10^6} = 10$$

$$A_s = 3\,\frac{\pi}{4}d^2 = 3\left(\frac{\pi}{4}\right)\left(\frac{7}{8}\right)^2 = 1.8040 \text{ in}^2$$

$$nA_s = 18.040 \text{ in}^2$$



Locate neutral axis.

$$8x\frac{x}{2} - (18.040)(14-x) = 0$$

$$4x^2 + 18.040\,x - 252.56 = 0$$

Solve for x $\quad x = \dfrac{-18.040 + \sqrt{18.040^2 + (4)(4)(252.56)}}{(2)(4)} = 6.005$ in.

$$14 - x = 7.995 \text{ in}$$

$$I = \frac{1}{3}\,8\,x^3 + nA_s(14-x)^2 = \frac{1}{3}(8)(6.005)^3 + (18.040)(7.995)^2$$

$$= 1730.4 \text{ in}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \quad \therefore \quad M = \frac{\sigma I}{ny}$$

Concrete: $\quad n = 1.0, \quad |y| = 6.005$ in, $\quad |\sigma| = 1350$ psi

$$M = \frac{(1350)(1730.5)}{(1.0)(6.005)} = 389\times10^3 \text{ lb}\cdot\text{in} = 389 \text{ kip}\cdot\text{in}$$

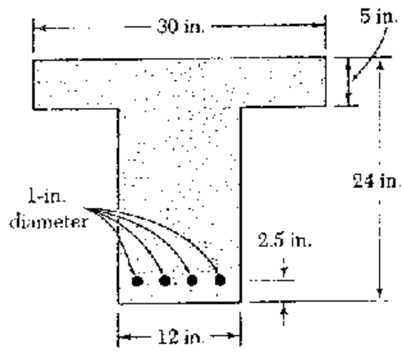
Steel: $\quad n = 10, \quad |y| = 7.995, \quad \sigma = 20\times10^3$ psi

$$M = \frac{(20\times10^3)(1730.5)}{(10)(7.995)} = 433\times10^3 \text{ lb}\cdot\text{in} = 433 \text{ kip}\cdot\text{in}$$

Choose the smaller value $\quad M = 389$ kip·in $= 32.4$ kip·ft ◄

Problem 4.50

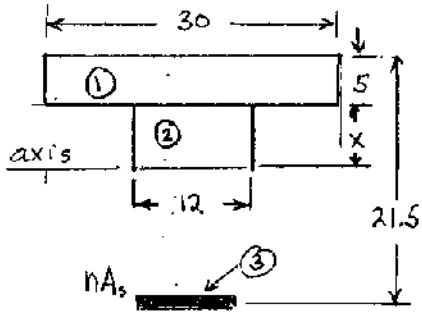
4.50 Knowing that the bending moment in the reinforced concrete beam is +150 kip · ft and that the modulus of elasticity is 3.75×10^6 psi for the concrete and 30×10^6 psi for the steel, determine (*a*) the stress in the steel, (*b*) the maximum stress in the concrete.



$$n = \frac{E_s}{E_c} = \frac{30\times10^6}{3.75\times10^6} = 8.0$$

$$A_s = 4\,\frac{\pi}{4}d^2 = 4\left(\frac{\pi}{4}\right)(1)^2 = 3.1416 \text{ in}^2$$

$$nA_s = 25.133 \text{ in}^2$$



Locate the neutral axis

$$(30)(5)(x+2.5) + 12\,x\,\frac{x}{2} - (25.133)(16.5-x) = 0$$

$$150x + 375 + 6x^2 - 414.69 + 25.133x = 0$$

$$6x^2 + 175.133x - 39.69 = 0$$

Solve for x $\quad x = \dfrac{-175.133 + \sqrt{(175.133)^2 + (4)(6)(39.69)}}{(2)(6)} = 0.225$ in.

$$16.5 - x = 16.275 \text{ in.}$$

$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(30)(5)^3 + (30)(5)(2.725)^2 = 1426.3 \text{ in}^4$$

$$I_2 = \frac{1}{3}b_2x^3 = \frac{1}{3}(12)(0.225)^3 = 0.1 \text{ in}^4$$

$$I_3 = nA_sd_3^2 = (25.133)(16.275)^2 = 6657.1 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 8083.5 \text{ in}^4$$

$\sigma = -\dfrac{nMy}{I}$ where $M = 150$ kip·ft $= 1800$ kip·in.

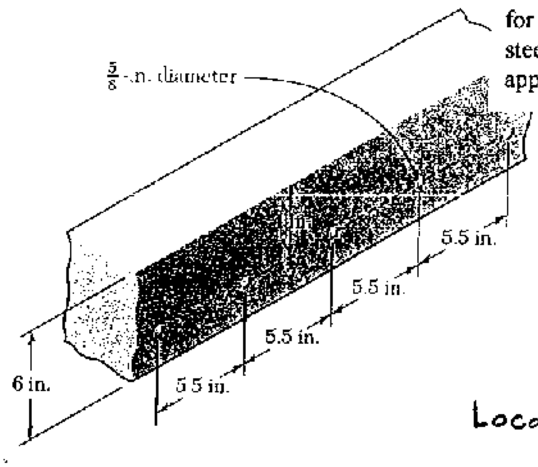
(a) Steel $\quad n = 8.0, \quad y = -16.275$ in

$$\sigma = -\frac{(8.0)(1800)(-16.275)}{8083.5} = 29.0 \text{ ksi}$$

(b) Concrete $\quad n = 1.0, \quad y = 5.225$ in

$$\sigma = -\frac{(1.0)(1800)(5.225)}{8083.5} = -1.163 \text{ ksi}$$

Problem 4.51



4.51 A concrete slab is reinforced by $\frac{5}{8}$-in.-diameter rods placed on 5.5-in. centers as shown. The modulus of elasticity is 3×10^6 psi for the concrete and 29×10^6 psi for the steel. Using an allowable stress of 1400 psi for the concrete and 20 ksi for the steel, determine the largest bending moment per foot of width that can be safely applied to the slab.

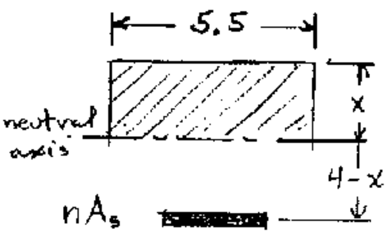
$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{3 \times 10^6} = 9.6667$$

Consider a section 5.5 in. wide.

$$A_s = \frac{\pi}{4} d_s^2 = \frac{\pi}{4}\left(\frac{5}{8}\right)^2 = 0.3068 \text{ in}^2$$

$$nA_s = 2.9657 \text{ in}^2$$

Locate the neutral axis.



$$5.5x \frac{x}{2} - (4-x)(2.9657) = 0$$

$$2.75x^2 + 2.9657x - 11.8628 = 0$$

Solving for x

$x = 1.6066$ in. $\qquad 4 - x = 2.3934$ in.

$$I = \frac{1}{3}(5.5)x^3 + (2.9657)(4-x)^2$$

$$= \frac{1}{3}(5.5)(1.6066)^3 + (2.9657)(2.3934)^2$$

$$= 24.591 \text{ in}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad M = \left|\frac{I\sigma}{ny}\right|$$

Concrete: $n = 1$, $y = 1.6066$ in., $\sigma = 1400$ psi

$$M = \frac{(24.591)(1400)}{(1.0)(1.6066)} = 21.429 \times 10^3 \text{ lb} \cdot \text{in}$$

Steel: $n = 9.6667$, $y = 2.3934$ in., $\sigma = 20$ ksi $= 20 \times 10^3$ psi

$$M = \frac{(24.591)(20 \times 10^3)}{(9.6667)(2.3934)} = 21.258 \times 10^3 \text{ lb} \cdot \text{in}$$

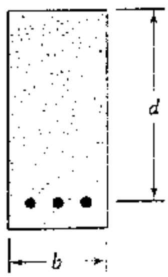
Choose the smaller value as the allowable moment for a 5.5 width.

$$M = 21.258 \times 10^3 \text{ lb} \cdot \text{in}$$

For a 1 ft = 12 in. width $\qquad M = \frac{12}{5.5}(21.258 \times 10^3) = 46.38 \times 10^3 \text{ lb} \cdot \text{in}$

$$M = 46.38 \text{ kip} \cdot \text{in} = 3.87 \text{ kip} \cdot \text{ft}$$

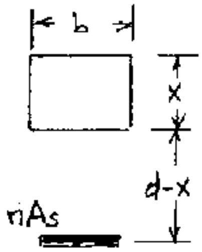
Problem 4.52



4.52 The design of a reinforced concrete bean is said to be *balanced* if the maximum stresses in the steel and concrete are equal, respectively, to the allowable stresses σ_s and σ_c. Show that to achieve a balanced design the distance x from the top of the beam to the neutral axis must be

$$x = \frac{d}{1 + \dfrac{\sigma_s E_c}{\sigma_c E_s}}$$

where E_c and E_s are the moduli of concrete and steel, respectively, and d is the distance is the distance from the top of the beam to the reinforcing steel.



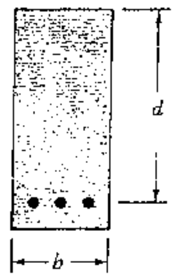
$$\sigma_s = \frac{n M (d-x)}{I} \qquad \sigma_c = \frac{Mx}{I}$$

$$\frac{\sigma_s}{\sigma_c} = \frac{n(d-x)}{x} = n \frac{d}{x} - n$$

$$\frac{d}{x} = 1 + \frac{1}{n}\frac{\sigma_s}{\sigma_c} = 1 + \frac{E_c \sigma_s}{E_s \sigma_c}$$

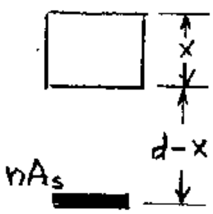
$$x = \frac{d}{1 + \dfrac{E_c \sigma_s}{E_s \sigma_c}}$$

Problem 4.53



4.53 For the concrete beam shown, the modulus of elasticity is 3.5×10^6 psi for the concrete and 29×10^6 psi for the steel. Knowing that $b = 8$ in. and $d = 22$ in., and using an allowable stress of 1800 psi for the concrete and 20 ksi for the steel, determine (*a*) the required area A_s of the steel reinforcement if the beam is to be balanced, (*b*) the largest allowable bending moment. (See Prob. 4.52 for definition of a balanced beam.)

The design of a reinforced concrete beam is said to be *balanced* if the maximum stresses in the steel and concrete are equal, respectively, to the allowable stresses σ_s and σ_c.



$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{3.5 \times 10^6} = 8.2857$$

$$\sigma_s = \frac{nM(d-x)}{I} \qquad \sigma_c = \frac{Mx}{I}$$

$$\frac{\sigma_s}{\sigma_c} = \frac{n(d-x)}{x} = n\frac{d}{x} - n$$

$$\frac{d}{x} = 1 + \frac{1}{n}\frac{\sigma_s}{\sigma_c} = 1 + \frac{1}{8.2857} \cdot \frac{20 \times 10^3}{1800} = 2.3410$$

$$x = 0.42717\, d = (0.42717)(22) = 9.398 \text{ in.}$$

$$d - x = 22 - 9.398 = 12.602 \text{ in.}$$

Locate neutral axis

$$bx\frac{x}{2} - nA_s(d-x) = 0$$

(a) $$A_s = \frac{bx^2}{2n(d-x)} = \frac{(8)(9.398)^2}{(2)(8.2857)(12.602)} = 3.3835 \text{ in}^2 \qquad A_s = 3.38 \text{ in}^2$$

$$I = \frac{1}{3}bx^3 + nA_s(d-x)^2 = \frac{1}{3}(8)(9.398)^3 + (8.2857)(3.3835)(12.602)^2$$

$$= 6665.6 \text{ in}^4$$

$$\sigma = \frac{nMy}{I} \qquad M = \frac{\sigma I}{ny}$$

Concrete: $n = 1.0$ $\quad y = 9.398$ in $\quad \sigma = 1800$ psi

$$M = \frac{(1800)(6665.6)}{(1.0)(9.398)} = 1.277 \times 10^6 \text{ lb}\cdot\text{in}$$

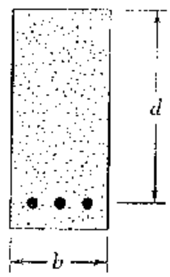
Steel: $n = 8.2857$ $\quad |y| = 12.602$ in $\quad \sigma = 20 \times 10^3$ psi

$$M = \frac{(20 \times 10^3)(6665.6)}{(8.2857)(12.602)} = 1.277 \times 10^6 \text{ lb}\cdot\text{in}$$

Note that both values are the same for balanced design

(b) $$M = 1.277 \times 10^3 \text{ kip}\cdot\text{in} = 106.4 \text{ kip}\cdot\text{ft}$$

Problem 4.54



4.54 For the concrete beam shown, the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel. Knowing that $b = 200$ mm and $d = 450$ mm and using an allowable stress of 12.5 MPa for the concrete and 140 MPa for the steel, determine (*a*) the required area A_s of the steel reinforcement if the beam is to be balanced, (*b*) the largest allowable bending moment. (See Prob. 4.52 for definition of a balanced beam.)

$$n = \frac{E_s}{E_c} = \frac{200 \times 10^9}{25 \times 10^9} = 8.0$$

$$\sigma_s = \frac{nM(d-x)}{I} \qquad \sigma_c = \frac{Mx}{I}$$

$$\frac{\sigma_s}{\sigma_c} = \frac{n(d-x)}{x} = n\frac{d}{x} - n$$

$$\frac{d}{x} = 1 + \frac{1}{n}\frac{\sigma_s}{\sigma_c} = 1 + \frac{1}{8.0}\cdot\frac{140 \times 10^6}{12.5 \times 10^6} = 2.40$$

$$x = 0.41667\, d = (0.41667)(450) = 187.5 \text{ mm}$$

Locate neutral axis

$$bx\frac{x}{2} - nA_s(d-x)$$

(a) $$A_s = \frac{bx^2}{2n(d-x)} = \frac{(200)(187.5)^2}{(2)(8.0)(262.5)} = 1674 \text{ mm}^2$$ (a) $A_s = 1674 \text{ mm}^2$

$$I = \tfrac{1}{3}bx^3 + nA_s(d-x)^2 = \tfrac{1}{3}(200)(187.5)^3 + (8.0)(1674)(262.5)^2$$

$$= 1.3623 \times 10^9 \text{ mm}^4 = 1.3623 \times 10^{-3} \text{ m}^4$$

$$\sigma = \frac{nMy}{I} \qquad M = \frac{I\sigma}{ny}$$

Concrete: $n = 1.0$ $\quad y = 187.5 \text{ mm} = 0.1875 \text{ m}$ $\quad \sigma = 12.5 \times 10^6$ Pa

$$M = \frac{(1.3623 \times 10^{-3})(12.5 \times 10^6)}{(1.0)(0.1875)} = 90.8 \times 10^3 \text{ N}\cdot\text{m}$$

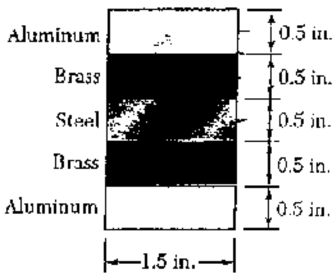
Steel: $n = 8.0$ $\quad y = 262.5 \text{ mm} = 0.2625 \text{ m}$ $\quad \sigma = 140 \times 10^6$ Pa

$$M = \frac{(1.3623 \times 10^{-3})(140 \times 10^6)}{(8.0)(0.2625)} = 90.8 \times 10^3 \text{ N}\cdot\text{m}$$

Note that both values are the same for balanced design.

(b) $M = 90.8$ kN·m

Problem 4.55



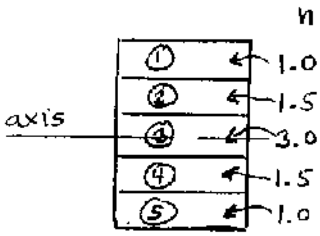
4.55 and 4.56 Five metal strips, each of 0.5 × 1.5-in. cross section, are bonded together to form the composite beam shown. The modulus of elasticity is 30×10^6 psi for the steel, 15×10^6 psi for the brass, and 10×10^6 psi for the aluminum. Knowing that the beam is bent about a horizontal axis by couple of moment 12 kip · in., determine (*a*) the maximum stress in each of the three metals, (*b*) the radius of curvature of the composite beam.

Use aluminum as the reference material

$$n = \frac{E_s}{E_a} = \frac{30 \times 10^6}{10 \times 10^6} = 3.0 \text{ in steel}$$

$$n = \frac{E_b}{E_a} = \frac{15 \times 10^6}{10 \times 10^6} = 1.5 \text{ in brass}$$

$$n = 1.0 \text{ in aluminum.}$$



For the transformed section

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1}{12}(1.5)(0.5)^3 + (0.75)(1.0)^2$$
$$= 0.7656 \text{ in}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1.5}{12}(1.5)(0.5)^3 + (1.5)(0.75)(0.5)^2 = 0.3047 \text{ in}^4$$

$$I_3 = \frac{n_3}{12} b_3 h_3^3 = \frac{3.0}{12}(1.5)(0.5)^3 = 0.0469 \text{ in}^4$$

$$I_4 = I_2 = 0.3047 \text{ in.}^4, \qquad I_5 = I_1 = 0.7656 \text{ in}^4$$

$$I = \sum_1^5 I_i = 2.1875 \text{ in}^4$$

(a) Aluminum: $\sigma = \dfrac{nMy}{I} = \dfrac{(1.0)(12)(1.25)}{2.1875} = 6.86 \text{ ksi}$ ◄

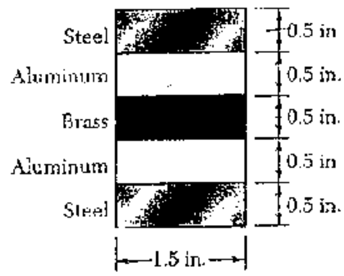
Brass: $\sigma = \dfrac{nMy}{I} = \dfrac{(1.5)(12)(0.75)}{2.1875} = 6.17 \text{ ksi}$ ◄

Steel: $\sigma = \dfrac{nMy}{I} = \dfrac{(3.0)(12)(0.25)}{2.1875} = 4.11 \text{ ksi}$ ◄

(b) $\dfrac{1}{\rho} = \dfrac{M}{E_a I} = \dfrac{12 \times 10^3}{(10 \times 10^6)(2.1875)} = 548.57 \times 10^{-6} \text{ in}^{-1}$

$\rho = 1823 \text{ in} = 151.9 \text{ ft}$ ◄

Problem 4.56



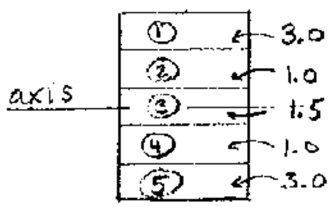
4.55 and 4.56 Five metal strips, each of 0.5 × 1.5-in. cross section, are bonded together to form the composite beam shown. The modulus of elasticity is 30×10^6 psi for the steel, 15×10^6 psi for the brass, and 10×10^6 psi for the aluminum. Knowing that the beam is bent about a horizontal axis by couple of moment 12 kip · in., determine (*a*) the maximum stress in each of the three metals, (*b*) the radius of curvature of the composite beam.

Use aluminum as the reference material

$$n = \frac{E_s}{E_a} = \frac{30\times10^6}{10\times10^6} = 3.0 \quad \text{in steel}$$

$$n = \frac{E_b}{E_a} = \frac{15\times10^6}{10\times10^6} = 1.5 \quad \text{in brass}$$

$$n = 1.0 \quad \text{in aluminum}$$



For the transformed section

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2$$

$$= \frac{3.0}{12}(1.5)(0.5)^3 + (3.0)(0.75)(1.0)^2 = 2.2969 \text{ in}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1.0}{12}(1.5)(0.5)^3 + (1.0)(0.75)(0.5)^2 = 0.2031 \text{ in}^4$$

$$I_3 = \frac{n_3}{12} b_3 h_3^3 = \frac{1.5}{12}(1.5)(0.5)^3 = 0.0234 \text{ in}^4$$

$$I_4 = I_2 = 0.2031 \text{ in}^4, \qquad I_5 = I_1 = 2.2969 \text{ in}^4$$

$$I = \sum_1^5 I_i = 5.0234 \text{ in}^4$$

(a) Steel: $\sigma = \dfrac{nMy}{I} = \dfrac{(3.0)(12)(1.25)}{5.0234} = 8.96 \text{ ksi}$ ◄

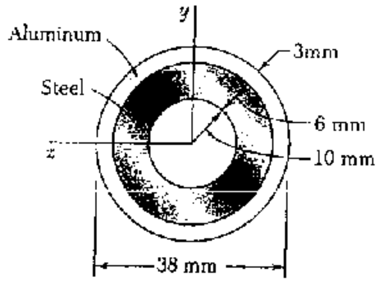
Aluminum: $\sigma = \dfrac{nMy}{I} = \dfrac{(1.0)(12)(0.75)}{5.0234} = 1.792 \text{ ksi}$ ◄

Brass: $\sigma = \dfrac{nMy}{I} = \dfrac{(1.5)(12)(0.25)}{5.0234} = 0.896 \text{ ksi}$ ◄

(b) $\dfrac{1}{\rho} = \dfrac{M}{E_a I} = \dfrac{12\times10^3}{(10\times10^6)(5.0234)} = 238.89\times10^{-6} \text{ in}^{-1}$

$\rho = 4186 \text{ in.} = 349 \text{ ft}$ ◄

Problem 4.57



4.57 A steel pipe and an aluminum pipe are securely bonded together to form the composite beam shown. The modulus of elasticity is 210 GPa for the steel and 70 GPa for the aluminum. Knowing that the composite beam is bent by couple of moment 500 N · m, determine the maximum stress (*a*) in the aluminum, (*b*) in the steel.

Use aluminum as the reference material

$n = 1.0$ in aluminum

$n = \frac{E_s}{E_a} = \frac{210}{70} = 3.0$ in steel

Steel: $I_1 = n_1 \frac{\pi}{4}(r_o^4 - r_i^4) = (3.0)\frac{\pi}{4}(16^4 - 10^4) = 130.85 \times 10^3 \text{ mm}^4$

Aluminum: $I_2 = n_2 \frac{\pi}{4}(r_o^4 - r_i^4) = (1.0)\frac{\pi}{4}(19^4 - 16^4) = 50.88 \times 10^3 \text{ mm}^4$

$I = I_1 + I_2 = 181.73 \times 10^3 \text{ mm}^4 = 181.73 \times 10^{-9} \text{ m}^4$

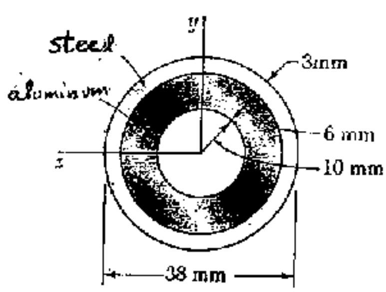
(a) Aluminum: $c = 19 \text{ mm} = 0.019 \text{ m}$

$$\sigma = \frac{nMc}{I} = \frac{(1.0)(500)(0.019)}{181.73 \times 10^{-9}} = 52.3 \times 10^6 \text{ Pa} = 52.3 \text{ MPa}$$

(b) Steel: $c = 16 \text{ mm} = 0.016 \text{ m}$

$$\frac{nMc}{I} = \frac{(3.0)(500)(0.016)}{181.73 \times 10^{-9}} = 132.1 \times 10^6 \text{ Pa} = 132.1 \text{ MPa}$$

Problem 4.58



4.58 Solve Prob. 4.57, assuming that the 6-mm-thick inner pipe is made of aluminum and that the 3-mm-thick outer pipe is made of steel.

4.57 A steel pipe and an aluminum pipe are securely bonded together to form the composite beam shown. The modulus of elasticity is 210 GPa for the steel and 70 GPa for the aluminum. Knowing that the composite beam is bent by couple of moment 500 N · m, determine the maximum stress (*a*) in the aluminum, (*b*) in the steel.

Use aluminum as the reference material

$n = 1.0$ in aluminum

$n = \frac{E_s}{E_a} = \frac{210}{70} = 3.0$ in steel

Steel: $I_1 = n_1 \frac{\pi}{4}(r_o^4 - r_i^4) = (3.0)\frac{\pi}{4}(19^4 - 16^4) = 152.65 \times 10^3 \text{ mm}^4$

Aluminum: $I_2 = n_2 \frac{\pi}{4}(r_o^4 - r_i^4) = (1.0)\frac{\pi}{4}(16^4 - 10^4) = 43.62 \times 10^3 \text{ mm}^4$

$I = I_1 + I_2 = 196.27 \times 10^3 \text{ mm}^4 = 196.27 \times 10^{-9} \text{ m}^4$

(a) Aluminum: $c = 16 \text{ mm} = 0.016 \text{ m}$

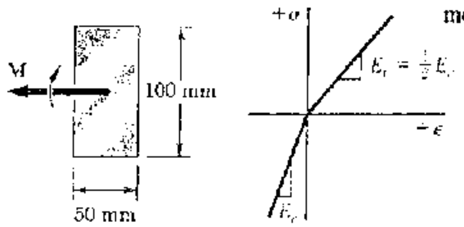
$$\sigma = \frac{nMc}{I} = \frac{(1.0)(500)(0.016)}{196.27 \times 10^{-9}} = 40.8 \times 10^6 \text{ Pa} = 40.8 \text{ MPa}$$

(b) Steel: $c = 19 \text{ mm} = 0.019 \text{ m}$

$$\sigma = \frac{nMc}{I} = \frac{(3.0)(500)(0.019)}{196.27 \times 10^{-9}} = 145.2 \times 10^6 \text{ Pa} = 145.2 \text{ MPa}$$

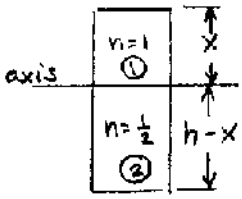
Problem 4.59

4.59 The rectangular beam shown is made of a plastic for which the value of the modulus of elasticity in tension is one-half of its value in compression. For a bending moment $M = 600\ \text{N}\cdot\text{m}$, determine the maximum (*a*) tensile stress, (*b*) compressive stress.



$n = \frac{1}{2}$ on the tension side of neutral axis

$n = 1$ on the compression side



Locate neutral axis.

$$n_1 b x \frac{x}{2} - n_2 b(h-x)\frac{h-x}{2} = 0$$

$$\frac{1}{2}bx^2 - \frac{1}{4}b(h-x)^2 = 0$$

$$x^2 = \frac{1}{2}(h-x)^2 \qquad x = \frac{1}{\sqrt{2}}(h-x)$$

$$x = \frac{1}{\sqrt{2}+1}h = 0.41421\ h = 41.421\ \text{mm}$$

$$h - x = 58.579\ \text{mm}$$

$$I_1 = n_1\frac{1}{3}bx^3 = (1)\left(\frac{1}{3}\right)(50)(41.421)^3 = 1.1844\times10^6\ \text{mm}^4$$

$$I_2 = n_2\frac{1}{3}b(h-x)^3 = \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)(50)(58.579)^3 = 1.6751\times10^6\ \text{mm}^4$$

$$I = I_1 + I_2 = 2.8595\times10^6\ \text{mm}^4 = 2.8595\times10^{-6}\ \text{m}^4$$

(a) tensile stress: $n = \frac{1}{2}$, $y = -58.579\ \text{mm} = -0.058579\ \text{m}$

$$\sigma = -\frac{nMy}{I} = -\frac{(0.5)(600)(-0.058579)}{2.8595\times10^{-6}} = 6.15\times10^6\ \text{Pa}$$

$$= 6.15\ \text{MPa}$$

(b) compressive stress: $n = 1$, $y = 41.421\ \text{mm} = 0.041421\ \text{m}$

$$\sigma = -\frac{nMy}{I} = -\frac{(1.0)(600)(0.041421)}{2.8595\times10^{-6}} = -8.69\times10^6\ \text{Pa}$$

$$= -8.69\ \text{MPa}$$

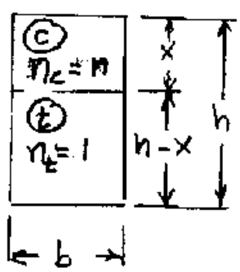
Problem 4.60

***4.60** A rectangular beam is made of material for which the modulus of elasticity is E_t in tension and E_c in compression. Show that the curvature of the beam in pure bending is

$$\frac{1}{\rho} = \frac{M}{E_r I}$$

where

$$E_r = \frac{4E_t E_c}{\left(\sqrt{E_t} + \sqrt{E_c}\right)^2}$$



Use E_t as the reference modulus.

Then $E_c = nE_t$

Locate neutral axis

$$nbx\frac{x}{2} - b(h-x)\frac{h-x}{2} = 0$$

$$nx^2 - (h-x)^2 = 0 \qquad \sqrt{n}\,x = (h-x)$$

$$x = \frac{h}{\sqrt{n}+1} \qquad h - x = \frac{\sqrt{n}\,h}{\sqrt{n}+1}$$

$$I_{trans} = \frac{n}{3}bx^3 + \frac{1}{3}b(h-x)^3 = \left[\frac{n}{3}\left(\frac{1}{\sqrt{n}+1}\right)^3 + \left(\frac{\sqrt{n}}{\sqrt{n}+1}\right)^3\right]bh^3$$

$$= \frac{1}{3}\frac{n+n^{3/2}}{(\sqrt{n}+1)^3}bh^3 = \frac{1}{3}\frac{n(1+\sqrt{n})}{(\sqrt{n}+1)^3}bh^3 = \frac{1}{3}\cdot\frac{n}{(\sqrt{n}+1)^2}bh^3$$

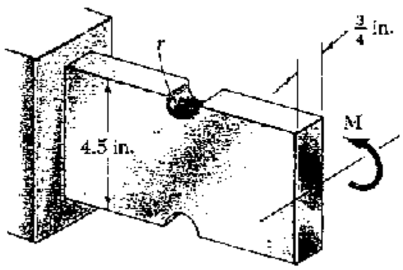
$$\frac{1}{\rho} = \frac{M}{E_t I_{trans}} = \frac{M}{E_r I} \qquad \text{where } I = \frac{1}{12}bh^3$$

$$E_r I = E_t I_{trans}$$

$$E_r = \frac{E_t I_{trans}}{I} = \frac{12}{bh^3}\cdot E_t\cdot\frac{n}{3(\sqrt{n}+1)^2}bh^3$$

$$= \frac{4E_t E_c/E_t}{\left(\sqrt{E_c/E_t}+1\right)^2} = \frac{4E_t E_c}{\left(\sqrt{E_c}+\sqrt{E_t}\right)^2}$$

Problem 4.61



4.61 Semicircular grooves of radius r must be milled as shown in the sides of a steel member. Using an allowable stress of 8 ksi, determine the largest bending moment that can be applied to the member when the radius r of the semicircular grooves is (a) $r = \frac{3}{8}$ in., (b) $r = \frac{3}{4}$ in.

(a) $d = D - 2r = 4.5 - (2)(\frac{3}{8}) = 3.75$ in.

$\frac{D}{d} = \frac{4.5}{3.75} = 1.20, \quad \frac{r}{d} = \frac{0.375}{3.75} = 0.10$

From Fig. 4.32 $K = 2.07$

$I = \frac{1}{12}bh^3 = \frac{1}{12}(\frac{3}{4})(3.75)^3 = 3.296 \text{ in}^4, \qquad c = \frac{1}{2} = 1.875$ in

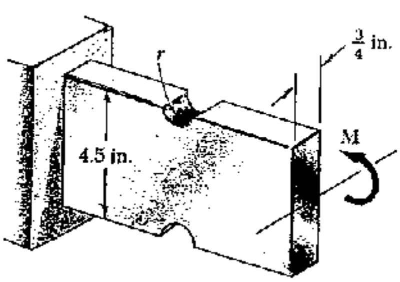
$\sigma = K\frac{Mc}{I} \therefore M = \frac{\sigma I}{Kc} = \frac{(8)(3.296)}{(2.07)(1.875)} = 6.79$ kip·in ◄

(b) $d = D - 2r = 4.5 - (2)(\frac{3}{4}) = 3.0, \quad \frac{D}{d} = \frac{4.5}{3.0} = 1.5, \quad \frac{r}{d} = \frac{0.75}{3.0} = 0.25$

From Fig. 4.32 $K = 1.61, \qquad I = \frac{1}{12}bh^3 = \frac{1}{12}(\frac{3}{4})(3.0)^3 = 1.6875 \text{ in}^4$

$c = \frac{1}{2}d = 1.5$ in. $\qquad M = \frac{\sigma I}{Kc} = \frac{(8)(1.6875)}{(1.61)(1.5)} = 5.59$ kip·in. ◄

Problem 4.62



4.62 Semicircular grooves of radius r must be milled as shown in the sides of a steel member. Knowing that $M = 4$ kip · in., determine the maximum stress in the member when (a) $r = \frac{3}{8}$ in., (b) $r = \frac{3}{4}$ in.

(a) $d = D - 2r = 4.5 - (2)(\frac{3}{8}) = 3.75$ in.

$\frac{D}{d} = \frac{4.5}{3.75} = 1.20, \quad \frac{r}{d} = \frac{0.375}{3.75} = 0.10$

From Fig. 4.32 $K = 2.07$

$I = \frac{1}{12}bh^3 = \frac{1}{12}(\frac{3}{4})(3.75)^3 = 3.296 \text{ in}^4, \qquad c = \frac{1}{2}d = 1.875$ in.

$\sigma = K\frac{Mc}{I} = \frac{(2.07)(4)(1.875)}{3.296} = 4.71$ ksi ◄

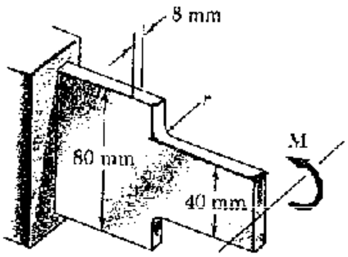
(b) $d = D - 2r = 4.5 - (2)(\frac{3}{4}) = 3.00$ in, $\frac{D}{d} = \frac{4.5}{3.00} = 1.50, \frac{r}{d} = \frac{0.75}{3.00} = 0.25$

From Fig 4.32 $K = 1.61$

$I = \frac{1}{12}bh^3 = \frac{1}{12}(\frac{3}{4})(3.00)^3 = 1.6875 \text{ in}^4, \qquad c = \frac{1}{2}d = 11.5$ in

$\sigma = K\frac{Mc}{I} = \frac{(1.61)(4)(1.5)}{1.6875} = 5.72.$ ksi ◄

Problem 4.63



4.63 Knowing that $M = 250$ N·m, determine the maximum stress in the beam shown when the radius r of the fillets is (a) 4 mm, (b) 8 mm.

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(8)(40)^3 = 42.667\times10^3\ \text{mm}^4 = 42.667\times10^{-9}\ \text{m}^4$$

$$c = 20\ \text{mm} = 0.020\ \text{m}$$

$$\frac{D}{d} = \frac{80\ \text{mm}}{40\ \text{mm}} = 2.00$$

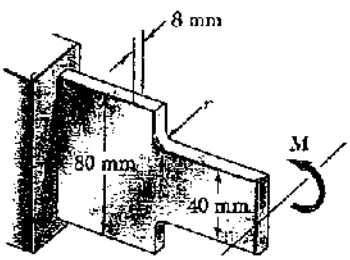
(a) $\frac{r}{d} = \frac{4\ \text{mm}}{40\ \text{mm}} = 0.10$ From Fig. 4.31 $K = 1.87$

$$\sigma_{max} = K\frac{Mc}{I} = \frac{(1.87)(250)(0.020)}{42.667\times10^{-9}} = 219\times10^6\ \text{Pa} = 219\ \text{MPa}$$

(b) $\frac{r}{d} = \frac{8\ \text{mm}}{40\ \text{mm}} = 0.20$ From Fig. 4.31 $K = 1.50$

$$\sigma_{max} = K\frac{Mc}{I} = \frac{(1.50)(250)(0.020)}{42.667\times10^{-9}} = 176\times10^6\ \text{Pa} = 176\ \text{MPa}$$

Problem 4.64



4.64 Knowing that the allowable stress for the beam shown is 90 MPa, determine (a) the allowable bending moment M when the radius r of the fillets is (a) 8 mm, (b) 12 mm.

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(8)(40)^3 = 42.667\times10^3\ \text{mm}^4 = 42.667\times10^{-9}\ \text{m}^4$$

$$c = 20\ \text{mm} = 0.020\ \text{m}$$

$$\frac{D}{d} = \frac{80\ \text{mm}}{40\ \text{mm}} = 2.00$$

(a) $\frac{r}{d} = \frac{8\ \text{mm}}{40\ \text{mm}} = 0.2$ From Fig. 4.31 $K = 1.50$

$$\sigma_{max} = K\frac{Mc}{I} \quad \therefore \quad M = \frac{\sigma_{max} I}{Kc} = \frac{(90\times10^6)(42.667\times10^{-9})}{(1.50)(0.020)}$$

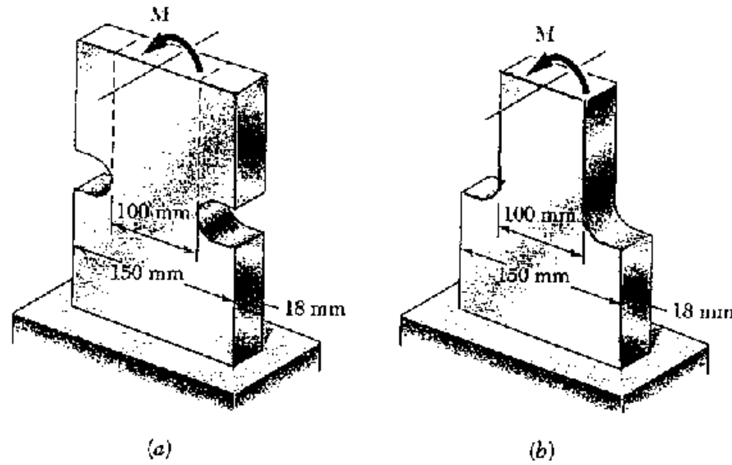
$$= 128\ \text{N·m}$$

(b) $\frac{r}{d} = \frac{12\ \text{mm}}{40\ \text{mm}} = 0.3$ From Fig 4.31 $K = 1.35$

$$M = \frac{(90\times10^6)(42.667\times10^{-9})}{(1.35)(0.020)} = 142\ \text{N·m}$$

Problem 4.65

4.65 The allowable stress used in the design of a steel bar is 80 MPa. Determine the largest couple **M** that can be applied to the bar (*a*) if the bar is designed with grooves having semicircular portions of radius $r = 15$ mm, as shown in Fig. 4.65*a*, (*b*) if the bar is redesigned by removing the material above the grooves as shown in Fig. 4.65*b*.



For both configuations

$D = 150$ mm, $d = 100$ mm.

$r = 15$ mm.

$$\frac{D}{d} = \frac{150}{100} = 1.50$$

$$\frac{r}{d} = \frac{15}{100} = 0.15$$

For configuration (a), Fig 4.32 gives $K_a = 1.92$.

For configuration (b) Fig 4.31 gives $K_b = 1.57$.

$$I = \frac{1}{12} b h^3 = \frac{1}{12}(18)(100)^3 = 1.5 \times 10^6 \text{ mm}^4 = 1.5 \times 10^{-6} \text{ m}^4$$

$$c = \frac{1}{2} d = 50 \text{ mm} = 0.050 \text{ m}$$

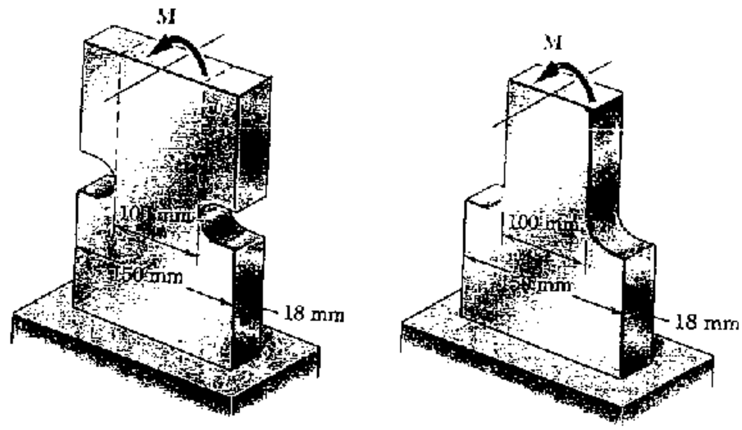
(a) $\sigma = \dfrac{KMc}{I} \quad \therefore \quad M = \dfrac{\sigma I}{Kc} = \dfrac{(80 \times 10^6)(1.5 \times 10^{-6})}{(1.92)(0.05)} = 1.25 \times 10^3 \text{ N}\cdot\text{m}$

$= 1.25 \text{ kN}\cdot\text{m}$ ◄

(b) $M = \dfrac{\sigma I}{Kc} = \dfrac{(80 \times 10^6)(1.5 \times 10^{-6})}{(1.57)(0.050)} = 1.53 \times 10^3 \text{ N}\cdot\text{m} = 1.53 \text{ kN}\cdot\text{m}$ ◄

Problem 4.66

4.66 A couple of moment $M = 2$ kN · m is to be applied to the end of a steel bar. Determine the maximum stress in the bar (*a*) if the bar is designed with grooves having semicircular portions of radius $r = 10$ mm, as shown in Fig. 4.65*a*, (*b*) if the bar is redesigned by removing the material above the grooves as shown in Fig. 4.65*b*.



(*a*) (*b*)

For both configurations

$D = 150$ mm, $d = 100$ mm

$r = 10$ mm

$\frac{D}{d} = \frac{150}{100} = 1.50$

$\frac{r}{d} = \frac{10}{100} = 0.10$

For configuration (a), Fig 4.32 give $K_a = 2.21$

For configuration (b), Fig. 4.31 gives $K_b = 1.79$

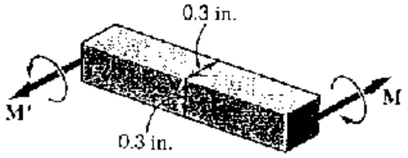
$I = \frac{1}{12} b h^3 = \frac{1}{12}(18)(100)^3 = 1.5 \times 10^6 \text{ mm}^4 = 1.5 \times 10^{-6} \text{ m}^4$

$c = \frac{1}{2} d = 50 \text{ mm} = 0.05 \text{ m}$

(a) $\sigma = \frac{KMc}{I} = \frac{(2.21)(2 \times 10^3)(0.05)}{1.5 \times 10^{-6}} = 147 \times 10^6 \text{ Pa} = 147 \text{ MPa}$ ◄

(b) $\sigma = \frac{KMc}{I} = \frac{(1.79)(2 \times 10^3)(0.05)}{1.5 \times 10^{-6}} = 119 \times 10^6 \text{ Pa} = 119 \text{ MPa}$ ◄

Problem 4.67



4.67 A bar having the cross section shown is made of a steel which is assumed to be elastoplastic with $E = 30 \times 10^6$ psi and $\sigma_Y = 48$ ksi. Determine the bending moment M at which (*a*) yield first occurs, (*b*) the plastic zones at the top and bottom of the bar are 0.09-in. thick.

(a) $I = \frac{1}{12} b h^3 = \frac{1}{12}(0.3)(0.3)^3 = 675 \times 10^{-6}$ in^4

$c = \frac{1}{2} h = 0.15$ in.

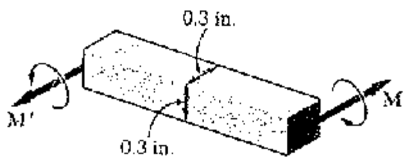
$M_Y = \frac{\sigma_Y I}{c} = \frac{(48)(675 \times 10^{-6})}{0.15} = 0.216$ kip·in

$M_Y = 216$ lb·in

(b) $y_Y = c - t_P = 0.15 - 0.09 = 0.06$ in

$M_P = \frac{3}{2} M_Y \left[1 - \frac{1}{3}\left(\frac{y_Y}{c}\right)^2\right] = \frac{3}{2}(216)\left[1 - \frac{1}{3}\left(\frac{0.06}{0.15}\right)^2\right]$ $\qquad M_P = 307$ lb·in

Problem 4.68



4.68 For the steel bar of Prob. 4.67, determine the thickness of the plastic zones at the top and bottom of the bar when (*a*) $M = 250$ lb · in., (*b*) $M = 300$ lb · in.

4.67 A bar having the cross section shown is made of a steel which is assumed to be elastoplastic with $E = 30 \times 10^6$ psi and $\sigma_Y = 48$ ksi. Determine the bending moment M at which (*a*) yield first occurs, (*b*) the plastic zones at the top and bottom of the bar are 0.09-in. thick.

$I = \frac{1}{12} b h^3 = \frac{1}{12}(0.3)(0.3)^3 = 675 \times 10^{-6}$ in^4

$c = \frac{1}{2} h = 0.15$ in.

$M_Y = \frac{\sigma_Y I}{c} = \frac{(48)(675 \times 10^{-6})}{0.15} = 0.216$ kip·in

$M_Y = 216$ lb·in

$M = \frac{3}{2} M_Y \left[1 - \frac{1}{3}\left(\frac{y_Y}{c}\right)^2\right]$

$\frac{y_Y}{c} = \sqrt{3 - 2\frac{M}{M_Y}}$

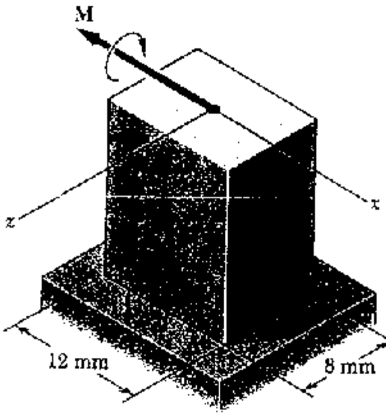
(a) $M = 250$ lb·in $\qquad \frac{y_Y}{c} = \sqrt{3 - 2\left(\frac{250}{216}\right)} = 0.82756$

$y_Y = (0.82215)(0.15) = 0.12416$ in $\qquad t_P = c - y_Y = 0.0258$ in.

(b) $M = 300$ lb·in $\qquad \frac{y_Y}{c} = \sqrt{3 - 2\left(\frac{300}{216}\right)} = 0.47140$

$y_Y = (0.47140)(0.15) = 0.07071$ in $\qquad t_P = c - y_Y = 0.0793$ in.

Problem 4.69



4.69 The prismatic bar shown is made of a steel which is assumed to be elastoplastic with $\sigma_Y = 300$ MPa and is subjected to a couple **M** parallel to the x axis. Determine the moment M of the couple for which (*a*) yield first occurs, (*b*) the elastic core of the bar is 4 mm thick.

(a) $I = \frac{1}{12}bh^3 = \frac{1}{12}(12)(8)^3 = 512 \text{ mm}^4$
$= 512 \times 10^{-12} \text{ m}^4$

$c = \frac{1}{2}h = 4 \text{ mm} = 0.004 \text{ m}$

$M_Y = \frac{\sigma_Y I}{c} = \frac{(300\times10^6)(512\times10^{-12})}{0.004}$

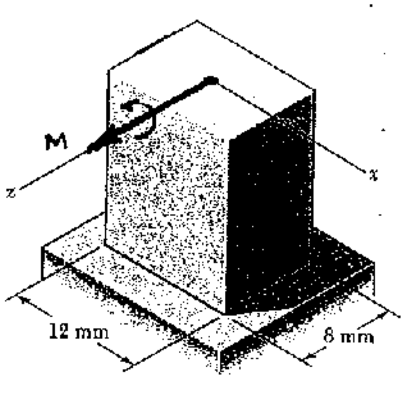
$= 38.4 \text{ N}\cdot\text{m}$ $\qquad M_Y = 38.4 \text{ N}\cdot\text{m}$ ◄

(b) $y_Y = \frac{1}{2}(4) = 2 \text{mm}$ $\qquad \frac{y_Y}{c} = \frac{2}{4} = 0.5$

$M = \frac{3}{2}M_Y\left[1 - \frac{1}{3}\left(\frac{y_Y}{c}\right)^2\right] = \frac{3}{2}(38.4)\left[1 - \frac{1}{3}(0.5)^2\right] = 52.8 \text{ N}\cdot\text{m}$

$M = 52.8 \text{ N}\cdot\text{m}$ ◄

Problem 4.70



4.70 Solve Prob. 4.69, assuming that the couple **M** is parallel to the z axis.

4.69 The prismatic bar shown is made of a steel which is assumed to be elastoplastic with $\sigma_Y = 300$ MPa and is subjected to a couple **M** parallel to the x axis. Determine the moment M of the couple for which (*a*) yield first occurs, (*b*) the elastic core of the bar is 4 mm thick.

(a) $I = \frac{1}{12}bh^3 = \frac{1}{12}(8)(12)^3 = 1.152\times10^3 \text{ mm}^4$
$= 1.152\times10^{-9} \text{ m}^4$

$c = \frac{1}{2}h = \text{ mm} = 0.006 \text{ m}$

$M_Y = \frac{\sigma_Y I}{c} = \frac{(300\times10^6)(1.152\times10^{-9})}{0.006}$

$= 57.6 \text{ N}\cdot\text{m}$ $\qquad M_Y = 57.6 \text{ N}\cdot\text{m}$ ◄

(b) $y_Y = \frac{1}{2}(4) = 2 \text{mm}$ $\qquad \frac{y_Y}{c} = \frac{2}{6} = \frac{1}{3}$

$M = \frac{3}{2}M_Y\left[1 - \frac{1}{3}\left(\frac{y_Y}{c}\right)^2\right] = \frac{3}{2}(57.6)\left[1 - \frac{1}{3}\left(\frac{1}{3}\right)^2\right] = 83.2 \text{ N}\cdot\text{m}$

$M = 83.2 \text{ N}\cdot\text{m}$ ◄

Problem 4.71

4.71 A solid square rod of side 0.6 in. is made of a steel that is assumed to be elastoplastic with $E = 29 \times 10^6$ psi and $\sigma_Y = 48$ ksi. Knowing that a couple **M** is applied and maintained about an axis parallel to a side of the cross section, determine the moment M of the couple for which the radius of curvature is 6 ft.

$$I = \frac{1}{12}(0.6)(0.6)^3 = 10.8 \times 10^{-3} \text{ in}^4 \qquad c = \frac{1}{2}h = 0.3 \text{ in.}$$

$$M_Y = \frac{I\sigma_Y}{c} = \frac{(10.8 \times 10^{-3})(48 \times 10^3)}{0.3} = 1728 \text{ lb}\cdot\text{in.} \qquad \rho = 6 \text{ ft} = 72 \text{ in.}$$

$$\varepsilon_{max} = \frac{c}{\rho} = \frac{0.3}{72} = 4.16667 \times 10^{-3} \qquad \varepsilon_Y = \frac{\sigma_Y}{E} = \frac{48 \times 10^3}{29 \times 10^6} = 1.65517 \times 10^{-3}$$

$$\frac{y_Y}{c} = \frac{\varepsilon_Y}{\varepsilon_m} = \frac{1.65517 \times 10^{-3}}{4.16667 \times 10^{-3}} = 0.39724$$

$$M = \frac{3}{2}M_Y\left[1 - \frac{1}{3}\left(\frac{y_Y}{c}\right)^2\right] = \frac{3}{2}(1728)\left[1 - \frac{1}{3}(0.39724)^2\right]$$

$$M = 2460 \text{ lb}\cdot\text{in}$$

Problem 4.72

4.72 For the solid square rod of Prob. 4.71, determine the moment M for which the radius of curvature is 3 ft.

4.71 A solid square rod of side 0.6 in. is made of a steel that is assumed to be elastoplastic with $E = 29 \times 10^6$ psi and $\sigma_Y = 48$ ksi Knowing that a couple **M** is applied and maintained about an axis parallel to a side of the cross section, determine the moment M of the couple for which the radius of curvature is 6 ft.

$$I = \frac{1}{12}(0.6)(0.6)^3 = 10.8 \times 10^{-3} \text{ in}^4 \qquad c = \frac{1}{2}h = 0.3 \text{ in}$$

$$M_Y = \frac{I\sigma_Y}{c} = \frac{(10.8 \times 10^{-3})(48 \times 10^3)}{0.3} = 1728 \text{ lb}\cdot\text{in} \qquad \rho = 3 \text{ ft} = 36 \text{ in.}$$

$$\varepsilon_{max} = \frac{c}{\rho} = \frac{0.3}{36} = 8.3333 \times 10^{-3} \qquad \varepsilon_Y = \frac{\sigma_Y}{E} = \frac{48 \times 10^3}{29 \times 10^6} = 1.65517 \times 10^{-3}$$

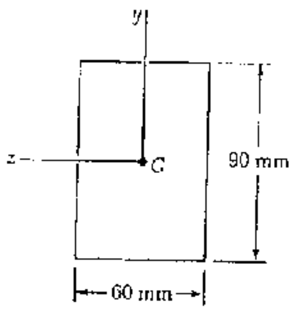
$$\frac{y_Y}{c} = \frac{\varepsilon_Y}{\varepsilon_{max}} = \frac{1.65517 \times 10^{-3}}{8.3333 \times 10^{-3}} = 0.19862$$

$$M = \frac{3}{2}M_Y\left[1 - \frac{1}{3}\left(\frac{y_Y}{c}\right)^2\right] = \frac{3}{2}(1728)\left[1 - \frac{1}{3}(0.19862)^2\right]$$

$$M = 2560 \text{ lb}\cdot\text{in}$$

Problem 4.73

4.73 and 4.74 A beam of the cross section shown is made of a steel which is assumed to be elastoplastic $E = 200$ GPa and $\sigma_Y = 240$ MPa. For bending about the z axis, determine the bending moment at which (*a*) yield first occurs, (*b*) the plastic zones at the top and bottom of the bar are 30-mm thick.

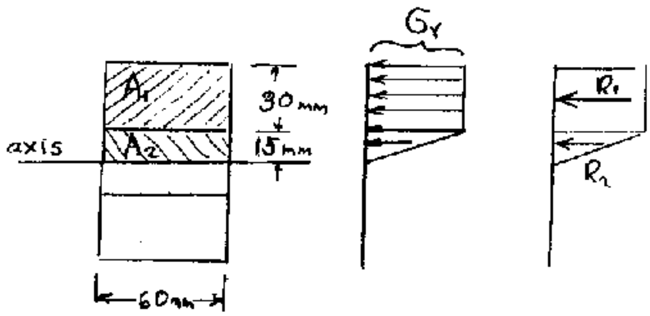


(a) $I = \frac{1}{12}bh^3 = \frac{1}{12}(60)(90)^3 = 3.645\times10^6\ mm^4 = 3.645\times10^{-6}\ m^4$

$c = \frac{1}{2}h = 45\ mm = 0.045\ m$

$M_Y = \frac{\sigma_Y I}{c} = \frac{(240\times10^6)(3.645\times10^{-6})}{0.045} = 19.44\times10\ N\cdot m$

$= 19.44\ kN\cdot m$



$R_1 = \sigma_Y A_1 = (240\times10^6)(0.060)(0.030)$
$= 432.\times10^3\ N$

$y_1 = 15\ mm + 15\ mm = 0.030\ m$

$R_2 = \frac{1}{2}\sigma_Y A_2 = (\frac{1}{2})(240\times10^6)(0.060)(0.015)$
$= 108\times10^3\ N$

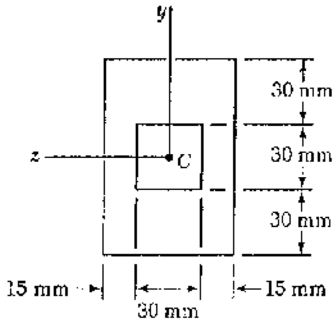
$y_2 = \frac{2}{3}(15\ mm) = 10\ mm = 0.010\ m$

(b) $M = 2(R_1y_1 + R_2y_2) = 2[(432\times10^3)(0.030) + (108\times10^3)(0.010)]$

$= 28.08\times10^3\ N\cdot m \qquad M = 28.1\ kN\cdot m$

Problem 4.74

4.73 and 4.74 A beam of the cross section shown is made of a steel which is assumed to be elastoplastic $E = 200$ GPa and $\sigma_Y = 240$ MPa. For bending about the z axis, determine the bending moment at which (*a*) yield first occurs, (*b*) the plastic zones at the top and bottom of the bar are 30-mm thick.



(a) $I_{rect} = \frac{1}{12} b h^3 = \frac{1}{12}(60)(90)^3 = 3.645 \times 10^6 \text{ mm}^4$

$I_{cutout} = \frac{1}{12} b h^3 = \frac{1}{12}(30)(30)^3 = 67.5 \times 10^3 \text{ mm}^4$

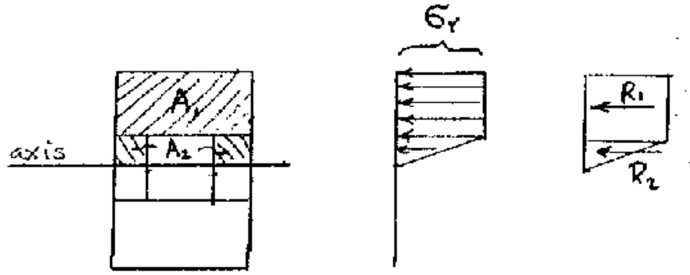
$I = 3.645 \times 10^3 - 67.5 \times 10^3 = 3.5775 \times 10^6 \text{ mm}^4$

$= 3.5775 \times 10^{-6} \text{ m}^4$

$c = \frac{1}{2} h = 45 \text{ mm} = 0.045 \text{ m}$

$M_Y = \frac{\sigma_Y I}{c} = \frac{(240 \times 10^6)(3.5775 \times 10^{-6})}{0.045}$

$= 19.08\ 10^3 \text{ N}\cdot\text{m} = 19.08 \text{ kN}\cdot\text{m}$



$R_1 = \sigma_Y A_1 = (240 \times 10^6)(0.060)(0.030) = 432 \times 10^3 \text{ N}$

$y_1 = 15 \text{ mm} + 15 \text{ mm} = 30 \text{ mm} = 0.030 \text{ m}$

$R_2 = \frac{1}{2} \sigma_Y A_2 = \frac{1}{2}(240 \times 10^6)(0.030)(0.015) = 54 \times 10^3 \text{ N}$

$y_2 = \frac{2}{3}(15 \text{ mm}) = 10 \text{ mm} = 0.010 \text{ m}$

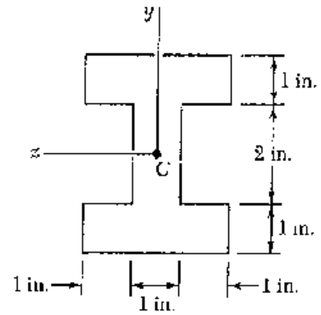
(b) $M = 2(R_1 y_1 + R_2 y_2)$

$= 2\left[(432 \times 10^3)(0.030) + (54 \times 10^3)(0.010)\right]$

$= 27.00 \times 10^3 \text{ N}\cdot\text{m} = 27.0 \text{ kN}\cdot\text{m}$

Problem 4.75

4.75 and 4.76 A beam of the cross section shown is made of a steel which is assumed to be elastoplastic $E = 29 \times 10^6$ psi and $\sigma_Y = 42$ ksi. For bending about the z axis, determine the bending moment and radius of curvature at which (a) yield first occurs, (b) the plastic zones at the top and bottom of the bar are 1-in. thick.



(a) $I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(3)(1)^3 + (3)(1)(1.5)^2 = 7.0 \text{ in}^4$

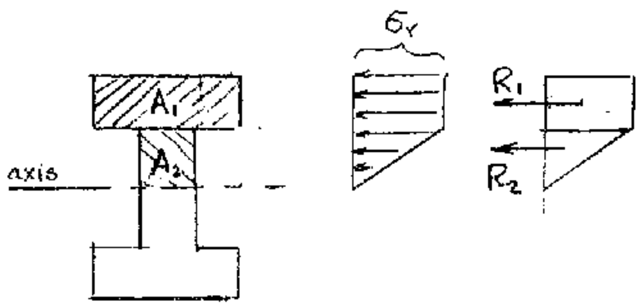
$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(1)(2)^3 = 0.6667 \text{ in}^4$

$I_3 = I_1 = 7.0 \text{ in}^4$

$I = I_1 + I_2 + I_3 = 14.6667 \text{ in}^4$

$c = 2 \text{ in.}$

$M_y = \frac{\sigma_Y I}{c} = \frac{(42)(14.6667)}{2} = 308 \text{ kip}\cdot\text{in}$



$R_1 = \sigma_Y A_1 = (42)(3)(1) = 126 \text{ kip}$

$y_1 = 1.0 + 0.5 = 1.5 \text{ in}$

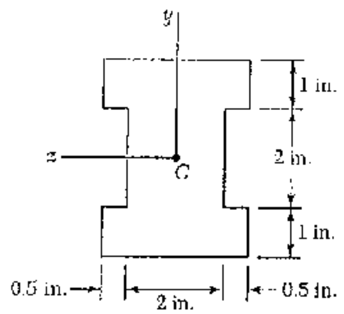
$R_2 = \frac{1}{2}\sigma_Y A_2 = \frac{1}{2}(42)(1)(1)$
$= 21 \text{ kip}$

$y_2 = \frac{2}{3}(1.0) = 0.6667 \text{ in.}$

(b) $M = 2(R_1 y_1 + R_2 y_2) = 2[(126)(1.5) + (21)(0.6667)] = 406 \text{ kip}\cdot\text{in}$

Problem 4.76

4.75 and 4.76 A beam of the cross section shown is made of a steel which is assumed to be elastoplastic $E = 29 \times 10^6$ psi and $\sigma_Y = 42$ ksi. For bending about the z axis, determine the bending moment and radius of curvature at which (a) yield first occurs, (b) the plastic zones at the top and bottom of the bar are 1-in. thick.



(a) $I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(3)(1)^3 + (3)(1)(1.5)^2 = 7.0 \text{ in}^4$

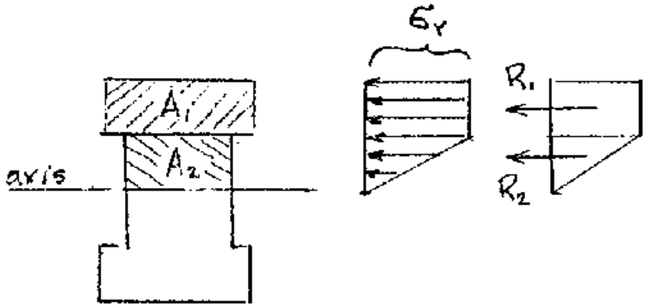
$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(2)(2)^3 = 1.3333 \text{ in}^4$

$I_3 = I_1 = 7.0 \text{ in}^4$

$I = I_1 + I_2 + I_3 = 15.3333 \text{ in}^4$

$c = 2 \text{ in.}$

$M_Y = \dfrac{\sigma_Y I}{c} = \dfrac{(42)(15.3333)}{2} = 322 \text{ kip}\cdot\text{in}$ ◄



$R_1 = \sigma_Y A_1 = (42)(3)(1) = 126 \text{ kip}$

$y_1 = 1.0 + 0.5 = 1.5 \text{ in.}$

$R_2 = \frac{1}{2}\sigma_Y A_2 = \frac{1}{2}(42)(2)(1)$
$= 42 \text{ kip}$

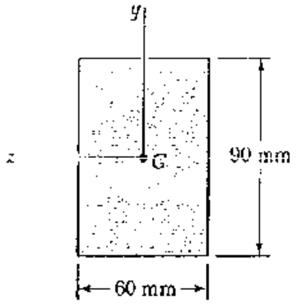
$y_2 = \frac{2}{3}(1) = 0.6667 \text{ in.}$

(b) $M = 2(R_1 y_1 + R_2 y_2) = 2[(126)(1.5) + (42)(0.6667)] = 434 \text{ kip}\cdot\text{in}$ ◄

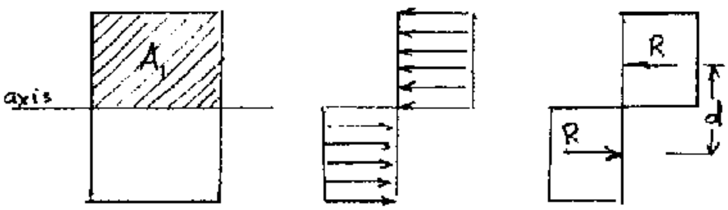
Problem 4.77

4.77 through 4.80 For the beam indicated, determine (*a*) the fully plastic moment M_p, (*b*) the shape factor of the cross section.

4.77 Beam of Prob. 4.73.



From PROBLEM 4.73 $E = 200$ GPa and $\sigma_Y = 240$ MPa.



$A_1 = (60)(45) = 2700 \text{ mm}^2$

$= 2700 \times 10^{-6} \text{ m}^2$

$R = \sigma_Y A_1$

$= (240 \times 10^6)(2700 \times 10^{-6})$

$= 648 \times 10^3$ N

$d = 45 \text{ mm} = 0.045$ m

(a) $M_p = R d = (648 \times 10^3)(0.045) = 29.16 \times 10^3 \text{ N}\cdot\text{m} = 29.2 \text{ kN}\cdot\text{m}$

(b) $I = \frac{1}{12} b h^3 = \frac{1}{12}(60)(90)^3 = 3.645 \times 10^6 \text{ mm}^4 = 3.645 \times 10^{-6} \text{ m}^4$

$c = 45 \text{ mm} = 0.045$ m

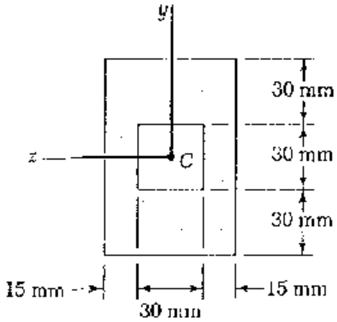
$M_Y = \frac{\sigma_Y I}{c} = \frac{(240 \times 10^6)(3.645 \times 10^{-6})}{0.045} = 19.44 \times 10^3 \text{ N}\cdot\text{m}$

$k = \frac{M_p}{M_Y} = \frac{29.16}{19.44} = 1.500$

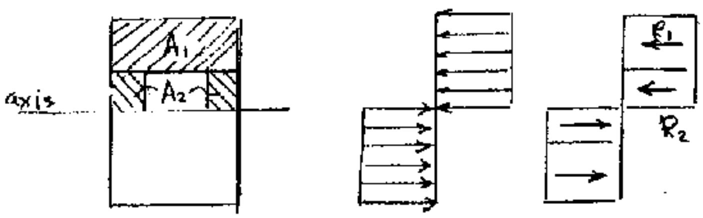
Problem 4.78

4.77 through 4.80 For the beam indicated, determine (*a*) the fully plastic moment M_p, (*b*) the shape factor of the cross section.

4.78 Beam of Prob. 4.74.



From PROBLEM 4.74 $E = 200$ GPa and $\sigma_Y = 240$ MPa.



$R_1 = \sigma_Y A_1$

$= (240\times10^6)(0.060)(0.030)$

$= 432\times10^3$ N

$y_1 = 15\text{ mm} + 15\text{ mm} = 30\text{ mm}$
$= 0.030$ m

$R_2 = \sigma_Y A_2$

$= (240\times10^6)(0.030)(0.015)$

$= 10.8\times10^3$ N

$y_2 = \frac{1}{2}(15) = 7.5\text{ mm} = 0.0075$ m

$$M_p = 2(R_1 y_1 + R_2 y_2) = 2\left[(432\times10^3)(0.030) + (10.8\times10^3)(0.0075)\right]$$

$$= 27.54\times10^3 \text{ N}\cdot\text{m} = 27.5 \text{ kN}\cdot\text{m}$$

(b) $I_{rect} = \frac{1}{12}bh^3 = \frac{1}{12}(60)(90)^3 = 3.645\times10^6 \text{ mm}^4$

$I_{cutout} = \frac{1}{12}bh^3 = \frac{1}{12}(30)(30)^3 = 67.5\times10^3 \text{ mm}^4$

$I = I_{rect} - I_{cutout} = 3.645\times10^6 - 67.5\times10^3 = 3.5775\times10^3 \text{ mm}^4$
$= 3.5775\times10^{-9} \text{ m}^4$

$c = \frac{1}{2}h = 45 \text{ mm} = 0.045$ m

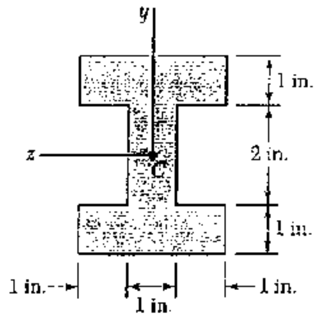
$$M_Y = \frac{\sigma_Y I}{c} = \frac{(240\times10^6)(3.5775\times10^{-9})}{0.045} = 19.08\times10^3 \text{ N}\cdot\text{m}$$

$$k = \frac{M_p}{M_Y} = \frac{27.54}{19.08} = 1.443$$

Problem 4.79

4.77 through 4.80 For the beam indicated, determine (*a*) the fully plastic moment M_p, (*b*) the shape factor of the cross section.

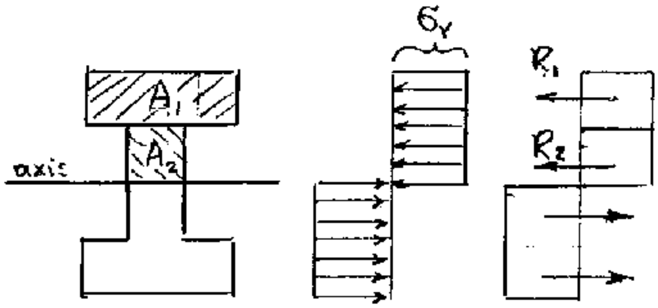
4.79 Beam of Prob. 4.75.



From PROBLEM 4.75 $E = 29 \times 10^6$ psi and $\sigma_Y = 42$ ksi.

$R_1 = \sigma_Y A_1 = (42)(3)(1) = 126$ kip

$\bar{y}_1 = 1.0 + 0.5 = 1.5$ in



$R_2 = \sigma_Y A_2 = (42)(1)(1) = 42$ kip.

$\bar{y}_2 = \frac{1}{2}(1) = 0.5$ in.

$$M_p = 2(R_1 y_1 + R_2 y_2) = 2\left[(126)(1.5) + (42)(0.5)\right] = 420 \text{ kip}\cdot\text{in}$$

(b) $I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(3)(1)^3 + (3)(1)(1.5)^2 = 7.0$ in^4

$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(1)(2)^3 = 0.6667$ in^4

$I_3 = I_1 = 7.0$ in^4

$I = I_1 + I_2 + I_3 = 14.6667$ in^4

$c = 2$ in

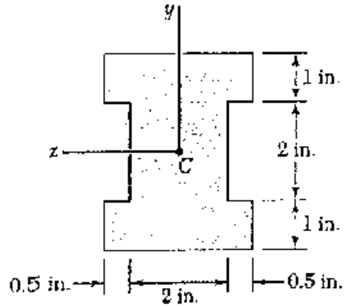
$$M_Y = \frac{\sigma_Y I}{c} = \frac{(42)(14.6667)}{2} = 308 \text{ kip}\cdot\text{in.}$$

$$k = \frac{M_p}{M_Y} = \frac{420}{308} = 1.364$$

Problem 4.80

4.77 through 4.80 For the beam indicated, determine (*a*) the fully plastic moment M_p, (*b*) the shape factor of the cross section.

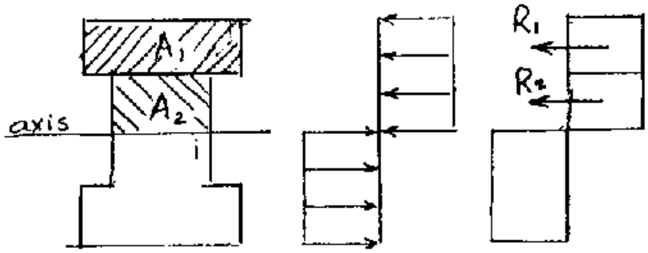
4.80 Beam of Prob. 4.76.



From PROBLEM 4.76 $E = 29 \times 10^6$ and $\sigma_Y = 42$ ksi.

$R_1 = \sigma_Y A_1 = (42)(3)(1) = 126$ kip

$y_1 = 1.0 + 0.5 = 1.5$ in.



$R_2 = \sigma_Y A_2 = (42)(2)(1) = 84$ kip

$y_2 = \frac{1}{2}(1.0) = 0.5$ in.

$M_p = 2(R_1 y_1 + R_2 y_2) = 2[(126)(1.5) + (84)(0.5)] = 462$ kip·in ◀

(b) $I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(3)(1)^3 + (3)(1)(1.5)^2 = 7.0$ in^4

$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(2)(2)^3 = 1.3333$ in^4

$I_3 = I_1 = 7.0$ in^4

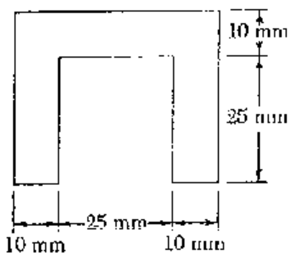
$I = I_1 + I_2 + I_3 = 15.3333$ in^4

$c = 2$ in.

$M_Y = \frac{\sigma_Y I}{c} = \frac{(42)(15.3333)}{2} = 322$ kip·in

$k = \frac{M_p}{M_Y} = \frac{462}{322} = 1.435$ ◀

Problem 4.81

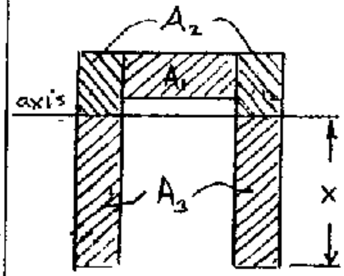


4.81 and 4.82 Determine the plastic moment M_p of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 240 MPa..

Total area $A = (25)(10) + (2)(10)(35) = 950\ mm^2$

$\frac{1}{2}A = 475\ mm^2$

$x = \frac{\frac{1}{2}A}{2b} = \frac{475}{20} = 23.75\ mm = 0.02375\ m$



$R_1 = \sigma_Y A_1 = (240\times10^6)(0.025)(0.010) = 60\times10^3\ N$

$\bar{y}_1 = 30 - 23.75 = 6.25\ m = 0.00625\ m$

$R_2 = \sigma_Y A_2 = (240\times10^6)(0.020)(0.01125) = 54\times10^3\ N$

$\bar{y}_2 = \frac{1}{2}(0.01125) = 0.005625\ m$

$R_3 = \sigma_Y A_3 = (240\times10^6)(0.020)(0.02375) = 114\times10^3\ N$

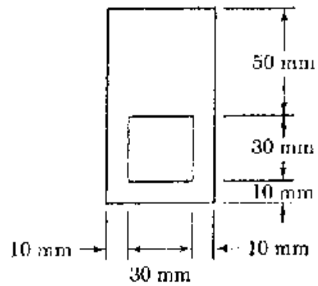
$\bar{y}_3 = \frac{1}{2}x = 0.011875\ m$

$M_p = R_1\bar{y}_1 + R_2\bar{y}_2 + R_3\bar{y}_3$

$= (60\times10^3)(0.00625) + (54\times10^3)(0.005625) + (114\times10^3)(0.011875)$

$= 2.0325\times10^3\ N\cdot m = 2.03\ kN\cdot m$

Problem 4.82

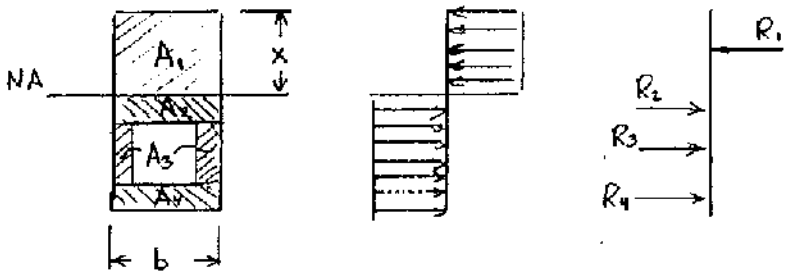


***4.81* and 4.82** Determine the plastic moment M_p of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 240 MPa..

Total area $A = (50)(90) - (30)(30) = 3600 \text{ mm}^2$

$\frac{1}{2}A = 1800 \text{ mm}^2$

$x = \frac{\frac{1}{2}A}{b} = \frac{1800}{50} = 36 \text{ mm}$



$A_1 = (50)(36) = 1800 \text{ mm}^2, \quad \bar{y}_1 = 18 \text{ mm} \quad A_1\bar{y}_1 = 32.4 \times 10^3 \text{ mm}^3$

$A_2 = (50)(14) = 700 \text{ mm}^2, \quad \bar{y}_2 = 7 \text{ mm} \quad A_2\bar{y}_2 = 4.9 \times 10^3 \text{ mm}^3$

$A_3 = (20)(30) = 600 \text{ mm}^2, \quad \bar{y}_3 = 29 \text{ mm} \quad A_3\bar{y}_3 = 17.4 \times 10^3 \text{ mm}^3$

$A_4 = (50)(10) = 500 \text{ mm}^2, \quad \bar{y}_4 = 49 \text{ mm} \quad A_4\bar{y}_4 = 24.5 \times 10^3 \text{ mm}^3$

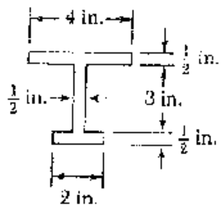
$A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3 + A_4\bar{y}_4 = 79.2 \times 10^3 \text{ mm}^3 = 79.2 \times 10^{-6} \text{ m}^3$

$M_p = \sigma_Y \sum A_i\bar{y}_i = (240 \times 10^6)(79.2 \times 10^{-6}) = 19.008 \times 10^3 \text{ N}\cdot\text{m}$

$= 19.01 \text{ kN}\cdot\text{m}$

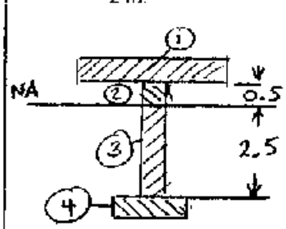
Problem 4.83

4.83 Determine the plastic moment M_p of the cross section shown assuming that the steel to be elastoplastic with a yield strength of 36 ksi.



Total area: $A = (4)(\frac{1}{2}) + (\frac{1}{2})(3) + (2)(\frac{1}{2}) = 4.5 \text{ in}^2$

$\frac{1}{2}A = 2.25 \text{ in}^2$



$A_1 = 2.00 \text{ in}^2, \quad \bar{y}_1 = 0.75, \quad A_1 y_1 = 1.50 \text{ in}^3$

$A_2 = 0.25 \text{ in}^2, \quad \bar{y}_2 = 0.25, \quad A_2 \bar{y}_2 = 0.0625 \text{ in}^3$

$A_3 = 1.25 \text{ in}^2, \quad \bar{y}_3 = 1.25, \quad A_3 \bar{y}_3 = 1.5625 \text{ in}^3$

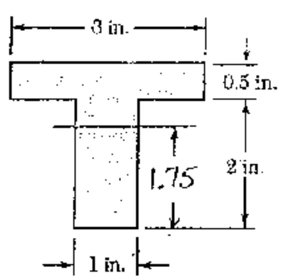
$A_4 = 1.00 \text{ in}^2, \quad \bar{y}_4 = 2.75, \quad A_4 \bar{y}_4 = 2.75 \text{ in}^3$

$M_p = \sigma_Y (A_1 \bar{y}_1 + A_2 \bar{y}_2 + A_3 \bar{y}_3 + A_4 \bar{y}_4)$

$= (36)(1.50 + 0.0625 + 1.5625 + 2.75) = 211.5 \text{ kip} \cdot \text{in}$

Problem 4.84

4.84 Determine the plastic moment M_p of the cross section shown, assuming that the steel to be elastoplastic with a yield strength of 48 ksi.

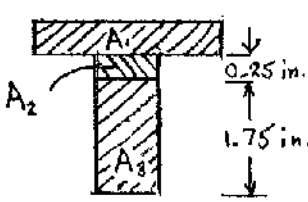


Total area: $A = (3)(0.5) + (1)(2) = 3.5 \text{ in}^2$

$\frac{1}{2}A = 1.75 \text{ in}^2 \qquad y_n = \frac{\frac{1}{2}A}{b} = \frac{1.75}{1} = 1.75 \text{ in.}$

$A_1 = (3)(0.5) = 1.5 \text{ in}^2, \quad \bar{y}_1 = 0.5 \text{ in.}, \quad A_1 \bar{y}_1 = 0.75 \text{ in}^3$

$A_2 = (1)(0.25) = 0.25 \text{ in}^2, \quad \bar{y}_2 = 0.125 \text{ in.} \quad A_2 \bar{y}_2 = 0.03125 \text{ in}^3$



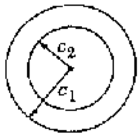
$A_3 = (1)(1.75) = 1.75 \text{ in}^2, \quad \bar{y}_3 = 0.875 \text{ in} \quad A_3 \bar{y}_3 = 1.53125 \text{ in}^3$

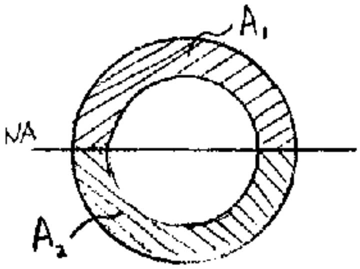
$M_p = \sigma_Y (A_1 \bar{y}_1 + A_2 \bar{y}_2 + A_3 \bar{y}_3)$

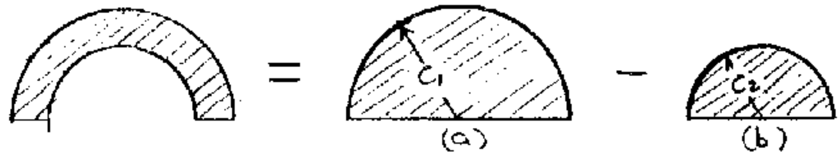
$= 48 (0.75 + 0.03125 + 1.53125) = 111.0 \text{ kip} \cdot \text{in}$

Problem 4.85

4.85 A thick-walled pipe of the cross section shown is made of a steel that is assumed to be elastoplastic with a yield strength σ_Y. Derive an expression for the plastic moment M_p of the pipe in terms of c_1, c_2, and σ_Y.







$$A_1\bar{y}_1 = A_a\bar{y}_a - A_b\bar{y}_b$$

$$= \left(\frac{\pi}{2}c_1^2\right)\left(\frac{4c_1}{3\pi}\right) - \left(\frac{\pi}{2}c_2^2\right)\left(\frac{4c_2}{3\pi}\right)$$

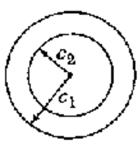
$$= \frac{2}{3}\left(c_1^3 - c_2\right)^3$$

$$A_2\bar{y}_2 = A_1\bar{y}_1 = \frac{2}{3}\left(c_1^3 - c_2^2\right)$$

$$M_p = \sigma_Y\left(A_1\bar{y}_1 + A_2\bar{y}_2\right) = \frac{4}{3}\sigma_Y\left(c_1^3 - c_2^3\right) \blacktriangleleft$$

Problem 4.86

4.86 Determine the plastic moment M_p of a thick-walled pipe of the cross section shown, knowing that $c_1 = 60$ mm, $c_2 = 40$ mm, and $\sigma_Y = 240$ MPa.



See the solution to PROBLEM 4.85 for derivation of the following expression for M_p.

$$M_p = \frac{4}{3}\sigma_Y\left(c_1^3 - c_2^3\right)$$

Data: $\sigma_Y = 240 \text{ MPa} = 240 \times 10^6 \text{ Pa}$

$c_1 = 60 \text{ mm} = 0.060 \text{ m}$

$c_2 = 40 \text{ mm} = 0.040 \text{ m}$

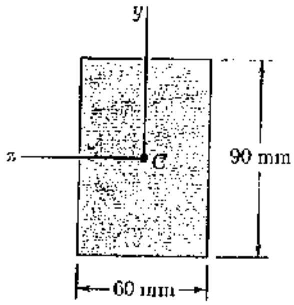
$$M_p = \frac{4}{3}\left(240 \times 10^6\right)\left(0.060^3 - 0.040^3\right) = 48.64 \times 10^3 \text{ N}\cdot\text{m}$$

$$= 48.6 \text{ kN}\cdot\text{m} \blacktriangleleft$$

Problem 4.87

4.87 and 4.88 For the beam indicated a couple of moment equal to the full plastic moment M_p is applied and then removed. Using a yield strength of 240 MPa, determine the residual stress at $y = 45$ mm.

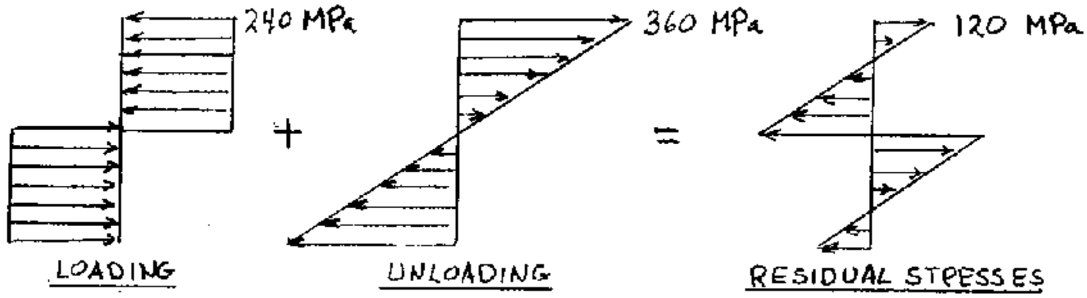
4.87 Beam of Prob. 4.73.



$M_p = 29.16 \times 10^3$ N·m (See solutions to Problems 4.73 and 4.77)

$I = 3.645 \times 10^{-6}\ m^4$, $\quad c = 0.045$ m

$\sigma' = \dfrac{M_{max}\, y}{I} = \dfrac{M_p c}{I}$ at $y = c = 45$ mm.



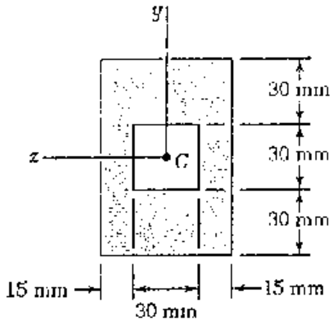
$\sigma' = \dfrac{(29.16 \times 10^3)(0.045)}{3.645 \times 10^{-6}} = 360 \times 10^6$ Pa

$\sigma_{res} = \sigma' - \sigma_Y = 360 \times 10^6 - 240 \times 10^6 = 120 \times 10^6$ Pa $= 120$ MPa

Problem 4.88

4.87 and 4.88 For the beam indicated a couple of moment equal to the full plastic moment M_p is applied and then removed. Using a yield strength of 240 MPa, determine the residual stress at $y = 45$ mm.

4.88 Beam of Prob. 4.74.

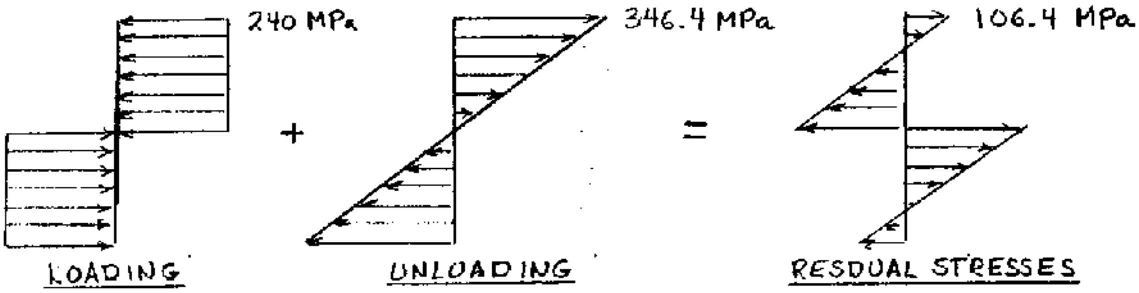


$M_p = 27.54 \times 10^3$ N·m (See solutions to Problems 4.74 and 4.78)

$I = 3.5775 \times 10^{-6}$ m^4, $\quad c = 0.045$ m

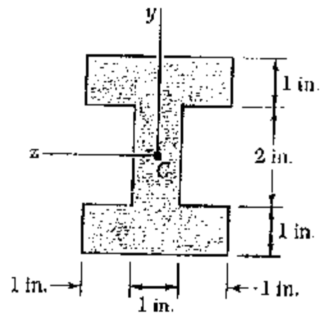
$\sigma' = \frac{M_{max} y}{I} = \frac{M_p c}{I}$ at $y = c$.

$\sigma' = \frac{(27.54 \times 10^3)(0.045)}{3.5775 \times 10^{-6}} = 346.4 \times 10^6$ Pa



$\sigma_{res} = \sigma' - \sigma_Y = 346.4 \times 10^6 - 240 \times 10^6 = 106.4 \times 10^6$ Pa $= 106.4$ MPa ◄

Problem 4.89



4.89 and 4.90 For the beam indicated a couple of moment equal to the full plastic moment M_p is applied and then removed. Using a yield strength of 42 ksi, determine the residual stress at (*a*) $y = 1$ in., (*b*) $y = 2$ in.

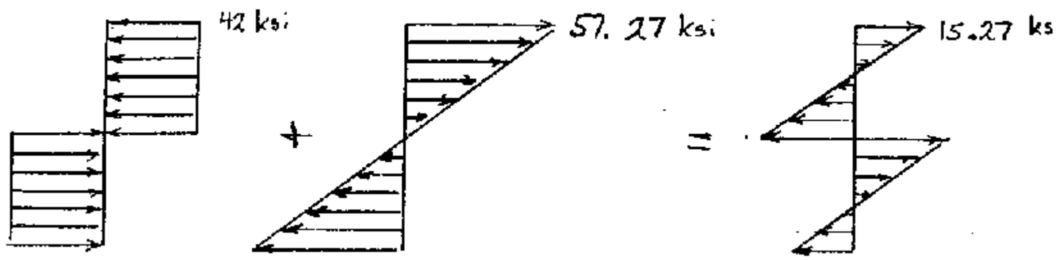
4.89 Beam of Prob. 4.75.

$M_p = 420$ kip·in (See solutions to Problems 4.75 and 4.79)

$I = 14.6667 \text{ in}^4$, $c = 2$ in.

$\sigma' = \frac{M_{max} y}{I} = \frac{M_p c}{I}$ at $y = c$.

$\sigma' = \frac{(420)(2)}{14.6667} = 57.27$ ksi



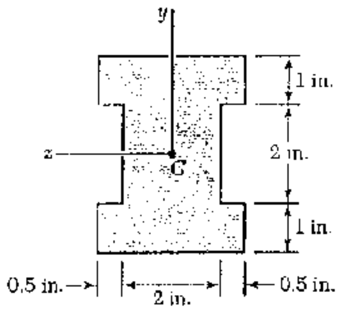
(a) At $y = 1 \text{ in.} = \frac{1}{2}c$ $\quad \sigma' = \frac{1}{2}(57.27) = 28.64$ ksi

$\sigma_{res} = -42 + 28.64 = -13.36$ ksi

(b) At $y = 2 \text{ in} = c$ $\quad \sigma' = 57.27$ ksi

$\sigma_{res} = -42 + 57.27 = 15.27$ ksi

Problem 4.90



4.89 and 4.90 For the beam indicated a couple of moment equal to the full plastic moment M_p is applied and then removed. Using a yield strength of 42 ksi, determine the residual stress at (*a*) $y = 1$ in., (*b*) $y = 2$ in.

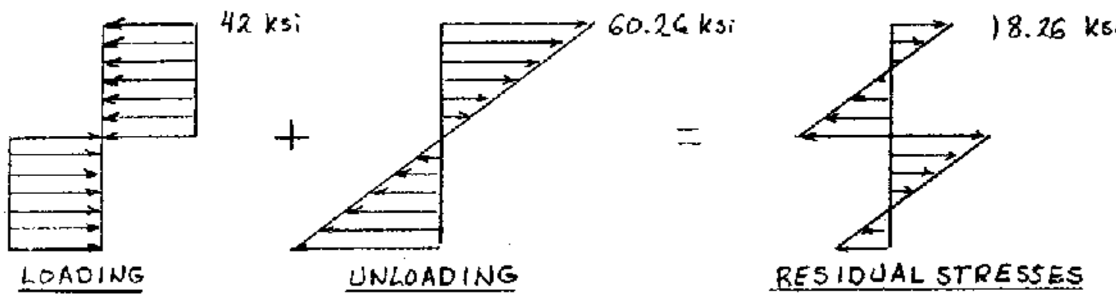
4.90 Beam of Prob. 4.76.

$M_p = 462$ kip·in (See solutions to Problems 4.76 and 4.80)

$I = 15.3333 \text{ in}^4, \qquad c = 2 \text{ in.}$

$$\sigma' = \frac{M_{max} y}{I} = \frac{M_p c}{I} \quad \text{for } y = c$$

$$\sigma' = \frac{(462)(2)}{15.3333} = 60.26 \text{ ksi}$$



(a) At $y = 1 \text{ in.} = \frac{1}{2}c \qquad \sigma' = \frac{1}{2}(60.26) = 30.13$ ksi

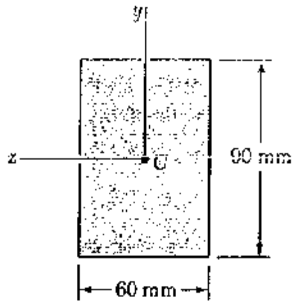
$$\sigma_{res} = -42 + 30.13 = -11.87 \text{ ksi}$$

(b) At $y = 2 \text{ in.} = c \qquad \sigma' = 60.26$ ksi

$$\sigma_{res} = -42 + 60.26 = 18.26 \text{ ksi}$$

Problem 4.91

4.91 A bending couple is applied to the beam of Prob. 4.73, causing plastic zones 30 mm thick to develop at the top and bottom of the beam. After the couple has been removed, determine (*a*) the residual stress at $y = 45$ mm, (*b*) the points where the residual stress is zero, (*c*) the radius of curvature corresponding to the permanent deformation of the beam.



See SOLUTION to PROBLEM 4.73 for bending couple and stress distribution during loading.

$M = 28.08\times10^3$ N·m $\quad y_Y = 15$ mm $= 0.015$ m

$E = 200$ GPa $\quad \sigma_Y = 240$ MPa

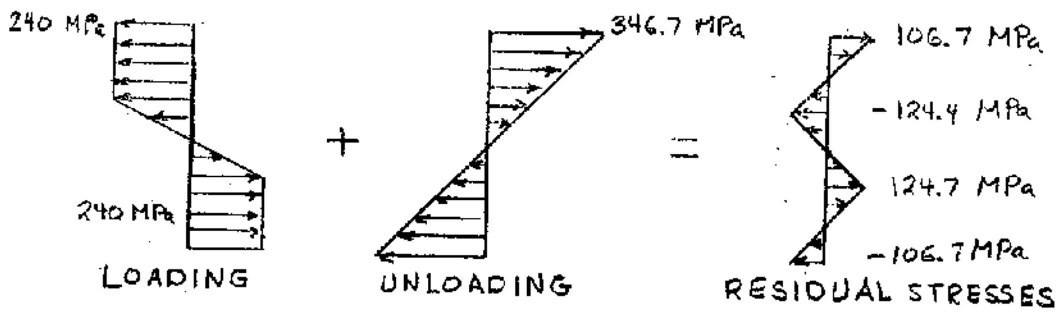
$I = 3.645\times10^{-6}$ m^4 $\quad c = 0.045$ m

(a) $\sigma' = \dfrac{Mc}{I} = \dfrac{(28.08\times10^3)(0.045)}{3.645\times10^{-6}} = 346.7\times10^6$ Pa $= 346.7$ MPa

$\sigma'' = \dfrac{My_Y}{I} = \dfrac{(28.08\times10^3)(0.015)}{3.645\times10^{-6}} = 115.6\times10^6$ Pa $= 115.6$ MPa

At $y = c$ $\quad \sigma_{res} = \sigma' - \sigma_Y = 346.7 - 240 = 106.7$ MPa

At $y = y_Y$ $\quad \sigma_{res} = \sigma'' - \sigma_Y = 115.6 - 240 = -124.4$ MPa



(b) $\sigma_{res} = 0 \quad \therefore \quad \dfrac{My_o}{I} - \sigma_Y = 0$

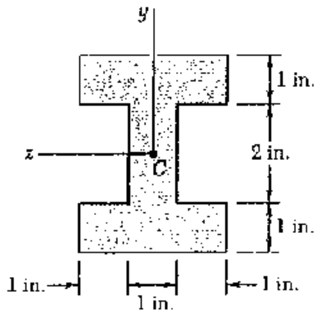
$y_o = \dfrac{I\sigma_Y}{M} = \dfrac{(3.645\times10^{-6})(240\times10^6)}{28.08\times10^3} = 31.15\times10^{-3}$ m $= 31.15$ mm

ans. $y_o = -31.15$ mm, 0, 31.15 mm

(c) At $y = y_Y$, $\quad \sigma_{res} = -124.4\times10^6$ Pa

$\sigma = -\dfrac{Ey}{\rho} \quad \therefore \quad \rho = -\dfrac{Ey}{\sigma} = \dfrac{(200\times10^9)(0.015)}{-124.4\times10^6} = 24.1$ m

Problem 4.92



4.92 A bending couple is applied to the beam of Prob. 4.75, causing plastic zones 2 in. thick to develop at the top and bottom of the beam. After the couple has been removed, determine (*a*) the residual stress at $y = 2$ in., (*b*) the points where the residual stress is zero, (*c*) the radius of curvature corresponding to the permanent deformation of the beam.

See SOLUTION to PROBLEM 4.75 for bending couple and stress distribution

$M = 406$ kip·in $\qquad y_Y = 1.0$ in

$E = 29 \times 10^6$ psi $= 29 \times 10^3$ ksi $\qquad \sigma_Y = 42$ ksi

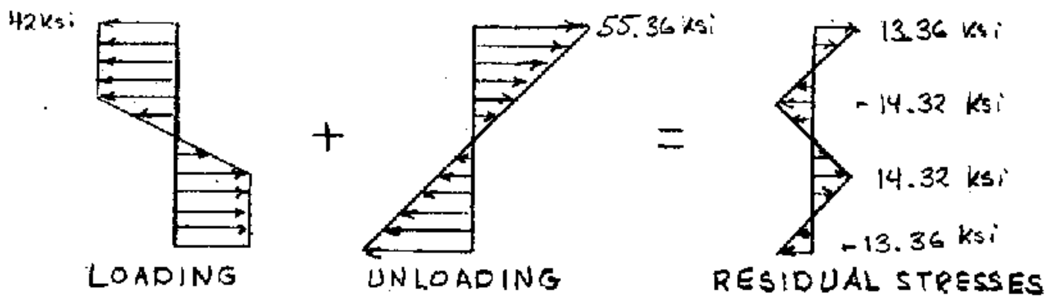
$I = 14.6667\ \text{in}^4 \qquad c = 2$ in.

(a) $\sigma' = \dfrac{Mc}{I} = \dfrac{(406)(2)}{14.6667} = 55.36$ ksi

$\sigma'' = \dfrac{My_Y}{I} = \dfrac{(406)(1.0)}{14.6667} = 27.68$ ksi

At $y = c \qquad \sigma_{res} = \sigma' - \sigma_Y = 55.36 - 42 = 13.36$ ksi

At $y = y_Y \qquad \sigma_{res} = \sigma'' - \sigma_Y = 27.68 - 42 = -14.32$ ksi



(b) $\sigma_{res} = 0 \quad \therefore \quad \dfrac{My_0}{I} - \sigma_Y = 0$

$y_0 = \dfrac{I\sigma_Y}{M} = \dfrac{(14.6667)(42)}{406} = 1.517$ in

ans. $\quad y_0 = -1.517$ in., 0, 1.517 in.

(c) At $y = y_Y$, $\quad \sigma_{res} = -14.32$ ksi

$\sigma = -\dfrac{Ey}{\rho} \quad \therefore \quad \rho = -\dfrac{Ey}{\sigma} = \dfrac{(29 \times 10^3)(1.0)}{14.32} = 2025$ in

$= 168.8$ ft

Problem 4.93

4.93 A rectangular bar which is straight and unstressed is bent into an arc of circle of radius ρ by two couples of moment M. After the couples are removed, it is observed that the radius of curvature of the bar is ρ_R. Denoting by ρ_Y the radius of curvature of the bar at the onset of yield, shown that the radii of curvature satisfy the following relation:

$$\frac{1}{\rho_R} = \frac{1}{\rho}\left(1 - \frac{3}{2}\frac{\rho}{\rho_Y}\left[1 - \frac{1}{3}\left(\frac{\rho}{\rho_Y}\right)^2\right]\right)$$

$$\frac{1}{\rho_Y} = \frac{M_Y}{EI}, \qquad M = \frac{3}{2}M_Y\left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right) \qquad \text{Let } m \text{ denote } \frac{M}{M_Y}$$

$$m = \frac{M}{M_Y} = \frac{3}{2}\left(1 - \frac{\rho^2}{\rho_Y^2}\right) \qquad \therefore \qquad \frac{\rho^2}{\rho_Y^2} = 3 - 2m$$

$$\frac{1}{\rho_R} = \frac{1}{\rho} - \frac{M}{EI} = \frac{1}{\rho} - \frac{mM_Y}{EI} = \frac{1}{\rho} - \frac{m}{\rho_Y}$$

$$= \frac{1}{\rho}\left\{1 - \frac{\rho}{\rho_Y}m\right\} = \frac{1}{\rho}\left\{1 - \frac{3}{2}\frac{\rho}{\rho_Y}\left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right)\right\}$$

Problem 4.94

4.94 A solid bar of rectangular cross section is made of a material that is assumed to be elastoplastic. Denoting by M_Y and ρ_Y, respectively, the bending moment and radius of curvature at the onset of yield, determine (*a*) the radius of curvature when a couple of moment $M = 1.25\,M_Y$ is applied to the bar, (*b*) the radius of curvature after the couple is removed. Check the results obtained by using the relation derived in Prob. 4.93.

(a) $$\frac{1}{\rho_Y} = \frac{M_Y}{EI}, \qquad M = \frac{3}{2}M_Y\left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right) \qquad \text{Let } m = \frac{M}{M_Y} = 1.25$$

$$m = \frac{M}{M_Y} = \frac{3}{2}\left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right) \qquad \frac{\rho}{\rho_Y} = \sqrt{3 - 2m} = 0.70711$$

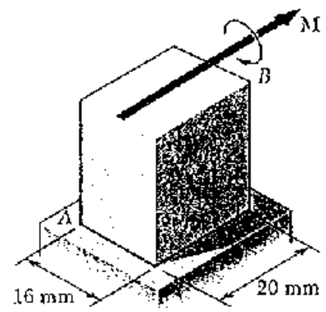
$$\rho = 0.70711\,\rho_Y$$

(b) $$\frac{1}{\rho_R} = \frac{1}{\rho} - \frac{M}{EI} = \frac{1}{\rho} - \frac{mM_Y}{EI} = \frac{1}{\rho} - \frac{m}{\rho_Y} = \frac{1}{0.70711\,\rho_Y} - \frac{1.25}{\rho_Y}$$

$$= \frac{0.16421}{\rho_Y} \qquad \rho_R = 6.09\,\rho_Y$$

Problem 4.95

4.95 The prismatic bar *AB* is made of a steel which is assumed to be elastoplastic and for which $E = 200$ GPa. Knowing that the radius of curvature of the bar is 2.4 m when a couple of moment $M = 350$ N · m is applied as shown, determine (*a*) the yield strength of the steel, (*b*) the thickness of the elastic core of the bar.



$$M = \frac{3}{2} M_Y \left(1 - \frac{1}{3} \frac{\rho^2}{\rho_Y^2} \right)$$
$$= \frac{3}{2} \frac{\sigma_Y I}{c} \left(1 - \frac{1}{3} \frac{\rho^2 \sigma_Y^2}{E^2 c^2} \right)$$
$$= \frac{3}{2} \frac{\sigma_Y b(2c)^3}{12 c} \left(1 - \frac{1}{3} \frac{\rho^2 \sigma_Y^2}{E^2 c^2} \right)$$
$$= \sigma_Y b c^2 \left(1 - \frac{1}{3} \frac{\rho^2 \sigma_Y^2}{E^2 c^2} \right)$$

(a) $bc^2 \sigma_Y \left(1 - \frac{\rho^2 \sigma_Y^2}{3 E^2 c^2} \right) = M$ Cubic equation for σ_Y

Data: $E = 200 \times 10^9$ Pa, $M = 420$ N·m $\rho = 2.4$ m

$b = 20$ mm $= 0.020$ m $c = \frac{1}{2} h = 8$ mm $= 0.008$ m

$(1.28 \times 10^{-6}) \sigma_Y \left[1 - 750 \times 10^{-21} \sigma_Y^2 \right] = 350$

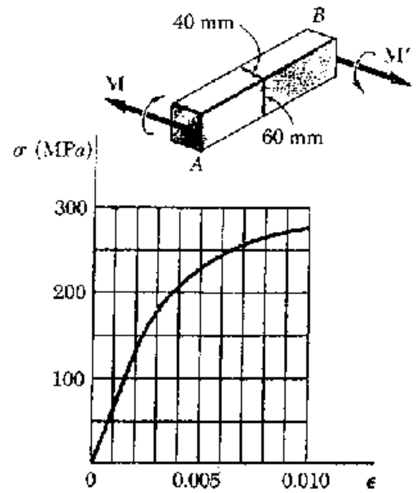
$\sigma_Y \left[1 - 750 \times 10^{-21} \sigma_Y^2 \right] = 273.44 \times 10^6$

Solving by trial $\sigma_Y = 292 \times 10^6$ Pa $= 292$ MPa ◄

(b) $y_Y = \frac{\sigma_Y \rho}{E} = \frac{(292 \times 10^6)(2.4)}{200 \times 10^9} = 3.504 \times 10^{-3}$ m $= 3.504$ mm

thickness of elastic core $= 2y_Y = 7.01$ mm ◄

Problem 4.96



4.96 The prismatic bar AB is made of an aluminum alloy for which the tensile stress-strain diagram is as shown. Assuming that the σ - ϵ diagram is the same in compression as in tension, determine (*a*) the radius of curvature of the bar when the maximum stress is 250 MPa, (*b*) the corresponding value of the bending moment. (*Hint*: For part *b*, plot σ versus y and use an approximate method of integration.)

(a) $\sigma_m = 250 \text{ MPa} = 250 \times 10^6 \text{ Pa}$

$\varepsilon_m = 0.0064$ from curve

$c = \frac{1}{2} h = 30 \text{ mm} = 0.030 \text{ m}$

$b = 40 \text{ mm} = 0.040 \text{ m}$

$\frac{1}{\rho} = \frac{\varepsilon_m}{c} = \frac{0.0064}{0.030} = 0.21333 \text{ m}^{-1}$

$\rho = 4.69 \text{ m}$

(b) Strain distribution $\quad \varepsilon = -\varepsilon_m \frac{y}{c} = -\varepsilon_m u \quad$ where $u = \frac{y}{c}$

Bending couple

$$M = -\int_{-c}^{c} y \sigma b\, dy = 2b \int_0^c y|\sigma|\, dy = 2bc^2 \int_0^1 u|\sigma|\, du = 2bc^2 J$$

where the integral J is given by $\int_0^1 u|\sigma|\, du$

Evaluate J using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$J = \frac{\Delta u}{3} \sum w u |\sigma|$$

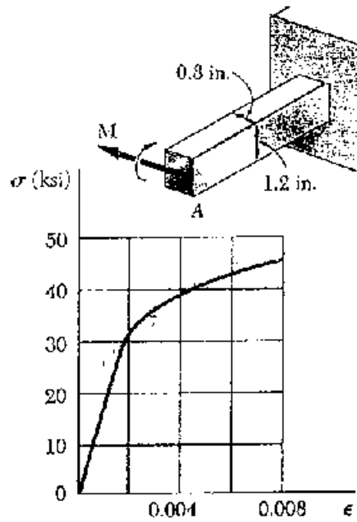
where w is a weighting factor, Using $\Delta u = 0.25$ we get the values given in the table below:

u	$\lvert\varepsilon\rvert$	$\lvert\sigma\rvert$, (MPa)	$u\lvert\sigma\rvert$, (MPa)	w	$wu\lvert\sigma\rvert$, (MPa)
0	0	0	0	1	0
0.25	0.0016	110	27.5	4	110
0.5	0.0032	180	90	2	180
0.75	0.0048	225	168.75	4	675
1.00	0.0064	250	250	1	250
					1215 ← $\sum wu\lvert\sigma\rvert$

$$J = \frac{(0.25)(1215)}{3} = 101.25 \text{ MPa} = 101.25 \times 10^6 \text{ Pa}$$

$$M = (2)(0.040)(0.030)^2(101.25 \times 10^6) = 7.29 \times 10^3 \text{ N·m} = 7.29 \text{ kN·m}$$

Problem 4.97



4.97 The prismatic bar AB is made of a bronze alloy for which the tensile stress-strain diagram is as shown. Assuming that the σ - ϵ diagram is the same in compression as in tension, determine (*a*) the maximum stress in the bar when the radius of curvature of the bar is 100 in., (*b*) the corresponding value of the bending moment. (See hint given in Prob. 4.96.)

(a) $\rho = 100$ in, $b = 0.8$ in, $c = 0.6$ in.

$$\varepsilon_m = \frac{c}{\rho} = \frac{0.6}{100} = 0.006$$

From the curve $\epsilon_m = 43$ ksi

(b) Strain distribution $\varepsilon = -\varepsilon_m \frac{y}{c} = -\varepsilon_m u$ where $u = \frac{y}{c}$

Bending couple

$$M = -\int_{-c}^{c} y\sigma b\,dy = 2b\int_0^c y|\sigma|\,dy = 2bc^2\int_0^1 u|\sigma|\,du = 2bc^2 J$$

where the integral J is given by $\int_0^1 u|\sigma|\,du$

Evaluate J using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$J = \frac{\Delta u}{3}\sum w u|\sigma|$$

where w is a weighting factor. Using $\Delta u = 0.25$ we get the values given in the table below:

u	$\lvert\varepsilon\rvert$	$\lvert\sigma\rvert$, ksi	$u\lvert\sigma\rvert$, ksi	w	$wu\lvert\sigma\rvert$, ksi
0	0	0	0	1	0
0.25	0.0015	25	6.25	4	25
0.5	0.003	36	18	2	36
0.75	0.0045	40	30	4	120
1.00	0.006	43	43	1	43
					224 ← $\sum wu\lvert\sigma\rvert$

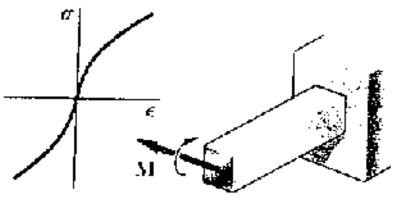
$$J = \frac{(0.25)(224)}{3} = 18.67 \text{ ksi}$$

$$M = (2)(0.8)(0.6)^2(18.67) = 10.75 \text{ kip-in.}$$

Problem 4.98

4.98 A prismatic bar of rectangular cross section is made of an alloy for which the stress-strain diagram can be represented by the relation $\epsilon = k\sigma^n$ for $\sigma > 0$ and $\epsilon = -|k\sigma^n|$ for $\sigma < 0$. If a couple **M** is applied to the bar show that the maximum stress is

$$\sigma_m = \frac{1+2n}{3n}\frac{Mc}{I}$$



Strain distribution $\quad \varepsilon = -\varepsilon_m \frac{y}{c} = -\varepsilon_m v \quad$ where $v = \frac{y}{c}$

Bending couple

$$M = -\int_{-c}^{c} y\sigma b\,dy = 2b\int_0^c y|\sigma|\,dy = 2bc^2\int_0^1 \frac{y}{c}|\sigma|\frac{dy}{c}$$

$$= 2bc^2\int_0^1 v|\sigma|\,dv$$

For $\quad \varepsilon = K\sigma^n, \qquad \varepsilon_m = K\sigma_m$

$$\frac{\varepsilon}{\varepsilon_m} = v = \left(\frac{\sigma}{\sigma_m}\right)^n \qquad \therefore\ |\sigma| = \sigma_m v^{\frac{1}{n}}$$

Then $\quad M = 2bc^2\int_0^1 v\,\sigma_m v^{\frac{1}{n}}\,dv = 2bc^2\sigma_m\int_0^1 v^{1+\frac{1}{n}}\,dv$

$$= 2bc^2\,\sigma_m\,\frac{v^{2+\frac{1}{n}}}{2+\frac{1}{n}}\Bigg|_0^1 = \frac{2n}{2n+1}\,bc^2\sigma_m$$

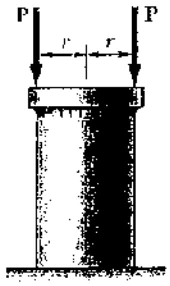
$$\sigma_m = \frac{2n+1}{2}\,\frac{M}{bc^2}$$

Recall $\quad \frac{I}{c} = \frac{1}{12}\frac{b(2c)^3}{c} = \frac{2}{3}bc^2 \qquad \therefore\ \frac{1}{bc^2} = \frac{2}{3}\frac{c}{I}$

$$\sigma_m = \frac{2n+1}{3n}\,\frac{Mc}{I}$$

Problem 4.99

4.99 Two forces **P** can be applied separately or at the same time to a plate which is welded to a solid circular bar of radius r. Determine the largest compressive stress in the circular bar, (a) when both forces are applied, (b) when only one of the forces is applied.



For a solid circular section $A = \pi r^2$, $I = \frac{\pi}{4} r^4$, $c = r$

Compressive stress $\sigma = -\frac{F}{A} - \frac{Mc}{I}$

$= -\frac{F}{\pi r^2} - \frac{4M}{\pi r^3}$

(a) Both forces applied. $F = 2P$, $M = 0$

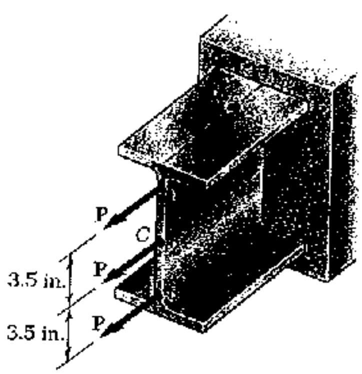
$\sigma = -2P/\pi r^2$ ◄

(b) One force applied. $F = P$, $M = Pr$

$\sigma = -\frac{F}{\pi r^2} - \frac{4Pr}{\pi r^3}$ $\sigma = -5P/\pi r^2$ ◄

Problem 4.100

4.100 As many as three axial loads each of magnitude $P = 10$ kips can be applied to the end of a W 8 × 21 rolled-steel shape. Determine the stress at point A, (a) for the loading shown, (b) if loads are applied at points 1 and 2 only.



For W 8 × 21 Appendix C gives

$A = 6.16 \text{ in}^2$, $d = 8.28$ in., $I_x = 75.3 \text{ in}^4$

At point A $y = \frac{1}{2} d = 4.14$ in.

$\sigma = \frac{F}{A} - \frac{My}{I}$

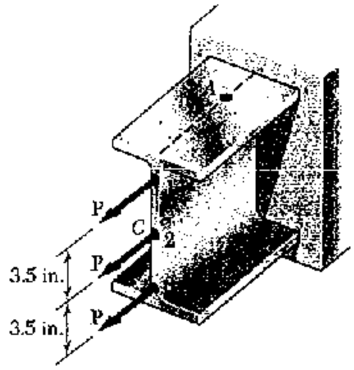
(a) Centric loading: $F = 30$ kips, $M = 0$

$\sigma = \frac{30}{6.16}$ $\sigma = 4.87$ ksi ◄

(b) Eccentric loading: $F = 2P = 20$ kips $M = -(10)(3.5) = -35$ kip·in.

$\sigma = \frac{20}{6.16} - \frac{(-35)(4.14)}{75.3}$ $\sigma = 5.17$ ksi ◄

Problem 4.101



4.101 As many as three axial loads each of magnitude $P = 10$ kips can be applied to the end of a W 8 × 21 rolled-steel shape. Determine the stress at point A, (a) for the loading shown, (b) if loads are applied at points 2 and 3 only.

For W8 × 21 Appendix C gives

$A = 6.16\ \text{in}^2, \quad d = 8.28\ \text{in.}, \quad I_x = 75.3\ \text{in}^4$

At point A $\quad y = \frac{1}{2}d = 4.14$ in.

$$\sigma = \frac{F}{A} - \frac{My}{I}$$

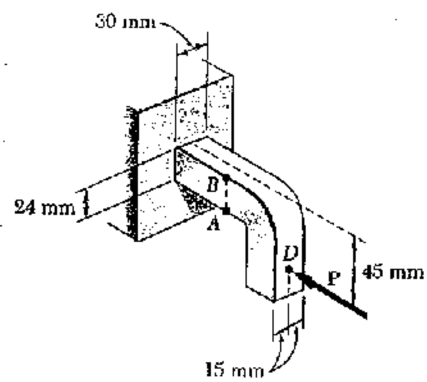
(a) Centric loading: $F = 30$ kips, $M = 0$

$\sigma = \frac{30}{6.16}$ $\qquad \sigma = 4.87$ ksi ◄

(b) Eccentric loading: $F = 2P = 20$ kips, $M = (10)(3.5) = 35$ kip·in

$\sigma = \frac{20}{6.16} - \frac{(35)(4.14)}{75.3}$ $\qquad \sigma = 1.322$ ksi ◄

Problem 4.102



4.102 Knowing that the magnitude of the horizontal force **P** is 8 kN, determine the stress at (a) point A, (b) point B.

$A = (30)(24) = 720\ \text{mm}^2 = 720 \times 10^{-6}\ \text{m}^2$

$e = 45 - 12 = 33\ \text{mm} = 0.033\ \text{m}$

$I = \frac{1}{12}bh^3 = \frac{1}{12}(30)(24)^3 = 34.56 \times 10^3\ \text{mm}^4 = 34.56 \times 10^{-9}\ \text{m}^4$

$c = 24\ \text{mm} = 0.12\ \text{m} \qquad P = 8 \times 10^3\ N$

$M = Pe = (8 \times 10^3)(0.033) = 264\ N\cdot m$

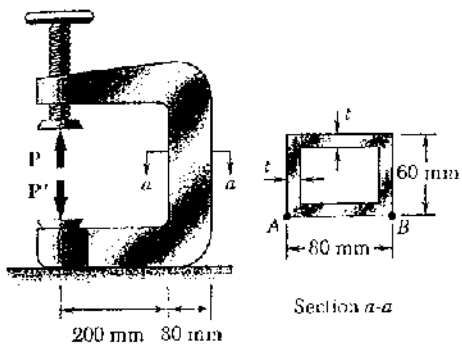
(a) $\quad \sigma_A = -\frac{P}{A} - \frac{Mc}{I} = -\frac{8 \times 10^3}{720 \times 10^{-6}} - \frac{(264)(0.12)}{34.56 \times 10^{-9}}$

$= -102.8 \times 10^6\ \text{Pa} = -102.8\ \text{MPa}$ ◄

(b) $\quad \sigma_B = -\frac{P}{A} + \frac{Mc}{I} = -\frac{8 \times 10^3}{720 \times 10^{-6}} + \frac{(264)(0.12)}{34.56 \times 10^{-9}} = 80.6 \times 10^6\ \text{Pa} = 80.6\ \text{MPa}$ ◄

Problem 4.103

4.103 The vertical portion of the press shown consists of a rectangular tube of wall thickness $t = 10$ mm. Knowing that the press has been tightened on wooden planks being glued together until $P = 20$ kN, determine the stress at (*a*) point *A*, (*b*) point *B*.



Rectangular cutout is 60 mm × 40 mm

$A = (80)(60) - (60)(40) = 2.4\times10^{3}\ \text{mm}^2 = 2.4\times10^{-3}\ \text{m}^2$

$I = \frac{1}{12}(60)(80)^3 - \frac{1}{12}(40)(60)^3 = 1.84\times10^{6}\ \text{mm}^4$

$= 1.84\times10^{-6}\ \text{m}^4$

$c = 40\ \text{mm} = 0.040\ \text{m} \qquad e = 200 + 40 = 240\ \text{mm} = 0.240\ \text{m}$

$M = Pe = (20\times10^3)(0.240) = 4.8\times10^{3}\ \text{N·m}$

(a) $\sigma_A = \frac{P}{A} + \frac{Mc}{I} = \frac{20\times10^3}{2.4\times10^{-3}} + \frac{(4.8\times10^3)(0.040)}{1.84\times10^{-6}} = 112.7\times10^{6}\ \text{Pa}$

$\sigma_A = 112.8\ \text{MPa}$ ◄

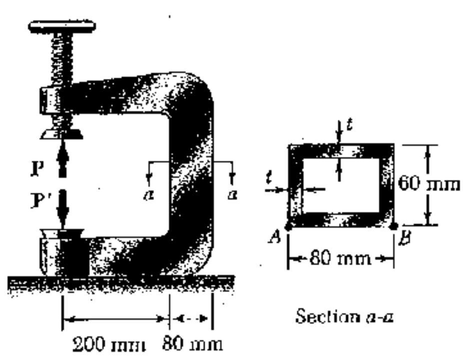
(b) $\sigma_B = \frac{P}{A} - \frac{Mc}{I} = \frac{20\times10^3}{2.4\times10^{-3}} - \frac{(4.8\times10^3)(0.040)}{1.84\times10^{-6}} = -96.0\times10^{6}\ \text{Pa}$

$\sigma_B = -96.0\ \text{MPa}$ ◄

Problem 4.104

4.104 Solve Prob. 4.103, assuming that $t = 8$ mm.

4.103 The vertical portion of the press shown consists of a rectangular tube of wall thickness $t = 10$ mm. Knowing that the press has been tightened on wooden planks being glued together until $P = 20$ kN, determine the stress at (*a*) point *A*, (*b*) point *B*.



Rectangular cutout is 64 mm × 44 mm

$A = (80)(60) - (64)(44) = 1.984\times10^{3}\ \text{mm}^2$

$= 1.984\times10^{-3}\ \text{m}^2$

$I = \frac{1}{12}(60)(80)^3 - \frac{1}{12}(44)(64)^3 = 1.59881\times10^{6}\ \text{mm}^2$

$= 1.59881\times10^{-6}\ \text{m}^4$

$c = 40\ \text{mm} = 0.004\ \text{m} \qquad e = 200 + 40 = 240\ \text{mm} = 0.240\ \text{m}$

$M = Pe = (20\times10^3)(0.240) = 4.8\times10^{8}\ \text{N·m}$

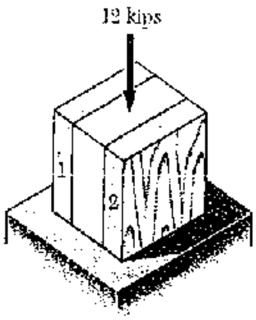
(a) $\sigma_A = \frac{P}{A} + \frac{Mc}{I} = \frac{20\times10^3}{1.984\times10^{-3}} + \frac{(4.8\times10^3)(0.040)}{1.59881\times10^{-6}} = 130.2\times10^{6}\ \text{Pa}$

$\sigma_A = 130.2\ \text{MPa}$ ◄

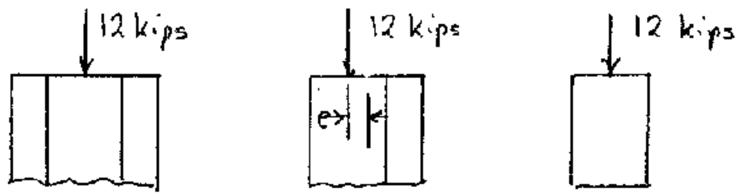
(b) $\sigma_B = \frac{P}{A} - \frac{Mc}{I} = \frac{20\times10^3}{1.984\times10^{-3}} - \frac{(4.8\times10^3)(0.040)}{1.59881\times10^{-6}} = -110.0\times10^{6}\ \text{Pa}$

$\sigma_B = -110.0\ \text{MPa}$ ◄

Problem 4.105



4.105 A short column is made by nailing two 1 × 4-in. planks to a 2 × 4-in. timber. Determine the largest compressive stress created in the column by a 12-kip load applied as shown at the center of the top section of the timber if (*a*) the column is as described, (*b*) plank 1 is removed, (*c*) both planks are removed.



(a) (b) (c)

(a) Centric loading: 4 in × 4 in cross section $\quad A = (4)(4) = 16 \text{ in}^2$

$$\sigma = -\frac{P}{A} = -\frac{12}{16} = -0.75 \text{ ksi}$$

(b) Eccentric loading: 4 in × 3 in cross section $\quad A = (4)(3) = 12 \text{ in}^2$

$$c = (\tfrac{1}{2})(3) = 1.5 \text{ in} \qquad e = 1.5 - 1.0 = 0.5 \text{ in.}$$

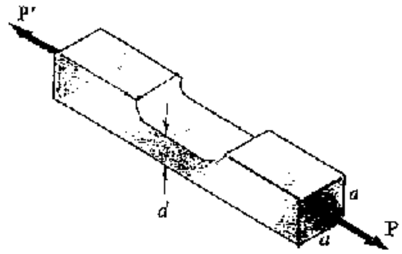
$$I = \frac{1}{12}bh^3 = \frac{1}{12}(4)(3)^3 = 9 \text{ in}^4$$

$$\sigma = -\frac{P}{A} - \frac{Pec}{I} = -\frac{12}{12} - \frac{(12)(0.5)(1.5)}{9} = -2.00 \text{ ksi}$$

(c) Centric loading: 4 in. × 2 in cross section $\quad A = (4)(2) = 8 \text{ in}^2$

$$\sigma = -\frac{P}{A} = -\frac{12}{8} = -1.50 \text{ ksi}$$

Problem 4.106



4.106 A milling operation was used to remove a portion of a solid bar of square cross section. Knowing that $a = 30$ mm, $d = 20$ mm, and $\sigma_{all} = 60$ MPa, determine the magnitude P of the largest forces that can be safely applied at the centers of the ends of the bar.

$$A = ad, \qquad I = \frac{1}{12}ad^3, \qquad c = \frac{1}{2}d$$

$$e = \frac{a}{2} - \frac{d}{2}$$

$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{ad} + \frac{6Ped}{ad^3}$$

$$\sigma = \frac{P}{ad} + \frac{3P(a-d)}{ad^2} = KP \qquad \text{where } K = \frac{1}{ad} + \frac{3(a-d)}{ad^2}$$

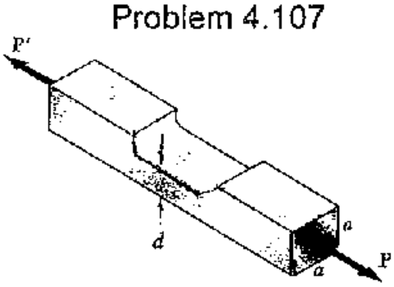
Data: $a = 30\text{ mm} = 0.030\text{ m}$ $\quad d = 20\text{ mm} = 0.020\text{ m}$

$$K = \frac{1}{(0.030)(0.020)} + \frac{(3)(0.010)}{(0.030)(0.020)^2} = 4.1667\times10^3\text{ m}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{60\times10^6}{4.1667\times10^3} = 14.40\times10^3\text{ N}$$

$P = 14.40$ kN ◄

Problem 4.107



4.107 A milling operation was used to remove a portion of a solid bar of square cross section. Forces of magnitude P are applied at the centers of the ends of the bar. Knowing that $a = 30$ mm and $\sigma_{all} = 135$ MPa, determine the smallest allowable depth d of the milled portion of the bar.

$$A = ad, \qquad I = \frac{1}{12}ad^3, \qquad c = \frac{1}{2}d$$

$$e = \frac{a}{2} - \frac{d}{2}$$

$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{ad} + \frac{Pec}{I} = \frac{P}{ad} + \frac{P\frac{1}{2}(a-d)\frac{1}{2}d}{\frac{1}{12}ad^3} = \frac{P}{ad} + \frac{3P(a-d)}{ad^2}$$

$$\sigma = \frac{3P}{d^2} - \frac{2P}{ad} \qquad \text{or} \qquad \sigma d^2 + \frac{2P}{a}d - 3P = 0$$

Solving for d $\quad d = \frac{1}{2\sigma}\left\{\sqrt{\left(\frac{2P}{a}\right)^2 + 12P\sigma} - \frac{2P}{a}\right\}$

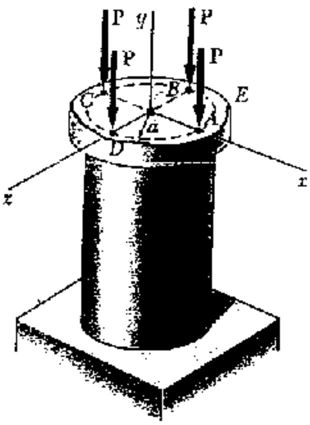
Data: $a = 0.030$ m, $\quad P = 18\times10^3$ N, $\quad \sigma = 135\times10^6$ Pa

$$d = \frac{1}{(2)(135\times10^6)}\left\{\sqrt{\left[\frac{(2)(18\times10^3)}{0.030}\right]^2 + 12(18\times10^3)(135\times10^6)} - \frac{(2)(18\times10^3)}{0.030}\right\}$$

$$= 16.04\times10^{-3}$$

$d = 16.04$ mm ◄

Problem 4.108



4.108 The four forces shown are applied to a rigid plate supported by a solid steel post of radius a. Knowing that $P = 100$ kN and $a = 40$ mm, determine the maximum stress in the post when (a) the force at D is removed, (b) the forces at C and D are removed.

For a solid circular section of radius a

$$A = \pi a^2 \qquad I = \frac{\pi}{4} a^4$$

(a) Centric force $F = 4P$, $M_x = M_z = 0$

$$\sigma = -\frac{F}{A} = -\frac{4P}{\pi a^2}$$

(b) Force at D is removed.

$F = 3P$, $M_x = -Pa$, $M_z = 0$

$$\sigma = -\frac{F}{A} - \frac{M_x z}{I} = -\frac{3P}{\pi a^2} - \frac{(-Pa)(-a)}{\frac{\pi}{4} a^2} = -\frac{7P}{\pi a^2}$$

(c) Forces at C and D are removed

$F = 2P$ $M_x = -Pa$, $M_z = -Pa$

Resultant bending couple $M = \sqrt{M_x^2 + M_z^2} = \sqrt{2}\,Pa$

$$\sigma = -\frac{F}{A} - \frac{Mc}{I} = -\frac{2P}{\pi a^2} - \frac{\sqrt{2}\,Pa\,a}{\frac{\pi}{4} a^2} = -\frac{2+4\sqrt{2}}{\pi}\frac{P}{a^2} = -2.437\,P/a^2$$

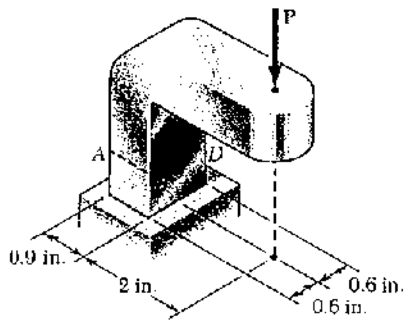
Numerical data: $P = 100 \times 10^3$ N, $a = 0.040$ m

Answers: (a) $\sigma = -\frac{(4)(100\times10^3)}{\pi(0.040)^2} = -79.6\times10^6$ Pa $\qquad -79.6$ MPa

(b) $\sigma = -\frac{(7)(100\times10^3)}{\pi(0.040)^2} = -139.3\times10^6$ Pa $\qquad -139.3$ MPa

(c) $\sigma = -\frac{(2.437)(100\times10^3)}{(0.040)^2} = -152.3\times10^6$ Pa $\qquad -152.3$ MPa

Problem 4.109



4.109 Knowing that the allowable stress in section *ABD* is 10 ksi, determine the largest force **P** which can be applied to the bracket shown.

$A = (1.2)(0.9) = 1.08 \text{ in}^2$

$I = \frac{1}{12}(1.2)(0.9)^3 = 72.9 \times 10^{-3} \text{ in}^4$

$c = \frac{1}{2}(0.9) = 0.45 \text{ in.}$

$e = 2 + 0.45 = 2.45 \text{ in.}$

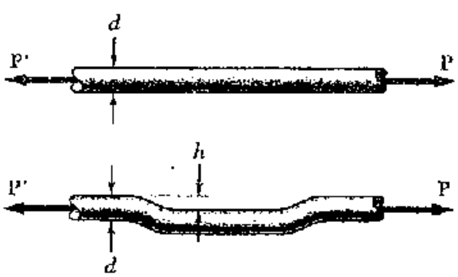
$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{Pec}{I} = PK$$

$$K = \frac{1}{A} + \frac{ec}{I} = \frac{1}{1.08} + \frac{(2.45)(0.45)}{72.9 \times 10^{-3}} = 16.049 \text{ in}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{10}{16.049} = 0.623 \text{ kip}$$

$P = 623 \text{ lb.}$ ◄

Problem 4.110



4.110 An offset *h* must be introduced into a solid circular rod of diameter *d*. Knowing that the maximum stress after the offset is introduced must not exceed 5 times the stress in the rod when it is straight, determine the largest offset that can be used.

For centric loading $\sigma_c = \frac{P}{A}$

For eccentric loading $\sigma_e = \frac{P}{A} + \frac{Phc}{I}$

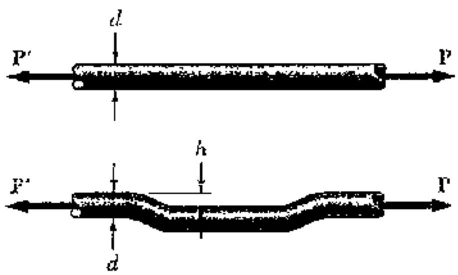
Given $\sigma_e = 5\sigma_c$

$$\frac{P}{A} + \frac{Phc}{I} = 5\frac{P}{A}$$

$$\frac{Phc}{I} = 4\frac{P}{A} \quad \therefore \quad h = \frac{4I}{cA} = \frac{\left(4\left(\frac{\pi}{64}d^4\right)\right)}{\left(\frac{d}{2}\right)\left(\frac{\pi}{4}d^2\right)} = \frac{1}{2}d = 0.500\,d$$ ◄

Problem 4.111

4.111 An offset h must be introduced into a metal tube of 0.75-in. outer diameter and 0.08-in. wall thickness. Knowing that the maximum stress after the offset is introduced must not exceed 4 times the stress in the rod when it is straight, determine the largest offset that can be used.



$$c = \frac{1}{2}d = 0.375 \text{ in.}$$

$$c_1 = c - t = 0.375 - 0.08 = 0.295 \text{ in}$$

$$A = \pi(c^2 - c_1^2) = \pi(0.375^2 - 0.295^2) = 0.168389 \text{ in}^2$$

$$I = \frac{\pi}{4}(c^4 - c_1^4) = \frac{\pi}{4}(0.375^4 - 0.295^4) = 9.5835 \times 10^{-3} \text{ in}^4$$

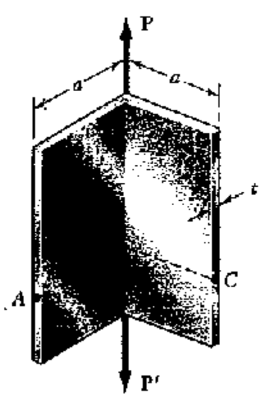
For centric loading $\quad \sigma_{cen} = \dfrac{P}{A}$ $\qquad$ For eccentric loading $\quad \sigma_{ecc} = \dfrac{P}{A} + \dfrac{Phc}{I}$

$$\sigma_{ecc} = 4\sigma_{cen} \quad \text{or} \quad \frac{P}{A} + \frac{Phc}{I} = 4\frac{P}{A}$$

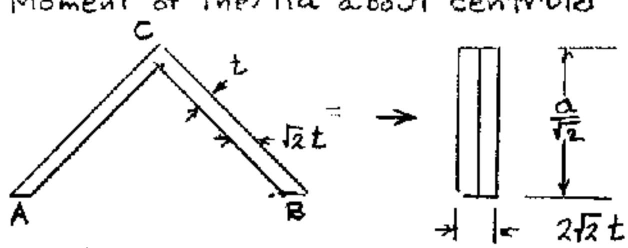
$$\frac{hc}{I} = \frac{3}{A} \qquad h = \frac{3I}{Ac} = \frac{(3)(9.5835\times10^{-3})}{(0.168389)(0.375)} \qquad h = 0.455 \text{ in.}$$

Problem 4.112

4.112 The shape shown was formed by bending a thin steel plate. Assuming that the thickness t is small compared to the length a of a side of the shape, determine the stress (a) at A, (b) at B, (c) at C.



Moment of inertia about centroid



$$I = \frac{1}{12}(2\sqrt{2}t)\left(\frac{a}{\sqrt{2}}\right)^3 = \frac{1}{12}ta^3$$

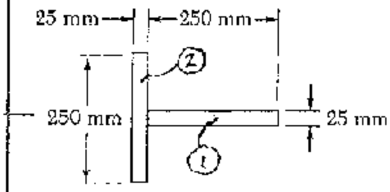
Area $\quad A = (2\sqrt{2}t)\left(\frac{a}{\sqrt{2}}\right) = 2at$, $\qquad c = \dfrac{a}{2\sqrt{2}}$

(a) $\quad \sigma_A = \dfrac{P}{A} - \dfrac{Pec}{I} = \dfrac{P}{2at} - \dfrac{P\left(\frac{a}{2\sqrt{2}}\right)\left(\frac{a}{2\sqrt{2}}\right)}{\frac{1}{12}ta^3} = -\dfrac{P}{2at}$

(b) $\quad \sigma_B = \dfrac{P}{A} + \dfrac{Pec}{I} = \dfrac{P}{2at} + \dfrac{P\left(\frac{a}{2\sqrt{2}}\right)\left(\frac{a}{2\sqrt{2}}\right)}{\frac{1}{12}ta^3} = \dfrac{2P}{at}$

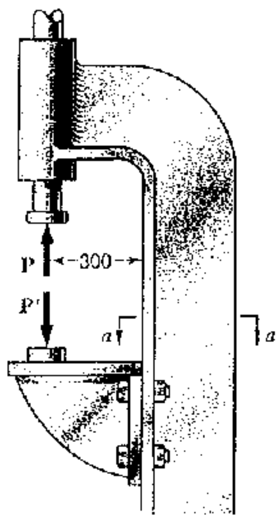
(c) $\quad \sigma_C = \sigma_A = -\dfrac{P}{2at}$

Problem 4.113

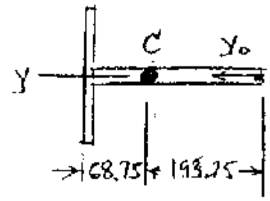


Section *a-a*

Dimensions in mm



4.113 Knowing that the allowable stress in section *a-a* of the hydraulic press shown is 40 MPa in tension and 80 MPa in compression, determine the largest force **P** that can be exerted by the press.



Locate centroid of cross section.

	A, mm^2	$\bar{y}_o, mm$	$A\bar{y}_o, mm^3$
①	6250	12.5	781.25×10^3
②	6250	262.5	1.640625×10^6
Σ	12500		2.421875×10^6

$$\bar{Y}_o = \frac{2.421875 \times 10^3}{12500} = 193.75 \text{ mm}$$

The centroid lies at point C.

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(25)(250)^3 + (6250)(68.75)^2$$
$$= 62.093 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A d_2^2 = \frac{1}{12}(250)(25)^3 + (6250)(68.75)^2$$
$$= 29.867 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 = 82.194 \times 10^6 \text{ mm}^4 = 91.960 \times 10^{-6} \text{ m}^4$$

$$A = 12500 \text{ mm}^2 = 12.5 \times 10^{-3} \text{ m}^2$$

Eccentricity $e = 300 + 25 + 250 - 193.75 = 381.25$ mm

The bending moment about the centroid is $M = -Pe$.

Stress: $\sigma = \frac{P}{A} - \frac{My}{I} = \frac{P}{A} + \frac{Pey}{I} = KP$ where $K = \frac{1}{A} + \frac{ey}{I}$

Then $P = \frac{\sigma}{K}$

Tensile stress: $\sigma = 40 \times 10^6$ Pa, $y = 81.25 \times 10^{-3}$ m

$$K = \frac{1}{12.5 \times 10^{-3}} + \frac{(381.25 \times 10^{-3})(81.25 \times 10^{-3})}{91.960 \times 10^{-6}} = 416.85 \text{ m}^{-2}$$

$$P = \frac{40 \times 10^6}{416.85} = 96.0 \times 10^3 \text{ N}$$

Compressive stress: $\sigma = -80 \times 10^6$ PA $\quad y = -193.75 \times 10^{-3}$ m

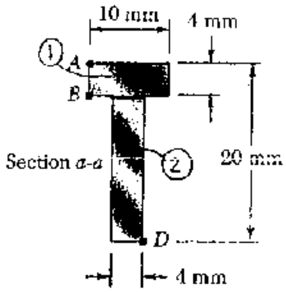
$$K = \frac{1}{12.5 \times 10^{-3}} + \frac{(381.25 \times 10^{-3})(-193.75 \times 10^{-3})}{91.960 \times 10^{-6}} = -723.25 \text{ m}^{-2}$$

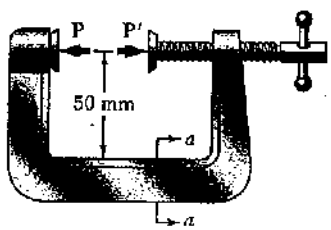
$$P = \frac{-80 \times 10^6}{-723.25} = 110.6 \times 10^3 \text{ N}$$

Choose the smaller value $P = 96.0 \times 10^3$ N $\qquad P = 96.0$ kN

Problem 4.114

4.114 Knowing that the clamp shown has been tightened on wooden planks being glued together until $P = 400$ N, determine in section *a-a* (*a*) the stress at point *A*, (*b*) the stress at point *D*, (*c*) the location of the neutral axis.





Locate centroid.

Part	A, mm^2	$\bar{y}_o$, mm	$A\bar{y}_o$, mm^3
1	40	18	720
2	64	8	512
	104		1232

$$\bar{Y}_o = \frac{1232}{104} = 11.846 \text{ mm}$$

The centroid lies 11.846 mm above point D.

Eccentricity $e = (50 + 20 - 11.846) = -58.154$ mm

Bending couple $M = Pe = (400)(-58.154 \times 10^{-3}) = -23.262$ N·m

$$A = 104 \text{ mm}^2 = 104 \times 10^{-6} \text{ m}^2$$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(10)(4)^3 + (40)(6.154)^2 = 1.5682 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(4)(16)^3 + (64)(3.846)^2 = 2.3120 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 3.8802 \times 10^3 \text{ mm}^4 = 3.8802 \times 10^{-9} \text{ m}^4$$

(a) Stress at point A: $y = 20 - 11.846 = 8.154$ mm $= 8.154 \times 10^{-3}$ m

$$\sigma_A = \frac{P}{A} - \frac{My}{I} = \frac{400}{104 \times 10^{-6}} - \frac{(-23.262)(8.154 \times 10^{-3})}{3.3802 \times 10^{-9}} = 52.7 \times 10^6 \text{ Pa}$$

$\sigma_A = 52.7$ MPa

(b) Stress at point D. $y = +11.846$ mm $= -11.846 \times 10^{-3}$ m

$$\sigma_B = \frac{P}{A} + \frac{My}{I} = \frac{400}{104 \times 10^{-6}} + \frac{(-23.262)(-11.846 \times 10^{-3})}{3.8802 \times 10^{-9}} = -67.2 \times 10^6 \text{ Pa}$$

$\sigma_B = -67.2$ MPa

(c) Location of neutral axis. $\sigma = 0$

$$\sigma = \frac{P}{A} - \frac{My}{I} = \frac{P}{A} + \frac{Pey}{I} = 0$$

$$y = \frac{I}{Ae} = \frac{3.8802 \times 10^{-9}}{(104 \times 10^{-6})(-58.154 \times 10^{-3})} = -0.642 \times 10^{-6} \text{ m}$$

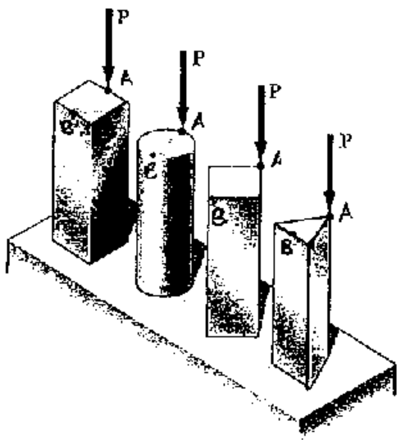
$= -0.642$ mm

Neutral axis lies $11.846 - 0.642 = 11.204$ mm

11.20 mm above D

Problem 4.115

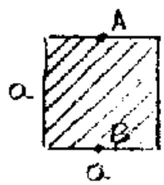
4.115 The four bars shown have the same cross-sectional area. For the given loadings, show that (*a*) the maximum compressive stresses are in the ratio 4:5:7:9, (*b*) the maximum tensile stresses are in the ratio 2:3:5:3. (*Note*: the cross section of the triangular bar is an equilateral triangle.)



Stresses

At A $$\sigma_A = -\frac{P}{A} - \frac{Pec_A}{I} = -\frac{P}{A}\left(1 + \frac{Aec_A}{I}\right)$$

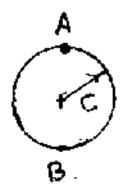
At B $$\sigma_B = -\frac{P}{A} + \frac{Pec_B}{I} = \frac{P}{A}\left(\frac{Aec_B}{I} - 1\right)$$



$$A_1 = a^2, \quad I_1 = \tfrac{1}{12}a^4, \quad c_A = c_B = \tfrac{1}{2}a, \quad e = \tfrac{1}{2}a$$

$$\sigma_A = -\frac{P}{A}\left(1 + \frac{(a^2)(\frac{1}{2}a)(\frac{1}{2}a)}{\frac{1}{12}a^2}\right) = -4\frac{P}{A_1}$$

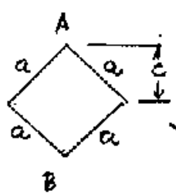
$$\sigma_B = \frac{P}{A}\left(\frac{(a^2)(\frac{1}{2}a)(\frac{1}{2}a)}{\frac{1}{12}a^2} - 1\right) = 2\frac{P}{A_1}$$



$$A_2 = \pi c^2 = a^2 \quad \therefore\ c = \frac{a}{\sqrt{\pi}}, \quad I_2 = \frac{\pi}{4}c^4$$

$$\sigma_A = -\frac{P}{A_2}\left(1 + \frac{(\pi c^2)(c)(c)}{\frac{\pi}{4}c^4}\right) = -5\frac{P}{A_2}$$

$$\sigma_B = \frac{P}{A_2}\left(\frac{(\pi c^2)(c)(c)}{\frac{\pi}{4}c^4} - 1\right) = 3\frac{P}{A_2}$$



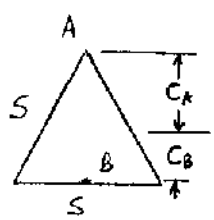
$$A_3 = a^2 \qquad c = \frac{\sqrt{2}}{2}a \qquad I_3 = \frac{1}{12}a^4 \qquad e = c$$

$$\sigma_A = -\frac{P}{A_3}\left(1 + \frac{(a^2)(\frac{\sqrt{2}}{2}a)(\frac{\sqrt{2}}{2}a)}{\frac{1}{12}a^4}\right) = -7\frac{P}{A_3}$$

$$\sigma_B = \frac{P}{A_3}\left(\frac{(a^2)(\frac{\sqrt{2}}{2}a)(\frac{\sqrt{2}}{2}a)}{\frac{1}{12}a^4} - 1\right) = 5\frac{P}{A_3}$$

$$A_4 = \tfrac{1}{2}(s)(\tfrac{\sqrt{3}}{2}s) = \tfrac{\sqrt{3}}{4}s^2 \qquad I_4 = \tfrac{1}{36}s(\tfrac{\sqrt{3}}{2}s)^3 = \tfrac{\sqrt{3}}{96}s^4$$

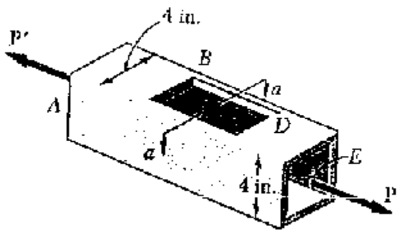
$$c_A = \tfrac{2}{3}\tfrac{\sqrt{3}}{2}s = \tfrac{s}{\sqrt{3}} = e \qquad c_B = \tfrac{s}{2\sqrt{3}}$$



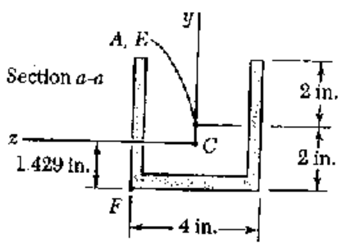
$$\sigma_A = -\frac{P}{A_4}\left(1 + \frac{(\frac{\sqrt{3}}{4}s^2)(\frac{s}{\sqrt{3}})(\frac{s}{\sqrt{3}})}{\frac{\sqrt{3}}{96}s^4}\right) = -9\frac{P}{A_4}$$

$$\sigma_B = \frac{P}{A_4}\left(\frac{(\frac{\sqrt{3}}{4}s^2)(\frac{s}{\sqrt{3}})(\frac{s}{2\sqrt{3}})}{\frac{\sqrt{3}}{96}s^4} - 1\right) = 3\frac{P}{A_4}$$

Problem 4.116



4.116 In order to provide access to the interior of a hollow square tube of 0.25-in. wall thickness, the portion CD of one side of the tube has been removed. Knowing that the loading of the tube is equivalent to two equal and opposite 15-kip forces acting at the geometric centers A and E of the ends of the tube, determine (*a*) the maximum stress in section *a-a*, (*b*) the stress at point F. *Given*: the centroid of the cross section is at C and $I_z = 4.81$ in^4.



Area $\quad A = (2)(4)(0.25) + (3.5)(0.25)$
$= 2.875 \text{ in}^2$

$M = Pe = (15)(2 - 1.429) = 8.565 \text{ kip-in.}$

(a) Maximum stress occurs at top of section where $y = 4 - 1.429 = 2.571$ in.

$$\sigma_{max} = \frac{P}{A} + \frac{My}{I} = \frac{15}{2.875} + \frac{(8.565)(2.571)}{4.81}$$

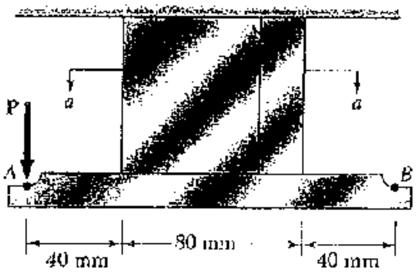
$\sigma_{max} = 9.80$ ksi ◀

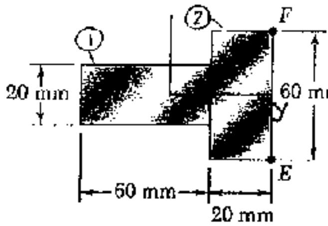
(b) At point F $\quad y = -1.429$ in.

$$\sigma_F = \frac{P}{A} + \frac{My}{I} = \frac{15}{2.875} + \frac{(8.565)(-1.429)}{4.81}$$

$\sigma_F = 2.67$ ksi ◀

Problem 4.117





Section *a-a*

4.117 Knowing that the allowable stress is 150 MPa in section *a-a* of the hanger shown, determine (*a*) the largest vertical force **P** that can be applied at point *A*, (*b*) the corresponding location of the neutral axis of section *a-a*.

Locate centroid

	A, mm^2	$\bar{y}_o$, mm	$A\bar{y}_o$, mm^3
①	1200	30	36×10^3
②	1200	70	84×10^3
Σ	2400		120×10^3

$$\bar{Y}_o = \frac{\Sigma A\bar{y}_o}{\Sigma A} = \frac{120\times10^3}{2400} = 50 \text{ mm}$$

The centroid lies 50 mm to the right of the left edge of the section.

Bending couple $M = Pe$

$e = 40 + 50 = 90$ mm $= 0.090$ m

$$I_1 = \tfrac{1}{12}(20)(60)^3 + (1200)(20)^2 = 840\times10^3 \text{ mm}^4$$

$$I_2 = \tfrac{1}{12}(60)(20)^3 + (1200)(20)^2 = 520\times10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 1.360\times10^6 \text{ mm}^4 = 1.360\times10^{-6} \text{ m}^4, \qquad A = 2400\times10^{-6} \text{ m}^2$$

(a) Based on tensile stress at left edge: $y = -50$ mm $= -0.050$ m

$$\sigma = \frac{P}{A} - \frac{Pey}{I} = KP$$

$$K = \frac{1}{A} - \frac{ey}{I} = \frac{1}{2400\times10^{-6}} - \frac{(0.090)(-0.050)}{1.360\times10^{-6}} = 3.7255\times10^3 \text{ m}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{150\times10^6}{3.7255\times10^3} = 40.3\times10^3 \text{ N} = 40.3 \text{ kN}$$

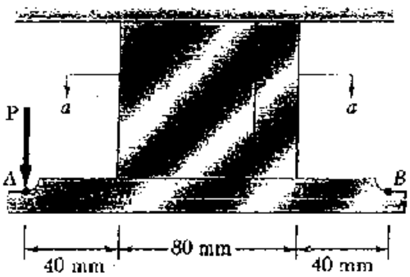
(b) Location of neutral axis: $\sigma = 0$

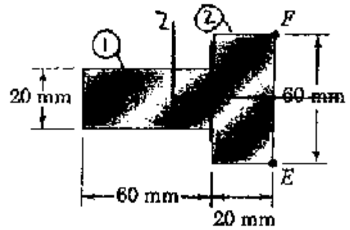
$$\sigma = \frac{P}{A} - \frac{Pey}{I} = 0 \qquad \frac{ey}{I} = \frac{1}{A}$$

$$y = \frac{I}{Ae} = \frac{1.360\times10^{-6}}{(2400\times10^{-6})(0.090)} = 6.30\times10^{-3} \text{ m} = 6.30 \text{ mm}$$

The neutral axis lies 6.30 mm to the right of the centroid or 56.30 mm from the left face.

Problem 4.118





Section a-a

4.118 Solve Prob 4.117, assuming that the vertical force **P** is applied at point *B*.

4.117 Knowing that the allowable stress is 150 MPa in section *a-a* of the hanger shown, determine (*a*) the largest vertical force **P** that can be applied at point *A*, (*b*) the corresponding location of the neutral axis of section *a-a*.

Locate centroid

	A, mm^2	$\bar{y}_o$, mm	$A\bar{y}_o$, mm^3
①	1200	30	36×10^3
②	1200	70	84×10^3
Σ	2400		120×10^3

$$\bar{Y}_o = \frac{\Sigma A\bar{y}_o}{\Sigma A} = \frac{120\times10^3}{2400} = 50 \text{ mm}$$

The centroid lies 50 mm to the right of the left edge of the section.

Bending couple $M = Pe$

$e = 50 - 120 = -70 \text{ mm} = -0.070 \text{ m}$

$$I_1 = \tfrac{1}{12}(20)(60)^3 + (1200)(20)^2 = 840\times10^3 \text{ mm}^4$$

$$I_2 = \tfrac{1}{12}(60)(20)^3 + (1200)(20)^2 = 520\times10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 1.360\times10^6 \text{ mm}^4 = 1.360\times10^{-6} \text{ m}^4, \quad A = 2400\times10^{-6} \text{ m}^2$$

(a) Based on stress at left edge of section: $y = -50 \text{ mm} = -0.050 \text{ m}$

$$\sigma = \frac{P}{A} - \frac{Pey}{I} = K_L P$$

$$K_L = \frac{1}{A} - \frac{ey}{I} = \frac{1}{2400\times10^{-6}} - \frac{(-0.070)(-0.050)}{1.360\times10^{-6}} = -2.1569\times10^3 \text{ m}^{-2}$$

$$P = \frac{\sigma}{K_L} = \frac{-150\times10^6}{-2.1569\times10^3} = 69.6\times10^3 \text{ N}$$

Based on stress at right edge of section: $y = 30 \text{ mm} = 0.030 \text{ m}$

$$\sigma = \frac{P}{A} - \frac{Pey}{I} = K_R P$$

$$K_R = \frac{1}{A} - \frac{ey}{I} = \frac{1}{2400\times10^{-6}} - \frac{(-0.070)(0.030)}{1.360\times10^{-6}} = 1.9608\times10^3 \text{ m}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{150\times10^6}{1.9608\times10^3} = 76.5\times10^3 \text{ N}$$

Choose the smaller value $P = 69.6\times10^3 \text{ N} = 69.6 \text{ kN}$ ◄

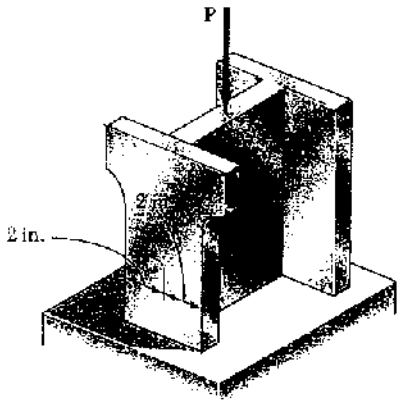
(b) Location of neutral axis: $\sigma = 0$

$$\sigma = \frac{P}{A} - \frac{Pey}{I} = 0 \qquad \frac{ey}{I} = \frac{1}{A}$$

$$y = \frac{I}{Ae} = \frac{1.360\times10^{-6}}{(2400\times10^{-6})(-0.070)} = -8.10\times10^{-3} \text{ m} = -8.10 \text{ mm}$$

Neutral axis lies $50 - 8.10 = 41.9$ mm from left face. ◄

Problem 4.119



4.119 Three steel plated, each of 1 × 6-in cross section, are welded together to form a short H-shaped column. Later, for architectural reasons, a 1-in. strip is removed from each side of one of the flanges. Knowing that the load remains centric with respect to the original cross section, and that the allowable stress is 15 ksi, determine the largest force **P**, (*a*) that could be applied to the original column, (*b*) that can be applied to the modified column.

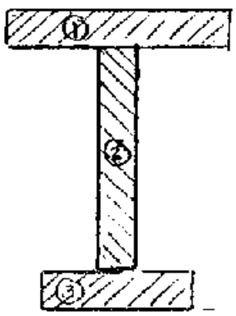
(a) Centric loading

$$A = (3)(1)(6) = 18 \text{ in}^2$$

$$\sigma = -\frac{P}{A} \quad \therefore \quad P = -\sigma A = -(-15)(18) = 270 \text{ kips}$$

(b) Eccentric loading

Reduced cross section



	A, in^2	$\bar{y}_o$, in.	$A\bar{y}_o$, in^3
1	6	3.5	21.0
2	6	0	0
3	4	-3.5	-14.0
Σ	16		7.0

$$\bar{Y}_o = \frac{\Sigma A\bar{y}_o}{\Sigma A} = \frac{7.0}{16} = 0.4375 \text{ in}$$

The centroid lies 0.4375 in. from the midpoint of the web.

$$I_1 = \frac{1}{12}(6)(1)^3 + (6)(3.0625)^2 = 56.773 \text{ in}^4$$

$$I_2 = \frac{1}{12}(1)(6)^3 + (6)(0.4375)^2 = 19.148 \text{ in}^4$$

$$I_3 = \frac{1}{12}(4)(1)^3 + (4)(3.9375)^2 = 62.349 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 138.27 \text{ in}^4, \qquad c = 4.4375 \text{ in}$$

$$M = Pe \quad \text{where} \quad e = 0.4375 \text{ in}$$

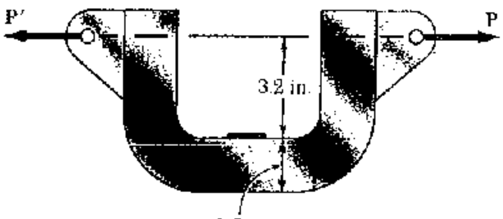
$$\sigma = -\frac{P}{A} - \frac{Mc}{I} = -\frac{P}{A} + \frac{Pec}{I} = -KP$$

$$K = \frac{1}{A} + \frac{ec}{I} = \frac{1}{16} + \frac{(0.4375)(4.4375)}{138.27} = 0.076541 \text{ in}^{-2}$$

$$P = -\frac{\sigma}{K} = -\frac{-15}{0.076541} = 196.0 \text{ kips}$$

Problem 4.120

4.120 The C-shaped steel bar is used as a dynamometer to determine the magnitude P of the forces shown. Knowing that the cross section of the bar is a square of side 1.6 in. and the strain on the inner edge was measured and found to be 450μ, determine the magnitude P of the forces. Use $E = 29 \times 10^6$ psi.



At the strain gage location

$$\sigma = E\varepsilon = (29\times10^6)(450\times10^{-6})$$
$$= 13.05\times10^3 \text{ psi}$$

$$A = (1.6)(1.6) = 2.56 \text{ in}^2$$

$$I = \tfrac{1}{12}(1.6)(1.6)^3 = 0.54613 \text{ in}^4$$

$$e = 3.2 + 0.8 = 4.0 \text{ in.}$$

$$c = 0.8 \text{ in.}$$

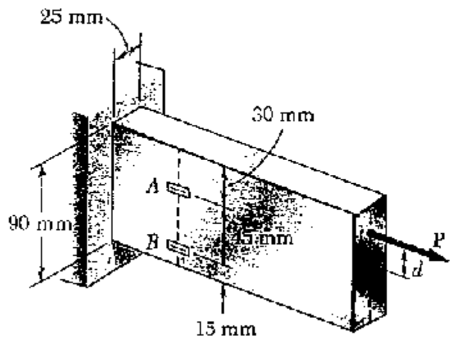
$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{Pec}{I} = KP$$

$$K = \frac{1}{A} + \frac{ec}{I} = \frac{1}{2.56} + \frac{(4.0)(0.8)}{0.54613} = 6.25 \text{ in}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{13.05\times10^3}{6.25} = 2.088\times10^3 \text{ lb.}$$

$P = 2.09$ kips

Problem 4.121



4.121 An eccentric force **P** is applied as shown to a steel bar of 25 × 90-mm cross section. The strains at A and B have been measured and found to be

$$\epsilon_A = +350\ \mu \qquad \epsilon_B = -70\ \mu$$

Knowing that $E = 200$ GPa, determine (*a*) the distance d, (*b*) the magnitude of the force **P**.

$h = 15 + 45 + 30 = 90$ mm

$b = 25$ mm $\qquad c = \frac{1}{2}h = 45\text{mm} = 0.045$ m

$A = bh = (25)(90) = 2.25\times10^3\ \text{mm}^2 = 2.25\times10^{-3}\ \text{m}^2$

$I = \frac{1}{12}bh^3 = \frac{1}{12}(25)(90)^3 = 1.51875\times10^6\ \text{mm}^4$

$= 1.51875\times10^{-6}\ \text{m}^4$

$y_A = 60 - 45 = 15\ \text{mm} = 0.015\text{m}, \quad y_B = 15 - 45 = -30\ \text{mm} = -0.030$ m

Stresses from strain gages at A and B

$\sigma_A = E\varepsilon_A = (200\times10^9)(350\times10^{-6}) = 70\times10^6$ Pa

$\sigma_B = E\varepsilon_B = (200\times10^9)(-70\times10^{-6}) = -14\times10^6$ Pa

$$\sigma_A = \frac{P}{A} - \frac{My_A}{I} \qquad (1)$$

$$\sigma_B = \frac{P}{A} - \frac{My_B}{I} \qquad (2)$$

Subtracting $\quad \sigma_A - \sigma_B = -\frac{M(y_A - y_B)}{I}$

$$M = -\frac{I(\sigma_A - \sigma_B)}{y_A - y_B} = -\frac{(1.51875\times10^{-6})(84\times10^6)}{0.045} = -2835\ \text{N}\cdot\text{m}$$

Multiplying (2) by y_A and (1) by y_B and subtracting

$$y_A\sigma_B - y_B\sigma_A = (y_A - y_B)\frac{P}{A}$$

$$P = \frac{A(y_A\sigma_B - y_B\sigma_A)}{y_A - y_B} = \frac{(2.25\times10^{-3})[(0.015)(-14\times10^6) - (-0.030)(70\times10^6)]}{0.045}$$

$$= 94.5\times10^3\ \text{N}$$

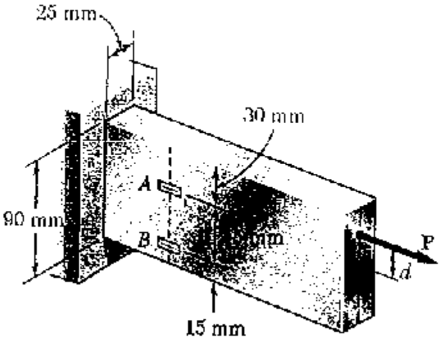
(a) $\quad M = -Pd \therefore \quad d = -\frac{M}{P} = -\frac{-2835}{94.5\times10^3} = 0.030\ \text{m} = 30$ mm ◄

(b) $\qquad P = 94.5$ kN ◄

Problem 4.122

4.122 Solve prob. 4.121, assuming that the measured strains are

$\epsilon_A = +600\ \mu \qquad \epsilon_B = +420\ \mu$



4.121 An eccentric force **P** is applied as shown to a steel bar of 25 × 90-mm cross section. The strains at A and B have been measured and found to be

$\epsilon_A = +350\ \mu \qquad \epsilon_B = -70\ \mu$

Knowing that $E = 200$ GPa, determine (*a*) the distance d, (*b*) the magnitude of the force **P**.

$h = 15 + 45 + 30 = 90$ mm

$b = 25$ mm $\qquad c = \frac{1}{2}h = 45$ mm $= 0.045$ m

$A = bh = (25)(90) = 2.25\times10^3\ \text{mm}^2 = 2.25\times10^{-3}\ \text{m}^2$

$I = \frac{1}{12}bh^3 = \frac{1}{12}(25)(90)^3 = 1.51875\times10^6\ \text{mm}^4 = 1.51875\times10^{-6}\ \text{m}^4$

$y_A = 60 - 45 = 15$ mm $= 0.015$ m, $\quad y_B = 15 - 45 = -30$ mm $= -0.030$ m

Stresses from strain gages at A and B

$\sigma_A = E\varepsilon_A = (200\times10^9)(600\times10^{-6}) = 120\times10^6$ Pa

$\sigma_B = E\varepsilon_B = (200\times10^9)(420\times10^{-6}) = 84\times10^6$ Pa

$$\sigma_A = \frac{P}{A} - \frac{My_A}{I} \qquad (1)$$

$$\sigma_B = \frac{P}{A} - \frac{My_B}{I} \qquad (2)$$

Subtracting $\qquad \sigma_A - \sigma_B = -\frac{M(y_A - y_B)}{I}$

$$M = -\frac{I(\sigma_A - \sigma_B)}{y_A - y_B} = -\frac{(1.51875\times10^{-6})(36\times10^6)}{0.045} = -1215\ \text{N}\cdot\text{m}$$

Multiplying (2) by y_A and (1) by y_B and subtracting

$$y_A\sigma_B - y_B\sigma_A = (y_A - y_B)\frac{P}{A}$$

$$P = \frac{A(y_A\sigma_B - y_B\sigma_A)}{y_A - y_B} = \frac{(2.25\times10^{-3})[(0.015)(84\times10^6) - (-0.030)(120\times10^6)]}{0.045}$$

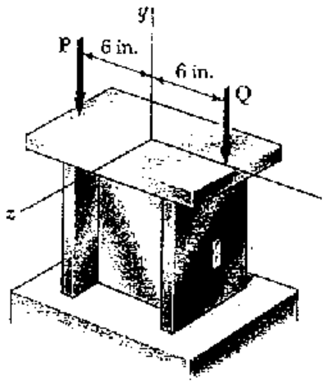
$$= 243\times10^3\ \text{N}$$

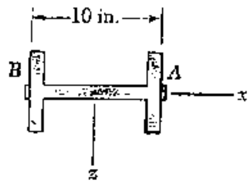
$M = -Pd$

(a) $\quad \therefore\ d = -\frac{M}{P} = -\frac{-1215}{243\times10^3} = 5\times10^{-3}$ m $= 5.00$ mm

(b) $\quad P = 243$ kN

Problem 4.123





$A = 10.0 \text{ in}^2$
$I_x = 273 \text{ in}^4$

4.123 A short length of a rolled-steel column supports a rigid plate on which two loads **P** and **Q** are applied as shown. The strains at two point A and B on the center line of the outer faces of the flanges have been measured and found to be

$$\epsilon_A = -400 \times 10^{-6} \text{ in./in.} \qquad \epsilon_B = -300 \times 10^{-6} \text{ in./in.}$$

Knowing that $E = 29 \times 10^6$ psi, determine the magnitude of each load.

Stresses at A and B from strain gages

$$\sigma_A = E\varepsilon_A = (29\times10^6)(-400\times10^{-6}) = -11.6\times10^3 \text{ psi}$$

$$\sigma_B = E\varepsilon_B = (29\times10^6)(-300\times10^{-6}) = -8.7\times10^3 \text{ psi}$$

Centric force $F = P + Q$

Bending couple $M = 6P - 6Q$

$c = 5$ in.

$$\sigma_A = -\frac{F}{A} + \frac{Mc}{I} = -\frac{P+Q}{10.0} + \frac{(6P-6Q)(5)}{273}$$

$$-11.6\times10^3 = +0.00989\,P - 0.20989\,Q \qquad (1)$$

$$\sigma_B = -\frac{F}{A} - \frac{Mc}{I} = -\frac{P+Q}{10.0} - \frac{(6P-6Q)(5)}{273}$$

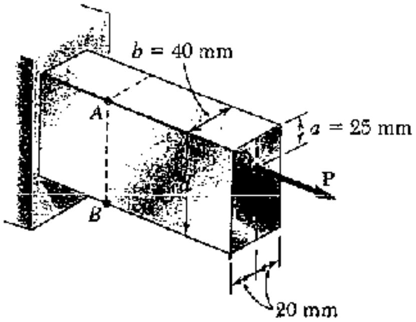
$$-8.7\times10^3 = -0.20989\,P + 0.00989\,Q \qquad (2)$$

Solving (1) and (2) simultaneously

$$P = 44.2\times10^3 \text{ lb} = 44.2 \text{ kips}$$

$$Q = 57.3\times10^3 \text{ lb.} = 57.3 \text{ kips}$$

Problem 4.124



*4.124 The eccentric axial force **P** acts at point D, which must be located 25 mm below the top surface of the steel bar shown. For $P = 60$ kN, determine (*a*) the depth d of the bar for which the tensile stress at point A is maximum, (*b*) the corresponding stress at point A.

$$A = bd \qquad I = \frac{1}{12} b d^3$$

$$c = \frac{1}{2} d \qquad e = \frac{1}{2} d - a$$

$$\sigma_A = \frac{P}{A} + \frac{Pec}{I}$$

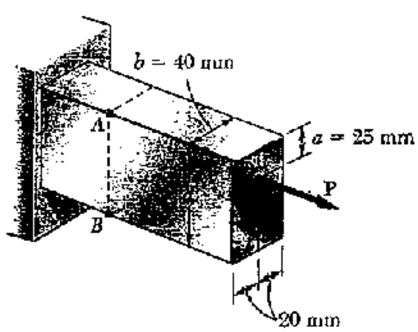
$$\sigma_A = \frac{P}{b}\left\{\frac{1}{d} + \frac{12\left(\frac{1}{2}d - a\right)\left(\frac{1}{2}d\right)}{d^3}\right\} = \frac{P}{b}\left\{\frac{4}{d} - \frac{6a}{d^2}\right\}$$

(a) Depth d for maximum σ_A. Differentiate with respect to d.

$$\frac{d\sigma_A}{dd} = \frac{P}{b}\left\{-\frac{4}{d^2} + \frac{12a}{d^3}\right\} = 0 \qquad d = 3a = 75 \text{ mm}$$

(b) $$\sigma_A = \frac{60\times 10^3}{40\times 10^{-3}}\left\{\frac{4}{75\times 10^{-3}} - \frac{(6)(25\times 10^{-3})}{(75\times 10^{-3})^2}\right\} = 40\times 10^6 \text{ Pa} = 40 \text{ MPa}$$

Problem 4.125



4.125 For the bar and loading or Prob. 4.124, determine (*a*) the depth d of the bar for which the compressive stress at point B is maximum, (*b*) the corresponding stress at point B.

*4.124 The eccentric axial force **P** acts at point D, which must be located 25 mm below the top surface of the steel bar shown. For $P = 60$ kN, determine (*a*) the depth d of the bar for which the tensile stress at point A is maximum, (*b*) the corresponding stress at point A.

$$A = bd \qquad I = \frac{1}{12} b d^3$$

$$c = \frac{1}{2} d \qquad e = \frac{1}{2} d - a$$

$$\sigma_B = \frac{P}{A} - \frac{Pec}{I}$$

$$\sigma_B = \frac{P}{b}\left\{\frac{1}{d} - \frac{(12)\left(\frac{1}{2}d - a\right)\left(\frac{1}{2}d\right)}{d^2}\right\} = \frac{P}{b}\left\{-\frac{2}{d} + \frac{6a}{d^2}\right\}$$

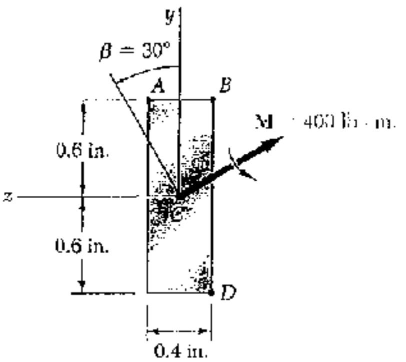
(a) Depth d for maximum σ_B: Differentiate with respect to d

$$\frac{d\sigma_B}{dd} = \frac{P}{b}\left\{\frac{2}{d^2} - \frac{12a}{d^3}\right\} = 0 \qquad d = 6a = 150 \text{ mm}$$

(b) $$\sigma_B = \frac{60\times 10^3}{40\times 10^{-3}}\left\{-\frac{2}{150\times 10^{-3}} + \frac{(6)(25\times 10^{-3})}{(150\times 10^{-3})^2}\right\} = -10\times 10^6 \text{ Pa} = -10 \text{ MPa}$$

Problem 4.126

4.126 through 4.128 The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.



$$I_z = \frac{1}{12}(0.4)(1.2)^3 = 57.6\times 10^{-3}\ \text{in}^4$$

$$I_y = \frac{1}{12}(1.2)(0.4)^3 = 6.40\times 10^{-3}\ \text{in}^4$$

$$y_A = y_B = -y_D = 0.6\ \text{in}$$

$$z_A = -z_B = -z_D = \left(\tfrac{1}{2}\right)(0.4) = 0.2\ \text{in.}$$

$$M_y = 400\cos 60^\circ = 200\ \text{lb}\cdot\text{in}\ ,\quad M_z = -400\sin 60^\circ = -346.41\ \text{lb}\cdot\text{in}$$

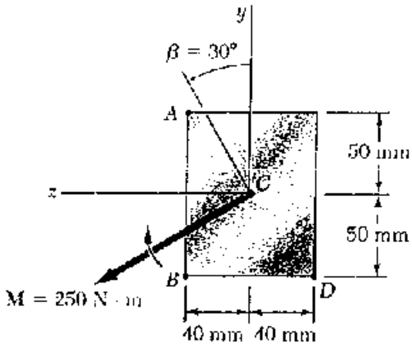
(a) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(-346.41)(0.6)}{57.6\times 10^{-3}} + \frac{(200)(0.2)}{6.40\times 10^{-3}}$$
$$= 9.86\times 10^3\ \text{psi} = 9.86\ \text{ksi}$$

(b) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(-346.41)(0.6)}{57.6\times 10^{-3}} + \frac{(200)(-0.2)}{6.4\times 10^{-3}}$$
$$= -2.64\times 10^3\ \text{psi} = -2.64\ \text{ksi}$$

(c) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(-346.41)(-0.6)}{57.6\times 10^{-3}} + \frac{(200)(-0.2)}{6.40\times 10^{-3}}$$
$$= -9.86\times 10^3\ \text{psi} = -9.86\ \text{ksi}$$

Problem 4.127

4.126 through 4.128 The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.



$$I_z = \frac{1}{12}(80)(100)^3 = 6.6667 \times 10^6 \text{ mm}^4 = 6.6667 \times 10^{-6} \text{ m}^4$$

$$I_y = \frac{1}{12}(100)(80)^3 = 4.2667 \times 10^6 \text{ mm}^4 = 4.2667 \times 10^{-6} \text{ m}^4$$

$$y_A = -y_B = -y_D = 50 \text{ mm}$$

$$z_A = z_B = -z_D = 40 \text{ mm}$$

$$M_y = -250 \sin 30° = -125 \text{ N}\cdot\text{m} \qquad M_z = 250 \cos 30° = 216.51 \text{ N}\cdot\text{m}$$

(a) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(216.51)(0.050)}{6.6667 \times 10^{-6}} + \frac{(-125)(0.040)}{4.2667 \times 10^{-6}}$$

$$= -2.80 \times 10^6 \text{ Pa}$$ -2.80 MPa ◀

(b) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(216.51)(-0.050)}{6.6667 \times 10^{-6}} + \frac{(-125)(0.040)}{4.2667 \times 10^{-6}}$$

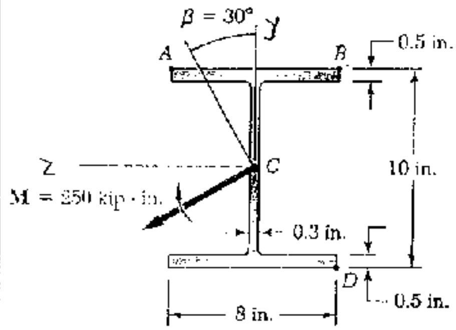
$$= 0.452 \times 10^3 \text{ Pa}$$ 0.452 MPa ◀

(c) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(216.51)(-0.050)}{6.6667 \times 10^{-6}} + \frac{(-125)(-0.040)}{4.2667 \times 10^{-6}}$$

$$= 2.80 \times 10^6 \text{ Pa}$$ 2.80 MPa ◀

Problem 4.128

4.126 through 4.128 The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.



Flange: $I_z = \frac{1}{12}(8)(0.5)^3 + (8)(0.5)(4.75)^2$

$= 90.333 \text{ in}^4$

$I_y = \frac{1}{12}(0.5)(8)^3 = 21.333 \text{ in}^4$

Web: $I_z = \frac{1}{12}(0.3)(9)^3 = 18.225 \text{ in}^4$

$I_y = \frac{1}{12}(9)(0.3)^3 = 0.02025 \text{ in}^4$

Total: $I_z = (2)(90.333) + 18.225 = 198.89 \text{ in}^4$

$I_y = (2)(21.333) + 0.02025 = 42.687 \text{ in}^4$

$y_A = y_B = -y_D = 5 \text{ in.}$; $z_A = -z_B = -z_c = 4 \text{ in.}$

$M_z = 250 \cos 30° = 216.51 \text{ kip-in}$

$M_y = -250 \sin 30° = -125 \text{ kip-in}$

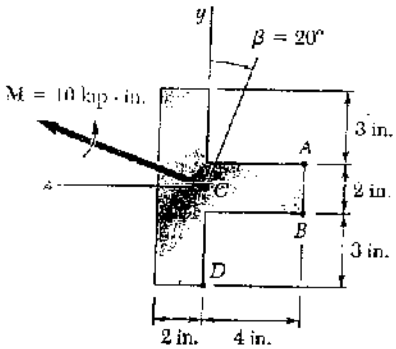
(a) $\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(216.51)(5)}{198.89} + \frac{(-125)(4)}{42.687} = -17.16 \text{ ksi}$

(b) $\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(216.51)(5)}{198.89} + \frac{(-125)(-4)}{42.687} = 6.27 \text{ ksi}$

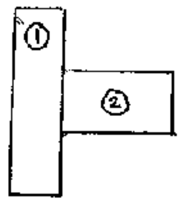
(c) $\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(216.51)(-5)}{198.89} + \frac{(-125)(-4)}{42.687} = 17.16 \text{ ksi}$

Problem 4.129

4.129 through 4.131 The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.



Locate centroid



	A, in^2	$\bar{z}, in$	$A\bar{z}, in^3$
①	16	−1	−16
②	8	2	16
Σ	24		0

The centroid lies at point C

$$I_z = \frac{1}{12}(2)(8)^3 + \frac{1}{12}(4)(2)^3 = 88 \text{ in}^4$$

$$I_y = \frac{1}{3}(8)(2)^3 + \frac{1}{3}(2)(4)^3 = 64 \text{ in}^4$$

$$y_A = -y_B = 1 \text{ in}, \qquad y_D = -4 \text{ in}$$

$$z_A = z_B = -4 \text{ in}, \qquad z_D = 0$$

$$M_z = 10 \cos 20° = 9.3969 \text{ kip·in}$$

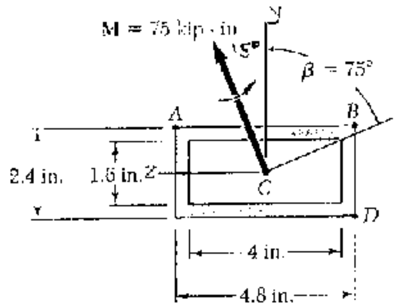
$$M_y = 10 \sin 20° = 3.4202 \text{ kip·in}$$

(a) $\sigma_A = -\dfrac{M_z y_A}{I_z} + \dfrac{M_y z_A}{I_y} = -\dfrac{(9.3969)(1)}{88} + \dfrac{(3.4202)(-4)}{64} = 0.321 \text{ ksi}$

(b) $\sigma_B = -\dfrac{M_z y_B}{I_z} + \dfrac{M_y z_B}{I_y} = -\dfrac{(9.3969)(-1)}{88} + \dfrac{(3.4202)(-4)}{64} = -0.107 \text{ ksi}$

(c) $\sigma_D = -\dfrac{M_z y_D}{I_z} + \dfrac{M_y z_D}{I_y} = -\dfrac{(9.3969)(-4)}{88} + \dfrac{(3.4202)(0)}{64} = 0.427 \text{ ksi}$

Problem 4.130



4.129 through 4.131 The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.

$$I_z = \frac{1}{12}(4.8)(2.4)^3 - \frac{1}{12}(4)(1.6)^3 = 4.1643 \text{ in}^4$$

$$I_y = \frac{1}{12}(2.4)(4.8)^3 - \frac{1}{12}(1.6)(4)^3 = 13.5851 \text{ in}^4$$

$$y_A = y_B = -y_D = 1.2 \text{ in.}$$

$$z_A = -z_B = -z_D = 2.4 \text{ in.}$$

$$M_z = 75 \sin 15° = 19.4114 \text{ kip} \cdot \text{in}$$

$$M_y = 75 \cos 15° = 72.444 \text{ kip} \cdot \text{in}$$

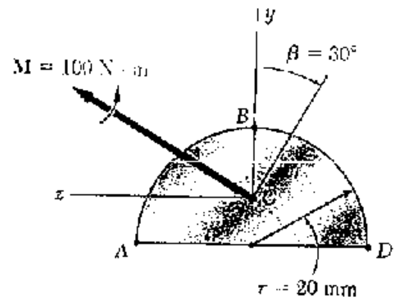
(a) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(19.4114)(1.2)}{4.1643} + \frac{(72.444)(2.4)}{13.5851} = 7.20 \text{ ksi}$$

(b) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(19.4114)(1.2)}{4.1643} + \frac{(72.444)(-2.4)}{13.5851} = -18.39 \text{ ksi}$$

(c) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(19.4114)(-1.2)}{4.1643} + \frac{(72.444)(-2.4)}{13.5851} = -7.20 \text{ ksi}$$

Problem 4.131

4.129 through 4.131 The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.



$$I_z = \frac{\pi}{8}r^4 - \left(\frac{\pi}{2}r^2\right)\left(\frac{4r}{3\pi}\right)^2 = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)r^4$$

$$= (0.109757)(20)^4 = 17.5611 \times 10^3 \text{ mm}^4 = 17.5611 \times 10^{-9} \text{ m}^4$$

$$I_y = \frac{\pi}{8}r^4 = \frac{\pi(20)^4}{8} = 62.832 \times 10^3 \text{ mm}^4 = 62.832 \times 10^{-9} \text{ m}^4$$

$$y_A = y_D = -\frac{4r}{3\pi} = -\frac{(4)(20)}{3\pi} = -8.4883 \text{ mm}$$

$$y_B = 20 - 8.4883 = 11.5117 \text{ mm}$$

$$z_A = -z_D = 20 \text{ mm} \qquad z_B = 0$$

$$M_z = 100 \cos 30° = 86.603 \text{ N}\cdot\text{m}$$

$$M_y = 100 \sin 30° = 50 \text{ N}\cdot\text{m}$$

(a) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(86.603)(-8.4883\times10^{-3})}{17.5611\times10^{-9}} + \frac{(50)(20\times10^{-3})}{62.832\times10^{-9}}$$

$$= 57.8 \times 10^6 \text{ Pa} \qquad 57.8 \text{ MPa}$$

(b) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(86.603)(11.5117\times10^{-3})}{17.5611\times10^{-9}} + \frac{(50)(0)}{62.832\times10^{-9}}$$

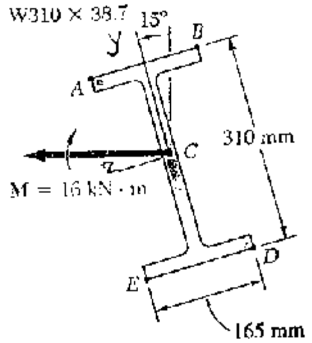
$$= -56.8 \times 10^6 \text{ Pa} \qquad -56.8 \text{ MPa}$$

(c) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(86.603)(-8.4883\times10^{-3})}{17.5611\times10^{-9}} + \frac{(50)(-20\times10^{-3})}{62.832\times10^{-9}}$$

$$= 25.9 \times 10^6 \text{ Pa} \qquad 25.9 \text{ MPa}$$

Problem 4.132

4.132 The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.



For W 310 × 38.7 rolled steel shape

$I_z = 85.1 \times 10^6\ mm^4 = 85.1 \times 10^{-6}\ m^4$

$I_y = 7.27 \times 10^6\ mm^4 = 7.27 \times 10^{-6}\ m^4$

$y_A = y_B = -y_D = -y_E = (\frac{1}{2})(310) = 155\ mm$

$z_A = z_E = -z_B = -z_D = (\frac{1}{2})(165) = 82.5\ mm$

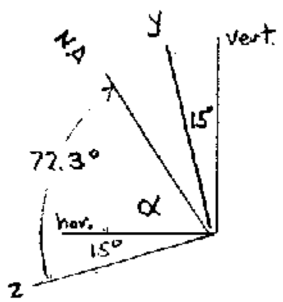
$M_z = (16 \times 10^3) \cos 15° = 15.455 \times 10^3\ N \cdot m$

$M_y = (16 \times 10^3) \sin 15° = 4.1411 \times 10^3\ N \cdot m$

(a) $\tan \varphi = \frac{I_z}{I_y} \tan \theta = \frac{85.1 \times 10^{-6}}{7.27 \times 10^{-6}} \tan 15° = 3.1365$

$\varphi = 72.3°$

$\alpha = 72.3 - 15 = 57.3°$ ◄

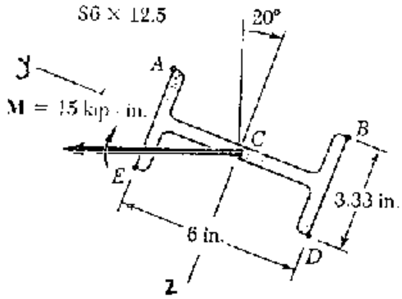


(b) Maximum tensile stress occurs at point E

$$\sigma_E = -\frac{M_z y_E}{I_z} + \frac{M_y z_E}{I_y} = -\frac{(15.455 \times 10^3)(-155 \times 10^{-3})}{85.1 \times 10^{-6}} + \frac{(4.1411 \times 10^3)(82.5 \times 10^{-3})}{7.27 \times 10^{-6}}$$

$$= 75.1 \times 10^6\ Pa = 75.1\ MPa$$ ◄

Problem 4.133



4.133 and 4.134 The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.

For S6×12.5 rolled steel shape

$I_z = 22.1 \text{ in}^4$

$I_y = 1.82 \text{ in}^4$

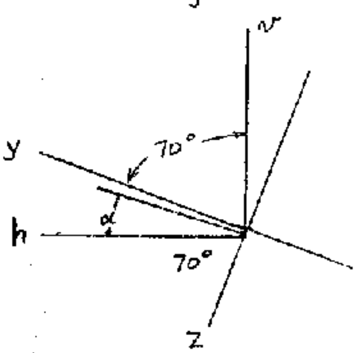
$z_E = -z_A = -z_B = z_D = \frac{1}{2}(3.33) = 1.665 \text{ in.}$

$y_A = y_B = -y_D = -y_E = \frac{1}{2}(6) = 3 \text{ in.}$

$M_z = 15 \sin 20° = 5.1303 \text{ kip}\cdot\text{in}$

$M_y = 15 \cos 20° = 14.095 \text{ kip}\cdot\text{in}$

(a) $\tan \varphi = \dfrac{I_z}{I_y} \tan\theta = \dfrac{22.1}{1.82} \tan(90° - 20°) = 33.36°$



$\varphi = 88.28°$

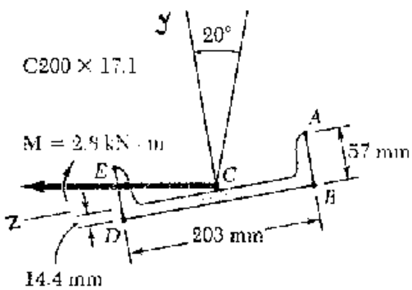
$\alpha = 88.28° - 70° = 18.28°$

(b) Maximum tensile stress occurs at point D.

$$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(5.1303)(-3)}{22.1} + \frac{(14.095)(1.665)}{1.82} = 13.59 \text{ ksi}$$

Problem 4.134

4.133 and 4.134 The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.



For C200 × 17.1 rolled steel shape

$I_z = 0.538 \times 10^6\ \text{mm}^4 = 0.538 \times 10^{-6}\ \text{m}^4$

$I_y = 13.4 \times 10^6\ \text{mm}^4 = 13.4 \times 10^{-6}\ \text{m}^4$

$z_E = z_D = -z_A = -z_B = \frac{1}{2}(203) = 101.5\ \text{mm}$

$y_D = y_B = -14.4\ \text{mm}$

$y_E = y_A = 57 - 14.4 = 42.6\ \text{mm}$

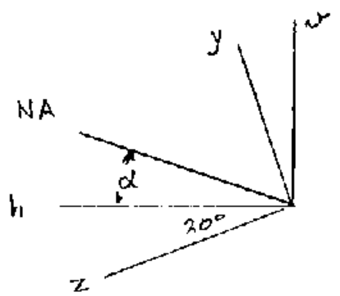
$M_z = (2.8 \times 10^3)\cos 20° = 2.6311 \times 10^3\ \text{N}\cdot\text{m}$

$M_y = (2.8 \times 10^3)\sin 20° = 957.66\ \text{N}\cdot\text{m}$

(a) $\tan\varphi = \dfrac{I_z}{I_y}\tan\theta = \dfrac{0.538}{13.4}\tan 20° = 0.014613$

$\varphi = 0.837°$

$\alpha = 20 - 0.837 = 19.16°$



(b) Maximum tensile stress occurs at point D

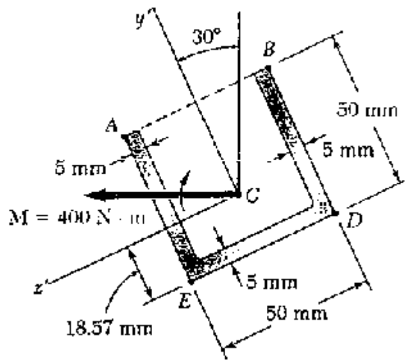
$$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(2.6311 \times 10^3)(-14.4 \times 10^{-3})}{0.538 \times 10^{-6}} + \frac{(957.66)(0.1015)}{13.4 \times 10^{-6}}$$

$$= 70.423 \times 10^6 + 7.254 \times 10^6 = 77.7 \times 10^6\ \text{Pa}$$

$$= 77.7\ \text{MPa}$$

Problem 4.135

4.135 through 4.137 The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.



$I_{y'} = 281 \times 10^3 \text{ mm}^4$
$I_{z'} = 176.9 \times 10^3 \text{ mm}^4$

$$I_{z'} = 176.9 \times 10^3 \text{ mm}^4 = 176.9 \times 10^{-9} \text{ m}^4$$

$$I_{y'} = 281 \times 10^3 \text{ mm}^4 = 281 \times 10^{-9} \text{ m}^4$$

$$y_E' = -18.57 \text{ mm}, \quad z_E = 25 \text{ mm}$$

$$M_{z'} = 400 \cos 30° = 346.41 \text{ N}\cdot\text{m}$$

$$M_{y'} = 400 \sin 30° = 200 \text{ N}\cdot\text{m}$$

(a) $$\tan\varphi = \frac{I_{z'}}{I_{y'}} \tan\theta = \frac{176.9 \times 10^{-9}}{281 \times 10^{-9}} \cdot \tan 30° = 0.36346$$

$$\varphi = 19.97°$$

$$\alpha = 30° - 19.97° = 10.03°$$

(b) Maximum tensile stress occurs at point E

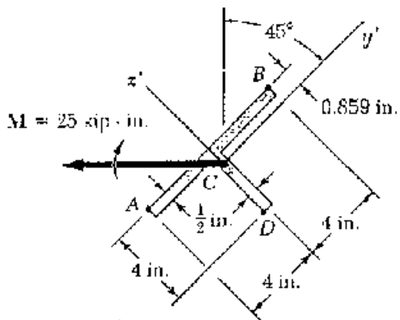
$$\sigma_D = -\frac{M_z y_E}{I_z} + \frac{M_y z_E}{I_y} = -\frac{(346.41)(-18.57 \times 10^{-3})}{176.9 \times 10^{-9}} + \frac{(200)(25 \times 10^{-3})}{281 \times 10^{-9}}$$

$$= 36.36 \times 10^6 + 17.79 \times 10^6 = 54.2 \times 10^6 \text{ Pa}$$

$$= 54.2 \text{ MPa}$$

Problem 4.136

4.135 through 4.137 The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.



$I_{y'} = 6.74 \text{ in}^4$
$I_{z'} = 21.4 \text{ in}^4$

$I_{z'} = 21.4 \text{ in}^4, \quad I_{y'} = 6.74 \text{ in}^4$

$z_A' = z_B' = 0.859 \text{ in}$

$z_D = -4 + 0.859 \text{ in} = -3.141 \text{ in}$

$y_A = -4 \text{ in}, \; y_B = 4 \text{ in}, \; y_D = -0.25 \text{ in}$

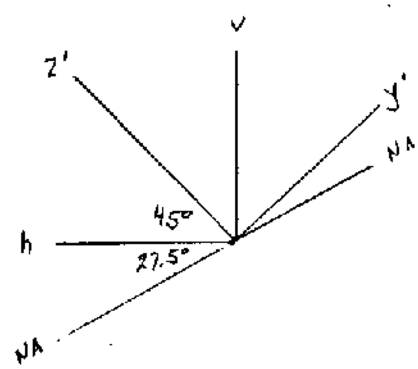
$M_{y'} = -25 \sin 45° = -17.678 \text{ kip} \cdot \text{in}$

$M_{z'} = 25 \cos 45° = 17.678 \text{ kip} \cdot \text{in}$

(a) $\tan \varphi = \frac{I_{z'}}{I_{y'}} \tan \theta = \frac{21.4}{6.74} \tan(-45°) = 3.1751$

$\varphi = -72.5°$

$\alpha = 72.5° - 45° = 27.5°$



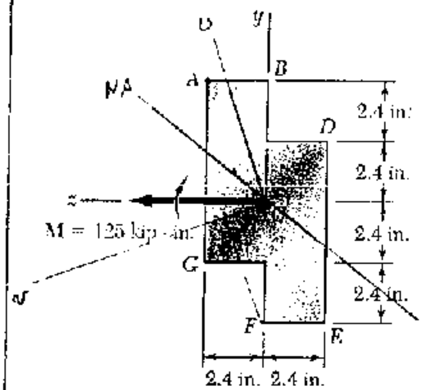
(b) Maximum tensile stress occurs at point D.

$$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(17.678)(-0.25)}{21.4} + \frac{(-17.678)(-3.141)}{6.74}$$

$$= 0.2065 + 8.238 = 8.44 \text{ ksi}$$

Problem 4.137

4.135 through 4.137 The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.



$$I_y = 2\left\{\tfrac{1}{3}(7.2)(2.4)^3\right\} = 66.355 \text{ in}^4$$

$$I_z = 2\left\{\tfrac{1}{12}(2.4)(7.2)^3 + (2.4)(7.2)(1.2)^2\right\} = 199.066 \text{ in}^4$$

$$I_{yz} = 2\left\{(2.4)(7.2)(1.2)(1.2)\right\} = 49.766 \text{ in}^4$$

Using Mohr's circle determine the principal axes and principal moments of inertia.

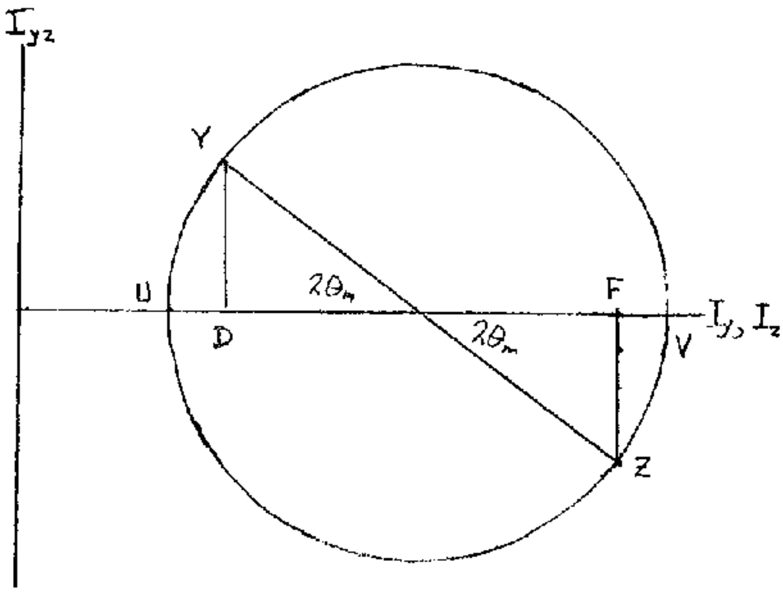
Y: $(66.355 \text{ in}^4,\ 49.766 \text{ in}^4)$

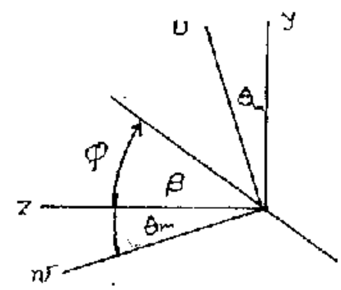
Z: $(199.066 \text{ in}^4,\ -49.766 \text{ in}^4)$

E: $(132.710 \text{ in}^4,\ 0)$

$$\tan 2\theta_m = \frac{DY}{DE} = \frac{49.766}{66.355}$$

$2\theta_m = 36.87°$ $\quad \theta_m = 18.435°$





$$R = \sqrt{\overline{DE}^2 + \overline{DY}^2} = 82.944 \text{ in}^4$$

$$I_u = 132.710 - 82.944 = 49.766 \text{ in}^4$$

$$I_v = 132.710 + 82.944 = 215.654 \text{ in}^4$$

(a) $M_u = 125 \sin 18.435° = 39.529$ kip · in.

$M_v = 125 \cos 18.435° = 118.585$ kip · in.

$$\tan\theta = \frac{M_u}{M_v} = \frac{39.529}{118.585}$$

$\theta = 18.435°$

$$\tan\varphi = \frac{I_v}{I_u}\tan\theta = \frac{215.645}{49.766}\tan 18.435°$$

$\varphi = 55.304°$

$\beta = 55.304° - 18.435° = 36.869°$ $\qquad \beta = 36.9°$ ◄

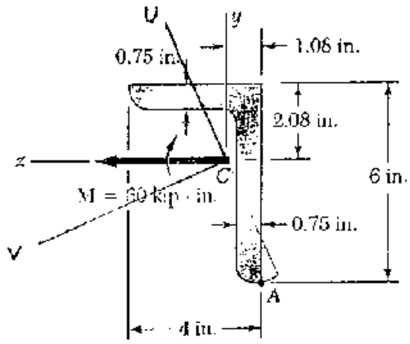
(b) Maximum tensile stress occurs at point F.

$u_E = -4.8\cos 18.435° = -4.5537$ in. $\qquad v_E = 4.8 \sin 18.435° = 1.5179$ in.

$$\sigma_F = -\frac{M_v u_E}{I_v} + \frac{M_u v_E}{I_u} = -\frac{(118.585)(-4.5537)}{215.654} + \frac{(39.529)(1.5179)}{49.766} = 3.71 \text{ ksi}$$ ◄

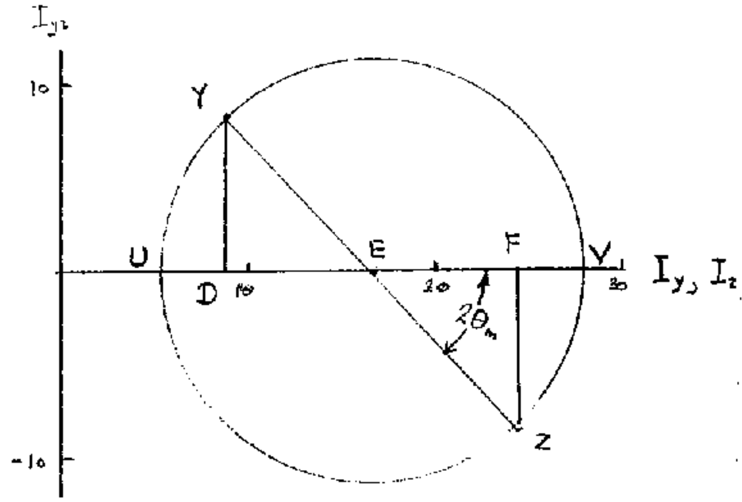
Problem 4.138

***4.138 and *4.139** The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine the stress at point *A*.



$I_y = 8.7 \text{ in}^4$
$I_z = 24.5 \text{ in}^4$
$I_{yz} = +8.3 \text{ in}^4$

Using Mohr's circle, determine the principal axes and principal moments of inertia



$Y\ (8.7,\ 8.3)\ \text{in}^4$
$Z\ (24.5,\ -8.3)\ \text{in}^4$
$E\ (16.6,\ 0)\ \text{in}^4$

$EF = 7.9\ \text{in}^4$
$FZ = 8.3\ \text{in}^4$

$R = \sqrt{7.9^2 + 8.3^2} = 11.46\ \text{in}^4 \qquad \tan 2\theta_m = \frac{FZ}{EF} = \frac{8.3}{7.9} = 1.0506$

$\theta_m = 23.2° \qquad I_u = 16.6 - 11.46 = 5.14\ \text{in}^4, \quad I_v = 16.6 + 11.46 = 28.06\ \text{in}^4$

$M_u = M \sin\theta_m = (60)\sin 23.2° = 23.64\ \text{kip-in}$

$M_v = M \cos\theta_m = (60)\cos 23.2° = 55.15\ \text{kip-in}$

$u_A = y_A \cos\theta_m + z_A \sin\theta_m = -3.92\cos 23.2° + 1.08 \sin 23.2° = -4.03\ \text{in.}$

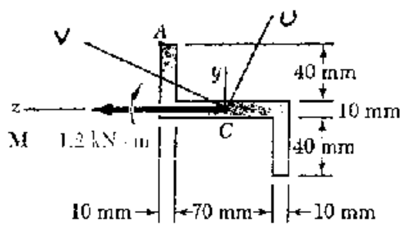
$v_A = z_A \cos\theta_m - y_A \sin\theta_m = -1.08\cos 23.2° + 3.92\sin 23.2° = 0.552\ \text{in.}$

$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u v_A}{I_u} = -\frac{(55.15)(-4.03)}{28.06} + \frac{(23.64)(0.552)}{5.14}$$

$$= 10.46\ \text{ksi}$$

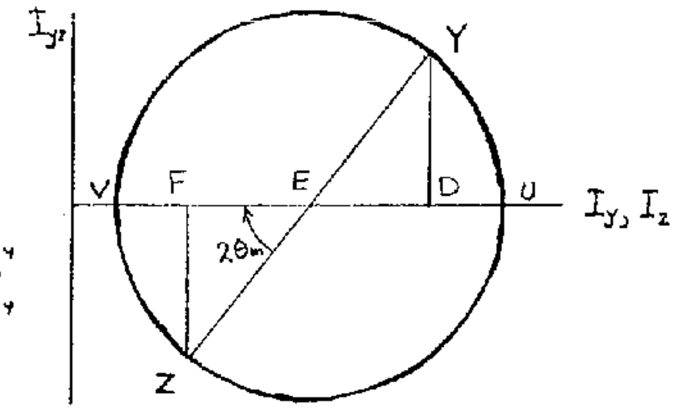
Problem 4.139

***4.138 and *4.139** The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine the stress at point A.



$I_y = 1.894 \times 10^6\ \text{mm}^4$
$I_z = 0.614 \times 10^6\ \text{mm}^4$
$I_{yz} = +0.800 \times 10^6\ \text{mm}^4$

Using Mohr's circle, determine the principal axes and the principal moments of inertia.



$Y\,(1.894, 0.800) \times 10^6\ \text{mm}^4$

$Z\,(0.614, 0.800) \times 10^6\ \text{mm}^4$

$E\,(1.254, 0\ ) \times 10^6\ \text{mm}^4$

$$R = \sqrt{\overline{EF}^2 + \overline{FZ}^2} = \sqrt{0.640^2 + 0.800^2} \times 10^{-6} = 1.0245 \times 10^6\ \text{mm}^4$$

$$I_V = (1.254 - 1.0245) \times 10^6\ \text{mm}^4 = 0.2295 \times 10^6\ \text{mm}^4 = 0.2295 \times 10^{-6}\ \text{m}^4$$

$$I_U = (1.254 + 1.0245) \times 10^6\ \text{mm}^4 = 2.2785 \times 10^6\ \text{mm}^4 = 2.2785 \times 10^{-6}\ \text{m}^4$$

$$\tan 2\theta_m = \frac{FZ}{FE} = \frac{0.800 \times 10^6}{0.640 \times 10^6} = 1.25 \qquad \theta_m = 25.67°$$

$$M_V = M \cos\theta_m = (1.2 \times 10^3)\cos 25.67° = 1.0816 \times 10^3\ \text{N·m}$$

$$M_U = -M \sin\theta_m = -(1.2 \times 10^3)\sin 25.67° = -0.5198 \times 10^3\ \text{N·m}$$

$$U_A = y_A \cos\theta_m - z_A \sin\theta_m = 45 \cos 25.67° - 45 \sin 25.67° = 21.07\ \text{mm}$$

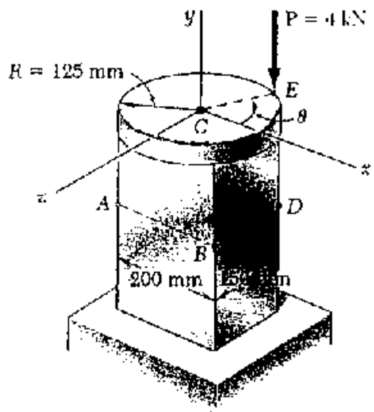
$$V_A = z_A \cos\theta_m + y_A \sin\theta_m = 45 \cos 25.67° + 45 \sin 25.67° = 60.05\ \text{mm}$$

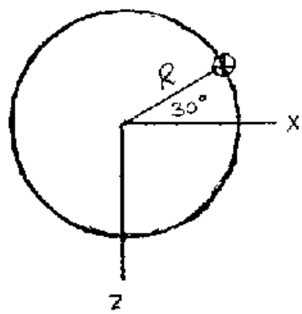
$$\sigma_A = -\frac{M_V U_A}{I_V} + \frac{M_U V_A}{I_U} = -\frac{(1.0816 \times 10^3)(21.07 \times 10^{-3})}{0.2295 \times 10^{-6}} + \frac{(-0.5198 \times 10^3)(60.05 \times 10^{-3})}{2.2785 \times 10^{-6}}$$

$$= 113.0 \times 10^6\ \text{Pa} = 113.0\ \text{MPa}$$

Problem 4.140

4.140 A rigid circular plate of 125-mm radius is attached to a solid 150 × 200-mm rectangular post, with the center of the plate directly above the center of the post. If a 4-kN force P is applied at E with $\theta = 30°$, determine (*a*) the stress at point A, (*b*) the stress at point B, (*c*) the point where the neutral axis intersects line ABD.

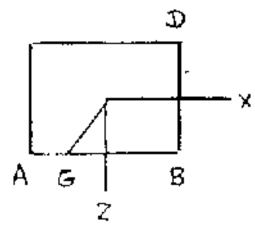




$P = 4 \times 10^3$ N (compression)

$M_x = -PR \sin 30°$
$= -(4\times10^3)(125\times10^{-3}) \sin 30°$
$= -250$ N·m

$M_z = -PR \cos 30°$
$= -(4\times10^3)(125\times10^{-3}) \cos 30°$
$= -433$ N·m



$I_x = \frac{1}{12}(200)(150)^3 = 56.25\times10^6 \text{ mm}^4 = 56.25\times10^{-6} \text{ m}^4$

$I_z = \frac{1}{12}(150)(200)^3 = 100\times10^6 \text{ mm}^4 = 100\times10^{-6} \text{ m}^4$

$-x_A = x_B = 100$ mm

$z_A = z_B = 75$ mm

$A = (200)(150) = 30\times10^3 \text{ mm}^2 = 30\times10^{-3} \text{ m}^2$

(a) $$\sigma_A = -\frac{P}{A} - \frac{M_x z_A}{I_x} + \frac{M_z x_A}{I_z} = -\frac{4\times10^3}{30\times10^{-3}} - \frac{(-250)(75\times10^{-3})}{56.25\times10^{-6}} + \frac{(-433)(-100\times10^{-3})}{100\times10^{-6}}$$

$$= 633\times10^3 \text{ Pa} = 633 \text{ kPa}$$

(b) $$\sigma_B = -\frac{P}{A} - \frac{M_x z_B}{I_x} + \frac{M_z x_B}{I_z} = -\frac{4\times10^3}{30\times10^{-3}} - \frac{(-250)(75\times10^{-3})}{56.25\times10^{-6}} + \frac{(-433)(100\times10^{-3})}{100\times10^{-6}}$$

$$= -233\times10^3 \text{ Pa} = -233 \text{ kPa}$$

(c) Let G be the point on AB where neutral axis intersects.

$\sigma_G = 0 \qquad z_G = 75 \text{ mm} \qquad x_G = ?$

$$\sigma_G = -\frac{P}{A} - \frac{M_x z_G}{I_x} + \frac{M_z x_G}{I_z} = 0$$

$$x_G = \frac{I_z}{M_z}\left\{\frac{P}{A} + \frac{M_x z_G}{I_x}\right\} = \frac{100\times10^{-6}}{-433}\left\{\frac{4\times10^3}{30\times10^{-3}} + \frac{(-250)(75\times10^{-3})}{56.25\times10^{-6}}\right\}$$

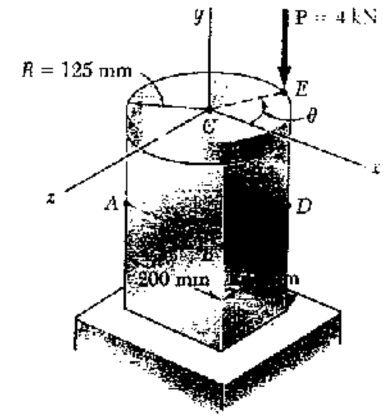
$$= 46.2\times10^{-3} \text{ m} = 46.2 \text{ mm}$$

Point G lies 146.2 mm from point A

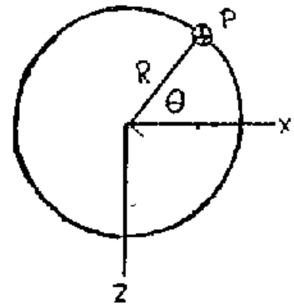
Problem 4.141

4.141 In Prob. 4.140, determine (*a*) the value of θ for which the stress at D reaches its largest value, (*b*) the corresponding values of the stress at A, B, C, and D.

4.140 A rigid circular plate of 125-mm radius is attached to a solid 150 × 200-mm rectangular post, with the center of the plate directly above the center of the post. If a 4-kN force **P** is applied at E with $\theta = 30°$, determine (*a*) the stress at point A, (*b*) the stress at point B, (*c*) the point where the neutral axis intersects line ABD.



(a)

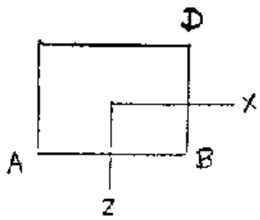


$P = 4 \times 10^3$ N

$PR = (4\times10^3)(125\times10^{-3}) = 500$ N·m

$M_x = -PR\sin\theta = -500\sin\theta$

$M_x = -PR\cos\theta = -500\cos\theta$



$I_x = \frac{1}{12}(200)(150)^3 = 56.25\times10^6\ \text{mm}^4 = 56.25\times10^{-6}\ \text{m}^4$

$I_z = \frac{1}{12}(150)(200)^3 = 100\times10^6\ \text{mm}^4 = 100\times10^{-6}\ \text{m}^4$

$x_D = 100$ mm $\qquad z_D = -75$ mm

$A = (200)(150) = 30\times10^3\ \text{mm}^2 = 30\times10^{-3}\ \text{m}^2$

$$\sigma = -\frac{P}{A} - \frac{M_x z}{I_x} + \frac{M_z x}{I_z} = -P\left\{\frac{1}{A} - \frac{Rz\sin\theta}{I_x} + \frac{Rx\cos\theta}{I_z}\right\}$$

For σ to be a maximum $\frac{d\sigma}{d\theta} = 0$ with $z = z_D$, $x = x_D$

$$\frac{d\sigma_D}{d\theta} = -P\left\{0 + \frac{Rz_D\cos\theta}{I_x} + \frac{Rx_D\sin\theta}{I_z}\right\} = 0$$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta = -\frac{I_z z_D}{I_x x_D} = -\frac{(100\times10^{-6})(-75\times10^{-3})}{(56.25\times10^{-6})(100\times10^{-3})} = \frac{4}{3}$$

$\sin\theta = 0.8$, $\quad \cos\theta = 0.6$, $\quad \theta = 53.1°$ ◄

(b) $$\sigma_A = -\frac{P}{A} - \frac{M_x z_A}{I_x} + \frac{M_z x_A}{I_z} = -\frac{4\times10^3}{30\times10^{-3}} + \frac{(500)(0.8)(75\times10^{-3})}{56.25\times10^{-6}} - \frac{(500)(0.6)(-100\times10^{-3})}{100\times10^{-6}}$$

$= (-0.13333 + 0.53333 + 0.300)\times10^6$ Pa $= 0.700\times10^6$ Pa $= 700$ kPa ◄

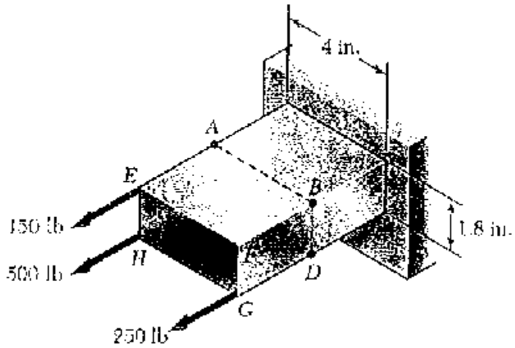
$\sigma_B = (-0.13333 + 0.53333 - 0.300)\times10^6$ Pa $= 0.100\times10^6$ Pa $= 100$ kPa ◄

$\sigma_C = (-0.13333 + 0 + 0)\times10^6$ Pa $= -133.3$ kPa ◄

$\sigma_D = (-0.13333 - 0.53333 - 0.300)\times10^6$ Pa $= -0.967\times10^6$ Pa $= -967$ kPa ◄

Problem 4.142

4.142 For the loading shown, determine (*a*) the stress at points *A* and *B*, (*b*) the point where the neutral axis intersects line *ABD*.

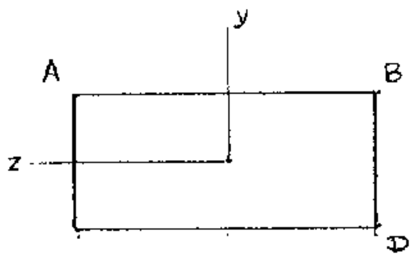


Add y and z axes as shown

$$A = (4)(1.8) = 7.2 \text{ in}^2$$

$$I_z = \frac{1}{12}(4)(1.8)^3 = 1.944 \text{ in}^4$$

$$I_y = \frac{1}{12}(1.8)(4)^3 = 9.6 \text{ in}^4$$



Resultant force and bending couples

$$P = 150 + 500 + 250 = 900 \text{ lb}$$

$$M_z = -(150)(0.9) + (500)(0.9) + (250)(0.9) = 540 \text{ lb}\cdot\text{in}$$

$$M_y = (150)(2) + (500)(2) - (250)(2) = 800 \text{ lb}\cdot\text{in}$$

(a) $\sigma_A = \frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{900}{7.2} - \frac{(540)(0.9)}{1.944} + \frac{(800)(2)}{9.6} =$ 41.7 psi

$\sigma_B = \frac{P}{A} - \frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = \frac{900}{7.2} - \frac{(540)(0.9)}{1.944} + \frac{(800)(-2)}{9.6} =$ 292 psi

(b) Intersection of neutral axis with line AB or its extension.

$\sigma = 0$, $y = 0.9$ in., $z = ?$

$$0 = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y y}{I_y} = \frac{900}{7.2} - \frac{(540)(0.9)}{1.944} + \frac{800z}{9.6}$$

$-125 + 83.333z = 0 \qquad z = 1.5$ in.

Intersects AB at 0.500 in. from A

Intersection of neutral axis with line BD or its extension.

$\sigma = 0 \qquad z = -2$ in. $\qquad y = ?$

$$0 = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y} = \frac{900}{7.2} - \frac{540y}{1.944} + \frac{(800)(-2)}{9.6}$$

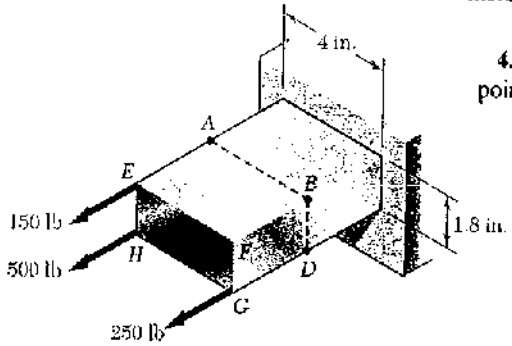
$-41.6667 - 277.778y = 0 \qquad y = -0.15$ in.

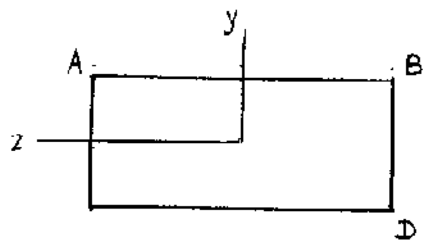
Intersects BD at 0.750 in. from D

Problem 4.143

4.143 Solve Prob. 4.142, assuming that the magnitude of the force applied at G is increased from 250 lb to 400 lb.

4.142 For the loading shown, determine (*a*) the stress at points A and B, (*b*) the point where the neutral axis intersects line ABD.





Add y and z axes as shown.

$A = (4)(1.8) = 7.2 \text{ in}^2$

$I_z = \frac{1}{12}(4)(1.8)^3 = 1.944 \text{ in}^4$

$I_y = \frac{1}{12}(1.8)(4)^3 = 9.6 \text{ in}^4$

Resultant force and bending couples

$P = 150 + 500 + 400 = 1050 \text{ lb}$

$M_z = -(150)(0.9) + (500)(0.9) + (400)(0.9) = 675 \text{ lb}\cdot\text{in}$

$M_y = (150)(2) + (500)(2) - (400)(2) = 500 \text{ lb}\cdot\text{in}$

(a) $\sigma_A = \frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{1050}{7.2} - \frac{(675)(0.9)}{1.944} + \frac{(500)(2)}{9.6} = -62.5 \text{ psi}$

$\sigma_B = \frac{P}{A} - \frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = \frac{1050}{7.2} - \frac{(675)(0.9)}{1.944} + \frac{(500)(-2)}{9.6} = -271 \text{ psi}$

(b) Intersection of neutral axis with line AB or its extension.

$\sigma = 0, \quad y = 0.9 \text{ in.} \quad z = ?$

$0 = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y} = \frac{1050}{7.2} - \frac{(675)(0.9)}{1.944} + \frac{500 z}{9.6}$

$-166.667 + 52.083 z = 0 \qquad z = 3.2 \text{ in.}$

Does not intersect AB

Intersection of neutral axis with line BD or its extension.

$\sigma = 0 \quad z = -2 \text{ in.} \quad y = ?$

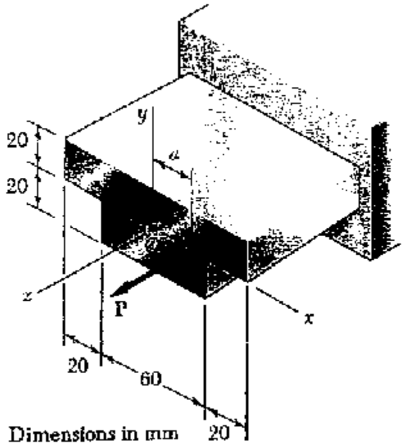
$0 = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y} = \frac{1050}{7.2} - \frac{675 y}{1.944} + \frac{(500)(-2)}{9.6}$

$41.6667 - 347.222 y = 0 \qquad y = 0.120 \text{ in.}$

Intersects BD at 0.780 in. from B

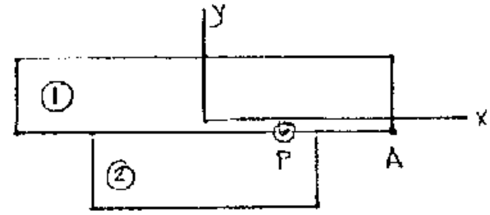
Problem 4.144

4.144 A horizontal load **P** of magnitude 100 kN is applied to the beam shown. Determine the largest distance a for which the maximum tensile stress in the beam does not exceed 75 MPa.



Dimensions in mm

Locate the centroid



	A, mm^2	$\bar{y}$, mm	$A\bar{y}$, mm^3
①	2000	10	20×10^3
②	1200	−10	-12×10^3
Σ	3200		8×10^3

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{8 \times 10^3}{3200} = 2.5 \text{ mm}$$

Move coordinate origin to the centroid

Coordinates of load point: $x_P = a$, $y_P = -2.5$ mm

Bending couples $M_x = y_P P$ $M_y = -aP$

$$I_x = \tfrac{1}{12}(100)(20)^3 + (2000)(7.5)^2 + \tfrac{1}{12}(60)(20)^3 + (1200)(12.5)^2 = 0.40667 \times 10^6 \text{ mm}^4 = 0.40667 \times 10^{-6} \text{ m}^4$$

$$I_y = \tfrac{1}{12}(20)(100)^3 + \tfrac{1}{12}(20)(60)^3 = 2.0267 \times 10^6 \text{ mm}^4 = 2.0267 \times 10^{-6} \text{ m}^4$$

$$\sigma = \frac{P}{A} + \frac{M_x y}{I_x} - \frac{M_y x}{I_y} \qquad \sigma_A = 75 \times 10^6 \text{ Pa}, \quad P = 100 \times 10^3 \text{ N}$$

$$M_y = \frac{I_y}{x}\left\{\frac{P}{A} + \frac{M_x y}{I_x} - \sigma\right\} \qquad \text{For point A} \quad x = 50 \text{ mm}, \ y = -2.5 \text{ mm}$$

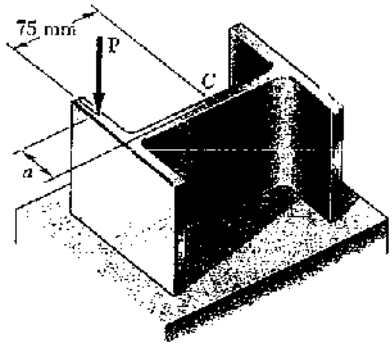
$$M_y = \frac{2.0267 \times 10^{-6}}{50 \times 10^{-3}}\left\{\frac{100 \times 10^3}{3200 \times 10^{-6}} + \frac{(-2.5)(100 \times 10^3)(-2.5 \times 10^{-3})}{0.40667 \times 10^{-6}} - 75 \times 10^6\right\}$$

$$= \frac{2.0267 \times 10^{-6}}{50 \times 10^{-3}}\left\{31.25 + 1.537 - 75\right\} \times 10^6 = -1.7111 \times 10^3 \text{ N·m}$$

$$a = -\frac{M_y}{P} = -\frac{(1.7111 \times 10^3)}{100 \times 10^3} = 17.11 \times 10^3 \text{ m} = 17.11 \text{ mm}$$

Problem 4.145

4.145 An axial load P of magnitude 50 kN is applied as shown to a short section of W150 × 24 rolled-steel member. Determine the largest distance a for which the maximum compressive stress does not exceed 90 MPa.



Add y- and z axes.

For W 150 × 24 rolled-steel section

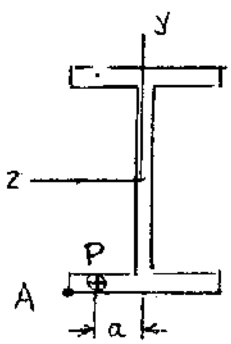
$A = 3060\ mm^2 = 3060 \times 10^{-6}\ m^2$

$I_z = 13.4 \times 10^6\ mm^4 = 13.4 \times 10^{-6}\ m^4$

$I_y = 1.83 \times 10^6\ mm^4 = 1.83 \times 10^{-6}\ m^4$

$d = 160\ mm, \qquad b_f = 102\ mm$

$y_A = -\frac{d}{2} = -80\ mm, \quad z_A = \frac{b_f}{2} = 51\ mm.$



$P = 50 \times 10^3\ N$

$M_z = -(50 \times 10^3)(75 \times 10^{-3}) = -3.75 \times 10^3\ N\cdot m$

$M_y = -Pa$

$\sigma_A = -\frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} \qquad \sigma_A = -90 \times 10^6\ Pa$

$$M_y = \frac{I_y}{z_A}\left\{ \frac{M_z y_A}{I_z} + \frac{P}{A} + \sigma_A \right\}$$

$$= \frac{1.83 \times 10^{-6}}{51 \times 10^{-3}}\left\{ \frac{(-3.75 \times 10^3)(-80 \times 10^{-3})}{13.4 \times 10^{-6}} + \frac{50 \times 10^3}{3060 \times 10^{-6}} + (-90 \times 10^6) \right\}$$

$$= \frac{1.83 \times 10^{-6}}{51 \times 10^{-3}}\left\{ +22.388 + 16.340 - 90 \right\} \times 10^6$$

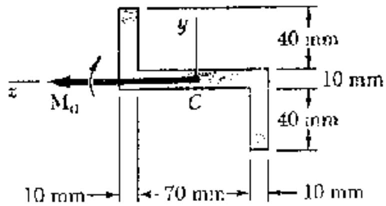
$$= -1.8398 \times 10^3\ N\cdot m$$

$$a = -\frac{M_y}{P} = -\frac{-1.8398 \times 10^3}{50 \times 10^3} = 36.8 \times 10^{-3}\ m = 36.8\ mm$$

Problem 4.146

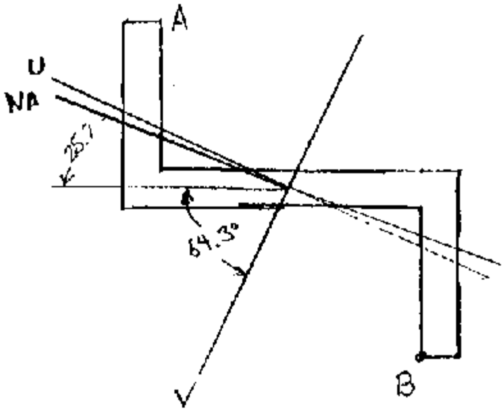
4.146 The Z section shown is subjected to a couple $\mathbf{M}_0$ acting in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress is not to exceed 80 MPa. *Given*: $I_{max} = 2.28 \times 10^{-6}\ \text{mm}^4$, $I_{min} = 0.23 \times 10^{-6}\ \text{mm}^4$, principal axes 25.7° and 64.3°.

[[down to the right and up to the right]]



$$I_v = I_{max} = 2.28 \times 10^6\ \text{mm}^4 = 2.28 \times 10^{-6}\ \text{m}^4$$

$$I_u = I_{min} = 0.23 \times 10^6\ \text{mm}^4 = 0.23 \times 10^{-6}\ \text{m}^4$$



$$M_v = M_o \cos 64.3°$$

$$M_u = M_o \sin 64.3°$$

$$\theta = 64.3°$$

$$\tan\varphi = \frac{I_v}{I_u}\tan\theta$$

$$= \frac{2.28 \times 10^{-6}}{0.23 \times 10^{-6}} \tan 64.3° = 20.597$$

$$\varphi = 87.22°$$

Points A and B are farthest from the neutral axis.

$$u_B = y_B \cos 64.3° + z_B \sin 64.3° = (-45)\cos 64.3° + (-35)\sin 64.3°$$

$$= -51.05\ \text{mm}$$

$$v_B = z_B \cos 64.3° - y_B \sin 64.3° = (-35)\cos 64.3° - (-45)\sin 64.3°$$

$$= +25.37\ \text{mm}$$

$$\sigma_B = -\frac{M_v u_B}{I_v} + \frac{M_u v_B}{I_u}$$

$$80 \times 10^6 = -\frac{(M_o \cos 64.3°)(-51.05 \times 10^{-3})}{2.28 \times 10^{-6}} + \frac{(M_o \sin 64.3°)(25.37 \times 10^{-3})}{0.23 \times 10^{-6}}$$

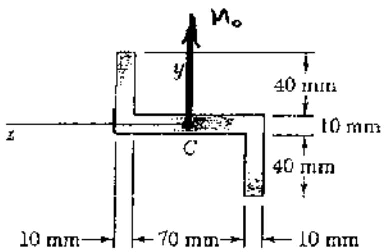
$$= 93.81 \times 10^3\ M_o$$

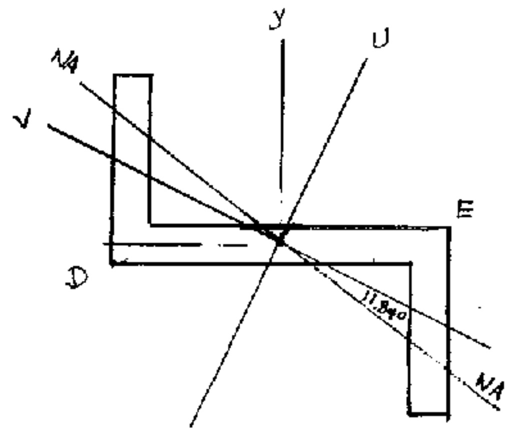
$$M_o = \frac{80 \times 10^6}{109.1 \times 10^3} = 733\ \text{N}\cdot\text{m}$$

Problem 4.147

4.147 Solve Prob. 4.146 assuming that the couple $\mathbf{M}_0$ act in a horizontal plane.

4.146 The Z section shown is subjected to a couple $\mathbf{M}_0$ acting in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress is not to exceed 80 MPa. *Given*: $I_{max} = 2.28 \times 10^{-6}\ \text{mm}^4$, $I_{min} = 0.23 \times 10^{-6}\ \text{mm}^4$, principal axes 25.7° and 64.3°
[[down to the right and up to the right]]





$$I_v = I_{min} = 0.23 \times 10^6\ \text{mm}^4 = 0.23 \times 10^6\ \text{m}^4$$

$$I_U = I_{max} = 2.23 \times 10^6\ \text{mm}^4 = 2.23 \times 10^6\ \text{m}^4$$

$$M_v = M_0 \cos 64.3°$$

$$M_U = M_0 \sin 64.3°$$

$$\theta = 64.3°$$

$$\tan\varphi = \frac{I_v}{I_U}\tan\theta$$

$$= \frac{0.23\times10^{-6}}{2.28\times10^{-6}}\tan 64.3° = 0.20961$$

$$\varphi = 11.84°$$

Points D and E are farthest from the neutral axis.

$$U_D = y_D \cos 25.7° - z_D \sin 25.7° = (-5)\cos 25.7° - 45 \sin 25.7°$$
$$= -24.02\ \text{mm}$$

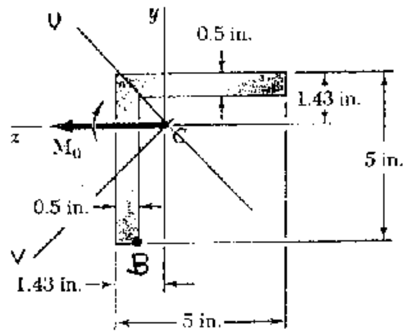
$$V_D = z_D \cos 25.7° + y_D \sin 25.7° = 45 \cos 25.7° + (-5)\sin 25.7°$$
$$= 38.38\ \text{mm}$$

$$\sigma_D = -\frac{M_V U_D}{I_V} + \frac{M_U V_D}{I_U} = -\frac{(M_0 \cos 64.3°)(-24.02\times10^{-3})}{0.23\times10^{-6}} + \frac{(M_0 \sin 64.3°)(38.38\times10^{-3})}{2.28\times10^{-6}}$$

$$80\times10^6 = 60.48\times10^3\ M_0$$

$$M_0 = 1.323\times10^3\ \text{N}\cdot\text{m} = 1.323\ \text{kN}\cdot\text{m}$$

Problem 4.148



4.148 A beam having the cross section shown is subjected to a couple $\mathbf{M}_0$ that acts in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress in the beam is not to exceed 12 ksi. *Given:* $I_y = I_z = 11.3\ \text{in}^4$, $A = 4.75\ \text{in}^2$, $k_{min} = 0.983$ in. (*Hint:* By reason of symmetry, the principal axes form an angle of 45° with the coordinate axes. Use the relations $I_{min} = Ak^2_{min}$ and $I_{min} + I_{max} = I_y + I_z$)

$$M_U = M_o \sin 45^\circ = 0.70711\ M_o$$

$$M_V = M_o \cos 45^\circ = 0.7071\ M_o$$

$$I_{min} = A k_{min}^2 = (4.75)(0.983)^2 = 4.59\ \text{in}^4$$

$$I_{max} = I_y + I_z - I_{min} = 11.3 + 11.3 - 4.59 = 18.01\ \text{in}^4$$

$$u_B = y_B \cos 45^\circ + z_B \sin 45^\circ = -3.57 \cos 45^\circ + 0.93 \sin 45^\circ = -1.866\ \text{in.}$$

$$v_B = z_B \cos 45^\circ - y_B \sin 45^\circ = -0.93 \cos 45^\circ - (-3.57)\sin 45^\circ = 3.182\ \text{in}$$

$$\sigma_B = -\frac{M_V u_B}{I_V} + \frac{M_U v_B}{I_U} = 0.70711\ M_o \left[-\frac{u_B}{I_{min}} + \frac{v_B}{I_{max}} \right]$$

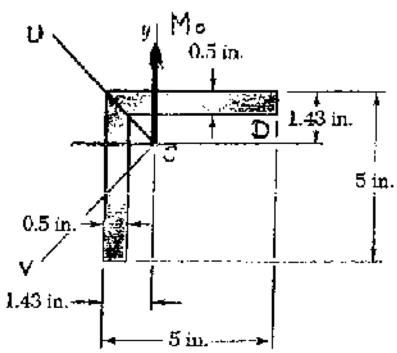
$$= 0.70711\ M_o \left[-\frac{(-1.866)}{4.59} + \frac{3.182}{18.01} \right] = 0.4124\ M_o$$

$$M_o = \frac{\sigma_B}{0.4124} = \frac{12}{0.4124} = 29.1\ \text{kip}\cdot\text{in}$$

Problem 4.149

4.149 Solve Prob. 4.148, assuming that the couple $\mathbf{M}_0$ acts in a horizontal plane.

4.148 A beam having the cross section shown is subjected to a couple $\mathbf{M}_0$ that acts in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress in the beam is not to exceed 12 ksi. *Given:* $I_y = I_z = 11.3\ \text{in}^4$, $A = 4.75\ \text{in}^2$, $k_{min} = 0.983$ in. (*Hint:* By reason of symmetry, the principal axes form an angle of 45° with the coordinate axes. Use the relations $I_{min} = Ak^2_{min}$ and $I_{min} + I_{max} = I_y + I_z$)



$$M_U = M_0 \cos 45° = 0.70711\, M_0$$

$$M_V = -M_0 \sin 45° = -0.70711\, M_0$$

$$I_{min} = A k_{min}^2 = (4.75)(0.983)^2 = 4.59\ \text{in}^4$$

$$I_{max} = I_y + I_z - I_{min} = 11.3 + 11.3 - 4.59 = 18.01\ \text{in}^4$$

$$U_D = y_D \cos 45° + z_D \sin 45° = 0.93 \cos 45° + (-3.57 \sin 45°) = -1.866\ \text{in.}$$

$$V_D = z_D \cos 45° - y_D \sin 45° = (-3.57)\cos 45° - (0.93)\sin 45° = 3.182\ \text{in.}$$

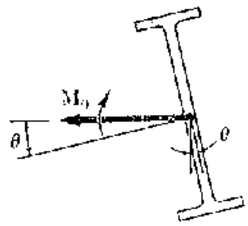
$$\sigma_D = -\frac{M_V U_D}{I_V} + \frac{M_U V_D}{I_U} = 0.70711\, M_0 \left[-\frac{U_D}{I_{min}} + \frac{V_D}{I_{max}} \right]$$

$$= 0.70711\, M_0 \left[-\frac{(-1.866)}{4.59} + \frac{3.182}{18.01} \right] = 0.4124\, M_0$$

$$M_0 = \frac{\sigma_D}{0.4124} = \frac{12}{0.4124} = 29.1\ \text{kip}\cdot\text{in}$$

Problem 4.150

4.150 A couple $\mathbf{M}_0$ acting in a vertical plane is applied to a W12 × 16 rolled-steel beam, whose web forms an angle θ with the vertical. Denoting by σ_0 the maximum stress in the beam when $\theta = 0$, determine the angle of inclination θ of the beam for which the maximum stress is $2\sigma_0$.



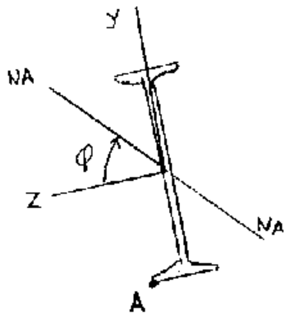
For W 12 × 16 rolled steel section

$I_z = 103 \text{ in}^4 \qquad I_y = 2.82 \text{ in}^4$

$d = 11.99 \text{ in} \qquad b_f = 3.990 \text{ in.}$

$y_A = -\frac{d}{2} \qquad z_A = \frac{b_f}{2}$

$$\tan\varphi = \frac{I_z}{I_y}\tan\theta = \frac{103}{2.82}\tan\theta = 36.52\tan\theta$$



Point A is farthest from the neutral axis.

$M_y = M_0 \sin\theta \qquad M_z = M_0\cos\theta$

$$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{M_0 d}{2I_z}\cos\theta + \frac{M_0 b_f}{2I_y}\sin\theta$$

$$= \frac{M_0 d}{2I_z}\left(1 + \frac{I_z b_f}{I_y d}\tan\theta\right)$$

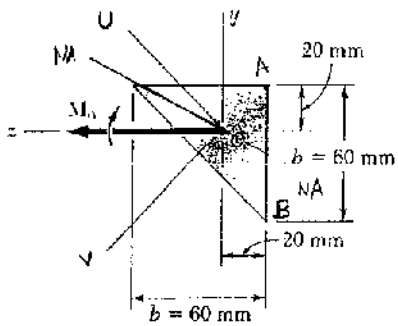
For $\theta = 0$ $\qquad \sigma_0 = \frac{M_0 d}{2I_z}$

$$\sigma_A = \sigma_0\left(1 + \frac{I_z b_f}{I_y d}\tan\theta\right) = 2\sigma_0$$

$$\tan\theta = \frac{I_y d}{I_z b_f} = \frac{(2.82)(11.99)}{(103)(3.990)} = 0.08273 \qquad \theta = 4.70°$$

Problem 4.151

4.151 A beam having the cross section shown is subjected to a couple $\mathbf{M}_0$ acting in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress is not to exceed 100 MPa. *Given:* $I_y = I_z = b^4/36$ and $I_{yz} = b^4/72$.

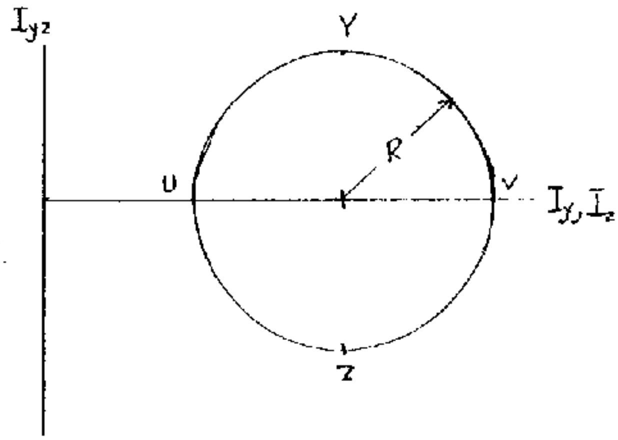


$$I_y = I_z = \frac{b^4}{36} = \frac{60^4}{36} = 0.360 \times 10^6 \text{ mm}^4$$

$$I_{yz} = \frac{b^4}{72} = \frac{60^4}{72} = 0.180 \times 10^6 \text{ mm}^4$$

Principal axes are symmetry axes.

Using Mohr's circle determine the principal moments of inertia



$$R = |I_{yz}| = 0.180 \times 10^6 \text{ mm}^4$$

$$I_v = \frac{I_y + I_z}{2} + R$$

$$= 0.540 \times 10^6 \text{ mm}^4 = 0.540 \times 10^{-6} \text{ m}^4$$

$$I_u = \frac{I_y + I_z}{2} - R$$

$$= 0.180 \times 10^6 \text{ mm}^4 = 0.180 \times 10^{-6} \text{ m}^4$$

$$M_u = M_o \sin 45° = 0.70711\, M_o, \qquad M_v = M_o \cos 45° = 0.70711\, M_o$$

$$\theta = 45° \qquad \tan\varphi = \frac{I_v}{I_u}\tan\theta = \frac{0.540 \times 10^{-6}}{0.180 \times 10^{-6}}\tan 45° = 3$$

$$\varphi = 71.56°$$

Point A: $u_A = 0$, $v_A = -20\sqrt{2}$ mm

$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u v_A}{I_u} = 0 + \frac{(0.70711\, M_o)(-20\sqrt{2} \times 10^{-3})}{0.180 \times 10^{-6}} = -111.11 \times 10^3\, M_o$$

$$M_o = -\frac{\sigma_A}{111.11 \times 10^3} = -\frac{-100 \times 10^6}{111.11 \times 10^3} = 900 \text{ N·m}$$

Point B: $u_B = -\frac{60}{\sqrt{2}}$ mm, $v_B = \frac{20}{\sqrt{2}}$ mm

$$\sigma_B = -\frac{M_v u_B}{I_v} + \frac{M_u v_B}{I_u} = -\frac{(0.70711\, M_o)(-\frac{60}{\sqrt{2}} \times 10^{-3})}{0.540 \times 10^{-6}} + \frac{(0.70711\, M_o)(\frac{20}{\sqrt{2}} \times 10^{-3})}{0.180 \times 10^{-6}}$$

$$= 111.11 \times 10^3\, M_o$$

$$M_o = \frac{\sigma_B}{111.11 \times 10^{-3}} = \frac{100 \times 10^6}{111.11 \times 10^{-3}} = 900 \text{ N·m}$$

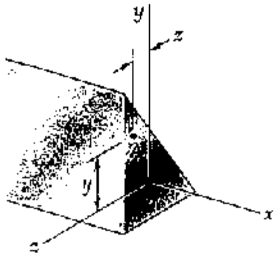
Choose the smaller value. $M_o = 900$ N·m

Problem 4.152

4.152 A beam of unsymmetric cross section is subjected to a couple $\mathbf{M}_0$ acting in the vertical plane xy. Show that the stress at point A, of coordinates y and z is

$$\sigma_A = -\frac{yI_y - zI_{yz}}{I_y I_z - I_{yz}^2} M_z$$

where I_y, I_z, and I_{yz} denote the moments and product or inertia of the cross section with respect to the coordinate axes, and M_z the moment of the couple.



The stress σ_A varies linearly with the coordinates y and z. Since the axial force is zero, the y- and z-axes are centroidal axes.

$\sigma_A = C_1 y + C_2 z$ where C_1 and C_2 are constants.

$$M_y = \int z\sigma_A \, dA = C_1 \int yz \, dA + C_2 \int z^2 dA$$

$$= I_{yz} C_1 + I_y C_2 = 0$$

$$C_2 = -\frac{I_{yz}}{I_y} C_1$$

$$M_z = -\int y \sigma_A \, dz = -C_1 \int y^2 \, dA + C_2 \int yz \, dA$$

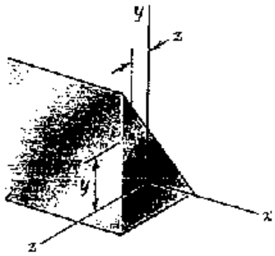
$$= -I_z C_1 - I_{yz} \frac{I_{yz}}{I_y} C_1$$

$$I_y M_z = -(I_y I_z - I_{yz}^2) C_1$$

$$C_1 = -\frac{I_y M_z}{I_y I_z - I_{yz}^2} \qquad C_2 = +\frac{I_{yz} M_z}{I_y I_z - I_{yz}^2}$$

$$\sigma_A = -\frac{I_y z - I_{yz} y}{I_y I_z - I_{yz}^2} M_z$$

Problem 4.153



4.153 A beam of unsymmetric cross section is subjected to a couple $\mathbf{M}_0$ acting in the horizontal plane xz. Show that the stress at point A, of coordinates y and z is

$$\sigma_A = -\frac{zI_z - yI_{yz}}{I_y I_z - I_{yz}^2} M_y$$

where I_y, I_z, and I_{yz} denote the moments and product or inertia of the cross section with respect to the coordinate axes, and M_y the moment of the couple.

The stress σ_A varies linearly with the coordinates y and z. Since the axial force is zero, the y- and z-axes are centroidal axes.

$\sigma_A = C_1 y + C_2 z$ where C_1 and C_2 are constants.

$$M_z = -\int y\sigma_A\,dA = -C_1\int y^2\,dA - C_2\int yz\,dA$$

$$= -I_z C_1 - I_{yz} C_2 = 0$$

$$C_1 = -\frac{I_{yz}}{I_z} C_2$$

$$M_y = \int z\sigma_A\,dA = C_1\int yz\,dA + C_2\int z^2\,dA$$

$$= I_{yz} C_1 + I_y C_2$$

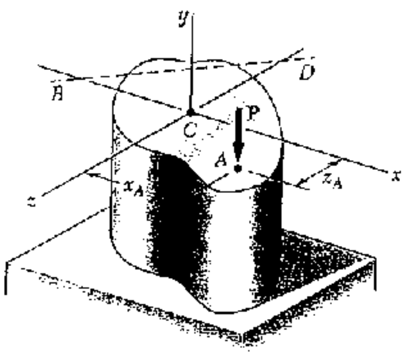
$$-I_{yz}\frac{I_{yz}}{I_z} C_2 + I_y C_2$$

$$I_z M_y = (I_y I_z - I_{yz}^2)\,C_2$$

$$C_2 = \frac{I_z M_y}{I_y I_z - I_{yz}^2} \qquad C_1 = -\frac{I_{yz} M_y}{I_y I_z - I_{yz}^2}$$

$$\sigma_A = \frac{I_z y - I_{yz} z}{I_y I_z - I_{yz}^2} M_y$$

Problem 4.154



4.154 (*a*) Show that, if a vertical force **P** is applied at point *A* of the section shown, the equation of the neutral axis *BD* is

$$\left(\frac{x_A}{k_z^2}\right)x + \left(\frac{z_A}{k_x^2}\right)z = -1$$

where k_z and k_x denote the radius of gyration of the cross section with respect to the *z* axis and the *x* axis, respectively. (*b*) Further shown that, if a vertical force **Q** is applied at any point located on line *BD*, the stress at point *A* will be zero.

Definitions $\quad k_x^2 = \dfrac{I_x}{A}, \quad k_z^2 = \dfrac{I_z}{A}$

(a) $\quad M_x = P z_A \qquad M_z = -P x_A$

$$\sigma_E = -\frac{P}{A} + \frac{M_z x_E}{I_z} - \frac{M_x z_E}{I_x} = -\frac{P}{A} - \frac{P x_A x_E}{A k_z^2} - \frac{P z_A z_E}{A k_x^2}$$

$$= -\frac{P}{A}\left[1 + \left(\frac{x_A}{k_z^2}\right)x_E + \left(\frac{z_A}{k_x^2}\right)z_E\right] = 0 \quad \text{if } E \text{ lies on neutral axis.}$$

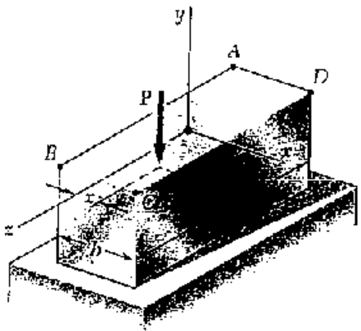
$$1 + \left(\frac{x_A}{k_z^2}\right)x + \left(\frac{z_A}{k_x^2}\right)z = 0, \qquad \left(\frac{x_A}{k_z^2}\right)x + \left(\frac{z_A}{k_x^2}\right)z = -1$$

(b) $\quad M_x = P z_E \qquad M_z = -P x_E$

$$\sigma_A = -\frac{P}{A} + \frac{M_z x_A}{I_z} - \frac{M_x z_A}{I_x} = -\frac{P}{A} - \frac{P x_E x_A}{A k_z^2} - \frac{P z_E z_A}{A k_x^2}$$

$= 0$ by equation from Part (a)

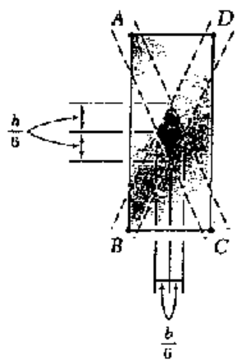
Problem 4.155



4.155 (*a*) Show that the stress at corner *A* of the prismatic member shown in Fig. P4.155*a* will be zero if the vertical force **P** is applied at a point located on the line

$$\frac{x}{b/6}+\frac{z}{h/6}=1$$

(*b*) Further show that, is no tensile stress is to occur in the member, the force **P** must be applied at a point located within the area bounded by the line found in part *a* and three similar lines corresponding to the condition of zero stress at *B*, *C*, and *D*, respectively. This area, shown in Fig. 4.155*b*, is known as the *kern* of the cross section.



$I_z = \frac{1}{12}hb^3 \qquad I_x = \frac{1}{12}bh^3 \qquad A = bh$

$z_A = -\frac{h}{2} \qquad x_A = -\frac{b}{2}$

Let P be the load point

$M_z = -Px_P \qquad M_x = Pz_P$

$$\sigma_A = -\frac{P}{A} + \frac{M_z x_A}{I_z} - \frac{M_x z_A}{I_x}$$

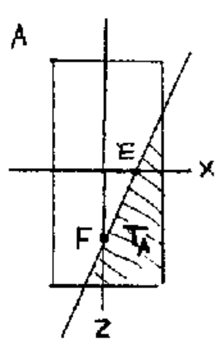
$$= -\frac{P}{bh} + \frac{(-Px_P)(-\frac{b}{2})}{\frac{1}{12}hb^3} - \frac{Pz_P(-\frac{h}{2})}{\frac{1}{12}bh^3}$$

$$= -\frac{P}{bh}\left[1 - \frac{x_P}{b/6} - \frac{z_P}{h/6}\right]$$

For $\sigma_A = 0$ $\qquad 1 - \frac{x}{b/6} - \frac{z}{h/6} = 0, \quad \frac{x}{b/6} + \frac{z}{h/6} = 1$

At point E $\quad z = 0 \quad \therefore x_E = b/6$

At point F $\quad x = 0 \quad \therefore z_F = h/6$

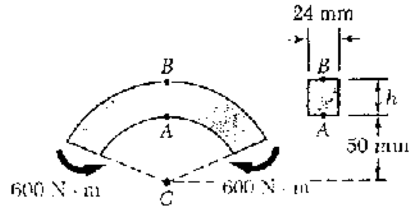


If the line of action (x_P, z_P) lies within the portion marked T_A, a tensile will occur at corner A.

By considering $\sigma_B = 0$, $\sigma_C = 0$, and $\sigma_D = 0$, the other portions producing tensile stresses are identified.

Problem 4.156

4.156 For the curved bar shown, determine the stress at point A when (*a*) $h = 50$ mm, (*b*) $h = 60$ mm.



(a) $h = 50\text{ mm}, \quad r_1 = 50\text{ mm}, \quad r_2 = 100\text{ mm}$

$$A = (24)(50) = 1.200\times10^3\text{ mm}$$

$$R = \frac{h}{\ln\frac{r_2}{r_1}} = \frac{50}{\ln\frac{100}{50}} = 72.13475\text{ mm}$$

$$\bar{r} = \tfrac{1}{2}(r_1 + r_2) = 75\text{ mm}$$

$$e = \bar{r} - R = 2.8652\text{ mm}$$

$$y_A = 72.13475 - 50 = 22.13475\text{ mm} \qquad r_A = 50\text{ mm}$$

$$\sigma_A = -\frac{My_A}{Aer_A} = -\frac{(600)(22.13475\times10^{-3})}{(1.200\times10^{-3})(2.8652\times10^{-3})(50\times10^{-3})} = -77.3\times10^6\text{ Pa}$$

-77.3 MPa ◄

(b) $h = 60\text{ mm}, \quad r_1 = 50\text{ mm}, \quad r_2 = 110\text{ mm}, \quad A = (24)(60) = 1.440\times10^3\text{ mm}^2$

$$R = \frac{h}{\ln\frac{r_2}{r_1}} = \frac{60}{\ln\frac{110}{50}} = 76.09796\text{ mm}$$

$$\bar{r} = \tfrac{1}{2}(r_1 + r_2) = 80\text{ mm} \qquad e = \bar{r} - R = 3.90204\text{ mm}$$

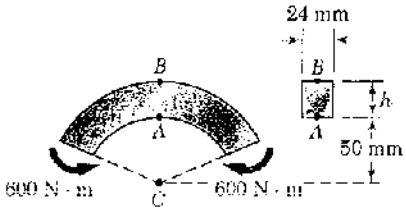
$$y_A = 76.09796 - 50 = 26.09796\text{ mm} \qquad r_A = 50\text{ mm}$$

$$\sigma_A = -\frac{My_A}{Aer_A} = -\frac{(600)(26.09796\times10^{-3})}{(1.440\times10^{-3})(3.90204\times10^{-3})(50\times10^{-3})} = -55.7\times10^6\text{ Pa}$$

-55.7 MPa ◄

Problem 4.157

4.157 For the curved bar shown, determine the stress at points A and B when h = 55 mm.



$h = 55$ mm $\qquad r_1 = 50$ mm $\qquad r_2 = 105$ mm

$A = (24)(55) = 1.320 \times 10^3\ \text{mm}^2$

$R = \dfrac{h}{\ln \frac{r_2}{r_1}} = \dfrac{55}{\ln \frac{105}{50}} = 74.13025$ mm

$\bar{r} = \frac{1}{2}(r_1 + r_2) = 77.5$ mm

$e = \bar{r} - R = 3.36975$ mm

$y_A = 74.13025 - 50 = 24.13025$ mm $\qquad r_A = 50$ mm

$$\sigma_A = -\frac{M y_A}{A e r_A} = -\frac{(600)(24.13025 \times 10^{-3})}{(1.320 \times 10^{-3})(3.36975 \times 10^{-3})(50 \times 10^{-3})} = -65.1 \times 10^6\ \text{Pa}$$

-65.1 MPa ◄

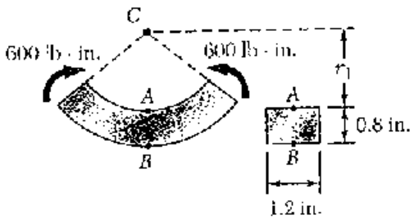
$y_B = 74.13025 - 105 = -30.86975$ mm $\qquad r_B = 105$ mm

$$\sigma_B = -\frac{M y_B}{A e r_B} = -\frac{(600)(-30.8697 \times 10^{-3})}{(1.320 \times 10^{-3})(3.36975 \times 10^{-3})(105 \times 10^{-3})} = 39.7 \times 10^6\ \text{Pa}$$

39.7 MPa ◄

Problem 4.158

4.158 For the curved bar and loading shown, determine the stress points A and B when $r_1 = 1.6$ in.



$h = 0.8$ in. $\quad r_1 = 1.6$ in. $\quad r_2 = 2.4$ in.

$A = (1.2)(0.8) = 0.96 \text{ in}^2$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{0.8}{\ln \frac{2.4}{1.6}} = 1.97304 \text{ in.}$$

$\bar{r} = \frac{1}{2}(r_1 + r_2) = 2.0$ in.

$e = \bar{r} - R = 0.02696$ in.

$y_A = 1.97304 - 1.6 = 0.37304$ in. $\quad r_A = 1.6$ in.

$$\sigma_A = -\frac{M y_A}{A e r_A} = -\frac{(600)(0.37304)}{(0.96)(0.02696)(1.6)} = -5.405 \times 10^3 \text{ psi}$$

-5.40 ksi ◄

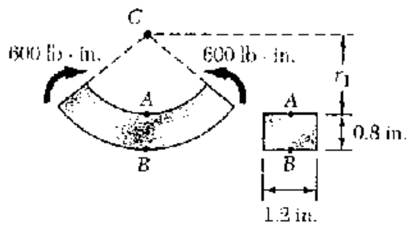
$y_B = 1.97304 - 2.4 = -0.42696$ in. $\quad r_B = 2.4$ in.

$$\sigma_B = -\frac{M y_B}{A e r_B} = -\frac{(600)(-0.42696)}{(0.96)(0.02696)(2.4)} = 4.124 \times 10^3 \text{ psi}$$

4.12 ksi ◄

Problem 4.159

4.159 For the curved bar and loading shown, determine the stress point A when (*a*) $r_1 = 1.2$ in., (*b*) $r_1 = 2$ in.



$h = 0.8$ in. $\qquad A = (1.2)(0.8) = 0.96\ in^2$

(a) $r_1 = 1.2$ in. $\qquad r_2 = 2.0$ in.

$$R = \frac{h}{\ln\frac{r_2}{r_1}} = \frac{0.8}{\ln\frac{2.4}{1.2}} = 1.56609 \text{ in.}$$

$$\bar{r} = \tfrac{1}{2}(r_1 + r_2) = 1.6 \text{ in.}$$

$$e = \bar{r} - R = 0.03391 \text{ in.}$$

$y_A = 1.97304 - 1.6 = 0.36609$ in. $\qquad r_A = r_1 = 1.2$ in.

$$\sigma_A = -\frac{My_A}{Aer_A} = -\frac{(600)(0.36609)}{(0.96)(0.03391)(1.2)} = -5.623 \times 10^3 \text{ psi}$$

-5.62 ksi ◄

(b) $r_1 = 2$ in. $\qquad r_2 = 2.8$ in.

$$R = \frac{h}{\ln\frac{r_2}{r_1}} = \frac{0.8}{\ln\frac{2.8}{2.0}} = 2.37761 \text{ in.}$$

$$\bar{r} = \tfrac{1}{2}(r_1 + r_2) = 2.4 \text{ in.}$$

$$e = \bar{r} - R = 0.02239 \text{ in.}$$

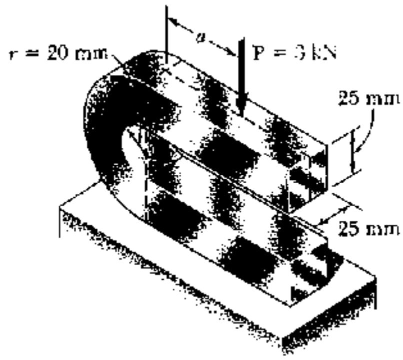
$y_A = 2.37761 - 2 = 0.37761$ in. $\qquad r_A = r_1 = 2$ in.

$$\sigma_A = -\frac{My_A}{Aer_A} = -\frac{(600)(0.37761)}{(0.96)(0.02239)(2)} = -5.270 \times 10^3 \text{ psi}$$

5.27 ksi ◄

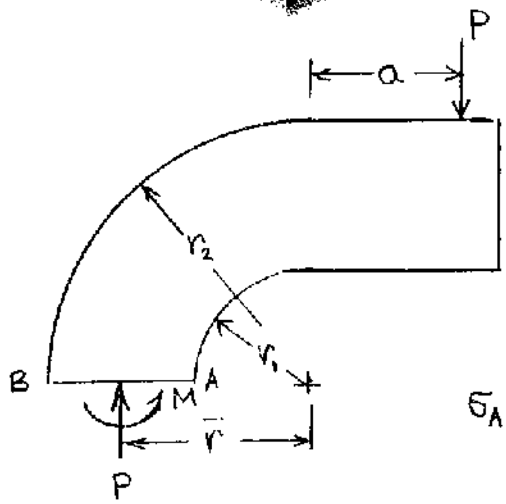
Problem 4.160

4.160 The curved portion of the bar shown has an inner radius of 20 mm. Knowing that the line of action of the 3-kN force is located at a distance $a = 60$ mm from the vertical plane containing the center of curvature of the bar, determine the largest compressive stress in the bar.



Reduce the internal forces transmitted across section AB to a force-couple system at the centroid of the section. The bending couple is

$$M = P(a + \bar{r})$$



For the rectangular section, the neutral axis for bending couple only lies at

$$R = \frac{h}{\ln \frac{r_2}{r_1}}. \qquad \text{Also} \quad e = \bar{r} - R$$

The maximum compressive stress occurs at point A. It is given by

$$\sigma_A = -\frac{P}{A} - \frac{M y_A}{A e r_1} = -\frac{P}{A} - \frac{P(a+\bar{r}) y_A}{A e r_1}$$

$$= -K\frac{P}{A} \qquad \text{with } y_A = R - r_1$$

Thus, $$K = 1 + \frac{(a+\bar{r})(R - r_1)}{e r_1}$$

Data: $h = 25$ mm, $r_1 = 20$ mm, $r_2 = 45$ mm, $\bar{r} = 32.5$ mm

$$R = \frac{25}{\ln \frac{45}{20}} = 30.8288 \text{ mm}, \quad e = 32.5 - 30.8288 = 1.6712 \text{ mm}$$

$b = 25$ mm, $A = bh = (25)(25) = 625 \text{ mm}^2 = 625 \times 10^{-6} \text{ m}^2$

$a = 60$ mm, $a + \bar{r} = 92.5$ mm, $R - r_1 = 10.8288$ mm

$$K = 1 + \frac{(92.5)(10.8288)}{(1.6712)(20)} = 30.968$$

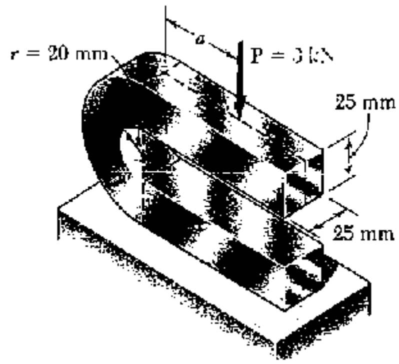
$P = 3 \times 10^3$ N

$$\sigma_A = -\frac{KP}{A} = -\frac{(30.968)(3\times10^3)}{625\times10^{-6}} = -148.6 \times 10^6 \text{ Pa}$$

$$= -148.6 \text{ MPa}$$

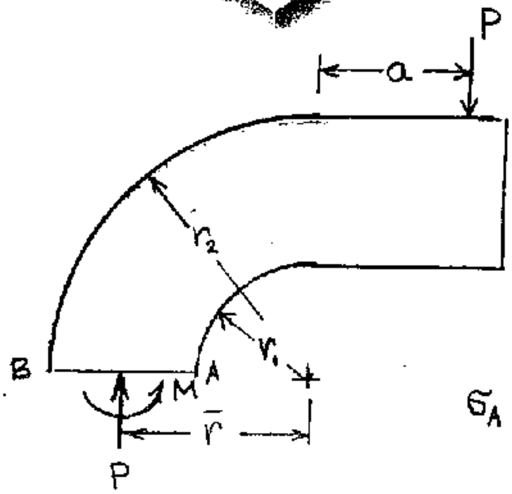
Problem 4.161

4.161 Knowing that the allowable stress in the bar is 150 MPa, determine the largest permissible distance a from the line of action of the 3-kN force to the vertical plane containing the center of curvature of the bar.



Reduce the internal forces transmitted across section AB to a force-couple system at the centroid of the section. The bending couple is

$$M = P(a + \bar{r})$$



For the rectangular section, the neut axis for bending couple only lies

$$R = \frac{h}{\ln \frac{r_2}{r_1}} \qquad \text{Also} \quad e = \bar{r} - R$$

The maximum compressive stress occurs at point A. It is given by

$$\sigma_A = -\frac{P}{A} - \frac{M y_A}{A e r_1} = -\frac{P}{A} - \frac{P(a+\bar{r})y_A}{A e r_1}$$

$$= -K\frac{P}{A} \qquad \text{with} \quad y_A = R - r_1$$

Thus, $\quad K = 1 + \dfrac{(a+\bar{r})(R - r_1)}{e r_1}$

Data: $h = 25\text{ mm}, \quad r_1 = 20\text{ mm}, \quad r_2 = 45\text{ mm}, \quad \bar{r} = 32.5\text{ mm}$

$$R = \frac{25}{\ln \frac{45}{20}} = 30.8288\text{ mm}, \quad e = 32.5 - 30.8288 = 1.6712\text{ mm}$$

$$b = 25\text{ mm}, \quad A = bh = (25)(25) = 625\text{ mm}^2 = 625 \times 10^{-6}\text{ m}^2$$

$$R - r_1 = 10.8288\text{ mm}$$

$$P = 3 \times 10^3\text{ N}\cdot\text{m} \qquad \sigma_A = -150 \times 10^6\text{ Pa}$$

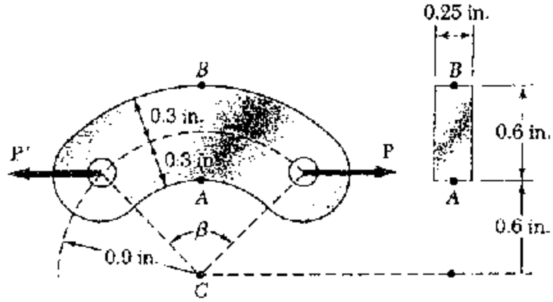
$$K = -\frac{\sigma_A A}{P} = -\frac{(-150 \times 10^6)(625 \times 10^{-6})}{3 \times 10^3} = 31.25$$

$$a + \bar{r} = \frac{(K-1)e r_1}{R - r_1} = \frac{(30.25)(1.6712)(20)}{10.8288} = 93.37\text{ mm}$$

$$a = 93.37 - 32.5 = 60.9\text{ mm}$$

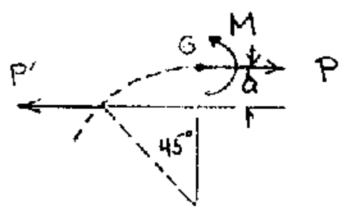
Problem 4.162

4.162 Steel links having the cross section shown are available with different central angles β. Knowing that the allowable stress is 15 ksi, determine the largest force **P** that can be applied to a link for which $\beta = 90°$.



Reduce section force to a force-couple system at G, the centroid of the cross section AB.

$$a = \bar{r}\left(1 - \cos\frac{\beta}{2}\right)$$



The bending couple is $M = -Pa$

For the rectangular section, the neutral axis for bending couple only lies at

$$R = \frac{h}{\ln\frac{r_2}{r_1}}. \qquad \text{Also} \quad e = \bar{r} - R$$

At point A the tensile stress is

$$\sigma_A = \frac{P}{A} - \frac{My_A}{Aer_1} = \frac{P}{A} + \frac{Pay_A}{Aer_1} = \frac{P}{A}\left(1 + \frac{ay_A}{er_1}\right) = K\frac{P}{A}$$

where $K = 1 + \dfrac{ay_A}{er_1}$ and $y_A = R - r_1$

$$P = \frac{A\sigma_A}{K}$$

Data: $\bar{r} = 0.9$ in, $r_1 = 0.6$ in, $r_2 = 1.2$ in, $h = 0.6$ in, $b = 0.25$ in.

$A = (0.25)(0.6) = 0.15\ \text{in}^2$, $R = \dfrac{0.6}{\ln\frac{1.2}{0.6}} = 0.86562$ in

$e = 0.9 - 0.86562 = 0.03438$ in., $y_A = 0.86562 - 0.6 = 0.26562$ in

$a = 0.9(1 - \cos 45°) = 0.26360$ in

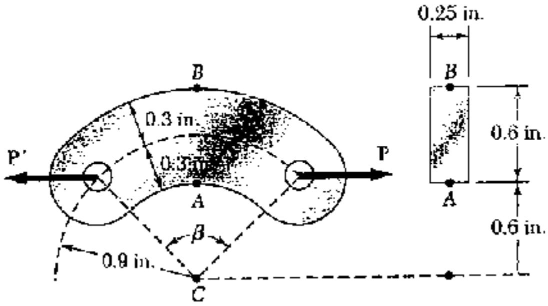
$$K = 1 + \frac{(0.26360)(0.26562)}{(0.03438)(0.6)} = 4.3943$$

$$P = \frac{(0.15)(15)}{4.3943} = 0.512 \text{ kips} = 512 \text{ lb}$$

Problem 4.163

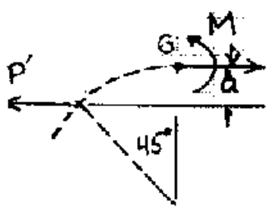
4.163 Solve Prob. 4.162, assuming that $\beta = 60°$.

4.162 Steel links having the cross section shown are available with different central angles β. Knowing that the allowable stress is 15 ksi, determine the largest force **P** that can be applied to a link for which $\beta = 90°$.



Reduce section force to a force-couple system at G, the centroid of the cross section AB.

$$a = \bar{r}\left(1 - \cos\frac{\beta}{2}\right)$$



The bending couple is $M = -Pa$

For the rectangular section, the neutral axis for bending couple only lies at

$$R = \frac{h}{\ln\frac{r_2}{r_1}}. \qquad \text{Also } e = \bar{r} - R$$

At point A the tensile stress is

$$\sigma_A = \frac{P}{A} + \frac{My_A}{Aer_1} = \frac{P}{A} + \frac{Pay_A}{Aer_1} = \frac{P}{A}\left(1 + \frac{ay_A}{er_1}\right) = K\frac{P}{A}$$

where $K = 1 + \frac{ay_A}{er_1}$ and $y_A = R - r_1$

$$P = \frac{A\sigma_A}{K}$$

Data: $\bar{r} = 0.9$ in, $r_1 = 0.6$ in, $r_2 = 1.2$ in, $h = 0.6$ in, $b = 0.25$

$A = (0.25)(0.6) = 0.15 \text{ in}^2$, $R = \frac{0.6}{\ln\frac{1.2}{0.6}} = 0.86562$ in.

$e = 0.9 - 0.86562 = 0.03438$ in., $y_A = 0.86562 - 0.6 = 0.26562$

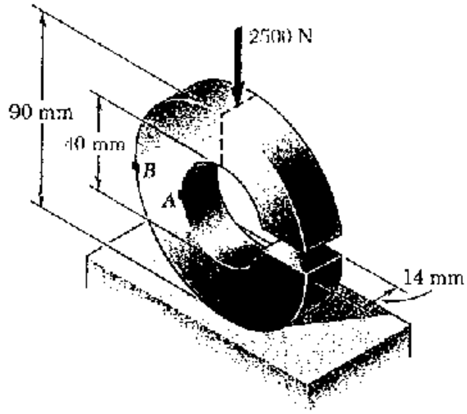
$a = 0.9(1 - \cos 30°) = 0.12058$ in.

$$K = 1 + \frac{(0.12058)(0.26562)}{(0.03438)(0.6)} = 2.5526$$

$$P = \frac{(0.15)(15)}{2.5526} = 0.881 \text{ kips} = 881 \text{ lb.}$$

Problem 4.164

4.164 For the split ring shown, determine the stress at (*a*) point *A*, (*b*) point *B*.



$r_1 = \frac{1}{2} 40 = 20 \text{ mm}, \quad r_2 = \frac{1}{2}(90) = 45 \text{ mm}$

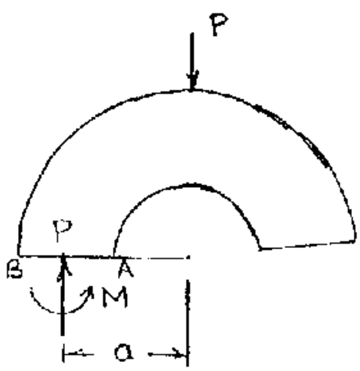
$h = r_2 - r_1 = 25 \text{ mm}$

$A = (14)(25) = 350 \text{ mm}^2$

$R = \dfrac{h}{\ln \frac{r_2}{r_1}} = \dfrac{25}{\ln \frac{45}{20}} = 30.8288$

$\bar{r} = \frac{1}{2}(r_1 + r_2) = 32.5 \text{ mm}$

$e = \bar{r} - R = 1.6712 \text{ mm}$



Reduce the internal forces transmitted across section AB to a force-couple system at the centroid of the cross section. The bending couple is

$M = Pa = P\bar{r}$

$= (2500)(32.5 \times 10^{-3}) = 81.25 \text{ N}\cdot\text{m}$

(a) Point A: $r_A = 20 \text{ mm}$

$y_A = 30.8288 - 20 = 10.8288 \text{ mm}$

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{AeR} = -\frac{2500}{350 \times 10^{-6}} - \frac{(81.25)(10.8288 \times 10^{-3})}{(350 \times 10^{-6})(1.6712 \times 10^{-3})(20 \times 10^{-3})}$$

$= -82.4 \times 10^6 \text{ Pa}$ $\qquad -82.4 \text{ MPa}$ ◄

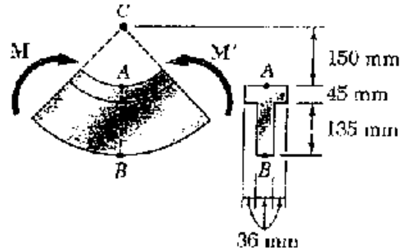
(b) Point B: $r_B = 45 \text{ mm}$ $\qquad y_B = 30.8288 - 45 = -14.1712 \text{ mm}$

$$\sigma_B = -\frac{P}{A} - \frac{My_B}{Aer_B} = -\frac{2500}{350 \times 10^{-6}} - \frac{(81.25)(-14.1712 \times 10^{-3})}{(350 \times 10^{-6})(1.6712 \times 10^{-3})(45 \times 10^{-3})}$$

$= 36.6 \times 10^6 \text{ Pa}$ $\qquad 36.6 \text{ MPa}$ ◄

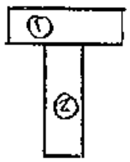
Problem 4.165

4.165 and 4.166 Knowing that $M = 20$ kN · m, determine the stress at (*a*) point *A*, (*b*) point *B*.



$$R = \frac{\Sigma A}{\Sigma \int \frac{1}{r} dA} = \frac{\Sigma b_i h_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\Sigma A_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\Sigma A_i \bar{r}_i}{\Sigma A_i}$$



r, mm	Part	b_i, mm	h, mm	A, mm^2	$b_i \ln \frac{r_{i+1}}{r_i}$, mm	$\bar{r}$, mm	$A\bar{r}$, mm^3
150							
	1	108	45	4860	28.3353	172.5	838.35×10^3
195							
	2	36	135	4860	18.9394	262.5	1275.75×10^3
330							
	Σ			9720	47.2747		2114.1×10^3

$R = \frac{9720}{47.2747} = 205.606$ mm $\qquad \bar{r} = \frac{2114.1 \times 10^3}{9720} = 217.5$ mm

$e = \bar{r} - R = 11.894$ mm. $\qquad M = 20 \times 10^3$ N·m

(a) $y_A = R - r_1 = 205.606 - 150 = 55.606$ mm

$$\sigma_A = -\frac{M y_A}{A e r_1} = -\frac{(20 \times 10^3)(55.606 \times 10^{-3})}{(9720 \times 10^{-6})(11.894 \times 10^{-3})(150 \times 10^{-3})}$$

$$= -64.1 \times 10^6 \text{ Pa} = -64.1 \text{ MPa}$$

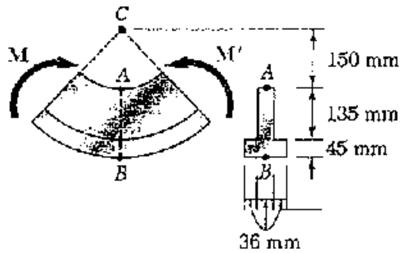
(b) $y_B = R - r_2 = 205.606 - 330 = -124.394$ mm

$$\sigma_B = -\frac{M y_B}{A e r_2} = -\frac{(20 \times 10^3)(-124.394 \times 10^{-3})}{(9720 \times 10^{-6})(11.894 \times 10^{-3})(330 \times 10^{-3})}$$

$$= 65.2 \times 10^6 \text{ Pa} = 65.2 \text{ MPa}$$

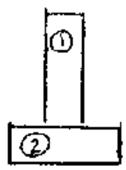
Problem 4.166

4.165 and 4.166 Knowing that $M = 20\text{ kN}\cdot\text{m}$, determine the stress at (*a*) point *A*, (*b*) point *B*.



$$R = \frac{\Sigma A}{\Sigma \int \frac{1}{r} dA} = \frac{\Sigma b_i h_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\Sigma A_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\Sigma A_i \bar{r}_i}{\Sigma A_i}$$



r, mm		b, mm	h, mm	A, mm^2	$b_i \ln \frac{r_{i+1}}{r_i}$, mm	$\bar{r}$, mm	$A\bar{r}$, mm^3
150							
	①	36	135	4860	23.1067	217.5	1.05705×10^6
285							
	②	108	45	4860	15.8332	307.5	1.49445×10^6
330							
	Σ			9720	38.9399		2.5515×10^6

$$R = \frac{9720}{38.9399} = 249.615 \text{ mm}, \qquad \bar{r} = \frac{2.5515\times10^6}{9720} = 262.5 \text{ mm}$$

$$e = \bar{r} - R = 12.885 \text{ mm}, \qquad M = 20\times10^3 \text{ N}\cdot\text{m}$$

(a) $y_A = R - r_1 = 249.615 - 150 = 99.615$ mm

$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(20\times10^3)(99.615\times10^{-3})}{(9720\times10^{-6})(12.885\times10^{-3})(150\times10^{-3})}$$

$$= -106.1\times10^6 \text{ Pa} = -106.1 \text{ MPa}$$

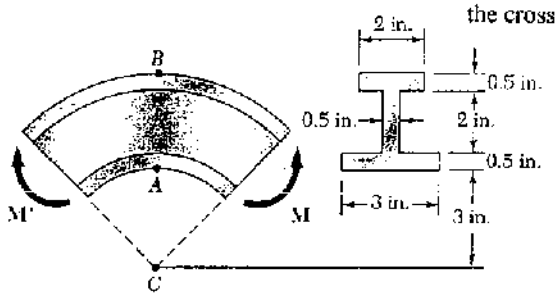
(b) $y_B = R - r_2 = 249.615 - 330 = -80.385$ mm

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(20\times10^3)(-80.385\times10^{-3})}{(9720\times10^{-6})(12.885\times10^{-3})(330\times10^{-6})}$$

$$= 38.9\times10^6 \text{ Pa} = 38.9 \text{ MPa}$$

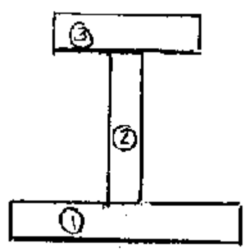
Problem 4.167

4.167 Three plates are welded together to form the curved beam shown. For the given loading, determine the distance e between the neutral axis and the centroid os the cross section.



$$R = \frac{\Sigma A}{\Sigma \int \frac{1}{r} dA} = \frac{\Sigma b_i h_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\Sigma A}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\Sigma A \bar{r}_i}{\Sigma A}$$



r	part	b	h	A	$b \ln \frac{r_{i+1}}{r_i}$	$\bar{r}$	$A\bar{r}$
3	①	3	0.5	1.5	0.462452	3.25	4.875
3.5	②	0.5	2	1.0	0.225993	4.5	4.5
5.5	③	2	0.5	1.0	0.174023	5.75	5.75
6	Σ			3.5	0.862468		15.125

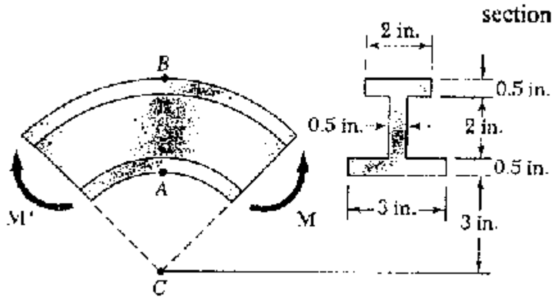
$R = \frac{3.5}{0.862468} = 4.05812$ in., $\bar{r} = \frac{15.125}{3.5} = 4.32143$ in.

$e = \bar{r} - R = 0.26331$ in.

0.263 in. ◄

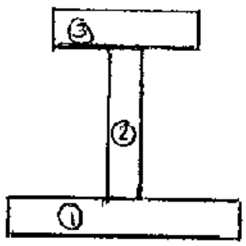
Problem 4.168

4.168 Three plates are welded together to form the curved bean shown. For $M = 8$ kip · in., determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) the centroid of the cross section



$$R = \frac{\Sigma A}{\Sigma \int \frac{1}{r} dA} = \frac{\Sigma b_i h_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\Sigma A}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\Sigma A \bar{r}_i}{\Sigma A}$$



r	part	b	h	A	$b \ln \frac{r_{i+1}}{r_i}$	$\bar{r}$	$A\bar{r}$
3	①	3	0.5	1.5	0.462452	3.25	4.875
3.5	②	0.5	2	1.0	0.225993	4.5	4.5
5.5	③	2	0.5	1.0	0.174023	5.75	5.75
6	Σ			3.5	0.862468		15.125

$R = \frac{3.5}{0.862468} = 4.05812$ in, $\bar{r} = \frac{15.125}{3.5} = 4.32143$ in.

$e = \bar{r} - R = 0.26331$ in. $\qquad M = -8$ kip · in

(a) $y_A = R - r_1 = 4.05812 - 3 = 1.05812$ in.

$$\sigma_A = -\frac{M y_A}{A e r_1} = -\frac{(-8)(1.05812)}{(3.5)(0.26331)(3)} = 3.06 \text{ ksi}$$

(b) $y_B = R - r_2 = 4.05812 - 6 = -1.94188$ in.

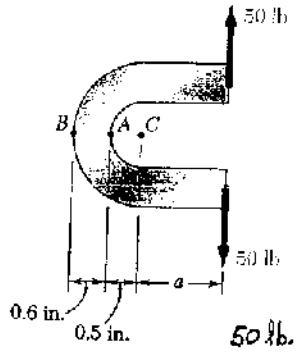
$$\sigma_B = -\frac{M y_B}{A e r_2} = -\frac{(-8)(-1.94188)}{(3.5)(0.26331)(6)} = -2.81 \text{ ksi}$$

(c) $y_C = R - \bar{r} = -e =$

$$\sigma_C = -\frac{M y_C}{A e \bar{r}} = -\frac{M e}{A e \bar{r}} = -\frac{M}{A \bar{r}} = -\frac{-8}{(3.5)(4.32143)} = 0.529 \text{ ksi}$$

Problem 4.169

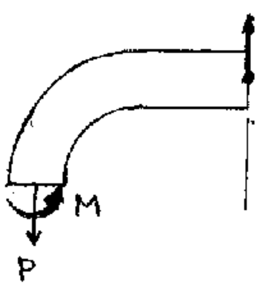
4.169 The bar shown has a *circular* cross section of 0.6-in. diameter. Knowing that $a = 1.2$ in., determine the stress at (a) point A, (b) point B.



$$c = \tfrac{1}{2}d = 0.3 \text{ in.} \qquad \bar{r} = 0.5 + 0.3 = 0.8 \text{ in.}$$

$$R = \tfrac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right] = \tfrac{1}{2}\left[0.8 + \sqrt{0.8^2 - 0.3^2}\right]$$

$$= 0.77081 \text{ in}$$



$$e = \bar{r} - R = 0.02919 \text{ in.}$$

$$A = \pi c^2 = \pi(0.3)^2 = 0.28274 \text{ in}^2$$

$$M = -P(a + \bar{r}) = -50\,(1.2 + 0.8) = -100 \text{ lb-in}$$

$$y_A = R - r_1 = 0.77081 - 0.5 = 0.27081 \text{ in.}$$

$$y_B = R - r_2 = 0.77081 - 1.1 = -0.32919 \text{ in.}$$

(a) $$\sigma_A = \frac{P}{A} - \frac{My_A}{Aer_1} = \frac{50}{0.28274} - \frac{(-100)(0.27081)}{(0.28274)(0.02919)(0.5)} = 6.74 \times 10^3 \text{ psi}$$

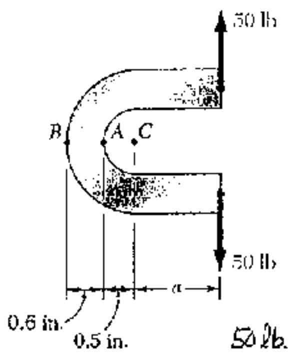
$$= 6.74 \text{ ksi}$$

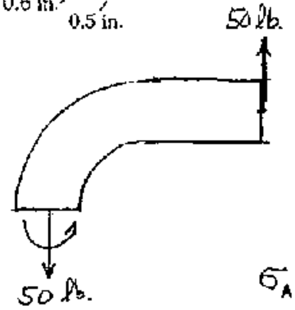
(b) $$\sigma_B = \frac{P}{A} - \frac{My_B}{Aer_2} = \frac{50}{0.28274} - \frac{(-100)(-0.32919)}{(0.28274)(0.02919)(1.1)} = -3.45 \times 10^3 \text{ psi}$$

$$= -3.45 \text{ ksi}$$

Problem 4.170

4.170 The bar shown has a *circular* cross section of 0.6-in. diameter. Knowing that the allowable stress is 8 ksi, determine the largest permissible distance a from the line of action of the 50-lb forces to the plane containing the center of curvature of the bar.





$$c = \tfrac{1}{2}d = 0.3 \text{ in}, \qquad \bar{r} = 0.5 + 0.3 = 0.8 \text{ in}$$

$$R = \tfrac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right] = \tfrac{1}{2}\left[0.8 + \sqrt{0.8^2 - 0.3^2}\right]$$

$$= 0.77081 \text{ in.} \qquad e = \bar{r} - R = 0.02919 \text{ in.}$$

$$A = \pi c^2 = \pi(0.3)^2 = 0.28274 \text{ in}^2$$

$$M = -P(a + \bar{r})$$

$$y_A = R - r_1 = 0.77081 - 0.5 = 0.27081 \text{ in.}$$

$$\sigma_A = \frac{P}{A} - \frac{My_A}{Aer_1} = \frac{P}{A} + \frac{P(a+\bar{r})y_A}{Aer_1} = \frac{P}{A}\left[1 + \frac{(a+\bar{r})y_A}{er_1}\right]$$

$$= \frac{KP}{A} \qquad \text{where} \quad K = 1 + \frac{(a+\bar{r})y_A}{er_1}$$

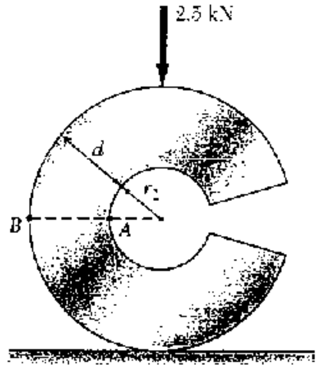
$$K = \frac{\sigma_A A}{P} = \frac{(8\times10^3)(0.28274)}{50} = 45.238$$

$$a + \bar{r} = \frac{(K-1)er_1}{y_A} = \frac{(44.238)(0.02919)(0.5)}{0.27081} = 2.384 \text{ in}$$

$$a = 2.384 - 0.8 = 1.584 \text{ in.}$$

Problem 4.171

4.171 The split ring shown has an inner radius $r_1 = 20$ mm and a *circular* cross section of diameter $d = 32$ mm. For the loading shown, determine the stress at (*a*) point *A*, (*b*) point *B*.



$c = \frac{1}{2}d = 16\text{ mm} \qquad r_1 = 20\text{ mm}, \quad r_2 = r_1 + d = 52\text{ mm}$

$\bar{r} = r_1 + c = 36\text{ mm}$

$R = \frac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right] = \frac{1}{2}\left[36 + \sqrt{36^2 - 16^2}\right]$

$= 34.1245\text{ mm}$

$e = \bar{r} - R = 1.8755\text{ mm}$

$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(32)^2 = 804.25\text{ mm}^2 = 804.25\times10^{-6}\text{ m}^2$

$P = 2.5\times10^3\text{ N} \qquad M = P\bar{r} = (2.5\times10^3)(36\times10^{-3}) = 90\text{ N}\cdot\text{m}$

(a) Point A: $\quad y_A = R - r_1 = 34.1245 - 20 = 14.1245\text{ mm}$

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{2.5\times10^3}{804.25\times10^{-6}} - \frac{(90)(14.1245\times10^{-3})}{(804.25\times10^{-6})(1.8755\times10^{-3})(20\times10^{-3})}$$

$= -45.2\times10^6\text{ Pa} = -45.2\text{ MPa}$ ◄

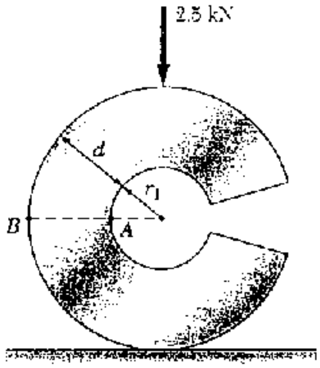
(b) Point B: $\quad y_B = R - r_2 = 34.1245 - 52 = -17.8755\text{ mm}$

$$\sigma_B = -\frac{P}{A} - \frac{My_B}{Aer_2} = -\frac{2.5\times10^3}{804.25\times10^{-6}} - \frac{(90)(-17.8755\times10^{-3})}{(804.25\times10^{-6})(1.8755\times10^{-3})(52\times10^{-3})}$$

$= 17.40\times10^6\text{ Pa} = 17.40\text{ MPa}$ ◄

Problem 4.172

4.172 The split ring shown has an inner radius $r_1 = 16$ mm and a *circular* cross section of diameter $d = 32$ mm. For the loading shown, determine the stress at (*a*) point *A*, (*b*) point *B*.



$c = \frac{1}{2}d = 16$ mm, $r_1 = 16$ mm, $r_2 = r_1 + d = 48$ mm

$\bar{r} = r_1 + c = 32$ mm

$R = \frac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right] = \frac{1}{2}\left[32 + \sqrt{32^2 - 16^2}\right]$

$= 29.8564$ mm

$e = \bar{r} - R = 2.1436$ mm

$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(32)^2 = 804.25\ \text{mm}^2 = 804.25 \times 10^{-6}\ \text{m}^2$

$P = 2.5 \times 10^3$ N $\qquad M = P\bar{r} = (2.5 \times 10^3)(32 \times 10^{-3}) = 80$ N·m

(a) Point A: $\quad y_A = R - r_1 = 29.8564 - 16 = 13.8564$ mm

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{2.5 \times 10^3}{804.25 \times 10^{-6}} - \frac{(80)(13.8564 \times 10^{-3})}{(804.25 \times 10^{-6})(2.1436 \times 10^{-3})(16 \times 10^{-3})}$$

$= -43.3 \times 10^6$ Pa $= -43.3$ MPa

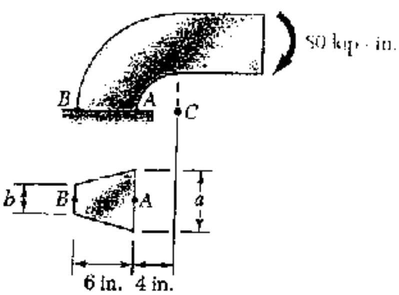
(b) Point B: $\quad y_B = R - r_2 = 29.8564 - 48 = -18.1436$ mm

$$\sigma_B = -\frac{P}{A} - \frac{My_B}{Aer_2} = -\frac{2.5 \times 10^6}{804.25 \times 10^{-6}} - \frac{(80)(-18.1436 \times 10^{-3})}{(804.25 \times 10^{-6})(2.1436 \times 10^{-3})(48 \times 10^{-3})}$$

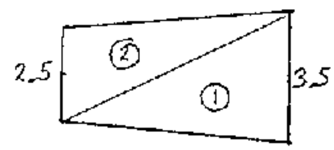
$= 14.43 \times 10^6$ Pa $= 14.43$ MPa

Problem 4.173

4.173 Knowing that the machine component shown has a trapezoidal cross section with $a = 3.5$ in. and $b = 2.5$ in., determine the stress at (*a*) point *A*, (*b*) point *B*.



Locate centroid



	A, in^2	$\bar{r}$, in	$A\bar{r}$, in^3
1	10.5	6	63
2	7.5	8	60
Σ	18		123

$$\bar{r} = \frac{123}{18} = 6.8333 \text{ in.}$$

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(6)^2(3.5 + 2.5)}{[(3.5)(10) - (2.5)(4)]\ln\frac{10}{4} - (6)(3.5 - 2.5)} = 6.3878 \text{ in}$$

$$e = \bar{r} - R = 0.4452 \text{ in} \qquad M = 80 \text{ kip·in.}$$

(a) $y_A = R - r_1 = 6.3878 - 4 = 2.3878$ in.

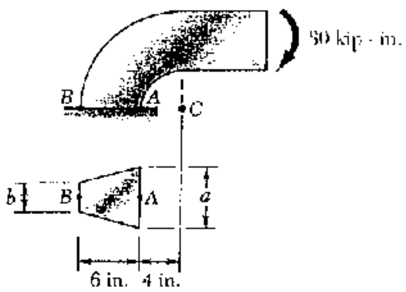
$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(80)(2.3878)}{(18)(0.4452)(4)} = -5.96 \text{ ksi}$$

(b) $y_B = R - r_2 = 6.3878 - 10 = -3.6122$ in

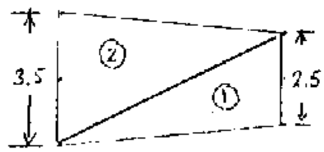
$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(80)(-3.6122)}{(18)(0.4452)(10)} = 3.61 \text{ ksi}$$

Problem 4.174

4.174 Knowing that the machine component shown has a trapezoidal cross section with $a = 2.5$ in. and $b = 3.5$ in., determine the stress at (*a*) point *A*, (*b*) point *B*.



Locate centroid



	A, in^2	$\bar{r}$, in.	$A\bar{r}$, in^3
①	7.5	6	45
②	10.5	8	84
Σ	18		129

$$\bar{r} = \frac{129}{18} = 7.1667 \text{ in.}$$

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(6)^2(2.5 + 3.5)}{[(2.5)(10) - (3.5)(4)]\ln\frac{10}{4} - (6)(2.5 - 3.5)} = 6.7168 \text{ in}$$

$e = \bar{r} - R = 0.4499$ in $\qquad\qquad M = 80$ kip·in.

(a) $y_A = R - r_1 = 2.7168$ in.

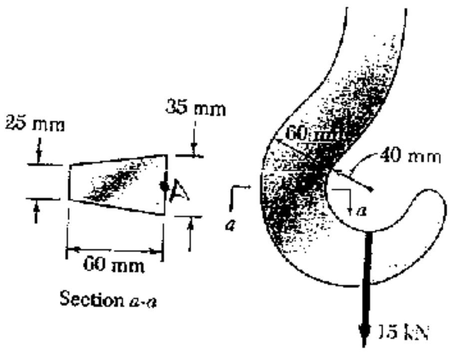
$$\sigma_A = -\frac{M y_A}{A e r_1} = -\frac{(80)(2.7168)}{(18)(0.4499)(4)} = -6.71 \text{ ksi}$$

(b) $y_B = R - r_2 = -3.2832$ in.

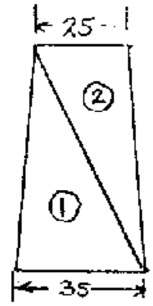
$$\sigma_B = -\frac{M y_B}{A e r_2} = -\frac{(80)(-3.2832)}{(18)(0.4499)(10)} = 3.24 \text{ ksi}$$

Problem 4.175

4.175 For the crane hook shown, determine the largest tensile stress in section a-a.



Locate centroid



	A, mm^2	$\bar{r}$, mm	$A\bar{r}$, mm^3
①	1050	60	63×10^3
②	750	80	60×10^3
Σ	1800		103×10^3

$$\bar{r} = \frac{103 \times 10^3}{1800} = 68.333 \text{ mm}$$

Foce-couple system at centroid: $P = 15 \times 10^3$ N

$$M = -P\bar{r} = -(15 \times 10^3)(68.333 \times 10^{-3}) = -1.025 \times 10^3 \text{ N}\cdot\text{m}$$

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(60)^2(35 + 25)}{[(35)(100) - (25)(40)]\ln\frac{100}{40} - (60)(35+25)} = 63.878 \text{ mm.}$$

$$e = \bar{r} - R = 4.452 \text{ mm.}$$

Maximum tensile stress occurs at point A

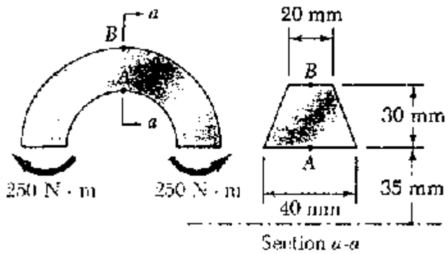
$$y_A = R - r_1 = 23.878 \text{ mm.}$$

$$\sigma_A = \frac{P}{A} - \frac{My_A}{Aer_1} = \frac{15 \times 10^3}{1800 \times 10^{-6}} - \frac{-(1.025 \times 10^3)(23.878 \times 10^{-3})}{(1800 \times 10^{-6})(4.452 \times 10^{-3})(40 \times 10^{-3})}$$

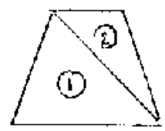
$$= 84.7 \times 10^6 \text{ Pa} = 84.7 \text{ MPa}$$

Problem 4.176

4.176 For the curved beam and loading shown, determine the stress at (*a*) point *A*, (*b*) point *B*.



Locate centroid.



	A, mm²	$\bar{r}$, mm	$A\bar{r}$, mm³
①	600	45	27×10^3
②	300	55	16.5×10^3
Σ	900		43.5×10^3

$$\bar{r} = \frac{43.5 \times 10^3}{900} = 48.333 \text{ mm.}$$

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_2 - b_1)}$$

$$= \frac{(0.5)(30)^2(40+20)}{[(40)(65) - (20)(35)]\ln\frac{65}{35} - (30)(40-20)} = 46.8608 \text{ mm}$$

$$e = \bar{r} - R = 1.4725 \text{ mm} \qquad M = -250 \text{ N·m}$$

(a) $y_A = R - r_1 = 11.8608$ mm.

$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(-250)(11.8608 \times 10^{-3})}{(900 \times 10^{-6})(1.4725 \times 10^{-3})(35 \times 10^{-3})} = 63.9 \times 10^6 \text{ Pa}$$

$$= 63.9 \text{ MPa}$$

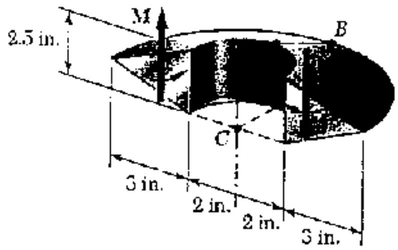
(b) $y_B = R - r_2 = -18.1392$ mm.

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(-250)(-18.1392 \times 10^{-3})}{(900 \times 10^{-6})(1.4725 \times 10^{-3})(65 \times 10^{-3})} = -52.6 \times 10^6 \text{ Pa}$$

$$= -52.6 \text{ MPa}$$

Problem 4.177

***4.177* and 4.178** Knowing that $M = 5$ kip · in., determine the stress at (*a*) point *A*, (*b*) point *B*.



$A = \frac{1}{2}bh = \frac{1}{2}(2.5)(3) = 3.75 \text{ in}^2$

$\bar{r} = 2 + 1 = 3.00000 \text{ in}$

$b_1 = 2.5 \text{ in.}, \; r_1 = 2 \text{ in}, \; b_2 = 0, \; r_2 = 5 \text{ in.}$

Use formula for trapezoid with $b_2 = 0$.

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(3)^2(2.5+0)}{\left[(2.5)(5) - (0)(2)\right]\ln\frac{5}{2} - (3)(2.5-0)} = 2.84548 \text{ in.}$$

$e = \bar{r} - R = 0.15452 \text{ in.} \qquad M = 5 \text{ kip} \cdot \text{in.}$

(a) $y_A = R - r_1 = 0.84548 \text{ in.}$

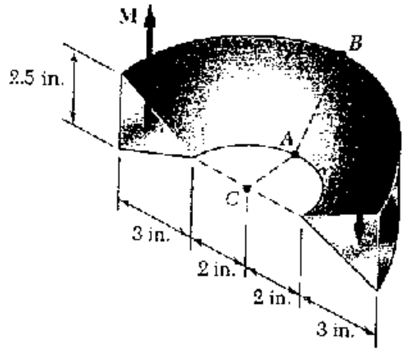
$$\sigma_A = -\frac{M y_A}{A e r_1} = -\frac{(5)(0.84548)}{(3.75)(0.15452)(2)} = -3.65 \text{ ksi}$$

(b) $y_B = R - r_2 = -2.15452 \text{ in.}$

$$\sigma_B = -\frac{M y_B}{A e r_2} = -\frac{(5)(-2.15452)}{(3.75)(0.15452)(5)} = 3.72 \text{ ksi}$$

Problem 4.178

4.177 **and 4.178** Knowing that $M = 5$ kip · in., determine the stress at (*a*) point *A*, (*b*) point *B*.



$A = \frac{1}{2}(2.5)(3) = 3.75 \text{ in}^2$

$\bar{r} = 2 + 2 = 4.00000 \text{ in}$

$b_1 = 0, \quad r_1 = 2 \text{ in.}, \quad b_2 = 2.5 \text{ in.} \quad r_2 = 5 \text{ in.}$

Use formula for trapezoid with $b_1 = 0$.

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(3)^2(0 + 2.5)}{[(0)(5) - (2.5)(2)]\ln\frac{5}{2} - (3)(0 - 2.5)} = 3.85466 \text{ in}$$

$e = \bar{r} - R = 0.14534 \text{ in.} \qquad M = 5 \text{ kip} \cdot \text{in.}$

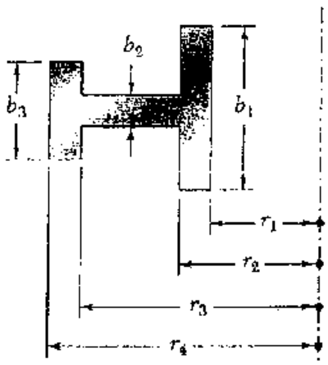
(a) $y_A = R - r_1 = 1.85466 \text{ in}$

$$\sigma_A = -\frac{M y_A}{A e r_1} = -\frac{(5)(1.85466)}{(3.75)(0.14534)(2)} = -8.51 \text{ ksi}$$

(b) $y_B = R - r_2 = -1.14534 \text{ in.}$

$$\sigma_B = -\frac{M y_B}{A e r_2} = -\frac{(5)(-1.14534)}{(3.75)(0.14534)(5)} = 2.10 \text{ ksi}$$

Problem 4.179



4.179 Show that if the cross section of a curved beam consists of two or more rectangles, the radius R of the neutral surface can be expressed as

$$R = \frac{A}{\ln\left[\left(\frac{r_2}{r_1}\right)^{b_1}\left(\frac{r_3}{r_2}\right)^{b_2}\left(\frac{r_4}{r_3}\right)^{b_3}\right]}$$

where A is the total area of the cross section.

$$R = \frac{\Sigma A}{\Sigma \int \frac{1}{r}\, dA} = \frac{A}{\Sigma\, b_i \ln \frac{r_{i+1}}{r_i}}$$

$$= \frac{A}{\Sigma \ln\left(\frac{r_{i+1}}{r_i}\right)^{b_i}} = \frac{A}{\ln\left[\left(\frac{r_2}{r_1}\right)^{b_1}\left(\frac{r_3}{r_2}\right)^{b_2}\left(\frac{r_4}{r_3}\right)^{b_3}\right]}$$

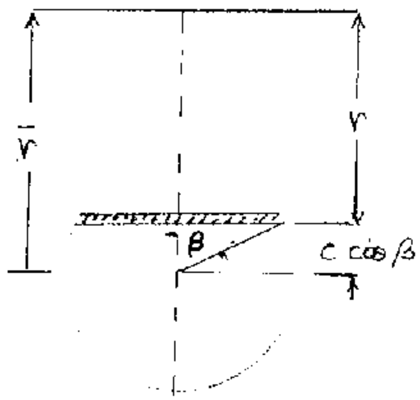
Note that for each rectangle $\int \frac{1}{r}\, dA = \int_{r_i}^{r_{i+1}} b_i \frac{dr}{r}$

$$= b_i \int_{r_i}^{r_{i+1}} \frac{dr}{r} = b_i \ln \frac{r_{i+1}}{r_i}$$

Problem 4.180

4.180 through 4.182 Using Eq. (4.66), derive the expression for R given in Fig. 4.79 for

4.180 A circular cross section



Use polar coordinate β as shown

width $\quad w = 2c\sin\beta$

$$r = \bar{r} - c\cos\beta$$

$$dr = c\sin\beta\, d\beta$$

$$dA = w\, dr = 2c^2\sin^2\beta\, d\beta$$

$$\int \frac{dA}{r} = \int_0^\pi \frac{2c^2\sin^2\beta}{\bar{r} - c\cos\beta}\, d\beta$$

$$\int \frac{dA}{r} = \int_0^\pi \frac{c^2(1-\cos^2\beta)}{\bar{r} - c\cos\beta}\, d\beta = 2\int_0^\pi \frac{\bar{r}^2 - c^2\cos^2\beta - (\bar{r}^2 - c^2)}{\bar{r} - c\cos\beta}\, d\beta$$

$$= 2\int_0^\pi (\bar{r} + c\cos\beta)\, d\beta - 2(\bar{r}^2 - c^2)\int_0^\pi \frac{dr}{\bar{r} - c\cos\beta}$$

$$= 2\bar{r}\,\beta\Big|_0^\pi + 2c\sin\beta\Big|_0^\pi$$

$$- 2(\bar{r}^2 - c^2)\frac{2}{\sqrt{\bar{r}^2 - c^2}}\tan^{-1}\frac{\sqrt{\bar{r}^2 - c^2}\tan\frac{1}{2}\beta}{\bar{r} + c}\Bigg|_0^\pi$$

$$= 2\bar{r}(\pi - 0) + 2c(0 - 0) - 4\sqrt{\bar{r}^2 - c^2}\cdot\left(\frac{\pi}{2} - 0\right)$$

$$2\pi\bar{r} - 2\pi\sqrt{\bar{r}^2 - c^2}$$

$$A = \pi c^2$$

$$R = \frac{A}{\int \frac{dA}{r}} = \frac{\pi c^2}{2\pi\bar{r} - 2\pi\sqrt{\bar{r}^2 - c^2}}$$

$$= \frac{1}{2}\,\frac{c^2}{\bar{r} - \sqrt{\bar{r}^2 - c^2}}\cdot\frac{\bar{r} + \sqrt{\bar{r}^2 - c^2}}{\bar{r} + \sqrt{\bar{r}^2 - c^2}}$$

$$= \frac{1}{2}\,\frac{c^2(\bar{r} + \sqrt{\bar{r}^2 - c^2})}{\bar{r}^2 - (\bar{r}^2 - c^2)} = \frac{1}{2}\,\frac{c^2(\bar{r} + \sqrt{\bar{r}^2 - c^2})}{c^2}$$

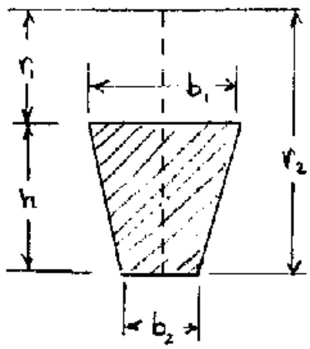
$$= \frac{1}{2}\left(\bar{r} + \sqrt{\bar{r}^2 - c^2}\right)$$

Problem 4.181

4.180 through 4.182 Using Eq. (4.66), derive the expression for R given in Fig. 4.79 for

4.181 A trapezoidal cross section

The section width w varies linearly with r



$$w = C_o + C_1 r$$

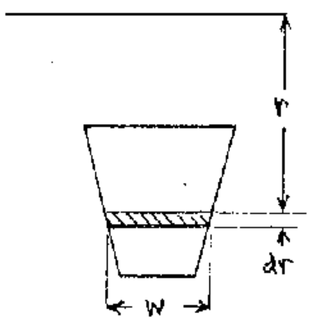
$w = b_1$ at $r = r_1$ and $w = b_2$ at $r = r_2$

$$b_1 = C_o + C_1 r_1$$

$$b_2 = C_o + C_1 r_2$$

$$b_1 - b_2 = C_1(r_1 - r_2) = -C_1 h$$

$$C_1 = -\frac{b_1 - b_2}{h}$$



$$r_2 b_1 - r_1 b_2 = (r_2 - r_1) C_o = h C_o$$

$$C_o = \frac{r_2 b_1 - r_1 b_2}{h}$$

$$\int \frac{dA}{r} = \int_{r_1}^{r_2} \frac{w}{r}\, dr = \int_{r_1}^{r_2} \frac{C_o + C_1 r}{r}\, dr$$

$$= C_o \ln r \Big|_{r_1}^{r_2} + C_1 r \Big|_{r_1}^{r_2}$$

$$= C_o \ln \frac{r_2}{r_1} + C_1 (r_2 - r_1)$$

$$= \frac{r_2 b_1 - r_1 b_2}{h} \ln \frac{r_2}{r_1} - \frac{b_1 - b_2}{h} h$$

$$= \frac{r_2 b_1 - r_1 b_2}{h} \ln \frac{r_2}{r_1} - (b_1 - b_2)$$

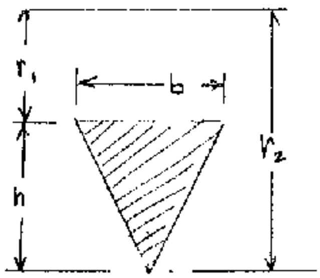
$$A = \frac{1}{2}(b_1 + b_2) h$$

$$R = \frac{A}{\int \frac{dA}{r}} = \frac{\frac{1}{2} h^2 (b_1 + b_2)}{(r_2 b_1 - r_1 b_2) \ln \frac{r_2}{r_1} - h(b_1 - b_2)}$$

Problem 4.182

4.180 through 4.182 Using Eq. (4.66), derive the expression for R given in Fig. 4.79 for

4.182 A triangular cross section



The section width w varies linearly with r

$$w = C_0 + C_1 r$$

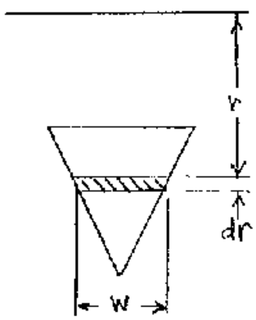
$w = b$ at $r = r_1$ and $w = 0$ at $r = r_2$

$$b = C_0 + C_1 r_1$$

$$0 = C_0 + C_1 r_2$$

$$b = C_1(r_1 - r_2) = -C_1 h$$

$$C_1 = -\frac{b}{h} \quad \text{and} \quad C_0 = -C_1 r_2 = \frac{b r_2}{h}$$



$$\int \frac{dA}{r} = \int_{r_1}^{r_2} \frac{w}{r}\,dr = \int_{r_1}^{r_2} \frac{C_0 + C_1 r}{r}\,dr$$

$$= C_0 \ln r \Big|_{r_1}^{r_2} + C_1 r \Big|_{r_1}^{r_2}$$

$$= C_0 \ln \frac{r_2}{r_1} + C_1(r_2 - r_1)$$

$$= \frac{b r_2}{h} \ln \frac{r_2}{r_1} - \frac{b}{h} h$$

$$= \frac{b r_2}{h} \ln \frac{r_2}{r_1} - b = b\left(\frac{r_2}{h} \ln \frac{r_2}{r_1} - 1\right)$$

$$A = \frac{1}{2} b h$$

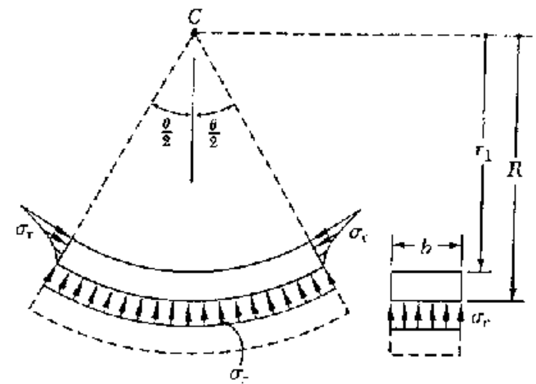
$$R = \frac{A}{\int \frac{dA}{r}} = \frac{\frac{1}{2} b h}{b\left(\frac{r_2}{h} \ln \frac{r_2}{r_1} - 1\right)} = \frac{\frac{1}{2} h}{\frac{r_2}{h} \ln \frac{r_2}{r_1} - 1}$$

Problem 4.183

4.183 For a curved bar of rectangular cross section subjected to a bending couple **M**, show that the radius stress at the neutral surface is

$$\sigma_r = \frac{M}{Ae}\left(1 - \frac{r_1}{R} - \ln\frac{R}{r_1}\right)$$

and compare the value of σ_r for the curved bar of Examples 4.10 and 4.11. (*Hint*: Consider the free-body diagram of the portion of the beam located above the neutral surface.)



At radial distance r

$$\sigma_\theta = \frac{M(r-R)}{Aer} = \frac{M}{Ae} - \frac{MR}{Aer}$$

For portion above the neutral axis, the resultant force is

$$H = \int \sigma_\theta \, dA = \int_{r_1}^{R} \sigma_\theta \, b \, dr$$

$$= \frac{Mb}{Ae}\int_{r_1}^{R} dr - \frac{MRb}{Ae}\int_{r_1}^{R}\frac{dr}{r}$$

$$= \frac{Mb}{Ae}(R - r_1) - \frac{MRb}{Ae}\ln\frac{R}{r_1} = \frac{MbR}{Ae}\left(1 - \frac{r_1}{R} - \ln\frac{R}{r_1}\right)$$

Resultant of σ_r

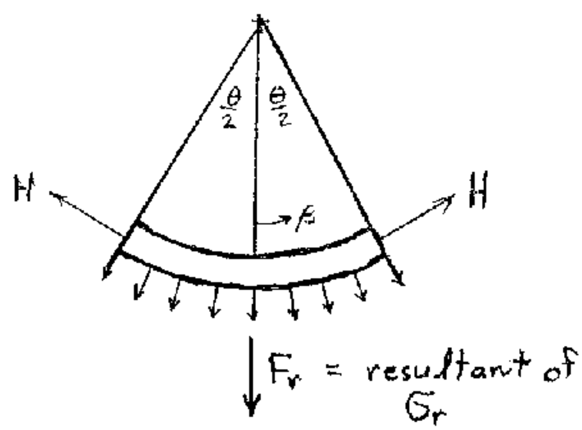
$$F_r = \int \sigma_r \cos\beta \, dA$$

$$= \int_{-\frac{\theta}{2}}^{\frac{\theta}{2}} \sigma_r \cos\beta \, b \, R\, d\beta$$

$$= \sigma_r bR \int_{-\frac{\theta}{2}}^{\frac{\theta}{2}} \cos\beta \, d\beta$$

$$= \sigma_r bR \sin\beta \Big|_{-\frac{\theta}{2}}^{\frac{\theta}{2}}$$

$$= 2\sigma_r bR \sin\frac{\theta}{2}$$



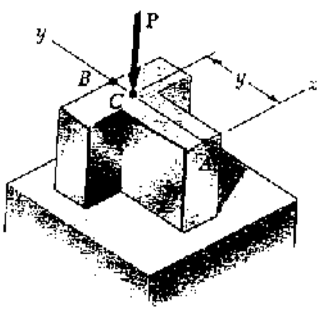
For equilibrium $\quad F_r - 2H\sin\frac{\theta}{2} = 0$

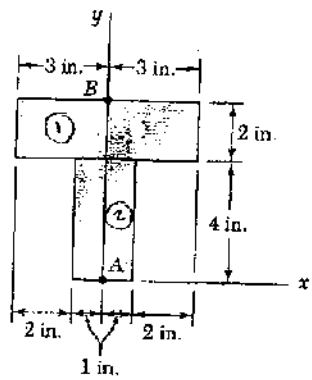
$$2\sigma_r bR\sin\frac{\theta}{2} - 2\frac{MbR}{Ae}\left(1 - \frac{r_1}{R} - \ln\frac{R}{r_1}\right) = 0$$

$$\sigma_r = \frac{M}{Ae}\left(1 - \frac{r_1}{R} - \ln\frac{R}{r_1}\right)$$

Problem 4.184

4.184 A vertical force **P** of magnitude 20 kips is applied at a point C located on the axis of symmetry of the cross section of a short column. Knowing that $y = 5$ in., determine (*a*) the stress at point *A*, (*b*) the stress at point *B*, (*c*) the location of the neutral axis.





(*b*)

Locate centroid

Part	A, in^2	$\bar{y}$, in	$A\bar{y}$, in^3
1	12	5	60
2	8	2	16
Σ	20		76

$$\bar{Y} = \frac{\Sigma A\bar{y}_i}{\Sigma A_i} = \frac{76}{20} = 3.8 \text{ in}$$

Eccentricity of load $e = 5 - 3.8 = 1.2$ in.

$$I_1 = \frac{1}{12}(6)(2)^3 + (12)(1.2)^2 = 21.28 \text{ in}^4$$

$$I_2 = \frac{1}{12}(2)(4)^3 + (8)(1.8)^2 = 36.587 \text{ in}^4$$

$$I = I_1 + I_2 = 57.867 \text{ in}^4$$

(a) Stress at A $\quad c_A = 3.8$ in

$$\sigma_A = -\frac{P}{A} + \frac{Pec_A}{I} = -\frac{20}{20} + \frac{(20)(1.2)(3.8)}{57.867} = 0.576 \text{ ksi}$$

(b) Stress at B $\quad c_B = 6 - 3.8 = 2.2$ in

$$\sigma_B = -\frac{P}{A} - \frac{Pec_B}{I} = -\frac{20}{20} - \frac{(20)(1.2)(2.2)}{57.867} = -1.912 \text{ ksi}$$

(c) Location of neutral axis: $\quad \sigma = 0$

$$\sigma = -\frac{P}{A} + \frac{Pea}{I} = 0 \quad \therefore \quad \frac{ea}{I} = \frac{1}{A}$$

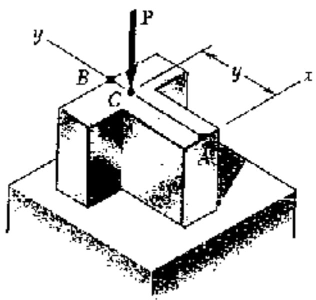
$$a = \frac{I}{Ae} = \frac{57.867}{(20)(1.2)} = 2.411 \text{ in}$$

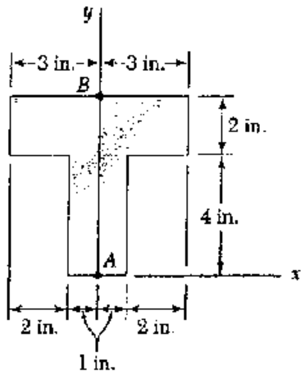
Neutral axis lies 2.411 in. below centroid or $3.8 - 2.411 = 1.389$ in above point A.

Answer 1.389 in from point A

Problem 4.185

4.185 A vertical force **P** of magnitude 20 kips is applied at a point C located on the axis of symmetry of the cross section of a short column. Determine the range of values of y for which tensile stresses do not occur in the column.





Locate centroid

	A, in^2	$\bar{y}, in$	$A\bar{y}, in^3$
①	12	5	60
②	8	2	16
Σ	20		76

$$\bar{y} = \frac{\Sigma A_i \bar{y}_i}{\Sigma A_i} = \frac{76}{20} = 3.8 \text{ in}$$

Eccentricity of load $e = y - 3.8$ in.

$y = e + 3.8$ in.

$$I_1 = \frac{1}{12}(6)(2)^3 + (12)(1.2)^2 = 21.28 \text{ in}^4$$

$$I_2 = \frac{1}{12}(2)(4)^3 + (8)(1.8)^2 = 36.587 \text{ in}^4$$

$$I = I_1 + I_2 = 57.867 \text{ in}^4$$

If stress at A equals zero. $c_A = 3.8$ in

$$\sigma_A = -\frac{P}{A} + \frac{Pec_A}{I} = 0 \quad \therefore \quad \frac{ec_A}{I} = \frac{1}{A}$$

$$e = \frac{I}{Ac_A} = \frac{57.867}{(20)(3.8)} = 0.761 \text{ in} \qquad y = 0.761 + 3.8 = 4.561 \text{ in.}$$

If stress at B equals zero. $c_B = 6 - 3.8 = 2.2$ in

$$\sigma_B = -\frac{P}{A} - \frac{Pec_B}{I} = 0 \quad \therefore \quad \frac{ec_B}{I} = -\frac{1}{A}$$

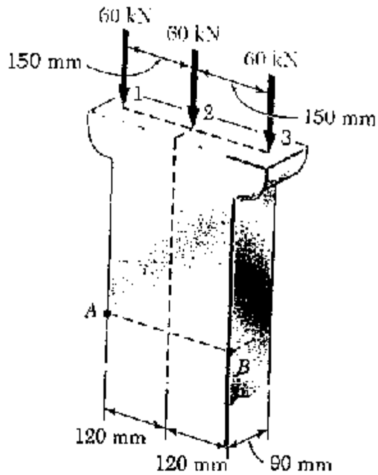
$$e = -\frac{I}{Ac_B} = -\frac{57.867}{(20)(2.2)} = -1.315 \text{ in}$$

$$y = -1.315 + 3.8 = 2.485 \text{ in.}$$

Answer: $2.485 \text{ in} < y < 4.561 \text{ in.}$

Problem 4.186

4.186 Determine the stress at points A and B, (a) for the loading shown, (b) if the 60-kN loads are applied at points 1 and 2 only.



(a) Loading is centric.

$P = 180\ kN = 180 \times 10^3\ N$

$A = (90)(240) = 21.6 \times 10^3\ mm^2 = 21.6 \times 10^{-6}\ m^2$

At A and B $\sigma = -\frac{P}{A} = -\frac{180 \times 10^3}{21.6 \times 10^{-3}} = -8.33 \times 10^6\ Pa$

$= -8.33\ MPa$

(b) Eccentric loading

$P = 120\ kN = 120 \times 10^3$

$M = (60 \times 10^3)(150 \times 10^{-3}) = 9.0 \times 10^3\ N \cdot m$

$I = \frac{1}{12} b h^3 = \frac{1}{12}(90)(240)^3 = 103.68 \times 10^6\ mm^4 = 103.68 \times 10^{-6}\ m^4$

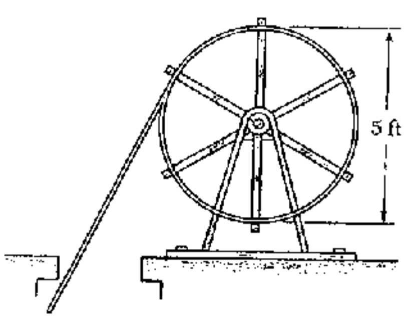
$c = 120\ mm = 0.120\ m$

At A $\sigma_A = -\frac{P}{A} - \frac{Mc}{I} = -\frac{120 \times 10^3}{21.6 \times 10^{-3}} - \frac{(9.0 \times 10^3)(0.120)}{103.68 \times 10^{-6}} = -15.97 \times 10^6\ Pa = -15.97\ MPa$

At B $\sigma_B = -\frac{P}{A} + \frac{Mc}{I} = -\frac{120 \times 10^3}{21.6 \times 10^{-3}} + \frac{(9.0 \times 10^3)(0.120)}{103.68 \times 10^{-6}} = 4.86 \times 10^6\ Pa = 4.86\ MPa$

Problem 4.187

4.187 Straight rods of 0.30-in. diameter and 200-ft length are sometimes used to clear underground conduits of obstructions or to thread wires through a new conduit. The rods are made of high-strength steel and, for storage and transportation, are wrapped on spools of 5-ft diameter. Assuming that the yield strength is not exceeded, determine (a) the maximum stress in a rod, when the rod, which was initially straight, is wrapped on a spool, (b) the corresponding bending moment in the for. Use $E = 29 \times 10^6$ psi.



$r = \frac{1}{2} d = \frac{1}{2}(0.30) = 0.15\ in$

$I = \frac{\pi}{4} r^4 = \frac{\pi}{4}(0.15)^4 = 397.61 \times 10^{-6}\ in^4$

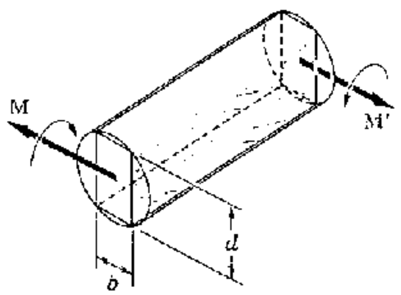
$D = 5\ ft = 60\ in$ $\qquad \rho = \frac{1}{2} D = 30\ in$

$c = r = 0.15\ in.$

(a) $\sigma_{max} = \frac{Ec}{\rho} = \frac{(29 \times 10^6)(0.15)}{30} = 145 \times 10^3\ psi = 145\ ksi$

(b) $M = \frac{EI}{\rho} = \frac{(29 \times 10^6)(397.61 \times 10^{-6})}{30} = 384\ lb \cdot in.$

Problem 4.188

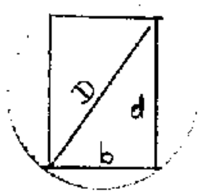


4.188 A couple **M** will be applied to a beam of rectangular cross section that is to be sawed from a log of circular cross section. Determine the ratio d/b for which (*a*) the maximum stress σ_m will be as small as possible, (*b*) the radius of curvature of the beam will be maximum.

Let D be the diameter of the log.

$D^2 = b^2 + d^2 \qquad d^2 = D^2 - b^2$

$I = \frac{1}{12} b d^3 \qquad c = \frac{1}{2} d \qquad \frac{I}{c} = \frac{1}{6} b d^2$



(a) σ_m is minimum when $\frac{I}{c}$ is maximum

$\frac{I}{c} = \frac{1}{6} b (D^2 - b^2) = \frac{1}{6} D^2 b - \frac{1}{6} b^3$

$\frac{d}{db}\left(\frac{I}{c}\right) = \frac{1}{6} D^2 - \frac{3}{6} b^2 = 0 \qquad b = \frac{1}{\sqrt{3}} D$

$d = \sqrt{D^2 - \frac{1}{3} D^2} = \sqrt{\frac{2}{3}} D \qquad \frac{d}{b} = \sqrt{2}$

$\rho = \frac{EI}{M} \qquad \rho$ is maximum when I is maximum.

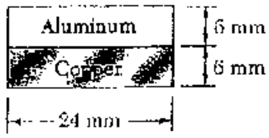
$\frac{1}{12} b d^3$ is maximum or $b^2 d^6$ is maximum

$(D^2 - d^2) d^6$ is maximum.

$6 D^2 d^5 - 8 d^7 = 0 \qquad d = \frac{\sqrt{3}}{2} D$

$b = \sqrt{D^2 - \frac{3}{4} D^2} = \frac{1}{2} D \qquad \frac{d}{b} = \sqrt{3}$

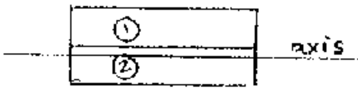
Problem 4.189



4.189 A copper strip (E = 105 GPa) and an aluminum strip (E = 75 GPa) are bonded together to form the composite bar shown. Knowing that the bar to bent about a horizontal axis by a couple of moment 35 N · m, determine the maximum stress in (*a*) the aluminum strip, (*b*) in the copper strip.

Use aluminum as the reference material

$n = 1.0$ in aluminum



$n = E_c / E_a = 105/75 = 1.4$ in copper

Transformed section

	A, mm^2	nA, mm^2	$\bar{y}_o$, mm	$nA\bar{y}_o$, mm^3
①	144	144	9	1296
②	144	201.6	3	604.8
Σ		345.6		1900.8

$$\bar{Y}_o = \frac{1900.8}{345.6} = 5.50 \text{ mm}$$

The neutral axis lies 5.50 mm above the bottom

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1.0}{12}(24)(6)^3 + (1.0)(24)(6)(3.5)^2 = 2196 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1.4}{12}(24)(6)^3 + (1.4)(24)(6)(2.5)^2 = 1864.8 \text{ mm}^4$$

$$I = I_1 + I_2 = 4060.8 \text{ mm}^4 = 4.0608 \times 10^{-9} \text{ m}^4$$

(a) Aluminum $\quad n = 1.0 \quad y = 12 - 5.5 = 6.5 \text{ mm} = 0.0065 \text{ m}$

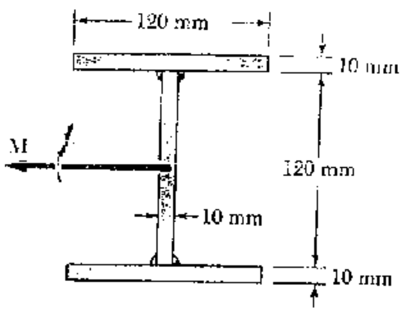
$$\sigma = -\frac{nMy}{I} = -\frac{(1.0)(35)(0.0065)}{4.0608 \times 10^{-9}} = -56.0 \times 10^6 \text{ Pa} = -56.0 \text{ MPa} \blacktriangleleft$$

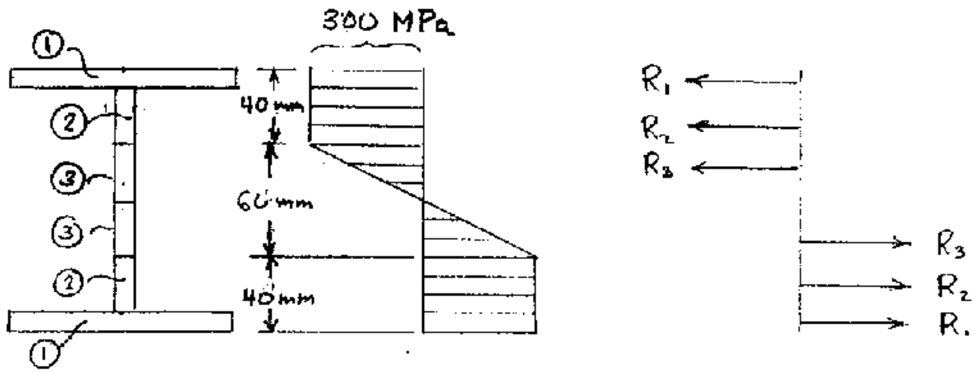
(b) Copper $\quad n = (1.4) \quad y = -5.5 \text{ mm} = -0.0055 \text{ m}$

$$\sigma = -\frac{nMy}{I} = -\frac{(1.4)(35)(-0.0055)}{4.0608 \times 10^{-9}} = 66.4 \times 10^6 \text{ Pa} = 66.4 \text{ MPa} \blacktriangleleft$$

Problem 4.190

4.190 Three 120 × 10-mm steel plates have been welded together to form the beam shown. Assuming that the steel is elastoplastic with $E = 200$ GPa and $\sigma_Y = 300$ MPa, determine (*a*) the bending moment for which the plastic zones at the top and bottom of the beam are 40 mm thick, (*b*) the corresponding radius of curvature of the beam.





$A_1 = (120)(10) = 1200 \text{ mm}^2 \qquad R_1 = \sigma_Y A_1 = (300\times10^6)(1200\times10^{-6}) = 360\times10^3$ N

$A_2 = (30)(10) = 300 \text{ mm}^2 \qquad R_2 = \sigma_Y A_2 = (300\times10^6)(300\times10^{-6}) = 90\times10^3$ N

$A_3 = (30)(10) = 300 \text{ mm}^2 \qquad R_3 = \frac{1}{2}\sigma_Y A_2 = \frac{1}{2}(300\times10^6)(300\times10^{-6}) = 45\times10^3$ N

$y_1 = 65 \text{ mm} = 65\times10^{-3}$ m

$y_2 = 45 \text{ mm} = 45\times10^{-3}$ m

$y_3 = 20 \text{ mm} = 20\times10^{-3}$ m

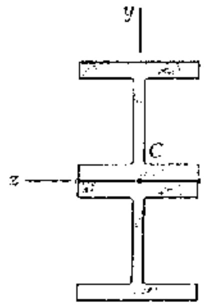
(a) $M = 2(R_1y_1 + R_2y_2 + R_3y_3) = 2\{(360)(65) + (90)(45) + (45)(20)\}$

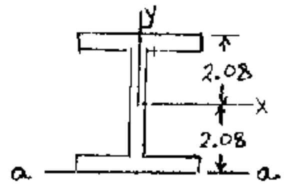
$= 56.7\times10^3 \text{ N}\cdot\text{m} = 56.7 \text{ kN}\cdot\text{m}$

(b) $\dfrac{y_Y}{\rho} = \dfrac{\sigma_Y}{E} \qquad \rho = \dfrac{E y_Y}{\sigma_Y} = \dfrac{(200\times10^9)(30\times10^{-3})}{300\times10^6} = 20$ m

Problem 4.191

4.191 and 4.192 Two W4 × 13 rolled sections are welded together as shown. Knowing that for the steel alloy used $\sigma_Y = 36$ ksi and $\sigma_U = 58$ ksi and using a factor of safety of 3.0, determine the largest couple that can be applied when the assembly is bent about the z axis.





Properties of W 4×13 rolled section
See Appendix B

Area = 3.83 in^2 Depth = 4.16 in
$I_x = 11.3$ in^4

For one rolled section, moment of inertia about axis a-a is

$$I_a = I_x + Ad^2 = 11.3 + (3.83)(2.08)^2 = 27.87 \text{ in}^4$$

For both sections $I_z = 2 I_a = 55.74$ in^4

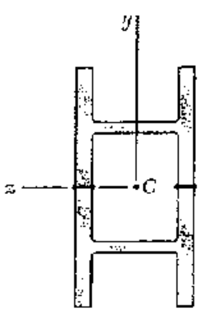
c = depth = 4.16 in

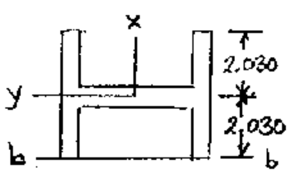
$$\sigma_{all} = \frac{\sigma_U}{F.S.} = \frac{58}{3.0} = 19.333 \text{ ksi} \qquad \sigma = \frac{Mc}{I}$$

$$M_{all} = \frac{\sigma_{all} I}{c} = \frac{(19.333)(55.74)}{4.16} = 259 \text{ kip}\cdot\text{in.}$$

Problem 4.192

4.191 and 4.192 Two W4 × 13 rolled sections are welded together as shown. Knowing that for the steel alloy used $\sigma_Y = 36$ ksi and $\sigma_U = 58$ ksi and using a factor of safety of 3.0, determine the largest couple that can be applied when the assembly is bent about the z axis.





Properties of W 4×13 rolled section
See Appendix B

Area = 3.83 in^2 Width = 4.060 in
$I_y = 3.86$ in^4

For one rolled section, moment of inertia about axis b-b is

$$I_b = I_y + Ad^2 = 3.86 + (3.83)(2.030)^2 = 19.643 \text{ in}^4$$

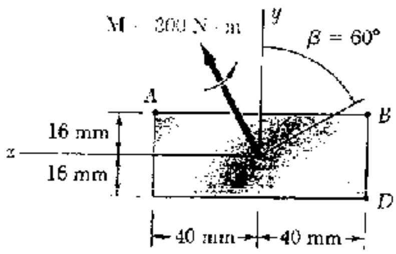
For both sections $I_z = 2 I_b = 39.286$ in^4

c = width = 4.060 in

$$\sigma_{all} = \frac{\sigma_U}{F.S.} = \frac{58}{3.0} = 19.333 \text{ ksi} \qquad \sigma = \frac{Mc}{I}$$

$$M_{all} = \frac{\sigma_{all} I}{c} = \frac{(19.333)(39.286)}{4.060} = 187.1 \text{ kip}\cdot\text{in}$$

Problem 4.193



4.193 The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.

$$I_z = \frac{1}{12}(80)(32)^3 = 218.45\times10^3\ \text{mm}^4 = 218.45\times10^{-9}\ \text{m}^4$$

$$I_y = \frac{1}{12}(32)(80)^3 = 1.36533\times10^6\ \text{mm}^4 = 1.36533\times10^{-6}\ \text{m}^4$$

$$y_A = y_B = -y_D = 16\ \text{mm}$$

$$z_A = -z_B = -z_D = 40\ \text{mm}$$

$$M_y = 300\cos 30^\circ = 259.81\ \text{N}\cdot\text{m}, \quad M_z = 300\sin 30^\circ = 150\ \text{N}\cdot\text{m}$$

(a) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(150)(16\times10^{-3})}{218.45\times10^{-9}} + \frac{(259.81)(40\times10^{-3})}{1.36533\times10^{-6}}$$

$$= -3.37\times10^6\ \text{Pa} = -3.37\ \text{MPa}$$

(b) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(150)(16\times10^{-3})}{218.45\times10^{-9}} + \frac{(259.81)(-40\times10^{-3})}{1.36533\times10^{-6}}$$

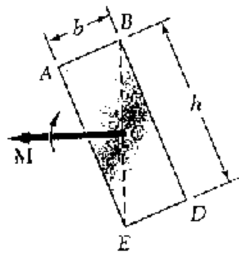
$$= -18.60\times10^6\ \text{Pa} = -18.60\ \text{MPa}$$

(c) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(150)(-16\times10^{-3})}{218.45\times10^{-9}} + \frac{(259.81)(-40\times10^{-3})}{1.36533\times10^{-6}}$$

$$= 3.37\times10^6\ \text{Pa} = 3.37\ \text{MPa}$$

Problem 4.194

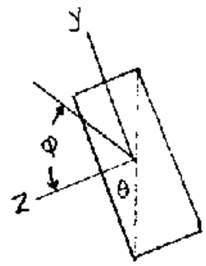
4.194 Show that, if a solid beam having a rectangular cross section is bent by a couple applied in a plane containing one of diagonal of the cross section, the neutral axis will lie along the other diagonal.



$$\tan\theta = \frac{b}{h}$$

$$M_z = M\cos\theta \quad , \qquad M_z = M\sin\theta$$

$$I_z = \frac{1}{12}bh^3 \qquad I_y = \frac{1}{12}hb^3$$

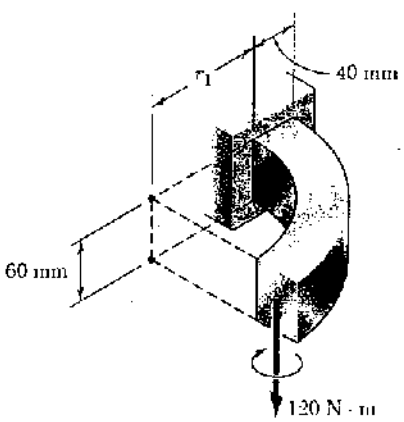


$$\tan\varphi = \frac{I_z}{I_y}\tan\theta = \frac{\frac{1}{12}bh^3}{\frac{1}{12}hb^3}\cdot\frac{b}{h} = \frac{h}{b}$$

Thus neutral axis passes through corner A ◄

Problem 4.195

4.195 The curved bar shown has a cross section of 40 × 60 mm and a inner radius $r_1 = 15$ mm. For the loading shown determine the largest tensile and compressive stresses in the bar.



$$h = 40\text{ mm}, \quad r_1 = 15\text{ mm}, \quad r_2 = 55\text{ mm}$$

$$A = (60)(40) = 2400\text{ mm}^2 = 2400\times10^{-6}\text{ m}^2$$

$$R = \frac{h}{\ln\frac{r_2}{r_1}} = \frac{40}{\ln\frac{55}{40}} = 30.786\text{ mm}$$

$$\bar{r} = \tfrac{1}{2}(r_1 + r_2) = 35\text{ mm}$$

$$e = \bar{r} - R = 4.214\text{ mm} \qquad \sigma = -\frac{My}{Aer}$$

At $r = 15$ mm $\quad y = 30.786 - 15 = 15.786$ mm

$$\sigma = -\frac{(120)(15.786\times10^{-3})}{(2400\times10^{-6})(4.214\times10^{-3})(15\times10^{-3})} = -12.49\times10^{6}\text{ Pa}$$

$$= -12.49\text{ MPa} \quad ◄$$

(compression)

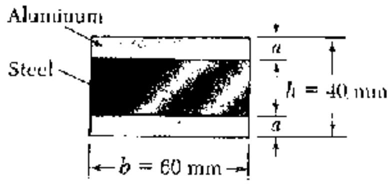
At $r = 55$ mm $\quad y = 30.786 - 55 = -24.214$ mm

$$\sigma = -\frac{(120)(-24.214\times10^{-3})}{(2400\times10^{-6})(4.214\times10^{-3})(55\times10^{-3})} = 5.22\times10^{6}\text{ Pa}$$

$$= 5.22\text{ MPa} \quad ◄$$

(tension)

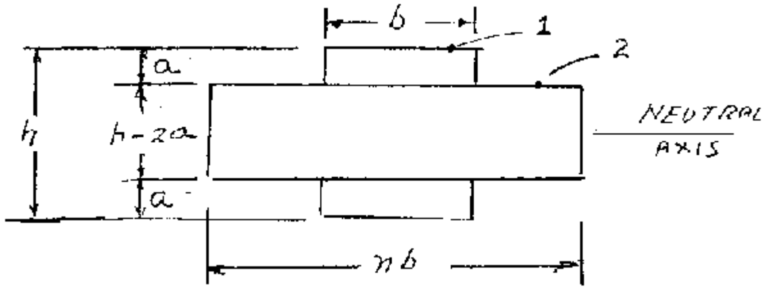
PROBLEM 4.C1

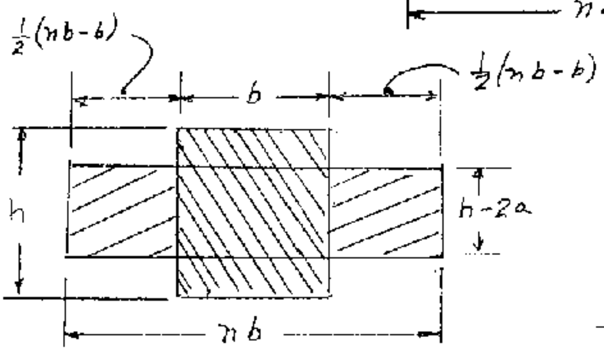


4.C1 Two aluminum strips and a steel strip are to be bonded together to form a composite member of width $b = 60$ mm and depth $h = 40$ mm. The modulus of elasticity is 200 GPa for the steel and 75 GPa for the aluminum. Knowing that $M = 1500\ \text{N}\cdot\text{m}$, write a computer program to calculate the maximum stress in the aluminum and in the steel for values of a from 0 to 20 mm using 2-mm increments. Using appropriate smaller increments, determine (*a*) the largest stress that can occur in the steel, (*b*) the corresponding value of a.

SOLUTION

TRANSFORMED SECTION (ALL STEEL) $n = \dfrac{E_{STEEL}}{E_{ALUM}}$





$$\bar{I} = \frac{1}{12} b h^3 + \frac{1}{12}\left[2\left(\frac{1}{2}\right)(nb-b)\right](h-2a)^3$$

AT POINT 1: $\sigma_{ALUM} = \dfrac{M\left(\frac{h}{2}\right)}{\bar{I}}$

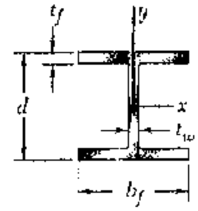
AT POINT 2: $\sigma_{STEEL} = n\,\dfrac{M\left(\frac{h}{2}-a\right)}{\bar{I}}$

FOR a = 0 TO 20 mm USING 2-mm INTERVALS COMPUTE: n, $\bar{I}$, σ_{ALUM}, σ_{STEEL}.

PROGRAM OUTPUT

```
b = 60  mm    h = 40  mm     M = 1500 N.m
 Moduli of elasticity:  Steel = 200 GPa  Aluminum = 75 GPa

    a        I          sigma        sigma
                       aluminum      steel
   mm    m^4/10^6        MPa          MPa

  0.000    0.8533      35.156       93.750
  2.000    0.7088      42.325      101.580
  4.000    0.5931      50.585      107.914
  6.000    0.5029      59.650      111.347
  8.000    0.4352      68.934      110.294
 10.000    0.3867      77.586      103.448
 12.000    0.3541      84.714       90.361
 14.000    0.3344      89.713       71.770
 16.000    0.3243      92.516       49.342
 18.000    0.3205      93.594       24.958
 20.000    0.3200      93.750        0.000
=====================================================
Find 'a' for max steel stress
    and the corresponding aluminum stress

  6.600    0.4804      62.447     111.572083
  6.610    0.4800      62.494     111.572159
  6.620    0.4797      62.540     111.572113

  Max Steel Stress = 111.6 MPa occurs when a = 6.61 mm
  Corresponding Aluminum stress = 62.5 MPa
```

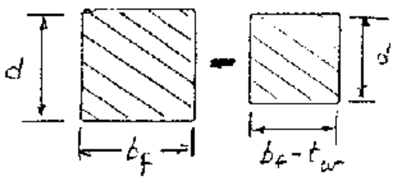
PROBLEM 4.C2



4.C2 A beam of the cross section shown, made of a steel that is assumed to be elastoplastic with a yield strength σ_Y and a modulus of elasticity E, is bent about the x axis. (a) Denoting by y_Y the half thickness of the elastic core, write a computer program to calculate the bending moment M and the radius of curvature ρ for values of y_Y from $\frac{1}{2}d$ to $\frac{1}{6}d$ using decrements equal to $\frac{1}{2}t_f$. Neglect the effect of fillets. (b) Use this program to solve Prob. 4.190

SOLUTION

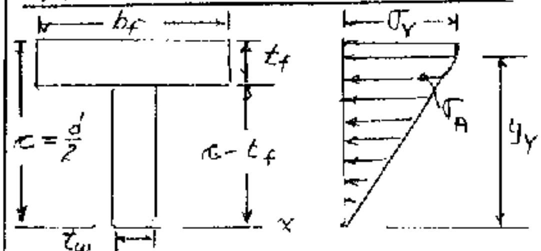
COMPUTE MOMENT OF INERTIA I_x



$$I_x = \frac{1}{12} b_f d^3 - \frac{1}{12}(b_f - t_w)(d - 2t_f)^3$$

MAXIMUM ELASTIC MOMENT: $M_Y = \sigma_Y \dfrac{I_x}{(d/2)}$

FOR YIELDING IN THE FLANGES: (CONSIDER UPPER HALF OF CROSS SECTION) $c = \frac{d}{2}$

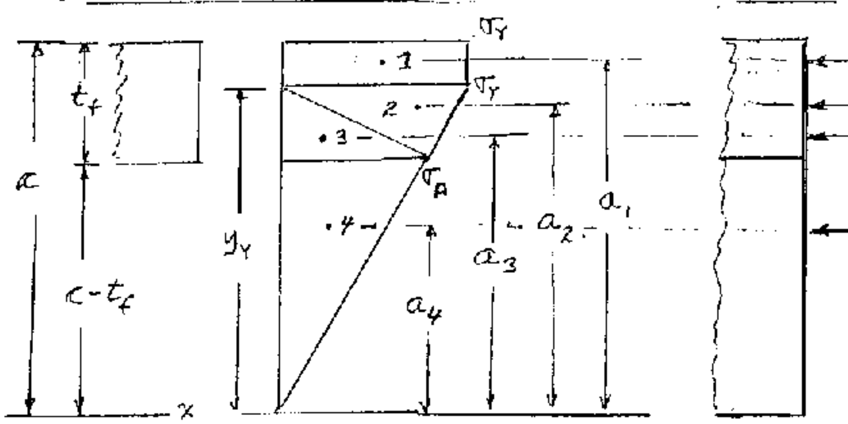


STRESS AT JUNCTION OF WEB AND FLANGE

$$\sigma_A = \frac{(d/2) - t_f}{y_Y}\sigma_Y$$

DETAIL OF STRESS DIAGRAM

RESULTANT FORCES



$R_1 = \sigma_Y[b_f(c - \sigma_Y)]$

$R_2 = \frac{1}{2}\sigma_Y\, b_f[y_Y - (c - t_f)]$

$R_3 = \frac{1}{2}\sigma_A\, b_f[y_Y - (c - t_f)]$

$R_3 = \frac{1}{2}\sigma_A\, t_w(c - t_f)$

$a_1 = \frac{1}{2}(c + y_Y)$

$a_2 = y_Y - \frac{1}{3}[y_Y - (c - t_f)]$

$a_3 = y_Y - \frac{2}{3}[y_Y - (c - t_f)]$

$a_4 = \frac{2}{3}(c - t_f)$

BENDING MOMENT

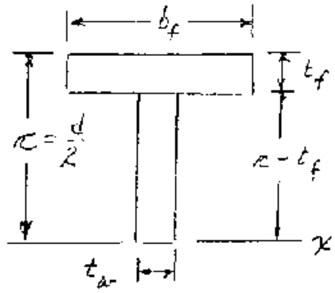
$$M = 2\sum_{n=1}^{4} R_n a_n$$

RADIUS OF CURVATURE

$$y_Y = \varepsilon_Y \rho = \frac{\sigma_Y}{E}\rho\,; \quad \rho = \frac{y_Y E}{\sigma_Y}$$

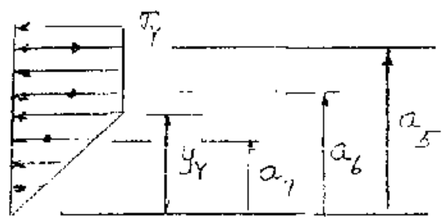
CONTINUED

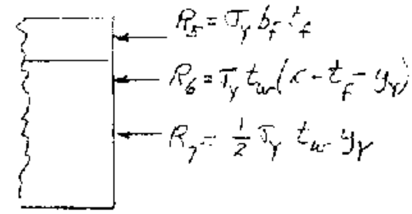
PROBLEM 4.C2 - CONTINUED



FOR YIELDING IN THE WEB (CONSIDER UPPER HALF OF CROSS SECTION)

$c = d/2$





$$a_5 = c - \tfrac{1}{2} t_f$$

$$a_6 = \tfrac{1}{2}\left[y_Y + (c - t_f)\right]$$

$$a_7 = \tfrac{2}{3}\, y_Y$$

BENDING MOMENT $\quad M = 2 \sum_{n=5}^{7} R_n a_n$

RADIUS OF CURVATURE $\quad y_Y = \epsilon_y \rho = \frac{\sigma_Y}{E}\rho \qquad \rho = \frac{y_Y E}{\sigma_Y}$

PROGRAM: KEY IN EXPRESSIONS FOR a_n and R_n FOR $n = 1$ TO 7

FOR $y_Y = c$ TO $(c - t_f)$ AT $-t_f/2$ DECREMENTS

COMPUTE $M = 2\sum R_n a_n$ FOR $n = 1$ TO 4 AND $\rho = \frac{y_Y E}{\sigma_Y}$, THEN PRINT

FOR $y_Y = (c - t_w)$ TO $c/3$ AT $-t_f/2$ DECREMENTS

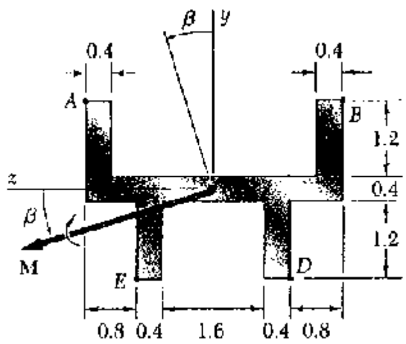
COMPUTE $M = 2\sum R_n a_n$ FOR $n = 5$ TO 7 AND $\rho = \frac{y_Y E}{\sigma_Y}$, THEN PRINT

INPUT NUMERICAL VALUES AND RUN PROGRAM

PROGRAM OUTPUT

```
 For a beam of Prob 4.190
Depth d = 140.00 mm                     Width of flange bf = 120.00 mm
Thickness of flange tf =  10.00 mm     Thickness of web tw =  10.00 mm

I = 0.000011600 m to the 4th
Yield strength of Steel sigmaY = 300 MPa
Yield Moment MY = 49.71 kip.in.

        yY(mm)          M(kN.m)     rho(m)
 For yielding still in the flange.
       70.000            49.71       46.67
       65.000            52.59       43.33
       60.000            54.00       40.00
 For yielding in the web
       60.000            54.00       40.00
       55.000            54.58       36.67
       50.000            55.10       33.33
       45.000            55.58       30.00
       40.000            56.00       26.67
       35.000            56.38       23.33
       30.000            56.70       20.00
       25.000            56.97       16.67
```

PROBLEM 4.C3



Dimensions in inches

4.C3 An 8 kip · in. couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Noting that the centroid of the cross section is located at C and that the y and z axes are principal axes, write a computer program to calculate the stress at A, B, C, and D for values of β from 0 to 180° using 10° increments. (*Given:* $I_y = 6.23\ \text{in}^4$ and $I_z = 1.481\ \text{in}^4$.)

SOLUTION INPUT COORDINATES OF A, B, C, D

$z_A = z(1) = 2 \qquad y_A = y(1) = 1.4$

$z_B = z(2) = -2 \qquad y_B = y(2) = 1.4$

$z_C = z(3) = -1 \qquad y_C = y(3) = -1.4$

$z_D = z(4) = 1 \qquad y_D = y(4) = -1.4$

COMPONENTS OF M.

$M_y = -M\sin\beta \qquad M_z = M\cos\beta$

Eq 4.55 page 273: $\sigma(n) = -\dfrac{M_z\, y(n)}{I_z} + \dfrac{M_y\, z(n)}{I_y}$

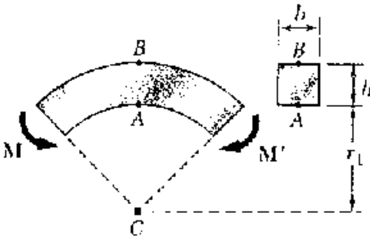
PROGRAM: FOR $\beta = 0$ TO $180°$ USING $10°$ INCREMENTS.
FOR $n = 1$ TO 4 USING UNIT INCREMENTS.
EVALUATE EQ 4.55 AND PRINT STRESSES
RETURN
RETURN

PROGRAM OUTPUT

```
Moment of couple M =  8.00 kip·in.
Moments of inertia: Iy = 6.23  in^4     Iz  = 1.481 in^4

         Coordinates of points A, B, D, and E
       Point A:  z(1) =  2:     y(1) =  1.4
       Point B:  z(2) = -2:     y(2) =  1.4
       Point D:  z(3) = -1:     y(3) = -1.4
       Point E:  z(4) =  1:     y(4) = -1.4
               - - - Stress at Points - - -
```

beta °	A ksi	B ksi	D ksi	E ksi
0	-7.565	-7.565	7.565	7.565
10	-7.896	-7.004	7.673	7.227
20	-7.987	-6.230	7.548	6.669
30	-7.836	-5.267	7.193	5.909
40	-7.446	-4.144	6.621	4.970
50	-6.830	-2.895	5.846	3.879
60	-6.007	-1.558	4.895	2.670
70	-5.001	-0.174	3.794	1.381
80	-3.843	1.216	2.578	0.049
90	-2.569	2.569	1.284	-1.284
100	-1.216	3.843	-0.049	-2.578
110	0.174	5.001	-1.381	-3.794
120	1.558	6.007	-2.670	-4.895
130	2.895	6.830	-3.879	-5.846
140	4.144	7.446	-4.970	-6.621
150	5.267	7.836	-5.909	-7.193
160	6.230	7.987	-6.669	-7.548
170	7.004	7.896	-7.227	-7.673
180	7.565	7.565	-7.565	-7.565

PROBLEM 4.C4



4.C4 Couples of moment $M = 2$ kN · m are applied as shown to a curved bar having a rectangular cross section with $h = 100$ mm and $b = 25$ mm. Write a computer program and use it to calculate the stresses at points A and B for values of the ratio r_1/h from 10 to 1 using decrements of 1, and from 1 to 0.1 using decrements of 0.1. Using appropriate smaller increments, determine the ratio r_1/h for which the maximum stress in the curved bar is 50 percent larger than the maximum stress in a straight bar of the same cross section.

SOLUTION INPUT: $h = 100$ mm, $b = 25$ mm, $M = 2$ kN·m

FOR STRAIGHT BAR: $\sigma_{STRAIGHT} = \frac{M}{S} = \frac{6M}{h^2 b} = 48$ MPa

FOLLOWING NOTATION OF SEC. 4.15, KEY IN THE FOLLOWING:

$r_2 = h + r_1$; $R = h/\ln(r_2 - r_1)$; $\bar{r} = r_1 + r_2$; $e = \bar{r} - R$; $A = bh = 2500$ (I)

STRESSES: $\sigma_A = \sigma_1 = M(r_1 - R)/(Aer_1)$ $\sigma_B = \sigma_2 = M(r_2 - R)/(Aer_2)$ (II)

SINCE $h = 100$ mm, FOR $r_1/h = 10$, $r_1 = 1000$ mm. ALSO $r_1/h = 10$, $r_1 = 100$

PROGRAM: FOR $r_1 = 1000$ TO 100 AT −100 DECREMENTS

USING EQUATIONS OF LINES I AND II EVALUATE $r_2, R, \bar{r}, e, \sigma_1$ AND σ_2

ALSO EVALUATE: ratio $= \sigma_1/\sigma_{STRAIGHT}$

RETURN AND REPEAT FOR $r_1 = 100$ TO 10 AT −10 DECREMENTS

PROGRAM OUTPUT

M = Bending Moment = 2. kN.m h = 100.000 in. A = 2500.00 mm^2
Stress in straight beam = 48.00 MPa

r1	rbar	R	e	sigma1	sigma2	r1/h	ratio
mm	mm	mm	mm	MPa	MPa	-	-
1000	1050	1049	0.794	-49.57	46.51	10.000	-1.033
900	950	949	0.878	-49.74	46.36	9.000	-1.036
800	850	849	0.981	-49.95	46.18	8.000	-1.041
700	750	749	1.112	-50.22	45.95	7.000	-1.046
600	650	649	1.284	-50.59	45.64	6.000	-1.054
500	550	548	1.518	-51.08	45.24	5.000	-1.064
400	450	448	1.858	-51.82	44.66	4.000	-1.080
300	350	348	2.394	-53.03	43.77	3.000	-1.105
200	250	247	3.370	-55.35	42.24	2.000	-1.153
100	150	144	5.730	-61.80	38.90	1.000	-1.288
100	150	144	5.730	-61.80	38.90	1.000	-1.288
90	140	134	6.170	-63.15	38.33	0.900	-1.316
80	130	123	6.685	-64.80	37.69	0.800	-1.350
70	120	113	7.299	-66.86	36.94	0.700	-1.393
60	110	102	8.045	-69.53	36.07	0.600	-1.449
50	100	91	8.976	-73.13	35.04	0.500	-1.523
40	90	80	10.176	-78.27	33.79	0.400	-1.631
30	80	68	11.803	-86.30	32.22	0.300	-1.798
20	70	56	14.189	-100.95	30.16	0.200	-2.103
10	60	42	18.297	-138.62	27.15	0.100	-2.888

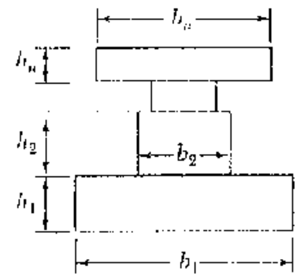
Find r1/h for (sigma max)/(sigma straight) = 1.5

52.70	103	94	8.703	-72.036	35.34	0.527	-1.501
52.80	103	94	8.693	-71.998	35.35	0.528	-1.500
52.90	103	94	8.683	-71.959	35.36	0.529	-1.499

Ratio of stresses is 1.5 for r1 = 52.8 mm or r1/h = 0.529

[Note: The desired ratio r1/h is valid for any beam
having a rectangular cross section.]

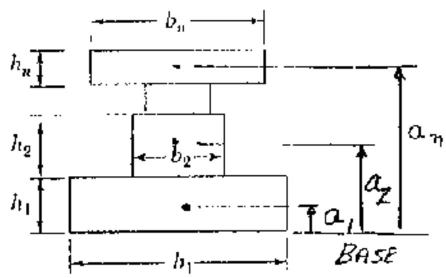
PROBLEM 4.C5



4.C5 The couple **M** is applied to a beam of the cross section shown. (*a*) Write a computer program that, for loads expressed in either SI or U.S. customary units, can be used to calculate the maximum tensile and compressive stresses in the beam. (*b*) Use this program to solve Probs. **4.7, 4.8, and 4.9.**

SOLUTION

INPUT: BENDING MOMENT M



FOR $n = 1$ TO n: ENTER b_n AND h_n (PRINT)

$\Delta AREA = b_n h_n$

$a_n = a_{n-1} + (h_{n-1})/2 + h_n/2$

[MOMENT OF RECTANGLE ABOUT BASE]

$\Delta m = (\Delta AREA)\, a_n$

[FOR WHOLE CROSS SECTION]

$m = m + \Delta m$; $AREA = AREA + \Delta AREA$

LOCATION OF CENTROID ABOVE BASE

$\bar{y} = m/AREA$ (PRINT)

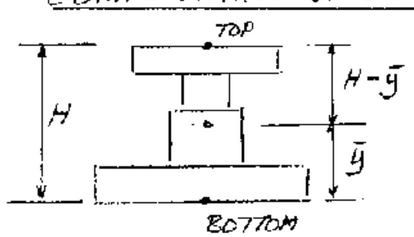
MOMENT OF INERTIA ABOUT HORIZONTAL CENTROIDAL AXIS

FOR $n = 1$ TO n: $a_n = a_{n-1} + (h_{n-1})/2 + h_n/2$

$\Delta I = b_n h_n^3/12 + (b_n h_n)(\bar{y} - a_n)^2$

$I = I + \Delta I$ (PRINT)

COMPUTATION OF STRESSES



TOTAL HEIGHT: FOR $n = 1$ TO n

$H = H + h_n$

STRESS AT TOP

$$M_{TOP} = -M \frac{H - \bar{y}}{I}$$ (PRINT)

STRESS AT BOTTOM

$$M_{BOTTOM} = M \frac{\bar{y}}{I}$$ (PRINT)

SEE NEXT PAGE FOR PRINTOUTS

CONTINUED

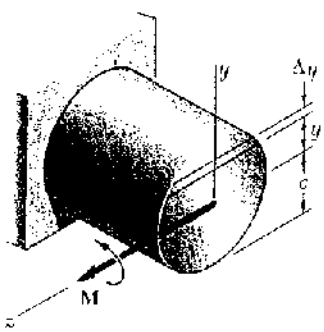
PROBLEM 4.C5 - CONTINUED

```
          Problem 4.7

Summary of Cross Section Dimensions
     Width (in.)       Height (in.)
        9.00             2.00
        3.00             6.00

Bending Moment = 600.000 kip.in.
Centroid is    3.000 in. above lower edge
Centroidal Moment of Inertia is   204.000 in^4

Stress at top of beam    =  -14.706 ksi
Stress at bottom of beam =    8.824 ksi
```

```
          Problem 4.8

Summary of Cross Section Dimensions
     Width (in.)       Height (in.)
        4.00             1.00
        1.00             6.00
        8.00             1.00

Bending Moment = 500.000 kip.in.
Centroid is    4.778 in. above lower edge
Centroidal Moment of Inertia is   155.111 in^4

Stress at top of beam    =  -10.387 ksi
Stress at bottom of beam =   15.401 ksi
```

```
          PROBLEM 4.9

Summary of Cross Section Dimensions
     Width (mm)        Height (mm)
       50               10
       20               50

Bending Moment =   1500.0000 N.m
Centroid is   25.000 mm above lower edge
Centroidal Moment of Inertia is     512500 mm^4

Stress at top of beam    = -102.439 MPa
Stress at bottom of beam =   73.171 MPa
```

PROBLEM 4.C6



4.C6 A solid rod of radius $c = 1.2$ in. is made of a steel that is assumed to be elastoplastic with $E = 29{,}000$ ksi and $\sigma_Y = 42$ ksi. The rod is subjected to a couple of moment M that increases from zero to the maximum elastic moment M_Y and then to the plastic moment M_p. Denoting by y_Y the half thickness of the elastic core, write a computer program and use it to calculate the bending moment M and the radius of curvature ρ for values of y_Y from 1.2 in. to 0 using 0.2-in. decrements. (*Hint:* Divide the cross section into 80 horizontal elements of 0.03-in. height.)

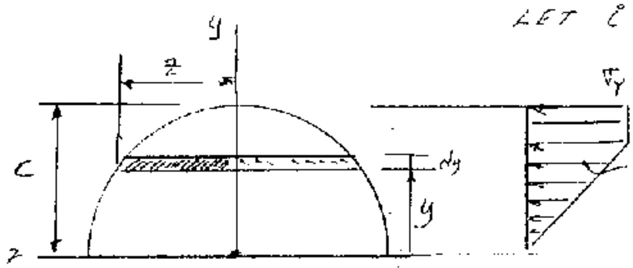
SOLUTION $M_Y = \sigma_Y \frac{\pi}{4} c^3 = (42\ \text{ksi})\frac{\pi}{4}(1.2\ \text{in.})^3 = 57\ \text{kip}\cdot\text{in}$

$M_p = \sigma_Y \frac{4}{3} c^3 = (42\ \text{ksi})\frac{4}{3}(1.2\ \text{in})^3 = 96.8\ \text{kip}\cdot\text{in.}$

CONSIDER TOP HALF OF ROD

LET ℓ = NUMBER OF ELEMENTS IN TOP HALF

Δh = HEIGTH OF EACH ELEMENT: $\Delta h = \dfrac{c}{\ell}$



σ_E = STRESS ON ELEMENT

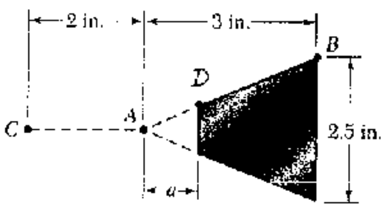
```
      FOR n = 0 TO l-1  STEP 1
        y = n(Δh)
        z = [c² - {(n+0.5)Δh}²]^½        ← z AT MIDHEIGHT OF ELEMENT
        IF y ≥ yY  GO TO 100
            σE = σY (n+0.5)Δh / σy         ← STRESS IN ELASTIC CORE
        GOTO 200
100         σE = σY                        ← STRESS IN PLASTIC ZONE
200     ΔAREA = 2z(Δh)
        ΔFORCE = σE (ΔAREA)
        ΔMOMENT = ΔFORCE (n+.5)Δh
        M = M + ΔMOMENT
        ρ = yY E/σY
        PRINT yY, M, AND ρ.
      NEXT
```

REPEAT FOR $y_Y = 1.2$ in, TO $y_Y = 0$ AT -0.2-in. DECREMENTS

PROGRAM OUTPUT

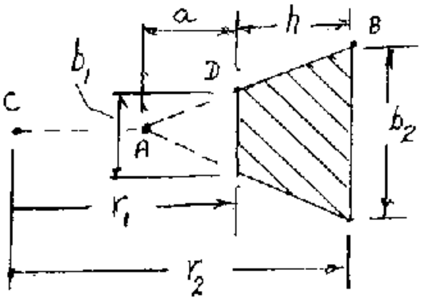
```
Radius of rod = 1.2 in.
Yield point of steel =  42 ksi
Yield moment = 57.0  kip·in        Plastic moment = 96.8  kip.in.
Number of elements in half of the rod =  40

For yY =  1.20 in.    M = 57.1 kip·in.    Radius of curvature = 828.57 in.
For yY =  1.00 in.    M = 67.2 kip·in.    Radius of curvature = 690.48 in.
For yY =  0.80 in.    M = 76.9 kip·in.    Radius of curvature = 552.38 in.
For yY =  0.60 in.    M = 85.2 kip·in.    Radius of curvature = 414.29 in.
For yY =  0.40 in.    M = 91.6 kip·in.    Radius of curvature = 276.19 in.
For yY =  0.20 in.    M = 95.5 kip·in.    Radius of curvature = 138.10 in.
For yY =  0.00 in.    M = infinite        Radius of curvature = zero
```

PROBLEM 4.C7



4.C7 The machine element of Prob. 4.178 to be redesigned by removing part of the triangular cross section. It is believed that the removal of a small triangular area of width a will lower the maximum stress in the element. In order to verify this design concept, write a computer program to calculate the maximum stress in the element for values of a from 0 to 1 in. using 0.1-in. increments. Using appropriate smaller increments, determine the distance a for which the maximum stress is as small as possible and the corresponding value of the maximum stress.

SOLUTION SEE FIG 4.79 PAGE 289



$M = 5$ kip·in, $r_2 = 5$ in, $b_2 = 2.5$ in

FOR $a = 0$ TO 1.0 AT 0.1 INTERVALS

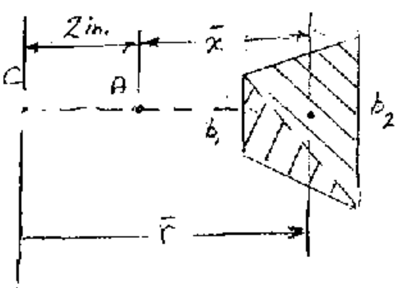
$h = 3 - a$

$r_1 = 2 + a$

$b_1 = b_2\,(a/(h+a))$

$AREA = (b_1 + b_2)(h/2)$

$\bar{x} = a + \left[\frac{1}{2}\, b_1 h (h/3) + \frac{1}{2}\, b_2 h (2h/3)\right] / AREA$



$\bar{r} = r_2 - (h - \bar{x})$

$$R = \frac{\frac{1}{2} h^2 (b_1 + b_2)}{(b_1 r_2 - b_2 r_1) \ln \frac{r_2}{r_1} - h(b_1 - b_2)}$$

$e = \bar{r} - R$

$\sigma_D = M(r_1 - R) / [AREA\,(e)(r_1)]$

$\sigma_B = M(r_2 - R) / [AREA\,(e)(r_2)]$

PRINT AND RETURN

PROGRAM OUTPUT

a	R	sigmaD	sigmaB	b1	rbar	e
in.	in.	ksi	ksi			
0.00	3.855	-8.5071	2.1014	0.00	4.00	0.145
0.10	3.858	-7.7736	2.1197	0.08	4.00	0.144
0.20	3.869	-7.2700	2.1689	0.17	4.01	0.140
0.30	3.884	-6.9260	2.2438	0.25	4.02	0.134
0.40	3.904	-6.7004	2.3423	0.33	4.03	0.127
0.50	3.928	-6.5683	2.4641	0.42	4.05	0.119
0.60	3.956	-6.5143	2.6102	0.50	4.07	0.111
0.70	3.985	-6.5296	2.7828	0.58	4.09	0.103
0.80	4.018	-6.6098	2.9852	0.67	4.11	0.094
0.90	4.052	-6.7541	3.2220	0.75	4.14	0.086
1.00	4.089	-6.9647	3.4992	0.83	4.17	0.078

Determination of the maximum compressive stress that is as small as possible

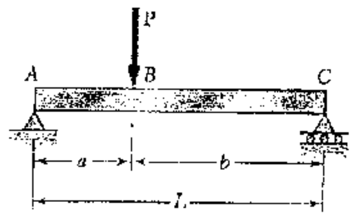
a	R	sigmaD	sigmaB	b1	rbar	e
in.	in.	ksi	ksi			
0.620	3.961	-6.51198	2.6425	0.52	4.07	0.109
0.625	3.963	-6.51185	2.6507	0.52	4.07	0.109
0.630	3.964	-6.51188	2.6591	0.52	4.07	0.109

ANSWER: When a = 625 in. the compressive stress is 6.51 ksi

Chapter 5

Problem 5.1

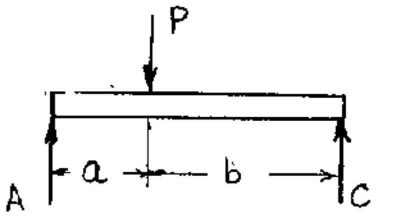
5.1 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.



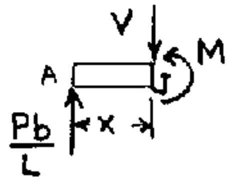
Reactions

$\Sigma M_C = 0 \quad LA - bP = 0 \quad A = \frac{Pb}{L}$

$\Sigma M_A = 0 \quad LC - aP = 0 \quad C = \frac{Pa}{L}$



From A to B $\quad 0 < x < a$

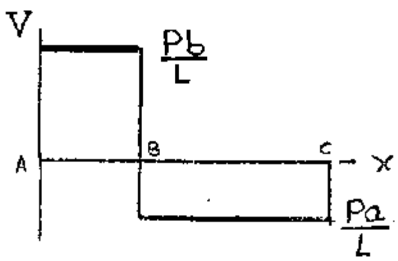


$+\uparrow \Sigma F_y = 0 \qquad \frac{Pb}{L} - V = 0$

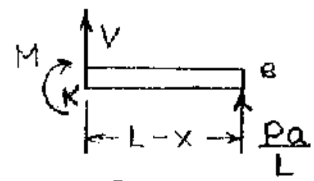
$V = \frac{Pb}{L}$

$\circlearrowleft \Sigma M_J = 0 \qquad M - \frac{Pb}{L}x = 0$

$M = \frac{Pbx}{L}$



From B to C $\quad a < x < L$

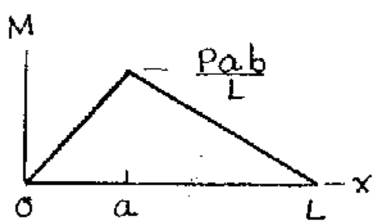


$+\uparrow \Sigma F_y = 0 \qquad V + \frac{Pa}{L} = 0$

$V = -\frac{Pa}{L}$

$\circlearrowleft \Sigma M_K = 0 \qquad -M + \frac{Pa}{L}(L - x) = 0$

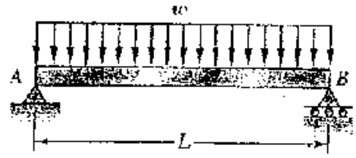
$M = \frac{Pa(L-x)}{L}$



At section B $\qquad M = \frac{Pab}{L}$

Problem 5.2

5.2 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.



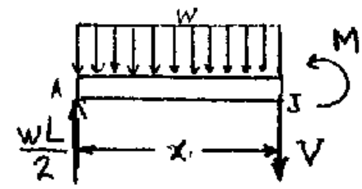
Reactions

$\circlearrowleft \Sigma M_B = 0 \qquad -AL + wL\cdot\frac{L}{2} = 0 \qquad A = \frac{wL}{2}$

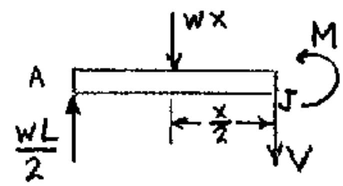
$\circlearrowleft \Sigma M_A = 0 \qquad BL - wL\cdot\frac{L}{2} = 0 \qquad B = \frac{wL}{2}$

Free body diagram for determining reactions.

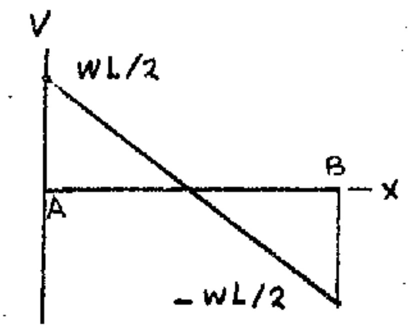
Over whole beam $\quad 0 < x < L$



Place section at x.



Replace distributed load by equivalent concentrated load.



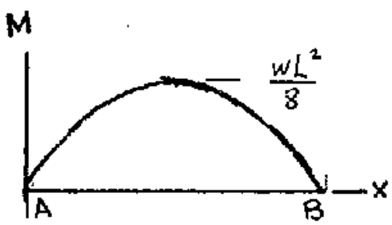
$+\uparrow \Sigma F_y = 0 \qquad \frac{wL}{2} - wx - V = 0$

$V = w\left(\frac{L}{2} - x\right)$ ◄

$\circlearrowleft \Sigma M_J = 0 \qquad -\frac{wL}{2}x + wx\frac{x}{2} + M = 0$

$M = \frac{w}{2}(Lx - x^2)$

$= \frac{w}{2}\,x(L - x)$ ◄

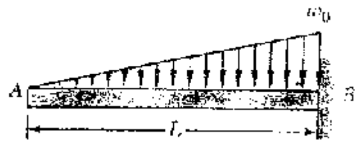


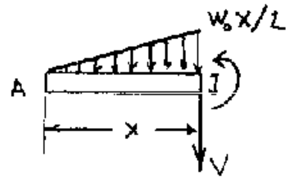
Maximum bending moment occurs at $x = \frac{L}{2}$.

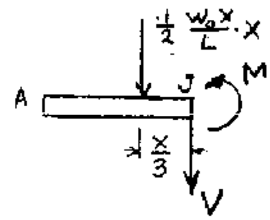
$M_{max} = \frac{wL^2}{8}$ ◄

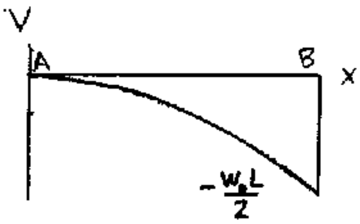
Problem 5.3

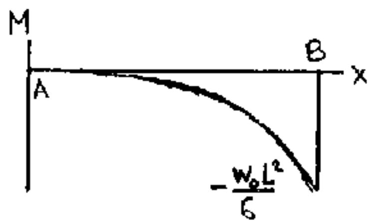
5.3 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.











$+\uparrow \Sigma F_y = 0 \qquad -\frac{1}{2}\frac{w_0 x}{L}\cdot x - V = 0$

$$V = -\frac{w_0 x^2}{2L}$$

$\circlearrowleft \Sigma M_J = 0 \qquad \frac{1}{2}\frac{w_0 x}{L}\cdot x \cdot \frac{x}{3} + M = 0$

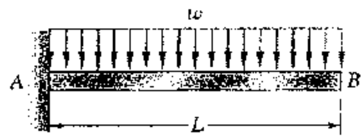
$$M = -\frac{w_0 x^3}{6L}$$

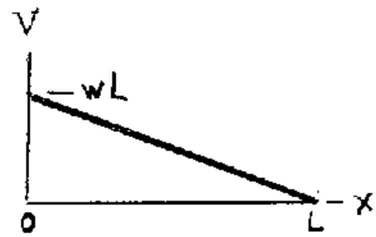
At $x = L$ $\qquad V = -\frac{w_0 L}{2} \qquad |V|_{max} = \frac{w_0 L}{2}$

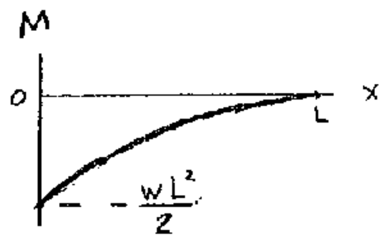
$\qquad M = -\frac{w_0 L^2}{6} \qquad |M|_{max} = \frac{w_0 L^2}{6}$

Problem 5.4

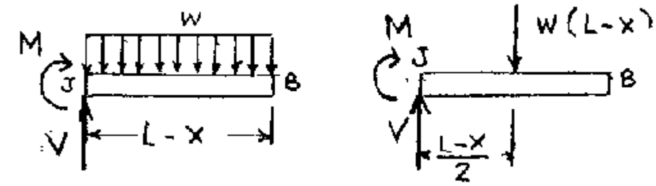
5.4 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.







Use portion to the right of the section as the free body.



Replace distributed load by equivalent concentred load.

$+\uparrow \Sigma F_y = 0 \qquad V - w(L-x) = 0$

$$V = w(L-x)$$

$+\circlearrowleft \Sigma M_J = 0 \qquad -M - w(L-x)\left(\frac{L-x}{2}\right) = 0$

$$M = -\frac{w}{2}(L-x)^2$$

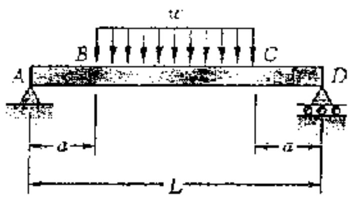
Largest negative bending moment occurs at $x = 0$.

$$M_{min} = -\frac{wL^2}{2}$$

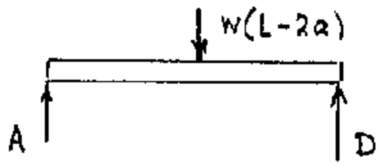
Thus, $\qquad |M|_{max} = \frac{wL^2}{2}$

Problem 5.5

5.5 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

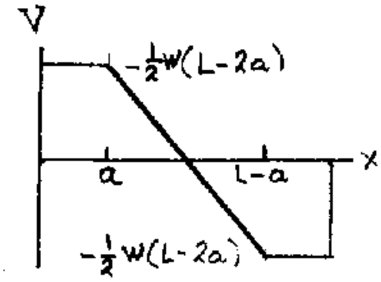


Calculate reactions after replacing distributed load by an equivalent concentrated load.

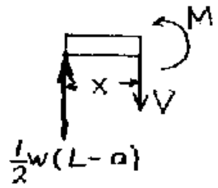


Reactions are

$$A = D = \tfrac{1}{2}w(L-2a)$$



From A to B $\quad 0 < x < a$

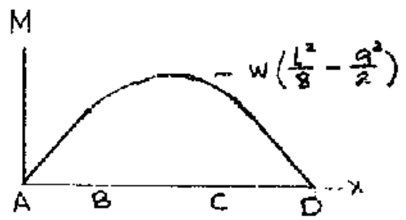


$+\uparrow \Sigma F_y = 0 \qquad \frac{1}{2}w(L-2a) - V = 0$

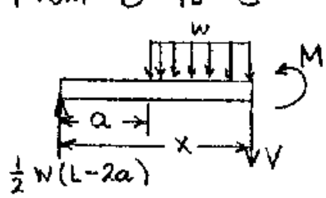
$V = \frac{1}{2}w(L-2a)$ ◄

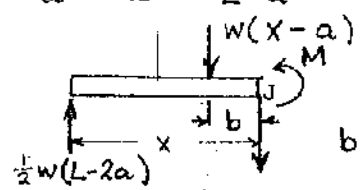
$\circlearrowleft \Sigma M = 0 \qquad -\frac{1}{2}w(L-2a) + M = 0$

$M = \frac{1}{2}w(L-2a)x$ ◄



From B to C $\quad a < x < L-a$





$b = \dfrac{x-a}{2}$

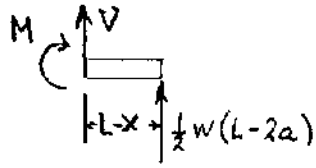
Place section cut at x. Replace distributed load by equiv. conc. load.

$+\uparrow \Sigma F_y = 0 \qquad \frac{1}{2}w(L-2a) - w(x-a) - V = 0 \qquad V = w\left(\frac{L}{2} - x\right)$ ◄

$\circlearrowleft M_J = 0 \qquad -\frac{1}{2}w(L-2a)x + w(x-a)\left(\frac{x-a}{2}\right) + M = 0$

$M = \frac{1}{2}w\left[(L-2a)x - (x-a)^2\right]$ ◄

From C to D $\quad L-a < x < L$



$+\uparrow \Sigma F_y = 0 \qquad V + \frac{1}{2}w(L-2a) = 0$

$V = -\frac{w}{2}(L-2a)$

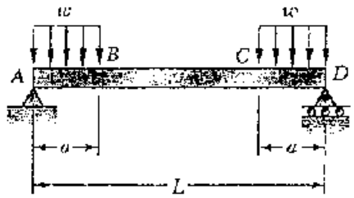
$\circlearrowleft \Sigma M_J = 0 \qquad -M + \frac{1}{2}w(L-2a)(L-x) = 0$

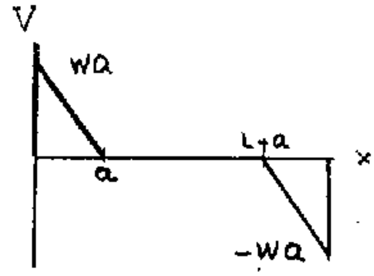
$M = \frac{1}{2}w(L-2a)(L-x)$ ◄

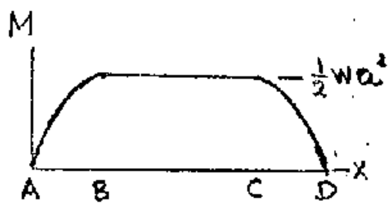
At $x = \frac{L}{2} \qquad M_{max} = w\left(\frac{L^2}{8} - \frac{a^2}{2}\right)$ ◄

Problem 5.6

5.6 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

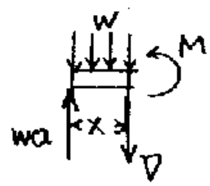


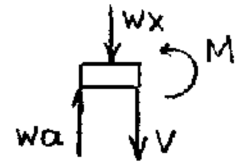




Reactions: $A = D = wa$

From A to B $\quad 0 < x < a$





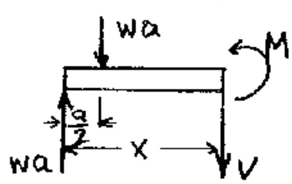
$+\uparrow \Sigma F_y = 0 \qquad wa - wx - V = 0$

$$V = w(a - x)$$

$\circlearrowleft \Sigma M_J = 0 \qquad -wax + (wx)\frac{x}{2} + M = 0$

$$M = w\left(ax - \frac{x^2}{2}\right)$$

From B to C $\quad a < x < L - a$



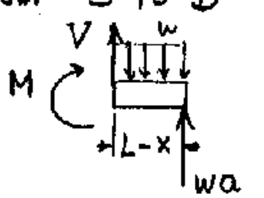
$\Sigma F_y = 0$

$wa - wa - V = 0$

$$V = 0$$

$\circlearrowleft \Sigma M_J = 0 \qquad -wax + wa\left(x - \frac{a}{2}\right) + M = 0 \qquad M = \frac{1}{2}wa^2$

From C to D $\quad L - a < x < L$

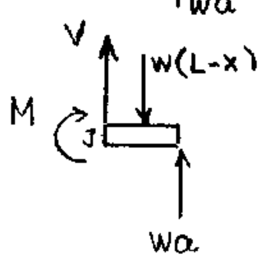


$+\uparrow \Sigma F_y = 0 \qquad V - w(L - x) + wa = 0$

$$V = w(L - x - a)$$

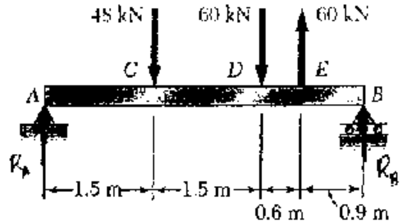
$\circlearrowleft \Sigma M_J = 0 \qquad -M - w(L-x)\left(\frac{L-x}{2}\right) + wa(L-x) = 0$

$$M = w\left[a(L-x) - \frac{1}{2}(L-x)^2\right]$$



Problem 5.7

5.7 and 5.8 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.



Reactions

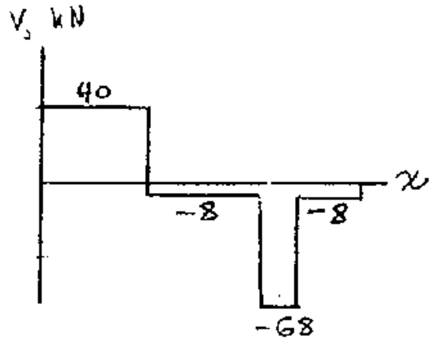
$+\circlearrowright \Sigma M_B = 0$

$-4.5\, R_A + (3.0)(48) + (1.5)(60) - (0.9)(60) = 0$

$R_A = 40 \text{ kN}$

$+\circlearrowright \Sigma M_A = 0$

$-(1.5)(48) - (3.0)(60) + (3.6)(60) + 4.5\, R_B = 0$

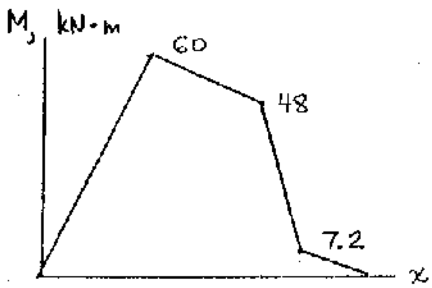


A to C $\quad V = 48 \text{ kN}$

C to D $\quad V = -8 \text{ kN}$

D to E $\quad V = -68 \text{ kN}$

E to B $\quad V = -8 \text{ kN}$

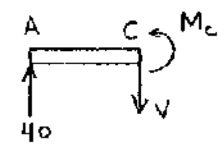


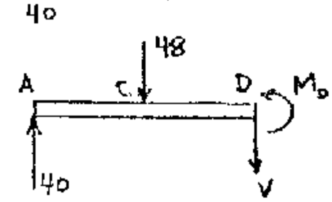
At A and B $\quad M = 0$

At C $\quad \circlearrowright \Sigma M_C = 0$

$-(1.5)(40) + M_C = 0$

$M_C = 60 \text{ kN·m}$





At D $\quad +\circlearrowright \Sigma M_D = 0$

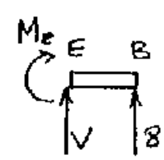
$-(3.0)(40) + (1.5)(48) + M_D = 0$

$M_D = 48 \text{ kN·m}$

At E $\quad +\circlearrowright \Sigma M_E = 0$

$-M_E + (0.9)(8) = 0$

$M_E = 7.2 \text{ kN·m}$

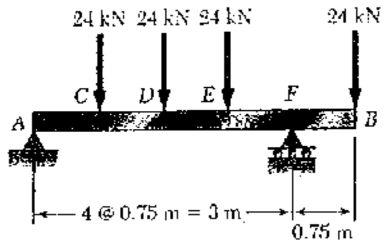


(a) $|V|_{max} = 68.0 \text{ kN}$ ◄

(b) $|M|_{max} = 60.0 \text{ kN·m}$ ◄

Problem 5.8

5.7 and 5.8 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.



Reactions at A and F.

$+\circlearrowleft \Sigma M_F = 0$

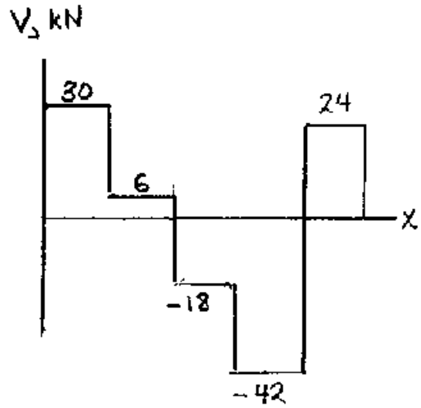
$-3R_A + (2.25)(24) + (1.50)(24) + (0.75)(24)$
$\quad - (0.75)(24) = 0$

$R_A = 30 \text{ kN} \uparrow$

$+\circlearrowleft \Sigma M_A = 0$

$-(0.75)(24) - (1.50)(24) - (2.25)(24) + 3R_F$
$\quad -(3.75)(24) = 0$

$R_F = 66 \text{ kN} \uparrow$



Shear diagram.

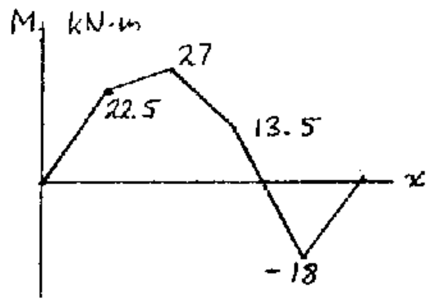
A to C $\quad V = 30 \text{ kN}$

C to D $\quad V = 30 - 24 = 6 \text{ kN}$

D to E $\quad V = 6 - 24 = -18 \text{ kN}$

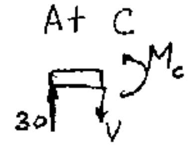
E to F $\quad V = -18 - 24 = -42 \text{ kN}$

F to B $\quad V = -42 + 66 = 24 \text{ kN}$



At A and B $\quad M_A = M_B = 0$

At C

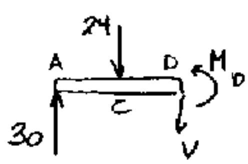


$+\circlearrowleft \Sigma M_c = 0$

$-(0.75)(30) + M_c = 0$

$M_c = 22.5 \text{ kN·m}$

At D

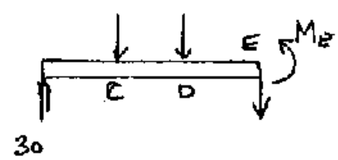


$+\circlearrowleft \Sigma M_D = 0$

$-(1.50)(30) + (0.75)(24) + M_D = 0$

$M_D = 27 \text{ kN·m}$

At E

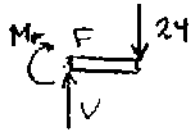


$+\circlearrowleft \Sigma M_E = 0$

$-(2.25)(30) + (1.50)(24) + (0.75)(24) + M_E = 0$

$M_E = 13.5 \text{ kN·m}$

At F



$+\circlearrowleft \Sigma M_F = 0$

$-M_F - (0.75)(24) = 0$

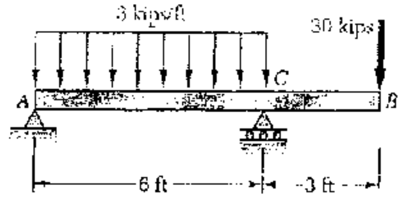
$M_F = -18 \text{ kN·m}$

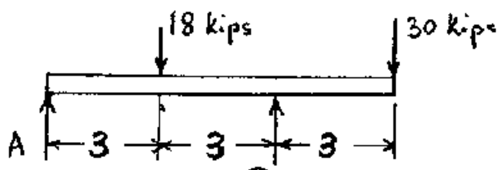
(a) $|V|_{max} = 42.0 \text{ kN}$ ◀

(b) $|M|_{max} = 27.0 \text{ kN·m}$ ◀

Problem 5.9

5.9 and 5.10 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.





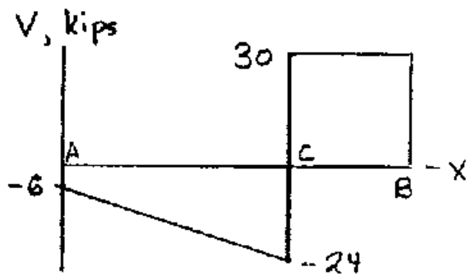
Reactions

$\circlearrowleft \Sigma M_C = 0 \qquad -6A + (3)(18) - (3)(30) = 0$

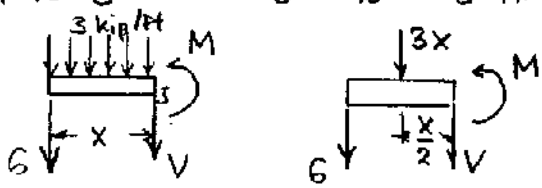
$A = -6$ kips i.e. 6 kips ↓

$\circlearrowleft \Sigma M_A = 0 \qquad 6C - (3)(18) - (9)(30) = 0$

$C = 54$ kips ↑



A to C $\qquad 0 < x < 6$ ft.

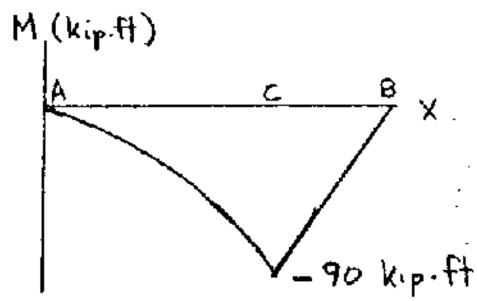


$+\uparrow \Sigma F_y = 0 \qquad -6 - 3x - V = 0$

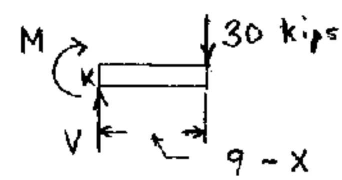
$V = -6 - 3x$ kips.

$\circlearrowleft \Sigma M_J = 0 \qquad -6x - (3x)\left(\frac{x}{2}\right) - M = 0$

$M = -6x - 1.5x^2$ kip·ft



C to B $\qquad 6$ ft $< x < 9$ ft



$+\uparrow \Sigma F_y = 0 \qquad V - 30 = 0$

$V = 30$ kips

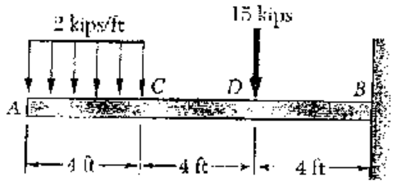
$\circlearrowleft \Sigma M_K = 0 \qquad -M - (9 - x)(30) = 0$

$M = 30x - 270$ kip·ft

From the diagrams $\qquad$ (a) $|V|_{max} = 30.0$ kips ◄

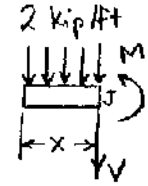
(b) $|M|_{max} = 90.0$ kip·ft ◄

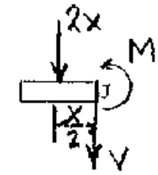
Problem 5.10



5.9 and 5.10 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

A to C
$0 < x < 4$ ft





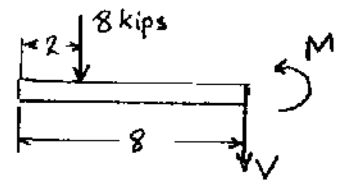
$+\uparrow \Sigma F_y = 0 \quad -V - 2x = 0 \quad V = -2x$ kips

$\circlearrowleft \Sigma M_J = 0 \quad M + (2x)(\frac{x}{2}) = 0$

$M = -x$ kip·ft

At C $\quad V = -8$ kips $\quad M = -16$ kip·ft

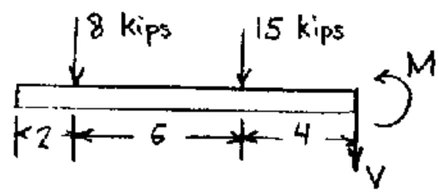
At D^-



$+\uparrow \Sigma F_y = 0 \quad -8 - V = 0 \quad V = -8$ kips

$\circlearrowleft \Sigma M_D = 0 \quad (6)(8) - M = 0 \quad M = -48$ kip ft

At B^-



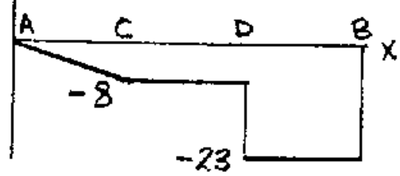
$+\uparrow \Sigma F_y = 0 \quad -8 - 15 - V = 0$

$V = -23$ kips

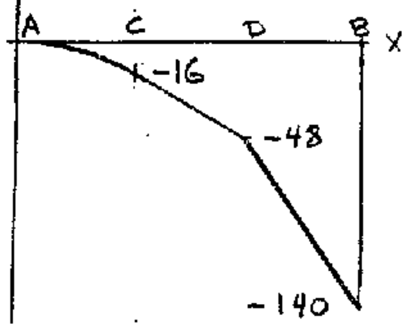
$\circlearrowleft \Sigma M_B = 0 \quad -(10)(8) - (4)(15) - M = 0$

$M = -140$ kip ft

V (kips)



M (kip·ft)

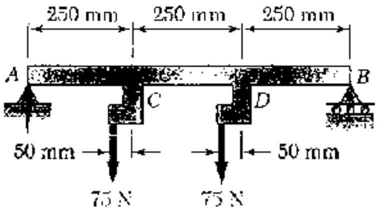


From the diagrams $\quad$ (a) $|V|_{max} = 23.0$ kips ◄

(b) $|M|_{max} = 140.0$ kip·ft ◄

Problem 5.11

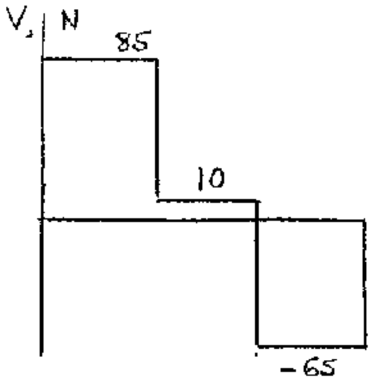
5.11 and *5.12* Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.



Reaction at A. $+\circlearrowleft M_B = 0$

$-0.750\,R_A + (0.550)(75) + (0.300)(75) = 0$

$R_A = 85 \text{ N} \uparrow$ Also $R_B = 65 \text{ N} \uparrow$

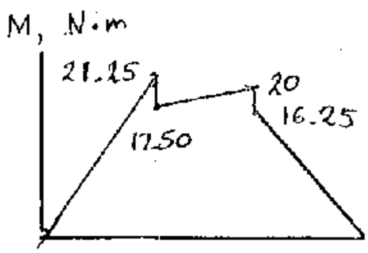


A to C $V = 85 \text{ N}$

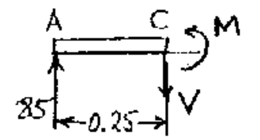
C to D $V = 10 \text{ N}$

D to B $V = -65 \text{ N}$

At A and B $M = 0$



Just to the left of C

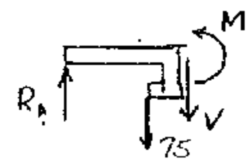


$+\circlearrowleft \Sigma M_C = 0$

$-(0.25)(85) + M = 0$

$M = 21.25 \text{ N}\cdot\text{m}$

Just to the right of C

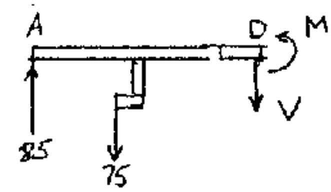


$+\circlearrowleft \Sigma M_C = 0$

$-(0.25)(85) + (0.050)(75) + M = 0$

$M = 17.50 \text{ N}\cdot\text{m}$

Just to the left of D

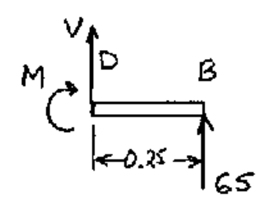


$+\circlearrowleft \Sigma M_D = 0$

$-(0.50)(85) + (0.300)(75) + M = 0$

$M = 20 \text{ N}\cdot\text{m}$

Just to the right of D



$+\circlearrowleft \Sigma M_D = 0$

$-M + (0.25)(65) = 0$

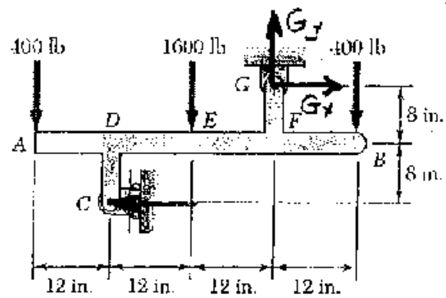
$M = 16.25 \text{ kN}$

(a) $|V|_{max} = 85.0 \text{ N}$ ◄

(b) $|M|_{max} = 21.25 \text{ N}\cdot\text{m}$ ◄

Problem 5.12

5.11 and 5.12 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.



$\circlearrowleft \Sigma M_G = 0$

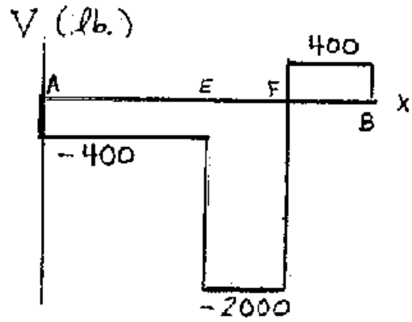
$-16\,C + (36)(400) + (12)(1600) - (12)(400) = 0$

$C = 1800$ lb.

$\xrightarrow{+} \Sigma F_x = 0 \qquad -C + G_x = 0 \qquad G_x = 1800$ lb.

$+\uparrow \Sigma F_y = 0 \qquad -400 - 1600 + G_y - 400 = 0$

$G_y = 2400$ lb.

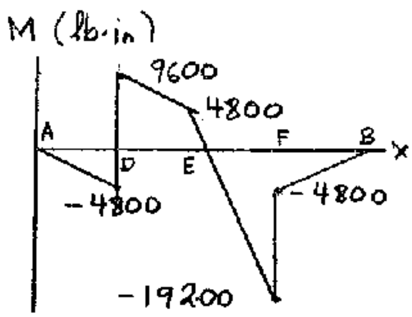


A to E $\qquad V = -400$ lb.

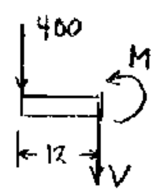
E to F $\qquad V = -2000$ lb.

F to B $\qquad V = 400$ lb.

At A and B $\qquad M = 0$



At D^-

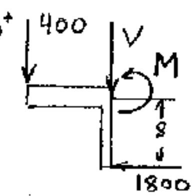


$\circlearrowleft \Sigma M_D = 0$

$(12)(400) + M = 0$

$M = -4800$ lb·in.

At D^+

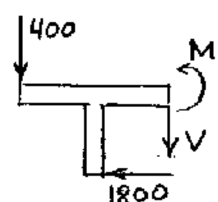


$\circlearrowleft \Sigma M_D = 0$

$(12)(400) - (8)(1800) + M = 0$

$M = 9600$ lb·in.

At E

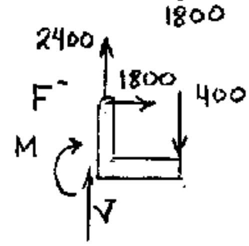


$\circlearrowleft \Sigma M_E = 0$

$(24)(400) - (8)(1800) + M = 0$

$M = 4800$ lb·in

At F^-

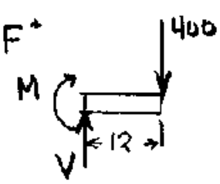


$\circlearrowleft \Sigma M_F = 0$

$-M - (8)(1800) - (12)(400) = 0$

$M = -19200$ lb·in

At F^+



$\Sigma M_F = 0$

$-M - (12)(400) = 0$

$M = -4800$ lb·in.

(a) Maximum $|V|$

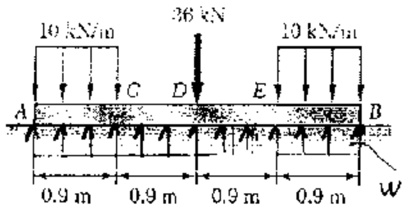
$= 2000$ lb ◄

(b) Maximum $|M|$

$= 19200$ lb·in. ◄

Problem 5.13

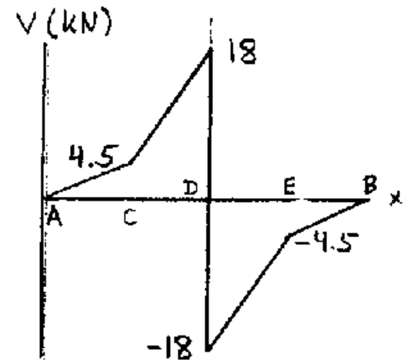
5.13 and *5.14* Assuming that the reaction of the ground to be uniformly distributed, draw the shear and bending-moment diagrams for the beam AB and determine the maximum absolute value of (*a*) of the shear, (*b*) of the bending moment.



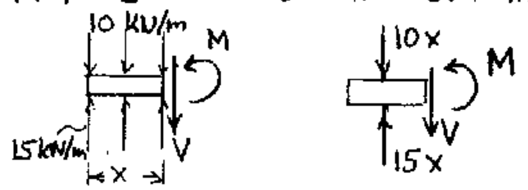
Over whole beam $+\uparrow \Sigma F_y = 0$

$$3.6\,w - (0.9)(10) - 36 - (0.9)(10) = 0$$

$$w = 15 \text{ kN/m}$$



A to C $\qquad 0 < x < 0.9$ m



$+\uparrow \Sigma F_y = 0 \qquad 15x - 10x - V = 0$

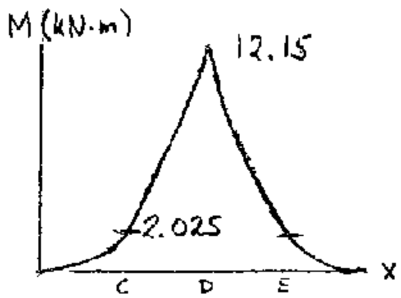
$$V = 5x$$

$\circlearrowleft \Sigma M_J = 0 \qquad -(15x)\frac{x}{2} + (10x)\frac{x}{2} + M = 0$

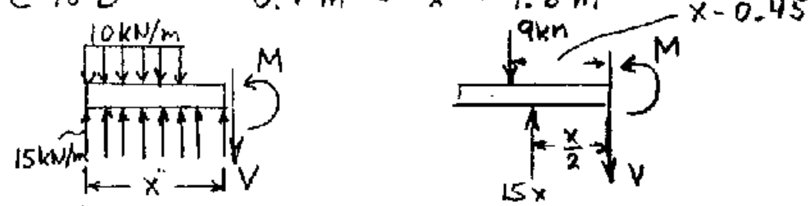
$$M = 2.5x^2$$

At $x = C$ $\qquad V = 4.5$ kN

$\qquad M = 2.025$ kN·m



C to D $\qquad 0.9 \text{ m} < x < 1.8 \text{ m}$



(a) Maximum $|V|$
$= 18$ kN ◄

$+\uparrow \Sigma F_y = 0 \qquad 15x - 9 - V = 0$

$$V = 15x - 9$$

(b) Maximum $|M|$
$= 12.15$ kN·m ◄

$\circlearrowleft \Sigma M_J = 0 \qquad -(15x)\left(\frac{x}{2}\right) + 9(x - 0.45) + M = 0$

$$M = 7.5x^2 - 9x + 4.05 = 0$$

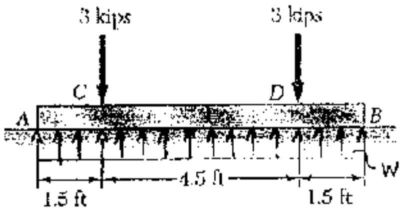
At D^- $\qquad V = 18$ kN

$\qquad M = 12.15$ kN·m

D to B $\qquad$ Use symmetry to calculate the shear and bending moment.

Problem 5.14

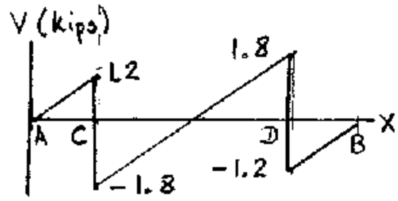
5.13 and *5.14* Assuming that the reaction of the ground to be uniformly distributed, draw the shear and bending-moment diagrams for the beam AB and determine the maximum absolute value of (a) of the shear, (b) of the bending moment.



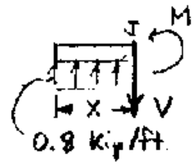
Over the whole beam

$+\uparrow \Sigma F_y = 0 \qquad 7.5w - 3 - 3 = 0$

$w = 0.8 \text{ kip/ft}$



A to C $\qquad 0 < x < 1.5$ ft



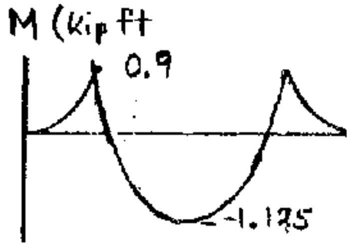
$+\uparrow \Sigma F_y = 0 \qquad 0.8x - V = 0$

$V = 0.8x$

$\circlearrowleft \Sigma M_J = 0$

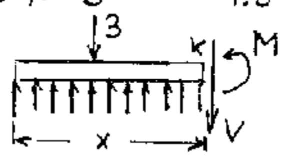
$-(0.8x)(\frac{x}{2}) + M = 0$

$M = 0.4x^2$



At C^- $\qquad V = 1.2$ kips, $\quad M = 0.9$ kip·ft

C to D $\qquad 1.5 \text{ ft} < x < 6 \text{ ft}$



$+\uparrow \Sigma F_y = 0$

$0.8x - 3 - V = 0$

$V = 0.8x - 3$

$\circlearrowleft \Sigma M_K = 0 \qquad -(0.8x)(\frac{x}{2}) + 3(x - 1.5) + M = 0$

$M = 0.4x^2 - 3x + 4.5$

At the center of the beam $\quad x = 3.75$ ft

$V = 0, \quad M = -1.125$ kip ft

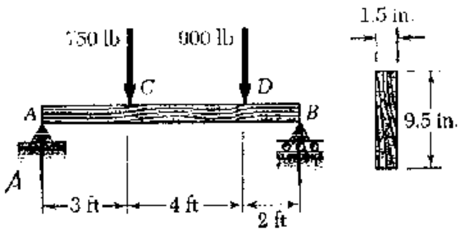
At C^+ $\qquad V = -1.8$ kip $\qquad M = 0.9$ kip ft

(a) Maximum $|V| = 1.8$ kips ◄

(b) Maximum $|M| = 1.125$ kip·ft ◄

Problem 5.15

5.15 and *5.16* For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.



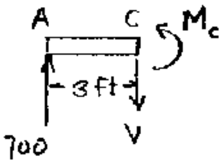
Reaction at A

$\circlearrowleft \Sigma M_B = 0$

$-9A + (6)(750) + (2)(900) = 0$

$A = 700$ lb.

Using free body. A to C.



$\circlearrowleft \Sigma M_C = 0 \qquad -(700)(3) + M_c = 0$

$M_c = 2100$ lb·ft $= 25.2 \times 10^3$ lb·in $= 25.2$ kip·in

For the cross section $I = \frac{1}{12}(1.5)(9.5)^3 = 107.172$ in^4

$c = 4.75$ in.

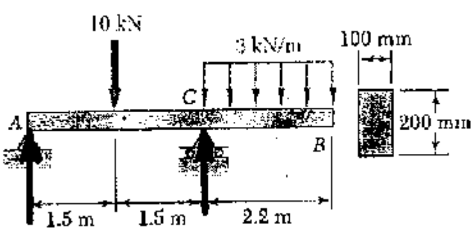
Maximum normal stress due to bending.

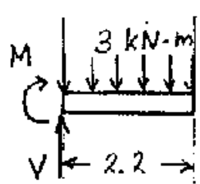
$$\sigma = \frac{Mc}{I} = \frac{(25.2)(4.75)}{107.172}$$

$\sigma = 1.117$ ksi

Problem 5.16

5.15 and *5.16* For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.





Using CB as a free body

$\circlearrowleft \Sigma M_c = 0$

$-M + (2.2)(3 \times 10^3)(1.1) = 0$

$M = 7.26 \times 10^5$ N·m

Section modulus for rectangle

$S = \frac{1}{6} b h^2$

$= \frac{1}{6}(100)(200)^2 = 666.7 \times 10^3$ mm^3

$= 666.7 \times 10^{-6}$ m^3

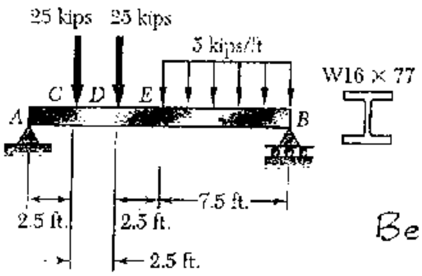
Normal stress

$$\sigma = \frac{M}{S} = \frac{7.26 \times 10^3}{666.7 \times 10^{-6}} = 10.89 \times 10^6 \text{ Pa}$$

$\sigma = 10.89$ MPa

Problem 5.17

5.17 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.



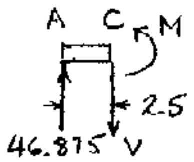
Reaction at A. $\circlearrowleft \Sigma M_B = 0$

$-15.0 R_A + (12.5)(25) + (10.0)(25)$
$+ (3.75)(7.5)(5) = 0 \qquad R_A = 46.875 \text{ kips}$

Bending moment at C. $\circlearrowleft \Sigma M_C = 0$

$-(2.5)(46.875) + M = 0$

$M = 117.1875 \text{ kip}\cdot\text{ft} = 1.40625 \times 10^3 \text{ kip}\cdot\text{in}$



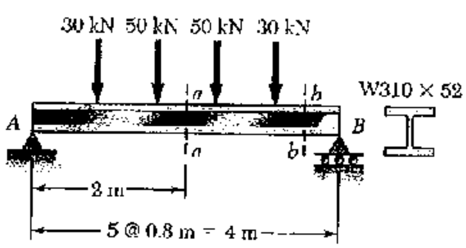
For W16×77 rolled steel section $\quad S = 134 \text{ in}^3$

Normal stress at C

$$\sigma = \frac{M}{S} = \frac{1.40625 \times 10^3}{134} = \qquad \sigma = 10.49 \text{ ksi}$$

Problem 5.18

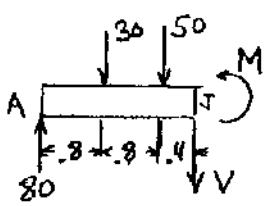
5.18 For the beam and loading shown, determine the maximum normal stress due to bending on section *a-a*.



Reactions: $\quad$ By symmetry $\quad A = B$

$+\uparrow \Sigma F_y = 0 \qquad A = B = 80 \text{ kN}$

Using left half of beam as free body



$\circlearrowleft \Sigma M_J = 0$

$-(80)(2) + (30)(1.2) + (50)(0.4) + M = 0$

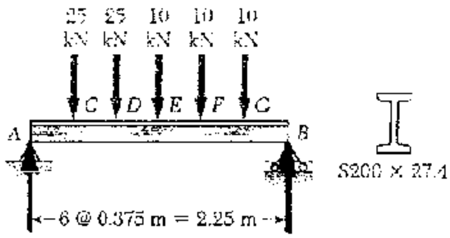
$M = 104 \text{ kN}\cdot\text{m} = 104 \times 10^3 \text{ N}\cdot\text{m}$

For W310 × 52 $\qquad S = 748 \times 10^3 \text{ mm}^3$
$= 748 \times 10^{-6} \text{ m}^3$

Normal stress $\quad \sigma = \dfrac{M}{S} = \dfrac{104 \times 10^3}{748 \times 10^{-6}} = 139.0 \times 10^6 \text{ Pa} = 139.0 \text{ MPa}$

Problem 5.19

5.19 and 5.20 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.

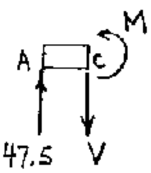


Use entire beam as free body

$\circlearrowleft \Sigma M_B = 0$

$$2.25A - (1.875)(25) - (1.5)(25) - (1.125)(10) - (0.75)(10) - (0.375)(10) = 0$$

$$A = 47.5 \text{ kN}$$



Use portion AC as free body

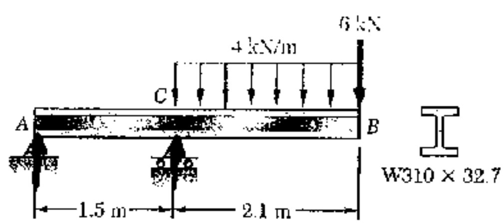
$$-(0.375)(47.5) + M = 0 \qquad M = 17.8125 \text{ kN}\cdot\text{m}$$

For S 200 × 27.4 $\quad S = 235 \times 10^3 \text{ mm}^3 = 235 \times 10^{-6} \text{ m}^3$

Normal stress $\quad \sigma = \dfrac{M}{S} = \dfrac{17.8125 \times 10^3}{235 \times 10^{-6}} = 75.8 \times 10^6 \text{ Pa} = 75.8 \text{ MPa}$ ◄

Problem 5.20

5.19 and 5.20 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.

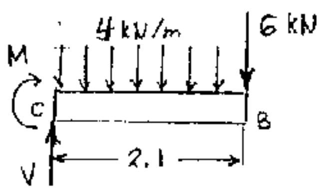


Use portion CB as free body.

$\circlearrowleft \Sigma M_C = 0$

$$-M + (4)(2.1)(1.05) + (6)(2.1) = 0$$

$$M = -21.42 \text{ kN}\cdot\text{m}$$

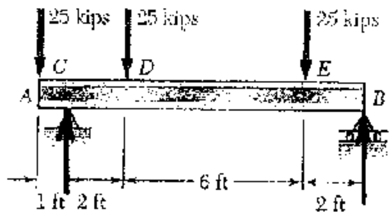


For W 310 × 32.7 $\quad S = 415 \times 10^3 \text{ mm}^3$
$\quad = 415 \times 10^{-6} \text{ m}^3$

Normal stress $\quad \sigma = \dfrac{|M|}{S} = \dfrac{21.42 \times 10^3}{415 \times 10^{-6}} = 51.6 \times 10^6 \text{ Pa}$
$= 51.6 \text{ MPa}$ ◄

Problem 5.21

5.21 and 5.22 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



S12 × 35

$\circlearrowleft + \; \sum M_B = 0$

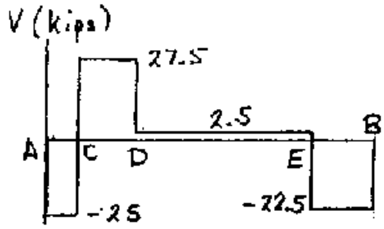
$(11)(25) - 10\,C + (8)(25) + (2)(25) = 0$

$C = 52.5$ kips

$\circlearrowleft + \; \sum M_C = 0$

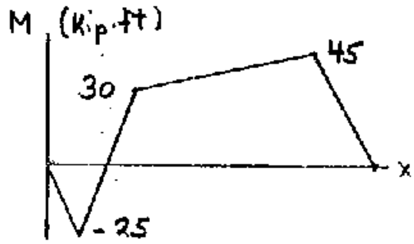
$(1)(25) - (2)(25) - (8)(25) + 10\,B = 0$

$B = 22.5$ kips



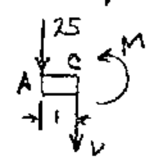
Shear

A to C^-	$V = -25$ kips
C^+ to D^-	$V = 27.5$ kips
D^+ to E^-	$V = 2.5$ kips
E^+ to B	$V = -22.5$ kips



Bending moments

At C

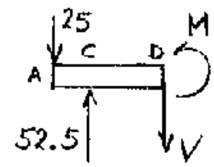


$\circlearrowleft + \; \sum M_C = 0$

$(1)(25) + M = 0$

$M = -25$ kip·ft

At D

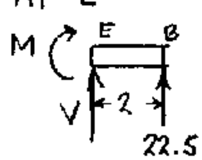


$\circlearrowleft + \; \sum M_D = 0$

$(3)(25) - (2)(52.5) + M = 0$

$M = 30$ kip·ft

At E



$\circlearrowleft + \; \sum M_E = 0 \qquad -M + (2)(22.5) = 0 \qquad M = 45$ kip·ft

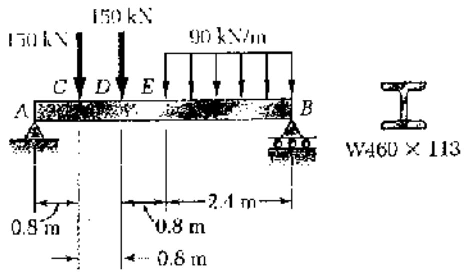
max $|M| = 45$ kip·ft $= 540$ kip·in

For S12 × 35 rolled steel section $\quad S = 38.2\ \text{in}^3$

Normal stress $\quad \sigma = \dfrac{|M|}{S} = \dfrac{540}{38.2} = 14.14$ ksi

Problem 5.22

5.21 and 5.22 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



Reaction at A $\quad +\circlearrowleft \Sigma M_B = 0$

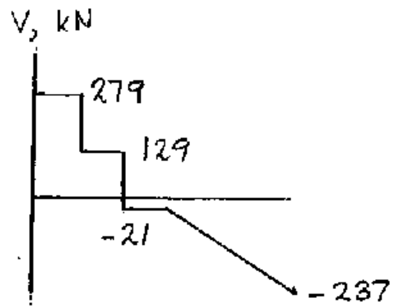
$$-4.8 R_A + (4.0)(150) + (3.2)(150) + (1.2)(2.4)(90) = 0$$

$$R_A = 279 \text{ kN}$$

Reaction at B $\quad +\circlearrowleft \Sigma M_A = 0$

$$-(0.8)(150) - (1.6)(150) - (3.6)(2.4)90) + 4.8 R_B = 0$$

$$R_B = 237 \text{ kN}$$

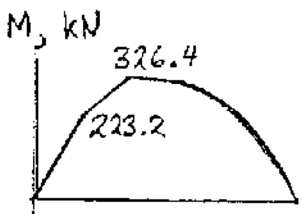


A to C $\quad V = 279$ kN

C to D $\quad V = 279 - 150 = 129$ kN

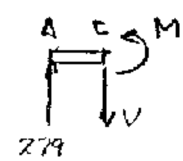
D to E $\quad V = 279 - 150 - 150 = -21$ kN

At B $\quad V = -237$ kN



At A and B $\quad M = 0$

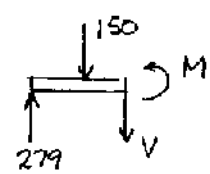
At C $\quad +\circlearrowleft \Sigma M_C = 0$



$$-(0.8)(279) + M = 0$$

$$M = 223.2 \text{ kN·m}$$

At D $\quad +\circlearrowleft M_D = 0$



$$-(1.6)(279) + (0.8)(150) + M = 0$$

$$M = 326.4 \text{ kN·m}$$

From diagram

$M_{max} = 326.4$ kN-m

$= 326.4 \times 10^3$ N·m

At E $\quad +\circlearrowleft M_E = 0$

$$-(2.4)(279) + (1.6)(150) + (0.8)(150) + M = 0$$

$$M = 309.6 \text{ kN·m}$$

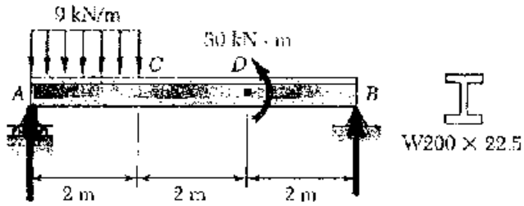
For W 460×113 rolled steel section $\quad S = 2400 \times 10^3 \text{ mm}^3 = 2400 \times 10^{-6} \text{ m}^3$

Normal stress $\quad \sigma_m = \dfrac{M_{max}}{S} = \dfrac{326.4 \times 10^3}{2400 \times 10^{-6}} = 136 \times 10^6$ Pa

$\sigma_m = 136.0$ MPa

Problem 5.23

5.23 and 5.24 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



$+\circlearrowleft \Sigma M_B = 0$

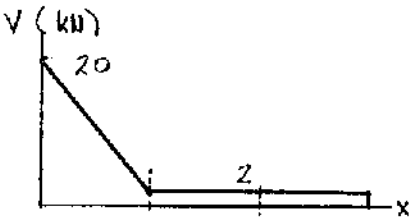
$-6A + (2)(9)(5) + 30 = 0$

$A = 20 \text{ kN}$

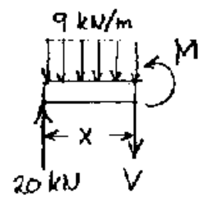
$+\circlearrowleft \Sigma M_A = 0$

$-(2)(9)(1) + 30 + 6B = 0$

$B = -2 \text{ kN}$ i.e. $2 \text{ kN} \downarrow$



A to C $\qquad 0 < x < 2 \text{ m}$



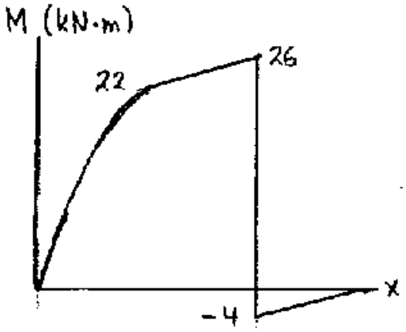
$+\uparrow \Sigma F_y = 0 \qquad 20 - 9x - V = 0$

$V = 20 - 9x$

$+\circlearrowleft \Sigma M_J = 0$

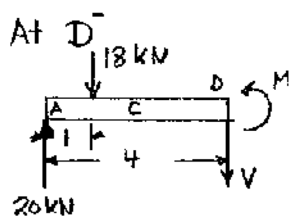
$-20x + (9x)\frac{x}{2} + M = 0$

$M = 20x - 4.5x^2$



At C $\qquad V = 2 \text{ kN} \qquad M = 22 \text{ kN·m}$

At D^-



$+\uparrow \Sigma F_y = 0$

$20 - 18 - V = 0$

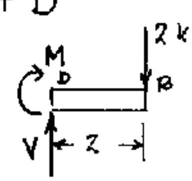
$V = 2 \text{ kN}$

$+\circlearrowleft \Sigma M_D = 0$

$-(4)(20) + (3)(18) + M = 0$

$M = 26 \text{ kN·m}$

At D^+



$+\uparrow \Sigma F_y = 0 \qquad V - 2 = 0$

$V = 2 \text{ kN}$

$+\circlearrowleft \Sigma M_D = 0$

$-M - (2)(2) = 0$

$M = -4 \text{ kN·m}$

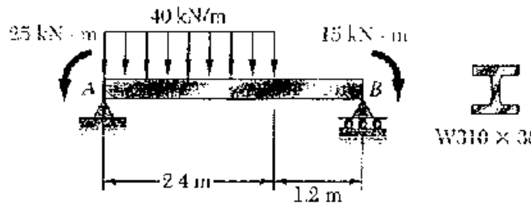
$\max |M| = 26 \text{ kN·m} = 26 \times 10^3 \text{ N·m}$

For rolled steel section W 200 × 22.5 $\qquad S = 194 \times 10^3 \text{ mm}^3 = 194 \times 10^{-6} \text{ m}^3$

Normal stress $\quad \sigma = \frac{|M|}{S} = \frac{26 \times 10^3}{194 \times 10^{-6}} = 134.0 \times 10^6 \text{ Pa} = 134.0 \text{ MPa}$ ◄

Problem 5.24

5.23* and *5.24 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



W310 × 38.7

Reaction at A $\circlearrowleft \Sigma M_B = 0$

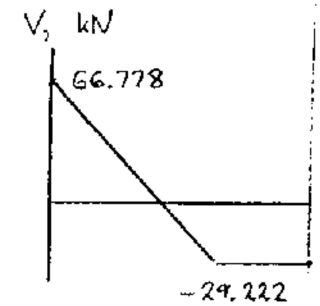
$-3.6 R_A + 2 \quad (2.4)(2.4)(40) - 15 = 0$

$R_A = 66.778 \text{ kN}$

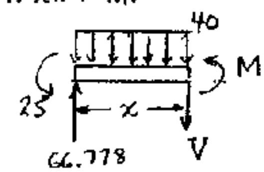
Reaction at B $+\circlearrowright M_A = 0$

$25 - (1.2)(2.4)(40) - 15 + 3.6 R_B = 0$

$R_B = 29.222 \text{ kN}$



$0 < x \leq 2.4 \text{ m}$

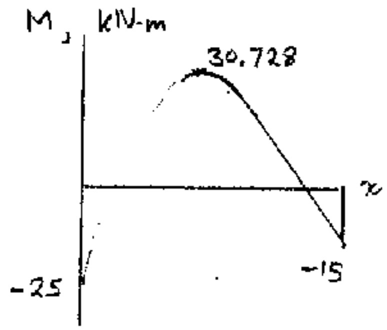


$+\uparrow \Sigma F_y = 0$

$66.778 - 40x - V = 0$

$V = 66.778 - 40x$

$V = 0$ at $x = 1.6944 \text{ m}$



$+\circlearrowright \Sigma M = 0$

$25 + (40x)\frac{x}{2} - 66.778x + M = 0$

$M = -20x^2 + 66.778x - 25$

$M = -(20)(1.6944)^2 + (66.778)(1.6944) - 25$

$= 30.728 \text{ kN-m}$ at $x = 1.6944 \text{ m}$

$2.4 \leq x < 3.6 \qquad V = -29.222 \text{ kN}$

$M = 29.222(3.6 - x) - 15$

$M = 20.067 \text{ kN} \cdot \text{m}$ at $x = 2.4 \text{ m}$

$M_{max} = 30.728 \text{ kN-m} = 30.728 \times 10^3 \text{ N} \cdot \text{m}$

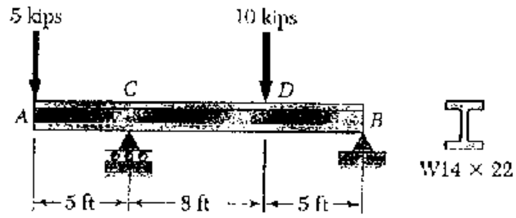
For W 310 × 38.7 rolled steel section $S = 549 \times 10^3 \text{ mm}^3$
$= 549 \times 10^{-6} \text{ m}^3$

$\sigma_m = \dfrac{M_{max}}{S} = \dfrac{30.728 \times 10^3}{549 \times 10^{-6}} = 55.97 \times 10^6 \text{ Pa}$

56.0 MPa ◄

Problem 5.25

5.25 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



Reaction at C $\circlearrowleft M_B = 0$

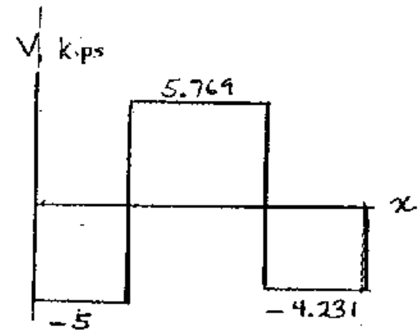
$(18)(5) - 13C + (5)(10) = 0$

$C = 10.769$ kips

Reaction at B $+\circlearrowleft M_C = 0$

$(5)(5) - (8)(10) + 13B = 0$

$B = 4.231$ kips



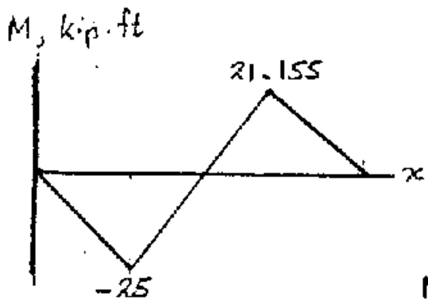
Shear diagram.

A to C^- $\quad V = -5$ kips

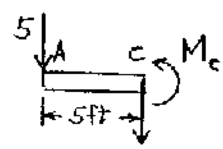
C^+ to D^- $\quad V = -5 + 10.769 = 5.769$ kips

D^+ to B $\quad V = 5.769 - 10 = -4.231$ kips

At A and B $\quad M = 0$



At C

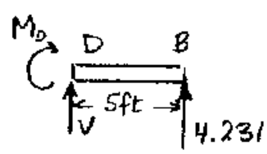


$+\circlearrowleft M_C = 0$

$(5)(5) + M_C = 0$

$M_C = -25$ kip·ft

At D



$+\circlearrowleft \Sigma M_D = 0$

$-M_D + (5)(4.231)$

$M_D = 21.155$ kip·ft

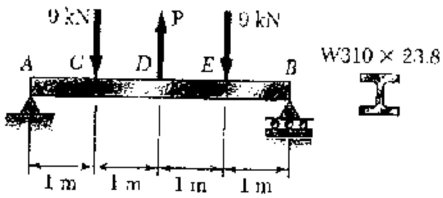
$|M|_{max}$ occurs at C $\qquad |M|_{max} = 25$ kip·ft $= 300$ kip·in

For W14×22 rolled steel section $\quad S = 29.0\ \text{in}^3$

Normal stress $\qquad \sigma = \frac{M}{S} = \frac{300}{29.0} = 10.34$ ksi

Problem 5.26

5.26 Knowing that $P = 10$ kN, draw the shear and bending-moment diagrams for beam AB and determine the maximum normal stress due to bending.

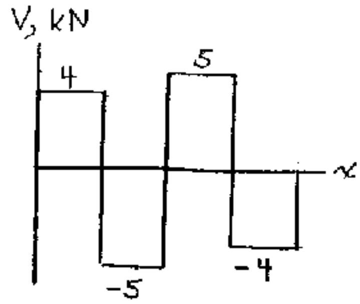


By symmetry the reactions at A and B are equal.

$A = B$

$+\uparrow \Sigma F_y = 0: \quad A + B - 9 + 10 - 9 = 0$

$A = B = 4$ kN



Shear diagram.

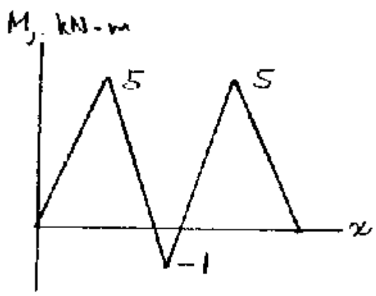
A to C $\quad V = 4$ kN

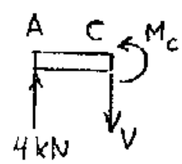
C to D $\quad V = 4 - 9 = -5$ kN

D to E $\quad V = -5 + 10 = 5$ kN

E to B $\quad V = 5 - 9 = -4$ kN

Bending moments. $\quad M_A = M_B = 0$

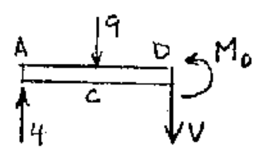




$+\circlearrowleft \Sigma M_c = 0$

$-(1)(4) + M_c = 0$

$M_c = 4$ kN·m



$+\circlearrowleft \Sigma M_D = 0$

$-(2)(4) + (1)(9) + M_D = 0$

$M_D = -1$ kN·m

By symmetry $\quad M_E = M_c = 4$ kN·m

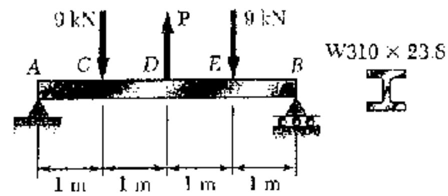
$|M|_{max} = M_c = 4 \text{ kN}\cdot\text{m} = 4\times 10^3 \text{ N}\cdot\text{m}$

For W 310 × 23.8 $\quad S = 280 \times 10^3 \text{ mm}^3 = 280 \times 10^{-6} \text{ m}^3$

Normal stress $\quad \sigma = \dfrac{M}{S} = \dfrac{4 \times 10^3}{280 \times 10^{-6}} = 14.29 \times 10^6$ Pa $\qquad$ 14.29 MPa

Problem 5.27

5.27 Determine (*a*) the magnitude of the upward force **P** for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (*Hint:* Draw the bending-moment diagram and the equate the absolute values of the largest positive and negative bending moments obtained..)

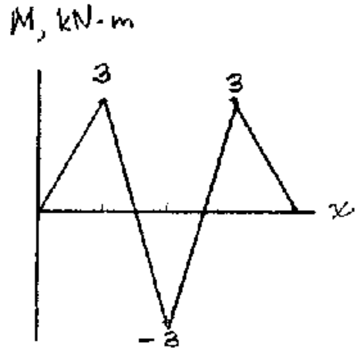


By symmetry the reactions at A and B are equal.

$$A = B$$

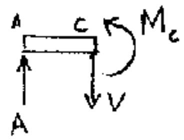
$$+\uparrow \Sigma F_y = 0: \quad A - 9 + P - 9 + B = 0$$

$$A = B = 9 - \tfrac{1}{2}P \qquad P = 18 - 2A$$



Also, by symmetry bending moments $M_C = M_E$.

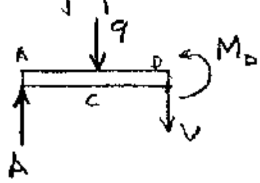
Using portion AC as a free body



$$+\circlearrowleft \Sigma M_C = 0$$

$$-1A + M_C = 0 \qquad M_C = 1A$$

Using portion ACD as a free body



$$+\circlearrowleft \Sigma M_D = 0$$

$$-2A + (1)(9) + M_D = 0$$

$$M_D = 2A - 9$$

Equate $M_C = -M_D$

$$1A = 9 - 2A \qquad A = 3 \text{ kN}$$

Then $P = 18 - (2)(3) = 12$ kN $\quad$ (a) 12.00 kN ◄

$$M_C = 3 \text{ kN}\cdot\text{m}$$

$$M_D = (2)(3) - 9 = -3 \text{ kN}\cdot\text{m}$$

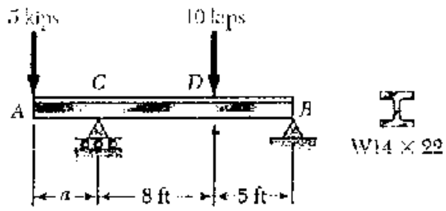
$$|M|_{max} = 3 \text{ kN}\cdot\text{m} = 3\times10^3 \text{ N}\cdot\text{m}$$

For W310 × 23.4 rolled steel section $S = 280\times10^3 \text{ mm}^3 = 280\times10^{-6} \text{ m}^3$

Normal stress $\sigma = \dfrac{M}{S} = \dfrac{3\times10^3}{280\times10^{-6}} = 10.71\times10^6$ Pa $\quad$ (b) 10.71 MPa ◄

Problem 5.28

5.28 Determine (*a*) the distance *a* for which the absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27) (*Hint:* Draw the bending-moment diagram and the equate the absolute values of the largest positive and negative bending moments obtained..)

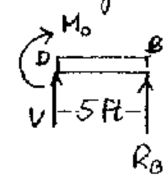


Reaction at B. $+\circlearrowleft \Sigma M_C = 0$

$$5a - (8)(10) + 13 R_B = 0$$

$$R_B = \frac{1}{13}(80 - 5a)$$

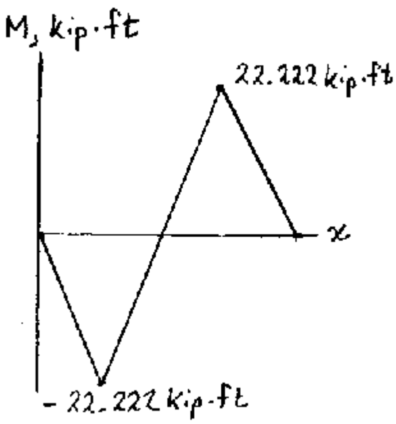
Bending moment at D.



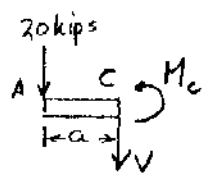
$+\circlearrowleft \Sigma M_D = 0$

$$-M_D + 5R_B = 0$$

$$M_D = 5R_B = \frac{5}{13}(80 - 5a)$$



Bending moment at C



$+\circlearrowleft M_C = 0$

$$5a + M_C = 0$$

$$M_C = -5a$$

Equate $-M_C = M_D$

$$5a = \frac{5}{13}(80 - 5a)$$

$a = 4.4444$ ft (a) $a = 4.44$ ft ◄

Then $-M_C = M_D = (5)(4.4444) = 22.222$ kip ft

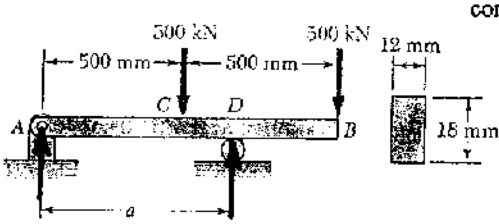
$|M|_{max} = 22.222$ kip·ft $= 266.67$ kip·in

For W14×22 rolled steel section $S = 29.0 \text{ in}^3$

Normal stress $\sigma = \frac{M}{S} = \frac{266.67}{29.0} = 9.20$ ksi (b) 9.20 ksi ◄

Problem 5.29

5.29 For the beam and loading shown, determine (*a*) the distance *a* for which the absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27)

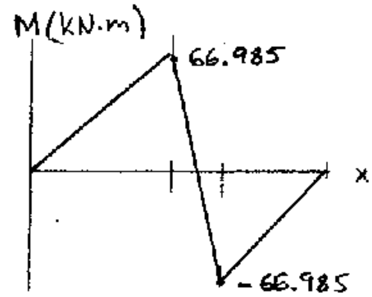


Reaction at A $\circlearrowleft \Sigma M_D = 0$

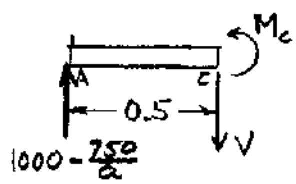
$$-Aa + (500)(a - 0.5) - 500(1 - a) = 0$$

$$Aa = 1000a - 750$$

$$A = 1000 - \frac{750}{a}$$



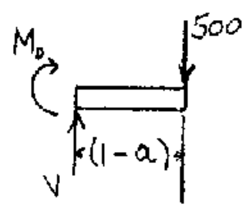
Bending moment at C $\circlearrowleft \Sigma M_C = 0$



$$-(0.5)\left(1000 - \frac{750}{a}\right) + M_C = 0$$

$$M_C = 500 - \frac{375}{a}$$

Bending moment at D $\circlearrowleft \Sigma M_D = 0$



$$-M_D - (500)(1 - a) = 0$$

$$M_D = -500(1 - a)$$

(a) Equate $\quad -M_D = M_C \qquad 500(1 - a) = 500 - \frac{375}{a}$

$$a = 0.86603 \text{ m} \qquad a = 866 \text{ mm}$$

$A = 133.98$ kN $\qquad M_C = 66.985$ kN·m $\qquad M_D = -66.985$ kN·m

For rectangular cross section $\quad S = \frac{1}{6} bh^3 = \frac{1}{6}(12)(18)^3 = 11.664 \times 10^3 \text{ mm}^3$

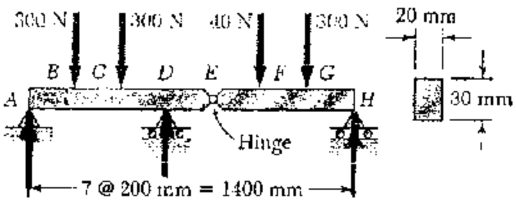
$$= 11.664 \times 10^{-6} \text{ m}^3$$

(b) Normal stress $\quad \sigma = \frac{|M|}{S} = \frac{66.985 \times 10^3}{11.664 \times 10^{-6}} = 5.74 \times 10^6 \text{ Pa}$

$$= 5.74 \text{ MPa}$$

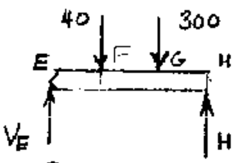
Problem 5.30

5.30 and 5.31 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



Free body EFGH

Note that $M_E = 0$ due to hinge.



$\circlearrowleft \Sigma M_E = 0$

$0.6\,H - (0.2)(40) - (0.4)(300) = 0$

$H = 213.33\ N$

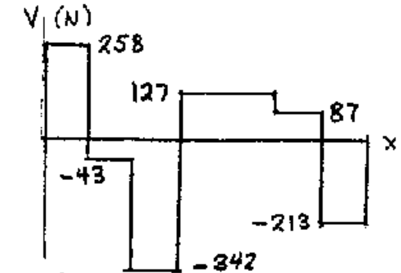
$+\uparrow \Sigma F_y = 0 \qquad V_E - 40 - 300 + 213.33 = 0$

$V_E = 126.67\ N$

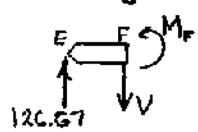
Shear: E to F $\quad V = 126.67$ N·m

F to G $\quad V = 86.67$ N·m

G to H $\quad V = -213.33$ N·m



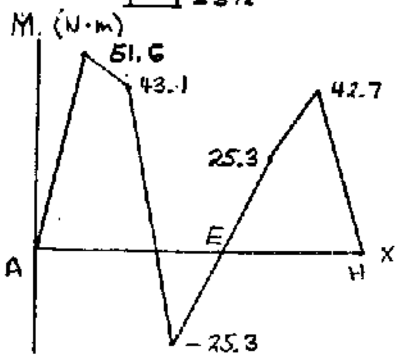
Bending moment at F



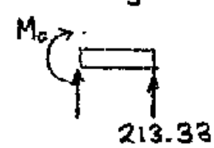
$\circlearrowleft \Sigma M_F = 0$

$M_F - (0.2)(126.67) = 0$

$M_F = 25.33$ N·m



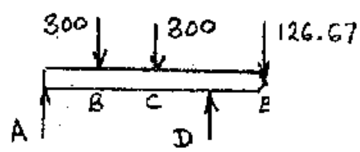
Bending moment at G



$\circlearrowleft \Sigma M_G = 0$

$-M_G + (0.2)(213.33) = 0$

$M_G = 42.67$ N·m



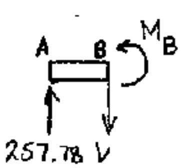
Free body ABCDE

$+\circlearrowleft \Sigma M_B = 0 \qquad -0.6\,A + (0.4)(300) + (0.2)(300) - (0.2)(126.67) = 0$

$A = 257.78\ N$

$+\circlearrowleft \Sigma M_A = 0 \qquad -(0.2)(300) - (0.4)(300) - (0.8)(126.67) + 0.6\,D = 0$

$D = 468.89\ N$

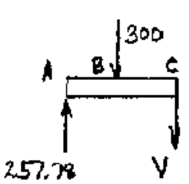


Bending moment at B

$\circlearrowleft \Sigma M_B = 0$

$-(0.2)(257.78) + M_B = 0$

$M_B = 51.56$ N·m

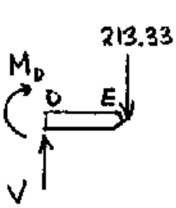


Bending moment at C

$+\circlearrowleft \Sigma M_C = 0$

$-(0.4)(257.78) + (0.2)(300) + M_C = 0$

$M_C = 43.11$ N·m



Bending moment at D

$+\circlearrowleft \Sigma M_D = 0$

$-M_D - (0.2)(213.33) = 0$

$M_D = -25.33$ N·m

max $|M| = 51.56$ N·m

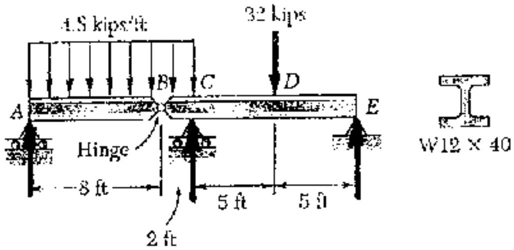
$S = \frac{1}{6} b h^2 = \frac{1}{6}(20)(30)^2 = 3 \times 10^3\ mm^3 = 3 \times 10^{-6}\ m^3$

Normal stress

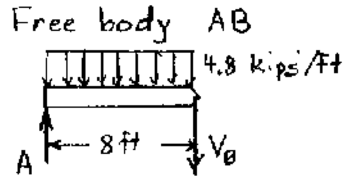
$$\sigma = \frac{51.56}{3 \times 10^{-6}} = 17.19 \times 10^6\ Pa = 17.19\ MPa$$

Problem 5.31

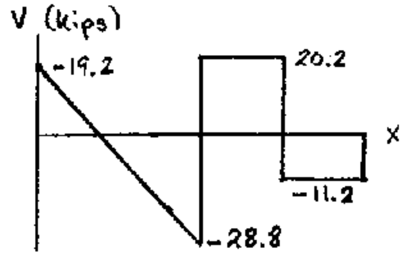
5.30 and 5.31 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



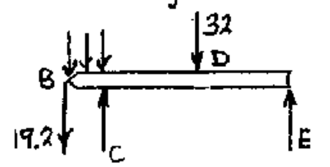
Free body AB



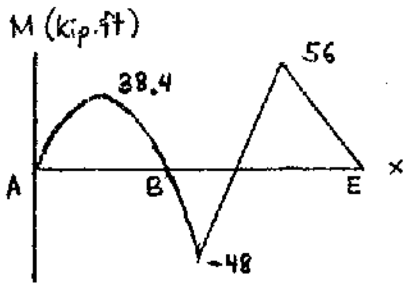
$\circlearrowleft \Sigma M_B = 0$
$(4.8)(8)(4) - 8A = 0$
$A = 19.2$ kips
$\circlearrowleft M_A = 0$
$-(4.8)(8)(4) - 8V_B = 0$
$V_B = -19.2$ kips



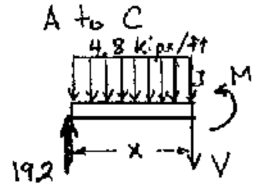
Free body BCDE



$+\circlearrowleft \Sigma M_E = 0$
$(19.2)(12) + (4.8)(2)(11)$
$- 10C + (32)(5) = 0$
$C = 49.2$ Kips
$+\circlearrowleft M_C = 0$
$(19.2)(2) + (4.8)(2)(1)$
$-(32)(5) + 10E = 0$
$E = 11.2$ kips



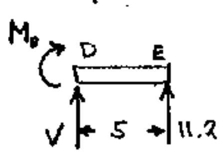
A to C



$0 < x < 10$ ft.
$+\uparrow \Sigma F_y = 0$
$19.2 - 4.8x - V = 0$
$V = 19.2 - 4.8x$ kips.
$\circlearrowleft M_J = 0$
$-19.2x + (4.8x)\frac{x}{2} + M = 0$
$M = 19.2x - 2.4x^2$ kip·ft

At C^- $x = 10$ $V = 19.2 - (4.8)(10) = -28.8$ Kips
At C $x = 10$ $M_C = (19.2)(10) - (2.4)(10)^2 = -48$ kip·ft
C to D $V = 19.2 - (4.8)(10) + 49.6 = 20.8$ Kips.
D to E $V = -11.2$ kips

Bending moment at D



$\circlearrowleft \Sigma M_D = 0$
$-M_D + (11.2)(5) = 0$
$M_D = 56$ kip·ft

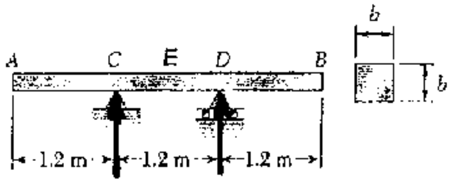
max $|M| = 56$ kip·ft $= 672$ kip·in

For W 12×40 rolled steel section $S = 51.9$ in^3

Normal stress $\sigma = \frac{|M|}{S} = \frac{672}{51.9} = 12.95$ ksi ◄

Problem 5.32

5.32 A solid steel bar has a square cross section of side b and is supported as shown. Knowing that for steel $\rho = 7860\ \text{kg/m}^3$, determine the dimension b for which the maximum normal stress due to bending is (*a*) 10 MPa, (*b*) 50 MPa.

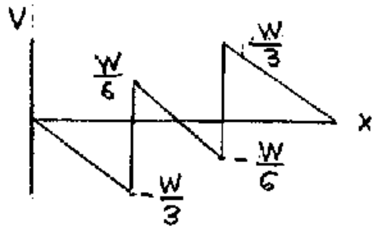


Weight density $\gamma = \rho g$

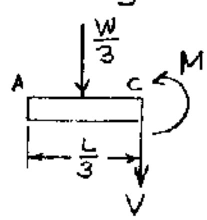
Let L = total length of beam

$$W = V\gamma = AL\rho g = b^2 L\rho g$$

Reactions at C and D $\quad C = D = \frac{W}{2}$



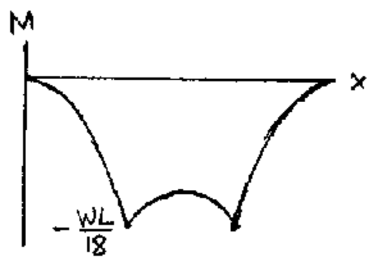
Bending moment at C



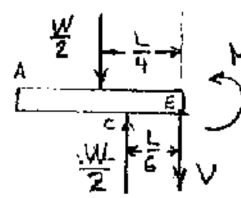
$$\circlearrowleft \Sigma M_C = 0$$

$$\left(\frac{L}{6}\right)\left(\frac{W}{3}\right) + M = 0$$

$$M = -\frac{WL}{18}$$



Bending moment at center of beam



$$\circlearrowleft \Sigma M_E = 0$$

$$\left(\frac{L}{4}\right)\left(\frac{W}{2}\right) - \left(\frac{L}{6}\right)\left(\frac{W}{2}\right) + M = 0$$

$$M = -\frac{WL}{24}$$

$$\max|M| = \frac{WL}{18} = \frac{b^2 L^2 \rho g}{18}$$

For a square section $\quad S = \frac{1}{6} b^3$

Normal stress $\quad \sigma = \frac{|M|}{S} = \frac{b^2 L^2 \rho g / 18}{b^3/6} = \frac{L^2 \rho g}{3b}$

Solve for b $\quad b = \frac{L^2 \rho g}{3\sigma}$

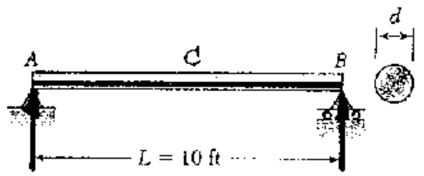
Data: $L = 3.6\ \text{m}$ $\quad \rho = 7860\ \text{kg/m}^3$ $\quad g = 9.81\ \text{m/s}^2$

(a) $\sigma = 10 \times 10^6\ \text{Pa}$ $\qquad$ (b) $\sigma = 50 \times 10^6\ \text{Pa}$

(a) $\quad b = \frac{(3.6)^2(7860)(9.81)}{(3)(10 \times 10^6)} = 33.3 \times 10^{-3}\ \text{m} = 33.3\ \text{mm}$ ◄

(b) $\quad b = \frac{(3.6)^2(7860)(9.81)}{(3)(50 \times 10^6)} = 6.66 \times 10^{-3}\ \text{m} = 6.66\ \text{mm}$ ◄

Problem 5.33



5.33 A solid steel rod of diameter d is supported as shown. Knowing that for steel $\gamma = 490\ \text{lb/ft}^3$, determine the smallest diameter d that can be used if the normal stress due to bending is not to exceed 4 ksi.

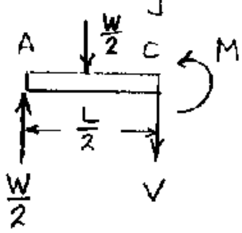
Let W = total weight

$$W = \mathcal{V}\gamma = AL\gamma = \frac{\pi}{4}d^2L\gamma$$

Reaction at A

$$A = \frac{1}{2}W$$

Bending moment at center of beam



$$+\circlearrowleft \Sigma M_C = 0$$

$$-\left(\frac{W}{2}\right)\left(\frac{L}{2}\right) + \left(\frac{W}{2}\right)\left(\frac{L}{4}\right) + M = 0$$

$$M = \frac{WL}{8} = \frac{\pi}{32}d^2L^2\gamma$$

For circular cross section $\left(c = \frac{1}{2}d\right)$

$$I = \frac{\pi}{4}c^4, \qquad S = \frac{I}{c} = \frac{\pi}{4}c^3 = \frac{\pi}{32}d^3$$

Normal stress

$$\sigma = \frac{M}{S} = \frac{\frac{\pi}{32}d^2L^2\gamma}{\frac{\pi}{32}d^3} = \frac{L^2\gamma}{d}$$

Solving for d $\qquad d = \dfrac{L^2\gamma}{\sigma}$

Data: $L = 10\ \text{ft} = (12)(10) = 120\ \text{in}$

$\gamma = 490\ \text{lb/ft}^3 = \dfrac{490}{12^3} = 0.28356\ \text{lb/in}^3$

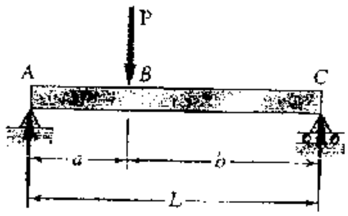
$\sigma = 4\ \text{ksi} = 4000\ \text{lb/in}^2$

$$d = \frac{(120)^2(0.28356)}{4000} = 1.021\ \text{in.}$$

Problem 5.34

5.34 Using the method of Sec. 5.3, solve Prob. 5.1*a*.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.



$\circlearrowleft \Sigma M_C = 0 \qquad LA - bP = 0 \qquad A = \frac{Pb}{L}$

$\circlearrowleft \Sigma M_A = 0 \qquad LC - aP = 0 \qquad C = \frac{Pa}{L}$

At A^+ $\quad V = A = \frac{Pb}{L} \qquad M = 0$

A to B^- $\quad 0 < x < a$

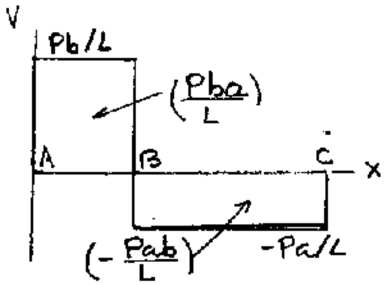
$w = 0 \qquad \int_0^x w\,dx = 0$

$V_B - V_A = 0 \qquad V_B = \frac{Pb}{L}$

$M_B - M_A = \int_0^a V\,dx = \int_0^a \frac{Pb}{L}\,dx = \frac{Pba}{L}$

$M_B = \frac{Pba}{L}$

At B^+ $\quad V = A - P = \frac{Pb}{L} - P = -\frac{Pa}{L}$



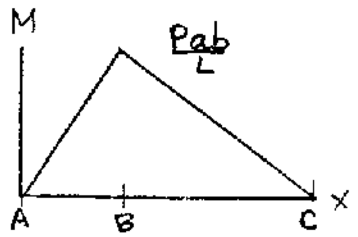
B^+ to C $\quad a < x < L$

$w = 0 \qquad \int_a^x w\,dx = 0$

$V_C - V_B = 0 \qquad V = -\frac{Pa}{L}$

$M_C - M_B = \int_a^L V\,dx = -\frac{Pa}{L}(L-a) = -\frac{Pab}{L}$

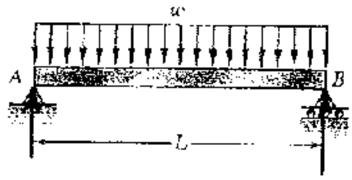
$M_C = M_B - \frac{Pab}{L} = \frac{Pba}{L} - \frac{Pab}{L} = 0$



Problem 5.35

5.35 Using the method of Sec. 5.3, solve Prob. 5.2*a*.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.



$\circlearrowleft \Sigma M_B = 0 \qquad -AL + wL\cdot\frac{L}{2} = 0 \qquad A = \frac{wL}{2}$

$\circlearrowleft \Sigma M_A = 0 \qquad BL - wL\cdot\frac{L}{2} = 0 \qquad B = \frac{wL}{2}$

$\frac{dV}{dx} = -w$

$V - V_A = -\int_0^x w\,dx = -wx$

$V = V_A - wx = A - wx = \frac{wL}{2} - wx$

$\frac{dM}{dx} = V$

$M - M_A = \int_0^x V\,dx = \int_0^x \left(\frac{wL}{2} - wx\right)dx$

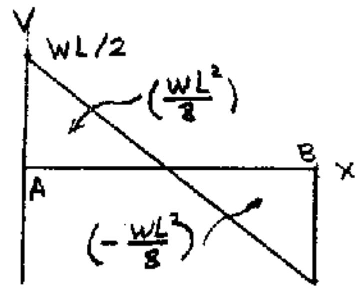
$= \frac{wLx}{2} - \frac{wx^2}{2}$

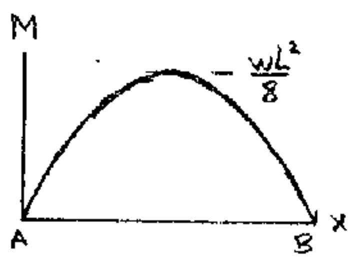
$M = M_A + \frac{wLx}{2} - \frac{wx^2}{2} = \frac{w}{2}(Lx - x^2)$

Maximum M occurs at $x = \frac{L}{2}$ where

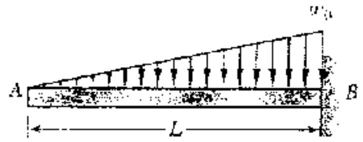
$V = \frac{dM}{dx} = 0$

$\text{Max } M = \frac{wL^2}{8}$





Problem 5.36



5.36 Using the method of Sec. 5.3, solve Prob. 5.3*a*.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown

$$w = w_0 \frac{x}{L}$$

$$V_A = 0 \,, \qquad M_A = 0$$

$$\frac{dV}{dx} = -w = -\frac{w_0 x}{L}$$

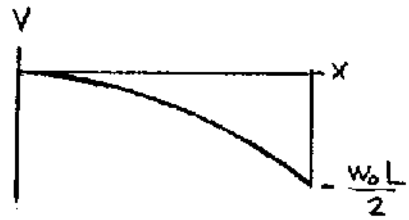
$$V - V_A = -\int_0^x \frac{w_0 x}{L} = -\frac{w_0 x^2}{2L}$$

$$V = -\frac{w_0 x^2}{2L}$$

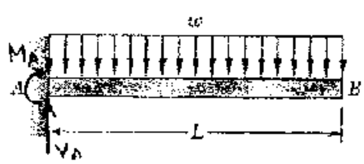
$$\frac{dM}{dx} = V = -\frac{w_0 x^2}{2L}$$

$$M - M_A = \int_0^x V\,dx = -\int_0^x \frac{w_0 x^2}{2L}\,dx$$

$$= -\frac{w_0 x^3}{6L}$$



Problem 5.37



5.37 Using the method of Sec. 5.3, solve Prob. 5.4*a*.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.

$$+\uparrow \Sigma F_y = 0 \qquad V_A - wL = 0 \qquad V_A = wL$$

$$\circlearrowleft \Sigma M_A = 0 \qquad -M - (wL)(\tfrac{L}{2}) = 0 \qquad M_A = -\frac{wL^2}{2}$$

$$\frac{dV}{dx} = -w$$

$$V - V_A = -\int_0^x w\,dx = -wx$$

$$V = wL - wx$$

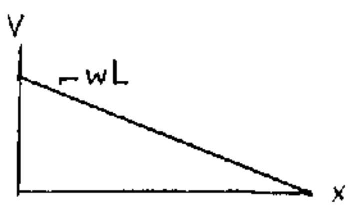
$$\frac{dM}{dx} = V = wL - wx$$

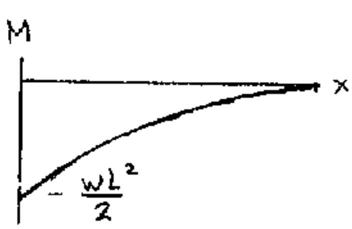
$$M - M_A = \int_0^x (wL - wx)\,dx = wLx - \frac{wx^2}{2}$$

$$M = -\frac{wL^2}{2} + wLx - \frac{wx^2}{2}$$

$$\max |V| = wL$$

$$\max |M| = \frac{wL^2}{2}$$

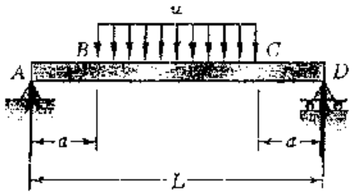


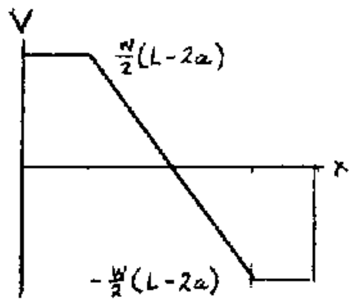


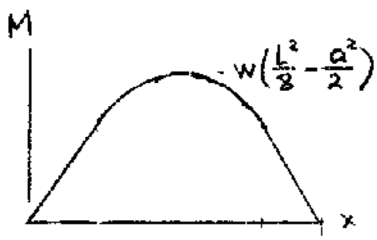
Problem 5.38

5.38 Using the method of Sec. 5.3, solve Prob. 5.5*a*.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.







Reactions $\quad A = D = \frac{1}{2}w(L-2a)$

At A $\quad V_A = A = \frac{1}{2}w(L-2a), \quad M_A = 0$

A to B $\quad 0 < x < a \qquad w = 0$

$$V_B - V_A = -\int_0^a w\,dx = 0$$

$$V_B = V_A = \frac{1}{2}w(L-2a)$$

$$M_B - M_A = \int_0^a V\,dx = \int_0^a \frac{1}{2}w(L-2a)\,dx$$

$$M_B = \frac{1}{2}w(L-2a)a$$

B to C $\quad a < x < L-a \qquad w = w$

$$V - V_B = -\int_a^x w\,dx = -w(x-a)$$

$$V = \frac{1}{2}w(L-2a) - w(x-a) = \frac{1}{2}w(L-2x)$$

$$\frac{dM}{dx} = V = \frac{1}{2}w(L-2x)$$

$$M - M_B = \int_a^x V\,dx = \frac{1}{2}w(Lx - x^2)\Big|_a^x$$

$$= \frac{1}{2}w(Lx - x^2 - La + a^2)$$

$$M = \frac{1}{2}w(L-2a)a + \frac{1}{2}w(Lx - x^2 - La + a^2)$$

$$= \frac{1}{2}w(Lx - x^2 - a^2)$$

At C $\quad x = L-a \qquad V_C = -\frac{1}{2}w(L-2a) \qquad M_C = \frac{1}{2}(L-2a)a$

C to D $\quad V = V_C = -\frac{1}{2}w(L-2a)$

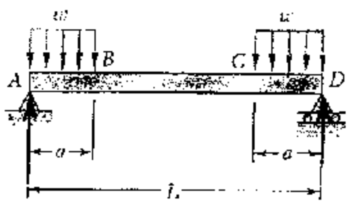
$$M_D = 0$$

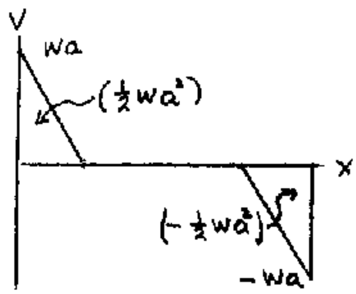
At $x = \frac{L}{2} \qquad M_{max} = w\left(\frac{L^2}{8} - \frac{a^2}{2}\right)$

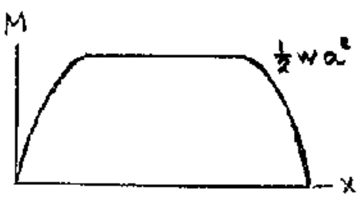
Problem 5.39

5.39 Using the method of Sec. 5.3, solve Prob. 5.6*a*.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.







Reactions $\quad A = D = wa$

A to B $\quad 0 < x < a \quad w = w$

$$V_A = A = wa, \qquad M_A = 0$$

$$V - V_A = -\int_0^x w\,dx = -wx$$

$$V = w(a - x) \qquad V_B = 0$$

$$\frac{dM}{dx} = V = wa - wx$$

$$M - M_A = \int_0^x V\,dx = \int_0^x (wa - wx)\,dx$$

$$= wax - \tfrac{1}{2}wx^2$$

$$M_B = \tfrac{1}{2}wa^2 \quad \text{at } x = a.$$

B to C $\quad a < x < L - a \quad V = 0$

$$\frac{dM}{dx} = V = 0$$

$$M - M_B = \int_a^x V\,dx = 0$$

$$M = M_B = \tfrac{1}{2}wa^2$$

C to D $\quad V - V_C = -\int_{L-a}^{x} w\,dx = -w\,[x - (L-a)]$

$$V = -w\,[x - (L-a)]$$

$$M - M_C = \int_{L-a}^{x} V\,dx = \int_{L-a}^{x} -w\,[x - (L-a)]\,dx$$

$$= -w\left[\frac{x^2}{2} - (L-a)x\right]\Big|_{L-a}^{x}$$

$$= -w\left[\frac{x^2}{2} - (L-a)x - \frac{(L-a)^2}{2} + (L-a)^2\right]$$

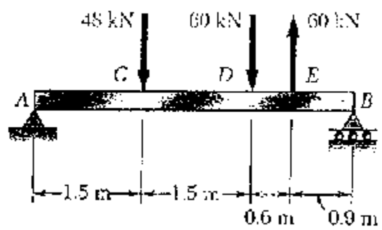
$$= -w\left[\frac{x^2}{2} - (L-a)x + \frac{(L-a)^2}{2}\right]$$

$$M = \tfrac{1}{2}wa^2 - w\left[\frac{x^2}{2} - (L-a)x + \frac{(L-a)^2}{2}\right]$$

Problem 5.40

5.40 Using the method of Sec. 5.3, solve Prob. 5.7.

5.7 and 5.8 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

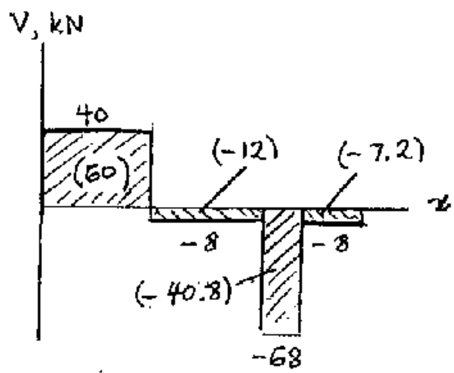


Reactions.

$+\circlearrowleft \Sigma M_B = 0$

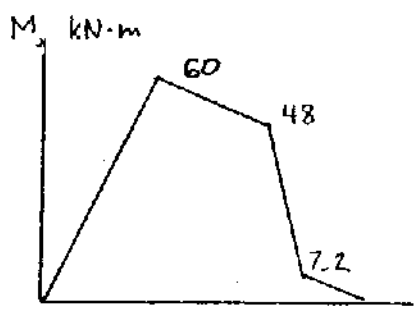
$-1.5 R_A + (3.0)(48) + (1.5)(60) - (0.9)(60) = 0$

$R_A = 40 \text{ kN}$



Shear

A to C^-	$V = 40 \text{ kN}$
C^+ to D^-	$V = 40 - 48 = -8 \text{ kN}$
D^+ to E^-	$V = -8 - 60 = -68 \text{ kN}$
E^+ to B	$V = -68 + 60 = -8 \text{ kN}$



Areas of Shear Diagram.

A to C	$A_{AC} = (1.5)(40) = 60 \text{ kN}\cdot\text{m}$
C to D	$A_{CD} = (1.5)(-8) = -12 \text{ kN}\cdot\text{m}$
D to E	$A_{DE} = (0.6)(-68) = -40.8 \text{ kN}\cdot\text{m}$
E to B	$A_{EB} = (0.9)(-8) = -7.2 \text{ kN}\cdot\text{m}$

Bending moments.

$M_A = 0$

$M_C = 0 + 60 = 60 \text{ kN}\cdot\text{m}$

$M_D = 60 - 12 = 48 \text{ kN}\cdot\text{m}$

$M_E = 48 - 40.8 = 7.2 \text{ kN}\cdot\text{m}$

$M_B = 7.2 - 7.2 = 0$

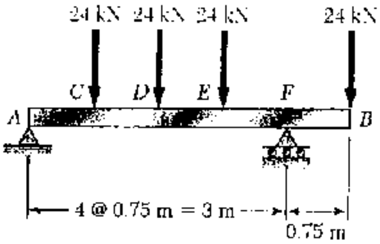
From the diagrams (a) $|V|_{max} = 68.0 \text{ kN}\cdot\text{m}$ ◄

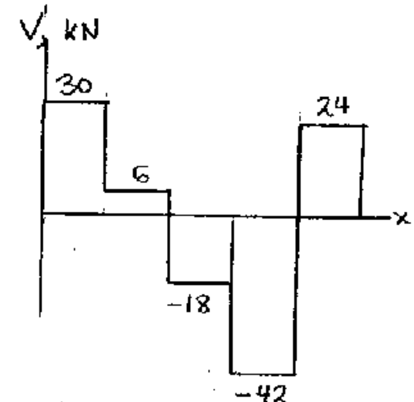
(b) $|M|_{max} = 60.0 \text{ kN}\cdot\text{m}$ ◄

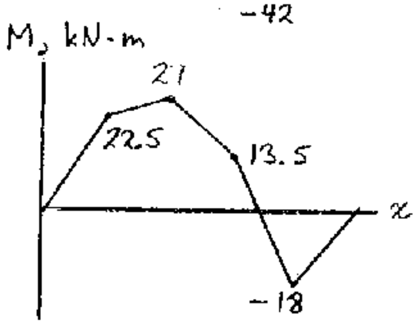
Problem 5.41

5.41 Using the method of Sec. 5.3, solve Prob. 5.8.

5.7 and 5.8 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.







Reactions at A and F.

$+\circlearrowleft \Sigma M_F = 0$

$-3R_A + (2.25)(24) + (1.50)(24) + (0.75)(24) - (0.75)(24) = 0$

$R_A = 30 \text{ kN} \uparrow$

$+\circlearrowleft \Sigma M_A = 0$

$-(0.75)(24) - (1.50)(24) - (2.25)(24) + 3R_F - (3.75)(24) = 0$

$R_F = 66 \text{ kN} \uparrow$

Shear diagram.

A to C $\quad V = 30 \text{ kN}$

C to D $\quad V = 30 - 24 = 6 \text{ kN}$

D to E $\quad V = 6 - 24 = -18 \text{ kN}$

E to F $\quad V = -18 - 24 = -42 \text{ kN}$

F to B $\quad V = -42 + 66 = 24 \text{ kN}$

Areas of shear diagram

A to C $\quad A_{AC} = (0.75)(30) = 22.5 \text{ kN·m}$

C to D $\quad A_{CD} = (0.75)(6) = 4.5 \text{ kN·m}$

D to E $\quad A_{DE} = (0.75)(-18) = -13.5 \text{ kN·m}$

E to F $\quad A_{EF} = (0.75)(-42) = -31.5 \text{ kN·m}$

F to B $\quad A_{FB} = (0.75)(24) = 18 \text{ kN·m}$

Bending moments.

$M_A = 0$

$M_C = 0 + 22.5 = 22.5 \text{ kN·m}$

$M_D = 22.5 + 4.5 = 27 \text{ kN·m}$

$M_E = 27 - 13.5 = 13.5 \text{ kN·m}$

$M_F = 13.5 - 31.5 = -18 \text{ kN·m}$

$M_B = -18 + 18 = 0$

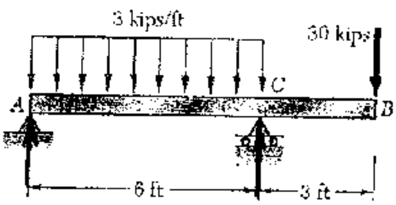
(a) $|V|_{max} = 42.0 \text{ kN}$ ◄

(b) $|M|_{max} = 27.0 \text{ kN·m}$ ◄

Problem 5.42

5.42 Using the method of Sec. 5.3, solve Prob. 5.9.

5.9 and 5.10 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment



$+\circlearrowleft \Sigma M_C = 0 \quad -6A + (3)(18) - (3)(30) = 0$

$A = -6$ kips i.e. 6 kips $\downarrow$

$+\circlearrowleft \Sigma M_A = 0 \quad 6C - (3)(18) - (9)(30) = 0$

$C = 54$ kips $\uparrow$

Shear

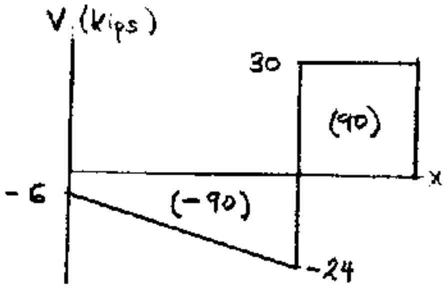
$V_A = -6$ kips

A to C $\quad 0 < x < 6$ ft. $\quad w = -3$ kips/ft

$V_B - V_A = -\int_0^6 w\,dx = -\int_0^6 3\,dx = -18$ kips

$V_B = -6 - 18 = -24$ kips

C to B $\quad V = -24 + 54 = 30$ kips.



Areas under shear diagram

A to C $\quad \int V dx = (\frac{1}{2})(-6-24)(6)$
$= -90$ kip ft.

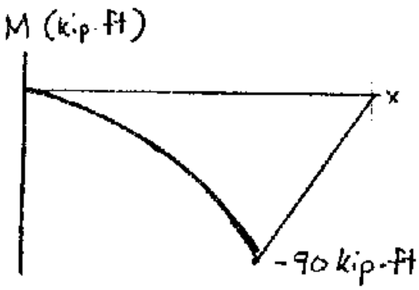
C to B $\quad \int V\,dx = (3)(30) = 90$ kip·ft

Bending moments

$M_A = 0$

$M_C = M_A + \int V dx = 0 - 90 = -90$ kip·ft

$M_B = M_C + \int V dx = -90 + 90 = 0$



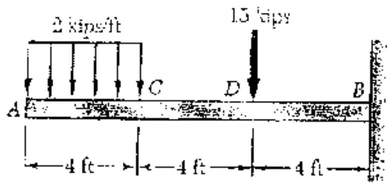
Maximum $|V| = 30$ kips ◄

Maximum $|M| = 90$ kip·ft ◄

Problem 5.43

5.43 Using the method of Sec. 5.3, solve Prob. 5.10.

5.9 and 5.10 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.



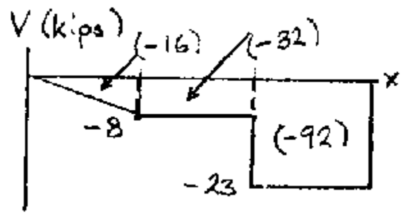
Shear

$V_A = 0$

$V_B = V_A - \int_A^B w\,dx = 0 - (4)(2) = -8$ kips

C to D $\quad V = -8$ kips

D to B $\quad V = -8 - 15 = -23$ kips



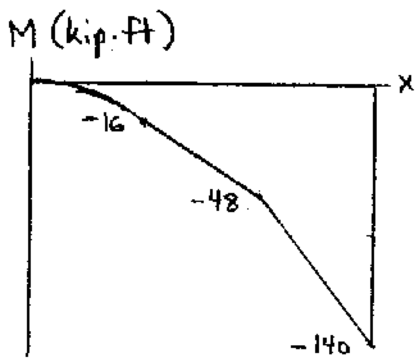
Areas under shear diagram

A to C $\quad \int V dx = (\frac{1}{2})(4)(-8) = -16$ kip-ft

C to D $\quad \int V dx = (4)(-8) = -32$ kip ft

D to B $\quad \int V dx = (4)(-23) = -92$ kip·ft

Bending moments



$M_A = 0$

$M_C = M_A + \int V dx = 0 - 16 = -16$ kip ft

$M_D = M_C + \int V dx = -16 - 32 = -48$ kip·ft

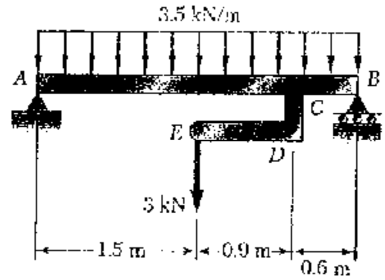
$M_B = M_D + \int V dx = -48 - 92 = -140$ kip·ft

Maximum $|V| = 23$ kips ◄

Maximum $|M| = 140$ kip·ft ◄

Problem 5.44

5.44 and 5.45 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.



Reaction at A.

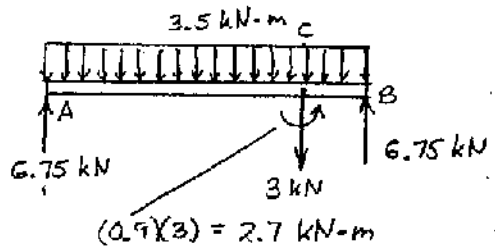
$+\circlearrowleft \sum M_B = 0$

$-3.0A + (1.5)(3.0)(3.5) + (1.5)(3) = 0$

$A = 6.75 \text{ kN}$

Reaction at B. $B = 6.75 \text{ kN}$

Beam ACB and loading. See sketch.



Areas of load diagram.

A to C $(2.4)(3.5) = 8.4 \text{ kN}$

C to B $(0.6)(3.5) = 2.1 \text{ kN}$

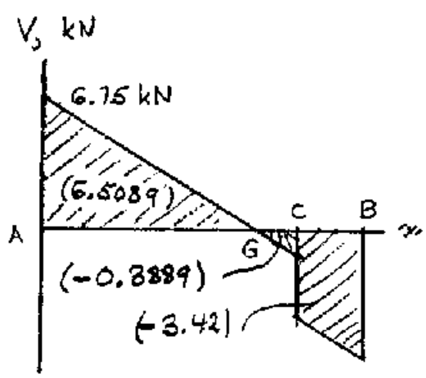
Shear diagram.

$V_A = 6.75 \text{ kN}$

$V_C^- = 6.75 - 8.4 = -1.65 \text{ kN}$

$V_C^+ = -1.65 - 3 = -4.65 \text{ kN}$

$V_B = -4.65 - 2.1 = -6.75 \text{ kN}$



Over A to C $V = 6.75 - 3.5x$

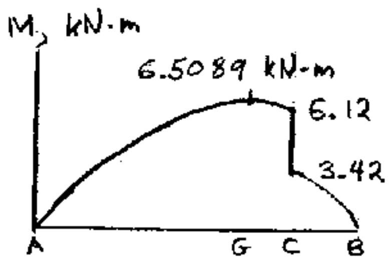
At G $V = 6.75 - 3.5x_G = 0$ $x_G = 1.9286 \text{ m}$

Areas of shear diagram.

A to G $\frac{1}{2}(1.9286)(6.75) = 6.5089 \text{ kN}\cdot\text{m}$

G to C $\frac{1}{2}(0.4714)(-1.65) = -0.3889 \text{ kN}\cdot\text{m}$

C to B $\frac{1}{2}(0.6)(-4.65 - 6.75) = -3.42 \text{ kN}\cdot\text{m}$



Bending moments.

$M_A = 0$

$M_G = 0 + 6.5089 = 6.5089 \text{ kN}\cdot\text{m}$

$M_C^- = 6.5089 - 0.3889 = 6.12 \text{ kN}\cdot\text{m}$

$M_C^+ = 6.12 - 2.7 = 3.42 \text{ kN}\cdot\text{m}$

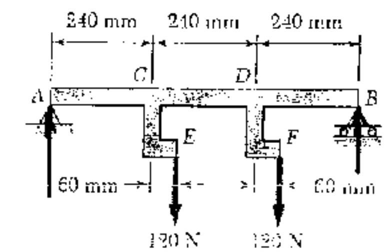
$M_B = 3.42 - 3.42 = 0$

(a) $|V|_{max} = 6.75 \text{ kN}$ ◄

(b) $|M|_{max} = 6.51 \text{ kN}\cdot\text{m}$ ◄

Problem 5.45

***5.44* and 5.45** Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

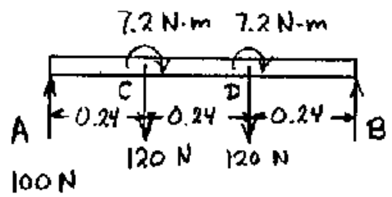


$\circlearrowleft + \Sigma M_B = 0 \qquad -0.72A + (0.48)(120) + (0.24)(120) - 7.2 - 7.2 = 0$

$A = 100 \text{ N}$

$\circlearrowleft + \Sigma M_A = 0 \qquad -(0.24)(120) - (0.48)(120) - 7.2 - 7.2 + 0.72B = 0$

$B = 140 \text{ N}$



Shear

A to C $\quad V = 100 \text{ N}$

C to D $\quad V = 100 - 120 = -20 \text{ N}$

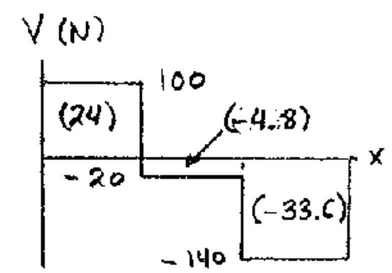
D to B $\quad V = -20 - 120 = -140 \text{ N}$

Areas under shear diagram

A to C $\quad \int V\,dx = (0.24)(100) = 24 \text{ N·m}$

C to D $\quad \int V\,dx = (0.24)(-20) = -4.8 \text{ N·m}$

D to B $\quad \int V\,dx = (0.24)(-140) = -33.6 \text{ N·m}$



Bending moments

$M_A = 0$

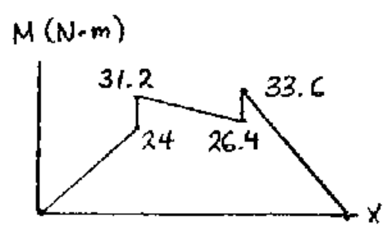
$M_C^- = 0 + 24 = 24 \text{ N·m}$

$M_C^+ = 24 + 7.2 = 31.2 \text{ N·m}$

$M_D^- = 31.2 - 4.8 = 26.4 \text{ N·m}$

$M_D^+ = 26.4 + 7.2 = 33.6 \text{ N·m}$

$M_B = 33.6 - 33.6 = 0$



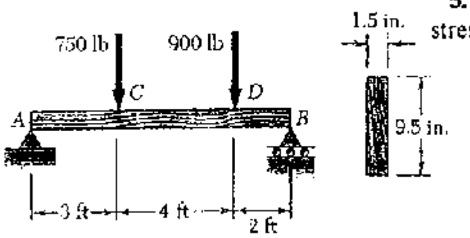
Maximum $|V| = 140 \text{ N}$ ◄

Maximum $|M| = 33.6 \text{ N·m}$ ◄

Problem 5.46

5.46 Using the method of Sec. 5.3, solve Prob. 5.15.

5.15 and ***5.16*** For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.

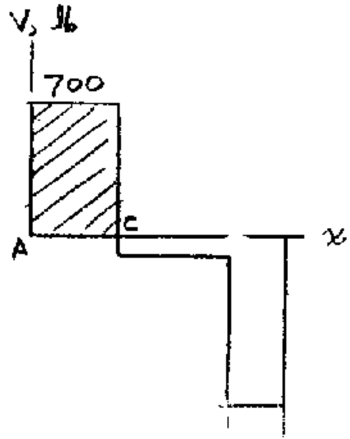


Reaction at A.

$+\circlearrowleft \Sigma M_B = 0$

$-9A + (6)(750) + (2)(900) = 0$

$A = 700$ lb.



Shear curve from A to C: $V = 700$ lb.

Area of shear curve from A to C

$A_{AC} = (3)(700) = 2100$ lb·ft

Bending moments. $M_A = 0$

$M_C = M_A + A_{AC} = 0 + 2100 = 2100$ lb·ft

$= 25.2 \times 10^3$ lb·in $= 25.2$ kip·in.

For the cross section $I = \frac{1}{12}(1.5)(9.5)^3 = 107.172$ in^4

$c = 4.75$ in.

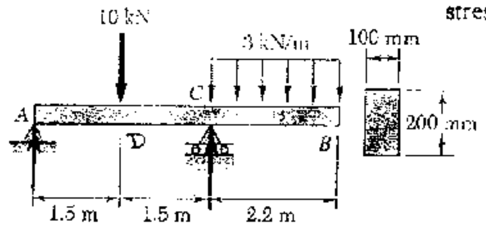
Maximum normal stress due to bending.

$\sigma = \frac{Mc}{I} = \frac{(25.2)(4.75)}{107.172}$ $\sigma = 1.117$ ksi

Problem 5.47

5.47 Using the method of Sec. 5.3, solve Prob. 5.16.

5.15 and ***5.16*** For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.



$\circlearrowleft \Sigma M_C = 0$

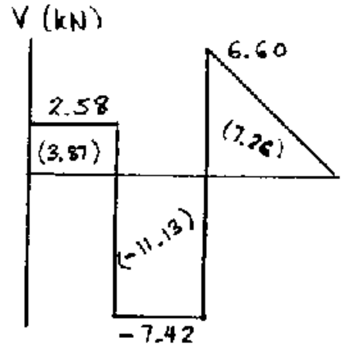
$-3A + (1.5)(10) - (1.1)(2.2)(3) = 0$

$A = 2.58 \text{ kN}$

$+\circlearrowleft \Sigma M_A = 0$

$-(1.5)(10) + 3C - (4.1)(2.2)(3) = 0$

$C = 14.02 \text{ kN}$



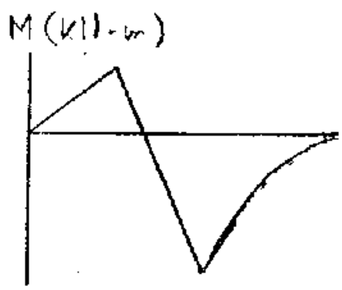
Shear

A to D^- $\quad V = 2.58 \text{ kN}$

D^+ to C^- $\quad V = 2.58 - 10 = -7.42 \text{ kN}$

C^+ $\quad V = -7.42 + 14.02 = 6.60 \text{ kN}$

B $\quad V = 6.60 - (2.2)(3) = 0$



Areas under shear diagram

A to D $\quad \int V dx = (1.5)(2.58) = 3.87 \text{ kN·m}$

D to C $\quad \int V dx = (1.5)(-7.42) = -11.13 \text{ kN·m}$

C to B $\quad \int V dx = (\frac{1}{2})(2.2)(6.60) = 7.26 \text{ kN·m}$

Bending moments

$M_A = 0$

$M_D = 0 + 3.87 = 3.87 \text{ kN·m}$

$M_C = 3.87 - 11.13 = -7.26 \text{ kN·m}$

$M_B = 7.26 - 7.26 = 0$

$|M_C| = 7.26 \text{ kN·m} = 7.26 \times 10^3 \text{ N·m}$

For rectangular cross section $\quad S = \frac{1}{6} b h^2 = (\frac{1}{6})(100)(200)^2$

$= 666.67 \times 10^3 \text{ mm}^3 = 666.67 \times 10^{-6} \text{ m}^3$

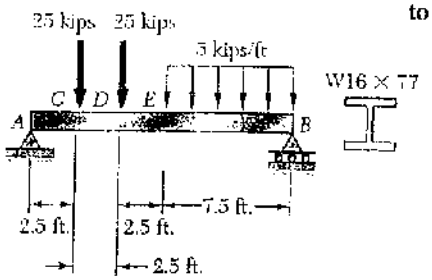
Normal stress $\quad \sigma = \frac{|M_C|}{S} = \frac{7.26 \times 10^3}{666.67 \times 10^{-6}} = 10.89 \times 10^6 \text{ Pa}$

$= 10.89 \text{ MPa}$

Problem 5.48

5.48 Using the method of Sec. 5.3, solve Prob. 5.17.

5.17 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.



$+\circlearrowleft \Sigma M_B = 0$

$-15A + (12.5)(25) + (10)(25) + (3.75)(7.5)(5) = 0$

$A = 46.875$ kips

Shear A to C $\quad V = 46.875$ kips

Area under shear curve A to C $\quad \int V dx = (2.5)(46.875)$

$= 117.1875$ kip·ft

$M_A = 0$

$M_C = 0 + 117.1875 = 117.1875$ kip·ft $= 1406.25$ kip·in

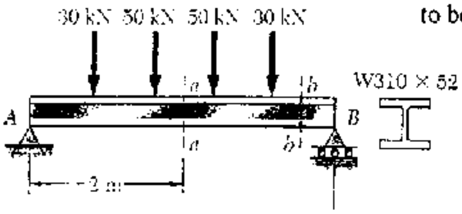
For W 16×77 rolled steel section $\quad S = 134\ in^3$

Normal stress $\quad \sigma = \frac{M}{S} = \frac{1406.25}{134} = 10.49$ ksi ◄

Problem 5.49

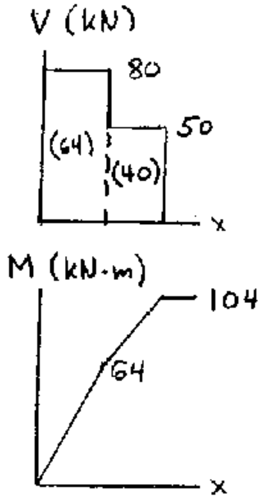
5.49 Using the method of Sec. 5.3, solve Prob. 5.18.

5.18 For the beam and loading shown, determine the maximum normal stress due to bending on section *a-a*.



Reactions: By symmetry $A = B$

$+\uparrow \Sigma F_y = 0 \qquad A = B = 80 \text{ kN}$



Shear

A to C $\quad V = 80 \text{ kN}$

C to D $\quad V = 80 - 30 = 50 \text{ kN}$

D to E $\quad V = 50 - 50 = 0$

Areas under shear diagram

A to C $\quad \int V dx = (80)(0.8) = 64 \text{ kN}\cdot\text{m}$

C to D $\quad \int V dx = (50)(0.8) = 40 \text{ kN}\cdot\text{m}$

D to E $\quad \int V dx = 0$

Bending moments

$M_A = 0$

$M_C = 0 + 64 = 64 \text{ kN}\cdot\text{m}$

$M_D = 64 + 40 = 104 \text{ kN}\cdot\text{m}$

$M_E = 104 + 0 = 104 \text{ kN}\cdot\text{m}$

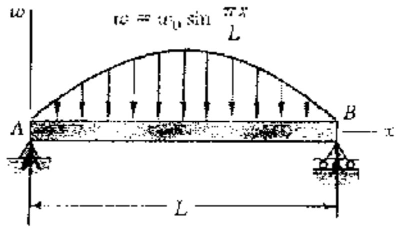
$M_E = 104 \text{ kN}\cdot\text{m} = 104 \times 10^3 \text{ N}\cdot\text{m}$

For W 310 × 52 $\quad S = 748 \times 10^3 \text{ mm}^3 = 748 \times 10^{-6} \text{ m}^3$

Normal stress $\quad \sigma = \dfrac{|M|}{S} = \dfrac{104 \times 10^3}{748 \times 10^{-6}} = 139.0 \times 10^6 \text{ Pa} = 139.0 \text{ MPa}$

Problem 5.50

5.50 and *5.51* Determine (*a*) the equations of the shear and bending-moment curves for the beam and loading shown, (*b*) the maximum absolute value of the bending moment in the beam.



$$\frac{dV}{dx} = -w = -w_0 \sin \frac{\pi x}{L}$$

$$V = \frac{w_0 L}{\pi} \cos \frac{\pi x}{L} + C_1 = \frac{dM}{dx}$$

$$M = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi x}{L} + C_1 x + C_2$$

$M = 0$ at $x = 0$ $\qquad C_2 = 0$

$M = 0$ at $x = L$ $\qquad 0 = 0 + C_1 L + 0 \qquad C_1 = 0$

$$V = \frac{w_0 L}{\pi} \cos \frac{\pi x}{L}$$

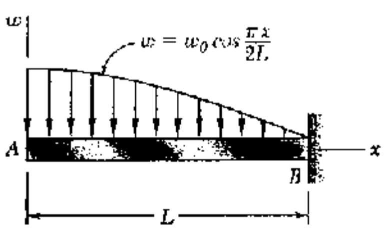
$$M = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi x}{L}$$

$$\frac{dM}{dx} = V = 0 \quad \text{at} \quad x = \frac{L}{2}$$

$$M_{max} = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi}{2} = \frac{w_0 L^2}{\pi^2}$$

Problem 5.51

5.50 and *5.51* Determine (*a*) the equations of the shear and bending-moment curves for the beam and loading shown, (*b*) the maximum absolute value of the bending moment in the beam.



$$\frac{dV}{dx} = -w = -w_0 \cos \frac{\pi x}{2L}$$

$$V = -\frac{2Lw_0}{\pi} \sin \frac{\pi x}{2L} + C_1 = \frac{dM}{dx}$$

$$M = \frac{4L^2 w_0}{\pi^2} \cos \frac{\pi x}{2L} + C_1 x + C_2$$

$V = 0$ at $x = 0$ $\qquad$ Hence $C_1 = 0$

$M = 0$ at $x = 0$. $\qquad$ Hence $C_2 = -\frac{4L^2 w_0}{\pi^2}$

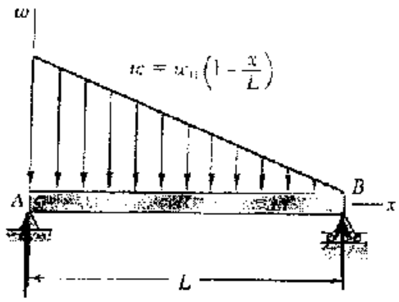
(a) $\qquad V = -(2Lw_0/\pi) \sin(\pi x/2L)$

$$M = -(4L^2 w_0/\pi^2)[1 - \cos(\pi x/2L)]$$

(b) $\qquad |M|_{max} = 4w_0 L^2/\pi^2$

Problem 5.52

5.52 Determine (*a*) the equations of the shear and bending-moment curves for the beam and loading shown, (*b*) the maximum absolute value of the bending moment in the beam.



$$w = w_0\left(1 - \frac{x}{L}\right)$$

$$\frac{dV}{dx} = -w = -w_0 + \frac{w_0 x}{L}$$

$$V = -w_0 x + \frac{w_0 x^2}{2L} + C_1 = \frac{dM}{dx}$$

$$M = -\frac{w_0 x^2}{2} + \frac{w_0 x^3}{6L} + C_1 x + C_2$$

$M = 0$ at $x = 0$ $\qquad C_2 = 0$

$M = 0$ at $x = L$ $\qquad 0 = -\frac{w_0 L^2}{2} + \frac{w_0 L^2}{6} + C_1 L \quad \therefore \quad C_1 = \frac{w_0 L}{3}$

$$V = -w_0 x + \frac{w_0 x^2}{2L} + \frac{w_0 L}{3}$$

$$M = -\frac{w_0 x^2}{2} + \frac{w_0 x^3}{6L} + \frac{w_0 L x}{3}$$

M is maximum where $\frac{dM}{dx} = V = 0$

$$0 = -w_0 x_m + \frac{w_0 x_m^2}{2L} + \frac{w_0 L}{3}$$

$$\frac{1}{2}x_m^2 - L x_m + \frac{1}{3}L^2 = 0 \qquad x_m = \frac{L \pm \sqrt{L^2 - (4)(\frac{1}{2})(\frac{1}{3}L^2)}}{(2)(\frac{1}{2})}$$

$$= \left(1 \pm \frac{\sqrt{3}}{3}\right)L$$

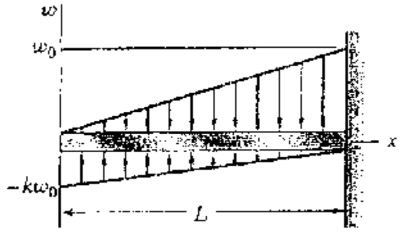
$$= \cancel{1.57735L},\ 0.42265\,L$$

$$M_{max} = \frac{-w_0(0.42265L)^2}{2} + \frac{w_0(0.42265L)^3}{6L} + \frac{w_0 L(0.42265L)}{3}$$

$$= 0.06415\ w_0 L^2$$

Problem 5.53

5.53 For the beam and loading shown, determine the equations of the shear and bending-moment curves and the maximum absolute value of the bending moment in the beam, knowing that (*a*) $k = 1$, (*b*) $k = 0.5$.



$$w = \frac{w_0 x}{L} - \frac{k w_0 (L-x)}{L} = (1+k)\frac{w_0 x}{L} - k w_0$$

$$\frac{dV}{dx} = -w = k w_0 - (1+k)\frac{w_0 x}{L}$$

$$V = k w_0 x - (1+k)\frac{w_0 x^2}{2L} + C_1$$

$V = 0$ at $x = 0$ $\qquad C_1 = 0$

$$\frac{dM}{dx} = V = k w_0 x - (1+k)\frac{w_0 x^2}{2L}$$

$$M = \frac{k w_0 x^2}{2} - (1+k)\frac{w_0 x^3}{6L} + C_2$$

$M = 0$ at $x = 0$ $\qquad C_2 = 0$

$$M = \frac{k w_0 x^2}{2} - \frac{(1+k) w_0 x^3}{6L}$$

(a) $k = 1$

$$V = w_0 x - \frac{w_0 x^2}{L}$$

$$M = \frac{w_0 x^2}{2} - \frac{w_0 x^3}{3L}$$

Maximum M occurs at $x = L$ $\qquad |M|_{max} = \frac{w_0 L^2}{6}$

(b) $k = \frac{1}{2}$

$$V = \frac{w_0 x}{2} - \frac{3 w_0 x^2}{4L}$$

$$M = \frac{w_0 x^2}{4} - \frac{w_0 x^3}{4L}$$

$V = 0$ at $x = \frac{2}{3}L$

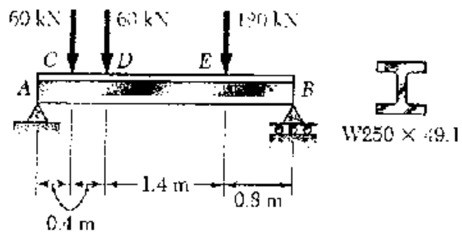
At $x = \frac{2}{3}L$ $\qquad M = \frac{w_0(\frac{2}{3}L)^2}{4} - \frac{w_0(\frac{2}{3}L)^3}{4L} = \frac{w_0 L^2}{27} = 0.03704\, w_0 L^2$

At $x = L$ $\qquad M = 0$

$$|M|_{max} = \frac{w_0 L^2}{27}$$

Problem 5.54

5.54 and 5.55 Draw the shear and bending- moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



$+\circlearrowleft \Sigma M_B = 0$

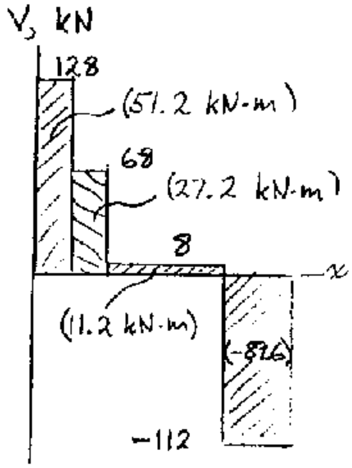
$-3.0\,A + (2.8)(60) + (2.2)(60) + (0.8)(120) = 0$

$A = 128 \text{ kN}$

$+\circlearrowleft \Sigma M_A = 0$

$-(0.4)(60) - (0.8)(60) - (2.2)(120) + 3.0\,B = 0$

$B = 112 \text{ kN}$

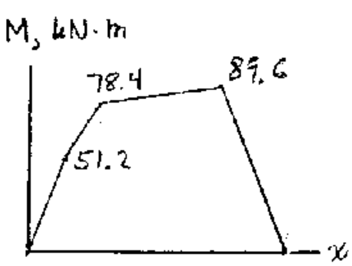


Shear diagram.

A to C^-	$V = 128 \text{ kN}$
C^+ to D^-	$V = 128 - 60 = 68 \text{ kN}$
D^+ to E^-	$V = 68 - 60 = 8 \text{ kN}$
E^+ to B	$V = 8 - 120 = -112 \text{ kN}$

Areas of shear diagram

A to C	$(0.4)(128) = 51.2 \text{ kN·m}$
C to D	$(0.4)(68) = 27.2 \text{ kN·m}$
D to E	$(1.4)(8) = 11.2 \text{ kN·m}$
E to B	$(0.8)(-112) = -89.6 \text{ kN·m}$



Bending moments

$M_A = 0$

$M_C = 0 + 51.2 = 51.2 \text{ kN·m}$

$M_D = 51.2 + 27.2 = 78.4 \text{ kN·m}$

$M_E = 78.4 + 11.2 = 89.6 \text{ kN·m}$

$M_B = 89.6 - 89.6 = 0$

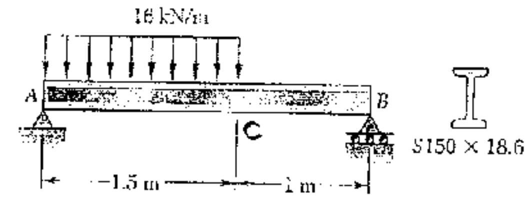
$|M|_{max} = 89.6 \text{ kN·m} = 89.6 \times 10^3 \text{ N·m}$

For W250 × 49.1 rolled steel section $S = 572 \times 10^3 \text{ mm}^3 = 572 \times 10^{-6} \text{ m}^3$

Normal stress $\sigma = \dfrac{|M|}{S} = \dfrac{89.6 \times 10^3}{572 \times 10^{-6}} = 156.6 \times 10^6 \text{ Pa} = 156.6 \text{ MPa}$ ◄

Problem 5.55

5.54 and 5.55 Draw the shear and bending- moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



$+\circlearrowleft \Sigma M_B = 0$

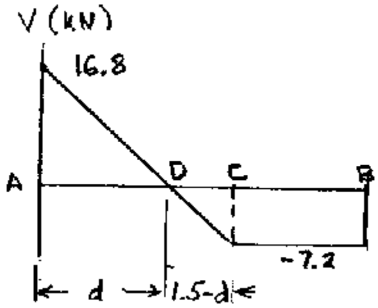
$-2.5A + (1.75)(1.5)(16) = 0$

$A = 16.8 \text{ kN}$

$+\circlearrowleft \Sigma M_A = 0$

$-(0.75)(1.5)(16) + 2.5B = 0$

$B = 7.2 \text{ kN}$

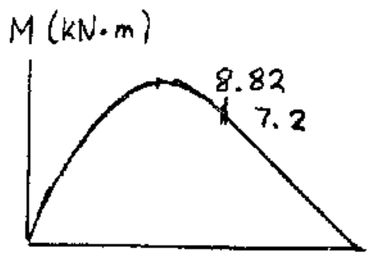


Shear

$V_A = 16.8 \text{ kN}$

$V_C = 16.8 - (1.5)(16) = -7.2 \text{ kN}$

$V_B = -7.2 \text{ kN}$



Locate point D where $V = 0$

$\frac{d}{16.8} = \frac{1.5-d}{7.2}$ $\qquad 24d = 25.2$

$d = 1.05 \text{ m}$ $\qquad 1.5 - d = 0.45 \text{ m}$

Areas under shear diagram

A to D $\quad \int V dx = (\frac{1}{2})(1.05)(16.8) = 8.82 \text{ kN}\cdot\text{m}$

D to C $\quad \int V dx = (\frac{1}{2})(0.45)(-7.2) = -1.62 \text{ kN}\cdot\text{m}$

C to B $\quad \int V dx = (1)(-7.2) = -7.2 \text{ kN}\cdot\text{m}$

Bending moments

$M_A = 0$

$M_D = 0 + 8.82 = 8.82 \text{ kN}\cdot\text{m}$

$M_C = 8.82 - 1.62 = 7.2 \text{ kN}\cdot\text{m}$

$M_B = 7.2 - 7.2 = 0$

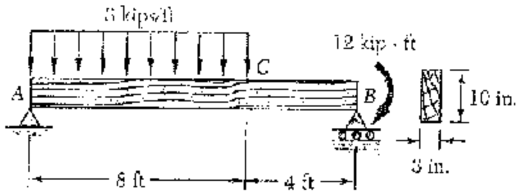
Maximum $|M| = 8.82 \text{ kN}\cdot\text{m} = 8.82 \times 10^3 \text{ N}\cdot\text{m}$

For S 150 × 18.6 rolled steel section $\quad S = 120 \times 10^3 \text{ mm}^3 = 120 \times 10^{-6} \text{ m}^3$

Normal stress $\quad \sigma = \frac{|M|}{S} = \frac{8.82 \times 10^3}{120 \times 10^{-6}} = 73.5 \times 10^6 \text{ Pa} = 73.5 \text{ MPa}$ ◀

Problem 5.56

5.56 and *5.57* Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



$\circlearrowleft \Sigma M_B = 0$

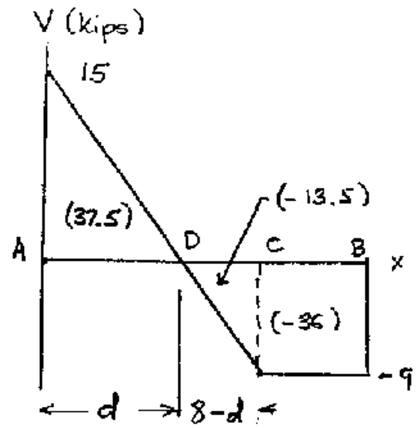
$-12A + (8)(8)(3) - 12 = 0$

$A = 15$ kips

$\circlearrowleft \Sigma M_A = 0$

$-(4)(8)(3) + 12B - 12 = 0$

$B = 9$ kips



Shear: $V_A = 15$ kips

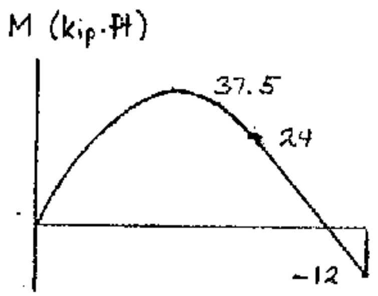
$V_C = 15 - (8)(3) = -9$ kips

C to B $V = -9$ kips.

Locate point D where $V = 0$

$\frac{d}{15} = \frac{8-d}{9}$ $\quad 24d = 120$

$d = 5$ ft $\quad 8 - d = 3$ ft



Areas under shear diagram

A to D $\int V\,dx = (\frac{1}{2})(5)(15) = 37.5$ kip·ft

D to C $\int V\,dx = (\frac{1}{2})(3)(-9) = -13.5$ kip ft

C to B $\int V\,dx = (4)(-9) = -36$ kip·ft

Bending moments: $M_A = 0$

$M_D = 0 + 37.5 = 37.5$ kip·ft

$M_C = 37.5 - 13.5 = 24$ kip·ft

$M_B = 24 - 36 = -12$ kip·ft

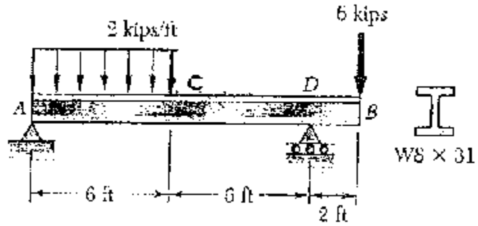
Maximum $|M| = 37.5$ kip ft $= 450$ kip·in

For rectangular cross section $S = \frac{1}{6}bh^2 = (\frac{1}{6})(3)(10)^2 = 50$ in^3

Normal stress $\sigma = \frac{|M|}{S} = \frac{450}{50} = 9$ ksi

Problem 5.57

5.56 and 5.57 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



$+\circlearrowleft M_D = 0$

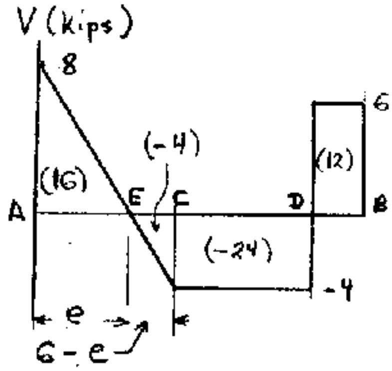
$-12A + (9)(6)(2) - (2)(6) = 0$

$A = 8$ kips

$+\circlearrowleft M_A = 0$

$-(3)(6)(2) + 12D - (14)(6) = 0$

$D = 10$ kips



Shear: $V_A = 8$ kips

$V_C = 8 - (6)(2) = -4$ kips

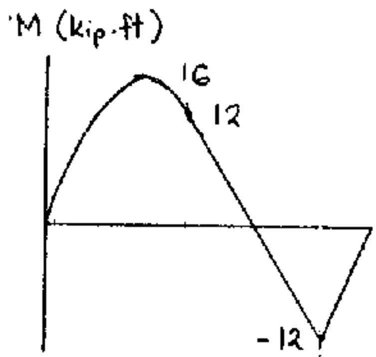
C to D $\quad V = -4$ kips

D to B $\quad V = -4 + 10 = 6$ kips.

Locate point E where $V = 0$

$\frac{e}{8} = \frac{6-e}{4} \qquad 12e = 48$

$e = 4$ ft $\qquad 6 - e = 3$ ft



Areas under shear diagram

A to E $\quad \int V dx = (\frac{1}{2})(4)(8) = 16$ kip·ft

E to C $\quad \int V dx = (\frac{1}{2})(2)(-4) = -4$ kip·ft

C to D $\quad \int V dx = (6)(-4) = -24$ kip·ft

D to B $\quad \int V dx = (2)(6) = 12$ kip·ft

Bending moments: $\quad M_A = 0$

$M_E = 0 + 16 = 16$ kip·ft

$M_C = 16 - 4 = 12$ kip·ft

$M_D = 12 - 24 = -12$ kip·ft

$M_B = -12 + 12 = 0$

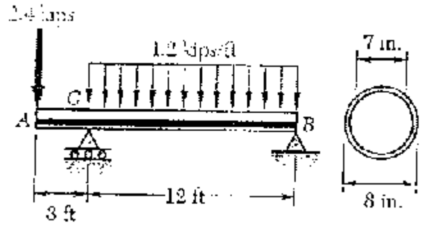
Maximum $|M| = 16$ kip·ft $= 192$ kip·in

For W 8×31 rolled steel section $\quad S = 27.5\ \text{in}^3$

Normal stress $\quad \sigma = \frac{|M|}{S} = \frac{192}{27.5} = 6.98$ ksi ◄

Problem 5.58

5.58 and 5.59 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



$+\circlearrowleft \Sigma M_C = 0$

$(3)(2.4) - (6)(12)(1.2) + 12B = 0$

$B = 6.6 \text{ kips}$

$+\circlearrowleft \Sigma M_B = 0$

$(15)(2.4) + (6)(12)(1.2) - 12C = 0$

$C = 10.2 \text{ kips}$

Shear: A to C $\quad V = -2.4 \text{ kips}$

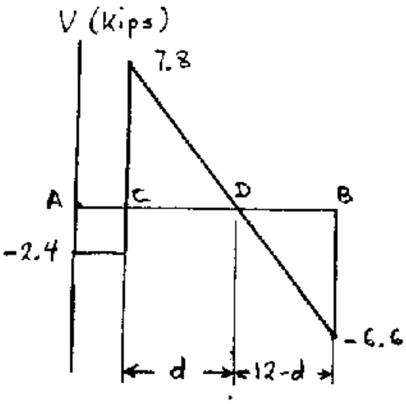
$C^+ \quad V = -2.4 + 10.2 = 7.8 \text{ kips}$

B $\quad V_B = 7.8 - (12)(1.2) = -6.6 \text{ kips}$

Locate point D where $V = 0$

$\frac{d}{7.8} = \frac{12-d}{6.6} \qquad 14.4 d = 93.6$

$d = 6.5 \text{ ft.} \qquad 12 - d = 5.5 \text{ ft.}$

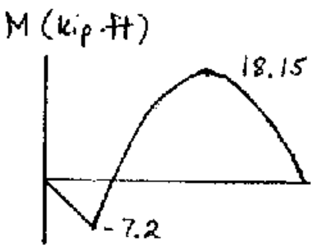


Areas under shear diagram

A to C $\quad \int V dx = (3)(-2.4) = -7.2 \text{ kip}\cdot\text{ft}$

C to D $\quad \int V dx = (\frac{1}{2})(6.5)(7.8) = 25.35 \text{ kip}\cdot\text{ft}$

D to B $\quad \int V dx = (\frac{1}{2})(5.5)(-6.6) = -18.15 \text{ kip}\cdot\text{ft}$



Bending moments $\quad M_A = 0$

$M_C = 0 - 7.2 = -7.2 \text{ kip}\cdot\text{ft}$

$M_D = -7.2 + 25.35 = 18.15 \text{ kip}\cdot\text{ft}$

$M_B = 18.15 - 18.15 = 0$

Maximum $|M| = 18.15 \text{ kip}\cdot\text{ft} = 217.8 \text{ kip}\cdot\text{in}$

For pipe $\quad c_o = \frac{d_o}{2} = \frac{8}{2} = 4 \text{ in} \qquad c_i = \frac{d_i}{2} = \frac{7}{2} = 3.5 \text{ in.}$

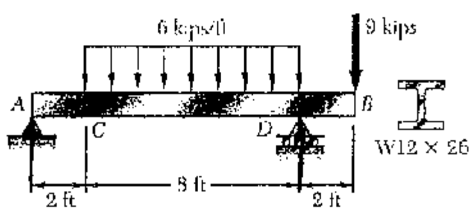
$I = \frac{\pi}{4}(c_o^4 - c_i^4) = \frac{\pi}{4}(4^4 - 3.5^4) = 82.20 \text{ in}^4$

$S = \frac{I}{c_o} = \frac{82.20}{4} = 20.80 \text{ in}^3$

Normal stress $\quad \sigma = \frac{|M|}{S} = \frac{217.8}{20.80} = 10.47 \text{ ksi}$ ◄

Problem 5.59

5.58* and *5.59 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



Reaction at A.

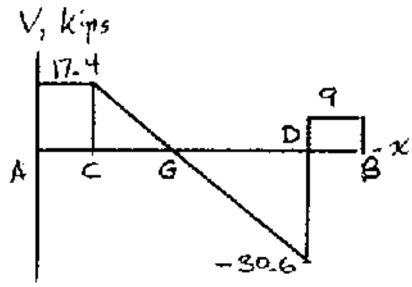
$+\circlearrowleft \Sigma M_D = 0$

$-10A + (4)(8)(6) - (2)(9) = 0 \qquad A = 17.4 \text{ kips} \uparrow$

Reaction at D.

$+\circlearrowleft M_A = 0$

$-(6)(8)(6) + 10D - (12)(9) = 0 \qquad D = 39.6 \text{ kips} \uparrow$



Areas of load diagram.

C to D $\quad (8)(6) = 48$ kip·ft

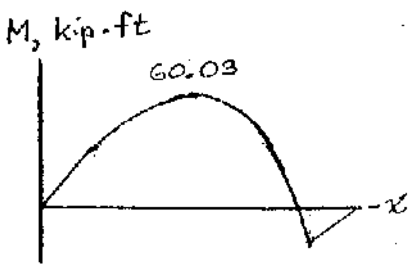
Shear diagram.

A to C $\qquad V = 17.4$ kips

$V_D^- = 17.4 - 48 = -30.6$ kips

$V_D^+ = -30.6 + 39.6 = 9$ kips

D to B $\qquad V = 9$ kips



Over CD $\quad V = 17.4 - 6(x-2)$

At G $\quad V = 17.4 - 6(x_G - 2) = 0 \quad x_G - 2 = 2.9$ ft

Areas of shear diagram

A to C $\quad (2)(17.4) = 34.8$ kip·ft

C to G $\quad \frac{1}{2}(2.9)(17.4) = 25.23$ kip·ft

G to D $\quad \frac{1}{2}(5.1)(-30.6) = -78.03$ kip·ft

D to B $\quad (2)(9) = 18$ kip·ft

Bending moments. $\qquad M_A = 0$

$M_C = 0 + 34.8 = 34.8$ kip·ft

$M_G = 34.8 + 25.23 = 60.03$ kip·ft

$M_D = 60.03 - 78.03 = -18$ kip·ft

$M_B = -18 + 18 = 0$

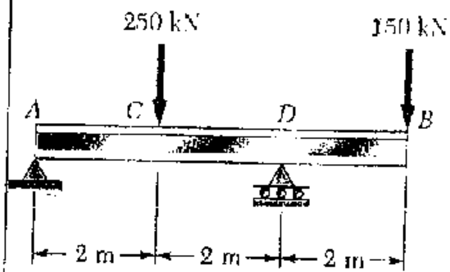
$|M|_{max} = M_G = 60.03$ kip·ft $= 720.36$ kip·in

For W12×26 rolled steel section $\qquad S = 33.4 \text{ in}^3$

Normal stress $\qquad \sigma_{max} = \dfrac{|M|_{max}}{S} = \dfrac{720.36}{33.4} \qquad \sigma_{max} = 21.6 \text{ ksi}$

Problem 5.60

5.60 and 5.61 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



W410 × 114

$w = 0$

$+\circlearrowleft \Sigma M_D = 0$

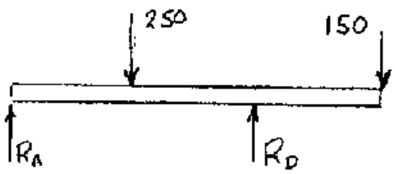
$-4R_A + (2)(250) - (2)(150) = 0$

$R_A = 50 \text{ kN} \uparrow$

$+\circlearrowleft \Sigma M_A = 0$

$4R_D - (2)(250) - (6)(150) = 0$

$R_D = 350 \text{ kN} \uparrow$

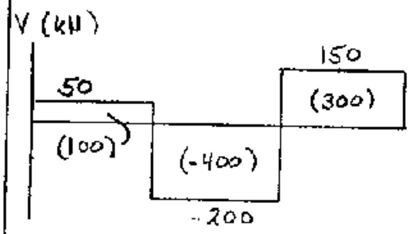


Shear: $V_A = 50$ kN

A to C $\quad V = 50$ kN

C to D $\quad V = 50 - 250 = -200$ kN

D to B $\quad V = -200 + 350 = 150$ kN

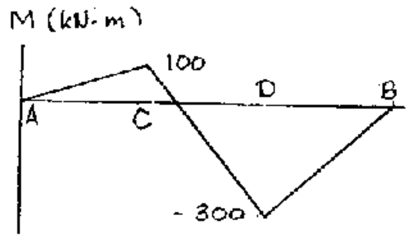


Areas of shear diagram

A to C $\quad \int V dx = (50)(2) = 100$ kN·m

C to D $\quad \int V dx = (-200)(2) = -400$ kN·m

D to B $\quad \int V dx = (150)(2) = 300$ kN·m



Bending moments: $M_A = 0$

$M_C = M_A + \int V dx = 0 + 100 = 100$ kN·m

$M_D = M_C + \int V dx = 100 - 400 = -300$ kN·m

$M_B = M_D + \int V dx = -300 + 300 = 0$

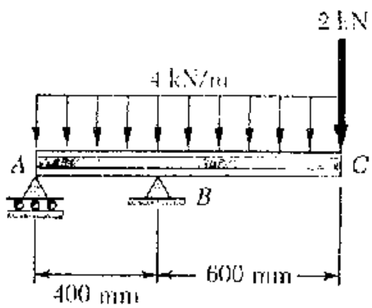
maximum $|M| = 300$ kN·m $= 300 \times 10^3$ N·m

For W410 × 114 rolled steel section $\quad S_x = 2200 \times 10^3 \text{ mm}^3 = 2200 \times 10^{-6} \text{ m}^3$

$$\sigma_m = \frac{|M|_{max}}{S_x} = \frac{300 \times 10^3}{2200 \times 10^{-6}} = 136.4 \times 10^6 \text{ Pa} = 136.4 \text{ MPa}$$

Problem 5.61

5.60 and 5.61 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



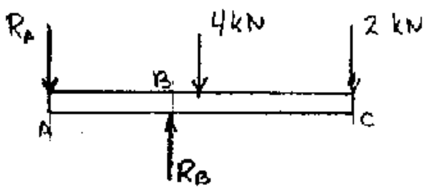
S100 × 11.5

$\sum M_B = 0 \quad (0.4)(R_A) - (0.1)(4) - (0.6)(2) = 0$

$R_A = 4 \text{ kN} \downarrow$

$\sum M_A = 0 \quad (0.4)(R_B) - (0.5)(4) - (1)(2) = 0$

$R_B = 10 \text{ kN} \uparrow$



A to B $\quad 0 < x < 0.4$ m

$\frac{dV}{dx} = -w = -4$ kN/m

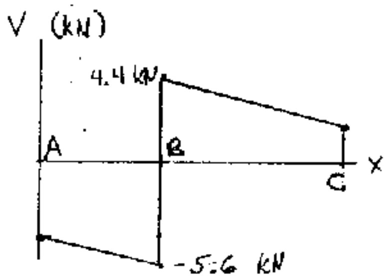
$V = -4x - 4$ kN

At $x = 0.4$ m $\quad V_B = -5.6$ kN

$\frac{dM}{dx} = -4x - 4$

$M = M_A - 2x^2 - 4x = 0 - 2x^2 - 4x$

At $x = 0.4$ m $\quad M_B = 1.92$ kN·m



B to C $\quad 0.4 \text{ m} < x < 1.0$ m

$\frac{dV}{dx} = -w = -4$ kN/m

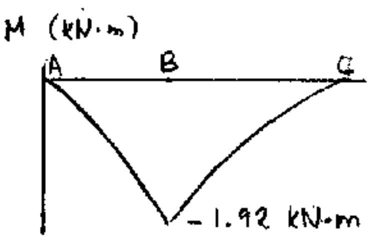
$V = 4.4 - 4(x - 0.4) = 6 - 4x$ kN

$\frac{dM}{dx} = 6 - 4x$

$M = 6x - 2x^2 + C_1$ kN·m

$M = 0$ at $x = 1 \quad \therefore \quad C_1 = 4$ kN·m

$M = 4 + 6x - 2x^2$ kN·m



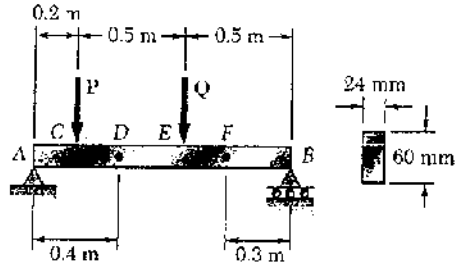
$|M|_{max} = 1.92 \text{ kN·m} = 1.92 \times 10^3 \text{ N·m}$

For S 100 × 11.5 rolled steel section $\quad S_x = 49.6 \times 10^3 \text{ mm}^3 = 49.6 \times 10^{-6} \text{ m}^3$

$\sigma_m = \frac{|M|_{max}}{S_x} = \frac{1.92 \times 10^3}{49.6 \times 10^{-6}} = 38.7 \times 10^6 \text{ Pa} = 38.7 \text{ MPa}$ ◄

Problem 5.62

***5.62** The beam AB supports two concentrated loads **P** and **Q**. The normal stress due to bending on the bottom edge of the beam is +55 MPa at D and +37.5 MPa at F. (*a*) Draw the shear and bending-moment diagrams for the beam. (*b*) Determine the maximum normal stress due to bending which occurs in the beam.



$I = \frac{1}{12}(24)(60)^3 = 432\times10^3\ mm^4$

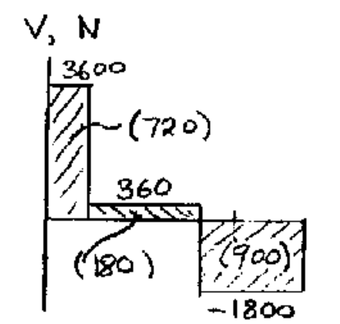
$c = 30\ mm$

$S = \frac{I}{c} = 14.4\times10^3\ mm^3 = 14.4\times10^{-6}\ m^3$

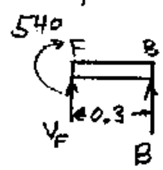
$M = S\sigma$

At D $\quad M_D = (14.4\times10^{-6})(55\times10^6) = 792\ N\cdot m$

At F $\quad M_F = (14.4\times10^{-6})(37.5\times10^6) = 540\ N\cdot m$



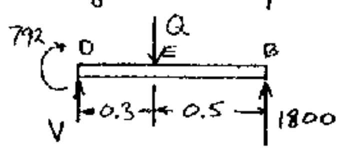
Using free body FB. $\circlearrowright \Sigma M_F = 0$



$-540 + 0.3B = 0$

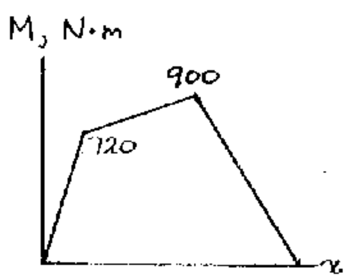
$B = \frac{540}{0.3} = 1800\ N$

Using free body DEFB $\quad +\circlearrowright \Sigma M_D = 0$

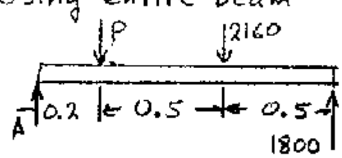


$-792 - 0.3Q + (0.8)(1800) = 0$

$Q = 2160\ N$



Using entire beam $\quad +\circlearrowright M_A = 0$



$-0.2P - (0.7)(2160)$
$+ (1.2)(1800) = 0$

$P = 3240\ N$

$+\uparrow \Sigma F_y = 0 \qquad A - 3240 - 2160 + 1800 = 0$

$A = 3600\ N$

Shear diagram and its areas.

A to C^-	$V = 3600\ N$	$A_{AC} = (0.2)(3600) = 720\ N\cdot m$
C^+ to E^-	$V = 3600 - 3240 = 360\ N$	$A_{CE} = (0.5)(360) = 180\ N\cdot m$
E^+ to B	$V = 360 - 2160 = -1800\ N$	$A_{EB} = (0.5)(-1800) = -900\ N\cdot m$

Bending moments

$M_A = 0$

$M_C = 0 + 720 = 720\ N\cdot m$

$M_E = 720 + 180 = 900\ N\cdot m$

$M_B = 900 - 900 = 0$

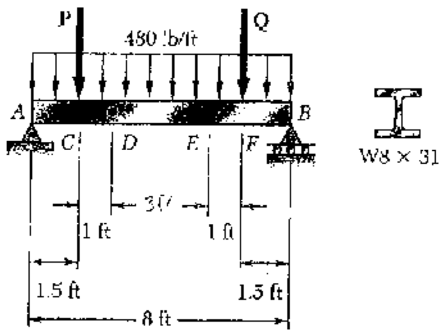
$|M|_{max} = 900\ N\cdot m$

$\sigma_{max} = \frac{|M|_{max}}{S} = \frac{900}{14.4\times10^{-6}} = 62.5\times10^6\ Pa$

(b) $\sigma_{max} = 62.5\ MPa$

Problem 5.63

*5.63 The beam AB supports a uniformly distributed load of 480 lb/ft and two concentrated loads **P** and **Q**. The normal stress due to bending on the bottom edge of the lower flange is +14.85 ksi at D and +10.65 ksi at E. (*a*) Draw the shear and bending-moment diagrams for the beam. (*b*) Determine the maximum normal stress due to bending which occurs in the beam.



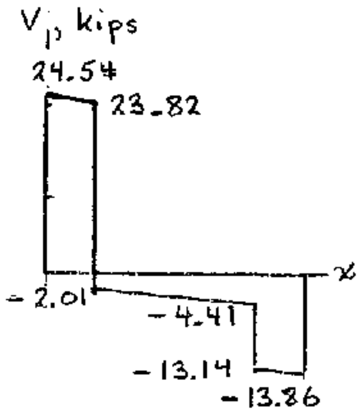
For W8×31 rolled steel section $S = 27.5 \text{ in}^3$

$M = S\sigma$

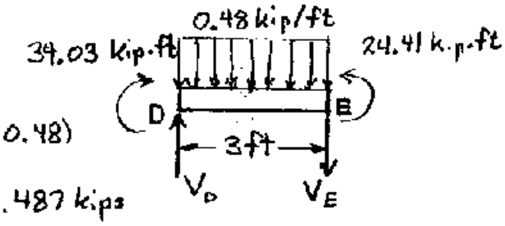
At D $\quad M_D = (27.5)(14.85) = 408.375 \text{ kip·in}$

At E $\quad M_E = (27.5)(10.65) = 292.875 \text{ kip·in.}$

$M_D = 34.03 \text{ kip·ft} \qquad M_E = 24.41 \text{ kip·ft}$



Use free body DE



$+\circlearrowleft \Sigma M_E = 0$

$-34.03 + 24.41 + (1.5)(3)(0.48) - 3V_D = 0$

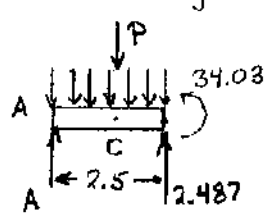
$V_D = -2.487 \text{ kips}$

$+\circlearrowleft M_D = 0$

$-34.03 + 24.41 - (1.5)(3)(0.48) - 3V_E = 0$

$V_E = -3.927 \text{ kip}$

Use free body ACD $\quad +\circlearrowleft \Sigma M_A = 0$



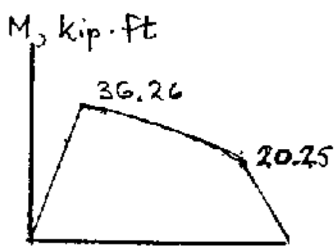
$-1.5\,P - (1.25)(2.5)(0.48) + (2.5)(2.487) + 34.03 = 0$

$P = 25.83 \text{ kips} \downarrow$

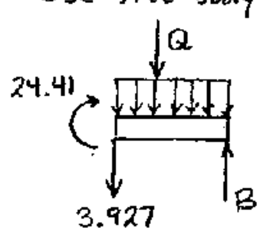
$+\uparrow \Sigma F_y = 0$

$A - (2.5)(0.48) + 2.487 - 25.83 = 0$

$A = 24.54 \text{ kips} \uparrow$



Use free body EFB $\quad +\circlearrowleft \Sigma M_B = 0$



$1.5\,Q + (1.25)(2.5)(0.48) + (2.5)(3.927) - 24.41 = 0$

$Q = 8.728 \text{ kips}$

$+\uparrow \Sigma F_y = 0$

$B - 3.927 - (2.5)(0.48) - 8.728 = 0$

$B = 13.855 \text{ kips.}$

continued

Problem 5.63 continued

Areas of load diagram

A to C $(1.5)(0.48) = 0.72$ kip ft
C to F $(5)(0.48) = 2.4$ kip ft
F to B $(1.5)(0.48) = 0.72$ kip-ft

Shear diagram

$V_A = 24.54$ kip
$V_C^- = 24.54 - 0.72 = 23.82$ kips
$V_C^+ = 23.82 - 25.83 = -2.01$ kips
$V_F^- = -2.01 - 2.4 = -4.41$ kips
$V_F^+ = -4.41 - 8.728 = -13.14$ kips
$V_B = -13.14 - 0.72 = -13.86$ kips

Areas of shear diagram

A to C $\frac{1}{2}(1.5)(24.52 + 23.82) = 36.26$ kip·ft
C to F $\frac{1}{2}(5)(-2.01 - 4.41) = -16.05$ kip-ft
F to B $\frac{1}{2}(1.5)(-13.14 - 13.86) = -20.25$ kip ft

Bending moments

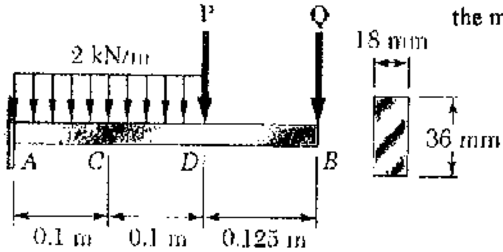
$M_A = 0$
$M_C = 0 + 36.26 = 36.26$ kip ft
$M_F = 36.26 - 16.05 = 20.21$ kip·ft
$M_B = 20.21 - 20.25 \approx 0$

Maximum $|M|$ occurs at C $\quad |M|_{max} = 36.26$ kip·ft
$= 435.1$ kip·in

Maximum stress $\quad \sigma = \dfrac{|M|_{max}}{S} = \dfrac{435.1}{27.5} = 15.82$ ksi ◄

Problem 5.64

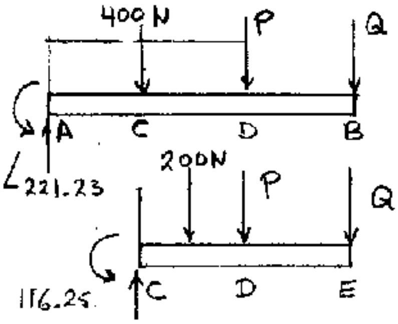
***5.64** Beam AB supports a uniformly distributed load of 2 kN/m and two concentrated loads **P** and **Q**. It has been experimentally determined that the normal stress due to bending in the bottom edge of the beam is -56.9 MPa at A and -29.9 MPa at C. Draw the shear and bending-moment diagrams for the beam and determine the magnitudes of the loads **P** and **Q**.



$I = \frac{1}{12}(18)(36)^3 = 69.984 \times 10^3 \text{ mm}^4$

$c = \frac{1}{2}d = 18 \text{ mm}$

$S = \frac{I}{c} = 3.888 \times 10^3 \text{ mm}^3 = 3.888 \times 10^{-6} \text{ m}^3$



At A $\quad M_A = S\sigma_A$

$M_A = (3.888 \times 10^{-6})(-56.9) = -221.25 \text{ N}\cdot\text{m}$

At C $\quad M_C = S\sigma_C$

$M_C = (3.888 \times 10^{-6})(-29.9) = -116.25 \text{ N}\cdot\text{m}$

$\circlearrowleft \Sigma M_A = 0$

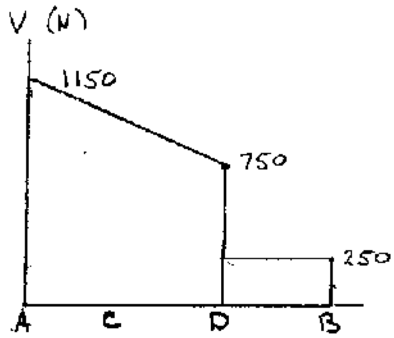
$221.23 - (0.1)(400) - 0.2P - 0.325Q = 0$

$0.2P + 0.325Q = 181.25 \qquad (1)$

$+\circlearrowleft M_C = 0$

$116.25 - (0.05)(200) - 0.1P - 0.225Q = 0$

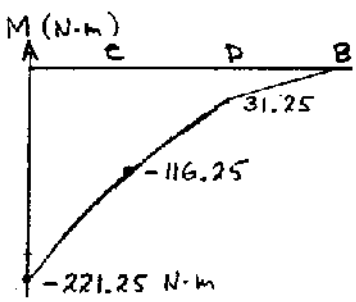
$0.1P + 0.225Q = 106.25 \qquad (2)$



Solving (1) and (2) simultaneously

$P = 500 \text{ N}$ ◀

$Q = 250 \text{ N}$ ◀



Reaction force at A

$R_A - 400 - 500 - 250 = 0$

$R_A = 1150 \text{ N}\cdot\text{m}$

$V_A = 1150 \text{ N} \qquad V_D = 250$

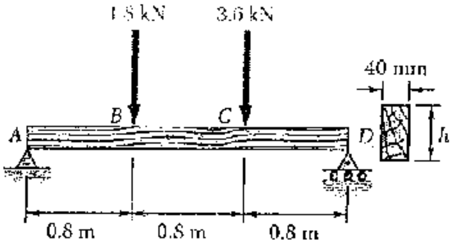
$M_A = -221.25 \text{ N}\cdot\text{m}$

$M_C = -116.25 \text{ N}\cdot\text{m}$

$M_D = -31.25 \text{ N}\cdot\text{m}$

Problem 5.65

5.65 and *5.66* For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.



$+\circlearrowleft \Sigma M_D = 0$

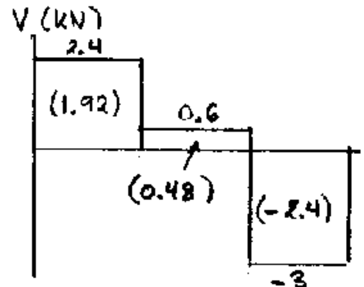
$-2.4A + (1.6)(1.8) + (0.8)(3.6) = 0$

$A = 2.4 \text{ kN}$

$+\circlearrowleft \Sigma M_A = 0$

$-(0.8)(1.8) - (1.6)(3.6) + 2.4D = 0$

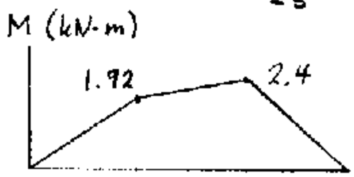
$D = 3 \text{ kN}$



Construct shear and bending moment diagrams

$$|M|_{max} = 2.4 \text{ kN}\cdot\text{m} = 2.4\times10^3 \text{ N}\cdot\text{m}$$

$$\sigma_{all} = 12 \text{ MPa} = 12\times10^6 \text{ Pa}$$



$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{2.4\times10^3}{12\times10^6} = 200\times10^{-6} \text{ m}^3$$

$$= 200\times10^3 \text{ mm}^3$$

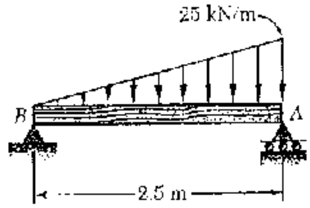
$$S = \frac{1}{6}bh^2 = \frac{1}{6}(40)h^2 = 200\times10^3$$

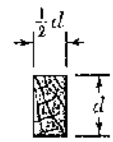
$$h^2 = \frac{(6)(200\times10^3)}{40} = 30\times10^3 \text{ mm}^2$$

$$h = 173.2 \text{ mm}$$

Problem 5.66

5.65 and 5.66 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.





Distributed load $w = \frac{25}{2.5}x = 10x$ kN/m

$$\frac{dV}{dx} = -w = -10x$$

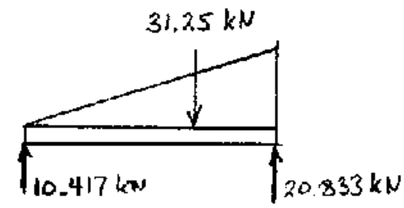
$$V = -5x^2 + C_1 = \frac{dM}{dx}$$

$$M = -\frac{5}{3}x^3 + C_1 x + C_2$$

$M = 0$ at $x = 0$ $\qquad C_2 = 0$

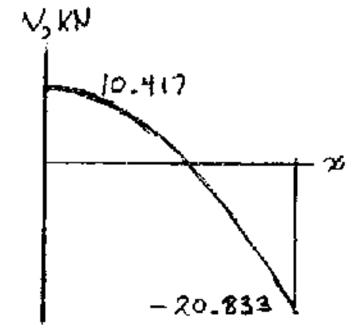
$M = 0$ at $x = 2.5$ m

$$-\frac{5}{3}(2.5)^3 + C_1(2.5) = 0 \qquad C_1 = 10.417 \text{ kN}$$



$$V = -5x^2 + 10.417$$

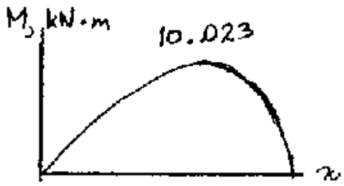
$V = 0 \qquad -5x^2 + 10.417 = 0 \qquad x = 1.4434$ m



$$M_{max} = -\frac{5}{3}(1.4434)^3 + (10.417)(1.4434)$$
$$= 10.023 \text{ kN-m} = 10.023 \times 10^3 \text{ N-m}$$

Required $S = \frac{M_{max}}{\sigma_{all}} = \frac{10.023 \times 10^3}{12 \times 10^6} = 835.29 \times 10^{-6}\ m^3$

$$= 835.29 \times 10^3\ mm^3$$



For the rectangular section $\quad I = \frac{1}{12}(\frac{1}{2}d)(d)^3$

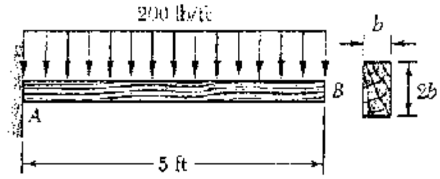
$c = \frac{1}{2}d \qquad S = \frac{I}{c} = \frac{d^3}{12}$

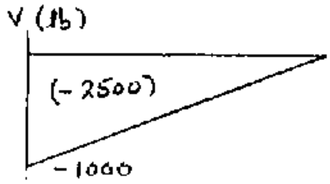
Equating expressions for S

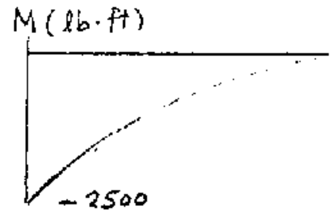
$$\frac{d^3}{12} = 835.29 \times 10^3 \qquad d = 216 \text{ mm}$$

Problem 5.67

5.67 and 5.68 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 1750 psi.







Construct shear and bending moment curves.

$$|M|_{max} = 2500 \text{ lb}\cdot\text{ft} = 2.5 \text{ kip}\cdot\text{ft}$$
$$= 30 \text{ kip}\cdot\text{in}$$

$$\sigma_{all} = 1.75 \text{ ksi}$$

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{30}{1.75} = 17.143 \text{ in}^3$$

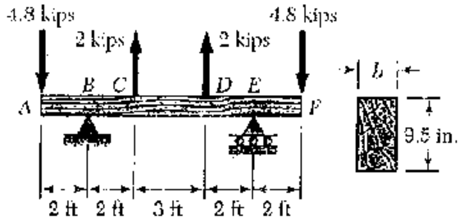
$$S = \frac{1}{6} b h^2 = \frac{1}{6} b (2b)^2 = \frac{2}{3} b^3 = 17.143$$

$$b^3 = \frac{(3)(17.143)}{2} = 25.7 \text{ in}^3$$

$$b = 2.95 \text{ in.}$$

Problem 5.68

5.67 and 5.68 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 1750 psi.



For equilibrium

$B = E = 2.8$ kips

Shear diagram

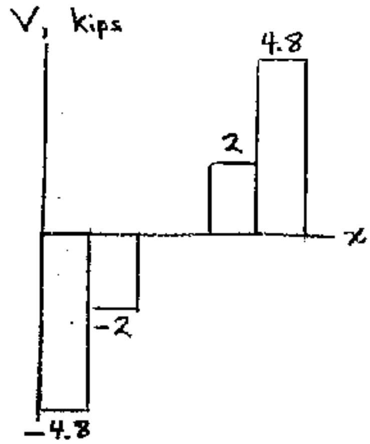
A to B^- $\quad V = -4.8$ kips

B^+ to C^- $\quad V = -4.8 + 2.8 = -2$ kips

C^+ to D^- $\quad V = -2 + 2 = 0$

D^+ to E^- $\quad V = 0 + 2 = 2$ kips

E^+ to F $\quad V = 2 + 2.8 = 4.8$ kips



Areas of shear diagram

A to B $\quad (2)(-4.8) = -9.6$ kip·ft

B to C $\quad (2)(-2) = -4$ kip·ft

C to D $\quad (3)(0) = 0$

D to E $\quad (2)(2) = 4$ kip·ft

E to F $\quad (2)(4.8) = 9.6$ kip·ft

Bending moments

$M_A = 0$

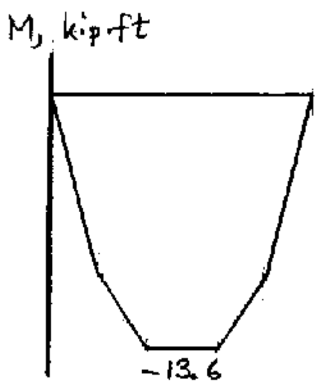
$M_B = 0 - 9.6 = -9.6$ kip·ft

$M_C = -9.6 - 4 = -13.6$ kip·ft

$M_D = -13.6 + 0 = -13.6$ kip·ft

$M_E = -13.6 + 4 = -9.6$ kip·ft

$M_F = -9.6 + 9.6 = 0$



$|M|_{max} = 13.6$ kip·ft $= 163.2$ kip·in $= 163.2 \times 10^3$ lb-in

Required value for S

$$S = \frac{|M|_{max}}{\sigma_{all}} = \frac{163.2 \times 10^3}{1750} = 93.257 \text{ in}^3$$

For a rectangular section $\quad I = \frac{1}{12} b h^3, \quad c = \frac{1}{2} h$

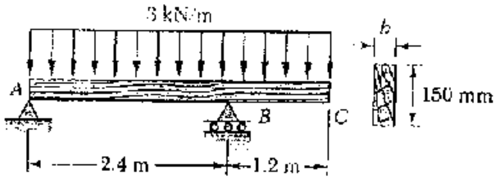
$$S = \frac{I}{c} = \frac{bh^2}{6} = \frac{(b)(9.5)^2}{6} = 15.0417\, b$$

Equating the two expressions for S

$15.0417\, b = 93.257 \qquad b = 6.20$ in. ◄

Problem 5.69

5.69 and 5.70 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.

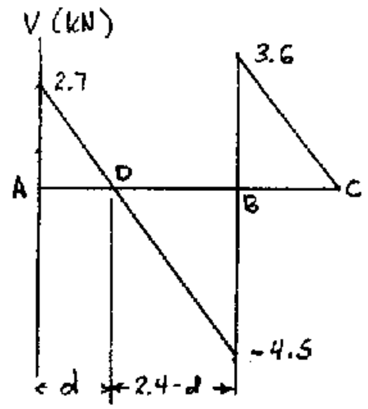


$+\circlearrowleft M_B = 0$

$-2.4A + (0.6)(3.6)(3) = 0 \qquad A = 2.7 \text{ kN}$

$+\circlearrowleft M_A = 0$

$-(1.8)(3.6)(3) + 2.4B = 0 \qquad B = 8.1 \text{ kN}$



Shear: $V_A = 2.7$ kN

$V_B^- = 2.7 - (2.4)(3) = -4.5$ kN

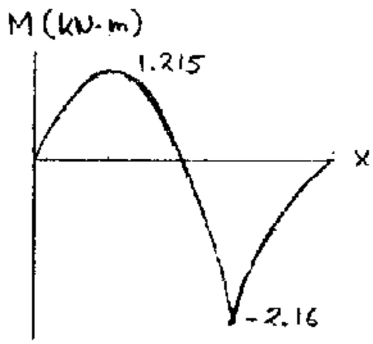
$V_B^+ = -4.5 + 8.1 = 3.6$ kN

$V_C = 3.6 - (1.2)(3) = 0$

Locate point D where $V = 0$

$$\frac{d}{2.7} = \frac{2.4-d}{4.5} \qquad 7.2d = 6.48$$

$$d = 0.9 \text{ m} \qquad 2.4 - d = 1.5 \text{ m}$$



Areas under shear curve

A to D $\quad \int V dx = (\frac{1}{2})(0.9)(2.7) = 1.215$ kN·m

D to B $\quad \int V dx = (\frac{1}{2})(1.5)(-4.5) = -3.375$ kN·m

B to C $\quad \int V dx = (\frac{1}{2})(1.2)(3.6) = 2.16$ kN·m

Bending moments: $M_A = 0$

$M_D = 0 + 1.215 = 1.215$ kN·m

$M_B = 1.215 - 3.375 = -2.16$ kN·m

$M_C = -2.16 + 2.16 = 0$

Maximum $|M| = 2.16 \text{ kN·m} = 2.16 \times 10^3 \text{ N·m}$

$\sigma_{all} = 12 \text{ MPa} = 12 \times 10^6 \text{ Pa}$

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{2.16 \times 10^3}{12 \times 10^6} = 180 \times 10^{-6} \text{ m}^3 = 180 \times 10^3 \text{ mm}^3$$

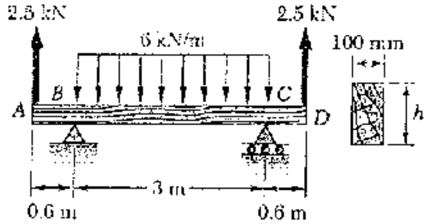
For rectangular section

$$S = \frac{1}{6} b h^2 = \frac{1}{6} b (150)^2 = 180 \times 10^3$$

$$b = \frac{(6)(180 \times 10^3)}{150^2} = 48 \text{ mm}$$

Problem 5.70

5.69 **and 5.70** For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.



By symmetry $B = C$

$+\uparrow \Sigma F_y = 0 \qquad B + C + 2.5 + 2.5 - (3)(6) = 0$

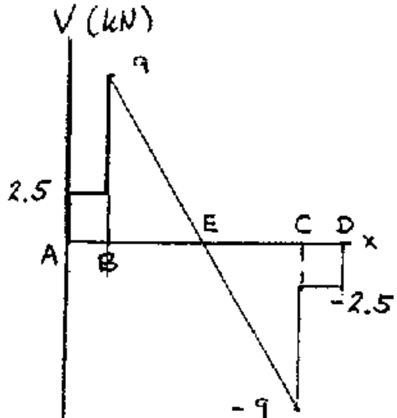
$B = C = 6.5 \text{ kN}$

Shear: A to B $\quad V = 2.5 \text{ kN}$

$V_{B^+} = 2.5 + 6.5 = 9 \text{ kN}$

$V_{C^-} = 9 - (3)(6) = -9 \text{ kN}$

C to D $\quad V = -9 + 6.5 = -2.5 \text{ kN}$



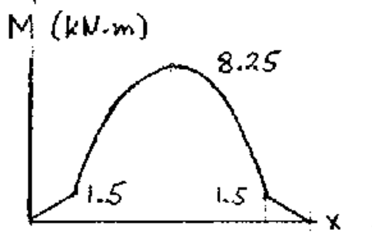
Areas under shear diagram

A to B $\quad \int V dx = (0.6)(2.5) = 1.5 \text{ kN}\cdot\text{m}$

B to E $\quad \int V dx = (\frac{1}{2})(1.5)(9) = 6.75 \text{ kN}\cdot\text{m}$

E to C $\quad \int V dx = -6.75 \text{ kN}\cdot\text{m}$

C to D $\quad \int V dx = -1.5 \text{ kN}\cdot\text{m}$



Bending moments $\quad M_A = 0$

$M_B = 0 + 1.5 = 1.5 \text{ kN}\cdot\text{m}$

$M_E = 1.5 + 6.75 = 8.25 \text{ kN}\cdot\text{m}$

$M_C = 8.25 - 6.75 = 1.5 \text{ kN}\cdot\text{m}$

$M_D = 1.5 - 1.5 = 0$

Maximum $|M| = 8.25 \text{ kN}\cdot\text{m} = 8.25 \times 10^3 \text{ N}\cdot\text{m}$

$\sigma_{all} = 12 \text{ MPa} = 12 \times 10^6 \text{ Pa}$

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{8.25 \times 10^3}{12 \times 10^6} = 687.5 \times 10^{-6} \text{ m}^3 = 687.5 \times 10^3 \text{ mm}^3$$

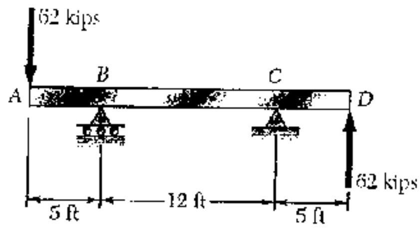
For a rectangular section $\quad S = \frac{1}{6} b h^2$

$687.5 \times 10^3 = (\frac{1}{6})(100) h^2 \qquad h^2 = \frac{(6)(687.5 \times 10^3)}{100} = 41.25 \times 10^3 \text{ mm}^2$

$h = 203 \text{ mm}$ ◄

Problem 5.71

5.71 and 5.72 Knowing that the allowable stress for the steel used is 24 ksi, select the most economical wide-flange beam to support the loading shown.



$\circlearrowleft + \sum M_C = 0$

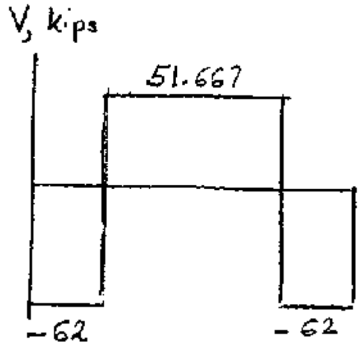
$(17)(62) - 12B + (5)(62) = 0$

$B = 113.667$ kips ↑

$\circlearrowleft + \sum M_B = 0$

$(5)(62) + 12C + (17)(62) = 0$

$C = -113.667$ kips or $C = 113.667$ kips ↓



Shear diagram.

A to B^- $\quad V = -62$ kips

B^+ to C^- $\quad V = -62 + 113.667 = 51.667$ kips

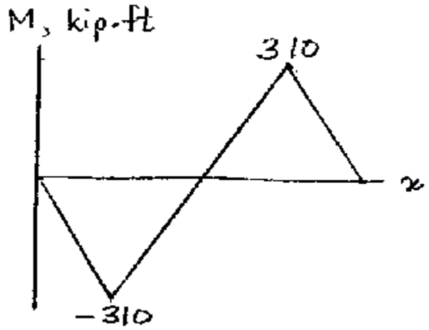
C^+ to D $\quad V = 51.667 - 113.667 = -62$ kips.

Areas of shear diagram.

A to B $\quad (5)(-62) = -310$ kip·ft

B to C $\quad (12)(51.667) = 620$ kip·ft

C to D $\quad (5)(-62) = -310$ kip·ft



Bending moments. $\quad M_A = 0$

$M_B = 0 - 310 = -310$ kip·ft

$M_C = -310 + 620 = 310$ kip·ft

$M_D = 310 - 310 = 0$

$|M|_{max} = 310$ kip·ft $= 3.72 \times 10^3$ kip·in

Required S_{min}

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{3.72 \times 10^3}{24} = 155 \text{ in}^3$$

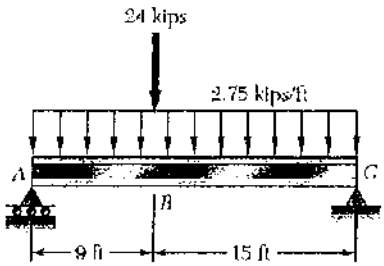
Shape	S, in³	
W 30 × 99	269	
W 27 × 84	213	←
W 24 × 104	258	
W 21 × 101	227	
W 18 × 106	204	
W 14 × 145	232	

Lightest wide flange beam

W 27 × 84 @ 84 lb/ft

Problem 5.72

5.71 and 5.72 Knowing that the allowable stress for the steel used is 24 ksi, select the most economical wide-flange beam to support the loading shown.



$\circlearrowleft \Sigma M_C = 0 \qquad -24A + (12)(24)(2.75) + (15)(24) = 0$

$A = 48 \text{ kips}$

$\circlearrowleft \Sigma M_A = 0 \qquad 24C - (12)(24)(2.75) - (9)(24) = 0$

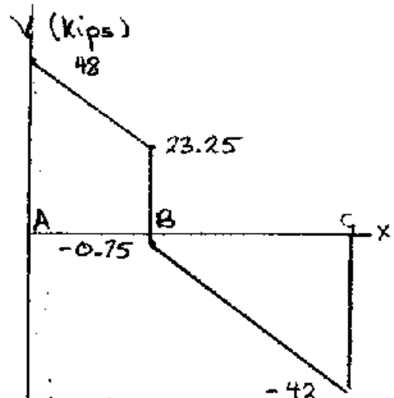
$C = 42 \text{ kips}$

Shear: $V_A = 48$

$V_{B^-} = 48 - (9)(2.75) = 23.25 \text{ kips}$

$V_{B^+} = 23.25 - 24 = -0.75 \text{ kips}$

$V_C = -0.75 - (15)(2.75) = -42 \text{ kips}$



Areas under shear diagram

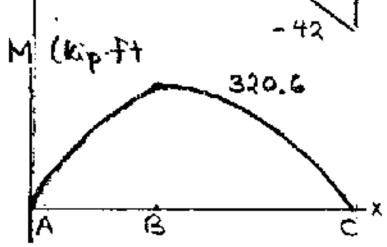
A to B $\int V dx = (\tfrac{1}{2})(9)(48 + 23.25) = 320.6 \text{ kip}\cdot\text{ft}$

B to C $\int V dx = (\tfrac{1}{2})(15)(-0.75 - 42) = -320.6 \text{ kip}\cdot\text{ft}$

Bending moments: $M_A = 0$

$M_B = 0 + 320.6 = 320.6 \text{ kip ft}$

$M_C = 320.6 - 320.6 = 0$



Maximum $|M| = 320.6 \text{ kip ft} = 3848 \text{ kip}\cdot\text{in}$

$\sigma_{all} = 24 \text{ ksi}$

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{3848}{24} = 160.3 \text{ in}^3$$

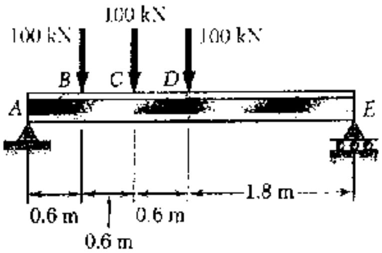
Shape	S (in^3)
W 30 × 99	269
W 27 × 84	213
W 24 × 104	258
W 21 × 101	227
W 18 × 106	204

Lightest wide flange beam

W 27 × 84 @ 84 lb/ft

Problem 5.73

5.73 and 5.74 Knowing that the allowable stress for the steel used is 160 MPa, select the most economical wide-flange beam to support the loading shown.



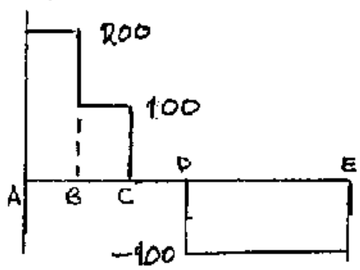
$+\circlearrowleft \Sigma M_E = 0 \quad -3.6A + (3)(100) + (2.4)(100) + (1.8)(100) = 0$

$A = 200 \text{ kN}$

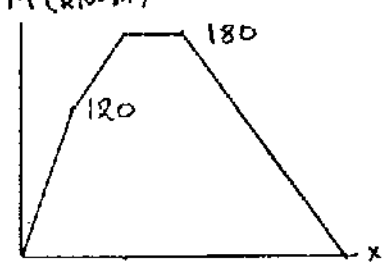
$+\circlearrowleft M_A = 0 \quad 3.6E - (1.8)(100) - (1.2)(100) - (0.6)(100) = 0$

$E = 100 \text{ kN}$

Shear:
- A to B $\quad V = 200 \text{ kN}$
- B to C $\quad V = 200 - 100 = 100 \text{ kN}$
- C to D $\quad V = 100 - 100 = 0$
- D to E $\quad V = 0 - 100 = -100 \text{ kN}$



Areas under shear diagram
- A to B $\quad \int V dx = (0.6)(200) = 120 \text{ kN}\cdot\text{m}$
- B to C $\quad \int V dx = (0.6)(100) = 60 \text{ kN}\cdot\text{m}$
- C to D $\quad \int V dx = 0$
- D to E $\quad \int V dx = (1.8)(-100) = -162 \text{ kN}\cdot\text{m}$



Bending moments $\quad M_A = 0$

$M_B = 0 + 120 = 120 \text{ kN}\cdot\text{m}$

$M_C = 120 + 60 = 180 \text{ kN}\cdot\text{m}$

$M_D = 180 + 0 = 180 \text{ kN}\cdot\text{m}$

$M_E = 180 - 180 = 0$

Maximum $|M| = 180 \text{ kN}\cdot\text{m} = 180 \times 10^3 \text{ N}\cdot\text{m}$

$\sigma_{all} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{180 \times 10^3}{160 \times 10^6} = 1.125 \times 10^{-3} \text{ m}^3 = 1125 \times 10^3 \text{ mm}^3$$

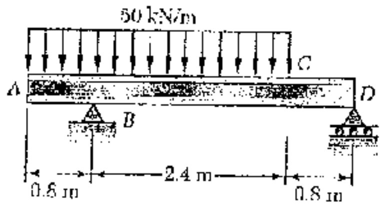
Shape	S (10^3 mm^3)
W 530 × 66	1340
W 460 × 74	1460
W 360 × 79	1280
W 250 × 101	1240

Lightest wide flange beam

W 530 × 66 @ 66 kg/m

Problem 5.74

5.73 and 5.74 Knowing that the allowable stress for the steel used is 160 MPa, select the most economical wide-flange beam to support the loading shown.

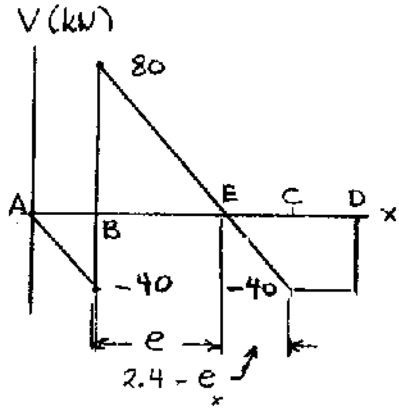


$\circlearrowleft \Sigma M_D = 0 \quad -3.2B + (2.4)(3.2)(50) = 0$

$B = 120 \text{ kN}$

$+\circlearrowleft \Sigma M_B = 0 \quad 3.2D - (0.8)(3.2)(50) = 0$

$D = 40 \text{ kN}$



Shear: $V_A = 0$

$V_{B^-} = 0 - (0.8)(50) = -40 \text{ kN}$

$V_{B^+} = -40 + 120 = 80 \text{ kN}$

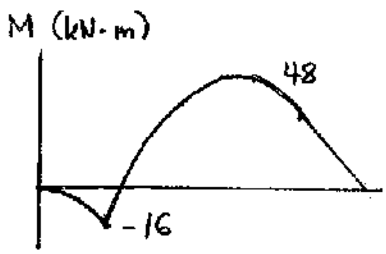
$V_C = 80 - (2.4)(50) = -40 \text{ kN}$

$V_D = -40 + 0 = -40 \text{ kN}$

Locate point E where $V = 0$

$\frac{e}{80} = \frac{2.4 - e}{40} \qquad 120e = 192$

$e = 1.6 \text{ m} \qquad 2.4 - e = 0.8 \text{ m}$



Areas: A to B, $\int V dx = (\frac{1}{2})(0.8)(-40) = -16 \text{ kN·m}$

B to E $\quad \int V dx = (\frac{1}{2})(1.6)(80) = 64 \text{ kN·m}$

E to C $\quad \int V dx = (\frac{1}{2})(0.8)(-40) = -16 \text{ kN·m}$

C to D $\quad \int V dx = (0.8)(-40) = -32 \text{ kN·m}$

Bending moments: $M_A = 0$

$M_B = 0 - 16 = -16 \text{ kN·m}$

$M_E = -16 + 64 = 48 \text{ kN·m}$

$M_C = 48 - 16 = 32 \text{ kN·m}$

$M_D = 32 - 32 = 0$

Maximum $|M| = 48 \text{ kN·m} = 48 \times 10^3 \text{ N·m}$

$\sigma_{all} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$

$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{48 \times 10^3}{160 \times 10^6} = 300 \times 10^{-6} \text{ m}^3 = 300 \times 10^3 \text{ mm}^3$

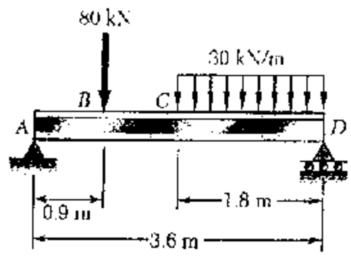
Shape	S (10^3 mm^3)
W 310 × 32.7	415
W 250 × 28.4	308
W 200 × 35.9	342

Lightest wide flange beam

W 250 × 28.4 @ 28.4 kg/m ◄

Problem 5.75

5.75 and 5.76 Knowing that the allowable stress for the steel used is 160 MPa, select the most economical S-shape beam to support the loading shown.



$+\circlearrowleft \Sigma M_D = 0 \quad -3.6A + (2.7)(80) + (0.9)(1.8)(30) = 0$

$A = 73.5 \text{ kN} \uparrow$

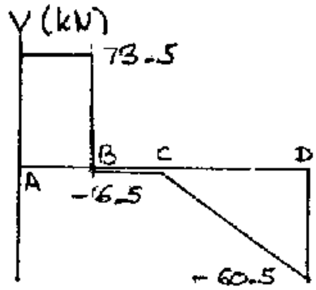
$+\circlearrowleft \Sigma M_A = 0 \quad 3.6D - (0.9)(80) - (2.7)(1.8)(30) = 0$

$D = 60.5 \text{ kN} \uparrow$

Shear: A to B $\quad V = 73.5$ kN

B to C $\quad V = 73.5 - 80 = -6.5$ kN

$V_D = -6.5 - (1.8)(30) = -60.5$ kN

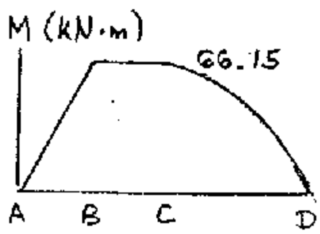


Areas under shear diagram

A to B $\quad \int V dx = (0.9)(73.5) = 66.15$ kN·m

B to C $\quad \int V dx = (0.9)(-6.5) = -5.85$ kN·m

C to D $\quad (\frac{1}{2})(1.8)(-6.5 - 60.5) = -60.30$ kN·m



Bending moments: $\quad M_A = 0$

$M_B = 0 + 66.15 = 66.15$ kN·m

$M_C = 66.15 - 5.85 = 60.30$ kN·m

$M_D = 66.30 - 66.30 = 0$

Maximum $|M| = 66.15$ kN·m $= 66.15 \times 10^3$ N·m

$\sigma_{all} = 160$ MPa $= 160 \times 10^6$ Pa

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{66.15 \times 10^3}{160 \times 10^6} = 413.4 \times 10^{-6} \text{ m}^3 = 413.4 \times 10^3 \text{ mm}^3$$

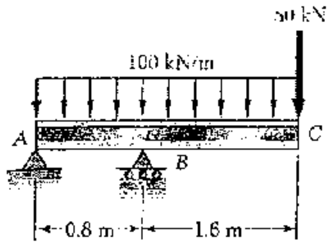
Shape	S (10^3 mm^3)
S 380 × 64	971
S 310 × 47.3	593
S 250 × 52	482

Lightest S-section

S 310 × 47.3 @ 47.3 kg/m ◄

Problem 5.76

5.75 and 5.76 Knowing that the allowable stress for the steel used is 160 MPa, select the most economical S-shape beam to support the loading shown.

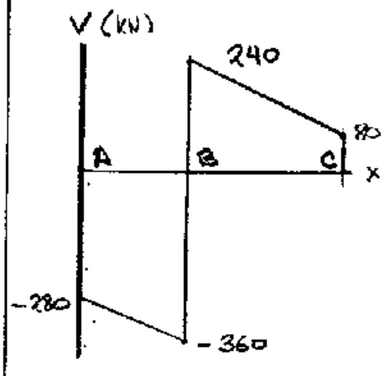


$+\circlearrowleft \Sigma M_B = 0 \qquad 0.8\,A - (0.4)(2.4)(100) - (1.6)(80) = 0$

$A = 280 \text{ kN} \downarrow$

$+\circlearrowleft \Sigma M_A = 0 \qquad 0.8\,B - (1.2)(2.4)(100) - (2.4)(80) = 0$

$B = 600 \text{ kN} \uparrow$



Shear: $V_A = -280$ kN

$V_{B^-} = -280 - (0.8)(100) = -360$ kN

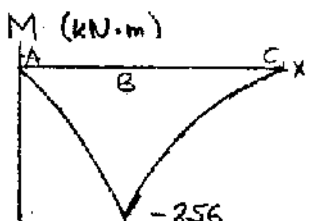
$V_{B^+} = -360 + 600 = 240$ kN

$V_C = 240 - (1.6)(100) = 80$ kN

Areas under shear diagram

A to B $\quad (\frac{1}{2})(0.8)(-280-360) = -256$ kN·m

B to C $\quad (\frac{1}{2})(1.6)(240+80) = 256$ kN·m



Bending moments: $\quad M_A = 0$

$M_B = 0 - 256 = -256$ kN·m

$M_C = -256 + 256 = 0$

Maximum $|M| = 256$ kN·m $= 256 \times 10^3$ N·m

$\sigma_{all} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{256 \times 10^3}{160 \times 10^6} = 1.6 \times 10^{-3}\ \text{m}^3 = 1600 \times 10^3\ \text{mm}^3$$

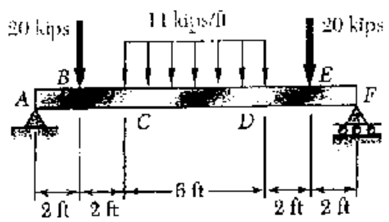
Shape	S (10^3 mm^3)
S 510 × 98.3	1950
S 460 × 104	1685

Lightest S-section

S 510 × 98.3 ◄

Problem 5.77

5.77 and 5.78 Knowing that the allowable stress for the steel used is 24 ksi, select the most economical S-shape beam to support the loading shown.



From statics the reactions at A and F are

A = 53 kips ↑ B = 53 kips ↑

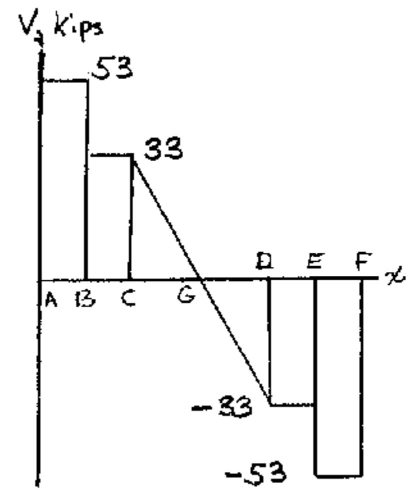
Area of load diagram over CD $(6)(11) = 66$ kips.

Shear diagram.

A to B^-	$V = 53$ kips
B^+ to C	$V = 53 - 20 = 33$ kips
At D	$V = 33 - 66 = -33$ kips
D to E^-	$V = -33$ kips
E^+ to F	$V = -33 - 20 = -53$ kips

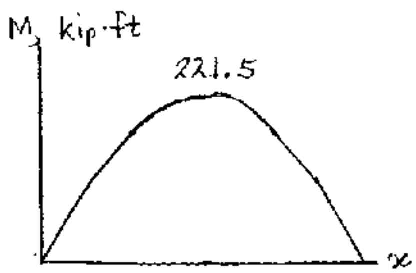
Over CD $V = 33 - 11(x - 4) = 0$

At G $V = 0$ $x_G = 7$ ft



Areas of shear diagram.

A to B	$(2)(53)$	$= 106$ kip·ft
B to C	$(2)(33)$	$= 66$ kip·ft
C to G	$\frac{1}{2}(3)(33)$	$= 49.5$ kip·ft
G to D	$\frac{1}{2}(3)(-33)$	$= -49.5$ kip·ft
D to E	$(2)(-33)$	$= -66$ kip·ft
E to F	$(2)(-53)$	$= -106$ kip·ft



Bending moments. $M_A = 0$

$M_B = 0 + 106 = 106$ kip·ft

$M_C = 106 + 66 = 172$ kip·ft

$M_G = 172 + 49.5 = 221.5$ kip·ft

$M_D = 221.5 - 49.5 = 172$ kip·ft

$M_E = 172 - 66 = 106$ kip·ft

$M_F = 106 - 106 = 0$

$|M|_{max} = 221.5$ kip·ft $= 2658$ kip·in.

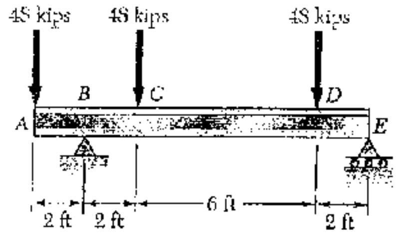
$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{2658}{24} = 110.75 \text{ in}^3$$

Shape	S, in^3
S 20 × 66	119

Use S 20 × 66

Problem 5.78

5.77 **and** *5.78* **Knowing that the allowable stress for the steel used is 24 ksi, select the most economical S-shape beam to support the loading shown.**

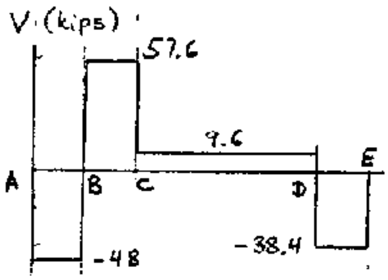


$+\sum M_E = 0 \quad (12)(48) - 10B + (8)(48) + (2)(48) = 0$

$B = 105.6$ kips

$+\sum M_B = 0 \quad (2)(48) - (2)(48) - (8)(48) + 10E = 0$

$E = 38.4$ kips



Shear: A to B $\quad V = -48$ kips

B to C $\quad V = -48 + 105.6 = 57.6$ kips

C to D $\quad V = 57.6 - 48 = 9.6$ kips

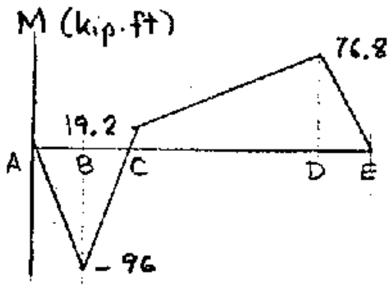
D to E $\quad V = 9.6 - 48 = -38.4$ kips

Areas: A to B $\quad (2)(-48) = -96$ kip·ft

B to C $\quad (2)(57.6) = 115.2$ kip·ft

C to D $\quad (6)(9.6) = 57.6$ kip·ft

D to E $\quad (2)(-38.4) = 76.8$ kip·ft



Bending moments: $\quad M_A = 0$

$M_B = 0 - 96 = -96$ kip·ft

$M_C = -96 + 115.2 = 19.2$ kip·ft

$M_D = 19.2 + 57.2 = 76.8$ kip·ft

$M_E = 76.8 - 76.8 = 0$

Maximum $|M| = 96$ kip·ft $= 1152$ kip·in

$\sigma_{all} = 24$ ksi

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{1152}{24} = 48 \text{ in}^3$$

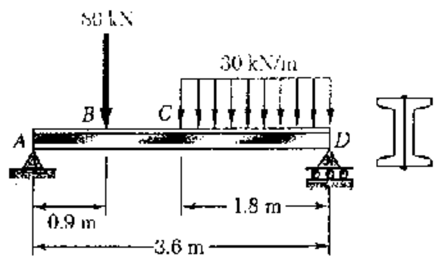
Shape	S (in³)
S 15 × 42.9	59.6
S 12 × 50	50.8

Lightest S-shaped beam

S 15 × 42.9

Problem 5.79

5.79 Two metric rolled-steel channels are to be welded back to back and used to support the loading shown. Knowing that the allowable normal stress for the steel used is 200 MPa, determine the most economical channels that can be used.

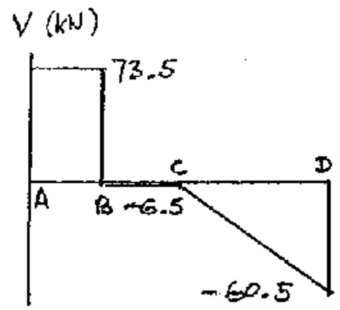


$\circlearrowleft + \Sigma M_D = 0 \quad -3.6A + (2.7)(80) + (0.9)(1.8)(30) = 0$

$A = 73.5 \text{ kN} \uparrow$

$\circlearrowleft + \Sigma M_A = 0 \quad 3.6D - (0.9)(80) - (2.7)(1.8)(30) = 0$

$D = 60.5 \text{ kN} \uparrow$



Shear: A to B $\quad V = 73.5$ kN

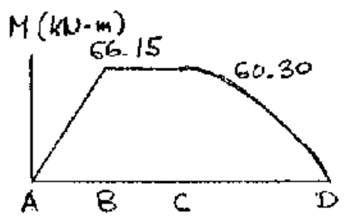
B to C $\quad V = 73.5 - 80 = -6.5$ kN

$V_D = -6.5 - (1.8)(30) = -60.5$ kN

Areas: A to B $\quad (0.9)(73.5) = 66.15$ kN·m

B to C $\quad (0.9)(-6.5) = -5.85$ kN·m

C to D $\quad (\frac{1}{2})(1.8)(-6.5 - 60.5) = -60.3$ kN·m



Bending moments: $\quad M_A = 0$

$M_B = 0 + 66.15 = 66.15$ kN·m

$M_C = 66.15 - 5.85 = 60.30$ kN·m

$M_D = 60.30 - 60.3 = 0$

Maximum $|M| = 66.15$ kN·m $= 66.15 \times 10^3$ N·m

$\sigma_{all} = 200$ MPa $= 200 \times 10^6$ Pa

For double channel $\quad S_{min} = \frac{|M|}{\sigma_{all}} = \frac{66.15 \times 10^3}{200 \times 10^6} = 330.75 \times 10^{-6} \text{ m}^3$

$= 330.75 \times 10^3 \text{ mm}^3$

For each channel $\quad S_{min} = (\frac{1}{2})(330.75 \times 10^3) = 165.4 \times 10^3 \text{ mm}^3$

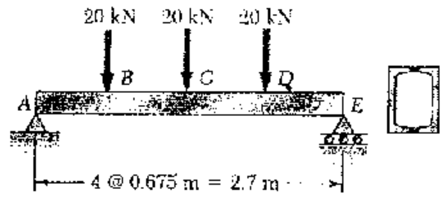
Shape	S (10^3 mm^3)
C 230 × 19.9	173
C 200 × 27.9	179

Lightest channel section

C 230 × 19.9

Problem 5.80

5.80 Two metric rolled-steel channels are to be welded along their edges and used to support the loading shown. Knowing that the allowable normal stress for the steel used is 150 MPa, determine the most economical channels that can be used.



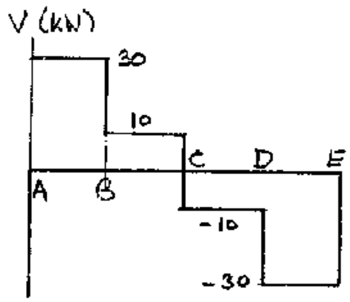
By symmetry $A = E$

$+\uparrow \Sigma F_y = 0 \quad A + E - 20 - 20 - 20 = 0$

$A = E = 30\ \text{kN}$

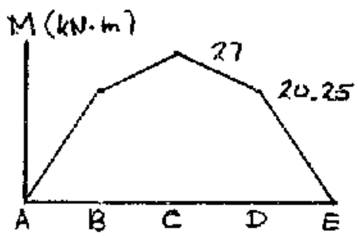
Shear:

A to B	$V = 30\ \text{kN}$	
B to C	$V = 30 - 20$	$= 10\ \text{kN}$
C to D	$V = 10 - 20$	$= -10\ \text{kN}$
D to E	$V = -10 - 20$	$= -30\ \text{kN}$



Areas:

A to B	$(0.675)(30) =$	$20.25\ \text{kN}\cdot\text{m}$
B to C	$(0.675)(10) =$	$6.75\ \text{kN}\cdot\text{m}$
C to D	$(0.675)(-10) =$	$-6.75\ \text{kN}\cdot\text{m}$
D to E	$(0.675)(-30) =$	$-20.25\ \text{kN}\cdot\text{m}$



Bending moments: $M_A = 0$

$M_B = 0 + 20.25 = 20.25\ \text{kN}\cdot\text{m}$

$M_C = 20.25 + 6.75 = 27\ \text{kN}\cdot\text{m}$

$M_D = 27 - 6.75 = 20.25\ \text{kN}\cdot\text{m}$

$M_E = 20.25 - 20.25 = 0$

Maximum $|M| = 27\ \text{kN}\cdot\text{m} = 27 \times 10^3\ \text{N}\cdot\text{m}$

$\sigma_{all} = 150\ \text{MPa} = 150 \times 10^6\ \text{Pa}$

For a section consisting of two channels

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{27 \times 10^3}{150 \times 10^6} = 180 \times 10^{-6}\ \text{m}^3 = 180 \times 10^3\ \text{mm}^3$$

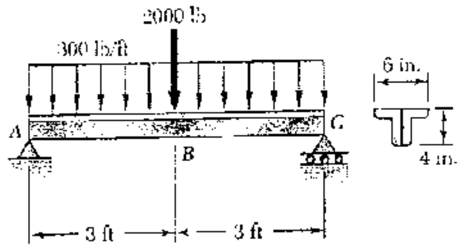
For each channel $\quad S_{min} = (\frac{1}{2})(180 \times 10^3) = 90 \times 10^3\ \text{mm}^3$

Shape	$S\ (10^3\ \text{mm}^3)$
C 180×14.6	99.2
C 150×19.3	93.6

Lightest channel section
C 180 × 14.6

Problem 5.81

5.81 Two L4 × 3 rolled-steel angles are bolted together and used to support the loading shown. Knowing that the allowable normal stress for the steel used is 24 ksi, determine the minimum angle thickness that can be used.



By symmetry $A = C$

$+\Sigma F_y = 0 \qquad A + C - 2000 - (6)(300) = 0$

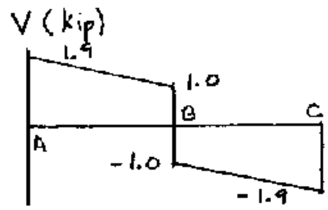
$A = C = 1900$ lb.

Shear: $V_A = 1900$ lb. $= 1.9$ kips

$V_{B^-} = 1900 - (3)(300) = 1000$ lb $= 1$ kip

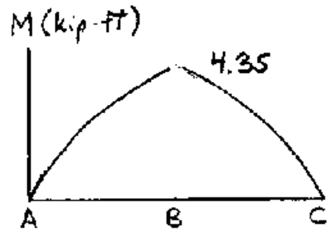
$V_{B^+} = 1000 - 2000 = -1000$ lb $= -1$ kip

$V_C = -1000 - (3)(300) = -1900$ lb $= -1.9$ kip



Areas: A to B $(\frac{1}{2})(3)(1.9 + 1) = 4.35$ kip·ft

B to C $(\frac{1}{2})(3)(-1 - 1.9) = -4.35$ kip·ft



Bending moments: $M_A = 0$

$M_B = 0 + 4.35 = 4.35$ kip·ft

$M_C = 4.35 - 4.35 = 0$

Maximum $|M| = 4.35$ kip·ft $= 52.2$ kip·in

$\sigma_{all} = 24$ ksi

For section consisting of two angles $\quad S_{min} = \frac{|M|}{\sigma_{all}} = \frac{52.2}{24} = 2.175 \text{ in}^3$

For each angle $\quad S_{min} = (\frac{1}{2})(2.175) = 1.0875 \text{ in}^3$

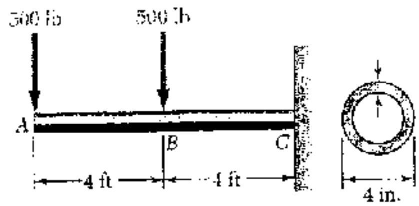
Angle section	S (in³)
L 4×3×½	1.89
L 4×3×⅜	1.46
L 4×3×¼	1.00

Smallest allowable thickness

$t = \frac{3}{8}$ in.

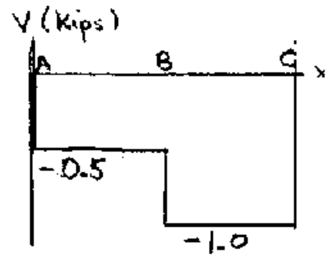
Problem 5.82

5.82 A steel pipe of 4-in. diameter is to support the loading shown. Knowing that the stock of pipes available has thicknesses varying from $\frac{1}{4}$ in. to 1 in. in $\frac{1}{8}$-in. increments, and that the allowable normal stress for the steel used is 24 ksi, determine the minimum wall thickness t that can be used.



Shear: A to B $\quad V = -500 \text{ lb} = -0.5 \text{ kip}$

B to C $\quad V = -500 - 500 = -1000 \text{ lb} = -1.0 \text{ kip}$



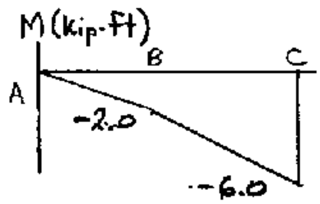
Areas: A to B $\quad (4)(-0.5) = -2.0 \text{ kip}\cdot\text{ft}$

B to C $\quad (4)(-1.0) = -4.0 \text{ kip}\cdot\text{ft}$

Bending moments: $\quad M_A = 0$

$M_B = 0 - 2.0 = -2.0 \text{ kip}\cdot\text{ft}$

$M_C = -2.0 - 4.0 = -6.0 \text{ kip}\cdot\text{ft}$



Maximum $|M| = 6.0 \text{ kip}\cdot\text{ft} = 72 \text{ kip}\cdot\text{in}$

$\sigma_{all} = 24 \text{ ksi}$

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{72}{24} = 3 \text{ in}^3$$

$I = \frac{\pi}{4}(c_2^4 - c_1^4) \qquad c = c_2 \qquad c_2 = \frac{1}{2}d = 2.0 \text{ in.}$

$$S = \frac{I}{c} = \frac{\pi}{4}\,\frac{c_2^4 - c_1^4}{c_2} = \frac{\pi}{4}\,\frac{2^4 - c_1^4}{2} = 3 \text{ in}^3$$

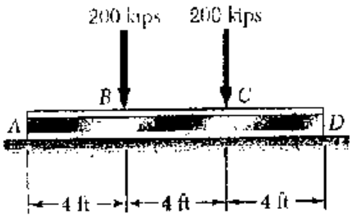
$$c_1^4 = 2^4 - \frac{(4)(2)(3)}{\pi} = 8.3606 \text{ in}^4 \qquad c_1 = 1.7004 \text{ in.}$$

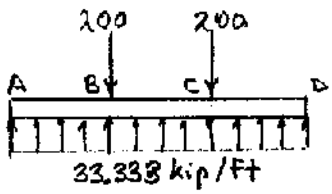
$$t_{min} = c_2 - c_1 = 2.0 - 1.7004 = 0.2996 \text{ in}$$

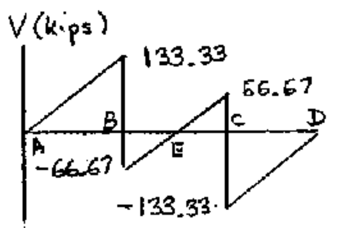
Using $\frac{1}{8}$ in. increments for design $\quad t = \frac{3}{8}$ in. ◄

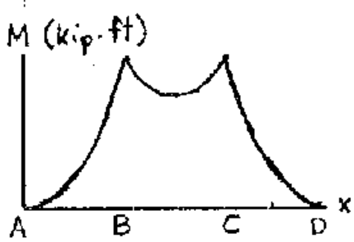
Problem 5.83

5.83 Assuming the upward reaction of the ground to be uniformly distributed and knowing that the allowable normal stress for the steel used is 24 ksi, select the most economical wide-flange beam to support the loading shown.









Distributed reaction $q = \frac{400}{12} = 33.333$ kip/ft

Shear: $V_A = 0$

$V_{B^-} = 0 + (4)(33.333) = 133.33$ kips

$V_{B^+} = 133.33 - 200 = -66.67$ kips

$V_{C^-} = -66.67 + 4(33.333) = 66.67$ kips

$V_{C^+} = 66.67 - 200 = -133.33$ kips

$V_D = -133.33 + (4)(33.333) = 0$ kips

Areas: A to B $(\frac{1}{2})(4)(133.33) = 266.67$ kip·ft

B to E $(\frac{1}{2})(2)(-66.67) = -66.67$ kip·ft

E to C $(\frac{1}{2})(2)(66.67) = 66.67$ kip·ft

C to D $(\frac{1}{2})(4)(-133.33) = -266.67$ kip·ft

Bending moments: $M_A = 0$

$M_B = 0 + 266.67 = 266.67$ kip·ft

$M_E = 266.67 - 66.67 = 200$ kip·ft

$M_C = 200 + 66.67 = 266.67$ kip·ft

$M_D = 266.67 - 266.67 = 0$

Maximum $|M| = 266.67$ kip·ft $= 3200$ kip·in

$\sigma_{all} = 24$ ksi

$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{3200}{24} = 133.3 \text{ in}^3$

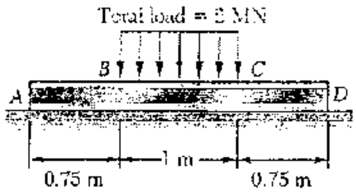
Shape	S (in³)
W 27 × 84	213
W 24 × 68	154 ←
W 21 × 101	227
W 18 × 76	146
W 16 × 77	134
W 14 × 145	232

Lightest W-shaped section

W 24 × 68

Problem 5.84

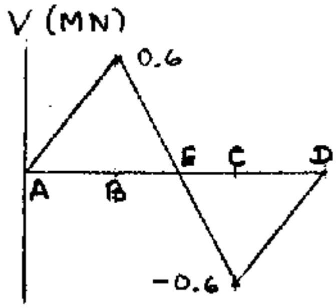
5.84 Assuming the upward reaction of the ground to be uniformly distributed and knowing that the allowable normal stress for the steel used is 170 MPa, select the most economical wide-flange beam to support the loading shown.



Downward distributed load $w = \frac{2}{1.0} = 2\ \text{MN/m}$

Upward distributed reaction $q = \frac{2}{2.5} = 0.8\ \text{MN/m}$

Net distributed load over BC $\quad 1.2\ \text{MN/m}$



Shear: $V_A = 0$

$V_B = 0 + (0.75)(0.8) = 0.6\ \text{MN}$

$V_C = 0.6 - (1.0)(1.2) = -0.6\ \text{MN}$

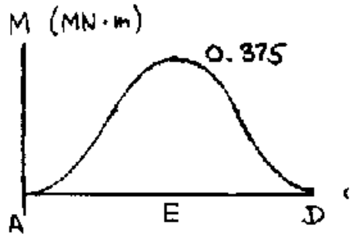
$V_D = -0.6 + (0.75)(0.8) = 0$

Areas: A to B $\quad (\frac{1}{2})(0.75)(0.6) = 0.225\ \text{MN}\cdot\text{m}$

B to E $\quad (\frac{1}{2})(0.5)(0.6) = 0.150\ \text{MN}\cdot\text{m}$

E to C $\quad (\frac{1}{2})(0.5)(-0.6) = -0.150\ \text{MN}\cdot\text{m}$

C to D $\quad (\frac{1}{2})(0.75)(-0.6) = -0.225\ \text{MN}\cdot\text{m}$



Bending moments: $\quad M_A = 0$

$M_B = 0 + 0.225 = 0.225\ \text{MN}\cdot\text{m}$

$M_E = 0.225 + 0.150 = 0.375\ \text{MN}\cdot\text{m}$

$M_C = 0.375 - 0.150 = 0.225\ \text{MN}\cdot\text{m}$

$M_D = 0.225 - 0.225 = 0$

Maximum $|M| = 0.375\ \text{MN}\cdot\text{m} = 375 \times 10^3\ \text{N}\cdot\text{m}$

$\sigma_{all} = 170\ \text{MPa} = 170 \times 10^6\ \text{Pa}$

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{375 \times 10^3}{170 \times 10^6} = 2.206 \times 10^{-3}\ \text{m}^3 = 2206 \times 10^3\ \text{mm}^3$$

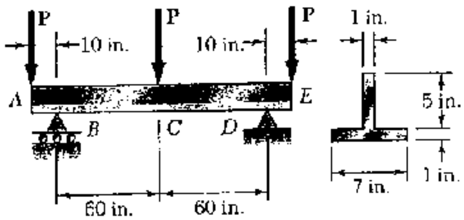
Shape	$S (10^3\ \text{mm}^3)$
W 690×125	3510
W 610×101	2530
W 530×150	3720
W 460×113	2400

Lightest wide flange section

W 610×101

Problem 5.85

5.85 Determine the allowable value of P for the loading shown, knowing that the allowable normal stress is + 8ksi in tension and − 18 ksi in compression.



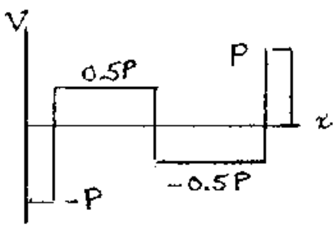
Reactions. $B = D = 1.5P \uparrow$

Shear diagram. A to B^- $V = -P$

B^+ to C^- $V = -P + 1.5P = 0.5P$

C^+ to D^- $V = 0.5P - P = -0.5P$

D^+ to E $V = -0.5P + 1.5P = P$

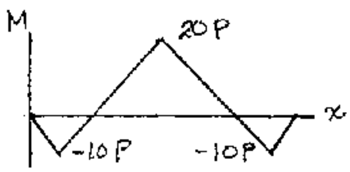


Areas. A to B $(10)(-P) = -10P$

B to C $(60)(0.5P) = 30P$

C to D $(60)(-0.5P) = -30P$

D to E $(10)(P) = 10P$



Bending moments. $M_A = 0$

$M_B = 0 - 10P = -10P$

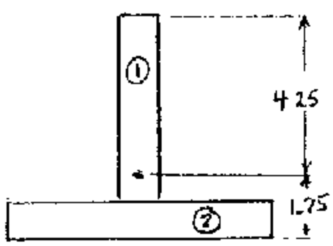
$M_C = -10P + 30P = 20P$

$M_D = 20P - 30P = -10P$

$M_E = -10P + 10P = 0$

Largest positive bending moment = $20P$

Largest negative bending moment = $-10P$



Centroid and moment of inertia.

Part	A, in^2	$\bar{y}_0$, in.	$A\bar{y}_0$, in^3	d, in.	Ad^2 in^4	$\bar{I}$, in^4
①	5	3.5	17.5	1.75	15.3125	10.417
②	7	0.5	3.5	1.25	10.9375	0.583
Σ	12		21		26.25	11.000

$\bar{Y} = \frac{21}{12} = 1.75$ in.

$I = \Sigma Ad^2 + \Sigma \bar{I} = 37.25$ in^4

Top: $y = 4.25$ in.

Bottom: $y = -1.75$ in

$\sigma = -\frac{My}{I}$

Top, Tension $8 = -\frac{(-10P)(4.25)}{37.25}$ $P = 7.01$ kips

Top, Comp. $-18 = -\frac{(20P)(4.25)}{37.25}$ $P = 7.89$ kips

Bot. Tension $8 = -\frac{(20P)(-1.75)}{37.25}$ $P = 8.51$ kips

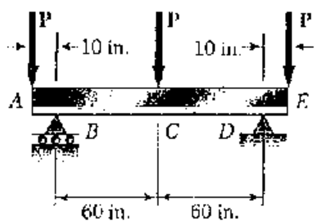
Bot. Comp $-18 = -\frac{(-10P)(-1.75)}{37.25}$ $P = 38.3$ kips

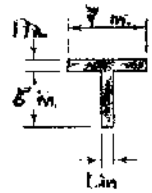
Smallest value of P is the allowable value $P = 7.01$ kips

Problem 5.86

5.86 Solve Prob. 5.85, assuming that the T-shaped beam is inverted.

5.85 Determine the allowable value of **P** for the loading shown, knowing that the allowable normal stress is + 8ksi in tension and − 18 ksi in compression.





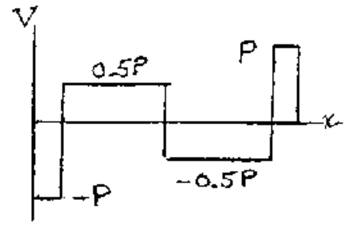
Reactions. $B = D = 1.5\,P \uparrow$

Shear diagram. A to B^- $V = -P$

B^+ to C^- $V = -P + 1.5\,P = 0.5\,P$

C^+ to D^- $V = 0.5P - P = -0.5P$

D^+ to E $V = -0.5P + 1.5P = P$

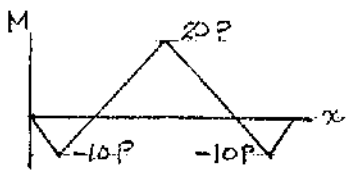


Areas. A to B $(10)(-P) = -10\,P$

B to C $(60)(0.5P) = 30\,P$

C to D $(60)(-0.5P) = -30\,P$

D to E $(10)(P) = 10\,P$



Bending moments. $M_A = 0$

$M_B = 0 - 10P = -10\,P$

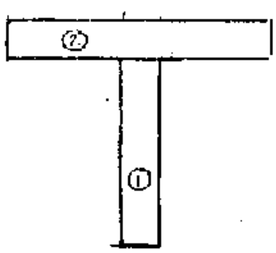
$M_C = -10\,P + 30\,P = 20\,P$

$M_D = 20\,P - 30\,P = -10\,P$

$M_E = -10\,P + 10\,P = 0$

Largest positive bending moment : $20\,P$

Largest negative bending moment : $-10\,P$



Centroid and moment of inertia.

Part	A, in^2	$\bar{y}_o$, in.	$A\bar{y}_o$, in^3	d, in.	Ad^2 in^4	$\bar{I}$, in^4
1	5	2.5	12.5	1.75	15.3125	10.417
2	7	5.5	38.5	1.25	10.9375	0.583
Σ	12		51		26.25	11.000

$\bar{Y} = \frac{51}{12} = 4.25$ in.

$I = \Sigma Ad^2 + \Sigma \bar{I} = 37.25$ in^4

Top: $y = 1.75$ in.

Bottom: $y = -4.25$ in.

$$\sigma = -\frac{My}{I}$$

Top, Tension $\quad 8 = -\dfrac{(-10P)(1.75)}{37.25} \qquad P = 17.03$ kips

Top, Comp. $\quad -18 = -\dfrac{(20P)(1.75)}{37.25} \qquad P = 19.16$ kips

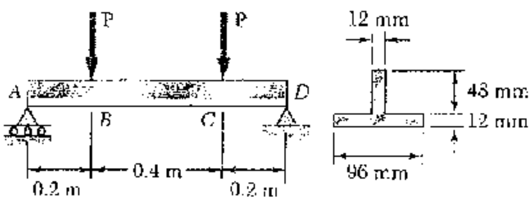
Bot. Tension $\quad 8 = -\dfrac{(20P)(-4.25)}{37.25} \qquad P = 3.51$ kips

Bot. Comp $\quad -18 = -\dfrac{(-10P)(-4.25)}{37.25} \qquad P = 15.78$ kips

Smallest value of P is the allowable value $\qquad P = 3.51$ kips ◄

Problem 5.87

5.87 and 5.88 Determine the largest permissible value of **P** for the beam and loading shown, knowing that the allowable normal stress is +80 MPa in tension and − 140 MPa in compression.

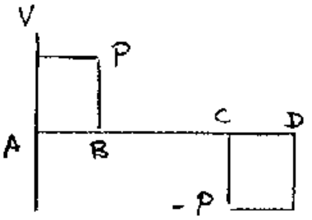


Reactions: $A = D = P$

Shear: A to B $V = P$

B to C $V = P - P = 0$

C to D $V = 0 - P = -P$



Areas: A to B $0.2P$

B to C 0

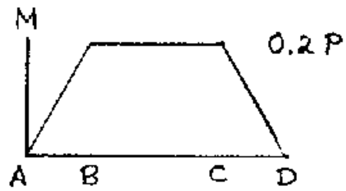
C to D $-0.2P$

Bending moments: $M_A = 0$

$M_B = 0 + 0.2P = 0.2P$

$M_C = 0.2P + 0 = 0.2P$

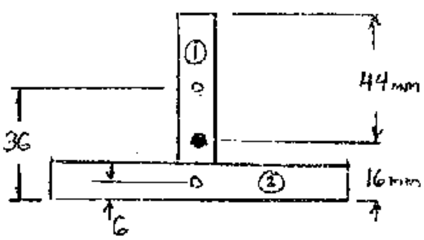
$M_D = 0.2P - 0.2P = 0$



Largest positive bending moment: $0.2P$

Largest negative bending moment: 0

Centroid and moment of inertia



Part	A (mm²)	$\bar{y}$ (mm)	$A\bar{y}$ (mm³)	d (mm)	Ad^2 (mm⁴)	$\bar{I}$ mm⁴
①	576	36	20736	20	230.4×10^3	110.6×10^3
②	1152	6	6912	10	115.2×10^3	13.8×10^3
Σ	1728		27648		345.6×10^3	124.4×10^3

$$\bar{Y} = \frac{27648}{1728} = 16 \text{ mm}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 470.0\times10^3 \text{ mm}^4$$

Top $c = 44$ mm $\quad S = \frac{I}{c} = \frac{470.0\times10^3}{44} = 10.68\times10^3 \text{ mm}^3 = 10.68\times10^{-6} \text{ m}^3$

Allowable pos. M. $\quad M = |\sigma_{comp}|\,S = (140\times10^6)(10.68\times10^{-6}) = 1495$ N·m

Bot. $c = 16$ mm $\quad S = \frac{I}{c} = \frac{470.0\times10^3}{16} = 29.38\times10^3 \text{ mm}^3 = 29.38\times10^{-6} \text{ m}^3$

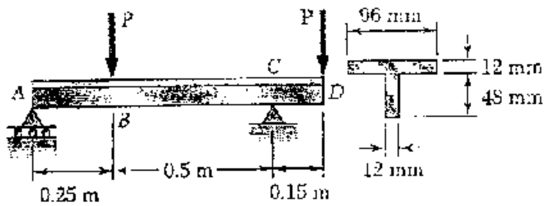
Allowable pos. M $\quad M = |\sigma_{tens}|\,S = (80\times10^6)(29.38\times10^{-6}) = 2350$ N·m

Smaller value $\quad M = 1495$ N·m

Allowable value of P $\quad 0.2P = 1495 \qquad P = 7475 \text{ N} = 7.48 \text{ kN}$ ◄

Problem 5.88

5.87 and 5.88 Determine the largest permissible value of P for the beam and loading shown, knowing that the allowable normal stress is +80 MPa in tension and −140 MPa in compression.

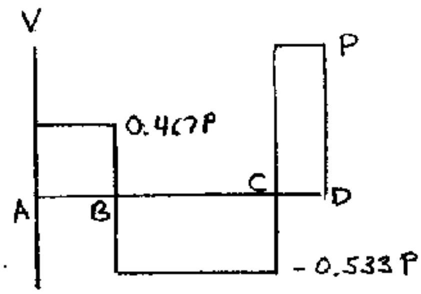


$+\circlearrowleft \Sigma M_C = 0 \quad -0.75 A + 0.5 P - 0.15 P = 0$

$A = 0.46667 \ P$

$+\circlearrowleft \Sigma M_A = 0 \quad 0.75 C - 0.25 P - 0.9 P = 0$

$C = 1.53333 \ P$



Shear: A to B $\quad V = 0.46667 \ P$

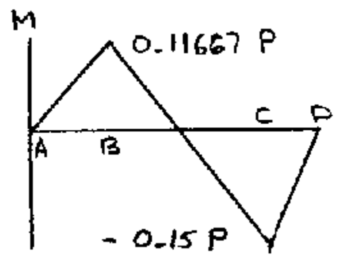
B to C $\quad V = 0.46667 P - P = -0.53333 \ P$

C to D $\quad V = -0.53333 P + 1.53333 P = P$

Areas: A to B $\quad (.25)(0.46667 P) = 0.11667 \ P$

B to C $\quad (0.5)(-0.53333 P) = -0.26667 \ P$

C to D $\quad (0.15) P = 0.15 P$



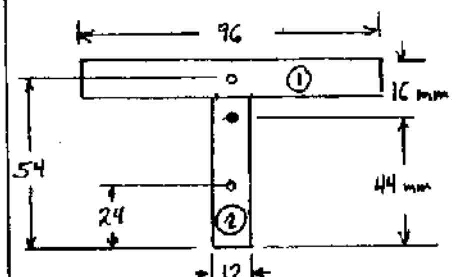
Bending moments: $\quad M_A = 0$

$M_B = 0 + 0.11667 P = 0.11667 \ P$

$M_C = 0.11667 P - 0.26667 P = -0.15 \ P$

$M_D = -0.15 P + 0.15 P = 0$

Centroid and moment of inertia.



Part	A (mm²)	$\bar{y}$ (mm)	$A\bar{y}$ (mm³)	d (mm)	Ad^2 (mm⁴)	$\bar{I}$ (mm⁴)
①	1152	54	62208	10	115200	13824
②	576	24	13824	20	230400	110592
Σ	1728		76032		345600	124416

$\bar{Y} = \frac{76032}{1728} = 44 \text{ mm}$

$I = \Sigma A d^2 + \Sigma \bar{I} = 470016 \text{ mm}^4$

Top: $\quad y = 16 \text{ mm} \qquad \frac{I}{y} = \frac{470016}{16} = 29.376 \times 10^3 \text{ mm}^3 = 29.376 \times 10^{-6} \text{ m}^3$

Bottom: $\quad y = -44 \text{ mm} \qquad \frac{I}{y} = \frac{470016}{44} = 10.682 \times 10^3 \text{ mm}^3 = -10.682 \times 10^{-6} \text{ m}^3$

Bending moment limits: $\quad M = -\frac{I\sigma}{y}$

Tension at B $\quad = (-10.682 \times 10^{-6})(80 \times 10^6) = 854.56 \text{ N·m} \quad \triangleleft B$

Comp. at B $\quad -(29.376 \times 10^{-6})(-140 \times 10^6) = 4.1126 \times 10^3 \text{ N·m}$

Tension at C $\quad -(29.376 \times 10^{-6})(80 \times 10^6) = -2.35 \times 10^3 \text{ N·m}$

Comp. at C $\quad -(-10.682 \times 10^{-6})(-140 \times 10^6) = -1.4955 \times 10^3 \text{ N·m} \quad \triangleleft C$

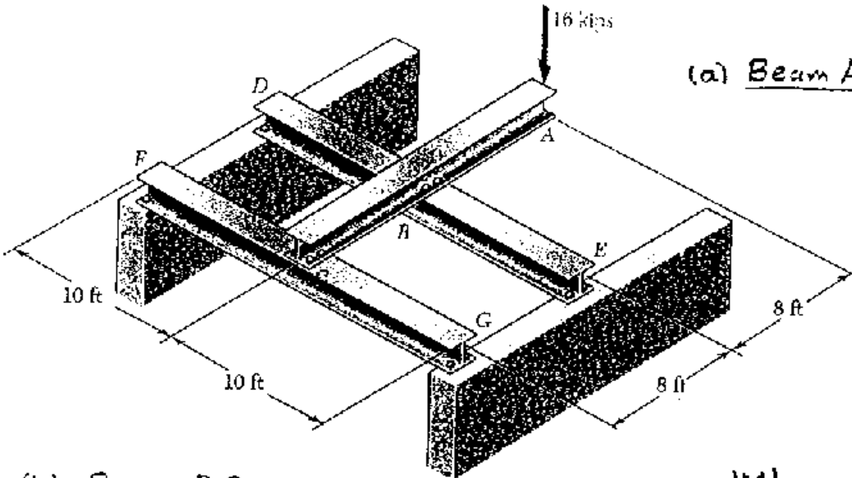
Allowable load: $\quad 0.11667 P = 854.56 \qquad P = 7.32 \times 10^3 \text{ N}$

$-0.15 P = -1.4955 \times 10^3 \qquad P = 9.97 \times 10^3 \text{ N}$

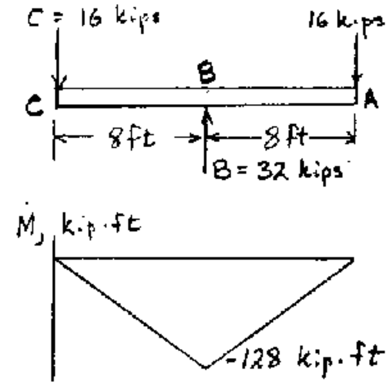
The smaller value is $\quad P = 7.32 \text{ kN}$

Problem 5.89

5.89 Beam *ABC* is bolted to beams *DBE* and *FCG*. Knowing that the allowable normal stress is 24 ksi, select the most economical wide-flange shape that can be used (*a*) for beam *ABC*, (*b*) for beam *DBE* (*c*) for beam *FCG*.



(a) Beam ABC



$M_B = -(8)(16) = -128$ kip·ft

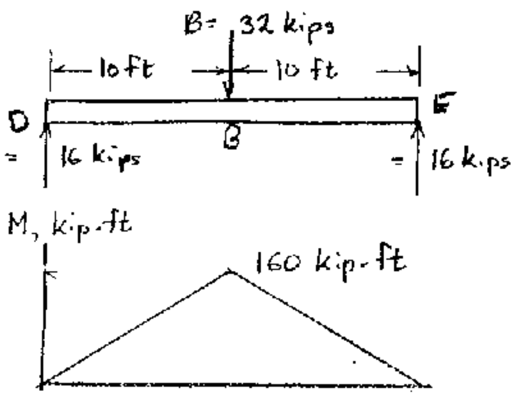
$|M|_{max} = 128$ kip·ft $= 1536$ kip·in.

$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{1536}{24} = 64 \text{ in}^3$

Shape	S, in³
W 21 × 44	81.6
W 18 × 50	88.9
W 16 × 40	64.7
W 14 × 53	77.8
W 12 × 50	64.7
W 10 × 68	75.7

← (a) W 16 × 40

(b) Beam DBE



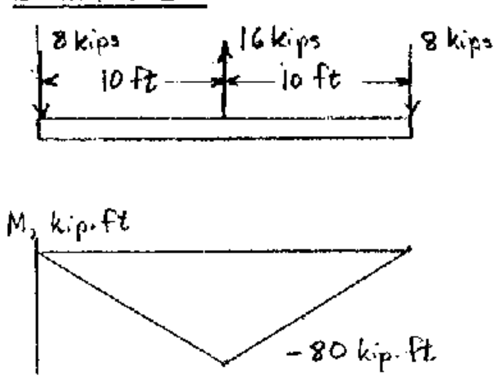
$|M|_{max} = 160$ kip·ft $= 1920$ kip·in

$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{1920}{24} = 80 \text{ in}^3$

Shape	S, in³
W 21 × 44	81.6
W 18 × 50	88.3
W 16 × 57	97.2
W 14 × 68	103
W 12 × 72	97.4
W 10 × 112	126

← (b) W 21 × 44

(c) Beam FCG



$|M|_{max} = 80$ kip·ft $= 960$ kip·in

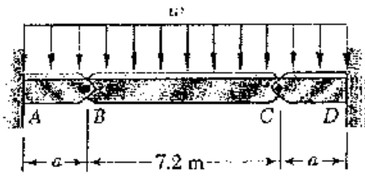
$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{960}{24} = 40 \text{ in}^3$

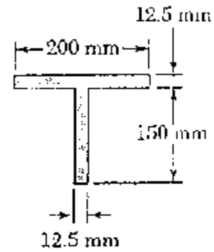
Shape	S, in³
W 18 × 35	57.6
W 16 × 31	47.2
W 14 × 30	42.0
W 12 × 35	45.6
W 10 × 39	42.1
W 8 × 48	43.3

← (c) W 14 × 30

Problem 5.90

5.90 Beams AB, BC, and CD have the cross section shown and are pin-connected at B and C. Knowing that the allowable normal stress is +110 MPa in tension and −150 MPa in compression, determine (*a*) the largest permissible value of w if beam BC is not to be overstressed, (*b*) the corresponding maximum distance a for which the cantilever beams AB and CD are not overstressed.





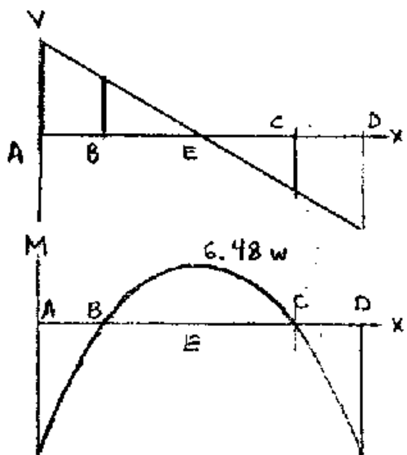
(a) $M_B = M_C = 0$

$V_B = -V_C = (\frac{1}{2})(7.2)w = 3.6\ w$

Area B to E of shear diagram

$(\frac{1}{2})(3.6)(3.6w) = 6.48\ w$

$M_E = 0 + 6.48\ w = 6.48\ w$

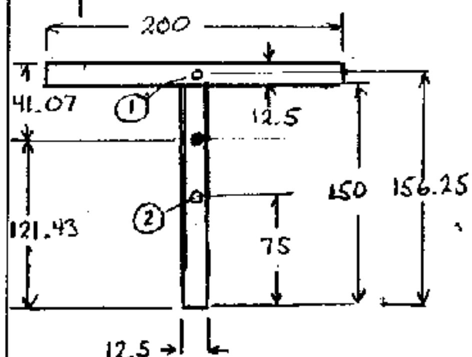


Centroid and moment of inertia

Part	A (mm²)	$\bar{y}$ (mm)	$A\bar{y}$ (mm³)	d (mm)	Ad^2 (mm⁴)	$\bar{I}$ (mm⁴)
①	2500	156.25	390625	34.82	3.031×10^6	0.0326×10^6
②	1875	75	140625	46.43	4.042×10^6	3.516×10^6
Σ	4375		531250		7.073×10^6	3.548×10^6

$\bar{Y} = \dfrac{531250}{4375} = 121.43$ mm

$I = \Sigma Ad^2 + \Sigma\bar{I} = 10.621\times10^6\ \text{mm}^4$



Location	y (mm)	I/y (10^3 mm³)
top	41.07	258.6
bottom	−121.43	−87.47

← also (10^{-6} m³)

Bending moment limits $M = -\sigma I/y$

Tension at E: $-(110\times10^6)(-87.47\times10^{-6}) = 9.622\times10^3$ N·m

Comp. at E: $-(-150\times10^{-6})(258.6\times10^{-6}) = 38.8\times10^3$ N·m

Tension at A & D: $-(110\times10^6)(258.6\times10^{-6}) = -28.45\times10^3$ N·m

Comp. at A & D: $-(-150\times10^6)(-87.47\times10^{-6}) = -13.121\times10^3$ N·m

(a) Allowable load w $\qquad 6.48w = 9.622\times10^3 \qquad w = 1.485\times10^3$ N/m
$= 1.485$ kN/m ◄

Shear at A $\qquad V_A = (a + 3.6)\,w$

Area A to B of shear diagram $\qquad \frac{1}{2}a(V_A + V_B) = \frac{1}{2}a(a+7.2)w$

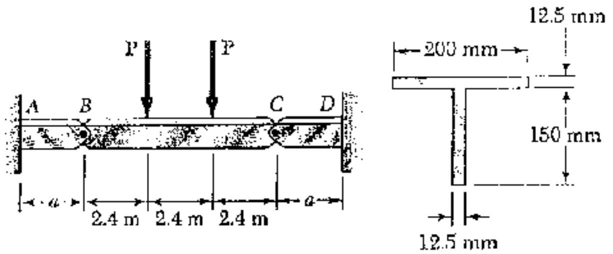
Bending moment at A (also D) $\qquad M_A = -\frac{1}{2}a(a+7.2)w$

$-\frac{1}{2}a(a+7.2)(1.485\times10^3) = -13.121\times10^3$

(b) $\frac{1}{2}a^2 + 3.6a - 8.837 = 0 \qquad a = 1.935$ m ◄

Problem 5.91

5.91 Beams *AB*, *BC*, and *CD* have the cross section shown and are pin-connected at *B* and *C*. Knowing that the allowable normal stress is +110 MPa in tension and −150 MPa in compression, determine (*a*) the largest permissible value of **P** if beam *BC* is not to be overstressed, (*b*) the corresponding maximum distance *a* for which the cantilever beams *AB* and *CD* are not overstressed.

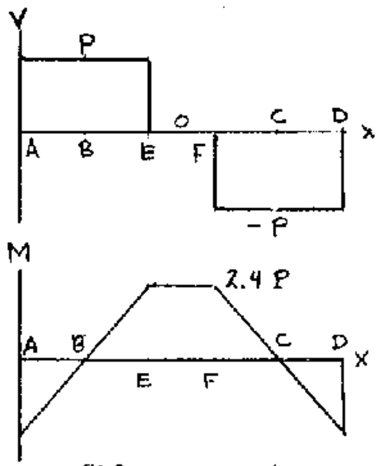


(a) $M_B = M_C = 0$

$V_B = -V_C = P$

Area B to E of shear diagram. $2.4\ P$

$M_E = 0 + 2.4\,P = 2.4\,P = M_F$

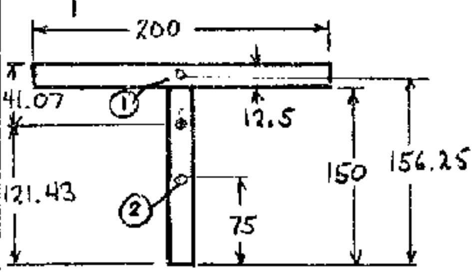


Centroid and moment of inertia

Part	A (mm²)	$\bar{y}$ (mm)	$A\bar{y}$ (mm³)	d (mm)	Ad² (mm⁴)	$\bar{I}$ (mm⁴)
①	2500	156.25	390625	34.82	3.031×10^6	0.0326×10^6
②	1875	75	140625	46.43	4.042×10^6	3.516×10^6
Σ	4375		531250		7.073×10^6	3.548×10^6

$$\bar{Y} = \frac{531250}{4375} = 121.43\ \text{mm}$$

$$I = \Sigma A d^2 + \Sigma \bar{I} = 10.621\times10^6\ \text{mm}^4$$



Location	y (mm)	I/y (10^3 mm³) ← also (10^{-6} m³)
top	41.07	258.6
bottom	−121.43	−87.47

Bending moment limits $M = -\sigma I/y$

Tension at E & F: $-(110\times10^6)(-87.47\times10^{-6}) = 9.622\times10^3$ N·m

Comp. at E & F: $-(-150\times10^6)(258.6\times10^{-6}) = 38.8\times10^3$ N·m

Tension at A & D: $-(110\times10^6)(258.6\times10^{-6}) = -28.45\times10^3$ N·m

Comp. at A & D: $-(-150\times10^6)(-87.47\times10^{-6}) = -13.121\times10^3$ N·m

(a) Allowable load P. $2.4\,P = 9.622\times10^3$ $P = 4.01\times10^3$ N $= 4.01$ kN

Shear at A $V_A = P$

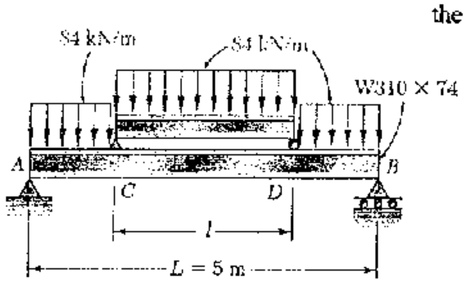
Area A to B of shear diagram $aV_A = aP$

Bending moment at A $M_A = -aP = -4.01\times10^3\,a$

(b) Distance a. $-4.01\times10^3\,a = -13.121\times10^3$ $a = 3.27$ m

Problem 5.92

5.92 A uniformly distributed load of 84 kN/m is to be supported over the 5-m span shown. Knowing that the allowable normal stress for the steel used is 165 MPa, determine (*a*) the smallest allowable length l of beam CD if the W 310 × 74 beam AB is not to be overstressed, (*b*) the W shape which should be used for beam CD. Neglect the weight of both beams.



For rolled steel section W 310×74 of beam AB

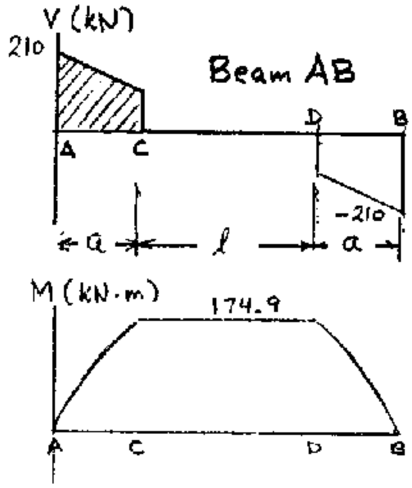
$S = 1060 \times 10^3\ mm^3 = 1060 \times 10^{-6}\ m^3$

$\sigma_{all} = 165\ MPa = 165 \times 10^6\ Pa$

Allowable bending moment

$M_{all} = S\,\sigma_{all} = (1060 \times 10^{-6})(165 \times 10^6) = 174.9 \times 10^3\ N\cdot m$

$= 174.9\ kN\cdot m$



By symmetry reactions A and B are equal.

$+\uparrow \Sigma F_y = 0 \quad A + B - (5)(84) = 0$

$A = B = 210\ kN$

By symmetry, reaction at C and D are equal.

$+\uparrow \Sigma F_y = 0 \quad C + D - 84\,l = 0$

$C = D = 42\,l$

Geometry $\quad a = \frac{1}{2}(5 - l)$

Beam AB: Area A to C of shear diagram

$\frac{1}{2}(a)(A + C) = \frac{1}{2}\cdot\frac{1}{2}(5 - l)(210 + 42\,l)$

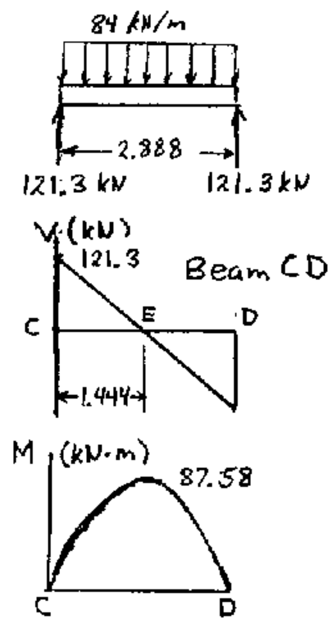
$= \frac{1}{4}(1050 - 42\,l^2)$

Bending moment at C $\quad \frac{1}{4}(1050 - 42\,l^2)$

$\frac{1}{4}(1050 - 42\,l^2) = 174.9 \qquad l^2 = 8.3429\ m^2$

(a) $\qquad l = 2.888\ m$ ◄

$C = D = 42\,l = 121.3\ kN$



Beam CD (midpoint E)

Area C to E of shear diagram

$\frac{1}{2}(1.444)(121.3) = 87.58\ kN\cdot m$

Bending moment at E $\qquad M = 87.58\ kN\cdot m = 87.58 \times 10^3\ N\cdot m$

$\sigma_{all} = 165 \times 10^6\ Pa$

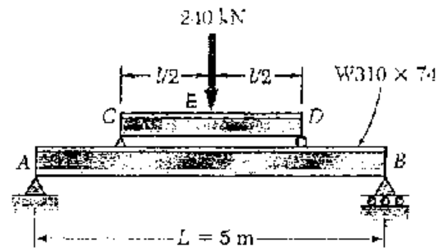
$S_{min} = \frac{M}{\sigma_{all}} = \frac{87.58 \times 10^3}{165 \times 10^6} = 531 \times 10^{-6}\ m^3 = 531 \times 10^3\ mm^3$

Shape	$S\ (10^3\ mm^3)$
W 460 × 52	942
W 410 × 38.8	637
W 360 × 39	578
W 310 × 38.7	549
W 250 × 44.8	535
W 200 × 59	582

(b) Use W 310 × 38.7 ◄

Problem 5.93

5.93 A 240-kN load is to be supported at the center of the 5-m span shown. Knowing that the allowable normal stress for the steel used is 165 MPa, determine (*a*) the smallest allowable length l of beam CD if the W310 × 74 beam AB is not to be over stressed, (*b*) the most economical W shape which can be used for beam CD. Neglect the weight of both beams.



For rolled steel section W 310 × 74 of beam AB

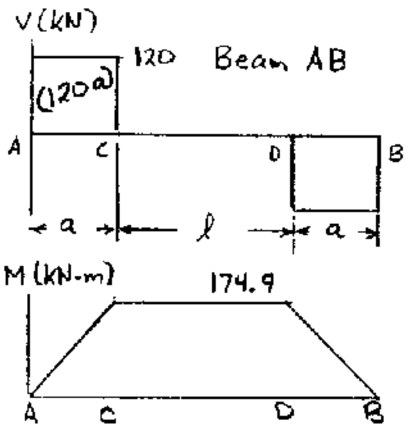
$S = 1060 \times 10^3\ mm^3 = 1060 \times 10^{-6}\ m^3$

$\sigma_{all} = 165\ MPa = 165 \times 10^6\ Pa$

Allowable bending moment

$M_{all} = S\sigma_{all} = (1060 \times 10^{-6})(165 \times 10^6) = 174.9 \times 10^3\ N\cdot m$

$= 174.9\ kN\cdot m$



(a) Beam AB

Area A to C of shear diagram $= 120\ a$

Bending moment at C $\quad 120\ a$

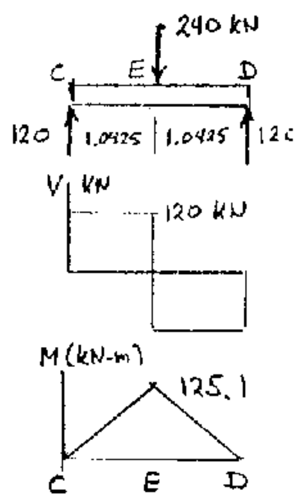
$120\ a = 174.9 \quad = 1.4575\ m$

Geometry: $2a + l = 5 \qquad l = 5 - 2a = 2.085\ m$ ◄

(b) Beam CD (midpoint E)

Area C to E of shear diagram

$= (1.0425)(120) = 125.1\ kN\cdot m$



Bending moment at E

$M = 125.1\ kN\cdot m = 125.1 \times 10^3\ N\cdot m$

$S_{min} = \dfrac{M}{\sigma_{all}} = \dfrac{125.1 \times 10^3}{165 \times 10^6} = 758.2 \times 10^{-6}\ m^3$

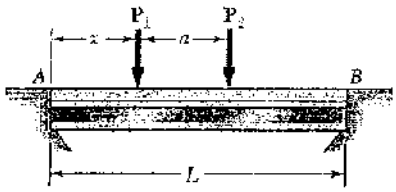
$= 758.2 \times 10^3\ mm^3$

Shape	S ($10^3\ mm^3$)
W 460 × 52	942
W 410 × 46.1	774 ←
W 360 × 57.8	899
W 310 × 60	851
W 250 × 67	809
W 200 × 86	853

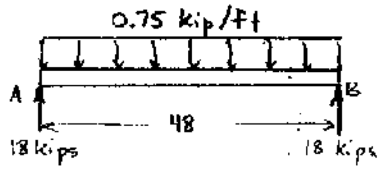
Answer W410 × 46.1 ◄

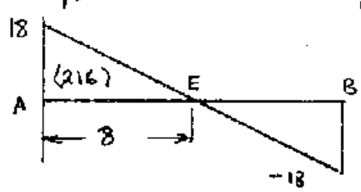
Problem 5.94

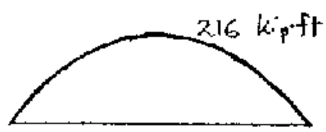
*5.94 A bridge of length $L = 48$ ft is to be built on a secondary road whose access to trucks is limited to two-axle vehicles of medium weight. It will consist of a concrete slab and of simply supported steel beams with an ultimate strength $\sigma_U = 60$ ksi. The combined weight of the slab and beams can be approximated by a uniformly distributed load $w = 0.75$ kips/ft on each beam. For the purpose of the design, it is assumed that a truck with axles located at a distance $a = 14$ ft from each other will be driven across the bridge and that the resulting concentrated loads $\mathbf{P}_1$ and $\mathbf{P}_2$ exerted on each beam could be as large as 24 kips and 6 kips, respectively. Determine the most economical wide-flange shape for the beams, using LRFD with the load factors $\gamma_D = 1.25$, $\gamma_L = 1.75$ and the resistance factor $\phi = 0.9$. [*Hint*. It can be shown that the maximum value of $|M_L|$ occurs under the larger load when that load is located to the left of the center of the beam at a distance equal to $aP_2(P_1 + P_2)$.]

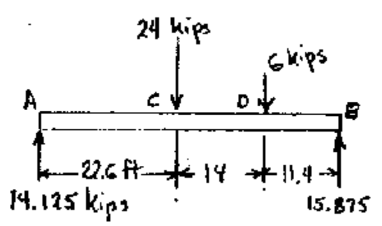


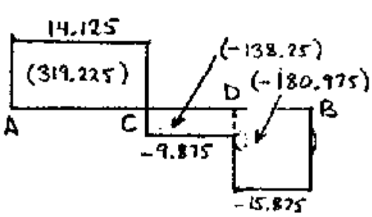
$L = 48$ ft
$a = 14$ ft
$P_1 = 24$ kips
$P_2 = 6$ kips
$W = 0.75$ kip/ft

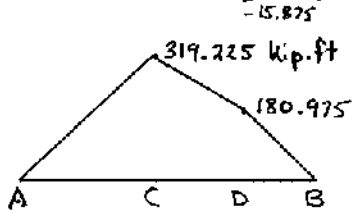












Dead load: $R_A = R_B = (\frac{1}{2})(48)(0.75) = 18$ kips

Area A to E of shear diagram $(\frac{1}{2})(8)(18) = 216$ kip·ft

$M_{max} = 216$ kip·ft $= 2592$ kip·in at point E

Live load: $u = \dfrac{aP_2}{2(P_1+P_2)} = \dfrac{(14)(6)}{(2)(30)} = 1.4$ ft

$x = \frac{L}{2} - u = 24 - 1.4 = 22.6$ ft
$x + a = 22.6 + 14 = 36.6$ ft
$L - x - a = 48 - 36.6 = 11.4$ ft

$\Sigma M_B = 0 \quad -48 R_A + (25.4)(24) + (11.4)(6) = 0$
$R_A = 14.125$ kips

Shear: A to C $\quad V = 14.125$ kips
C to D $\quad V = 14.125 - 24 = -9.875$ kips
D to B $\quad V = -15.875$ kips

Area A to C $(22.6)(14.125) = 319.225$ kip·ft

Bending moment: $M_C = 319.225$ kip·ft $= 3831$ kip·in.

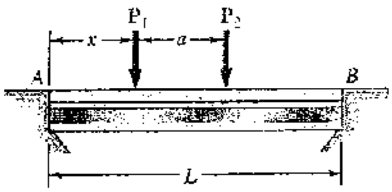
Design: $\gamma_D M_D + \gamma_L M_L = \phi M_U = \phi \sigma_{all} S_{min}$

$$S_{min} = \frac{\gamma_D M_D + \gamma_L M_L}{\phi \sigma_{all}} = \frac{(1.25)(2592) + (1.75)(3831)}{(0.9)(60)}$$
$$= 184.2 \text{ in}^3$$

Shape	S (in³)
W 30×99	269
W 27×84	213 ←
W 24×104	258
W 21×101	227
W 18×106	204

W27 × 84 ◄

Problem 5.95



$L = 48$ ft
$a = 14$ ft
$P_1 = 24$ kips
$P_2 = 6$ kips
$w = 0.75$ kip/ft

***5.95** Assuming that the front and read axle loads remain in the same ratio as for the truck of Prob. 5.94, determine how much heavier a truck could safely cross the bridge designed in that problem.

***5.94** A bridge of length $L = 48$ ft is to be built on a secondary road whose access to trucks is limited to two-axle vehicles of medium weight. It will consist of a concrete slab and of simply supported steel beams with an ultimate strength $\sigma_U = 60$ ksi. The combined weight of the slab and beams can be approximated by a uniformly distributed load $w = 0.75$ kips/ft on each beam. For the purpose of the design, it is assumed that a truck with axles located at a distance $a = 14$ ft from each other will be driven across the bridge and that the resulting concentrated loads $\mathbf{P}_1$ and $\mathbf{P}_2$ exerted on each beam could be as large as 24 kips and 6 kips, respectively. Determine the most economical wide-flange shape for the beams, using LRFD with the load factors $\gamma_D = 1.25$, $\gamma_L = 1.75$ and the resistance factor $\phi = 0.9$. [*Hint.* It can be shown that the maximum value of $|M_L|$ occurs under the larger load when that load is located to the left of the center of the beam at a distance equal to $aP_2(P_1 + P_2)$.]

See solution to Problem 5.94 for calculation of the following:

$M_D = 2592$ kip·in $\qquad M_L = 3831$ kip·in

For rolled steel section W 27×84 $\qquad S = 213$ in^3

Allowable live load moment M_L^*

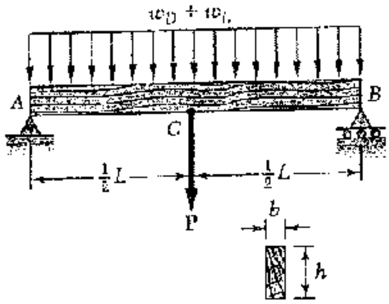
$$\gamma_D M_D + \gamma_L M_L^* = \phi M_U = \phi \sigma_U S$$

$$M_L^* = \frac{\phi \sigma_U S - \gamma_D M_D}{\gamma_L} = \frac{(0.9)(60)(213) - (1.25)(2592)}{1.75} = 4721 \text{ kip·in}$$

Ratio $\dfrac{M_L^*}{M_L} = \dfrac{4721}{3831} = 1.232 = 1 + 0.232$

Increase $\qquad$ 23.2 %

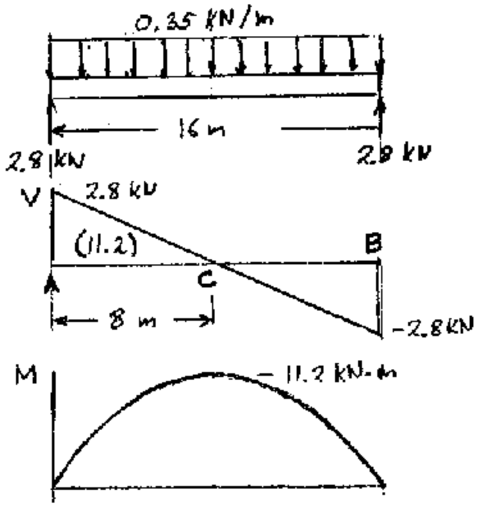
Problem 5.96



*5.96 A roof structure consists of plywood and roofing material supported by several timber beams of length $L = 16$ m. The dead load carried by each beam, including the estimated wright of the beam, can be represented by a uniformly distributed load $w_D = 350$ N/m. The live load consists of a snow load, represented by a uniformly distributed load $w_L = 600$ N/m, and a 6-kN concentrated load **P** applied at the midpoint C of each beam. Knowing that the ultimate strength for the timber used is $\sigma_U = 50$ MPa and that the width of the beam is $b = 75$ mm, determine the minimum allowable depth h of the beams, using LRFD with the load factors $\gamma_D = 1.2$, $\gamma_L = 1.6$ and the resistance factor $\phi = 0.9$.

$L = 16$ m, $w_D = 350$ N/m $= 0.35$ kN/m

$w_L = 600$ N/m $= 0.6$ kN/m, $P = 6$ kN



Dead load: $R_A = (\frac{1}{2})(16)(0.35) = 2.8$ kN

Area A to C of shear diagram

$(\frac{1}{2})(8)(2.8) = 11.2$ kN·m

Bending moment at C: 11.2 kN·m $= 11.2 \times 10^3$ N·m

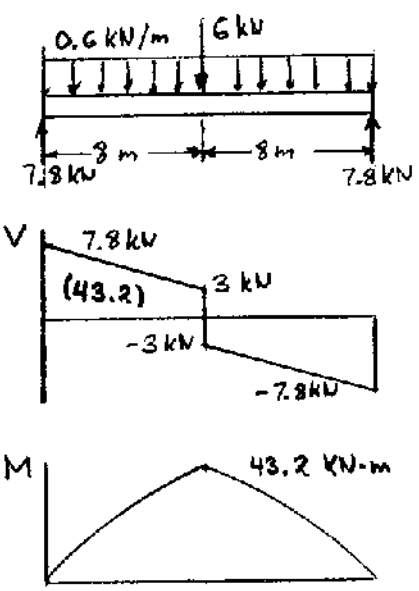
Live load: $R_A = \frac{1}{2}[(16)(0.6) + 6] = 7.8$ kN

Shear at C^- $V = 7.8 - (8)(0.6) = 3$ kN

Area A to C of shear diagram

$(\frac{1}{2})(8)(7.8 + 3) = 43.2$ kN·m

Bending moment at C: 43.2 kN·m $= 43.2 \times 10^3$ N·m



Design $\gamma_D M_D + \gamma_L M_L = \phi M_U = \phi \sigma_U S$

$$S = \frac{\gamma_D M_D + \gamma_L M_L}{\phi \sigma_U} = \frac{(1.2)(11.2 \times 10^3) + (1.6)(43.2 \times 10^3)}{(0.9)(50 \times 10^6)}$$

$$= 1.8347 \times 10^{-3} \text{ m}^3 = 1.8347 \times 10^6 \text{ mm}^3$$

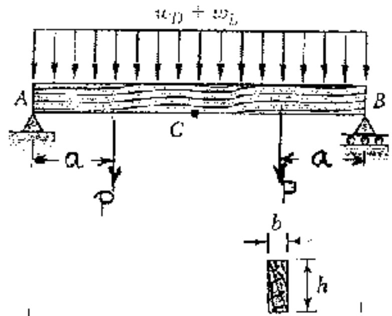
For a rectangular section $S = \frac{1}{6} b h^2$

$$h = \sqrt{\frac{6S}{b}} = \sqrt{\frac{(6)(1.8347 \times 10^6)}{75}} = 383 \text{ mm}$$

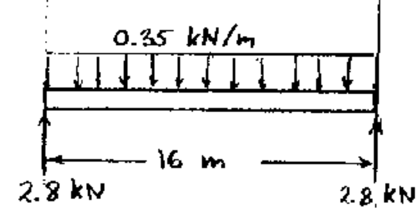
Problem 5.97

***5.97** Solve Prob. 5.96, assuming that the 6-kN concentrated load **P** applied to each beam is replaced by 3-kN concentrated loads $\mathbf{P}_1$ and $\mathbf{P}_2$ applied at a distance of 4 m from each end of the beams.

***5.96** A roof structure consists of plywood and roofing material supported by several timber beams of length $L = 16$ m. The dead load carried by each beam, including the estimated wright of the beam, can be represented by a uniformly distributed load $w_D = 350$ N/m. The live load consists of a snow load, represented by a uniformly distributed load $w_L = 600$ N/m, and a 6-kN concentrated load **P** applied at the midpoint C of each beam. Knowing that the ultimate strength for the timber used is $\sigma_U = 50$ MPa and that the width of the beam is $b = 75$ mm, determine the minimum allowable depth h of the beams, using LRFD with the load factors $\gamma_D = 1.2$, $\gamma_L = 1.6$ and the resistance factor $\phi = 0.9$.

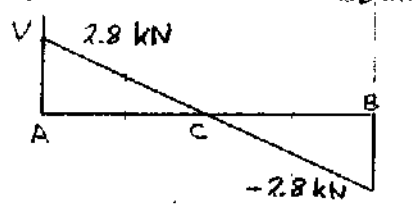


$L = 16$ m, $a = 4$ m, $w_D = 350$ N/m $= 0.35$ kN/m
$w_L = 600$ N/m $= 0.6$ kN/m $P = 3$ kN

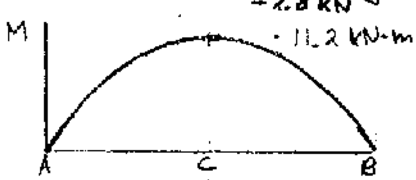


<u>Dead load:</u> $R_A = (\frac{1}{2})(16)(0.35) = 2.8$ kN

Area A to C of shear diagram
$(\frac{1}{2})(8)(2.8) = 11.2$ kN·m

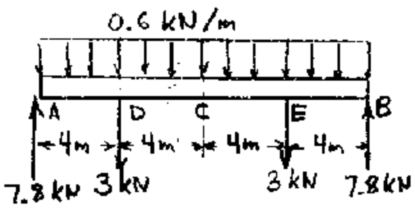


Bending moment at C: 11.2 kN·m $= 11.2 \times 10^3$ N·m



<u>Live load:</u> $R_A = \frac{1}{2}[(16)(0.6) + 3 + 3] = 7.8$ kN

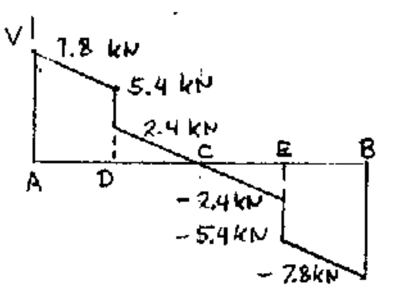
Shear at D^- $7.8 - (4)(0.6) = 5.4$ kN
Shear at D^+ $5.4 - 3 = 2.4$ kN



Area A to D $(\frac{1}{2})(4)(7.8 + 5.4) = 26.4$ kN·m

Area D to C $(\frac{1}{2})(4)(2.4) = 4.8$ kN·m

Bending moment at C $= 26.4 + 4.8 = 31.2$ kN·m
$= 31.2 \times 10^3$ N·m



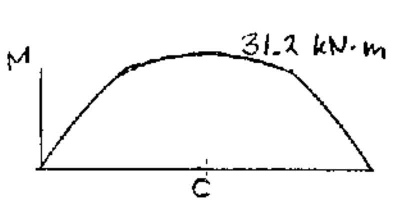
<u>Design</u> $\gamma_D M_D + \gamma_L M_L = \phi M_U = \phi \sigma_U S$

$$S = \frac{\gamma_D M_D + \gamma_L M_L}{\phi \sigma_U} = \frac{(1.2)(11.2\times10^3) + (1.6)(31.2\times10^3)}{(0.9)(50\times10^6)}$$

$$= 1.408 \times 10^{-3} \text{ m}^3 = 1.408 \times 10^6 \text{ mm}^3$$

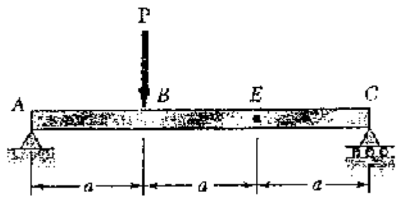
For a rectangular section $S = \frac{1}{6} b h^2$

$$h = \sqrt{\frac{6S}{b}} = \sqrt{\frac{(6)(1.408\times10^6)}{75}} = 336 \text{ mm}$$

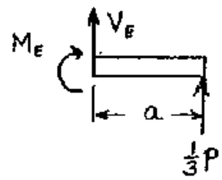


Problem 5.98

5.98 through 5.100 (*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment at point *E* and check your answer by drawing the free-body diagram of the portion of the beam to the right of *E*.



$+\circlearrowleft \Sigma M_C = 0 \qquad -3aA + 2aP = 0 \qquad A = \frac{2}{3}P$



(a) $V = \frac{2}{3}P - P\langle x-a\rangle^0$

$M = \frac{2}{3}Px - P\langle x-a\rangle^1$

At point $E \qquad x = 2a$

(b) $M_E = \frac{2}{3}P(2a) - Pa = \frac{1}{3}Pa$

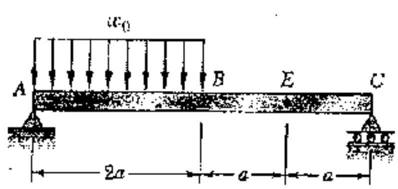
$\Sigma M_A = 0 \qquad 3aC - aP = 0 \qquad C = \frac{1}{3}P$

$+\circlearrowleft \Sigma M_E = 0 \qquad -M_E + (a)(\frac{1}{3}P) = 0$

$M_E = \frac{1}{3}Pa$

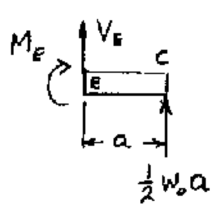
Problem 5.99

5.98 through 5.100 (*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment at point *E* and check your answer by drawing the free-body diagram of the portion of the beam to the right of *E*.



$+\circlearrowleft \Sigma M_C = 0 \qquad -4aA + (3a)(2aw_0) = 0 \qquad A = \frac{3}{2}w_0 a$

$w = w_0 - w_0\langle x-2a\rangle^0 = -\frac{dV}{dx}$



(a) $V = -w_0 x + w_0\langle x-2a\rangle^1 + \frac{3}{2}w_0 a = \frac{dM}{dx}$

$M = -\frac{1}{2}w_0 x^2 + \frac{1}{2}w_0\langle x-2a\rangle^2 + \frac{3}{2}w_0 a x + 0$

At point $E \qquad x = 3a$

(b) $M_E = -\frac{1}{2}w_0(3a)^2 + \frac{1}{2}w_0 a^2 + \frac{3}{2}w_0 a(3a)$

$= \frac{1}{2}w_0 a^2$

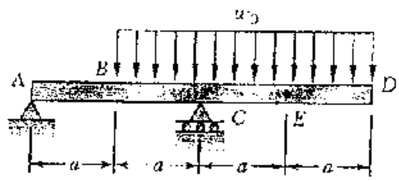
$+\circlearrowleft \Sigma M_A = 0 \qquad 4aC - (a)(2aw_0) = 0 \qquad C = \frac{1}{2}w_0 a$

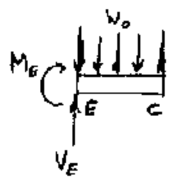
$+\circlearrowleft \Sigma M_E = 0 \qquad -M_E + (a)(\frac{1}{2}w_0 a) = 0$

$M_E = \frac{1}{2}w_0 a^2$

Problem 5.100

5.98 through 5.100 (*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment at point *E* and check your answer by drawing the free-body diagram of the portion of the beam to the right of *E*.





$+\circlearrowleft \Sigma M_C = 0 \quad -2a\,A - (\frac{a}{2})(3a\,w_o) = 0 \quad A = -\frac{3}{4} w_o a$

$+\circlearrowleft \Sigma M_A = 0 \quad 2a\,C + (\frac{5a}{2})(3a\,w_o) = 0 \quad C = \frac{15}{4} w_o a$

$w = w_o \langle x-a \rangle^0 = -\frac{dV}{dx}$

(a) $V = -w_o \langle x-a \rangle^1 - \frac{3}{4} w_o a + \frac{15}{4} w_o a \langle x-2a \rangle^0 = \frac{dM}{dx}$

$M = -\frac{1}{2} w_o \langle x-a \rangle^2 - \frac{3}{4} w_o a x + \frac{15}{4} w_o a \langle x-2a \rangle^1 + 0$

At point E $\quad x = 3a$

(b) $M_E = -\frac{1}{2} w_o (2a)^2 - \frac{3}{4} w_o a (3a) + \frac{15}{4} w_o a (a)$

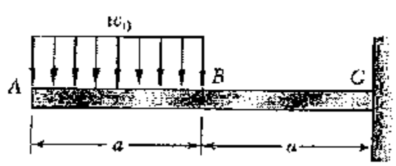
$= -\frac{1}{2} w_o a^2$

Check: $+\Sigma M_E = 0 \quad -M_E - \frac{a}{2}(w_o a) = 0$

$M_E = -\frac{1}{2} w_o a^2$

Problem 5.101

5.101 through 5.103 (*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment at point *C* and check your answer by drawing the free-body diagram of the entire beam.



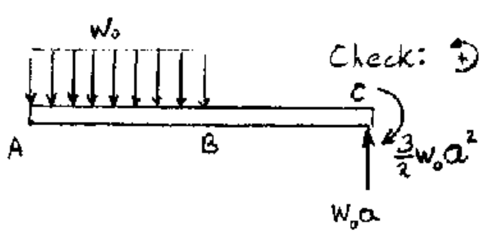
$w = w_o - w_o \langle x-a \rangle^0 = -\frac{dV}{dx}$

(a) $V = -w_o x + w_o \langle x-a \rangle^1 = \frac{dM}{dx}$

$M = -\frac{1}{2} w_o x^2 + \frac{1}{2} w_o \langle x-a \rangle^2$

At point C $\quad x = 2a$

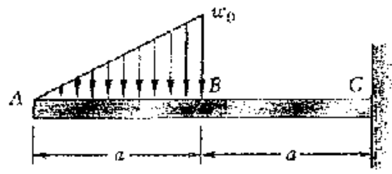
(b) $M_C = -\frac{1}{2} w_o (2a)^2 + \frac{1}{2} w_o a^2 = -\frac{3}{2} w_o a^2$



Check: $+\circlearrowleft \Sigma M_C = 0 \quad (\frac{3a}{2})(w_o a) + M_C = 0$

$M_C = -\frac{3}{2} w_o a^2$

Problem 5.102



5.101 through *5.103* (*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment at point *C* and check your answer by drawing the free-body diagram of the entire beam.

$$w = \frac{w_0 x}{a} - w_0\langle x-a\rangle^0 - \frac{w_0}{a}\langle x-a\rangle^1 = -\frac{dV}{dx}$$

(a) $$V = -\frac{w_0 x^2}{2a} + w_0\langle x-a\rangle^1 + \frac{w_0}{2a}\langle x-a\rangle^2 = \frac{dM}{dx}$$

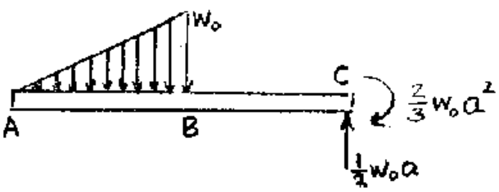
$$M = -\frac{w_0 x^3}{6a} + \frac{w_0}{2}\langle x-a\rangle^2 + \frac{w_0}{6a}\langle x-a\rangle^3$$

At point C $x = 2a$

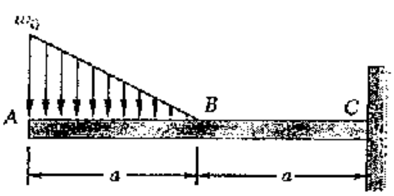
(b) $$M_C = -\frac{w_0(2a)^3}{6a} + \frac{w_0 a^2}{2} + \frac{w_0 a^3}{6a} = -\frac{2}{3}w_0 a^2$$

Check: $+\circlearrowleft \Sigma M_C = 0$ $\quad \left(\frac{4a}{3}\right)\left(\frac{1}{2}w_0 a\right) + M_C = 0$

$$M_C = -\frac{2}{3}w_0 a^2$$



Problem 5.103



5.101 through *5.103* (*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment at point *C* and check your answer by drawing the free-body diagram of the entire beam.

$$w = w_0 - \frac{w_0 x}{a} + \frac{w_0}{a}\langle x-a\rangle^1 = -\frac{dV}{dx}$$

(a) $$V = -w_0 x + \frac{w_0 x^2}{2a} - \frac{w_0}{2a}\langle x-a\rangle^2 = \frac{dM}{dx}$$

$$M = -\frac{w_0 x^2}{2} + \frac{w_0 x^3}{6a} - \frac{w_0}{6a}\langle x-a\rangle^3$$

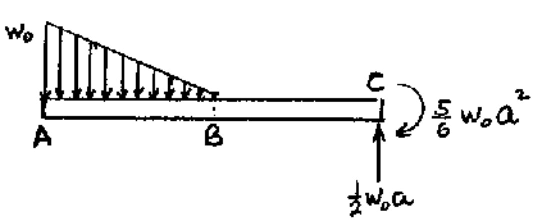
At point C $x = 2a$

(b) $$M_C = -\frac{w_0(2a)^2}{2} + \frac{w_0(2a)^3}{6a} - \frac{w_0 a^3}{6a} = -\frac{5}{6}w_0 a^2$$

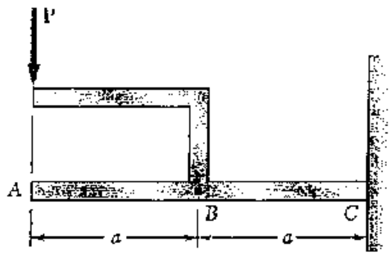
Check:

$+\circlearrowleft \Sigma M_C = 0$ $\quad \left(\frac{5}{3}a\right)\left(\frac{1}{2}w_0 a\right) + M_C = 0$

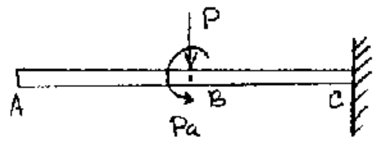
$$M_C = -\frac{5}{6}w_0 a^2$$



Problem 5.104



5.104 (*a*) Using singularity functions, write the equations for the shear and bending moment for beam *ABC* under the loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment just to the right of point *B*.



(a) $V = -P\langle x-a\rangle^0$

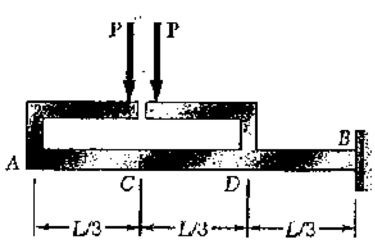
$\frac{dM}{dx} = -P\langle x-a\rangle^0$

$M = -P\langle x-a\rangle^1 - Pa\langle x-a\rangle^0$

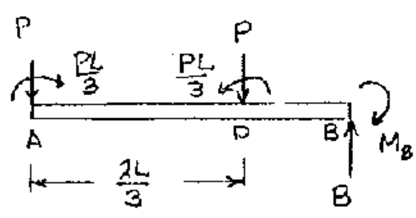
Just to the right of B $x = a^+$

(b) $M = -0 - Pa = -Pa$

Problem 5.105



5.105 (*a*) Using singularity functions, write the equations for the shear and bending moment for beam *ABC* under the loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment just to the right of point *D*.



(a) $V = -P - P\langle x-\frac{2L}{3}\rangle^0 = \frac{dM}{dx}$

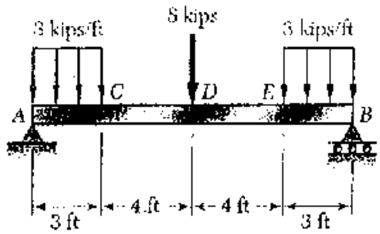
$M = -Px + \frac{PL}{3} - P\langle x-\frac{2L}{3}\rangle^1 - \frac{PL}{3}\langle x-\frac{2L}{3}\rangle^0$

Just to the right of D $x = \frac{2L}{3}$

(b) $M_D^+ = -P(\frac{2L}{3}) + \frac{PL}{3} - P(0) - \frac{PL}{3} = -\frac{4PL}{3}$

Problem 5.106

5.106 through 5.109 (*a*) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum value of the bending moment in the beam.



$\circlearrowleft \Sigma M_B = 0$

$-14A + (12.5)(3)(3) + (7)(8) + (1.5)(3)(3) = 0$

$A = 13 \text{ kips} \uparrow$

$w = 3 - 3\langle x-3\rangle^0 + 3\langle x-11\rangle^0 = -\frac{dV}{dx}$

(a) $V = 13 - 3x + 3\langle x-3\rangle^1 - 8\langle x-7\rangle^0 - 3\langle x-11\rangle^1$ kips

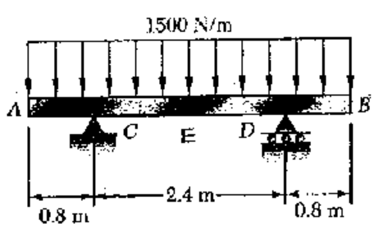
$M = 13x - 1.5x^2 + 1.5\langle x-3\rangle^2 - 8\langle x-7\rangle^1 - 1.5\langle x-11\rangle^2$ kip·ft

$V_C = 13 - (3)(3) = 4$ kips
$V_D^- = 13 - (3)(7) + (3)(4) = 4$ kips ← Changes sign
$V_D^+ = 13 - (3)(7) + (3)(4) - 8 = -4$ kips
$V_E = 13 - (3)(11) + (3)(8) - 8 = -4$ kips
$V_B = 13 - (3)(14) + (3)(11) - 8 - (3)(3) = -13$ kips

(b) $|M|_{max} = M_D = (13)(7) - (1.5)(7)^2 + (1.5)(4)^2 - 0 - 0 = 41.5$ kip·ft

Problem 5.107

5.106 through 5.109 (*a*) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum value of the bending moment in the beam.



$w = 1.5$ kN/m

By statics $C = D = 3$ kN ↑

(a) $V = -1.5x + 3\langle x-0.8\rangle^0 + 3\langle x-3.2\rangle^0$ kN

$M = -0.75x^2 + 3\langle x-0.8\rangle^1 + 3\langle x-3.2\rangle^1$ kN·m

Locate point E where $V = 0$ Assume $x_C < x_E < x_D$

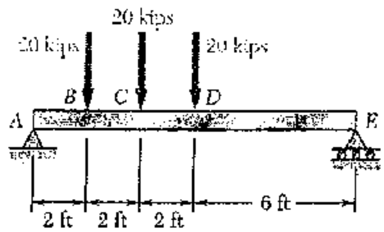
$0 = -1.5x_E + 3(x_E - 0.8) + 0$ $x_E = 2.0$ m

$M_C = -(0.75)(0.8)^2 + 0 + 0 = -0.480$ kN·m
$M_E = -(0.75)(2.0)^2 + (3)(1.2) + 0 = 0.600$ kN·m
$M_D = -(0.75)(3.2)^2 + (3)(2.4) + 0 = -0.480$ kN·m

(b) $|M|_{max} = 0.600$ kN·m (b) 600 N·m

Problem 5.108

5.106 through 5.109 (*a*) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum value of the bending moment in the beam.



$+\sum M_E = 0 \quad -12A + (10)(20) + (8)(20) + (6)(20) = 0$

$A = 40$ kips.

(a) $V = 40 - 20\langle x-2\rangle^0 - 20\langle x-4\rangle^0 - 20\langle x-6\rangle^0$ kips

$M = 40x - 20\langle x-2\rangle^1 - 20\langle x-4\rangle^1 - 20\langle x-6\rangle^1$ kip·ft

Values of V

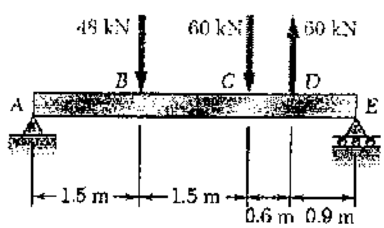
A to B	$V = 40$ kip
B to C	$V = 40 - 20 = 20$ kips
C to D	$V = 40 - 20 - 20 = 0$
D to E	$V = 40 - 20 - 20 - 20 = -20$ kip

Bending moment is constant and maximum over C to D.

(b) At C $\quad x = 4$ ft $\quad M = (40)(4) - (20)(2) - 0 - 0 = 120$ kip·ft

Problem 5.109

5.106 through 5.109 (*a*) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum value of the bending moment in the beam.



$+\circlearrowleft \sum M_E = 0 \quad -4.5R_A + (3.0)(48) + (1.5)(60) - (0.9)(60) = 0$

$R_A = 40$ kN

(a) $V = 40 - 48\langle x-1.5\rangle^0 - 60\langle x-3.0\rangle^0 + 60\langle x-3.6\rangle^0$ kN

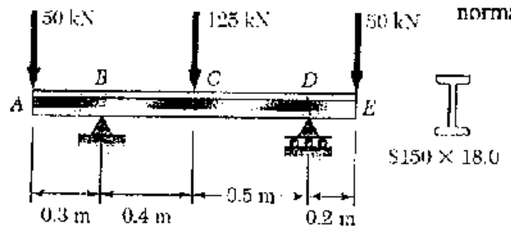
$M = 40x - 48\langle x-1.5\rangle^1 - 60\langle x-3.0\rangle^1 + 60\langle x-3.6\rangle^1$ kN·m

Pt.	x (m)	M (kN·m)
A	0	0
B	1.5	$(40)(1.5) = 60$ kN·m
C	3.0	$(40)(3.0) - (48)(1.5) = 48$ kN·m
D	3.6	$(40)(3.6) - (48)(2.1) - (60)(0.6) = 7.2$ kN·m
E	4.5	$(40)(4.5) - (48)(3.0) - (60)(1.5) + (60)(0.9) = 0$

(b) $M_{max} = 60$ kN·m

Problem 5.110

5.110 and 5.111 (*a*) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum normal stress due to bending.



$+\circlearrowleft \Sigma M_D = 0$

$(1.2)(50) - 0.9B + (0.5)(125) - (0.2)(50) = 0$

$B = 125 \text{ kN} \uparrow$

$+\circlearrowleft \Sigma M_B = 0$

$(0.3)(50) - (0.4)(125) + 0.9D - (1.1)(50) = 0$

$D = 100 \text{ kN} \uparrow$

(a) $V = -50 + 125\langle x-0.3\rangle^0 - 125\langle x-0.7\rangle^0 + 100\langle x-1.2\rangle^0 \quad \text{kN}$ ◀

$M = -50x + 125\langle x-0.3\rangle^1 - 125\langle x-0.7\rangle^1 + 100\langle x-1.2\rangle^1 \quad \text{kN}\cdot\text{m}$ ◀

Point	x (m)	M (kN-m)
B	0.3	$-(50)(0.3) + 0 - 0 + 0 = -15 \text{ kN}\cdot\text{m}$
C	0.7	$-(50)(0.7) + (125)(0.4) - 0 + 0 = 15 \text{ kN}\cdot\text{m}$
D	1.2	$-(50)(1.2) + (125)(0.9) - (125)(0.5) + 0 = -10 \text{ kN}\cdot\text{m}$
E	1.4	$-(50)(1.4) + (125)(1.1) - (125)(0.7) + (100)(0.2) = 0$ checks.

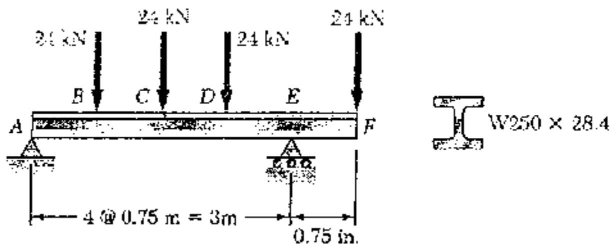
Maximum $|M| = 15 \text{ kN-m} = 15 \times 10^3 \text{ N-m}$

For S 150 × 18.0 rolled steel section $S = 120 \times 10^3 \text{ mm}^3 = 120 \times 10^{-6} \text{ m}^3$

(b) Normal stress $\sigma = \dfrac{|M|}{S} = \dfrac{15 \times 10^3}{120 \times 10^{-6}} = 125 \times 10^6 \text{ Pa}$ 125.0 MPa ◀

Problem 5.111

5.110 **and** *5.111* (*a*) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum normal stress due to bending.



$+\circlearrowleft \Sigma M_E = 0$

$-3R_A + (2.25)(24) - (1.5)(24) - (0.75)(24) + (0.75)(24) = 0$

$R_A = 30$ kips

$+\circlearrowleft \Sigma M_A = 0 \quad -(0.75)(24) - (1.5)(24) - (2.25)(24) + 3R_E - (3.75)(24) = 0$

$R_E = 66$ kips

(a) $V = 30 - 24\langle x-0.75\rangle^0 - 24\langle x-1.5\rangle^0 - 24\langle x-2.25\rangle^0 + 66\langle x-3\rangle^0$ kN

$M = 30x - 24\langle x-0.75\rangle^1 - 24\langle x-1.5\rangle^1 - 24\langle x-2.25\rangle^1 + 66\langle x-3\rangle^1$ kN·m

Pt	x (m)	M (kN·m)
B	0.75	$(30)(0.75) = 22.5$ kN·m
C	1.5	$(30)(1.5) - (24)(0.75) = 27$ kN·m
D	2.25	$(30)(2.25) - (24)(1.5) - (24)(0.75) = 13.5$ kN·m
E	3.0	$(30)(3.0) - (24)(2.25) - (24)(1.5) - (24)(0.75) = -18$ kN·m
F	3.75	$(30)(3.75) - (24)(3.0) - (24)(2.25) - (24)(1.5) + (66)(0.75) = 0$

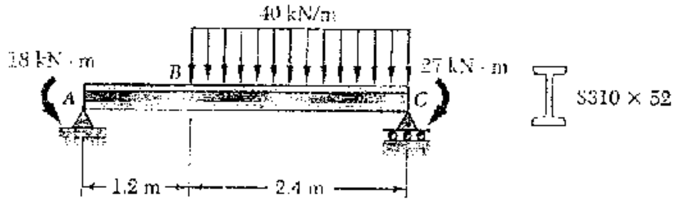
Maximum $|M| = 27$ kN·m $= 27 \times 10^3$ N·m

For rolled steel section W250 × 28.4 $\quad S = 308 \times 10^3\ mm^3 = 308 \times 10^{-6}\ m^3$

(b) Normal stress $\quad \sigma = \dfrac{|M|}{S} = \dfrac{27 \times 10^3}{308 \times 10^{-6}} = 87.7 \times 10^6$ Pa $= 87.7$ MPa

Problem 5.112

5.112 **and 5.113** (*a*) Using singularity functions, find the magnitude and location of the maximum bending moment for the beam and loading shown. (*b*) Determine the maximum normal stress due to bending.



$\circlearrowleft + \; M_C = 0$

$18 - 3.6 R_A + (1.2)(2.4)(40) - 27 = 0$

$R_A = 29.5 \text{ kN}$

$V = 29.5 - 40\langle x - 1.2\rangle^1 \text{ kN}$

Point D $\quad V = 0 \qquad 29.5 - 40(x_D - 1.2) = 0$

$x_D = 1.9375 \text{ m}$

$M = -18 + 29.5x - 20\langle x - 1.2\rangle^2 \text{ kN-m}$

$M_A = -18 \text{ kN·m}$

$M_D = -18 + (29.5)(1.9375) - (20)(0.7375)^2 = 28.278 \text{ kN-m}$

$M_E = -18 + (29.5)(3.6) - (20)(2.4)^2 = -27 \text{ kN·m}$

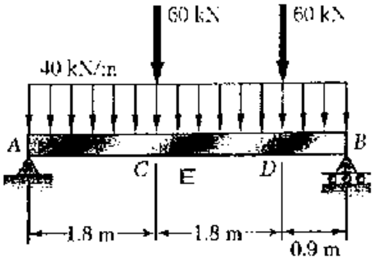
(a) Maximum $|M| = 28.278 \text{ kN·m}$ at $x = 1.9375 \text{ m}$ ◄

For S310 × 52 rolled steel section $\quad S = 625 \times 10^3 \text{ mm}^3 = 625 \times 10^{-6} \text{ m}^3$

(b) Normal stress $\quad \sigma = \dfrac{|M|}{S} = \dfrac{28.278 \times 10^3}{625 \times 10^{-6}} = 45.2 \times 10^6 \text{ Pa} = 45.2 \text{ MPa}$ ◄

Problem 5.113

***5.112* and 5.113** (*a*) Using singularity functions, find the magnitude and location of the maximum bending moment for the beam and loading shown. (*b*) Determine the maximum normal stress due to bending.



W530 × 66

$+\circlearrowleft \Sigma M_B = 0$

$-4.5\,A + (2.25)(4.5)(40) + (2.7)(60) + (0.9)(60) = 0$

$A = 138 \text{ kN} \uparrow$

$+\circlearrowleft \Sigma M_A = 0$

$-(2.25)(4.5)(40) - (1.8)(60) - (3.6)(60) + 4.5\,B = 0$

$B = 162 \text{ kN} \uparrow$

$w = 40 \text{ kN} = \frac{dV}{dx}$

$V = -40x + 138 - 60\langle x-1.8\rangle^0 - 60\langle x-3.6\rangle^0 = \frac{dM}{dx}$

$M = -20x^2 - 138x - 60\langle x-1.8\rangle^1 - 60\langle x-3.6\rangle^1$

$V_C^+ = -(40)(1.8) + 138 - 60 = 6 \text{ kN}$

$V_D^- = -(40)(3.6) + 138 - 60 = -66 \text{ kN}$

Locate point E where $V = 0$. It lies between C and D.

$V_E = -40x_E + 138 - 60 + 0 = 0 \qquad x_E = 1.95 \text{ m}$

$M_E = -(20)(1.95)^2 + (138)(1.95) - (60)(1.95-1.8) = 184 \text{ kN}\cdot\text{m}$

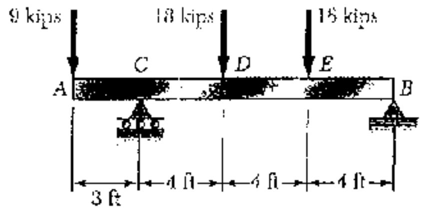
(a) $|M|_{max} = 184 \text{ kN}\cdot\text{m} = 184 \times 10^3 \text{ N}\cdot\text{m}$ at $x = 1.950 \text{ m}$ ◄

For W 530 × 66 rolled steel section $S = 1340 \times 10^3 \text{ mm}^3 = 1340 \times 10^{-6} \text{ m}^3$

(b) Normal stress $\sigma = \frac{|M|_{max}}{S} = \frac{184 \times 10^3}{1340 \times 10^{-6}} = 137.3 \times 10^6 \text{ Pa}$ 137.3 MPa ◄

Problem 5.114

5.114 and 5.115 A beam is being designed to be supported and loaded as shown. (*a*) Using singularity functions, find the magnitude and location of the maximum bending moment in the beam. (*b*) Knowing that the allowable normal stress for the steel to be used is 24 ksi, find the most economical wide-flange shape that can be used.



$+\circlearrowleft \Sigma M_B = 0$

$(15)(9) - 12C + (8)(18) + (4)(18) = 0$

$C = 29.25$ kips

$+\circlearrowleft \Sigma M_C = 0$

$(3)(9) - (4)(18) - (8)(18) + 12B = 0$

$B = 15.75$

$$V = -9 + 29.25\langle x-3\rangle^0 - 18\langle x-7\rangle^0 - 18\langle x-11\rangle^0$$

$$M = -9x + 29.25\langle x-3\rangle^1 - 18\langle x-7\rangle^1 - 18\langle x-11\rangle^1$$

At A $\quad x = 0 \quad M_A = 0$

At C $\quad x = 3$ ft $\quad M_C = -(9)(3) + 0 - 0 - 0 = -27$ kip·ft

At D $\quad x = 7$ ft $\quad M_D = -(9)(7) + (29.25)(4) - 0 - 0 = 54$ kip·ft

At E $\quad x = 11$ ft $\quad M_E = -(9)(11) + (29.25)(8) - (18)(4) - 0 = 63$ kip·ft

At B $\quad x = 15$ ft $\quad M_B = -(9)(15) + (29.25)(12) - (18)(8) - (18)(4) = 0$ checks

(a) $|M|_{max} = 63$ kip·ft at E ◄

$|M|_{max} = 758$ kip·in

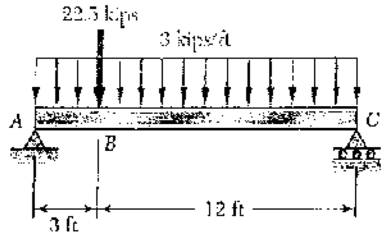
$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{758}{24} = 31.5 \text{ in}^3$$

(b)

Shape	S (in³)
W 16 × 26	38.4
W 14 × 26	35.3
W 12 × 26	33.4
W 10 × 33	35.0

W16×26 or W14×26 or W12×26 ◄

Problem 5.115



5.114 and 5.115 A beam is being designed to be supported and loaded as shown. (*a*) Using singularity functions, find the magnitude and location of the maximum bending moment in the beam. (*b*) Knowing that the allowable normal stress for the steel to be used is 24 ksi, find the most economical wide-flange shape that can be used.

$$\circlearrowleft \Sigma M_C = 0 \quad -15 R_A + (7.5)(15)(3) + (12)(22.5) = 0$$

$$R_A = 40.5 \text{ kips.}$$

$$w = 3 \text{ kips/ft} = -\frac{dV}{dx}$$

$$V = 40.5 - 3x - 22.5\langle x-3\rangle^0 \quad \text{kips}$$

Location of point D where $V = 0$. Assume $3 < x_D < 12$

$$0 = 40.5 - 3x_D - 22.5 \qquad x_D = 6 \text{ ft}$$

$$M = 40.5x - 1.5x^2 - 22.5\langle x-3\rangle^1 \quad \text{kip}\cdot\text{ft}$$

At point D $(x = 6 \text{ ft})$ $\quad M = (40.5)(6) - (1.5)(6)^2 - (22.5)(3)$

$$= 121.5 \text{ kip}\cdot\text{ft} = 1458 \text{ kip}\cdot\text{in}$$

(a) Maximum $|M| = 121.5$ kip·ft at $x = 6$ ft

$$S_{min} = \frac{M}{\sigma_{all}} = \frac{1458}{24} = 60.75 \text{ in}^3$$

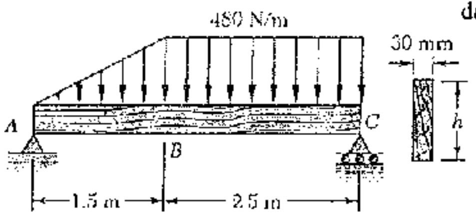
(b)

Shape	S (in³)
W 21 × 44	81.6
W 18 × 50	88.9
W 16 × 40	64.7 ←
W 14 × 43	62.7
W 12 × 50	64.7
W 10 × 68	75.7

Answer W16 × 40

Problem 5.116

5.116 and 5.117 A timber beam is being designed to be supported and loaded as shown. (*a*) Using singularity functions, find the magnitude and location of the maximum bending moment in the beam. (*b*) Knowing that the available stock consists of beams with an allowable stress of 12 MPa and a rectangular cross section of 30-mm width and depth *h* varying from 80 mm to 160 mm in 10-mm increments, determine the most economical cross section that can be used.



$480 \text{ N/m} = 0.48 \text{ kN/m}$

$+\circlearrowleft \Sigma M_C = 0$

$-4R_A + (3)(\tfrac{1}{2})(1.5)(0.48) + (1.25)(2.5)(0.48) = 0$

$R_A = 0.645 \text{ kN}$

$$w = \frac{0.48}{1.5}x - \frac{0.48}{1.5}\langle x-1.5\rangle^1 = 0.32x - 0.32\langle x-1.5\rangle^1 \text{ kN/m} = -\frac{dV}{dx}$$

$$V = 0.645 - 0.16x^2 + 0.16\langle x-1.5\rangle^2 \quad \text{kN}$$

Locate point D where $V = 0$. Assume $1.5 \text{ m} < x_D < 4 \text{ m}$

$$0 = 0.645 - 0.16x_D^2 + 0.16(x_D - 1.5)^2$$

$$= 0.645 - \cancel{0.16x_D^2} + \cancel{0.16x_D^2} - 0.48x_D + 0.36$$

$$x_D = 2.09375 \text{ m}$$

$$M = 0.645x - 0.05333x^3 + 0.05333\langle x-1.5\rangle^3 \quad \text{kN-m}$$

(a) At point D $\quad M_D = (0.645)(2.09375) - (0.05333)(2.09375)^3 + (0.05333)(0.59375)^3$

$= 0.87211 \text{ kN}\cdot\text{m}$

$$S_{min} = \frac{M_D}{\sigma_{all}} = \frac{0.87211\times10^3}{12\times10^6} = 72.6758\times10^{-6}\text{ m}^3 = 72.6758\times10^3 \text{ mm}^3$$

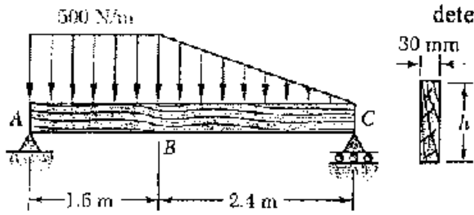
For a rectangular cross section $\quad S = \frac{1}{6}bh^2 \qquad h = \sqrt{\frac{6S}{b}}$

$$h_{min} = \sqrt{\frac{(6)(72.6758\times10^3)}{30}} = 120.56 \text{ mm}$$

(b) At next larger 10-mm increment $\qquad h = 130 \text{ mm}$

Problem 5.117

5.116 and *5.117* A timber beam is being designed to be supported and loaded as shown. (*a*) Using singularity functions, find the magnitude and location of the maximum bending moment in the beam. (*b*) Knowing that the available stock consists of beams with an allowable stress of 12 MPa and a rectangular cross section of 30-mm width and depth *h* varying from 80 mm to 160 mm in 10-mm increments, determine the most economical cross section that can be used.



$500\ N/m = 0.5\ kN/m$

$+\circlearrowleft \Sigma M_C = 0$

$-4R_A + (3.2)(1.6)(0.5) + (1.6)(\frac{1}{2})(2.4)(0.5) = 0$

$R_A = 0.880\ kN$

$w = 0.5 - \frac{0.5}{2.4}\langle x-1.6\rangle^1 = 0.5 - 0.20833\langle x-1.6\rangle^1\ kN/m \qquad -\frac{dV}{dx}$

$V = 0.880 - 0.5x + 0.104167\langle x-1.6\rangle^2\ kN$

$V_A = 0.880\ kN$

$V_B = 0.880 - (0.5)(1.6) = 0.080\ kN$

$V_C = 0.880 - (0.5)(4) + (0.104167)(2.4)^2 = -0.520\ kN$

(Sign change for V_B, V_C)

Locate point D (between B and C) where $V = 0$.

$0 = 0.880 - 0.5x_D + 0.104167(x_D - 1.6)^2$

$0.104167x_D^2 - 0.83333x_D + 1.14667 = 0$

$$x_D = \frac{0.83333 \pm \sqrt{(0.83333)^2 - (4)(0.104167)(1.14667)}}{(2)(0.104167)}$$

$= 4.0 \pm 2.2342 = \cancel{6.2342},\ 1.7658\ m$

$M = 0.880x - 0.25x^2 + 0.347222\langle x-1.6\rangle^3\ kN\text{-}m$

$M_D = (0.880)(1.7658) - (0.25)(1.7658)^2 + (0.34722)(0.1658)^3 = 0.776\ kN\text{-}m$

(a) $M_{max} = 0.776\ kN\text{-}m$ at $x = 1.7658\ m$

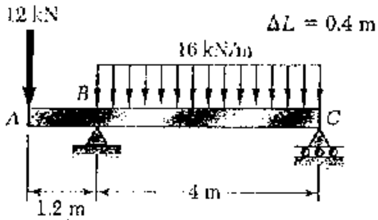
$S_{min} = \frac{M_{max}}{\sigma_{all}} = \frac{0.776\times 10^3}{12\times 10^6} = 64.66\times 10^{-6}\ m^3 = 64.66\times 10^3\ mm^3$

For a rectangular cross section $S = \frac{1}{6}bh^2 \qquad h = \sqrt{\frac{6S}{b}}$

$h_{min} = \sqrt{\frac{(6)(64.66\times 10^3)}{30}} = 113.7\ mm$

(b) At next higher 10-mm increment $h = 120\ mm$

Problem 5.118



5.118 through 5.121 Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increment ΔL, starting at point A and ending at the right-hand support.

$$+\circlearrowleft \Sigma M_C = 0$$

$$(5.2)(12) - 4B + (2)(4)(16) = 0$$

$$B = 47.6 \text{ kN} \uparrow$$

$$+\circlearrowleft \Sigma M_B = 0$$

$$(1.2)(12) - (2)(4)(16) + 4C = 0$$

$$C = 28.4 \text{ kN} \uparrow$$

$$w = 16\langle x-1.2\rangle^0 = -\frac{dV}{dx}$$

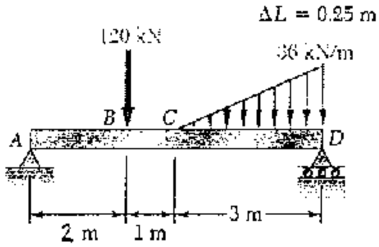
$$V = -16\langle x-1.2\rangle^1 - 12 + 47.6\langle x-1.2\rangle^0$$

$$M = -8\langle x-1.2\rangle^2 - 12x + 47.6\langle x-1.2\rangle^1$$

x m	V kN	M kN m
0.0	-12.0	0.00
0.4	-12.0	-4.80
0.8	-12.0	-9.60
1.2	35.6	-14.40
1.6	29.2	-1.44
2.0	22.8	8.96
2.4	16.4	16.80
2.8	10.0	22.08
3.2	3.6	24.80
3.6	-2.8	24.96
4.0	-9.2	22.56
4.4	-15.6	17.60
4.8	-22.0	10.08
5.2	-28.4	-0.00

Problem 5.119

5.118 through 5.121 Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increment ΔL, starting at point A and ending at the right-hand support.



$+\circlearrowleft \Sigma M_D = 0$

$-6\ R_A + (4)(120) + (1)(\tfrac{1}{2})(3)(36) = 0$

$R_A = 89\ kN$

$w = \frac{36}{3}\langle x-3\rangle^1 = 12\langle x-3\rangle^1$

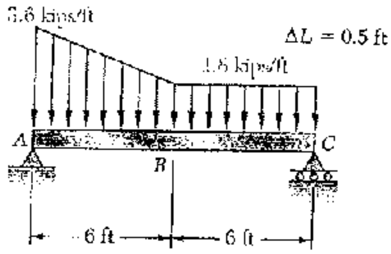
$V = 89 - 120\langle x-2\rangle^0 - 6\langle x-3\rangle^2$ kN ◄

$M = 89x - 120\langle x-2\rangle^1 - 2\langle x-3\rangle^3$ kN·m ◄

x m	V kN	M kN·m
0.0	89.0	0.0
0.3	89.0	22.3
0.5	89.0	44.5
0.8	89.0	66.8
1.0	89.0	89.0
1.3	89.0	111.3
1.5	89.0	133.5
1.8	89.0	155.8
2.0	-31.0	178.0
2.3	-31.0	170.3
2.5	-31.0	162.5
2.8	-31.0	154.8
3.0	-31.0	147.0
3.3	-31.4	139.2
3.5	-32.5	131.3
3.8	-34.4	122.9
4.0	-37.0	114.0
4.3	-40.4	104.3
4.5	-44.5	93.8
4.8	-49.4	82.0
5.0	-55.0	69.0
5.3	-61.4	54.5
5.5	-68.5	38.3
5.8	-76.4	20.2
6.0	-85.0	-0.0

Problem 5.120

5.118 through 5.121 Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increment ΔL, starting at point A and ending at the right-hand support.



$\curvearrowleft \Sigma M_C = 0$

$-12 R_A + (6)(12)(1.8) + (10)(\frac{1}{2})(6)(1.8) = 0$

$R_A = 15.3$ Kips

$w = 3.6 - \frac{1.8}{6}x + \frac{1.8}{6}\langle x-6\rangle^1$

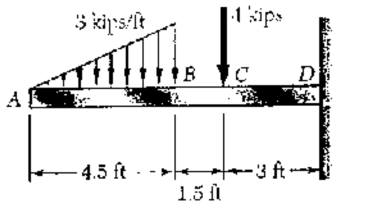
$= 3.6 - 0.3x + 0.3\langle x-6\rangle^1$

$V = 15.3 - 3.6x + 0.15x^2 - 0.15\langle x-6\rangle^2$ kips

$M = 15.3x - 1.8x^2 + 0.05x^3 - 0.05\langle x-6\rangle^3$ kip·ft

x ft	V kips	M kip·ft
0.0	15.30	0.0
0.5	13.54	7.2
1.0	11.85	13.6
1.5	10.24	19.1
2.0	8.70	23.8
2.5	7.24	27.8
3.0	5.85	31.1
3.5	4.54	33.6
4.0	3.30	35.6
4.5	2.14	37.0
5.0	1.05	37.8
5.5	0.04	38.0
6.0	-0.90	37.8
6.5	-1.80	37.1
7.0	-2.70	36.0
7.5	-3.60	34.4
8.0	-4.50	32.4
8.5	-5.40	29.9
9.0	-6.30	27.0
9.5	-7.20	23.6
10.0	-8.10	19.8
10.5	-9.00	15.5
11.0	-9.90	10.8
11.5	-10.80	5.6
12.0	-11.70	0.0

Problem 5.121



5.118 through 5.121 Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increment ΔL, starting at point A and ending at the right-hand support.

$$w = \frac{3}{4.5}x - 3\langle x-4.5\rangle^0 - \frac{3}{4.5}\langle x-4.5\rangle^1$$

$$= \frac{2}{3}x - 3\langle x-4.5\rangle^0 - \frac{2}{3}\langle x-4.5\rangle^1 = -\frac{dV}{dx}$$

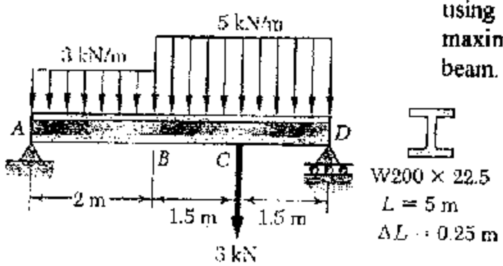
$$V = -\frac{1}{3}x^2 + 3\langle x-4.5\rangle^1 + \frac{1}{3}\langle x-4.5\rangle^2 - 4\langle x-6\rangle^0$$

$$M = -\frac{1}{9}x^3 + \frac{3}{2}\langle x-4.5\rangle^2 + \frac{1}{9}\langle x-4.5\rangle^3 - 4\langle x-6\rangle^1$$

x ft	V kips	M kip ft
0.0	0.00	0.00
0.5	-0.08	-0.01
1.0	-0.33	-0.11
1.5	-0.75	-0.38
2.0	-1.33	-0.89
2.5	-2.08	-1.74
3.0	-3.00	-3.00
3.5	-4.08	-4.76
4.0	-5.33	-7.11
4.5	-6.75	-10.13
5.0	-6.75	-13.50
5.5	-6.75	-16.88
6.0	-10.75	-20.25
6.5	-10.75	-25.63
7.0	-10.75	-31.00
7.5	-10.75	-36.38
8.0	-10.75	-41.75
8.5	-10.75	-47.13
9.0	-10.75	-52.50

Problem 5.122

5.122 and 5.123 For the beam and loading shown, and using a computer and step functions, (*a*) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔL indicated, (*b*) using smaller increments if necessary, determine with a 2 percent accuracy the maximum normal stress in the beam. Place the origin of the x axis at end A of the beam.



$$\circlearrowleft \Sigma M_D = 0$$

$$-5R_A + (4.0)(2.0)(3) + (1.5)(3)(5) + (1.5)(3) = 0$$

$$R_A = 10.2 \text{ kN}$$

$$w = 3 + 2\langle x-2\rangle^0 \text{ kN/m} = -\frac{dV}{dx}$$

$$V = 10.2 - 3x - 2\langle x-2\rangle^1 - 3\langle x-3.5\rangle^0 \quad \text{kN}$$

$$M = 10.2x - 1.5x^2 - \langle x-2\rangle^2 - 3\langle x-3.5\rangle^1 \quad \text{kN}\cdot\text{m}$$

x m	V kN	M kN·m	sigma MPa
0.00	10.20	0.00	0.0
0.25	9.45	2.46	12.7
0.50	8.70	4.72	24.4
0.75	7.95	6.81	35.1
1.00	7.20	8.70	44.8
1.25	6.45	10.41	53.6
1.50	5.70	11.92	61.5
1.75	4.95	13.26	68.3
2.00	4.20	14.40	74.2
2.25	2.95	15.29	78.8
2.50	1.70	15.88	81.8
2.75	0.45	16.14	83.2
3.00	-0.80	16.10	83.0
3.25	-2.05	15.74	81.2
3.50	-6.30	15.07	77.7
3.75	-7.55	13.34	68.8
4.00	-8.80	11.30	58.2
4.25	-10.05	8.94	46.1
4.50	-11.30	6.27	32.3
4.75	-12.55	3.29	17.0
5.00	-13.80	-0.00	-0.0
2.83	0.05	16.164	83.3
2.84	0.00	16.164	83.3 ←
2.85	-0.05	16.164	83.3

For rolled steel section W 200 × 22.5

$$S = 194 \times 10^3 \text{ mm}^3$$

$$S = 194 \times 10^3 \text{ mm}^3 = 194 \times 10^{-6} \text{ m}^3$$

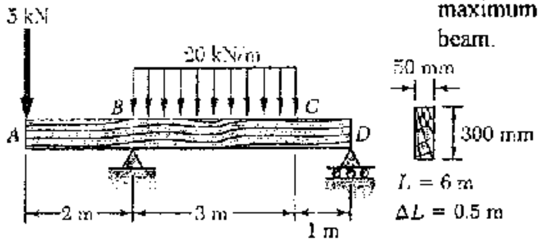
$$\sigma_{max} = \frac{M_{max}}{S} = \frac{16.164 \times 10^3}{194 \times 10^{-6}}$$

$$= 83.3 \times 10^6 \text{ Pa}$$

$$= 83.3 \text{ MPa}$$

Problem 5.123

5.122 and 5.123 For the beam and loading shown, and using a computer and step functions, (*a*) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔL indicated, (*b*) using smaller increments if necessary, determine with a 2 percent accuracy the maximum normal stress in the beam. Place the origin of the x axis at end A of the beam.



$L = 6$ m

$\Delta L = 0.5$ m

$+\circlearrowleft \Sigma M_D = 0$

$-4R_B + (6)(5) + (2.5)(3)(20) = 0$

$R_B = 45$ kN

$w = 20\langle x-2\rangle^0 - 20\langle x-5\rangle^0$ kN/m $= -\frac{dV}{dx}$

$V = -5 + 45\langle x-2\rangle^0 - 20\langle x-2\rangle^1 + 20\langle x-5\rangle^1$ kN

$M = -5x + 45\langle x-2\rangle^1 - 10\langle x-2\rangle^2 + 10\langle x-5\rangle^2$ kN-m

x m	V kN	M kN·m	sigma MPa
0.00	-5	0.00	0.0
0.50	-5	-2.50	-3.3
1.00	-5	-5.00	-6.7
1.50	-5	-7.50	-10.0
2.00	40	-10.00	-13.3
2.50	30	7.50	10.0
3.00	20	20.00	26.7
3.50	10	27.50	36.7
4.00	0	30.00	40.0
4.50	-10	27.50	36.7
5.00	-20	20.00	26.7
5.50	-20	10.00	13.3
6.00	-20	0.00	0.0

Maximum $|M| = 30$ kN·m at $x = 4.0$ m

For rectangular cross section

$S = \frac{1}{6}bh^2 = (\frac{1}{6})(50)(300)^2$

$= 750 \times 10^3$ mm^3

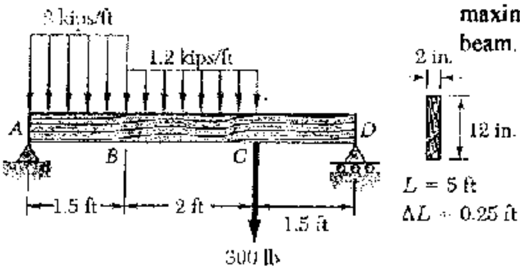
$= 750 \times 10^{-6}$ m^3

$\sigma_{max} = \frac{M_{max}}{S} = \frac{30 \times 10^3}{750 \times 10^{-6}}$

$= 40 \times 10^6$ Pa $= 40$ MPa

Problem 5.124

5.124 and 5.125 For the beam and loading shown, and using a computer and step functions, (*a*) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔL indicated, (*b*) using smaller increments if necessary, determine with a 2 percent accuracy the maximum normal stress in the beam. Place the origin of the x axis at end A of the beam.



$300 \text{ lb} = 0.3 \text{ kips}$

$+\circlearrowleft \Sigma M_D = 0$

$-5R_A + (4.25)(1.5)(2) + (2.5)(2)(1.2) + (1.5)(0.3) = 0$

$R_A = 3.84$ kips.

$$w = 2 - 0.8\langle x - 1.5\rangle^0 - 1.2\langle x - 3.5\rangle^0 \text{ kip/ft}$$

$$V = 3.84 - 2x + 0.8\langle x - 1.5\rangle^1 + 1.2\langle x - 3.5\rangle^1 - 0.3\langle x - 3.5\rangle^0 \text{ kips}$$

$$M = 3.84x - x^2 + 0.4\langle x - 1.5\rangle^2 + 0.6\langle x - 3.5\rangle^2 - 0.3\langle x - 3.5\rangle^1 \text{ kip}\cdot\text{ft}$$

x ft	V kips	M kip·ft	sigma ksi
0.00	3.84	0.00	0.000
0.25	3.34	0.90	0.224
0.50	2.84	1.67	0.417
0.75	2.34	2.32	0.579
1.00	1.84	2.84	0.710
1.25	1.34	3.24	0.809
1.50	0.84	3.51	0.877
1.75	0.54	3.68	0.921
2.00	0.24	3.78	0.945
2.25	-0.06	3.80	0.951
2.50	-0.36	3.75	0.937
2.75	-0.66	3.62	0.906
3.00	-0.96	3.42	0.855
3.25	-1.26	3.14	0.786
3.50	-1.86	2.79	0.697
3.75	-1.86	2.32	0.581
4.00	-1.86	1.86	0.465
4.25	-1.86	1.39	0.349
4.50	-1.86	0.93	0.232
4.75	-1.86	0.46	0.116
5.00	-1.86	-0.00	-.000
2.10	0.12	3.80	0.949
2.20	0.00	3.80	0.951 ←
2.30	-0.12	3.80	0.949

Maximum $|M| = 3.809$ kip·ft
$= 45.648$ kip·in
at $x = 2.20$ ft

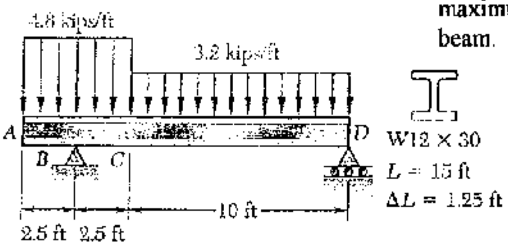
Rectangular section
2 in × 12 in.

$S = \frac{1}{6}bh^2 = (\frac{1}{6})(2)(12)^2$
$= 48 \text{ in}^3$

$\sigma = \dfrac{M}{S} = \dfrac{45.648}{48}$
$= 0.951$ ksi

Problem 5.125

5.124 and 5.125 For the beam and loading shown, and using a computer and step functions, (*a*) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔL indicated, (*b*) using smaller increments if necessary, determine with a 2 percent accuracy the maximum normal stress in the beam. Place the origin of the x axis at end A of the beam.



$+\circlearrowleft \Sigma M_D = 0$

$-12.5\, R_B + (12.5)(5.0)(4.8) + (5)(10)(3.2) = 0$

$R_B = 36.8$ kips.

$w = 4.8 - 1.6\langle x-5\rangle^0$ kips/ft

$V = -4.8x + 36.8\langle x-2.5\rangle^0 + 1.6\langle x-5\rangle^1$ kips ◄

$M = -2.4x^2 + 36.8\langle x-2.5\rangle^1 + 0.8\langle x-5\rangle^2$ kip·ft ◄

x ft	V kips	M kip·ft	sigma ksi
0.00	0.0	0.00	0.00
1.25	-6.0	-3.75	-1.17
2.50	24.8	-15.00	-4.66
3.75	18.8	12.25	3.81
5.00	12.8	32.00	9.95
6.25	8.8	45.50	14.15
7.50	4.8	54.00	16.79
8.75	0.8	57.50	17.88
10.00	-3.2	56.00	17.41
11.25	-7.2	49.50	15.39
12.50	-11.2	38.00	11.81
13.75	-15.2	21.50	6.68
15.00	-19.2	0.00	0.00
8.90	0.32	57.58	17.90
9.00	-0.00	57.60	17.91 ←
9.10	-0.32	57.58	17.90

Maximum $M = 57.6$ kip·ft
$= 691.2$ kip·in
at $x = 9.0$ ft.

For rolled steel section W12×30

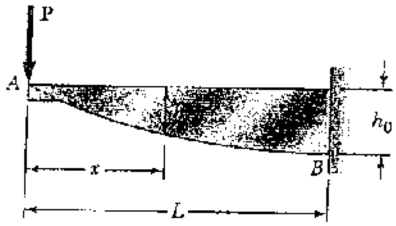
$S = 38.6\ \text{in}^3$

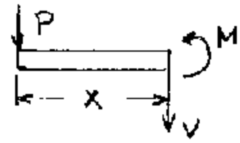
Maximum normal stress

$\sigma = \dfrac{M}{S} = \dfrac{691.2}{38.6} = 17.91$ ksi ◄

Problem 5.126

5.126 and 5.127 The beam AB, consisting of a cast-iron plate of uniform thickness b and length L, is to support the load shown. (*a*) Knowing that the beam is to be of constant strength, express h in terms of x, L, and h_0. (*b*) Determine the maximum allowable load if $L = 36$ in., $h_0 = 12$ in., $b = 1.25$ in., and $\sigma_{all} = 24$ ksi.





$$V = -P$$

$$M = -Px \qquad |M| = Px$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{P}{\sigma_{all}}x$$

For a rectangular cross section $\quad S = \frac{1}{6}bh^2$

Equating $\quad \frac{1}{6}bh^2 = \frac{Px}{\sigma_{all}} \qquad h = \left(\frac{6Px}{\sigma_{all}b}\right)^{\frac{1}{2}} \qquad (1)$

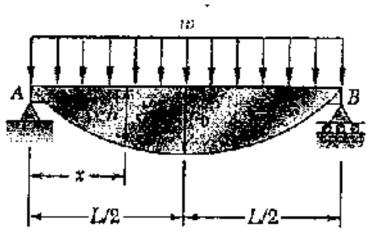
At $x = L \qquad h = h_0 = \left\{\frac{6PL}{\sigma_{all}b}\right\}^{1/2} \qquad (2)$

(a) Divide Eq. (1) by Eq. (2) and solve for $h \qquad h = h_0(x/L)^{1/2}$ ◀

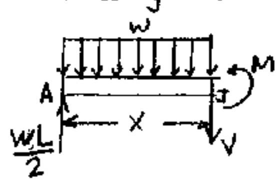
(b) Solving for P $\qquad P = \frac{\sigma_{all}bh_0^2}{6L} = \frac{(24)(1.25)(12)^2}{(6)(36)} = 20$ kips ◀

Problem 5.127

5.126 and 5.127 The beam AB, consisting of a cast-iron plate of uniform thickness b and length L, is to support the load shown. (*a*) Knowing that the beam is to be of constant strength, express h in terms of x, L, and h_0. (*b*) Determine the maximum allowable load if $L = 36$ in., $h_0 = 12$ in., $b = 1.25$ in., and $\sigma_{all} = 24$ ksi.



$+\uparrow \Sigma F_y = 0 \qquad R_A + R_B - wL = 0 \qquad R_A = R_B = \frac{wL}{2}$



$$\circlearrowleft \Sigma M_J = 0$$

$$\frac{wL}{2}x - wx\frac{x}{2} + M = 0$$

$$M = \frac{w}{2}x(L-x)$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{wx(L-x)}{2\sigma_{all}}$$

For a rectangular cross section $\quad S = \frac{1}{6}bh^2$

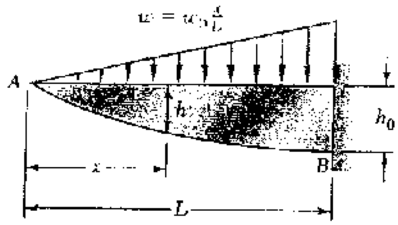
Equating $\quad \frac{1}{6}bh^2 = \frac{wx(L-x)}{2\sigma_{all}} \qquad h = \left\{\frac{3wx(L-x)}{\sigma_{all}b}\right\}^{1/2}$ ◀

(a) At $x = \frac{L}{2} \qquad h = h_0 = \left\{\frac{3wL^2}{4\sigma_{all}b}\right\}^{1/2} \qquad h = h_0\left[\frac{x}{L}\left(1 - \frac{x}{L}\right)\right]^{\frac{1}{2}}$ ◀

(b) Solving for $w \qquad w = \frac{4\sigma_{all}bh_0^2}{3L^2} = \frac{(4)(24)(1.25)(12)^2}{(3)(36)^2} = 4.44$ kip/in ◀

Problem 5.128

5.128 and *5.129* The beam AB, consisting of a cast-iron plate of uniform thickness b and length L, is to support the distributed load $w(x)$ shown. (*a*) Knowing that the beam is to be of constant strength, express h in terms of x, L, and h_0. (*b*) Determine the smallest value of h_0 if $L = 750$ mm., $b = 30$ mm, $w_0 = 300$ kN/m, and $\sigma_{all} = 200$ MPa.



$$\frac{dV}{dx} = -w = -\frac{w_0 x}{L}$$

$$V = -\frac{w_0 x^2}{2L} = \frac{dM}{dx}$$

$$M = -\frac{w_0 x^3}{6L} \qquad |M| = \frac{w_0 x^3}{6L}$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{w_0 x^3}{6L\sigma_{all}}$$

For a rectangular cross section $\quad S = \frac{1}{6} b h^2$

Equating $\quad \frac{1}{6} b h^2 = \frac{w_0 x^3}{6L\sigma_{all}} \qquad h = \sqrt{\frac{w_0 x^3}{\sigma_{all} b L}}$

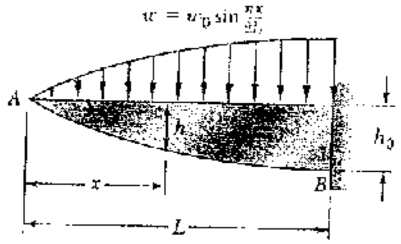
At $x = L \qquad h = h_0 = \sqrt{\frac{w_0 L^2}{\sigma_{all} b}} \qquad$ (a) $\quad h = h_0 \left(\frac{x}{L}\right)^{3/2}$ ◄

Data: $L = 750$ mm $= 0.75$ m, $\quad b = 30$ mm $= 0.030$ m
$w_0 = 300$ kN/m $= 300 \times 10^3$ N/m, $\quad \sigma_{all} = 200$ MPa $= 200 \times 10^6$ Pa

(b) $\quad h_0 = \sqrt{\frac{(300\times 10^3)(0.75)^2}{(200\times 10^6)(0.030)}} = 167.7 \times 10^{-3}$ m $= 167.7$ mm ◄

Problem 5.129

5.128 and *5.129* The beam AB, consisting of a cast-iron plate of uniform thickness b and length L, is to support the distributed load $w(x)$ shown. (*a*) Knowing that the beam is to be of constant strength, express h in terms of x, L, and h_0. (*b*) Determine the smallest value of h_0 if $L = 750$ mm., $b = 30$ mm, $w_0 = 300$ kN/m, and $\sigma_{all} = 200$ MPa.



$$\frac{dV}{dx} = -w = -w_0 \sin \frac{\pi x}{2L}$$

$$V = \frac{2 w_0 L}{\pi} \cos \frac{\pi x}{2L} + C_1$$

$$V = 0 \text{ at } x = 0 \rightarrow \quad C_1 = -\frac{2 w_0 L}{\pi}$$

$$\frac{dM}{dx} = V = -\frac{2 w_0 L}{\pi}\left(1 - \cos \frac{\pi x}{2L}\right)$$

$$M = -\frac{2 w_0 L}{\pi}\left(x - \frac{2L}{\pi} \sin \frac{\pi x}{2L}\right) \qquad |M| = \frac{2 w_0 L}{\pi}\left(x - \frac{2L}{\pi} \sin \frac{\pi x}{L}\right)$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{2 w_0 L}{\pi \sigma_{all}}\left(x - \frac{2L}{\pi} \sin \frac{\pi x}{2L}\right)$$

For a rectangular cross section $\quad S = \frac{1}{6} b h^2$

Equating $\quad \frac{1}{6} b h^2 = \frac{2 w_0 L}{\pi \sigma_{all}}\left(x - \frac{2L}{\pi} \sin \frac{\pi x}{2L}\right)$

$$h = \left\{\frac{12\, w_0 L}{\pi \sigma_{all} b}\left(x - \frac{2L}{\pi} \sin \frac{\pi x}{2L}\right)\right\}^{1/2}$$

At $x = L \qquad h = h_0 = \left\{\frac{12\, w_0 L^2}{\pi \sigma_{all} b}\left(1 - \frac{2}{\pi}\right)\right\}^{1/2} = 1.178\sqrt{\frac{w_0 L^2}{\sigma_{all} b}}$

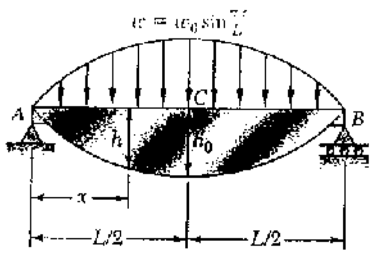
(a) $\quad h = h_0\left[\left(\frac{x}{L} - \frac{2}{\pi} \sin \frac{\pi x}{2L}\right) / \left(1 - \frac{2}{\pi}\right)\right]^{1/2} = 1.659\, h_0 \left[\frac{x}{L} - \frac{2}{\pi} \sin \frac{\pi x}{2L}\right]^{1/2}$ ◄

Data: $\quad L = 750 \text{ mm} = 0.75 \text{ m}, \quad b = 30 \text{ mm} = 0.030 \text{ m}$
$\qquad w_0 = 300 \text{ kN/m} = 300 \times 10^3 \text{ N/m}, \quad \sigma_{all} = 200 \text{ MPa} = 200 \times 10^6 \text{ Pa}$

(b) $\quad h_0 = 1.178 \sqrt{\frac{(300 \times 10^3)(0.75)^2}{(200 \times 10^6)(0.030)}} = 197.6 \times 10^{-3} \text{ m} = 197.6 \text{ mm}$ ◄

Problem 5.130

5.130* and *5.131 The beam AB, consisting of an aluminum plate of uniform thickness b and length L, is to support the load shown. (a) Knowing that the beam is to be of constant strength, express h in terms of x, L, and h_0 for portion AC of the beam. (b) Determine the maximum allowable load if L = 800 mm., h_0 = 200 mm, b = 25 mm., and σ_{all} = 72 MPa.



$$\frac{dV}{dx} = -w = -w_0 \sin \frac{\pi x}{L}$$

$$\frac{dM}{dx} = V = \frac{w_0 L}{\pi} \cos \frac{\pi x}{L} + C_1$$

$$M = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi x}{L} + C_1 x + C_2$$

At A, $x = 0$ $M = 0$ $\quad 0 = 0 + 0 + C_2 \quad C_2 = 0$

At B, $x = L$ $M = 0$ $\quad 0 = \frac{w_0 L^2}{\pi^2} \sin \pi + C_1 L \quad C_1 = 0$

$$M = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi x}{L}$$

For constant strength $\quad S = \frac{|M|}{\sigma_{all}} = \frac{w_0 L^2}{\pi^2 \sigma_{all}} \sin \frac{\pi x}{L}$

For a rectangular section $\quad I = \frac{1}{12} b h^3, \ c = \frac{h}{2}, \ S = \frac{I}{c} = \frac{1}{6} b h^2$

Equating the two expressions for S $\quad \frac{1}{6} b h^2 = \frac{w_0 L^2}{\pi^2 \sigma_{all}} \sin \frac{\pi x}{L} \qquad (1)$

At $x = \frac{L}{2}$ $\quad h = h_0 \quad \frac{1}{6} b h_0^2 = \frac{w_0 L^2}{\pi^2 \sigma_{all}} \qquad (2)$

(a) Dividing Eq(1) by Eq. 2 $\quad \frac{h^2}{h_0^2} = \sin \frac{\pi x}{L} \qquad h = h_0 \left(\sin \frac{\pi x}{L} \right)^{1/2}$ ◄

(b) Solving Eq. (1) for w_0 $\quad w_0 = \frac{\pi^2 \sigma_{all} b h_0^2}{6 L^2}$

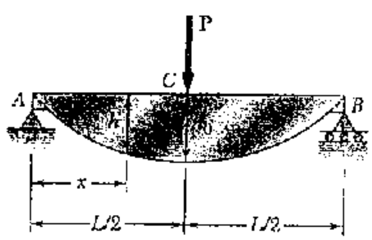
Data: $\sigma_{all} = 72 \times 10^6$ Pa, $L = 800$ mm $= 0.800$ m, $h_0 = 200$ mm $= 0.200$ m, $b = 25$ mm $= 0.025$ m

$$w_0 = \frac{\pi^2 (72 \times 10^6)(0.025)(0.200)^2}{(6)(0.800)^2} = 185.1 \times 10^3 \text{ N/m}$$

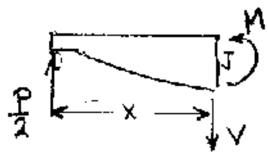
185.1 kN/m ◄

Problem 5.131

5.130* and *5.131 The beam *AB*, consisting of an aluminum plate of uniform thickness *b* and length *L*, is to support the load shown. (*a*) Knowing that the beam is to be of constant strength, express *h* in terms of *x*, *L*, and h_0 for portion *AC* of the beam. (*b*) Determine the maximum allowable load if $L = 800$ mm., $h_0 = 200$ mm, $b = 25$ mm., and $\sigma_{all} = 72$ MPa.



$R_A = R_B = \frac{P}{2}$



$\circlearrowleft \Sigma M_J = 0$

$-\frac{P}{2}x + M = 0$

$M = \frac{Px}{2} \qquad (0 < x < \frac{L}{2})$

$S = \frac{M}{\sigma_{all}} = \frac{Px}{2\sigma_{all}}$

For a rectangular cross section $\quad S = \frac{1}{6}bh^2$

Equating $\quad \frac{1}{6}bh^2 = \frac{Px}{2\sigma_{all}} \qquad h = \sqrt{\frac{3Px}{\sigma_{all}b}}$

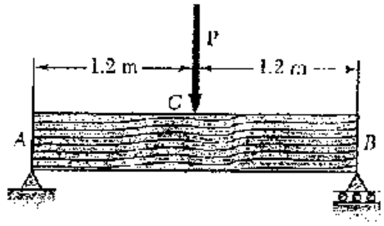
(a) At $x = \frac{L}{2} \quad h = h_0 = \sqrt{\frac{3PL}{2\sigma_{all}b}} \qquad h = h_0\sqrt{\frac{2x}{L}}\ , \ 0 < x < \frac{L}{2}$ ◄

For $x > \frac{L}{2}$ replace x by $L - x$

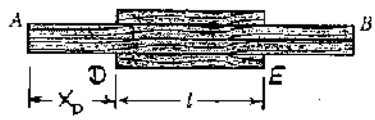
(b) Solving for P $\quad P = \frac{2\sigma_{all}bh_0^2}{3L} = \frac{(2)(72\times10^6)(0.025)(0.200)^2}{(3)(0.8)} = 60\times10^3$ N

$= 60$ kN ◄

Problem 5.132



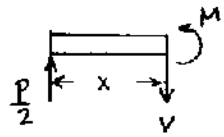
(a)



(b)

5.132 and 5.133 A preliminary design on the use of a simply supported prismatic timber beam indicated that a beam with a rectangular cross section 50 mm wide and 200 mm deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, four pieces of the same timber as the original beam and of 50 × 50-mm cross section. Determine the length *l* of the two outer pieces of timber that will yield the same factor of safety as the original design.

$R_A = R_B = \frac{P}{2}$

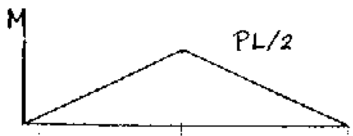


$0 < x < \frac{L}{2}$

$\circlearrowright \Sigma M_J = 0 \qquad -\frac{P}{2}x + M = 0$

$M = \frac{Px}{2} \quad \text{or} \quad M = \frac{M_{max}x}{1.2}$

Bending moment diagram is two straight lines.



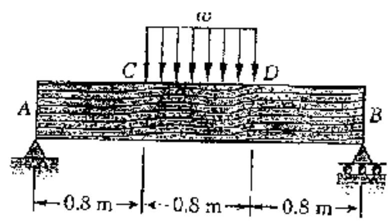
At C $\qquad S_C = \frac{1}{6}bh_C^2 \qquad M_C = M_{max}$

At D $\qquad S_D = \frac{1}{6}bh_D^2 \qquad M_D = \frac{M_{max}x_D}{1.2}$

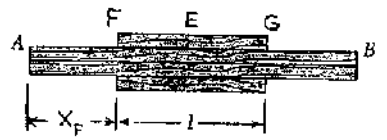
$$\frac{S_D}{S_C} = \frac{h_D^2}{h_C^2} = \left(\frac{100\text{ mm}}{200\text{ mm}}\right)^2 = \frac{1}{4} = \frac{M_D}{M_C} = \frac{x_D}{1.2} \qquad x_D = 0.3 \text{ m}$$

$$\frac{l}{2} = 1.2 - x_D = 0.9 \qquad l = 1.800 \text{ m}$$

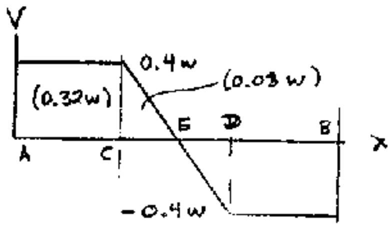
Problem 5.133

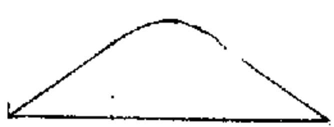


(a)



(b)





5.132 and 5.133 A preliminary design on the use of a simply supported prismatic timber beam indicated that a beam with a rectangular cross section 50 mm wide and 200 mm deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, four pieces of the same timber as the original beam and of 50 × 50-mm cross section. Determine the length *l* of the two outer pieces of timber that will yield the same factor of safety as the original design.

$$R_A = R_B = \frac{0.8\ w}{2} = 0.4\ w$$

Shear: A to C $\quad V = 0.4\ w$

D to B $\quad V = -0.4\ w$

Areas: A to C $\quad (0.8)(0.4)w = 0.32\ w$

C to E $\quad (\frac{1}{2})(0.4)(0.4)w = 0.08\ w$

Bending moments.

At C $\quad M_C = 0.40\ w$

A to C $\quad M = 0.40\ wx$

At C $\quad S_c = \frac{1}{6} b h_c^2 \qquad M_c = M_{max} = 0.40\ w$

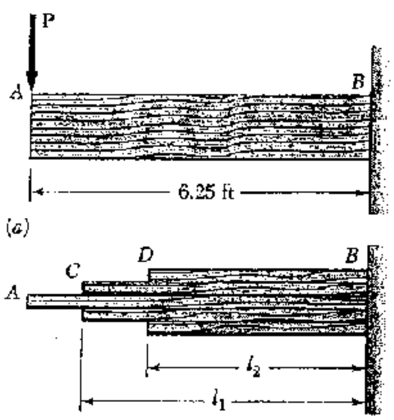
At F $\quad S_F = \frac{1}{6} b h_F^2 \qquad M_F = 0.40\ w x_F$

$$\frac{S_F}{S_c} = \frac{h_F^2}{h_c^2} = \left(\frac{100\ mm}{200\ mm}\right)^2 = \frac{1}{4} = \frac{M_F}{M_c} = \frac{0.40\ w x_F}{0.40\ w}$$

$x_F = 0.25\ m \qquad \frac{l}{2} = 1.2 - x_F = 0.95\ m$

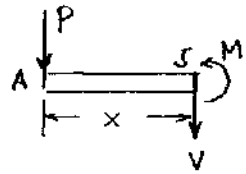
$$l = 1.900\ m$$

Problem 5.134



(a) (b)

5.134 and 5.135 A preliminary design on the use of a cantilever prismatic timber beam indicated that a beam with a rectangular cross section 2 in. wide and 10 in. deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, five pieces of the same timber as the original beam and of 2 × 10-in. cross section. Determine the respective lengths l_1 and l_2 of the two inner and outer pieces of timber that will yield the factor of safety as the original design.



$+\circlearrowleft \Sigma M_J = 0$

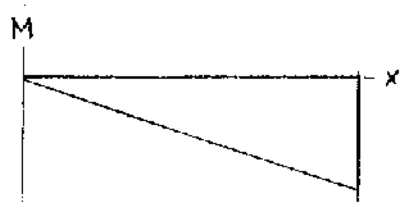
$Px + M = 0 \qquad M = -Px$

$|M| = Px$

At B $\quad |M|_B = M_{max}$

At C $\quad |M|_C = M_{max} x_C / 6.25$

At D $\quad |M|_D = M_{max} x_D / 6.25$



$$S_B = \frac{1}{6} b h^2 = \frac{1}{6} \cdot b (5b)^2 = \frac{25}{6} b^3$$

A to C $\quad S_C = \frac{1}{6} \cdot b (b)^2 = \frac{1}{6} b^3$

C to D $\quad S_D = \frac{1}{6} b (3b)^2 = \frac{9}{6} b^3$

$$\frac{|M|_C}{|M|_B} = \frac{x_C}{6.25} = \frac{S_C}{S_B} = \frac{1}{25} \qquad x_C = \frac{(1)(6.25)}{25} = 0.25 \text{ ft}$$

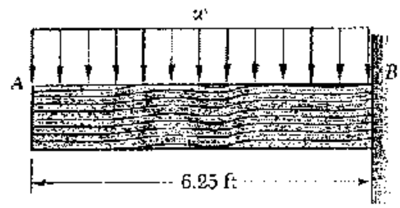
$$l_1 = 6.25 - 0.25 = 6.00 \text{ ft}$$

$$\frac{|M|_D}{|M|_B} = \frac{x_D}{6.25} = \frac{S_D}{S_B} = \frac{9}{25} \qquad x_D = \frac{(9)(6.25)}{25} = 2.25 \text{ ft}$$

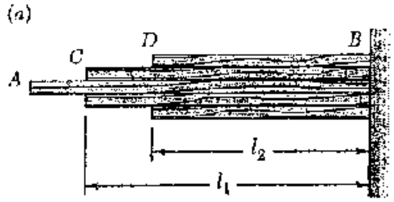
$$l_2 = 6.25 - 2.25 = 4.00 \text{ ft}$$

Problem 5.135

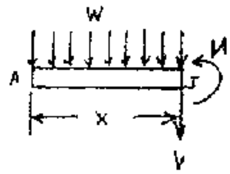
5.134 and *5.135* A preliminary design on the use of a cantilever prismatic timber beam indicated that a beam with a rectangular cross section 2 in. wide and 10 in. deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, five pieces of the same timber as the original beam and of 2 × 10-in. cross section. Determine the respective lengths l_1 and l_2 of the two inner and outer pieces of timber that will yield the factor of safety as the original design.



(a)



(b)



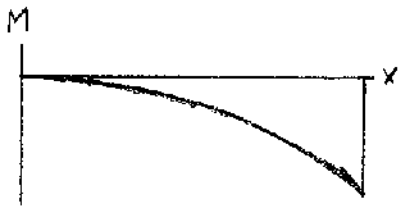
$+\circlearrowleft \Sigma M_J = 0 \quad wx\frac{x}{2} + M = 0$

$M = -\frac{wx^2}{2} \qquad |M| = \frac{wx^2}{2}$

At B $\quad |M|_B = |M|_{max}$

At C $\quad |M|_C = |M|_{max}(x_C/6.25)^2$

At D $\quad |M|_D = |M|_{max}(x_D/6.25)^2$



At B $\quad S_B = \frac{1}{6}bh^2 = \frac{1}{6}b(5b)^2 = \frac{25}{6}b^3$

A to C $\quad S_C = \frac{1}{6}bh^2 = \frac{1}{6}b(b)^2 = \frac{1}{6}b^3$

C to D $\quad S_D = \frac{1}{6}bh^2 = \frac{1}{6}b(3b)^2 = \frac{9}{6}b^3$

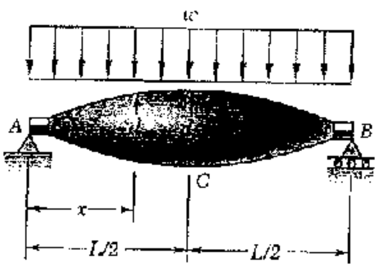
$$\frac{|M|_C}{|M|_B} = \left(\frac{x_C}{6.25}\right)^2 = \frac{S_C}{S_B} = \frac{1}{25} \qquad x_C = \frac{6.25}{\sqrt{25}} = 1.25 \text{ ft}$$

$l_1 = 6.25 - 1.25 \text{ ft} = 5.00 \text{ ft}$ ◄

$$\frac{|M|_D}{|M|_B} = \left(\frac{x_D}{6.25}\right)^2 = \frac{S_D}{S_B} = \frac{9}{25} \qquad x_D = \frac{6.25\sqrt{9}}{\sqrt{25}} = 3.75 \text{ ft}$$

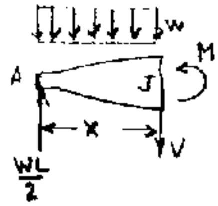
$l_2 = 6.25 - 3.75 \text{ ft} = 2.50 \text{ ft}$ ◄

Problem 5.136



5.136 and *5.137* A machine element of cast aluminum and in the shape of a solid of revolution of variable diameter d is being designed to support the load shown. Knowing that the machine element is to be of constant strength, express d in terms of x, L, and d_0.

$$R_A = R_B = \frac{wL}{2}$$



$$+\circlearrowleft \Sigma M_J = 0$$

$$-\frac{wL}{2}x + wx\frac{x}{2} + M = 0$$

$$M = \frac{w}{2}x(L-x)$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{wx(L-x)}{2\sigma_{all}}$$

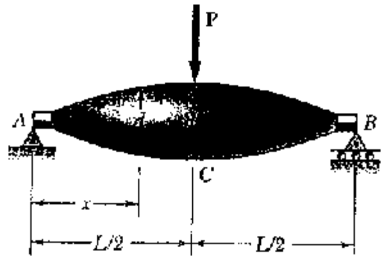
For a solid circular cross section $c = \frac{d}{2}$ $I = \frac{\pi}{4}c^3$ $S = \frac{I}{c} = \frac{\pi d^3}{32}$

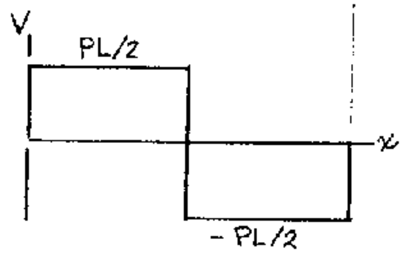
Equating $\frac{\pi d^3}{32} = \frac{wx(L-x)}{2\sigma_{all}}$ $d = \left\{\frac{16\,wx(L-x)}{\pi\sigma_{all}}\right\}^{1/3}$

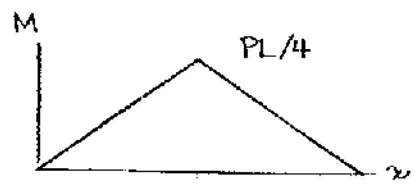
At $x = \frac{L}{2}$ $d = d_0 = \left\{\frac{4wL^2}{\pi\sigma_{all}}\right\}^{1/3}$ $d = d_0\left\{4\frac{x}{L}\left(1-\frac{x}{L}\right)\right\}^{1/3}$

Problem 5.137

5.136 and *5.137* A machine element of cast aluminum and in the shape of a solid of revolution of variable diameter d is being designed to support the load shown. Knowing that the machine element is to be of constant strength, express d in terms of x, L, and d_0.







Draw shear and bending moment diagrams.

$$0 \leq x \leq \frac{L}{2} \qquad M = \frac{Px}{2}$$

$$\frac{L}{2} \leq x \leq L \qquad M = \frac{P(L-x)}{2}$$

For a solid circular section $\quad c = \frac{1}{2}d$

$$I = \frac{\pi}{4}c^4 = \frac{\pi}{64}d^4 \qquad S = \frac{I}{c} = \frac{\pi}{32}d^3$$

For constant design, σ = constant

$$S = \frac{M}{\sigma}$$

For $0 \leq x \leq \frac{L}{2}$ $\qquad \frac{\pi}{32}d^3 = \frac{Px}{2} \qquad$ (1a)

For $\frac{L}{2} \leq x \leq L$ $\qquad \frac{\pi}{32}d^3 = \frac{P(L-x)}{2} \qquad$ (1b)

At point C $\qquad \frac{\pi}{32}d_0^3 = \frac{PL}{4} \qquad$ (2)

Dividing Eq. (1a) by Eq. (2)

$$0 \leq x \leq \frac{L}{2} \qquad \frac{d^3}{d_0^3} = \frac{2x}{L}$$

$$d = d_0(2x/L)^{1/3}$$

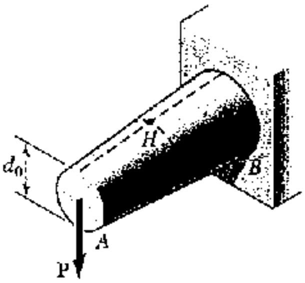
Dividing Eq. (1b) by Eq. (2)

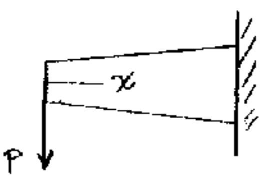
$$\frac{L}{2} \leq x \leq L \qquad \frac{d^3}{d_0^3} = \frac{2(L-x)}{L}$$

$$d = d_0[2(L-x)/L]^{1/3}$$

Problem 5.138

5.138 A transverse force **P** is applied as shown at end A of the conical taper AB. Denoting by d_0 the diameter of the taper at the A, show that the maximum normal stress occurs at point H, which is contained in a transverse section of diameter $d = 1.5\, d_0$.





$$V = -P = \frac{dM}{dx} \qquad M = -Px$$

Let $d = d_0 + kx$

For a solid circular section $I = \frac{\pi}{4}c^4 = \frac{\pi}{64}d^3$

$$c = \frac{d}{2} \qquad S = \frac{I}{c} = \frac{\pi}{32}d^3 = \frac{\pi}{32}(d_0 + kx)^3$$

$$\frac{dS}{dx} = \frac{3\pi}{32}(d_0 + kx)^2 k = \frac{3\pi}{32}d^2 k$$

Stress $\sigma = \frac{|M|}{S} = \frac{Px}{S}$

At H $\quad \frac{d\sigma}{dx} = \frac{1}{S^2}\left(PS - Px_H\frac{dS}{dx}\right) = 0$

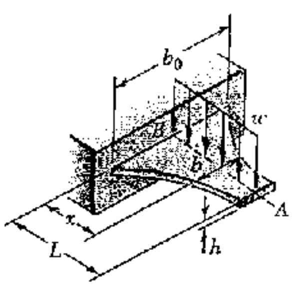
$$S - x_H\frac{dS}{dx} = \frac{\pi}{32}d^3 - x_H\frac{3\pi}{32}d^2 k$$

$$kx_H = \frac{1}{3}d = \frac{1}{3}(d_0 + k_H x_H) \qquad kx_H = \frac{1}{2}d_0$$

$$d = d_0 + \frac{1}{2}d_0 = \frac{3}{2}d_0 \qquad d = 1.5\, d_0$$

Problem 5.139

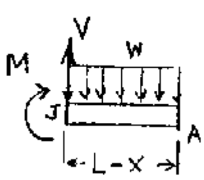
5.139 A cantilever beam AB consisting of a steel plate of uniform depth h and variable width b is to support the distributed load w along its center line AB. (a) knowing that the beam is to be of constant strength, express b in terms of x, L, and b_0. (b) Determine the maximum allowable value of w if $L = 15$ in., $b_0 = 8$ in., $h = 0.75$ in., and $\sigma_{all} = 24$ ksi.



$$+\circlearrowleft \Sigma M_J = 0 \qquad -M - w(L-x)\frac{L-x}{2} = 0$$

$$M = -\frac{w(L-x)^2}{2} \qquad |M| = \frac{w(L-x)^2}{2}$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{w(L-x)^2}{2\sigma_{all}}$$



For a rectangular cross section $S = \frac{1}{6}bh^2$

$$\frac{1}{6}bh^2 = \frac{w(L-x)^2}{2\sigma_{all}} \qquad b = \frac{3w(L-x)^2}{\sigma_{all}h^2}$$

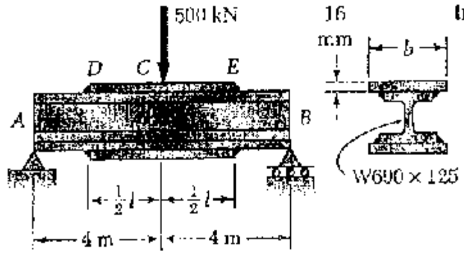
At $x = 0$ $\quad b = b_0 = \frac{3wL^2}{\sigma_{all}h^2} \qquad b = b_0\left(1 - \frac{x}{L}\right)^2$

Solving for w $\quad w = \frac{\sigma_{all}b_0h^2}{3L^2} = \frac{(24)(8)(0.75)^2}{(3)(15)^2} = 0.160$ kip/in

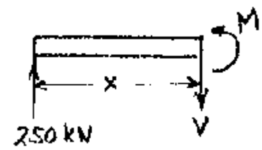
$= 160$ lb/in

Problem 5.140

5.140 Assuming that the length and width of the cover plates used with the beam of Sample Prob. 5.12 are, respectively, $l = 4$ m and $b = 285$ mm, and recalling that the thickness of each plate is 16 mm, determine the maximum normal stress on a transverse section (*a*) through the center of the beam, (*b*) just to the left of *D*.



$R_A = R_B = 250$ kN



$\circlearrowleft \Sigma M_J = 0$

$-250\,x + M = 0$

$M = 250\,x \quad \text{kN}\cdot\text{m}$

At center of beam $\quad x = 4\,\text{m} \qquad M_c = (250)(4) = 1000 \text{ kN}\cdot\text{m}$

At D $\quad x = \frac{1}{2}(8 - l) = \frac{1}{2}(8-4) = 2\,\text{m} \qquad M_D = 500 \text{ kN}\cdot\text{m}$

At center of beam $\quad I = I_{beam} + 2 I_{plate}$

$$= 1190\times10^6 + 2\left\{(285)(16)\left(\frac{678}{2} + \frac{16}{2}\right)^2 + \frac{1}{12}(285)(16)^3\right\}$$

$$= 2288 \times 10^6 \text{ mm}^4$$

$$c = \frac{678}{2} + 16 = 355 \text{ mm} \qquad S = \frac{I}{c} = 6445\times10^3 \text{ mm}^3 = 6445\times10^{-6} \text{ m}^3$$

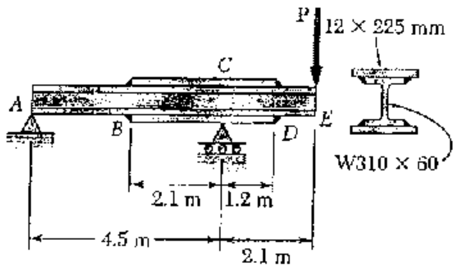
(a) $\quad$ Normal stress $\quad \sigma = \dfrac{M}{S} = \dfrac{1000\times10^3}{6445\times10^{-6}} = 155.2\times10^6 \text{ Pa} = 155.2 \text{ MPa}$ ◄

At D $\quad S = 3510\times10^3 \text{ mm}^3 = 3510\times10^{-6} \text{ m}^3$

(b) $\quad$ Normal stress $\quad \sigma = \dfrac{M}{S} = \dfrac{500\times10^3}{3510\times10^{-6}} = 142.4\times10^6 \text{ Pa} = 142.4 \text{ MPa}$ ◄

Problem 5.141

5.141 Knowing that $\sigma_{all} = 150$ MPa, determine the largest concentrated load **P** that can be applied at end E of the beam shown.



$+\circlearrowleft \Sigma M_C = 0 \quad -4.5 R_A - 2.1 P = 0$

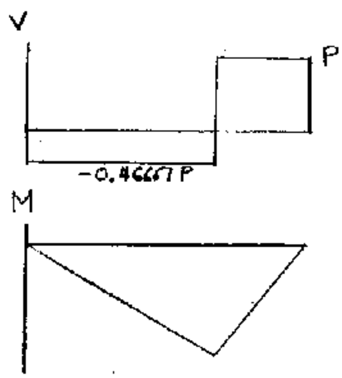
$R_A = -0.46667 P$ ie $0.46667 P \downarrow$

$+\circlearrowleft \Sigma M_A = 0 \quad 4.5 R_C - 6.6 P = 0$

$R_C = 1.46667 P$

Shear: A to C $\quad V = -0.46667 P$

C to E $\quad V = P$



Bending moments:

$M_C = -(4.5)(0.46667 P) = -2.10 P$ kN·m

$M_B = \frac{2.4}{4.5} M_C = -1.12 P$ kN·m

$M_D = \frac{0.9}{2.1} M_C = -0.9 P$ kN·m

At B and D $\quad S = 851 \times 10^3 \text{ mm}^3 = 851 \times 10^{-6} \text{ m}^3$

$\sigma_{all} = \frac{|M|}{S} = \frac{1.120 P_{all}}{851 \times 10^{-6}} = 150 \times 10^6 \quad$ at B

$P_{all} = 114.0$ kN

At C $\quad I = I_{beam} + 2 I_{plate}$

$= 129 \times 10^6 + 2\left\{(225)(12)\left(\frac{310}{2} + \frac{12}{2}\right)^2 + \frac{1}{12}(225)(12)^3\right\}$

$= 269 \times 10^6 \text{ mm}^4$

$c = \frac{310}{2} + 12 = 167$ mm $\qquad S = \frac{I}{c} = 1611 \times 10^3 \text{ mm}^3$

$= 1611 \times 10^{-6} \text{ m}^3$

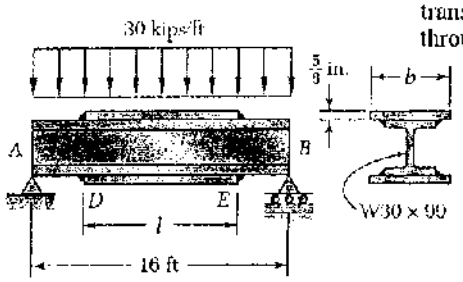
$\sigma_{all} = \frac{|M|}{S} = \frac{2.10 P}{1611 \times 10^{-6}} = 150 \times 10^6$

$P_{all} = 115.1$ kN

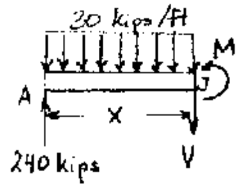
Allowable load is the smaller value $\qquad P = 114.0$ kN

Problem 5.142

5.142 Two cover plates, each $\frac{5}{8}$-in. thick, are welded to a W30 × 99 beam as shown. Knowing that l = 9 ft and b = 12 in., determine the maximum normal stress on a transverse section (*a*) through the center of the beam, (*b*) just to the left of *D*.



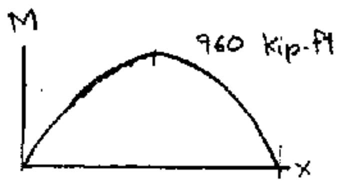
$R_A = R_B = 240$ kips



$\circlearrowleft \Sigma M_J = 0$

$-240x + 30x\frac{x}{2} + M = 0$

$M = 240x - 15x^2$ kip·ft



At center of beam $x = 8$ ft

$M_c = 960$ kip·ft $= 11520$ kip·in

At point D, $x = \frac{1}{2}(16-9) = 3.5$ ft

$M_D = 656.25$ kip·ft $= 7875$ kip·in.

At center of beam $I = I_{beam} + 2I_{plate}$

$$I = 3990 + 2\left\{(12)(0.625)\left(\frac{29.65}{2} + \frac{0.625}{2}\right)^2 + \frac{1}{12}(12)(0.625)^3\right\} = 7428 \text{ in}$$

$$c = \frac{29.65}{2} + 0.625 = 15.45 \text{ in}$$

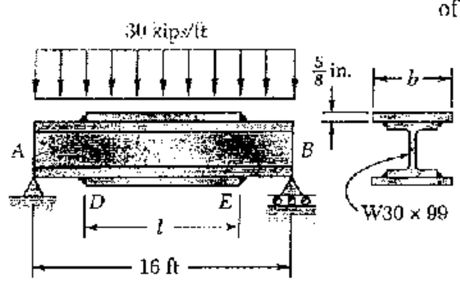
(a) Normal stress $\sigma = \frac{Mc}{I} = \frac{(11520)(15.45)}{7428} = 24.0$ ksi ◄

At point D $S = 269$ in^3

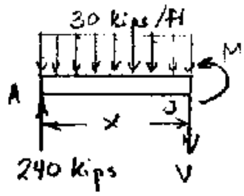
(b) Normal stress $\sigma = \frac{M}{S} = \frac{7875}{269} = 29.3$ ksi ◄

Problem 5.143

5.143 Two cover plates, each $\frac{5}{8}$-in. thick, are welded to a W30 × 99 beam as shown. Knowing that $\sigma_{all} = 22$ ksi for both the beam and the plates, determine the required value of (*a*) the length of the plates, (*b*) the width of the plates.



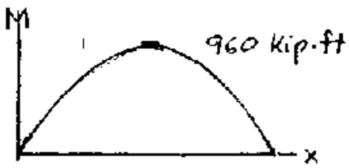
$R_A = R_B = 240$ Kips



$\circlearrowleft \sum M_J = 0$

$-240x + 30x\frac{x}{2} + M = 0$

$M = 240x - 15x^2$ kip·ft



For W 30 × 99 rolled steel section

$S = 269$ in^3

Allowable bending moment

$M_{all} = \sigma_{all} S = (22)(269) = 5918$ kip·in

$= 493.167$ kip·ft

To locate points D and E, set $M = M_{all}$

$240x - 15x^2 = 493.167 \qquad 15x^2 - 240x + 493.167 = 0$

$$x = \frac{240 \pm \sqrt{(240)^2 - (4)(15)(493.167)}}{(2)(15)} = 2.42 \text{ ft},\ 13.58 \text{ ft}$$

(a) $\quad l = x_E - x_D = 13.58 - 2.42 = 11.16$ ft ◄

Center of beam $\quad M = 960$ kip·ft $= 11520$ kip·in

$S = \frac{M}{\sigma_{all}} = \frac{11520}{22} = 523.64$ in^3

$c = \frac{29.65}{2} + 0.625 = 15.45$ in

Required moment of inertia $\quad I = Sc = 8090$ in^4

But $\quad I = I_{beam} + 2I_{plate}$

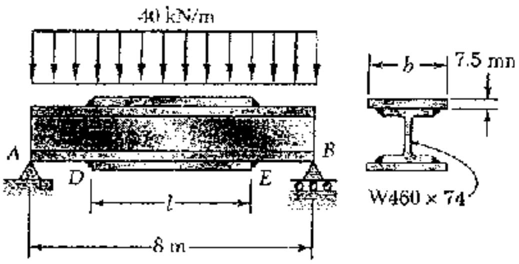
$$8090 = 3990 + 2\left\{(b)(0.625)\left(\frac{29.65}{2} + \frac{0.625}{2}\right)^2 + \frac{1}{12}(b)(0.625)^3\right\}$$

$= 3990 + 286.47\,b$

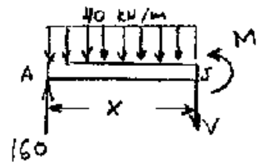
(b) $\quad b = 14.31$ in. ◄

Problem 5.144

5.144 Two cover plates, each 7.5 mm thick, are welded to a W460 × 74 beam as shown. Knowing that $l = 5$ m and $b = 200$ mm, determine the maximum normal stress on a transverse section (*a*) through the center of the beam, (*b*) just to the left of *D*.



$R_A = R_B = 160$ kN



$+\circlearrowleft \Sigma M_J = 0$

$-160x + (40x)\frac{x}{2} + M = 0$

$M = 160x - 20x^2$ kN·m

At center of beam $x = 4$m $\quad M_c = 320$ kN·m

At D $\quad x = \frac{1}{2}(8 - l) = 1.5$ m $\quad M_D = 195$ kN·m

At center of beam $\quad I = I_{beam} + 2I_{plate}$

$$= 333 \times 10^6 + 2\left\{(200)(7.5)\left(\frac{457}{2} + \frac{7.5}{2}\right)^2 + \frac{1}{12}(200)(7.5)^3\right\}$$

$$= 494.8 \times 10^6 \text{ mm}^4$$

$c = \frac{457}{2} + 7.5 = 236$ mm $\qquad S = \frac{I}{c} = 2097 \times 10^3$ mm^3

$= 2097 \times 10^{-6}$ m^3

(a) Normal stress $\quad \sigma = \frac{M}{S} = \frac{320 \times 10^3}{2097 \times 10^{-6}} = 152.6 \times 10^6$ Pa

$= 152.6$ MPa ◄

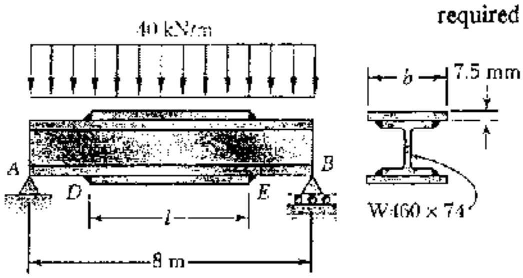
At D $\quad S = 1460 \times 10^3$ mm^3 $= 1460 \times 10^{-6}$ m^3

(b) Normal stress $\quad \sigma = \frac{M}{S} = \frac{195 \times 10^3}{1460 \times 10^{-6}} = 133.6 \times 10^6$ Pa

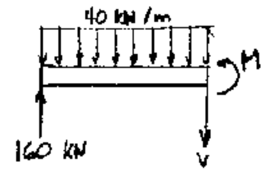
$= 133.6$ MPa ◄

Problem 5.145

5.145 Two cover plates, each 7.5 mm thick, are welded to a W460 × 74 beam as shown. Knowing that $\sigma_{all} = 150$ MPa for both the beam and the plates, determine the required value of (*a*) the length of the plates, (*b*) the width of the plates.



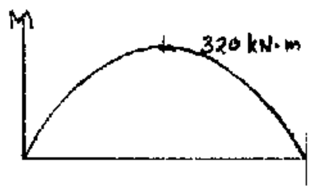
$R_A = R_B = 160$ kN



$+\circlearrowleft \Sigma M_J = 0$

$-160x + (40x)\left(\frac{x}{2}\right) + M = 0$

$M = 160x - 20x^2$ kN·m



For W 460×74 rolled steel beam

$S = 1460 \times 10^3\ mm^3 = 1460 \times 10^{-6}\ m^3$

Allowable bending moment

$M_{all} = \sigma_{all} S = (150 \times 10^6)(1460 \times 10^{-6})$

$= 219 \times 10^3$ N·m $= 219$ kN·m

To locate points D and E, set $M = M_{all}$

$160x - 20x^2 = 219 \qquad 20x^2 - 160x + 219 = 0$

$$x = \frac{160 \pm \sqrt{160^2 - (4)(20)(219)}}{(2)(20)} = \begin{matrix} 1.753\ m \\ 6.247\ m \end{matrix}$$

(a) $x_D = 1.753$ ft. $\quad x_E = 6.247$ ft $\quad l = x_E - x_D = 4.49$ m ◄

At center of beam $\quad M = 320$ kN·m $= 320 \times 10^3$ N·m

$S = \frac{M}{\sigma_{all}} = \frac{320 \times 10^3}{150 \times 10^6} = 2133 \times 10^{-6}\ m^3 = 2133 \times 10^3\ mm^3$

$c = \frac{457}{2} + 7.5 = 236\ mm^4$

Required moment of inertia $\quad I = Sc = 503.4 \times 10^6\ mm^4$

But $\quad I = I_{beam} + 2I_{plate}$

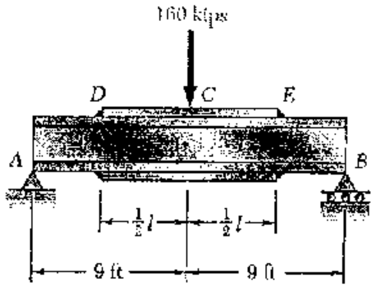
$503.4 \times 10^6 = 333 \times 10^6 + 2\left\{(b)(7.5)\left(\frac{457}{2} + \frac{7.5}{2}\right)^2 + \frac{1}{12}(b)(7.5)^3\right\}$

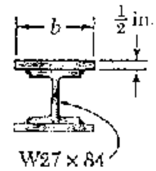
$= 333 \times 10^6 + 809.2 \times 10^3\ b$

(b) $\quad b = 211$ mm ◄

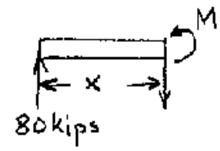
Problem 5.146

5.146 Two cover plates, each $\frac{1}{2}$-in. thick, are welded to a W27 × 84 beam as shown. Knowing that l = 10 ft and b = 10.5 in., determine the maximum normal stress on a transverse section (*a*) through the center of the beam, (*b*) just to the left of *D*.





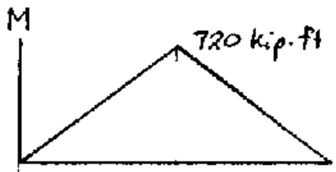
$R_A = R_B = 80$ kips



$+\circlearrowleft \Sigma M_J = 0$

$-80x + M = 0$

$M = 80x$ kip·ft



At C $\quad x = 9$ ft $\quad M_C = 720$ kip·ft $= 8640$ kip·in

At D $\quad x = 9 - 5 = 4$ ft

$M_D = (80)(4) = 320$ kip·ft $= 3840$ kip·in

At center of beam $\quad I = I_{beam} + 2I_{plate}$

$$I = 2850 + 2\left\{(10.5)(0.500)\left(\frac{26.71}{2} + \frac{0.500}{2}\right)^2 + \frac{1}{12}(10.5)(0.500)^3\right\}$$

$= 4794$ in^3

$c = \frac{26.71}{2} + 0.500 = 13.855$ in

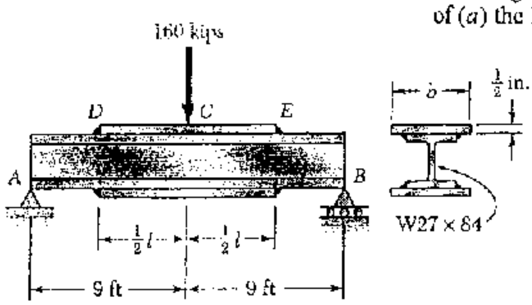
(a) Normal stress $\quad \sigma = \frac{Mc}{I} = \frac{(8640)(13.855)}{4794} = 25.0$ ksi ◄

At point D $\quad S = 213$ in^3

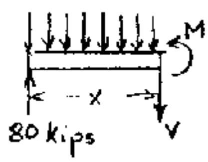
(b) Normal stress $\quad \sigma = \frac{M}{S} = \frac{3840}{213} = 18.03$ ksi ◄

Problem 5.147

5.147 Two cover plates, each $\frac{1}{2}$-in. thick, are welded to a W27 × 84 beam as shown. Knowing that $\sigma_{all} = 24$ ksi for both the beam and the plates, determine the required value of (*a*) the length of the plates, (*b*) the width of the plates.



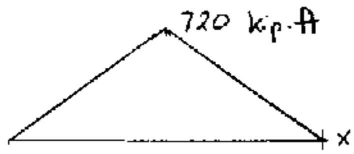
$R_A = R_B = 80$ kips



$+\circlearrowleft \Sigma M_J = 0$

$-80x + M = 0$

$M = 80x$ kip·ft



At D $\qquad S = 213 \text{ in}^3$

Allowable bending moment

$M_{all} = \sigma_{all} S = (24)(213) = 5112$ kip·in

$= 426$ kip.

Set $M_D = M_{all}$ $\qquad 80x_D = 426 \qquad x_D = 5.325$ ft

(a) $\qquad \ell = 18 - 2x_D = 7.35$ ft ◄

At center of beam $\qquad M = (80)(9) = 720$ kip·ft $= 8640$ kip·in.

$$S = \frac{M}{\sigma_{all}} = \frac{8640}{24} = 360 \text{ in}^3$$

$$c = \frac{26.71}{2} + 0.500 = 13.855 \text{ in}$$

Required moment of inertia $\qquad I = Sc = 4987.8 \text{ in}^4$

But $\qquad I = I_{beam} + 2 I_{plate}$

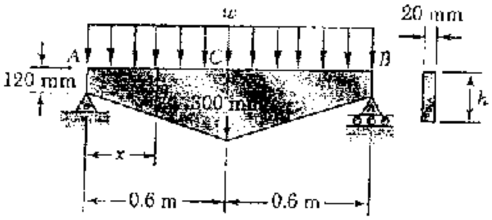
$$4987.8 = 2850 + 2\left\{(b)(0.500)\left(\frac{26.71}{2} + \frac{0.500}{2}\right)^2 + \frac{1}{12}(b)(0.500)^3\right\}$$

$$= 2850 + 185.12\, b$$

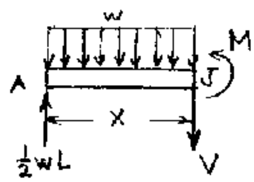
(b) $\qquad b = 11.55$ in. ◄

Problem 5.148

5.148 For the tapered beam shown, determine (*a*) the transverse section in which the maximum normal stress occurs. (*b*) the largest distributed load w that can be applied, knowing that $\sigma_{all} = 140$ MPa.



$R_A = R_B = \frac{1}{2}wL \qquad L = 1.2\ \text{m}$

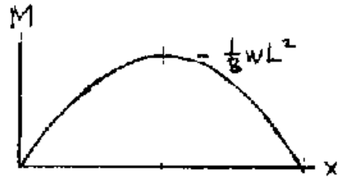


$\circlearrowleft \Sigma M_J = 0$

$-\frac{1}{2}wL + wx\frac{x}{2} + M = 0$

$M = \frac{w}{2}(Lx - x^2)$

$= \frac{w}{2}x(L-x)$



For the tapered beam $\quad h = a + kx$

$a = 120$ mm $\qquad k = \frac{300-120}{0.6} = 300$ mm/m

For rectangular cross section $\quad S = \frac{1}{6}bh^2 = \frac{1}{6}b(a+kx)^2$

Bending stress $\quad \sigma = \frac{M}{S} = \frac{3w}{b}\frac{Lx - x^2}{(a+kx)^2}$

To find location of maximum bending stress set $\frac{d\sigma}{dx} = 0$

$$\frac{d\sigma}{dx} = \frac{3w}{b}\frac{d}{dx}\left\{\frac{Lx - x^2}{(a+kx)^2}\right\} = \frac{3w}{b}\left\{\frac{(a+kx)^2(L-2x) - (Lx-x^2)2(a+kx)k}{(a+kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{(a+kx)(L-2x) - 2k(Lx-x^2)}{(a+kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL + kLx - 2ax - \cancel{2kx^2} - 2kLx + \cancel{2kx^2}}{(a+kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL - (2a+kL)x}{(a+kx)^3}\right\} = 0$$

(a) $\quad x_m = \frac{aL}{2a+kL} = \frac{(120)(1.2)}{(2)(120) + (300)(1.2)} = 0.24$ m

$h_m = a + kx_m = 120 + (300)(0.24) = 192$ mm

$S_m = \frac{1}{6}bh_m^2 = \frac{1}{6}(20)(192)^2 = 122.88\times10^3\ \text{mm}^3 = 122.88\times10^{-6}\ \text{m}^3$

Allowable value of M_m $\qquad M_m = S_m\sigma_{all} = (122.88\times10^{-6})(140\times10^6)$

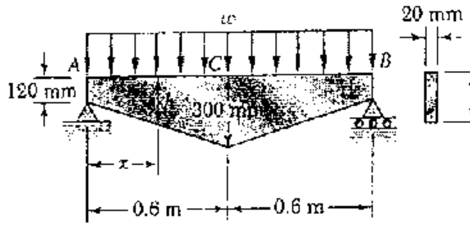
$= 17.2032\times10^3$ N-m

(b) Allowable value of w $\qquad w = \frac{2M_m}{x_m(L-x_m)} = \frac{(2)(17.2032\times10^3)}{(0.24)(0.96)}$

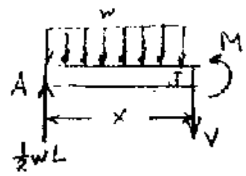
$= 149.3\times10^3$ N/m $= 149.3$ kN/m

Problem 5.149

5.149 For the tapered beam shown, knowing that $w = 160$ kN/m, determine (*a*) the transverse section in which the maximum normal stress occurs, (*b*) the corresponding value of the normal stress.



$R_A = R_B = \frac{1}{2} wL$

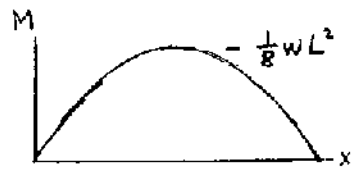


$\circlearrowleft \Sigma M_J = 0$

$-\frac{1}{2} wLx + wx\frac{x}{2} + M = 0$

$M = \frac{w}{2}(Lx - x^2)$

$= \frac{w}{2} x(L - x)$



where $w = 160$ kN/m and $L = 1.2$ m.

For the tapered beam $h = a + kx$

$a = 120$ mm $\quad k = \frac{300 - 120}{0.6} = 300$ mm/m

For a rectangular cross section $S = \frac{1}{6} b h^2 = \frac{1}{6} b (a + kx)^2$

Bending stress $\sigma = \frac{M}{S} = \frac{3w}{b} \cdot \frac{Lx - x^2}{(a + kx)^2}$

To find location of maximum bending stress set $\frac{d\sigma}{dx} = 0$

$$\frac{d\sigma}{dx} = \frac{3w}{b}\frac{d}{dx}\left\{\frac{Lx - x^2}{(a + kx)^2}\right\} = \frac{3w}{b}\left\{\frac{(a+kx)^2(L-2x) - (Lx - x^2)2(a+kx)k}{(a+kx)^4}\right\}$$

$$= \frac{3w}{b}\left\{\frac{(a+kx)(L-2x) - 2k(Lx - x^2)}{(a+kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL + kLx - 2ax - \cancel{2kx^2} - 2kLx + \cancel{2kx^2}}{(a+kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL - 2ax - kLx}{(a+kx)^3}\right\} = 0$$

(a) $\quad x_m = \frac{aL}{2a + kL} = \frac{(120)(1.2)}{(2)(120) + (300)(1.2)} = 0.24$ m

$h_m = a + kx_m = 120 + (300)(0.24) = 192$ mm

$S_m = \frac{1}{6} b h_m^2 = \frac{1}{6}(20)(192)^2 = 122.88 \times 10^3$ mm^3 $= 122.88 \times 10^{-6}$ m^3

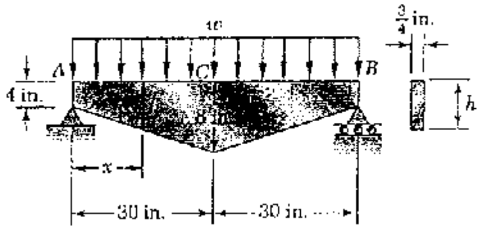
$M_m = \frac{w}{2} x_m (L - x_m) = \frac{160 \times 10^3}{2}(0.24)(0.96) = 18.432 \times 10^3$ N·m

(b) Maximum bending stress $\quad \sigma_m = \frac{M_m}{S_m} = \frac{18.432 \times 10^3}{122.88 \times 10^{-6}} = 150 \times 10^6$ Pa

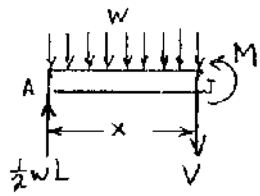
$= 150$ MPa

Problem 5.150

5.150 For the tapered beam shown, determine (*a*) the transverse section in which the maximum normal stress occurs. (*b*) the largest distributed load w that can be applied, knowing that $\sigma_{all} = 24$ ksi.



$$R_A = R_B = \tfrac{1}{2} wL \qquad L = 60 \text{ in}$$

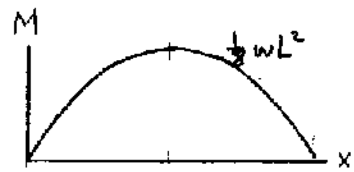


$$\circlearrowleft \Sigma M_J = 0$$

$$-\tfrac{1}{2}wLx + wx\tfrac{x}{2} + M = 0$$

$$M = \frac{w}{2}(Lx - x^2)$$

$$= \frac{w}{2}x(L-x)$$



For the tapered beam $\quad h = a + kx$

$$a = 4 \text{ in} \qquad k = \frac{8-4}{30} = \frac{2}{15} \text{ in/in.}$$

For a rectangular cross section $\quad S = \frac{1}{6}bh^2 = \frac{1}{6}b(a+kx)^2$

Bending stress $\quad \sigma = \dfrac{M}{S} = \dfrac{3w}{b} \cdot \dfrac{Lx - x^2}{(a+kx)^2}$

To find location of maximum bending stress set $\dfrac{d\sigma}{dx} = 0$

$$\frac{d\sigma}{dx} = \frac{3w}{b}\frac{d}{dx}\left\{\frac{Lx - x^2}{(a+kx)^2}\right\} = \frac{3w}{b}\left\{\frac{(a+kx)^2(L-2x) - (Lx-x^2)2(a+kx)k}{(a+kx)^4}\right\}$$

$$= \frac{3w}{b}\left\{\frac{(a+kx)(L-2x) - 2k(Lx-x^2)}{(a+kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL + kLx - 2ax - 2kx^2 - 2kLx + 2kx^2}{(a+kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL - (2a+kL)x}{(a+kx)^3}\right\} = 0$$

(a) $\quad x_m = \dfrac{aL}{2a + kL} = \dfrac{(4)(60)}{(2)(4) + (\frac{2}{15})(60)} = 15$ in. ◄

$$h_m = a + kx_m = 4 + (\tfrac{2}{15})(15) = 6.00 \text{ in.}$$

$$S_m = \tfrac{1}{6}bh_m^3 = (\tfrac{1}{6})(\tfrac{3}{4})(6.00)^2 = 4.50 \text{ in}^3$$

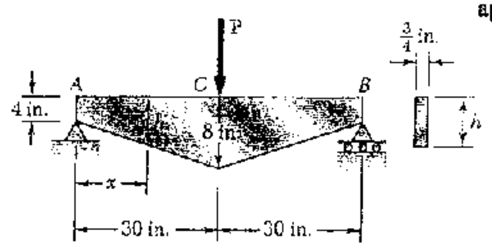
Allowable value of $M_m = S_m\sigma_{all} = (4.50)(24) = 180.0$ kip·in

(b) Allowable value of $w \qquad w = \dfrac{2M_m}{x_m(L-x_m)} = \dfrac{(2)(108.0)}{(15)(45)} = 0.320$ kip/in

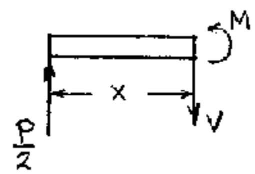
$$= 320 \text{ lb/in} \quad ◄$$

Problem 5.151

5.151 For the tapered beam shown, determine (*a*) the transverse section in which the maximum normal stress occurs, (*b*) the largest concentrated load **P** that can be applied, knowing that $\sigma_{all} = 24$ ksi.



$$R_A = R_B = \frac{P}{2}$$



$$\circlearrowleft \Sigma M_J = 0$$

$$-\frac{Px}{2} + M = 0$$

$$M = \frac{Px}{2} \qquad (0 < x < \tfrac{L}{2})$$

For a tapered beam $\qquad h = a + kx$

For a rectangular cross section $\qquad S = \frac{1}{6} b h^2 = \frac{1}{6} b (a + kx)^2$

Bending stress $\qquad \sigma = \frac{M}{S} = \frac{3Px}{b(a+kx)^2}$

To find location of maximum bending stress set $\frac{d\sigma}{dx} = 0$

$$\frac{d\sigma}{dx} = \frac{3P}{b}\frac{d}{dx}\left\{\frac{x}{(a+kx)^2}\right\} = \frac{3P}{b}\,\frac{(a+kx)^2 - x\cdot 2(a+kx)k}{(a+kx)^4}$$

$$= \frac{3P}{b}\,\frac{a-kx}{(a+kx)^3} = 0 \qquad\qquad x_m = \frac{a}{k}$$

Data: $\quad a = 4$ in., $\quad k = \frac{8-4}{30} = 0.13333$ in/in

(a) $\qquad x_m = \frac{4}{0.13333} = 30$ in.

$$h_m = a + kx_m = 8 \text{ in}$$

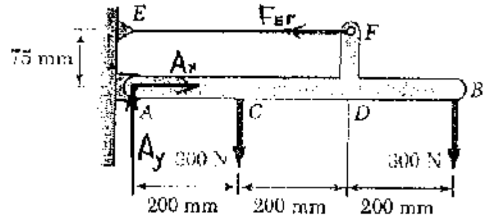
$$S_m = \frac{1}{6} b h_m^2 = \left(\frac{1}{6}\right)\left(\frac{3}{4}\right)(8)^2 = 8 \text{ in}^3$$

$$M_m = \sigma_{all} S_m = (24)(8) = 192 \text{ kip}\cdot\text{in}$$

(b) $\qquad P = \frac{2M_m}{x_m} = \frac{(2)(192)}{30} = 12.8$ kips

Problem 5.152

5.152 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.



$+\circlearrowleft \Sigma M_A = 0$

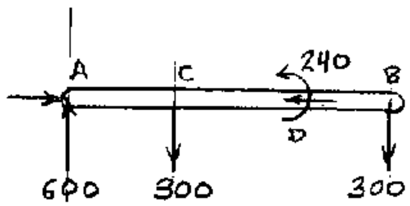
$0.075\ F_{EF} - (0.2)(300) - (0.6)(300) = 0$

$F_{EF} = 3.2 \times 10^3\ N$

$\xrightarrow{+} \Sigma F_x = 0 \qquad A_x - F_{EF} = 0 \qquad A_x = 3.2 \times 10^3\ N$

$+\uparrow \Sigma F_y = 0 \qquad A_y - 300 - 300 = 0$

$A_y = 600\ N$



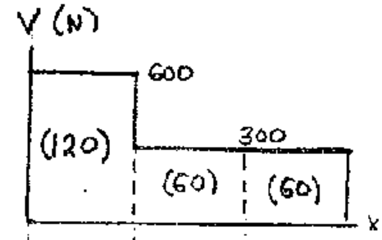
Couple at D $\quad M_D = (0.075)(3.2 \times 10^3)$

$= 240\ N \cdot m$

Shear

A to C $\quad V = 600\ N$

C to B $\quad V = 600 - 300 = 300\ N$

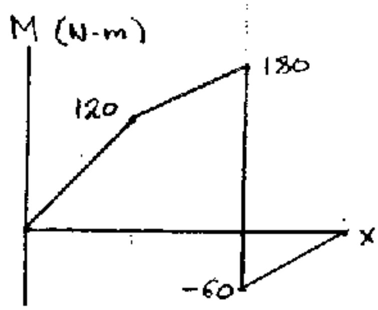


Areas under shear diagram

A to C $\quad \int V dx = (0.2)(600) = 120\ N \cdot m$

C to D $\quad \int V dx = (0.2)(300) = 60\ N \cdot m$

D to B $\quad \int V dx = (0.2)(300) = 60\ N \cdot m$



Bending moments

$M_A = 0$

$M_C = 0 + 120 = 120\ N \cdot m$

$M_D^- = 120 + 60 = 180\ N \cdot m$

$M_D^+ = 180 - 240 = -60\ N \cdot m$

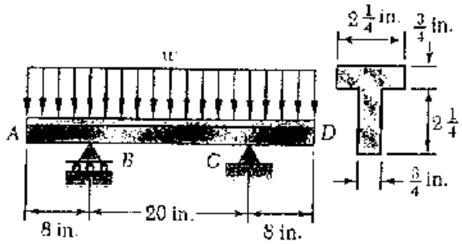
$M_B = -60 + 60 = 0$

(a) $\qquad$ Maximum $|V| = 600\ N$

(b) $\qquad$ Maximum $|M| = 180\ N \cdot m$

Problem 5.153

5.153 Determine the largest permissible distributed load w for the beam shown, knowing that the allowable normal stress is +12 ksi in tension and −29.5 ksi in compression.



Reactions: $B + C - 36\,w = 0$ $\quad B = C = 18\,w$

Shear: $V_A = 0$

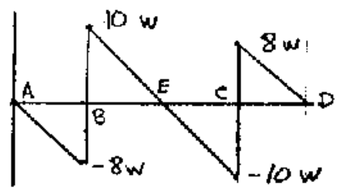
$V_{B^-} = 0 - 8w = -8w$

$V_{B^+} = -8w + 18\,w = 10\,w$

$V_{C^-} = 10\,w - 20\,w = -10\,w$

$V_{C^+} = -10\,w + 18\,w = 8w$

$V_D = 8w - 8w = 0$



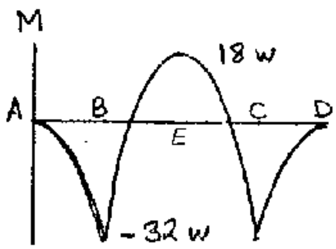
Areas: A to B $(\frac{1}{2})(8)(-8w) = -32\,w$

B to E $(\frac{1}{2})(10)(10\,w) = 50\,w$

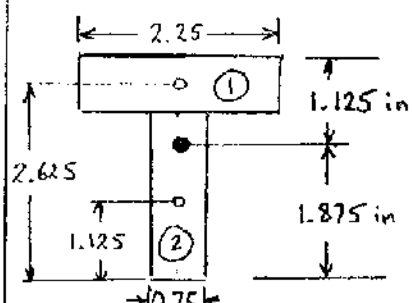
Bending moments: $M_A = 0$

$M_B = 0 - 32\,w = -32\,w$

$M_E = -32\,w + 50\,w = 18\,w$



Centroid and moment of inertia



Part	A (in²)	$\bar{y}$ (in)	$A\bar{y}$ (in³)	d in²	Ad^2 (in⁴)	$\bar{I}$ (in⁴)
①	1.6875	2.625	4.4297	0.75	0.9492	0.0791
②	1.6875	1.125	1.8984	0.75	0.9492	0.7119
Σ	3.375		6.3281		1.8984	0.7910

$\bar{Y} = \frac{6.3281}{3.375} = 1.875$ in

$I = \Sigma A d^2 + \Sigma \bar{I} = 2.6894$ in⁴

Top: $y = 1.125$ $\quad I/y = 2.3906$ in³

Bottom: $y = -1.875$ $\quad I/y = -1.4343$ in³

Bending moment limits $\quad M = -\sigma I/y$

Tension at B and C $\quad -(12)(2.3906) = \quad -28.687$ kip·in

Comp. at B and C $\quad -(-19.5)(-1.4343) = \quad -27.969$ kip·in

Tension at E $\quad -(12)(-1.4343) = \quad 17.212$ kip·in

Compression at E $\quad -(-19.5)(2.3906) = \quad 46.6$ kip·in

Allowable load w

B & C $\quad -32\,w = -27.969 \quad w = 0.874$ kip/in

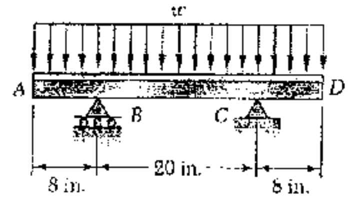
E $\quad 18\,w = 17.212 \quad w = 0.956$ kip/in

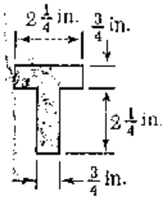
Smallest $\quad w = 0.874$ kip/in $= 10.49$ kip/ft.

Problem 5.154

5.154 Solve Prob. 5.153, assuming that the cross section of the beam is reversed, with the flange of the beam resting on the supports at B and C.

5.153 Determine the largest permissible distributed load w for the beam shown, knowing that the allowable normal stress is +12 ksi in tension and −29.5 ksi in compression.





Reactions $B + C - 36w = 0$ $\quad B = C = 18w$

Shear: $V_A = 0$

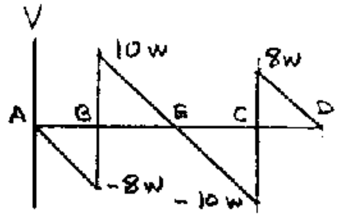
$V_B^- = 0 - 8w = -8w$

$V_B^+ = -8w + 18w = 10w$

$V_C^- = 10w - 20w = -10w$

$V_C^+ = -10w + 18w = 8w$

$V_D = 8w - 8w = 0$



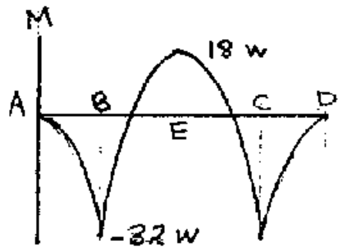
Areas: A to B $\quad (\frac{1}{2})(8)(-8w) = -32w$

B to E $\quad (\frac{1}{2})(10)(10w) = 50w$

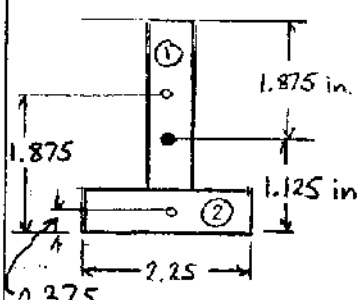
Bending moments: $M_A = 0$

$M_B = 0 - 32w = -32w$

$M_E = -32w + 50w = 18w$



Centroid and moment of inertia



Part	A (in²)	$\bar{y}$ (in)	$A\bar{y}$ (in³)	d (in)	Ad^2 (in⁴)	$\bar{I}$ (in⁴)
①	1.6875	1.875	3.1641	0.75	0.9492	0.7119
②	1.6875	0.375	0.6328	0.75	0.9492	0.0791
Σ	3.375		3.7969		1.8984	0.7910

$\bar{Y} = \frac{3.7969}{3.375} = 1.125$ in.

$I = \Sigma Ad^2 + \Sigma \bar{I} = 2.6894$ in⁴

Top: $y = 1.875$ in $\qquad I/y = 1.4343$ in³

Bottom $y = -1.125 \qquad -I/y = -2.3906$ in³

Bending moment limits $\qquad M = -\sigma I/y$

Tension at B and C $\quad -(12)(1.4343) = -17.212$ kip·in ←

Comp. at B and C $\quad -(-19.5)(-2.3906) = -46.6$ kip·in

Tension at E $\quad -(12)(-2.3906) = 28.687$ kip·in

Compression at E $\quad -(-19.5)(1.4343) = 27.969$ kip·in ←

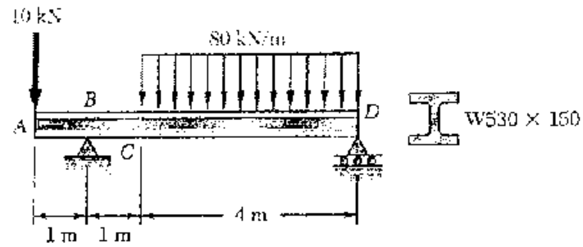
Allowable load w: $\quad$ B & C $\quad -32w = -17.212 \quad w = 0.539$ kip/in

E $\quad 18w = 27.969 \quad w = 1.554$ kip/in

Smallest $\quad w = 0.539$ kip/in $= 6.45$ kip/ft ←

Problem 5.155

5.155 (*a*) Using singularity functions, find the magnitude and location of the maximum bending moment for the beam and loading shown. (*b*) Determine the maximum normal stress due to bending.



$+\circlearrowleft M_D = 0$

$(6)(10) - 5R_B + (2)(4)(80) = 0$

$R_B = 140 \text{ kN}$

$w = 80\langle x-2\rangle^0 \text{ kN/m} = -dV/dx$

$V = -10 + 140\langle x-1\rangle^0 - 80\langle x-2\rangle^1 \quad \text{kN}$

A to B $\quad V = -10 \text{ kN}$

B to C $\quad V = -10 + 140 = 130 \text{ kN}$

D $\quad (x = 6) \quad V = -10 + 140 - 80(4) = -190 \text{ kN}$

V changes sign at B and at point E $(x = x_E)$ between C and D.

$V = 0 = -10 + 140\langle x_E - 1\rangle^0 - 80\langle x_E - 2\rangle^1$

$= -10 + 140 - 80(x_E - 2) \qquad x_E = 3.625 \text{ m}$

$M = -10x + 140\langle x-1\rangle^1 - 40\langle x-2\rangle^2 \quad \text{kN}\cdot\text{m}$

At pt. B $\quad x = 1 \qquad M_B = -(10)(1) = -10 \text{ kN}\cdot\text{m}$

At pt. E $\quad x = 3.625$

$M_E = -(10)(3.625) + (140)(2.625) - (40)(1.625)^2 = 225.6 \text{ kN}\cdot\text{m}$

(a) $|M|_{max} = 225.6 \text{ kN}\cdot\text{m} \quad$ at $\quad x = 3.625 \text{ m}$

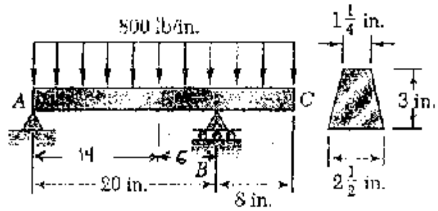
For W 530 × 150 $\qquad S = 3720 \times 10^3 \text{ mm}^3 = 3720 \times 10^{-6} \text{ m}^3$

(b) Normal stress $\qquad \sigma = \dfrac{|M|}{S} = \dfrac{225.6 \times 10^3}{3720 \times 10^{-6}} = 60.6 \times 10^6 \text{ Pa}$

$= 60.6 \text{ MPa}$

Problem 5.156

5.156 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

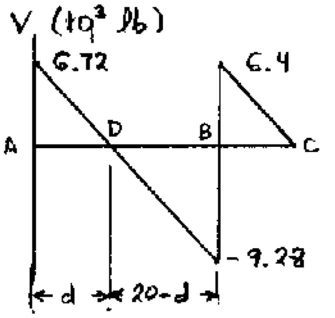


$\circlearrowleft \Sigma M_B = 0 \qquad -20A + (6)(28)(800) = 0$

$A = 6.72 \times 10^3$ lb.

$\circlearrowleft \Sigma M_A = 0 \qquad 20B - (14)(28)(800) = 0$

$B = 15.68 \times 10^3$ lb.



Shear: $V_A = 6.72 \times 10^3$ lb.

B^- $\quad V_{B^-} = 6.72\times10^3 - (20)(800) = -9.28\times10^3$ lb.

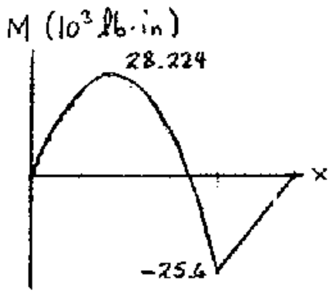
B^+ $\quad V_{B^+} = -9.28\times10^3 + (15.68\times10^3) = 6.4\times10^3$ lb

C $\quad V_C = 6.4\times10^3 - (8)(800) = 0$

Locate point D where V = 0

$\frac{d}{6.72} = \frac{20-d}{9.28} \qquad 16\,d = 134.4$

$d = 8.4$ in $\qquad 20 - d = 11.6$ in.



Areas under shear diagram

A to D $\quad \int V dx = (\frac{1}{2})(8.4)(6.72\times10^3) = 28.224\times10^3$ lb·in

D to B $\quad \int V dx = (\frac{1}{2})(11.6)(-9.28\times10^3) = -53.824\times10^3$ lb·in

B to C $\quad \int V dx = (\frac{1}{2})(8)(6.4\times10^3) = 25.6\times10^3$ lb·in

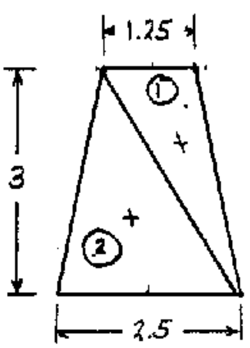
Bending moments: $\quad M_A = 0$

$M_D = 0 + 28.224\times10^3 = 28.224\times10^3$ lb·in

$M_B = 28.224\times10^3 - 53.824\times10^3 = -25.6\times10^3$ lb·in

$M_C = -25.6\times10^3 + 25.6\times10^3 = 0$

Maximum $|M| = 28.224\times10^3$ lb·in



Locate centroid of cross section

$\bar{Y} = \frac{7.5}{5.625} = 1.3333$ in. from bottom

For each triangle $\bar{I} = \frac{1}{36} b h^3$

Part	A, in²	$\bar{y}$, in	$A\bar{y}$, in³	d, in	Ad^2, in⁴	$\bar{I}$ in⁴
1	1.875	2	3.75	0.6667	0.8333	0.9375
2	3.75	1	3.75	0.3333	0.4167	1.875
Σ	5.625		7.5		1.25	2.8125

Moment of inertia

$I = \Sigma \bar{I} + \Sigma A d^2$

$= 1.25 + 2.8125 = 4.0625$ in⁴

Normal stress

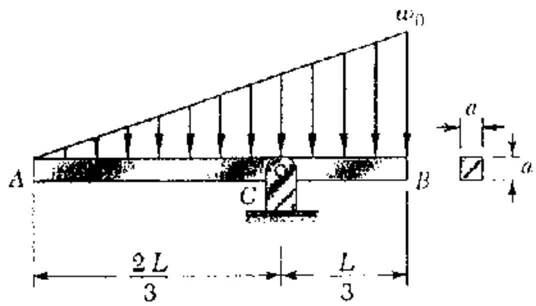
$\sigma = \frac{Mc}{I} = \frac{(28.224\times10^3)(1.6667)}{4.0625}$

$= 11.58\times10^3$ psi

$= 11.58$ ksi

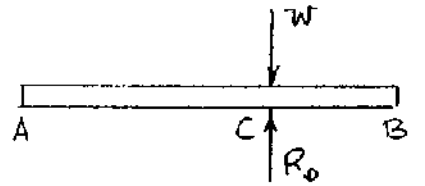
Problem 5.157

5.157 Beam AB, of length L and square cross section of side a, is supported by a pivot at C and loaded as shown. (a) Check that the beam is in equilibrium. (b) Show that the maximum stress due to bending occurs at C and is equal to $w_0L^2/(1.5a)^3$.



Replace distributed load by equivalent concentrated load at the centroid of the area of the load diagram.

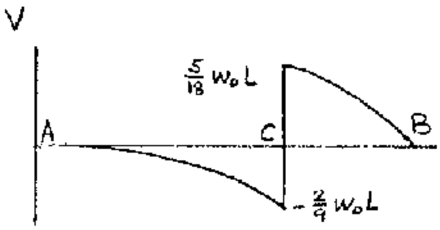
For the triangular distribution the centroid lies at $x = \frac{2L}{3}$.



$$W = \frac{1}{2} w_0 L$$

$$\Sigma F_y = 0 \qquad R_D - W = 0 \qquad R_D = \frac{1}{2} w_0 L$$

$$\Sigma M_C = 0 \qquad 0 = 0 \qquad \text{equilibrium}$$



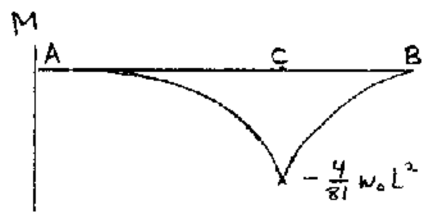
$$V = 0, \quad M = 0 \quad \text{at } x = 0$$

$0 < x < \frac{2L}{3}$

$$\frac{dV}{dx} = -w = -\frac{w_0 x}{L}$$

$$\frac{dM}{dx} = V = -\frac{w_0 x^2}{2L} + C_1 = -\frac{w_0 x^2}{2L}$$

$$M = -\frac{w_0 x^3}{6L} + C_2 = -\frac{w_0 x^3}{6L}$$



Just to the left of C

$$V = -\frac{w_0 (2L/3)^2}{2L} = -\frac{2}{9} w_0 L$$

Just to the right of C

$$V = -\frac{2}{9} w_0 L + R_D = \frac{5}{18} w_0 L$$

Note sign change. Maximum $|M|$ occurs at C.

$$M_C = -\frac{w_0 (2L/3)^3}{6L} = -\frac{4}{81} w_0 L^2$$

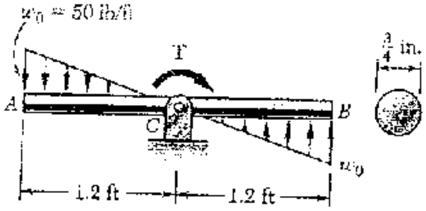
Maximum $|M| = \frac{4}{81} w_0 L^2$

For square cross section $\qquad I = \frac{1}{12} a^4 \qquad c = \frac{1}{2} a$

$$\sigma_m = \frac{|M|_{max} c}{I} = \frac{4}{81} \frac{w_0 L^2 \, 6}{a^3} = \frac{8}{27} \frac{w_0 L^2}{a^3} = \left(\frac{2}{3}\right)^3 \frac{w_0 L^2}{a^3} = \frac{w_0 L^2}{(1.5 a)^3}$$

Problem 5.158

5.158 Knowing that rod AB is in equilibrium under the loading shown, draw the shear and bending-moment diagrams and determine the maximum normal stress due to bending.



A to C $\qquad 0 < x < 1.2$ ft

$$w = 50\left(1 - \frac{x}{1.2}\right) = 50 - 41.667x$$

$$\frac{dV}{dx} = -w = 41.667x - 50$$

$$V = V_A + \int_0^x (41.667x - 50)\,dx$$

$$= 0 + 20.833x^2 - 50x = \frac{dM}{dx}$$

$$M = M_A + \int_0^x V\,dx$$

$$= 0 + \int_0^x (20.833x^2 - 50x)\,dx$$

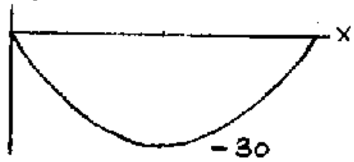
$$= 6.944x^3 - 25x^2$$

At $x = 1.2$ ft, $\quad V = -30$ lb.
$\qquad\qquad\qquad\quad M = -24$ lb·in.

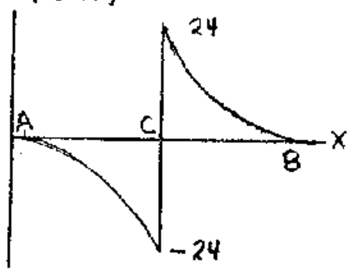
C to B $\qquad$ Use symmetry conditions.

Maximum $|M| = 24$ lb·ft $= 288$ lb·in.

V (lb.)



M (lb·ft)



Cross section $\qquad c = \frac{d}{2} = \left(\frac{1}{2}\right)(0.75) = 0.375$ in

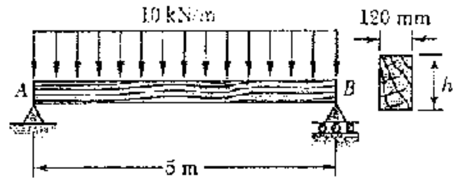
$$I = \frac{\pi}{4}c^4 = \left(\frac{\pi}{4}\right)(0.375) = 15.532 \times 10^{-3} \text{ in}^4$$

Normal stress $\qquad \sigma = \frac{|M|c}{I} = \frac{(288)(0.375)}{15.532 \times 10^{-3}} = 6.95 \times 10^3$ psi

$$= 6.95 \text{ ksi}$$

Problem 5.159

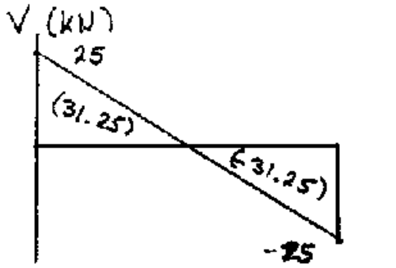
5.159 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.



Reactions: $A = B$ by symmetry

$+\uparrow \Sigma F_y = 0 \qquad A + B - (5)(10) = 0$

$A = B = 25$ kN



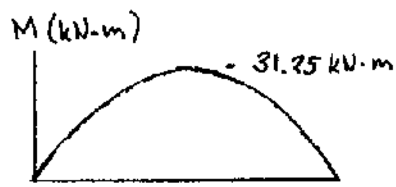
From bending moment diagram

$|M|_{max} = 31.25$ kN·m $= 31.25 \times 10^3$ N·m

$\sigma_{all} = 12$ MPa $= 12 \times 10^6$ Pa

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{31.25 \times 10^3}{12 \times 10^6} = 2.604 \times 10^{-3}\ m^3$$

$$= 2.604 \times 10^6\ mm^3$$



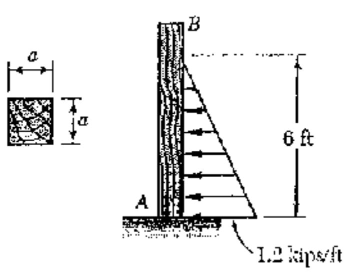
$$S = \frac{1}{6} b h^2 = \left(\frac{1}{6}\right)(120) h^2 = 2.604 \times 10^6$$

$$h^2 = \frac{(6)(2.064 \times 10^6)}{120} = 130.21 \times 10^3\ mm^2$$

$h = 361$ mm ◄

Problem 5.160

5.160 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 1750 psi.

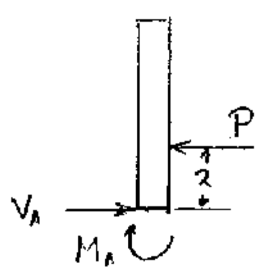


Equivalent concentrated load

$P = \left(\frac{1}{2}\right)(6)(1.2) = 3.6$ kips

Bending moment at A

$M_A = (2)(3.6) = 7.2$ kip·ft $= 86.4$ kip·in



$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{86.4}{1.75} = 49.37\ in^3$$

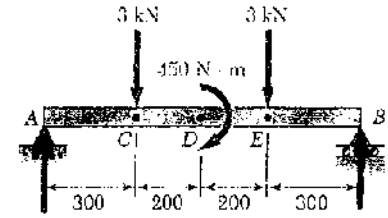
For a square section $\quad S = \frac{1}{6} a^3$

$$a = \sqrt[3]{6S}$$

$$a_{min} = \sqrt[3]{(6)(49.37)} = 6.67\ in.$$ ◄

Problem 5.161

5.161 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

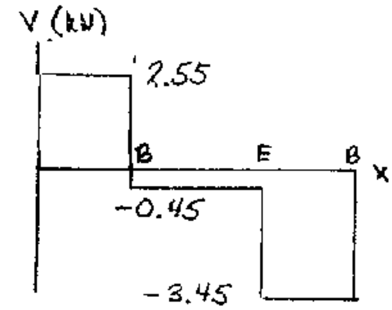


$\circlearrowleft \Sigma M_B = 0 \qquad (700)(3) - 450 + (300)(3) - 1000A = 0$

$A = 2.55 \text{ kN} \uparrow$

$\circlearrowleft \Sigma M_A = 0 \qquad -(300)(3) - 450 - (700)(3) + 1000B = 0$

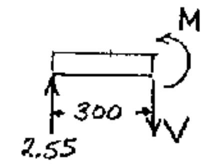
$B = 3.45 \text{ kN} \uparrow$



At A $\quad V = 2.55$ kN $\quad M = 0$

A to C $\quad V = 2.55$ kN

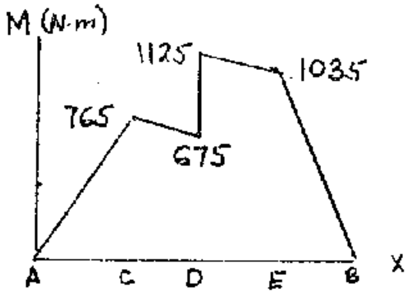
At C



$\circlearrowleft \Sigma M_C = 0$

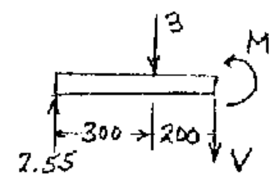
$-(300)(2.55) + M = 0$

$M = 765$ N·m



C to E $\quad V = -0.45$ N·m

At D^-

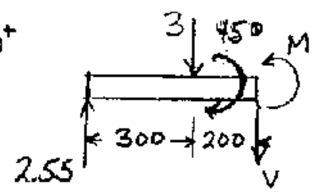


$\circlearrowleft \Sigma M_D = 0$

$-(500)(2.55) + (200)(3) + M = 0$

$M = 675$ N·m

At D^+



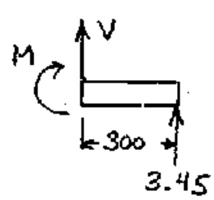
$\circlearrowleft \Sigma M_D = 0$

$-(500)(2.55) + (200)(3) - 450 + M = 0$

$M = 1125$ N·m

E to B $\quad V = -3.45$ kN

At E



$\circlearrowleft \Sigma M_E = 0$

$-M + (300)(3.45) = 0$

$M = 1035$ N·m.

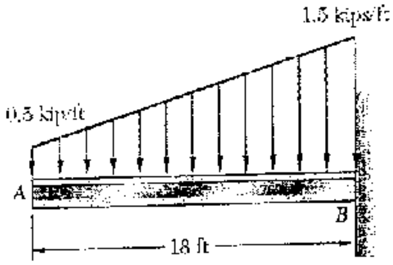
At B $\quad V = 3.45$ kN, $\quad M = 0$

(a) $\quad$ Maximum $|V| = 3.45$ kN ◄

(b) $\quad$ Maximum $|M| = 1125$ N·m ◄

Problem 5.162

5.162 Knowing that the allowable normal stress for the steel used is 24 ksi, select the most economical wide-flange beam to support the loading shown.



$$w = 0.5 + \frac{(1.5-0.5)x}{18} = 0.5 + 0.0555\ x$$

$$\frac{dV}{dx} = -w = -0.5 - 0.05556\ x$$

$$V = 0 - 0.5x - 0.02778\ x^2 = \frac{dM}{dx}$$

$$M = 0 - 0.25x^2 - 0.009259\ x^3$$

Maximum $|M|$ occurs at $x = 18$ ft

$$|M|_{max} = (0.25)(18)^2 + (0.009259)(18)^3 = 135 \text{ kip-ft} = 1620 \text{ kip}\cdot\text{in}$$

$\sigma_{all} = 24$ ksi $\qquad S_{min} = \dfrac{|M|_{max}}{\sigma_{all}} = \dfrac{1620}{24} = 67.5\ in^3$

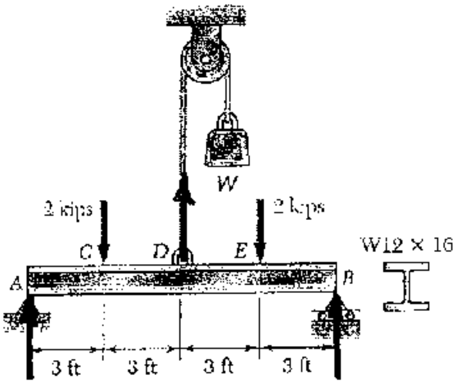
Shape	S (in³)
W21 × 44	81.6
W18 × 50	88.9 ←
W16 × 57	92.2
W14 × 53	77.8
W12 × 72	97.4
W10 × 68	75.7

Lightest wide flange beam

W 18 × 50 @ 50 lb/ft

Problem 5.163

5.163 Determine (*a*) the magnitude of the counterweight W for which the maximum value of the bending moment in the beam is a small as possible, (*b*) the corresponding maximum stress due to bending. (See hint of Prob. 5.27.)

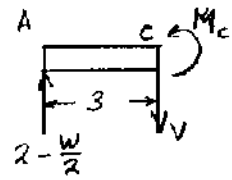


By symmetry $A = B$

$+\uparrow \Sigma F_y = 0 \qquad A - 2 + W - 2 + B = 0$

$A = B = 2 - \frac{W}{2}$

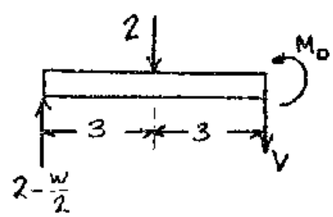
Bending moment at C $\quad \circlearrowleft \Sigma M_C = 0$



$-(3)(2 - \frac{W}{2}) + M_C = 0$

$M_C = 6 - 1.5\,W$

Bending moment at D $\quad \circlearrowleft \Sigma M_D = 0$



$-(6)(2 - \frac{W}{2}) + (3)(2) + M_D = 0$

$M_D = 6 - 3W$

(a) Equate $\quad -M_D = M_C$

$3W - 6 = 6 - 1.5W \qquad W = 2.667$ kips

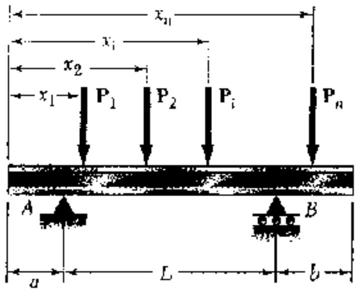
$M_C = 2.0$ kip ft $\qquad M_D = -2.0$ kip ft

max $|M| = 2.0$ kip·ft $= 24$ kip·in

For W 12×16 rolled steel section $\quad S = 17.1\ \text{in}^3$

(b) Normal stress $\quad \sigma = \frac{|M|}{S} = \frac{24}{17.1} = 1.404$ ksi

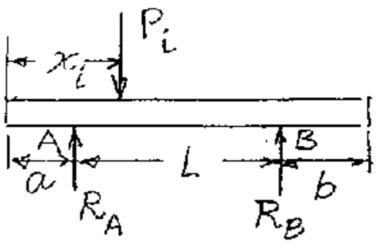
PROBLEM 5.C1



5.C1 Several concentrated loads P_i ($i = 1, 2, \ldots, n$) can be applied to a beam as shown. Write a computer program that can be used to calculate the shear, bending moment, and normal stress at any point of the beam for a given loading of the beam and a given value of its section modulus. Use this program to solve Probs. 5.18, 5.21, and 5.25. (*Hint:* Maximum values will occur at a support or under a load.

SOLUTION

REACTIONS AT A AND B



$$+\circlearrowleft \sum M_A = 0: \quad R_B L - \sum_i P_i (x_i - a)$$

$$R_B = (1/L) \sum_i P_i (x_i - a)$$

$$R_A = \sum_i P_i - R_B$$

WE USE STEP FUNCTIONS (See bottom of page 348 of text.)

WE DEFINE: IF $x \geq a$ THEN STPA=1 ELSE STPA = 0

IF $x \geq a+L$ THEN STPB=1 ELSE STPB = 0

IF $x \geq x_i$ THEN STP(I)=1 ELSE STP(I) = 0

$$V = R_A\,STPA + R_B\,STPB - \sum_i P_i\,STP(I)$$

$$M = R_A (x-a) STPA + R_B (x-a-L) STPB - \sum_i P_i (x - x_i) STP(I)$$

$\sigma = M/S$, where S is obtained from Appendix C.

PROGRAM OUTPUTS

Problem 5.18

RA=80.0 kN RB=80.0 kN

X m	V kN	M kN.m	Sigma MPa
2.00	0.00	104.00	139.0

Problem 5.25

R1 = 10.77 kips R2 = 4.23 kips

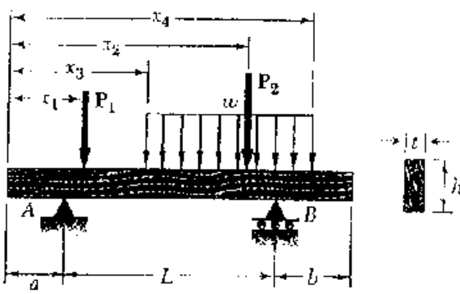
X ft	V kips	M kip in.	Sigma ksi
0	-5.00	0	0
5	5.69	-300.00	-10.34
13	-4.23	253.8	8.75
18	-4.23	0	0

Problem 5.21

R1 = 52.5 kips R2 = 22.5 kips

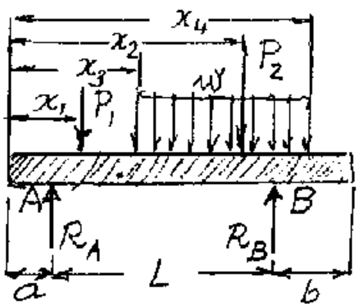
X ft	V kips	M kip.ft	Sigma ksi
0.00	-25.00	0.00	0.00
1.00	27.50	-25.00	-7.85
3.00	2.50	30.00	9.42
9.00	-22.50	45.00	14.14
11.00	0.00	0.00	0.00

PROBLEM 5.C2



5.C2 A timber beam is to be designed to support a distributed load and up to two concentrated loads as shown. One of the dimensions of its uniform rectangular cross section has been specified and the other is to be determined so that the maximum normal stress in the beam will not exceed a given allowable value σ_{all}. Write a computer program that can be used to calculate at given intervals ΔL the shear, the bending moment, and the smallest acceptable value of the unknown dimension. Apply this program to solve the following problems, using the intervals ΔL indicated: (*a*) Prob. 5.65 ($\Delta L = 0.1$ m), (*b*) Prob. 5.69 ($\Delta L = 0.2$ m), (*c*) Prob. 5.80 ($\Delta L = 0.3$ m).

SOLUTION



REACTIONS AT A AND B

$$+\circlearrowleft \Sigma M_A = 0: \; R_B L - P_1(x_1 - a) - P_2(x_2 - a) - w(x_4 - x_3)\left(\frac{x_4 + x_3}{2} - a\right) = 0$$

$$R_B = \frac{1}{L}\left[P_1(x_1 - a) + P_2(x_2 - a) + \frac{1}{2}w(x_4 - x_3)(x_4 + x_3 - 2a)\right]$$

$$R_A = P_1 + P_2 + w(x_4 - x_3) - R_B$$

WE USE STEP FUNCTIONS (See bottom of page 348 of text)

SET $n = (a + b + L)/\Delta L$

FOR $i = 0$ TO n: $x = (\Delta L)i$

WE DEFINE: IF $x \geq a$ THEN STPA = 1 ELSE STPA = 0

IF $x \geq a + L$, THEN STPB = 1 ELSE, STB = 0

IF $x \geq x_1$ THEN STP1 = 1 ELSE STP1 = 0

IF $x \geq x_2$ THEN STP2 = 1 ELSE STP2 = 0

IF $x \geq x_3$ THEN STP3 = 1 ELSE STP3 = 0

IF $x \geq x_4$ THEN STP4 = 1 ELSE STP4 = 0

$$V = R_A\,\text{STPA} + R_B\,\text{STPB} - P_1\,\text{STP1} - P_2\,\text{STP2} - w(x - x_3)\,\text{STP3} + w(x - x_4)\,\text{STP4}$$

$$M = R_A(x - a)\,\text{STPA} + R_B(x - a - L)\,\text{STPB} - P_1(x - x_1)\,\text{STP1} - P_2(x - x_2)\,\text{STP2} - \frac{1}{2}w(x - x_3)^2\,\text{STP3} + \frac{1}{2}w(x - x_4)^2\,\text{STP4}$$

$$S_{min} = |M|/\sigma_{all}$$

IF UNKNOWN DIMENSION IS h:

From $S = \frac{1}{6}th^2$, we have $h = \sqrt{6S/t}$

IF UNKNOWN DIMENSION IS t:

From $S = \frac{1}{6}th^2$, we have $t = 6S/h^2$

(CONTINUED)

PROBLEM 5.C2 CONTINUED

PROGRAM OUTPUTS

Problem 5.65

RA = 2.40 kN RB = 3.00 kN

X m	V kN	M kN.m	H mm
0.00	2.40	0.000	0.00
0.10	2.40	0.240	54.77
0.20	2.40	0.480	77.46
0.30	2.40	0.720	94.87
0.40	2.40	0.960	109.54
0.50	2.40	1.200	122.47
0.60	2.40	1.440	134.16
0.70	2.40	1.680	144.91
0.80	0.60	1.920	154.92
0.90	0.60	1.980	157.32
1.00	0.60	2.040	159.69
1.10	0.60	2.100	162.02
1.20	0.60	2.160	164.32
1.30	0.60	2.220	166.58
1.40	0.60	2.280	168.82
1.50	0.60	2.340	171.03
1.60	-3.00	2.400	173.21
1.70	-3.00	2.100	162.02
1.80	-3.00	1.800	150.00
1.90	-3.00	1.500	136.93
2.00	-3.00	1.200	122.47
2.10	-3.00	0.900	106.07
2.20	-3.00	0.600	86.60
2.30	-3.00	0.300	61.24
2.40	0.00	0.000	0.05

Problem 5.69

RA = 2.70 kN RB = 8.10 kN

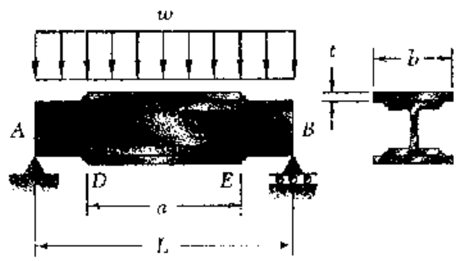
X m	V kN	M kN.m	T mm
0.00	2.70	0.000	0.00
0.20	2.10	0.480	10.67
0.40	1.50	0.840	18.67
0.60	0.90	1.080	24.00
0.80	0.30	1.200	26.67
1.00	-0.30	1.200	26.67
1.20	-0.90	1.080	24.00
1.40	-1.50	0.840	18.67
1.60	-2.10	0.480	10.67
1.80	-2.70	0.000	0.00
2.00	-3.30	-0.600	13.33
2.20	-3.90	-1.320	29.33
2.40	3.60	-2.160	48.00
2.60	3.00	-1.500	33.33
2.80	2.40	-0.960	21.33
3.00	1.80	-0.540	12.00
3.20	1.20	-0.240	5.33
3.40	0.60	-0.060	1.33
3.60	0.00	-0.000	0.00

Problem 5.70

RA = 6.50 kN RB = 6.50 kN

X m	V kN	M kN.m	H mm
0.00	2.50	0.000	0.00
0.30	2.50	0.750	61.24
0.60	9.00	1.500	86.60
0.90	7.20	3.930	140.18
1.20	5.40	5.820	170.59
1.50	3.60	7.170	189.34
1.80	1.80	7.980	199.75
2.10	-0.00	8.250	203.10
2.40	-1.80	7.980	199.75
2.70	-3.60	7.170	189.34
3.00	-5.40	5.820	170.59
3.30	-7.20	3.930	140.18
3.60	-2.50	1.500	86.60
3.90	-2.50	0.750	61.24
4.20	0.00	0.000	0.06

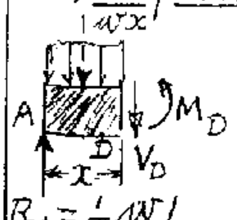
PROBLEM 5.C3



5.C3 Two cover plates, each of thickness t, are to be welded to a wide-flange beam of length L, which is to support a uniformly distributed load w. Denoting by σ_{all} the allowable normal stress in the beam and in the plates, by d the depth of the beam, and by I_b and S_b, respectively, the moment of inertia and the section modulus of the cross section of the unreinforced beam about a horizontal centroidal axis, write a computer program that can be used to calculate the required value of (a) the length a of the plates, (b) the width b of the plates. Use this program to solve Probs. 5.143 and 5.145.

SOLUTION

(a) Required Length of Plates



FB = AD: $+\circlearrowleft \Sigma M_D = 0$: $M_D + wx\left(\frac{x}{2}\right) - R_A x = 0$

But: $R_A = \frac{1}{2}wL$ and $M_D = S\sigma_{all}$ Divide by $\frac{1}{2}w$:

$$x^2 - Lx + (2S\sigma_{all}/w) = 0 \qquad \text{Set } k = \frac{2S\sigma_{all}}{w}:$$

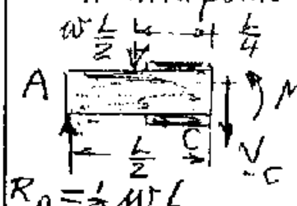
$$x^2 - Lx + k = 0$$

Solving the quadratic, $x = \dfrac{L - \sqrt{L^2 - 4k}}{2}$

Compute x and $a = L - 2x$

(b) Required Width of Plates

At midpoint C of beam:

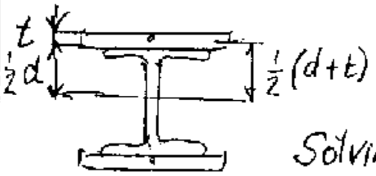


FB = AC: $+\circlearrowleft \Sigma M_C = 0$: $M_C + \frac{wL}{2}\frac{L}{4} - \frac{wL}{2}\frac{L}{2} = 0$

Compute $M_C = \frac{1}{8}wL^2$

Compute: $c = t + \frac{1}{2}d$

From $\sigma_{all} = \dfrac{M_C c}{I}$ compute $I = \dfrac{M_C c}{\sigma_{all}}$



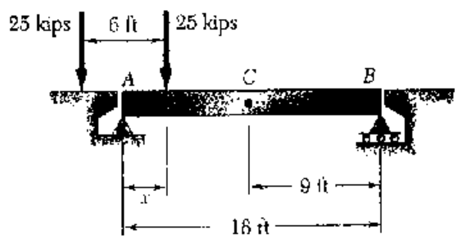
But $I = I_{beam} + I_{plates} = I_b + 2\left[\frac{1}{12}bt^3 + bt\left(\frac{d+t}{2}\right)^2\right]$

Solving for b: $b = \dfrac{6(I - I_b)}{t\left[t^2 + 3(d+t)^2\right]}$

PROGRAM OUTPUTS

PROB. 5.155: W 460×74, σ_{all} = 150 MPa w = 40 kN/m, L = 8 m, t = 7.5 mm d = 457 mm, I_b = 333×10⁶ mm⁴, S = 1460×10³ mm³	PROB. 5.157: W 30×99, σ_{all} = 22 ksi w = 30 kips/ft, L = 16 ft, t = 5/8 in. d = 29.65 in., I_b = 3990 in⁴, S = 269 in³
Problem 5.145 a = 4.49 m b = 211 mm	**Problem 5.143** a = 11.16 ft b = 14.31 in.

PROBLEM 5.C4



5.C4 Two 25-kip loads are maintained 6 ft apart as they are moved slowly across the 18-ft beam AB. Write a computer program and use it to calculate the bending moment under each load and at the midpoint C of the beam for values of x from 0 to 24 ft at intervals $\Delta x = 1.5$ ft.

SOLUTION

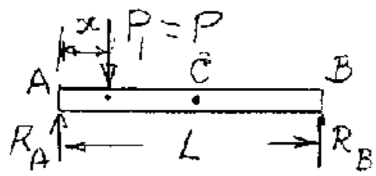
NOTATION: Length of beam $= L = 18$ ft

Loads: $P_1 = P_2 = P = 25$ kips

Distance between loads $= d = 6$ ft

We note that $d < L/2$

(1)



(1) FROM $x = 0$ TO $x = d$:

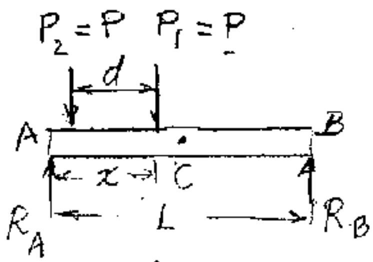
$+\circlearrowleft \Sigma M_B = 0$: $P(L-x) - R_A L = 0$.

$$R_A = P(L-x)/L$$

Under P_1: $M_1 = R_A x$

At C: $M_C = R_A\left(\frac{L}{2}\right) - P\left(\frac{L}{2} - x\right)$

(2 A)

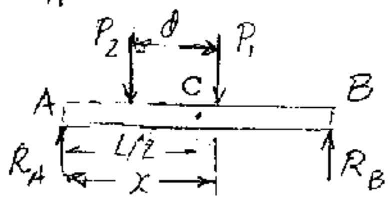


(2) FROM $x = d$ TO $x = L$:

$+\circlearrowleft \Sigma M_B = 0$: $P(L-x) + P(L-x+d) - R_A L = 0$.

$$R_A = P(2L - 2x + d)/L$$

(2 B)



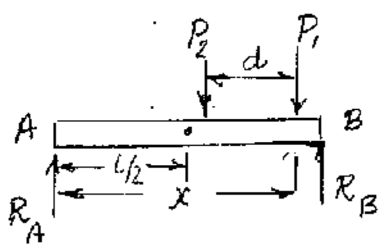
Under P_1: $M_1 = R_A x - Pd$

Under P_2: $M_2 = R_A(x-d)$

(2A) FROM $x = d$ TO $x = L/2$:

$$M_C = R_A\left(\frac{L}{2}\right) - P\left(\frac{L}{2} - x\right) - P\left(\frac{L}{2} - x + d\right)$$
$$= R_A(L/2) - P(L - 2x + d)$$

(2 C)



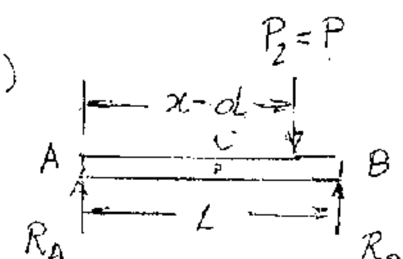
(2B) FROM $x = L/2$ TO $x = L/2 + d$:

$$M_C = R_A(L/2) - P\left(\frac{L}{2} - x + d\right)$$

(2C) FROM $x = L/2 + d$ TO $x = L$:

$$M_C = R_A L/2$$

(3)



(3) FROM $x = L$ TO $x = L + d$:

$+\circlearrowleft \Sigma M_B = 0$: $P(L - x + d) - R_A L = 0$

$$R_A = P(L - x + d)/L$$

Under P_2: $M_2 = R_A(x - d)$

At C: $M_C = R_A(L/2)$

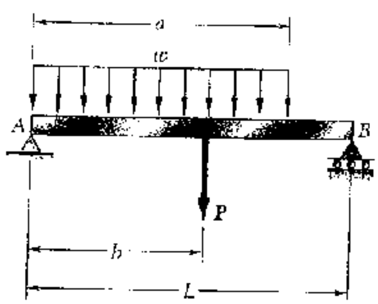
(CONTINUED)

PROGRAM OUTPUT

P = 25 kips, L = 18 ft, D = 6 ft

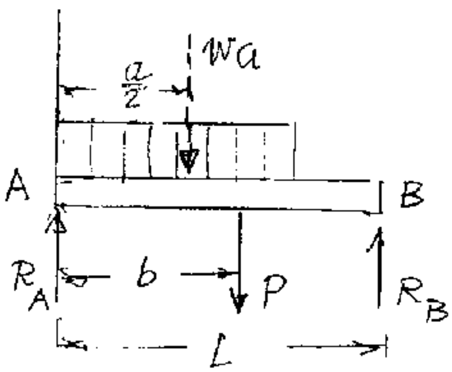
x ft	MC kip.ft	M1 kip.ft	M2 kip.ft
0.0	0.00	0.00	0.00
1.5	18.75	34.38	0.00
3.0	37.50	62.50	0.00
4.5	56.25	84.38	0.00
6.0	75.00	100.00	0.00
7.5	112.50	131.25	56.25
9.0	150.00	150.00	100.00
10.5	150.00	156.25	131.25
12.0	150.00	150.00	150.00
13.5	150.00	131.25	156.25
15.0	150.00	100.00	150.00
16.5	112.50	56.25	131.25
18.0	75.00	0.00	100.00
19.5	56.25	0.00	84.38
21.0	37.50	0.00	62.50
22.5	18.75	0.00	34.38
24.0	0.00	0.00	0.00

PROBLEM 5.C5



5.C5 Write a computer program that can be used to plot the shear and bending-moment diagrams for the beam and loading shown. Apply this program with a plotting interval $\Delta L = 0.2$ ft to the beam and loading of (*a*) Prob. 5.72, (*b*) Prob. 5.115.

SOLUTION



REACTIONS AT A AND B

USING FB DIAGRAM OF BEAM:

$+\circlearrowleft \Sigma M_A = 0: \quad R_B L - Pb - wa(a/2) = 0$

$$R_B = (1/L)(Pb + \tfrac{1}{2} wa^2)$$

$$R_A = P + wa - R_B$$

WE USE STEP FUNCTIONS (See bottom of page 348 of text).

SET $n = L/\Delta L$. FOR $i = 0$ TO n: $x = (\Delta L)\, i$

WE DEFINE: IF $x \geq a$ THEN STPA=1 ELSE STPA=0

IF $x \geq b$ THEN STPB=1 ELSE STPB=0.

$$V = R_A - wx + w(x-a)\,STPA - P\,STPB$$

$$M = R_A x - \tfrac{1}{2}wx^2 + \tfrac{1}{2}w(x-a)^2\,STPA - P(x-b)\,STPB$$

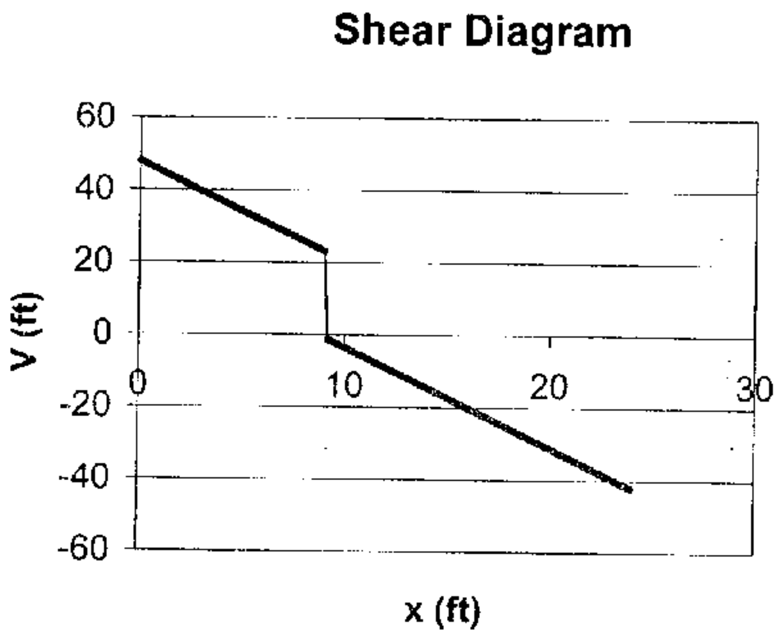
LOCATE AND PRINT (x, V) AND (x, M)

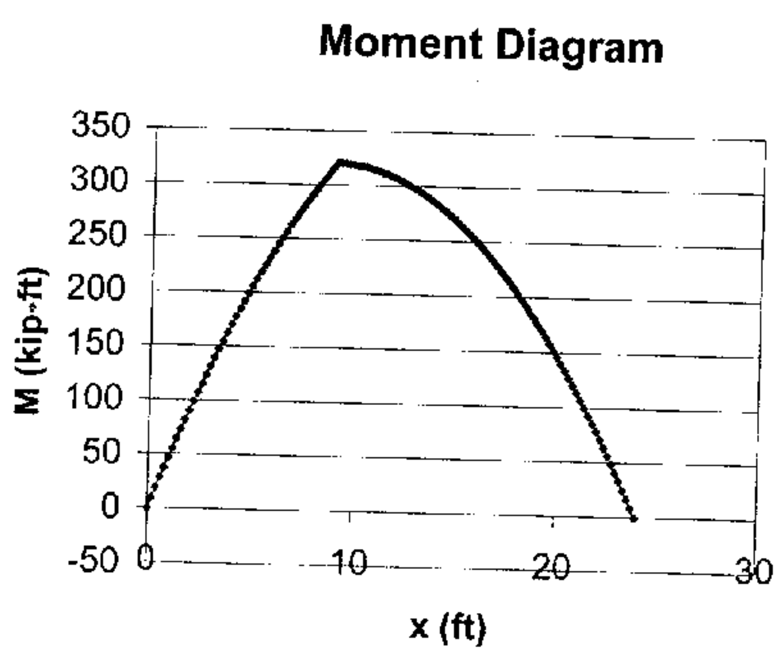
SEE NEXT PAGES FOR PROGRAM OUTPUTS

(CONTINUED)

PROGRAM OUTPUT

```
Problem 5.72

RA =    48.00 kips   RB =    42.00 kips
```





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PROGRAM OUTPUT FOR

```
Problem 5.115

RA =    40.50 kips  RB =     27.00 kips
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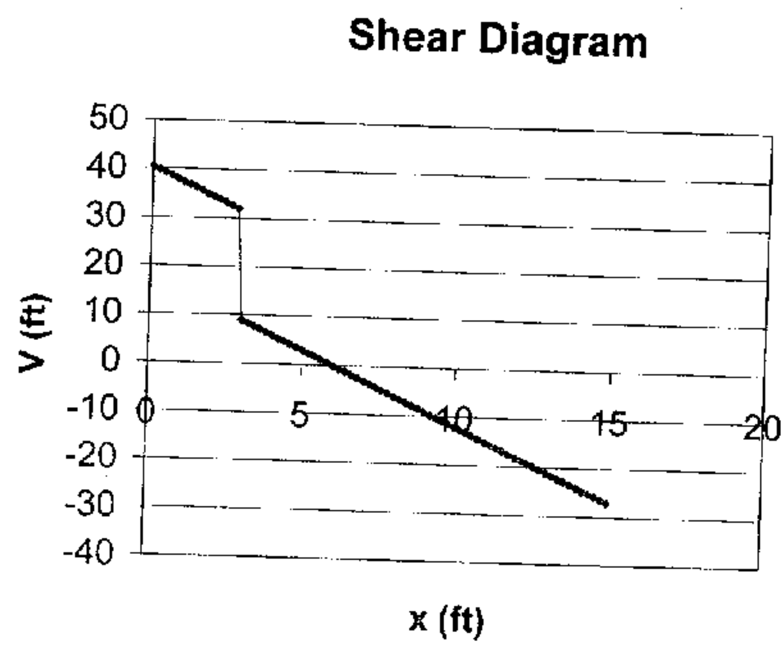
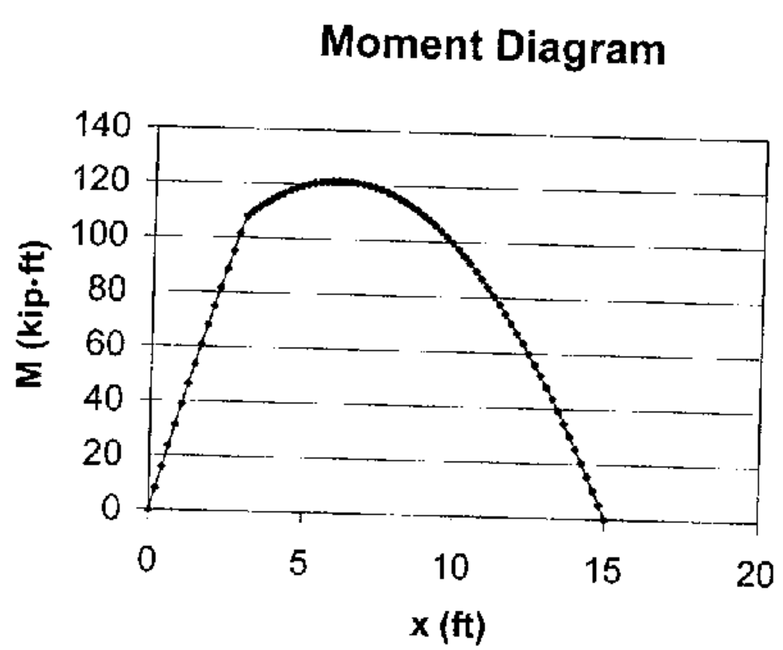
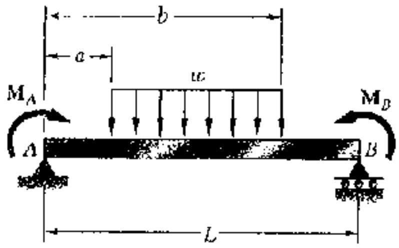




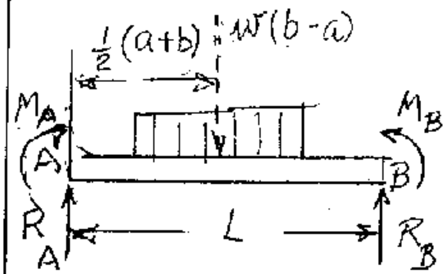
PROBLEM 5.C6



5.C6 Write a computer program that can be used to plot the shear and bending-moment diagrams for the beam and loading shown. Apply this program with a plotting interval $\Delta L = 0.025$ m to the beam and loading of Prob. 5.112

SOLUTION

REACTIONS AT A AND B



$+\circlearrowleft \sum M_A = 0:$

$R_B L + M_B - M_A - w(b-a)\frac{1}{2}(a+b) = 0$

$R_B = (1/L)\left[M_A - M_B + \frac{1}{2}w(b^2 - a^2)\right]$

$R_A = w(b-a) - R_B$

WE USE STEP FUNCTIONS (See bottom of page 348 of text)

SET $n = L/\Delta L$, FOR $i = 0$ TO n: $x = (\Delta L)i$

WE DEFINE: IF $x \geq a$ THEN STPA = 1 ELSE STPA = 0

IF $x \geq b$ THEN STPB = 1 ELSE STPB = 0

$V = R_A - w(x-a)\,\text{STPA} + w(x-b)\,\text{STPB}$

$M = M_A + R_A x - \frac{1}{2}w(x-a)^2\,\text{STPA} + \frac{1}{2}w(x-b)^2\,\text{STPB}$

LOCATE AND PRINT (x, V) AND (x, M)

PROGRAM OUTPUT ON NEXT PAGE

(CONTINUED)

PROGRAM OUTPUT

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Problem 5.112

RA =    29.50 kips   RB =     66.50 kips
```
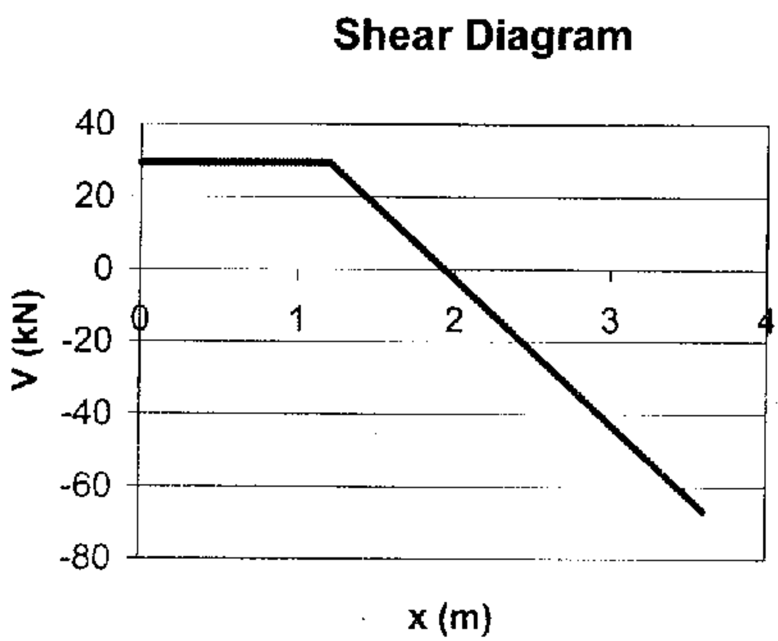
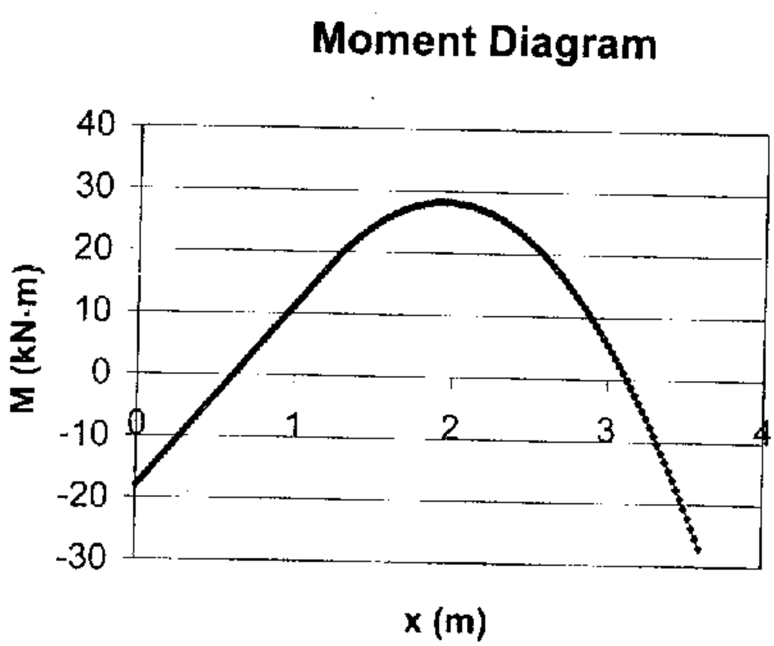
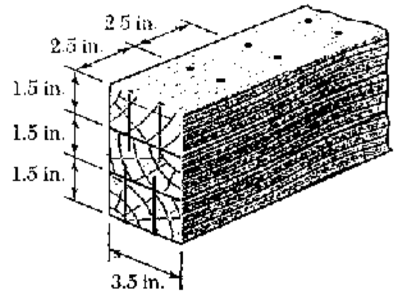





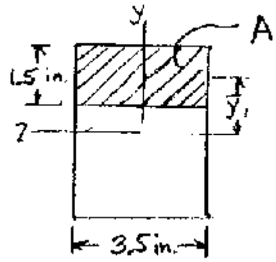
Chapter 6

Problem 6.1



6.1 Three boards, each of 1.5×3.5-in. rectangular cross section, are nailed together to form a beam that is subjected to a vertical shear of 250 lb. Knowing that the spacing between each pair of nails is 2.5 in., determine the shearing force in each nail.

$$I = \frac{1}{12} b h^3 = \frac{1}{12}(3.5)(4.5)^3 = 26.578 \text{ in}^4$$



$A = (3.5)(1.5) = 5.25 \text{ in}^2$

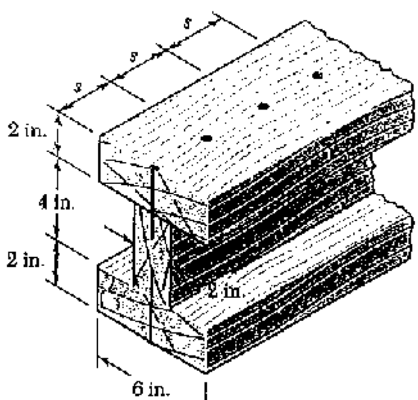
$\bar{y}_1 = 1.5 \text{ in.}$

$Q = A\bar{y}_1 = 7.875 \text{ in}^3$

$$q = \frac{VQ}{I} = \frac{(250)(7.875)}{26.578} = 74.074 \text{ lb/in}$$

$$qs = 2F_{nail} \qquad F_{nail} = \frac{qs}{2} = \frac{(74.074)(2.5)}{2} = 92.6 \text{ lb.}$$

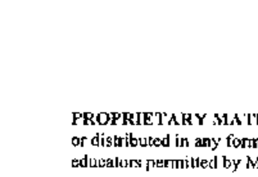
Problem 6.2



6.2 Three boards, each 2 in. thick, are nailed together to form a beam that is subjected to a vertical shear. Knowing that the allowable shearing force in each nail is 150 lb, determine the allowable shear if the spacing s between the nails is 3 in.

$$I_1 = \frac{1}{12} b h^3 + A d^2$$
$$= \frac{1}{12}(6)(2)^3 + (6)(2)(3)^2 = 112 \text{ in}^4$$
$$I_2 = \frac{1}{12} b h^3 = \frac{1}{12}(2)(4)^3 = 10.667 \text{ in}^4$$
$$I_3 = I_1 = 112 \text{ in}^4$$
$$I = I_1 + I_2 + I_3 = 234.667 \text{ in}^4$$

$$Q = A_1 \bar{y}_1 = (6)(2)(3) = 36 \text{ in}^3$$



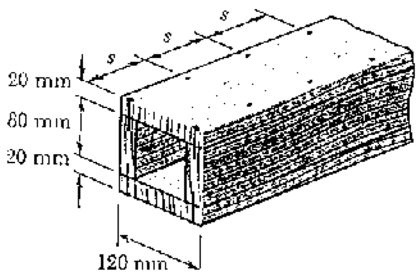
$$qs = F_{nail} \quad (1) \qquad q = \frac{VQ}{I} \quad (2)$$

Dividing Eq (2) by Eq (1) $\quad \frac{1}{s} = \frac{VQ}{F_{nail} I}$

$$V = \frac{F_{nail} I}{Q s} = \frac{(150)(234.667)}{(36)(3)} = 326 \text{ lb.}$$

Problem 6.3

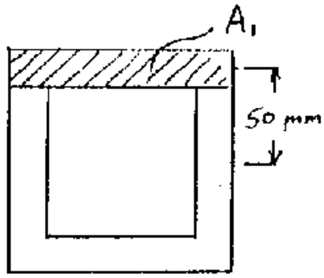
6.3 A square box beam is made of two 20 × 80-mm planks and two 20 × 120-mm planks nailed together as shown. Knowing that the spacing between the nails is s = 50 mm and that the allowable shearing force in each nail is 300 N, determine (*a*) the largest allowable vertical shear in the beam, (*b*) the corresponding maximum shearing stress in the beam.



$$I = \frac{1}{12} b_2 h_2^3 - \frac{1}{12} b_1 h_1^3$$

$$= \frac{1}{12}(120)(120)^3 - \frac{1}{12}(80)(80)^3 = 13.8667 \times 10^6 \text{ mm}^4$$

$$= 13.8667 \times 10^{-6} \text{ m}^4$$



(a) $A_1 = (120)(20) = 2400 \text{ mm}^2$

$\bar{y}_1 = 50 \text{ mm}$

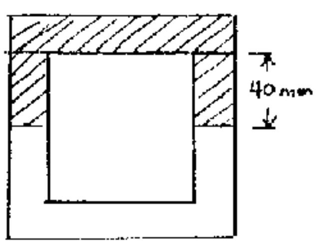
$Q_1 = A_1 \bar{y}_1 = 120 \times 10^3 \text{ mm}^3 = 120 \times 10^{-6} \text{ m}^3$

$q_{all} = \dfrac{2F_{nail}}{s} = \dfrac{(2)(300)}{50 \times 10^{-3}} = 12 \times 10^3 \text{ N}$

$q = \dfrac{VQ}{I}$

$V = \dfrac{qI}{Q} = \dfrac{(12 \times 10^3)(13.8667 \times 10^{-6})}{120 \times 10^{-6}}$

$= 1.38667 \times 10^3 \text{ N} = 1.387 \text{ kN}$ ◄



(b) $Q = Q_1 + (2)(20)(40)(20)$

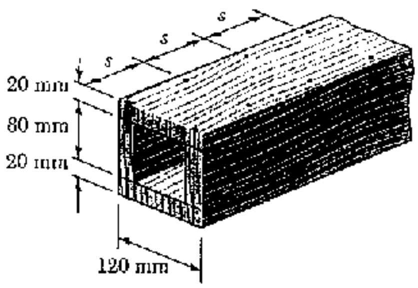
$= 120 \times 10^3 + 32 \times 10^3 = 152 \times 10^3 \text{ mm}^3$

$= 152 \times 10^{-6} \text{ m}^3$

$\tau_{max} = \dfrac{VQ}{It} = \dfrac{(1.38667 \times 10^3)(152 \times 10^{-6})}{(13.8667 \times 10^{-6})(2 \times 20 \times 10^{-3})}$

$= 380 \times 10^3 \text{ Pa}$ 380 kPa ◄

Problem 6.4

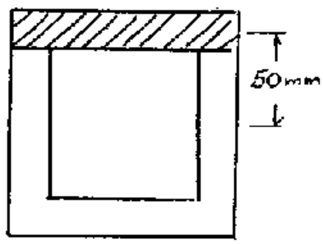


6.4 A square box beam is made of two 20 × 80-mm planks and two 20 × 120-mm planks nailed together as shown. Knowing that the spacing between the nails is s = 30 mm and that the vertical shear in the beam is V = 1200 N, determine (*a*) the shearing force in each nail, (*b*) the maximum shearing stress in the beam.

$$I = \frac{1}{12} b_2 h_2^3 - \frac{1}{12} b_1 h_1^3$$

$$= \frac{1}{12}(120)(120)^3 - \frac{1}{12}(80)(80)^3 = 13.8667 \times 10^6 \text{ mm}^4$$

$$= 13.8667 \times 10^{-6} \text{ m}^4$$



(a) $A_1 = (120)(20) = 2400 \text{ mm}^2$

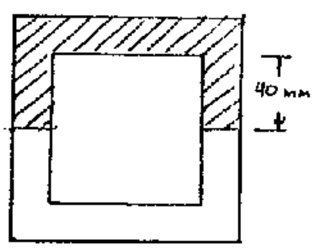
$\bar{y}_1 = 50 \text{ mm}$

$Q_1 = A_1 \bar{y}_1 = 120 \times 10^3 \text{ mm}^3 = 120 \times 10^{-6} \text{ m}^3$

$$q = \frac{VQ}{I} = \frac{(1200)(120 \times 10^{-6})}{13.8667 \times 10^{-6}} = 10.385 \times 10^3 \text{ N/m}$$

$qs = 2 F_{nail}$

$$F_{nail} = \frac{qs}{2} = \frac{(10.385 \times 10^3)(30 \times 10^{-3})}{2} = 155.8 \text{ N}$$



(b) $Q = Q_1 + (2)(20)(40)(20)$

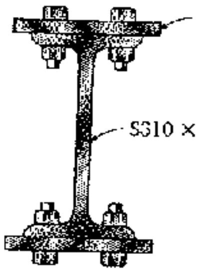
$= 120 \times 10^3 + 32 \times 10^3 = 152 \times 10^3 \text{ mm}^3$

$= 152 \times 10^{-6} \text{ m}^3$

$$\tau_{max} = \frac{VQ}{It} = \frac{(1200)(152 \times 10^{-6})}{(13.8667 \times 10^{-6})(2 \times 20 \times 10^{-3})}$$

$= 329 \times 10^3 \text{ Pa}$ 329 kPa

Problem 6.5



6.5 The American Standard rolled-steel beam shown has been reinforced by attaching to it two 16 × 200-mm plates, using 18-mm-diameter bolts spaced longitudinally every 120 mm. Knowing that the average allowable shearing stress in the bolts is 90 MPa, determine the largest permissible vertical shearing force.

Calculate moment of inertia.

Part	A (mm²)	d (mm)	Ad^2 (10^6 mm⁴)	$\bar{I}$ (10^6 mm⁴)
Top plate	3200	*160.5	82.43	0.07
S310×52	6650	0		95.3
Bot. plate	3200	*160.5	82.43	0.07
Σ			164.86	95.44

$$* \; d = \frac{305}{2} + \frac{16}{2} = 160.5 \text{ mm}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 260.3 \times 10^6 \text{ mm}^4 = 260.3 \times 10^{-6} \text{ m}^4$$

$$Q = A_{plate}\, d_{plate} = (3200)(160.5) = 513.6 \times 10^3 \text{ mm}^3 = 513.6 \times 10^{-6} \text{ m}^3$$

$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4}(18 \times 10^{-3})^2 = 254.47 \times 10^{-6} \text{ m}^2$$

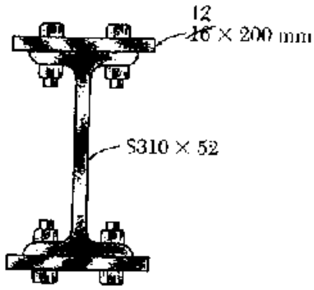
$$F_{bolt} = \tau_{all} A_{bolt} = (90 \times 10^6)(254.47 \times 10^{-6}) = 22.90 \times 10^3 \text{ N}$$

$$q s = 2 F_{bolt} \qquad q = \frac{2F_{bolt}}{s} = \frac{(2)(22.90 \times 10^3)}{120 \times 10^{-3}} = 381.7 \times 10^3 \text{ N/m}$$

$$q = \frac{VQ}{I} \qquad V = \frac{Iq}{Q} = \frac{(260.3 \times 10^{-6})(381.7 \times 10^3)}{513.6 \times 10^{-6}} = 193.5 \times 10^3 \text{ N}$$

$$= 193.5 \text{ kN} \quad \blacktriangleleft$$

Problem 6.6



6.6 Solve Prob. 6.5, assuming that the reinforcing plates are only 12 mm thick.

6.5 The American Standard rolled-steel beam shown has been reinforced by attaching to it two 16 × 200-mm plates, using 18-mm-diameter bolts spaced longitudinally every 120 mm. Knowing that the average allowable shearing stress in the bolts is 90 MPa, determine the largest permissible vertical shearing force.

Calculate moment of inertia.

Part	A (mm^2)	d (mm)	Ad^2 ($10^6\ mm^4$)	$\bar{I}$ ($10^6\ mm^4$)
Top plate	2400	* 158.5	60.29	0.03
S 310 × 52	6650	0	0	95.3
Bot. plate	2400	158.5	60.29	0.03
Σ			120.58	95.36

$* \ d = \frac{305}{2} + \frac{12}{2}$

$= 158.5$ mm

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 215.94 \times 10^6\ mm^4 = 215.94 \times 10^{-6}\ m^4$$

$$Q = A_{plate}\, d_{plate} = (200)(12)(158.5) = 380.4 \times 10^3\ mm^3 = 380.4 \times 10^{-6}\ m^3$$

$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4}(18 \times 10^{-3})^2 = 254.47 \times 10^{-6}\ m^2$$

$$F_{bolt} = \tau_{all} A_{bolt} = (90 \times 10^6)(254.47 \times 10^{-6}) = 22.902 \times 10^3\ N$$

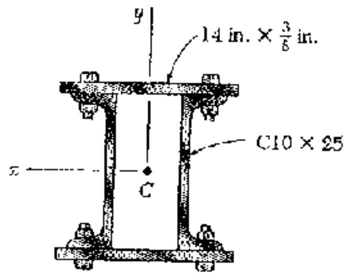
$$qs = 2F_{bolt} \qquad q = \frac{2F_{bolt}}{s} = \frac{(2)(22.903 \times 10^3)}{120 \times 10^{-3}} = 381.7 \times 10^3\ N/m$$

$$q = \frac{VQ}{I} \qquad V = \frac{Iq}{Q} = \frac{(215.94 \times 10^{-6})(381.7 \times 10^3)}{380.4 \times 10^{-6}} = 217 \times 10^3\ N$$

$$= 217\ kN$$

Problem 6.7

6.7 and **6.8** A column is fabricated by connecting the rolled-steel members shown by bolts of $\frac{3}{4}$-in. diameter spaced longitudinally every 5 in. Determine the average shearing stress in the bolts caused by a shearing force of 30 kips parallel to the y axis.



Calculate moment of inertia.

Part	A (in²)	d (in)	Ad² (in⁴)	$\bar{I}$ (in⁴)
Top plate	5.25	*5.1875	141.28	0.06
C10×25	7.35	0		91.2
C10×25	7.35	0		91.2
Bot. plate	5.25	*5.1875	141.28	0.06
Σ			282.56	182.52

$$* \ d = \frac{10}{2} + \frac{1}{2}\left(\frac{3}{8}\right) = 5.1875 \text{ in.} = \bar{y}_1$$

$$I = \Sigma A d^2 + \Sigma \bar{I} = 282.56 + 182.52 = 465.08 \text{ in}^4$$

$$Q = A_{plate}\bar{y}_1 = (5.25)(5.1875) = 27.234 \text{ in}^3$$

$$q = \frac{VQ}{I} = \frac{(30)(27.234)}{465.08} = 1.7567 \text{ kips/in}$$

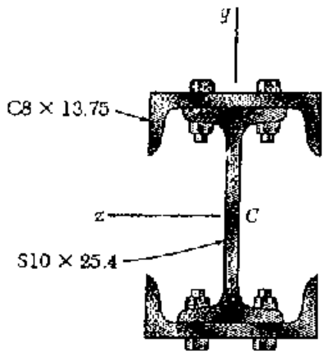
$$F_{bolt} = \frac{1}{2} q s = \left(\frac{1}{2}\right)(1.7567)(5) = 4.392 \text{ kips}$$

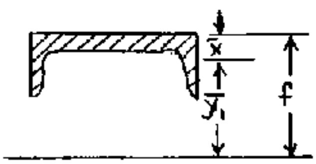
$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4}\left(\frac{3}{4}\right)^2 = 0.4418 \text{ in}^2$$

$$\tau_{bolt} = \frac{F_{bolt}}{A_{bolt}} = \frac{4.392}{0.4418} = 9.94 \text{ ksi}$$

Problem 6.8

6.7 and *6.8* A column is fabricated by connecting the rolled-steel members shown by bolts of $\frac{3}{4}$-in. diameter spaced longitudinally every 5 in. Determine the average shearing stress in the bolts caused by a shearing force of 30 kips parallel to the y axis.





Geometry

$$f = \left(\frac{d}{2}\right)_s + (t_w)_c$$

$$= \frac{10}{2} + 0.303 = 5.303 \text{ in.}$$

$$\bar{x} = 0.533 \text{ in}$$

$$\bar{y}_1 = f - \bar{x} = 5.303 - 0.533 = 4.770 \text{ in.}$$

Determine moment of inertia.

Part	A (in²)	d (in)	A d² (in⁴)	$\bar{I}$ (in⁴)
C8 × 13.75	4.04	4.770	91.92	1.53
S10 × 25.4	7.46	0	0	124
C8 × 13.75	4.04	4.770	91.92	1.53
Σ			183.84	127.06

$$I = \Sigma A d^2 + \Sigma \bar{I} = 183.84 + 127.06 = 310.9 \text{ in}^4$$

$$Q = A\bar{y}_1 = (4.04)(4.770) = 19.271 \text{ in}^3$$

$$q = \frac{VQ}{I} = \frac{(30)(19.271)}{310.9} = 1.8595 \text{ kip/in}$$

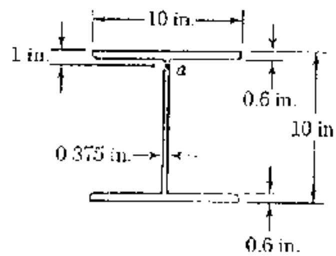
$$F_{bolt} = \frac{1}{2} q s = \left(\frac{1}{2}\right)(1.8595)(5) = 4.649 \text{ kip}$$

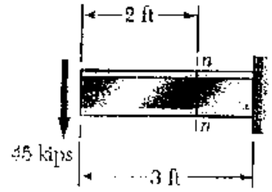
$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4}\left(\frac{3}{4}\right)^2 = 0.4418 \text{ in}^2$$

$$\tau_{bolt} = \frac{F_{bolt}}{A_{bolt}} = \frac{4.649}{0.4418} = 10.52 \text{ ksi}$$

Problem 6.9

6.9 through 6.12 For the beam and loading shown, consider section *n-n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.



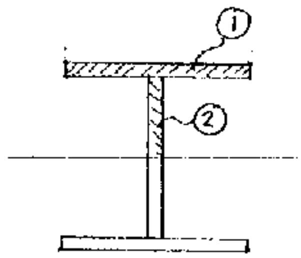


$V = 45$ kips

Part	A (in²)	d (in.)	Ad^2 (in⁴)	$\bar{I}$ (in⁴)
Flange	6.00	4.7	132.54	0.18
Web	3.30	0	0	21.296
Flange	6.00	4.7	132.54	0.18
Σ			265.08	21.656

$$I = \Sigma Ad^2 + \Sigma \bar{I}$$
$$= 265.08 + 21.656$$
$$= 286.736 \text{ in}^4$$

(a)

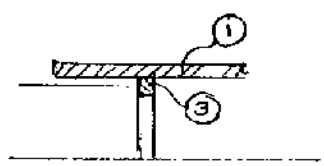


$$Q = A_1\bar{y}_1 + A_2\bar{y}_2$$
$$= (6.00)(4.7) + (0.375)(4.4)(2.2) = 31.83 \text{ in}^3$$

$t = 0.375$ in.

$$\tau_{max} = \frac{VQ}{It} = \frac{(45)(31.83)}{(286.736)(0.375)} = 13.32 \text{ ksi} \blacktriangleleft$$

(b)



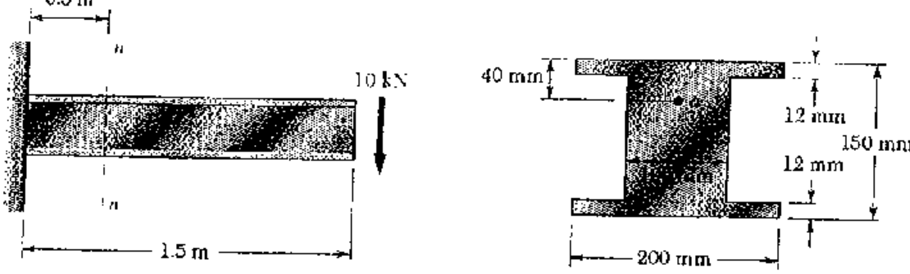
$$Q_a = A_1\bar{y}_1 + A_3\bar{y}_3$$
$$= (6.00)(4.7) + (0.375)(0.4)\left(\frac{4.4+4.0}{2}\right)$$
$$= 28.83 \text{ in}^3$$

$t = 0.375$ in.

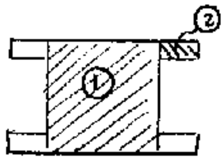
$$\tau_a = \frac{VQ_a}{It} = \frac{(45)(28.83)}{(286.736)(0.375)} = 12.07 \text{ ksi} \blacktriangleleft$$

Problem 6.10

6.9 through 6.12 For the beam and loading shown, consider section *n-n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.

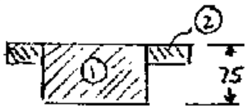


At section n-n
$V = 10$ kN



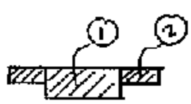
$$I = I_1 + 4 I_2$$
$$= \frac{1}{12} b_1 h_1^3 + 4\left(\frac{1}{12} b_2 h_2^3 + A_2 d_2^2\right)$$
$$= \frac{1}{12}(100)(150)^3 + 4\left[\left(\frac{1}{12}\right)(50)(12)^3 + (50)(12)(69)^2\right]$$
$$= 28.125\times10^6 + 4\left[0.0072\times10^6 + 2.8566\times10^6\right]$$
$$= 39.58\times10^6 \text{ mm}^4 = 39.58\times10^{-6} \text{ m}^4$$

(a)



$$Q = A_1\bar{y}_1 + 2A_2\bar{y}_2$$
$$= (100)(75)(37.5) + (2)(50)(12)(69)$$
$$= 364.05\times10^3 \text{ mm}^3 = 364.05\times10^{-6} \text{ m}^3$$
$$t = 100 \text{ mm} = 0.100 \text{ m}$$
$$\tau_{max} = \frac{VQ}{It} = \frac{(10\times10^3)(364.05\times10^{-6})}{(39.58\times10^{-6})(0.100)} = 920\times10^3 \text{ Pa} = 920 \text{ kPa}$$

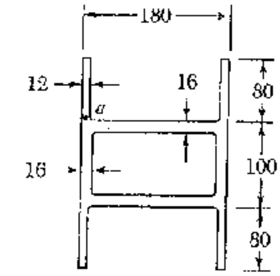
(b)



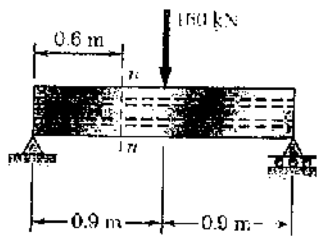
$$Q = A_1\bar{y}_1 + 2A_2\bar{y}_2$$
$$= (100)(40)(55) + (2)(50)(12)(69)$$
$$= 302.8\times10^3 \text{ mm}^3 = 302.8\times10^{-6} \text{ m}^3$$
$$t = 100 \text{ mm} = 0.100 \text{ m}$$
$$\tau = \frac{VQ}{It} = \frac{(10\times10^3)(302.8\times10^{-6})}{(39.58\times10^{-6})(0.100)} = 765\times10^3 \text{ Pa} = 765 \text{ kPa}$$

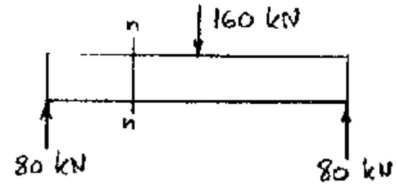
Problem 6.11

6.9 through 6.12 For the beam and loading shown, consider section *n-n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.

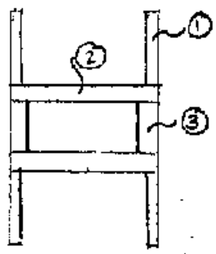


Dimensions in mm





At section n-n $\quad V = 80\ \text{kN}$



Consider cross section as composed of rectangles of types ①, ②, and ③.

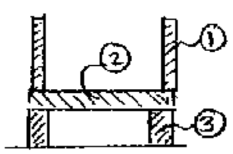
$$I_1 = \frac{1}{12}(12)(80)^3 + (12)(80)(90)^2 = 8.288\times10^6\ \text{mm}^4$$

$$I_2 = \frac{1}{12}(180)(16)^3 + (180)(16)(42)^2 = 5.14176\times10^6\ \text{mm}^4$$

$$I_3 = \frac{1}{12}(16)(68)^3 = 419.24\times10^3\ \text{mm}^4$$

$$I = 4I_1 + 2I_2 + 2I_1 = 44.274\times10^6\ \text{mm}^4$$
$$= 44.274\times10^{-6}\ \text{m}^4$$

(a) Calculate Q at neutral axis.



$$Q_1 = (12)(80)(90) = 86.4\times10^3\ \text{mm}^4$$
$$Q_2 = (180)(16)(42) = 120.96\times10^3\ \text{mm}^4$$
$$Q_3 = (16)(34)(17) = 9.248\times10^3\ \text{mm}^4$$

$$Q = 2Q_1 + Q_2 + 2Q_3 = 312.256\times10^3\ \text{mm}^3 = 312.256\times10^{-6}\ \text{m}^3$$

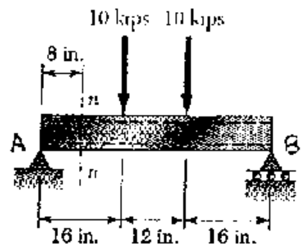
$$\tau = \frac{VQ}{It} = \frac{(80\times10^3)(312.256\times10^{-6})}{(44.274\times10^{-6})(2\times16\times10^{-3})} = 17.63\times10^6\ \text{Pa} \qquad 17.63\ \text{MPa} \blacktriangleleft$$

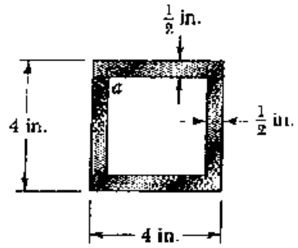
(b) At point a, $\quad Q = Q_1 = 86.4\times10^3\ \text{mm}^4 = 86.4\times10^{-6}\ \text{m}^4$

$$\tau = \frac{VQ}{It} = \frac{(80\times10^3)(86.4\times10^{-6})}{(44.274\times10^{-6})(12\times10^{-3})} = 13.01\times10^6\ \text{Pa} \qquad 13.01\ \text{MPa} \blacktriangleleft$$

Problem 6.12

6.9 through 6.12 For the beam and loading shown, consider section *n-n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.

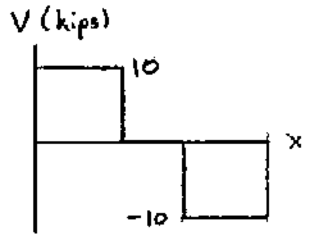




By symmetry $R_A = R_B$

$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - 10 - 10 = 0$

$R_A = R_B = 10$ kips

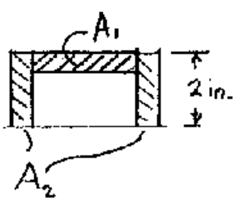


From the shear diagram, $V = 10$ kips at n-n.

$I = \frac{1}{12} b_2 h_2^3 - \frac{1}{12} b_1 h_1^3$

$= \frac{1}{12}(4)(4)^3 - \frac{1}{12}(3)(3)^3 = 14.583 \text{ in}^4$

(a)



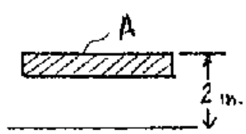
$Q = A_1 \bar{y}_1 + A_2 \bar{y}_2 = (3)(\frac{1}{2})(1.75) + (2)(\frac{1}{2})(2)(1)$

$= 4.625 \text{ in}^3$

$t = \frac{1}{2} + \frac{1}{2} = 1$ in.

$\tau_{max} = \frac{VQ}{It} = \frac{(10)(4.625)}{(14.583)(1)} = 3.17$ ksi ◄

(b)



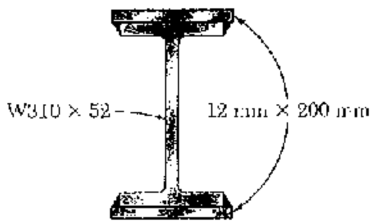
$Q = A\bar{y} = (4)(\frac{1}{2})(1.75) = 3.5 \text{ in}^3$

$t = \frac{1}{2} + \frac{1}{2} = 1$ in.

$\tau = \frac{VQ}{It} = \frac{(10)(3.5)}{(14.583)(1)} = 2.40$ ksi ◄

Problem 6.13

6.13 Two steel plates of 12 × 200-mm rectangular cross section are welded to the W 310 × 52 beam as shown. Determine the largest allowable vertical shear if the shearing stress in the beam is not to exceed 90 MPa.

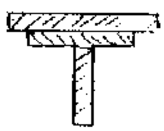


Calculate moment of inertia.

Part	A (mm²)	d (mm)	Ad² (10⁶ mm⁴)	Ī (10⁶ mm⁴)
Top plate	2400	*165	65.34	0.0288
W 310 × 52	6670	0	0	119.
Bot. plate	2400	*165	65.34	0.0288
Σ			130.68	119.0576

$$^{*}d = \frac{318}{2} + \frac{12}{2} = 165 \text{ mm}$$

$$I = \Sigma A d^2 + \Sigma \bar{I} = 249.74 \times 10^{6} \text{ mm}^4 = 249.74 \times 10^{-6} \text{ m}^4$$



Web dimensions
7.6 mm × 145.8 mm

Part	A (mm²)	$\bar{y}$ (mm)	$A\bar{y}$ (10³ mm³)
Plate	2400	165.	396.
Top flange	2204.4	152.4	335.95
Half web	1108.08	72.9	80.78
Σ			812.73

$$Q = \Sigma A\bar{y} = 812.73 \times 10^{3} \text{ mm}^3 = 812.73 \times 10^{-6} \text{ m}^3$$

$$t = 7.6 \text{ mm} = 7.6 \times 10^{-3} \text{ m}$$

$$\tau = \frac{VQ}{It} \qquad V = \frac{It\tau}{Q} = \frac{(249.74 \times 10^{-6})(7.6 \times 10^{-3})(90 \times 10^{6})}{812.73 \times 10^{-6}}$$

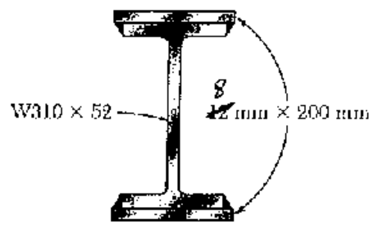
$$= 210 \times 10^{3} \text{ N}$$

210 kN ◄

Problem 6.14

6.14 Solve Prob. 6.13, assuming that the two steel plates are (*a*) replaced by 8 × 200-mm plates, (*b*) removed.

6.13 Two steel plates of 12 × 200-mm rectangular cross section are welded to the W 310 × 52 beam as shown. Determine the largest allowable vertical shear if the shearing stress in the beam is not to exceed 90 MPa.

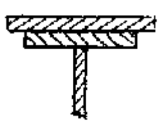


(a) Calculate moment of inertia.

$$* \; d = \frac{318}{2} + \frac{8}{2} = 163 \text{ mm}$$

Part	A (mm²)	d (mm)	Ad² (10⁶ mm⁴)	Ī (10⁶ mm⁴)
Top plate	1600	* 163	42.5104	0.008
W310×52	6670	0		119.
Bot. plate	1600	* 163	42.5104	0.008
			85.0208	119.016

$$I = \Sigma A d^2 + \Sigma \bar{I} = 204.04 \times 10^6 \text{ mm}^4 = 204.04 \times 10^{-6} \text{ m}^4$$



Web dimensions
7.6 mm × 145.8

Part	A (mm²)	$\bar{y}$ (mm)	$A\bar{y}$ (10³ mm³)
Plate	1600	163	260.8
Top flange	2204.4	152.4	335.95
Half web	1108.08	72.9	80.78
Σ			677.53

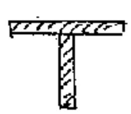
$$Q = \Sigma A\bar{y} = 677.53 \times 10^3 \text{ mm}^3 = 677.53 \times 10^{-6} \text{ m}^3$$

$$t = 7.6 \text{ mm} = 7.6 \times 10^{-3} \text{ m}$$

$$\tau = \frac{VQ}{It} \qquad V = \frac{It\tau}{Q} = \frac{(204.04 \times 10^{-6})(7.6 \times 10^{-3})(90 \times 10^6)}{677.53 \times 10^{-6}}$$

$$= 206 \times 10^3 \text{ N} \qquad 206 \text{ kN} \blacktriangleleft$$

(b) $I = 119 \times 10^6 \text{ mm}^4 = 119 \times 10^{-6} \text{ m}^4$



$$Q = (335.93 + 80.78) \times 10^3 = 416.71 \times 10^3 \text{ mm}^3 = 416.71 \times 10^{-6} \text{ m}^3$$

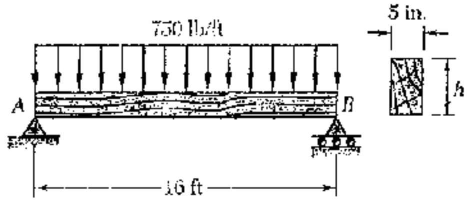
$$t = 7.6 \times 10^{-3} \text{ m}$$

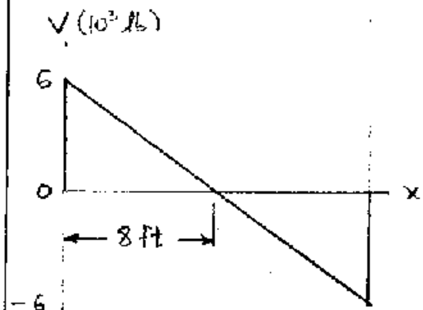
$$V = \frac{It\tau}{Q} = \frac{(119 \times 10^{-6})(7.6 \times 10^{-3})(90 \times 10^6)}{416.71 \times 10^{-6}}$$

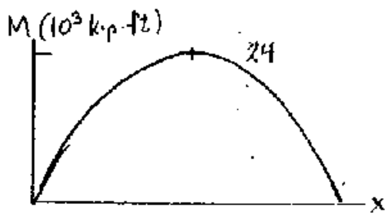
$$= 195.3 \times 10^3 \text{ N} \qquad 195.3 \text{ kN} \blacktriangleleft$$

Problem 6.15

6.15 For the beam and loading shown, determine the minimum required depth h, knowing that for the grade of timber used, $\sigma_{all} = 1750$ psi and $\tau_{all} = 130$ psi.







Total load $(750\ \text{lb/ft})(16\ \text{ft}) = 12 \times 10^3$ lb

Reaction at A $\quad R_A = 6 \times 10^3$ lb

$V_{max} = 6 \times 10^3$ lb

$M_{max} = \frac{1}{2}(8\ \text{ft})(6 \times 10^3) = 24 \times 10^3$ lb·ft

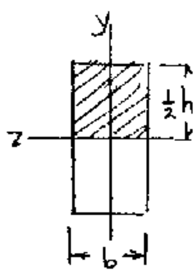
$= 288 \times 10^3$ lb·in

Bending. $\quad S = \frac{1}{6} b h^2$ for rectangular section.

$$S = \frac{M_{max}}{\sigma_{all}} = \frac{288 \times 10^3}{1750} = 164.57\ \text{in}^3$$

$$h = \sqrt{\frac{6S}{b}} = \sqrt{\frac{(6)(164.57)}{5}} = 14.05\ \text{in}$$

Shear. $\quad I = \frac{1}{12} b h^3$ for rectangular section.



$A = \frac{1}{2} b h$

$\bar{y} = \frac{1}{4} h$

$Q = A\bar{y} = \frac{1}{8} b h^2$

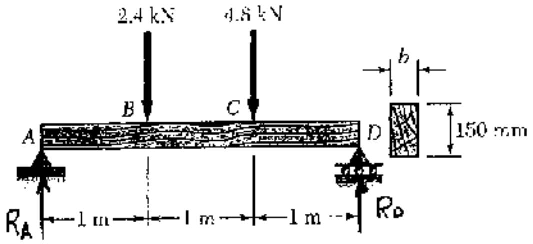
$$\tau_{max} = \frac{VQ}{Ib} = \frac{3V_{max}}{2bh}$$

$$h_r = \frac{3V_{max}}{2b\tau_{max}} = \frac{(3)(6 \times 10^3)}{(2)(5)(130)} = 13.85\ \text{in.}$$

The larger value of h is the minimum required depth. $\quad h = 14.05$ in. ◄

Problem 6.16

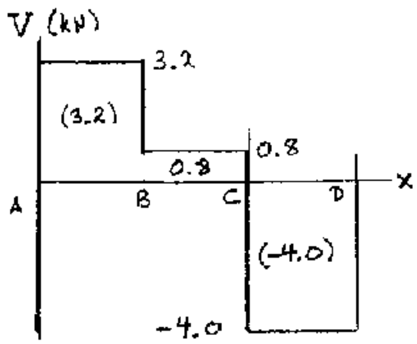
6.16 For the beam and loading shown, determine the minimum required width b, knowing that for the grade of timber used $\sigma_{all} = 12$ MPa and $\tau_{all} = 825$ kPa.



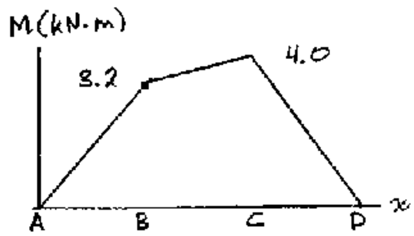
$+\circlearrowleft M_D = 0: \ -3R_A + (2)(2.4) + (1)(4.8) = 0$

$R_A = 3.2 \text{ kN}$

Draw shear and bending moment diagrams.



$|V|_{max} = 4.0 \text{ kN} \qquad |M|_{max} = 4.0 \text{ kN·m}$



Bending. $\quad S = \dfrac{|M|_{max}}{\sigma_{all}} = \dfrac{4.0 \times 10^3}{12 \times 10^6}$

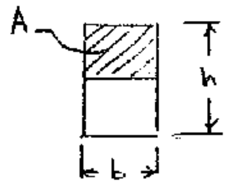
$= 333.33 \times 10^{-6} \text{ m}^3 = 333.33 \times 10^3 \text{ mm}^3$

For a rectangular cross section,

$S = \dfrac{I}{c} = \dfrac{\frac{1}{12} bh^3}{\frac{1}{2}h} = \dfrac{1}{6} bh^2$

$b = \dfrac{6S}{h^2} = \dfrac{(6)(333.33 \times 10^3)}{150^2} = 88.9 \text{ mm}$

Shear. $\quad A = \frac{1}{2} bh, \quad \bar{y} = \frac{1}{4} h$



$Q = A\bar{y} = \frac{1}{8} bh^2, \quad I = \frac{1}{12} bh^3$

$\tau = \dfrac{VQ}{It} = \dfrac{3V}{2bh}$

$bh = \dfrac{3}{2}\dfrac{V}{\tau} = \dfrac{3}{2} \dfrac{4.0 \times 10^3}{825 \times 10^3}$

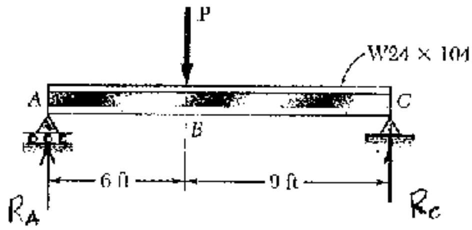
$= 7.2727 \times 10^{-3} \text{ m}^2 = 7.2727 \times 10^3 \text{ mm}^2$

$b = \dfrac{bh}{h} = \dfrac{7.2727 \times 10^3}{150} = 48.5 \text{ mm}$

The required value for b is the larger one. $\qquad b = 88.9 \text{ mm}$ ◀

Problem 6.17

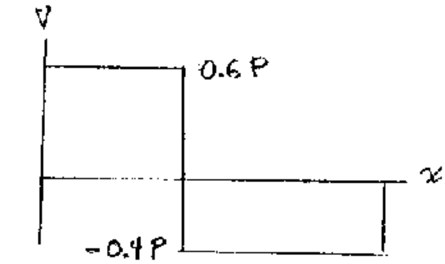
6.17 For the wide-flange beam with the loading shown, determine the largest load **P** that can be applied, knowing that the maximum normal stress is 24 ksi and the largest shearing stress using the approximation $\tau_m = V/A_{web}$ is 14.5 ksi.



$+\circlearrowleft \Sigma M_C = 0: \quad -15 R_A + 9P = 0$

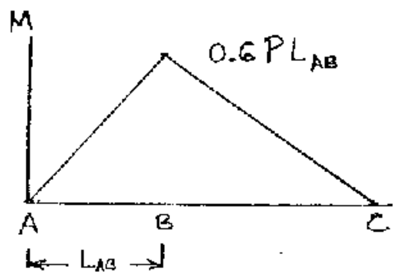
$R_A = 0.6\,P$

Draw shear and bending moment diagrams.



$|V|_{max} = 0.6\,P \qquad |M|_{max} = 0.6\,P L_{AB}$

$L_{AB} = 6 \text{ ft} = 72 \text{ in.}$



Bending. For W24×104, $S = 258 \text{ in}^3$

$$S = \frac{|M|_{max}}{\sigma_{all}} = \frac{0.6\,P L_{AB}}{\sigma_{all}}$$

$$P = \frac{\sigma_{all} S}{0.6\,L_{AB}} = \frac{(24)(258)}{(0.6)(72)} = 143.3 \text{ kips}$$

Shear. $A_{web} = d\,t_w$

$= (24.06)(0.500)$

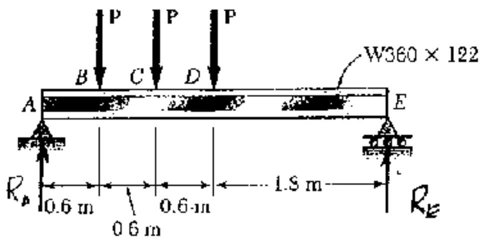
$= 12.03 \text{ in}^2$

$$\tau = \frac{|V|_{max}}{A_{web}} = \frac{0.6\,P}{A_{web}}$$

$$P = \frac{\tau A_{web}}{0.6} = \frac{(14.5)(12.03)}{0.6} = 291 \text{ kips}$$

The smaller value of P is the allowable value. $P = 143.3$ kips ◄

Problem 6.18

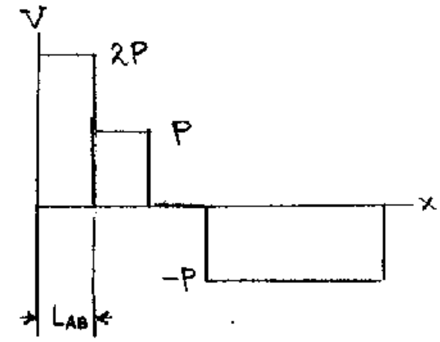


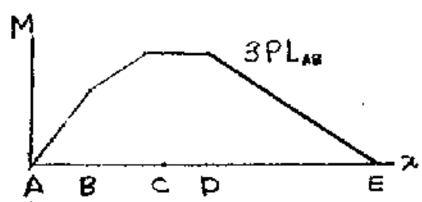
6.18 For the wide-flange beam with the loading shown, determine the largest load **P** that can be applied, knowing that the maximum normal stress is 160 MPa and the largest shearing stress using the approximation $\tau_m \quad V/A_{web}$ is 100 MPa.

$$+\circlearrowleft \Sigma M_E = 0: -3.6R_A + 3.0P + 2.4P + 1.8P = 0$$

$$R_A = 2P$$

Draw shear and bending moment diagrams.





$$M_B = 2PL_{AB}, \quad M_C = M_D = 3PL_{AB}$$

$$|V|_{max} = 2P \qquad |M|_{max} = 3PL_{AB}$$

Bending. For W360 × 122 $\quad S = 2010 \times 10^3\ mm^3 = 2010 \times 10^{-6}\ m^3$

$$\frac{|M|_{max}}{\sigma_{all}} = \frac{3PL_{AB}}{\sigma_{all}} = S$$

$$P = \frac{\sigma_{all} S}{3L_{AB}} = \frac{(160\times10^6)(2010\times10^{-6})}{(3)(0.6)} = 178.7\times10^3\ N$$

Shear. $\quad A_{web} = d\,t_w = (363)(13.0) = 4.719\times10^3\ mm^2 = 4.719\times10^{-3}\ m^2$

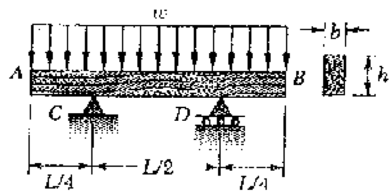
$$\tau = \frac{|V|_{max}}{A_{web}} = \frac{2P}{A_{web}}$$

$$P = \frac{\tau A_{web}}{2} = \frac{(100\times10^6)(4.719\times10^{-3})}{2} = 236\times10^3\ N$$

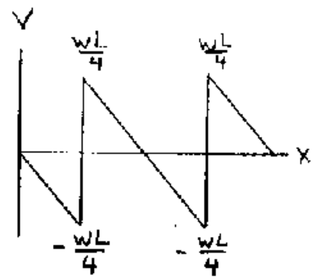
The smaller value of P is the allowable one. $\qquad P = 178.7\ kN$ ◄

Problem 6.19

6.19 A timber beam AB of length L and rectangular cross section carries a uniformly distributed load w and is supported as shown. (*a*) Show that the ratio τ_m/σ_m of the maximum values of the shearing and normal stresses in the beam is equal to $2h/L$, where h and L are, respectively, the depth and the length of the beam. (*b*) Determine the depth h and the width b of the beam, knowing that $L = 5$ m, $w = 8$ kN/m, $\tau_m = 1.08$ MPa, and $\sigma_m = 12$ MPa.



$$R_A = R_B = \frac{wL}{2}$$



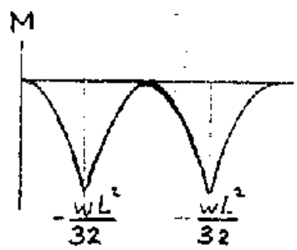
From shear diagram $\quad |V|_m = \dfrac{wL}{4} \quad (1)$

For rectangular section $\quad A = bh \quad (2)$

$$\tau_m = \frac{3}{2}\frac{V_m}{A} = \frac{3wL}{8bh} \quad (3)$$

From bending moment diagram

$$|M|_m = \frac{wL^2}{32} \quad (4)$$



For a rectangular cross section

$$S = \frac{1}{6}bh^2 \quad (5)$$

$$\sigma_m = \frac{|M|_m}{S} = \frac{3wL^2}{16bh^2} \quad (6)$$

(a) Dividing eq. (3) by eq. (6) $\quad \dfrac{\tau_m}{\sigma_m} = \dfrac{2h}{L}$ ◄

(b) Solving for h $\quad h = \dfrac{L\tau_m}{2\sigma_m} = \dfrac{(5)(1.08\times10^6)}{(2)(12\times10^6)} = 225\times10^{-3}$ m

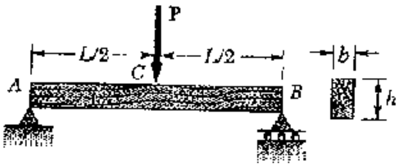
$= 225$ mm ◄

Solving eq. (3) for b $\quad b = \dfrac{3wL}{8h\tau_m} = \dfrac{(3)(8\times10^3)(5)}{(8)(225\times10^{-3})(1.08\times10^6)}$

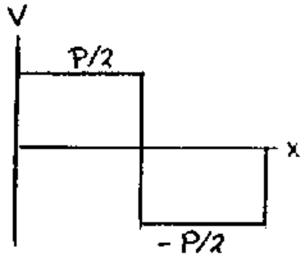
$= 61.7\times10^{-5}$ m $= 61.7$ mm ◄

Problem 6.20

6.20 A timber beam AB of length L and rectangular cross section carries a single concentrated load **P** at its midpoint C. (*a*) Show that the ratio τ_m/σ_m of the maximum values of the shearing and normal stresses in the beam is equal to $2h/L$, where h and L are, respectively, the depth and the length of the beam. (*b*) Determine the depth h and the width b of the beam, knowing that $L = 2$ m, $P = 40$ kN, $\tau_m = 960$ kPa, and $\sigma_m = 12$ MPa.



Reactions: $R_A = R_B = P/2$

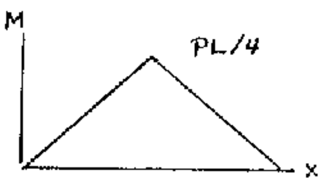


(1) $V_{max} = R_A = \frac{P}{2}$

(2) $A = bh$ for rectangular section.

(3) $\tau_m = \frac{3}{2}\frac{V_{max}}{A} = \frac{3P}{4bh}$ for rectangular section.

(4) $M_{max} = \frac{PL}{4}$



(5) $S = \frac{1}{6}bh^2$ for rectangular section.

(6) $\sigma_m = \frac{M_{max}}{S} = \frac{3PL}{2bh^2}$

(a) $\frac{\tau_m}{\sigma_m} = \frac{h}{2L}$ ◄

(b) Solving for h, $h = \frac{2L\tau_m}{\sigma_m} = \frac{(2)(2)(960\times10^3)}{12\times10^6} = 320\times10^{-3}$ m

$= 320$ mm ◄

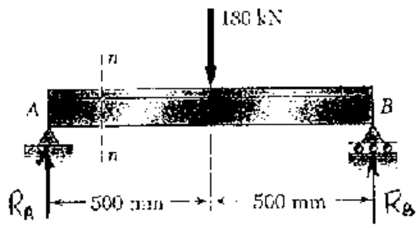
Solving equation (3) for b,

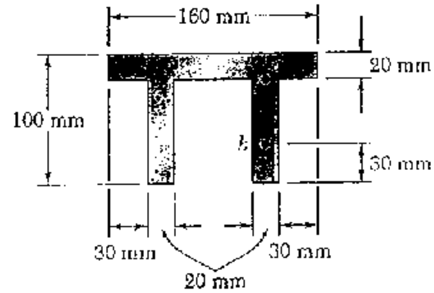
$$b = \frac{3P}{4h\tau_m} = \frac{(3)(40\times10^3)}{(4)(320\times10^{-3})(960\times10^3)} = 97.7\times10^{-3} \text{ m}$$

$= 97.7$ mm ◄

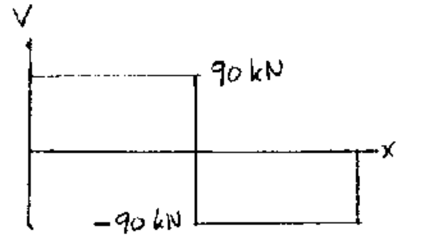
Problem 6.21

6.21 and 6.22 For the beam and loading shown, consider section *n-n* and determine the shearing stress at (*a*) point *a*, (*b*) point *b*.

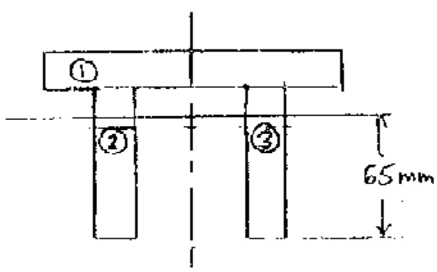




Draw the shear diagram. $|V|_{max} = 90$ kN

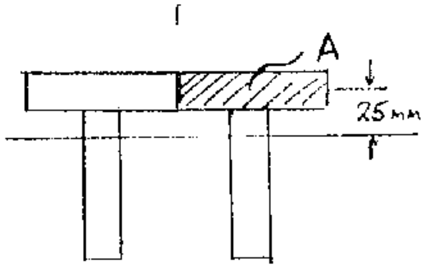


Part	$A(mm^2)$	$\bar{y}(mm)$	$A\bar{y}\,(10^3 mm^3)$	$d(mm)$	$Ad^2\,(10^6 mm^4)$	$\bar{I}\,(10^6 mm^4)$
①	3200	90	288	25	2.000	0.1067
②	1600	40	64	−25	1.000	0.8533
③	1600	40	64	−25	1.000	0.8533
Σ	6400		416		4.000	1.8133



$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{416\times10^3}{6400} = 65 \text{ mm}$$

$$I = \Sigma A d^2 + \Sigma \bar{I} = (4.000 + 1.8133)\times10^6 \text{ mm}^4$$
$$= 5.8133\times10^6 \text{ mm}^4 = 5.8133\times10^{-6} \text{ m}^4$$



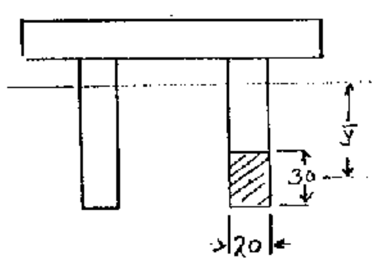
(a) $A = (80)(20) = 1600 \text{ mm}^2$

$\bar{y} = 25$ mm

$Q_a = A\bar{y} = 40\times10^3 \text{ mm}^3 = 40\times10^{-6} \text{ m}^3$

$$\tau_a = \frac{VQ_a}{It} = \frac{(90\times10^3)(40\times10^{-6})}{(5.8133\times10^{-6})(20\times10^{-3})} = 31.0\times10^6 \text{ Pa}$$

31.0 MPa ◄



(b) $A = (30)(20) = 600 \text{ mm}^2$

$\bar{y} = 65 - 15 = 50$ mm

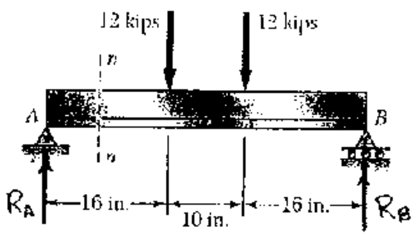
$Q_b = A\bar{y} = 30\times10^3 \text{ mm}^3 = 30\times10^{-6} \text{ m}^3$

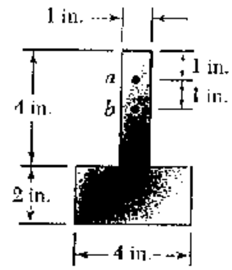
$$\tau_b = \frac{VQ_b}{It} = \frac{(90\times10^3)(30\times10^{-6})}{(5.8133\times10^{-6})(20\times10^{-3})} = 23.2\times10^6 \text{ Pa}$$

23.2 MPa ◄

Problem 6.22

6.21 and 6.22 For the beam and loading shown, consider section *n-n* and determine the shearing stress at (*a*) point *a*, (*b*) point *b*.

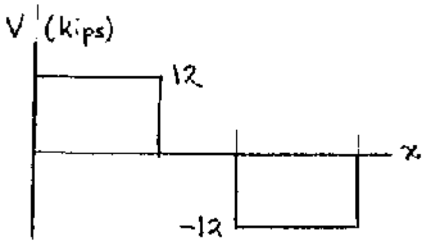




$R_A = R_B = 12$ kips.

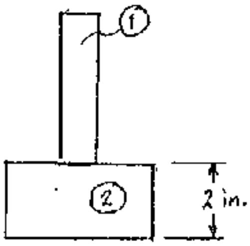
Draw shear diagram.

$V = 12$ kips



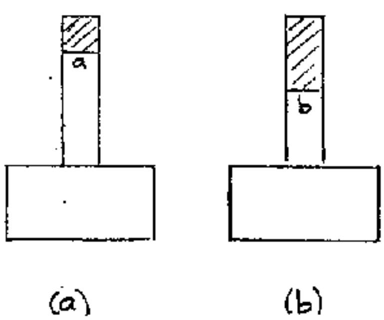
Determine section properties.

Part	$A\,(in^2)$	$\bar{y}\,(in.)$	$A\bar{y}\,(in^3)$	$d\,(in.)$	$Ad^2\,(in^4)$	$\bar{I}\,(in^4)$
①	4	4	16	2	16	5.333
②	8	1	8	-1	8	2.667
Σ	12		24		24	8.000



$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{24}{12} = 2\ in.$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 32\ in^4$$



(a) $A = 1\ in^2$ $\quad \bar{y} = 3.5\ in.$ $\quad Q_a = A\bar{y} = 3.5\ in^3$

$t = 1\ in.$

$$\tau_a = \frac{VQ_a}{It} = \frac{(12)(3.5)}{(32)(1)} = 1.3125\ ksi$$ ◄

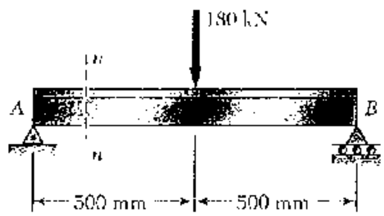
(b) $A = 2\ in^2$ $\quad \bar{y} = 3\ in.$ $\quad Q_b = A\bar{y} = 6\ in^3$

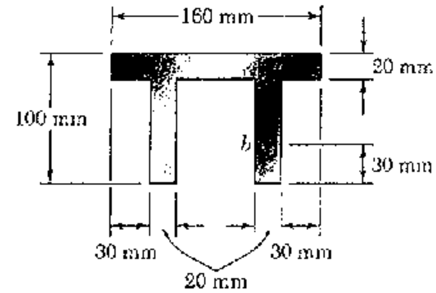
$t = 1\ in.$

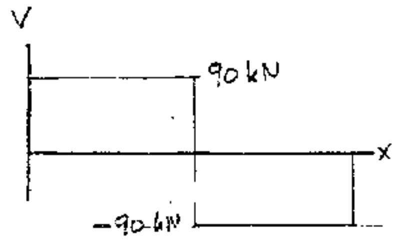
$$\tau_b = \frac{VQ_b}{It} = \frac{(12)(6)}{(32)(1)} = 2.25\ ksi$$ ◄

Problem 6.23

6.23 and 6.24 For the beam and loading shown, determine the largest shearing stress in section *n-n*.

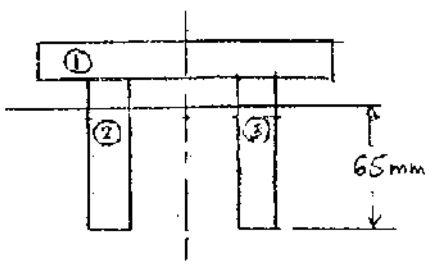






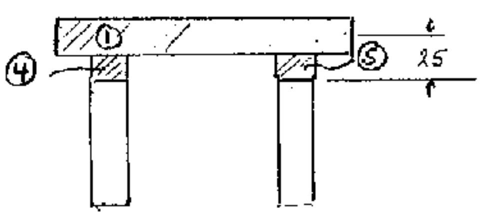
Draw the shear diagram. $|V|_{max} = 90$ kN

Part	$A\,(mm^2)$	$\bar{y}\,(mm)$	$A\bar{y}\,(10^3 mm^3)$	$d\,(mm)$	$Ad^2\,(10^6 mm^4)$	$\bar{I}\,(10^6 mm^4)$
①	3200	90	288	25	2.000	0.1067
②	1600	40	64	−25	1.000	0.8533
③	1600	40	64	−25	1.000	0.8533
Σ	6400		416		4.000	1.8133



$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{416 \times 10^3}{6400} = 65 \text{ mm}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = (4.000 + 1.8133) \times 10^6 \text{ mm}^4$$
$$= 5.8133 \times 10^6 \text{ mm}^4 = 5.8133 \times 10^{-6} \text{ m}^4$$



Part	$A\,(mm^2)$	$\bar{y}\,(mm)$	$A\bar{y}\,(10^3 mm^3)$
①	3200	25	80
④	300	7.5	2.25
⑤	300	7.5	2.25
Σ			84.5

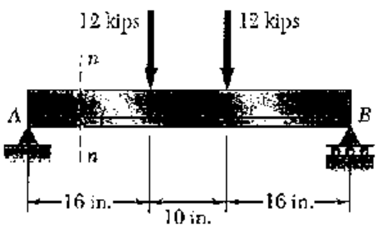
$$Q = \Sigma A\bar{y} = 84.5 \times 10^3 \text{ mm}^3 = 84.5 \times 10^{-6} \text{ m}^3$$

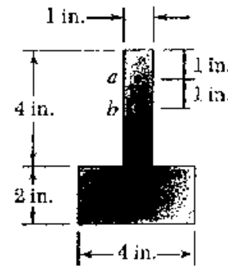
$$t = (2)(20) = 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$$

$$\tau_{max} = \frac{VQ}{It} = \frac{(90 \times 10^3)(84.5 \times 10^{-6})}{(5.8133 \times 10^{-6})(40 \times 10^{-3})} = 32.7 \times 10^6 \text{ Pa} = 32.7 \text{ MPa}$$

Problem 6.24

6.23 and 6.24 For the beam and loading shown, determine the largest shearing stress in section *n-n*.

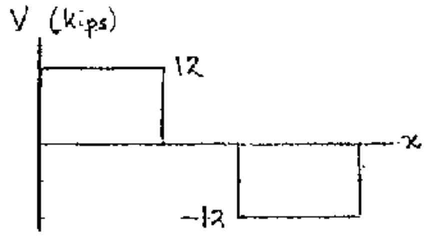




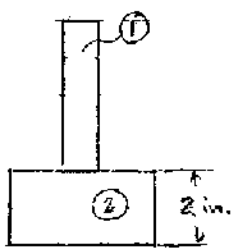
$R_A = R_B = 12$ kips.

Draw shear diagram.

$V = 12$ kips



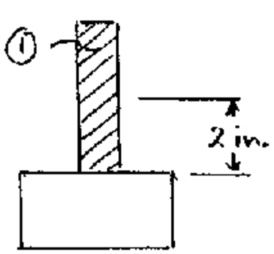
Determine section properties.



Part	$A(in^2)$	$\bar{y}(in.)$	$A\bar{y}(in^3)$	$d(in.)$	$Ad^2(in^4)$	$\bar{I}(in^4)$
①	4	4	16	2	16	5.333
②	8	1	8	-1	8	2.667
Σ	12		24		24	8.000

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{24}{12} = 2 \text{ in.}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 32 \text{ in}^4$$



$$Q = A_1 \bar{y}_1 = (4)(2) = 8 \text{ in}^3$$

$$t = 1 \text{ in.}$$

$$\tau_{max} = \frac{VQ}{It} = \frac{(12)(8)}{(32)(1)} = 3.00 \text{ ksi}$$

Problem 6.25

6.25 through 6.28 A beam having the cross section shown is subjected to a vertical shear **V**. Determine (*a*) the horizontal line along which the shearing stress is maximum, (*b*) the constant k in the following expression for the maximum shearing stress

$$\tau_{\max} = k\frac{V}{A}$$

where A is the cross-sectional area of the beam.

$I = \frac{\pi}{4}c^4$ and $A = \pi c^2$

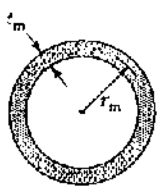
For semicircle, $A_s = \frac{\pi}{2}c^2 \quad \bar{y} = \frac{4c}{3\pi}$

$$Q = A\bar{y} = \frac{\pi}{2}c^2 \cdot \frac{4c}{3\pi} = \frac{2}{3}c^3$$

(a) τ_{max} occurs at center where $t = 2c.$ ◄

(b) $\tau_{max} = \frac{VQ}{It} = \frac{V \cdot \frac{2}{3}c^3}{\frac{\pi}{4}c^4 \cdot 2c} = \frac{4V}{3\pi c^2} = \frac{4}{3}\frac{V}{A} \qquad k = \frac{4}{3} = 1.333$ ◄

Problem 6.26



6.25 through 6.28 A beam having the cross section shown is subjected to a vertical shear **V**. Determine (*a*) the horizontal line along which the shearing stress is maximum, (*b*) the constant k in the following expression for the maximum shearing stress

$$\tau_{\max} = k\frac{V}{A}$$

where A is the cross-sectional area of the beam.

For a thin walled circular section, $A = 2\pi r_m t_m$

$J = Ar_m^2 = 2\pi r_m^3 t_m, \qquad I = \frac{1}{2}J = \pi r_m^3 t_m$

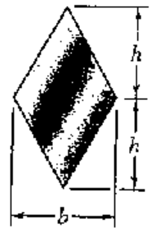
For a semicircular arc, $\bar{y} = \frac{2r_m}{\pi}$

$A_s = \pi r_m t_m \qquad Q = A_s\bar{y} = \pi r_m t_m \frac{2r_m}{\pi} = 2r_m^2 t_m$

(a) $t = 2t_m$ at neutral axis where maximum occurs. ◄

(b) $\tau_{max} = \frac{VQ}{It} = \frac{V(2r_m^2 t_m)}{(\pi r_m^3 t_m)(2t_m)} = \frac{V}{\pi r_m t_m} = \frac{2V}{A} \qquad k = 2.00$ ◄

Problem 6.27

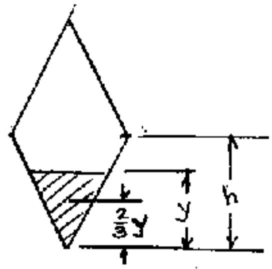


6.25 through 6.28 A beam having the cross section shown is subjected to a vertical shear **V**. Determine (*a*) the horizontal line along which the shearing stress is maximum, (*b*) the constant k in the following expression for the maximum shearing stress

$$\tau_{\max} = k\frac{V}{A}$$

where A is the cross-sectional area of the beam.

$A = 2\left(\frac{1}{2}bh\right) = bh \qquad I = 2\left(\frac{1}{12}bh^3\right) = \frac{1}{6}bh^3$



For a cut at location y, where $y \le h$

$A(y) = \frac{1}{2}\left(\frac{by}{h}\right)y = \frac{by^2}{2h}$

$\bar{y}(y) = h - \frac{2}{3}y$

$Q(y) = A\bar{y} = \frac{by^2}{2} - \frac{by^3}{3h}$

$t(y) = \frac{by}{h}$

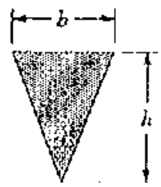
$$\tau(y) = \frac{VQ}{It} = V\,\frac{6}{bh^3}\cdot\frac{h}{by}\cdot\left(\frac{by^2}{2} - \frac{by^3}{3h}\right) = \frac{V}{bh}\left[3\left(\frac{y}{h}\right) - 2\left(\frac{y}{h}\right)^2\right]$$

(a) To find the location of maximum of τ, set $\frac{d\tau}{dy} = 0$.

$\frac{d\tau}{dy} = \frac{V}{bh^2}\left[3 - 4\frac{y_m}{h}\right] = 0 \qquad \frac{y_m}{h} = \frac{3}{4}$ i.e. $\pm\frac{1}{4}h$ from neutral axis. ◄

(b) $\tau(y_m) = \frac{V}{bh}\left[3\left(\frac{3}{4}\right) - 2\left(\frac{3}{4}\right)^2\right] = \frac{9}{8}\frac{V}{bh} = 1.125\frac{V}{A} \qquad k = 1.125$ ◄

Problem 6.28

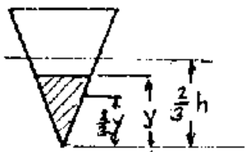


***6.25* through 6.28** A beam having the cross section shown is subjected to a vertical shear V. Determine (*a*) the horizontal line along which the shearing stress is maximum. (*b*) the constant *k* in the following expression for the maximum shearing stress

$$\tau_{max} = k\frac{V}{A}$$

where A is the cross-sectional area of the beam.

$$A = \frac{1}{2}bh \qquad I = \frac{1}{36}bh^3$$



For a cut at location y,

$$A(y) = \frac{1}{2}\left(\frac{by}{h}\right)y = \frac{by^2}{2h}$$

$$\bar{y}(y) = \frac{2}{3}h - \frac{2}{3}y$$

$$Q(y) = A\bar{y} = \frac{by^2}{3}(h-y)$$

$$t(y) = \frac{by}{h}$$

$$\tau(y) = \frac{VQ}{It} = \frac{V\frac{by^2}{3}(h-y)}{\left(\frac{1}{36}bh^3\right)\frac{by}{h}} = \frac{12\,Vy(h-y)}{bh^3} = \frac{12V}{bh^3}(hy - y^2)$$

(a) To find location of maximum of τ, set $\frac{d\tau}{dy} = 0$.

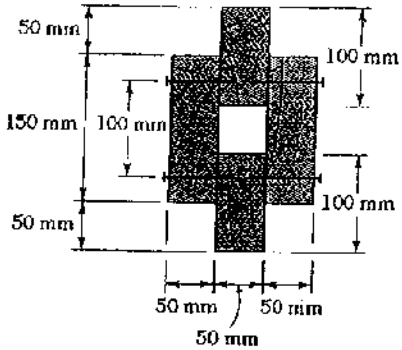
$$\frac{d\tau}{dy} = \frac{12V}{bh^3}(h - 2y_m) = 0 \qquad y_m = \frac{1}{2}h \quad \text{i.e. at mid-height}$$

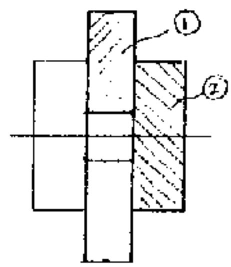
(b) $$\tau_m = \frac{12V}{bh^3}(hy_m - y_m^2) = \frac{12V}{bh^3}\left[\frac{1}{2}h^2 - \left(\frac{1}{2}h\right)^2\right] = \frac{3V}{bh^2} = \frac{3}{2}\frac{V}{A}$$

$$k = \frac{3}{2} = 1.500$$

Problem 6.29

6.29 The built-up timber beam is subjected to a 6-kN vertical shear. Knowing that the longitudinal spacing of the nails is $s = 60$ mm and that each nail is 90 mm long, determine the shearing force in each nail.





$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2$$
$$= \frac{1}{12}(50)(100)^3 + (50)(100)(75)^2$$
$$= 32.292 \times 10^6 \text{ mm}^4$$
$$I_2 = \frac{1}{12} b h^3 = \frac{1}{12}(50)(150)^3$$
$$= 14.0625 \times 10^6 \text{ mm}^4$$

$$I = 2I_1 + 2I_2 = \quad 92.71 \times 10^6 \text{ mm}^4 = 92.71 \times 10^{-6} \text{ m}^4$$

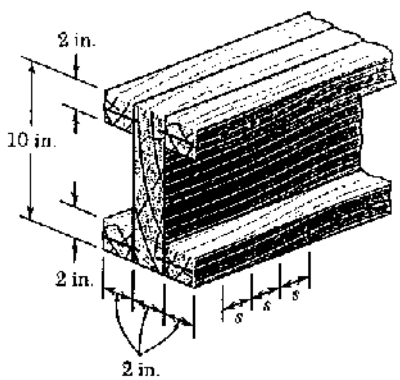
$$Q = Q_1 = A_1 \bar{y}_1 = (50)(100)(75) = 375 \times 10^3 \text{ mm}^3 = 375 \times 10^{-6} \text{ m}^3$$

$$q = \frac{VQ}{I} = \frac{(6 \times 10^3)(375 \times 10^{-6})}{92.71 \times 10^{-6}} = 24.27 \times 10^3 \text{ N/m} \qquad s = 60 \text{ mm} = 60 \times 10^{-3} \text{ m}$$

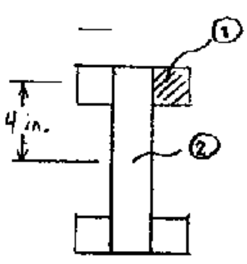
$$2F_{nail} = qs \qquad F_{nail} = \frac{1}{2} qs = \frac{1}{2}(24.27 \times 10^3)(60 \times 10^{-3}) = 728 \text{ N}$$ ◄

Problem 6.30

6.30 The built-up timber beam is subjected to a vertical shear of 1200 lb. Knowing that the allowable shearing force in the nails is 75 lb, determine the largest permissible spacing s of the nails.

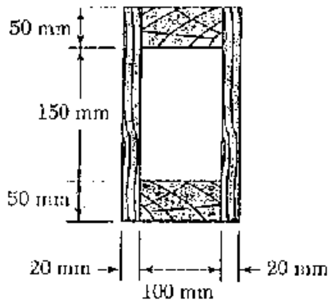


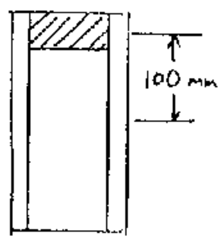
$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2$$
$$= \frac{1}{12}(2)(2)^3 + (2)(2)(4)^2 = 65.333 \text{ in}^4$$
$$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(2)(10)^3 = 16.667 \text{ in}^4$$
$$I = 4I_1 + I_2 = 428 \text{ in}^4$$
$$Q = Q_1 = A_1 \bar{y}_1 = (2)(2)(4) = 16 \text{ in}^3$$
$$q = \frac{VQ}{I} = \frac{(1200)(16)}{428} = 44.86 \text{ lb/in}$$



$$F_{nail} = qs \qquad s = \frac{F_{nail}}{q} = \frac{75}{44.86} = 1.672 \text{ in.}$$ ◄

Problem 6.31





6.31 The built-up beam shown is made by gluing together two 20 × 250-mm plywood strips and two 50 × 100-mm planks. Knowing that the allowable average shearing stress in the glued joints is 350 kPa, determine the largest permissible vertical shear in the beam.

$$I = \frac{1}{12}(140)(250)^3 - \frac{1}{12}(100)(150)^3 = 154.167 \times 10^6 \text{ mm}^4$$
$$= 154.167 \times 10^{-6} \text{ m}^4$$

$$Q = A\bar{y} = (100)(50)(100) = 500 \times 10^3 \text{ mm}^3$$
$$= 500 \times 10^{-6} \text{ m}^3$$

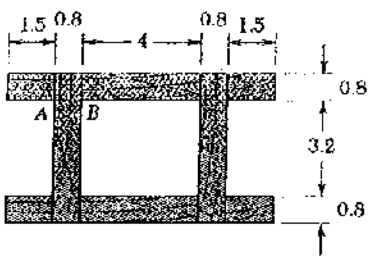
$$t = 50 \text{ mm} + 50 \text{ mm} = 100 \text{ mm} = 100 \times 10^{-3} \text{ m}$$

$$\tau = \frac{VQ}{It}$$

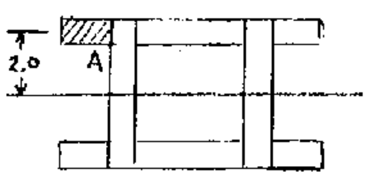
$$V = \frac{It\tau}{Q} = \frac{(154.167 \times 10^{-6})(100 \times 10^{-3})(350 \times 10^3)}{500 \times 10^{-6}}$$

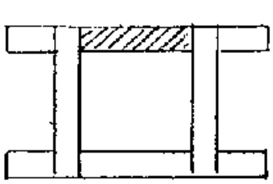
$$= 10.79 \times 10^3 \text{ N} \qquad V = 10.79 \text{ kN}$$

Problem 6.32



Dimensions in inches





6.32 The built-up beam was made by gluing together several wooden planks. Knowing that the beam is subjected to a 1200-lb shear, determine the average shearing stress in the glued joint (*a*) at *A*, (*b*) at *B*.

$$I = 2\left[\frac{1}{12}(0.8)(4.8)^3 + \frac{1}{12}(7)(0.8)^3 + (7)(0.8)(2.0)^2\right]$$
$$= 60.143 \text{ in}^4$$

(a) $A_a = (1.5)(0.8) = 1.2 \text{ in}^2 \qquad \bar{y}_a = 2.0 \text{ in.}$

$Q_a = A_a\bar{y}_a = 2.4 \text{ in}^3$

$t_a = 0.8 \text{ in.}$

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(1200)(2.4)}{(60.143)(0.8)} = 59.9 \text{ psi}$$

(b) $A_b = (4)(0.8) = 3.2 \text{ in}^2 \qquad \bar{y}_b = 2.0 \text{ in.}$

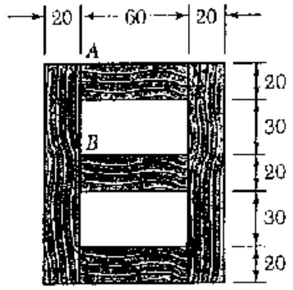
$Q_b = A_b\bar{y}_b = (3.2)(2.0) = 6.4 \text{ in}^3$

$t_b = (2)(0.8) = 1.6 \text{ in.}$

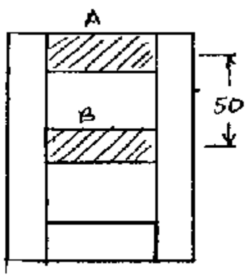
$$\tau_b = \frac{VQ_b}{It_b} = \frac{(1200)(6.4)}{(60.143)(1.6)} = 79.8 \text{ psi}$$

Problem 6.33

6.33 Several planks are glued together to form the box beam shown. Knowing that the beam is subjected to a vertical shear of 3 kN, determine the average shearing stress in the glued joint (*a*) at *A*, (*b*) at *B*.



Dimensions in mm



$$I_A = \frac{1}{12}bh^3 + Ad^2 = \frac{1}{12}(60)(20)^3 + (60)(20)(50)^2$$
$$= 3.04 \times 10^6 \text{ mm}^4$$

$$I_B = \frac{1}{12}bh^3 = \frac{1}{12}(60)(20)^3 = 0.04 \times 10^6 \text{ mm}^4$$

$$I_C = \frac{1}{12}bh^3 = \frac{1}{12}(20)(120)^3 = 2.88 \times 10^6 \text{ mm}^4$$

$$I = 2I_A + I_B + 2I_C = 11.88 \times 10^6 \text{ mm}^4$$
$$= 11.88 \times 10^{-6} \text{ m}^4$$

$$Q_A = A\bar{y} = (60)(20)(50) = 60 \times 10^3 \text{ mm}^3 = 60 \times 10^{-6} \text{ m}^3$$

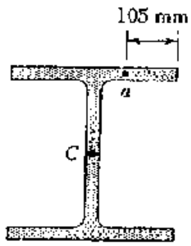
$$t = 20 \text{ mm} + 20 \text{ mm} = 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$$

(a) $$\tau_A = \frac{VQ}{It} = \frac{(3 \times 10^3)(60 \times 10^{-6})}{(11.88 \times 10^{-6})(40 \times 10^{-3})} = 379 \times 10^3 \text{ Pa}$$
$$= 379 \text{ kPa}$$ ◀

$Q_B = 0$ (b) $$\tau_B = \frac{VQ_B}{It} = 0$$ ◀

Problem 6.34

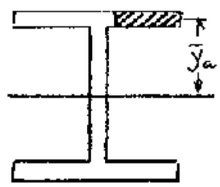
6.34 Knowing that a W360 × 122 rolled-steel beam is subjected to a 250-kN vertical shear, determine the shearing stress (*a*) at point *A*, (*b*) at the centroid *C* of the section.



For W360 × 122, $d = 363$ mm, $b_f = 257$ mm, $t_f = 21.70$ mm, $t_w = 13.0$ mm

$I = 365 \times 10^6 \text{ mm}^4 = 365 \times 10^{-6} \text{ m}^4$

(a)



$A_a = (105)(21.70) = 2278.5 \text{ mm}^2$

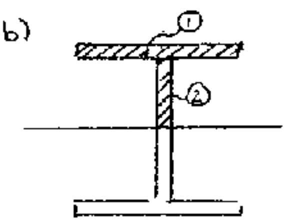
$\bar{y}_a = \frac{d}{2} - \frac{t_f}{2} = \frac{363}{2} - \frac{21.70}{2} = 170.65 \text{ mm}$

$Q_a = A_a \bar{y}_a = 388.8 \times 10^3 \text{ mm}^3 = 388.8 \times 10^{-6} \text{ m}^3$

$t_a = t_f = 21.70 \text{ mm} = 21.7 \times 10^{-3} \text{ m}$

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(250 \times 10^3)(388.8 \times 10^{-6})}{(365 \times 10^{-6})(21.7 \times 10^{-3})} = 12.27 \times 10^6 \text{ Pa} = 12.27 \text{ MPa}$$ ◄

(b)



$A_1 = b_f t_f = (257)(21.70) = 5577 \text{ mm}^2$

$\bar{y}_1 = \frac{d}{2} - \frac{t_f}{2} = \frac{363}{2} - \frac{21.70}{2} = 170.65 \text{ mm}$

$A_2 = t_w\left(\frac{d}{2} - t_f\right) = (13.0)(159.8) = 2077 \text{ mm}^2$

$\bar{y}_2 = \frac{1}{2}\left(\frac{d}{2} - t_f\right) = 79.9 \text{ mm}$

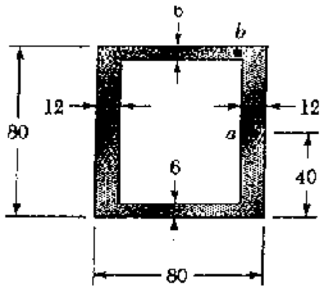
$Q_c = \Sigma A\bar{y} = (5577)(170.65) + (2077)(79.9) = 1117.7 \times 10^3 \text{ mm}^3$
$= 1117.7 \times 10^{-6} \text{ m}^3$

$t_c = t_w = 13.0 \text{ mm} = 13 \times 10^{-3} \text{ m}$

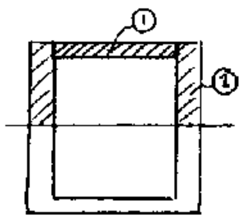
$$\tau_c = \frac{VQ_c}{It_c} = \frac{(250 \times 10^3)(1117.7 \times 10^{-6})}{(365 \times 10^{-6})(13 \times 10^{-3})} = 58.9 \times 10^6 \text{ Pa} = 58.9 \text{ MPa}$$ ◄

Problem 6.35

6.35 An extruded aluminum beam has the cross section shown. Knowing that the vertical shear in the beam is 150 kN, determine the shearing stress at (*a*) point *a*, (*b*) point *b*.



Dimesions in mm



$$I = \frac{1}{12}(80)(80)^3 - \frac{1}{12}(56)(68)^3 = 1.9460 \times 10^6 \text{ mm}^4$$
$$= 1.946 \times 10^{-6} \text{ m}^4$$

(a) $Q_a = A_1\bar{y}_1 + 2A_2\bar{y}_2$

$$= (56)(6)(37) + (2)(12)(40)(20) = 31.632 \times 10^3 \text{ mm}^3$$
$$= 31.632 \times 10^{-6} \text{ m}^3$$

$t_a = (2)(12) = 24 \text{ mm} = 0.024 \text{ m}$

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(150\times10^3)(31.632\times10^{-6})}{(1.946\times10^{-6})(0.024)} = 101.6 \times 10^6 \text{ Pa}$$
$$= 101.6 \text{ MPa}$$

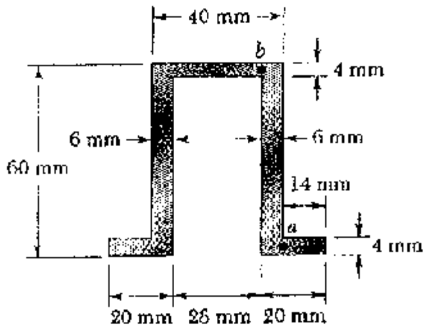
(b) $Q_b = A_1\bar{y}_1 = (56)(6)(37) = 12.432 \times 10^3 \text{ mm}^3$
$$= 12.432 \times 10^{-6} \text{ m}^3$$

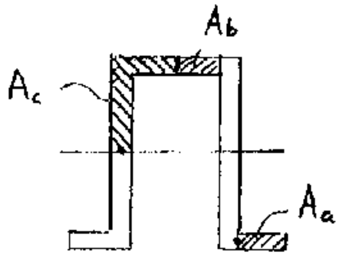
$t_b = (2)(6) = 12 \text{ mm} = 0.012 \text{ m}$

$$\tau_b = \frac{VQ_b}{It_b} = \frac{(150\times10^3)(12.432\times10^{-6})}{(1.946\times10^{-6})(0.012)} = 79.9 \times 10^6 \text{ Pa} = 79.9 \text{ MPa}$$

Problem 6.36

6.36 Knowing that a given vertical shear **V** causes a maximum shearing stress of 75 MPa in the hat-shaped extrusion shown, determine the corresponding shearing stress at (*a*) point *a*, (*b*) point *b*.





Neutral axis lies 30 mm above bottom.

$$\tau_c = \frac{VQ_c}{It}, \quad \tau_a = \frac{VQ_a}{It_a}, \quad \tau_b = \frac{VQ_b}{It_b}$$

$$\frac{\tau_a}{\tau_c} = \frac{Q_a t_c}{Q_c t_a}, \quad \frac{\tau_b}{\tau_c} = \frac{Q_b t_c}{Q_c t_b}$$

$Q_c = (6)(30)(15) + (14)(4)(28) = 4260 \text{ mm}^3$

$t_c = 6 \text{ mm}$

$Q_a = (14)(4)(28) = 1568 \text{ mm}^3$

$t_a = 4 \text{ mm}$

$Q_b = (14)(4)(28) = 1568 \text{ mm}^3$

$t_b = 4 \text{ mm}$

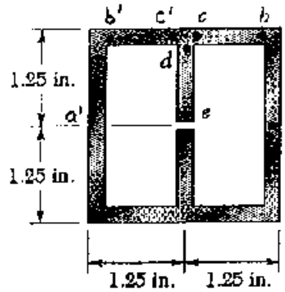
$\tau_c = 75 \text{ MPa}$

$$\tau_a = \frac{Q_a}{Q_c}\cdot\frac{t_c}{t_a}\tau_c = \frac{1568}{4260}\cdot\frac{6}{4}\cdot 75 = 41.4 \text{ MPa}$$ ◄

$$\tau_b = \frac{Q_b}{Q_c}\cdot\frac{t_c}{t_b}\tau_c = \frac{1568}{4260}\cdot\frac{6}{4}\cdot 75 = 41.4 \text{ MPa}$$ ◄

Problem 6.37

6.37 and 6.38 The extruded beam shown has a uniform wall thickness of $\frac{1}{8}$ in. Knowing that the vertical shear in the beam is 2 kips, determine the shearing stress at each of the five points indicated.



$$I = \frac{1}{12}(2.50)(2.50)^3 - \frac{1}{12}(2.125)(2.25)^3 = 1.2382 \text{ in}^4$$

Add symmetric points c′, b′, and a′.

$Q_e = 0$

$Q_d = (0.125)(1.125)\left(\frac{1.125}{2}\right) = 0.07910 \text{ in}^3$ $\quad t_d = 0.125$ in.

$Q_c = Q_e + (0.125)^2(1.1875) = 0.09765 \text{ in}^3$ $\quad t_c = 0.25$ in.

$Q_b = Q_c + (2)(1.0625)(0.125)(1.1875) = 0.41308 \text{ in}^3$ $\quad t_b = 0.25$ in.

$Q_a = Q_b + (2)(0.125)(1.25)\left(\frac{1.25}{2}\right) = 0.60839 \text{ in}^3$ $\quad t_a = 0.25$ in.

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(2)(0.60839)}{(1.2382)(0.25)} = 3.93 \text{ ksi}$$

$$\tau_b = \frac{VQ_b}{It_b} = \frac{(2)(0.41308)}{(1.2382)(0.25)} = 2.67 \text{ ksi}$$

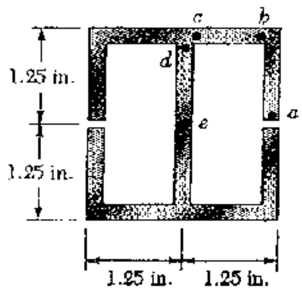
$$\tau_c = \frac{VQ_c}{It_c} = \frac{(2)(0.09765)}{(1.2382)(0.25)} = 0.63 \text{ ksi}$$

$$\tau_d = \frac{VQ_d}{It_d} = \frac{(2)(0.07910)}{(1.2382)(0.125)} = 1.02 \text{ ksi}$$

$$\tau_e = \frac{VQ_e}{It_e} = 0$$

Problem 6.38

6.37 and 6.38 The extruded beam shown has a uniform wall thickness of $\frac{1}{8}$ in. Knowing that the vertical shear in the beam is 2 kips, determine the shearing stress at each of the five points indicated.



$$I = \frac{1}{12}(2.50)(2.50)^3 - \frac{1}{12}(2.125)(2.25)^3 = 1.2382 \text{ in}^4$$

$t = 0.125$ in. at all sections.

$V = 2$ kips

$Q_a = 0 \qquad \tau_a = \frac{VQ_a}{It} = 0$

$Q_b = (0.125)(1.25)\left(\frac{1.25}{2}\right) = 0.09766 \text{ in}^3$

$$\tau_b = \frac{VQ_b}{It} = \frac{(2)(0.09766)}{(1.2382)(0.125)} = 1.26 \text{ ksi}$$

$Q_c = Q_b + (1.0625)(0.125)(1.1875) = 0.25537 \text{ in}^3$

$$\tau_c = \frac{VQ_c}{It} = \frac{(2)(0.25537)}{(1.2382)(0.125)} = 3.30 \text{ ksi}$$

$Q_d = 2Q_c + (0.125)^2(1.1875) = 0.52929$

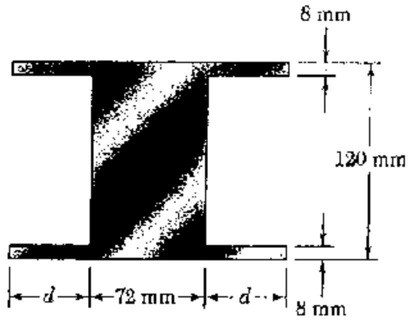
$$\tau_d = \frac{VQ_d}{It} = \frac{(2)(0.52929)}{(1.2382)(0.125)} = 6.84 \text{ ksi}$$

$Q_e = Q_d + (0.125)(1.125)\left(\frac{1.125}{2}\right) = 0.60839$

$$\tau_e = \frac{VQ}{It} = \frac{(2)(0.60839)}{(1.2382)(0.125)} = 7.86 \text{ ksi}$$

Problem 6.39

6.39 The vertical shear is 25 kN in a beam having the cross section shown. Knowing that $d = 50$ mm, determine the shearing stress at (*a*) point *a*, (*b*) point *b*.

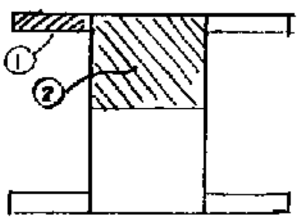


$$I_1 = \frac{1}{12}(50)(8)^3 + (50)(8)(56)^2 = 1.25653 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{3}(72)(60)^3 = 5.184 \times 10^6 \text{ mm}^4$$

$$I = 4I_1 + 2I_2 = 15.3933 \times 10^6 \text{ mm}^4 = 15.3933 \times 10^{-6} \text{ m}^4$$

$$Q_1 = A_1\bar{y}_1 = (50)(8)(56) = 22.4 \times 10^3 \text{ mm}^3 = 22.4 \times 10^{-6} \text{ m}^3$$



$$Q_2 = A_2\bar{y}_2 = (72)(60)(30) = 129.6 \times 10^3 \text{ mm}^3 = 129.6 \times 10^{-6} \text{ m}^3$$

(a) $Q_a = Q_1$ $\qquad t_a = 8 \text{ mm} = 8 \times 10^{-3} \text{ m}$

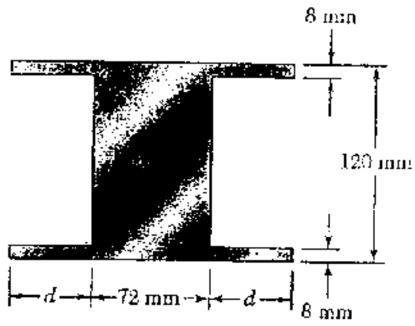
$$\tau_a = \frac{VQ_a}{It_a} = \frac{(25 \times 10^3)(22.4 \times 10^{-6})}{(15.3933 \times 10^{-6})(8 \times 10^{-3})} = 4.55 \times 10^6 \text{ Pa} = 4.55 \text{ MPa}$$

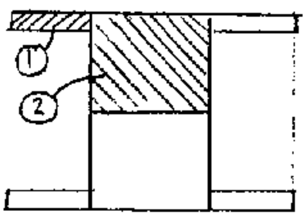
(b) $Q_b = 2Q_1 + Q_2 = 174.4 \times 10^{-6} \text{ m}^3$

$t_b = 72 \text{ mm} = 72 \times 10^{-3} \text{ m}$

$$\tau_b = \frac{VQ_b}{It_b} = \frac{(25 \times 10^3)(174.4 \times 10^{-6})}{(15.3933 \times 10^{-6})(72 \times 10^{-3})} = 3.93 \times 10^6 \text{ Pa} = 3.93 \text{ MPa}$$

Problem 6.40





6.40 The vertical shear is 25 kN in a beam having the cross section shown. Determine (*a*) the distance *d* for which $\tau_a = \tau_b$, (*b*) the corresponding shearing stress at points *a* and *b*.

$$Q_1 = (d)(8)(56) = 448\, d \ \text{mm}^3$$

$$Q_2 = (72)(60)(30) = 129.6\times 10^3 \ \text{mm}^3$$

$$Q_a = Q_1 = 448\, d \qquad t_a = 8 \ \text{mm}$$

$$Q_b = 2Q_1 + Q_2 = 896\, d + 129.6\times 10^3 \ \text{mm}^3$$

$$t_b = 72 \ \text{mm}$$

$$\tau_a = \frac{VQ_a}{I t_a} \qquad \tau_b = \frac{VQ_b}{I t_b}$$

Since $\tau_a = \tau_b$ $\quad \dfrac{Q_a}{t_a} = \dfrac{Q_b}{t_b}$

(a) $$\frac{448\, d}{8} = \frac{896\, d + 129.4\times 10^3}{72} \qquad d = 41.3 \ \text{mm} \blacktriangleleft$$

(b) $$I_1 = \tfrac{1}{12}(d)(8)^3 + (d)(8)(56)^2 = 25.131\, d = 1.03696\times 10^6 \ \text{mm}^4$$

$$I_2 = \tfrac{1}{3}(72)(60)^3 = 5.184\times 10^6 \ \text{mm}^4$$

$$I = 4I_1 + 2I_2 = 14.5158\times 10^6 \ \text{mm}^4 = 14.5158\times 10^{-6} \ \text{m}^4$$

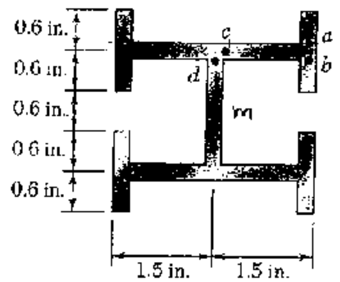
$$Q_a = 448\, d = (448)(41.263) = 18.4857\times 10^3 \ \text{mm}^3 = 18.4857\times 10^{-6} \ \text{m}^3$$

$$\tau_a = \frac{VQ_a}{I t_a} = \frac{(25\times 10^3)(18.4857\times 10^{-6})}{(14.5158\times 10^{-6})(8\times 10^{-3})} = 3.98\times 10^6 \ \text{Pa}$$

$$= 3.98 \ \text{MPa} \blacktriangleleft$$

Problem 6.41

6.41 An extruded beam has the cross section shown and a uniform wall thickness of 0.20 in. Knowing that a given vertical shear **V** causes a maximum shearing stress τ = 9 ksi, determine the shearing stress at the four points indicated.



$$Q_a = (0.2)(0.5)(1.5 - 0.25) = 0.125 \text{ in}^3$$

$$Q_b = (0.2)(0.5)(0.3 + 0.25) = 0.055 \text{ in}^3$$

$$Q_c = Q_a + Q_b + (1.4)(0.2)(0.9) = 0.432 \text{ in}^3$$

$$Q_d = 2Q_a + 2Q_b + (3.0)(0.2)(0.9) = 0.900 \text{ in}^3$$

$$Q_m = Q_d + (0.2)(0.8)(0.4) = 0.964 \text{ in}^3$$

$\tau = \dfrac{VQ}{It}$ =

Since V, I, and t are constant, τ is proportional to Q.

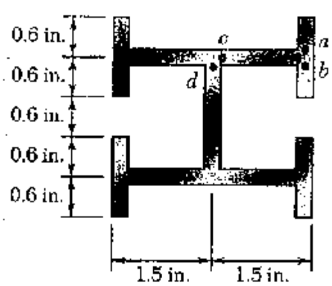
$$\frac{\tau_a}{0.125} = \frac{\tau_b}{0.055} = \frac{\tau_c}{0.432} = \frac{\tau_d}{0.900} = \frac{\tau_m}{0.964} = \frac{9}{0.964}$$

$\tau_a = 1.167$ ksi ; $\tau_b = 0.513$ ksi ; $\tau_c = 4.03$ ksi ; $\tau_d = 8.40$ ksi ◀

Problem 6.42

6.42 Solve Prob. 6.41 assuming that the beam is subjected to a horizontal shear **V**.

6.41 An extruded beam has the cross section shown and a uniform wall thickness of 0.20 in. Knowing that a given vertical shear **V** causes a maximum shearing stress τ = 9 ksi, determine the shearing stress at the four points indicated.



$$Q_a = (0.5)(0.2)(1.4) = 0.140 \text{ in}^3$$

$$Q_b = (0.5)(0.2)(1.4) = 0.140 \text{ in}^2$$

$$Q_c = Q_a + Q_b + (0.2)(1.4)(0.8) = 0.504 \text{ in}^3$$

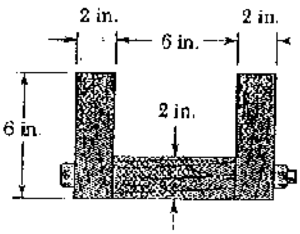
$Q_d = 0$ $\qquad$ $Q_m = Q_c = 0.504 \text{ in}^3$

$\tau = \dfrac{VQ}{It}$ $\qquad$ Since V, I, and t are constant, τ is proportional to Q.

$$\frac{\tau_a}{0.140} = \frac{\tau_b}{0.140} = \frac{\tau_c}{0.504} = \frac{\tau_d}{0} = \frac{\tau_m}{Q_m} = \frac{9}{0.504}$$

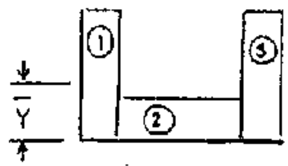
$\tau_a = 2.50$ ksi ; $\tau_b = 2.50$ ksi , $\tau_c = 9.00$ ksi , $\tau_d = 0$ ◀

Problem 6.43



6.43 A beam consists of three planks connected as shown by $\frac{3}{8}$-in.-diameter bolts spaced every 12 in. along the longitudinal axis of the beam. Knowing that the beam is subjected to a 2500-lb vertical shear, determine the average shearing stress in the bolts.

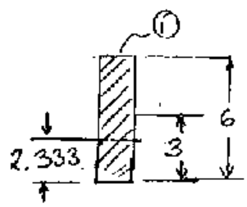
Locate neutral axis and compute moment of inertia.



Part	A (in²)	$\bar{y}$ (in.)	$A\bar{y}$ in³	d (in)	Ad^2 (in⁴)	$\bar{I}$ (in⁴)
1	12	3	36	0.667	5.333	36
2	12	1	12	1.333	21.333	4
3	12	3	36	0.667	5.333	36
Σ	36		84		32	76

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{84}{36} = 2.333 \text{ in.}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 108 \text{ in}^4$$



$$Q = A_1 \bar{y}_1 = (2)(6)(3 - 2.333) = 8 \text{ in}^3$$

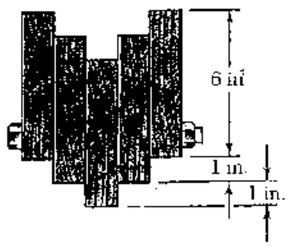
$$q = \frac{VQ}{I} = \frac{(2500)(8)}{108} = 185.2 \text{ lb/in}$$

$$F_{bolt} = qs = (185.2)(12) = 2.222 \times 10^3 \text{ lb.}$$

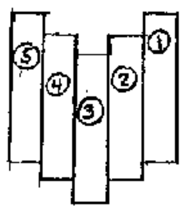
$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4}\left(\frac{3}{8}\right)^2 = 0.1104 \text{ in}^2$$

$$\tau_{bolt} = \frac{F_{bolt}}{A_{bolt}} = \frac{2.222 \times 10^3}{0.1104} = 20.1 \times 10^3 \text{ psi} = 20.1 \text{ ksi}$$

Problem 6.44



6.44 A beam consists of five planks of 1.5 × 6-in. cross section connected by steel bolts with a longitudinal spacing of 9 in. Knowing that the shear in the beam is vertical and equal to 2000 lb. and that the allowable average shearing stress in each bolt is 7500 psi, determine the smallest permissible bolt diameter that may be used.



Part	A (in^2)	$\bar{y}_0$ (in)	$A\bar{y}_0$ (in^3)	$\bar{y}$ (in.)	$A\bar{y}^2$ (in^4)	$\bar{I}$ in^4
①	9	5	45	0.8	5.76	27
②	9	4	36	−0.2	0.36	27
③	9	3	27	−1.2	12.96	27
④	9	4	36	−0.2	0.36	27
⑤	9	5	45	0.8	5.76	27
Σ	45		189		25.20	135

$$\bar{Y}_0 = \frac{\Sigma A y}{\Sigma A} = \frac{189}{45} = 4.2 \text{ in.}$$

$$I = \Sigma A d^2 + \Sigma \bar{I} = 160.2 \text{ in}^4$$

Between ① and ②: $\quad Q_{12} = Q_1 = A\bar{y}_1 = (9)(0.8) = 7.2 \text{ in}^3$

Between ② and ③: $\quad Q_{23} = Q_1 + A\bar{y}_2 = 7.2 + (9)(-0.2) = 5.4 \text{ in}^3$

$q = \dfrac{VQ}{I}$ $\qquad$ Maximum q is based on $Q_{12} = 7.2 \text{ in}^3$

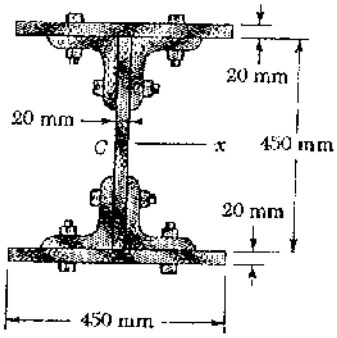
$$q = \frac{(2000)(7.2)}{160.2} = 89.888 \text{ lb/in}$$

$$F_{bolt} = qs = (89.888)(9) = 809 \text{ lb}$$

$\tau_{bolt} = \dfrac{F_{bolt}}{A_{bolt}}$ $\qquad A_{bolt} = \dfrac{F_{bolt}}{\tau_{bolt}} = \dfrac{809}{7500} = 0.1079 \text{ in}^2$

$A_{bolt} = \dfrac{\pi}{4} d_{bolt}^2$ $\qquad d_{bolt} = \sqrt{\dfrac{4A_{bolt}}{\pi}} = \sqrt{\dfrac{(4)(0.1079)}{\pi}} = 0.371 \text{ in.}$ ◄

Problem 6.45



6.45 Two 20 × 450-mm steel plates are bolted to four L 152 × 152 × 19.0 angles to form a beam with the cross section shown. The bolts have a 22-mm diameter and are spaced longitudinally every 125 mm. Knowing that the allowable average shearing stress in the bolts is 90 MPa, determine the largest permissible vertical shear in the beam. (*Given:* $I_x = 1896 \times 10^6\ \text{mm}^4$).

Flange: $I_f = \frac{1}{12}(450)(20)^3 + (450)(20)(235)^2 = 497.3 \times 10^6\ \text{mm}^4$

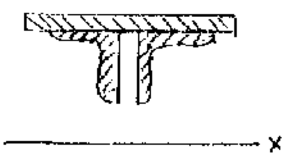
Web: $I_w = \frac{1}{12}(20)(450)^3 = 151.9 \times 10^6\ \text{mm}^4$

Angle: $\bar{I} = 11.6 \times 10^6\ \text{m}^4$, $A = 5420\ \text{mm}^2$

$y = 44.9\ \text{mm}$ $\quad d = 225 - 44.9 = 180.1\ \text{mm}$

$$I_a = \bar{I} + Ad^2$$
$$= 11.6 \times 10^6 + (5420)(180.1)^2 = 187.4 \times 10^6\ \text{mm}^4$$

$$I = 2I_f + I_w + 4I_a = 1896 \times 10^6\ \text{mm}^4 = 1896 \times 10^{-6}\ \text{m}^4$$



$$Q_f = (450)(20)(235) = 2115 \times 10^3\ \text{mm}^3$$

$$Q_a = (5420)(180.1) = 976 \times 10^3\ \text{mm}^3$$

$$Q = Q_f + 2Q_a = 4067 \times 10^3\ \text{mm}^3 = 4067 \times 10^{-6}\ \text{m}^3$$

$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4}(22)^2 = 380.1\ \text{mm}^2 = 380.1 \times 10^{-6}\ \text{m}^2$$

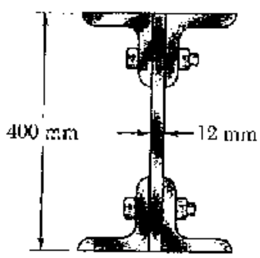
$$F_{bolt} = 2\tau_{bolt} A_b = (2)(90 \times 10^6)(380.1 \times 10^{-6}) = 68.42 \times 10^3\ \text{N}$$

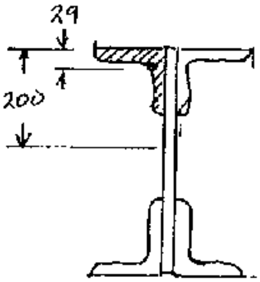
$$q_{all} = \frac{F_{bolt}}{s} = \frac{68.42 \times 10^3}{0.125} = 547.36 \times 10^3\ \text{N/m}$$

$$q = \frac{VQ}{I} \qquad V_{all} = \frac{I q_{all}}{Q} = \frac{(1896 \times 10^{-6})(547.36 \times 10^3)}{4067 \times 10^{-6}} = 255 \times 10^3\ \text{N}$$

$$= 255\ \text{kN} \blacktriangleleft$$

Problem 6.46





6.46 Four 102 × 102 × 9.5 steel angles shapes and a 12 × 400-mm steel plate are bolted together to form a beam with the cross section shown. The bolts are of 22-mm diameter and are spaced longitudinally every 120 mm. Knowing that the beam is subjected to a vertical shear of 240 kN, determine the average shearing stress in each bolt.

Angle: $A = 1850 \text{ mm}^2, \quad \bar{I} = 1.83 \times 10^6 \text{ mm}^4, \quad y = 29 \text{ mm}$

$d = 200 - 29 = 171 \text{ mm}$

$I_a = \bar{I} + Ad^2 = 56.726 \times 10^6 \text{ mm}^4$

Plate: $I_p = \frac{1}{12}(12)(400)^3 = 64 \times 10^6 \text{ mm}^4$

$I = 4I_a + I_p = 287.7 \times 10^6 \text{ mm}^4 = 287.7 \times 10^{-6} \text{ m}^4$

$Q = (1850)(171) = 316.35 \times 10^3 \text{ mm}^3 = 316.35 \times 10^{-6} \text{ m}^3$

$q = \dfrac{VQ}{I} = \dfrac{(240 \times 10^3)(316.35 \times 10^{-6})}{287.7 \times 10^{-6}} = 263.9 \times 10^3 \text{ N/m}$

$F_{bolt} = qs = (263.9 \times 10^3)(120 \times 10^{-3}) = 31.668 \times 10^3 \text{ N}$

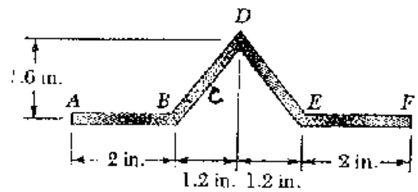
$A_{bolt} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(22)^2 = 380.13 \text{ mm}^2 = 380.13 \times 10^{-6} \text{ m}^2$

$\tau_{bolt} = \dfrac{F_{bolt}}{A_{bolt}} = \dfrac{31.668 \times 10^3}{380.13 \times 10^{-6}} = 83.3 \times 10^6 \text{ Pa}$

83.3 MPa ◄

Problem 6.47

6.47 A plate of $\frac{1}{4}$-in. thickness is corrugated as shown and then used as a beam. For a vertical shear of 1.2 kips, determine (*a*) the maximum shearing stress in the section, (*b*) the shearing stress at point *B*. Also sketch the shear flow in the cross section.



$l_{BD} = \sqrt{(1.2)^2 + (1.6)^2} = 2.0 \text{ in.}$

$A_{BD} = (0.25)(2.0) = 0.5 \text{ in}^2$

Locate neutral axis and compute moment of inertia.

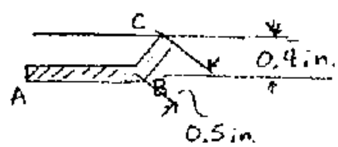
Part	A (in²)	$\bar{y}$ (in)	$A\bar{y}$ (in³)	d (in)	Ad^2 (in⁴)	$\bar{I}$ (in⁴)
AB	0.5	0	0	0.4	0.080	neglect
BD	0.5	0.8	0.4	0.4	0.080	*0.1067
DE	0.5	0.8	0.4	0.4	0.080	*0.1067
EF	0.5	0	0	0.4	0.080	neglect
Σ	2.0		0.8		0.320	0.2133

$\bar{Y} = \dfrac{\Sigma A\bar{y}}{\Sigma A} = \dfrac{0.8}{2.0} = 0.4 \text{ in.}$

$* \ \frac{1}{12} A_{BD} h^2 = \frac{1}{12}(0.5)(1.6)^2 = 0.1067 \text{ in}^4$

$I = \Sigma Ad^2 + \Sigma \bar{I} = 0.5333 \text{ in}^4$

(a)



$Q_m = Q_{AB} + Q_{BC}$

$Q_{AB} = (2)(0.25)(0.4) = 0.2 \text{ in}^3$

$Q_{BC} = (0.5)(0.25)(0.2) = 0.025 \text{ in}^3$

$Q_m = 0.225 \text{ in}^3$

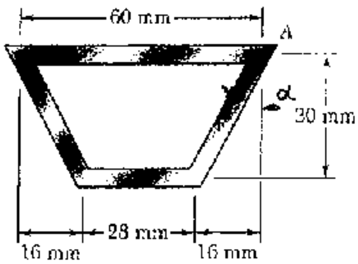
$$\tau_m = \frac{VQ_m}{It} = \frac{(1.2)(0.225)}{(0.5333)(0.25)} = 2.025 \text{ ksi}$$

(b) $Q_B = Q_{AB} = 0.2 \text{ in}^3$

$$\tau_B = \frac{VQ_B}{It} = \frac{(1.2)(0.2)}{(0.5333)(0.25)} = 1.80 \text{ ksi}$$

$\tau_D = 0$

Problem 6.48



6.48 An extruded beam has the cross section shown and a uniform wall thickness of 3 mm. For a vertical shear of 10 kN, determine (*a*) the shearing stress at point *A*, (*b*) the maximum shearing stress in the beam. Also sketch the shear flow in the cross section.

$\tan\alpha = \frac{16}{30} \qquad \alpha = 28.07°$

Side: $A = (3\sec\alpha)(30) = 102 \text{ mm}^2$

$\bar{I} = \frac{1}{12}(3\sec\alpha)(30)^3 = 7.6498 \times 10^3 \text{ mm}^4$

Part	$A\,(\text{mm}^2)$	$\bar{y}_o\,(\text{mm})$	$A\bar{y}\,(10^3\,\text{mm}^3)$	$d\,(\text{mm})$	$Ad^2\,(10^3\,\text{mm}^4)$	$\bar{I}\,(10^3\,\text{mm}^4)$
Top	180	30	5.4	11.932	25.627	neglect
Side	102	15	1.53	3.077	0.966	7.6498
Side	102	15	1.53	3.077	0.966	7.6498
Bot	84	0	0	18.077	27.449	neglect
Σ	468		8.46		55.008	15.2996

$$Y_o = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{8.46\times10^3}{468} = 18.077 \text{ mm}$$

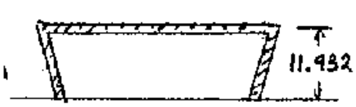
$$I = \Sigma Ad^2 + \Sigma\bar{I} = 70.31\times10^3 \text{ mm}^4 = 70.31\times10^{-9} \text{ m}^4$$

(a) $Q_A = (180)(11.932) = 2.14776\times10^3 \text{ mm}^3 = 2.14776\times10^{-6} \text{ m}^3$

$t = (2)(3\times10^{-3}) = 6\times10^{-3} \text{ m}$

$$\tau_A = \frac{VQ}{It} = \frac{(10\times10^3)(2.14776\times10^{-6})}{(70.31\times10^{-9})(6\times10^{-6})} = 50.9\times10^6 \text{ Pa} = 50.9 \text{ MPa}$$

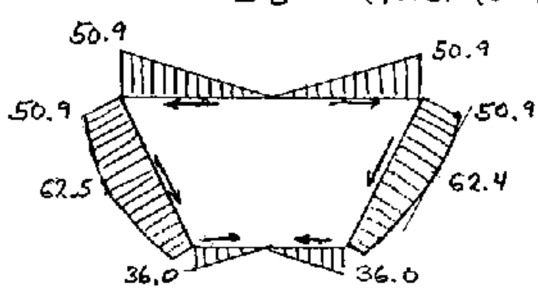
(b)



$$Q_m = Q_A + (2)(3\sec\alpha)(11.932)(\tfrac{1}{2}\times11.932)$$
$$= 2.14776\times10^3 + 484.06 = 2.6318\times10^3 \text{ mm}^3$$
$$= 2.6318\times10^{-6} \text{ m}^3$$

$t = 6\times10^{-3} \text{ m}$

$$\tau_m = \frac{VQ_m}{It} = \frac{(10\times10^3)(2.6318\times10^{-6})}{(70.31\times10^{-9})(6\times10^{-3})} = 62.4\times10^6 \text{ Pa} = 62.4 \text{ MPa}$$

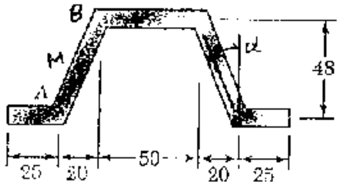


Shearing stress (MPa)

$Q_B = (28)(3)(18.077) = 1.51847\times10^3 \text{ mm}^3$

$$\tau_B = \frac{Q_B}{Q_A}\tau_A = \frac{1.51847\times10^3}{2.14776\times10^3}(50.9)$$
$$= 36.0 \text{ MPa}$$

Problem 6.49



Dimensions in mm

6.49 A plate of 4-mm thickness is bent as shown and then used as a beam. For a vertical shear of 12 kN, determine (*a*) the shearing stress at point *A*, (*b*) the maximum shearing stress in the beam. Also sketch the shear flow in the cross section.

$\tan\alpha = \frac{20}{48} \qquad \alpha = 22.62°$

Slanted side: $A_S = (4\sec\alpha)(48) = 208\ \text{mm}^2$

$\bar{I}_S = \frac{1}{12}(4\sec\alpha)(48)^3 = 39.936\times10^3\ \text{mm}^4$

Top: $I_T = \frac{1}{12}(50)(4)^3 + (50)(4)(24)^2 = 115.46\times10^3\ \text{mm}^4$

Bottom: $I_B = I_T = 115.46\times10^3\ \text{mm}^4$

$I = 2\bar{I}_S + I_T + I_B = 310.8\times10^3\ \text{mm}^4 = 310.8\times10^{-9}\ \text{m}^4$

(a) $Q_A = (25)(4)(24) = 2.4\times10^3\ \text{mm}^3 = 2.4\times10^{-6}\ \text{m}^3$

$t = 4\ \text{mm} = 4\times10^{-3}\ \text{m}$

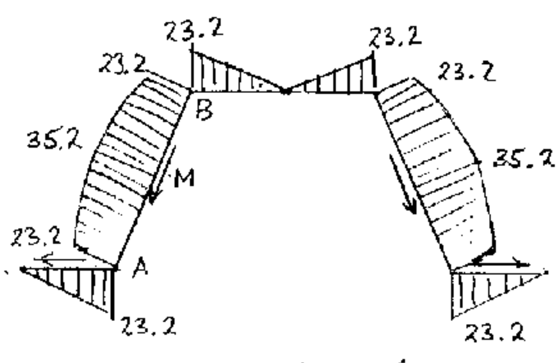
$\tau_A = \frac{VQ_A}{It} = \frac{(12\times10^3)(2.4\times10^{-6})}{(310.8\times10^{-9})(4\times10^{-3})} = 23.2\times10^6\ \text{Pa} = 23.2\ \text{MPa}$ ◄

(b) Maximum shearing occurs at point M, 24 mm above the bottom.

$Q_M = Q_A + (4\sec\alpha)(24)(12) = 2.4\times10^3 + 1.248\times10^3 = 3.648\times10^3\ \text{mm}^3 = 3.648\times10^{-6}\ \text{m}^3$

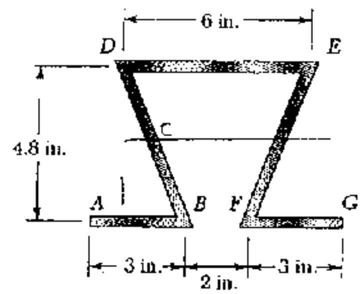
$\tau_M = \frac{VQ_M}{It} = \frac{(12\times10^3)(3.648\times10^{-6})}{(310.8\times10^{-9})(4\times10^{-3})} = 35.2\times10^6\ \text{Pa} = 35.2\ \text{MPa}$ ◄

$Q_B = Q_A \qquad \tau_B = \tau_A = 23.2\ \text{MPa}$



Shearing stress (MPa)

Problem 6.50



6.50 A plate of thickness t is bent as shown and then used as a beam. For a vertical shear of 600 lb, determine (*a*) the thickness t for which the maximum shearing stress is 300 psi, (*b*) the corresponding shearing stress at point E. Also sketch the shear flow in the cross section.

$$L_{BD} = L_{EF} = \sqrt{4.8^2 + 2^2} = 5.2 \text{ in.}$$

Neutral axis lies at 2.4 in. above AB.

Calculate I.

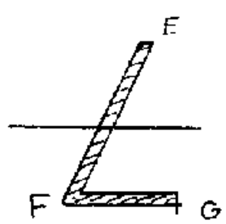
$$I_{AB} = (3t)(2.4)^2 = 17.28\,t$$
$$I_{BD} = \tfrac{1}{12}(5.2\,t)(4.8)^2 = 9.984\,t$$
$$I_{DE} = (6\,t)(2.4)^2 = 34.56\,t$$
$$I_{EF} = I_{DB} = 9.984\,t$$
$$I_{FG} = I_{AB} = 17.28\,t$$

$$I = \Sigma I = 89.09\,t$$

(a) At point C: $Q_c = Q_{AB} + Q_{BC} = (3t)(2.4) + (2.6\,t)(1.2) = 10.32\,t$

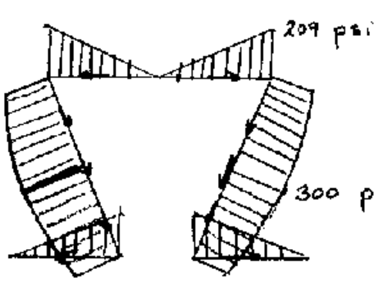
$$\tau = \frac{VQ_c}{It} \quad \therefore \quad t = \frac{VQ}{\tau I} = \frac{(600)(10.32\,t)}{(300)(89.09\,t)} = 0.23168 \text{ in.}$$

(b) $I = (89.09)(0.23168) = 20.64 \text{ in}^3$



$$Q_E = Q_{EF} + Q_{FG}$$
$$= 0 + (3)(0.23168)(2.4) = 1.668 \text{ in}^3$$

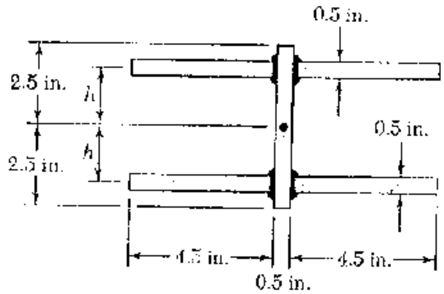
$$\tau_E = \frac{VQ_E}{It} = \frac{(600)(1.668)}{(20.64)(0.23168)} = 209 \text{ psi}$$



Shearing stress (psi)

Problem 6.51

6.51 The design of a beam requires welding four horizontal plates to a vertical 0.5 × 5-in. plate as shown. For a vertical shear **V**, determine the dimension *h* for which the shear flow through the welded surface is maximum.



Horizontal plate:

$$I_h = \frac{1}{12}(4.5)(0.5)^3 + (4.5)(0.5)h^2$$

$$= 0.046875 + 2.25\,h^2$$

Vertical plate: $I_v = \frac{1}{12}(0.5)(5)^3 = 5.2083 \text{ in}^4$

Whole section: $I = 4I_h + I_v = 9h^2 + 5.39583 \text{ in}^4$

For one horizontal plate $Q = (4.5)(0.5)h = 2.25\,h \text{ in}^3$

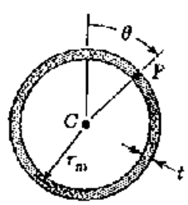
$$q = \frac{VQ}{I} = \frac{2.25\,Vh}{9h^2 + 5.39583}$$

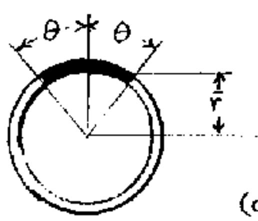
To maximize q set $\frac{dq}{dh} = 0$

$$2.25V\frac{(9h^2+5.39583) - 18h^2}{(9h^2+5.39583)^2} = 0 \qquad h = 0.774 \text{ in.}$$

Problem 6.52

6.52 (*a*) Determine the shearing stress at point *P* of a thin-walled pipe of the cross section shown caused by a vertical shear **V**. (*b*) Show that the maximum shearing stress occurs for $\theta = 90°$ and is equal to $2V/A$, where A is the cross-sectional area of the pipe.





$$A = 2\pi r_m t \qquad J = Ar_m^2 = 2\pi r_m^3 t \qquad I = \tfrac{1}{2}J = \pi r_m^3 t$$

$$\bar{r} = \frac{\sin\theta}{\theta} \quad \text{for a circular arc}$$

$$A_p = 2r\theta t$$

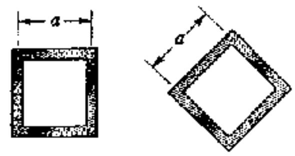
$$Q_p = A_p\bar{r} = 2rt\sin\theta$$

(a) $$\tau_p = \frac{VQ_p}{I(2t)} = \frac{(V)(2rt\sin\theta)}{(\pi r_m^3 t)(2t)} = \frac{V\sin\theta}{\pi r_m t}$$

(b) $$\tau_m = \frac{2V\sin\frac{\pi}{2}}{2\pi r_m t} = \frac{2V}{A}$$

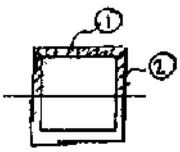
Problem 6.53

6.53 and 6.54 An extruded beam has a uniform wall thickness t. Denoting by **V** the vertical shear and by A the cross-sectional area of the beam, express the maximum shearing stress as $\tau_{max} = k(V/A)$ and determine the constant k for each of the two orientations shown.



(a) (b)

(a)



$I_1 = (at)\left(\frac{a}{2}\right)^2 = \frac{1}{4}a^3t$

$I_2 = \frac{1}{3}t\left(\frac{a}{2}\right)^3 = \frac{1}{24}a^3t$

$I = 2I_1 + 4I_2 = \frac{2}{3}a^3t$

$Q_1 = (at)\left(\frac{a}{2}\right) = \frac{1}{2}a^2t$

$Q_2 = \left(\frac{1}{2}at\right)\left(\frac{a}{4}\right) = \frac{1}{8}a^2t$

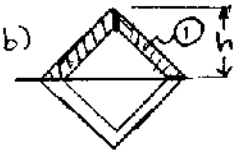
$Q = Q_1 + 2Q_2 = \frac{3}{4}a^2t$

$\tau_{max} = \frac{VQ}{I(2t)} = \frac{V\left(\frac{3}{4}a^2t\right)}{\left(\frac{2}{3}a^3t\right)(2t)} =$

$= \frac{9}{16}\frac{V}{at} = \frac{9}{4}\frac{V}{4at} = \frac{9}{4}\frac{V}{A}$

$= k\frac{V}{A} \quad \therefore \quad k = \frac{9}{4} = 2.25$ ◄

(b)



$h = \frac{1}{2}\sqrt{2}\,a$

$I_1 = \frac{1}{3}A_1h^2 = \left(\frac{1}{3}at\right)\left(\frac{\sqrt{2}}{2}a\right)^2$

$= \frac{1}{6}a^3t$

$I = 4I_1 = \frac{2}{3}a^3t$

$Q_1 = at\left(\frac{h}{2}\right) = \frac{1}{4}\sqrt{2}\,a^2t$

$Q = 2Q_1 = \frac{1}{2}\sqrt{2}\,a^2t$

$\tau_{max} = \frac{VQ}{I(2t)} = \frac{V\left(\frac{1}{2}\sqrt{2}\,a^2t\right)}{\left(\frac{2}{3}a^3t\right)(2t)}$

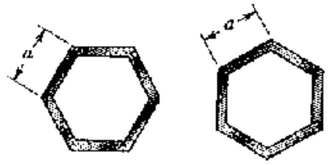
$= \frac{3\sqrt{2}}{8}\frac{V}{at} = \frac{3\sqrt{2}}{2}\frac{V}{4at}$

$= \frac{3\sqrt{2}}{2}\frac{V}{A} = k\frac{V}{A}$

$k = \frac{3\sqrt{2}}{2} = 2.12$ ◄

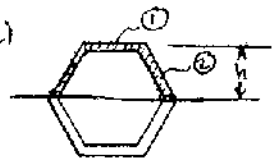
Problem 6.54

6.53 and 6.54 An extruded beam has a uniform wall thickness t. Denoting by **V** the vertical shear and by A the cross-sectional area of the beam, express the maximum shearing stress as $\tau_{max} = k(V/A)$ and determine the constant k for each of the two orientations shown.



(a) (b)

(a)



$h = \frac{\sqrt{3}}{2} a$

$A_1 = A_2 = at$

$I_1 = A_1 h^2 = a t h^2 = \frac{3}{4} a^3 t$

$I_2 = \frac{1}{3} A_2 h^2 = \frac{1}{3} a t \frac{3}{4} a^2 = \frac{1}{4} a^3 t$

$I = 2 I_1 + 4 I_2 = \frac{5}{2} a^3 t$

$Q_1 = A_1 h = \frac{\sqrt{3}}{2} a^2 t$

$Q_2 = A_2 \frac{h}{2} = \frac{\sqrt{3}}{4} a^2 t$

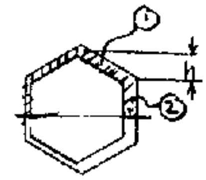
$Q_m = Q_1 + 2 Q_2 = \sqrt{3}\, a^2 t$

$\tau_m = \frac{VQ}{I(2t)} = \frac{V \sqrt{3} a^2 t}{(\frac{5}{2} a^3 t) 2t} = \frac{\sqrt{3}}{5} \frac{V}{at}$

$= \frac{6\sqrt{3}}{5} \frac{V}{6at} = \frac{6\sqrt{3}}{5} \frac{V}{A} = k \frac{V}{A}$

$k = \frac{6\sqrt{3}}{5} = 2.08$ ◄

(b)



$h = \frac{a}{2}$

$A_1 = at \qquad A_2 = \frac{1}{2} at$

$I_1 = \bar{I}_1 + A_1 d^2$

$= \frac{1}{12} a t h^2 + a t \left(\frac{a}{2} + \frac{h}{2}\right)^2$

$= \frac{1}{48} a^3 t + \frac{9}{16} a^3 t = \frac{7}{12} a^3 t$

$I_2 = \frac{1}{3} t \left(\frac{a}{2}\right)^3 = \frac{1}{24} a^3 t$

$I = 4 I_1 + 4 I_2 = \frac{5}{2} a^3 t$

$Q_1 = at \left(\frac{a}{2} + \frac{h}{2}\right) = \frac{3}{4} a^2 t$

$Q_2 = \left(\frac{1}{2} at\right)\left(\frac{a}{4}\right) = \frac{1}{8} a^2 t$

$Q = 2 Q_1 + 2 Q_2 = \frac{7}{4} a^3 t$

$\tau_m = \frac{VQ}{I(2t)} = \frac{V \cdot \frac{7}{4} a^3 t}{(\frac{5}{2} a^3 t)(2t)}$

$= \frac{7}{20} \frac{V}{at} = \frac{42}{20} \frac{V}{6at} = \frac{21}{10} \frac{V}{A}$

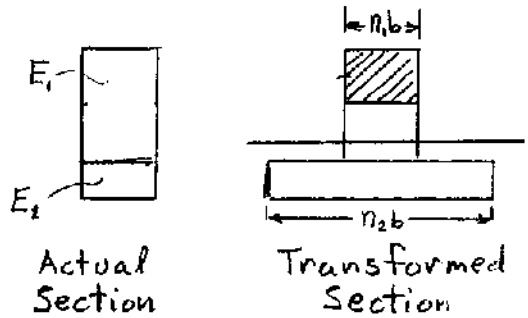
$= k \frac{V}{A} \qquad k = \frac{21}{10} = 2.10$ ◄

Problem 6.55

6.55 For a beam made of two or more materials with different moduli of elasticity, show that Eq. (6.6)

$$\tau_{ave} = \frac{VQ}{It}$$

remains valid provided that both Q and I are computed by using the transformed section of the beam (see Sect 4.6) and provided further that t is the actual width of the beam where τ_{ave} is computed.



Let E_{ref} be a reference modulus of elasticity

$$n_1 = \frac{E_1}{E_{ref}}, \quad n_2 = \frac{E_2}{E_{ref}}, \quad \text{etc.}$$

Widths b of actual section are multiplied by n's to obtain the transformed section. The bending stress distribution in the cross section is given by

$$\sigma_x = -\frac{nMy}{I}$$

where I is the moment of inertia of the transformed cross section and y is measured from the centroid of the transformed section.

The horizontal shearing force over length Δx is

$$\Delta H = -\int(\Delta\sigma_x)\,dA = \int \frac{n(\Delta M)y}{I}\,dA = \frac{(\Delta M)}{I}\int ny\,dA = \frac{Q(\Delta M)}{I}$$

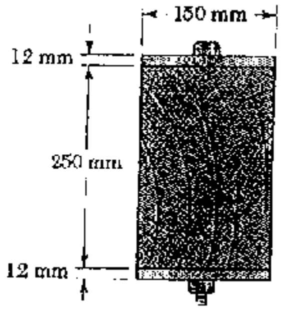
$Q = \int ny\,dA$ = first moment of transformed section.

Shear flow: $\quad q = \frac{\Delta H}{\Delta x} = \frac{\Delta M}{\Delta x}\frac{Q}{I} = \frac{VQ}{I}$

q is distributed over actual width t, thus $\tau = \frac{q}{t}$.

$$\tau = \frac{VQ}{It}$$

Problem 6.56



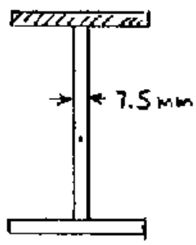
6.56 A composite beam is made by attaching the timber and steel portions shown with bolts of 12-mm diameter spaced longitudinally every 200 mm. The modulus of elasticity is 10 GPa for the wood and 200 GPa for the steel. For a vertical shear of 4 kN, determine (*a*) the average shearing stress in the bolts, (*b*) the shearing stress at the center of the cross section. (*Hint*: Use the method indicated in Prob. 6.55.)

Let $E_{ref} = E_s = 200 \text{ GPa}$

$n_s = 1 \qquad n_w = \frac{E_w}{E_s} = \frac{10 \text{ GPa}}{200 \text{ GPa}} = \frac{1}{20}$

Widths of transformed section:

$b_s = 150 \text{ mm} \qquad b_w = (\frac{1}{20})(150) = 7.5 \text{ mm}$



$$I = 2\left[\tfrac{1}{12}(150)(12)^3 + (150)(12)(125+6)^2\right]$$
$$\tfrac{1}{12}(7.5)(250)^3$$
$$= 2\left[0.0216\times10^6 + 30.890\times10^6\right] + 9.766\times10^6$$
$$71.589\times10^6 \text{ mm}^4 = 71.589\times10^{-6} \text{ m}^4$$

(a) $Q_1 = (150)(12)(125+6) = 235.8\times10^3 \text{ mm}^3$
$= 235.8\times10^{-6} \text{ m}^3$

$$q = \frac{VQ_1}{I} = \frac{(4\times10^3)(235.8\times10^{-6})}{71.589\times10^{-6}} = 13.175\times10^3 \text{ N/m}$$

$$F_{bolt} = qs = (23.187\times10^3)(200\times10^{-3}) = 2.635\times10^3 \text{ N}$$

$$A_{bolt} = \frac{\pi}{4}d_{bolt}^2 = (\frac{\pi}{4})(12)^2 = 113.1 \text{ mm}^2 = 113.1\times10^{-6} \text{ m}^2$$

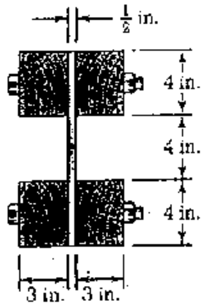
$$\tau_{bolt} = \frac{F_{bolt}}{A_{bolt}} = \frac{2.635\times10^3}{113.1\times10^{-6}} = 23.3\times10^6 \text{ Pa} = 23.3 \text{ MPa}$$

(b) $Q_2 = Q_1 + (7.5)(125)(62.5) = 235.8\times10^3 + 58.594\times10^3 = 294.4\times10^3 \text{ mm}^3$
$= 294.4\times10^{-6} \text{ m}^3$

$t = 150 \text{ mm} = 150\times10^{-3} \text{ m}$

$$\tau_c = \frac{VQ_2}{It} = \frac{(4\times10^3)(294.4\times10^{-6})}{(71.589\times10^{-6})(150\times10^{-3})} = 109.7\times10^3 \text{ Pa} = 109.7 \text{ kPa}$$

Problem 6.57



6.57 A composite beam is made by attaching the timber and steel portions shown with bolts of $\frac{5}{8}$-in. diameter spaced longitudinally every 8 in. The modulus of elasticity is 1.9×10^6 psi for the wood and 29×10^6 psi for the steel. For a vertical shear of 4000 lb, determine (*a*) the average shearing stress in the bolts, (*b*) the shearing stress at the center of the cross section. (*Hint*: Use the method indicated in Prob. 6.55.)

Let $E_{ref} = E_w = 1.9 \times 10^6$ psi

$n_s = \frac{E_s}{E_w} = \frac{29}{1.9} = 15.263$

For one timber, $I_w = \frac{1}{12}(3)(4)^3 + (3)(4)(4)^2 = 208 \text{ in}^4$

For steel plate, $I_s = \frac{1}{12}(\frac{1}{2})(12)^3 = 72 \text{ in}^4$

For transformed section, $I = n_s I_s + 4 I_w = 1930.9 \text{ in}^4$

(a) $Q_1 = (3)(4)(4) = 48 \text{ in}^3$

$q = \frac{VQ_1}{I} = \frac{(4000)(48)}{1930.9} = 99.435 \text{ lb/in}$

$F_{bolt} = qs = (99.435)(8) = 795.5 \text{ lb}$

$A_{bolt} = \frac{\pi}{4} d^2 = \frac{\pi}{4}\left(\frac{5}{8}\right)^2 = 0.30678 \text{ in}^2$

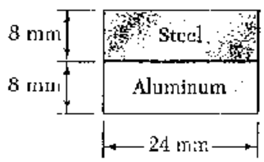
$\tau_{bolt} = \frac{F_{bolt}}{A_{bolt}} = \frac{795.5}{0.30678} = 2590 \text{ psi} = 2.59 \text{ ksi}$ ◄

(b) $Q_2 = 2Q_1 + (15.263)(\frac{1}{2})(6)(3) = 96 + 137.367 = 233.367 \text{ in}^3$

$t = \frac{1}{2} \text{ in.} = 0.5 \text{ in.}$

$\tau_c = \frac{VQ_2}{It} = \frac{(4 \times 10^3)(233.367)}{(1930.9)(0.5)} = 967 \text{ psi}$ ◄

Problem 6.58



6.58 A steel bar and an aluminum bar are bonded together as shown to form a composite beam. Knowing that the vertical shear in the beam is 6 kN and that the modulus of elasticity is 210 GPa for the steel and 70 GPa for the aluminum, determine (*a*) the average stress at the bonded surface, (*b*) the maximum shearing stress in the beam. (*Hint:* Use the method indicated in Prob. 6.55.)

$n = 1$ in aluminum $\qquad n = \frac{210}{70} = 3$ in steel

Part	A (mm^2)	nA (mm^2)	$\bar{y}_o$ (mm)	$nA\bar{y}$ (mm^3)	d (mm)	nAd^2 (mm^4)	$n\bar{I}$ (mm^4)
Steel	192	576	12	6912	2	2304	3072
Alum.	192	192	4	768	−6	6912	1024
Σ		768		7680		9216	4096

For transformed section, $\bar{Y}_o = \dfrac{\Sigma nA\bar{y}}{\Sigma nA} = \dfrac{7680}{768} = 10$ mm above bottom.

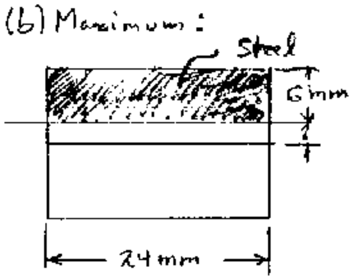
$$\bar{I} = \Sigma nAd^2 + \Sigma n\bar{I} = 13312 \text{ mm}^4 = 13312 \times 10^{-12} \text{ m}^4$$

(a) At bond: $\quad Q_1 = (576)(2) = 1152 \text{ mm}^3 = 1152 \times 10^{-9} \text{ m}^3$

$$t = 24 \text{ mm} = 24 \times 10^{-3} \text{ m}$$

$$\tau = \frac{VQ_1}{It} = \frac{(6 \times 10^3)(1152 \times 10^{-9})}{(13312 \times 10^{-12})(24 \times 10^{-3})} = 21.6 \times 10^6 \text{ Pa} = 21.6 \text{ MPa} \blacktriangleleft$$

(b) Maximum:



$$nA = (3)(24)(6) = 432 \text{ mm}^2$$

$$\bar{y} = 3 \text{ mm}$$

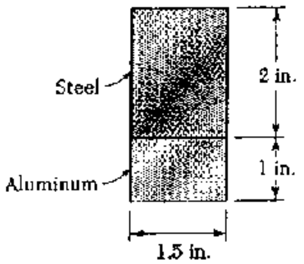
$$Q_2 = nA\bar{y} = (432)(3) = 1296 \text{ mm}^3 = 1296 \times 10^{-9} \text{ m}^3$$

$$t = 24 \text{ mm} = 24 \times 10^{-3} \text{ m}$$

$$\tau = \frac{VQ_2}{It} = \frac{(6 \times 10^3)(1296 \times 10^{-9})}{(13312 \times 10^{-12})(24 \times 10^{-3})}$$

$$= 24.3 \times 10^6 \text{ Pa} = 24.3 \text{ MPa} \blacktriangleleft$$

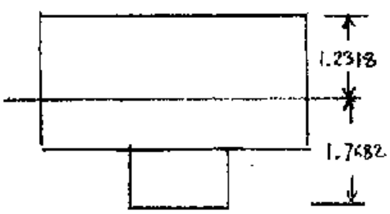
Problem 6.59



6.59 A steel bar and an aluminum bar are bonded together as shown to form a composite beam. Knowing that the vertical shear in the beam is 4 kips and that the modulus of elasticity is 29×10^6 psi for the steel and 10.6×10^6 psi for the aluminum, determine (*a*) the average stress at the bonded surface, (*b*) the maximum shearing stress in the beam. (*Hint:* Use the method indicated in Prob. 6.55.)

$n = 1$ in aluminum $\qquad n = \dfrac{29 \times 10^6 \text{ psi}}{10.6 \times 10^6 \text{ psi}} = 2.7358$ in steel

Part	$nA\ (in^2)$	$\bar{y}\ (in)$	$nA\bar{y}\ (in^3)$	$d\ (in)$	$nAd^2\ (in^4)$	$n\bar{I}\ (in^4)$
Steel	8.2074	2.0	16.4148	0.2318	0.4410	2.7358
Alum	1.5	0.5	0.75	1.2682	2.4125	0.1250
Σ	9.7074		17.1648		2.8535	2.8608



$$\bar{Y} = \frac{\Sigma nA\bar{y}}{\Sigma A} = \frac{17.1648}{9.7074} = 1.7682 \text{ in}$$

$$I = \Sigma nAd^2 + \Sigma n\bar{I} = 5.7143 \text{ in}^4$$

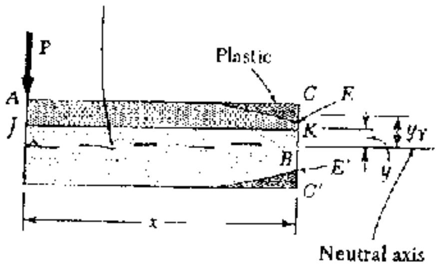
(a) At the bonded surface: $\quad Q = (1.5)(1.2682) = 1.9023 \text{ in}^3$

$$\tau = \frac{VQ}{It} = \frac{(4)(1.9023)}{(5.7143)(1.5)} = 0.888 \text{ ksi}$$

(b) At the neutral axis: $\quad Q = (2.7358)(1.5)(1.2318)\left(\frac{1.2318}{2}\right) = 3.1133 \text{ in}^3$

$$\tau_{max} = \frac{VQ}{It} = \frac{(4)(3.1133)}{(5.7143)(1.5)} = 1.453 \text{ ksi}$$

Problem 6.60



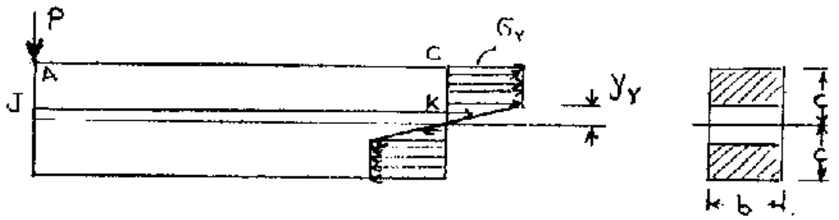
6.60 Consider the cantilever beam AB discussed in Sec. 6.8 and the portion $ACKJ$ of the beam that is located to the left of the transverse section CC' and above the horizontal plane JK, where K is a point at a distance $y < y_Y$ above the neutral axis (Fig. 6.60). (*a*) Recalling that $\sigma_x = \sigma_Y$ between C and E and $\sigma_x = (\sigma_Y / y_Y) y$ between E and K, show that the magnitude of the horizontal shearing force **H** exerted on the lower face of the portion of beam $ACKJ$ is

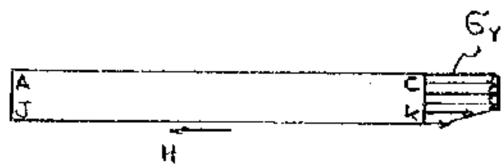
$$H = \frac{1}{2} b \sigma_Y \left(2c - y_Y - \frac{y^2}{y_Y} \right)$$

(*b*) Observing that the shearing stress at K is

$$\tau_{xy} = \lim_{\Delta A \to 0} \frac{\Delta H}{\Delta A} = \lim_{\Delta x \to 0} \frac{1}{b} \frac{\Delta H}{\Delta x} = \frac{1}{b} \frac{\partial H}{\partial x}$$

and recalling that y_Y is a function of x defined by Eq. (6.14), derive Eq. (6.15).





Point K is located a distance y above the neutral axis.

The stress distribution is given by

$\sigma = \sigma_Y \dfrac{y}{y_Y}$ for $0 \le y < y_Y$ and $\sigma = \sigma_Y$ for $y_Y \le y \le c$.

For equilibrium of horizontal forces acting on ACKJ,

$$H = \int \sigma \, dA = \int_y^{y_Y} \frac{\sigma_Y y b}{y_Y} dy + \int_{y_Y}^{c} \sigma_Y b \, dy = \frac{\sigma_Y b}{y_Y} \left(\frac{y_Y^2 - y^2}{2} \right) + \sigma_Y b (c - y_Y)$$

$$= \frac{1}{2} b \sigma_Y \left(2c - y_Y - \frac{y^2}{y_Y} \right) \qquad \blacktriangleleft \text{(a)}$$

Note that y_Y is a function of x.

$$\tau_{xy} = \frac{1}{b} \frac{\partial H}{\partial x} = \frac{1}{2} \sigma_Y \left(-\frac{\partial y_Y}{\partial x} + \frac{y^2}{y_Y^2} \frac{d y_Y}{dx} \right) = -\frac{1}{2} \sigma_Y \left(1 - \frac{y^2}{y_Y^2} \right) \frac{d y_Y}{dx}$$

But $\quad M = Px = \frac{3}{2} M_Y \left(1 - \frac{1}{3} \frac{y_Y^2}{c^2} \right)$

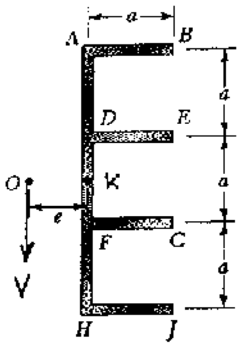
Differentiating, $\dfrac{dM}{dx} = P = \frac{3}{2} M_Y \left(-\frac{2}{3} \frac{y_Y}{c^2} \frac{d y_Y}{dx} \right)$

$$\frac{d y_Y}{dx} = -\frac{P c^2}{y_Y M_Y} = -\frac{P c^2}{y_Y \frac{2}{3} \sigma_Y b c^2} = -\frac{3}{2} \frac{P}{\sigma_Y b y_Y}$$

Then, $\tau_{xy} = \frac{1}{2} \sigma_Y \left(1 - \frac{y^2}{y_Y^2} \right) \frac{3}{2} \frac{P}{\sigma_Y b \sigma_Y} = \dfrac{3P}{4 b y_Y} \left(1 - \dfrac{y^2}{y_Y^2} \right) \qquad \blacktriangleleft \text{(b)}$

Problem 6.61

6.61 through 6.64 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

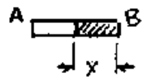


$$I_{AB} = I_{HJ} = at\left(\frac{3a}{2}\right)^2 + \frac{1}{12}at^3 \approx \frac{9}{4}ta^3$$

$$I_{DE} = I_{FG} = at\left(\frac{a}{2}\right)^2 + \frac{1}{12}at^3 \approx \frac{1}{4}ta^3$$

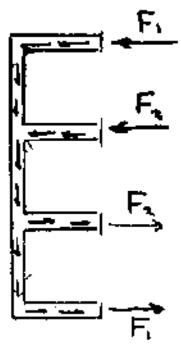
$$I_{AH} = \frac{1}{12}t(3a)^3 = \frac{9}{4}ta^3 \qquad I = \Sigma I = \frac{29}{4}ta^3$$

Part AB: $A = tx \quad \bar{y} = \frac{3a}{2} \quad Q = \frac{3}{2}atx$

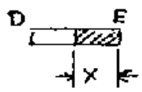


$$\tau = \frac{VQ}{It} = \frac{V\cdot\frac{3}{2}atx}{\frac{29}{4}ta^3t} = \frac{6Vx}{29a^2t}$$

$$F_1 = \int \tau\,dA = \int_0^a \frac{6Vx}{29a^2t}\,t\,dx = \frac{6V}{29a^2}\int_0^a x\,dx = \frac{3}{29}V$$



Part DE: $A = tx \quad \bar{y} = \frac{a}{2} \quad Q = \frac{1}{2}atx$



$$\tau = \frac{VQ}{It} = \frac{V\cdot\frac{1}{2}atx}{\frac{29}{4}ta^3t} = \frac{2Vx}{29a^2t}$$

$$F_2 = \int \tau\,dA = \int_0^a \frac{2Vx}{29a^2t}\,t\,dx = \frac{2V}{29a^2}\int_0^a x\,dx = \frac{1}{29}V$$

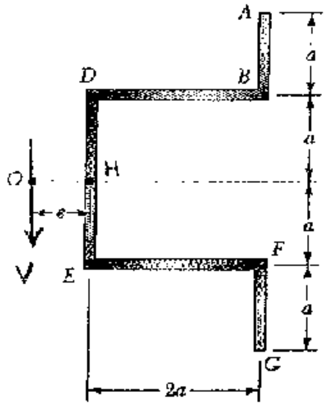
$$\circlearrowright\Sigma M_K = \circlearrowright\Sigma M_K:$$

$$Ve = F_1(3a) + F_2(a) = \frac{9}{29}Va + \frac{1}{29}Va = \frac{10}{29}Va$$

$$e = \frac{10}{29}a = 0.345\,a$$

Problem 6.62

6.61 through 6.64 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

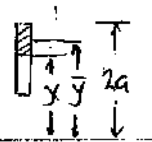


$$I_{AB} = I_{FG} = \frac{1}{12}ta^3 + (ta)\left(\frac{3a}{2}\right)^2 = \frac{7}{3}ta^3$$

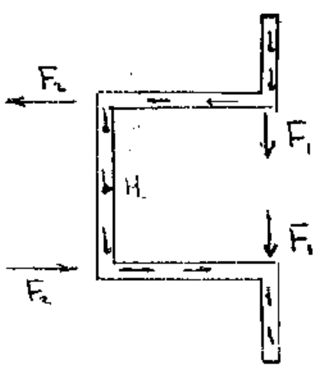
$$I_{DB} = I_{EF} = (2at)a^2 + \frac{1}{12}(2a)t^3 \approx 2a^3t$$

$$I_{DE} = \frac{1}{12}t(2a)^3 = \frac{2}{3}ta^3 \qquad I = \Sigma I = \frac{28}{3}ta^3$$

Part AB: $\quad A = t(2a-y), \quad \bar{y} = \frac{2a+y}{2}$



$$Q = A\bar{y} = \frac{1}{2}t(2a-y)(2a+y) = \frac{1}{2}t(4a^2-y^2)$$



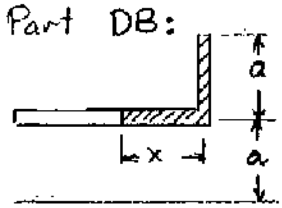
$$\tau = \frac{VQ}{It} = \frac{V}{2I}(4a^2-y^2)$$

$$F_1 = \int \tau\, dA = \int_a^{2a} \frac{V}{2I}(4a^2-y^2)\,t\,dy$$

$$= \frac{Vt}{2I}\left(4a^2y - \frac{y^3}{3}\right)\Big|_a^{2a} = \frac{Vta^3}{2I}\left[(4)(2) - \frac{(2)^3}{3} - (4)(1) + \left(\frac{1}{3}\right)\right]$$

$$= \frac{5}{6}\frac{Vta^3}{I} = \frac{5}{56}V$$

Part DB:



$$Q = (ta)\frac{3a}{2} + txa = ta\left(\frac{3a}{2} + x\right)$$

$$\tau = \frac{VQ}{It} = \frac{Va}{I}\left(\frac{3a}{2} + x\right)$$

$$F_2 = \int \tau\, dA = \int_0^{2a} \frac{Va}{I}\left(\frac{3a}{2} + x\right)t\,dx = \frac{Vta}{I}\int_0^{2a}\left(\frac{3a}{2} + x\right)dx$$

$$= \frac{Vta}{I}\left(\frac{3ax}{2} + \frac{x^2}{2}\right)\Big|_0^{2a} = \frac{Vta^3}{I}\left[\frac{(3)(2)}{2} + \frac{(2)^2}{2}\right]$$

$$= 5\frac{Vta^3}{I} = \frac{15}{28}V$$

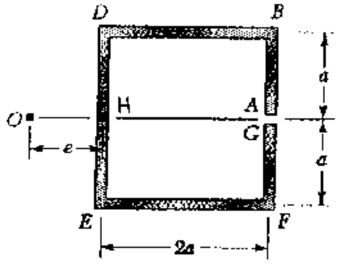
$$\circlearrowright \Sigma M_H = \circlearrowright \Sigma M_H:$$

$$Ve = F_2(2a) - 2F_1(2a) = \frac{30}{28}Va - \frac{20}{56}Va = \frac{5}{7}Va$$

$$e = \frac{5}{7}a$$

Problem 6.63

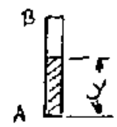
6.61 through 6.64 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.



$$I_{AB} = I_{FG} = \tfrac{1}{3}ta^3 \qquad I_{DB} = I_{EF} = 2ata^3 + \tfrac{1}{12}2att^3 \approx 2ta^3$$

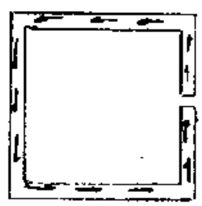
$$I_{DE} = \tfrac{1}{12}t(2a)^3 = \tfrac{2}{3}ta^3 \qquad \bar{I} = \Sigma I = \tfrac{16}{3}ta^3$$

Part AB: $A = ty \quad \bar{y} = \frac{y}{2} \quad Q = \frac{1}{2}ty^2$

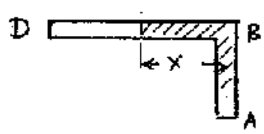


$$\tau = \frac{VQ}{It} = \frac{V\cdot\frac{1}{2}ty^2}{\frac{16}{3}ta^3t} = \frac{3Vy^2}{32a^3t}$$

$$F_1 = \int \tau\, dA = \int_0^a \tau t\, dy = \frac{3V}{32a^3}\int_0^a y^2 dt = \frac{1}{32}V$$

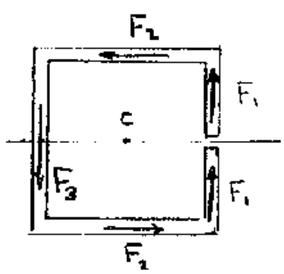


Part BD:



$$Q = Q_B + txa = \tfrac{1}{2}ta^2 + tax$$

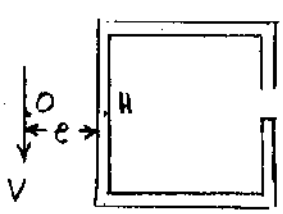
$$\tau = \frac{VQ}{It} = \frac{Vt}{\frac{16}{3}a^3t}\left(\tfrac{1}{2}a^2 + ax\right) = \frac{3V}{32a^2}(a + 2x)$$



$$F_2 = \int \tau\, dA = \int_0^{2a} \frac{3V}{32a^2}(a + 2x)\, dx$$

$$= \frac{3V}{32a^2}(ax + x^2)\Big|_0^{2a} = \frac{3V}{32a^2}(2a^2 + 4a^2) = \frac{9}{16}V$$

$$\circlearrowleft \Sigma M_H = \circlearrowleft \Sigma M_H :$$



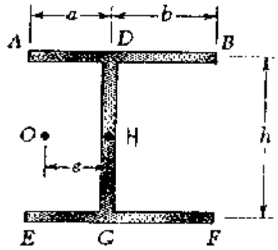
$$Ve = (2a)(2F_1) + (2a)(F_2)$$

$$= \tfrac{1}{8}Va + \tfrac{9}{8}Va = \tfrac{5}{4}Va \qquad e = \tfrac{5}{4}a$$

$$= 1.250\,a$$

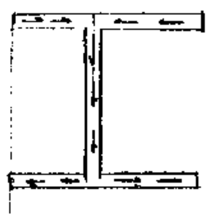
Problem 6.64

6.61 through 6.64 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

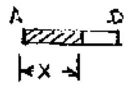


$$I_{AB} = I_{EF} = (a+b)t\left(\frac{h}{2}\right)^2 + \frac{1}{12}(a+b)t^3 \approx \frac{1}{4}t(a+b)h^2$$

$$I_{DG} = \frac{1}{12}th^3 \qquad I = \Sigma I = \frac{1}{12}t(6a+6b+h)h^2$$



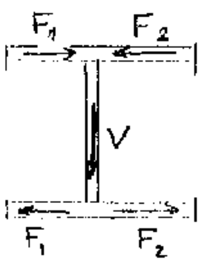
Part AD: $\quad Q = tx\frac{h}{2} = \frac{1}{2}thx$



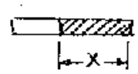
$$\tau = \frac{VQ}{It} = \frac{Vhx}{2I}$$

$$F_1 = \int \tau\, dA = \int_0^a \frac{Vhx}{2I}t\,dx = \frac{Vht}{2I}\int_0^a x\,dx$$

$$= \frac{Vht}{2I}\left.\frac{x^2}{2}\right|_0^a = \frac{Vhta^2}{4I}$$



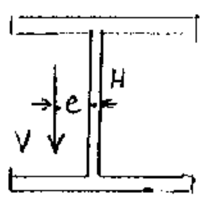
Part BD: $\quad Q = tx\frac{h}{2} = \frac{1}{2}thx$



$$\tau = \frac{VQ}{It} = \frac{Vhx}{2I}$$

$$F_2 = \int \tau\, dA = \int_0^b \frac{Vhx}{2I}t\,dx = \frac{Vht}{2I}\int_0^b x^2\,dx$$

$$= \frac{Vht}{2I}\left.\frac{x^2}{2}\right|_0^b = \frac{Vhtb^2}{4I}$$



$$\circlearrowleft \Sigma M_H = \circlearrowleft \Sigma M_H:$$

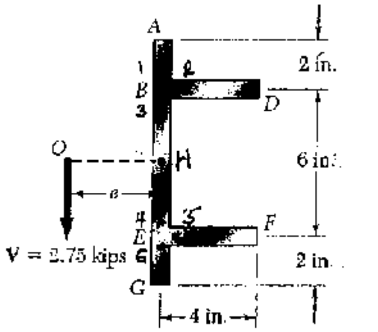
$$Ve = F_2 h - F_1 h = \frac{Vh^2t(b^2-a^2)}{4I}$$

$$= \frac{Vh^2t(b^2-a^2)}{4 \cdot \frac{1}{12}t(6a+6b+h)h^2} = \frac{3V(b^2-a^2)}{6a+6b+h}$$

$$e = \frac{3(b^2-a^2)}{6(a+b)+h}$$

Problem 6.65

6.65* and *6.66 An extruded beam has the cross section shown. Determine (*a*) the location of the shear center *O*, (*b*) the distribution of the shearing stresses caused by the 2.75-kip vertical shearing force applied at *O*.

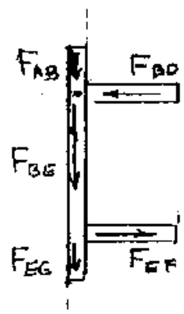


$t = \frac{1}{8}$ in. $= 0.125$ in.

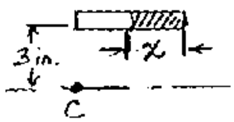
Part	A (in^2)	d (in.)	Ad^2 (in^4)	$\bar{I}$ (in^4)
BD	0.50	3	4.50	≈ 0
ABEG	1.25	0	0	10.417
EF	0.50	3	4.50	≈ 0
Σ	2.25		9.00	10.417

$$I = \Sigma A d^2 + \Sigma \bar{I} = 19.417 \text{ in}^4$$

(a)



Part BD



$$Q(x) = 3tx$$

$$q(x) = \frac{VQ(x)}{I} = \frac{V}{I}(3tx)$$

$$F_{BD} = \frac{3Vt}{I}\int_0^4 x\,dx = \frac{3Vt}{I}(8) = \frac{24Vt}{I}$$

Its moment about H: $(M_{BD})_H = 3F_{BD} = \frac{72Vt}{I}$

Part EF: By same method, $F_{EF} = \frac{24Vt}{I}$ $\quad (M_{EF})_H = \frac{72Vt}{I}$

Moments of F_{AB}, F_{BE}, and F_{EG} about H are zero.

$$Ve = \Sigma M_H = \frac{72Vt}{I} + \frac{72Vt}{I} = \frac{144Vt}{I}$$

$$e = \frac{144t}{I} = \frac{(144)(0.125)}{19.417} = 0.927 \text{ in.}$$

(b) At A, D, F, and G: $\quad Q = 0 \qquad \tau_A = \tau_D = \tau_F = \tau_G = 0$

Just above B: $Q_1 = Q_{AB} = (2t)(4) = 8t$

$$\tau_1 = \frac{VQ_1}{It} = \frac{(2.75)(8t)}{(19.417)t} = 1.133 \text{ ksi}$$

Just to the right of B: $Q_2 = Q_{BD} = (3)t(4) = 12t$

$$\tau_2 = \frac{VQ_2}{It} = \frac{(2.75)(12t)}{(19.417)t} = 1.700 \text{ ksi}$$

Just below B: $Q_3 = Q_1 + Q_2 = 20t$

$$\tau_3 = \frac{VQ_3}{It} = \frac{(2.75)(20t)}{(19.417)t} = 2.83 \text{ ksi}$$

Continued on next page.

Problem 6.65 continued.

At H (neutral axis): $Q_H = Q_3 + Q_{BH} = 20t + t(3)(1.5) = 24.5t$

$$\tau_H = \frac{VQ_H}{It} = \frac{(2.75)(24.5t)}{(19.417)t} = 3.47 \text{ ksi}$$

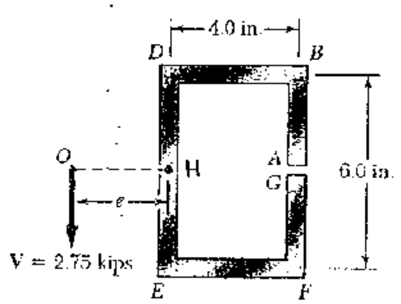
By symmetry: $\tau_4 = \tau_3 = 2.83$ ksi

$\tau_5 = \tau_2 = 1.700$ ksi

$\tau_6 = \tau_1 = 1.133$ ksi

Problem 6.66

6.65 and 6.66 An extruded beam has the cross section shown. Determine (*a*) the location of the shear center *O*, (*b*) the distribution of the shearing stresses caused by the 2.75-kip vertical shearing force applied at *O*.



$t = \frac{1}{8}$ in. = 0.125 in.

$I_{AB} = \frac{1}{3}(0.125)(3)^3 = 1.125 \text{ in}^4$

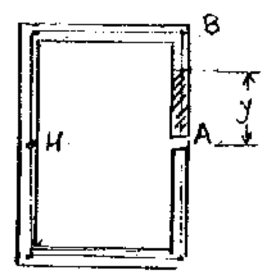
$I_{BD} = \frac{1}{12}(4)(0.125)^3 + (4)(0.125)(3)^2 = 4.50065 \text{ in}^4$

$I_{DE} = \frac{1}{12}(0.125)(6)^3 = 2.25 \text{ in}^4$

$I_{EF} = I_{BD} = 4.50065 \text{ in}^4$

$I_{FG} = I_{AB} = 1.125 \text{ in}^4$

$I = \Sigma I = 13.50 \text{ in}^4$

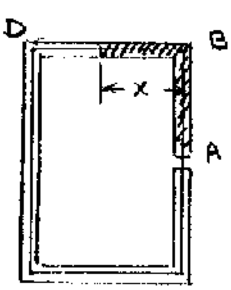


(a) Part AB: $Q(y) = ty\frac{y}{2} = 0.5\,ty^2$

$$q(y) = \frac{VQ(y)}{I} = \frac{0.5\,Vt}{I}y^2$$

$$F_{AB} = \int_0^3 q(y)\,dy = \frac{0.5\,Vt}{I}\int_0^3 y^2\,dy = 4.5\frac{Vt}{I} \uparrow$$

Its moment about H is $4F_{AB} = 18\frac{Vt}{I}$



$Q_B = (0.5)(t)(3)^2 = 4.5t$

Part BD: $Q(x) = Q_B + xt(3) = (4.5 + 3x)t$

$$q(x) = \frac{VQ(x)}{I} = \frac{Vt}{I}(4.5 + 3x)$$

$$F_{BD} = \int_0^4 q(x)\,dx = \frac{Vt}{I}\int_0^4 (4.5 + 3x)\,dx = 42\frac{Vt}{I} \leftarrow$$

Its moment about H: $3F_{BD} = 126\frac{Vt}{I}$

$Q_D = [4.5 + (3)(4)]t = 16.5t$

Continued on next page.

Problem 6.66 continued.

Part EF: By symmetry with part BD, $F_{EF} = 42 \dfrac{Vt}{I} \rightarrow$

Its moment about H is $3F_{EF} = 126 \dfrac{Vt}{I}$ ↺

Part FG: By symmetry with part AB, $F_{FG} = 4.5 \dfrac{Vt}{I} \uparrow$

Its moment about H is $4F_{FG} = 18 \dfrac{Vt}{I}$ ↺

Moment about H of force in part DE is zero.

$$Ve = \Sigma M_H = \frac{Vt}{I}(18 + 126 + 0 + 126 + 18) = \frac{144\,Vt}{I}$$

$$e = \frac{144\,t}{I} = \frac{(2.88)(0.125)}{13.50} = 2.67 \text{ in.}$$

(b) $Q_A = Q_G = 0$ $\qquad\qquad \tau_A = \tau_G = 0$

$Q_B = Q_D = 4.5t$

$$\tau_B = \tau_F = \frac{VQ_B}{It} = \frac{(2.75)(4.5t)}{13.50\,t} = 0.917 \text{ ksi}$$

$Q_D = Q_E = 16.5t$

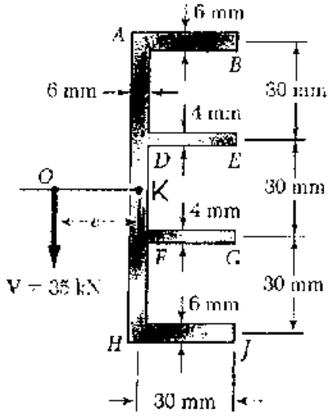
$$\tau_D = \tau_E = \frac{VQ_D}{It} = \frac{(2.75)(16.5t)}{13.50\,t} = 3.36 \text{ ksi}$$

At H (neutral axis): $Q_H = Q_D + t(3)(1.5) = 21t$

$$\tau_H = \frac{VQ_H}{It} = \frac{(2.75)(21t)}{13.50\,t} = 4.28 \text{ ksi}$$

Problem 6.67

6.67 and 6.68 For an extruded beam having the cross section shown, determine (*a*) the location of the shear center *O*, (*b*) the distribution of the shearing stresses caused by the vertical shearing force **V** shown applied at *O*.



$I_z = 1.149 \times 10^6\ \text{mm}^4$

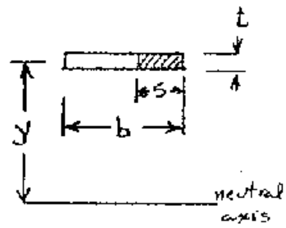
$I_{AB} = I_{HJ} = \frac{1}{12}(30)(6)^3 + (30)(6)(45)^2 = 0.365 \times 10^6\ \text{mm}^4$

$I_{DE} = I_{FG} = \frac{1}{12}(30)(4)^3 + (30)(4)(15)^2 = 0.02716 \times 10^6\ \text{mm}^4$

$I'_{AH} = \frac{1}{12}(6)(90)^3 = 0.3645 \times 10^6\ \text{mm}^4$

$I = \Sigma I = 1.14882 \times 10^6\ \text{mm}^4$

(a) For a typical flange,

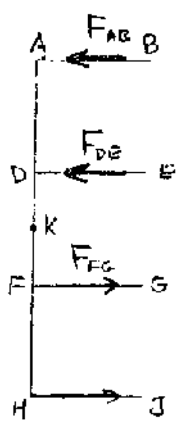


$A(s) = ts$

$Q(s) = yts$

$q(s) = \dfrac{VQ(s)}{I} = \dfrac{Vyts}{I}$

$F = \int_0^b q(s)\,ds = \dfrac{Vytb^2}{2I}$



Flange AB: $F_{AB} = \dfrac{V(45)(6)(30^2)}{(2)(1.14882 \times 10^6)} = 0.10576\ V \leftarrow$

Flange DE: $F_{DE} = \dfrac{V(15)(4)(30)^2}{(2)(1.14882 \times 10^6)} = 0.023502\ V \leftarrow$

Flange FG: $F_{FG} = 0.023502\ V \rightarrow$

Flange HJ: $F_{HJ} = 0.10576\ V \rightarrow$

$+\circlearrowright \Sigma M_K = +\circlearrowright \Sigma M_K$:

$Ve = 45 F_{AB} + 15 F_{DE} + 15 F_{FG} + 45 F_{HJ} = 10.223\ V$

Dividing by V, $e = 10.22\ \text{mm}$ ◀

(b) Calculation of shearing stresses.

$V = 35 \times 10^3\ \text{N}$ $\quad I = 1.14882 \times 10^{-6}\ \text{m}^4$

At B, E, G, and J: $\tau = 0$ ◀

At A and H: $Q = (30)(6)(45) = 8.1 \times 10^3\ \text{mm}^3 = 8.1 \times 10^{-6}\ \text{m}^3$

$t = 6 \times 10^{-3}\ \text{m}$

$\tau = \dfrac{VQ}{It} = \dfrac{(35 \times 10^3)(8.1 \times 10^{-6})}{(1.14882 \times 10^{-6})(6 \times 10^{-3})} = 41.1 \times 10^6\ \text{Pa} = 41.1\ \text{MPa}$ ◀

Continued on next page.

Problem 6.67 continued

Just above D and just below F:

$$Q = 8.1\times10^3 + (6)(30)(30) = 13.5\times10^3\ \text{mm}^3 = 13.5\times10^{-6}\ \text{m}^3$$

$$t = 6\times10^{-3}\ \text{m}$$

$$\tau = \frac{VQ}{It} = \frac{(35\times10^3)(13.5\times10^{-6})}{(1.14882\times10^{-6})(6\times10^{-3})} = 68.5\times10^6\ \text{Pa} = 68.5\ \text{MPa}$$

Just to right of D and just to the right of F:

$$Q = (30)(4)(15) = 1.8\times10^3\ \text{mm}^3 = 1.8\times10^{-6}\ \text{m}^3 \qquad t = 4\times10^{-3}\ \text{m}$$

$$\tau = \frac{VQ}{It} = \frac{(35\times10^3)(1.8\times10^{-6})}{(1.14882\times10^{-6})(4\times10^{-3})} = 13.71\times10^6\ \text{Pa} = 13.71\ \text{MPa}$$

Just below D and just above F:

$$Q = 13.5\times10^3 + 1.8\times10^3 = 15.3\times10^3\ \text{mm}^3 = 15.3\times10^{-6}\ \text{m}^3$$

$$t = 6\times10^{-3}\ \text{m}$$

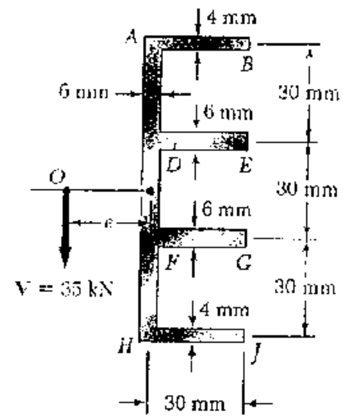
$$\tau = \frac{VQ}{It} = \frac{(35\times10^3)(15.3\times10^{-6})}{(1.14882\times10^{-6})(6\times10^{-3})} = 77.7\times10^6\ \text{Pa} = 77.7\ \text{MPa}$$

At K: $\quad Q = 15.3\times10^3 + (6)(15)(7.5) = 15.975\times10^3\ \text{mm}^3 = 15.975\times10^{-6}\ \text{m}^3$

$$\tau = \frac{VQ}{It} = \frac{(35\times10^3)(15.975\times10^{-6})}{(1.14882\times10^{-6})(6\times10^{-3})} = 81.1\times10^6\ \text{Pa} = 81.1\ \text{MPa}$$

Problem 6.68

6.67 and 6.68 For an extruded beam having the cross section shown, determine (*a*) the location of the shear center *O*, (*b*) the distribution of the shearing stresses caused by the vertical shearing force **V** shown applied at *O*.



$I_z = 0.933 \times 10^6$ mm^4

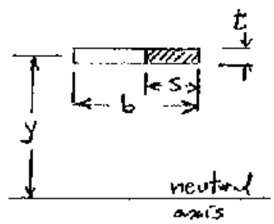
$I_{AB} = I_{HJ} = \frac{1}{12}(30)(4)^3 + (30)(4)(45)^2 = 0.24316 \times 10^6$ mm^4

$I_{DE} = I_{FG} = \frac{1}{12}(30)(6)^3 + (30)(6)(15)^2 = 0.04109 \times 10^6$ mm^4

$I_{AH} = \frac{1}{12}(6)(90)^3 = 0.3645 \times 10^6$ mm^3

$I = \Sigma I = 0.9329 \times 10^6$ mm^4

(a) For a typical flange, $A(s) = ts$

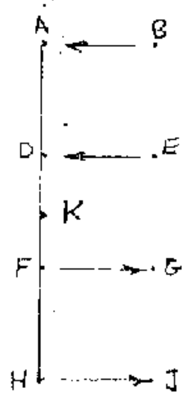


$Q(s) = yts$

$q(s) = \frac{VQ(s)}{I} = \frac{Vyts}{I}$

$F = \int_0^b q(s)\,ds$

$= \frac{Vytb^2}{2I}$



Flange AB : $F_{AB} = \frac{V(45)(4)(30)^2}{(2)(0.9329 \times 10^6)} = 0.086826\ V \leftarrow$

Flange DE : $F_{DE} = \frac{V(15)(6)(30)^2}{(2)(0.9329 \times 10^6)} = 0.043413\ V \leftarrow$

Flange FG : $F_{FG} = 0.043413\ V \rightarrow$

Flange HJ : $F_{HJ} = 0.086826\ V \rightarrow$

$+\circlearrowright \Sigma M_K = +\circlearrowright \Sigma M_K$:

$Ve = 45\,F_{AB} + 15\,F_{DE} + 15\,F_{FG} + 45\,F_{HJ} = 9.1167\ V$

Dividing by V, $e = 9.12$ mm ◄

(b) Calculation of shearing stresses.

$V = 35 \times 10^3$ N $\qquad I = 0.9329 \times 10^{-6}$ m^4

At B, E, G, and J : $\tau = 0$ ◄

At A and H: $Q = (30)(4)(45) = 5.4 \times 10^3$ mm^3 $= 5.4 \times 10^{-6}$ m^3

$q = \frac{VQ}{I}\ \frac{(35 \times 10^3)(5.4 \times 10^{-6})}{0.9329 \times 10^{-6}} = 202.59 \times 10^3$ N/m

Just to the right of A and H: $t = 4 \times 10^{-3}$ m

$\tau = \frac{q}{t} = \frac{202.59 \times 10^3}{4 \times 10^{-3}} = 50.6 \times 10^6$ Pa $= 50.6$ MPa ◄

Continued on next page.

Problem 6.68 continued

Just below A and just above H: $t = 6\times10^{-3}$ m

$$\tau = \frac{q}{t} = \frac{202.59\times10^{3}}{6\times10^{-3}} = 33.8\times10^{6}\ \text{Pa} = 33.8\ \text{MPa}$$

Just above D and just below F: $t = 6\times10^{-3}$ m

$$Q = 5.4\times10^{3} + (6)(30)(30) = 10.8\times10^{3}\ \text{mm}^3 = 10.8\times10^{-6}\ \text{m}^3$$

$$\tau = \frac{VQ}{It} = \frac{(35\times10^{3})(10.8\times10^{-6})}{(0.9329\times10^{-6})(6\times10^{-6})} = 67.5\times10^{6}\ \text{Pa} = 67.5\ \text{MPa}$$

Just to the right of D and E: $t = 6\times10^{-3}$ m

$$Q = (30)(6)(15) = 2.7\times10^{3}\ \text{mm}^3 = 2.7\times10^{-6}\ \text{m}^3$$

$$\tau = \frac{VQ}{It} = \frac{(35\times10^{3})(2.7\times10^{-6})}{(0.9329\times10^{-6})(6\times10^{-3})} = 16.88\times10^{6}\ \text{Pa} = 16.88\ \text{MPa}$$

Just below D and just above F: $t = 6\times10^{-3}$

$$Q = 10.8\times10^{3} + 2.7\times10^{3} = 13.5\times10^{3}\ \text{mm}^3 = 13.5\times10^{-6}\ \text{m}^3$$

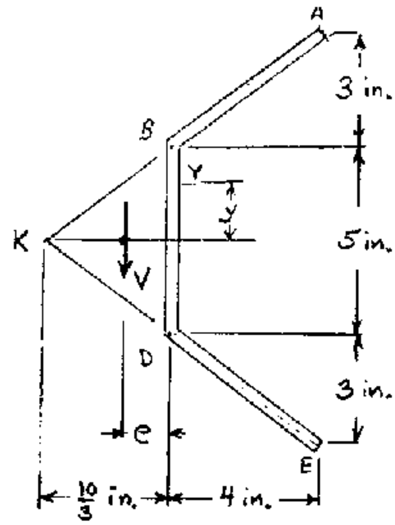
$$\tau = \frac{VQ}{It} = \frac{(35\times10^{3})(13.5\times10^{-6})}{(0.9329\times10^{-6})(6\times10^{-6})} = 84.4\times10^{6}\ \text{Pa} = 84.4\ \text{MPa}$$

At K: $Q = 13.5\times10^{3} + (6)(15)(7.5) = 14.175\times10^{3}\ \text{mm}^3 = 14.175\times10^{-6}\ \text{m}^3$

$$\tau = \frac{VQ}{It} = \frac{(35\times10^{3})(14.175\times10^{-6})}{(0.9329\times10^{-6})(6\times10^{-3})} = 88.6\times10^{6}\ \text{Pa} = 88.6\ \text{MPa}$$

Problem 6.69

6.69 through 6.74 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.



$L_{AB} = \sqrt{4^2 + 3^2} = 5 \text{ in.} \qquad A_{AB} = 5t$

$I_{AB} = \frac{1}{12} A_{AB} h^2 + A_{AB} d^2 = \frac{1}{12}(5t)(3)^2 + (5t)(4)^2$

$= 83.75 \; t \; \text{in}^4$

$I_{BD} = \frac{1}{12}(t)(5)^3 = 10.417 \; t \; \text{in}^4$

$I = 2 I_{AB} + I_{BD} = 177.917 \; t \; \text{in.}$

In part BD: $\quad Q = Q_{AB} + Q_{BY}$

$Q = (5t)(4) + (2.5 - y)t(\frac{1}{2})(2.5 + y)$

$= 20t + 3.125t - \frac{1}{2} t y^2$

$= (23.125 - \frac{1}{2} y^2)\, t$

$\tau = \dfrac{VQ}{It} \qquad F_{BD} = \int \tau \, dA = \int_{-2.5}^{2.5} \dfrac{V(23.125 - \frac{1}{2}y^2)t}{I\,t} \cdot t\,dy$

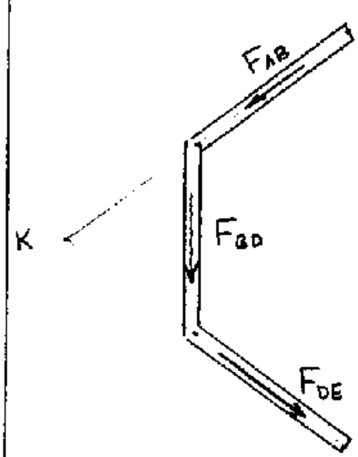
$= \dfrac{Vt}{I} \int_{-2.5}^{2.5} (23.125 - \frac{1}{2}y^2)\,dy = \dfrac{Vt}{I}\left[23.125\,y - \frac{1}{6}y^3\right]_{-2.5}^{2.5}$

$= \dfrac{Vt}{I} \cdot 2\left[(23.125)(2.5) - \dfrac{(2.5)^3}{6}\right] = \dfrac{Vt\,(110.417)}{177.917\,t}$

$= 0.62061 \, V$

$\circlearrowleft \Sigma M_K = \circlearrowleft \Sigma M_K: \qquad -V\left(\frac{10}{3} - e\right) = -\frac{10}{3}(0.62061 V)$

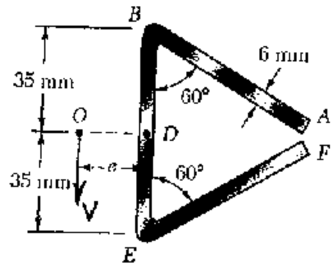
$e = \frac{10}{3}\left[1 - 0.62061\right] = 1.265 \text{ in.}$



Note that the lines of action of F_{AB} and F_{DE} pass through point K. Thus, these forces have zero moment about point K.

Problem 6.70

6.69 through 6.74 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

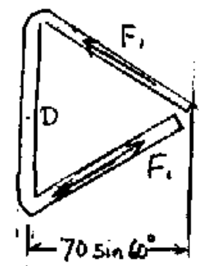


$$I_{DB} = \tfrac{1}{3}(6)(35)^3 = 85.75\times10^3\ \text{mm}^4$$

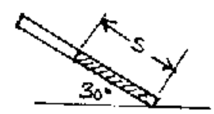
$$L_{AB} = 70\ \text{mm} \qquad A_{AB} = (70)(6) = 420\ \text{mm}^2$$

$$I_{AB} = \tfrac{1}{3}A_{AB}h^2 = (\tfrac{1}{3})(420)(35)^2 = 171.5\times10^3\ \text{mm}^4$$

$$I = (2)(85.75\times10^3) + (2)(171.5\times10^3) = 514.5\times10^3\ \text{mm}^4$$



Part AB: $A = ts = 6s$



$$\bar{y} = \tfrac{1}{2}s\sin 30° = \tfrac{1}{4}s$$

$$Q = A\bar{y} = \tfrac{3}{2}s^2$$

$$\tau = \frac{VQ}{It} = \frac{3Vs^2}{It}$$

$$F_1 = \int \tau\, dA = \int_0^{70}\frac{3Vs^2}{2It}\cancel{t}\,ds = \frac{3V}{I}\int_0^{70}s^2\,ds$$

$$= \frac{(3)(70)^3}{(2)(3)I}V = \tfrac{1}{3}V$$

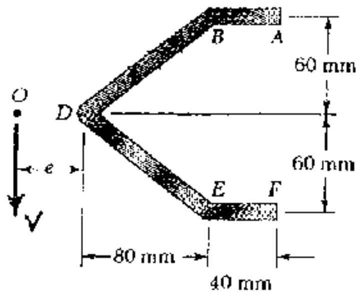
$$+\circlearrowleft\Sigma M_D = +\circlearrowleft\Sigma M_D:\quad Ve = 2(F_1\cos 60°)(70\sin 60°)]$$

$$= 20.2\ V$$

Dividing by V, $\qquad e = 20.2\ \text{mm}$

Problem 6.71

6.69 through *6.74* Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

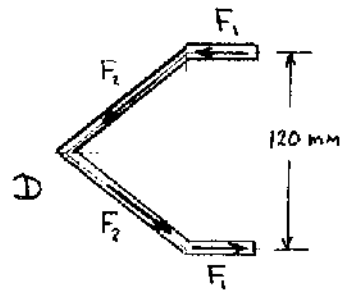


$I_{AB} = (40\,t)(60)^2 = 144\times 10^3\ t$

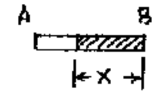
$L_{DB} = \sqrt{80^2 + 60^2} = 100\ \text{mm} \qquad A_{DB} = 100\ t$

$I_{DB} = \frac{1}{3} A_{DB} h^2 = \frac{1}{3}(100\,t)(60)^2 = 120\times 10^3\ t$

$I = 2 I_{AB} + 2 I_{DB} = 528\times 10^3\ t$



Part AB: $\quad A = t x \quad \bar{y} = 60\ \text{mm}$



$Q = A\bar{y} = 60\,t x \quad \text{mm}^3$

$\tau = \frac{VQ}{It} = \frac{V(60\,t x)}{It} = \frac{60\,V x}{I}$

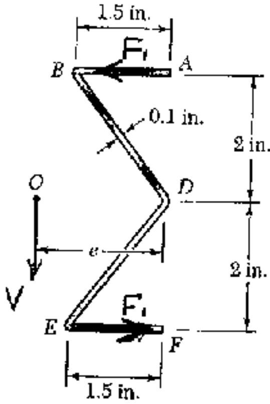
$F_1 = \int \tau\, dA = \int_0^{40} \frac{60\,V\,x}{I}\, t\, dx = \frac{60\,Vt}{I}\int_0^{40} x\, dx$

$= \frac{60\,Vt}{I}\left.\frac{x^2}{2}\right|_0^{30} = \frac{(60)(30)^2\,V\cancel{t}}{(2)(528\times 10^3)\cancel{t}} = 0.051136\ V$

$\circlearrowright \Sigma M_D = \circlearrowright \Sigma M_D: \quad Ve = (0.051136\ V)(120) \qquad e = (0.051136)(120) = 6.14\ \text{mm}$

Problem 6.72

6.69 through 6.74 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.



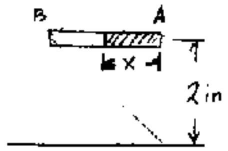
$I_{AB} = \frac{1}{12}(1.5)(0.1)^3 + (1.5)(0.1)(2)^2 = 0.600125 \text{ in}^4$

$L_{BD} = \sqrt{1.5^2 + 2^2} = 2.5 \text{ in.}$ $\quad A_{BD} = (2.5)(0.1) = 0.25 \text{ in}^2$

$I_{BD} = \frac{1}{3}A_{BD}h^2 = \frac{1}{3}(0.25)(2)^2 = 0.33333 \text{ in}^4$

$I = 2I_{AB} + 2I_{BD} = 1.86692 \text{ in}^4$

Part AB:



$A(x) = tx = 0.1x, \quad \bar{y} = 2 \text{ in.}$

$Q(x) = A(x)\bar{y} = 0.2x \text{ in}^3$

$q(x) = \frac{VQ(x)}{I} = \frac{0.2Vx}{I}$

$F_1 = \int_0^{1.5} q(x)\,dx = \frac{0.2V}{I}\int_0^{1.5} x\,dx$

$= \frac{(0.2)(1.5)^2}{2}\frac{V}{I} = 0.225\frac{V}{I}$

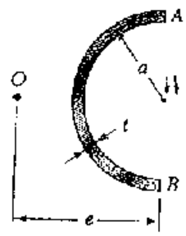
Likewise, by symmetry, in part EF: $F_1 = 0.225\frac{V}{I}$

$+\circlearrowleft \Sigma M_D = +\circlearrowleft \Sigma M_D: \quad Ve = 4F_1 = 9\frac{V}{I} = 0.482V$

Dividing by V, $\quad e = 0.482 \text{ in.}$

Problem 6.73

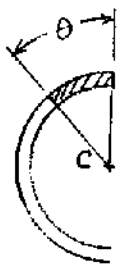
6.69 through 6.74 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.



For a thin-walled hollow circular cross section $A = 2\pi a t$

$J = a^2 A = 2\pi a^3 t \qquad I = \frac{1}{2} J = \pi a^3 t$

For the half-pipe section $I = \frac{\pi}{2} a^3 t$



Use polar coordinate θ for partial cross section

$A = st = a\theta t \qquad s = \text{arc length}$

$\bar{r} = a \frac{\sin\alpha}{\alpha}$ where $\alpha = \frac{\theta}{2}$

$\bar{y} = \bar{r}\cos\alpha = a\frac{\sin\alpha\cos\alpha}{\alpha}$

$Q = A\bar{y} = a\theta t\, a\frac{\sin\alpha\cos\alpha}{\alpha} = a^2 t\,(2\sin\alpha\cos\alpha)$

$= a^2 t \sin 2\alpha = a^2 t \sin\theta$

$\tau = \frac{VQ}{It} = \frac{Va^2}{I}\sin\theta$

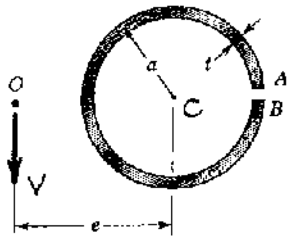
$M_H = \int a\tau\, dA = \int_0^\pi a\frac{Va^2}{I}\sin\theta\; t a\, d\theta = \frac{Va^4 t}{I}\left. -\cos\theta\right|_0^\pi$

$= 2\frac{Va^4 t}{I} = \frac{4}{\pi} Va$

But $M_H = Ve$, hence $e = \frac{4}{\pi}a = 1.273\, a$ ◄

Problem 6.74

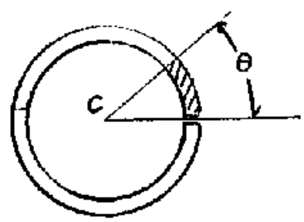
6.69 through 6.74 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.



For whole cross section, $A = 2\pi a t$

$J = A a^2 = 2\pi a^3 t \qquad I = \frac{1}{2}J = \pi a^3 t$

Use polar coordinate θ for partial cross section.



$A = st = a\theta t \qquad s = \text{arc length}$

$\bar{r} = a\,\frac{\sin\alpha}{\alpha}$ where $\alpha = \frac{1}{2}\theta$

$\bar{y} = \bar{r}\sin\alpha = a\,\frac{\sin^2\alpha}{\alpha}$

$Q = A\bar{y} = a\theta t\, a\,\frac{\sin^2\alpha}{\alpha} = a^2 t\; 2\sin^2\alpha$

$= a^2 t\; 2\sin^2\frac{\theta}{2} = a^2 t(1-\cos\theta)$

$\tau = \frac{VQ}{It} = \frac{Va^2}{I}(1-\cos\theta)$

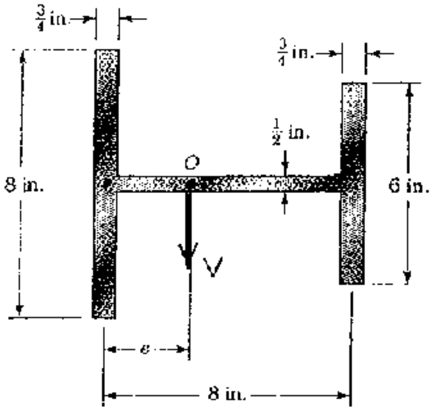
$M_c = \int a\tau\, dA = \int_0^{2\pi}\frac{Va^3}{I}(1-\cos\theta)\,t\,a\,d\theta = \frac{Va^4 t}{I}(\theta-\sin\theta)\Big|_0^{2\pi}$

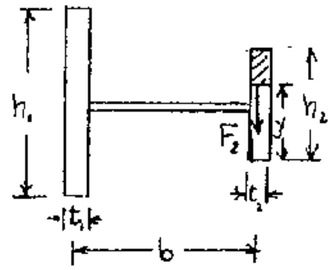
$= \frac{2\pi V a^4 t}{\pi a^3 t} = 2aV$

But $M_c = Ve$; hence $e = 2a$ ◀

Problem 6.75

6.75 and 6.76 A thin-walled beam has the cross section shown. Determine the location of the shear center O of the cross section.





$$I = \frac{1}{12} t_1 h_1^3 + \frac{1}{12} t_2 h_2^3$$

Right flange:

$$A = \left(\tfrac{1}{2}h_2 - y\right)t_2$$

$$\bar{y} = \tfrac{1}{2}\left(\tfrac{1}{2}h_2 + y\right)t_2$$

$$Q = A\bar{y}$$

$$= \tfrac{1}{2}\left(\tfrac{1}{2}h_2 - y\right)\left(\tfrac{1}{2}h_2 + y\right)t_2$$

$$= \tfrac{1}{2}\left(\tfrac{1}{4}h_2^2 - y^2\right)t_2$$

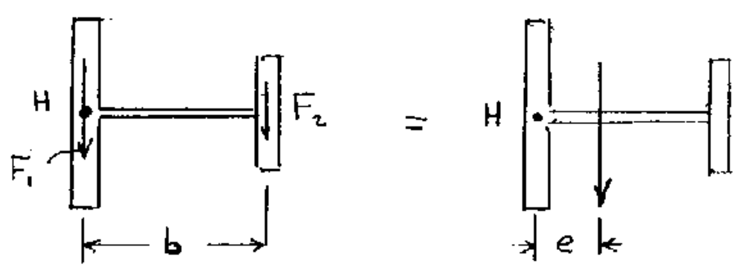
$$\tau = \frac{VQ}{It_2} = \frac{V}{2It_2}\left(\tfrac{1}{4}h_2^2 - y^2\right)t_2$$

$$F_2 = \int \tau\, dA = \int_{-h_2/2}^{h_2/2} \frac{Vt_2}{2It_2}\left(\tfrac{1}{4}h_2^2 - y^2\right)t_2\, dy = \frac{Vt_2}{2I}\left(\tfrac{1}{4}h_2^2 y - \frac{y^3}{3}\right)\Big|_{-h_2/2}^{h_2/2}$$

$$= \frac{Vt_2}{2I}\left\{\tfrac{1}{4}h_2^2\,\frac{h_2}{2} - \tfrac{1}{3}\left(\frac{h_2}{2}\right)^3 + \tfrac{1}{4}h_2^2\,\frac{h_2}{2} - \tfrac{1}{3}\left(\frac{h_2}{2}\right)^3\right\} = \frac{Vt_2h_2^3}{12I} = \frac{V\,t_2h_2^3}{t_1h_1^3 + t_2h_2^3}$$

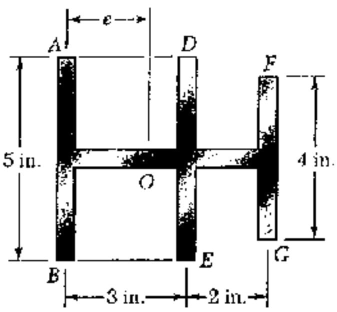
$$\circlearrowleft\Sigma M_H = \circlearrowleft\Sigma M_H: \quad -Ve = -F_2 b = -V\frac{t_2h_2^3 b}{t_1h_1^3 + t_2h_2^3}$$

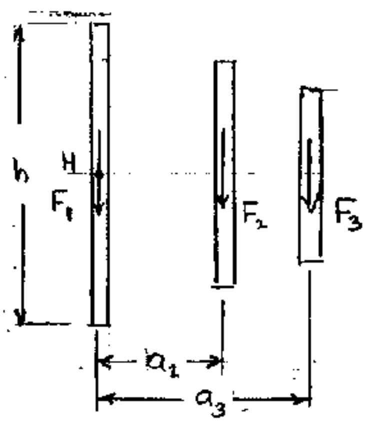
$$e = \frac{t_2h_2^3 b}{t_1h_1^3 + t_2h_2^3} = \frac{(0.75)(6)^3(8)}{(0.75)(8)^3 + (0.75)(6)^3} = 2.37 \text{ in.}$$



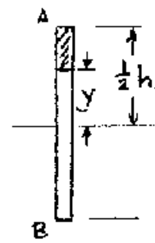
Problem 6.76

6.75 and 6.76 A thin-walled beam has the cross section shown. Determine the location of the shear center O of the cross section.





Let $h_1 = \overline{AB}$, $h_2 = \overline{DE}$, and $h_3 = \overline{FG}$.



$$I = \frac{1}{12} t \left(h_1^3 + h_2^3 + h_3^3\right)$$

Part AB: $A = (\frac{1}{2}h_1 - y)\,t$

$$\bar{y} = \frac{1}{2}\left(\frac{1}{2}h_1 + y\right)$$

$$Q = A\bar{y} = \frac{1}{2}t\left(\frac{1}{2}h_1 - y\right)\left(\frac{1}{2}h_1 + y\right) = \frac{1}{2}t\left(\frac{1}{4}h_1^2 - y^2\right)$$

$$\tau = \frac{VQ}{It} = \frac{V}{2I}\left(\frac{1}{4}h_1^2 - y^2\right)$$

$$F_1 = \int \tau\, dA = \int_{-\frac{1}{2}h_1}^{\frac{1}{2}h_1} \frac{V}{2I}\left(\frac{1}{4}h_1^2 - y^2\right) t\, dy$$

$$= \frac{Vt}{2I}\left(\frac{1}{4}h_1^2 y - \frac{y^3}{3}\right)\Bigg|_{-\frac{1}{2}h_1}^{\frac{1}{2}h_1}$$

$$= \frac{Vt}{I}\left(\frac{1}{4}h_1^2 \cdot \frac{1}{2}h_1 - \frac{1}{3}\left(\frac{h_1}{2}\right)^3\right) = \frac{Vt\,h_1^3}{12\,I}$$

$$= \frac{h_1^3\, V}{h_1^3 + h_2^3 + h_3^3}$$

Likewise, for Part DE: $\quad F_2 = \dfrac{h_2^3\, V}{h_1^3 + h_2^3 + h_3^3}$

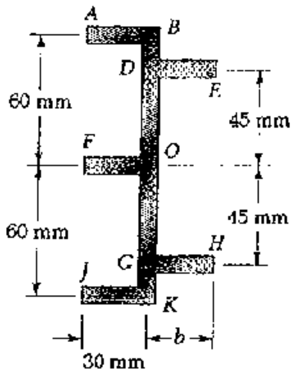
and for Part FG: $\quad F_3 = \dfrac{h_3^3\, V}{h_1^3 + h_2^3 + h_3^3}$

$+\circlearrowleft \Sigma M_H = +\circlearrowleft \Sigma M_H$: $\quad Ve = a_2 F_2 + a_3 F_3 = \dfrac{a_2 h_2^3 + a_3 h_3^3}{h_1^3 + h_2^3 + h_3^3} V$

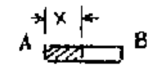
Dividing by V, $\quad e = \dfrac{a_2 h_2^3 + a_3 h_3^3}{h_1^3 + h_2^3 + h_3^3} = \dfrac{(3)(5)^3 + (5)(4)^3}{5^3 + 5^3 + 4^3} = 2.21$ in. ◀

Problem 6.77

6.77 and 6.78 A thin-walled beam of uniform thickness has the cross section shown. Determine the dimension b for which the shear center O of the cross section is located at the point indicated.



Part AB: $A = tx \quad \bar{y} = 60\text{ mm}$



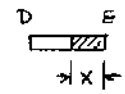
$$Q = A\bar{y} = 60\,t\,x\ \text{mm}^3$$

$$\tau = \frac{VQ}{It} = \frac{60\,Vx}{I}$$

$$F_1 = \int \tau\,dA = \int_0^{30} \frac{60\,Vx}{I}\,t\,dx = \frac{60\,Vt}{I}\int_0^{30} x\,dx$$

$$= \frac{60\,Vt}{I}\,\frac{x^2}{2}\Big|_0^{30} = \frac{(60)(30)^2}{2}\,\frac{Vt}{I} = 27\times10^3\,\frac{Vt}{I}$$

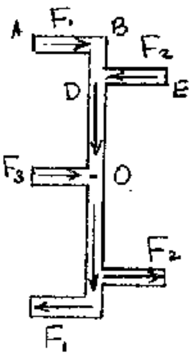
Part DE: $A = tx \qquad \bar{y} = 45\text{ mm}$



$$Q = A\bar{y} = 45\,tx$$

$$\tau = \frac{VQ}{It} = \frac{45\,Vx}{I}$$

$$F_2 = \int \tau\,dA = \int_0^{b} \frac{45\,Vx}{I}\,t\,dx = \frac{45\,Vt}{I}\int_0^{b} x\,dx = \frac{45\,b^2\,Vt}{2I}$$



$$+\circlearrowleft\Sigma M_o = +\circlearrowleft\Sigma M_o \qquad 0 = (2)(45)F_2 - (2)(60)F_1$$

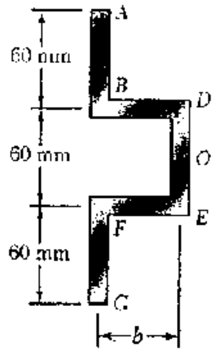
$$\left[(45)^2 b^2 - (2)(60)(27\times10^3)\right]\frac{Vt}{I} = 0$$

$$b^2 = \frac{(2)(60)(27\times10^3)}{45^2} = 1600\text{ mm}^2 \qquad b = 40\text{ mm}$$

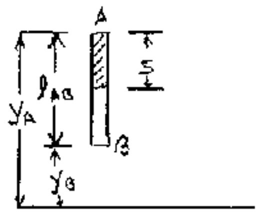
Note that the pair of F_1 forces form a couple. Likewise, the pair of F_2 forces form a couple. The lines of action of the forces in BDOGK pass through point O.

Problem 6.78

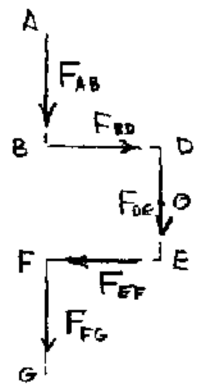
6.77 and 6.78 A thin-walled beam of uniform thickness has the cross section shown. Determine the dimension b for which the shear center O of the cross section is located at the point indicated.



Part AB :



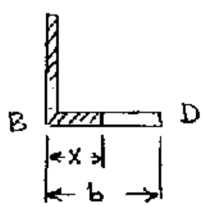
$$A(s) = ts$$
$$\bar{y}(s) = y_A - \tfrac{1}{2}s$$
$$Q(s) = A(s)\bar{y}(s) = ty_A s - \tfrac{1}{2}ts^2$$
$$q(s) = \frac{VQ(s)}{I} = \frac{Vt}{I}(y_A s - \tfrac{1}{2}s^2)$$
$$F_{AB} = \int_0^{l_{AB}} q(s)\,ds$$
$$= \frac{Vt}{I}\left(\frac{y_A l_{AB}^2}{2} - \frac{l_{AB}^3}{6}\right) \downarrow$$



At B: $Q_B = ty_A l_{AB} - \tfrac{1}{2}t\,l_{AB}^2$

By symmetry, $F_{FG} = F_{AB}$

Part BD :



$$A(x) = tx$$
$$Q(x) = Q_B + y_B A(x) = Q_B + t y_B x$$
$$q(x) = \frac{VQ(x)}{I} = \frac{V}{I}(Q_B + t y_B x)$$
$$F_{BD} = \int_0^b q(x)\,dx = \frac{V}{I}(Q_B b + \tfrac{1}{2}t y_B b^2) \rightarrow$$

By symmetry, $F_{EF} = F_{BD}$

F_{DE} is not required, since its moment about is zero.

$$\circlearrowleft \Sigma M_o = 0: \qquad b(F_{AB} + F_{FG}) - y_B F_{BD} + y_F F_{EF} = 0$$
$$2b\,F_{AB} - 2y_B F_{BD} = 0$$
$$2b\cdot\frac{Vt}{I}\left(\frac{y_A l_{AB}^2}{2} - \frac{l_{AB}^3}{6}\right) - 2y_B\frac{V}{I}(Q_B b + \tfrac{1}{2}t y_B b^2)$$
$$\frac{2Vt}{I}\left\{\tfrac{1}{2}y_A l_{AB}^2 - \tfrac{1}{6}l_{AB}^3\right\}b - \frac{2Vt}{I}\left\{(y_A l_{AB} - \tfrac{1}{2}l_{AB}^2)y_B b - \tfrac{1}{2}y_B^2 b^2\right. = 0$$

Dividing by $\frac{2Vt}{I}$ and substituting numerical data,

$$\left\{\tfrac{1}{2}(90)(60)^2 - \tfrac{1}{6}(60)^3\right\}b - \left[(90)(60) - \tfrac{1}{2}(60)^2\right\}(30)b + \tfrac{1}{2}(30)^2 b^2 = 0$$
$$126\times10^3\,b - 108\times10^3\,b + 450\,b^2 = 0$$
$$18\times10^3\,b - 450\,b^2 = 0 \qquad\qquad b = 0 \text{ and } b = 40.0 \text{ mm}$$

Problem 6.79

6.79 For the angle shape and loading of Sample Prob. 6.5, check that $\int q\,dz = 0$ along the horizontal leg of the angle and $\int q\,dy = P$ along its vertical leg.

Refer to Sample Prob. 6.5.

Along horizontal leg: $\tau_f = \dfrac{3P(a-z)(a-3z)}{4ta^3} = \dfrac{3P}{4ta^3}(a^2 - 4az + 3z^2)$

$$\int q\,dz = \int_0^a \tau_f t\,dz = \frac{3P}{4a^3}\int_0^a (a^2 - 4az + 3z^2)\,dz = \frac{3P}{4a^3}\left(a^2 z - 4a\frac{z^2}{2} + 3\frac{z^3}{3}\right)\Big|_0^a$$

$$= \frac{3P}{4a^3}(a^3 - 2a^3 + a^3) = 0$$

Along vertical leg: $\tau_e = \dfrac{3P(a-y)(a+5y)}{4ta^3} = \dfrac{3P}{4ta^3}(a^2 + 4ay - 5y^2)$

$$\int q\,dy = \int_0^a \tau_e t\,dy = \frac{3P}{4a^3}\int_0^a (a^2 + 4ay - 5y^2)\,dy = \frac{3P}{4a^3}\left(a^2 y + 4a\frac{y^2}{2} - 5\frac{y^3}{3}\right)\Big|_0^a$$

$$= \frac{3P}{4a^3}\left(a^3 + 2a^3 - \frac{5}{3}a^3\right) = \frac{3P}{4a^3}\cdot\frac{4}{3}a^3 = P$$

Problem 6.80

6.80 For the angle shape and loading of Sample Prob. 6.5, (*a*) determine the points where the shearing stress is maximum and the corresponding values of the stress, (*b*) verify that the points obtained are located on the neutral axis corresponding to the given loading.

Refer to Sample Prob. 6.5.

(a) Along vertical leg: $\tau_e = \dfrac{3P(a-y)(a+5y)}{4ta^3} = \dfrac{3P}{4ta^3}(a^2 + 4ay - 5y^2)$

$$\frac{d\tau_e}{dy} = \frac{3P}{4ta^3}(4a - 10y) = 0 \qquad y = \frac{2}{5}a$$

$$\tau_m = \frac{3P}{4ta^3}\left[a^2 + (4a)(\tfrac{2}{5}a) - (5)(\tfrac{2}{5}a)^2\right] = \frac{3P}{4ta^3}(\tfrac{9}{5}a^2) = \frac{27}{20}\frac{P}{ta}$$

Along horizontal leg: $\tau_f = \dfrac{3P(a-z)(a-3z)}{4ta^3} = \dfrac{3P}{4ta^3}(a^2 - 4az + 3z^2)$

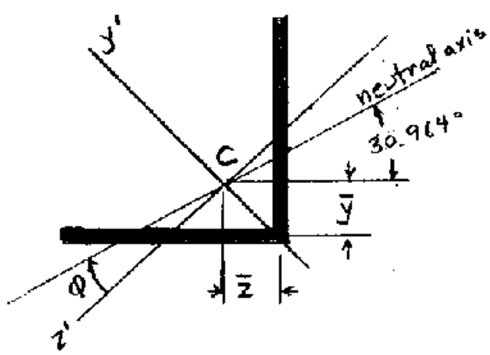
$$\frac{d\tau_f}{dz} = \frac{3P}{4ta^3}(-4a + 6z) = 0 \qquad z = \frac{2}{3}a$$

$$\tau_m = \frac{3P}{4ta^3}\left[a^2 - (4a)(\tfrac{2}{3}a) + (3)(\tfrac{2}{3}a)^2\right] = \frac{3P}{4ta^3}(-\tfrac{5}{3}a^2) = -\frac{1}{4}\frac{P}{ta}$$

At the corner: $y = 0,\ z = 0 \qquad \tau = \dfrac{3}{4}\dfrac{P}{ta}$

(b) $I_{y'} = \frac{1}{3}ta^3 \qquad I_{z'} = \frac{1}{12}ta^3 \qquad \theta = 45°$

$$\tan\varphi = \frac{I_{z'}}{I_{y'}}\tan\theta = \frac{1}{4} \qquad \varphi = 14.036°$$



$$\theta - \varphi = 45 - 14.036 = 30.964°$$

$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{at(a/2)}{2at} = \frac{1}{4}a$$

$$\bar{z} = \frac{\Sigma A\bar{z}}{\Sigma A} = \frac{at(a/2)}{2at} = \frac{1}{4}a$$

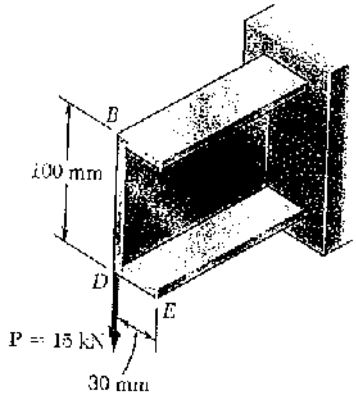
Neutral axis intersects vertical leg at

$$y = \bar{y} + \bar{z}\tan 30.964°$$
$$= (\tfrac{1}{4} + \tfrac{1}{4}\tan 30.964°)a = 0.400a$$
$$= \tfrac{2}{5}a$$

Neutral axis intersects horizontal leg at

$$z = \bar{z} + \bar{y}\tan(45° + \varphi)$$
$$= (\tfrac{1}{4} + \tfrac{1}{4}\tan 59.036°)a = 0.6667a$$
$$= \tfrac{2}{3}a$$

Problem 6.81



***6.81** A steel plate, 160 mm wide and 8 mm thick, is bent to form the channel shown. Knowing that the vertical load **P** acts at a point in the midplane of the web of the channel, determine (*a*) the torque **T** that would cause the channel to twist in the same way that it does under the load **P**, (*b*) the maximum shearing stress in the channel caused by the load **P**.

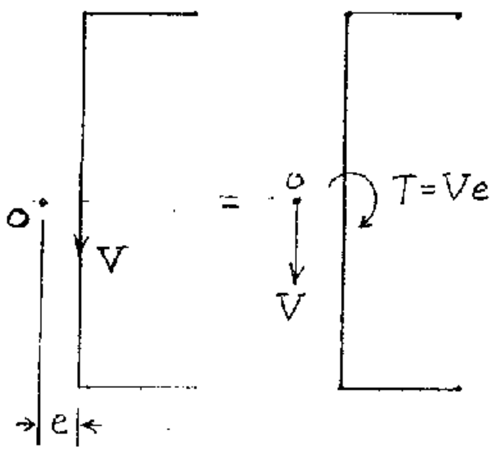
Use results of Example 6.06 with $b = 30$ mm, $h = 100$ mm, and $t = 8$ mm.

$$e = \frac{b}{2 + \frac{h}{3b}} = \frac{30}{2 + \frac{100}{(3)(30)}} = 9.6429 \text{ mm} = 9.6429 \times 10^{-3} \text{ m}$$

$$I = \frac{1}{12} t h^2 (6b + h) = \frac{1}{12}(8)(100)^2[(6)(30) + 100]$$

$$= 1.86667 \times 10^6 \text{ mm}^4 = 1.86667 \times 10^{-6} \text{ m}^4$$

$$V = 15 \times 10^3 \text{ N}$$



(a) $T = Ve = (15 \times 10^3)(9.6429 \times 10^{-3}) = 144.64$ N·m ◄

Stress at neutral axis due to V

$$Q = bt\frac{h}{2} + t\left(\frac{h}{2}\right)\left(\frac{h}{4}\right)$$

$$= \frac{1}{8} t h (h + 4b)$$

$$= \frac{1}{8}(8)(100)[100 + (4)(30)]$$

$$= 22 \times 10^3 \text{ mm}^3 = 22 \times 10^{-6} \text{ m}^3$$

$$t = 8 \times 10^{-3} \text{ m}$$

$$\tau_V = \frac{VQ}{It} = \frac{(15 \times 10^3)(22 \times 10^{-6})}{(1.86667 \times 10^{-6})(8 \times 10^{-3})} = 22.10 \times 10^6 \text{ Pa} = 22.10 \text{ MPa}$$

Stress due to T $\quad a = 2b + h = 160 \text{ mm} = 0.160 \text{ m}$

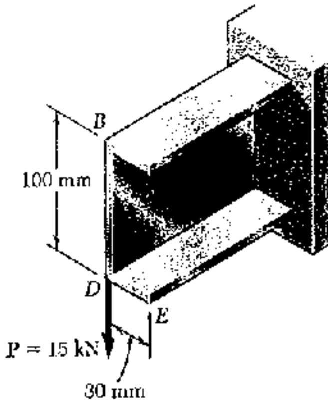
$$c_1 = \frac{1}{3}\left(1 - 0.630\frac{t}{a}\right) = \frac{1}{3}\left[1 - (0.630)\frac{8}{160}\right] = 0.32281$$

$$\tau_V = \frac{T}{c_1 a t^2} = \frac{144.64}{(0.32281)(0.160)(8 \times 10^{-3})^2} = 43.76 \times 10^6 \text{ Pa}$$

$$= 43.76 \text{ MPa}$$

(b) By superposition, $\tau_{max} = \tau_V + \tau_T = 65.9$ MPa ◄

Problem 6.82



***6.82** Solve Prob. 6.81, assuming that a 6-mm-thick plate is bent to form the channel shown.

***6.81** A steel plate, 160 mm wide and 8 mm thick, is bent to form the channel shown. Knowing that the vertical load **P** acts at a point in the midplane of the web of the channel, determine (*a*) the torque **T** that would cause the channel to twist in the same way that it does under the load **P**, (*b*) the maximum shearing stress in the channel caused by the load **P**.

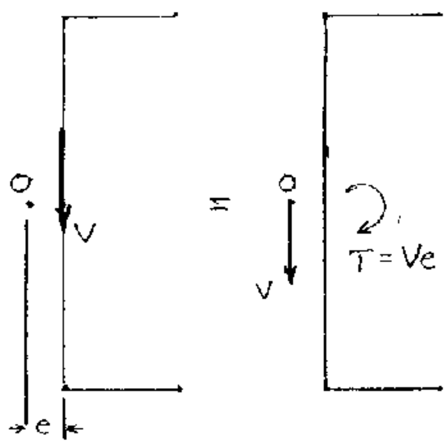
Use results of Example 6.06 with $b = 30$ mm

$h = 100$ mm, and $t = 6$ mm

$$e = \frac{b}{2 + \frac{h}{3b}} = \frac{30}{2 + \frac{100}{(3)(30)}} = 9.6429 \text{ mm} = 9.6429 \times 10^{-3} \text{ m}$$

$$I = \frac{1}{12} th^2 (6b + h) = \frac{1}{12}(6)(100)^2 [(6)(30) + 100]$$
$$= 1.400 \times 10^6 \text{ mm}^4 = 1.400 \times 10^{-6} \text{ m}^4$$

$$V = 15 \times 10^3 \text{ N}$$



(a) $T = Ve = (15 \times 10^3)(9.6429 \times 10^{-3}) = 144.64 \text{ N}\cdot\text{m}$

Stress at neutral axis due to V

$$Q = bt\frac{h}{2} + t\left(\frac{h}{2}\right)\left(\frac{h}{4}\right)$$
$$= \frac{1}{8} th(h + 4b)$$
$$= \frac{1}{8}(6)(100)[100 + (4)(30)]$$
$$= 16.5 \times 10^3 \text{ mm}^3 = 16.5 \times 10^{-6} \text{ m}^3$$

$$t = 6 \times 10^{-3} \text{ m}$$

$$\tau_v = \frac{VQ}{It} = \frac{(15 \times 10^3)(16.5 \times 10^{-6})}{(1.400 \times 10^{-6})(6 \times 10^{-3})} = 29.46 \times 10^6 \text{ Pa} = 29.46 \text{ MPa}$$

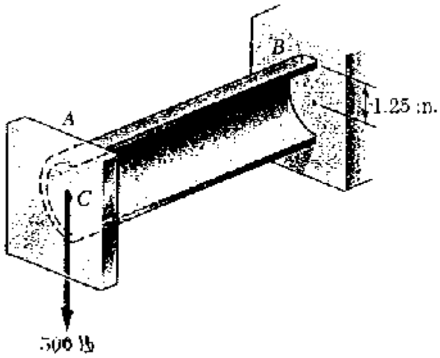
Stress due to T $\quad a = 2b + h = 160 \text{ mm} = 0.160 \text{ m}$

$$c_1 = \frac{1}{3}\left(1 - 0.630\frac{t}{a}\right) = \frac{1}{3}\left[1 - (0.630)\frac{6}{160}\right] = 0.32546$$

$$\tau_v = \frac{T}{c_1 a t^2} = \frac{144.64}{(0.32546)(0.160)(6 \times 10^{-3})^2} = 77.16 \times 10^6 \text{ Pa} = 77.16 \text{ MPa}$$

(b) By superposition $\quad \tau_{max} = \tau_v + \tau_T = 106.6 \text{ MPa}$

Problem 6.83

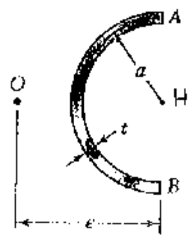


***6.83** The cantilever beam AB, consisting of half of a thin-walled pipe of 1.25-in. mean radius and $\frac{3}{8}$-in. wall thickness, is subjected to a 500-lb vertical load. Knowing that the line of action of the load passes through the centroid C of the cross section of the beam, determine (*a*) the equivalent force-couple system at the shear center of the cross section, (*b*) the maximum shearing stress in the beam. (*Hint*: The shear center O of this cross section was shown in Prob. 6.73 to be located twice as far from its vertical diameter as its centroid C.)

From the solution to Problem 6.73,

$$I = \frac{\pi}{2} a^3 t \qquad Q = a^2 t \sin\theta$$

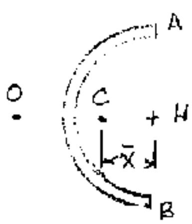
$$e = \frac{4}{\pi} a \qquad Q_{max} = a^2 t$$



For a half-pipe section, the distance from the center of the semi-circle to the centroid is

$$\bar{x} = \frac{2}{\pi} a$$

At each section of the beam, the shearing force V is equal to P. Its line of action passes through the centroid C. The moment arm of its moment about the shear center O is



$$d = e - \bar{x} = \frac{4}{\pi}a - \frac{2}{\pi}a = \frac{2}{\pi}a$$

(a) Equivalent force-couple system at O.

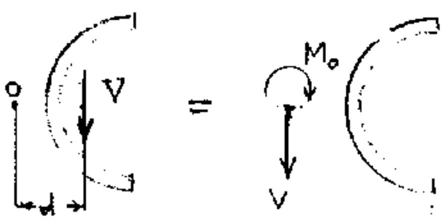
$$V = P \qquad M_o = Vd = \frac{2}{\pi} Pa$$

Data: $P = 500$ lb $\quad a = 1.25$ in.

$$M_o = \left(\tfrac{2}{\pi}\right)(500)(1.25)$$

$V = 500$ lb ◄

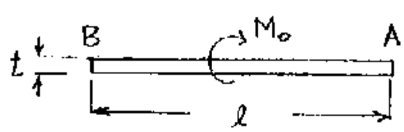
$M_o = 398$ lb-in ◄



(b) Shearing stresses

(1) Due to V. $\qquad \tau_V = \dfrac{V Q_{max}}{I t}$

$$\tau_V = \frac{(P)(a^2 t)}{(\frac{\pi}{2}a^3 t)(t)} = \frac{2P}{\pi a t} = \frac{(2)(500)}{\pi(1.25)(0.375)} = 679 \text{ psi}$$



(2) Due to M_o, the torque.

For a long rectangular section of length l and width t the shearing stress due to torque M_o is

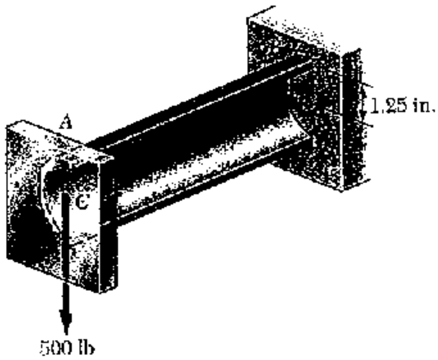
$$\tau_M = \frac{M_o}{c_1 l t^2} \qquad \text{where} \qquad c_1 = \frac{1}{3}\left(1 - 0.630\frac{t}{l}\right)$$

Data: $l = \pi a = \pi(1.25) = 3.927$ in. $\quad t = 0.375$ in. $\quad c_1 = 0.31328$

$$\tau_M = \frac{397.9}{(0.31328)(3.927)(0.375)^2} = 2300 \text{ psi}$$

By superposition $\quad \tau = \tau_V + \tau_M = 679 \text{ psi} + 2300 \text{ psi} = 2980$ psi

Problem 6.84



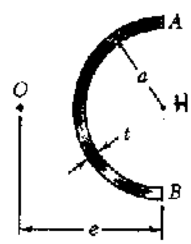
*6.84 Solve Prob. 6.83, assuming that the thickness of the beam is reduced to $\frac{1}{4}$ in.

*6.83 The cantilever beam AB, consisting of half of a thin-walled pipe of 1.25-in. mean radius and $\frac{3}{8}$-in. wall thickness, is subjected to a 500-lb vertical load. Knowing that the line of action of the load passes through the centroid C of the cross section of the beam, determine (*a*) the equivalent force-couple system at the shear center of the cross section, (*b*) the maximum shearing stress in the beam. (*Hint:* The shear center O of this cross section was shown in Prob. 6.73 to be located twice as far from its vertical diameter as its centroid C.)

From the solution to Problem 6.73

$$I = \frac{\pi}{2} a^3 t \qquad Q = a^2 t \sin\theta$$

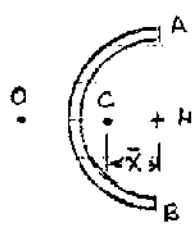
$$e = \frac{4}{\pi} a \qquad Q_{max} = a^2 t$$



For a half-pipe section, the distance from the center of the semi-circle to the centroid is

$$\bar{x} = \frac{2}{\pi} a$$

At each section of the beam, the shearing force V is equal to P. Its line of action passes through the centroid C. The moment arm of its moment about the shear center O is



$$d = e - \bar{x} = \frac{4}{\pi} a - \frac{2}{\pi} a = \frac{2}{\pi} a$$

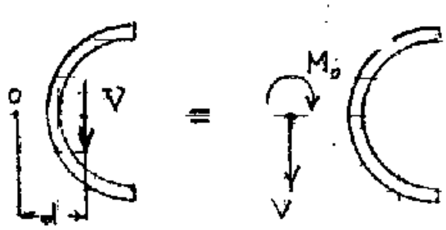
(a) Equivalent force-couple system at O.

$$V = P \qquad M_o = Vd = \frac{2}{\pi} Pa$$

Data: $P = 500$ lb $\quad a = 1.25$ in.

$V = 500$ lb ◄

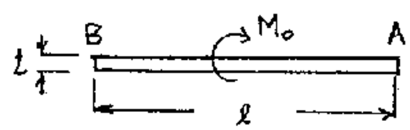
$M_o = 398$ lb-in ◄



(b) Shearing stresses

(1) Due to V. $\quad \tau_v = \dfrac{V Q_{max}}{It}$

$$\tau_v = \frac{(P)(a^2 t)}{(\frac{\pi}{2} a^3 t)(t)} = \frac{2P}{\pi a t} = \frac{(2)(500)}{\pi(1.25)(0.250)} = 1019 \text{ psi}$$



(2) Due to M_o, the torque.

For a long rectangular section of length l and width t the shearing stress due to torque M_o is

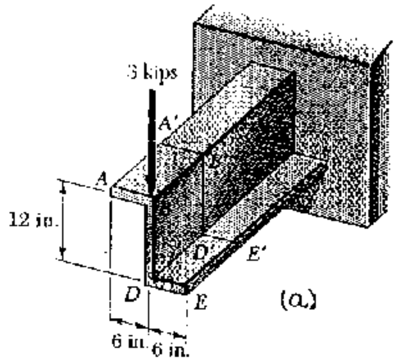
$$\tau_M = \frac{M_o}{c_1 l t^2} \qquad \text{where} \quad c_1 = \frac{1}{3}\left(1 - 0.630\frac{t}{l}\right)$$

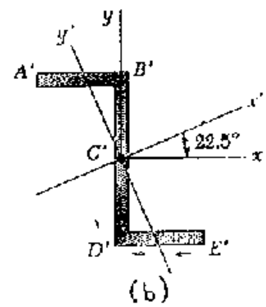
Data: $l = \pi a = \pi(1.25) = 3.927$ in. $\quad t = 0.375$ in $\quad c_1 = 0.31996$

$$\tau_M = \frac{392.9}{(0.31996)(3.927)(0.250)^2} = 5067 \text{ psi}$$

By superposition $\quad \tau = \tau_v + \tau_M = 1019 \text{ psi} + 5067 \text{ psi} = 6090$ psi

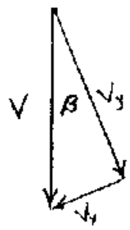
Problem 6.85





***6.85** The cantilever beam shown consists of a Z shape of $\frac{1}{4}$-in. thickness. For the given loading, determine the distribution of the shearing stresses along line $A'B'$ in the upper horizontal leg of the Z shape. The x' and y' axes are the principal centroidal axes of the cross section and the corresponding moments of inertia are $I_{x'} = 166.3 \text{ in}^4$ and are $I_{y'} = 13.61 \text{ in}^4$.

$V = 3$ kips $\qquad \beta = 22.5°$



$V_{x'} = V \sin\beta \qquad V_{y'} = V\cos\beta$

In upper horizontal leg use coordinate x $\quad (-6 \text{ in} \leq x \leq 0)$

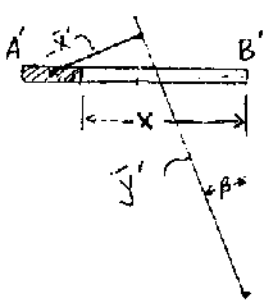
$A = \frac{1}{4}(6 + x)$ in.

$\bar{x} = \frac{1}{2}(-6 + x)$ in.

$\bar{y} = 6$ in.

$\bar{x}' = \bar{x}\cos\beta + \bar{y}\sin\beta$

$\bar{y}' = \bar{y}\cos\beta - \bar{x}\sin\beta$



Due to $V_{x'}$: $\quad \tau_1 = \dfrac{V_{x'} A \bar{x}'}{I_y t}$

$$\tau_1 = \frac{(V\sin\beta)(\frac{1}{4})(6+x)\left[\frac{1}{2}(-6+x)\cos\beta + 6\sin\beta\right]}{(13.61)(\frac{1}{4})}$$

$$= 0.084353(6+x)(-0.47554 + 0.46194x)$$

Due to $V_{y'}$: $$\tau_2 = \frac{V_{y'} A \bar{y}'}{I_x t} = \frac{(V\cos\beta)(\frac{1}{4})(6+x)\left[6\cos\beta - \frac{1}{2}(-6+x)\sin\beta\right]}{(166.3)(\frac{1}{4})}$$

$$= 0.0166665(6+x)\left[6.69132 - 0.19134x\right]$$

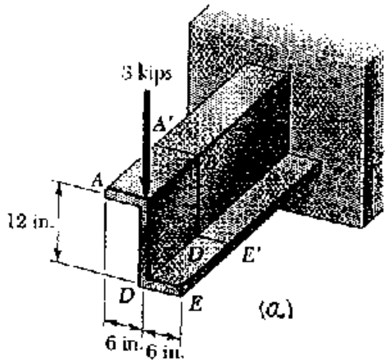
Total: $\tau_1 + \tau_2 = (6+x)\left[0.07141 + 0.085396x\right]$

x (in)	−6	−5	−4	−3	−2	−1	0
τ (ksi)	0	−0.105	−0.140	−0.104	0.003	0.180	0.428

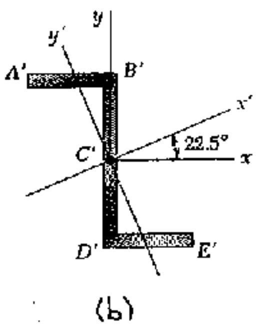
Problem 6.86

*6.86 For the cantilever beam and loading of Prob. 6.85, determine the distribution of the shearing stress along line $B'D'$ in the vertical web of the Z shape.

*6.85 The cantilever beam shown consists of a Z shape of $\frac{1}{4}$-in. thickness. For the given loading, determine the distribution of the shearing stresses along line $A'B'$ in the upper horizontal leg of the Z shape. The x' and y' axes are the principal centroidal axes of the cross section and the corresponding moments of inertia are $I_{x'} = 166.3\ \text{in}^4$ and are $I_{y'} = 13.61\ \text{in}^4$.

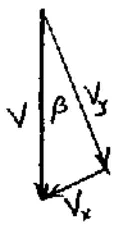


(a)



(b)

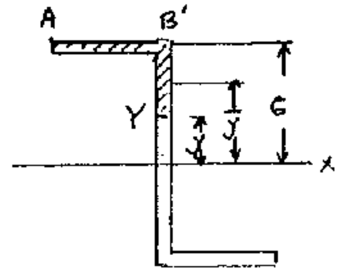
$V = 3$ kips $\qquad \beta = 22.5°$



$V_{y'} = V\sin\beta \qquad V_{y'} = V\cos\beta$

For part AB' $\quad A = (\frac{1}{4})(6) = 1.5\ \text{in}^2$

$\bar{x} = -3$ in, $\bar{y} = 6$ in.



For part B'Y

$A = \frac{1}{4}(6-y)$

$\bar{x} = 0 \qquad \bar{y} = \frac{1}{2}(6+y)$

$x' = x\cos\beta + y\sin\beta$

$y' = y\cos\beta - x\sin\beta$

Due to $V_{x'}$: $\quad \tau_1 = \dfrac{V_{x'}(A_{AB}\bar{x}'_{AB} + A_{BY}\bar{x}'_{BY})}{I_{y'}\,t}$

$$\tau_1 = \frac{(V\sin\beta)\left[(1.5)(-3\cos\beta + 6\sin\beta) + \frac{1}{4}(6-y)\frac{1}{2}(6+y)\sin\beta\right]}{(13.61)(\frac{1}{4})}$$

$$= \frac{(V\sin\beta)\left[-0.7133 + 1.7221 - 0.047835\,y^2\right]}{3.4025} = 0.3404 - 0.01614\,y^2$$

Due to $V_{y'}$: $\quad \tau_2 = \dfrac{V_{y'}(A_{AB}\bar{y}'_{AB} + A_{BY}\bar{y}')}{I_{x'}\,t}$

$$\tau_2 = \frac{(V\cos\beta)\left[(1.5)(6\cos\beta + 3\sin\beta) + \frac{1}{4}(6-y)\frac{1}{2}(6+y)\cos\beta\right]}{(166.3)(\frac{1}{4})}$$

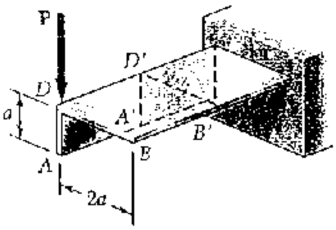
$$= \frac{(V\cos\beta)\left[10.037 + 4.1575 - 0.11548\,y^2\right]}{(166.3)(\frac{1}{4})} = 0.9463 - 0.00770\,y^2$$

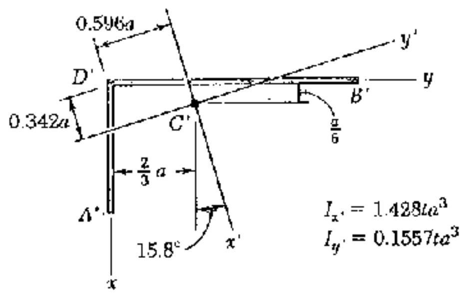
Total: $\quad \tau_1 + \tau_2 = 1.2867 - 0.02384\,y^2$

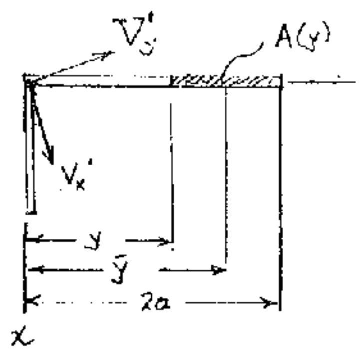
y (in)	0	± 2	± 4	± 6
τ (ksi)	1.287	1.191	0.905	0.428

Problem 6.87

*6.87 Determine the distribution of the shearing stresses along line $D'B'$ in the horizontal leg of the angle shape for the loading shown. The x' and y' axes are the principal centroidal axes of the cross section.







$\beta = 15.8°$ $\quad V_x' = P\cos\beta \quad V_y' = -P\sin\beta$

$A(y) = (2a-y)t \qquad \bar{y} = \frac{1}{2}(2a+y), \ \bar{x} = 0$

Coordinate transformation.

$y' = (y - \frac{2}{3}a)\cos\beta - (x - \frac{1}{6}a)\sin\beta$

$x' = (x - \frac{1}{6}a)\cos\beta + (y - \frac{2}{3}a)\sin\beta$

In particular,

$\bar{y}' = (\bar{y} - \frac{2}{3}a)\cos\beta - (\bar{x} - \frac{1}{6}a)\sin\beta$

$= (\frac{1}{2}y + \frac{1}{3}a)\cos\beta - (-\frac{1}{6}a)\sin\beta$

$= 0.48111\,y + 0.36612\,a$

$\bar{x}' = (\bar{x} - \frac{1}{6}a)\cos\beta + (\bar{y} - \frac{2}{3}a)\sin\beta$

$= (-\frac{1}{6}a)\cos\beta + (\frac{1}{2}y + \frac{1}{3}a)\sin\beta$

$= 0.13614\,y - 0.06961\,a$

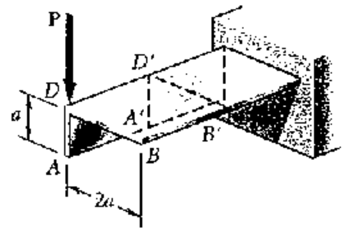
$$+\!\!\rightarrow \tau = \frac{V_x' A \bar{x}'}{I_y' t} + \frac{V_y' A \bar{y}'}{I_x' t}$$

$$= \frac{(P\cos\beta)(2a-y)(t)(0.13614y - 0.06961a)}{(0.1557\ ta^3)(t)} + \frac{(-P\sin\beta)(2a-y)(0.48111y + 0.36612a)}{(1.428\,a^3t)(t)}$$

$$= \frac{P(2a-y)(0.750y - 0.500a)}{ta^3}$$

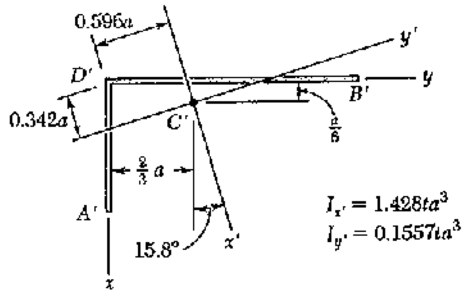
$y\ (a)$	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
$-\tau\ (\frac{P}{at})$	−1.000	−0.417	0	0.250	0.333	0.250	0

Problem 6.88



***6.88** For the angle shape and loading of Prob. 6.87, determine the distribution of the shearing stresses along line $D'A'$ in the vertical leg.

***6.87** Determine the distribution of the shearing stresses along line $D'B'$ in the horizontal leg of the angle shape for the loading shown. The x' and y' axes are the principal centroidal axes of the cross section.



$\beta = 15.8°$ $\quad V_x' = P\cos\beta$ $\quad V_y' = -P\sin\beta$

$A(x) = (a-x)t$ $\quad \bar{x} = \frac{1}{2}(a+x),\ \bar{y} = 0$

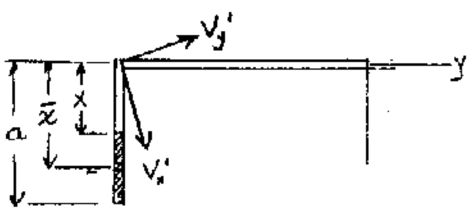
Coordinate transformation.

$$y' = (y - \tfrac{2}{3}a)\cos\beta - (x - \tfrac{1}{6}a)\sin\beta$$

$$x' = (x - \tfrac{1}{6}a)\cos\beta + (y - \tfrac{2}{3}a)\sin\beta$$

In particular,

$$\bar{y}' = (\bar{y} - \tfrac{2}{3}a)\cos\beta - (\bar{x} - \tfrac{1}{6}a)\sin\beta$$
$$= (-\tfrac{2}{3}a)\cos\beta - (\tfrac{1}{2}x + \tfrac{1}{3}a)\sin\beta$$
$$= -0.13614x - 0.73224a$$



$$\bar{x}' = (\bar{x} - \tfrac{1}{6}a)\cos\beta + (\bar{y} - \tfrac{2}{3}a)\sin\beta$$
$$= (\tfrac{1}{2}x + \tfrac{1}{3}a)\cos\beta + (-\tfrac{2}{3}a)\sin\beta$$
$$= 0.48111x + 0.13922a$$

$$+\downarrow\ \tau = \frac{V_x' A(x)\bar{x}'}{I_y' t} - \frac{V_y' A(x)\bar{y}'}{I_{x'} t}$$

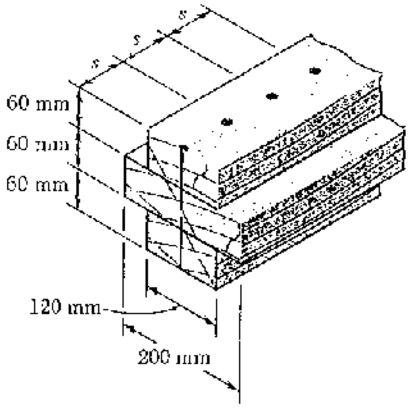
$$= \frac{(P\cos\beta)(a-x)(t)(0.48111x + 0.13922a)}{(0.1557ta^3)(t)} + \frac{(-P\sin\beta)(a-x)(t)(-0.13614x - 0.73224a)}{(1.428ta^3)(t)}$$

$$= \frac{P(a-x)(3.00x + 1.000a)}{ta^3}$$

$x\ (a)$	0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{6}$	1
$\tau\downarrow$	1.000	1.250	1.333	1.250	1.000	0.583	0

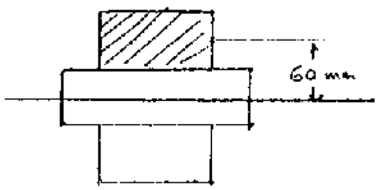
Problem 6.89

6.89 Three boards are nailed together to form the beam shown, which is subjected to a vertical shear. Knowing that the spacing between the nails is $s = 75$ mm and that the allowable shearing force in each nail is 400 N, determine the allowable shear.



Part	A (mm²)	d (mm)	Ad^2 (10^6 mm⁴)	$\bar{I}$ (10^6 mm⁴)
Top plank	7200	60	25.92	2.16
Middle plank	12000	0	0	3.60
Bottom plank	7200	60	25.92	2.16
Σ			51.84	7.92

$$I = \Sigma A d^2 + \Sigma \bar{I} = 59.76 \times 10^6 \text{ mm}^4 = 59.76 \times 10^{-6} \text{ m}^4$$



$$Q = (7200)(60) = 432 \times 10^3 \text{ mm}^3 = 432 \times 10^{-6} \text{ m}^3$$

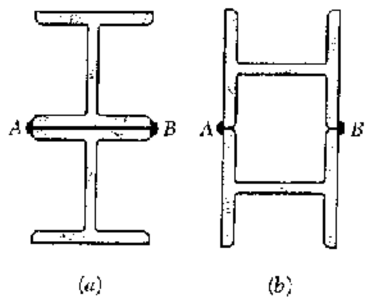
$$q = \frac{VQ}{I} \qquad F_{nail} = qs$$

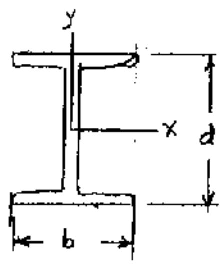
$$q = \frac{F_{nail}}{s} \qquad V = \frac{Iq}{Q} = \frac{I F_{nail}}{Qs}$$

$$V = \frac{(59.76 \times 10^{-6})(400)}{(432 \times 10^{-6})(75 \times 10^{-3})} = 738 \text{ N}$$

Problem 6.90

6.90 Two W 8 × 31 rolled sections may be welded at A and B in either of the two ways shown in order to form a composite beam. Knowing that for each weld the allowable horizontal shearing force is 3000 lb per inch of weld, determine the maximum allowable vertical shear in the composite beam for each of the two arrangements shown.





Since there are two welds, the allowable shear flow on a beam is 6000 lb/in.

$$q = \frac{VQ}{I} \qquad V = \frac{Iq}{Q}$$

For W8×31 rolled steel section

$A = 9.13 \text{ in}^2, \quad d = 8.00 \text{ in.}, \quad b = 7.995 \text{ in.}$

$I_x = 110 \text{ in}^4 \qquad I_y = 37.1 \text{ in}^4$

(a) $I = 2\left[110 + (9.13)\left(\frac{8.00}{2}\right)^2\right] = 512.16 \text{ in}^4$

$Q = (9.13)\left(\frac{8.00}{2}\right) = 36.52 \text{ in}^3$

$V = \frac{(512.16)(6000)}{36.52} = 84.1 \times 10^3 \text{ lb} \qquad 84.1 \text{ kips}$ ◄

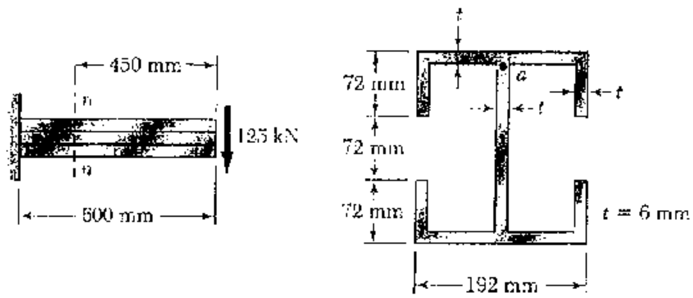
(b) $I = 2\left[37.1 + (9.13)\left(\frac{7.995}{2}\right)^2\right] = 365.99 \text{ in}^4$

$Q = (9.13)\left(\frac{7.995}{2}\right) = 36.50 \text{ in}^3$

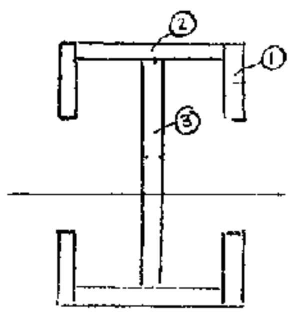
$V = \frac{(365.99)(6000)}{36.50} = 60.2 \times 10^3 \text{ lb} \qquad 60.2 \text{ kips}$ ◄

Problem 6.91

6.91 For the beam and loading shown, consider section *n-n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.



At section n-n $V = 125$ kN



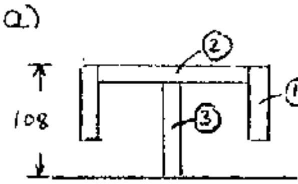
$$I_1 = \frac{1}{12}(6)(72)^3 + (6)(72)(72)^2 = 2.4261 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12}(180)(6)^3 + (180)(6)(105)^2 = 11.910 \times 10^6 \text{ mm}^4$$

$$I_3 = \frac{1}{12}(6)(204)^3 = 4.2448 \times 10^6 \text{ mm}^4$$

$$I = 4I_1 + 2I_2 + I_3 = 37.77 \times 10^6 \text{ mm}^4$$
$$= 37.77 \times 10^{-6} \text{ m}^4$$

(a)



$$Q = 2A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3$$
$$= (2)(6)(72)(72) + (180)(6)(105) + (6)(102)(51)$$
$$= 206.82 \times 10^3 \text{ mm}^3 = 206.82 \times 10^{-6} \text{ m}^4$$

$$t = 6 \text{ mm} = 6 \times 10^{-3} \text{ m}$$

$$\tau_{max} = \frac{VQ}{It} = \frac{(125 \times 10^3)(206.82 \times 10^{-6})}{(37.77 \times 10^{-6})(6 \times 10^{-3})} = 114.1 \times 10^6 \text{ Pa} = 114.1 \text{ MPa}$$

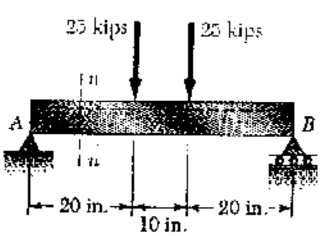
(b) $Q = 2A_1\bar{y}_1 + A_2\bar{y}_2$
$$= (2)(6)(72)(72) + (180)(6)(105) = 175.61 \times 10^3 \text{ mm}^3 = 175.61 \times 10^{-6} \text{ m}^3$$

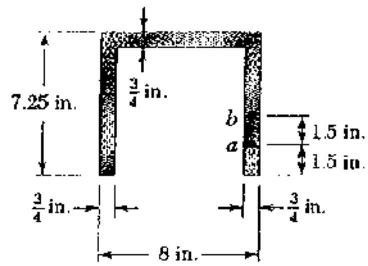
$$t = 6 \text{ mm} = 6 \times 10^{-3} \text{ m}$$

$$\tau = \frac{VQ}{It} = \frac{(125 \times 10^3)(175.61 \times 10^{-6})}{(37.77 \times 10^{-6})(6 \times 10^{-3})} = 96.9 \times 10^6 \text{ Pa} = 96.9 \text{ MPa}$$

Problem 6.92

6.92 For the beam and loading shown, consider section *n-n* and determine the shearing stress at (*a*) point *a*, (*b*) the shearing stress at point *b*.

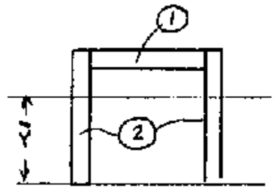




$R_A = R_B = 25$ kips

At section n-n $V = 25$ kips.

Locate centroid and calculate moment of inertia.

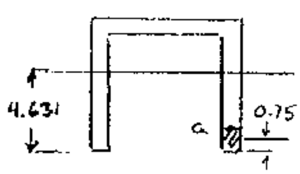


Part	A (in²)	$\bar{y}$ (in.)	$A\bar{y}$ (in³)	d (in.)	Ad^2 (in⁴)	$\bar{I}$ (in⁴)
①	4.875	6.875	33.52	2.244	24.55	0.23
②	10.875	3.625	39.42	1.006	11.01	47.68
Σ	15.75		72.94		35.56	47.86

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{72.94}{15.75} = 4.631 \text{ in.}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 35.56 + 47.86 = 83.42 \text{ in}^4$$

(a)

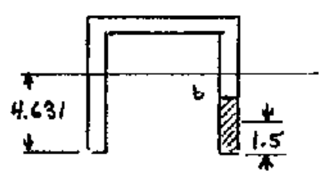


$$Q_a = A\bar{y} = \left(\tfrac{3}{4}\right)(1.5)(4.631 - 0.75) = 4.366 \text{ in}^3$$

$$t = \tfrac{3}{4} = 0.75 \text{ in.}$$

$$\tau_a = \frac{VQ}{It} = \frac{(25)(4.366)}{(83.42)(0.75)} = 1.745 \text{ ksi}$$

(b)



$$Q_b = A\bar{y} = \left(\tfrac{3}{4}\right)(3)(4.631 - 1.5) = 7.045 \text{ in}^3$$

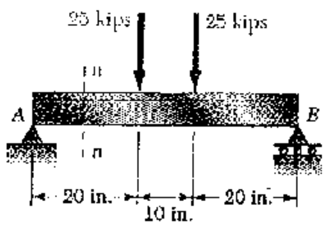
$$t = 0.75 \text{ in.}$$

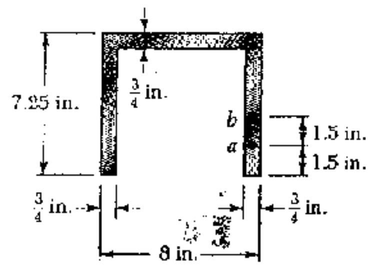
$$\tau_b = \frac{VQ}{It} = \frac{(25)(7.045)}{(83.42)(0.75)} = 2.82 \text{ ksi}$$

Problem 6.93

6.93 For the beam and loading shown in Prob. 6.92, determine the largest shearing stress in section *n-n*.

6.92 For the beam and loading shown, consider section *n-n* and determine the shearing stress at (*a*) point *a*, (*b*) the shearing stress at point *b*.

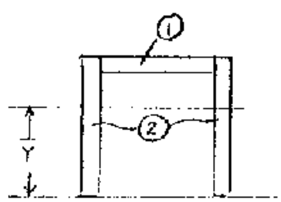




$R_A = R_B = 25$ kips

At section n-n $V = 25$ kips

Locate centroid and calculate moment of inertia.

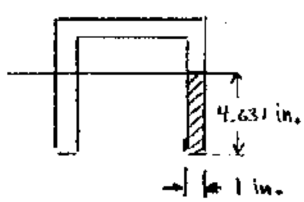


Part	A (in²)	$\bar{y}$ (in.)	$A\bar{y}$ (in³)	d (in.)	Ad^2 (in⁴)	$\bar{I}$ (in⁴)
①	4.875	6.875	33.52	2.244	24.55	0.23
②	10.875	3.625	39.42	1.006	11.01	47.68
Σ	15.75		72.94		35.56	47.86

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{72.94}{15.75} = 4.631 \text{ in.}$$

$$I = \Sigma A d^2 + \Sigma \bar{I} = 35.56 + 47.86 = 83.42 \text{ in}^4$$

Largest shearing stress occurs on section through centroid of entire cross section.

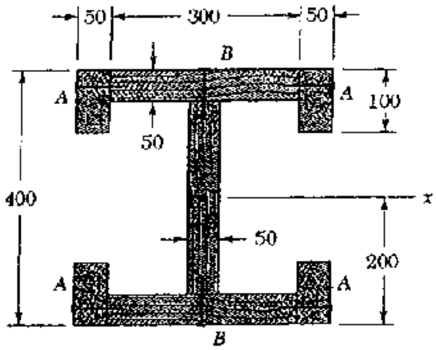


$$Q = A\bar{y} = \left(\tfrac{3}{4}\right)(4.631)\left(\frac{4.631}{2}\right) = 8.042 \text{ in}^3$$

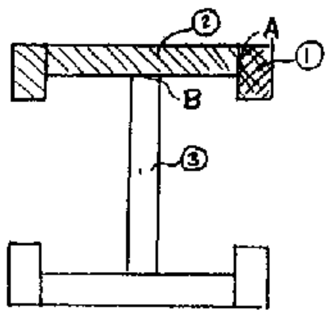
$$t = \tfrac{3}{4} = 0.75 \text{ in.}$$

$$\tau = \frac{VQ}{It} = \frac{(25)(8.042)}{(83.42)(0.75)} = 3.21 \text{ ksi}$$

Problem 6.94



Dimensions in mm



6.94 The built-up wooden beam shown is subjected to a vertical shear of 8 kN. Knowing that the nails are spaced longitudinally every 60 mm at A and every 25 mm at B, determine the shearing force in the nails (a) at A, (b) at B. (*Given*: I_x $1.504 \times 10^9\ \text{mm}^4$.)

$$I_x = 1.504 \times 10^9\ \text{mm}^4 = 1504 \times 10^{-6}\ \text{m}^4$$

$$s_A = 60\ \text{mm} = 0.060\ \text{m}$$

$$s_B = 25\ \text{mm} = 0.025\ \text{m}$$

(a) $Q_A = Q_1 = A_1 \bar{y}_1 = (50)(100)(150) = 750 \times 10^3\ \text{mm}^3$
$= 750 \times 10^{-6}\ \text{m}^3$

$$F_A = q_A s_A$$

$$= \frac{V Q_1 s_A}{I} = \frac{(8 \times 10^3)(750 \times 10^{-6})(0.060)}{1504 \times 10^{-6}}$$

$$= 239\ \text{N}$$

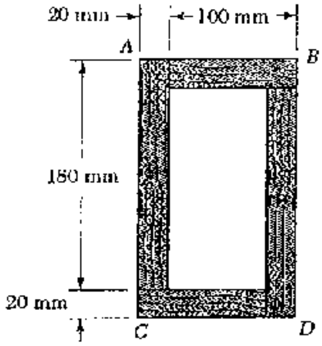
(b) $Q_2 = A_2 \bar{y}_2 = (300)(50)(175) = 2625 \times 10^3\ \text{mm}^3$

$Q_B = 2Q_1 + Q_2 = 4125 \times 10^3\ \text{mm}^3$
$= 4125 \times 10^{-6}\ \text{m}^3$

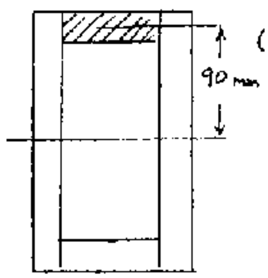
$$F_B = q_B s_B = \frac{V Q_B s_B}{I} = \frac{(8 \times 10^3)(4125 \times 10^{-6})(0.025)}{1504 \times 10^{-6}} = 549\ \text{N}$$

Problem 6.95

6.95 Two 20 × 100-mm and two 20 × 180-mm boards are glued together as shown to form a 120 × 200-mm box beam. Knowing that the beam is subjected to a vertical shear of 3.5 kN, determine the average shearing stress in the glued joint (*a*) at *A*, (*b*) at *B*.



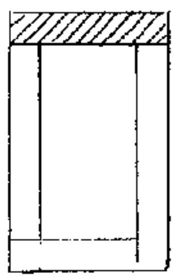
$$I = \frac{1}{12}(120)(200)^3 - \frac{1}{12}(80)(160)^3 = 52.693 \times 10^6 \text{ mm}^4 = 52.693 \times 10^{-6} \text{ m}^4$$



(a) $Q_A = (80)(20)(90) = 144 \times 10^3 \text{ mm}^3 = 144 \times 10^{-6} \text{ m}^3$

$t_A = (2)(20) = 40 \text{ mm} = 0.040 \text{ m}$

$$\tau_A = \frac{VQ_A}{I t_A} = \frac{(3.5\times10^3)(144\times10^{-6})}{(52.693\times10^{-6})(0.040)} = 239 \times 10^3 \text{ Pa} = 239 \text{ kPa}$$



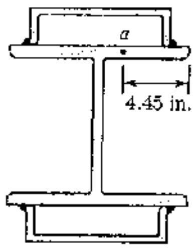
(b) $Q_B = (120)(20)(90) = 216 \times 10^3 \text{ mm}^3 = 216 \times 10^{-6} \text{ m}^3$

$t_B = (2)(20) = 40 \text{ mm} = 0.040 \text{ m}$

$$\tau_B = \frac{VQ_B}{I t_B} = \frac{(3.5\times10^3)(216\times10^{-6})}{(52.693\times10^{-6})(0.040)} = 359\times10^3 \text{ Pa} = 359 \text{ kPa}$$

Problem 6.96

6.96 The composite beam shown is made by welding C 8 × 11.5 rolled-steel channels to the flanges of a W 10 × 54 wide-flange rolled-steel shape. Knowing that the beam is subjected to a vertical shear of 45 kips, determine (*a*) the horizontal shearing force per inch at each weld, (*b*) the shearing stress at point *a* of the flange of the wide-flange shape.



For W10×54 rolled steel shape

$d = 10.09$ in., $t_f = 0.615$ in., $I_x = 303$ in^4

For C8×11.5 rolled steel shape

$A = 3.38$ in^2, $b_f = 2.260$ in., $t_f = 0.390$ in.

$I_y = 1.32$ in^4, $\bar{x} = 0.571$

Compute moment of inertia of the composite beam.

$$I = 303 + 2\left[1.32 + (3.38)\left(\frac{10.09}{2} + 2.260 - 0.571\right)^2\right]$$

$$= 612.18 \text{ in}^4$$

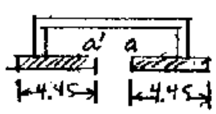
(a) For one channel $\quad \bar{y} = \frac{10.09}{2} + 2.260 - 0.571 = 6.734$ in.

$$Q = A\bar{y} = (3.38)(6.734) = 22.761 \text{ in}^3$$

$$q = \frac{VQ}{I} = \frac{(45)(22.761)}{612.18} = 1.673 \text{ kips/in.}$$

Since there are two welds, the horizontal shearing force per inch at each weld is

$$\frac{q}{2} = \frac{1.673}{2} = 0.837 \text{ kips/in} \blacktriangleleft$$



(b) For cuts at a and a',

$$Q = 22.761 + 2\left[(4.45)(0.615)\left(\frac{10.09}{2} - \frac{0.615}{2}\right)\right]$$

$$= 48.692 \text{ in}^3$$

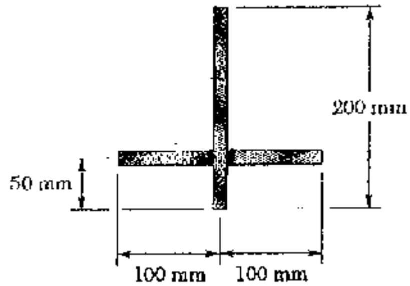
Since there are cuts at points a and a',

$$t = 2t_f = (2)(0.615) = 1.230 \text{ in.}$$

$$\tau = \frac{VQ}{It} = \frac{(45)(48.629)}{(612.18)(1.230)} = 2.91 \text{ ksi} \blacktriangleleft$$

Problem 6.97

6.97 Three plates, each 12 mm thick, are welded together to form the section shown. For a vertical shear of 100 kN, determine the shear flow through the welded surfaces and sketch the shear flow in the cross section.

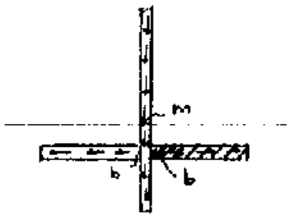


Locate neutral axis.

$$\Sigma A = (12)(200) + (2)(94)(12) = 4656 \text{ mm}^3$$

$$\Sigma A\bar{y} = (12)(200)(100) + (2)(94)(12)(50) = 352.8 \times 10^3 \text{ mm}^3$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = 75.77 \text{ mm}$$



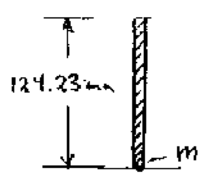
$$I = \frac{1}{12}(12)(200)^3 + (12)(200)(24.23)^2 + 2\left[\frac{1}{12}(94)(12)^3 + (94)(12)(25.77)^2\right]$$

$$= 10.934 \times 10^6 \text{ mm}^3 = 10.934 \times 10^{-6} \text{ m}^3$$

$$Q = (94)(12)(25.77) = 29.07 \times 10^3 \text{ mm}^3 = 29.07 \times 10^{-6} \text{ m}^3$$

$$q = \frac{VQ}{I} = \frac{(100 \times 10^3)(29.07 \times 10^{-6})}{10.934 \times 10^{-6}} = 266 \times 10^3 \text{ N/m} = 266 \text{ kN/m}$$

The maximum shear flow in the cross section occurs at the neutral axis.



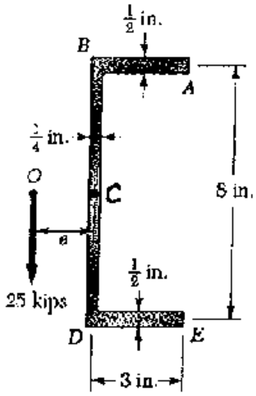
$$Q_m = (12)(124.33)\left(\frac{124.33}{2}\right) = 92.748 \times 10^3 \text{ mm}^3 = 92.748 \times 10^{-6} \text{ m}^3$$

$$q_m = \frac{VQ_m}{I} = \frac{(100 \times 10^3)(92.748 \times 10^{-6})}{(10.934 \times 10^{-6})} = 848 \times 10^3 \text{ N/m}$$

848 kN/m

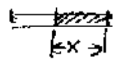
Problem 6.98

6.98 An extruded beam has the cross section shown. Determine (*a*) the location of the shear center O, (*b*) the distribution of the shearing stresses caused by a 25-kip vertical shearing force applied at O.



$$I = 2\left[\tfrac{1}{12}(3)(\tfrac{1}{2})^3 + (3)(\tfrac{1}{2})(4)^2\right] + \tfrac{1}{12}(\tfrac{1}{4})(8)^3 = 58.729 \text{ in}^4$$

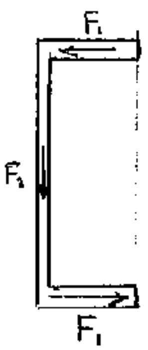
Part AB: $\quad A = \frac{1}{2}x, \quad \bar{y} = 4, \quad Q = A\bar{y} = 2x$



$$\tau = \frac{VQ}{It} = \frac{(25)(2x)}{(58.729)(\frac{1}{2})} = 1.7027x$$

Point A: $\quad x = 0 \qquad \tau = 0$

Point B: $\quad x = 3 \text{ in.} \qquad \tau = 5.11 \text{ ksi}$



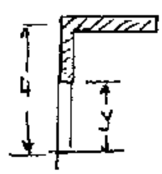
$$F_1 = \int \tau\, dA = \int_0^3 1.7027x \cdot \tfrac{1}{2}\, dx = \frac{1.7027}{4}x^2\Big|_0^3$$

$$= \frac{(1.7027)(3)^2}{4} = 3.8311 \text{ kips}$$

$$\circlearrowright \Sigma M_H = \circlearrowright \Sigma M_H: \qquad 25e = (F_1)(8)$$

$$e = \frac{(3.8311)(8)}{25} = 1.226 \text{ in.}$$

Part BD: $\quad Q = (2)(3) + (\frac{1}{4})(4-y)(\frac{4+y}{2})$



$$= 6 + \tfrac{1}{8}(16 - y^2) = 8 - \tfrac{1}{8}y^2$$

$$\tau = \frac{VQ}{It} = \frac{25(8 - \frac{1}{8}y^2)}{(58.729)(\frac{1}{4})}$$

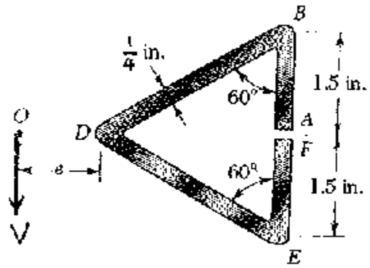
$$= 13.622 - 0.2128y^2$$

Point B: $y = 4$ in. $\qquad \tau = 10.22$ ksi

Point C: $y = 0 \qquad \tau = 13.62$ ksi

Problem 6.99

6.99 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.



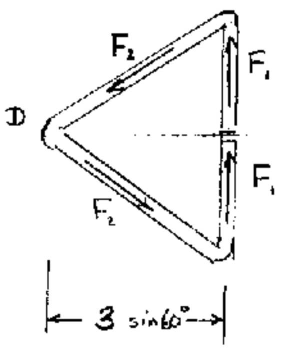
$$I_{AB} = \frac{1}{3}\left(\frac{1}{4}\right)(1.5)^3 = 0.28125 \text{ in}^4$$

$$L_{BD} = 3 \text{ in.} \quad A_{BD} = (3)\left(\tfrac{1}{4}\right) = 0.75 \text{ in}^2$$

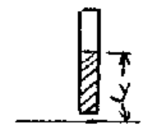
$$I_{BD} = \frac{1}{3} A_{BD} h^2 = \frac{1}{3}(0.75)(1.5)^2 = 0.5625 \text{ in}^4$$

$$I = (2)(0.28125) + (2)(0.5625) = 1.6875 \text{ in}^4$$

Part AB: $\quad A = \frac{1}{4} y \quad \bar{y} = \frac{1}{2} y \quad Q = A\bar{y} = \frac{1}{8} y^2$



$$\tau = \frac{VQ}{It} = \frac{V\,y^2}{(8)(1.6875)(0.25)} = \frac{Vy^2}{3.375}$$



$$F_1 = \int \tau\, dA = \int_0^{1.5} \frac{Vy^2}{3.375} \cdot (0.25\, dy)$$

$$= \frac{(0.25)V}{3.375} \left.\frac{y^3}{3}\right|_0^{1.5} = \frac{(0.25)(1.5)^3}{(3.375)(3)}$$

$$= 0.08333\ V$$

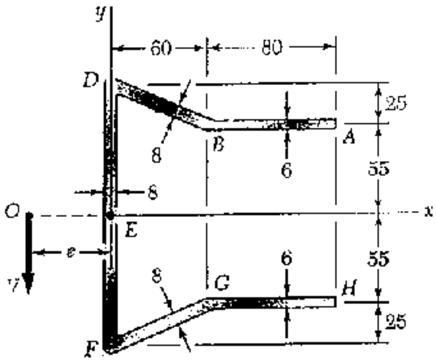
$$\circlearrowleft M_D = \circlearrowleft M_D: \quad Ve = 2\ F_1\ (3 \sin 60°)$$

$$Ve = (2)(0.08333)V(3 \sin 60°)$$

$$e = (2)(0.08333)(3 \sin 60°) = 0.433 \text{ in.}$$

Problem 6.100

6.100 For an extruded beam having the cross section shown, determine (*a*) the location of the shear center *O*, (*b*) the distribution of the shearing stresses caused by a vertical 2.5-kN force **V** applied at *O*.



$I_{AB} = I_{GH} = \frac{1}{12}(80)(6)^3 + (80)(6)(55)^2$
$= 1.45344 \times 10^6 \text{ mm}^4$

$I_{DE} = \frac{1}{12}(8)(160)^3 = 2.73067 \times 10^6 \text{ mm}^4$

Parts BD and FG:

$l_{BD} = l_{FG} = \sqrt{60^2 + 25^2} = 65 \text{ mm}$

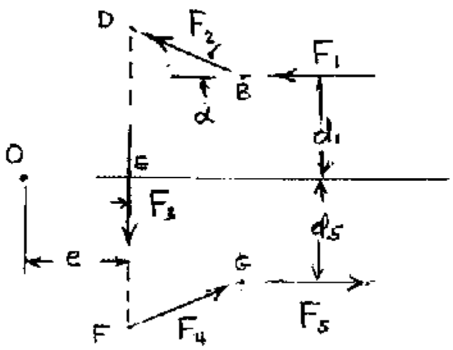
$\sin\alpha = \frac{25}{65} \qquad \cos\alpha = \frac{60}{65}$

$A_{BD} = A_{FG} = (65)(8) = 520 \text{ mm}^2$

$\bar{y}_{BD} = -\bar{y}_{FG} = \frac{55+80}{2} = 67.5 \text{ mm}$

$\bar{I}_{BD} = \bar{I}_{FG} = \frac{1}{12}\frac{8}{\sin\alpha}(25)^3 = 27.083 \times 10^3 \text{ mm}^4$

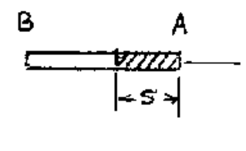
$I_{BD} = I_{FG} = (520)(67.5)^2 + 27.083 \times 10^3$
$= 2.39633 \times 10^6 \text{ mm}^4$



$I = \sum I = 10.4302 \times 10^6 \text{ mm}^4$

By symmetry, $F_5 = F_1$ and $F_4 = F_2$.

Calculation of F_1 and F_5.



$A(s) = t_{AB}\, s$

$Q(s) = \bar{y}_{AB}\, t_{AB}\, s$

$q(s) = \frac{V}{I} Q(s) = V \frac{\bar{y}_{AB} t_{AB}\, s}{I}$

$F_1 = \int_0^l q(s)\,ds = V \frac{\bar{y}_{AB} t_{AB}}{I} \int_0^{l_{AB}} s\,ds = V \frac{\bar{y}_{AB}\, t_{AB}\, l_{AB}^2}{2I}$

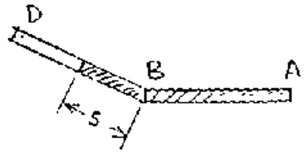
$= V \frac{(55)(6)(80)^2}{(2)(10.430 \times 10^6)} = 0.101244\, V \leftarrow$

$F_5 = 0.101244\, V \rightarrow$

At point B: $Q_B = \bar{y}_{AB}\, t_{AB}\, l_{AB} = (55)(6)(80) = 26.4 \times 10^3 \text{ mm}^3$

Continued on next page.

Calculation of F_2 and F_4.



In part BD $\quad A(s) = t_{BD}\, s$

$$\bar{y}(s) = \bar{y}_{AB} + \tfrac{1}{2} s \sin\alpha$$

$$Q(s) = Q_B + A(s)\,\bar{y}(s) = Q_B + t_{BD}\, s\,(\bar{y}_{AB} + \tfrac{1}{2} s \sin\alpha)$$

$$= Q_B + t_{DB}\,\bar{y}_{AB}\, s + \tfrac{1}{2} s^2 \sin\alpha$$

$$q(s) = \frac{V\,Q(s)}{I} = \frac{V}{I}\,(Q_B + t_{BD}\,\bar{y}_{AB}\, s + \tfrac{1}{2} t_{BD}\, s^2 \sin\alpha$$

$$F_2 = \int_0^{l_{BD}} = \frac{V}{I}\left\{ Q_B\, l_{BD} + \tfrac{1}{2} t_{BD}\,\bar{y}_{AB}\, l_{BD}^2 + \tfrac{1}{6} t_{BD}\, l_{BD}^3 \sin\alpha \right\}$$

$$= \frac{V}{10.4302\times10^6}\left\{ (26.4\times10^3)(65) + \tfrac{1}{2}(8)(55)(65)^2 + \tfrac{1}{6}(8)(65)^3\left(\tfrac{25}{65}\right) \right\}$$

$$= 0.267141\ V \measuredangle\alpha$$

$$F_4 = 0.267141\ V \measuredangle\alpha$$

At point D $\quad Q_D = 26.4\times10^3 + (520)(67.5) = 61.5\times10^3\ \text{mm}^3$

F_3 need not be calculated since its moment about E is zero.

$+\circlearrowleft \Sigma M_E = +\circlearrowleft \Sigma M_E$ by statics.

$$Ve = 55\,F_1 + 80\,F_2 \cos\alpha + 80\,F_4 \cos\alpha + 55\,F_5$$

$$= 110\,F_1 + 160\,F_2 \cos\alpha$$

$$= \left[(110)(0.101244) + (160)(0.267141)\tfrac{60}{65}\right] V = 50.6\ V$$

Dividing by V, $\qquad e = 50.6$ mm

At point E: $\quad Q_E = Q_D + A_{DE}\,\bar{y}_{DE} = 61.5\times10^3 + (8)(80)(40) = 87.1\times10^3\ \text{mm}^3$

Continued on next page.

Calculation of shearing stresses.

$V = 2.5\times10^3$ N $\qquad I = 10.4302\times10^{-6}$ m^4

At points A and H: $\qquad \tau = 0$ ◄

At points B and G: $\qquad Q = 26.4\times10^{-6}$ m^3

$$q = \frac{VQ}{I} = \frac{(2.5\times10^3)(26.4\times10^{-6})}{10.4302\times10^{-6}} = 6.3278\times10^3 \text{ N/m}$$

On the right: $t = 6\times10^{-3}$ m

$$\tau = \frac{q}{t} = \frac{6.3278\times10^3}{6\times10^{-3}} = 1.054\times10^6 \text{ Pa} = 1.054 \text{ MPa}$$ ◄

On the left: $t = 8\times10^{-3}$ m

$$\tau = \frac{q}{t} = \frac{6.3278\times10^3}{8\times10^{-3}} = 0.791\times10^6 \text{ Pa} = 0.791 \text{ MPa}$$ ◄

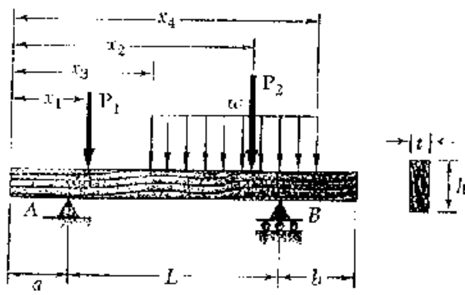
At points D and F: $Q = 61.5\times10^{-6}$ m^3, $t = 8\times10^{-3}$ m

$$\tau = \frac{VQ}{It} = \frac{(2.5\times10^3)(61.5\times10^{-6})}{(10.4302\times10^{-6})(8\times10^{-3})} = 1.843\times10^6 \text{ Pa} = 1.843 \text{ MPa}$$ ◄

At point E: $Q = 87.1\times10^{-6}$ m^3, $t = 8\times10^{-3}$ m

$$\tau = \frac{VQ}{It} = \frac{(2.5\times10^3)(87.1\times10^{-6})}{(10.4302\times10^{-6})(8\times10^{-3})} = 2.61\times10^6 \text{ Pa} = 2.61 \text{ MPa}$$ ◄

PROBLEM 6.C1



6.C1 A timber beam is to be designed to support a distributed load and up to two concentrated loads as shown. One of the dimensions of its uniform rectangular cross section has been specified and the other is to be determined so that the maximum normal stress and the maximum shearing stress in the beam will not exceed given allowable values σ_{all} and τ_{all}. Measuring x from end A and using either SI or U.S. customary units, write a computer program to calculate for successive cross sections, from $x = 0$ to $x = L$ and using given increments Δx, the shear, the bending moment, and the smallest value of the unknown dimension that satisfies in that section (1) the allowable normal stress requirement, (2) the allowable shearing stress requirement. Use this program to design the beams of uniform cross section of the following problems, assuming $\sigma_{all} = 12$ MPa and $\tau_{all} = 825$ kPa, and using the increments indicated: (*a*) Prob. 5.65 ($\Delta x = 0.1$ m), (*b*) Prob. 5.159 ($\Delta x = 0.2$ m).

SOLUTION

See solution of P5.C2 for the determination of R_A, R_B, $V(x)$, and $M(x)$

We recall that

$$V(x) = R_A\,STPA + R_B\,STPB - P_1\,STP1 - P_2\,STP2 - w(x - x_3)\,STP3 + w(x - x_4)\,STP4$$

$$M(x) = R_A(x-a)\,STPA + R_B(x-a-L)\,STPB - P_1(x-x_1)\,STP1 - P_2(x-x_2)\,STP2 - \tfrac{1}{2}w(x-x_3)^2\,STP3 + \tfrac{1}{2}w(x-x_4)^2\,STP4$$

where STPA, STPB, STP1, STP2, STP3, and STP4 are step functions defined in P5.C2.

(1) TO SATISFY THE ALLOWABLE NORMAL STRESS REQUIREMENT:

If unknown dimension is h:

$S_{min} = |M|/\sigma_{all}$. From $S = \frac{1}{6}th^2$, we have $h_\sigma = h = \sqrt{6S/t}$

If unknown dimension is t:

$S_{min} = |M|/\sigma_{all}$. From $S = \frac{1}{6}th^2$, we have $t_\sigma = t = 6S/h^2$

(2) TO SATISFY THE ALLOWABLE SHEARING STRESS REQUIREMENT:

We use Eq. (6.10), page 378: $\tau_{max} = \dfrac{3}{2}\dfrac{|V|}{A} = \dfrac{3}{2}\dfrac{|V|}{th}$

If unknown dimension is h: $\quad h_\tau = h = \dfrac{3|V|}{2t\tau_{all}}$

If unknown dimension is t: $\quad t_\tau = t = \dfrac{3|V|}{2h\tau_{all}}$

(CONTINUED)

PROGRAM OUTPUTS

Problem 5.65

RA = 2.40 kN   RB = 3.00 kN

X m	V kN	M kN.m	HSIG mm	HTAU mm
0.00	2.40	0.000	0.00	109.09
0.10	2.40	0.240	54.77	109.09
0.20	2.40	0.480	77.46	109.09
0.30	2.40	0.720	94.87	109.09
0.40	2.40	0.960	109.54	109.09
0.50	2.40	1.200	122.47	109.09
0.60	2.40	1.440	134.16	109.09
0.70	2.40	1.680	144.91	109.09
0.80	0.60	1.920	154.92	27.27
0.90	0.60	1.980	157.32	27.27
1.00	0.60	2.040	159.69	27.27
1.10	0.60	2.100	162.02	27.27
1.20	0.60	2.160	164.32	27.27
1.30	0.60	2.220	166.58	27.27
1.40	0.60	2.280	168.82	27.27
1.50	0.60	2.340	171.03	27.27
1.60	-3.00	2.400	173.21	136.36 ◄
1.70	-3.00	2.100	162.02	136.36
1.80	-3.00	1.800	150.00	136.36
1.90	-3.00	1.500	136.93	136.36
2.00	-3.00	1.200	122.47	136.36
2.10	-3.00	0.900	106.07	136.36
2.20	-3.00	0.600	86.60	136.36
2.30	-3.00	0.300	61.24	136.36
2.40	0.00	0.000	0.05	0.00

Problem 5.159

RA = 25.00 kN   RB = 25.00 kN

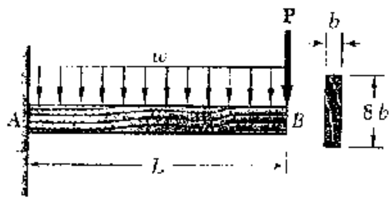
X m	V kN	M kN.m	HSIG mm	HTAU mm
0.00	25.00	0.000	0.00	378.79 ◄
0.20	23.00	4.800	141.42	348.48
0.40	21.00	9.200	195.79	318.18
0.60	19.00	13.200	234.52	287.88
0.80	17.00	16.800	264.58	257.58
1.00	15.00	20.000	288.68	227.27
1.20	13.00	22.800	308.22	196.97
1.40	11.00	25.200	324.04	166.67
1.60	9.00	27.200	336.65	136.36
1.80	7.00	28.800	346.41	106.06
2.00	5.00	30.000	353.55	75.76
2.20	3.00	30.800	358.24	45.45
2.40	1.00	31.200	360.56	15.15
2.60	-1.00	31.200	360.56	15.15
2.80	-3.00	30.800	358.24	45.45
3.00	-5.00	30.000	353.55	75.76
3.20	-7.00	28.800	346.41	106.06
3.40	-9.00	27.200	336.65	136.36
3.60	-11.00	25.200	324.04	166.67
3.80	-13.00	22.800	308.22	196.97
4.00	-15.00	20.000	288.68	227.27
4.20	-17.00	16.800	264.58	257.58
4.40	-19.00	13.200	234.52	287.88
4.60	-21.00	9.200	195.79	318.18
4.80	-23.00	4.800	141.42	348.48
5.00	0.00	0.000	0.00	0.00

The smallest allowable value of h is the largest of the values shown in the last two columns.

For Prob. 5.65, $h = h_{\sigma} = 173.2$ mm. ◄

For Prob. 5.159 $h = h_{\tau} = 379$ mm ◄

PROBLEM 6.C2



6.C2 A cantilever timber beam AB of length L and of uniform rectangular section shown supports a concentrated load **P** at its free end and a uniformly distributed load w along its entire length. Write a computer program to determine the length L and the width b of the beam for which both the maximum normal stress and the maximum shearing stress in the beam reach their largest allowable values. Assuming $\sigma_{all} = 1.8$ ksi and $\tau_{all} = 120$ psi, use this program to determine the dimensions L and b when (*a*) $P = 1000$ lb and $w = 0$, (*b*) $P = 0$ and $w = 12.5$ lb/in., (*c*) $P = 500$ lb and $w = 12.5$ lb/in.

SOLUTION

Both the maximum shear and the maximum bending moment occur at A. We have

$$V_A = P + wL \qquad M_A = PL + \frac{1}{2} wL^2$$

TO SATISFY THE ALLOWABLE NORMAL STRESS REQUIREMENT:

$$\sigma_{all} = \frac{M_A}{S} = \frac{M_A}{\frac{1}{6} b(8b)^2} = \frac{3M_A}{32b^3} \qquad b_\sigma = b = \left[\frac{3}{32}\frac{M_A}{\sigma_{all}}\right]^{1/3}$$

TO SATISFY THE ALLOWABLE SHEARING STRESS REQUIREMENT:

We use Eq. (6.10), page 378:

$$\tau_{all} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{V_A}{b(8b)} = \frac{3V_A}{16b^2} \qquad b_\tau = b = \left[\frac{3}{16}\frac{V_A}{\tau_{all}}\right]^{1/2}$$

PROGRAM

For $L = 0$, $V_A = P$ and $b_\tau > 0$, while $M_A = 0$ and $b_\sigma = 0$.

Starting with $L = 0$ and using increments $\Delta L = 0.001$ in., we increase L until b_σ and b_τ become equal. We then print L and b.

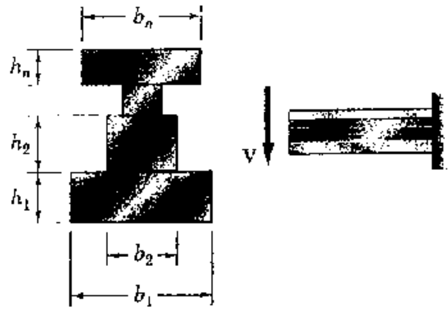
PROGRAM OUTPUTS

```
For P = 1000 lb, w =  0.0 lb/in.
Increment = 0.0010 in.
L = 37.5 in., b = 1.250 in.
```

```
For P =    0 lb, w = 12.5 lb/in.
Increment = 0.0010 in.
L = 70.3 in., b = 1.172 in.
```

```
For P =  500 lb, w = 12.5 lb/in.
Increment = 0.0010 in.
L = 59.8 in., b = 1.396 in.
```

PROBLEM 6.C3



6.C3 A beam having the cross section shown is subjected to a vertical shear **V**. Write a computer program that, for loads and dimensions expressed in either SI or U.S. customary units, can be used to calculate the shearing stress along the line between any two adjacent rectangular areas forming the cross section. Use this program to solve (*a*) Prob. 6.10, (*b*) Prob. 6.12, (*c*) Prob. 6.21.

SOLUTION

1. Enter V and the number n of rectangles.
2. For $i = 1$ to n, enter the dimensions b_i and h_i.
3. Determine the area $A_i = b_i h_i$ of each rectangle.
4. Determine the elevation of the centroid of each rectangle:

$$\bar{y}_i = \sum_{k=1}^{i} h_k - 0.5 h_i$$

and the elevation $\bar{y}$ of the centroid of the entire section:

$$\bar{y} = \left(\sum_i A_i \bar{y}_i\right) / \left(\sum_i A_i\right)$$

5. Determine the centroidal moment of inertia of the entire section:

$$I = \sum_i \left[\frac{1}{12} b_i h_i^3 + A_i (\bar{y}_i - \bar{y})^2\right]$$

6. For each surface separating two rectangles i and $i+1$, determine Q_i of the area below that surface:

$$Q_i = \sum_{k=1}^{i} = A_k (\bar{y}_k - \bar{y})$$

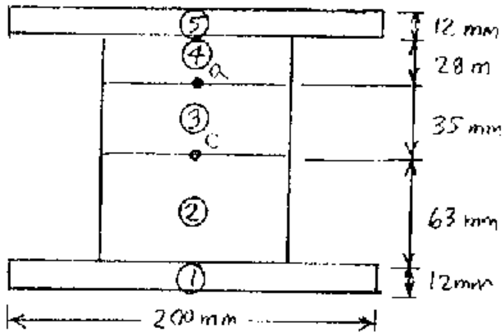
7. Select for t_i the smaller of b_i and b_{i+1}.

The shearing stress on the surface between the rectangles i and $i+1$ is

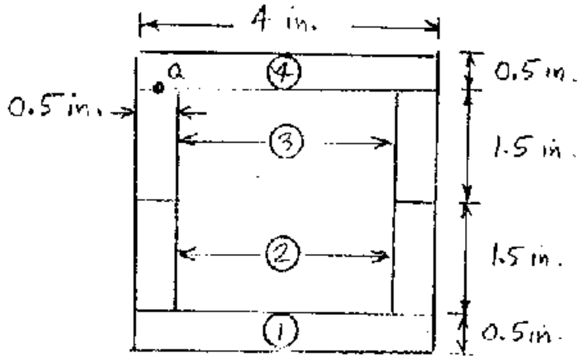
$$\tau_i = \frac{V Q_i}{I t_i}$$

(CONTINUED)

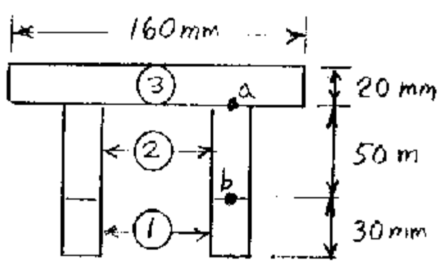
PROGRAM OUTPUTS




```
Problem 6.10

Shearing force =  10 kN
YBAR =  75.000 mm above base
I =  39.580*10^-6 mm^4
Between elements 1 and 2:
TAU =      418.39 kPA
Between elements 2 and 3:
TAU =      919.78 kPA      (a)
Between elements 3 and 4:
TAU =      765.03 kPA      (b)
Between elements 4 and 5:
TAU =      418.39 kPA
```

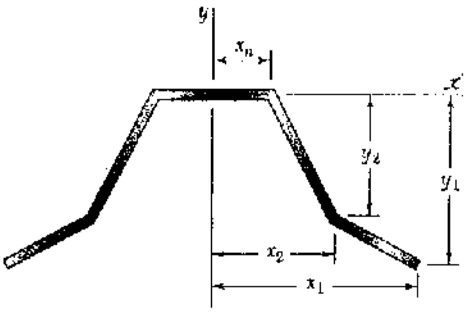



```
Problem 6.12

Shearing force = 10 kips
YBAR =  2.000 in.
I =  14.58 in^4
Between elements 1 and 2:
TAU =      2.400 ksi
Between elements 2 and 3:
TAU =      3.171 ksi      (a)
Between elements 3 and 4:
TAU =      2.400 ksi      (b)
```




```
Problem 6.21

Shearing force =  90 kN
YBAR = 65.000 mm
I = 58.133*10^-6 mm^4
Between elements 1 and 2:
TAU =      23.222 MPA      (b)
Between elements 2 and 3:
TAU =      30.963 MPA      (a)
```

PROBLEM 6.C4



6.C4 A plate of uniform thickness t is bent as shown into a shape with a vertical plane of symmetry and is then used as a beam. Write a computer program that, for loads and dimensions expressed in either SI or U.S. customary units, can be used to determine the distribution of shearing stresses caused by a vertical shear **V**. Use this program (a) to solve Prob. 6.47, (b) to find the shearing stress at a point E for the shape and load of Prob. 6.50, assuming a thickness $t = \frac{1}{4}$ in.

SOLUTION

For each element on the right-hand side, we compute (for $i=1$ to n):

Length of element $= L_i = \sqrt{(x_i - x_{i+1})^2 + (y_i - y_{i+1})^2}$

Area of element $= A_i = t L_i$ where $t = \frac{1}{4}$ in.

Distance from x axis to centroid of element $= \bar{y}_i = \frac{1}{2}(y_i + y_{i+1})$

Distance from x axis to centroid of section:

$$\bar{y} = (\Sigma A_i \bar{y}_i) / \Sigma A_i$$

Note that $y_n = 0$ and that $x_{n+1} = y_{n+1} = 0$

Moment of inertia of section about centroidal axis:

$$\bar{I} = 2 \Sigma A_i \left[\frac{1}{12}(y_i - y_{i+1})^2 + (\bar{y}_i - \bar{y})^2\right]$$

Computation of Q at point P where stress is desired

$Q = \Sigma A_i (\bar{y}_i - \bar{y})$ where sum extends to the areas located between one end of section and point P.

Shearing stress at P:

$$\tau = \frac{VQ}{It}$$

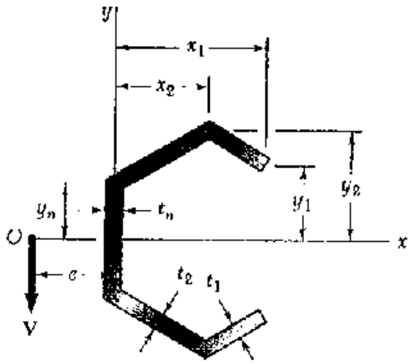
NOTE: τ_{max} occurs on neutral axis, i.e., for $y_P = \bar{y}$

PROGRAM OUTPUTS

```
Part (a):

I = 0.5333 in^4
Taumax = 2.02 ksi
TauB = 1.800 ksi
```

```
Part (b):

I = 22.27 in^4
TauE = 194.0 psi
```

PROBLEM 6.C5



6.C5 The cross section of an extruded beam is symmetric with respect to the x axis and consists of several straight segments as shown. Write a computer program that, for loads and dimensions expressed in either SI or U.S. customary units, can be used to determine (*a*) the location of the shear center O, (*b*) the distribution of shearing stresses caused by a vertical force applied at O. Use this program to solve Probs. 6.66 and 6.70.

SOLUTION

SINCE SECTION IS SYMMETRIC WITH X AXIS, COMPUTATIONS WILL BE DONE FOR TOP HALF.

FOR $i = 1$ TO $n+1$ (NOTE: $n+1$ IS THE ORIGIN)

ENTER t_i, x_i, y_i

COMPUTE LENGTH OF EACH SEGMENT

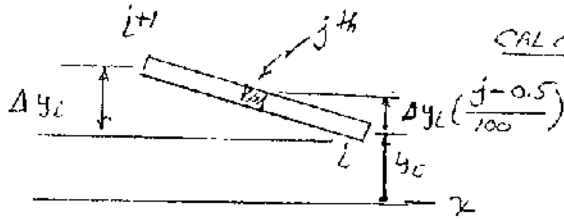
FOR $i = 1$ TO n

$\Delta x_i = x_{i+1} - x_i$

$\Delta y_i = y_{i+1} - y_i$

$L = (\Delta x_i^2 + \Delta y_i^2)^{1/2}$

CALCULATE MOMENT OF INERTIA I_x



CONSIDER EACH SEGMENT AS MADE OF 100 EQUAL PARTS

FOR $i = 1$ TO n

$\Delta AREA = L_i t_i / 100$

FOR $j = 1$ TO 100

$y = y_i + \Delta y_i (j - 0.5)/100$

$\Delta I = (\Delta AREA)\, y^2$

$I_x = I_x + \Delta I$

SINCE ONLY TOP HALF WAS USED

$I_x = 2 I_x$

CALCULATE SHEARING STRESS AT ENDS OF SEGMENTS AND SHEAR FORCES IN SEGMENTS

FOR $i = 1$ TO n

$\Delta AREA = L_i t_i / 100$, $\tau_{new} = \tau_{next}$

FOR $j = 1$ TO 100

$y = y_i + \Delta y_i (j - 0.5)/100$

$\Delta Q = (\Delta AREA)\, y$

$\tau_{old} = \tau_{new}$, $Q = Q + \Delta Q$

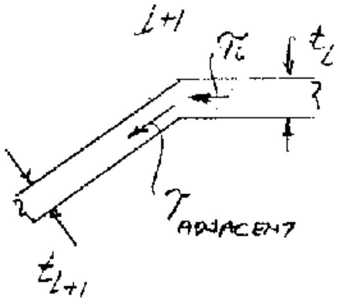
$\tau_{new} = VQ / I_x t_i$

$\tau_{ave} = 0.5(\tau_{old} + \tau_{new})$

$\tau = \tau + \tau_{ave}$

CONTINUED

PROBLEM 6.C5 - CONTINUED



$$FORCE_i = \tau(\Delta AREA)$$
$$\tau_i = VQ/I_x t_i$$
$$(\tau_{ADJACENT})_i = VQ/I_x t_{i+1}$$
$$Q_i = Q$$
$$\tau_{next} = (\tau_{ADJACENT})_i$$

LOCATION OF SHEAR CENTER

CALCULATE MOMENT OF SHEAR FORCES ABOUT ORIGIN

FOR $i = 1$ TO n

$$(F_x)_i = FORCE_i\,(\Delta x_i)/L_i$$
$$(F_y)_i = FORCE_i\,(\Delta y_i)/L_i$$
$$MOMENT_i = -(F_x)_i\, y_i + (F_y)_i\, x_i$$
$$MOMENT = MOMENT + MOMENT_i$$

FOR WHOLE SECTION $\quad MOMENT = 2(MOMENT)$

SHEAR CENTER IS AT

$$e = MOMENT/V$$

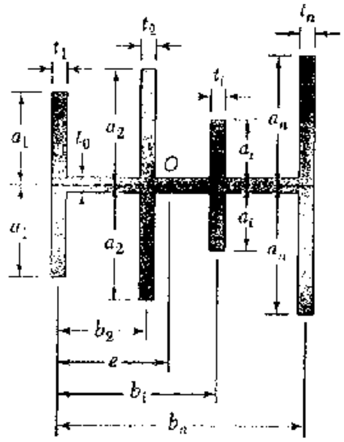
PROGRAM OUTPUT

```
Prob. 6.66
              T(K)          X(K)          Y(K)          L(K)
              in.           in.           in.           in.

 1            .13           4.00          .00           3.00

 2            .13           4.00          3.00          4.00

 3            .13           .00           3.00          3.00

 4            .13           .00           .00
Moment of inertia: Ix =     13.4999 in**4   Shear =     2.750
kips

Junction       Q          Tau       Tau       Force in
of segments               Before    After     segment
              in**3       ksi       ksi       kips

1 and 2        .56        .92       .92        .12
2 and 3       2.06       3.36      3.36       1.07
3 and 4       2.63       4.28      4.28       1.49

Moment of shear forces about origin:   M =     7.338 kip·in.
    + counterclockwise

Distance from origin to shear center:   e =   2.6684 in.
```

CONTINUED

```
Prob. 6.70
              T(K)          X(K)          Y(K)          L(K)
              mm            mm            mm            mm

 1            6.00          60.62         .00           70.00

 2            6.00          .00           35.00         35.00

 3            6.00          .00           .00
Moment of inertia: Ix =     514487. mm**4   Shear = 1000.000 N

Junction        Q        Tau       Tau      Force in
of segments              Before    After    segment
              mm**3      MPa       MPa        kN

1 and 2     7350.00      2.38      2.38     335.01
2 and 3    11025.00      3.57      3.57     666.27

Moment of shear forces about origin:   M =     20.309 N·m
    + counterclockwise

Distance from origin to shear center:   e =    20.309 mm
```

PROBLEM 6.C6



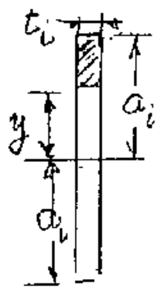
6.C6 A thin-walled beam has the cross section shown. Write a computer program that, for loads and dimensions expressed in either SI or U.S. customary units, can be used to determine the location of the shear center O of the cross section. Use the program to solve Prob. 6.75.

SOLUTION

Distribution of shearing stresses in element i

Let V = shear in cross section

$\bar{I}$ = Centroidal moment of inertia of section



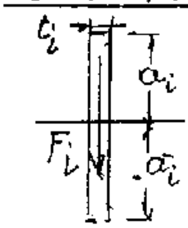
We have for shaded area

$$Q = A\bar{y} = t_i(a_i - y)\,\frac{a_i + y}{2}$$

$$= \frac{1}{2}t_i\,(a_i^2 - y^2)$$

$$\tau = \frac{QV}{It_i} = \frac{V}{2I}(a_i^2 - y^2)$$

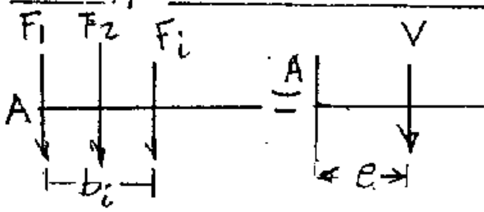
Force exerted on element i



$$F_i = \int_{-a_i}^{a_i} \tau\,(t_i\,dy) = \frac{Vt_i}{2I}\int_{-a_i}^{a_i}(a_i^2 - y^2)\,dy$$

$$= \frac{Vt_i}{I}\int_0^{a_i}(a_i^2 - y^2)\,dy = \frac{Vt_i}{I}\left(a_i^3 - \frac{1}{3}a_i^3\right) = \frac{2}{3}\frac{V}{I}t_i a_i^3$$

The system of the forces F_i must be equivalent to V at shear center.



$$\Sigma F = \Sigma F: \quad \frac{2}{3}\frac{V}{I}\Sigma t_i a_i^3 = V \qquad (1)$$

$$\Sigma M_A = \Sigma M_A: \quad \frac{2}{3}\frac{V}{I}\Sigma t_i a_i^3 b_i = eV \qquad (2)$$

Divide (2) by (1): $\quad e = \dfrac{\Sigma t_i a_i^3 b_i}{\Sigma t_i a_i^3}$ ◀

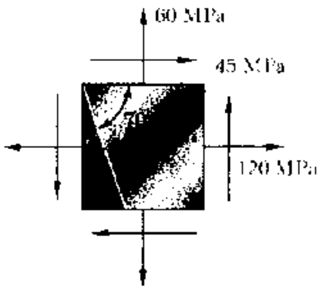
PROGRAM OUTPUT:

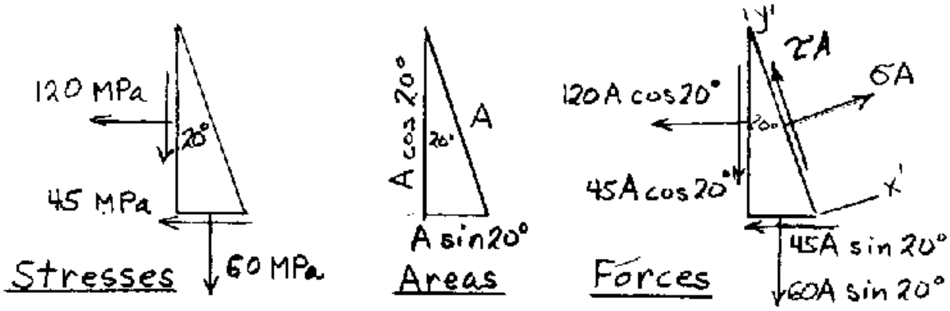
```
Problem 6.75

For element 1:
t = 0.75 in., a = 4 in., b = 0

For element 2:
t = 0.75 in., a = 3 in., b= 8 in.

Answer: e = 2.37 in.
```
◀

Chapter 7

Problem 7.1

7.1 through 7.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.





$+\searrow \Sigma F = 0$

$\sigma A - 120A\cos 20^\circ \cos 20^\circ - 45A\cos 20^\circ \sin 20^\circ - 45A\sin 20^\circ \cos 20^\circ - 60A\sin 20^\circ \sin 20^\circ = 0$

$\sigma = 120\cos^2 20^\circ + 45\cos 20^\circ \sin 20^\circ + 45\sin 20^\circ \cos 20^\circ + 60\sin^2 20^\circ = 14.19 \text{ MPa}$

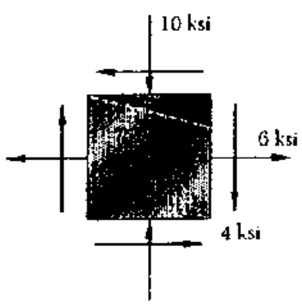
$+\nearrow \Sigma F = 0$

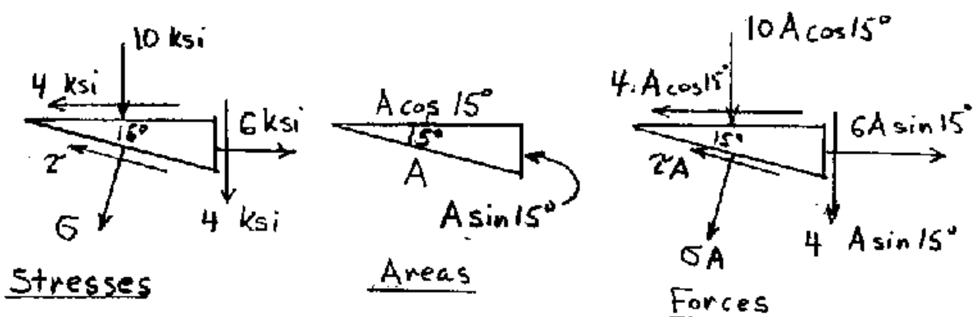
$\tau A + 120A\cos 20^\circ \sin 20^\circ - 45A\cos 20^\circ \cos 20^\circ + 45A\sin 20^\circ \sin 20^\circ - 60A\sin 20^\circ \cos 20^\circ = 0$

$\tau = -120\cos 20^\circ \sin 20^\circ + 45(\cos^2 20^\circ - \sin^2 20^\circ) + 60\sin 20^\circ \cos 20^\circ = 15.19 \text{ MPa}$

Problem 7.2

7.1 through 7.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.





$+\swarrow \Sigma F = 0$

$\sigma A + 4A\cos 15^\circ \sin 15^\circ + 10A\cos 15^\circ \cos 15^\circ - 6A\sin 15^\circ \sin 15^\circ + 4A\sin 15^\circ \cos 15^\circ = 0$

$\sigma = -4\cos 15^\circ \sin 15^\circ - 10\cos^2 15^\circ + 6\sin^2 15^\circ - 4\sin 15^\circ \cos 15^\circ = 10.93 \text{ ksi}$

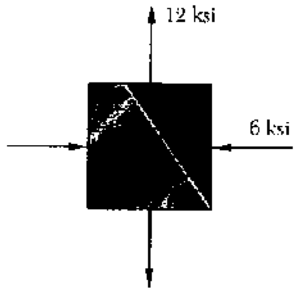
$+\nwarrow \Sigma F = 0$

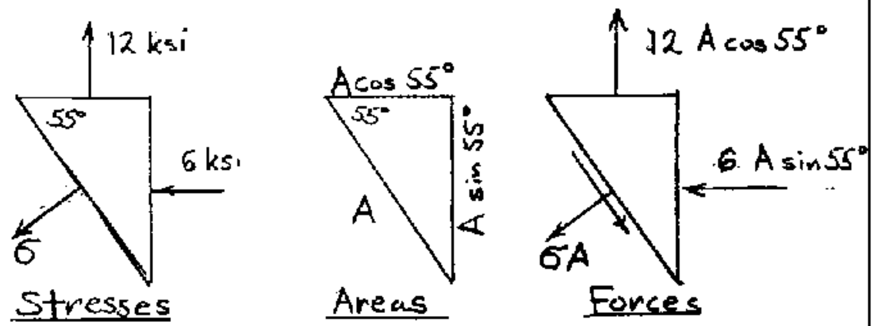
$\tau A + 4A\cos 15^\circ \cos 15^\circ - 10A\cos 15^\circ \sin 15^\circ - 6A\sin 15^\circ \cos 15^\circ - 4A\sin 15^\circ \sin 15^\circ = 0$

$\tau = -4(\cos^2 15^\circ - \sin^2 15^2) + (10+6)\cos 15^\circ \sin 15^\circ = 0.536 \text{ ksi}$

Problem 7.3

7.1 through 7.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.





$+\swarrow \Sigma F = 0$

$\sigma A - 12 A \cos 55° \cos 55° + 6 A \sin 55° \sin 55° = 0$

$\sigma = 12 \cos^2 55° - 6 \sin^2 55° = -0.078 \text{ ksi}$ ◄

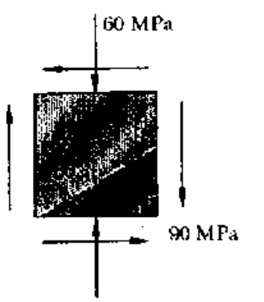
$+\searrow \Sigma F = 0$

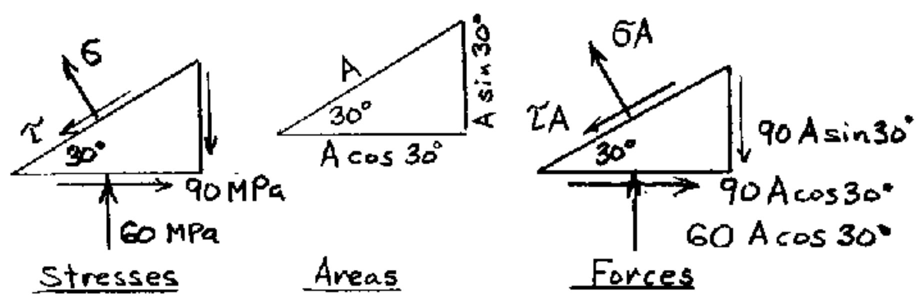
$\tau A - 12 A \cos 55° \sin 55° - 6 A \sin 55° \cos 55°$

$\tau = 18 \cos 55° \sin 55° = 8.46 \text{ ksi}$ ◄

Problem 7.4

7.1 through 7.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.





$+\nwarrow \Sigma F = 0$

$\sigma A - 90 A \sin 30° \cos 30° - 90 A \cos 30° \sin 30° + 60 A \cos 30° \cos 30° = 0$

$\sigma = 180 \sin 30° \cos 30° - 60 \cos^2 30° = 32.9 \text{ MPa}$ ◄

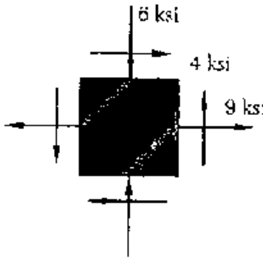
$+\swarrow \Sigma F = 0$

$\tau A + 90 A \sin 30° \sin 30° - 90 A \cos 30° \cos 30° - 60 A \cos 30° \sin 30° = 0$

$\tau = 90 (\cos^2 30° - \sin^2 30°) + 60 \cos 30° \sin 30° = 71.0 \text{ MPa}$ ◄

Problem 7.5

7.5 through 7.8 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.



$\sigma_x = 9$ ksi $\qquad \sigma_y = -6$ ksi $\qquad \tau_{xy} = 4$ ksi

(a) $\tan 2\theta_p = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y} = \dfrac{(2)(4)}{9+6} = 0.5333$

$2\theta_p = 28.07^\circ \qquad \theta_p = 14.04^\circ,\ 104.04^\circ$ ◄

(b) $\sigma_{max,min} = \dfrac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \dfrac{9-6}{2} \pm \sqrt{\left(\dfrac{9+6}{2}\right)^2 + (8)^2}$

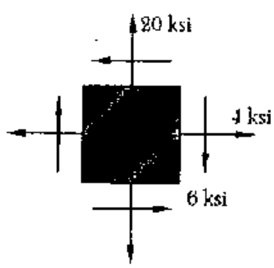
$= 1.5 \pm 8.5$ ksi

$\sigma_{max} = 10$ ksi ◄

$\sigma_{min} = -7$ ksi ◄

Problem 7.6

7.5 through 7.8 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.



$\sigma_x = 4$ ksi $\qquad \sigma_y = 20$ ksi $\qquad \tau_{xy} = -6$ ksi

(a) $\tan 2\theta_p = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y} = \dfrac{(2)(-6)}{4-20} = 0.750$

$2\theta_p = 36.87^\circ \qquad \theta_p = 18.43^\circ,\ 108.43^\circ$ ◄

(b) $\sigma_{max,min} = \dfrac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \dfrac{4+20}{2} \pm \sqrt{\left(\dfrac{4-20}{2}\right)^2 + (-6)^2}$

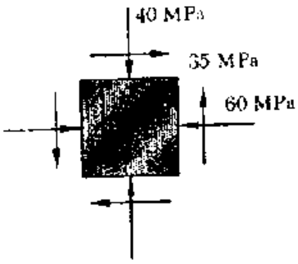
$= 12 \pm 10$ ksi

$\sigma_{max} = 22$ ksi ◄

$\sigma_{min} = 2$ ksi ◄

Problem 7.7

7.5 through 7.8 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.



$\sigma_x = -60$ MPa $\qquad \sigma_y = -40$ MPa $\qquad \tau_{xy} = 35$ MPa

(a) $\tan 2\theta_p = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y} = \dfrac{(2)(35)}{-60+40} = -3.50$

$2\theta_p = -74.05° \qquad \theta_p = -37.03°,\ 52.97°$

(b) $\sigma_{max,min} = \dfrac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \dfrac{-60-40}{2} \pm \sqrt{\left(\dfrac{-60+40}{2}\right)^2 + (35)^2}$

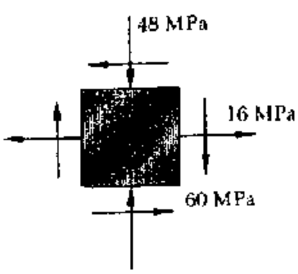
$= -50 \pm 36.4$ MPa

$\sigma_{max} = -13.60$ MPa

$\sigma_{min} = -86.4$ MPa

Problem 7.8

7.5 through 7.8 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.



$\sigma_x = 16$ MPa $\qquad \sigma_y = -48$ MPa $\qquad \tau_{xy} = -60$ MPa

(a) $\tan 2\theta_p = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y} = \dfrac{(2)(-60)}{16+48} = -1.875$

$2\theta_p = -61.93° \qquad \theta_p = -30.96°,\ 59.04°$

(b) $\sigma_{max,min} = \dfrac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

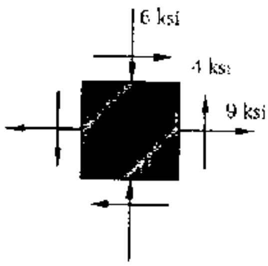
$= \dfrac{16-48}{2} \pm \sqrt{\left(\dfrac{16+48}{2}\right)^2 + (-60)^2}$

$= -16 \pm 68$

$\sigma_{max} = 52$ MPa

$\sigma_{min} = -84$ MPa

Problem 7.9



7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

$\sigma_x = 9 \text{ ksi} \qquad \sigma_y = -6 \text{ ksi} \qquad \tau_{xy} = 4 \text{ ksi}$

(a) $\tan 2\theta_s = -\dfrac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\dfrac{9+6}{(2)(4)} = -1.875$

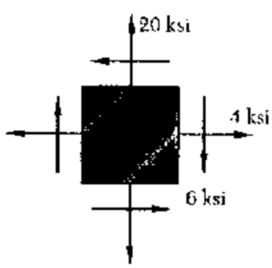
$2\theta_s = -61.93° \qquad \theta_s = -30.96°, \; 59.04°$

(b) $\tau_{max} = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{\left(\dfrac{9+6}{2}\right)^2 + (4)^2} = 8.5 \text{ ksi}$

(c) $\sigma' = \sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = \dfrac{9-6}{2} = 1.5 \text{ ksi}$

Problem 7.10



7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

$\sigma_x = 4 \text{ ksi} \qquad \sigma_y = 20 \text{ ksi} \qquad \tau_{xy} = -6 \text{ ksi}$

(a) $\tan 2\theta_s = -\dfrac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\dfrac{4-20}{(2)(-6)} = -1.3333$

$2\theta_s = -53.13° \qquad \theta_s = -26.57°, \; 63.43°$

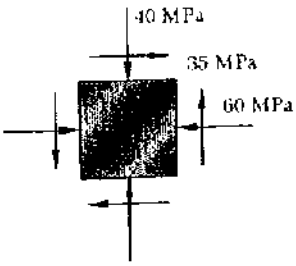
(b) $\tau_{max} = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{\left(\dfrac{4-20}{2}\right)^2 + (-6)^2} = 10 \text{ ksi}$

(c) $\sigma' = \sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = \dfrac{4+20}{2} = 12 \text{ ksi}$

Problem 7.11

7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.



$\sigma_x = -60 \text{ MPa} \qquad \sigma_y = -40 \text{ MPa} \qquad \tau_{xy} = 35 \text{ MPa}$

(a) $\tan 2\theta_s = -\dfrac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\dfrac{-60+40}{(2)(35)} = 0.2857$

$2\theta_s = 15.95° \qquad \theta_s = 7.97°,\ 97.97°$ ◄

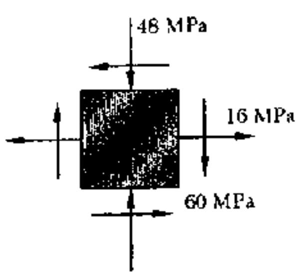
(b) $\tau_{max} = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{\left(\dfrac{-60+40}{2}\right)^2 + (35)^2} = 36.4 \text{ MPa}$ ◄

(c) $\sigma' = \sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = \dfrac{-60-40}{2} = -50 \text{ MPa}$ ◄

Problem 7.12

7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.



$\sigma_x = 16 \text{ MPa} \qquad \sigma_y = -48 \text{ MPa} \qquad \tau_{xy} = -60 \text{ MPa}$

(a) $\tan 2\theta_s = -\dfrac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\dfrac{16+48}{(2)(-60)} = 0.5333$

$2\theta_s = 28.07° \qquad \theta_s = 14.04°,\ 104.04°$ ◄

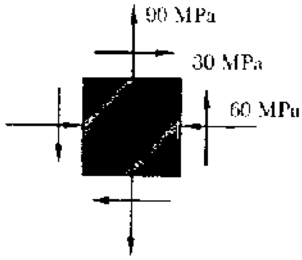
(b) $\tau_{max} = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{\left(\dfrac{16+48}{2}\right)^2 + (-60)^2} = 68 \text{ MPa}$ ◄

(c) $\sigma' = \sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = \dfrac{16-48}{2} = -16 \text{ MPa}$ ◄

Problem 7.13

7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.



$$\sigma_x = -60 \text{ MPa} \qquad \sigma_y = 90 \text{ MPa} \qquad \tau_{xy} = 30 \text{ MPa}$$

$$\frac{\sigma_x + \sigma_y}{2} = 15 \text{ MPa} \qquad \frac{\sigma_x - \sigma_y}{2} = -75 \text{ MPa}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{xy} = \quad -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

(a) $\theta = -25^\circ \qquad 2\theta = -50^\circ$

$$\sigma_{x'} = 15 - 75\cos(-50^\circ) + 30\sin(-50^\circ) = -56.2 \text{ MPa}$$

$$\tau_{x'y'} = \quad +75\sin(-50^\circ) + 30\cos(-50^\circ) = -38.2 \text{ MPa}$$

$$\sigma_{y'} = 15 + 75\cos(-50^\circ) - 30\sin(-50^\circ) = 86.2 \text{ MPa}$$

(b) $\theta = 10^\circ \qquad 2\theta = 20^\circ$

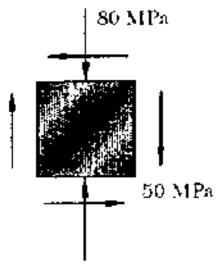
$$\sigma_{x'} = 15 - 75\cos(20^\circ) + 30\sin(20^\circ) = -45.2 \text{ MPa}$$

$$\tau_{x'y'} = \quad +75\sin(20^\circ) + 30\cos(20^\circ) = 53.8 \text{ MPa}$$

$$\sigma_{y'} = 15 + 75\cos(20^\circ) - 30\sin(20^\circ) = 75.2 \text{ MPa}$$

Problem 7.14

7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.



$\sigma_x = 0 \qquad \sigma_y = -80 \text{ MPa} \qquad \tau_{xy} = -50 \text{ MPa}$

$\dfrac{\sigma_x + \sigma_y}{2} = -40 \text{ MPa} \qquad \dfrac{\sigma_x - \sigma_y}{2} = 40 \text{ MPa}$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

(a) $\theta = -25^\circ \qquad 2\theta = -50^\circ$

$\sigma_{x'} = -40 + 40\cos(-50^\circ) - 50\sin(-50^\circ) = 24.0 \text{ MPa}$ ◄

$\tau_{x'y'} = -40\sin(-50^\circ) - 50\cos(-50^\circ) = -1.5 \text{ MPa}$ ◄

$\sigma_{y'} = -40 - 40\cos(-50^\circ) + 50\sin(-50^\circ) = -104.0 \text{ MPa}$ ◄

(b) $\theta = 10^\circ \qquad 2\theta = 20^\circ$

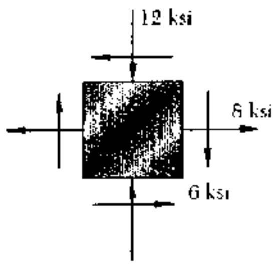
$\sigma_{x'} = -40 + 40\cos(20^\circ) - 50\sin(20^\circ) = -19.5 \text{ MPa}$ ◄

$\tau_{x'y'} = -40\sin(20^\circ) - 50\cos(20^\circ) = -60.7 \text{ MPa}$ ◄

$\sigma_{y'} = -40 - 40\cos(20^\circ) + 50\sin(20^\circ) = -60.5 \text{ MPa}$ ◄

Problem 7.15

7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.



$\sigma_x = 8 \text{ ksi} \qquad \sigma_y = -12 \text{ ksi} \qquad \tau_{xy} = -6 \text{ ksi}$

$\dfrac{\sigma_x + \sigma_y}{2} = -2 \text{ ksi} \qquad \dfrac{\sigma_x - \sigma_y}{2} = 10 \text{ ksi}$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

(a) $\theta = -25^\circ \qquad 2\theta = -50^\circ$

$\sigma_{x'} = -2 + 10\cos(-50^\circ) - 6\sin(-50^\circ) = 9.02 \text{ ksi}$ ◀

$\tau_{x'y'} = -10\sin(-50^\circ) - 6\cos(-50^\circ) = 3.80 \text{ ksi}$ ◀

$\sigma_{y'} = -2 - 10\cos(-50^\circ) + 6\sin(-50^\circ) = -13.02 \text{ ksi}$ ◀

(b) $\theta = 10^\circ \qquad 2\theta = 20^\circ$

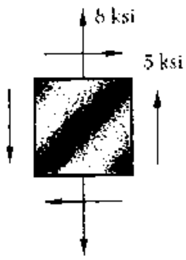
$\sigma_{x'} = -2 + 10\cos(20^\circ) - 6\sin(20^\circ) = 5.34 \text{ ksi}$ ◀

$\tau_{x'y'} = -10\sin(20^\circ) - 6\cos(20^\circ) = -9.06 \text{ ksi}$ ◀

$\sigma_{y'} = -2 - 10\cos(20^\circ) + 6\sin(20^\circ) = -9.34 \text{ ksi}$ ◀

Problem 7.16

7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.



$\sigma_x = 0 \qquad \sigma_y = 8 \text{ ksi} \qquad \tau_{xy} = 5 \text{ ksi}$

$\dfrac{\sigma_x + \sigma_y}{2} = 4 \text{ ksi} \qquad \dfrac{\sigma_x - \sigma_y}{2} = -4 \text{ ksi}$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

(a) $\theta = -25° \qquad 2\theta = -50°$

$\sigma_{x'} = 4 - 4\cos(-50°) + 5\sin(-50°) = -2.40 \text{ ksi}$ ◄

$\tau_{x'y'} = 4\sin(-50°) + 5\cos(-50°) = 0.15 \text{ ksi}$ ◄

$\sigma_{y'} = 4 + 4\cos(-50°) - 5\sin(-50°) = 10.40 \text{ ksi}$ ◄

(b) $\theta = 10° \qquad 2\theta = 20°$

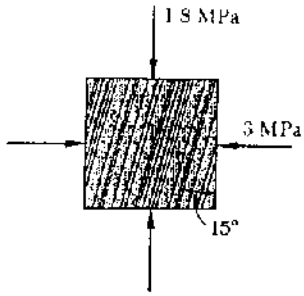
$\sigma_{x'} = 4 - 4\cos(20°) + 5\sin(20°) = 1.95 \text{ ksi}$ ◄

$\tau_{x'y'} = 4\sin(20°) + 5\cos(20°) = 6.07 \text{ ksi}$ ◄

$\sigma_{y'} = 4 + 4\cos(20°) - 5\cos(20°) = 6.05 \text{ ksi}$ ◄

Problem 7.17

7.17 and 7.18 The grain of a wooden member forms an angle of 15° with the vertical.. For the state of stress shown, determine (*a*) the in-plane shearing stress parallel to the grain, (*b*) the normal stress perpendicular to the grain.



$\sigma_x = -3 \text{ MPa} \qquad \sigma_y = -1.8 \text{ MPa} \qquad \tau_{xy} = 0$

$\theta = -15° \qquad 2\theta = -30°$

(a) $\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\sin 2\theta$

$= -\frac{-3 + 1.8}{2}\sin(-30°) + 0$

$= -0.300 \text{ MPa}$ ◄

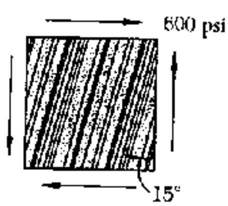
(b) $\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$

$= \frac{-3 - 1.8}{2} + \frac{-3 + 1.8}{2}\cos(-30°) + 0$

$= -2.92 \text{ MPa}$ ◄

Problem 7.18

7.17 and 7.18 The grain of a wooden member forms an angle of 15° with the vertical.. For the state of stress shown, determine (*a*) the in-plane shearing stress parallel to the grain, (*b*) the normal stress perpendicular to the grain.



$\sigma_x = 0 \qquad \sigma_y = 0 \qquad \tau_{xy} = 600 \text{ psi}$

$\theta = -15° \qquad 2\theta = -30°$

(a) $\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$

$= -0 + 600\cos(-30°)$

$= 520 \text{ psi}$ ◄

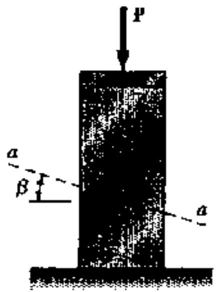
(b) $\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$

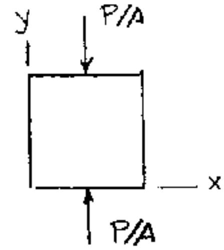
$= 0 + 0 + 600\sin(-30°)$

$= -300 \text{ psi}$ ◄

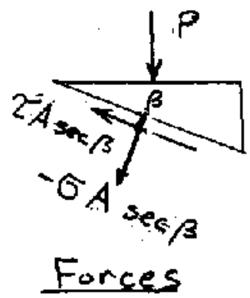
Problem 7.19

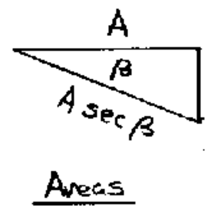
7.19 The centric force **P** is applied to a short post as shown. Knowing that the stresses on plane *a-a* are $\sigma = -15$ ksi and $\tau = 5$ ksi, determine (*a*) the angle β that plane *a-a* forms with the horizontal, (*b*) the maximum compressive stress in the post.

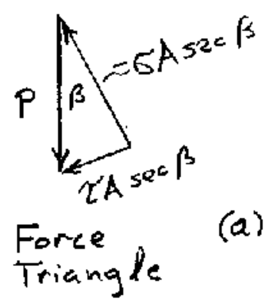




$$\sigma_x = 0$$
$$\tau_{xy} = 0$$
$$\sigma_y = \sigma_{max\ comp.} = -\frac{P}{A}$$







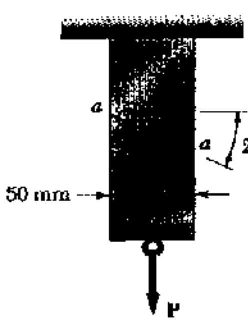
$$\tan\beta = \frac{\tau \cancel{A \sec\beta}}{-\sigma \cancel{A \sec\beta}} = \frac{5}{15} = \frac{1}{3}$$

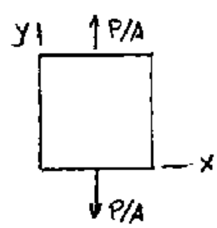
(a) $\beta = \arctan\frac{1}{3} = 18.4°$

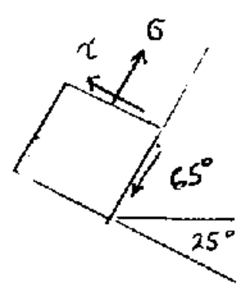
(b) $P = (-\sigma A \sec\beta)(\sec\beta)$ $\qquad \frac{P}{A} = \frac{-\sigma}{\cos^2\beta} = \frac{15}{\cos^2 18.4°} = 16.67$ ksi

Problem 7.20

7.20 Two members of uniform cross section 50 × 80 mm are glued together along plane *a-a*, that forms an angle or 25° with the horizontal. Knowing that the allowable stresses for the glued joint are $\sigma = 800$ kPa and $\tau = 600$ kPa, determine the largest centric load **P** that can be applied.







For plane a-a $\theta = 65°$

$$\sigma_x = 0 \qquad \tau_{xy} = 0 \qquad \sigma_y = \frac{P}{A}$$

$$\sigma = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta = 0 + \frac{P}{A}\sin^2 65° + 0$$

$$P = \frac{A\sigma}{\sin^2 65°} = \frac{(50\times10^{-3})(80\times10^{-3})(800\times10^{3})}{\sin^2 65°} = 3.90\times10^{3}\ \text{N}$$

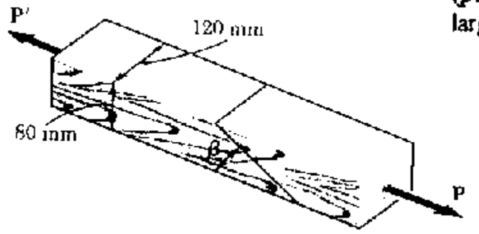
$$\tau = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta) = \frac{P}{A}\sin 65°\cos 65° + 0$$

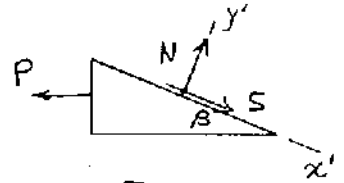
$$P = \frac{A\tau}{\sin 65°\cos 65°} = \frac{(50\times10^{-3})(80\times10^{-3})(600\times10^{3})}{\sin 65°\cos 65°} = 6.27\times10^{3}\ N$$

Allowable value of P is the smaller one. $P = 3.90\times10^{3} N = 3.90\ kN$

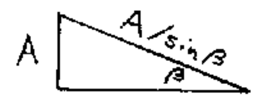
Problem 7.21

7.21 Two wooden members of 80 × 120-mm uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $\beta = 22°$ and that the maximum allowable stresses in the joint are, respectively, 400 kPa in tension (perpendicular to the splice) and 600 kPa in shear (parallel to the splice), determine the largest centric load **P** that can be applied.





Forces



Areas

$$A = (80)(120) = 9.6\times10^{3}\ \text{mm}^2 = 9.6\times10^{-3}\ \text{m}^2$$

$$N_{all} = \sigma_{all} A/\sin\beta = \frac{(400\times10^{3})(9.6\times10^{-3})}{\sin 22°} = 10.251\times10^{3}\ \text{N}$$

$$+\nearrow \Sigma F_{y'} = 0: \quad N - P\sin\beta = 0 \qquad P = \frac{N}{\sin\beta} = \frac{10.251\times10^{3}}{\sin 22°} = 27.4\times10^{3}\ \text{N}$$

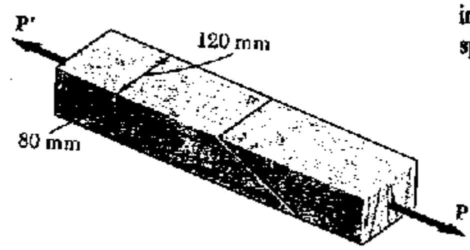
$$S_{all} = \tau_{all} A/\sin\beta = \frac{(600\times10^{3})(9.6\times10^{-3})}{\sin 22°} = 15.376\times10^{3}\ \text{N}$$

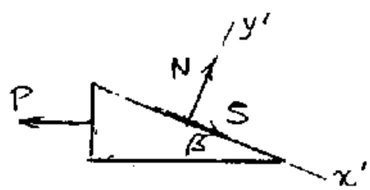
$$+\searrow \Sigma F_{x'} = 0: \quad S - P\cos\beta = 0 \qquad P = \frac{S}{\cos\beta} = \frac{15.376\times10^{3}}{\cos 22°} = 16.58\times10^{3}\ \text{N}$$

The smaller value for P governs. $P = 16.58\times10^{3}\ \text{N} = 16.58\ \text{kN}$ ◄

Problem 7.22

7.22 Two wooden members of 80 × 120-mm uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $\beta = 25°$ and that centric loads of magnitude $P = 10$ kN are applied to the members as shown, determine (*a*) the in-plane shearing stress parallel to the splice, (*b*) the normal stress perpendicular to the splice.





Forces

Areas

$$A = (80)(120) = 9.6\times10^{3}\ \text{mm}^2 = 9.6\times10^{-3}\ \text{m}^2$$

(b) $+\nearrow \Sigma F_{y'} = 0: \quad N - P\sin\beta = 0 \qquad N = P\sin\beta = (10\times10^{3})\sin 25° = 4.226\times10^{3}\ \text{N}$

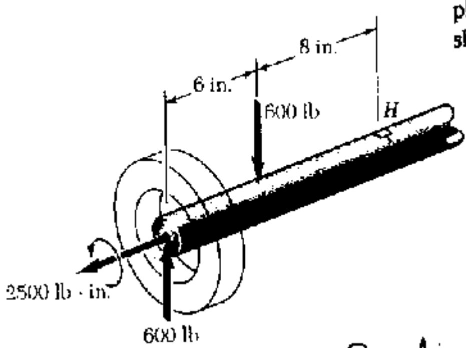
$$\sigma = \frac{N}{A/\sin\beta} = \frac{(4.226\times10^{3})\sin 25°}{9.6\times10^{3}} = 186.0\times10^{3}\ \text{Pa} = 186.0\ \text{kPa}$$ ◄

(a) $+\searrow \Sigma F_{x'} = 0: \quad S - P\cos\beta = 0 \qquad S = P\cos\beta = (10\times10^{3})\cos 25° = 9.063\times10^{3}\ \text{N}$

$$\tau = \frac{N}{A/\sin\beta} = \frac{(9.063\times10^{3})\sin 25°}{9.6\times10^{-3}} = 399\times10^{3}\ \text{Pa} = 399\ \text{kPa}$$ ◄

Problem 7.23

7.23 The axle of an automobile is acted upon by the forces and couple shown. Knowing that the diameter of the solid axle is 1.25 in., determine (*a*) the principal planes and principal stresses at point *H* located on top of the axle, (*b*) the maximum shearing stress at the same point.



$c = \frac{1}{2}d = \frac{1}{2}(1.25) = 0.625 \text{ in}^4$

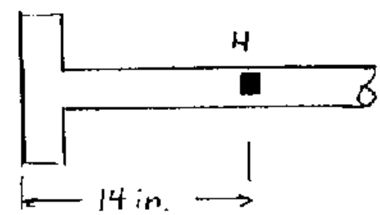
Torsion: $\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

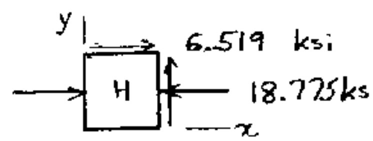
$\tau = \frac{(2)(2500)}{\pi(0.625)^3} = 6.519 \times 10^3 \text{ psi} = 6.519 \text{ ksi}$

Bending: $I = \frac{\pi}{4}c^4 = 119.842 \times 10^{-3} \text{ in}^4$

$M = (6)(600) = 3600 \text{ lb·in}$ $\quad \sigma = -\frac{My}{I} = -\frac{(3600)(0.625)}{119.842 \times 10^{-3}} = -18.775 \times 10^3 \text{ psi} = -18.775 \text{ ksi}$

Top view





Stresses

$\sigma_x = -18.775 \text{ ksi} \qquad \sigma_y = 0 \qquad \tau_{xy} = 6.519 \text{ ksi}$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -9.387 \text{ ksi}$

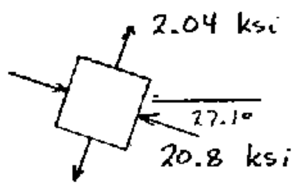
$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(-9.387)^2 + (6.519)^2} = 11.429 \text{ ksi}$

(a) $\sigma_1 = \sigma_{ave} + R = -9.387 + 11.429 = 2.04 \text{ ksi}$ ◄

$\sigma_2 = \sigma_{ave} - R = -9.387 - 11.429 = -20.8 \text{ ksi}$ ◄

$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(6.519)}{-9.387} = -1.3889$

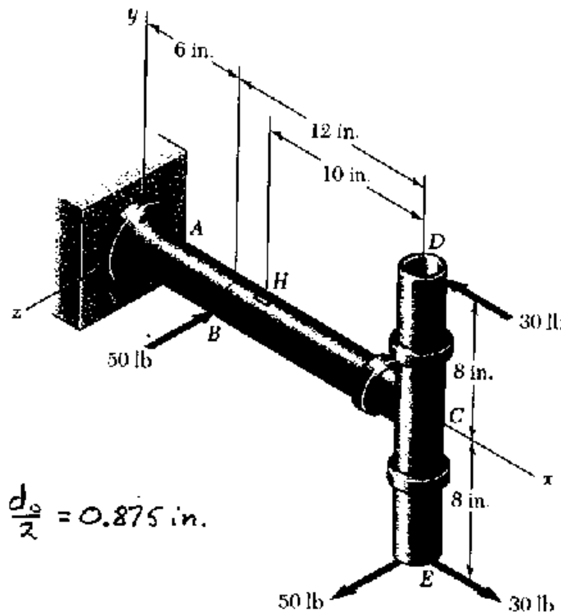
$\theta_p = -27.1° \text{ and } 62.9°$ ◄



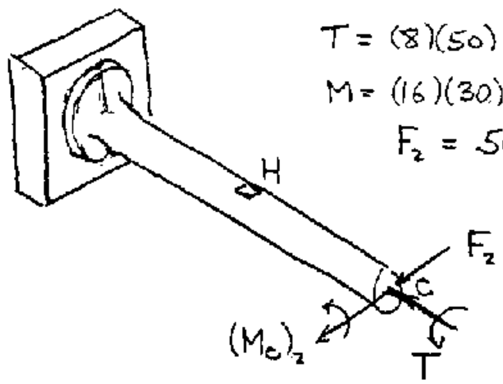
(b) $\tau_{max} = R = 11.43 \text{ ksi}$ ◄

Problem 7.24

7.24 Several forces are applied to the pipe assembly shown. Knowing that the inner and outer diameters of the pipe are equal to 1.50 in. and 1.75 in, respectively, determine (*a*) the principal planes and the principal stresses at point *H* located at the top of the outside surface of the pipe, (*b*) the maximum shearing stress at the same point.



Replace forces on pipe DCE by an equivalent force-couple system at C.



$T = (8)(50) = 400$ lb·in

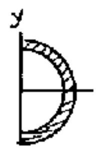
$M = (16)(30) = 480$ lb·in

$F_z = 50$ lb

Cross section. $c_1 = \frac{d_i}{2} = 0.750$ in. $\quad c_2 = \frac{d_o}{2} = 0.875$ in.

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = 0.42376 \text{ in}^4$

$I = \frac{1}{2}J = 0.21188$ in.



$Q_y = \frac{2}{3}(c_2^3 - c_1^3) = \frac{2}{3}(0.875^3 - 0.750^3) = 0.16536 \text{ in}^3$

$t = c_2 - c_1 = 0.125$ in.

At the section containing element H

$T = 400$ lb·in, $\quad M_z = 480$ lb·in, $\quad V_z = 50$ lb.

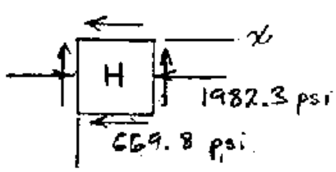
Stresses.

Torsion: $\tau_{zx} = -\frac{Tc}{J} = -\frac{(400)(0.875)}{0.42376} = -825.9$ psi

Bending: $\sigma_x = -\frac{My}{I} = -\frac{(480)(0.875)}{0.21188} = -1982.3$ psi

Transverse Shear: $\tau_{zx} = \frac{VQ}{I(2t)} = \frac{(50)(0.16536)}{(0.21188)(0.250)} = 156.1$ psi

Total: $\sigma_z = 0$, $\sigma_x = -1982.3$ psi, $\tau_{zx} = -825.9 + 156.1 = -669.8$ psi

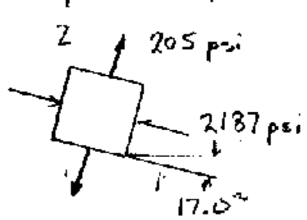


(a) $\tan 2\theta_p = \frac{2\tau_{zx}}{\sigma_z - \sigma_x} = \frac{(2)(-669.8)}{0 + 1982.3} = -0.67578$

$\theta_p = -17.0°$ and $73.0°$ from z-axis. ◄

$\sigma_{ave} = \frac{1}{2}(\sigma_z + \sigma_x) = \frac{1}{2}(0 - 1982.3) = -991.1$ psi

$R = \sqrt{\left(\frac{\sigma_z - \sigma_x}{2}\right)^2 + \tau_{zx}^2} = \sqrt{991.1^2 + 669.8^2} = 1196.2$ psi



$\sigma_1 = \sigma_{ave} + R = 205.$ psi at $17.0°$ ↻ from z-axis. ◄

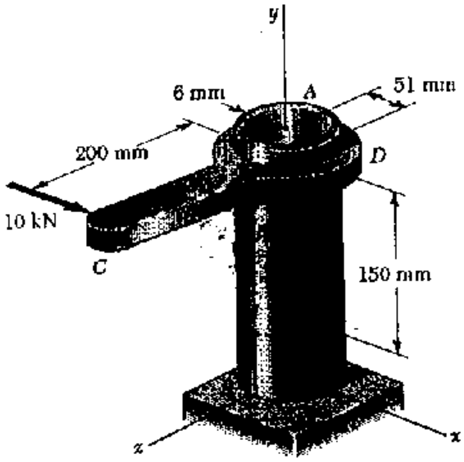
$\sigma_2 = \sigma_{ave} - R = -2187$ psi at $73.0°$ ↺ from z-axis. ◄

(b) $\tau_{max} = R$

$\tau_{max} = 1196$ psi ◄

Problem 7.25

7.25 The steel pipe *AB* has a 102-mm outer diameter and a 6-mm wall thickness. Knowing that arm *CD* is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point *H*.



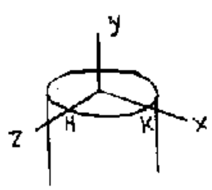
$r_o = \frac{d_o}{2} = \frac{102}{2} = 51 \text{ mm} \qquad r_i = r_o - t = 45 \text{ mm}$

$J = \frac{\pi}{2}(r_o^4 - r_i^4) = 4.1855 \times 10^6 \text{ mm}^4$
$= 4.1855 \times 10^{-6} \text{ m}^4$

$I = \frac{1}{2}J = 2.0927 \text{ m}^4$

Force-couple system at center of tube in the plane containing points H and K.

$F_x = 10 \times 10^3 \text{ N}$
$M_y = (10\times10^3)(200\times10^{-3}) = 2000 \text{ N}\cdot\text{m}$
$M_z = -(10\times10^3)(150\times10^{-3}) = -1500 \text{ N}\cdot\text{m}$



Torsion

$T = M_y = 2000 \text{ N}\cdot\text{m}$
$c = r_o = 51\times10^{-3} \text{ m}$

$\tau_{xy} = \frac{Tc}{J} = \frac{(2000)(51\times10^{-3})}{4.1855\times10^{-6}} = 24.37\times10^6 \text{ Pa}$

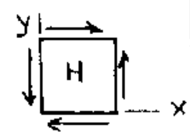
Transverse Shear

For semicircle $\qquad A = \frac{\pi}{2}r^2 \quad \bar{y} = \frac{4}{3\pi}r$
$Q = A\bar{y} = \frac{2}{3}r^3$

$Q = Q_o - Q_i = \frac{2}{3}r_o^3 - \frac{2}{3}r_i^3 = 27.684\times10^3 \text{ mm}^3$
$= 27.684\times10^{-6} \text{ m}^3$

$V = F_x = 10\times10^3 \text{ N} \qquad t = (2)(6 \text{ mm}) = 12 \text{ mm} = 12\times10^{-3} \text{ m}$

$\tau_{xy} = \frac{VQ}{It} = \frac{(10\times10^3)(27.684\times10^{-6})}{(2.0927\times10^{-6})(12\times10^{-3})} = 11.02\times10^6 \text{ Pa}$



Bending: Point H lies on neutral axis. $\quad \sigma_y = 0$

Total stresses at point H: $\quad \sigma_x = 0, \ \sigma_y = 0$
$\tau_{xy} = 24.37\times10^6 + 11.02\times10^6 = 35.39\times10^6 \text{ Pa}$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 0 \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 35.39\times10^6 \text{ Pa}$

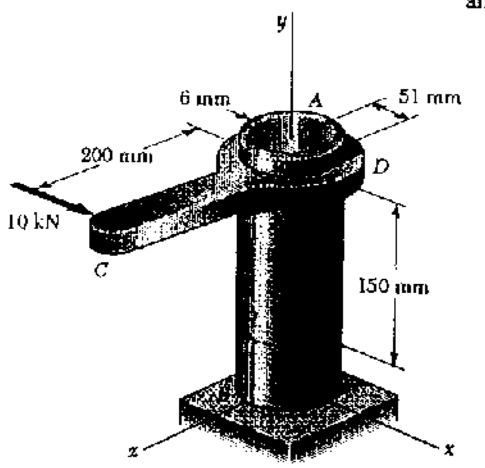
$\sigma_{max} = \sigma_{ave} + R = 35.39\times10^6 \text{ Pa} = 35.4 \text{ MPa}$ ◄

$\sigma_{min} = \sigma_{ave} - R = -35.39\times10^6 \text{ Pa} = -35.4 \text{ MPa}$ ◄

$\tau_{max} = R = 35.4 \text{ MPa}$ ◄

Problem 7.26

7.26 The steel pipe AB has a 102-mm outer diameter and a 6-mm wall thickness. Knowing that arm CD is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point K.



$r_o = \frac{d_o}{2} = \frac{102}{2} = 51\ \text{mm} \qquad r_i = r_o - t = 45\ \text{mm}$

$J = \frac{\pi}{2}(r_o^4 - r_i^4) = 4.1855\times10^6\ \text{mm}^4 = 4.1855\times10^{-6}\ \text{m}^4$

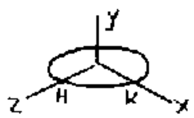
$I = \frac{1}{2}J = 2.0927\times10^{-6}\ \text{m}^4$

Force-couple system at center of tube in the plane containing points H and K

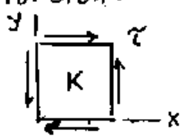
$F_x = 10\ \text{kN} = 10\times10^3\ \text{N}$

$M_y = (10\times10^3)(200\times10^{-3}) = 2000\ \text{N}\cdot\text{m}$

$M_z = -(10\times10^3)(150\times10^{-3}) = -1500\ \text{N}\cdot\text{m}$



Torsion:



At point K, place local x-axis in negative global z-direction

$T = M_y = 2000\ \text{N}\cdot\text{m} \qquad c = r_o = 51\times10^{-3}\ \text{m}$

$\tau_{xy} = \frac{Tc}{J} = \frac{(2000)(51\times10^{-3})}{4.1855\times10^{6}} = 24.37\times10^6\ \text{Pa} = 24.37\ \text{MPa}$

Transverse Shear: Stress due to transverse shear $V = F_x$ is zero at pt. K.

Bending: $|\sigma_y| = \frac{|M_z|c}{I} = \frac{(1500)(51\times10^{-3})}{2.0927\times10^{-6}} = 36.56\times10^6\ \text{Pa} = 36.56\ \text{MPa}$

Point K lies on compression side of neutral axis: $\sigma_y = -36.56\ \text{MPa}$

Total stresses at point K $\qquad \sigma_x = 0 \qquad \sigma_y = -36.56\ \text{MPa},\ \tau_{xy} = 24.37\ \text{MPa}$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -18.28\ \text{MPa}$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_x}{2}\right)^2 + \tau_{xy}^2} = 30.46\ \text{MPa}$

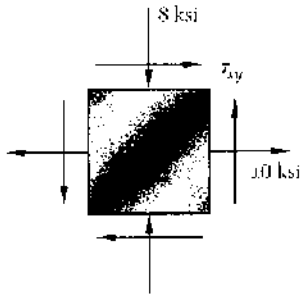
$\sigma_{max} = \sigma_{ave} + R = -18.28 + 30.46 = +12.18\ \text{MPa}$ ◄

$\sigma_{min} = \sigma_{ave} - R = -18.28 - 30.46 = -48.74\ \text{MPa}$ ◄

$\tau_{max} = R = 30.46\ \text{MPa}$ ◄

Problem 7.27

7.27 For the state of plane stress shown, determine (*a*) the largest value of τ_{xy} for which the maximum in-plane shearing stress is equal to or less than 12 ksi, (*b*) the corresponding principal stresses.



$\sigma_x = 10$ ksi, $\sigma_y = -8$ ksi, $\tau_{xy} = ?$

$$\tau_{max} = R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{10-(-8)}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{9^2 + \tau_{xy}^2} = 12 \text{ ksi}$$

(a) $\tau_{xy} = \sqrt{12^2 - 9^2} = 7.94$ ksi

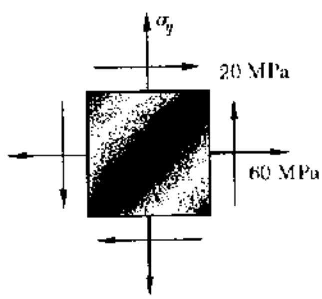
(b) $\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 1$ ksi

$\sigma_a = \sigma_{ave} + R = 1 + 12 = 13$ ksi

$\sigma_b = \sigma_{ave} - R = 1 - 12 = -11$ ksi

Problem 7.28

7.28 For the state of plane stress shown, determine the largest value of σ_y for which the maximum in-plane shearing stress is equal to of less than 75 MPa.



$\sigma_x = 60$ MPa, $\sigma_y = ?$, $\tau_{xy} = 20$ MPa

Let $u = \frac{\sigma_x - \sigma_y}{2}$. Then $\sigma_y = \sigma_x - 2u$

$R = \sqrt{u^2 + \tau_{xy}^2} = 75$ MPa

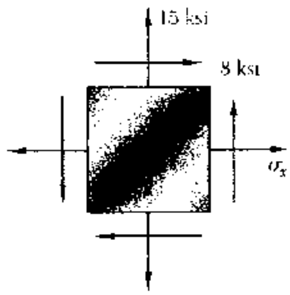
$u = \pm\sqrt{R^2 - \tau_{xy}^2} = \pm\sqrt{75^2 - 20^2} = 72.284$ MPa

$\sigma_y = \sigma_x - 2u = 60 \mp (2)(72.284) = -84.6$ MPa or 205 MPa

Largest value of σ_y is required. $\sigma_y = 205$ MPa

Problem 7.29

7.29 Determine the range of values of σ_x for which the maximum in-plane shearing stress is equal to or less than 10 ksi.



$\sigma_x = ?$, $\sigma_y = 15$ ksi , $\tau_{xy} = 8$ ksi

Let $u = \dfrac{\sigma_x - \sigma_y}{2}$ $\qquad \sigma_x = \sigma_y + 2u$

$R = \sqrt{u^2 + \tau_{xy}^2} = \tau_{max} = 10$ ksi

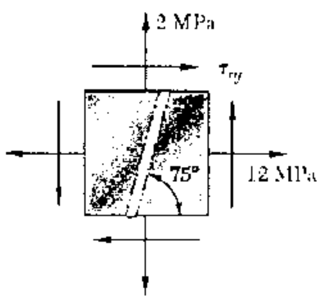
$u = \pm\sqrt{R^2 - \tau_{xy}^2} = \pm\sqrt{10^2 - 8^2} = \pm 6$ ksi

$\sigma_x = \sigma_y + 2u = 15 \pm (2)(6) = 27$ ksi or 3 ksi

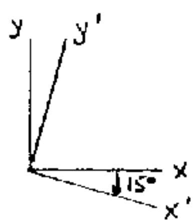
Allowable range $\quad 3 \text{ ksi} \le \sigma_x \le 27 \text{ ksi}$

Problem 7.30

7.30 For the state of plane stress shown, determine (*a*) the value of τ_{xy} for which the in-plane shearing stress parallel to the weld is zero, (*b*) the corresponding principal stresses.



$\sigma_x = 12$ MPa , $\sigma_y = 2$ MPa , $\tau_{xy} = ?$



Since $\tau_{x'y'} = 0$, x'-direction is a principal direction.

$\theta_p = -15°$

$\tan 2\theta_p = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y}$

(a) $\tau_{xy} = \frac{1}{2}(\sigma_x - \sigma_y)\tan 2\theta_p = \frac{1}{2}(12-2)\tan(-30°) = -2.89$ MPa

$R = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{5^2 + 2.89^2} = 5.7735$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 7$ MPa

(b) $\sigma_a = \sigma_{ave} + R = 7 + 5.7735 = 12.77$ MPa

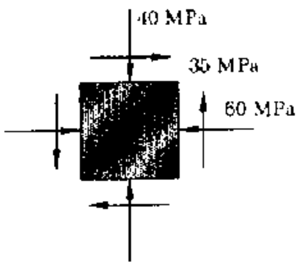
$\sigma_b = \sigma_{ave} - R = 7 - 5.7735 = 1.226$ MPa

Problem 7.31

7.31 Solve Probs. 7.7 and 7.11, using Mohr's circle.

7.5 through 7.8 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.



$\sigma_x = -60\ \text{MPa} \qquad \sigma_y = -40\ \text{MPa} \qquad \tau_{xy} = 35\ \text{MPa}$

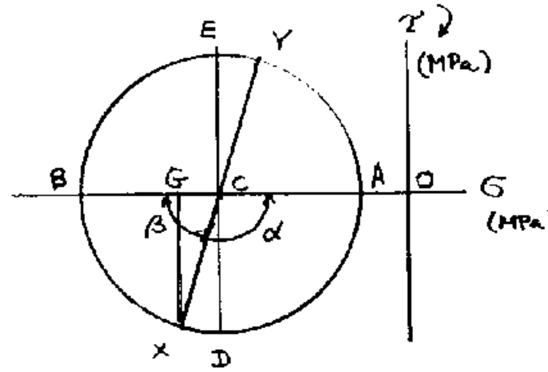
$\sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = -50\ \text{MPa}$

Points

$X: (\sigma_x, -\tau_{xy}) = (-60\ \text{MPa}, -35\ \text{MPa})$

$Y: (\sigma_y, \tau_{xy}) = (-40\ \text{MPa}, 35\ \text{MPa})$

$C: (\sigma_{ave}, 0) = (-50\ \text{MPa}, 0)$



$\tan\beta = \dfrac{GX}{CG} = \dfrac{35}{10} = 3.500$

$\beta = 74.05°$

$\theta_B = -\frac{1}{2}\beta = -37.03°$ ◄

$\alpha = 180° - \beta = 105.95°$

$\theta_A = \frac{1}{2}\alpha = 52.97°$ ◄

$R = \sqrt{\overline{CG}^2 + \overline{GX}^2} = \sqrt{10^2 + 35^2} = 36.4\ \text{MPa}$

$\sigma_{min} = \sigma_{ave} - R = -50 - 36.4 = -86.4\ \text{MPa}$ ◄

$\sigma_{max} = \sigma_{ave} + R = -50 + 36.4 = -13.6\ \text{MPa}$ ◄

$\theta_D = \theta_B + 45° = 7.97°$ ◄

$\theta_E = \theta_A + 45° = 97.97°$ ◄

$\tau_{max} = R = 36.4\ \text{MPa}$ ◄

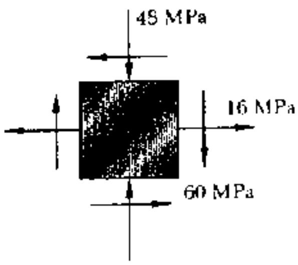
$\sigma' = \sigma_{ave} = -50\ \text{MPa}$ ◄

Problem 7.32

7.32 Solve Probs. 7.8 and 7.12, using Mohr's circle.

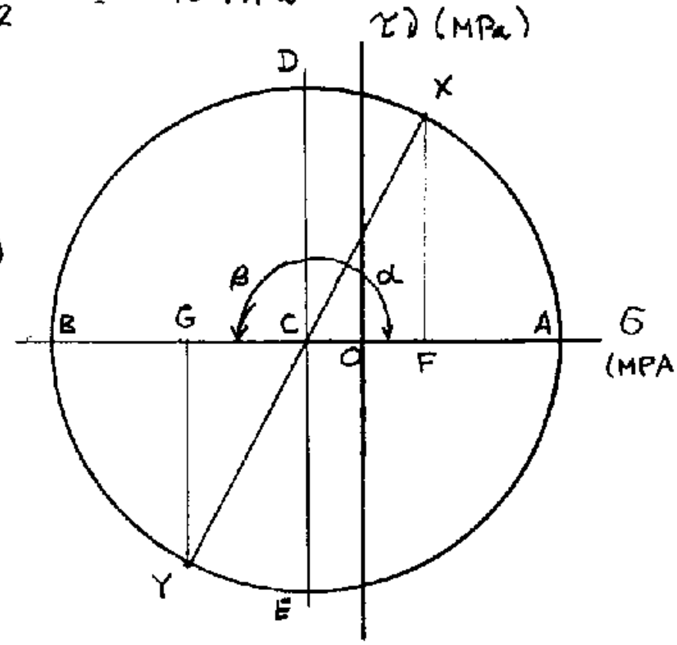
7.5 through 7.8 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.



$\sigma_x = 16 \text{ MPa} \qquad \sigma_y = -48 \text{ MPa} \qquad \tau_{xy} = -60 \text{ MPa}$

$\sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = -16 \text{ MPa}$



Points:

X: $(\sigma_x, -\tau_{xy}) = (16 \text{ MPa}, 60 \text{ MPa})$

Y: $(\sigma_y, \tau_{xy}) = (-48 \text{ MPa}, -60 \text{ MPa})$

C: $(\sigma_{ave}, 0) = (-16 \text{ MPa}, 0)$

$\tan\alpha = \dfrac{FX}{CF} = \dfrac{60}{32} = 1.875$

$\alpha = 61.93°$

$\theta_A = -\frac{1}{2}\alpha = -30.96°$ ◄

$\beta = 180° - \alpha = 118.07°$

$\theta_B = \frac{1}{2}\beta = 59.04°$ ◄

$R = \sqrt{\overline{CF}^2 + \overline{FX}^2} = \sqrt{32^2 + 60^2} = 68 \text{ MPa}$

$\sigma_a = \sigma_{max} = \sigma_{ave} + R = -16 + 68 = 52 \text{ MPa}$ ◄

$\sigma_b = \sigma_{min} = \sigma_{ave} - R = -16 - 68 = -84 \text{ MPa}$ ◄

$\theta_d = \theta_a + 45° = 14.04°$ ◄

$\theta_e = \theta_b + 45° = 104.04°$ ◄

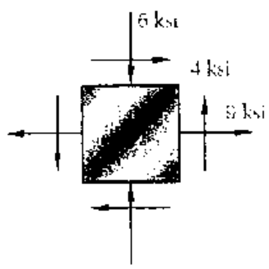
$\tau_{max} = R = 68 \text{ MPa}$ ◄

$\sigma' = \sigma_{ave} = -16 \text{ MPa}$ ◄

Problem 7.33

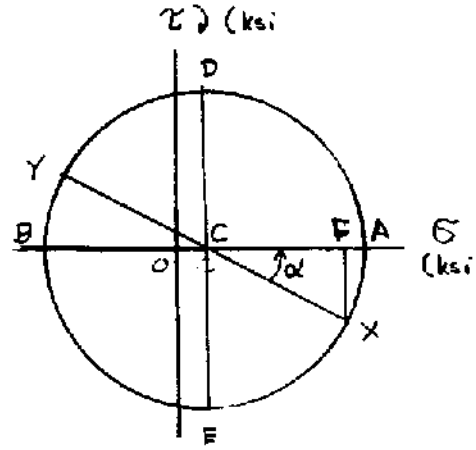
7.33 Solve Prob. 7.9, using Mohr's circle.

7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.



$\sigma_x = 9 \text{ ksi} \qquad \sigma_y = -6 \text{ ksi} \qquad \tau_{xy} = 4 \text{ ksi}$

$\sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = 1.5 \text{ ksi}$



Points

X: $(\sigma_x, -\tau_{xy}) = (9 \text{ ksi}, -4 \text{ ksi})$

Y: $(\sigma_y, \tau_{xy}) = (-6 \text{ ksi}, 4 \text{ ksi})$

C: $(\sigma_{ave}, 0) = (1.5 \text{ ksi}, 0)$

$\tan\alpha = \dfrac{FX}{CF} = \dfrac{4}{7.5} = 0.5333$

$\alpha = 28.07°$

$\theta_a = \frac{1}{2}\alpha = 14.04°$

$\theta_d = \theta_a + 45° = 59.04°$ ◄

$\theta_e = \theta_a - 45° = -30.96°$ ◄

$R = \sqrt{\overline{CF}^2 + \overline{FX}^2} = \sqrt{7.5^2 + 4^2} = 8.5 \text{ ksi}$

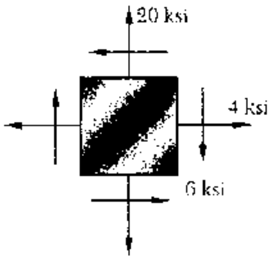
$\tau_{max} = R = 8.5 \text{ ksi}$ ◄

$\sigma' = \sigma_{ave} = 1.5 \text{ ksi}$ ◄

Problem 7.34

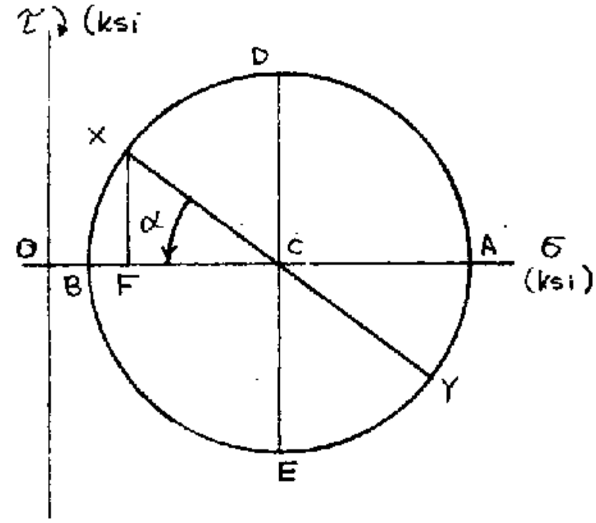
7.34 Solve Prob. 7.10, using Mohr's circle.

7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.



$\sigma_x = 4$ ksi $\qquad \sigma_y = 20$ ksi $\qquad \tau_{xy} = -6$ ksi

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = \frac{4+20}{2} = 12 \text{ ksi}$$



Points

X: $(\sigma_x, -\tau_{xy}) = (4 \text{ ksi}, 6 \text{ ksi})$

Y: $(\sigma_y, \tau_{xy}) = (20 \text{ ksi}, -6 \text{ ksi})$

C: $(\sigma_{ave}, 0) = (12 \text{ ksi}, 0)$

$$\tan\alpha = \frac{FX}{FC} = \frac{6}{8} = 0.75$$

$\alpha = 36.87°$

$\theta_b = \frac{1}{2}\alpha = 18.43°$

$\theta_d = \theta_b - 45° = -26.57°$ ◄

$\theta_e = \theta_b + 45° = 63.43°$ ◄

$R = \sqrt{\overline{CF}^2 + \overline{FX}^2} = \sqrt{8^2 + 6^2} = 10$ ksi

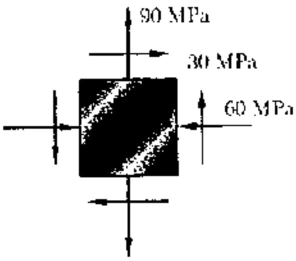
$\tau_{max} = R = 10$ ksi ◄

$\sigma' = \sigma_{ave} = 12$ ksi ◄

Problem 7.35

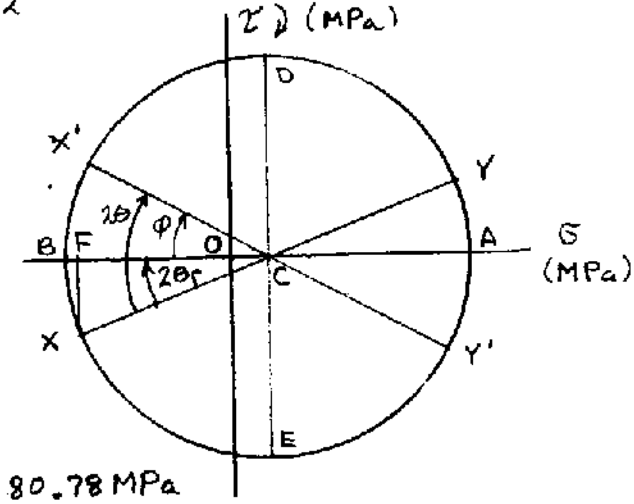
7.35 Solve Prob. 7.13, using Mohr's circle.

7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.



$\sigma_x = -60$ MPa $\quad \sigma_y = 90$ MPa $\quad \tau_{xy} = 30$ MPa

$\sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = 15$ MPa



Points

X: (−60 MPa, −30 MPa)

Y: (90 MPa, 30 MPa)

C: (15 MPa, 0)

$\tan 2\theta_p = \dfrac{FX}{FC} = \dfrac{30}{75} = 0.4$

$2\theta_p = 21.80° \qquad \theta_p = 10.90°$

$R = \sqrt{\overline{FC}^2 + \overline{FX}^2} = \sqrt{75^2 + 30^2} = 80.78$ MPa

(a) $\theta = 25° \qquad 2\theta = 50°$

$\varphi = 2\theta - 2\theta_p = 50° - 21.80° = 28.20°$

$\sigma_{x'} = \sigma_{ave} - R\cos\varphi = -56.2$ MPa ◄

$\tau_{x'y'} = -R\sin\varphi = -38.2$ MPa ◄

$\sigma_{y'} = \sigma_{ave} + R\cos\varphi = 86.2$ MPa ◄

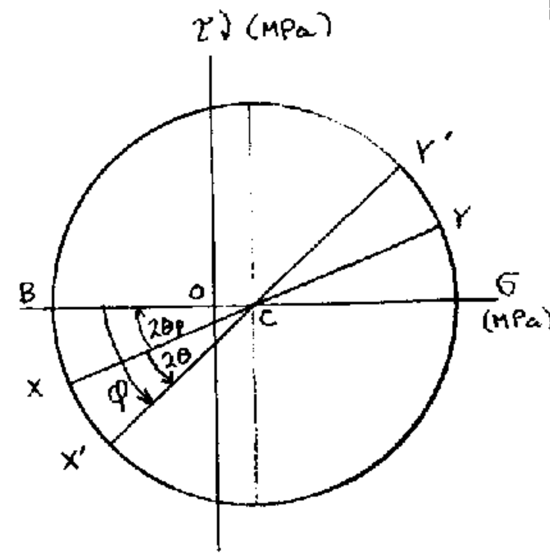
(b) $\theta = 10° \qquad 2\theta = 20°$

$\varphi = 2\theta_p + 2\theta = 21.80° + 20° = 41.80°$

$\sigma_{x'} = \sigma_{ave} - R\cos\varphi = -45.2$ MPa ◄

$\tau_{x'y'} = R\sin\varphi = 53.8$ MPa ◄

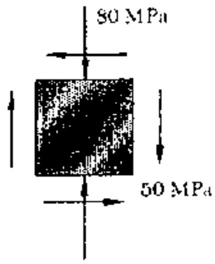
$\sigma_{y'} = \sigma_{ave} + R\cos\varphi = 75.2$ MPa ◄



Problem 7.36

7.36 Solve Prob. 7.14, using Mohr's circle.

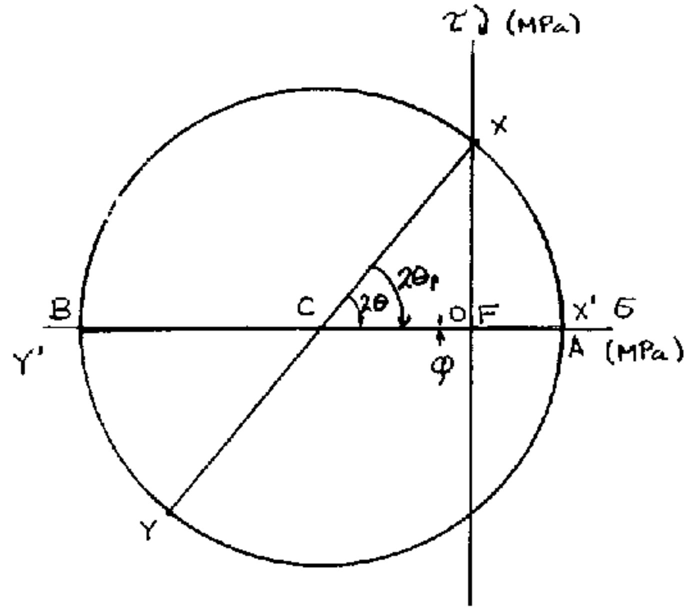
7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.



$\sigma_x = 0 \qquad \sigma_y = -80 \text{ MPa} \qquad \tau_{xy} = -50 \text{ MPa}$

$\sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = -40 \text{ MPa}$

Points
X: (0, 50 MPa)
Y: (-80 MPa, -50 MPa)
C: (-40 MPa, 0)



$\tan 2\theta_p = \dfrac{FX}{CF} = \dfrac{50}{40} = 1.25$

$2\theta_p = 51.34°$

$R = \sqrt{\overline{CF}^2 + \overline{FX}^2} = \sqrt{40^2 + 50^2}$
$= 64.03 \text{ MPa}$

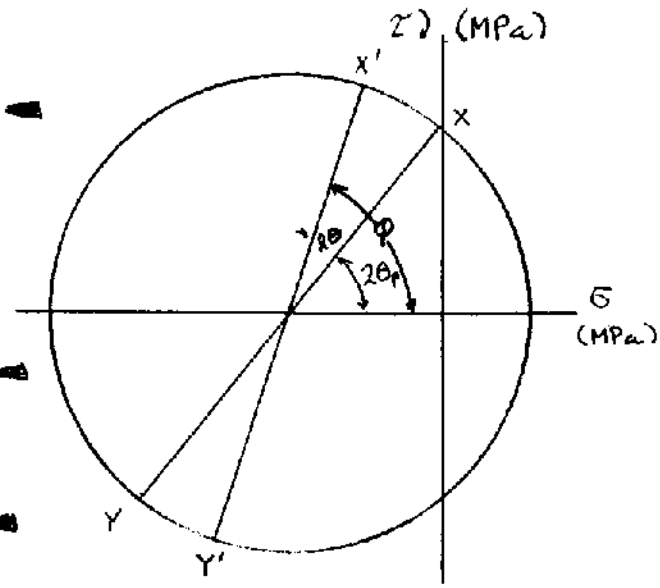
(a) $\theta = 25°$ ↻ $\qquad 2\theta = 50°$ ↻

$\varphi = 51.34° - 50° = 1.34°$ ↺

$\sigma_{x'} = \sigma_{ave} + R\cos\varphi = 24.0 \text{ MPa}$ ◄

$\tau_{x'y'} = -R\sin\varphi = -1.5 \text{ MPa}$ ◄

$\sigma_{y'} = \sigma_{ave} - R\cos\varphi = -104.0 \text{ MPa}$ ◄



(b) $\theta = 10°$ ↺ $\qquad 2\theta = 20°$ ↺

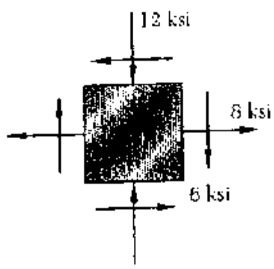
$\varphi = 51.34° + 20° = 71.34°$

$\sigma_{x'} = \sigma_{ave} + R\cos\varphi = -19.5 \text{ MPa}$ ◄

$\tau_{x'y'} = \mp R\sin\varphi = -60.7 \text{ MPa}$ ◄

$\sigma_{y'} = \sigma_{ave} - R\cos\varphi = -60.5 \text{ MPa}$ ◄

Problem 7.37

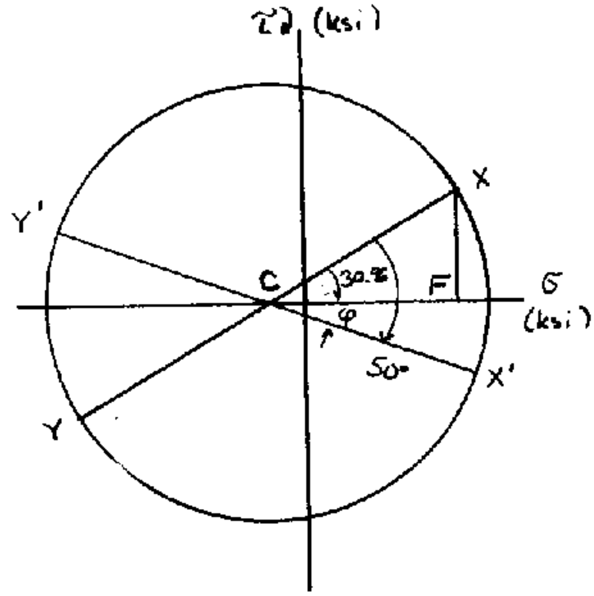


7.37 Solve Prob. 7.15, using Mohr's circle.

7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.

$\sigma_x = 8$ ksi $\quad \sigma_y = -12$ ksi $\quad \tau_{xy} = -6$ ksi

$\sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = -2$ ksi



Points

X: (8 ksi, 6 ksi)

Y: (-12 ksi, -6 ksi)

C: (-2 ksi, 0)

$\tan 2\theta_p = \dfrac{FX}{CF} = \dfrac{6}{10} = 0.6$

$2\theta_p = 30.96°$

$R = \sqrt{\overline{CF}^2 + \overline{FX}^2} = \sqrt{10^2 + 6^2} = 11.66$ ksi

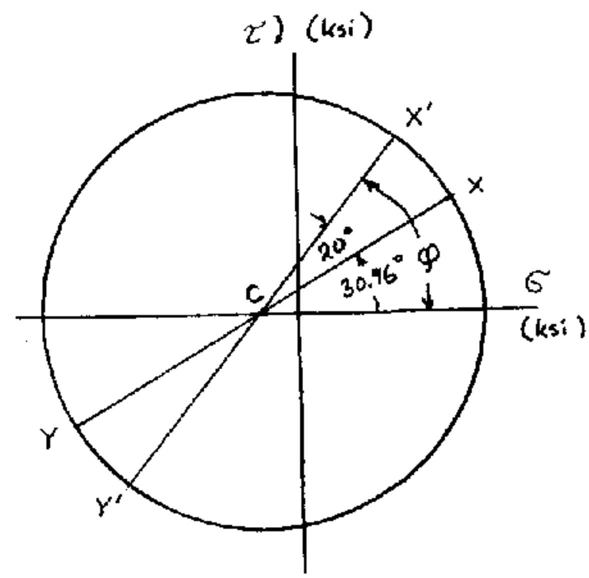
(a) $\theta = 25°$ ↻ $\quad 2\theta = 50°$ ↻

$\varphi = 50° - 30.96° = 19.04°$

$\sigma_{x'} = \sigma_{ave} + R\cos\varphi = 9.02$ ksi ◄

$\tau_{x'y'} = R\sin\varphi = 3.80$ ksi ◄

$\sigma_{y'} = \sigma_{ave} - R\cos\varphi = -13.02$ ksi ◄



(b) $\theta = 10°$ ↺ $\quad 2\theta = 20°$ ↺

$\varphi = 30.96° + 20° = 50.96°$

$\sigma_{x'} = \sigma_{ave} + R\cos\varphi = 5.34$ ksi ◄

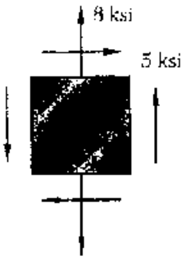
$\tau_{x'y'} = -R\sin\varphi = -9.06$ ksi ◄

$\sigma_{y'} = \sigma_{ave} - R\cos\varphi = -9.34$ ksi ◄

Problem 7.38

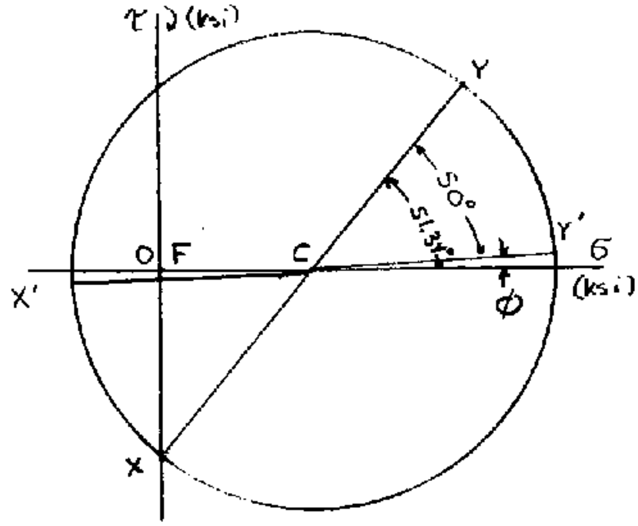
7.38 Solve Prob. 7.16, using Mohr's circle.

7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.



$\sigma_x = 0 \qquad \sigma_y = 8 \text{ ksi} \qquad \tau_{xy} = 5 \text{ ksi}$

$\sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = 4 \text{ ksi}$



Points:

X: (0, -5 ksi)

Y: (8 ksi, 5 ksi)

C: (4 ksi, 0)

$\tan 2\theta_p = \dfrac{FX}{FC} = \dfrac{5}{4} = 1.25$

$2\theta_p = 51.34°$

$R = \sqrt{\overline{FC}^2 + \overline{FX}^2} = \sqrt{4^2 + 5^2}$

$= 6.40 \text{ ksi}$

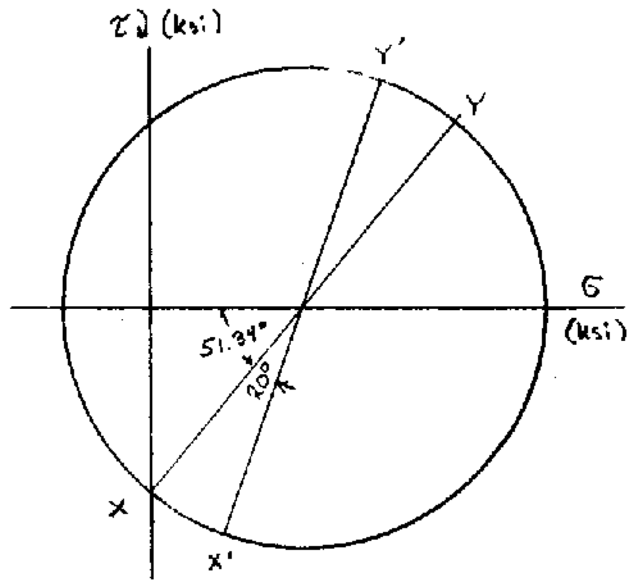
(a) $\theta = 25°$ ↻ $\quad 2\theta = 50°$ ↻

$\varphi = 51.34° - 50° = 1.34°$

$\sigma_{x'} = \sigma_{ave} - R\cos\varphi = -2.40 \text{ ksi}$ ◄

$\tau_{x'y'} = R\sin\varphi = 0.15 \text{ ksi}$ ◄

$\sigma_{y'} = \sigma_{ave} + R\cos\varphi = 10.40 \text{ ksi}$ ◄



(b) $\theta = 10°$ ↺ $\quad 2\theta = 20°$ ↺

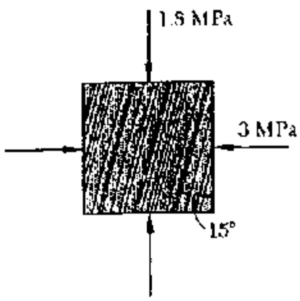
$\varphi = 51.34° + 20° = 71.34°$

$\sigma_{x'} = \sigma_{ave} - R\cos\varphi = 1.95 \text{ ksi}$ ◄

$\tau_{x'y'} = R\sin\varphi = 6.07 \text{ ksi}$ ◄

$\sigma_{y'} = \sigma_{ave} + R\cos\varphi = 6.05 \text{ ksi}$ ◄

Problem 7.39



7.39 Solve Prob. 7.17, using Mohr's circle.

7.17 and 7.18 The grain of a wooden member forms an angle of 15° with the vertical.. For the state of stress shown, determine (*a*) the in-plane shearing stress parallel to the grain, (*b*) the normal stress perpendicular to the grain.

$\sigma_x = -3$ MPa $\qquad \sigma_y = -1.8$ MPa $\qquad \tau_{xy} = 0$

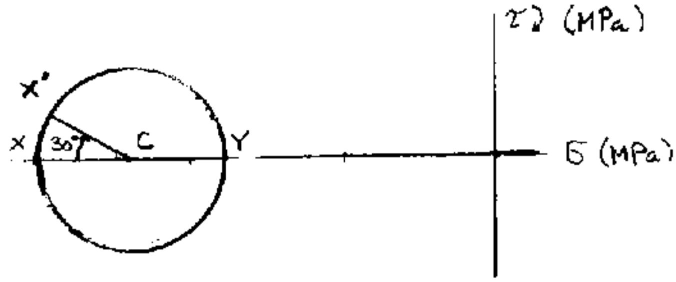
$\sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = -2.4$ MPa

Points

X: $(\sigma_x, -\tau_{xy}) = (-3\text{ Mpa}, 0)$

Y: $(\sigma_y, \tau_{xy}) = (-1.8\text{ MPa}, 0)$

C: $(\sigma_{ave}, 0) = (-2.4\text{ MPa}, 0)$



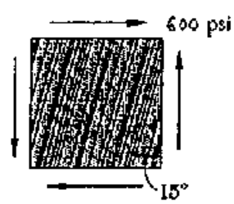
$\theta = -15°$ $\qquad 2\theta = -30°$

$\overline{CX} = 0.6$ MPa $\qquad R = 0.6$ MPa

(a) $\tau_{x'y'} = -\overline{C}X' \sin 30 = -R \sin 30° = -0.6 \sin 30° = -0.3$ MPa

(b) $\sigma_{x'} = \sigma_{ave} - \overline{C}X' \cos 30° = -2.4 - 0.6 \cos 30° = -2.92$ MPa

Problem 7.40



7.40 Solve Prob. 7.18, using Mohr's circle.

7.17 and 7.18 The grain of a wooden member forms an angle of 15° with the vertical.. For the state of stress shown, determine (*a*) the in-plane shearing stress parallel to the grain, (*b*) the normal stress perpendicular to the grain.

$\sigma_x = \sigma_y = 0 \qquad \tau_{xy} = 400$ psi

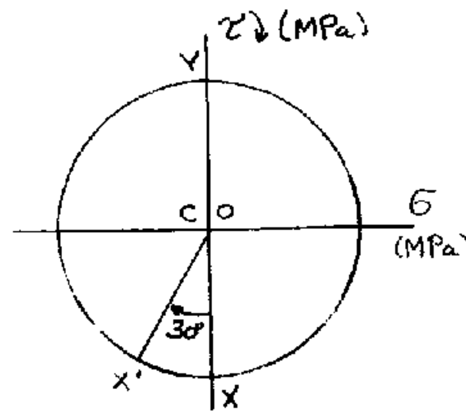
$\sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = 0$

Points

X: $(\sigma_x, -\tau_{xy}) = (0, -600\text{ psi})$

Y: $(\sigma_y, \tau_{xy}) = (0, 600\text{ psi})$

C: $(\sigma_{ave}, 0) = (0, 0)$



$\theta = -15°$ $\qquad 2\theta = -30°$

$\overline{CX} = R = 600$ psi

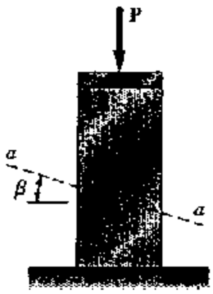
(a) $\tau_{x'y'} = R \cos 30° = 600 \cos 30° = 520$ psi ,

(b) $\sigma_{x'} = \sigma_{ave} - R \sin 30° = -600 \sin 30° = -300$ psi

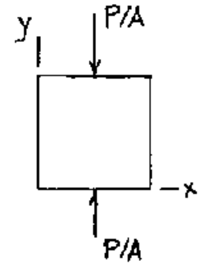
Problem 7.41

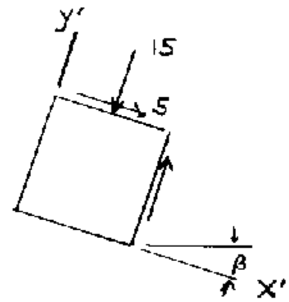
7.41 Solve Prob. 7.19, using Mohr's circle.

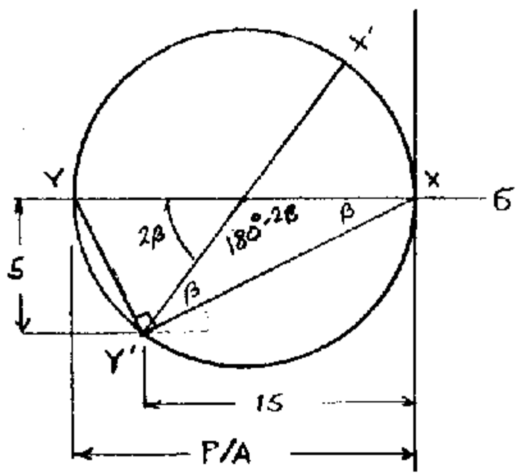
7.19 The centric force **P** is applied to a short post as shown. Knowing that the stresses on plane *a-a* are $\sigma = -15$ ksi and $\tau = 5$ ksi, determine (*a*) the angle β that plane *a-a* forms with the horizontal, (*b*) the maximum compressive stress in the post.



$\sigma_x = 0$
$\tau_{xy} = 0$
$\sigma_y = -P/A$







From the Mohr's circle

$$\tan\beta = \frac{5}{15} = 0.3333 \qquad \beta = 18.4^\circ$$

$$-\sigma = \frac{P}{2A} + \frac{P}{2A}\cos 2\beta$$

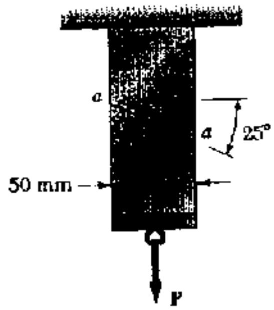
$$\frac{P}{A} = \frac{2(-\sigma)}{1+\cos 2\beta} = \frac{(2)(15)}{1+\cos 2\beta}$$

$$= 16.67 \text{ ksi}$$

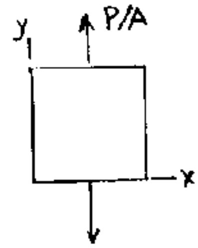
Problem 7.42

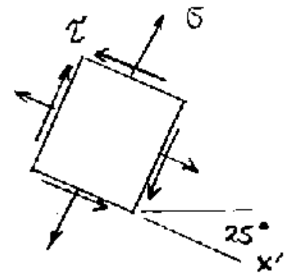
7.42 Solve Prob. 7.20, using Mohr's circle.

7.20 Two members of uniform cross section 50 × 80 mm are glued together along plane *a*-*a*, that forms an angle or 25° with the horizontal. Knowing that the allowable stresses for the glued joint are $\sigma = 800$ kPa and $\tau = 600$ kPa, determine the largest centric load **P** that can be applied.

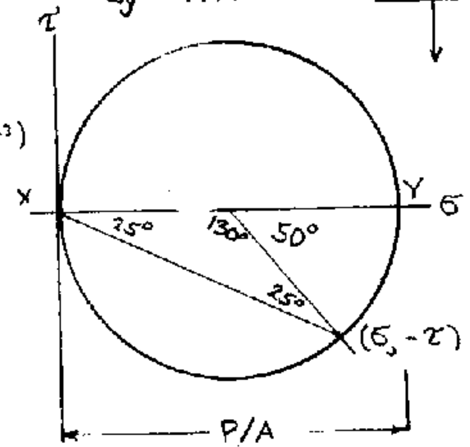


$\sigma_x = 0$
$\tau_{xy} = 0$
$\sigma_y = P/A$





$A = (50\times10^{-3})(80\times10^{-3})$
$= 4\times10^{-3}\ m^2$



$\sigma = \frac{P}{2A}(1 + \cos 50°)$

$P = \frac{2A\sigma}{1 + \cos 50°}$

$P \le \frac{(2)(4\times10^{-3})(800\times10^{3})}{1 + \cos 50°}$

$P \le 3.90\times10^{3}$ N

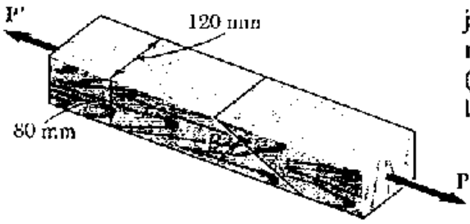
$\tau = \frac{P}{2A}\sin 50°$ $\qquad P = \frac{2A\tau}{\sin 50°} \le \frac{(2)(4\times10^{-3})(600\times10^{3})}{\sin 50°} = 6.27\times10^{3}$ N

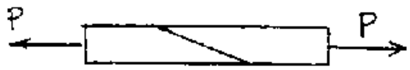
Choosing the smaller value $\quad P \le 3.90\times10^{3}\ N = 3.90\ kN$

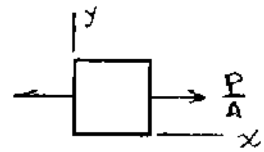
Problem 7.43

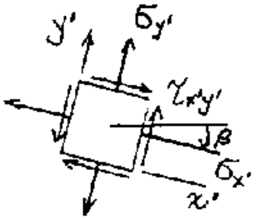
7.43 Solve Prob. 7.21, using Mohr's circle.

7.21 Two wooden members of 80 × 120-mm uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $\beta = 22°$ and that the maximum allowable stresses in the joint are, respectively, 400 kPa in tension (perpendicular to the splice) and 600 kPa in shear (parallel to the splice), determine the largest centric load **P** that can be applied.









$$\sigma_x = \frac{P}{A}, \quad \sigma_y = 0, \quad \tau_{xy} = 0$$

Plotted points

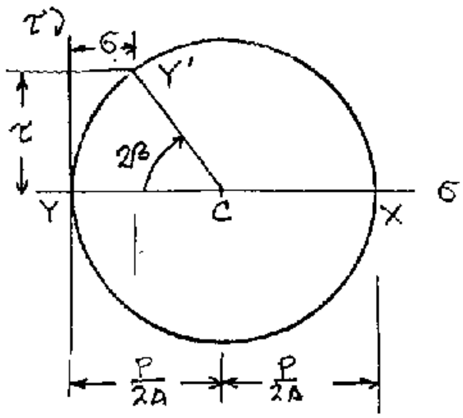
X : $(P/A, 0)$, Y : $(0, 0)$

C : $(P/2A, 0)$.

$$R = \overline{CX} = \frac{P}{2A}$$

Coordinates of point Y'

$$\sigma = \frac{P}{2A}(1 - \cos 2\beta), \quad \tau = \frac{P}{2A}\sin 2\beta$$



Data: $A = (80)(120) = 9.6 \times 10^3\ \text{mm}^2 = 9.6 \times 10^{-3}\ \text{m}^2$

If $\sigma = 400\ \text{kPa} = 400 \times 10^3\ \text{Pa}$,

$$P = \frac{2A\sigma}{1 - \cos 2\beta} = \frac{(2)(9.6 \times 10^{-3})(400 \times 10^3)}{(1 - \cos 44°)}$$

$$= 27.4 \times 10^3\ \text{N} = 27.4\ \text{kN}$$

If $\tau = 600\ \text{kPa} = 600 \times 10^3\ \text{Pa}$, $\quad P = \frac{2A\tau}{\sin 2\beta} = \frac{(2)(9.6 \times 10^{-3})(600 \times 10^3)}{\sin 44°}$

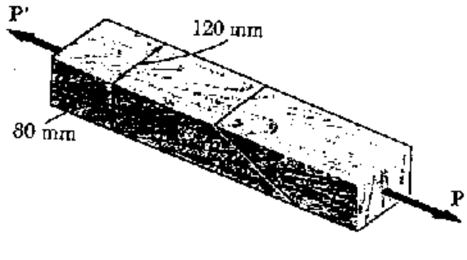
$$= 16.58 \times 10^3\ \text{N} = 16.58\ \text{kN}$$

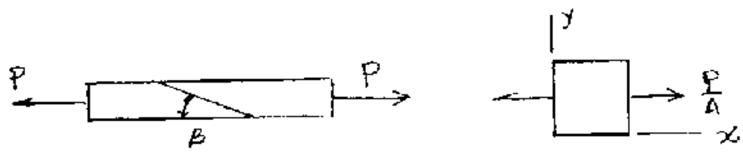
The smaller value of P governs. $\quad P = 16.58\ \text{kN}$ ◄

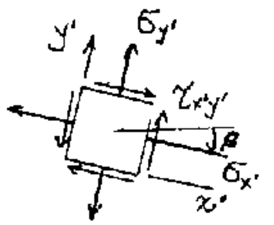
Problem 7.44

7.44 Solve Prob. 7.22, using Mohr's circle.

7.22 Two wooden members of 80 × 120-mm uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $\beta = 25°$ and that centric loads of magnitude $P = 10$ kN are applied to the members as shown, determine (*a*) the in-plane shearing stress parallel to the splice, (*b*) the normal stress perpendicular to the splice.







$\sigma_x = \frac{P}{A}$, $\sigma_y = 0$, $\tau_{xy} = 0$

Plotted points

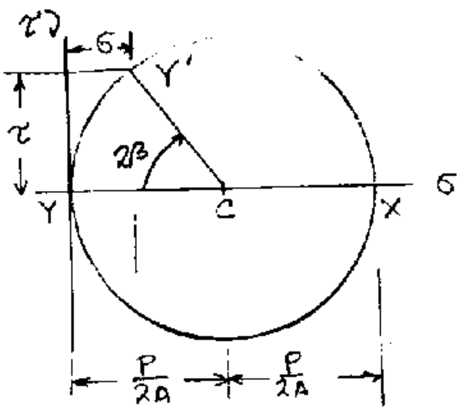
X : $(P/A, 0)$, Y: $(0, 0)$

C : $(P/2A, 0)$

$R = \overline{CX} = \frac{P}{2A}$

Coordinates of point Y'

$\sigma = \frac{P}{2A}(1 - \cos 2\beta)$, $\tau = \frac{P}{2A}\sin 2\beta$



Data: $A = (80)(120) = 9.6 \times 10^3\ \text{mm}^2 = 9.6 \times 10^{-3}\ \text{m}^2$

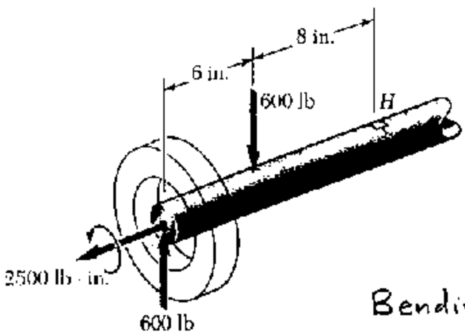
(a) $\tau = \frac{(10 \times 10^3)\sin 50°}{(2)(9.6 \times 10^{-3})} = 399 \times 10^3\ \text{Pa} = 399\ \text{kPa}$ ◄

(b) $\sigma = \frac{(10 \times 10^3)(1 - \cos 50°)}{(2)(9.6 \times 10^{-3})} = 186.0 \times 10^3\ \text{Pa} = 186.0\ \text{kPa}$ ◄

Problem 7.45

7.45 Solve Prob. 7.23, using Mohr's circle.

7.23 The axle of an automobile is acted upon by the forces and couple shown. Knowing that the diameter of the solid axle is 1.25 in., determine (*a*) the principal planes and principal stresses at point *H* located on top of the axle, (*b*) the maximum shearing stress at the same point.



$c = \frac{1}{2}d = \frac{1}{2}(1.25) = 0.625 \text{ in}$

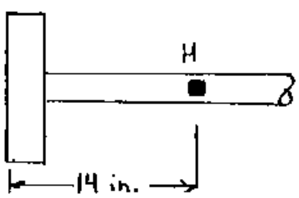
Torsion: $\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$= \frac{(2)(2500)}{\pi(0.625)^3} = 6.519 \times 10^3 \text{ psi} = 6.519 \text{ ksi}$

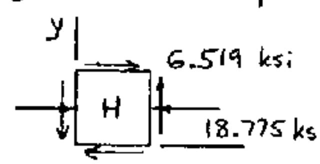
Bending: $I = \frac{\pi}{4}c^4 = 119.842 \times 10^{-3} \text{ in}^4$

$M = (6)(600) = 3600 \text{ lb-in.}$ $\quad \sigma = -\frac{My}{I} = -\frac{(3600)(0.625)}{119.842 \times 10^{-3}} = -18.775 \times 10^3 \text{ psi} = -18.775 \text{ ksi}$

Top view

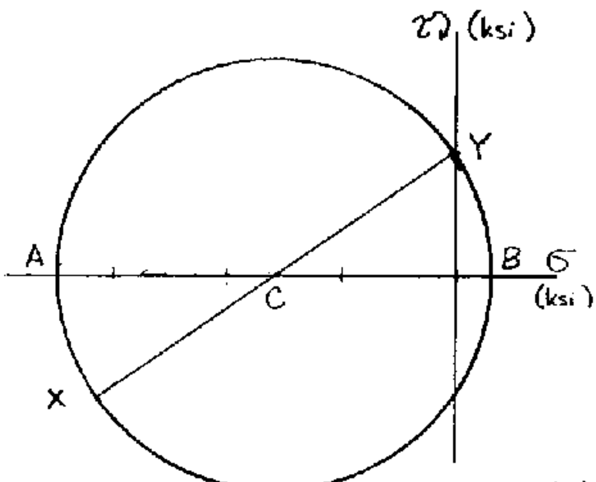


Stresses



$\sigma_x = -18.775 \text{ ksi}, \quad \sigma_y = 0, \quad \tau_{xy} = 6.519 \text{ ksi}$

Plotted points. $X: (-18.775, -6.519)$; $Y: (0, 6.519)$; $C(-9.3875, 0)$



$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -9.3875 \text{ ksi}$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{\left(\frac{-18.775}{2}\right)^2 + (6.519)^2}$

$= 11.429 \text{ ksi}$

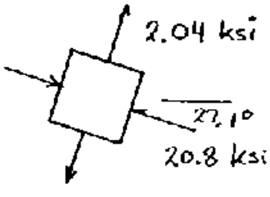
$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(6.519)}{-18.775}$

$= -1.3889$

(a) $\quad \theta_a = 27.1° \circlearrowleft, \quad \theta_b = 62.9° \circlearrowright$ ◄

$\sigma_a = \sigma_{ave} - R = -9.3875 - 11.429 = -20.8 \text{ ksi}$ ◄

$\sigma_b = \sigma_{ave} + R = -9.3875 + 11.429 = 2.04 \text{ ksi}$ ◄

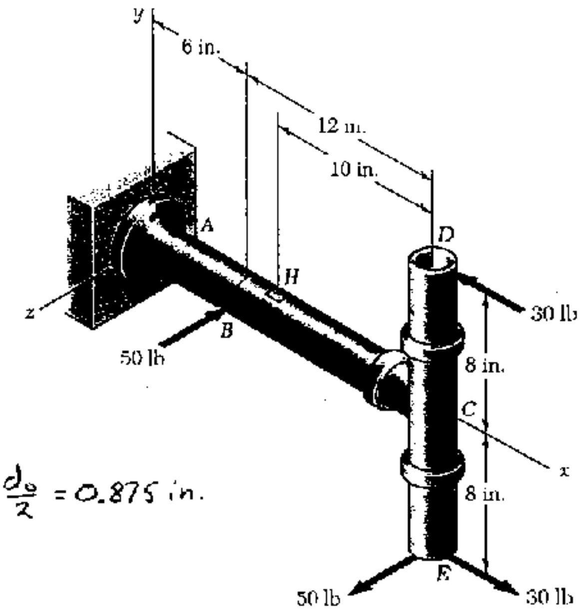


(b) $\tau_{max} = R = 11.43 \text{ ksi}$ ◄

Problem 7.46

7.46 Solve Prob. 7.24, using Mohr's circle.

7.24 Several forces are applied to the pipe assembly shown. Knowing that the inner and outer diameters of the pipe are equal to 1.50 in. and 1.75 in, respectively, determine (*a*) the principal planes and the principal stresses at point *H* located at the top of the outside surface of the pipe, (*b*) the maximum shearing stress at the same point.

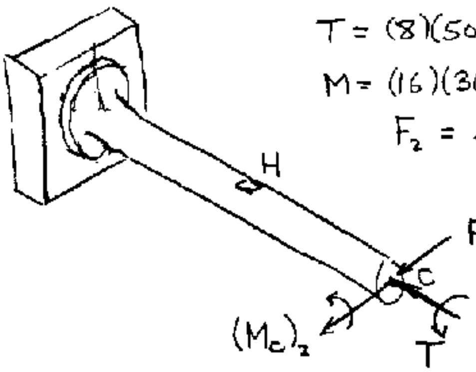


Replace forces on pipe DCE by an equivalent force-couple system at C.

$T = (8)(50) = 400$ lb·in

$M = (16)(30) = 480$ lb·in

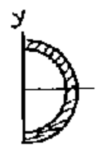
$F_z = 50$ lb



Cross section. $c_1 = \frac{d_i}{2} = 0.750$ in. $c_2 = \frac{d_o}{2} = 0.875$ in.

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = 0.42376$ in^4

$I = \frac{1}{2}J = 0.21188$ in.



$Q_y = \frac{2}{3}(c_2^3 - c_1^3) = \frac{2}{3}(0.875^3 - 0.750^3) = 0.16536$ in^3

$t = c_2 - c_1 = 0.125$ in.

At the section containing element H

$T = 400$ lb·in, $M_z = 480$ lb·in, $V_z = 50$ lb.

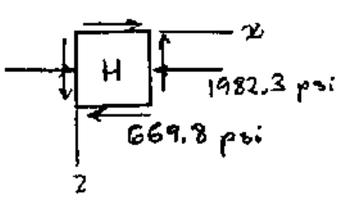
Stresses.

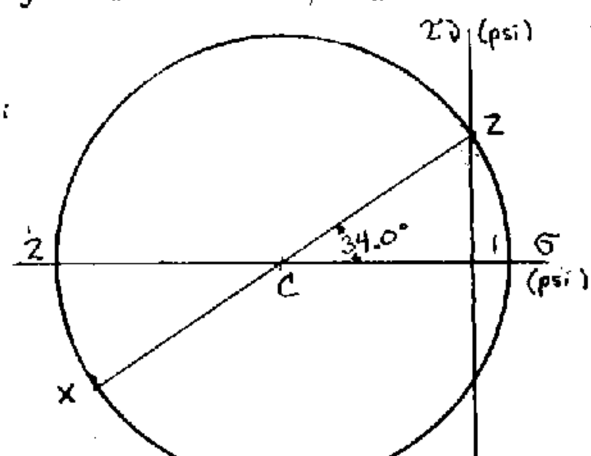
Torsion: $\tau_{zx} = -\frac{Tc}{J} = -\frac{(400)(0.875)}{0.42376} = -825.9$ psi

Bending: $\sigma_x = -\frac{M_z y}{I} = -\frac{(480)(0.875)}{0.21188} = -1982.3$ psi

Transverse Shear: $\tau_{zx} = \frac{V_z Q_y}{I(2t)} = \frac{(50)(0.16536)}{(0.21188)(0.250)} = 156.1$ psi

Total: $\sigma_z = 0$, $\sigma_x = -1982.3$, $\tau_{zx} = -825.9 + 156.1 = -669.8$ psi





Plotted points

Z: (0, 669.8)

X: (-1982.8, -669.8)

C: (-991.1, 0)

$\theta_1 = 17.0°$ ◄

$\theta_2 = 73.0°$ ◄

$\sigma_1 = 205$ psi ◄

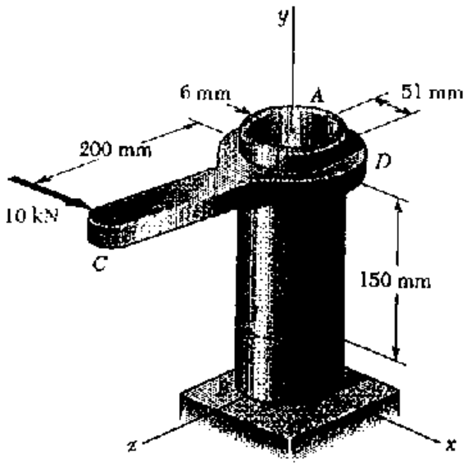
$\sigma_2 = 2187$ psi ◄

$\tau_{max} = R = 1196$ psi ◄

Problem 7.47

7.47 Solve Prob. 7.25, using Mohr's circle.

7.25 The steel pipe AB has a 102-mm outer diameter and a 6-mm wall thickness. Knowing that arm CD is rigidly attached tot he pipe, determine the principal stresses and the maximum shearing stress at point H.



$r_o = \frac{d_o}{2} = \frac{102}{2} = 51\ mm \qquad r_i = r_o - t = 45\ mm$

$J = \frac{\pi}{2}(r_o^4 - r_i^4) = 4.1855\times10^6\ mm^4 = 4.1855\times10^{-6}\ m^4$

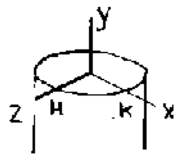
$I = \frac{1}{2}J = 2.0927\times10^{-6}\ m^4$

Force-couple system at center of tube in the plane containing points H and K

$F_x = 10\times10^3\ N$

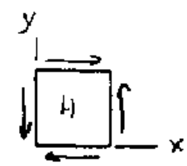
$M_y = (10\times10^3)(200\times10^{-3}) = 2000\ N\cdot m$

$M_z = -(10\times10^3)(150\times10^{-3}) = -1500\ N\cdot m$



Torsion: $T = M_y = 2000\ N\cdot m$

$c = r_o = 51\times10^{-3}\ m$



$$\tau_{xy} = \frac{Tc}{J} = \frac{(2000)(51\times10^{-3})}{4.1855\times10^{-6}} = 24.37\ MPa$$

Transverse Shear:

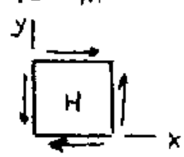
For semicircle $\quad A = \frac{\pi}{2}r^2 \qquad \bar{y} = \frac{4}{3\pi}r$

$Q = A\bar{y} = \frac{2}{3}r^3$

For pipe $\quad Q = Q_o - Q_i = \frac{2}{3}r_o^3 - \frac{2}{3}r_i^3 = 27.684\times10^3\ mm^3 = 27.684\times10^{-6}\ m^3$

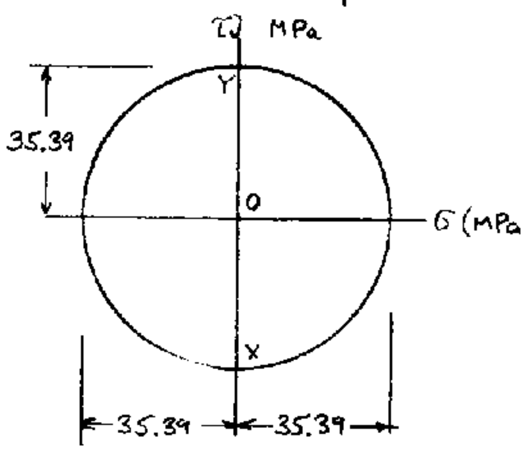
$V = F_x = 10\times10^3\ N \qquad t = (2)(6) = 12\ mm = 12\times10^{-3}\ m$

$$\tau_{xy} = \frac{VQ}{It} = \frac{(10\times10^3)(27.684\times10^{-6})}{(2.0927\times10^{-6})(12\times10^{-3})} = 11.02\ MPa$$



Bending: Point H lies on neutral axis $\quad \sigma_y = 0$

Total stresses at point H $\quad \sigma_x = 0,\ \sigma_y = 0 \qquad \tau_{xy} = 24.37 + 11.02 = 35.39\ MPa$



$\sigma_{ave} = 0$

$R = 35.39\ MPa$

$\sigma_{max} = \sigma_{ave} + R = 35.39\ MPa$

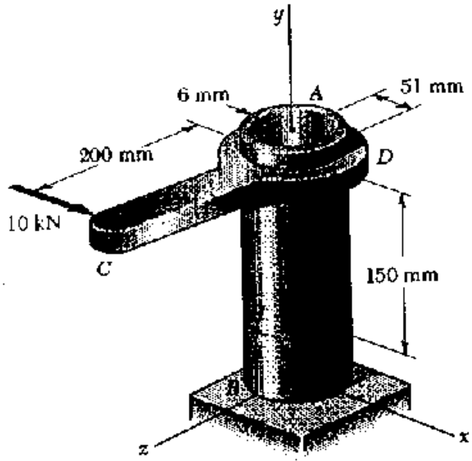
$\sigma_{min} = \sigma_{ave} - R = -35.39\ MPa$

$\tau_{max} = R = 35.39\ MPa$

Problem 7.48

7.48 Solve Prob. 7.26, using Mohr's circle.

7.26 The steel pipe AB has a 102-mm outer diameter and a 6-mm wall thickness. Knowing that arm CD is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point K.



$r_o = \frac{d_o}{2} = \frac{102}{2} = 51 \text{ mm} \qquad r_i = r_o - t = 45 \text{ mm}$

$J = \frac{\pi}{2}(r_o^4 - r_i^4) = 4.1855 \times 10^6 \text{ mm}^4 = 4.1855 \times 10^{-6} \text{ m}^4$

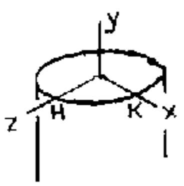
$I = \frac{1}{2} J = 2.0927 \times 10^{-6} \text{ m}^4$

Force-couple system at center of tube in the plane containing points H and K

$F_x = 10 \times 10^3 \text{ N}$

$M_y = (10 \times 10^3)(200 \times 10^{-3}) = 2000 \text{ N}\cdot\text{m}$

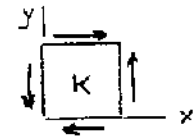
$M_z = -(10 \times 10^3)(150 \times 10^{-3}) = -1500 \text{ N}\cdot\text{m}$



Torsion: $T = M_y = 2000 \text{ N}\cdot\text{m}$

$c = r_o = 51 \times 10^{-3} \text{ m}$

$\tau_{xy} = \frac{Tc}{J} = \frac{(2000)(51 \times 10^{-3})}{4.1855 \times 10^{-6}} = 24.37 \text{ MPa}$



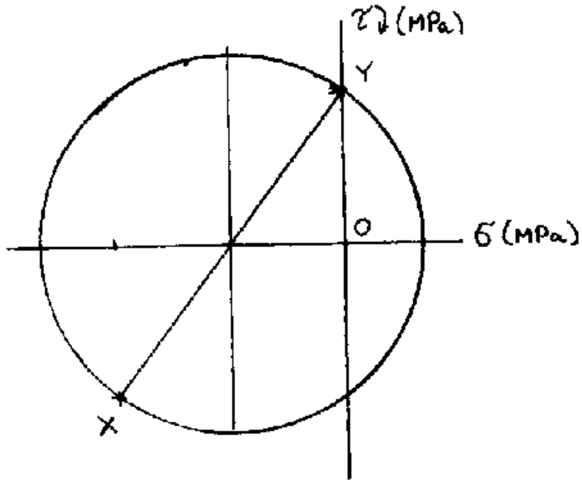
Note that local x-axis is taken along negative global z-direction.

Transverse Shear: Stress due to $V = F_x$ is zero at point K.

Bending: $|\sigma_y| = \frac{|M_z| c}{I} = \frac{(1500)(51 \times 10^{-3})}{2.0927 \times 10^{-6}} = 36.56 \text{ MPa}$

Point K lies on compression side of neutral axis. $\sigma_y = -36.56 \text{ MPa}$

Total stresses at point K $\quad \sigma_x = 0, \ \sigma_y = -36.56 \text{ MPa}, \ \tau_{xy} = 24.37 \text{ MPa}$



$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -18.28 \text{ MPa}$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 30.46 \text{ MPa}$

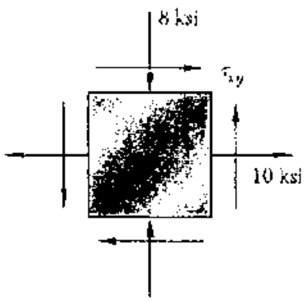
$\sigma_{max} = \sigma_{ave} + R = -18.28 + 30.46$
$= 12.18 \text{ MPa}$ ◄

$\sigma_{min} = \sigma_{ave} - R = -18.28 - 30.46$
$= -48.74 \text{ MPa}$ ◄

$\tau_{max} = R = 30.46 \text{ MPa}$ ◄

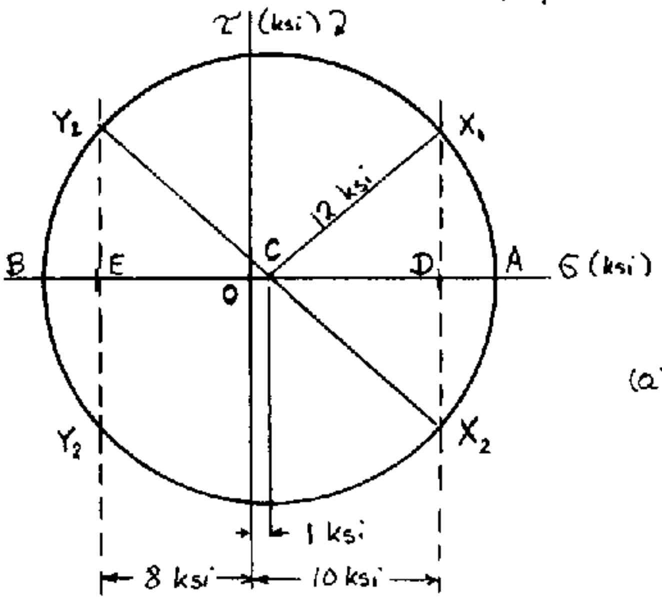
Problem 7.49

7.49 Solve Prob. 7.27, using Mohr's circle.



7.27 For the state of plane stress shown, determine (*a*) the largest value of τ_{xy} for which the maximum in-plane shearing stress is equal to or less than 12 ksi, (*b*) the corresponding principal stresses.

The center of the Mohr's circle lies at point C with coordinates $\left(\frac{\sigma_x + \sigma_y}{2}, 0\right) = \left(\frac{10-8}{2}, 0\right) = (1.0 \text{ ksi}, 0)$. The radius of the circle is $\tau_{max(in\text{-}plane)} = 12$ ksi.



The stress point $(\sigma_x, -\tau_{xy})$ lie along the line X_1X_2 of th Mohr circle diagram. The extreme points with $R \leq 12$ ksi are X_1 and X_2.

(a) The largest allowable value of τ_{xy} is obtained from triangle CDX_1

$$\overline{DX_1}^2 = \overline{DX_2}^2 = \sqrt{\overline{CX_1}^2 - \overline{CD}^2}$$

$$\tau_{xy}^2 = \sqrt{12^2 - 9^2} = 7.94 \text{ ksi}$$

(b) The principal stresses are

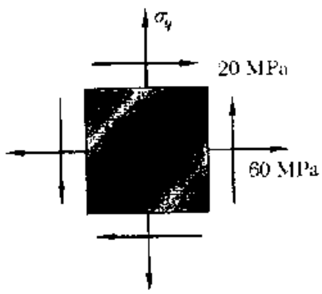
$$\sigma_a = 1 + 12 = 13 \text{ ksi}$$

$$\sigma_b = 1 - 12 = -11 \text{ ksi}$$

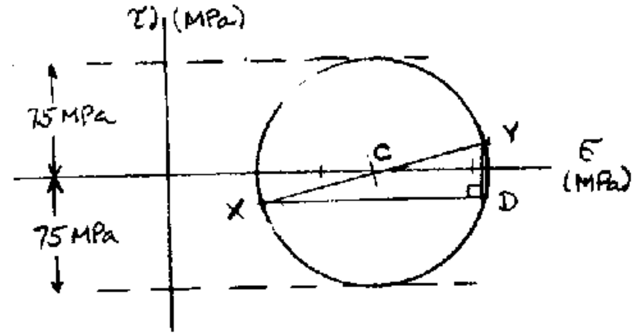
Problem 7.50

7.50 Solve Prob. 7.28, using Mohr's circle.

7.28 For the state of plane stress shown, determine the largest value of σ_y for which the maximum in-plane shearing stress is equal to of less than 75 MPa.



$\sigma_x = 60$ MPa, $\sigma_y = ?$, $\tau_{xy} = 20$ MPa



Given: $\tau_{max} = R = 75$ MPa

$\overline{XY} = 2R = 150$ MPa

$\overline{DY} = (2)(\tau_{xy}) = 40$ MPa

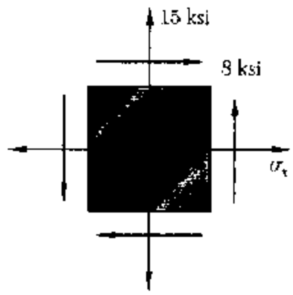
$\overline{XD} = \sqrt{\overline{XY}^2 - \overline{DY}^2} = \sqrt{150^2 - 40^2} = 144.6$ MPa

$\sigma_y = \sigma_x + \overline{XD} = 60 + 144.6 = 205$ MPa

Problem 7.51

7.51 Solve Prob. 7.29, using Mohr's circle.

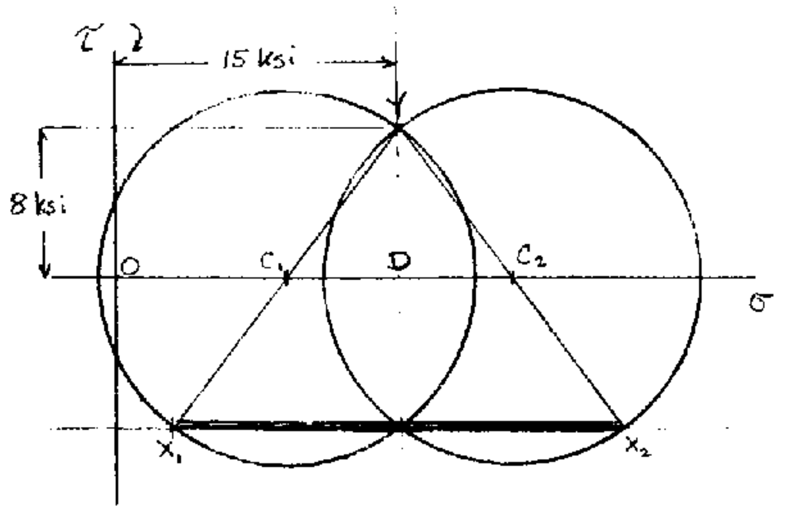
7.29 Determine the range of values of σ_x for which the maximum in-plane shearing stress is equal to or less than 10 ksi.



For the Mohr's circle, point Y lies at (15 ksi, 8 ksi).

The radius of limiting circles is R = 10 ksi.

Let C_1 be the location of the left most limiting circle and C_2 be that of the right most one.



$\overline{C_1Y} = 10$ ksi

$\overline{C_2Y} = 10$ ksi

Noting right triangles C_1DY and C_2DY

$\overline{C_1D}^2 + \overline{DY}^2 = \overline{C_1Y}^2 \qquad \overline{C_1D}^2 + 8^2 = 10^2 \qquad \overline{C_1D} = 6$ ksi

Coordinates of point C_1 are (0, 15 − 6) = (0, 9 ksi).

Likewise, coordinates of point C_2 are = (0, 15 + 6) = (0, 21 ksi)

Coordinates of point X_1 (9 − 6, − 8) = (3 ksi, − 8 ksi)

Coordinates of point X_2 (21 + 6, − 8) = (27 ksi, − 8 ksi)

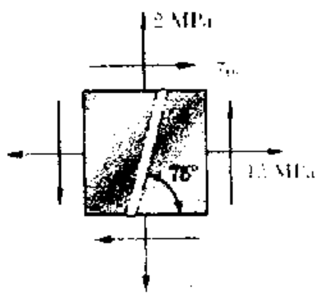
The point $(\sigma_x, -\tau_{xy})$ must lie on the line X_1X_2

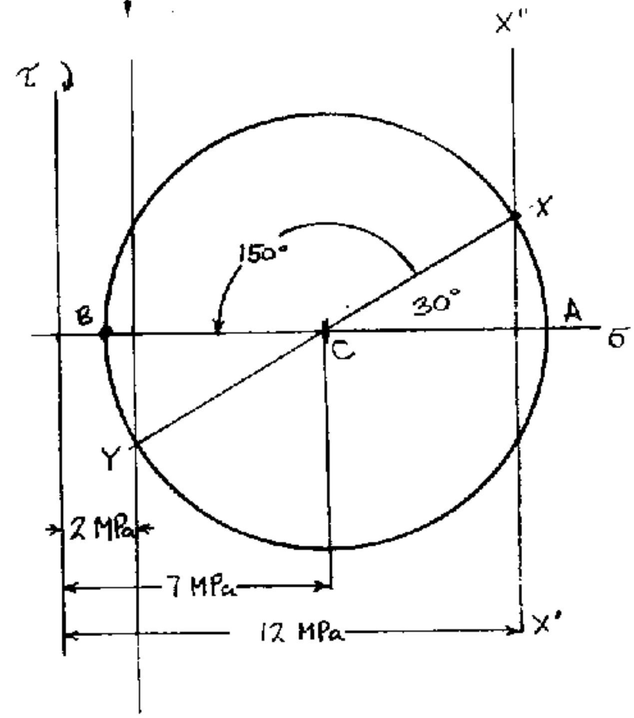
Thus $\qquad 3 \text{ ksi} \le \sigma_x \le 27 \text{ ksi}$ ◄

Problem 7.52

7.52 Solve Prob. 7.30, using Mohr's circle.

7.30 For the state of plane stress shown, determine (*a*) the value of τ_{xy} for which the in-plane shearing stress parallel to the weld is zero, (*b*) the corresponding principal stresses.





Point X of Mohr's circle must lie on X'X'' so that $\sigma_x = 12$ MPa. Likewise, point Y lies on line Y'Y'' so that $\sigma_y = 2$ MPa. The coordinates of C are

$$\frac{2+12}{2},\ 0 = (7\text{ MPa}, 0).$$

Counterclockwise rotation through 150° brings line CX to CB, where $\tau = 0$.

$$R = \frac{\sigma_x - \sigma_y}{2}\sec 30°$$

$$= \frac{12-2}{2}\sec 30°$$

$$= 5.77\text{ MPa}$$

$$\tau_{xy} = -\frac{\sigma_x - \sigma_y}{2}\tan 30°$$

$$= -\frac{12-2}{2}\tan 30°$$

$$= -2.89\text{ MPa}$$

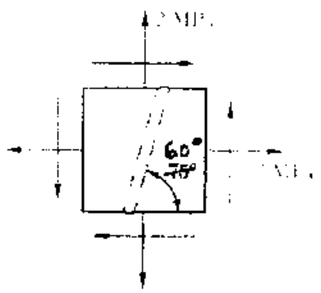
$$\sigma_A = \sigma_{ave} + R = 7 + 5.77 = 12.77\text{ MPa}$$

$$\sigma_B = \sigma_{ave} - R = 7 - 5.77 = 1.23\text{ MPa}$$

Problem 7.53

7.53 Solve Prob. 7.30, using Mohr's circle and assuming that the weld forms an angle of 60° with the horizontal.

7.30 For the state of plane stress shown, determine (*a*) the value of τ_{xy} for which the in-plane shearing stress parallel to the weld is zero, (*b*) the corresponding principal stresses.



Locate point C

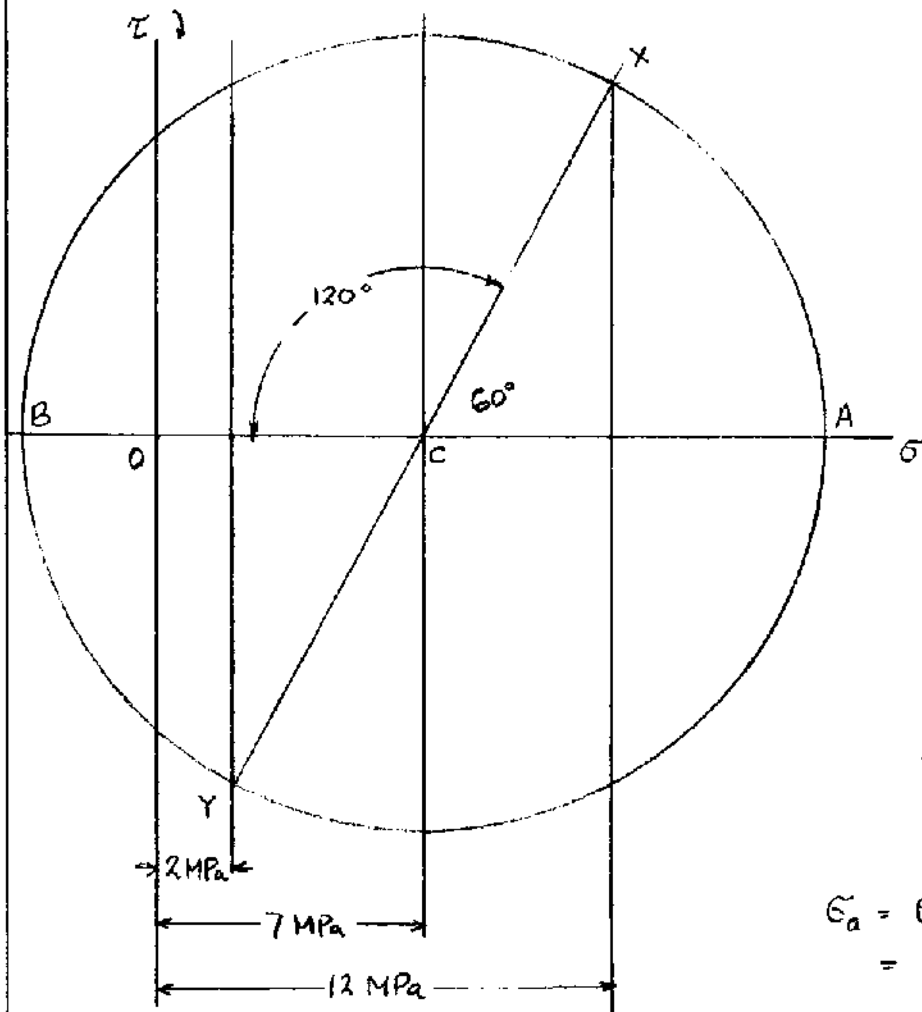
at $\sigma = \frac{12+2}{2} = 7$ MPa

with $\tau = 0$.

Angle XCB = 120°

$\frac{\sigma_x - \sigma_y}{2} = \frac{12-2}{2}$

$= 5$ MPa



$R = 5 \sec 60°$

$= 10$ MPa

$\tau_{xy} = -5 \tan 60°$

$= -8.66$ MPa ◄

$\sigma_a = \sigma_{ave} + R$

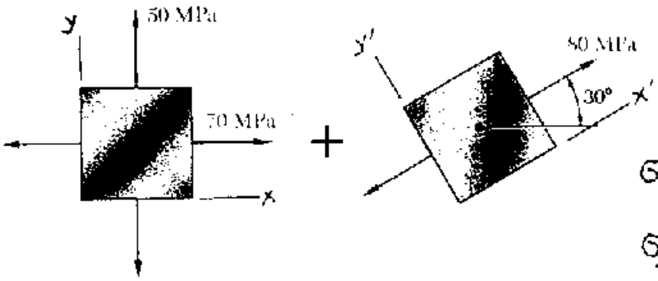
$= 7 + 10 = 17$ MPa ◄

$\sigma_b = \sigma_{ave} - R$

$= 7 - 10 = -3$ MPa ◄

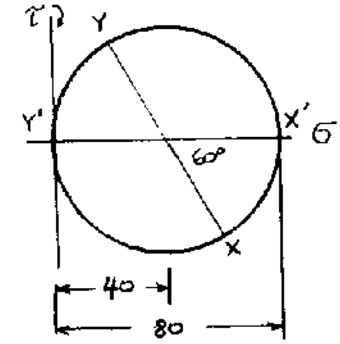
Problem 7.54

7.54 and 7.55 Determine the principal planes and the principal stress for the state of plane stress resulting from the superposition of the two states of stress shown.



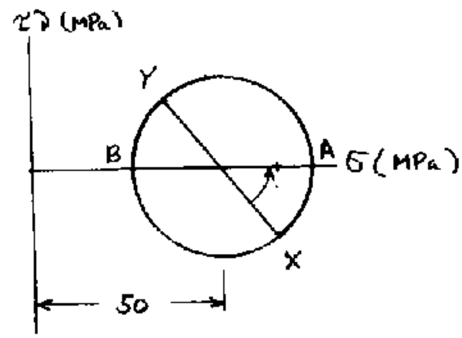
Mohr's circle for 2nd stress state

$\sigma_x = 40 + 40 \cos 60°$
$= 60$ MPa
$\sigma_y = 40 - 40 \cos 60°$
$= 20$ MPa
$\tau_{xy} = 40 \sin 60°$
$= 34.64$ MPa



Resultant stresses

$\sigma_x = 70 + 60 = 130$ MPa
$\sigma_y = 50 + 20 = 70$ MPa
$\tau_{xy} = 0 + 34.64 = 34.64$ MPa



$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}(130 + 70) = 100$ MPa

$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(34.64)}{130 - 70} = 1.1547$

$2\theta_p = 49.11°$ $\theta_a = 24.6°$ $\theta_b = 114.6°$

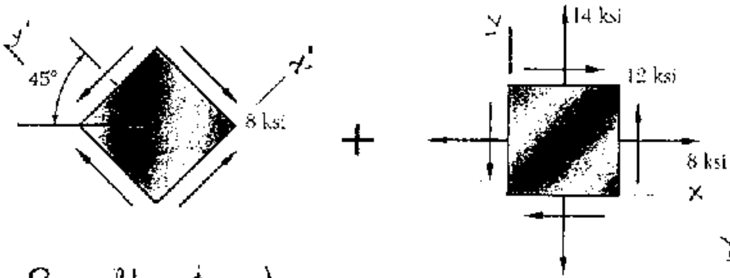
$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 45.8$ MPa

$\sigma_a = \sigma_{ave} + R = 145.8$ MPa

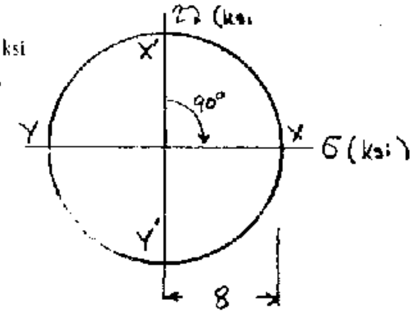
$\sigma_b = \sigma_{ave} - R = 54.2$ MPa

Problem 7.55

7.54 and 7.55 Determine the principal planes and the principal stress for the state of plane stress resulting from the superposition of the two states of stress shown.



Mohr's circle for 1st stress state.



$\sigma_x = 8$ ksi

$\sigma_y = -8$ ksi

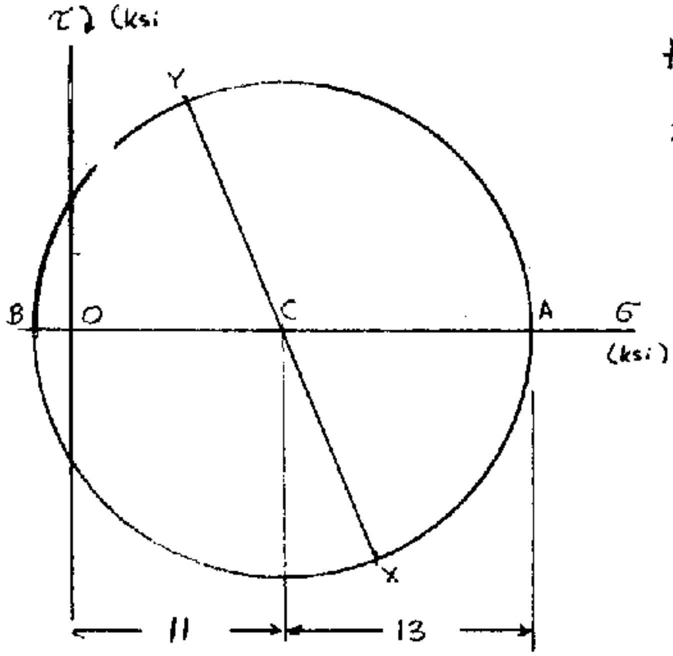
$\tau_{xy} = 0$

Resultant stresses

$\sigma_x = 8 + 8 = 16$ ksi

$\sigma_y = -8 + 14 = 6$ ksi

$\tau_{xy} = 12 + 0 = 12$ ksi



$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 11$ ksi

$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(12)}{10} = 2.4$

$2\theta_p = 67.38°$ $\quad \theta_a = 33.69°$ ◀

$\theta_b = 123.69°$ ◀

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

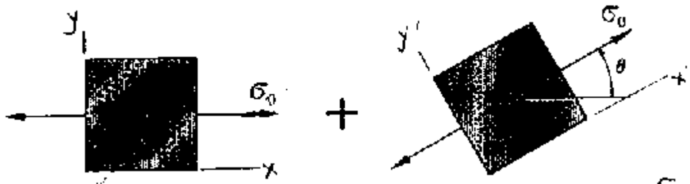
$= \sqrt{5^2 - 12^2} = 13$ ksi

$\sigma_a = \sigma_{ave} + R = 24$ ksi ◀

$\sigma_b = \sigma_{ave} - R = -2$ ksi ◀

Problem 7.56

7.56 **and** *7.57* Determine the principal planes and the principal stress for the state of plane stress resulting from the superposition of the two states of stress shown.

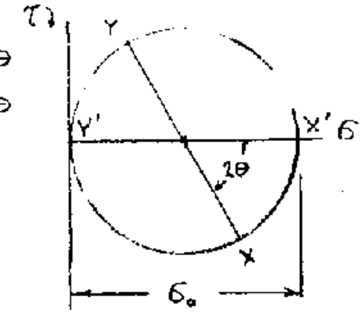


Mohr's circle for 2nd stress state

$$\sigma_x = \tfrac{1}{2}\sigma_0 + \tfrac{1}{2}\sigma_0\cos 2\theta$$

$$\sigma_y = \tfrac{1}{2}\sigma_0 - \tfrac{1}{2}\sigma_0\cos 2\theta$$

$$\tau_{xy} = \tfrac{1}{2}\sigma_0 \sin 2\theta$$



Resultant stresses

$$\sigma_x = \sigma_0 + \tfrac{1}{2}\sigma_0 + \tfrac{1}{2}\sigma_0\cos 2\theta = \tfrac{3}{2}\sigma_0 + \tfrac{1}{2}\sigma_0\cos 2\theta$$

$$\sigma_y = 0 + \tfrac{1}{2}\sigma_0 - \tfrac{1}{2}\sigma_0\cos 2\theta = \tfrac{1}{2}\sigma_0 - \tfrac{1}{2}\sigma_0\cos 2\theta$$

$$\tau_{xy} = 0 + \tfrac{1}{2}\sigma_0\sin 2\theta = \tfrac{1}{2}\sigma_0 \sin 2\theta$$

$$\sigma_{ave} = \tfrac{1}{2}(\sigma_x + \sigma_y) = \sigma_0$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{\sigma_0 \sin 2\theta}{\sigma_0 + \sigma_0\cos 2\theta}$$

$$= \frac{\sin 2\theta}{1 + \cos 2\theta} = \tan\theta$$

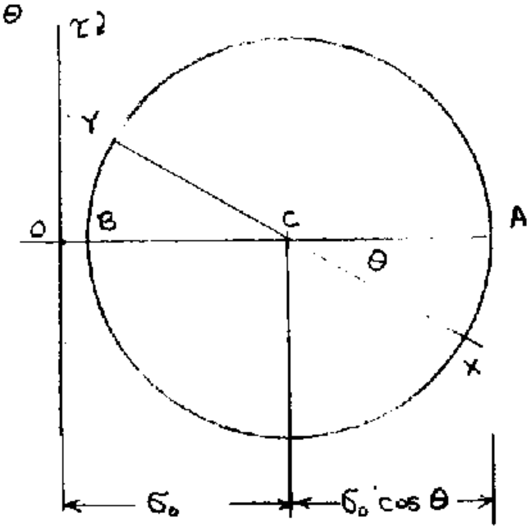
$$\theta_p = \tfrac{1}{2}\theta \quad \blacktriangleleft$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} =$$

$$= \sqrt{\left(\tfrac{1}{2}\sigma_0 + \tfrac{1}{2}\sigma_0\cos 2\theta\right)^2 + \left(\tfrac{1}{2}\sigma_0 \sin 2\theta\right)^2}$$

$$= \tfrac{1}{2}\sigma_0\sqrt{1 + 2\cos 2\theta + \cos^2 2\theta + \sin^2 2\theta}$$

$$= \frac{\sqrt{2}}{2}\sigma_0\sqrt{1 + \cos 2\theta} = \sigma_0 |\cos\theta|$$

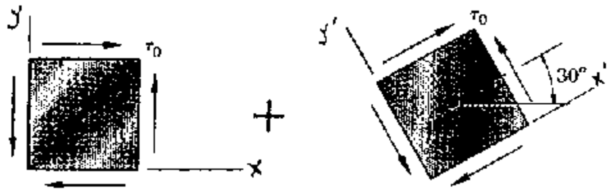


$$\sigma_a = \sigma_{ave} + R = \sigma_0 + \sigma_0\cos\theta \quad \blacktriangleleft$$

$$\sigma_b = \sigma_{ave} - R = \sigma_0 - \sigma_0\cos\theta \quad \blacktriangleleft$$

Problem 7.57

7.56 and 7.57 Determine the principal planes and the principal stress for the state of plane stress resulting from the superposition of the two states of stress shown.

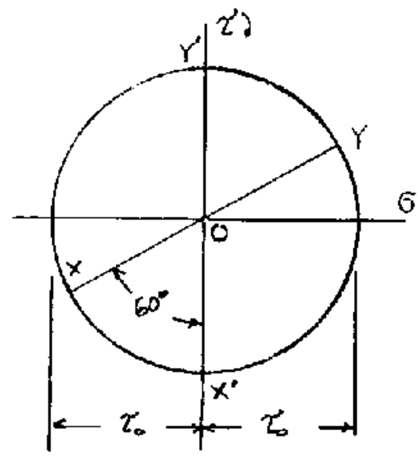


Mohr's circle for 2nd state of stress

$\sigma_{x'} = 0$

$\sigma_{y'} = 0$

$\tau_{x'y'} = \tau_o$



$\sigma_x = -\tau_o \sin 60° = -\frac{\sqrt{3}}{2}\tau_o$

$\sigma_y = \tau_o \sin 60° = \frac{\sqrt{3}}{2}\tau_o$

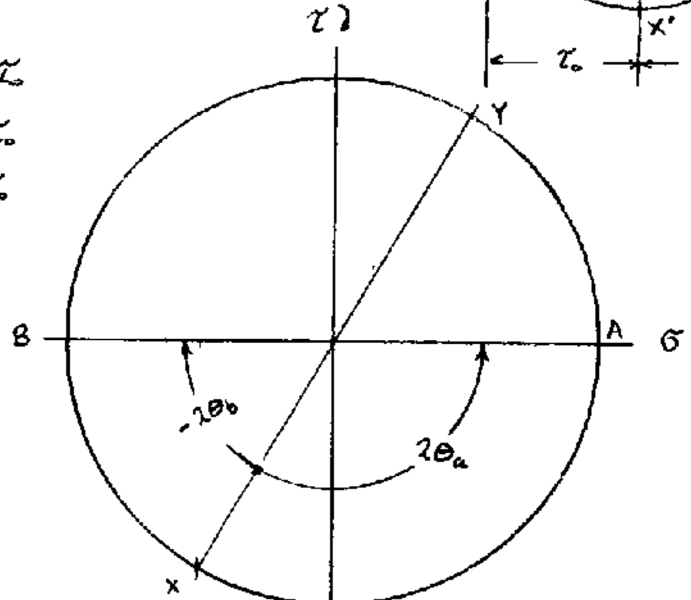
$\tau_{xy} = \tau_o \cos 60° = \frac{1}{2}\tau_o$

Resultant stresses

$\sigma_x = 0 - \frac{\sqrt{3}}{2}\tau_o = -\frac{\sqrt{3}}{2}\tau_o$

$\sigma_y = 0 + \frac{\sqrt{3}}{2}\tau_o = \frac{\sqrt{3}}{2}\tau_o$

$\tau_{xy} = \tau_o + \frac{1}{2}\tau_o = \frac{3}{2}\tau_o$



$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 0$

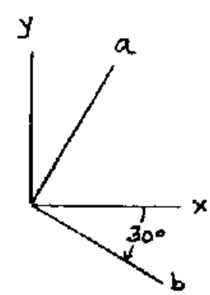
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{\sqrt{3}}{2}\tau_o\right)^2 + \left(\frac{3}{2}\tau_o\right)^2}$$

$$= \sqrt{3}\,\tau_o$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2\,\frac{3}{2}}{-\sqrt{3}} = -\sqrt{3}$$

$2\theta_p = -60°$ $\theta_b = -30°$ $\theta_a = 60°$ ◄

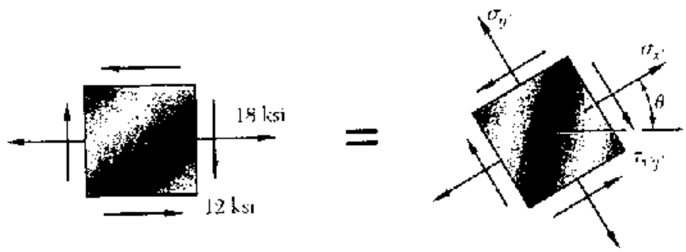


$\sigma_a = \sigma_{ave} + R = \sqrt{3}\,\tau_o$ ◄

$\sigma_b = \sigma_{ave} - R = -\sqrt{3}\,\tau_o$ ◄

Problem 7.58

7.58 For the state of stress shown, determine the range of values of θ for which the normal stress $\sigma_{x'}$ is equal to or less than 20 ksi.



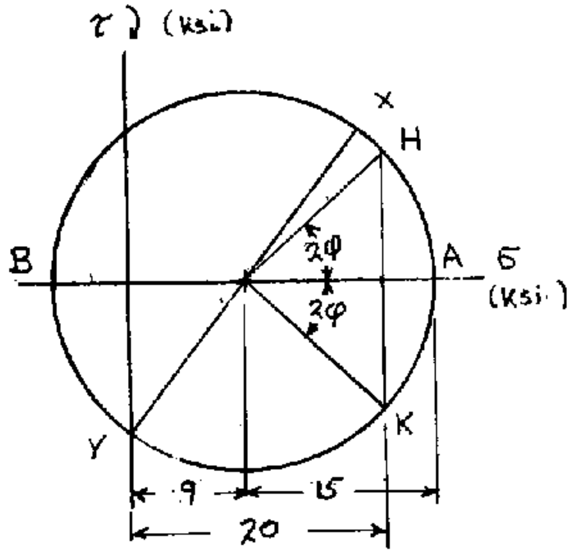
$\sigma_x = 18$ ksi , $\sigma_y = 0$

$\tau_{xy} = -12$ ksi

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 9$ ksi

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{9^2 + 12^2} = 15$ ksi



$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-12)}{18} = -\frac{4}{3}$

$2\theta_p = -53.13°$

$\theta_a = -26.565°$

$\sigma_{x'} \le 20$ ksi for states of stress corresponding to arc HBK of Mohr's circle. From the circle

$R \cos 2\varphi = 20 - 9 = 11$ ksi

$\cos 2\varphi = \frac{11}{15} = 0.73333$

$2\varphi = 42.833°$ $\quad\quad \varphi = 21.417°$

$\theta_H = \theta_a + \varphi = -26.565° + 21.417° = -5.15°$

$2\theta_K = 2\theta_H + 360° - 4\varphi = -10.297° + 360° - 85.666° = 264.037°$

$\theta_K = 132.02°$

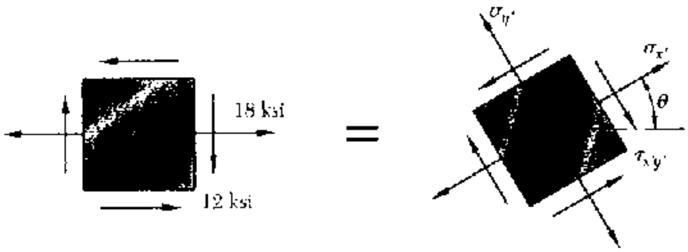
Permissibe range of θ is $\quad \theta_H \le \theta \le \theta_K$

$$-5.15° \le \theta \le 132.02°$$

Also $174.85° \le \theta \le 312.02°$

Problem 7.59

7.59 For the state of stress shown, determine the range of values of θ for which the normal stress $\sigma_{x'}$ is equal to or less than 10 ksi.



$\sigma_x = 18$ ksi, $\sigma_y = 0$
$\tau_{xy} = -12$ ksi

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 9$ ksi

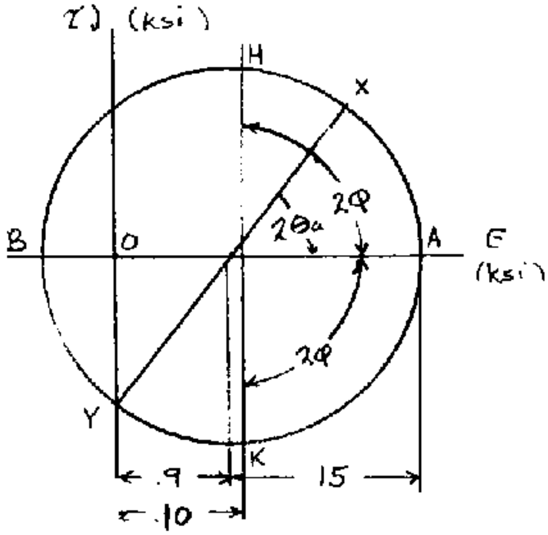
$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{9^2 + 12^2} = 15$ ksi

$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-12)}{18} = -\frac{4}{3}$

$2\theta_p = -53.13°$

$\theta_a = -26.565°$



$\sigma_{x'} \leq 10$ ksi for states of stress corresponding to the arc HBK of Mohr's circle. From the circle

$R\cos 2\varphi = 10 - 9 = 1$ ksi

$\cos 2\varphi = \frac{1}{15} = 0.066667$

$2\varphi = 86.177°$ $\qquad \varphi = 43.089°$

$\theta_H = \theta_a + \varphi = -26.565° + 43.089° = 16.524°$

$2\theta_K = 2\theta_H + 360° - 4\varphi = 32.524° + 360° - 172.355° = 220.169°$

$\theta_K = 110.085°$

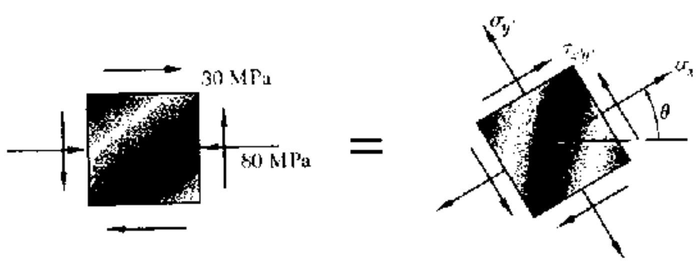
Permissible range of θ is $\theta_H \leq \theta \leq \theta_K$

$16.524° \leq \theta \leq 110.085°$

Also $196.524° \leq \theta \leq 290.085°$

Problem 7.60

7.60 For the state of stress shown, determine the range of values of θ for which the magnitude of the shearing stress $\tau_{x'y'}$ is eaual to or less than 40 MPa.



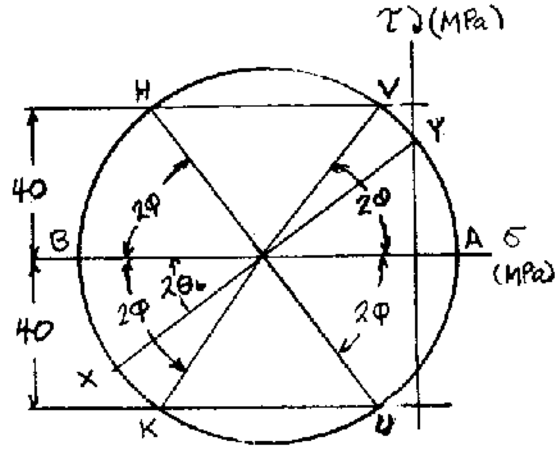
$\sigma_x = -80$ MPa, $\sigma_y = 0$

$\tau_{xy} = 30$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -40$ MPa

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{(-40)^2 + 30^2} = 50$ MPa



$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(30)}{-80} = -0.75$

$2\theta_p = -36.870°$

$\theta_b = -18.435°$

$|\tau_{x'y'}| \le 40$ MPa for states of stress corresponding to arcs HBK and UAV of Mohr's circle. The angle φ is calculated from

$R \sin 2\varphi = 40$

$\sin 2\varphi = \frac{40}{50} = 0.8$

$2\varphi = 53.130°$ $\qquad \varphi = 26.565°$

$\theta_H = \theta_b - \varphi = -18.435° - 26.565° = -45°$

$\theta_K = \theta_b + \varphi = -18.435 + 26.565° = 8.13°$

$\theta_U = \theta_H + 90° = 45°$

$\theta_V = \theta_K + 90° = 98.13°$

Permissible ranges of θ $\qquad \theta_H \le \theta \le \theta_K$

$-45° \le \theta \le 8.13°$

$\theta_U \le \theta \le \theta_V$

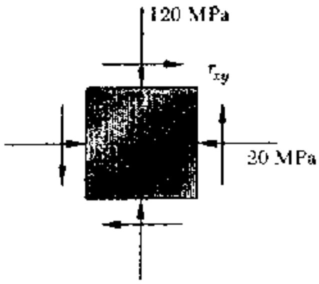
$45° \le \theta \le 98.13°$

Also $\qquad 135° \le \theta \le 188.13°$

$225° \le \theta \le 278.13°$

Problem 7.61

7.61 For the element shown, determine the range of values of τ_{xy} for which the maximum tensile stress is equal to or less than 60 MPa.



$\sigma_x = -20$ MPa $\quad \sigma_y = -120$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -70$ MPa

Set $\sigma_{max} = 60$ MPa $= \sigma_{ave} + R$

$R = \sigma_{max} - \sigma_{ave} = 130$ MPa

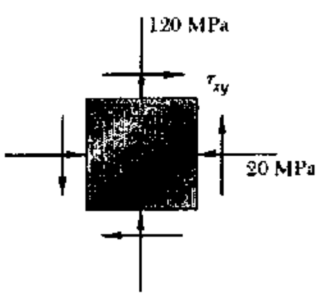
But $R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$|\tau_{xy}| = \sqrt{R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} = \sqrt{130^2 - 50^2} = 120$ MPa

Range of τ_{xy} $\qquad -120 \text{ MPa} \le \tau_{xy} \le 120 \text{ MPa}$ ◀

Problem 7.62

7.62 For the element shown, determine the range of values of τ_{xy} for which the maximum in-plane shearing stress is equal to or less than 150 MPa.



$\sigma_x = -20$ MPa $\quad \sigma_y = -120$ MPa

$\frac{1}{2}(\sigma_x - \sigma_y) = 50$ MPa

Set $\tau_{max(in\text{-}plane)} = R = 150$ MPa

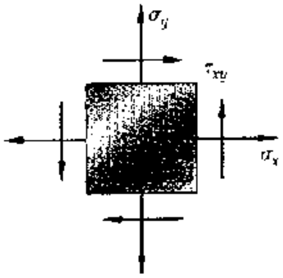
But $R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$|\tau_{xy}| = \sqrt{R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} = \sqrt{150^2 - 50^2} = 141.4$ MPa

Range of τ_{xy} $\qquad -141.4 \text{ MPa} \le \tau_{xy} \le 141.4 \text{ MPa}$ ◀

Problem 7.63

7.63 For the state of stress shown it is known that the normal and shearing stresses are directed as shown and that $\sigma_x = 14$ ksi, $\sigma_y = 9$ ksi, and $\sigma_{min} = 5$ ksi. Determine (*a*) the orientation of the principal planes, (*b*) the principal stress σ_{max}, (*c*) the maximum in-plane shearing stress.



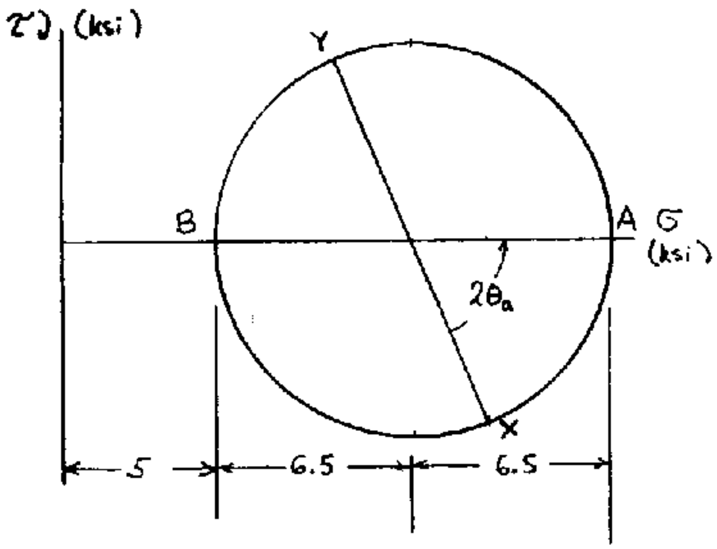
$\sigma_x = 14$ ksi, $\sigma_y = 9$ ksi, $\quad \sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 11.5$ ksi

$\sigma_{min} = \sigma_{ave} - R \quad \therefore \quad R = \sigma_{ave} - \sigma_{min}$
$= 11.5 - 5 = 6.5$ ksi

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{xy} = \pm\sqrt{R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} = \pm\sqrt{6.5^2 - 2.5^2} = \pm 6 \text{ ksi}$$

But it is given that τ_{xy} is positive, thus $\tau_{xy} = +6$ ksi



(a) $\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$
$= \frac{(2)(6)}{5} = 2.4$

$2\theta_p = 67.38°$

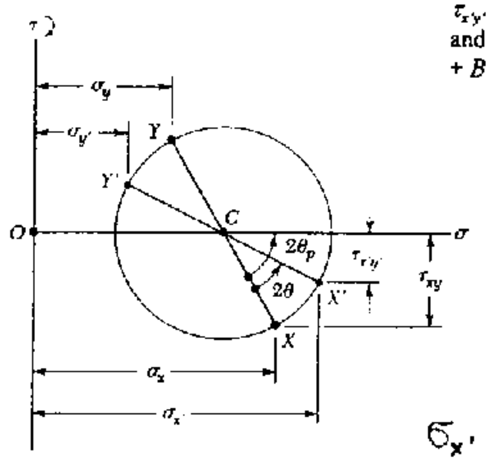
$\theta_a = 33.69°$

$\theta_b = 123.69°$

(b) $\sigma_{max} = \sigma_{ave} + R$
$= 18$ ksi

(c) $\tau_{max(in\text{-}plane)} = R$
$= 6.5$ ksi

Problem 7.64



7.64 The Mohr circle shown corresponds to the state of stress given in Fig. 7.5*a* and *b*. Noting that $\sigma_{x'} = OC + (CX')\cos(2\theta_p - 2\theta)$ and that $\tau_{x'y'} = (CX')\sin(2\theta_p - 2\theta)$, derive the expressions for $\sigma_{x'}$ and $\tau_{x'y'}$ given in Eqs. (7.5) and (7.6), respectively. [*Hint:* Use $\sin(A + B) = \sin A\cos B + \cos A\sin B$ and $\cos(A + B) = \cos A\cos B - \sin A\sin B$.]

$$\overline{OC} = \tfrac{1}{2}(\sigma_x + \sigma_y) \qquad \overline{CX'} = \overline{CX}$$

$$\overline{CX'}\cos 2\theta_p = \overline{CX}\cos 2\theta_p = \frac{\sigma_x - \sigma_y}{2}$$

$$\overline{CX'}\sin 2\theta_p = \overline{CX}\sin 2\theta_p = \tau_{xy}$$

$$\sigma_{x'} = \overline{OC} + \overline{CX}'\cos(2\theta_p - 2\theta)$$

$$= \overline{OC} + \overline{CX}'(\cos 2\theta_p \cos 2\theta + \sin 2\theta_p \sin 2\theta)$$

$$= \overline{OC} + \overline{CX}'\cos 2\theta_p \cos 2\theta + \overline{CX}'\sin 2\theta_p \sin 2\theta$$

$$= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

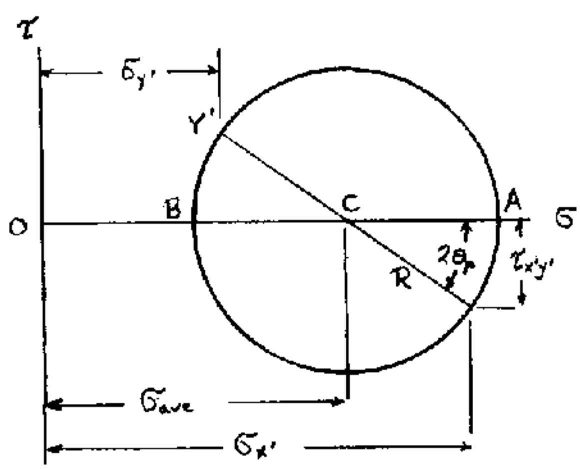
$$\tau_{x'y'} = \overline{CX}'\sin(2\theta_p - 2\theta) = \overline{CX}'(\sin 2\theta_p \cos 2\theta - \cos 2\theta_p \sin 2\theta)$$

$$= \overline{CX}'\sin 2\theta_p \cos 2\theta - \overline{CX}'\cos 2\theta_p \sin 2\theta$$

$$= \tau_{xy}\cos 2\theta - \frac{\sigma_x - \sigma_y}{2}\sin 2\theta$$

Problem 7.65

7.65 (*a*) Prove that the expression $\sigma_{x'}\sigma_{y'} - \tau_{x'y'}^2$. Where $\sigma_{x'}$, $\sigma_{y'}$, and $\tau_{x'y'}$ are components of the stress along the rectangular axes x' and y', is independent of the orientation of these axes. Also, show that the given expression represents the square of the tangent drawn from the origin of coordinates of Mohr's circle. (*b*) Using the invariance property established in part *a*, express the shearing stress τ_{xy} om terms of σ_x, σ_y, and the principal stresses σ_{max} and σ_{min}



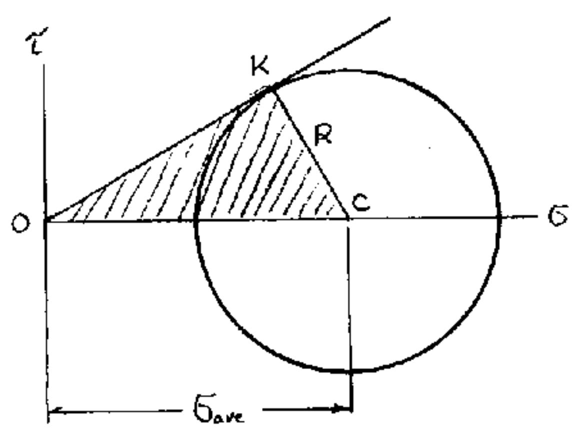
(a) From Mohr's circle

$$\tau_{x'y'} = R \sin 2\theta_p$$

$$\sigma_{x'} = \sigma_{ave} + R\cos 2\theta_p$$

$$\sigma_{y'} = \sigma_{ave} - R\cos 2\theta_p$$

$$\sigma_{x'}\sigma_{y'} - \tau_{x'y'}^2$$
$$= \sigma_{ave}^2 - R^2\cos^2 2\theta_p - R^2\sin^2 2\theta_p$$
$$= \sigma_{ave}^2 - R^2;$$ independent of θ_p.



Draw line $\overline{OK}$ from origin tangent to the circle at K. Triangle OCK is a right triangle

$$\overline{OC}^2 = \overline{OK}^2 + \overline{CK}^2$$

$$\overline{OK}^2 = \overline{OC}^2 - \overline{CK}^2$$
$$= \sigma_{ave}^2 - R^2$$
$$= \sigma_{x'}\sigma_{y'} - \tau_{x'y'}$$

(b) Applying above to σ_x, σ_y, and τ_{xy} and to σ_a, σ_b,

$$\sigma_x\sigma_y - \tau_{xy}^2 = \sigma_a\sigma_b - \tau_{ab}^2 = \sigma_{ave}^2 - R^2$$

But $\tau_{ab} = 0$, $\sigma_a = \sigma_{max}$, $\sigma_b = \sigma_{min}$

$$\sigma_x\sigma_y - \tau_{xy}^2 = \sigma_{max}\sigma_{min}$$

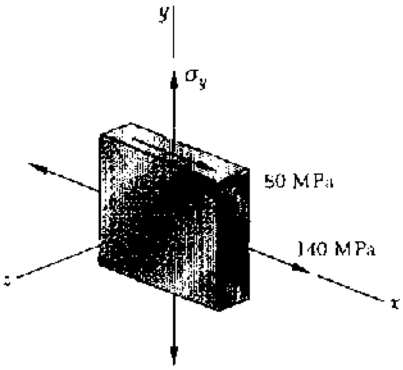
$$\tau_{xy}^2 = \sigma_x\sigma_y - \sigma_{max}\sigma_{min}$$

$$\tau_{xy} = \pm\sqrt{\sigma_x\sigma_y - \sigma_{max}\sigma_{min}}$$

The sign cannot be determined from above equations.

Problem 7.66

7.66 For the state of stress shown, determine the maximum shearing stress when (*a*) $\sigma_y = 20$ MPa, (*b*) $\sigma_y = 140$ MPa. (*Hint*: Consider both in-plane and out-of-plane shearing stresses.)

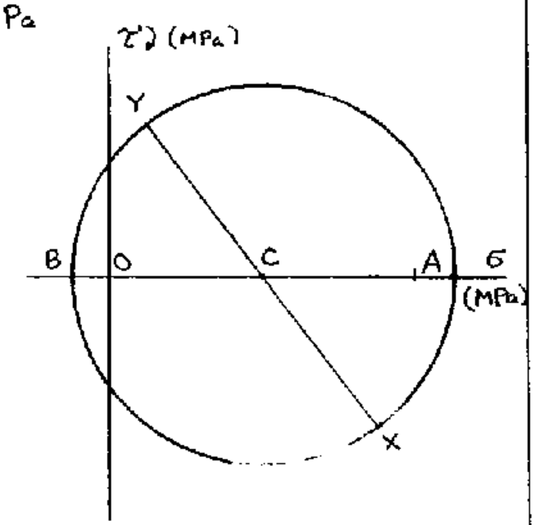


(a) $\sigma_x = 140$ MPa, $\sigma_y = 20$ MPa

$\tau_{xy} = 80$ MPa

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 80 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{60^2 + 80^2} = 100 \text{ MPa}$$



$\sigma_a = \sigma_{ave} + R = 80 + 100 = 180$ MPa (max)

$\sigma_b = \sigma_{ave} - R = 80 - 100 = -20$ MPa (min)

$\sigma_c = 0$

$\tau_{max(in\,plane)} = \frac{1}{2}(\sigma_a - \sigma_b) = 100$ MPa

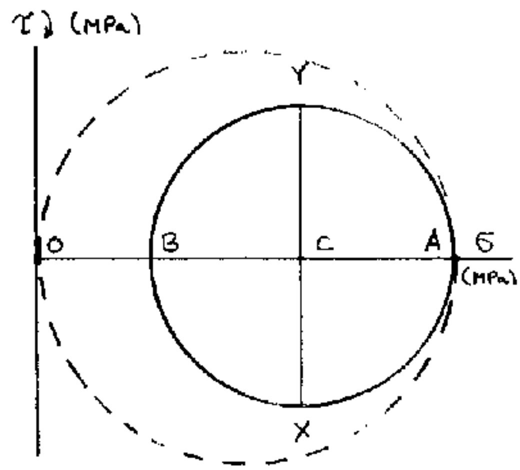
$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 100$ MPa ◄

(b) $\sigma_x = 140$ MPa, $\sigma_y = 140$ MPa

$\tau_{xy} = 80$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 140$ MPa

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{0 + 80^2} = 80 \text{ MPa}$$



$\sigma_a = \sigma_{ave} + R = 220$ MPa (max)

$\sigma_b = \sigma_{ave} - R = 60$ MPa

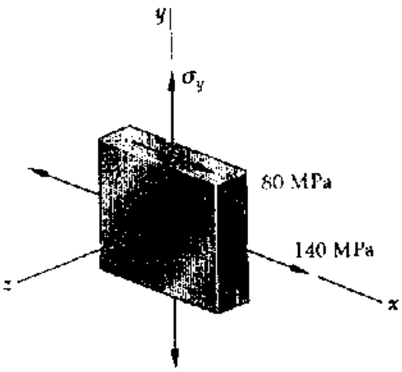
$\sigma_c = 0$ (min)

$\tau_{max(in\,plane)} = \frac{1}{2}(\sigma_a - \sigma_b) = 80$ MPa

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 110$ MPa ◄

Problem 7.67

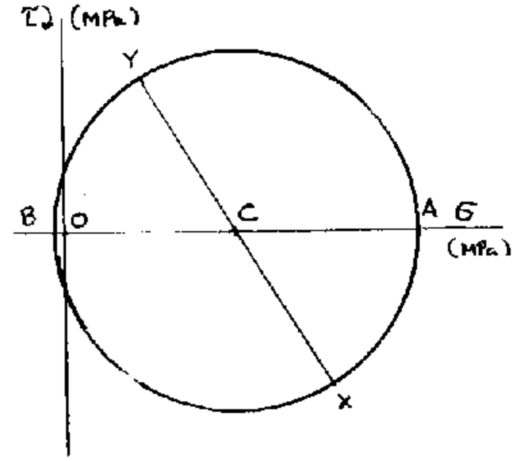
7.67 For the state of stress shown, determine the maximum shearing stress when (*a*) $\sigma_y = 40$ MPa, (*b*) $\sigma_y = 120$ MPa. (*Hint*: Consider both in-plane and out-of-plane shearing stresses.)



(a) $\sigma_x = 140$ MPa $\quad \sigma_y = 40$ MPa $\quad \tau_{xy} = 80$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y)$
$= 90$ MPa

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$
$= \sqrt{50^2 + 80^2}$
$= 94.34$ MPa



$\sigma_a = \sigma_{ave} + R = 184.34$ MPa (max)

$\sigma_b = \sigma_{ave} - R = -4.34$ MPa (min)

$\sigma_c = 0$

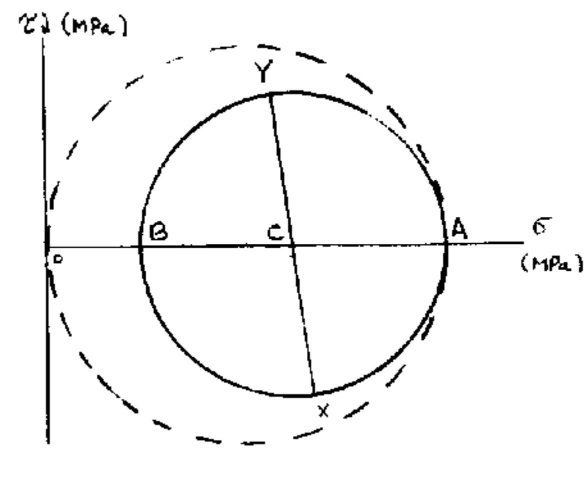
$\tau_{max(in\text{-}plane)} = \frac{1}{2}(\sigma_a - \sigma_b) = R = 94.34$ MPa

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = \frac{1}{2}(\sigma_a - \sigma_b) = 94.34$ MPa ◄

(b) $\sigma_x = 140$ MPa, $\sigma_y = 120$ MPa, $\tau_{xy} = 80$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 130$ MPa

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$
$= \sqrt{10^2 + 80^2} = 80.62$ MPa



$\sigma_a = \sigma_{ave} + R = 210.62$ MPa (max)

$\sigma_b = \sigma_{ave} - R = 49.38$ MPa

$\sigma_c = 0$ (min)

$\sigma_{max} = \sigma_a = 210.62$ MPa

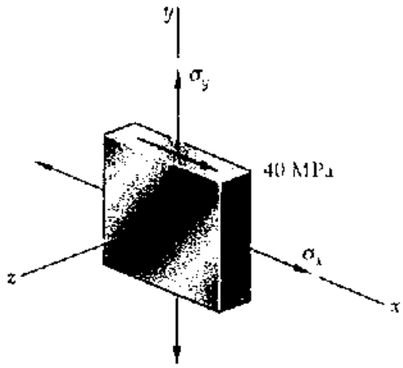
$\sigma_{min} = \sigma_c = 0$

$\tau_{max(in\text{-}plane)} = R = 86.62$ MPa

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 105.31$ MPa ◄

Problem 7.68

7.68 For the state of plane stress shown, determine the maximum shearing stress when (*a*) $\sigma_x = 30$ MPa and $\sigma_y = 90$ MPa, (*b*) $\sigma_x = 70$ MPa and $\sigma_y = 10$ MPa.. (*Hint*: Consider both in-plane and out-of-plane shearing stresses.)



(a) $\sigma_x = 30$ MPa $\quad \sigma_y = 90$ MPa $\quad \tau_{xy} = 40$ MPa

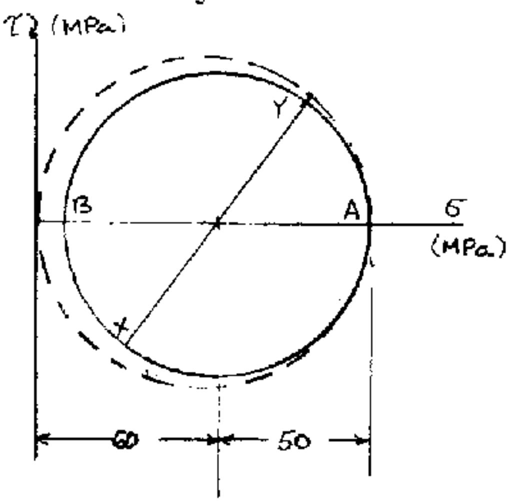
$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y)$

$= 60$ MPa

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{30^2 + 40^2}$

$= 50$ MPa



$\sigma_a = \sigma_{ave} + R = 60 + 50 = 110$ MPa (max)

$\sigma_b = \sigma_{ave} - R = 60 - 50 = 10$ MPa

$\sigma_c = 0$ (min)

$\tau_{max(in\text{-}plane)} = R = 50$ MPa

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 55$ MPa ◄

(b) $\sigma_x = 70$ MPa $\quad \sigma_y = 10$ MPa $\quad \tau_{xy} = 40$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 40$ MPa

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{30^2 + 40^2} = 50$ MPa

$\sigma_a = \sigma_{ave} + R = 90$ MPa (max)

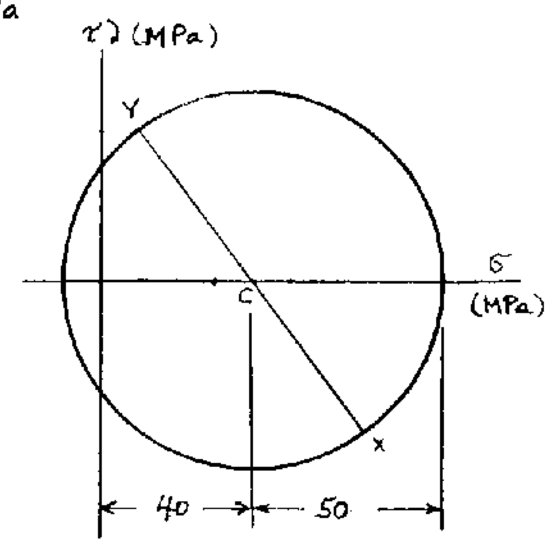
$\sigma_b = \sigma_{ave} - R = -10$ MPa (min)

$\sigma_c = 0$

$\sigma_{max} = 90$ MPa

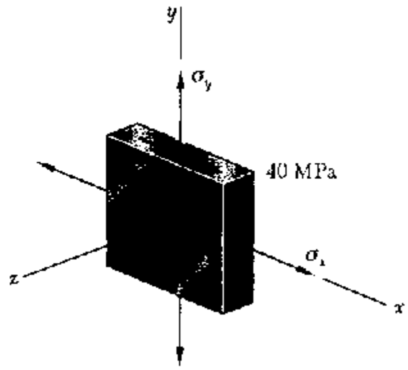
$\sigma_{min} = -10$ MPa

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 50$ MPa ◄



Problem 7.69

7.69 For the state of plane stress shown, determine the maximum shearing stress when (*a*) $\sigma_x = 0$ and $\sigma_y = 60$ MPa, (*b*) $\sigma_x = 105$ MPa and $\sigma_y = 45$ MPa. (*Hint*: Consider both in-plane and out-of-plane shearing stresses.)



(a) $\sigma_x = 0$, $\sigma_y = 60$ MPa, $\tau_{xy} = 40$ MPa

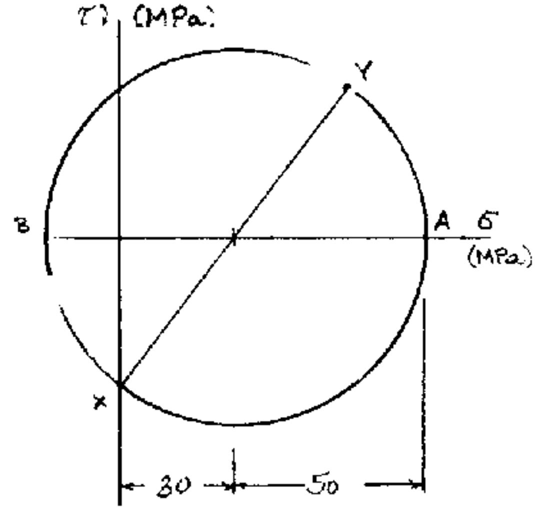
$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y)$

$= 30$ MPa

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{(-30)^2 + 40^2}$

$= 50$ MPa



$\sigma_a = \sigma_{ave} + R = 80$ MPa (max)

$\sigma_b = \sigma_{ave} - R = -20$ MPa (min)

$\sigma_c = 0$

$\sigma_{max} = 80$ MPa $\qquad \sigma_{min} = -20$ MPa

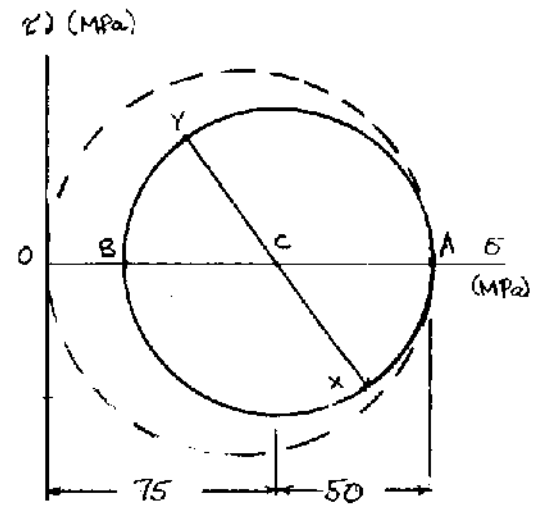
$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 50$ MPa ◄

(b) $\sigma_x = 105$ MPa $\quad \sigma_y = 45$ MPa $\quad \tau_{xy} = 40$ MPa

$\sigma_{ave} = 75$ MPa

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{(-30)^2 + 40^2} = 50$ MPa



$\sigma_a = \sigma_{ave} + R = 125$ MPa (max)

$\sigma_b = \sigma_{ave} - R = 25$ MPa

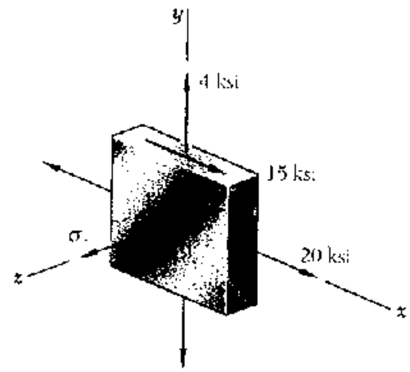
$\sigma_c = 0$ (min)

$\sigma_{max} = 125$ MPa, $\quad \sigma_{min} = 0$

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 62.5$ MPa ◄

Problem 7.70

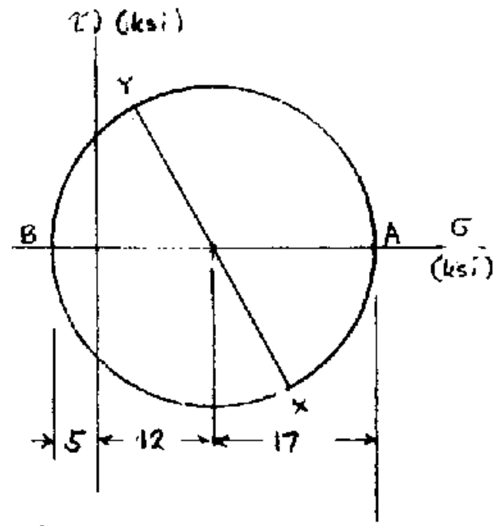
7.70 and 7.71 For the state of stress shown, determine the maximum shearing stress when (a) $\sigma_z = 0$, (b) $\sigma_z = +9$ ksi, (c) $\sigma_z = -9$ ksi.



$\sigma_x = 20$ ksi $\quad \sigma_y = 4$ ksi $\quad \tau_{xy} = 15$ ksi

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y)$
$= 12$ ksi

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$
$= \sqrt{8^2 + 15^2}$
$= 17$ ksi



$\sigma_a = \sigma_{ave} + R = 29$ ksi
$\sigma_b = \sigma_{ave} - R = -5$ ksi

(a) $\sigma_z = 0$, $\quad \sigma_a = 29$ ksi, $\quad \sigma_b = -5$ ksi

$\sigma_{max} = 29$ ksi, $\quad \sigma_{min} = -5$ ksi, $\quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 17$ ksi ◄

(b) $\sigma_z = +9$ ksi, $\quad \sigma_a = 29$ ksi, $\quad \sigma_b = -5$ ksi

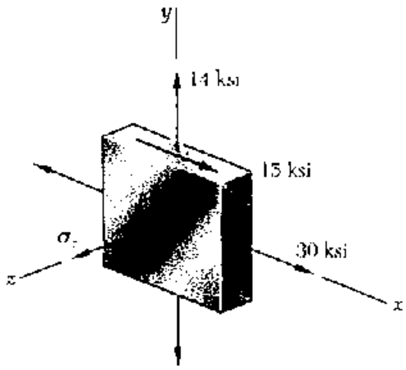
$\sigma_{max} = 29$ ksi, $\quad \sigma_{min} = -5$ ksi, $\quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 17$ ksi ◄

(c) $\sigma_z = -9$ ksi, $\quad \sigma_a = 29$ ksi, $\quad \sigma_b = -5$ ksi

$\sigma_{max} = 29$ ksi, $\quad \sigma_{min} = -9$ ksi, $\quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 19$ ksi ◄

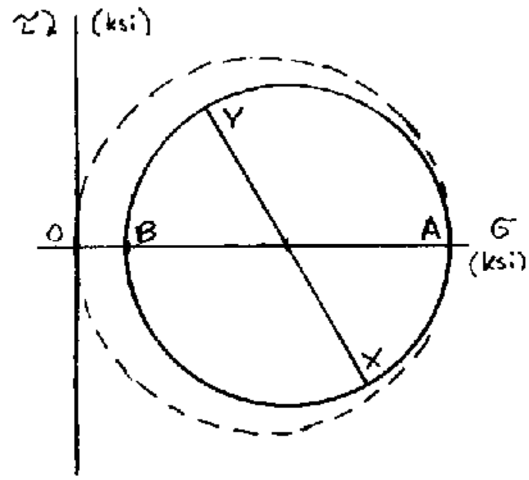
Problem 7.71

7.70 and 7.71 For the state of stress shown, determine the maximum shearing stress when (*a*) $\sigma_z = 0$, (*b*) $\sigma_z = +9$ ksi, (*c*) $\sigma_z = -9$ ksi.



$\sigma_x = 30$ ksi, $\sigma_y = 14$ ksi, $\tau_{xy} = 15$ ksi

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 22 \text{ ksi}$$



$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{30-14}{2}\right)^2 + 15^2} = 17 \text{ ksi}$$

$\sigma_a = \sigma_{ave} + R = 39$ ksi

$\sigma_b = \sigma_{ave} - R = 5$ ksi

(a) $\sigma_z = 0$, $\sigma_a = 39$ ksi, $\sigma_b = 5$ ksi

$\sigma_{max} = 39$ ksi, $\sigma_{min} = 0$, $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 19.5$ ksi ◄

(b) $\sigma_z = 9$ ksi, $\sigma_a = 39$ ksi, $\sigma_b = 5$ ksi

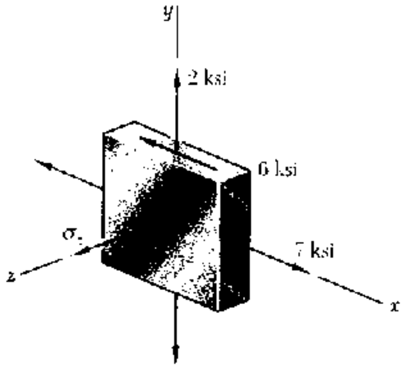
$\sigma_{max} = 39$ ksi, $\sigma_{min} = 5$ ksi, $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 17$ ksi ◄

(c) $\sigma_z = -9$ ksi, $\sigma_a = 39$ ksi, $\sigma_b = 5$ ksi

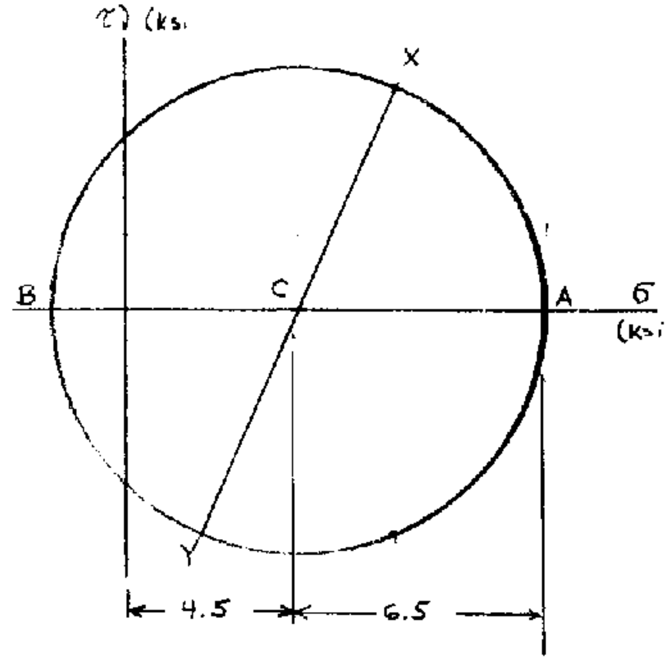
$\sigma_{max} = 39$ ksi, $\sigma_{min} = -9$ ksi, $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 24$ ksi ◄

Problem 7.72

7.72 and 7.73 For the state of stress shown, determine the maximum shearing stress when (*a*) $\sigma_z = +4$ ksi, (*b*) $\sigma_z = -4$ ksi, (c) $\sigma_z = 0$.



$\sigma_x = 7$ ksi , $\sigma_y = 2$ ksi , $\tau_{xy} = -6$ ksi



$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 4.5$ ksi

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{2.5^2 + (-6)^2} = 6.5$ ksi

$\sigma_a = \sigma_{ave} + R = 11$ ksi

$\sigma_b = \sigma_{ave} - R = -2$ ksi

(a) $\sigma_z = 4$ ksi , $\sigma_a = 11$ ksi , $\sigma_b = -2$ ksi

$\sigma_{max} = 11$ ksi , $\sigma_{min} = -2$ ksi , $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 6.5$ ksi

(b) $\sigma_z = -4$ ksi , $\sigma_a = 11$ ksi , $\sigma_b = -2$ ksi

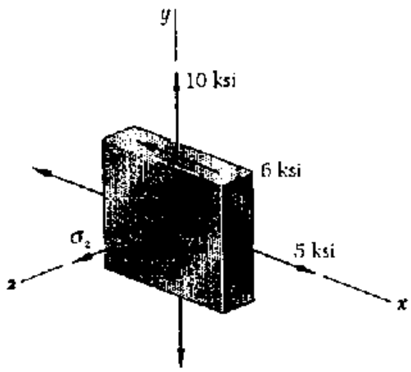
$\sigma_{max} = 11$ ksi , $\sigma_{min} = -4$ ksi , $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 7.5$ ksi

(c) $\sigma_z = 0$, $\sigma_a = 11$ ksi , $\sigma_b = -2$ ksi

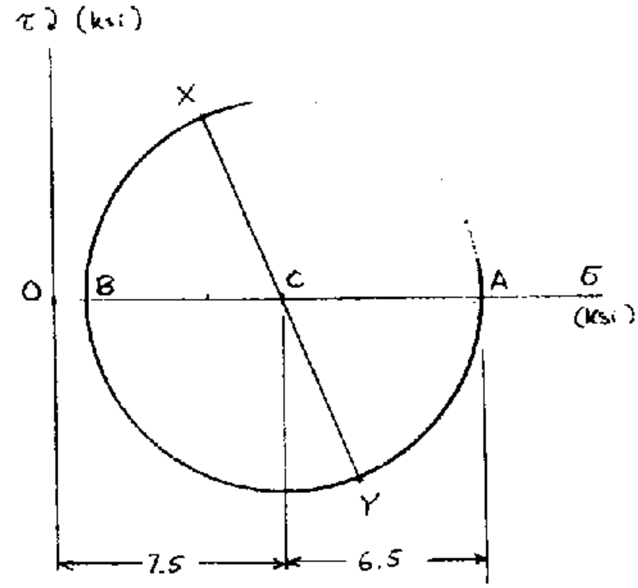
$\sigma_{max} = 11$ ksi , $\sigma_{min} = -2$ ksi , $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 6.5$ ksi

Problem 7.73

7.72 and 7.73 For the state of stress shown, determine the maximum shearing stress when (*a*) $\sigma_z = +4$ ksi, (*b*) $\sigma_z = -4$ ksi, (c) $\sigma_z = 0$.



$\sigma_x = 5$ ksi, $\sigma_y = 10$ ksi, $\tau_{xy} = -6$ ksi



$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 7.5$ ksi

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{(-2.5)^2 + (-6)^2} = 6.5$ ksi

$\sigma_a = \sigma_{ave} + R = 14$ ksi

$\sigma_b = \sigma_{ave} - R = 1$ ksi

(a) $\sigma_z = +4$ ksi, $\sigma_a = 14$ ksi, $\sigma_b = 1$ ksi

$\sigma_{max} = 14$ ksi, $\sigma_{min} = 1$ ksi, $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 6.5$ ksi ◄

(b) $\sigma_z = -4$ ksi, $\sigma_a = 14$ ksi, $\sigma_b = 1$ ksi

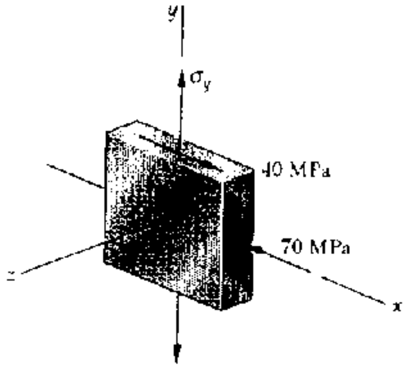
$\sigma_{max} = 14$ ksi, $\sigma_{min} = -4$ ksi, $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 9$ ksi ◄

(c) $\sigma_z = 0$, $\sigma_a = 14$ ksi, $\sigma_b = 1$ ksi

$\sigma_{max} = 14$ ksi, $\sigma_{min} = 0$, $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 7$ ksi ◄

Problem 7.74

7.74 For the state of stress shown, determine two values of σ_y for which the maximum shearing stress is 75 MPa.



$\sigma_x = -70$ MPa, $\tau_{xy} = 40$ MPa

Let $u = \frac{\sigma_y - \sigma_x}{2}$ $\qquad \sigma_y = 2u + \sigma_x$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \sigma_x + u$

$R = \sqrt{u^2 + \tau_{xy}^2}$ $\qquad u = \pm\sqrt{R^2 - \tau_{xy}^2}$

Case 1 $\quad \tau_{max} = R = 75$ MPa, $\quad u = \pm\sqrt{75^2 - 40^2} = \pm 63.44$ MPa

(1a) $u = +63.44$ MPa $\quad \sigma_y = 2u + \sigma_x = 56.88$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -6.56$ MPa

$\sigma_a = \sigma_{ave} + R = 68.44$ MPa, $\quad \sigma_b = \sigma_{ave} - R = -81.56$ MPa

$\sigma_c = 0 \quad \sigma_{max} = 68.44$ MPa, $\sigma_{min} = -81.56$ MPa $\quad \tau_{max} = 75$ MPa

(1b) $u = -63.44$ MPa $\quad \sigma_y = 2u + \sigma_x = -196.88$ MPa (reject)

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -133.44$ MPa $\qquad \sigma_a = \sigma_{ave} + R = -58.44$ MPa

$\sigma_b = \sigma_{ave} - R = -208.44$ MPa, $\quad \sigma_c = 0$, $\quad \sigma_{max} = 0$

$\sigma_{min} = -208.44$ MPa, $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 104.22$ MPa $\neq 75$ MPa

Case (2) Assume $\sigma_{max} = 0$, $\quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 75$ MPa

$\sigma_{min} = -150$ MPa $= \sigma_b$

$\sigma_b = \sigma_{ave} - R = \sigma_x + u - \sqrt{u^2 + \tau_{xy}^2}$

$\sqrt{u^2 + \tau_{xy}^2} = \sigma_x + u - \sigma_b$

$\cancel{u^2} + \tau_{xy}^2 = (\sigma_x - \sigma_b)^2 + 2(\sigma_x - \sigma_b)u + \cancel{u^2}$

$2u = \frac{\tau_{xy}^2 - (\sigma_x - \sigma_b)^2}{\sigma_x - \sigma_b} \qquad \frac{(40)^2 - (-70 + 150)^2}{-70 + 150} = -60$ MPa

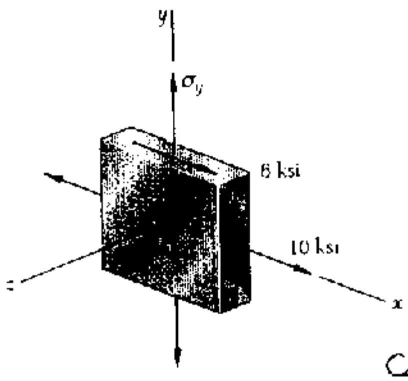
$u = -30$ MPa $\qquad \sigma_y = 2u + \sigma_x = -130$ MPa

$R = \sqrt{u^2 + \tau_{xy}^2} = 50$ MPa

$\sigma_a = \sigma_b + 2R = -150 + 100 = -50$ MPa $\qquad$ O.K.

Problem 7.75

7.75 For the state of stress shown, determine two values of σ_y for which the maximum shearing stress is 7.5 ksi.



$\sigma_x = 10$ ksi, $\tau_{xy} = 6$ ksi, $\tau_{max} = 7.5$ ksi

Let $u = \dfrac{\sigma_y - \sigma_x}{2}$ $\quad \sigma_y = 2u + \sigma_x$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \sigma_x + u$

$R = \sqrt{u^2 + \tau_{xy}^2}$ $\quad u = \pm\sqrt{R^2 - \tau_{xy}^2}$

Case 1 $\quad \tau_{max} = R = 7.5$ ksi, $u = \pm 4.5$ ksi

(1a) $u = +4.5$ ksi $\quad \sigma_y = 2u + \sigma_x = 19$ ksi $\quad$ reject

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 14.5$ ksi, $\sigma_a = \sigma_{ave} + R = 22$ ksi, $\sigma_b = \sigma_{ave} - R = 7$ ksi

$\sigma_{max} = 22$ ksi, $\sigma_{min} = 0$, $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 11$ ksi $\neq 7.5$ ksi

(1b) $u = -4.5$ ksi $\quad \sigma_y = 2u + \sigma_x = 1$ ksi ◄

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 5.5$ ksi, $\sigma_a = \sigma_{ave} + R = 13$ ksi, $\sigma_b = \sigma_{ave} - R = -2$ ksi

$\sigma_{max} = 13$ ksi, $\sigma_{min} = -2$ ksi, $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 7.5$ ksi $\quad$ O.K.

Case 2 $\quad$ Assume $\sigma_{min} = 0$ $\quad \sigma_{max} = 2\tau_{max} = 15$ ksi $= \sigma_a$

$\sigma_a = \sigma_{ave} + R = \sigma_x + u + \sqrt{u^2 + \tau_{xy}^2}$

$\sigma_a - \sigma_x - u = \sqrt{u^2 + \tau_{xy}^2}$

$(\sigma_a - \sigma_x - u)^2 = u^2 + \tau_{xy}^2$

$(\sigma_a - \sigma_x)^2 - 2(\sigma_a - \sigma_x)u + \cancel{u^2} = \cancel{u^2} + \tau_{xy}^2$

$2u = \dfrac{(\sigma_a - \sigma_x)^2 - \tau_{xy}^2}{\sigma_a - \sigma_x} = \dfrac{(15 - 10)^2 - 6^2}{15 - 10} = -2.2$ ksi

$u = -1.1$ ksi $\quad \sigma_y = 2u + \sigma_x = 7.8$ ksi ◄

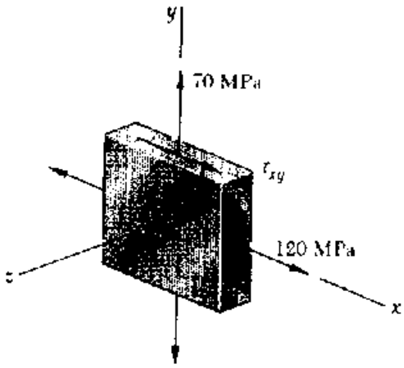
$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 8.9$ ksi $\quad R = \sqrt{u^2 + \tau_{xy}^2} = 6.1$ ksi

$\sigma_a = \sigma_{ave} + R = 15$ ksi ✓ $\quad \sigma_b = \sigma_{ave} - R = 2.8$ ksi

$\sigma_{max} = 15$ ksi, $\sigma_{min} = 0$ $\quad \tau_{max} = 7.5$ ksi ✓

Problem 7.76

7.76 For the state of stress shown, determine the value of τ_{xy} for which the maximum shearing stress is 80 MPa.



$\sigma_x = 120$ MPa $\quad \sigma_y = 70$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 95$ MPa

$\frac{\sigma_x - \sigma_y}{2} = \frac{120 - 70}{2} = 25$ MPa

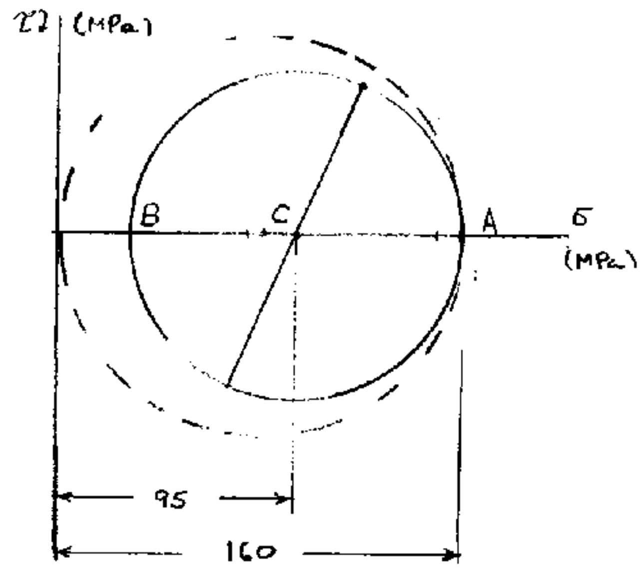
Assume $\quad \sigma_{min} = 0 \quad \sigma_{max} = 2\tau_{max} = 160$ MPa

$\sigma_a = \sigma_{max} = \sigma_{ave} + R \qquad R = \sigma_{max} - \sigma_{ave} = 160 - 95 = 65$ MPa

$R^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2 \qquad \tau_{xy}^2 = R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 = 65^2 - 25^2 = 60^2$

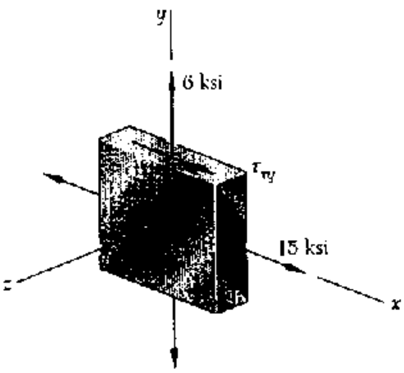
$\tau_{xy} = \pm 60$ ksi ◄

$\sigma_b = \sigma_a - 2R = 160 - 130 = 30$ MPa $\geq 0 \quad$ O.K.



Problem 7.77

7.77 For the state of stress shown, determine the value of τ_{xy} for which the maximum shearing stress is (*a*) 9 ksi, (*b*) 12 ksi.



$\sigma_x = 15$ ksi $\quad \sigma_y = 6$ ksi

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 10.5$ ksi

$u = \frac{\sigma_x - \sigma_y}{2} = 4.5$ ksi

(a) For $\tau_{max} = 9$ ksi

Center of Mohrs circle lies at point C. Lines marked (a) show the limits on τ_{max}. Limit on σ_{max} is $\sigma_{max} = 2\tau_{max} = 18$ ksi. For the Mohr's circle $\sigma_a = \sigma_{max}$ corresponds to point A_a.

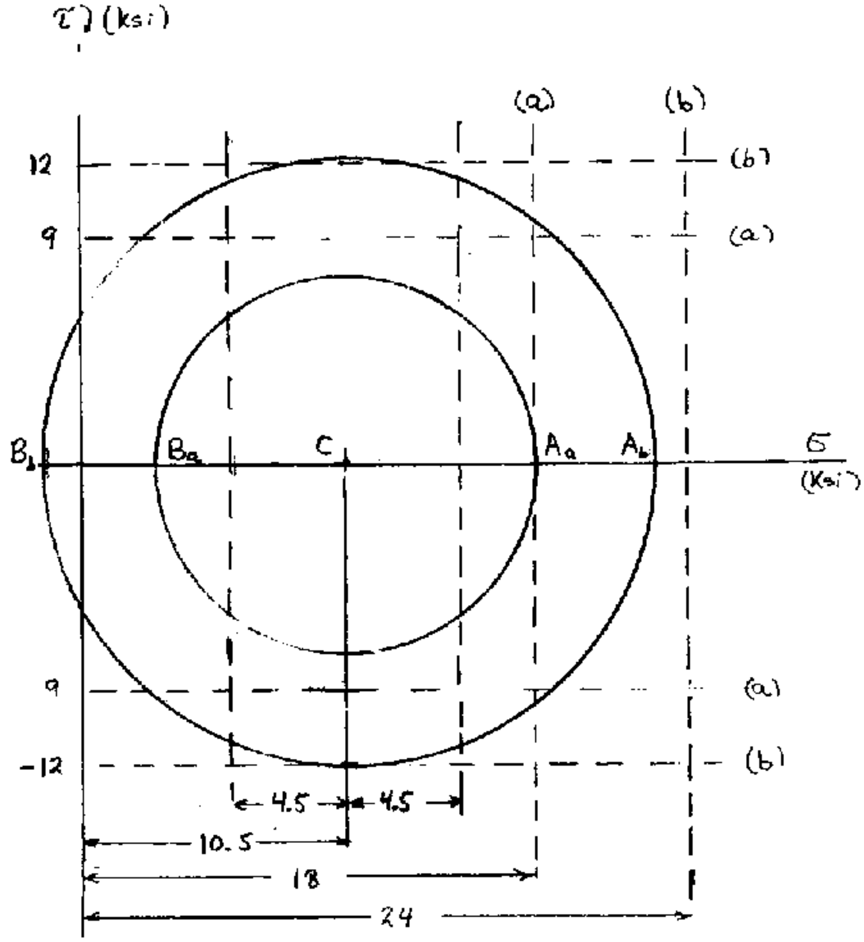
$R = \sigma_a - \sigma_{ave}$
$= 18 - 10.5 = 7.5$ ksi

$R = \sqrt{u^2 + \tau_{xy}^2}$

$\tau_{xy} = \pm\sqrt{R^2 - u^2}$

$= \pm\sqrt{7.5^2 - 4.5^2}$

$= \pm\ 6$ ksi ◀



(b) For $\tau_{max} = 12$ ksi.

Center of Mohr's circle lies at point C. $\quad R = 12$ ksi

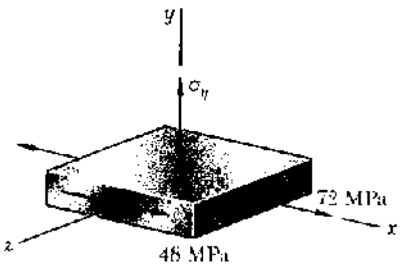
$\tau_{xy} = \pm\sqrt{R^2 - u^2} = \pm\ 11.24$ ksi ◀

Checking $\quad \sigma_a = 10.5 + 12 = 22.5$ ksi $\quad \sigma_b = 10.5 - 12 = -1.5$ ksi
$\sigma_c = 0$

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 12$ ksi $\quad$ O.K

Problem 7.78

7.78 For the state of stress shown, determine two values of σ_y for which the maximum shearing stress is 64 MPa.



$\sigma_x = 72$ MPa $\quad \sigma_z = 0 \quad \tau_{xz} = 48$ MPa

Mohr's circle for stresses in zx-plane

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_z) = 36$ MPa

$R = \sqrt{\left(\frac{\sigma_z - \sigma_x}{2}\right)^2 + \tau_{xz}^2} = \sqrt{36^2 + 48^2} = 60$ MPa

$\sigma_a = \sigma_{ave} + R = 96$ MPa, $\quad \sigma_b = \sigma_{ave} - R = -24$ MPa

Assume $\sigma_{max} = \sigma_a = 96$ MPa

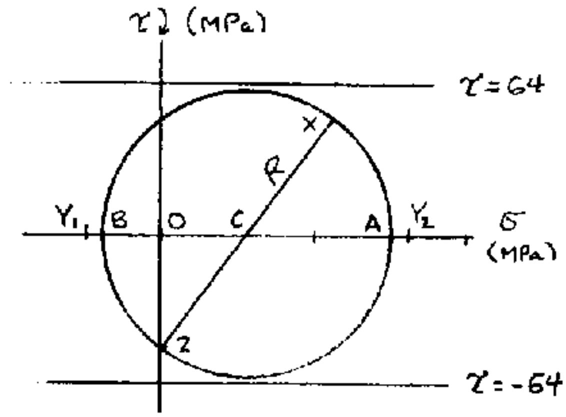
$\sigma_y = \sigma_{min} = \sigma_{max} - 2\tau_{max}$

$= 96 - (2)(64) = -32$ MPa ◄

Assume $\sigma_{min} = \sigma_b = -24$ MPa

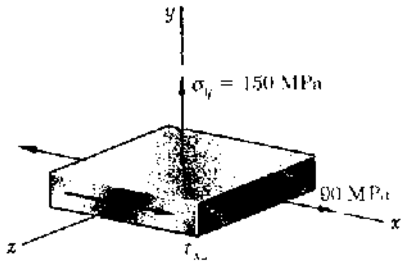
$\sigma_y = \sigma_{max} = \sigma_{min} + 2\tau_{max}$

$= -24 + (2)(64) = 104$ MPa ◄



Problem 7.79

7.79 For the state of stress shown, determine the range of values of τ_{xz} for which the maximum shearing stress is equal to or less than 90 MPa.



$\sigma_x = 90$ MPa, $\sigma_z = 0$, $\sigma_y = 150$ MPa

For Mohr's circle of stresses in zx-plane

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_z) = 45$ MPa

$U = \frac{\sigma_x - \sigma_z}{2} = 45$

Assume $\sigma_{max} = \sigma_y = 150$ MPa

$\sigma_{min} = \sigma_b = \sigma_{max} - 2\tau_{max}$
$= 150 - (2)(90) = -30$ MPa

$R = \sigma_{ave} - \sigma_b$
$= 45 - (-30) = 75$ MPa

$\sigma_a = \sigma_{ave} + R$
$= 45 + 75 = 120$ MPa $< \sigma_y$

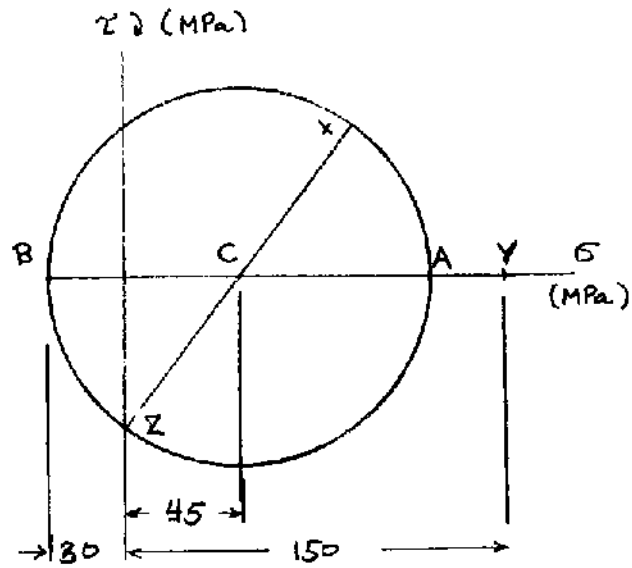
o.k.

$R = \sqrt{U^2 + \tau_{xz}^2}$

$\tau_{xz} = \pm\sqrt{R^2 - U^2}$

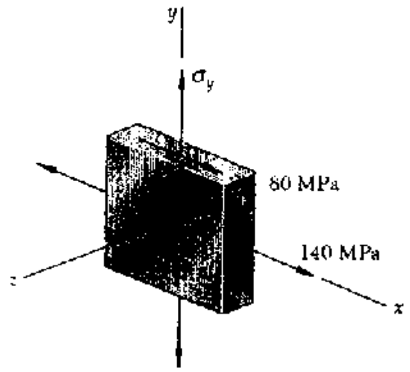
$= \pm\sqrt{75^2 - 45^2} = \pm 60$ MPa

$-60 \text{ MPa} \leq \tau_{xz} \leq 60 \text{ MPa}$ ◄



Problem 7.80

***7.80** For the state of stress of Prob 7.66, determine (*a*) the value of σ_y for which the maximum shearing stress is as small as possible, (*b*) the corresponding value of the shearing stress.



Let $\quad u = \frac{\sigma_x - \sigma_y}{2} \qquad \sigma_y = \sigma_x - 2u$

$$\sigma_{ave} = \tfrac{1}{2}(\sigma_x + \sigma_y) = \sigma_x - u$$

$$R = \sqrt{u^2 + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{ave} + R = \sigma_x - u + \sqrt{u^2 + \tau_{xy}^2}$$

$$\sigma_b = \sigma_{ave} - R = \sigma_x - u - \sqrt{u^2 + \tau_{xy}^2}$$

Assume τ_{max} is the in-plane shearing stress $\qquad \tau_{max} = R$

Then $\tau_{max(in\text{-}plane)}$ is minimum if $u = 0$

$$\sigma_y = \sigma_x - 2u = \sigma_x = 140 \text{ MPa}, \qquad \sigma_{ave} = \sigma_x - u = 140 \text{ MPa}$$

$$R = |\tau_{xy}| = 80 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 140 + 80 = 220 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = 140 - 80 = 60 \text{ MPa}$$

$$\sigma_{max} = 220 \text{ MPa}, \quad \sigma_{min} = 0, \quad \tau_{max} = \tfrac{1}{2}(\sigma_{max} - \sigma_{min}) = 110 \text{ MPa}$$

Assumption is incorrect

Assume $\sigma_{max} = \sigma_a = \sigma_{ave} + R = \sigma_x - u + \sqrt{u^2 + \tau_{xy}^2}$

$$\sigma_{min} = 0 \qquad \tau_{max} = \tfrac{1}{2}(\sigma_{max} - \sigma_{min}) = \tfrac{1}{2}\sigma_a$$

$$\frac{d\sigma_a}{du} = -1 + \frac{u}{\sqrt{u^2 + \tau_{xy}^2}} \neq 0 \qquad \text{(no minimum)}$$

Optimum value for u occurs when $\tau_{max(out\text{-}of\text{-}plane)} = \tau_{max(in\text{-}plane)}$

$$\therefore \tfrac{1}{2}(\sigma_a + R) = R \quad \text{or} \quad \sigma_a = R \quad \text{or} \quad \sigma_x - u = \sqrt{u^2 + \tau_{xy}^2}$$

$$(\sigma_x - u)^2 = \sigma_x^2 - 2u\sigma_x + \cancel{u^2} = \cancel{u^2} + \tau_{xy}^2$$

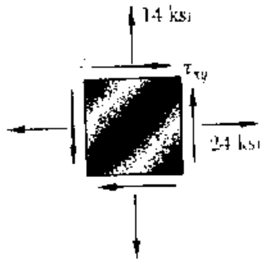
$$2u = \frac{\sigma_x^2 - \tau_{xy}^2}{\sigma_x} = \frac{140^2 - 80^2}{140} = 94.3 \text{ MPa} \qquad u = 47.14 \text{ MPa}$$

(a) $\quad \sigma_y = \sigma_x - 2u = 140 - 94.3 = 45.7 \text{ MPa}$ ◄

(b) $\quad R = \sqrt{u^2 + \tau_{xy}^2} = \tau_{max} = 92.9 \text{ MPa}$ ◄

Problem 7.81

7.81 The state of plane stress shown occurs in a machine component made of a steel with $\sigma_Y = 30$ ksi. Using the maximum-distortion-energy criterion, determine whether yield occurs when (*a*) $\tau_{xy} = 6$ ksi, (*b*) $\tau_{xy} = 12$ ksi, (*c*) $\tau_{xy} = 14$ ksi. If yield does not occur, determine the corresponding factor of safety.



$\sigma_x = 24$ ksi $\qquad \sigma_y = 14$ ksi $\qquad \sigma_z = 0$

For stresses in xy-plane $\quad \sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 19$ ksi

$\frac{\sigma_x - \sigma_y}{2} = 5$ ksi

(a) $\tau_{xy} = 6$ ksi $\qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(5)^2 + (6)^2} = 7.810$ ksi

$\sigma_a = \sigma_{ave} + R = 26.810$ ksi, $\qquad \sigma_b = \sigma_{ave} - R = 11.190$ ksi

$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 23.324$ ksi < 30 ksi (No yielding)

$F.S. = \frac{30}{23.324} = 1.286$

(b) $\tau_{xy} = 12$ ksi $\qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(5)^2 + (12)^2} = 13$ ksi

$\sigma_a = \sigma_{ave} + R = 32$ ksi, $\qquad \sigma_b = \sigma_{ave} - R = 6$ ksi

$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 29.462$ ksi < 30 ksi (No yielding)

$F.S. = \frac{30}{29.462} = 1.018$

(c) $\tau_{xy} = 14$ ksi $\qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(5)^2 + (14)^2} = 14.866$ ksi

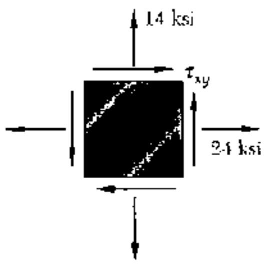
$\sigma_a = \sigma_{ave} + R = 33.866, \qquad \sigma_b = \sigma_{ave} - R = 4.134$ ksi

$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 32.00$ ksi > 30 ksi (Yielding occurs)

Problem 7.82

7.82 Solve Prob. 7.81, using the maximum-shearing-stress criterion.

7.81 The state of plane stress shown occurs in a machine component made of a steel with $\sigma_Y = 30$ ksi. Using the maximum-distortion-energy criterion, determine whether yield occurs when (*a*) $\tau_{xy} = 6$ ksi, (*b*) $\tau_{xy} = 12$ ksi, (*c*) $\tau_{xy} = 14$ ksi. If yield does not occur, determine the corresponding factor of safety.



$\sigma_x = 24$ ksi $\quad \sigma_y = 14$ ksi $\quad \sigma_z = 0$

For stresses in xy-plane $\quad \sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 19$ ksi

$\frac{\sigma_x - \sigma_y}{2} = 5$ ksi

(a) $\tau_{xy} = 6$ ksi $\quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 7.810$ ksi

$\sigma_a = \sigma_{ave} + R = 26.810$ ksi, $\quad \sigma_b = \sigma_{ave} - R = 11.190$ ksi

$\sigma_{max} = 26.810$ ksi, $\quad \sigma_{min} = 0$

$2\tau_{max} = \sigma_{max} - \sigma_{min} = 26.810$ ksi < 30 ksi $\quad$ (No yielding)

$F.S. = \frac{30}{26.810} = 1.119$ ◄

(b) $\tau_{xy} = 12$ ksi $\quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 13$ ksi

$\sigma_a = \sigma_{ave} + R = 32$ ksi, $\quad \sigma_b = \sigma_{ave} - R = 6$ ksi

$\sigma_{max} = 32$ ksi $\quad \sigma_{min} = 0$

$2\tau_{max} = \sigma_{max} - \sigma_{min} = 32$ ksi > 30 ksi $\quad$ (Yielding occurs)

(c) $\tau_{xy} = 14$ ksi $\quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 14.866$ ksi

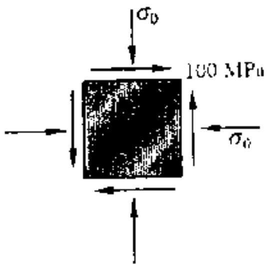
$\sigma_a = \sigma_{ave} + R = 33.866$ ksi $\quad \sigma_b = \sigma_{ave} - R = 4.134$ ksi

$\sigma_{max} = 33.866$ ksi $\quad \sigma_{min} = 0$

$2\tau_{max} = \sigma_{max} - \sigma_{min} = 33.866$ ksi > 30 ksi $\quad$ (Yielding occurs)

Problem 7.83

7.83 The state of plane stress shown occurs in a machine component made of a steel with $\sigma_Y = 325$ MPa. Using the maximum-distortion-energy criterion, determine whether yield will occur when (*a*) $\sigma_0 = 200$ MPa, (*b*) $\sigma_0 = 240$ MPa, (*c*) $\sigma_0 = 280$ MPa. If yield does not occur, determine the corresponding factor of safety.



$$\sigma_{ave} = -\sigma_0 \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 100 \text{ MPa}$$

(a) $\sigma_0 = 200$ MPa $\qquad \sigma_{ave} = -200$ MPa

$\sigma_a = \sigma_{ave} + R = -100$ MPa, $\qquad \sigma_b = \sigma_{ave} - R = -300$ MPa

$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 264.56$ MPa < 325 MPa (No yielding)

$$F.S. = \frac{325}{264.56} = 1.228$$

(b) $\sigma_0 = 240$ MPa $\qquad \sigma_{ave} = -240$ MPa

$\sigma_a = \sigma_{ave} + R = -140$ MPa, $\qquad \sigma_b = \sigma_{ave} - R = -340$ MPa

$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 295.97$ MPa < 325 MPa (No yielding)

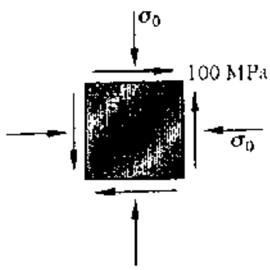
$$F.S. = \frac{325}{295.97} = 1.098$$

(c) $\sigma_0 = 280$ MPa $\qquad \sigma_{ave} = -280$ MPa

$\sigma_a = \sigma_{ave} + R = -180$ MPa, $\qquad \sigma_b = \sigma_{ave} - R = -380$ MPa

$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 329.24$ MPa > 325 MPa (Yielding occurs)

Problem 7.84



7.84 Solve Prob. 7.83, using the maximum-shearing-stress criterion.

7.83 The state of plane stress shown occurs in a machine component made of a steel with $\sigma_Y = 325$ MPa. Using the maximum-distortion-energy criterion, determine whether yield will occur when (*a*) $\sigma_0 = 200$ MPa, (*b*) $\sigma_0 = 240$ MPa, (*c*) $\sigma_0 = 280$ MPa. If yield does not occur, determine the corresponding factor of safety.

$\sigma_{ave} = -\sigma_0 \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 100 \text{ MPa}$

(a) $\sigma_0 = 200$ MPa, $\quad \sigma_{ave} = -200$ MPa

$\sigma_a = \sigma_{ave} + R = -100$ MPa $\qquad \sigma_b = \sigma_{ave} - R = -300$ MPa

$\sigma_{max} = 0$, $\quad \sigma_{min} = -300$ MPa

$2\tau_{max} = \sigma_{max} - \sigma_{min} = 300 \text{ MPa} < 325 \text{ MPa}$ (No yielding)

$$F.S. = \frac{325}{300} = 1.083$$

(b) $\sigma_0 = 240$ MPa, $\quad \sigma_{ave} = -240$ MPa

$\sigma_a = \sigma_{ave} + R = -140$ MPa , $\quad \sigma_b = \sigma_{ave} - R = -340$ MPa

$\sigma_{max} = 0$, $\quad \sigma_{min} = -340$ MPa

$2\tau_{max} = \sigma_{max} - \sigma_{min} = 340 \text{ MPa} > 325 \text{ MPa}$ (Yielding occurs)

(c) $\sigma_0 = 280$ MPa, $\quad \sigma_{ave} = -280$ MPa

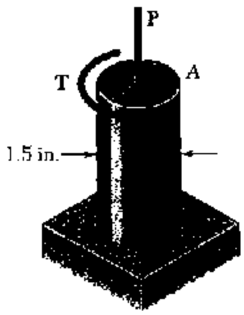
$\sigma_a = \sigma_{ave} + R = -180$ MPa , $\quad \sigma_b = \sigma_{ave} - R = -380$ MPa

$\sigma_{max} = 0$, $\quad \sigma_{min} = -380$ MPa

$2\tau_{max} = \sigma_{max} - \sigma_{min} = 380 \text{ MPa} > 325 \text{ MPa}$ (Yielding occurs)

Problem 7.85

7.85 The 1.5-in.-diameter shaft *AB* is made of a grade of steel for which the yield strength is σ_Y is 42 ksi. Using the maximum-shearing-stress criterion, determine the magnitude of the torque **T** for which yield occurs when $P = 60$ kips.



$P = 60 \text{ kips} \qquad A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(1.5)^2 = 1.7671 \text{ in}^2$

$\sigma_x = -\frac{P}{A} = -\frac{60}{1.7671} = -33.953 \text{ ksi}$

$\sigma_y = 0 \qquad \sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}\sigma_x$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2}$

$2\tau_{max} = 2R = \sqrt{\sigma_x^2 + 4\tau_{xy}^2} = \sigma_Y$

$4\tau_{xy}^2 = \sigma_Y^2 - \sigma_x^2 \qquad \tau_{xy} = \frac{1}{2}\sqrt{\sigma_Y^2 - \sigma_x^2} = \frac{1}{2}\sqrt{42^2 - 33.953^2}$

$= 12.361 \text{ ksi}$

From torsion $\quad \tau_{xy} = \frac{Tc}{J} \qquad T = \frac{J\tau_{xy}}{c}$

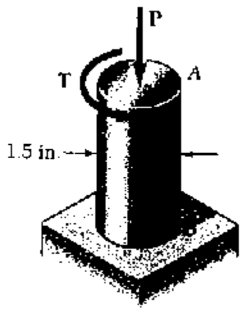
$c = \frac{1}{2}d = 0.75 \text{ in} \qquad J = \frac{\pi}{2}c^4 = 0.49701 \text{ in}^4$

$T = \frac{(0.49701)(12.361)}{0.75} = 8.19 \text{ kip}\cdot\text{in}$

Problem 7.86

7.86 Solve Prob. 7.85, using the maximum-distortion-energy criterion.

7.85 The 1.5-in.-diameter shaft *AB* is made of a grade of steel for which the yield strength is σ_Y is 42 ksi. Using the maximum-shearing-stress criterion, determine the magnitude of the torque T for which yield occurs when $P = 60$ kips.



$P = 60$ kips $\quad A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(1.5)^2 = 1.7671 \text{ in}^2$

$\sigma_x = -\frac{P}{A} = -\frac{60}{1.7671} = -33.953$ ksi

$\sigma_y = 0 \qquad \sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}\sigma_x$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2}$

$\sigma_a = \sigma_{ave} + R \qquad \sigma_b = \sigma_{ave} - R$

$\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b = (\sigma_{ave} + R)^2 + (\sigma_{ave} - R)^2 - (\sigma_{ave} + R)(\sigma_{ave} - R)$

$= \sigma_{ave}^2 + \cancel{2\sigma_{ave}R} + R^2 + \sigma_{ave}^2 - \cancel{2\sigma_{ave}R} + R^2 - \sigma_{ave}^2 + R^2$

$= \sigma_{ave}^2 + 3R^2$

$= \frac{1}{4}\sigma_x^2 + 3\left(\frac{1}{4}\sigma_x^2 + \tau_{xy}^2\right) = \sigma_x^2 + 3\tau_{xy}^2 = \sigma_Y^2$

$3\tau_{xy}^2 = \sigma_Y^2 - \sigma_x^2 \qquad \tau_{xy} = \frac{1}{\sqrt{3}}(\sigma_Y^2 - \sigma_x^2) = \frac{1}{\sqrt{3}}\sqrt{42^2 - 33.953^2}$

$= 14.273$ ksi

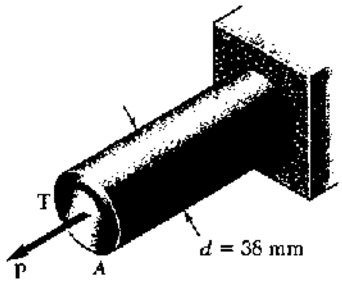
From torsion $\quad \tau_{xy} = \frac{Tc}{J} \qquad T = \frac{J\tau_{xy}}{c}$

$c = \frac{1}{2}d = 0.75$ in. $\qquad J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.75)^4 = 0.49701 \text{ in}^4$

$T = \frac{(0.49701)(14.273)}{0.75} = 9.46$ kip·in ◄

Problem 7.87

7.87 The 38-mm-diameter shaft AB is made of a grade of steel for which the yield strength is $\sigma_Y = 250$ MPa. Using the maximum-shearing-stress criterion, determine the magnitude of the torque T for which yield occurs when $P = 240$ kN.



$P = 240 \times 10^3$ N

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(38)^2 = 1.1341 \times 10^3 \text{ mm}^2 = 1.1341 \times 10^{-3} \text{ m}^2$

$\sigma_x = \frac{P}{A} = \frac{240 \times 10^3}{1.1341 \times 10^{-3}} = 211.6 \times 10^6 \text{ Pa} = 211.6 \text{ MPa}$

$\sigma_y = 0 \qquad \sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}\sigma_x$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2}$

$2\tau_{max} = 2R = \sqrt{\sigma_x^2 + 4\tau_{xy}^2} = \sigma_Y$

$4\tau_{xy}^2 = \sigma_Y^2 - \sigma_x^2 \qquad \tau_{xy} = \frac{1}{2}\sqrt{\sigma_Y^2 - \sigma_x^2} = \frac{1}{2}\sqrt{250^2 - 211.6^2}$

$= 66.568 \text{ MPa} = 65.568 \times 10^6 \text{ Pa}$

From torsion $\quad \tau_{xy} = \frac{Tc}{J} \qquad T = \frac{J\tau_{xy}}{c}$

$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}\left(\frac{38}{2}\right)^4 = 204.71 \times 10^3 \text{ mm}^4 = 204.71 \times 10^{-9} \text{ m}^4$

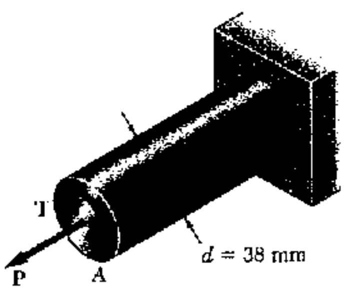
$c = \frac{1}{2}d = 19 \times 10^{-3}$ m

$T = \frac{(204.71 \times 10^{-9})(65.668 \times 10^6)}{19 \times 10^{-3}} = 717 \text{ N} \cdot \text{m}$

Problem 7.88

7.88 Solve Prob. 7.87, using the maximum-distortion-energy criterion.

7.87 The 38-mm-diameter shaft AB is made of a grade of steel for which the yield strength is $\sigma_Y = 250$ MPa. Using the maximum-shearing-stress criterion, determine the magnitude of the torque **T** for which yield occurs when $P = 240$ kN.



$P = 240 \times 10^3$ N

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(38)^2 = 1.1341 \times 10^3 \text{ mm}^2 = 1.1341 \times 10^{-3} \text{ m}^2$

$\sigma_x = \frac{P}{A} = \frac{240 \times 10^3}{1.1341 \times 10^{-3}} = 211.6 \times 10^6 \text{ Pa} = 211.6 \text{ MPa}$

$\sigma_y = 0 \qquad \sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}\sigma_x$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2}$

$\sigma_a = \sigma_{ave} + R = \frac{1}{2}\sigma_x + \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2}$

$\sigma_b = \sigma_{ave} - R = \frac{1}{2}\sigma_x - \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2}$

$$\begin{aligned}\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b = {} & \frac{1}{4}\sigma_x^2 + \sigma_x\sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2} + \frac{1}{4}\sigma_x^2 + \tau_{xy}^2 \\ & + \frac{1}{4}\sigma_x^2 - \sigma_x\sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2} + \frac{1}{4}\sigma_x^2 + \tau_{xy}^2 \\ & - \frac{1}{4}\sigma_x^2 + \frac{1}{4}\sigma_x^2 + \tau_{xy}^2 \\ = {} & \sigma_x^2 + 3\tau_{xy}^2 = \sigma_Y^2\end{aligned}$$

$\tau_{xy}^2 = \frac{1}{3}(\sigma_Y^2 - \sigma_x^2)$

$\tau_{xy} = \frac{1}{\sqrt{3}}\sqrt{250^2 - 211.6^2} = 76.867 \text{ MPa} = 76.867 \times 10^6 \text{ Pa}$

From torsion $\qquad \tau_{xy} = \frac{Tc}{J} \qquad T = \frac{J\tau_{xy}}{c}$

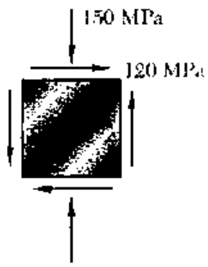
$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}\left(\frac{38}{2}\right)^4 = 204.71 \times 10^3 \text{ mm}^4 = 204.71 \times 10^{-9} \text{ m}^4$

$c = \frac{1}{2}d = 19 \times 10^{-3}$ m

$T = \frac{(204.71 \times 10^{-9})(76.876 \times 10^6)}{19 \times 10^{-3}} = 828 \text{ N}\cdot\text{m}$

Problem 7.89

7.89 and 7.90 The state of plane stress shown is expected in a cast-iron base. Knowing that for the grade of cast iron used $\sigma_{UT} = 160$ MPa and $\sigma_{UC} = 320$ MPa and using Mohr's criterion, determine whether rupture of the component will occur.



$\sigma_x = 0 \qquad \sigma_y = -150 \text{ MPa} \qquad \tau_{xy} = 120 \text{ MPa}$

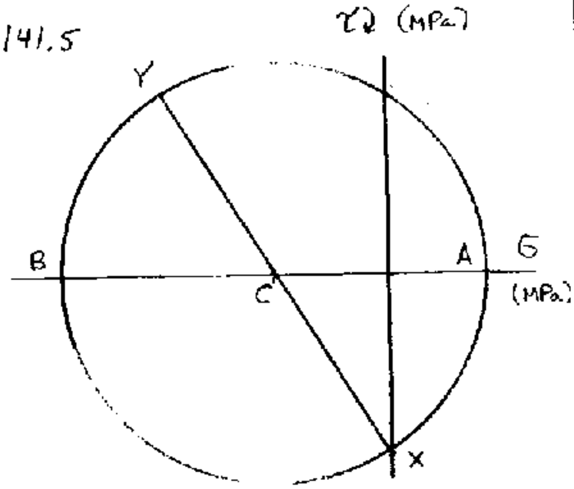
$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -75 \text{ MPa}$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 141.5 \text{ MPa}$

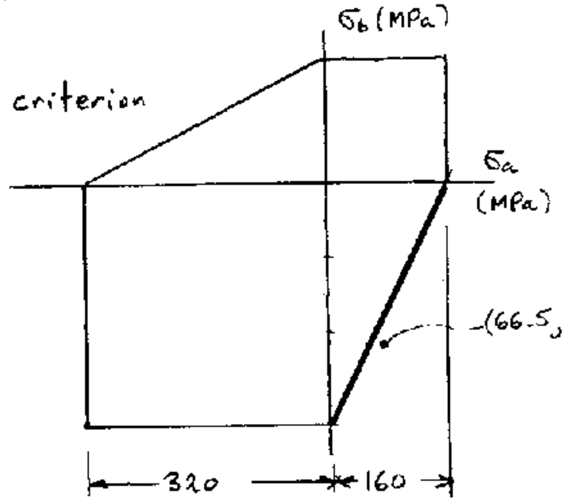
$= \sqrt{75^2 + 120^2} = 141.5$

$\sigma_a = \sigma_{ave} + R = 66.5 \text{ MPa}$

$\sigma_b = \sigma_{ave} - R = -216.5 \text{ MPa}$



Mohr's criterion

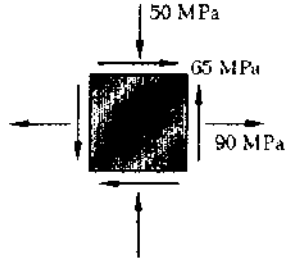


Equation of 4th quadrant boundary is $\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$

$\frac{66.5}{160} - \frac{-216.5}{320} = 1.092 > 1$ Rupture occurs.

Problem 7.90

7.89 and 7.90 The state of plane stress shown is expected in a cast-iron base. Knowing that for the grade of cast iron used $\sigma_{UT} = 160$ MPa and $\sigma_{UC} = 320$ MPa and using Mohr's criterion, determine whether rupture of the component will occur.

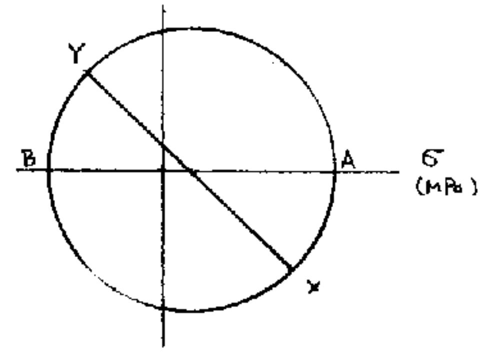


$\sigma_x = 90$ MPa , $\sigma_y = -50$ MPa , $\tau_{xy} = 65$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 20$ MPa

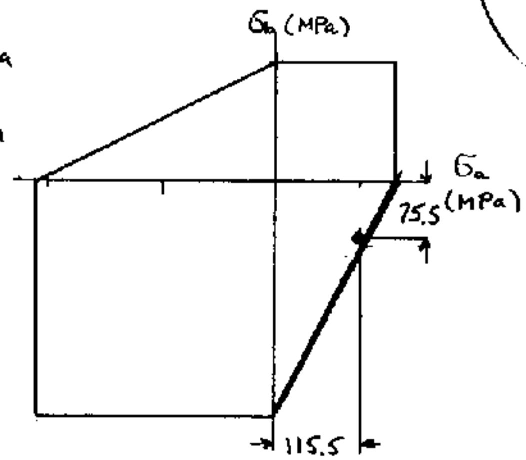
$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{70^2 + 65^2} = 95.5$ MPa



$\sigma_a = \sigma_{ave} + R = 115.5$ MPa

$\sigma_b = \sigma_{ave} - R = -75.5$ MPa

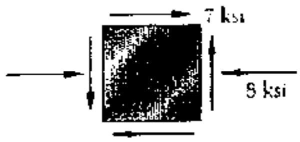


Equation of 4th quadrant boundary $\quad \frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$

$\frac{115.5}{160} - \frac{(-75.5)}{320} = 0.958 < 1 \qquad$ No rupture

Problem 7.91

7.91 and *7.92* The state of plane stress shown is expected to occur in an aluminum casting. Knowing that for the aluminum alloy used $\sigma_{UT} = 10$ ksi and $\sigma_{UC} = 30$ ksi and using Mohr's criterion, determine whether rupture of the component will occur.



$\sigma_x = -8$ ksi, $\sigma_y = 0$, $\tau_{xy} = 7$ ksi

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -4$ ksi

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{4^2 + 7^2} = 8.062$ ksi

$\sigma_a = \sigma_{ave} + R = -4 + 8.062 = 4.062$ ksi

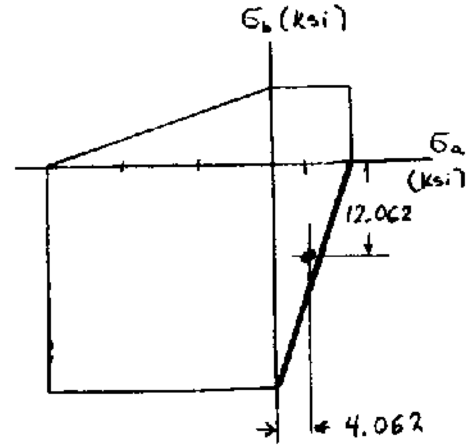
$\sigma_b = \sigma_{ave} - R = -4 - 8.062 = -12.062$ ksi

Equation of 4 th quadrant of boundary

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

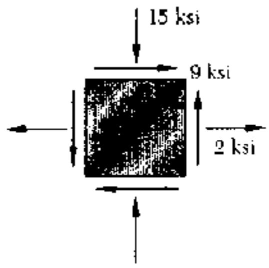
$$\frac{4.062}{10} - \frac{(-12.062)}{30} = 0.808 < 1$$

(No rupture)



Problem 7.92

7.91 and *7.92* The state of plane stress shown is expected to occur in an aluminum casting. Knowing that for the aluminum alloy used $\sigma_{UT} = 10$ ksi and $\sigma_{UC} = 30$ ksi and using Mohr's criterion, determine whether rupture of the component will occur.



$\sigma_x = 2$ ksi $\quad \sigma_y = -15$ ksi $\quad \tau_{xy} = 9$ ksi

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -6.5$ ksi

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{8.5^2 + 9^2} = 12.379$ ksi

$\sigma_a = \sigma_{ave} + R = 5.879$ ksi

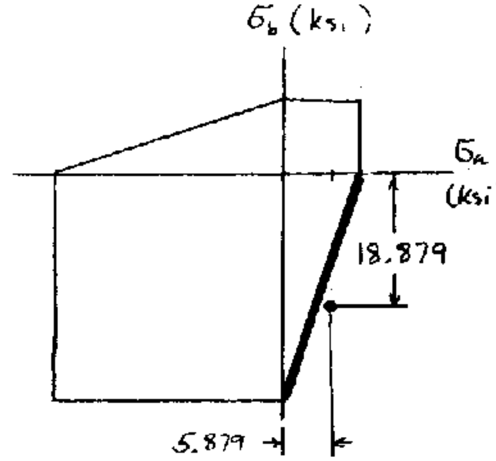
$\sigma_b = \sigma_{ave} - R = -18.879$ ksi

Equation of 4th quadrant of boundary

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

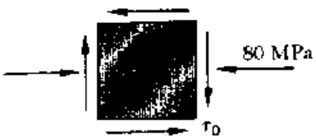
$$\frac{5.879}{10} - \frac{(-18.879)}{30} = 1.217 > 1$$

Rupture will occur. ◄



Problem 7.93

7.93 The state of plane stress shown will occur at a critical point in a pipe made of an aluminum alloy for which $\sigma_{UT} = 75$ MPa and $\sigma_{UC} = 150$ MPa. Using Mohr's criterion, determine the shearing stress τ_0 for which failure should be expected.



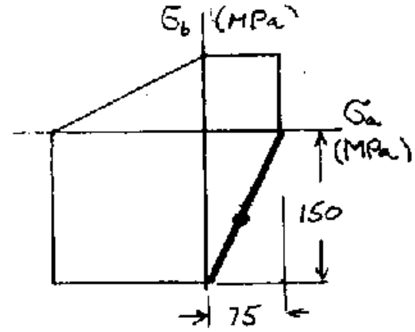
$\sigma_x = -80$ MPa $\quad \sigma_y = 0 \quad \tau_{xy} = -\tau_0$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -40$ MPa

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{40^2 + \tau_0^2}$ MPa

$\sigma_a = \sigma_{ave} + R, \quad \sigma_b = \sigma_{ave} - R, \quad \tau_0 = \pm\sqrt{R^2 - 40^2}$

Since $|\sigma_{ave}| < R$, stress point lies in 4th quadrant. Equation of 4th quadrant boundary is



$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

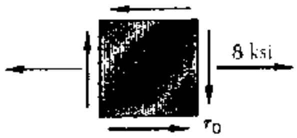
$$\frac{-40 + R}{75} - \frac{-40 - R}{150} = 1$$

$$\frac{R}{75} + \frac{R}{150} = 1 + \frac{40}{75} - \frac{40}{150} = 1.2667$$

$R = 63.33$ MPa, $\quad \tau_0 = \pm\sqrt{63.33^2 - 40^2} = \pm 49.1$ MPa ◄

Problem 7.94

7.94 The state of plane stress shown will occur at a critical point in an aluminum casting that is made of an alloy for which $\sigma_{UT} = 10$ ksi and $\sigma_{UC} = 25$ ksi. Using Mohr's criterion, determine the shearing stress τ_0 for which failure should be expected.



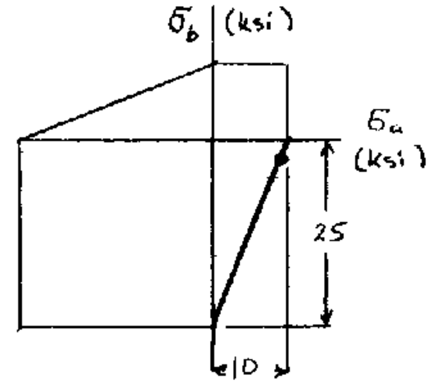
$\sigma_x = 8$ ksi , $\sigma_y = 0$, $\tau_{xy} = \tau_0$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 4$ ksi

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{4^2 + \tau_0^2}$, $\tau_0 = \pm\sqrt{R^2 - 4^2}$

$\sigma_a = \sigma_{ave} + R = (4 + R)$ ksi $\qquad \sigma_b = \sigma_{ave} - R = (4 - R)$ ksi

Since $|\sigma_{ave}| < R$, stress point lies in 4th quadrant. Equation of 4th quadrant boundary is



$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

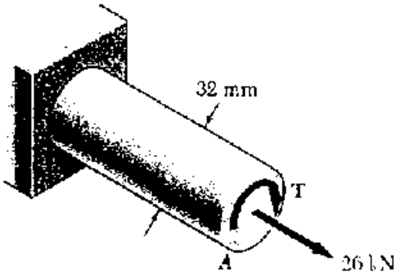
$$\frac{4+R}{10} - \frac{4-R}{25} = 1$$

$$\left(\frac{1}{10} + \frac{1}{25}\right)R = 1 - \frac{4}{10} + \frac{4}{25}$$

$R = 5.429$ ksi $\qquad \tau_0 = \pm\sqrt{5.429^2 - 4^2} = \pm\ 3.67$ ksi ◀

Problem 7.95

7.95 The case-aluminum rod shown is made of an alloy for which $\sigma_{UT} = 60$ MPa and $\sigma_{UC} = 120$ MPa. Using Mohr's criterion, determine the magnitude of the torque **T** for which failure should be expected.



$$P = 26 \times 10^3 \text{ N}$$

$$A = \frac{\pi}{4}(32)^2 = 804.25 \text{ mm}^2 = 804.25 \times 10^{-6} \text{ m}^2$$

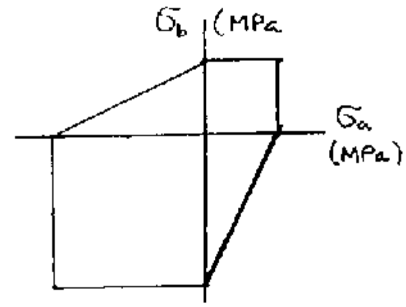
$$\sigma_x = \frac{P}{A} = \frac{26 \times 10^3}{804.25 \times 10^{-6}} = 32.328 \times 10^6 \text{ Pa} = 32.328 \text{ MPa}$$

$$\sigma_{ave} = \tfrac{1}{2}(\sigma_x + \sigma_y) = \tfrac{1}{2}(32.328 + 0) = 16.164 \text{ MPa}$$

$$\frac{\sigma_x - \sigma_y}{2} = \tfrac{1}{2}(32.328 - 0) = 16.164 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 16.164 + R \text{ MPa}, \quad \sigma_b = \sigma_{ave} - R = 16.164 - R \text{ MPa}$$

Since $|\sigma_{ave}| < R$, stress point lies in the 4th quadrant. Equation of the 4th quadrant is



$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

$$\frac{16.164 + R}{60} - \frac{16.164 - R}{120} = 1$$

$$\left(\frac{1}{60} + \frac{1}{120}\right)R = 1 - \frac{16.164}{60} + \frac{16.164}{120} \qquad R = 34.612 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right) + \tau_{xy}^2}$$

$$\tau_{xy} = \sqrt{R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} = \sqrt{34.612^2 - 16.164^2} = 30.606 \text{ MPa}$$

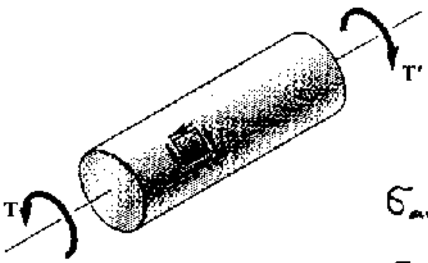
$$= 30.606 \times 10^6 \text{ Pa}$$

For torsion $\quad \tau_{xy} = \dfrac{Tc}{J} = \dfrac{2T}{\pi c^3} \qquad$ where $\quad c = \tfrac{1}{2}d = 16 \text{ mm} = 16 \times 10^{-3} \text{ m}$

$$T = \frac{\pi}{2} c^3 \tau_{xy} = \frac{\pi}{2}(16 \times 10^{-3})^3(30.606 \times 10^6) = 196.9 \text{ N} \cdot \text{m} \quad ◄$$

Problem 7.96

7.96 The case-aluminum rod shown is made of an alloy for which $\sigma_{UT} = 70$ MPa and $\sigma_{UC} = 175$ MPa. Knowing that the magnitude T of the applied torques is slowly increased and using Mohr's criterion, determine the shearing stress τ_0 that should be expected at rupture.



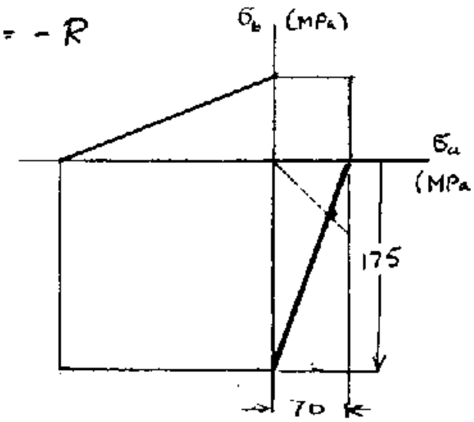
$\sigma_x = 0 \quad , \quad \sigma_y = 0 \qquad \tau_{xy} = -\tau_0$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 0$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{0 + \tau_{xy}^2} = |\tau_{xy}|$

$\sigma_a = \sigma_{ave} + R = R \qquad \sigma_b = \sigma_{ave} - R = -R$

Since $|\sigma_{ave}| < R$, stress point lies in 4th quadrant. Equation of boundary of 4th quadrant is



$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$

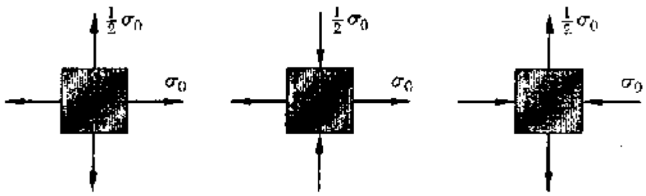
$\frac{R}{70} - \frac{-R}{175} = 1$

$\left(\frac{1}{70} + \frac{1}{175}\right) R = 1$

$R = 50$ MPa $\qquad |\tau_{xy}| = \tau_0 = 50$ MPa

Problem 7.97

7.97 A machine component is made of a grade of cast iron for which $\sigma_{UT} = 8$ ksi and $\sigma_{UC} = 20$ ksi. For each of the states of stress shown, and using Mohr's criterion, determine the normal stress σ_0 at which repture of the component should be expected.

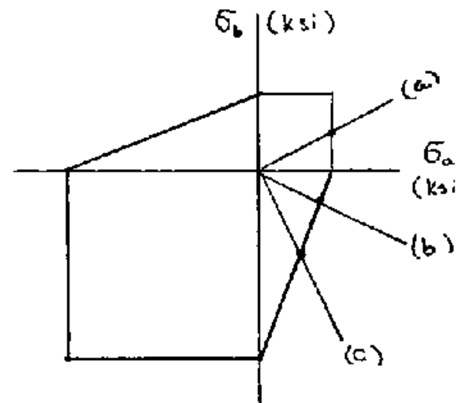


(a) *(b)* *(c)*

(a) $\sigma_a = \sigma_o$, $\sigma_b = \frac{1}{2}\sigma_o$

Stress point lies in 1st quadrant.

$\sigma_a = \sigma_o = \sigma_{UT} = 8$ ksi ◄



(b) $\sigma_a = \sigma_o$, $\sigma_b = -\frac{1}{2}\sigma_o$

Stress point lies in 4th quadrant.
Equation of 4th quadrant boundary is

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

$$\frac{\sigma_o}{8} - \frac{-\frac{1}{2}\sigma_o}{20} = 1 \qquad \sigma_o = 6.67 \text{ ksi} ◄$$

(c) $\sigma_a = \frac{1}{2}\sigma_o$, $\sigma_b = -\sigma_o$, 4th quadrant

$$\frac{\frac{1}{2}\sigma_o}{8} - \frac{-\sigma_o}{20} = 1 \qquad \sigma_o = 8.89 \text{ ksi} ◄$$

Problem 7.98

7.98 A spherical gas container made of steel has a 6-m outer diameter and a wall thickness of 9 mm. Knowing that the internal pressure is 500 kPa, determine the maximum normal stress and the maximum shearing stress in the container.

$$r = \tfrac{1}{2}d - t = \frac{6}{2} - 9\times10 = 2.991 \text{ m} \qquad t = 9\times10^{-3} \text{ m}$$

$$p = 500\times10^{3} \text{ Pa}$$

$$\sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{(500\times10^3)(2.991)}{(2)(9\times10^{-3})} = 83.1\times10^{6} \text{ Pa}$$

$$\sigma_{max} = \sigma_1 \qquad \sigma_{max} = 83.1 \text{ MPa}$$

$$\sigma_{min} \approx -p = 0.5\times10^{6} \text{ Pa}$$

$$\tau_{max} = \tfrac{1}{2}(\sigma_{max} - \sigma_{min}) = 41.8\times10^{6} \text{ Pa} \qquad \tau_{max} = 41.8 \text{ MPa}$$

Problem 7.99

7.99 The maximum gage pressure is known to be 10 MPa in a spherical steel pressure vessel having a 200-mm outer diameter and a 6-mm wall thickness. Knowing that the ultimate stress in the steel used is $\sigma_U = 400$ MPa, determine the factor of safety with respect to tensile failure.

$$r = \tfrac{1}{2}d - t = \tfrac{1}{2}d - t = \tfrac{1}{2}(200) - 6 = 94 \text{ mm} = 94\times10^{-3} \text{ m}$$

$$t = 6\times10^{-3} \text{ m}$$

$$\sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{(10\times10^6)(94\times10^{-3})}{(2)(6)} = 78.3\times10^{6} \text{ Pa} = 78.3 \text{ MPa}$$

$$\sigma_{max} = \sigma_1 = 78.3 \text{ MPa}$$

$$F.S. = \frac{\sigma_U}{\sigma_{max}} = \frac{400}{78.3} = 5.11$$

Problem 7.100

7.100 Determine the normal stress in a basketball of 9.5-in. outer diameter and 0.125-in. wall thickness that is inflated to a gage pressure of 9 psi.

$$r = \tfrac{1}{2}d - t = \tfrac{1}{2}(9.5) - 0.125 = 4.625 \text{ in.}$$

$$\sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{(9)(4.625)}{(2)(0.125)} = 166.5 \text{ psi}$$

Problem 7.101

7.101 A spherical gas container having an outer diameter of 15 ft and a wall thickness of 0.90 in. is made of a steel for which $E = 29 \times 10^6$ psi and $\nu = 0.29$. Knowing that the gage pressure in the container is increased from zero to 250 psi, determine (a) the maximum normal stress in the container, (*b*) the increase in the diameter of the container.

$r = \frac{1}{2}d - t = \frac{1}{2}(15)(12) - 0.90 = 89.1$ in. $\qquad t = 0.90$ in.

(a) $\sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{(250)(89.1)}{(2)(0.90)} = 12.38 \times 10^3$ psi $\qquad$ 12.38 ksi

(b) $\varepsilon_1 = \varepsilon_2 = \frac{1}{E}(\sigma_1 - \nu\sigma_2) = \frac{1-\nu}{E}\sigma_1$

$= \frac{1 - 0.29}{29 \times 10^6}(12.38 \times 10^3) = 302.97 \times 10^{-6}$

$\Delta d = (d)(\varepsilon_1) = (15)(12)(302.97 \times 10^{-6}) = 0.0545$ in.

Problem 7.102

7.102 A spherical pressure vessel has an outer diameter of 3 m and a wall thickness of 12 mm. Knowing that for the steel used $\sigma_{all} = 80$ MPa, $E = 200$ GPa, and $\nu = 0.29$, determine (*a*) the allowable gage pressure, (*b*) the corresponding increase in the diameter of the vessel.

$r = \frac{1}{2}d - t = \frac{1}{2}(3000) - 12 = 1488$ mm

$\sigma_1 = \sigma_2 = \sigma_{all} = 8$ MPa

(a) $\sigma_1 = \sigma_2 = \frac{pr}{2t} \qquad p = \frac{2t\sigma_1}{r} = \frac{(2)(12)(80)}{1488} = 1.290$ MPa

$\varepsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2) = \frac{1-\nu}{E}\sigma_1 = \frac{1 - 0.29}{200 \times 10^9}(8 \times 10^6) = 28.4\ \mu$

(b) $\Delta d = d\,\varepsilon_1 = (3000)(28.4 \times 10^{-6}) = 85.2 \times 10^{-3}$ mm $= 0.0852$ mm

Problem 7.103

7.103 A spherical pressure vessel of 750-mm outer diameter is to be fabricated from a steel having an ultimate stress $\sigma_U = 400$ MPa. Knowing that a factor of safety of 4 is desired and that the gage pressure can reach 4.2 MPa, determine the smallest wall thickness that should be used.

$p = 4.2 \times 10^6$ Pa $\qquad r = \frac{1}{2}d - t = \frac{0.750}{2} - t = 0.375 - t$ m

$\sigma_{max} = \sigma_1 = \sigma_2 = \frac{pr}{2t}$

$F.S. = \frac{\sigma_U}{\sigma_{max}} = \frac{2t\sigma_U}{pr}$

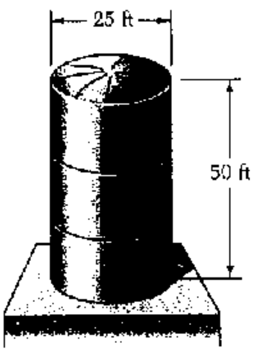
$2\sigma_U t = (F.S.)\,pr$

$(2)(400 \times 10^6)\,t = (4)(4.2 \times 10^6)(0.375 - t)$

$816.8 \times 10^6\, t = 6.3 \times 10^6 \qquad t = 7.71 \times 10^{-3}$ m $\qquad t = 7.71$ mm

Problem 7.104

7.104 When filled to capacity, the unpressurized storage tank shown contains water to a height of 48 ft above its base. Knowing that the lower portion of the tank has a wall thickness of 0.625 in., determine the maximum normal stress and the maximum shearing stress in the tank. (Specific weight of water = 62.4 lb/ft³.)



$$p_{max} = \gamma h = (62.4\ lb/ft^3)(48\ ft) = 2995.2\ lb/ft^2$$
$$= 20.8\ lb/in^2 = 20.8\ psi$$
$$r = \tfrac{1}{2}d - t = \tfrac{1}{2}(25)(12) - 0.625 = 149.375\ in.$$
$$\sigma_1 = \frac{pr}{t} = \frac{(20.8)(149.375)}{0.625} = 4.97 \times 10^3\ psi$$
$$\sigma_{max} = \sigma_1 = 4.97 \times 10^3\ psi \qquad \sigma_{max} = 4.97\ ksi$$
$$\sigma_{min} \approx -p = -250\ psi, \quad \tau_{max} = 2.496 \times 10^3\ psi \qquad \tau_{max} = 2.50\ ksi$$

Problem 7.105

7.105 Determine the largest internal pressure that can be applied to a cylindrical tank of 1.75-m outer diameter and 16-mm wall thickness if the ultimate normal stress of the steel used is 450 MPa and a factor of safety of 5.0 is desired.

$$\sigma_1 = \frac{\sigma_U}{F.S.} = \frac{450\ MPa}{5} = 90\ MPa = 90 \times 10^6\ Pa$$
$$r = \tfrac{1}{2}d - t = \frac{1.75}{2} - 16 \times 10^{-3} = 0.859\ m$$
$$\sigma_1 = \frac{pr}{t} \qquad p = \frac{t\sigma_1}{r} = \frac{(16\times10^{-3})(90\times10^6)}{0.859} = 1.676 \times 10^6\ Pa$$
$$1.676\ MPa$$

Problem 7.106

7.106 A storage tank contains liquified propane under a pressure of 210 psi at a temperature of 100°F. Knowing that the tank has an outer diameter of 12.6 in and a wall thickness of 0.11 in. Determine the maximum normal stress and the maximum shearing stress in the tank

$$r = \tfrac{1}{2}d - t = \tfrac{1}{2}(12.6) - 0.11 = 6.19\ in. \qquad t = 0.11\ in.$$
$$\sigma_1 = \frac{pr}{t} = \frac{(210)(6.19)}{0.11} = 11.82 \times 10^3\ psi$$
$$\sigma_{max} = \sigma_1 = 11.82 \times 10^3\ psi \qquad \sigma_{max} = 11.82\ ksi$$
$$\sigma_{min} \approx -p = -210\ psi$$
$$\tau_{max} = \tfrac{1}{2}(\sigma_{max} - \sigma_{min}) = 6.01 \times 10^3\ psi \qquad \tau_{max} = 6.01\ ksi$$

Problem 7.107

7.107 The bulk storage tank shown in Fig. 7.49 has an outer diameter of 3.5 m and a wall thickness of 20 mm. At a time when the internal pressure of the tank is 1.2 MPa, determine the maximum normal stress and the maximum shearing stress in the tank.

$r = \frac{1}{2}d - t = \frac{1}{2}(3.5) - 20\times10^{-3} = 1.73 \text{ m}$ $\qquad t = 20\times10^{-3} \text{ m}$

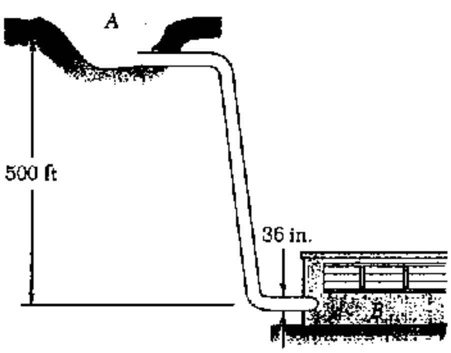
$\sigma_1 = \frac{pr}{t} = \frac{(1.2\times10^6)(1.73)}{20\times10^{-3}} = 103.8\times10^6 \text{ Pa}$

$\sigma_{max} = \sigma_1 = 103.8\times10^6 \text{ Pa}$ $\qquad \sigma_{max} = 103.8 \text{ MPa}$ ◄

$\sigma_{min} \approx -p = 1.2\times10^6 \text{ Pa}$

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 52.5\times10^6 \text{ Pa}$ $\qquad \tau_{max} = 52.5 \text{ MPa}$ ◄

Problem 7.108



7.108 A steel penstock has a 36-in. outer diameter, a 0.5-in. wall thickness, and connects a reservoir at A with a generating station at B. Knowing that the specific weight of water is 62.4 lb/ft³, determine the maximum normal stress and the maximum shearing stress in the penstock under static conditions.

$r = \frac{1}{2}d - t = \frac{1}{2}(36) - 0.5 = 17.5 \text{ in.}$

$p = \gamma h = (62.4 \text{ lb/ft}^3)(500 \text{ ft}) = 31.2\times10^3 \text{ lb/ft}^2$

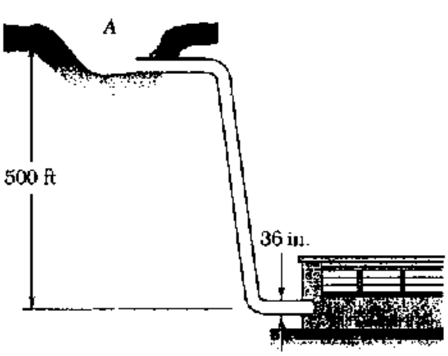
$= 216.67 \text{ psi}$

$\sigma_1 = \frac{pr}{t} = \frac{(216.67)(17.5)}{0.5} = 7583 \text{ psi}$

$\sigma_{max} = \sigma_1 = 7583 \text{ psi}$ $\qquad \sigma_{max} = 7.58 \text{ ksi}$ ◄

$\sigma_{min} \approx -p = -217 \text{ psi}$ $\qquad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 3900 \text{ psi}$ $\qquad \tau_{max} = 3.90 \text{ ksi}$ ◄

Problem 7.109



7.109 A steel penstock has a 36-in. outer diameter, a 0.5-in. wall thickness, and connects a reservoir at A with a generating station at B. Knowing that the specific weight of water is 62.4 lb/ft³ and that the allowable normal stress in the steel is 12.5 ksi, determine the smallest thickness that can be used for the penstock.

$p = \gamma h = (62.4 \text{ lb/ft}^3)(500 \text{ ft}) = 31.2\times10^3 \text{ lb/ft}^2$

$= 216.67 \text{ psi}$

$\sigma_1 = 12.5 \text{ ksi} = 12.5\times10^3 \text{ psi}$

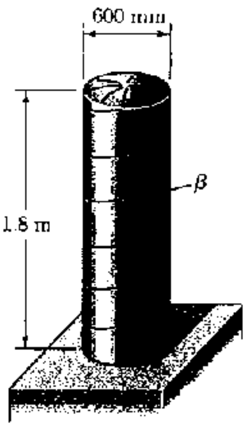
$r = \frac{1}{2}d - t = 18 - t$

$\sigma_1 = \frac{pr}{t}, \quad \frac{r}{t} = \frac{\sigma_1}{p}, \qquad \frac{18 - t}{t} = \frac{12.5\times10^3}{216.67} = 57.692$

$\frac{18}{t} = 58.692 \qquad t = 0.307 \text{ in}$ ◄

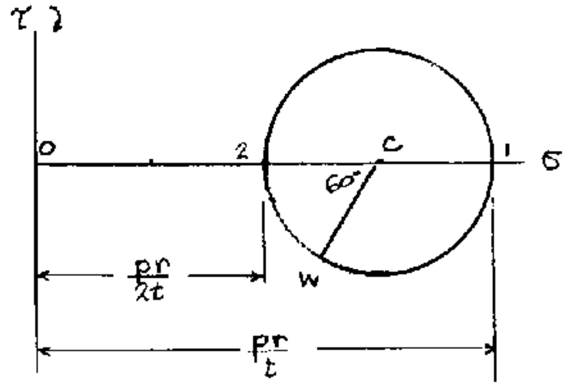
Problem 7.110

7.110 The cylindrical portion of the compressed air tank shown is fabricated of 8-mm-thick plate welded along a helix forming an angle $\beta = 30°$ with the horizontal. Knowing that the allowable stress normal to the weld is 75 MPa, determine the largest gage pressure that can be used in the tank.



$r = \frac{1}{2}d - t = \frac{1}{2}(600) - 6 = 292 \text{ mm}$

$\sigma_1 = \frac{pr}{t} \qquad \sigma_2 = \frac{1}{2}\frac{pr}{t}$



$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4}\frac{pr}{t}$

$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4}\frac{pr}{t}$

$\sigma_w = \sigma_{ave} + R\cos 60°$

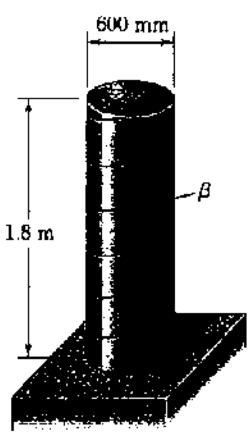
$= \frac{5}{8}\frac{pr}{t}$

$p = \frac{8}{5}\frac{\sigma_w t}{r}$

$p = \frac{8}{5}\frac{(75)(8)}{292} = 3.29 \text{ MPa}$ ◀

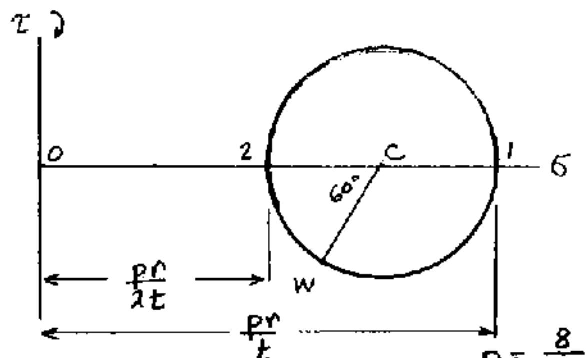
Problem 7.111

7.111 The cylindrical portion of the compressed air tank shown is fabricated of 8-mm-thick plate welded along a helix forming an angle $\beta = 30°$ with the horizontal. Determine the gage pressure that will cause a shearing stress parallel to the weld of 30 MPa.



$r = \frac{1}{2}d - t = \frac{1}{2}(600) - 8 = 292 \text{ mm}$

$\sigma_1 = \frac{pr}{t}, \qquad \sigma_2 = \frac{1}{2}\frac{pr}{t}$



$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4}\frac{pr}{t}$

$\tau_w = R \sin 60°$

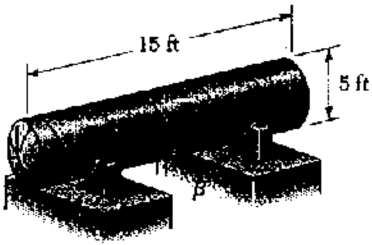
$= \frac{\sqrt{3}}{8}\frac{pr}{t}$

$p = \frac{8}{\sqrt{3}}\frac{\tau_w t}{R}$

$p = \frac{8}{\sqrt{3}}\frac{(30)(8)}{294} = 3.80 \text{ MPa}$ ◀

Problem 7.112

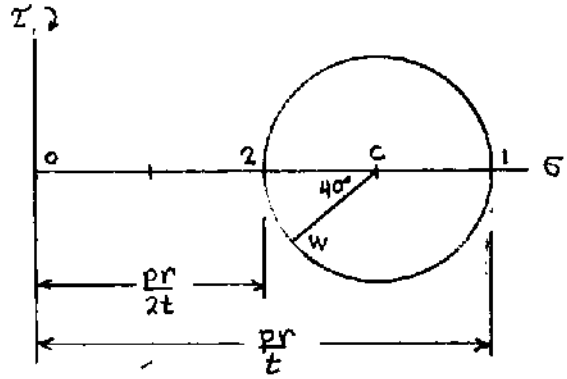
7.112 The pressure tank shown has a 0.375-in. wall thickness and butt welded seams forming an angle $\beta = 20°$ with a transverse plane. For a gage pressure of 85 psi, determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.



$d = 5\text{ ft} = 60\text{ in.} \qquad r = \frac{1}{2}d - t = 30 - 0.375 = 29.625\text{ in.}$

$\sigma_1 = \frac{pr}{t} = \frac{(85)(29.625)}{0.375} = 6715\text{ psi}$

$\sigma_2 = \frac{1}{2}\sigma_1 = 3357.5\text{ psi}$



$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = 5036.25\text{ psi}$

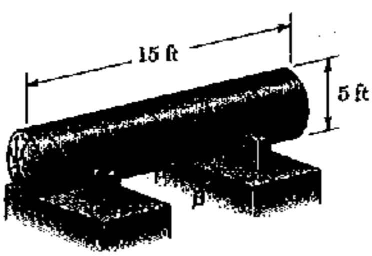
$R = \frac{\sigma_1 - \sigma_2}{2} = 1678.75\text{ psi}$

(a) $\sigma_w = \sigma_{ave} - R\cos 40° = 3750\text{ psi}$

(b) $\tau_w = R\sin 40° = 1079\text{ psi}$

Problem 7.113

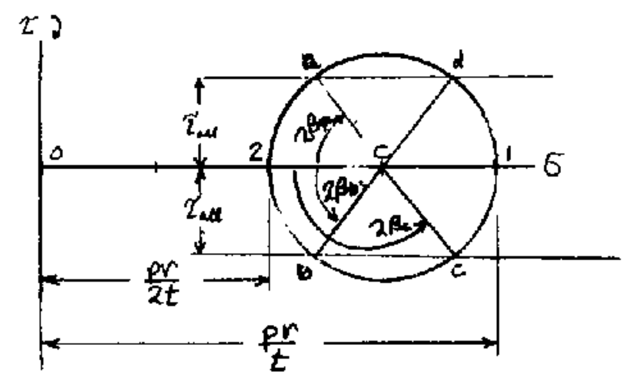
7.113 The pressure tank shown has a 0.375-in. wall thickness and butt welded seams forming an angle β with a transverse plane. Determine the range of values of β that can be used if the shearing stress parallel to the weld is not to exceed 1350 psi when the gage pressure is 85 psi.



$d = 5\text{ ft} = 60\text{ in} \qquad r = \frac{1}{2}d - t = 30 - \frac{3}{8} = 29.625\text{ in}$

$\sigma_1 = \frac{pr}{t} = \frac{(85)(29.625)}{0.375} = 6715\text{ psi}$

$\sigma_2 = \frac{1}{2}\sigma_1 = 3357.5\text{ psi}$



$R = \frac{\sigma_1 - \sigma_2}{2} = 1678.75$

$\tau_w = R\sin 2\beta = \tau_{all}$

$\sin 2\beta_a = \frac{\tau_w}{R} = \frac{1350}{1678.75} = 0.80417$

$2\beta_a = -53.53°$ $\qquad \beta_a = -26.8°$

$2\beta_b = +53.53°$ $\qquad \beta_b = 26.8°$

$\left.\right\} \quad -26.8° \le \beta \le 26.8°$

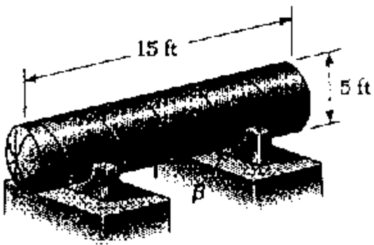
$2\beta_c = -53.53° + 180° = 126.47°$ $\qquad \beta_c = 63.2°$

$2\beta_d = 53.53° + 180° = 233.53°$ $\qquad \beta_d = 116.8°$

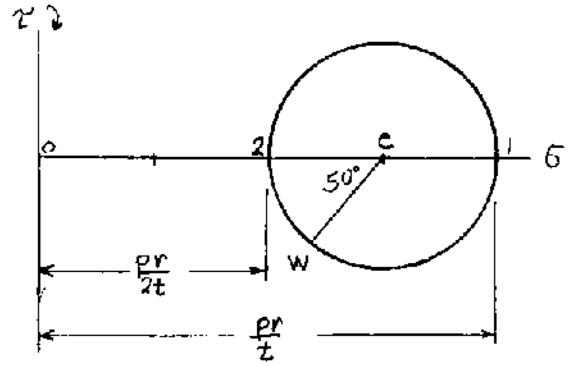
$\left.\right\} \quad 63.2° \le \beta \le 116.8°$

Problem 7.114

7.114 The pressure tank shown has a 0.375-in. wall thickness and butt welded seams forming an angle $\beta = 25°$ with a transverse plane. Determine the largest allowable gage pressure, knowing that the allowable normal stress perpendicular to the weld is 18 ksi and the allowable shearing stress parallel to the weld is 10 ksi.



$d = 5\text{ ft} = 60\text{ in.} \qquad r = \frac{1}{2}d - t = 30 - 0.375 = 29.625\text{ in.}$



$$\sigma_1 = \frac{pr}{t}$$

$$\sigma_2 = \frac{pr}{2t}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4}\frac{pr}{t}$$

$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4}\frac{pr}{t}$$

$$\sigma_w = \sigma_{ave} - R\cos 50°$$

$$= \left(\frac{3}{4} - \frac{1}{4}\cos 50°\right)\frac{pr}{t}$$

$$= 0.5893\frac{pr}{t}$$

$$p = \frac{\sigma_w t}{0.5893\, r} = \frac{(18)(0.375)}{(0.5893)(29.625)} = 0.387\text{ ksi} = 387\text{ psi}$$

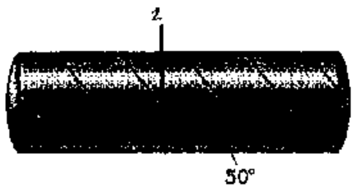
$$\tau_w = R\sin 50° = 0.19151\frac{pr}{t}$$

$$p = \frac{\tau_w t}{0.19151\, r} = \frac{(10)(0.375)}{(0.19151)(29.625)} = 0.661\text{ ksi} = 661\text{ psi}$$

Allowable gage pressure is the smaller value. $\quad p = 387\text{ psi}$ ◄

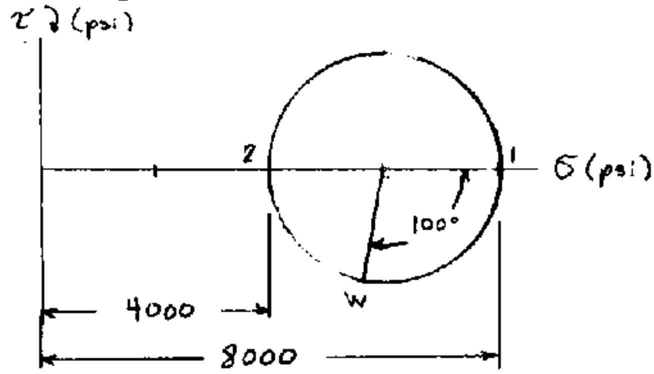
Problem 7.115

7.115 The steel pressure tank shown has a 30-in. inner diameter and a 0.375-in. wall thickness. Knowing that the butt welded seams form an angle $\beta = 50°$ with the longitudinal axis of the tank and that the gage pressure in the tank is 200 psi, determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.



$r = \frac{1}{2}d = 15$ in.

$\sigma_1 = \frac{pr}{t} = \frac{(200)(15)}{0.375} = 8000$ psi $\qquad \sigma_2 = \frac{1}{2}\sigma_1 = 4000$ psi



$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = 6000$ psi

$R = \frac{\sigma_1 - \sigma_2}{2} = 2000$ psi

(a) $\sigma_w = \sigma_{ave} + R\cos 100°$

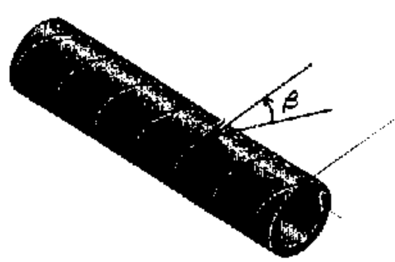
$= 5652$ psi

(b) $\tau_w = R \sin 100°$

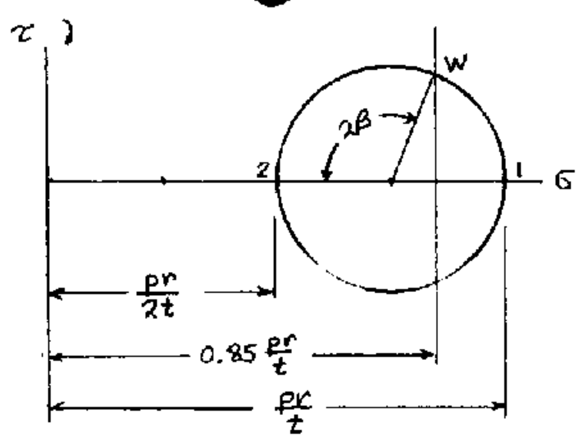
$= 1970$ psi

Problem 7.116

7.116 The pressurized tank shown was fabricated by welding strips of plate along a helix forming an angle β with a transverse plane. Determine the largest value of β that can be used if the normal stress perpendicular to the weld is not be larger than 85 percent of the maximum stress in the tank.



$\sigma_1 = \frac{pr}{t} \qquad \sigma_2 = \frac{pr}{2t}$



$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4}\frac{pr}{t}$

$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4}\frac{pr}{t}$

$\sigma_w = \sigma_{ave} - R\cos 2\beta$

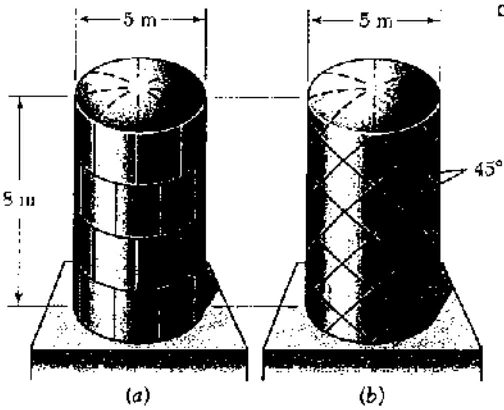
$0.85\frac{pr}{t} = \left(\frac{3}{4} - \frac{1}{4}\cos 2\beta\right)\frac{pr}{t}$

$\cos 2\beta = -4\left(0.85 - \frac{3}{4}\right) = -0.4$

$2\beta = 113.6° \qquad \beta = 56.8°$

Problem 7.117

7.117 Square plates, each of 16-mm thickness, can be bent and welded together in either of the two ways shown to form the cylindrical portion of a compressed air tank. Knowing that the allowable normal stress perpendicular to the weld is 65 MPa, determine the largest allowable gage pressure in each case.



$r = \frac{1}{2}d - t = \frac{1}{2}(5) - 16\times10^{-3} = 2.484 \text{ m}$

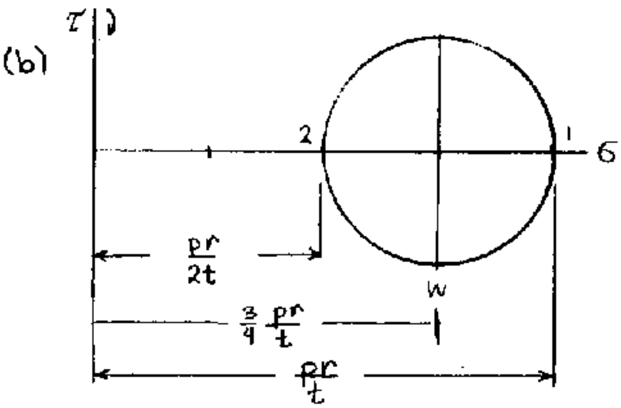
$\sigma_1 = \frac{pr}{t} \qquad \sigma_2 = \frac{pr}{2t}$

(a) $\sigma_1 = 65 \text{ MPa} = 65\times10^6 \text{ Pa}$

$p = \frac{\sigma_1 t}{r} = \frac{(65\times10^6)(16\times10^{-3})}{2.484} = 419\times10^3 \text{ Pa}$

419 kPa ◄

(b)



$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4}\frac{pr}{t}$

$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4}\frac{pr}{t}$

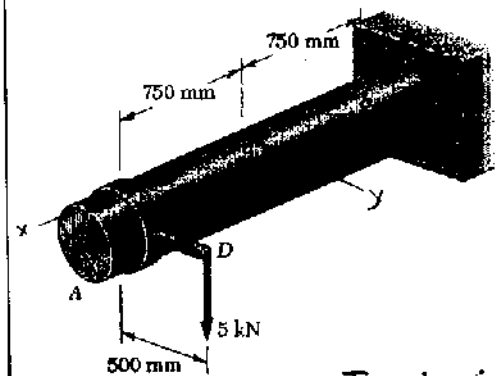
$\beta = \pm 45°$

$\sigma_w = \sigma_{ave} + R\cos\beta$

$= \frac{3}{4}\frac{pr}{t}$

$p = \frac{4\sigma_w t}{3r} = \frac{(4)(65\times10^6)(16\times10^{-3})}{(3)(2.484)} = 558\times10^3 \text{ Pa} \qquad 558 \text{ kPa}$ ◄

Problem 7.118



7.118 The compressed-air tank AB has an inner diameter of 450 mm and a uniform wall thickness of 6 mm. Knowing that the gage pressure in the tank is 1.2 MPa, determine the maximum normal stress and the maximum in-plane shearing stress at point a on the top of the tank.

$r = \frac{1}{2}d = 225$ mm $\quad t = 6$ mm

$\sigma_1 = \frac{pr}{t} = \frac{(1.2)(225)}{6} = 45$ MPa

$\sigma_2 = \frac{pr}{2t} = 22.5$ MPa

Torsion: $c_1 = 225$ mm, $\quad c_2 = 225 + 6 = 231$ mm

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = 446.9 \times 10^6 \text{ mm}^4 = 446.9 \times 10^{-6} \text{ m}^4$

$T = (5 \times 10^3)(500 \times 10^{-3}) = 2500$ N-m

$\tau = \frac{Tc}{J} = \frac{(2500)(231 \times 10^{-3})}{446.9 \times 10^{-6}} = 1.292 \times 10^6$ Pa $= 1.292$ MPa

Transverse shear: $\tau = 0$ at point a.

Bending: $I = \frac{1}{2}J = 223.45 \times 10^{-6} \text{ m}^4$, $\quad c = 231 \times 10^{-3}$ m.

At point a $\quad M = (5 \times 10^3)(750 \times 10^{-3}) = 3750$ N-m

$\sigma = \frac{Mc}{I} = \frac{(3750)(231 \times 10^{-3})}{223.45 \times 10^{-6}} = 3.88$ MPa

Total stresses (MPa)

Longitudinal $\quad \sigma_x = 22.5 + 3.88 = 26.38$

Circumferential $\quad \sigma_y = 45$

Shear $\quad \tau_{xy} = 1.292$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 35.69$ MPa

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 9.40$ MPa

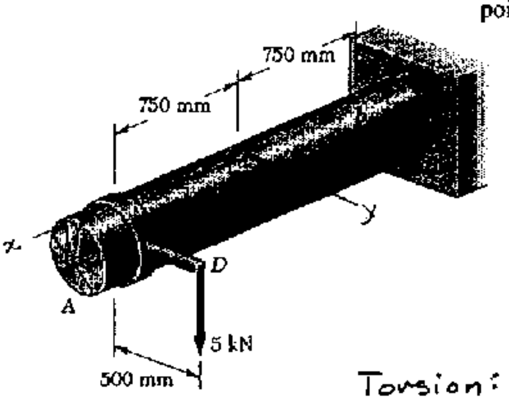
$\sigma_{max} = \sigma_{ave} + R = 45.1$ MPa ◄

$\tau_{max\,(in\text{-}plane)} = R = 9.40$ MPa ◄

Problem 7.119

7.119 For the compressed-air tank and loading of Prob. 7.118, determine the maximum normal stress and the maximum in-plane shearing stress at point *b* on the top of the tank.

7.118 The compressed-air tank *AB* has an inner diameter of 450 mm and a uniform wall thickness of 6 mm. Knowing that the gage pressure in the tank is 1.2 MPa, determine the maximum normal stress and the maximum in-plane shearing stress at point *a* on the top of the tank.



$r = \frac{1}{2}d = 225$ mm $\quad t = 6$ mm

$\sigma_1 = \frac{pr}{t} = \frac{(1.2)(225)}{6} = 45$ MPa

$\sigma_2 = \frac{pr}{2t} = 22.5$ MPa

Torsion: $c_1 = 225$ mm, $\quad c_2 = 225 + 6 = 231$ mm

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = 446.9 \times 10^6$ mm^4 $= 446.9 \times 10^{-6}$ m^4

$T = (5 \times 10^3)(500 \times 10^{-3}) = 2500$ N·m

$\tau = \frac{Tc}{J} = \frac{(2500)(231 \times 10^{-3})}{446.9 \times 10^{-6}} = 1.292 \times 10^6$ Pa $= 1.292$ MPa

Transverse shear: $\tau = 0$ at point

Bending: $I = \frac{1}{2}J = 223.45 \times 10^{-6}$ m^4, $\quad c = 231 \times 10^{-3}$ m

At point b $\quad M = (5 \times 10^3)(2 \times 750 \times 10^{-3}) = 7500$ N-m

$\sigma = \frac{Mc}{I} = \frac{(7500)(231 \times 10^{-3})}{223.45 \times 10^{-6}} = 7.75$ MPa

Total Stresses (MPa)

Longitudinal: $\sigma_x = 22.5 + 7.75 = 30.25$ MPa

Circumferential: $\sigma_y = 45$ MPa

Shear $\quad \tau_{xy} = 1.292$ MPa

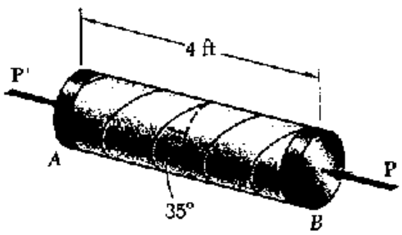
$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 37.625$ MPa

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 7.487$ MPa

$\sigma_{max} = \sigma_{ave} + R = 45.1$ MPa ◄

$\tau_{max(in\text{-}plane)} = R = 7.49$ MPa ◄

Problem 7.120



7.120 A pressure vessel of 10-in. inner diameter and 0.25 in. wall thickness is fabricated from a 4-ft section of spirally welded pipe AB and is equipped with two rigid end plates. The gage pressure inside the vessel is 300 psi and 10-kip centric axial forces **P** and **P'** are applied to the end plates. Determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.

$r = \frac{1}{2}d = \frac{1}{2}(10) = 5 \text{ in.} \qquad t = 0.25 \text{ in.}$

$\sigma_1 = \frac{pr}{t} = \frac{(300)(5)}{0.25} = 6000 \text{ psi} = 6 \text{ ksi}$

$\sigma_2 = \frac{pr}{2t} = \frac{(300)(5)}{(2)(0.25)} = 3000 \text{ psi} = 3 \text{ ksi}$

$r_o = r + t = 5 + 0.25 = 5.25 \text{ in.}$

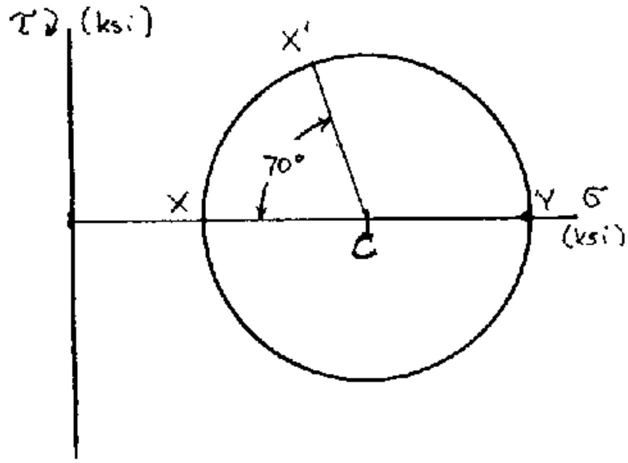
$A = \pi(r_o^2 - r^2) = \pi(5.25^2 - 5.00^2) = 8.0503 \text{ in.}$

$\sigma = -\frac{P}{A} = -\frac{10 \times 10^3}{18.0803} = -1242 \text{ psi} = -1.242 \text{ ksi}$

Total stresses. Longitudinal $\sigma_x = 3 - 1.242 = 1.758 \text{ ksi}$

Circumferential $\sigma_y = 6 \text{ ksi}$

Shear $\tau_{xy} = 0$



Plotted points for Mohr's circle.

$X: (1.758, 0)$

$Y: (6, 0)$

$C: (2.121, 0)$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 3.879 \text{ ksi}$

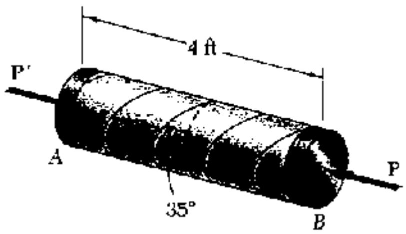
$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{\left[\frac{(1.758 - 6)}{2}\right]^2 + 0} = 2.121 \text{ ksi}$

(a) $\sigma_{x'} = \sigma_{ave} + R\cos 70° = 3.879 - 2.121 \cos 70° = 3.15 \text{ ksi}$ ◄

(b) $|\tau_{xy}| = R \sin 70° = 2.121 \sin 70° = 1.993 \text{ ksi}$ ◄

Problem 7.121



7.121 Solve Prob. 7.120, assuming that the magnitude P of the two forces is increased to 30 kips.

7.120 A pressure vessel of 10-in. inner diameter and 0.25 in. wall thickness is fabricated from a 4-ft section of spirally welded pipe AB and is equipped with two rigid end plates. The gage pressure inside the vessel is 300 psi and 10-kip centric axial forces **P** and **P'** are applied to the end plates. Determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.

$r = \frac{1}{2}d = \frac{1}{2}(10) = 5$ in. $\qquad t = 0.25$ in.

$\sigma_1 = \frac{pr}{t} = \frac{(300)(5)}{0.25} = 6000$ psi $= 6$ ksi

$\sigma_2 = \frac{pr}{2t} = \frac{(300)(5)}{(2)(0.25)} = 3000$ psi $= 3$ ksi

$r_o = r + t = 5 + 0.25 = 5.25$ in.

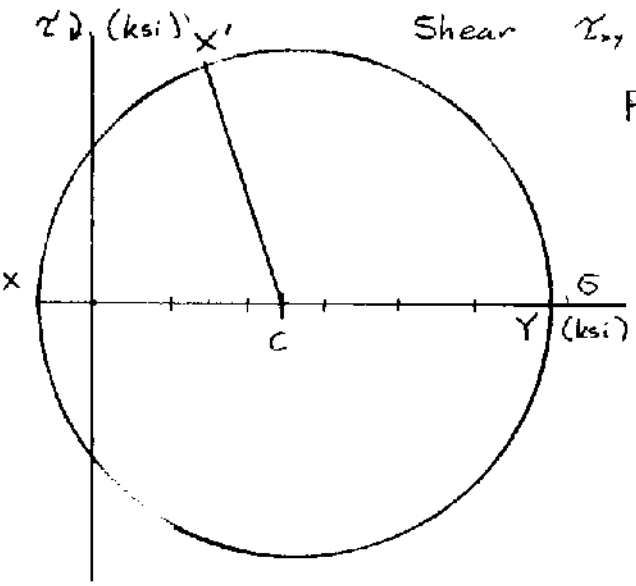
$A = \pi(r_o^2 - r^2) = \pi(5.25^2 - 5^2) = 8.0503$ in^2

$\sigma = -\frac{P}{A} = -\frac{30\times10^3}{8.0503} = -3727$ psi $= -3.727$ ksi

Total stresses. Longitudinal $\sigma_x = 3 - 3.727 = -0.727$ ksi

Circumferential $\sigma_y = 6$ ksi

Shear $\tau_{xy} = 0$



Plotted points for Mohr's circle.

X: $(-0.727, 0)$

Y: $(6, 0)$

C: $(2.6365, 0)$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 2.6365$ ksi

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

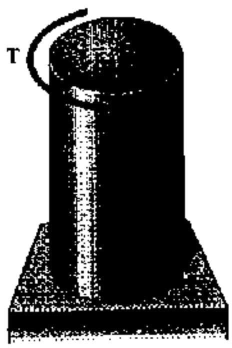
$= \sqrt{\left(\frac{-0.727 - 6}{2}\right)^2 + 0} = 3.3635$ ksi

(a) $\sigma_{x'} = \sigma_{ave} - R\cos 70° = 2.6365 - 3.3635\cos 70° = 1.486$ ksi ◄

(b) $|\tau_{x'y'}| = R\sin 70° = 3.3635\sin 70° = 3.16$ ksi ◄

Problem 7.122

7.122 A torque of magnitude $T = 12\ \text{kN}\cdot\text{m}$ is applied to the end of a tank containing compressed air under a pressure to 8 MPa. Knowing that the tank has a 180-mm inner diameter and a 12-mm wall thickness,. Determine the maximum normal stress and the maximum shearing stress in the tank.



$d = 180\ \text{mm} \qquad r = \frac{1}{2}d = 90\ \text{mm} \qquad t = 12\ \text{mm}$

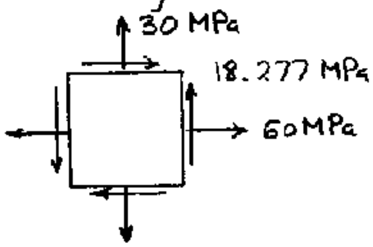
Torsion: $c_1 = 90\ \text{mm} \qquad c_2 = 90 + 12 = 102\ \text{mm}$

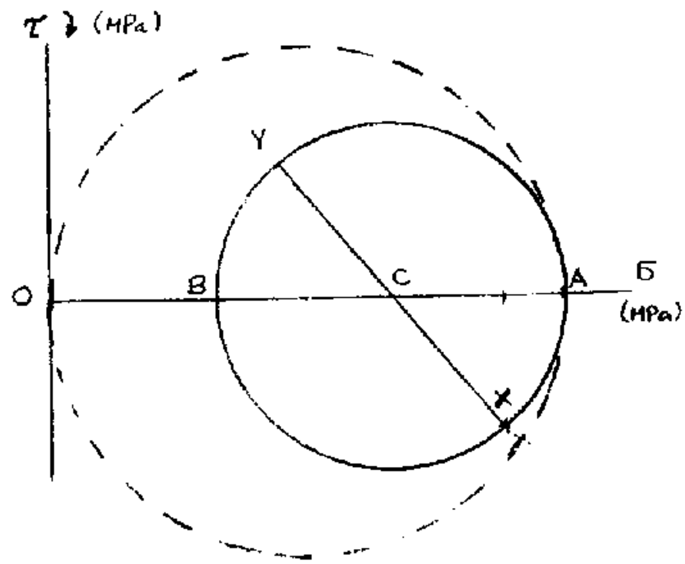
$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = 66.968\times 10^6\ \text{mm}^4 = 66.968\times 10^{-6}\ \text{m}^4$$

$$\tau = \frac{Tc}{J} = \frac{(12\times 10^3)(102\times 10^{-3})}{66.968\times 10^{-6}} = 18.277\ \text{MPa}$$

Pressure: $\sigma_1 = \frac{pr}{t} = \frac{(8)(90)}{12} = 60\ \text{MPa} \qquad \sigma_2 = \frac{pr}{2t} = 30\ \text{MPa}$

Summary of stresses $\qquad \sigma_x = 60\ \text{MPa},\ \sigma_y = 30\ \text{MPa},\ \tau_{xy} = 18.277\ \text{MPa}$





$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 45\ \text{MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 23.64\ \text{MPa}$$

$$\sigma_a = \sigma_{ave} + R = 68.64\ \text{MPa}$$

$$\sigma_b = \sigma_{ave} - R = 21.36\ \text{MPa}$$

$$\sigma_c \approx 0$$

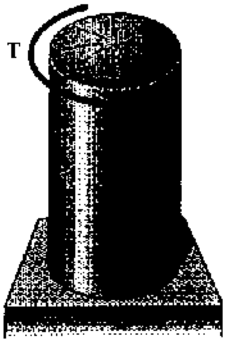
$$\sigma_{max} = 68.64\ \text{MPa}$$

$$\sigma_{min} = 0$$

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 34.32\ \text{MPa}$$

Problem 7.123

7.123 The tank shown has a 180-mm inner diameter and a 12-mm wall thickness. Knowing that the tank contains compressed air under a pressure of 8 MPa, determine the magnitude T of the applied torque for which the maximum normal stress is 75 MPa.



$r = \frac{1}{2}d = (\frac{1}{2})(180) = 90 \text{ mm} \qquad t = 12 \text{ mm}$

$\sigma_1 = \frac{pr}{t} = \frac{(8)(90)}{12} = 60 \text{ MPa} \qquad \sigma_2 = \frac{pr}{2t} = 30 \text{ MPa}$

$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = 45 \text{ MPa}$

$\sigma_{max} = 75 \text{ MPa} \qquad R = \sigma_{max} - \sigma_{ave} = 30 \text{ MPa}$

$R = \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau_{xy}^2} = \sqrt{15^2 + \tau_{xy}^2}$

$\tau_{xy} = \sqrt{R^2 - 15^2} = \sqrt{30^2 - 15^2} = 25.98 \text{ MPa}$

$= 25.98 \times 10^6 \text{ Pa}$

Torsion: $c_1 = 90 \text{ mm} \qquad c_2 = 90 + 12 = 102 \text{ mm}$

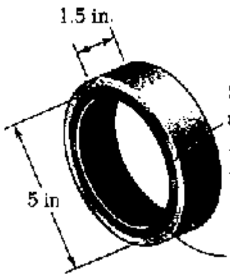
$J = \frac{\pi}{2}(c_2^4 - c_1^4) = 66.968 \times 10^6 \text{ mm}^4 = 66.968 \times 10^{-6} \text{ m}^4$

$\tau_{xy} = \frac{Tc}{J} \qquad T = \frac{J\tau_{xy}}{c} = \frac{(66.968 \times 10^{-6})(25.98 \times 10^6)}{102 \times 10^{-3}} = 17.06 \times 10^3 \text{ N}\cdot\text{m}$

$= 17.06 \text{ kN}\cdot\text{m}$ ◄

Problem 7.124

7.124 A brass ring of 5-in. outer diameter and 0.25-in. thickness fits exactly inside a steel ring of 5-in. inner diameter and 0.125-in. thickness when the temperature of both rings is 50°F. Knowing that the temperature of both rings is then raised to 125°F, determine (*a*) the tensile stress in the steel ring, (*b*) the corresponding pressure exerted by the brass ring on the steel ring.



STEEL
$t_s = \frac{1}{8}$ in.
$E_s = 29 \times 10^6$ psi
$\alpha_s = 6.5 \times 10^{-6}/°F$

BRASS
$t_b = \frac{1}{4}$ in.
$E_b = 15 \times 10^6$ psi
$\alpha_b = 11.6 \times 10^{-6}/°F$

Let p be the contact pressure between the rings. Subscript s refers to the steel ring. Subscript b refers to the brass ring.

Steel ring: Internal pressure p, $\quad \sigma_s = \frac{pr}{t_s} \qquad (1)$

Corresponding strain $\quad \varepsilon_{sp} = \frac{\sigma_s}{E_s} = \frac{pr}{E_s t_s}$

Strain due to temperature change $\quad \varepsilon_{sT} = \alpha_s \Delta T$

Total strain $\quad \varepsilon_s = \frac{pr}{E_s t_s} + \alpha_s \Delta T$

Change in length of circumference

$$\Delta L_s = 2\pi r \varepsilon_s = 2\pi r\left(\frac{pr}{E_s t_s} + \alpha_s \Delta T\right)$$

Brass ring: External pressure p, $\quad \sigma_b = -\frac{pr}{t_b}$

Corresponding strains $\quad \varepsilon_{bp} = -\frac{pr}{E_b t_b}, \quad \varepsilon_{bT} = \alpha_b \Delta T$

Change in length of circumference

$$\Delta L_b = 2\pi r \varepsilon_b = 2\pi r\left(-\frac{pr}{E_b t_b} + \alpha_b \Delta T\right)$$

Equating ΔL_s to ΔL_b $\qquad \frac{pr}{E_s t_s} + \alpha_s \Delta T = -\frac{pr}{E_b t_b} + \alpha_b \Delta T$

$$\left(\frac{r}{E_s t_s} + \frac{r}{E_b t_b}\right) p = (\alpha_b - \alpha_s)\Delta T \qquad (2)$$

Data: $\quad \Delta T = 125°F - 50°F = 75°F$

$$r = \tfrac{1}{2} d = \tfrac{1}{2}(5) = 2.5 \text{ in.}$$

From Eq. (2) $\quad \left[\frac{2.5}{(29\times10^6)(0.125)} + \frac{2.5}{(15\times10^6)(0.25)}\right] p = (11.6 - 6.5)(10^{-6})(75)$

$$1.35632\times10^{-6}\, p = 382.5\times10^{-6} \qquad p = 282.0 \text{ psi}$$

From Eq. (1) $\quad \sigma_s = \frac{pr}{t_s} = \frac{(282.0)(2.5)}{0.125} = 5.64\times10^3$ psi

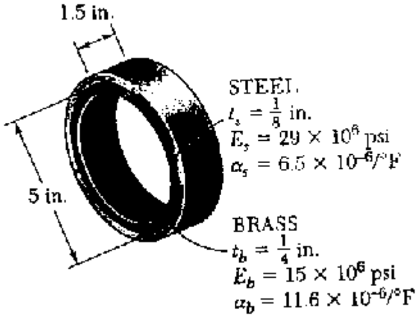
(a) $\sigma_s = 5.64$ ksi ◀

(b) $p = 282$ psi ◀

Problem 7.125

7.125 Solve Prob. 7.124, assuming that the brass ring is 0.125 in. thick and the steel ring is 0.25 in. thick.

7.124 A brass ring of 5-in. outer diameter and 0.25-in. thickness fits exactly inside a steel ring of 5-in. inner diameter and 0.125-in. thickness when the temperature of both rings is 50°F. Knowing that the temperature of both rings is then raised to 125°F, determine (*a*) the tensile stress in the steel ring, (*b*) the corresponding pressure exerted by the brass ring on the steel ring.



Let p be the contact pressure between the rings. Subscript s refers to the steel ring. Subscript b refers to the brass ring.

Steel ring: Internal pressure p, $\sigma_s = \dfrac{pr}{t_s}$ (1)

Corresponding strain $\varepsilon_{sp} = \dfrac{\sigma_s}{E_s} = \dfrac{pr}{E_s t_s}$

Strain due to temperature change $\varepsilon_{sT} = \alpha_s \Delta T$

Total strain $\varepsilon_s = \dfrac{pr}{E_s t_s} + \alpha_s \Delta T$

Change in length of circumference

$$\Delta L_s = 2\pi r \varepsilon_s = 2\pi r \left(\frac{pr}{E_s t_s} + \alpha_s \Delta T \right)$$

Brass ring: External pressure p, $\sigma_b = -\dfrac{pr}{t_b}$

Corresponding strains $\varepsilon_{bp} = -\dfrac{pr}{E_b t_b}$, $\varepsilon_{bT} = \alpha_b \Delta T$

Change in length of circumference

$$\Delta L_b = 2\pi r \varepsilon_b = 2\pi r \left(-\frac{pr}{E_b t_b} + \alpha_b \Delta T \right)$$

Equating ΔL_s to ΔL_b $\quad \dfrac{pr}{E_s t_s} + \alpha_s \Delta T = -\dfrac{pr}{E_b t_b} + \alpha_b \Delta T$

$$\left(\frac{r}{E_s t_s} + \frac{r}{E_b t_b} \right) p = (\alpha_b - \alpha_s)\, \Delta T \qquad (2)$$

Data: $\Delta T = 125°F - 50°F = 75°F$

$r = \frac{1}{2} d = \frac{1}{2}(5) = 2.5$ in.

From Eq. (2) $\left[\dfrac{2.5}{(29 \times 10^6)(0.25)} + \dfrac{2.5}{(15 \times 10^6)(0.125)} \right] p = (11.6 - 6.5)(10^{-6})(75)$

$1.67816 \times 10^{-6}\, p = 382.5 \times 10^{-6}$ $\qquad p = 227.93$ psi

From Eq. (1) $\sigma_s = \dfrac{pr}{t_s} = \dfrac{(227.93)(2.5)}{0.25} = 2279$ psi

(a) $\sigma_s = 2.28$ ksi ◄

(b) $p = 228$ psi ◄

Problem 7.126

7.126 through 7.129 For the given state of plane strain, use the methods of Sec. 7.10 to determine the state of strain associated with axes x' and y' rotated through the given angle θ.

7.126 $\varepsilon_x = +240\mu$, $\varepsilon_y = +160\mu$, $\gamma_{xy} = +150\mu$, $\theta = 60°\circlearrowright = -60°$

$$\frac{\varepsilon_x + \varepsilon_y}{2} = +200\mu \qquad \frac{\varepsilon_x - \varepsilon_y}{2} = 40\mu$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left\{200 + 40\cos(-120°) + \frac{150}{2}\sin(-120°)\right\}\mu = 115.0\mu$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left\{200 - 40\cos(-120°) - \frac{150}{2}\sin(-120°)\right\}\mu = 285\mu$$

$$\gamma_{x'y'} = -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

$$= \left\{-(240 - 160)\sin(-120°) + 150\cos(-120°)\right\}\mu = -5.72\mu$$

Problem 7.127

7.126 through 7.129 For the given state of plane strain, use the methods of Sec. 7.10 to determine the state of strain associated with axes x' and y' rotated through the given angle θ.

7.127 $\varepsilon_x = 0$, $\varepsilon_y = +320\mu$, $\gamma_{xy} = -100\mu$, $\theta = 30°\circlearrowleft = +30°$

$$\frac{\varepsilon_x + \varepsilon_y}{2} = 160\mu \qquad \frac{\varepsilon_x - \varepsilon_y}{2} = -160\mu$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left\{160 - 160\cos 60° - \frac{100}{2}\sin 60°\right\}\mu = +36.7\mu$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left\{160 + 160\cos 60° + \frac{100}{2}\sin 60°\right\}\mu = +283\mu$$

$$\gamma_{x'y'} = -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

$$= \left\{-(0 - 320)\sin 60° - 100\cos 60°\right\}\mu = +227\mu$$

Problem 7.128

7.126 through 7.129 For the given state of plane strain, use the methods of Sec. 7.10 to determine the state of strain associated with axes x' and y' rotated through the given angle θ

7.128 $\varepsilon_x = -800\,\mu,\ \ \varepsilon_y = +450\,\mu,\ \ \gamma_{xy} = +200\,\mu,\ \ \theta = 25°\circlearrowright = -25°$

$$\frac{\varepsilon_x + \varepsilon_y}{2} = -175\,\mu \qquad \frac{\varepsilon_x - \varepsilon_y}{2} = -625\,\mu$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left\{-175 - 625\cos(-50°) + \frac{200}{2}\sin(-50°)\right\}\mu = -653\,\mu$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left\{-175 + 625\cos(-50°) - \frac{200}{2}\sin(-50°)\right\}\mu = +303\,\mu$$

$$\gamma_{x'y'} = -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

$$= \left\{-(-800 - 450)\sin(-50°) + 200\cos(-50°)\right\}\mu = -829\,\mu$$

Problem 7.129

7.126 through 7.129 For the given state of plane strain, use the methods of Sec. 7.10 to determine the state of strain associated with axes x' and y' rotated through the given angle θ

7.129 $\varepsilon_x = +500\,\mu,\ \ \varepsilon_y = -300\,\mu,\ \ \gamma_{xy} = 0,\ \ \theta = 25°\circlearrowleft = +25°$

$$\frac{\varepsilon_x + \varepsilon_y}{2} = 100\,\mu \qquad \frac{\varepsilon_x - \varepsilon_y}{2} = 400\,\mu$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \{100 + 400\cos 50° + 0\}\mu = 357\,\mu$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \{100 - 400\cos 50° - 0\}\mu = -157.1\,\mu$$

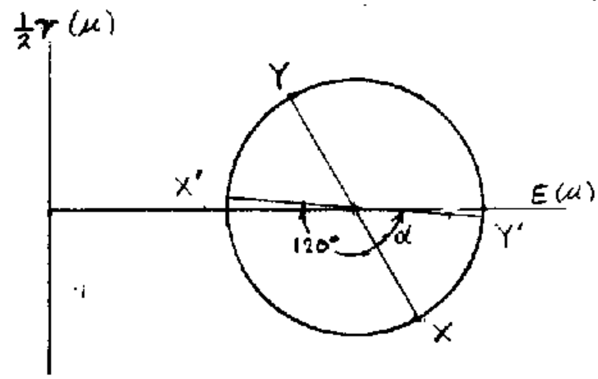
$$\gamma_{x'y'} = -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

$$= \{-(500 - (-300))\sin 50° + 0\}\mu = -613\,\mu$$

Problem 7.130

7.130 through 7.133 For the given state of plane strain, Use Mohr's circle to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ.

7.130 $\varepsilon_x = +240\mu,\ \varepsilon_y = +160\mu,\ \gamma_{xy} = +150\mu,\ \theta = 60°\circlearrowleft$



Plotted points

X: $(+240\mu, -75\mu)$

Y: $(+160\mu, 75\mu)$

C: $(+200\mu, 0)$

$\tan\alpha = \frac{75}{40} = 1.875 \qquad \alpha = 61.93°$

$R = \sqrt{(40\mu)^2 + (75\mu)^2} = 85\mu$

$\beta = 2\theta - \alpha = -120° - 61.93° = -181.93°$

$\varepsilon_{x'} = \varepsilon_{ave} + R\cos\beta = 200\mu + (85\mu)\cos(-181.93°)$
$= 115.0\ \mu$ ◄

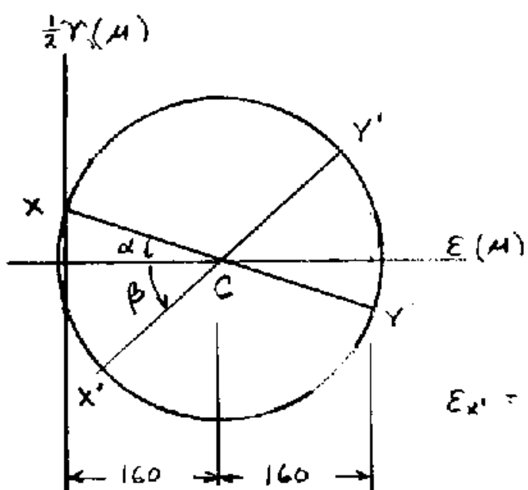
$\varepsilon_{y'} = \varepsilon_{ave} - R\cos\beta = 200\mu - (85\mu)\cos(-181.93°)$
$= 285\ \mu$ ◄

$\frac{1}{2}\gamma_{x'y'} = -R\sin\beta = -85\sin(-181.93) = -2.86\mu$
$\gamma_{x'y'} = -5.7\ \mu$ ◄

Problem 7.131

7.130 through 7.133 For the given state of plane strain, Use Mohr's circle to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ.

7.131 $\varepsilon_x = 0,\quad \varepsilon_y = +320\mu,\quad \gamma_{xy} = -100\mu,\ \theta = 30°\circlearrowright$



Plotted points

X: $(0, 50\mu)$

Y: $(320\mu, -50\mu)$

C: $(160\mu, 0)$

$\tan\alpha = \frac{50}{160} \qquad \alpha = 17.35°$

$R = \sqrt{(160\mu)^2 + (50\mu)^2} = 167.63\ \mu$

$\beta = 2\theta - \alpha = 60° - 17.35° = 42.65°$

$\varepsilon_{x'} = \varepsilon_{ave} - R\cos\beta = 160\mu - 167.63\mu\cos 42.65°$
$= 36.7\ \mu$ ◄

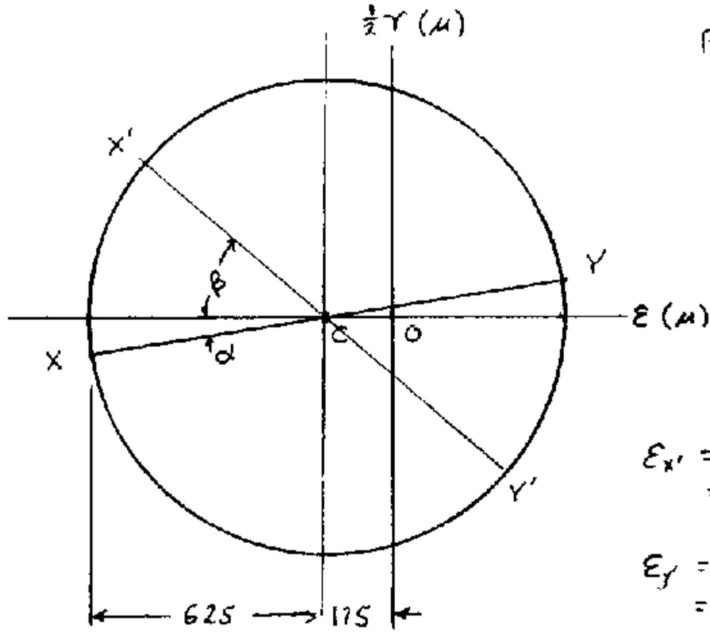
$\varepsilon_{y'} = \varepsilon_{ave} + R\cos\beta = 160\mu + 167.63\mu\cos 42.65°$
$= 283\ \mu$ ◄

$\frac{1}{2}\gamma_{x'y'} = R\sin\beta = 167.63\mu\sin 42.65°$
$\gamma_{x'y'} = 227\ \mu$ ◄

Problem 7.132

7.130 through 7.133 For the given state of plane strain, Use Mohr's circle to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ.

7.132 $\varepsilon_x = -800\mu \quad \varepsilon_y = 450\mu \quad \gamma_{xy} = +200\mu \quad \theta = 25°$



Plotted points
X: $(-800\mu, -100\mu)$
Y: $(+450\mu, +100\mu)$
C: $(-175\mu, 0)$

$\tan\alpha = \dfrac{100}{625} \qquad \alpha = 9.09°$

$R = \sqrt{(625\mu)^2 + (100\mu)^2} = 632.95\mu$

$\beta = 2\theta - \alpha = 50° - 9.09° = 40.91°$

$\varepsilon_{x'} = \varepsilon_{ave} - R\cos\beta = -175\mu - 632.95\mu \cos 40.91°$
$= -653\mu$ ◀

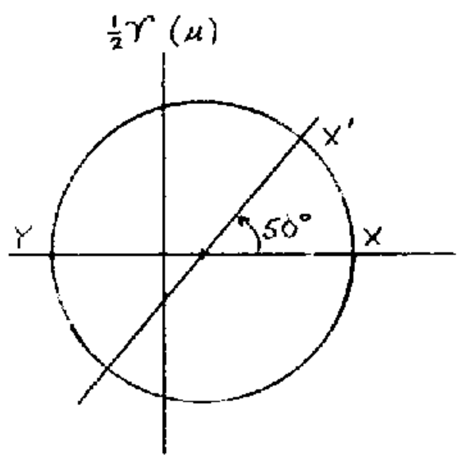
$\varepsilon_{y'} = \varepsilon_{ave} + R\cos\beta = -175$
$= +303\mu$ ◀

$\frac{1}{2}\gamma_{x'y'} = -R\sin\beta = -632.95\mu \sin 40.91° \qquad \gamma_{x'y'} = -829\mu$ ◀

Problem 7.133

7.130 through 7.133 For the given state of plane strain, Use Mohr's circle to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ.

7.133 $\varepsilon_x = +500\mu, \quad \varepsilon_y = -300\mu, \quad \gamma_{xy} = 0, \quad \theta = 25°$



Plotted points
X: $(500\mu, 0)$
Y: $(-300\mu, 0)$
C: $(100\mu, 0)$

$\tan\alpha = 0 \qquad \alpha = 0$

$R = 400\mu \qquad \beta = 2\theta - \alpha = 50°$

$\varepsilon_{x'} = \varepsilon_{ave} + R\cos\beta = 100\mu + (400\mu)\cos 50°$
$= 357\mu$ ◀

$\varepsilon_{y'} = \varepsilon_{ave} - R\cos\beta = 100\mu - (400\mu)\cos 50°$
$= -157.1\mu$ ◀

$\frac{1}{2}\gamma_{x'y'} = -R\sin\beta = -400\sin 50° = -306.4\mu \qquad \gamma_{x'y'} = -613\mu$ ◀

Problem 7.134

7.134 through 7.137 The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (*a*) the direction and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain. (Use $\nu = \frac{1}{3}$)

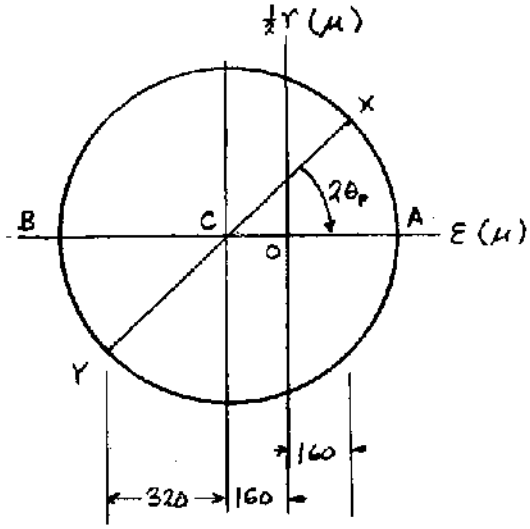
7.134 $\varepsilon_x = +160\,\mu \qquad \varepsilon_y = -480\,\mu \qquad \gamma_{xy} = -600\,\mu$

(a) For Mohr's circle of strain, plot points

X: $(160\,\mu,\ 300\,\mu)$

Y: $(-480\,\mu,\ -300\,\mu)$

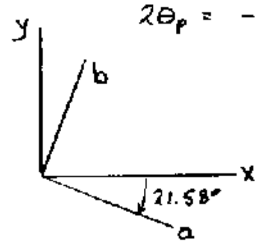
C: $(-160\,\mu,\ 0)$



(a) $\tan 2\theta_p = \dfrac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \dfrac{-300}{320} = -0.9375$

$2\theta_p = -43.15^\circ \qquad \theta_p = -21.58^\circ$

and $-21.58 + 90 = 68.42^\circ$



$\theta_a = -21.58^\circ$ ◄

$\theta_b = 68.42^\circ$ ◄

$R = \sqrt{(320\,\mu)^2 + (300\,\mu)^2} = 438.6\,\mu$

$\varepsilon_a = \varepsilon_{ave} + R = -160\,\mu + 438.6\,\mu = +278.6\,\mu$ ◄

$\varepsilon_b = \varepsilon_{ave} - R = -160\,\mu - 438.6\,\mu = -598.6\,\mu$ ◄

(b) $\frac{1}{2}\gamma_{(max,\ in\text{-}plane)} = R \qquad \gamma_{(max,\ in\text{-}plane)} = 2R = 877\,\mu$ ◄

(c) $\varepsilon_c = -\dfrac{\nu}{1-\nu}(\varepsilon_a + \varepsilon_b) = -\dfrac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = -\dfrac{1/3}{2/3}(160\,\mu - 480\,\mu)$

$= 160\,\mu$ ◄

$\varepsilon_{max} = 278.6\,\mu \qquad \varepsilon_{min} = -598.6\,\mu$

$\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 278.6\,\mu + 598.6\,\mu = 877\,\mu$ ◄

Problem 7.135

7.134 through 7.137 The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (*a*) the direction and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain. (Use $\nu = \frac{1}{3}$)

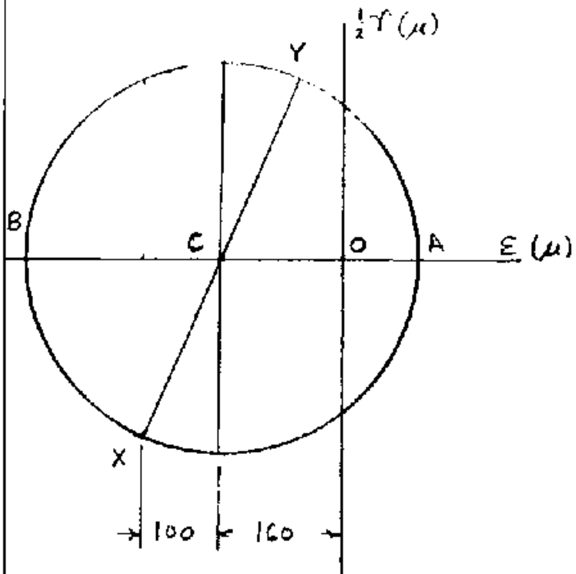
7.135 $\varepsilon_x = -260\mu \qquad \varepsilon_y = -60\mu \qquad \gamma_{xy} = +480\mu$

For Mohr's circle of strain plot points

$X: (-260\mu, -240\mu)$

$Y: (-60\mu, 240\mu)$

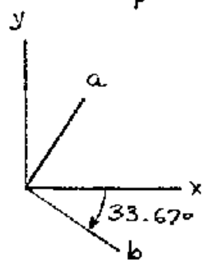
$C: (-160\mu, 0)$



$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{480}{-260+60} = -2.4$$

$2\theta_p = -67.38°$ $\qquad \theta_b = -33.67°$

$\theta_a = 56.31°$



$$R = \sqrt{(100\mu)^2 + (240\mu)^2}$$

$R = 260\mu$

(a) $\varepsilon_a = \varepsilon_{ave} + R = -160\mu + 260\mu = 100\mu$

$\varepsilon_b = \varepsilon_{ave} - R = -160\mu - 260\mu = -420\mu$

(b) $\frac{1}{2}\gamma_{max\,(in\text{-}plane)} = R \qquad \gamma_{max\,(in\text{-}plane)} = 2R = 520\mu$

$$\varepsilon_c = -\frac{\nu}{1-\nu}(\varepsilon_a + \varepsilon_b) = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = -\frac{1/3}{2/3}(-260 - 60)$$
$$= 160\mu$$

$\varepsilon_{max} = 160\mu \qquad \varepsilon_{min} = -420\mu$

(c) $\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 160\mu + 420\mu = 580\mu$

Problem 7.136

7.134 through 7.137 The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (*a*) the direction and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain. (Use $\nu = 1/3$)

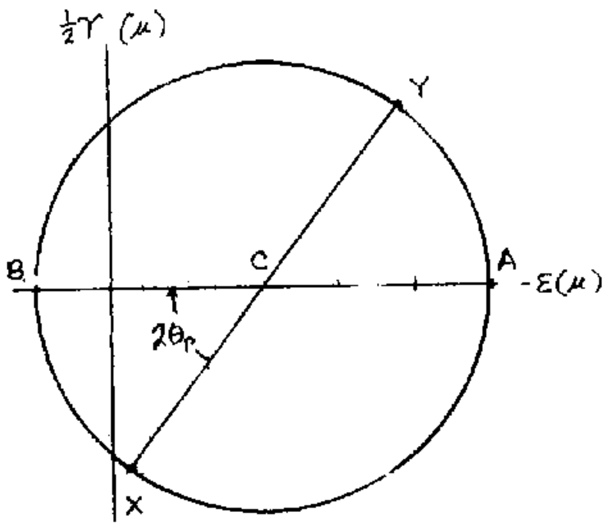
7.136 $\varepsilon_x = +30\,\mu, \quad \varepsilon_y = +570\,\mu, \quad \gamma_{xy} = +720\,\mu$

Plotted points.

X : $(30\,\mu, -360\,\mu)$

Y : $(570\,\mu, +360\,\mu)$

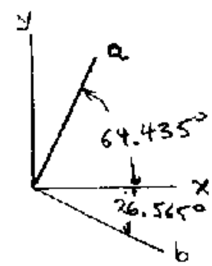
C : $(300\,\mu, 0)$



$$\tan 2\theta_p = \frac{-360}{270} = -1.3333$$

$$2\theta_p = -53.13^\circ$$

(a) $\theta_b = -26.565^\circ$, $\theta_a = 64.435^\circ$



$$R = \sqrt{(270\,\mu)^2 + (360\,\mu)^2} = 450\,\mu$$

$$\varepsilon_a = \varepsilon_{ave} + R = 300\,\mu + 450\,\mu = 750\,\mu$$

$$\varepsilon_b = \varepsilon_{ave} - R = 300\,\mu - 450\,\mu = -150\,\mu$$

(b) $\gamma_{max\,(in\text{-}plane)} = 2R = 900\,\mu$

$$\varepsilon_c = -\frac{\nu}{1-\nu}(\varepsilon_a + \varepsilon_b) = -\frac{1/3}{2/3}(750\,\mu - 150\,\mu) = -300\,\mu$$

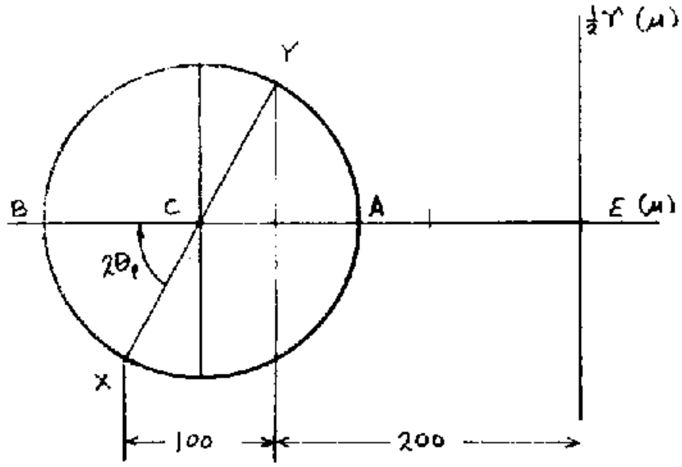
$$\varepsilon_{max} = \varepsilon_a = 750\,\mu, \qquad \varepsilon_{min} = \varepsilon_c = -300\,\mu$$

(c) $\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 750\,\mu - (-300\,\mu) = 1050\,\mu$

Problem 7.137

7.134 through 7.137 The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (*a*) the direction and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain. (Use $\nu = \frac{1}{3}$)

7.137 $\varepsilon_x = -600\,\mu \qquad \varepsilon_y = -400\,\mu \qquad \gamma_{xy} = +350\,\mu$



Plotted points

X: $(-600\,\mu,\ -175\,\mu)$

Y: $(-400\,\mu,\ +175\,\mu)$

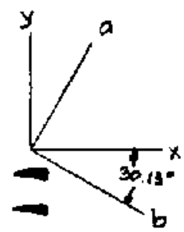
C: $(-500\,\mu,\ 0)$

$\tan 2\theta_p = -\frac{175}{100}$

$2\theta_p = -60.26$

$\theta_b = -30.13°$

$\theta_a = 59.87°$



$R = \sqrt{(100\,\mu)^2 + (175\,\mu)^2} = 201.6\,\mu$

(a) $\varepsilon_a = \varepsilon_{ave} + R = -500\mu + 201.6\mu = -298\,\mu$

$\varepsilon_b = \varepsilon_{ave} - R = -500\mu - 201.6\mu = -702\,\mu$

(b) $\gamma_{max\,(in\text{-}plane)} = 2R = 403\,\mu$

$\varepsilon_c = -\frac{\nu}{1-\nu}(\varepsilon_a + \varepsilon_b) = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = -\frac{1/3}{2/3}(-600\mu - 400\mu) = +500\,\mu$

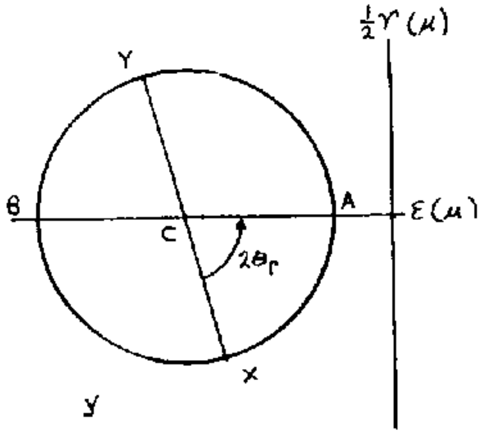
$\varepsilon_{max} = 500\mu \qquad \varepsilon_{min} = -702\,\mu$

(c) $\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 500\mu + 702\mu = 1202\,\mu$

Problem 7.138

7.138 through 7.141 The for given state of plane strain, use Mohr's circle to determine (*a*) the orientation and magnitude of the principal strains, (*b*) the maximum in-plane strain, (*c*) the maximum shearing strain.

7.138 $\varepsilon_x = -180\mu, \quad \varepsilon_y = -260\mu \quad \gamma_{xy} = +315\mu$



Plot points

X: $(-180\mu, -157.5\mu)$ Y: $(-260\mu, +157.5\mu)$

C: $(-220\mu, 0)$

(a) $\tan 2\theta_p = \dfrac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \dfrac{315}{80} = 3.9375$

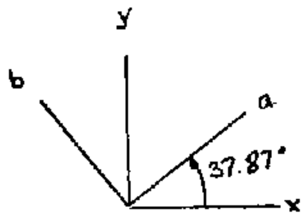
$2\theta_p = 75.75°$ $\quad \theta_a = 37.87°$

$\theta_b = 127.87°$

$R = \sqrt{(40\mu)^2 + (157.5\mu)^2} = 162.5\mu$

$\varepsilon_a = \varepsilon_{ave} + R = -220\mu + 162.5\mu = -57.5\mu$

$\varepsilon_b = \varepsilon_{ave} - R = -220\mu - 162.5 = -382.5\mu$



(b) $\gamma_{max(inplane)} = 2R = 325\ \mu$

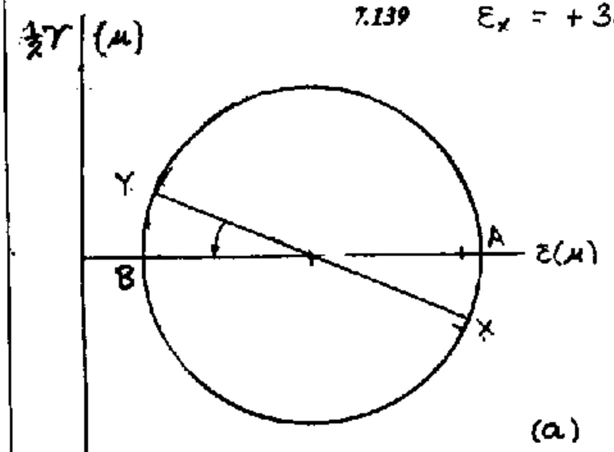
(c) $\varepsilon_c = 0 \qquad \varepsilon_{max} = 0, \quad \varepsilon_{min} = -382.5\mu$

$\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 0 + 382.5\mu = 382.5\mu$

Problem 7.139

7.138 through 7.141 The for given state of plane strain, use Mohr's circle to determine (*a*) the orientation and magnitude of the principal strains, (*b*) the maximum in-plane strain, (*c*) the maximum shearing strain.

7.139 $\varepsilon_x = +300\mu \quad \varepsilon_y = +60\mu \quad \gamma_{xy} = +100\mu$



X: $(300\mu, -50\mu)$ Y: $(60\mu, 50\mu)$

C: $(180\mu, 0)$

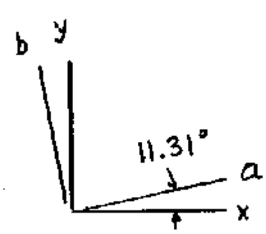
$\tan 2\theta_p = \dfrac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \dfrac{100}{300-60} = 22.62°$

$\theta_a = 11.31° \qquad \theta_b = 101.31°$

$R = \sqrt{(120\mu)^2 + (50\mu)^2} = 130\mu$

(a) $\varepsilon_a = \varepsilon_{ave} + R = 180\mu + 130\mu = 310\mu$

$\varepsilon_b = \varepsilon_{ave} - R = 180\mu - 130\mu = 50\mu$



(b) $\gamma_{max\,(in\text{-}plane)} = 2R = 260\mu$

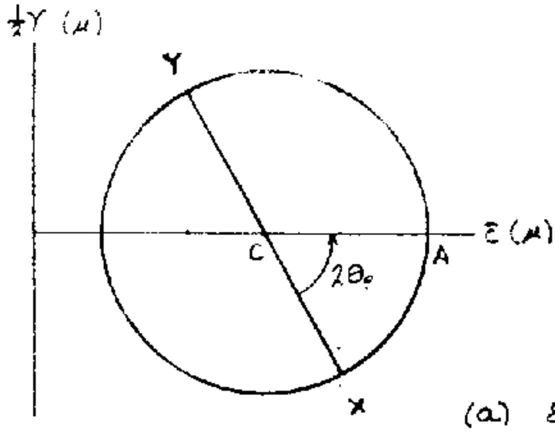
(c) $\varepsilon_c = 0 \qquad \varepsilon_{max} = 310\mu \quad \varepsilon_{min} = 0$

$\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 310\mu$

Problem 7.140

7.138 through 7.141 The for given state of plane strain, use Mohr's circle to determine (*a*) the orientation and magnitude of the principal strains, (*b*) the maximum in-plane strain, (*c*) the maximum shearing strain.

7.140 $\varepsilon_x = +400\,\mu \qquad \varepsilon_y = +200\,\mu \qquad \gamma_{xy} = +375\,\mu$



Plotted points

X: $(+400\mu,\ -187.5\mu)$

Y: $(+200\mu,\ +187.5\mu)$

C: $(+300\mu,\ 0)$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{375}{400-200} = 1.875$$

$2\theta_p = 61.93°$ $\quad \theta_a = 30.96°$, $\theta_b = 120.96°$

$$R = \sqrt{(100\mu)^2 + (187.5\mu)^2} = 212.5\mu$$

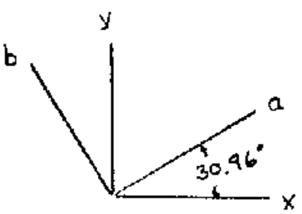
(a) $\varepsilon_a = \varepsilon_{ave} + R = 300\mu + 212.5\mu = 512.5\mu$

$\varepsilon_b = \varepsilon_{ave} - R = 300\mu - 212.5\mu = 87.5\mu$

(b) $\gamma_{max\,(in\text{-}plane)} = 2R = 425\mu$

(c) $\varepsilon_c = 0 \qquad \varepsilon_{max} = 512.5\mu \qquad \varepsilon_{min} = 0$

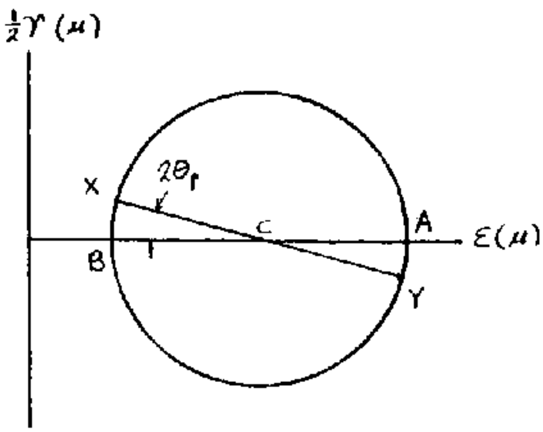
$\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 512.5\mu$



Problem 7.141

7.138 through 7.141 The for given state of plane strain, use Mohr's circle to determine (*a*) the orientation and magnitude of the principal strains, (*b*) the maximum in-plane strain, (*c*) the maximum shearing strain.

7.141 $\varepsilon_x = +60\,\mu \qquad \varepsilon_y = +240\,\mu \qquad \gamma_{xy} = -50\,\mu$



Plotted points X: $(60\mu,\ 25\mu)$

Y: $(240\mu,\ -25\mu)$ $\quad$ C: $(150\mu,\ 0)$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-50}{60-240} = 0.277778$$

$2\theta_p = 15.52°$ $\quad \theta_b = 7.76°$ $\quad \theta_a = 97.76°$

$$R = \sqrt{(90\mu)^2 + (25\mu)^2} = 93.4\ \mu$$

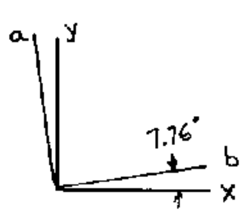
(a) $\varepsilon_a = \varepsilon_{ave} + R = 150\mu + 93.4\mu = 243.4\mu$

$\varepsilon_b = \varepsilon_{ave} - R = 150\mu - 93.4\mu = 56.6\mu$

(b) $\gamma_{max\,(in\text{-}plane)} = 2R = 186.8\mu$

(c) $\varepsilon_c = 0 \qquad \varepsilon_{max} = 243.4\mu \qquad \varepsilon_{min} = 0$

$\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 243.4$

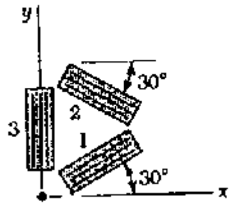


Problem 7.142

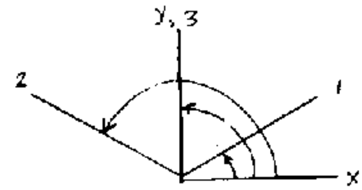
7.142 The strains determined by the use of the rosette shown during the test of a machine element are"

$\epsilon_1 = +600\mu$ $\quad$ $\epsilon_2 = +450\mu$ $\quad$ $\epsilon_3 = -75\mu$

Determine (*a*) the in-plane principal strains, (*b*) the in-plane maximum shearing strain.



$\theta_1 = 30°$
$\theta_2 = 150°$
$\theta_3 = 90°$



$$\varepsilon_x \cos^2\theta_1 + \varepsilon_y \sin^2\theta_1 + \gamma_{xy}\sin\theta_1\cos\theta_1 = \varepsilon_1$$
$$0.75\,\varepsilon_x + 0.25\,\varepsilon_y + 0.43301\,\gamma_{xy} = 600\mu \quad (1)$$
$$\varepsilon_x \cos^2\theta_2 + \varepsilon_y \sin^2\theta_2 + \gamma_{xy}\sin\theta_2\cos\theta_2 = \varepsilon_2$$
$$0.75\,\varepsilon_x + 0.25\,\varepsilon_y - 0.43301\,\gamma_{xy} = 450\mu \quad (2)$$
$$\varepsilon_x \cos^2\theta_3 + \varepsilon_y \sin^2\theta_3 + \gamma_{xy}\sin\theta_3\cos\theta_3 = \varepsilon_3$$
$$0 + \varepsilon_y \quad 0 = -75\mu \quad (3)$$

Solving (1), (2), and (3) simultaneously

$$\varepsilon_x = 725\,\mu, \quad \varepsilon_y = -75\,\mu, \quad \gamma_{xy} = 173.21\,\mu$$

$$\varepsilon_{ave} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = 325\,\mu$$

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \sqrt{\left(\frac{725 + 75}{2}\right)^2 + \left(\frac{173.21}{2}\right)^2} = 409.3\,\mu$$

(a) $\varepsilon_a = \varepsilon_{ave} + R = 734\,\mu$ ◄

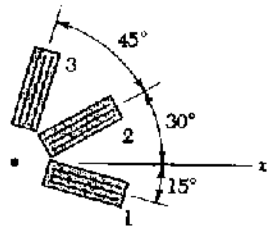
$\varepsilon_b = \varepsilon_{ave} - R = -84.3\,\mu$ ◄

(b) $\gamma_{max(in\text{-}plane)} = 2R = 819\,\mu$ ◄

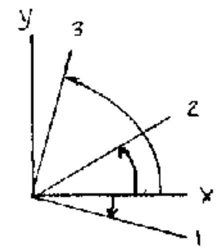
Problem 7.143

7.143 Determine the strain ϵ_x knowing that the following strains have been determined by use of the rosette shown:

$\epsilon_1 = +480 \times 10^{-6}$ in./in. $\quad \epsilon_2 = -120 \times 10^{-6}$ in./in. $\quad \epsilon_3 = +80 \times 10^{-6}$ in./in.



$\theta_1 = -15°$
$\theta_2 = 30°$
$\theta_3 = 75°$



$$\varepsilon_x \cos^2\theta_1 + \varepsilon_y \sin^2\theta_1 + \gamma_{xy} \sin\theta_1 \cos\theta_1 = \varepsilon_1$$

$$0.9330\,\varepsilon_x + 0.06699\,\varepsilon_y - 0.25\,\gamma_{xy} = 480 \times 10^{-6} \qquad (1)$$

$$\varepsilon_x \cos^2\theta_2 + \varepsilon_y \sin^2\theta_2 + \gamma_{xy} \sin\theta_2 \cos\theta_2 = \varepsilon_2$$

$$0.75\,\varepsilon_x + 0.25\,\varepsilon_y + 0.4330\,\gamma_{xy} = -120 \times 10^{-6} \qquad (2)$$

$$\varepsilon_x \cos^2\theta_3 + \varepsilon_y \sin^2\theta_3 + \gamma_{xy} \sin\theta_3 \cos\theta_3 = \varepsilon_3$$

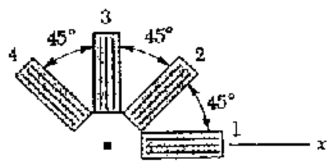
$$0.06699\,\varepsilon_x + 0.9330\,\varepsilon_y + 0.25\,\gamma_{xy} = 80 \times 10^{-6} \qquad (3)$$

Solving (1), (2), and (3) simultaneously

$\varepsilon_x = 253 \times 10^{-6}$ in/in , $\varepsilon_y = 307 \times 10^{-6}$ in/in , $\gamma_{xy} = -893 \times 10^{-6}$ in/in

$\varepsilon_x = 253 \times 10^{-6}$ in/in

Problem 7.144



7.144 The rosette shown has been used to determine the following strains at a point on the surface of a crane hook:

$\epsilon_1 = +420\ \mu$ $\quad\epsilon_2 = -45\ \mu$ $\quad\epsilon_4 = +165\ \mu$

(*a*) What should be the reading of gage 3? (*b*) Determine the principal strains and the maximum in-plane shearing strain.

(a) Gages 2 and 4 are 90° apart $\quad \varepsilon_{ave} = \frac{1}{2}(\varepsilon_2 + \varepsilon_4)$

$$\varepsilon_{ave} = \frac{1}{2}(-45\mu + 165\mu) = 60\mu$$

Gages 1 and 3 are also 90° apart $\quad \varepsilon_{ave} = \frac{1}{2}(\varepsilon_1 + \varepsilon_3)$

$$\varepsilon_3 = 2\varepsilon_{ave} - \varepsilon_1 = (2)(60\mu) - 420\mu = -300\mu$$

(b) $\varepsilon_x = \varepsilon_1 = 420\ \mu \qquad \varepsilon_y = \varepsilon_3 = -300\mu$

$$\gamma_{xy} = 2\varepsilon_2 - \varepsilon_x - \varepsilon_1 = (2)(-45\mu) - 420\mu + 300\mu$$

$$= -210\mu$$

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \sqrt{\left(\frac{420\mu + 300\mu}{2}\right)^2 + \left(\frac{-210\mu}{2}\right)^2}$$

$$= 375\mu$$

$$\varepsilon_a = \varepsilon_{ave} + R = 60\mu + 375\mu = 435\mu$$

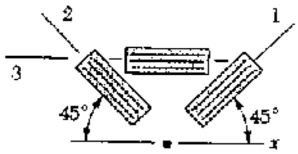
$$\varepsilon_b = \varepsilon_{ave} - R = 60\mu - 375\mu = -315\mu$$

$$\gamma_{max(in\text{-}plane)} = 2R = 750\mu$$

Problem 7.145

7.145 Determine the largest in-plane normal strain, knowing that the following strains have been obtained by the used of the rosette shown:

$\epsilon_1 = -50 \times 10^{-6}$ in./in. $\quad \epsilon_2 = +360 \times 10^{-6}$ in./in. $\quad \epsilon_3 = +315 \times 10^{-6}$ in./in.



$\theta_1 = 45° \qquad \theta_2 = -45° \qquad \theta_3 = 0$

$$\varepsilon_x \cos^2\theta_1 + \varepsilon_y \sin^2\theta_1 + \gamma_{xy}\sin\theta_1\cos\theta_1 = \varepsilon_1$$

$$0.5\,\varepsilon_x + 0.5\,\varepsilon_y + 0.5\,\gamma_{xy} = -50\times10^{-6} \qquad (1)$$

$$\varepsilon_x \cos^2\theta_2 + \varepsilon_y \sin^2\theta_2 + \gamma_{xy}\sin\theta_2\cos\theta_2 = \varepsilon_2$$

$$0.5\,\varepsilon_x + 0.5\,\varepsilon_y - 0.5\,\gamma_{xy} = 360\times10^{-6} \qquad (2)$$

$$\varepsilon_x \cos^2\theta_3 + \varepsilon_y \sin^2\theta_3 + \gamma_{xy}\sin\theta_3\cos\theta_3 = \varepsilon_3$$

$$\varepsilon_x + 0 + 0 = 315\times10^{-6} \qquad (3)$$

From (3) $\qquad \varepsilon_x = 315\times10^{-6}$ in/in.

Eq. (1) − Eq. (2) $\qquad \gamma_{xy} = -50\times10^{-6} - 360\times10^{-6} = -410\times10^{-6}$ in/in

Eq (1) + Eq (2) $\qquad \varepsilon_x + \varepsilon_y = \varepsilon_1 + \varepsilon_2$

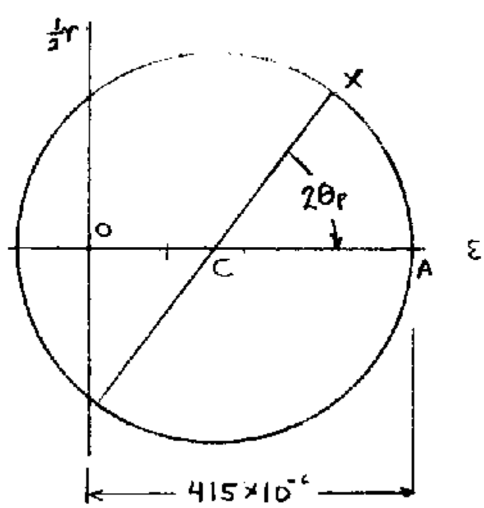
$$\varepsilon_y = \varepsilon_1 + \varepsilon_2 - \varepsilon_x = -50\times10^{-6} + 360\times10^{-6} - 315\times10^{-6} = -5\times10^{-6} \text{ in/in.}$$

$$\varepsilon_{ave} = \tfrac{1}{2}(\varepsilon_x + \varepsilon_y) = 155\times10^{-6} \text{ in/in}$$

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \sqrt{\left(\frac{315\times10^{-6} + 5\times10^{-6}}{2}\right)^2 + \left(\frac{-410\times10^{-6}}{2}\right)^2}$$

$$= 260\times10^{-6} \text{ in/in.}$$

$$\varepsilon_{max} = \varepsilon_{ave} + R = 415\times10^{-6} \text{ in/in} \quad ◄$$



$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = -1.323$$

$$2\theta_p = -52.0°$$

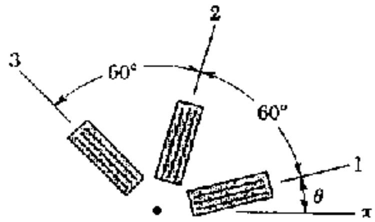
$$\theta_p = -26.0°$$

Problem 7.146

7.146 Show that the sum of the three strain measurements made with a 60° rosette is independent of the orientation of the rosette and equal to

$$\epsilon_1 + \epsilon_2 + \epsilon_3 = 3\epsilon_{avg}$$

where ϵ_{avg} is the abscissa of the center of the corresponding Mohr's circle.



$$\varepsilon_1 = \varepsilon_{ave} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta \qquad (1)$$

$$\varepsilon_2 = \varepsilon_{ave} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos(2\theta + 120°) + \frac{\gamma_{xy}}{2}\sin(2\theta + 120°)$$

$$= \varepsilon_{ave} + \frac{\varepsilon_x - \varepsilon_y}{2}(\cos 120°\cos 2\theta - \sin 120°\sin 2\theta)$$
$$+ \frac{\gamma_{xy}}{2}(\cos 120°\sin 2\theta + \sin 120°\cos 2\theta)$$

$$= \varepsilon_{ave} + \frac{\varepsilon_x - \varepsilon_y}{2}\left(-\frac{1}{2}\cos 2\theta - \frac{\sqrt{3}}{2}\sin 2\theta\right)$$
$$+ \frac{\gamma_{xy}}{2}\left(-\frac{1}{2}\sin 2\theta + \frac{\sqrt{3}}{2}\cos 2\theta\right) \qquad (2)$$

$$\varepsilon_3 = \varepsilon_{ave} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos(2\theta + 240°) + \frac{\gamma_{xy}}{2}\sin(2\theta + 240°)$$

$$= \varepsilon_{ave} + \frac{\varepsilon_x - \varepsilon_y}{2}(\cos 240°\cos 2\theta - \sin 240°\sin 2\theta)$$
$$+ \frac{\gamma_{xy}}{2}(\cos 240°\sin 2\theta + \sin 240°\cos 2\theta)$$

$$= \varepsilon_{ave} + \frac{\varepsilon_x - \varepsilon_y}{2}\left(-\frac{1}{2}\cos 2\theta + \frac{\sqrt{3}}{2}\sin 2\theta\right)$$
$$+ \frac{\gamma_{xy}}{2}\left(-\frac{1}{2}\sin 2\theta - \frac{\sqrt{3}}{2}\cos 2\theta\right) \qquad (3)$$

Adding (1), (2), and (3)

$$\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 3\varepsilon_{ave} + 0 + 0$$

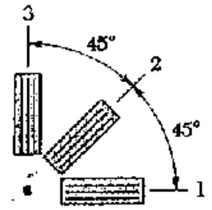
$$3\varepsilon_{ave} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$$

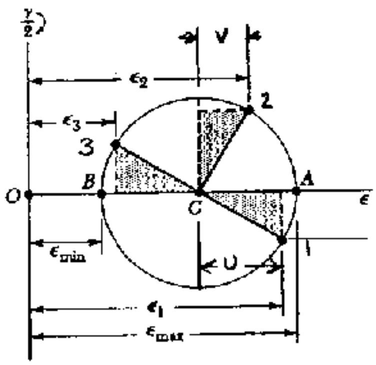
Problem 7.147

7.147 Using a 45° rosette, the strains ϵ_1, ϵ_2, and ϵ_3 have been determined at a given point. Using Mohr's circle, show that the principal strains are:

$$\varepsilon_{\max,\min} = \frac{1}{2}(\varepsilon_1 + \varepsilon_3) \pm \frac{1}{\sqrt{2}}\left[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2\right]^{1/2}$$

(*Hint:* The shaded triangles are congruent.)





Since gage directions 1 and 3 are 90° apart

$$\varepsilon_{ave} = \tfrac{1}{2}(\varepsilon_1 + \varepsilon_3)$$

Let $\quad u = \varepsilon_1 - \varepsilon_{ave} = \tfrac{1}{2}(\varepsilon_1 - \varepsilon_3)$

$$v = \varepsilon_2 - \varepsilon_{ave} = \varepsilon_2 - \tfrac{1}{2}(\varepsilon_1 + \varepsilon_3)$$

$$R^2 = u^2 + v^2$$

$$= \tfrac{1}{4}(\varepsilon_1 - \varepsilon_3)^2 + \varepsilon_2^2 - \varepsilon_2(\varepsilon_1 + \varepsilon_3) + \tfrac{1}{4}(\varepsilon_1 + \varepsilon_3)^2$$

$$= \tfrac{1}{4}\varepsilon_1^2 - \cancel{\tfrac{1}{2}\varepsilon_1\varepsilon_3} + \tfrac{1}{4}\varepsilon_3^2$$
$$+ \varepsilon_2^2 - \varepsilon_2\varepsilon_1 - \varepsilon_2\varepsilon_3$$
$$+ \tfrac{1}{4}\varepsilon_1^2 + \cancel{\tfrac{1}{2}\varepsilon_1\varepsilon_3} + \tfrac{1}{4}\varepsilon_3^2$$

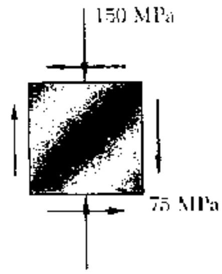
$$= \tfrac{1}{2}\varepsilon_1^2 - \varepsilon_2\varepsilon_1 + \varepsilon_2^2 - \varepsilon_2\varepsilon_3 + \tfrac{1}{2}\varepsilon_3^2$$

$$= \tfrac{1}{2}(\varepsilon_1 - \varepsilon_2)^2 + \tfrac{1}{2}(\varepsilon_2 - \varepsilon_3)^2$$

$$R = \frac{1}{\sqrt{2}}\left[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2\right]$$

$\varepsilon_{max, min} = \varepsilon_{ave} \pm R$ gives the required formula.

Problem 7.148



7.148 The given state of plane stress is known to exist on the surface of a machine component. Knowing that $E = 200$ GPa and $G = 77$ GPa, determine the direction and magnitude of the three principal strains (*a*) by determining the corresponding state of strain [use Eq. (2.43) and Eq. (2.38)] and then using Mohr's circle for strain, (*b*) by using Mohr's circle for stress to determine the principal planes and principal stresses and then determining the corresponding strains.

(a) $\sigma_x = 0$, $\sigma_y = -150 \times 10^6$ Pa, $\tau_{xy} = -75 \times 10^6$ Pa

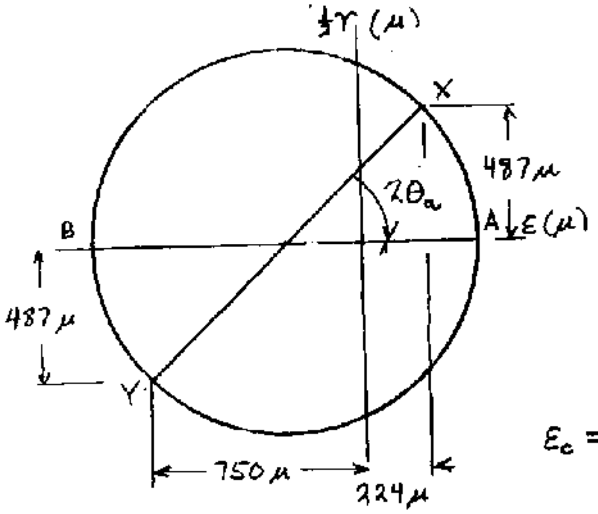
$E = 200 \times 10^9$ Pa $\quad G = 77 \times 10^9$ Pa

$$G = \frac{E}{2(1+\nu)} \qquad \nu = \frac{E}{2G} - 1 = 0.2987$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{1}{200 \times 10^9}\left[0 + (0.2987)(150 \times 10^6)\right] = 224\ \mu$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = \frac{1}{200 \times 10^9}\left[(-150 \times 10^6) - 0\right] = -750\ \mu$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{-75 \times 10^6}{77 \times 10^9} = -974\ \mu \qquad \frac{\gamma_{xy}}{2} = -487.0\ \mu$$



$$\varepsilon_{ave} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = -263\ \mu$$

$$\varepsilon_x - \varepsilon_y = 974\ \mu$$

$$\tan 2\theta_a = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-974}{974} = -1.000$$

$$2\theta_a = -45.0° \qquad \theta_a = -22.5°$$

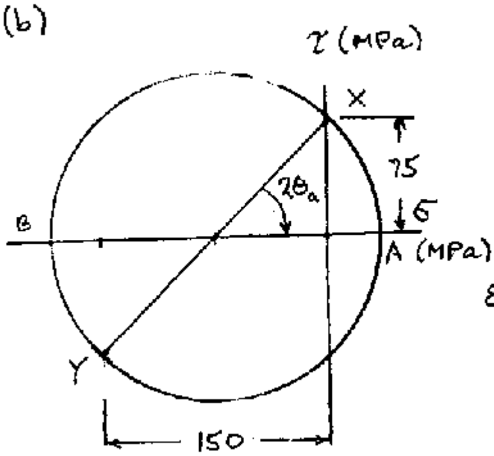
$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = 689\ \mu$$

$$\varepsilon_a = \varepsilon_{ave} + R = 426\ \mu$$

$$\varepsilon_b = \varepsilon_{ave} - R = -952\ \mu$$

$$\varepsilon_c = -\frac{\nu}{E}(\sigma_x + \sigma_y) = -\frac{(0.2987)(0 - 150 \times 10^6)}{200 \times 10^9} = -224\ \mu$$

(b)



$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -75\ \text{MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{0 + 150}{2}\right)^2 + 75^2} = 106.07\ \text{MPa}$$

$$\sigma_a = \sigma_{ave} + R = 31.07\ \text{MPa}$$

$$\sigma_b = \sigma_{ave} - R = -181.07\ \text{MPa}$$

$$\varepsilon_a = \frac{1}{E}(\sigma_a - \nu\sigma_b)$$

$$= \frac{1}{200 \times 10^9}\left[31.07 \times 10^6 - (0.2987)(-181.07 \times 10^6)\right]$$

$$= 426 \times 10^{-6} = 426\ \mu$$

$$\tan 2\theta_a = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -1.000 \qquad 2\theta_a = -45° \qquad \theta_a = -22.5°$$

Problem 7.149

7.149 The following state of strain has been determined on the surface of a cast-iron machine part:

$\epsilon_1 = -720 \times 10^{-6}$ in./in. $\quad \epsilon_2 = -400 \times 10^{-6}$ in./in. $\quad \epsilon_3 = +660 \times 10^{-6}$ in./in.

Knowing that $E = 10 \times 10^6$ psi and $G = 4 \times 10^6$ psi, determine the principal planes and principal stresses (*a*) by determining the corresponding state of plane stress [use Eq. (2.36), Eq.(2.43), and the first two equations of Prob.2.74] and then using Mohr's circle for stress, (*b*) by using Mohr's circle for strain to determine the orientation and magnitude of the principal strains and the determining the corresponding stresses.

$$G = \frac{E}{2(1+\nu)} \qquad \nu = \frac{E}{2G} - 1 = \frac{10}{(2)(4)} - 1 = 0.25$$

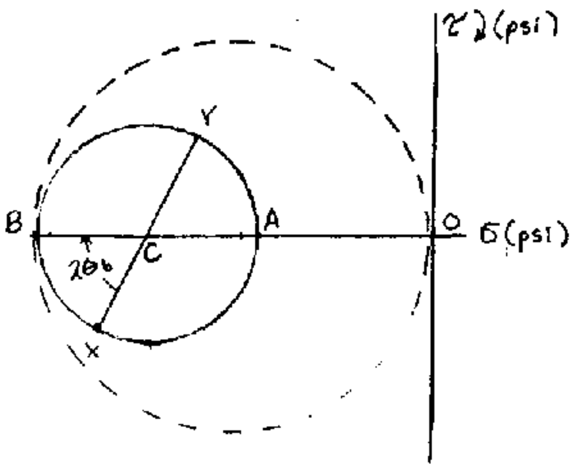
$$\frac{E}{1-\nu^2} = \frac{10\times10^6}{1-0.25^2} = 10.667\times10^6 \text{ psi}$$

Note that the 3rd principal stress $\sigma_c = 0$

(a) $$\sigma_1 = \frac{E}{1-\nu^2}(\varepsilon_1 + \nu\varepsilon_2) = 10.667\times10^6\left[-720\times10^{-6} + (0.25)(-400\times10^{-6})\right] = -8746.7 \text{ psi}$$

$$\sigma_2 = \frac{E}{1-\nu^2}(\varepsilon_2 + \nu\varepsilon_1) = 10.667\times10^6\left[-400\times10^{-6} + (0.25)(-720\times10^{-6})\right] = -6186.7 \text{ psi}$$

$$\tau = G\gamma = (4\times10^6)(660\times10^{-6}) = 2640 \text{ psi}$$



$$\sigma_{ave} = \tfrac{1}{2}(\sigma_1 + \sigma_2) = -7466.7 \text{ psi}$$

$$\tan 2\theta_b = \frac{2\tau}{\sigma_1 - \sigma_2} = -2.0625$$

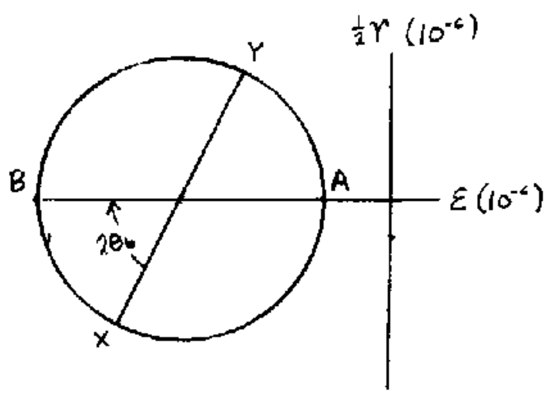
$2\theta_b = -64.1°$ $\quad \theta_b = -32.1°$ $\quad \theta_a = 57.9°$

$$R = \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2} = 2934 \text{ psi}$$

$$\sigma_a = \sigma_{ave} + R = -4533 \text{ psi}$$

$$\sigma_b = \sigma_{ave} - R = -10400 \text{ psi}$$

(b) $$\varepsilon_{ave} = \tfrac{1}{2}(\varepsilon_1 + \varepsilon_2) = -560\times10^{-6}$$



$$\tan 2\theta_b = \frac{\gamma}{\varepsilon_1 - \varepsilon_2} = \frac{660}{-720+400} = -2.0625$$

$2\theta_b = -64.1°$ $\quad \theta_b = -32.1°$ $\quad \theta_a = 57.9°$

$$R = \sqrt{\left(\frac{\varepsilon_1 - \varepsilon_2}{2}\right)^2 + \left(\frac{\gamma}{2}\right)^2} = 366.74\times10^{-6}$$

$$\varepsilon_a = \varepsilon_{ave} + R = -193.26\times10^{-6}$$

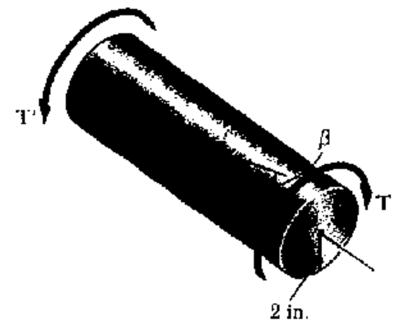
$$\varepsilon_b = \varepsilon_{ave} - R = -926.74\times10^{-6}$$

$$\sigma_a = \frac{E}{1-\nu^2}(\varepsilon_a + \nu\varepsilon_b) = -4533 \text{ psi}$$

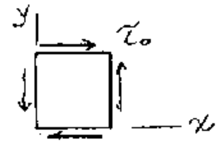
$$\sigma_b = \frac{E}{1-\nu^2}(\varepsilon_b + \nu\varepsilon_a) = -10400 \text{ psi}$$

Problem 7.150

7.150 A single gage is cemented to a solid 4-in.-diameter steel shaft at an angle $\beta = 25°$ with a line parallel to the axis of the shaft Knowing that $G = 11.5 \times 10^6$ psi, determine the torque **T** indicated by a gage reading of 300×10^{-6} in./in



For torsion, $\sigma_x = \sigma_y = 0$, $\tau = \tau_o$

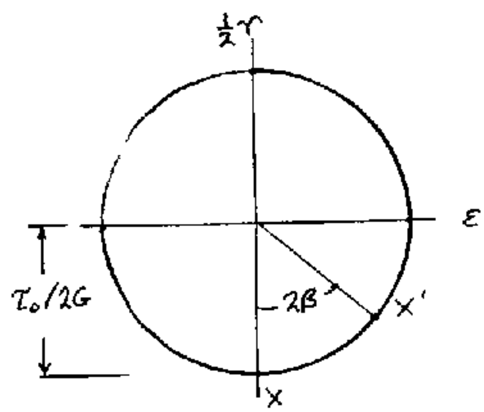


$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = 0$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = 0$$

$$\gamma_{xy} = \frac{\tau_o}{G} \qquad \frac{1}{2}\gamma_{xy} = \frac{\tau_o}{2G}$$

Draw the Mohr's circle for strain



$$R = \frac{\tau_o}{2G}$$

$$\varepsilon_{x'} = R \sin 2\beta = \frac{\tau_o}{2G}\sin 2\beta$$

But $\tau_o = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2G\varepsilon_{x'}}{\sin 2\beta}$

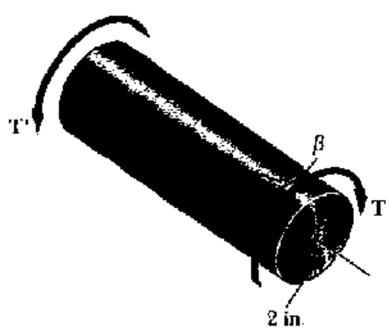
$$T = \frac{\pi c^3 G \varepsilon_{x'}}{\sin 2\beta} = \frac{\pi (2)^3 (11.5 \times 10^6)(300 \times 10^{-6})}{\sin 50°}$$

$$= 113.2 \times 10^3 \text{ lb} \cdot \text{in} = 113.2 \text{ kip} \cdot \text{in}$$

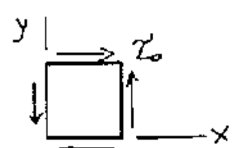
Problem 7.151

7.151 Solve Prob. 7.150, assuming that the gage forms an angle $\beta = 35°$ with a line parallel to the axis of the shaft.

7.150 A single gage is cemented to a solid 4-in.-diameter steel shaft at an angle $\beta = 25°$ with a line parallel to the axis of the shaft. Knowing that $G = 11.5 \times 10^6$ psi, determine the torque **T** indicated by a gage reading of 300×10^{-6} in./in.



For torsion $\sigma_x = 0$, $\sigma_y = 0$, $\tau_{xy} = \tau_o$

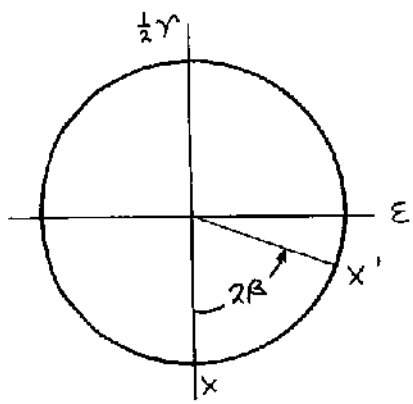


$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = 0$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = 0$$

$$\gamma_{xy} = \frac{\tau_o}{G} \qquad \frac{1}{2}\gamma_{xy} = \frac{\tau_o}{2G}$$

Draw Mohr's circle for strain



$$R = \frac{\tau_o}{2G} \qquad \varepsilon_{x'} = R \sin 2\beta = \frac{\tau_o}{2G}\sin 2\beta$$

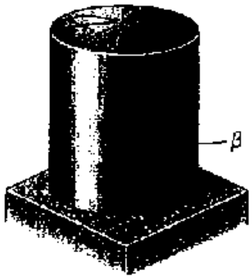
But $\tau_o = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2G\varepsilon_{x'}}{\sin 2\beta}$

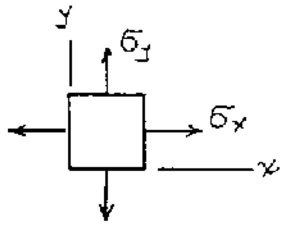
$$T = \frac{\pi c^3 G \varepsilon_{x'}}{\sin 2\beta} = \frac{\pi (2)^3 (11.5 \times 10^6)(300 \times 10^{-6})}{\sin 70°}$$

$$= 92.3 \times 10^3 \text{ lb} \cdot \text{in} = 92.3 \text{ kip} \cdot \text{in}$$

Problem 7.152

7.152 A single strain gage forming an angle $\beta = 18°$ with a horizontal plane is used to determine the gage pressure in the cylindrical steel tank shown. The cylindrical wall of the tank is 6 mm thick, has a 600-mm inside diameter, and is made of a steel with $E = 200$ GPa and $\nu = 0.30$. Determine the pressure in the tank indicated by a strain gage reading of 280μ.



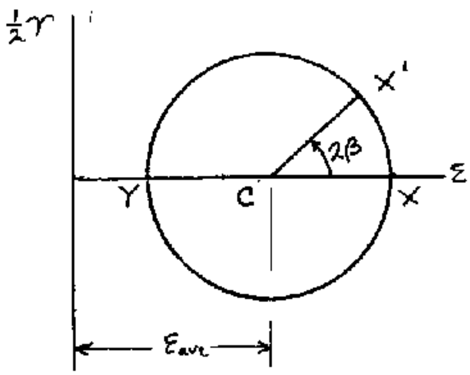


$$\sigma_x = \sigma_1 = \frac{pr}{t}, \quad \sigma_y = \tfrac{1}{2}\sigma_x, \quad \sigma_z \approx 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \left(1 - \frac{\nu}{2}\right)\frac{\sigma_x}{E} = 0.85\frac{\sigma_x}{E}$$

$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = \left(\tfrac{1}{2} - \nu\right)\frac{\sigma_x}{E} = 0.20\frac{\sigma_x}{E}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = 0$$



Draw Mohr's circle for strain

$$\varepsilon_{ave} = \tfrac{1}{2}(\varepsilon_x + \varepsilon_y) = 0.525\frac{\sigma_x}{E}$$

$$R = \tfrac{1}{2}(\varepsilon_x - \varepsilon_y) = 0.325\frac{\sigma_x}{E}$$

$$\varepsilon_{x'} = \varepsilon_{ave} + R\cos 2\beta = (0.525 + 0.325\cos 2\beta)\frac{\sigma_x}{E}$$

$$p = \frac{t\sigma_x}{r} = \frac{tE\varepsilon_{x'}}{r(0.525 + 0.325\cos 2\beta)}$$

Data: $r = \tfrac{1}{2}d = \tfrac{1}{2}(600) = 300 \text{ mm} = 0.300 \text{ m}$

$t = 6\times10^{-3}$ mm $\qquad E = 200\times10^9$ Pa, $\qquad \varepsilon_{x'} = 280\times10^{-6}$

$\beta = 18°$

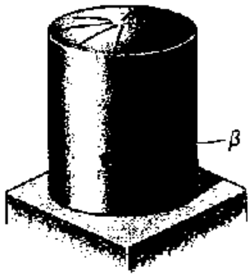
$$p = \frac{(6\times10^{-3})(200\times10^9)(280\times10^{-6})}{(0.300)(0.525 + 0.325\cos 36°)} = 1.421\times10^6 \text{ Pa}$$

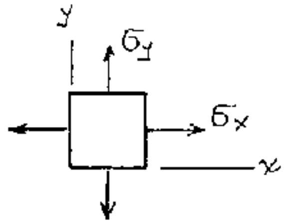
$p = 1.421$ MPa ◀

Problem 7.153

7.153 Solve Prob. 7.152, assuming that the gage forms an angle $\beta = 35°$ with a horizontal plane.

7.152 A single strain gage forming an angle $\beta = 18°$ with a horizontal plane is used to determine the gage pressure in the cylindrical steel tank shown. The cylindrical wall of the tank is 6 mm thick, has a 600-mm inside diameter, and is made of a steel with $E = 200$ GPa and $\nu = 0.30$. Determine the pressure in the tank indicated by a strain gage reading of 280μ.





$\sigma_x = \sigma_1 = \frac{pr}{t}, \quad \sigma_y = \frac{1}{2}\sigma_x, \sigma_z \approx 0$

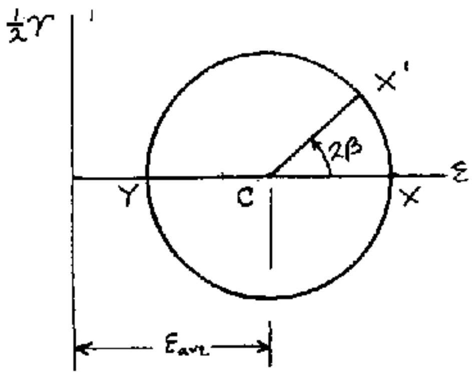
$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = (1 - \frac{\nu}{2})\frac{\sigma_x}{E}$

$= 0.85\frac{\sigma_x}{E}$

$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = (\frac{1}{2} - \nu)\frac{\sigma_x}{E}$

$= 0.20\frac{\sigma_x}{E}$

$\gamma_{xy} = \frac{\tau_{xy}}{G} = 0$



Draw Mohr's circle for strain

$\varepsilon_{ave} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = 0.525\frac{\sigma_x}{E}$

$R = \frac{1}{2}(\varepsilon_x - \varepsilon_y) = 0.325\frac{\sigma_x}{E}$

$\varepsilon_{x'} = \varepsilon_{ave} + R\cos 2\beta = (0.525 + 0.325\cos 2\beta)\frac{\sigma_x}{E}$

$$p = \frac{t\sigma_x}{r} = \frac{tE\varepsilon_{x'}}{r(0.525 + 0.325\cos 2\beta)}$$

Data: $r = \frac{1}{2}d = \frac{1}{2}(600) = 300\text{ mm} = 0.300\text{ m}$

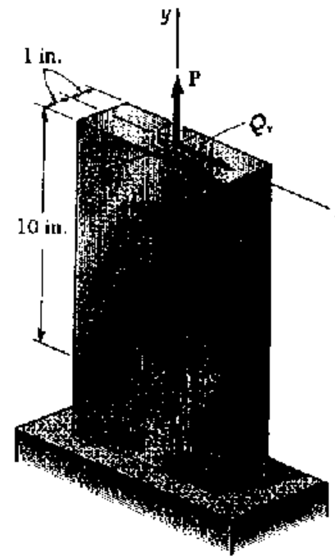
$t = 6 \times 10^{-3}\text{ m} \qquad E = 200 \times 10^9\text{ Pa} \qquad \varepsilon_{x'} = 280 \times 10^{-6}$

$\beta = 35°$

$$p = \frac{(6\times10^{-3})(200\times10^9)(280\times10^{-6})}{(0.300)(0.525 + 0.325\cos 70°)} = 1.761\times10^6\text{ Pa}$$

$p = 1.761$ MPa ◄

Problem 7.154



7.154 A centric axial force **P** and a horizontal force $\mathbf{Q}_x$ are both applied at point C of the rectangular bar shown. A 45° strain rosette on the surface of the bar at point A indicates the following strains:

$\epsilon_1 = -75 \times 10^{-6}$ in./in. $\quad \epsilon_2 = +300 \times 10^{-6}$ in./in. $\quad \epsilon_3 = +250 \times 10^{-6}$ in./in.

Knowing that $E = 29 \times 10^6$ psi and $\nu = 0.30$, determine the magnitudes of **P** and $\mathbf{Q}_x$.

$$\varepsilon_x = \varepsilon_1 = -75 \times 10^{-6} \qquad \varepsilon_y = \varepsilon_3 = 250 \times 10^{-6}$$

$$\gamma_{xy} = 2\varepsilon_2 - \varepsilon_1 - \varepsilon_3 = 425 \times 10^{-6}$$

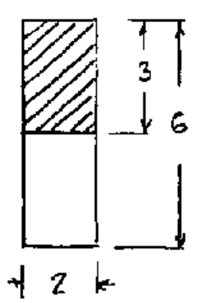
$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y) = \frac{29}{1-0.3^2}\left[-75 + (0.3)(250)\right] = 0$$

$$\sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x) = \frac{29}{1-0.3^2}\left[250 + (0.3)(-75)\right] = 7.25 \times 10^3 \text{ psi}$$

$$\frac{P}{A} = \sigma_y \qquad P = A\sigma_y = (2)(6)(7.25 \times 10^3) = 87.0 \times 10^3 \text{ lb} = 87.0 \text{ kips}$$

$$G = \frac{E}{2(1+\nu)} = \frac{29 \times 10^6}{(2)(1.3)} = 11.154 \times 10^6 \text{ psi}$$

$$\tau_{xy} = G\gamma_{xy} = (11.154)(425) = 4.740 \times 10^3 \text{ psi}$$



$$I = \frac{1}{12} bh^3 = \frac{1}{12}(2)(6)^3 = 36 \text{ in}^4$$

$$Q = A\bar{y} = (2)(3)(1.5) = 9 \text{ in}^3 \qquad t = 2 \text{ in}$$

$$\tau_{xy} = \frac{VQ}{It}$$

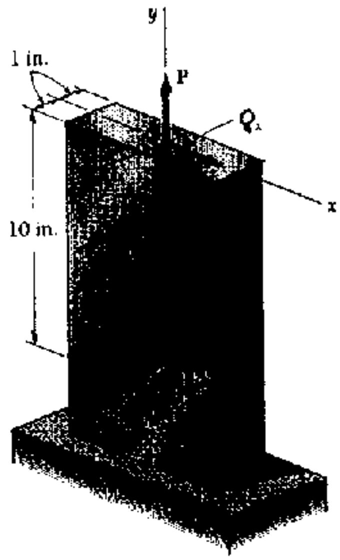
$$V = \frac{It\tau_{xy}}{Q} = \frac{(36)(2)(4.74 \times 10^3)}{9} = 37.9 \times 10^3 \text{ lb.}$$

$$Q_x = V = 37.9 \times 10^3 \text{ lb} = 37.9 \text{ kips}$$

Problem 7.155

7.155 Solve Prob. 7.154, assuming that the rosette at point A indicates the following strains:
$\epsilon_1 = -60 \times 10^{-6}$ in./in. $\quad \epsilon_2 = +410 \times 10^{-6}$ in./in. $\quad \epsilon_3 = +200 \times 10^{-6}$ in./in.

7.154 A centric axial force **P** and a horizontal force $\mathbf{Q}_x$ are both applied at point C of the rectangular bar shown. A 45° strain rosette on the surface of the bar at point A indicates the following strains:
$\epsilon_1 = -75 \times 10^{-6}$ in./in. $\quad \epsilon_2 = +300 \times 10^{-6}$ in./in. $\quad \epsilon_3 = +250 \times 10^{-6}$ in./in.
Knowing that $E = 29 \times 10^6$ psi and $\nu = 0.30$, determine the magnitudes of **P** and $\mathbf{Q}_x$.



$\varepsilon_x = \varepsilon_1 = -60 \times 10^{-6} \qquad \varepsilon_y = \varepsilon_3 = 200 \times 10^{-6}$

$\gamma_{xy} = 2\varepsilon_2 - \varepsilon_1 - \varepsilon_3 = 680 \times 10^{-6}$

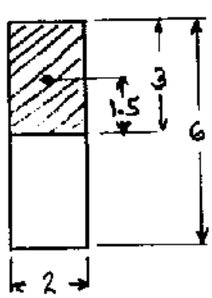
$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y) = \frac{29}{1-0.3^2}[-60 + (0.3)(200)]$
$= 0$

$\sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x) = \frac{29}{1-0.3^2}[200 + (0.3)(-60)]$
$= 5.800 \times 10^3$ psi

$\frac{P}{A} = \sigma_y \qquad P = A\sigma_y = (2)(6)(5.800 \cdot 10^3)$
$= 69.6 \times 10^3$ lb $= 69.6$ kips

$G = \frac{E}{2(1+\nu)} = \frac{29 \times 10^6}{(2)(1.3)} = 11.154 \times 10^6$ psi

$\tau_{xy} = G\gamma_{xy} = (11.154)(680) = 7.585 \times 10^3$ psi



$I = \frac{1}{12} b h^3 = \frac{1}{12}(2)(6)^3 = 36$ in^4

$Q = A\bar{y} = (2)(3)(1.5) = 9$ in$^3 \qquad t = 2$ in.

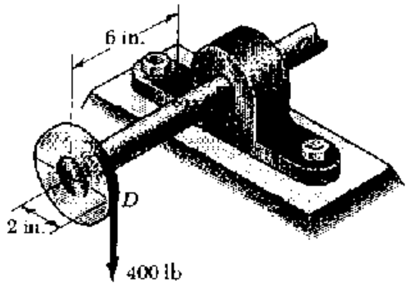
$\tau_{xy} = \frac{VQ}{It}$

$V = \frac{It\tau_{xy}}{Q} = \frac{(36)(2)(7.585 \times 10^3)}{9} = 60.7 \times 10^3$ lb.

$Q_x = V = 60.7 \times 10^3$ lb. $= 60.7$ kips

Problem 7.156

7.156 A 400-lb vertical force is applied at D to a gear attached to the solid one-inch diameter shaft AB. Determine the principal stresses and the maximum shearing stress at point H located as shown on top of the shaft.



Equivalent force-couple system at center of shaft in section at point H.

$V = 400 \text{ lb.} \qquad M = (400)(6) = 2400 \text{ lb}\cdot\text{in}$

$T = (400)(2) = 800 \text{ lb}\cdot\text{in.}$

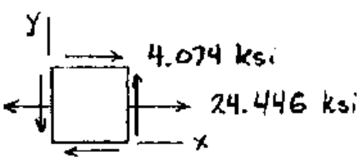
Shaft cross section. $\quad d = \text{in} \quad c = \frac{1}{2}d = 0.5 \text{ in}$

$J = \frac{\pi}{2}c^4 = 0.098175 \text{ in}^4 \qquad I = \frac{1}{2}J = 0.049087 \text{ in}^4$

Torsion: $\quad \tau = \dfrac{Tc}{J} = \dfrac{(800)(0.5)}{0.098175} = 4.074\times10^3 \text{ psi} = 4.074 \text{ ksi}$

Bending: $\quad \sigma = \dfrac{Mc}{I} = \dfrac{(2400)(0.5)}{0.049087} = 24.446\times10^3 \text{ psi} = 24.446 \text{ ksi}$

Transverse shear: Stress at point H is zero.



$\sigma_x = 24.446 \text{ ksi} \qquad \sigma_y = 0 \qquad \tau_{xy} = 4.074 \text{ ksi}$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 12.223 \text{ ksi}$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(12.223)^2 + (4.074)^2}$

$= 12.884 \text{ ksi}$

$\sigma_a = \sigma_{ave} + R = 25.107 \text{ ksi}$ ◀

$\sigma_b = \sigma_{ave} - R = -0.661 \text{ ksi}$ ◀

$\tau_{max} = R = 12.884 \text{ ksi}$ ◀

Problem 7.157

7.157 Determine the largest internal pressure that can be applied to a cylindrical tank of 5.5-ft outer-diameter and $\frac{5}{8}$-in. wall thickness if the ultimate normal stress of the steel used is 65 ksi and a factor of safety of 5.0 is desired.

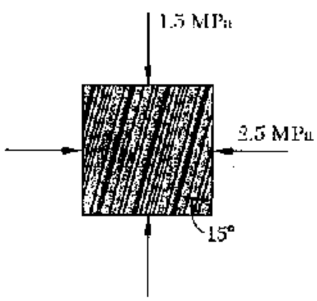
$$\sigma_1 = \frac{\sigma_U}{F.S.} = \frac{65}{5.0} = 13 \text{ ksi} \qquad d = 5.5 \text{ ft} = 66 \text{ in}$$

$$r = \tfrac{1}{2}d - t = \tfrac{1}{2}(66) - 0.625 = 32.375 \text{ in}$$

$$\sigma_1 = \frac{pr}{t} \qquad p = \frac{\sigma_1 t}{r} = \frac{(13)(0.625)}{32.375} = 0.251 \text{ ksi} = 251 \text{ psi}$$

Problem 7.158

7.158 The grain of a wooden member forms an angle of 15° with the vertical For the state of stress shown, determine (*a*) the in-plane shearing stress parallel to the grain, (*b*) the normal stress perpendicular to the grain.



$$\sigma_x = -2.5 \text{ MPa} \qquad \sigma_y = -1.5 \text{ MPa} \qquad \tau_{xy} = 0$$

$$\theta = -15^\circ \qquad 2\theta = -30^\circ$$

$$(a)\ \tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{-2.5-(-1.5)}{2}\sin(-30^\circ) + 0$$

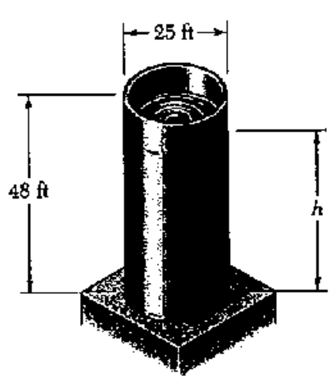
$$= -0.250 \text{ MPa}$$

$$(b)\ \sigma_{x'} = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{-2.5+(-1.5)}{2} + \frac{-2.5-(-1.5)}{2}\cos(-30^\circ) + 0 = -2.43 \text{ MPa}$$

Problem 7.159

7.159 The unpressurized cylindrical storage tank shown has a $\frac{3}{16}$-in. wall thickness and is made of a steel having a 60-ksi ultimate strength in tension. Determine the maximum height *h* to which it can be filled with water if a factor of safety of 4.0 is desired. (Specific weight of water = 62.4 lb/ft^3.)



$$d_o = (25)(12) = 300 \text{ in.}$$

$$r = \tfrac{1}{2}d - t = 150 - \tfrac{3}{16} = 149.81 \text{ in.}$$

$$\sigma_{all} = \frac{\sigma_U}{F.S.} = \frac{60 \text{ ksi}}{4.0} = 15 \text{ ksi} = 15\times10^3 \text{ psi}$$

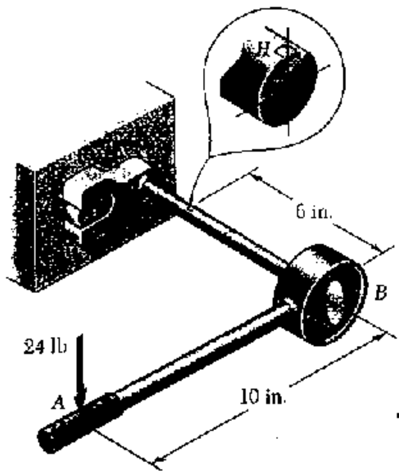
$$\sigma_{all} = \frac{pr}{t}$$

$$p = \frac{t\,\sigma_{all}}{r} = \frac{(\frac{3}{16})(15\times10^3)}{149.81} = 18.77 \text{ psi} = 2703 \text{ lb/ft}^2$$

But $p = \gamma h$ $\qquad h = \dfrac{p}{\gamma} = \dfrac{2703 \text{ lb/ft}^2}{62.4 \text{ lb/ft}^3} = 43.3 \text{ ft}$

Problem 7.160

7.160 A mechanic uses a crowfoot wrench to loosen a bolt at E. Knowing that the mechanic applies a vertical force of 24-lb at A, determine the principal stresses and the maximum shearing stress at point H located as shown on top of the 0.75-in. diameter shaft.



Equivalent force-couple system at center of shaft in section at point H.

$V = 24$ lb. $\quad M = (24)(6) = 144$ lb·in

$T = (24)(10) = 240$ lb·in

Shaft cross section: $d = 0.75$ in, $c = \frac{1}{2}d = 0.375$ in.

$J = \frac{\pi}{2}c^4 = 0.031063$ in^4 $\quad I = \frac{1}{2}J = 0.015532$ in^4

Torsion: $\tau = \dfrac{Tc}{J} = \dfrac{(240)(0.375)}{0.031063} = 2.897 \times 10^3$ psi $= 2.897$ ksi

Bending: $\sigma = \dfrac{Mc}{I} = \dfrac{(144)(0.375)}{0.015532} = 3.477 \times 10^3$ psi $= 3.477$ ksi

Transverse Shear: At point H stress due to transverse shear is zero.

Resultant stresses: $\sigma_x = 3.477$ ksi, $\quad \sigma_y = 0$, $\quad \tau_{xy} = 2.897$ ksi

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 1.738$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{1.738^2 + 2.897^2} = 3.378$ ksi

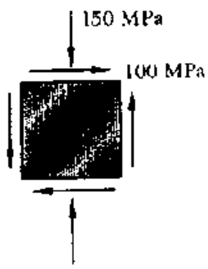
$\sigma_a = \sigma_{ave} + R = 5.116$ ksi ◄

$\sigma_b = \sigma_{ave} - R = -1.640$ ksi ◄

$\tau_{max} = R = 3.378$ ksi ◄

Problem 7.161

7.161 The state of plane stress shown is expected to occur in a cast-iron machine base. Knowing that for the grade of cast iron used $\sigma_{UT} = 160$ MPa and $\sigma_{UC} = 320$ MPa and using Mohr's criterion, determine whether rupture of the component will occur.

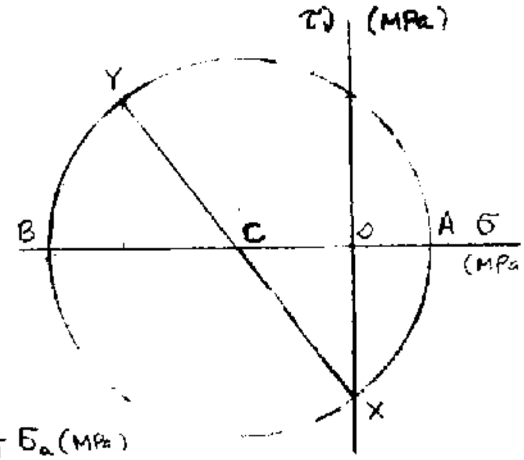


$\sigma_x = 0 \qquad \sigma_y = -150 \text{ MPa} \qquad \tau_{xy} = 100 \text{ MPa}$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -75 \text{ MPa}$

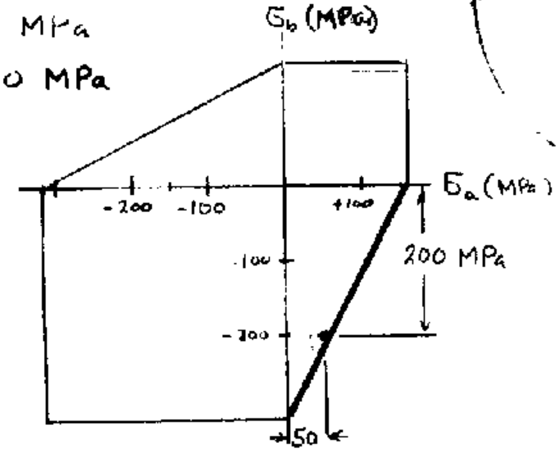
$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{75^2 + 100^2} = 125 \text{ MPa}$



$\sigma_a = \sigma_{ave} + R = 50 \text{ MPa}$

$\sigma_b = \sigma_{ave} - R = -200 \text{ MPa}$

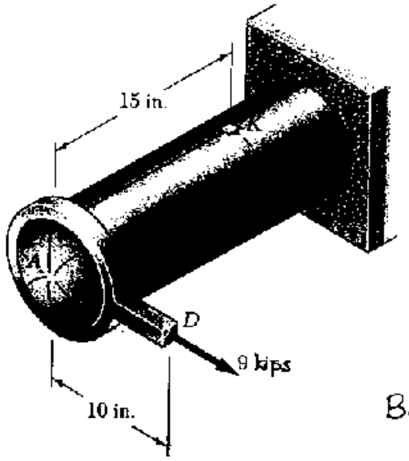


Equation of the 4th quadrant boundary is $\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$

$\frac{50}{160} - \frac{(-200)}{320} = 0.9375 < 1$, No rupture.

Problem 7.162

7.162 The cylindrical tank AB has an 8-in. inner diameter and a 0.32-in. wall thickness. Knowing that the pressure inside the tank is 600 psi, determine the maximum normal stress and the maximum shearing stress at point K located on the top of the tank.



$r_i = \frac{d_i}{2} = 4 \text{ in} \qquad r_o = r_i + t = 4.32 \text{ in.}$

$\sigma_1 = \frac{p r_i}{t} = \frac{(600)(4)}{0.32} = 7500 \text{ psi} = 7.50 \text{ ksi}$

$\sigma_2 = \frac{1}{2}\sigma_1 = 3.75 \text{ ksi}$

Torsion: No applied torque

Bending: Point K lies on neutral axis.

Transverse shear: $V = 9$ kips

For semicircle
$A = \frac{\pi}{2} r^2$
$\bar{y} = \frac{4r}{3\pi}$
$Q = \frac{2}{3} r^3$

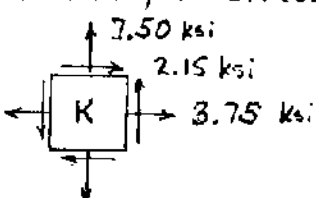
$Q = Q_o - Q_i = \frac{2}{3} r_o^3 - \frac{2}{3} r_i^3 = \frac{2}{3}(4.32^3 - 4^3) = 11.081 \text{ in}^3$

$t = (2)(0.32) = 0.64 \text{ in}$

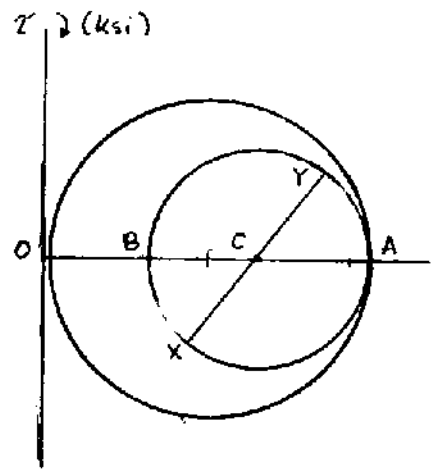
$I = \frac{\pi}{4}(r_o^4 - r_i^4) = \frac{\pi}{4}(4.32^4 - 4^4) = 72.481 \text{ in}^4$

$\tau = \frac{VQ}{It} = \frac{(9)(11.081)}{(72.481)(0.64)} = 2.15 \text{ ksi}$

Summary of stresses:



Longitudinal $\sigma_x = \sigma_1 = 3.75$ ksi
Circumferential $\sigma_y = \sigma_2 = 7.50$ ksi
Shear $\tau_{xy} = 2.15$ ksi



$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 5.625 \text{ ksi}$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= 2.853 \text{ ksi}$

$\sigma_a = \sigma_{ave} + R = 8.48 \text{ ksi}$

$\sigma_b = \sigma_{ave} - R = 2.77 \text{ ksi}$

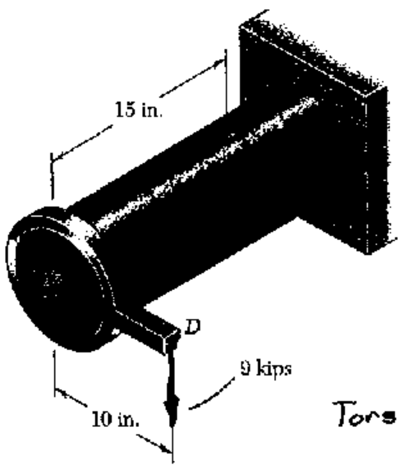
$\sigma_z = 0$

$\sigma_{max} = 8.48 \text{ ksi}$ ◄

$\sigma_{min} = 0$

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 4.24 \text{ ksi}$ ◄

Problem 7.163



7.163 Solve Prob. 7.162, assuming that the 9-kip force applied at point D is directed vertically downward.

7.162 The cylindrical tank AB has an 8-in. inner diameter and a 0.32-in. wall thickness. Knowing that the pressure inside the tank is 600 psi, determine the maximum normal stress and the maximum shearing stress at point K located on the top of the tank.

$r_i = \frac{d_i}{2} = 4 \text{ in} \qquad r_o = r_i + t = 4.32 \text{ in}$

$\sigma_1 = \frac{pr_i}{t} = \frac{(600)(4)}{0.32} = 7500 \text{ psi} = 7.50 \text{ ksi}$

$\sigma_2 = \frac{1}{2}\sigma_1 = 3.75 \text{ ksi}$

Torsion: $J = \frac{\pi}{2}(r_o^4 - r_i^4) = 144.96 \text{ in}^4 \qquad c = r_o = 4.32 \text{ in}$

$T = (9)(10) = 90 \text{ kip}\cdot\text{in}$

$\tau = \frac{Tc}{J} = \frac{(90)(4.32)}{144.96} = 2.68 \text{ ksi}$

Bending: $I = \frac{1}{2}J = 72.48 \text{ in}^4 \qquad c = r_o = 4.32 \text{ in}$

$M = (9)(15) = 135 \text{ kip}\cdot\text{in} \qquad \sigma_m = \frac{Mc}{I} = \frac{(135)(4.32)}{72.48} = 8.05 \text{ ksi}$

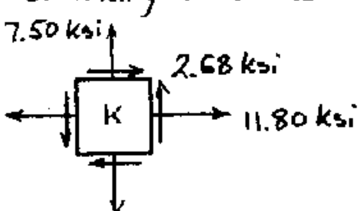
Transverse shear: At point K, $VQ/It = 0$

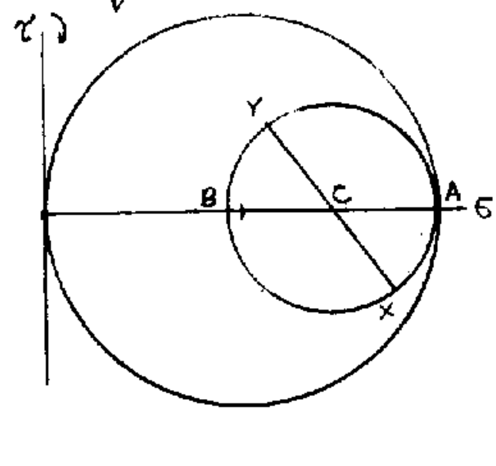
Summary of stresses:

Longitudinal $\sigma_x = \sigma_1 = 3.75 + 8.05 = 11.80 \text{ ksi}$

Circumferential $\sigma_y = \sigma_2 = 7.50 \text{ ksi}$

Shear $\tau_{xy} = 2.68 \text{ ksi}$





$\sigma_{ave} = \frac{1}{2}(11.80 + 7.50) = 9.65 \text{ ksi}$

$R = \sqrt{\left(\frac{11.80 - 7.50}{2}\right)^2 + (2.68)^2} = 3.44 \text{ ksi}$

$\sigma_a = \sigma_{ave} + R = 13.09 \text{ ksi}$

$\sigma_b = \sigma_{ave} - R = 6.21 \text{ ksi}$

$\sigma_z = 0$

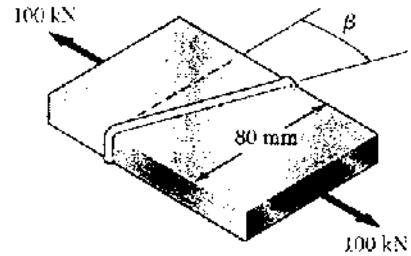
$\sigma_{max} = 13.09 \text{ ksi}$ ◄

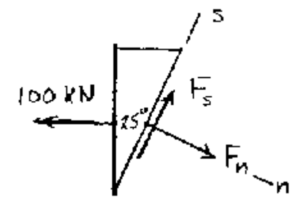
$\sigma_{min} = 0$

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 6.54 \text{ ksi}$ ◄

Problem 7.164

7.164 Two steel plates of uniform cross section 10 × 80 mm are welded together as shown. Knowing that centric 100-kN forces are applied to the welded plates and that $\beta = 25°$, determine (*a*) the in-plane shearing stress parallel to the weld, (*b*) the normal stress perpendicular to the weld.





Forces

Area of weld

$$A_w = \frac{(10\times10^{-3})(80\times10^{-3})}{\cos 25°} = 882.7 \quad 10^{-6}\ m^2$$

(a) $\Sigma F_s = 0 \qquad F_s - 100\sin 25° = 0 \qquad F_s = 42.26\ kN$

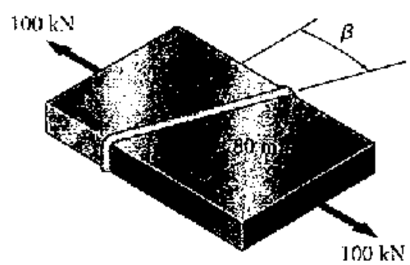
$$\tau_w = \frac{F_s}{A_w} = \frac{42.26\times10^3}{882.7\times10^{-6}} = 47.9\times10^6\ Pa = 47.9\ MPa$$

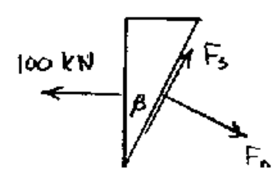
(b) $\Sigma F_n = 0 \qquad F_N - 100\cos 25° = 0 \qquad F_n = 90.63\ kN$

$$\sigma_w = \frac{F_n}{A_w} = \frac{90.63\times10^3}{882.7\times10^{-6}} = 102.7\times10^6\ Pa = 102.7\ MPa$$

Problem 7.165

7.165 Two steel plates of uniform cross section 10 × 80 mm are welded together as shown. Knowing that centric 100-kN forces are applied to the welded plates and that the in-plane shearing stress parallel to the weld is 30 MPa, determine (*a*) the angle β, (*b*) the corresponding normal stress perpendicular to the weld.





Forces

Area of weld

$$A_w = \frac{(10\times10^{-3})(80\times10^{-3})}{\cos\beta} = \frac{800\times10^{-6}}{\cos\beta}\ m^2$$

(a) $\Sigma F_s = 0 \qquad F_s - 100\sin\beta = 0 \qquad F_s = 100\sin\beta\ kN = 100\times10^3\sin\beta\ N$

$$\tau_w = \frac{F_s}{A_w} \qquad 30\times10^6 = \frac{100\times10^3\sin\beta}{800\times10^{-6}/\cos\beta} = 125\times10^6\ \sin\beta\cos\beta$$

$$\sin\beta\cos\beta = \frac{1}{2}\sin 2\beta = \frac{30\times10^6}{125\times10^6} = 0.240 \qquad \beta = 14.34°$$

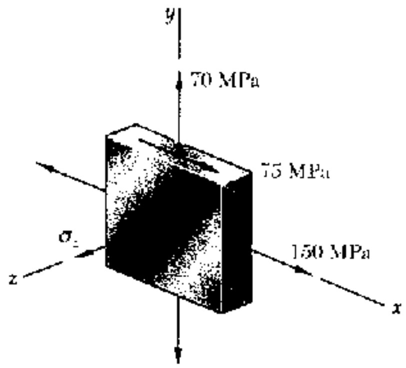
(b) $\Sigma F_n = 0 \qquad F_n - 100\cos\beta = 0 \qquad F_n = 100\cos 14.34° = 96.88\ kN$

$$A_w = \frac{800\times10^{-6}}{\cos 14.34} = 825.74\times10^{-6}\ m^2$$

$$\sigma = \frac{F_n}{A_w} = \frac{96.88\times10^3}{825.74\times10^{-6}} = 117.3\times10^6\ Pa = 117.3\ MPa$$

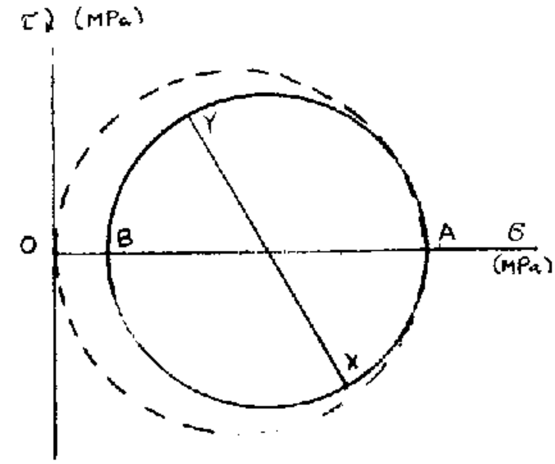
Problem 7.166

7.166 For the state of stress shown, determine the maximum shearing stress when (*a*) $\sigma_z = 0$, (*b*) $\sigma_z = +45$ MPa, (*c*) $\sigma_z = -45$ MPa.



$\sigma_x = 150$ MPa , $\sigma_y = 70$ MPa, $\tau_{xy} = 75$ MPa

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y)$$
$$= 110 \text{ MPa}$$
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$
$$= \sqrt{40^2 + 75^2}$$
$$= 85 \text{ MPa}$$



$\sigma_a = \sigma_{ave} + R = 195$ MPa

$\sigma_b = \sigma_{ave} - R = 25$ MPa

(a) $\sigma_z = 0$, $\sigma_a = 195$ MPa , $\sigma_b = 25$ MPa

$\sigma_{max} = 195$ MPa , $\sigma_{min} = 0$, $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 97.5$ MPa ◄

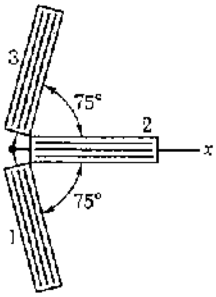
(b) $\sigma_z = +45$ MPa , $\sigma_a = 195$ MPa , $\sigma_b = 25$ MPa

$\sigma_{max} = 195$ MPa, $\sigma_{min} = 25$ MPa , $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 85$ MPa ◄

(c) $\sigma_z = -45$ MPa, $\sigma_a = 195$ MPa, $\sigma_b = 25$ MPa

$\sigma_{max} = 195$ MPa, $\sigma_{min} = -45$ MPa , $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 120$ MPa ◄

Problem 7.167



7.167 The strains determined by the use of a rosette attached as shown to the surface of a machine element are

$\epsilon_1 = \ 93.1 \times 10^{-6}$ in./in. $\quad \epsilon_2 = +385 \times 10^{-6}$ in./in. $\quad \epsilon_3 = +210 \times 10^{-6}$ in./in.

Determine (*a*) the orientation and magnitude of the principal strains in the plane of the rosette, (*b*) the maximum in-plane shearing stress.

Use $\varepsilon_{x'} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$

where $\theta = -75°$ for gage 1, $\theta = 0$ for gage 2, and $\theta = +75°$ for gage 3.

$$\varepsilon_1 = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_y - \varepsilon_y)\cos(-150°) + \frac{\gamma_{xy}}{2}\sin(-150°) \quad (1)$$

$$\varepsilon_2 = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 0 + \frac{\gamma_{xy}}{2}\sin 0 \quad (2)$$

$$\varepsilon_3 = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos(150°) + \frac{\gamma_{xy}}{2}\sin(150°) \quad (3)$$

From Eq. (2) $\quad \varepsilon_x = \varepsilon_2 = 385 \times 10^{-6}$ in/in

Adding Eq's (1) and (3)

$$\varepsilon_1 + \varepsilon_3 = (\varepsilon_x + \varepsilon_y) + (\varepsilon_x - \varepsilon_y)\cos 150°$$

$$= \varepsilon_x(1 + \cos 150°) + \varepsilon_y(1 - \cos 150°)$$

$$\varepsilon_y = \frac{\varepsilon_1 + \varepsilon_3 - \varepsilon_x(1 + \cos 150°)}{(1 - \cos 150°)} = \frac{-93.1\times10^{-6} + 210\times10^{-6} - 385\times10^{-6}(1 + \cos 150°)}{1 - \cos 150°}$$

$$= 35.0 \times 10^{-6} \text{ in/in}$$

Subtracting Eq (1) from Eq (3)

$$\varepsilon_3 - \varepsilon_1 = \gamma_{xy}\sin 150°$$

$$\gamma_{xy} = \frac{\varepsilon_3 - \varepsilon_1}{\sin 150°} = \frac{210\times10^{-6} - (-93.1\times10^{-6})}{\sin 150°} = 606.2 \times 10^{-6} \text{ in/in.}$$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{606.2\times10^{-6}}{385\times10^{-6} - 35.0\times10^{-6}} = 1.732 \qquad \text{(a)}\ \theta_a = 30.0°,\ \theta_b = 120.0°$$

$$\varepsilon_{ave} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = \frac{1}{2}(385\times10^{-6} + 35.0\times10^{-6}) = 210\times10^{-6} \text{ in/in}$$

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \sqrt{\left(\frac{385\times10^{-6} - 35.0\times10^{-6}}{2}\right)^2 + \left(\frac{606.2}{2}\right)^2} = 350.0\times10^{-6}$$

$$\varepsilon_a = \varepsilon_{ave} + R = 210\times10^{-6} + 350.0\times10^{-6} = 560\times10^{-6} \text{ in/in}$$

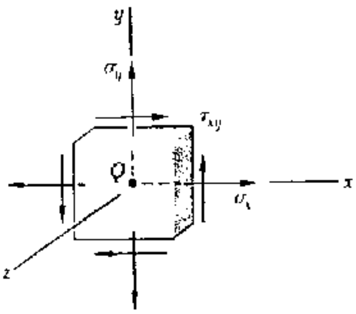
$$\varepsilon_b = \varepsilon_{ave} - R = 210\times10^{-6} + 350.0\times10^{-6} = -140.0\times10^{-6} \text{ in/in}$$

(b) $\dfrac{\gamma_{max(in\,plane)}}{2} = R = 350.0\times10^{-6}$ in/in

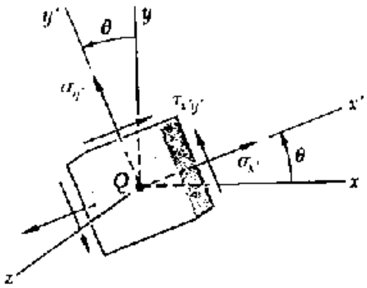
$$\gamma_{max(in\,plane)} = 700 \times 10^{-6} \text{ in/in}$$

PROBLEM 7.C1

7.C1 A state of plane stress is defined by the stress components σ_x, σ_y, and τ_{xy} associated with the element shown in Fig. P7.C1*a*. (*a*) Write a computer program that can be used to calculate the stress components $\sigma_{x'}$, $\sigma_{y'}$, and $\tau_{x'y'}$ associated with the element after it has rotated through an angle θ about the z axis (Fig. P7.C1*b*). (*b*) Use this program to solve Probs. 7.13 through 7.16.



(*a*)



(*b*)

SOLUTION PROGRAM FOLLOWING EQUATIONS

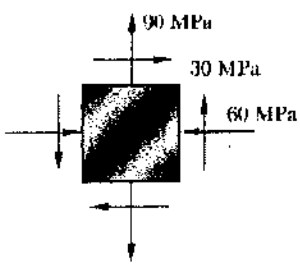
EQ(7.5), p427: $\sigma_{x'} = \dfrac{\sigma_x+\sigma_y}{2} + \dfrac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$

EQ(7.7), p427: $\sigma_{y'} = \dfrac{\sigma_x+\sigma_y}{2} - \dfrac{\sigma_x-\sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$

EQ(7.6), p427: $\tau_{x'y'} = -\dfrac{\sigma_x-\sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$

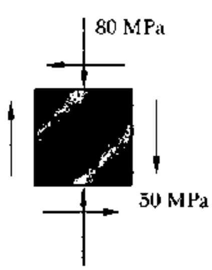
ENTER σ_x, σ_y, τ_{xy} AND θ

PRINT VALUES OBTAINED FOR $\sigma_{x'}$, $\sigma_{y'}$ and $\tau_{x'y'}$




```
Problem 7.13a

Sigma x = -60 MPa
Sigma y =  90 MPa
Tau xy  =  30 MPa

Rotation of element
(+ counterclockwise)
theta = -25 degrees

     Sigma x'  =  -56.19 MPa
     Sigma y'  =   86.19 MPa
     Tau x'y'  =  -38.17 MPa
```

```
Problem 7.13b

Sigma x = -60 MPa
Sigma y =  90 MPa
Tau xy  =  30 MPa

Rotation of element
(+ counterclockwise)
theta =  10 degrees

     Sigma x'  =  -45.22 MPa
     Sigma y'  =   75.22 MPa
     Tau x'y'  =   53.84 MPa
```



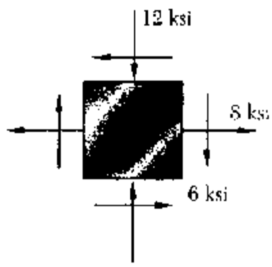

```
Problem 7.14a

  Sigma x =   0 MPa
  Sigma y = -80 MPa
  Tau xy  = -50 MPa

Rotation of element
(+ counterclockwise)
  theta = -25 degrees

  Sigma x' =   24.01 MPa
  Sigma y' = -104.01 MPa
  Tau x'y' =   -1.50 MPa
```

```
Problem 7.14b

  Sigma x =   0 MPa
  Sigma y = -80 MPa
  Tau xy  = -50 MPa

Rotation of element
(+ counterclockwise)
  theta =  10 degrees

  Sigma x' = -19.51 MPa
  Sigma y' = -60.49 MPa
  Tau x'y' = -60.67 MPa
```

CONTINUED

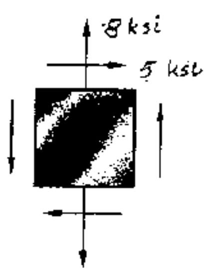
PROBLEM 7.C1 - CONTINUED

PROGRAM OUTPUT




```
Problem 7.15a

  Sigma x =    8 ksi
  Sigma y = -12 ksi
  Tau xy  =   -6 ksi

Rotation of element
(+ counterclockwise)
  theta = -25 degrees

  Sigma x'  =    9.02 ksi
  Sigma y'  = -13.02 ksi
  Tau x'y'  =    3.80 ksi
```

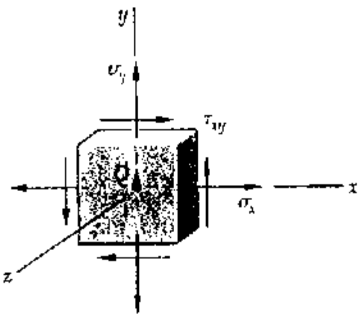
```
Problem 7.15b

  Sigma x =    8 ksi
  Sigma y = -12 ksi
  Tau xy  =   -6 ksi

Rotation of element
(+ counterclockwise)
  theta =  10 degrees

  Sigma x'  =    5.34 ksi
  Sigma y'  =   -9.34 ksi
  Tau x'y'  =   -9.06 ksi
```




```
Problem 7.16a

Sigma x =    0 ksi
Sigma y =    8 ksi
Tau xy  =    5 ksi

Rotation of element
(+ counterclockwise)
theta = -25 degrees

      Sigma x'  =    -2.40 ksi
      Sigma y'  =    10.40 ksi
      Tau x'y'  =     0.15 ksi
```

```
Problem 7.16b

Sigma x =    0 ksi
Sigma y =    8 ksi
Tau xy  =    5 ksi

Rotation of element
(+ counterclockwise)
theta =  10 degrees

      Sigma x'  =     1.95 ksi
      Sigma y'  =     6.05 ksi
      Tau x'y'  =     6.07 ksi
```

PROBLEM 7.C2



7.C2 A state of plane stress is defined by the stress components σ_x, σ_y, and τ_{xy} associated with the element shown in Fig. P7.C1*a*. (*a*) Write a computer program that can be used to determine the principal axes, the principal stresses, the maximum in-plane shearing stress, and the maximum shearing stress. (*b*) Use this program to solve Probs. 7.7, 7.11, 7.66, and 7.67.

SOLUTION PROGRAM [illegible]IONS

EQ. (7.10) $\sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2}$; $R = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

EQ. (7.14) $\sigma_{max} = \sigma_{ave} + R$

$\sigma_{min} = \sigma_{ave} - R$

EQ. (7.12) $\theta_p = \tan^{-1} \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y}$

EQ. (7.15) $\theta_s = \tan^{-1} - \dfrac{\sigma_x - \sigma_y}{2\tau_{xy}}$

SHEARING STRESS IF $\sigma_{max} > 0$ and $\sigma_{min} < 0$:

THEN $\tau_{max\,(in\text{-}plane)} = R$; $\tau_{max\,(out\text{-}of\text{-}plane)} = R$

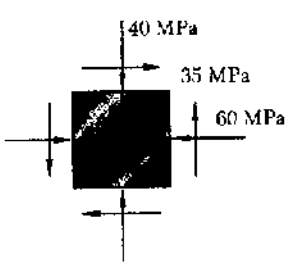
IF $\sigma_{max} > 0$ and $\sigma_{min} > 0$:

THEN $\tau_{max\,(in\text{-}plane)} = R$; $\tau_{max\,(out\text{-}of\text{-}plane)} = \frac{1}{2}\sigma_{max}$

IF $\sigma_{max} < 0$ and $\sigma_{min} < 0$:

THEN $\tau_{max\,(in\text{-}plane)} = R$; $\tau_{max\,(out\text{-}of\text{-}plane)} = \frac{1}{2}|\sigma_{min}|$

PROGRAM OUTPUT




```
Problems 7.7 AND 7.11

  Sigma x = -60.00 MPa
  Sigma y = -40.00 MPa
  Tau xy  =  35.00 MPa

Angle between xy axes and principal axes
            ( + counterclockwise )
      Theta p = -37.03 deg.  and  52.97 deg.
  Sigma max = -13.60 MPa
  Sigma min = -86.40 MPa
Angle between xy axis and planes of maximum in-plane shearing stress
            ( + counterclockwise )
      Theta s =   7.97 deg.  and  97.97 deg.
  Tau max (in plane) =  36.40 MPa
  Tau max    =  43.20 MPa
```

CONTINUED

PROBLEM 7.C2 - CONTINUED

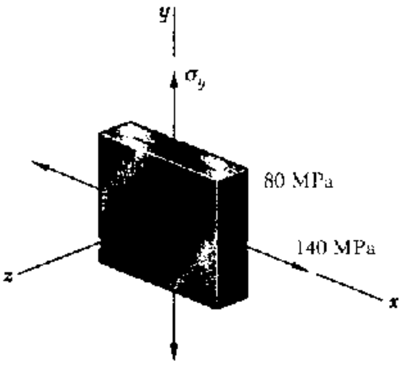


Fig. P7.66 and P7.67

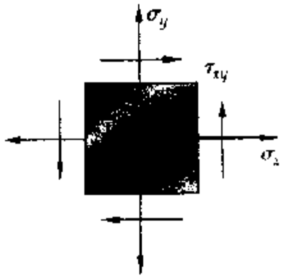
```
Problem 7.66a:  Sigma x = 140.00 MPa
                Sigma y =  20.00 MPa
                Tau xy  =  80.00 MPa

Angle between xy axes and principal axes
            ( + counterclockwise )
      Theta p =  26.57 deg.  and 116.57 deg.
  Sigma max = 180.00 MPa
  Sigma min = -20.00 MPa
Angle between xy axis and planes of maximum in-plane
    in-plane shearing stress ( + counterclockwise )
      Theta s =  71.57 deg.  and 161.57 deg.
  Tau max (in-plane)       = 100.00 MPa
  Tau max (out-of-plane)   = 100.00 MPa
```

```
Problem 7.66b:  Sigma x = 140.00 MPa
                Sigma y = 140.00 MPa
                Tau xy  =  80.00 MPa

Angle between xy axes and principal axes
            ( + counterclockwise )
      Theta p =  45.00 deg.  and 135.00 deg.
  Sigma max = 220.00 MPa
  Sigma min =  60.00 MPa
Angle between xy axis and planes of maximum in-plane
    in-plane shearing stress ( + counterclockwise )
      Theta s =  90.00 deg.  and 180.00 deg.
  Tau max (in-plane)       =  80.00 MPa
  Tau max (out-of-plane)   = 110.00 MPa
```

```
Problem 7.67a:  Sigma x = 140.00 MPa
                Sigma y =  40.00 MPa
                Tau xy  =  80.00 MPa

Angle between xy axes and principal axes
            ( + counterclockwise )
      Theta p =  29.00 deg.  and 119.00 deg.
  Sigma max = 184.34 MPa
  Sigma min =  -4.34 MPa
Angle between xy axis and planes of maximum in-plane
    in-plane shearing stress ( + counterclockwise )
      Theta s =  74.00 deg.  and 164.00 deg.
  Tau max (in-plane)       =  94.34 MPa
  Tau max (out-of-plane)   =  94.34 MPa
```

```
Problem 7.67b:  Sigma x = 140.00 MPa
                Sigma y = 120.00 MPa
                Tau xy  =  80.00 MPa

Angle between xy axes and principal axes
            ( + counterclockwise )
      Theta p =  41.44 deg.  and 131.44 deg.
  Sigma max = 210.62 MPa
  Sigma min =  49.38 MPa
Angle between xy axis and planes of maximum in-plane
    in-plane shearing stress ( + counterclockwise )
      Theta s =  86.44 deg.  and 176.44 deg.
  Tau max (in-plane)       =  80.62 MPa
  Tau max (out-of-plane)   = 105.31 MPa
```

PROBLEM 7.C3



7.C3 (*a*) Write a computer program that, for a given state of plane stress and a given yield strength of a ductile material, can be used to determine whether the material will yield. The program should use both the maximum-shearing-strength criterion and the maximum-distortion-energy criterion. It should also print the values of the principal stresses and, if the material does not yield, calculate the factor of safety. (*b*) Use this program to solve Probs. 7.81 through 7.84.

SOLUTION

PRINCIPAL STRESSES

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} \quad ; \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{ave} + R$$

$$\sigma_b = \sigma_{ave} - R$$

MAXIMUM-SHEARING-STRESS CRITERION $\tau_Y = \frac{1}{2}\sigma_Y$

IF σ_a AND σ_B HAVE SAME SIGN, $\tau_{max} = \frac{1}{2}\sigma_a$

IF $\tau_{max} > \tau_Y$, YIELDING OCCURS

IF $\tau_{max} < \tau_Y$, NO YIELDING OCCURS, AND

$$\text{FACTOR OF SAFETY} = \frac{\tau_Y}{\tau_{max}}$$

MAXIMUM-DISTORTION-ENERGY CRITERION

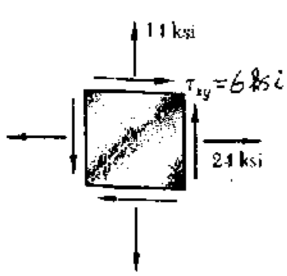
COMPUTE $\text{RADICAL} = \sqrt{\sigma_a^2 - \sigma_a\sigma_b + \sigma_b^2}$

IF RADICAL $> \sigma_Y$, YIELDING OCCURS

IF RADICAL $< \sigma_Y$, NO YIELDING OCCURS, AND

$$\text{FACTOR OF SAFETY} = \frac{\sigma_Y}{\text{RADICAL}}$$

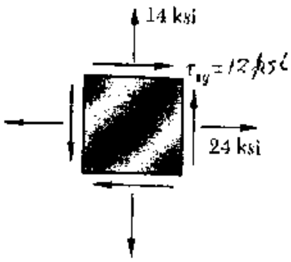
PROGRAM OUTPUT




```
Problems 7.81a and 7.82a    Yield strength = 30 ksi

   Sigma x = 24.00 ksi
   Sigma y = 14.00 ksi
   Tau xy  =  6.00 ksi
        Sigma max =   26.81 ksi
        Sigma min =   11.19 ksi
    (a) Using the maximum-shearing-stress criterion:
         Material will not yield
         F.S. =  1.119
    (b) Using the maximum-distortion-energy criterion:
         Material will not yield
         F.S. =  1.286
```

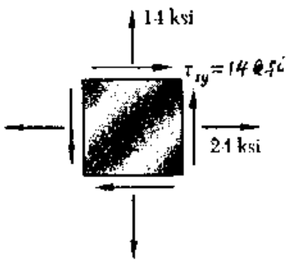
CONTINUED

PROBLEM 7.C3 - CONTINUED



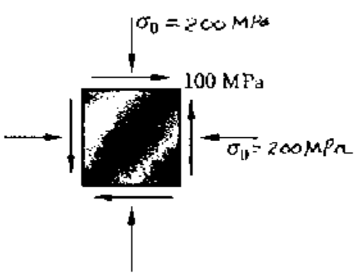
Problems 7.81b and 7.82b   Yield strength = 30 ksi

```
Sigma x = 24.00 ksi
Sigma y = 14.00 ksi
Tau xy  = 12.00 ksi
      Sigma max =   32.00 ksi
      Sigma min =    6.00 ksi
 (a) Using the maximum-shearing-stress criterion:
       Material will yield
 (b) Using the maximum-distortion-energy criterion:
       Material will not yield
       F.S. =  1.018
```



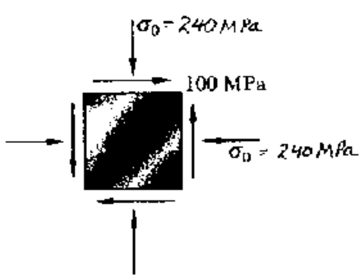
Problems 7.81c and 7.82c   Yield strength = 30 ksi

```
Sigma x = 24.00 ksi
Sigma y = 14.00 ksi
Tau xy  = 14.00 ksi
      Sigma max =   33.87 ksi
      Sigma min =    4.13 ksi
 (a) Using the maximum-shearing-stress criterion:
       Material will yield
 (b) Using the maximum-distortion-energy criterion:
       Material will yield
```



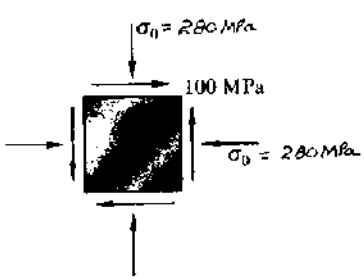
Problems 7.83a and 7.84a   Yield strength = 325 MPa

```
                       Sigma x = -200.00 MPa
                       Sigma y = -200.00 MPa
                       Tau xy  =  100.00 MPa
               Sigma max = -100.00 MPa
               Sigma min = -300.00 MPa
         Using the maximum-shearing-stress criterion:
           Material will not yield
           F.S. =  1.083
         Using the maximum-distortion-energy criterion:
           Material will not yield
           F.S. =  1.228
```



Problems 7.83b and 7.84b   Yield strength = 325 MPa

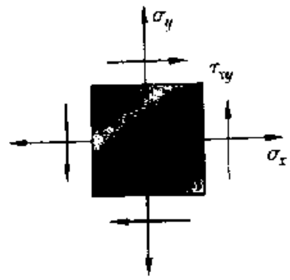
```
                       Sigma x = -240.00 MPa
                       Sigma y = -240.00 MPa
                       Tau xy  =  100.00 MPa
               Sigma max = -140.00 MPa
               Sigma min = -340.00 MPa
         Using the maximum-shearing-stress criterion:
           Material will yield
         Using the maximum-distortion-energy criterion:
           Material will not yield
           F.S. =  1.098
```



Problems 7.83c and 7.84c   Yield strength = 325 MPa

```
                       Sigma x = -280.00 MPa
                       Sigma y = -280.00 MPa
                       Tau xy  =  100.00 MPa
               Sigma max = -180.00 MPa
               Sigma min = -380.00 MPa
         Using the maximum-shearing-stress criterion:
           Material will yield
         Using the maximum-distortion-energy criterion:
           Material will yield
```

PROBLEM 7.C4



7.C4 (*a*) Write a computer program based on Mohr's fracture criterion for brittle materials that, for a given state of plane stress and given values of the ultimate strength of the material in tension and in compression, can be used to determine whether rupture will occur. The program should also print the values of the principal stresses. (*b*) Use this program to solve Probs. 7.91 and 7.92 and to check the answers given for Probs. 7.93 and 7.94.

SOLUTION

PRINCIPAL STRESSES

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{ave} + R$$

$$\sigma_b = \sigma_{ave} - R$$

MOHR'S FRACTURE CRITERION

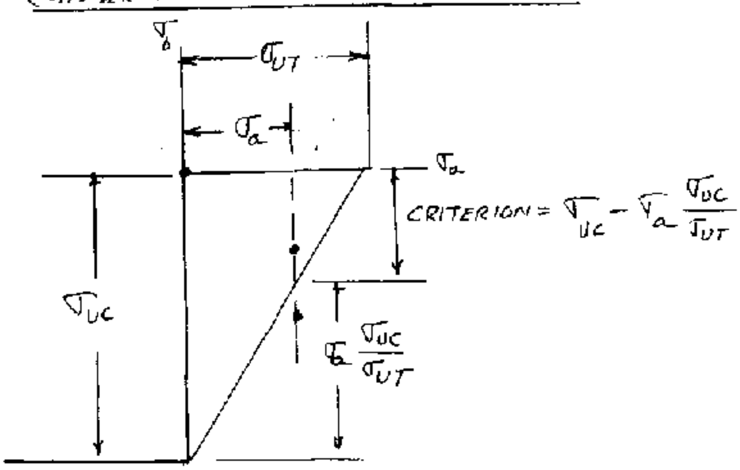
IF σ_a AND σ_b HAVE SAME SIGN, AND

$\sigma_a < \sigma_{UT}$ AND $\sigma_b < \sigma_{UC}$, NO FAILURE

$\sigma_a > \sigma_{UT}$ OR $\sigma_b > \sigma_{UC}$, FAILURE

IF $\sigma_a > 0$ AND $\sigma_b < 0$:

CONSIDER FOURTH QUADRANT OF FIG. 7.47



FOR NO RUPTURE TO OCCUR:
POINT (σ_a, σ_b) MUST LIE WITHIN MOHR'S ENVELOPE (FIG. 7.47)

IF σ_b > CRITERION,
THEN RUPTURE OCCURS

IF σ_b < CRITERION,
THEN NO RUPTURE OCCURS

PROGRAM OUTPUT

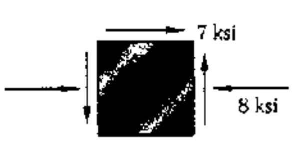


Fig. P7.91

```
Problem 7.91      Sigma x =  -8.00 ksi
                  Sigma y =   0.00 ksi
                   Tau xy =   7.00 ksi
Ultimate strength in tension     = 10 ksi
Ultimate strength in compression = 30 ksi

  Sigma max = Sigma a    4.06 ksi
  Sigma min = Sigma b  -12.06 ksi
Rupture will not occur
```

CONTINUED

PROBLEM 7.C4 - CONTINUED

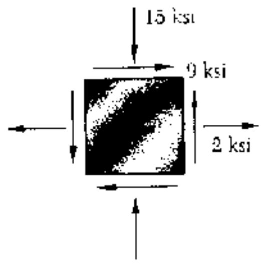


Fig. *P7.92*

```
Problem 7.92      Sigma x =   2.00 ksi
                  Sigma y = -15.00 ksi
                   Tau xy =   9.00 ksi
Ultimate strength in tension     = 10 ksi
Ultimate strength in compression = 30 ksi

  Sigma max = Sigma a    5.88 ksi
  Sigma min = Sigma b  -18.88 ksi
Rupture will occur
```

TO CHECK ANSWERS TO FOLLOWING PROBLEMS, WE CHECK FOR RUPTURE WITH GIVEN ANSWERS AND AN ADJACENT VALUE.

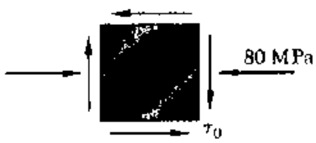
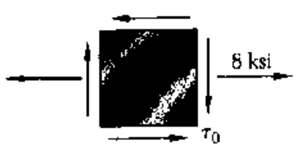


Fig. P7.93

ANSWER:
RUPTURE OCCURS FOR $\tau_0 = 49.1$ MPa

```
Problem 7.93      Sigma x = -80.00 MPa
                  Sigma y =   0.00 MPa
                   Tau xy =  49.10 MPa
Ultimate strength in tension     =  75 MPa
Ultimate strength in compression = 150 MPa
  Sigma max = Sigma a =   23.33 MPa
  Sigma min = Sigma b = -103.33 MPa
     Rupture will not occur
```

```
Problem 7.93      Sigma x = -80.00 MPa
                  Sigma y =   0.00 MPa
                   Tau xy =  49.20 MPa
Ultimate strength in tension     =  75 MPa
Ultimate strength in compression = 150 MPa
  Sigma max = Sigma a =   23.41 MPa
  Sigma min = Sigma b = -103.41 MPa
     Rupture will occur
```



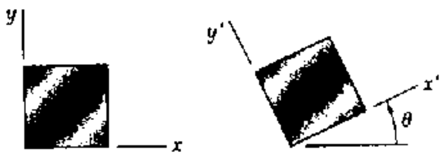
ANSWER:
RUPTURE OCCURS FOR $\tau_0 = 3.67$ ksi

Fig. *P7.94*

```
Problem 7.94      Sigma x =   8.00 ksi
                  Sigma y =   0.00 ksi
                   Tau xy =   3.67 ksi
Ultimate strength in tension     = 10 ksi
Ultimate strength in compression = 25 ksi
  Sigma max = Sigma a    9.43 ksi
  Sigma min = Sigma b   -1.43 ksi
     Rupture will not occur
```

```
Problem 7.94      Sigma x =   8.00 ksi
                  Sigma y =   0.00 ksi
                   Tau xy =   3.68 ksi
Ultimate strength in tension     = 10 ksi
Ultimate strength in compression = 25 ksi
  Sigma max = Sigma a    9.44 ksi
  Sigma min = Sigma b   -1.44 ksi
     Rupture will occur
```

PROBLEM 7.C5



7.C5 A state of plane strain is defined by the strain components ϵ_x, ϵ_y, and γ_{xy} associated with the x and y axes. (*a*) Write a computer program that can be used to calculate the strain components $\epsilon_{x'}$, $\epsilon_{y'}$, and $\gamma_{x'y'}$ associated with the frame of reference $x'y'$ obtained by rotating the x and y axes through an angle θ. (*b*) Use this program to solve Probs. 7.126 through 7.129.

SOLUTION <u>PROGRAM FOLLOWS EQUATIONS</u>

EQ (7.44) $\varepsilon_x' = \dfrac{\varepsilon_x+\varepsilon_y}{2} + \dfrac{\varepsilon_x-\varepsilon_y}{2}\cos 2\theta + \frac{1}{2}\gamma_{xy}\sin 2\theta$

EQ (7.45) $\varepsilon_y' = \dfrac{\varepsilon_x+\varepsilon_y}{2} - \dfrac{\varepsilon_x-\varepsilon_y}{2}\sin 2\theta - \frac{1}{2}\gamma_{xy}\cos 2\theta$

EQ. (7.46) $\gamma_{xy}' = -(\varepsilon_x-\varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$

<u>ENTER</u> ε_x, ε_y, γ_{xy} AND θ

<u>PRINT</u> VALUES OBTAINED FOR ε_x', ε_y', AND γ_{xy}'

<u>PROGRAM OUTPUT</u>

```
Problem 7.126        Epsilon x =  240 micro meters
                     Epsilon y =  160 micro meters
                     Gamma xy  =  150 micro radians
Rotation of element, in degrees  ( + counterclockwise )
                     Theta = -60 degrees

          Epsilon x'=  115.05 micro meters
          Epsilon y'=  284.95 micro meters
          Gamma x'y'=   -5.72 micro radians


Problem 7.127        Epsilon x =    0 micro meters
                     Epsilon y =  320 micro meters
                     Gamma xy  = -100 micro radians
Rotation of element, in degrees  ( + counterclockwise )
                     Theta =  30 degrees

          Epsilon x'=   36.70 micro meters
          Epsilon y'=  283.30 micro meters
          Gamma x'y'=  227.13 micro radians


Problem 7.128        Epsilon x = -800 micro meters
                     Epsilon y =  450 micro meters
                     Gamma xy  =  200 micro radians
Rotation of element, in degrees  ( + counterclockwise )
                     Theta = -25 degrees

          Epsilon x'= -653.35 micro meters
          Epsilon y'=  303.35 micro meters
          Gamma x'y'= -829.00 micro radians


Problem 7.129        Epsilon x =  500 micro meters
                     Epsilon y = -300 micro meters
                     Gamma xy  =    0 micro radians
Rotation of element, in degrees  ( + counterclockwise )
                     Theta =  25 degrees

          Epsilon x'=  357.12 micro meters
          Epsilon y'= -157.12 micro meters
          Gamma x'y'= -612.84 micro radians
```

PROBLEM 7.C6

7.C6 A state of strain is defined by the strain components ϵ_x, ϵ_y, and γ_{xy}, associated with the x and y axes. (a) Write a computer program that can be used to determine the orientation and magnitude of the principal strains, the maximum in-plane shearing strain, and the maximum shearing strain. (b) Use this program to solve Probs. 7.134 through 7.137.

SOLUTION PROGRAM FOLLOWING EQUATIONS

EQ(7.50) $\varepsilon_{ave} = \dfrac{\varepsilon_x + \varepsilon_y}{2}$ $\quad R = \sqrt{\left(\dfrac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\dfrac{\gamma_{xy}}{2}\right)^2}$

EQ(7.51) $\varepsilon_{max} = \varepsilon_{ave} + R$ $\quad \varepsilon_{min} = \varepsilon_{ave} - R$

EQ(7.52) $\theta_p = \tan^{-1} \dfrac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$

SHEARING STRAINS

MAXIMUM IN-PLANE SHEARING STRAIN

$$\gamma_{max(in\text{-}plane)} = 2R$$

CALCULATE OUT-OF-PLANE SHEARING STRAIN AND CHECK WHETHER IT IS THE MAXIMUM SHEARING STRAIN

LET $\varepsilon_a = \varepsilon_{max}$

$\varepsilon_b = \varepsilon_{min}$

CALCULATE $\varepsilon_c = -\dfrac{\nu}{1-\nu}(\varepsilon_a + \varepsilon_b)$

IF $\varepsilon_a > \varepsilon_b > \varepsilon_c$: $\gamma_{out\text{-}of\text{-}plane} = \varepsilon_a - \varepsilon_c$

IF $\varepsilon_a > \varepsilon_c > \varepsilon_b$: $\gamma_{out\text{-}of\text{-}plane} = \varepsilon_a - \varepsilon_b = 2R$

IF $\varepsilon_c > \varepsilon_a > \varepsilon_b$: $\gamma_{out\text{-}of\text{-}plane} = \varepsilon_c - \varepsilon_b$

PROGRAM PRINTOUT

```
Problem 7.134        Epsilon x =  160 micro meters
                     Epsilon y = -480 micro meters
                     Gamma xy  = -600 micro radians
                     nu = 0.333

Angle between xy axes and principal axes (+ = counterclockwise)
       Theta p   =  -21.58 degrees
       Epsilon a =  278.63 micro meters
       Epsilon b = -598.63 micro meters
       Epsilon c =  159.98 micro meters

       Gamma max (in plane) =  877.27 micro radians
       Gamma max =  877.27 micro radians
```

CONTINUED

```
Problem 7.135      Epsilon x = -260 micro meters
                   Epsilon y =  -60 micro meters
                   Gamma xy  =  480 micro radians
                   nu = 0.333

Angle between xy axes and principal axes (+ = counterclockwise)
       Theta p   =  -33.69 degrees
       Epsilon a =  100.00 micro meters
       Epsilon b = -420.00 micro meters
       Epsilon c =  159.98 micro meters

       Gamma max (in plane) =  520.00 micro radians
       Gamma max =  579.98 micro radians


Problem 7.136      Epsilon x =   30 micro meters
                   Epsilon y =  570 micro meters
                   Gamma xy  =  720 micro radians
                   nu = 0.333

Angle between xy axes and principal axes (+ = counterclockwise)
       Theta p   =   -26.57 degrees
       Epsilon a =   750.00 micro meters
       Epsilon b =  -150.00 micro meters
       Epsilon c =  -300.00 micro meters

       Gamma max (in plane) =  900.00 micro radians
       Gamma max = 1050.00 micro radians


Problem 7.137      Epsilon x = -600 micro meters
                   Epsilon y = -400 micro meters
                   Gamma xy  =  350 micro radians
                   nu = 0.333

Angle between xy axes and principal axes (+ = counterclockwise)
       Theta p   =   -30.13 degrees
       Epsilon a =  -298.44 micro meters
       Epsilon b =  -701.56 micro meters
       Epsilon c =   500.00 micro meters

       Gamma max (in plane) =  403.11 micro radians
       Gamma max = 1201.56 micro radians
```

PROBLEM 7.C7

7.C7 A state of plane strain is defined by the strain components ϵ_x, ϵ_y, and γ_{xy} measured at a point. (*a*) Write a computer program that can be used to determine the orientation and magnitude of the principal strains, the maximum in-plane shearing strain, and the maximum shearing strain. (*b*) Use this program to solve Probs. 7.138 through 7.141.

SOLUTION

PROGRAM FOLLOWING EQUATIONS

EQ(7.50) $\varepsilon_{ave} = \dfrac{\varepsilon_x + \varepsilon_y}{2}$ $\qquad R = \sqrt{\left(\dfrac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\dfrac{\gamma_{xy}}{2}\right)^2}$

EQ(7.51) $\varepsilon_{max} = \varepsilon_{ave} + R \qquad \varepsilon_{min} = \varepsilon_{ave} - R$

EQ(7.52) $\theta_p = \tan^{-1} \dfrac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$

SHEARING STRAINS

MAXIMUM IN-PLANE SHEARING STRAIN

$$\gamma_{xy\,(in-plane)} = 2R$$

CALCULATE OUT-OF-PLANE SHEARING STRAIN AND CHECK WHETHER IT IS THE MAXIMUM SHEARING STRAIN

LET $\varepsilon_a = \varepsilon_{max}$

$\varepsilon_b = \varepsilon_{min}$

$\varepsilon_c = 0$ (PLAIN STRAIN)

IF $\varepsilon_a > \varepsilon_b > \varepsilon_c$: $\gamma_{OUT-OF-PLANE} = \varepsilon_a - \varepsilon_c$

IF $\varepsilon_a > \varepsilon_c > \varepsilon_b$: $\gamma_{OUT-OF-PLANE} = \varepsilon_a - \varepsilon_b = 2R$

IF $\varepsilon_c > \varepsilon_a > \varepsilon_b$: $\gamma_{OUT-OF-PLANE} = \varepsilon_c - \varepsilon_b$

PROGRAM PRINTOUT

```
Problem 7.138        Epsilon x = -180 micro meters
                     Epsilon y = -260 micro meters
                     Gamma xy  =  315 micro radians
                     nu = 0.000

Angle between xy axes and principal axes (+ = counterclockwise)
       Theta p   =    37.87 and -52.13 degrees
       Epsilon a =   -57.50 micro meters
       Epsilon b =  -382.50 micro meters
       Epsilon c =     0.00 micro meters

       Gamma max (in plane) =  325.00 micro radians
       Gamma max =  382.50 micro radians
```

CONTINUED

```
Problem 7.139          Epsilon x =  300 micro meters
                       Epsilon y =   60 micro meters
                       Gamma xy  =  100 micro radians
                       nu = 0.000

Angle between xy axes and principal axes (+ = counterclockwise)
       Theta p   =     11.31 and -78.69 degrees
       Epsilon a =   310.00 micro meters
       Epsilon b =    50.00 micro meters
       Epsilon c =     0.00 micro meters

       Gamma max (in plane) =  260.00 micro radians
       Gamma max =  310.00 micro radians


Problem 7.140          Epsilon x =  400 micro meters
                       Epsilon y =  200 micro meters
                       Gamma xy  =  375 micro radians
                       nu = 0.000

Angle between xy axes and principal axes (+ = counterclockwise)
       Theta p   =     30.96 and -59.04 degrees
       Epsilon a =   512.50 micro meters
       Epsilon b =    87.50 micro meters
       Epsilon c =     0.00 micro meters

       Gamma max (in plane) =  425.00 micro radians
       Gamma max =  512.50 micro radians


Problem 7.141          Epsilon x =   60 micro meters
                       Epsilon y =  240 micro meters
                       Gamma xy  =  -50 micro radians
                       nu = 0.000

Angle between xy axes and principal axes (+ = counterclockwise)
       Theta p   =      7.76 and -82.24 degrees
       Epsilon a =   243.41 micro meters
       Epsilon b =    56.59 micro meters
       Epsilon c =     0.00 micro meters

       Gamma max (in plane) =  186.82 micro radians
       Gamma max =  243.41 micro radians
```

PROBLEM 7.C8

7.C8 A rosette consisting of three gages forming, respectively, angles θ_1, θ_2, and θ_3 with the x axis is attached to the free surface of a machine component made of a material with a given Poisson's ratio ν. (*a*) Write a computer program that, for given readings ϵ_1, ϵ_2, and ϵ_3 of the gages, can be used to calculate the strain components associated with the x and y axes and to determine the orientation and magnitude of the three principal strains, the maximum in-plane shearing strain, and the maximum shearing strain. (*b*) Use this program to solve Probs. 7.142 through 7.145.

SOLUTION

FOR $n = 1$ TO 3, ENTER θ_n and ϵ_n

ENTER: NU = ν

SOLVE EQS. (7.60) FOR ϵ_x, ϵ_y, AND γ_{xy} USING METHOD OF DETERMINATES OR ANY OTHER METHOD,

ENTER $\epsilon_{ave} = \dfrac{\epsilon_x + \epsilon_y}{2}$; $R = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \gamma_{xy}^2}$

$$\epsilon_a = \epsilon_{max} = \epsilon_{ave} + R$$

$$\epsilon_b = \epsilon_{min} = \epsilon_{ave} - R$$

$$\epsilon_c = -\frac{\nu}{1-\nu}\left(\epsilon_a + \epsilon_b\right)$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\epsilon_x - \epsilon_y}$$

SHEARING STRAINS

MAXIMUM IN-PLANE SHEARING STRAIN

$$\gamma_{max\,(in\text{-}plane)} = 2R$$

CALCULATE OUT-OF-PLANE SHEARING STRAIN, AND CHECK WHETHER IT IS THE MAXIMUM SHEARING STRAIN,

IF $\epsilon_c < \epsilon_b$: $\gamma_{OUT\text{-}OF\text{-}PLANE} = \epsilon_a - \epsilon_c$

IF $\epsilon_c > \epsilon_a$: $\gamma_{OUT\text{-}OF\text{-}PLANE} = \epsilon_c - \epsilon_b$

OTHERWISE: $\gamma_{OUT\text{-}OF\text{-}PLANE} = 2R$

PROGRAM OUTPUT

```
Problem 7.142
                         Gage     theta      epsilon
                                  degrees    micro meters

                          1         30          600
                          2        -30          450
                          3         90          -75

          Epsilon x =   725.000 micro meters
          Epsilon y =   -75.000 micro meters
          Gamma xy  =   173.205 micro radians

          Epsilon a =   734.268 micro meters
          Epsilon b =   -84.268 micro meters
          Gamma max (in plane) =  818.535 micro radians
```

CONTINUED

Problem 7.143

Gage	theta degrees	epsilon in./in.
1	-15	480
2	30	-120
3	75	80

```
Epsilon x =   253.21 micro meters
Epsilon y =   306.79 micro meters
Gamma xy  =  -892.82 micro radians

Epsilon a =   727.21 micro meters
Epsilon b =  -167.21 micro meters
Gamma max (in plane) =  894.43 micro radians
```

Problem 7.144

OBSERVE THAT GAGE 3 IS ORIENTATED ALONG THE y AXIS. THEREFORE

ENTER θ_4 AND ε_4 AS θ_3 AND ε_3,

THE VALUE OF ε_y THAT IS OBTAINED IS ALSO THE EXPECTED READING OF GAGE 3.

Gage	theta degrees	epsilon micro meters
1	0	420
2	45	-45
$4 \rightarrow \cancel{3}$	135	165

```
Epsilon x =   420.00 micro meters
Epsilon y =  -300.00 micro meters
Gamma xy  =  -210.00 micro radians

Epsilon a =   435.00 micro meters
Epsilon b =  -315.00 micro meters
Gamma max (in plane) =  750.00 micro radians
```

Problem 7.145

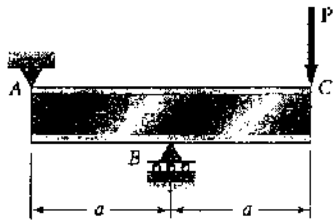
Gage	theta degrees	epsilon in./in.
1	45	-50
2	-45	360
3	0	315

```
Epsilon x =   315.000 in./in.
Epsilon y =    -5.000 in./in.
Gamma xy  =  -410.000 micro radians

Epsilon a =   415.048 in./in.
Epsilon b =  -105.048 in./in.
Gamma max (in plane) =  520.096 micro radians
```

Chapter 8

Problem 8.1

8.1 An overhanging W920 × 446 rolled-steel beam supports a load **P** as shown. Knowing that P = 1420 kN, a = 2.5 m, and σ_{all} = 200 MPa, determine (a) the maximum value of the normal stress σ_m in the beam, (b) the maximum value of the principal stress σ_{max} at the junction of the flange and web, (c) whether the specified shape is acceptable as far as these two stresses are concerned.



$|V|_{max} = 1420 \text{ kN} = 1420 \times 10^3 \text{ N}$

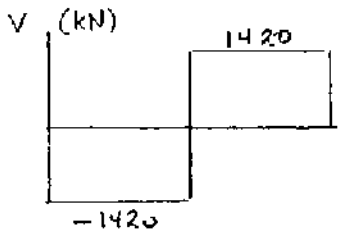
$|M|_{max} = (1420 \times 10^3)(2.5) = 3.55 \times 10^6 \text{ N}\cdot\text{m}$

For W920 × 446 rolled steel beam

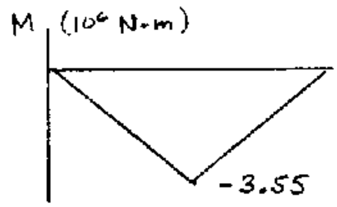
$d = 933 \text{ mm} \quad b_f = 423 \text{ mm} \quad t_f = 42.70 \text{ mm}$

$t_w = 24.0 \text{ mm} \quad I_x = 8470 \times 10^6 \text{ mm}^4 \quad S_x = 18200 \times 10^3 \text{ mm}^3$

$c = \frac{1}{2} d = 466.5 \text{ mm} \quad y_b = c - t_f = 423.8 \text{ mm}$



(a) $\sigma_m = \dfrac{|M|_{max}}{S_x} = \dfrac{3.55 \times 10^6}{18200 \times 10^{-6}} = 195.1 \text{ MPa}$



$\sigma_b = \dfrac{y_b}{c} \sigma_m = \dfrac{423.8}{466.5}(195.1) = 177.2 \text{ MPa}$

$A_f = b_f t_f = 18.062 \times 10^3 \text{ mm}^2$

$\bar{y}_f = \frac{1}{2}(c + y_b) = 445.15 \text{ mm}$

$Q_b = A_f \bar{y}_f = 8040.3 \times 10^3 \text{ mm}^3 = 8040.3 \times 10^{-6} \text{ m}^3$

$\tau_{xy} = \dfrac{|V|_{max} Q_b}{I_x t_w} = \dfrac{(1420 \times 10^3)(8040.3 \times 10^{-6})}{(8470 \times 10^{-6})(24.0 \times 10^{-3})} = 56.2 \text{ MPa}$

$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{177.2}{2}\right)^2 + 56.2^2} = 104.9 \text{ MPa}$

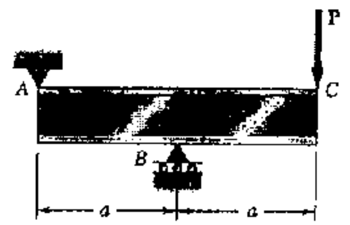
(b) $\sigma_{max} = \dfrac{\sigma_b}{2} + R = 193.5 \text{ MPa}$

(c) Since 193.5 MPa < σ_{all}, W920 × 446 is <u>acceptable</u>.

Problem 8.2

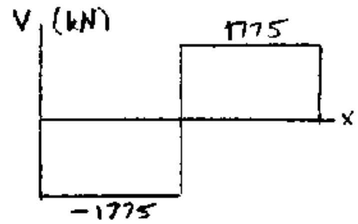
8.2 Solve Prob. 8.1, assuming that $P = 1775$ kN and $a = 2.0$ m.

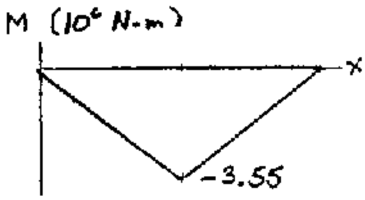
8.1 An overhanging W920 × 446 rolled-steel beam supports a load **P** as shown. Knowing that $P = 1420$ kN, $a = 2.5$ m, and $\sigma_{all} = 200$ MPa, determine (*a*) the maximum value of the normal stress σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of the flange and web, (*c*) whether the specified shape is acceptable as far as these two stresses are concerned.



$|V|_{max} = 1775 \text{ kN} = 1775 \times 10^3 \text{ N}$

$|M|_{max} = (1775 \times 10^3)(2.0) = 3.55 \times 10^6 \text{ N·m}$





For W920×446 rolled steel section

$d = 933$ mm $\quad b_f = 423$ mm $\quad t_f = 42.70$ mm

$t_w = 24.0$ mm $\quad I_x = 8470 \times 10^6 \text{ mm}^4 \quad S_x = 18200 \times 10^3 \text{ mm}^3$

$c = \frac{1}{2} d = 466.5 \text{ mm} \quad y_b = c - t_f = 423.8 \text{ mm}$

(a) $\sigma_m = \dfrac{|M|_{max}}{S_x} = \dfrac{3.55 \times 10^6}{18200 \times 10^{-6}} = 195.1 \text{ MPa}$ ◄

$\sigma_b = \dfrac{y_b}{c} \sigma_m = \dfrac{423.8}{466.5}(195.1) = 177.2 \text{ MPa}$

$A_f = b_f t_f = 18.062 \times 10^3 \text{ mm}^2$

$\bar{y}_f = \frac{1}{2}(c + y_b) = 445.5 \text{ mm}$

$Q_b = A_f \bar{y}_f = 8040.3 \times 10^3 \text{ mm}^3 = 8040.3 \times 10^{-6} \text{ m}^3$

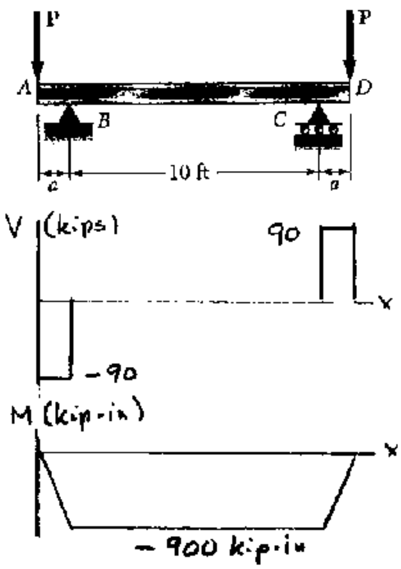
$$\tau_{xy} = \frac{|V|_{max} Q_b}{I_x t_w} = \frac{(1775 \times 10^3)(8040.3 \times 10^{-6})}{(8470 \times 10^{-6})(24.0 \times 10^{-3})} = 70.2 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{177.2}{2}\right)^2 + 70.2^2} = 113.0 \text{ MPa}$$

(b) $\sigma_{max} = \dfrac{\sigma_b}{2} + R = 201.6 \text{ MPa}$ ◄

(c) Since $201.6 \text{ MPa} > \sigma_{all}$ W920×446 is <u>not acceptable</u> ◄

Problem 8.3



8.3 An overhanging W10 × 39 rolled-steel beam supports a load **P** as shown. Knowing that $P = 90$ kips, $a = 10$ in., and $\sigma_{all} = 36$ ksi, determine (*a*) the maximum value of the normal stress σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of the flange and web, (*c*) whether the specified shape is acceptable as far as these two stresses are concerned.

$|V|_{max} = 90$ kips

$|M|_{max} = (90)(10) = 900$ kip-in

For W 10 × 39 rolled steel section

$d = 9.92$ in. $\quad b_f = 7.985$ in. $\quad t_f = 0.530$ in.

$t_w = 0.315$ in. $\quad I_x = 209$ in^4 $\quad S_x = 42.1$ in^3

$c = \frac{1}{2}d = 4.96$ in. $\quad y_b = c - t_f = 4.43$ in.

(a) $\sigma_m = \dfrac{|M|_{max}}{S_x} = \dfrac{900}{42.1} = 21.37$ ksi ◄

$\sigma_b = \dfrac{y_b}{c}\sigma_m = \left(\dfrac{4.43}{4.96}\right)(21.37) = 19.09$ ksi

$A_f = b_f t_f = 4.23205$ in^2

$\bar{y}_f = \frac{1}{2}(c + y_b) = 4.695$ in.

$Q_b = A_f \bar{y}_f = 19.869$ in^3

$\tau_{xy} = \dfrac{|V|_{max} Q_b}{I_x t_w} = \dfrac{(90)(19.869)}{(209)(0.315)} = 27.16$ ksi

$R = \sqrt{\left(\dfrac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = 28.79$ ksi

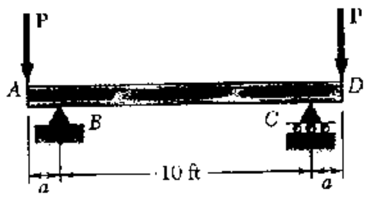
(b) $\sigma_{max} = \dfrac{\sigma_b}{2} + R = 38.3$ ksi ◄

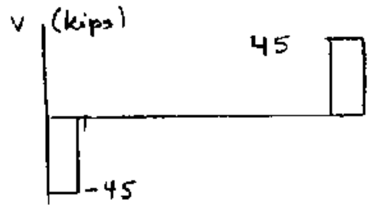
(c) Since $\sigma_{max} > 36$ ksi, W 10 × 39 is not acceptable. ◄

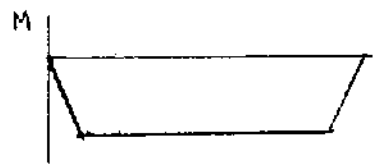
Problem 8.4

8.4 Solve Prob. 8.3, assuming that $P = 45$ kips, $a = 20$ in..

8.3 An overhanging W10 × 39 rolled-steel beam supports a load **P** as shown. Knowing that $P = 90$ kips, $a = 10$ in., and $\sigma_{all} = 36$ ksi, determine (*a*) the maximum value of the normal stress σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of the flange and web, (*c*) whether the specified shape is acceptable as far as these two stresses are concerned.







$|V|_{max} = 45$ kips

$|M|_{max} = (45)(20) = 900$ kip·in

For W 10 × 39 rolled steel section

$d = 9.92$ in. $\quad b_f = 7.985$ in. $\quad t_f = 0.530$ in.

$t_w = 0.315$ in. $\quad I_x = 209$ in^4 $\quad S_x = 42.1$ in^3

$c = \frac{1}{2} d = 4.96$ in. $\quad y_b = c - t_f = 4.43$ in.

(a) $\sigma_m = \dfrac{|M|_{max}}{S_x} = \dfrac{900}{42.1} = 21.37$ ksi

$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{4.43}{4.96}\right)(21.37) = 19.09 \text{ ksi}$$

$A_f = b_f t_f = 4.23205$ in^2

$\bar{y}_f = \frac{1}{2}(c + y_b) = 4.695$ in.

$Q_b = A_f \bar{y}_f = 19.869$ in^3

$$\tau_{xy} = \frac{|V|_{max} Q_b}{I_x t_w} = \frac{(45)(19.869)}{(209)(0.315)} = 13.58 \text{ ksi}$$

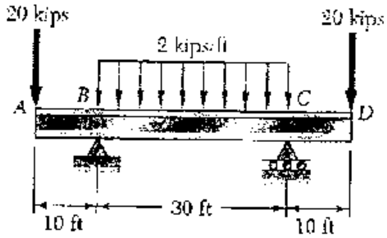
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = 16.60$$

(b) $\sigma_{max} = \dfrac{\sigma_b}{2} + R = 26.14$ ksi

(c) Since $\sigma_{max} < 36$ ksi, W 10 × 39 is <u>acceptable</u>.

Problem 8.5

8.5 and 8.6 (*a*) Knowing that $\sigma_{all} = 24$ ksi and $\tau_{all} = 14.5$ ksi, select the most economical wide-flange shape that should be used to support the loading shown. (*b*) Determine the values to be expected for σ_m, τ_m, and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.

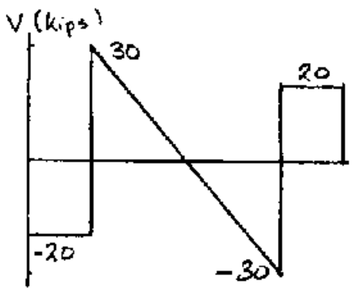


$R_A = 50$ kips ↑ $\quad$ $R_D = 50$ kips ↑

$|V|_{max} = 30$ kips

$|M|_{max} = 200$ kip·ft $= 2400$ kip·in.

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{2400}{24} = 100 \text{ in}^3$$



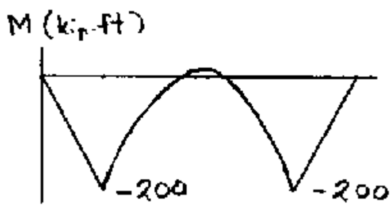
Shape	S (in³)
W 24 × 68	154
→ W 21 × 62	127
W 18 × 76	146
W 16 × 77	134
W 12 × 96	103
W 10 × 112	131

(a) Use W 21 × 62.

$d = 20.99$ in.

$t_f = 0.615$ in.

$t_w = 0.400$ in.



$$\sigma_m = \frac{|M|_{max}}{S} = \frac{2400}{127} = 18.90 \text{ ksi}$$

$$\tau_m = \frac{|V|_{max}}{d\,t_w} = \frac{30}{(20.99)(0.400)} = 3.57 \text{ ksi}$$

$c = \frac{1}{2}d = \frac{20.99}{2} = 10.495$ in. $\quad$ $y_b = c - t_f = 10.495 - 0.615 = 9.88$ in.

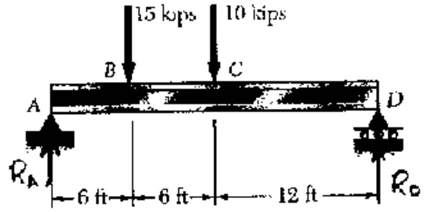
$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{9.88}{10.495}\right)(18.90) = 17.79 \text{ ksi}$$

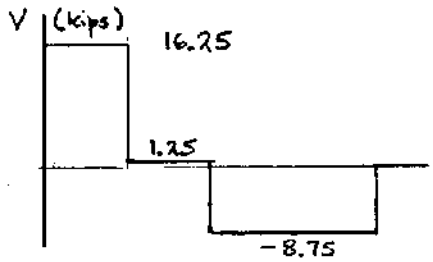
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_m^2} = \sqrt{(8.896)^2 + (3.57)^2} = 9.586 \text{ ksi}$$

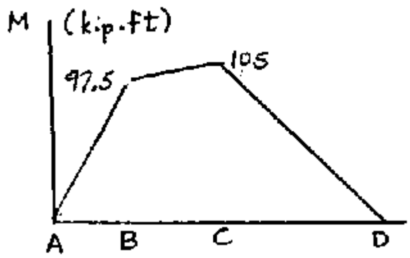
$$\sigma_{max} = \frac{\sigma_b}{2} + R = 8.896 + 9.586 = 18.48 \text{ ksi}$$

Problem 8.6

8.5 and 8.6 (*a*) Knowing that $\sigma_{all} = 24$ ksi and $\tau_{all} = 14.5$ ksi, select the most economical wide-flange shape that should be used to support the loading shown. (*b*) Determine the values to be expected for σ_m, τ_m, and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.







$+\circlearrowleft \Sigma M_D = 0: \quad -24 R_A + (18)(15) + (12)(10) = 0$

$R_A = 16.25 \text{ kips} \uparrow$

$|V|_{max} = 16.25 \text{ kips}$

$|M|_{max} = 105 \text{ kip}\cdot\text{ft} = 1260 \text{ kip}\cdot\text{in}$

$S_{min} = \dfrac{|M|_{max}}{\sigma_{all}} = \dfrac{1260}{24} = 52.5 \text{ in}^3$

Shape	S_x (in^3)
W 21 × 44	81.6
W 18 × 35	57.6
W 16 × 40	64.7
W 14 × 38	54.6
W 12 × 50	64.7
W 10 × 54	60.0

(a) Use W 18 × 35
$d = 17.70$ in.
$t_f = 0.425$ in.
$t_w = 0.300$ in.

(a) Use W 18×35 with $S = 57.6 \text{ in}^3$

$c = \frac{1}{2}d = 8.85$ in.; $y_b = c - t_f = 8.425$ in.

At C: $M = (105)(12) = 1260$ kip·in. $\quad \sigma_m = \dfrac{M}{S} = \dfrac{1260}{57.6} = 21.875$ ksi

$|V| = 8.75$ kips $\quad \tau_m = \dfrac{V}{dt_w} = \dfrac{8.75}{(17.70)(0.300)} = 1.65$ ksi

$\sigma_b = \dfrac{y_b}{c}\sigma_m = \left(\dfrac{8.425}{8.85}\right)(21.875) = 20.82$ ksi

$R = \sqrt{\left(\dfrac{\sigma_b}{2}\right)^2 + \tau_m^2} = \sqrt{\left(\dfrac{19.33}{2}\right)^2 + 1.65^2} = 10.54$ ksi

$\sigma_{max} = \dfrac{\sigma_b}{2} + R = 20.95$ ksi

At B: $M = (97.5)(12) = 1170$ kip·in $\quad \sigma_m = \dfrac{1170}{57.6} = 20.31$ ksi

$|V| = 16.25$ kips $\quad \tau_m = \dfrac{V}{dt_w} = \dfrac{16.25}{(17.70)(0.300)} = 3.06$ ksi

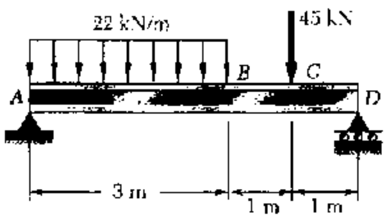
$\sigma_b = \dfrac{y_b}{c}\sigma_m = \left(\dfrac{8.425}{8.85}\right)(20.31) = 19.34$ ksi

$R = \sqrt{\left(\dfrac{\sigma_b}{2}\right)^2 + \tau_m^2} = \sqrt{\left(\dfrac{19.34}{2}\right)^2 + 3.06^2} = 10.14$ ksi

$\sigma_{max} = \dfrac{\sigma_b}{2} + R = 19.81$ ksi

Problem 8.7

8.7 and 8.8 (*a*) Knowing that $\sigma_{all} = 160$MPa and $\tau_{all} = 100$ MPa, select the most economical metric wide-flange shape that should be used to support the loading shown. (*b*) Determine the values to be expected for σ_m, τ_m, and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.



$+\circlearrowleft \Sigma M_D = 0 \quad -5R_A + (3.5)(3)(22) + (1)(45) = 0$

$R_A = 55.2$ kN ↑. $\qquad R_D = 55.8$ kN ↑

Draw shear and bending moment diagrams.

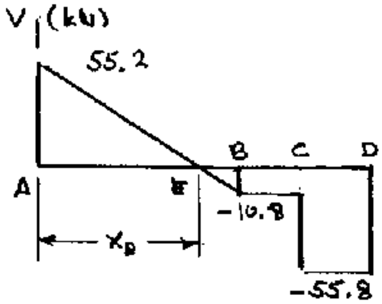
Locate point where $V = 0$ and M_E.

$55.2 - 22x_E = 0 \qquad x_E = 2.5091$ m.

$M_E = \frac{1}{2}(55.2)(2.5091) = 69.25$ kN·m

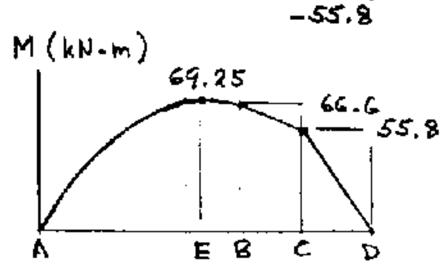
$S_{min} = \frac{|M_{max}|}{\sigma_{all}} = \frac{69.25 \times 10^3}{160 \times 10^6} = 432.8 \times 10^{-6}\ m^3$

$= 432.8 \times 10^3\ mm^3$



Shape	$S\ (10^3\ mm^3)$
W 410 × 38.8	637
→ W 360 × 32.9	474
W 310 × 38.7	549
W 250 × 44.8	535
W 200 × 46.1	448

(a) Use W 360 × 32.9 ◄



For W360 × 32.9 $\quad S = 474 \times 10^3\ mm^3 = 474 \times 10^{-6}\ m^3$

$A_{web} = d\,t_w = (349)(5.8) = 2024.2\ mm^2 = 2024.2 \times 10^{-6}\ m^2$

At point E: $\quad \sigma_m = \frac{M_E}{S} = \frac{69.25 \times 10^3}{474 \times 10^{-6}} = 146.1$ MPa ◄

At point C: $\quad \sigma_m = \frac{M_C}{S} = \frac{55.8 \times 10^3}{474 \times 10^{-6}} = 117.7$ MPa ◄

$\tau_m = \frac{|V|}{A_{web}} = \frac{55.8 \times 10^3}{2024.2 \times 10^{-6}} = 27.6$ MPa ◄

$c = \frac{1}{2}d = \frac{1}{2}(349) = 174.5$ mm $\qquad y_b = c - t_f = 166$ mm

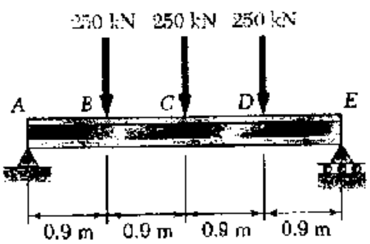
$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{166}{174.5}\right)(117.7) = 112.0$ MPa

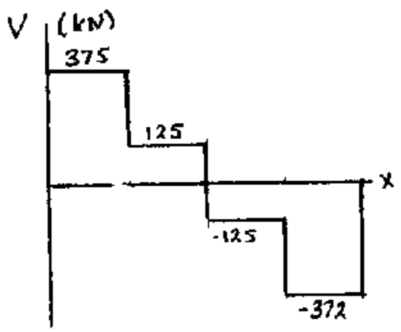
$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_m^2} = 62.4$ MPa

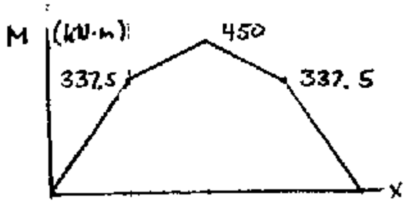
$\sigma_{max} = \frac{\sigma_b}{2} + R = 118.4$ MPa ◄

Problem 8.8

8.7 and 8.8 (*a*) Knowing that $\sigma_{all} = 160$MPa and $\tau_{all} = 100$ MPa, select the most economical metric wide-flange shape that should be used to support the loading shown. (*b*) Determine the values to be expected for σ_m, τ_m, and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.







Reactions: $R_A = 375$ kN ↑ , $R_E = 375$ kN ↑

$|V|_{max} = 375$ kN

$|M|_{max} = 450$ kN·m

$|V|$ at point C : 125 kN

$$S_{min} = \frac{M_{max}}{\sigma_{all}} = \frac{450 \times 10^3}{160 \times 10^6} = 2.8125 \times 10^{-3}\ m^3$$
$$= 2812.5 \times 10^3\ mm^3$$

Shape	S_x $(10^3 mm^4)$
W 840 × 176	5890
W 760 × 147	4410
→ W 690 × 125	3510
W 610 × 155	4220
W 530 × 150	3720
W 460 × 158	3340
W 360 × 216	3800

(a) Use W 690 × 125.

$d = 678$ mm

$t_f = 16.30$ mm

$t_w = 11.7$ mm

(b) $$\sigma_m = \frac{|M|_{max}}{S_x} = \frac{450 \times 10^3}{3510 \times 10^{-6}} = 128.2 \times 10^6\ Pa = 128.2\ MPa$$

$$\tau_m = \frac{|V|_{max}}{A_w} = \frac{|V|_{max}}{d t_w} = \frac{375 \times 10^3}{(678 \times 10^{-3})(11.7 \times 10^{-3})} = 47.3 \times 10^6\ Pa = 47.3\ MPa$$

At point C, $$\tau_w = \frac{|V|}{A_w} = \frac{125 \times 10^3}{(678 \times 10^{-3})(11.7 \times 10^{-3})} = 15.76 \times 10^6\ Pa$$
$$= 15.76\ MPa$$

$$c = \tfrac{1}{2} d = \frac{678}{2} = 339\ mm \qquad y_b = c - t_f = 339 - 16.30 = 322.7\ mm$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{322.7}{339}\right)(128.2) = 122.0\ MPa$$

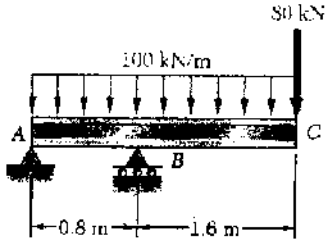
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_w^2} = \sqrt{(61.0)^2 + (15.76)^2} = 63.0\ MPa$$

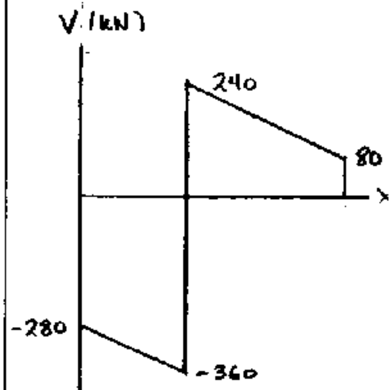
$$\sigma_{max} = \frac{\sigma_b}{2} + R = 61.0 + 63.0 = 124.0\ MPa$$

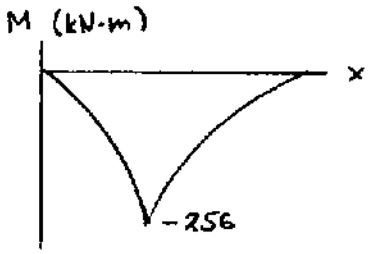
Problem 8.9

8.9 through *8.14* Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{all}$. For the selected design, determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

8.9 Loading of Prob. 5.76 and selected S510 × 98.3 shape.







From Problem 5.76 $\quad \sigma_{all} = 160$ MPa

$|M|_{max} = 256$ kN·m at point B

$|V| = 360$ kN at B

For S 510 × 98.3 rolled steel section

$d = 508$ mm, $b_f = 159$ mm, $t_f = 20.2$ mm

$t_w = 12.8$ mm, $I_z = 495 \times 10^6$ mm^4, $S_z = 1950 \times 10^3$ mm^3

$c = \frac{1}{2} d = 254$ mm

(a) $\sigma_m = \dfrac{|M|_{max}}{S_z} = \dfrac{256 \times 10^3}{1950 \times 10^{-6}} = 131.3$ MPa ◄

$y_b = c - t_f = 233.8$

$\sigma_b = \dfrac{y_b}{c}\sigma_m = 120.9$ MPa $\qquad \dfrac{\sigma_b}{2} = 60.45$ MPa

$A_f = b_f t_f = 3212$ mm^2

$\bar{y} = \frac{1}{2}(c + y_b) = 243.9$ mm

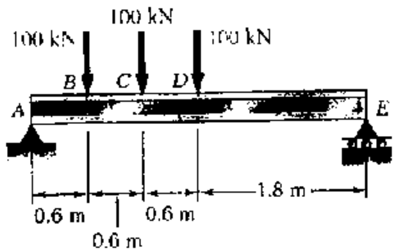
$Q = A_f \bar{y} = 783.4 \times 10^3$ mm^3

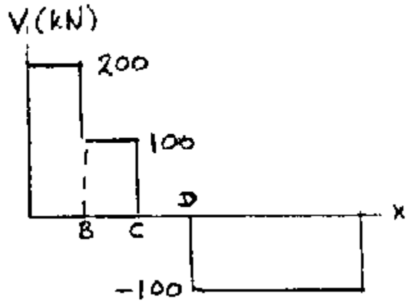
$$\tau_b = \frac{VQ}{I t_w} = \frac{(360 \times 10^3)(783.4 \times 10^{-6})}{(495 \times 10^{-6})(12.8 \times 10^{-3})} = 44.5 \text{ MPa}$$

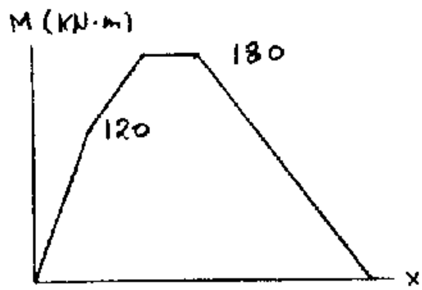
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{60.45^2 + 44.5^2} = 75.06 \text{ MPa}$$

(b) $\sigma_{max} = \dfrac{\sigma_b}{2} + R = 60.45 + 75.06 = 135.5$ MPa ◄

Problem 8.10







8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{all}$. For the selected design, determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

8.10 Loading of Prob. 5.73 and selected W460 × 74 shape.

From Problem 5.73 $\sigma_{all} = 160$ MPa

$|M|_{max} = 180$ kN-m at C and D

$|V| = 100$ kN at C and D

For W 460 × 74 rolled steel section

$d = 457$ mm, $b_f = 190$ mm, $t_f = 14.50$ mm

$t_w = 9.0$ mm, $I_z = 333 \times 10^6$ mm^4, $S_z = 1460 \times 10^3$ mm^3

$c = \frac{1}{2} d = 228.5$ mm

(a) $\sigma_m = \dfrac{|M|_{max}}{S} = \dfrac{180 \times 10^3}{1460 \times 10^{-6}} = 123.3$ MPa

$y_b = c - t_f = 214$ mm

$\sigma_b = \dfrac{y_b}{c} \sigma_m = 115.5$ MPa

$A_f = b_f t_f = 2755$ mm^2

$\bar{y} = \frac{1}{2}(c + y_b) = 221.25$ mm

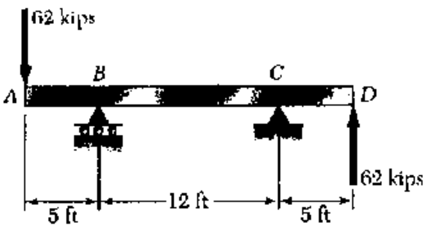
$Q = A_f \bar{y} = (2755)(221.25) = 609.5 \times 10^3$ mm^3

$$\tau_b = \frac{VQ}{I t_w} = \frac{(100 \times 10^3)(609.5 \times 10^{-6})}{(333 \times 10^{-6})(9.0 \times 10^{-3})} = 20.3 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{57.75^2 + 20.3^2} = 61.21 \text{ MPa}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 57.75 + 61.21 = 119.0 \text{ MPa}$$

Problem 8.11



8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{all}$. For the selected design, determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

8.11 Loading of Prob. 5.71 and selected W27 × 84 shape.

From Problem 5.71 $\quad \sigma_{all} = 24$ ksi.

$|M|_{max} = 310$ kip·ft $= 3720$ kip·in at section B.

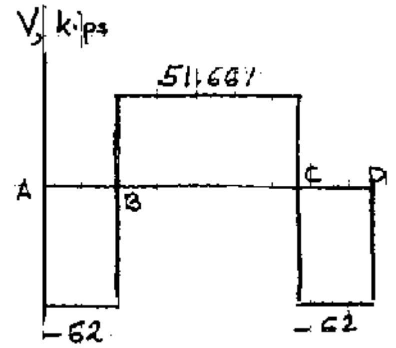
$|V| = 62$ kips at B.

For W 27 × 84 rolled steel shape

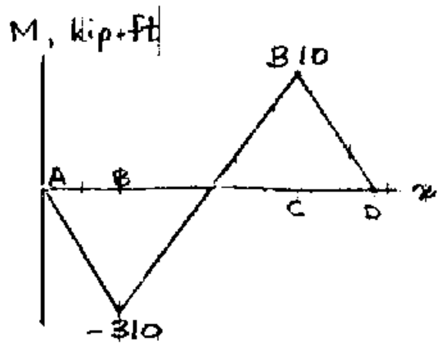
$d = 26.71$ in. $\quad b_f = 9.960$ in. $\quad t_f = 0.640$ in.

$t_w = 0.460$ in. $\quad I_z = 2850$ in^4 $\quad S_z = 213$ in^3

$c = \frac{1}{2}d = 13.355$ in.



(a) $\quad \sigma_m = \dfrac{|M|_{max}}{S} = \dfrac{3720}{213} = 17.46$ ksi ◄



$y_b = c - t_f = 12.715$ in.

$\sigma_b = \dfrac{y_b}{c}\sigma_m = 16.628$ ksi

$A_f = b_f t_f = 6.3744$ in^2

$\bar{y} = \frac{1}{2}(c + y_b) = 13.035$ in

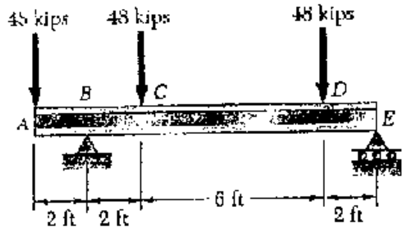
$Q = A_f \bar{y} = (6.3744)(13.035) = 83.09$ in^3

$$\tau_b = \frac{VQ}{It_w} = \frac{(62)(83.09)}{(2850)(0.460)} = 3.930 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{8.314^2 + 3.930^2} = 9.196 \text{ ksi}$$

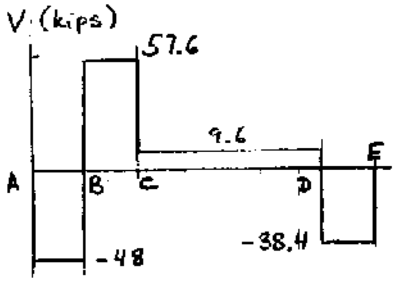
(b) $\quad \sigma_{max} = \dfrac{\sigma_b}{2} + R = 17.51$ ksi ◄

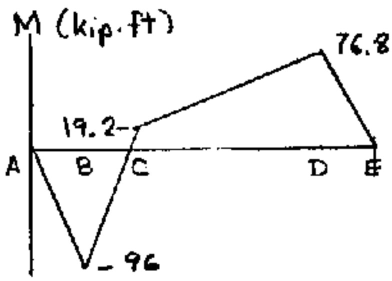
Problem 8.12



8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{all}$. For the selected design, determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

8.12 Loading of Prob. 5.78 and selected S15 × 42.9 shape.





From Problem 5.78 $\sigma_{all} = 24$ ksi

$|M|_{max} = 96$ kip·ft $= 1152$ kip·in at section B.

$|V| = 57.6$ kips at B.

For S 15 × 42.9 rolled steel shape

$d = 15.00$ in. $\quad b_f = 5.501$ in. $\quad t_f = 0.622$ in.

$t_w = 0.411$ in. $\quad I_z = 447\ \text{in}^4 \quad S = 59.6\ \text{in}^3$

$c = \frac{1}{2}d = 7.50$ in.

(a) $\sigma_m = \dfrac{|M|_{max}}{S} = \dfrac{1152}{59.6} = 19.33$ ksi

$y_b = c - t_f = 6.878$ in.

$\sigma_b = \dfrac{y_b}{c}\sigma_m = 17.726$ ksi

$A_f = b_f t_f = 3.4216\ \text{in}^2$

$\bar{y} = \frac{1}{2}(c + y_b) = 7.189$ in.

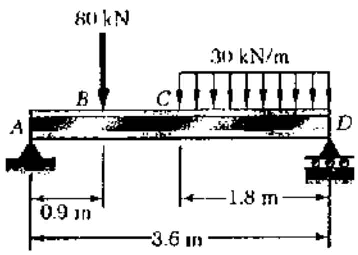
$Q = A_f \bar{y} = (3.4216)(7.189) = 24.60\ \text{in}^3$

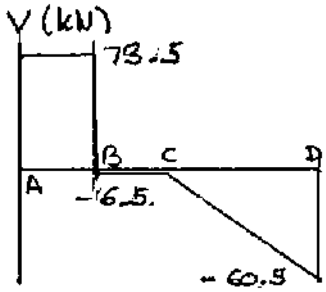
$$\tau_b = \frac{VQ}{It_w} = \frac{(57.6)(24.60)}{(447)(0.411)} = 7.712 \text{ ksi}$$

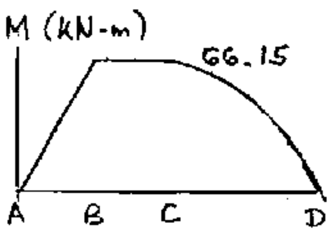
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{8.863^2 + 7.712^2} = 11.749 \text{ ksi}$$

(b) $\sigma_{max} = \dfrac{\sigma_b}{2} + R = 20.6$ ksi

Problem 8.13







8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \leq \sigma_{all}$. For the selected design, determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

8.13 Loading of Prob. 5.75 and selected S310 × 47.3 shape.

From Problem 5.75 $\sigma_{all} = 160$ MPa

$|M|_{max} = 66.15$ kN·m at section B.

$|V| = 73.5$ kN at B

For S 310 × 47.3 rolled steel section

$d = 305$ mm $\quad b_f = 127$ mm $\quad t_f = 13.8$ mm

$t_w = 8.9$ mm $\quad I = 90.6 \times 10^6\ mm^4 \quad S = 593 \times 10^3\ mm^3$

$c = \frac{1}{2}d = 152.5$ mm

(a) $\sigma_m = \dfrac{|M|_{max}}{S} = \dfrac{66.15 \times 10^3}{593 \times 10^{-6}} = 111.6$ MPa ◄

$y_b = c - t_f = 138.7$ mm

$\sigma_b = \dfrac{y_b}{c}\sigma_m = 101.5$ MPa

$A_f = b_f t_f = 1752.6\ mm^2$

$\bar{y} = \frac{1}{2}(c + y_b) = 145.6$ mm

$Q = A_f \bar{y} = 255.18 \times 10^3\ mm^3 = 255.18 \times 10^{-6}\ m^3$

$$\tau_b = \frac{VQ}{I t_w} = \frac{(73.5 \times 10^3)(255.18 \times 10^{-6})}{(90.6 \times 10^{-6})(8.9 \times 10^{-3})} = 23.26 \text{ MPa}$$

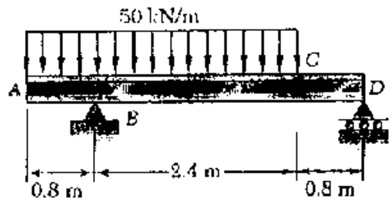
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{50.75^2 + 23.26^2} = 55.83 \text{ MPa}$$

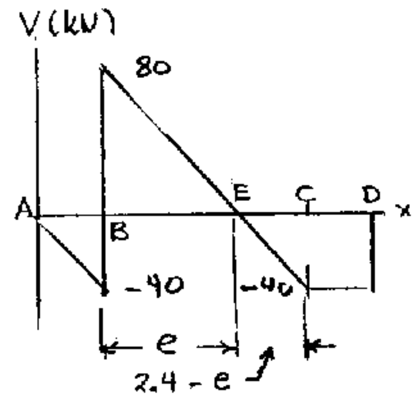
$$\sigma_{max} = \frac{\sigma_b}{2} + R = 106.6 \text{ MPa}$$ ◄

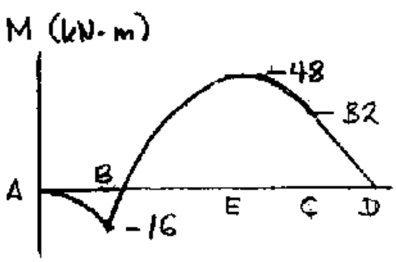
Problem 8.14

8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{all}$. For the selected design, determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

8.14 Loading of Prob. 5.74 and selected W250 × 28.4 shape.







From Problem 5.74 $\sigma_{all} = 160$ MPa

$|M|_{max} = 48$ kN·m at section E, which lies 1.6 m to the right of B.

$|V| = 0$

For W 250 × 28.4 rolled steel section

$d = 260$ mm $\quad b_f = 102$ mm $\quad t_f = 10.0$ mm

$t_w = 6.4$ mm $\quad I = 40.0 \times 10^6\ \text{mm}^4 \quad S = 308 \times 10^3\ \text{mm}^3$

$c = \frac{1}{2}d = 130$ mm

(a) $\sigma_m = \dfrac{|M|_{max}}{S} = \dfrac{48 \times 10^3}{308 \times 10^{-6}} = 155.8$ MPa ◄

$y_b = c - t_f = 120$ mm

$A_f = b_f t_f = 1020\ \text{mm}^2$

$\bar{y} = \frac{1}{2}(c + y_b) = 125$ mm

$Q = A_f \bar{y} = 127.5 \times 10^3\ \text{mm}^3 = 127.5 \times 10^{-6}\ \text{m}^3$

At section E $\quad V = 0 \quad M = M_{max}$

$\sigma_b = \dfrac{y_b}{c}\sigma_m = 143.8$ MPa

$\tau_b = \dfrac{VQ}{It_w} = 0 \qquad$ (b) $\sigma_{max} = \sigma_b = 143.8$ MPa ◄

At section C. $\quad |V| = 40$ kN $\quad M = 32$ kN·m

$\sigma_b = \dfrac{M y_b}{I} = \dfrac{(32 \times 10^3)(120 \times 10^{-3})}{40.0 \times 10^{-6}} = 96$ MPa

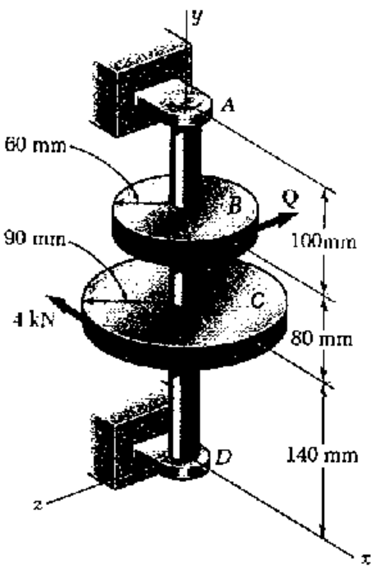
$\tau_b = \dfrac{VQ}{It_w} = \dfrac{(40 \times 10^3)(127.5 \times 10^{-6})}{(40.0 \times 10^{-6})(6.4 \times 10^{-3})} = 19.92$ MPa

$R = \sqrt{\left(\dfrac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{48^2 + 19.92^2} = 51.97$ MPa

$\sigma_{max} = \dfrac{\sigma_b}{2} + R = 100.0$ MPa, which is less than σ_{max} at E.

Problem 8.15

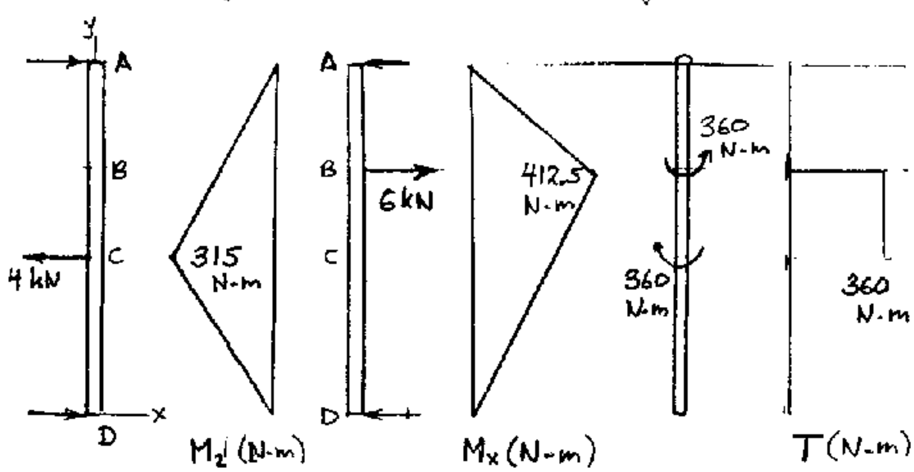
8.15 The 4-kN force is parallel to the x axis, and the force $\mathbf{Q}$ is parallel to the z axis. The shaft AD is hollow. Knowing that the inner diameter is half the outer diameter and that $\tau_{all} = 60$ MPa, determine the smallest permissible outer diameter of the shaft.



$$\Sigma M_y = 0: \quad 60\times10^{-3}\, Q - (90\times10^{-3})(4\times10^3) = 0$$

$$Q = 6\times10^3 \text{ N} = 6 \text{ kN}$$

Bending moment and torque diagrams.



In xy plane $(M_z)_{max} =$ 315 N·m at C.

In yz plane $(M_x)_{max} =$ 412.5 N·m at B.

About z-axis $T_{max} =$ 360 N·m between B and C.

At B $\quad M_z = \left(\frac{100}{180}\right)(315) = 175$ N·m

$$\sqrt{M_x^2 + M_z^2 + T^2} = \sqrt{175^2 + 412.5^2 + 360^2} = 574.79 \text{ N·m}$$

At C $\quad M_x = \left(\frac{140}{220}\right)(412.5) = 262.5$ N·m

$$\sqrt{M_x^2 + M_z^2 + T^2} = \sqrt{315^2 + 262.5^2 + 360^2} = 545.65 \text{ N·m}$$

Largest value is 574.79 N·m

$$\tau_{max} = \frac{\max\sqrt{M_x^2 + M_z^2 + T^2}\; c}{J}$$

$$\frac{J}{c} = \frac{\max\sqrt{M_x^2 + M_z^2 + T^2}}{\tau_{max}} = \frac{574.79}{60\times10^6} = 9.5798\times10^{-6} \text{ m}^3 = 9.5798\times10^3 \text{ mm}^3$$

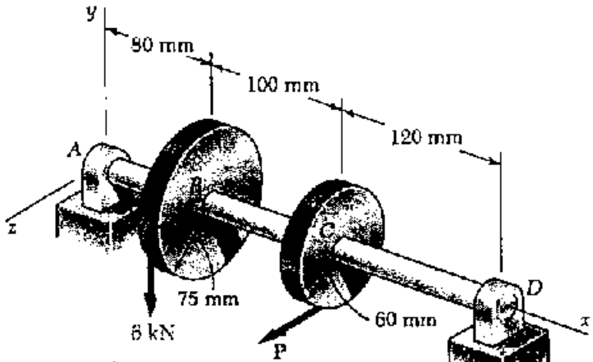
$$\frac{J}{c} = \frac{\frac{\pi}{2}(c_o^4 - c_i^4)}{c_o} = \frac{\pi}{2} c_o^3\left(1 - \frac{c_i^4}{c_o^4}\right) = \frac{\pi}{2} c_o^3\left[1 - \left(\tfrac{1}{2}\right)^4\right] = 1.47262\, c_o^3$$

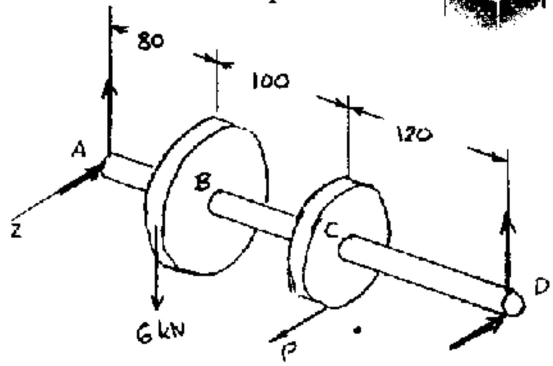
$1.47262\, c_o^3 = 9.5798\times10^3 \qquad c_o = 18.67$ mm

$$d_o = 2c_o = 37.3 \text{ mm}$$

Problem 8.16

8.16 The 6-kN force is vertical and the force **P** is parallel to the z axis. Knowing that $\tau_{all} = 60$ MPa, determine the smallest permissible diameter of the solid shaft AD.





$\Sigma M_x = 0$

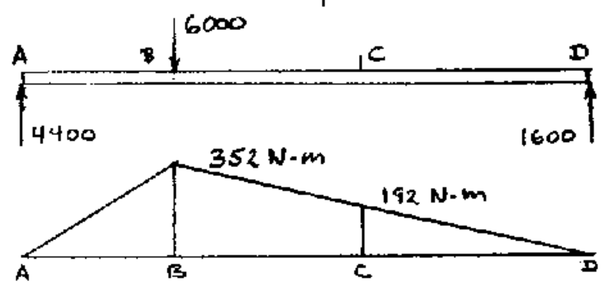
$(6\times10^3)(75\times10^{-3}) - (60\times10^{-3})P = 0$

$P = 7.5\times10^3$ N

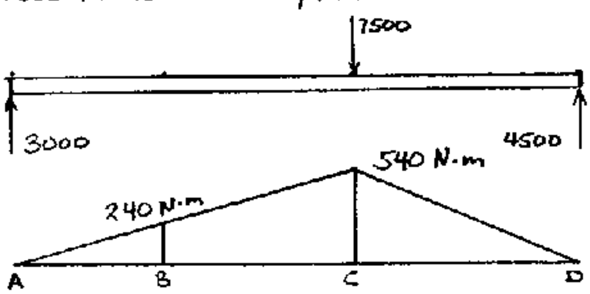
Over portion BC

$T = (6\times10^3)(75\times10^{-3}) = 450$ N·m

Forces in vertical plane:



Forces in horizontal plane:



Bending moments:

At B: $M = \sqrt{352^2 + 240^2}$
$= 426.0$ N·m

At C: $M = \sqrt{540^2 + 192^2}$
$= 573.1$ N·m

Critical section is just to the left of gear C.

$M = 573.1$ N·m $\quad T = 450$ N·m $\quad \sqrt{M^2+T^2} = 728.67$ N·m

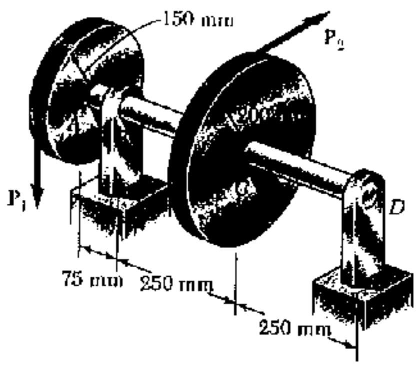
$$\tau_{all} = \frac{c}{J}\left(\sqrt{M^2+T^2}\right)_{max}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\left(\sqrt{M^2+T^2}\right)_{max}}{\tau_{all}} = \frac{728.67}{60\times10^6} = 12.145\times10^{-6}\ \text{m}^3$$

$c = 19.77\times10^{-3}$ m $\qquad d = 2c = 39.5\times10^{-3}$ m $= 39.5$ mm

Problem 8.17

8.17 The vertical force $\mathbf{P}_1$ and the horizontal force $\mathbf{P}_2$ are applied as shown to disks welded to the solid shaft AD. Knowing that the diameter of the shaft is 40 mm and that $\tau_{all} = 55$ MPa, determine the largest permissible magnitude of the force $\mathbf{P}_2$.

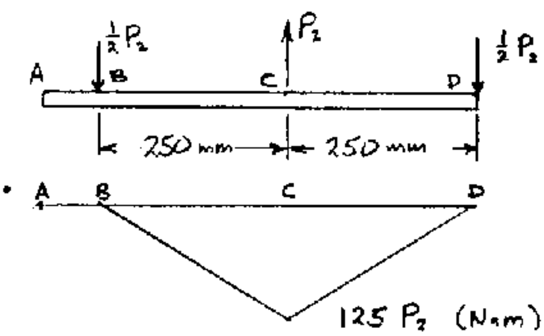


Let the dimensions of P_2 be kN.

$\Sigma M_{shaft} = 0$: $150 P_1 - 200 P_2 = 0 \qquad P_1 = \frac{4}{3} P_2$

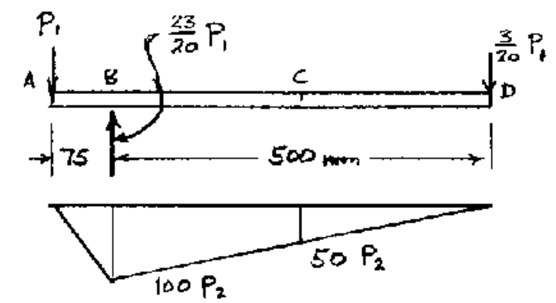
Torque over portion ABC: $T = 200 P_2$ in N·m.

Bending in horizontal plane.



$(M_C)_y = (250)(\frac{1}{2}P_2) = 125 P_2$ in N·m.

Bending in vertical plane.



$(M_B)_z = 75 P_1 = 100 P_2$ in N·m

At B: $\sqrt{M_y^2 + M_z^2 + T^2} = \sqrt{(100 P_2)^2 + 0 + (200 P_2)^2} = 223.61 P_2$

At C: $\sqrt{M_y^2 + M_z^2 + T^2} = \sqrt{(125 P_2)^2 + (50 P_2)^2 + (200 P_2)^2} = 241.09 P_2$

$d = 40$ mm $= 40 \times 10^{-3}$ m $\qquad c = \frac{1}{2}d = 20 \times 10^{-3}$ m

$J = \frac{\pi}{2} c^4 = \frac{\pi}{2}(20 \times 10^{-3})^4 = 251.33 \times 10^{-9}$ m^4

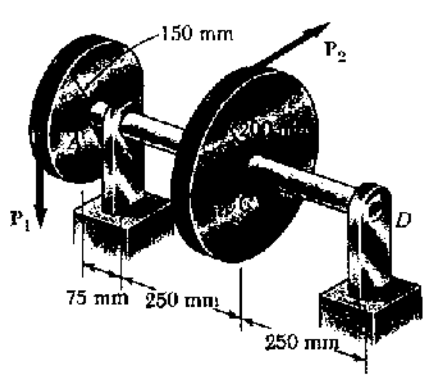
$$\tau_{max} = \frac{\max \sqrt{M_y^2 + M_z^2 + T^2}\, c}{J}$$

$$55 \times 10^6 = \frac{(241.09 P_2)(20 \times 10^{-3})}{251.33 \times 10^{-9}} \qquad P_2 = 2.87 \text{ kN}$$

Problem 8.18

8.18 Solve Prob. 8.17, assuming that the solid shaft AD has been replaced by a hollow shaft of the same material and of inner diameter 35 mm and outer diameter 45 mm.

8.17 The vertical force $\mathbf{P}_1$ and the horizontal force $\mathbf{P}_2$ are applied as shown to disks welded to the solid shaft AD. Knowing that the diameter of the shaft is 40 mm and that $\tau_{all} = 55$ MPa, determine the largest permissible magnitude of the force $\mathbf{P}_2$.

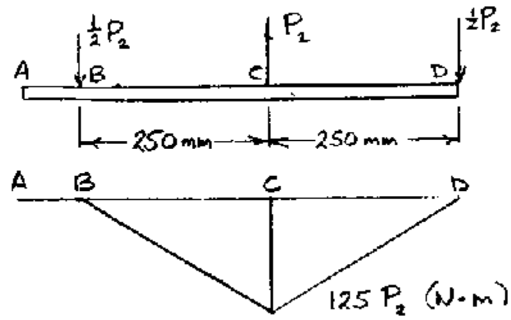


Let the dimensions of P_2 be kN.

$\Sigma M_{shaft} = 0: \quad 150 P_1 - 200 P_2 = 0 \qquad P_1 = \frac{4}{3} P_1$

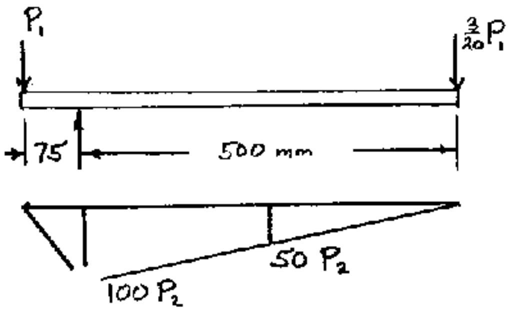
Torque of portion ABC: $T = 200 P_2$ in N·m.

Bending in horizontal plane.



$(M_C)_y = (250)(\frac{1}{2}P_2)$
$\doteq 125\, P_2$
in N·m.

Bending in vertical plane.



$(M_B)_z = 75\ P_1$
$= 100\ P_2$
in N·m

$$\text{At B:} \quad \sqrt{M_y^2 + M_z^2 + T^2} = \sqrt{0 + (100P_2)^2 + (200P_2)^2} = 223.61\ P_2$$

$$\text{At C:} \quad \sqrt{M_y^2 + M_z^2 + T^2} = \sqrt{(125P_2)^2 + (50P_2)^2 + (200P_2)^2} = 241.09\ P_2$$

$$c_o = \tfrac{1}{2} d_o = 22.5 \text{ mm} = 22.5 \times 10^{-3} \text{ m} \qquad c_i = \tfrac{1}{2} d_i = 17.5 \text{ mm} = 17.5 \times 10^{-3} \text{ m}$$

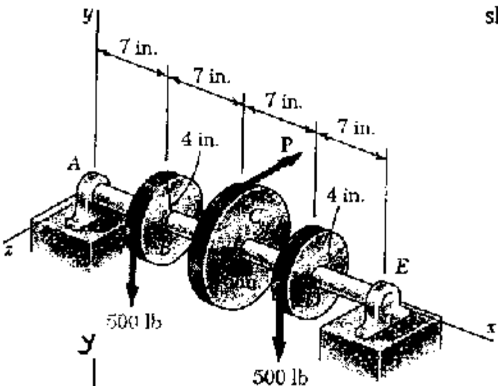
$$J = \frac{\pi}{2}(c_o^4 - c_i^4) = \frac{\pi}{2}(22.5^4 - 17.5^4) = 255.25 \times 10^3 \text{ mm}^4 = 255.25 \times 10^{-9} \text{ m}^4$$

$$\tau_{max} = \frac{\max \sqrt{M_y^2 + M_z^2 + T^2}\ c_o}{J}$$

$$55 \times 10^6 = \frac{(241.09\, P_2)(22.5 \times 10^{-3})}{255.25 \times 10^{-9}} \qquad P_2 = 2.59 \text{ kN} \blacktriangleleft$$

Problem 8.19

8.19 The two 500-lb forces are vertical and the force **P** is parallel to the z axis. Knowing that $\tau_{all} = 8$ ksi, determine the smallest permissible diameter of the solid shaft AE.



$$\Sigma M_x = 0 \qquad (4)(500) - 6P + (4)(500) = 0$$

$$P = 666.67 \text{ lb}$$

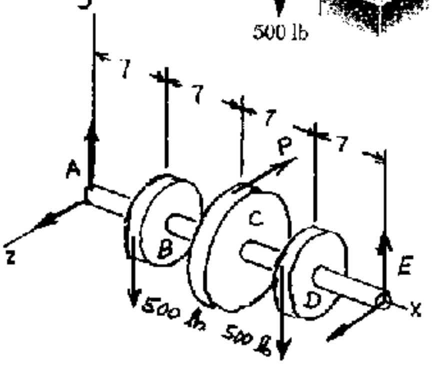
Torques:

AB: $T = 0$

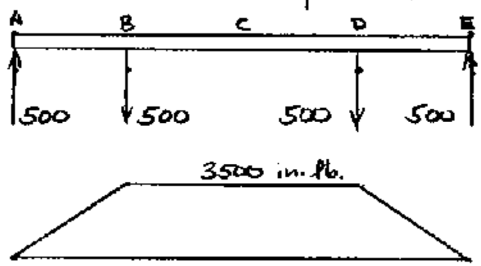
BC: $T = -(4)(500) = -2000$ in·lb

CD: $T = (4)(500) = 2000$ in·lb

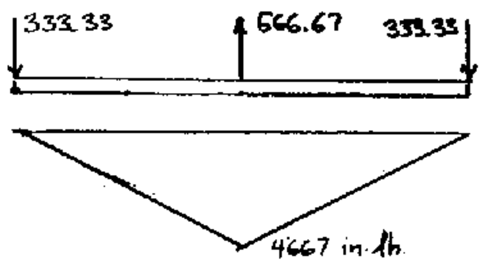
DE: $T = 0$



Forces in vertical plane:



Forces in horizontal plane:



Critical sections are either side of disk C.

$T = 2000$ in·lb $\qquad M_z = 3500$ in·lb

$M_y = 4667$ in lb

$$\tau_{all} = \frac{c}{J}\sqrt{M_y^2 + M_z^2 + T^2}$$

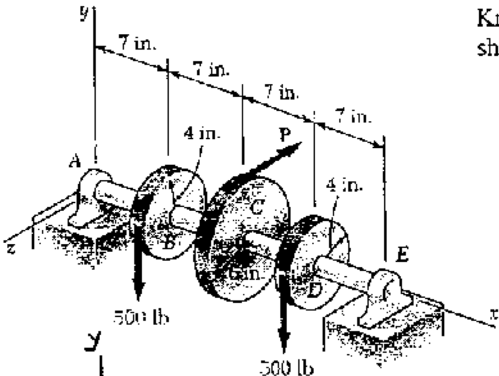
$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M_y^2 + M_z^2 + T^2}}{\tau_{all}} = \frac{\sqrt{4667^2 + 3500^2 + 2000^2}}{8 \times 10^3} = 0.77083 \text{ in}^3$$

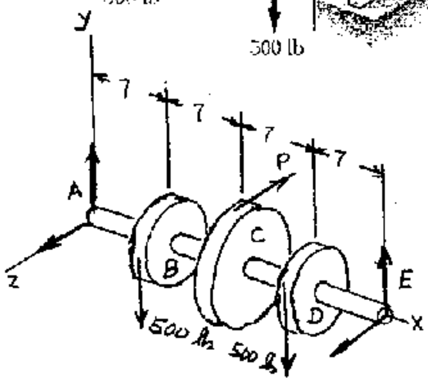
$c = 0.7888$ in. $\qquad d = 2c = 1.578$ in.

Problem 8.20

8.20 For the gear-and-shaft system and loading of Prob. 8.19, determine the smallest permissible diameter of shaft AE, knowing that the shaft is hollow and has an inner diameter that is $\frac{2}{3}$ the outer diameter.

8.19 The two 500-lb forces are vertical and the force **P** is parallel to the z axis. Knowing that τ_{all} = 8 ksi, determine the smallest permissible diameter of the solid shaft AE.





$$\Sigma M_x = 0 \qquad (4)(500) - 6P + (4)(500) = 0$$

$$P = 666.67 \text{ lb.}$$

Torques:

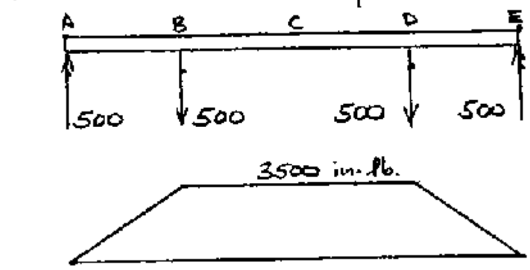
AB: $T = 0$

BC: $T = -(4)(500) = -2000$ in lb

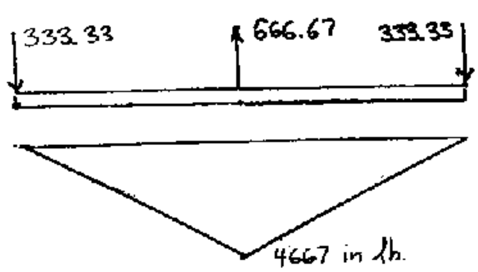
CD: $T = (4)(500) = 2000$ in lb

DE: $T = 0$

Forces in vertical plane:



Forces in horizontal plane:



Critical sections are either side of disk C.

$T = 2000$ in.lb. $\quad M_z = 3500$ in.lb

$M_y = 4667$ in lb.

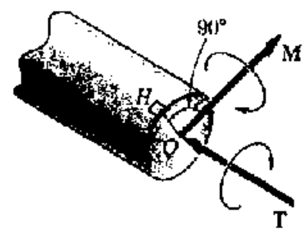
$$\tau_{all} = \frac{c}{J}\sqrt{M_y^2 + M_z^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2c}\left[c_o^4 - c_i^4\right] = \frac{\pi}{2c}\left[c^4 - \left(\tfrac{2}{3}c\right)^4\right] = \frac{\pi}{2c}\,\frac{65c^4}{81} = \frac{65\pi}{162}c^3$$

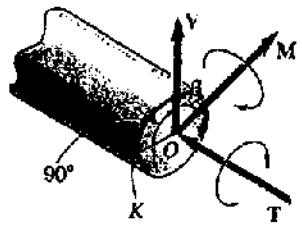
$$\frac{65\pi}{162}c^3 = \frac{\sqrt{M_y^2 + M_z^2 + T^2}}{\tau_{all}} = \frac{\sqrt{4667^2 + 3500^2 + 2000^2}}{8\times 10^3} = 0.77083 \text{ in}^3$$

$c = 0.8488$ in. $\qquad d = 2c = 1.698$ in.

Problem 8.21



(a)



(b)

8.21 It was stated in Sec. 8.3 that the shearing stresses produced in a shaft by the transverse loads are usually much smaller than those produced by the torques. In the preceding problems their effect was ignored and it was assumed that the maximum shearing stress in a given section occurred at point H (Fig. P8.21a) and was equal to the expression obtained in Eq. (8.5), namely,

$$\tau_H = \frac{c}{J}\sqrt{M^2 + T^2}$$

Show that the maximum shearing stress at point K (Fig. P8.21b), where the effect of the shear V is greatest, can be expressed as

$$\tau_K = \frac{c}{J}\sqrt{(M\cos\beta)^2 + (\tfrac{2}{3}cV + T)^2}$$

where β is the angle between the vectors **V** and **M**. It is clear that the effect of the shear V cannot be ignored when $\tau_K \geq \tau_H$. (*Hint*: Only the component of **M** along **V** contributes to the shearing stress at K.)

Shearing stress at point K.

Due to V: For a semicircle $Q = \frac{2}{3}c^3$

For a circle cut across its diameter $t = d = 2c$

For a circular section $I = \frac{1}{2}J$

$$\tau_{xy} = \frac{VQ}{It} = \frac{(V)(\frac{2}{3}c^3)}{(\frac{1}{2}J)(2c)} = \frac{2}{3}\frac{Vc^2}{J}$$

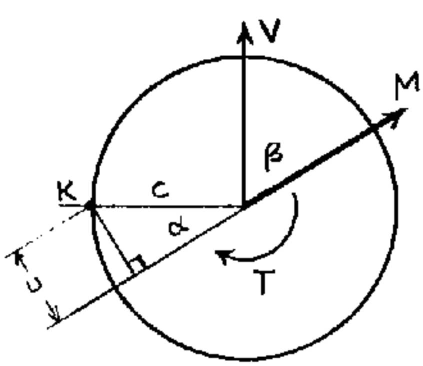
Due to T: $\tau_{xy} = \frac{Tc}{J}$

Since these shearing stresses have the same orientation,

$$\tau_{xy} = \frac{c}{J}\left(\frac{2}{3}Vc + T\right)$$

Bending stress at point K: $\sigma_x = \frac{Mu}{I} = \frac{2Mu}{J}$

where u is distance between point K and the neutral axis,



cross section

$$u = c\sin\alpha = c\sin(\tfrac{\pi}{2} - \beta) = c\cos\beta$$

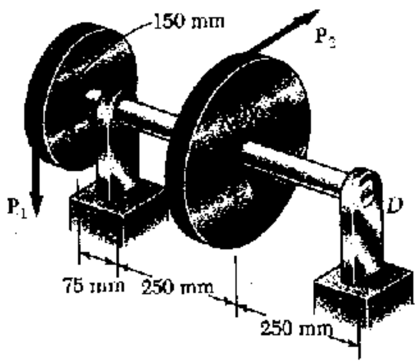
$$\sigma_x = \frac{2Mc\cos\beta}{J}$$

Using Mohr's circle.

$$\tau_K = R = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{c}{J}\sqrt{(M\cos\beta)^2 + (\tfrac{2}{3}Vc + T)^2}$$

Problem 8.22



8.22 Assuming that the magnitudes of the forces applied to disks A and C of Prob. 8.17 are, respectively $P_1 = 4.8$ kN and $P_2 = 3.6$ kN, and using the expressions given in Prob. 8.21, determine the values of τ_K and τ_H in a section (a) just to the left of B, (b) just to the left of C.

8.17 The vertical force $\mathbf{P}_1$ and the horizontal force $\mathbf{P}_2$ are applied as shown to disks welded to the solid shaft AD. Knowing that the diameter of the shaft is 40 mm and that $\tau_{all} = 55$ MPa, determine the largest permissible magnitude of the force $\mathbf{P}_2$.

From Problem 8.17 $\quad d = 40 \text{ mm} = 40\times10^{-3}$ m

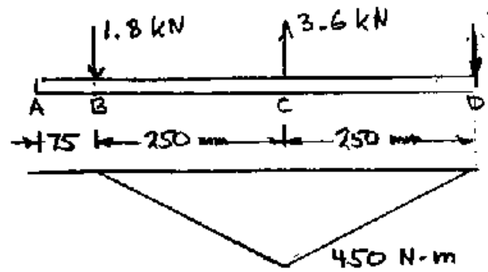
$c = \frac{1}{2}d = 20\times10^{-3}$ m

$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(20\times10^{-3})^4 = 251.33\times10^{-9}$ m^4

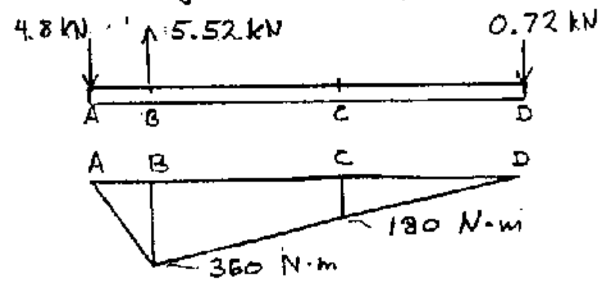
Torque over portion ABC of shaft.

$T = (150\times10^{-3})(4.8\times10^{3}) = 720$ N·m

Bending in horizontal plane.

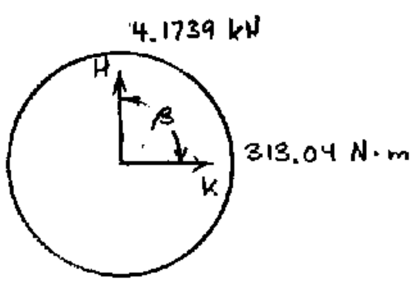


Bending in vertical plane.



(a) Just to the left of section B. $\quad V = 4.8\times10^{3}$ N, $M = 360$ N·m

$\beta = 90°$ $\quad T = 720$ N·m



$\tau_H = \frac{c}{J}\sqrt{M^2 + T^2} = \frac{20\times10^{-3}}{251.33\times10^{-9}}\sqrt{360^2 + 720^2}$

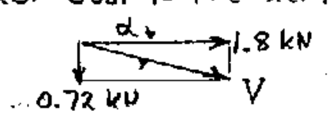
$= 64.1\times10^{6}$ Pa $\qquad$ 64.1 MPa

$\tau_K = \frac{c}{J}\sqrt{(M\cos\beta)^2 + (\frac{2}{3}Vc + T)^2} = \frac{c}{J}|Vc + T|$

$= \frac{20\times10^{-3}}{251.33\times10^{-9}}\left[\frac{2}{3}(4.8\times10^{3})(20\times10^{-3}) + 720\right]$

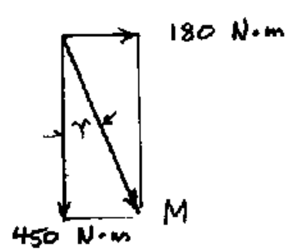
$= 62.4\times10^{6}$ Pa $\qquad$ 62.4 MPa

(b) Just to the left of section C. $\quad V = \sqrt{(4.8)^2 + (0.72)^2} = 1.9387$ kN



Shear

$\alpha = \tan^{-1}\left(\frac{0.72}{1.8}\right) = 21.8°$



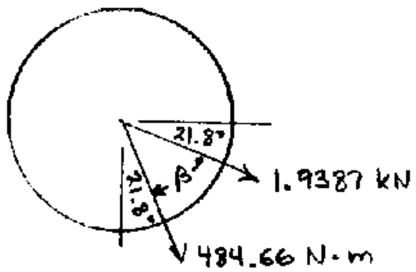
Bending moment

$M = \sqrt{450^2 + 180^2} = 484.66$ N·m

$\gamma = \tan^{-1}\left(\frac{180}{450}\right) = 21.8°$

continued.

Problem 8.22 continued



$$\beta = 90° - \alpha - \gamma = 90° - 21.8° - 21.8° = 46.4°$$

$$\tau_H = \frac{c}{J}\sqrt{M^2 + T^2} = \frac{20 \times 10^{-3}}{251.33 \times 10^{-9}}\sqrt{484.66^2 + 720^2}$$

$$= 69.1 \times 10^6 \text{ Pa} \qquad 69.1 \text{ MPa} \blacktriangleleft$$

$$M\cos\beta = 334.23 \text{ N}\cdot\text{m}$$

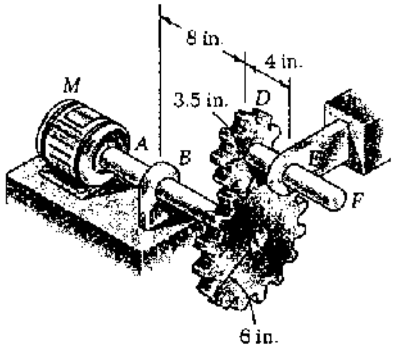
$$\tfrac{2}{3}Vc + T = \tfrac{2}{3}(1.9387 \times 10^3)(20 \times 10^{-3}) + 720$$

$$= 745.85 \text{ N}\cdot\text{m}$$

$$\tau_K = \frac{c}{J}\sqrt{(M\cos\beta)^2 + (\tfrac{2}{3}Vc + T)^2} = \frac{20 \times 10^{-3}}{251.33 \times 10^{-9}}\sqrt{(334.23)^2 + (745.85)^2}$$

$$= 65.0 \times 10^6 \text{ Pa} \qquad 65.0 \text{ MPa} \blacktriangleleft$$

Problem 8.23



8.23 The solid shafts *ABC* and *DEF* and the gears shown are used to transmit 20 hp from the motor *M* to a machine tool connected to shaft *DEF*. Knowing that the motor rotates at 240 rpm and that $\tau_{all} = 7.5$ ksi, determine the smallest permissible diameter of (*a*) shaft *ABC*, (*b*) shaft *DEF*.

$$20 \text{ hp} = (20)(6600) = 132 \times 10^3 \text{ in}\cdot\text{lb/s}$$

$$240 \text{ rpm} = \frac{240}{60} = 4 \text{ Hz}$$

(a) Shaft ABC: $T = \dfrac{P}{2\pi f} = \dfrac{132 \times 10^3}{(2\pi)(4)} = 5252 \text{ lb}\cdot\text{in}$

Gear C $\quad F_{CD} = \dfrac{T}{r_C} = \dfrac{5252}{6} = 875.4 \text{ lb.}$

Bending moment at B: $\quad M_B = (8)(875.4) = 7003 \text{ lb}\cdot\text{in}$

$$\tau_{all} = \frac{c}{J}\sqrt{M^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2+T^2}}{\tau_{all}} = \frac{\sqrt{(5252)^2 + (7003)^2}}{7500} = 1.1671 \text{ in}^3$$

$c = 0.9057$ in $\qquad d = 2c = 1.811$ in ◄

(b) Shaft DEF: $\quad T = r_D F_{CD} = (3.5)(875.4) = 3064 \text{ lb}\cdot\text{in}$

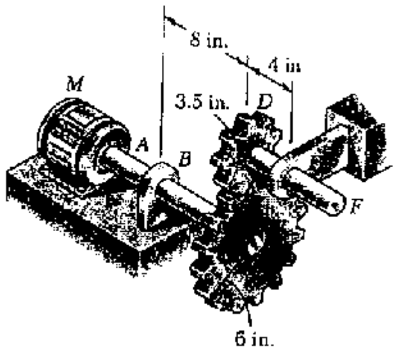
Bending moment at E: $\quad M_E = (4)(875.4) = 3502 \text{ lb}\cdot\text{in}$

$$\tau_{all} = \frac{c}{J}\sqrt{M^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2+T^2}}{\tau_{all}} = \frac{\sqrt{(3502)^2 + (3064)^2}}{7500} = 0.6204 \text{ in}^3$$

$c = 0.7337$ in. $\qquad d = 2c = 1.467$ in. ◄

Problem 8.24



8.24 Solve Prob. 8.23, assuming that the motor rotates at 360 rpm.

8.23 The solid shafts *ABC* and *DEF* and the gears shown are used to transmit 20 hp from the motor *M* to a machine tool connected to shaft *DEF*. Knowing that the motor rotates at 240 rpm and that $\tau_{all} = 7.5$ ksi, determine the smallest permissible diameter of (*a*) shaft *ABC*, (*b*) shaft *DEF*.

$20 \text{ hp} = (20)(6600) = 132 \times 10^3 \text{ in}\cdot\text{lb/s}$

$360 \text{ rpm} = \frac{360}{60} = 6 \text{ Hz}$

(a) Shaft ABC: $T = \frac{P}{2\pi f} = \frac{132 \times 10^3}{(2\pi)(6)} = 3501 \text{ lb}\cdot\text{in}$

Gear C $F_{CD} = \frac{T}{r_C} = \frac{3501}{6} = 583.6 \text{ lb.}$

Bending moment at B: $M_B = (8)(583.6) = 4669 \text{ lb}\cdot\text{in}$

$\tau_{all} = \frac{c}{J}\sqrt{M^2 + T^2}$

$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{\sqrt{4669^2 + 3501^2}}{7500} = 0.77806 \text{ in}^3$

$c = 0.791 \text{ in.}$ $d = 2c = 1.582 \text{ in.}$ ◄

(b) Shaft DEF: $T = r_D F_{CD} = (3.5)(583.6) = 2043 \text{ lb}\cdot\text{in}$

Bending moment at E: $M_E = (4)(583.6) = 2334 \text{ lb}\cdot\text{in}$

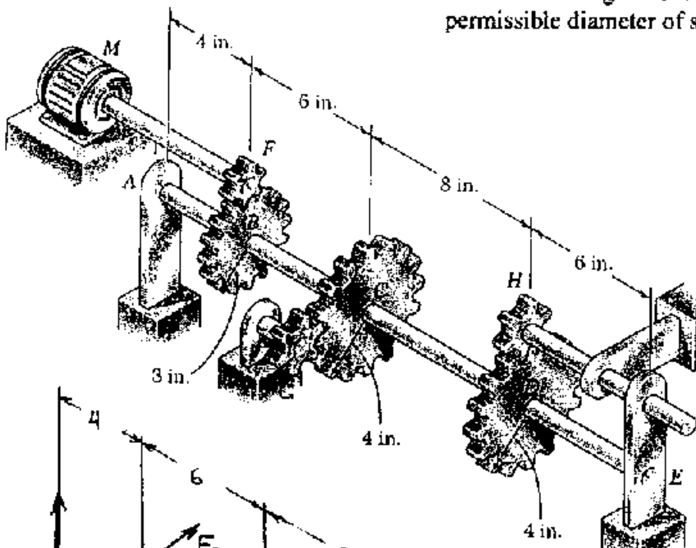
$\tau_{all} = \frac{c}{J}\sqrt{M^2 + T^2}$

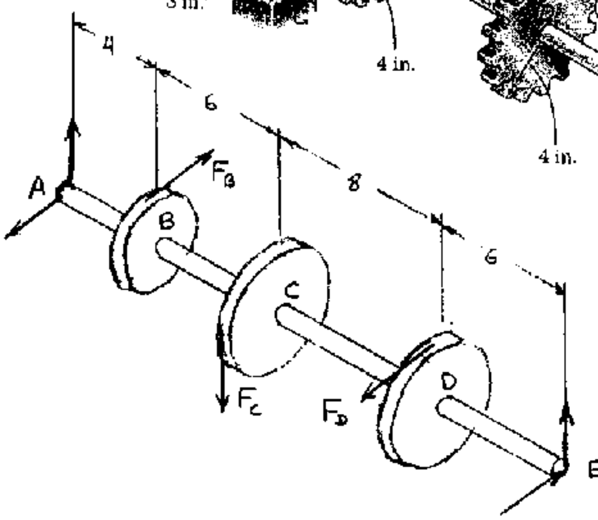
$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{\sqrt{2334^2 + 2043^2}}{7500} = 0.41362 \text{ in}^3$

$c = 0.6410 \text{ in.}$ $d = 2c = 1.282 \text{ in.}$ ◄

Problem 8.25

8.25 The solid shaft AE rotates at 600 rpm and transmits 60 hp from the motor M to machine tools connected to gears G and H. Knowing that $\tau_{all} = 8$ ksi and that 40 hp is taken off at gear G and 20 hp is taken off at gear H, determine the smallest permissible diameter of shaft AE.





$$60 \text{ hp} = (60)(6600) = 396\times10^3 \text{ in}\cdot\text{lb/sec}$$

$$f = \frac{600 \text{ rpm}}{60 \text{ sec/min}} = 10 \text{ Hz}$$

Torque on gear B.

$$T_B = \frac{P}{2\pi f} = \frac{396\times10^3}{2\pi(10)} = 6302.5 \text{ lb}\cdot\text{in}$$

Torques on gears C and D:

$$T_C = \frac{40}{60}T_B = 4201.7 \text{ lb}\cdot\text{in}$$

$$T_D = \frac{20}{60}T_B = 2100.8 \text{ lb}\cdot\text{in}$$

Shaft torques:

AB: $T_{AB} = 0$
BC: $T_{BC} = 6302.5$ lb·in
CD: $T_{CD} = 2100.8$ lb·in
DE: $T_{DE} = 0$

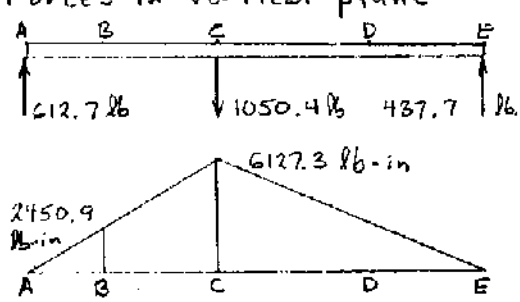
Gear forces:

$$F_B = \frac{T_B}{r_B} = \frac{6302.5}{3} = 2100.8 \text{ lb.}$$

$$F_C = \frac{T_C}{r_C} = \frac{4201.7}{4} = 1050.4 \text{ lb}$$

$$F_D = \frac{T_D}{r_D} = \frac{2100.8}{4} = 525.2 \text{ lb.}$$

Forces in vertical plane



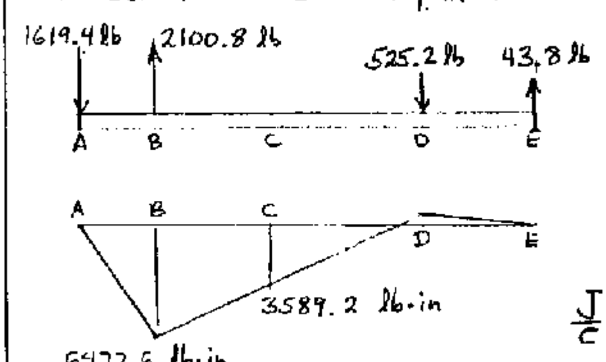
At B^+, $\sqrt{M_z^2 + M_y^2 + T^2}$

$$= \sqrt{2450.9^2 + 6477.6^2 + 6302.5^2} = 9364 \text{ lb}\cdot\text{in}$$

At C^-, $\sqrt{M_z^2 + M_y^2 + T^2}$

$$= \sqrt{6127.3^2 + 3589.2^2 + 6302.5^2} = 9495 \text{ lb}\cdot\text{in} \quad \text{(maximum)}$$

Forces in horizontal plane



$$\tau_{all} = \frac{c}{J}\left(\sqrt{M_z^2 + M_y^2 + T^2}\right)_{max}$$

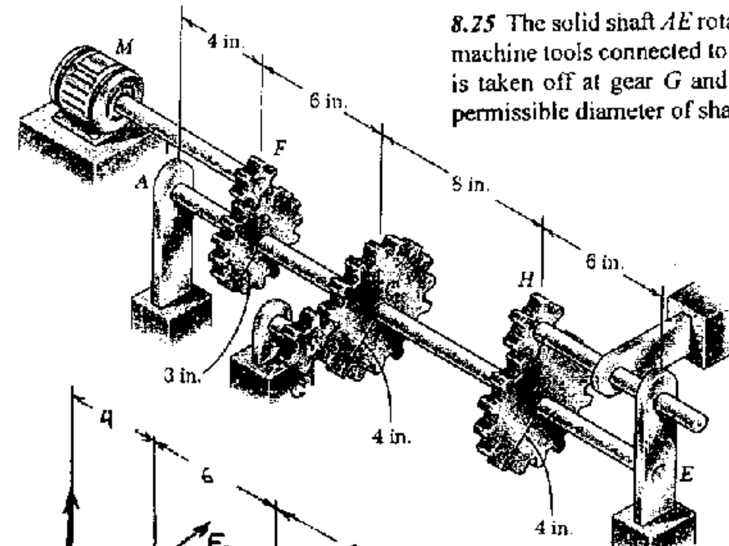
$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\left(\sqrt{M_z^2 + M_y^2 + T^2}\right)}{\tau_{all}} = \frac{9495}{8\times10^3} = 1.1868 \text{ in}^3$$

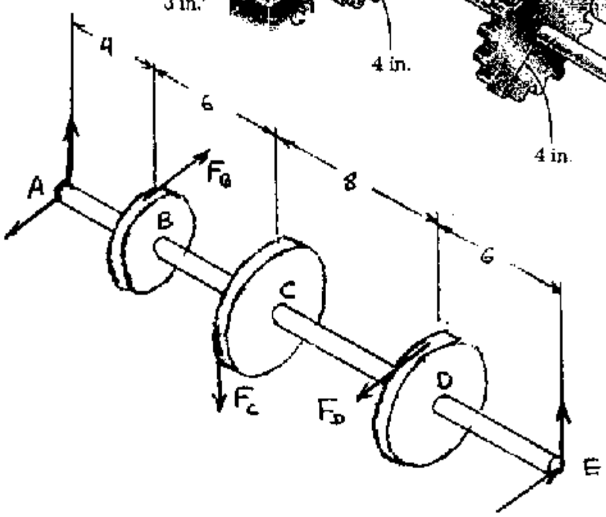
$c = 0.911$ in. $\quad d = 2c = 1.822$ in.

Problem 8.26

8.26 Solve Prob. 8.25, assuming that 30 hp is taken off at gear G and 30 hp is taken off at gear H.

8.25 The solid shaft AE rotates at 600 rpm and transmits 60 hp from the motor M to machine tools connected to gears G and H. Knowing that $\tau_{all} = 8$ ksi and that 40 hp is taken off at gear G and 20 hp is taken off at gear H, determine the smallest permissible diameter of shaft AE.





$60 \text{ hp} = (60)(6600)$
$= 396 \times 10^3 \text{ in}\cdot\text{lb/sec}$

$f = \frac{600 \text{ rpm}}{60 \text{ sec/min}} = 10 \text{ Hz}$

Torque on gear B

$T_B = \frac{P}{2\pi f} = \frac{396\times10^3}{2\pi(10)}$
$= 6302.5 \text{ lb}\cdot\text{in}$

Torques on gears C and D

$T_C = \frac{30}{60} T_B = 3151.3 \text{ lb}\cdot\text{in}$

$T_D = \frac{30}{60} T_B = 3151.3 \text{ lb}\cdot\text{in}$

Shaft torques

AB: $T_{AB} = 0$
BC: $T_{BC} = 6302.5$ lb·in
CD: $T_{CD} = 3151.3$ lb·in
DE: $T_{DE} = 0$

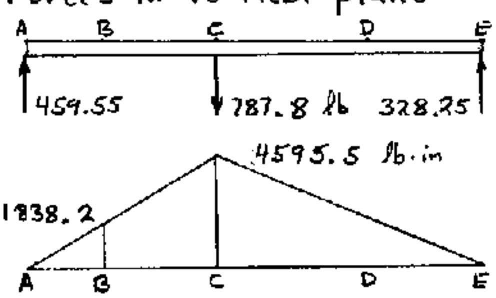
Gear forces

$F_B = \frac{T_B}{r_B} = \frac{6302.5}{3} = 2100.8 \text{ lb.}$

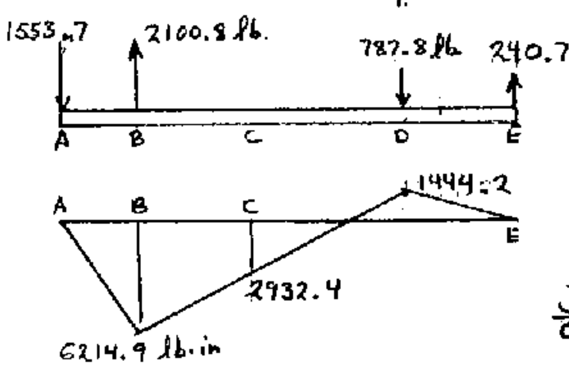
$F_C = \frac{T_C}{r_C} = \frac{3151.3}{4} = 787.8 \text{ lb.}$

$F_D = \frac{T_D}{r_D} = \frac{3151.3}{4} = 787.8 \text{ lb.}$

Forces in vertical plane



Forces in horizontal plane



At B^+ $\sqrt{M_z^2 + M_y^2 + T^2}$
$= \sqrt{1838.2^2 + 6214.9^2 + 6302.5^2}$
$= 9040.2 \text{ lb}\cdot\text{in}$ (maximum)

At C^- $\sqrt{M_z^2 + M_y^2 + T^2}$
$= \sqrt{4595.5^2 + 2932.4^2 + 6302.5^2}$
$= 8333.0 \text{ lb}\cdot\text{in}$

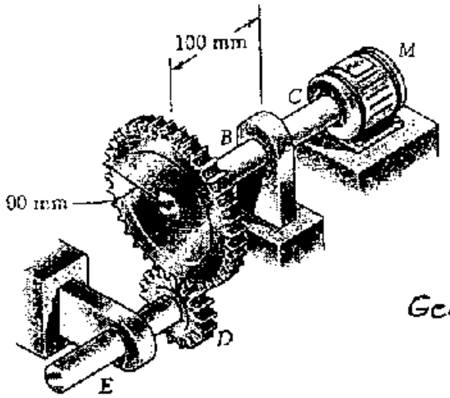
$\tau_{all} = \frac{c}{J}\left(\sqrt{M_z^2 + M_y^2 + T^2}\right)_{max}$

$\frac{J}{c} = \frac{\pi}{2} c^3 = \frac{\left(\sqrt{M_z^2 + M_y^2 + T^2}\right)}{\tau_{all}} = \frac{9040.3}{8\times10^3} = 1.1300 \text{ in}^3$

$c = 0.8960 \text{ in.}$ $\quad d = 2c = 1.792 \text{ in.}$

Problem 8.27

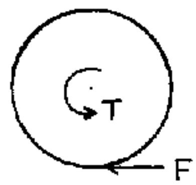
8.27 The solid shaft ABC and the gears shown are used to transmit 10 kW from the motor M to a machine tool connected to gear D. Knowing that the motor rotates at 240 rpm and that $\tau_{all} = 60$ MPa, determine the smallest permissible diameter of shaft ABC.



$$f = \frac{240 \text{ rpm}}{60 \text{ sec/min}} = 4 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{10 \times 10^3}{(2\pi)(4)} = 397.89 \text{ N·m}$$

Gear A.



$$F r_A - T = 0$$

$$F = \frac{T}{r_A} = \frac{397.89}{90 \times 10^{-3}} = 4421 \text{ N}$$

Bending moment at B: $M_B = L_{AB} F = (100 \times 10^{-3})(4421) = 442.1$ N·m

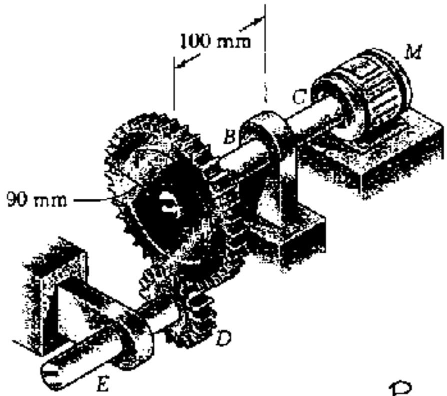
$$\tau_{all} = \frac{c}{J}\sqrt{M^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2} c^3 = \frac{\sqrt{M^2 + T^2}}{\tau_{all}}$$

$$c^3 = \frac{2}{\pi} \frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{(2)\sqrt{442.1^2 + 397.89^2}}{\pi (60 \times 10^6)} = 6.3108 \times 10^{-6} \text{ m}^3$$

$c = 18.479 \times 10^{-3}$ m $\qquad d = 2c = 37.0 \times 10^{-3}$ m $= 37.0$ mm ◄

Problem 8.28



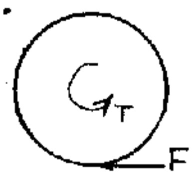
8.28 Assuming that shaft *ABC* of Prob. 8.27 is hollow and has an outer diameter of 50 mm, determine the largest permissible inner diameter of the shaft.

8.27 The solid shaft *ABC* and the gears shown are used to transmit 10 kW from the motor *M* to a machine tool connected to gear *D*. Knowing that the motor rotates at 240 rpm and that $\tau_{all} = 60$ MPa, determine the smallest permissible diameter of shaft *ABC*.

$$f = \frac{240 \text{ rpm}}{60 \text{ sec/min}} = 4 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{10\times10^3}{(2\pi)(4)} = 397.89 \text{ N-m}$$

Gear A.



$$F r_A - T = 0$$

$$F = \frac{T}{r_A} = \frac{397.89}{90\times10^{-3}} = 4421 \text{ N}$$

Bending moment at B: $M_B = L_{AB} F = (100\times10^{-3})(4421) = 442.1$ N·m

$$\tau_{all} = \frac{c_o}{J}\sqrt{M^2+T^2} \qquad c_o = \tfrac{1}{2}d_o = 25\times10^{-3} \text{ m}$$

$$\frac{J}{c_o} = \frac{\pi}{2}\frac{(c_o^4 - c_i^4)}{c_o} = \frac{\sqrt{M^2+T^2}}{\tau_{all}}$$

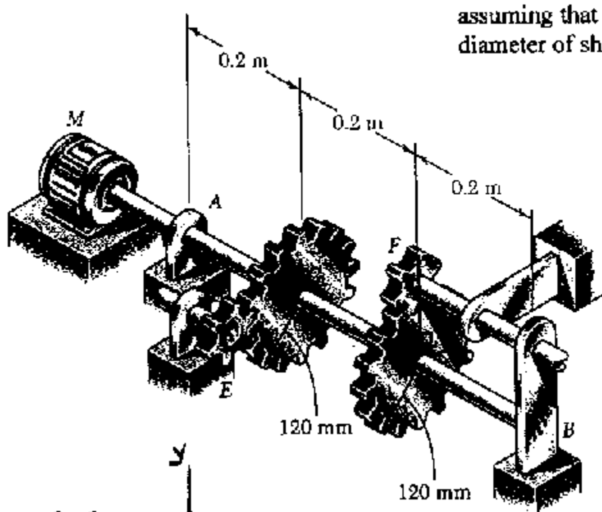
$$c_i^4 = c_o^4 - \frac{2c_o\sqrt{M^2+T^2}}{\pi\,\tau_{all}} = (25\times10^{-3})^4 - \frac{(2)(25\times10^{-3})\sqrt{442.1^2+397.89^2}}{\pi\,(60\times10^6)}$$

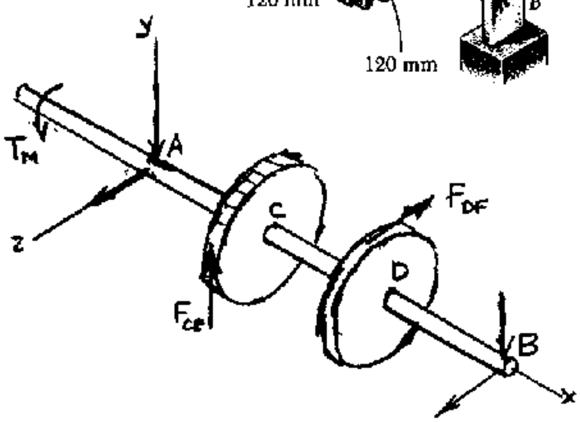
$$= 390.625\times10^{-9} - 157.772\times10^{-9} = 232.85\times10^{-9}$$

$$c_i = 21.967\times10^{-3} \text{ m} \qquad d_i = 2c_i = 43.93\times10^{-3} \text{ m} = 43.9 \text{ mm}$$

Problem 8.29

8.29 The solid shaft AB rotates at 360 rpm and transmits 20 kW from the motor M to machine tools connected to gears E and F. Knowing that $\tau_{all} = 45$ MPa and assuming that 10 kW is taken off at each gear, determine the smallest permissible diameter of shaft AB.





$$f = 360 \text{ rpm} = \frac{360}{60} = 6 \text{ Hz}$$

$$T_M = \frac{\mathcal{P}_M}{2\pi f} = \frac{20\times10^3}{2\pi(6)} = 530.5 \text{ N}\cdot\text{m}$$

$$T_C = \frac{\mathcal{P}_C}{2\pi f} = \frac{10\times10^3}{2\pi(6)} = 265.26 \text{ N}\cdot\text{m}$$

$$T_D = \frac{\mathcal{P}_D}{2\pi f} = \frac{10\times10^3}{2\pi(6)} = 265.26 \text{ N}\cdot\text{m}$$

$$F_{CE} = \frac{T_C}{r_C} = \frac{265.26}{120\times10^{-3}} = 2.210\times10^3 \text{ N}$$

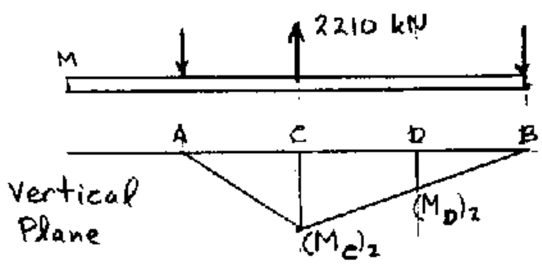
$$F_{DF} = \frac{T_D}{r_D} = \frac{265.26}{120\times10^{-3}} = 2.210\times10^3 \text{ N}$$

$$(M_C)_z = \frac{(0.2)(0.4)(2.210\times10^3)}{0.6} = 294.7 \text{ N}\cdot\text{m}$$

$$(M_D)_z = \tfrac{1}{2}(M_B)_z = 147.37 \text{ N}\cdot\text{m}$$

$$(M_C)_y = \frac{(0.4)(0.2)(2.210\times10^3)}{0.6} = 294.7 \text{ N}\cdot\text{m}$$

$$(M_B)_y = \tfrac{1}{2}(M_C)_y = 147.37 \text{ N}\cdot\text{m}$$



Torques in shaft.

$T_{MAC} = 530.5$ N·m

$T_{CD} = 265.26$ N·m

$T_{DB} = 0$

Just to the left of gear C.

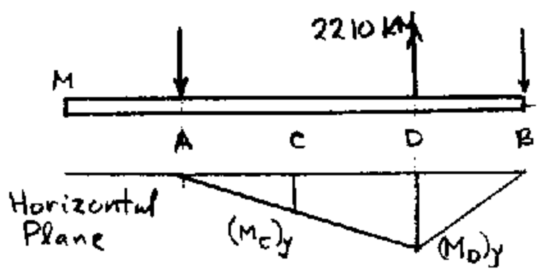
$$\max \sqrt{M_y^2 + M_z^2 + T^2} = 624.5 \text{ N}\cdot\text{m}$$

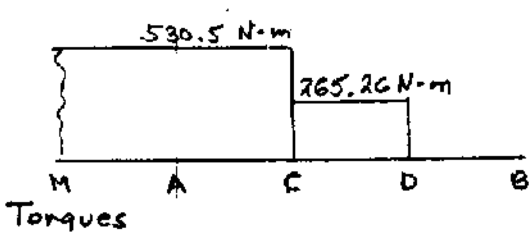
$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\max\sqrt{M_y^2+M_z^2+T^2}}{\tau_{max}}$$

$$= \frac{624.5}{45\times10^6} = 13.878\times10^{-6} \text{ m}^3$$

$$c = 20.67\times10^{-3} \text{ m} = 20.67 \text{ mm}$$

$$d = 2c = 41.3 \text{ mm}$$

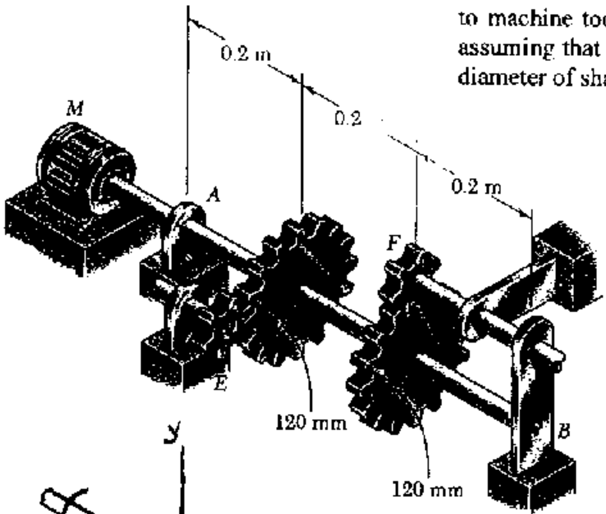


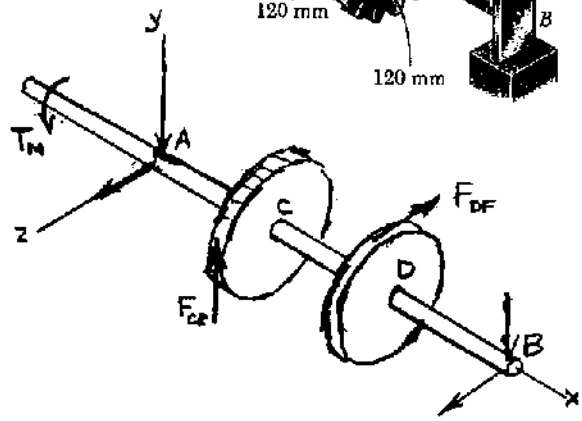


Problem 8.30

8.30 Solve Prob. 8.29, assuming that the entire 20 kW is taken off at gear E.

8.29 The solid shaft AB rotates at 360 rpm and transmits 20 kW from the motor M to machine tools connected to gears E and F. Knowing that τ_{all} = 45 MPa and assuming that 10 kW is taken off at each gear, determine the smallest permissible diameter of shaft AB.





$$f = 360\text{ rpm} = \frac{360}{60} = 6\text{ Hz}$$

$$T_M = \frac{\mathcal{P}_M}{2\pi f} = \frac{20\times10^3}{2\pi(6)} = 530.5\text{ N}\cdot\text{m}$$

$$T_C = \frac{\mathcal{P}_C}{2\pi f} = \frac{20\times10^3}{2\pi(6)} = 530.5\text{ N}\cdot\text{m}$$

$$T_D = \frac{\mathcal{P}_E}{2\pi f} = 0$$

$$F_{CE} = \frac{T_C}{r_C} = \frac{530.5}{120\times10^{-3}} = 4.421\times10^3\text{ N}$$

$$F_{DF} = \frac{T_D}{r_D} = 0$$

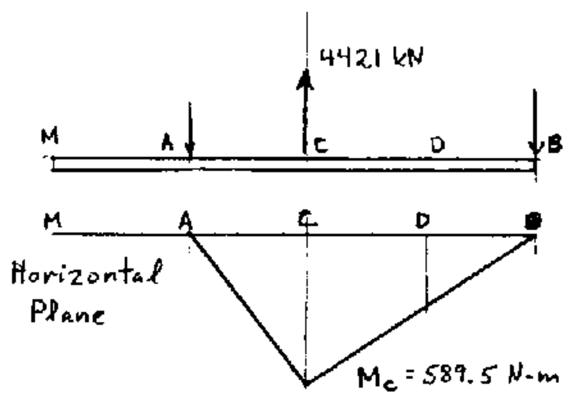
Since $F_{DF} = 0$, there is no bending in the horizontal plane.

$$M_C = \frac{(0.2)(0.4)(4.421\times10^3)}{0.6} = 589.5\text{ N}\cdot\text{m}$$

Torques in shaft

$$T_{MAC} = 530.5\text{ N}\cdot\text{m}$$

$$T_{CDE} = 0$$



Just to the left of gear C

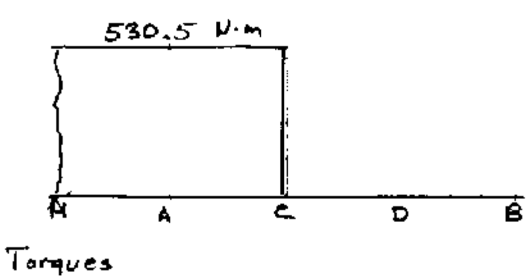
$$\max\sqrt{M_y^2 + M_z^2 + T^2}$$
$$= \sqrt{0 + 589.5^2 + 530.5^2}$$
$$= 793.1\text{ N}\cdot\text{m}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\max\sqrt{M_y^2 + M_z^2 + T^2}}{\tau_{all}}$$

$$= \frac{793.1}{45\times10^6} = 17.624\times10^{-6}\text{ m}^3$$

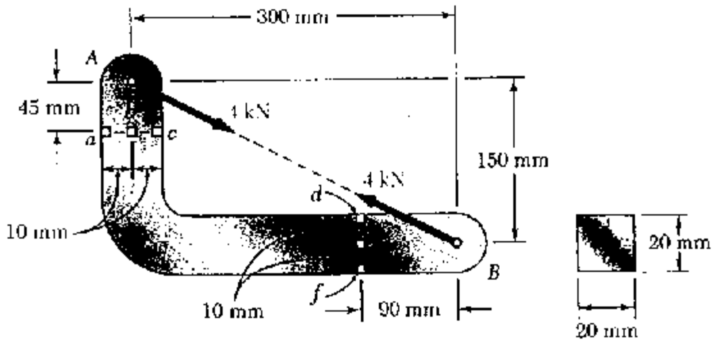
$$c = 22.39\times10^{-3} = 22.39\text{ mm}$$

$$d = 2c = 44.8\text{ mm}$$



Problem 8.31

8.31 Two 4-kN forces are applied to an L-shaped machine element AB as shown. Determine the normal and shearing stresses at (a) point a, (b) point b, (c) point c.

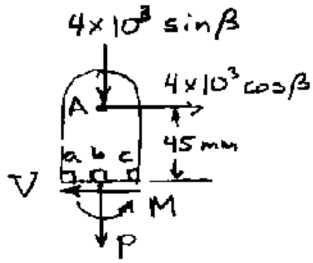


Let β be the slope angle of line AB.

$\tan\beta = \frac{150}{300}$ $\qquad \beta = 26.565°$

Draw free body sketch of the portion of the machine element lying above section $\underline{abc}$.

$$P = -(4\times10^3)\sin\beta = -1.78885\times10^3 \text{ N}$$



$$V = (4\times10^3)\cos\beta = 3.5777\times10^3 \text{ N}$$

$$M = (45\times10^{-3})(4\times10^3)\cos\beta = 160.997 \text{ N·m}$$

Section properties: $A = (20)(20) = 400 \text{ mm}^2 = 400\times10^{-6} \text{ m}^2$

$$I = \tfrac{1}{12}(20)(20)^3 = 13.3333\times10^3 \text{ mm}^4 = 13.3333\times10^{-9} \text{ m}^4$$

(a) Point $\underline{a}$. $\quad \sigma = \frac{P}{A} - \frac{Mx}{I} = -\frac{1.78885\times10^3}{400\times10^{-6}} - \frac{(160.997)(-10\times10^{-3})}{13.3333\times10^{-9}}$

$$= 116.3 \text{ MPa}$$

$$\tau = 0$$

(b) Point $\underline{b}$. $\quad \sigma = \frac{P}{A} = -\frac{1.78885\times10^3}{400\times10^{-6}} \qquad = -4.47 \text{ MPa}$

$$Q = (20)(10)(5) = 1000 \text{ mm}^3 = 10^{-6} \text{ m}^3$$

$$\tau = \frac{VQ}{It} = \frac{(3.5777)(10^{-6})}{(13.3333\times10^{-9})(20\times10^{-3})} = 13.42 \text{ MPa}$$

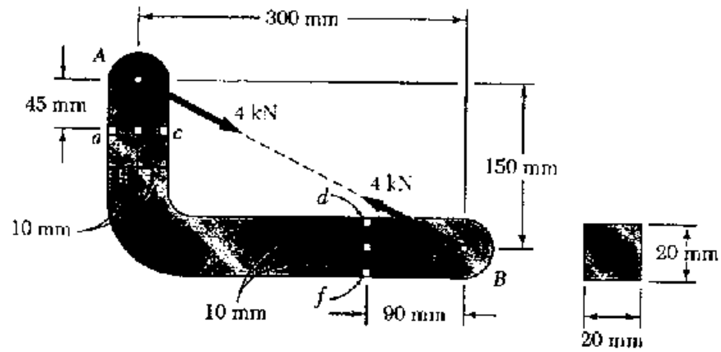
(c) Point $\underline{c}$. $\quad \sigma = \frac{P}{A} - \frac{Mx}{I} = -\frac{1.78885\times10^3}{400\times10^{-6}} - \frac{(160.997)(10\times10^{-3})}{13.3333\times10^{-9}}$

$$= -125.2 \text{ MPa}$$

$$\tau = 0$$

Problem 8.32

8.32 Two 4-kN forces are applied to an L-shaped machine element AB as shown. Determine the normal and shearing stresses at (*a*) point *d*, (*b*) point e, (*c*) point *f*.

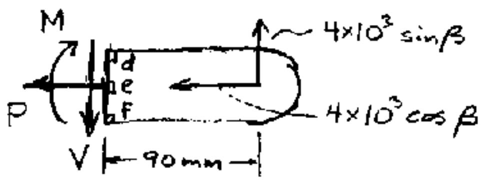


Let β be the slope angle of line AB.

$\tan\beta = \frac{150}{300} \qquad \beta = 26.565°$

Draw free body sketch of the portion of the machine element lying to the right of section $\underline{def}$.

$P = -(4\times10^3)\cos\beta$
$= -3.5777\times10^3$ N



$V = (4\times10^3)\sin\beta = 1.78885\times10^3$ N

$M = (90\times10^{-3})(4\times10^{-3})\sin\beta = 160.997$ N-m

Section properties. $A = (20)(20) = 400\text{ mm}^2 = 400\times10^{-6}\text{ m}^2$

$I = \frac{1}{12}(20)(20)^3 = 13.3333\times10^3\text{ mm}^4 = 13.3333\times10^{-9}\text{ m}^4$

(a) Point $\underline{d}$. $\sigma = \frac{P}{A} - \frac{My}{I} = \frac{-3.5777\times10^3}{400\times10^{-6}} - \frac{(160.997)(10\times10^{-3})}{13.3333\times10^{-9}}$

$= -129.7$ MPa ◄

$\tau = 0$ ◄

(b) Point $\underline{e}$. $\sigma = \frac{P}{A} = \frac{-3.5777\times10^3}{400\times10^{-6}} = -8.94$ MPa ◄

$Q = (20)(10)(5) = 1000\text{ mm}^3 = 10^{-6}\text{ m}^3$

$\tau = \frac{VQ}{It} = \frac{(1.78885\times10^3)(10^{-6})}{(13.3333\times10^{-9})(20\times10^{-3})} = 6.71$ MPa ◄

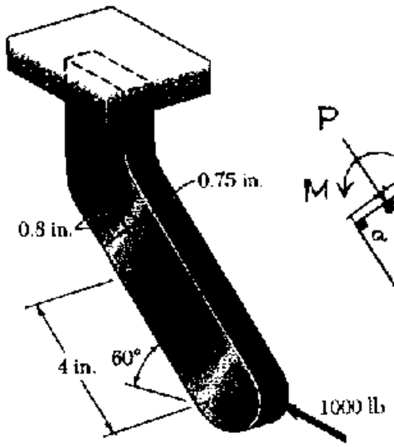
(c) Point $\underline{f}$. $\sigma = \frac{P}{A} - \frac{My}{I} = \frac{-3.5777\times10^3}{400\times10^{-6}} - \frac{(160.997)(-10\times10^{-3})}{13.3333\times10^{-9}}$

$= 111.8$ MPa ◄

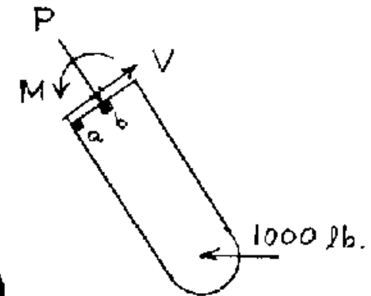
$\tau = 0$ ◄

Problem 8.33

8.33 For the bracket and loading shown, determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*.



Draw free body diagram of portion below section $\underline{ab}$.



From statics,

$P = 1000 \cos 60° = 500$ lb

$V = 1000 \sin 60° = 866$ lb.

$M = (4)(1000)\sin 60°$
$= 3464$ lb·in.

Section properties.

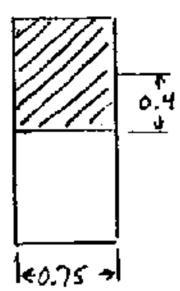
$A = (0.75)(1.6) = 1.2 \text{ in}^2$

$I = \frac{1}{12}(0.75)(1.6)^3 = 0.256 \text{ in}^4$

(a) Point $\underline{a}$ $\quad \sigma = -\frac{P}{A} - \frac{Mc}{I} = -\frac{500}{1.2} - \frac{(3464)(0.8)}{0.256} = -11.24$ ksi ◄

$\tau = 0$ ◄

(b) Point $\underline{b}$ $\quad \sigma = -\frac{P}{A} = -\frac{500}{1.2} = -0.417$ ksi ◄



$A = (0.75)(0.8) = 0.6 \text{ in}^2$

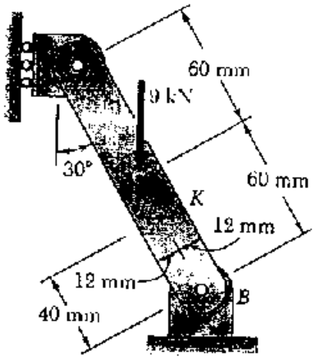
$\bar{y} = 0.4$ in.

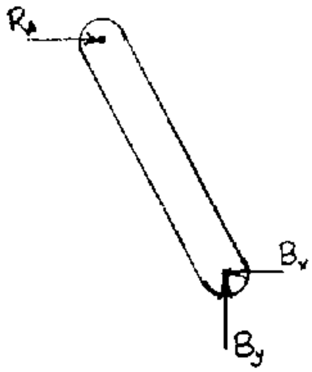
$Q = A\bar{y} = (0.6)(0.4) = 0.24 \text{ in}^3$

$\tau = \frac{VQ}{It} = \frac{(866)(0.24)}{(0.256)(0.75)} = 1.083$ ksi ◄

Problem 8.34

8.34 **and** *8.35* Member AB has a uniform rectangular cross section of 10×24 mm. For the loading shown, determine the normal and shearing stresses (a) point H, (b) point K.





$\circlearrowleft \Sigma M_B = 0$

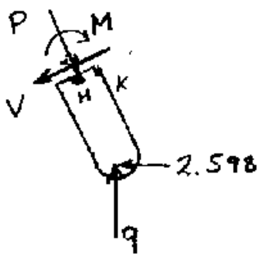
$(120 \cos 30°) R_A - (60 \sin 30°)(9) = 0$

$R_A = 2.598$ kN

$+\uparrow \Sigma F_y = 0 \qquad B_y - 9 = 0 \qquad B_y = 9 \text{ kN}\uparrow$

$+\rightarrow \Sigma F_x = 0 \qquad 2.598 - B_x = 0$

$B_x = 2.598 \text{ kN} \leftarrow$



At the section containing points H and K,

$P = 9 \cos 30° + 2.598 \sin 30° = 9.093$ kN

$V = 9 \sin 30° - 2.598 \cos 30° = 2.25$ kN

$M = (9 \times 10^3)(40 \times 10^{-3} \sin 30°) - (2.598 \times 10^3)(40 \times 10^{-3} \cos 30°) = 90 \text{ N} \cdot \text{m}$

$A = 10 \times 240 = 240 \text{ mm}^2 = 240 \times 10^{-6} \text{ m}^2$

$I = \frac{1}{12}(10)(24)^3 = 11.52 \times 10^3 \text{ mm}^4 = 11.52 \times 10^{-9} \text{ m}^4$

(a) At point H,

$$\sigma_x = -\frac{P}{A} = -\frac{9.093 \times 10^3}{240 \times 10^{-6}} = -37.9 \text{ MPa}$$

$$\tau_{xy} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{2.25 \times 10^3}{240 \times 10^{-6}} = 14.06 \text{ MPa}$$

(b) At point K,

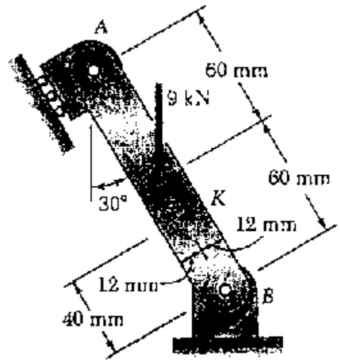
$$\sigma_x = -\frac{P}{A} - \frac{Mc}{I} = -\frac{9.093 \times 10^3}{240 \times 10^{-6}} - \frac{(90)(12 \times 10^{-3})}{11.52 \times 10^{-9}}$$

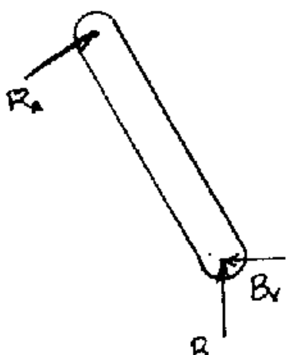
$$= -131.6 \text{ MPa}$$

$$\tau_{xy} = 0$$

Problem 8.35

8.34 and 8.35 Member *AB* has a uniform rectangular cross section of 10 × 24 mm. For the loading shown, determine the normal and shearing stress at (*a*) point *H*, (*b*) point *K*.





$\overset{+}{\circlearrowleft} \Sigma M_B = 0$

$(9)(60 \sin 30^\circ) - 120 R_A = 0$

$R_A = 2.25$ kN

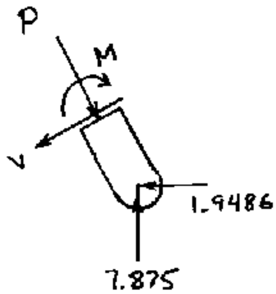
$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad 2.25 \cos 30^\circ - B_x = 0$

$B_x = 1.9486$ kN ←

$+\uparrow \Sigma F_y = 0$

$2.25 \sin 30^\circ - 9 + B_y = 0$

$B_y = 7.875$ kN ↑



At the section containing points H and K,

$P = 7.875 \cos 30^\circ + 1.9486 \sin 30^\circ = 7.794$ kN

$V = 7.875 \sin 30^\circ - 1.9486 \cos 30^\circ = 2.25$ kN

$M = (7.875 \times 10^3)(40 \times 10^{-3} \sin 30^\circ) - (1.9486 \times 10^3)(40 \times 10^{-3} \cos 30^\circ) = 90$ N·m

$A = 10 \times 24 = 240 \text{ mm}^2 = 240 \times 10^{-6} \text{ m}^2$

$I = \frac{1}{12}(10)(24)^3 = 11.52 \times 10^3 \text{ mm}^4 = 11.52 \times 10^{-9} \text{ m}^4$

(a) At point H,
$\sigma_x = -\frac{P}{A} = -\frac{7.794 \times 10^3}{240 \times 10^{-6}} = -32.5$ MPa ◄

$\tau_{xy} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{2.25 \times 10^3}{240 \times 10^{-6}} = 14.06$ MPa ◄

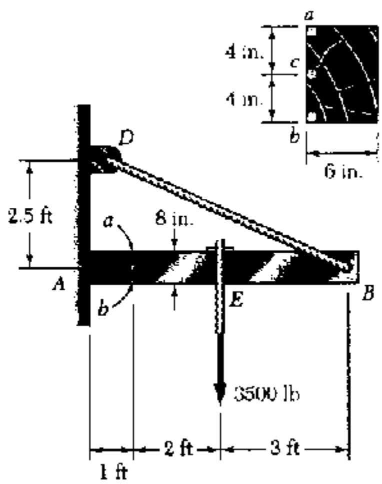
(b) At point K,
$\sigma_x = -\frac{P}{A} - \frac{Mc}{I} = -\frac{7.794 \times 10^3}{240 \times 10^{-6}} - \frac{(90)(12 \times 10^{-3})}{11.52 \times 10^{-9}}$

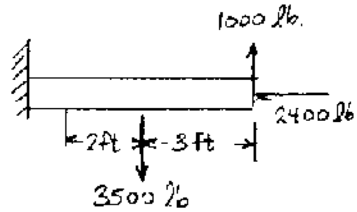
$= -126.2$ MPa ◄

$\tau_{xy} = 0$ ◄

Problem 8.36

8.36 The cantilever beam AB has a rectangular cross section of 6 × 8 in. Knowing that the tension in the cable BD is 2600 lb and neglecting the weight of the beam, determine the normal and shearing stresses at the three points indicated.





$DE = \sqrt{(6)^2 + (2.5)^2} = 6.5 \text{ ft}$

Vertical component of T_{DE}

$\left(\frac{2.5}{6.5}\right)(2600) = 1000 \text{ lb.}$

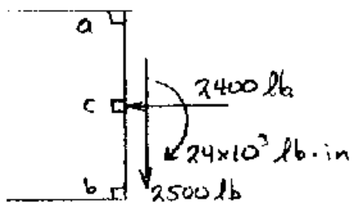
Horizontal component of $T_{DE} = \left(\frac{6}{6.5}\right)(2600) = 2400 \text{ lb.}$

At the section containing points $\underline{a}$, $\underline{b}$, and $\underline{c}$.

$P = 2400 \text{ lb.} \qquad V = 3500 - 1000 = 2500 \text{ lb}$

$M = (2+3)(1000) - (2)(3500) = -2000 \text{ lb}\cdot\text{ft}$

$= -24 \times 10^3 \text{ lb}\cdot\text{in}$



Section properties

$A = (6)(8) = 48 \text{ in}^3$

$I = \frac{1}{12}(6)(8)^3 = 256 \text{ in}^4$

$c = 4 \text{ in.}$

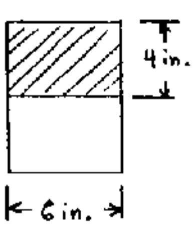
At point $\underline{a}$. $\quad \sigma_x = -\frac{P}{A} - \frac{Mc}{I} = -\frac{2400}{48} - \frac{(-24\times10^3)(4)}{256} = 325 \text{ psi}$ ◄

$\tau_{xy} = 0$ ◄

At point $\underline{b}$ $\quad \sigma_x = -\frac{P}{A} + \frac{Mc}{I} = -\frac{2400}{48} + \frac{(-24\times10^3)(4)}{256} = -425 \text{ psi}$ ◄

$\tau_{xy} = 0$

At point $\underline{c}$ $\quad \sigma_x = -\frac{P}{A} = -\frac{2400}{48} = -50 \text{ psi}$ ◄

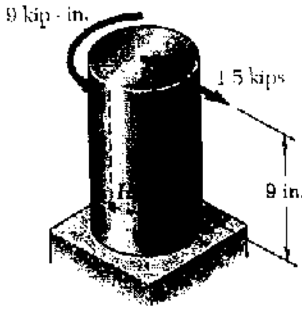


$A = (6)(4) = 24 \text{ in}^2, \quad \bar{y} = 2 \text{ in.} \quad Q = A\bar{y} = 48 \text{ in}^3$

$\tau_{xy} = -\frac{VQ}{It} = -\frac{(2500)(48)}{(256)(6)} = -78.1 \text{ psi}$ ◄

Problem 8.37

8.37 A 1.5-kip force and a 9-kip in. couple are applied at the top of the 2.5-in.-diameter cast-iron post shown. Determine the normal and shearing stresses at (*a*), point *H*, (*b*) point *K*.



diameter = 2.5 in.

At the section containing points H and K.

$P = 0 \qquad V = 1.5$ kips

$T = 9$ kip·in $\qquad M = (1.5)(9) = 13.5$ kip·in

$d = 2.5$ in. $\qquad c = \frac{1}{2}d = 1.25$ in.

$A = \pi c^2 = 4.909\ in^2 \qquad I = \frac{\pi}{4}c^4 = 1.9175\ in^4 \qquad J = 2I = 3.835\ in^4$

For a semicircle, $Q = \frac{2}{3}c^3 = 1.3021\ in^3$

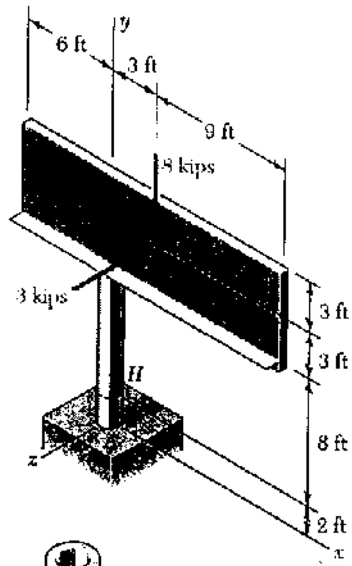
(a) At point H, $\qquad \sigma_H = 0$

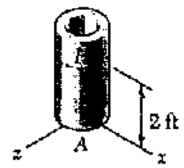
$$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(9)(1.25)}{3.835} + \frac{(1.5)(1.3021)}{(1.9175)(2.5)} = 2.934 + 0.407$$
$$= 3.34 \text{ ksi}$$

(b) At point K, $\qquad \sigma_K = -\frac{Mc}{I} = -\frac{(13.5)(1.25)}{1.9175} = -8.80$ ksi

$$\tau_K = \frac{Tc}{J} = \frac{(9)(1.25)}{3.835} = 2.93 \text{ ksi}$$

Problem 8.38





8.38 The billboard shown weights 8000 lb and is supported by a structural tube that has a 15-in. outer diameter and a 0.5-in. wall thickness. At a time when the resultant of the wind pressure is 3 kips located at the center C of the billboard, determine the normal and shearing stresses at point H.

At section containing point H,

$P = 8$ kips (compression)

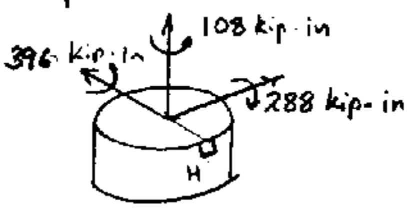
$T = (3)(3) = 9 \text{ kip}\cdot\text{ft} = 108 \text{ kip}\cdot\text{in}$

$M_x = -(11)(3) = -33 \text{ kip}\cdot\text{ft} = -396 \text{ kip}\cdot\text{in}$

$M_z = -(3)(8) = -24 \text{ kip}\cdot\text{ft} = -288 \text{ kip}\cdot\text{in}$

$V = 3$ kip

Couples



Section properties.

$d_o = 15$ in. $\quad c_o = \frac{1}{2}d_o = 7.5$ in. $\quad c_i = c_o - t = 7.0$ in.

$A = \pi(c_o^2 - c_i^2) = 22.777 \text{ in}^2$

$I = \frac{\pi}{4}(c_o^4 - c_i^4) = 599.31 \text{ in}^4$

$J = 2I = 1198.62 \text{ in}^4$

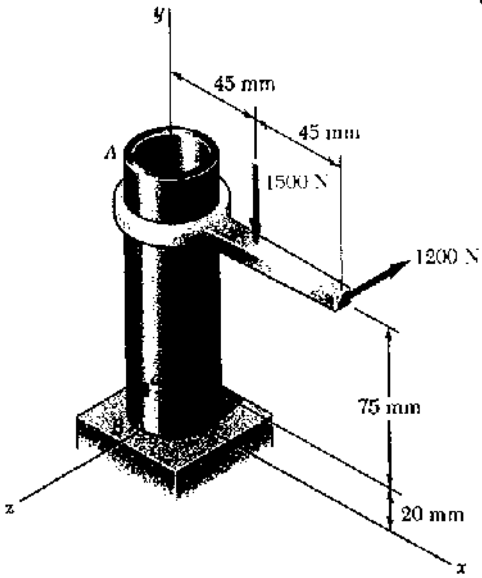
$Q = \frac{2}{3}(c_o^3 - c_i^3) = 52.583 \text{ in}^3$

$$\sigma = -\frac{P}{A} - \frac{Mc}{I} = -\frac{8}{22.777} - \frac{(288)(7.5)}{599.31} = -0.351 - 3.604 = -3.96 \text{ ksi}$$

$$\tau = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(108)(7.5)}{1198.62} + \frac{(3)(52.583)}{(599.31)(1.0)} = 0.675 + 0.268 = 0.938 \text{ ksi}$$

Problem 8.39

8.39 Two forces are applied to the pipe AB as shown. Knowing that the pipe has inner and outer diameters equal to 35 and 42 mm, respectively, determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*.

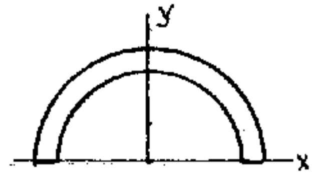


$$c_o = \frac{d_o}{2} = 21 \text{ mm}, \quad c_i = \frac{d_i}{2} = 17.5 \text{ mm}$$

$$A = \pi(c_o^2 - c_i^2) = 423.33 \text{ mm}^2$$

$$J = \frac{\pi}{2}(c_o^4 - c_i^4) = 158.166 \times 10^3 \text{ mm}^4$$

$$I = \frac{1}{2}J = 79.083 \times 10^3 \text{ mm}^4$$



For semicircle with semicircular cutout

$$Q = \frac{2}{3}(c_o^3 - c_i^3)$$

$$Q = 2.6011 \times 10^3 \text{ mm}^3$$

At the section containing points $\underline{a}$ and $\underline{b}$

$$P = -1500 \text{ N} \qquad V_z = -1200 \text{ N} \qquad V_x = 0$$

$$M_z = -(45 \times 10^{-3})(1500) = -67.5 \text{ N}\cdot\text{m}$$

$$M_x = -(75 \times 10^{-3})(1200) = -90 \text{ N}\cdot\text{m}$$

$$T = (90 \times 10^{-3})(1200) = 108 \text{ N}\cdot\text{m}$$

(a) $$\sigma = \frac{P}{A} - \frac{M_x c}{I} = \frac{-1500}{423.33 \times 10^{-6}} - \frac{(-90)(21 \times 10^{-3})}{79.083 \times 10^{-9}} = 20.4 \text{ MPa}$$ ◄

$$\tau = \frac{Tc}{J} + \frac{V_x Q}{It} = \frac{(108)(21 \times 10^{-3})}{158.166 \times 10^{-9}} + 0 = 14.34 \text{ MPa}$$ ◄

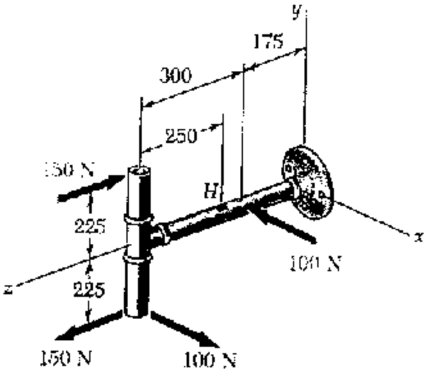
(b) $$\sigma = \frac{P}{A} + \frac{M_z c}{I} = \frac{-1500}{423.33 \times 10^{-6}} + \frac{(-67.5)(21 \times 10^{-3})}{79.083 \times 10^{-9}} = -21.5 \text{ MPa}$$ ◄

$$\tau = \frac{Tc}{J} + \frac{|V_z| Q}{It} = \frac{(108)(21 \times 10^{-3})}{158.166 \times 10^{-9}} + \frac{(1200)(2.6011 \times 10^{-6})}{(79.083 \times 10^{-9})(7 \times 10^{-3})}$$

$$= 19.98 \text{ MPa}$$ ◄

Problem 8.40

8.40 Several forces are applied to the pipe assembly shown. Knowing that each section of pipe has inner and outer diameters equal to 36 and 42 mm, respectively, determine the normal and shearing stresses at point H located at the top of the outer surface of the pipe.



Dimensions in mm

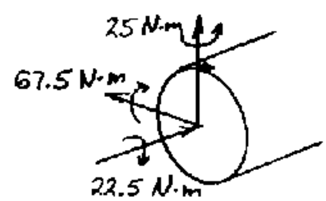
At the section containing point H,

$P = 0, \quad V_x = 100 \text{ N}, \quad V_y = 0$

$M_x = -(0.450)(150) = -67.5 \text{ N}\cdot\text{m}$

$M_y = (0.250)(100) = 25 \text{ N}\cdot\text{m}$

$M_z = -(0.225)(100) = -22.5 \text{ N}\cdot\text{m}$



$d_o = 42 \text{ mm} \qquad d_i = 32 \text{ mm}$

$c_o = 21 \text{ mm} \qquad c_i = 18 \text{ mm}$

$t = c_o - c_i = 3 \text{ mm}$

$A = \pi(c_o^2 - c_i^2) = 367.57 \text{ mm}^2 = 367.57 \times 10^{-6} \text{ m}^2$

$I = \frac{\pi}{4}(c_o^4 - c_i^4) = 70.30 \times 10^3 \text{ mm}^4 = 70.30 \times 10^{-9} \text{ m}^4, \quad J = 2I = 140.59 \times 10^{-9} \text{ m}^4$

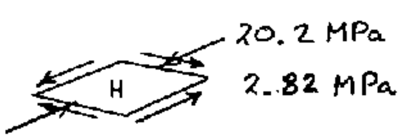
For half-pipe, $Q = \frac{2}{3}(c_o^3 - c_i^3) = 2.286 \times 10^3 \text{ mm}^3 = 2.286 \times 10^{-6} \text{ m}^3$

$\sigma_H = \frac{M_x y}{I_x} = \frac{(-67.5)(21 \times 10^{-3})}{70.30 \times 10^{-9}} = -20.2 \text{ MPa}$ ◄

Due to torque: $(\tau_H)_T = \frac{Tc}{J} = \frac{(22.5)(21 \times 10^{-3})}{140.59 \times 10^{-9}} = 3.36 \text{ MPa}$

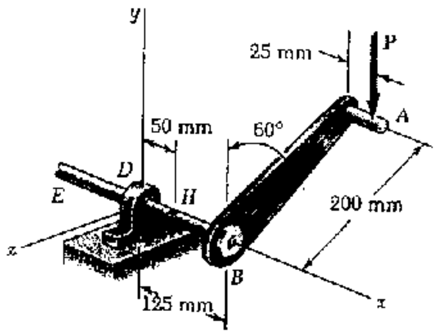
Due to shear: $(\tau_H)_V = \frac{VQ}{It} = \frac{(100)(2.286 \times 10^{-6})}{(70.30 \times 10^{-9})(6 \times 10^{-3})} = 0.54 \text{ MPa}$

Net: $\tau_H = 3.36 - 0.54 = 2.82 \text{ MPa}$ ◄



Problem 8.41

8.41 A vertical force **P** of magnitude 250 N is applied to the crank at point A. Knowing that the shaft BDE has a diameter of 18 mm, determine the principal stresses and the maximum shearing stress at point H located at the top of the shaft, 50 mm to the right of support D.

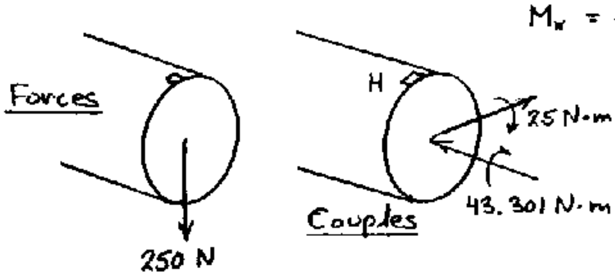


Force-couple system at the centroid of the section containing point H.

$F_x = 0, \quad V_y = -250 \text{ N}, \quad V_z = 0$

$M_z = -(125 - 50 + 25)(10^{-3})(250) = -25 \text{ N}\cdot\text{m}$

$M_x = -(200 \sin 60°)(10^{-3})(250) = -43.301 \text{ N}\cdot\text{m}$



$d = 18 \text{ mm} \qquad c = \frac{1}{2}d = 9 \text{ mm}$

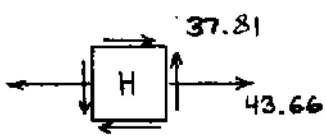
$I = \frac{\pi}{4}c^4 = 5.153 \times 10^3 \text{ mm}^4 = 5.153 \times 10^{-9} \text{ m}^4$

$J = 2I = 10.306 \times 10^{-9} \text{ m}^4$

At point H, $\quad \sigma_H = -\frac{M_z y}{I_z} = -\frac{(-25)(9 \times 10^{-3})}{5.153 \times 10^{-9}} = 43.66 \text{ MPa}$

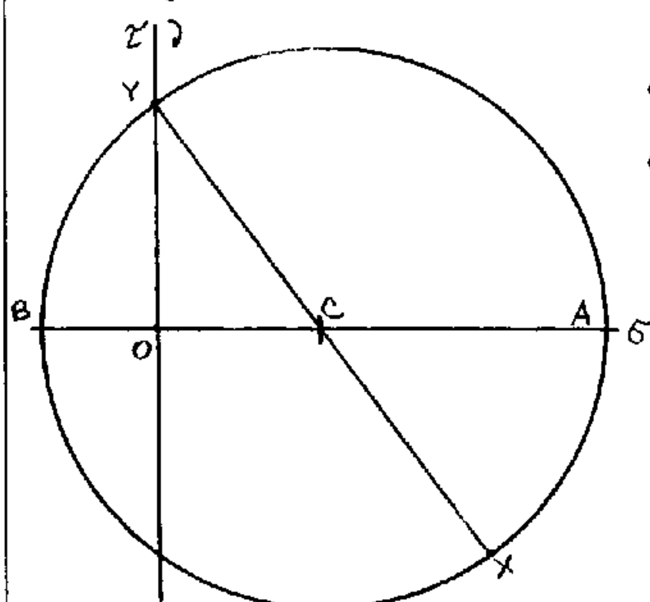
$\tau_H = \frac{Tc}{J} = \frac{(43.301)(9 \times 10^{-3})}{10.306 \times 10^{-9}} = 37.81 \text{ MPa}$

Use Mohr's circle.



$\sigma_c = \frac{1}{2}\sigma_H = 21.83 \text{ MPa}$

$R = \sqrt{\left(\frac{43.66}{2}\right)^2 + (37.81)^2} = 43.66 \text{ MPa}$



$\sigma_a = \sigma_c + R = 65.5 \text{ MPa}$ ◄

$\sigma_b = \sigma_c - R = -21.8 \text{ MPa}$ ◄

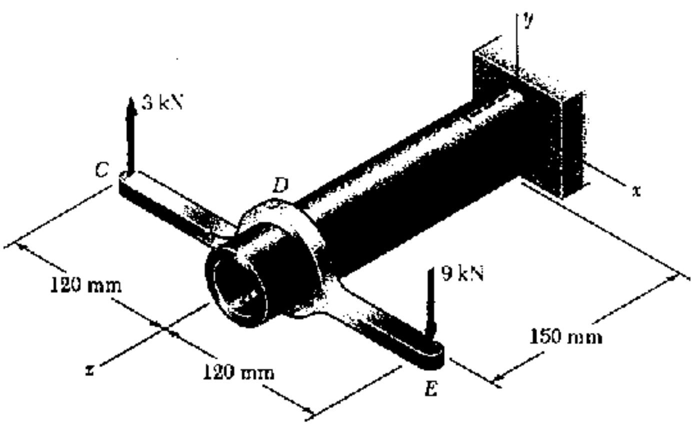
$\tan 2\theta_p = \frac{2\tau_H}{\sigma_H} = \frac{75.62}{43.66} = 1.7320$

$\theta_a = 30°, \qquad \theta_b = 120°$

$\tau_{max} = R = 43.7 \text{ MPa}$ ◄

Problem 8.42

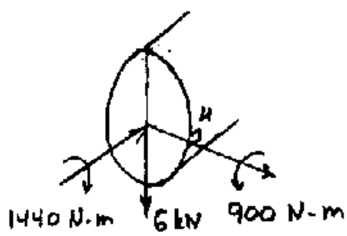
8.42 The steel pipe AB has a 72-mm outer diameter and a 5-mm wall thickness. Knowing that the arm CDE is rigidly attached to the pipe, determine the principal stresses, principal planes, and the maximum shearing stress at point H.



Replace the forces at C and E by an equivalent force-couple system at D.

$$F_D = 9 - 3 = 6 \text{ kN} \downarrow$$

$$T_D = (9\times10^3)(120\times10^{-3}) + (3\times10^3)(120\times10^{-3}) = 1440 \text{ N}\cdot\text{m}$$



At the section containing point H,

$$P = 0, \quad V = 6 \text{ kN}, \quad T = 1440 \text{ N}\cdot\text{m}$$

$$M = (6\times10^3)(150\times10^{-3}) = 900 \text{ N}\cdot\text{m}$$

Section properties: $d_o = 72$ mm $\quad c_o = \frac{1}{2} d_o = 36$ mm $\quad c_i = c_o - t = 31$ mm

$$A = \pi(c_o^2 - c_i^2) = 1.0524 \times 10^3 \text{ mm}^2 = 1.0524 \times 10^{-3} \text{ m}^2$$

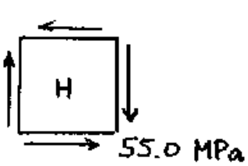
$$I = \frac{\pi}{4}(c_o^4 - c_i^4) = 593.84 \times 10^{-3} \text{ mm}^4 = 593.84 \times 10^{-9} \text{ m}^4$$

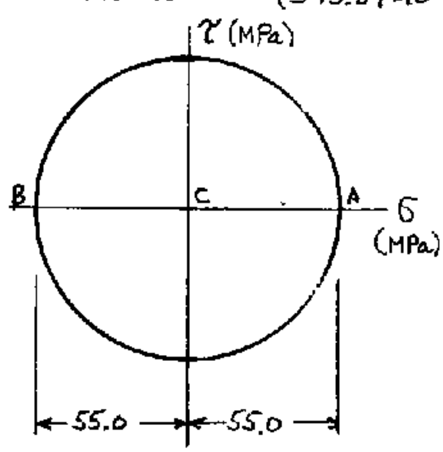
$$J = 2I = 1.1877 \times 10^{-6} \text{ m}^4$$

For half-pipe, $Q = \frac{2}{3}(c_o^3 - c_i^3) = 11.243 \times 10^3 \text{ mm}^3 = 11.243 \times 10^{-6} \text{ m}^3$

At point H. Point H lies on the neutral axis of bending. $\sigma_H = 0$.

$$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(1440)(36\times10^{-3})}{1.1877\times10^{-6}} + \frac{(6\times10^3)(11.243\times10^{-6})}{(593.84\times10^{-9})(10\times10^{-3})} = 55.0 \text{ MPa}$$





Use Mohr's circle.

$\sigma_c = 0$

$R = 55.0$ MPa

$\sigma_a = \sigma_c + R = 55.0$ MPa ◄

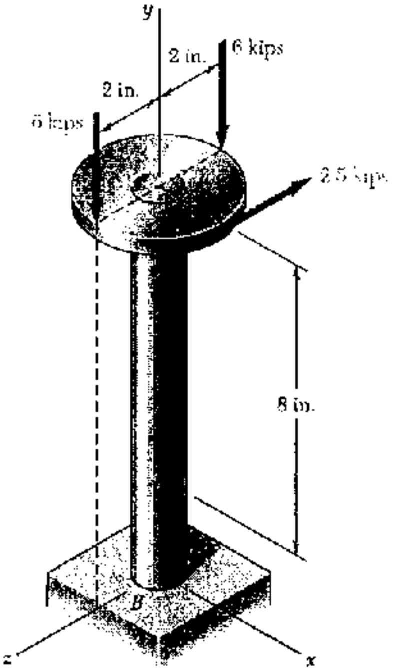
$\sigma_b = \sigma_c - R = -55.0$ MPa ◄

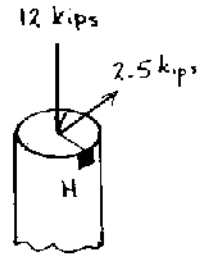
$\theta_a = -45°, \quad \theta_b = +45°$ ◄

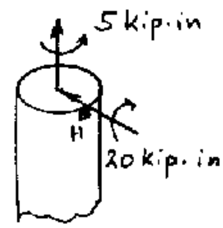
$\tau_{max} = R = 55.0$ MPa ◄

Problem 8.43

8.43 Three forces are applied to a 4-in.-diameter plate that is attached to the solid 1.8-in. diameter shaft AB. At point H, determine (*a*) the principal stresses and principal planes, (*b*) the maximum shearing stress.







At the section containing point H,

$P = 12$ kips (compression)

$V = 2.5$ kips

$T = (2)(2.5) = 5$ kip·in

$M = (8)(2.5) = 20$ kip·in.

$d = 1.8$ in $\qquad c = \frac{1}{2}d = 0.9$ in.

$A = \pi c^2 = 2.545$ in^2

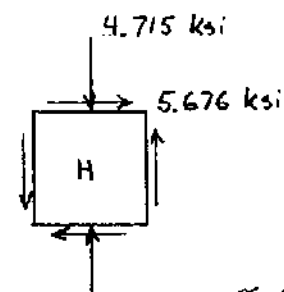
$I = \frac{\pi}{4}c^4 = 0.5153$ in^4

$J = 2I = 1.0306$ in^4

For a semicircle, $\quad Q = \frac{2}{3}c^3 = 0.486$ in^3

Point H lies on neutral axis of bending. $\quad \sigma_H = \frac{P}{A} = -\frac{12}{2.545} = -4.715$ ksi

$$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(5)(0.9)}{1.0306} + \frac{(2.5)(0.486)}{(0.5153)(1.8)} = 5.676 \text{ ksi}$$



Use Mohr's circle.

$\sigma_c = \frac{1}{2}(-4.715) = -2.3575$ ksi

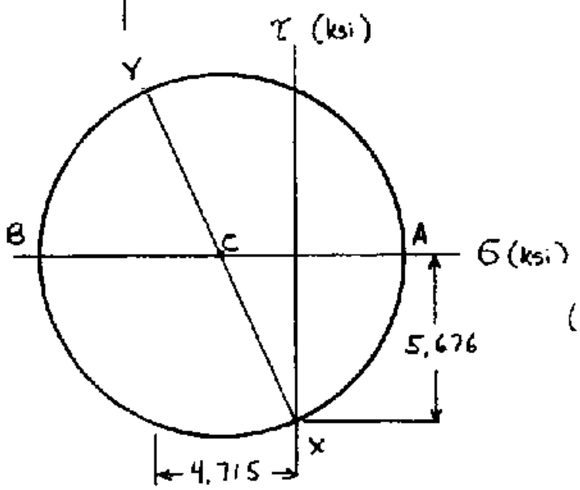
$R = \sqrt{\left(\frac{4.715}{2}\right)^2 + 5.676^2} = 6.1461$

(a) $\sigma_a = \sigma_c + R = 3.79$ ksi

$\sigma_b = \sigma_c - R = -8.50$ ksi

$\tan 2\theta_p = \frac{(2)(5.676)}{4.715} = 2.408$

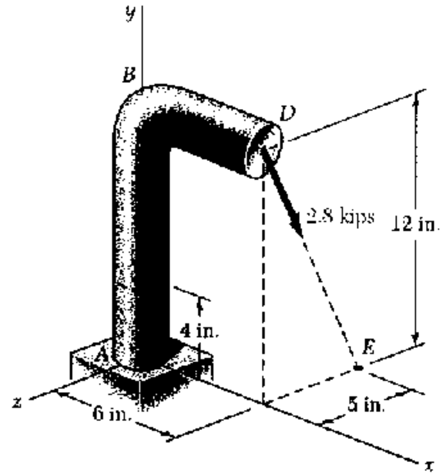
$\theta_a = 33.7°$ $\qquad \theta_b = 123.7°$



(b) $\tau_{max} = R = 6.15$ ksi

Problem 8.44

8.44 A 2.8-kip force is applied as shown to the 2.4-in.-diameter cast-iron post *ABD*. At point *H*, determine (*a*) the principal stresses and principal planes, (*b*) the maximum shearing stress.



$DE = \sqrt{5^2 + 12^2} = 13$ in.

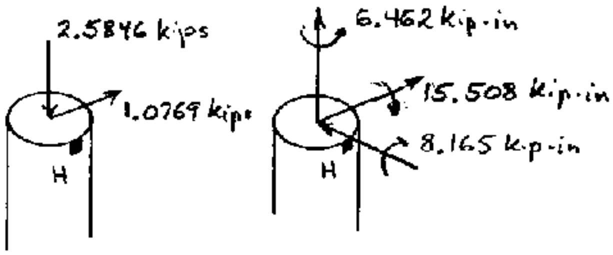
At point D $\qquad F_x = 0$

$F_y = -\left(\frac{12}{13}\right)(2.8) = -2.5846$ kips

$F_z = -\left(\frac{5}{13}\right)(2.8) = -1.0769$ kips

Moment of equivalent force-couple system at C, the centroid of the section containing point H

$$\vec{M} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 6 & 8 & 0 \\ 0 & -2.5846 & -1.0769 \end{vmatrix} = 8.165\,\vec{i} + 6.462\,\vec{j} - 15.508\,\vec{k} \quad \text{kip}\cdot\text{in}$$



Section properties

$d = 2.4$ in. $\qquad c = \frac{1}{2}d = 1.2$ in.

$A = \pi c^2 = 4.5239$ in^2

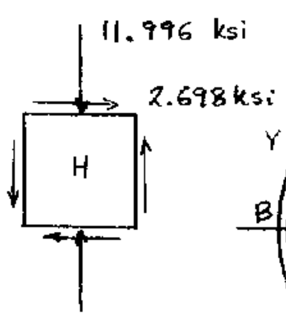
$I = \frac{\pi}{4}c^4 = 1.6286$ in^4

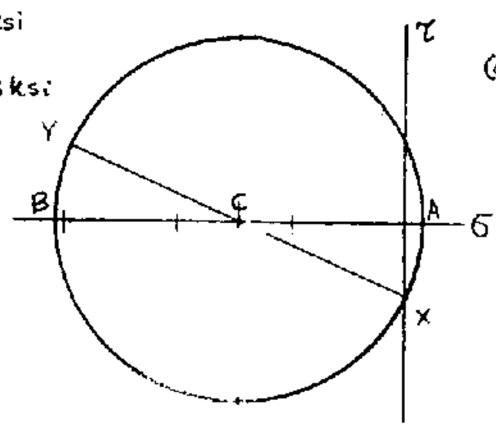
$J = 2I = 3.2572$ in^4

For a semicircle $\quad Q = \frac{2}{3}c^3 = 1.152$ in^3

At point H $\quad \sigma_H = -\frac{P}{A} - \frac{Mc}{I} = -\frac{2.5846}{4.5239} - \frac{(15.508)(1.2)}{1.6286} = -11.996$ ksi

$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(6.462)(1.2)}{3.2572} + \frac{(1.0769)(1.152)}{(1.6286)(2.4)} = 2.698$ ksi





(a) $\sigma_c = \frac{\sigma_H}{2} = -5.998$ ksi

$R = \sqrt{\left(\frac{\sigma_H}{2}\right)^2 + \tau_H^2} = 6.677$ ksi

$\sigma_a = \sigma_c + R = 0.579$ ksi

$\sigma_b = \sigma_c - R = -12.58$ ksi

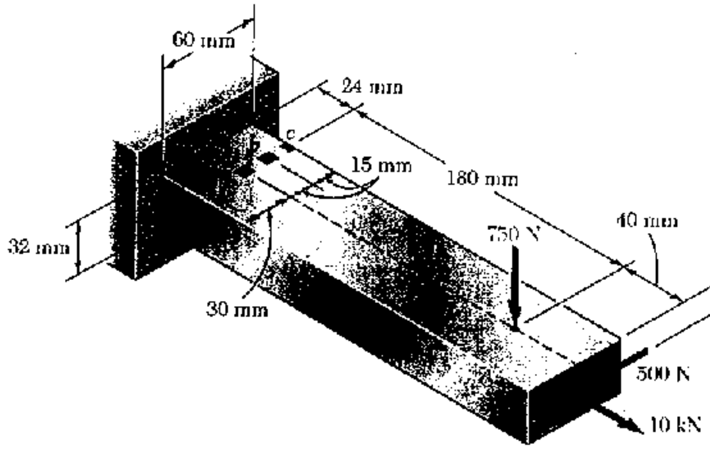
$\tan 2\theta_p = \frac{2\tau_H}{|\sigma_H|} = 0.4497$

$\theta_a = 12.1°, \quad \theta_b = 102.1°$

(b) $\tau_{max} = R = 6.58$ ksi

Problem 8.45

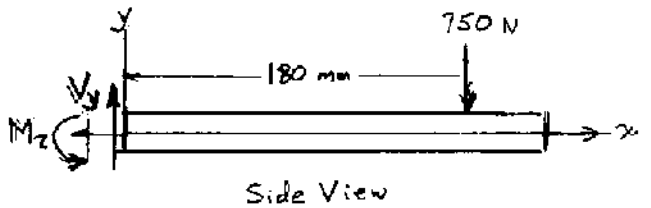
8.45 Three forces are applied to the bar shown. Determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*, (*c*) point *c*.



$A = (60)(32) = 1920 \text{ mm}^2 = 1920 \times 10^{-6} \text{ m}^2$

$I_z = \frac{1}{12}(60)(32)^3 = 163.84 \times 10^3 \text{ mm}^4 = 163.84 \times 10^{-9} \text{ m}^4$

$I_y = \frac{1}{12}(32)(60)^3 = 576 \times 10^3 \text{ mm}^4 = 576 \times 10^{-9} \text{ m}^4$



At the section containing points $\underline{a}$, $\underline{b}$, and $\underline{c}$

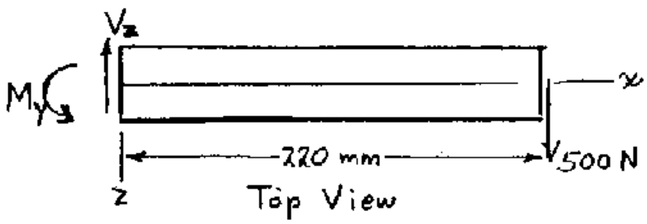
$P = 10 \text{ kN}$

$V_y = 750 \text{ N}, \quad V_z = 500 \text{ N}$

$M_z = (180 \times 10^{-3})(750) = 135 \text{ N}\cdot\text{m}$

$M_y = (220 \times 10^{-3})(500) = 110 \text{ N}\cdot\text{m}$

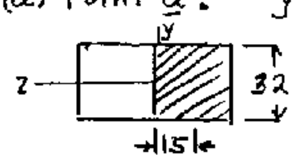
$T = 0$



$\sigma = \frac{P}{A} + \frac{M_z y}{I_z} - \frac{M_y z}{I_y}$

$\tau = \frac{V_z Q}{I_y t}$

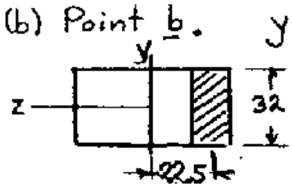
(a) Point $\underline{a}$. $y = 16 \text{ mm}, \ z = 0, \quad Q = A\bar{z} = (32)(30)(15) = 14.4 \times 10^3 \text{ mm}^3$



$\sigma = \frac{10 \times 10^3}{1920 \times 10^{-6}} + \frac{(135)(16 \times 10^{-3})}{163.84 \times 10^{-9}} - 0 = 18.39 \text{ MPa}$

$\tau = \frac{(500)(14.4 \times 10^{-6})}{(576 \times 10^{-9})(32 \times 10^{-3})} = 0.391 \text{ MPa}$

(b) Point $\underline{b}$. $y = 16 \text{ mm}, \ z = -15 \text{ mm}, \quad Q = A\bar{z} = (32)(15)(22.5) = 10.8 \times 10^3 \text{ mm}^3$



$\sigma = \frac{10 \times 10^3}{1920 \times 10^{-6}} + \frac{(135)(16 \times 10^{-3})}{163.84 \times 10^{-9}} - \frac{(110)(-15 \times 10^{-3})}{576 \times 10^{-9}} = 21.3 \text{ MPa}$

$\tau = \frac{(500)(10.8 \times 10^{-6})}{(576 \times 10^{-9})(32 \times 10^{-3})} = 0.293 \text{ MPa}$

(c) Point $\underline{c}$. $y = 16 \text{ mm}, \ z = -30 \text{ mm}, \quad Q = 0$

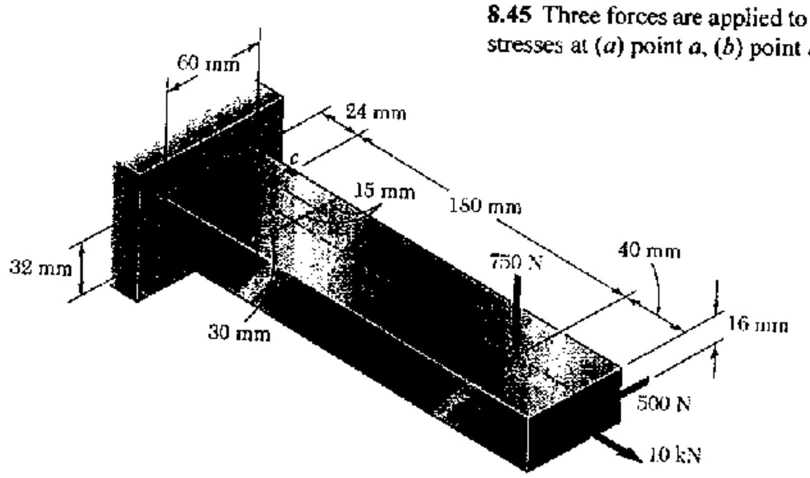
$\sigma = \frac{10 \times 10^3}{1920 \times 10^{-6}} + \frac{(135)(16 \times 10^{-3})}{163.84 \times 10^{-9}} - \frac{(110)(-30 \times 10^{-3})}{576 \times 10^{-9}} = 24.1 \text{ MPa}$

$\tau = 0$

Problem 8.46

8.46 Solve Prob. 8.45, assuming that the 750-N force is directed vertically upward.

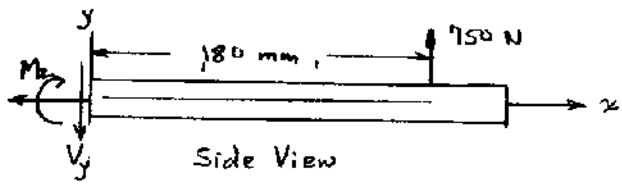
8.45 Three forces are applied to the bar shown. Determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*, (*c*) point *c*.



$A = (60)(32) = 1920\ \text{mm}^2 = 1920 \times 10^{-6}\ \text{m}^2$

$I_z = \frac{1}{12}(60)(32)^3 = 163.84 \times 10^3\ \text{mm}^4 = 163.84 \times 10^{-9}\ \text{m}^4$

$I_y = \frac{1}{12}(32)(60)^3 = 576 \times 10^3\ \text{mm}^4 = 576 \times 10^{-9}\ \text{m}^4$

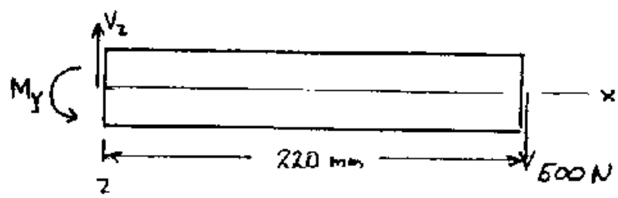


Side View

At the section containing points $\underline{a}$, $\underline{b}$, and $\underline{c}$

$P = 10\ \text{kN}, \quad T = 0$

$V_y = 750\ \text{N}, \quad V_z = 500\ \text{N}$



Top View

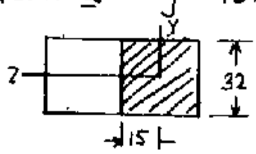
$M_z = (180 \times 10^{-3})(750) = 135\ \text{N}\cdot\text{m}$

$M_y = (220 \times 10^{-3})(500) = 110\ \text{N}\cdot\text{m}$

$\sigma = \frac{P}{A} - \frac{M_z y}{I_z} - \frac{M_y z}{I_y}$

$\tau = \frac{V_z Q}{I_z t}$

(a) Point $\underline{a}$ $\quad y = 16\ \text{mm}, \quad z = 0, \quad Q = A\bar{z} = (32)(30)(15) = 14.4 \times 10^3\ \text{mm}^3$



$$\sigma = \frac{10 \times 10^3}{1920 \times 10^{-6}} - \frac{(135)(16 \times 10^{-3})}{163.84 \times 10^{-9}} - 0 = -7.98\ \text{MPa}$$

$$\tau = \frac{(500)(14.4 \times 10^{-6})}{(163.84 \times 10^{-9})(32 \times 10^{-3})} = 0.391\ \text{MPa}$$

(b) Point $\underline{b}$ $\quad y = 16\ \text{mm}, \quad z = -15\ \text{mm}, \quad Q = A\bar{z} = (32)(15)(22.5) = 10.8 \times 10^3\ \text{mm}^3$

$$\sigma = \frac{10 \times 10^3}{1920 \times 10^{-6}} - \frac{(135)(16 \times 10^{-3})}{163.84 \times 10^{-9}} - \frac{(110)(-15 \times 10^{-3})}{576 \times 10^{-9}} = -5.11\ \text{MPa}$$

$$\tau = \frac{(500)(10.8 \times 10^{-6})}{(163.84 \times 10^{-9})(32 \times 10^{-3})} = 0.293\ \text{MPa}$$

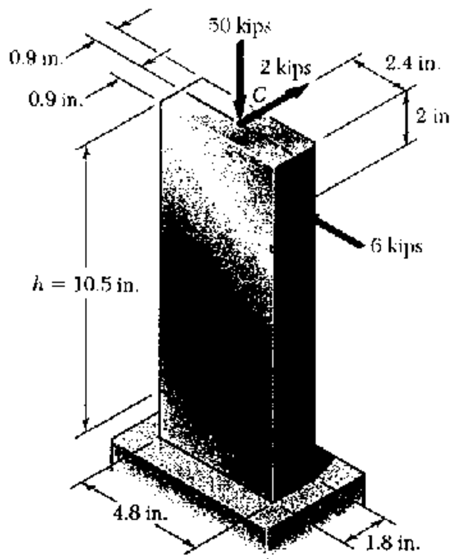
(c) Point $\underline{c}$ $\quad y = 16\ \text{mm}, \quad z = -30\ \text{mm}, \quad Q = 0$

$$\sigma = \frac{10 \times 10^3}{1920 \times 10^{-6}} - \frac{(135)(16 \times 10^{-3})}{163.84 \times 10^{-9}} - \frac{(110)(-30 \times 10^{-3})}{576 \times 10^{-9}} = -2.25\ \text{MPa}$$

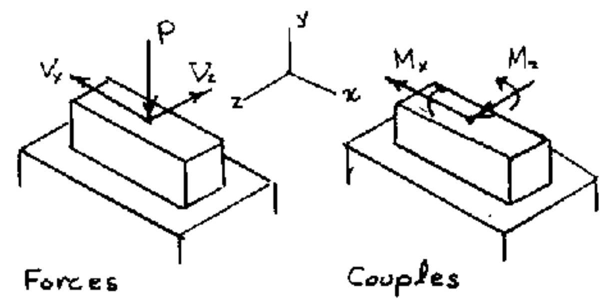
$$\tau = 0$$

Problem 8.47

8.47 Three forces are applied to the bar shown. Determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*, (*c*) point *c*.



Calculate forces and couples at section containing points $\underline{a}$, $\underline{b}$, and $\underline{c}$. $h = 10.5$ in.



$P = 50$ kips $\qquad V_x = 6$ kips $\qquad V_z = 2$ kips

$M_z = (10.5 - 2)(6) = 51$ kip·in

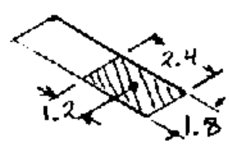
$M_x = (10.5)(2) = 21$ kip·in

Section properties. $A = (1.8)(4.8) = 8.64\ \text{in}^2$

$I_x = \frac{1}{12}(4.8)(1.8)^3 = 2.3328\ \text{in}^4 \qquad I_z = \frac{1}{12}(1.8)(4.8)^3 = 16.5888\ \text{in}^4$

Stresses $\qquad \sigma = -\dfrac{P}{A} + \dfrac{M_z x}{I_z} + \dfrac{M_x z}{I_x} \qquad \tau = \dfrac{V_x Q}{I_z t}$

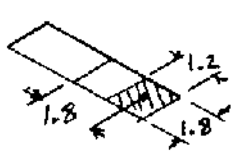
(a) Point $\underline{a}$: $\quad x = 0,\ z = 0.9$ in., $\quad Q = (1.8)(2.4)(1.2) = 5.184\ \text{in}^3$



$$\sigma = -\frac{50}{8.64} + 0 + \frac{(21)(0.9)}{2.3328} = 2.31\ \text{ksi}$$

$$\tau = \frac{(6)(5.184)}{(16.5888)(1.8)} = 1.042\ \text{ksi}$$

(b) Point $\underline{b}$: $\quad x = 1.2$ in., $z = 0.9$ in., $\quad Q = (1.8)(1.2)(1.8) = 3.888\ \text{in}^3$



$$\sigma = -\frac{50}{8.64} + \frac{(51)(1.2)}{16.5888} + \frac{(21)(0.9)}{2.3328} = 6.00\ \text{ksi}$$

$$\tau = \frac{(6)(3.888)}{(16.5888)(1.8)} = 0.781\ \text{ksi}$$

(c) Point $\underline{c}$: $\quad x = 2.4$ in., $z = 0.9$ in., $\quad Q = 0$

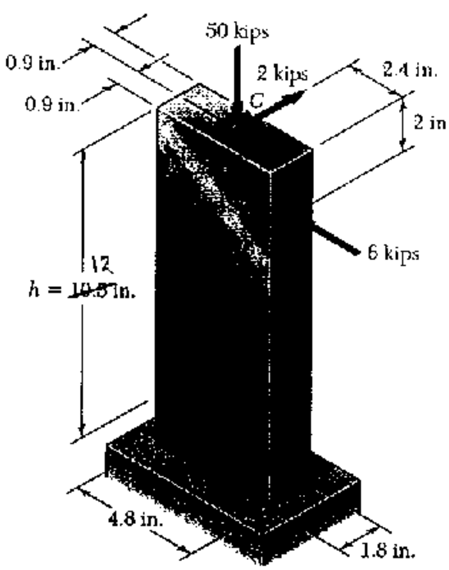
$$\sigma = -\frac{50}{8.64} + \frac{(51)(2.4)}{16.5888} + \frac{(21)(0.9)}{2.3328} = 9.69\ \text{ksi}$$

$$\tau = 0$$

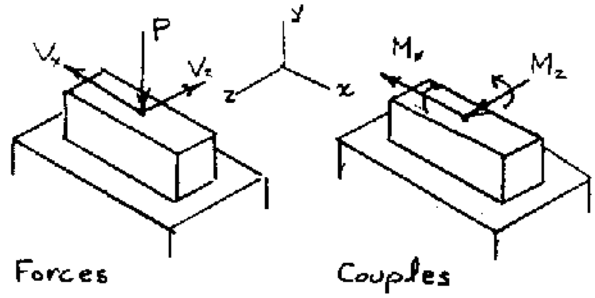
Problem 8.48

8.48 Solve Prob. 8.47, assuming that $h = 12$ in.

8.47 Three forces are applied to the bar shown. Determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*, (*c*) point *c*.



Calculate forces and couples at section containing points $\underline{a}$, $\underline{b}$, and $\underline{c}$. $h = 12$ in



$P = 50$ kips $\quad V_x = 6$ kips $\quad V_z = 2$ kips

$M_z = (12 - 2)(6) = 60$ kip·in

$M_x = (12)(2) = 24$ kip·in

Section properties. $A = (1.8)(4.8) = 8.64\ in^2$

$I_x = \frac{1}{12}(4.8)(1.8)^3 = 2.3328\ in^4 \qquad I_z = \frac{1}{12}(1.8)(4.8)^3 = 16.5888\ in^4$

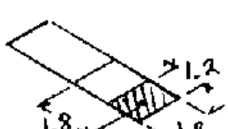
Stresses $\quad \sigma = -\frac{P}{A} + \frac{M_z x}{I_z} + \frac{M_x z}{I_x} \qquad \tau = \frac{V_x Q}{I_z t}$

(a) Point $\underline{a}$: $\quad x = 0,\ z = 0.9$ in., $\quad Q = (1.8)(2.4)(1.2) = 5.184\ in^3$

$$\sigma = -\frac{50}{8.64} + 0 + \frac{(24)(0.9)}{2.3328} = 3.47 \text{ ksi}$$

$$\tau = \frac{(6)(5.184)}{(16.5888)(1.8)} = 1.042 \text{ ksi}$$

(b) Point $\underline{b}$: $\quad x = 1.2$ in. $\ z = 0.9$ in, $\quad Q = (1.8)(1.2)(1.8) = 3.888\ in^3$



$$\sigma = -\frac{50}{8.64} + \frac{(60)(1.2)}{16.5888} + \frac{(24)(0.9)}{2.3328} = 7.81 \text{ ksi}$$

$$\tau = \frac{(6)(3.888)}{(16.5888)(1.8)} = 0.781 \text{ ksi}$$

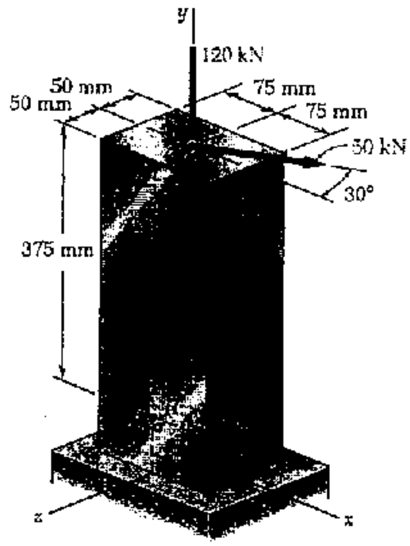
(c) Point $\underline{c}$: $\quad x = 2.4$ in., $\ z = 0.9$ in., $\quad Q = 0$

$$\sigma = -\frac{50}{8.64} + \frac{(60)(2.4)}{16.5888} + \frac{(24)(0.9)}{2.3328} = 12.15 \text{ ksi}$$

$$\tau = 0$$

Problem 8.49

8.49 For the post and loading shown, determine the principal stresses, principal planes, and maximum shearing stress at point H.



Components of force at point C.

$F_x = 50 \cos 30° = 43.301$ kN

$F_z = -50 \sin 30° = -25$ kN, $\quad F_y = -120$ kN

Section forces and couples at the section containing points H and K.

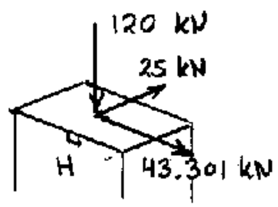
$P = 120$ kN (compression)

$V_x = 43.301$ kN, $\quad V_z = -25$ kN

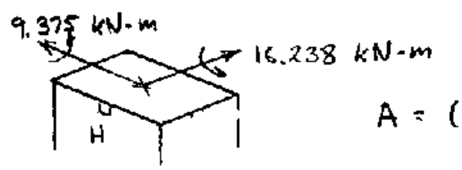
$M_x = -(25)(0.375) = -9.375$ kN-m.

$M_y = 0$

$M_z = -(43.301)(0.375) = -16.238$ kN-m



Forces



Couples

$A = (100)(150) = 15 \times 10^3$ mm^2
$= 15 \times 10^{-3}$ m^2

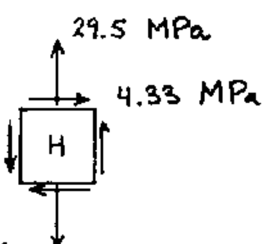
$I_x = \frac{1}{12}(150)(100)^3 = 12.5 \times 10^6$ mm^4
$= 12.5 \times 10^{-6}$ m^4

Stresses at point H

$$\sigma_H = -\frac{P}{A} - \frac{M_x z}{I_x} = -\frac{(120\times10^3)}{15\times10^{-3}} - \frac{(-9.375\times10^3)(50\times10^{-3})}{12.5\times10^{-6}} = 29.5 \text{ MPa}$$

$$\tau_H = \frac{3}{2}\frac{V_x}{A} = \frac{3}{2}\frac{43.301\times10^3}{15\times10^{-3}} = 4.33 \text{ MPa}$$

Use Mohr's circle.



$\sigma_c = \frac{1}{2}\sigma_H = 14.75$ MPa

$R = \sqrt{\left(\frac{29.5}{2}\right)^2 + 4.33^2} = 15.37$ MPa

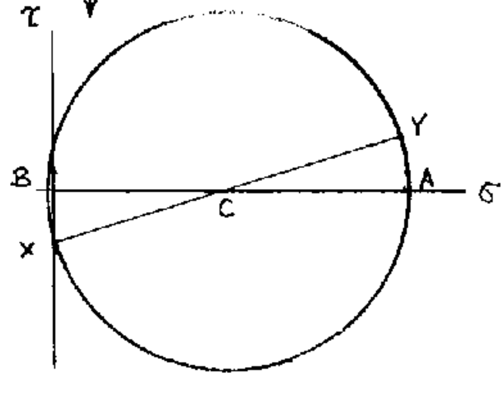
$\sigma_a = \sigma_c + R = 30.1$ MPa ◀

$\sigma_b = \sigma_c - R = -0.62$ MPa ◀

$\tan 2\theta_p = \frac{2\tau_H}{-\sigma_H} = -0.2936$

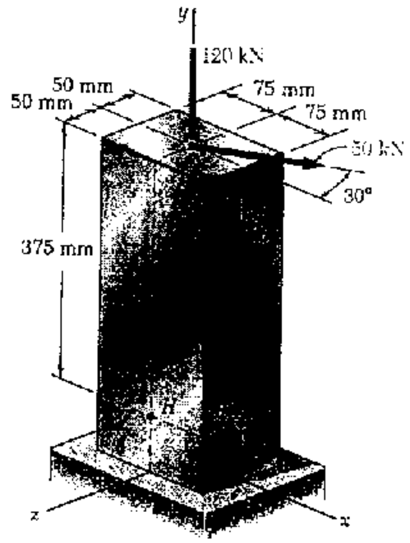
$\theta_a = -8.2°$ $\quad \theta_b = 81.8°$ ◀

$\tau_{max} = R = 15.37$ MPa ◀



Problem 8.50

8.50 For the post and loading shown, determine the principal stresses, principal planes, and maximum shearing stress at point K.



Components of force at point C.

$F_x = 50 \cos 30° = 43.301$ kN

$F_z = -50 \sin 30° = -25$ kN $\qquad F_y = -120$ kN

Section forces and couples at the section containing points H and K.

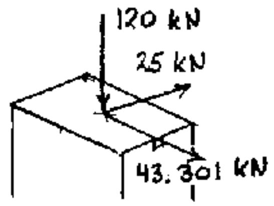
$P = 120$ kN (compression)

$V_x = 43.301$ kN, $\qquad V_z = -25$ kN

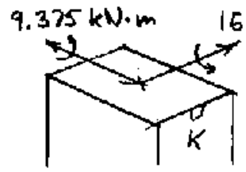
$M_x = -(25)(0.375) = -9.375$ kN·m

$M_y = 0$

$M_z = -(43.301)(0.375) = -16.238$ kN·m



Forces



Couples

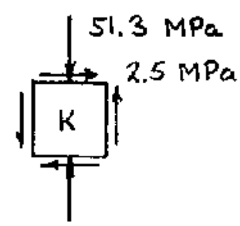
$A = (100)(150) = 15 \times 10^3$ mm^2 $= 15 \times 10^{-3}$ m^2

$I_z = \frac{1}{12}(100)(150)^3 = 28.125 \times 10^6$ mm^4 $= 28.125 \times 10^{-6}$ m^4

Stresses at point K.

$$\sigma_K = -\frac{P}{A} + \frac{M_z x}{I_z} = -\frac{120 \times 10^3}{15 \times 10^{-3}} + \frac{(-16.238 \times 10^3)(75 \times 10^{-3})}{28.125 \times 10^{-6}} = -51.3 \text{ MPa}$$

$$\tau_K = \frac{3}{2}\frac{V_z}{A} = \frac{3}{2}\frac{25 \times 10^3}{15 \times 10^{-3}} = 2.5 \text{ MPa}$$



Use Mohr's circle.

$\sigma_c = \frac{1}{2}\sigma_K = -25.65$ MPa

$R = \sqrt{\left(\frac{51.3}{2}\right)^2 + (2.5)^2} = 25.77$ MPa

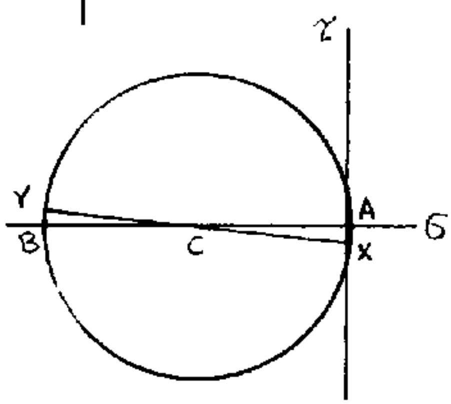
$\sigma_a = \sigma_c + R = 0.12$ MPa ◄

$\sigma_b = \sigma_c - R = -51.4$ MPa ◄

$\tan 2\theta_p = \frac{2\tau_K}{-\sigma_K} = 0.09747$

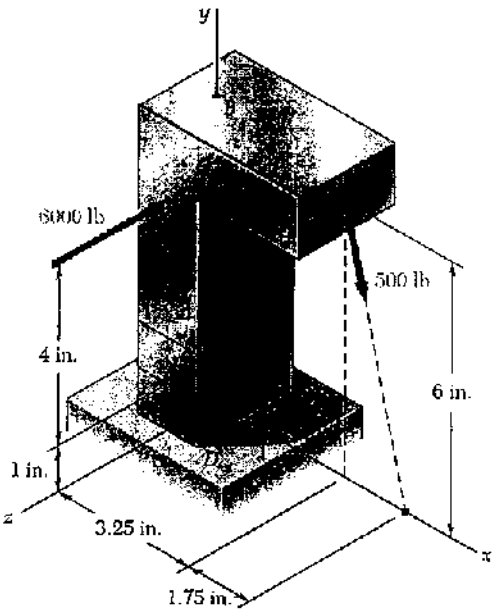
$\theta_a = 2.8°$ $\qquad \theta_b = 92.8°$ ◄

$\tau_{max} = R = 25.8$ MPa ◄



Problem 8.51

8.51 Two forces are applied to the small post *BD* as shown. Knowing that the vertical portion of the post has a cross section of 1.5 × 2.4 in., determine the principal stresses, principal planes, and maximum shearing stress at point *H*.



Components of 500 lb-force.

$$F_x = \frac{(500)(1.75)}{6.25} = 140 \text{ lb.}$$

$$F_y = -\frac{(500)(6)}{6.25} = -480 \text{ lb.}$$

Moment arm of 500 lb force.

$$\vec{r} = 3.25\,\vec{i} + (6-1)\,\vec{j}$$

Moment of 500 lb-force.

$$\vec{M} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3.25 & 5 & 0 \\ 140 & -480 & 0 \end{vmatrix} = -2260\,\vec{k} \text{ lb}\cdot\text{in}$$

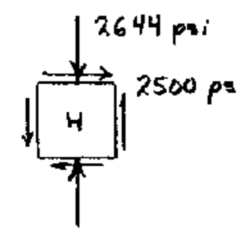
At the section containing point H, $P = -480$ lb. $V_x = 140$ lb.

$V_z = -6000$ lb., $M_z = -2260$ lb·in, $M_x = -(4)(6000) = -24000$ lb·in.

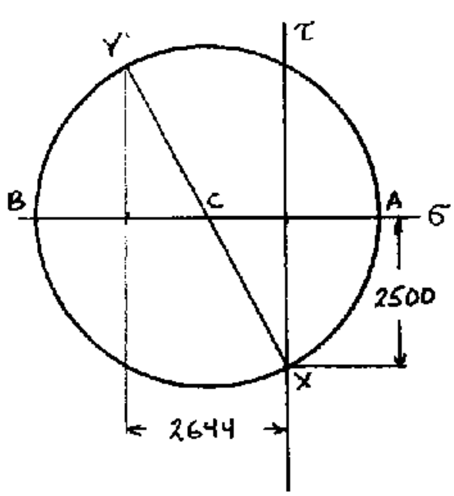
$A = (1.5)(2.4) = 3.6 \text{ in}^2$ $\quad I_z = \frac{1}{12}(2.4)(1.5)^3 = 0.675 \text{ in}^4$

$$\sigma_H = \frac{P}{A} + \frac{M_z x}{I_z} = -\frac{480}{3.6} + \frac{(-2260)(0.75)}{0.675} = -2644 \text{ psi}$$

$$\tau_H = \frac{3}{2}\frac{V_z}{A} = \frac{3}{2}\frac{6000}{3.6} = 2500 \text{ psi}$$



Use Mohr's circle.



$$\sigma_c = -\frac{2644}{2} = -1322 \text{ psi}$$

$$R = \sqrt{\left(\frac{2644}{2}\right)^2 + (2500)^2} = 2828 \text{ psi}$$

$$\sigma_a = \sigma_c + R = 1506 \text{ psi} \quad \blacktriangleleft$$

$$\sigma_b = \sigma_c - R = -4150 \text{ psi} \quad \blacktriangleleft$$

$$\tan 2\theta_p = \frac{2\tau_H}{|\sigma_H|} = \frac{(2)(2500)}{2644} = 1.891$$

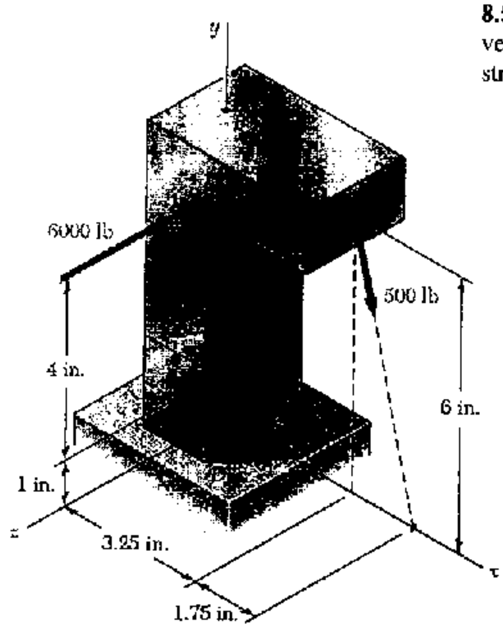
$$\theta_a = 31.1^\circ, \quad \theta_b = 121.1^\circ \quad \blacktriangleleft$$

$$\tau_{max} = R = 2828 \text{ psi} \quad \blacktriangleleft$$

Problem 8.52

8.52 Solve Prob. 8.51, assuming that the magnitude of the 6000-lb force is reduced to 1500 lb.

8.51 Two forces are applied to the small post *BD* as shown. Knowing that the vertical portion of the post has a cross section of 1.5 × 2.4 in., determine the principal stresses, principal planes, and maximum shearing stress at point *H*.



Components of 500 lb. force.

$$F_x = \frac{(500)(1.75)}{6.25} = 140 \text{ lb}$$

$$F_y = -\frac{(500)(6)}{6.25} = -480 \text{ lb.}$$

Moment arm of 500 lb. force.

$$\vec{r} = 3.25\,\vec{i} + (6-1)\,\vec{j}$$

$$\vec{M} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3.25 & 5 & 0 \\ 140 & -480 & 0 \end{vmatrix} = -2260\,\vec{k} \text{ lb}\cdot\text{in}$$

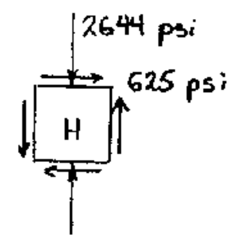
At the section containing point H, $\quad P = -480$ lb. $\quad V_x = 140$ lb.

$V_z = -1500$ lb, $\quad M_z = -2260$ lb·in, $\quad M_x = -(4)(1500) = -6000$ lb·in.

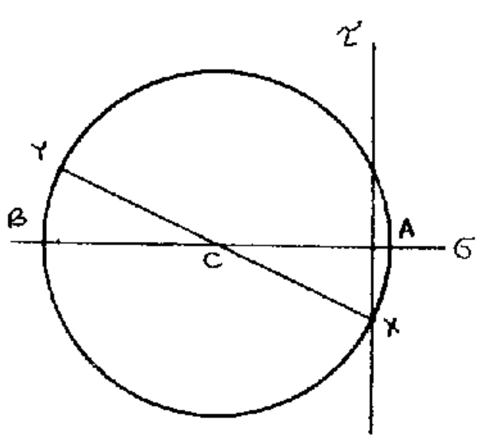
$$A = (1.5)(2.4) = 3.6 \text{ in}^2 \qquad I_z = \tfrac{1}{12}(2.4)(1.5)^3 = 0.675 \text{ in}^4$$

$$\sigma_H = \frac{P}{A} + \frac{M_z x}{I_z} = -\frac{480}{3.6} + \frac{(-2260)(0.75)}{0.675} = -2644 \text{ psi}$$

$$\tau_H = \frac{3}{2}\frac{V_z}{A} = \frac{3}{2}\frac{1500}{3.6} = 625 \text{ psi}$$



Use Mohr's circle.



$$\sigma_c = \tfrac{1}{2}\sigma_H = -1322 \text{ psi}$$

$$R = \sqrt{\left(\frac{2644}{2}\right)^2 + (625)^2} = 1462 \text{ psi}$$

$$\sigma_a = \sigma_c + R = 140 \text{ psi}$$

$$\sigma_b = \sigma_c - R = -2784 \text{ psi}$$

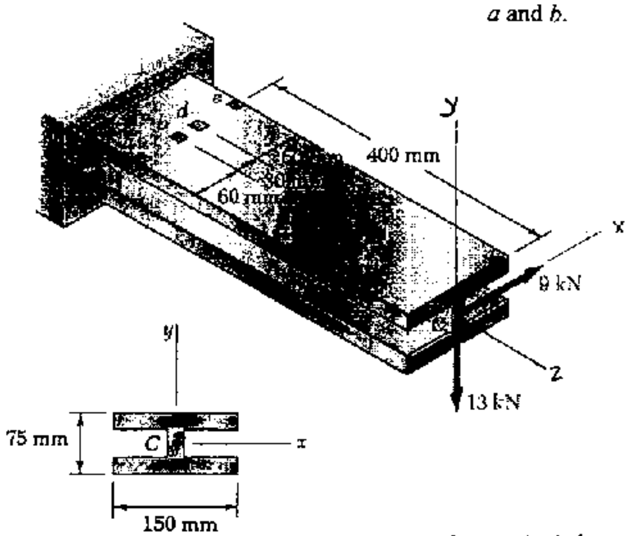
$$\tan 2\theta_p = \frac{2\tau_H}{|\sigma_H|} = \frac{(2)(625)}{2644} = 0.4728$$

$$\theta_a = 12.7^\circ \qquad \theta_b = 102.7^\circ$$

$$\tau_{max} = R = 1462 \text{ psi}$$

Problem 8.53

8.53 Three steel plates, each 13 mm thick, are welded together to form a cantilever beam. For the loading shown, determine the normal and shearing stresses at points a and b.



t = 13 mm

Equivalent force-couple system at section containing points a and b.

$F_x = 9\ \text{kN}, \quad F_y = -13\ \text{kN}, \quad F_z = 0$

$M_x = (0.400)(13\times10^3) = 5200\ \text{N}\cdot\text{m}$

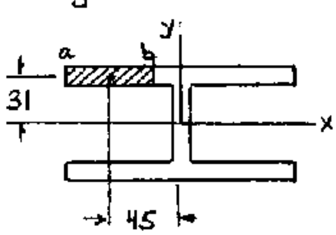
$M_y = 0.400)(9\times10^3) = 3600\ \text{N}\cdot\text{m}$

$M_z = 0$

$A = (2)(150)(13) + (13)(75-26) = 4537\ \text{mm}^2 = 4537\times10^{-6}\ \text{m}^2$

$I_x = 2\left[\frac{1}{12}(150)(13)^3 + (150)(13)(37.5-6.5)^2\right] + \frac{1}{12}(13)(75-26)^3 = 3.9303\times10^6\ \text{mm}^4 = 3.9303\times10^{-6}\ \text{m}^4$

$I_y = 2\cdot\frac{1}{12}(13)(150)^3 + \frac{1}{12}(75-26)(13)^3 = 7.3215\times10^6\ \text{mm}^4 = 7.3215\times10^{-6}\ \text{m}^4$



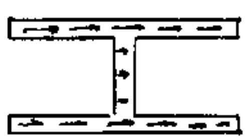
For point $\underline{a}$, $\quad Q_x = 0 \qquad Q_y = 0$

For point $\underline{b}$, $\quad A^* = (60)(13) = 780\ \text{mm}^2$

$\bar{x} = -45\ \text{mm} \qquad \bar{y} = 31\ \text{mm}$

$Q_x = A^*\bar{y} = 24.18\times10^3\ \text{mm}^3 = 24.18\times10^{-6}\ \text{m}^3$

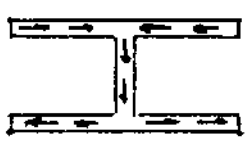
$Q_y = A^*\bar{x} = -35.1\times10^{-3}\ \text{mm}^3 = -35.1\times10^{-6}\ \text{m}^3$



At point $\underline{a}$, $\sigma_a = \dfrac{M_x y}{I_x} - \dfrac{M_y x}{I_y}$

$$= \frac{(5200)(37.5\times10^{-3})}{3.9303\times10^{-6}} - \frac{(3600)(-75\times10^{-3})}{7.3215\times10^{-6}} = 86.5\ \text{MPa}$$

$$\tau_a = 0$$



At point $\underline{b}$, $\sigma_b = \dfrac{M_x y}{I_x} - \dfrac{M_y x}{I_y}$

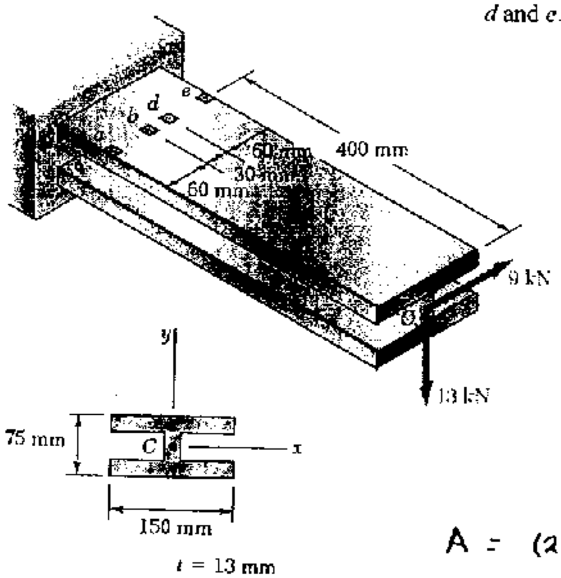
$$= \frac{(5200)(37.5\times10^{-3})}{3.9303\times10^{-6}} - \frac{(3600)(-15\times10^{-3})}{7.3215\times10^{-6}} = 57.0\ \text{MPa}$$

$$\tau_b = \frac{|V_x||Q_y|}{I_y t} + \frac{|V_y||Q_x|}{I_x t} = \frac{(9\times10^3)(35.1\times10^{-6})}{(7.3215\times10^{-6})(13\times10^{-3})} + \frac{(13\times10^3)(24.18\times10^{-6})}{(3.9303\times10^{-6})(13\times10^{-3})}$$

$$= 3.32\ \text{MPa} + 6.15\ \text{MPa} = 9.47\ \text{MPa}$$

Problem 8.54

8.54 Three steel plates, each 13 mm thick, are welded together to form a cantilever beam. For the loading shown, determine the normal and shearing stresses at points d and e.



Equivalent force-couple system at section containing points a and b.

$F_x = 9\text{ kN}, \quad F_y = -13\text{ kN}, \quad F_z = 0$

$M_x = (0.400)(13\times10^3) = 5200\ \text{N}\cdot\text{m}$

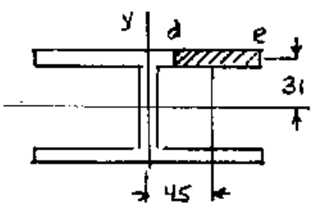
$M_y = (0.400)(9\times10^3) = 3600\ \text{N}\cdot\text{m}$

$M_z = 0$

$A = (2)(150)(13) + (13)(75-26) = 4537\ \text{mm}^2 = 4537\times10^{-6}\ \text{m}^2$

$I_x = 2\left[\frac{1}{12}(150)(13)^3 + (150)(13)(37.5-6.5)^2\right] + \frac{1}{12}(13)(75-26)^3 = 3.9303\times10^6\ \text{mm}^4 = 3.9303\times10^{-6}\ \text{m}^4$

$I_y = 2\left[\frac{1}{12}(13)(150)^3\right] + \frac{1}{12}(75-26)(13)^3 = 7.3215\times10^6\ \text{mm}^4 = 7.3215\times10^{-6}\ \text{m}^4$

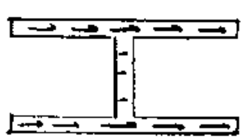


For point $\underline{d}$, $\quad A^* = (60)(13) = 780\ \text{mm}^2$

$\bar{x} = 45\text{ mm} \qquad \bar{y} = 31\text{ mm}$

$Q_x = A^*\bar{y} = 24.18\times10^3\ \text{mm}^3 = 24.18\times10^{-6}\ \text{m}^3$

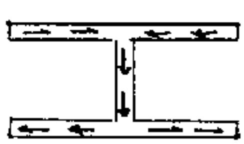
$Q_y = A^*\bar{x} = 35.1\times10^3\ \text{mm}^3 = 35.1\times10^{-6}\ \text{m}^3$



For point $\underline{e}$, $\quad Q_x = 0 \quad , \quad Q_y = 0$

At point $\underline{d}$, $\quad \sigma_d = \dfrac{M_x y}{I_x} - \dfrac{M_y x}{I_y}$

$= \dfrac{(5200)(37.5\times10^{-3})}{3.9303\times10^{-6}} - \dfrac{(3600)(15\times10^{-3})}{7.3215\times10^{-6}} = 42.2\ \text{MPa}$



Due to V_x: $\quad \tau_d = \dfrac{|V_x||Q_y|}{I_y t} = \dfrac{(9000)(35.1\times10^{-6})}{(7.3215\times10^{-6})(13\times10^{-3})} = 3.32\ \text{MPa} \rightarrow$

Due to V_y: $\quad \tau_d = \dfrac{|V_y||Q_x|}{I_x t} = \dfrac{(13000)(24.18\times10^{-6})}{(3.9303\times10^{-6})(13\times10^{-3})} = 6.15\ \text{MPa} \leftarrow$

Net $\qquad \tau_d = 2.83\ \text{MPa}$

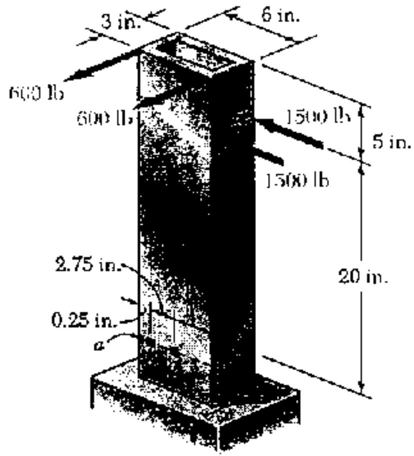
At point $\underline{e}$, $\quad \sigma_e = \dfrac{M_x y}{I_x} - \dfrac{M_y x}{I_y} = \dfrac{(5200)(37.5\times10^{-3})}{3.9303\times10^{-6}} - \dfrac{(3600)(75\times10^{-3})}{7.3215\times10^{-6}}$

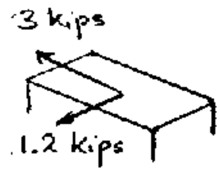
$= 12.74\ \text{MPa}$

$\tau_e = 0$

Problem 8.55

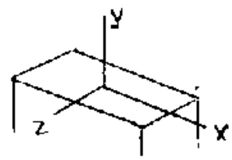
8.55 Knowing that the structural tube shown has a uniform wall thickness of 0.25 in., determine the normal and shearing stresses at the three points indicated.





Shear

Bending moment



$b_o = 6$ in. $\quad b_i = b_o - 2t = 5.5$ in.
$h_o = 3$ in. $\quad h_i = h_o - 2t = 2.5$ in.

$I_x = \frac{1}{12}(b_o h_o^3 - b_i h_i^3) = 6.3385 \text{ in}^4 \qquad I_z = \frac{1}{12}(h_o b_o^3 - h_i b_i^3) = 19.3385 \text{ in}^4$

Normal stresses

$$\sigma = \frac{M_z x}{I_z} - \frac{M_x z}{I_x}$$

$$\frac{(60)(-3)}{19.3385} - \frac{(30)(1.5)}{6.3385} = -16.41 \text{ ksi}$$

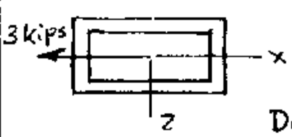
$$\frac{(60)(-2.75)}{19.3385} - \frac{(30)(1.5)}{6.3385} = -15.63 \text{ ksi}$$

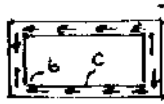
$$\frac{(60)(0)}{19.3385} - \frac{(30)(1.5)}{6.3385} = -7.10 \text{ ksi}$$

Shearing stresses

Point $\underline{a}$ is an outside corner; $\tau_a = 0$

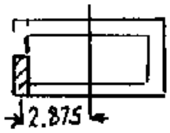
Direction of shearing stresses





Due to V_x

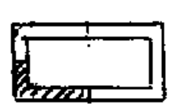
At point $\underline{b}$



$Q_{zb} = (1.5)(0.25)(2.875) = 1.0781 \text{ in}^3$

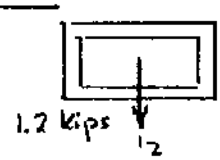
$$\tau_{b,V_x} = \frac{V_x Q_z}{I_z t} = \frac{(3)(1.0781)}{(19.3385)(0.25)} = 0.669 \text{ ksi}$$

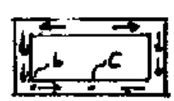
At point $\underline{c}$



$Q_{zc} = Q_{zb} + (2.75)(0.25)(\frac{2.75}{2})$
$= 2.0234 \text{ in}^3$

$$\tau_{c,V_x} = \frac{V_x Q_z}{I_z t} = \frac{(3)(2.0234)}{(19.3385)(0.25)} = 1.256 \text{ ksi}$$





Due to V_z

At point $\underline{b}$

$Q_{xb} = (2.75)(0.25)(1.375) = 0.9453 \text{ in}^3$

$$\tau_{b,V_z} = \frac{V_z Q_x}{I_x t} = \frac{(1.2)(0.9453)}{(6.3385)(0.25)} = 0.716 \text{ ksi}$$

At point $\underline{c}$ (symmetry axis)

$\tau_{c,V_z} = 0$

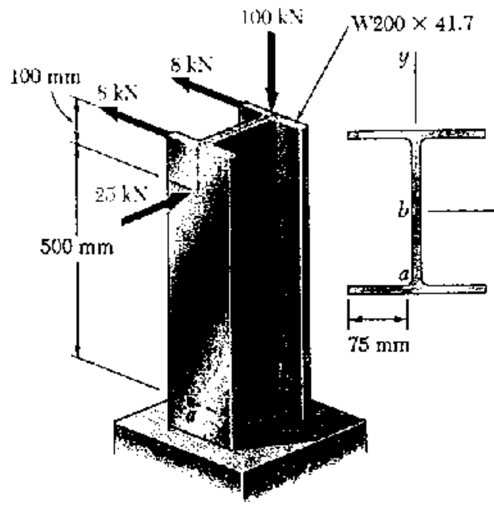
Net shearing stress at points $\underline{b}$ and $\underline{c}$

$\tau_b = 0.716 - 0.669 = 0.047$ ksi

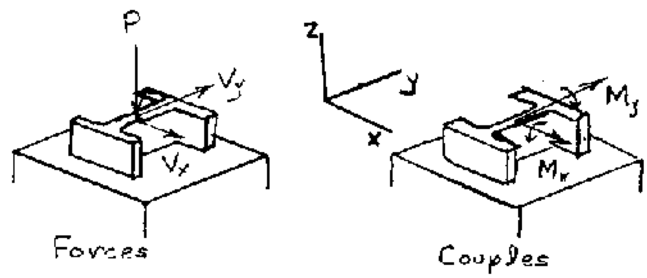
$\tau_c = 1.256$ ksi

Problem 8.56

8.56 Four forces are applied to a W 200 × 41.7 rolled beam as shown. Determine the principal stresses and maximum shearing stress at point *a*.



Calculate forces and couples at section containing point $\underline{a}$.



$P = 100$ kN $\quad V_x = -16$ kN $\quad V_y = 25$ kN

$$M_x = -(500\times10^{-3})(25\times10^{3}) - \left(\tfrac{205}{2}\times10^{-3}\right)(100\times10^{3}) = -122.75\times10^{3}\ \text{N}\cdot\text{m}$$

$$M_y = (500+100)(10^{-3})(16\times10^{3}) = 9.6\times10^{3}\ \text{N}\cdot\text{m}$$

Section properties.

$A = 5310\ \text{mm}^2 \qquad d = 205\ \text{mm}$

$b_f = 166\ \text{mm} \qquad t_f = 11.8\ \text{mm}$

$t_w = 7.2\ \text{mm}$

$I_x = 40.9\times10^{6}\ \text{mm}^4 = 40.9\times10^{-6}\ \text{m}^4, \quad I_y = 9.01\times10^{6}\ \text{mm}^4 = 9.01\times10^{-6}\ \text{m}^4$

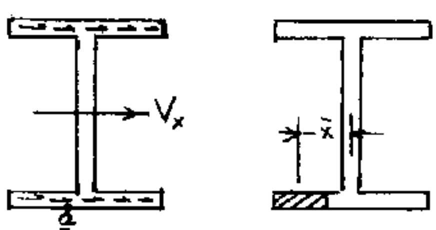
Point $\underline{a}$: $\quad x_a = -\frac{166}{2} + 75 = -8\ \text{mm}, \quad y_a = -\frac{205}{2} = -102.5\ \text{mm}$

$$\sigma_a = -\frac{P}{A} + \frac{M_x y_a}{I_x} + \frac{M_y x_a}{I_y}$$

$$= -\frac{100\times10^{3}}{5310\times10^{-6}} + \frac{(-122.75\times10^{3})(-102.5\times10^{-3})}{40.9\times10^{-6}} + \frac{(9.6\times10^{3})(-8\times10^{-3})}{9.01\times10^{-6}}$$

$$= -18.83\times10^{6} + 57.01\times10^{6} - 8.52\times10^{6} = 29.66\ \text{MPa}$$

Shearing stress at point $\underline{a}$ due to V_x



$A = (75)(11.8) = 885\ \text{mm}^2$

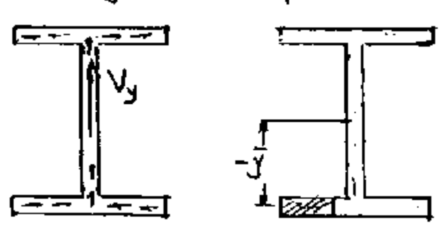
$\bar{x} = -\frac{166}{2} + \frac{75}{2} = -45.5\ \text{mm}$

$Q = -A\bar{x} = 40.2675\times10^{3}\ \text{mm}^3$

$$\tau_{xz} = \frac{V_x Q}{I_y t} = \frac{(-16\times10^{3})(40.2675\times10^{-6})}{(9.01\times10^{-6})(11.8\times10^{-3})}$$

$$= -6.060\ \text{MPa}$$

Shearing stress at point $\underline{a}$ due to V_y



$A = (75)(11.8) = 885\ \text{mm}^2$

$\bar{y} = -\frac{205}{2} + \frac{11.8}{2} = -96.6\ \text{mm}$

$Q = -A\bar{y} = -85.491\times10^{3}\ \text{mm}^3$

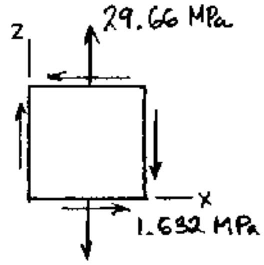
$$\tau_{xz} = -\frac{V_y Q}{I_x t} = -\frac{(25\times10^{3})(-85.491\times10^{-6})}{(40.9\times10^{-6})(11.8\times10^{-3})}$$

$$= 4.428\ \text{MPa}$$

continued

Problem 8.56 continued

Combined shearing stress $\tau_d = -6.060 + 4.428 = -1.632$ MPa



$$\sigma_{ave} = \frac{29.66 + 0}{2} = 14.83 \text{ MPa}$$

$$R = \sqrt{\left(\frac{29.66 - 0}{2}\right)^2 + (1.632)^2} = 14.92 \text{ MPa}$$

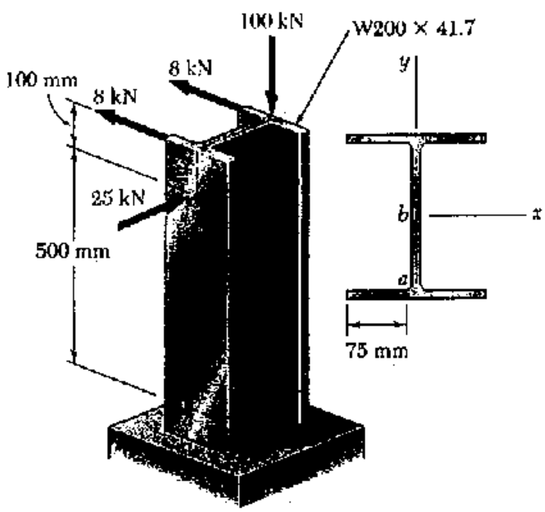
$$\sigma_{max} = \sigma_{ave} + R = 29.75 \text{ MPa}$$

$$\sigma_{min} = \sigma_{ave} - R = -0.09 \text{ MPa}$$

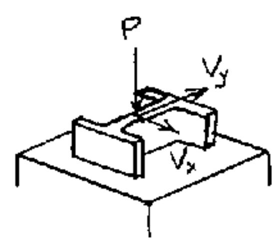
$$\tau_{max} = R = 14.92 \text{ MPa}$$

Problem 8.57

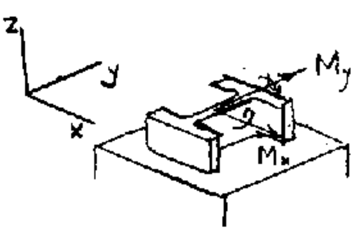
8.57 Four forces are applied to a W 200 × 41.7 rolled beam as shown. Determine the principal stresses and maximum shearing stress at point *b*.



Calculate forces and couples at section containing point b.



Forces



Couples

$P = 100$ kN $\quad V_x = -16$ kN $\quad V_y = 25$ kN

$$M_x = -(500 \times 10^{-3})(25 \times 10^3) - \left(\frac{205}{2} \times 10^{-3}\right)(100 \times 10^3) = -22.75 \times 10^3 \text{ N·m}$$

$$M_y = (500 + 100)(10^{-3})(16 \times 10^3) = 9.6 \times 10^3 \text{ N·m}$$

Section properties.

$A = 5310$ mm² $\quad d = 205$ mm

$b_f = 166$ mm $\quad t_f = 11.8$ mm

$t_w = 7.2$ mm

$I_x = 40.9 \times 10^6 \text{ mm}^4 = 40.9 \times 10^{-6} \text{ m}^4, \quad I_y = 9.01 \times 10^6 \text{ mm}^4 = 9.01 \times 10^{-6} \text{ m}^4$

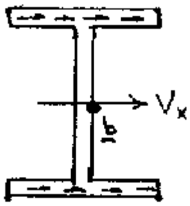
Point b: $\quad x_b = 0, \quad y_b = 0$

$$\sigma_b = -\frac{P}{A} + \frac{M_x y_b}{I_x} + \frac{M_y x_b}{I_y} = -\frac{100 \times 10^3}{5310 \times 10^{-6}} = -18.83 \text{ MPa}$$

continued

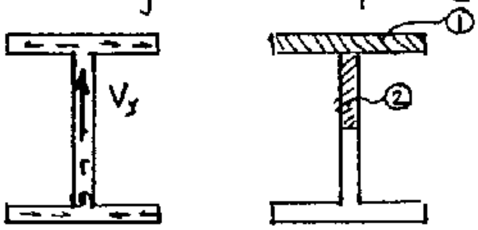
Problem 8.57 continued

Shearing stress at point b due to V_x.



$\tau_{xz} = 0$

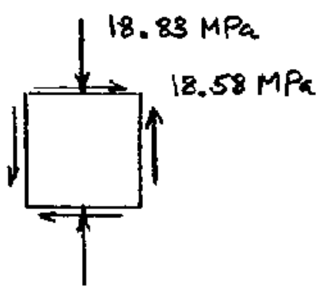
Shearing stress at point b due to V_y



	A (mm²)	$\bar{y}$ (mm)	$A\bar{y}$ (10^3 mm³)
1	1958.8	96.6	189.220
2	653.04	45.35	29.615
Σ			218.835

$$Q = \Sigma A\bar{y} = 218.835 \times 10^3 \text{ mm}^3 = 218.835 \times 10^{-6} \text{ m}^3$$

$$\tau_b = \frac{V_y Q}{I_x t_w} = \frac{(25 \times 10^3)(218.835 \times 10^{-6})}{(40.9 \times 10^{-6})(7.2 \times 10^{-3})} = 18.58 \text{ MPa}$$



$$\sigma_{ave} = \frac{-18.83 + 0}{2} = -9.415 \text{ MPa}$$

$$R = \sqrt{\left(\frac{-18.83 - 0}{2}\right)^2 + (18.58)^2} = 20.829 \text{ MPa}$$

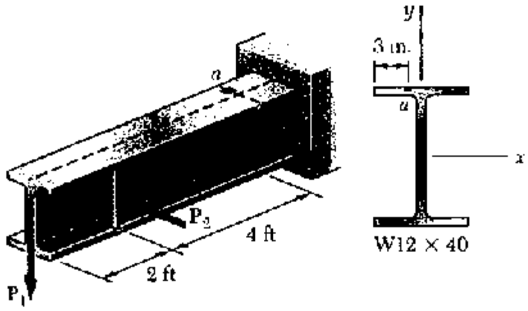
$$\sigma_{max} = \sigma_{ave} + R = 11.41 \text{ MPa}$$ ◄

$$\sigma_{min} = \sigma_{ave} - R = -30.2 \text{ MPa}$$ ◄

$$\tau_{max} = R = 20.8 \text{ MPa}$$ ◄

Problem 8.58

8.58 Two forces $\mathbf{P}_1$ and $\mathbf{P}_2$, are applied as shown in directions perpendicular to the longitudinal axis of a W 12 × 40 beam. Knowing that $P_1 = 5$ kips and $P_2 = 3$ kips, determine the principal stresses and the maximum shearing stress at point a.



At the section containing point $\underline{a}$

$M_x = (6)(5) = 30 \text{ kip·ft} = 360 \text{ kip·in}$

$M_y = -(4)(3) = -12 \text{ kip·ft} = -144 \text{ kip·in}$

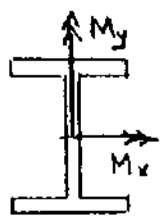
$V_x = -3 \text{ kips} \qquad V_y = -5 \text{ kips}$

For W 12 × 40 rolled steel section

$d = 11.94 \text{ in.} \qquad b_f = 8.005 \text{ in.} \qquad t_f = 0.515 \text{ in.} \qquad t_w = 0.295 \text{ in.}$

$I_x = 310 \text{ in}^4 \qquad I_y = 44.1 \text{ in}^4$

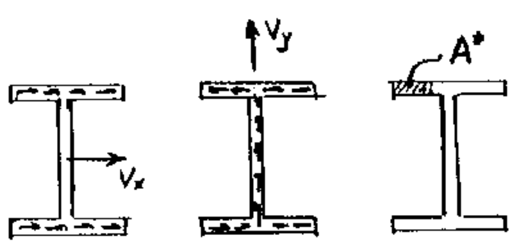
Normal stress at point $\underline{a}$. $\quad x = -\frac{b_f}{2} + 3 = -1.0025 \text{ in.} \quad y = \frac{d}{2} = 5.97 \text{ in}$



$$\sigma_z = \frac{M_x y}{I_x} - \frac{M_y x}{I_y} = \frac{(360)(5.97)}{310} - \frac{(-144)(-1.0025)}{44.1}$$

$$= 6.933 - 3.273 = 3.660 \text{ ksi}$$

Shearing stress at point $\underline{a}$.



$$\tau_{xz} = \frac{V_x A^* \bar{x}}{I_y t_f} - \frac{V_y A^* \bar{y}}{I_x t_f}$$

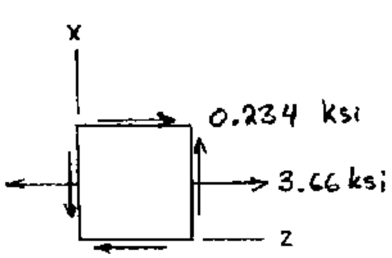
$A^* = (3)(0.515) = 1.545 \text{ in}$

$\bar{x} = -\frac{8.005}{2} + \frac{3}{2} = -2.5025 \text{ in.}$

$\bar{y} = \frac{11.94}{2} - \frac{0.515}{2} = 5.7125 \text{ in.}$

$$\tau_{xz} = \frac{(-3)(1.545)(-2.5025)}{(44.1)(0.515)} - \frac{(-5)(1.545)(5.7125)}{(310)(0.515)}$$

$$= 0.5107 - 0.2764 = 0.234 \text{ ksi}$$



$\sigma_{ave} = \frac{3.66 + 0}{2} = 1.830 \text{ ksi}$

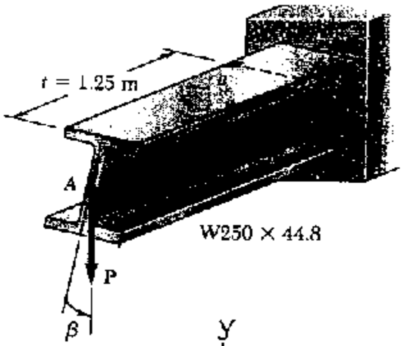
$R = \sqrt{\left(\frac{3.66 - 0}{2}\right)^2 + 0.234^2} = 1.845 \text{ ksi}$

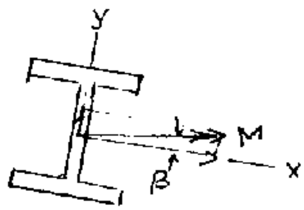
$\sigma_{max} = \sigma_{ave} + R = 1.830 + 1.845 = 3.675 \text{ ksi}$ ◄

$\sigma_{min} = \sigma_{ave} - R = 1.830 - 1.845 = -0.015 \text{ ksi}$ ◄

$\tau_{max} = R = 1.845 \text{ ksi}$ ◄

Problem 8.59





8.59 A vertical force **P** is applied at the center of the free end of cantilever beam AB. (*a*) If the beam is installed with the web vertical ($\beta = 0$) and with its longitudinal axis AB horizontal, determine the magnitude of the force **P** for which the normal stress at point a is ± 120 MPa. (*b*) Solve part *a*, assuming that the beam is installed with $\beta = 3°$.

For W250 × 44.8 rolled steel section

$S_x = 535 \times 10^3\ mm^3 = 535 \times 10^{-6}\ m^3$

$S_y = 95.0 \times 10^3\ mm = 95.0 \times 10^{-6}\ m^3$

At the section containing point a.

$M_x = Pl\cos\beta$, $M_y = Pl\sin\beta$

Stress at a

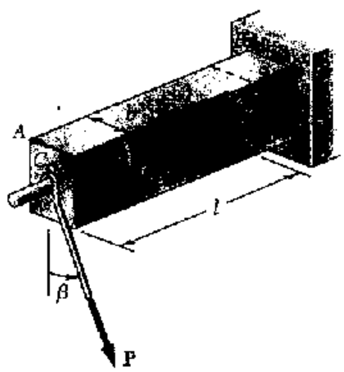
$$\sigma = \frac{M_x}{S_x} + \frac{M_y}{S_y} = \frac{Pl\cos\beta}{S_x} + \frac{Pl\sin\beta}{S_y}$$

Allowable load. $P_{all} = \frac{\sigma_{all}}{l}\left[\frac{\cos\beta}{S_x} + \frac{\sin\beta}{S_y}\right]^{-1}$

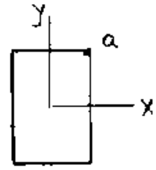
(a) $\beta = 0$: $P_{all} = \frac{120 \times 10^6}{1.25}\left[\frac{1}{535 \times 10^{-6}} + 0\right]^{-1} = 51.4 \times 10^3\ N = 51.4\ kN$ ◄

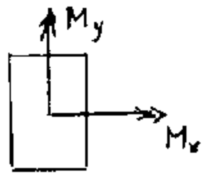
(b) $\beta = 3°$: $P_{all} = \frac{120 \times 10^6}{1.25}\left[\frac{\cos 3°}{535 \times 10^{-6}} + \frac{\sin 3°}{95.0 \times 10^{-6}}\right]^{-1} = 39.7\ kN$ ◄

Problem 8.60



8.60 A force **P** is applied to a cantilever beam by means of a cable attached to a bolt located at the center of the free end of the beam. Knowing that P acts in a direction perpendicular to the longitudinal axis of the beam, determine (*a*) the normal stress at point *a* in terms of P, b, h, l, and β, (*b*) the values of β for which the normal stress at a is zero.





$$I_x = \frac{1}{12} b h^3 \qquad I_y = \frac{1}{12} h b^3$$

$$\sigma = \frac{M_x (h/2)}{I_x} - \frac{M_y (b/2)}{I_y}$$

$$= \frac{6 M_x}{b h^2} - \frac{6 M_y}{h b^2}$$

$$\vec{P} = P \sin\beta \, \vec{i} - P \cos\beta \, \vec{j} \qquad \vec{r} = l \, \vec{k}$$

$$\vec{M} = \vec{r} \times \vec{P} = l \vec{k} \times (P \sin\beta \, \vec{i} - P\cos\beta \, \vec{j}) = Pl \cos\beta \, \vec{i} + Pl \sin\beta \, \vec{j}$$

$$M_x = Pl \cos\beta \qquad M_y = Pl \sin\beta$$

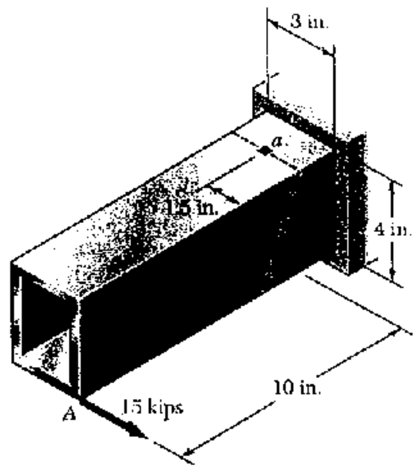
(a) $$\sigma = \frac{6Pl\cos\beta}{b h^2} - \frac{6Pl \sin\beta}{h b^2} = \frac{6Pl}{bh}\left[\frac{\cos\beta}{h} - \frac{\sin\beta}{b}\right]$$ ◄

(b) $$\sigma = 0 \qquad \frac{\cos\beta}{h} - \frac{\sin\beta}{b} = 0 \qquad \tan\beta = \frac{b}{h}$$

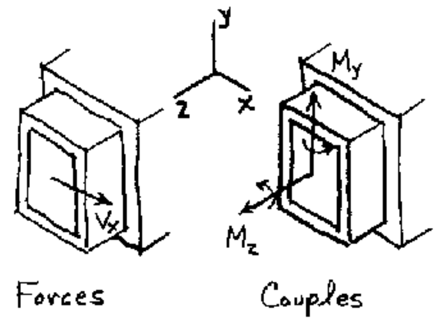
$$\beta = \tan^{-1}\left(\frac{b}{h}\right)$$ ◄

Problem 8.61

***8.61** The structural tube shown has a uniform wall thickness of 0.3 in. Knowing that the 15-kip load is applied 0.15 in. above the base of the tube, determine the shearing stress at (*a*) point *a*, (*b*) point *b*.



Calculate forces and couples at section containing points $\underline{a}$ and $\underline{b}$.



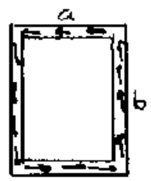
$V_x = 15$ kips

$M_z = (2-0.15)(15) = 27.75$ kip-in

$M_y = (10)(15) = 150$ kip-in

Shearing stresses due to torque $T = M_z$

$$\mathcal{a} = [3-(2)(0.15)][4-(2)(0.15)] = 9.99 \text{ in}^2$$

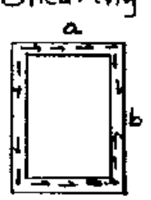


$$q = \frac{M_z}{2\mathcal{a}} = \frac{27.75}{(2)(9.99)} = 1.3889 \text{ kip/in}$$

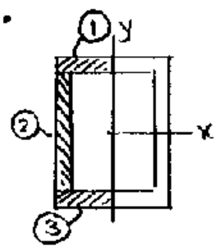
At point $\underline{a}$. $t = 0.3$ in. $\quad \tau_a = \frac{q}{t} = \frac{1.3889}{0.3} = 4.630$ ksi $\leftarrow$

At point $\underline{b}$. $t = 0.3$ in. $\quad \tau_b = \frac{q}{t} = \frac{1.3889}{0.3} = 4.630$ ksi $\uparrow$

Shearing stresses due to V_x.



At point $\underline{a}$



Part	A (in²)	$\bar{x}$ (in.)	A $\bar{x}$ (in³)
①	0.45	-0.75	-0.3375
②	1.02	-1.35	-1.377
③	0.45	-0.75	-0.3375
Σ			-2.052

$Q = |\Sigma A\bar{x}| = 2.052 \text{ in}^3 \qquad t = (2)(0.3) = 0.6$ in.

$$I_y = \frac{1}{12}(4)(3)^3 - \frac{1}{12}(3.4)(2.4)^3 = 5.0832$$

$$\tau_a = \frac{V_x Q}{I_y t} = \frac{(15)(2.052)}{(5.0832)(0.6)} = 10.092 \text{ ksi}$$

At point $\underline{b}$ $\qquad \tau_b = 0$

Combined shearing stresses.

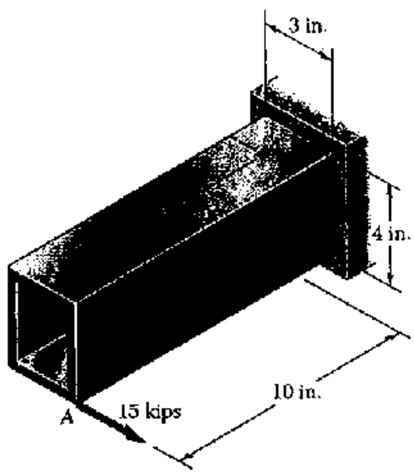
At point $\underline{a}$. $\quad \tau_a = 4.630 \leftarrow + 10.092 \rightarrow \; = \; 5.46$ ksi $\rightarrow$

At point $\underline{b}$. $\quad \tau_b = 4.630 \uparrow + 0 \; = \; 4.63$ ksi $\uparrow$

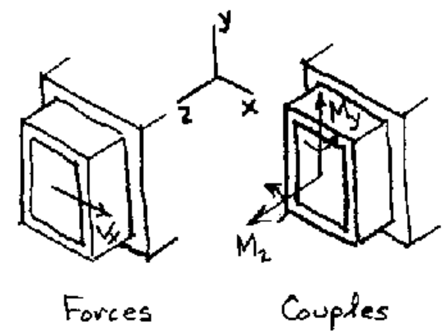
Problem 8.62

***8.62** For the tube and loading of Prob. 8.61, determine the principal stresses and the maximum shearing stress at point *b*.

8.61 Knowing that the structural tube shown has a uniform wall thickness of 0.3 in., determine the shearing stress at points *a* and *b*.



Calculate forces and couples at section containing point $\underline{b}$.



$V_x = 15$ kips

$M_z = (2 - 0.15)(15) = 27.75$ kip·in

$M_y = (10)(15) = 150$ kip·in

$$I_y = \frac{1}{12}(4)(3)^3 - \frac{1}{12}(3.4)(2.4)^3 = 5.0832 \text{ in}^4$$

$$\sigma_b = -\frac{M_y x_b}{I_y} = -\frac{(150)(1.5)}{5.0832} = -44.26 \text{ ksi}$$

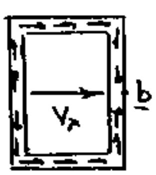
Shearing stress at point $\underline{b}$ due to torque M_z.

$$a = [3-(2)(0.15)][4-(2)(0.15)] = 9.99 \text{ in}^2$$

$$q = \frac{M_z}{2a} = \frac{27.75}{(2)(9.99)} = 1.38889 \text{ kip/in}$$

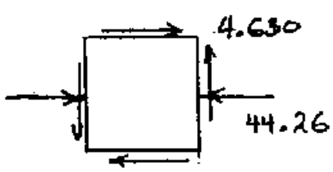
$$\tau = \frac{q}{t} = \frac{1.38889}{0.3} = 4.630 \text{ ksi}$$

Shearing at point $\underline{b}$ due to V_x.



$$\tau = 0$$

Calculation of principal stresses and maximum shearing stress.



$$\sigma_{ave} = \frac{-44.26 + 0}{2} = -22.13 \text{ ksi}$$

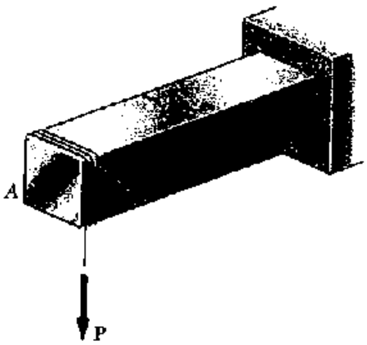
$$R = \sqrt{\left(\frac{-44.26 - 0}{2}\right)^2 + (4.630)^2} = 22.61 \text{ ksi}$$

$$\sigma_{max} = \sigma_{ave} + R = 0.48 \text{ ksi}$$

$$\sigma_{min} = \sigma_{ave} - R = -44.7 \text{ ksi}$$

$$\tau_{max} = R = 22.6 \text{ ksi}$$

Problem 8.63



***8.63** A 5-kN force **P** is applied to a wire that is wrapped around bar AB as shown. Knowing that the cross section of the bar is a square of side $d = 40$ mm, determine the principal stresses and the maximum shearing stress at point a.

Bending: Point a lies on the neutral axis.

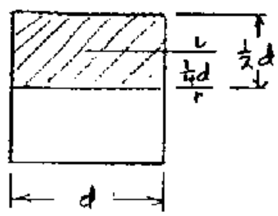
$\sigma = 0$

Torsion: $\tau = \dfrac{T}{c_1 a b^2}$ where $a = b = d$ and

$c_1 = 0.208$ for a square section.

Since $T = \dfrac{Pd}{2}$, $\tau_T = \dfrac{P}{0.416\, d^2} = 2.404 \dfrac{P}{d^2}$

Transverse shear: $V = P$ $\quad I = \frac{1}{12} d^4$



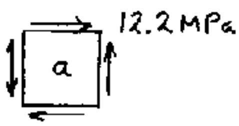
$A = \frac{1}{2} d^2 \quad \bar{y} = \frac{1}{4} d \quad Q = A\bar{y} = \frac{1}{8} d^3$

$t = d$

$\tau_V = \dfrac{VQ}{It} = 1.5 \dfrac{P}{d^2}$

By superposition $\quad \tau = \tau_T + \tau_V = 3.904 \dfrac{P}{d^2}$

$\tau = \dfrac{(3.904)(5 \times 10^3)}{(40 \times 10^{-3})^2} = 12.2 \times 10^6$ Pa $\qquad$ 12.2 MPa.



By Mohr's circle,

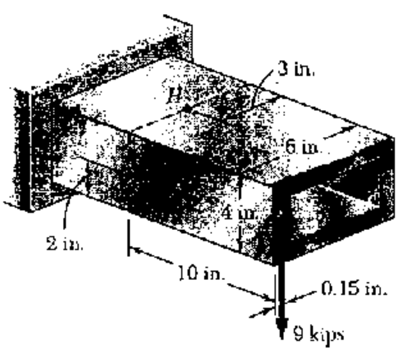
$\sigma_{max} = 12.2$ MPa

$\sigma_{min} = -12.2$ MPa

$\tau_{max} = 12.2$ MPa

Problem 8.64

***8.64** Knowing that the structural tube shown has a uniform wall thickness of 0.3 in., determine the principal stresses, principal planes, and maximum shearing stress at (*a*) point *H*, (*b*) point *K*.



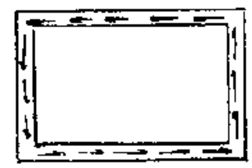
At the section containing points H and K

$V = 9$ kips $\qquad M = (9)(10) = 90$ kip·in.

$T = (9)(3 - 0.15) = 25.65$ kip·in

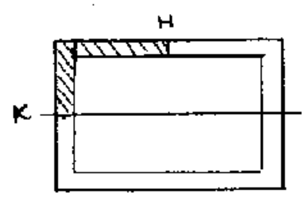
Torsion:

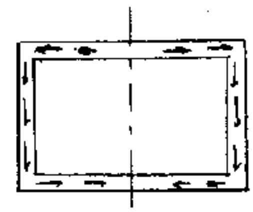
$a = (5.7)(3.7) = 21.09$ in^2



$$\tau = \frac{T}{2ta} = \frac{25.65}{(2)(0.3)(21.09)} = 2.027 \text{ ksi}$$

Transverse shear:





$Q_H = 0$

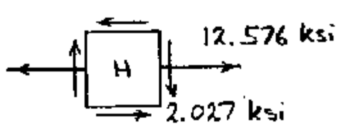
$Q_K = (3)(2)(1) - (2.7)(1.7)(0.85)$
$\quad = 2.0985$ in^3

$I = \frac{1}{12}(6)(4)^3 - \frac{1}{12}(5.4)(3.4)^3 = 14.3132$ in^4

$\tau_H = 0 \qquad \tau_K = \dfrac{VQ_a}{It} = \dfrac{(9)(2.0985)}{(14.3132)(0.3)} = 4.398$ ksi

Bending: $\quad \sigma_H = \dfrac{Mc}{I} = \dfrac{(90)(2)}{14.3132} = 12.576$ ksi , $\qquad \sigma_K = 0$

(a) Point H:



$\sigma_c = \dfrac{12.576}{2} = 6.288$ ksi

$R = \sqrt{\left(\frac{12.576}{2}\right)^2 + (2.027)^2} = 6.607$ ksi

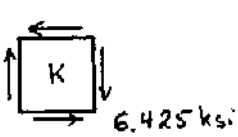
$\sigma_{max} = \sigma_c + R = 12.90$ ksi ◄

$\sigma_{min} = \sigma_c - R = -0.32$ ksi ◄

$\tan 2\theta_p = \dfrac{2\tau}{\sigma} = -0.3224 \qquad \theta_p = -8.9°, 81.1°$ ◄

$\tau_{max} = R = 6.61$ ksi ◄

(b) Point K: $\qquad \sigma = 0 \qquad \tau = 2.027 + 4.398 = 6.425$ ksi



$\sigma_{max} = 6.43$ ksi ◄

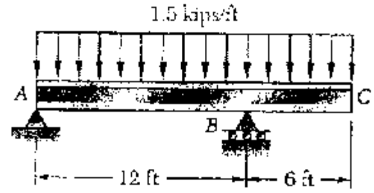
$\sigma_{min} = -6.43$ ksi ◄

$\theta_p = \pm 45°$ ◄

$\tau_{max} = 6.43$ ksi ◄

Problem 8.65

8.65 (*a*) Knowing that $\sigma_{all} = 24$ ksi and $\tau_{all} = 14.5$ ksi, select the most economical wide-flange shape that should be used to support the loading shown. (*b*) Determine the values to be expected for σ_m, τ_m, and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.



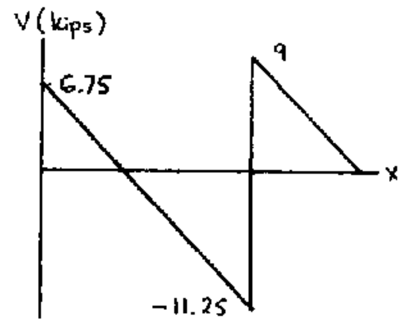
$\circlearrowleft \Sigma M_B = 0 \quad -12R_A + (1.5)(18)(3) = 0 \quad R_A = 6.75 \text{ kips} \uparrow$

$\circlearrowleft \Sigma M_A = 0 \quad 12 R_B + (1.5)(18)(9) = 0 \quad R_B = 20.25 \text{ kips} \uparrow$

$|V|_{max} = 11.25$ kips

$|M|_{max} = 27 \text{ kip·ft} = 324 \text{ kip·in}$

$S_{min} = \dfrac{|M|_{max}}{\sigma_{all}} = \dfrac{324}{24} = 13.5 \text{ in}^3$



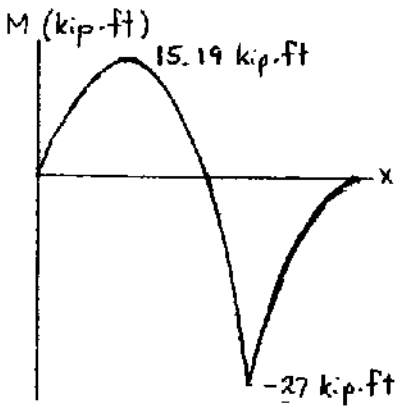
Shape	S (in³)
W 12 × 16	17.1
→ W 10 × 15	13.8
W 8 × 18	15.2
W 6 × 20	13.4

(a) Use W 10 × 15 ◀

$d = 9.99$ in.

$t_f = 0.270$ in.

$t_w = 0.230$ in.



(b) $\sigma_m = \dfrac{|M|_{max}}{S} = \dfrac{324}{13.8} = 23.5$ ksi ◀

$\tau_m = \dfrac{|V|_{max}}{d\,t_w} = \dfrac{11.25}{(9.99)(0.230)} = 4.90$ ksi ◀

$c = \frac{1}{2} d = \dfrac{9.99}{2} = 4.995$ in.

$y_b = c - t_f = 4.995 - 0.270 = 4.725$ in.

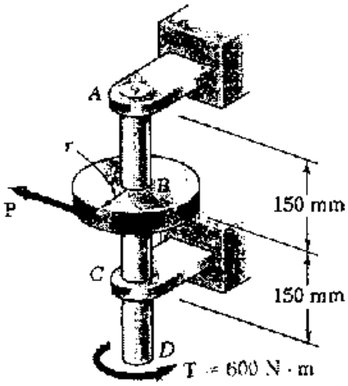
$\sigma_b = \dfrac{y_b}{c}\,\sigma_m = \left(\dfrac{4.725}{4.995}\right)(23.5) = 22.2$ ksi

$R = \sqrt{\left(\dfrac{\sigma_b}{2}\right)^2 + \tau_m^2} = \sqrt{\left(\dfrac{22.2}{2}\right)^2 + (4.90)^2} = 12.1$ ksi

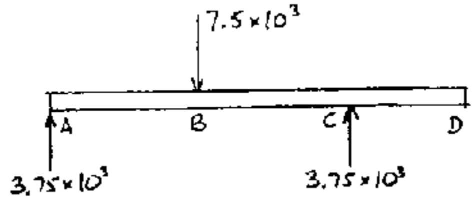
$\sigma_{max} = \dfrac{\sigma_b}{2} + R = \dfrac{22.2}{2} + 12.1 = 23.2$ ksi ◀

Problem 8.66

8.66 Determine the smallest allowable diameter of the solid shaft $ABCD$, knowing that $\tau_{all} = 60$ MPa and that the radius of disk B is $r = 80$ mm.

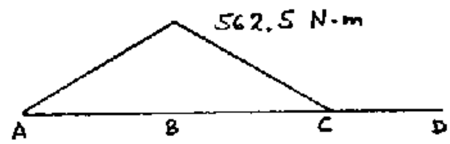


$\Sigma M_{axis} = 0 \qquad T - Pr = 0 \qquad P = \frac{T}{r} = \frac{600}{80\times10^{-3}} = 7.5\times10^{3}$ N

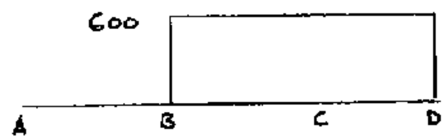


$R_A = R_C = \frac{1}{2}P$
$= 3.75\times10^{3}$ N

$M_B = (3.75\times10^{3})(150\times10^{-3})$
$= 562.5$ N-m



Bending moment.



Torque.

Critical section lies at point B. $\quad M = 562.5$ N-m , $T = 600$ N-m

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{(\sqrt{M^2+T^2})_{max}}{\tau_{all}}$$

$$c^3 = \frac{2}{\pi}\,\frac{\sqrt{M^2+T^2}}{\tau_{all}} = \frac{2}{\pi}\,\frac{\sqrt{(562.5)^2+(600)^2}}{60\times10^{6}} = 8.726\times10^{-6}\ \text{m}^3$$

$c = 20.58\times10^{-3}$ m $\qquad d = 2c = 41.2\times10^{-3}$ m $= 41.2$ mm

Problem 8.67

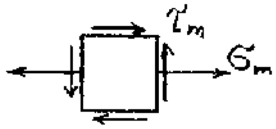
8.67 Using the notation Sec. 8.3 and neglecting the effect of shearing stresses caused by transverse loads, show that the maximum normal stress in a cylindrical shaft can be expressed as

$$\sigma_{\max} = \frac{c}{J}[(M_y^2 + M_z^2)^{\frac{1}{2}} + (M_y^2 + M_z^2 + T^2)^{\frac{1}{2}}]_{\max}$$

Maximum bending stress $\quad \sigma_m = \dfrac{|M|c}{I} = \dfrac{\sqrt{M_y^2 + M_z^2}\, c}{I}$

Maximum torsional stress $\quad \tau_m = \dfrac{Tc}{J}$

$$\frac{\sigma_m}{2} = \frac{\sqrt{M_y^2 + M_z^2}\, c}{2I} = \frac{c}{J}\sqrt{M_y^2 + M_z^2}$$



Using Mohr's circle,

$$R = \sqrt{\left(\frac{\sigma_m}{2}\right)^2 + \tau_m^2} = \sqrt{\frac{c^2}{J^2}(M_y^2 + M_z^2) + \frac{T^2 c^2}{J^2}}$$

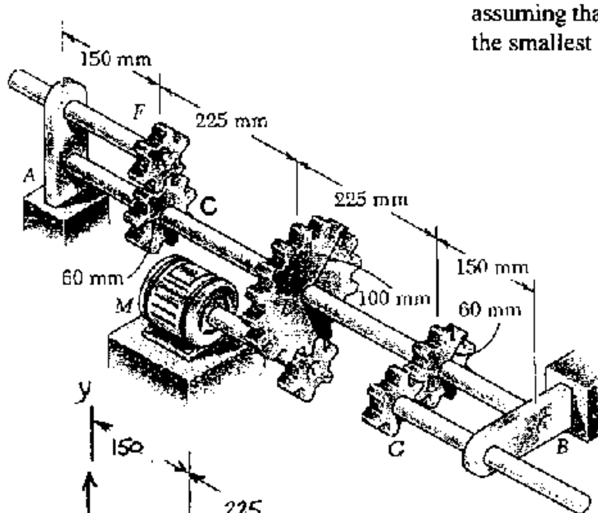
$$= \frac{c}{J}\sqrt{M_y^2 + M_z^2 + T^2}$$

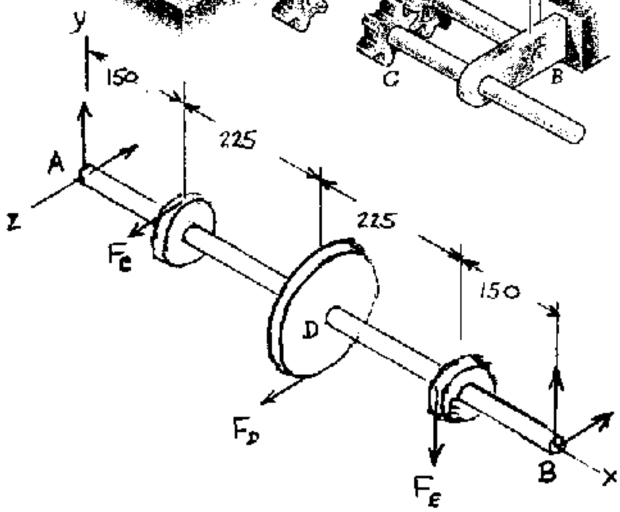
$$\sigma_{\max} = \frac{\sigma_m}{2} + R = \frac{c}{J}\sqrt{M_y^2 + M_z^2} + \frac{c}{J}\sqrt{M_y^2 + M_z^2 + T^2}$$

$$= \frac{c}{J}\left[(M_y^2 + M_z^2)^{\frac{1}{2}} + (M_y^2 + M_z^2 + T^2)^{\frac{1}{2}}\right]$$

Problem 8.68

8.68 The solid shaft AB rotates at 450 rpm and transmits 20 kW from the motor M to machine tools connected to gears F and G. Knowing that $\tau_{all} = 55$ MPa and assuming that 8 kW is taken off at gear F and 12 kW is taken off at gear G, determine the smallest permissible diameter of shaft AB.





$f = \frac{450}{60} = 7.5$ Hz

Torque applied at D:

$T_D = \frac{P}{2\pi f} = \frac{20\times10^3}{(2\pi)(7.5)} = 424.41$ N·m

Torques on gears C and E:

$T_C = \frac{8}{20} T_D = 169.76$ N·m

$T_E = \frac{12}{20} T_D = 254.65$ N·m

Forces on gears:

$F_D = \frac{T_D}{r_D} = \frac{424.41}{100\times10^{-3}} = 4244$ N

$F_C = \frac{T_C}{r_C} = \frac{169.76}{60\times10^{-3}} = 2829$ N

$F_E = \frac{T_E}{r_E} = \frac{254.65}{60\times10^{-3}} = 4244$ N

Torques in various parts

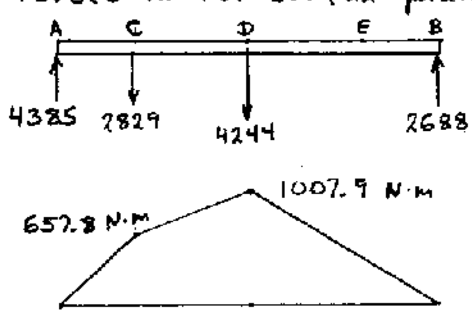
AC : $T = 0$

CD : $T = 169.76$ N·m

DE : $T = 254.65$ N·m

EB : $T = 0$

Forces in horizontal plane.



Critical point lies just the right of D.

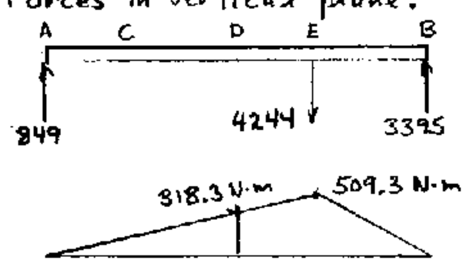
$T = 254.65$ N·m

$M_y = 1007.9$ N·m

$M_z = 318.3$ N·m

$\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{max} = 1087.2$ N·m

Forces in vertical plane.



$\tau_{all} = \frac{c}{J}\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{max}$

$\frac{J}{c} = \frac{\pi}{2} c^3 = \frac{\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{max}}{\tau_{all}} = \frac{1087.2}{55\times10^6}$

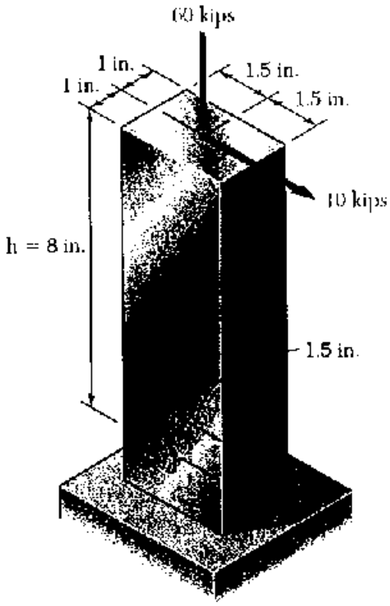
$= 19.767\times10^{-3}$ m

$c = 23.26\times10^{-3}$ m

$d = 2c = 46.5\times10^{-3}$ m $= 46.5$ mm

Problem 8.69

8.69 Two forces are applied to the bar shown. At point a, determine (a) the principal stresses and principal planes, (b) the maximum shearing stresses.



At the section containing point $\underline{a}$ and $\underline{b}$.

$V = 10$ kips $\qquad P = 60$ kips (compression)

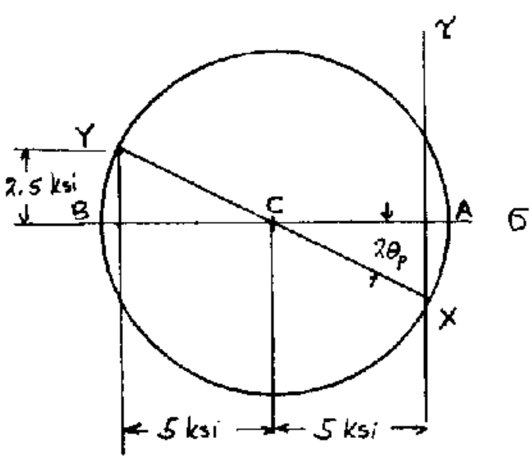
$M = (8)(10) = 80$ kip-in

$A = (2)(3) = 6 \text{ in}^2$

$I = \frac{1}{12}(2)(3)^3 = 4.5$ kip-in

At point a $\quad \sigma_y = -\frac{P}{A} = -\frac{60}{6} = -10$ ksi

$\tau = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{(10)}{6} = 2.5$ ksi , $\sigma_x = 0$



Use Mohr's circle.

$\sigma_c = -5$ ksi

$R = \sqrt{5^2 + 2.5^2} = 5.590$ ksi

$\sigma_a = \sigma_c + R = 0.590$ ksi

$\sigma_b = \sigma - R = -10.59$ ksi

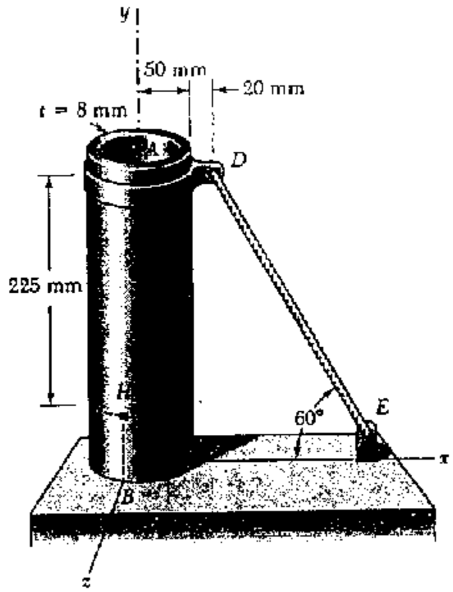
$\tan 2\theta_p = \frac{2.5}{5} = 0.5$

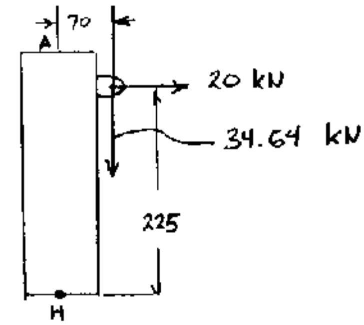
$\theta_p = 13.3°, \ 103.3°$

$\tau_{max} = R = 5.59$ ksi

Problem 8.70

8.70 The steel pile *AB* has a 100-mm outer diameter and an 8-mm wall thickness. Knowing that the tension in the cable is 40 kN, determine the normal and shearing stress at point *H*.





Vertical force
$40 \cos 30° = 34.64$ kN

Horizontal Force
$40 \sin 30° = 20$ kN

Point H lies on neutral axis of bending

Section properties.

$d_o = 100$ mm $\quad c_o = \frac{1}{2} d_o = 50$ mm $\quad c_i = c_o - t = 42$ mm

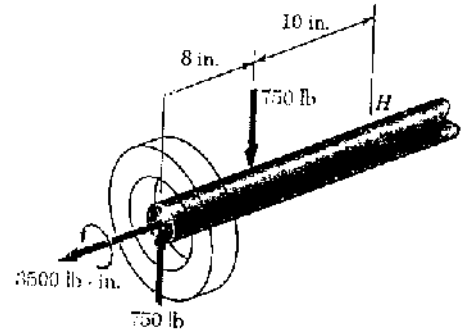
$A = \pi(c_o^2 - c_i^2) = 2.312 \times 10^3 \text{ mm}^2 = 2.312 \times 10^{-3} \text{ m}^2$

$\sigma = -\frac{P}{A} = -\frac{34.64 \times 10^3}{2.312 \times 10^{-3}} = -14.98$ MPa

For thin pipe, $\tau = 2\frac{V}{A} = \frac{(2)(20 \times 10^3)}{2.314 \times 10^{-3}} = 17.29$ MPa

Problem 8.71

8.71 The axle of a small truck is acted upon by the forces and couple shown. Knowing that the diameter of the axle is 1.42 in., determine the normal and shearing stresses at point H located on the top of the axle.



The bending moment causing normal stress at point H is

$$M = (8)(750) = 6000 \text{ lb} \cdot \text{in.}$$

$$c = \tfrac{1}{2}d = 0.71 \text{ in.}$$

$$I = \tfrac{\pi}{4}c^4 = 0.19958 \text{ in}^4, \quad J = 2I = 0.39916 \text{ in}^4$$

Normal stress at H:

$$\sigma_H = -\frac{Mc}{I} = -\frac{(6000)(0.71)}{0.19958} = -21.3 \times 10^3 \text{ psi}$$

$$= -21.3 \text{ ksi}$$

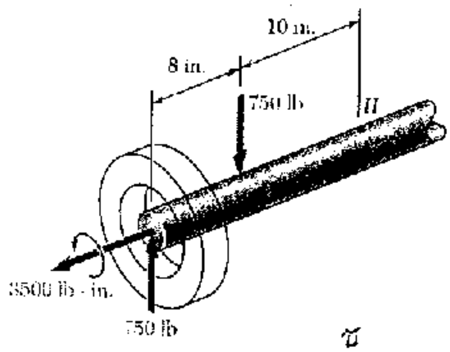
At the section containing point H, $V = 0$, $T = 3500$ lb in

$$\tau_H = \frac{Tc}{J} = \frac{(3500)(0.71)}{0.39916} = 6.23 \text{ ksi}$$

Problem 8.72

8.72 For the truck axle and loading of Prob. 8.71, determine the principal stresses and the maximum shearing stress at point H.

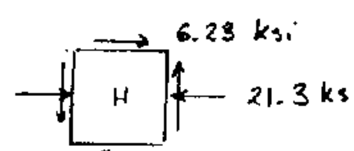
8.71 The axle of a small truck is acted upon by the forces and couple shown. Knowing that the diameter of the axle is 1.42 in., determine the normal and shearing stresses at point H located on the top of the axle.



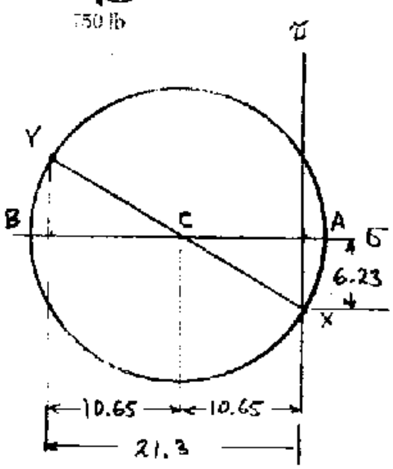
From the solution of Prob. 8.71

$$\sigma_H = -21.3 \text{ ksi}$$

$$\tau_H = 6.23 \text{ ksi}$$



Use Mohr's circle.



$$\sigma_c = -\frac{21.3}{2} = -10.65 \text{ ksi}$$

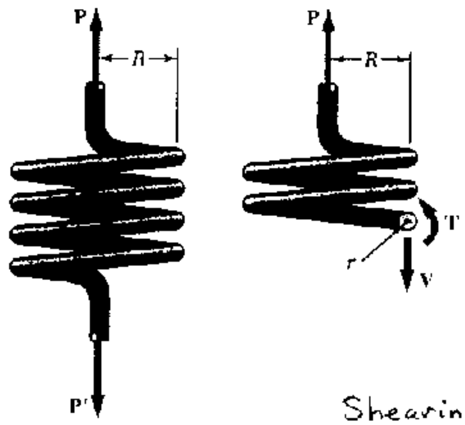
$$R = \sqrt{\left(\frac{21.3}{2}\right)^2 + (6.23)^2} = 12.34 \text{ ksi}$$

$$\sigma_a = \sigma_c + R = 1.69 \text{ ksi}$$

$$\sigma_b = \sigma_c - R = -23.0 \text{ ksi}$$

$$\tau_{max} = R = 12.34 \text{ ksi}$$

Problem 8.73



8.73 A close-coiled spring is made of a circular wire of radius r that is formed into a helix of radius R. Determine the maximum shearing stress produced by the two equal and opposite forces **P** and **P'**. (*Hint*: First determine the shear **V** and the torque **T** in a transverse cross section.)

$+\uparrow \Sigma F_y = 0: \qquad P - V = 0 \qquad V = P$

$+\circlearrowright \Sigma M_c = 0: \qquad T - PR = 0 \qquad T = PR$

Shearing stress due to T

$$\tau_T = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2PR}{\pi r^3}$$

Shearing stress due to V

For semicircle, $\quad Q = \frac{2}{3} r^3, \quad t = d = 2r$

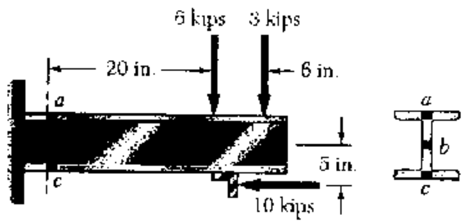
For solid circular section, $\quad I = \frac{1}{2} J = \frac{\pi}{4} r^4$

$$\tau_V = \frac{VQ}{It} = \frac{V(\frac{2}{3} r^3)}{\frac{\pi}{4} r^4 (2r)} = \frac{4V}{3\pi r^2} = \frac{4P}{3\pi r^2}$$

By superposition $\qquad \tau_{max} = \tau_T + \tau_V = P(2R + 4r/3)/\pi r^3$ ◄

Problem 8.74

8.74 Three forces are applied to a W 6 ×20 rolled beam as shown. Determine the normal and shearing stresses at points a, b, and c. (*Note*: Points a and c are located at the top and bottom surfaces of the flanges, respectively).

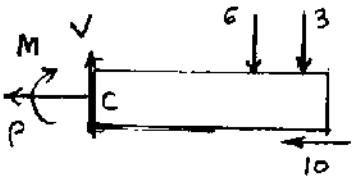


$+\Sigma M_c = 0: \quad M + (20)(6) + (26)(3) + (5)(10) = 0$

$M = -248 \text{ kip in}$

$\underrightarrow{+}\ \Sigma F_x = 0: \quad -P - 10 = 0 \qquad P = -10 \text{ kips}$

$+\uparrow \Sigma F_y = 0 \quad V - 6 - 3 = 0 \qquad V = 9 \text{ kips}$

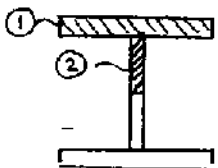


For W6×20 rolled steel shape, $A = 5.87 \text{ in}^2$, $d = 6.20$ in., $b_f = 6.020$ in., $t_f = 0.365$ in. $t_w = 0.260$ in., $I_x = 41.4 \text{ in}^4$, $S_x = 13.4 \text{ in}^3$.

Point a. $\sigma = \frac{P}{A} - \frac{M}{S_x} = \frac{-10}{5.87} - \frac{-248}{13.4} = 16.80 \text{ ksi}$ ◄

$\tau = 0$ ◄

Point b. $\sigma = \frac{P}{A} = \frac{-10}{5.87} = -1.704 \text{ ksi}$ ◄



Part	A (in²)	$\bar{y}$ (in.)	$A\bar{y}$ (in³)
①	2.1973	2.9175	6.4106
②	0.7111	1.3675	0.9724
Σ			7.383

$Q = 7.383 \text{ in}^3$

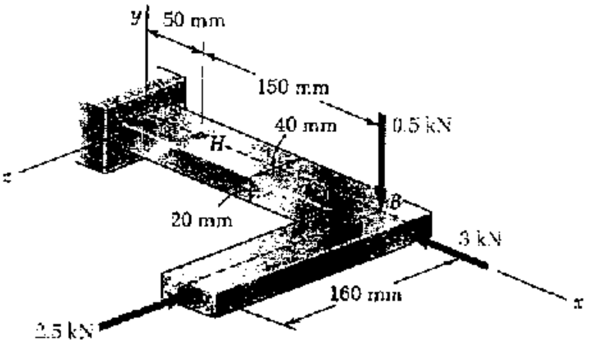
$\tau = \frac{VQ}{I_x t_w} = \frac{(9)(7.383)}{(41.4)(0.260)} = 6.17 \text{ ksi}$ ◄

Point c. $\sigma = \frac{P}{A} + \frac{M}{S} = \frac{-10}{5.87} + \frac{-248}{13.4} = -20.2 \text{ ksi}$ ◄

$\tau = 0$ ◄

Problem 8.75

8.75 Three forces are applied to the machine component ABD as shown. Knowing that the cross section containing point H is a 20×40-mm rectangle, determine the principal stresses and the maximum shearing stress at point H.

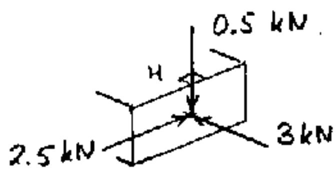


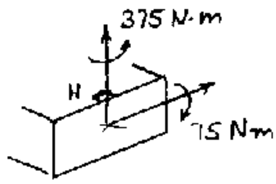
Equivalent force-couple system at section containing point H:

$F_x = -3\text{ kN}, \quad F_y = -0.5\text{ kN}, \quad F_z = -2.5\text{ kN}$

$M_x = 0, \quad M_y = (0.150)(2500) = 375\text{ N}\cdot\text{m}$

$M_z = -(0.150)(500) = -75\text{ N}\cdot\text{m}$





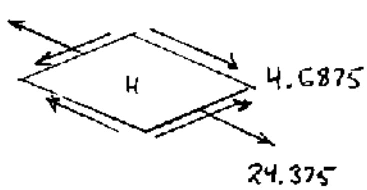
$A = (20)(40) = 800\text{ mm}^2 = 800 \times 10^{-6}\text{ m}^2$

$I_z = \frac{1}{12}(40)(20)^3 = 26.667 \times 10^3\text{ mm}^4 = 26.667 \times 10^{-9}\text{ m}^4$

$$\sigma_H = \frac{P}{A} - \frac{M_z y}{I_z} = \frac{-3000}{800 \times 10^{-6}} - \frac{(-75)(10 \times 10^{-3})}{26.667 \times 10^{-9}} = 24.375\text{ MPa}$$

$$\tau_H = \frac{3}{2}\frac{|V_z|}{A} = \frac{3}{2} \cdot \frac{2500}{800 \times 10^{-6}} = 4.6875\text{ MPa}$$

Use Mohr's circle.

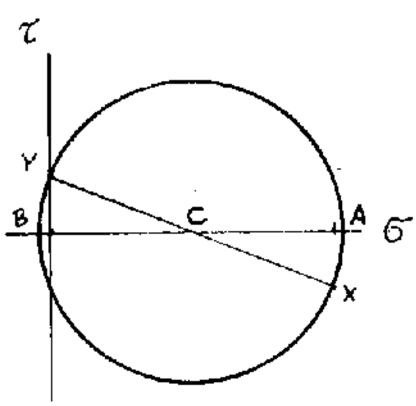


$\sigma_c = \frac{1}{2}\sigma_H = 12.1875\text{ MPa}$

$R = \sqrt{\left(\frac{24.375}{2}\right)^2 + (4.6875)^2} = 13.0579\text{ MPa}$

$\sigma_a = \sigma_c + R = 25.2\text{ MPa}$ ◄

$\sigma_b = \sigma_c - R = -0.87\text{ MPa}$ ◄

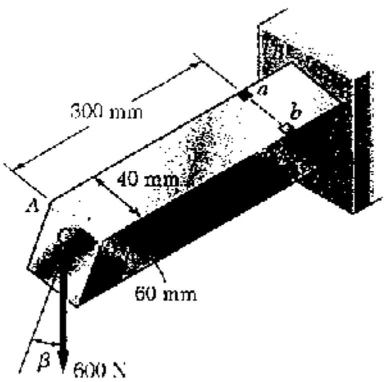


$\tan 2\theta_p = \frac{2\tau_H}{\sigma_H} = \frac{(2)(4.6875)}{24.375} = 0.3846$

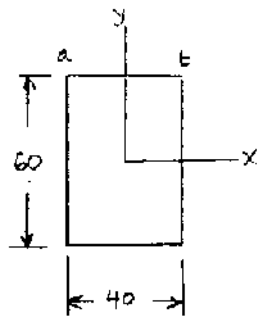
$\theta_a = 10.5°, \quad \theta_b = 100.5°$

$\tau_{max} = R = 13.06\text{ MPa}$ ◄

Problem 8.76

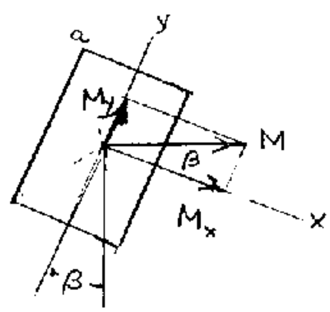


8.76 The cantilever beam AB will be installed so that the 60-mm side forms an angle β between 0 and 90° with the vertical. Knowing that the 600-N vertical forces is applied at the center of the free end of the beam, determine the normal stress at point a when (*a*) $\beta = 0$, (*b*) $\beta = 90°$. (*c*) Also, determine the value of β for which the normal stress at point a is a maximum and the corresponding value of that stress.



$S_x = \frac{1}{6}(40)(60)^2 = 24 \times 10^3 \text{ mm}^3$
$= 24 \times 10^{-6} \text{ m}^3$

$S_y = \frac{1}{6}(60)(40)^2 = 16 \times 10^3 \text{ mm}^3$
$= 16 \times 10^{-3} \text{ m}^3$



$M = P\ell = (600)(300 \times 10^{-3}) = 180 \text{ N}\cdot\text{m}$

$M_x = M\cos\beta = 180\cos\beta$

$M_y = M\sin\beta = 180\sin\beta$

$$\sigma_a = \frac{M_x}{S_x} + \frac{M_y}{S_y} = \frac{180\cos\beta}{24 \times 10^{-6}} + \frac{180\sin\beta}{16 \times 10^{-6}}$$
$$= (7.5 \times 10^6)(\cos\beta + \tfrac{3}{2}\sin\beta) \text{ Pa}$$
$$= 7.5(\cos\beta + \tfrac{3}{2}\sin\beta) \text{ MPa}$$

(a) $\beta = 0$ $\qquad \sigma_a = 7.50 \text{ MPa}$ ◄

(b) $\beta = 90°$ $\qquad \sigma_a = 11.25 \text{ MPa}$ ◄

(c) $\frac{d\sigma_a}{d\beta} = 7.5(-\sin\beta + \tfrac{3}{2}\cos\beta) = 0$

$\sin\beta = \frac{3}{2}\cos\beta \qquad \tan\beta = \frac{3}{2} \qquad \beta = 56.3°$ ◄

$\sigma_a = 7.5(\cos 56.3° + \tfrac{3}{2}\sin 56.3°) = 13.52 \text{ MPa}$ ◄

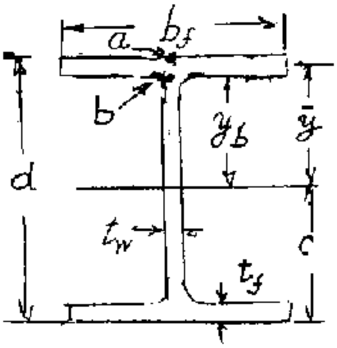
PROBLEM 8.C1

8.C1 Let us assume that the shear V and the bending moment M have been determined in a given section of a rolled-steel beam. Write a computer program to calculate in that section, from the data available in Appendix C, (a) the maximum normal stress σ_m, (b) the principal stress σ_{max} at the junction of a flange and the web. Use this program to solve parts a and b of the following problems:

(1) Prob. 8.1 (Use V = 1420 kN and M = 3550 kN·m)
(2) Prob. 8.2 (Use V = 1775 kN and M = 3550 kN·m)
(3) Prob. 8.3 (Use V = 90 kips and M = 900 kip·in.)
(4) Prob. 8.4 (Use V = 45 kips and M = 900 kip·in.)

SOLUTION

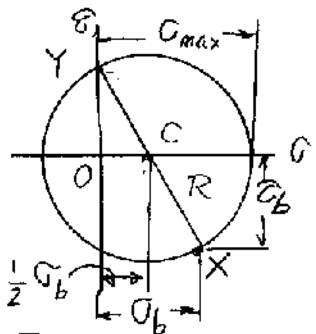
We enter the given values of V and M obtain from Appendix C the values of d, b_f, t_f, t_w, I, and S for the given WF shape.



We compute $c = d/2$, $\quad y_b = c - t_f$

$\bar{y} = c - \frac{1}{2}t_f$, $\quad \sigma_a = M/S$, $\quad \sigma_b = \sigma_a(y_b/c)$

$Q = b_f t_f \bar{y}$, $\quad \tau_b = \dfrac{VQ}{It_w}$



From Mohr's circle:

$$\sigma_{max} = \frac{1}{2}\sigma_b + R$$

$$\sigma_{max} = \frac{1}{2}\sigma_b + \sqrt{\left(\frac{1}{2}\sigma_b\right)^2 + \tau_b^2}$$

PROGRAM OUTPUTS

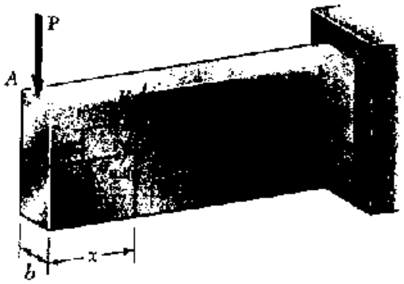
```
Problem 8.1

Given Data:
V =  1420 kN, M =  3550 kN.m
d =  930 mm, bf =  423 mm
tf =  43 mm, tw =  24 mm
I =  8470 (10^6 mm^4)
S =  18200 (10^3 mm^3)
Answers:
(a) SIGA = 195.05 MPa
(b) SIGM = 193.50 MPa
```

```
Problem 8.2

Given Data:
V =  1775 kN, M = 3550 kN.m
d =  930 mm, bf =  423 mm
tf =  43 mm, tw =  24 mm
I =  8470 (10^6 mm^4)
S =  18200 (10^3 mm^3)
Answers:
(a) SIGA = 195.05 MPa
(b) SIGM = 201.64 MPa
```

```
Problem 8.3

Given Data:
V =  90 kips, M =  900 kip.in.
d =  9.92 in., bf =  7.985 in.
tf =  .530 in., tw =  .315 in.
I =  209.00 in^4, S =  42.10 in^4
Answers:
(a) SIGA =  21377.67 psi
(b) SIGM =  38338.15 psi
```

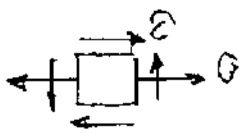
```
Problem 8.4

Given Data:
V =  45 kips, M =  900 kip.in.
d =  9.92 in., bf =  7.985 in.
tf =  .530 in., tw =  .315 in.
I =  209.00 in^4, S =  42.10 in^4
Answers:
(a) SIGA =  21377.67 psi
(b) SIGM =  26147.63 psi
```

PROBLEM 8.C2



8.C2 A cantilever beam AB with a rectangular cross section of width b and depth $2c$ supports a single concentrated load **P** at its end A. Write a computer program to calculate, for any values of x/c and y/c, (*a*) the ratios σ_{max}/σ_m and σ_{min}/σ_m, where σ_{max} and σ_{min} are the principal stresses at point $K(x, y)$ and σ_m the maximum normal stress in the same transverse section, (*b*) the angle θ_p that the principal planes at K form with a transverse and a horizontal plane through K. Use this program to check the values shown in Fig. 8.8 and to verify that σ_{max} exceeds σ_m if $x \le 0.544c$, as indicated in the second footnote on page 499.

SOLUTION



Since the distribution of the normal stresses is linear, we have $\sigma = \sigma_m (y/c)$ (1)

where $\sigma_m = \dfrac{Mc}{I} = \dfrac{Pxc}{I}$ (2)

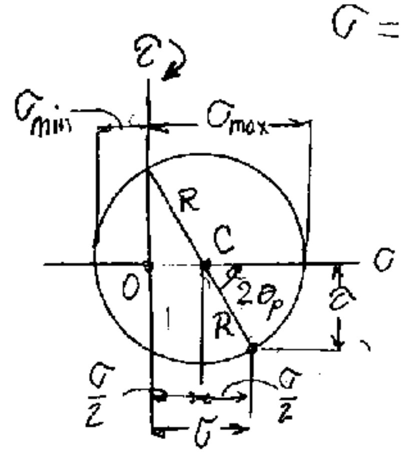
We use Eq. (8.4), page 498: $\tau = \dfrac{3}{2}\dfrac{P}{A}\left(1 - \dfrac{y^2}{c^2}\right)$ (3)

Dividing (3) by (2): $\dfrac{\tau}{\sigma_m} = \dfrac{3}{2}\dfrac{I}{A}\;\dfrac{1-(y/c)^2}{xc}$

or, since $\dfrac{I}{A} = \dfrac{\frac{1}{12}b(2c)^3}{b(2c)} = \dfrac{1}{3}c^2$; $\dfrac{\tau}{\sigma_m} = \dfrac{1}{2}\;\dfrac{1-(y/c)^2}{x/c}$ (4)

Letting $X = x/c$ and $Y = y/c$, Eqs. (1) and (4) yield

$$\sigma = \sigma_m Y \qquad \tau = \sigma_m \frac{1-Y^2}{2X}$$



Using Mohr's circle, we calculate

$$R = \sqrt{\left(\tfrac{1}{2}\sigma\right)^2 + \tau^2}$$

$$= \frac{1}{2}\sigma_m\sqrt{Y^2 + \left(\frac{1-Y^2}{X}\right)^2}$$

$$\frac{\sigma_{max}}{\sigma_m} = \frac{1}{2}Y + R \qquad \frac{\sigma_{min}}{\sigma_m} = \frac{1}{2}Y - R$$

$$\tan 2\theta_p = \frac{\tau}{\sigma/2} = \frac{1-Y^2}{2X(Y/2)} = \frac{1-Y^2}{XY} \qquad \theta_p = \frac{1}{2}\tan^{-1}\left(\frac{1-Y^2}{XY}\right)$$

NOTE

For $y > 0$, the angle θ_p is ↻, which is opposite to what was arbitrarily assumed in Fig. P8.C2.

(CONTINUED)

PROGRAM OUTPUTS

For x/c = 2 :

y/c	Sigmin/Sigm	Sigmax/Sigm	Theta
1.0	0.000	1.000	0.00
0.8	-0.010	0.810	6.34
0.6	-0.040	0.640	14.04
0.4	-0.090	0.490	23.20
0.2	-0.160	0.360	33.69
0.0	-0.250	0.250	45.00
-0.2	-0.360	0.160	33.69
-0.4	-0.490	0.090	-23.20
-0.6	-0.640	0.040	-14.04
-0.8	-0.810	0.010	-6.34
-1.0	-1.000	0.000	-0.00

For x/c = 8 :

y/c	Sigmin/Sigm	Sigmax/Sigm	Theta
1.0	0.000	1.000	0.00
0.8	-0.001	0.801	1.61
0.6	-0.003	0.603	3.80
0.4	-0.007	0.407	7.35
0.2	-0.017	0.217	15.48
0.0	-0.062	0.063	45.00
-0.2	-0.217	0.017	-15.48
-0.4	-0.407	0.007	-7.35
-0.6	-0.603	0.003	-3.80
-0.8	-0.801	0.001	-1.61
-1.0	-1.000	0.000	-0.00

To check that $\sigma_{max} > \sigma_m$ if $x \leq 0.544c$, we run the program for $x/c = 0.544$ and for $x/c = 0.545$ and observe that σ_{max}/σ_m exceeds 1 for several values of y/c in the first case, but does not exceed 1 in the second case.

For x/c = 0.544 :

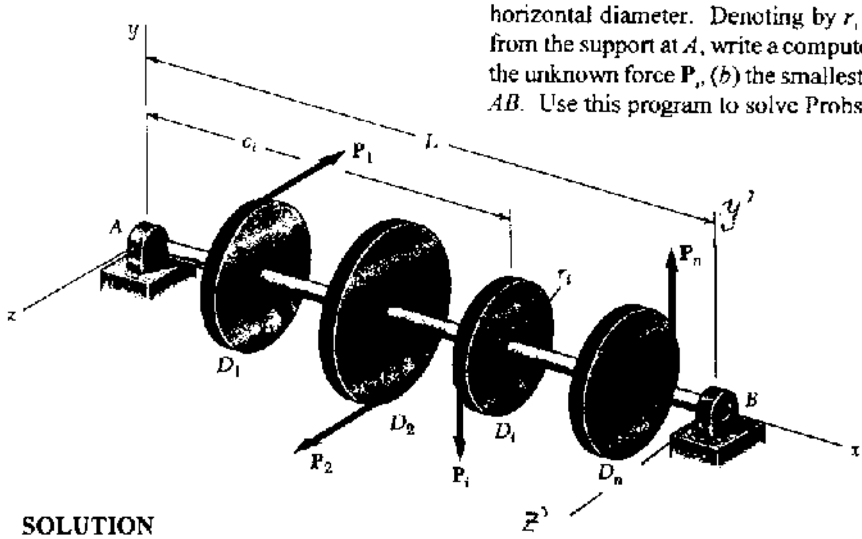
y/c	Sigmin/Sigm	Sigmax/Sigm	Theta
0.30	-0.700	0.9997	39.92
0.31	-0.690	1.0001	39.72
0.32	-0.680	1.0004	39.51
0.33	-0.670	1.0005	39.30
0.34	-0.660	1.0005	39.09
0.35	-0.650	1.0003	38.88
0.36	-0.640	1.0000	38.66
0.37	-0.630	0.9996	38.44
0.38	-0.619	0.9990	38.21
0.39	-0.608	0.9983	37.98
0.40	-0.598	0.9975	37.74

For x/c = 0.545 :

y/c	Sigmin/Sigm	Sigmax/Sigm	Theta
0.30	-0.698	0.9982	39.91
0.31	-0.689	0.9986	39.71
0.32	-0.679	0.9989	39.50
0.33	-0.669	0.9990	39.29
0.34	-0.659	0.9990	39.08
0.35	-0.649	0.9988	38.87
0.36	-0.639	0.9986	38.65
0.37	-0.628	0.9982	38.42
0.38	-0.618	0.9976	38.20
0.39	-0.607	0.9970	37.96
0.40	-0.596	0.9962	37.73

PROBLEM 8.C3

8.C3 Disks $D_1, D_2, \ldots, D_n$ are attaches as shown in Fig. 8.C3 to the solid shaft AB of length L, uniform diameter d, and allowable shearing stress τ_{all}. Forces $\mathbf{P}_1, \mathbf{P}_2, \ldots, \mathbf{P}_n$ of known magnitude (except for one of them) are applied to the disks, either at the top or bottom of its vertical diameter, or at the left or right end of its horizontal diameter. Denoting by r_i the radius of disk D_i and by c_i its distance from the support at A, write a computer program to calculate (*a*) the magnitude of the unknown force $\mathbf{P}_i$, (*b*) the smallest permissible value of the diameter d of shaft AB. Use this program to solve Probs. 8.15 and 8.19.



SOLUTION

1. Determine the unknown force P_i by equating to zero the sum of their torques T_i about the x axis.

2. Determine the components $(F_y)_i$ and $(F_z)_i$ of all forces.

3. Determine the components A_y and A_z of reaction at A by summing moments about axes $Bz' /\!/ z$ and $By' /\!/ y$:

$$\Sigma M_{z'} = 0: \quad -A_y L - \Sigma (F_y)_i (L - c_i) = 0, \qquad A_y = -\frac{1}{L} \Sigma (F_y)_i (L - c_i)$$

$$\Sigma M_{y'} = 0: \quad A_z L + \Sigma (F_z)_i (L - c_i) = 0, \qquad A_z = -\frac{1}{L} \Sigma (F_z)_i (L - c_i)$$

4. Determine $(M_y)_i$, $(M_z)_i$, and torque T_i just to the left of disk D_i:

$$(M_y)_i = A_z c_i + \sum_k (F_z)_k < c_i - c_k >^1$$

$$(M_z)_i = -A_y c_i - \sum_k (F_y)_k < c_i - c_k >^1$$

$$T_i = \sum_k T_k < c_i - c_k >^0$$

where < > indicates a singularity function.

5. The minimum diameter d required to the left of D_i is obtained by first computing $(J/c)_i$ from Eq. (8.7):

$$\left(\frac{J}{c}\right)_i = \frac{\sqrt{(M_y)_i^2 + (M_z)_i^2 + T_i^2}}{\tau_{all}}$$

(CONTINUED)

PROBLEM 8.C3 CONTINUED

6. Recalling that $J = \frac{1}{2}\pi c^4$ and, thus, that $\left(\frac{J}{c}\right)_i = \frac{1}{2}\pi c_i^3$, we have $c_i = \frac{2}{\pi}\left(\frac{J}{c}\right)_i^{1/3}$ and $d_i = \frac{4}{\pi}\left(\frac{J}{c}\right)_i^{1/3}$

This is the required diameter just to the left of disk D_i

7. The required diameter just to the right of disk D_i is obtained by replacing T_i with T_{i+1} in the above computation.

8. The smallest permissible value of the diameter of the shaft is the <u>largest</u> of the values obtained for d_i.

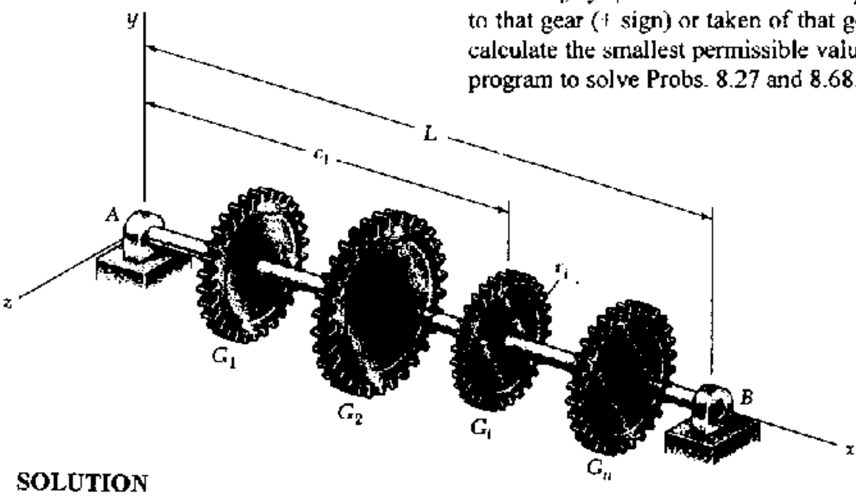
PROGRAM OUTPUTS

```
Problem  8.15

Length of shaft =  300 mm
TAU = 60 MPa
For Disk 1
Force= 6.000 kN
Radius of disk =  75 mm
Distance from A in mm =  80
For Disk 2
Force= 0.000 kN
Radius of disk =  60 mm
Distance from A in mm = 180
Unknown force=  -7.500 kN
AY=  4.400 kN, AZ= -3.000 kN
BY=  1.600 kN, BZ= -4.500 kN
Just to the left of Disk 1
MY=-240.00 Nm
MZ=-352.00 Nm
T=    0.00 Nm
Diameter must be at least  33.07 mm
Just to the right of Disk 1
T= 450.00 Nm
Diameter must be at least  37.47 mm
Just to the left of Disk 2
MY=-540.00 Nm
MZ=-192.00 Nm
T= 450.00 Nm
Diameter must be at least  39.55 mm
Just to the right of Disk 2
T=    0.00 Nm
Diameter must be at least  36.51 mm
```

```
Problem  8.19

Length of shaft=   28 in.
TAU (ksi)=      8
For Disk 1
Force =  0.500 kips
Radius of disk =   4.0 in.
Distance from A=   7.0 in.
For Disk 2
Force =  0.000 kips
Radius of disk =   6.0 in.
Distance from A=  14.0 in.
For Disk 3
Force =  0.500 kips
Radius of disk =   4.0 in.
Distance from A=  21.0 in.
Unknown force= -0.667 kips
AY=  0.500 kips, AZ=  0.333 kips
BY=  0.500 kips, BZ=  0.333 kips
Just to the left of Disk 1
MY=  2.3333 kip.in.
MZ=  -3.5000 kip.in.
T=   0.0000 kip.in.
Diameter must be at least  1.389 in.
Just to the right of Disk 1
T=   2.00 kip.in.
Diameter must be at least  1.437 in.
Just to the left of Disk 2
MY=  4.6667 kip.in.
MZ=  -3.5000 kip.in.
T=   2.0000 kip.in.
Diameter must be at least  1.578 in.
Just to the right of Disk 2
T=  -2.00 kip.in.
Diameter must be at least  1.578 in.
Just to the left of Disk 3
MY=  2.3333 kip.in.
MZ=  -3.5000 kip.in.
T=  -2.0000 kip.in.
Diameter must be at least  1.437 in.
Just to the right of Disk 3
T=   0.00 kip.in.
Diameter must be at least  1.389 in.
```

PROBLEM 8.C4

8.C4 The solid shaft AB of length L, uniform diameter d, and allowable shearing stress τ_{all} rotates at a given speed expressed in rpm (Fig. 8.C4). Gears G_1, G_2, . . . , G_n are attached to the shaft and each of these gears meshes with another gear (not shown), either at the top or bottom of its vertical diameter, or at the left or right end of its horizontal diameter. One of these other gears is connected to a motor and the rest of them to various machine tools. Denoting by r_i the radius of disk G_i, by c_i its distance from the support at A, and by P_i the power transmitted to that gear (+ sign) or taken of that gear (- sign), write a computer program to calculate the smallest permissible value of the diameter d of shaft AB. Use this program to solve Probs. 8.27 and 8.68.



SOLUTION

1. Enter ω in rpm and determine frequency $f = \omega/60$.

2. For each gear, determine the torque $T_i = P_i/2\pi f$, where P_i is the power input (+) or output (−) at the gear.

3. For each gear, determine the force $F_i = T_i/r_i$ exerted on the gear and its components $(F_y)_i$ and $(F_z)_i$.

4. Determine the components A_y and A_z of reaction at A by summing moments about axes $Bz' \parallel z$ and $By' \parallel y$:

$$\Sigma M_{z'} = 0: \; -A_y L - \Sigma (F_y)_i (L - c_i) = 0, \quad A_y = -\frac{1}{L} \Sigma (F_y)_i (L - c_i)$$

$$\Sigma M_{y'} = 0: \; A_z L + \Sigma (F_z)_i (L - c_i) = 0, \quad A_z = -\frac{1}{L} \Sigma (F_z)_i (L - c_i)$$

5. Determine $(M_y)_i$, $(M_z)_i$, and torque T_i just to the left of gear G_i:

$$(M_y)_i = A_z c_i + \sum_k (F_z)_k \langle c_i - c_k \rangle^1$$

$$(M_z)_i = -A_y c_i - \sum_k (F_y)_k \langle c_i - c_k \rangle^1$$

$$T_i = \sum_k T_k \langle c_i - c_k \rangle^0$$

Where $\langle \; \rangle$ indicates a singularity function.

(CONTINUED)

6. The minimum diameter d required to the left of G_i is obtained by first computing $(J/c)_i$ from Eq. (8.7):

$$\left(\frac{J}{c}\right)_i = \frac{\sqrt{(M_y)_i^2 + (M_z)_i^2 + T_i^2}}{\tau_{all}}$$

7. Recalling that $J = \frac{1}{2}\pi c^4$ and, thus, that $\left(\frac{J}{c}\right)_i = \frac{1}{2}\pi c_i^3$, we have $c_i = \frac{2}{\pi}\left(\frac{J}{c}\right)_i^{1/3}$ and $d_i = \frac{4}{\pi}\left(\frac{J}{c}\right)_i^{1/3}$

This is the required diameter just to the left of gear G_i.

8. The required diameter just to the right of gear G_i is obtained by replacing T_i with T_{i+1} in the above computation.

9. The smallest permissible value of the diameter of the shaft is the <u>largest</u> of the values obtained for d_i.

(CONTINUED)

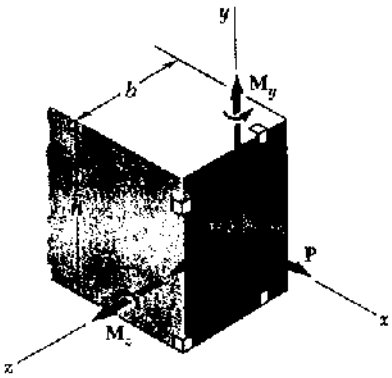
PROBLEM 8.C4 CONTINUED

PROGRAM OUTPUTS

```
Problem  8.27

Omega = 600 rpm
Number of Gears: 3
Length of shaft =   24 in.
Tau =   8 ksi
For Gear 1
Power input =  60.00 hp
Radius of gear= 3.00 in.
Distance from A in inches =    4.0
FY=             0
FZ =            2.100845
For Gear 2
Power input = -40.00 hp
Radius of gear= 4.00 in.
Distance from A in inches =  10.0
FY=             1.050423
FZ =            0
For Gear 3
Power input = -20.00 hp
Radius of gear= 4.00 in.
Distance from A in inches =  18.0
FY=             0
FZ =           -.5252113
AY=-0.6127 kips, AZ=-1.6194 kips
BY=-0.4377 kips, BZ= 0.0438 kips
Just to the left of Gear 1
MY= -6.478 kip.in.
MZ=  2.451 kip.in.
T=  0.000 kip.in.
Diameter must be at least  1.640 in.
Just to the right of Gear 1
T=  6.3025 kip.in.
Diameter must be at least  1.813 in.
Just to the left of Gear 2
MY= -3.589 kip.in.
MZ=  6.127 kip.in.
T=  6.303 kip.in.
Diameter must be at least  1.822 in.
Just to the right of Gear 2
T=  2.1008 kip.in.
Diameter must be at least  1.677 in.
Just to the left of Gear 3
MY=  0.263 kip.in.
MZ=  2.626 kip.in.
T=  2.101 kip.in.
Diameter must be at least  1.290 in.
Just to the right of Gear 3
T=  0.0000 kip.in.
Diameter must be at least  1.189 in.
```

```
Problem  8.68

Omega = 450 rpm
Number of Gears: 3
Length of shaft =  750 mm
Tau =  55 MPa
For Gear 1
Power input =  -8.00 kW
Radius of gear= 60 mm
Distance from A in mm = 150
For Gear 2
Power input =  20.00 kW
Radius of gear=100 mm
Distance from A in mm = 375
For Gear 3
Power input = -12.00 kW
Radius of gear= 60 mm
Distance from A in mm = 600
AY= -0.849 kN, AZ=   4.386
BY= -3.395 kN, BZ=   2.688
Just to the left of Gear 1
MY= 657.84 Nm
MZ= 127.32 Nm
T=   0.00 Nm
Diameter must be at least  39.59 mm
Just to the right of Gear 1
T=-169.77 Nm
Diameter must be at least  40.00 mm
Just to the left of Gear 2
MY=1007.98 Nm
MZ= 318.31 Nm
T=-169.77 Nm
Diameter must be at least  46.28 mm
Just to the right of Gear 2
T= 254.65 Nm
Diameter must be at least  46.52 mm
Just to the left of Gear 3
MY= 403.19 Nm
MZ= 509.30 Nm
T= 254.65 Nm
Diameter must be at least  40.13 mm
Just to the right of Gear 3
T=   0.00 Nm
Diameter must be at least  39.18 mm
```

PROBLEM 8.C5



8.C5 Write a computer program that can be used to calculate the normal and shearing stresses at points with given coordinates y and z located on the surface of a machine part having a rectangular cross section. The internal forces are known to be equivalent to the force-couple system shown. Write the program so that the loads and dimensions can be expressed in either SI or U.S. customary units. Use this program to solve (*a*) Prob. 8.45*a*, (*b*) Prob. 8.47*b*.

SOLUTION

ENTER: b AND h

PROGRAM: $A = bh \qquad I_y = b^3h/12 \qquad I_z = bh^3/12$

FOR POINT ON SURFACE, ENTER y AND z

NOTE y AND z MUST SATISFY ONE OF FOLLOWING:

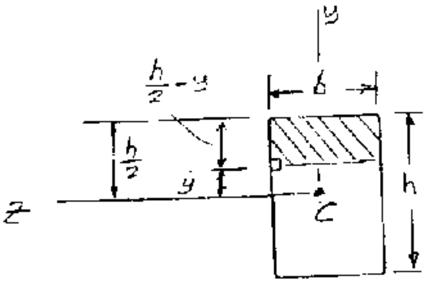
$$y^2 = h^2/4 \quad \text{AND} \quad z^2 \le b^2/4 \qquad (1)$$

OR

$$z^2 = b^2/4 \quad \text{AND} \quad y^2 \le h^2/4 \qquad (2)$$

IF EITHER (1) OR (2) ARE SATISFIED, COMPUTE

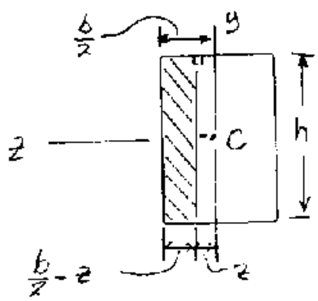
$$\sigma = \frac{P}{A} + \frac{M_y z}{I_y} - \frac{M_z y}{I_z}$$



IF $z^2 = b^2/4$, THEN POINT IS ON VERTICAL SURFACE AND

$$Q_z = b\left(\frac{h}{2} - y\right)\left(\frac{h}{2} + y\right)\frac{1}{2} = b\left(\frac{h^2}{8} - \frac{y^2}{2}\right)$$

$$\tau = \frac{V_y Q_z}{I_z b}$$



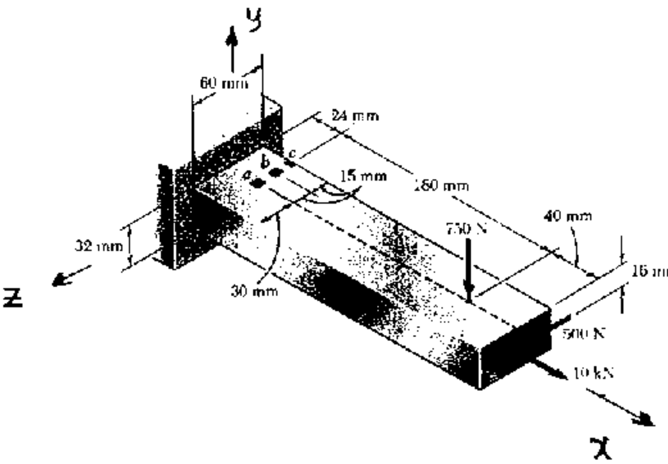
IF $y^2 = h^2/4$, THE POINT IS ON HORIZONTAL SURFACE, AND

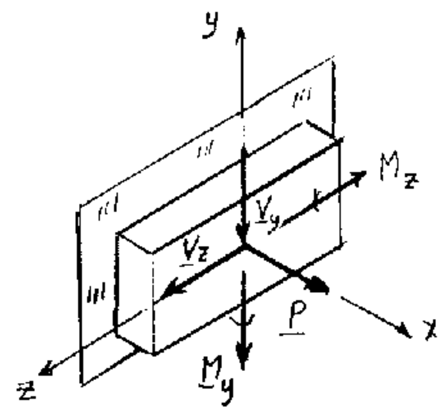
$$Q_y = h\left(\frac{b}{2} - z\right)\left(\frac{b}{2} + z\right)\frac{1}{2} = h\left(\frac{b^2}{8} - \frac{z^2}{2}\right)$$

$$\tau = \frac{V_z Q_y}{I_y h}$$

CONTINUED

PROBLEM 8.C5 CONTINUED



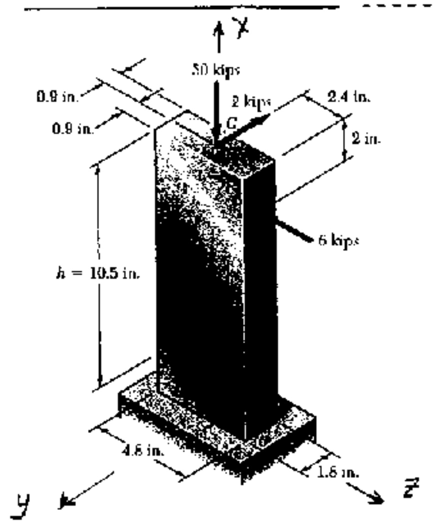


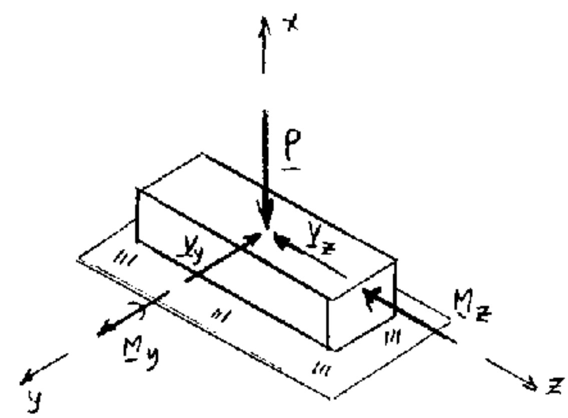
FORCE-COUPLE SYSTEM

$P = 10\ kN \quad V_y = -750\ N \quad V_z = 500\ N$

$M_y = (500\ N)(220\ mm) = 110\ N\cdot m \quad M_z = (750\ N)(180mm) = 135\ N\cdot m$

```
Problem 8.45 a          Force-Couple at Centroid
                  P =   10000.00 N
                 MY =      110.00 N·m      MZ =    -135.00 N·m
                 VY =      750.00 N        VZ =     500.00 N
                 + + + + + + + + + + + + + + + + + + + + + + + + + + +
                At point of coordinates:     y  =  16.00 mm   z =   0.00 mm
                      sigma =  18.392 MPa
                        tau =   0.391 MPa
```





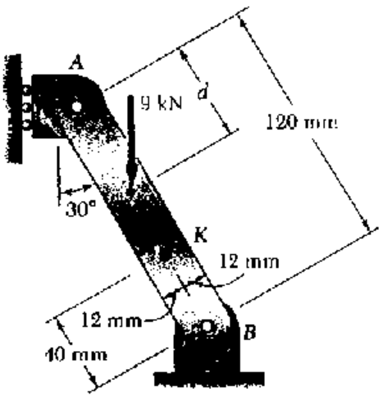
FORCE-COUPLE SYSTEM $\quad P = -50\ kips \quad V_z = -6\ Kips \quad V_y = -2\ Kips$

$M_y = (6Kips)(8.5in.) = 51\ kip\cdot in.$

$M_z = (2Kips)(10.5in.) = 21\ kip\cdot in$

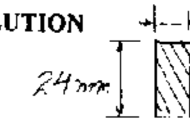
```
Problem 8.47 b          Force-Couple at Centroid
                  P =      -50.00 kips
                 MY =       51.00 kip·in.     MZ =     -21.00 kip·in.
                 VY =       -2.00 kips        VZ =      -6.00 kips
                 + + + + + + + + + + + + + + + + + + + + + + + + + + +
                At point of coordinates:     y  =   0.90 in.    z =    1.20 in.
                      sigma =   6.004 ksi
                        tau =  -0.781 ksi
```

PROBLEM 8.C6



8.C6 Member AB has a rectangular cross section of 10×24 mm. For the loading shown, write a computer program that can be used to determine the normal and shearing stresses at points H and K for values of d from 0 to 120 mm, using 15-mm increments. Use this program to solve Prob. 8.34.

SOLUTION



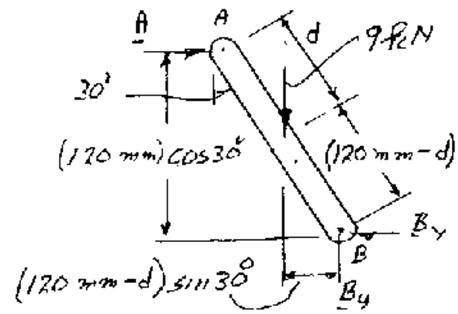
ENTER

$A = (0.010\ m)(0.024\ m) = 240 \times 10^{-6}\ m^2$

$I = (0.010\ m)(0.024\ m)^3/12 = 138.24 \times 10^{-9}\ m^4$

$c = 0.5(0.024\ m) = 12 \times 10^{-3}\ m$

COMPUTE REACTION AT A.

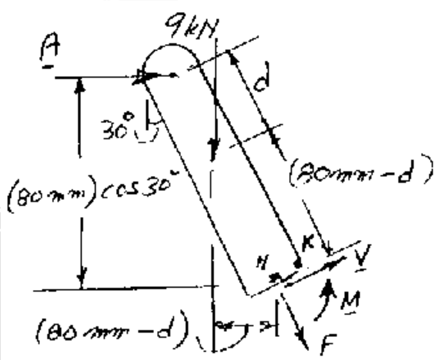


$+\circlearrowleft \Sigma M_B = 0$:

$(9\ kN)(120 - d)\sin 30° - A(120)\cos 30° = 0$

$A = (9\ kN)\dfrac{(120\ mm - d)}{120\ mm}\tan 30°$

FREE BODY FROM A TO SECTION CONTAINING POINTS H AND K.



DEFINE: IF $d < 80$ mm THEN STP = 1 ELSE STP = 0

PROGRAM FORCE-COUPLE SYSTEM

$F = -A\sin 30° - (9\ kN)\cos 30°\,(STP)$

$V = -A\cos 30° + (9\ kN)\sin 30°\,(STP)$

$M = A(80\ mm)\cos 30° - (9\ kN)(80\ mm - d)\sin 30°\,(STP)$

AT POINT H:

$\sigma_H = +F/A \qquad \tau_H = \frac{3}{2}V/A$

AT POINT K:

$\sigma_K = +F/A - Mc/I \qquad \tau_K = 0$

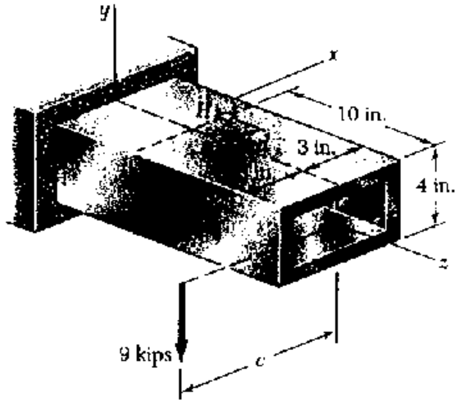
PROGRAM OUTPUT

Problem 8.34

Stresses in MPa

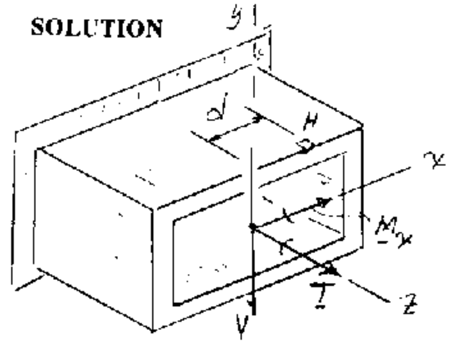
d mm	SigmaH	TauH	SigmaK	TauK
0.0	-43.30	0.00	-43.30	0.00
15.0	-41.95	3.52	-65.39	0.00
30.0	-40.59	7.03	-87.47	0.00
45.0	-39.24	10.55	-109.55	0.00
60.0	-37.89	14.06	-131.64	0.00 ◄
75.0	-36.54	17.58	-153.72	0.00
90.0	-2.71	-7.03	-96.46	0.00
105.0	-1.35	-3.52	-48.23	0.00
120.0	0.00	0.00	0.00	0.00

PROBLEM 8.C7



***8.C7** The structural tube shown has a uniform wall thickness of 0.3 in. A 9-kip force is applied at a bar (not shown) that is welded to the end of th tube. Write a computer program that can be used to determine, for any given value of c, the principal stresses, principal planes, and maximum shearing stress at point H for values of d from -3 in. to 3 in., using one-inch increments. Use this program to solve Prob. 8.64a.

SOLUTION



FORCE - COUPLE SYSTEM

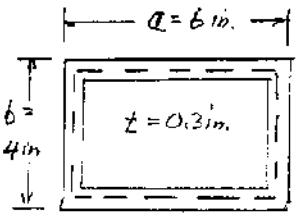
ENTER:

$V = 9\text{ kips} \downarrow$

$M_x = (9\text{ kips})(10\text{ in.})$

$= 90\text{ kip}\cdot\text{in.}$

$T = (9\text{ kips})\,c$

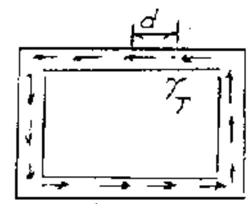


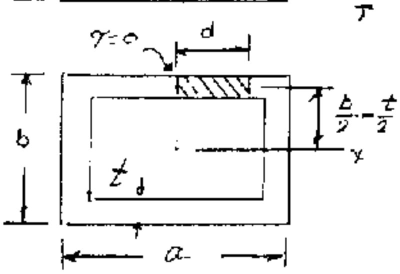
AREA ENCLOSED

$\mathcal{Q} = (a-t)(b-t)$

$\tau_T = \dfrac{T}{2t\mathcal{Q}} = \dfrac{9c}{2t\mathcal{Q}}$

τ_T = SHEARING STRESS DUE TO TORSION



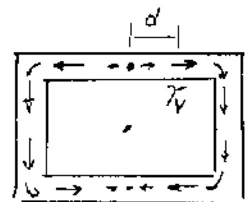


$Q = dt\left(\frac{b}{2} - \frac{t}{2}\right)$

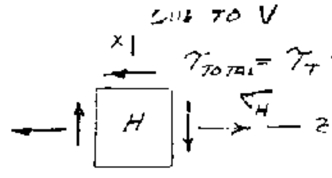
$\bar{I} = ab^3/12 - (a-2t)(b-2t)^3/12$

$\tau_V = \dfrac{VQ}{It}$

τ_V = SHEARING STRESS DUE TO V



$\tau_{TOTAL} = \tau_T - \tau_V$



BENDING: $\sigma_H = \dfrac{M_x\left(\frac{b}{2}\right)}{I}$

PRINCIPAL STRESSES

$\sigma_{ave} = \frac{1}{2}\sigma_H \;;\; R = \sqrt{\left(\frac{\sigma_H}{2}\right)^2 + \tau_{TOTAL}^2}$

$\sigma_{max} = \sigma_{ave} + R \;;\; \sigma_{min} = \sigma_{ave} - R \;;\; \theta_p = \frac{1}{2}\tan^{-1}\left(\frac{\tau_{TOTAL}}{\sigma_{ave}}\right) \;;\; \tau_{max} = \sqrt{\left(\frac{\sigma_H}{2}\right)^2 + \tau_{TOTAL}^2}$

```
Rectangular tube of uniform thickness t = 0.3 in,
Outside dimensions
  Horizontal width a = 6 in.
  Vertical depth   b = 4 in.
Vertical load      P = 9 kips; line of action at x = -c
Find normal and shearing streses at
     Point H (x = d, y = b/2)
```

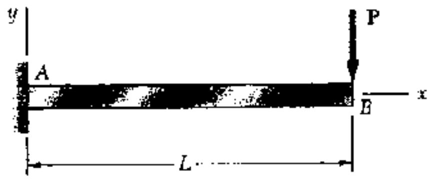
Problem 8.64a Program Output for Value of c = 2.85 in.

d in.	sigma ksi	tauV ksi	tauT ksi	tauTotal ksi	sigmaMax ksi	sigmaMin ksi	tauMax ksi	theta p degrees
-3.00	12.58	-3.49	-2.03	-5.52	14.65	-2.08	8.36	-18.49
-2.00	12.58	-2.33	-2.03	-4.35	13.94	-1.36	7.65	-16.00
-1.00	12.58	-1.16	-2.03	-3.19	13.34	-0.76	7.05	-12.78
0.00	12.58	0.00	-2.03	-2.03	12.89	-0.32	6.61	-8.73
1.00	12.58	1.16	-2.03	-0.86	12.63	-0.06	6.35	-3.89
2.00	12.58	2.33	-2.03	0.30	12.58	-0.01	6.30	1.36
3.00	12.58	3.49	-2.03	1.46	12.74	-0.17	6.46	6.46

Chapter 9

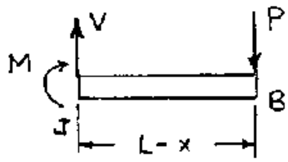
Problem 9.1

9.1 through 9.4 For the loading shown, determine (*a*) the equation of the elastic curve for the cantilever beam *AB*, (*b*) the deflection at the free end, (*c*) the slope at the free end.



$[x = 0,\ y = 0]$

$[x = 0,\ \frac{dy}{dx} = 0]$



$\circlearrowleft \Sigma M_J = 0: \quad -M - P(L-x) = 0$

$$M = -P(L-x)$$

$$EI\frac{d^2y}{dx^2} = -P(L-x) = -PL + Px$$

$$EI\frac{dy}{dx} = -PLx + \tfrac{1}{2}Px^2 + C_1$$

$[x = 0,\ \frac{dy}{dx} = 0] \qquad 0 = -0 + 0 + C_1$

$$C_1 = 0$$

$$EIy = -\tfrac{1}{2}PLx^2 + \tfrac{1}{6}Px^3 + C_1x + C_2$$

$[x = 0,\ y = 0] \qquad 0 = -0 + 0 + 0 + C_2 \qquad C_2 = 0$

(a) Elastic curve. $\quad y = -\dfrac{Px^2}{6EI}(3L - x)$ ◄

$$\frac{dy}{dx} = -\frac{Px}{2EI}(2L - x)$$

(b) y @ $x = L$. $\quad y_B = -\dfrac{PL^2}{6EI}(3L - L) = -\dfrac{PL^3}{3EI}$

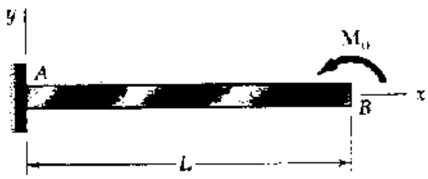
$$y_B = \frac{PL^3}{3EI}\ \downarrow$$ ◄

(c) $\frac{dy}{dx}$ @ $x = L$. $\quad \left.\dfrac{dy}{dx}\right|_B = -\dfrac{PL}{2EI}(2L - L) = -\dfrac{PL^2}{2EI}$

$$\theta_B = \frac{PL^2}{2EI}$$ ◄

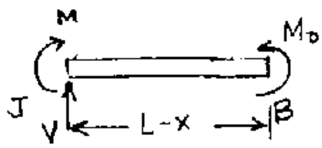
Problem 9.2

9.1 through 9.4 For the loading shown, determine (*a*) the equation of the elastic curve for the cantilever beam *AB*, (*b*) the deflection at the free end, (*c*) the slope at the free end.



$[x=0, y=0]$

$[x=0, \frac{dy}{dx}=0]$



$+\circlearrowleft \sum M_J = 0: \quad M_0 - M = 0 \qquad M = M_0$

$$EI\frac{d^2y}{dx^2} = M_0$$

$$EI\frac{dy}{dx} = M_0 x + C_1$$

$$EI\,y = \frac{1}{2}M_0 x^2 + C_1 x + C_2$$

$[x=0, \frac{dy}{dx} = 0] \qquad 0 = 0 + C_1$

$C_1 = 0$

$$EIy = \frac{1}{2}M_0 x^2 + C_2$$

$[x=0, y=0] \qquad 0 = 0 + C_2 \qquad C_2 = 0$

(a) Elastic curve. $\quad y = \dfrac{M_0 x^2}{2EI}$

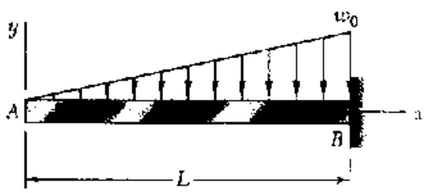
$\dfrac{dy}{dx} = \dfrac{M_0 x}{EI}$

(b) y @ $x = L$. $\quad y_B = \dfrac{M_0 L^2}{2EI} \qquad y_B = \dfrac{M_0 L^2}{2EI} \uparrow$

(c) $\dfrac{dy}{dx}$ @ $x = L$. $\quad \left.\dfrac{dy}{dx}\right|_B = \dfrac{M_0 L}{EI} \qquad \theta_B = \dfrac{M_0 L}{EI} \measuredangle$

Problem 9.3

9.1 through 9.4 For the loading shown, determine (*a*) the equation of the elastic curve for the cantilever beam *AB*, (*b*) the deflection at the free end, (*c*) the slope at the free end.



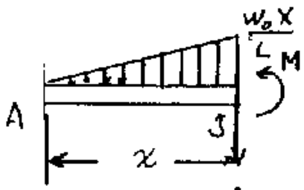
Use free body AJ

$$+\circlearrowleft \Sigma M_J = 0: \quad M + \frac{w_0 x^2}{2L} \cdot \frac{x}{3} = 0$$

$[x = L,\ y = 0]$

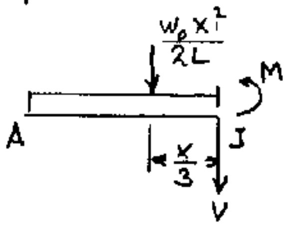
$[x = L,\ \frac{dy}{dx} = 0]$

$$M = -\frac{1}{6} \frac{w_0 x^3}{L}$$



$$\frac{d^2 y}{dx} = -\frac{1}{6} \frac{w_0 x^3}{L}$$

$$EI \frac{dy}{dx} = -\frac{1}{24} \frac{w_0 x^4}{L} + C_1$$



$$EI\,y = -\frac{1}{120} \frac{w_0 x^5}{L} + C_1 x + C_2$$

$[x = L,\ \frac{dy}{dx} = 0]$: $\quad -\frac{1}{24} w_0 L^3 + C_1$

$$C_1 = \frac{1}{24} w_0 L^3$$

$[x = L,\ y = 0]$ $\quad EI\,y = -\frac{1}{120} w_0 L^4 + \frac{1}{24} w_0 L^4 + C_2 = 0$

$$C_2 - \frac{1}{30} w_0 L^4$$

(a) Elastic curve. $\quad y = -\frac{w_0}{120 EI L}(x^5 - 5L^4 x + 4L^5)$ ◄

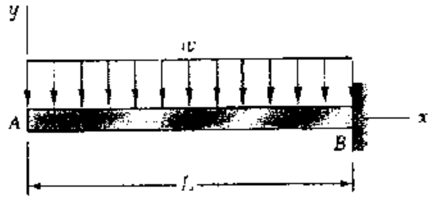
$$\frac{dy}{dx} = \frac{w_0}{24 EIL}(-x^4 + L^4)$$

(b) y @ $x = 0$: $\quad y_A = -\frac{w_0 L^4}{30 EI} \qquad y_A = \frac{w_0 L^4}{30 EI} \downarrow$ ◄

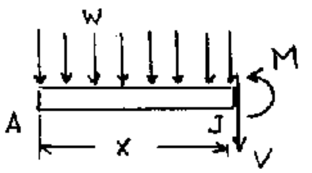
(c) $\frac{dy}{dx}$ @ $x = 0$: $\quad \left.\frac{dy}{dx}\right|_A = \frac{w_0 L^3}{24 EI} \qquad \theta_A = \frac{w_0 L^4}{24 EI}$ ◄

Problem 9.4

9.1 through 9.4 For the loading shown, determine (*a*) the equation of the elastic curve for the cantilever beam *AB*, (*b*) the deflection at the free end, (*c*) the slope at the free end.



[x = L, y = 0]
[x = L, $\frac{dy}{dx}$ = 0]



$$\circlearrowleft \Sigma M_J = 0: \quad (wx)\frac{x}{2} + M = 0$$

$$M = -\frac{1}{2}wx^2$$

$$EI\frac{d^2y}{dx^2} = M = -\frac{1}{2}wx^2$$

$$EI\frac{dy}{dx} = -\frac{1}{6}wx^3 + C_1$$

$$\left[x = L, \frac{dy}{dx} = 0\right] \qquad 0 = -\frac{1}{6}wL^3 + C_1$$

$$C_1 = \frac{1}{6}wL^3$$

$$EI\frac{dy}{dx} = -\frac{1}{6}wx^3 + \frac{1}{6}wL^3$$

$$EIy = -\frac{1}{24}wx^4 + \frac{1}{6}wL^3x + C_2$$

$$[x = L, y = 0] \qquad 0 = -\frac{1}{24}wL^4 + \frac{1}{6}wL^4 + C_2 = 0$$

$$C_2 = \left(\frac{1}{24} - \frac{1}{6}\right)wL^4 = -\frac{3}{24}wL^4$$

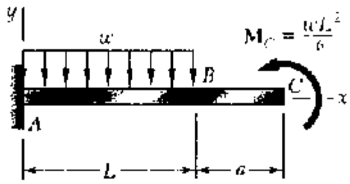
(a) Elastic curve. $\quad y = -\frac{w}{24EI}\left(x^4 - 4L^3x + 3L^4\right)$ ◄

(b) y @ $x = 0$: $\quad y_A = -\frac{3wL^4}{24EI} = -\frac{wL^4}{8EI} \qquad y_A = \frac{wL^4}{8EI}\downarrow$ ◄

(c) $\frac{dy}{dx}$ @ $x = 0$: $\quad \left.\frac{dy}{dx}\right|_A = \frac{wL^3}{6EI} \qquad \theta_A = \frac{wL^3}{6EI}$ ◄

Problem 9.5

9.5 and 9.6 For the cantilever beam and loading shown, determine (*a*) the equation of the elastic curve for portion *AB* of the beam, (*b*) the deflection at *B*, (*c*) the slope at *B*.



$[x = 0, y = 0]$

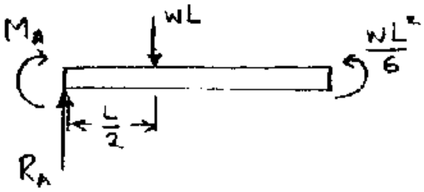
$[x = 0, \frac{dy}{dx} = 0]$

Using ABC as a free body

$+\uparrow \Sigma F_y = 0: \quad R_A - wL = 0 \qquad R_A = wL$

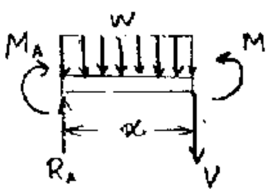
$+\circlearrowleft \Sigma M_A = 0: \quad -M_A - (wL)(\frac{L}{2}) + \frac{wL^2}{6} = 0$

$M_A = -\frac{1}{3}wL$



Using AJ as a free body (Portion AB only)

$+\circlearrowleft M_J = 0: \quad M + (wx)(\frac{x}{2}) - R_A x - M_A = 0$



$$M = -\frac{1}{2}wx^2 + R_A x + M_A$$

$$= -\frac{1}{2}wx^2 + wLx - \frac{1}{3}wL^2$$

$$EI\frac{d^2y}{dx^2} = -\frac{1}{2}wx^2 + wLx - \frac{1}{3}wL^2$$

$$EI\frac{dy}{dx} = -\frac{1}{6}wx^3 + \frac{1}{2}wLx^2 - \frac{1}{3}wLx + C_1$$

$[x = 0, \frac{dy}{dx} = 0]: \quad -0 + 0 - 0 + C_1 = 0 \qquad C_1 = 0$

$$EIy = -\frac{1}{24}wx^4 + \frac{1}{6}wLx^3 - \frac{1}{6}wLx^2 + C_2$$

$[x = 0, y = 0] \quad -0 + 0 - 0 + C_2 = 0 \qquad C_2 = 0$

(a) Elastic curve over AB. $\quad y = \frac{w}{24EI}(-x^4 + 4Lx^3 - 4L^2x^2)$ ◄

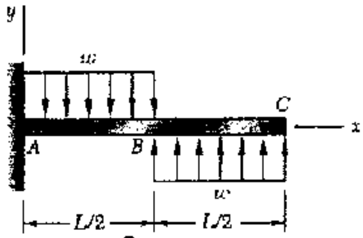
$$\frac{dy}{dx} = \frac{w}{6EI}(-x^3 + 2Lx^2 - L^2x)$$

(b) y at $x = L$: $\quad y_B = -\frac{wL^4}{24EI} \qquad y_B = \frac{wL^4}{24EI} \downarrow$ ◄

(c) $\frac{dy}{dx}$ at $x = L$: $\quad \left.\frac{dy}{dx}\right|_B = 0 \qquad \theta_B = 0$ ◄

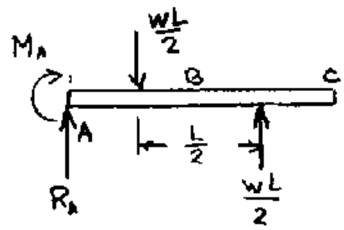
Problem 9.6

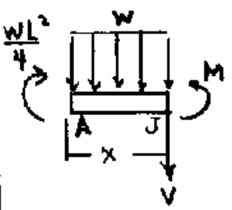
9.5 and 9.6 For the cantilever beam and loading shown, determine (*a*) the equation of the elastic curve for portion *AB* of the beam, (*b*) the deflection at *B*, (*c*) the slope at *B*.



$[x=0,\ y=0]$

$[x=0,\ \frac{dy}{dx}=0]$





Using ABC as a free body.

$+\uparrow \Sigma F_y = 0: \quad R_A - \frac{wL}{2} + \frac{wL}{2} = 0 \qquad R_A = 0$

$+\circlearrowleft M_A = 0: \quad -M_A + \left(\frac{wL}{2}\right)\left(\frac{L}{2}\right) = 0 \qquad M_A = \frac{wL^2}{4}$

Using AJ as a free body (Portion AB only)

$\circlearrowleft \Sigma M_J = 0: \quad -\frac{wL^2}{4} + (wx)\frac{x}{2} + M = 0$

$M = \frac{1}{4}wL^2 - \frac{1}{2}wx^2$

$EI\frac{d^2y}{dx^2} = \frac{1}{4}wL^2 - \frac{1}{2}wx^2$

$EI\frac{dy}{dx} = \frac{1}{4}wL^2x - \frac{1}{6}wx^3 + C_1$

$[x=0,\ \frac{dy}{dx}=0] \qquad 0 = 0 - 0 + C_1 \qquad C_1 = 0$

$EI\,y = \frac{1}{8}wL^2x^2 - \frac{1}{24}wx^4 + C_1x + C_2$

$[x=0,\ y=0] \qquad 0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$

(a) Elastic curve. $\quad y = \frac{w}{EI}\left(\frac{1}{8}L^2x^2 - \frac{1}{24}x^4\right)$ ◄

$\frac{dy}{dx} = \frac{w}{EI}\left(\frac{1}{4}L^2x - \frac{1}{6}x^3\right)$

(b) y at $x = \frac{L}{2}$: $\quad y_B = \frac{w}{EI}\left\{\frac{1}{8}L^2\left(\frac{L}{2}\right)^2 - \frac{1}{24}\left(\frac{L}{2}\right)^4\right\} = \frac{wL^4}{EI}\left\{\frac{1}{32} - \frac{1}{384}\right\}$

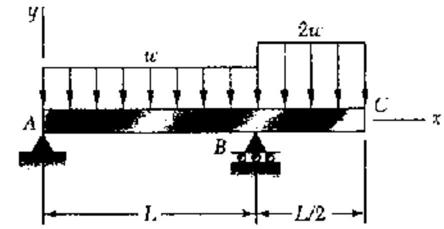
$= \frac{11\,wL^4}{384\,EI} \qquad y_B = \frac{11\,wL^4}{384\,EI}\uparrow$ ◄

(c) $\frac{dy}{dx}$ at $x = \frac{L}{2}$: $\quad \theta_B = \frac{w}{EI}\left\{\frac{1}{4}L^2\left(\frac{L}{2}\right) - \frac{1}{6}\left(\frac{L}{2}\right)^3\right\} = \frac{wL^3}{EI}\left\{\frac{1}{8} - \frac{1}{48}\right\}$

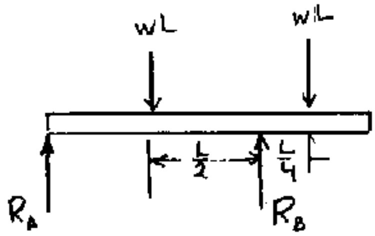
$= \frac{5\,wL^3}{48\,EI} \qquad \theta_B = \frac{5\,wL^3}{48\,EI}$ ◄

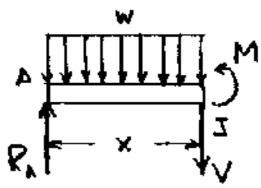
Problem 9.7

9.7 For the beam and loading shown, determine (*a*) the equation of the elastic curve for portion *AB* of the beam, (*b*) the slope at *A*, (*c*) the slope at *B*.



$[x=0,\ y=0]$ $\quad$ $[x=L,\ y=0]$





Using free body ABC

$$+\circlearrowleft \Sigma M_B = 0: \quad -R_A L + (wL)\left(\frac{L}{2}\right) - (wL)\left(\frac{L}{4}\right) = 0$$

$$R_A = \frac{1}{4}wL$$

For portion AB $\quad (0 < x < L)$

$$+\circlearrowleft \Sigma M_J = 0 \qquad M - R_A x + (wx)\left(\frac{x}{2}\right) = 0$$

$$M = \frac{1}{4}wLx - \frac{1}{2}wx^2$$

$$EI\frac{d^2y}{dx^2} = \frac{1}{4}wLx - \frac{1}{2}wx^2$$

$$EI\frac{dy}{dx} = \frac{1}{8}wLx^2 - \frac{1}{6}wx^3 + C_1$$

$$EI\,y = \frac{1}{24}wLx^3 - \frac{1}{24}wx^4 + C_1 x + C_2$$

$[x=0,\ y=0]$: $\quad 0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$

$[x=L,\ y=0]$ $\quad 0 = \frac{1}{24}wL^4 - \frac{1}{24}wL^4 + C_1 L + 0 = 0 \qquad C_1 = 0$

(a) Elastic curve $(0 \le x \le L)$. $\qquad y = \frac{w}{24EI}(Lx^3 - x^4)$ ◄

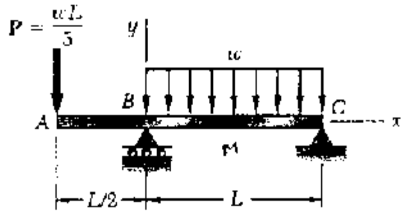
$$\frac{dy}{dx} = \frac{w}{24EI}\left(3Lx^2 - 4x^3\right)$$

(b) $\frac{dy}{dx}$ at $x=0$ $\qquad \left.\frac{dy}{dx}\right|_A = 0 \qquad \theta_A = 0$ ◄

(c) $\frac{dy}{dx}$ at $x = L$ $\qquad \left.\frac{dy}{dx}\right|_B = -\frac{wL^3}{24EI} \qquad \theta_B = \frac{wL^3}{24EI}$ ◄

Problem 9.8

9.8 For the beam and loading shown, determine (*a*) the equation of the elastic curve for portion *BC* of the beam, (*b*) the deflection at midspan, (*c*) the slope at *B*.

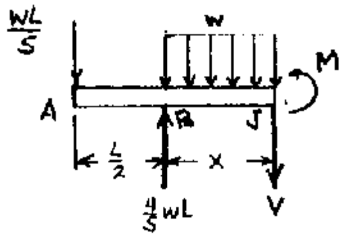


$x = -\frac{L}{2}$ $\begin{bmatrix} x = 0 \\ y = 0 \end{bmatrix}$ $\begin{bmatrix} x = L \\ y = 0 \end{bmatrix}$

Using ABC as a free body

$$+\circlearrowleft \Sigma M_C = 0: \quad \left(\frac{wL}{5}\right)\left(\frac{3L}{2}\right) - R_B L + (wL)\left(\frac{L}{2}\right) = 0$$

$$R_B = \frac{4}{5} wL$$



For portion BC only $(0 < x < L)$

$$+\circlearrowleft \Sigma M_J = 0: \quad \frac{wL}{5}\left(\frac{L}{2} + x\right) - \frac{4}{5} wLx + (wx)\frac{x}{2} + M = 0$$

$$M = \frac{3}{5} wLx - \frac{1}{2} wx^2 - \frac{1}{10} wL^2$$

$$EI \frac{d^2y}{dx^2} = \frac{3}{5} wLx - \frac{1}{2} wx^2 - \frac{1}{10} wL^2$$

$$EI \frac{dy}{dx} = \frac{3}{10} wLx^2 - \frac{1}{6} wx^3 - \frac{1}{10} wL^2 x + C_1$$

$$EI\, y = \frac{1}{10} wLx^3 - \frac{1}{24} wx^4 - \frac{1}{20} wL^2 x^2 + C_1 x + C_2$$

$[x = 0,\ y = 0]$ $\quad 0 = 0 - 0 - 0 + 0 + C_2 \qquad C_2 = 0$

$[x = L,\ y = 0]$ $\quad 0 = \left(\frac{1}{10} - \frac{1}{24} - \frac{1}{20}\right) wL^4 + C_1 L + 0 \qquad C_1 = -\frac{1}{120} wL^3$

(a) Elastic curve.

$$y = \frac{w}{EI}\left(\frac{1}{10} Lx^3 - \frac{1}{24} x^4 - \frac{1}{20} L^2 x^2 - \frac{1}{120} L^3 x\right)$$

$$\frac{dy}{dx} = \frac{w}{EI}\left(\frac{3}{10} Lx^2 - \frac{1}{6} x^3 - \frac{1}{10} L^2 x - \frac{1}{120} L^3\right)$$

(b) y @ $x = \frac{L}{2}$:

$$y_M = \frac{w}{EI}\left\{\frac{1}{10} L\left(\frac{L}{2}\right)^3 - \frac{1}{24}\left(\frac{L}{2}\right)^4 - \frac{1}{20} L^2\left(\frac{L}{2}\right)^2 - \frac{1}{120} L^3\left(\frac{L}{2}\right)\right\}$$

$$= \frac{wL^4}{EI}\left\{\frac{1}{80} - \frac{1}{384} - \frac{1}{80} - \frac{1}{240}\right\} = -\frac{13\, wL^4}{1920\, EI}$$

$$y_M = \frac{13\, wL^4}{1920\, EI} \downarrow$$

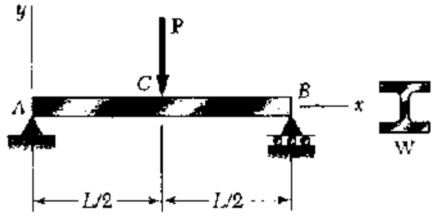
(c) $\frac{dy}{dx}$ @ $x = 0$:

$$\left.\frac{dy}{dx}\right|_B = \frac{w}{EI}\left(0 - 0 - 0 - \frac{1}{120} L^3\right) = -\frac{wL^3}{120\, EI}$$

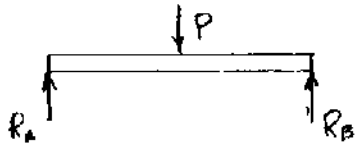
$$\theta_B = \frac{wL^3}{120\, EI}$$

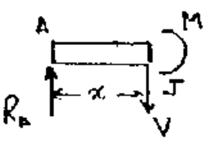
Problem 9.9

9.9 Knowing that beam AB is a W 130 × 23.8 rolled shape and that $P = 50$ kN, $L = 1.25$ m, and $E = 200$ GPa, determine (*a*) the slope at A, (*b*) the deflection at C.



$[x=0,\ y=0]$ $[x=L,\ y=0]$ $[x=\frac{L}{2},\ \frac{dy}{dx}=0]$





Use symmetry boundary condition at C.

By symmetry $\quad R_A = R_B = \frac{1}{2}P$

Using free body AJ $\quad 0 \le x \le \frac{L}{2}$

$+\circlearrowleft \Sigma M_J = 0 \qquad M - R_A x = 0$

$$M = R_A x = \tfrac{1}{2}Px$$

$$EI\frac{d^2y}{dx^2} = \tfrac{1}{2}Px$$

$$EI\frac{dy}{dx} = \tfrac{1}{4}Px^2 + C_1$$

$$EIy = \tfrac{1}{12}Px^3 + C_1 x + C_2$$

$[x=0,\ y=0] \qquad 0 = 0 + 0 + C_2 \qquad C_2 = 0$

$[x=\frac{L}{2},\ \frac{dy}{dx}=0] \qquad 0 = \frac{1}{4}P\left(\frac{L}{2}\right)^2 + C_1 \qquad C_1 = -\frac{1}{16}PL^2$

Elastic curve. $\quad y = \dfrac{PL}{48EI}(4x^3 - 3L^2x)$

$$\frac{dy}{dx} = \frac{PL}{16EI}(4x^2 - L^2)$$

Slope at $x = 0$. $\quad \left.\dfrac{dy}{dx}\right|_A = -\dfrac{PL^3}{16EI} \qquad \theta_A = \dfrac{PL^2}{16EI}$

Deflection at $x = \frac{L}{2}$. $\quad y_C = -\dfrac{PL^3}{48EI} \qquad y_C = \dfrac{PL^3}{48EI} \downarrow$

Data: $\quad P = 50\times10^3$ N, $\quad I = 8.80\times10^6\ \text{mm}^4 = 8.80\times10^{-6}\ \text{m}^4$

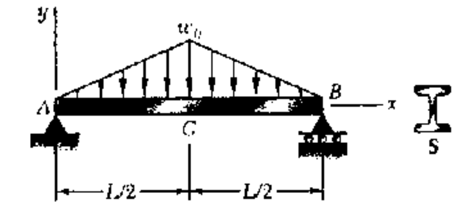
$E = 200\times10^9$ Pa $\qquad EI = 1.76\times10^6\ \text{N}\cdot\text{m}^2 \qquad L = 1.25$ m

(a) $\quad \theta_A = \dfrac{(50\times10^3)(1.25)^2}{(16)(1.76\times10^6)} = 2.77\times10^{-3}$ rad ◄

(b) $\quad y_C = \dfrac{(50\times10^3)(1.25)^3}{(48)(1.76\times10^6)} = 1.156\times10^{-3}$ m $= 1.156$ mm $\downarrow$ ◄

Problem 9.10

9.10 Knowing that beam AB is an S 8 × 18.4 rolled shape and that w_0 = 4 kips/ft, L = 9 ft, and $E = 29 \times 10^6$ psi, determine (*a*) the slope at A, (*b*) the deflection at C.



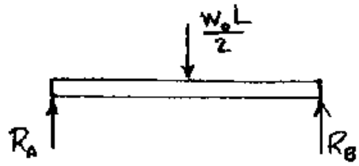
$[x=0, y=0]$ $\quad$ $[x=L, y=0]$

$[x=\frac{L}{2}, \frac{dy}{dx}=0]$

Use symmetry boundary condition at C.

Using free body ACB and symmetry

$$R_A = R_B = \tfrac{1}{4} w_0 L$$

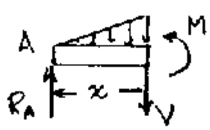


For $0 < x < \frac{L}{2}$ $\qquad w = \dfrac{2w_0 x}{L}$

$$\frac{dV}{dx} = -w = -\frac{2w_0 x}{L}$$

$$\frac{dM}{dx} = V = -\frac{w_0 x^2}{L} + R_A = \frac{w_0}{L}\left(\tfrac{1}{4}L^2 - x^2\right)$$

$$M = \frac{w_0}{L}\left(\tfrac{1}{4}L^2 x - \tfrac{1}{3}x^3\right) + C_M$$



But $M = 0$ at $x = 0$; hence $C_M = 0$

$$EI\frac{d^2y}{dx^2} = \frac{w_0}{L}\left(\tfrac{1}{4}L^2 x - \tfrac{1}{3}x^3\right)$$

$$EI\frac{dy}{dx} = \frac{w_0}{L}\left(\tfrac{1}{8}L^2x^2 - \tfrac{1}{12}x^4\right) + C_1$$

$[x=\frac{L}{2}, \frac{dy}{dx}=0]$ $\qquad 0 = \dfrac{w_0}{L}\left(\tfrac{1}{32}L^4 - \tfrac{1}{192}L^4\right) + C_1 = 0 \qquad C_1 = -\tfrac{5}{192}w_0 L^3$

$$EIy = \frac{w_0}{L}\left(\tfrac{1}{24}L^2x^3 - \tfrac{1}{120}x^5\right) - \tfrac{5}{192}w_0L^3x + C_2$$

$[x=0, y=0]$ $\qquad 0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$

Elastic curve. $\qquad y = \dfrac{w_0}{EIL}\left(\tfrac{1}{24}L^2x^3 - \tfrac{1}{60}x^5 - \tfrac{5}{192}L^4x\right)$

$$\frac{dy}{dx} = \frac{w_0}{EIL}\left(\tfrac{1}{8}L^2x^2 - \tfrac{1}{12}x^4 - \tfrac{5}{192}L^4\right)$$

Data: $w_0 = 4$ kips/ft $\qquad E = 29\times10^6$ psi, $\qquad I = 57.6$ in^4

$EI = (29\times10^6)(57.6) = 1.6704\times10^9$ lb·in^2 = 11.6×10^3 kip·ft^2

$L = 9$ ft

(a) Slope at $x = 0$. $\qquad \dfrac{dy}{dx} = \dfrac{4}{(11.6\times10^3)(9)}\left[-\left(\tfrac{5}{192}\right)(9)^4\right] = -6.55\times10^{-3}$

$$\theta_A = 6.55\times10^{-3} \text{ rad} \measuredangle$$

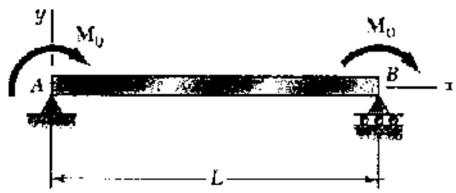
(b) Deflection at $x = 4.5$ ft.

$$y = \frac{4}{(11.6\times10^3)(9)}\left[\left(\tfrac{1}{24}\right)(9)^2(4.5)^3 - \tfrac{1}{60}(4.5)^5 - \tfrac{5}{192}(9)^4(4.5)\right] = -18.853\times10^{-3} \text{ ft}$$

$$= 0.226 \text{ in.} \downarrow$$

Problem 9.11

9.11 (*a*) Determine the location and magnitude of the maximum absolute deflection in AB between A and the center of the beam. (*b*) Assuming that beam AB is a W 460 × 113, $M_0 = 224$ kN·m and $E = 200$ GPa, determine the maximum allowable length L so that the maximum deflection does not exceed 1.2 mm.

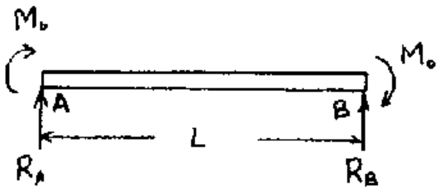


$[x=0,\ y=0]$ $\qquad$ $[x=L,\ y=0]$

Using AB as a free body

$$\Sigma M_B = 0: \quad -2M_0 - R_A L = 0$$

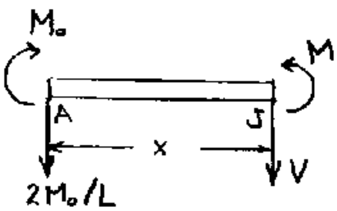
$$R_A = -\frac{2M_0}{L}$$



Using portion AJ as a free body

$$\Sigma M_J = 0: \quad -M_0 + \frac{2M_0}{L}x + M = 0$$

$$M = \frac{M_0}{L}(L - 2x)$$



$$EI\frac{d^2y}{dx^2} = \frac{M_0}{L}(L-2x)$$

$$EI\frac{dy}{dx} = \frac{M_0}{L}(Lx - x^2) + C_1$$

$$EI\,y = \frac{M_0}{L}\left(\tfrac{1}{2}Lx^2 - \tfrac{1}{3}x^3\right) + C_1 x + C_2$$

$[x=0,\ y=0]$ $\qquad 0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$

$[x=L,\ y=0]$ $\qquad 0 = \frac{M_0}{L}\left(\tfrac{1}{2}L^3 - \tfrac{1}{3}L^3\right) + C_1 L + 0 \qquad C_1 = -\tfrac{1}{6}M_0 L^2$

$$y = \frac{M_0}{EIL}\left(\tfrac{1}{2}Lx^2 - \tfrac{1}{3}x^3 - \tfrac{1}{6}L^2 x\right) \qquad \frac{dy}{dx} = \frac{M_0}{EIL}\left(Lx - x^2 - \tfrac{1}{6}L^2\right)$$

To find location of maximum deflection set $\frac{dy}{dx} = 0$

$$x_m^2 - Lx_m - \tfrac{1}{6}L^2 = 0 \qquad x_m = \frac{L - \sqrt{L^2 - (4)(\frac{1}{6}L^2)}}{2} = \tfrac{1}{2}\left(1 - \tfrac{\sqrt{3}}{3}\right)L = 0.21132\ L$$

$$y_m = \frac{M_0 L^2}{EI}\left\{(\tfrac{1}{2})(0.21132)^2 - (\tfrac{1}{3})(0.21132)^3 - (\tfrac{1}{6})(0.21132)\right\} = -0.0160375\frac{M_0 L^2}{EI}$$

$$|y_m| = 0.0160375\frac{M_0 L^2}{EI}$$

Solving for L, $\qquad L = \left\{\dfrac{EI|y_m|}{0.0160375 M_0}\right\}^{1/2}$

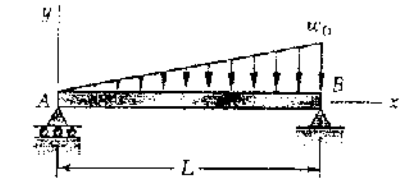
Data: $E = 200 \times 10^9$ Pa, $\quad I = 556 \times 10^6\ \text{mm}^4 = 556 \times 10^{-6}\ \text{m}^4$

$|y_m| = 1.2\ \text{mm} = 1.2 \times 10^{-3}\ \text{m}, \qquad M_0 = 224 \times 10^3\ \text{N}\cdot\text{m}$

$$L = \left\{\frac{(200\times10^9)(556\times10^{-6})(1.2\times10^{-3})}{(0.0160375)(224\times10^3)}\right\}^{1/2} = 6.09\ \text{m}$$

Problem 9.12

9.12 For the beam and loading shown, (*a*) express the magnitude and location of the maximum deflection in terms of w_0, L, E, and I. (*b*) Calculate the value of the maximum deflection, assuming that beam AB is a W 18 × 50 rolled shape and that $w_0 = 4.5$ kips/ft, $L = 18$ ft, and $E = 29 \times 10^6$ psi.

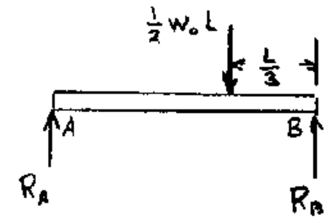


$[x=0, y=0]$ $[x=L, y=0]$

Using entire beam as a free body,

$\circlearrowleft \Sigma M_B = 0: \quad -R_A L + (\frac{1}{2} w_0 L)(\frac{L}{3}) = 0$

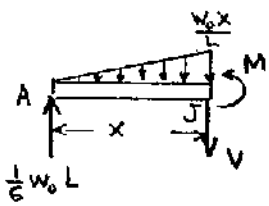
$R_A = \frac{1}{6} w_0 L$



Using AJ as a free body, $\circlearrowleft \Sigma M_J = 0$:

$-\frac{1}{6} w_0 L x + (\frac{1}{2} \frac{w_0 x^2}{L})(\frac{x}{3}) + M = 0$

$M = \frac{1}{6} w_0 L x - \frac{1}{6} \frac{w_0}{L} x^3$



$$EI \frac{d^2y}{dx^2} = \frac{1}{6} w_0 L x - \frac{1}{6} \frac{w_0}{L} x^3$$

$$EI \frac{dy}{dx} = \frac{1}{12} w_0 L x^2 - \frac{1}{24} \frac{w_0}{L} x^4 + C_1$$

$$EI y = \frac{1}{36} w_0 L x^3 - \frac{1}{120} \frac{w_0}{L} x^5 + C_1 x + C_2$$

$[x=0, y=0]$ $\quad 0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$

$[x=L, y=0]$ $\quad 0 = \frac{1}{36} w_0 L^4 - \frac{1}{120} w_0 L^4 + C_1 L + 0 \qquad C_1 = -\frac{7 w_0 L^3}{360}$

Elastic curve. $\quad y = \frac{w_0}{EI} \left\{ \frac{1}{36} L x^3 - \frac{1}{120} \frac{x^5}{L} - \frac{7}{360} L^3 x \right\}$

$$\frac{dy}{dx} = \frac{w_0}{EI} \left\{ \frac{1}{12} L x^2 - \frac{1}{24} \frac{x^4}{L} - \frac{7}{360} L^3 \right\}$$

To find location of maximum deflection, set $\frac{dy}{dx} = 0$

$15 x_m^4 - 30 L^2 x_m^2 + 7 L^4 = 0 \qquad x_m^2 = \frac{30 L^2 - \sqrt{900 L^4 - 420 L^4}}{30}$

$x_m^2 = (1 - \sqrt{\frac{8}{15}}) L^2 = 0.2697 L^2 \qquad x_m = 0.5193 L$

$y_m = \frac{w_0}{EI} \left\{ \frac{1}{36} L (0.5193 L)^3 - \frac{1}{120} \frac{(0.5193 L^5)}{L} - \frac{7}{360} L^3 (0.5193 L) \right\}$

$= -0.00652 \frac{w_0 L^4}{EI}$ or $0.00652 \frac{w_0 L^4}{EI} \downarrow$

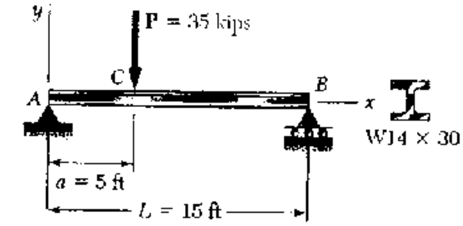
Data: $w_0 = 4.5$ kips/ft $= \frac{4500}{12} = 375$ lb/in, $\quad L = 18$ ft $= 216$ in.

$I = 800$ in^4 for W18×50

$y_m = \frac{(0.00652)(375)(216)^4}{(29 \times 10^6)(800)} = 0.229$ in. $\downarrow$

Problem 9.13

9.13 For the beam and loading shown, determine the deflection at point C. Use $E = 29 \times 10^6$ psi.



$[x=0,\ y=0]$ $\quad$ $[x=L,\ y=0]$

$[x=a,\ y=y]$

$[x=a,\ \frac{dy}{dx}=\frac{dy}{dx}]$

Let $b = L - a$

Reactions: $R_A = \frac{Pb}{L}\uparrow$, $R_B = \frac{Pa}{L}\uparrow$

Bending moments

$0 < x < a \qquad M = \frac{Pb}{L}x$

$a < x < L \qquad M = \frac{P}{L}[bx - L(x-a)]$

$0 < x < a$

$$EI\frac{d^2y}{dx^2} = \frac{P}{L}(bx)$$

$$EI\frac{dy}{dx} = \frac{P}{L}\left(\frac{1}{2}bx^2\right) + C_1 \qquad (1)$$

$$EI\,y = \frac{P}{L}\left(\frac{1}{6}bx^3\right) + C_1x + C_2 \qquad (2)$$

$a < x < L$

$$EI\frac{d^2y}{dx^2} = \frac{P}{L}[bx - L(x-a)]$$

$$EI\frac{dy}{dx} = \frac{P}{L}\left[\frac{1}{2}bx^2 - \frac{1}{2}L(x-a)^2\right] + C_3 \qquad (2)$$

$$EI\,y = \frac{P}{L}\left[\frac{1}{6}bx^3 - \frac{1}{6}L(x-a)^3\right] + C_3x + C_4 \qquad (4)$$

$[x=0,\ y=0]$ Eq (2) $\quad 0 = 0 + 0 + C_2 \qquad C_2 = 0$

$[x=a,\ \frac{dy}{dx}=\frac{dy}{dx}]$ Eqs. (1) and (3) $\quad \frac{P}{L}\left(\cancel{\frac{1}{2}ba^2}\right) + C_1 = \frac{P}{L}\left[\cancel{\frac{1}{2}ba^2} + 0\right] + C_3 \therefore\ C_3 = C_1$

$[x=a,\ y=y]$ Eqs (2) and (4) $\quad \frac{P}{L}\left(\cancel{\frac{1}{6}ba^3}\right) + C_1a + C_2$

$= \frac{P}{L}\left[\cancel{\frac{1}{6}ba^3} + 0\right] + \cancel{C_1a} + C_4 \qquad C_4 = C_2 = 0$

$[x=L,\ y=0]$ Eq. (4) $\quad \frac{P}{L}\left[\frac{1}{6}bL^3 - \frac{1}{6}L(L-a)^3\right] + C_3L = 0$

$$C_1 = C_3 = \frac{P}{L}\left[\frac{1}{6}(L-a)^3 - \frac{1}{6}bL^2\right] = \frac{P}{L}\left(\frac{1}{6}b^3 - \frac{1}{6}bL^2\right)$$

Make $x = a$ in Eq. (2).

$$y_C = \frac{P}{EIL}\left[\frac{1}{6}ba^3 + \frac{1}{6}b^3a - \frac{1}{6}bL^2a\right] = \frac{P(ba^3 + b^3a - L^2ab)}{6EIL}$$

Data: $P = 35$ kips, $E = 29 \times 10^6$ psi $= 29 \times 10^3$ kips/in²

$L = 15$ ft, $a = 5$ ft, $b = 10$ ft.

$I = 291$ in⁴, $EI = 8.439 \times 10^6$ kip·in² $= 58.604 \times 10^3$ kip·ft²

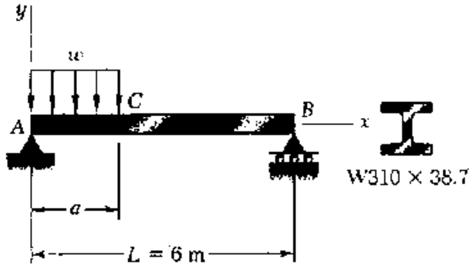
$$y_C = \frac{35}{(6)(58.604\times10^3)(15)}\left[(10)(5)^3 + (10^3)(5) - (15)^2(5)(10)\right]$$

$$= -33.179 \times 10^{-3}\text{ ft} = -0.398\text{ in.}$$

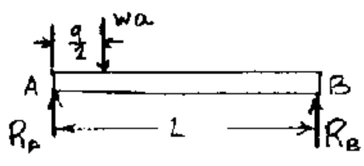
$y_C = 0.398$ in. $\downarrow$

Problem 9.14

9.14 For the beam and loading shown, knowing that $a = 2$ m, $w = 50$ kN/m, and $E = 200$ GPa, determine (*a*) the slope at support *A*, (*b*) the deflection at point *C*.



Using ACB as a free body and noting that $L = 3a$,



$+\circlearrowleft \Sigma M_A = 0: \quad R_B L - (wa)\left(\frac{a}{2}\right) = 0$

$R_B = (wa)\frac{a}{2L} = \frac{1}{6}wa$

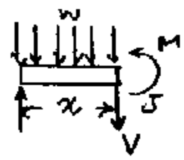
$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - wa = 0 \quad R_A = \frac{5}{6}wa$

$[x=0,\ y=0] \quad [x=L,\ y=0]$

$[x=a,\ y=y]$

$[x=a,\ \frac{dy}{dx}=\frac{dy}{dx}]$

$0 \le x \le a$



$+\circlearrowleft \Sigma M_J = 0:$

$M - R_A x + (wx)\left(\frac{x}{2}\right) = 0$

$M = R_A x - \frac{1}{2}wx^2$

$EI\frac{d^2y}{dx^2} = R_A x - \frac{1}{2}wx^2$

$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{6}wx^3 + C_1$

$EI\,y = \frac{1}{6}R_A x^3 - \frac{1}{24}wx^4 + C_1 x + C_2$

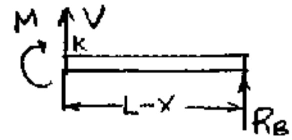
$[x=0,\ y=0] \quad 0 = 0 - 0 + 0 + C_2$

$C_2 = 0$

$EI\,y = \frac{1}{6}R_A x^3 - \frac{1}{24}wx^4 + C_1 x$

$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{6}wx^3 + C_1$

$a \le x \le L$



$+\circlearrowleft \Sigma M_K = 0$

$-M + R_B(L-x) = 0$

$M = R_B(L-x)$

$EI\frac{d^2y}{dx^2} = R_B(L-x)$

$EI\frac{dy}{dx} = -\frac{1}{2}R_B(L-x)^2 + C_3$

$EI\,y = \frac{1}{6}R_B(L-x)^3 + C_3 x + C_4$

$[x=L,\ y=0] \quad 0 = 0 + C_3 L + C_4$

$C_4 = -C_3 L$

$EI\,y = \frac{1}{6}R_B(L-x)^3 - C_3(L-x)$

$EI\frac{dy}{dx} = -\frac{1}{2}R_B(L-x)^2 + C_3$

$[x=a,\ \frac{dy}{dx}=\frac{dy}{dx}] \quad \frac{1}{2}R_A a^2 - \frac{1}{6}wa^3 + C_1 = -\frac{1}{2}R_B(2a)^2 + C_3$

$C_3 = C_1 + \frac{1}{2}R_A a^2 - \frac{1}{6}wa^3 + \frac{1}{2}R_B(2a)^2 = C_1 + \frac{7}{12}wa^3$

$[x=a,\ y=y] \quad \frac{1}{6}R_A a^3 - \frac{1}{24}wa^4 + C_1 a = \frac{1}{6}R_B(2a)^3 - (C_1 + \frac{7}{12}wa^3)(2a)$

$3C_1 a = -\frac{1}{6}R_A a^3 + \frac{1}{24}wa^4 + \frac{1}{6}R_B(2a)^3 - \frac{7}{12}wa^2(2a) = -\frac{25}{24}wa^4$

$C_1 = -\frac{25}{72}wa^3$

For $0 \le x \le a$, $\quad EI\,y = \frac{5}{36}wax^3 - \frac{1}{24}wx^4 - \frac{25}{72}wa^3 x$

$EI\frac{dy}{dx} = \frac{5}{12}wax^2 - \frac{1}{6}wx^3 - \frac{25}{72}wa^3$

continued

Problem 9.14 continued

Data: $w = 50 \times 10^3$ N·m, $a = 2$ m, $E = 200 \times 10^9$ Pa

$I = 85.1 \times 10^6 \text{ mm}^4 = 85.1 \times 10^{-6} \text{ m}^4$, $EI = 17.02 \times 10^6 \text{ N}\cdot\text{m}^2$

(a) Slope at $x = 0$.

$$17.02 \times 10^6 \left.\frac{dy}{dx}\right|_A = 0 - 0 - \frac{25}{72}(50 \times 10^3)(2)^3$$

$$\left.\frac{dy}{dx}\right|_A = \theta_A = -8.16 \times 10^{-3}$$

$\theta_A = 8.16 \times 10^{-3}$ rad

(b) Deflection at $x = 2$m.

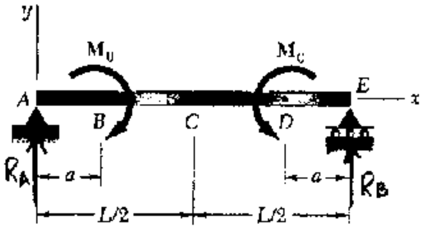
$$EI y_C = \frac{5}{36} w a^4 - \frac{1}{24} w a^4 - \frac{25}{72} w a^4 = -\frac{1}{4} w a^4$$

$$17.02 \times 10^6 y_C = -\frac{1}{4}(50 \times 10^3)(2)^4 \qquad y_C = -11.75 \times 10^{-3} \text{ m}$$

$y_C = 11.75$ mm ↓

Problem 9.15

9.15 Knowing that beam AE is a W 360 × 101 rolled shape and that $M_0 = 310$ kN·m, $L = 2.4$ m, $a = 0.5$ m and $E = 200$ GPa, determine (*a*) the equation of the elastic curve for portion BD, (*b*) the deflection at point C.



Use continuity boundary condition at B and symmetry boundary condition at C.

From Statics, $R_A = R_B = 0$

$[x=0, y=0]$

$[x=a, y=y]$

$[x=a, \frac{dy}{dx}=\frac{dy}{dx}]$

$[x=\frac{L}{2}, \frac{dy}{dx}=0]$

$0 \le x \le a$	$a \le x \le L-a$
$M = 0$	$M = M_0$
$EI\frac{d^2y}{dx^2} = 0$	$EI\frac{d^2y}{dx^2} = M_0$
$EI\frac{dy}{dx} = C_1$	$EI\frac{dy}{dx} = M_0x + C_3$
$EIy = C_1x + C_2$	$EIy = \frac{1}{2}M_0x^2 + C_3x + C_4$

$[x=0, y=0]$ $\quad 0 = 0 + C_2 \quad C_2 = 0$

$[x=\frac{L}{2}, \frac{dy}{dx}=0]$ $\quad 0 = \frac{1}{2}M_0L + C_3 \quad C_3 = -\frac{1}{2}M_0L$

$[x=a, \frac{dy}{dx}=\frac{dy}{dx}]$ $\quad C_1 = M_0a + C_3 = -M_0(\frac{1}{2}L - a)$

$[x=a, y=y]$ $\quad -M_0(\frac{1}{2}L-a)a + 0 = \frac{1}{2}M_0a^2 - \frac{1}{2}M_0La + C_4$

$$C_4 = \frac{1}{2}M_0a^2$$

(a) Elastic curve $(a \le x \le L-a)$. $\quad EIy = \frac{1}{2}M_0x^2 - \frac{1}{2}M_0Lx + \frac{1}{2}M_0a^2$

$$y = \frac{M_0}{2EI}(x^2 - Lx + a^2)$$

(b) Deflection at $x = \frac{L}{2}$.

$$y_C = \frac{M_0}{2EI}\left[\left(\frac{L}{2}\right)^2 - (L)\left(\frac{L}{2}\right) + a^2\right] = -\frac{M_0}{8EI}(L^2 - 4a^2)$$

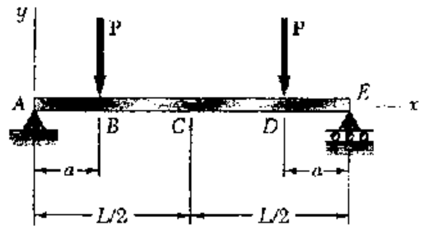
Data: $M_0 = 310\times10^3$ N·m, $L = 2.4$ m, $a = 0.5$ m, $E = 200\times10^9$ Pa

$I = 302\times10^6\ \text{mm}^4 = 302\times10^{-6}\ \text{m}^4 \qquad EI = 60.4\times10^6\ \text{N}\cdot\text{m}^2$

$$y_C = -\frac{310\times10^3}{(8)(60.4\times10^6)}\left[(2.4)^2 - (4)(0.5)^2\right] = -3.05\times10^{-3}$$

$$y_C = 3.05\ \text{mm} \downarrow$$

Problem 9.16



9.16 Knowing that beam AE is an S 200×27.4 rolled shape and that $P = 17.5$ kN, $L = 2.5$ m, $a = 0.8$ m and $E = 200$ GPa, determine (*a*) the equation of the elastic curve for portion BD, (*b*) the deflection at the center C of the beam.

Consider portion ABC only, and consider symmetry about C.

Reactions $R_A = R_E = P$

Boundary conditions: $[x = 0,\ y = 0]$, $[x = a,\ y = y]$, $[x = a,\ \frac{dy}{dx} = \frac{dy}{dx}]$, $[x = \frac{L}{2},\ \frac{dy}{dx} = 0]$

$0 < x < a$

$$EI\frac{d^2y}{dx^2} = M = Px$$

$$EI\frac{dy}{dx} = \frac{1}{2}Px^2 + C_1 \qquad (1)$$

$$EIy = \frac{1}{6}Px^3 + C_1x + C_2 \qquad (2)$$

$[x = 0,\ y = 0] \rightarrow C_2 = 0$

$a < x < L - a$

$$EI\frac{d^2y}{dx^2} = M = Pa$$

$$EI\frac{dy}{dx} = Pax + C_3$$

$$EIy = \frac{1}{2}Pax^2 + C_3x + C_4$$

$[x = \frac{L}{2},\ \frac{dy}{dx} = 0] \rightarrow C_3 = -\frac{1}{2}PaL$

$[x = \frac{L}{2},\ \frac{dy}{dx} = \frac{dy}{dx}]$ $\qquad \frac{1}{2}Pa^2 + C_1 = Pa^2 - \frac{1}{2}PaL \qquad C_1 = \frac{1}{2}Pa^2 - \frac{1}{2}PaL$

$[x = \frac{L}{2},\ y = y]$ $\qquad \frac{1}{6}Pa^3 + (\frac{1}{2}Pa^2 - \frac{1}{2}PaL)a = \frac{1}{2}Pa^3 - \frac{1}{2}Pa^2L + C_4$

$$C_4 = \frac{1}{6}Pa^3$$

(a) Elasic curve for portion BD.

$$y = \frac{1}{EI}\left(\frac{1}{2}Pax^2 + C_3x + C_4\right)$$

$$= \frac{P}{EI}\left(\frac{1}{2}ax^2 - \frac{1}{2}aLx + \frac{1}{6}a^3\right)$$

For deflection at C. set $x = \frac{L}{2}$.

$$y_C = \frac{P}{EI}\left(\frac{1}{8}aL^2 - \frac{1}{4}aL^2 + \frac{1}{6}a^3\right) = -\frac{Pa}{EI}\left(\frac{1}{8}L^2 - \frac{1}{6}a^2\right)$$

Data: $I = 23.9 \times 10^6\ \text{mm}^4 = 23.9 \times 10^{-6}\ \text{m}^4$, $E = 200 \times 10^9$ Pa

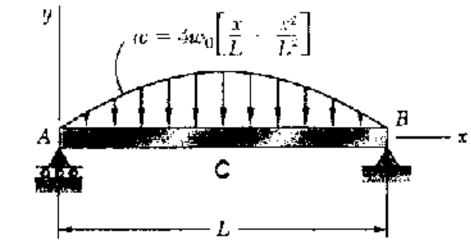
$P = 17.5 \times 10^3$ N, $L = 2.5$ m, $a = 0.8$ m

(b) $$y_C = -\frac{(17.5 \times 10^3)(0.8)}{(200 \times 10^9)(23.9 \times 10^{-6})}\left\{\frac{2.5^2}{8} - \frac{0.8^2}{6}\right\} = -1.976 \times 10^{-3}\ \text{m}$$

$$y_C = 1.976\ \text{mm} \downarrow$$

Problem 9.17

9.17 For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the slope at end *A*, (*c*) the deflection at the midpoint of the span.



$[x=0,\ M=0]$ $[x=L,\ M=0]$

$[x=0,\ y=0]$ $[x=L,\ y=0]$

Boundary conditions at A and B are noted.

$$w = \frac{w_0}{L^2}(4Lx - 4x^2)$$

$$\frac{dV}{dx} = -w = \frac{w_0}{L^2}(4x^2 - 4Lx)$$

$$\frac{dM}{dx} = V = \frac{w_0}{L^2}\left(\frac{4}{3}x^3 - 2Lx^2\right) + C_1$$

$$M = \frac{w_0}{L^2}\left(\frac{1}{3}x^4 - \frac{2}{3}Lx^3\right) + C_1 x + C_2$$

$[x=0,\ M=0]$ $\quad 0 = 0 + 0 + 0 + C_2 \qquad C_2 = 0$

$[x=L,\ M=0]$ $\quad 0 = \frac{w_0}{L^2}\left(\frac{1}{3}L^4 - \frac{2}{3}L^4\right) + C_1 L + 0 \qquad C_1 = \frac{1}{3}w_0 L^2$

$$EI\frac{d^2y}{dx^2} = M = \frac{w_0}{L^2}\left(\frac{1}{3}x^4 - \frac{2}{3}Lx^3 + \frac{1}{3}L^3 x\right)$$

$$EI\frac{dy}{dx} = \frac{w_0}{L^2}\left(\frac{1}{15}x^5 - \frac{1}{6}Lx^4 + \frac{1}{6}L^3x^2\right) + C_3$$

$$EI\,y = \frac{w_0}{L^2}\left(\frac{1}{90}x^6 - \frac{1}{30}Lx^5 + \frac{1}{18}L^3x^3\right) + C_3 x + C_4$$

$[x=0,\ y=0]$ $\quad 0 = 0 + 0 + 0 + 0 + C_4 \qquad C_4 = 0$

$[x=L,\ y=0]$ $\quad 0 = \frac{w_0}{L^2}\left(\frac{1}{90}L^6 - \frac{1}{30}L^6 + \frac{1}{18}L^6\right) + C_3 L + 0 \qquad C_3 = -\frac{1}{30}w_0 L^3$

(a) Elastic curve. $\quad y = \frac{w_0}{EIL^2}\left(\frac{1}{90}x^6 - \frac{1}{30}Lx^5 + \frac{1}{18}L^3x^3 - \frac{1}{30}L^5 x\right)$

$$\frac{dy}{dx} = \frac{w_0}{EIL^2}\left(\frac{1}{15}x^5 - \frac{1}{6}Lx^4 + \frac{1}{6}L^3x^2 - \frac{1}{30}L^5\right)$$

(b) Slope at end A. Set $x = 0$ in $\frac{dy}{dx}$. $\quad \left.\frac{dy}{dx}\right|_A = -\frac{1}{30}\frac{w_0 L^3}{EI}$

$$\theta_A = \frac{1}{30}\frac{w_0 L^3}{EI}$$

(c) Deflection at midpoint. Set $x = \frac{L}{2}$ in y.

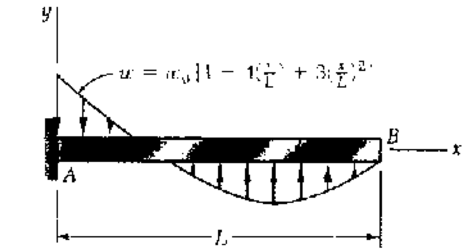
$$y_C = \frac{w_0 L^4}{EI}\left\{\left(\frac{1}{90}\right)\left(\frac{1}{2}\right)^6 - \left(\frac{1}{30}\right)\left(\frac{1}{2}\right)^5 + \frac{1}{18}\left(\frac{1}{2}\right)^3 - \frac{1}{30}\left(\frac{1}{2}\right)\right\}$$

$$= \frac{w_0 L^4}{EI}\left\{\frac{1}{5760} - \frac{1}{960} + \frac{1}{144} - \frac{1}{60}\right\} = -\frac{61}{5760}\frac{w_0 L^4}{EI}$$

$$y_C = \frac{61}{5760}\frac{w_0 L^4}{EI} \downarrow$$

Problem 9.18

9.18 For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the deflection at the free end.



$[x=0, y=0]$ $\quad$ $[x=L, V=0]$

$[x=0, \frac{dy}{dx}=0]$ $\quad$ $[x=L, M=0]$

Boundary conditions are shown at left.

$$\frac{dV}{dx} = -w = -w_0\left[1 - 4\left(\frac{x}{L}\right) + 3\left(\frac{x}{L}\right)^2\right]$$

$$V = -w_0\left[x - \frac{2x^2}{L} + \frac{x^3}{L^2}\right] + C_V$$

$[x = L, V = 0]$:

$$0 = -w_0[L - 2L + L] + C_V = 0 \qquad C_V = 0$$

$$\frac{dM}{dx} = V = -w_0\left[x - \frac{2x^2}{L} + \frac{x^3}{L^2}\right]$$

$$M = -w_0\left[\frac{x^2}{2} - \frac{2x^3}{3L} + \frac{x^4}{4L^2}\right] + C_M$$

$$[x = L, M = 0]: \quad 0 = -w_0\left[\frac{1}{2}L^2 - \frac{2}{3}L^2 + \frac{1}{4}L^2\right] + C_M \qquad C_M = \frac{1}{12}w_0L^2$$

$$EI\frac{d^2y}{dx^2} = M = -w_0\left[\frac{1}{2}x^2 - \frac{2}{3}\frac{x^3}{L} + \frac{1}{4}\frac{x^4}{L^2} - \frac{1}{12}L^2\right]$$

$$EI\frac{dy}{dx} = -w_0\left[\frac{1}{6}x^3 - \frac{1}{6}\frac{x^4}{L} + \frac{1}{20}\frac{x^5}{L^2} - \frac{1}{12}L^2x\right] + C_1$$

$$\left[x = 0, \frac{dy}{dx} = 0\right] \longrightarrow \qquad C_1 = 0$$

$$EIy = -w_0\left[\frac{1}{24}x^4 - \frac{1}{30}\frac{x^5}{L} + \frac{1}{120}\frac{x^6}{L^2} - \frac{1}{24}L^2x^2\right] + C_2$$

$$[x = 0, y = 0] \longrightarrow \qquad C_2 = 0$$

(a) Elastic curve. $y = -\frac{w_0}{EIL^2}\left(\frac{1}{24}L^2x^4 - \frac{1}{30}Lx^5 + \frac{1}{120}x^6 - \frac{1}{24}L^4x^2\right)$ ◄

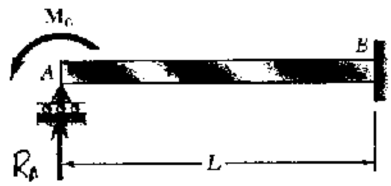
(b) Deflection at $x = L$.

$$y_B = -\frac{w_0}{EIL^2}\left(\frac{1}{24}L^6 - \frac{1}{30}L^6 + \frac{1}{120}L^6 - \frac{1}{24}L^6\right) = -\frac{w_0L^4}{40EI}$$

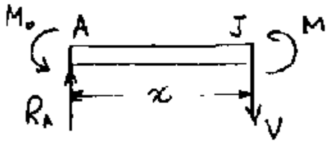
$$y_B = \frac{w_0L^4}{40EI} \downarrow$$ ◄

Problem 9.19

***9.19* through 9.22** For the beam and loading shown, determine the reaction at the roller support.



$[x=0, y=0]$ $[x=L, y=0]$
$[x=L, \frac{dy}{dx}=0]$



Reactions are statically indeterminate.

Boundary conditions are shown at left.

Using free body AJ

$+\circlearrowleft \Sigma M_J = 0 \qquad M_0 - R_A x + M = 0$

$$M = R_A x - M_0$$

$$EI\frac{d^2y}{dx^2} = R_A x - M_0$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - M_0 x + C_1$$

$[x=L, \frac{dy}{dx}=0] \qquad 0 = \frac{1}{2}R_A L^2 - M_0 L + C_1 \qquad C_1 = M_0 L - \frac{1}{2}R_A L^2$

$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{2}M_0 x^2 + C_1 x + C_2$$

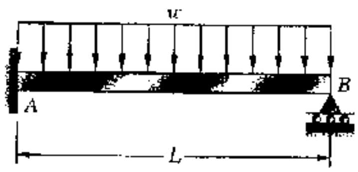
$[x=0, y=0] \quad \rightarrow \quad C_2 = 0$

$[x=L, y=0] \qquad 0 = \frac{1}{6}R_A L^3 - \frac{1}{2}M_0 L^2 + (M_0 L - \frac{1}{2}R_A L^2)L + 0$

$$R_A = \frac{3}{2}\frac{M_0}{L} \uparrow$$ ◄

Problem 9.20

***9.19* through 9.22** For the beam and loading shown, determine the reaction at the roller support.



$[x=0, y=0]$ $[x=0, \frac{dy}{dx}=0]$ $[x=0, y=0]$

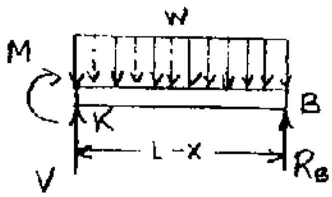
Reactions are statically indeterminate.

Boundary conditions are shown at left.

Using free body KB

$$+\circlearrowleft M_K = 0: \quad R_B(L-x) - w(L-x)\left(\frac{L-x}{2}\right) - M = 0$$

$$M = R_B(L-x) - \tfrac{1}{2}w(L-x)^2$$



$$EI\frac{d^2y}{dx^2} = R_B(L-x) - \tfrac{1}{2}w(L-x)^2$$

$$EI\frac{dy}{dx} = -\tfrac{1}{2}R_B(L-x)^2 + \tfrac{1}{6}w(L-x)^3 + C_1$$

$[x=0, \frac{dy}{dx}=0]$: $\quad 0 = -\frac{1}{2}R_B L^2 + \frac{1}{6}wL^3 + C_1 \qquad C_1 = \frac{1}{2}R_B L^2 - \frac{1}{6}wL^3$

$$EIy = \tfrac{1}{6}R_B(L-x)^3 - \tfrac{1}{24}w(L-x)^4 + C_1 x + C_2$$

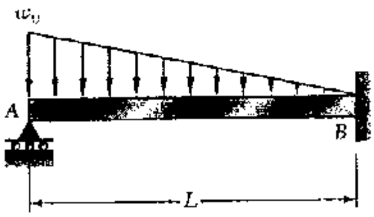
$[x=0, y=0]$: $\quad 0 = \frac{1}{6}R_B L^3 - \frac{1}{24}wL^4 + C_2 \qquad C_2 = -\frac{1}{6}R_B L^3 + \frac{1}{24}wL^4$

$[x=L, y=0]$ $\quad 0 = 0 - 0 + C_1 L + C_2$

$$\tfrac{1}{2}R_B L^3 - \tfrac{1}{6}wL^4 - \tfrac{1}{6}R_B L^3 + \tfrac{1}{24}wL^4 = 0 \qquad R_B = \tfrac{3}{8}wL \uparrow$$

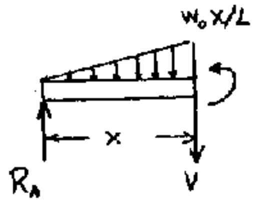
Problem 9.21

***9.19* through 9.22** For the beam and loading shown, determine the reaction at the roller support.



Reactions are statically indeterminate.
Boundary conditions are shown at left.

$[x = 0,\ y = 0]$ $\qquad$ $[x = L,\ y = 0]$ $\qquad$ $[x = L,\ \frac{dy}{dx} = 0]$



$$w = \frac{w_0}{L}(L - x)$$

$$\frac{dV}{dx} = -w = -\frac{w_0}{L}(L - x)$$

$$\frac{dM}{dx} = V = -\frac{w_0}{L}\left(Lx - \frac{1}{2}x^2\right) + R_A$$

$$M = -\frac{w_0}{L}\left(\frac{1}{2}Lx^2 - \frac{1}{6}x^3\right) + R_A x$$

$$EI\frac{d^2y}{dx^2} = -\frac{w_0}{L}\left(\frac{1}{2}Lx^2 - \frac{1}{6}x^3\right) + R_A x$$

$$EI\frac{dy}{dx} = -\frac{w_0}{L}\left(\frac{1}{6}Lx^3 - \frac{1}{24}x^4\right) + \frac{1}{2}R_A x^2 + C_1$$

$$EI\,y = -\frac{w_0}{L}\left(\frac{1}{24}Lx^3 - \frac{1}{120}x^5\right) + \frac{1}{6}R_A x^3 + C_1 x + C_2$$

$[x = 0,\ y = 0]$ $\qquad 0 = 0 + 0 + 0 + C_2 \qquad\qquad C_2 = 0$

$[x = L,\ \frac{dy}{dx} = 0]$ $\qquad -\frac{w_0}{L}\left(\frac{1}{6}L^4 - \frac{1}{24}L^4\right) + \frac{1}{2}R_A L^2 + C_1 = 0$

$$C_1 = \frac{1}{8}w_0 L^3 - \frac{1}{2}R_A L^2$$

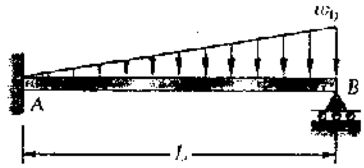
$[x = L,\ y = 0]$ $\qquad -\frac{w_0}{L}\left(\frac{1}{24}L^4 - \frac{1}{120}L^4\right) + \frac{1}{6}R_A L^3 + \left(\frac{1}{8}w_0 L^3 - \frac{1}{2}R_A L^2\right)L = 0$

$$\left(\frac{1}{2} - \frac{1}{6}\right)R_A = \left(\frac{1}{8} - \frac{1}{24} + \frac{1}{120}\right)w_0 L$$

$$\frac{1}{3}R_A = \frac{11}{120}w_0 L \qquad\qquad R_A = \frac{11}{40}w_0 L \uparrow$$

Problem 9.22

***9.19* through 9.22** For the beam and loading shown, determine the reaction at the roller support.

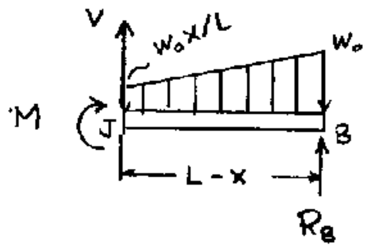


$[x=0, y=0]$ $[x=0, \frac{dy}{dx}=0]$ $[x=L, y=0]$

Reactions are statically indeterminate.

Boundary conditions are shown at left.

Using free body JB $\circlearrowright \Sigma M_J = 0$:



$$-M + R_B(L-x) + \tfrac{1}{2}w_0(L-x)\tfrac{2}{3}(L-x) + \tfrac{1}{2}\frac{w_0 x}{L}(L-x)\tfrac{1}{3}(L-x) = 0$$

$$M = R_B(L-x) - \frac{w_0}{6L}\left[2L(L-x)^2 + x(L-x)^2\right]$$

$$= R_B(L-x) - \frac{w_0}{6L}\left[2L^3 - 4L^2x + 2Lx^2 + xL^2 - 2Lx^2 + x^3\right]$$

$$= R_B(L-x) - \frac{w_0}{6L}(x^3 - 3L^2x + 2L^3)$$

$$EI\frac{d^2y}{dx^2} = R_B(L-x) - \frac{w_0}{6L}(x^3 - 3L^2x + 2L^3)$$

$$EI\frac{dy}{dx} = R_B(Lx - \tfrac{1}{2}x^2) - \frac{w_0}{6L}\left(\tfrac{1}{4}x^4 - \tfrac{3}{2}L^2x^2 + 2L^3x\right) + C_1$$

$$EIy = R_B(\tfrac{1}{2}Lx^2 - \tfrac{1}{6}x^3) - \frac{w_0}{6L}\left(\tfrac{1}{20}x^5 - \tfrac{1}{2}L^2x^3 + L^3x^2\right) + C_1x + C_2$$

$[x=0, y=0] \quad \rightarrow \quad C_2 = 0$

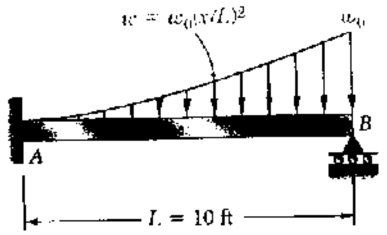
$[x=0, \frac{dy}{dx}=0] \quad \rightarrow \quad C_1 = 0$

$[x=L, y=0] \quad 0 = R_B L^3(\tfrac{1}{2} - \tfrac{1}{6}) - \frac{w_0 L^4}{6}(\tfrac{1}{20} - \tfrac{1}{2} + 1)$

$\tfrac{1}{3}R_B = (\tfrac{1}{6})(\tfrac{11}{20})w_0 L \qquad R_B = \tfrac{11}{40}w_0 L \uparrow$ ◀

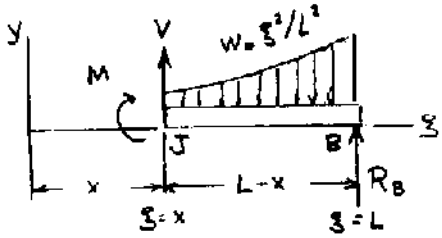
Problem 9.23

9.23 For the beam shown determine the reaction at the roller support when $w_0 = 1.4$ kips/ft.



$[x=0,\ y=0]$ $\quad$ $[x=L,\ y=0]$

$[x=0,\ \frac{dy}{dx}=0]$



Reactions are statically indeterminate.

Boundary conditions are shown at left.

Using free body JB, $\circlearrowleft \Sigma M_J = 0:$

$$-M + \int_x^L \frac{w_0}{L^2}\xi^2(\xi - x)\,d\xi + R_B(L-x) = 0$$

$$M = \frac{w_0}{L^2}\int_x^L \xi^2(\xi - x)\,d\xi - R_B(L-x)$$

$$= \frac{w_0}{L^2}\left(\frac{1}{4}\xi^4 - \frac{1}{3}x\xi^3\right)\Big|_x^L - R_B(L-x)$$

$$= \frac{w_0}{L^2}\left(\frac{1}{4}L^4 - \frac{1}{3}L^3x + \frac{1}{12}x^4\right) - R_B(L-x)$$

$$EI\frac{d^2y}{dx^2} = \frac{w_0}{L^2}\left(\frac{1}{4}L^4 - \frac{1}{3}L^3x + \frac{1}{12}x^4\right) - R_B(L-x)$$

$$EI\frac{dy}{dx} = \frac{w_0}{L^2}\left(\frac{1}{4}L^4x - \frac{1}{6}L^3x^2 + \frac{1}{60}x^5\right) - R_B\left(Lx - \frac{1}{2}x^2\right) + C_1$$

$$EI\,y = \frac{w_0}{L^2}\left(\frac{1}{8}L^4x^2 - \frac{1}{18}L^3x^3 + \frac{1}{360}x^6\right) - R_B\left(\frac{1}{2}Lx^2 - \frac{1}{6}x^3\right) + C_1x + C_2$$

$[x=0,\ \frac{dy}{dx}=0]$ $\qquad 0 = 0 + 0 + C_1 \qquad\qquad C_1 = 0$

$[x=0,\ y=0]$ $\qquad 0 = 0 + 0 + 0 + C_2 \qquad\qquad C_2 = 0$

$[x=L,\ y=0]$ $\qquad \left(\frac{1}{8} - \frac{1}{18} + \frac{1}{360}\right)w_0L^4 - \left(\frac{1}{2} - \frac{1}{6}\right)R_BL^3 = 0$

$$\frac{13}{180}w_0L^4 - \frac{1}{3}R_BL^3 = 0 \qquad\qquad R_B = \frac{13}{60}w_0L$$

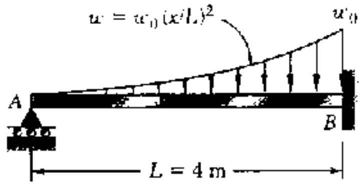
Data: $w_0 = 1.4$ kips/ft, $\quad L = 10$ ft,

$$R_B = \frac{13}{60}(1.4)(10) = 3.03 \text{ kips}$$

3.03 kips ↑ ◄

Problem 9.24

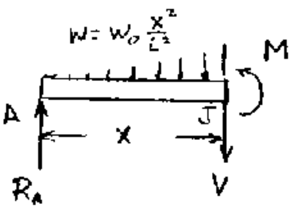
9.24 For the beam shown determine the reaction at the roller support when $w_0 = 65$ kN/m.



Reactions are statically indeterminate.

Boundary conditions are shown at left.

$[x = 0, y = 0]$ $\quad$ $[x = L, y = 0]$ $\quad$ $[x = L, \frac{dy}{dx} = 0]$



$$w = w_0 \frac{x^2}{L^2}$$

$$\frac{dV}{dx} = -w = -\frac{w_0}{L^2} x^2$$

$$\frac{dM}{dx} = V = -\frac{w_0}{L^2}\frac{x^3}{3} + R_A$$

$$M = -\frac{w_0}{L^2}\frac{x^4}{12} + R_A x$$

$$EI \frac{d^2y}{dx^2} = -\frac{w_0}{L^2}\frac{x^4}{12} + R_A x$$

$$EI \frac{dy}{dx} = -\frac{w_0}{L^2}\frac{x^5}{60} + \frac{1}{2} R_A x^2 + C_1$$

$$EI\, y = -\frac{w_0}{L^2}\frac{x^6}{360} + \frac{1}{6} R_A x^3 + C_1 x + C_2$$

$[x = 0, y = 0]$ $\quad 0 = 0 + 0 + 0 + C_2 \qquad C_2 = 0$

$[x = L, \frac{dy}{dx} = 0]$ $\quad -\frac{1}{60} w_0 L^3 + \frac{1}{2} R_A L^2 + C_1 = 0 \qquad C_1 = \frac{1}{60} w_0 L^3 - \frac{1}{2} R_A L^2$

$[x = L, y = 0]$ $\quad -\frac{1}{360} w_0 L^4 + \frac{1}{6} R_A L^3 + \left(\frac{1}{60} w_0 L^3 - \frac{1}{2} R_A L^2\right) L = 0$

$$\left(\frac{1}{2} - \frac{1}{6}\right) R_A = \left(\frac{1}{60} - \frac{1}{360}\right) w_0 L$$

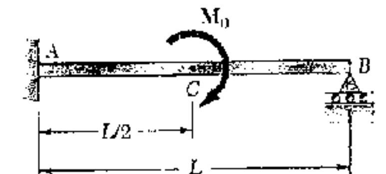
$$\frac{1}{3} R_A = \frac{1}{72} w_0 L \qquad R_A = \frac{1}{18} w_0 L$$

Data: $w_0 = 65$ kN/m, $\quad L = 4$ m

$$R_A = \frac{1}{18}(65)(4) = 14.44 \text{ kN} \uparrow$$ ◄

Problem 9.25

9.25 through 9.28 Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.



$[x=0, y=0]$ $\quad [x=L, y=0]$

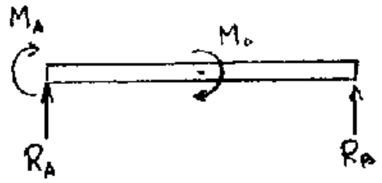
$[x=0, \frac{dy}{dx}=0]$

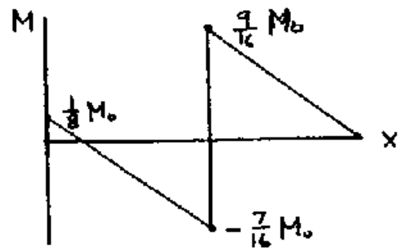
Reactions are statically indeterminate.

$+\uparrow \Sigma F_y = 0: \quad R_A + R_B = 0 \qquad R_A = -R_B$

$\circlearrowleft \Sigma M_A = 0: \quad -M_A - M_0 + R_B L = 0$

$$M_A = R_B L - M_0$$





$0 < x < \frac{L}{2}$

$$M = R_A x + M_A = -M_0 + R_B L - R_B x$$

$$EI\frac{d^2y}{dx^2} = -M_0 + R_B(L-x)$$

$$EI\frac{dy}{dx} = -M_0 x + R_B\left(Lx - \tfrac{1}{2}x^2\right) + C_1$$

$$EI\,y = -\tfrac{1}{2}M_0 x^2 + R_B\left(\tfrac{1}{2}Lx^2 - \tfrac{1}{6}x^3\right) + C_1 x + C_2$$

$\frac{L}{2} < x < L \qquad M = R_B(L-x)$

$$EI\frac{d^2y}{dx^2} = R_B(L-x)$$

$$EI\frac{dy}{dx} = R_B\left(Lx - \tfrac{1}{2}x^2\right) + C_3$$

$$EI\,y = R_B\left(\tfrac{1}{2}Lx^2 - \tfrac{1}{6}x^3\right) + C_3 x + C_4$$

$[x=0, \frac{dy}{dx}=0] \qquad 0 + 0 + C_1 = 0 \qquad C_1 = 0$

$[x=0, y=0] \qquad 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x=\frac{L}{2}, \frac{dy}{dx}=\frac{dy}{dx}] \qquad -M_0\frac{L}{2} + \cancel{R_B\left(\tfrac{1}{2}L^2 - \tfrac{1}{6}L^2\right)} = \cancel{R_B\left(\tfrac{1}{2}L^2 - \tfrac{1}{6}L^2\right)} + C_3 \qquad C_3 = -\frac{M_0 L}{2}$

$[x=\frac{L}{2}, y=y] \qquad -\frac{1}{2}M_0\left(\frac{L}{2}\right)^2 + \cancel{R_B\left(\tfrac{1}{8}L^3 - \tfrac{1}{48}L^3\right)} = \cancel{R_B\left(\tfrac{1}{8}L^3 - \tfrac{1}{48}L^3\right)} + C_3\frac{L}{2} + C_4$

$$C_4 = -\tfrac{1}{8}M_0 L^2 - \tfrac{1}{2}C_3 L = \left(-\tfrac{1}{8} + \tfrac{1}{4}\right)M_0 L^2 = \tfrac{1}{8}M_0 L^2$$

$[x=L, y=0] \qquad R_B\left(\tfrac{1}{2}L^3 - \tfrac{1}{6}L^3\right) + \frac{M_0 L}{2}L + \tfrac{1}{8}M_0 L^2 = 0$

$$\left(\tfrac{1}{2} - \tfrac{1}{6}\right)R_B L^3 = \left(\tfrac{1}{2} - \tfrac{1}{8}\right)M_0 L^2 \qquad \tfrac{1}{3}R_B = \tfrac{3}{8}\frac{M_0}{L}$$

$$R_B = \frac{9}{8}\frac{M_0}{L}\uparrow$$

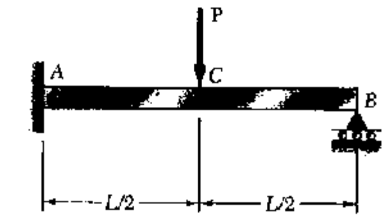
$$M_A = \tfrac{9}{8}M_0 - M_0 = \tfrac{1}{8}M_0$$

$$M_{C^-} = -M_0 + \tfrac{9}{8}\frac{M_0}{L}\frac{L}{2} = -\tfrac{7}{16}M_0$$

$$M_{C^+} = R_B\left(L - \tfrac{L}{2}\right) = \tfrac{9}{8}\frac{M_0}{L}\left(\frac{L}{2}\right) = \tfrac{9}{16}M_0$$

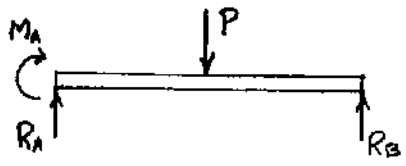
Problem 9.26

9.25 through 9.28 Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.



$[x=0,\ y=0]$ $\quad [x=L,\ y=0]$

$[x=0,\ \frac{dy}{dx}=0]$



Reactions are statically indeterminate.

$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - P = 0 \qquad R_A = P - R_B$

$+\circlearrowleft \Sigma M_A = 0: \quad -M_A + \frac{1}{2}PL - R_B L = 0$

$M_A = R_B L - \frac{1}{2}PL$

$0 < x < \frac{1}{2}L \qquad M = M_A + R_A x$

$$EI\frac{d^2y}{dx^2} = M_A + R_A x$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 + C_1$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 + C_1 x + C_2$$

$\frac{1}{2}L < x < L \qquad M = M_A + R_A x - P(x - \frac{1}{2}L)$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - P(x - \frac{1}{2}L)$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - \frac{1}{2}P(x - \frac{1}{2}L)^2 + C_3$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{1}{6}P(x - \frac{L}{2})^3 + C_3 x + C_4$$

$[x=0,\ \frac{dy}{dx}=0] \qquad 0 + 0 + C_1 = 0 \qquad C_1 = 0$

$[x=0,\ y=0] \qquad 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x=\frac{L}{2},\ \frac{dy}{dx}=\frac{dy}{dx}] \qquad \frac{1}{2}M_A L + \frac{1}{8}R_A L^2 + 0 = \frac{1}{2}M_A + \frac{1}{8}R_A L - 0 + C_3 \qquad C_3 = 0$

$[x=\frac{L}{2},\ y=y] \qquad \frac{1}{8}M_A L^2 + \frac{1}{48}R_A L^3 + 0 + 0$

$\qquad = \frac{1}{8}M_A L^2 + \frac{1}{48}R_A L^3 - 0 + 0 + C_4 \qquad C_4 = 0$

$[x=L,\ y=0] \qquad \frac{1}{2}M_A L^2 + \frac{1}{6}R_A L^3 - \frac{1}{48}PL^3 + 0 + 0 = 0$

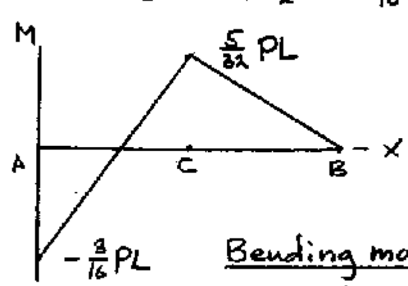
$\frac{1}{2}(R_B L - \frac{1}{2}P)L^3 + \frac{1}{6}(P - R_B)L^3 - \frac{1}{48}PL^3 = 0 \qquad R_B = \frac{5}{16}P \uparrow$ ◄

$R_A = P - \frac{5}{16}P \qquad R_A = \frac{7}{16}P \uparrow$

$M_A = \frac{5}{16}PL - \frac{1}{2}PL \qquad M_A = -\frac{3}{16}PL$ ◄

$M_C = R_B(\frac{L}{2}) = (\frac{5}{16}P)(\frac{L}{2}) \qquad M_C = \frac{5}{32}PL$ ◄

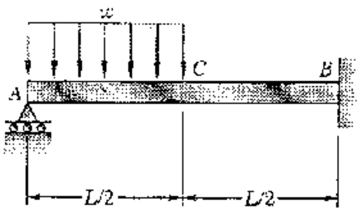
$M_B = 0$ ◄



Bending moment diagram

Problem 9.27

9.25 through 9.28 Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.



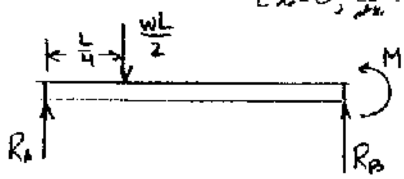
Reactions are statically indeterminate.

$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - \frac{1}{2}wL = 0 \qquad R_B = \frac{1}{2}wL - R_A$

$+\circlearrowleft \Sigma M_A = 0: \quad -R_A L + \left(\frac{wL}{2}\right)\left(\frac{3L}{4}\right) + M_B = 0$

$M_B = R_A L - \frac{3}{8}wL^2$

$[x=0, y=0] \qquad [x=0, y=0] \qquad [x=0, \frac{dy}{dx}=0]$



$0 < x \le \frac{L}{2} \qquad M = R_A x - \frac{1}{2}wx^2$

$EI\frac{d^2y}{dx^2} = R_A x - \frac{1}{2}wx^2$

$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{6}wx^3 + C_1$

$EI\,y = \frac{1}{6}R_A x^3 - \frac{1}{24}wx^4 + C_1 x + C_2$

$\frac{L}{2} \le x < L \qquad M = R_A x - \frac{wL}{2}\left(x - \frac{L}{4}\right)$

$EI\frac{d^2y}{dx^2} = R_A x - wL\left(\frac{1}{2}x - \frac{1}{8}L\right)$

$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - wL\left(\frac{1}{4}x^2 - \frac{1}{8}Lx\right) + C_3$

$EI\,y = \frac{1}{6}R_A x^3 - wL\left(\frac{1}{12}x^3 - \frac{1}{16}Lx^2\right) + C_3 x + C_4$

$[x=0, y=0] \qquad 0 - 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x=\frac{L}{2}, \frac{dy}{dx}=\frac{dy}{dx}] \qquad \frac{1}{8}R_A L^2 - \frac{1}{48}wL^3 + C_1 = \frac{1}{8}R_A L^2 - 0 + C_3 \qquad C_3 = C_1 - \frac{1}{48}wL^3$

$[x=\frac{L}{2}, y=y \qquad \frac{1}{48}R_A L^3 - \frac{1}{384}wL^4 + \frac{1}{2}C_1 L + 0$

$= \frac{1}{48}R_A L^3 - wL\left(\frac{1}{96}L^3 - \frac{1}{64}L^3\right) + \left(C_1 - \frac{1}{48}wL^3\right)\left(\frac{L}{2}\right) + C_4$

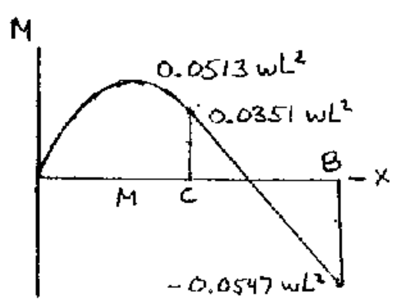
$C_4 = \frac{1}{384}wL^4$

$[x=L, \frac{dy}{dx}=0] \qquad \frac{1}{2}R_A L^2 - wL\left(\frac{1}{4}L^2 - \frac{1}{8}L^2\right) + C_3 = 0$

$C_3 = \frac{1}{8}wL^3 - \frac{1}{2}R_A L^2$

$[x=L, y=0] \qquad \frac{1}{6}R_A L^3 - wL\left(\frac{1}{12}L^3 - \frac{1}{16}L^3\right) + \left(\frac{1}{8}wL^3 - \frac{1}{2}R_A L^2\right)L + \frac{1}{384}wL^4 = 0$

$R_A = \frac{41}{128}wL \uparrow$



Bending moment diagram

$R_B = \frac{1}{2}wL - \frac{41}{128}wL \qquad R_B = \frac{23}{128}wL \uparrow$

$M_B = \frac{41}{128}wL^2 - \frac{3}{8}wL^2 \qquad M_B = -\frac{7}{128}wL^2 = -0.0547\,wL^2$

Over $0 < x < \frac{L}{2}, \qquad V = R_A - wx = \frac{41}{128}wL - wx$

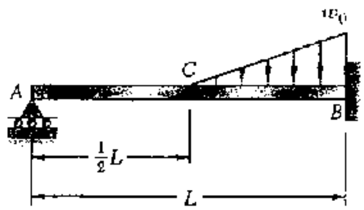
$V = 0$ at $x = x_m = \frac{41}{128}L$

$M = \frac{41}{128}wLx - \frac{1}{2}wx^2$

$M_C = \frac{41}{256}wL^2 - \frac{1}{8}wL^2 = 0.0351\,wL^2, \quad M_m = \frac{41}{128}wL\left(\frac{41}{128}L\right) - \frac{1}{2}w\left(\frac{41}{128}L\right)^2 = 0.0513\,wL^2$

Problem 9.28

9.25 through 9.28 Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.



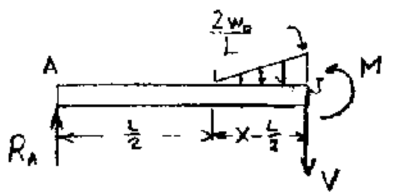
Reactions are statically indeterminate.

$0 \le x \le \frac{L}{2}$

$$EI\frac{d^2y}{dx^2} = M = R_A x \qquad (1)$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 + C_1 \qquad (2)$$

$$EI\,y = \frac{1}{6}R_A x^3 + C_1 x + C_2 \qquad (3)$$



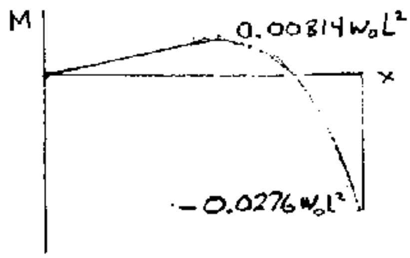
$\frac{L}{2} \le x \le L \qquad \circlearrowright \sum M_J = 0$

$$-R_A x + \frac{1}{2}\frac{2w_0}{L}(x-\tfrac{L}{2})(x-\tfrac{L}{2})\tfrac{1}{3}(x-\tfrac{L}{2}) + M = 0$$

$$EI\frac{d^2y}{dx^2} = M = R_A x - \frac{1}{3}\frac{w_0}{L}(x-\tfrac{L}{2})^3 \qquad (4)$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{12}\frac{w_0}{L}(x-\tfrac{L}{2})^4 + C_3 \qquad (5)$$

$$EI\,y = \frac{1}{6}R_A x^3 - \frac{1}{60}\frac{w_0}{L}(x-\tfrac{L}{2})^5 + C_3 x + C_4 \qquad (6)$$



$[x=0,\ y=0] \qquad 0 = 0 + 0 + C_2 \qquad C_2 = 0$

$[x=\frac{L}{2},\ \frac{dy}{dx}=\frac{dy}{dx}] \qquad \frac{1}{2}R_A(\frac{L}{2})^2 + C_1 = \frac{1}{2}R_A(\frac{L}{2})^2 - 0 + C_3 \qquad C_1 = C_3$

$[x=\frac{L}{2},\ y=y] \qquad \cancel{\frac{1}{6}R_A(\frac{L}{2})^3} + \cancel{C_1\frac{L}{2}} + C_2 = \cancel{\frac{1}{6}R_A(\frac{L}{2})^3} - 0 + \cancel{C_3\frac{L}{2}} + C_4 \qquad C_4 = C_2 = 0$

$[x=L,\ \frac{dy}{dx}=0] \qquad \frac{1}{2}R_A L^2 - \frac{1}{12}\frac{w_0}{L}(\frac{L}{2})^4 + C_3 = 0 \qquad C_3 = \frac{1}{192}w_0 L^3 - \frac{1}{2}R_A L^2$

$[x=L,\ y=0] \qquad \frac{1}{6}R_A L^3 - \frac{1}{60}\frac{w_0}{L}(\frac{L}{2})^5 + \frac{1}{96}w_0 L^4 - \frac{1}{2}R_A L^3 + 0 = 0$

$$(\tfrac{1}{2} - \tfrac{1}{6})R_A L^3 = (\tfrac{1}{192} - \tfrac{1}{1920})w_0 L^4 \qquad \tfrac{1}{3}R_A = \frac{3}{640}w_0 L \qquad R_A = \frac{9}{640}w_0 L \uparrow$$

From (1), with $x = \frac{L}{2}$ $\qquad M_C = R_A\frac{L}{2} = \frac{9}{1280}w_0 L^2 = 0.007031\ w_0 L^2$

From (4), with $x = L$ $\qquad M_B = \frac{9}{640}w_0 L^2 - \frac{1}{3}\frac{w_0}{L}(\frac{L}{2})^3 = -\frac{53}{1920}w_0 L^2$

$$= -0.02761\ w_0 L^2$$

Location of maximum positive M in portion CB.

$$\frac{dM}{dx} = R_A - \frac{w_0}{L}(x_m - \tfrac{L}{2})^2 = 0 \qquad x_m - \frac{L}{2} = \sqrt{\frac{R_A L}{w_0}} = \sqrt{\frac{9}{640}}\,L = 0.1186\ L$$

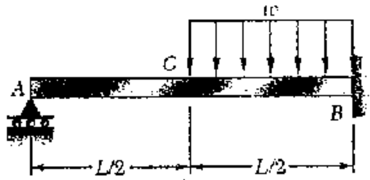
$$x_m = 0.5L + 0.1186\,L = 0.6186\ L$$

From (4), with $x = x_m$, $\qquad M_m = R_A(0.6186\,L) - \frac{1}{3}\frac{w_0}{L}(0.1186\,L)^3$

$$= 0.008143\ w_0 L^2$$

Problem 9.29

9.29 and 9.30 Determine the reaction at the roller support and the deflection at point C.



$[x=0, y=0]$ $\quad$ $[x=L, y=0]$

$[x=L, \frac{dy}{dx}=0]$

$[x=\frac{L}{2}, y=y]$

$[x=\frac{L}{2}, \frac{dy}{dx}=\frac{dy}{dx}]$

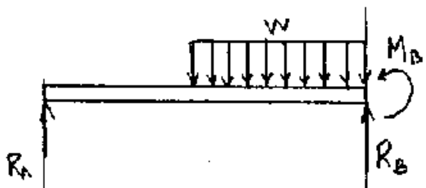
Reactions are statically indeterminate.

$0 < x \le \frac{L}{2} \qquad M = R_A x$

$EI \frac{d^2y}{dx^2} = R_A x$

$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 + C_1$

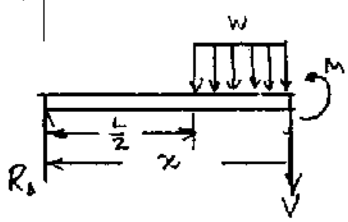
$EI\, y = \frac{1}{6} R_A x^3 + C_1 x + C_2$



$\frac{L}{2} \le x < L$

$M = R_A x - w(x-\frac{L}{2})\frac{1}{2}(x-\frac{L}{2}) = R_A x - \frac{1}{2} w (x-\frac{L}{2})^2$

$EI \frac{d^2y}{dx^2} = R_A x - \frac{1}{2} w (x-\frac{L}{2})^2$



$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - \frac{1}{6} w (x-\frac{L}{2})^3 + C_3$

$EI\, y = \frac{1}{6} R_A x^3 - \frac{1}{24} w (x-\frac{L}{2})^4 + C_3 x + C_4$

$[x=0, y=0] \qquad 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x=\frac{L}{2}, \frac{dy}{dx}=\frac{dy}{dx}] \qquad \frac{1}{8} R_A L^2 + C_1 = \frac{1}{8} R_A x^2 + C_3 \qquad C_3 = C_1$

$[x=\frac{L}{2}, y=y] \qquad \frac{1}{48} R_A L^3 + \frac{1}{2} C_1 L + 0 = \frac{1}{48} R_A x^3 - 0 + \frac{1}{2} C_1 L + C_4$

$C_4 = 0$

$[x=L, \frac{dy}{dx}=0] \qquad \frac{1}{2} R_A L^2 - \frac{1}{24} w L^3 + C_3 = 0 \qquad C_3 = -(\frac{1}{2} R_A L^2 - \frac{1}{48} w L^3)$

$[x=L, y=0] \qquad \frac{1}{6} R_A L^3 - \frac{1}{384} w L^4 - (\frac{1}{2} R_A - \frac{1}{48} w L^3) L + 0$

$R_A = \frac{7}{128} wL \uparrow$

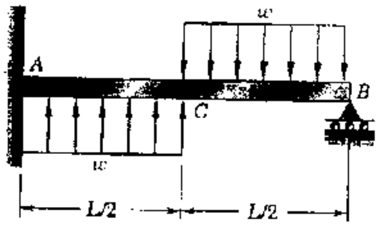
$C_1 = C_3 = -(\frac{7}{256} wL^3 - \frac{1}{48} wL^3) = -\frac{5}{768} wL^3$

At $x=\frac{L}{2}$ $\qquad EI y_C = \frac{1}{6} R_A (\frac{L}{2})^3 + (-\frac{5}{768} wL^3)\frac{L}{2} + 0 = -\frac{13}{6144} wL^4$

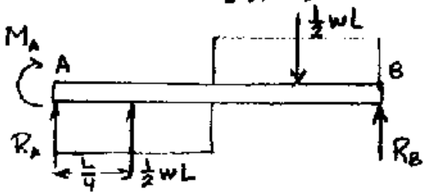
$y_C = \frac{13}{6144} \frac{wL^4}{EI} \downarrow$

Problem 9.30

9.29 and 9.30 Determine the reaction at the roller support and the deflection at point *C*.



$[x=0,\ y=0]$
$[x=0,\ \frac{dy}{dx}=0]$
$[x=L,\ y=0]$
$[x=\frac{L}{2},\ y=y]$
$[x=\frac{L}{2},\ \frac{dy}{dx}=\frac{dy}{dx}]$



Reactions are statically indeterminate

$+\uparrow \sum F_y = 0 \qquad R_A + \frac{1}{2}wL - \frac{1}{2}wL + R_B = 0 \qquad R_A = -R_B$

$\circlearrowleft \sum M_A = 0 \qquad -M_A - (\frac{1}{2}wL)\frac{L}{2} + R_B L = 0$

$M_A = R_B L - \frac{1}{4}wL^2$

From A to C $\qquad 0 < x \le \frac{L}{2}$

$EI\frac{d^2y}{dx^2} = M = M_A + R_A x + \frac{1}{2}wx^2$

$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 + \frac{1}{6}wx^3 + C_1$

$EI\,y = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 + \frac{1}{24}wx^4 + C_1 x + C_2$

From C to B $\qquad \frac{L}{2} \le x < L$

$EI\frac{d^2y}{dx^2} = M = M_A + R_A x + \frac{1}{2}wL(x-\frac{L}{4}) - \frac{1}{2}w(x-\frac{L}{2})^2$

$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 + \frac{1}{4}wL(x-\frac{L}{4})^2 - \frac{1}{6}w(x-\frac{L}{2})^3 + C_3$

$EI\,y = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 + \frac{1}{12}wL(x-\frac{L}{4})^3 - \frac{1}{24}(x-\frac{L}{2})^4 + C_3 x + C_4$

$[x=0,\ \frac{dy}{dx}=0] \qquad 0+0+0+C_1=0 \qquad C_1 = 0$

$[x=0,\ y=0] \qquad 0+0+0+0+C_2=0 \qquad C_2 = 0$

$[x=\frac{L}{2},\ \frac{dy}{dx}=\frac{dy}{dx}] \qquad \cancel{M_A\frac{L}{2}} - \frac{1}{2}R_A(\frac{L}{2})^2 + \frac{1}{6}w(\frac{L}{2})^3 = \cancel{M_A\frac{L}{2}} + \frac{1}{2}R_A(\frac{L}{2})^2 + \frac{1}{4}wL(\frac{L}{4})^2 - 0 + C_3$

$C_3 = (\frac{1}{48} - \frac{1}{64})wL^3 = \frac{1}{192}wL^3$

$[x=\frac{L}{2},\ y=y] \qquad \cancel{\frac{1}{2}M_A(\frac{L}{2})^2} + \cancel{\frac{1}{6}R_A(\frac{L}{2})^3} + \frac{1}{24}w(\frac{L}{2})^4$

$= \cancel{\frac{1}{2}M_A(\frac{L}{2})^2} + \frac{1}{6}R_A(\frac{L}{2})^3 + \frac{1}{12}wL(\frac{L}{4})^3 - 0 + \frac{1}{192}wL^3(\frac{L}{2}) + C_4$

$C_4 = (\frac{1}{384} - \frac{1}{768} + \frac{1}{384})wL^4 = -\frac{1}{768}wL^4$

$[x=L,\ y=0] \qquad \frac{1}{2}M_A L^2 + \frac{1}{6}R_A L^3 + \frac{1}{12}wL(\frac{3L}{4})^3 - \frac{1}{24}w(\frac{L}{2})^4 + \frac{1}{192}wL^3(L) - \frac{1}{768}wL^4 = 0$

$\frac{1}{2}(R_B L - \frac{1}{4}wL^2)L^2 + \frac{1}{6}(-R_B)L^3 + (\frac{27}{768} - \frac{1}{384} + \frac{1}{192} - \frac{1}{768})wL^4 = 0$

$(\frac{1}{2} - \frac{1}{6})R_B L^3 = (\frac{1}{8} - \frac{7}{192})wL^4 \qquad \frac{1}{3}R_B = \frac{17}{192}wL \qquad R_B = \frac{17}{64}wL\uparrow$

$R_A = -R_B = -\frac{17}{64}wL$

$M_A = R_B L - \frac{1}{4}wL^2 = (\frac{17}{64} - \frac{1}{4})wL^2 = \frac{1}{64}wL^2$

(b) Deflection at C. $\qquad$ (y at $x = \frac{L}{2}$)

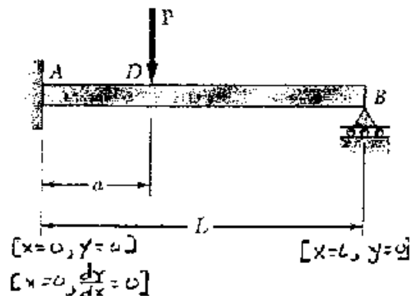
$EI\,y_C = \frac{1}{2}M_A(\frac{L}{2})^2 + \frac{1}{6}R_A(\frac{L}{2})^3 + \frac{1}{24}w(\frac{L}{2})^4 = \frac{1}{2}(\frac{1}{64}wL^2)(\frac{L}{2})^2 + \frac{1}{6}(-\frac{17}{64}wL)(\frac{L}{2})^3 + \frac{1}{24}w(\frac{L}{2})^4$

$= (\frac{1}{512} - \frac{17}{3072} + \frac{1}{384})wL^4 = -\frac{1}{1024}wL^4$

$y_C = \frac{1}{1024}\frac{wL^4}{EI}\downarrow$

Problem 9.31

9.31 and 9.32 Determine the reaction at the roller support and the deflection at point D if a is equal to $L/3$.

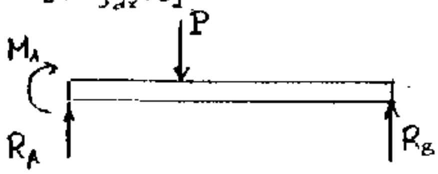


Reactions are statically indeterminate.

$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - P = 0 \qquad R_A = P - R_B$

$+\circlearrowleft \Sigma M_A = 0: \quad -M_A - Pa - R_B L = 0$

$M_A = R_B L - Pa$



$0 < x < a \qquad M = M_A + R_A x$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 + C_1$$

$$EI\,y = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 + C_1 x + C_2$$

$a < x < L \qquad M = M_A + R_A x - P(x-a)$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - P(x-a)$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - \frac{1}{2}P(x-a)^2 + C_3$$

$$EI\,y = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{1}{6}P(x-a)^3 + C_3 x + C_4$$

$[x=0, \frac{dy}{dx}=0] \qquad 0 + 0 + C_1 = 0 \qquad C_1 = 0$

$[x=0, y=0] \qquad 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x=a, \frac{dy}{dx}=\frac{dy}{dx}] \qquad \cancel{M_A a} + \cancel{\frac{1}{2}R_A a^2} + C_1 = \cancel{M_A a} + \cancel{\frac{1}{2}R_A a^2} - 0 + C_3 \qquad C_3 = C_1 = 0$

$[x=a, y=y] \qquad \cancel{\frac{1}{2}M_A a^2} + \cancel{\frac{1}{6}R_A a^3} + \cancel{C_1 a} + C_2$

$= \cancel{\frac{1}{2}M_A a^2} + \cancel{\frac{1}{6}R_A a^3} - 0 + \cancel{C_3 a} + C_4 \qquad C_4 = C_2 = 0$

$[x=L, y=0] \qquad \frac{1}{2}M_A L^2 + \frac{1}{6}R_A L^3 - \frac{1}{6}P(L-a)^3 + 0 + 0 = 0$

$$\frac{1}{2}(R_B L - Pa)L^2 + \frac{1}{6}(P - R_B)L^3 - \frac{1}{6}P(L-a)^3 = 0$$

$$\left(\frac{1}{2} - \frac{1}{6}\right)R_B L^3 = P\left[\frac{1}{2}aL^2 - \frac{1}{6}L^3 + \frac{1}{6}(L-a)^3\right]$$

$$\frac{1}{3}R_B L^3 = P\left[\cancel{\frac{1}{2}aL^2} - \cancel{\frac{1}{6}L^3} + \cancel{\frac{1}{6}L^3} - \cancel{\frac{1}{2}L^2 a} + \frac{1}{2}La^2 - \frac{1}{6}a^3\right]$$

$$= Pa^2\left(\frac{1}{2}L - \frac{1}{6}a\right)$$

$$R_B = \frac{Pa^2}{2L^3}(3L - a) = \frac{P(L/3)^2}{2L^3}\left(3L - \frac{L}{3}\right) = \frac{4}{27}P\uparrow$$

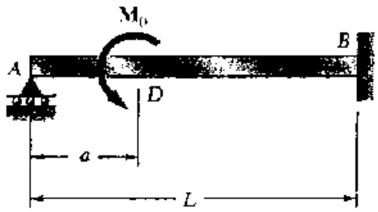
Deflection at D. $\quad (y \text{ at } x = a = \frac{L}{3})$

$$y_D = \frac{1}{EI}\left\{\frac{1}{2}M_A\left(\frac{L}{3}\right)^2 + \frac{1}{6}R_A\left(\frac{L}{3}\right)^3\right\} = \frac{1}{EI}\left\{\frac{1}{18}\left(R_B L - P\frac{L}{3}\right)L^2 + \frac{1}{162}(P - R_B)L^3\right\}$$

$$= \frac{PL^3}{EI}\left\{\frac{1}{18}\left(\frac{4}{27} - \frac{1}{3}\right) + \frac{1}{162}\left(1 - \frac{4}{27}\right)\right\} = -\frac{11}{2187}\frac{PL^3}{EI}, \qquad y_D = \frac{11}{2187}\frac{PL^3}{EI}\downarrow$$

Problem 9.32

9.31* and *9.32 Determine the reaction at the roller support and the deflection at point D if a is equal to $L/3$.



Reactions are statically indeterminate.

$[x=0, y=0]$ $\quad [x=L, y=0]$ $\quad [x=L, \frac{dy}{dx}=0]$ $\quad [x=a, \frac{dy}{dx}=\frac{dy}{dx}]$ $\quad [x=a, y=y]$

$0 < x < a \qquad M = R_A x$

$$EI\frac{d^2y}{dx^2} = M = R_A x$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 + C_1$$

$$EI\,y = \frac{1}{6}R_A x^3 + C_1 x + C_2$$

$a < x < L \qquad M = R_A x - M_0$

$$EI\frac{d^2y}{dx^2} = R_A x - M_0$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - M_0(x-a) + C_3$$

$$EI\,y = \frac{1}{6}R_A x^3 - \frac{1}{2}M_0(x-a)^2 + C_3 x + C_4$$

$[x=0, y=0] \qquad 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x=a, \frac{dy}{dx}=\frac{dy}{dx}] \qquad \frac{1}{2}R_A a^2 + C_1 = \frac{1}{2}R_A a^2 - 0 + C_3 \qquad C_1 = C_3$

$[x=a, y=y] \qquad \frac{1}{6}R_A a^3 + C_1 a + C_2 = \frac{1}{6}R_A a^3 + 0 + C_3 a + C_4 \qquad C_2 = C_4 = 0$

$[x=L, \frac{dy}{dx}=0] \qquad \frac{1}{2}R_A L^2 - M_0(L-a) + C_3 = 0 \qquad C_3 = M_0(L-a) - \frac{1}{2}R_A L^2$

$[x=L, y=0] \qquad \frac{1}{6}R_A L^3 - \frac{1}{2}M_0(L-a)^2 + [M_0(L-a) - \frac{1}{2}R_A L^2]L + 0 = 0$

$$\left(\frac{1}{2}-\frac{1}{6}\right)R_A L^3 = M_0\left[(L-a)L - \frac{1}{2}(L-a)^2\right]$$

$$\frac{1}{3}R_A L^3 = M_0\left[L^2 - aL - \frac{1}{2}L^2 + La - \frac{1}{2}a^2\right] = \frac{1}{2}M_0(L^2 - a^2)$$

$$R_A = \frac{3}{2}\frac{M_0}{L^3}(L^2 - a^2) = \frac{3}{2}\frac{M_0}{L^3}\left[L^2 - \left(\frac{L}{3}\right)^2\right] = \frac{4}{3}\frac{M_0}{L}\uparrow$$

Deflection at D. $\quad (y \text{ at } x = a = \frac{L}{3})$

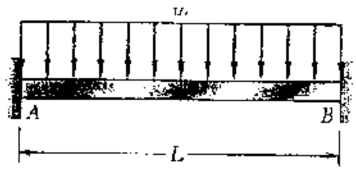
$$y_D = \frac{1}{EI}\left\{\frac{1}{6}R_A\left(\frac{L}{3}\right)^3 + C_1\left(\frac{L}{3}\right)\right\} = \frac{1}{EI}\left\{\frac{1}{6}\left(\frac{4}{3}\frac{M_0}{L}\right)\left(\frac{L}{3}\right)^3 + C_3\left(\frac{L}{3}\right)\right\}$$

$$= \frac{1}{EI}\left\{\frac{4}{486}M_0 L^2 + \left[M_0\left(L-\frac{L}{3}\right) - \frac{1}{2}\cdot\frac{4}{3}\frac{M_0}{L}L^2\right]\frac{L}{3}\right\}$$

$$= \frac{M_0 L^2}{EI}\left(\frac{4}{486} + \frac{2}{9} - \frac{4}{18}\right) = \frac{2}{243}\frac{M_0 L^2}{EI}\uparrow$$

Problem 9.33

9.33 and 9.34 Determine the reaction at A and draw the bending moment diagram for the beam and loading shown.



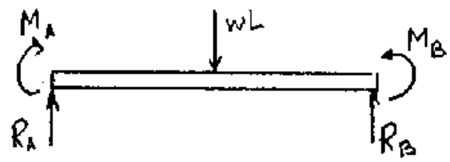
$[x=0, y=0]$ $\quad$ $[x=L, y=0]$

$[x=0, \frac{dy}{dx}=0]$ $\quad$ $[x=L, \frac{dy}{dx}=0]$

Reactions are statically indeterminate.

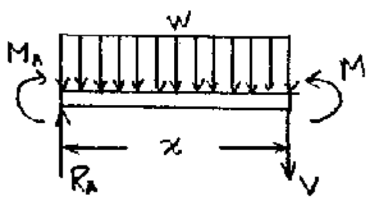
By symmetry, $R_B = R_A$; $M_B = M_A$

$$\frac{dy}{dx} = 0 \quad \text{at } x = \frac{L}{2}$$



$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - wL = 0$$

$$R_B = R_A = \tfrac{1}{2} wL \uparrow$$



Over entire beam, $M = M_A + R_A x - \frac{1}{2} w x^2$

$$EI \frac{d^2y}{dx^2} = M_A + \tfrac{1}{2} wLx - \tfrac{1}{2} wx^2$$

$$EI \frac{dy}{dx} = M_A x + \tfrac{1}{4} wLx^2 - \tfrac{1}{6} wx^3 + C_1$$

$$[x=0, \tfrac{dy}{dx} = 0] \quad 0 + 0 - 0 + C_1 = 0$$

$$C_1 = 0$$

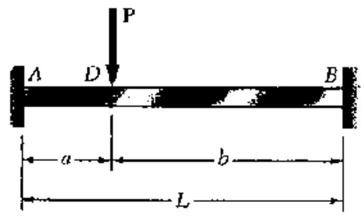
$$[x = \tfrac{L}{2}, \tfrac{dy}{dx} = 0] \quad \tfrac{1}{2} M_A L + \tfrac{1}{16} wL^3 - \tfrac{1}{48} wL^3 + 0 = 0$$

$$M_A = -\tfrac{1}{12} wL^2$$

$$M = -\tfrac{1}{12} wL^2 + \tfrac{1}{2} wLx - \tfrac{1}{2} wx^2 \qquad M = \frac{wL}{12}[6x(L-x) - L^2]$$

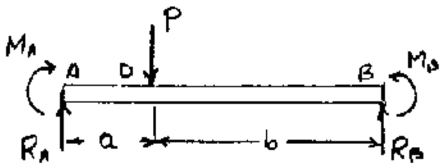
Problem 9.34

9.33 and 9.34 Determine the reaction at A and draw the bending moment diagram for the beam and loading shown.



$[x=0, y=0]$ $\quad$ $[x=L, y=0]$

$[x=0, \frac{dy}{dx}=0]$ $\quad$ $[x=L, \frac{dy}{dx}=0]$



Reactions are statically indeterminate.

$0 \le x \le a$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 + C_1$$

$$EI y = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 + C_1 x + C_2$$

$[x=0, \frac{dy}{dx}=0] \longrightarrow \quad C_1 = 0$

$[x=0, y=0] \longrightarrow \quad C_2 = 0$

$a \le x \le L$

$$EI\frac{d^2y}{dx^2} = M_A + R_A x - P(x-a)$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - \frac{1}{2}P(x-a)^2 + C_3$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{1}{6}P(x-a)^3 + C_3 x + C_4$$

$[x=a, \frac{dy}{dx}=\frac{dy}{dx}]$ $\quad M_A a + \frac{1}{2}R_A a^2 = M_A a + \frac{1}{2}R_A a^2 - 0 + C_3$ $\quad C_3 = 0$

$[x=a, y=y]$ $\quad \frac{1}{2}M_A a^2 + \frac{1}{6}R_A a^3 = \frac{1}{2}M_A a^2 + \frac{1}{6}R_A a^3 - 0 + 0 + C_4$ $\quad C_4 = 0$

$[x=L, \frac{dy}{dx}=0]$ $\quad M_A L + \frac{1}{2}R_A L^2 - \frac{1}{2}Pb^2 = 0$ $\quad (1)$

$[x=L, y=0]$ $\quad \frac{1}{2}M_A L^2 + \frac{1}{6}R_A L^3 - \frac{1}{6}Pb^3 = 0$ $\quad (2)$

Solving (1) and (2) simultaneously, $\quad M_A = -\frac{Pb^2(L-b)}{L^2} = -\frac{Pab^2}{L^2}$ ◄

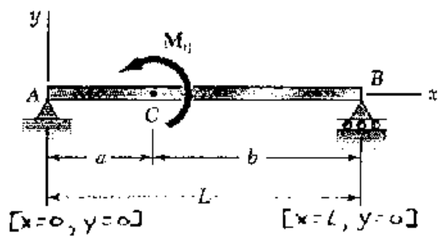
$$R_A = \frac{Pb^2}{L^2} - \frac{2M_A}{L} = \frac{Pb^2}{L^2}\left(1 + 2\frac{a}{L}\right) = \frac{Pb^2(3a+b)}{L^3} \uparrow$$ ◄

$$M_B = -M_A + R_A L - Pb = -\frac{Pab^2}{L^2} + \frac{Pb^2(3a+b)(a+b)}{L^3} - Pb = -\frac{Pba^2}{L^2}$$ ◄

$$M_D = M_A + R_A a = -\frac{Pab^2}{L^2} + \frac{Pb^2(3a+b)a}{L^3} = \frac{2Pa^2b^2}{L^3}$$ ◄

Problem 9.35

9.35 and 9.36 For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the slope at end *A*, (*c*) the deflection of point *C*.



Reactions $R_A = \frac{M_0}{L}\uparrow$, $R_B = \frac{M_0}{L}\downarrow$

$0 < x < a \qquad M = R_A x$

$a < x < L \qquad M = R_A x - M_0$

Using singularity functions

$$EI\frac{d^2y}{dx^2} = M = R_A x - M_0\langle x-a\rangle^0$$

$$EI\frac{dy}{dx} = \tfrac{1}{2}R_A x^2 - M_0\langle x-a\rangle^1 + C_1$$

$$EI\,y = \tfrac{1}{6}R_A x^3 - \tfrac{1}{2}M_0\langle x-a\rangle^2 + C_1 x + C_2$$

$[x=0,\ y=0] \qquad 0 = 0 - 0 + 0 + C_2 \qquad\qquad C_2 = 0$

$[x=L,\ y=0] \qquad \tfrac{1}{6}R_A L^3 - \tfrac{1}{2}M_0(L-a)^2 + C_1 L + 0 = 0$

$$C_1 L = -\frac{1}{6}\frac{M_0}{L}L^3 + \frac{1}{2}M_0 b^2 \qquad\qquad C_1 = \frac{M_0}{6L}(3b^2 - L^2)$$

(a) Elastic curve. $$y = \frac{1}{EI}\left\{\frac{1}{6}\frac{M_0}{L}x^3 - \frac{1}{2}M_0\langle x-a\rangle^2 + \frac{M_0}{6L}(3b^2-L^2)x\right\}$$

$$= \frac{M_0}{6EIL}\left\{x^3 - 3L\langle x-a\rangle^2 + (3b^2-L^2)x\right\}$$

$$\frac{dy}{dx} = \frac{M_0}{6EIL}\left\{3x^2 - 6L\langle x-a\rangle^1 + (3b^2-L^2)\right\}$$

(b) Slope at A. $\left(\frac{dy}{dx} \text{ at } x=0\right)$

$$\theta_A = \frac{M_0}{6EIL}\left\{0 - 0 + 3Lb^2 - L^3\right\} = \frac{M_0}{6EIL}(3b^2 - L^2)$$

(c) Deflection at C. (y at $x = a$)

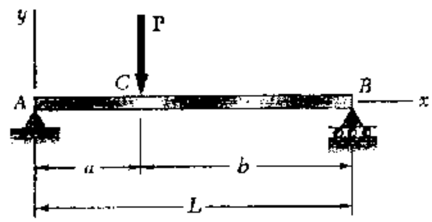
$$y_C = \frac{M_0}{6EIL}\left\{a^3 - 0 + (3b^2 - L^2)a\right\} = \frac{M_0 a}{6EIL}\left\{a^2 + 3b^2 - (a+b)^2\right\}$$

$$= \frac{M_0 a}{6EIL}\left\{\cancel{a^2} + 3b^2 - \cancel{a^2} - 2ab - b^2\right\} = \frac{M_0 a}{6EIL}\left\{2b^2 - 2ab\right\}$$

$$= \frac{M_0 ab}{3EIL}(b-a)\uparrow$$

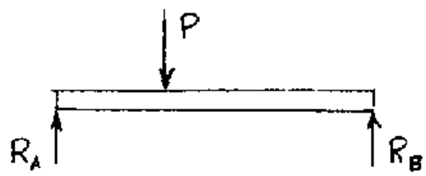
Problem 9.36

9.35 and 9.36 For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the slope at end *A*, (*c*) the deflection of point *C*.



$[x=0, M=0]$ $[x=L, M=0]$

$[x=0, y=0]$ $[x=L, y=0]$



$$+\circlearrowleft \Sigma M_B = 0 \qquad -R_A L + Pb = 0 \qquad R_A = \frac{Pb}{L}$$

$$\frac{dM}{dx} = V = R_A - P\langle x-a\rangle^0 = \frac{Pb}{L} - P\langle x-a\rangle^0$$

$$M = \frac{Pb}{L}x - P\langle x-a\rangle^1 + \cancel{M_A}$$

$$EI\frac{d^2y}{dx^2} = \frac{Pb}{L}x - P\langle x-a\rangle$$

$$EI\frac{dy}{dx} = \frac{Pb}{2L}x^2 - \frac{1}{2}P\langle x-a\rangle^2 + C_1$$

$$EI\,y = \frac{Pb}{6L}x^3 - \frac{1}{6}P\langle x-a\rangle^3 + C_1x + C_2$$

$$[x=0, y=0] \qquad C_2 = 0$$

$$[x=L, y=0] \qquad \frac{Pb}{6L}L^3 - \frac{1}{6}P(L-a)^3 + C_1L = 0$$

$$C_1 = -\frac{1}{6}\frac{P}{L}(bL^2 - b^3) = -\frac{1}{6}\frac{Pb}{L}(L^2-b^2)$$

(a) Elastic curve.

$$y = \frac{P}{EI}\left\{\frac{b}{6L}x^3 - \frac{1}{6}\langle x-a\rangle^3 - \frac{1}{6}\frac{b}{L}(L^2-b^2)x\right\}$$

$$y = \frac{P}{6EIL}\left\{bx^3 - L\langle x-a\rangle^3 - b(L^2-b^2)\right\}$$

(b) Slope at end A.

$$EI\left.\frac{dy}{dx}\right|_{x=0} = C_1 = -\frac{Pb}{6L}(L^2-b^2)$$

$$\theta_A = -\frac{Pb}{6EIL}(L^2-b^2) \qquad \theta_A = \frac{Pb}{6EIL}(L^2-b^2)\measuredangle$$

(c) Deflection at C.

$$EI\,y_C = \frac{Pb}{6L}a^3 + C_1a = \frac{Pba^3}{6L} - \frac{Pb}{6L}(L^2-b^2)a$$

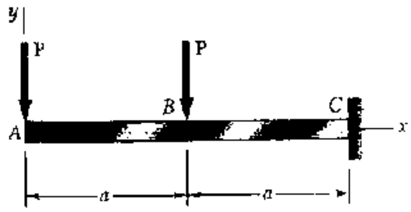
$$= \frac{Pba}{6L}(a^2 - L^2 - b^2)$$

$$y_C = -\frac{Pab}{6EIL}(L^2-a^2-b^2) = -\frac{Pab}{6EIL}\{\cancel{a^2}+2ab+\cancel{b^2}-\cancel{a^2}-\cancel{b^2}\}$$

$$= -\frac{Pa^2b^2}{3EIL} \qquad y_C = \frac{Pa^2b^2}{3EIL}\downarrow$$

Problem 9.37

9.37 and 9.38 For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the slope at the free end, (*c*) the deflection of the free end.



$$\frac{dM}{dx} = V = -P - P\langle x-a\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M = -Px - P\langle x-a\rangle$$

$$EI\frac{dy}{dx} = -\tfrac{1}{2}Px^2 - \tfrac{1}{2}P\langle x-a\rangle^2 + C_1$$

$$EIy = -\tfrac{1}{6}Px^3 - \tfrac{1}{6}P\langle x-a\rangle^3 + C_1x + C_2$$

$\left[x = 2a,\ \frac{dy}{dx} = 0\right]$ $\quad -\tfrac{1}{2}P(2a)^2 - \tfrac{1}{2}P(a)^2 + C_1 = 0 \qquad C_1 = \tfrac{5}{2}Pa^2$

$[x = 2a,\ y = 0]$ $\quad -\tfrac{1}{6}P(2a)^3 - \tfrac{1}{6}P(a)^3 + \tfrac{5}{2}Pa^2(2a) + C_2 = 0$

$$C_2 = -\tfrac{7}{2}Pa^3$$

$$EIy = -\tfrac{1}{6}Px^3 - \tfrac{1}{6}P\langle x-a\rangle^3 + \tfrac{5}{2}Pa^2x - \tfrac{7}{2}Pa^3$$

(a) Elastic curve. $\quad y = \frac{P}{EI}\left\{-\tfrac{1}{6}x^3 - \tfrac{1}{6}\langle x-a\rangle^3 + \tfrac{5}{2}a^2x - \tfrac{7}{2}a^3\right\}$

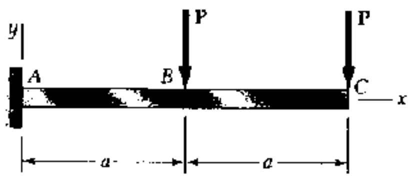
$$\frac{dy}{dx} = \frac{P}{EI}\left\{-\tfrac{1}{2}x^2 - \tfrac{1}{2}\langle x-a\rangle^2 + \tfrac{5}{2}a^2\right\}$$

(b) Slope at A. $\left(\frac{dy}{dx} \text{ at } x = 0\right)$ $\qquad \left.\frac{dy}{dx}\right|_A = \frac{5Pa^2}{2EI} \qquad \theta_A = \frac{5Pa^2}{2EI}$ ∠

(c) Deflection at A. (y at $x = 0$) $\qquad y_A = -\frac{7Pa^3}{2EI} \qquad y_A = \frac{7Pa^3}{2EI}\downarrow$

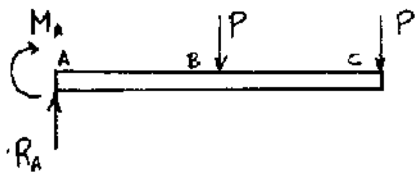
Problem 9.38

9.37 and 9.38 For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the slope at the free end, (*c*) the deflection of the free end.



$+\uparrow \Sigma F_y = 0: \quad R_A - P - P = 0 \qquad R_A = 2P$

$+\circlearrowleft \Sigma M_A = 0: \quad -M_A - Pa - P(2a) = 0 \qquad M_A = -3Pa$



$$\frac{dM}{dx} = V = 2P - P\langle x-a\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M = 2Px - P\langle x-a\rangle^1 - 3Pa$$

$$EI\frac{dy}{dx} = Px^2 - \frac{1}{2}P\langle x-a\rangle^2 - 3Pax + C_1$$

$[x=0,\ \frac{dy}{dx}=0] \qquad 0 - 0 - 0 + C_1 = 0 \qquad C_1 = 0$

$$EIy = \frac{1}{3}Px^3 - \frac{1}{6}P\langle x-a\rangle^3 - \frac{3}{2}Pax^2 + C_2$$

$[x=0,\ y=0] \qquad 0 - 0 - 0 + C_2 = 0 \qquad C_2 = 0$

$$EIy = \frac{1}{3}Px^3 - \frac{1}{6}P\langle x-a\rangle^3 - \frac{3}{2}Pax^2$$

(a) Elastic curve. $\quad y = \frac{P}{EI}\left\{\frac{1}{3}x^3 - \frac{1}{6}\langle x-a\rangle^3 - \frac{3}{2}ax^2\right\}$

$$\frac{dy}{dx} = \frac{P}{EI}\left\{x^2 - \frac{1}{2}\langle x-a\rangle^2 - 3ax\right\}$$

(b) Slope at end C.

$\frac{dy}{dx}$ at $x = 2a$

$$\left.\frac{dy}{dx}\right|_A = \frac{P}{EI}\left\{(2a)^2 - \frac{1}{2}a^2 - (3a)(2a)\right\} = -\frac{5Pa^2}{2EI} \qquad \theta_c = \frac{5Pa^2}{2EI}$$

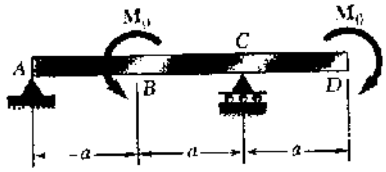
(c) Deflection at end C.

(y at $x = 2a$)

$$y_A = \frac{P}{EI}\left\{\frac{1}{3}(2a)^3 - \frac{1}{6}a^3 - \left(\frac{3}{2}a\right)(2a)^2\right\} = -\frac{7Pa^3}{2EI} \qquad y_c = \frac{7Pa^3}{2EI}\downarrow$$

Problem 9.39

9.39* and *9.40 For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at point *B*, (*c*) the deflection at end *D*.



Loading is self-equilibrated. $R_A = R_C = 0$

$$EI\frac{d^2y}{dx^2} = M = -M_0\langle x-a\rangle^0$$

$$EI\frac{dy}{dx} = -M_0\langle x-a\rangle^1 + C_1$$

$$EIy = -\tfrac{1}{2}M_0\langle x-a\rangle^2 + C_1x + C_2$$

$[x=0,\ y=0]$ $\quad 0 + 0 + C_2 = 0 \quad\quad C_2 = 0$

$[x=2a,\ y=0]$ $\quad -\tfrac{1}{2}M_0a^2 + 0 + C_1(2a) \quad\quad C_1 = \tfrac{1}{4}M_0a$

$$EIy = -\tfrac{1}{2}M_0\langle x-a\rangle^2 + \tfrac{1}{4}M_0ax$$

Elastic curve. $\quad y = \dfrac{M_0}{EI}\left\{-\tfrac{1}{2}\langle x-a\rangle^2 + \tfrac{1}{4}ax\right\}$

$$\frac{dy}{dx} = \frac{M_0}{EI}\left\{-\langle x-a\rangle^1 + \tfrac{1}{4}a\right\}$$

(a) Slope at end A. $\left(\frac{dy}{dx} \text{ at } x=0\right)$

$$\left.\frac{dy}{dx}\right|_A = \frac{M_0a}{4EI} \qquad\qquad \theta_A = \frac{M_0a}{4EI}\ \measuredangle$$

(b) Deflection at point B. $(y \text{ at } x=a)$

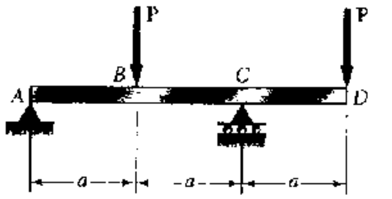
$$y_B = \frac{M_0}{EI}\left\{-0 + (\tfrac{1}{4}a)(a)\right\} \qquad\qquad y_B = \frac{M_0a^2}{4EI}\uparrow$$

(c) Deflection at end D. $(y \text{ at } x=3a)$

$$y_D = \frac{M_0}{EI}\left\{-\tfrac{1}{2}(2a)^2 + (\tfrac{1}{4}a)(3a)\right\} = -\frac{5M_0a^2}{4EI} \qquad y_D = \frac{5M_0a^2}{4EI}\downarrow$$

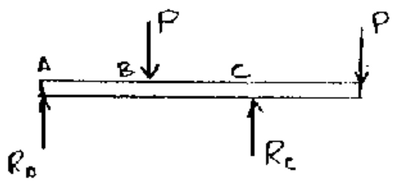
Problem 9.40

9.39* and *9.40 For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at point *B*, (*c*) the deflection at end *D*.



$$+\circlearrowleft \Sigma M_C = 0: \quad -2aR_A + aP - aP = 0 \qquad R_A = 0$$

$$+\uparrow \Sigma F_y = 0: \quad R_A + R_C - P - P = 0 \qquad R_C = 2P \uparrow$$

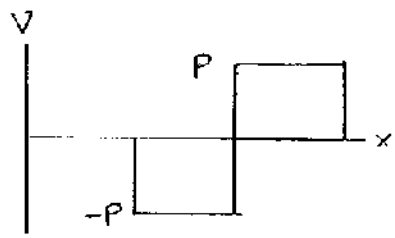


$$\frac{dM}{dx} = V = -P\langle x-a\rangle^0 + 2P\langle x-2a\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M = -P\langle x-a\rangle^1 + 2P\langle x-2a\rangle^1$$

$$EI\frac{dy}{dx} = -\frac{1}{2}P\langle x-a\rangle^2 + P\langle x-2a\rangle^2 + C_1$$

$$EIy = -\frac{1}{6}P\langle x-a\rangle^3 + \frac{1}{3}P\langle x-2a\rangle^3 + C_1x + C_2$$



$$[x=0,\ y=0] \qquad -0+0+0+C_2 = 0 \qquad C_2 = 0$$

$$[x=2a,\ y=0] \qquad -\frac{1}{6}Pa^3 + 0 + 0 + C_1(2a) + 0 = 0$$

$$C_1 = \frac{1}{12}Pa^2$$

$$EIy = -\frac{1}{6}P\langle x-a\rangle^3 + \frac{1}{3}P\langle x-2a\rangle^3 + \frac{1}{12}Pa^2x$$

Elastic curve.
$$y = \frac{P}{EI}\left\{-\frac{1}{6}\langle x-a\rangle^3 + \frac{1}{3}\langle x-2a\rangle^3 + \frac{1}{12}a^2x\right\}$$

$$\frac{dy}{dx} = \frac{P}{EI}\left\{-\frac{1}{2}\langle x-a\rangle^2 + \langle x-2a\rangle^2 + \frac{1}{12}a^2\right\}$$

(a) Slope at end A. $\left(\frac{dy}{dx}\text{ at } x=0\right)$

$$\left.\frac{dy}{dx}\right|_A = \frac{P}{EI}\left\{-0+0+\frac{1}{12}a^2\right\} = \frac{Pa^2}{12EI} \qquad \theta_A = \frac{Pa^2}{12EI}$$

(b) Deflection at point B. (y at $x=a$)

$$y_B = \frac{P}{EI}\left\{-0+0+\frac{1}{12}a^3\right\} = \frac{Pa^3}{12EI} \qquad y_B = \frac{Pa^3}{12EI}\uparrow$$

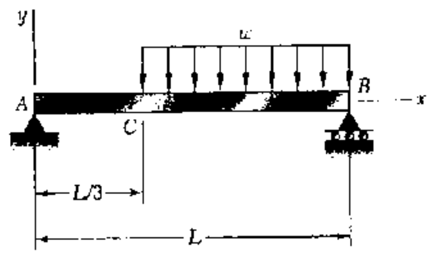
(c) Deflection at end D. (y at $x=3a$)

$$y_D = \frac{P}{EI}\left\{-\frac{1}{6}(2a)^3 + \frac{1}{3}a^3 + \left(\frac{1}{12}a^2\right)(3x)\right\} = -\frac{3Pa^3}{4EI}$$

$$y_D = \frac{3Pa^3}{4EI}\downarrow$$

Problem 9.41

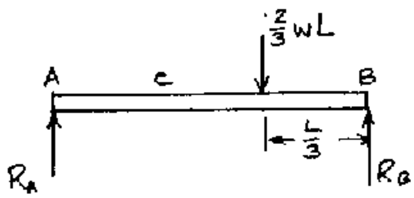
9.41 For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the slope at point *A*, (*c*) the deflection at point *C*.



"Use free body ACB with the distributed load replaced by equivalent concentrated load."

$$+\circlearrowleft M_B = 0: \quad -R_A L + \left(\tfrac{2}{3}wL\right)\left(\tfrac{L}{3}\right) = 0$$

$$R_A = \tfrac{2}{9}wL$$



$$\frac{dV}{dx} = -w = -w\langle x - \tfrac{L}{3}\rangle^0$$

$$\frac{dM}{dx} = V = \tfrac{2}{9}wL - w\langle x - \tfrac{L}{3}\rangle^1$$

$$EI\frac{d^2y}{dx^2} = M = \tfrac{2}{9}wLx - \tfrac{1}{2}w\langle x - \tfrac{L}{3}\rangle^2$$

$$EI\frac{dy}{dx} = \tfrac{1}{9}wLx^2 - \tfrac{1}{6}w\langle x - \tfrac{L}{3}\rangle^3 + C_1$$

$$EIy = \tfrac{1}{27}wLx^3 - \tfrac{1}{24}w\langle x - \tfrac{L}{3}\rangle^4 + C_1x + C_2$$

$[x = 0, y = 0]$ $\quad 0 - 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x = L, y = 0]$ $\quad \left(\tfrac{1}{27}wL\right)L^3 - \tfrac{1}{24}\left(\tfrac{2L}{3}\right)^4 + C_1L + 0 = 0$

$$C_1 = -\tfrac{7}{243}wL^3$$

$$EIy = \tfrac{1}{27}wLx^3 - \tfrac{1}{24}w\langle x - \tfrac{L}{3}\rangle^4 - \tfrac{7}{243}wL^3x$$

(a) Elastic curve. $\quad y = \dfrac{w}{EI}\left\{\tfrac{1}{27}Lx^3 - \tfrac{1}{24}\langle x - \tfrac{L}{3}\rangle^4 - \tfrac{7}{243}L^3x\right\}$ ◄

$$\frac{dy}{dx} = \frac{w}{EI}\left\{\tfrac{1}{9}Lx^2 - \tfrac{1}{6}\langle x - \tfrac{L}{3}\rangle^3 - \tfrac{7}{243}L^3\right\}$$

(b) Slope at point A. $\quad \left(\dfrac{dy}{dx} \text{ at } x = 0\right)$

$$\left.\frac{dy}{dx}\right|_A = \frac{w}{EI}\left\{0 - 0 - \tfrac{7}{243}L^3\right\} = -\frac{7wL^3}{243EI} \qquad \theta_A = \frac{7wL^3}{243EI} ◄$$

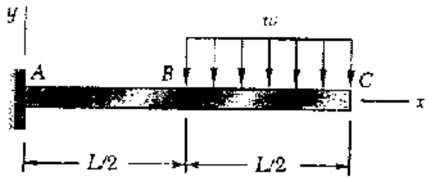
(c) Deflection at point C. $\quad \left(y \text{ at } x = \tfrac{L}{3}\right)$

$$y_C = \frac{w}{EI}\left\{\left(\tfrac{1}{27}L\right)\left(\tfrac{L}{3}\right)^3 - 0 - \left(\tfrac{7}{243}L^3\right)\left(\tfrac{L}{3}\right)\right\} = -\frac{2wL^4}{243EI}$$

$$y_C = \frac{2wL^4}{243EI} \downarrow ◄$$

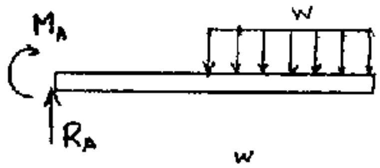
Problem 9.42

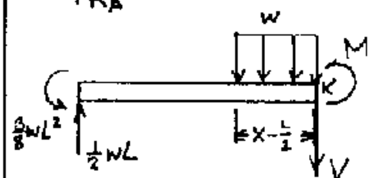
9.42 For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the deflection at point *B*, (*c*) the deflection at point *C*.



$[x=0,\ \frac{dy}{dx}=0]$

$[x=0,\ y=0]$





$+\uparrow \Sigma F_y = 0: \quad R_A - \frac{1}{2}wL = 0 \qquad R_A = \frac{1}{2}wL$

$\circlearrowleft \Sigma M_A = 0: \quad -M_A - (\frac{1}{2}wL)(\frac{3}{4}L) = 0$

$M_A = -\frac{3}{8}wL^2$

$0 < x < \frac{L}{2} \qquad M = -\frac{3}{8}wL^2 + \frac{1}{2}wLx$

$\frac{L}{2} < x < L$ (See free body diagram)

$\circlearrowleft \Sigma M_K = 0$

$\frac{3}{8}wL^2 - \frac{1}{2}wLx + \frac{1}{2}w(x-\frac{L}{2})^2 + M = 0$

$M = -\frac{3}{8}wL^2 + \frac{1}{2}wLx - \frac{1}{2}w(x-\frac{L}{2})^2$

Using singularity functions,

$$EI\frac{d^2y}{dx^2} = M = -\frac{3}{8}wL^2 + \frac{1}{2}wLx - \frac{1}{2}w\langle x-\frac{L}{2}\rangle^2$$

$$EI\frac{dy}{dx} = -\frac{3}{8}wL^2x + \frac{1}{4}wLx^2 - \frac{1}{6}w\langle x-\frac{L}{2}\rangle^3 + C_1$$

$[x=0,\ \frac{dy}{dx}=0] \qquad -0+0-0+C_1 = 0 \qquad C_1 = 0$

$$EIy = -\frac{3}{16}wL^2x^2 + \frac{1}{12}wLx^3 - \frac{1}{24}w\langle x-\frac{L}{2}\rangle^4 + C_1x + C_2$$

$[x=0,\ y=0] \qquad -0+0-0+0+C_2 = 0 \qquad C_2 = 0$

(a) Elastic curve.
$$y = \frac{w}{EI}\left\{-\frac{3}{16}L^2x^2 + \frac{1}{12}Lx^3 - \frac{1}{24}\langle x-\frac{L}{2}\rangle^4\right\}$$

$$\frac{dy}{dx} = \frac{w}{EI}\left\{-\frac{3}{8}L^2x + \frac{1}{4}Lx^2 - \frac{1}{6}\langle x-\frac{L}{2}\rangle^3\right\}$$

(b) Deflection at B. (y at $x = \frac{L}{2}$)

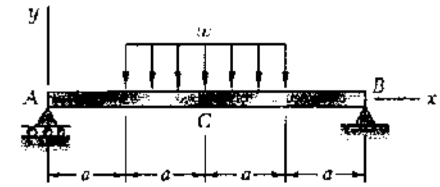
$$y_B = \frac{wL^4}{EI}\left\{-\frac{3}{16}(\frac{1}{2})^2 + \frac{1}{12}(\frac{1}{2})^3 - 0\right\} = -\frac{7}{192}\frac{wL^4}{EI}, \qquad y_B = \frac{7}{192}\frac{wL^4}{EI}\downarrow$$

(c) Deflection at C. (y at $x = L$)

$$y_C = \frac{wL^4}{EI}\left\{-\frac{3}{16}(1)^2 + \frac{1}{12}(1)^3 - \frac{1}{24}(\frac{1}{2})^4\right\} = -\frac{41}{384}\frac{wL^4}{EI}, \qquad y_C = \frac{41}{384}\frac{wL^4}{EI}\downarrow$$

Problem 9.43

9.43 For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the deflection at the midpoint *C*.



$[x=0, y=0]$ $[x=4a, y=0]$

$[x=0, M=0]$ $[x=4a, M=0]$

By symmetry, $R_A = R_B$

$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - 2wa = 0 \qquad R_A = wa$

$$w(x) = w\langle x-a\rangle^0 - w\langle x-3a\rangle^0$$

$$\frac{dV}{dx} = -w(x) = -w\langle x-a\rangle^0 + w\langle x-3a\rangle^0$$

$$\frac{dM}{dx} = V = R_A - w\langle x-a\rangle^1 + w\langle x-3a\rangle^1$$

$$M = M_A + R_A x - \tfrac{1}{2}w\langle x-a\rangle^2 + \tfrac{1}{2}\langle x-3a\rangle^2 \qquad \text{with } M_A = 0$$

$$EI\frac{d^2y}{dx^2} = M = wax - \tfrac{1}{2}w\langle x-a\rangle^2 + \tfrac{1}{2}w\langle x-3a\rangle^2$$

$$EI\frac{dy}{dx} = \tfrac{1}{2}wax^2 - \tfrac{1}{6}w\langle x-a\rangle^3 + \tfrac{1}{6}w\langle x-3a\rangle^3 + C_1$$

$$EI\,y = \tfrac{1}{6}wax^3 - \tfrac{1}{24}w\langle x-a\rangle^4 + \tfrac{1}{24}w\langle x-3a\rangle^4 + C_1x + C_2$$

$[x=0, y=0] \qquad 0 - 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x=4a, y=0] \qquad \tfrac{1}{6}wa(4a)^3 - \tfrac{1}{24}w(3a)^4 + \tfrac{1}{24}(a)^4 + C_1(4a) = 0$

$$4C_1 = wa^3\left\{-\tfrac{64}{6} + \tfrac{81}{24} - \tfrac{1}{24}\right\} = -\tfrac{22}{3}wa^3 \qquad C_1 = -\tfrac{11}{6}wa^3$$

(a) Equation of elastic curve.

$$y = \frac{w}{EI}\left\{\tfrac{1}{6}ax^3 - \tfrac{1}{24}\langle x-a\rangle^4 + \tfrac{1}{24}\langle x-3a\rangle^4 - \tfrac{11}{6}a^3x\right\}$$

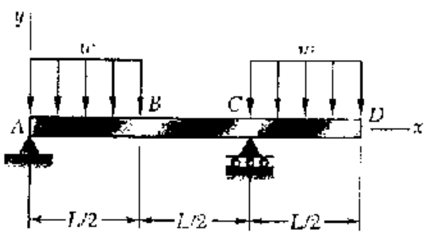
(b) Deflection at C. (y at $x = 2a$)

$$y_C = \frac{wa^4}{EI}\left\{\tfrac{1}{6}(2)^3 - \tfrac{1}{24}(1)^4 + 0 - \tfrac{11}{6}(2)\right\} = -\frac{19}{8}\frac{wa^4}{EI}$$

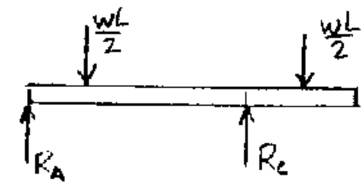
$$y_C = \frac{19}{8}\frac{wa^4}{EI}\downarrow$$

Problem 9.44

9.44 For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the deflection at point *B*, (*c*) the deflection at point *D*.



Use free body ABCD with the distributed loads replaced by equivalent concentrated loads.



$$+\circlearrowleft \Sigma M_C = 0: \quad -R_A L + \left(\frac{wL}{2}\right)\left(\frac{3L}{4}\right) - \left(\frac{wL}{2}\right)\left(\frac{L}{4}\right) = 0$$

$$R_A = \frac{1}{4} wL$$

$$+\circlearrowleft \Sigma M_A = 0: \quad R_C L - \left(\frac{wL}{2}\right)\left(\frac{L}{4}\right) - \left(\frac{wL}{2}\right)\left(\frac{5L}{4}\right) = 0$$

$$R_C = \frac{3}{4} wL$$

$$\frac{dV}{dx} = -w = -w + w\langle x - \tfrac{L}{2}\rangle^0 - w\langle x - L\rangle^0$$

Integrating and adding terms to account for the reactions,

$$\frac{dM}{dx} = V = -wx + w\langle x - \tfrac{L}{2}\rangle^1 - w\langle x - L\rangle^1 + R_A + R_C\langle x - L\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M = -\tfrac{1}{2}wx^2 + \tfrac{1}{2}w\langle x - \tfrac{L}{2}\rangle^2 - \tfrac{1}{2}w\langle x - L\rangle^2 + R_A x + R_C\langle x - L\rangle^1$$

$$EI\frac{dy}{dx} = -\tfrac{1}{6}wx^3 + \tfrac{1}{6}w\langle x - \tfrac{L}{2}\rangle^3 - \tfrac{1}{6}w\langle x - L\rangle^3 + \tfrac{1}{2}R_A x^2 + \tfrac{1}{2}R_C\langle x - L\rangle^2 + C_1$$

$$EIy = -\tfrac{1}{24}wx^4 + \tfrac{1}{24}w\langle x - \tfrac{L}{2}\rangle^4 - \tfrac{1}{24}w\langle x - L\rangle^4 + \tfrac{1}{6}R_A x^3 + \tfrac{1}{6}R_C\langle x - L\rangle^3 + C_1 x + C_2$$

$[x = 0, y = 0]$ $\quad -0 + 0 - 0 + 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x = L, y = 0]$

$$-\tfrac{1}{24}wL^4 + \tfrac{1}{24}w\left(\tfrac{L}{2}\right)^4 - 0 + \tfrac{1}{6}\left(\tfrac{wL}{4}\right)L^3 + 0 + C_1 L + 0 = 0 \qquad C_1 = -\tfrac{1}{384}wL^3$$

$$EIy = -\tfrac{1}{24}wx^4 + \tfrac{1}{24}w\langle x - \tfrac{L}{2}\rangle^4 + \tfrac{1}{24}w\langle x - L\rangle^4 + \tfrac{1}{6}\left(\tfrac{wL}{4}\right)x^3 + \tfrac{1}{6}\left(\tfrac{3wL}{4}\right)\langle x - L\rangle^3 - \tfrac{1}{384}wL^3 x$$

(a) Elastic curve.

$$y = \frac{w}{24EI}\left\{-x^4 + \langle x - \tfrac{L}{2}\rangle^4 - \langle x - L\rangle^4 + Lx^3 + 3L\langle x - L\rangle^3 - \tfrac{1}{16}L^3 x\right\} \quad ◄$$

(b) Deflection at B. $\quad$ (y at $x = \frac{L}{2}$)

$$y_B = \frac{w}{24EI}\left\{-\left(\tfrac{L}{2}\right)^4 + 0 - 0 + (L)\left(\tfrac{L}{2}\right)^3 + 0 - \left(\tfrac{1}{16}L^3\right)\left(\tfrac{L}{2}\right)\right\} = \frac{wL^4}{768EI} \uparrow \quad ◄$$

(c) Deflection at D. $\quad$ (y at $x = \frac{3L}{2}$)

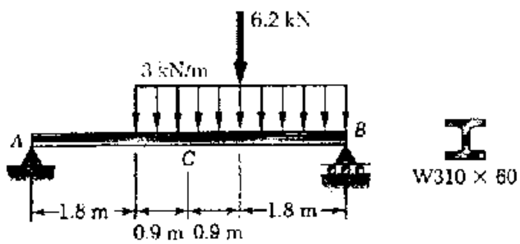
$$y_D = \frac{w}{24EI}\left\{-\left(\tfrac{3L}{2}\right)^4 + L^4 - \left(\tfrac{L}{2}\right)^4 + (L)\left(\tfrac{3L}{2}\right)^3 + (3L)\left(\tfrac{L}{2}\right)^3 - \left(\tfrac{1}{16}L\right)\left(\tfrac{3L}{2}\right)\right\}$$

$$= -\frac{5wL^4}{256EI}$$

$$y_D = \frac{5wL^4}{256EI} \downarrow \quad ◄$$

Problem 9.45

9.45 For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the midpoint *C*. Use $E = 200$ GPa.

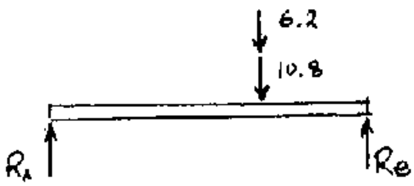


Units: Forces in kN, lengths in meters.

$+\circlearrowleft \Sigma M_B = 0:$

$-5.4 R_A - (1.8)(6.2 + 10.8) = 0$

$R_A = 5.6667$ kN



$w(x) = 3\langle x - 1.8\rangle^0$

$\frac{dV}{dx} = -w(x) = -3\langle x - 1.8\rangle^0$

$\frac{dM}{dx} = V = 5.6667 - 3\langle x - 1.8\rangle^1 - 6.2\langle x - 3.6\rangle^0$

$EI\frac{d^2y}{dx^2} = M = 5.6667x - \frac{3}{2}\langle x - 1.8\rangle^2 - 6.2\langle x - 3.6\rangle^1$ kN·m

$EI\frac{dy}{dx} = 2.8333x^2 - \frac{1}{2}\langle x - 1.8\rangle^3 - 3.1\langle x - 3.6\rangle^2 + C_1$ kN·m²

$EI\,y = 0.9444x^3 - \frac{1}{8}\langle x - 1.8\rangle^4 - 1.0333\langle x - 3.6\rangle^3 + C_1x + C_2$ kN·m³

$[x = 0, y = 0]$ $\quad 0 - 0 - 0 + 0 + C_2 = 0 \quad C_2 = 0$

$[x = 5.4, y = 0]$ $\quad (0.9444)(5.4)^3 - \frac{1}{8}(3.6)^4 - 1.0333(1.8)^3 + C_1(5.4) + 0 = 0$

$C_1 = -22.535$ kN·m²

Data: $E = 200 \times 10^9$ Pa, $I = 129 \times 10^6\ \text{mm}^4 = 129 \times 10^{-6}\ \text{m}^4$

$EI = (200 \times 10^9)(129 \times 10^{-6}) = 25.8 \times 10^6\ \text{N·m}^2 = 25.8 \times 10^3\ \text{kN·m}^2$

(a) Slope at A. $\left(\frac{dy}{dx}\text{ at } x = 0\right)$

$EI\frac{dy}{dx} = 0 - 0 - 0 - 22.535$ kN·m²

$\theta_A = -\frac{22.535}{25.8 \times 10^3} = -873 \times 10^{-6} = 0.873 \times 10^{-3}$ rad $\circlearrowright$

(b) Deflection at C. (y at $x = 2.7$ m)

$EI\,y_C = (0.9444)(2.7)^3 - \frac{1}{8}(0.9)^4 - 0 - (22.535)(2.7) + 0$

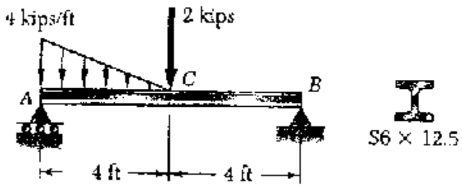
$= -42.337$ kN·m³

$y_C = -\frac{42.337}{25.8 \times 10^3} = -1.641 \times 10^{-3}$ m

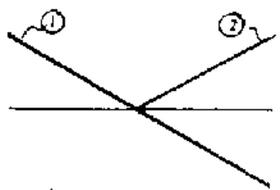
$y_C = 1.641$ mm $\downarrow$

Problem 9.46

9.46 For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the midpoint *C*. Use $E = 29 \times 10^6$ psi.

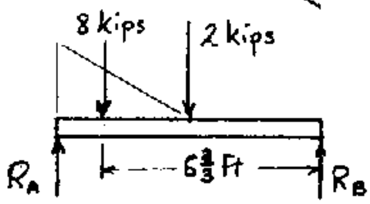


Distributed loads: ① $w_1(x) = w_0 - kx$
② $w_2(x) = kx$



Data: $a = 4$ ft, $w_0 = 4$ kips/ft, $k = 1$ kip/ft^2
$P = 2$ kips.

$$+\circlearrowleft \Sigma M_B = 0: \quad -8R_A + (8)(6\tfrac{2}{3}) + (2)(4) = 0 \qquad R_A = \tfrac{23}{3} \text{ kips}$$



$$w(x) = w_0 - kx + k\langle x-4\rangle^1$$
$$= 4 - x + \langle x-4\rangle^1$$

$$\frac{dV}{dx} = -w = -4 + x - \langle x-4\rangle^1$$

$$\frac{dM}{dx} = V = \frac{23}{3} - 4x + \frac{1}{2}x^2 - \frac{1}{2}\langle x-4\rangle^2 - 2\langle x-4\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M = \frac{23}{3}x - 2x^2 + \frac{1}{6}x^3 - \frac{1}{6}\langle x-4\rangle^3 - 2\langle x-4\rangle^1 \quad \text{kip}\cdot\text{ft}$$

$$EI\frac{dy}{dx} = \frac{23}{6}x^2 - \frac{2}{3}x^3 + \frac{1}{24}x^4 - \frac{1}{24}\langle x-4\rangle^4 - \langle x-4\rangle^2 + C_1 \quad \text{kip}\cdot\text{ft}^2$$

$$EIy = \frac{23}{18}x^3 - \frac{1}{6}x^4 + \frac{1}{120}x^5 - \frac{1}{120}\langle x-4\rangle^5 - \frac{1}{3}\langle x-4\rangle^3 + C_1x + C_2 \quad \text{kip}\cdot\text{ft}^3$$

$[x=0,\ y=0]$ $\quad 0 - 0 + 0 + 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x=8,\ y=0]$ $\quad (\frac{23}{18})(8)^3 - \frac{1}{6}(8)^4 + \frac{1}{120}(8)^5 - \frac{1}{120}(4)^5 - \frac{1}{3}(4)^3 + C_1(8) = 0$

$$C_1 = -26.844 \quad \text{kip}\cdot\text{ft}^2$$

Data: $E = 29\times10^6$ psi $= 29\times10^3$ ksi $\qquad I = 22.1$ in^4

$$EI = (29\times10^3)(22.1) = 640.9\times10^3 \text{ kip}\cdot\text{in}^2 = 4451 \text{ kip}\cdot\text{ft}^2$$

(a) Slope at A, $\quad (\frac{dy}{dx}$ at $x = 0)$

$$EI\theta_A = 0 + 0 + 0 + 0 + 0 - 26.844 \quad \text{kip}\cdot\text{ft}^2$$

$$\theta_A = -\frac{26.844}{4451} = -6.03\times10^{-5} \text{ rad} \qquad 6.03\times10^{-3} \text{ rad}$$

(b) Deflection at C. $\quad (y$ at $x = 4$ ft.)

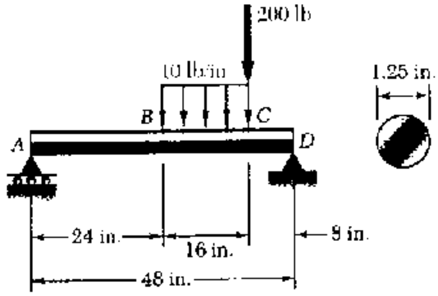
$$EIy_C = \frac{23}{18}(4)^3 - \frac{1}{6}(4)^4 + \frac{1}{120}(4)^5 - 0 - 0 - (26.844)(4) + 0$$
$$= -59.73 \quad \text{kip}\cdot\text{ft}^3$$

$$y_C = -\frac{59.73}{4451} = -13.42\times10^{-3} \text{ ft}$$

$$= 0.1610 \text{ in.} \downarrow$$

Problem 9.47

9.47 For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at point *B*. Use $E = 29 \times 10^6$ psi.



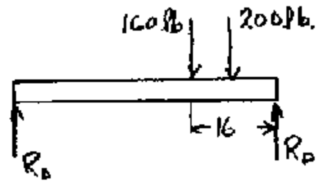
Units: Forces in lb. Lengths in inches

$c = \frac{1}{2}d = (\frac{1}{2})(1.25) = 0.625$ in.

$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(0.625)^4 = 119.84 \times 10^{-3}$ in^4

$EI = (29 \times 10^6)(119.84 \times 10^{-3}) = 3.4754 \times 10^6$ lb-in^2

Use entire beam ABCD as free body.



$+\circlearrowleft \Sigma M_D = 0: \quad -48 R_A + (16)(160) + (8)(200) = 0$

$R_A = 86.667$ lb $\uparrow$

$w(x) = 10\langle x-24\rangle^0 - 10\langle x-40\rangle^0$ lb/in.

$\frac{dV}{dx} = -w = -10\langle x-24\rangle^0 + 10\langle x-40\rangle^0$ lb/in

$\frac{dM}{dx} = V = -10\langle x-24\rangle^1 + 10\langle x-40\rangle^1 + 86.667 - 200\langle x-40\rangle^0$ lb-in

$EI\frac{d^2y}{dx^2} = M = -5\langle x-24\rangle^2 + 5\langle x-40\rangle^2 + 86.667x - 200\langle x-40\rangle^1$ lb-in

$EI\frac{dy}{dx} = -\frac{5}{3}\langle x-24\rangle^3 + \frac{5}{3}\langle x-40\rangle^3 + 43.333x^2 - 100\langle x-40\rangle^2 + C_1$ lb-in^2

$EIy = -\frac{5}{12}\langle x-24\rangle^4 + \frac{5}{12}\langle x-40\rangle^4 + 14.4444x^3 - \frac{100}{3}\langle x-40\rangle^3 + C_1x + C_2$ lb-in^3

$[x=0,\ y=0] \qquad -0 + 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x=48,\ y=0] \qquad -(\frac{5}{12})(24)^4 + (\frac{5}{12})(8)^4 + (14.4444)(48)^3 - (\frac{100}{3})(8)^3 + 48C_1 = 0$

$C_1 = -30.08 \times 10^3$ lb-in^2

(a) Slope at end A. $\left(\frac{dy}{dx}\text{ at } x=0\right)$

$EI\left(\frac{dy}{dx}\right)_A = -0 + 0 + 0 + C_1$

$\left(\frac{dy}{dx}\right)_A = \frac{C_1}{EI} = \frac{-30.08 \times 10^3}{3.4754 \times 10^6} = -8.66 \times 10^{-3} \qquad \theta_A = 8.66 \times 10^{-3}$ rad $\circlearrowright$

(b) Deflection at point B. (y at $x = 24$ in.)

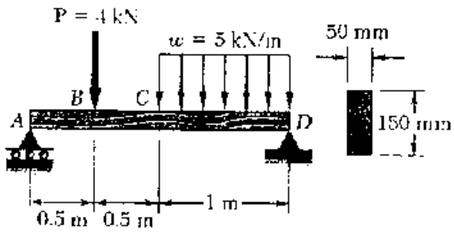
$EIy_B = -0 + 0 + (14.4444)(24)^3 - 0 + (-30.08 \times 10^3)(24)$

$= -522.24 \times 10^3$ lb-in^3

$y_B = \frac{-522.24 \times 10^3}{3.4754 \times 10^6} = -0.1503$ in. $\qquad y_B = 0.1503$ in. $\downarrow$

Problem 9.48

9.48 For the timber beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the midpoint *C*. Use E = 12 GPa.



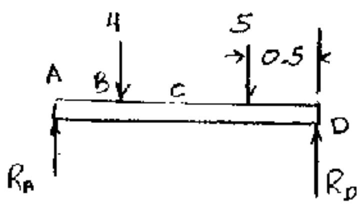
Units: Forces in kN, lengths in meters.

$I = \frac{1}{12}(50)(150)^3 = 14.0625 \times 10^6 \text{ mm}^4$

$= 14.0625 \times 10^{-6} \text{ m}^4$

$EI = (12 \times 10^9)(14.0625 \times 10^{-6}) =$

$168.75 \times 10^3 \text{ N}\cdot\text{m}^2 = 168.75 \text{ kN}\cdot\text{m}^2$



$+\circlearrowleft M_D = 0: \quad -2R_A + (1.5)(4) + (0.5)(5) = 0$

$R_A = 4.25 \text{ kN}$

$w(x) = 5\langle x-1\rangle^0 \qquad \text{kN}\cdot\text{m}$

$\frac{dV}{dx} = -w = -5\langle x-1\rangle^0 \quad \text{kN/m}$

$\frac{dM}{dx} = V = -5\langle x-1\rangle^1 + 4.25 - 4\langle x-0.5\rangle^0 \qquad \text{kN}$

$EI\frac{d^2y}{dx^2} = M = -\frac{5}{2}\langle x-1\rangle^2 + 4.25x - 4\langle x-0.5\rangle^1 \qquad \text{kN}\cdot\text{m}$

$EI\frac{dy}{dx} = -\frac{5}{6}\langle x-1\rangle^3 + 2.125x^2 - 2\langle x-0.5\rangle^2 + C_1 \qquad \text{kN}\cdot\text{m}^2$

$EIy = -\frac{5}{24}\langle x-1\rangle^4 + \frac{2.125}{3}x^3 - \frac{2}{3}\langle x-0.5\rangle^3 + C_1x + C_2 \qquad \text{kN}\cdot\text{m}^3$

$[x=0, y=0] \qquad -0+0-0+0+C_2 = 0 \qquad C_2 = 0$

$[x = 2\text{m}, y=0] \quad -(\frac{5}{24})(1)^4 + (\frac{2.125}{3})(2)^3 - (\frac{2}{3})(1.5)^3 + 2C_1 = 0$

$C_1 = -1.60417 \text{ kN}\cdot\text{m}^2$

(a) Slope at end A. ($\frac{dy}{dx}$ at $x=0$)

$EI\left(\frac{dy}{dx}\right)_A = -0+0-0+C_1$

$\left(\frac{dy}{dx}\right)_A = \frac{C_1}{EI} = \frac{-1.60417}{168.75} = -9.51 \times 10^{-3}$ $\qquad \theta_A = 9.51 \times 10^{-3} \text{ rad}$

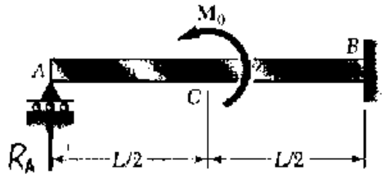
(b) Deflection at midpoint C. (y at $x = 1$ m)

$EIy_C = -0 + (\frac{2.125}{3})(1)^3 - (\frac{2}{3})(0.5)^3 + (-1.60417)(1) = -979.17 \times 10^{-3} \text{ kN}\cdot\text{m}^3$

$y_C = \frac{-979.17 \times 10^{-3}}{168.75} = -5.80 \times 10^{-3} \text{ m} \qquad y_C = 5.80 \text{ mm} \downarrow$

Problem 9.49

9.49 and 9.50 For the beam and loading shown, determine (*a*) the reaction at the roller support, (*b*) the deflection at point *C*.



For $0 \le x \le \frac{L}{2}$, $\quad M = R_A x$

For $\frac{L}{2} \le x \le L$, $\quad M = R_A x - M_0$

Then $\quad EI\frac{d^2y}{dx^2} = M = R_A x - M_0\langle x - \frac{L}{2}\rangle^0$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - M_0\langle x - \frac{L}{2}\rangle^1 + C_1$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{2}M_0\langle x - \frac{L}{2}\rangle^2 + C_1 x + C_2$$

$[x=0, y=0] \qquad 0 - 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x=L, \frac{dy}{dx}=0] \qquad \frac{1}{2}R_A L^2 - M_0(\frac{L}{2}) + C_1 = 0 \qquad C_1 = \frac{1}{2}(M_0 L - R_A L^2)$

$[x=L, y=0] \qquad \frac{1}{6}R_A L^3 - \frac{1}{2}M_0(\frac{L}{2})^2 + \frac{1}{2}(M_0 L - R_A L^2)L + 0 = 0$

$$-\frac{1}{3}R_A L^3 + \frac{3}{8}M_0 L^2 = 0$$

(a) Reaction at A. $\qquad R_A = \frac{9M_0}{8L}, \quad M_A = 0$ ◄

$$C_1 = \frac{1}{2}\left[M_0 L - \left(\frac{9M_0}{8L}\right)(L^2)\right] = -\frac{1}{16}M_0 L$$

$$EIy = \frac{1}{6}\left(\frac{9M_0}{8L}\right)x^3 - \frac{1}{2}M_0\langle x - \frac{L}{2}\rangle^2 - \frac{1}{16}M_0 L x + 0$$

Elastic curve. $\qquad y = \frac{M_0}{EIL}\left\{\frac{9}{8}x^3 - \frac{1}{2}L\langle x - \frac{L}{2}\rangle^2 - \frac{1}{16}L^2 x\right\}$

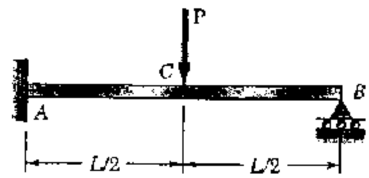
(b) Deflection at point C. $\quad$ (y at $x = \frac{L}{2}$)

$$y_C = \frac{M_0}{EIL}\left\{\left(\frac{1}{6}\right)\left(\frac{9}{8}\right)\left(\frac{L}{2}\right)^3 - 0 - \left(\frac{1}{16}L^2\right)\left(\frac{L}{2}\right)\right\} = -\frac{M_0 L^2}{128\,EI}$$

$$y_C = \frac{M_0 L^2}{128\,EI} \downarrow$$ ◄

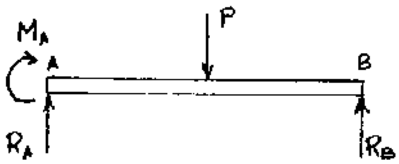
Problem 9.50

9.49 and 9.50 For the beam and loading shown, determine (*a*) the reaction at the roller support, (*b*) the deflection at point *C*.



$[x=0, y=0]$ $\quad [x=L, y=0]$

$[x=0, \frac{dy}{dx}=0]$



$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - P = 0 \qquad R_A = P - R_B$

$+\circlearrowleft \Sigma M_A = 0: \quad -M_A - P\frac{L}{2} + R_B L = 0 \qquad M_A = R_B L - \frac{1}{2}PL$

Reactions are statically indeterminate.

$\frac{dM}{dx} = V = R_A - P\langle x - \frac{L}{2}\rangle^0$

$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - P\langle x - \frac{L}{2}\rangle^1$

$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - \frac{1}{2}P\langle x - \frac{L}{2}\rangle^2 + C_1$

$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{1}{6}P\langle x - \frac{L}{2}\rangle^3 + C_1 x + C_2$

$[x=0, \frac{dy}{dx}=0] \quad 0 + 0 + 0 + C_1 = 0 \qquad C_1 = 0$

$[x=0, y=0] \quad 0 + 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x=L, y=0] \qquad \frac{1}{2}M_A L^2 + \frac{1}{6}R_A L^3 - \frac{1}{6}P(\frac{L}{2})^3 + 0 + 0 = 0$

$\frac{1}{2}(R_B L - \frac{1}{2}PL)L^2 + \frac{1}{6}(P - R_B)L^3 - \frac{1}{48}PL^3 = 0$

$(\frac{1}{2} - \frac{1}{6})R_B L^3 = (\frac{1}{4} - \frac{1}{6} + \frac{1}{48})PL^3 \qquad \frac{1}{3}R_B = \frac{5}{48}P \qquad R_B = \frac{5}{16}P \uparrow$

$R_A = P - \frac{5}{16}P = \frac{11}{16}P$

$M_A = \frac{5}{16}PL - \frac{1}{2}PL = -\frac{3}{16}PL$

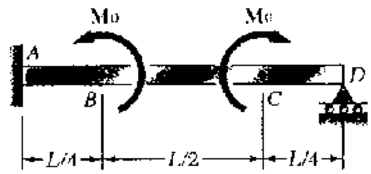
(b) Deflection at C. $\quad$ (y at $x = \frac{L}{2}$)

$y_C = \frac{1}{EI}\left\{\frac{1}{2}M_A(\frac{L}{2})^2 + \frac{1}{6}R_A(\frac{L}{2})^3 + 0 + 0 + 0\right\}$

$= \frac{PL^3}{EI}\left\{(\frac{1}{2})(-\frac{3}{16})(\frac{1}{4}) + (\frac{1}{6})(\frac{11}{16})(\frac{1}{8})\right\} = -\frac{7}{168}\frac{PL^3}{EI} \qquad y_C = \frac{7}{168}\frac{PL^3}{EI} \downarrow$

Problem 9.51

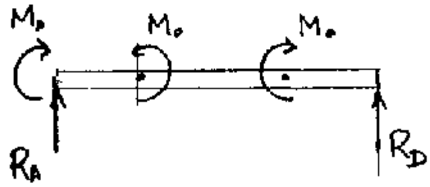
9.51 and 9.52 For the beam and loading shown, determine (*a*) the reaction at the roller support, (*b*) the deflection at point *B*.



$$+\uparrow \Sigma F_y = 0: \quad R_A + R_D = 0 \qquad R_A = -R_D$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A + M_o - M_o + R L = 0$$

$$M_A = R_D L$$



$$M(x) = M_A + R_A x - M_o\langle x - \tfrac{L}{4}\rangle^0 + M_o\langle x - \tfrac{3L}{4}\rangle^0$$

$$EI\frac{d^2y}{dx^2} = R_D L - R_D x - M_o\langle x - \tfrac{L}{4}\rangle^0 + M_o\langle x - \tfrac{3L}{4}\rangle^0$$

$$EI\frac{dy}{dx} = R_D L x - \tfrac{1}{2}R_D x^2 - M_o\langle x - \tfrac{L}{4}\rangle^1 + M_o\langle x - \tfrac{3L}{4}\rangle^1 + C_1$$

$$\left[x = 0,\ \frac{dy}{dx} = 0\right] \qquad 0 - 0 - 0 + 0 + C_1 = 0$$

$$C_1 = 0$$

$$EIy = \tfrac{1}{2}R_D L x^2 - \tfrac{1}{6}R_D x^3 - \tfrac{1}{2}M_o\langle x - \tfrac{L}{4}\rangle^1 + \tfrac{1}{2}M_o\langle x - \tfrac{3L}{4}\rangle^1 + C_2$$

$$[x = 0,\ y = 0] \qquad 0 - 0 - 0 + 0 + C_2 = 0 \qquad C_2 = 0$$

$$[x = L,\ y = 0] \qquad \tfrac{1}{2}R_D L^3 - \tfrac{1}{6}R_D L^3 - \tfrac{1}{2}M_o\left(\tfrac{3L}{4}\right)^2 + \tfrac{1}{2}M_o\left(\tfrac{L}{4}\right)^2 = 0$$

(a) Reaction at D. $\qquad R_D = \dfrac{3M_o}{4L}\uparrow$

$$EIy = \tfrac{1}{2}\left(\tfrac{3M_o}{4L}\right)L x^2 - \tfrac{1}{6}\left(\tfrac{3M_o}{4L}\right)x^3 - \tfrac{1}{2}M_o\langle x - \tfrac{L}{4}\rangle^2 + \tfrac{1}{2}M_o\langle x - \tfrac{3L}{4}\rangle^2$$

Elastic curve.

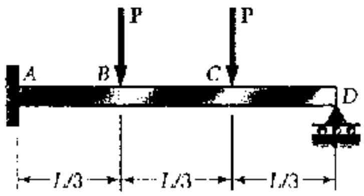
$$y = \frac{M_o}{EIL}\left\{\tfrac{3}{8}L x^2 - \tfrac{1}{8}x^3 - \tfrac{1}{2}L\langle x - \tfrac{L}{4}\rangle^2 + \tfrac{1}{2}L\langle x - \tfrac{3L}{8}\rangle^2\right\}$$

(b) Deflection at point B. $\quad (y \text{ at } x = \tfrac{L}{4})$

$$y_B = \frac{M_o}{EIL}\left\{\left(\tfrac{3}{8}L\right)\left(\tfrac{L}{4}\right)^2 - \left(\tfrac{1}{8}\right)\left(\tfrac{L}{4}\right)^3 - 0 + 0\right\} = \frac{11\,M_o L^2}{512\,EI}\uparrow$$

Problem 9.52

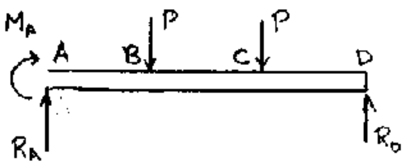
9.51 and 9.52 For the beam and loading shown, determine (*a*) the reaction at the roller support, (*b*) the deflection at point *B*.



$$+\uparrow \Sigma F_y = 0: \quad R_A - P - P + R_D = 0 \qquad R_A = 2P - R_D$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A - \frac{PL}{3} - \frac{2PL}{3} + R_D L = 0$$

$$M_A = R_D L - PL$$



$$\frac{dM}{dx} = V = R_A - P\langle x - \tfrac{L}{3}\rangle^0 - P\langle x - \tfrac{2L}{3}\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - P\langle x - \tfrac{L}{3}\rangle^1 - P\langle x - \tfrac{2L}{3}\rangle^1$$

$$EI\frac{dy}{dx} = M_A x + \tfrac{1}{2}R_A x^2 - \tfrac{1}{2}P\langle x - \tfrac{L}{3}\rangle^2 - \tfrac{1}{2}P\langle x - \tfrac{2L}{3}\rangle^2 + C_1$$

$$\left[x = 0,\ \frac{dy}{dx} = 0\right] \qquad 0 + 0 - 0 - 0 + C_1 = 0 \qquad C_1 = 0$$

$$EI\,y = \tfrac{1}{2}M_A x^2 + \tfrac{1}{6}R_A x^3 - \tfrac{1}{6}P\langle x - \tfrac{L}{3}\rangle^3 - \tfrac{1}{6}P\langle x - \tfrac{2L}{3}\rangle^3 + C_2$$

$$[x = 0,\ y = 0] \qquad 0 + 0 - 0 - 0 + C_2 = 0 \qquad C_2 = 0$$

$$EIy = \tfrac{1}{2}M_A x^2 + \tfrac{1}{6}R_A x^3 - \tfrac{1}{6}P\langle x - \tfrac{L}{3}\rangle^3 - \tfrac{1}{6}P\langle x - \tfrac{2L}{3}\rangle^3$$

$$[x = L,\ y = 0] \qquad \tfrac{1}{2}(R_D L - PL)L^2 + \tfrac{1}{6}(2P - R_D)L^3 - \tfrac{1}{6}P\left(\tfrac{2L}{3}\right)^3 - \tfrac{1}{6}P\left(\tfrac{L}{3}\right)^3 = 0$$

$$\tfrac{1}{3}R_D L^3 - \tfrac{2}{9}PL^3 = 0$$

(a) Reaction at D. $\qquad R_D = \frac{2}{3}P \uparrow$ ◄

$$M_A = \tfrac{2}{3}PL - PL = -\tfrac{1}{3}PL \qquad R_A = 2P - \tfrac{2}{3}P = \tfrac{4}{3}P$$

$$EIy = \tfrac{1}{2}\left(-\tfrac{1}{3}PL\right)x^2 + \tfrac{1}{6}\left(\tfrac{4}{3}P\right)x^3 - \tfrac{1}{6}P\langle x - \tfrac{L}{3}\rangle^3 - \tfrac{1}{6}\langle x - \tfrac{2L}{3}\rangle^3$$

Elastic curve.

$$y = \frac{P}{EI}\left\{ -\tfrac{1}{6}Lx^2 + \tfrac{2}{9}x^3 - \tfrac{1}{6}\langle x - \tfrac{L}{3}\rangle^3 - \tfrac{1}{6}\langle x - \tfrac{2L}{3}\rangle^3 \right\}$$

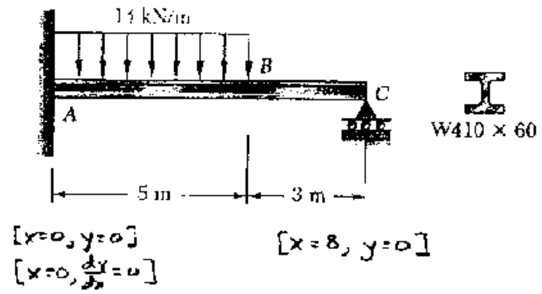
(b) Deflection at B. $\quad$ (y at $x = \frac{L}{3}$)

$$y_B = \frac{P}{EI}\left\{ -\left(\tfrac{1}{6}L\right)\left(\tfrac{L}{3}\right)^2 + \tfrac{2}{9}\left(\tfrac{L}{3}\right)^3 - 0 - 0 \right\} = -\frac{5PL^3}{486EI}$$

$$y_B = \frac{5PL^3}{486EI} \downarrow$$ ◄

Problem 9.53

9.53 For the beam and loading shown, determine (*a*) the reaction at point *C*, (*b*) the deflection at point *B*. Use $E = 200$ GPa.



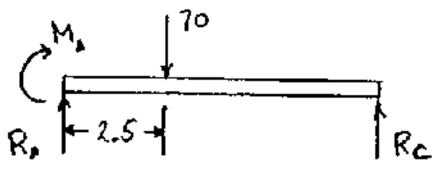
Units: Forces in kN, lengths in m.

$+\uparrow \Sigma F_y = 0: \quad R_A - 70 + R_C = 0$

$R_A = 70 - R_C \quad$ kN

$+\circlearrowleft M_A = 0: \quad -M_A - (70)(2.5) + 8R_C = 0$

$M_A = 8R_C - 175 \quad$ kN·m



Reactions are statically indeterminate.

$w(x) = 14 - 14\langle x-5\rangle^0 \quad$ kN/m

$\frac{dV}{dx} = -w = -14 + 14\langle x-5\rangle^0 \quad$ kN/m

$\frac{dM}{dx} = V = R_A - 14x + 14\langle x-5\rangle^1 \quad$ kN

$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - 7x^2 + 7\langle x-5\rangle^2 \quad$ kN·m

$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - \frac{7}{3}x^3 + \frac{7}{3}\langle x-5\rangle^3 + C_1 \quad$ kN·m²

$EI\,y = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{7}{12}x^4 + \frac{7}{12}\langle x-5\rangle^4 + C_1 x + C_2 \quad$ kN·m³

$[x=0,\ \frac{dy}{dx}=0] \quad 0+0+0+0+C_1 = 0 \qquad C_1 = 0$

$[x=0,\ y=0] \quad 0+0+0+0+0+C_2 = 0 \qquad C_2 = 0$

$[x=8,\ y=0] \quad \frac{1}{2}M_A(8)^2 + \frac{1}{6}R_A(8)^3 - \frac{7}{12}(8)^4 + \frac{7}{12}(3)^4 + 0 + 0 = 0$

$32(8R_C - 175) + \frac{512}{6}(70 - R_C) - \frac{28105}{12} = 0$

$170.667\,R_C = 5600 - \frac{35840}{6} + \frac{28105}{12} = 1968.75 \qquad R_C = 11.536 \text{ kN}\uparrow$

$M_A = (8)(11.536) - 175 = -82.715 \quad$ kN·m

$R_A = 70 - 11.536 = 58.464 \quad$ kN

Data: $E = 200\times10^9$ Pa $\qquad I = 216\times10^6\ \text{mm}^4 = 216\times10^{-6}\ \text{m}^4$

$EI = (200\times10^9)(216\times10^{-6}) = 43.2\times10^6\ \text{N·m}^2 = 43200\ \text{kN·m}^2$

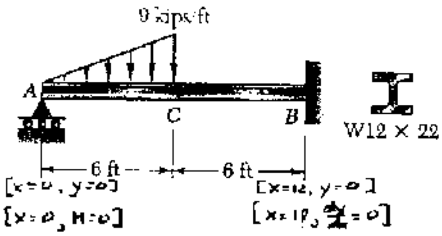
(b) Deflection at B. (y at $x = 5$ m)

$EI\,y_B = \frac{1}{2}(-82.715)(5)^2 + \frac{1}{6}(58.464)(5)^3 - \frac{7}{12}(5)^4 = -180.52 \quad$ kN·m³

$y_B = -\frac{180.52}{43200} = -4.18\times10^{-3}$ m $\qquad y_B = 4.18$ mm $\downarrow$

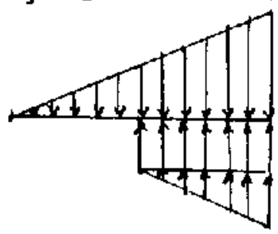
Problem 9.54

9.54* and *9.55 For the beam and loading shown, determine (*a*) the reaction at point *A*, (*b*) the deflection at point *C*. Use $E = 29 \times 10^6$ psi.



Units: Forces in kips, lengths in ft.

$$k = \frac{9 \text{ kips/ft}}{6 \text{ ft}} = 1.5 \text{ kips/ft}^2$$



$$w(x) = 1.5x - 9\langle x-6\rangle^0 - 1.5\langle x-6\rangle^1$$

$$\frac{dV}{dx} = -w(x) = -1.5x + 9\langle x-6\rangle^0 + 1.5\langle x-6\rangle^1$$

$$\frac{dM}{dx} = V = R_A - 0.75x^2 + 9\langle x-6\rangle^1 + 0.75\langle x-6\rangle^2$$

$$EI\frac{d^2y}{dx^2} = M = R_A x - 0.25x^3 + 4.5\langle x-6\rangle^2 + 0.25\langle x-6\rangle^3$$

$$EI\frac{dy}{dx} = \tfrac{1}{2}R_A x^2 - 0.0625x^4 + 1.5\langle x-6\rangle^3 + 0.0625\langle x-6\rangle^4 + C_1 \qquad \text{kip}\cdot\text{ft}^2$$

$$EIy = \tfrac{1}{6}R_A x^3 - 0.0125x^5 + 0.375\langle x-6\rangle^4 + 0.0125\langle x-6\rangle^5 + C_1 x + C_2 \qquad \text{kip}\cdot\text{ft}^3$$

$[x=0, y=0]$ $\quad 0 + 0 + 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x=12, \frac{dy}{dx}=0]$ $\quad \tfrac{1}{2}(R_A)(12)^2 - (0.0625)(12)^4 + (1.5)(6)^3 + (0.0625)(6)^4 + C_1 = 0$

$$C_1 = 891 - 72R_A = 0 \qquad \text{kip}\cdot\text{ft}^2$$

$[x=12, y=0]$ $\quad \tfrac{1}{6}R_A(12)^3 - (0.0125)(12)^5 + (0.375)(6)^4 + (0.0125)(6)^5$
$\quad + (891 - 72R_A)(12) + 0 = 0$

$$(864 - 288)R_A = 8164.8 \qquad R_A = 14.175 \text{ kips} \uparrow$$

$$C_1 = 891 - (72)(14.175) = -129.6 \text{ kip}\cdot\text{ft}^2$$

Data: $E = 29 \times 10^6$ psi $= 29 \times 10^3$ ksi $\qquad I = 156 \text{ in}^4$

$$EI = (29\times10^3)(156) = 4.524\times10^6 \text{ kip}\cdot\text{in}^2 = 31417 \text{ kip ft}^2$$

(b) Deflection at C. (y at $x = 6$)

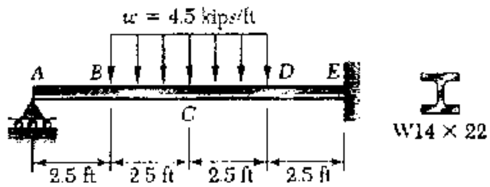
$$EIy_C = \tfrac{1}{6}(14.175)(6)^3 - (0.0125)(6)^5 + 0 + 0 - (129.6)(6) + 0$$
$$= -364.5 \text{ kip}\cdot\text{ft}^3$$

$$y_C = -\frac{364.5}{31417} = -11.60\times10^{-3} \text{ ft}$$

$$y_C = 0.1392 \text{ in.} \downarrow$$

Problem 9.55

9.54* and *9.55 For the beam and loading shown, determine (*a*) the reaction at point A, (*b*) the deflection at point C. Use $E = 29 \times 10^6$ psi.



$[x=0,\ y=0]$ $[x=0,\ M=0]$ $[x=10,\ y=0]$ $[x=10,\ \frac{dy}{dx}=0]$

Units: Forces in kips, lengths in ft.

$$w(x) = 4.5\langle x-2.5\rangle^0 - 4.5\langle x-7.5\rangle^0$$

$$\frac{dV}{dx} = -w(x) = -4.5\langle x-2.5\rangle^0 + 4.5\langle x-7.5\rangle^0 \quad \text{kip/ft}$$

$$\frac{dM}{dx} = V = R_A - 4.5\langle x-2.5\rangle^1 + 4.5\langle x-7.5\rangle^1 \quad \text{kips}$$

$$EI\frac{d^2y}{dx^2} = M = R_A x - 2.25\langle x-2.5\rangle^2 + 2.25\langle x-7.5\rangle^2 \quad \text{kip}\cdot\text{ft}$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{2.25}{3}\langle x-2.5\rangle^3 + \frac{2.25}{3}\langle x-7.5\rangle^3 + C_1 \quad \text{kip}\cdot\text{ft}^2$$

$$EI\,y = \frac{1}{6}R_A x^3 - \frac{2.25}{12}\langle x-2.5\rangle^4 + \frac{2.25}{12}\langle x-2.5\rangle^4 + C_1 x + C_2 \quad \text{kip}\cdot\text{ft}^3$$

$[x=0,\ y=0]$ $\quad 0+0+0+0+C_2 = 0 \qquad C_2 = 0$

$[x=10,\ \frac{dy}{dx}=0]$ $\quad \frac{1}{2}R_A(10)^2 - \frac{2.25}{3}(7.5)^3 + \frac{2.25}{3}\langle 2.5\rangle^3 + C_1 = 0$

$$C_1 = 304.69 - 50R_A \quad \text{kip}\cdot\text{ft}^2$$

$[x=10,\ y=0]$ $\quad \frac{1}{6}R_A(10)^3 - \frac{2.25}{12}(7.5)^4 + \frac{2.25}{12}(2.5)^4 + (304.69-50R_A)(10) + 0 = 0$

$$\left(500 - \frac{1000}{6}\right)R_A = 24609 \qquad R_A = 7.3833 \text{ kips} \uparrow$$

$$C_1 = 304.69 - (50)(7.3833) = -64.45 \quad \text{kip}\cdot\text{ft}^2$$

Data: $E = 29\times10^6$ psi $= 29\times10^3$ ksi, $\quad I = 199$ in^4

$$EI = (29\times10^3)(199) = 5.771\times10^6 \text{ kip}\cdot\text{in}^2 = 40076 \text{ kip}\cdot\text{ft}^2$$

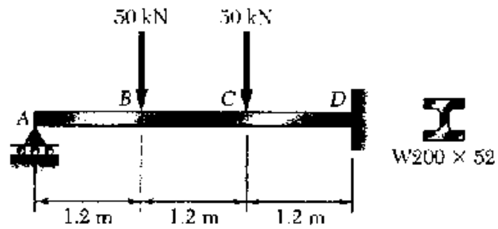
(b) Deflection at C. (y at $x = 5$ ft)

$$EI\,y_C = \frac{1}{6}(7.3833)(5)^3 - \frac{2.25}{12}(2.5)^4 + 0 - (64.45)(5) + 0 = -175.76 \text{ kip}\cdot\text{ft}^3$$

$$y_C = -\frac{175.76}{40076} = -4.3856\times10^{-3} \text{ ft} \qquad y_C = 0.0526 \text{ in.} \downarrow$$

Problem 9.56

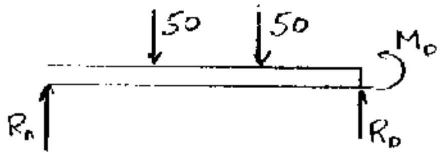
9.56 For the beam and loading shown, determine (*a*) the reaction at point *A*, (*b*) the deflection at point *B*. Use $E = 200$ GPa.



Units: Forces in kN, lengths in meters.

$I = 52.7 \times 10^6 \text{ mm}^4 = 52.7 \times 10^{-6} \text{ m}^4$

$EI = (200 \times 10^9)(52.7 \times 10^{-6}) = 10.54 \times 10^6 \text{ N}\cdot\text{m}^2$
$= 10540 \text{ kN}\cdot\text{m}^2$



$$\frac{dM}{dx} = V = R_A - 50\langle x-1.2\rangle^0 - 50\langle x-2.4\rangle^0 \quad \text{kN}$$

$$EI\frac{d^2y}{dx^2} = M = R_A x - 50\langle x-1.2\rangle^1 - 50\langle x-2.4\rangle^1 \quad \text{kN-m}$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - 25\langle x-1.2\rangle^2 - 25\langle x-2.4\rangle^2 + C_1 \quad \text{kN-m}^2$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{25}{3}\langle x-1.2\rangle^3 - \frac{25}{3}\langle x-2.4\rangle^3 + C_1 x + C_2 \quad \text{kN-m}^3$$

$[x=0, y=0]$ $\quad 0 - 0 - 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x=3.6, \frac{dy}{dx}=0]$ $\quad \frac{1}{2}R_A(3.6)^2 - (25)(2.4)^2 - (25)(1.2)^2 + C_1 = 0$

$$C_1 = 180 - 6.48 R_A \quad \text{kN}\cdot\text{m}^2$$

$[x=3.6, y=0]$ $\quad \frac{1}{6}R_A(3.6)^3 - \frac{25}{3}(2.4)^3 - \frac{25}{3}(1.2)^3 + (180 - 6.48 R_A)(3.6) = 0$

$$-15.552 R_A + 518.4 = 0$$

(a) Reaction at A. $\quad R_A = \frac{100}{3}$ kN $\qquad R_A = 33.3 \text{ kN} \uparrow$ ◄

$$C_1 = 180 - (6.48)(33.3333) = -36 \quad \text{kN-m}^2$$

$$EIy = \frac{1}{6}\left(\frac{100}{3}\right)x^3 - \frac{25}{3}\langle x-1.2\rangle^3 - \frac{25}{3}\langle x-2.4\rangle^3 - 36x \quad \text{kN}\cdot\text{m}^3$$

(b) Deflection at point B. $\quad$ (y at $x = 1.2$ m)

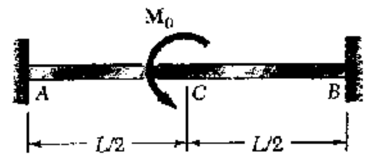
$$EIy_B = \left(\frac{1}{6}\right)\left(\frac{100}{3}\right)(1.2)^3 - 0 - 0 - (36)(1.2) = -33.6 \quad \text{kN}\cdot\text{m}^3$$

$$y_B = \frac{-33.6}{10540} = -3.19 \times 10^{-3} \text{ m}$$

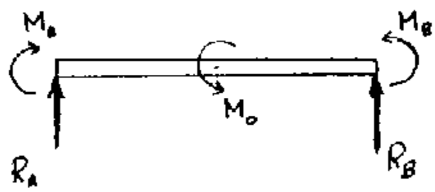
$y_B = 3.19 \text{ mm} \downarrow$ ◄

Problem 9.57

9.57 For the beam and loading shown, determine (*a*) the reaction at point *A*, (*b*) the slope at point *C*.



Reactions are statically indeterminate.



$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - M_0\langle x - \tfrac{L}{2}\rangle^0$$

$$EI\frac{dy}{dx} = M_A x + \tfrac{1}{2}R_A x^2 - M_0\langle x - \tfrac{L}{2}\rangle^1 + C_1$$

$$EI\,y = \tfrac{1}{2}M_A x^2 + \tfrac{1}{6}R_A x^3 - \tfrac{1}{2}M_0\langle x - \tfrac{L}{2}\rangle^2 + C_1 x + C_2$$

$[x = 0,\ \frac{dy}{dx} = 0]$ $\qquad C_1 = 0$

$[x = 0,\ y = 0]$ $\qquad C_2 = 0$

$[x = L,\ \frac{dy}{dx} = 0]$ $\qquad M_A L + \frac{1}{2}R_A L^2 - M_0\frac{L}{2} = 0$ $\qquad M_A = \frac{1}{2}M_0 - \frac{1}{2}R_A L$

$[x = L,\ y = 0]$ $\qquad \frac{1}{2}M_A L^2 + \frac{1}{6}R_A L^3 - \frac{1}{2}M_0\left(\frac{L}{2}\right)^2 = 0$

$$\tfrac{1}{2}\left(\tfrac{1}{2}M_0 - \tfrac{1}{2}R_A L\right)L^2 + \tfrac{1}{6}R_A L^3 - \tfrac{1}{8}M_0 L^2 = 0$$

$$\left(\tfrac{1}{4} - \tfrac{1}{6}\right)R_A L^3 = \left(\tfrac{1}{4} - \tfrac{1}{8}\right)M_0 L^2 \qquad R_A = \frac{3}{2}\frac{M_0}{L}\uparrow$$

$$M_A = \tfrac{1}{2}M_0 - \tfrac{1}{2}\cdot\tfrac{3}{2}\frac{M_0}{L}L = -\tfrac{1}{4}M_0$$

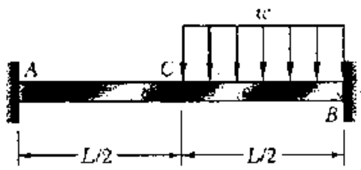
(b) Slope at C. $\left(\frac{dy}{dx} \text{ at } x = \frac{L}{2}\right)$

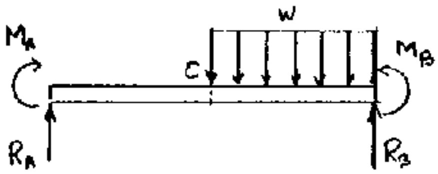
$$\theta_C = \frac{1}{EI}\left\{\left(-\tfrac{1}{4}M_0\right)\frac{L}{2} + \tfrac{1}{2}\left(\tfrac{3}{2}\frac{M_0}{L}\right)\left(\frac{L}{2}\right)^2 + 0 + 0 = \right\} -\frac{1}{16}\frac{M_0 L}{EI}$$

$$\theta_C = \frac{1}{16}\frac{M_0 L}{EI}$$

Problem 9.58

9.58 For the beam and loading shown, determine (*a*) the reaction at point *A*, (*b*) the deflection at midpoint *C*.





$$w(x) = w\langle x - \tfrac{L}{2}\rangle^0$$

$$\frac{dV}{dx} = -w(x) = -w\langle x - \tfrac{L}{2}\rangle^0$$

$$\frac{dM}{dx} = V = R_A - w\langle x - \tfrac{L}{2}\rangle^1$$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - \tfrac{1}{2}w\langle x - \tfrac{L}{2}\rangle^2$$

$$EI\frac{dy}{dx} = M_A x + \tfrac{1}{2}R_A x^2 - \tfrac{1}{6}w\langle x - \tfrac{L}{2}\rangle^3 + C_1$$

$$EIy = \tfrac{1}{2}M_A x^2 + \tfrac{1}{6}R_A x^3 - \tfrac{1}{24}w\langle x - \tfrac{L}{2}\rangle^4 + C_1 x + C_2$$

$[x = 0,\ \frac{dy}{dx} = 0]$ $\quad 0 + 0 - 0 + C_1 = 0 \quad\quad C_1 = 0$

$[x = 0,\ y = 0]$ $\quad 0 + 0 - 0 + 0 + C_2 = 0 \quad\quad C_2 = 0$

$[x = L,\ \frac{dy}{dx} = 0]$ $\quad M_A L + \frac{1}{2}R_A L^2 - \frac{1}{6}w\left(\frac{L}{2}\right)^3 = 0 \quad\quad (1)$

$[x = L,\ y = 0]$ $\quad \frac{1}{2}M_A L^2 + \frac{1}{6}R_A L^2 - \frac{1}{24}w\left(\frac{L}{2}\right)^4 = 0 \quad\quad (2)$

Solving equations (1) and (2) simultaneously,

(a) $\quad R_A = \frac{3wL}{32} \qquad M_A = -\frac{5wL^2}{192}$

$$R_A = \frac{3wL}{32} \uparrow$$

$$M_A = \frac{5wL^2}{192} \circlearrowright$$

$$EIy = -\frac{5}{384}wL^2x^2 + \frac{3}{192}wLx^3 - \frac{1}{24}w\langle x - \tfrac{L}{2}\rangle^4$$

Elastic curve.

$$y = \frac{w}{EI}\left\{-\frac{5}{384}L^2x^2 + \frac{3}{192}Lx^3 - \frac{1}{24}\langle x - \tfrac{L}{2}\rangle^4\right\}$$

(b) Deflection at midpoint C. $\quad$ (y at $x = \frac{L}{2}$)

$$y_C = \frac{w}{EI}\left\{\left(-\frac{5}{384}L^2\right)\left(\frac{L}{2}\right)^2 + \left(\frac{3}{192}L\right)\left(\frac{L}{2}\right)^3 - 0\right\}$$

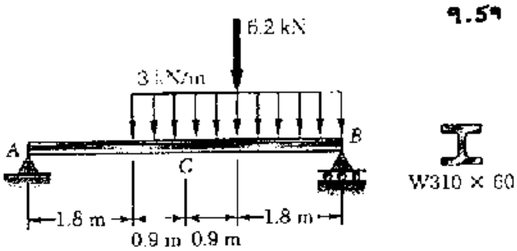
$$= -\frac{wL^4}{768\,EI}$$

$$y_B = \frac{wL^4}{768\,EI} \downarrow$$

Problem 9.59

9.59 through 9.62 For the beam and loading indicated, determine the magnitude and location of the largest downward deflection.

9.59 Beam and loading of Prob. 9.45.



See solution to Prob. 9.45 for the derivation of the equations used in the following.

$EI = 25.8 \times 10^3 \text{ kN}\cdot\text{m}$

$EI\frac{dy}{dx} = 2.8333x^2 - \frac{1}{2}\langle x-1.8\rangle^2 - 3.1\langle x-3.6\rangle^2 - 22.535$

$EIy = 0.9444x^3 - \frac{1}{6}\langle x-1.8\rangle^3 - 1.03333\langle x-3.6\rangle^3 - 22.535x$

To find location of maximum $|y|$, set $\frac{dy}{dx} = 0$. Assume $1.8 < x_m < 3.6$

$EI\frac{dy}{dx} = 2.8333x_m^2 - \frac{1}{2}(x_m - 1.8)^2 - 22.535 = 0$

Solving by iteration: $x_m = 3,\ 2.86,\ 2.855$ $\quad df/dx = 15.8,\ 15.15$ $\qquad x_m = 2.855 \text{ m}$ ◄

$EIy_m = 0.9444x_m^3 - \frac{1}{6}(x_m - 1.8)^3 - 22.535x_m$

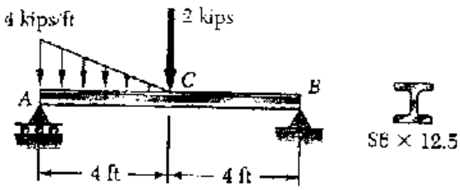
$= (0.9444)(2.855)^3 - \frac{1}{6}(2.855-1.8)^3 - (22.535)(2.855) = -42.507 \text{ kN}\cdot\text{m}^3$

$y_m = -\frac{42.507}{25.8\times10^3} = -1.648\times10^{-3} \text{ m}$ $\qquad y_m = 1.648 \text{ mm} \downarrow$ ◄

Problem 9.60

9.59 through 9.62 For the beam and loading indicated, determine the magnitude and location of the largest downward deflection.

9.60 Beam and loading of Prob. 9.46.



See solution to Prob. 9.46 for the derivation of the equations used in the following.

$EI = 4451 \text{ kip}\cdot\text{ft}^2$

$$EI\frac{dy}{dx} = \frac{23}{6}x^2 - \frac{2}{3}x^3 + \frac{1}{24}x^4 - \frac{1}{24}\langle x-4\rangle^4 - \langle x-4\rangle^2 - 26.844 \qquad \text{kip}\cdot\text{ft}^2$$

$$EIy = \frac{23}{18}x^3 - \frac{1}{6}x^4 + \frac{1}{120}x^5 - \frac{1}{120}\langle x-4\rangle^5 - \frac{1}{3}\langle x-4\rangle^3 - 26.844\,x \qquad \text{kip}\cdot\text{ft}^3$$

To find location of maximum $|y|$, set $\frac{dy}{dx} = 0$. Assume $0 < x < 4$ ft.

$$EI\frac{dy}{dx} = \frac{23}{6}x_m^2 - \frac{2}{3}x_m^3 + \frac{1}{24}x_m^4 - 26.844 = 0 \qquad f.$$

Solve by iteration: $x_m = 4.0 \quad 3.73 \quad 3.735$ $\qquad x_m = 3.735$ ft. ◄

$df/dx = 9.33 \quad 9.42$

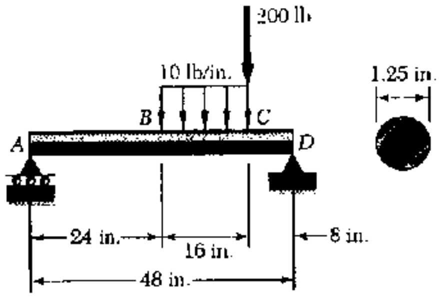
$$EIy_m = \frac{23}{18}(3.735)^3 - \frac{1}{6}(3.735)^4 + \frac{1}{120}(3.735)^5 - (26.844)(3.735) = -60.06 \text{ kip}\cdot\text{ft}^3$$

$$y_m = -\frac{60.06}{4451} = -13.49\times10^{-3} \text{ ft.} \qquad y_m = 0.1619 \text{ in.} \downarrow$$ ◄

Problem 9.61

9.59 through 9.62 For the beam and loading indicated, determine the magnitude and location of the largest downward deflection.

9.61 Beam and loading of Prob. 9.47.



See solution to Prob. 9.47 for the derivation of equations used in the following.

$EI = 3.4754 \times 10^6$ lb·in²

$C_1 = -30.08 \times 10^3$ lb·in², $C_2 = 0$

$$EI\frac{dy}{dx} = -\frac{5}{3}\langle x-24\rangle^3 + \frac{5}{3}\langle x-40\rangle^3 + 43.333\,x^2 - 100\langle x-40\rangle^2 + C_1 \quad \text{lb·in}^2$$

$$EI\,y = -\frac{5}{12}\langle x-24\rangle^4 + \frac{5}{12}\langle x-40\rangle^4 + 14.4444\,x^3 - \frac{100}{3}\langle x-40\rangle^3 + C_1 x + C_2 \quad \text{lb·in}^3$$

To find location of maximum $|y|$, set $\frac{dy}{dx} = 0$. Assume $24 < x_m < 40$.

$$EI\frac{dy}{dx} = -\frac{5}{3}(x_m-24)^3 + 0 + 43.3333\,x^2 - 0 - 30.08\times10^3 = 0$$

$$f = -\frac{5}{3}(x_m-24)^3 + 43.3333\,x^2 - 30.08\times10^3$$

$$\frac{df}{dx_m} = -5(x_m-24)^2 + 86.667\,x$$

Solve by iteration:

$x_m =$	24	26.46	26.35
$f =$	-5.12×10^3	249	-14
$df/dx =$	2.08×10^3	2.26×10^3	

$x_m = 26.4$ in. ◄

$$EI\,y_m = -\frac{5}{12}(26.35-24)^4 + 0 + (14.4444)(26.35)^3 - (30.08\times10^3)(26.35)$$

$$= -528.35\times10^3 \quad \text{lb·in}^3$$

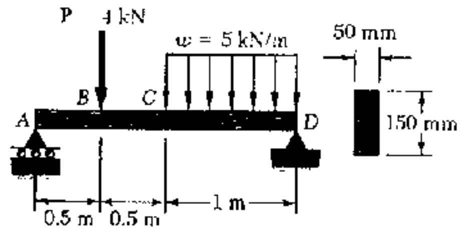
$$y_m = \frac{-528.35\times10^3}{3.4754\times10^6} = -0.1520 \text{ in.}$$

$y_m = 0.1520$ in. ↓ ◄

Problem 9.62

9.59 through 9.62 For the beam and loading indicated, determine the magnitude and location of the largest downward deflection.

9.62 Beam and loading of Prob. 9.48.



See solution to Prob. 9.48 for the derivation of the equations used in the following.

$EI = 168.75\ \text{kN}\cdot\text{m}^2 \qquad C_1 = -1.60417\ \text{kN}\cdot\text{m}^2 \qquad C_2 = 0$

$$EI\frac{dy}{dx} = -\frac{5}{6}\langle x-1\rangle^3 + 2.125x^2 - 2\langle x-0.5\rangle^2 + C_1 \qquad \text{kN}\cdot\text{m}^2$$

$$EIy = -\frac{5}{24}\langle x-1\rangle^4 + \frac{2.125}{3}x^3 - \frac{2}{3}\langle x-0.5\rangle^3 + C_1x + C_2 \qquad \text{kN}\cdot\text{m}^3$$

Compute slope at C. ($\frac{dy}{dx}$ at $x = 1$ m)

$$EI\left(\frac{dy}{dx}\right)_C = 0 + (2.125)(1)^2 - 2(0.5)^2 - 1.60417 = 20.83\times10^{-3}\ \text{kN}\cdot\text{m}^2$$

Since the slope at C is positive, the largest deflection occurs in portion BC, where

$$EI\frac{dy}{dx} = 2.125x^2 - 2(x-0.5)^2 - 1.60417$$

$$EIy = \frac{2.125}{3}x^3 - \frac{2}{3}(x-0.5)^3 - 1.60417x$$

To find the location of the largest downward deflection, set $\frac{dy}{dx} = 0$.

$$2.125x_m^2 - 2(x_m^2 - x_m + 0.25) - 1.60417$$
$$= 0.125x_m^2 + 2x_m - 2.10417 = 0$$

$$x = 1.0521 - 0.0625x^2$$

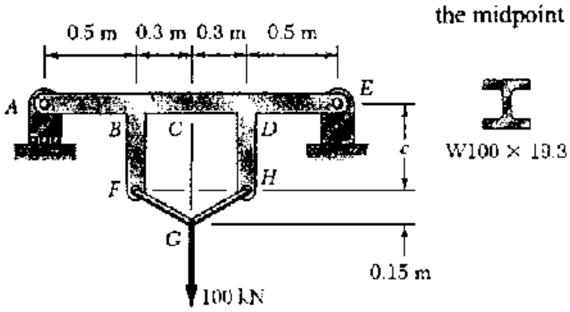
Solve by iteration. $x_m = 1,\ 0.989,\ 0.991,$ $\qquad x_m = 0.991\ \text{m}$ ◄

$$EIy_m = \left(\frac{2.125}{3}\right)(0.991)^3 - \frac{2}{3}(0.991-0.5)^3 - (1.60417)(0.991)$$
$$= -0.97927\ \text{kN}\cdot\text{m}^3$$

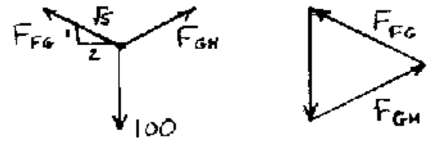
$$y_m = \frac{-0.97927}{168.75} = -5.80\times10^{-3}\ \text{m} \qquad y_m = 5.80\ \text{mm}\downarrow$$ ◄

Problem 9.63

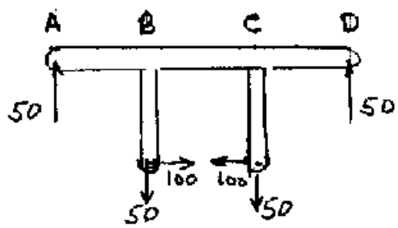
9.63 The rigid bars BF and DH are welded to the rolled-steel beam AE as shown. Determine for the loading shown (a) the deflection at point B, (b) the deflection at the midpoint C of the beam. Use $E = 200$ GPa.



Using joint G as a free body,



By symmetry $F_{GH} = F_{FG}$



$2F_{GHy} - 100 = 0 \qquad F_{GHy} = 50$ kN

$F_{GHx} = 2F_{GHx} = 100$ kN.

Forces in kN. Lengths in m.

$V = 50 - 50\langle x-0.5\rangle^0 - 50\langle x-1.1\rangle^0$ kN

$M = 50x - 50\langle x-0.5\rangle^1 - 50\langle x-1.1\rangle^0$
$\quad + 40\langle x-0.5\rangle^0 - 40\langle x-1.1\rangle$ kN·m

$EI\frac{dy}{dx} = 25x^2 - 25\langle x-0.5\rangle^2 - 25\langle x-1.1\rangle^2 - 40\langle x-0.5\rangle^1 + 40\langle x-1.1\rangle^1 + C_1$ kN·m²

$EI\,y = \frac{25}{3}x^3 - \frac{25}{3}\langle x-0.5\rangle^3 - \frac{25}{3}\langle x-1.1\rangle^3 - 20\langle x-0.5\rangle^2 + 20\langle x-1.1\rangle^2 + C_1x + C_2$ kN·m³

$[x=0,\ y=0] \qquad C_2 = 0$

$[x=1.6,\ y=0]$

$\left(\frac{25}{3}\right)(1.6)^3 - \left(\frac{25}{3}\right)(1.1)^3 - \left(\frac{25}{3}\right)(0.5)^3 - (20)(1.1)^2 + (20)(0.5)^2 + C_1(1.6) + 0 = 0$

$C_1 = -1.75$ kN·m³

For $EI\,y_B$, $x = 0.5$ m

$EI\,y_B = \left(\frac{25}{3}\right)(0.5)^3 - 0 - 0 + 0 - 0 - (1.75)(0.5) = 0.1667$ kN·m³

For $EI\,y_C$, $x = 0.8$ m

$EI\,y_C = \left(\frac{25}{3}\right)(0.8)^3 - \left(\frac{25}{3}\right)(0.3)^3 - 0 - (20)(0.3)^2 - 0 - (1.75)(0.8) + 0$
$= -0.8417$ kN·m³

For W 100 × 19.3 rolled steel section $I = 4.77\times10^6\ \text{mm}^4 = 4.77\times10^{-6}\ \text{m}^4$

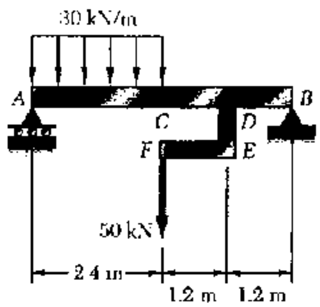
$EI = (200\times10^9)(4.77\times10^{-6}) = 954\times10^3\ \text{N·m}^2 = 954\ \text{kN·m}^2$

(a) $y_B = \frac{0.1667}{954} = 0.175\times10^{-3}$ m $= 0.175$ mm ↑ ◄

(b) $y_C = \frac{0.8417}{954} = 0.882\times10^{-3}$ m $= 0.882$ mm ↑ ◄

Problem 9.64

9.64 The rigid bar *DEF* is welded at point *D* to the rolled-steel beam *AB*. For the loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at midpoint *C* of the beam. Use $E = 200$ GPa.

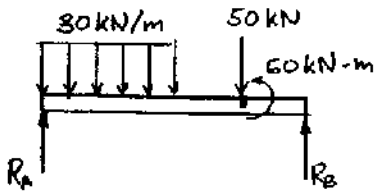


Units: Forces in kN; lengths in meters.

$+\circlearrowleft \Sigma M_B = 0:$

$-4.8 R_A + (30)(2.4)(3.6) + (50)(2.4) = 0$

$R_A = 79$ kN ↑



$I = 212 \times 10^6 \text{ mm}^4 = 212 \times 10^{-6} \text{ m}^4$

$EI = (200 \times 10^9)(212 \times 10^{-6}) = 42.4 \times 10^6 \text{ N}\cdot\text{m}^2$
$= 42400 \text{ kN}\cdot\text{m}^2$

$w(x) = 30 - 30\langle x-2.4\rangle^0$

$\frac{dV}{dx} = -w = -30 + 30\langle x-2.4\rangle^0 \quad \text{kN/m}$

$\frac{dM}{dx} = V = 79 - 30x + 30\langle x-2.4\rangle^1 - 50\langle x-3.6\rangle^0 \quad \text{kN}$

$EI\frac{d^2y}{dx^2} = M = 79x - 15x^2 + 15\langle x-2.4\rangle^2 - 50\langle x-3.6\rangle^1 - 60\langle x-3.6\rangle^0 \quad \text{kN}\cdot\text{m}$

$EI\frac{dy}{dx} = \frac{79}{2}x^2 - 5x^3 + 5\langle x-2.4\rangle^3 - 25\langle x-3.6\rangle^2 - 60\langle x-3.6\rangle^1 + C_1 \quad \text{kN}\cdot\text{m}^2$

$EIy = \frac{79}{6}x^3 - \frac{5}{4}x^4 + \frac{5}{4}\langle x-2.4\rangle^4 - \frac{25}{3}\langle x-3.6\rangle^3 - 30\langle x-3.6\rangle^2 + C_1x + C_2 \quad \text{kN}\cdot\text{m}^3$

$[x=0,\ y=0] \quad 0 - 0 + 0 - 0 - 0 + 0 + C_2 = 0 \quad C_2 = 0$

$[x=4.8,\ y=0] \quad (\frac{79}{6})(4.8)^3 - (\frac{5}{4})(4.8)^4 + (\frac{5}{4})(2.4)^4 - (\frac{25}{3})(1.2)^3 - (30)(1.2)^2 + 4.8C_1 = 0$

$C_1 = -161.76 \text{ kN}\cdot\text{m}^2$

(a) Slope at point A. ($\frac{dy}{dx}$ at $x=0$)

$EI\left(\frac{dy}{dx}\right)_A = 0 - 0 + 0 - 0 - 0 - 161.76 = -161.76 \text{ kN}\cdot\text{m}^2$

$\left(\frac{dy}{dx}\right)_A = \frac{-161.76}{42400} = -3.82 \times 10^{-3}$ $\qquad \theta_A = 3.82 \times 10^{-3}$ rad ◄

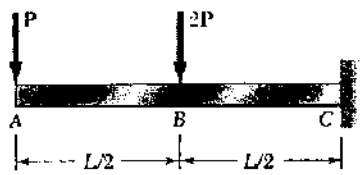
(b) Deflection at midpoint C. (y at $x = 2.4$)

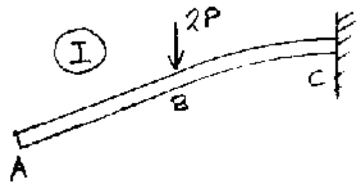
$EIy_C = (\frac{79}{6})(2.4)^3 - (\frac{5}{4})(2.4)^4 + 0 - 0 - 0 - (161.76)(2.4) + 0 = -247.68 \text{ kN}\cdot\text{m}^3$

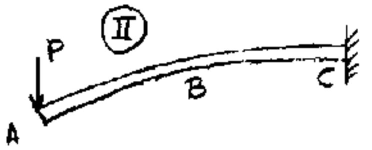
$y_C = \frac{-247.68}{42400} = -5.84 \times 10^{-3}$ m $\qquad y_C = 5.84$ mm ↓ ◄

Problem 9.65

9.65 and 9.66 For the cantilever beam and loading shown, determine the slope and deflection at the free end.







Loading I: 2P downward at B.

Case 1 of Appendix D applied to portion BC.

$$\theta_B' = \frac{(2P)(L/2)^2}{2EI} = \frac{1}{4}\frac{PL^2}{EI}$$

$$y_B' = \frac{(2P)(L/2)^3}{3EI} = \frac{1}{12}\frac{PL^3}{EI}$$

AB remains straight.

$$\theta_A' = \theta_B' = \frac{1}{4}\frac{PL^2}{EI}$$

$$y_A' = y_B' - \left(\frac{L}{2}\right)\theta_B' = -\frac{1}{12}\frac{PL^3}{EI} - \frac{1}{8}\frac{PL^3}{EI} = -\frac{5}{24}\frac{PL^3}{EI}$$

Loading II: P downward at A. Case 1 of Appendix D.

$$\theta_A'' = \frac{PL^2}{2EI}, \quad y_A'' = -\frac{PL^3}{3EI}$$

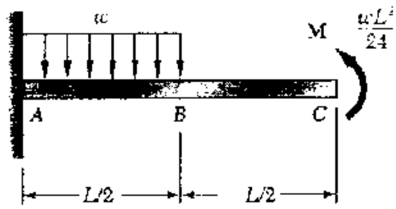
By superposition,

$$\theta_A = \theta_A' + \theta_A'' = \frac{1}{4}\frac{PL^2}{EI} + \frac{1}{2}\frac{PL^2}{EI} = \frac{3}{4}\frac{PL^2}{EI} \qquad \frac{3PL^2}{4EI} \measuredangle$$

$$y_A = y_A' + y_A'' = -\frac{5}{24}\frac{PL^3}{EI} - \frac{1}{3}\frac{PL^3}{EI} = -\frac{13}{24}\frac{PL^3}{EI} \qquad \frac{13PL^3}{24EI} \downarrow$$

Problem 9.66

9.65 and 9.66 For the cantilever beam and loading shown, determine the slope and deflection at the free end.



Loading I: Downward distributed load w applied to portion AB.

Case 2 of Appendix D applied to portion AB.

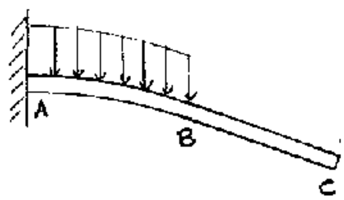
$$\theta_B' = -\frac{w(L/2)^3}{6EI} = -\frac{1}{48}\frac{wL^3}{EI}$$

$$y_B' = -\frac{w(L/2)^4}{8EI} = -\frac{1}{128}\frac{wL^4}{EI}$$

Portion BC remains straight.

$$\theta_C' = \theta_B' = -\frac{1}{48}\frac{wL^3}{EI}$$

$$y_C' = y_B' + \left(\frac{L}{2}\right)\theta_B' = -\frac{1}{128}\frac{wL^4}{EI} - \frac{1}{96}\frac{wL^4}{EI} = -\frac{7}{384}\frac{wL^4}{EI}$$



Loading II: Counterclockwise couple $\frac{wL^2}{24}$ applied at C.

Case 3 of Appendix D

$$\theta_C'' = \frac{(wL^2/24)L}{EI} = \frac{1}{24}\frac{wL^3}{EI}$$

$$y_C'' = \frac{(wL^2/24)L^2}{2EI} = \frac{1}{48}\frac{wL^4}{EI}$$

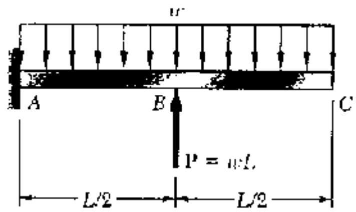
By superposition,

$$\theta_C = \theta_C' + \theta_C'' = -\frac{1}{48}\frac{wL^3}{EI} + \frac{1}{24}\frac{wL^3}{EI} = \frac{1}{48}\frac{wL^3}{EI}$$

$$y_C = y_C' + y_C'' = -\frac{7}{384}\frac{wL^4}{EI} + \frac{1}{48}\frac{wL^4}{EI} = \frac{1}{384}\frac{wL^4}{EI} \uparrow$$

Problem 9.67

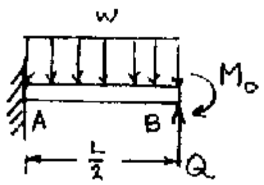
9.67 and 9.68 For the cantilever beam and loading shown, determine the slope and deflection at point *B*.



Consider portion AB as a cantilever beam subjected to the three loadings shown.

By statics: $Q = P - \frac{wL}{2} = \frac{wL}{2}$

$$M_o = \left(\frac{wL}{2}\right)\left(\frac{L}{2} \cdot \frac{L}{2}\right) = \frac{wL^2}{8}$$



Slope at B. $\theta_B = (\theta_B)_w + (\theta_B)_Q + (\theta_B)_M$

Case 2 of App. D. $(\theta_B)_w = -\frac{w}{6EI}\left(\frac{L}{2}\right)^3 = -\frac{wL^3}{48EI}$

Case 1 of App. D. $(\theta_B)_Q = \frac{+(wL/2)}{2EI}\left(\frac{L}{2}\right)^2 = \frac{wL^3}{16EI}$

Case 3 of App. D. $(\theta_B)_M = -\frac{(wL^2/8)}{EI}\left(\frac{L}{2}\right) = -\frac{wL^3}{16EI}$

$$\theta_B = -\frac{wL^3}{48EI} \qquad \theta_B = \frac{wL^3}{48EI}$$

Deflection at B. $y_B = (y_B)_w + (y_B)_Q + (y_B)_M$

Case 2 of App. D. $(y_B)_w = -\frac{w}{8EI}\left(\frac{L}{2}\right)^4 = -\frac{wL^4}{128EI}$

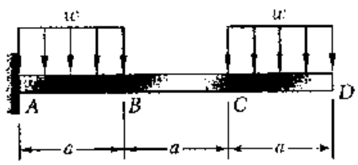
Case 1 of App. D. $(y_B)_Q = +\frac{(wL/2)}{3EI}\left(\frac{L}{2}\right)^3 = \frac{wL^4}{48EI}$

Case 3 of App. D $(y_B)_M = -\frac{(wL^2/8)}{2EI}\left(\frac{L}{2}\right)^2 = -\frac{wL^4}{64EI}$

$$y_B = -\frac{wL^4}{384EI} \qquad y_B = \frac{wL^4}{384EI} \downarrow$$

Problem 9.68

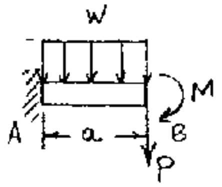
9.67 and 9.68 For the cantilever beam and loading shown, determine the slope and deflection at point B.



Consider portion AB as a cantilever beam subjected to the three loadings shown.

By statics: $\quad P = wa$

$$M = (wa)(a + \tfrac{a}{2}) = \tfrac{3}{2} wa^2$$



Slope at B. $\quad \theta_B = (\theta_B)_w + (\theta_B)_P + (\theta_B)_M$

Case 2 of App. D. $\quad (\theta_B)_w = -\dfrac{wa^3}{6EI}$

Case 1 of App. D. $\quad (\theta_B)_P = -\dfrac{(wa)a^2}{2EI} = -\dfrac{wa^3}{2EI}$

Case 3 of App. D $\quad (\theta_B)_M = -\dfrac{(\frac{3}{2}wa^2)a}{EI} = -\dfrac{3wa^3}{2EI}$

$$\theta_B = -\frac{13wa^3}{6EI} \qquad\qquad \theta_B = \frac{13wa^3}{6EI}$$

Deflection at B. $\quad y_B = (y_B)_w + (y_B)_P + (y_B)_M$

Case 2 of App. D. $\quad (y_B)_w = -\dfrac{wa^4}{8EI}$

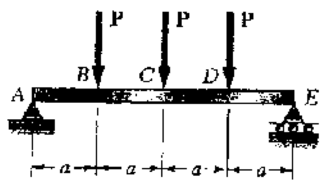
Case 1 of App. D. $\quad (y_B)_P = -\dfrac{(wa)a^3}{3EI} = -\dfrac{wa^4}{3EI}$

Case 3 of App. D. $\quad (y_B)_M = -\dfrac{(\frac{3}{2}wa^2)a}{2EI} = -\dfrac{3wa^3}{4EI}$

$$y_B = -\frac{29wa^4}{24EI} \qquad\qquad y_B = \frac{29wa^4}{24EI}\downarrow$$

Problem 9.69

***9.69* through 9.72** For the beam and loading shown, determine (*a*) the deflection at the *C*, (*b*) the slope at end *A*.



Loading I: Load at B

Case 5 in Appendix D.

$L = 4a, \quad a = a, \quad b = 3a, \quad x = 2a$

For $x > a$, replace x by $L-x$ and interchange a and b in expression for elastic curve given.

$$y = \frac{Pa}{6EIL}\left[(L-x)^3 - (L^2-a^2)(L-x)\right]$$

$$y_c = \frac{Pa}{6EI(4a)}\left[(2a)^3 - (16a^2-a^2)(2a)\right] = -\frac{11}{12}\frac{Pa^3}{EI}$$

$$\theta_A = -\frac{Pb(L^2-b^2)}{6EIL} = -\frac{P(3a)(16a^2-9a^2)}{6EI(4a)} = -\frac{7}{8}\frac{Pa^2}{EI}$$

Loading II: Load at C. Case 4 of Appendix D with $L = 4a$.

$$y_c = -\frac{PL^3}{48EI} = -\frac{P(4a)^3}{48EI} = -\frac{4}{3}\frac{Pa^3}{EI}$$

$$\theta_A = -\frac{PL^2}{16EI} = -\frac{P(4a)^2}{16EI} = -\frac{Pa^2}{EI}$$

Loading III: Load at D. Case 5 of Appendix D.

$L = 4a, \quad a = 3a, \quad b = a, \quad x = 2a$ at point C.

$$y_c = \frac{Pb}{6EIL}\left[x^3 - (L^2-b^2)x\right] = \frac{Pa}{6EI(4a)}\left[(2a)^3 - (16a^2-a^2)(2a)\right]$$

$$= -\frac{11}{12}\frac{Pa^3}{EI}$$

$$\theta_A = -\frac{Pb(L^2-b^2)}{6EIL} = -\frac{Pa(16a^2-a^2)}{6EI(4a)} = -\frac{5}{8}\frac{Pa^3}{EI}$$

(a) Deflection at C. $\quad y_c = -\frac{11}{12}\frac{Pa^3}{EI} - \frac{4}{3}\frac{Pa^3}{EI} - \frac{11}{12}\frac{Pa^3}{EI} = -\frac{19}{6}\frac{Pa^3}{EI}$

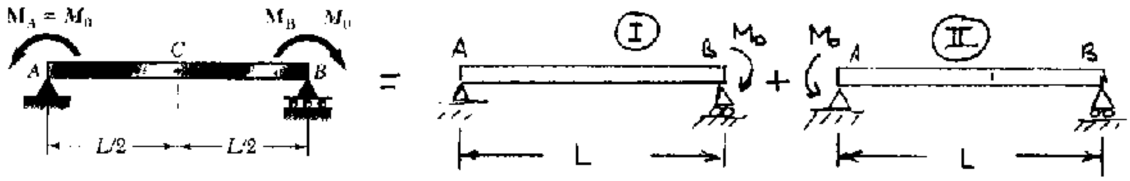
$$y_c = \frac{19}{6}\frac{Pa^3}{EI} \downarrow$$

(b) Slope at A. $\quad \theta_A = -\frac{7}{8}\frac{Pa^2}{EI} - \frac{Pa^2}{EI} - \frac{5}{8}\frac{Pa^2}{EI} = -\frac{5}{2}\frac{Pa^2}{EI}$

$$\theta_A = \frac{5}{2}\frac{Pa^2}{EI}$$

Problem 9.70

***9.69* through 9.72** For the beam and loading shown, determine (*a*) the deflection at the *C*, (*b*) the slope at end *A*.



Loading I: Case 7 of Appendix D. $\quad y = -\frac{M_0}{6EIL}(x^3 - L^2 x)$

At C, $x = \frac{L}{2}$ $\quad (y_C)_1 = -\frac{M_0}{6EIL}\left[\left(\frac{L}{2}\right)^3 - L^2\left(\frac{L}{2}\right)\right] = \frac{M_0 L^2}{16EI}$

$(\theta_A)_1 = \frac{M_0 L}{6EI} \qquad (\theta_B)_1 = -\frac{M_0 L}{3EI}$

Loading II: Mirror image of Loading I.

$(y_C)_2 = (y_C)_1 \ ; \quad (\theta_A)_2 = -(\theta_B)_1$

(a) Deflection at C.

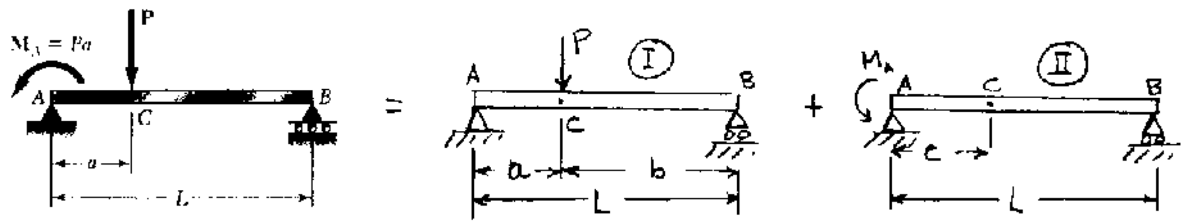
By superposition, $y_C = (y_C)_1 + (y_C)_2 = \frac{M_0 L^2}{8EI}$ $\qquad y_C = \frac{M_0 L^2}{8EI} \uparrow$ ◄

(b) Slope at A.

By superposition, $\theta_A = (\theta_A)_1 + (\theta_A)_2 = -\frac{M_0 L}{2EI}$ $\qquad \theta_A = \frac{M_0 L}{2EI} \measuredangle$ ◄

Problem 9.71

9.69 through 9.72 For the beam and loading shown, determine (*a*) the deflection at the C, (*b*) the slope at end A.



Loading I: Case 5 of Appendix D. $\quad b = L - a$

For $x = a$: $\quad (y_C)_1 = -\dfrac{Pa^2b^2}{3EIL} = -\dfrac{Pa^2(L-a)^2}{3EIL}$

For end A: $\quad (\theta_A)_1 = -\dfrac{Pb(L^2-b^2)}{6EIL} = -\dfrac{P(L-a)(2La-a^2)}{6EIL}$

$$= -\frac{Pa(2L^2 - 3La + a^2)}{6EIL}$$

Loading II: Mirror image of Case 5 of Appendix D.

Replace x by $(L-x)$ in expression for y.

$$y = -\frac{M_A}{6EIL}\left[(L-x)^3 - L^2(L-x)\right] = -\frac{M_A x}{6EIL}(3Lx - 2L^2 - x^2)$$

With $x = a$, and $M_A = Pa$, $\quad (y_C)_2 = -\dfrac{Pa^2}{6EIL}(3La - 2L^2 - a^2)$

$$(\theta_A)_2 = \frac{M_A L}{3EI} = \frac{PaL}{3EI}$$

(a) Deflection at C. $\quad y_C = (y_C)_1 + (y_C)_2$

$$y_C = -\frac{Pa^2}{6EIL}\left[2(L-a)^2 + 3La - 2L^2 - a^2\right] = \frac{Pa^3(L-a)}{6EIL} \uparrow$$

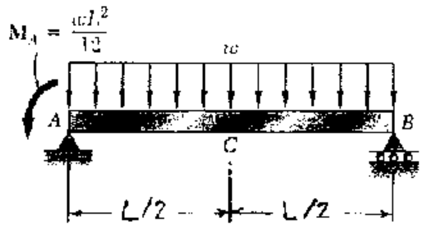
(b) Slope at end A. $\quad \theta_A = (\theta_A)_1 + (\theta_A)_2$

$$\theta_A = -\frac{Pa}{6EIL}\left[2L^2 - 3La + a^2 - 2L^2\right] = +\frac{Pa^2(3L-a)}{6EIL}$$

$$\theta_A = \frac{Pa^2(3L-a)}{6EIL}$$

Problem 9.72

9.69 through 9.72 For the beam and loading shown, determine (*a*) the deflection at the C, (*b*) the slope at end A.



Loading I: Case 6 in Appendix D.

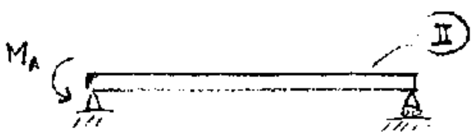
$$y_C = -\frac{5}{384}\frac{wL^4}{EI} \ ; \quad \theta_A = -\frac{1}{24}\frac{wL^3}{EI}$$

Loading II: Case 7 of Appendix D.

Note that center deflection is

$$y_C = -\frac{M_A}{6EIL}\left[\left(\frac{L}{2}\right)^3 - L^2\left(\frac{L}{2}\right)\right]$$

$$= \frac{1}{16}\frac{M_A L}{EI}$$



$$\theta_A = \frac{M_A L}{3EI}$$

with $M_A = \frac{wL^2}{12}$, $\qquad y_C = \frac{1}{192}\frac{wL^4}{EI}$, $\quad \theta_A = \frac{1}{36}\frac{wL^3}{EI}$

(a) Deflection at C. $\quad y_C = -\frac{5}{384}\frac{wL^4}{EI} + \frac{1}{192}\frac{wL^4}{EI} = -\frac{1}{128}\frac{wL^4}{EI}$

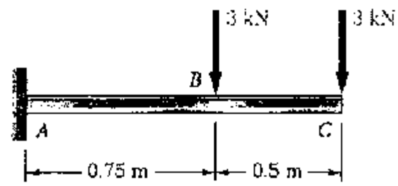
$$y_C = \frac{1}{128}\frac{wL^4}{EI} \downarrow$$

(b) Slope at A. $\quad \theta_A = -\frac{1}{24}\frac{wL^3}{EI} + \frac{1}{36}\frac{wL^3}{EI} = -\frac{1}{72}\frac{wL^3}{EI}$

$$\theta_A = \frac{1}{72}\frac{wL^3}{EI}$$

Problem 9.73

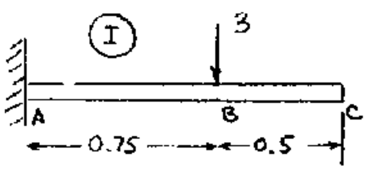
9.73 For the cantilever beam and loading shown, determine the slope and deflection at end C. Use $E = 200$ GPa.



S100 × 11.5

Units: Forces in kN, lengths in m.

Loading I: Concentrated load at B



Case 1 of Appendix D applied to portion AB.

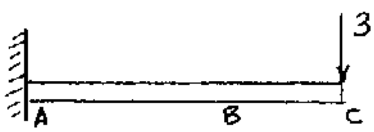
$$\theta_B' = -\frac{PL^2}{2EI} = -\frac{(3)(0.75)^2}{2EI} = -\frac{0.84375}{EI}$$

$$y_B' = -\frac{PL^3}{3EI} = -\frac{(3)(0.75)^3}{3EI} = -\frac{0.421875}{EI}$$

Portion BC remains straight

$$\theta_C' = \theta_B' = -\frac{0.84375}{EI}$$

$$y_C' = y_B' - (0.5)\theta_B' = -\frac{0.84375}{EI}$$



Loading II: Concentrated load at C. Case 1 of Appendix D.

$$\theta_A'' = -\frac{PL^2}{2EI} = -\frac{(3)(1.25)^2}{2EI} = -\frac{2.34375}{EI}$$

$$y_A'' = -\frac{PL^3}{3EI} = -\frac{(3)(1.25)^3}{3EI} = -\frac{1.953125}{EI}$$

By superposition,

$$\theta_A = \theta_A' + \theta_A'' = -\frac{3.1875}{EI}$$

$$y_A = y_A' + y_A'' = -\frac{2.796875}{EI}$$

Data: $E = 200 \times 10^9$ Pa, $I = 2.53 \times 10^6\ \text{mm}^4 = 2.53 \times 10^{-6}\ \text{m}^4$

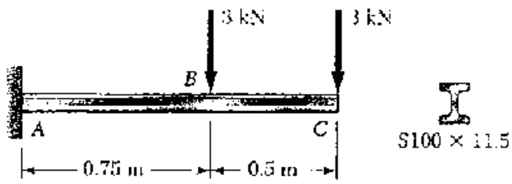
$$EI = (200 \times 10^9)(2.53 \times 10^{-6}) = 506 \times 10^3\ \text{N}\cdot\text{m}^2 = 506\ \text{kN}\cdot\text{m}^2$$

Slope at C. $\theta_C = -\dfrac{3.1875}{506} = -6.30 \times 10^{-3}$ rad $= 6.30 \times 10^{-3}$ rad ◄

Deflection at C. $y_C = -\dfrac{2.796875}{506} = -5.53 \times 10^{-3}$ m $= 5.53$ mm ↓ ◄

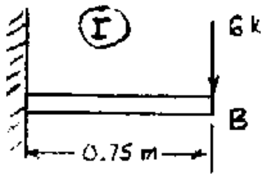
Problem 9.74

9.74 For the cantilever beam and loading shown, determine the slope and deflection at point B. Use $E = 200$ GPa.

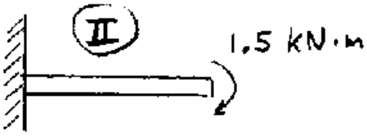


Units: Forces in kN, lengths in m.

The slope and deflection at B depend only on the deformation of portion AB.



Reducing the force at C to an equivalent force-couple system at B and adding the force already at B gives the loadings I and II shown.



Loading I: Case 1 of Appendix D

$$\theta_B' = -\frac{PL^2}{2EI} = -\frac{(6)(0.75)^2}{2EI} = -\frac{1.6875}{EI}$$

$$y_B' = -\frac{PL^3}{3EI} = -\frac{(6)(0.75)^3}{3EI} = -\frac{0.84375}{EI}$$

Loading II: Case 3 of Appendix D

$$\theta_B'' = -\frac{ML}{EI} = -\frac{(1.5)(0.75)}{EI} = -\frac{1.125}{EI}$$

$$y_B'' = -\frac{ML^2}{2EI} = -\frac{(1.5)(0.75)^2}{EI} = -\frac{0.421875}{EI}$$

By superposition,

$$\theta_B = \theta_B' + \theta_B'' = -\frac{2.8125}{EI}$$

$$y_B = y_B' + y_B'' = -\frac{1.265625}{EI}$$

Data: $E = 200 \times 10^9$ Pa, $\quad I = 2.53 \times 10^6 \text{ mm}^4 = 2.53 \times 10^{-6} \text{ m}^4$

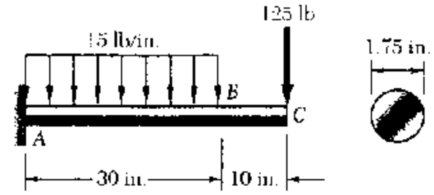
$$EI = (200 \times 10^9)(2.53 \times 10^{-6}) = 506 \times 10^3 \text{ N}\cdot\text{m}^2 = 506 \text{ kN}\cdot\text{m}^2$$

Slope at B. $\quad \theta_B = -\frac{2.8125}{506} = -5.56 \times 10^{-3} \text{ rad} = 5.56 \times 10^{-3} \text{ rad}$ ◄

Deflection at B. $\quad y_B = -\frac{1.265625}{506} = -2.50 \times 10^{-3} \text{ m} = 2.50 \text{ mm} \downarrow$ ◄

Problem 9.75

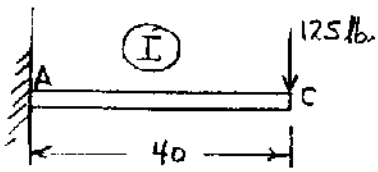
9.75 For the cantilever beam and loading shown, determine the slope and deflection at end C. Use $E = 29 \times 10^6$ psi.



$c = \frac{1}{2}d = \frac{1}{2}(1.75) = 0.875$ in.

$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(0.875)^4 = 0.460386$ in^4

$EI = (29\times10^6)(0.460386) = 13.351\times10^6$ lb$\cdot$in^2

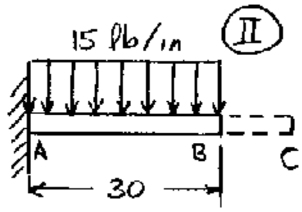


Loading I. Case 1 of Appendix D.

$P = 125$ lb, $L = 40$ in.

$$(y_C)_1 = -\frac{PL^3}{3EI} = -\frac{(125)(40)^3}{(3)(13.351\times10^6)} = -0.199735 \text{ in.}$$

$$(\theta_C)_1 = -\frac{PL^2}{2EI} = -\frac{(125)(40)^2}{(2)(13.351\times10^6)} = -7.4901\times10^{-3}$$



Loading II. Treat portion AB as a cantilever beam. (Case 2)

$w = 15$ lb/in, $L = 30$ in.

$$(y_B)_2 = -\frac{wL^4}{8EI} = -\frac{(15)(30)^4}{(8)(13.351\times10^6)} = -0.113755 \text{ in.}$$

$$(\theta_B)_2 = -\frac{wL^3}{6EI} = -\frac{(15)(30)^3}{(6)(13.351\times10^6)} = -5.0558\times10^{-3}$$

Portion BC remains straight for loading II.

$L_{BC} = 10$ in.

$(y_C)_2 = (y_B)_2 + L_{BC}(\theta_B)_2 = -0.164313$ in.

$(\theta_C)_2 = (\theta_B)_2 = -5.0558\times10^{-3}$

Slope at end C. By superposition, $\theta_C = (\theta_C)_1 + (\theta_C)_2$

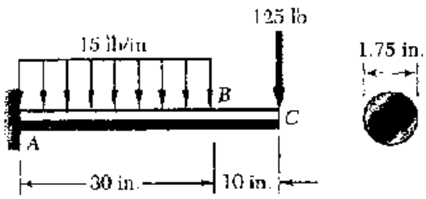
$\theta_C = -12.5459\times10^{-3}$ $\qquad$ $\theta_C = 12.55\times10^{-3}$ rad ◄

Deflection at end C. By superposition, $y_C = (y_C)_1 + (y_C)_2$

$y_C = -0.364048$ in. $\qquad$ $y_C = 0.364$ in. $\downarrow$ ◄

Problem 9.76

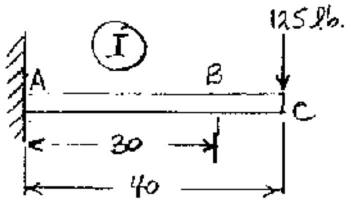
9.76 For the cantilever beam and loading shown, determine the slope and deflection at point B. Use $E = 29 \times 10^6$ psi.



$$c = \tfrac{1}{2}d = \tfrac{1}{2}(1.75) = 0.875 \text{ in.}$$

$$I = \tfrac{\pi}{4}c^4 = \tfrac{\pi}{4}(0.875)^4 = 0.460386 \text{ in}^4$$

$$EI = (29\times10^6)(0.460386) = 13.351\times10^6 \text{ lb}\cdot\text{in}^2$$



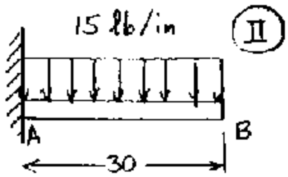
Loading I: Case 1 of Appendix D.

$$P = 125 \text{ lb.} \quad L = 40 \text{ in.} \quad x = 30 \text{ in.}$$

$$y_1 = \frac{P}{6EI}(x^3 - 3Lx^2)$$

$$\theta_1 = \frac{dy}{dx} = \frac{P}{2EI}(x^2 - 2Lx)$$

$$(y_B)_1 = \frac{125}{(6)(13.351\times10^6)}\left[(30)^3 - (3)(40)(30)^2\right] = -0.126395 \text{ in.}$$



$$(\theta_B)_1 = \frac{125}{(2)(13.351\times10^6)}\left[(30)^2 - (2)(40)(30)\right] = -7.0219\times10^{-3}$$

Loading II. Case 2 of Appendix D.

$$w = 15 \text{ lb/in}, \quad L = 30 \text{ in}$$

$$(y_B)_2 = -\frac{wL^4}{8EI} = -\frac{(15)(30)^4}{(8)(13.351\times10^6)} = -0.113755 \text{ in.}$$

$$(\theta_B)_2 = -\frac{wL^3}{6EI} = -\frac{(15)(30)^3}{(6)(13.351\times10^6)} = -5.0558\times10^{-3}$$

Slope at point B. $\quad \theta_B = (\theta_B)_1 + (\theta_B)_2$

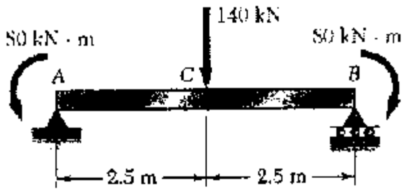
$$\theta_B = -12.0777\times10^{-3} \qquad \theta_B = 12.08\times10^{-3} \text{ rad} \ \blacktriangleleft$$

Deflection at point B. $\quad y_B = (y_B)_1 + (y_B)_2$

$$y_B = -0.24015 \text{ in.} \qquad y_B = 0.240 \text{ in.} \downarrow \ \blacktriangleleft$$

Problem 9.77

9.77 and 9.78 For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at point *C*. Use $E = 200$ GPa.



Units: Forces in kN, lengths in m.

Loading I: Moment at B.

Case 7 of Appendix D. $M = 80$ kN·m, $L = 5.0$ m, $x = 2.5$ m

$$\theta_A = \frac{ML}{6EI} = \frac{(80)(5.0)}{6EI} = \frac{66.667}{EI}$$

$$y_C = -\frac{M}{6EIL}(x^3 - L^2 x) = -\frac{80}{6EI(5.0)}\left[2.5^3 - (5.0)^2(2.5)\right] = \frac{125}{EI}$$

Loading II: Moment at A. Case 7 of Appendix D.

$M = 80$ kN·m, $L = 5.0$ m, $x = 2.5$ m

$$\theta_A = \frac{ML}{3EI} = \frac{(80)(5.0)}{3EI} = \frac{133.333}{EI}$$

$$y_C = \frac{125}{EI} \quad \text{(Same as loading I)}$$

Loading III: 140 kN concentrated load at C. $P = 140$ kN

$$\theta_A = -\frac{PL^2}{16EI} = -\frac{(140)(5.0)^2}{16EI} = -\frac{218.75}{EI}$$

$$y_C = -\frac{PL^3}{48EI} = -\frac{(140)(5.0)^3}{48EI} = -\frac{364.583}{EI}$$

Data: $E = 200 \times 10^9$ Pa, $I = 156 \times 10^6$ mm^4 $= 156 \times 10^{-6}$ m^4

$$EI = (200 \times 10^9)(156 \times 10^{-6}) = 31.2 \times 10^6 \text{ N·m}^2 = 31200 \text{ kN·m}^2$$

(a) Slope at A. $\theta_A = \dfrac{67.667 + 133.333 - 218.75}{31200} = -0.601 \times 10^{-3}$ rad

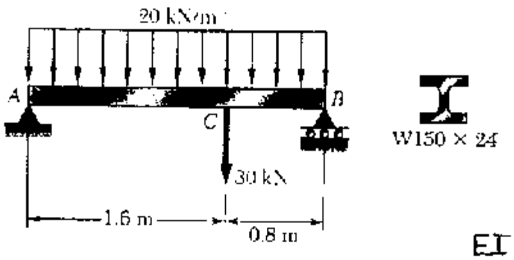
$$\theta_A = 0.601 \times 10^{-3} \text{ rad} \;\curvearrowright \quad ◄$$

(b) Deflection at C. $y_C = \dfrac{125 + 125 - 364.583}{31200} = -3.67 \times 10^{-3}$ m

$$y_C = 3.67 \text{ mm} \downarrow \quad ◄$$

Problem 9.78

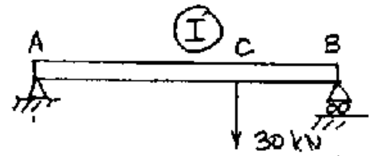
9.77* and *9.78 For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at point *C*. Use $E = 200$ GPa.

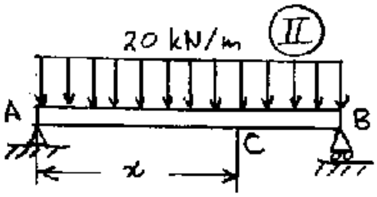


Units: Forces in kN. Lengths in meters.

For W150×24 $\quad I = 13.4 \times 10^6 \text{ mm}^4$
$= 13.4 \times 10^{-6} \text{ m}^4$

$EI = (200 \times 10^9)(13.4 \times 10^{-6}) = 2.68 \times 10^6 \text{ N}\cdot\text{m}$
$= 2680 \text{ kN}\cdot\text{m}^2$





Loading I: Case 5 of Appendix D.

$P = 30 \text{ kN}, \; L = 2.4 \text{ m}, \; a = 1.6 \text{ m}, \; b = 0.8 \text{ m}$

$$(\theta_A)_1 = -\frac{Pb(L^2 - b^2)}{6EIL} = -\frac{(30)(0.8)(2.4^2 - 0.8^2)}{(6)(2680)(2.4)}$$
$$= -3.1841 \times 10^{-3}$$

$$(y_C)_1 = -\frac{Pa^2b^2}{3EIL} = -\frac{(30)(1.6)^2(0.8)^2}{(3)(2680)(2.4)}$$
$$= -2.54726 \times 10^{-3} \text{ m}$$

Loading II: Case 6 of Appendix D.

$w = 20 \text{ kN/m}, \quad L = 2.4 \text{ m}, \quad x = 1.6 \text{ m at point C.}$

$$(\theta_A)_2 = -\frac{wL^3}{24EI} = -\frac{(20)(2.4)^3}{(24)(2680)} = -4.2985 \times 10^{-3}$$

$$y_2 = -\frac{w}{24EI}(x^4 - 2Lx^3 + L^3x)$$

$$(y_C)_2 = -\frac{20}{(24)(2680)}\left[(1.6)^4 - (2)(2.4)(1.6)^3 + (2.4)^3(1.6)\right] = -2.80199 \times 10^{-3} \text{ m}$$

(a) Slope at end A. $\quad \theta_A = (\theta_A)_1 + (\theta_A)_2$

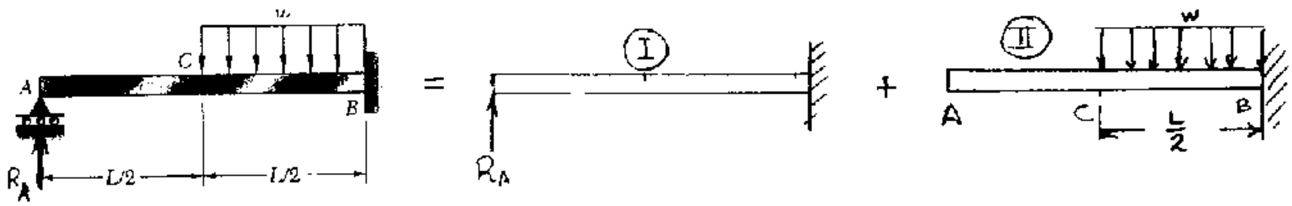
$\theta_A = -7.4826 \times 10^{-3}$ $\qquad\qquad \theta_A = 7.48 \times 10^{-3} \text{ rad}$ ◀

(b) Deflection at point C. $\quad y_C = (y_C)_1 + (y_C)_2$

$y_C = -5.34925 \times 10^{-3} \text{ m}$ $\qquad\qquad y_C = 5.35 \text{ mm} \downarrow$ ◀

Problem 9.79

9.79 and 9.80 For the uniform beam shown, determine (*a*) the reaction at *A*, (*b*) the reaction at *B*.



Consider R_A as redundant and replace loading system by I and II.

Loading I. (Case 1 of Appendix D) $\quad (y_A)_1 = \dfrac{R_A L^3}{3EI}$

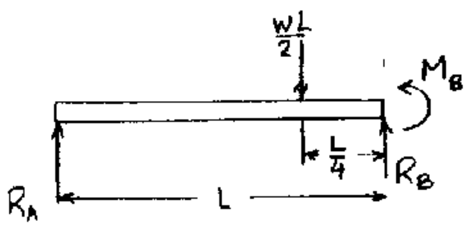
Loading II. Portion CB. (Case 2 of Appendix D)

$$(y_B)_2 = -\frac{w(L/2)^4}{8EI} = -\frac{wL^4}{128\,EI}, \qquad (\theta_B)_2 = \frac{w(L/2)^3}{6EI} = \frac{wL^3}{48\,EI}$$

Portion AC is straight. $\quad (y_A)_2 = (y_B)_2 - \frac{L}{2}(\theta_B)_2 = -\dfrac{7wL^4}{384\,EI}$

Superposition and constraint. $\quad y_A = (y_A)_1 + (y_A)_2 = 0$

(a) $$\frac{R_A L^3}{3EI} - \frac{7wL^4}{384\,EI} = 0 \qquad R_A = \frac{7wL}{128}\uparrow$$



Using entire beam as a free body

$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - \frac{wL}{2} = 0$

$$R_B = \frac{wL}{2} - \frac{7wL}{128} \qquad R_B = \frac{57wL}{128}\uparrow$$

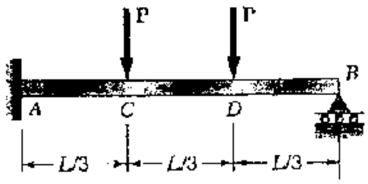
$+\circlearrowleft \Sigma M_B = 0: \quad M_B - R_A L - \frac{wL}{2}\cdot\frac{L}{4} = 0$

$$M_B = R_A L - \frac{wL^2}{8} = -\frac{9wL^2}{128}$$

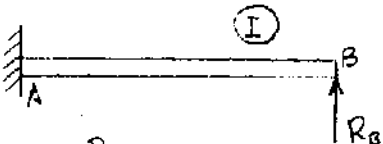
$$M_B = \frac{9wL^2}{128} \circlearrowright$$

Problem 9.80

9.79 and 9.80 For the uniform beam shown, determine (*a*) the reaction at *A*, (*b*) the reaction at *B*.

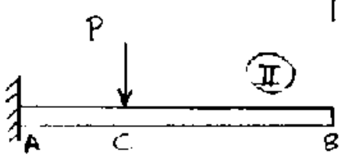


Consider R_B as redundant and replace loading system by I, II, and III.



Loading I. Case 1 of Appendix D applied to AB.

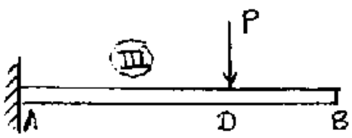
$$(y_B)_I = \frac{R_B L^3}{3EI}$$



Loading II. Case 1 applied to portion AC.

$$(\theta_C)_{II} = -\frac{P(L/3)^2}{2EI} = -\frac{1}{18}\frac{PL^2}{EI}$$

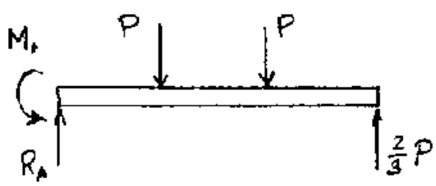
$$(y_C)_{II} = -\frac{P(L/3)^3}{3EI} = -\frac{1}{81}\frac{PL^3}{EI}$$



Portion CB remains straight.

$$(y_B)_{II} = (y_C)_{II} + \frac{2L}{3}(\theta_C)_{II} = -\frac{4}{81}\frac{PL^3}{EI}$$

Loading III. Case 1 applied to portion AD.



$$(\theta_D)_{III} = \frac{P(2L/3)^2}{2EI} = -\frac{2}{9}\frac{PL^2}{EI}$$

$$(y_D)_{III} = \frac{P(2L/3)^3}{3EI} = -\frac{8}{81}\frac{PL^3}{EI}$$

Portion DB remains straight.

$$(y_C)_{III} = (y_D)_{III} + \frac{L}{3}(\theta_D)_{III} = -\frac{14}{81}\frac{PL^3}{EI}$$

Superposition and constraint: $\quad y_B = (y_B)_I + (y_B)_{II} + (y_B)_{III} = 0$

$$\frac{1}{3}R_B L^3 - \frac{4}{81}\frac{PL^3}{EI} - \frac{14}{81}\frac{PL^3}{EI} = \frac{1}{3}\frac{R_B L^3}{EI} - \frac{2}{9}\frac{PL^3}{EI} = 0 \qquad \text{(b)}\ R_B = \frac{2}{3}P\uparrow$$

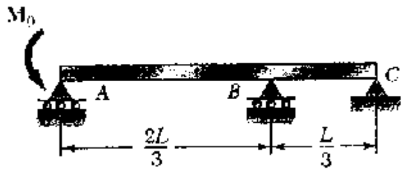
Statics:

$+\uparrow \Sigma F_y = 0: \quad R_A - P - P + \frac{2}{3}P = 0 \qquad$ (a) $R_A = \frac{4}{3}P\uparrow$

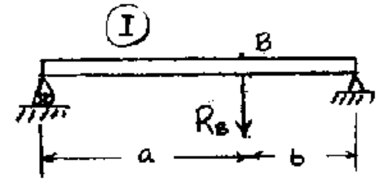
$\circlearrowleft \Sigma M_A = 0: \quad M_A - P\left(\frac{L}{3}\right) - P\left(\frac{2L}{3}\right) + \left(\frac{2}{3}P\right)(L) = 0 \qquad M_A = \frac{1}{3}PL \circlearrowleft$

Problem 9.81

9.81 and 9.82 For the uniform beam shown, determine the reaction at each of the three supports.

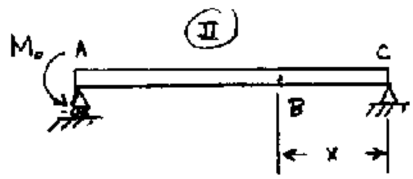


Beam is statically indeterminate to first degree. Consider R_B to be the redundant reaction, and replace the loading by loadings I and II.



Loading I: Case 5 of Appendix D.

$$(y_B)_I = -\frac{R_B a^2 b^2}{3EIL} = -\frac{R_B(2L/3)^2(L/3)^2}{3EIL} = -\frac{4}{243}\frac{R_B L^3}{EI}$$



Loading II: Case 7 of Appendix D.

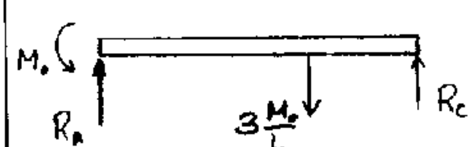
$$(y_B)_{II} = -\frac{M_o}{6EIL}(x^3 - L^2x) = -\frac{M_o}{6EIL}\left[\left(\frac{L}{3}\right)^3 - L^2\left(\frac{L}{3}\right)\right]$$

$$= \frac{4}{81}\frac{M_o L^2}{EI}$$

Superposition and constraint.

$$y_B = (y_B)_I + (y_B)_{II} = 0$$

$$-\frac{4}{243}\frac{R_B L^3}{EI} + \frac{4}{81}\frac{M_o L^2}{EI} = 0 \qquad R_B = 3\frac{M_o}{L}\downarrow$$



Statics:

$\circlearrowleft \Sigma M_c = 0:$

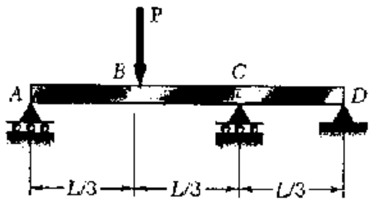
$$-R_A L + M_o + 3\frac{M_o}{L}\cdot\frac{L}{3} = 0 \qquad R_A = 2\frac{M_o}{L}\uparrow$$

$+\uparrow \Sigma F_y = 0:$

$$2\frac{M_o}{L} - 3\frac{M_o}{L} + R_c = 0 \qquad R_c = \frac{M_o}{L}\uparrow$$

Problem 9.82

9.81 and 9.82 For the uniform beam shown, determine the reaction at each of the three supports.



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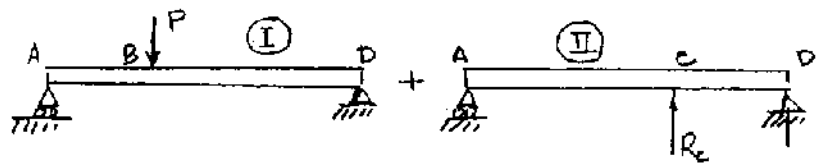


Consider R_C as redundant and replace loading system by I and II.

Loading I. (Case 5 of Appendix D) $a = \frac{2L}{3}$, $b = \frac{L}{3}$, $x = \frac{L}{3}$ at C.

$$(y_C)_1 = \frac{Pb}{6EIL}\left[x^3 - (L^2 - b^2)x\right] = \frac{P(L/3)}{6EIL}\left[\left(\frac{L}{3}\right)^3 - \left\{L^2 - \left(\frac{L}{3}\right)^2\right\}\frac{L}{3}\right]$$

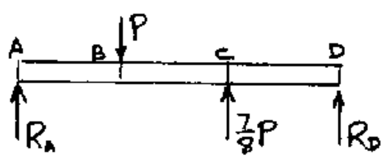
$$= -\frac{7PL^3}{486\,EI}$$

Loading II. (Case 5 of Appendix D) $a = \frac{2L}{3}$, $b = \frac{L}{3}$

$$(y_C)_2 = \frac{R_C a^2 b^2}{3EIL} = \frac{R_C (L/3)^2 (2L/3)^2}{3EIL} = \frac{4R_C L^3}{243\,EI}$$

Superposition and constraint. $y_C = (y_C)_1 + (y_C)_2 = 0$

$$-\frac{7PL^3}{486EI} + \frac{4R_C}{243EI} = 0 \qquad R_C = \frac{7}{8}P \uparrow$$



$+\circlearrowleft \Sigma M_D = 0:$

$$-R_A L + P\left(\frac{2L}{3}\right) - \left(\frac{7}{8}P\right)\left(\frac{L}{3}\right) = 0$$

$$R_A = \frac{3}{8}P \uparrow$$

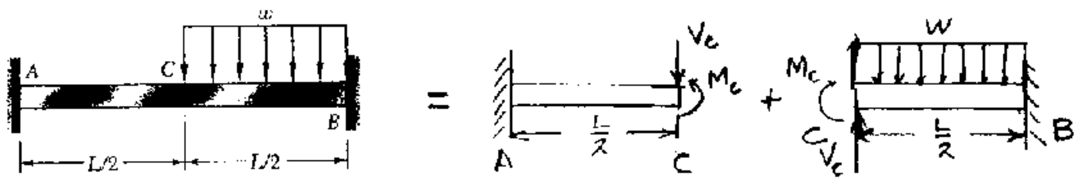
$+\uparrow \Sigma F_y = 0: \quad R_A + R_D - P + \frac{7}{8}P = 0$

$$R_D = P - \frac{7}{8}P - \frac{3}{8}P = -\frac{1}{4}P$$

$$R_D = \frac{1}{4}P \downarrow$$

Problem 9.83

9.83* and *9.84 For the beam shown, determine the reaction at *B*.



Portion AC: Superposition of Cases 3 and 1 of Appendix D.

$$y_C = \frac{M_C(L/2)^2}{2EI} - \frac{V_C(L/2)^3}{3EI} = \frac{M_C L^2}{8EI} - \frac{V_C L^3}{24EI}$$

$$\theta_C = \frac{M_C(L/2)}{EI} - \frac{V_C(L/2)^2}{2EI} = \frac{M_C L}{2EI} - \frac{V_C L^3}{8EI}$$

Portion CB: Superposition of Cases 3, 1, and 2 of Appendix D.

$$y_C = \frac{M_C(L/2)^2}{2EI} + \frac{V_C(L/2)^3}{3EI} - \frac{w(L/2)^4}{8EI}$$

$$= \frac{M_C L^2}{8EI} + \frac{V_C L^3}{24EI} - \frac{wL^4}{128EI}$$

$$\theta_C = -\frac{M_C(L/2)}{EI} - \frac{V_C(L/2)^3}{2EI} + \frac{w(L/2)^3}{6EI}$$

$$= -\frac{M_C L}{2EI} - \frac{V_C L^3}{8EI} + \frac{wL^3}{48EI}$$

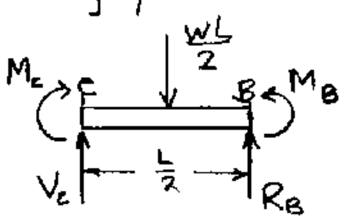
Matching expressions for y_C,

$$\cancel{\frac{M_C L^2}{8EI}} - \frac{V_C L^3}{24EI} = \cancel{\frac{M_C L^2}{8EI}} + \frac{V_C L^3}{24EI} - \frac{wL^4}{128EI} \qquad V_C = \frac{3}{32}wL$$

Matching expressions for θ_C,

$$\frac{M_C L}{2EI} - \cancel{\frac{V_C L^3}{8EI}} = -\frac{M_C L}{2EI} - \cancel{\frac{V_C L^3}{8EI}} + \frac{wL^3}{48EI} \qquad M_C = \frac{1}{48}wL^2$$

Using portion CB as a free body,



$$+\uparrow \Sigma F_y = 0: \quad R_B + V_C - \frac{wL}{2} = 0$$

$$R_B = \frac{wL}{2} - \frac{3}{32}wL = \frac{13}{32}wL \uparrow \quad ◄$$

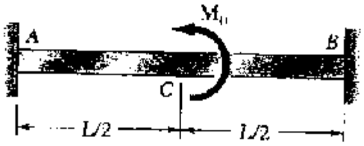
$$+\Sigma M_B = 0: \quad M_B - M_C - V_C\frac{L}{2} + \frac{wL}{2}\cdot\frac{L}{4} = 0$$

$$M_B = \frac{1}{48}wL^2 + \left(\frac{3}{32}wL\right)\left(\frac{L}{2}\right) - \frac{wL^2}{8}$$

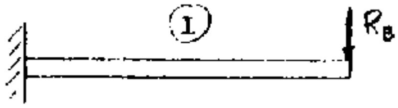
$$M_B = -\frac{11}{192}wL^2 \qquad M_B = \frac{11}{192}wL^2 \circlearrowright \quad ◄$$

Problem 9.84

9.83* and *9.84 For the beam shown, determine the reaction at B.

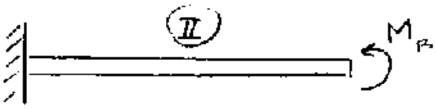


Beam is second degree indeterminate. Choose R_B and M_B as redundant reactions.



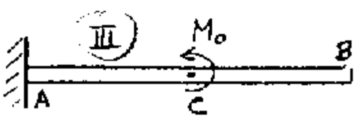
Loading I. Case 1 of Appendix D.

$$(y_B)_I = -\frac{R_B L^3}{3EI}, \quad (\theta_B)_I = -\frac{R_B L^2}{2EI}$$



Loading II. Case 3 of Appendix D.

$$(y_B)_{II} = \frac{M_B L^2}{2EI}, \quad (\theta_B)_{II} = \frac{M_B L}{EI}$$



Loading III. Case 3 applied to portion AC.

$$(y_C)_{III} = \frac{M_0 (L/2)^2}{2EI} = \frac{M_0 L^2}{8EI}$$

$$(\theta_C)_{III} = \frac{M_0 (L/2)}{EI} = \frac{M_0 L}{2EI}$$

Portion CB remains straight.

$$(y_B)_{III} = (y_C)_{III} + \frac{L}{2}(\theta_C)_{III} = \frac{3}{8}\frac{M_0 L^2}{EI}$$

$$(\theta_B)_{III} = (\theta_C)_{III} = \frac{1}{2}\frac{M_0 L^2}{EI}$$

Superposition and constraint:

$$y_B = (y_B)_I + (y_B)_{II} + (y_B)_{III} = 0$$

$$-\frac{L^3}{3EI}R_B + \frac{L^2}{2EI}M_B + \frac{3}{8}\frac{M_0 L^2}{EI} = 0 \qquad (1)$$

$$\theta_B = (\theta_B)_I + (\theta_B)_{II} + (\theta_B)_{III} = 0$$

$$-\frac{L^2}{2EI}R_B + \frac{L}{EI}M_B + \frac{1}{2}\frac{M_0 L}{EI} = 0 \qquad (2)$$

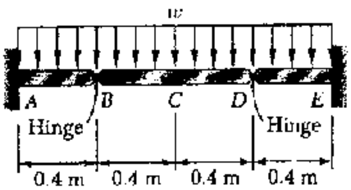
Solving (1) and (2) simultaneously,

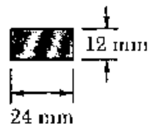
$$R_B = \frac{3}{2}\frac{M_0}{L} \downarrow$$ ◄

$$M_B = \frac{1}{4}M_0 \circlearrowright$$ ◄

Problem 9.85

9.85 A central beam *BD* is joined at hinges to two cantilever beams *AB* and *DE*. All beams have the cross section shown. For the loading shown, determine the largest w so that the deflection at C does not exceed 3 mm. Use $E = 200$ GPa.





Let $a = 0.4$ m

Cantilever beams AB and CD.

Cases 1 and 2 of Appendix D. $\quad y_C = -\frac{(wa)a^3}{3EI} - \frac{wa^4}{8EI} = \frac{11}{24}\frac{wa^4}{EI}$

Beam BCD, with $L = 0.8$ m; assuming that points B and D do not move.

Case 6 of Appendix D.

$$y_C' = -\frac{5wL^4}{384EI}$$

Additional deflection due to movement of points B and D.

$$y_C'' = y_B = y_D = -\frac{11}{24}\frac{wa^4}{EI}$$

Total deflection at C. $\quad y_C = y_C' + y_C''$

$$y_C = -\frac{w}{EI}\left\{\frac{5L^4}{384} + \frac{11a^4}{24}\right\}$$

Data: $E = 200\times10^9$ Pa, $\quad I = \frac{1}{12}(24)(12)^3 = 3.456\times10^{-3}\ \text{mm}^4 = 3.456\times10^{-9}\ \text{m}^4$

$EI = (200\times10^9)(3.456\times10^{-9}) = 691.2\ \text{N}\cdot\text{m}^2$

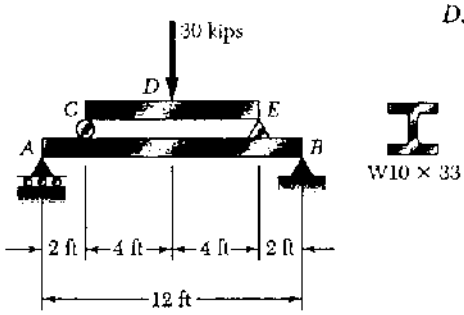
$y_C = -3\times10^{-3}$ m

$$-3\times10^{-3} = -\frac{w}{691.2}\left\{\frac{(5)(0.8)^4}{384} + \frac{(11)(0.4)^4}{24}\right\} = -24.69\times10^{-6}\ w$$

$w = 121.5$ N/m

Problem 9.86

9.86 Beam CE rests on beam AB, as shown. Knowing that a W 10 × 33 rolled-steel shape is used for each beam, determine for the loading shown the deflection at point D. Use $E = 29 \times 10^6$ psi.



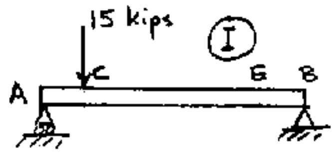
For W10×33, $I = 170 \text{ in}^4$

$EI = (29\times10^6)(170) = 4.93\times10^9 \text{ lb}\cdot\text{in}^2$

$= 34236 \text{ kip}\cdot\text{ft}^2$

Beam AB: 15 kip downward loads at C and E.

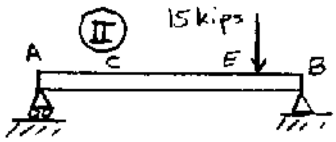
Refer to Case 5 of Appendix D.



Loading I. $(y_C)_1 = -\dfrac{Pa^2b^2}{3EIL}$

with $a = 2$ ft, $b = 10$ ft, $L = 12$ ft

$(y_C)_1 = -\dfrac{(15)(2)^2(10)^2}{(3)(34236)(12)} = -4.8682\times10^{-3}$ ft



Loading II. $(y_C)_2 = \dfrac{Pb[x^3-(L^2-b^2)x]}{6EIL}$

with $b = 2$ ft, $x = 2$ ft, $L = 12$ ft

$(y_C)_2 = \dfrac{(15)(2)[2^3-(12^2-2^2)(2)]}{(6)(34236)(12)}$

$= -3.3104\times10^{-3}$ ft

$y_C = (y_C)_1 + (y_C)_2 = -8.1786\times10^{-3}$ ft

By symmetry $y_E = y_C$

Beam CDE. 30 kip downward load at D.

Refer to Case 4 of Appendix D.

$y_{D/C} = -\dfrac{PL^3}{48EI}$

with $P = 30$ kips and $L = 8$ ft.

$y_{D/C} = -\dfrac{(30)(8)^3}{(48)(34236)} = -9.3469\times10^{-3}$ ft

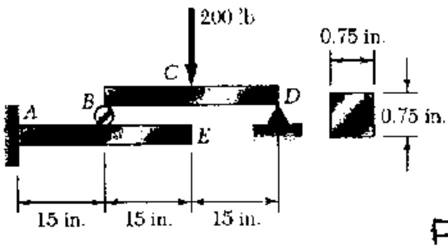
Total deflection at D.

$y_D = y_C + y_{D/C} = 17.5255\times10^{-3}$ ft

$= 0.210$ in. ↓ ◀

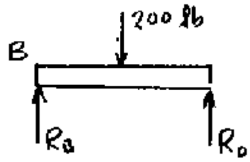
Problem 9.87

9.87 Beam *BD* rests on the cantilever beam *AE*, as shown. Knowing that a square rod of side 0.75 in. is used for each beam, determine for the loading shown (*a*) the deflection at point *C*, (*b*) the deflection at point *E*. Use $E = 29 \times 10^6$ psi.



$$I = \frac{1}{12}(0.75)(0.75)^3 = 26.367 \times 10^{-3}\ \text{in}^4$$

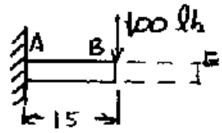
$$EI = (29 \times 10^6)(26.367 \times 10^{-3}) = 764.65 \times 10^3\ \text{lb} \cdot \text{in}^2$$



For free body BCD, $+\circlearrowleft M_D = 0$:

$$-30 R_B + (15)(200) = 0 \qquad R_B = 100\ \text{lb.}$$

Cantilever ABE. Portion AB.



Case 1 of Appendix D.

$$y_B = -\frac{PL^3}{3EI} = -\frac{(100)(15)^3}{(3)(764.65 \times 10^3)} = -0.14713\ \text{in.}$$

$$\theta_B = -\frac{PL^2}{2EI} = -\frac{(100)(15)^2}{(2)(764.65 \times 10^3)} = -14.713 \times 10^{-3}$$

Deflection at point C if point B does not move.

Apply Case 4 of Appendix D to beam BCD.

$$(y_C)_1 = -\frac{PL^3}{48EI} = -\frac{(200)(30)^3}{(48)(764.65 \times 10^3)} = -0.14713\ \text{in}$$

Addition deflection at point C due to movement of point B.

$$(y_C)_2 = \frac{15}{30} y_B = -0.07356\ \text{in.}$$

(a) Total deflection at point C. $\quad y_C = (y_C)_1 + (y_C)_2 = -0.22069$ in.

$y_C = 0.221$ in. ↓ ◄

(b) Deflection at point E. $\quad y_E = y_B + L_{BE}\theta_B$

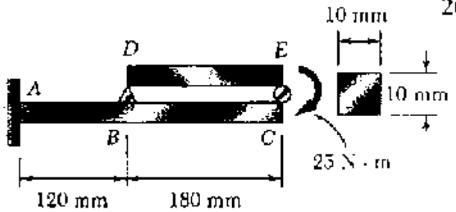
$$= -0.14713 + (15)(-14.713 \times 10^{-3})$$

$$= -0.3678\ \text{in.}$$

$y_E = 0.368$ in. ↓ ◄

Problem 9.88

9.88 Beam DE rests on the cantilever beam AC as shown. Knowing that a square rod of side 10 mm is used for each beam, determine the deflection at end C if the 25-N·m couple is applied (a) to end E of beam DE, (b) to end C of beam AC. Use E = 200 GPa.

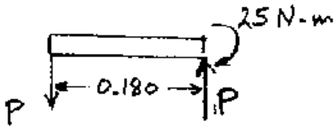


$$E = 200 \times 10^9 \text{ Pa}$$

$$I = \tfrac{1}{12}(10)(10)^3 = 833.33 \text{ mm}^4 = 833.33 \times 10^{-12} \text{ m}^4$$

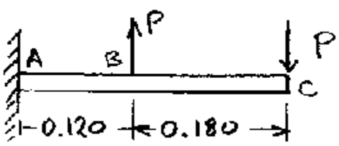
$$EI = 166.667 \text{ N·m}^2$$

(a) Couple applied to beam DE.



Free body DE. $+\circlearrowleft \Sigma M = 0:$

$$0.180\,P - 25 = 0 \qquad P = 138.889 \text{ N}$$



Loads on cantilever beam ABC are P↑ at point B and P↓ at point C as shown.

Due to P↑ at point B.

Using portion AB and applying Case 1 of Appendix D,

$$(y_B)_1 = \frac{PL^3}{3EI} = \frac{(138.889)(0.120)^3}{(3)(166.667)} = 0.480 \times 10^{-3} \text{ m}$$

$$(\theta_B)_1 = \frac{PL^2}{2EI} = \frac{(138.889)(0.120)^2}{(2)(166.667)} = 6.00 \times 10^{-3}$$

$$(y_C)_1 = (y_B)_1 + L_{BC}(\theta_B)_1 = 0.480 \times 10^{-3} + (0.180)(6.00 \times 10^{-3}) = 1.56 \times 10^{-3} \text{ m}$$

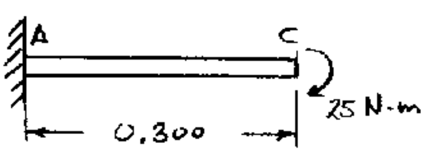
Due to load P↓ at point C. Case 1 of App. D applied to ABC.

$$(y_C)_2 = -\frac{PL^3}{3EI} = -\frac{(138.889)(0.120 + 0.180)^3}{(3)(166.667)} = -7.50 \times 10^{-3} \text{ m}$$

Total deflection at point C. $y_C = (y_C)_1 + (y_C)_2 = -5.94 \times 10^{-3}$ m

$y_C = 5.94$ mm ↓ ◄

(b) Couple applied to beam AC.



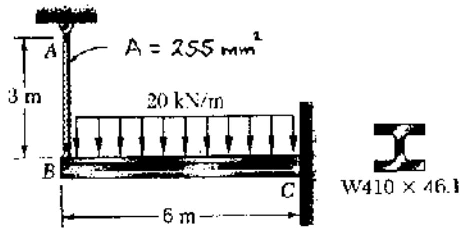
Case 3 of Appendix D.

$$y_C = -\frac{ML^2}{2EI} = -\frac{(25)(0.300)^2}{(2)(166.667)} = -6.75 \times 10^{-3} \text{ m}$$

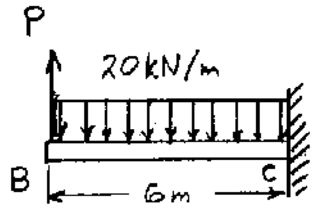
$y_C = 6.75$ mm ↓ ◄

Problem 9.89

9.89 The cantilever beam BC is attached to the steel cable AB as shown. Knowing that the cable is initially taught, determine the tension in the cable caused by the distributed load shown. Use E = 200 GPa.



Let P be the tension developed in member AB and δ_B be the elongation of that member.



$$\delta = \frac{PL}{EA} = \frac{(P)(3)}{(200\times10^9)(254.47\times10^{-6})}$$
$$= 58.946\times10^{-9}\,P$$

Beam BC: $\quad I = 156\times10^6\ \text{mm}^4 = 156\times10^{-6}\ \text{m}^4$

$$EI = (200\times10^9)(156\times10^{-6}) = 31.2\times10^6\ \text{N}\cdot\text{m}^2$$

Loading I. 20 kN/m downward.

Refer to Case 2 of Appendix D.

$$(y_B)_1 = -\frac{wL^4}{8EI} = -\frac{(20\times10^3)(6)^4}{(8)(31.2\times10^6)}$$
$$= -103.846\times10^{-3}\ \text{m}$$

Loading II. Upward force P at point B.

Refer to Case 1 of Appendix D.

$$(y_B)_2 = \frac{PL^3}{3EI} = \frac{P(6)^3}{(3)(31.2\times10^6)} = 2.3077\times10^{-6}\,P$$

By superposition, $\quad y_B = (y_B)_1 + (y_B)_2$

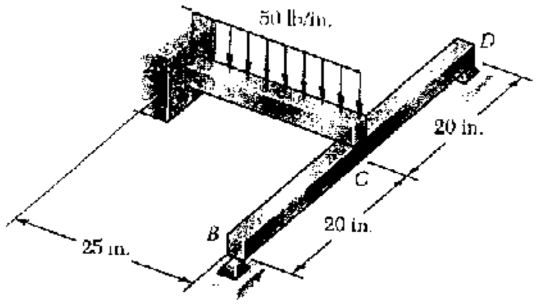
Also, matching the deflection at B, $\quad y_B = -\delta$

$$-103.846\times10^{-3} + 2.3077\times10^{-6}\,P = -58.946\times10^{-9}\,P$$
$$2.3666\times10^{-6}\,P = -103.846\times10^{-3} \qquad P = 43.9\times10^3\ \text{N}$$

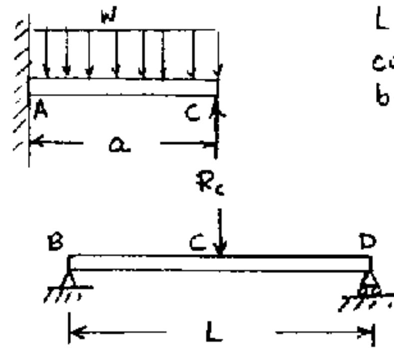
$$P = 43.9\ \text{kN}$$

Problem 9.90

9.90 For the loading shown, knowing that beams AC and BD have the same flexural rigidity, determine the reaction at B.



Consider the two beams shown below.



Let R_c be the contact force between beams AC and BCD.

Applying Cases 1 and 2 of Appendix D to cantilever beam AC,

$$y_c = \frac{R_c a^3}{3EI} - \frac{wa^4}{8EI}$$

Applying Case 4 of Appendix D to simply supported beam BCD,

$$y_c = -\frac{R_c L^3}{48EI}$$

Equating expressions for y_c,

$$\frac{R_c a^3}{3EI} - \frac{wa^4}{8EI} = -\frac{R_c L^3}{48EI}$$

$$(16a^3 + L^3)R_c = 6wa^4$$

$$R_c = \frac{6wa}{16 + L^3/a^3}$$

Data: $w = 50$ lb/in, $a = 25$ in., $L = 20 + 20 = 40$ in.

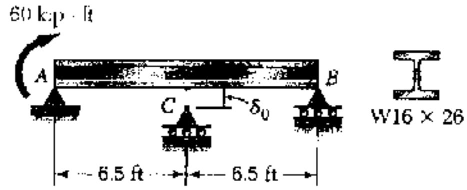
$$R_c = \frac{(6)(50)(25)}{16 + (40/25)^3} = 373.21 \text{ lb}$$

Using beam BCD as a free body,

$$+\circlearrowleft \Sigma M_D = 0: \quad -R_B L + R_c \frac{L}{2} = 0 \qquad R_B = \tfrac{1}{2}R_c = 186.6 \text{ lb} \uparrow$$

Problem 9.91

9.91 Before the 60 kip·ft load is applied, a gap, $\delta_0 = 0.05$ in. existed between the W 16 × 26 beam and the support at C. Knowing that $E = 29 \times 10^6$ psi, determine the reaction at each support after the couple is applied.



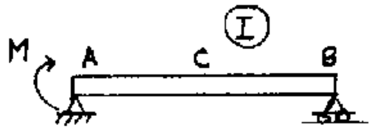
Units: Forces in kips, lengths in ft.

$\delta_0 = 0.05 \text{ in} = 4.1667 \times 10^{-3}$ ft

$E = 29 \times 10^6$ psi $= 29 \times 10^3$ ksi

$I = 301 \text{ in}^4$

$EI = 8.729 \times 10^6 \text{ kip-in}^2 = 60618 \text{ kip-ft}^2$

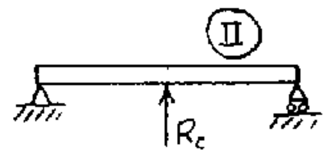


Loading I. Case 7 of Appendix D.

$$y = -\frac{M}{6EIL}(x^3 - L^2 x)$$

with $M = 60$ kip·ft, $L = 13$ ft, $x = 6.5$ ft

$$(y_c)_1 = -\frac{(60)[6.5^3 - (13)^2(6.5)]}{(6)(60618)(13)} = -10.454 \times 10^{-3} \text{ ft}$$



Loading II. Case 4 of Appendix D.

$$(y_c)_2 = \frac{R_c L^3}{48 EI} = \frac{(13)^3 R_c}{(48)(60618)}$$

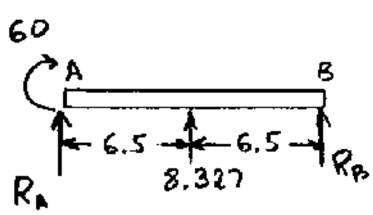
$$= 755.07 \times 10^{-6} R_B$$

Deflection at C.

$$y_c = (y_c)_1 + (y_c)_2 = -\delta_0$$

$$-10.454 \times 10^{-3} + 755.07 \times 10^{-6} R_c = -4.1667 \times 10^{-3}$$

$R_c = 8.327$ kips ↑



Statics:

$\circlearrowleft \sum M_B = 0$:

$$-13 R_A - 60 - (6.5)(8.327) = 0$$

$R_A = -8.779$ kips $\qquad R_A = 8.779$ kips ↓

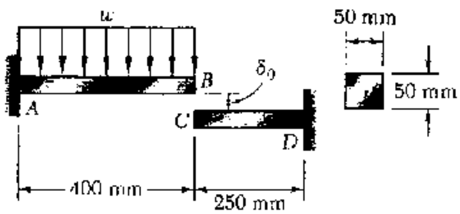
$\circlearrowleft \sum M_A = 0$:

$$13 R_A - 60 + (6.5)(8.327) = 0$$

$R_B = 0.452$ kips ↑

Problem 9.92

9.92 Before the uniformly distributed load w is applied, a gap, $\delta_0 = 1.2$ mm, exists between the ends of the cantilever bars AB and CD. Knowing that $E = 105$ GPa and $w = 30$ kN/m, determine (*a*) the reaction at A, (*b*) the reaction at D.



$I = \frac{1}{12}(50)(50)^3 = 520.833\times10^3\ \text{mm}^3 = 520.833\times10^{-9}\ \text{m}$

$EI = (105\times10^9)(520.833\times10^{-9}) = 54.6875\times10^3\ \text{N}\cdot\text{m}^2$

$= 54.6875\ \text{kN}\cdot\text{m}^2$

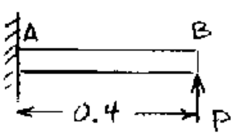
Units: Forces in kN. Lengths in meters.

Compute deflection at B due to w. Case 8 of Appendix D.

$$(y_B)_1 = -\frac{wL^4}{8EI} = -\frac{(30)(0.400)^4}{(8)(54.6875)} = -1.75543\times10^{-3} = -1.7553\ \text{mm}$$

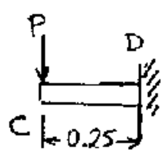
The displacement is more than δ_0, the gap closes.

Let P be the contact force between points B and C.



Compute deflection of B due to P. Use Case 1 of Appendix D.

$$(y_B)_2 = \frac{PL^3}{3EI} = \frac{P(0.4)^3}{(3)(54.6875)} = 390.095\times10^{-6}\ P\ \ \text{m}$$



Compute deflection of C due to P.

$$y_C = -\frac{PL^3}{3EI} = -\frac{P(0.25)^3}{(3)(54.6875)} = -95.238\times10^{-6}\ P\ \ \text{m}$$

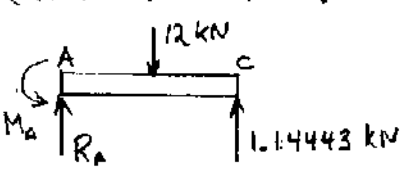
Displacement condition. $\quad y_B + \delta_0 = y_C$

Using superposition, $\quad (y_B)_1 + (y_B)_2 - \delta_0 = y_C$

$$-1.75543\times10^{-3} + 390.095\times10^{-6}P + 1.2\times10^{-3} = -95.238\times10^{-6}P$$

$$485.333\times10^{-6}\ P = 0.55543\times10^{-3} \qquad P = 1.14443\ \text{kN}.$$

(a) Reaction at A.



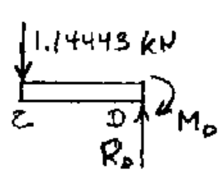
$+\uparrow \Sigma F_y = 0: \quad R_A - 12 + 1.14443 = 0$

$R_A = 10.86\ \text{kN}\uparrow$ ◄

$+\circlearrowleft \Sigma M_A = 0: \quad M_A - (0.2)(12) + (0.4)(1.14443) = 0$

$M_A = 1.942\ \text{kN}\cdot\text{m}\circlearrowleft$ ◄

(b) Reaction at D.



$+\uparrow \Sigma F_y = 0: \quad R_D - 1.14443 = 0$

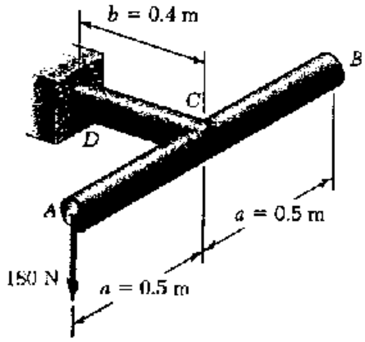
$R_D = 1.144\ \text{kN}\uparrow$ ◄

$+\circlearrowleft \Sigma M_D = 0: \quad -M_D + (0.25)(1.14443) = 0$

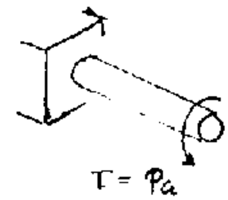
$M_D = 0.286\ \text{kN}\cdot\text{m}\circlearrowleft$ ◄

Problem 9.93

9.93 Two 24-mm-diameter aluminum rods are welded together to form the T-shaped hanger shown. Knowing that $E = 70$ GPa and $G = 26$ GPa, determine the deflection at (*a*) end *A*, (*b*) end *B*.



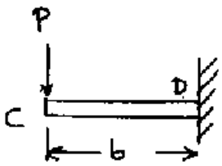
Consider torsion of rod CD. (180 N = P)



$$\varphi_C = \frac{TL}{GJ} = \frac{(Pa)b}{GJ}$$

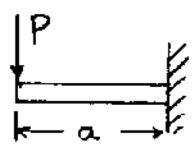
$$(y_A)_I = -a\varphi_C = -\frac{Pa^2b}{GJ}$$

$$(y_B)_I = a\varphi_C = \frac{Pa^2b}{GJ}$$



Consider bending of rod CD.

$$(y_A)_{II} = (y_B)_{II} = (y_C)_{II} = -\frac{Pb^3}{3EI} \qquad \text{(Case 1, App D.)}$$



Consider bending of rod portion AC.

$$(y_A)_{III} = -\frac{Pa^3}{3EI}$$

By superposition,

$$y_A = (y_A)_I + (y_A)_{II} + (y_A)_{III}$$
$$= P\left\{-\frac{a^2b}{GJ} - \frac{b^3}{3EI} - \frac{a^3}{3EI}\right\}$$

$$y_B = (y_B)_I + (y_B)_{II}$$
$$= P\left\{\frac{a^2b}{GJ} - \frac{b^3}{3EI}\right\}$$

Data: $G = 26 \times 10^9$ Pa, $J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(12)^4 = 32.572 \times 10^3\ \text{mm}^4 = 32.572 \times 10^{-9}\ \text{m}^4$

$E = 70 \times 10^9$ Pa, $I = \frac{1}{2}J = 16.286 \times 10^{-9}\ \text{m}^4$

$GJ = 846.87\ \text{N}\cdot\text{m}^2$ $\qquad EI = 1140.02\ \text{N}\cdot\text{m}^2$

$a = 0.5$ m, $b = 0.4$ m

$$y_A = 180\left\{-\frac{(0.5)^2(0.4)}{846.87} - \frac{(0.4)^3}{(3)(1140.02)} - \frac{(0.5)^3}{(3)(1140.02)}\right\} = -31.2 \times 10^{-3}\ \text{m}$$

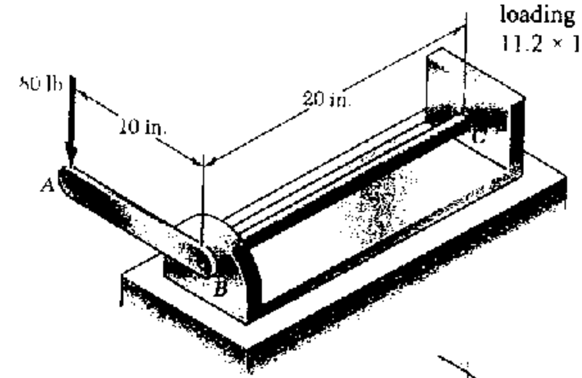
$$= 31.2\ \text{mm} \downarrow \quad (a)$$

$$y_B = 180\left\{\frac{(0.5)^2(0.4)}{846.87} - \frac{(0.4)^3}{(3)(1140.02)}\right\} = 17.89 \times 10^{-3}\ \text{m}$$

$$17.89\ \text{mm} \uparrow \quad (b)$$

Problem 9.94

9.94 A $\frac{7}{8}$-in.-diameter rod BC is attached to the lever AB and to the fixed support at C. Lever AB has a uniform cross section $\frac{3}{8}$ in. thick and 1 in. deep. For the loading shown, determine the deflection of point A. Use $E = 29 \times 10^6$ psi and $G = 11.2 \times 10^6$ psi.



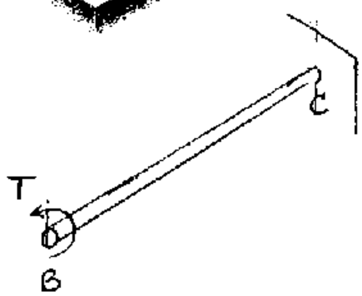
Deformation of rod BC. (Torsion)

$$c = \tfrac{1}{2}d = \tfrac{1}{2}\left(\tfrac{7}{8}\right) = 0.4375 \text{ in}$$

$$J = \tfrac{\pi}{2}c^4 = 57.548 \times 10^{-3} \text{ in}^4$$

$$T = Pa = (80)(10) = 800 \text{ lb}\cdot\text{in}.$$

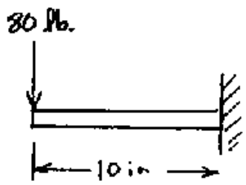
$$L = 20 \text{ in}.$$



$$\varphi_B = \frac{TL}{GJ} = \frac{(800)(20)}{(11.2\times10^6)(57.548\times10^{-3})}$$

$$= 24.824 \times 10^{-3} \text{ rad}.$$

Deflection of point A assuming lever AB to be rigid.



$$(y_A)_1 = a\,\varphi_B = (10)(24.824\times10^{-3})$$

$$= 0.24824 \text{ in} \downarrow$$

Additional deflection due to bending of lever AB.
Refer to Case 1 of Appendix D.

$$I = \tfrac{1}{12}\left(\tfrac{3}{8}\right)(1)^3 = 31.25\times10^{-3} \text{ in}^4$$

$$(y_A) = \frac{PL^3}{3EI} = \frac{(80)(10)^3}{(3)(29\times10^6)(31.25\times10^{-3})}$$

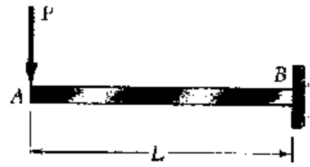
$$= 0.02943 \text{ in.} \downarrow$$

Total deflection at point A.

$$y_A = (y_A)_1 + (y_A)_2 = 0.278 \text{ in.} \downarrow$$

Problem 9.95

9.95 through 9.98 For the uniform cantilever beam and loading shown, determine (*a*) the slope at the free end, (*b*) the deflection at the free end.



Place reference tangent at B.

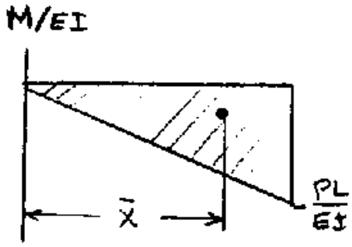
Draw $\frac{M}{EI}$ diagram.

$$A = \frac{1}{2}\left(-\frac{PL}{EI}\right)L = -\frac{PL^2}{2EI}$$

$$\bar{x} = \frac{2}{3}L$$

$$\theta_{B/A} = A = -\frac{PL^2}{2EI}$$

$$t_{A/B} = A\bar{x} = \left(-\frac{PL^3}{2EI}\right)\left(\frac{2}{3}L\right) = -\frac{PL^3}{3EI}$$



(a) Slope at end A.

$$\theta_B = \theta_A + A$$

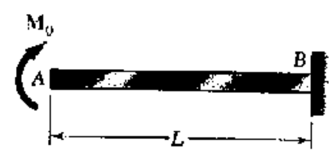
$$0 = \theta_A - \frac{PL^2}{2EI} \qquad \theta_A = \frac{PL^2}{2EI} \measuredangle$$

(b) Deflection at A.

$$y_A = t_{A/B} = -\frac{PL^3}{3EI} \qquad y_A = \frac{PL^3}{3EI} \downarrow$$

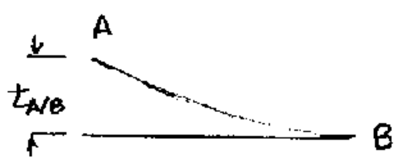
Problem 9.96

9.95 through 9.98 For the uniform cantilever beam and loading shown, determine (*a*) the slope at the free end, (*b*) the deflection at the free end.



Place reference tangent at B.

Draw $\frac{M}{EI}$ diagram.

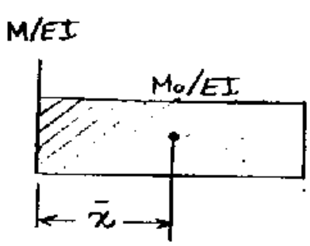


$$A = \left(\frac{M_0}{EI}\right)L = \frac{M_0L}{EI}$$

$$\bar{x} = \frac{1}{2}L$$

$$\theta_{B/A} = A = \frac{M_0L}{EI}$$

$$t_{B/A} = A\bar{x} = \left(\frac{M_0L}{EI}\right)\left(\frac{1}{2}L\right) = \frac{M_0L^2}{2EI}$$



(a) Slope at end A.

$$\theta_B = \theta_A + A \qquad 0 = \theta_A + \frac{M_0L}{EI}$$

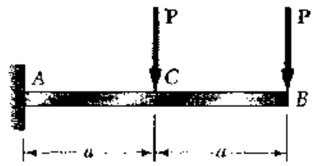
$$\theta_A = -\frac{M_0L}{EI} \qquad \theta_A = \frac{M_0L}{EI}$$

(b) Deflection at A.

$$y_A = t_{A/B} = \frac{M_0L^2}{2EI} \qquad y_A = \frac{M_0L^2}{2EI} \uparrow$$

Problem 9.97

9.95 through 9.98 For the uniform cantilever beam and loading shown, determine (*a*) the slope at the free end, (*b*) the deflection at the free end.



Place reference tangent at A. $\theta_A = 0$

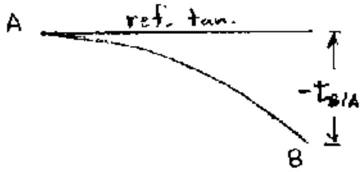
Draw $\frac{M}{EI}$ diagram by parts (two triangles).

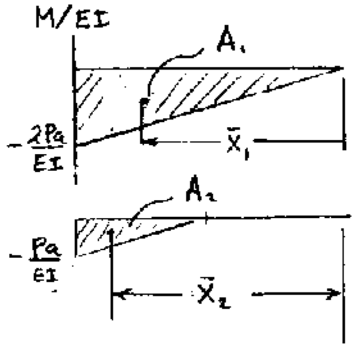
$$A_1 = \frac{1}{2}\left(-\frac{2Pa}{EI}\right)(2a) = -\frac{2Pa^2}{EI}$$

$$\bar{x}_1 = \frac{2}{3}(2a) = \frac{4}{3}a$$

$$A_2 = \frac{1}{2}\left(-\frac{Pa}{EI}\right)a = -\frac{1}{2}\frac{Pa^2}{EI}$$

$$\bar{x}_2 = a + \frac{2}{3}a = \frac{5}{3}a$$





By first moment-area theorem,

$$\theta_{B/A} = A_1 + A_2 = -\frac{2Pa^2}{EI} - \frac{1}{2}\frac{Pa^2}{EI} = -\frac{5}{2}\frac{Pa^2}{EI}$$

$$\theta_B = \theta_A + \theta_{B/A} = -\frac{5}{2}\frac{Pa^2}{EI} = \frac{5Pa^2}{2EI}$$ ◄

By second moment area theorem,

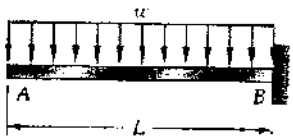
$$t_{B/A} = A_1\bar{x}_1 + A_2\bar{x}_2$$

$$= \left(-\frac{2Pa^2}{EI}\right)\left(\frac{4}{3}a\right) + \left(-\frac{1}{2}\frac{Pa^2}{EI}\right)\left(\frac{5}{3}a\right) = -\frac{7}{2}\frac{Pa^3}{EI}$$

$$y_B = t_{B/A} = -\frac{7}{2}\frac{Pa^3}{EI} \qquad = \frac{7Pa^2}{2EI}\downarrow$$ ◄

Problem 9.98

9.95 through 9.98 For the uniform cantilever beam and loading shown, determine (*a*) the slope at the free end, (*b*) the deflection at the free end.

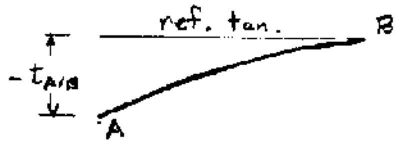


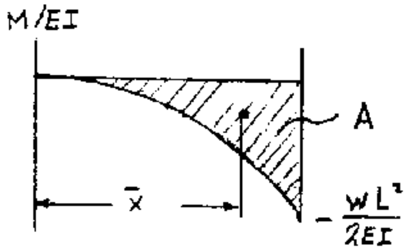
Place reference tangent at B. $\theta_B = 0$

Draw $\frac{M}{EI}$ curve as parabola.

$$A = -\frac{1}{3}\left(\frac{wL^2}{2EI}\right)L = -\frac{1}{6}\frac{wL^3}{EI}$$

$$\bar{x} = L - \frac{1}{4}L = \frac{3}{4}L$$





By first moment-area theorem,

$$\theta_{B/A} = A = -\frac{1}{6}\frac{wL^3}{EI}$$

$$\theta_B = \theta_A + \theta_{B/A}$$

$$\theta_A = \theta_B - \theta_{B/A} = 0 + \frac{1}{6}\frac{wL^3}{EI} = \frac{1}{6}\frac{wL^3}{EI} \measuredangle$$

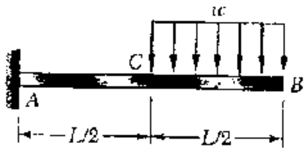
By second moment-area theorem,

$$t_{A/B} = \bar{x}A = \left(\frac{3}{4}L\right)\left(-\frac{1}{6}\frac{wL^3}{EI}\right) = -\frac{1}{8}\frac{wL^4}{EI}$$

$$y_A = t_{A/B} = -\frac{1}{8}\frac{wL^4}{EI} = \frac{wL^4}{8EI}\downarrow$$

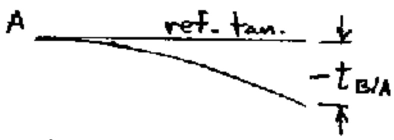
Problem 9.99

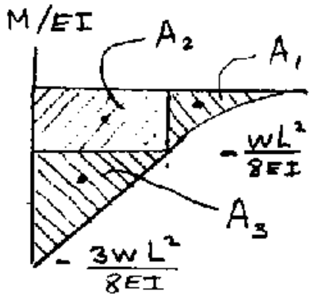
9.99* and *9.100 For the uniform cantilever beam and loading shown, determine (*a*) the slope and deflection at (*a*) point *B*, (*b*) point *C*.



Place reference tangent at A.

Draw $\frac{M}{EI}$ diagram.





$$A_1 = \frac{1}{3}\left(-\frac{wL^2}{8EI}\right)\left(\frac{L}{2}\right) = -\frac{wL^3}{48EI}$$

$$A_2 = \left(-\frac{wL^2}{8EI}\right)\left(\frac{L}{2}\right) = -\frac{wL^3}{16EI}$$

$$A_3 = \frac{1}{2}\left(-\frac{2wL^2}{8EI}\right)\left(\frac{L}{2}\right) = -\frac{wL^3}{16EI}$$

$$\theta_A = 0 \quad , \quad y_A = 0$$

(a) Slope at B.

$$\theta_B = \theta_A + A_1 + A_2 + A_3$$
$$= -\frac{7wL^3}{48EI} \qquad \theta_B = \frac{7wL^3}{48EI}$$

Deflection at B

$$y_B = t_{B/A} = A_1\left(\frac{3}{4}\cdot\frac{L}{2}\right) + A_2\left(\frac{L}{2} + \frac{1}{2}\frac{L}{2}\right) + A_3\cdot\left(\frac{L}{2} + \frac{2}{3}\cdot\frac{L}{2}\right)$$
$$= -\frac{wL^4}{128EI} - \frac{3wL^4}{64EI} - \frac{5wL^4}{96EI} = -\frac{41wL^4}{384EI}$$

$$y_B = \frac{41wL^4}{384EI} \downarrow$$

(b) Slope at C.

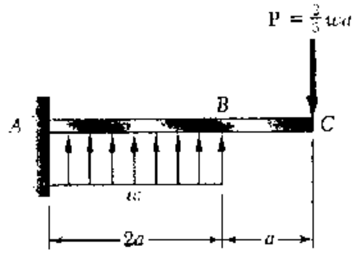
$$\theta_C = \theta_A + A_2 + A_3 = -\frac{wL^3}{8EI} \qquad \theta_C = \frac{wL^3}{8EI}$$

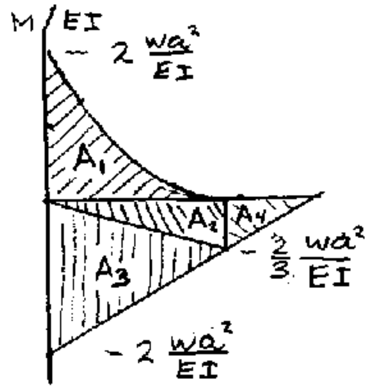
Deflection at C.

$$y_C = t_{C/A} = A_2\left(\frac{1}{2}\cdot\frac{L}{2}\right) + A_3\left(\frac{2}{3}\cdot\frac{L}{2}\right)$$
$$= -\frac{7wL^4}{192EI} \qquad y_C = \frac{7wL^4}{192EI} \downarrow$$

Problem 9.100

9.99* and *9.100 For the uniform cantilever beam and loading shown, determine (*a*) the slope and deflection at (*a*) point *B*, (*b*) point *C*.





Draw $\frac{M}{EI}$ diagram by parts and divide into areas A_1, A_2, A_3, and A_4.

$$A_1 = \frac{1}{3}(2a)\left(2\frac{wa^2}{EI}\right) = \frac{4}{3}\frac{wa^3}{EI}$$

$$A_2 = \frac{1}{2}(2a)\left(-\frac{2wa^2}{3EI}\right) = -\frac{2}{3}\frac{wa^3}{EI}$$

$$A_3 = \frac{1}{2}(2a)\left(-\frac{2wa^2}{EI}\right) = -2\frac{wa^3}{EI}$$

$$A_4 = \frac{1}{2}(a)\left(-\frac{2}{3}\frac{wa^2}{EI}\right) = -\frac{1}{3}\frac{wa^3}{EI}$$

(a) Slope at B.

$$\theta_B = A_1 + A_2 + A_3 = -\frac{4}{3}\frac{wa^3}{EI} = \frac{4wa^3}{3EI} \measuredangle$$

Deflection at B. $\quad y_B = t_{B/A}$

$$y_B = t_{B/A} = A_1\left(\frac{3}{4}\cdot 2a\right) + A_2\left(\frac{1}{3}\cdot 2a\right) + A_3\left(\frac{2}{3}\cdot 2a\right)$$

$$= 2\frac{wa^4}{EI} - \frac{4}{9}\frac{wa^4}{EI} - \frac{8}{3}\frac{wa^4}{EI} = -\frac{10}{9}\frac{wa^4}{EI}$$

(b) Slope at C.

$$\theta_C = \theta_B + A_4 = -\frac{4}{3}\frac{wa^3}{EI} - \frac{1}{3}\frac{wa^3}{EI} = -\frac{5}{3}\frac{wa^3}{EI}$$

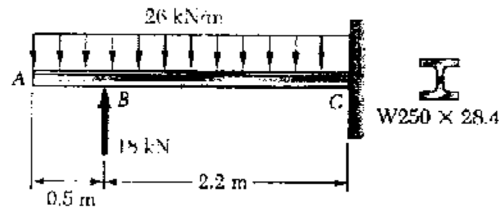
Deflection at C.

$$y_C = y_B + \theta_B a + t_{C/B}$$

$$= -\frac{10}{9}\frac{wa^4}{EI} + \left(-\frac{4}{3}\frac{wa^3}{EI}\right)(a) + \left(-\frac{1}{3}\frac{wa^3}{EI}\right)\left(\frac{2a}{3}\right) = -\frac{8}{3}\frac{wa^4}{EI}$$

Problem 9.101

9.101 For the cantilever beam and loading shown, determine (*a*) the slope at point A, (*b*) the deflection at point A. Use $E = 200$ GPa.



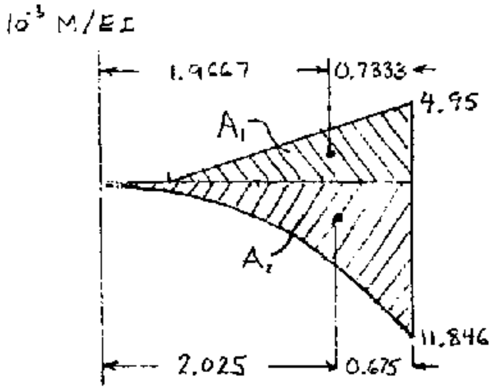
Units: Forces in kN, lengths in m.

$E = 200 \times 10^9$ Pa

$I = 40.0 \times 10^6 \text{ mm}^4 = 40.0 \times 10^{-6} \text{ m}^4$

$EI = (200 \times 10^9)(40.0 \times 10^{-6}) = 8.00 \times 10^6 \text{ N}\cdot\text{m}^2$

$= 8000 \text{ kN}\cdot\text{m}^2$



Draw M/EI diagram by parts.

$\frac{M_1}{EI} = \frac{(18)(2.2)}{8000} = 4.95 \times 10^{-3} \text{ m}^{-1}$

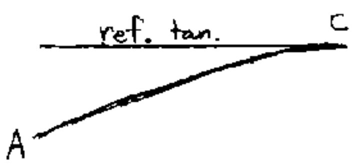
$A_1 = \frac{1}{2}(4.95 \times 10^{-3})(2.2) = 5.445 \times 10^{-3}$

$\bar{x}_1 = \frac{1}{3}(2.2) = 0.7333 \text{ m}$

$\frac{M_2}{EI} = -\frac{(26)(2.7)^2}{(2)(8000)} = -11.846 \times 10^{-3} \text{ m}^{-1}$

$A_2 = \frac{1}{3}(-11.846 \times 10^{-3})(2.7) = -10.662 \times 10^{-3}$

$\bar{x}_2 = \frac{1}{4}(2.7) = 0.675 \text{ m}$



Draw reference tangent at C.

$\theta_C = \theta_A + \theta_{C/A} = \theta_A + A_1 + A_2 = 0$

$\theta_A = -A_1 - A_2 = -5.445 \times 10^{-3} + 10.662 \times 10^{-3}$

$= 5.22 \times 10^{-3}$ rad

$\theta_A = 5.22 \times 10^{-3} \measuredangle$ ◄

$y_A = y_C - \theta_C L + t_{A/C}$

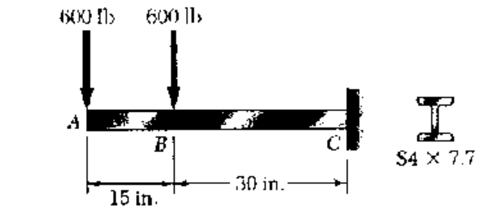
$= 0 - 0 + A_1 \bar{x}_1 + A_2 \bar{x}_2$

$= 0 - 0 + (5.445 \times 10^{-3})(1.9667) - (10.662 \times 10^{-3})(2.025)$

$= -10.881 \times 10^{-3}$ m $=$ 10.88 mm ↓ ◄

Problem 9.102

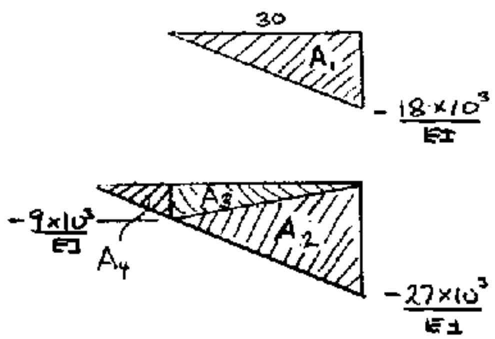
9.102 For the cantilever beam and loading shown, determine the slope and deflection at (*a*) end *A*, (*b*) point *B*. Use $E = 29 \times 10^6$ psi.



$I = 6.08 \text{ in}^4$

$EI = (29\times10^6)(6.08) = 176.32\times10^6 \text{ lb}\cdot\text{in}^2$

Draw $\frac{M}{EI}$ diagram by parts.



$A_1 = -\frac{1}{2}(30)\frac{18\times10^3}{EI} = -1.5313\times10^{-3}$

$A_2 = -\frac{1}{2}(30)\frac{27\times10^3}{EI} = -2.2970\times10^{-3}$

$A_3 = -\frac{1}{2}(30)\frac{9\times10^3}{EI} = -0.7657\times10^{-3}$

$A_4 = -\frac{1}{2}(15)\frac{9\times10^3}{EI} = -0.3828\times10^{-3}$

Place reference tangent at C.

$\theta_C = 0 \qquad y_C = 0$

(a) Slope at A. $\theta_A = -\theta_{C/A} = -A_1 - A_2 - A_3 - A_4 = 4.98\times10^{-3}$

Deflection at A. $y_A = t_{A/C}$

$y_A = A_1(15+20) + A_2(15+20) + A_3(15+10) + A_4(10) = -0.1570 \text{ in.}$

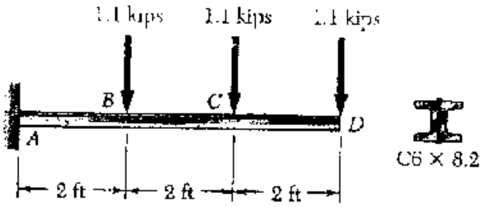
(b) Slope at B. $\theta_B = -\theta_{B/A} = -A_1 + A_2 + A_3 = 4.59\times10^{-3}$

Deflection at B. $y_B = t_{B/C}$

$y_B = A_1(20) + (A_2(20) + A_3(10) = -0.0842 \text{ in.}$

Problem 9.103

9.103 Two C 6 × 8.2 channels are welded back to back and loaded as shown. Knowing that $E = 29 \times 10^6$ psi., determine (*a*) the slope at point *D*, (*b*) the deflection at point *D*.



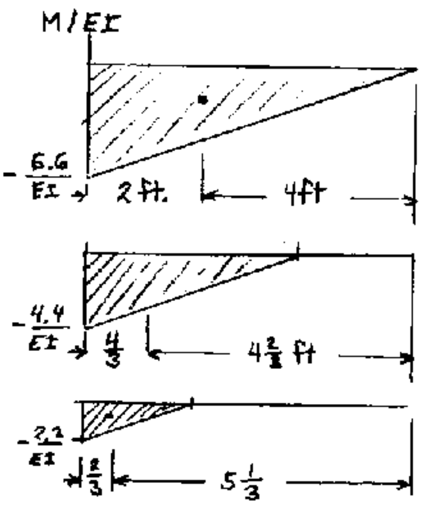
Units: Forces in kips; lengths in ft.

$E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi}$

$I = (2)(13.1) = 26.2 \text{ in}^4$

$EI = (29 \times 10^3)(26.2) = 759.8 \times 10^3 \text{ kip} \cdot \text{in}^2$

$= 5276 \text{ kip} \cdot \text{ft}^2$



Draw $\frac{M}{EI}$ diagram by parts.

$$\frac{M_1}{EI} = -\frac{(1.1)(6)}{EI} = -\frac{6.6}{EI} \quad \text{ft}^{-1}$$

$$A_1 = \frac{1}{2}\left(\frac{6.6}{EI}\right)(6) = -\frac{19.8}{EI}$$

$$\bar{x}_1 = \frac{1}{3}(6) = 2 \text{ ft.}$$

$$\frac{M_2}{EI} = -\frac{(1.1)(4)}{EI} = -\frac{4.4}{EI} \quad \text{ft}^{-1}$$

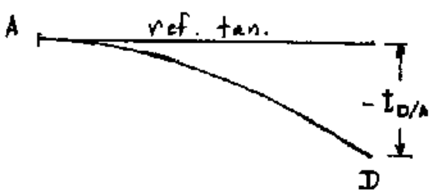
$$A_2 = \frac{1}{2}\left(-\frac{4.4}{EI}\right)(4) = -\frac{8.8}{EI}$$

$$\bar{x}_2 = \frac{1}{3}(4) = \frac{4}{3} \text{ ft}$$

$$\frac{M_3}{EI} = -\frac{(1.1)(2)}{EI} = -\frac{2.2}{EI} \quad \text{ft}^{-1}$$

$$A_3 = \frac{1}{2}\left(-\frac{2.2}{EI}\right)(2) = -\frac{2.2}{EI}$$

$$\bar{x}_3 = \frac{1}{3}(2) = \frac{2}{3} \text{ ft}$$



Place reference tangent at A. $\theta_A = 0$

$$\theta_{D/A} = A_1 + A_2 + A_3 = -\frac{30.8}{EI} = -\frac{30.8}{5276} = -5.84 \times 10^{-3} \text{ rad.}$$

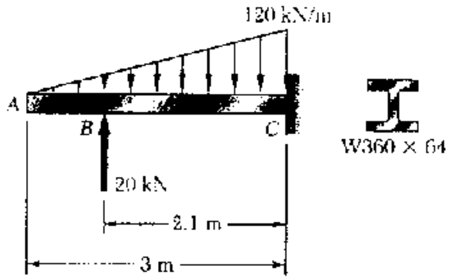
$$\theta_D = \theta_A + \theta_{D/A} = -5.84 \times 10^{-3} \text{ rad.}$$

$$t_{D/A} = \left(-\frac{19.8}{EI}\right)(4) + \left(-\frac{8.8}{EI}\right)\left(4\tfrac{2}{3}\right) + \left(-\frac{2.2}{EI}\right)\left(5\tfrac{1}{3}\right) = -\frac{132.0}{EI} = -\frac{132.0}{5276} = 25.02 \times 10^{-3} \text{ ft}$$

$$y_D = t_{D/A} = 25.02 \times 10^{-3} \text{ ft} = 0.300 \text{ in.} \downarrow$$

Problem 9.104

9.104 For the cantilever beam and loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *A*. Use $E = 200$ GPa.

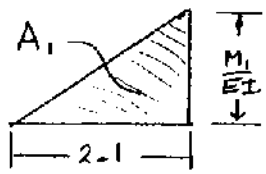


Units: Forces in kN; lengths in meters.

$I = 178 \times 10^6 \text{ mm}^4 = 178 \times 10^{-6}$

$EI = (200 \times 10^9)(178 \times 10^{-6}) = 35600 \text{ kN-m}^2$

Draw $\frac{M}{EI}$ diagram by parts.



$\frac{M_1}{EI} = \frac{(20)(2.1)}{35600} = 1.17978 \times 10^{-3} \text{ m}^{-1}$

$A_1 = (\frac{1}{2})(2.1)(1.17978 \times 10^{-3}) = 1.23876 \times 10^{-3}$

$M_2 = -\frac{(\frac{1}{2})(120)(3)(1)}{35600} = -5.0562 \times 10^{-3} \text{ m}^{-1}$

$A_2 = (\frac{1}{4})(3)(-5.0562 \times 10^{-3}) = -3.7921 \times 10^{-3}$

Place reference tangent at C. $\theta_C = 0$

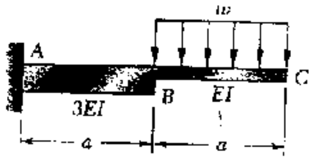
(a) Slope at A. $\theta_A = -\theta_{C/A} = -A_1 - A_2 = 2.55 \times 10^{-3}$ ◄

(b) Deflection at A. $y_A = t_{A/C}$

$y_C = A_1(3 - 0.7) + A_2(3)(\frac{3}{5}) = -6.25 \times 10^{-3} \text{ m} = 6.25 \text{ mm} \downarrow$ ◄

Problem 9.105

9.105 For the cantilever beam ABC, determine the deflection at (*a*) point B, (*b*) end C.

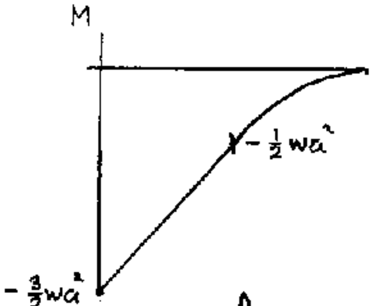


Draw $\frac{M}{EI}$ diagram.

$$A_1 = \frac{1}{2}\left(-\frac{1}{6}\frac{wa^2}{EI}\right)a = -\frac{1}{12}\frac{wa^3}{EI}$$

$$A_2 = \frac{1}{2}\left(-\frac{1}{2}\frac{wa}{EI}\right)a = -\frac{1}{4}\frac{wa^3}{EI}$$

$$A_3 = \frac{1}{3}\left(-\frac{1}{2}\frac{wa^2}{EI}\right)a = -\frac{1}{6}\frac{wa^3}{EI}$$



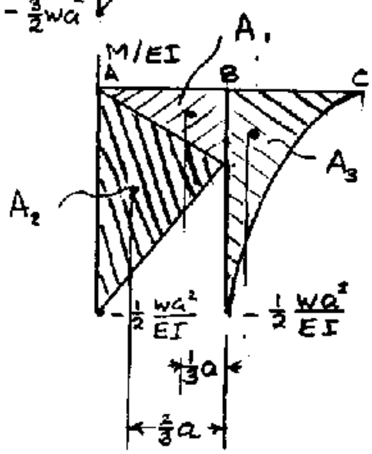
Place reference tangent at A.

(a) Deflection at B.

$$t_{B/A} = A_1\left(\tfrac{1}{3}a\right) + A_2\left(\tfrac{2}{3}a\right)$$

$$= \left(-\frac{1}{12}\frac{wa^3}{EI}\right)\left(\tfrac{1}{3}a\right) + \left(-\frac{1}{4}\frac{wa^3}{EI}\right)\left(\tfrac{2}{3}a\right)$$

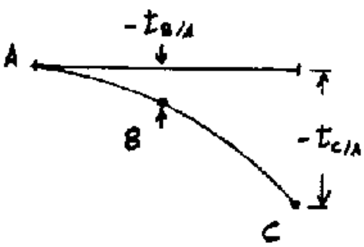
$$= -\frac{7}{36}\frac{wa^4}{EI}$$



$$y_B = t_{B/A} = -\frac{7}{36}\frac{wa^4}{EI}$$ ◄

(b) Deflection at C.

$$t_{C/A} = A_1\left(a+\tfrac{1}{3}a\right) + A_2\left(a+\tfrac{2}{3}a\right) + A_3\left(a-\tfrac{1}{4}a\right)$$



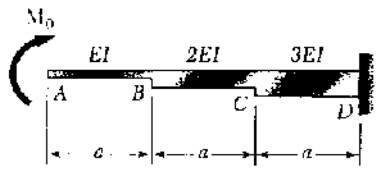
$$= \left(-\frac{1}{12}\frac{wa^2}{EI}\right)\left(\tfrac{4}{3}a\right) + \left(-\frac{1}{4}\frac{wa^2}{EI}\right)\left(\tfrac{5}{3}a\right)$$

$$-\left(\frac{1}{6}\frac{wa^2}{EI}\right)\left(\tfrac{3}{4}a\right) = -\frac{47}{72}\frac{wa^4}{EI}$$

$$y_C = t_{C/A} = -\frac{47}{72}\frac{wa^4}{EI}$$ ◄

Problem 9.106

9.106 For the cantilever beam and loading shown, determine the deflection and slope at end A caused by the moment $\mathbf{M}_0$.

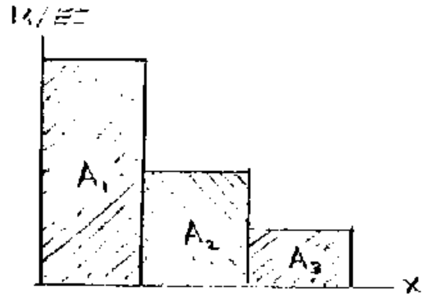


Draw $\frac{M}{EI}$ diagram.

$$A_1 = +\frac{M_0 a}{EI}$$

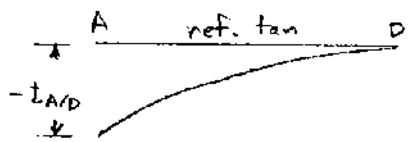
$$A_2 = +\frac{M_0 a}{2EI}$$

$$A_3 = +\frac{M_0 a}{3EI}$$



$\theta_D = 0$, $y_D = 0$

Place reference tangent at D.



Deflection at A. $\qquad y_A = t_{A/D}$

$$y_A = A_1\left(\tfrac{1}{2}a\right) + A_2\left(\tfrac{3}{2}a\right) + A_3\left(\tfrac{5}{2}a\right) = \frac{25 M_0 a^2}{12EI} \uparrow$$

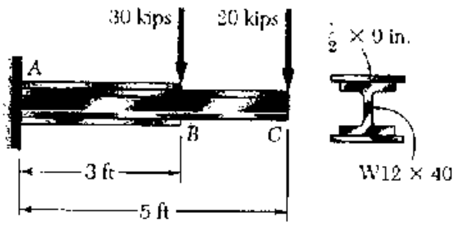
Slope at A. $\qquad \theta_A = -\theta_{C/A}$

$$\theta_A = -A_1 - A_2 - A_3 = -\frac{11 M_0 a}{6EI}$$

$$\theta_A = \frac{11 M_0 a}{6EI}$$

Problem 9.107

9.107 Two cover plates are welded to the rolled-steel beam as shown. Using $E = 29 \times 10^6$ psi., determine the slope and deflection at end C.



Use units of kips and ft.

Over portion BC $\qquad I = 310 \text{ in}^4$

$EI = (29\times10^6)(310) = 8.99 \times 10^9 \text{ lb}\cdot\text{in}$

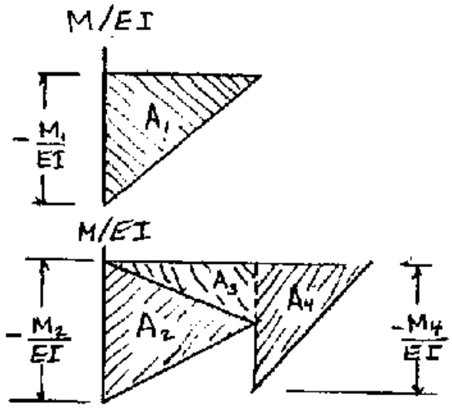
$= 62431 \text{ kip}\cdot\text{ft}^2$

Portion AB	$A(\text{in}^2)$	$d(\text{in.})$	$Ad^2(\text{in}^4)$	$\bar{I}(\text{in}^4)$
Top plate	4.5	6.22	174.1	0.05
W12×40		0	0	310
Bot. plate	4.5	6.22	174.1	0.05
Σ			348.2	310.1

$I = 348.2 + 310.1 = 658.3 \text{ in}^4$

$EI = (29\times10^6)(658.3) = 19.091\times10^9 \text{ lb}\cdot\text{in}^2$

$= 132574 \text{ kip}\cdot\text{ft}^2$



Draw $\frac{M}{EI}$ diagram.

$$\frac{M_1}{EI} = -\frac{(30)(3)}{132574} = -678.86\times10^{-6} \text{ ft}^{-1}$$

$$\frac{M_2}{EI} = -\frac{(20)(5)}{132574} = -754.30\times10^{-6} \text{ ft}^{-1}$$

$$\frac{M_4}{EI} = -\frac{(20)(2)}{62431} = -640.71\times10^{-6} \text{ ft}^{-1}$$

$A_1 = (\frac{1}{2})(3)(-678.86\times10^{-6}) = -1.01829\times10^{-3}$

$A_2 = (\frac{1}{2})(3)(-754.30\times10^{-6}) = -1.13145\times10^{-3}$

$A_3 = \frac{2}{3} A_2 \qquad -0.45248\times10^{-3}$

$A_4 = (\frac{1}{2})(2)(-640.71\times10^{-6}) = -0.64071\times10^{-3}$

Place reference tangent at A.

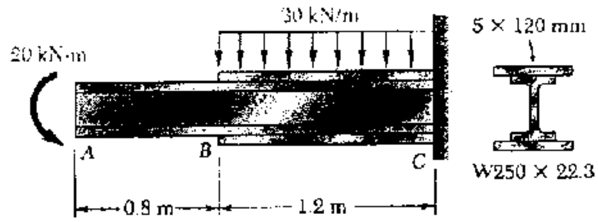
Slope at C. $\qquad \theta_C = \theta_{C/A} = A_1 + A_2 + A_3 + A_4 = -3.24\times10^{-3}$

Deflection at C. $\quad y_C = t_{C/A}$

$y_C = (4)(A_1) + (4)(A_2) + (3)(A_3) + (\frac{4}{3})(A_4) = -10.81\times10^{-3} \text{ ft} = -0.1297 \text{ in.}$

Problem 9.108

9.108 Two cover plates are welded to the rolled-steel beam as shown. Using E = 200 GPa, determine the (a) slope at end A, (b) the deflection at end A.

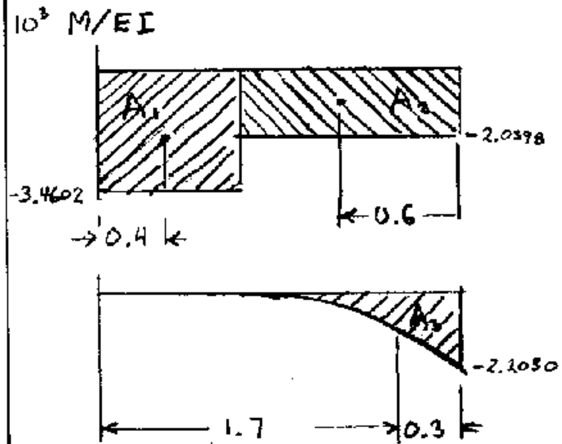


Units: Forces in kN, lengths in m.

$E = 200 \times 10^9$ Pa

From A to B $\quad I = 28.9 \times 10^6\ \text{mm}^4 = 28.9 \times 10^{-6}\ \text{m}^4$

$EI = (200 \times 10^9)(28.9 \times 10^{-6}) = 5.78 \times 10^6\ \text{N·m}^2 = 5780\ \text{kN·m}^2$



From B to C $\quad I = I_W + 2A_p d^2 + 2\bar{I}_p$

$A_p = 5 \times 120 = 600\ \text{mm}^2$

$d = \frac{254}{2} + \frac{5}{2} = 129.5\ \text{mm}$

$Ad^2 = 10.062 \times 10^6\ \text{mm}^4$

$\bar{I}_p = \frac{1}{12}(120)(5)^3 = 0.00125 \times 10^6\ \text{mm}^4$

$I = [28.9 + (2)(10.062) + (2)(0.00125)] \times 10^6\ \text{mm}^4 = 49.03 \times 10^6\ \text{mm}^4 = 49.03 \times 10^{-6}\ \text{m}^4$

$EI = (200 \times 10^9)(49.03 \times 10^{-6}) = 9.805 \times 10^6\ \text{N·m}^2 = 9805\ \text{kN·m}^2$

Draw M/EI diagram by parts.

A to B $\quad \frac{M_1}{EI} = -\frac{20}{5780} = -3.4602 \times 10^{-3}\ \text{m}^{-1}$

B to C $\quad \frac{M_2}{EI} = -\frac{20}{9805} = -2.0398 \times 10^{-3}\ \text{m}^{-1}$

$\frac{M_3}{EI} = -\frac{(30)(1.2)^2}{(2)(9805)} = -2.2030 \times 10^{-3}\ \text{m}^{-1}$

$A_1 = (-3.4602 \times 10^{-3})(0.8) = -2.7682 \times 10^{-3}$

$A_2 = (-2.0398 \times 10^{-3})(1.2) = -2.4478 \times 10^{-3}$

$A_3 = \frac{1}{3}(-2.2030 \times 10^{-3})(1.2) = -0.8812 \times 10^{-3}$

Place reference tangent at C. $\quad \theta_C = 0$

(a) Slope at A. $\quad \theta_A = \theta_C - \theta_{A/C} = 0 - (A_1 + A_2 + A_3) = 6.10 \times 10^{-3}$ rad ◄

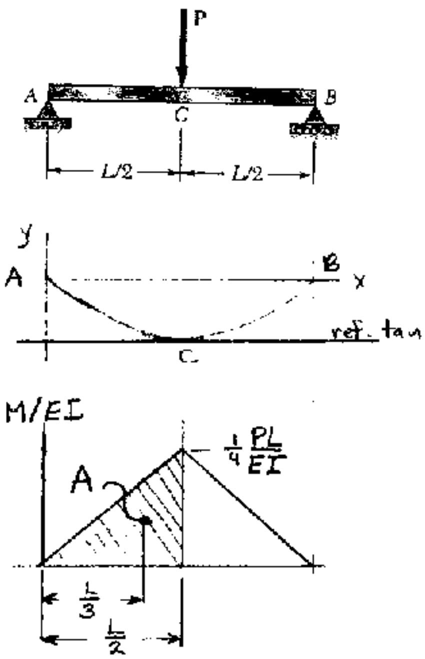
(b) Deflection at A.

$y_A = t_{A/C} = (-2.7682 \times 10^{-3})(0.4) + (-2.4478 \times 10^{-3})(1.4) + (-0.8812 \times 10^{-3})(1.7)$

$= -6.03 \times 10^{-3}\ \text{m} = 6.03\ \text{mm} \downarrow$ ◄

Problem 9.109

9.109 through 9.114 For the prismatic beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.



Symmetric beam and loading.

Place reference tangent at C.

$\theta_C = 0 \quad ; \quad y_C = -t_{A/C}$

Reactions $R_A = R_B = \frac{1}{2}P$

Bending moment at C. $M_C = \frac{1}{4}PL$

$A = \frac{1}{2}\left(\frac{1}{4}\frac{PL}{EI}\right)\left(\frac{L}{2}\right) = \frac{1}{16}\frac{PL^2}{EI}$

(a) Slope at A. $\theta_A = \theta_C - \theta_{C/A}$

$\theta_A = 0 - \frac{1}{16}\frac{PL^2}{EI} = -\frac{1}{16}\frac{PL^2}{EI}$ ◄

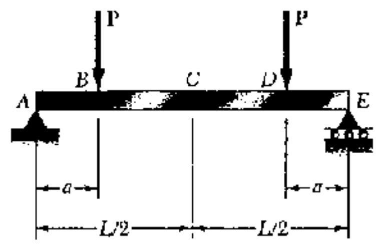
(b) Deflection at C.

$y_C = -t_{A/C} = -A\left(\frac{L}{3}\right) = -\left(\frac{1}{16}\frac{PL^2}{EI}\right)\left(\frac{L}{3}\right)$

$y_C = \frac{1}{48}\frac{PL^3}{EI}$ ◄

Problem 9.110

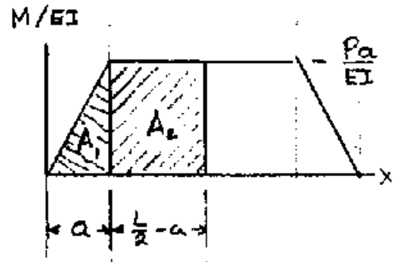
9.109 through 9.114 For the prismatic beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.



Symmetric beam with symmetric loading.

$\theta_C = 0$ Place reference tangent at C.

Draw $\frac{M}{EI}$ diagram.

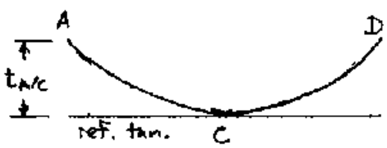


$$A_1 = \frac{1}{2} a \left(\frac{Pa}{EI}\right) = \frac{Pa^2}{2EI}$$

$$A_2 = \left(\frac{L}{2} - a\right)\left(\frac{Pa}{EI}\right) = \frac{Pa(L-2a)}{2EI}$$

Slope at end A. $\theta_A = -\theta_{C/A}$

$$\theta_A = A_1 + A_2 = -\frac{Pa(L-a)}{2EI}$$



Deflection at C. $y_C = -t_{A/C}$

$$t_{A/C} = \left(\frac{2}{3}a\right)A_1 + \left[\frac{1}{2}\left(\frac{L}{2}+a\right)\right]A_2$$

$$= \frac{2}{3}\frac{Pa^2}{2EI} + \left(\frac{L+2a}{4}\right)\frac{Pa(L-2a)}{2EI}$$

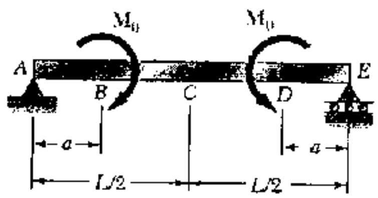
$$= \frac{Pa}{EI}\left\{\frac{1}{3}a^2 + \frac{1}{8}(L^2 - 4a^2)\right\}$$

$$= \frac{Pa}{24EI}(3L^2 - 4a^2)$$

$$y_C = -\frac{Pa}{24EI}(3L^2 - 4a^2)$$

Problem 9.111

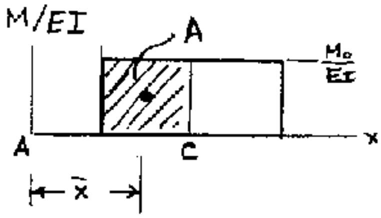
9.109 through 9.114 For the prismatic beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.



Symmetric beam and loading.

Place reference tangent at C. $\theta_C = 0$.

Draw $\frac{M}{EI}$ diagram.

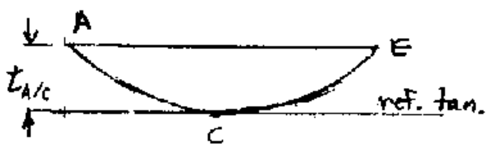


(a) Slope at A. θ_A

$$A = \frac{M_0}{EI}\left(\frac{L}{2} - a\right) = \frac{1}{2}\frac{M_0}{EI}(L - 2a)$$

$$\theta_A = \theta_C - \theta_{C/A} = 0 - A =$$

$$= -\frac{1}{2}\frac{M_0}{EI}(L - 2a)$$



(b) Deflection at C.

$$\bar{x} = a + \frac{1}{2}\left(\frac{L}{2} - a\right) = \frac{1}{4}(L + 2a)$$

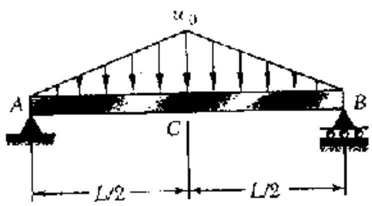
$$y_C = -t_{C/A} = A\bar{x}$$

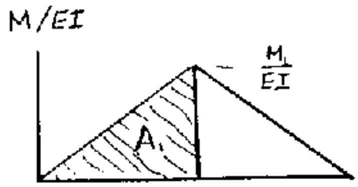
$$= -\frac{1}{2}\frac{M_0}{EI}(L - 2a)\frac{1}{4}(L + 2a)$$

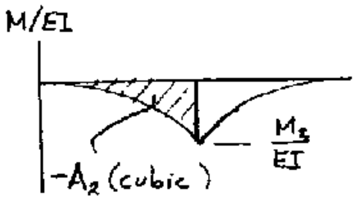
$$= -\frac{1}{8}\frac{M_0}{EI}(L^2 - 4a^2)$$

Problem 9.112

9.109 through 9.114 For the prismatic beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.







Symmetric beam and loading.

$\theta_C = 0$ Place reference tangent at C.

Reactions. $R_A = R_B = \frac{w_0 L}{4}$

Draw $\frac{M}{EI}$ diagram by parts.

$$\frac{M_1}{EI} = \frac{R_A L}{2} = \frac{w_0 L^2}{8EI}$$

$$A_1 = \frac{1}{2}\left(\frac{L}{2}\right)\left(\frac{M_1}{EI}\right) = \frac{w_0 L^3}{32EI}$$

$$\frac{M_2}{EI} = \frac{1}{EI}\cdot\frac{1}{2}\left(\frac{w_0 L}{2}\right)\left(\frac{1}{3}\cdot\frac{L}{2}\right) = -\frac{w_0 L^2}{24EI}$$

$$A_2 = \frac{1}{4}\left(\frac{L}{2}\right)\left(-\frac{w_0 L^2}{24EI}\right) = -\frac{w_0 L^3}{192EI}$$

Slope at A. $\theta_A = -\theta_{C/A}$

$$\theta_A = -A_1 - A_2 = \left(-\frac{1}{32} + \frac{1}{192}\right)\frac{w_0 L^3}{EI}$$

$$= -\frac{5wL^3}{192EI}$$

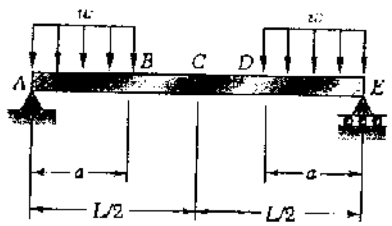
Deflection at C. $y_C = t_{A/C}$

$$t_{A/C} = A_1\left[\left(\frac{2}{3}\right)\left(\frac{L}{2}\right)\right] + A_2\left[\left(\frac{4}{5}\right)\left(\frac{L}{2}\right)\right] = \left[\left(\frac{1}{3}\right)\left(\frac{1}{32}\right) - \left(\frac{2}{5}\right)\left(\frac{1}{192}\right)\right]\frac{w_0 L^4}{EI} = \frac{w_0 L^4}{120EI}$$

$$y_C = -\frac{w_0 L^4}{120EI}$$

Problem 9.113

9.109 through 9.114 For the prismatic beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.



Symmetric beam and loading.

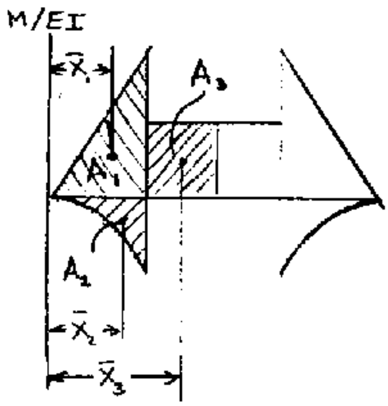
Place reference tangent at C. $\theta_C = 0$

Reactions. $R_A = R_E = wa$

Bending moment

Over AB $\quad M = wax - \frac{1}{2}wa^2$

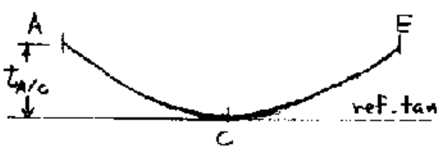
Over BD $\quad M = \frac{1}{2}wa^2$



Draw $\frac{M}{EI}$ diagram by parts.

$$\frac{M_1}{EI} = \frac{wa^2}{EI} \qquad \frac{M_2}{EI} = -\frac{1}{2}\frac{wa^2}{EI}$$

$$\frac{M_3}{EI} = \frac{1}{2}\frac{wa^2}{EI}$$



$$A_1 = \frac{1}{2}\frac{M_1}{EI}a = \frac{1}{2}\frac{wa^3}{EI}$$

$$A_2 = \frac{1}{3}\frac{M_2}{EI}a = -\frac{1}{6}\frac{wa^3}{EI}$$

$$A_3 = \frac{M_3}{EI}\left(\frac{L}{2}-a\right) = \frac{1}{4}\frac{wa^2}{EI}(L-2a)$$

(a) Slope at A. $\quad \theta_A = \theta_C - \theta_{C/A} = 0 - (A_1 + A_2 + A_3)$

$$= -\frac{1}{2}\frac{wa^3}{EI} + \frac{1}{6}\frac{wa^3}{EI} - \frac{1}{4}\frac{wa^2}{EI}(L-2a) = -\frac{wa^2}{EI}\left(\frac{1}{4}L - \frac{1}{6}a\right)$$

$$= -\frac{1}{12}\frac{wa^2}{EI}(3L-2a)$$

(b) Deflection at C. $\quad y_C = -t_{C/A}$

$$\bar{x}_1 = \frac{2}{3}a, \quad \bar{x}_2 = \frac{3}{4}a, \quad \bar{x}_3 = a + \frac{1}{2}\left(\frac{L}{2}-a\right) = \frac{1}{4}(L+2a)$$

$$y_C = -t_{C/A} = -A_1\bar{x}_1 - A_2\bar{x}_2 - A_3\bar{x}_3$$

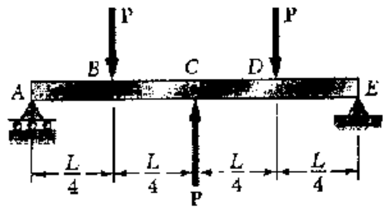
$$= -\left(\frac{1}{2}\frac{wa^3}{EI}\right)\left(\frac{2}{3}a\right) + \left(\frac{1}{6}\frac{wa^3}{EI}\right)\left(\frac{3}{4}a\right) - \frac{1}{4}\left(\frac{wa^2}{EI}\right)(L-2a)\frac{1}{4}(L+2a)$$

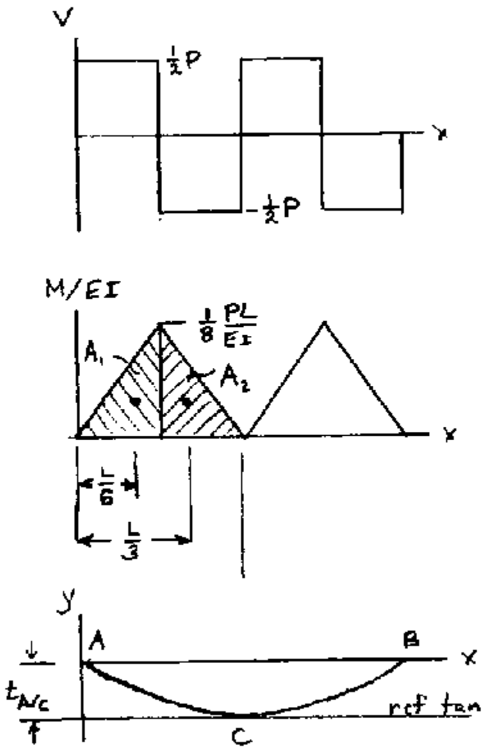
$$= -\frac{1}{3}\frac{wa^3}{EI} + \frac{1}{8}\frac{wa^3}{EI} - \frac{1}{16}\frac{wa^2}{EI}(L^2-4a^2)$$

$$= -\frac{wa^2}{EI}\left(\frac{1}{16}L^2 - \frac{1}{24}a^2\right) = -\frac{1}{48}\frac{wa^2}{EI}(3L^2-2a^2)$$

Problem 9.114

9.109 through 9.114 For the prismatic beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.





Symmetric beam and loading.

Place reference tangent at C. $\theta_C = 0$

Reactions $R_A = R_E = \frac{1}{2}P$

Draw V (shear) and M/EI diagrams.

$$A_1 = A_2 = \frac{1}{2}\left(\frac{1}{8}\frac{PL}{EI}\right)\frac{L}{4} = \frac{1}{64}\frac{PL^2}{EI}$$

(a) Slope at A.

$$\theta_A = \theta_C - \theta_{A/C} = 0 - A_1 - A_2$$

$$= -\frac{1}{32}\frac{PL^2}{EI}$$

(b) Deflection at C.

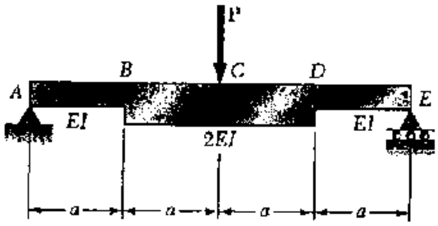
$$y_C = -t_{A/C} = -\left(A_1 \frac{L}{6} + A_2 \frac{L}{3}\right)$$

$$= -\left(\frac{1}{64}\frac{PL^3}{EI}\cdot\frac{L}{6} + \frac{1}{64}\frac{PL^3}{EI}\frac{L}{3}\right)$$

$$= -\frac{1}{128}\frac{PL^3}{EI}$$

Problem 9.115

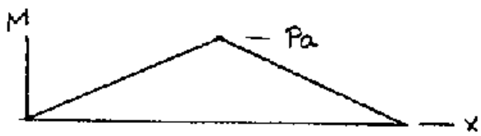
9.115 and 9.116 For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.



Symmetric beam and loading. $R_A = R_E = \frac{1}{2}P$

$$M_{max} = (\tfrac{1}{2}P)(2a) = Pa$$

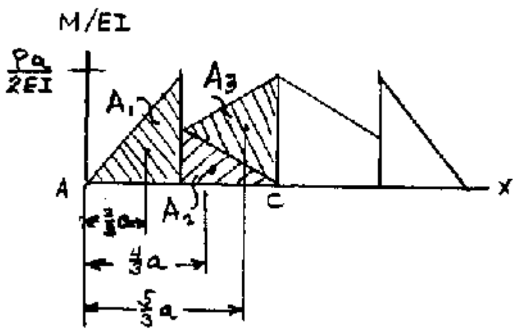
Draw M and $\frac{M}{EI}$ diagrams.



$$A_1 = \tfrac{1}{2}\left(\frac{Pa}{2EI}\right)a = \frac{1}{4}\frac{Pa^2}{EI}$$

$$A_2 = \tfrac{1}{2}\left(\frac{Pa}{4EI}\right)a = \frac{1}{8}\frac{Pa^2}{EI}$$

$$A_3 = \tfrac{1}{2}\left(\frac{Pa}{2EI}\right)a = \frac{1}{4}\frac{Pa^2}{EI}$$



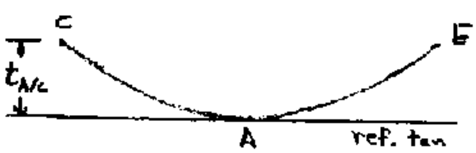
Place reference tangent at C. $\theta_C = 0$

(a) Slope at A.

$$\theta_A = \theta_C - \theta_{C/A} = 0 - (A_1 + A_2 + A_3)$$

$$= -\frac{5}{8}\frac{Pa^2}{EI}$$

(b) Deflection at C.



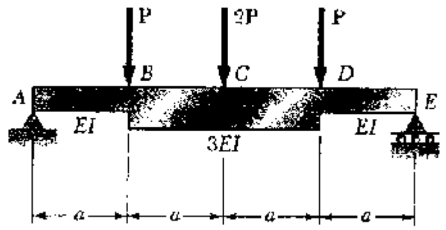
$$|y_C| = t_{A/C} = A_1(\tfrac{2}{3}a) + A_2(\tfrac{4}{3}a) + A_3(\tfrac{5}{3}a)$$

$$\frac{1}{6}\frac{Pa^3}{EI} + \frac{1}{6}\frac{Pa^3}{EI} + \frac{5}{12}\frac{Pa^3}{EI}$$

$$= \frac{3}{4}\frac{Pa^3}{EI} \downarrow$$

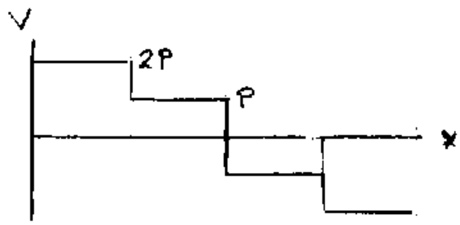
Problem 9.116

9.115* and *9.116 For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.



Symmetric beam and loading. $R_A = R_E = 2P$.

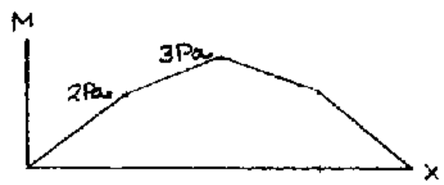
Draw V, M, and $\frac{M}{EI}$ diagrams.



$$A_1 = \frac{1}{2}\left(\frac{2Pa}{EI}\right)a = \frac{Pa^2}{EI}$$

$$A_2 = \frac{1}{2}\left(\frac{2}{3}\frac{Pa}{EI}\right)a = \frac{1}{3}\frac{Pa^2}{EI}$$

$$A_3 = \frac{1}{2}\left(\frac{Pa}{EI}\right)a = \frac{1}{2}\frac{Pa^2}{EI}$$

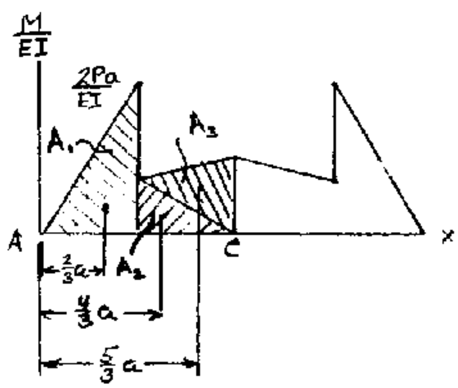


Place reference tangent at C. $\theta_C = 0$

(a) Slope at A.

$$\theta_A = \theta_C - \theta_{C/A} = 0 - (A_1 + A_2 + A_3)$$

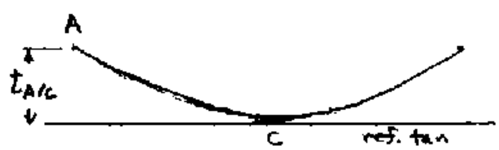
$$= -\frac{11}{6}\frac{Pa^2}{EI}$$



(b) Deflection at C.

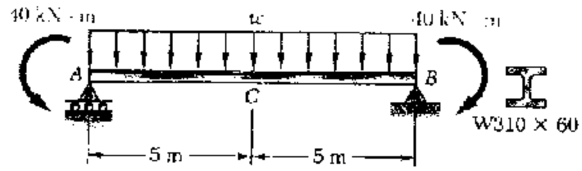
$$|y_C| = t_{A/C} = A_1\left(\tfrac{2}{3}a\right) + A_2\left(\tfrac{4}{3}a\right) + A_3\left(\tfrac{5}{3}a\right)$$

$$= \frac{35}{18}\frac{Pa^3}{EI} \downarrow$$



Problem 9.117

9.117 For the beam and loading shown and knowing that $w = 8$ kN/m, determine (*a*) the slope at end *A*, (*b*) the deflection at midpoint *C*. Use $E = 200$ GPa.

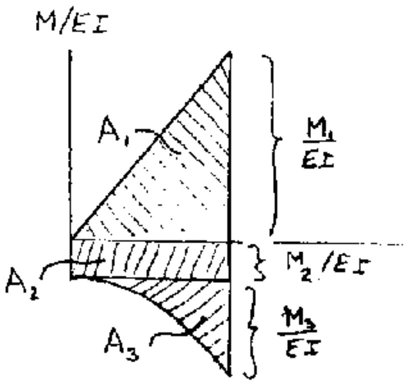


$E = 200 \times 10^9$ Pa

$I = 129 \times 10^6 \text{ mm}^4 = 129 \times 10^{-6} \text{ m}^4$

$EI = (200 \times 10^9)(129 \times 10^{-6}) = 25.8 \times 10^6 \text{ N}\cdot\text{m}^2$

$= 25800 \text{ kN}\cdot\text{m}^2$



Symmetrical beam and loading.

$R_A = R_B = \frac{1}{2}(8)(10) = 40$ kN

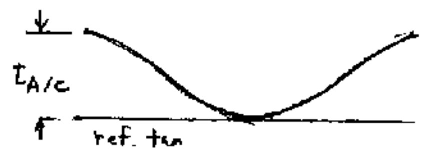
Bending moment

$M = 40x - 40 - \frac{1}{2}(8)x^2$

At $x = 5$

$M = 200 - 40 - 100$

Draw $\frac{M}{EI}$ diagram by parts.



$\frac{M_1}{EI} = \frac{200}{25800} = 7.7519 \times 10^{-3} \text{ m}^{-1}$

$\frac{M_2}{EI} = \frac{-40}{25800} = -1.5504 \times 10^{-3} \text{ m}^{-1}$

$\frac{M_3}{EI} = \frac{-100}{25800} = -3.8760 \times 10^{-3} \text{ m}^{-1}$

$A_1 = \frac{1}{2}(7.7519 \times 10^{-3})(5) = 19.380 \times 10^{-3}$ $\qquad \bar{x}_1 = (\frac{2}{3})(5) = 3.3333$ m

$A_2 = -(1.5504)(5) = -7.7520 \times 10^{-3}$ $\qquad \bar{x}_2 = (\frac{1}{2})(5) = 2.5$ m

$A_3 = -\frac{1}{3}(3.8760)(5) = -6.4600 \times 10^{-3}$ $\qquad \bar{x}_3 = (\frac{3}{4})(5) = 3.75$ m

Place reference tangent at C. $\theta_C = 0$

(a) Slope at A. $\theta_A = \theta_C - \theta_{C/A} = 0 - (A_1 + A_2 + A_3)$

$\theta_A = -(19.380 \times 10^{-3} - 7.7520 \times 10^{-3} - 6.4600 \times 10^{-3}) = -5.17 \times 10^{-3}$ rad ◄

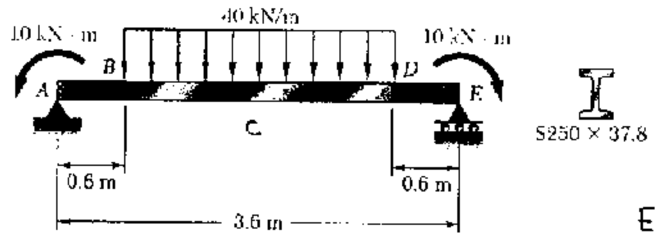
(b) Deflection at C. $|y_C| = t_{A/C}$

$= (19.380 \times 10^{-3})(3.3333) - (7.7520 \times 10^{-3})(2.5) - (6.4600 \times 10^{-3})(3.75)$

$= 21.0 \times 10^{-3}$ m $\qquad = 21.0$ mm ↓ ◄

Problem 9.118

9.118 and 9.119 For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the midpoint of the beam. Use $E = 200$ GPa.

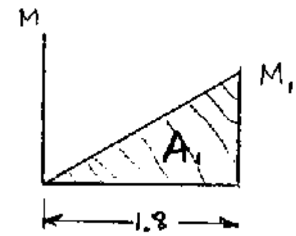


Use units of kN and m.

For S250 × 37.8

$I = 51.1 \times 10^6\ \text{mm}^4 = 51.1 \times 10^{-6}\ \text{m}^4$

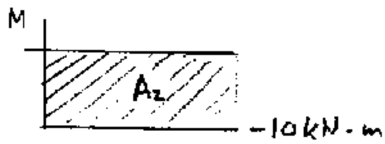
$EI = (200 \times 10^9)(51.1 \times 10^{-6})$
$= 10.22 \times 10^6\ \text{N}\cdot\text{m}^2 = 10220\ \text{kN}\cdot\text{m}^2$



Place reference tangent at midpoint C.

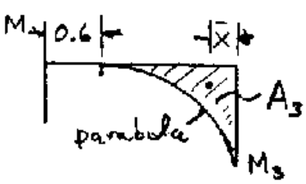
Reactions: $R_A = R_E = \frac{1}{2}(40)(3.6 - 1.2) = 48\ \text{kN} \uparrow$

Draw bending moment diagram of left half of beam by parts.



$M_1 = (48)(1.8) = 86.4\ \text{kN}\cdot\text{m}$

$A_1 = \frac{1}{2}(1.8)(86.4) = 77.76\ \text{kN}\cdot\text{m}^2$

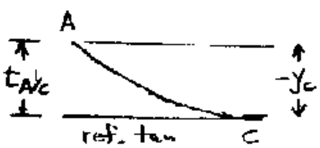


$A_2 = (1.8)(-10) = -18\ \text{kN}\cdot\text{m}^2$

$M_3 = \frac{1}{2}(40)(1.8 - 0.6)^2 = -28.8\ \text{kN}\cdot\text{m}$

$A_3 = \frac{1}{3}(1.2)(-28.8) = -11.52\ \text{kN}\cdot\text{m}^2$

$\bar{x} = \frac{1}{4}(1.2) = 0.30\ \text{m}$



Slope at end A. $\quad \theta_A = -\theta_{A/C}$

$$\theta_A = \frac{1}{EI}\left\{-A_1 - A_2 - A_3\right\}$$

$$= \frac{-77.76 + 18 + 11.52}{10220} = -4.72 \times 10^{-3}$$

Deflection at midpoint C. $\quad y_C = -t_{A/C}$

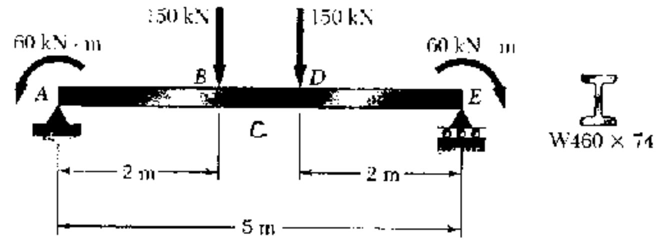
$$t_{A/C} = \frac{1}{EI}\left\{1.2 A_1 + 0.9 A_2 + (1.8 - 0.3) A_3\right)$$

$$= \frac{(1.2)(77.76) - (0.9)(18) - (1.5)(11.52)}{10220} = 5.85 \times 10^{-3}\ \text{m}$$

$y_C = -5.85 \times 10^{-3}\ \text{m} \qquad y_C = 5.85\ \text{mm} \downarrow$

Problem 9.119

9.118 and 9.119 For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the midpoint of the beam. Use $E = 200$ GPa.

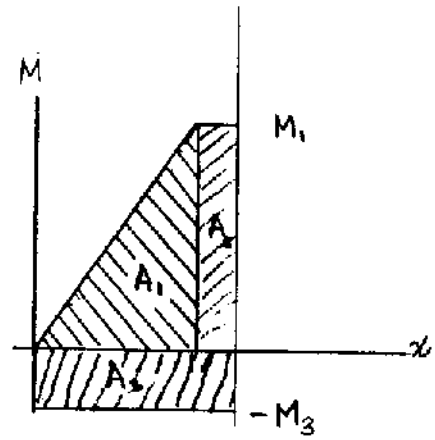


Use units of kN and m.

For W460 × 74

$I = 333 \times 10^6\ mm^4 = 333 \times 10^{-6}\ m^4$

$EI = (200 \times 10^9)(333 \times 10^{-6})$
$= 66.6 \times 10^6\ N \cdot m^2 = 66600\ kN \cdot m^2$



Symmetric beam and loading. Place reference tangent at midpoint C where $\theta_C = 0$.

Reactions. $R_A = R_E = 150$ kN ↑

Draw bending moment diagram of left half of beam by parts

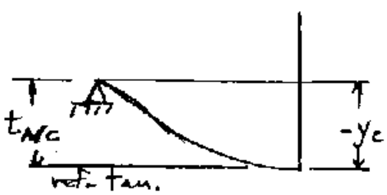
$M_1 = (2)(150) = 300$ kN·m

$A_1 = (\frac{1}{2})(2)(300) = 300\ kN \cdot m^2$

$A_2 = (0.5)(300) = 150\ kN \cdot m^2$

$M_3 = -60$ kN·m

$A_3 = (2.5)(-60) = -150\ kN \cdot m^2$



Slope at end A. $\theta_A = -\theta_{C/A}$

$$\theta_A = \frac{1}{EI}\left\{-A_1 - A_2 - A_3\right\}$$

$$= \frac{-300 - 150 + 150}{66600} = -4.50 \times 10^{-3}$$

Deflection at midpoint C. $y_C = -t_{A/C}$

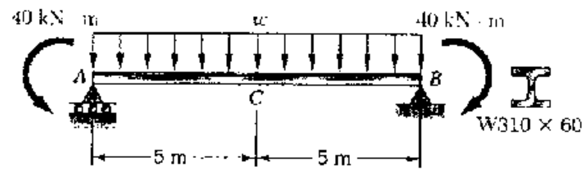
$$t_{A/C} = \frac{1}{EI}\left\{\left(\tfrac{2}{3} \cdot 2\right) A_1 + \left(2 + \tfrac{0.5}{2}\right) A_2 + \left(\tfrac{2.5}{2}\right) A_3\right\}$$

$$= \frac{400 + 337.5 - 187.5}{66600} = 8.26 \times 10^{-3}\ m$$

$y_C = -8.26 \times 10^{-3}$ m $\qquad$ $y_C = 8.26$ mm ↓

Problem 9.120

9.120 For the beam and loading of Prob. 9.117, determine the value of w for which the deflection is zero at the midpoint C of the beam. Use $E = 200$ GPa.



Symmetric beam and loading.

$R_A = R_B = 5w \qquad (w \text{ in kN/m})$

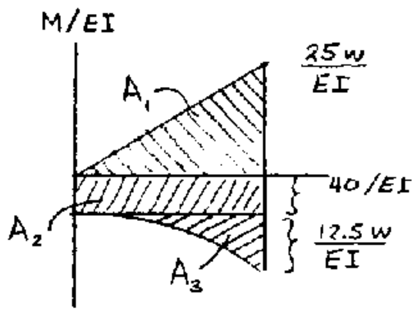
Bending moment in kN·m.

$M = 5wx - 40 - \frac{1}{2}wx^2$

At $x = 5$ m

$M = 25w - 40 - 12.5w$

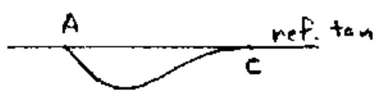
Draw $\frac{M}{EI}$ diagram by parts.



$A_1 = \frac{1}{2}\left(\frac{25w}{EI}\right)(5) = \frac{62.5w}{EI}$

$A_2 = -\frac{(40)(5)}{EI} = -\frac{200}{EI}$

$A_3 = -\frac{1}{3}\left(\frac{12.5w}{EI}\right)(5) = -\frac{20.833w}{EI}$



$\bar{x}_1 = \frac{2}{3}(5) = 3.3333$ m

$\bar{x}_2 = \frac{1}{2}(5) = 2.5$ m

$\bar{x}_3 = \frac{3}{4}(5) = 3.75$ m

Place reference tangent at C.

Deflection at C is zero. $\qquad t_{A/C} = y_A - y_C = 0$

$A_1\bar{x}_1 + A_2\bar{x}_2 + A_3\bar{x}_3 = 0$

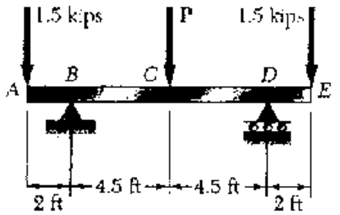
$\left(\frac{62.5w}{EI}\right)(3.3333) - \left(\frac{200}{EI}\right)(2.5) - \left(\frac{20.833w}{EI}\right)(3.75) = 0$

$\frac{130.21w}{EI} - \frac{500}{EI} = 0$

$w = \frac{500}{130.21} = 3.84 \text{ kN/m}$

Problem 9.121

9.121 Knowing that the magnitude of the load **P** is 7 kips, determine (*a*) the slope at end *A*, (*b*) the deflection at end *A*, (*c*) the deflection at midpoint *C* of the beam. Use $E = 29 \times 10^6$ psi.

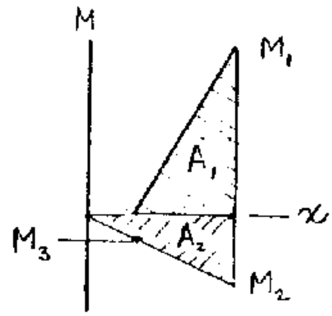


S6 × 12.5

Use units of kips and ft. $P = 7$ kips

For S6 × 12.5 $I = 22.1 \text{ in}^4$

$EI = (29\times10^6)(22.1) = 640.9\times10^6 \text{ lb}\cdot\text{in}^2$
$= 4451 \text{ kip}\cdot\text{ft}^2$



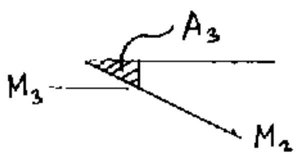
Symmetric beam with symmetric loading. Place reference tangent at midpoint C where $\theta_C = 0$.

$R_B = R_D = \frac{1}{2}(1.5 + 7 + 1.5) = 5 \text{ kips} \uparrow$

Draw the bending moment diagram by parts for the left half of the beam.

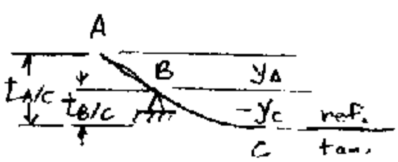
$M_1 = (4.5)(5) = 22.5 \text{ kip}\cdot\text{ft}$

$A_1 = \frac{1}{2}(4.5)(22.5) = 50.625 \text{ kip}\cdot\text{ft}^2$



$M_2 = -(2+4.5)(1.5) = -9.75 \text{ kip}\cdot\text{ft}$

$A_2 = \frac{1}{2}(6.5)(-9.75) = -31.6875 \text{ kip}\cdot\text{ft}^2$



$M_3 = -(2)(1.5) = -3 \text{ kip}\cdot\text{ft}$

$A_3 = \frac{1}{2}(2)(-3) = -3 \text{ kip}\cdot\text{ft}^2$

Formulas. $\theta_A = -\theta_{C/A}$, $y_A - y_C = t_{A/C}$ $y_C = y_A - t_{A/C}$

$y_B = y_A - 2\,\theta_A + t_{B/A} = 0$ $y_A = -2\theta_A - t_{B/A}$

$\theta_{C/A} = \frac{1}{EI}\{A_1 + A_2) = \frac{50.625 - 31.6875}{4451} = 4.25466\times10^{-3}$

$t_{A/C} = \frac{1}{EI}\{(2+3)A_1 + \frac{2}{3}(6.5)A_2\} = \frac{115.8125}{4451} = 26.0194\times10^{-3} \text{ ft}$

$t_{B/A} = \frac{1}{EI}\{\frac{1}{3}(2)A_3\} = \frac{-2}{4451} = -0.44934\times10^{-3} \text{ ft}$

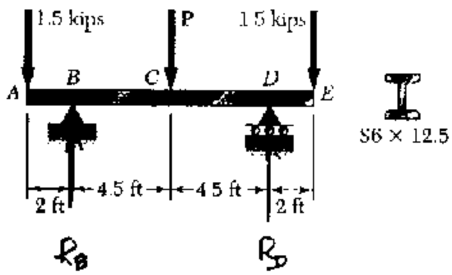
(a) Slope at end A. $\theta_A = -4.25\times10^{-3}$ ◄

(b) Deflection at A. $y_A = -(2)(-4.25466\times10^{-3}) - (-0.44934\times10^{-3})$
$= 8.95866\times10^{-3} \text{ ft}$ $y_A = 0.1075 \text{ in.}\uparrow$ ◄

(c) Deflection at C $y_C = 8.95866\times10^{-3} - 26.0194\times10^{-3} = -17.06074\times10^{-3} \text{ ft}$
$y_C = -0.205 \text{ in.}$ $y_C = 0.205 \text{ in.}\downarrow$ ◄

Problem 9.122

9.122 For the beam and loading shown, determine (*a*) the load **P** for which the deflection is zero at the midpoint *C* of the beam, (*b*) the corresponding deflection at end *A*. Use $E = 29 \times 10^6$ psi.

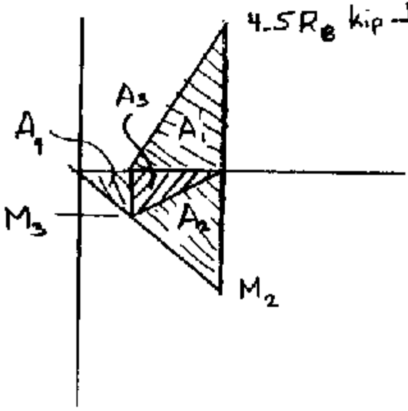


Use units of kips and ft.

For S6 × 12.5 $\quad I = 22.1$ in^4

$EI = (29 \times 10^6)(22.1) = 640.9 \times 10^6$ lb·in^2
$= 4451$ kip·ft^2

Symmetric beam with symmetric loading. Place reference tangent at midpoint C where $\theta_C = 0$.



Draw the bending moment diagram by parts for the left half of the beam.

$A_1 = \frac{1}{2}(4.5)(4.5)R_B = 10.125\, R_B$ kip·ft^2

$M_2 = -(2 + 4.5)(1.5) = -9.75$ kip·ft

$A_2 = \frac{1}{2}(4.5)(-9.75) = -21.9375$ kip·ft^2

$M_3 = -(2)(1.5) = -3$ kip·ft

$A_3 = \frac{1}{2}(4.5)(-3) = -6.75$ kip·ft^2

$A_4 = \frac{1}{2}(2)(-3) = -3$ kip·ft^2

(a) $\quad t_{B/C} = 0: \quad \frac{1}{EI}\left\{\frac{2}{3}(4.5)A_1 + \frac{2}{3}(4.5)A_2 + \frac{1}{3}(4.5)A_3\right\}$

$= \dfrac{30.375\, R_B - 75.9375}{EI} = 0 \qquad R_B = 2.5$ kips

By statics, $-(2)(1.5) + 2R_B - P = 0 \qquad P = 2.00$ kips ◄

$A_1 = (10.125)(2.5) = 25.3125$ kip·ft^2

(b) $\quad y_A = t_{A/C} = \frac{1}{EI}\left\{(2+3)A_1 + (2+3)A_2 + (2+1.5)A_3 + (\frac{2}{3}\cdot 2)A_4\right\}$

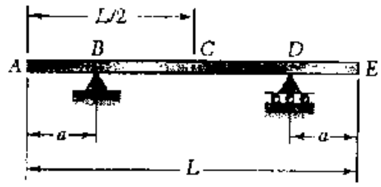
$= \dfrac{(5)(25.3125) + (5)(-21.9375) + (3.5)(-6.75) + (\frac{4}{3})(-3)}{4451}$

$= -2.4152 \times 10^{-3}$ ft $= -0.0290$ in.

$y_C = 0.0290$ in. ↓ ◄

Problem 9.123

*9.123 A uniform rod AE is to be supported at two points B and D. Determine the distance a for which the slope at ends A and E is zero



Let w = weight per unit length of rod.

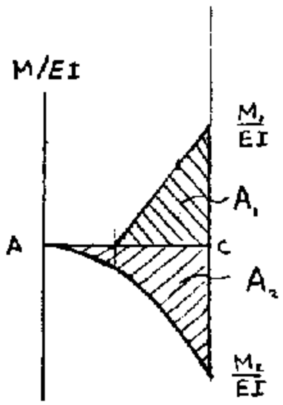
Symmetric beam and loading.

$$R_B = R_D = \frac{1}{2} wL$$

Bending moment

Over AB $\quad M = -\frac{1}{2}wx^2$

Over BCD $\quad M = -\frac{1}{2}wx^2 + \frac{1}{2}wL(x-a)$

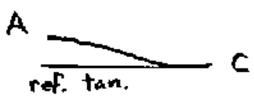


Draw $\frac{M}{EI}$ diagram by parts.

$$\frac{M_1}{EI} = \frac{1}{2}\frac{wL\left(\frac{L}{2}-a\right)}{EI} = \frac{1}{4}\frac{wL(L-2a)}{EI}$$

$$\frac{M_2}{EI} = \frac{1}{2}\frac{w\left(\frac{L}{2}\right)^2}{EI} = -\frac{1}{8}\frac{wL^2}{EI}$$

$$A_1 = \frac{1}{2}\frac{M_1}{EI}\left(\frac{L}{2}-a\right) = \frac{1}{16}\frac{wL(L-2a)^2}{EI}$$



$$A_2 = \frac{1}{3}\left(\frac{M_2}{EI}\right)\frac{L}{2} = -\frac{1}{48}\frac{wL^3}{EI}$$

Place reference tangent at C. $\quad \theta_C = 0$

$$\theta_A = \theta_C - \theta_{C/A} = 0 - (A_1 + A_2) = 0$$

$$-\frac{1}{16}\frac{wL(L-2a)^2}{EI} + \frac{1}{48}\frac{wL^3}{EI} = 0$$

Let $u = \frac{a}{L}$ and divide by $\frac{wL^3}{48EI}$.

$$1 - 3(1-2u)^2 = 0$$

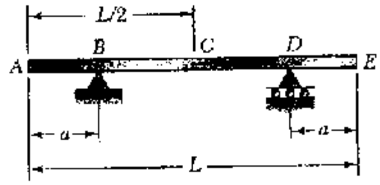
$$1 - 2u = \frac{\sqrt{3}}{3}$$

$$u = \frac{1}{2}\left(1 - \frac{\sqrt{3}}{3}\right) = 0.21132$$

$$\frac{a}{L} = 0.211 \qquad a = 0.211\,L$$

Problem 9.124

*9.124 A uniform rod AE is to be supported at two points B and D. Determine the distance a from the ends of the rod to the points of support, if the downward deflections of points A, C, and E are to be equal.



Let w = weight per unit length of rod.

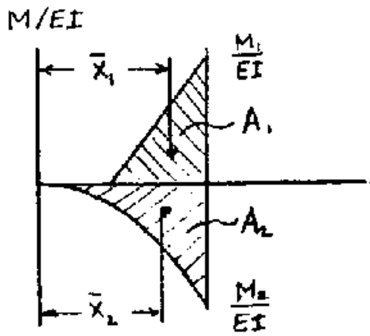
Symmetric beam and loading.

$$R_B = R_D = \frac{1}{2}wL$$

Bending moment:

Over AB $\quad M = -\frac{1}{2}wx^2$

Over BCD $\quad M = -\frac{1}{2}wx^2 + \frac{1}{2}wL(x-a)$



Draw $\frac{M}{EI}$ diagram by parts.

$$\frac{M_1}{EI} = \frac{1}{2}\frac{wL(\frac{L}{2}-a)}{EI} = \frac{1}{4}\frac{wL(L-2a)}{EI}$$

$$\frac{M_2}{EI} = -\frac{1}{2}\frac{w(\frac{L}{2})^2}{EI} = -\frac{1}{8}\frac{wL^2}{EI}$$

$$A_1 = \frac{1}{2}\frac{M_1}{EI}\left(\frac{L}{2}-a\right) = \frac{1}{16}\frac{wL(L-2a)^2}{EI}$$

$$A_2 = \frac{1}{3}\left(\frac{M_2}{EI}\right)\left(\frac{L}{2}\right) = -\frac{1}{48}\frac{wL^3}{EI}$$

$$\bar{x}_1 = a + \frac{2}{3}\left(\frac{L}{2}-a\right) = \frac{1}{3}(L+a)$$

$$\bar{x}_2 = \frac{L}{2} - \frac{1}{4}\left(\frac{L}{2}\right) = \frac{3}{8}L$$

Place reference tangent at C.

$$y_c - y_c = t_{A/C} = 0$$

$$A_1\bar{x}_1 + A_2\bar{x}_2 = 0$$

$$\frac{1}{16}\frac{wL(L-2a)^2}{EI}\frac{1}{3}(L+a) - \frac{1}{48}\frac{wL^3}{EI}\frac{3}{8}L = 0$$

Let $u = a/L$. Divide by $\frac{wL^4}{48EI}$.

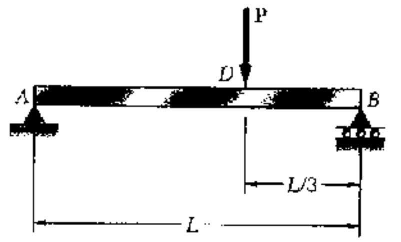
$$(1-2u)^2(1+u) - \frac{3}{8} = 0$$

$$4u^3 - 3u + \frac{5}{8} = 0$$

Solving for u, $\quad u = 0.22315 \quad \frac{a}{L} = 0.223 \quad a = 0.223\,L$ ◄

Problem 9.125

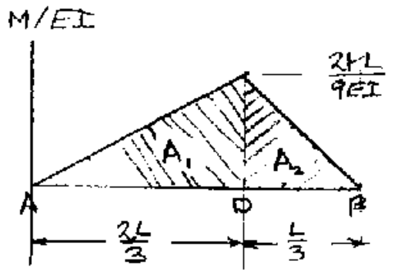
9.125 through *9.128* For the prismatic beam and loading shown, determine (*a*) the deflection at point *D*, (*b*) the slope at end *A*.



From statics $\quad R_A = \frac{1}{3}P \uparrow, \quad R_B = \frac{2}{3}P\uparrow$

$M_D = R_A\left(\frac{2}{3}L\right) = \frac{2}{9}PL$

Draw $\frac{M}{EI}$ diagram.



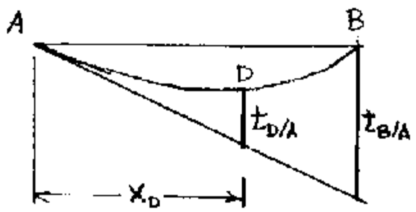
$$A_1 = \frac{1}{2}\left(\frac{2L}{3}\right)\left(\frac{2PL}{9EI}\right) = \frac{2PL^2}{27EI}$$

$$A_2 = \frac{1}{2}\left(\frac{L}{3}\right)\left(\frac{2PL}{9EI}\right) = \frac{PL^2}{27EI}$$

Place reference tangent at A.

$$t_{B/A} = A_1\left(\frac{L}{3} + \frac{1}{3}\cdot\frac{2L}{3}\right) + A_2\left(\frac{2}{3}\cdot\frac{L}{3}\right) = \frac{4PL^3}{81EI}$$

$$t_{D/A} = A_1\left(\frac{1}{3}\cdot\frac{2L}{3}\right) = \frac{4PL^3}{243EI}$$



(a) Deflection at D.

$$y_D = t_{D/A} - \frac{x_D}{L}t_{B/A}$$

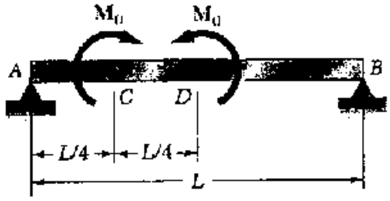
$$y_D = \frac{4PL^3}{243EI} - \left(\frac{2}{3}\right)\frac{4PL^3}{81EI}$$

$$= -\frac{4PL^3}{243EI} \qquad y_D = \frac{4PL^3}{243EI}\downarrow$$

(b) Slope at end A. $\quad \theta_A = -\frac{t_{B/A}}{L} = -\frac{4PL^2}{81EI} \qquad \theta_0 = \frac{4PL^2}{81EI}$

Problem 9.126

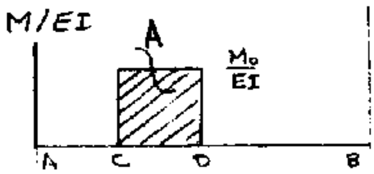
9.125 through *9.128* For the prismatic beam and loading shown, determine (*a*) the deflection at point *D*, (*b*) the slope at end *A*.



From Statics $R_A = R_B = 0$.

Draw $\frac{M}{EI}$ diagram.

$$A = \left(\frac{M_0}{EI}\right)\left(\frac{L}{4}\right) = \frac{1}{4}\frac{M_0 L}{EI}$$



Place reference tangent at A.

$$t_{B/A} = A\left(\frac{L}{2} + \frac{L}{8}\right) = \frac{5}{32}\frac{M_0 L^2}{EI}$$

$$t_{D/A} = A\left(\frac{L}{8}\right) = \frac{1}{32}\frac{M_0 L^2}{EI}$$

(b) Slope at A.

$$\theta_A = -\frac{t_{B/A}}{L} = -\frac{5}{32}\frac{M_0 L}{EI} = \frac{5 M_0 L}{32 EI}$$

(a) Deflection at D.

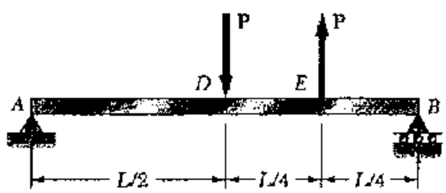
$$y_D = t_{D/A} - \frac{x_D}{L} t_{B/A} = t_{D/A} - \frac{1}{2} t_{B/A}$$

$$= \frac{1}{32}\frac{M_0 L^2}{EI} - \frac{5}{64}\frac{M_0 L^2}{EI} = -\frac{3}{64}\frac{M_0 L^2}{EI}$$

$$= \frac{3 M_0 L^2}{64 EI} \downarrow$$

Problem 9.127

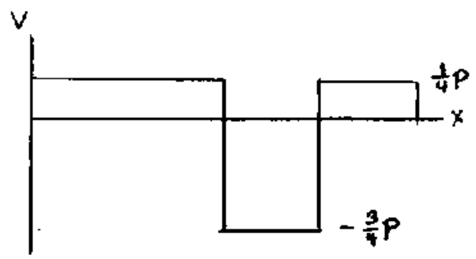
9.125 through 9.128 For the prismatic beam and loading shown, determine (*a*) the deflection at point *D*, (*b*) the slope at end *A*.

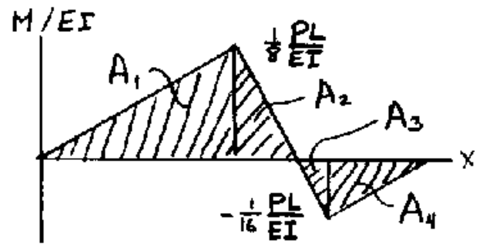


$$+\circlearrowleft \Sigma M_B = 0 \qquad -R_A L + \frac{PL}{2} - \frac{PL}{4} = 0 \qquad R_A = \frac{1}{4}P \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0 \qquad -\frac{PL}{2} + P\frac{3L}{4} + R_B L = 0 \qquad R_B = \frac{1}{4}P \downarrow$$

Draw V (shear) diagram and $\frac{M}{EI}$ diagram.





$$A_1 = \frac{1}{2}\left(\frac{1}{8}\frac{PL}{EI}\right)\left(\frac{L}{2}\right) = \frac{1}{32}\frac{PL^2}{EI}$$

$$A_2 = \frac{1}{2}\left(\frac{1}{8}\frac{PL}{EI}\right)\left(\frac{L}{6}\right) = \frac{1}{96}\frac{PL^2}{EI}$$

$$A_3 = \frac{1}{2}\left(-\frac{1}{16}\frac{PL}{EI}\right)\left(\frac{L}{12}\right) = -\frac{1}{384}\frac{PL^2}{EI}$$

$$A_4 = \frac{1}{2}\left(-\frac{1}{16}\frac{PL}{EI}\right)\left(\frac{L}{4}\right) = -\frac{1}{128}\frac{PL^2}{EI}$$

Place reference tangent at A.

$$\begin{aligned} t_{B/A} &= \left(\frac{1}{32}\frac{PL^2}{EI}\right)\left(\frac{2L}{3}\right) \\ &\quad + \left(\frac{1}{96}\frac{PL^2}{EI}\right)\left(\frac{L}{2} - \frac{1}{3}\cdot\frac{L}{6}\right) \\ &\quad + \left(-\frac{1}{384}\frac{PL^2}{EI}\right)\left(\frac{L}{4} + \frac{1}{3}\frac{L}{12}\right) \\ &\quad + \left(-\frac{1}{128}\frac{PL^2}{EI}\right)\left(\frac{2}{3}\cdot\frac{L}{4}\right) \\ &= \frac{1}{48}\frac{PL^3}{EI} + \frac{1}{216}\frac{PL^3}{EI} \\ &\quad - \frac{5}{6912}\frac{PL^3}{EI} - \frac{1}{768}\frac{PL^3}{EI} \\ &= \frac{3}{128}\frac{PL^3}{EI} \end{aligned}$$

$$t_{D/A} = \left(\frac{1}{32}\frac{PL^2}{EI}\right)\left(\frac{1}{3}\cdot\frac{L}{2}\right) = \frac{1}{192}\frac{PL^3}{EI}$$

(a) Deflection at D.

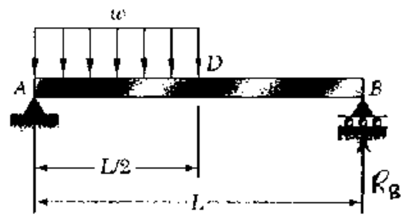
$$y_D = t_{D/A} - \frac{x_D}{L}t_{B/A} = \frac{1}{192}\frac{PL^3}{EI} - \frac{1}{2}\left(\frac{3}{128}\frac{PL^3}{EI}\right) = -\frac{5}{768}\frac{PL^3}{EI} \quad ◄$$

(b) Slope at A.

$$\theta_A = -\frac{t_{B/A}}{L} = -\frac{3}{128}\frac{PL^2}{EI} \quad ◄$$

Problem 9.128

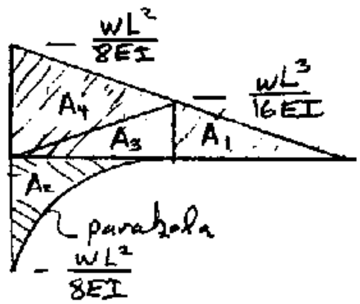
9.125 through *9.128* For the prismatic beam and loading shown, determine (*a*) the deflection at point *D*, (*b*) the slope at end *A*.



$$+\circlearrowleft \Sigma M_A = 0: \quad R_B L - \frac{wL}{2}\left(\frac{L}{4}\right) = 0$$

$$R_B = \frac{1}{8} w \uparrow$$

Draw $\frac{M}{EI}$ diagram by parts.



$$\frac{M_1}{EI} = \frac{R_B L}{EI} = \frac{wL^2}{8EI}$$

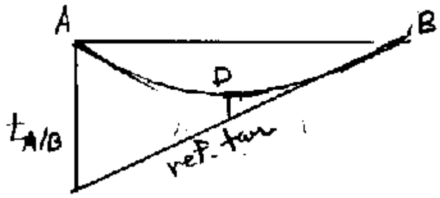
$$\frac{M_2}{EI} = -\frac{wL^3}{8EI}$$

$$A_1 = \left(\frac{1}{2}\right)\left(\frac{L}{2}\right)\frac{wL^2}{16EI} = \frac{wL^3}{64EI}$$

$$A_2 = \left(\frac{1}{3}\right)\left(\frac{L}{2}\right)\frac{wL^2}{8EI} = -\frac{wL^3}{48EI}$$

$$A_3 = \left(\frac{1}{2}\right)\left(\frac{L}{2}\right)\frac{wL^2}{16EI} = \frac{wL^3}{64EI}$$

$$A_4 = \left(\frac{1}{2}\right)\left(\frac{L}{2}\right)\frac{wL^2}{8EI} = \frac{wL^3}{32EI}$$



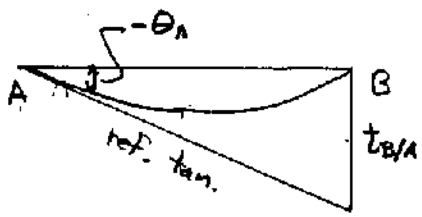
(a) Deflection at D.

Place reference tangent at B.

$$y_D = t_{D/B} - \frac{1}{2} t_{A/B}$$

$$t_{D/A} = \left(\frac{1}{3}\cdot\frac{L}{2}\right)A_1 = \frac{wL^4}{384EI}$$

$$t_{B/A} = \frac{L}{3}(A_1 + A_3 + A_4) + \left(\frac{L}{4}\cdot\frac{L}{2}\right)A_4 = \frac{7wL^4}{384EI}$$



$$y_D = \frac{wL^4}{384EI} - \frac{1}{2}\cdot\frac{7wL^4}{384EI} = -\frac{5wL^4}{768EI}$$

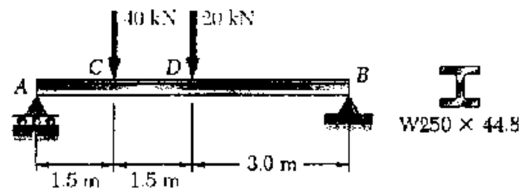
(b) Slope at A. Place reference tangent at A.

$$\theta_A = -\frac{1}{L} t_{B/A} = -\left(\frac{1}{L}\right)\left\{\left(\frac{2L}{3}\right)(A_1 + A_3 + A_4) + \left(L - \frac{1}{4}\cdot\frac{L}{2}\right)A_2\right\}$$

$$= -\frac{3wL^2}{128EI}$$

Problem 9.129

9.129 For the beam and loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *D*. Use $E = 200$ GPa.

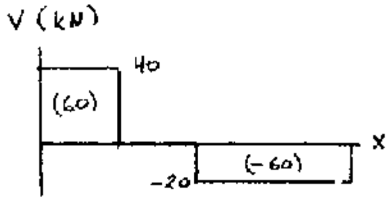


$E = 200 \times 10^9$ Pa

$I = 71.1 \times 10^6\ \text{mm}^4 = 71.1 \times 10^{-6}\ \text{m}^4$

$EI = (200 \times 10^9)(71.1 \times 10^{-6}) = 14.22 \times 10^6\ \text{N}\cdot\text{m}^2$

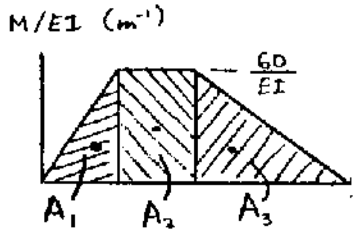
$= 14220\ \text{kN}\cdot\text{m}^2$



$+\circlearrowleft \Sigma M_B = 0: \quad -6R_A + (4.5)(40) + (3)(20) = 0$

$R_A = 40$ kN.

Draw shear and $\frac{M}{EI}$ diagrams.

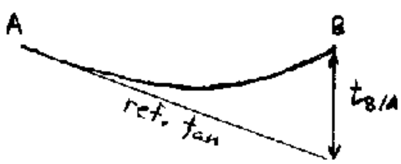


$A_1 = \frac{1}{2}\left(\frac{60}{EI}\right)(1.5) = \frac{45}{EI}$

$A_2 = \left(\frac{60}{EI}\right)(1.5) = \frac{90}{EI}$

$A_3 = \frac{1}{2}\left(\frac{60}{EI}\right)(3) = \frac{90}{EI}$

Place reference tangent at A.



$t_{B/A} = A_1(4.5 + 0.5) + A_2(3 + 0.75)$

$+ A_3(2.0) = \frac{742.5}{EI}$ m.

$t_{D/A} = A_1(1.5 + 0.5) + A_2(0.75)$

$= \frac{157.5}{EI}$ m

(a) Slope at A. $\quad \theta_A = -\frac{t_{B/A}}{L} = -\frac{742.5}{6EI} = -\frac{123.75}{EI} = -\frac{123.75}{14220}$

$= -8.70 \times 10^{-3}$ rad. ◀

(b) Deflection at D.

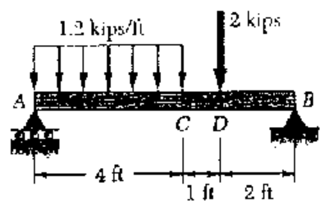
$y_D = t_{D/A} - \frac{x_D}{L} t_{B/A} = \frac{157.5}{EI} - \left(\frac{3}{6}\right)\left(\frac{742.5}{EI}\right) = -\frac{213.75}{EI}$

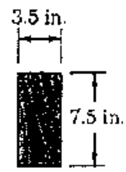
$= -\frac{213.75}{14220} = -15.03 \times 10^{-3}$ m

$= 15.03$ mm ↓ ◀

Problem 9.130

9.130 For the timber beam and loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *D*. Use $E = 1.5 \times 10^6$ psi.

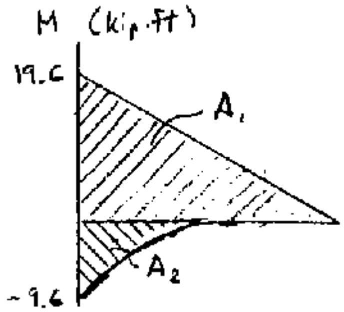




$I = \frac{1}{12}(3.5)(7.5)^3 = 123.047 \text{ in}^4$

$E = 1.5 \times 10^6 \text{ psi} = 1.5 \times 10^3 \text{ ksi}$

$EI = 184.57 \times 10^3 \text{ kip}\cdot\text{in}^2 = 1281.7 \text{ kip}\cdot\text{ft}^2$



$\circlearrowleft \Sigma M_A = 0 \qquad 7R_B - (2)(5) - (1.2)(4)(2) = 0$

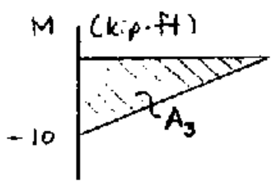
$R_B = 2.8 \text{ kip}$

Draw bending moment diagram by parts.

$M_1 = (2.8)(7) = 19.6 \text{ kip}\cdot\text{ft}$

$M_2 = -(1.2)(4)(2) = -9.6 \text{ kip}\cdot\text{ft}$

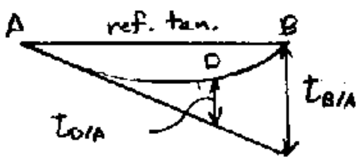
$M_3 = -(2)(5) = -10 \text{ kip}\cdot\text{ft}$



$A_1 = \frac{1}{2}(7)(19.6) = 68.6 \text{ kip}\cdot\text{ft}^2$

$A_2 = \frac{1}{3}(4)(-9.6) = -12.8 \text{ kip}\cdot\text{ft}^2$

$A_3 = \frac{1}{2}(5)(-10) = -25.0 \text{ kip}\cdot\text{ft}^2$



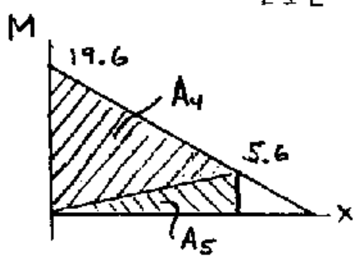
Draw reference tangent at A.

$\theta_A = -\frac{t_{B/A}}{L}$

$y_D = t_{D/A} - \frac{x_D}{L} t_{B/A}$

$EI\, t_{B/A} = A_1\left(7 - \frac{7}{3}\right) + A_2(7-1) + A_3\left(7 - \frac{5}{3}\right) = 110.0 \text{ kip}\cdot\text{ft}^3$

(a) $\quad \theta_A = -\frac{EI\, t_{B/A}}{EI\, L} = -\frac{110.0}{(1281.7)(7)} = -12.26 \times 10^{-3} \text{ rad}$



$A_4 = \frac{1}{2}(19.6)(5) = 49 \text{ kip}\cdot\text{ft}^2$

$A_5 = \frac{1}{2}(5.6)(5) = 14 \text{ kip}\cdot\text{ft}^2$

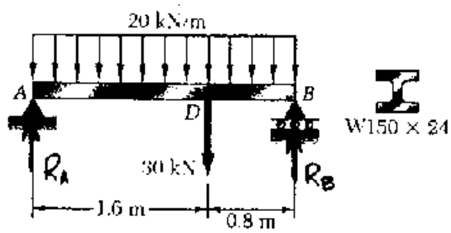
$EI\, t_{D/A} = A_4\left(5 - \frac{5}{3}\right) + A_5\left(\frac{5}{3}\right) + A_2(5-1) + A_3\left(5 - \frac{5}{3}\right) = 52.133 \text{ kip}\cdot\text{ft}^3$

$EI y_D = 52.133 - \frac{5}{7}(110.0) = -26.438 \text{ kip}\cdot\text{ft}^3$

(b) $\quad y_D = -\frac{26.438}{1281.7} = -20.63 \times 10^{-3} \text{ ft} = 0.248 \text{ in.} \downarrow$

Problem 9.131

9.131 For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at point *D*. Use $E = 200$ GPa.



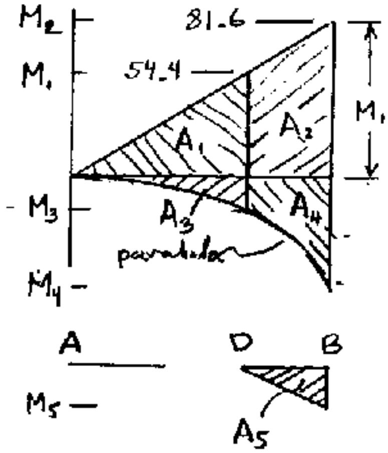
Units: Forces in kN. Lengths in meters.

For W150×24 $\quad I = 13.4 \times 10^6 \text{ mm}^4$

$= 13.4 \times 10^{-6} \text{ m}^4$

$EI = (200 \times 10^9)(13.4 \times 10^{-6}) = 2.68 \times 10^6 \text{ N}\cdot\text{m}^2$

$= 2680 \text{ kN}\cdot\text{m}^2$



$+\circlearrowleft \Sigma M_B = 0: \; -2.4 R_A + (0.8)(30) + (1.2)(2.4)(20) = 0$

$R_A = 34 \text{ kN} \uparrow$

Draw bending moment diagram by parts.

$M_1 = (1.6)(34) = 54.4 \text{ kN}\cdot\text{m}$
$M_2 = (2.4)(34) = 81.6 \text{ kN}\cdot\text{m}$
$M_3 = -\frac{1}{2}(20)(1.6)^2 = -25.6 \text{ kN}\cdot\text{m}$
$M_4 = -\frac{1}{2}(20)(2.4)^2 = -57.6 \text{ kN}\cdot\text{m}$
$M_5 = -(0.8)(30) = -24 \text{ kN}\cdot\text{m}$

$A_1 = \frac{1}{2}(1.6)(54.4) = 43.52 \text{ kN}\cdot\text{m}^2$
$A_1 + A_2 = \frac{1}{2}(2.4)(81.6) = 97.92 \text{ kN}\cdot\text{m}^2$
$A_3 = \frac{1}{3}(1.6)(-25.6) = -13.6533 \text{ kN}\cdot\text{m}^2$
$A_3 + A_4 = \frac{1}{3}(2.4)(-57.6) = -46.08 \text{ kN}\cdot\text{m}^2$
$A_5 = \frac{1}{2}(0.8)(-24) = -9.6 \text{ kN}\cdot\text{m}^2$

(a) Slope at θ. $\quad$ Place reference tangent at A. $\quad \theta_A = -\frac{1}{L} t_{B/A}$

$$t_{B/A} = \frac{1}{EI}\left\{(A_1 + A_2)(\tfrac{1}{3})(2.4) + (A_3 + A_4)(\tfrac{1}{4})(2.4) + A_5(\tfrac{1}{3})(0.8)\right\}$$

$$= \frac{48.128}{2680} = 17.9582 \times 10^{-3} \text{ m}$$

$$\theta_A = -\frac{17.9582 \times 10^{-3}}{2.4} = -7.48258 \times 10^{-3} \qquad \theta_A = 7.48 \times 10^{-3} \text{ rad}$$

(b) Deflection at point D. $\quad y_D = t_{D/A} + \theta_A x_D$

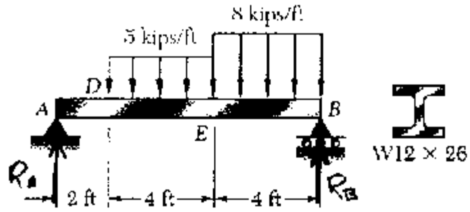
$$t_{D/A} = \frac{1}{EI}\left\{A_1(\tfrac{1}{3})(1.6) + A_2(\tfrac{1}{4})(1.6)\right\} = \frac{17.7493}{2680} = 6.62289 \times 10^{-3} \text{ m}$$

$$y_D = 6.62289 \times 10^{-3} + (-7.48258 \times 10^{-3})(1.6) = -5.3492 \times 10^{-3} \text{ m}$$

$$y_D = 5.35 \text{ mm} \downarrow$$

Problem 9.132

9.132 For the beam and loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *E*. Use $E = 29 \times 10^6$ psi.

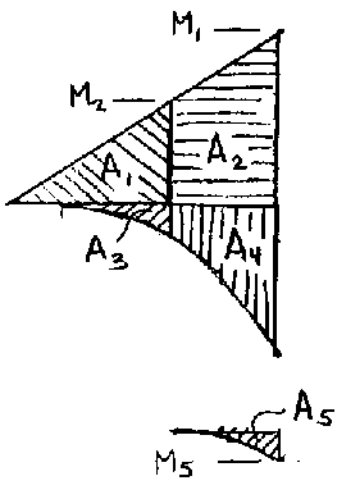


Units: Forces in kips, lengths in ft.

For W 12 × 26, $I = 204 \text{ in}^4$

$$EI = (29 \times 10^6)(204) = 5.916 \times 10^9 \text{ lb}\cdot\text{in}^2 = 41083 \text{ kip}\cdot\text{ft}^2$$

$$+\circlearrowleft \Sigma M_B = 0: \quad -10 R_A + (6)(4)(5) + (2)(4)(8) = 0 \qquad R_A = 18.4 \text{ kips} \uparrow$$



Consider loading as 5 kips/ft from D to B plus 3 kips/ft from E to B. Draw bending moment diagram by parts.

$$M_1 = 10 R_A = 184 \text{ kip}\cdot\text{ft}$$
$$M_2 = 6 R_A = 110.4 \text{ kip}\cdot\text{ft}$$
$$M_3 = -\tfrac{1}{2}(5)(8)^2 = -160 \text{ kip}\cdot\text{ft}$$
$$M_4 = -\tfrac{1}{2}(5)(4)^2 = -40 \text{ kip}\cdot\text{ft}$$
$$M_5 = -\tfrac{1}{2}(3)(4)^2 = -24 \text{ kip}\cdot\text{ft}$$

$$A_1 + A_2 = \tfrac{1}{2}(10)(184) = 920 \text{ kip}\cdot\text{ft}^2$$
$$A_1 = \tfrac{1}{2}(6)(110.4) = 331.2 \text{ kip}\cdot\text{ft}^2$$
$$A_3 + A_4 = \tfrac{1}{3}(8)(-160) = -426.667 \text{ kip}\cdot\text{ft}^2$$
$$A_3 = \tfrac{1}{3}(4)(-40) = -53.333 \text{ kip}\cdot\text{ft}^2$$
$$A_5 = \tfrac{1}{3}(4)(-24) = -32 \text{ kip}\cdot\text{ft}^2$$

(a) Slope at A. $\quad y_B = y_A + \theta_A L + t_{B/A}. \quad y_A = y_B = 0$

$$\theta_A = -t_{B/A}/L$$

$$t_{B/A} = \frac{1}{EI}\left\{(A_1 + A_2)(\tfrac{1}{3})(10) + (A_3 + A_4)(\tfrac{1}{4})(8) + (A_5)(\tfrac{1}{4})(4)\right\}$$
$$= \frac{2181.33}{41083} = 53.096 \times 10^{-3} \text{ ft}$$

$$\theta_A = -\frac{53.096 \times 10^{-3}}{10} = -5.3096 \times 10^{-3} \qquad \theta_A = -5.31 \text{ rad} \circlearrowright$$

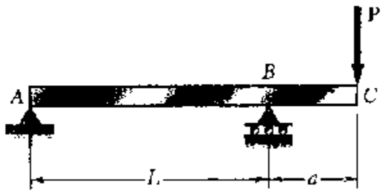
(b) Deflection at E. $\quad y_E = x_E \theta_A + t_{E/A}$

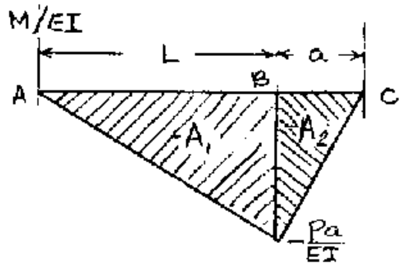
$$t_{E/A} = \frac{1}{EI}\left\{(A_1)(\tfrac{1}{3})(6) + (A_3)(\tfrac{1}{4})(4)\right\} = \frac{609.067}{41083} = 14.8253 \times 10^{-3} \text{ ft}$$

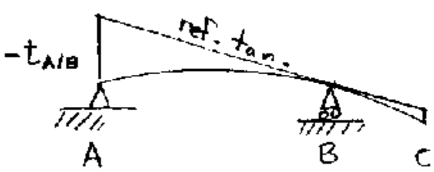
$$y_E = (6)(-5.3096 \times 10^{-3}) + 14.8253 \times 10^{-3} = -17.0323 \times 10^{-3} \text{ ft}$$
$$= 0.204 \text{ in.} \downarrow$$

Problem 9.133

9.133 For the beam and loading shown, determine (*a*) the slope at point *C*, (*b*) the deflection at point *C*.







$$A_1 = -\frac{PaL}{2EI},$$

$$A_2 = -\frac{Pa^2}{2EI}$$

$$t_{A/B} = A_1\left(\frac{2}{3}L\right) = -\frac{PaL^2}{3EI}$$

$$\theta_B = \frac{t_{A/B}}{L} = -\frac{PaL}{3EI}$$

(a) Slope at C. $\theta_C = \theta_B + \theta_{C/B}$

$$\theta_C = -\frac{PaL}{3EI} - \frac{Pa^2}{2EI}$$

$$= -\frac{Pa(2L+3a)}{6EI}$$

$$\theta_C = \frac{Pa(2L+3a)}{6EI} \ \searrow$$

Deflection at point C.

$$y_C = a\theta_B + t_{C/B}$$

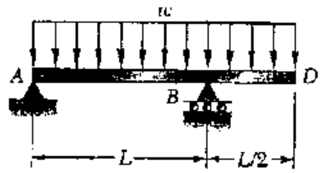
$$y_C = -\frac{Pa^2L}{3EI} + \left(-\frac{Pa^2}{2EI}\right)\left(\frac{2}{3}a\right)$$

$$= -\frac{Pa^2(L+a)}{3EI}$$

$$y_C = \frac{Pa^2(L+a)}{3EI} \downarrow$$

Problem 9.134

9.134 For the beam and loading shown, determine (*a*) the slope at point A, (*b*) the deflection at point D.



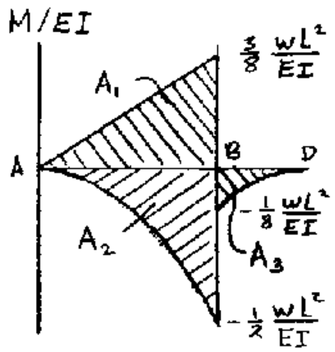
$$+\circlearrowleft \Sigma M_B = 0: \quad -R_A L + \left(\tfrac{3}{2} wL\right)\left(\tfrac{1}{4} L\right) = 0 \qquad R_A = \tfrac{3}{8} wL$$

Draw $\dfrac{M}{EI}$ diagram by parts.

$$A_1 = \tfrac{1}{2}\left(\tfrac{3}{8}\dfrac{wL^2}{EI}\right)L = \tfrac{3}{16}\dfrac{wL^3}{EI}$$

$$A_2 = -\tfrac{1}{3}\left(\tfrac{1}{2}\dfrac{wL^2}{EI}\right)L = -\tfrac{1}{6}\dfrac{wL^3}{EI}$$

$$A_3 = -\tfrac{1}{3}\left(\tfrac{1}{8}\dfrac{wL^2}{EI}\right)\dfrac{L}{2} = -\tfrac{1}{48}\dfrac{wL^3}{EI}$$

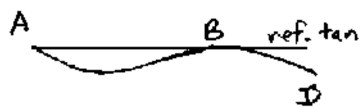


Place reference tangent at A.

$$t_{B/A} = A_1 \frac{L}{3} + A_2 \frac{L}{4}$$

$$= \frac{1}{16}\frac{wL^4}{EI} - \frac{1}{24}\frac{wL^4}{EI} = \frac{1}{48}\frac{wL^4}{EI}$$

(a) Slope at A.



$$\theta_A = -\frac{t_{B/A}}{L} = -\frac{1}{48}\frac{wL^3}{EI}$$

$$t_{D/A} = A_1\left(\frac{L}{3}+\frac{L}{2}\right) + A_2\left(\frac{L}{4}+\frac{L}{2}\right) + A_3\left(\frac{3}{4}\cdot\frac{L}{2}\right)$$

$$= \frac{5}{32}\frac{wL^4}{EI} - \frac{1}{8}\frac{wL^4}{EI} - \frac{1}{128}\frac{wL^4}{EI} = \frac{3}{128}\frac{wL^4}{EI}$$

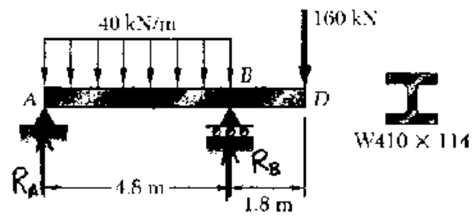
(b) Deflection at D.

$$y_D = t_{D/A} - \frac{x_D}{L} t_{B/A} = \frac{3}{128}\frac{wL^4}{EI} - \frac{3}{2}\cdot\frac{1}{48}\frac{wL^4}{EI} = -\frac{1}{128}\frac{wL^4}{EI}$$

$$y_D = \frac{1}{128}\frac{wL^4}{EI} \downarrow$$

Problem 9.135

9.135 For the beam and loading shown, determine (*a*) the slope at point *B*, (*b*) the deflection at point *D*. Use $E = 200$ GPa.



Units: Forces in kN; lengths in meters.

$I = 462 \times 10^6 \text{ mm}^4 = 462 \times 10^{-6} \text{ m}^4$

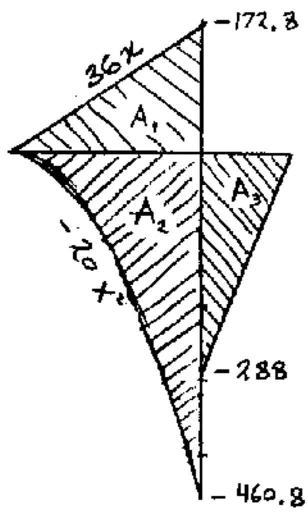
$EI = (200 \times 10^9)(462 \times 10^{-6})$

$\quad = 92.4 \times 10^6 \text{ N}\cdot\text{m}^2 = 92400 \text{ kN}\cdot\text{m}^2$

$+\circlearrowleft \Sigma M_B = 0: \quad -4.8 R_A + (40)(4.8)(2.4) - (160)(1.8) = 0$

$R_A = 36 \text{ kN}$

Draw bending moment diagram by parts.



$A_1 = \frac{1}{2}(4.8)(172.8) = 414.72 \text{ kN}\cdot\text{m}^2$

$A_2 = \frac{1}{3}(4.8)(-460.8) = -737.28 \text{ kN}\cdot\text{m}^2$

$A_3 = \frac{1}{2}(1.8)(-288) = -259.2 \text{ kN}\cdot\text{m}^2$

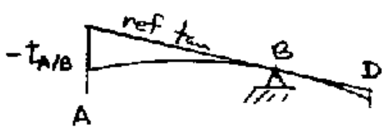
Place reference tangent at B.

(a) Slope at B. $\qquad y_A = y_B - L\theta_B + t_{A/B}$

$$\theta_B = \frac{t_{B/A}}{L} = \frac{1}{EIL}\left\{ A_1\left(\tfrac{2}{3}\right)(4.8) + A_2\left(\tfrac{3}{4}\right)(4.8) \right\}$$

$$= \frac{-1327.104}{(92400)(4.8)} = -2.9922 \times 10^{-3}$$

$\theta_B = 2.99 \times 10^{-3} \text{ rad}$ ◄



(b) Deflection at D. $\qquad y_D = y_B + a\theta_B + t_{D/B}$

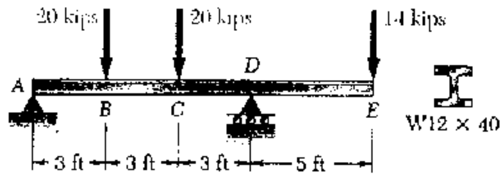
$$= 0 + (1.8)(-2.9922 \times 10^{-3}) - \frac{1}{EI}\left\{ A_3\left(\tfrac{2}{3}\right)(1.8) \right\}$$

$$= -5.3860 \times 10^{-3} - \frac{311.04}{92400}$$

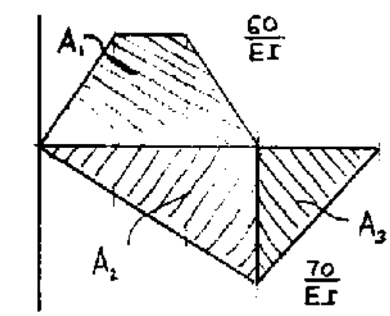
$= -8.75 \times 10^{-3}$ m $\qquad y_D = 8.75$ mm ↓ ◄

Problem 9.136

9.136 For the beam and loading shown, determine (*a*) the slope at point *D*, (*b*) the deflection at point *E*. Use $E = 29 \times 10^6$ psi.



Draw bending moment diagram as the sum of two diagrams: one for the pair of 20 kip loads and one for the 14 kip load.



$A_1 = [2 \cdot \frac{1}{2}(3)(60) + (3)(60)]/EI = 360/EI$

$A_2 = \frac{1}{2}(9)(70)/EI = -315/EI$

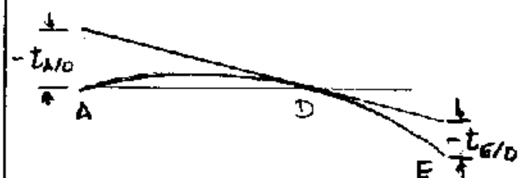
$A_3 = \frac{1}{2}(5)(70) = -175/EI$

$E = 29 \times 10^6$ psi $= 29 \times 10^3$ ksi

$I = 310$ in^4

$EI = (29 \times 10^3)(310) = 8.99 \times 10^6$ kip·in^2

$= 62430$ kip·ft^2



Place reference tangent at D.

$t_{A/D} = A_1(4.5) + A_2(6) = -270/EI$ ft.

(a) Slope at D. $\theta_D = \frac{t_{A/D}}{L} = -\frac{270}{9EI} = -\frac{30}{EI}$

$= -0.48054 \times 10^{-3}$ $= 0.481 \times 10^{-3}$ rad ◄

$t_{E/D} = A_3(\frac{2}{3} \cdot 5) = -583.333/EI = -9.3438 \times 10^{-3}$ ft

(b) Deflection at E.

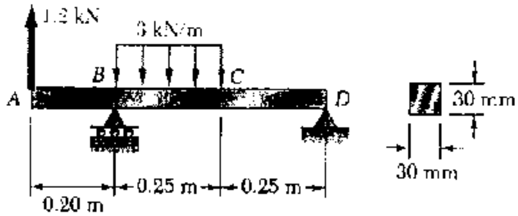
$y_E = L_{DE}\theta_D + t_{E/D}$

$= -(5)(0.48054 \times 10^{-3}) - 9.3438 \times 10^{-3} = -11.75 \times 10^{-3}$ ft

$= 0.1410$ in. ↓ ◄

Problem 9.137

9.137 Knowing the beam *AD* is made of a solid steel bar, determine the (a) slope at point *B*, (c) the deflection at point *A*. Use $E = 200$ GPa.



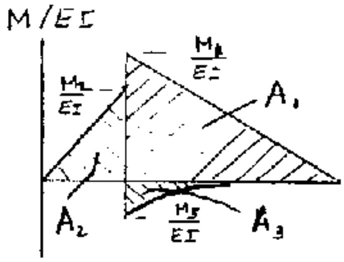
$E = 200 \times 10^9$ Pa

$I = \frac{1}{12}(30)(30)^3 = 67.5 \times 10^3 \text{ mm}^4$

$= 67.5 \times 10^{-9} \text{ m}^4$

$EI = (200 \times 10^9)(67.5 \times 10^{-9}) = 13500 \text{ N}\cdot\text{m}^2$

$= 13.5 \text{ kN}\cdot\text{m}^2$



$\circlearrowleft \Sigma M_B = 0 \quad -(0.2)(1.2) - (3)(0.25)(0.125) + .5R_D = 0$

$R_D = 0.6675$ kN

Draw $\frac{M}{EI}$ diagram by parts.

$M_1 = (0.6675)(0.5) = 0.33375$ kN·m

$M_2 = (1.2)(0.2) = 0.240$ kN·m

$M_3 = -\frac{1}{2}(3)(0.25)^2 = -0.09375$ kN·m

$A_1 = \frac{1}{2}(0.33375)(0.5)/EI = 0.0834375/EI$

$A_2 = \frac{1}{2}(0.240)(0.2)/EI = 0.024/EI$

$A_3 = \frac{1}{3}(-0.09375)(0.25)/EI = -0.0078125/EI$

Place reference tangent at B.

$t_{D/B} = A_1(\frac{2}{3}\cdot 0.5) + A_3(\frac{3}{4}\cdot(0.25) + 0.25) = 0.024395/EI$

(a) Slope at B. $\quad \theta_B = -\frac{t_{D/B}}{L} = -\frac{0.024395}{0.5\ EI} = -\frac{0.048789}{EI}$

$= -3.6140 \times 10^{-3}$ $\qquad 3.61 \times 10^{-3}$ rad ◄

$t_{A/B} = A_2(\frac{2}{3}(0.20)) = 0.0032/EI = 0.23704 \times 10^{-3}$ m

(b) Deflection at A.

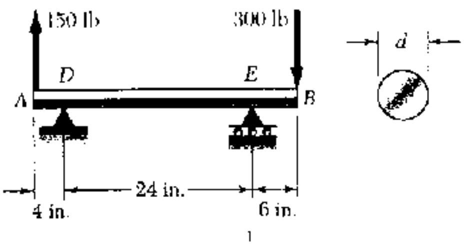
$y_A = t_{A/B} - L_{AB}\theta_B$

$= 0.23704 \times 10^{-3} - (0.2)(-3.6140 \times 10^{-3}) = 0.960 \times 10^{-3}$ m

$= 0.960$ mm ↑ ◄

Problem 9.138

9.138 Knowing that the beam AB is made of a solid steel rod of diameter $d = 0.75$ in., determine for the loading shown (a) the slope at point D, (b) the deflection at point A. Use $E = 29 \times 10^6$ psi.

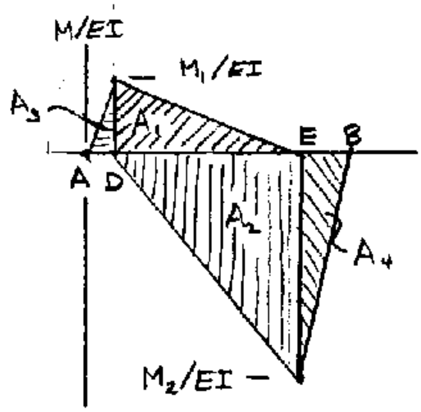


Units: Forces in lb; lengths in inches.

$$c = \frac{1}{2} = \frac{1}{2}(0.75) = 0.375 \text{ in.}$$

$$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(0.375)^4 = 0.0155316 \text{ in}^4$$

$$EI = (29\times10^6)(0.0155316) = 450.4\times10^3 \text{ lb}\cdot\text{in}^2$$



Draw $\frac{M}{EI}$ diagram by parts by considering the bending moment diagram due to each of the applied loads.

$$\frac{M_1}{EI} = \frac{(150)(4)}{450.4\times10^3} = 1.33215\times10^{-3} \text{ in}^{-1}$$

$$\frac{M_2}{EI} = -\frac{(300)(6)}{450.4\times10^3} = -3.99645\times10^{-3} \text{ in}^{-1}$$

$$A_1 = \frac{1}{2}(24)(1.33215\times10^{-3}) = 15.9858\times10^{-3}$$

$$A_2 = \frac{1}{2}(24)(-3.99645\times10^{-3}) = -47.9574\times10^{-3}$$

$$A_3 = \frac{1}{2}(4)(1.33215\times10^{-3}) = 2.6643\times10^{-3}$$

Place reference tangent at D.

(a) Slope at point D. $\quad y_E = y_D + L\theta_D + t_{E/D} \qquad \theta_D = -t_{E/D}/L$

$$t_{E/A} = 16A_1 + 8A_2 = -127.8864\times10^{-3} \text{ in.}$$

$$\theta_D = -\frac{-127.8864\times10^{-3}}{24} = 5.3286\times10^{-3} \qquad \theta_D = 5.33\times10^{-3} \text{ rad} \measuredangle$$

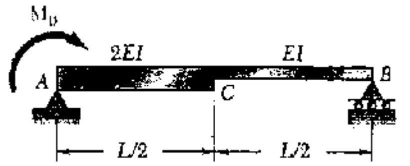
(b) Deflection at A. $\quad y_A = y_D - a\theta_D + t_{A/D} = t_{A/D} - a\theta_D$

$$y_A = A_3\left(\tfrac{2}{3}\right)(4) - (4)(5.3286\times10^{-3}) = -14.21\times10^{-3} \text{ in.}$$

$$y_A = -0.01421 \text{ in.} \downarrow$$

Problem 9.139

9.139 and 9.140 For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the slope at end *B*, (*c*) the deflection at the midpoint *C*.

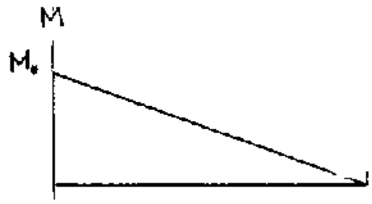


Draw bending moment and $\frac{M}{EI}$ diagrams.

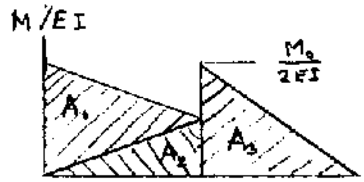
$$A_1 = \frac{1}{2}\left(\frac{M_0}{2EI}\right)\left(\frac{L}{2}\right) = \frac{1}{8}\frac{M_0 L}{EI}$$

$$A_2 = \frac{1}{2}\left(\frac{M_0}{4EI}\right)\left(\frac{L}{2}\right) = \frac{1}{16}\frac{M_0 L}{EI}$$

$$A_3 = \frac{1}{2}\left(\frac{M_0}{2EI}\right)\left(\frac{L}{2}\right) = \frac{1}{8}\frac{M_0 L}{EI}$$



Place reference tangent at A.



$$t_{B/A} = A_1\left(\frac{L}{2} + \frac{2}{3}\frac{L}{2}\right) + A_2\left(\frac{L}{2} + \frac{1}{3}\frac{L}{2}\right) + A_3\left(\frac{2}{3}\frac{L}{2}\right)$$

$$= \left(\frac{1}{8}\frac{M_0 L}{EI}\right)\left(\frac{5}{6}L\right) + \left(\frac{1}{16}\frac{M_0 L}{EI}\right)\left(\frac{2}{3}L\right) + \left(\frac{1}{8}\frac{M_0 L}{EI}\right)\left(\frac{1}{3}L\right)$$

$$= \frac{3}{16}\frac{M_0 L^2}{EI}$$

(a) Slope at A.

$$\theta_A = -\frac{t_{B/A}}{L} = -\frac{3}{16}\frac{ML}{EI}$$

(b) Slope at B.

$$\theta_B = \theta_A + \theta_{B/A} = \theta_A + A_1 + A_2 + A_3$$

$$= -\frac{3}{16}\frac{M_0 L}{EI} + \frac{1}{8}\frac{M_0 L}{EI} + \frac{1}{16}\frac{M_0 L}{EI} + \frac{1}{8}\frac{M_0 L}{EI}$$

$$= \frac{1}{8}\frac{M_0 L}{EI}$$

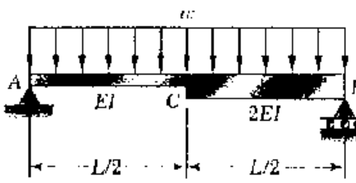
$$t_{C/A} = A_1\left(\frac{2}{3}\frac{L}{2}\right) + A_2\left(\frac{1}{3}\frac{L}{2}\right) = \left(\frac{1}{8}\frac{M_0 L}{EI}\right)\left(\frac{1}{3}L\right) + \left(\frac{1}{16}\frac{M_0 L}{EI}\right)\left(\frac{1}{6}L\right) = \frac{5}{96}\frac{M_0 L^2}{EI}$$

(c) Defection at C.

$$y_C = t_{C/A} + \frac{L}{2}\theta_A = \frac{5}{96}\frac{M_0 L^2}{EI} - \frac{3}{32}\frac{M_0 L^2}{EI} = -\frac{1}{24}\frac{M_0 L^2}{EI} = \frac{1}{24}\frac{M_0 L^2}{EI}\downarrow$$

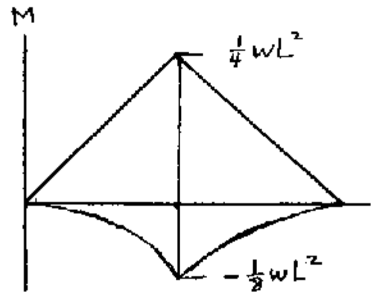
Problem 9.140

9.139 and 9.140 For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the slope at end *B*, (*c*) the deflection at the midpoint *C*.



Reactions: $R_A = R_B = \frac{1}{2}wL$

Draw bending moment and M/EI diagrams by parts as shown.



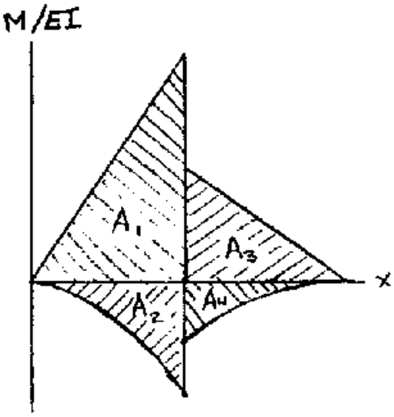
$$A_1 = \frac{1}{2}\cdot\frac{L}{2}\cdot\frac{wL^2}{4EI} = \frac{wL^3}{16EI}$$

$$A_2 = -\frac{1}{3}\cdot\frac{L}{2}\cdot\frac{wL^2}{8EI} = -\frac{wL^3}{48EI}$$

$$A_3 = \frac{1}{2}\cdot\frac{L}{2}\cdot\frac{wL^2}{8EI} = \frac{wL^3}{32EI}$$

$$A_4 = -\frac{1}{3}\cdot\frac{L}{2}\cdot\frac{wL^2}{16EI} = -\frac{wL^3}{96EI}$$

Place reference tangent at A.



(a) Slope at end A. $\quad y_B = y_A + L\theta_A + t_{B/A}$

$$\theta_A = -t_{B/A}/L$$

$$t_{B/A} = \left(\frac{L}{2}+\frac{L}{6}\right)A_1 + \left(\frac{L}{2}+\frac{L}{8}\right)A_2 + \frac{L}{3}A_3 + \frac{3L}{8}A_4$$

$$= \frac{wL^4}{EI}\left(\frac{1}{24} - \frac{5}{384} + \frac{1}{96} - \frac{1}{256}\right) = \frac{9wL^4}{256EI}$$

$$\theta_A = -\frac{9wL^4}{256EI}\cdot\frac{1}{L} = -\frac{9wL^3}{256EI} \qquad \theta_A = \frac{9wL^3}{256EI} ◄$$

(b) Slope at end B.

$$\theta_B = \theta_A + \theta_{B/A} = -\frac{9wL^3}{256EI} + A_1 + A_2 + A_3 + A_4$$

$$\theta_B = \frac{7wL^3}{256EI} \qquad \theta_B = \frac{7wL^3}{256EI} ◄$$

(c) Deflection at midpoint C. $\quad y_C = y_C + \frac{L}{2}\theta_A + t_{C/A}$

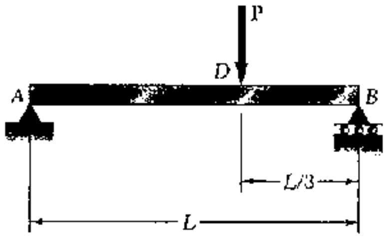
$$t_{C/A} = \left(\frac{L}{6}\right)A_1 + \left(\frac{L}{8}\right)A_2 = \frac{wL^4}{128EI}$$

$$y_C = 0 + \left(\frac{L}{2}\right)\left(-\frac{9wL^3}{256EI}\right) + \frac{wL^4}{128EI} = -\frac{5wL^4}{512EI} \qquad y_C = \frac{5wL^4}{512EI} \downarrow ◄$$

Problem 9.141

***9.141* through 9.144** For the beam and loading shown, determine the magnitude and location of the largest downward deflection.

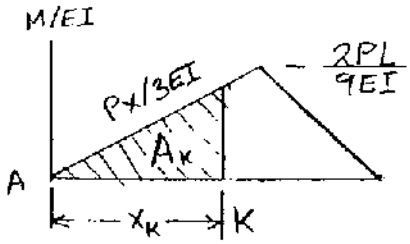
9.141 Beam and loading of Prob. 9.125



From the solution to Problem 9.125

$R_A = \frac{1}{3}P\uparrow, \quad \theta_A = -\frac{4PL^2}{81EI}$

Over portion AD, $\frac{M}{EI} = \frac{R_A x}{EI} = \frac{Px}{3EI}$



$$A_K = \frac{1}{2}x_K\left(\frac{Px_K}{3EI}\right) = \frac{Px_K^2}{6EI}$$

$$\theta_K = \theta_A + A_K = 0$$

$$-\frac{4PL^2}{81EI} + \frac{Px_K^2}{6EI} = 0$$

$$x_K^2 = \frac{8}{27}L^2$$

$$x_K = 0.54433\ L$$

$$A_K = -\theta_A = \frac{4PL^2}{81EI}$$

$$t_{A/K} = \left(\frac{2}{3}x_K\right)A_K = 0.01792\frac{PL^3}{EI}$$

$$y_A = y_K + t_{A/K} = 0$$

$$y_K = -t_{A/K} = -0.01792\frac{PL^3}{EI}$$

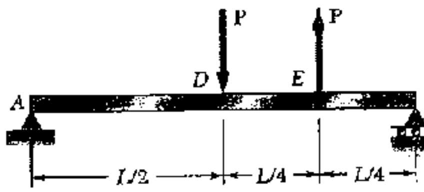
$y_K = 0.01792\frac{PL^3}{EI}\downarrow$ ◄

$x_K = 0.544\ L$ ◄

Problem 9.142

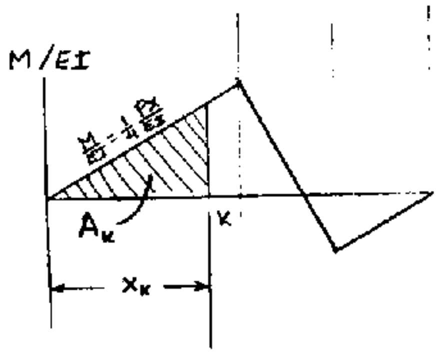
***9.141* through 9.144** For the beam and loading shown, determine the magnitude and location of the largest downward deflection.

9.142 Beam and loading of Prob. 9.127



Referring to the solution of Prob. 9.127

$$R_A = \frac{1}{4}P, \quad t_{B/A} = \frac{3}{128}\frac{PL^3}{EI}, \quad \theta_A = -\frac{3}{128}\frac{PL^2}{EI}$$



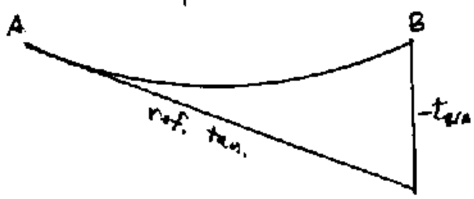
$$\theta_K = \theta_A + \theta_{K/A}$$

$$= -\frac{3}{128}\frac{PL^2}{EI} + A_K$$

$$= -\frac{3}{128}\frac{PL^2}{EI} + \frac{1}{2}\left(\frac{1}{4}\frac{Px_K}{EI}\right)x_K$$

$$= \frac{P}{EI}\left(-\frac{3}{128}L^2 + \frac{1}{8}x_K^2\right) = 0$$

$$x_K = \sqrt{\frac{3}{16}}\,L = \frac{1}{4}\sqrt{3}\,L = 0.433\,L$$



$$t_{K/A} = A_K\left(\frac{1}{3}x_K\right) = \frac{1}{2}\left(\frac{1}{4}\frac{Px_K^2}{EI}\right)\frac{x_K}{3}$$

$$= \frac{1}{24}\frac{Px_K^3}{EI} = \frac{\sqrt{3}}{512}\frac{PL^3}{EI}$$

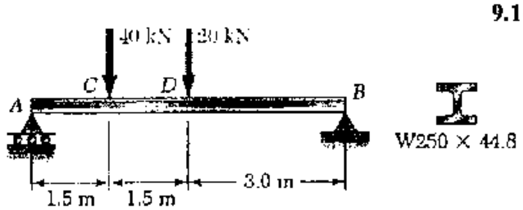
$$y_K = t_{K/A} - \frac{x_K}{L}t_{B/A} = \frac{\sqrt{3}}{512}\frac{PL^3}{EI} - \left(\frac{1}{4}\sqrt{3}\right)\frac{3}{128}\frac{PL^3}{EI} = -\frac{\sqrt{3}}{256}\frac{PL^3}{EI}$$

$$= 0.00677\frac{PL^3}{EI}\downarrow$$

Problem 9.143

***9.141* through 9.144** For the beam and loading shown, determine the magnitude and location of the largest downward deflection.

9.143 Beam and loading of Prob. 9.129



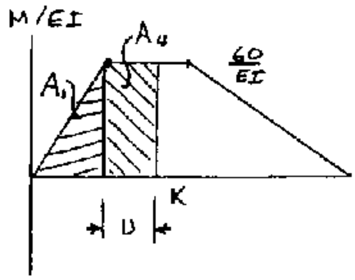
Referring to the solution to Prob. 9.129,

$$EI = 14220 \text{ kN}\cdot\text{m}^2$$

$$R_A = 40 \text{ kN}, \quad A_1 = \frac{45}{EI}$$

$$t_{B/A} = \frac{742.5}{EI} \text{ m}$$

$$\theta_A = -\frac{123.75}{EI}$$



Let K be the location of the maximum deflection. Assume that K lies between C and D.

$$\theta_K = \theta_A + \theta_{K/A}$$

$$= -\frac{123.75}{EI} + A_1 + A_4$$

$$= -\frac{123.75}{EI} + \frac{45}{EI} + \frac{60\,u}{EI} = 0$$

$$u = \frac{123.75-45}{60} = 1.3125 \text{ m.}$$

$$x_K = 1.5 + u = 2.8125 \text{ m}$$

$$t_{K/A} = A_1\,(u + 0.5) + A_4\left(\tfrac{1}{2}u\right)$$

$$= \frac{45}{EI}(1.8125) + \frac{(60)(1.3125)(\frac{1}{2})(1.3125)}{EI} = \frac{133.242}{EI}$$

$$y_K = t_{K/A} - \frac{x_K}{L}\,t_{B/A}$$

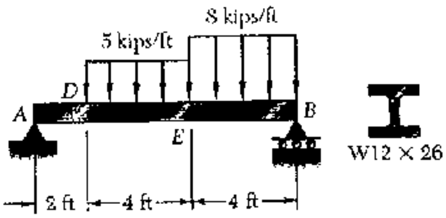
$$= \frac{133.242}{EI} - \frac{2.8125}{6}\left(\frac{742.5}{EI}\right) = -\frac{214.80}{EI} = -\frac{214.80}{14220}$$

$$= -15.11\times10^{-3} \text{ m} = 1.511 \text{ mm}\downarrow$$

Problem 9.144

9.141 through 9.144 For the beam and loading shown, determine the magnitude and location of the largest downward deflection.

9.144 Beam and loading of Prob. 9.132



From the solution to Problem 9.132

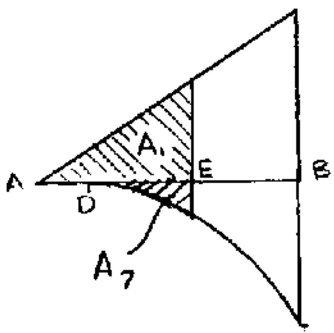
$EI = 41083 \text{ kip-ft}^2$

$R_A = 18.4 \text{ kips}$

$A_1 = 331.2 \text{ kip-ft}^2$

$A_3 = -53.333 \text{ kip-ft}^2$

$\theta_A = -5.3096 \times 10^{-3}$



Slope at E. $\quad \theta_E = \theta_A + \theta_{E/A}$

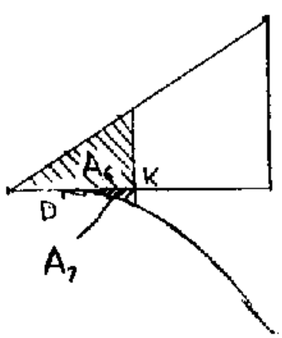
$$\theta_{E/A} = \frac{1}{EI}\{A_1 + A_3\} = \frac{278.767}{41083} = 6.7855 \times 10^{-3}$$

$\theta_E = 1.4759 \times 10^{-3}$

Since $\theta_E > 0$, the point K of zero slope lies to the left of point E. Let x_K be the coordinate of point K.

$$A_6 = \frac{1}{2} R_A x_K^2 = 9.2\, x_K^2$$

$$A_7 = -\frac{1}{6}(5)(x_K - 2)^3$$



$$\theta_K = \theta_A + \theta_{K/A} = \theta_A + \frac{1}{EI}\{A_6 + A_7\} = 0$$

$$A_6 + A_7 + EI\,\theta_A = 0$$

$$f(x_K) = 9.2\, x_K^2 - \frac{5}{6}(x_K - 2)^3 - 218.134 = 0$$

$$\frac{df}{dx_K} = 18.4\, x_K - 2.5\,(x_K - 2)^2$$

Solve for x_K by iteration. $\quad x_K = (x_K)_0 - \dfrac{f}{df/dx_K}$

x_K	5	5.1473	5.1525
f	−10.634	−0.362	0.001
df/dx_K	72.2	70.131	

$x_K = 5.1525$ ft

$A_6 = 244.244 \text{ kip-ft}^2, \qquad A_7 = -26.108 \text{ kip-ft}^2$

Maximum deflection. $\quad y_A \pm y_K + t_{A/K} = 0 \qquad y_K = -t_{A/K}$

$$\bar{x}_6 = \frac{2}{3}x_K, \qquad \bar{x}_7 = 2 + \frac{3}{4}(x_K - 2) = \frac{3x_K + 2}{4}$$

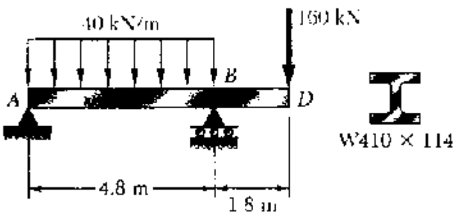
$x_K = 5.15$ ft ◄

$$y_7 = -\frac{1}{EI}\{A_6\bar{x}_6 + A_7\bar{x}_7\} = -\frac{725.033}{41083} = -17.648 \times 10^{-3} \text{ ft}$$

$y_K = 0.212$ in. ↓ ◄

Problem 9.145

9.145 For the beam and loading of Prob. 9.135, determine the largest upward deflection in span *AB*.



Units: Forces in kN. Lengths in meters.

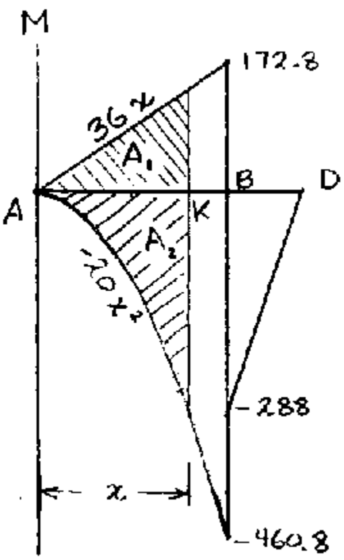
$I = 462 \times 10^6\ mm^4 = 462 \times 10^{-6}\ m^4$

$EI = (200 \times 10^9)(462 \times 10^{-6})$

$= 92.4 \times 10^6\ N\cdot m^2 = 92400\ kN\cdot m$

$+\circlearrowleft M_B = 0: \quad -4.8 R_A + (40)(4.8)(2.4) - (160)(1.8) = 0$

$R_A = 36\ kN$



$A_1 = \frac{1}{2}x(36x) = 18x^2$

$A_2 = \frac{1}{3}x(-20x^2) = -\frac{20}{3}x^3$

Place reference tangent at A.

$y_B = y_A + L\theta_A + t_{B/A} = 0$

$\theta_A = -\frac{t_{B/A}}{L}$

$(A_1)_B = (18)(4.8)^2 = 414.72\ kN\text{-}m^2$

$(A_2)_B = (\frac{20}{3})(4.8)^3 = -737.28\ kN\cdot m^2$

$\theta_A = -\frac{1}{EIL}\left\{(A_1)_B(\frac{1}{3})(4.8) + (A_2)_B(\frac{1}{4})(4.8)\right\}$

$= -\frac{-221.184}{(92400)(4.8)} = 0.49870 \times 10^{-3}$

Locate point K of maximum deflection. $\quad \theta_K = \theta_A + \theta_{K/A} = 0$

$EI\theta_A + A_1 + A_2 = 0$

$f = 46.08 + 18x_K^2 - \frac{20}{3}x_K^3 = 0 \qquad \frac{df}{dx} = 36x_K - 20x_K^2$

Solve by iteration.

$x_K = (x_K)_0 - \frac{f}{df/dx}$

x_K	3	3.39	3.327	3.3251	3.32514 ←
df/dx	-72	-107.8	-101.6	-101.42	
f	28.08	-6.78	-0.188	0.005	

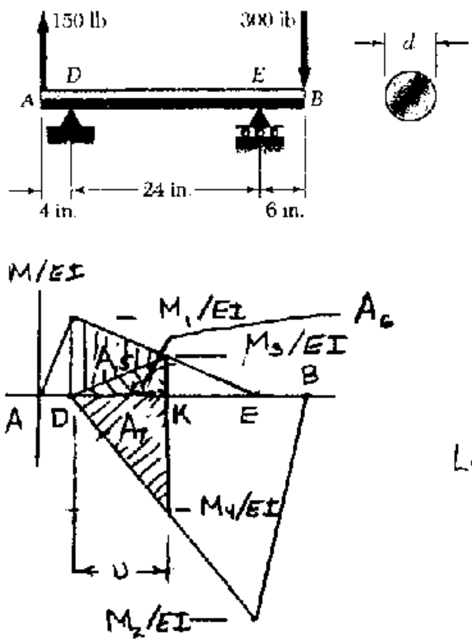
Place reference tangent at K. $\quad y_A = y_K + t_{A/K}$

$y_A - y_K = -t_{A/K} = -\frac{1}{EI}\left\{(A_1)(\frac{2}{3}x_K) + A_2(\frac{3}{4}x_K)\right\} = -\frac{1}{EI}\left\{12x_K^3 - 5x_K^4\right\}$

$= -\frac{170.064}{92400} = -1.841 \times 10^{-3}\ m \qquad y_K = 1.841\ mm \uparrow$ ◄

Problem 9.146

9.146 For the beam and loading of Prob. 9.138, determine the largest upward deflection in span *DE*.



Units: Forces in lb; lengths in inches.

From the solution to Problem 9.138

$$EI = 450.4\times10^3 \text{ lb}\cdot\text{in}$$

$$\frac{M_1}{EI} = 1.33215\times10^{-3} \text{ in}^{-1}$$

$$\frac{M_2}{EI} = -3.99645\times10^{-3} \text{ in}^{-1}$$

$$\theta_D = 5.3286\times10^{-3}$$

Location of maximum deflection.

$$\frac{M_3}{EI} = \frac{M_1}{EI}\left(1-\frac{u}{24}\right)$$

$$\frac{M_4}{EI} = \frac{M_2}{EI}\frac{u}{24}$$

$$A_5 = \frac{1}{2}\frac{M_1}{EI}\cdot u = 0.666075\times10^{-3}\,u$$

$$A_6 = \frac{1}{2}\frac{M_3}{EI}\left(\frac{u}{24}\right) = 0.666075\times10^{-3}\left(1-\frac{u}{24}\right)u$$

$$A_7 = \frac{1}{2}\frac{M_4}{EI}\frac{u}{24} = -1.998225\times10^{-3}\left(\frac{u}{24}\right)u$$

$$\theta_K = \theta_D + A_5 + A_6 + A_7 = 0 \qquad \text{Multiply by } 10^3.$$

$$5.3286 + 0.666075\,u + 0.666075\left(1-\frac{u}{24}\right)u - (1.998225)\frac{u}{24}\,u = 0$$

$$5.3286 + 1.33215\,u - 0.1110125\,u^2 = 0$$

$$u = 15.16515 \text{ in.}$$

$$A_5 = 10.10113\times10^{-3},\quad A_6 = 3.71842\times10^{-3},\quad A_7 = -19.14814\times10^{-3}$$

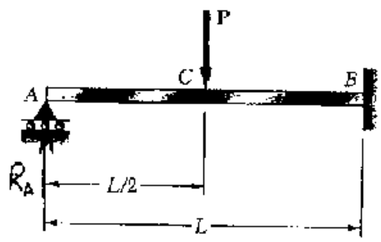
Maximum deflection in portion DE. $\qquad y_D = y_K + t_{D/K} = 0$

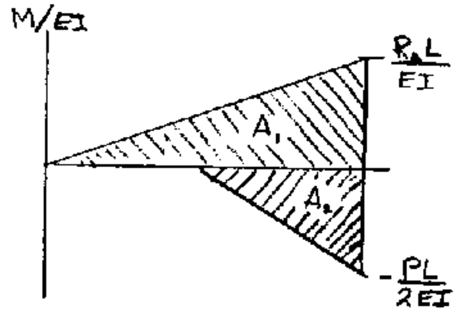
$$y_K = -t_{D/K} = -\left\{A_5\left(\frac{u}{3}\right) + A_6\left(\frac{2u}{3}\right) + A_7\left(\frac{2u}{3}\right)\right\}$$

$$= -\{-0.1049\} = 0.1049 \text{ in.} \uparrow$$

Problem 9.147

9.147 through 1.150 For the beam and loading shown, determine the reaction at the roller support.





Remove support A and treat R_A as redundant.

Draw the $\frac{M}{EI}$ diagram by parts.

$$A_1 = \frac{1}{2} L \frac{R_A L}{EI} = \frac{R_A L^2}{2EI}$$

$$A_2 = -\frac{1}{2} \frac{L}{2} \frac{PL}{2} = -\frac{PL^2}{8EI}$$

Place reference tangent at B.

$$y_A = y_B - \theta_B L + t_{A/B} = 0$$

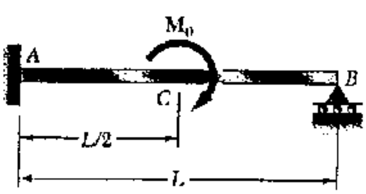
$$t_{A/B} = 0$$

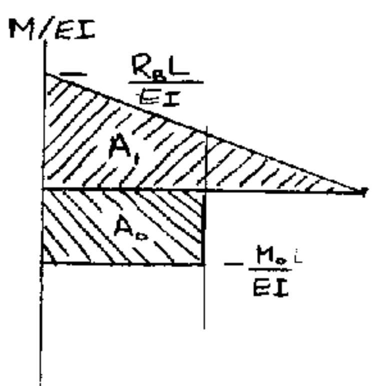
$$A_1\left(\frac{2L}{3}\right) + A_2\left(\frac{L}{2} + \frac{L}{3}\right) = 0$$

$$\frac{R_A L^3}{3EI} - \frac{5PL^3}{48EI} = 0 \qquad R_A = \frac{5}{16} P \uparrow$$

Problem 9.148

9.147 through 1.150 For the beam and loading shown, determine the reaction at the roller support.





Remove support B and treat R_B as redundant.

Draw $\frac{M}{EI}$ diagram.

$$A_1 = \frac{1}{2} L \frac{R_A L}{EI} = \frac{R_A L^2}{2EI}$$

$$A_2 = \frac{L}{2} \cdot \frac{M_0 L}{EI} = \frac{M_0 L^2}{2EI}$$

Place reference tangent at A.

$$y_B = y_A + L\theta_A + t_{B/A} = 0$$

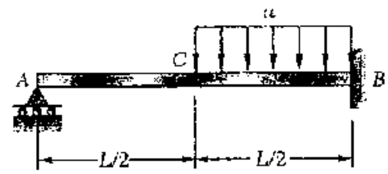
$$t_{B/A} = 0$$

$$A_1\left(\frac{2L}{3}\right) + A_2\left(\frac{L}{2} + \frac{L}{4}\right) = 0$$

$$\frac{R_A L^3}{3EI} - \frac{3M_0 L^2}{8EI} = 0 \qquad R_A = \frac{9}{8}\frac{M_0}{L} \uparrow$$

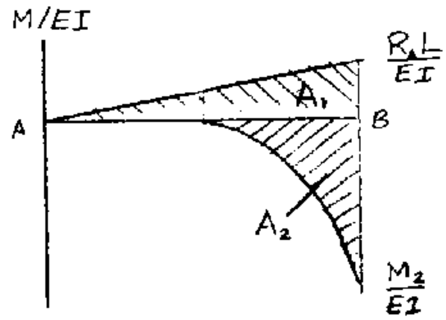
Problem 9.149

9.147 through 1.150 For the beam and loading shown, determine the reaction at the roller support.



Remove support A and treat R_A as redundant.

Draw M/EI diagram for loads R_A and w.



$$M_2 = -\frac{1}{2} w \left(\frac{L}{2}\right)^2 = -\frac{1}{8} wL^2$$

$$A_1 = \frac{1}{2}\left(\frac{R_A L}{EI}\right) L = \frac{1}{2}\frac{R_A L^2}{EI}$$

$$A_2 = \frac{1}{3}\left(-\frac{1}{8}\frac{wL^2}{EI}\right)\left(\frac{L}{2}\right) = -\frac{1}{48}\frac{wL^3}{EI}$$

Place reference tangent at B.

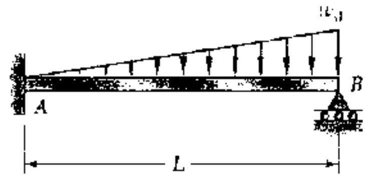
$$t_{A/B} = A_1\left(\frac{2}{3}L\right) + A_2\left(\frac{L}{2} + \frac{3}{4}\frac{L}{2}\right)$$

$$= \frac{1}{3}\frac{R_A L^3}{EI} - \frac{7}{384}\frac{wL^4}{EI} = 0$$

$$R_A = \frac{7}{128} wL \uparrow$$

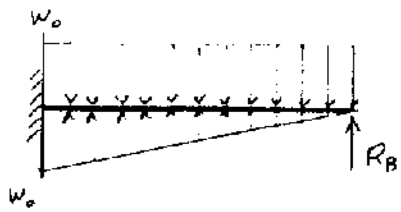
Problem 9.150

9.147 through 1.150 For the beam and loading shown, determine the reaction at the roller support.



Remove support B and treat R_B as redundant.

Replace loading by equivalent shown at left.



Draw M/EI diagram for load w_0 and R_B.

Use parts as shown.

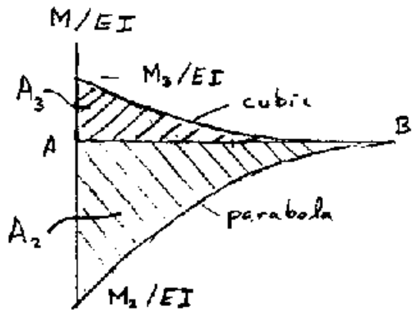
$$A_1 = \frac{1}{2}\left(\frac{R_B L}{EI}\right)(L) = \frac{1}{2}\frac{R_B L^2}{EI}$$

$$M_2 = -\frac{1}{2} w_0 L^2$$

$$A_2 = \frac{1}{3}\left(-\frac{1}{2}\frac{w_0 L^2}{EI}\right)L = -\frac{1}{6}\frac{w_0 L^3}{EI}$$

$$M_3 = \frac{1}{6}\frac{w_0}{L} L^3 = \frac{1}{6} w_0 L^2$$

$$A_3 = \frac{1}{4}\left(\frac{1}{6}\frac{w_0 L^2}{EI}\right)L = \frac{1}{24}\frac{w_0 L^3}{EI}$$

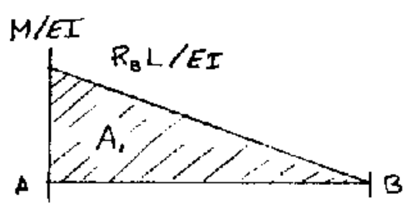


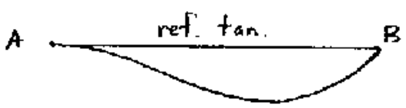
Place reference tangent at A.

$$t_{B/A} = A_1\left(\frac{2}{3}L\right) + A_2\left(\frac{3}{4}L\right) + A_3\left(\frac{4}{5}L\right)$$

$$= \frac{1}{3}\frac{R_B L^3}{EI} - \frac{1}{8}\frac{w_0 L^4}{EI} + \frac{1}{30}\frac{w_0 L^4}{EI} = 0$$

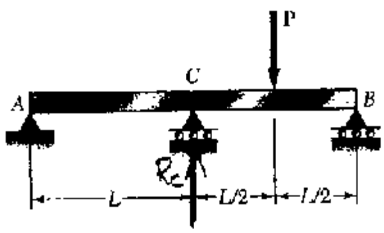
$$R_B = \frac{11}{40} w_0 L = 0.275\, w_0 L \uparrow$$





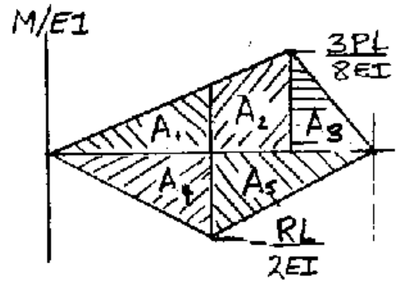
Problem 9.151

9.151 **and** *9.152* For the beam and loading shown, determine the reaction at each support.



Remove support C and add reaction R_c.

Draw $\frac{M}{EI}$ due to each of the loads (P and R_c.



$$A_1 = \tfrac{1}{2}\cdot L \cdot \tfrac{2}{3}\cdot \frac{3PL}{8EI} = \frac{PL^2}{8EI}$$

$$A_1 + A_2 = \tfrac{1}{2}\cdot \tfrac{3L}{2}\cdot \frac{3PL}{8EI} = \frac{9PL^2}{32EI}$$

$$A_3 = \tfrac{1}{2}\cdot \tfrac{L}{2}\cdot \frac{3PL}{8EI} = \frac{3PL^2}{32EI}$$

$$A_4 = \tfrac{1}{2}\cdot L\left(-\frac{R_cL}{2EI}\right) = -\frac{R_cL^2}{4EI}$$

$$A_4 + A_5 = \tfrac{1}{2}(2L)\left(-\frac{R_cL}{2EI}\right) = -\frac{R_cL^2}{2EI}$$

Place reference tangent at A. $\quad y_A = 0$

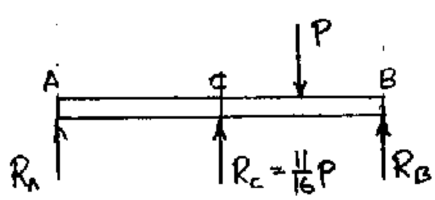
$$y_C = L\theta_A + t_{C/A} = 0 \qquad \theta_A = -\frac{t_{C/A}}{L}$$

$$y_B = 2L\theta_A + t_{B/A} = 0 \qquad -2t_{C/A} + t_{B/A} = 0$$

$$-2\left[A_1\tfrac{L}{3} + A_4\tfrac{L}{3}\right] + \left[(A_1+A_2)\left(\tfrac{L}{2}+\tfrac{2}{3}\cdot\tfrac{3L}{2}\right) + A_3\cdot\tfrac{2}{3}\tfrac{L}{2} + (A_4+A_5)\cdot L\right] = 0$$

$$-2\left[\frac{PL^3}{24EI} - \frac{R_cL^3}{12EI}\right] + \left[\frac{9PL^3}{32EI} + \frac{PL^3}{32EI} - \frac{R_cL^3}{2EI}\right] = 0$$

$$-\frac{1.R_cL^3}{3EI} + \frac{11PL^3}{48EI} = 0 \qquad R_c = \tfrac{11}{16}P\uparrow$$



$+\circlearrowleft \Sigma M_B = 0:$

$$-2LR_A - LR_c + \tfrac{L}{2}P = 0$$

$$R_A = \tfrac{P}{4} - \tfrac{1}{2}R_c = -\tfrac{3}{32}P$$

$$R_A = \tfrac{3}{32}P\downarrow$$

$+\circlearrowleft \Sigma M_A = 0:$

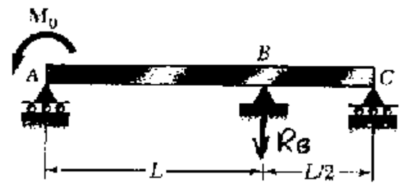
$$2LR_B + LR_c - \tfrac{3L}{2}P = 0$$

$$R_B = \tfrac{3P}{4} - \tfrac{1}{2}R_c = \tfrac{13}{32}P$$

$$R_B = \tfrac{13}{32}P\uparrow$$

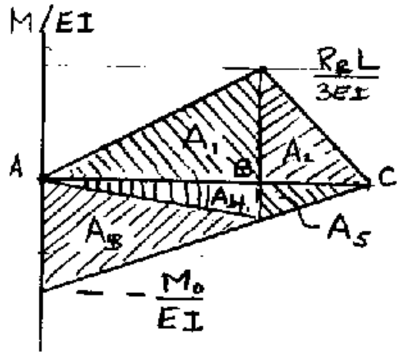
Problem 9.152

9.151 **and** ***9.152*** For the beam and loading shown, determine the reaction at each support.



Choose $R_B \downarrow$ as the redundant reaction.

Draw $\frac{M}{EI}$ diagram for the loads R_B and M_0.



$$A_1 = \frac{1}{2}(L)\left(\frac{R_B L}{3EI}\right) = \frac{R_B L^2}{6EI}$$

$$A_2 = \frac{1}{2}\left(\frac{L}{2}\right)\left(\frac{R_B L}{3EI}\right) = \frac{R_B L^2}{12EI}$$

$$A_3 = \frac{1}{2}(L)\left(-\frac{M_0}{EI}\right) = -\frac{M_0 L}{2EI}$$

$$A_4 = \frac{1}{2}(L)\left(\frac{1}{3}\right)\left(-\frac{M_0}{EI}\right) = -\frac{M_0 L}{6EI}$$

$$A_3 + A_4 + A_5 = \frac{1}{2}\left(\frac{3L}{2}\right)\left(-\frac{M_0}{EI}\right) = -\frac{3M_0 L}{4EI}$$

$$y_B = y_A + L\theta_A + t_{B/A} \qquad \theta_A = -t_{B/A}/L$$

$$y_C = y_A + \frac{3L}{2}\theta_A + t_{C/A} = 0 \qquad -\frac{3}{2}t_{B/A} + t_{C/A} = 0$$

$$t_{B/A} = (A_1)\left(\frac{L}{3}\right) + A_3\left(\frac{2L}{3}\right) + A_4\left(\frac{L}{3}\right) = \frac{R_B L^3}{18EI} - \frac{7M_0 L^2}{18EI}$$

$$t_{C/A} = (A_1)\left(\frac{L}{2} + \frac{L}{3}\right) + A_2\left(\frac{L}{3}\right) + (A_3 + A_4 + A_5)(L) = \frac{R_B L^3}{6EI} - \frac{3M_0 L^2}{4EI}$$

$$-\frac{3}{2}t_{B/A} + t_{C/A} = \frac{R_B L^3}{12EI} - \frac{M_0 L^2}{6EI} = 0 \qquad R_B = \frac{2M_0}{L} \downarrow$$

$$+\circlearrowleft \Sigma M_C = 0: \quad M_0 + \frac{L}{2}R_B - \frac{3L}{2}R_A = 0$$

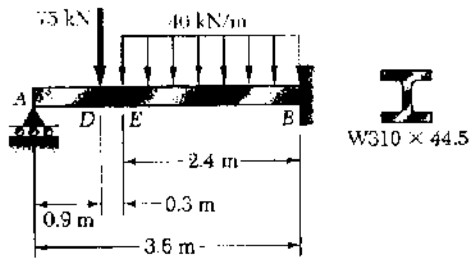
$$R_A = \frac{2}{3L}\left[M_0 + M_0\right] \qquad R_A = \frac{4M_0}{3L} \uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B + R_C = 0 \qquad \frac{4}{3}\frac{M_0}{L} - \frac{2M_0}{L} + R_C = 0$$

$$R_C = \frac{2M_0}{3L} \uparrow$$

Problem 9.153

9.153 and 9.154 Determine the reaction at the roller support and draw the bending-moment diagram for the beam and loading shown.

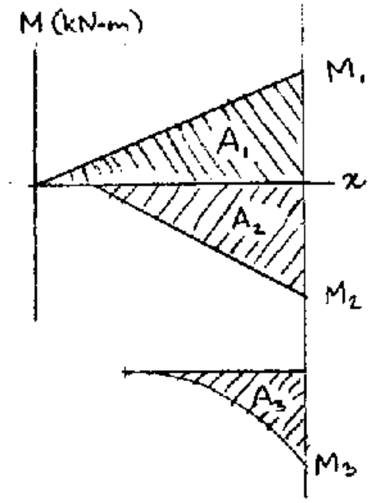


Units: Forces in kN. Lengths in meters.

Let R_A be the redundant reaction.

Remove support at A and add reaction $R_A \uparrow$.

Draw bending moment diagram by parts.



$$M_1 = 3.6 R_A \quad \text{kN}\cdot\text{m}$$

$$M_2 = -(75)(0.3 + 2.4) = -202.5 \text{ kN}\cdot\text{m}$$

$$M_3 = -\tfrac{1}{2}(40)(2.4)^2 = -115.2 \text{ kN}\cdot\text{m}$$

$$A_1 = \tfrac{1}{2}(3.6)(3.6 R_A) = 6.48 \text{ kN}\cdot\text{m}^2$$

$$A_2 = \tfrac{1}{2}(2.7)(-202.5) = -273.375 \text{ kN}\cdot\text{m}^2$$

$$A_3 = \tfrac{1}{3}(2.4)(-115.2) = -92.16 \text{ kN}\cdot\text{m}^2$$

Place reference tangent at B, where

$$\theta_B = 0 \quad \text{and} \quad y_B = 0$$

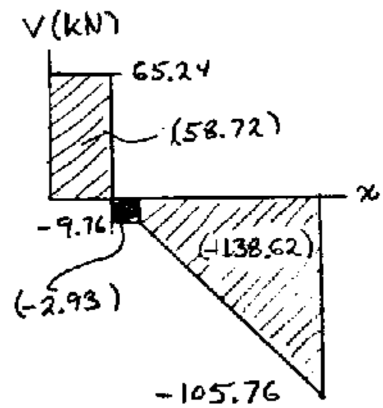
Then, $y_A = t_{A/B} = 0$

$$t_{A/B} = \frac{1}{EI}\left\{\left(\tfrac{2}{3}\cdot 3.6\right)A_1 + \left(0.9 + \tfrac{2}{3}\cdot 2.7\right)A_2 + \left(0.9 + 0.3 + \tfrac{3}{4}\cdot 2.4\right)A_3\right\}$$

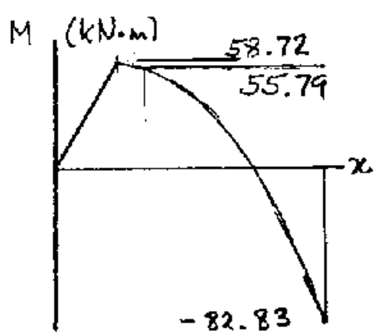
$$= \frac{1}{EI}\left\{15.552 R_A - 1014.5925\right\} = 0$$

$$R_A = 65.24 \text{ kN} \uparrow$$

Draw shear diagram.



A to D	$V = R_A = 65.24$ kN
D to E	$V = 65.24 - 75 = -9.76$ kN
E to B	$V = -9.76 - 40(x - 1.2)$ kN
At B	$V_B = -105.76$ kN.



Bending moment diagram. $M_A = 0$

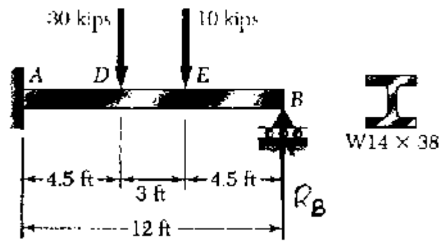
$$M_D = M_A + 58.72 = 58.72 \text{ kN}\cdot\text{m}$$

$$M_E = 58.72 - 2.93 = 55.79 \text{ kN}\cdot\text{m}$$

$$M_B = 55.79 - 138.62 = -82.83 \text{ kN}\cdot\text{m}$$

Problem 9.154

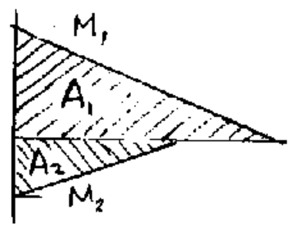
9.153 and 9.154 Determine the reaction at the roller support and draw the bending-moment diagram for the beam and loading shown.



Units. Forces in kips. Lengths in feet.

Let R_B be the redundant reaction.
Remove support B and add load R_B.

Draw bending moment diagram by parts.



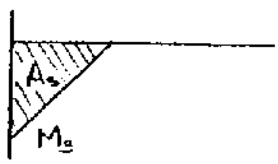
$M_1 = 12 R_B$ kip·ft

$M_2 = -(4.5+3)(10) = -75$ kip·ft

$M_3 = -(4.5)(30) = -135$ kip·ft

$A_1 = \frac{1}{2}(12)(12R_B) = 72 R_B$ kip·ft²

$A_2 = \frac{1}{2}(7.5)(-75) = -281.25$ kip·ft²

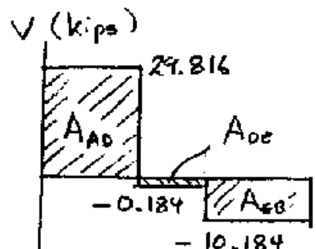


$A_3 = \frac{1}{2}(4.5)(-135) = -303.75$ kip·ft²

$y_B = y_A + 12\,\theta_A + t_{B/A} = 0$

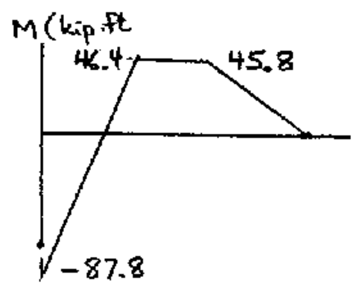
$t_{B/A} = 0$

$$t_{B/A} = \frac{1}{EI}\{(72R_B)(8) + (-281.25)(4.5+5) + (-303.75)(7.5+3)\} = 0$$



$576\,R_B - 5861.25 = 0 \qquad R_B = 10.18$ kips ↑

Draw shear diagram working from right to left.



B to E $\qquad V = -R_B = -10.176$ kips

E to D $\qquad V = -10.184 + 10 = -0.176$ kips

D to A $\qquad V = -0.176 + 30 = 29.824$ kips

Areas of shear diagram.

$A_{AD} = (4.5)(29.824) = 134.21$ kip·ft

$A_{DE} = (3)(-0.176) = -0.53$ kip·ft

$A_{EB} = (4.5)(-10.176) = 45.79$ kip·ft

Bending moments.

$M_A = M_1 + M_2 + M_3 = -87.89$ kip·ft

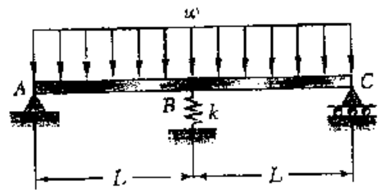
$M_D = M_A + A_{AD} = 46.32$ kip·ft

$M_E = M_D + A_{DE} = 45.79$ kip·ft

$M_B = M_E + A_{EB} = 0$

Problem 9.155

9.155 For the beam and loading shown, determine the spring constant k for which the force in the spring is equal to one-third of the total load on the beam.

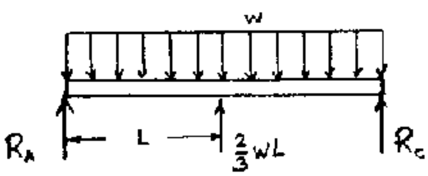


Symmetric beam and loading. $R_C = R_A$

Spring force: $F = \frac{1}{3}(2wL) = \frac{2}{3}wL$

$+\uparrow \Sigma F_y = 0: \quad R_A + F - 2wL + R_C = 0$

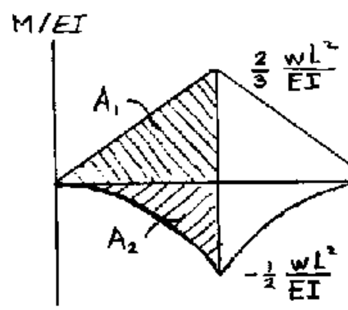
$$R_A = R_C = \frac{2}{3}wL$$



Draw $\frac{M}{EI}$ diagram by parts.

$$A_1 = \frac{1}{2}\left(\frac{2}{3}\frac{wL^2}{EI}\right)L = \frac{1}{3}\frac{wL^3}{EI}$$

$$A_2 = -\frac{1}{3}\left(\frac{1}{2}\frac{wL^2}{EI}\right)L = -\frac{1}{6}\frac{wL^3}{EI}$$



Place reference tangent at B. $\theta_B = 0$

$$y_B = -t_{A/B}$$

$$= -\left(A_1 \cdot \frac{2}{3}L + A_2 \cdot \frac{3}{4}L\right)$$

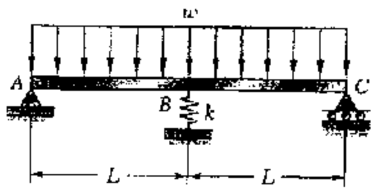
$$= -\frac{7}{72}\frac{wL^4}{EI}$$

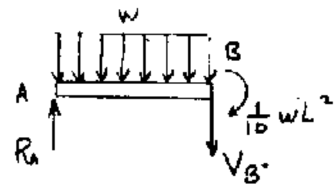
$$F = -k\,y_B$$

$$k = -\frac{F}{y_B} = \frac{\frac{2}{3}wL}{\frac{7}{72}\frac{wL^4}{EI}} = \frac{48}{7}\frac{EI}{L^3}$$ ◄

Problem 9.156

9.156 For the beam and loading shown, determine the spring constant k for which the bending moment at B is $M_B = -wL^2/10$.



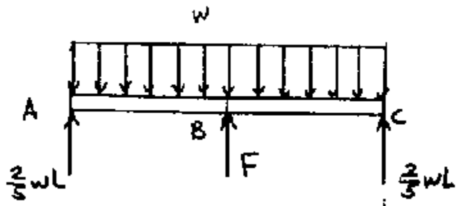


Using free body AB, $+\circlearrowleft M_B = 0$:

$$-R_A L + (wL)\left(\frac{L}{2}\right) - \frac{1}{10}wL^2 = 0$$

$$R_A = \frac{2}{5}wL \uparrow$$

Symmetric beam and loading. $R_C = R_A$

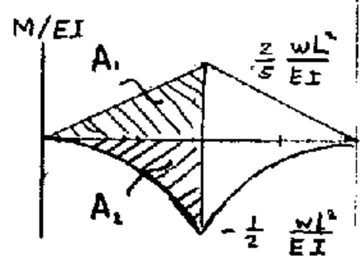


Using free body ABC, $+\uparrow \Sigma F_y = 0$:

$$\frac{2}{5}wL + F + \frac{3}{5}wL - 2wL = 0$$

$$F = \frac{6}{5}wL$$

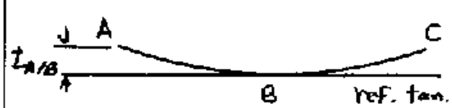
Draw $\frac{M}{EI}$ diagram by parts.



$$A_1 = \frac{1}{2}\left(\frac{2}{5}\frac{wL^2}{EI}\right)L = \frac{1}{5}\frac{wL^3}{EI}$$

$$A_2 = -\frac{1}{3}\left(\frac{1}{2}\frac{wL^2}{EI}\right)L = -\frac{1}{6}\frac{wL^3}{EI}$$

Place reference tangent at B. $\theta_B = 0$



$$y_B = -t_{A/B}$$

$$= -\left(A_1 \cdot \frac{2}{3}L + A_2 \cdot \frac{3}{4}L\right)$$

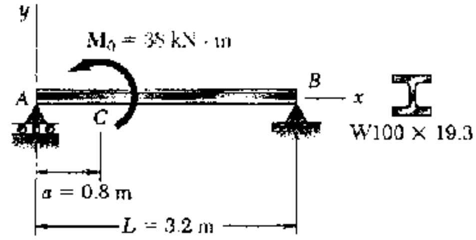
$$= -\frac{1}{120}\frac{wL^4}{EI}$$

$$F = -ky_B$$

$$k = -\frac{F}{y_B} = \frac{\frac{6}{5}wL}{\frac{1}{120}\frac{wL^4}{EI}} = 144\frac{EI}{L^3}$$

Problem 9.157

9.157 For the beam and loading shown, determine the deflection at point C. Use $E = 200$ GPa.



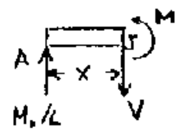
$[x=0, y=0]$ $[x=L, y=0]$

$[x=a, y=y]$

$[x=a, \frac{dy}{dx}=\frac{dy}{dx}]$

Reactions: $R_A = M_o/L \uparrow$, $R_B = M_o/L \downarrow$

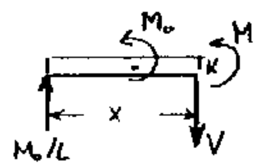
$0 < x < a$ $\quad \circlearrowleft \Sigma M_J = 0:$



$$-\frac{M_o}{L}x + M = 0$$

$$M = \frac{M_o}{L}x$$

$a < x < L$ $\quad +\circlearrowleft M_K = 0:$



$$-\frac{M_o}{L}x + M_o + M = 0$$

$$M = \frac{M_o}{L}(x - L)$$

$0 < x < a$		$a < x < L$	
$EI\frac{d^2y}{dx^2} = \frac{M_o}{L}x$		$EI\frac{d^2y}{dx^2} = \frac{M_o}{L}(x-L)$	
$EI\frac{dy}{dx} = \frac{M_o}{L}(\frac{1}{2}x^2) + C_1$	(1)	$EI\frac{dy}{dx} = \frac{M_o}{L}(\frac{1}{2}x^2 - Lx) + C_3$	(3)
$EI\,y = \frac{M_o}{L}(\frac{1}{6}x^3) + C_1 x + C_2$	(2)	$EI\,y = \frac{M_o}{L}(\frac{1}{6}x^3 - \frac{1}{2}Lx^2) + C_3 x + C_4$	(4)

$[x=0, y=0]$ Eq. (2): $\quad 0 = 0 + 0 + C_2 \qquad C_2 = 0$

$[x=a, \frac{dy}{dx}=\frac{dy}{dx}]$ Eqs. (1) & (3): $\quad \frac{M_o}{L}(\frac{1}{2}a^2) + C_1 = \frac{M_o}{L}(\frac{1}{2}a^2 - La) + C_3$

$$C_3 = C_1 + M_o a$$

$[x=a, y=y]$ Eqs (2) & (4) $\quad \frac{M_o}{L}(\cancel{\tfrac{1}{6}a^3}) + \cancel{C_1 a} = \frac{M_o}{L}(\cancel{\tfrac{1}{6}a^3} - \frac{1}{2}La^2) + (\cancel{C_1} + M_o a)a + C_4$

$$C_4 = -\frac{1}{2}M_o a^2$$

$[x=L, y=0]$ Eq (4) $\quad \frac{M_o}{L}(\frac{1}{6}L^3 - \frac{1}{2}L^3) + (C_1 + M_o a)L - \frac{1}{2}M_o a^2 = 0$

$$C_1 = \frac{M_o}{L}(\frac{1}{3}L^2 + \frac{1}{2}a^2 - aL)$$

Elastic curve for $0 < x < a$. $\quad y = \frac{M_o}{EIL}\left[\frac{1}{6}x^3 + (\frac{1}{3}L^2 + \frac{1}{2}a^2 - aL)x\right]$

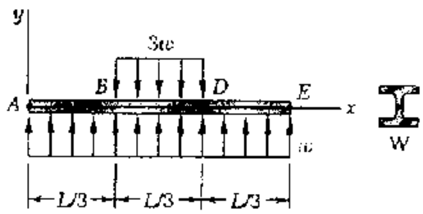
Make $x = a$. $\quad y_C = \frac{M_o}{EIL}\left[\frac{1}{6}a^3 + \frac{1}{3}L^2 a + \frac{1}{2}a^3 - a^2 L\right] = \frac{M_o}{EIL}\left[\frac{2}{3}a^3 + \frac{1}{3}L^2 a - La^2\right]$

Data: $E = 200 \times 10^9$ Pa, $I = 4.77 \times 10^6\ \text{mm}^4 = 4.77 \times 10^{-6}$ m, $M_o = 38 \times 10^3$ N·m

$$y_C = \frac{(38\times10^3)\left[(2)(0.8)^3/3 + (3.2)^2(0.8)/3 - (3.2)(0.8)^2\right]}{(200\times10^9)(4.77\times10^{-6})(3.2)} = 12.75 \times 10^{-3}\ \text{m}$$

$$= 12.75\ \text{mm} \uparrow$$

Problem 9.158



9.158 Uniformly distributed loads are applied to beam AE as shown. (*a*) Selecting the x axis through the centers A and E of the end sections of the beam, determine the equation of the elastic curve for portion AB of the beam. (*b*) Knowing that the beam is a W 200 × 35.9 rolled shape and that $L = 3$ m, $w = 5$ kN/m, and $E = 200$ GPa, determine the distance of the center of the beam from the x axis.

$0 < x < \frac{L}{3}$ $\circlearrowleft \Sigma M_J = 0:$

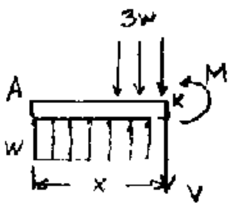
$-(wx)(\frac{x}{2}) + M = 0$

$M = \frac{1}{2}wx^2$

$EI\frac{d^2y}{dx^2} = \frac{1}{2}wx^2$

$EI\frac{dy}{dx} = \frac{1}{6}wx^3 + C_1$

$EIy = \frac{1}{24}wx^4 + C_1x + C_2$



$\frac{L}{3} \le x \le \frac{2L}{3}$ $+\circlearrowleft \Sigma M_K = 0: \quad -(wx)(\frac{x}{2}) + 3w(x-\frac{L}{3})(\frac{x-\frac{L}{3}}{2}) + M = 0$

$$M = \frac{1}{2}wx^2 - \frac{3}{2}w(x-\frac{L}{3})^2$$

$EI\frac{d^2y}{dx^2} = \frac{1}{2}wx^2 - \frac{3}{2}w(x-\frac{L}{3})^2$

$EI\frac{dy}{dx} = \frac{1}{6}wx^3 - \frac{1}{2}w(x-\frac{L}{3})^3 + C_3$

$EIy = \frac{1}{24}wx^4 - \frac{1}{8}w(x-\frac{L}{3})^4 + C_3x + C_4$

$[x=0, y=0]$ $0 = 0 + 0 + C_2$ $C_2 = 0$

$[x=\frac{L}{3}, \frac{dy}{dx}=\frac{dy}{dx}]$ $\cancel{\frac{1}{6}w(\frac{L}{3})^3} + C_1 = \cancel{\frac{1}{6}w(\frac{L}{3})^3} + 0 + C_3$ $C_1 = C_3$

$[x=\frac{L}{3}, y=y]$ $\cancel{\frac{1}{24}w(\frac{L}{3})^4} + \cancel{C_1\frac{L}{3}} + C_2 = \cancel{\frac{1}{24}w(\frac{L}{3})^4} + 0 + \cancel{C_3\frac{L}{3}} + C_4$

$$C_4 = C_2 = 0$$

Symmetry boundary condition $[x=\frac{L}{2}, \frac{dy}{dx}=0]$

$\frac{1}{6}w(\frac{L}{2})^3 - \frac{1}{2}w(\frac{L}{2}-\frac{L}{3})^3 + C_3 = 0$ $C_3 = -(\frac{1}{48}-\frac{1}{432})wL^3 = -\frac{1}{54}wL^3$

(a) Elastic curve for portion AB.

$$y = \frac{1}{EI}\left\{\frac{1}{24}wx^4 + C_1x + C_2\right\} = \frac{w}{EI}\left(\frac{1}{24}x^4 - \frac{1}{54}L^3x\right)$$

(b) Deflection at center. $y_C = \frac{1}{EI}\left\{\frac{1}{24}w(\frac{L}{2})^4 - \frac{1}{8}w(\frac{L}{2}-\frac{L}{3})^4 - \frac{1}{54}wL^3(\frac{L}{2}) + 0\right\}$

$$= \frac{wL^4}{EI}\left\{\frac{1}{384} - \frac{1}{10368} - \frac{1}{108}\right\} = -\frac{35}{5184}\frac{wL^4}{EI}$$

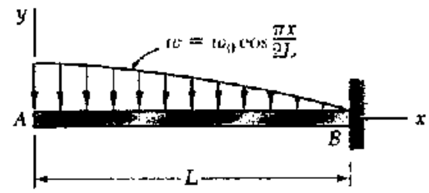
Data: $I = 34.4\times10^6\ mm^4 = 34.4\times10^{-6}\ m^4$, $E = 200\times10^9$ Pa, $L = 3$ m

$w = 5\times10^3$ N/m

$$y_C = -\frac{35}{5184}\frac{(5\times10^3)(3)^4}{(200\times10^9)(34.4\times10^{-6})} = -397\times10^{-6}\ m, \text{ ie } 0.397\ mm \downarrow$$

Problem 9.159

9.159 For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the slope at the free end, (*c*) the deflection at the free end.



$[x=0, V=0]$ $\quad [x=L, \frac{dy}{dx}=0]$

$[x=0, M=0]$ $\quad [x=L, y=0]$

$$\frac{dV}{dx} = -w = -w_0 \cos\frac{\pi x}{2L}$$

$$V = -\frac{2w_0 L}{\pi}\sin\frac{\pi x}{2L} + C_1$$

$[x=0, V=0]$ $\quad 0 = 0 + C_1 \quad C_1 = 0$

$$\frac{dM}{dx} = V = -\frac{2w_0 L}{\pi}\sin\frac{\pi x}{2L}$$

$$M = \frac{4w_0 L^2}{\pi^2}\cos\frac{\pi x}{2L} + C_2$$

$[x=0, M=0]$ $\quad C_2 = -\frac{4w_0 L^2}{\pi^2}$

$$EI\frac{d^2y}{dx^2} = M = \frac{4w_0L^2}{\pi^2}\left(\cos\frac{\pi x}{2L} - 1\right)$$

$$EI\frac{dy}{dx} = \frac{4w_0L^2}{\pi^2}\left(\frac{2L}{\pi}\sin\frac{\pi x}{2L} - x\right) + C_3$$

$[x=L, \frac{dy}{dx}=0]$ $\quad \frac{4w_0L^2}{\pi^2}\left(\frac{2L}{\pi} - L\right) + C_3 = 0 \quad C_3 = \frac{4w_0L^3}{\pi^3}(\pi - 2)$

$$EIy = \frac{4w_0L^2}{\pi^2}\left[-\frac{4L^2}{\pi^2}\cos\frac{\pi x}{2L} - \frac{1}{2}x^2\right] + C_3 x + C_4$$

$[x=L, y=0]$ $\quad \frac{4w_0L^2}{\pi^2}\left(-\frac{1}{2}L^2\right) + C_3 L + C_4 = 0$

$$C_4 = \frac{2w_0L^4}{\pi^2} - C_3 L$$

(a) Elastic curve. $\quad y = \frac{w_0}{EI}\left\{-\frac{16L^4}{\pi^4}\cos\frac{\pi x}{2L} - \frac{2L^2x^2}{\pi^2} + \frac{4L^3}{\pi^3}(\pi-2)(x-L) + \frac{2L^4}{\pi^2}\right\}$

$$y = \frac{2w_0L^4}{\pi^4 EI}\left\{-8\cos\frac{\pi x}{2L} - \pi^2\frac{x^2}{L^2} + 2\pi(\pi-2)\frac{x}{L} + \pi(4-\pi)\right\}$$ ◄

(b) Slope at free end. $(x=0)$

$$EI\left.\frac{dy}{dx}\right|_{x=0} = C_3 = \frac{4(\pi-2)}{\pi^3}w_0L^3$$

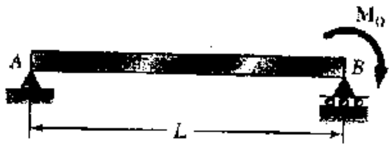
$$\left.\frac{dy}{dx}\right|_A = \frac{4(\pi-2)}{\pi^3}\frac{w_0L^3}{EI} = 0.14727\frac{w_0L^3}{EI}$$ ◄

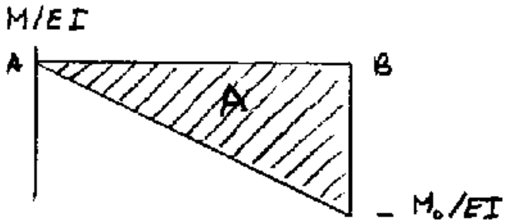
(c) Deflection at free end. $(x=0)$

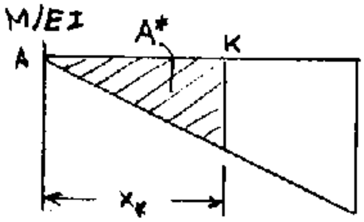
$$y_A = \frac{2w_0L^4}{\pi^4 EI}\{-8 + \pi(4-\pi)\} = -0.10889\frac{w_0L^4}{EI} = 0.1089\frac{w_0L^4}{EI}\downarrow$$ ◄

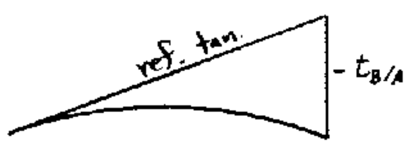
Problem 9.160

9.160 For the beam and loading shown, determine the magnitude and location of the maximum deflection.









Draw $\frac{M}{EI}$ diagram.

Place reference tangent at A.

$$A = \frac{1}{2}\left(-\frac{M_0}{EI}\right)L = -\frac{1}{2}\frac{M_0 L}{EI}$$

$$t_{B/A} = A\left(\frac{L}{3}\right) = -\frac{1}{6}\frac{M_0 L^2}{EI}$$

$$\theta_A = -\frac{t_{B/A}}{L} = \frac{1}{6}\frac{M_0 L}{EI}$$

$$A^* = \frac{1}{2}\left(\frac{M_0}{EI}\frac{x_K}{L}\right)x_K = -\frac{1}{2}\frac{M_0 x_K^2}{EIL}$$

$$\theta_K = \theta_A + \theta_{K/A} = \theta_A + A^* = 0$$

$$\frac{1}{6}\frac{M_0 L}{EI} - \frac{1}{2}\frac{M_0 x_K^2}{EIL} = 0$$

$$x_K = \frac{\sqrt{3}}{3}L = 0.577\,L$$

$$t_{K/A} = A^*\left(\frac{1}{3}x_K\right)$$

$$= -\frac{1}{2}\frac{M_0 x_K^2}{EIL}\left(\frac{1}{3}x_K\right)$$

$$= -\frac{1}{6}\frac{M_0 x_K^3}{EIL}$$

Maximum deflection.

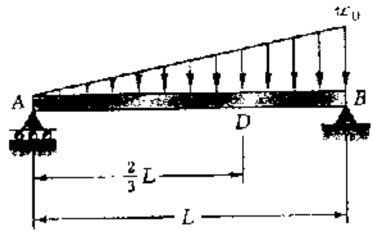
$$y_K = t_{K/A} - \frac{x_K}{L}t_{B/A} = -\frac{1}{6}\frac{M_0 x_K^3}{EIL} - \frac{x_K}{L}\left(-\frac{M_0 L^2}{6EI}\right)$$

$$= \frac{M_0 x_K}{6EIL}\left(L^2 - x_K^2\right) = \frac{\sqrt{3}}{18}\frac{M_0}{EI}\left(L^2 - \frac{1}{3}L^2\right) = \frac{\sqrt{3}}{27}\frac{M_0 L^2}{EI}$$

$$= 0.0642\frac{M_0 L^2}{EI}\uparrow$$

Problem 9.161

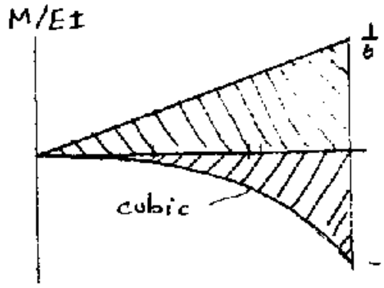
9.161 For the prismatic beam and loading shown, determine the magnitude and location of the largest downward deflection.



$+\circlearrowleft M_B = 0 \quad -R_A L + (\frac{1}{2}w_0 L)(\frac{1}{3}L) = 0 \qquad R_A = \frac{1}{6}w_0 L$

Bending moment $\quad M = R_A x - \frac{1}{6}\frac{w_0}{L}x^3$

$= \frac{1}{6}\frac{w_0}{L}(L^2 x - x^3)$

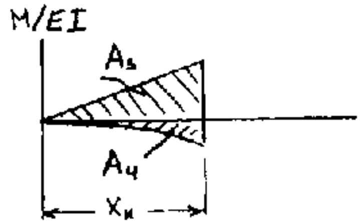


At $x = L \qquad M = \frac{1}{6}w_0 L^2 - \frac{1}{6}w_0 L^2$

Draw $\frac{M}{EI}$ diagram by parts.

$A_1 = \frac{1}{2}\left(\frac{1}{6}\frac{w_0 L^2}{EI}\right)L = \frac{1}{12}\frac{w_0 L^3}{EI} \qquad \bar{x}_1 = \frac{1}{3}L$

$A_2 = \frac{1}{4}\left(-\frac{1}{6}\frac{w_0 L^2}{EI}\right)L = -\frac{1}{24}\frac{w_0 L^3}{EI} \qquad \bar{x}_2 = \frac{1}{5}L$



Place reference tangent at A.

$t_{B/A} = A_1\bar{x}_1 + A_2\bar{x}_2$

$= \frac{1}{36}\frac{w_0 L^4}{EI} - \frac{1}{120}\frac{wL^4}{EI} = \frac{7}{360}\frac{w_0 L^4}{EI}$

Slope at A. $\quad \theta_A = -\frac{t_{B/A}}{L} = -\frac{7}{360}\frac{w_0 L^3}{EI}$

$A_3 = A_1\left(\frac{x_K}{L}\right)^2 = \frac{1}{12}\frac{w_0 L^3}{EI}u^2$

$A_4 = A_2\left(\frac{x_K}{L}\right)^4 = -\frac{1}{24}\frac{w_0 L^3}{EI}u^4$

$\theta_{K/A} = A_3 + A_4 = \frac{w_0 L^3}{EI}\left(\frac{1}{12}u^2 - \frac{1}{24}u^4\right) = -\theta_A = \frac{7}{360}\frac{w_0 L^3}{EI}$

$u^4 - 2u^2 + \frac{7}{15} = 0 \qquad$ Solving for $u \qquad u = 0.51933$

$x_K = 0.51933 L$

$A_3 = \frac{1}{12}\frac{w_0 L^3}{EI}(0.51933)^2 = 0.0224753\frac{w_0 L^3}{EI}, \qquad \bar{x}_3 = \frac{1}{3}(0.51933)L$

$A_4 = -\frac{1}{24}\frac{w_0 L^3}{EI}(0.51933)^4 = -0.0030308\frac{w_0 L^3}{EI}, \qquad \bar{x}_4 = \frac{1}{5}(0.51933)L$

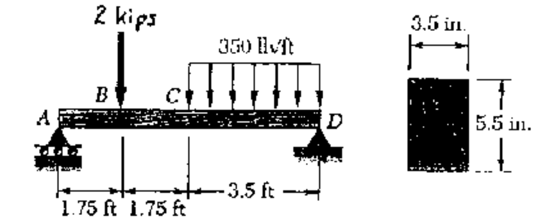
$t_{K/A} = A_3\bar{x}_3 + A_4\bar{x}_4 = 0.0035759\frac{w_0 L^4}{EI}$

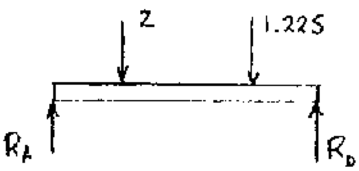
$y_K = t_{K/A} - \frac{x_K}{L}t_{B/A} = 0.0035759\frac{w_0 L^4}{EI} - (0.51933)\left(\frac{7}{360}\frac{w_0 L^4}{EI}\right)$

$= -0.00652\frac{w_0 L^4}{EI} = 0.00652\frac{w_0 L^4}{EI} \downarrow$

Problem 9.162

9.162 For the timber beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the midpoint *C*. Use $E = 1.6 \times 10^6$ psi.





Units: Forces in kips; lengths in ft.

$+\circlearrowleft \Sigma M_D = 0:$

$$-7R_A + (2)(5.25) + (1.225)(1.75) = 0$$

$$R_A = 1.80625 \text{ kips}$$

$$w(x) = 0.350\langle x-3.5\rangle^0$$

$$\frac{dV}{dx} = -w = -0.35\langle x-3.5\rangle^0$$

$$\frac{dM}{dx} = V = 1.05625 - 1\langle x-1.75\rangle^0 - 0.35\langle x-3.5\rangle^1$$

$$EI\frac{d^2y}{dx^2} = M = 1.80625x - 2\langle x-1.75\rangle^1 - 0.175\langle x-3.5\rangle^2 \quad \text{kip}\cdot\text{ft}$$

$$EI\frac{dy}{dx} = 0.903125x^2 - 1\langle x-1.75\rangle^2 - 0.05833\langle x-3.5\rangle^3 + C_1 \quad \text{kip}\cdot\text{ft}^2$$

$$EI\,y = 0.301042x^3 - \tfrac{1}{3}\langle x-1.75\rangle^3 - 0.014583\langle x-3.5\rangle^4 + C_1x + C_2 \quad \text{kip}\cdot\text{ft}^3$$

$[x=0,\ y=0]$ $\qquad C_2 = 0$

$[x=7,\ y=0]$ $\quad (0.301042)(7)^3 - \tfrac{1}{3}(5.25)^3 - 0.014583(3.5)^4 + C_1(7) + 0 = 0$

$$C_1 = -7.54779 \text{ kip}\cdot\text{ft}^2$$

Data: $E = 1.6\times10^6$ psi $= 1.6\times10^3$ ksi

$$I = \tfrac{1}{12}(3.5)(5.5)^3 = 48.526 \text{ in}^3$$

$$EI = (1.6\times10^3)(48.526) = 77.6417 \text{ kip}\cdot\text{in}^2 = 539.18 \text{ kip}\cdot\text{ft}^2$$

(a) Slope at A. $\left(\frac{dy}{dx}\text{ at } x=0\right)$

$$EI\frac{dy}{dx} = 0 - 0 - 0 - 7.54779 \text{ kip}\cdot\text{ft}^2$$

$$\theta_A = -\frac{7.54779}{539.18} = -14.00\times10^{-3} \text{ rad} = 14.00\times10^{-3} \text{ rad} \nwarrow$$

(b) Deflection at C. (y at $x = 3.5$ ft.)

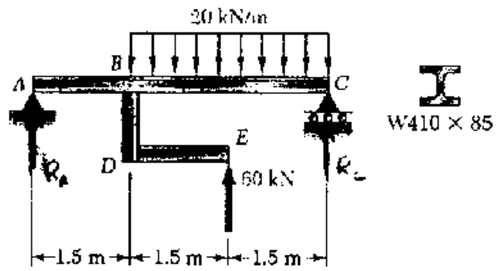
$$EI\,y_C = (0.301042)(3.5)^3 - \tfrac{1}{3}(1.75)^3 - 0 - (7.54779)(3.5) + 0$$

$$= -15.297 \text{ kip}\cdot\text{ft}^3$$

$$y_C = -\frac{15.297}{539.18} = -28.37\times10^{-3} \text{ ft} = 0.340 \text{ in.}\downarrow$$

Problem 9.163

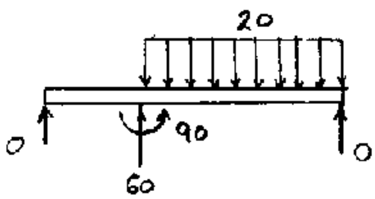
9.163 The rigid bar *BDE* is welded at point *B* to the rolled steel beam *AC*. For the loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *B*. Use $E = 200$ GPa.



$$+\circlearrowleft \Sigma M_C = 0: \quad -4.5 R_A + (20)(3)(1.5) - (60)(1.5) = 0$$

$$R_A = 0$$

Units: Forces in kN; lengths in m.



$$EI \frac{d^2y}{dx^2} = M = 60\langle x-1.5\rangle^1 - 90\langle x-1.5\rangle^0 - \frac{1}{2}(20)\langle x-1.5\rangle^2$$

$$EI \frac{dy}{dx} = 30\langle x-1.5\rangle^2 - 90\langle x-1.5\rangle^1 - (\tfrac{1}{6})(20)\langle x-1.5\rangle^3 + C_1$$

$$EI\, y = 10\langle x-1.5\rangle^3 - 45\langle x-1.5\rangle^2 - \tfrac{1}{24}(20)\langle x-1.5\rangle^4 + C_1 x + C_2$$

$[x = 0, y = 0]$ $\quad 0 + 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x = 4.5, y = 0]$ $\quad (10)(3)^3 - (45)(3)^2 - \frac{1}{24}(20)(3)^4 + 4.5C_1 + 0 = 0$

$$C_1 = 45 \text{ kN}\cdot\text{m}^2$$

Data: $E = 200 \times 10^9$ Pa, $I = 315 \times 10^6 \text{ mm}^4 = 315 \times 10^{-6} \text{ m}^4$

$$EI = (200 \times 10^9)(315 \times 10^{-6}) = 63 \times 10^6 \text{ N}\cdot\text{m}^2 = 63000 \text{ kN}\cdot\text{m}^2$$

(a) Slope at A. $\quad \left(\frac{dy}{dx} \text{ at } x = 0\right)$

$$EI\,\theta_A = C_1 = 45 \text{ kN}\cdot\text{m}^2$$

$$\theta_A = \frac{45}{63000} = 0.714 \times 10^{-3} \text{ rad} \qquad \theta_A = 0.714 \times 10^{-3} \text{ rad} \measuredangle$$

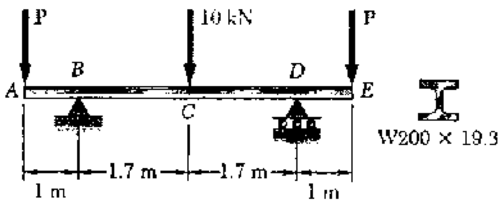
(b) Deflection at B. $\quad (y \text{ at } x = 1.5)$

$$EI\,y_B = (C_1)(1.5) = (45)(1.5) = 67.5 \text{ kN}\cdot\text{m}^3$$

$$y_B = \frac{67.5}{63000} = 1.071 \times 10^{-3} \text{ m} = 1.071 \text{ mm} \uparrow$$

Problem 9.164

9.164 Knowing that $P = 8$ kN, determine (*a*) the slope at end *A*, (*b*) the deflection at the midpoint *C*. Use $E = 200$ GPa.

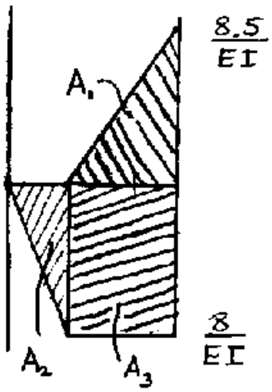


$E = 200 \times 10^9$ Pa

$I = 16.6 \times 10^6\ \text{mm}^4 = 16.6 \times 10^{-6}\ \text{m}^4$

$EI = (200 \times 10^9)(16.6 \times 10^{-6}) = 3.32 \times 10^6\ \text{N}\cdot\text{m}^2$

$= 3320\ \text{kN}\cdot\text{m}^2$



Symmetric beam and loading.

$R_A = R_B = P + 5 = 8 + 5 = 13$ kN.

Bending moment

Over AB $\quad M = -Px = -8x$

Over BC $\quad M = -8x + 13(x-1)$

$= 5(x-1) - 8$

Draw $\frac{M}{EI}$ diagram by parts.

$A_1 = \frac{1}{2}\left(\frac{8.5}{EI}\right)(1.7) = \frac{7.225}{EI}$

$A_2 = -\frac{1}{2}\left(\frac{8}{EI}\right)(1) = -\frac{4}{EI}$

$A_3 = -\left(\frac{8}{EI}\right)(1.7) = -\frac{13.600}{EI}$

Place reference tangent at C $\quad \theta_C = 0$

(a) Slope at A. $\quad \theta_A = \theta_C - \theta_{C/A} = 0 - (A_1 + A_2 + A_3)$

$$\theta_A = -\left(\frac{7.225}{EI} - \frac{4}{EI} - \frac{13.600}{EI}\right) = \frac{10.375}{EI} = \frac{10.375}{3320} = 3.125 \times 10^{-3}\ \text{rad}$$

(b) Deflection at C. $\quad y_C = -t_{B/C}$

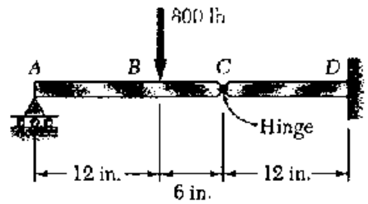
$= -(A_1\bar{x}_1 + A_3\bar{x}_3)$

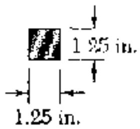
$= -\left[\left(\frac{7.225}{EI}\right)\left(\frac{2}{3}(1.7)\right) - \left(\frac{13.600}{EI}\right)\left(\frac{1.7}{2}\right)\right] = \frac{3.3717}{EI} = \frac{3.3717}{3320}$

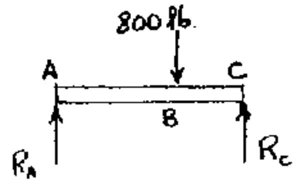
$= 1.016 \times 10^{-3}\ \text{m} = 1.016\ \text{mm} \uparrow$

Problem 9.165

9.165 The two beams shown have the same cross section and are joined by a hinge at C. For the loading shown, determine (*a*) the slope at point A, (*b*) the deflection at point B. Use $E = 29 \times 10^6$ psi.







Using free body ABC,

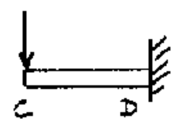
$\circlearrowleft \Sigma M_A = 0: \quad 18 R_C - (12)(800) = 0$

$R_C = 533.33$ lb.

$E = 29 \times 10^6$ psi

$I = \frac{1}{12} b h^3 = \frac{1}{12}(1.25)(1.25)^3 = 0.20345 \text{ in}^4$

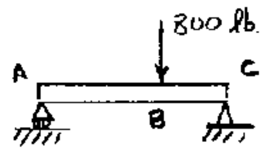
$EI = (29 \times 10^6)(0.20345) = 5.900 \times 10^6 \text{ lb} \cdot \text{in}^2$



Using cantilever beam CD with load R_C,

Case 1 of Appendix D.

$$y_C = -\frac{R_C L_{CD}^3}{3EI} = -\frac{(533.33)(12)^3}{(3)(5.900 \times 10^6)} = -52.067 \times 10^{-3} \text{ in.}$$



Calculation of θ_A' and y_B' assuming that point C does not move.

Case 5 of Appendix D $\qquad P = 800$ lb. $L = 18$ in., $a = 12$ in., $b = 6$ in.

$$\theta_A' = -\frac{Pb(L^2 - b^2)}{6EIL} = -\frac{(800)(6)(18^2 - 6^2)}{(6)(5.900 \times 10^6)(18)} = -2.1695 \times 10^{-3} \text{ rad.}$$

$$y_B' = -\frac{Pb^2a^2}{3EIL} = -\frac{(800)(6)^2(12)^2}{(3)(5.900 \times 10^6)(18)} = -13.017 \times 10^{-3} \text{ in.}$$

Additional slope and deflection due to movement of point C.

$$\theta_A'' = \frac{y_C}{L_{AC}} = -\frac{52.067 \times 10^{-3}}{18} = -2.8926 \times 10^{-3} \text{ rad.}$$

$$y_B'' = \frac{a}{L} y_C = -\frac{(12)(52.067 \times 10^{-3})}{18} = -34.711 \times 10^{-3} \text{ in.}$$

(a) Slope at A. $\qquad \theta_A = \theta_A' + \theta_A'' = -2.1695 \times 10^{-3} - 2.8926 \times 10^{-3}$

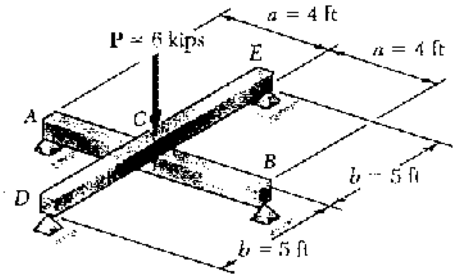
$= -5.06 \times 10^{-3}$ rad $= 5.06 \times 10^{-3}$ rad ◄

(b) Deflection at B. $\qquad y_B = y_B' + y_B'' = -13.017 \times 10^{-3} - 34.711 \times 10^{-3}$

$= -47.7 \times 10^{-3}$ in. $= 0.0477$ in. $\downarrow$ ◄

Problem 9.166

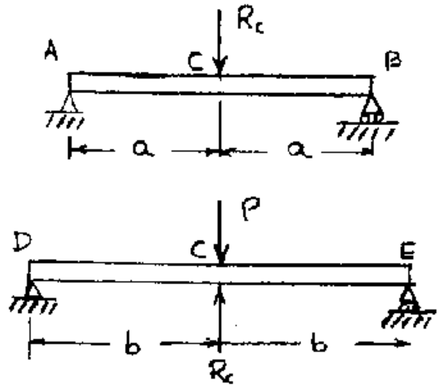
9.166 For the loading shown, and knowing that beams *AB* and *DE* have the same flexural rigidity, determine the reaction (*a*) at *B*, (*b*) at *E*.



Units: Forces in kips; lengths in ft.

For beam ACB, using Case 4 of Appendix D.

$$(y_C)_1 = -\frac{R_C(2a)^3}{48EI}$$



For beam DCE, using Case 4 of Appendix D.

$$(y_C)_2 = \frac{(R_C - P)(2b)^3}{48EI}$$

Matching deflections at C,

$$-\frac{R_C(2a)^3}{48EI} = \frac{(R_C - P)(2b)^3}{48EI}$$

$$R_C = \frac{Pb^3}{a^3 + b^3} = \frac{(6)(5)^3}{4^3 + 5^3} = 3.968 \text{ kips}$$

$$P - R_C = 6 - 3.968 = 2.032 \text{ kips}$$

Using free body ACB, $+\circlearrowleft M_A = 0: \quad 2aR_B - aR_C = 0$

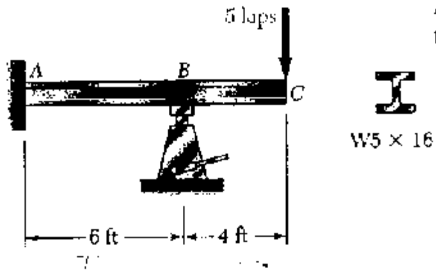
(a) $R_B = \frac{1}{2}R_C = 1.984 \text{ kips} \uparrow$ ◄

Using free body DCE, $+\circlearrowleft M_D = 0: \quad 2bR_E - b(P - R_C) = 0$

(b) $R_E = \frac{1}{2}(P - R_C) = 1.016 \text{ kips} \uparrow$ ◄

Problem 9.167

9.167 A hydraulic jack can be used to raise point B of the cantilever beam ABC. The beam was originally straight, horizontal, and unloaded. A 5-kip load was then applied at point C, causing this point to move down. Determine (*a*) how much point B should be raised to return point C to its original position, (*b*) the final value of the reaction at B. Use $E = E = 29 \times 10^6$ psi.

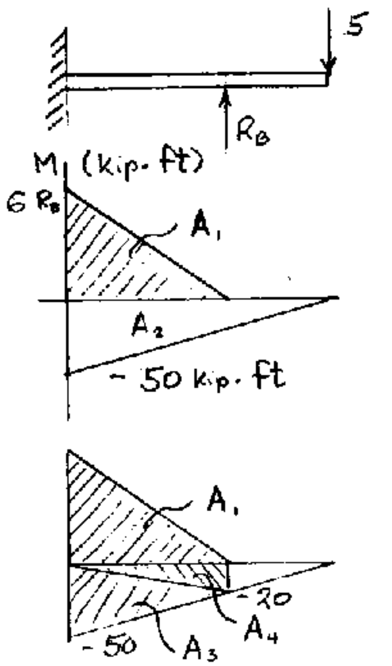


For W 5 × 16 $\qquad I_x = 21.3 \text{ in}^4$

$E = 29 \times 10^6$ psi

$EI = (29\times10^6)(21.3) = 617.7 \times 10^6$ lb·in²

$= 4.2896 \times 10^3$ kip·ft²



Let R_B be the jack force in kips.

$A_1 = \frac{1}{2}(6R_B)(6) = 18 R_B$

$A_2 = \frac{1}{2}(-50)(10) = -250$ kip·ft²

$EI\, t_{C/A} = (4 + 4)A_1 + (\frac{2}{3}\cdot 10)A_2$

$= 144 R_B - 1666.67$

$R_B = \dfrac{1666.67}{144} = 11.5741$ kips

$A_1 = 208.333$ kip·ft²

$A_3 = \frac{1}{2}(-50)(6) = -150$

$A_4 = \frac{1}{2}(-20)(6) = -60$

$EI\, t_{B/A} = 4A_1 + 4A_3 + 2A_4$

$= 113.333$ kip·ft³

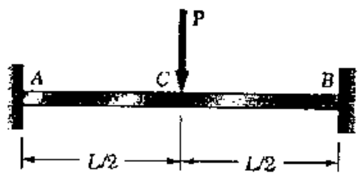
(a) $y_B = t_{B/A} = \dfrac{EI\, t_{B/A}}{EI} = \dfrac{113.333}{4.2896\times10^3} = 26.42\times10^{-3}$

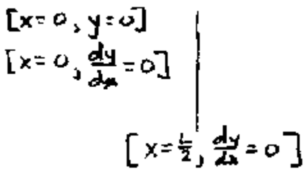
$= 26.42 \times 10^{-3}$ ft $= 0.317$ in. ↑

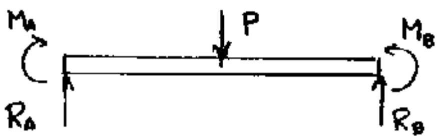
(b) $R_B = 11.57$ kips ↑

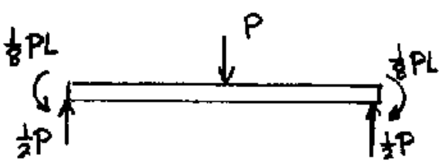
Problem 9.168

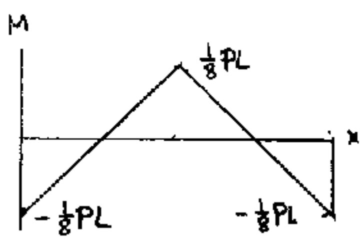
9.168 Determine the reaction at A and draw the bending moment diagram for the beam and loading shown.











By symmetry, $R_A = R_B$ and $\frac{dy}{dx} = 0$ at $x = \frac{L}{2}$.

$+\uparrow \Sigma F_y = 0 \quad R_A + R_B - P = 0 \qquad R_A = R_B = \frac{1}{2}P$ ◄

Moment reaction is statically indeterminate.

$0 < x < \frac{L}{2} \qquad M = M_A + R_A x = M_A + \frac{1}{2}Px$

$EI\frac{d^2y}{dx^2} = M_A + \frac{1}{2}Px$

$EI\frac{dy}{dx} = M_A x + \frac{1}{4}Px^2 + C_1$

$[x=0, \frac{dy}{dx}=0] \qquad 0 - 0 + C_1 = 0 \qquad C_1 = 0$

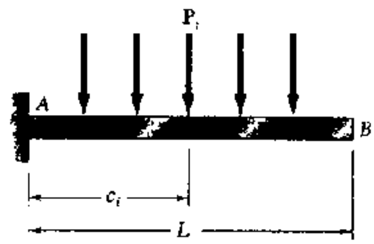
$[x=\frac{L}{2}, \frac{dy}{dx}=0] \qquad M_A\frac{L}{2} + \frac{1}{4}P\left(\frac{L}{2}\right)^2 + 0 = 0$

$M_A = -\frac{1}{8}PL \qquad M_A = \frac{1}{8}PL \circlearrowleft$ ◄

By symmetry, $M_B = M_A = \frac{1}{8}PL \circlearrowright$ ◄

$M_C = M_A + \frac{1}{2}P\frac{L}{2} = -\frac{1}{8}PL + \frac{1}{4}PL = \frac{1}{8}PL$ ◄

PROBLEM 9.C1



9.C1 Several concentrate loads can be applied to the cantilever beam *AB*. Write a computer program to calculate the slope and deflection of beam *AB* from $x = 0$ to $x = L$, using given increments Δx. Apply this program with increments $\Delta x = 50$ mm to the beam and loading of Probs. 9.73 and Prob. 9.74.

SOLUTION

FOR EACH LOAD, ENTER

P_i, c_i

COMPUTE REACTION AT A

FOR $i = 1$ TO NUMBER LOADS

$R_A = R_A + P_i$

$M_A = M_A - P_i c_i$

COMPUTE SLOPE AND DEFLECTION

USE METHOD OF INTEGRATION STARTING WITH $x = 0$ AND UPDATING THROUGH INCREMENTS, SUPERPOSE:

(1) DUE TO REACTION AT A:

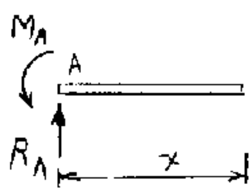
$\theta = (1/EI)(R_A x^2/2.0 + M_A x)$

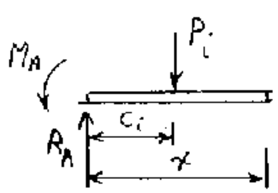
$y = (1/EI)(R_A x^3/6.0 + M_A x^2/2.0)$

(2) DUE TO EACH LOAD WITH $c_i < x$:

$\theta = -(1/EI)(P_i/2.0)(x - c_i)^2$

$y = -(1/EI)(P_i/6.0)(x - c_i)^3$





AT $x = 0$, $y = \dfrac{dy}{dx} = 0$

∴ THE CONSTANTS OF INTEGRATION EQUAL ZERO

CONTINUED

PROBLEM 9.C1 CONTINUED

PROGRAM OUTPUT

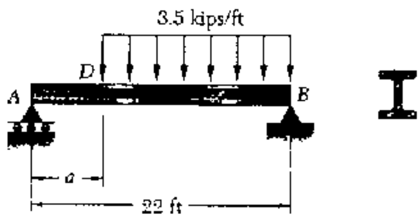
```
Problem  9.73 and 9.74

 At A:  Force =   6.0 kN     Couple = -6.0 kN·m

     x        Slope          Deflection
     m       radians             m

   .00       .000000          .000000
   .05     -.000578         -.000015
   .10     -.001126         -.000057
   .15     -.001645         -.000127
   .20     -.002134         -.000221
   .25     -.002594         -.000340
   .30     -.003024         -.000480
   .35     -.003424         -.000642
   .40     -.003794         -.000822
   .45     -.004135         -.001021
   .50     -.004447         -.001235
   .55     -.004728         -.001465
   .60     -.004980         -.001708
   .65     -.005203         -.001962
   .70     -.005395         -.002227
   .75     -.005558         -.002501  ◀
   .80     -.005699         -.002783
   .85     -.005825         -.003071
   .90     -.005936         -.003365
   .95     -.006033         -.003664
  1.00     -.006114         -.003968
  1.05     -.006181         -.004275
  1.10     -.006233         -.004586
  1.15     -.006270         -.004898
  1.20     -.006292         -.005213
  1.25     -.006299         -.005527  ◀
```

PROBLEM 9.C2



9.C2 The 22-ft beam AB consists of a W 21× 62 rolled-steel shape and supports a 3.5-kip/ft distributed load as shown. Write a computer program and use it to calculate for values of a from 0 to 22 ft, using 1-ft increments, (a) the slope and deflection at D, (b) the location and magnitude of the maximum deflection. Use $E = 29 \times 10^6$ psi.

SOLUTION

ENTER LOAD w, LENGTH L, a

COMPUTE REACTION AT A

$$R_A = w\,(L-a)^2/(2.0\,L)$$

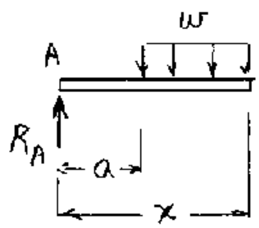
COMPUTE SLOPE AND DEFLECTION AT D

USING SINGULARITY FUNCTIONS:

$$C_1 = -\frac{w}{24L}(L-a)^4 - \frac{1}{6}R_A L^2$$

$$\theta = (1/EI)(R_A a^2/2.0 + C_1)$$

$$y = (1/EI)(R_A a^3/6.0 + C_1 a)$$



$$EI\frac{d^2y}{dx^2} = R_A x - \frac{w}{2}\langle x-a\rangle^2$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{w}{6}\langle x-a\rangle^3 + C_1$$

$$EI\,y = \frac{1}{6}R_A x^3 - \frac{w}{24}\langle x-a\rangle^4 + C_1 x + C_2$$

FROM BOUNDARY CONDITIONS:

$$C_2 = 0$$

$$C_1 = -\frac{w}{24L}(L-a)^4 - \frac{1}{6}R_A L^2$$

COMPUTE LOCATION AND MAGNITUDE OF MAXIMUM DEFLECTION

MAXIMUM y AT $\theta = 0$:

$$0 = \frac{1}{2}R_A x^2 - \frac{w}{6}\langle x-a\rangle^3 + C_1$$

IF $x_{max} \le a$

$$\frac{1}{2}R_A x^2 + C_1 = 0$$

$$x_{max} = \sqrt{\frac{-2.0\,C_1}{R_A}}$$

$$y_{max} = \frac{1}{6}R_A x_{max}^3 + C_1 x_{max}$$

ASSUME $x < A$:

$$x_{max} = (-2.0\,C_1/R_A)^{\frac{1}{2}}$$

IF $x_{max} < a$, THEN

$$y_{max} = (1/EI)\left(\frac{1}{6}R_A x_{max}^3 + C_1 x_{max}\right)$$

IF $x_{max} > a$, THEN

BEGIN WITH $x = a$

$$\theta = (1/EI)\left(\frac{1}{2}R_A x - \frac{1}{6}(x-a)^3 + C_1\right)$$

INCREASE x BY SMALL AMOUNT UNTIL θ IS APPROXIMATELY 0

$$y_{max} = (1/EI)\left(\frac{1}{6}R_A x^3 - \frac{w}{24}(x-a)^4 + C_1 x\right)$$

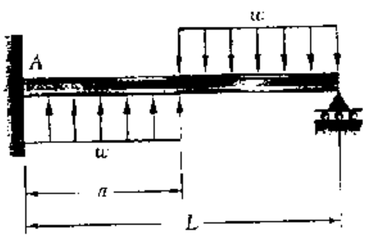
CONTINUED

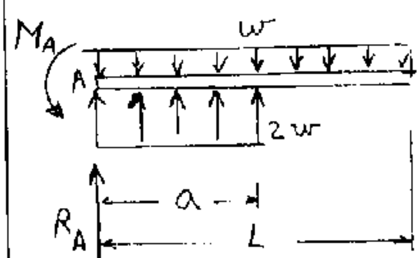
PROBLEM 9.C2 CONTINUED

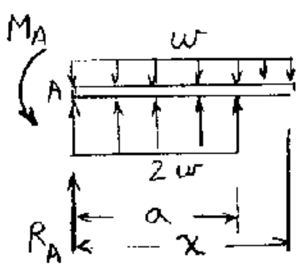
PROGRAM OUTPUT

a ft	theta D radians	yD in.	xm ft	ym in.
0.	-.00580	.000000	11.000	-.478290
1.	-.00569	-.068758	11.008	-.475922
2.	-.00539	-.133047	11.030	-.468860
3.	-.00494	-.189440	11.068	-.457231
4.	-.00439	-.235551	11.121	-.441245
5.	-.00378	-.269927	11.189	-.421192
6.	-.00314	-.291944	11.272	-.397443
7.	-.00250	-.301695	11.370	-.370441
8.	-.00188	-.299889	11.481	-.340699
9.	-.00131	-.287738	11.606	-.308795
10.	-.00080	-.266855	11.742	-.275364
11.	-.00036	-.239145	11.885	-.241090
12.	-.00001	-.206699	12.028	-.206700
13.	.00025	-.171684	12.159	-.172954
14.	.00043	-.136240	12.275	-.140603
15.	.00052	-.102374	12.376	-.110339
16.	.00054	-.071846	12.463	-.082792
17.	.00049	-.046069	12.537	-.056515
18.	.00039	-.026001	12.596	-.037987
19.	.00027	-.012036	12.643	-.021604
20.	.00014	-.003896	12.675	-.009677
21.	.00004	-.000530	12.695	-.002431
22.	.00000	.000000	12.702	.000000

PROBLEM 9.C3







AT $x = 0$, $y = \frac{dy}{dx} = 0$

$\therefore$ THE CONSTANTS OF INTEGRATION ARE ZERO

9.C3 The cantilever beam AB caries the distributed loads shown. Write a computer program to calculate the slope and deflection of beam AB from $x = 0$ to $x = L$ using given increments Δx. Apply this program with increments $\Delta x = 100$ mm, assuming that $L = 2.4$ m, $w = 36$ kN/m, and (*a*) $a = 0.6$ m, (*b*) $a = 1.2$ m, (*c*) $a = 1.8$ m. Use $E = 200$ GPa.

SOLUTION

ENTER w, a, L

COMPUTE REACTION AT A

$$R_A = wL - 2.0wa$$
$$M_A = \frac{1}{2}wL^2 - \frac{1}{2}wa^2$$

COMPUTE SLOPE AND DEFLECTION

USE EQUATION OF ELASTIC CURVE

STARTING WITH $x=0$ AND UPDATING THROUGH INCREMENTS; SUPERPOSE:

(1) DUE TO REACTIONS AT A

$$\theta = (1/EI)\left(\frac{1}{2}R_A x^2 + M_A x\right)$$
$$y = (1/EI)\left(\frac{1}{6}R_A x^3 + \frac{1}{2}M_A x\right)$$

(2) DUE TO LOAD w

$$\theta = -(1/EI)\left(\frac{1}{6}wx^3\right)$$
$$y = -(1/EI)\left(\frac{1}{24}wx^4\right)$$

(3) DUE TO LOAD $2w$

IF $x \le a$

$$\theta = (1/EI)\left(\frac{1}{3}wx^3\right)$$
$$y = (1/EI)\left(\frac{1}{12}wx^4\right)$$

IF $x > a$

$$\theta = (1/EI)\left(\frac{1}{3}wx^3 - \frac{1}{3}w(x-a)^3\right)$$
$$y = (1/EI)\left(\frac{1}{12}wx^4 - \frac{1}{12}w(x-a)^4\right)$$

CONTINUED

PROGRAM OUTPUT

Problem 9.C3 (a) a = 0.6 m

At A: Force = 43.2 kN Couple = -90.7 kN·m

x m	slope radians	deflection m
.00	.000000	.000000
.10	-.000905	-.000046
.20	-.001762	-.000179
.30	-.002567	-.000396
.40	-.003318	-.000691
.50	-.004009	-.001058
.60	-.004638	-.001491
.70	-.005202	-.001983
.80	-.005703	-.002529
.90	-.006145	-.003122
1.00	-.006533	-.003756
1.10	-.006868	-.004427
1.20	-.007156	-.005128
1.30	-.007399	-.005856
1.40	-.007602	-.006607
1.50	-.007769	-.007376
1.60	-.007902	-.008160
1.70	-.008006	-.008955
1.80	-.008083	-.009760
1.90	-.008139	-.010571
2.00	-.008177	-.011387
2.10	-.008199	-.012206
2.20	-.008211	-.013027
2.30	-.008215	-.013848
2.40	-.008216	-.014669

Problem 9.C3 (b) a = 1.2 m

At A: Force = 0.0 kN Couple = -51.8 kN·m

x m	slope radians	deflection m
.00	.000000	.000000
.10	-.000529	-.000026
.20	-.001055	-.000106
.30	-.001574	-.000237
.40	-.002081	-.000420
.50	-.002574	-.000653
.60	-.003048	-.000934
.70	-.003500	-.001262
.80	-.003926	-.001633
.90	-.004323	-.002046
1.00	-.004687	-.002497
1.10	-.005014	-.002982
1.20	-.005301	-.003498
1.30	-.005544	-.004041
1.40	-.005747	-.004606
1.50	-.005913	-.005189
1.60	-.006047	-.005787
1.70	-.006150	-.006398
1.80	-.006228	-.007017
1.90	-.006284	-.007642
2.00	-.006321	-.008273
2.10	-.006344	-.008906
2.20	-.006356	-.009541
2.30	-.006360	-.010177
2.40	-.006361	-.010813

CONTINUED

PROBLEM 9.C3 PROGRAM OUTPUTS CONTINUED

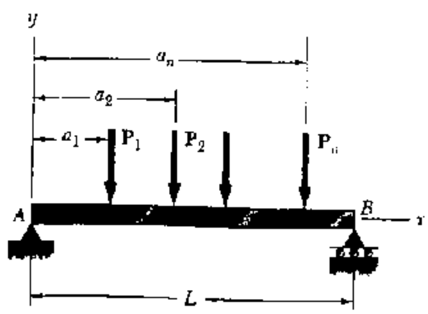
Problem 9.C3 (c) a = 1.8 m

At A: Force = -43.2 kN Couple = 13.0 kN·m

x m	slope radians	deflection m
.00	.000000	.000000
.10	.000111	.000006
.20	.000182	.000021
.30	.000215	.000041
.40	.000216	.000063
.50	.000187	.000083
.60	.000133	.000099
.70	.000056	.000109
.80	-.000039	.000110
.90	-.000149	.000101
1.00	-.000270	.000080
1.10	-.000398	.000046
1.20	-.000530	.000000
1.30	-.000662	-.000060
1.40	-.000790	-.000132
1.50	-.000911	-.000217
1.60	-.001021	-.000314
1.70	-.001116	-.000421
1.80	-.001193	-.000537
1.90	-.001248	-.000659
2.00	-.001286	-.000786
2.10	-.001309	-.000916
2.20	-.001320	-.001047
2.30	-.001325	-.001179
2.40	-.001325	-.001312

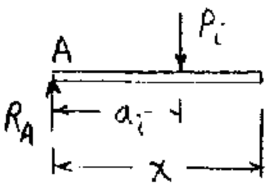
PROBLEM 9.C4

9.C4 The simple beam *AB* is of constant flexural rigidity *EI* and carries several concentrated loads as shown. Using the *Method of Integration*, write a computer program that can be used to calculate the slope and deflection at points along the beam from $x = 0$ to $x = L$ using given increments Δx. Apply this program to the beam and loading of (*a*) Prob. 9.13 with $\Delta x = 1$ ft, (*b*) Prob. 9.16 with $\Delta x = 0.05$ m, (*c*) Prob. 9.129 $\Delta x = 0.25$ m.



SOLUTION

FOR LOAD P_i:



FOR $x < a_i$

$$EI \frac{d^2y}{dx^2} = R_A x$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 + C_1$$

$$EI\, y = \frac{1}{6} R_A x^3 + C_1 x + C_2$$

FOR $x > a_i$

$$EI \frac{d^2y}{dx^2} = R_A x - P_i(x - a_i)$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - \frac{1}{2} P_i (x - a_i)^2 + C_3$$

$$EI\, y = \frac{1}{6} R_A x^3 - \frac{1}{6} (x - a_i)^3 + C_3 x + C_4$$

FROM BOUNDARY CONDITIONS

$$C_2 = C_4 = 0$$

$$C_1 = C_3 = \frac{P_i}{6L} (L - a_i)^3 - \frac{1}{6} R_A L^2$$

NOTE: R_A FOR LOAD P_i

FOR EACH LOAD, ENTER P_i, a_i

COMPUTE REACTION AT A

FOR $i = 1$ TO NUMBER LOADS:

$M_A = M_A + P_i\, a_i$

$LOAD = LOAD + P_i$

THEN:

$R_B = M_A / L$

$R_A = LOAD - R_B$

COMPUTE SLOPE AND DEFLECTION

STARTING WITH $x = 0$ AND UPDATING THROUGH INCREMENTS, SUPERPOSE:

(1) DUE TO REACTION AT A

$\theta = (1/EI)\left(\frac{1}{2} R_A x^2\right)$

$y = (1/EI)\left(\frac{1}{6} R_A x^3\right)$

(2) DUE TO LOADS – CONSTANT PART

$CONST_1 = -\frac{1}{6} R_A L^2$

FOR 1 TO NUMBER LOADS

$CONST_2 = \frac{1}{6L} P_i (L - a_i)^3 + CONST_2$

THEN, TOTAL CONTRIBUTION FOR CONSTANT

$CONST = (1/EI)(CONST_1 + CONST_2)$

(3) DUE TO LOADS – REMAINING PART

IF $x \leq a_i$

$\theta = (1/EI)\left(\frac{1}{2.0} R_A x^2\right)$

$y = (1/EI)\left(\frac{1}{6.0} R_A x^3\right)$

IF $x > a_i$

$\theta = (1/EI)\left(\frac{1}{2.0} R_A x^2 - \frac{1}{2.0} P_i (x - a_i)^2\right)$

$y = (1/EI)\left(\frac{1}{6.0} R_A x^3 - \frac{1}{6.0} P_i (x - a_i)^3\right)$

CONTINUED

PROBLEM 9.C4 CONTINUED

PROGRAM OUTPUT

Problem 9.13

x in.	theta rad*10**3	y in.
.00	-8.2948	.000000
12.00	-8.0957	-.098742
24.00	-7.4985	-.192705
36.00	-6.5031	-.277113
48.00	-5.1096	-.347188
60.00	-3.3179	-.398151
72.00	-1.4267	-.426420
84.00	.2654	-.433189
96.00	1.7585	-.420846
108.00	3.0525	-.391781
120.00	4.1474	-.348383
132.00	5.0433	-.293039
144.00	5.7400	-.228141
156.00	6.2377	-.156075
168.00	6.5363	-.079232
180.00	6.6359	.000000

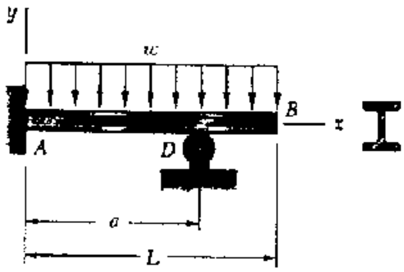
Problem 9.129

x m	theta rad*10**3	y mm	
.000	-8.703	.000	(a)
.250	-8.615	-2.168	
.500	-8.351	-4.293	
.750	-7.911	-6.329	
1.000	-7.296	-8.234	
1.250	-6.505	-9.962	
1.500	-5.538	-11.472	
1.750	-4.483	-12.724	
2.000	-3.428	-13.713	
2.250	-2.373	-14.438	
2.500	-1.319	-14.900	
2.750	-.264	-15.098	
3.000	.791	-15.032	(b)
3.250	1.802	-14.706	
3.500	2.725	-14.138	
3.750	3.560	-13.350	
4.000	4.307	-12.365	
4.250	4.967	-11.204	
4.500	5.538	-9.889	
4.750	6.021	-8.442	
5.000	6.417	-6.886	
5.250	6.725	-5.241	
5.500	6.944	-3.531	
5.750	7.076	-1.776	
6.000	7.120	.000	

Problem 9.16

x m	theta rad*10**3	y mm
.000	-2.490	.000
.050	-2.485	-.124
.100	-2.471	-.248
.150	-2.448	-.371
.200	-2.416	-.493
.250	-2.375	-.613
.300	-2.325	-.730
.350	-2.265	-.845
.400	-2.197	-.957
.450	-2.119	-1.065
.500	-2.032	-1.168
.550	-1.936	-1.268
.600	-1.831	-1.362
.650	-1.716	-1.451
.700	-1.593	-1.533
.750	-1.460	-1.610
.800	-1.318	-1.679
.850	-1.172	-1.741
.900	-1.025	-1.796
.950	-.879	-1.844
1.000	-.732	-1.884
1.050	-.586	-1.917
1.100	-.439	-1.943
1.150	-.293	-1.961
1.200	-.146	-1.972
1.250	.000	-1.976
1.300	.146	-1.972
1.350	.293	-1.961
1.400	.439	-1.943
1.450	.586	-1.917
1.500	.732	-1.884
1.550	.879	-1.844
1.600	1.025	-1.796
1.650	1.172	-1.741
1.700	1.318	-1.679
1.750	1.460	-1.610
1.800	1.593	-1.533
1.850	1.716	-1.451
1.900	1.831	-1.362
1.950	1.936	-1.268
2.000	2.032	-1.168
2.050	2.119	-1.065
2.100	2.197	-.957
2.150	2.265	-.845
2.200	2.325	-.730
2.250	2.375	-.613
2.300	2.416	-.493
2.350	2.448	-.371
2.400	2.471	-.248
2.450	2.485	-.124
2.500	2.490	.000

PROBLEM 9.C5



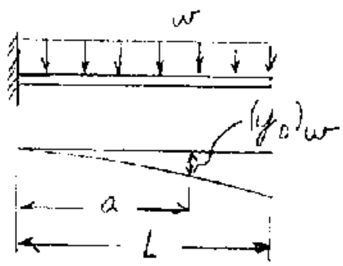
9.C5 The supports of beam AB consist of a fixed support at end A and a roller support located at point D. Write a computer program that can be used to calculate the slope and deflection at the free end of the beam for values of a from 0 to L using given increments Δa. Apply this program to calculate the slope and deflection at point B for each of the following cases:

	L	ΔL	w	E	Shape
(a)	12 ft	0.5 ft	1.6 k/ft	29×10^6 psi	W 16×57
(b)	3 m	0.2m	18 kN/m	200 GPa	W460×113

SOLUTION

BEAM IS INDETERMINATE

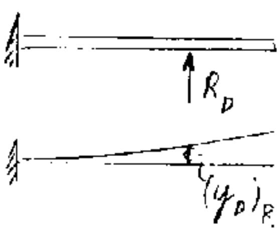
USE APPENDIX D AND SUPERPOSITION



DETERMINE REACTION AT D

DUE TO DISTRIBUTED LOAD

$$(y_D)_w = -\frac{w}{24EI}\left(a^4 - 4La^3 + 6L^2a^2\right)$$



DUE TO REDUNDANT LOAD:

$$(y_D)_R = \frac{R_D L^3}{3EI}$$

REDUNDANT REACTION:

SINCE $(y_D)_w + (y_D)_R = 0$:

$$R_D = \frac{3EI}{L^3}(y_D)_w$$

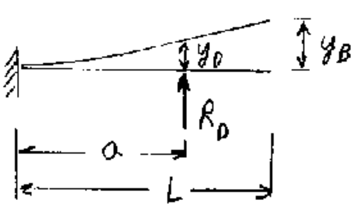
COMPUTE SLOPE AND DEFLECTION AT B

SUPERPOSE:

DUE TO DISTRIBUTED LOAD:

$$\theta_B = -\frac{wL^3}{6EI}$$

$$y_B = -\frac{wL^4}{8EI}$$



DUE TO R_D:

$$\theta_B = \frac{Pa^2}{2EI}$$

$$y_B = \frac{Pa^3}{3EI} + (L-a)\frac{Pa^2}{2EI}$$

$$\theta_B = \theta_D$$

$$y_B = y_D + (L-a)\theta_D$$

CONTINUED

PROBLEM 9.C5 CONTINUED

PROGRAM OUTPUT

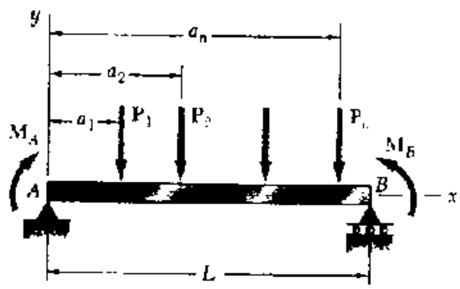
Problem 9.C5 (a)

a ft	theta B rad*10^-3	y at B in.
.0	-3.019	-.3260
.5	-2.743	-.2869
1.0	-2.483	-.2511
1.5	-2.238	-.2183
2.0	-2.007	-.1885
2.5	-1.790	-.1614
3.0	-1.586	-.1369
3.5	-1.395	-.1149
4.0	-1.216	-.0953
4.5	-1.049	-.0778
5.0	-.893	-.0624
5.5	-.748	-.0490
6.0	-.613	-.0374
6.5	-.488	-.0274
7.0	-.373	-.0191
7.5	-.266	-.0122
8.0	-.168	-.0067
8.5	-.077	-.0025
9.0	.006	.0006
9.5	.082	.0027
10.0	.152	.0037
10.5	.216	.0039
11.0	.274	.0033
11.5	.328	.0020
12.0	.377	.0000

Problem 9.C5 (b)

a m	theta B rad*10^-3	y at B mm
.0	-.728	-1.6389
.2	-.624	-1.3324
.4	-.529	-1.0663
.6	-.442	-.8374
.8	-.364	-.6426
1.0	-.293	-.4789
1.2	-.230	-.3435
1.4	-.174	-.2338
1.6	-.124	-.1472
1.8	-.079	-.0813
2.0	-.040	-.0337
2.2	-.006	-.0024
2.4	.023	.0149
2.6	.049	.0198
2.8	.072	.0143

PROBLEM 9.C6

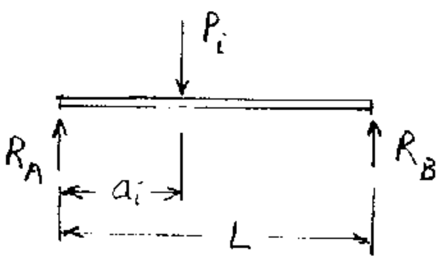


9.C6 For the beam and loading shown, use the *Moment-Area Method* to write a computer to calculate the slope and deflection at points along the beam from $x = 0$ to $x = L$ using given increments Δx. Apply this program to calculate the slope and deflection at each concentrated load for the beam of (*a*) Prob. 9.77 with $\Delta x = 0.5$ m, (*b*) Prob. 9.119 with $\Delta x - 0.5$ m.

SOLUTION

ENTER M_A AND M_B

FOR EACH LOAD ENTER P_i AND a_i



DETERMINE REACTION AT A

DUE TO MOMENTS AT ENDS:

$(R_A)_1 = -(M_A - M_B)/L$

DUE TO LOADS P_i:

FOR $i = 1$ TO NUMBER OF LOADS

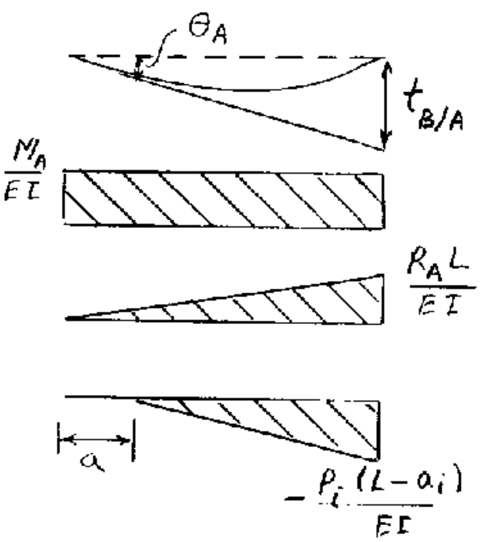
$R_B = R_B + P_i\, a_i/L$

LOAD = LOAD + P_i

$(R_A)_2 = \text{LOAD} - R_B$

$R_A = (R_A)_1 + (R_A)_2$

DETERMINE SLOPE AT A



USE SECOND MOMENT-AREA THEOREM TO GET TANGENTIAL DEVIATION AT B

DUE TO M_A:

$t_{B/A} = M_A L^2/(2.0\,EI)$

DUE TO R_A:

$t_{B/A} = R_A L^3/(6.0\,EI)$

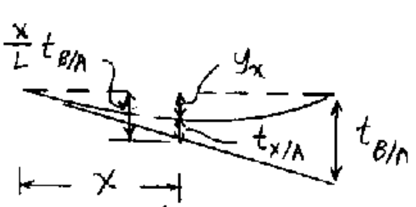
DUE TO LOADS P_i:

FOR $i = 1$ TO NUMBER OF LOADS

$t_{B/A} = -P_i(L - a_i)^3/(6.0\,EI)$

SUM $t_{B/A}$:

$\theta_A = -t_{B/A}/L$



$y = \frac{x}{L} t_{B/A} - t_{x/A}$

DETERMINE SLOPE AND DEFLECTIONS

FOR $x = 0$ TO L, SUPERPOSE:

DUE TO M_A AND R_A:

$\theta_x = \theta_A + (M_A x + R_A x^2/2.0)/EI$

CONTINUED

PROBLEM 9.C6 CONTINUED

$$y_x = \frac{x}{L} t_{B/A} - M_A x^2/(2.0EI) - R_A x^3/(6.0EI)$$

DUE TO LOADS P_i :

DO FOR ALL LOADS WITH $a_i < x$

$$\theta_x = P_i (x - a_i)^2/(2.0\,EI)$$

$$y_x = P_i (x - a_i)^3/((6.0\,EI)$$

PROGRAM OUTPUT

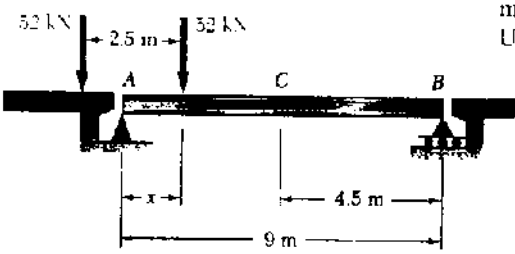
Problem 9.77

x m	theta rad*1000	y at x mm	
.000	-.600962	.000000	◄ (a)
.500	-1.602564	.574252	
1.000	-2.043269	1.509081	
1.500	-1.923077	2.524039	
2.000	-1.241987	3.338675	
2.500	.000000	3.672543	◄ (b)
3.000	1.241987	3.338676	
3.500	1.923077	2.524039	
4.000	2.043269	1.509082	
4.500	1.602564	.574253	
5.000	.600962	.000000	

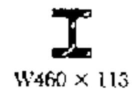
Problem 9.119

x m	theta rad*1000	y at x mm	
.000	-4.504505	.000000	◄ (a)
.500	-4.673423	2.317943	
1.000	-4.279279	4.579579	
1.500	-3.322072	6.503378	
2.000	-1.801802	7.807808	
2.500	.000000	8.258258	◄ (b)
3.000	1.801802	7.807808	
3.500	3.322072	6.503378	
4.000	4.279279	4.579579	
4.500	4.673423	2.317943	
5.000	4.504505	.000000	

PROBLEM 9.C7



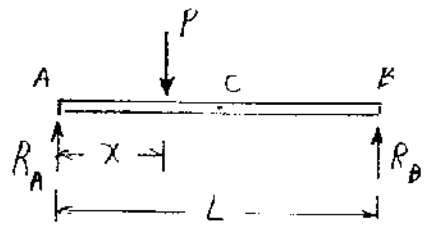
9.C7 Two 52 kN loads are maintained 2.5 m apart as they are moved slowly across beam AB. Write a computer program to calculate the deflection at the midpoint C of the beam for values of x from 0 to 9 m, using 0.5-m increments. Use $E = 200$ GPa.



SOLUTION

ENTER LOAD P, BEAM LENGTH L AND SPACE BETWEEN LOADS D

WILL SOLVE WITH MOMENT-AREA METHOD



DETERMINE DEFLECTION AT C

FOR $x = 0$ TO L

IF $0 \le x \le D$:

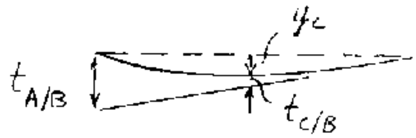
HAVE ONE LOAD TO LEFT OF C

$$R_B = Px/L$$

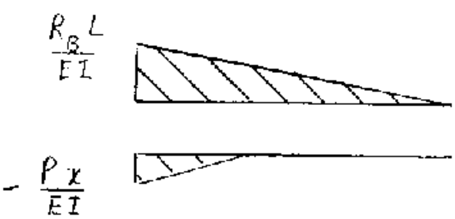
$$t_{A/B} = (R_B L^3 - Px^3)/(6.0EI)$$

$$t_{C/B} = R_B L^3/(48.0EI)$$

$$y_C = \frac{1}{2} t_{A/B} - t_{C/B}$$



$$y_C = \frac{1}{2} t_{A/B} - t_{C/B}$$



IF $D < x \le L/2$

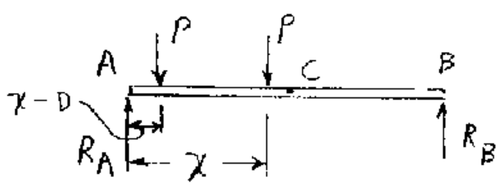
HAVE TWO LOADS TO LEFT OF C

$$R_B = Px/L + P(x-D)/L$$

$$t_{A/B} = (R_B L^3 - Px^3 - P(x-D)^3)/(6.0EI)$$

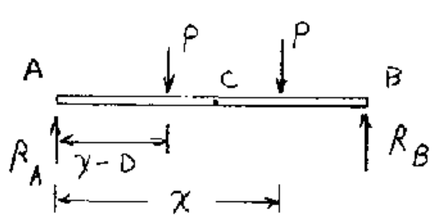
$$t_{C/B} = R_B L^3/(48.0EI)$$

$$y_C = \frac{1}{2} t_{A/B} - t_{C/B}$$



IF $L/2 < x \le (L/2 + D)$

HAVE ONE LOAD TO LEFT OF C AND ONE TO RIGHT OF C OR AT C



CONTINUED

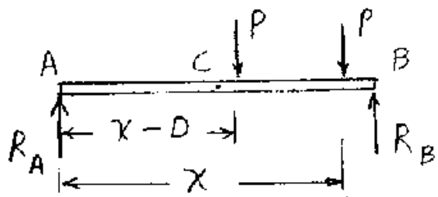
PROBLEM 9.C7 CONTINUED

$$R_B = Px/L + P(x-D)/L$$

$$t_{A/B} = (R_B L^3 - Px^3 - P(x-D)^3)/(6.0EI)$$

$$t_{C/B} = (R_B L^3/48.0 - P(x-\tfrac{L}{2})^3/6.0)/EI$$

$$y_C = \frac{1}{2} t_{A/B} - t_{C/B}$$



IF $(L/2 + D) < x < L$

HAVE BOTH LOADS TO RIGHT OF C

$$R_B = Px/L + P(x-D)/L$$

$$t_{A/B} = (R_B L^3 - Px^3 - P(x-D)^3)/(6.0EI)$$

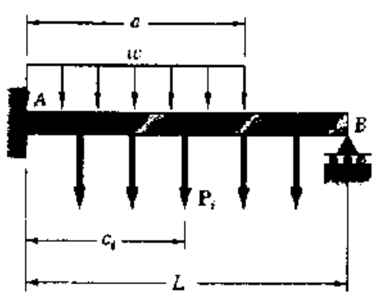
$$t_{C/B} = (R_B L^3/48.0 - P(x-\tfrac{L}{2})^3/6.0 - P(x-D-\tfrac{L}{2})^3/6.0)/EI$$

$$y_C = \frac{1}{2} t_{A/B} - t_{C/B}$$

PROGRAM OUTPUT

x m	RB kN	ThetaB rad	YC mm
.0	.000	.00000	.00000
.5	2.889	.00315	1.17881
1.0	5.778	.00624	2.32839
1.5	8.667	.00921	3.41951
2.0	11.556	.01200	4.42296
2.5	14.444	.01456	5.30950
3.0	20.222	.01998	7.22872
3.5	26.000	.02499	8.94335
4.0	31.778	.02947	10.39493
4.5	37.556	.03331	11.52503
5.0	43.333	.03639	12.28492
5.5	49.111	.03859	12.66487
6.0	54.889	.03980	12.66487
6.5	60.667	.03989	12.28492
7.0	66.444	.03876	11.52503
7.5	72.222	.03629	10.39493
8.0	78.000	.03235	8.94335
8.5	83.778	.02684	7.22872
9.0	89.556	.01963	5.30950

PROBLEM 9.C8



9.C8 A uniformly distributed load w and several distributed loads P_i may be applied to the cantilever beam AB. Write a computer program to determine the reaction at the roller support and apply this program to the beam and loading of (*a*) Prob. 9.53*a*, (*b*) Prob. 9.154.

SOLUTION

THE BEAM IS INDETERMINATE

USE EQUATION OF ELASTIC CURVE

ENTER w AND FOR EACH LOAD P_i AND c_i

COMPUTE DISPLACEMENT AT B DUE TO LOADS

REACTION AT A:

DUE TO w

$$R_A = wa$$

$$M_A = \frac{1}{2} w a^2$$

FOR $i = 1$ TO NUMBER LOADS P_i

$$R_A = R_A - P_i$$

$$M_A = M_A - P_i c_i$$

FOR DISPLACEMENT AT B, SUPERPOSE:

DUE TO REACTION AT A

$$EI\, y_B = \frac{1}{6} R_A L^3 + \frac{1}{2} M_A L^2$$

DUE TO DISTRIBUTED LOADS

$$EI\, y_B = \frac{1}{24} \left(-wL^4 + w(L-a)^4\right)$$

DUE TO P_i

FOR $i = 1$ TO NUMBER LOADS

$$EI\, y_B = \frac{1}{6} P_i (L - c_i)^3$$

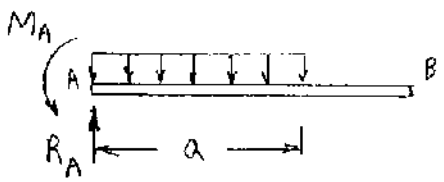
COMPUTE DISPLACEMENT AT B DUE TO UNIT R_B

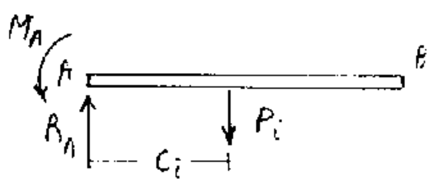
$$EI(y_B)_{UNIT} = \frac{1}{3} L^3$$

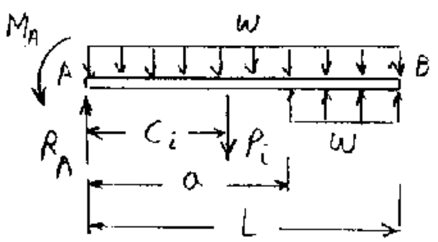
COMPUTE REACTION AT B

FROM $EI\, y_B + R_B EI (y_B)_{UNIT} = 0$

$$R_B = -y_B/(y_B)_{UNIT}$$

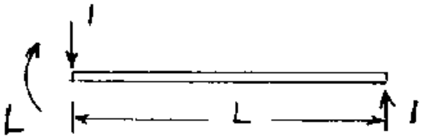






AT $x = 0$, $y = \frac{dy}{dx} = 0$

$\therefore$ THE CONSTANTS OF INTEGRATION ARE ZERO



$$EI \frac{d^2y}{dx^2} = -x + L$$

$$EI \frac{dy}{dx} = -\frac{1}{2}x^2 + Lx + C_1$$

$$EI\, y = -\frac{1}{6}x^3 + \frac{1}{2}Lx^2 + C_1 x + C_2$$

BOUNDARY CONDITIONS GIVE $C_1 = C_2 = 0$

CONTINUED

PROGRAM OUTPUT

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Problem 9.53 (a)

    Reaction at Roller Support = 11.5356 kN

Problem 9.154

    Reaction at Roller Support = 10.1758 kN
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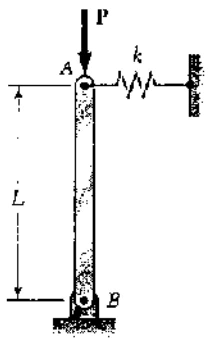
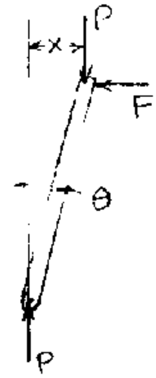
Chapter 10

Problem 10.1

10.1 Knowing that the spring at A is of constant k and that the bar AB is rigid, determine the critical load P_{cr}.





Let θ be the angle change of bar AB.

$$F = kx = kL\sin\theta$$

$$\circlearrowleft \Sigma M_B = 0: \quad FL\cos\theta - Px = 0$$

$$kL^2\sin\theta\cos\theta - PL\sin\theta = 0$$

Using $\sin\theta \approx \theta$ and $\cos\theta \approx 1$, $\quad kL^2\theta - PL\theta = 0$

$$(kL^2 - PL)\theta = 0 \qquad P_{cr} = kL$$

Problem 10.2

10.2 Knowing that the torsional spring at B is of constant K and that the bar AB is rigid, determine the critical load P_{cr}.

Let θ be the angle change of bar AB.

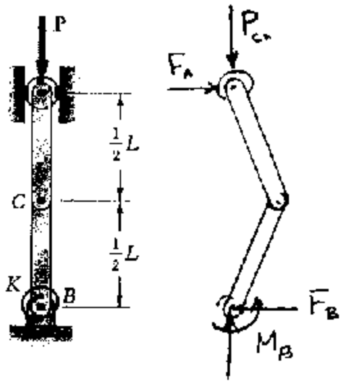
$$M = K\theta, \qquad x = L\sin\theta \approx L\theta$$

$$\circlearrowleft M_B = 0 \qquad M - Px = 0 \qquad K\theta - PL\theta = 0$$

$$(K - PL)\theta = 0 \qquad P_{cr} = K/L$$

Problem 10.3

10.3 Two rigid bars AC and BC are connected by a pin at C as shown. Knowing that the torsional spring at B is of constant K, determine the critical load P_{cr} for the system.

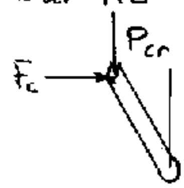


Let θ be the angle change of each bar.

$M_B = K\theta$

$+\circlearrowleft M_B = 0 \qquad K\theta - F_A L = 0 \qquad F_A = \dfrac{K\theta}{L}$

Bar AC $\quad +\circlearrowleft \Sigma M_C = 0:$

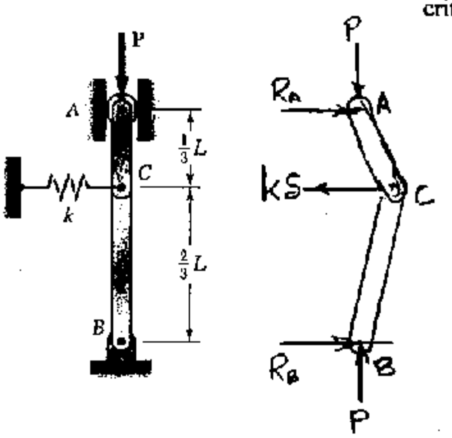


$$P_{cr}\,\tfrac{1}{2}L\theta - \tfrac{1}{2}L F_A = 0$$

$$P_{cr} = \frac{F_A}{\theta} = \frac{K}{L}$$

Problem 10.4

10.4 Two rigid bars AC and BC are connected as shown to a spring of constant k. Knowing that the spring can act in either tension or compression, determine the critical load P_{cr} for the system.



Let δ be the deflection of point C.

Using free body AC and $+\circlearrowleft \Sigma M_C = 0:$

$$-\tfrac{1}{3}LR_A + P\delta = 0 \qquad R_A = \frac{3P\delta}{L}$$

Using free body BC and $+\circlearrowleft \Sigma M_C = 0:$

$$\tfrac{2}{3}LR_B - P\delta = 0 \qquad R_B = \frac{3P\delta}{2L}$$

Using both free bodies together,

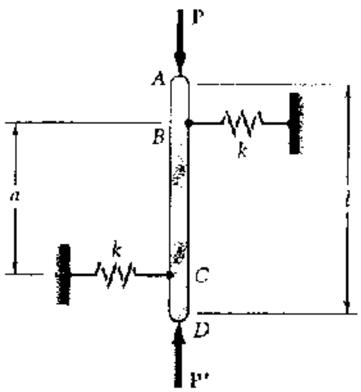
$$\xrightarrow{+} \Sigma F_x = 0: \quad R_A + R_B - k\delta = 0$$

$$\frac{3P\delta}{L} + \frac{3P\delta}{2L} - k\delta = 0$$

$$\left(\frac{9}{2}\frac{P}{L} - k\right)\delta = 0$$

$$P_{cr} = \frac{2kL}{9}$$

Problem 10.5



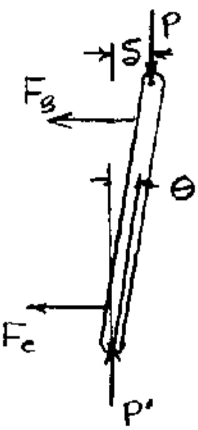
10.5 The rigid bar AD is attached to two springs of constant k and is in equilibrium in the position shown. Knowing that the equal and opposite loads **P** and **P'** *remain vertical*, determine the magnitude P_{cr} of the critical load for the system. Each spring can act in either tension or compression.

Let x_B and x_C be the deflections of points B and C, positive $\rightarrow$.

Then, $F_B = -k x_B$ and $F_C = -k x_C$

$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad -F_B - F_C = 0 \qquad F_C = -F_B$

$x_C = -x_B \qquad F_B$ and F_C form a couple $\circlearrowleft$.



Let θ be the angle change. $y_B = -y_C = \frac{1}{2} a \sin\theta$

$\delta = l \sin\theta$

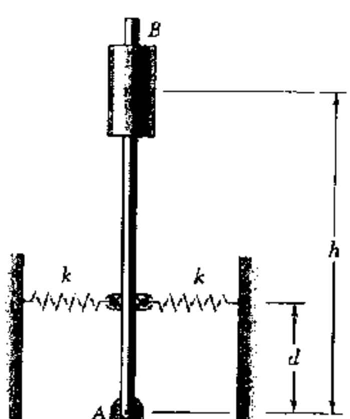
P and P' form a couple $P\delta$ $\circlearrowright$

$+\circlearrowleft \Sigma M = 0: \quad k\left(\frac{1}{2} a \sin\theta\right) a \cos\theta - P l \sin\theta = 0$

$$P = \frac{ka^2 \cos\theta}{2l}$$

Let $\theta \rightarrow 0$ $\qquad P_{cr} = \dfrac{ka^2}{2l}$ ◄

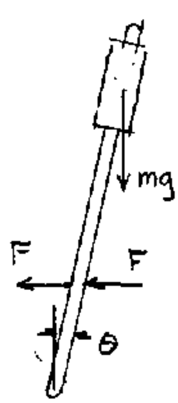
Problem 10.6



10.6 If $m = 125$ kg, $h = 700$ mm, and the constant of each spring is $k = 2.8$ kN/m, determine the range of values of the distance d for which the equilibrium of the rigid rod AB is stable in the position shown. Each spring can act in either tension or compression.

$h = 700 \text{ mm.} = 700 \times 10^{-3}$ m

Let θ be the small rotation of AB.



$x = d\theta \qquad F = kx = kd\theta$

$+\circlearrowleft \Sigma M_A = 0: \quad 2Fd - mgh\theta = 0$

$2kd^2\theta - mgh = 0$

$$d_{cr}^2 = \frac{mgh}{2k}$$

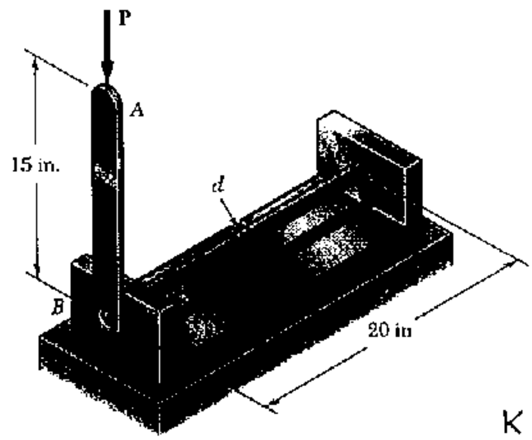
$$d_{cr} = \sqrt{\frac{mgh}{k}} = \sqrt{\frac{(125)(9.81)(700 \times 10^{-3})}{(2)(2.8 \times 10^3)}}$$

$= 0.392$ m $= 392$ mm

$d > 392$ mm for stability ◄

Problem 10.7

10.7 The steel rod BC is attached to the rigid bar AB and to the fixed support at C. Knowing that $G = 11.2 \times 10^6$ psi, determine the critical load P_{cr} of the system when $d = \frac{1}{2}$ in.



Look at torsion spring BC.

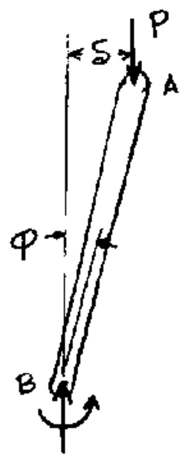
$$\varphi = \frac{TL}{GJ} \qquad T = \frac{GJ}{L}\varphi = K\varphi$$

$$G = 11.2 \times 10^6 \text{ psi}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi}{2}\left(\frac{1}{4}\right)^4 = 6.1359 \times 10^{-3} \text{ in}^4$$

$$L = 20 \text{ in.}$$

$$K = \frac{GJ}{L} = \frac{(11.2\times10^6)(6.1359\times10^{-3})}{20} = 3436.1 \text{ lb}\cdot\text{in/rad}$$



$$\delta = l \sin\varphi = 15 \sin\varphi$$

$$+\circlearrowleft \Sigma M_B = 0: \ -P\delta + T = 0$$

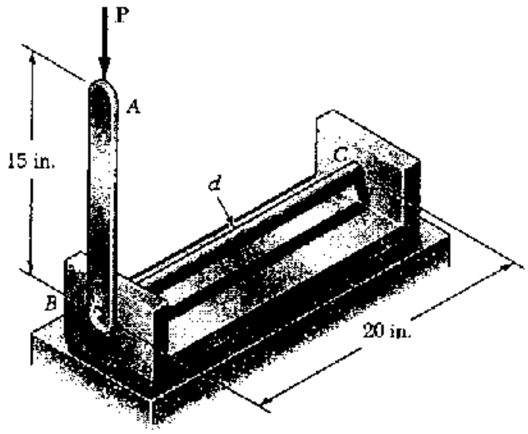
$$P = \frac{T}{\delta} = \frac{K\varphi}{l \sin\varphi} = \frac{3436.1\ \varphi}{15 \sin\varphi} = 229\frac{\varphi}{\sin\varphi} \text{ lb.}$$

Let $\varphi \to 0$.

$$P_{cr} = 229 \text{ lb.}$$

Problem 10.8

10.8 The steel rod BC is attached to the rigid bar AB and to the fixed support at C. Knowing that $G = 11.2 \times 10^6$ psi, determine the diameter of rod BC for which the critical load P_{cr} of the system is 80 lb.



Look at torsion spring BC.

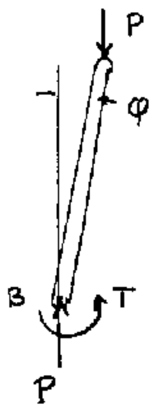
$$\varphi = \frac{TL}{GJ} \qquad T = \frac{GJ}{L}\varphi = K\varphi$$

$$G = 11.2 \times 10^6 \text{ psi}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{32}$$

$$L = 20 \text{ in}$$

$$K = \frac{(11.2 \times 10^6)\pi d^4}{(20)(32)} = 54978\ d^4$$



$$\circlearrowleft \Sigma M_B = 0:$$

$$T - P\ell \sin\varphi = 0$$

$$K\varphi - P\ell \sin\varphi = 0$$

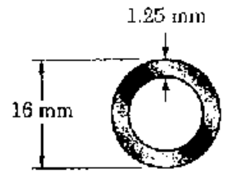
$$P = \frac{K\varphi}{\ell \sin\varphi} \qquad P_{cr} = \frac{K}{\ell}$$

$$K = 54978\ d^4 = P_{cr}\ell = (80)(15) = 1200$$

$$d = \sqrt[4]{\frac{1200}{54978}} = 0.384 \text{ in.}$$

Problem 10.9

10.9 Determine the critical load of an aluminum tube that is 1.5 m long and has a 16-mm outer diameter and a 1.25 mm wall thickness. Use $E = 70$ GPa.



$c_o = \frac{1}{2} d_o = \frac{1}{2}(16) = 8$ mm $\quad c_i = c_o - t = 8 - 1.25 = 6.75$ mm

$$I = \frac{\pi}{4}(c_o^4 - c_i^4) = \frac{\pi}{4}(8^4 - 6.75^4) = 1.58655 \times 10^3 \text{ mm}^4$$
$$= 1.58655 \times 10^{-9} \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2(70 \times 10^9)(1.58655 \times 10^{-9})}{(1.5)^2}$$
$$= 487$$

$P_{cr} = 487$ N

Problem 10.10

10.10 Determine the critical load of a round wooden dowel that is 48 in. long and has a diameter of (*a*) 0.375 in., (*b*) 0.5 in. Use $E = 1.6 \times 10^6$ psi.

(a) $c = \frac{1}{2} d = 0.1875$ in $\quad I = \frac{\pi}{4} c^4 = 970.7 \times 10^{-6}$ in^4

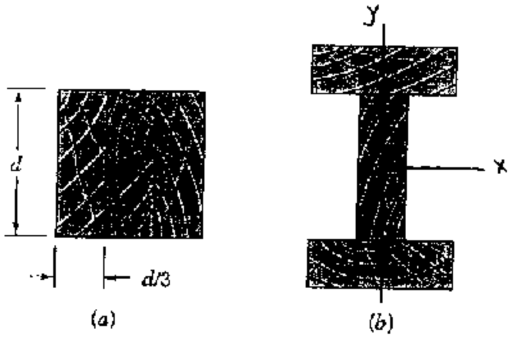
$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2(1.6 \times 10^6)(970.7 \times 10^{-6})}{(48)^2} = 6.65 \text{ lb.}$$

(b) $c = \frac{1}{2} d = 0.25$ in $\quad I = \frac{\pi}{4} c^4 = 3.068 \times 10^{-3}$ in^4

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2(1.6 \times 10^6)(3.068 \times 10^{-3})}{(48)^2} = 21.0 \text{ lb.}$$

Problem 10.11

10.11 A column of effective length L can be made by gluing together identical planks in either of the arrangements shown. Determine the ratio of the critical load using the arrangement a to the critical load using the arrangement b.



Arrangement (a).

$$I_a = \frac{1}{12} d^4$$

$$P_{cr,a} = \frac{\pi^2 E I}{L_e^2} = \frac{\pi^2 E d^4}{12 L_e^2}$$

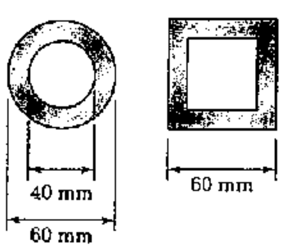
Arrangement (b). $I_{min} = I_y = \frac{1}{12}\left(\frac{d}{3}\right)(d^3) + \frac{1}{12}(d)\left(\frac{d}{3}\right)^3 + \frac{1}{12}\left(\frac{d}{3}\right)(d)^3 = \frac{19}{324} d^4$

$$P_{cr,b} = \frac{\pi^2 E I}{L_e^2} = \frac{19 \pi^2 E d^4}{324 L_e^2}$$

$$\frac{P_{cr,a}}{P_{cr,b}} = \frac{1}{12} \cdot \frac{324}{19} = \frac{27}{19} = 1.421$$

Problem 10.12

10.12 Two brass rods used as compression members, each of 3-m effective length, have the cross sections shown. (a) Determine the wall thickness of the hollow square rod for which the rods have the same cross-sectional area. (b) Using E = 105 GPa, determine the critical load of each rod.



(a) Same area: $\frac{\pi}{4}(d_o^2 - d_i^2) = b_o^2 - b_i^2$

$$b_i^2 = b_o^2 - \frac{\pi}{4}(d_o^2 - d_i^2)$$

$$= 60^2 - \frac{\pi}{4}(60^2 - 40^2) = 2.0292 \text{ mm}^2$$

$b_i = 45.047$ mm $\qquad t = \frac{1}{2}(b_o - b_i) = 7.48$ mm

(b) Circular: $I = \frac{\pi}{64}(d_o^4 - d_i^4) = 510.51 \times 10^3 \text{ mm}^4 = 510.51 \times 10^{-9} \text{ m}^4$

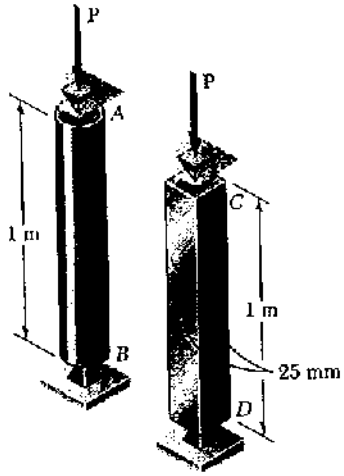
$$P_{cr} = \frac{\pi^2 E I}{L^2} = \frac{\pi^2 (105 \times 10^9)(510.51 \times 10^{-9})}{(3.0)^2} = 58.8 \times 10^3 \text{ N} = 58.8 \text{ kN}$$

Square: $I = \frac{1}{12}(b_o^4 - b_i^4) = 736.85 \times 10^3 \text{ mm}^4 = 736.85 \times 10^{-9} \text{ m}^4$

$$P_{cr} = \frac{\pi^2 E I}{L^2} = \frac{\pi^2 (105 \times 10^9)(736.85 \times 10^{-9})}{(3.0)^2} = 84.8 \times 10^3 \text{ N} = 84.8 \text{ kN}$$

Problem 10.13

10.13 Determine the radius of the round strut so that the round and square struts have the same cross-sectional area and compute the critical load for each strut. Use $E = 200$ GPa.



For square strut $A = 25^2 = 625 \text{ mm}^2$

$I = \frac{1}{12}(25)^4 = 32.552 \times 10^3 \text{ mm}^4 = 32.552 \times 10^{-9} \text{ m}^4$

For round strut $\pi d^2 = A$

$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(625)}{\pi}} = 28.2 \text{ mm}$

$c = \frac{1}{2} d = 14.10 \text{ mm}$

$I = \frac{\pi}{4} c^4 = 31.085 \times 10^3 \text{ mm}^3 = 31.085 \times 10^{-9} \text{ m}^4$

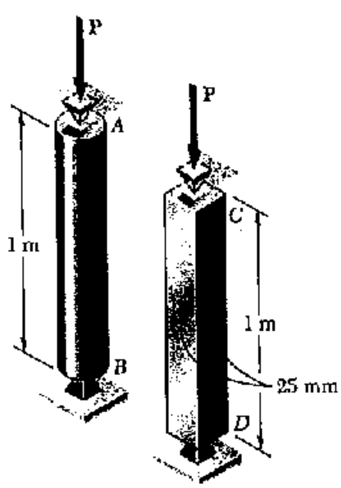
Critical loads $P_{cr} = \frac{\pi^2 EI}{L^2}$

Round strut. $P_{cr} = \frac{\pi^2 (200 \times 10^9)(31.085 \times 10^{-9})}{(1)^2} = 61.4 \times 10^3 \text{ N} = 61.4 \text{ kN}$

Square strut. $P_{cr} = \frac{\pi^2 (200 \times 10^9)(32.552 \times 10^{-9})}{(1)^2} = 64.3 \times 10^3 \text{ N} = 64.3 \text{ kN}$

Problem 10.14

10.14 Determine (*a*) the critical load for the square strut, (*b*) the radius of the round strut for which both struts have the same critical load. (*c*) Express the cross-sectional area of the square strut as a percentage of the cross-sectional area of the round strut. Use $E = 200$ GPa.



(a) Square strut. $A = (25)^2 = 625 \text{ mm}^2$

$I = \frac{1}{12}(25)^4 = 32.552 \times 10^3 \text{ mm}^4 = 32.552 \times 10^{-9} \text{ m}^4$

$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (200 \times 10^9)(32.552 \times 10^{-9})}{(1)^2}$

$= 64.3 \times 10^3 \text{ N} = 64.3 \text{ kN}$

(b) Round strut. For the same critical load, modulus of elasticity, and length, the moments of inertia must be equal.

$\frac{\pi}{4} c^4 = I$

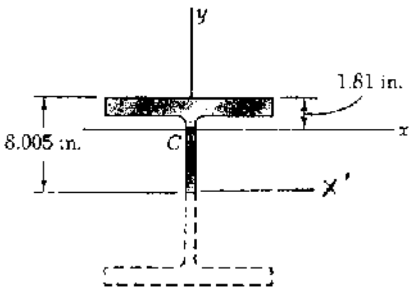
$c = \sqrt[4]{\frac{4I}{\pi}} = \sqrt[4]{\frac{(4)(32.552 \times 10^3)}{\pi}} = 14.27 \text{ mm}$

$d = 2c = 28.5 \text{ mm}$

(c) Area. $A = \pi c^2 = 639.58 \text{ mm}$, Ratio $= \frac{625}{639.58} = 0.977 = 97.7\%$

Problem 10.15

10.15 A column of 26-ft effective length is made from half of a W16 × 40 rolled-steel shape. Knowing that the centroid of the cross section is located as shown, determine the factor of safety if the allowable centric load is 20 kips. Use $E = 29 \times 10^6$ psi.



Full W 16 × 40: $A = 11.8 \text{ in}^2$

$I_x = 518 \text{ in}^4, \quad I_y = 28.9 \text{ in}^4$

Half W 16 × 40:

$A = (\frac{1}{2})(11.8) = 5.90 \text{ in}^2$

$I_x = \frac{1}{2}(518) - (5.90)(8.005 - 1.81)^2 = 32.57 \text{ in}^4$

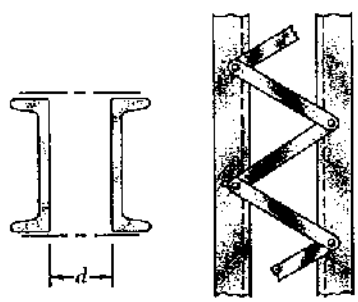
$I_y = \frac{1}{2}(28.9) = 14.45 \text{ in}^4 = I_{min}$

$$P_{cr} = \frac{\pi^2 E I_{min}}{L_e^2} = \frac{\pi^2 (29 \times 10^6)(14.45)}{(26 \times 12)^2} = 42.5 \times 10^3 \text{ lb} = 42.5 \text{ kips.}$$

$$P_{all} = \frac{P_{cr}}{F.S.} \qquad F.S. = \frac{P_{cr}}{P_{all}} = \frac{42.5}{20} = 2.125$$

Problem 10.16

10.16 A single compression member of 27-ft effective length is obtained by connecting two C 8 × 11.5 steel channels with lacing bars as shown. Knowing that the factor of safety is 1.85, determine the allowable centric load for the member. Use $E = 29 \times 10^6$ psi and $d = 4$ in.



For C 8 × 11.5 rolled steel shape, $A = 3.38 \text{ in}^2$

$\bar{I}_x = 32.6 \text{ in}^4, \quad \bar{I}_y = 1.32 \text{ in}^4, \quad \bar{x} = 0.571 \text{ in.}$

For the fabricated column

$I_x = 2\bar{I}_x = (2)(32.6) = 65.2 \text{ in}^4$

$I_y = 2[\bar{I}_y + A(\frac{d}{2} + \bar{x})^2]$

$= 2[1.32 + (3.38)(2 + 0.571)^2] = 47.32 \text{ in}^4$

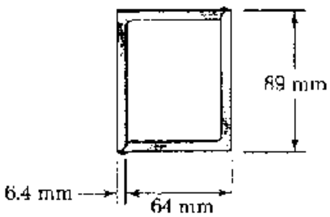
$I_{min} = I_y = 47.32 \text{ in}^4, \qquad L_e = 27 \text{ ft} = 324 \text{ in.}$

$$P_{cr} = \frac{\pi^2 E I_{min}}{L_e^2} = \frac{\pi^2 (29 \times 10^6)(47.32)}{(324)^2} = 129 \times 10^3 \text{ lb}$$

$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{129 \times 10^3}{1.85} = 69.7 \times 10^3 \text{ lb} = 69.7 \text{ kips}$$

Problem 10.17

10.17 A column of 4-m effective length is made by welding together two 89 × 64 × 6.4-mm angles as shown. Using E = 200 GPa, determine the factor of safety with respect to buckling for a centric load of 80 kN.



$b_o = 89$ mm $\qquad h_o = 64 + 6.4 = 70.4$ mm

$b_i = b_o - 2t = 76.2$ mm $\qquad h_i = h_o - 2t = 57.6$ mm

$I_{min} = \frac{1}{12}(b_o h_o^3 - b_i h_i^3) = 1.37427 \times 10^6 \text{ mm}^4$

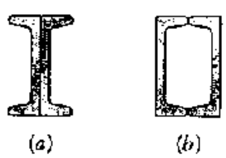
$= 1.37427 \times 10^{-6} \text{ m}^4$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (200\times10^9)(1.37427\times10^{-6})}{(4)^2} = 169.54\times10^3 \text{ N} = 169.54 \text{ kN}$$

$$F.S. = \frac{P_{cr}}{P} = \frac{169.54}{80} = 2.12$$

Problem 10.18

10.18 A column of 3-m effective length is to be made by welding together two C130 × 13 rolled-steel channels. Using E = 200 GPa, determine for each arrangement shown the allowable centric load if a factor of safety of 2.4 is required.



For channel C 130 × 13 $\qquad A = 1710 \text{ mm}^2 \qquad b_f = 48$ mm

$I_x = 3.70\times10^6 \text{ mm}^4 \qquad I_y = 0.264\times10^6 \text{ mm}^4 \qquad \bar{x} = 12.2$ mm

Arrangement (a). $\qquad I_x = (2)(3.70\times10^6) = 7.40\times10^6 \text{ mm}^4$

$I_y = 2[0.264\times10^6 + (1710)(12.2)^2] = 1.0370\times10^6 \text{ mm}^2$

$I_{min} = I_y = 1.0370\times10^6 \text{ mm} = 1.0370\times10^{-6} \text{ m}^4$

$$P_{cr} = \frac{\pi^2 E I_{min}}{L_e^2} = \frac{\pi^2(200\times10^9)(1.0370\times10^{-6})}{(3.0)^2} = 227\times10^3 \text{ N} = 227 \text{ kN}$$

$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{227}{2.4} = 94.8 \text{ kN}$$

Arrangement (b). $\qquad I_x = (2)(3.70\times10^6) \text{ mm}^4$

$I_y = 2[0.264\times10^6 + (1710)(48 - 12.2)^2] = 4.911\times10^6 \text{ mm}^4$

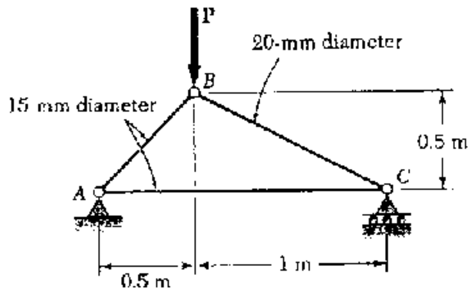
$I_{min} = I_y = 4.911\times10^6 \text{ mm}^4 = 4.911\times10^{-6} \text{ m}^4$

$$P_{cr} = \frac{\pi^2 E I_{min}}{L_e^2} = \frac{\pi^2(200\times10^9)(4.911\times10^{-6})}{(3.0)^2} = 1077\times10^3 \text{ N} = 1077 \text{ kN}$$

$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{1077}{2.4} = 449 \text{ kN}$$

Problem 10.19

10.19 Knowing that a factor of safety of 2.6 is required, determine the largest load **P** that can be applied to the structure shown. Use $E = 200$ GPa and consider only buckling in the plane of the structure.



BC: $L_{BC} = \sqrt{1^2 + 0.5^2} = 1.1180 \text{ m}$

$I = \frac{\pi}{64} d_{BC}^4 = \frac{\pi}{64}(20)^4 = 7.854 \times 10^3 \text{ mm}^4$
$= 7.854 \times 10^{-9} \text{ m}^4$

$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (200 \times 10^9)(7.854 \times 10^{-9})}{(1.1180)^2}$
$= 12.403 \times 10^3 \text{ N} = 12.403 \text{ kN}$

$F_{BC,all} = \frac{P_{cr}}{F.S.} = \frac{12.403}{2.6} = 4.770 \text{ kN}$

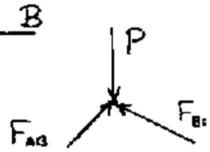
AB: $L_{AB} = \sqrt{0.5^2 + 0.5^2} = 0.70711 \text{ m}$

$I = \frac{\pi}{64} d^4 = \frac{\pi}{64}(15)^4 = 2.485 \times 10^3 \text{ mm}^4 = 2.485 \times 10^{-9} \text{ m}^4$

$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (200 \times 10^9)(2.485 \times 10^{-9})}{(0.70711)^2} = 9.8106 \times 10^3 \text{ N} = 9.8106 \text{ kN}$

$F_{AB,all} = \frac{P_{cr}}{F.S.} = \frac{9.8106}{2.6} = 3.773 \text{ kN}$

Joint B



$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{0.5}{0.70711} F_{AB} - \frac{1.0}{1.1180} F_{BC} = 0$

$F_{BC} = 0.79057 F_{AB}$

$+\uparrow \Sigma F_y = 0: \quad \frac{.5}{0.70711} F_{AB} + \frac{0.5}{1.1180} F_{BC} - P = 0$

$0.70711 F_{AB} + (0.44721)(0.79057 F_{AB}) - P = 0$

$P = 1.06066 F_{AB}$

$P = (1.06066) \frac{F_{BC}}{0.79057} = 1.3416 \ F_{BC}$

Allowable value for P.

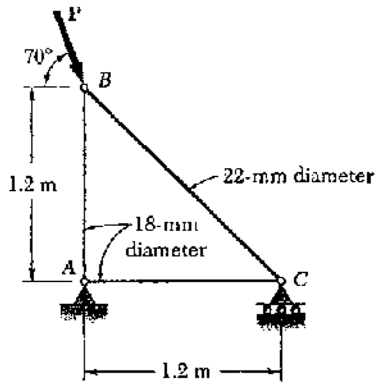
$P < 1.06066 \ F_{AB,all} = (1.06066)(3.773) = 4.00 \text{ kN}$

$P < 1.3416 \ F_{BC,all} = (1.3416)(4.770) = 6.40 \text{ kN}$

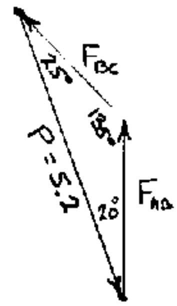
$P_{all} = 4.00 \text{ kN}$

Problem 10.20

10.20 Knowing that $P = 5.2$ kN, determine the factor of safety for the structure shown. Use $E = 200$ GPa and consider only buckling in the plane of the structure.



Joint B: From force triangle,



$$\frac{F_{AB}}{\sin 25^\circ} = \frac{F_{BC}}{\sin 20^\circ} = \frac{5.2}{\sin 135^\circ}$$

$$F_{AB} = 3.1079 \text{ kN (comp)}$$

$$F_{BC} = 2.5152 \text{ kN (comp)}$$

Member AB: $I_{AB} = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{4}\left(\frac{18}{2}\right)^4 = 5.153 \times 10^3 \text{ mm}^4 = 5.153 \times 10^{-9} \text{ m}^4$

$$F_{AB,cr} = \frac{\pi^2 E I_{AB}}{L_{AB}^2} = \frac{\pi^2 (200 \times 10^9)(5.153 \times 10^{-9})}{(1.2)^2} = 7.0636 \times 10^3 \text{ N} = 7.0636 \text{ kN}$$

$$\text{F.S.} = \frac{F_{AB,cr}}{F_{AB}} = \frac{7.0636}{3.1079} = 2.27$$

Member BC: $I_{BC} = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{4}\left(\frac{22}{2}\right)^4 = 11.499 \times 10^3 \text{ mm}^4 = 11.499 \times 10^{-9} \text{ m}^4$

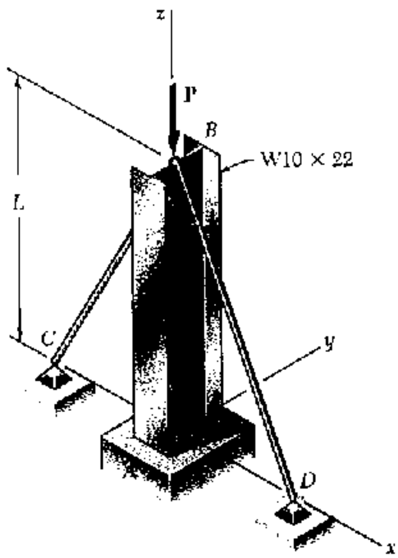
$$L_{BC}^2 = 1.2^2 + 1.2^2 = 2.88 \text{ m}^2$$

$$F_{BC,cr} = \frac{\pi^2 E I_{BC}}{L_{BC}^2} = \frac{\pi^2 (200 \times 10^9)(11.499 \times 10^{-9})}{2.88} = 7.8813 \times 10^3 \text{ N} = 7.8813 \text{ kN}$$

$$\text{F.S.} = \frac{F_{BC,cr}}{F_{BC}} = \frac{7.8813}{2.5152} = 3.13$$

Smallest F.S. governs. $\text{F.S.} = 2.27$ ◀

Problem 10.21



10.21 Column AB carries a centric load $\mathbf{P}$ of magnitude 15 kips. Cables BC and BD are taut and prevent motion of point B in the xz plane. Using Euler's formula and a factor of safety of 2.2, and neglecting the tension in the cables, determine the maximum allowable length L. Use $E = 29 \times 10^6$ psi.

W 10 × 22 : $I_x = 118 \text{ in}^4$, $I_y = 11.4 \text{ in}^4$

$P = 15 \times 10^3$ lb.

$P_{cr} = (F.S.)P = (2.2)(15\times10^3) = 33\times10^3$ lb

Buckling in xz-plane. $L_e = 0.7L$

$$P_{cr} = \frac{\pi^2 E I_y}{(0.7L)^2} \qquad L = \frac{\pi}{0.7}\sqrt{\frac{EI_y}{P_{cr}}}$$

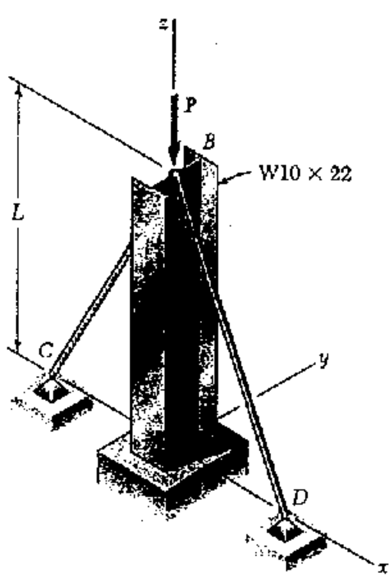
$$L = \frac{\pi}{0.7}\sqrt{\frac{(29\times10^6)(11.4)}{33\times10^3}} = 449.2 \text{ in.}$$

Buckling in yz-plane. $L_e = 2L$

$$P_{cr} = \frac{\pi^2 E I_x}{(2L)^2} \qquad L = \frac{\pi}{2}\sqrt{\frac{EI_x}{P_{cr}}} = \frac{\pi}{2}\sqrt{\frac{(29\times10^6)(118)}{33\times10^3}} = 505.8 \text{ in.}$$

Smaller value for L governs. $L = 449.2 \text{ in.} = 37.4 \text{ ft}$ ◄

Problem 10.22



10.22 A W 8 × 21 rolled-steel shape is used with the support and cable arrangement shown in Prob. 10.21. Knowing that $L = 24$ ft, determine the allowable centric load $\mathbf{P}$ if a factor of safety of 2.2 is required. Use $E = 29 \times 10^6$ psi.

10.21 Column AB carries a centric load $\mathbf{P}$ of magnitude 15 kips. Cables BC and BD are taut and prevent motion of point B in the xz plane. Using Euler's formula and a factor of safety of 2.2, and neglecting the tension in the cables, determine the maximum allowable length L. Use $E = 29 \times 10^6$ psi.

W 8 × 21 : $I_x = 75.3 \text{ in}^4$, $I_y = 9.77 \text{ in}^4$

$L = 24 \text{ ft} = 288 \text{ in.}$

Buckling in xz-plane: $L_e = 0.7 = 201.6$ in.

$$P_{cr} = \frac{\pi^2 E I_y}{L_e^2} = \frac{\pi^2(29\times10^6)(9.77)}{(201.6)^2} = 68.80\times10^3 \text{ lb.}$$

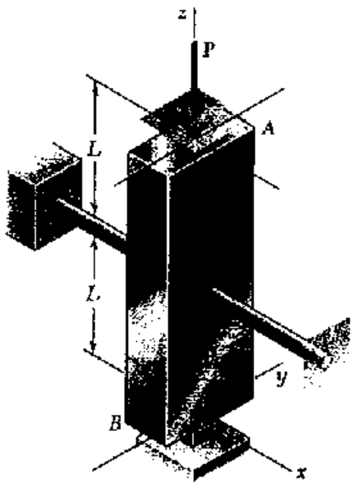
Buckling in yz-plane: $L_e = 2L = 576$ in.

$$P_{cr} = \frac{\pi^2 E I_x}{L_e^2} = \frac{\pi^2(29\times10^6)(75.3)}{(576)^2} = 64.96\times10^3 \text{ lb.}$$

The smaller value governs. $P_c = 64.96\times10^3$ lb. $= 64.96$ kips

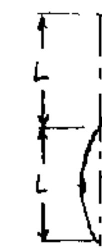
$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{64.96}{2.2} = 29.5 \text{ kips}$$ ◄

Problem 10.23



10.23 Column *ABC* has a uniform rectangular cross section and is braced in the *xz* plane at its midpoint *C*. (*a*) Determine the ratio b/d for which the factor of safety is the same with respect to buckling in the *xz* and *yz* planes. (*b*) Using the ratio found in part *a*, design the cross section of the column so that the factor of safety will be 3.0 when $P = 4.4$ kN, $L = 1$ m, and $E = 200$ GPa.

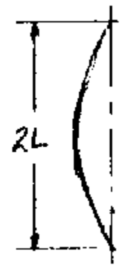
Buckling in xz-plane. $L_e = L$, $I = \dfrac{db^3}{12}$



$$(P_{cr})_1 = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 E d b^3}{12 L^2}$$

$$(F.S.)_1 = \frac{(P_{cr})_1}{P} = \frac{\pi^2 E d b^3}{12 P L^2}$$

Buckling in yz-plane. $L_e = 2L$, $I = \dfrac{bd^3}{12}$



$$(P_{cr})_2 = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 E b d^3}{12(2L)^2}$$

$$(F.S.)_2 = \frac{(P_{cr})_2}{P} = \frac{\pi^2 E b d^3}{48 P L^2}$$

(a) Equating the two factors of safety,
$$\frac{\pi^2 E d b^3}{12 P L^2} = \frac{\pi^2 E b d^3}{48 P L^2}$$

$$b^2 = \tfrac{1}{4} d^2 \qquad b/d = 1/2$$

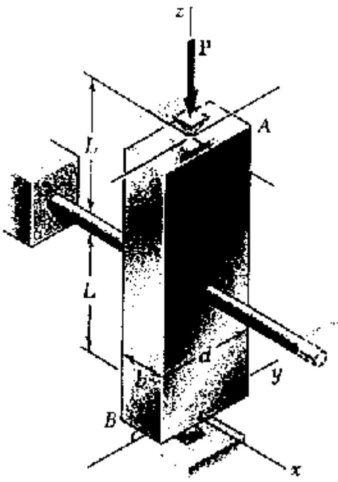
Then, $(F.S.) = \dfrac{\pi^2 E d^4}{96 P L^2}$

$$d^4 = \frac{96 (F.S.) P L^2}{\pi^2 E} = \frac{(96)(3.0)(4.4\times 10^3)(1)^2}{\pi^2 (200\times 10^9)} = 641.97 \times 10^{-9}\ \text{m}^4$$

(b) $d = 28.3 \times 10^{-3}$ m $\qquad d = 28.3$ mm

$b = 14.15$ mm

Problem 10.24

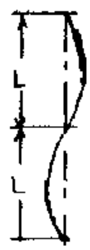


10.24 Column ABC has a uniform rectangular cross section with $b = 12$ mm and $d = 22$ mm. The column is braced in the xz plane at its midpoint C and carries a centric load **P** of magnitude 3.8 kN. Knowing that a factor of safety of 3.2 is required, determine the largest allowable length L. Use $E = 200$ GPa.

$$P_{cr} = (F.S.)P = (3.2)(3.8 \times 10^3) = 12.16 \times 10^3 \text{ N}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} \qquad L_e = \pi\sqrt{\frac{EI}{P_{cr}}}$$

<u>Buckling in xz-plane.</u> $\quad L = L_e = \pi\sqrt{\frac{EI}{P_{cr}}}$

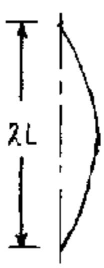


$$I = \frac{1}{12} d b^3 = \frac{1}{12}(22)(12)^3 = 3.168 \times 10^3 \text{ mm}^4$$

$$= 3.168 \times 10^{-9} \text{ m}^4$$

$$L = \pi\sqrt{\frac{(200 \times 10^9)(3.168 \times 10^{-9})}{12.16 \times 10^3}} = 0.717 \text{ m}$$

<u>Buckling in yz-plane.</u> $\quad L_e = 2L \qquad L = \frac{L_e}{2} = \frac{\pi}{2}\sqrt{\frac{EI}{P_{cr}}}$



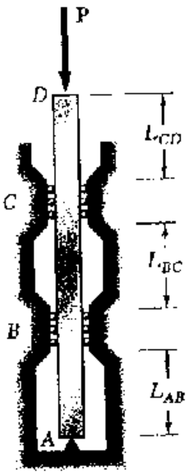
$$I = \frac{1}{12} b d^3 = \frac{1}{12}(12)(22)^3 = 10.648 \times 10^3 \text{ mm}^4 = 10.648 \times 10^{-9} \text{ m}^4$$

$$L = \frac{\pi}{2}\sqrt{\frac{(200 \times 10^9)(10.648 \times 10^{-9})}{12.16 \times 10^3}} = 0.657 \text{ m}$$

The smaller length governs. $\quad L = 0.657 \text{ m} = 657 \text{ mm}$ ◄

Problem 10.25

10.25 A 1-in.-square aluminum strut is maintained in the position shown by a pin support at A and by sets of rollers at B and C that prevent rotation of the strut in the plane of the figure. Knowing that $L_{AB} = 3$ ft, $L_{BC} = 4$ ft, and $L_{CD} = 1$ ft, determine the allowable load **P** using a factor of safety with respect to buckling of 3.2. Consider only buckling in the plane of the figure and use $E = 10.4 \times 10^6$ psi.



$$I = \frac{1}{12}bh^3 = \frac{1}{12}(1)(1)^3 = 0.083333 \text{ in}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} \qquad P_{all} = \frac{(P_{cr})_{min}}{F.S.} = \frac{\pi^2 EI}{(F.S.)(L_e)_{max}^2}$$

Portion AB: $L_e = 0.7 L_{AB} = (0.7)(3) = 2.1$ ft

Portion BC: $L_e = 0.5 L_{BC} = (0.5)(4) = 2.0$ ft

Portion CD $L_e = 2 L_e = (2)(1) = 2.0$ ft

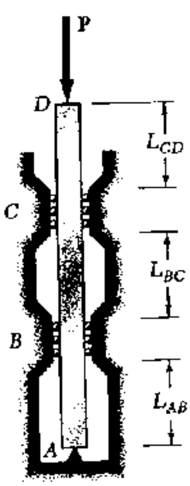
$(L_e)_{max} = 2.1$ ft $= 25.2$ in.

$$P_{all} = \frac{\pi^2(10.4\times10^6)(0.083333)}{(3.2)(25.2^2)} = 4.21\times10^3 \text{ lb} = 4.21 \text{ kips}$$

Problem 10.26

10.26 For the strut of Prob. 10.25, knowing that $L_{AB} = 3$ ft, determine (*a*) the largest values of L_{BC} and L_{CD} that may be used if the allowable load **P** is to be as large as possible, (*b*) the magnitude of the corresponding allowable load.

10.25 A 1-in.-square aluminum strut is maintained in the position shown by a pin support at A and by sets of rollers at B and C that prevent rotation of the strut in the plane of the figure. Knowing that $L_{AB} = 3$ ft, $L_{BC} = 4$ ft, and $L_{CD} = 1$ ft, determine the allowable load **P** using a factor of safety with respect to buckling of 3.2. Consider only buckling in the plane of the figure and use $E = 10.4 \times 10^6$ psi.



$$I = \frac{1}{12}bh^3 = \frac{1}{12}(1)(1)^3 = 0.083333 \text{ in}^4$$

(a) Equivalent lengths: AB $L_e = 0.7 L_{AB} = 2.1$ ft $= 25.2$ in.

BC $L_e = 0.5 L_{BC}$

$L_{BC} = \frac{2.1}{0.5} = 4.2$ ft

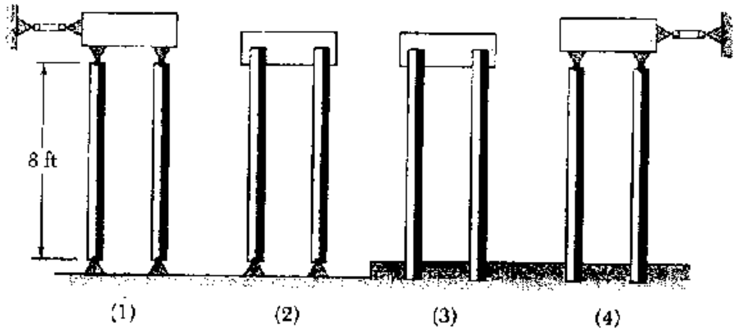
CD $L_e = 2 L_{CD}$

$L_{CD} = \frac{2.1}{2} = 1.05$ ft

(b) $$P_{all} = \frac{P_{cr}}{F.S.} = \frac{\pi^2 EI}{(F.S.)L_e^2} = \frac{\pi^2(10.4\times10^6)(0.083333)}{(3.2)(25.2)^2} = 4.21\times10^3 \text{ lb} = 4.21 \text{ kips}$$

Problem 10.27

10.27 Two columns are used to support a block weighing 3.25 kips in each of the four ways shown. (*a*) Knowing that the column of Fig. (1) is made of steel with a 1.25-in.-diameter, determine the factor of safety with respect to buckling for the loading shown. (*b*) Determine the diameter of each of the other columns for which the factor of safety is the same as the factor of safety obtained in part *a*. Use $E = 29 \times 10^6$ psi.



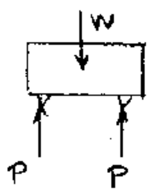
(a) $I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{4}\left(\frac{1.25}{2}\right)^4$

$= 0.119842 \text{ in}^4$

$L = 8 \text{ ft} = 96 \text{ in}$

$P_{cr} = \frac{\pi^2 EI}{L^2}$

$$P_{cr} = \frac{\pi^2 (29\times10^6)(0.119842)}{(96)^2} = 3722 \text{ lb} = 3.722 \text{ kip} \quad \text{for one column.}$$



$P = \frac{1}{2}W = \frac{3.25}{2} = 1.625 \text{ kip}.$

$F.S. = \frac{P_{cr}}{P} = \frac{3.722}{1.625} = 2.29$

$P_{cr(1)} = \frac{\pi^2 EI}{L^2} \qquad P_{cr(\alpha)} = \frac{\pi^2 E I_\alpha}{(L_{e,\alpha})^2}$

$\frac{P_{cr(\alpha)}}{P_{cr(1)}} = 1 \qquad \frac{I_\alpha}{I_{(1)}}\cdot\frac{L^2}{L_e^2} = 1 \qquad \left(\frac{d_\alpha}{d_{(1)}}\right)^4\left(\frac{L_e}{L}\right)^2 = 1$

$d_\alpha = d_{(1)}\sqrt{\frac{L_{e(\alpha)}}{L}}$

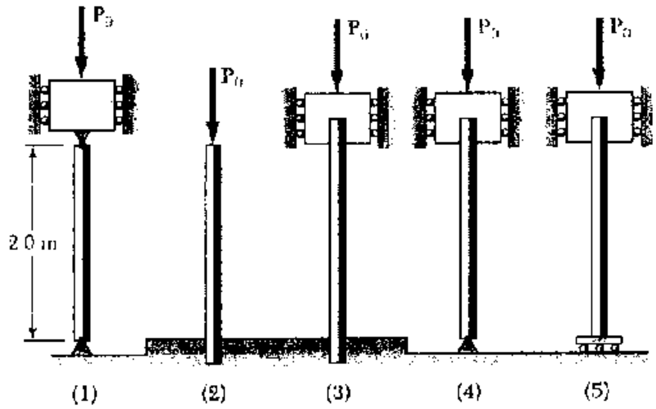
(b) (2) $L_{e(2)}/L = 2.0 \qquad d_{(2)} = 1.25\sqrt{2.0} = 1.768 \text{ in.}$

(3) $L_{e(3)}/L = 1.0 \qquad d_{(3)} = 1.25 \text{ in.}$

(4) $L_{e(4)}/L = 0.7 \qquad d_{(4)} = 1.25\sqrt{0.7} = 1.046 \text{ in.}$

Problem 10.28

10.28 Each of the five struts consists of an aluminum tube that has a 32-mm outer diameter and a 4-mm wall thickness. Using $E = 70$ GPa and a factor of safety of 2.3, determine the allowable load $\mathbf{P}_0$ for each support condition shown.



$c_o = \frac{1}{2} d_o = \frac{1}{2}(32) = 16$ mm

$c_i = c_o - t = 16 - 4 = 12$ mm

$I = \frac{\pi}{4}(c_o^4 - c_i^4) = 35.1858 \times 10^3$ mm^4

$= 35.1858 \times 10^{-9}$ m^4

$\pi^2 EI = \pi^2 (70 \times 10^9)(35.1858 \times 10^{-9})$

$= 24309$ N·m^2

$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{24309}{L_e^2}$

$P_{all} = \frac{P_{cr}}{F.S.} = \frac{10569}{L_e^2}$

(1) $L_e = (1)(2.0) = 2.0$ m, $P_{all} = 2642$ N $= 2.64$ kN ◄

(2) $L_e = (2)(2.0) = 4.0$ m, $P_{all} = 661$ N $= 0.661$ kN ◄

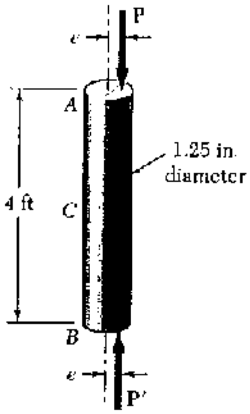
(3) $L_e = (\frac{1}{2})(2.0) = 1.0$ m, $P_{all} = 10569$ N $= 10.57$ kN ◄

(4) $L_e = (0.7)(2.0) = 1.4$ m, $P_{all} = 5392$ N $= 5.39$ kN ◄

(5) $L_e = (1.0)(2.0) = 2.0$ m, $P_{all} = 2642$ N $= 2.64$ kN ◄

Problem 10.29

10.29 An axial load **P** is applied to the 1.25-in.-diameter steel rod AB as shown. For $P = 8.6$ kips and $e = \frac{1}{16}$ in., determine (*a*) the deflection at the midpoint C of the rod, (*b*) the maximum stress in the rod. Use $E = 29 \times 10^6$ psi.



$$c = \frac{1}{2}d = \frac{1.25}{2} = 0.625 \text{ in.}$$

$$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(0.625)^4 = 0.119842 \text{ in}^4$$

$$L_e = L = 4 \text{ ft} = 48 \text{ in.}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2(29\times10^6)(0.119842)}{(48)^2} = 14.8876\times10^3 \text{ lb}$$

$$\frac{P}{P_{cr}} = \frac{8.6\times10^3}{14.8876\times10^3} = 0.57766$$

(a) $$y_{max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right] = \frac{1}{16}\left[1.71691\right]$$

$$= 0.1073 \text{ in.}$$

$$M_{max} = P(e + y_{max}) = (8.6\times10^3)\left(\frac{1}{16} + 0.1073\right) = 1.46033\times10^3 \text{ lb}\cdot\text{in}$$

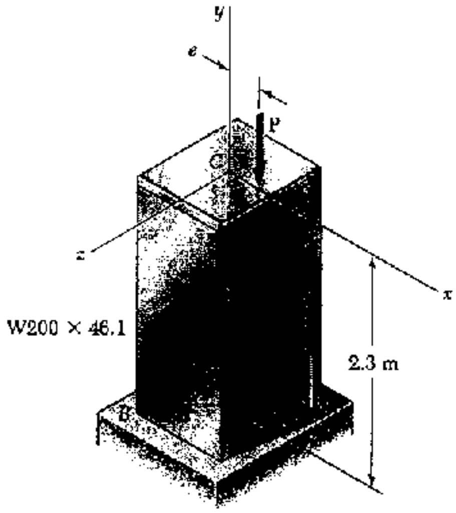
$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(1.25)^2 = 1.22718 \text{ in}^2$$

(b) $$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{8.6\times10^3}{1.22718} + \frac{(1.46033\times10^3)(0.625)}{0.119842} = 14.62\times10^3 \text{ psi}$$

$$= 14.62 \text{ ksi}$$

Problem 10.30

10.30 An axial load **P** of magnitude 560 kN is applied at a point on the x axis at a distance $e = 8$ mm from the geometric axis of the W 200 × 46.1 rolled-steel column BC. Using $E = 200$ GPa, determine (a) the horizontal deflection of end C, (b) the maximum stress in the column.



$$L_e = 2L = (2)(2.3) = 4.6 \text{ m} \qquad e = 8 \times 10^{-3} \text{ m}$$

$$\text{W } 200 \times 46.1 \qquad A = 5860 \text{ mm}^2 = 5860 \times 10^{-6} \text{ m}^2$$

$$I_y = 15.3 \times 10^6 \text{ mm}^4 = 15.3 \times 10^{-6} \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 E I}{L_e^2} = \frac{\pi^2 (200 \times 10^9)(15.3 \times 10^{-6})}{(4.6)^2}$$

$$= 1.42727 \times 10^6 \text{ N}$$

$$\frac{P}{P_{cr}} = \frac{560 \times 10^3}{1.42727 \times 10^6} = 0.39236$$

(a) $$y_m = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right] = (8 \times 10^{-3})\left[\sec\left(\frac{\pi}{2}\sqrt{0.39236}\right) - 1\right]$$

$$= (8 \times 10^{-3})\left[\sec(0.98393) - 1\right] = (8 \times 10^{-3})\left[1.8058 - 1\right]$$

$$= 6.447 \times 10^{-3} \text{ m} \qquad = 6.45 \text{ mm}$$

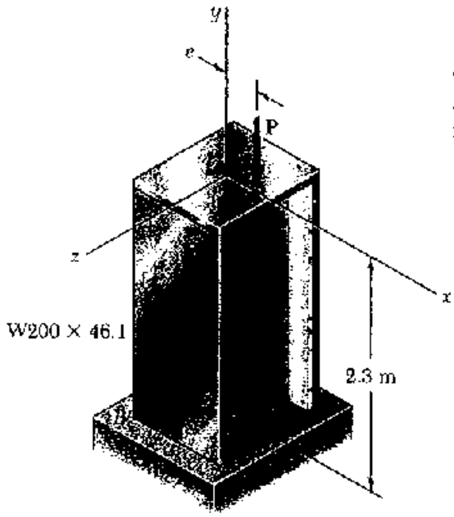
$$M_{max} = P(y_m + e) = (560 \times 10^3)(8 \times 10^{-3} + 6.447 \times 10^{-3}) = 8.090 \times 10^3 \text{ N} \cdot \text{m}$$

$$S_y = 151 \times 10^3 \text{ mm}^3 = 151 \times 10^{-6} \text{ m}^3$$

(b) $$\sigma_{max} = \frac{P}{A} + \frac{M}{S_y} = \frac{560 \times 10^3}{5860 \times 10^{-6}} + \frac{8.090 \times 10^3}{151 \times 10^{-6}} = 149.1 \times 10^6 \text{ Pa} = 149.1 \text{ MPa}$$

Problem 10.31

10.31 Solve Prob. 10.30 if the load **P** is applied parallel to the geometric axis of the column AB so that it intersects the x axis at $e = 6$ mm.



10.30 An axial load **P** of magnitude 560 kN is applied at a point on the x axis at a distance $e = 8$ mm from the geometric axis of the W 200 × 46.1 rolled-steel column BC. Using $E = 200$ GPa, determine (*a*) the horizontal deflection of end C, (*b*) the maximum stress in the column.

$$L_e = 2L = (2)(2.3) = 4.6 \text{ m} \qquad e = 6\times10^{-3} \text{ m}$$

$$W\,200\times46.1 \qquad A = 5860 \text{ mm}^2 = 5860\times10^{-6} \text{ m}^2$$

$$I_y = 15.3\times10^{6} \text{ mm}^4 = 15.3\times10^{-6} \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2(200\times10^9)(15.3\times10^{-6})}{(4.6)^2}$$

$$= 1.42727\times10^6 \text{ N}$$

$$\frac{P}{P_{cr}} = \frac{560\times10^3}{1.42727\times10^6} = 0.39236$$

(a) $$y_m = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right] = (6\times10^{3})\left[\sec\left(\frac{\pi}{2}\sqrt{0.39236}\right) - 1\right]$$

$$= (6\times10^{-3})\left[\sec(0.98393) - 1\right] = (6\times10^{-3})(1.8058 - 1)$$

$$= 4.835\times10^{-3} \text{ m}$$

$$y_m = 4.84 \text{ mm}$$

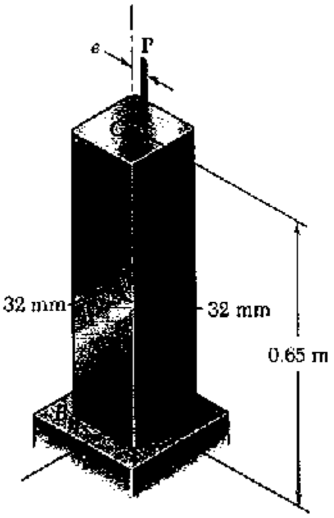
$$M_{max} = P(y_m + e) = (560\times10^3)(4.835\times10^{-3} + 6\times10^{-3}) = 6.0676\times10^3 \text{ N}\cdot\text{m}$$

$$S_y = 151\times10^3 \text{ mm}^3 = 151\times10^{-6} \text{ m}^3$$

(b) $$\sigma_{max} = \frac{P}{A} + \frac{M}{S_y} = \frac{560\times10^3}{5860\times10^{-6}} + \frac{6.0676\times10^3}{151\times10^{-6}} = 135.7\times10^6 \text{ Pa} = 135.7 \text{ MPa}$$

Problem 10.32

10.32 An axial load **P** is applied to the 32-mm-square aluminum bar BC as shown. When $P = 24$ kN, the horizontal deflection at end C is 4 mm. Using $E = 70$ GPa, determine (*a*) the eccentricity e of the load, (*b*) the maximum stress in the rod.



$$I = \frac{1}{12}(32)^4 = 87.3813\times10^3\ \text{mm}^4 = 87.3813\times10^{-9}\ \text{m}^4$$

$$A = (32)^2 = 1.024\times10^3\ \text{mm}^2 = 1.024\times10^{-3}\ \text{m}^2$$

$$L_e = 2L = (2)(0.65) = 1.30\ \text{m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2(70\times10^9)(87.3813\times10^{-9})}{(1.30)^2}$$

$$= 35.7215\times10^3\ N = 35.7215\ \text{kN}$$

$$\frac{P}{P_{cr}} = \frac{24}{35.7215} = 0.67186$$

(a) $$y_{max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right] = e\left[\sec\left(\frac{\pi}{2}\sqrt{0.67186}\right) - 1\right]$$

$$= e\left[\sec 1.28754 - 1\right] = 2.5780\ e$$

$$e = \frac{y_{max}}{2.5780} = \frac{4}{2.5780} = 1.552\ \text{mm}$$ ◄

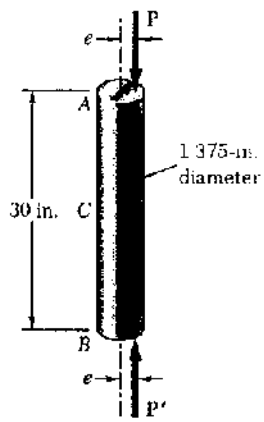
(b) $$M_{max} = P(e + y_{max}) = (24\times10^3)\left[1.552\times10^{-3} + 4\times10^{-3}\right] = 133.24\ N\cdot m$$

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max}c}{I} = \frac{24\times10^3}{1.024\times10^{-3}} + \frac{(133.24)(0.016)}{87.3813\times10^{-9}} = 47.8\times10^6\ Pa$$

$$= 47.8\ \text{MPa}$$ ◄

Problem 10.33

10.33 An axial load P is applied to the 1.375-in.-diameter steel rod AB as shown. When $P = 21$ kips, it is observed that the horizontal deflection of the midpoint C is 0.03 in. Using $E = 29 \times 10^6$ psi, determine (*a*) the eccentricity e of the load, (*b*) the maximum stress in the rod.



$$c = \tfrac{1}{2}d = 0.6875 \text{ in.} \qquad A = \pi c^2 = 1.4849 \text{ in}^2$$

$$I = \tfrac{\pi}{4}c^4 = 0.175461 \text{ in}^4$$

$$L_e = L = 30 \text{ in.}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2(29\times10^6)(0.175461)}{(30)^2} = 55.800\times10^3 \text{ lb}$$

$$\frac{P}{P_{cr}} = \frac{21\times10^3}{55.8\times10^3} = 0.37634$$

(a) $$y_{max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)-1\right] = e[\sec 0.96363 - 1]$$

$$= 0.75272\, e$$

$$e = \frac{y_{max}}{0.75272} = \frac{0.03}{0.75272} = 0.0399 \text{ in.}$$

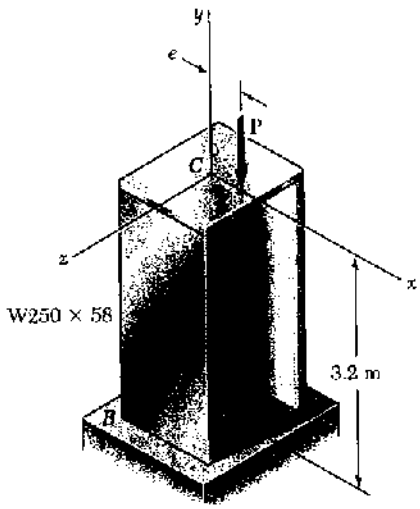
(b) $$M_{max} = P(e + y_{max}) = (21\times10^3)(0.0399 + 0.03) = 1.467\times10^3 \text{ lb-in}$$

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max}c}{I} = \frac{21\times10^3}{1.4849} + \frac{(1.467\times10^3)(0.6875)}{0.175461} = 19.89\times10^3 \text{ psi}$$

$$= 19.89 \text{ ksi}$$

Problem 10.34

10.34 The axial load **P** is applied at a point located on the x axis at a distance e from the geometric axis of the rolled-steel column BC. When $P = 350$ kN, the horizontal deflection of the top of the column is 5 mm. Using $E = 200$ GPa, determine (*a*) the eccentricity e of the load, (*b*) the maximum stress in the column.



W 250 × 58 $\quad A = 7420 \text{ mm}^2 = 7420 \times 10^{-6} \text{ m}^2$

$\bar{I}_y = 18.8 \times 10^6 \text{ mm}^4 = 18.8 \times 10^{-6} \text{ m}^4$

$S_y = 185 \times 10^3 \text{ mm}^3 = 185 \times 10^{-6} \text{ m}^3$

$L_e = 2L = (2)(3.2) = 6.4 \text{ m}$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9)(18.8 \times 10^{-6})}{(6.4)^2}$$

$$= 906 \times 10^3 \text{ N} = 906 \text{ kN}$$

$$\frac{P}{P_{cr}} = \frac{350}{906} = 0.38631$$

(a) $$y_{max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right] = e\left[\sec 0.97632 - 1\right] = 0.78546\, e$$

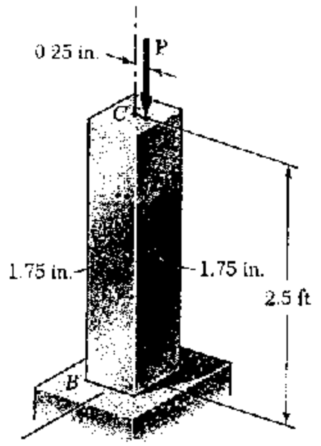
$$e = \frac{y_{max}}{0.78546} = \frac{5}{0.78546} = 6.37 \text{ mm}$$

(b) $$M_{max} = P(e + y_{max}) = (350 \times 10^3)(6.37 \times 10^{-3} + 5 \times 10^{-3}) = 3.978 \times 10^3 \text{ N}\cdot\text{m}$$

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max}}{S_y} = \frac{350 \times 10^3}{7420 \times 10^{-6}} + \frac{3.978 \times 10^3}{185 \times 10^{-6}} = 68.7 \times 10^6 \text{ Pa} = 68.7 \text{ MPa}$$

Problem 10.35

10.35 An axial load **P** is applied at a point D that is 0.25 in. from the geometric axis of the square aluminum bar BC. Using $E = 10.1 \times 10^6$ psi, determine (*a*) the load **P** for which the horizontal deflection of end C is 0.50 in., (*b*) the corresponding maximum stress in the column.



$$I = \frac{1}{12} b h^3 = \frac{1}{12}(1.75)(1.75)^3 = 0.78157 \text{ in}^4$$

$$A = (1.75)^2 = 3.0625 \text{ in}^2 \qquad c = \frac{1}{2}(1.75) = 0.875 \text{ in.}$$

$$L = 2.5 \text{ ft} = 30 \text{ in} \qquad L_e = 2L = 60 \text{ in.}$$

$$P_{cr} = \frac{\pi^2 E I}{L_e^2} = \frac{\pi^2 (10.1 \times 10^3)(0.78157)}{(60)^2} = 21.641 \text{ kips.}$$

$$y_{max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_c}}\right) - 1\right]$$

$$\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{y_{max} + e}{e}, \qquad \cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{e}{y_{max} + e}$$

(a) $$\frac{P}{P_{cr}} = \left[\frac{2}{\pi} \arccos \frac{e}{e + y_{max}}\right]^2 = \left[\frac{2}{\pi} \arccos \frac{0.25}{0.25 + 0.50}\right]^2$$

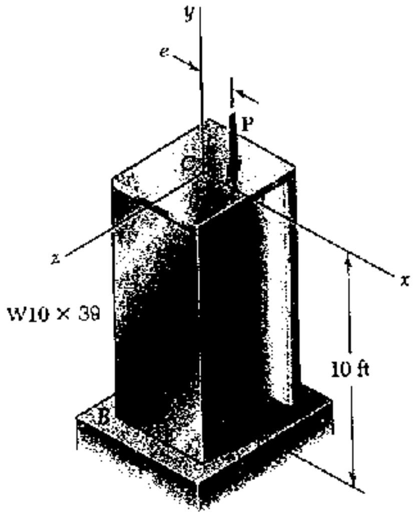
$$= 0.61411 \qquad P = 0.61411\, P_{cr} = 13.29 \text{ kips} \blacktriangleleft$$

(b) $$M_{max} = P(e + y_{max}) = (13.29)(0.25 + 0.50) = 9.9675 \text{ kip}\cdot\text{in.}$$

$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{13.29}{3.0625} + \frac{(9.9675)(0.875)}{0.78157} = 15.50 \text{ ksi} \blacktriangleleft$$

Problem 10.36

10.36 An axial load **P** is applied at a point located on the x axis at a distance $e = 0.5$ in. from the geometric axis of the W 10 × 39 rolled-steel column BC. Using $E = 29 \times 10^6$ psi, determine (a) the load **P** for which the horizontal deflection of the top of the column is 0.6 in., (b) the corresponding maximum stress in the column.



W 10 × 39 $\quad A = 11.5\ \text{in}^2, \quad I_y = 45.0\ \text{in}^4, \quad S_y = 11.3\ \text{in}^3$

$$L_e = 2L = (2)(10) = 20\ \text{ft} = 240\ \text{in.}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2(29\times10^6)(45.0)}{(240)^2}$$

$$= 223.61\times10^3\ \text{lb} = 223.61\ \text{kips}$$

$$y_{max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right] \qquad \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{y_{max}+e}{e}$$

$$\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{e}{y_{max}+e}$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi}\arccos\left(\frac{e}{y_{max}+e}\right)\right]^2 = \left[\frac{2}{\pi}\arccos\left(\frac{0.5}{0.6+0.5}\right)\right]^2$$

$$= 0.48944$$

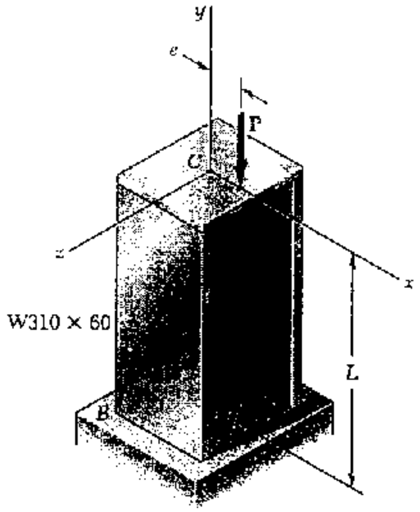
(a) $P = (0.48944)(223.61) = 109.4\ \text{kips}$ ◄

(b) $M_{max} = P(e + y_{max}) = (109.4)(0.5 + 0.6) = 120.39\ \text{kip}\cdot\text{in}$

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max}}{S_y} = \frac{109.4}{11.5} + \frac{120.39}{11.3} = 20.2\ \text{ksi}$$ ◄

Problem 10.37

10.37 An axial load **P** is applied at a point located on the x axis at a distance $e = 12$ mm from the geometric axis of the W310 × 60 rolled-steel column BC. Assuming that $L = 3.5$ m and using $E = 200$ GPa, determine (a) the load **P** for which the horizontal deflection at end C is 15 mm, (b) the corresponding maximum stress in the column.



W310×60 $\quad A = 7590 \text{ mm}^2 = 7590\times10^{-6} \text{ m}^2$

$I_y = 18.3\times10^6 \text{ mm}^4 = 18.3\times10^{-6} \text{ m}^4$

$S_y = 180\times10^3 \text{ mm}^3 = 180\times10^{-6} \text{ m}^3$

$L = 3.5 \text{ m} \qquad L_e = 2L = 7.0 \text{ m}$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2(200\times10^9)(18.3\times10^{-6})}{(7.0)^2}$$

$$= 737.2\times10^3 \text{ N} = 737.2 \text{ kN}$$

$$y_{max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right] \qquad \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{y_{max}+e}{e} \qquad \cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{e}{y_{max}+e}$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi}\arccos\frac{e}{y_{max}+e}\right]^2 = \left[\frac{2}{\pi}\arccos\frac{12}{15+12}\right]^2 = 0.49957$$

(a) $\qquad P = 0.49957\, P_{cr} = 368.28 \text{ kN} \qquad\qquad 368 \text{ kN}$ ◄

$$M_{max} = P(e + y_{max}) = (368.28\times10^3)(12+15)(10^{-3}) = 9944 \text{ N}\cdot\text{m}$$

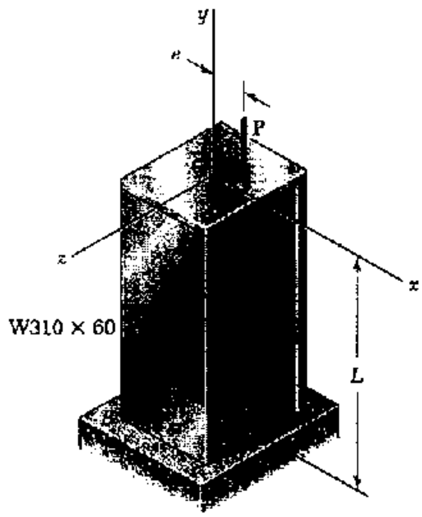
(b) $\qquad \sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{M}{S_y} = \frac{368.28\times10^3}{7590\times10^{-6}} + \frac{9944}{180\times10^{-6}} = 103.8\times10^6 \text{ Pa}$

$= 103.8 \text{ MPa}$ ◄

Problem 10.38

10.38 Solve Prob. 10.37, assuming that L is 4.5 m.

10.37 An axial load **P** is applied at a point located on the x axis at a distance $e = 12$ mm from the geometric axis of the W310 × 60 rolled-steel column BC. Assuming that $L = 3.5$ m and using $E = 200$ GPa, determine (a) the load **P** for which the horizontal deflection at end C is 15 mm, (b) the corresponding maximum stress in the column.



W 310 × 60 $\quad A = 7590\ \text{mm}^2 = 7590 \times 10^{-6}\ \text{m}^2$

$I_y = 18.3 \times 10^6\ \text{mm}^4 = 18.3 \times 10^{-6}\ \text{m}^4$

$S_y = 180 \times 10^3\ \text{mm}^3 = 180 \times 10^{-6}\ \text{m}^3$

$L = 4.5\ \text{m} \qquad L_e = 2L = 9.0\ \text{m}$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2(200 \times 10^9)(18.3 \times 10^{-6})}{(9.0)^2}$$

$$= 445.96 \times 10^3\ \text{N} = 445.96\ \text{kN}$$

$$y_{max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right] \qquad \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{y_{max} + e}{e} \qquad \cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{e}{y_{max} + e}$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi}\arccos\frac{e}{y_{max} + e}\right]^2 = \left[\frac{2}{\pi}\arccos\frac{12}{15 + 12}\right]^2 = 0.49957$$

(a) $\quad P = 0.49957\, P_{cr} = 222.79\ \text{kN} \qquad 223\ \text{kN}$ ◄

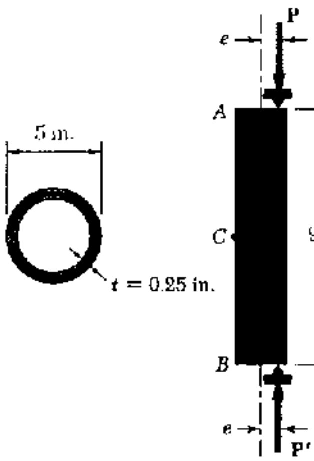
$$M_{max} = P(e + y_{max}) = (222.79 \times 10^3)(12 + 15)(10^{-3}) = 6015\ \text{N}\cdot\text{m}$$

(b) $$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{M}{S_y} = \frac{222.79 \times 10^3}{7590 \times 10^{-6}} + \frac{6015}{180 \times 10^{-6}} = 62.8 \times 10^6\ \text{Pa}$$

$$= 62.8\ \text{MPa}$$ ◄

Problem 10.39

10.39 A brass pipe having the cross section shown has an axial load **P** applied 0.15 in. from its geometric axis. Using $E = 17 \times 10^6$ psi, determine (*a*) the load **P** for which the horizontal deflection at the midpoint *C* is 0.20 in., (*b*) the corresponding maximum stress in the column.



$$c_o = \frac{1}{2} d_o = \frac{1}{2}(5) = 2.5 \text{ in.} \qquad c_i = c_o - t = 2.25 \text{ in.}$$

$$I = \frac{\pi}{4}(c_o^4 - c_i^4) = \frac{\pi}{4}(2.5^4 - 2.25^4) = 10.551 \text{ in}^4$$

$$L_e = L = 9.0 \text{ ft} = 108 \text{ in.}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2(17\times10^6)(10.551)}{(108)^2} = 151.77\times10^3 \text{ lb}$$

$$= 151.77 \text{ kips}$$

(a) $y_{max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right] \qquad \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{y_{max}+e}{e}$

$$\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{e}{e+y_{max}} \qquad \frac{P}{P_{cr}} = \left[\frac{2}{\pi}\arccos\frac{e}{y_{max}+e}\right]^2$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi}\arccos\frac{0.15}{0.20+0.15}\right]^2 = 0.51557 \qquad P = 0.51557\, P_{cr} = 78.249 \text{ kips}$$

(b) $M_{max} = P(e + y_{max}) = (78.249)(0.15 + 0.20) = 27.387 \text{ kip}\cdot\text{in}$

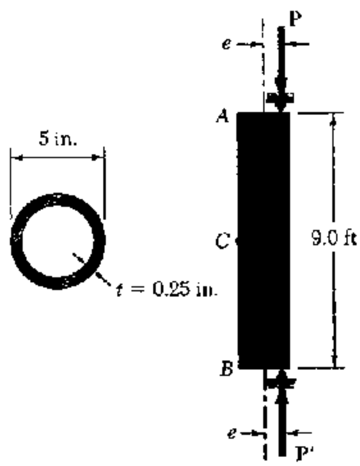
$$A = \pi(c_o^2 - c_i^2) = \pi(2.5^2 - 2.25^2) = 3.7306 \text{ in}^2$$

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max}c}{I} = \frac{78.249}{3.7306} + \frac{(27.387)(2.5)}{10.551} = 27.5 \text{ ksi}$$

Problem 10.40

10.40 Solve Prob. 10.39, assuming that the axial load **P** is applied 0.3 in. from the geometric axis of the column.

10.39 A brass pipe having the cross section shown has an axial load **P** applied 0.15 in. from its geometric axis. Using $E = 17 \times 10^6$ psi, determine (*a*) the load **P** for which the horizontal deflection at the midpoint *C* is 0.20 in., (*b*) the corresponding maximum stress in the column.



$c_o = \frac{1}{2} d_o = \frac{1}{2}(5) = 2.5 \text{ in.} \qquad c_i = c_o - t = 2.25 \text{ in.}$

$I = \frac{\pi}{4}(c_o^4 - c_i^4) = \frac{\pi}{4}(2.5^4 - 2.25^4) = 10.551 \text{ in}^4$

$L_e = L = 9.0 \text{ ft} = 108 \text{ in}$

$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2(17\times10^6)(10.551)}{(108)^2} = 151.77\times10^3 \text{ lb}$

$= 151.77 \text{ kips}$

(a) $y_{max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right] \qquad \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{y_{max}+e}{e} \qquad \cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{e}{y_{max}+e}$

$\frac{P}{P_{cr}} = \left[\frac{2}{\pi}\arccos\frac{e}{y_{max}+e}\right]^2 = \left[\frac{2}{\pi}\arccos\frac{0.3}{0.20+0.3}\right]^2 = 0.34849$

$P = 0.34849\, P_{cr} = 52.891 \text{ kips}$ ◄

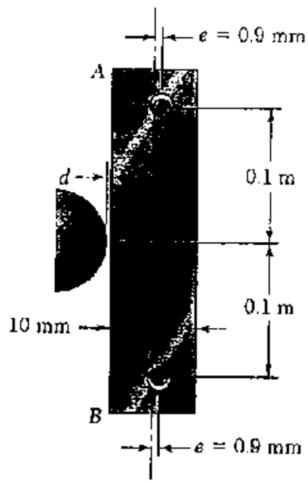
(b) $M_{max} = P(e + y_{max}) = (52.891)(0.3 + 0.20) = 26.446 \text{ kip-in}$

$A = \pi(c_o^2 - c_i^2) = \pi(2.5^2 - 2.25^2) = 3.7306 \text{ in}^2$

$\sigma_{max} = \frac{P}{A} + \frac{M_{max}c}{I} = \frac{52.891}{3.7306} + \frac{(26.446)(2.5)}{10.551} = 20.4 \text{ ksi}$ ◄

Problem 10.41

10.41 The steel bar AB has a 10×10-mm square cross section and is held by pins that are a fixed distance apart and are located at a distance $e = 0.9$ mm from the geometric axis of the bar. Knowing that at temperature T_0 the pins are in contact with the bar and that the force in the bar is zero, determine the increase in temperature for which the bar will just make contact with point C if $d = 0.3$ mm. Use $E = 200$ GPa and the coefficient of thermal expansion $\alpha = 11.7 \times 10^{-6}/°C$.



$$A = (10)^2 = 100 \text{ mm}^2 = 100 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12}(10)^4 = 833.33 \text{ mm}^4 = 833.33 \times 10^{-12} \text{ m}^4$$

$$EI = (200 \times 10^9)(833.33 \times 10^{-12}) = 166.667 \text{ N}\cdot\text{m}^2$$

$$L_e = L = 0.2 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2(166.667)}{(0.2)^2} = 41.1234 \times 10^3 \text{ N}$$

Calculate P using the secant formula

$$y_{max} = d = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right] \quad \text{or} \quad \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = 1 + \frac{d}{e} =$$

$$\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{e}{e+d} = \frac{0.9}{0.9 + 0.3} = 0.75$$

$$\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}} = \cos^{-1}(0.75) = 0.72273$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi}(0.72273)\right]^2 = 0.21170 \qquad P = 0.21170\, P_{cr} = 8.7058 \times 10^3 \text{ N}$$

If the effects of eccentricity and the shortening due to bending are neglected, the shortening of the bar, if not constrained by the pins would be

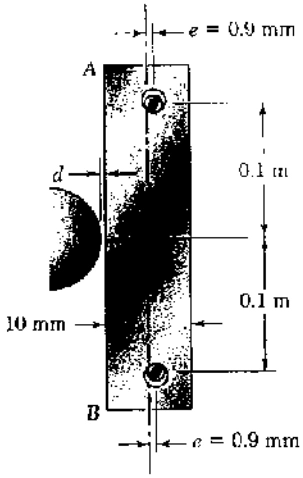
$$\delta = \frac{PL}{EA} = \frac{(8.7058 \times 10^3)(0.2)}{(200 \times 10^9)(100 \times 10^{-6})} = 87.058 \times 10^{-6} \text{ m}$$

This mechanical shortening is compensated by thermal expansion

$$L\alpha(\Delta T) - \delta = 0$$

$$\Delta T = \frac{\delta}{L\alpha} = \frac{87.058 \times 10^{-6}}{(0.2)(11.7 \times 10^{-6})} = 37.2\ °C$$

Problem 10.42



10.42 For the bar of Prob. 10.41, determine the required distance d for which the bar will just make contact with point C when the temperature increases by 60 °C.

10.41 The steel bar AB has a 10 × 10-mm square cross section and is held by pins that are a fixed distance apart and are located at a distance $e = 0.9$ mm from the geometric axis of the bar. Knowing that at temperature T_0 the pins are in contact with the bar and that the force in the bar is zero, determine the increase in temperature for which the bar will just make contact with point C if $d = 0.3$ mm. Use $E = 200$ GPa and the coefficient of thermal expansion $\alpha = 11.7 \times 10^{-6}/°C$.

$$A = (10)^2 = 100 \text{ mm}^2 = 100 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12}(10)^4 = 833.33 \text{ mm}^4 = 833.33 \times 10^{-12} \text{ m}^4$$

$$EI = (200 \times 10^9)(833.33 \times 10^{-12}) = 166.667 \text{ N}\cdot\text{m}^2$$

$$L_e = L = 0.2 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (166.667)}{(0.2)^2} = 41.1234 \times 10^3 \text{ N}$$

Free thermal expansion of the bar:

$$\delta = L\alpha(\Delta T) = (0.2)(11.7 \times 10^{-6})(60) = 140.4 \times 10^{-6} \text{ m}$$

The free expansion is prevented by constraint of the pins.

If eccentricity and shortening due to bending are neglected, the constraining force P is given by

$$P = \frac{EA\delta}{L} = \frac{(200 \times 10^9)(100 \times 10^{-6})(140.4 \times 10^{-6})}{0.2} = 14.04 \times 10^3 \text{ N}$$

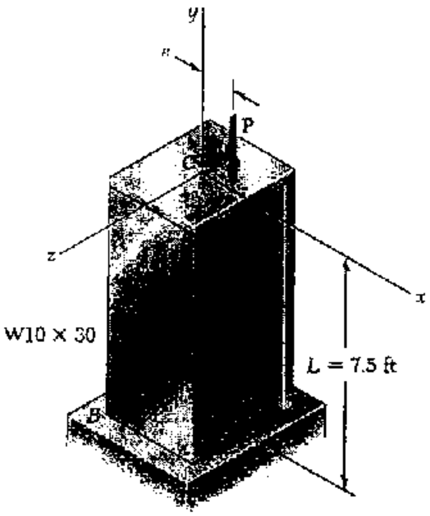
Using the secant formula,

$$y_{max} = d = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right]$$

$$= (9 \times 10^{-3})\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{14.04 \times 10^3}{41.1234 \times 10^3}}\right) - 1\right] = 5.81 \times 10^{-3} \text{ m}$$

$$d = 5.81 \text{ mm}$$

Problem 10.43



10.43 An axial load **P** is applied to the W10 × 30 rolled-steel column BC that is free at its top C and fixed at its base B. Knowing that the eccentricity of the load is e = 0.5 in. and that for the grade of steel used σ_Y = 36 ksi and $E = 29 \times 10^6$ psi., determine (*a*) the magnitude of P of the allowable load when a factor of safety of 2.4 with respect to permanent deformation is required, (*b*) the ratio of the load found in part *a* to the magnitude of the allowable centric load for the column. (*Hint:* Since the factor of safety must be applied to the load **P**, not to the stress, use Fig. 10.24 to determine P_Y).

W 10 × 30 $\quad A = 8.84 \text{ in}^2 \quad r_y = 1.37$ in.

$c = \frac{b_f}{2} = \frac{5.810}{2} = 2.905$ in $\quad I_y = 16.7 \text{ in}^4$

$L = 7.5$ ft $= 90$ in. $\quad L_e = 2L = 180$ in

$\frac{L_e}{r} = \frac{180}{1.37} = 131.39 \quad e = 0.5$ in

$\frac{ec}{r^2} = \frac{(0.5)(2.905)}{(1.37)^2} = 0.7739$

Using Fig 10.24 $\quad \frac{P}{A} = 10.47$ ksi

$P = (10.47)(8.84) = 92.6$ kips

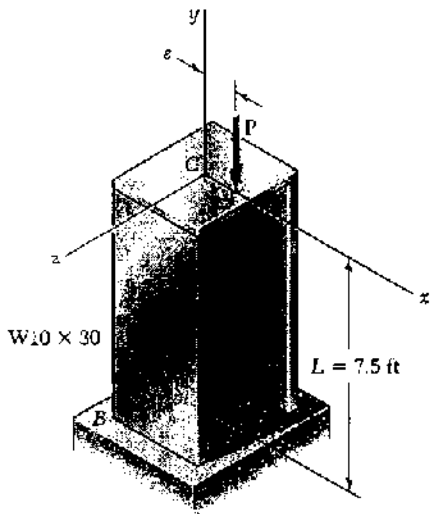
(a) Using factor of safety, $\quad P_{all} = \frac{92.6}{2.4} = 38.6$ kips ◄

$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2(29000)(16.7)}{(180)^2} = 147.5$ kips

Using factor of safety, $\quad P_{all} = \frac{147.5}{2.4} = 61.5$ kips

(b) $\quad$ ratio $= \frac{38.6}{61.5} = 0.628$ ◄

Problem 10.44



10.44 Solve Prob. 10.43, assuming that the length of the column is reduced to 5.0 ft.

10.43 An axial load **P** is applied to the W10 × 30 rolled-steel column *BC* that is free at its top *C* and fixed at its base *B*. Knowing that the eccentricity of the load is e = 0.5 in. and that for the grade of steel used σ_Y = 36 ksi and $E = 29 \times 10^6$ psi., determine (*a*) the magnitude of *P* of the allowable load when a factor of safety of 2.4 with respect to permanent deformation is required, (*b*) the ratio of the load found in part *a* to the magnitude of the allowable centric load for the column. (*Hint:* Since the factor of safety must be applied to the load **P**, not to the stress, use Fig. 10.24 to determine P_Y).

W10 × 30 $\quad A = 8.84\text{ in}^2 \quad I_y = 16.7\text{ in}^4$

$r_y = 1.37$ in $\qquad c = \frac{b_f}{2} = \frac{5.810}{2} = 2.905$ in

$L = 5.0$ ft $= 60$ in $\qquad L_e = 2L = 120$ in

$$\frac{L_e}{r} = \frac{120}{1.37} = 87.6$$

$$\frac{ec}{r^2} = \frac{(0.5)(2.905)}{(1.37)^2} = 0.7739$$

Using Fig 10.24 $\quad \frac{P}{A} = 14.90$ ksi $\qquad P = (14.90)(8.84) = 131.7$ kips

(a) Using factor of safety, $\quad P_{all} = \frac{131.7}{2.4} = 54.9$ kips ◄

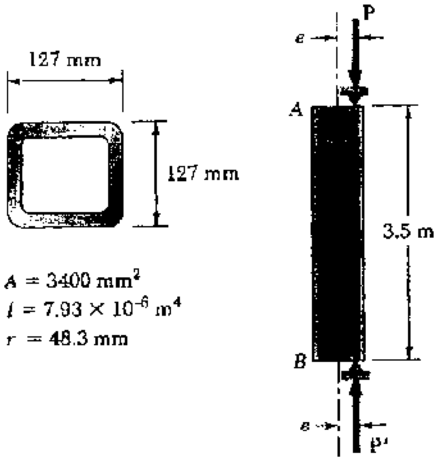
$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2(29000)(16.7)}{(120)^2} = 332 \text{ kips}$$

Using factor of safety, $\quad P_{all} = \frac{332}{2.4} = 138.3$ kips

(b) $\quad$ ratio $= \frac{54.9}{138.3} = 0.397$ ◄

Problem 10.45

10.45 A 3.5-m-long steel tube having the cross section and properties shown is used as a column. For the grade of steel used $\sigma_Y = 250$ MPa and $E = 200$ GPa. Knowing that a factor of safety of 2.6 with respect to permanent deformation is required, determine the allowable load **P** when the eccentricity e is (*a*) 15 mm, (*b*) 7.5 mm. (See hint of Prob. 10.43).



$A = 3400\text{ mm}^2$
$I = 7.93 \times 10^{-6}\text{ m}^4$
$r = 48.3\text{ mm}$

$A = 3400 \times 10^{-6}\ \text{m}^2 \qquad r = 48.3 \times 10^{-3}\ \text{m}$

$L_e = 3.5\ \text{m} \qquad \frac{L_e}{r} = \frac{3.5}{48.3 \times 10^{-3}} = 72.46$

$c = \frac{127}{2} = 63.5\ \text{mm}.$

(a) $\underline{e = 15\,\text{mm}}$. $\quad \frac{ec}{r^2} = \frac{(15)(63.5)}{(48.3)^2} = 0.40829$

Using Fig. 10.24 with $L_e/r = 72.46$ and $ec/r^2 = 0.40829$,

$P/A = 144.75\ \text{MPa} = 144.75 \times 10^6\ \text{Pa}$

$P = (144.75 \times 10^6)(3400 \times 10^{-6}) = 492 \times 10^3\ \text{N}$

Using factor of safety, $\quad P_{all} = \frac{492 \times 10^3}{2.6} = 189 \times 10^3\ \text{N} = 189\ \text{kN}$ ◄

(b) $\underline{e = 7.5\ \text{mm}}$. $\quad \frac{ec}{r^2} = \frac{(7.5)(63.5)}{(48.3)^2} = 0.20415$

Using Fig. 10.24 with $L_e/r = 72.46$ and $ec/r^2 = 0.20415$,

$P/A = 175.2\ \text{MPa} = 175.2 \times 10^6\ \text{Pa}$

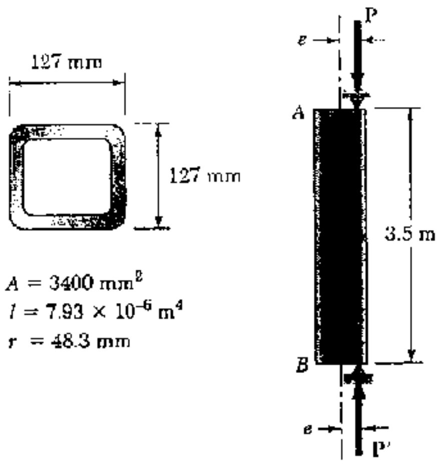
$P = (175.2 \times 10^6)(3400 \times 10^{-6}) = 596 \times 10^3\ \text{N}$

Using factor of safety, $\quad P_{all} = \frac{596 \times 10^3}{2.6} = 229 \times 10^3\ \text{N} = 229\ \text{kN}$ ◄

Problem 10.46

10.46 Solve Prob. 10.45, assuming that the length of the steel tube is increased to 5 m.

10.45 A 3.5-m-long steel tube having the cross section and properties shown is used as a column. For the grade of steel used $\sigma_Y = 250$ MPa and $E = 200$ GPa. Knowing that a factor of safety of 2.6 with respect to permanent deformation is required, determine the allowable load P when the eccentricity e is (*a*) 15 mm, (*b*) 7.5 mm. (See hint of Prob. 10.43).



$A = 3400$ mm^2
$I = 7.93 \times 10^{-6}$ m^4
$r = 48.3$ mm

$A = 3400 \times 10^{-6}$ m^2 $\qquad r = 48.3 \times 10^{-3}$ m

$L_e = 5$ m $\qquad \frac{L_e}{r} = \frac{5}{48.3 \times 10^{-3}} = 103.52$

$c = \frac{127}{2} = 63.5$ mm

(a) $\underline{e = 15 \text{mm}}$. $\qquad \frac{ec}{r^2} = \frac{(15)(63.5)}{(48.3)^2} = 0.40829$

Using Fig. 10.24 with $\frac{L_e}{r} = 103.52$

and $\frac{ec}{r^2} = 0.40829$ gives $\frac{P}{A} = 112.75$ MPa $= 112.75 \times 10^{-6}$ Pa

$P = (112.75 \times 10^6)(3400 \times 10^6) = 383 \times 10^3$ N

Using factor of safety $\quad P_{all} = \frac{383 \times 10^3}{2.6} = 147 \times 10^3 \text{ N} = 147 \text{ kN}$

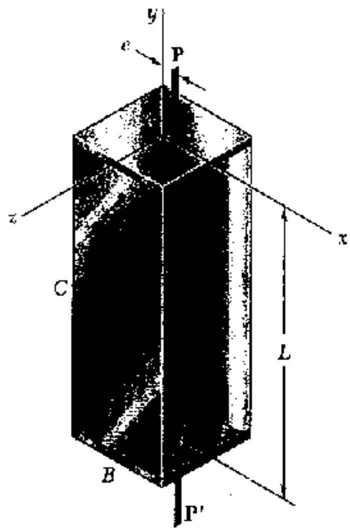
(b) $\underline{e = 7.5 \text{ mm}}$. $\qquad \frac{ec}{r^2} = \frac{(7.5)(63.5)}{(48.3)^2} = 0.20415$

Using Fig. 10.24 gives $\frac{P}{A} = 133.2$ MPa $= 133.2 \times 10^6$ Pa

$P = (133.2 \times 10^6)(3400 \times 10^{-6}) = 453 \times 10^3$ N

Using factor of safety, $\quad P_{all} = \frac{453 \times 10^3}{2.6} = 174 \times 10^3 \text{ N} = 174 \text{ kN}$

Problem 10.47



10.47 Axial loads of magnitude $P = 20$ kips are applied parallel to the geometric axis of the W 8 × 15 rolled-steel column AB and intersect the x axis at a distance e from the geometric axis. Knowing that $\sigma_{all} = 12$ ksi and $E = 29 \times 10^6$ psi, determine the largest permissible length L when (a) $e = 0.25$ in., (b) $e = 0.5$ in.

Data: $P = 20$ kips $\quad E = 29 \times 10^3$ ksi

W 8 × 15 $\quad A = 4.44\ in^2, \quad c = \frac{b_f}{2} = 2.0075$ in.

$I_y = 3.41\ in^4, \quad r_y = 0.876\ in^4$

$\sigma_{all} = \sigma_{max} = 12$ ksi

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)\right]$$

$$\frac{A\sigma_{max}}{P} - 1 = \frac{ec}{r^2}\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)$$

$$\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{r^2}{ec}\left[\frac{A\sigma_{max}}{P} - 1\right]$$

(a) $\underline{e = 0.25\text{ in.}}$ $\quad \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{(0.876)^2}{(0.25)(2.0075)}\left[\frac{(4.44)(12)}{20} - 1\right] = 2.5443$

$\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = 0.39304 \qquad \frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}} = 1.16686$

$\frac{P}{P_{cr}} = \left[\frac{2}{\pi}(1.16686)\right]^2 = 0.55182$

$P_{cr} = \frac{P}{0.55182} = \frac{\pi^2 EI}{L_e^2}$

$L_e^2 = \frac{0.55182\,\pi^2 EI}{P} = \frac{0.55182\,\pi^2(29\times10^3)(3.41)}{20} = 26.926\times10^3\ in^2$

$L_e = 164.1$ in. $\qquad L = L_e = 164.1\text{ in.} = 13.68$ ft ◄

(b) $\underline{e = 0.5\text{ in.}}$ $\quad \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{(0.876)^2}{(0.5)(2.0075)}\left[\frac{(4.44)(12)}{20} - 1\right] = 1.27214$

$\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = 0.78608 \qquad \frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}} = 0.66636$

$\frac{P}{P_{cr}} = \left[\frac{2}{\pi}(0.66636)\right]^2 = 0.179962$

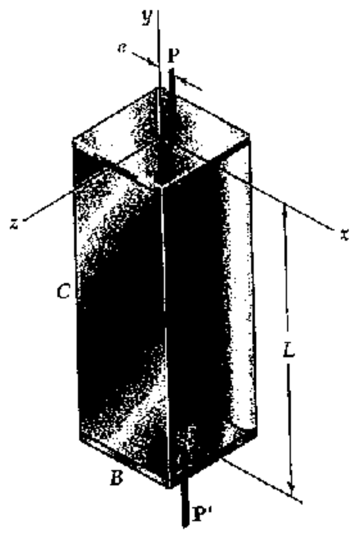
$P_{cr} = \frac{P}{0.179962} = \frac{\pi^2 EI}{L_e^2}$

$L_e^2 = \frac{0.179962\,\pi^2 EI}{P} = \frac{0.179964\,\pi^2(29\times10^3)(3.41)}{20} = 8.7822\times10^3\ in^2$

$L_e = 93.713$ in. $\qquad L = L_e = 93.713\text{ in.} = 7.81$ ft ◄

Problem 10.48

10.48 Axial loads of magnitude $P = 135$ kips are applied parallel to the geometric axis of the W 10 × 54 rolled-steel column AB and intersect the x axis at a distance e from the geometric axis. Knowing that $\sigma_{all} = 12$ ksi and $E = 29 \times 10^6$ psi, determine the largest permissible length L when (a) $e = 0.25$ in., (b) $e = 0.5$ in.



Data: $P = 135$ kips $\quad E = 29 \times 10^3$ ksi

W10×54 $\quad A = 15.8 \text{ in}^2 \quad c = \frac{b_f}{2} = 5.015$ in.

$I_y = 103 \text{ in}^4 \quad r_y = 2.56$ in.

$\sigma_{all} = \sigma_{max} = 12$ ksi

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)\right]$$

$$\frac{A\sigma_{max}}{P} - 1 = \frac{ec}{r^2}\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)$$

$$\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{r^2}{ec}\left[\frac{A\sigma_{max}}{P} - 1\right]$$

(a) $\underline{e = 0.25 \text{ in.}}$ $\quad \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{(2.56)^2}{(0.25)(5.015)}\left[\frac{(15.8)(12)}{135} - 1\right] = 2.11411$

$\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = 0.47301 \qquad \frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}} = 1.07809$

$\frac{P}{P_{cr}} = \left[\frac{2}{\pi}(1.07809)\right]^2 = 0.47105$

$P_{cr} = \frac{P}{0.47105} = \frac{\pi^2 EI}{L_e^2}$

$L_e^2 = \frac{0.47105\,\pi^2 EI}{P} = \frac{0.47105\,\pi^2(29\times10^3)(103)}{135} = 102.866\times10^3 \text{ in}^2$

$L_e = 320.73$ in. $\qquad L = L_e = 320.73 \text{ in.} = 26.7 \text{ ft}$ ◄

(b) $\underline{e = 0.5 \text{ in.}}$ $\quad \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{(2.56)^2}{(0.5)(5.015)}\left[\frac{(15.8)(12)}{135} - 1\right] = 1.05706$

$\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = 0.94602 \qquad \frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}} = 0.33006$

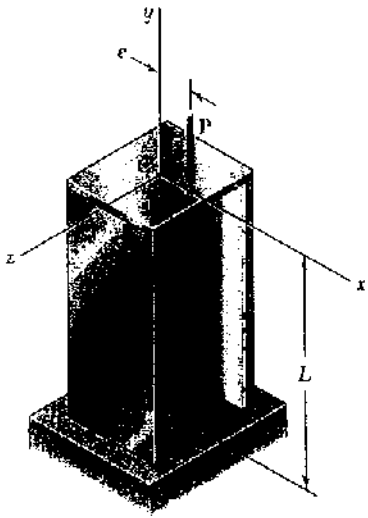
$\frac{P}{P_{cr}} = \left[\frac{2}{\pi}(0.33006)\right]^2 = 0.044151$

$P_{cr} = \frac{P}{0.044151} = \frac{\pi^2 EI}{L_e^2}$

$L_e^2 = \frac{0.044151\,\pi^2 EI}{P} = \frac{0.044151\,\pi^2(29\times10^3)(103)}{135} = 9.6414\times10^3 \text{ in}^2$

$L_e = 98.19$ in. $\qquad L = L_e = 98.19 \text{ in.} = 8.18 \text{ ft}$ ◄

Problem 10.49



10.49 A 250-kN axial load **P** is applied to the W200 × 35.9 rolled-steel column BC which is free at its top C and fixed at its base B. Knowing that the eccentricity of the load is e = 6mm, determine the largest permissible length L if the allowable stress in the column is 80 MPa. Use E = 200 GPa.

W200 × 35.9 $\quad A = 4580\ \text{mm}^2 = 4580\times10^{-6}\ \text{m}^2$

$b_f = 165\ \text{mm} \qquad c = \frac{b_f}{2} = 82.5\ \text{mm}$

$I_y = 7.64\times10^6\ \text{mm}^4 = 7.64\times10^{-6}\ \text{m}^4 \qquad r_y = 40.8\ \text{mm}$

$\sigma_{max} = 80\times10^6\ \text{Pa} \qquad P = 250\times10^3\ \text{N}$

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)\right]$$

$$\frac{A\sigma_{max}}{P} - 1 = \frac{ec}{r^2}\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)$$

$$\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{r_y^2}{ec}\left(\frac{A\sigma_{max}}{P} - 1\right) = \frac{(40.8)^2}{(6)(82.5)}\left[\frac{(4580\times10^{-6})(80\times10^6)}{250\times10^3} - 1\right]$$

$$= 1.56657$$

$\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = 0.63866 \qquad \frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}} = 0.87804$

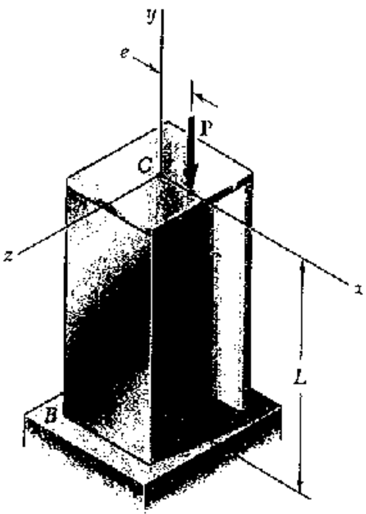
$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi}(0.87804)\right]^2 = 0.31245$$

$$P_{cr} = \frac{P}{0.31245} = \frac{\pi^2 EI}{L_e^2}$$

$$L_e^2 = \frac{0.31245\,\pi^2 EI}{P} = \frac{0.31245\,\pi^2(200\times10^9)(7.64\times10^{-6})}{250\times10^3} = 18.848\ \text{m}^2$$

$L_e = 4.34\ \text{m} = 2L \qquad\qquad L = 2.17\ \text{m}$ ◄

Problem 10.50



10.50 A 100-kN axial load **P** is applied to the W150 × 18 rolled-steel column BC which is free at its top C and fixed at its base B. Knowing that the eccentricity of the load is $e = 6$mm, determine the largest permissible length L if the allowable stress in the column is 80 MPa. Use $E = 200$ GPa.

W150 × 18 $\quad A = 2290\ \text{mm}^2 = 2290 \times 10^{-6}\ \text{m}^2$

$b_f = 102\ \text{mm} \qquad c = \frac{b_f}{2} = 51\ \text{mm}$

$I_y = 1.26 \times 10^6\ \text{mm}^4 = 1.26 \times 10^{-6}\ \text{m}^4 \qquad r_y = 23.5\ \text{mm}$

$\sigma_{max} = 80 \times 10^6\ \text{Pa} \qquad P = 100 \times 10^3\ \text{N}$

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r_y^2}\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)\right]$$

$$\frac{A\sigma_{max}}{P} - 1 = \frac{ec}{r_y^2}\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)$$

$$\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{r_y^2}{ec}\left(\frac{A\sigma_{max}}{P} - 1\right) = \frac{(23.5)^2}{(6)(51)}\left[\frac{(2290\times10^{-6})(80\times10^6)}{100\times10^3} - 1\right]$$

$$= 1.50154$$

$$\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = 0.66598 \qquad \frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}} = 0.84199$$

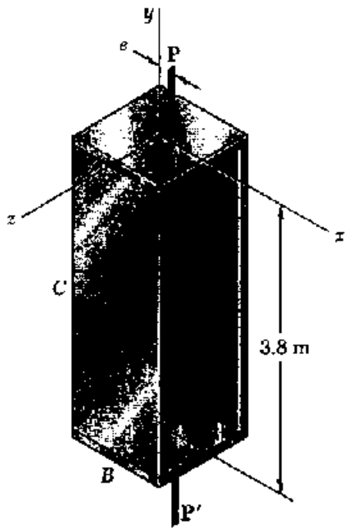
$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi}(0.84198)\right]^2 = 0.28732$$

$$P_{cr} = \frac{P}{0.28732} = \frac{\pi^2 EI}{L_e^2}$$

$$L_e^2 = \frac{0.28732\,\pi^2 EI}{P} = \frac{0.28732\,\pi^2(200\times10^9)(1.26\times10^{-6})}{100\times10^3} = 7.1461\ \text{m}^2$$

$L_e = 2.6732 = 2L \qquad\qquad L = 1.337\ \text{m}$ ◄

Problem 10.51



10.51 Axial loads of magnitude $P = 175$ kN are applied parallel to the geometric axis of a W250 × 44.8 rolled-steel column AB and intersect the axis at a distance $e = 12$ mm from its geometric axis. Knowing that $\sigma_Y = 250$ MPa and $E = 200$ GPa, determine the factor of safety with respect to yield. (*Hint:* Since the factor of safety must be applied to the load **P**, not to the stresses, use Fig. 10.24 to determine P_Y.)

For W 250 × 44.8 $A = 5720\ \text{mm}^2$, $r_y = 35.1$ mm

$L_e = 3800$ mm $L_e/r = 108.26$

$c = \frac{b_f}{2} = \frac{148}{2} = 74$ mm $e = 12$ mm

$$\frac{ec}{r^2} = \frac{(12)(74)}{(35.1)^2} = 0.72077$$

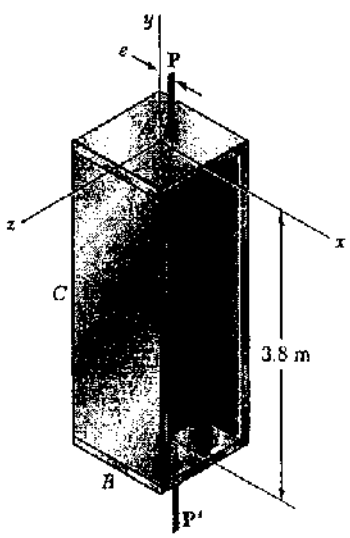
Using Fig 10.24 with $L_e/r = 108.26$ and $\frac{ec}{r^2} = 0.72077$,

$$P_Y/A = 90.37\ \text{MPa} = 90.37 \times 10^6\ \text{N/m}^2$$

$$P_Y = A\,P_Y/A = (5720 \times 10^{-6})(90.37 \times 10^6) = 517 \times 10^3\ \text{N} = 517\ \text{kN}$$

$$\text{F.S.} = \frac{P_Y}{P} = \frac{517}{175} = 2.95$$

Problem 10.52



10.52 Solve Prob. 10.51, assuming that $e = 0.16$ mm and $P = 155$ kN.

10.51 Axial loads of magnitude $P = 175$ kN are applied parallel to the geometric axis of a W250 × 44.8 rolled-steel column AB and intersect the axis at a distance $e = 12$ mm from its geometric axis. Knowing that $\sigma_Y = 250$ MPa and $E = 200$ GPa, determine the factor of safety with respect to yield. (*Hint:* Since the factor of safety must be applied to the load **P**, not to the stresses, use Fig. 10.24 to determine P_Y.)

For W 250 × 44.8 $A = 5720\ \text{mm}^2$, $r_y = 35.1$ mm

$L_e = 3800$ mm $L_e/r = 108.26$

$c = \frac{b_f}{2} = \frac{148}{2} = 74$ mm $e = 16$ mm

$$\frac{ec}{r^2} = \frac{(16)(74)}{(35.1)^2} = 0.96103$$

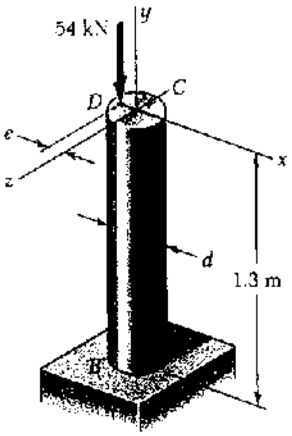
Using Fig 10.24 with $L_e/r = 108.26$ and $\frac{ec}{r^2} = 0.96103$,

$$P_Y/A = 81.17\ \text{MPa} = 81.17\ \text{N/m}^2$$

$$P_Y = A(P_Y/A) = (5720 \times 10^{-6})(81.17 \times 10^6) = 464 \times 10^3\ \text{N} = 464\ \text{kN}$$

$$\text{F.S.} = \frac{P_Y}{P} = \frac{464}{155} = 3.00$$

Problem 10.53



10.53 A 54 kN axial load is applied with an eccentricity $e = 10$ mm to the circular steel rod BC that is free at its top C and fixed at its base B. Knowing that the stock of rods available for use have diameters in increments of 4 mm from 44 mm to 72 mm, determine the lightest rod that may be used if $\sigma_{all} = 110$ MPa. Use $E = 200$ GPa.

For a solid circular section $\quad c = \frac{1}{2}d$

$$A = \pi c^2 = \frac{\pi}{4}d^2 \qquad I = \frac{\pi}{4}c^4 = \frac{\pi}{64}d^4$$

$$r^2 = \frac{I}{A} = \frac{1}{16}d^2 \qquad \frac{ec}{r^2} = \frac{8e}{d}$$

$$P = 54\times10^3 \text{ N} \qquad e = 10\text{mm} = 0.010 \text{ m}$$

$$L_e = 2L = (2)(1.3) = 2.6 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2}$$

$$\frac{P}{P_{cr}} = \frac{PL_e^2}{\pi^2 EI} = \frac{(54\times10^3)(2.6)^2(64)}{\pi^2(200\times10^9)\pi d^4} = 3.7674\times10^{-6}\,d^{-4}$$

$$\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}} = 3.0489\times10^{-3}\,d^{-2} \qquad \frac{P}{A} = \frac{(54\times10^3)(4)}{\pi d^2} = 68.755\times10^3\,d^{-2}$$

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)\right] \qquad \frac{ec}{r^2} = \frac{0.08}{d}$$

$$\sigma_{max} = 68.755\times10^3\,d^{-2}\left[1 + 0.08\,d^{-1}\sec\left(3.0489\times10^{-3}\,d^{-2}\right)\right]$$

d (mm)	σ_{max} (MPa)
44	-1599 $(P/P_{cr} > 1)$
48	232
52	116.6
56	77.5 ←
60	57.5
64	45.3
68	37.0
72	31.0

The lightest section with $\sigma_{max} < 110$ MPa is $d = 56$ mm.

Checking: $\quad d = 56 \text{ mm} = 0.056 \text{ m} \qquad c = 0.028 \text{ m}$

$$A = \frac{\pi}{4}d^2 = 2.4630\times10^{-3} \text{ m}^2$$

$$I = \frac{\pi}{64}d^4 = 482.75\times10^{-9} \text{ m}^4$$

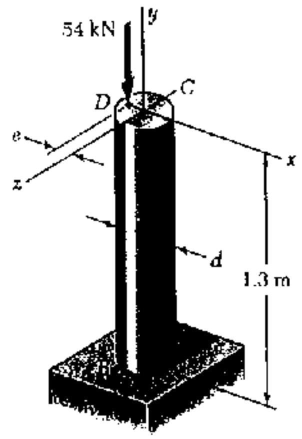
$$r^2 = \frac{I}{A} = 196\times10^{-6} \qquad \frac{ec}{r^2} = \frac{(0.010)(0.028)}{196\times10^{-6}} = 1.4286$$

$$P_{cr} = \frac{\pi^2(200\times10^9)(482.75\times10^{-9})}{(2.6)^2} = 140.96\times10^3 \text{ N}$$

$$\frac{P}{P_{cr}} = \frac{54\times10^3}{140.96\times10^3} = 0.38308$$

$$\sigma_{max} = \frac{54\times10^3}{2.4630\times10^{-3}}\left[1 + 1.4286\sec\left(\frac{\pi}{2}\sqrt{0.38308}\right)\right] = 77.5\times10^6 \text{ Pa}$$

Problem 10.54



10.54 Solve Prob. 10.53, assuming that the 54 kN axial load will be applied to the rod with an eccentricity $e = \frac{1}{2}\ d$.

10.53 A 54 kN axial load is applied with an eccentricity $e = 10$ mm to the circular steel rod BC that is free at its top C and fixed at its base B. Knowing that the stock of rods available for use have diameters in increments of 4 mm from 44 mm to 72 mm, determine the lightest rod that may be used if $\sigma_{all} = 110$ MPa. Use $E = 200$ GPa.

For a solid circular section $\quad c = \frac{1}{2}d$

$$A = \pi c^2 = \frac{\pi}{4}d^2 \qquad I = \frac{\pi}{4}c^4 = \frac{\pi}{64}d^4$$

$$r^2 = \frac{I}{A} = \frac{1}{16}d^2$$

Use $e = \frac{1}{2}d \qquad \frac{ec}{r^2} = (\frac{1}{2})(\frac{1}{2})(16) = 4$

$$P = 54\times10^3\ \text{N}$$

$$L_e = 2L = (2)(1.3) = 2.6\ \text{m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2}$$

$$\frac{P}{P_{cr}} = \frac{PL_e^2}{\pi^2 EI} = \frac{(54\times10^3)(2.6)^2(64)}{\pi^2(200\times10^9)\ \pi\ d^4} = 3.7674\times10^{-6}\ d^{-4}$$

$$\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}} = 3.0489\times10^{-3}\ d^{-2} \qquad \frac{P}{A} = \frac{(54\times10^{-3})(4)}{\pi d^2} = 68.755\times10^3\ d^{-2}$$

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)\right]$$

$$\sigma_{max} = 68.755\times10^3\ d^{-2}\left[1 + 4\sec(3.0489\times10^{-3}\ d^{-2})\right]$$

d (mm)	σ_{max} (MPa)
44	−35.1
48	517
52	263
56	177.6
60	134.4
64	108.1 ←
68	90.1
72	77.0

The lightest section with $\sigma_{max} < 110$ MPa is $d = 64$ mm

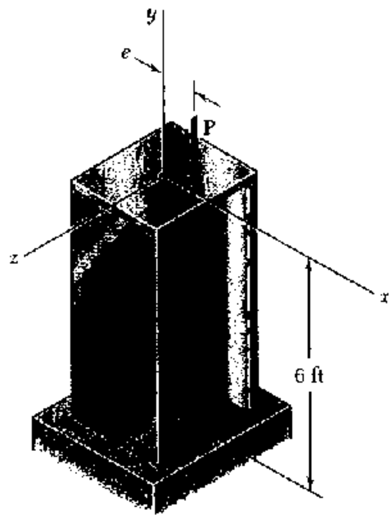
Checking $\quad A = \frac{\pi}{4}(0.064)^2 = 3.217\times10^{-3}\ \text{m}^2, \quad I = \frac{\pi}{64}(0.064)^4 = 823.55\times10^{-9}\ \text{m}^4$

$$P_{cr} = \frac{\pi^2(200\times10^9)(823.55\times10^{-9})}{(2.6)^2} = 240.48\times10^3\ \text{N}$$

$$\frac{P}{P_{cr}} = \frac{54\times10^3}{240.48\times10^3} = 0.22455$$

$$\sigma_{max} = \frac{54\times10^3}{3.217\times10^{-3}}\left[1 + 4\sec\left(\frac{\pi}{2}\sqrt{0.22455}\right)\right] = 108.1\times10^6\ \text{Pa}$$

Problem 10.55



10.55 An axial load of magnitude $P = 50$ kips is applied at a point located on the x axis at a distance $e = 0.25$ in. from the geometric axis of the wide-flange column BC. Knowing that $E = 29 \times 10^6$ psi, chose the lightest W8 shape that can be used if $\sigma_{all} = 18$ ksi.

$P = 50$ kips $\quad L = 6$ ft $\quad E = 29 \times 10^3$ ksi

$L_e = 2L = 12$ ft $= 144$ in.

$$P_{cr} = \frac{\pi^2 E I_y}{L_e^2} = \frac{\pi^2 (29\times10^3) I_y}{(144)^2}$$

$$= 13.803\ I_y$$

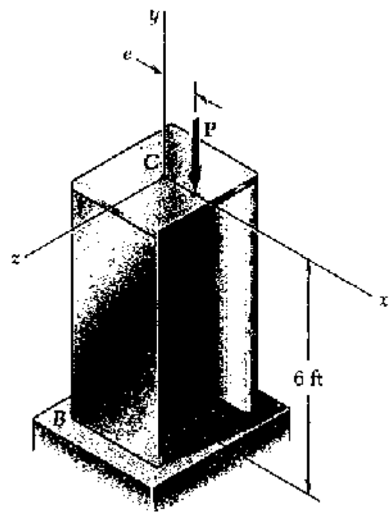
$$c = \frac{1}{2} b_f \qquad \frac{ec}{r^2} = \frac{(0.25)\, b_f}{(r_y)^2 (2)} = \frac{0.125\, b_f}{r_y^2}$$

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r_y^2} \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)\right]$$

Shape	A (in²)	b_f (in.)	I_y (in⁴)	r_y (in.)	P_{cr} (kips)	$\frac{ec}{r_y^2}$	σ_{max} (ksi)	
W8 × 58	17.1	8.220	75.1	2.10	1037	0.233	3.65	
W8 × 48	14.1	8.110	60.9	2.08	841	0.234	4.44	
W8 × 40	11.7	8.070	49.1	2.04	678	0.243	5.41	
W8 × 35	10.3	8.020	42.6	2.03	588	0.243	6.17	
W8 × 31	9.13	7.995	37.1	2.02	512	0.245	7.00	
W8 × 28	8.25	6.535	21.7	1.62	299.5	0.311	8.41	
W8 × 24	7.08	6.495	18.5	1.61	255.4	0.313	9.94	
W8 × 21	6.16	5.270	9.77	1.26	134.85	0.415	13.96	
W8 × 18	5.26	5.250	7.97	1.23	110.01	0.484	17.93	←
W8 × 15	4.44	4.015	3.41	0.876	47.07	0.654	×	
W8 × 13	3.84	4.000	2.73	0.843	37.68	0.704	×	

Use W8 × 18 $\quad \sigma_{max} = 17.93$ ksi ◄

Problem 10.56



10.56 Solve Prob. 10.55, assuming that the magnitude of the axial load is $P = 78$ kips.

10.55 An axial load of magnitude $P = 50$ kips is applied at a point located on the x axis at a distance $e = 0.25$ in. from the geometric axis of the wide-flange column BC. Knowing that $E = 29 \times 10^6$ psi, chose the lightest W8 shape that can be used if $\sigma_{all} = 18$ ksi.

$P = 78$ kips $\qquad L = 6$ ft $\qquad E = 29 \times 10^3$ ksi

$L_e = 2L = 12$ ft $= 144$ in.

$$P_{cr} = \frac{\pi^2 E I}{L_e^2} = \frac{\pi^2 (29 \times 10^3) I_y}{(144)^2}$$

$$= 13.803\ I_y$$

$$c = \tfrac{1}{2} b_F \qquad \frac{ec}{r^2} = \frac{(0.25)\, b_F}{(r_y)^2 (2)} = \frac{0.125\, b_F}{r_y^2}$$

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r_y^2} \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)\right]$$

Shape	A (in²)	b_F (in.)	I_y (in⁴)	r_y (in.)	P_{cr} (kips)	$\frac{ec}{r_y^2}$	σ_{max} (ksi)
W 8×58	17.1	8.220	75.1	2.10	1037	0.233	5.73
W 8×48	14.1	8.110	60.9	2.08	841	0.234	6.99
W 8×40	11.7	8.070	49.1	2.04	678	0.243	8.55
W 8×35	10.3	8.020	42.6	2.03	588	0.243	9.76
W 8×31	9.13	7.995	37.1	2.02	512	0.245	11.10
W 8×28	8.25	6.535	21.7	1.62	299.5	0.311	13.68
W 8×24	7.08	6.495	18.3	1.61	252.6	0.313	16.38
W 8×21	6.16	5.270	9.77	1.26	134.85	0.415	27.0
W 8×18	5.26	5.250	7.97	1.23	110.01	0.434	41.0
W 8×15	4.44	4.015	3.41	0.876	47.07	0.654	×
W 8×13	3.84	4.000	2.73	0.843	37.68	0.704	×

Use W 8×24 $\quad \sigma_{max} = 16.38$ ksi ◀

Problem 10.57

10.57 A W8 × 31 rolled-steel shape is used to form a column of 21-ft effective length. Using allowable stress design, determine the allowable centric load if the yield strength of the grade of steel used is (*a*) $\sigma_Y = 36$ ksi, (*b*) $\sigma_Y = 50$ ksi. Use $E = 29 \times 10^6$ psi.

Steel: $E = 29000$ ksi W 8×31 $A = 9.13\ \text{in}^2$ $r_{min} = 2.02$ in

$L_e = 21$ ft $= 252$ in $L_e/r = 124.75$

(a) $\sigma_Y = 36$ ksi $C_c = \sqrt{\dfrac{2\pi^2 E}{\sigma_Y}} = \sqrt{\dfrac{2\pi^2(29000)}{36}} = 126.10$

$L_e/r < C_c$ $\dfrac{L/r}{C_c} = 0.98932$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.98932) - \frac{1}{8}(0.98932)^3 = 1.91662$$

$$\sigma_{all} = \frac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{36}{1.91662}\left[1 - \frac{1}{2}(0.98932)^2\right] = 9.59\ \text{ksi}$$

$$P_{all} = \sigma_{all} A = (9.59)(9.13) = 87.6\ \text{kips}$$

(b) $\sigma_Y = 50$ ksi $C_c = \sqrt{\dfrac{2\pi^2(29000)}{50}} = 107.00$

$L_e/r > C_c$ $\sigma_{all} = \dfrac{\pi^2 E}{\frac{23}{12}(L/r)^2} = 9.59$ ksi

$$P_{all} = \sigma_{all} A = (9.59)(9.13) = 87.6\ \text{kips}$$

Problem 10.58

10.58 Using allowable stress design, determine the allowable centric load for a column of 6-m effective length that is made from the following rolled-steel shape: (*a*) W200 × 35.9, (*b*) W200 × 86. Use $\sigma_Y = 250$ MPa and $E = 200$ GPa.

Steel: $C_c = \sqrt{\dfrac{2\pi^2 E}{\sigma_Y}} = \sqrt{\dfrac{2\pi^2(200\times10^9)}{250\times10^6}} = 125.664$

(a) W200×35.9 $A = 4580\times10^{-6}\ \text{m}^2$ $r_{min} = 40.8\times10^{-3}$ m

$$\frac{L_e}{r} = \frac{6}{40.8\times10^{-3}} = 147.06 > C_c$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L/r)^2} = \frac{\pi^2(200\times10^9)}{(1.92)(147.06)^2} = 47.539\times10^6\ \text{Pa}$$

$$P_{all} = A\sigma_{all} = (4580\times10^{-6})(47.539\times10^6) = 218\times10^3\ \text{N} = 218\ \text{kN}$$

(b) W 200 × 86 $A = 11000\times10^{-6}\ \text{m}^2$ $r_{min} = 53.2\times10^{-3}$ m

$$\frac{L_e}{r} = \frac{6}{53.2\times10^{-3}} = 112.782 < 125.664 \qquad \frac{L_e/r}{C_c} = 0.89749$$

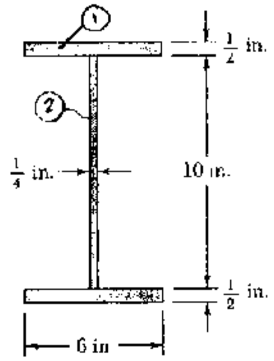
$$F.S. = \frac{5}{3} + \frac{3}{8}(0.89749) - \frac{1}{8}(0.89749)^3 = 1.91219$$

$$\sigma_{all} = \frac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\frac{L_e/r}{C_c}\right)^2\right] = \frac{250\times10^6}{1.91219}\left[1 - 0.89749^2\right] = 78.058\times10^6\ \text{Pa}$$

$$P_{all} = A\sigma_{all} = (11000\times10^{-6})(78.058\times10^6) = 859\times10^3\ \text{N} = 859\ \text{kN}$$

Problem 10.59

10.59 A column with the cross section shown has a 13.5-ft effective length. Using allowable stress design, determine the largest centric load that can be applied to the column. Use $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi.



$$A = 2A_1 + A_2 = (2)(\tfrac{1}{2})(6) + (10)(\tfrac{1}{4}) = 8.5 \text{ in}^2$$

$$I_y = 2I_1 + I_2 = (2)(\tfrac{1}{12})(\tfrac{1}{2})(6)^3 + (\tfrac{1}{12})(10)(\tfrac{1}{4})^3 = 18.013 \text{ in}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{18.013}{8.5}} = 1.4557 \text{ in.}$$

$$L_e = 13.5 \text{ ft} = 162 \text{ in} \qquad \frac{L_e}{r} = 111.29 < C_c$$

Steel: $E = 29000$ ksi, $\sigma_Y = 36$ ksi $\qquad C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29000)}{36}} = 126.10$

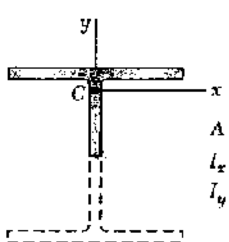
$$\frac{L_e/r}{C_c} = 0.8826 \qquad F.S. = \frac{5}{3} + \frac{3}{8}(0.8826) - \frac{1}{8}(0.8826)^3 = 1.912$$

$$\sigma_{all} = \frac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\frac{L_e/r}{C_c}\right)^2\right] = \frac{36}{1.912}\left[1 - \frac{1}{2}(0.8826)^2\right] = 11.49 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (11.49)(8.5) = 97.7 \text{ kips}$$

Problem 10.60

10.60 A column is made from half of a W360 × 216 rolled-steel shape, with the geometric properties as shown. Using allowable stress design, determine the allowable centric load if the effective length of the column is (*a*) 4.0 m, (*b*) 6.5 m. Use $\sigma_Y = 345$ MPa and $E = 200$ GPa.



$A = 13.8 \times 10^3$ mm²
$I_x = 26.0 \times 10^6$ mm⁴
$I_y = 142.0 \times 10^6$ mm⁴

$$r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{26.0 \times 10^6}{13.8 \times 10^3}} = 43.406 \text{ mm} = 43.406 \times 10^{-3} \text{ m}$$

$$A = 13.8 \times 10^{-3} \text{ m}^2$$

Steel $\quad C_c = \frac{2\pi^2 E}{\sigma_Y} = \sqrt{\frac{2\pi^2(200 \times 10^9)}{345 \times 10^6}} = 106.97$

(a) $L_e = 4.0$ m. $\qquad \frac{L_e}{r} = 92.153 < C_c \qquad \frac{L_e/r}{C_c} = 0.86149$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.86149) - \frac{1}{8}(0.86149)^3 = 1.9098$$

$$\sigma_{all} = \frac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{345 \times 10^6}{1.9098}\left[1 - \frac{1}{2}(0.86149)^2\right] = 113.61 \times 10^6 \text{ Pa}$$

$$P_{all} = \sigma_{all} A = (113.61 \times 10^6)(13.8 \times 10^{-3}) = 1568 \times 10^3 \text{ N} = 1568 \text{ kN}$$

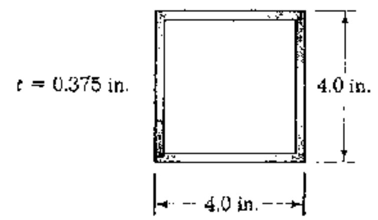
(b) $L_e = 6.5$ m. $\qquad \frac{L_e}{r} = 149.75 > C_c$

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L/r)^2} = \frac{\pi^2(200 \times 10^9)}{(1.92)(149.75)^2} = 45.845 \times 10^6 \text{ Pa}$$

$$P_{all} = \sigma_{all} A = (45.845 \times 10^6)(13.8 \times 10^{-3}) = 633 \times 10^3 \text{ N} = 633 \text{ kN}$$

Problem 10.61

10.61 A compression member has the cross section shown and an effective length of 5 ft. Knowing that the aluminum alloy used is 2014-T6, determine the allowable centric load.



$b_o = 4.0 \text{ in.} \quad b_i = b_o - 2t = 3.25 \text{ in.}$

$A = (4.0)^2 - (3.25)^2 = 5.4375 \text{ in}^2$

$I = \frac{1}{12}[(4.0)^4 - (3.25)^4] = 12.036 \text{ in}^4$

$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{12.036}{5.4375}} = 1.488 \text{ in.} \qquad L_e = 5 \text{ ft} = 60 \text{ in.}$

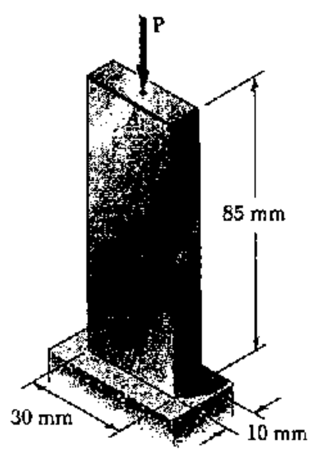
$\frac{L}{r} = \frac{60}{1.488} = 40.33 < 55$ for 2014-T6 aluminum alloy.

$\sigma_{all} = 30.7 - 0.23(L/r) = 30.7 - (0.23)(40.33) = 21.42 \text{ ksi}$

$P_{all} = \sigma_{all} A = (21.42)(5.4375) = 116.5 \text{ kips}$

Problem 10.62

10.62 Bar *AB* is free at its end *A* and fixed at its base *B*. Determine the allowable centric load **P** if the aluminum alloy is (*a*) 6061-T6, (*b*) 2014-T6.



$A = (30)(10) = 300 \text{ mm}^2 = 300 \times 10^{-6} \text{ m}^2$

$I_{min} = \frac{1}{12}(30)(10)^3 = 2.50 \times 10^3 \text{ mm}^4$

$r_{min} = \sqrt{\frac{I}{A}} = \sqrt{\frac{2.50 \times 10^3}{300}} = 2.887 \text{ mm}$

$L_e = 2L = (2)(85) = 170 \text{ mm} \qquad \frac{L_e}{r_m} = 58.88$

(a) 6061-T6 $\quad L/r < 66$

$\sigma_{all} = 139 - 0.868(L/r) = 139 - (0.868)(58.88)$
$= 87.9 \text{ MPa}$

$P_{all} = \sigma_{all} A = (87.9 \times 10^6)(300 \times 10^{-6}) = 26.4 \times 10^3 \text{ N}$
$= 26.4 \text{ kN}$

(b) 2014-T6 $\quad L/r > 55$

$\sigma_{all} = \frac{372 \times 10^3}{(L/r)^2} = \frac{372 \times 10^3}{(58.88)^2} = 107.3 \text{ MPa}$

$P_{all} = \sigma_{all} A = (107.3 \times 10^6)(300 \times 10^{-6}) = 32.2 \times 10^3 \text{ N} = 32.2 \text{ kN}$

Problem 10.63

10.63 A sawn lumber column with a 7.5 × 5.5-in. cross section has an 18-ft effective length. Knowing that for the grade of wood used the adjusted allowable stress for compression parallel to the grain is $\sigma_C = 1220$ psi and that $E = 1.3 \times 10^6$ psi, determine the maximum allowable centric load for the column.

Sawn lumber: $c = 0.8$, $\sigma_C = 1220$ psi $E = 1.3 \times 10^6$ psi $K_{cE} = 0.3$

$A = (7.5)(5.5) = 41.25 \text{ in}^2$ $d = 5.5$ in. $L = 18$ ft $= 216$ in

$L/d = 216/5.5 = 39.273$

$\sigma_{cE} = \dfrac{K_{cE} E}{(L/d)^2} = \dfrac{(0.3)(1.3\times10^6)}{(39.273)^2} = 252.86$ ps $\dfrac{\sigma_{cE}}{\sigma_C} = 0.20726$

$u = \dfrac{1+\sigma_{cE}/\sigma_C}{2c} = \dfrac{1.20726}{(2)(0.8)} = 0.754537$ $v = \dfrac{\sigma_E/\sigma_C}{c} = 0.259075$

$C_P = u - \sqrt{u^2 - v} = 0.197535$

$\sigma_{all} = \sigma_C C_P = (1220)(0.197535) = 241.0$ psi

$P_{all} = \sigma_{all} A = (241.0)(41.25) = 9.94 \times 10^3$ lb $= 9.94$ kips

Problem 10.64

10.64 A column having a 3.5-m effective length is made of sawn lumber with a 114 × 140-mm cross section. Knowing that for the grade of wood used the adjusted allowable stress for compression parallel to the grain is $\sigma_C = 7.6$ MPa and $E = 10$ GPa, determine the maximum allowable centric load for the column.

Sawn lumber: $c = 0.8$, $\sigma_C = 7.6$ MPa $K_{cE} = 0.3$ $E = 10000$ MPa

$A = (114)(140) = 15960 \text{ mm}^2 = 15960 \times 10^{-6} \text{ m}^2$

$d = 114 \text{ mm} = 114 \times 10^{-3}$ m

$L/d = 3.5/114\times10^{-3} = 30.70$

$\sigma_{cE} = \dfrac{K_{cE} E}{(L/d)^2} = \dfrac{(0.3)(10000)}{(30.70)^2} = 3.1827$ MPa $\dfrac{\sigma_{cE}}{\sigma_C} = 0.41878$

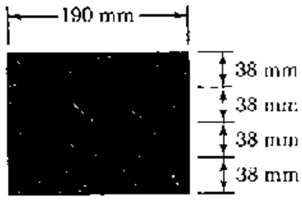
$u = \dfrac{1+\sigma_{cE}/\sigma_C}{2c} = \dfrac{1.41878}{(2)(0.8)} = 0.88673$ $v = \dfrac{\sigma_{cE}/\sigma_C}{c} = 0.523475$

$C_P = u - \sqrt{u^2 - v} = 0.37408$

$\sigma_{all} = \sigma_C C_P = (7.6)(0.37408) = 2.84$ MPa

$P_{all} = \sigma_{all} A = (2.84\times10^6)(15960\times10^{-6}) = 45.4\times10^3$ N $= 45.4$ kN

Problem 10.65



10.65 The glue laminated column shown is made from four planks, each of 38 × 190-mm cross section. Knowing that for the grade of wood used the adjusted allowable stress for compression parallel to the grain is σ_C = 10 MPa and E = 12 GPa, determine the maximum allowable centric load if the effective length of the column is (*a*) 7 m, (*b*) 3 m.

Glued laminated column. $c = 0.9$, $K_{cE} = 0.418$

$\sigma_C = 10$ MPa $\qquad E = 12000$ MPa

$4 \times 38 = 152 \text{ mm} = d \qquad b = 190 \text{ mm}$

$A = (152)(190) = 28.88 \times 10^3 \text{ mm}^2 = 28.88 \times 10^{-3} \text{ m}^2$

(a) $\underline{L = 7 \text{ m}} = 7000$ mm $\qquad L/d = 46.053$

$$\sigma_{cE} = \frac{K_{cE}E}{(L/d)^2} = \frac{(0.418)(12000)}{(46.053)^2} = 2.36510 \text{ MPa} \qquad \frac{\sigma_{cE}}{\sigma_C} = 0.236510$$

$$u = \frac{1 + \sigma_{cE}/\sigma_C}{2c} = \frac{1.19709}{(2)(0.9)} = 0.68695 \qquad \frac{\sigma_{cE}/\sigma_C}{c} = 0.262788$$

$$C_P = u - \sqrt{u^2 - v} = 0.22966$$

$$\sigma_{all} = \sigma_C C_P = (10)(0.22966) = 2.2966 \text{ MPa}$$

$$P_{all} = \sigma_{all} A = (2.2966 \times 10^6)(28.88 \times 10^{-3}) = 66.3 \times 10^3 \text{ N} = 66.3 \text{ kN}$$

(b) $\underline{L = 3 \text{ m}} = 3000$ mm $\qquad L/d = 19.7368$

$$\sigma_{cE} = \frac{K_{cE}E}{(L/d)^2} = \frac{(0.418)(12000)}{(19.7368)^2} = 12.8766 \text{ MPa} \qquad \frac{\sigma_{cE}}{\sigma_C} = 1.28766$$

$$u = \frac{1 + \sigma_{cE}/\sigma_C}{2c} = \frac{2.28766}{(2)(0.9)} = 1.27092 \qquad v = \frac{\sigma_{cE}/\sigma_C}{c} = 1.43074$$

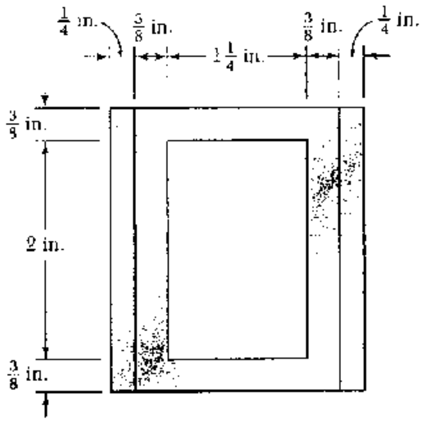
$$C_P = u - \sqrt{u^2 - v} = 0.84138$$

$$\sigma_{all} = \sigma_C C_P = (10)(0.84138) = 8.4138 \text{ MPa}$$

$$P_{all} = \sigma_{all} A = (8.4138 \times 10^6)(28.88 \times 10^{-3}) = 243 \times 10^3 \text{ N} = 243 \text{ kN}$$

Problem 10.66

10.66 An aluminum structural tube is reinforced by riveting two plates to it as shown for use as a column of 5.6 ft effective length. Knowing that all material is aluminum alloy 2014-T6, determine the maximum allowable centric load.



$b_o = \frac{1}{4} + \frac{3}{8} + 1\frac{1}{4} + \frac{3}{8} + \frac{1}{4} = 2\frac{1}{2}$ in. $= 2.5$ in.

$b_i = 1\frac{1}{4}$ in. $= 1.25$ in.

$h_o = \frac{3}{8} + 2 + \frac{3}{8} = 2\frac{3}{4} = 2.75$ in.

$h_i = 2$ in.

$A = b_o h_o - b_i h_i = (2.5)(2.75) - (1.25)(2)$
$= 4.375 \text{ in}^2$

$I_x = \frac{1}{12}\left[b_o h_o^3 - b_i h_i^3\right] = \frac{1}{12}\left[(2.5)(2.75)^3 - (1.25)(2)^3\right]$
$= 3.49935 \text{ in}^4$

$I_y = \frac{1}{12}\left[h_o b_o^3 - h_i b_i^3\right] = \frac{1}{12}\left[(2.75)(2.5)^3 - (2)(1.25)^3\right] = 3.25521 \text{ in}^4 = I_{min}$

$r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{3.25521}{4.375}} = 0.86258$ in. $\qquad L_e = 5.6$ ft $= 67.2$ in.

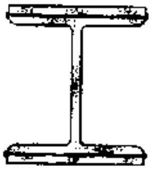
$\frac{L_e}{r} = \frac{67.2}{0.86258} = 77.906 \geq 55$

$\sigma_{all} = \frac{54000}{(L_e/r)^2} = 8.8972$ ksi

$P_{all} = \sigma_{all} A = (8.8972)(4.375) = 38.9$ kips

Problem 10.67

10.67 and *10.68* A compression member of 9-m effective length is obtained by welding two 10-mm-thick steel plates to a W250 × 80 rolled-steel shape as shown. Knowing that $\sigma_Y = 345$ MPa and $E = 200$ GPa and using allowable stress design, determine the allowable centric load for the compression member.



For W 250 × 80 $\quad A = 10200\ \text{mm}^2,\quad d = 256\ \text{mm},\quad b_f = 255\ \text{mm}$

$I_x = 126 \times 10^6\ \text{mm}^4,\quad I_y = 43.1 \times 10^6\ \text{mm}^4$

For one plate: $\quad A = (255)(10) = 2550\ \text{mm}^2$

$$I_x = \frac{1}{12}(255)(10)^3 + (2550)\left(\frac{256}{2} + \frac{10}{2}\right)^2 = 45.128 \times 10^6\ \text{mm}^4$$

$$I_y = \frac{1}{12}(10)(255)^3 = 13.818 \times 10^6\ \text{mm}^4$$

For column: $\quad A = 10200 + (2)(2550) = 15300\ \text{mm}^2 = 15.3 \times 10^{-3}\ \text{m}^2$

$$I_x = 126 \times 10^6 + (2)(45.128 \times 10^6) = 216.256 \times 10^6\ \text{mm}^4$$

$$I_y = 43.1 \times 10^6 + (2)(13.818 \times 10^6) = 70.736 \times 10^6\ \text{mm}^4 = I_{min}$$

$$r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{70.736 \times 10^6}{15300}} = 67.995\ \text{mm} = 67.995 \times 10^{-3}\ \text{m}$$

$$\frac{L_e}{r} = \frac{9}{67.995 \times 10^{-3}} = 132.363$$

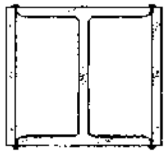
Steel: $\quad C_c = \sqrt{\dfrac{2\pi^2 E}{\sigma_Y}} = \sqrt{\dfrac{2\pi^2(200 \times 10^9)}{345 \times 10^6}} = 106.972 < \dfrac{L_e}{r}$

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L_e/r)^2} = \frac{\pi^2(200 \times 10^9)}{(1.92)(132.363)^2} = 58.681 \times 10^6\ \text{Pa}$$

$$P_{all} = \sigma_{all} A = (58.681 \times 10^6)(15.3 \times 10^{-3}) = 898 \times 10^3\ \text{N} = 898\ \text{kN}$$

Problem 10.68

10.67 and *10.68* A compression member of 9-m effective length is obtained by welding two 10-mm-thick steel plates to a W250 × 80 rolled-steel shape as shown. Knowing that $\sigma_Y = 345$ MPa. and $E = 200$ GPa and using allowable stress design, determine the allowable centric load for the compression member.



For W250 × 80 $\quad A = 10200 \text{ mm}^2, \; d = 256 \text{ mm}, \; b_f = 255 \text{ mm}$

$I_x = 126 \times 10^6 \text{ mm}^4, \quad I_y = 43.1 \times 10^6 \text{ mm}^4$

For one plate: $A = (256)(10) = 2560 \text{ mm}^4$

$$I_x = \frac{1}{12}(10)(256)^3 = 13.981 \times 10^6 \text{ mm}^4$$

$$I_y = \frac{1}{12}(256)(10)^3 + (2560)\left(\frac{255}{2} + \frac{10}{2}\right)^2 = 44.965 \times 10^6 \text{ mm}^4$$

For column: $\quad A = 10200 + (2)(2560) = 15.32 \times 10^3 \text{ mm}^2 = 15.32 \times 10^{-3} \text{ m}^2$

$$I_x = 126 \times 10^6 + (2)(13.981 \times 10^6) = 153.962 \times 10^6 \text{ mm}^4$$

$$I_y = 43.1 \times 10^6 + (2)(44.965 \times 10^6) = 133.03 \times 10^6 \text{ mm}^4 = I_{min}$$

$$r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{133.03 \times 10^6}{15.32 \times 10^3}} = 93.185 \text{ mm} = 93.185 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r} = \frac{9}{93.185 \times 10^{-3}} = 96.582$$

Steel: $\quad C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(200 \times 10^9)}{345 \times 10^6}} = 106.972 \quad > \frac{L_e}{r}$

$$\frac{L_e/r}{C_c} = \frac{96.582}{106.972} = 0.90287$$

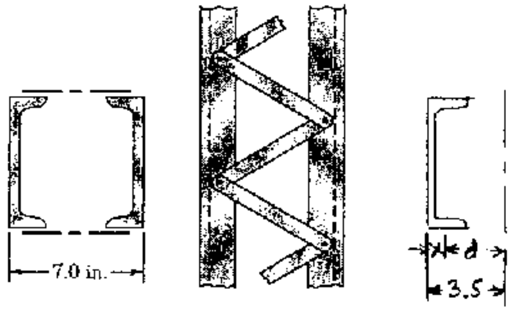
$$F.S. = \frac{5}{3} + \frac{3}{8}(0.90287) - \frac{1}{8}(90287)^3 = 1.91324$$

$$\sigma_{all} = 345\left[1 - \frac{1}{2}(0.90287)^2\right] = 204.382 \text{ MPa} = 204.382 \times 10^6 \text{ Pa}$$

$$P_{all} = \frac{\sigma_{all} A}{F.S.} = \frac{(204.382 \times 10^6)(15.32 \times 10^{-3})}{1.91324} = 1.637 \times 10^6 \text{ N} = 1637 \text{ kN} \blacktriangleleft$$

Problem 10.69

10.69 A column of 21-ft effective length is obtained by connecting two C10 × 20 steel channels with lacing bars as shown. Using allowable stress design, determine the allowable centric load for the column. Use $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi.



C10×20 $\quad A = 5.88\ in^2 \quad \bar{x} = 0.606\ in$

$I_x = 78.9\ in^4 \quad I_y = 2.81\ in^4$

$d = 3.5 - \bar{x} = 2.894\ in$

For the column: $A = (2)(5.88) = 11.76\ in^2$

$I_x = (2)(78.9) = 157.8\ in^4$

$I_y = 2[2.81 + (5.88)(2.894)^2] = 104.11\ in^4$

$r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{104.11}{11.76}} = 2.975\ in. \qquad L_e = 21\ ft = 252\ in.$

$\frac{L_e}{r} = 84.69 \qquad C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29000)}{36}} = 126.10$

$\frac{L_e}{r} < C_c \qquad \frac{L_e/r}{C_c} = 0.67165$

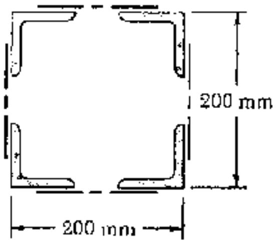
$F.S. = \frac{5}{3} + \frac{3}{8}(0.67165) - \frac{1}{8}(0.67165)^3 = 1.8807$

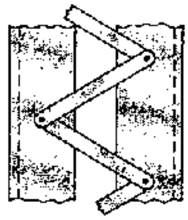
$\sigma_{all} = \frac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\frac{L_e/r}{C_c}\right)^2\right] = \frac{36}{1.8807}\left[1 - \frac{1}{2}(0.67165)^2\right] = 14.82\ ksi$

$P_{all} = \sigma_{all} A = (14.82)(11.76) = 174.3\ kips$

Problem 10.70

10.70 A column of 6.4-m effective length is obtained by connecting four 89 × 89 × 9.5-mm steel angles with lacing bars as shown. Using allowable stress design, determine the allowable centric load for the column. Use $\sigma_Y = 345$ MPa and $E = 200$ GPa.





Steel: $C_c = \sqrt{\dfrac{2\pi^2 E}{\sigma_Y}} = \sqrt{\dfrac{2\pi^2(200\times10^9)}{345\times10^6}} = 106.97$

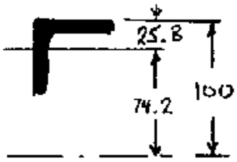
89×89×9.5 mm angle

$A_L = 1600\ \text{mm}^2$

$x = 25.8\ \text{mm}$

$I_x = 1.19\times10^6\ \text{mm}^4$

$d = 100 - x = 74.2\ \text{mm}$



$I = 4(Ad^2 + I_x) = 4[(1600)(74.2)^2 + 1.19\times10^6]$

$= 39.996\times10^6\ \text{mm}^4$

$A = 4A_L = 6400\ \text{mm}^2 = 6400\times10^{-6}\ \text{m}^2$

$r = \sqrt{\dfrac{I}{A}} = 79.053\ \text{mm} = 79.053\times10^{-3}\ \text{m}$

$\dfrac{L_e}{r} = \dfrac{6.4}{79.053\times10^{-3}} = 80.958 < C_c \qquad \dfrac{L_e/r}{C_c} = 0.75683$

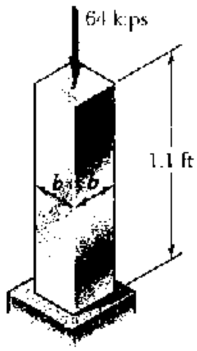
$F.S. = \frac{5}{3} + \frac{3}{8}(0.75683) - \frac{1}{8}(0.75683)^3 = 1.8963$

$\sigma_{all} = \dfrac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \dfrac{345\times10^6}{1.8963}\left[1 - \frac{1}{2}(0.75683)^2\right] = 129.83\times10^6\ \text{Pa}$

$P_{all} = \sigma_{all}A = (129.83\times10^6)(6400\times10^{-6}) = 831\times10^3\ \text{N} = 831\ \text{kN}$ ◄

Problem 10.71

10.71 A 64-kip centric load is applied to the column shown, which is free at its top A and fixed at its base B. Using aluminum alloy 2014-T6, select the smallest square cross section that can be used.



$$L_e = 2L = (2)(1.1) = 2.2 \text{ ft} = 26.4 \text{ in.}$$

$$A = b^2 \qquad I = \frac{1}{12}b^4 \qquad r = \sqrt{\frac{I}{A}} = \frac{b}{\sqrt{12}}$$

$$\frac{L_e}{r} = \frac{26.4\sqrt{12}}{b} = \frac{91.452}{b}$$

2014-T6 aluminum alloy.

Assume $\frac{L_e}{r} < 55.$

$$\sigma_{all} = 30.7 - 0.23\,(L_e/r) = 30.7 - \frac{21.034}{b} \quad \text{ksi}$$

$$P_{all} = \sigma_{all} A = \left(30.7 - \frac{21.034}{b}\right) b^2 = 64$$

$$30.7\,b^2 - 21.034\,b - 64 = 0$$

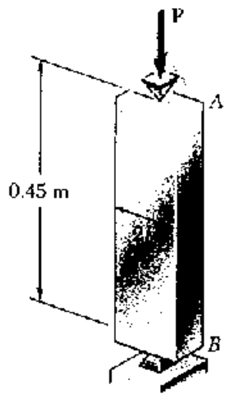
$$b = \frac{21.034 + \sqrt{(21.034)^2 - (4)(30.7)(-64)}}{(2)(30.7)} = 1.8265 \text{ in.}$$

$$\frac{L_e}{r} = \frac{91.452}{1.8265} = 50.069 < 55 \text{ as assumed.}$$

$b = 1.827$ in. ◄

Problem 10.72

10.72 A 72-kN centric load must be supported by an aluminum column as shown. Using the aluminum alloy 6061-T6, determine the minimum dimension b that can be used.



$L_e = L = 0.45\ m, \quad A = 2b^2 \quad I_{min} = \frac{1}{12}(2b)(b)^3 = \frac{1}{6}b^4$

$r = \sqrt{\frac{I_{min}}{A}} = \frac{b}{\sqrt{12}} \quad \frac{L_e}{r} = \frac{0.45\sqrt{12}}{b} = \frac{1.55885}{b}$

6061-T6 aluminum alloy. Assume $\frac{L_e}{r} < 66.$

$\sigma_{all} = 139 - 0.868(L_e/r) = 139 - \frac{(0.868)(1.55885)}{b}$

$= 139 - \frac{1.35308}{b}$ MPa

$P_{all} = \sigma_{all} A = \left(139 - \frac{1.35308}{b}\right)(10^6)(2b^2)$

$72 \times 10^3 = 278 \times 10^6\, b^2 - 2.70616 \times 10^6\, b$

$278\, b^2 - 2.70616\, b - 0.072 = 0$

$b = \frac{2.70616 + \sqrt{(2.70616)^2 - (4)(278)(-0.072)}}{(2)(278)} = 21.680 \times 10^{-3}\ m$

$r = 6.2586 \times 10^{-3}\ m$

$\frac{L_e}{r} = \frac{0.45}{6.2586 \times 10^{-3}} = 71.9$ Assumption is not verified.

Assume $\frac{L_e}{r} > 66. \quad \sigma_{all} = \frac{351 \times 10^3}{(L_e/r)^2} = \frac{351 \times 10^3\, b^2}{(1.55885)^2} = 144.444 \times 10^3\, b^2$ MPa

$P_{all} = \sigma_{all} A = (144.444 \times 10^3\, b^2)(10^6)(2b^2) = 288.89 \times 10^9\, b^4$ N

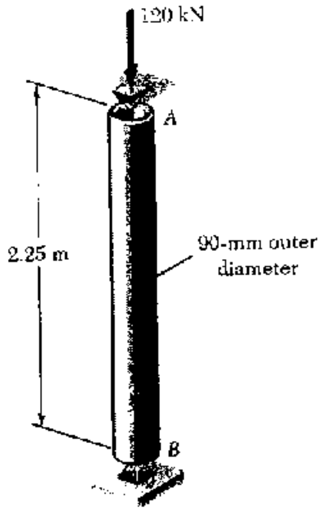
$72 \times 10^3 = 288.89 \times 10^9\, b^4 \quad b^4 = 249.23 \times 10^{-9}\ m^4$

$b = 22.3 \times 10^{-3}\ m \quad r = 6.45 \times 10^{-3}\ m \quad \frac{L_e}{r} = 69.77 > 66$

Assumption is verified. $b = 22.3 \times 10^{-3}\ m = 22.3\ mm$ ◀

Problem 10.73

10.73 An aluminum tube of 90-mm outer diameter is to carry a centric load of 120 kN. Knowing that the stock of tubes available for use are made of alloy 2014-T6 and with wall thicknesses in increments of 3 mm from 6 mm to 15 mm, determine the lightest tube that can be used.



$L = 2250 \text{ mm}, \quad P = 120 \times 10^3 \text{ N} \qquad r_o = 45 \text{ mm}$

$r_i = r_o - t \qquad A = \pi(r_o^2 - r_i^2) \qquad I = \frac{\pi}{4}(r_o^4 - r_i^4)$

$r = \sqrt{I/A}$

For 2014-T6 aluminum alloy

$$\sigma_{all} = 212 - 1.585(L/r) \text{ MPa} \quad \text{if } L/r < 55$$

$$\sigma_{all} = \frac{372 \times 10^3}{(L/r)^2} \text{ MPa} \quad \text{if } L/r > 55$$

$$P_{all} = \sigma_{all} A$$

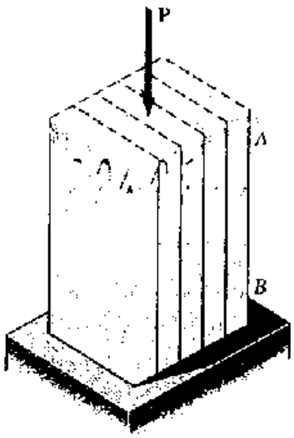
Calculate P_{all} for each thickness.

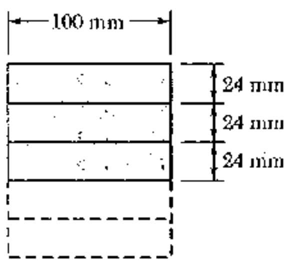
t mm	r_i mm	A mm^2	I 10^6 mm^4	r mm	L/r	σ_{all} MPa	P_{all} kN
6	39	1583	1.404	29.78	75.56	65.16	103.1
→ 9	36	2290	1.901	28.82	78.08	61.01	139.7 ←
12	33	2941	2.289	27.90	80.65	57.20	168.2
15	30	3534	2.584	27.04	83.20	53.74	189.9

Since P_{all} must be greater than 120 kN, use $t = 9$ mm ◄

Problem 10.74

10.74 A glue laminated column of 3-m effective length is to be made from boards of 24 × 100-mm cross section. Knowing that for the grade of wood used, $E = 11$ GPa and the adjusted allowable stress for compression parallel to the grain is $\sigma_C = 9$ MPa, determine the number of boards that must be used to support the centric load shown when (*a*) $P = 34$ kN, (*b*) $P = 17$ kN.





One board: $A_1 = (100)(24) = 2400 \text{ mm}^2 = 2.4 \times 10^{-3} \text{ m}^2$

n boards: $A = (2.4 \times 10^{-3})n$

1 to 4 boards: $d = 24n \text{ mm} = 24 \times 10^{-3} n \text{ m}$

$$\frac{L}{d} = \frac{3}{24 \times 10^{-3}} = \frac{3}{24 \times 10^{-3}} = \frac{125}{n}$$

Since $\frac{L}{d} < 50$, $n \geq 3$

5 or more boards, $d = 100 \text{ mm} = 0.1 \text{ m}$

$$\frac{L}{d} = \frac{3}{0.1} = 30$$

Since $n \geq 3$, the column is laminated.

$c = 0.9 \qquad K_{CE} = 0.418$

For 3 or 4 boards,

$$\sigma_{CE} = \frac{K_{CE} E}{(L/d)^2} = \frac{(0.418)(11 \times 10^9) n^2}{(125)^2} = 294.272 \times 10^3 n^2 \text{ Pa}$$

$$\sigma_{CE}/\sigma_C = \frac{294.272 \times 10^3 n^2}{9 \times 10^6} = 32.697 \times 10^{-3} n^2$$

For 5 or more boards, $\sigma_{CE} = \frac{(0.418)(11 \times 10^9)}{(30)^2} = 5.1089 \times 10^6$ Pa

$$\sigma_{CE}/\sigma_C = \frac{5.1089 \times 10^6}{9 \times 10^6} = 0.56765$$

Let $u = \frac{1 + \sigma_{CE}/\sigma_C}{2c}$ and $v = \frac{\sigma_{CE}/\sigma_C}{c}$.

Then, $C_P = u - \sqrt{u^2 - v}$ $\qquad \sigma_{all} = \sigma_C C_P = 9 \times 10^6 C_P$

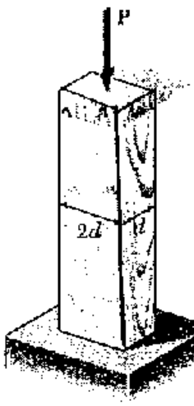
n	u	v	C_P	σ_{all} (MPa)	A (m^2)	P_{all} (kN)
3	0.71904	0.36330	0.32697	2.9427	7.2×10^{-3}	21.19
4	0.84619	0.58128	0.47910	4.3119	9.6×10^{-3}	41.39
5 or more	0.87092	0.63072	0.51345	4.6210	$2.4 \times 10^{-3} n$	11.09 n

(a) $\underline{P = 34 \text{ kN}}$ Use 4 boards ◄

(b) $\underline{P = 17 \text{ kN}}$ Use 3 boards ◄

Problem 10.75

10.75 An 18-kip centric load is applied to a rectangular sawn lumber column of 22-ft effective length. Using sawn lumber for which the adjusted allowable stress for compression parallel to the grain is $\sigma_C = 1050$ psi and knowing that $E = 1.5 \times 10^6$ psi, determine the smallest cross section that may be used. Use $b = 2d$.



Sawn lumber: $c = 0.8$ $\quad K_{cE} = 0.3$

$\sigma_c = 1050$ psi $\quad E = 10 \times 10^6$ psi

$A = 2d^2 \qquad L = 22 \text{ ft} = 264 \qquad L/d = \dfrac{264}{d}$

Assumed $C_p = 0.5$

$\sigma_{all} = \sigma_c C_p = (1050)(0.5) = 525$ psi

$P_{all} = \sigma_{all} A = 2\sigma_{all} d^2$

$d = \sqrt{\dfrac{P_{all}}{2\sigma_{all}}} = \sqrt{\dfrac{18\,000}{2\sigma_{all}}} = \dfrac{94.868}{\sqrt{\sigma_{all}}} = 4.14$ in

$L/d = 63.76 \qquad \sigma_{cE} = \dfrac{K_{cE}E}{(L/d)^2} = \dfrac{(0.3)(1.5\times10^6)}{(L/d)^2} = \dfrac{450\times10^3}{(L/d)^2} = 110.69$ psi

$\sigma_{cE}/\sigma_c = 0.1054$

Checked: $C_p = \dfrac{1+\sigma_{cE}/\sigma_c}{2c} - \sqrt{\left(\dfrac{1+(\sigma_{cE}/\sigma_c)}{2c}\right)^2 - \dfrac{\sigma_{cE}/\sigma_c}{c}} = 0.0966$

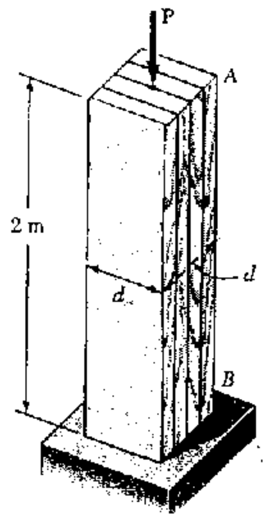
Results of similar trials are summarized in the table below:

Assumed C_p	σ_{all} (psi)	d (in)	L/d	σ_{cE} (psi)	σ_{cE}/σ_c	Checked C_p	ΔC_p
0.5	525	4.14	63.76	110.68	0.1054	0.0966	−0.4034
0.2983	313.2	5.36	49.25	185.52	0.1767	0.1697	−0.1286
0.2340	245.7	6.05	43.62	236.50	0.2252	0.2136	−0.0204
0.2238	235.0	6.19	42.66	247.28	0.2355	0.2227	−0.0011
0.2233	234.4	6.20	42.61	247.87	0.2361	0.2232	−0.0001

$d = 6.20$ in. ◄

Problem 10.76

10.76 The glued laminated column shown is free at its top A and fixed at its base B. Using wood that has an adjusted allowable stress for compression parallel to the grain $\sigma_C = 9.2$ MPa and a modulus of elasticity $E = 12$ GPa, determine the smallest cross section that can support a centric load of 62 kN.



Glued laminated column: $c = 0.9$ $\quad K_{cE} = 0.418$

$L_e = 2L = (2)(2.0) = 4$ m $= 4000$ mm

$\sigma_c = 9.2$ MPa $\qquad E = 12000$ MPa

$A = d^2 \qquad P_{all} = 62000$ N

$L_e/d = 4000/d \quad$ with d in mm

$\sigma_{all} = \sigma_c C_p$

Assume C_p. $\qquad \sigma_{all} = \sigma_c C_p = 9.2\, C_p$ MPa

$$d = \sqrt{\frac{P_{all}}{\sigma_{all}}} = \sqrt{\frac{62000}{\sigma_{all}}} = \frac{249}{\sqrt{\sigma_{all}}}$$

$$\sigma_{cE} = \frac{K_{cE}E}{(L/d)^2} = \frac{(0.418)(12000)}{(L/d)^2} = \frac{5016}{(L/d)^2}$$

Checking, $\quad C_p = \dfrac{1+\sigma_{cE}/\sigma_c}{2c} - \sqrt{\left(\dfrac{1+\sigma_{cE}/\sigma_c}{2c}\right)^2 - \dfrac{\sigma_{cE}/\sigma_c}{c}}$

Calculations are carried out in the following table:

C_p (assumed)	σ_{all} (MPa)	d mm	L/d	σ_{cE} (MPa)	σ_{cE}/σ_c	C_p	ΔC_p
0.5	4.6	116.1	34.45	4.225	0.4593	0.4274	−0.0726
0.4	3.68	129.8	30.82	5.282	0.5741	0.5183	+0.1183
0.46	4.232	121.0	33.05	4.593	0.4992	0.4600	+0.0000

$\sigma_{all} = (0.4600)(9.2) = 4.232$ MPa

$$d = \frac{249}{\sqrt{4.232}} = 121 \text{ mm}$$

Problem 10.77

10.77 A column of 4.5-m effective length must carry a centric load of 900 kN. Knowing that $\sigma_Y = 345$ MPa and $E = 200$ GPa, use allowable stress design to select the wide-flange shape of 250-mm nominal depth that should be used.

Steel: $E = 200000$ MPa $\quad \sigma_Y = 345$ MPa $\quad C_c = \sqrt{\dfrac{2\pi^2 E}{\sigma_Y}} = 106.97$

$$P < \frac{\sigma_Y}{(F.S.)A} \qquad A > \frac{(F.S.)P}{\sigma_Y} = \frac{(5/3)(900\times10^3)}{345\times10^6} = 4348\times10^{-6}\ m^2 = 4348\ mm^2$$

$$P < \frac{\pi^2 EI}{(F.S.)L_e^2} \qquad I > \frac{(F.S.)PL_e^2}{\pi^2 E} = \frac{(1.92)(900\times10^3)(4.5)^2}{\pi^2\ 200\times10^9} = 17.73\times10^{-6}\ m^4$$
$$= 17.73\times10^{6}\ mm^4$$

Try W250×58. $\quad A = 7420\ mm^2 \quad r_y = 50.3$ mm

$$\frac{L_e}{r_y} = \frac{4.5}{50.3\times10^{-3}} = 89.46 < 106.97 \qquad \frac{L_e/r_y}{C_c} = 0.8363$$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.8363) - \frac{1}{8}(0.8363)^3 = 1.907$$

$$\sigma_{all} = \frac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\frac{L_e/r}{C_c}\right)^2\right] = \frac{345}{1.907}\left[1 - \frac{1}{2}(0.8363)^2\right] = 117.6\ MPa$$

$$P_{all} = A\sigma_{all} = (7420\times10^{-6})(117.6\times10^6) = 873\times10^3\ N = 873\ kN < 900\ kN$$

(not acceptable)

Try W250×67. $\quad A = 8580\ mm^2 \quad r_y = 51.0$ mm

$$\frac{L_e}{r_y} = \frac{4.5}{51.0\times10^{-3}} = 88.24 < 106.97 \qquad \frac{L_e/r_y}{C_c} = 0.8249$$

$$F.S. = 1.906 \qquad \sigma_{all} = \frac{345}{1.906}\left[1 - \frac{1}{2}(0.8249)^2\right] = 119.4\ MPa$$

$$P_{all} = A\sigma_{all} = (8580\times10^{-6})(119.4\times10^6) = 1025\times10^3\ N = 1025\ kN > 900\ kN$$

(acceptable)

Use W250×67.

Problem 10.78

10.78 A column of 22.5-ft effective length must carry a centric load of 288 kips. Using allowable stress design, select the wide-flange shape of 14-in. nominal depth that should be used. Use $\sigma_Y = 50$ ksi and $E = 29 \times 10^6$ psi.

$$P < \frac{\sigma_Y A}{F.S.} \qquad A > \frac{(F.S.)P}{\sigma_Y} = \frac{(5/3)(288)}{50} = 9.6 \text{ in}^2$$

$$L_e = 22.5 \text{ ft} = 270 \text{ in} \qquad E = 29 \times 10^6 \text{ psi} = 29000 \text{ ksi}$$

$$P < \frac{\pi^2 EI}{1.92 L_e^2} \qquad I > \frac{1.92 P L_e^2}{\pi^2 E} = \frac{(1.92)(288)(270)^2}{\pi^2 (29000)} = 140.8 \text{ in}^4$$

Try W 14 × 82. $A = 24.1 \text{ in}^2$, $I_{min} = 148 \text{ in}^4$, $r = 2.48$ in

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29000)}{50}} = 107.00$$

$$\frac{L_e}{r} = \frac{270}{2.48} = 108.87 > 107.00$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92 (L/r)^2} = \frac{\pi^2 (29000)}{(1.92)(108.87)^2} = 12.58 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (12.58)(24.1) = 303 \text{ kips} > 288 \text{ kips}$$

Use W 14 × 82.

Problem 10.79

10.79 A column of 17-ft effective length must carry a centric load of 235 kips. Using allowable stress design, select the wide-flange shape of 10-in. nominal depth that should be used. Use $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi.

$$P < \frac{\sigma_Y A}{F.S.} \qquad A > \frac{(F.S.)P}{\sigma_Y} = \frac{(5/3)(235)}{36} = 10.88 \text{ in}^2$$

$$L_e = 17 \text{ ft} = 204 \text{ in}^2 \qquad E = 29 \times 10^6 \text{ psi} = 29000 \text{ ksi}$$

$$P < \frac{\pi^2 EI}{1.92 L^2} \qquad I > \frac{1.92 P L_e^2}{\pi^2 E} = \frac{(1.92)(235)(204)^2}{\pi^2 (29000)} = 65.6 \text{ in}^4$$

Try W 10 × 54. $A = 15.8 \text{ in}^2$ $I_y = 103 \text{ in}^4$ $r = 2.56$ in

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29000)}{36}} = 126.10$$

$$\frac{L_e}{r} = \frac{204}{2.56} = 79.69 < C_c \qquad \frac{L_e/r}{C_c} = \frac{79.69}{126.10} = 0.63194$$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.63194) - \frac{1}{8}(0.63194)^3 = 1.8721$$

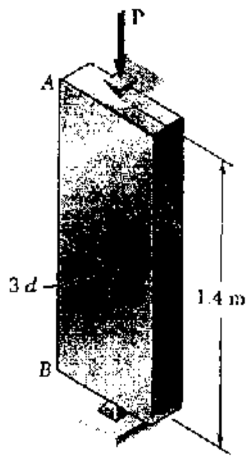
$$\sigma_{all} = \frac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{36}{1.8721}\left[1 - \frac{1}{2}(0.63194)^2\right] = 15.39 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (15.39)(15.8) = 243 \text{ kips} > 235 \text{ kips}$$

Use W 10 × 54.

Problem 10.80

10.80 A centric load P must be supported by the steel bar AB. Using allowable stress design, determine the smallest dimension d of the cross section that can be used when (a) $P = 108$ kN, (b) $P = 166$ kN. Use $\sigma_y = 250$ MPa and $E = 200$ GPa.



$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(200\times10^9)}{250}} = 125.66$$

$$L_e = L = 1.4 \text{ m}$$

$$A = (3d)(d) = 3d^2$$

$$I = \tfrac{1}{12}(3d)(d)^3 = \tfrac{1}{4}d^4$$

$$r = \sqrt{\frac{I}{A}} = \frac{d}{\sqrt{12}} = 0.288675\, d$$

(a) $\underline{P = 108\times10^3 \text{ N}}$. Assume $\frac{L_e}{r} > C_c$.

$$P_{all} = \frac{\pi^2 EI}{1.92\, L_e^2} \qquad I = \frac{(1.92)P_{all}L_e^2}{\pi^2 E} = \frac{1}{4}d^4$$

$$d^4 = \frac{(4)(1.92)P L_e^2}{\pi^2 E} = \frac{(4)(1.92)(108\times10^3)(1.4)^2}{\pi^2(200\times10^9)} = 823.59\times10^{-9} \text{ m}^4$$

$$d = 30.125\times10^{-3} \text{ m} \qquad r = 8.696\times10^{-3} \text{ m}$$

$$\frac{L_e}{r} = \frac{1.4}{8.696\times10^{-3}} = 160.99 > 125.66 \qquad d = 30.1 \text{ mm}$$

(b) $\underline{P = 166\times10^3 \text{ N}}$. Assume $\frac{L_e}{r} > C_c$

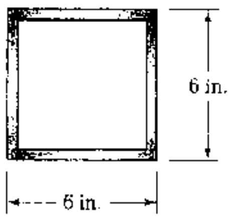
$$d^4 = \frac{(4)(1.92)P L_e^2}{\pi^2 E} = \frac{(4)(1.92)(166\times10^3)(1.4)^2}{\pi^2(200\times10^9)} = 1.26588\times10^{-6} \text{ m}^4$$

$$d = 33.543\times10^{-3} \text{ m} \qquad r = 9.68295\times10^{-3}$$

$$\frac{L_e}{r} = \frac{1.4}{9.68295\times10^{-3}} = 144.58 > 125.66 \qquad d = 33.5 \text{ mm}$$

Problem 10.81

10.81 A square structural tube having the cross section shown is used as a column of 26-ft effective length to carry a centric load of 65 kips. Knowing that the tubes available for use are made with wall thicknesses of $\frac{1}{16}$ in. from $\frac{1}{4}$ in. to $\frac{3}{4}$ in., use allowable stress design to determine the lightest tube that can be used. Use $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi.



$b_o = 6 \text{ in.} \qquad b_i = b_o - 2t \qquad A = b_o^2 - b_i^2$

$I = \frac{1}{12}(b_o^4 - b_i^4)$

$L_e = 26 \text{ ft} = 312 \text{ in.} \qquad P = 65 \text{ kips}$

$\sigma_Y = 36 \text{ ksi} \qquad E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi}$

Steel: $C_c = \sqrt{\dfrac{2\pi^2 E}{\sigma_Y}} = \sqrt{\dfrac{2\pi^2\, 29\times 10^3}{36}} = 126.099$

Try $t = \frac{1}{2} \text{ in.} = 0.5 \text{ in.} \qquad b_i = 5.0 \text{ in.} \qquad A = 6^2 - 5.0^2 = 11 \text{ in}^2$

$I = \frac{1}{12}\left[(6)^4 - (5.0)^4\right] = 55.9167 \text{ in}^4$

$r = \sqrt{\dfrac{I}{A}} = 2.2546 \text{ in.}$

$\dfrac{L_e}{r} = \dfrac{312}{2.2546} = 138.382 > C_c$

$\sigma_{all} = \dfrac{\pi^2 E}{1.92(L_e/r)^2} = 7.7846 \text{ ksi}$

$P_{all} = \sigma_{all} A = (7.7846)(11) = 85.7 \text{ kips.}$

P_{all} is approximately proportional to t.

$\dfrac{t}{0.5} = \dfrac{65}{85.7} \qquad t \approx 0.3795 \text{ in.}$

Try $t = \frac{3}{8} \text{ in.} = 0.375 \text{ in.} \qquad b_i = 5.25 \text{ in.} \qquad A = 8.4375 \text{ in}^2$

$I = \frac{1}{12}\left[(6)^4 - (5.25)^4\right] = 44.6924 \text{ in}^4$

$r = \sqrt{\dfrac{I}{A}} = 2.3015 \text{ in.}$

$\dfrac{L_e}{r} = \dfrac{312}{2.3015} = 135.564 > C_c$

$\sigma_{all} = \dfrac{\pi^2 E}{1.92(L_e/r)^2} = 8.1116 \text{ ksi}$

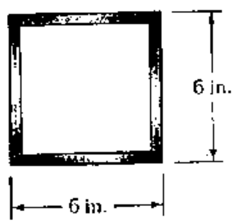
$P_{all} = \sigma_{all} A = (8.1116)(8.4375) = 68.4 \text{ kips.} > 65 \text{ kips}$

Use $t = 3/8$ in. ◀

Problem 10.82

10.82 Solve Prob. 10.81, assuming that the effective length of the column is decreased to 20 ft.

10.81 A square structural tube having the cross section shown is used as a column of 26-ft effective length to carry a centric load of 65 kips. Knowing that the tubes available for use are made with wall thicknesses of $\frac{1}{16}$ in. from $\frac{1}{4}$ in. to $\frac{3}{4}$ in., use allowable stress design to determine the lightest tube that can be used. Use $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi.



$b_o = 6$ in. $\quad b_i = b_o - 2t \quad A = b_o^2 - b_i^2$

$I = \frac{1}{12}(b_o^4 - b_i^4)$

$L_e = 20$ ft $= 240$ in. $\quad P = 65$ kips

$\sigma_Y = 36$ ksi $\quad E = 29 \times 10^6$ psi $= 29 \times 10^3$ ksi

Steel: $\quad C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29\times10^3)}{36}} = 126.099$

Try $t = \frac{1}{2}$ in. $= 0.5$ in. $\quad b_i = 5.0$ in. $\quad A = 6^2 - 5.0^2 = 11$ in^2

$I = \frac{1}{12}[(6)^4 - (5.0)^4] = 55.9167$ in^4

$r = \sqrt{\frac{I}{A}} = 2.2546$ in.

$\frac{L_e}{r} = \frac{240}{2.2546} = 106.449 < C_c \quad \frac{L_e/r}{C_c} = 0.84417$

F.S. $= \frac{5}{3} + \frac{3}{8}(0.84417) - \frac{1}{8}(0.84417)^3 = 1.908$

$\sigma_{all} = \frac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\frac{L_e/r}{C_c}\right)^2\right] = \frac{36}{1.908}\left[1 - \frac{1}{2}(0.84417)^2\right] = 12.145$ ksi

$P_{all} = \sigma_{all} A = (12.145)(11) = 133.6$ kips

P_{all} is approximately proportional to t.

$\frac{t}{0.5} \approx \frac{65}{133.6} \qquad t \approx 0.243$ in.

Try $t = \frac{1}{4}$ in. $= 0.25$ in. $\quad b_i = 5.5$ in. $\quad A = 5.75$ in.

$I = \frac{1}{12}[(6)^4 - (5.5)^4] = 31.745$ in$^4 \quad r = \sqrt{\frac{I}{A}} = 2.3496$ in.

$\frac{L_e}{r} = \frac{240}{2.3496} = 102.143 < C_c \quad \frac{L_e/r}{C_c} = 0.81002$

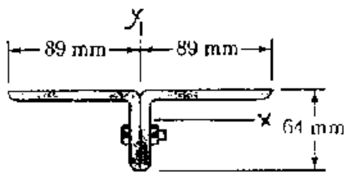
F.S. $= \frac{5}{3} + \frac{3}{8}(0.81002) - \frac{1}{8}(0.81002)^3 = 1.904$

$\sigma_{all} = \frac{36}{1.904}\left[1 - \frac{1}{2}(0.81002)^2\right] = 12.705$ ksi

$P_{all} = \sigma_{all} A = (12.705)(5.75) = 73.1$ kips > 65 kips

Use $t = 1/4$ in. ◄

Problem 10.83



10.83 Two 89 × 64-mm angles are bolted together as shown for use as a column of 2.4-m effective length to carry a centric load of 180 kN. Knowing that the angles available have thicknesses of 6.4 mm, 9.5 mm, and 12.7 mm, use allowable stress design to determine the lightest angles that can be used. Use $\sigma_Y = 250$ MPa and $E = 200$ GPa.

Steel: $\sigma_Y = 250 \times 10^6$ Pa, $E = 200 \times 10^9$ Pa

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(200\times10^9)}{250\times10^6}} = 125.664$$

Use x and y axes as added above.

Try L 89 × 64 × 9.5 $\quad A = (2)(1360) = 2720 \text{ mm}^2 = 2720\times10^{-6}$ mm

$r_x = 18.6 \text{ mm} = 18.6\times10^{-3}$ m $\quad$ (r_y in Appendix C)

$L_e/r = 2.4/18.6\times10^{-3} = 129.032 > C_c$

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L/r)^2} = \frac{\pi^2(200\times10^9)}{(1.92)(129.032)^2} = 61.749\times10^6 \text{ Pa}$$

$P_{all} = A\sigma_{all} = (2720\times10^{-6})(61.749\times10^6) = 168.0\times10^3 \text{ N} = 168.0$ kN

$P_{all} < P = 180$ kN. Do not use.

Try L 89 × 64 × 12.7 $\quad A = (2)(1780) = 3960 \text{ mm}^2 = 3960\times10^{-6} \text{ m}^2$

$r_x = 18.1 \text{ mm} = 18.1\times10^{-3}$ m

$L_e/r = 2.4/18.1\times10^{-3} = 132.597 > C_c$

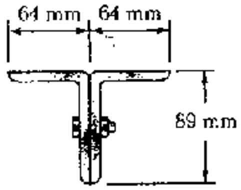
$$\sigma_{all} = \frac{\pi^2 E}{1.92(L_e/r)^2} = \frac{\pi^2(200\times10^9)}{(1.92)(132.597)^2} = 58.474\times10^6 \text{ Pa}$$

$P_{all} = A\sigma_{all} = (3960\times10^{-6})(58.474\times10^6) = 232\times10^3 \text{ N} = 232$ kN

$P_{all} > P$ $\qquad$ Use L 89 × 64 × 12.7.

Problem 10.84

10.84 Two 89 × 64-mm angles are bolted together as shown for use as a column of 2.4-m effective length to carry a centric load of 325 kN. Knowing that the angles available have thicknesses of 6.4 mm, 9.5 mm, and 12.7 mm, use allowable stress design to determine the lightest angles that can be used. Use $\sigma_Y = 250$ MPa and $E = 200$ GPa.



Steel: $\sigma_Y = 250\times10^6$ Pa $\quad E = 200\times10^9$ Pa

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(200\times10^9)}{250\times10^6}} = 125.664$$

Try L $89\times64\times9.5$.

$$A = (2)(1360) = 2720\ \text{mm}^2 = 2720\times10^{-6}\ \text{m}^2$$

$$I_x = (2)(1.07\times10^6) = 2.14\times10^6\ \text{mm}^4$$

$$I_y = (2)[0.463\times10^6 + (1360)(16.9)^2] = 1.70285\times10^6\ \text{mm}^4 = I_{min}$$

$$r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{1.70285\times10^6}{2720}} = 25.021\ \text{mm} = 25.021\times10^{-3}\ \text{m}$$

$$\frac{L_e}{r} = \frac{2.4}{25.021\times10^{-3}} = 95.919 < C_c \qquad \frac{L_e/r}{C_c} = 0.76330$$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.76330) - \frac{1}{8}(0.76330)^3 = 1.8973$$

$$\sigma_{all} = \frac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\frac{L_e/r}{C_c}\right)^2\right] = \frac{250\times10^6}{1.8973}\left[1 - \frac{1}{2}(0.76330)^2\right] = 93.38\times10^6\ \text{Pa}$$

$$P_{all} = \sigma_{all}A = (93.38\times10^6)(2720\times10^{-6}) = 254\times10^3\ \text{N} = 254\ \text{kN} < 325\ \text{kN}$$

Do not use.

Try L $89\times64\times12.7$.

$$A = (2)(1780) = 3560\ \text{mm}^2 = 3560\times10^{-6}\ \text{m}^2$$

$$I_x = (2)(1.36\times10^6) = 2.72\times10^6\ \text{mm}^4$$

$$I_y = (2)[0.581\times10^6 + (1780)(18.1)^2] = 2.3283\times10^6\ \text{mm}^4 = I_{min}$$

$$r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{2.3283\times10^6}{3560}} = 25.574\ \text{mm} = 25.574\times10^{-3}\ \text{m}$$

$$\frac{L_e}{r} = \frac{2.4}{25.574\times10^{-3}} = 93.846 < C_c \qquad \frac{L_e/r}{C_c} = 0.7468$$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.7468) - \frac{1}{8}(0.7468)^3 = 1.89465$$

$$\sigma_{all} = \frac{250\times10^6}{1.89465}\left[1 - \left(\frac{1}{2}\right)(0.7468)^2\right] = 95.155\times10^6\ \text{Pa}$$

$$P_{all} = \sigma_{all}A = (95.155\times10^6)(3560\times10^{-6}) = 339\times10^3\ \text{N} = 339\ \text{kN} > 325\ \text{kN}$$

Use L $89\times64\times12.7$. ◄

Problem 10.85

*10.85 A column with a 5.8-m effective length supports a centric load, with ratio of dead to live load equal to 1.35. The dead load factor is $\gamma_D = 1.2$, the live load factor $\gamma_L = 1.6$, and the resistance factor $\phi = 0.85$. Use load and resistance factor design to determine the allowable centric dead and live loads if the column is made of the following rolled-steel shape: (*a*) W250 × 67, (*b*) W 360 × 101. Use $\sigma_Y = 345$ MPa and $E = 200$ GPa.

(a) W250 × 67 $\quad A = 8580\ \text{mm}^2 = 8580 \times 10^{-6}\ \text{m}^2, \quad r_y = 51.0\ \text{mm} = 51.0 \times 10^{-3}\ \text{m}$

$$L_e/r_y = \frac{5.8}{51.0 \times 10^{-3}} = 113.725$$

$$\lambda_c = \frac{L_e/r}{\pi}\sqrt{\frac{\sigma_Y}{E}} = \frac{113.725}{\pi}\sqrt{\frac{345 \times 10^6}{200 \times 10^9}} = 1.5035 > 1.5$$

$$P_u = A\left(\frac{0.877}{\lambda_c^2}\right)\sigma_Y = (8580 \times 10^{-6})\left(\frac{0.877}{1.5035^2}\right)(345 \times 10^6) = 1.1484 \times 10^6\ \text{N}$$

$$\gamma_D P_D + \gamma_L P_L = \varphi P_u$$

$$(1.2)P_D + \left(\frac{1.6}{1.35}\right)P_D = (0.85)(1.1484 \times 10^6) \qquad P_D = 409.26 \times 10^3\ \text{N}$$

$$P_L = \frac{409.26 \times 10^3}{1.35} = 303.16 \times 10^3\ \text{N}$$

$$P_D = 409\ \text{kN}, \quad P_L = 303\ \text{kN}$$

(b) W 360 × 101 $\quad A = 12900\ \text{mm}^2 = 12900 \times 10^{-6}\ \text{m}^2, \quad r_y = 62.6\ \text{mm} = 62.6 \times 10^{-3}\ \text{m}$

$$L_e/r_y = \frac{5.8}{62.6 \times 10^{-3}} = 92.652$$

$$\lambda_c = \frac{L_e/r}{\pi}\sqrt{\frac{\sigma_Y}{E}} = \frac{92.652}{\pi}\sqrt{\frac{345 \times 10^6}{200 \times 10^9}} = 1.22489 < 1.5$$

$$P_u = A(0.658^{\lambda_c^2})\sigma_Y = (12900 \times 10^{-6})(0.658)^{1.22489^2}(345 \times 10^6)$$

$$= 2.3751 \times 10^6\ \text{N}$$

$$\gamma_D P_D + \gamma_L P_L = \varphi P_u$$

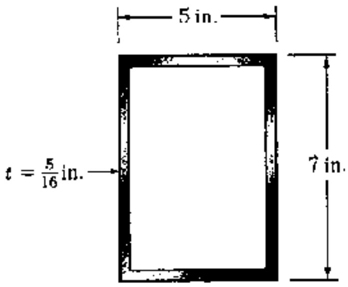
$$(1.2)P_D + \left(\frac{1.6}{1.35}\right)P_D = (0.85)(2.3751 \times 10^6) \qquad P_D = 846.41 \times 10^3\ \text{N}$$

$$P_L = \frac{846.41 \times 10^3}{1.35} = 626.97 \times 10^3\ \text{N}$$

$$P_D = 846\ \text{kN}, \quad P_L = 627\ \text{kN}$$

Problem 10.86

*10.86 A rectangular tube having the cross section shown is used as a column of 14.5-ft effective length. Knowing that $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi., use load and resistance factor design to determine the largest centric live load that can be applied if the centric dead load is 54 kips. Use a dead load factor $\gamma_D = 1.2$, a live load factor $\gamma_L = 1.6$ and the resistance factor $\phi = 0.85$.



$L = 14.5 \text{ ft} = 174 \text{ in.}$

$b_o = 7 \text{ in.} \qquad b_i = 7 - (2)(\frac{5}{16}) = 6\frac{3}{8} \text{ in.}$

$h_o = 5 \text{ in.} \qquad h_i = 5 - (2)(\frac{5}{16}) = 4\frac{3}{8} \text{ in.}$

$A = (7)(5) - (6\frac{3}{8})(4\frac{3}{8}) = 7.109375 \text{ in}^2$

$I = \frac{1}{12}\left[(7)(5)^3 - (6\frac{3}{8})(4\frac{3}{8})^3\right] = 28.42967 \text{ in}^4$

$r = \sqrt{\frac{I}{A}} = 1.99972 \text{ in.}$

$\frac{L_e}{r} = \frac{174}{1.99972} = 87.012$

$\lambda_c = \frac{L_e/r}{\pi}\sqrt{\frac{\sigma_Y}{E}} = \frac{87.012}{\pi}\sqrt{\frac{36}{29000}} = 0.97585 < 1.5$

$\lambda_c^2 = 0.952276$

$P_U = A(0.658)^{\lambda_c^2}\sigma_Y = (7.109375)(0.658)^{0.952276}(36) = 171.805 \text{ kips.}$

$\gamma_D P_D + \gamma_L P_L = \varphi P_U$

$(1.2)(54) + (1.6)P_L = (0.85)(171.805) \qquad P_L = 50.8 \text{ kips}$

Problem 10.87

*10.87 A column of 5.5-m effective length must carry a centric dead load of 310 kN and a centric live load of 375 kN. Knowing that $\sigma_Y = 250$ MPa and $E = 200$ GPa, use load and resistance factor design to select the wide-flange shape of 310-mm nominal depth that should be used. The dead load factor $\gamma_D = 1.2$, the live load factor $\gamma_L = 1.6$ and the resistance factor $\phi = 0.85$.

$$\gamma_D P_D + \gamma_L P_L = \phi P_U$$

Required. $P_U = \dfrac{\gamma_D P_D + \gamma_L P_L}{\phi} = \dfrac{(1.2)(310) + (1.6)(375)}{0.85} = 1143$ kN

Preliminary calculations.

$$P_U < \sigma_Y A \quad \therefore \quad A > \frac{P_U}{\sigma_Y} = \frac{1143 \times 10^3}{250 \times 10^6} = 4.572 \times 10^{-3}\ \text{m}^2 = 4572\ \text{mm}^2$$

$$P_U < \frac{\pi^2 EI}{L^2} \quad \therefore \quad I > \frac{P_U L_e^2}{\pi^2 E} = \frac{(1143 \times 10^3)(5.5)^2}{\pi^2 (200 \times 10^9)} = 17.52 \times 10^{-6}\ \text{m}^4 = 17.52 \times 10^6\ \text{mm}^4$$

Try W 310×60. $A = 7590\ \text{mm}^2 = 7590 \times 10^{-6}\ \text{m}^2$

$I_y = 18.3 \times 10^6\ \text{mm}^4$, $\quad r_y = 49.1\ \text{mm} = 49.1 \times 10^{-3}\ \text{m}$

$$\lambda_c = \frac{L_e}{\pi r}\sqrt{\frac{\sigma_Y}{E}} = \frac{5.5}{\pi (49.1 \times 10^{-3})}\sqrt{\frac{250 \times 10^6}{200 \times 10^9}} = 1.2606 < 1.5$$

$$\lambda_c^2 = 1.5892$$

$$P_U = A(0.658)^{\lambda_c^2}\sigma_Y = (7590 \times 10^{-6})(0.658)^{1.5892}(250 \times 10^6)$$

$$= 975 \times 10^3\ \text{N} = 975\ \text{kN} < 1143\ \text{kN}$$

Too light. Do not use.

Try W310 × 74. $A = 9480\ \text{mm}^2 = 9480 \times 10^{-6}\ \text{m}^2$

$r_y = 49.7\ \text{mm} = 49.7 \times 10^{-3}\ \text{m}$

$$\lambda_c = \frac{5.5}{\pi (49.7 \times 10^{-3})}\sqrt{\frac{250 \times 10^6}{200 \times 10^9}} = 1.2454 \qquad \lambda_c^2 = 1.5510$$

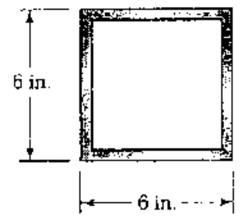
$$P_U = (9480 \times 10^{-6})(0.658)^{1.5510}(250 \times 10^6) = 1238 \times 10^3\ \text{N}$$

$$= 1238\ \text{kN} > 1143\ \text{kN}$$

Use W310×74. ◄

Problem 10.88

*10.88 The structural tube having the cross section shown is used as a column of 15-ft effective length to carry a centric dead load of 51 kips and a centric live load of 58 kips. Knowing that the tubes available for use are made with wall thicknesses in increments of $\frac{1}{16}$ in. from $\frac{3}{16}$ in. to $\frac{3}{8}$ in., use load and resistance factor design to determine the lightest tube that can be used. Use $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi. The dead load factor $\gamma_D = 1.2$, the live load factor $\gamma_L = 1.6$ and the resistance factor $\phi = 0.85$.



$L_e = 15 \text{ ft} = 180 \text{ in}$

$\gamma_D P_D + \gamma_L P_L = \varphi P_U$

Required $P_U = \dfrac{\gamma_D P_D + \gamma_L P_L}{\varphi} = \dfrac{(1.2)(51) + (1.6)(58)}{0.85} = 181.2$ kips

Try $t = \frac{1}{4}$ in $= 0.25$ in. $\quad b_o = 6.0$ in. $\quad b_i = b_o - 2t = 5.5$ in

$A = b_o^2 - b_i^2 = (6)^2 - (5.5)^2 = 5.75 \text{ in}^2$

$I = \frac{1}{12}(b_o^4 - b_i^4) = \frac{1}{12}\left[(6)^4 - (5.5)^4\right] = 31.74 \text{ in}^4$

$r = \sqrt{\dfrac{I}{A}} = \sqrt{\dfrac{31.74}{5.75}} = 2.3496$ in $\qquad \dfrac{L_e}{r} = \dfrac{180}{2.3496} = 76.61$

$\lambda_c = \dfrac{L_e/r}{\pi}\sqrt{\dfrac{\sigma_Y}{E}} = \dfrac{76.61}{\pi}\sqrt{\dfrac{36}{29000}} = 0.85916 < 1.5 \qquad \lambda_c^2 = 0.73815$

$P_U = A\,(0.658)^{\lambda_c}\sigma_Y = (5.75)(0.658)^{0.73815}(36) = 152.0 \text{ kips} < 181.2 \text{ kips}$

Thickness is to small.

Since P_U is approximately proportional to thickness, the required thickness is approximately

$$\frac{t_{req}}{0.25} \approx \frac{P_{U(req)}}{152} = \frac{181.18}{152} \qquad t_{req} \approx 0.296 \text{ in.}$$

Try $t = \frac{5}{16}$ in. $= 0.3125$ in. $\quad b_i = 5.375$ in.

$A = 7.1094 \text{ in}^2, \quad I = 38.44 \text{ in}^4, \quad r = 2.3254 \text{ in} \qquad \dfrac{L_e}{r} = 77.41$

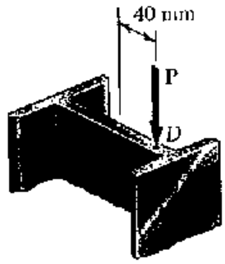
$\lambda_c = \dfrac{77.41}{\pi}\sqrt{\dfrac{36}{29000}} = 0.86811 < 1.5 \qquad \lambda_c^2 = 0.75361$

$P_U = (7.1094)(0.658)^{0.75361}(36) = 186.7 \text{ kips} > 181.2 \text{ kips}$

Use $t = \frac{5}{16}$ in. ◄

Problem 10.89

10.89 A steel compression member of 2.75-m effective length supports an eccentric load as shown. Using the allowable-stress method and assuming $e = 40$ mm, determine the load P. Use $\sigma_Y = 250$ MPa and $E = 200$ GPa.



Steel: $E = 200 \times 10^9$ Pa , $\sigma_Y = 250 \times 10^6$ Pa

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(200\times10^9)}{250\times10^6}} = 125.664$$

S130 × 15: $A = 1890 \text{ mm}^2 = 1890\times10^{-6} \text{ m}^2$

$r_{min} = r_y = 16.3 \text{ mm} = 16.3\times10^{-3} \text{ m}$

$S_x = 79.8\times10^3 \text{ mm}^3 = 79.8\times10^{-6} \text{ m}^3$

$e_x = 0$

$e_y = 40 \text{ mm} = 40\times10^{-3} \text{ m}$

$$\frac{L_e}{r_{min}} = \frac{2.75}{16.3\times10^{-3}} = 168.712 > C_c$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L_e/r)^2} = \frac{\pi^2(200\times10^9)}{(1.92)(168.713)^2} = 36.119\times10^6 \text{ Pa}$$

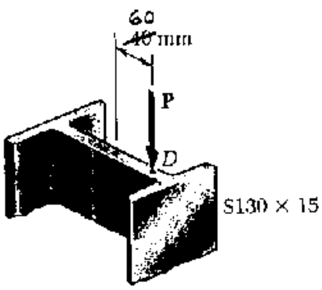
$$\frac{P}{A} + \left|\frac{M_x}{S_x}\right| + \left|\frac{M_y}{S_y}\right| = \frac{P}{A} + \frac{Pe_y}{S_x} + \frac{Pe_x}{S_y} = P\left(\frac{1}{A} + \frac{e_y}{S_x} + \frac{e_x}{S_y}\right) = KP$$

$$KP \le \sigma_{all} \qquad P \le \frac{\sigma_{all}}{K}$$

$$K = \frac{1}{1890\times10^{-6}} + \frac{40\times10^{-3}}{79.8\times10^{-6}} + 0 = 1.03035\times10^3 \text{ m}^{-2}$$

$$P \le \frac{36.119\times10^6}{1.03035\times10^3} = 35.1\times10^3 \text{ N} = 35.1 \text{ kN}$$

Problem 10.90



10.90 Solve Prob. 10.89, using $e = 60$ mm.

10.89 A steel compression member of 2.75-m effective length supports an eccentric load as shown. Using the allowable-stress method and assuming $e = 40$ mm, determine the load **P**. Use $\sigma_Y = 250$ MPa and $E = 200$ GPa.

Steel: $E = 200 \times 10^9$ Pa, $\sigma_Y = 250 \times 10^6$ Pa

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(200\times10^9)}{250\times10^6}} = 125.664$$

S 130×15 $\quad A = 1890 \text{ mm}^2 = 1890\times10^{-6} \text{ m}^2$

$r_{min} = r_y = 16.3 \text{ mm} = 16.3\times10^{-3} \text{ m}$

$S_x = 79.8\times10^3 \text{ mm}^3 = 79.8\times10^{-6} \text{ m}^3$

$e_x = 0$

$e_y = 60 \text{ mm} = 60\times10^{-3} \text{ m}$

$$\frac{L_e}{r_{min}} = \frac{2.75}{16.3\times10^{-3}} = 168.712 > C_c$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L_e/r)^2} = \frac{\pi^2(200\times10^9)}{(1.92)(168.712)^2} = 36.119\times10^6 \text{ Pa}$$

$$\frac{P}{A} + \left|\frac{M_x}{S_x}\right| + \left|\frac{M_y}{S_y}\right| = \frac{P}{A} + \frac{Pe_y}{S_x} + \frac{Pe_x}{S_y} = P\left(\frac{1}{A} + \frac{e_y}{S_x} + \frac{e_x}{S_y}\right) = KP$$

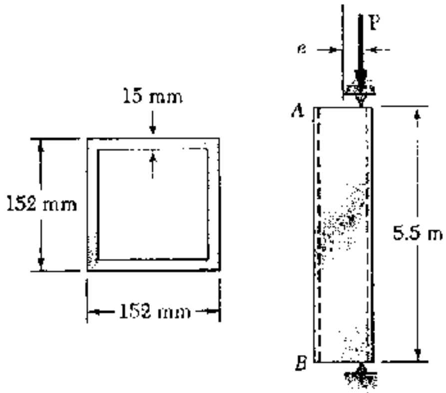
$$KP \le \sigma_{all} \qquad P \le \frac{\sigma_{all}}{K}$$

$$K = \frac{1}{1890\times10^{-6}} + \frac{60\times10^{-3}}{79.8\times10^{-6}} + 0 = 1.28098\times10^3 \text{ m}^{-2}$$

$$P \le \frac{36.119\times10^6}{1.28098\times10^3} = 28.2\times10^3 \text{ N} = 28.2 \text{ kN}$$

Problem 10.91

10.91 A column of 5.5-m effective length is made of the aluminum alloy 2014-T6, for which the allowable stress in bending is 220 MPa. Using the interaction method, determine the allowable load **P**, knowing that the eccentricity is (*a*) $e = 0$, (*b*) $e = 40$ mm.



$b_o = 152\ \text{mm} \qquad b_i = b_o - 2t = 122\ \text{mm}$

$A = b_o^2 - b_i^2 = 8220\ \text{mm}^2 = 8220 \times 10^{-6}\ \text{m}^2$

$I = \frac{1}{12}(b_o^4 - b_i^4) = 26.02 \times 10^6\ \text{mm}^4$

$r = \sqrt{\frac{I}{A}} = 56.26\ \text{mm} = 56.26 \times 10^{-3}\ \text{m}$

$\frac{L}{r} = \frac{5.5}{56.26 \times 10^{-3}} = 97.76 > 55$

$\sigma_{all,c} = \frac{372 \times 10^3}{(L/r)^2} = \frac{372 \times 10^3}{(97.76)^2} = 38.92\ \text{MPa}$ for centric loading

$$\frac{P}{A\sigma_{all,c}} + \frac{Pec}{I\sigma_{all,b}} = 1$$

(a) $\underline{e = 0}$ $\qquad P = A\sigma_{all,c} = (8220 \times 10^{-6})(38.92 \times 10^6) = 320 \times 10^3\ \text{N} = 320\ \text{kN}$ ◄

(b) $\underline{e = 40 \times 10^{-3}\ \text{m}}$ $\qquad c = \frac{1}{2}(152) = 76\ \text{mm} = 76 \times 10^{-3}\ \text{m}$

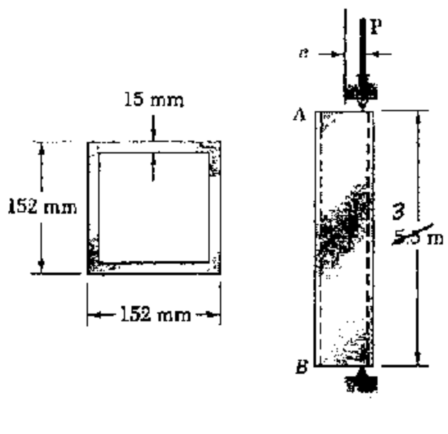
$$\frac{P}{(8220 \times 10^{-6})(38.92 \times 10^6)} + \frac{P(40 \times 10^{-3})(76 \times 10^{-3})}{(26.02 \times 10^{-6})(220 \times 10^6)} = 3.6568 \times 10^{-6} P = 1$$

$P = 273 \times 10^3\ \text{N} = 273\ \text{kN}$ ◄

Problem 10.92

10.92 Solve Prob. 10.91, assuming that the effective length of the column is 3.0 m.

10.91 A column of 5.5-m effective length is made of the aluminum alloy 2014-T6, for which the allowable stress in bending is 220 MPa. Using the interaction method, determine the allowable load **P**, knowing that the eccentricity is (*a*) $e = 0$, (*b*) $e = 40$ mm.



$b_o = 152 \text{ mm} \qquad b_i = b_o - 2t = 122 \text{ mm}$

$A = b_o^2 - b_i^2 = 8220 \text{ mm}^2 = 8200 \times 10^{-6} \text{ m}^2$

$I = \frac{1}{12}(b_o^4 - b_i^4) = 26.02 \times 10^6 \text{ mm}^4$

$r = \sqrt{\frac{I}{A}} = 56.26 \text{ mm} = 52.26 \times 10^{-3} \text{ m}$

$$\frac{L}{r} = \frac{3.0}{56.26 \times 10^{-3}} = 53.32 < 55$$

$$\sigma_{all,c} = 212 - 1.585(L/r) = 212 - (1.585)(53.32) = 127.5 \text{ MPa}$$

$$\frac{P}{A\sigma_{all,c}} + \frac{Pec}{I\sigma_{all,b}} = 1$$

(a) $\underline{e = 0}$ $\qquad P = A\sigma_{all} = (8220 \times 10^{-6})(127.5 \times 10^6) = 1048 \times 10^3 \text{ N} = 1048 \text{ kN}$ ◄

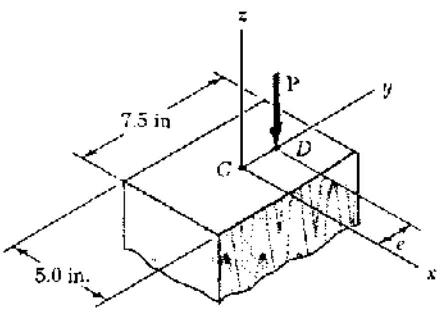
(b) $\underline{e = 40 \text{ mm}} = 40 \times 10^{-3} \text{ m} \qquad c = (\frac{1}{2})(152) = 76 \text{ mm} = 76 \times 10^{-3} \text{ m}$

$$\frac{P}{(8220 \times 10^{-6})(127.5 \times 10^6)} + \frac{P(40 \times 10^{-3})(76 \times 10^{-3})}{(26.02 \times 10^{-6})(220 \times 10^6)} = 1.4852 \times 10^{-6} P = 1$$

$$P = 673 \times 10^3 \text{ N} = 673 \text{ kN}$$ ◄

Problem 10.93

10.93 A sawn-lumber column of 5.0 × 7.5-in. cross section has an effective length of 8.5 ft. The grade of wood used has an adjusted allowable stress for compression parallel to the grain $\sigma_C = 1180$ psi and a modulus of elasticity $E = 1.2 \times 10^6$ psi. Using the allowable-stress method, determine the largest eccentric load P that can be applied when (*a*) $e = 0.5$ in., (*b*) $e = 1.0$ in.



Sawn lumber: $\sigma_c = 1180$ psi $\quad E = 1.2\times10^6$ psi

$c = 0.8 \quad K_{cE} = 0.300$

$L_e = 8.5$ ft $= 102$ in

$b = 7.5$ in., $d = 5.0$ in., $c = \frac{b}{2} = 3.75$ in.

$A = bd = (7.5)(5.0) = 37.5 \text{ in}^2 \qquad I_x = \frac{1}{12}(5.0)(7.5)^3 = 175.78 \text{ in}^4$

$$\sigma_{cE} = \frac{K_{cE}E}{(L/d)^2} = \frac{K_{cE}Ed^2}{L^2} = \frac{(0.300)(1.2\times10^6)(5.0)^2}{(102)^2} = 865 \text{ psi}$$

$$\sigma_{cE}/\sigma_c = 865/1180 = 0.7331$$

$$C_p = \frac{1+\sigma_{cE}/\sigma_c}{2c} - \sqrt{\left(\frac{1+\sigma_{cE}/\sigma_c}{2c}\right)^2 - \frac{\sigma_{cE}/\sigma_c}{c}} = 0.5763$$

$$\sigma_{all} = \sigma_c C_p = (1180)(0.5763) = 680 \text{ psi}$$

$$\frac{P_{all}}{A} + \frac{P_{all}ec}{I_x} = \sigma_{all}$$

$$P_{all} = \frac{\sigma_{all}}{\frac{1}{A} + \frac{ec}{I_x}}$$

(a) $e = 0.5$ in.

$$P_{all} = \frac{680}{\frac{1}{37.5} + \frac{(0.5)(3.75)}{175.78}} = 18210 \text{ lb.} = 18.21 \text{ kips}$$

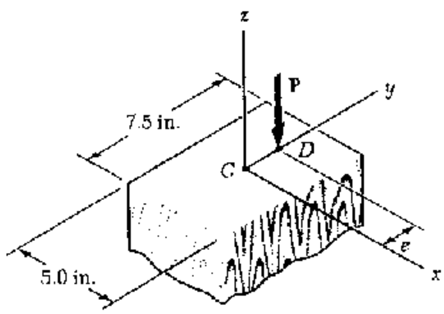
(b) $e = 1.0$ in.

$$P_{all} = \frac{680}{\frac{1}{37.5} + \frac{(1.0)(3.75)}{175.78}} = 14170 \text{ lb.} = 14.17 \text{ kips}$$

Problem 10.94

10.94 Solve Prob. 10.93 using the interaction method and an allowable stress in bending of 1300 psi.

10.93 A sawn-lumber column of 5.0 × 7.5-in. cross section has an effective length of 8.5 ft. The grade of wood used has an adjusted allowable stress for compression parallel to the grain σ_C = 1180 psi and a modulus of elasticity E = 1.2 × 10^6 psi. Using the allowable-stress method, determine the largest eccentric load **P** that can be applied when (*a*) e = 0.5 in., (*b*) e = 1.0 in.



Sawn lumber: $\sigma_C = 1180$ psi $E = 1.2 \times 10^6$ psi

$c = 0.8$ $K_{cE} = 0.300$

$L_e = 8.5$ ft $= 102$ in.

$b = 7.5$ in., $d = 5.0$ in., $c = \frac{b}{2} = 3.75$ in.

$A = bd = (7.5)(5.0) = 37.5\ \text{in}^2$ $I_x = \frac{1}{12}(5.0)(7.5)^3 = 175.78\ \text{in}^4$

$$\sigma_{CE} = \frac{K_{cE}E}{(L/d)^2} = \frac{K_{cE}Ed^2}{L^2} = \frac{(0.300)(1.2\times10^6)(5.0)^2}{(102)^2} = 865\ \text{psi}$$

$$\sigma_{CE}/\sigma_C = 865/1180 = 0.7331$$

$$C_p = \frac{1+\sigma_{CE}/\sigma_C}{2c} - \sqrt{\left(\frac{1+\sigma_{CE}/\sigma_C}{2c}\right)^2 - \frac{\sigma_{CE}/\sigma_C}{c}} = 0.5763$$

$$\sigma_{all} = \sigma_C C_p = (1180)(0.5763) = 680\ \text{psi}$$

$$\frac{P_{all}}{A\sigma_{all}} + \frac{Pec}{I_x(\sigma_{all})_b} = BP \le 1 \qquad P \le \frac{1}{B}$$

(a) $e = 0.5$ in.

$$B = \frac{1}{(37.5)(680)} + \frac{(0.5)(3.75)}{(175.78)(1300)} = 47.421\times10^{-6}\ \text{lb}^{-1}$$

$$P_{all} = \frac{1}{47.421\times10^{-6}} = 21.1\times10^3\ \text{lb} = 21.1\ \text{kips}$$

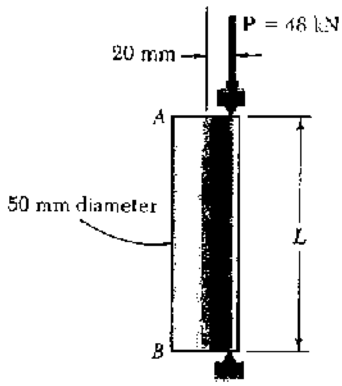
(b) $e = 1.0$ in.

$$B = \frac{1}{(37.5)(680)} + \frac{(1.0)(3.75)}{(175.78)(1300)} = 55.626\times10^{-6}\ \text{lb}^{-1}$$

$$P_{all} = \frac{1}{55.626\times10^{-6}} = 17.98\times10^3\ \text{lb} = 17.98\ \text{kips}$$

Problem 10.95

10.95 An eccentric load $P = 48$ kN is applied at a point 20 mm from the geometric axis of a 50-mm-diameter rod made of the aluminum alloy 6061-T6. Using the interaction method and an allowable stress in bending of 145 MPa, determine the largest allowable effective length L that can be used.



$c = \frac{1}{2}d = 25$ mm $\qquad A = \pi c^2 = 1.9635\times10^3$ mm^2

$I = \frac{\pi}{4}c^4 = 306.8\times10^3$ mm^4 $\qquad r = \sqrt{\frac{I}{A}} = 12.5$ mm

$e = 20$ mm $\qquad \sigma_{all,b} = 145\times10^6$ Pa

$$\frac{P}{A\sigma_{all,c}} + \frac{Pec}{I\sigma_{all,b}} = 1$$

$$\frac{P}{A\sigma_{all,c}} = 1 - \frac{Pec}{I\sigma_{all,b}}$$

$$\frac{1}{\sigma_{all,c}} = \frac{A}{P}\left[1 - \frac{Pec}{I\sigma_{all,b}}\right] = \frac{1.9635\times10^{-3}}{48\times10^3}\left[1 - \frac{(48\times10^3)(20\times10^{-3})(25\times10^{-3})}{(306.8\times10^{-9})(145\times10^6)}\right]$$

$$= 18.838\times10^{-9}\ \text{Pa}^{-1} \qquad \sigma_{all,c} = 53.086\times10^6\ \text{Pa} = 53.086\ \text{MPa}$$

Assume $\frac{L}{r} > 66$. $\qquad \sigma_{all,c} = \frac{351\times10^3}{(L/r)^2}$

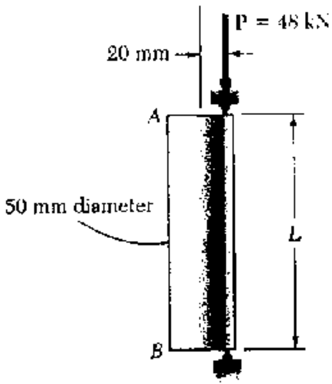
$$\frac{L}{r} = \sqrt{\frac{351\times10^3}{\sigma_{all,c}}} = \sqrt{\frac{351\times10^3}{53.086}} = 81.314 > 66$$

$$L = 81.314\,r = (81.314)(12.5\times10^{-3}) = 1.016\ \text{m}$$

Problem 10.96

10.96 Solve Prob. 10.95, assuming that the aluminum alloy used is 2014-T6 and that the allowable stress in bending is 180 MPa.

10.95 An eccentric load $P = 48$ kN is applied at a point 20 mm from the geometric axis of a 50-mm-diameter rod made of the aluminum alloy 6061-T6. Using the interaction method and an allowable stress in bending of 145 MPa, determine the largest allowable effective length L that can be used.



$c = \frac{1}{2}d = 25 \text{ mm} \qquad A = \pi c^2 = 1.9635 \times 10^3 \text{ mm}^2$

$I = \frac{\pi}{4}c^4 = 306.8 \times 10^3 \text{ mm}^4 \qquad r = \sqrt{\frac{I}{A}} = 12.5 \text{ mm}$

$e = 20 \text{ mm} \qquad \sigma_{all,b} = 180 \times 10^6 \text{ Pa}$

$$\frac{P}{A\sigma_{all,c}} + \frac{Pec}{I\sigma_{all,b}} = 1$$

$$\frac{P}{A\sigma_{all,c}} = 1 - \frac{Pec}{I\sigma_{all,b}}$$

$$\frac{1}{\sigma_{all,c}} = \frac{A}{P}\left[1 - \frac{Pec}{I\sigma_{all,b}}\right] = \frac{1.9635 \times 10^{-3}}{48 \times 10^3}\left[1 - \frac{(48 \times 10^3)(20 \times 10^{-3})(25 \times 10^{-3})}{(306.8 \times 10^{-9})(180 \times 10^6)}\right]$$

$$= 23.129 \times 10^{-9} \text{ Pa}^{-1} \qquad \sigma_{all,c} = 43.236 \times 10^6 \text{ Pa} = 43.236 \text{ MPa}$$

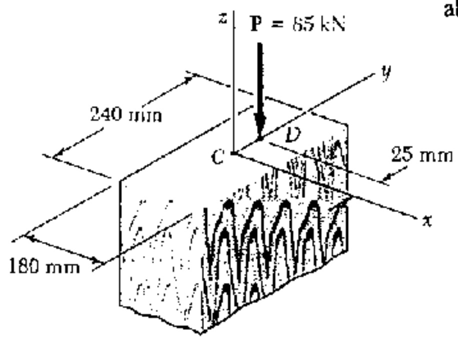
Assume $\frac{L}{r} > 55$. $\qquad \sigma_{all,c} = \frac{372 \times 10^3}{(L/r)^2}$

$$\frac{L}{r} = \sqrt{\frac{372 \times 10^3}{\sigma_{all,c}}} = \sqrt{\frac{372 \times 10^3}{43.236}} = 92.757 > 55$$

$$L = 92.757\, r = (92.757)(12.5 \times 10^{-3}) = 1.159 \text{ m}$$

Problem 10.97

10.97 A rectangular column is made of a grade of sawn wood that has an adjusted allowable stress for compression parallel to the grain $F_C = 8.3$ MPa and a modulus of elasticity $E = 11.1$ GPa. Using the allowable-stress method, determine the largest allowable effective length L that can used.



$d = 180\text{ mm} = 0.180\text{ m} \qquad b = 240\text{ mm} = 0.240\text{ m}$

$A = bd = 43.2\times10^{-3}\text{ m}^2 \qquad E = 11100\text{ MPa}$

$I_x = \frac{1}{12} d b^3 = \frac{1}{12}(0.180)(0.240)^3 = 207.36\times10^{-6}\text{ m}^4$

$e = 25\text{ mm} = 0.025\text{ m} \qquad c = \frac{b}{2} = 0.120\text{ m}$

$\frac{P}{A} + \frac{Pec}{I} \le \sigma_{all} \qquad \sigma_{all} \ge \frac{85\times10^3}{43.2\times10^{-3}} + \frac{(85\times10^3)(0.025)(0.120)}{207.36\times10^{-6}} = 3.9496\times10^6\text{ Pa}$

$= \qquad \text{MPa}$

$C_P = \frac{\sigma_{all}}{\sigma_C} = \frac{3.1973}{8.3} = 0.38522 = y \qquad \text{Let } x = \sigma_{CE}/\sigma_C$

$y = \frac{1+x}{2c} - \sqrt{\left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}} \qquad \text{where } c = 0.8 \text{ for sawn lumber}$

$\frac{1+x}{2c} - y = \sqrt{\left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}}$

$\cancel{\left(\frac{1+x}{2c}\right)^2} - y\left(\frac{1+x}{c}\right) + y^2 = \cancel{\left(\frac{1+x}{2c}\right)^2} - \frac{x}{c}$

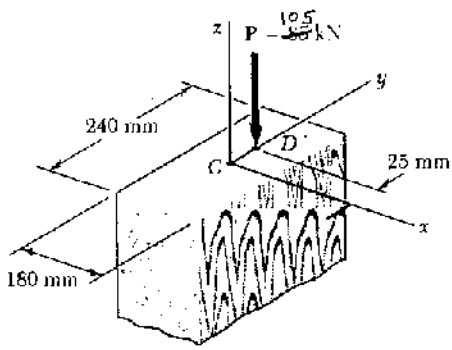
$x = y\,\frac{(1-cy)}{1-y} = (0.38522)\frac{1-(0.8)(0.38522)}{1-0.38522} = 0.43350$

$\sigma_{CE} = \sigma_C(0.43350) = (8.3)(0.43350) = 3.598\text{ MPa}$

$\sigma_{CE} = \frac{K_{CE}E}{(L/d)^2} \qquad L^2 = \frac{K_{CE}Ed^2}{\sigma_{CE}} \qquad \text{where } K_{CE} = 0.300$

$L = d\sqrt{\frac{K_{CE}E}{\sigma_{CE}}} = (0.180)\sqrt{\frac{(0.300)(11100)}{3.598}} = 5.48\text{ m}$

Problem 10.98



10.98 Solve Prob. 10.97, assuming that $P = 105$ kN.

10.97 A rectangular column is made of a grade of sawn wood that has an adjusted allowable stress for compression parallel to the grain F_C 8.3 MPa and a modulus of elasticity $E = 11.1$ GPa. Using the allowable-stress method, determine the largest allowable effective length L that can used.

$d = 180 \text{ mm} = 0.180 \text{ m} \qquad b = 240 \text{ mm} = 0.240 \text{ m}$

$A = bd = 43.2\times10^{-3} \text{ m}^2 \qquad E = 11100 \text{ MPa}$

$I_y = \frac{1}{12} d b^3 = \frac{1}{12}(0.180)(0.240)^3 = 207.36\times10^{-6} \text{ m}^4$

$e = 25 \text{ mm} = 0.025 \text{ m} \qquad c = \frac{b}{2} = 0.120 \text{ m}$

$\frac{P}{A} + \frac{Pec}{I_x} \le \sigma_{all} \qquad \sigma_{all} \ge \frac{105\times10^3}{43.2\times10^{-3}} + \frac{(105\times10^3)(0.025)(0.120)}{207.36\times10^{-6}} = 3.9496\times10^6 \text{ Pa}$

$= 3.9496 \text{ MPa}$

$C_p = \frac{\sigma_{all}}{\sigma_c} = \frac{3.9496}{8.3} = 0.47586 = y \qquad \text{Let } x = \sigma_{cE}/\sigma_c$

$y = \frac{1+x}{c} - \sqrt{\left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}}$ where $c = 0.8$ for sawn lumber

$\frac{1+x}{2c} - y = \sqrt{\left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}}$

$\left(\frac{1+x}{2c} - y\right)^2 = \left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}$

$\cancel{\left(\frac{1+x}{2c}\right)^2} - \frac{1+x}{c} y + y^2 = \cancel{\left(\frac{1+x}{2c}\right)^2} - \frac{x}{c}$

$x = y\,\frac{1-cy}{1-y} = (0.47586)\,\frac{1-(0.8)(0.47586)}{1-0.47586} = 0.56227$

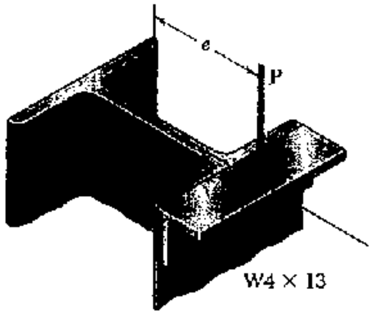
$\sigma_{cE} = \sigma_c(0.56227) = (8.3)(0.56227) = 4.6668 \text{ MPa}$

$\sigma_{cE} = \frac{K_{cE} E}{(L/d)^2} \qquad L^2 = \frac{K_{cE} E d^2}{\sigma_{cE}} \qquad \text{where } K_{cE} = 0.300$

$L = d\sqrt{\frac{K_{cE} E}{\sigma_{cE}}} = 0.180\sqrt{\frac{(0.300)(11100)}{4.6668}} = 4.81 \text{ m}$

Problem 10.99

10.99 A steel compression member of 9-ft effective length supports an eccentric load as shown. Using the allowable-stress method, determine the maximum allowable eccentricity e if (a) $P = 30$ kips, (b) $P = 18$ kips. Use $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi.



W 4×13 $\quad A = 3.83 \text{ in}^2, \quad S_x = 5.46 \text{ in}^3$

$r_y = 1.00$ in. $\quad L_e = 9$ ft $= 108$ in. $\quad L_e/r_y = 108$

Steel: $C_c = \sqrt{\dfrac{2\pi^2 E}{\sigma_Y}} = \sqrt{\dfrac{2\pi^2(29\times10^3)}{36}} = 126.10$

$$\frac{L_e/r}{C_c} = \frac{108}{126.10} = 0.85647$$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.85647) - \frac{1}{8}(0.85647)^3 = 1.9093$$

$$\sigma_{all} = \frac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\frac{L_e/r}{C_c}\right)^2\right] = \frac{36}{1.9093}\left[1 - \left(\tfrac{1}{2}\right)(0.85647)^2\right] = 11.9396 \text{ ksi}$$

$$\frac{P}{A} + \frac{Pec}{I_x} = \frac{P}{A} + \frac{Pe}{S_x} = \sigma_{all} \qquad e = \frac{S_x}{P}\left(\sigma_{all} - \frac{P}{A}\right)$$

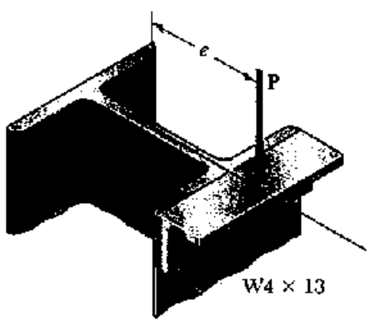
(a) $P = 30$ kips $\qquad e = \dfrac{5.46}{30}\left(11.9396 - \dfrac{30}{3.83}\right) = 0.747$ in. ◄

(b) $P = 18$ kips $\qquad e = \dfrac{5.46}{18}\left(11.9396 - \dfrac{18}{3.83}\right) = 2.20$ in. ◄

Problem 10.100

10.100 Solve Prob. 10.99, assuming that the effective length of the column is increased to 12 ft and that (a) $P = 20$ kips, (b) $P = 15$ kips.

10.99 A steel compression member of 9-ft effective length supports an eccentric load as shown. Using the allowable-stress method, determine the maximum allowable eccentricity e if (a) $P = 30$ kips, (b) $P = 18$ kips. Use $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi.



W 4×13 $\quad A = 3.83 \text{ in}^2, \quad S_x = 5.46 \text{ in}^3$

$r_y = 1.00$ in. $\quad L_e = 12$ ft $= 144$ in. $\quad L_e/r = 144$

Steel: $C_c = \sqrt{\dfrac{2\pi^2 E}{\sigma_Y}} = \sqrt{\dfrac{2\pi^2(29\times10^3)}{36}} = 126.10$

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L_e/r)^2} = \frac{\pi^2(29\times10^3)}{(1.92)(144)^2} = 7.1891 \text{ ksi}$$

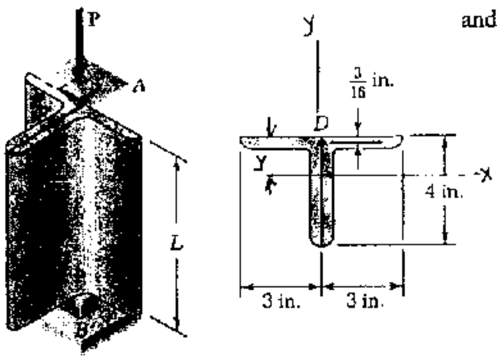
$$\frac{P}{A} + \frac{Pec}{I_x} = \frac{P}{A} + \frac{Pe}{S_x} = \sigma_{all} \qquad e = \frac{S_x}{P}\left(\sigma_{all} - \frac{P}{A}\right)$$

(a) $P = 20$ kips $\qquad e = \dfrac{5.46}{20}\left(7.1891 - \dfrac{20}{3.83}\right) = 0.537$ in. ◄

(b) $P = 15$ kips $\qquad e = \dfrac{5.46}{15}\left(7.1891 - \dfrac{15}{3.83}\right) = 1.191$ in. ◄

Problem 10.101

10.101 Two $4 \times 3 \times \frac{3}{8}$-in. steel angles are welded together to form the column *AB*. An axial load **P** of magnitude 14 kips is applied at point *D*. Using the allowable-stress method, determine the largest allowable length *L*. Assume $E = 29 \times 10^6$ psi. and $\sigma_Y = 36$ ksi.



One angle: $L\ 4 \times 3 \times \frac{3}{8}$ $\quad A = 2.48\ \text{in}^2$

$I_x = 3.96\ \text{in}^4,\ S_x = 1.46\ \text{in}^3,\ r_x = 1.26\ \text{in.}\quad y = 1.28\ \text{in.}$

$I_y = 1.92\ \text{in}^4,\quad r_y = 0.879\ \text{in},\ x = 0.782\ \text{in}$

Two angles: $\quad A = (2)(2.48) = 4.96\ \text{in}^2$

$I_x = (2)(3.96) = 7.92\ \text{in}^4,\quad S_x = (2)(1.46) = 2.92\ \text{in}^3,\ r_x = 1.26,\quad y = 1.28\ \text{in.}$

$I_y = 2\left[I_{y(1)} + Ax^2\right] = (2)\left[1.92 + (2.48)(0.782)^2\right] = 6.873\ \text{in}^4 = I_{min}$

$r_{min} = \sqrt{\frac{I_{min}}{A}} = 1.177\ \text{in.}\qquad e = y - \frac{3}{16} = 1.28 - \frac{3}{16} = 1.0925\ \text{in}$

$P = 14\ \text{kips}\qquad \sigma_{all} = \frac{P}{A} + \frac{Pey}{I_x} = \frac{14}{4.96} + \frac{(14)(1.0925)(1.28)}{7.92} = 5.294\ \text{ksi}$

$E = 29000\ \text{ksi}\qquad C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29000)}{36}} = 126.1$

Assume $\frac{L_e}{r} > C_c.\qquad \sigma_{all} = \frac{\pi^2 E}{1.92(L/r_{min})^2}\qquad \left(\frac{L}{r_{min}}\right)^2 = \frac{\pi^2 E}{1.92\,\sigma_{all}}$

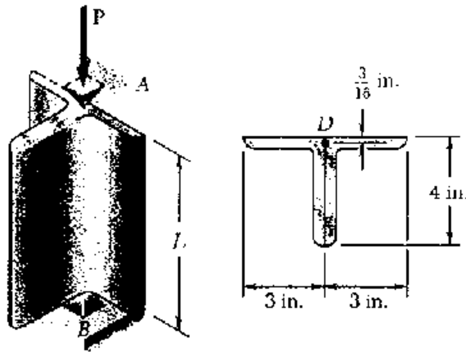
$\frac{L}{r_{min}} = \sqrt{\frac{\pi^2 E}{1.92\,\sigma_{all}}} = \sqrt{\frac{\pi^2(29000)}{(1.92)(5.294)}} = 167.8 > C_c$

$L = 167.8\ r_{min} = (167.8)(1.177) = 197.5\ \text{in.} = 16.46\ \text{ft}$ ◄

Problem 10.102

10.102 Solve Prob. 10.101, using the interaction method with $P = 18$ kips and an allowable stress in bending of 22 ksi.

10.101 Two $4 \times 3 \times \frac{3}{8}$-in. steel angles are welded together to form the column AB. An axial load **P** of magnitude 14 kips is applied at point D. Using the allowable-stress method, determine the largest allowable length L. Assume $E = 29 \times 10^6$ psi. and $\sigma_Y = 36$ ksi.



One angle $L\ 4\times3\times\frac{3}{8}$ $\quad A = 2.48\ in^2$

$I_x = 3.96\ in^4,\ S_x = 1.46\ in^3,\ r_x = 1.26\ in,\ y = 1.28\ in.$

$I_y = 1.92\ in^4,\qquad r_y = 0.879\ in.,\ x = 0.782\ in.$

Two angles $\quad A = (2)(2.48) = 4.96\ in^2$

$I_x = (2)(3.96) = 7.92\ in^4,\quad S_x = (2)(1.46) = 2.92\ in^3,\quad r_x = 1.26\ in.,\ y = 1.28\ in.$

$I_y = 2\left[I_{y(1)} + Ax^2\right] = (2)\left[1.92 + (2.48)(0.782)^2\right] = 6.873\ in^4 = I_{min}$

$r_{min} = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{6.873}{4.96}} = 1.177\ in. \qquad e = y - \frac{3}{16} = 1.28 - \frac{3}{16} = 1.0925\ in.$

$P = 18\ kips \qquad \frac{P}{A\sigma_{all,c}} + \frac{Pey}{I_x\sigma_{all,b}} = 1 \qquad \frac{1}{\sigma_{all,c}} = \frac{A}{P}\left(1 - \frac{Pey}{I_x\sigma_{all,b}}\right)$

$$\frac{1}{\sigma_{all,c}} = \frac{4.96}{18}\left[1 - \frac{(18)(1.0925)(1.28)}{(7.92)(22)}\right] = 0.23575(ksi)^{-1}$$

$\sigma_{all,c} = 4.2418\ ksi \qquad E = 29000\ ksi$

$$C_c = \sqrt{\frac{2\pi^2E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29000)}{36}} = 126.1$$

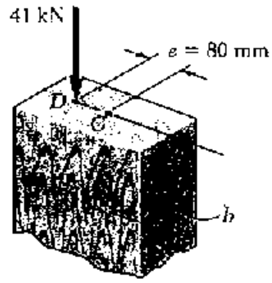
Assume $\frac{L_e}{r} > C_c. \qquad \sigma_{all,c} = \frac{\pi^2E}{1.92(L_e/r)^2}$

$$\frac{L_e}{r} = \sqrt{\frac{\pi^2E}{1.92\,\sigma_{all,c}}} = \sqrt{\frac{\pi^2(29000)}{(1.92)(4.2418)}} = 187.466 > C_c$$

$L_e = 187.466\ r_{min} = (187.466)(1.177) = 220.65\ in. = 18.39\ ft$ ◄

Problem 10.103

10.103 A sawn lumber column of rectangular cross section has a 2.2-m effective length and supports a 41 kN load as shown. The sizes available for use have b equal to 90 mm, 140 mm, 190 mm and 240 mm. The grade of wood has an adjusted allowable stress for compression parallel to the grain $\sigma_C = 8.1$ MPa and $E = 8.3$ GPa. Use the allowable-stress method to determine the lightest section that can be used.



Sawn lumber: $\sigma_C = 8.1$ MPa $E = 8.3$ GPa

$c = 0.8$ $K_{cE} = 0.300$

$L_e = 2.2$ m

$$\frac{P_{all}}{A} + \frac{P_{all}ec}{I_x} = \sigma_{all}$$

$e = 80\times10^{-3}$ m, $c = \frac{1}{2}(190) = 95$ mm $= 95\times10^{-3}$ m

$A = 0.190\ b$ m^2 $I_x = \frac{1}{12}b(0.190)^3 = 571.58\times10^{-6}\ b$ m^4

$$\frac{P_{all}}{0.190\,b} + \frac{P_{all}(80\times10^{-3})(95\times10^{-3})}{571.58\times10^{-6}\ b} = \frac{18.56\,P_{all}}{b} = \sigma_{all}$$

$P_{all} = 0.05388\ \sigma_{all}\, b$

$d = 0.190$ m or b, whichever is smaller.

$$\sigma_{cE} = \frac{K_{cE}E}{(L/d)^2} = \frac{(0.300)(8300)}{(2.2/d)^2} = 514.5\ d^2 \quad \text{MPa}$$

$$\sigma_{cE}/\sigma_C = \frac{514.5\,d^2}{8.1} = 63.51\,d^2$$

$$C_P = \frac{1+\sigma_{cE}/\sigma_C}{2c} - \sqrt{\left(\frac{1+\sigma_{cE}/\sigma_C}{2c}\right)^2 - \frac{\sigma_{cE}/\sigma_C}{c}}$$

$$= \frac{1+\sigma_{cE}/\sigma_C}{1.6} - \sqrt{\left(\frac{1+\sigma_{cE}/\sigma_C}{1.6}\right)^2 - \frac{\sigma_{cE}/\sigma_C}{0.8}}$$

$\sigma_{all} = \sigma_C\, C_P = (8.1\times10^6)\,C_P$

$P_{all} = (0.05388\,b)(8.1\times10^6)\ C_P = 472.47\times10^3\ b\,C_P$

Calculate P_{all} for all four values of b. See table below.

b (m)	d (m)	σ_{cE}/σ_C	C_P	P_{all} (kN)
0.090	0.090	0.51443	0.44367	18.87
→ 0.140	0.140	1.24480	0.76081	50.3
0.190	0.190	2.2927	0.8878	79.7
0.240	0.190	2.2927	0.8878	100.7

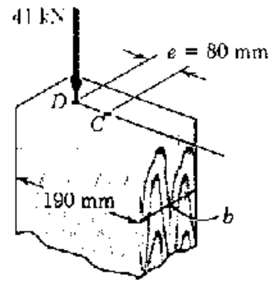
← P = 41 kN

Use $b = 0.140$ m $= 140$ mm ◄

Problem 10.104

10.104 Solve Prob. 10.103, assuming that e = 40 mm.

10.103 A sawn lumber column of rectangular cross section has a 2.2-m effective length and supports a 41 kN load as shown. The sizes available for use have b equal to 90 mm, 140 mm, 190 mm and 240 mm. The grade of wood has an adjusted allowable stress for compression parallel to the grain σ_C = 8.1 MPa and E = 8.3 GPa. Use the allowable-stress method to determine the lightest section that can be used.



Sawn lumber: $\sigma_C = 8.1$ MPa $\quad E = 8.3$ GPa

$c = 0.8 \quad K_{cE} = 0.300$

$L_e = 2.2$ m

$$\frac{P_{all}}{A} + \frac{P_{all}ec}{I_x} = \sigma_{all}$$

$e = 40\times10^{-3}$ m, $\quad c = \frac{1}{2}(190) = 95$ mm $= 95\times10^{-3}$ m

$A = 0.190\ b\ \text{m}^2 \quad I_x = \frac{1}{12}b(0.190)^3 = 571.58\times10^{-6}\, b\ \text{m}^4$

$$\frac{P_{all}}{0.190b} + \frac{P_{all}(40\times10^{-3})(95\times10^{-3})}{571.58\times10^{-6}\,b} = \frac{11.911\ P_{all}}{b} = \sigma_{all}$$

$P_{all} = 0.083953\ \sigma_{all} b$

$d = 0.190$ m or b, whichever is smaller.

$$\sigma_{cE} = \frac{K_{cE}E}{(L/d)^2} = \frac{(0.300)(8300)}{(2.2/d)^2} = 514.5\ d^2 \text{ MPa}$$

$$\sigma_{cE}/\sigma_c = \frac{514.5\ d^2}{8.1} = 63.51\ d^2$$

$$C_p = \frac{1+\sigma_{cE}/\sigma_c}{2c} - \sqrt{\left(\frac{1+\sigma_{cE}/\sigma_c}{2c}\right)^2 - \frac{\sigma_{cE}/\sigma_c}{c}}$$

$$= \frac{1+\sigma_{cE}/\sigma_c}{1.6} - \sqrt{\left(\frac{1+\sigma_{cE}/\sigma_c}{1.6}\right)^2 - \frac{\sigma_{cE}/\sigma_c}{0.8}}$$

$\sigma_{all} = \sigma_c C_p = (8.1\times10^6)\,C_p$

$P_{all} = (0.083953\ b)(8.1\times10^6)\,C_p = 680.02\times10^3\ b\,C_p$

Calculate P_{all} for all four values of b. See table below.

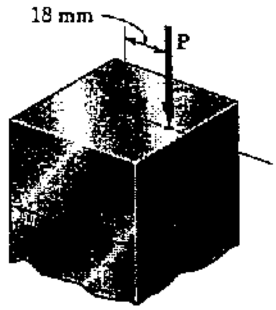
b(m)	d(m)	σ_{cE}/σ_c	C_p	P_{all} (kN)
0.090	0.090	0.51443	0.44367	27.2
→ 0.140	0.140	1.24480	0.76081	72.4
0.190	0.190	2.2927	0.8878	114.7
0.240	0.190	2.2927	0.8878	144.9

← P = 41 kN

Use $b = 0.140$ m $= 140$ mm

Problem 10.105

10.105 A compression member made of steel has a 1.62 m effective length and must support the 128-kN load **P** as shown. For the material used, $\sigma_Y = 250$ MPa and $E = 200$ GPa. Using the interaction method with an allowable bending stress equal to 150 MPa, determine the smallest dimension d of the cross section that can be used.



Using dimensions in meters,

$A = 40\times10^{-3}\,d \qquad L_e = 1.62\text{ m}$

$I_x = \frac{1}{12}(40\times10^{-3})^3 d = 5.3333\times10^{-6}\,d$

$I_y = \frac{1}{12}(40\times10^{-3})d^3 = 3.3333\times10^{-3}\,d^3$

$|x| = \frac{1}{2}d, \qquad |y| = 20\text{ mm} = 20\times10^{-3}\text{ m} \qquad |e_x| = 18\text{ mm} = 18\times10^{-3}\text{ m}$

Steel: $\sigma_Y = 250$ MPa $\quad E = 200000$ MPa

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(200000)}{250}} = 125.66$$

Assume $d > 40\text{ mm} = 40\times10^{-3}\text{ m}$. $\qquad$ Then, $I_{min} = I_x$

$$r = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{5.3333\times10^{-6}d}{3.3333\times10^{-3}d}} = 11.547\times10^{-3}\text{ m} \qquad \frac{L_e}{r} = 140.29 > C_c$$

$$\sigma_{all,centric} = \frac{\pi^2 E}{1.92(L_e/r)^2} = \frac{\pi^2(200000)}{(1.92)(140.29)^2} = 52.236\text{ MPa} \qquad \sigma_{all,bending} = 150\text{ MPa}$$

$$\frac{P}{A\sigma_{all,centric}} + \frac{Pe_x x}{I_y\sigma_{all,bending}} = 1$$

$$\frac{128\times10^3}{(40\times10^{-3}d)(52.236\times10^6)} + \frac{(128\times10^3)(18\times10^{-3})(\frac{1}{2}d)}{(3.3333\times10^{-3}d^3)(150\times10^6)} = 1$$

$$\frac{61.260\times10^{-3}}{d} + \frac{2.304\times10^{-3}}{d^2} = 1$$

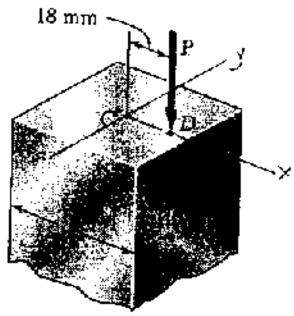
$$d^2 - 61.260\times10^{-3}d - 2.304\times10^{-3} = 0$$

$$d = \frac{1}{2}\left\{61.260\times10^{-3} + \sqrt{(61.260\times10^{-3})^2 + (4)(2.304\times10^{-3})}\right\}$$

$$= 87.6\times10^{-3}\text{ m} > 40\times10^{-3}\text{ m}$$

$$d = 87.6\text{ mm}$$

Problem 10.106



10.106 Solve Prob. 10.105, assuming that the effective length is 720 mm and that the magnitude P of the eccentric load is 198 kN.

10.105 A compression member made of steel has a 1.62 m effective length and must support the 128-kN load **P** as shown. For the material used, $\sigma_Y = 250$ MPa and $E =$ 200 GPa. Using the interaction method with an allowable bending stress equal to 150 MPa, determine the smallest dimension d of the cross section that can be used.

Using dimensions in meters,

$$A = 40\times10^{-3}\ d \qquad L_e = 720\text{ mm} = 0.720\text{ m}$$

$$I_x = \tfrac{1}{12}(40\times10^{-3})^3 d = 5.3333\times10^{-6}\ d$$

$$I_y = \tfrac{1}{12}(40\times10^{-3})\, d^3 = 3.3333\times10^{-3}\ d^3$$

$$|x| = \frac{d}{2}, \qquad |y| = 20\text{ mm} = 0.020\text{ m} \qquad |e_x| = 18\text{ mm} = 18\times10^{-3}\text{ m}$$

Steel: $\sigma_Y = 250$ MPa $\quad E = 200000$ MPa $\quad C_c = \sqrt{\dfrac{2\pi^2 E}{\sigma_Y}}$

$$C_c = \sqrt{\frac{2\pi^2(200000)}{250}} = 125.66$$

Assume $d > 40\text{ mm} = 40\times10^{-3}$ m. Then, $I_{min} = I_x$

$$r = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{5.3333\times10^{-6}\ d}{40\times10^{-3}\ d}} = 11.547\times10^{-3}\text{ m}, \quad \frac{L_e}{r} = 62.35 < C_c$$

$$\frac{L_e/r}{C_c} = 0.49621 \qquad F.S. = \tfrac{5}{3} + \tfrac{3}{8}(0.49621) - \tfrac{1}{8}(0.49621)^3 = 1.83747$$

$$\sigma_{all,\,centric} = \frac{\sigma_Y}{F.S.}\left[1 - \tfrac{1}{2}\left(\frac{L_e/r}{C_c}\right)^2\right] = \frac{250}{1.83747}\left[1 - \tfrac{1}{2}(0.49621)^2\right] = 119.31\text{ MPa}$$

$$\sigma_{all,\,bending} = 150\text{ MPa}$$

$$\frac{P}{A\,\sigma_{all,\,centric}} + \frac{P e_x x}{I_y\, \sigma_{all,\,bending}} = 1$$

$$\frac{198\times10^3}{(40\times10^{-3} d)(119.31\times10^6)} + \frac{(198\times10^3)(18\times10^{-3})(\tfrac{1}{2}d)}{(3.3333\times10^{-3} d^3)(150\times10^6)} = 1$$

$$\frac{41.489\times10^{-3}}{d} + \frac{3.5640\times10^{-3}}{d^2} = 1$$

$$d^2 - 41.489\times10^{-3}\, d - 3.5640\times10^{-3} = 0$$

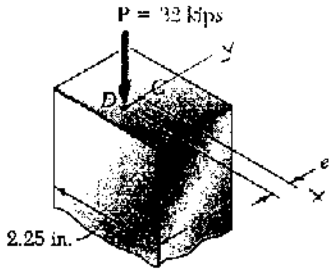
$$d = \tfrac{1}{2}\left\{41.489\times10^{-3} + \sqrt{(41.489\times10^{-3})^2 + (4)(3.5640\times10^{-3})}\right\}$$

$$= 83.9\times10^{3}\text{ m} \quad > 40\times10^{-3}\text{ m}$$

$$d = 83.9\text{ mm}$$

Problem 10.107

10.107 A compression member of rectangular cross section has an effective length of 36 in. and is made of the aluminum alloy 2014-T6 for which the allowable stress in bending is 24 ksi. Using the interaction method, determine the smallest dimension d of the cross section that can be used when $e = 0.4$ in.



$A = 2.25\ d \qquad c = \frac{1}{2}d \qquad e = 0.4$ in. $\qquad L_e = 36$ in

$\sigma_{all,b} = 24$ ksi $\qquad P = 32$ kips

$I_x = \frac{1}{12}(2.25)d^3 \qquad r_x = \frac{d}{\sqrt{12}}$

Assume $r_x = r_{min}$, i.e. $d < 2.25$ in.

$L_e/r_{min} = \sqrt{12}\ L_e/d$

Assume $L_e/r_{min} > 55$. $\quad \sigma_{all,c} = \frac{54000}{(L_e/r_x)^2} = \frac{54000\ d^2}{12\ L_e^2} = \frac{54000}{(12)(36)^2}d^2 = 3.47222\ d^2$

$$\frac{P}{A\sigma_{all,c}} + \frac{Pec}{I\sigma_{all,b}} = \frac{32}{(2.25d)(3.47222d^2)} + \frac{(12)(32)(0.4)(\frac{1}{2}d)}{(2.25d^3)(24)} = 1$$

$\frac{4.096}{d^3} + \frac{1.42222}{d^2} = 1 \qquad$ Let $x = \frac{1}{d} \qquad 4.096x^3 + 1.42222x^2 = 1$

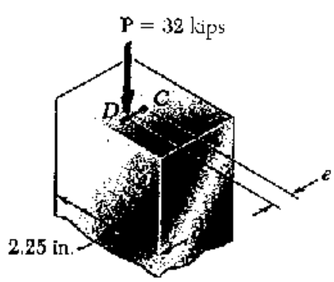
Solving for x, $\quad x = 0.528118$, $\quad d = \frac{1}{x} = 1.894$ in < 2.25 in.

$L/r_x = (\sqrt{12})(36)/1.894 = 65.8 > 55$ $\qquad d = 1.894$ in. ◄

Problem 10.108

10.108 Solve Prob. 10.107, assuming that $e = 0.2$ in.

10.107 A compression member of rectangular cross section has an effective length of 36 in. and is made of the aluminum alloy 2014-T6 for which the allowable stress in bending is 24 ksi. Using the interaction method, determine the smallest dimension d of the cross section that can be used when $e = 0.4$ in.



$A = 2.25\ d \qquad c = \frac{1}{2}d \qquad e = 0.2$ in. $\qquad L_e = 36$ in.

$\sigma_{all,b} = 24$ ksi $\qquad P = 32$ kips

$I_x = \frac{1}{12}(2.25)d^3 \qquad r_x = \frac{d}{\sqrt{12}}$

Assume $r_x = r_{min}$, ie. $d < 2.25$ in.

$L_e/r_{min} = \sqrt{12}\ L_e/d$

Assume $L_e/r_{min} > 55$. $\quad \sigma_{all,c} = \frac{54000}{(L_e/r_x)^2} = \frac{54000\ d^2}{12\ L_e^2} = \frac{54000\ d^2}{(12)(36)^2} = 3.47222 d^2$

$$\frac{P}{A\sigma_{all,c}} + \frac{Pec}{I\sigma_{all,b}} = \frac{32}{(2.25d)(3.47222d^2)} + \frac{(12)(32)(0.2)(\frac{1}{2}d)}{(2.25d^3)(24)}$$

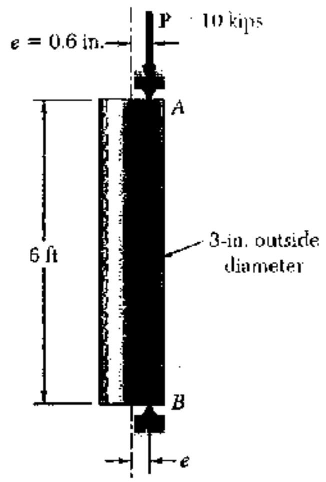
$\frac{4.096}{d^3} + \frac{0.71111}{d^2} = 1 \qquad$ Let $x = \frac{1}{d}$. $\quad 4.096x^3 + 0.71111x^2 = 1$

Solving for x, $\quad x = 0.57216\ \text{in}^{-1} \quad d = \frac{1}{x} = 1.748$ in. < 2.25 in

$L_e/r_x = (\sqrt{12})(36)/1.748 = 71.4 > 55$

$d = 1.748$ in. ◄

Problem 10.109



10.109 An aluminum tube of 3-in. outside diameter is to carry a load of 10 kips having an eccentricity $e = 0.6$ in. Knowing that the stock of tubes available for use are made of alloy 2014-T6 and have wall thicknesses in increments of $\frac{1}{16}$ in. up to $\frac{1}{2}$ in., determine the lightest tube that may be used. Use the allowable-stress method of design.

$L_e = 6 \text{ ft} = 72 \text{ in.} \qquad c_o = \frac{1}{2} d_o = 1.500 \text{ in.} = c$

$c_i = c_o - t = 1.500 - t$

$A = \pi (c_o^2 - c_i^2) \qquad I = \frac{\pi}{4}(c_o^4 - c_i^4)$

Try $t = \frac{1}{4} \text{ in.} = 0.25 \text{ in.} \qquad c_i = 1.250 \text{ in.}$

$A = 2.1598 \text{ in}^2 \qquad I = 2.0586 \text{ in}^4$

$r = \sqrt{\frac{I}{A}} = 0.97629 \text{ in}$

$\frac{L}{r} = \frac{72}{0.97629} = 73.748 > 55.$

$\sigma_{all} = \frac{54000}{(L/r)^2} = \frac{54000}{(73.748)^2} = 9.9285 \text{ ksi}$

$\frac{P_{all}}{A} + \frac{Mc}{I} = \frac{P_{all}}{A} + \frac{P_{all} ec}{I} = \left[\frac{1}{2.1598} + \frac{(0.6)(1.500)}{2.0586}\right] P_{all} = 0.9002 P_{all}$

$P_{all} = \frac{\sigma_{all}}{0.9002} = \frac{9.9285}{0.9002} = 11.03 \text{ kips.} > 10 \text{ kips}$

Since P_{all} is nearly proportional to the thickness t,

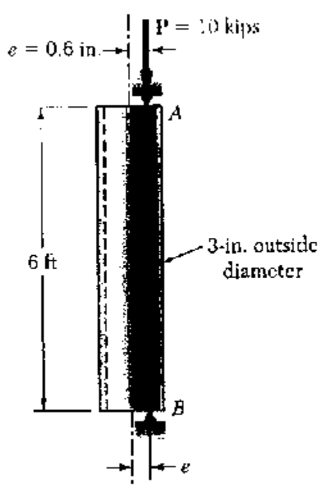
$\frac{t}{0.25} \approx \frac{10}{11.03} \qquad t \approx 0.227 \text{ in} > \frac{3}{16} \text{ in.} = 0.1875 \text{ in.}$

$\frac{3}{16}$ in. thickness would be to light. Use $t = \frac{1}{4}$ in. ◄

Problem 10.110

10.110 Solve Prob. 10.109, using the interaction method of design with an allowable stress in bending of 25 ksi.

10.109 An aluminum tube of 3-in. outside diameter is to carry a load of 10 kips having an eccentricity $e = 0.6$ in. Knowing that the stock of tubes available for use are made of alloy 2014-T6 and have wall thicknesses in increments of $\frac{1}{16}$ in. up to $\frac{1}{2}$ in., determine the lightest tube that may be used. Use the allowable-stress method of design.



$L_e = 6\text{ ft} = 72\text{ in.} \qquad c_o = \frac{1}{2}d_o = 1.500\text{ in.} = c$

$c_i = c_o - t = 1.500 - t$

$A = \pi(c_o^2 - c_i^2) \qquad I = \frac{\pi}{4}(c_o^4 - c_i^4)$

Try $t = \frac{1}{4}\text{ in.} = 0.25\text{ in.} \qquad c_i = 1.250\text{ in.}$

$A = 2.1598\text{ in}^2 \qquad I = 2.0586\text{ in}^4$

$r = \sqrt{\frac{I}{A}} = 0.97629\text{ in.}$

$\frac{L}{r} = \frac{72}{0.97629} = 73.748 > 55$

$$\sigma_{all,c} = \frac{54000}{(L/r)^2} = \frac{54000}{(73.748)^2} = 9.9285\text{ ksi}$$

$$\frac{P_{all}}{A\sigma_{all,c}} + \frac{Mc}{I\sigma_{all,b}} = \frac{P_{all}}{A\sigma_{all}} + \frac{P_{all}ec}{I\sigma_{all,b}}$$

$$= \left[\frac{1}{(2.1598)(9.9285)} + \frac{(0.6)(1.500)}{(2.0586)(25)}\right]P_{all} = 0.06412\,P_{all} = 1$$

$$P_{all} = \frac{1}{0.06412} = 15.60\text{ kips} > 10\text{ kips.}$$

Since P_{all} is nearly proportional to the thickness t,

$\frac{t}{0.25} \approx \frac{10}{15.60} \qquad t \approx 0.1603\text{ in} < \frac{3}{16}\text{ in.} = 0.1875\text{ in.}$

Try $t = \frac{3}{16} = 0.1875\text{ in.} \qquad c_i = 1.3125\text{ in.}$

$A = 1.6567\text{ in}^2, \quad I = 1.64537\text{ in}^4, \quad r = 0.99658\text{ in.}$

$\frac{L}{r} = 72.247 > 55 \qquad \sigma_{all,c} = 10.345\text{ ksi}$

$$\left[\frac{1}{(1.6567)(10.345)} + \frac{(0.6)(1.500)}{(1.64537)(25)}\right]P_{all} = 0.080225\,P_{all} = 1$$

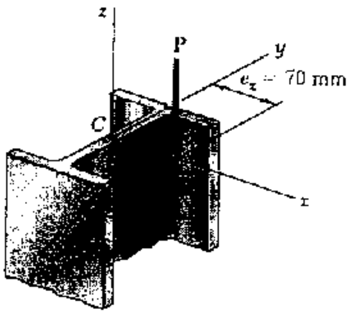
$$P_{all} = \frac{1}{0.080225} = 12.46\text{ kips.}$$

$\frac{t}{0.1875} \approx \frac{10}{12.46} \qquad t \approx 0.1504\text{ in.} < \frac{3}{16}\text{ in.}$

Use $t = \frac{3}{16}$ in. ◀

Problem 10.111

10.111 A steel column of 7.2-m effective length is to support an 83-kN eccentric load **P** at a point *D* located on the *x* axis as shown. Using the allowable-stress method, select the wide-flange shape of 250-mm nominal depth that should be used. Use $E = 200$ GPa, $\sigma_Y = 250$ MPa.



Steel: $E = 200000$ MPa $\quad \sigma_Y = 250$ MPa

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(200000)}{250}} = 125.66$$

$L_e = 7.2$ m

Try W 250 × 49.1. $\quad A = 6250 \times 10^{-6}$

$b_f = 202 \times 10^{-3}$ m, $\quad c = 101 \times 10^{-3}$ m, $\quad I_y = 15.1 \times 10^{-6}$ m^4, $\quad r_y = 49.2 \times 10^{-3}$ m

$$\frac{L_e}{r_y} = \frac{7.2}{49.2 \times 10^{-3}} = 146.34 > C_c$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92 (L/r_y)^2} = \frac{\pi^2(200000)}{(1.92)(146.34)^2} = 48.01 \text{ MPa}$$

$$\frac{P}{A} + \frac{Pe_x c}{I_y} = \frac{83 \times 10^3}{6250 \times 10^{-6}} + \frac{(83 \times 10^3)(70 \times 10^{-3})(101 \times 10^{-3})}{15.1 \times 10^{-6}}$$

$$= 13.28 \times 10^6 + 38.86 \times 10^6 = 52.14 \text{ MPa} > 48.01 \text{ MPa}$$

(not allowed)

Required area $\quad A \approx \left(\frac{52.14}{48.01}\right)(6250 \text{ mm}^2) = 6788 \text{ mm}^2$

Try W 250 × 58.

$$\frac{L_e}{r_y} = \frac{7.2}{50.3 \times 10^{-3}} = 143.14 \qquad \sigma_{all} = \frac{\pi^2(200000)}{(1.92)(143.14)^2} = 50.18 \text{ MPa}$$

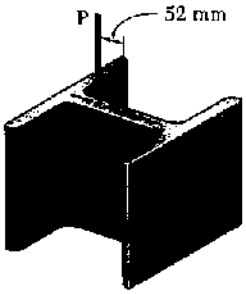
$$\frac{P}{A} + \frac{Pe_x c}{I_y} = \frac{83 \times 10^3}{7420 \times 10^{-6}} + \frac{(83 \times 10^3)(70 \times 10^{-3})(101.5 \times 10^{-3})}{18.8 \times 10^{-6}}$$

$$= 11.19 \times 10^6 + 31.37 \times 10^6 = 42.56 \text{ MPa} < 50.18 \text{ MPa}$$

Use W250 × 58.

Problem 10.112

10.112 A steel column of 6.3-m effective length must carry a load of 360 kN with an eccentricity of 52 mm as shown. Using the interaction method, select the wide-flange shape of 310-mm nominal depth that should be used. Use $E = 200$ GPa, $\sigma_Y = 250$ MPa and $\sigma_{all} = 160$ MPa in bending.



Steel: $C_c = \sqrt{\dfrac{2\pi^2 E}{\sigma_Y}} = \sqrt{\dfrac{2\pi^2(200\times10^9)}{250\times10^6}} = 125.66$

$$\frac{P}{A\sigma_{all,c}} + \frac{Pec}{I_x \sigma_{all,b}} < 1 \qquad (1)$$

Assume $\dfrac{L}{r_y} > C_c = 125.66$.

Then $\sigma_{all,c} = \dfrac{\pi^2 E}{1.92(L/r_y)^2} = \dfrac{\pi^2 E r_y^2}{1.92\, L^2}$

The first term of inequality (1) becomes

$$\frac{1.92\,PL^2}{\pi^2 E A r_y^2} = \frac{1.92\,PL^2}{\pi^2 E I_y} = \frac{\alpha}{I_y} \quad \text{where } \alpha = \frac{1.92\,PL^2}{\pi^2 E}$$

$$\alpha = \frac{(1.92)(360\times10^3)(6.3)^2}{\pi^2(200\times10^9)} = 13.898\times10^{-6}\ \text{m}^4 = 13.898\times10^6\ \text{mm}^4$$

The second term of inequality (1) is $\dfrac{Pe}{S_x \sigma_{all,b}} = \dfrac{\beta}{S_x}$ where

$$\beta = \frac{Pe}{\sigma_{all,b}} = \frac{(360\times10^3)(52\times10^{-3})}{160\times10^6} = 117\times10^{-6}\ \text{m}^3 = 117\times10^3\ \text{mm}^4$$

Inequality (1) is then $\dfrac{\alpha}{I_y} + \dfrac{\beta}{S_x} < 1$

Obviously, $I_y > \alpha = 13.898\times10^6\ \text{mm}^4$

$S_x > \beta = 117\times10^3\ \text{mm}^4$

Try W310×60. $I_y = 18.3\times10^6\ \text{mm}^4$, $S_x = 851\times10^3\ \text{mm}^2$

$$\left.\begin{aligned} &\frac{\alpha}{I_y} + \frac{\beta}{S_x} = \frac{13.898}{18.3} + \frac{117}{851} = 0.897 < 1 \\ &r_y = 49.1\ \text{mm} \qquad \frac{L}{r_y} = \frac{6.3}{49.1\times10^{-3}} = 128.3 > C_c \end{aligned}\right\}\ \text{safe}$$

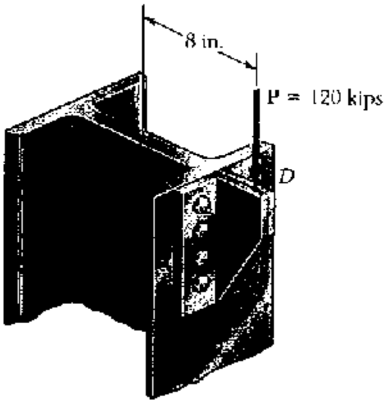
Try W310×52. $I_y = 10.3\times10^6\ \text{mm}^4$, $S_x = 748\times10^3\ \text{mm}^3$

$$\left.\begin{aligned} &\frac{\alpha}{I_y} + \frac{\beta}{S_x} = \frac{13.898}{10.3} + \frac{117}{748} = 1.506 > 1 \\ &r_y = 39.3\ \text{mm} \qquad \frac{L}{r_y} = \frac{6.3}{39.3\times10^{-3}} = 160.3 > C_c \end{aligned}\right\}\ \text{unsafe}$$

Use W310×60. ◄

Problem 10.113

10.113 A steel column having a 24-ft effective length is loaded eccentrically as shown. Using the allowable stress method, select the wide-flange shape of 14-in. nominal depth that should be used. Use $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi.



Steel: $C_c = \sqrt{\dfrac{2\pi^2 E}{\sigma_Y}} = \sqrt{\dfrac{2\pi^2(29000)}{36}} = 126.1$

$L = 24 \text{ ft} = 288 \text{ in.}$

If column is short and loading is centric

$F.S = \frac{5}{3} \qquad \sigma_{all} = \dfrac{\sigma_Y}{F.S} = \dfrac{36}{5/3} = 21.6 \text{ ksi}$

$A > \dfrac{P}{\sigma_{all}} = \dfrac{120}{21.6} = 5.56 \text{ in}^2$

If column is long and loading is centric, $\quad F.S. = 1.92$

$P < \dfrac{P_{cr}}{1.92} = \dfrac{\pi^2 E I_y}{1.92 L^2}, \qquad I_y > \dfrac{1.92 PL^2}{\pi^2 E} = \dfrac{(1.92)(120)(288)^2}{\pi^2(29000)} = 66.8 \text{ in}^4$

$\dfrac{P}{A} + \dfrac{Pec}{I_x} = \dfrac{P}{A} + \dfrac{Pe}{S_x} < \sigma_{all}$

Try W14×68. $\quad A = 20.0 \text{ in}^2, \ I_y = 121 \text{ in}^4, \ r_y = 2.46 \text{ in.} \quad S_x = 103 \text{ in}^3$

$\dfrac{L}{r_y} = \dfrac{288}{2.46} = 117.07 < C_c \qquad \dfrac{L/r_y}{C_c} = 0.9284$

$F.S. = \frac{5}{3} + \frac{3}{8}(0.9284) - \frac{1}{8}(0.9284)^2 = 1.9148$

$\sigma_{all} = \dfrac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\dfrac{L/r_y}{C_c}\right)^2\right] = \dfrac{36}{1.9148}\left[1 - \frac{1}{2}(0.9248)^2\right] = 10.698 \text{ ksi}$

$\dfrac{P}{A} + \dfrac{Pe}{S_x} = \dfrac{120}{20.0} + \dfrac{(120)(8)}{103} = 15.32 \text{ ksi} > \sigma_{all}$ $\quad$ unsafe

Assuming that σ_{max} is approximately proportional to $\frac{1}{A}$, the required area is

$A = (20.0)\dfrac{15.32}{10.698} = 28.64 \text{ in}^2$

Try W14×145. $\quad A = 42.7 \text{ in}^2, \ S_x = 232 \text{ in}^3, \ r_y = 3.98 \text{ in}$

$\dfrac{L}{r_y} = 72.36 \qquad \dfrac{L/r_y}{C_c} = 0.5738 \qquad F.S. = 1.8582 \qquad \sigma_{all} = 16.18 \text{ ksi}$

$\dfrac{P}{A} + \dfrac{Pe}{S_x} = 6.95 \text{ ksi} < 16.18 \text{ ksi}$ $\quad$ safe

Try W14×82. $\quad A = 24.1 \text{ in}^2, \ S_x = 123 \text{ in}^3, \ r_y = 2.48 \text{ in.}$

$\dfrac{L}{r_y} = 116.13 \qquad \dfrac{L/r_y}{C_c} = 0.9209 \qquad F.S. = 1.9144 \qquad \sigma_{all} = 10.83 \text{ ksi}$

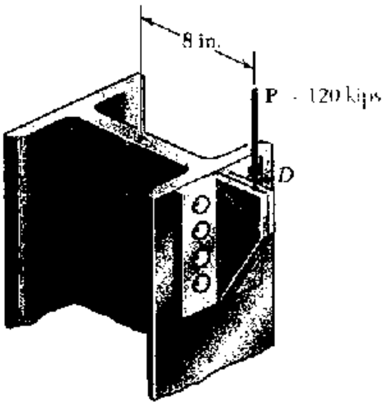
$\dfrac{P}{A} + \dfrac{Pe}{S_x} = 12.78 \text{ ksi} > 10.83 \text{ ksi}$ $\quad$ unsafe

Use W14×145. ◄

Problem 10.114

10.114 Solve Prob. 10.113 using the interaction method, assuming that $\sigma_Y = 50$ ksi and an allowable stress in bending of 30 ksi.

10.113 A steel column having a 24-ft effective length is loaded eccentrically as shown. Using the allowable stress method, select the wide-flange shape of 14-in. nominal depth that should be used. Use $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi.



Steel: $C_c = \sqrt{\dfrac{2\pi^2 E}{\sigma_Y}} = \sqrt{\dfrac{2\pi^2(29000)}{50}} = 107.1$

$L_e = 24 \text{ ft} = 288 \text{ in.}$

Assume $\dfrac{L}{r_y} > 107.1$

$r_y < \dfrac{L}{107.1} = \dfrac{288}{107.1} = 2.69 \text{ in.}$

This criterion is met by all W14's except for W14×370 and W14×145.

$$\sigma_{all,c} = \frac{\pi^2 E}{1.92(L/r_y)^2} = \frac{\pi^2 E r_y^2}{1.92 L^2}$$

Interaction formula: $\dfrac{P}{A\sigma_{all,c}} + \dfrac{Pec}{I_x \sigma_{all,b}} < 1$ (1)

The first term of inequality (1) becomes

$$\frac{1.92 PL^2}{\pi^2 E A r_y^2} = \frac{1.92 PL^2}{\pi^2 E I_y} = \frac{(1.92)(120)(288)^2}{\pi^2(29000) I_y} = \frac{66.768}{I_y}$$

The second term of inequality (1) becomes

$$\frac{Pe}{S_x \sigma_{all,b}} = \frac{(120)(8)}{(S_x)(30)} = \frac{32}{S_x}$$

Then inequality (1) becomes $\dfrac{66.768}{I_y} + \dfrac{32}{S_x} = \beta < 1$

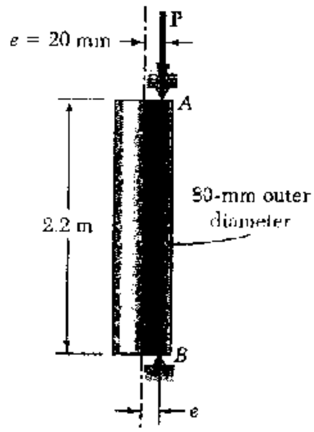
Obviously $I_y > 66.768 \text{ in}^4$ and $S_x > 32 \text{ in}^4$

Shape	I_y (in.^{4})	S_x (in^3)	β
W14×82	148	128	0.701
W14×68	121	103	0.862 ←
W14×53	57.7	77.8	2.49

W14×68 is the lightest shape with $\beta < 1$

Use W14×68.

Problem 10.115



10.115 A steel tube of 80-mm outer diameter is to carry a 93-kN load **P** with an eccentricity of 20 mm. The tubes available for use are made with wall thicknesses in increments of 3 mm from 6 mm to 15 mm. Using the allowable stress method, determine the lightest tube that can be used. Assume E = 200 GPa and σ_Y = 250 MPa.

$r_o = \frac{1}{2} d_o = 40 \text{ mm}, \quad r_i = r_o - t$

$A = \pi (r_o^2 - r_i^2), \qquad I = \frac{\pi}{4}(r_o^4 - r_i^4) \qquad r = \sqrt{\frac{I}{A}}$

t mm	r_i mm	A mm^2	I 10^6 mm^4	r mm
3	37	726	0.539	27.24
6	34	1395	0.961	26.25
9	31	2007	1.285	25.31
12	28	2564	1.528	24.41
15	25	3063	1.704	23.59

$L_e = 2.2 \text{ m}$

$P = 93 \times 10^3 \text{ N}$

Steel: $E = 200000$ MPa $\qquad C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2 (200000)}{250}} = 125.66$

Try $t = 9$ mm. $\quad \frac{L_e}{r} = \frac{2.2}{25.31 \times 10^{-3}} = 86.92 < C_c \qquad \frac{L_e/r}{C_c} = 0.6917$

$F.S. = \frac{5}{3} + \frac{3}{8}(0.6917) - \frac{1}{8}(0.6917)^3 = 1.885$

$\sigma_{all} = \frac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\frac{L_e/r}{C_c}\right)^2\right] = \frac{250}{1.885}\left[1 - \frac{1}{2}(0.6917)^2\right] = 100.9 \text{ MPa}$

$\frac{P}{A} + \frac{Pec}{I} = \frac{93 \times 10^3}{2007 \times 10^{-6}} + \frac{(93 \times 10^3)(20 \times 10^{-3})(40 \times 10^{-3})}{1.285 \times 10^{-6}} = 104.2 \text{ MPa} > 100.9 \text{ MPa}$ (not allowed)

Approximate required area: $\left(\frac{104.2}{100.9}\right)(2007 \times 10^{-6}) = 2073 \times 10^{-6} \text{ m}^2 = 2073 \text{ mm}^2$

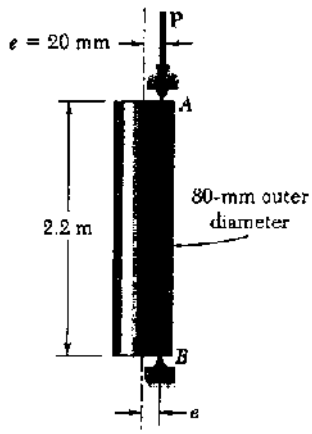
Try $t = 12$ mm. $\quad \frac{L_e}{r} = \frac{2.2}{24.41 \times 10^{-3}} = 90.12 < C_c \qquad \frac{L_e/r}{C_c} = 0.7172$

$F.S. = 1.890 \qquad \sigma_{all} = 98.3 \text{ MPa}$

$\frac{P}{A} + \frac{Pec}{I} = \frac{93 \times 10^3}{2564 \times 10^{-6}} + \frac{(93 \times 10^3)(20 \times 10^{-3})(40 \times 10^{-3})}{1.528 \times 10^{-6}} = 85.0 \text{ MPa} < 98.3 \text{ MPa}$

Use $t = 12$ mm. ◄

Problem 10.116



10.116 Solve Prob. 10.115, using the interaction method, with $P = 165$ kN, $e = 15$ mm, and an allowable stress in bending of 150 MPa.

10.115 A steel tube of 80-mm outer diameter is to carry a 93-kN load **P** with an eccentricity of 20 mm. The tubes available for use are made with wall thicknesses in increments of 3 mm from 6 mm to 15 mm. Using the allowable stress method, determine the lightest tube that can be used. Assume $E = 200$ GPa and $\sigma_Y = 250$ MPa.

$r_o = \frac{1}{2} d_o = 40$ mm $\qquad r_i = r_o - t$

$A = \pi(r_o^2 - r_i^2) \qquad I = \frac{\pi}{4}(r_o^4 - r_i^4) \qquad r = \sqrt{\frac{I}{A}}$

t mm	r_i mm	A mm^2	I $10^6 mm^4$	r mm
3	37	726	0.539	27.24
6	34	1395	0.961	26.25
9	31	2007	1.285	25.31
12	28	2564	1.528	24.41
15	25	3063	1.704	23.59

$L_e = 2.2$ m

$P = 165 \times 10^3$ N

$\sigma_{all, bending} = 150$ MPa

Steel: $E = 200000$ MPa $\qquad C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(200000)}{250}} = 125.66$

Try $t = 9$ mm. $\quad \frac{L_e}{r} = \frac{2.2}{25.31 \times 10^{-3}} = 86.92 < C_c \qquad \frac{L_e/r}{C_c} = 0.6917$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.6917) - \frac{1}{8}(0.6917)^3 = 1.885$$

$$\sigma_{all, centric} = \frac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\frac{L_e/r}{C_c}\right)^2\right] = \frac{250}{1.885}\left[1 - \frac{1}{2}(0.6917)^2\right] = 100.9 \text{ MPa}$$

$$\frac{P}{A\sigma_{all, centric}} + \frac{Pec}{I\sigma_{all, bending}} = \frac{165 \times 10^3}{(2007 \times 10^{-6})(100.9 \times 10^6)} + \frac{(165 \times 10^3)(15 \times 10^{-3})(40 \times 10^{-3})}{(1.285 \times 10^{-6})(150 \times 10^6)}$$

$$= 0.815 + 0.514 = 1.329 > 1 \qquad \text{(not allowed)}$$

Approximate required area: $\quad A = (1.329)(2007) = 2667 \text{ mm}^2$

Try $t = 12$ mm. $\quad \frac{L_e}{r} = \frac{2.2}{24.41 \times 10^{-3}} = 90.12 < C_c \qquad \frac{L_e/r}{C_c} = 0.7172$

$F.S = 1.890 \qquad \sigma_{all, centric} = 98.3$ MPa

$$\frac{P}{A\sigma_{all, centric}} + \frac{Pec}{I\sigma_{all, bending}} = \frac{165 \times 10^3}{(2564 \times 10^{-6})(98.3 \times 10^6)} + \frac{(165 \times 10^3)(15 \times 10^{-3})(40 \times 10^{-3})}{(1.528 \times 10^{-6})(150 \times 10^6)}$$

$$= 0.655 + 0.432 = 1.087 > 1 \qquad \text{(not allowed)}$$

Try $t = 15$ mm. $\quad \frac{L_e}{r} = \frac{2.2}{23.59 \times 10^{-3}} = 93.26 < C_c \qquad (L_e/r)/C_c = 0.7422$

$F.S. = 1.894 \qquad \sigma_{all, centric} = 95.64$ MPa

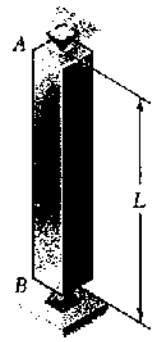
$$\frac{P}{A\sigma_{all, centric}} + \frac{Pec}{I\sigma_{all, bending}} = \frac{165 \times 10^3}{(3063 \times 10^{-6})(95.64 \times 10^6)} + \frac{(165 \times 10^3)(15 \times 10^{-3})(40 \times 10^{-3})}{(1.704 \times 10^{-6})(150 \times 10^6)}$$

$$= 0.563 + 0.387 = 0.950 < 1 \qquad \text{(allowed)}$$

Use $t = 15$ mm.

Problem 10.117

10.117 Supports A and B of the pin-ended column shown are at a fixed distance L from each other. Knowing that at a temperature T_0 the force in the column is zero and that buckling occurs when the temperature is $T_1 = T_0 + \Delta T$, express ΔT in terms of b, L, and the coefficient of thermal expansion α.



Let P be the compressive force in the column.

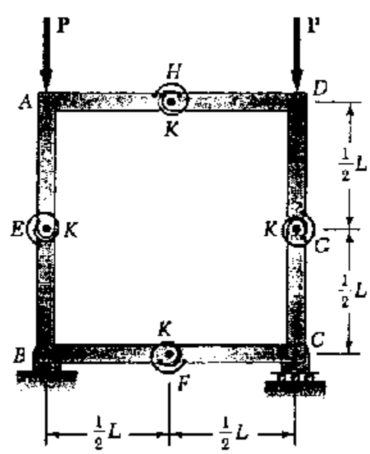
$$L\alpha(\Delta T) - \frac{PL}{EA} = 0 \qquad P = EA\alpha(\Delta T)$$

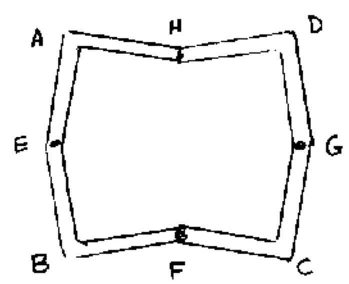
$$P_{cr} = \frac{\pi^2 EI}{L^2} = P = EA\alpha(\Delta T)$$

$$\Delta T = \frac{\pi^2 EI}{L^2 EA\alpha} = \frac{\pi^2 E b^4/12}{L^2 E b^2} = \frac{\pi^2 b^2}{12 L^2 \alpha}$$

Problem 10.118

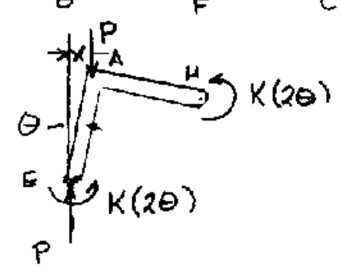
10.118 A frame consists of four L-shaped members connected by four torsional springs, each of constant K. Knowing that equal loads **P** are applied at points A and D as shown, determine the critical value P_{cr} of the loads applied to the frame.





Let θ be the rotation of each L-shaped member.

Angle change across each torsional spring is 2θ.



$$x = \tfrac{1}{2}L\sin\theta \approx \tfrac{1}{2}L\theta$$

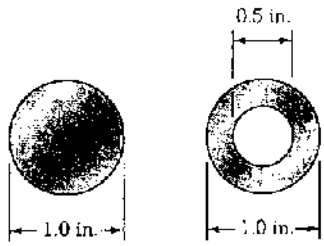
$$\Sigma M_E = 0:$$

$$K(2\theta) + K(2\theta) - Px = 0$$

$$P_{cr} = \frac{4K\theta}{x} = \frac{8K}{L}$$

Problem 10.119

10.119 A compression member of 20-in. effective length consists of a solid 1.0-in.-diameter aluminum rod. In order to reduce the weight of the member by 25%, the solid rod is replaced by a hollow rod of the cross section shown. Determine (*a*) the percent reduction in the critical load, (*b*) the value of the critical load for the hollow rod. Use $E = 10.6 \times 10^6$ psi.



Solid: $A_S = \frac{\pi}{4} d_o^2$ $\quad I_S = \frac{\pi}{4}\left(\frac{d_o}{2}\right)^4 = \frac{\pi}{64} d_o^4$

Hollow: $A_H = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{3}{4} A_S = \frac{3}{4}\frac{\pi}{4} d_o^2$

$d_i^2 = \frac{1}{4} d_o^2 \quad d_i = \frac{1}{2} d_o = 0.5 \text{ in.}$

Solid rod: $I_s = \frac{\pi}{64}(1.0)^4 = 0.049087 \text{ in}^4$

$$P_{cr} = \frac{\pi^2 E I_s}{L^2} = \frac{\pi^2 (10.6\times10^6)(0.049087)}{(20)^2} = 12.839\times10^3 \text{ lb}$$

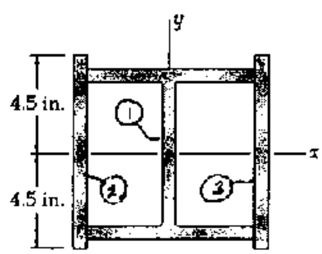
Hollow rod: $I_H = \frac{\pi}{64}(d_o^4 - d_i^4) = \frac{\pi}{64}\left[(1)^4 - (\tfrac{1}{2})^4\right] = 0.046019 \text{ in}^4$

(b) $$P_{cr} = \frac{\pi^2 E I_H}{L^2} = \frac{\pi^2(10.6\times10^6)(0.046019)}{(20)^2} = 12.036\times10^3 \text{ lb} = 12.04 \text{ kips}$$

(a) $$\frac{P_S - P_H}{P_S} = \frac{12.839\times10^3 - 12.036\times10^3}{12.839\times10^3} = 0.0625 = 6.25\%$$

Problem 10.120

10.120 A column of 22-ft effective length is to be made by welding two 9 × 0.5 in. plates to a W8 × 35 as shown. Determine the allowable centric load if a factor of safety of 2.3 is required. Use $E = 29 \times 10^6$ psi.



① W 8 × 35 $\quad I_x = 127 \text{ in}^4 \quad I_y = 42.6 \text{ in}^4$

$b_f = 8.02 \text{ in}$

② and ③ For each plate: $A = (0.5)(9.0) = 4.5 \text{ in}^2$

$I_x = \frac{1}{12}(0.5)(9)^3 = 30.375 \text{ in}^4$

$I_y = \frac{1}{12}(9)(0.5)^3 + (4.5)\left(\frac{8.02}{2} + \frac{0.5}{2}\right)^2] = 81.758 \text{ in}^4$

Column: $I_x = 127 + (2)(30.375) = 187.75 \text{ in}^4 = I_{min}$

$I_y = 42.6 + (2)(81.758) = 206.12 \text{ in}$

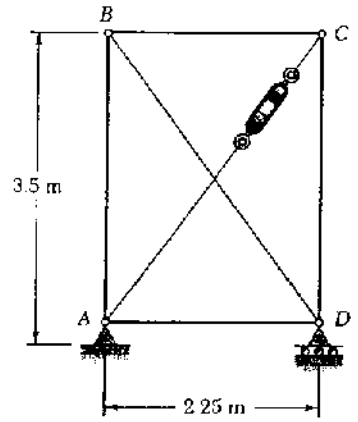
$L = 22 \text{ ft} = 264 \text{ in.}$

$$P_{cr} = \frac{\pi^2 E I}{L_e^2} = \frac{\pi^2(29\times10^6)(187.75)}{264^2} = 771.0\times10^3 \text{ lb} = 771 \text{ kips}$$

$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{771}{2.3} = 335 \text{ kips}$$

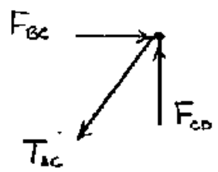
Problem 10.121

10.121 Members AB and CD are 30-mm-diameter steel rods, and members BC and AD are 22-mm-diameter steel rods. When the turnbuckle is tightened, the diagonal member AC is put in tension. Knowing that a factor of safety with respect to buckling of 2.75 is required, determine the largest allowable tension in AC. Use E = 200 GPa and consider only buckling in the plane of the structure.



$$L_{AC} = \sqrt{(3.5)^2 + (2.25)^2} = 4.1608 \text{ m}$$

Joint C



$$\pm \Sigma F_x = 0: \quad F_{BC} - \frac{2.25}{4.1608} T_{AC} = 0$$

$$T_{AC} = 1.84926\, F_{BC}$$

$$+\Sigma F_y = 0: \quad F_{CD} - \frac{3.5}{4.1608} T_{AC} = 0$$

$$T_{AC} = 1.1888\, F_{CD}$$

Members BC and AD: $\quad I_{BC} = \frac{\pi}{4}\left(\frac{d_{BC}}{2}\right)^4 = \frac{\pi}{4}\left(\frac{22}{2}\right)^4 = 11.499 \times 10^3 \text{ mm}^4 = 11.499 \times 10^{-9} \text{ m}^4$

$L_{BC} = 2.25 \text{ m} \qquad F_{BC,cr} = \dfrac{\pi^2 E I_{BC}}{L_{BC}^2} = \dfrac{\pi^2 (200 \times 10^9)(11.499 \times 10^{-9})}{(2.25)^2} = 4.4836 \times 10^3 \text{ N}$

$F_{BC,al} = \dfrac{F_{BC,cr}}{F.S.} = 1.6304 \times 10^3 \text{ N} \qquad T_{AC,al} = 3.02 \times 10^3 \text{ N}$

Members AB and CD: $\quad I_{CD} = \frac{\pi}{4}\left(\frac{d_{CD}}{2}\right)^4 = \frac{\pi}{4}\left(\frac{30}{2}\right)^4 = 39.761 \times 10^3 \text{ mm}^4 = 39.761 \times 10^{-9} \text{ m}^4$

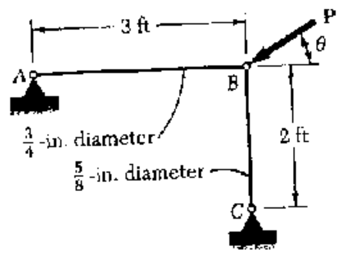
$L_{CD} = 3.5 \text{ m} \qquad F_{CD,cr} = \dfrac{\pi^2 E I_{CD}}{L_{CD}^2} = \dfrac{\pi^2 (200 \times 10^9)(39.761 \times 10^{-9})}{(3.5)^2} = 6.4069 \times 10^3 \text{ N}$

$F_{CD,al} = \dfrac{F_{CD,cr}}{F.S.} = 2.3298 \times 10^3 \text{ N} \qquad T_{AC,al} = 2.77 \times 10^3 \text{ N}$

Smaller value for $T_{AC,al}$ governs. $\qquad T_{AC,al} = 2.77 \times 10^3 \text{ N} = 2.77 \text{ kN}$ ◄

Problem 10.122

10.122 (*a*) Considering only buckling in the plane of the structure shown and using Euler's formula, determine the value of θ between 0 and 90° for which the allowable magnitude of the load **P** is maximum. (*b*) Determine the corresponding maximum value of P knowing that a factor of safety of 3.2 is required. Use $E = 29 \times 10^6$ psi.



Strut AB. $L = 3\text{ ft} = 36\text{ in.}$

$c = \frac{1}{2}d = \frac{1}{2}\left(\frac{3}{4}\right) = 0.375\text{ in}$

$I = \frac{\pi}{4}c^4 = 15.5316 \times 10^{-3}\text{ in}^4$

$$(P_{AB})_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2(29\times10^6)(15.5316\times10^{-3})}{(36)^2}$$

$= 3.4301\times10^3\text{ lb.}$

<u>Strut BC</u>. $L = 2\text{ ft} = 24\text{ in.},\quad c = \frac{1}{2}\left(\frac{5}{8}\right) = 0.3125\text{ in.},\quad I = 7.4901\times10^{-3}\text{ in}^4$

$$(P_{BC})_{cr} = \frac{\pi^2(29\times10^6)(7.4901\times10^{-3})}{(24)^2} = 3.7219\times10^3\text{ lb.}$$

For the structure, P is maximum of both struts buckle simultaneously.

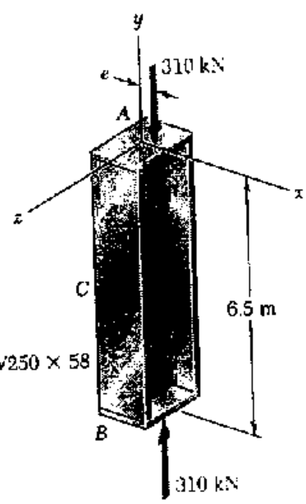
(a) $\tan\theta = \frac{(P_{BC})_{cr}}{(P_{AB})_{cr}} = 1.08507 \qquad \theta = 47.2°$

(b) $P_{cr} = \sqrt{(P_{AB})_{cr}^2 + (P_{BC})_{cr}^2} = 5.0614\times10^3\text{ lb} = 5.0614\text{ kips}$

With F.S. = 3.2, $\quad P = \frac{5.0614}{3.2} = 1.582\text{ kips}$

Problem 10.123

10.123 The line of action of the 310-kN axial load is parallel to the geometric axis of the column AB and intersects the x axis at $x = e$. Using $E = 200$ GPa, determine (*a*) the eccentricity e when the deflection of the midpoint C of the column is 9 mm, (*b*) the corresponding maximum stress in the column.



For W250 × 58 $\quad A = 7420\text{ mm}^2 = 7420\times10^{-6}\text{ m}^2$

$I_y = 18.8\times10^6\text{ mm}^3 = 18.8\times10^{-6}\text{ m}^4$

$S_y = 185\times10^3\text{ mm}^3 = 185\times10^{-6}\text{ m}^3$

$L = 6.5\text{ m} \qquad L_e = 6.5\text{ m}$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2(200\times10^9)(18.8\times10^{-6})}{(6.5)^2} = 878.3\times10^3\text{ N}$$

$$\frac{P}{P_{cr}} = \frac{310\times10^3}{878.3\times10^3} = 0.35294$$

$$y_{max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right] = 0.67990\,e$$

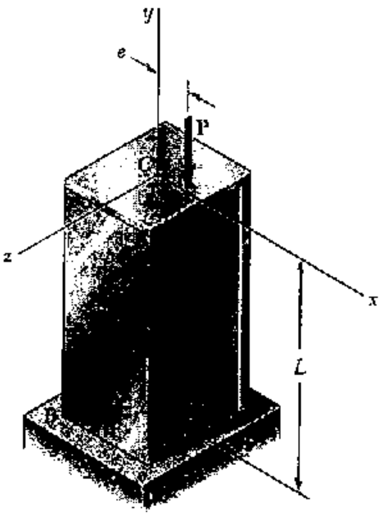
(a) $e = \frac{y_{max}}{0.67990} = \frac{9\times10^{-3}}{0.67990} = 13.24\times10^{-3}\text{ m} = 13.24\text{ mm}$

(b) $M_{max} = P(e + y_{max}) = (310\times10^3)(9 + 13.24)(10^{-3}) = 6893.5\text{ N}\cdot\text{m}$

$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{M}{S_y} = \frac{310\times10^3}{7420\times10^{-6}} + \frac{6893.5}{185\times10^{-6}}$$

$$= 41.78\times10^6 + 37.26\times10^6 = 79.04\times10^6\text{ Pa} = 79.0\text{ MPa}$$

Problem 10.124



10.124 A 26-kip axial load **P** is applied to a W6 × 12 rolled-steel column *BC* that is free at its top *C* and fixed at its base *B*. Knowing that the eccentricity of the load is $e = 0.25$ in., determine the largest permissible length *L* if the allowable stress in the column is 14 ksi. Use $E = 29 \times 10^6$ psi.

Data: $P = 26$ kips, $e = 0.25$ in

$E = 29 \times 10^6$ psi $= 29000$ ksi

W 6 × 12: $A = 3.55\ \text{in}^2 \quad b_f = 4.000$ in

$c = \frac{b_f}{2} = 2.000$ in, $I_y = 2.99\ \text{in}^4$, $r_y = 0.918$ in.

$\sigma_{max} = 14$ ksi

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)\right]$$

$$\frac{A\sigma_{max}}{P} - 1 = \frac{ec}{r^2}\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)$$

$$\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{r^2}{ec}\left(\frac{A\sigma_{max}}{P} - 1\right) = \frac{(0.918)^2}{(0.25)(2.000)}\left[\frac{(3.55)(14)}{26} - 1\right] = 1.53635$$

$$\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = 0.65089 \qquad \frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}} = 0.86204$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi}(0.86204)\right]^2 = 0.30117$$

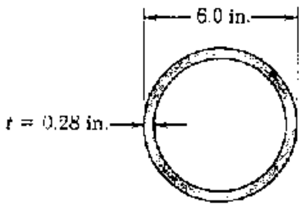
$$P_{cr} = \frac{P}{0.30117} = \frac{\pi^2 EI}{L_e^2}$$

$$L_e^2 = \frac{0.30117\,\pi^2 EI}{P} = \frac{0.30117\,\pi^2(29000)(2.99)}{26} = 9.913 \times 10^3\ \text{in}^2$$

$L_e = 99.56$ in $= 2L \qquad L = 49.78$ in. $= 4.15$ ft ◄

Problem 10.125

10.125 A steel pipe having the cross section shown is used as a column. Using the AISC allowable stress design formulas, determine the allowable centric load if the effective length of the column is (*a*) 18 ft, (*b*) 26 ft. Use $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi.



$$c_o = \frac{d_o}{2} = 3.0 \text{ in} \qquad c_i = c_o - t = 2.72 \text{ in}$$

$$A = \pi(c_o^2 - c_i^2) = 5.0316 \text{ in}^2 \qquad r = \sqrt{\frac{I}{A}} = 2.0247 \text{ in}$$

$$I = \frac{\pi}{4}(c_o^4 - c_i^4) = 20.627 \text{ in}^4$$

Steel: $E = 29000$ ksi $\qquad C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29000)}{36}} = 126.10$

(a) $L_e = 18 \text{ ft} = 216 \text{ in.} \qquad L_e/r = 106.68 < C_c \qquad \frac{L_e/r}{C_c} = 0.84601$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.84601) - \frac{1}{8}(0.84601)^3 = 1.9082$$

$$\sigma_{all} = \frac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{36}{1.9082}\left[1 - \frac{1}{2}(0.84601)^2\right] = 12.11 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (12.11)(5.0316) = 61.0 \text{ kips}$$

(b) $L_e = 26 \text{ ft} = 312 \text{ in} \qquad L_e/r = 154.097 > C_c$

$$\sigma_{all} = \frac{\pi^2 E}{1.92\,(L/r)^2} = \frac{\pi^2(29000)}{(1.92)(154.097)^2} = 6.28 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (6.28)(5.0316) = 31.6 \text{ kips}$$

Problem 10.126

10.126 A column of 4.6-m effective length must carry a centric load of 525 kN. Knowing that $\sigma_Y = 345$ MPa and $E = 200$ GPa, use the AISC allowable stress design formulas to select the wide-flange shape of 200-mm nominal depth that should be used.

$$P < \frac{\sigma_Y A}{F.S.}$$

$$A > \frac{(F.S.)P}{\sigma_Y} = \frac{(5/3)(525\times10^3)}{345\times10^6} = 2.54\times10^{-3} \text{ m}^2 = 2540 \text{ mm}^2$$

$$P < \frac{\pi^2 EI}{1.92\,L_e^2}$$

$$I > \frac{1.92\,PL_e^2}{\pi^2 E} = \frac{(1.92)(525\times10^3)(4.6)^2}{\pi^2(200\times10^9)} = 10.89\times10^{-6} \text{ m}^4 = 10.89\times10^6 \text{ mm}^4$$

Try W 200 × 46.1. $\quad A = 5860 \text{ mm}^2, \quad I_{min} = 15.3\times10^6 \text{ mm}^4, \quad r = 51.1\times10^{-3}$ m

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(200\times10^9)}{345\times10^6}} = 106.97$$

$$\frac{L_e}{r} = \frac{4.6}{51.1\times10^{-3}} = 90.02 < C_c \qquad \frac{L_e/r}{C_c} = 0.84154$$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.84154) - \frac{1}{8}(0.84154)^3 = 1.9077$$

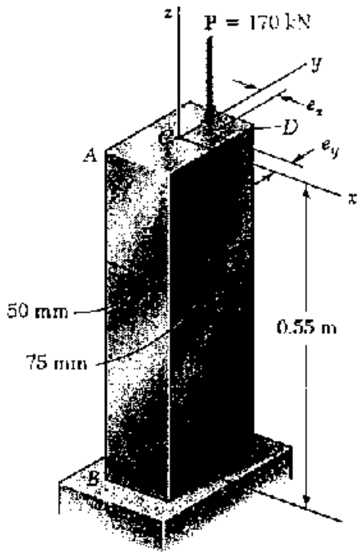
$$\sigma_{all} = \frac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{345\times10^6}{1.9077}\left[1 - \frac{1}{2}(0.84154)^2\right] = 116.8\times10^6 \text{ Pa}$$

$$P_{all} = \sigma_{all} A = (116.8\times10^6)(5860\times10^{-6}) = 684 \text{ kN} > 525 \text{ kN}$$

Use W 200 × 46.1.

Problem 10.127

10.127 The compression member AB is made of a steel for which $\sigma_Y = 250$ MPa and $E = 200$ GPa. It is free at its top A and fixed at its base B. Using the interaction method with an allowable bending stress equal to 120 MPa and knowing that the eccentricities e_x and e_y are equal, determine their largest allowable common value.



Steel: $\sigma_Y = 250$ MPa $\quad E = 200000$ MPa

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(200000)}{250}} = 125.66$$

$$A = (75\times10^{-3})(50\times10^{-3}) = 3750\times10^{-6}\ \text{m}^2$$

$$I_y = \tfrac{1}{12}(75\times10^{-3})(50\times10^{-3})^3 = 781.25\times10^{-9}\ \text{m}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = 14.434\times10^{-3}\ \text{m}$$

$$I_x = \tfrac{1}{12}(50\times10^{-3})(75\times10^{-3})^3 = 1.7578\times10^{-6}\ \text{m}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = 21.651\times10^{-3}\ \text{m}$$

$$L_e = 2L = (2)(0.55) = 1.10\ \text{m} \qquad L_e/r_{min} = 1.10/14.434\times10^{-3} = 76.21 < C_c$$

$$\frac{L_e/r_{min}}{C_c} = \frac{76.21}{125.66} = 0.6065 \qquad \text{F.S.} = \frac{5}{3} + \frac{3}{8}(0.6065) - \frac{1}{8}(0.6065)^3 = 1.8662$$

$$\sigma_{all(centric)} = \frac{\sigma_Y}{\text{F.S.}}\left[1 - \frac{1}{2}\left(\frac{L_e/r_{min}}{C_c}\right)^2\right] = \frac{250}{1.8662}\left[1 - \frac{1}{2}(0.6065)^2\right] = 109.32\ \text{MPa}$$

$$\sigma_{all(bending)} = 120\ \text{MPa}$$

$$\frac{P}{A\sigma_{all(centric)}} + \frac{Pe_x y}{I_x \sigma_{all(bending)}} + \frac{Pe_y x}{I_y \sigma_{all(bending)}} = 1 \qquad \text{with } e_x = e_y$$

$$\frac{P}{\sigma_{all(bending)}}\left(\frac{y}{I_x} + \frac{x}{I_y}\right)e = 1 - \frac{P}{A\sigma_{all(centric)}}$$

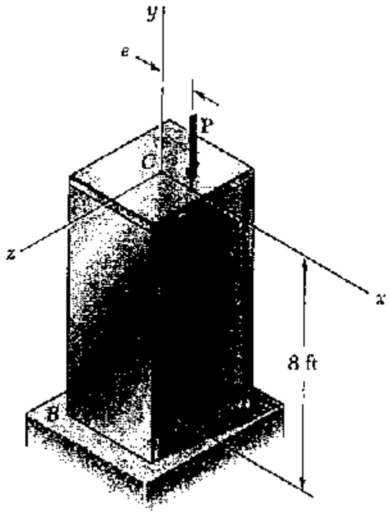
$$\frac{170\times10^3}{120\times10^6}\left(\frac{37.5\times10^{-3}}{1.7578\times10^{-6}} + \frac{25\times10^{-3}}{781.25\times10^{-9}}\right)e = 1 - \frac{170\times10^3}{(3750\times10^{-6})(109.32\times10^6)}$$

$$75.556\,e = 1 - 0.41468$$

$$e = 7.75\times10^{-3}\ \text{m} = 7.75\ \text{mm}$$

Problem 10.128

10.128 A 43-kip axial load **P** is applied to the rolled-steel column BC at a point on the x axis at a distance $e = 2.5$ in. from the geometric axis of the column. Using the allowable-stress method, select the wide-flange shape of 8-in. nominal depth that should be used. Use $E = 29 \times 10^6$ psi. and $\sigma_Y = 36$ ksi.



Steel: $E = 29 \times 10^3$ ksi $\quad \sigma_Y = 36$ ksi

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29000)}{36}} = 126.10$$

$L = 8$ ft $= 96$ in $\quad L_e = 2L = 192$ in

Try W 8×31. $\quad r_y = 2.02$ in, $\quad \frac{L_e}{r_y} = 95.05 < C_c$

$$\frac{L_e/r_y}{C_c} = 0.754$$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.754) - \frac{1}{8}(0.754)^3 = 1.896$$

$$\sigma_{all} = \frac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\frac{L_e/r_y}{C_c}\right)^2\right] = \frac{36}{1.896}\left[1 - \frac{1}{2}(0.754)^2\right]$$

$$= 13.59 \text{ ksi}$$

$$\frac{P}{A} + \frac{Pec}{I_y} = \frac{43}{9.13} + \frac{(43)(2.5)(\frac{1}{2}\cdot 7.995)}{37.1} = 4.71 + 11.58 = 16.29 \text{ ksi}$$

> 13.59 ksi (not allowed)

Approximate required area. $\left(\frac{16.29}{13.59}\right)(9.13) = 10.9$ in^2

Try W 8×35. $\quad r_y = 2.03 \quad \frac{L_e}{r_y} = 94.58 < C_c \quad \frac{L_e/r_y}{C_c} = 0.750$

F.S. $= 1.895 \quad \sigma_{all} = 13.65$ ksi

$$\frac{P}{A} + \frac{Pec}{I_y} = \frac{43}{10.3} + \frac{(43)(2.5)(\frac{1}{2}\cdot 8.020)}{42.6} = 14.29 \text{ ksi} > 13.65 \text{ ksi} \quad \text{(not allowed)}$$

Try W8×40. $\quad r_y = 2.04 \quad \frac{L_e}{r_y} = 94.12 < C_c \quad \frac{L_e/r_y}{C_c} = 0.746$

F.S. $= 1.895 \quad \sigma_{all} = 13.71$ ksi

$$\frac{P}{A} + \frac{Pec}{I_y} = \frac{43}{11.7} + \frac{(43)(2.5)(\frac{1}{2}\cdot 8.07)}{49.1} = 12.51 \text{ ksi} < 13.71 \text{ ksi} \quad \text{(allowed)}$$

Use W 8×40. ◄

PROBLEM 10.C1

10.C1 A solid steel rod having an effective length of 500 mm is to be used as a compression strut to carry a centric load **P**. For the grade of steel used $E = 200$ GPa and $\sigma_Y = 245$ MPa. Knowing that a factor of safety of 2.8 is required and using Euler's formula, write a computer program and use it to calculate the allowable centric load P_{all} for values of the radius of the rod from 6 mm to 24 mm, using 2-mm increments.

SOLUTION

ENTER RADIUS RAD, EFFECTIVE LENGTH L_e AND FACTOR OF SAFETY FS

COMPUTE RADIUS OF GYRATION

$$A = \pi\, RAD^2$$

$$I = \frac{1}{4}\pi\, RAD^4$$

$$r = \sqrt{\frac{I}{A}}$$

DETERMINE ALLOWABLE CENTRIC LOAD

CRITICAL STRESS:

$$\sigma_{cr} = \frac{\pi^2 E}{(L_e/r)^2}$$

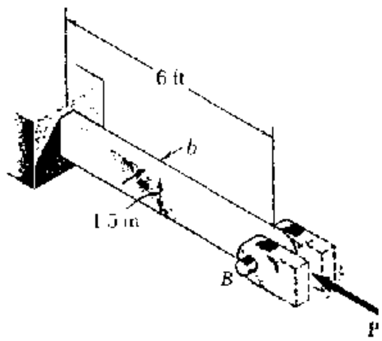
LET σ EQUAL SMALLER OF σ_{cr} AND σ_Y

$$P_{all} = \frac{\sigma A}{FS}$$

PROGRAM OUTPUT

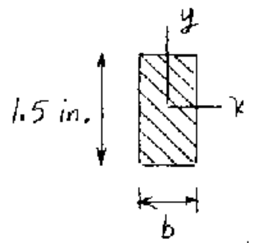
```
Radius      Critical    Allowable
of rod       stress       load
  m           MPa          kN

.006          71.1         2.87
.008         126.3         9.07
.010         197.4        22.15
- - - - - - - - - - - - - - - - -
.012         284.2        39.58
.014         386.9        53.88
.016         505.3        70.37
.018         639.6        89.06
.020         789.6       109.96
.022         955.4       133.05
.024        1137.0       158.34

Below the dashed line we have:
   critical stress > yield strength
```

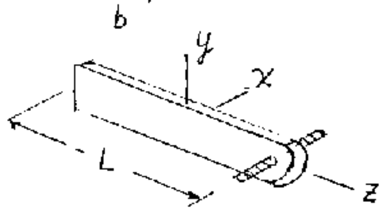
PROBLEM 10.C2



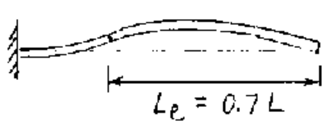
10.C2 An aluminum bar is fixed at end A and supported at end B so that it is free to rotate about a horizontal axis through the pin. Rotation about a vertical axis at end B is prevented by the brackets. Knowing that $E = 10.1 \times 10^6$ psi, use Euler's formula with a factor of safety of 2.5 to determine the allowable centric load **P** for values of b from 0.75 in. to 1.5 in., using 0.125-in. increments.

SOLUTION

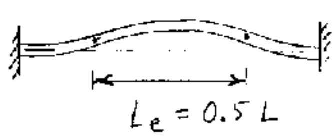




BUCKLING IN YZ PLANE



BUCKLING IN XZ PLANE



ENTER E, LENGTH L AND FACTOR OF SAFETY FS

FOR b = 0.75 TO 1.5 WITH 0.125 INCREMENTS

COMPUTE RADIOS OF GYRATION

$$A = 1.5\, b$$

$$I_x = \frac{1}{12}\, b\, 1.5^3 \qquad r_x = \sqrt{\frac{I_x}{A}}$$

$$I_y = \frac{1}{8}\, b^3 \qquad r_y = \sqrt{\frac{I_y}{A}}$$

COMPUTE CRITICAL STRESSES

$$(\sigma_{cr})_x = \frac{\pi^2 E}{(0.7L/r_x)^2}$$

$$(\sigma_{cr})_y = \frac{\pi^2 E}{(0.5L/r_y)^2}$$

LET σ_{cr} EQUAL SMALLER STRESS

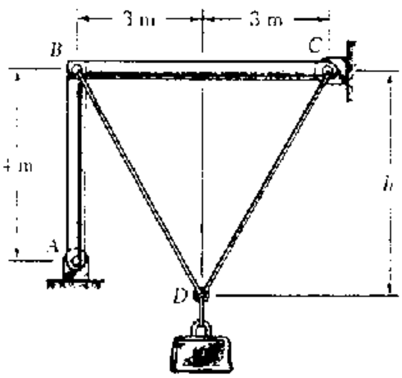
COMPUTE ALLOWABLE CENTRIC LOAD

$$P_{all} = \frac{\sigma_{cr} A}{FS}$$

PROGRAM OUTPUT

b in.	Critical stress x axis ksi	Critical stress y axis ksi	Allowable load kips
.750	7.358	3.6	1.62
.875	7.358	4.9	2.58
1.000	7.358	6.4	3.85
1.125	7.358	8.1	4.97
1.250	7.358	10.0	5.52
1.375	7.358	12.1	6.07
1.500	7.358	14.4	6.62

PROBLEM 10.C3



10.C3 The pin-ended members AB and BC consist of sections of aluminum pipe of 120-mm outer diameter and 10-mm wall thickness. Knowing that a factor of safety of 3.5 is required, determine the mass m of the largest block that can be supported by the cable arrangement shown for values of h from 4 m to 8 m, using 0.25-m increments. Use $E = 70$ GPa and consider only buckling in the plane of the structure.

SOLUTION

COMPUTE MOMENT OF INERTIA

$$I = \frac{\pi}{4}\left(0.06^4 - 0.05^4\right)$$

FOR h = 4 TO 8 USING 0.25 INCREMENTS

COMPUTE ALLOWABLE LOADS IN MEMBERS

$$(F_{AB})_{cr} = \frac{\pi^2 EI}{3.5(4)^2}\ ; \quad (F_{BC})_{cr} = \frac{\pi^2 EI}{3.5(6)^2}$$

DETERMINE ALLOWABLE W

$$(W_{all})_1 = 2\,(F_{AB})_{cr}\ ; \quad (W_{all})_2 = \frac{h}{1.5}\,(F_{BC})_{cr}$$

W_{all} EQUALS SMALLER VALUE

COMPUTE MASS m

$$m = \frac{W_{all}}{9.81}$$

JOINT D:

h, 3, T, h, 3, T, W

$\Sigma F_y = 0$ YIELDS

$$T_Y = \frac{1}{2}W$$

$$\frac{T_x}{T_y} = \frac{3}{h} \text{ YIELDS}$$

$$T_x = \frac{1.5W}{h}$$

JOINT B:

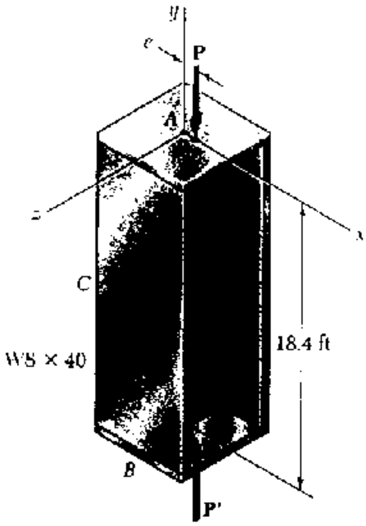
$$F_{BC} = \frac{1.5W}{h}$$

$$F_{AB} = \frac{1}{2}W \qquad T_x = \frac{1.5W}{h} \qquad T_Y = \frac{1}{2}W$$

PROGRAM OUTPUT

h m	Weight critical stress AB kN	Weight critical stress BC kN	mass kg
4.00	455.11	269.7	7854.88
4.25	455.11	286.6	8345.80
4.50	455.11	303.4	8836.74
4.75	455.11	320.3	9327.66
5.00	455.11	337.1	9818.59
5.25	455.11	354.0	10309.52
5.50	455.11	370.8	10800.45
5.75	455.11	387.7	11291.38
6.00	455.11	404.5	11782.31
6.25	455.11	421.4	12273.24
6.50	455.11	438.3	12764.17
6.75	455.11	455.1	13255.10
7.00	455.11	472.0	13255.10
7.25	455.11	488.8	13255.10
7.50	455.11	505.7	13255.10
7.75	455.11	522.5	13255.10
8.00	455.11	539.4	13255.10

PROBLEM 10.C4



10.C4 An axial load **P** is applied at a point located on the x axis at a distance $e = 0.5$ in. from the geometric axis of the W8 × 40 rolled-steel column AB. Using $E = 29 \times 10^6$ psi, write a computer program and use it to calculate for values of P from 25 to 75 kips, using 5-kip increments, (a) the horizontal deflection at the midpoint C, (b) the maximum stress in the column.

SOLUTION

ENTER LENGTH L, ECCENTRICITY e

ENTER PROPERTIES A, I_y, r_y, b_f

COMPUTE CRITICAL LOAD

$$P_{cr} = \frac{\pi^2 E I_y}{L^2}$$

FOR P = 25 TO 75 IN INCREMENTS OF 5

COMPUTE HORIZONTAL DEFLECTION AT C

$$y_c = e\left(\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1.0\right)$$

COMPUTE MAXIMUM STRESS

$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{e\, b_f}{2\, r_y^2}\sec\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)$$

PROGRAM OUTPUT

Load kip	maximum deflection in.	maximum stress kips
25.0	.059	3.29
30.0	.072	3.99
35.0	.086	4.69
40.0	.100	5.41
45.0	.115	6.14
50.0	.130	6.88
55.0	.146	7.65
60.0	.163	8.43
65.0	.181	9.22
70.0	.199	10.04
75.0	.219	10.88

PROBLEM 10.C5

10.C5 A column of effective length L is made from a rolled-steel shape and carries a centric axial load **P**. The yield strength for the grade of steel used is denoted by σ_Y, the modulus of elasticity by E, the cross-sectional area of the selected shape by A, and its smallest radius of gyration by r. Using the AISC design formulas for allowable stress design, write a computer program that can be used with either SI or U.S. customary units to determine the allowable load **P**. Use this program to solve (*a*) Prob. 10.57, (*b*) Prob. 10.58, (*c*) Prob. 10.60.

SOLUTION

ENTER L, E, σ_Y

ENTER PROPERTIES A, r_y

DETERMINE ALLOWABLE STRESS

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}}$$

IF $L/r_y \geq C_c$

$$\sigma_{all} = \frac{\pi^2 E}{1.92\,(L/r_y)^2}$$

IF $L/r_y < C_c$

$$FS = \frac{5}{3} + \frac{3}{8}\left(\frac{L/r_y}{C_c}\right) - \frac{1}{8}\left(\frac{L/r_y}{C_c}\right)^3$$

$$\sigma_{all} = \frac{\sigma_Y}{FS}\left(1 - \frac{(L/r_y)^2}{2\,C_c^2}\right)$$

CACULATE ALLOWABLE LOAD:

$$P_{all} = \sigma_{al}\, A$$

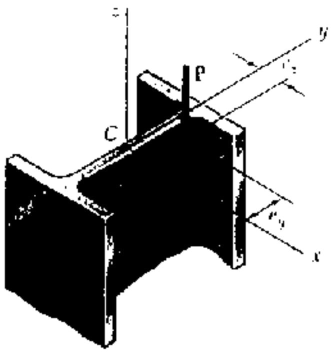
CONTINUED

PROBLEM 10.C5 CONTINUED

PROGRAM OUTPUT

```
Problem 10.57 a

 Effective Length =  21.00  ft
                A =  9.130 in**2
               ry =  2.020 in.
   Yield strength =  36.0 ksi
                E =  29000 ksi
 ---------------------------------------
 Allowable centroid load: P =  87.566 kips

Problem 10.57 b

 Effective Length =  21.00  ft
                A =  9.130 in**2
               ry =  2.020 in.
   Yield strength =  50.0 ksi
                E =  29000 ksi
 ---------------------------------------
 Allowable centroid load: P =  87.452 kips

Problem 10.58 a

 Effective Length =  6.00  m
                A =  4580.0 mm**2
               ry =  40.8 mm
   Yield strength =  250.0 MPa
                E =  200 GPa
 ---------------------------------------
 Allowable centroid load: P =  217.727 kN

Problem 10.58 b

 Effective Length =  6.00  m
                A =  11000.0 mm**2
               ry =  53.2 mm
   Yield strength =  250.0 MPa
                E =  200 GPa
 ---------------------------------------
 Allowable centroid load: P =  858.637 kN

Problem 10.60 a

 Effective Length =  4.00  m
                A =  13.8 mm**2
               ry =  43.4 mm
   Yield strength =  345.0 MPa
                E =  200 GPa
 ---------------------------------------
 Allowable centroid load: P =    1568 kN

Problem 10.60 b

 Effective Length =  6.50  m
                A =  13800.0 mm**2
               ry =  43.4 mm
   Yield strength =  345.0 MPa
                E =  200 GPa
 ---------------------------------------
 Allowable centroid load: P =  632.667 kN
```

PROBLEM 10.C6



10.C6 A column of effective length L is made from a rolled-steel shape and is loaded eccentrically as shown. The yield strength of the grade of steel used is denoted by σ_Y, the allowable stress in bending by σ_{all}, the modulus of elasticity by E, the cross-sectional area of the selected shape by A, and its smallest radius of gyration by r. Write a computer program that can be used with either SI or U.S. customary units to determine the allowable load **P**, using either the allowable-stress method or the interaction method. Use this program to check the given answer for (*a*) Prob. 10.113, (*b*) Prob. 10.114.

SOLUTION

ENTER L, E, σ_Y, $(\sigma_{all})_{bending}$, e_x, e_y

ENTER PROPERTIES A, S_x, S_y, r_y

DETERMINE ALLOWABLE STRESS

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}}$$

IF $L/r_y \geq C_c$

$$\sigma_{all} = \frac{\pi^2 E}{1.92\,(L/r)^2}$$

IF $L/r_y < C_c$

$$FS = \frac{5}{3} + \frac{3}{8}\left(\frac{L/r_y}{C_c}\right) - \frac{1}{8}\left(\frac{L/r_y}{C_c}\right)^3$$

$$\sigma_{all} = \frac{\sigma_Y}{FS}\left(1 - \frac{(L/r_y)^2}{2\,C_c^2}\right)$$

FOR ALLOWABLE-STRESS METHOD

$$COEF = \frac{1}{A} + \frac{e_x}{S_x} + \frac{e_y}{S_y}$$

$$P_{all} = \frac{\sigma_{all}}{COEF}$$

FOR INTERACTION METHOD

$$COFF = \frac{1}{A\,\sigma_{all}} + \frac{(e_x/S_x) + (e_y/S_y)}{(\sigma_{all})_{bending}}$$

$$P_{all} = \frac{1.0}{COEF}$$

CONTINUED

PROBLEM 10.C6 CONTINUED

PROGRAM OUTPUT

```
Problem 10.113

 Effective Length =  24.00 ft
                A =  42.700 in**2
               RY =  3.980 in.
               SX =  232.00 in**3
   Yield strength =  36.0 ksi
                E =  29 x 10^3 ksi
 ----------------------------------------
 Using Allowable-Stress Method
 Allowable load: P = 279.496 kips
 ----------------------------------------

Problem 10.114
 Effective Length = 24.00 ft
                A = 20.000 in**2
               RY =  2.460 in.
               SX =  103.00 in**3
   Yield strength = 50.0 ksi
                E = 29 x 10^3 ksi
 ----------------------------------------
 Using Interaction Method
 Allowable load: P = 139.157 kips
 ----------------------------------------
```

Chapter 11

Problem 11.1

11.1 Determine the modulus of resilience for each of the following grades of structural steel:

(*a*) ASTM A709 Grade 50: $\sigma_Y = 50$ ksi
(*b*) ASTM A913 Grade 65: $\sigma_Y = 65$ ksi
(*c*) ASTM A709 Grade 100: $\sigma_Y = 100$ ksi

Structural steel $\quad E = 29 \times 10^6$ psi for all three steels given.

(a) $\sigma_Y = 50$ ksi $= 50 \times 10^3$ psi

$$u_Y = \frac{\sigma_Y^2}{2E} = \frac{(50 \times 10^3)^2}{(2)(29 \times 10^6)} = 43.1 \text{ in} \cdot \text{lb/in}^3$$

(b) $\sigma_Y = 65$ ksi $= 65 \times 10^3$ psi

$$u_Y = \frac{\sigma_Y^2}{2E} = \frac{(65 \times 10^6)^2}{(2)(29 \times 10^6)} = 72.8 \text{ in} \cdot \text{lb/in}^3$$

(c) $\sigma_Y = 100$ ksi $= 100 \times 10^3$ psi

$$u_Y = \frac{\sigma_Y^2}{2E} = \frac{(100 \times 10^3)^2}{(2)(29 \times 10^6)} = 172.4 \text{ in} \cdot \text{lb/in}^3$$

Problem 11.2

11.2 Determine the modulus of resilience for each of the following aluminum alloys:

(*a*) 1100-H14:	$E = 70$ GPa	$\sigma_Y = 55$ MPa
(*b*) 2014-T6	$E = 75$ GPa:	$\sigma_Y = 220$ MPa
(*c*) 6061-T6	$E = 70$ GPa:	$\sigma_Y = 150$ MPa

Aluminum alloys

(a) $E = 70 \times 10^9$ Pa, $\sigma_Y = 55 \times 10^6$ Pa

$$u_Y = \frac{\sigma_Y^2}{2E} = \frac{(55 \times 10^6)^2}{(2)(70 \times 10^9)} = 21.6 \times 10^3 \text{ N} \cdot \text{m/m}^2 = 21.6 \text{ kJ/m}^3$$

(b) $E = 75 \times 10^9$ Pa, $\sigma_Y = 220 \times 10^6$ Pa

$$u_Y = \frac{\sigma_Y^2}{2E} = \frac{(220 \times 10^6)^2}{(2)(75 \times 10^9)} = 323 \times 10^3 \text{ N} \cdot \text{m/m}^2 = 323 \text{ kJ/m}^3$$

(c) $E = 70 \times 10^9$ Pa, $\sigma_Y = 150 \times 10^6$ Pa

$$u_Y = \frac{\sigma_Y^2}{2E} = \frac{(150 \times 10^6)^2}{(2)(70 \times 10^9)} = 160.7 \times 10^3 \text{ N} \cdot \text{m/m}^3 = 160.7 \text{ kJ/m}^3$$

Problem 11.3

11.3 Determine the modulus of resilience for each of the following metals:

(*a*) Stainless steel
AISI 302 (annealed): $E = 190$ GPa $\sigma_Y = 260$ MPa

(*b*) Stainless steel2014-T6
AISI 302 (cold-rolled): $E = 190$ GPa $\sigma_Y = 520$ MPa

(*c*) Malleable cast iron: $E = 165$ GPa $\sigma_Y = 230$ MPa

(a) $E = 190\times10^{9}$ Pa, $\sigma_Y = 260\times10^{6}$ Pa

$$u_Y = \frac{\sigma_Y^2}{2E} = \frac{(260\times10^{6})^2}{(2)(190\times10^{9})} = 177.9\times10^{3}\ \mathrm{N\cdot m/m^3} = 177.9\ \mathrm{kJ/m^3}$$

(b) $E = 190\times10^{9}$ Pa, $\sigma_Y = 520\times10^{6}$ Pa

$$u_Y = \frac{\sigma_Y^2}{2E} = \frac{(520\times10^{6})^2}{(2)(190\times10^{9})} = 712\times10^{3}\ \mathrm{N\cdot m/m^3} = 712\ \mathrm{kJ/m^3}$$

(c) $E = 165\times10^{9}$ Pa, $\sigma_Y = 230\times10^{6}$ Pa

$$u_Y = \frac{\sigma_Y^2}{2E} = \frac{(230\times10^{6})^2}{(2)(165\times10^{9})} = 160.3\times10^{3}\ \mathrm{N\cdot m/m^3} = 160.3\ \mathrm{kJ/m^3}$$

Problem 11.4

11.4 Determine the modulus of resilience for each of the following alloys:

(*a*) Titanium: $E = 16.5\times10^6$ psi $\sigma_Y = 125$ ksi

(*b*) Magnesium: $E = 6.5\times10^6$ psi $\sigma_Y = 30$ ksi

(*c*) Cupronickel (annealed) $E = 20\times10^6$ psi $\sigma_Y = 18$ ksi

(a) $E = 16.5\times10^{6}$ psi, $\sigma_Y = 120\times10^{3}$ psi

$$u_Y = \frac{\sigma_Y^2}{2E} = \frac{(125\times10^{3})^2}{(2)(16.5\times10^{6})} = 473\ \mathrm{in\cdot lb/in^3}$$

(b) $E = 6.5\times10^{6}$ psi, $\sigma_Y = 29\times10^{3}$ psi

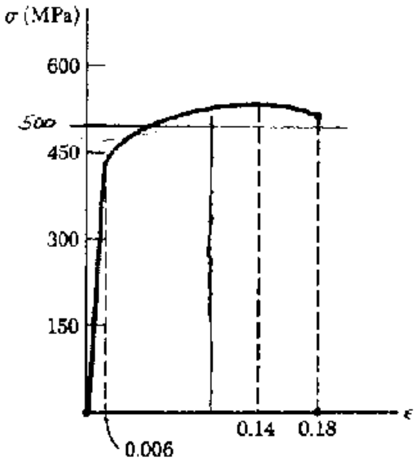
$$u_Y = \frac{\sigma_Y^2}{2E} = \frac{(30\times10^{3})^2}{(2)(6.5\times10^{6})} = 69.2\ \mathrm{in\cdot lb/in^3}$$

(c) $E = 20\times10^{6}$ psi, $\sigma_Y = 16\times10^{3}$ psi

$$u_Y = \frac{\sigma_Y^2}{2E} = \frac{(18\times10^{3})^2}{(2)(20\times10^{6})} = 8.10\ \mathrm{in\cdot lb/in^3}$$

Problem 11.5

11.5 The stress-strain diagram shown has been drawn from data obtained during a tensile test of an aluminum alloy. Using $E = 72$ GPa, determine (*a*) the modulus of resilience of the alloy, (*b*) the modulus of toughness of the alloy.



(a) $\sigma_Y = E\varepsilon_Y$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{1}{2}E\varepsilon_Y^2 = \frac{1}{2}(72\times10^9)(0.006)^2$$

$$= 1296\times10^3\ \text{N}\cdot\text{m/m}^3 = 1296\ \text{kJ/m}^3$$

(b) Modulus of toughness = total area under the stress-strain curve

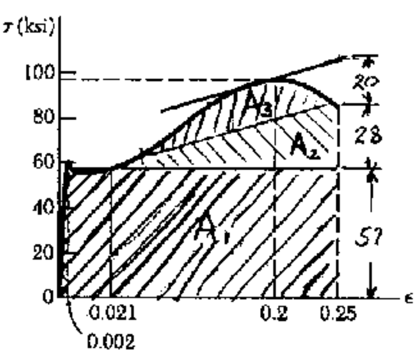
The average ordinate of the stress-strain curve is 500 MPa = 500×10^6 N/m^2

The area under the curve is $A = (500\times10^6)(0.18) = 90\times10^6$ N/m^2

modulus of toughness $= 90\times10^6$ J/m^3 = 90 MJ/m^3

Problem 11.6

11.6 The stress-strain diagram shown has been drawn from data obtained during a tensile test of a specimen of structural steel. Using $E = 29\times10^6$ psi, determine (*a*) the modulus of resilience of the steel, (*b*) the modulus of toughness of the steel.



(a) $\sigma_Y = E\varepsilon_Y$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{1}{2}E\varepsilon_Y^2 = \frac{1}{2}(29\times10^6)(0.002)^2$$

$$= 58.0\ \text{in}\cdot\text{lb/in}^3$$

(b) Modulus of toughness = total area under the stress-strain curve

$A_1 = (57)(0.25 - 0.002) = 14.14$ kips/in^2 = 14.14 in·kip/in^3

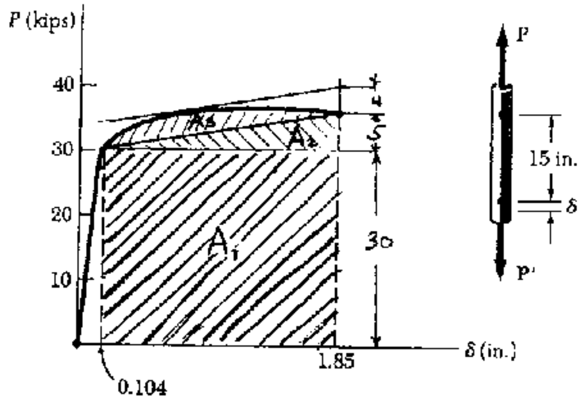
$A_2 = \frac{1}{2}(28)(0.25 - 0.021) = 3.21$ kips/in^2 = 3.21 in·kip/in^3

$A_3 = \frac{2}{3}(20)(0.25 - 0.075) = 2.33$ kips/in^2 = 2.33 in·kip/in^3

modulus of toughness $= U_Y + A_1 + A_2 + A_3 \approx 20$ in·kip/in^3

Problem 11.7

11.7 The load-deformation diagram shown has been drawn from data obtained during a tensile test of a 0.875-in.-diameter rod of an aluminum alloy. Knowing that the deformation was measured using a 15-in. gage length, determine (*a*) the modulus of resilience of the alloy, (*b*) the modulus of toughness of the alloy.



Volume of stressed material involved in the measurement:

$$V = \frac{\pi}{4} d^2 L$$

$$= \frac{\pi}{4}(0.875)^2(15) = 9.0198 \text{ in}^3$$

(a) Modulus of resilience.

$P_Y = 30$ kips, $\delta_Y = 0.104$ in

$U_Y = \frac{1}{2} P_Y \delta_Y = \frac{1}{2}(30)(0.104) = 1.56$ in·kip $= 1560$ in·lb

modulus of resilience $\quad u_Y = \frac{U_Y}{V} = \frac{1560}{9.0198} = 173.0$ in·lb/in^3 ◄

(b) Modulus of toughness.

$A_1 = (30)(1.85 - 0.104) = 52.38$ kip·in $= 52380$ in·lb/in^3

$A_2 = \frac{1}{2}(5)(1.85 - 0.104) = 4.365$ kip·in $= 4365$ in·lb/in^3

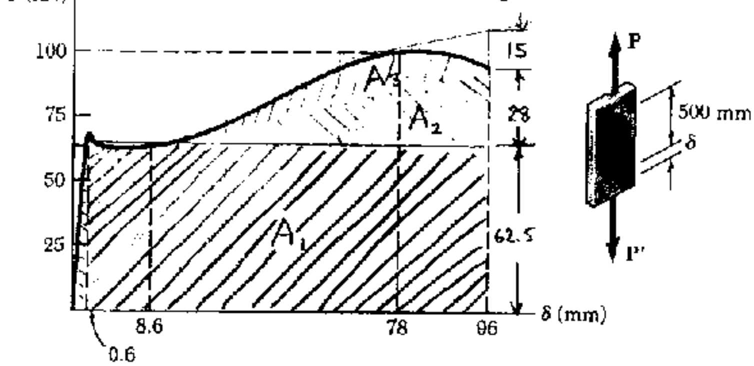
$A_3 = \frac{2}{3}(4)(1.85 - 0.104) = 4.656$ kip·in $= 4656$ in·lb/in.

$U = U_Y + A_1 + A_2 + A_3 = 62961$ in·lb/in^3

modulus of toughness $\quad \frac{U}{V} = \frac{62961}{9.0198} = 6980$ in·lb/in^3 ◄

Problem 11.8

11.8 The load-deformation diagram shown has been drawn from data obtained during a tensile test of structural steel. Knowing that the cross-sectional area of the specimen is 250 mm² and that the deformation was measured using a 500-mm gage length, determine (*a*) the modulus of resilience of the steel, (*b*) the modulus of toughness of the steel.



Assuming that yielding occurs at $P = 62.5$ kN and $\delta = 0.6$ mm

$$U_Y = \tfrac{1}{2}(62.5\times10^3)(0.6\times10^{-3})$$
$$= 18.75 \text{ N}\cdot\text{m}$$
$$= 18.75 \text{ J}$$

Volume of stressed material: $V = AL = (250)(500) = 125\times10^3 \text{ mm}^3$
$$= 125\times10^{-6} \text{ m}^3$$

$$u_Y = \frac{U_Y}{V} = \frac{18.75}{125\times10^{-6}} = 150\times10^3 = 150 \text{ kJ/m}^3$$

$$A_1 = (62.5\times10^6)(96\times10^{-3}) = 6\times10^3 \text{ N-m} = 6\times10^3 \text{ J}$$

$$A_2 = \tfrac{1}{2}(28\times10^3)(96-8.6)\times10^{-3} = 1.22\times10^3 \text{ N}\cdot\text{m} = 1.22\times10^3 \text{ J}$$

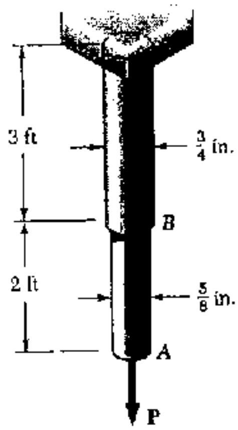
$$A_3 = \tfrac{2}{3}(15\times10^6)(61\times10^{-3}) = 0.61\times10^3 \text{ N}\cdot\text{m} = 0.61\times10^3 \text{ J}$$

Total energy $U = U_Y + A_1 + A_2 + A_3 = 7.85\times10^3$ J

modulus of toughness $= \dfrac{U}{V} = \dfrac{7.85\times10^3}{125\times10^{-6}} = 63\times10^6 \text{ J/m}^3$
$$= 63 \text{ MJ/m}^3$$

Problem 11.9

11.9 Using $E = 29 \times 10^6$ psi, determine (*a*) the strain energy of the steel rod *ABC* when $P = 8$ kips, (*b*) the corresponding strain energy density in portions *AB* and *BC* of the rod.



$P = 8$ kips, $E = 29 \times 10^3$ ksi

$A = \frac{\pi}{4} d^2$, $V = AL$, $\sigma = \frac{P}{A}$, $u = \frac{\sigma^2}{2E}$

$U = uV$

Portion	d in.	L in.	A in²	V in³	σ ksi	u in·kip/in³	U in·kip
AB	0.625	24	0.3608	7.363	26.08	11.72×10^{-3}	86.32×10^{-3}
BC	0.75	36	0.4418	15.904	18.11	5.65×10^{-3}	89.92×10^{-3}
Σ							176.24×10^{-3}

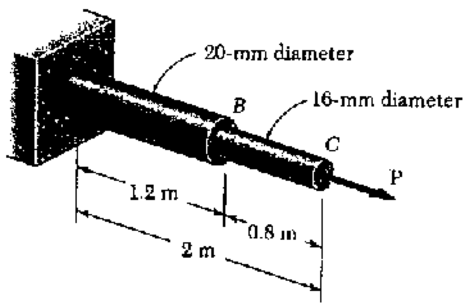
(a) $U = 176.2 \; 10^{-3}$ in·kip $= 176.2$ in·lb

(b) In AB, $u = 11.72 \times 10^{-3}$ in kip/in³ $= 11.72$ in·lb/in³

In BC, $u = 5.65 \times 10^{-3}$ in·kip/in³ $= 5.65$ in·lb/in³

Problem 11.10

11.10 Using $E = 200$ GPa, determine (*a*) the strain energy of the steel rod *ABC* when $P = 25$ kN, (*b*) the corresponding strain-energy density in portions *AB* and *BC* of the rod.



$$A_{AB} = \frac{\pi}{4}(20)^2 = 314.16 \text{ mm}^2 = 314.16 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4}(16)^2 = 201.06 \text{ mm}^2 = 201.06 \times 10^{-6} \text{ m}^2$$

$$P = 25 \times 10^3 \text{ N}$$

$$U = \Sigma \frac{P^2 L}{2EA}$$

$$= \frac{(25 \times 10^3)^2 (1.2)}{(2)(200 \times 10^9)(314.16 \times 10^{-6})} + \frac{(25 \times 10^3)^2 (0.8)}{(2)(200 \times 10^9)(201.06 \times 10^{-6})}$$

(a) $U = 5.968 + 6.213 = 12.18 \text{ N·m} = 12.18 \text{ J}$

(b) $\sigma_{AB} = \frac{P}{A_{AB}} = \frac{25 \times 10^3}{314.16 \times 10^{-6}} = 79.58 \times 10^6 \text{ Pa}$

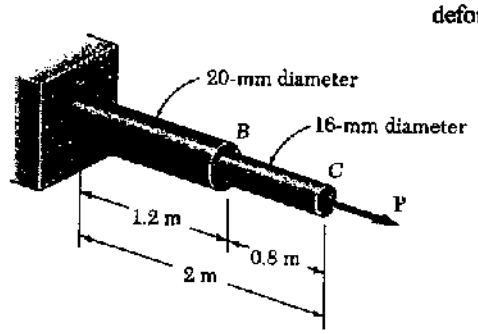
$$u_{AB} = \frac{\sigma_{AB}^2}{2E} = \frac{(79.58 \times 10^6)^2}{(2)(200 \times 10^9)} = 15.83 \times 10^3 = 15.83 \text{ kJ/m}^3$$

$$\sigma_{BC} = \frac{P}{A_{AB}} = \frac{25 \times 10^3}{201.16 \times 10^{-6}} = 124.28 \times 10^6 \text{ Pa}$$

$$u_{BC} = \frac{\sigma_{BC}^2}{2E} = \frac{(124.28 \times 10^6)^2}{(2)(200 \times 10^9)} = 38.6 \times 10^3 = 38.6 \text{ kJ/m}^3$$

Problem 11.11

11.11 Rod *ABC* is made of a steel for which the yield strength is $\sigma_Y = 250$ MPa and the modulus of elasticity is $E = 200$ GPa. Determine, for the loading shown, the maximum strain energy that can be acquired by the rod without causing any permanent deformation.



$$A_{AB} = \frac{\pi}{4}(20)^2 = 314.16 \text{ mm}^2 = 314.16 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4}(16)^2 = 201.06 \text{ mm}^2 = 201.06 \times 10^{-6} \text{ m}^2$$

$$P = \sigma_Y A_{min} = (250 \times 10^6)(201.06 \times 10^{-6})$$

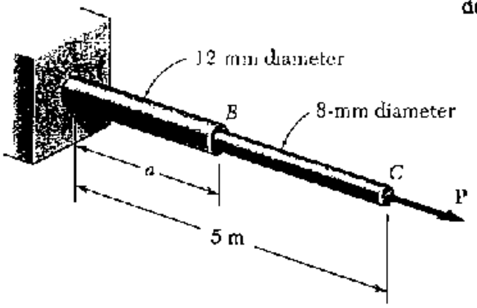
$$= 50.265 \times 10^3 \text{ Pa}$$

$$U = \Sigma \frac{P^2 L}{2EA} = \frac{(50265)^2 (1.2)}{(2)(200 \times 10^9)(314.16 \times 10^{-6})} + \frac{(50265)^2 (0.8)}{(2)(200 \times 10^9)(201.06 \times 10^{-6})}$$

$$= 24.13 + 25.13 = 49.3 \text{ J}$$

Problem 11.12

11.12 Rods *AB* and *BC* are made of a steel for which the yield strength is σ_Y = 300 MPa and the modulus of elasticity is E = 200 GPa. Determine the maximum strain energy that can be acquired by the assembly without causing permanent deformation when the length a of rod *AB* is (*a*) 2 m, (*b*) 4 m.



$$A_{AB} = \frac{\pi}{4}(12)^2 = 113.097\ mm^2 = 113.097\times10^{-6}\ m^2$$

$$A_{BC} = \frac{\pi}{4}(8)^2 = 50.265\ mm^2 = 50.265\times10^{-6}\ m^2$$

$$P = \sigma_Y A_{min} = (300\times10^6)(50.265\times10^{-6}) = 15.08\times10^3\ N$$

$$U = \Sigma\frac{P^2 l}{2EA} = \frac{P^2}{2E}\Sigma\frac{l}{A}$$

(a) $a = 2m, \quad L-a = 5-2 = 3m$

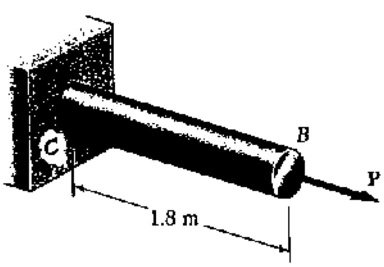
$$U = \frac{(15.08\times10^3)^2}{(2)(200\times10^9)}\left[\frac{2}{113.097\times10^{-6}} + \frac{3}{50.265\times10^{-6}}\right] = 44.0\ N\cdot m = 44.0\ J$$

(b) $a = 4m \quad L-a = 1\ m$

$$U = \frac{(15.08\times10^3)^2}{(2)(200\times10^9)}\left[\frac{4}{113.097\times10^{-6}} \quad \frac{1}{50.265\times10^{-6}}\right] = 31.4\ N\cdot m = 31.4\ J$$

Problem 11.13

11.13 Rod *BC* is made of a steel for which the yield strength is σ_Y = 300 MPa and the modulus of elasticity is E = 200 GPa. Knowing that a strain energy of 10 J must be acquired by the rod when the axial load **P** is applied, determine the diameter of the rod for which the factor of safety with respect to permanent deformation is six.



For factor of safety of six on the energy,

$$U_Y = (6)(10) = 60\ J$$

$$u_Y = \frac{\sigma_Y^2}{2E} = \frac{(300\times10^6)^2}{(2)(200\times10^9)} = 225\times10^3\ J/m^3$$

$$U_Y = A L u_Y$$

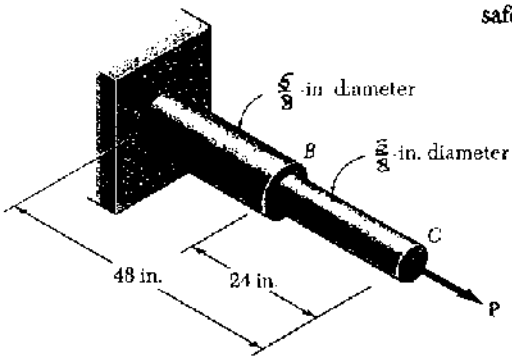
$$A = \frac{\pi}{4}d^2$$

$$A = \frac{U_Y}{L u_Y} = \frac{60}{(1.8)(225\times10^3)} = 148.148\times10^{-6}\ m^2$$

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(148.148\times10^{-6})}{\pi}} = 13.73\times10^{-3}\ m = 13.73\ mm$$

Problem 11.14

11.14 Rod AB is made of a steel for which the yield strength is $\sigma_Y = 65$ ksi and the modulus of elasticity is $E = 29 \times 10^6$ psi. Knowing that a strain energy of 60 in · lb must be acquired by the rod as the axial load **P** is applied, determine the factor of safety of the rod with respect to permanent deformation.



$$A_{AB} = \frac{\pi}{4}\left(\frac{5}{8}\right)^2 = 0.3068 \text{ in}^2$$

$$A_{BC} = \frac{\pi}{4}\left(\frac{3}{8}\right)^2 = 0.11045 \text{ in}^2$$

$$P_Y = \sigma_Y A_{min} = (65\times10^3)(0.11045) = 7185 \text{ lb.}$$

$$U_Y = \sum \frac{P_Y^2 \ell}{EA} = \frac{P_Y^2}{2E}\sum\frac{\ell}{A}$$

$$U_Y = \frac{(7185)^2}{(2)(29\times10^6)}\left[\frac{48-24}{0.3068} + \frac{24}{0.11045}\right] = 263 \text{ in. lb}$$

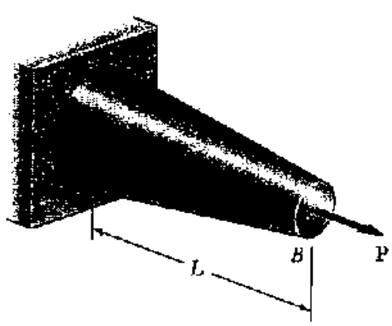
$$F.S. = \frac{U_Y}{U_{design}} = \frac{263}{60} = 4.38$$

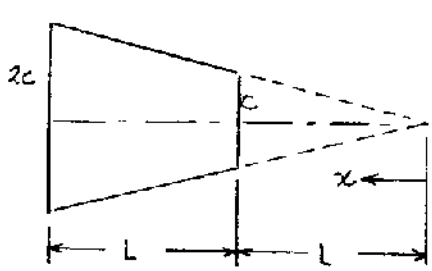
Problem 11.15

11.15 Show by integration that the strain energy of the tapered rod AB is

$$U = \frac{1}{4}\frac{P^2 L}{EA_{min}}$$

where A_{min} is the cross-sectional area at end B.





$$\text{radius} \quad r = \frac{cx}{L} \qquad A_{min} = \pi c^2$$

$$A = \pi r^2 = \frac{\pi c^2}{L^2}x^2$$

$$U = \int_L^{2L}\frac{P^2\,dx}{2EA} = \frac{P^2}{2E}\int_L^{2L}\frac{L^2}{\pi c^2}\frac{dx}{x^2}$$

$$= \frac{P^2L^2}{2E\pi c^2}\left(-\frac{1}{x}\right)\Big|_L^{2L}$$

$$= \frac{P^2L^2}{2EA_{min}}\left(-\frac{1}{2L}+\frac{1}{L}\right) = \frac{P^2L^2}{4EA_{min}}$$

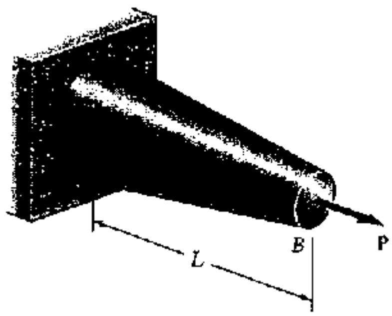
Problem 11.16

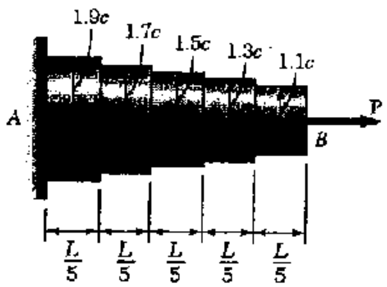
11.16 Solve, Prob. 11.15, using the stepped rod shown as an approximation of the tapered rod. What is the percentage error in the answer obtained.

11.15 Show by integration that the strain energy of the tapered rod AB is

$$U = \frac{1}{4}\frac{P^2 L}{EA_{min}}$$

where A_{min} is the cross-sectional area at end B.





$$A_i = \pi r_i^2 \qquad A_{min} = \pi c^2$$

$$U = \sum \frac{P^2 l_i}{2EA_i} = \frac{P^2 (L/5)}{2E} \sum \frac{1}{A_i}$$

$$= \frac{P^2 L}{10 \pi E} \sum \frac{1}{r_i^2}$$

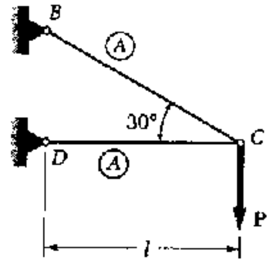
$$= \frac{P^2 L}{10 \pi E}\left\{\frac{1}{(1.9c)^2} + \frac{1}{(1.7c)^2} + \frac{1}{(1.5c)^2} + \frac{1}{(1.3c)^2} + \frac{1}{(1.1c)^2}\right\}$$

$$= \frac{P^2 L}{10 E(\pi c^2)}\left\{2.4856\right\} = 0.24856 \frac{P^2 L}{EA_{min}}$$

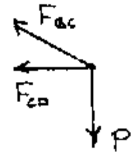
$$\% \text{ error} = \frac{0.24856 - 0.25}{0.25} \times 100\% = -0.575\%$$

Problem 11.17

11.17 through 11.20 In the truss shown, all members are made of the same material and have the uniform cross-sectional area indicated. Determine the strain energy of the truss when the load **P** is applied.



Joint C



$+\uparrow \Sigma F_y = 0:$

$\frac{1}{2} F_{BC} - P = 0 \qquad F_{BC} = 2P$

$+\rightarrow \Sigma F_x = 0$

$-F_{CD} - \frac{\sqrt{3}}{2} F_{CD} = 0 \qquad F_{BC} = -\sqrt{3}\, P$

$$U = \Sigma \frac{F^2 L}{2EA} = \frac{1}{2E} \Sigma \frac{F^2 L}{A}$$

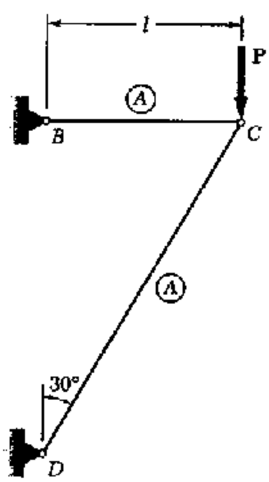
Member	F	L	A	F^2L/A
BC	$2P$	$\frac{2}{\sqrt{3}}\ell$	A	$\frac{8}{\sqrt{3}} P^2\ell/A$
CD	$-\sqrt{3}P$	ℓ	A	$3\, P^2\ell/A$
Σ				$7.62 P^2\ell/A$

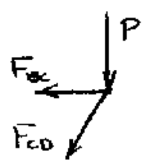
$$U = \frac{1}{2E}\left(7.62 \frac{P^2\ell}{A}\right)$$

$$= 3.81 \frac{P^2\ell}{EA} \blacktriangleleft$$

Problem 11.18

11.17 through 11.20 In the truss shown, all members are made of the same material and have the uniform cross-sectional area indicated. Determine the strain energy of the truss when the load **P** is applied.





Joint C

$+\uparrow \Sigma F_y = 0:$

$-\frac{\sqrt{3}}{2} F_{CD} - P = 0 \qquad F_{CD} = -\frac{2}{\sqrt{3}} P$

$+\rightarrow \Sigma F_x = 0$

$-F_{BC} - \frac{1}{2} F_{CD} = 0 \qquad F_{BC} = \frac{1}{\sqrt{3}} P$

$$U = \Sigma \frac{F^2 L}{2EA} = \frac{1}{2E} \Sigma \frac{F^2 L}{A}$$

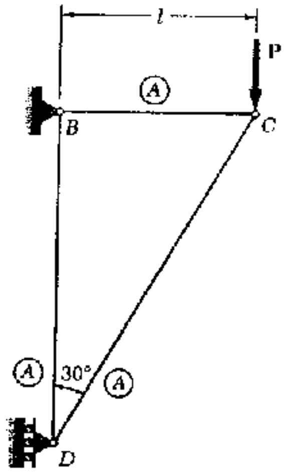
Member	F	L	A	F^2L/A
BC	$\frac{1}{\sqrt{3}}P$	ℓ	A	$\frac{1}{3} P^2\ell/A$
CD	$-\frac{2}{\sqrt{3}}P$	2ℓ	A	$\frac{8}{3} P^2\ell/A$
Σ				$3 P^2\ell/A$

$$U = \frac{1}{2E}\left(3 \frac{P^2\ell}{A}\right)$$

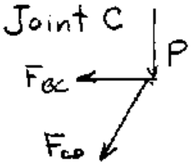
$$= 1.5 \frac{P^2\ell}{EA} \blacktriangleleft$$

Problem 11.19

11.17 through 11.20 In the truss shown, all members are made of the same material and have the uniform cross-sectional area indicated. Determine the strain energy of the truss when the load **P** is applied.



Joint C



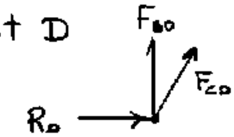
$+\uparrow\Sigma F_y = 0: \quad -\frac{\sqrt{3}}{2}F_{CD} - P = 0$

$F_{CD} = -\frac{2}{\sqrt{3}}P$

$\xrightarrow{+}\Sigma F_x = 0 \quad -F_{BC} - \frac{1}{2}F_{CD} = 0$

$F_{BC} = \frac{1}{\sqrt{3}}P$

Joint D



$+\uparrow\ \Sigma F_y = 0:$

$F_{BD} + \frac{\sqrt{3}}{2}F_{CD} = 0$

$F_{BD} = P$

Member	F	L	A	F^2L/A
BC	$\frac{1}{\sqrt{3}}P$	l	A	$\frac{1}{3}P^2l/A$
CD	$-\frac{2}{\sqrt{3}}P$	$2l$	A	$\frac{8}{3}P^2l/A$
BD	P	$\sqrt{3}l$	A	$\sqrt{3}P^2l/A$
Σ				$4.732\,P^2l/A$

$U = \Sigma\frac{1}{2}\frac{F^2L}{EA}$

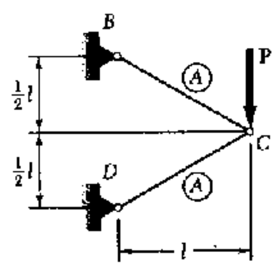
$= \frac{1}{2E}\Sigma\frac{F^2L}{A}$

$= \frac{1}{2E}\left(4.732\,\frac{P^2l}{A}\right)$

$= 2.37\,\frac{P^2l}{EA}$ ◄

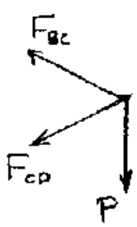
Problem 11.20

11.17 through 11.20 In the truss shown, all members are made of the same material and have the uniform cross-sectional area indicated. Determine the strain energy of the truss when the load **P** is applied.



$L_{BC} = L_{CD} = \sqrt{l^2 + (\frac{1}{2}l)^2} = \frac{\sqrt{5}}{2}l$

Joint C



$\xrightarrow{+}\Sigma F_x = 0: \quad -\frac{2}{\sqrt{5}}F_{BC} - \frac{2}{\sqrt{5}}F_{CD} = 0$

$F_{CD} = -F_{BC}$

$+\uparrow\ \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}}F_{BC} - \frac{1}{\sqrt{5}}F_{CD} - P = 0$

$F_{BC} = \frac{\sqrt{5}}{2}P \qquad F_{CD} = -\frac{\sqrt{5}}{2}P$

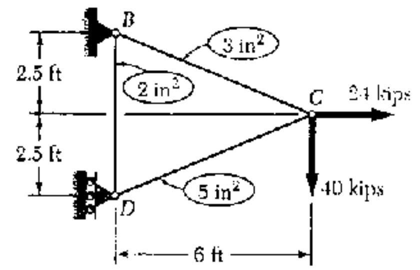
$U = \Sigma\frac{F^2L}{2EA} = \frac{1}{2EA}\left[F_{BC}^2L_{BC} + F_{CD}^2L_{CD}\right]$

$= \frac{1}{2EA}\left[\left(\frac{\sqrt{5}}{2}P\right)^2\left(\frac{\sqrt{5}}{2}l\right) + \left(\frac{\sqrt{5}}{2}P\right)^2\left(\frac{\sqrt{5}}{2}l\right)\right]$

$= 1.398\,\frac{P^2l}{EA}$ ◄

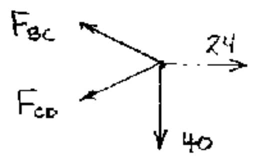
Problem 11.21

11.21 Each member of the truss shown is made of aluminum and has the cross-sectional area shown. Using $E = 10.5 \times 10^6$ psi, determine the strain energy of the truss for the loading shown.



$$L_{BC} = L_{CD} = \sqrt{6^2 + 2.5^2} = 6.5 \text{ ft} = 78 \text{ in.}$$

Joint C



$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{6}{6.5} F_{BC} - \frac{6}{6.5} F_{CD} + 24 = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{2.5}{6.5} F_{BC} - \frac{2.5}{6.5} F_{CD} - 40 = 0 \qquad (2)$$

Solving (1) and (2) simultaneously

$$F_{BC} = 65 \text{ kips} \qquad F_{CD} = -39 \text{ kips}$$

Joint D

$$+\uparrow \Sigma F_y = 0: \quad F_{BD} + \frac{2.5}{6.5} F_{CD} = 0 \qquad F_{BD} = 15 \text{ kips}$$

$$U = \Sigma \frac{F^2 L}{2EA} = \frac{1}{2E} \Sigma \frac{F^2 L}{A}$$

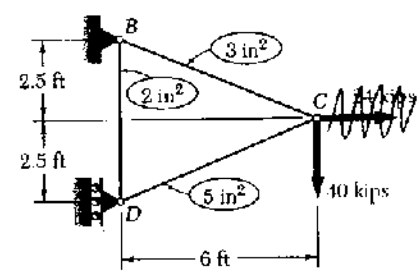
Member	F (10^3 lb)	L (in.)	A (in²)	F^2L/A (10^9 lb²/in)
BC	65	78	3	109.85
BD	15	60	2	6.75
CD	−39	78	5	23.73
Σ				140.33

$$U = \frac{140.33 \times 10^9}{(2)(10.5 \times 10^6)} = 6682 \text{ lb·in} = 6.68 \text{ kip·in}$$

Problem 11.22

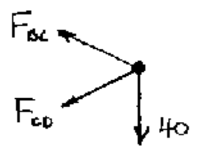
11.22 Solve Prob. 11.21, assuming that the 24-kip load is removed.

11.21 Each member of the truss shown is made of aluminum and has the cross-sectional area shown. Using $E = 10.5 \times 10^6$ psi, determine the strain energy of the truss for the loading shown.



$$L_{BC} = L_{CD} = \sqrt{6^2 + 2.5^2} = 6.5 \text{ ft} = 78 \text{ in.}$$

<u>Joint C</u>



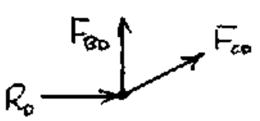
$$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad -\frac{6}{6.5} F_{BC} - \frac{6}{6.5} F_{CD} = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{2.5}{6.5} F_{BC} - \frac{2.5}{6.5} F_{CD} - 40 = 0 \qquad (2)$$

Solving (1) and (2) simultaneously,

$$F_{BC} = 52 \text{ kips} \qquad F_{CD} = -52 \text{ kips}$$

<u>Joint D</u>



$$+\uparrow \Sigma F_y = 0: \quad F_{BD} + \frac{2.5}{6.5} F_{CD} = 0 \qquad F_{BD} = 20 \text{ kips}$$

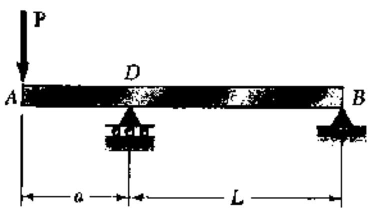
$$U = \Sigma \frac{F^2 L}{2EA} = \frac{1}{2E} \Sigma \frac{F^2 L}{A}$$

Member	F (10^3 lb)	L (in.)	A (in²)	F^2L/A (10^9 lb²/in)
BC	52	78	3	70.304
BD	20	60	2	12.0
CD	-52	78	5	42.1824
Σ				124.4864

$$U = \frac{124.4864 \times 10^9}{(2)(10.5 \times 10^6)} = 5928 \text{ lb} \cdot \text{in} = 5.93 \text{ kip} \cdot \text{in} \blacktriangleleft$$

Problem 11.23

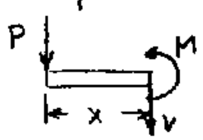
11.23 through 11.26 Taking into account only the effect of normal stresses, determine the strain energy of the prismatic beam AB for the loading shown.



$\circlearrowleft \Sigma M_D = 0:$

$aP + LR_B = 0 \qquad R_B = -\frac{aP}{L} = \frac{aP}{L}\downarrow$

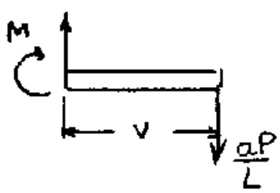
Over portion AD : $M = -Px$



$$U_{AD} = \int_0^a \frac{M^2}{2EI}dx = \frac{1}{2EI}\int_0^a P^2x^2dx = \frac{P^2}{2EI}\left.\frac{x^3}{3}\right|_0^a$$

$$= \frac{P^2a^3}{6EI}$$

Over portion DB : $M = -\frac{aP}{L}v$



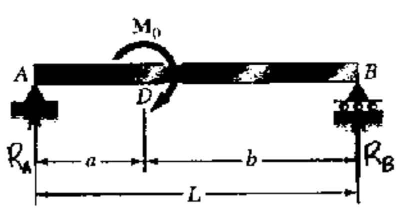
$$U_{DB} = \int_0^L \frac{M^2}{2EI}dv = \frac{1}{2EI}\int_0^L \frac{a^2P^2}{L^2}v^2dv$$

$$= \frac{P^2a^2}{2EIL^2}\int_0^L v^2dv = \frac{P^2a^2}{2EIL^2}\left.\frac{v^3}{3}\right|_0^L = \frac{P^2a^2L}{6EI}$$

Total $\quad U = U_{AD} + U_{DB} = \frac{P^2a^2}{6EI}(a+L)$ ◄

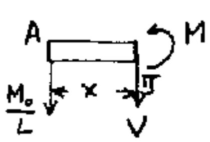
Problem 11.24

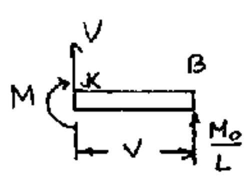
11.23 through 11.26 Taking into account only the effect of normal stresses, determine the strain energy of the prismatic beam AB for the loading shown.



$+\circlearrowleft \Sigma M_B = 0: \quad -R_AL - M_0 = 0 \qquad R_A = \frac{M_0}{L}\downarrow$

$+\circlearrowleft \Sigma M_A = 0: \quad R_BL - M_0 = 0 \qquad R_B = \frac{M_0}{L}\uparrow$





A to D $\quad +\circlearrowleft \Sigma M_J = 0: \quad \frac{M_0x}{L} + M = 0$

$$M = -\frac{M_0x}{L}$$

$$U_{AD} = \int_0^a \frac{M^2dx}{2EI} = \frac{M_0^2}{2EIL^2}\int_0^a x^2dx = \frac{M_0^2a^3}{6EIL^2}$$

D to B $\quad +\circlearrowleft \Sigma M_K = 0: \quad -M + \frac{M_0v}{L}$

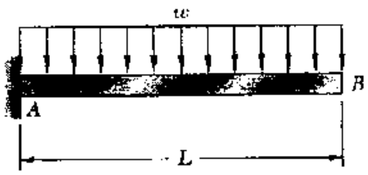
$$M = \frac{M_0v}{L}$$

$$U_{DB} = \int_0^b \frac{M^2dv}{2EI} = \frac{M_0^2}{2EIL^2}\int_0^b v^2dv = \frac{M_0^2b^3}{6EIL^2}$$

Total $\quad U = U_{AD} + U_{DB} = \frac{M_0^2(a^3+b^3)}{6EIL^2}$ ◄

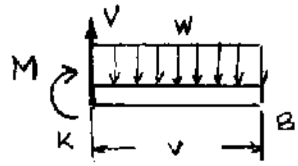
Problem 11.25

11.23 through 11.26 Taking into account only the effect of normal stresses, determine the strain energy of the prismatic beam AB for the loading shown.



$$\circlearrowleft \Sigma M_K = 0: \quad -M - (wv)\left(\frac{v}{2}\right) = 0$$

$$M = -\frac{1}{2}wv^2$$



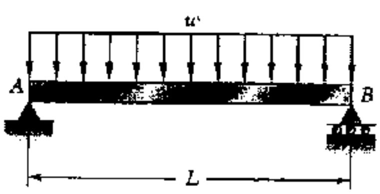
$$U = \int_0^L \frac{M^2}{2EI}\,dv = \frac{1}{2EI}\int_0^L \left(\frac{1}{2}wv^2\right)^2 dv$$

$$= \frac{w^2}{8EI}\int_0^L v^4\,dv = \frac{w^2}{8EI}\,\frac{v^5}{5}\Big|_0^L$$

$$= \frac{w^2L^5}{40EI}$$

Problem 11.26

11.23 through 11.26 Taking into account only the effect of normal stresses, determine the strain energy of the prismatic beam AB for the loading shown.



$$\circlearrowleft \Sigma M_B = 0; \quad -R_A L + (wL)\left(\frac{L}{2}\right) = 0 \qquad R_A = \frac{wL}{2}$$

Bending moment $\quad M = R_A x - \frac{1}{2}wx^2$

$$= \frac{w}{2}(Lx - x^2)$$

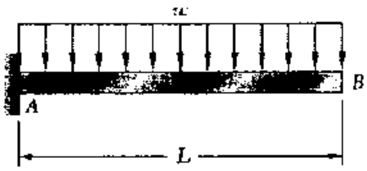
$$U = \int_0^L \frac{M^2}{2EI}\,dx = \frac{w^2}{8EI}\int_0^L (Lx - x^2)^2\,dx$$

$$= \frac{w^2}{8EI}\int_0^L (L^2x^2 - 2Lx^3 + x^4)\,dx = \frac{w^2}{8EI}\left[\frac{L^2x^3}{3} - \frac{2Lx^4}{4} + \frac{x^5}{5}\right]\Big|_0^L$$

$$= \frac{w^2L^5}{8EI}\left[\frac{1}{3} - \frac{1}{2} + \frac{1}{5}\right] = \frac{w^2L^5}{240EI}$$

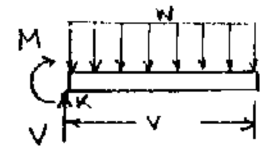
Problem 11.27

11.27 Assuming that the prismatic beam AB has a rectangular cross section, show that for the given loading the maximum value of the strain-energy density in the beam is



$$u_{max} = 15\frac{U}{V}$$

where U is the strain energy of the beam and V is its volume.



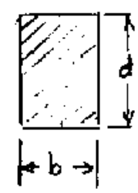
$+\circlearrowleft \Sigma M_K = 0: \quad -M - (wv)\frac{v}{2} = 0$

$$M = -\frac{1}{2}wv^2$$

$$U = \int_0^L \frac{M^2}{2EI}\,dv = \frac{1}{2EI}\int_0^L \left(\frac{1}{2}wv^2\right)^2 dv = \frac{w^2}{8EI}\int_0^L v^4\,dv = \frac{w^2}{8EI}\,\frac{x^5}{5}\Big|_0^L = \frac{w^2L^5}{40EI}$$

$$M_{max} = \frac{1}{2}wL^2 \qquad \sigma_{max} = \frac{M_{max}c}{I}$$

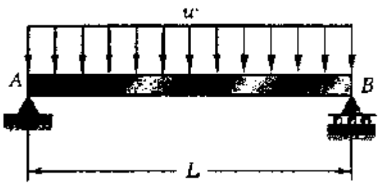
$$u_{max} = \frac{\sigma_{max}^2}{2E} = \frac{M_{max}^2c^2}{2EI^2} = \frac{\frac{1}{4}w^2L^4c^2}{2EI^2} = \frac{w^2L^4c^2}{8EI^2}$$



$$\frac{U}{u_{max}} = \frac{LI}{5c^2} = \frac{L\left(\frac{1}{12}bd^3\right)}{5\left(\frac{d}{2}\right)^2} = \frac{1}{15}Lbd = \frac{1}{15}V$$

$$u_{max} = 15\frac{U}{V}$$

Problem 11.28



11.28 Assuming that the prismatic beam AB has a rectangular cross section, show that for the given loading the maximum value of the strain-energy density in the beam is

$$u_{max} = \frac{45}{8}\frac{U}{V}$$

where U is the strain energy of the beam and V is its volume.

$$+\circlearrowleft M_B = 0: \quad -R_A L + (wL)\frac{L}{2} = 0 \qquad R_A = \frac{1}{2}wL$$

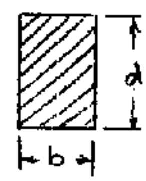
$$M = R_A x - \frac{1}{2}wL^2 = \frac{1}{2}w(Lx - x^2)$$

$$U = \int_0^L \frac{M^2}{2EI}dx = \frac{w^2}{8EI}\int_0^L (L^2x^2 - 2Lx^3 + x^4)\,dx = \frac{w^2}{8EI}\left[\frac{L^2x^3}{3} - \frac{2Lx^4}{4} + \frac{x^5}{5}\right]\Bigg|_0^L$$

$$= \frac{w^2L^5}{8EI}\left[\frac{1}{3} - \frac{1}{2} + \frac{1}{5}\right] = \frac{w^2L^5}{240EI}$$

$$M_{max} = \frac{1}{2}w\left[L\cdot\frac{L}{2} - \left(\frac{L}{2}\right)^2\right] = \frac{1}{8}wL^2 \qquad \sigma_{max} = \frac{M_{max}c}{I} = \frac{wL^2c}{8I}$$

$$u_{max} = \frac{\sigma_{max}^2}{2E} = \frac{w^2L^4c^2}{128EI^2}$$

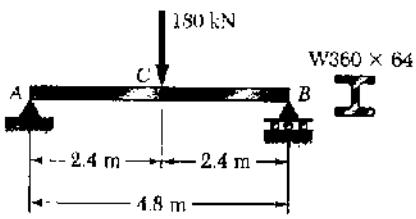


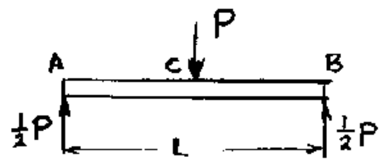
$$\frac{U}{u_{max}} = \frac{8LI}{15c^2} = \frac{8L(\frac{1}{12}bd^3)}{15(\frac{d}{2})^2} = \frac{8}{45}Lbd = \frac{8}{45}V$$

$$u_{max} = \frac{45}{8}\frac{U}{V}$$

Problem 11.29

11.29 and 11.30 Using $E = 200$ GPa, determine the strain energy due to bending for the steel beam and loading shown.





Over portion AC: $\quad M = \frac{1}{2}Px$

$$U_{AC} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI}\,dx = \frac{P^2}{8EI}\int_0^{\frac{L}{2}} x^2\,dx$$

$$= \frac{P^2}{8EI}\,\frac{x^3}{3}\Big|_0^{\frac{L}{2}} = \frac{P^2L^3}{192\,EI}$$

By symmetry, $U_{CB} = U_{AC} = \frac{P^2L^3}{192\,EI}$

Total: $U = U_{AC} + U_{CB} = \frac{P^2L^3}{96\,EI}$

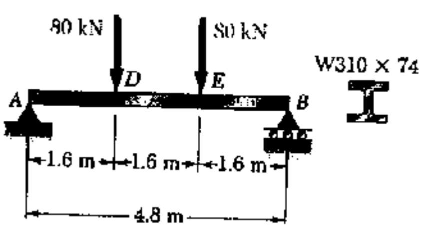
Data: $P = 180\times10^3$ N, $L = 4.8$ m, $E = 200\times10^9$ Pa

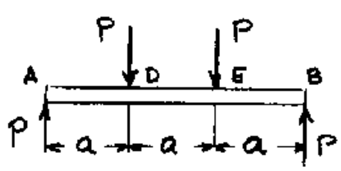
$I = 178\times10^6\ \text{mm}^4 = 178\times10^{-6}\ \text{m}^4$

$$U = \frac{(180\times10^3)^2(4.8)^3}{(96)(200\times10^9)(178\times10^{-6})} = 1048\ \text{N}\cdot\text{m} = 1048\ \text{J}$$

Problem 11.30

11.29 and 11.30 Using $E = 200$ GPa, determine the strain energy due to bending for the steel beam and loading shown.





Over portion AD: $\quad M = Px$

$$U_{AD} = \int_0^a \frac{M^2}{2EI}\,dx = \frac{1}{2EI}\int_0^a (Px)^2\,dx$$

$$= \frac{P^2}{2EI}\,\frac{x^3}{3}\Big|_0^a = \frac{P^2a^3}{6EI}$$

Over portion DE: $\quad M = Pa$

$$U_{DE} = \frac{(Pa)^2 a}{2EI} = \frac{P^2a^3}{2EI}$$

By symmetry, $U_{EB} = U_{AD} = \frac{P^2a^3}{6EI}$

$$U = U_{AD} + U_{DE} + U_{EB} = \frac{5}{6}\frac{P^2a^3}{EI}$$

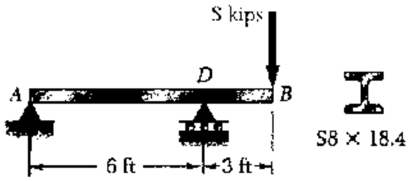
Data: $P = 80\times10^3$ N, $a = 1.6$ m, $E = 200\times10^9$ Pa

$I = 165\times10^6\ \text{mm}^4 = 165\times10^{-6}\ \text{m}^4$

$$U = \frac{5}{6}\,\frac{(80\times10^3)^2(1.6)^3}{(200\times10^9)(165\times10^{-6})} = 662\ \text{N}\cdot\text{m} = 662\ \text{J}$$

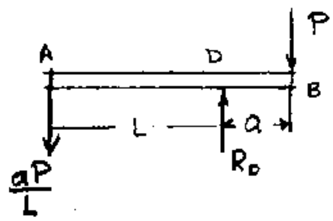
Problem 11.31

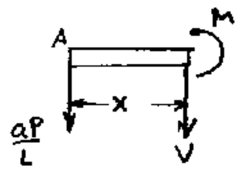
11.31 **and 11.32** Using $E = 29 \times 10^6$ psi, determine the strain energy due to bending for the steel beam and loading shown.



$\circlearrowleft \Sigma M_D = 0 \qquad -R_A L - aP = 0 \qquad R_A = \frac{aP}{L} \downarrow$

Over portion AD: $\quad M = -\frac{aP}{L}x$



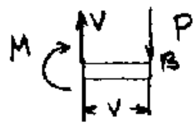


$$U_{AD} = \int_0^L \frac{M^2}{2EI}dx = \frac{1}{2EI}\int_0^L \left(\frac{aP}{L}x\right)^2 dx$$

$$= \frac{P^2a^2}{2EIL^2}\int_0^L x^2 dx$$

$$= \frac{P^2a^2L}{6EI}$$

Over portion DB: $\quad M = -Pv$



$$U_{DB} = \int_0^a \frac{M^2}{2EI}dv = \frac{1}{2EI}\int_0^a P^2x^2 dx = \frac{P^2a^3}{6EI}$$

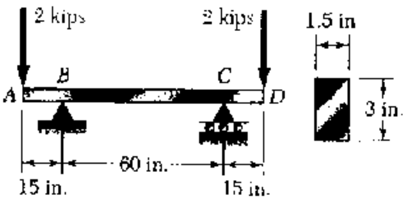
Total: $\quad U = U_{AD} + U_{DB} = \frac{P^2a^2}{6EI}(a + L)$

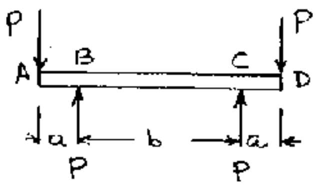
Data: $P = 8000$ lb., $L = 6$ ft. $= 72$ in, $a = 3$ ft $= 36$ in, $E = 29 \times 10^6$ psi
$I = 57.6$ in^4

$$U = \frac{(8000)^2(36)^2(72 + 36)}{(6)(29 \times 10^6)(57.6)} = 894 \text{ in}\cdot\text{lb.}$$

Problem 11.32

11.31 and 11.32 Using $E = 29 \times 10^6$ psi, determine the strain energy due to bending for the steel beam and loading shown.





Over A to B: $\quad M = -Px$

$$U_{AB} = \int_0^a \frac{M^2 dx}{2EI} = \frac{P^2}{2EI}\int_0^a x^2 dx = \frac{P^2 a^3}{6EI}$$

Over B to C: $\quad -M = Pa = \text{constant}$

$$U_{BC} = \frac{M^2 b}{2EI} = \frac{P^2 a^2 b}{2EI}$$

By symmetry, $\quad U_{CD} = U_{AB} = \dfrac{P^2 a^3}{6EI}$

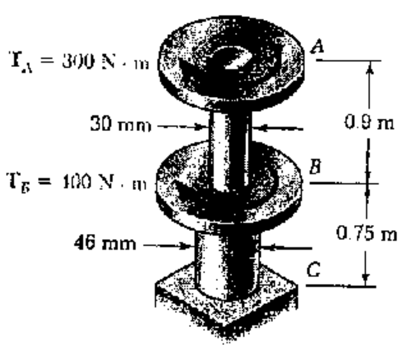
Total $\quad U = U_{AB} + U_{BC} + U_{CD} = \dfrac{P^2 a^2 (2a + 3b)}{6EI}$

Data: $P = 2 \times 10^3$ lb, $\quad a = 15$ in., $\quad b = 60$ in. $\quad I = \frac{1}{12}(1.5)(3)^3 = 3.375 \text{ in}^4$

$$U = \frac{(2\times10^3)^2(15)^2[(2)(15)+(3)(60)]}{(6)(29\times10^6)(3.375)} = 322 \text{ in}\cdot\text{lb}$$

Problem 11.33

11.33 In the assembly shown torques $\mathbf{T}_A$ and $\mathbf{T}_B$ are exerted on disks A and B respectively. Knowing that both shafts are solid and made of aluminum ($G = 73$ GPa), determine the total strain energy acquired by the assembly.



Over portion AB:

$T_{AB} = T_A = 300$ N·m

$J_{AB} = \frac{\pi}{2}c^4 = \frac{\pi}{2}\left(\frac{30}{2}\right)^4 = 79.52 \times 10^3 \text{ mm}^4 = 79.52 \times 10^{-9} \text{ m}^4$

$L_{AB} = 0.9$ m

$$U_{AB} = \frac{T_{AB}^2 L_{AB}}{2GJ_{AB}} = \frac{(300)^2(0.9)}{(2)(73\times10^9)(79.52\times10^{-9})} = 6.977 \text{ J}$$

Over portion BC: $\quad T_{BC} = T_A + T_B = 300 + 400 = 700$ N·m, $\quad L_{BC} = 0.75$ m

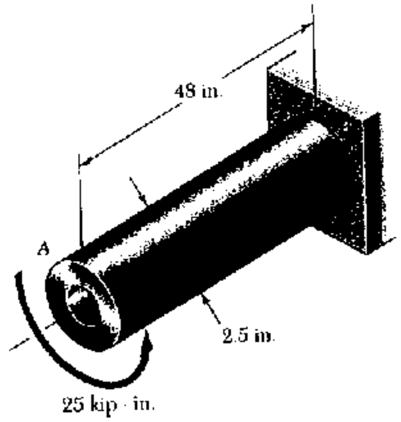
$J_{BC} = \frac{\pi}{2}\left(\frac{46}{2}\right)^4 = 439.57 \times 10^3 \text{ mm}^4 = 439.57 \times 10^{-9} \text{ m}^4$

$$U_{BC} = \frac{T_{BC}^2 L_{BC}}{2GJ_{BC}} = \frac{(700)^2(0.75)}{(2)(73\times10^9)(439.57\times10^{-9})} = 5.726 \text{ J}$$

Total, $\quad U = U_{AB} + U_{BC} = 6.977 + 5.726 = 12.70$ J

Problem 11.34

11.34 The design specifications for the steel shaft AB require that the shaft acquire a strain energy of 400 in · lb as the 25-kip · in. torque is applied. Using $G = 11.2 \times 10^6$ psi, determine (a) the largest inner diameter of the shaft that can be used, (b) the corresponding maximum shearing stress in the shaft.



$U = 400$ in·lb $\qquad T = 25$ kip·in $= 25\times10^3$ lb·in

$L = 48$ in

$$U = \frac{T^2 L}{2GJ}$$

$$J = \frac{T^2 L}{2GU} = \frac{(25\times10^3)^2 (48)}{(2)(11.2\times10^6)(400)} = 3.3482 \text{ in}^4$$

But $J = \frac{\pi}{2}\left[\left(\frac{d_o}{2}\right)^4 - \left(\frac{d_i}{2}\right)^4\right] = \frac{\pi}{32}(d_o^4 - d_i^4)$

(a) $d_i^4 = d_o^4 - \frac{32}{\pi}J = 2.5^4 - \frac{32}{\pi}(3.3482) = 4.95787 \text{ in}^4$

$d_i = 1.492$ in.

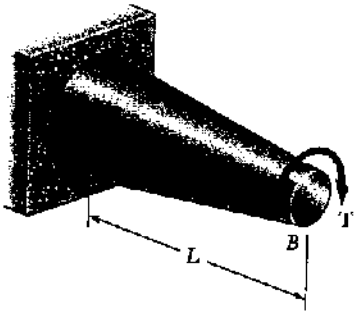
(b) $\tau = \frac{Tc_o}{J} = \frac{(25\times10^3)(1.25)}{2.5112} = 9.33\times10^3$ psi $= 9.33$ ksi

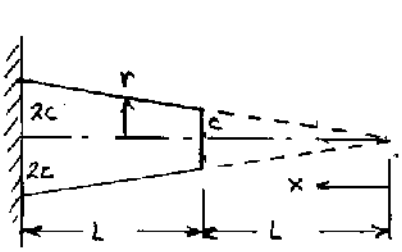
Problem 11.35

11.35 Show by integration that the strain energy in the tapered rod AB is

$$U = \frac{7}{48}\frac{T^2 L}{GJ_{min}}$$

where J_{min} is the polar moment of inertia of the rod at end B.





$$r = \frac{cx}{L}$$

$$J = \frac{\pi}{2}r^4 = \frac{\pi}{2}\frac{c^4}{L^4}x^4, \qquad J_{min} = \frac{\pi}{2}c^4$$

$$U = \int_L^{2L} \frac{T^2\,dx}{2GJ} = \int_L^{2L} \frac{T^2}{2G\left(\frac{\pi}{2}\frac{c^4}{L^4}x^4\right)}dx$$

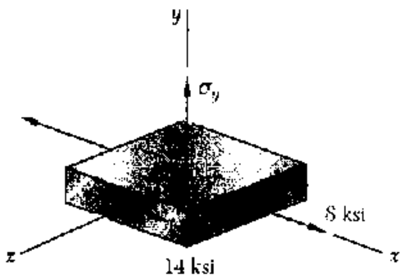
$$= \frac{T^2 L^4}{2GJ_{min}}\int_L^{2L}\frac{dx}{x^4}$$

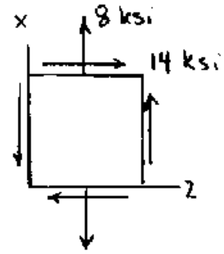
$$= \frac{T^2L^4}{2GJ_{min}}\left(-\frac{1}{3x^3}\right)\Bigg|_L^{2L}$$

$$U = \frac{T^2L^2}{2GJ_{min}}\left(-\frac{1}{3(2L)^3} + \frac{1}{3L^3}\right) = \frac{7}{48}\frac{T^2L}{GJ_{min}}$$

Problem 11.36

11.36 The state of stress shown occurs in a machine component made of a grade of steel for which $\sigma_Y = 65$ ksi. Using the maximum-distortion-energy criterion, determine the range of values of σ_y for which the factor of safety associated with the yield strength is equal to or larger then 2 2.





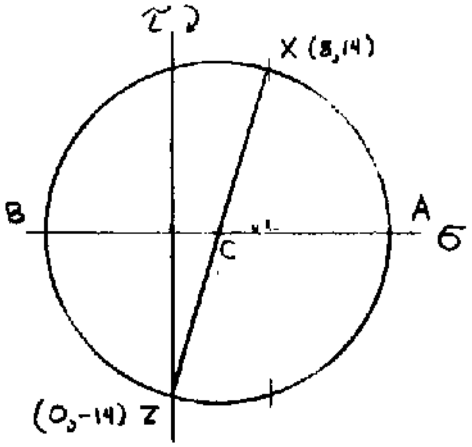
$\sigma_{ave} = \frac{1}{2}(0 + 8) = 4$ ksi

$\frac{\sigma_x - \sigma_z}{2} = \frac{8 - 0}{2} = 4$ ksi

$\tau_{xz} = 14$ ksi

$R = \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2}$

$= \sqrt{4^2 + 14^2} = 14.56$ ksi



$\sigma_a = \sigma_{ave} + R = 18.56$ ksi

$\sigma_b = \sigma_{ave} - R = -10.56$ ksi

$\sigma_c = \sigma_y$

$$(\sigma_a - \sigma_b)^2 + (\sigma_b - \sigma_c)^2 + (\sigma_c - \sigma_a)^2 = 2\left(\frac{\sigma_Y}{F.S.}\right)^2$$

$$(18.56 + 10.56)^2 + (-10.56 - \sigma_y)^2 + (\sigma_y - 18.56)^2 = 2\left(\frac{65}{2.2}\right)^2$$

$$847.97 + (111.51 + 21.12\sigma_y + \sigma_y^2) + (\sigma_y^2 - 37.12\sigma_y + 344.47 = 1745.87$$

$$2\sigma_y^2 - 16\sigma_y - 441.92 = 0$$

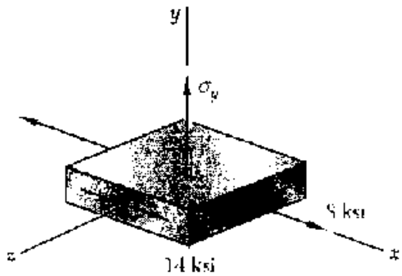
$$\sigma_y = \frac{16 \pm \sqrt{16^2 + (4)(2)(441.92)}}{(2)(2)} = 4 \pm 15.39$$

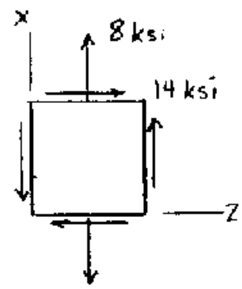
$$\sigma_y = 19.39 \text{ ksi}, \ -11.39 \text{ ksi}$$

$$-11.39 \text{ ksi} \leq \sigma_y \leq 19.39 \text{ ksi}$$

Problem 11.37

11.37 The state of stress shown occurs in a machine component made of a grade of steel for which $\sigma_Y = 65$ ksi. Using the maximum-distortion-energy criterion, determine the factor of safety associated with the yield strength when (*a*) $\sigma_y = +16$ ksi, (*b*) $\sigma_y = -16$ ksi.



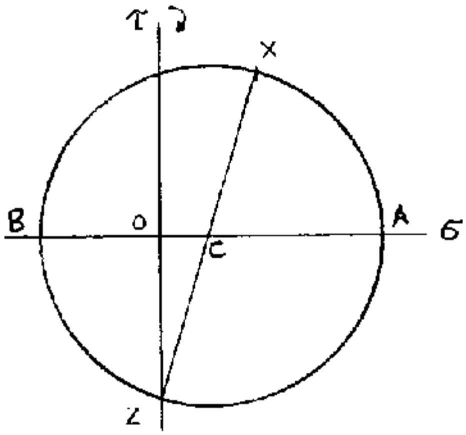


$$\sigma_{ave} = \tfrac{1}{2}(0+8) = 4 \text{ ksi}$$

$$\frac{\sigma_x - \sigma_z}{2} = \frac{8-0}{2} = 4 \text{ ksi}$$

$$\tau_{xz} = 14 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2} = \sqrt{4^2 + 14^2} = 14.56 \text{ ksi}$$



$$\sigma_a = \sigma_{ave} + R = 18.56$$

$$\sigma_b = \sigma_{ave} - R = -10.56$$

$$\sigma_c = \sigma_y$$

$$(\sigma_a - \sigma_b)^2 + (\sigma_b - \sigma_c)^2 + (\sigma_c - \sigma_a)^2 = 2\left(\frac{\sigma_Y}{F.S.}\right)^2$$

(a) $\sigma_c = \sigma_y = 16$ ksi

$$(18.56 + 10.56)^2 + (-10.56 - 16)^2 + (16 - 18.56)^2 = 2\left(\frac{65}{F.S.}\right)^2$$

$$847.97 + 705.43 + 6.55 = \frac{8450}{(F.S.)^2} \qquad F.S. = 2.33$$

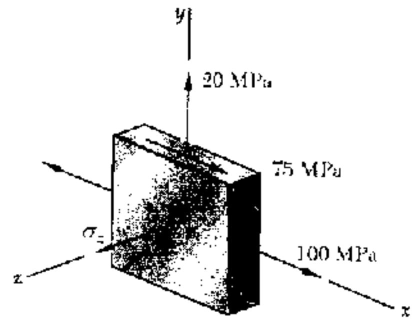
(b) $\sigma_c = \sigma_y = -16$ ksi

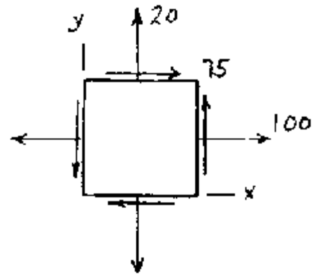
$$(18.56 + 10.56)^2 + (-10.56 + 16)^2 + (-16 - 18.56)^2 = 2\left(\frac{65}{F.S.}\right)^2$$

$$847.97 + 29.59 + 1194.39 = \frac{8450}{(F.S.)^2} \qquad F.S. = 2.02$$

Problem 11.38

11.38 The state of stress shown occurs in a machine component made of a brass for which $\sigma_Y = 160$ MPa. Using the maximum-distortion-energy criterion, determine whether yield occurs when (*a*) $\sigma_z = +45$ MPa, (*b*) $\sigma_z = -45$ MPa.





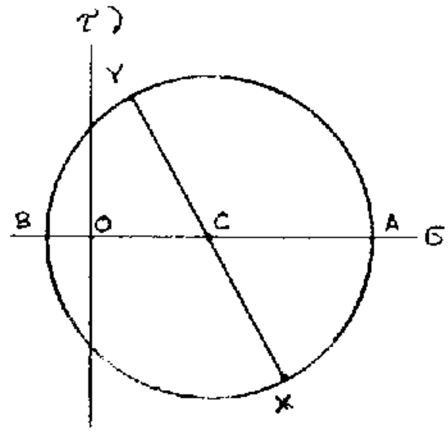
$$\sigma_{ave} = \tfrac{1}{2}(100 + 20) = 60 \text{ MPa}$$

$$\frac{\sigma_x - \sigma_y}{2} = \frac{100 - 20}{2} = 40 \text{ MPa}$$

$$\tau_{xy} = 75 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{40^2 + 75^2} = 85 \text{ MPa}$$



$$\sigma_a = \sigma_{ave} + R = 145 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = -25 \text{ MPa}$$

$$\sigma_c = \sigma_z$$

$$(\sigma_a - \sigma_b)^2 + (\sigma_b - \sigma_c)^2 + (\sigma_c - \sigma_a)^2 \overset{?}{\lessgtr} 2\sigma_Y^2$$

(a) $\sigma_c = \sigma_z = +45$ MPa

$$(145 + 25)^2 + (-25 - 45)^2 + (45 - 145)^2 \overset{?}{<} 2(160)^2 = 51200$$

$$28900 + 4900 + 10000 = 43800 < 51200$$ (No yield)

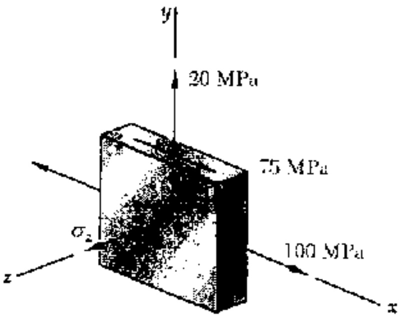
(b) $\sigma_c = \sigma_z = -45$ MPa

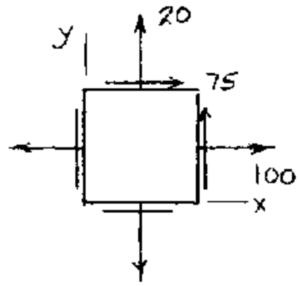
$$(145 + 25)^2 + (-25 + 45)^2 + (-45 - 145)^2 \overset{?}{<} 51200$$

$$28900 + 400 + 36100 = 65400 > 51200$$ (Yield occurs)

Problem 11.39

11.39 The state of stress shown occurs in a machine component made of a brass for which $\sigma_Y = 160$ MPa. Using the maximum-distortion-energy criterion, determine the range of values of σ_z for which yield does not occur.





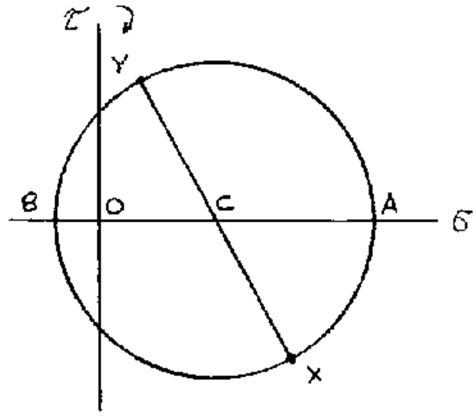
$$\sigma_{ave} = \frac{1}{2}(100 + 20) = 60 \text{ MPa}$$

$$\frac{\sigma_x - \sigma_y}{2} = \frac{100 - 20}{2} = 40 \text{ MPa}$$

$$\tau_{xy} = 75 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{40^2 + 75^2} = 85 \text{ MPa}$$



$$\sigma_a = \sigma_{ave} + R = 145 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = -25 \text{ MPa}$$

$$\sigma_c = \sigma_z$$

$$(\sigma_a - \sigma_b)^2 + (\sigma_b - \sigma_c)^2 + (\sigma_c - \sigma_a)^2 = 2\sigma_Y^2$$

$$(145 + 25)^2 + (-25 - \sigma_z)^2 + (\sigma_z - 145)^2 = (2)(160)^2$$

$$28900 + (625 + 50\sigma_z + \sigma_z^2) + (\sigma_z^2 - 290\sigma_z + 21025 = 51200$$

$$2\sigma_z^2 - 240\sigma_z - 650 = 0$$

$$\sigma_z = \frac{240 \pm \sqrt{240^2 + (4)(2)(650)}}{(2)(2)} = 60 \pm 62.65$$

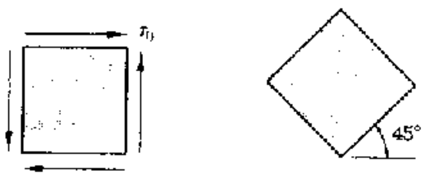
$$\sigma_z = 122.65 \text{ MPa}, \quad -2.65 \text{ MPa}$$

$$-2.65 \text{ MPa} < \sigma_z < 122.65 \text{ MPa}$$

Problem 11.40

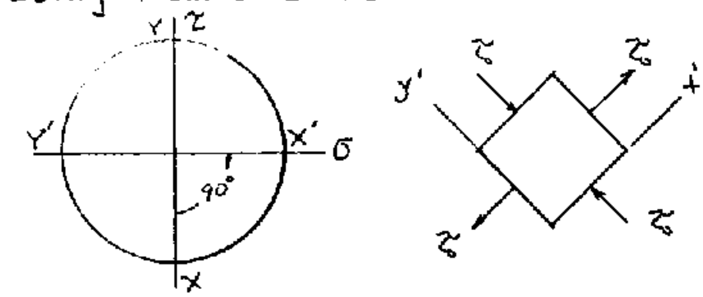
11.40 For the state of stress shown in Fig. *a*, determine the stresses in an element oriented as shown in Fig. *b*. Compare the strain energy density in the given state by first using Fig. *a* and then by using Fig. *b*. Equating the two results obtained, show that

$$G = \frac{E}{2(1+\nu)}$$



(a) (b)

Using Mohr's circle



(a) $\sigma_x = 0, \quad \sigma_y = 0, \quad \tau_{xy} = \tau_0$

$$U = \frac{1}{2E}\left(\sigma_x^2 + \sigma_y^2 - 2\nu\sigma_x\sigma_y\right) + \frac{1}{2G}\tau_{xy}^2 = \frac{\tau_0^2}{2G}$$

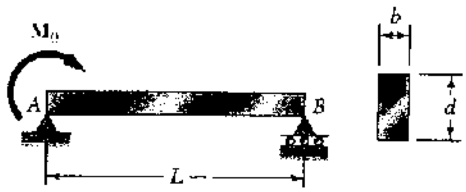
(b) $\sigma_{x'} = \tau_0, \quad \sigma_{y'} = -\tau_0, \quad \tau_{x'y'} = 0$

$$U = \frac{1}{2E}\left(\sigma_{x'}^2 + \sigma_{y'}^2 - 2\nu\sigma_{x'}\sigma_{y'}\right) + \frac{1}{2G}\tau_{x'y'}^2 = \frac{(2+2\nu)\tau_0^2}{2E}$$

Equate $\quad \dfrac{\tau_0^2}{2G} = \dfrac{(2+2\nu)\tau_0^2}{2E} \qquad G = \dfrac{E}{2(1+\nu)}$

Problem 11.41

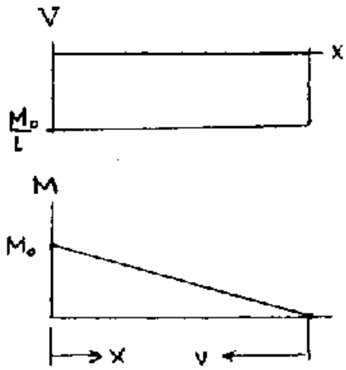
11.41 Determine the strain energy of the prismatic beam AB, taking into account the effect of both normal and shearing stresses.



Reactions $\quad R_A = \frac{M_o}{L}\downarrow \;,\; R_B = \frac{M_o}{L}\uparrow$

Shear: $\quad V = -\frac{M_o}{L}$

Bending moment: $\quad M = \frac{M_o}{L} v$



<u>For bending</u>

$$U_1 = \int_0^L \frac{M^2}{2EI}\,dv = \frac{M_o^2}{2EIL^2}\int_0^L v^2\,dv = \frac{M_o L^3}{6EIL^2}$$

$$= \frac{M_o^2 L}{6EI}$$

<u>For shear</u>

$$\tau_{xy} = \frac{3}{2}\frac{V}{A}\left(1 - \frac{y^2}{c^2}\right) \qquad c = \tfrac{1}{2}d$$

$$u = \frac{\tau_{xy}^2}{2G} = \frac{9V^2}{8GA^2}\left(1 - \frac{y^2}{c^2}\right)^2 = \frac{9M_o^2}{8G(bd)^2L^2}\left(1 - 2\frac{y^2}{c^2} + \frac{y^4}{c^4}\right)$$

$$U_2 = \int u\,d\mathcal{V} = \int_0^L\int_{-c}^{c} u\; b\,dy\,dx = \frac{9M_o^2 b}{8Gb^2d^2L^2}\int_0^L\int_{-c}^{c}\left(1 - 2\frac{y^2}{c^2} + \frac{y^4}{c^4}\right)dy\,dx$$

$$= \frac{9M_o^2}{8Gbd^2L^2}\int_0^L\left(y - \frac{2}{3}\frac{y^3}{c^2} + \frac{1}{5}\frac{y^5}{c^4}\right)\Bigg|_{-c}^{c} dx = \frac{9M_o^2}{8Gbd^2L^2}\int_0^L\left(2c - \frac{4}{3}c + \frac{2}{5}c\right)dx$$

$$\frac{9M_o^2}{8Gbd^2L^2}\left(\frac{16}{15}c\right)L = \frac{6}{5}\frac{M_o^2 c}{Gbd^2L} = \frac{3}{5}\frac{M_o^2}{GbdL}$$

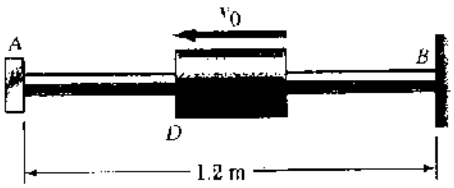
Total $\quad U = U_1 + U_2 = \dfrac{M_o^2 L}{6EI} + \dfrac{3}{5}\dfrac{M_o^2}{GbdL}$

With $\quad I = \frac{1}{12}bd^3$

$$U = \frac{2M_o^2L}{Ebd^3} + \frac{3}{5}\frac{M_o^2}{GbdL} = \frac{2M_o^2L}{Ebd^3}\left\{1 + \frac{3}{10}\cdot\frac{E}{G}\frac{d^2}{L^2}\right\} \blacktriangleleft$$

Problem 11.42

11.42 A 5-kg collar D moves along the uniform rod AB and has a speed $v_0 = 6$ m/s when it strikes a small plate attached to end A of the rod.. Using $E = 200$ GPa and knowing that the allowable stress in the rod is 250 MPa, determine the smallest diameter which can be used for the rod.



$$U_m = \tfrac{1}{2} m v_0^2 = \tfrac{1}{2}(5)(6)^2 = 90 \text{ J}$$

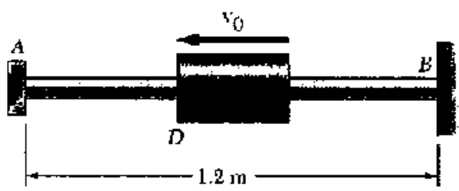
$$U_m = \frac{P_m^2 L}{2EA} = \frac{(A\sigma_{max})^2 L}{2EA}$$

$$A = \frac{2EU_m}{\sigma_{max}^2 L} = \frac{(2)(200\times10^9)(90)}{(250\times10^6)^2(1.2)} = 480\times10^{-6} \text{ m}^2$$

$$\frac{\pi}{4}d^2 = A \qquad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(480\times10^{-6})}{\pi}} = 24.7\times10^{-3} \text{ m} = 24.7 \text{ mm}$$

Problem 11.43

11.43 A 6-kg collar has a speed $v_0 = 4.5$ m/s when it strikes a small plate attached to end A of the 20-mm-diameter rod AB. Using $E = 200$ GPa, determine (*a*) the equivalent static load, (*b*) the maximum stress in the rod (*c*) the maximum deflection of the A.



$$U_m = \tfrac{1}{2} m v_0^2 = \tfrac{1}{2}(6)(4.5)^2 = 60.75 \text{ J}$$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(20\times10^{-3})^2 = 314.159\times10^{-6} \text{ m}^2$$

$$U_m = \frac{P^2 L}{2EA}$$

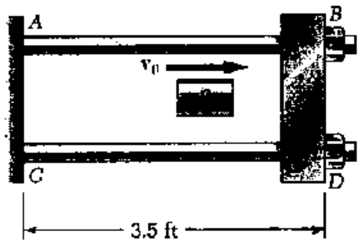
(a) $$P = \sqrt{\frac{2EAU_m}{L}} = \sqrt{\frac{(2)(200\times10^9)(314.159\times10^{-6})(60.75)}{1.2}}$$

$$= 79.76\times10^3 \text{ N} \qquad P = 79.8 \text{ kN}$$

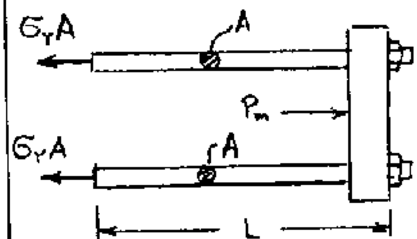
(b) $$\sigma_{max} = \frac{P}{A} = \frac{79.76\times10^3}{314.159\times10^{-6}} = 254\times10^6 \text{ Pa} \qquad \sigma_{max} = 254 \text{ MPa}$$

(c) $$\delta = \frac{PL}{EA} = \frac{(79.76\times10^3)(1.2)}{(200\times10^9)(314.159\times10^{-6})} = 1.523\times10^{-3} = 1.523 \text{ mm}$$

Problem 11.44



11.44 The cylindrical block E has a speed $v_0 = 16$ ft/s when it strikes squarely the yoke BD that is attached to the $\frac{7}{8}$-in.-diameter rods AB and CD. Knowing that the rods are made of a steel for which $\sigma_Y = 50$ ksi and $E = 29 \times 10^6$ psi, determine the weight of the block E for which the factor of safety is five with respect to permanent deformation of the rods.



At the onset of yielding the force in each rod is

$$F = \sigma_Y A$$

Corresponding strain energy.

$$U_{AB} = \frac{F_{AB}^2 L_{AB}}{2 E A_{AB}} = \frac{\sigma_Y^2 A^2 L}{2 E A} = \frac{\sigma_Y^2 A L}{2E}$$

$$U_{CD} = \text{same} = \frac{\sigma_Y^2 A L}{2E}$$

$$U_m = U_{AB} + U_{CD} = \frac{\sigma_Y^2 A L}{E}$$

$$U_m = \left(\tfrac{1}{2} m v_0^2\right)(F.S.) = \left(\tfrac{1}{2}\frac{W}{g} v_0^2\right)(F.S.)$$

Solving for W; $\quad W = \dfrac{2g U_m}{v_0^2 (F.S.)} = \dfrac{2g \sigma_Y^2 A L}{v_0^2 (F.S.) E}$

Data: $g = 32.17 \text{ ft/sec}^2 = 386 \text{ in/sec}^2$; $\sigma_Y = 50 \times 10^3$ psi;

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}\left(\frac{7}{8}\right)^2 = 0.60132 \text{ in}^2 \qquad E = 29 \times 10^6$ psi

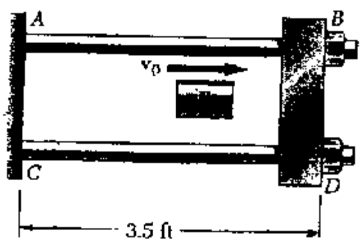
$L = 3.5 \text{ ft} = 42 \text{ in} \qquad F.S. = 5$

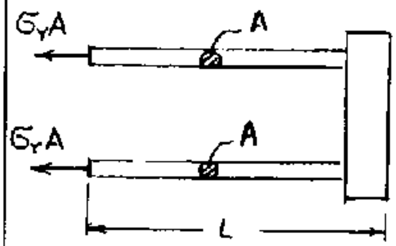
$v_0 = 16 \text{ ft/sec} = 192 \text{ in/sec}$

$$W = \frac{(2)(386)(50 \times 10^3)^2 (0.60132)(42)}{(192)^2 (5)(29 \times 10^6)} = 9.12 \text{ lb.}$$

Problem 11.45

11.45 The 18-lb cylindrical block E has a horizontal velocity v_0 when it strikes squarely the yoke BD that is attached to the $\frac{7}{8}$-in.-diameter rods AB and CD. Knowing that the rods are made of a steel for which $\sigma_Y = 50$ ksi and $E = 29 \times 10^6$ psi, determine the maximum allowable speed v_0 if the rods are not to be permanently deformed.





At the onset of yielding the force in each rod is

$$F = \sigma_Y A$$

Corresponding strain energy.

$$U_{AB} = \frac{F_{AB}^2 L_{AB}}{2EA_{AB}} = \frac{\sigma_Y^2 A^2 L}{2EA} = \frac{\sigma_Y^2 A L}{2E}$$

$$U_{CD} = \text{same} = \frac{\sigma_Y^2 A L}{2E}$$

$$\text{Total} \quad U_m = U_{AB} + U_{CD} = \frac{\sigma_Y^2 A L}{E}$$

$$U_m = \frac{1}{2} m v_0^2 = \frac{1}{2}\frac{W}{g} v_0^2$$

Solving for v_0^2, $\quad v_0^2 = \dfrac{2gU_m}{W} = \dfrac{2g\sigma_Y^2 AL}{EW}$

$$v_0 = \sqrt{\frac{2g\sigma_Y^2 AL}{EW}}$$

Data: $g = 32.17 \text{ ft/sec}^2 = 386 \text{ in/sec}^2$, $\quad \sigma_Y = 50 \times 10^3$ psi

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}\left(\frac{7}{8}\right)^2 = 0.60132 \text{ in}^2$, $\quad E = 29 \times 10^6$ psi

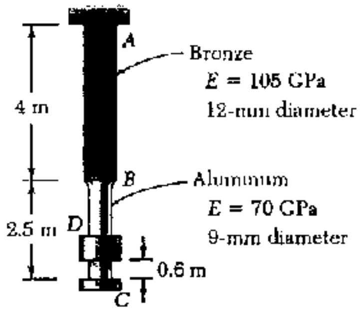
$L = 3.5 \text{ ft} = 42 \text{ in}$ $\quad W = 18$ lb.

$$v_0 = \sqrt{\frac{(2)(386)(50\times10^3)^2(0.60132)(42)}{(29\times10^6)(18)}} = 305.6 \text{ in/sec}$$

$$= 25.5 \text{ ft./sec} \quad \blacktriangleleft$$

Problem 11.46

11.46 Collar D is released from rest in the position shown and is stopped by a small plate attached at end C of the vertical rod ABC. Determine the mass of the collar for which the maximum normal stress in portion BC is 125 MPa.



Portion BC: $\sigma_m = 125 \times 10^6$ Pa

$A_{BC} = \frac{\pi}{4}(9)^2 = 63.617 \text{ mm}^2 = 63.617 \times 10^{-6} \text{ m}^2$

$P_m = \sigma_m A_{BC} = 7952$ N

Corresponding strain energy.

$$U_{BC} = \frac{P_m^2 L_{BC}}{2E_{BC}A_{BC}} = \frac{(7952)^2(2.5)}{(2)(70\times10^9)(63.617\times10^{-6})} = 17.750 \text{ J}$$

$$A_{AB} = \frac{\pi}{4}(12)^2 = 113.907 \text{ mm}^2 = 113.907\times10^{-6} \text{ m}^2$$

$$U_{AB} = \frac{P_m^2 L_{AB}}{2E_{AB}A_{AB}} = \frac{(7952)^2(4)}{(2)(105\times10^9)(113.907\times10^{-6})} = 10.574 \text{ J}$$

$$U_m = U_{BC} + U_{AB} = 28.324 \text{ J}$$

Corresponding elongation Δ_m. $\quad \frac{1}{2}P_m\Delta_m = U_m$

$$\Delta_m = \frac{2U_m}{P_m} = \frac{(2)(28.324)}{7952} = 7.12\times10^{-3} \text{ m}$$

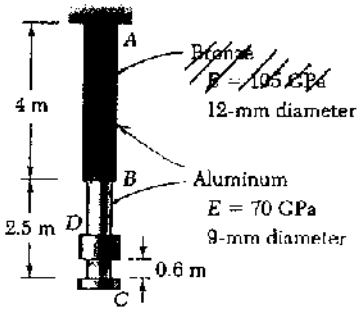
Falling distance $\quad h = 0.6 + 7.12\times10^{-3} = 0.60712$ m

Work of weight $= U_m \qquad Wh = mgh = U_m$

$$m = \frac{U_m}{gh} = \frac{28.324}{(9.81)(0.60712)} = 4.76 \text{ kg}$$

Problem 11.47

11.47 Solve Prob. 11.46, assuming that both portions of rod *ABC* are made of aluminum.



11.46 Collar *D* is released from rest in the position shown and is stopped by a small plate attached at end *C* of the vertical rod *ABC*. Determine the mass of the collar for which the maximum normal stress in portion *BC* is 125 MPa.

Portion BC: $\sigma_m = 125 \times 10^6$ Pa

$A_{BC} = \frac{\pi}{4}(9)^2 = 63.617 \text{ mm}^2 = 63.617 \times 10^{-6} \text{ m}^2$

$P_m = \sigma_m A_{BC} = 7952 \text{ N}$

Corresponding strain energy.

$$U_{BC} = \frac{P_m^2 L_{BC}}{2 E A_{BC}} = \frac{(7952)^2(2.5)}{(2)(70 \times 10^9)(63.617 \times 10^{-6})} = 17.750 \text{ J}$$

$$A_{AB} = \frac{\pi}{4}(12)^2 = 113.907 \text{ mm}^2 = 113.907 \times 10^{-6} \text{ m}^2$$

$$U_{AB} = \frac{P_m^2 L_{AB}}{2 E A_{AB}} = \frac{(7952)^2(4)}{(2)(70 \times 10^9)(113.907 \times 10^{-6})} = 15.861 \text{ J}$$

Total $\quad U_m = U_{BC} + U_{AB} = 33.611 \text{ J}$

Corresponding elongation Δ_m. $\quad \frac{1}{2} P_m \Delta_m = U_m$

$$\Delta_m = \frac{2U_m}{P_m} = \frac{(2)(33.611)}{7952} = 8.45 \times 10^{-3} \text{ m}$$

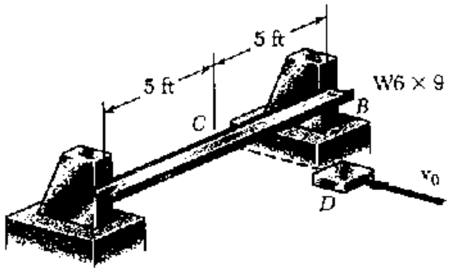
Falling distance. $\quad h = 0.6 + \Delta_m = 0.60845 \text{ m}$

Work of weight $= U_m$. $\quad Wh = mgh = U_m$

$$m = \frac{U_m}{gh} = \frac{33.611}{(9.81)(0.60845)} = 5.63 \text{ kg}$$

Problem 11.48

11.48 The steel beam AB is struck squarely at its midpoint C by a 100-lb block moving horizontally with a speed $v_0 = 7$ ft/s. Using $E = 29 \times 10^6$ psi, determine (a) the equivalent static load, (b) the maximum normal stress in the beam, (c) the maximum deflection of the midpoint C of the beam.



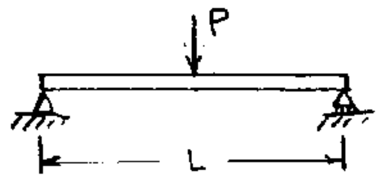
From Appendix C for W6×9

$I_x = 16.4 \text{ in}^4 \qquad S_x = 5.56 \text{ in}^3$

For moving block, $m = \frac{W}{g}$

Kinetic energy: $T = \frac{1}{2} m v_0^2 = \frac{1}{2}\frac{W}{g} v_0^2$

$$T = \frac{1}{2}\frac{(100 \text{ lb})(7 \text{ ft/s})^2}{32.2 \text{ ft/sec}^2} = 76.087 \text{ ft}\cdot\text{lb} = 913.04 \text{ in}\cdot\text{lb}.$$



From Appendix D, Case 4

$$|y_m| = \frac{PL^3}{48EI} \qquad M_{max} = \frac{PL}{4}$$

$$U_m = \frac{1}{2}P_m|y_m| = \frac{P_m^2 L^3}{96EI} = T$$

$L = 5 \text{ ft} + 5 \text{ ft} = 10 \text{ ft} = 120 \text{ in.}$

(a) $$P_m = \sqrt{\frac{96EIT}{L^3}} = \sqrt{\frac{(96)(29\times10^6)(16.4)(913.04)}{(120)^3}} = 4.9117\times10^3 \text{ lb}$$

$$= 4.91 \text{ kips}$$

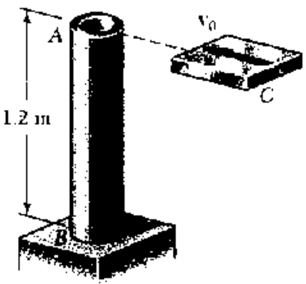
(b) $$M_{max} = \frac{(4.9117\times10^3)(120)}{4} = 147.35\times10^3 \text{ lb}\cdot\text{in}$$

$$\sigma_{max} = \frac{M_{max}}{S_x} = \frac{147.35\times10^3}{5.56} = 26.5\times10^3 \text{ psi} = 26.5 \text{ ksi}$$

(c) $$y_m = \frac{2U_m}{P_m} = \frac{(2)(913.04)}{4.9117\times10^3} = 0.372 \text{ in.}$$

Problem 11.49

11.49 The post AB consists of a steel pipe of 80-mm-diameter and 6-mm wall thickness. A 6-kg block C moving horizontally with at velocity $\mathbf{v}_0$ hits the post squarely at A. Using E = 200 GPa, determine the largest speed v_0 for which the maximum normal stress in the pipe does not exceed 180 MPa.

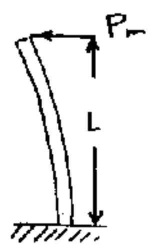


$$c_o = \tfrac{1}{2}d_o = \tfrac{1}{2}(80) = 40 \text{ mm} \qquad c_i = c_o - t = 20 - 6 = 34 \text{ mm}$$

$$I = \tfrac{\pi}{4}(c_o^4 - c_i^4) = \tfrac{\pi}{4}(40^4 - 34^4) = 961.06 \times 10^3 \text{ mm}^4$$

$$= 961.06 \times 10^{-9} \text{ m}^4$$

$$\sigma_m = 180 \times 10^6 \text{ Pa} = \frac{M_m c}{I}$$



$$M_m = \frac{I\sigma_m}{c} = \frac{(961.04 \times 10^{-9})(180 \times 10^6)}{40 \times 10^{-3}} = 4.325 \times 10^3 \text{ N}\cdot\text{m}$$

$$P_m = \frac{M_m}{L} = \frac{4.325 \times 10^3}{1.2} = 3.604 \times 10^3 \text{ N}$$

By Appendix D, Case 1

$$y_m = \frac{P_m L^3}{3EI} = \frac{(3.604 \times 10^3)(1.2)^3}{(3)(200 \times 10^9)(961.06 \times 10^{-9})} = 10.8 \times 10^{-3} \text{ m}$$

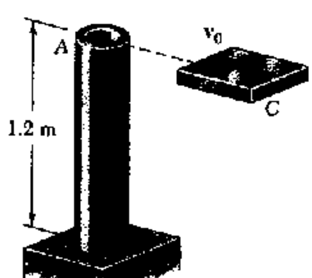
$$U_m = \tfrac{1}{2}P_m y_m = \tfrac{1}{2}(3.604 \times 10^3)(10.8 \times 10^{-3}) = 19.4616 \text{ J}$$

$$\tfrac{1}{2}mv_o^2 = U_m \qquad v_o = \sqrt{\frac{2U_m}{m}} = \sqrt{\frac{(2)(19.4616)}{6}} = 2.55 \text{ m/s}$$

Problem 11.50

11.50 Solve Prob. 11.49, assuming that the post AB consists of a solid steel rod of 80-mm diameter.

11.49 The post AB consists of a steel pipe of 80-mm-diameter and 6-mm wall thickness. A 6-kg block C moving horizontally with at velocity $\mathbf{v}_0$ hits the post squarely at A. Using E = 200 GPa, determine the largest speed v_0 for which the maximum normal stress in the pipe does not exceed 180 MPa.



$$c = \tfrac{1}{2}d = \tfrac{1}{2}(80) = 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$$

$$I = \tfrac{\pi}{4}c^4 = \tfrac{\pi}{4}(40 \times 10^{-3})^4 = 2.01062 \times 10^{-6} \text{ m}^4$$

$$\sigma_m = 180 \times 10^6 \text{ Pa} = \frac{M_m c}{I}$$

$$M_m = \frac{I\sigma_m}{c} = \frac{(2.01062 \times 10^{-6})(180 \times 10^6)}{40 \times 10^{-3}} = 9.048 \times 10^3 \text{ N}\cdot\text{m}$$

$$P_m = \frac{M_m}{L} = \frac{9.048 \times 10^3}{1.2} = 7.5398 \times 10^3 \text{ N}$$

By Appendix D, Case 1 $\quad y_m = \dfrac{P_m L^3}{3EI}$

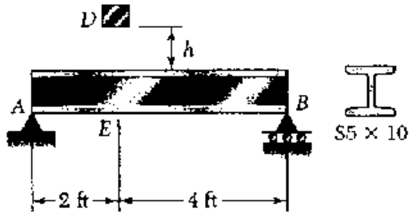
$$y_m = \frac{(7.5398 \times 10^3)(1.2)^3}{(3)(200 \times 10^9)(2.01062 \times 10^{-6})} = 10.8 \times 10^{-3} \text{ m}$$

$$U_m = \tfrac{1}{2}P_m y_m = \tfrac{1}{2}(7.5398 \times 10^{-3})(10.8 \times 10^{-3}) = 40.715 \text{ J}$$

$$\tfrac{1}{2}mv_o^2 = U_m \qquad v_o = \sqrt{\frac{2U_m}{m}} = \sqrt{\frac{(2)(40.715)}{6}} = 3.68 \text{ m/s}$$

Problem 11.51

11.51 The 45-lb block D is dropped from a height $h = 0.6$ ft onto the steel beam AB. Knowing that $E = 29\times 10^6$ psi, determine (*a*) the maximum deflection at point E, (*b*) the maximum normal stress in the beam.

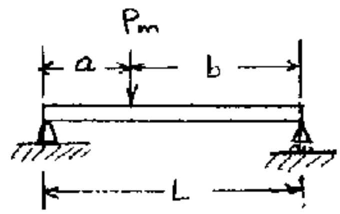


$I_x = 12.3 \text{ in}^4 \qquad S_x = 4.92 \text{ in}^4$

$a = 2\text{ft} = 24 \text{ in.} \qquad b = 4 \text{ ft} = 48 \text{ in}$

$L = 6 \text{ ft} = 72 \text{ in.} \qquad h = 0.6 \text{ ft} = 7.2 \text{ in.}$

$R_A = \frac{P_m b}{L}\uparrow, \quad R_B = \frac{P_m a}{L}\uparrow, \quad M_m = M_E = \frac{P_m ab}{L}$



From Appendix D Case 5

$$y_E = \frac{P_m a^2 b^2}{3EIL} = \frac{P_m (24)^2(48)^2}{(3)(29\times10^6)(12.3)(72)} = 17.2245\times10^{-6}\, P_m$$

$P_m = 58057\, y_E$

$U_m = \frac{1}{2} P_m y_m = 29028\, y_E^2$

Work of falling weight: $\quad W(h + y_E) = 45(7.2 + y_E) = 324 + 45 y_E$

Equating work and energy, $\quad 324 + 45 y_E = 29028\, y_E^2$

$$y_E^2 - 1.5502\times10^{-3}\, y_E - 11.1615\times10^{-3} = 0$$

(a) $y_E = \frac{1}{2}\left[1.5502\times10^{-3} + \sqrt{(1.5502\times10^{-3})^2 - (4)(-11.1615\times10^{-3})}\right] = 0.1064 \text{ in.}$ ◄

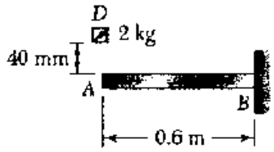
(b) $P_m = (58057)(0.1064) = 6.179\times10^3 \text{ lb.}$

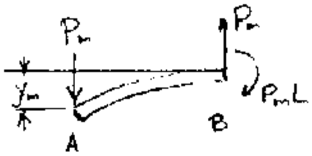
$$M_m = \frac{(6.179\times10^3)(24)(48)}{72} = 98.86\times10^3 \text{ lb}\cdot\text{in}$$

$$\sigma_m = \frac{M_m}{S_x} = \frac{98.86\times10^3}{4.92} = 20.1\times10^3 \text{ psi} = 20.1 \text{ ksi}$$ ◄

Problem 11.52

11.52 and 11.53 The 2-kg block D is dropped from the position shown onto the end of a 16-mm-diameter rod. Knowing that E = 200 GPa, determine (a) the maximum deflection of end A, (b) the maximum bending moment in the rod, (c) the maximum normal stress in the rod.





$$I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{4}\left(\frac{16}{2}\right)^4 = 3.2170\times10^3\ mm^4 = 3.2170\times10^{-9}\ m^4$$

$c = \frac{d}{2} = 8\ mm = 8\times10^{-3}\ m \qquad L_{AB} = 0.6\ m$

Appendix D, Case 1

$$y_m = \frac{P_m L_{AB}^3}{3EI} \qquad M_m = P_m L_{AB}$$

$$P_m = \frac{3EI}{L_{AB}^3} y_m = \frac{(3)(200\times10^9)(3.217\times10^{-9})}{(0.6)^3} = 8.9361\times10^3\ y_m$$

$$U_m = \frac{1}{2}P_m y_m = \frac{1}{2}(8.9361\times10^3)\,y_m^2 = 4.4681\times10^3\,y_m^2$$

Work of dropped weight: $mg(h+y_m) = (2)(9.81)(0.040+y_m)$
$= 0.7848 + 19.62\,y_m$

Equating work and energy,

$$0.7848 + 19.62\,y_m = 4.4681\times10^3\,y_m^2$$

$$y_m^2 - 4.3911\times10^{-3}\,y_m - 175.645\times10^{-6} = 0$$

(a) $$y_m = \frac{1}{2}\left\{4.3911\times10^{-3} + \sqrt{(4.3911\times10^{-3})^2 + (4)(175.645\times10^{-6})}\right\}$$
$$= 15.629\times10^{-3}\ m = 15.63\ mm$$ ◄

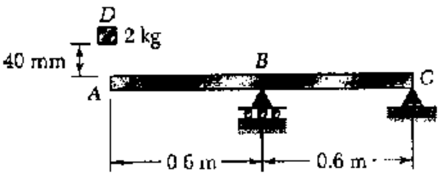
$$P_m = (8.9361\times10^3)(15.629\times10^{-3}) = 139.66\ N$$

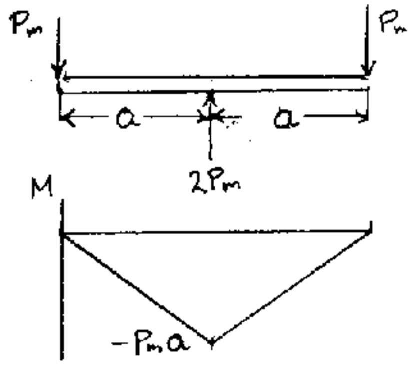
(b) $M_m = -P_m L_{AB} = -(139.66)(0.6) = -83.8\ N\cdot m$ ◄

(c) $\sigma_m = \frac{|M_m|c}{I} = \frac{(83.8)(8\times10^{-3})}{3.2170\times10^{-9}} = 208\times10^6\ Pa = 208\ MPa$ ◄

Problem 11.53

11.52 and 11.53 The 2-kg block D is dropped from the position shown onto the end of a 16-mm-diameter rod. Knowing that $E = 200$ GPa, determine (*a*) the maximum deflection of end A, (*b*) the maximum bending moment in the rod, (*c*) the maximum normal stress in the rod.





$I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{4}\left(\frac{16}{2}\right)^4 = 3.2170 \times 10^3\ mm^4 = 3.2170 \times 10^{-9}\ m^4$

$c = \frac{d}{2} = 8\ mm = 8 \times 10^{-3}\ m \qquad a = 0.6\ m.$

Over AB; $\quad M = -P_m x \qquad M_m = -P_m a$

$$U_{AB} = \int_0^a \frac{P_m^2 x^2}{2EI}dx = \frac{P_m^2 a^3}{6EI}$$

$$= \frac{(0.6)^3}{(6)(200 \times 10^9)(3.2170 \times 10^{-9})} P_m^2$$

$$= 55.953 \times 10^{-6}\ P_m^2$$

By symmetry of bending moment diagram,

$U_{BC} = U_{AB} = 55.953 \times 10^{-6}\ P_m^2$

$U_m = U_{AB} + U_{BC} = 111.906 \times 10^{-6}\ P_m^2$

$\frac{1}{2} P_m y_m = U_m = 111.906 \times 10^{-6}\ P_m^2 \qquad P_m = 4.4681 \times 10^3\ y_m$

$U_m = \frac{1}{2} P_m y_m = 2.2340 \times 10^3\ y_m^2$

Work of dropped weight: $\quad mg(h + y_m) = (2)(9.81)(0.040 + y_m)$

$= 0.7848 + 19.62\ y_m$

Equating work and energy,

$0.7848 + 19.62\ y_m = 2.2340 \times 10^3\ y_m^2$

$y_m^2 - 8.7825 \times 10^{-3}\ y_m - 351.298 \times 10^{-6} = 0$

(a) $\quad y_m = \frac{1}{2}\left\{ 8.7825 \times 10^{-3} + \sqrt{(8.7282 \times 10^{-3})^2 + (4)(351.298 \times 10^{-6})} \right\}$

$= 23.636 \times 10^{-3}\ m = 23.6\ mm$ ◄

$P_m = (4.4681 \times 10^3)(23.636 \times 10^{-6}) = 105.61\ N$

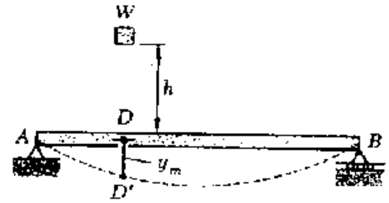
(b) $\quad M_m = -(105.61)(0.6) = -64.4\ N \cdot m$ ◄

(c) $\quad \sigma_m = \frac{|M_m| c}{I} = \frac{(64.4)(8 \times 10^{-3})}{3.2170 \times 10^{-9}} = 157.6 \times 10^6\ Pa$

$= 157.6\ MPa$ ◄

Problem 11.54

11.54 A block of weight W is placed in contact with a beam at some given point D and released. Show that the resulting maximum deflection at point D is twice as large as the deflection due to a static load W applied at D.



Consider dropping the weight from a height h above the beam. The work done by the weight is

$$\text{Work} = W(h + y_m)$$

Strain energy: $U = \frac{1}{2} P_m y_m = \frac{1}{2} k y_m^2$

where k is the spring constant of the beam for loading at point D.

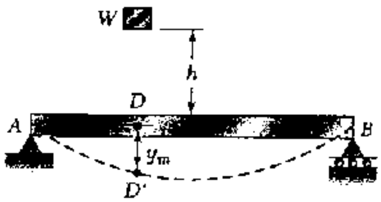
Equating work and energy, $W(h + y_m) = \frac{1}{2} k y_m^2$

Setting $h = 0$, $\quad W y_m = \frac{1}{2} k y_m^2, \quad y_m = \frac{2W}{k}$

The static deflection at point D due to weight applied at D is

$\delta_{st} = \frac{W}{k}$. $\quad$ Thus, $\quad y_m = 2\,\delta_{st}$

Problem 11.55

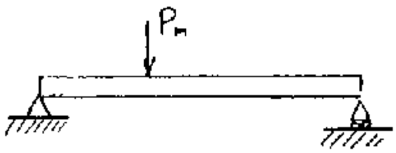


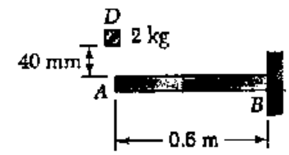
11.55 A block of weight W is dropped from a height h onto the horizontal beam AB and hits it at point D. (*a*) Show that the maximum deflection y_m at point D can be expressed as

$$y_m = y_{st}\left(1+\sqrt{1+\frac{2h}{y_{st}}}\right)$$

where y_{st} represents the deflection at D caused by a static load W applied at that point and where the quantity in parenthesis is referred to at the *impact factor*. (*b*) Compute the impact factor for the beam and the impact of Prob. 11.52.

11.52 and 11.53 The 2-kg block D is dropped from the position shown onto the end of a 16-mm-diameter rod. Knowing that E = 200 GPa, determine (*a*) the maximum deflection of end A, (*b*) the maximum bending moment in the rod, (*c*) the maximum normal stress in the rod.





Work of falling weight: $\quad$ Work $= W(h + y_m)$

Strain energy: $\quad U = \frac{1}{2} P y_m = \frac{1}{2} k y_m^2$

where k is the spring constant for a load applied at point D.

Equating work and energy,

$$W(h + y_m) = \frac{1}{2} k y_m^2$$

$$y_m^2 - \frac{2W}{k} y_m - \frac{2W}{k} h = 0$$

$$y_m^2 - 2 y_{st} y_m - 2 y_{st} h = 0 \qquad \text{where} \quad y_{st} = \frac{W}{k}$$

(a) $$y_m = \frac{2 y_{st} + \sqrt{4 y_{st}^2 + 8 y_{st} h}}{2} = y_{st}\left(1 + \sqrt{1 + \frac{2h}{y_{st}}}\right)$$

For Prob. 11.52 $\qquad W = mg = (2)(9.81) = 19.62 \text{ N}$

$E = 200 \times 10^9$ Pa $\qquad I = \frac{\pi}{4}\left(\frac{16}{2}\right)^4 = 3.217 \times 10^3 \text{ mm}^4 = 3.217 \times 10^{-9} \text{ m}^4$

$L = 0.6$ m $\qquad h = 40 \text{ mm} = 40 \times 10^{-3}$ m

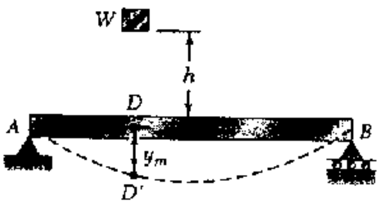
Using Appendix D Case 1 $\qquad y_{st} = \frac{WL^3}{3EI}$

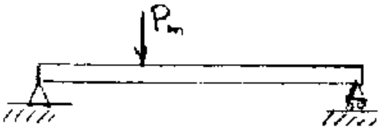
$$y_{st} = \frac{(19.62)(0.6)^3}{(3)(200 \times 10^9)(3.217 \times 10^{-9})} = 2.196 \times 10^{-3} \text{ m}$$

$$\frac{2h}{y_{st}} = \frac{(2)(40 \times 10^{-3})}{2.196 \times 10^{-3}} = 36.44$$

(b) impact factor $= 1 + \sqrt{1 + 36.44} = 7.12$

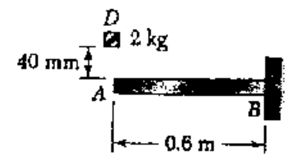
Problem 11.56





11.56 A block of weight W is dropped from a height h onto the horizontal beam AB and hits it at point D. (*a*) Denoting by y_m the exact value of the maximum deflection at D and by y'_m the value obtained by neglecting the effect of this deflection on the change in potential energy of the block, show that the absolute value of the relative error is $(y'_m - y_m)/y_m$ never exceeds $y'_m/2h$. (*b*) Check the result obtained in part *a* by solving part *A* of Prob. 11.52 without taking y_m into account when determining the change in potential energy of the load, and comparing the answer obtained in this way with the exact answer to that problem.

11.52 and 11.53 The 2-kg block D is dropped from the position shown onto the end of a 16-mm-diameter rod. Knowing that E = 200 GPa, determine (*a*) the maximum deflection of end A, (*b*) the maximum bending moment in the rod, (*c*) the maximum normal stress in the rod.



$U = \frac{1}{2}P_m y_m = \frac{1}{2}k y_m^2$ where k is the spring constant for a load at point D.

Work of falling weight: exact: $\text{Work} = W(h+y_m)$
approximate: $\text{Work} \approx Wh$

Equating work and energy, $\frac{1}{2}k y_m^2 = W(h+y_m)$ (1) exact
$\frac{1}{2}k y_m'^2 = W h$ (2) approximate

where y_m' is the approximate value for y_m.

Subtracting $\frac{1}{2}k(y_m^2 - y_m'^2) = W y_m$

$$y_m^2 - \tilde{y}_m^2 = (y_m - y_m')(y_m + y_m') = \frac{2W}{k} y_m$$

Relative error: $\frac{y_m - y_m'}{y_m} = \frac{2W}{k(y_m + \tilde{y}_m)}$

But, $\frac{2W}{k} = \frac{y_m'^2}{h}$ from equation (2).

(a) Relative error $= \frac{y_m - y_m'}{y_n} = \frac{y_m'^2}{h(y_m + y_m')} < \frac{y_m'}{2h}$

(b) From the solution to Prob. 11.62 $y_m = 15.63$ mm

Approximate solution: $W = mg = (2)(9.81) = 19.62$ N

$E = 200 \times 10^9$ Pa $\quad I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{4}\left(\frac{16}{2}\right)^4 = 3.217 \times 10^3 \text{ mm}^4 = 3.217 \times 10^{-9} \text{ m}^4$

$L = 0.6$ m, $\quad h = 40$ mm $= 40 \times 10^{-3}$ m

$$k = \frac{3EI}{L^3} = \frac{(3)(200\times10^9)(3.217\times10^{-9})}{(0.6)^3} = 8.936 \times 10^3 \text{ N/m}$$

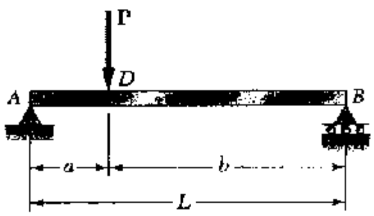
$$y_m'^2 = \frac{2Wh}{k} = \frac{(2)(19.62)(40\times10^{-3})}{8.936\times10^3} = 175.65 \times 10^{-6} \text{ m}^2$$

$y_m' = 13.25 \times 10^{-3}$ m $= 13.25$ mm

relative error $= \frac{15.63 - 13.25}{15.63} = 0.152$ ◄ $\quad \frac{y_m'}{2h} = 0.166$ ◄

Problem 11.57

11.57 and 11.58 Using the method of work and energy, determine the deflection at point D caused by the load **P**.

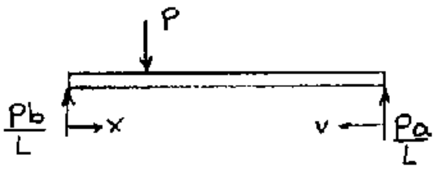


Reactions: $R_A = \frac{Pb}{L}$, $R_B = \frac{Pa}{L}$

Over AD: $M = R_A x = \frac{Pbx}{L}$

$$U_{AD} = \int_0^a \frac{M^2}{2EI}dx = \frac{P^2b^2}{2EIL^2}\int_0^a x^2\,dx$$

$$= \frac{P^2b^2a^3}{6EIL^2}$$



Over DB: $M = R_B v = \frac{Pav}{L}$

$$U_{DB} = \int_0^b \frac{M^2}{2EI}dv = \frac{P^2a^2}{2EIL^2}\int_0^b v^2\,dv$$

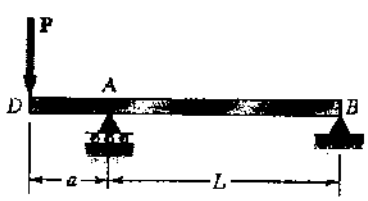
$$= \frac{P^2a^2b^3}{6EIL^2}$$

Total $U = U_{AB} + U_{BC} = \frac{P^2a^2b^2(a+b)}{6EIL^2} = \frac{P^2a^2b^2}{6EIL}$

$\frac{1}{2}P\delta_D = U \qquad \delta_D = \frac{2U}{P} = \frac{Pa^2b^2}{3EIL}\ \downarrow$

Problem 11.58

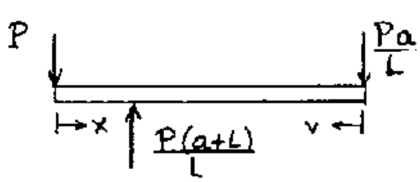
11.57 and 11.58 Using the method of work and energy, determine the deflection at point D caused by the load **P**.



$\circlearrowleft \Sigma M_A = 0 \qquad Pa + R_B L = 0 \qquad R_B = -\frac{Pa}{L}$

Over portion DA: $M = -Px$

$$U_{DA} = \int_0^a \frac{M^2}{2EI}dx = \frac{P^2}{2EI}\int_0^a x^2\,dx = \frac{P^2a^3}{6EI}$$



Over portion AB: $M = -\frac{Pav}{L}$

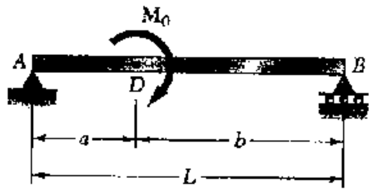
$$U_{AB} = \int_0^L \frac{M^2}{2EI}dv = \frac{P^2a^2}{2EIL^2}\int_0^L v^2\,dv = \frac{Pa^2L}{6EI}$$

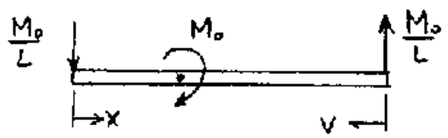
Total $U = U_{DA} + U_{AB} = \frac{P^2a^2(a+L)}{6EI}$

$\frac{1}{2}P\delta_D = U \qquad \delta_D = \frac{2U}{P} = \frac{Pa^2(a+L)}{3EI}\ \downarrow$

Problem 11.59

11.59 and *11.60* Using the method of work and energy, determine the slope at point D caused by the couple $\mathbf{M}_0$.





Reactions $R_A = \frac{M_o}{L}\downarrow \qquad R_B = \frac{M_o}{L}\uparrow$

Over portion AD $\quad M = -\frac{M_o x}{L}$

$$U_{AD} = \int_0^a \frac{M^2}{2EI}dx = \frac{M_o^2}{2EIL^2}\int_0^a x^2 dx$$

$$= \frac{M_o a^3}{6EIL^2}$$

Over portion DB $\quad M = \frac{M_o v}{L}$

$$U_{DB} = \int_0^b \frac{M^2}{2EI}dv = \frac{M_o^2}{2EIL^2}\int_0^b v^2 dv$$

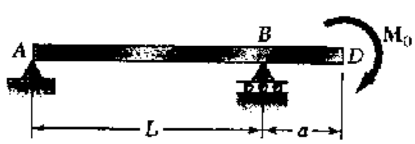
$$= \frac{M_o^2 b^3}{6EIL^2}$$

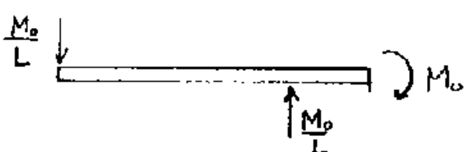
Total $\quad U = U_{AD} + U_{DB} = \frac{M_o^2(a^3+b^3)}{6EIL^2}$

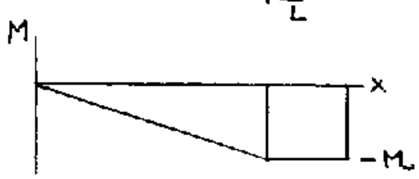
$$\tfrac{1}{2}M_o\theta_D = U \qquad \theta_D = \frac{2U}{M_o} = \frac{M_o(a^3+b^3)}{3EIL^2}$$

Problem 11.60

11.59 and *11.60* Using the method of work and energy, determine the slope at point D caused by the couple $\mathbf{M}_0$.







Reactions $R_A = \frac{M_o}{L}\downarrow \qquad R_B = \frac{M_o}{L}\uparrow$

Over portion AB $\quad M = -\frac{M_o x}{L}$

$$U_{AB} = \int_0^L \frac{M^2}{2EI}dx = \frac{M_o^2}{2EIL^2}\int_0^L x^2 dx$$

$$= \frac{M_o^2 L}{6EI}$$

Over portion BD $\quad M = -M_o$

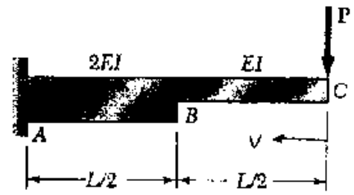
$$U_{BD} = \frac{M_o^2 a}{2EI}$$

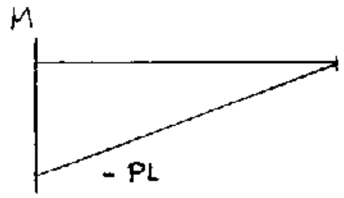
Total $\quad U = U_{AB} + U_{BD} = \frac{M_o^2(L+3a)}{6EI}$

$$\tfrac{1}{2}M_o\theta_D = U \qquad \theta_D = \frac{2U}{M_o} = \frac{M_o(L+3a)}{3EI}$$

Problem 11.61

11.61 and 11.62 Using the method of work and energy, determine the deflection at point C caused by the load **P**.





Bending moment. $M = -Pv$

Over AB:

$$U_{AB} = \int_{\frac{L}{2}}^{L} \frac{M^2}{4EI}\,dv = \frac{P^2}{4EI}\int_{\frac{L}{2}}^{L} v^2\,dv$$

$$= \frac{P^2}{12EI}\left[L^3 - \left(\tfrac{L}{2}\right)^3\right] = \frac{7}{96}\frac{P^2L^3}{EI}$$

Over BC $\quad U_{BC} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI}\,dv = \frac{P^2}{2EI}\int_0^{\frac{L}{2}} v^3\,dv$

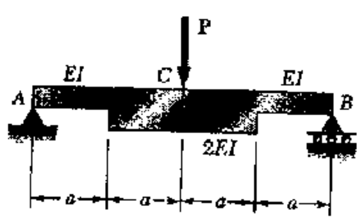
$$= \frac{1}{48}\frac{P^2L^3}{EI}$$

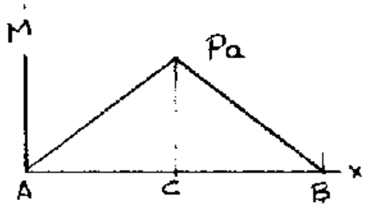
Total $\quad U = U_{AB} + U_{BC} = \frac{3}{32}\frac{P^2L^3}{EI}$

$\frac{1}{2}P\delta_c = 0 \qquad \delta_c = \frac{2U}{P} = \frac{3}{16}\frac{PL^3}{EI}\downarrow$

Problem 11.62

11.61 and 11.62 Using the method of work and energy, determine the deflection at point C caused by the load **P**.





Symmetric beam and loading: $R_A = R_B = \frac{1}{2}P$

From A to C $\qquad M = R_A x = \frac{1}{2}Px$

$$U_{AC} = \int_0^a \frac{M^2}{2EI}\,dx + \int_a^{2a}\frac{M^2}{4EI}\,dx$$

$$= \frac{P^2}{8EI}\int_0^a x^2\,dx + \frac{P^2}{16EI}\int_a^{2a} x^2\,dx$$

$$\frac{P^2a^3}{24EI} + \frac{P^2}{48EI}\left[(2a)^3 - a^3\right] = \frac{3}{16}\frac{P^2a^3}{EI}$$

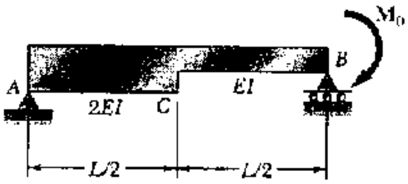
By symmetry $\quad U_{CB} = U_{AB} = \frac{3}{16}\frac{P^2a^3}{EI}$

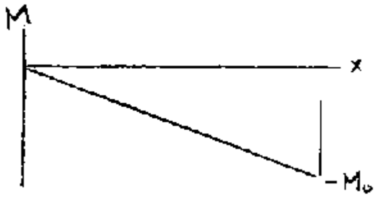
Total $\quad U = U_{AB} + U_{BC} = \frac{3}{8}\frac{P^2a^3}{EI}$

$\frac{1}{2}P\delta_c = U \qquad \delta_c = \frac{2U}{P} = \frac{3}{4}\frac{Pa^3}{EI}\downarrow$

Problem 11.63

11.63 Using the method of work and energy, determine the slope at point B caused by the couple $\mathbf{M}_0$.





$$\circlearrowleft \Sigma M_B = 0: \quad -R_A L - M_0 = 0 \qquad R_A = -\frac{M_0}{L}$$

$$M = R_A x = -\frac{M_0}{L}x$$

Over portion AC: $U_{AC} = \int_0^{L/2} \frac{M^2}{2(2EI)}\,dx$

$$U_{AC} = \frac{M_0^2}{4EIL^2}\int_0^{L/2} x^2\,dx = \frac{1}{96}\frac{M_0^2 L}{EI}$$

Over portion CB: $U_{CB} = \int_{L/2}^{L} \frac{M^2}{2EI}\,dx$

$$U_{CB} = \frac{M_0^2}{2EIL^2}\int_{L/2}^{L} x^2\,dx = \frac{M_0^2}{6EIL^2}\left[L^3 - \left(\frac{L}{2}\right)^3\right]$$

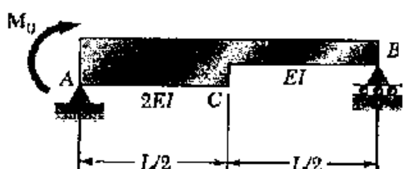
$$= \frac{7}{48}\frac{M_0^2 L}{EI}$$

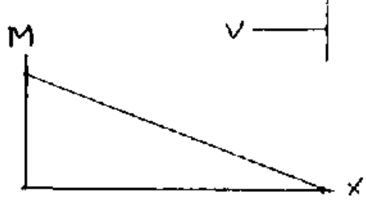
Total $U = U_{AC} + U_{CB} = \frac{5}{32}\frac{M_0^2 L}{EI}$

$$\frac{1}{2}M_0\theta_B = U \qquad \theta_B = \frac{2U}{M_0} = \frac{5}{16}\frac{M_0 L}{EI}$$

Problem 11.64

11.64 Using the method of work and energy, determine the slope at point A caused by the couple $\mathbf{M}_0$.





$$R_B = \frac{M_0}{L}$$

$$M = R_B v = \frac{M_0}{L}v$$

Over AC: $U_{AC} = \int_{L/2}^{L} \frac{M^2}{2(2EI)}\,dv$

$$U_{AC} = \frac{M_0^2}{4EIL^2}\int_{L/2}^{L} v^2\,dv = \frac{M_0^2}{12EIL^2}\left[L^3 - \left(\frac{L}{2}\right)^3\right]$$

$$= \frac{7}{96}\frac{M_0^2 L}{EI}$$

Over CB: $U_{CB} = \int_0^{L/2} \frac{M^2}{2EI}\,dv$

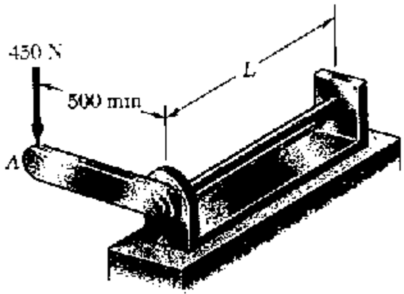
$$U_{CB} = \frac{M_0^2}{2EIL^2}\int_0^{L/2} v^2\,dv = \frac{1}{48}\frac{M_0^2 L}{EI}$$

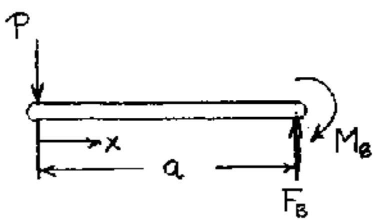
Total $U = U_{AC} + U_{CB} = \frac{3}{32}\frac{M_0^2 L}{EI}$

$$\frac{1}{2}M_0\theta_A = U \qquad \theta_A = \frac{2U}{M_0} = \frac{3}{16}\frac{M_0 L}{EI}$$

Problem 11.65

11.65 The 20-mm-diameter steel rod BC is attached to the lever AB and to the fixed support C. The uniform steel lever is 10 mm thick and 30 mm deep. Using the method of work and energy, determine the deflection of point A when $L = 600$ mm. Use $E = 200$ GPa and $G = 77.2$ GPa.





Member AB. (Bending)

$I = \frac{1}{12}(10)(30)^3 = 22.5\times10^3\ \text{mm}^4 = 22.5\times10^{-9}\ \text{m}^4$

$a = 500\ \text{mm} = 0.500\ \text{m}$

$M_B = Pa = (450)(0.500) = 225\ \text{N}\cdot\text{m}$

$M = Px$

$$U_{AB} = \int_0^a \frac{M^2}{2EI}dx = \int_0^a \frac{P^2x^2dx}{2EI} = \frac{P^2a^3}{6EI}$$

$$= \frac{(450)^2(0.500)^3}{(6)(200\times10^9)(22.5\times10^{-9})} = 0.9375\ \text{J}$$

Member BC (Torsion)

$T = M_B = 225\ \text{N}\cdot\text{m}$ $\quad c = \frac{1}{2}d = 10\ \text{mm}$ $\quad J = \frac{\pi}{2}c^4 = 15.708\times10^3\ \text{mm}^4 = 15.708\times10^{-9}\ \text{m}^4$

$L = 600\ \text{mm} = 0.600\ \text{m}$

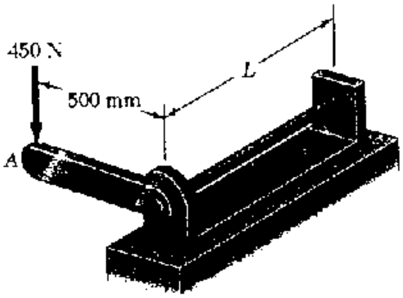
$$U_{BC} = \frac{T^2L}{2GJ} = \frac{(225)^2(0.600)}{(2)(77.2\times10^9)(15.708\times10^{-9})} = 12.5242\ \text{J}$$

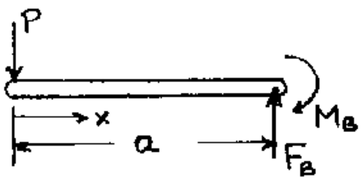
Total $\quad U = U_{AB} + U_{BC} = 0.9375 + 12.5242 = 13.4617\ \text{J}$

$\frac{1}{2}P\delta_A = U \qquad \delta_A = \frac{2U}{P} = \frac{(2)(13.4617)}{450} = 59.8\times10^{-3}\ \text{m} = 59.8\ \text{mm}$ ◄

Problem 11.66

11.66 The 20-mm-diameter steel rod BC is attached to the lever AB and to the fixed support C. The uniform steel lever is 10 mm thick and 30 mm deep. Using the method of work and energy, determine the length L of the rod BC for which the deflection at point A is 40 mm. Use $E = 200$ GPa and $G = 77.2$ GPa.





Member AB. (Bending)

$$I = \frac{1}{12}(10)(30)^3 = 22.5 \times 10^3 \text{ mm}^4 = 22.5 \times 10^{-9} \text{ m}^4$$

$$a = 500 \text{ mm} = 0.500 \text{ m}$$

$$M_B = Pa = (450)(0.500) = 225 \text{ N}\cdot\text{m}$$

$$M = Px$$

$$U_{AB} = \int_0^a \frac{M^2}{2EI} dx = \int_0^a \frac{P^2 x^2 \, dx}{2EI} = \frac{P^2 a^3}{6EI}$$

$$= \frac{(450)^2 (0.500)^3}{(6)(200 \times 10^9)(22.5 \times 10^{-9})} = 0.9375 \text{ J}$$

Member BC (Torsion) $\qquad T = M_B = 225 \text{ N}\cdot\text{m}$

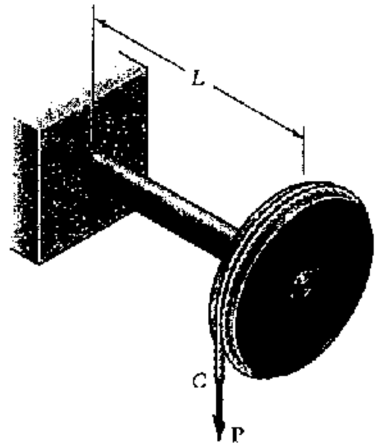
$$c = \frac{1}{2}d = 10 \text{ mm} \qquad J = \frac{\pi}{2}c^4 = 15.708 \times 10^3 \text{ mm}^4 = 15.708 \times 10^{-9} \text{ m}^4$$

$$U_{BC} = \frac{T^2 L}{2GJ} = \frac{(225)^2 L}{(2)(77.2 \times 10^9)(15.708 \times 10^{-9})} = 20.874 \, L$$

$$\frac{1}{2} P \delta_A = U_{AB} + U_{BC}$$

$$\frac{1}{2}(450)(40 \times 10^{-3}) = 0.9375 + 20.874 \, L \qquad L = 0.386 \text{ m} = 386 \text{ mm}$$

Problem 11.67

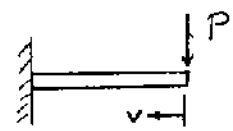


11.67 A disk of radius a has been welded to end B of the solid steel shaft AB. A cable is the wrapped around the disk and a vertical force **P** is applied to end C of the cable. Knowing that the radius of the shaft is r and neglecting the deformations of the disk and of the cable, show that the deflection of point C caused by the application of **P** is

$$\delta_C = \frac{PL^3}{3EI}\left(1 + 1.5\frac{Er^2}{GL^2}\right)$$

Torsion: $T = Pa$

$$U_t = \frac{T^2 L}{2GJ} = \frac{P^2 a^2 L}{2GJ}$$



Bending: $M = Pv$

$$U_b = \int_0^L \frac{M^2\,dv}{2EI} = \int_0^L \frac{P^2 v^2\,dv}{2EI} = \frac{P^2 L^3}{6EI}$$

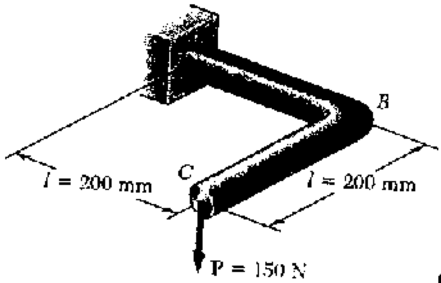
Total $\quad U = \dfrac{P^2 a^2 L}{2GJ} + \dfrac{P^2 L^3}{6EI} = \dfrac{1}{2} P \delta_c$

$$\delta_c = \frac{Pa^2 L}{GJ} + \frac{PL^3}{3EI} = \frac{PL^3}{3EI}\left(1 + \frac{3EIa^2}{GJL^2}\right)$$

Since $J = 2I \qquad \delta_c = \dfrac{PL^3}{3EI}\left(1 + \dfrac{3}{2}\dfrac{Ea^2}{GL^2}\right)$

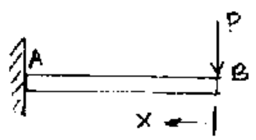
Problem 11.68

11.68 The 12-mm-diameter steel rod *ABC* has been bent into the shape shown. Knowing that E = 200 GPa and G = 77.2 GPa, determine the deflection of end C caused by the 150-N force.



$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}\left(\frac{12}{2}\right)^4 = 2.0358\times10^3\ \text{mm}^4 = 2.0358\times10^{-9}\ \text{m}^4$$

$$I = \tfrac{1}{2}J = 1.0179\times10^{-9}\ \text{m}^4$$

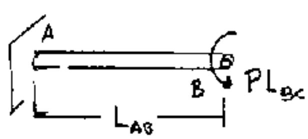


Portion AB: bending $M = -Px$

$$U_{AB,b} = \int_0^{L_{AB}} \frac{M^2}{2EI}\,dx = \frac{P^2}{2EI}\int_0^{L_{AB}} x^2\,dx$$

$$= \frac{P^2 L_{AB}^3}{6EI} = \frac{(150)^2(200\times10^{-3})^3}{(6)(200\times10^9)(1.0179\times10^{-9})}$$

$$= 0.14736\ \text{J}$$

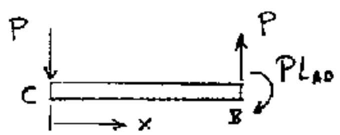


torsion $T = PL_{BC}$

$$U_{AB,t} = \frac{T^2 L_{AB}}{2GJ} = \frac{P^2 L_{BC}^2 L_{AB}}{2GJ}$$

$$= \frac{(150)^2(200\times10^{-3})^2(200\times10^{-3})}{(2)(77.2\times10^9)(2.0358\times10^{-9})}$$

$$= 0.57265\ \text{J}$$



Portion BC: $M = -Px$

$$U_{BC} = \int_0^{L_{BC}} \frac{M^2}{2EI}\,dx = \frac{P^2}{3EI}\int_0^{L_{BC}} x^2\,dx = \frac{P^2 L_{BC}^3}{6EI}$$

$$= \frac{(150)^2(200\times10^{-3})^3}{(6)(200\times10^9)(1.0179\times10^9)} = 0.14736\ \text{J}$$

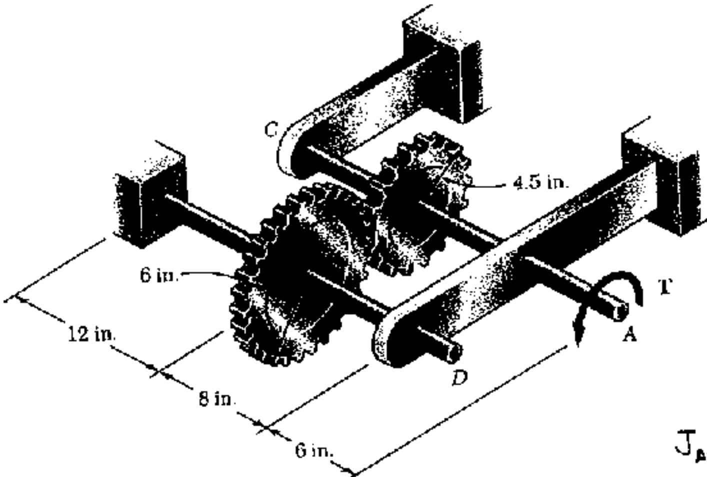
Total: $U = U_{AB,b} + U_{AB,t} + U_{BC} = 0.86737\ \text{J}$

Work-energy: $\tfrac{1}{2}P\delta = U \qquad \delta = \frac{2U}{P} = \frac{(2)(0.86737)}{150}$

$$= 11.57\times10^{-3}\ \text{m} = 11.57\ \text{mm}\downarrow$$

Problem 11.69

11.69 Two steel shafts, each of $\frac{7}{8}$-in diameter, are connected by the gears shown. Knowing that $G = 11.2 \times 10^6$ psi and that shaft DF is fixed at F, determine the angle through which end A rotates when a 1.2 kip · in. torque is applied at A. Ignore the strain energy due to the bending of the shafts.



Work-energy equation:

$$\frac{1}{2} T_A \varphi_A = U$$

$$\varphi_A = \frac{2U}{T_A}$$

Portion AB of shaft ABC:

$$T_{AB} = T_A = 1200 \text{ lb}\cdot\text{in}$$

$$L_{AB} = 8 + 6 = 14 \text{ in.}$$

$$J_{AB} = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi}{2}\left(\frac{7}{16}\right)^4 = 57.548 \times 10^{-3} \text{ in}^4$$

$$U_{AB} = \frac{T_{AB}^2 L_{AB}}{2GJ_{AB}} = \frac{(1200)^2(14)}{(2)(11.2\times10^6)(57.548\times10^{-3})} = 15.639 \text{ in}\cdot\text{lb}$$

Portion BC of shaft ABC: $\quad U_{BC} = 0$

Gear B: $\quad F_{BE} = \frac{T_B}{r_B} = \frac{T_{AB}}{r_B} = \frac{1200}{4.5} = 266.67 \text{ lb}$

Gear E: $\quad T_E = r_E F_{BE} = (6)(266.67) = 1600 \text{ lb}\cdot\text{in}$

Portion DE of shaft DEF: $\quad U_{DE} = 0$

Portion EF of shaft DEF: $\quad T_{EF} = T_E = 1600 \text{ lb}\cdot\text{in}$

$$L_{EF} = 12 \text{ in.} \qquad J_{EF} = J_{AB} = 57.548\times10^{-3} \text{ in}^4$$

$$U_{EF} = \frac{T_{EF}^2 L_{EF}}{2GJ_{EF}} = \frac{(1600)^2(12)}{(2)(11.2\times10^6)(57.548\times10^{-3})} = 23.831 \text{ in}\cdot\text{lb}$$

Total: $\quad U = U_{AB} + U_{BC} + U_{DE} + U_{EF} = 39.47 \text{ in}\cdot\text{lb}$

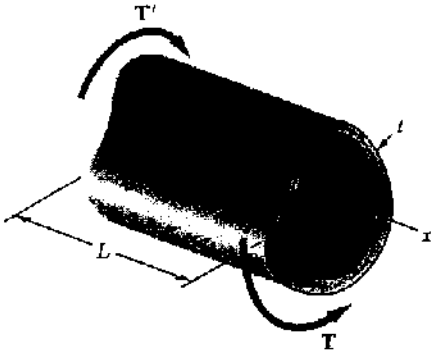
$$\varphi_A = \frac{2U}{T_A} = \frac{(2)(39.47)}{1200} = 65.78\times10^{-3} \text{ rad} = 3.77°$$

Problem 11.70

11.70 The thin-walled hollow cylindrical member AB has a noncircular cross section of nonuniform thickness. Using the expression given in Eq. (3.53) of Sec. 3.13, and the expression for the strain-energy density given in Eq. (11.19), show that the angle of twist of member AB is

$$\phi = \frac{TL}{4\mathfrak{a}^2 G}\oint \frac{ds}{t}$$

where ds is an element of the center line of the wall cross section and $\mathfrak{a}$ is the area enclosed by that center line.



From equation (3.53), $\tau = \frac{T}{2t\mathfrak{a}}$

Strain energy density:

$$u = \frac{\tau^2}{2G} = \frac{T^2}{8Gt^2\mathfrak{a}^2}$$

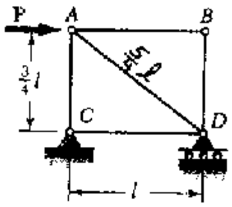
$$U = \int_0^L \oint u\, t\, ds\, dx$$

$$= \int_0^L \frac{T^2}{8G\mathfrak{a}^2}\oint \frac{ds}{t}\, dx = \frac{T^2 L}{8G\mathfrak{a}^2}\oint \frac{ds}{t}$$

Work of torque $= \frac{1}{2}T\varphi = \frac{T^2 L}{8G\mathfrak{a}^2}\oint \frac{ds}{t}$ $\qquad \varphi = \frac{TL}{4G\mathfrak{a}^2}\oint \frac{ds}{t}$

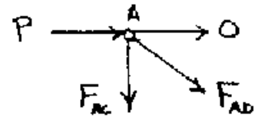
Problem 11.71

11.71 and 11.72 Each member of the truss shown has a uniform cross-sectional area A. Using the method of work and energy, determine the horizontal deflection of the point of application of the load **P**.



Members AB and BD are zero force members.

Joint A



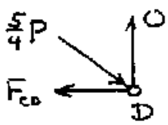
$+\rightarrow \Sigma F_x = 0$

$\frac{4}{5} F_{AD} + P = 0 \qquad F_{AD} = -\frac{5}{4}P$

$+\uparrow \Sigma F_y = 0$

$-F_{AC} - \frac{3}{5} F_{AD} = 0 \qquad F_{AC} = \frac{3}{4}P$

Joint D $\quad +\rightarrow \Sigma F_x = 0$



$\frac{4}{5} \cdot \frac{5}{4} P - F_{CD} = 0$

$F_{CD} = P$

Member	F	L	F^2L
AB	0	l	0
BD	0	$\frac{3}{4}l$	0
AD	$-\frac{5}{4}P$	$\frac{5}{4}l$	$\frac{125}{64}P^2l$
CD	P	l	P^2l
AC	$\frac{3}{4}P$	$\frac{3}{4}L$	$\frac{27}{64}P^2l$
Σ			$\frac{27}{8}P^2l$

$$U = \Sigma \frac{F^2L}{2EA} = \frac{1}{2EA}\Sigma F^2L$$

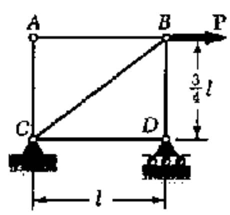
$$= \frac{27}{16}\frac{P^2l}{EA}$$

Work of $P = \frac{1}{2}P\Delta = U$

$$\Delta = \frac{2U}{P} = \frac{27}{8}\frac{Pl}{EA} = 3.375\frac{Pl}{EA} \rightarrow$$

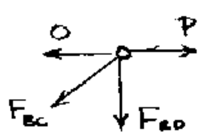
Problem 11.72

11.71 and 11.72 Each member of the truss shown has a uniform cross-sectional area A. Using the method of work and energy, determine the horizontal deflection of the point of application of the load **P**.



Members AB, AC, and CD are zero force members.

Joint B



$+\rightarrow \Sigma F_x = 0$

$P - \frac{4}{5}F_{BC} = 0 \qquad F_{BC} = \frac{5}{4}P$

$+\uparrow \Sigma F_y = 0$

$-F_{BD} - \frac{3}{5}F_{BC} = 0 \qquad F_{BD} = -\frac{3}{4}P$

$$U = \Sigma \frac{F^2L}{2EA} = \frac{1}{2EA}\Sigma F^2L$$

$$= \frac{19}{16}\frac{P^2l}{EA}$$

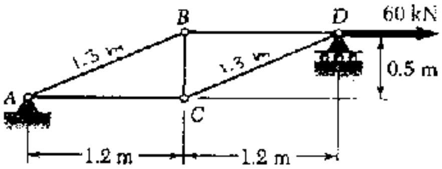
Member	F	L	F^2L
AB	0	l	0
AC	0	$\frac{3}{4}l$	0
CD	0	l	0
BC	$\frac{5}{4}P$	$\frac{5}{4}l$	$\frac{125}{64}P^2l$
BD	$-\frac{3}{4}P$	$\frac{3}{4}l$	$\frac{27}{64}P^2l$
Σ			$\frac{19}{8}P^2l$

Work of $P = \frac{1}{2}P\Delta = U$

$$\Delta = \frac{2U}{P} = \frac{19}{8}\frac{Pl}{EA} = 2.375\frac{Pl}{EA} \rightarrow$$

Problem 11.73

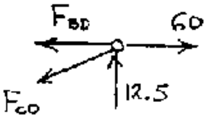
11.73 Each member of the truss shown is made of steel; the cross-sectional area of member *BC* is 800 mm² and for all other members the cross-sectional area is 400 mm². Using $E = 200$ GPa, determine the deflection of point *D* caused by the 60-kN load.



Entire truss $\circlearrowleft \sum M_A = 0$

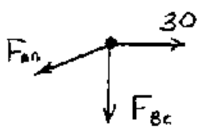
$2.4 R_D - (0.5)(60) = 0 \qquad R_D = 12.5 \text{ kN}$

Joint D $\quad +\uparrow \sum F_y = 0 \qquad 12.5 - \frac{0.5}{1.3} F_{CD} = 0 \qquad F_{CD} = 32.5 \text{ kN}$



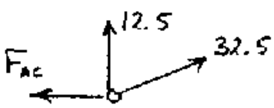
$\xrightarrow{+} \sum F_x = 0 \qquad 60 - F_{BD} - \frac{1.2}{1.3} F_{CD} = 0 \qquad F_{BD} = 30 \text{ kN}$

Joint B $\quad \xrightarrow{+} \sum F_x = 0 \qquad 30 - \frac{1.2}{1.3} F_{AB} = 0 \qquad F_{AB} = 32.5 \text{ kN}$



$+\uparrow \sum F_y = 0 \qquad -\frac{0.5}{1.3} F_{AB} + F_{BC} = 0 \qquad F_{BC} = 12.5 \text{ kN}$

Joint C $\quad \xrightarrow{+} \sum F_x = 0 \qquad -F_{AC} + \frac{1.2}{1.3}(32.5) = 0 \qquad F_{AC} = 30 \text{ kN}$



$$U = \sum \frac{F^2 L}{2EA} = \frac{1}{2E} \sum \frac{F^2 L}{A}$$

Member	F (kN)	L (m)	A (10^{-6} m²)	F^2L/A (N²/m)
CD	32.5	1.3	400	3.4328×10^{12}
BD	30	1.2	400	2.7×10^{12}
AB	32.5	1.3	400	3.4328×10^{12}
BC	12.5	0.5	800	0.0977×10^{12}
AC	30	1.2	400	2.7×10^{12}
Σ				12.3633×10^{12}

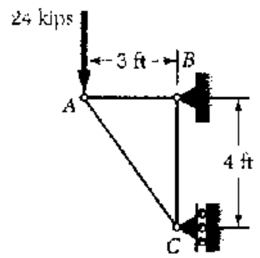
$$U = \frac{12.3633 \times 10^{12}}{(2)(200 \times 10^{9})} = 30.908 \text{ J}$$

$$\frac{1}{2} P \Delta = U \qquad \Delta = \frac{2U}{P} = \frac{(2)(30.908)}{60 \times 10^{3}} = 1.030 \times 10^{-3} \text{ m}$$

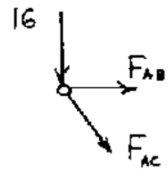
$$= 1.030 \text{ mm} \rightarrow$$

Problem 11.74

11.74 Each member of the truss shown is made of steel and has a uniform cross-sectional area of 3 in^2. Using $E = 29 \times 10^6$ psi, determine the vertical deflection of joint *A* caused by the application of the 24-kip load.



Joint A



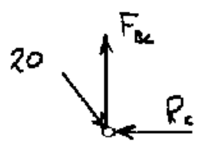
$+\uparrow \Sigma F_y = 0$

$-16 - \frac{4}{5} F_{AC} = 0 \qquad F_{AC} = -20$ kips

$+\rightarrow \Sigma F_x = 0$

$\frac{3}{5} F_{AC} + F_{AB} = 0 \qquad F_{AB} = 12$ kips

Joint C $\qquad +\uparrow \Sigma F_y = 0 \qquad F_B - \frac{4}{5}(20) = 0 \qquad F_{BC} = 16$ kips



$$U = \Sigma \frac{F^2 L}{2EA} = \frac{1}{2EA} \Sigma F^2 L$$

$E = 29 \times 10^3$ ksi

$A = 3$ in^2

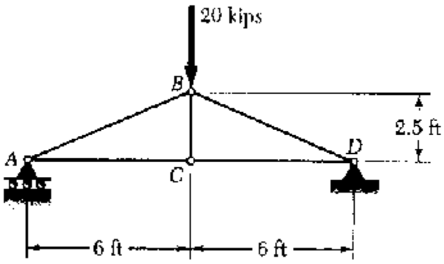
Member	F (kips)	L (in)	F^2L (kip^2·in)
AB	18	36	11664
AC	-30	60	54000
BC	24	48	27648
Σ			93312

$$U = \frac{93312}{(2)(29 \times 10^3)(3)} = 0.53628 \text{ kip}\cdot\text{in.}$$

$$\tfrac{1}{2} P\Delta = U \qquad \Delta = \frac{2U}{P} = \frac{(2)(0.53628)}{24} = 0.0447 \text{ in.} \downarrow$$

Problem 11.75

11.75 Each member of the truss shown is made of steel and has a cross-sectional area of 5 in². Using $E = 29 \times 10^6$ psi, determine the vertical deflection of point B caused by the 20-kip load.



$R_A = R_B = 10$ kips ↑

$L_{AC} = L_{CD} = 6$ ft $= 72$ in.

$L_{BC} = 2.5$ ft $= 30$ in.

$L_{AB} = L_{BC} = \sqrt{6^2 + 2.5^2} = 6.5$ ft $= 78$ in.

Joint A

$+\uparrow \Sigma F_y = 0: \; \frac{2.5}{6.5} F_{AB} + 10 = 0$

$F_{AB} = -26$ kips

$\overset{+}{\rightarrow} \Sigma F_x = 0: \; \frac{6}{6.5} F_{AB} + F_{AC} = 0$

$F_{AC} = 24$ kips

Joint C: $F_{BC} = 0$, $F_{CD} = 24$ kips

By symmetry, $F_{BD} = F_{AB} = -26$ kips

$$U = \Sigma \frac{F^2 L}{2EA} = \frac{1}{2EA} \Sigma F^2 L$$

Member	F (kips)	L (in.)	F^2L (kip²·in)
AB	-26	78	52728
AC	24	72	41472
BC	0	30	0
CD	24	72	41472
BD	-26	78	52728
Σ			188400

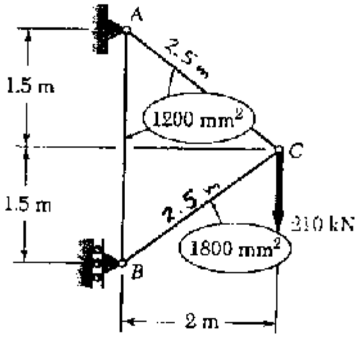
$$U = \frac{188400}{(2)(29 \times 10^3)(5)} = 0.64966 \text{ kip} \cdot \text{in}$$

$$\frac{1}{2} P \delta_B = U$$

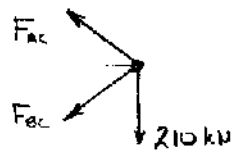
$$\delta_B = \frac{2U}{P} = \frac{(2)(0.64966)}{20} = 0.0650 \text{ in.} \downarrow$$

Problem 11.76

11.76 Members of the truss shown are made of steel and have the cross-sectional areas shown. Using $E = 200$ GPa, determine the vertical deflection of joint C caused by the application of the 210-kN load.



Joint C



$\stackrel{+}{\rightarrow}\Sigma F_x = 0:$

$$-\frac{4}{5}F_{AC} - \frac{4}{5}F_{BC} = 0$$

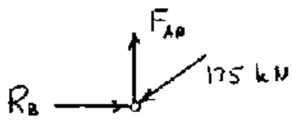
$+\uparrow \Sigma F_y = 0:$

$$\frac{3}{5}F_{AC} - \frac{3}{5}F_{BC} - 210 = 0$$

Solving simultaneously,

$F_{AC} = 175$ kN $\qquad F_{BC} = -175$ kN

Joint B



$+\uparrow \Sigma F_y = 0$

$$F_{AB} - \left(\frac{3}{5}\right)(175) = 0$$

$$F_{AB} = 105 \text{ kN}$$

$$U = \Sigma \frac{F^2 L}{2EA}$$

Member	F (kN)	L (m)	A (10^{-6} m²)	F^2L/A (N²/m)
AB	105	3.0	1200	27.5625×10^{12}
AC	175	2.5	1200	63.8021×10^{12}
BC	−175	2.5	1800	42.5347×10^{12}
Σ				133.8993×10^{12}

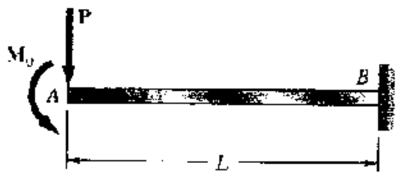
$$U_m = \frac{1}{2E}\Sigma\frac{F^2L}{A} = \frac{133.8993 \times 10^{12}}{(2)(200 \times 10^9)} = 334.75 \text{ J}$$

$$\frac{1}{2}P_m \Delta_m = U_m$$

$$\Delta_m = \frac{2U_m}{P_m} = \frac{(2)(334.75)}{210 \times 10^3} = 3.19 \times 10^{-3} \text{ m} = 3.19 \text{ mm} \downarrow$$

Problem 11.77

11.77 through 11.79 Using the information in Appendix D, compute the work of the loads as they are applied to the beam (*a*) if the load **P** is applied first, (b) if the couple **M** is applied first.



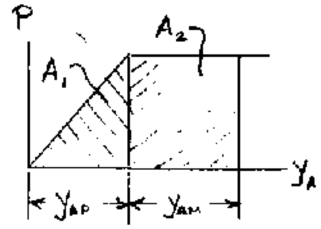
From Appendix D, Case 1

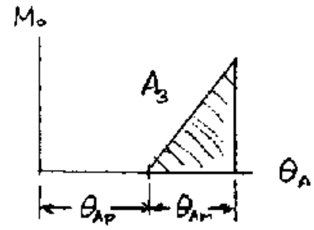
$$y_{AP} = \frac{PL^3}{3EI} \qquad \theta_{AP} = \frac{PL^2}{2EI}$$

From Appendix D, Case 3

$$y_{AM} = \frac{M_0L^2}{2EI} \qquad \theta_{AM} = \frac{M_0L}{EI}$$

(a) First P, then M_0.



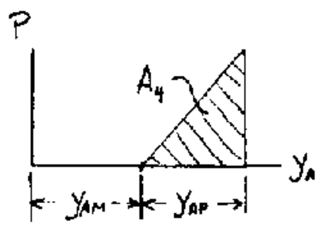


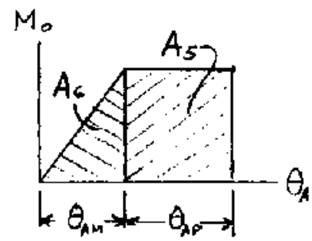
$$U = A_1 + A_2 + A_3$$

$$= \tfrac{1}{2}Py_{AP} + Py_{AM} + \tfrac{1}{2}M_0\theta_{AM}$$

$$= \frac{P^2L^3}{6EI} + \frac{PM_0L^2}{2EI} + \frac{M_0^2L}{2EI}$$

(b) First M_0, then P.





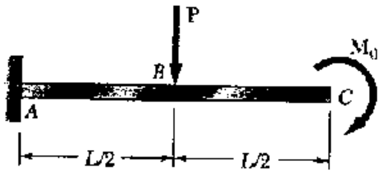
$$U = A_4 + A_5 + A_6$$

$$= \tfrac{1}{2}Py_{AP} + M_0\theta_{AP} + \tfrac{1}{2}M_0\theta_{AM}$$

$$= \frac{P^2L^3}{6EI} + \frac{M_0PL^2}{2EI} + \frac{M_0^2L}{2EI}$$

Problem 11.78

11.77 through 11.79 Using the information in Appendix D, compute the work of the loads as they are applied to the beam (*a*) if the load **P** is applied first, (b) if the couple **M** is applied first.



Appendix D Cases 1 and 3

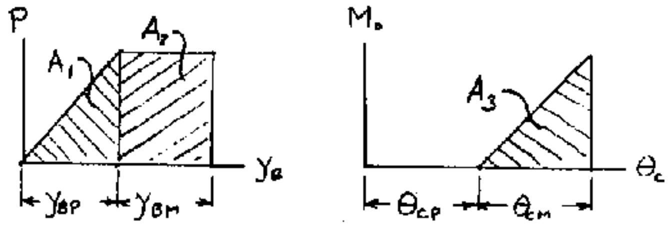
$$y_{BP} = \frac{P(L/2)^3}{3EI} = \frac{PL^3}{24EI}$$

$$\theta_{CP} = \frac{P(L/2)^2}{2EI} = \frac{PL^2}{8EI}$$

$$y_{BM} = \frac{M_0(L/2)^2}{2EI} = \frac{M_0L^2}{8EI}$$

$$\theta_{BM} = \frac{M_0L}{EI}$$

(a) First P, then M_0.

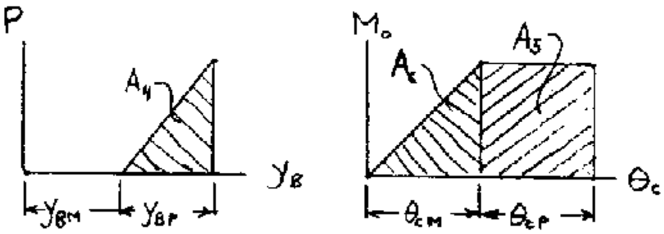


$$U = A_1 + A_2 + A_3$$

$$= \tfrac{1}{2}Py_{BP} + Py_{BM} + \tfrac{1}{2}M_0\theta_{CM}$$

$$= \frac{P^2L^3}{48EI} + \frac{PM_0L^2}{8EI} + \frac{M_0^2L}{2EI}$$

(b) First M_0, then P.



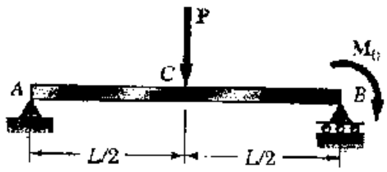
$$U = A_4 + A_5 + A_6$$

$$= \tfrac{1}{2}Py_{BP} + M_0\theta_{CP} + \tfrac{1}{2}M_0\theta_{CM}$$

$$= \frac{P^2L^3}{48EI} + \frac{M_0PL^2}{8EI} + \frac{M_0^2L}{2EI}$$

Problem 11.79

11.77 through 11.79 Using the information in Appendix D, compute the work of the loads as they are applied to the beam (*a*) if the load **P** is applied first, (b) if the couple **M** is applied first.



From Appendix D, Case 4

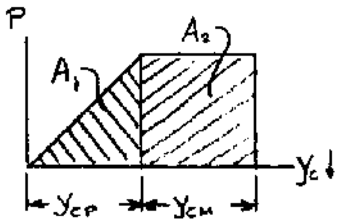
$$\downarrow y_C = \frac{PL^3}{48EI} \qquad \circlearrowleft \theta_B = -\frac{PL^2}{16EI}$$

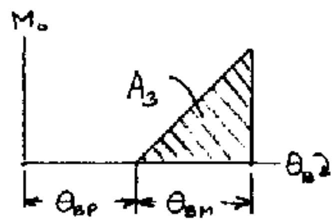
From Appendix D, Case 7

$$\downarrow y_C = \frac{M_0}{6EIL}\left((L/2)^3 - L^2(L/2)\right) = -\frac{M_0L^2}{16EI}$$

$$\circlearrowright \theta_B = \frac{M_0L}{3EI}$$

(a) First P, then M_0.



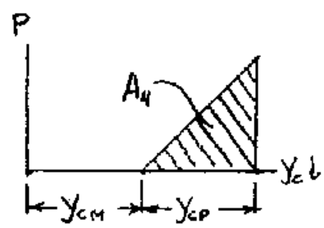


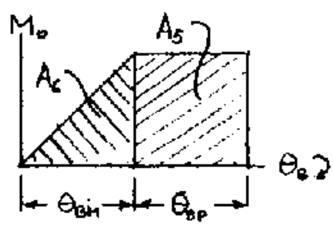
$$U = A_1 + A_2 + A_3$$

$$= \tfrac{1}{2}Py_{CP} + Py_{CM} + \tfrac{1}{2}M_0\theta_{BM}$$

$$= \frac{P^2L^3}{96EI} - \frac{PM_0L^2}{16EI} + \frac{M_0^2L}{6EI} \blacktriangleleft$$

(b) First M_0, then P.





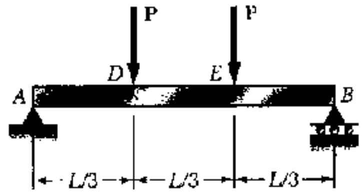
$$U = A_4 + A_5 + A_6$$

$$= \tfrac{1}{2}Py_{CP} + M_0\theta_{BP} + \tfrac{1}{2}M_0\theta_{BM}$$

$$= \frac{P^2L^3}{96EI} - \frac{M_0PL^2}{16EI} + \frac{M_0^2L}{6EI} \blacktriangleleft$$

Problem 11.80

11.80 through 11.82 For the beam and loading shown, (*a*) compute the work of the loads as they are applied successively to the beam, using the information provided in Appendix D, (*b*) compute the strain energy of the beam by the method of Sec. 11.4 and show that it is equal to the work obtained in part *a*.



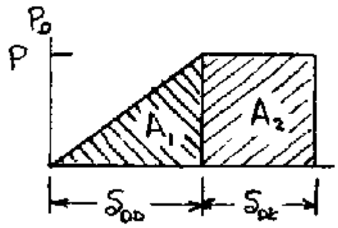
(a) Label the forces P_D and P_E.

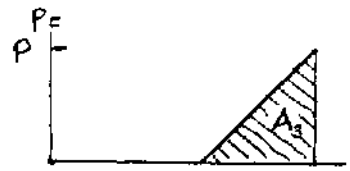
Using Appendix D, Case 5

$$\delta_{EE} = \frac{P_E a^2 b^2}{3EIL} = \frac{P_E (\frac{2}{3}L)^2(\frac{1}{3}L)^2}{3EIL} = \frac{4}{243}\frac{P_E L^3}{EI}$$

$$\delta_{DE} = \frac{P_E b}{6EIL}\left[(L^2-b^2)x - x^3\right] = \frac{P_E(\frac{1}{3}L)}{6EIL}\left[(L^2-(\tfrac{L}{3})^2)(\tfrac{L}{3}) - (\tfrac{L}{3})^3\right] = \frac{7}{486}\frac{P_E L^3}{EI}$$

Likewise, $\delta_{DD} = \frac{4}{243}\frac{P_D L^3}{EI}$ and $\delta_{ED} = \frac{7}{486}\frac{P_D L^3}{EI}$





Let P_D be applied first.

$U = A_1 + A_2 + A_3$

$$U = \tfrac{1}{2}P_D\delta_{DD} + P_D\delta_{DE} + \tfrac{1}{2}P_E\delta_{EE} = \frac{2}{243}\frac{P_D^2 L^3}{EI} + \frac{7}{486}\frac{P_D P_E L^3}{EI} + \frac{2}{243}\frac{P_E^2 L^3}{EI}$$

With $P_D = P_E = P$ $\qquad U = \frac{5}{162}\frac{P^2L^3}{EI}$ ◄

(b) Reactions: $R_A = R_B = P\uparrow$

Over portion AD. $(0 \le x \le \frac{L}{3})$ $\quad M = Px$

$$U_{AD} = \int_0^{L/3}\frac{M^2}{2EI}dx = \frac{P^2}{2EI}\int_0^{L/3}x^2dx = \frac{P^2}{6EI}\left(\frac{L}{3}\right)^3 = \frac{1}{162}\frac{P^2L^3}{EI}$$

Over portion DE. $(\frac{L}{3} \le x \le \frac{2L}{3})$ $\quad M = \frac{PL}{3}$

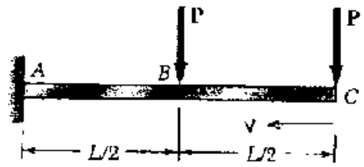
$$U_{DE} = \int_{L/3}^{2L/3}\frac{M^2}{2EI}dx = \frac{(PL/3)^2}{2EI}(L/3) = \frac{1}{54}\frac{P^2L^3}{EI}$$

Over portion ED. By symmetry, $U_{EB} = U_{AD} = \frac{1}{162}\frac{P^2L^3}{EI}$

Total. $\quad U = U_{AD} + U_{DE} + U_{EB} = \frac{5}{162}\frac{P^2L^3}{EI}$ ◄

Problem 11.81

11.80 through 11.82 For the beam and loading shown, (*a*) compute the work of the loads as they are applied successively to the beam, using the information provided in Appendix D, (*b*) compute the strain energy of the beam by the method of Sec. 11.4 and show that it is equal to the work obtained in part *a*.



(a) Label the forces P_B and P_C

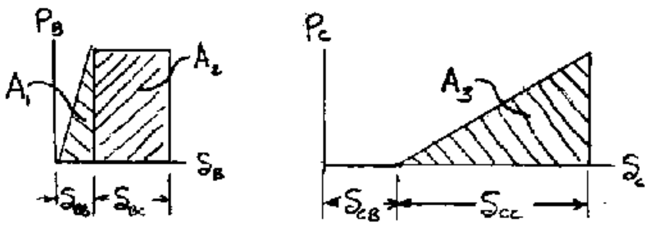
Using Appendix D Case 1

$$\delta_{BB} = \frac{P_B (L/2)^3}{3EI} = \frac{1}{24}\frac{P_B L^3}{EI}$$

$$\delta_{CB} = \delta_{BB} + \frac{L}{2}\theta_B = \frac{1}{24}\frac{P_B L^3}{3EI} + \frac{L}{2}\frac{P_B (L/2)^2}{2EI} = \left(\frac{1}{24} + \frac{1}{16}\right)\frac{P_B L^3}{EI} = \frac{5}{48}\frac{P_B L^3}{EI}$$

$$\delta_{CC} = \frac{1}{3}\frac{P_C L^3}{EI}$$

$$\delta_{BC} = \frac{P_C}{6EI}\left(3Lx^2 - x^3\right) = \frac{P_C}{6EI}\left(3L\left(\frac{L}{2}\right)^2 - \left(\frac{L}{2}\right)^3\right) = \frac{5}{48}\frac{P_C L^3}{EI}$$



Apply P_B first, then P_C

$U = A_1 + A_2 + A_3$

$$U = \frac{1}{2}P_B\delta_{BB} + P_B\delta_{BC} + \frac{1}{2}P_C\delta_{CC} = \frac{1}{48}\frac{P_B^2 L^3}{EI} + \frac{5}{48}\frac{P_B P_C L^3}{EI} + \frac{1}{6}\frac{P_C^2 L^3}{EI}$$

With $P_B = P_C = P$ $\qquad U = \left(\frac{1}{48} + \frac{5}{48} + \frac{1}{6}\right)\frac{P^2 L^3}{EI} = \frac{7}{24}\frac{P^2 L^3}{EI}$ ◄

Over AB: $\quad M = Pv + P\left(v - \frac{L}{2}\right) = P\left(2v - \frac{L}{2}\right)$

$$U_{AB} = \int_{\frac{L}{2}}^{L}\frac{M^2}{2EI}dv = \frac{P^2}{2EI}\int_{\frac{L}{2}}^{L}\left(4v^2 - 2Lv + \frac{1}{4}L^2\right)dv$$

$$= \frac{P^2}{2EI}\left\{\frac{4}{3}\left[L^3 - \left(\frac{L}{2}\right)^3\right] - 2L\cdot\frac{1}{2}\left[L^2 - \left(\frac{L}{2}\right)^2\right] + \frac{1}{4}L^2\left[L - \frac{L}{2}\right]\right\}$$

$$= \frac{P^2}{2EI}\left\{\frac{7}{6}L^3 - \frac{3}{4}L^3 + \frac{1}{8}L^3\right\} = \frac{13}{48}\frac{P^2L^3}{EI}$$

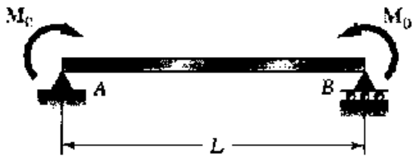
Over BC: $\quad M = Pv \qquad U_{BC} = \int_0^{\frac{L}{2}}\frac{M^2}{2EI}dv = \frac{P^2}{2EI}\int_0^{\frac{L}{2}}v^2 dv = \frac{P^2}{2EI}\cdot\frac{1}{3}\left(\frac{L}{2}\right)^3$

$$= \frac{P^2L^3}{48EI}$$

Total $\quad U = U_{AB} + U_{BC} = \left(\frac{13}{48} + \frac{1}{48}\right)\frac{P^2L^3}{EI} = \frac{7}{24}\frac{P^2L^3}{EI}$ ◄

Problem 11.82

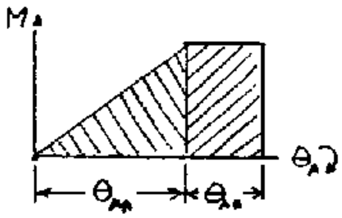
11.80 through 11.82 For the beam and loading shown, (*a*) compute the work of the loads as they are applied successively to the beam, using the information provided in Appendix D, (*b*) compute the strain energy of the beam by the method of Sec. 11.4 and show that it is equal to the work obtained in part *a*.

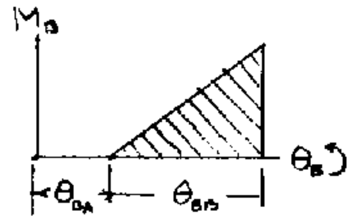


(a) Label the couples M_A and M_B.

Using Appendix D, Case 7

$$\theta_{AA} = \frac{M_A L}{3EI} \qquad \theta_{BA} = \frac{M_A L}{6EI} \qquad \theta_{BB} = \frac{M_B L}{3EI} \qquad \theta_{AB} = \frac{M_B L}{6EI}$$





Apply M_A first, then M_B.

$$U = A_1 + A_2 + A_3$$

$$U = \tfrac{1}{2}M_A\theta_{AA} + M_A\theta_{AB} + \tfrac{1}{2}M_B\theta_{BB} = \frac{1}{6}\frac{M_A^2 L}{EI} + \frac{1}{6}\frac{M_A M_B L}{EI} + \frac{1}{6}\frac{M_B^2 L}{EI}$$

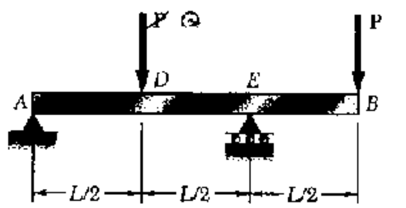
With $M_A = M_B = M_0$ $\qquad U = \dfrac{1}{2}\dfrac{M_0^2 L}{EI}$ ◄

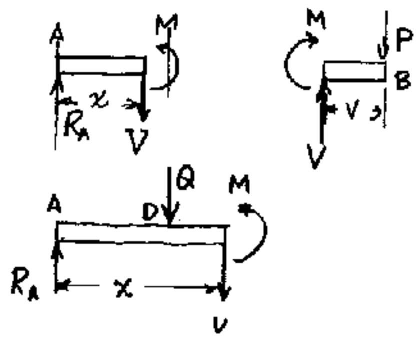
(b) Bending moment $\qquad M = M_0$

$$U = \int_0^L \frac{M^2}{2EI}\,dx = \frac{M_0^2 L}{2EI}$$ ◄

Problem 11.83

11.83 and 11.85 For the prismatic beam shown, determine the deflection of point D.





Change force at D from P to Q.

Reactions $R_A = \frac{1}{2}(Q-P)\uparrow, \quad R_E = \frac{1}{2}(3P+Q)\uparrow$

Strain energy $\quad U = U_{AD} + U_{DE} + U_{EB}$

Deflection of point D.

$$\delta_D = \frac{\partial U}{\partial Q} = \frac{\partial U_{AD}}{\partial Q} + \frac{\partial U_{DE}}{\partial Q} + \frac{\partial U_{EB}}{\partial Q}$$

Portion AD. $\quad U_{AD} = \int_0^{L/2} \frac{M^2}{2EI}\,dx$

$$M = R_A x = \frac{1}{2}(Q-P)x$$

$$\frac{\partial M}{\partial Q} \quad \frac{1}{2}x \qquad \text{With } Q = P, \quad M = 0$$

$$\frac{\partial U_{AD}}{\partial Q} = \int_0^{L/2} \frac{M}{EI}\frac{\partial M}{\partial Q}\,dx = 0$$

Portion DE: $\quad U_{DE} = \int_{L/2}^{L} \frac{M^2}{2EI}\,dx$

$$M = R_A x - Q\left(x - \frac{L}{2}\right) = \frac{1}{2}(Q-P)x - Q\left(x-\frac{L}{2}\right) = \frac{1}{2}Q(L-x) - \frac{1}{2}Px$$

$$\frac{\partial M}{\partial Q} = \frac{1}{2}(L-x)$$

With $Q = P, \qquad M = -\frac{1}{2}P(2x-L)$

$$\frac{\partial U_{DE}}{\partial Q} = \int_{L/2}^{L} \frac{M}{EI}\frac{\partial M}{\partial Q}\,dx = -\frac{P}{4EI}\int_{L/2}^{L}(2x-L)(L-x)\,dx$$

$$= -\frac{P}{4EI}\int_{L/2}^{L}(3Lx - 2x^2 - L^2)\,dx$$

$$= -\frac{P}{4EI}\left(\frac{3}{2}Lx^2 - \frac{2}{3}x^3 - L^2x\right)\Big|_{L/2}^{L}$$

$$= -\frac{PL^3}{4EI}\left[\frac{3}{2} - \frac{2}{3} - 1 - \left(\frac{3}{2}\right)\left(\frac{1}{4}\right) + \left(\frac{2}{3}\right)\left(\frac{1}{8}\right) + \frac{1}{2}\right) = -\frac{PL^3}{96EI}$$

Portion EB: $\quad U_{EB} = \int_0^{L/2} \frac{M^2}{2EI}\,dv$

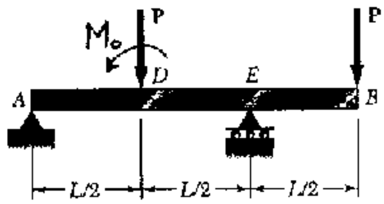
$$M = -Pv \qquad \frac{\partial M}{\partial Q} = 0 \qquad \frac{\partial U_{EB}}{\partial Q} = \int_0^{L/2} \frac{M}{EI}\frac{\partial M}{\partial Q}\,dv = 0$$

Deflection of Point D. $\quad \delta_D = 0 - \frac{PL^3}{96EI} + 0 = -\frac{PL^3}{96EI}\downarrow$

$$\delta_D = \frac{PL^3}{96EI}\uparrow$$

Problem 11.84

11.84 and ***11.86*** For the prismatic beam shown, determine the slope at point D.



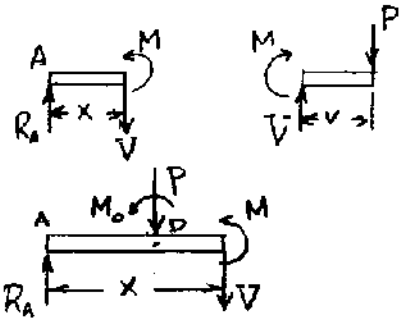
Add couple M_0 at point D as shown.

Reactions. $R_A = \frac{M_0}{L}\uparrow, \quad R_E = 2P - \frac{M_0}{L}\uparrow$

Strain energy. $U = U_{AD} + U_{DE} + U_{EB}$

Slope at point D.

$$\theta_D = \frac{\partial U}{\partial M_0} = \frac{\partial U_{AD}}{\partial M_0} + \frac{\partial U_{DE}}{\partial M_0} + \frac{\partial U_{EB}}{\partial M_0}$$



Portion AD. $U_{AD} = \int_0^{L/2} \frac{M^2}{2EI}\,dx$

$$M = R_A x = \frac{M_0}{L}x$$

$$\frac{\partial M}{\partial M_0} = \frac{x}{L} \qquad \text{With } M_0 = 0, \quad M = 0$$

$$\frac{\partial U_{AD}}{\partial M_0} = \int_0^{L/2} \frac{M}{EI}\frac{\partial M}{\partial M_0}\,dx = 0$$

Portion DE: $\quad U_{DE} = \int_{L/2}^{L} \frac{M^2}{2EI}\,dx$

$$M = R_A x - P\left(x - \frac{L}{2}\right) - M_0 = \frac{M_0}{L}x - P\left(x - \frac{L}{2}\right) - M_0$$

$$\frac{\partial M}{\partial M_0} = \left(\frac{x}{L} - 1\right) \qquad \text{With } M_0 = 0, \qquad M = -P\left(x - \frac{L}{2}\right)$$

$$\frac{\partial U_{DE}}{\partial M_0} = \int_{L/2}^{L} \frac{M}{EI}\frac{\partial M}{\partial M_0}\,dx = -\frac{P}{EI}\int_{L/2}^{L}\left(\frac{x}{L} - 1\right)\left(x - \frac{L}{2}\right)dx$$

$$= \frac{P}{2EIL}\int_{L/2}^{L}(L - x)(2x - L)\,dx = \frac{P}{2EIL}\int_{L/2}^{L}(3Lx - 2x^2 - L^2)\,dx$$

$$= \frac{P}{2EIL}\left(\frac{3}{2}Lx^2 + \frac{2}{3}x^3 - L^2x\right)\Big|_{L/2}^{L}$$

$$= \frac{PL^3}{2EIL}\left[\frac{3}{2} - \frac{2}{3} - 1 - \left(\frac{3}{2}\right)\left(\frac{1}{4}\right) + \left(\frac{2}{3}\right)\left(\frac{1}{8}\right) + \frac{1}{2}\right] = \frac{PL^2}{48EI}$$

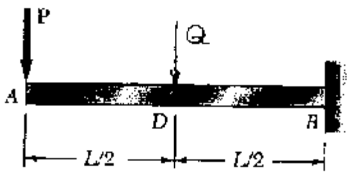
Portion EB: $\quad U_{EB} = \int_0^{L/2} \frac{M^2}{2EI}\,dv$

$$M = -Pv \qquad \frac{\partial M}{\partial M_0} = 0 \qquad \frac{\partial U_{EB}}{\partial M_0} = \int_0^{L/2}\frac{M}{EI}\frac{\partial M}{\partial M_0}\,dv = 0$$

Slope at point D. $\quad \theta_D = 0 + \frac{PL^2}{4EI} + 0 \qquad = \frac{PL^2}{48EI}\ \measuredangle$

Problem 11.85

11.83 and 11.85 For the prismatic beam shown, determine the deflection of point *D*.



Add force Q at point D.

$$U = \int_0^L \frac{M^2}{2EI}\,dx$$

$$\delta_D = \frac{\partial U}{\partial Q} = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial Q}\,dx = \frac{1}{EI}\int_0^L M\frac{\partial M}{\partial Q}\,dx$$

Over portion AD $\quad 0 < x < \frac{L}{2} \qquad M = -Px, \qquad \frac{\partial M}{\partial Q} = 0$

Over portion DB $\quad \frac{L}{2} < x < L \qquad M = -Px - Q\left(x - \frac{L}{2}\right), \qquad \frac{\partial M}{\partial Q} = -\left(x - \frac{L}{2}\right)$

Set $Q = 0$.

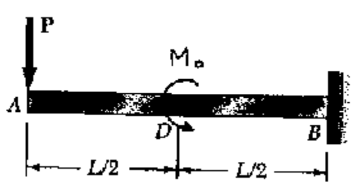
$$\delta_D = \frac{1}{EI}\int_0^{\frac{L}{2}}(-Px)(0)\,dx + \frac{1}{EI}\int_{\frac{L}{2}}^{L}(-Px)\left[-\left(x - \frac{L}{2}\right)\right]dx$$

$$= \frac{P}{EI}\int_{\frac{L}{2}}^{L}\left(x^2 - \frac{L}{2}x\right)dx = \frac{P}{EI}\left\{\frac{1}{3}L^3 - \frac{1}{3}\left(\frac{L}{2}\right)^3 - \left(\frac{L}{2}\right)\frac{1}{2}L^2 + \frac{L}{2}\,\frac{1}{2}\left(\frac{L}{2}\right)^2\right\}$$

$$= \left(\frac{1}{3} - \frac{1}{24} - \frac{1}{4} + \frac{1}{16}\right)\frac{PL^3}{EI} = \frac{5}{48}\frac{PL^3}{EI}\downarrow$$

Problem 11.86

11.84 and *11.86* For the prismatic beam shown, determine the slope at point *D*.



Add couple M_0 at point D.

$$U = \int_0^L \frac{M^2}{2EI}\,dx$$

$$\circlearrowleft\theta_D = \frac{\partial U}{\partial M_0} = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial M_0}\,dx = \frac{1}{EI}\int_0^L M\frac{\partial M}{\partial M_0}\,dx$$

Over portion AD: $\quad 0 < x < \frac{L}{2} \qquad M = -Px \qquad \frac{\partial M}{\partial M_0} = 0$

Over portion DB: $\quad \frac{L}{2} < x < L \qquad M = -Px - M_0 \qquad \frac{\partial M}{\partial M_0} = -1$

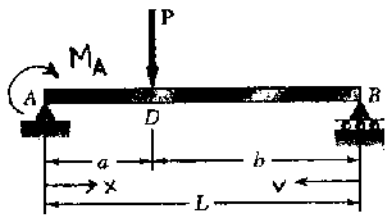
Set $M_0 = 0$, $\quad \theta_D = \frac{1}{EI}\int_{\frac{L}{2}}^{L}(-Px)(0)\,dx + \frac{1}{EI}\int_{\frac{L}{2}}^{L}(-Px)(-1)\,dx$

$$= \frac{P}{EI}\int_{\frac{L}{2}}^{L} x\,dx = \frac{P}{EI}\left[\frac{1}{2}L^2 - \frac{1}{2}\left(\frac{L}{2}\right)^2\right]$$

$$= \left(\frac{1}{2} - \frac{1}{8}\right)\frac{PL^2}{EI} = \frac{3}{8}\frac{PL^2}{EI}\;\measuredangle$$

Problem 11.87

11.87 For the prismatic beam shown, determine the slope at point A.



Add couple M_A at point A.

Reactions: $R_A = \frac{Pb}{L} - \frac{M_A}{L}$, $R_B = \frac{Pa}{L} + \frac{M_A}{L}$

$$U = \int_0^L \frac{M^2}{2EI}\,dx = \frac{1}{2EI}\int_0^a M^2\,dx + \frac{1}{2EI}\int_0^b M^2\,dv$$

$$\theta_A = \frac{\partial U}{\partial M_A} = \frac{1}{EI}\int_0^a M\frac{\partial M}{\partial M_A}\,dx + \frac{1}{EI}\int_0^b M\frac{\partial M}{\partial M_A}\,dv$$

Over portion AD $(0<x<a)$ $\quad M = M_A + R_A x = M_A\left(1-\frac{x}{L}\right) + \frac{Pbx}{L}$, $\quad \frac{\partial M}{\partial M_A} = 1 - \frac{x}{L}$

Over portion DB $(0<v<b)$ $\quad M = R_B v = \frac{Pav}{L} + \frac{M_A v}{L}$, $\quad \frac{\partial M}{\partial M_A} = \frac{v}{L}$

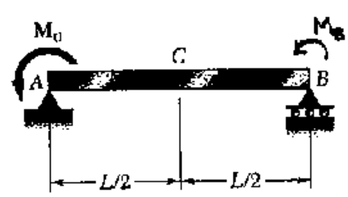
Set $M_A = 0$ $\quad \theta_A = \frac{1}{EI}\int_0^a \left(\frac{Pbx}{L}\right)\left(1-\frac{x}{L}\right)dx + \frac{1}{EI}\int_0^b \frac{Pav}{L}\,\frac{v}{L}\,dv$

$$= \frac{P}{EIL^2}\left(\tfrac{1}{2}bLa^2 - \tfrac{1}{3}ba^3 + \tfrac{1}{3}ab^3\right)$$

$$= \frac{Pab}{6EIL^2}\left(3La - 2a^2 + 2b^2\right)$$

Problem 11.88

11.88 For the prismatic beam shown, determine the slope at point B.



Add couple M_B at point B as shown.

Reactions. $R_A = \frac{1}{L}(M_0 + M_B)\uparrow$

Strain energy. $U = \int_0^L \frac{M^2}{2EI}\,dx$

Slope at point B. $\theta_B = \frac{\partial U}{\partial M_B}$

$$M = R_A x - M_0 = (M_0 + M_B)\frac{x}{L} - M_0$$

$\frac{\partial M}{\partial M_B} = \frac{x}{L}$ $\quad$ With $M_B = 0$ $\quad M = M_0\left(\frac{x}{L} - 1\right)$

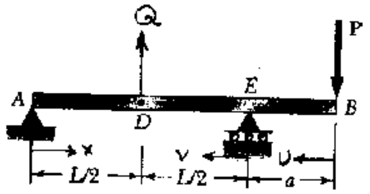
$$\frac{\partial U}{\partial M_B} = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial M_B}\,dx = \frac{M_0}{EI}\int_0^L\left(\frac{x}{L}-1\right)\frac{x}{L}\,dx = \frac{M_0}{EIL^2}\int_0^L (x-L)x\,dx$$

$$= \frac{M_0}{EIL^2}\int_0^L (x^2 - Lx)\,dx = \frac{M_0}{EI}\left(\frac{x^3}{3} - \frac{Lx^2}{2}\right)\Big|_0^L = -\frac{M_0 L}{6EI}$$

$$\theta_B = -\frac{M_0 L}{6EI} = \frac{M_0 L}{6EI}$$

Problem 11.89

11.89 and 11.90 For the prismatic beam shown, determine the deflection at point D.



Add force Q at point D.

Reactions $R_A = -\frac{Pa}{L} - \frac{1}{2}Q$, $R_E = \frac{P(a+L)}{L} - \frac{1}{2}Q$

$U = U_{AD} + U_{DE} + U_{EB}$; $\delta_D = \frac{\partial U}{\partial Q}$

Over portion AD: $U_{AD} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI}\,dx$, $M = R_A x = -\frac{Pa}{L}x - \frac{1}{2}Qx$, $\frac{\partial M}{\partial Q} = -\frac{1}{2}x$

Set $Q = 0$. $\frac{\partial U_{AD}}{\partial Q} = \frac{1}{EI}\int_0^{\frac{L}{2}}\left(-\frac{Pa}{L}x\right)\left(-\frac{1}{2}x\right)dx = \frac{Pa}{2EIL}\int_0^{\frac{L}{2}} x^2\,dx$

$$= \frac{Pa}{2EIL}\,\frac{1}{3}\left(\frac{L}{2}\right)^3 = \frac{PaL^2}{48EI}$$

Over portion DE: $U_{DE} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI}\,dv$ $\qquad \frac{\partial M}{\partial Q} = -\frac{1}{2}v$

$$M = R_E v - P(a+v) = \frac{P(a+L)}{L}v - \frac{1}{2}Qv - P(a+v) = \frac{Pa}{L}v - Pa - \frac{1}{2}Qv$$

Set $Q = 0$. $\frac{\partial U_{DE}}{\partial Q} = \frac{1}{EI}\int_0^{\frac{L}{2}}\left(\frac{Pa}{L}v - Pa\right)\left(-\frac{1}{2}v\right) = \frac{Pa}{2EIL}\int_0^{\frac{L}{2}}(-v^2 + Lv)\,dv$

$$= \frac{Pa}{2EIL}\left[-\frac{1}{3}\left(\frac{L}{2}\right)^3 + (L)\frac{1}{2}\left(\frac{L}{2}\right)^2\right] = \frac{Pa}{2EIL}\left[-\frac{L^3}{24} + \frac{L^3}{8}\right]$$

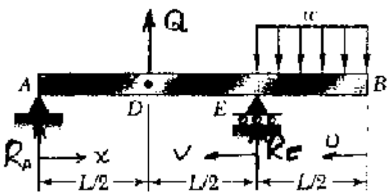
$$= \frac{1}{24}\,\frac{PaL^2}{EI}$$

Over portion EB: $M = -Pu$ $\qquad \frac{\partial M}{\partial Q} = 0$ $\qquad \frac{\partial U_{EB}}{\partial Q} = 0$

$$\delta_D = \frac{\partial U_{AD}}{\partial Q} + \frac{\partial U_{DE}}{\partial Q} + \frac{\partial U_{EB}}{\partial Q} = \frac{PaL^2}{48EI} + \frac{PaL^2}{24EI} + 0 = \frac{1}{16}\,\frac{PaL^2}{EI}\ \uparrow$$

Problem 11.90

11.89 and 11.90 For the prismatic beam shown, determine the deflection at point D.



Add force Q at point D.

Reactions: $R_A = -\frac{1}{8}wL - \frac{1}{2}Q$, $R_E = \frac{5}{8}wL - \frac{1}{2}Q$

$U = U_{AD} + U_{DE} + U_{EB}$; $\quad \delta_D = \frac{\partial U}{\partial Q}$

Over portion AD: $U_{AD} = \int_0^{L/2} \frac{M^2}{2EI}\,dx$, $\quad M = R_A x = -2wLx - \frac{1}{2}Qx$, $\quad \frac{\partial M}{\partial x} = -\frac{1}{2}x$

Set $Q = 0$. $\quad M = -\frac{1}{8}wLx$

$$\frac{\partial U_{AD}}{\partial Q} = \int_0^{L/2} \frac{M}{EI}\frac{\partial M}{\partial Q}\,dx = \int_0^{L/2}\left(-\frac{wLx}{8EI}\right)\left(-\frac{1}{2}x\right)dx = \frac{wL}{16EI}\int_0^{L/2} x^2\,dx = \frac{wL^4}{384EI}$$

Over portion DE: $U_{DE} = \int_0^{L/2} \frac{M^2}{2EI}\,dv$

$M = R_E v - \frac{wL}{2}\left(v + \frac{L}{4}\right) = \frac{1}{8}wL(v - L) - \frac{1}{2}Qv \qquad \frac{\partial M}{\partial Q} = -\frac{1}{2}v$

Set $Q = 0 \qquad M = -\frac{wL}{8}(L - v)$

$$\frac{\partial U}{\partial Q} = \int_0^{L/2} \frac{M}{EI}\frac{\partial M}{\partial Q}\,dv = \frac{wL}{16EI}\int_0^{L/2}(L - v)\,v\,dv = \frac{wL}{16EI}\left(\frac{Lv^2}{2} - \frac{v^3}{3}\right)_0^{L/2}$$

$$= \frac{wL^4}{16EI}\left(\frac{1}{8} - \frac{1}{24}\right) = \frac{wL^4}{192EI}$$

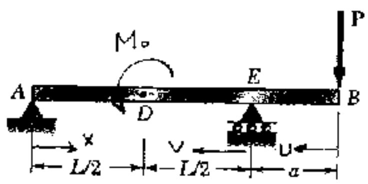
Over portion EB: $M = -\frac{1}{2}wu^2 \qquad \frac{\partial M}{\partial Q} = 0 \qquad U_{EB} = \int_0^{L/2} \frac{M^2}{2EI}\,du$

$$\frac{\partial U_{EB}}{\partial Q} = \int_0^{L/2} \frac{M}{EI}\frac{\partial M}{\partial Q}\,du = 0$$

$$\delta_D = \frac{\partial U_{AD}}{\partial Q} + \frac{\partial U_{DE}}{\partial Q} + \frac{\partial U_{EB}}{\partial Q} = \frac{wL^4}{384EI} + \frac{wL^4}{192EI} + 0 = \frac{wL^4}{128EI} \uparrow$$

Problem 11.91

11.91 and 11.92 For the prismatic beam shown, determine the slope at point D.



Add couple M_o at point D.

Reactions: $R_A = -\frac{Pa}{L} + \frac{M_o}{L}$, $R_E = \frac{P(a+L)}{L} - \frac{M_o}{L}$

$U = U_{AD} + U_{DE} + U_{EB}$ $\qquad \theta_D = \frac{\partial U}{\partial M_o}$

Over portion AD: $U_{AD} = \int_0^{L/2} \frac{M^2}{2EI}\,dx$, $M = R_A x = -\frac{Pa}{L}x + \frac{M_o}{L}x$, $\frac{\partial M}{\partial M_o} = \frac{1}{L}x$

Set $M_o = 0$. $\frac{\partial U_{AD}}{\partial M_o} = \frac{1}{EI}\int_0^{L/2}\left(-\frac{Pa}{L}x\right)\left(\frac{1}{L}x\right) = -\frac{Pa}{EIL^2}\int_0^{L/2} x^2\,dx$

$= -\frac{Pa}{EIL^2}\,\frac{1}{3}\left(\frac{L}{2}\right)^3 = -\frac{PaL}{24EI}$

Over portion DE: $U_{DE} = \int_0^{L/2}\frac{M^2}{2EI}\,dv$ $\qquad \frac{\partial M}{\partial M_o} = -\frac{1}{L}v$

$M = R_E v - P(a+v) = \frac{P(a+L)}{L}v - \frac{M_o}{L}v - P(a+v) = \frac{Pa}{L}v - Pa - \frac{M_o}{L}v$

Set $M_o = 0$. $\frac{\partial U_{DE}}{\partial M_o} = \frac{1}{EI}\int_0^{L/2}\left(\frac{Pa}{L}v - Pa\right)\left(-\frac{1}{L}v\right)dv = -\frac{Pa}{EIL^2}\int_0^{L/2}(v^2 - Lv)\,dv$

$= -\frac{Pa}{EIL^2}\left[\frac{1}{3}\left(\frac{L}{2}\right)^3 - L\cdot\frac{1}{2}\left(\frac{L}{2}\right)^2\right] = -\frac{Pa}{EIL^2}\left[\frac{1}{24}L^3 - \frac{1}{8}L^3\right]$

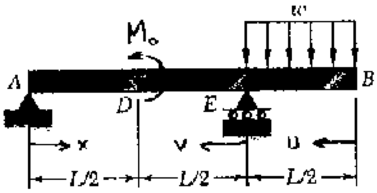
$= \frac{1}{12}\frac{PaL}{EI}$

Over portion EB: $M = -Pu$ $\qquad \frac{\partial M}{\partial M_o} = 0$ $\qquad \frac{\partial U_{EB}}{\partial M_o} = 0$

Total $\quad \theta_D = \frac{\partial U_{AD}}{\partial M_o} + \frac{\partial U_{DE}}{\partial M_o} + \frac{\partial U_{EB}}{\partial M_o} = -\frac{1}{24}\frac{PaL}{EI} + \frac{1}{12}\frac{PaL}{EI} + 0 = \frac{1}{24}\frac{PaL}{EI}$ ∠ ◄

Problem 11.92

***11.91* and 11.92** For the prismatic beam shown, determine the slope at point D.



Add couple M_0 at point D.

Reactions. $R_A = \frac{M_0}{L} - \frac{wL}{8}$, $R_E = -\frac{M_0}{L} + \frac{5wL}{8}$

$U = U_{AD} + U_{DE} + U_{EB}$; $\quad \theta_D = \frac{\partial U}{\partial M_0}$

Over portion AD: $U_{AD} = \int_0^{L/2} \frac{M^2}{2EI}\,dx$, $\quad M = R_A x = \frac{M_0 x}{L} - \frac{wLx}{8} \qquad \frac{\partial M}{\partial M_0} = \frac{x}{L}$

Set $M_0 = 0$. $\quad M = -\frac{wLx}{8}$

$$\frac{\partial U_{AD}}{\partial M_0} = \int_0^{L/2} \frac{M}{EI}\frac{\partial M}{\partial M_0}\,dx = \int_0^{L/2}\left(-\frac{wLx}{8EI}\right)\left(\frac{x}{L}\right) = -\frac{w}{8EI}\int_0^{L/2} x^2\,dx = -\frac{wL^3}{192\,EI}$$

Over portion DE: $U_{DE} = \int_0^{L/2} \frac{M^2}{2EI}\,dv$

$$M = R_E v - \frac{wL}{2}\left(v + \frac{L}{4}\right) = -\frac{M_0 v}{L} + \frac{wL}{8}(v - L) \qquad \frac{\partial M}{\partial M_0} = -\frac{v}{L}$$

Set $M_0 = 0$. $\quad M = -\frac{1}{8}wL(L - v)$

$$\frac{\partial U_{DE}}{\partial M_0} = \frac{w}{8EI}\int_0^{L/2}(L - v)v\,dv = \frac{w}{8EI}\left(\frac{Lv^2}{2} - \frac{v^3}{3}\right)\Big|_0^{L/2} = \frac{wL^3}{96\,EI}$$

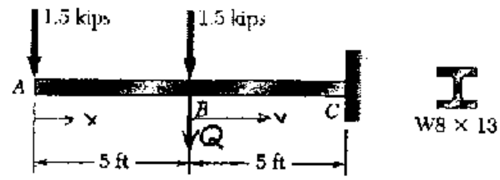
Over portion EB $\quad U_{EB} = \int_0^{L/2}\frac{M^2}{2EI}\,du \qquad M = -\frac{1}{2}wu^2 \qquad \frac{\partial M}{\partial M_0} = 0$

$$\frac{\partial U_{EB}}{\partial M_0} = \int_0^{L/2}\frac{M}{EI}\frac{\partial M}{\partial M_0}\,du = 0$$

$$\theta_D = \frac{\partial U_{AD}}{\partial M_0} + \frac{\partial U_{DE}}{\partial M_0} + \frac{\partial U_{EB}}{\partial M_0} = -\frac{wL^3}{192\,EI} + \frac{wL^3}{96\,EI} + 0 = \frac{wL^3}{192\,EI}$$

Problem 11.93

11.93 For the beam and loading shown, determine the deflection of point B. Use $E = 29 \times 10^6$ psi.



Add force Q at point B

Units: forces in kips, lengths in ft.

$E = 29 \times 10^3$ ksi $\quad I = 39.6$ in^4

$EI = (29 \times 10^3)(39.6) = 1.148 \times 10^6$ kip·in^2 $= 7975$ kip·ft^2

$U = \int_0^5 \frac{M^2}{2EI} dx + \int_0^5 \frac{M^2}{2EI} dv \qquad \delta_B = \frac{\partial U}{\partial Q} = \frac{1}{EI}\left\{ \int_0^5 M \frac{\partial M}{\partial Q} dx + \int_0^5 M \frac{\partial M}{\partial Q} dv \right\}$

Over AB: $M = -1.5x, \quad \frac{\partial M}{\partial Q} = 0 \qquad \int_0^5 M \frac{\partial M}{\partial Q} dx = 0$

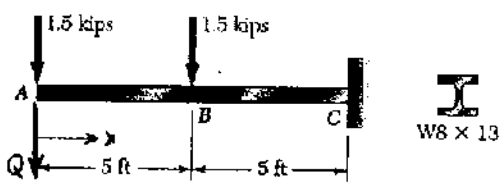
Over BC: $M = -1.5(v+5) - 1.5v - Qv = -3v - 7.5 - Qv, \quad \frac{\partial M}{\partial Q} = -v$

$$\int_0^5 M \frac{\partial M}{\partial Q} dv = \int_0^5 (3v^2 + 7.5\, v)\, dv = (3)(\tfrac{1}{3})(5)^3 + (7.5)(\tfrac{1}{2})(5)^2 = 218.75$$

$$\delta_B = \frac{1}{EI}\{0 + 143.75\} = \frac{218.75}{7975} = 27.43 \times 10^{-3} \text{ ft} = 0.329 \text{ in.} \downarrow$$

Problem 11.94

11.94 For the beam and loading shown, determine the deflection of point A. Use $E = 29 \times 10^6$ psi.



Add force Q at point A.

Units: forces in kips, lengths in ft.

$E = 29 \times 10^3$ ksi, $\quad I = 39.6$ in^4

$EI = (29 \times 10^3)(39.6) = 1.148 \times 10^6$ kip·in^2 $= 7975$ kip·ft^2

$U = \int_0^{10} \frac{M^2}{2EI} dx \qquad \delta_A = \frac{\partial U}{\partial Q} = \frac{1}{EI}\int_0^{10} M \frac{\partial M}{\partial Q} dx$

Over portion AB $\quad 0 < x < 5, \quad M = -1.5x - Qx \qquad \frac{\partial M}{\partial Q} = -x$

$$\int_0^5 M \frac{\partial M}{\partial Q} dx = \int_0^5 (1.5x)(x)\,dx = 1.5\int_0^5 x^2 dx = (1.5)(\tfrac{1}{3})(5)^3 = 62.5$$

Over portion BC $\quad 5 < x < 10 \qquad M = -1.5x - 1.5(x-5) - Qx$

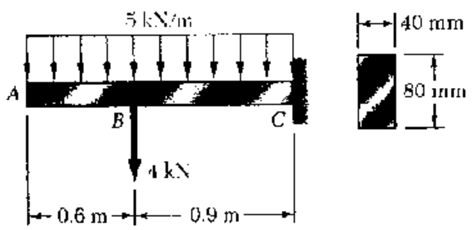
$M = -3x + 7.5 - Qx \qquad \frac{\partial M}{\partial Q} = -x$

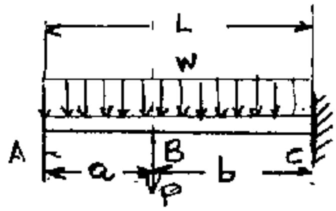
$$\int_5^{10} M \frac{\partial M}{\partial Q} dx = \int_5^{10} (3x^2 - 7.5x)\,dx = (3)(\tfrac{1}{3})(10^3 - 5^3) - (7.5)(\tfrac{1}{2})(10^2 - 5^2)$$
$$= 593.75$$

$$\delta_A = \frac{1}{EI}\{62.5 + 593.75\} = \frac{656.29}{7975} = 82.29 \times 10^{-3} \text{ ft} = 0.987 \text{ in.} \downarrow$$

Problem 11.95

11.95 For the beam and loading shown, determine the deflection at point B. Use E = 200 GPa.





$$U = \int_0^a \frac{M^2}{2EI}\,dx + \int_a^L \frac{M^2}{2EI}\,dx$$

$$\delta_B = \frac{\partial U}{\partial P} = \int_0^a \frac{M}{EI}\frac{\partial M}{\partial P}\,dx + \int_a^L \frac{M}{EI}\frac{\partial M}{\partial P}\,dx$$

Portion AB $(0 \le x \le a)$

$$M = -\tfrac{1}{2}wx^2 \qquad \frac{\partial M}{\partial P} = 0$$

$$\int_0^a \frac{M}{EI}\frac{\partial M}{\partial P}\,dx = 0$$

Portion BC $(a < x \le L)$

$$M = -\tfrac{1}{2}wx^2 - P(x-a)$$

$$\frac{\partial M}{\partial P} = -(x-a)$$

$$\int_a^L \frac{M}{EI}\frac{\partial M}{\partial P}\,dx = \frac{w}{2EI}\int_a^L x^2(x-a)\,dx + \frac{P}{EI}\int_a^L (x-a)^2\,dx$$

$$= \frac{w}{2EI}\int_a^L (x^3 - ax^2)\,dx + \frac{P}{EI}\int_0^b v^2\,dv$$

$$= \frac{w}{2EI}\left(\frac{L^4}{4} - \frac{aL^3}{3} - \frac{a^4}{4} + \frac{a^4}{3}\right) + \frac{Pb^3}{3EI}$$

$$\delta_B = 0 + \frac{w}{2EI}\left(\frac{L^4}{4} - \frac{aL^3}{3} + \frac{a^4}{12}\right) + \frac{Pb^3}{3EI}$$

Data: $a = 0.6\text{ m}$, $b = 0.9\text{ m}$, $L = a + b = 1.5\text{ m}$

$w = 5\times10^3\text{ N/m} \qquad P = 4\times10^3\text{ N}$

$I = \frac{1}{12}(40)(80)^3 = 1.70667\times10^6\text{ mm}^4 = 1.70667\times10^{-6}\text{ m}^4$

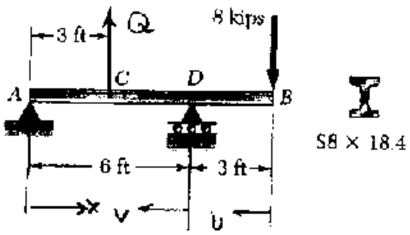
$EI = (200\times10^9)(1.70667\times10^{-6}) = 341\,333\text{ N}\cdot\text{m}^2$

$$\delta_B = 0 + \frac{5\times10^3}{(2)(341333)}\left[\frac{(1.5)^4}{4} - \frac{(0.6)(1.5)^3}{3} + \frac{(0.6)^4}{12}\right] + \frac{(4\times10^3)(0.9)^3}{(3)(341333)}$$

$$= 7.25\times10^{-3}\text{ m} \qquad = 7.25\text{ mm}\downarrow$$

Problem 11.96

11.96 For the beam and loading shown, determine the deflection at point C. Use $E = 29 \times 10^6$ psi.



Units: Forces in kip, lengths in ft.

$E = 29 \times 10^3$ ksi $\quad I = 57.6$ in^4

$EI = (29 \times 10^3)(57.6) = 1.6704 \times 10^6$ kip·in^2 $= 11600$ kip·ft^2

Add dummy force Q at point C. Reactions $R_A = 4 + \frac{1}{2}Q \downarrow$, $R_D = 12 - \frac{1}{2}Q \uparrow$

$U = U_{AC} + U_{CD} + U_{DB} \qquad \delta_c = \frac{\partial U}{\partial Q} = \frac{\partial U_{AC}}{\partial Q} + \frac{\partial U_{CD}}{\partial Q} + \frac{\partial U_{DB}}{\partial Q}$

Over AC $\quad 0 < x < 3 \qquad M = -(4 + \frac{1}{2}Q)x \qquad \frac{\partial M}{\partial Q} = -\frac{1}{2}x \qquad$ Set $Q = 0$.

$$\frac{\partial U_{AC}}{\partial Q} = \frac{1}{EI}\int_0^3 (4x)(\tfrac{1}{2}x)\,dx = \frac{2}{EI}\int_0^3 x^2\,dx = \frac{(2)(3)^3}{3EI} = \frac{18}{EI}$$

Over CD $\quad 0 < v < 3 \qquad M = R_B v - 8(v+3) = 12v - \frac{1}{2}Q - 8v - 24 = 4v - 24 - \frac{1}{2}Qv$

$\frac{\partial M}{\partial Q} = -\frac{1}{2}v \qquad$ Set $Q = 0$

$$\frac{\partial U_{CD}}{\partial Q} = \frac{1}{EI}\int_0^3 (24 - 4v)(\tfrac{1}{2}v)\,dv = \frac{1}{EI}\int_0^3 (12v - 2v^2)\,dv = \frac{1}{EI}\left\{(12)\frac{(3)^2}{2} - (2)\frac{(3)^3}{3}\right\}$$

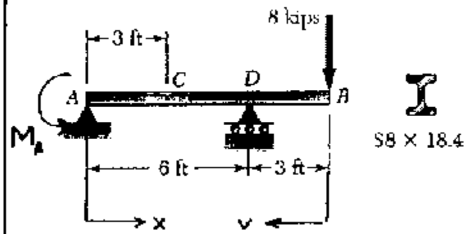
$$= \frac{36}{EI}$$

Over DB $\quad 0 < u < 3 \qquad M = -8u \qquad \frac{\partial M}{\partial Q} = 0 \qquad \frac{\partial U_{DB}}{\partial Q} = 0$

$$\delta_c = \frac{18}{EI} + \frac{36}{EI} + 0 = \frac{54}{11600} = 4.655 \times 10^{-3} \text{ ft} = 0.0559 \text{ in.} \uparrow$$

Problem 11.97

11.97 For the beam and loading shown, determine the slope at end A. Use $E = 29 \times 10^3$ ksi



Units: Forces in kips, lengths in ft.

$E = 29 \times 10^3$ ksi, $I = 57.6$ in^4

$EI = (29 \times 10^3)(57.6) = 1.6704 \times 10^6$ kip·in^2 = 11600 kip·ft^2

Add dummy couple M_A at end A. Reactions: $R_A = -4 + \frac{M_A}{6}$, $R_B = 12 - \frac{M_A}{6}$

$$U = U_{AD} + U_{DB} \qquad \theta_A = \frac{\partial U}{\partial M_A} = \frac{\partial U_{AD}}{\partial M_A} + \frac{\partial U_{DB}}{\partial M_A}$$

Over AD $\quad 0 < x < 6 \qquad M = -M_A + R_A x = -M_A - 4x + \frac{M_A}{6}x$

$$\frac{\partial M}{\partial M_A} = -\left(1 - \frac{x}{6}\right) \qquad \text{Set } M_A = 0.$$

$$\frac{\partial U_{AD}}{\partial M_A} = \frac{1}{EI}\int_0^6 (4x)\left(1 - \frac{x}{6}\right)dx = \frac{1}{EI}\int_0^6 \left(4x - \frac{2}{3}x^2\right)dx = \frac{1}{EI}\left\{(4)\frac{6^2}{2} - \frac{2}{3}\,\frac{6^3}{3}\right\}$$

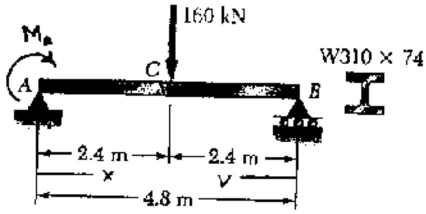
$$= \frac{24}{EI}$$

Over DB $\quad 0 < v < 3 \qquad M = -8v \qquad \frac{\partial M}{\partial M_A} = 0 \qquad \frac{\partial U_{DB}}{\partial M_A} = 0$

$$\theta_A = \frac{24}{EI} + 0 = \frac{24}{11600} = 2.07 \times 10^{-3} \text{ rad}$$

Problem 11.98

11.98 For the beam and loading shown, determine the slope at end A. Use $E = 200$ GPa.



Add couple M_A at point A.

Units: forces in kN, lengths in m.

$E = 200 \times 10^9$ Pa, $I = 165 \times 10^6$ mm^4 $= 165 \times 10^{-6}$ m^4

$EI = (200 \times 10^9)(165 \times 10^{-6}) = 33 \times 10^6$ N·m^2 $= 33000$ kN·m^2

Reactions: $R_A = 80 - \frac{M_A}{4.8}$ $\qquad R_B = 80 + \frac{M_A}{L}$

$$U = U_{AB} + U_{BC} = \int_0^{2.4} \frac{M^2}{2EI}dx + \int_0^{2.4} \frac{M^2}{2EI}dv \qquad \theta_A = \frac{\partial U}{\partial M_A} = \frac{\partial U_{AB}}{\partial M_A} + \frac{\partial U_{BC}}{\partial M_A}$$

Over AB: $\quad M = M_A + R_A x = M_A + 80x - \frac{M_A}{4.8}x \qquad \frac{\partial M}{\partial M_A} = \left(1 - \frac{x}{4.8}\right)$

Set $M_A = 0$. $\quad \frac{\partial U_{AB}}{\partial M_A} = \frac{1}{EI}\int_0^{2.4}(80x)\left(1 - \frac{x}{4.8}\right)dx = \frac{1}{EI}\int_0^{2.4}(80x - 16.6667x^2)dx$

$$= \frac{1}{EI}\left\{(80)\left(\tfrac{1}{2}\right)(2.4)^2 - (16.6667)\left(\tfrac{1}{3}\right)(2.4)^3\right\} = \frac{153.6}{EI}$$

Over BC: $\quad M = R_B v = 80v + \frac{M_A}{4.8}v, \qquad \frac{\partial M}{\partial M_A} = \frac{1}{4.8}v$

Set $M_A = 0$. $\quad \frac{\partial U_{BC}}{\partial M_A} = \frac{1}{EI}\int_0^{2.4}(80v)\left(\frac{1}{4.8}v\right)dv = \frac{16.6667}{EI}\int_0^{2.4}v^2\,dv$

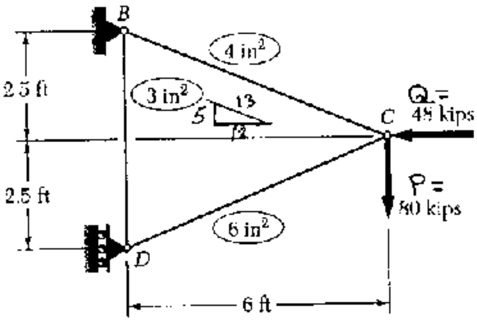
$$= \frac{(16.6667)(2.4)^3}{3\,EI} = \frac{76.8}{EI}$$

$$\theta_A = \frac{1}{EI}\{153.6 + 76.8\} = \frac{230.4}{33000} = 6.98 \times 10^{-3} \text{ rad.}$$

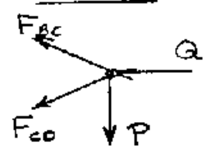
Problem 11.99

11.99 and 11.100 Each member of the truss shown is made of steel and has the cross-sectional area shown. Using $E = 29 \times 10^6$ psi, determine the deflection indicated.

11.99 Vertical deflection of joint C.



Joint C



$\xrightarrow{+} \Sigma F_x = 0:$

$$-\frac{12}{13}F_{BC} - \frac{12}{13}F_{CD} - Q = 0$$

$$F_{BC} + F_{CD} = -\frac{13}{12}Q \qquad (1)$$

$+\uparrow \Sigma F_y = 0:$

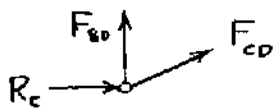
$$\frac{5}{13}F_{BC} - \frac{5}{13}F_{CD} - P = 0$$

$$F_{BC} - F_{CD} = \frac{13}{5}P \qquad (2)$$

Solving (1) and (2) simultaneously,

$$F_{BC} = \frac{13}{10}P - \frac{13}{24}Q \qquad F_{CD} = -\frac{13}{10}P - \frac{13}{24}Q$$

Joint D



$+\uparrow \Sigma F_y = 0: \quad \frac{5}{13}F_{CD} + F_B = 0$

$$F_{BD} = -\frac{5}{13}F_{CD} = \frac{1}{2}P + \frac{5}{24}Q$$

Lengths of members: $L_{BC} = 78$ in, $L_{CD} = 78$ in., $L_{BC} = 60$ in.

$$U = \Sigma \frac{F^2 L}{2EA}$$

$$\delta_P = \frac{\partial U}{\partial P} = \Sigma \frac{FL}{EA}\frac{\partial F}{\partial P} = \frac{1}{E}\Sigma \frac{FL}{A}\frac{\partial F}{\partial P}$$

Member	F	L (in.)	$\frac{\partial F}{\partial P}$	A (in²)	$\frac{FL}{A}\frac{\partial F}{\partial P}$
BC	$\frac{13}{10}P - \frac{13}{24}Q$	78	$\frac{13}{10}$	4	$32.955P - 13.73125Q$
CD	$-\frac{13}{10}P - \frac{13}{24}Q$	78	$-\frac{13}{10}$	6	$21.97P + 9.15417Q$
BD	$\frac{1}{2}P + \frac{5}{24}Q$	60	$\frac{1}{2}$	3	$5.00P + 2.08333Q$
Σ					$59.975P - 2.49375Q$

Further data: $E = 29 \times 10^6$ psi $= 29000$ ksi

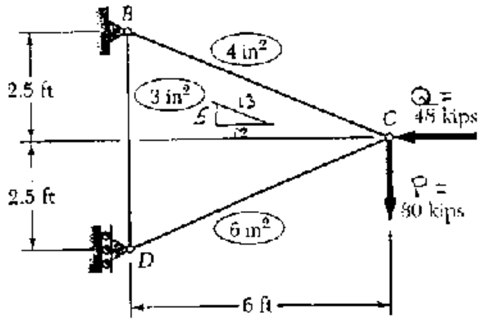
$P = 80$ kips, $Q = 48$ kips

$$\delta_P = \frac{(59.975)(80) + (-2.49375)(48)}{29000} = 0.1613 \text{ in.} \downarrow$$

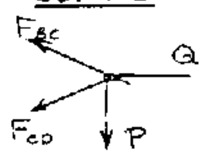
Problem 11.100

11.99 and 11.100 Each member of the truss shown is made of steel and has the cross-sectional area shown. Using $E = 29 \times 10^6$ psi, determine the deflection indicated.

11.100 Horizontal deflection of point C



Joint C



$\overset{+}{\rightarrow} \Sigma F_x = 0:$

$$-\frac{12}{13} F_{BC} - \frac{12}{13} F_{CD} - Q = 0$$

$$F_{BC} + F_{CD} = -\frac{13}{12} Q \qquad (1)$$

$+\uparrow \Sigma F_y = 0:$

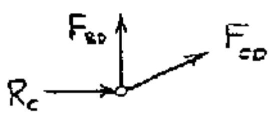
$$\frac{5}{13} F_{BC} - \frac{5}{13} F_{CD} - P = 0$$

$$F_{BC} - F_{CD} = \frac{13}{5} P \qquad (2)$$

Solving (1) and (2) simultaneously,

$$F_{BC} = \frac{13}{10} P - \frac{13}{24} Q \qquad F_{CD} = -\frac{13}{10} P - \frac{13}{24} Q$$

Joint D



$+\uparrow \Sigma F_y = 0: \quad \frac{5}{13} F_{CD} + F_{B} = 0$

$$F_{BD} = -\frac{5}{13} F_{CD} = \frac{1}{2} P + \frac{5}{24} Q$$

Lengths of members: $L_{BC} = 78$ in., $L_{CD} = 78$ in., $L_{BC} = 60$ in.

$$U = \Sigma \frac{F^2 L}{2EA}$$

$$\delta_Q = \frac{\partial U}{\partial Q} = \Sigma \frac{FL}{EA} \frac{\partial F}{\partial Q} = \frac{1}{E} \Sigma \frac{FL}{A} \frac{\partial F}{\partial Q}$$

Member	F	L (in.)	$\frac{\partial F}{\partial Q}$	A (in²)	$\frac{FL}{A} \frac{\partial F}{\partial Q}$
BC	$\frac{13}{10}P - \frac{13}{24}Q$	78	$-\frac{13}{24}$	4	$-13.73125P + 5.72135Q$
CD	$-\frac{13}{10}P - \frac{13}{24}Q$	78	$-\frac{13}{24}$	6	$9.15467P + 3.81424Q$
BD	$\frac{1}{2}P + \frac{5}{24}Q$	60	$\frac{5}{24}$	3	$2.08333P + 0.86806Q$
Σ					$-2.49325P + 10.40365Q$

Further data: $E = 29 \times 10^6$ psi $= 29000$ ksi

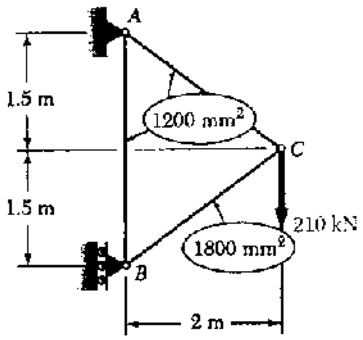
$P = 80$ kips, $Q = 48$ kips

$$\delta_Q = \frac{(-2.49325)(80) + (10.40365)(48)}{29000} = 0.01034 \text{ in.} \leftarrow$$

Problem 11.101

11.101 and *11.102* Each member of the truss shown is made of steel and has the cross-sectional area shown. Using $E = 200$ GPa, determine the deflection indicated.

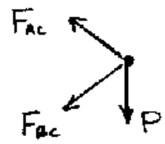
11.101 Vertical deflection of joint C.



Call the vertical load P. The vertical deflection of joint C is δ_P.

$$\delta_P = \frac{\partial U}{\partial P} = \frac{\partial}{\partial P}\Sigma\frac{F^2L}{2EA} = \frac{1}{E}\Sigma\frac{FL}{A}\frac{\partial F}{\partial P}$$

Joint C: $\xrightarrow{+}\Sigma F_x = 0: \quad -\frac{4}{5}F_{AC} - \frac{4}{5}F_{BC} = 0$

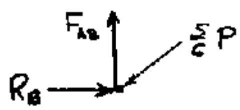


$+\uparrow\Sigma F_y \qquad \frac{3}{5}F_{AC} + \frac{3}{5}F_{BC} - P = 0$

Solving simultaneously,

$F_{AC} = \frac{5}{6}P \qquad F_{BC} = -\frac{5}{6}P$

Joint B $\quad +\uparrow\Sigma F_y = 0:$



$F_{AB} - \frac{3}{5}\cdot\frac{5}{6}P = 0$

$F_{AB} = \frac{1}{2}P$

Member	F	$\partial F/\partial P$	L (m)	A (10^{-6} m²)	$F(\partial F/\partial P)L/A$
AB	$\frac{1}{2}P$	$\frac{1}{2}$	3	1200	625 P
AC	$\frac{5}{6}P$	$\frac{5}{6}$	2.5	1200	1446.76 P
BC	$-\frac{5}{6}P$	$-\frac{5}{6}$	2.5	1800	964.51 P
Σ					3036.27 P

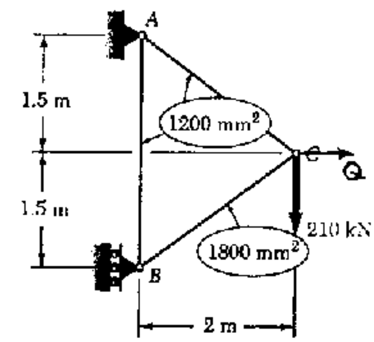
$$\delta_P = \frac{1}{E}(3036.27\,P) = \frac{(3036.27)(210\times10^3)}{200\times10^9} = 3.19\times10^{-3}\text{ m}$$

$$= 3.19 \text{ mm} \downarrow$$

Problem 11.102

11.101 and *11.102* Each member of the truss shown is made of steel and has the cross-sectional area shown. Using $E = 200$ GPa, determine the deflection indicated.

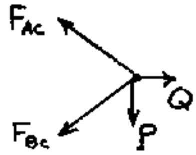
11.102 Horizontal deflection of point C.



Call the vertical force P. Add a dummy horizontal force Q (positive →) at joint C. The horizontal deflection of joint C is δ_Q.

$$\delta_Q = \frac{\partial U}{\partial Q} = \frac{\partial}{\partial Q}\sum \frac{F^2 L}{2EI} = \frac{1}{E}\sum \frac{FL}{A}\frac{\partial F}{\partial Q}$$

Joint C $\quad \stackrel{+}{\rightarrow} \Sigma F_x = 0:$



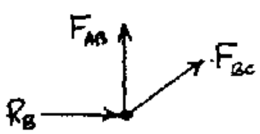
$$-\tfrac{4}{5}F_{AC} - \tfrac{4}{5}F_{BC} + Q = 0$$

$+\uparrow \Sigma F_y = 0:$

$$\tfrac{3}{5}F_{AC} - \tfrac{3}{5}F_{BC} - P = 0$$

Solving simultaneously $\quad F_{AC} = \frac{5}{6}P + \frac{5}{8}Q \qquad F_{BC} = -\frac{5}{6}P + \frac{5}{8}Q$

Joint B $\quad +\uparrow \Sigma F_y = 0:$



$$F_{AB} + \tfrac{3}{5}F_{BC} = 0$$

$$F_{AB} = -\tfrac{3}{5}F_{BC} = \tfrac{1}{2}P - \tfrac{3}{8}Q$$

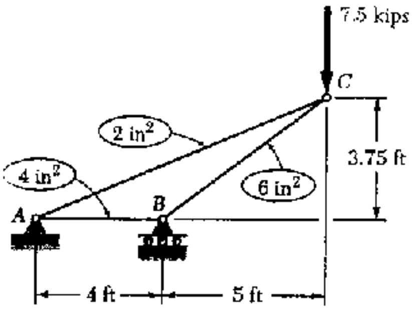
Member	F	$\partial F/\partial Q$	L (m)	A (10^{-6} m²)	with Q = 0 $F(\partial F/\partial Q)L/A$
AB	$\frac{1}{2}P - \frac{3}{8}Q$	$-\frac{3}{8}$	3	1200	−468.75 P
AC	$\frac{5}{6}P + \frac{5}{8}Q$	$+\frac{5}{8}$	2.5	1200	1085.07 P
BC	$-\frac{5}{6}P + \frac{5}{8}Q$	$+\frac{5}{8}$	2.5	1800	−723.38 P
					−107.06 P

$$\delta_Q = \frac{1}{E}(-107.06\,P) = -\frac{(107.06)(210\times10^3)}{200\times10^9} = -0.1124\times10^{-3}\text{ m}$$

$$= 0.1124 \text{ mm} \leftarrow$$

Problem 11.103

***11.103* and 11.104** Each member of the truss shown is made of steel and has the cross-sectional area shown. Using $E = 29 \times 10^6$ psi, determine the deflection indicated.
11.103 Vertical deflection of joint *C*.



Call the vertical load P. The vertical deflection of joint C is δ_P.

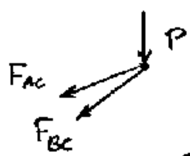
$$\delta_P = \frac{\partial U}{\partial P} = \frac{\partial}{\partial P}\sum \frac{F^2 L}{2EA} = \frac{1}{E}\sum \frac{FL}{A}\frac{\partial F}{\partial P}$$

Geometry $\overline{AC} = \sqrt{9^2 + 3.75^2} = 9.75 \text{ ft} = 117 \text{ in}$

$\overline{BC} = \sqrt{5^2 + 3.75^2} = 6.25 \text{ ft} = 75 \text{ in}$

$4 \text{ ft} = 48 \text{ in.}, \quad 5 \text{ ft} = 60 \text{ in}, \quad 3.75 \text{ ft} = 45 \text{ in.}$

Joint C

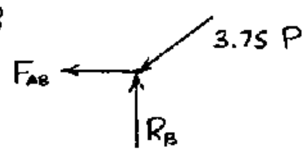


$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{108}{117}F_{AC} - \frac{60}{75}F_{BC} = 0$

$+\uparrow \Sigma F_y = 0: \quad -\frac{45}{117}F_{AC} - \frac{45}{75}F_{BC} - P = 0$

Solving simultaneously, $F_{AC} = 3.25\,P$, $F_{BC} = -3.75\,P$

Joint B



$\xrightarrow{+} \Sigma F_x = 0: \quad -F_{AB} - \frac{60}{75}F_{AC} = 0$

$F_{AB} = -3.00\,P.$

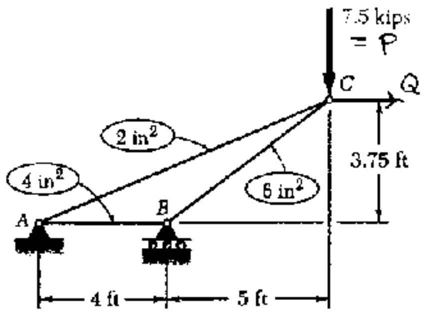
Member	F	∂F/∂P	L (in)	A (in²)	F(∂F/∂P)L/A
AB	-3.00 P	-3.00	48	4	108.00 P
AC	3.25 P	3.25	117	2	617.91 P
BC	-3.75 P	-3.75	75	6	175.78 P
Σ					901.69 P

$$\delta_P = \frac{901.69\,P}{E} = \frac{(901.69)(7.5 \times 10^3)}{29 \times 10^6} = 0.233 \text{ in.} \downarrow$$

Problem 11.104

11.103 and 11.104 Each member of the truss shown is made of steel and has the cross-sectional area shown. Using $E = 29 \times 10^6$ psi, determine the deflection indicated.

11.104 Horizontal deflection of point C.



Call the vertical load P. Add horizontal dummy load Q at joint C. The horizontal deflection of joint C is δ_Q.

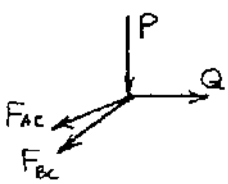
$$\delta_Q = \frac{\partial U}{\partial Q} = \frac{\partial}{\partial Q}\sum \frac{F^2 L}{2EA} = \frac{1}{E}\sum \frac{FL}{A}\frac{\partial F}{\partial Q}$$

Geometry: $\overline{AC} = \sqrt{9^2 + 3.75^2} = 9.75 \text{ ft} = 117 \text{ in.}$

$\overline{BC} = \sqrt{5^2 + 3.75^2} = 6.25 \text{ ft} = 75 \text{ in.}$

$4 \text{ ft} = 48 \text{ in.}$ $5 \text{ ft} = 60 \text{ in.}$ $3.75 \text{ ft} = 45 \text{ in.}$

Joint C



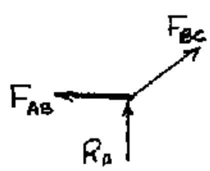
$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{108}{117}F_{AC} - \frac{60}{75}F_{BC} + Q = 0$

$+\uparrow \Sigma F_y = 0: \quad -\frac{45}{117}F_{AC} - \frac{45}{75}F_{BC} - P = 0$

Solving simultaneously, $F_{AC} = 3.25\,P + 2.4375\,Q$

$F_{BC} = -3.75\,P - 1.5625\,Q$

Joint B



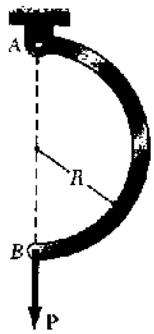
$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{4}{5}F_{AC} - F_{AB} = 0$

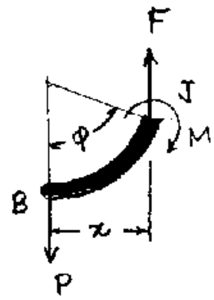
$F_{AB} = \frac{4}{5}F_{BC} = -3.00\,P - 1.25\,Q$

Member	F	∂F/∂Q	L (in.)	A (in²)	Q = 0 F(∂F/∂Q)L/A
AB	−3.00 P − 1.25 Q	−1.25	48	4	45.00 P
AC	3.25 P + 2.4375 Q	2.4375	117	2	463.43 P
BC	−3.75 P − 1.5625 Q	−1.5625	75	6	73.24 P
Σ					581.67 P

$$\delta_Q = \frac{581.67\,P}{E} = \frac{(581.67)(7.5\times10^3)}{29\times10^6} = 0.1504 \text{ in.} \rightarrow$$

Problem 11.105 **11.105** For the uniform rod and loading shown and using Castigliano's theorem, determine the deflection of point B.





Use polar coordinate φ.

Calculate the bending moment $M(\varphi)$ using free body BJ.

$$+\circlearrowleft \Sigma M_J = 0: \quad Px - M = 0$$

$$M = Px = PR\sin\varphi$$

Strain energy: $U = \int_0^L \frac{M^2}{2EI}\,ds$

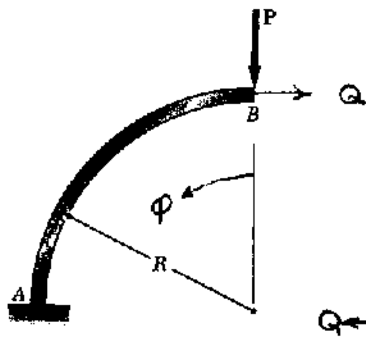
$$U = \int_0^{\pi} \frac{(PR\sin\varphi)^2}{2EI}(R\,d\varphi) = \frac{P^2R^3}{2EI}\int_0^{\pi}\sin^2\varphi\,d\varphi$$

$$= \frac{P^2R^3}{2EI}\int_0^{\pi}\frac{1-\cos 2\varphi}{2}\,d\varphi$$

$$= \frac{P^2R^2}{2EI}\left(\frac{1}{2}\varphi\Big|_0^{\pi} - \frac{1}{4}\sin 2\varphi\Big|_0^{\pi}\right) = \frac{\pi P^2 R}{4EI}$$

By Castigliano's theorem, $\delta = \frac{\partial U}{\partial P} = \frac{\pi P R^3}{2EI} \downarrow$ ◄

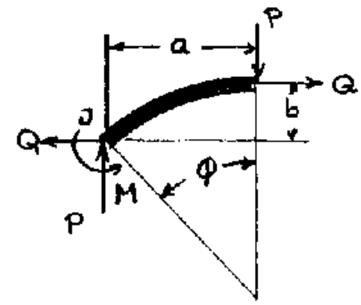
Problem 11.106 **11.106** For the beam and loading shown and using Castigliano's theorem, determine (*a*) the horizontal deflection of point *B*, (*b*) the vertical deflection of point *B*.



Add dummy load Q at point B.

Use polar coordinate φ.

$$U = \int_0^{\frac{\pi}{2}} \frac{M^2}{2EI} R\,d\varphi$$



Bending moment

$$+\circlearrowleft \Sigma M_J = 0: \quad M - Pa - Qb = 0$$

$$M = Pa + Qb$$
$$= PR\sin\varphi + QR(1-\cos\varphi)$$

$$\frac{\partial M}{\partial P} = R\sin\varphi \qquad \frac{\partial M}{\partial Q} = R(1-\cos\varphi) \qquad \text{Set } Q = 0$$

(a) $$\delta_Q = \frac{\partial U}{\partial Q} = \frac{1}{EI}\int_0^{\frac{\pi}{2}} M\frac{\partial M}{\partial Q} R\,d\varphi = \frac{1}{EI}\int_0^{\frac{\pi}{2}} PR\sin\varphi\; R(1-\cos\varphi)\,R\,d\varphi$$

$$= \frac{PR^3}{EI}\int_0^{\frac{\pi}{2}}(\sin\varphi - \sin\varphi\cos\varphi)\,d\varphi = \frac{PR^3}{EI}\left(-\cos\varphi - \frac{1}{2}\sin^2\varphi\right)\Big|_0^{\frac{\pi}{2}}$$

$$= \frac{PR^3}{EI}\left(-\cos\frac{\pi}{2} + \cos 0 - \frac{1}{2}\sin^2\frac{\pi}{2} + \frac{1}{2}\sin^2 0\right)$$

$$= \frac{PR^3}{EI}\left(0 + 1 - \frac{1}{2} + 0\right) = \frac{1}{2}\frac{PR^3}{EI} \rightarrow$$

(b) $$\delta_P = \frac{\partial U}{\partial P} = \frac{1}{EI}\int_0^{\frac{\pi}{2}} M\frac{\partial M}{\partial P} R\,d\varphi = \frac{1}{EI}\int_0^{\frac{\pi}{2}} PR\sin\varphi\; R\sin\varphi\; R\,d\varphi$$

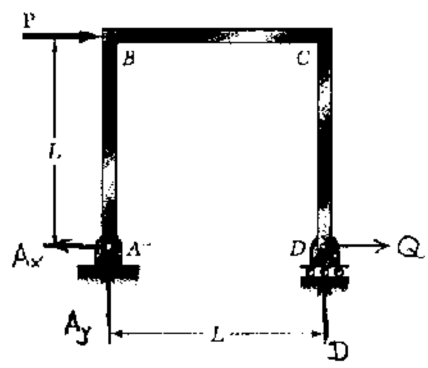
$$= \frac{PR^3}{EI}\int_0^{\frac{\pi}{2}} \sin^2\varphi\,d\varphi = \frac{PR^3}{EI}\int_0^{\frac{\pi}{2}} \frac{1}{2}(1-\cos 2\varphi)\,d\varphi$$

$$= \frac{PR^3}{EI}\left(\frac{1}{2}\varphi - \frac{1}{4}\sin 2\varphi\right)\Big|_0^{\frac{\pi}{2}} = \frac{PR^3}{EI}\left(\frac{1}{2}\cdot\frac{\pi}{2} - \frac{1}{2}\cdot 0 - \frac{1}{4}\sin\pi + \frac{1}{2}\cdot\sin 0\right)$$

$$= \frac{PR^3}{EI}\left(\frac{\pi}{4} - 0 - 0 + 0\right) = \frac{\pi}{4}\frac{PR^3}{EI} \downarrow$$

Problem 11.107

11.107 Three rods, each of the same flexural rigidity EI, are welded to form the frame $ABCD$. For the loading shown, determine the deflection at point D.



Add dummy force Q at point D as shown.

Statics $\circlearrowleft \Sigma M_A = 0: \; DL - PL = 0 \qquad D = P \uparrow$

$\xrightarrow{+} \Sigma F_x = 0: \; -A_x + P + Q = 0 \qquad A_x = (P+Q) \leftarrow$

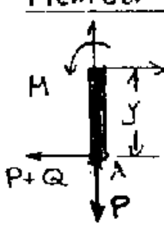
$+\uparrow \Sigma F_y = 0: \; A_y + D = 0 \qquad A_y = P \downarrow$

$$U = U_{AB} + U_{BC} + U_{CD}$$

By Castigliano's theorem, $\delta_D = \frac{\partial U}{\partial Q}$

$$\delta_D = \frac{\partial U_{AB}}{\partial Q} + \frac{\partial U_{BC}}{\partial Q} + \frac{\partial U_{CD}}{\partial Q}$$

Member AB

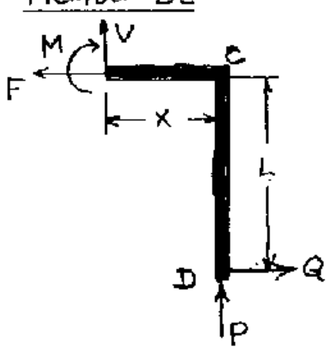


$M = (P+Q)y \qquad \frac{\partial M}{\partial Q} = y \qquad$ Set $Q = 0 \qquad M = Py$

$$U_{AB} = \int_0^L \frac{M^2 dy}{2EI}$$

$$\frac{\partial U_{AB}}{\partial Q} = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial Q} dy = \frac{P}{EI}\int_0^L y^2 dy = \frac{PL^3}{3EI}$$

Member BC



$M = Px + QL \qquad \frac{\partial M}{\partial Q} = L \qquad$ Set $Q = 0 \qquad M = Px$

$$U_{BC} = \int_0^L \frac{M^2 dx}{2EI}$$

$$\frac{\partial U_{BC}}{\partial Q} = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial Q} dx = \frac{PL}{EI}\int_0^L x\, dx = \frac{PL^3}{2EI}$$

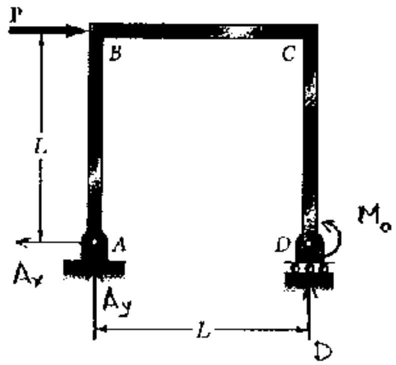
Member CD $\qquad M = Qy \qquad \frac{\partial M}{\partial Q} = y \qquad$ Set $Q = 0 \qquad M = 0$

$$U_{CD} = \int_0^L \frac{M^2 dy}{2EI} \qquad \frac{\partial U_{CD}}{\partial Q} = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial Q} dy = 0$$

$$\delta_D = \frac{PL^3}{3EI} + \frac{PL^3}{2EI} + 0 = \frac{5PL^3}{6EI} \rightarrow$$

Problem 11.108

11.108 Three rods, each of the same flexural rigidity EI, are welded to form the frame $ABCD$. For the loading shown, determine the angle formed by the frame at point D.



Add dummy couple M_o at point D.

Statics $\circlearrowleft + \Sigma M_A = 0: \quad M_o + DL - PL = 0$

$$D = P - \frac{M_o}{L} \uparrow$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad -A_x + P = 0 \qquad A_x = P \leftarrow$$

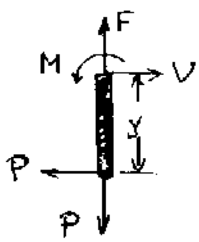
$$+\uparrow \Sigma F_y = 0: \quad A_y + D = 0 \qquad A_y = P - \frac{M_o}{L} \downarrow$$

$$U = U_{AB} + U_{BC} + U_{CD}$$

By Castigliano's theorem, $\theta_D = \frac{\partial U}{\partial M_o}$

$$\theta_D = \frac{\partial U_{AB}}{\partial M_o} + \frac{\partial U_{BC}}{\partial M_o} + \frac{\partial U_{CD}}{\partial M_o}$$

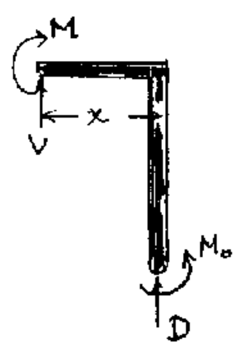
Member AB



$$M = Py \qquad \frac{\partial M}{\partial M_o} = 0 \qquad U_{AB} = \int_0^L \frac{M^2}{2EI}\,dy$$

$$\frac{\partial U_{AB}}{\partial M_o} = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial M_o}\,dy = 0$$

Member BC



$$M = M_o + Dx = M_o + Px - \frac{M_o x}{L}$$

Set $M_o = 0 \qquad M = Px$

$$U_{BC} = \int_0^L \frac{M^2}{2EI}\,dx$$

$$\frac{\partial U_{BC}}{\partial M_o} = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial M_o}\,dx = \frac{P}{EI}\int_0^L x\left(1 - \frac{x}{L}\right)dy = \frac{PL^2}{6EI}$$

Member CD $\qquad M = M_o \qquad \frac{\partial M}{\partial M_o} = 0$

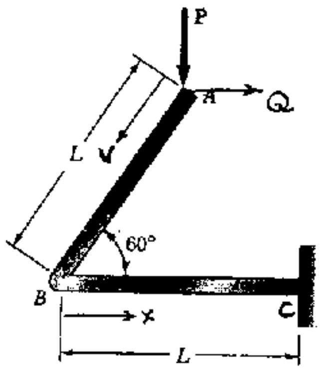
Set $M_o = 0 \qquad M = 0$

$$U = \int_0^L \frac{M^2}{2EI}\,dy \qquad \frac{\partial U}{\partial M_o} = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial M_o}\,dx = 0$$

$$\theta_D = 0 + \frac{PL^2}{6EI} + 0 = \frac{PL^2}{6EI} \measuredangle$$

Problem 11.109

11.109 A uniform rod of flexural rigidity EI is bent and loaded as shown. Determine (*a*) the vertical deflection of point A, (*b*) the horizontal deflection of point A



Add dummy horizontal force Q at point A.

Over AB $\quad M = \frac{1}{2}Pv + \frac{\sqrt{3}}{2}Qv$

$$\frac{\partial M}{\partial P} = \frac{1}{2}v \qquad \frac{\partial M}{\partial Q} = \frac{\sqrt{3}}{2}v$$

$$U_{AB} = \int_0^L \frac{M^2}{2EI}\,dx \qquad \text{Set } Q = 0$$

$$\frac{\partial U_{AB}}{\partial P} = \frac{1}{EI}\int_0^L M\frac{\partial M}{\partial P}\,dv = \frac{1}{EI}\int_0^L \left(\tfrac{1}{2}Pv\right)\left(\tfrac{1}{2}v\right)dv$$

$$= \frac{1}{12}\frac{PL^3}{EI}$$

$$\frac{\partial U_{AB}}{\partial Q} = \frac{1}{EI}\int_0^L M\frac{\partial M}{\partial Q}\,dv = \frac{1}{EI}\int_0^L \left(\tfrac{1}{2}Pv\right)\frac{\sqrt{3}}{2}\,dv$$

$$= \frac{\sqrt{3}}{12}\frac{PL^3}{EI}$$

Over BC $\quad M = -P\left(x - \frac{L}{2}\right) + \frac{\sqrt{3}}{2}QL, \quad \frac{\partial M}{\partial P} = -\left(x - \frac{L}{2}\right), \quad \frac{\partial M}{\partial Q} = \frac{\sqrt{3}}{2}L$

$$U_{BC} = \int_0^L \frac{M^2}{2EI}\,dx \qquad \text{Set } Q = 0.$$

$$\frac{\partial U_{BC}}{\partial P} = \frac{1}{EI}\int_0^L M\frac{\partial M}{\partial P}\,dx = \frac{1}{EI}\int_0^L P\left(x-\frac{L}{2}\right)^2 dx = \frac{P}{3EI}\left(x-\frac{L}{2}\right)^3\Big|_0^L = \frac{1}{12}\frac{PL^3}{EI}$$

$$\frac{\partial U_{BC}}{\partial Q} = \frac{1}{EI}\int_0^L M\frac{\partial M}{\partial Q}\,dx = \frac{-1}{EI}\int_0^L P\left(x-\frac{L}{2}\right)\left(\frac{\sqrt{3}}{2}\right)L\,dx = -\frac{\sqrt{3}P}{4EI}\left(x-\frac{L}{2}\right)^2\Big|_0^L = 0$$

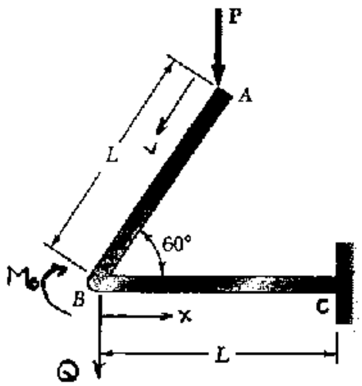
(a) vertical deflection of point A.

$$\delta_P = \frac{\partial U_{AB}}{\partial P} + \frac{\partial U_{BC}}{\partial P} = \frac{1}{6}\frac{PL^3}{EI}\ \downarrow$$

(b) horizontal deflection of point A.

$$\delta_Q = \frac{\partial U_{AB}}{\partial Q} + \frac{\partial U_{BC}}{\partial Q} = \frac{\sqrt{3}}{12}\frac{PL^3}{EI} = 0.1443\frac{PL^3}{EI} \rightarrow$$

Problem 11.110 *11.110* A uniform rod of flexural rigidity EI is bent and loaded as shown. Determine (*a*) the vertical deflection of point B, (*b*) the slope of BC at point B.



Add dummy vertical Q and dummy couple M_0 at B.

Over AB $\quad M = \frac{1}{2}Pv, \quad \frac{\partial M}{\partial Q} = 0, \quad \frac{\partial M}{\partial M_0} = 0$

$$U_{AB} = \int_0^L \frac{M^2}{2EI}\,dv$$

$$\frac{\partial U_{AB}}{\partial Q} = 0 \qquad \frac{\partial U_{AB}}{\partial M_0} = 0$$

Over BC $\quad M = -P(x - \frac{L}{2}) - Qx + M_0 \qquad \frac{\partial M}{\partial Q} = -x, \quad \frac{\partial M}{\partial M_0} = 1$

$$U_{BC} = \int_0^L \frac{M^2}{2EI}\,dx \qquad \text{Set } Q = 0 \text{ and } M_0 = 0.$$

$$\frac{\partial U_{BC}}{\partial Q} = \frac{1}{EI}\int_0^L M\frac{dM}{\partial Q}\,dx = \frac{1}{EI}\int_0^L P(x - \tfrac{L}{2})x\,dx = \frac{P}{EI}\left[\frac{L^3}{3} - \left(\frac{L}{2}\right)\frac{L^2}{2}\right]$$

$$= \frac{1}{12}\frac{PL^3}{EI}$$

$$\frac{\partial U_{BC}}{\partial M_0} = \frac{1}{EI}\int_0^L M\frac{\partial M}{\partial M_0}\,dx = \frac{-1}{EI}\int_0^L P(x - \tfrac{L}{2})\,dx = 0$$

(a) vertical deflection of point B.

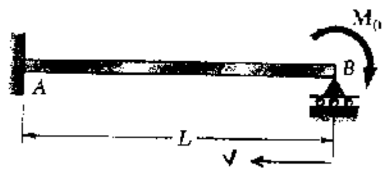
$$\delta_Q = \frac{\partial U_{AB}}{\partial Q} + \frac{\partial U_{BC}}{\partial Q} = \frac{1}{12}\frac{PL^3}{EI} \downarrow$$

(b) slope of BC at point B.

$$\theta_B = \frac{\partial U_{AB}}{\partial M_0} + \frac{\partial U_{BC}}{\partial M_0} = 0$$

Problem 11.111

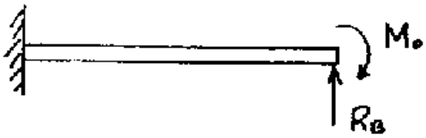
11.111 through 11.114 Determine the reaction at the roller support and the draw the bending-moment diagram for the beam and loading shown.



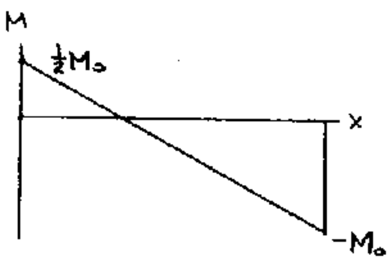
Remove support B and add reaction R_B as a load.

$$U = \int_0^L \frac{M^2}{2EI}\,dv$$

$$y_B = \frac{\partial U}{\partial R_B} = \frac{1}{EI}\int_0^L M\frac{\partial M}{\partial R_B}\,dv = 0$$



$$M = R_B v - M_o \qquad \frac{\partial M}{\partial R_B} = v$$



$$y_B = \frac{1}{EI}\int_0^L (R_B v - M_o)\,v\,dv$$

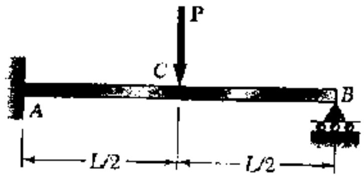
$$= \frac{R_B}{EI}\int_0^L v^2\,dv - \frac{M_o}{EI}\int_0^L v\,dv$$

$$= \frac{R_B L^3}{3EI} - \frac{M_o L^2}{2EI} = 0 \qquad R_B = \frac{3}{2}\frac{M_o}{L}\uparrow$$

$$M_A = R_B - M_o = \frac{3}{2}M_o - M_o = \frac{1}{2}M_o$$

Problem 11.112

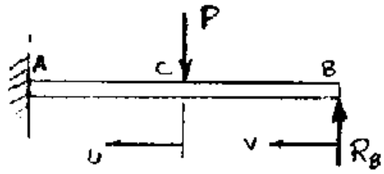
11.111 through 11.114 Determine the reaction at the roller support and the draw the bending-moment diagram for the beam and loading shown.



Remove support B and add reaction R_B as a load.

$$U = U_{AC} + U_{CB} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI}\,du + \int_0^{\frac{L}{2}} \frac{M^2}{2EI}\,dv$$

$$y_B = \frac{\partial U}{\partial R_B} = \frac{\partial U_{AB}}{\partial R_B} + \frac{\partial U_{CB}}{\partial R_B} = 0$$



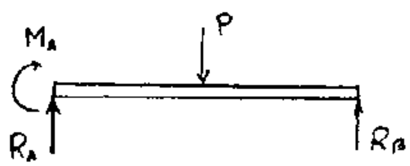
Over AC: $M = R_B\left(u + \frac{L}{2}\right) - Pu$, $\quad \frac{\partial M}{\partial R_B} = \left(u + \frac{L}{2}\right)$

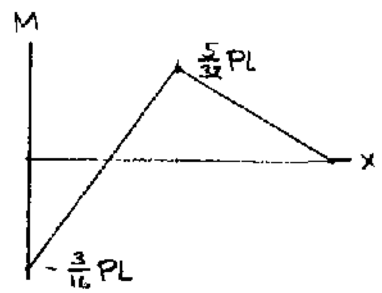
$$\frac{\partial U_{AB}}{\partial R_B} = \frac{1}{EI}\int_0^{\frac{L}{2}} \left[R_B\left(u + \frac{L}{2}\right) - Pu\right]\left(u + \frac{L}{2}\right)du$$

$$= \frac{R_B}{EI}\int_0^{\frac{L}{2}} \left(u + \frac{L}{2}\right)^2 du - \frac{P}{EI}\int_0^{\frac{L}{2}} u\left(u + \frac{L}{2}\right)du$$

$$= \frac{R_B}{3EI}\left[L^3 - \left(\frac{L}{2}\right)^3\right] - \frac{P}{EI}\left[\frac{1}{3}\left(\frac{L}{2}\right)^3 + \frac{L}{2}\cdot\frac{1}{2}\left(\frac{L}{2}\right)^2\right]$$

$$\frac{7}{24}\frac{R_B L^3}{EI} - \frac{5}{48}\frac{PL^3}{EI}$$





Over CB: $M = R_B v \qquad \frac{\partial M}{\partial R_B} = v$

$$\frac{\partial U_{CB}}{\partial R_B} = \frac{1}{EI}\int_0^{\frac{L}{2}} (R_B v)v\,dv = \frac{R_B}{3EI}\left(\frac{L}{2}\right)^3 = \frac{1}{24}\frac{R_B L^3}{EI}$$

$$y_B = \left(\frac{7}{24} + \frac{1}{24}\right)\frac{R_B L^3}{EI} - \frac{5}{48}\frac{PL^3}{EI} = 0$$

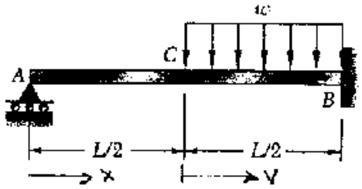
$$R_B = \frac{5}{16}P \uparrow$$

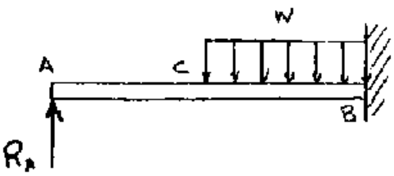
$$M_C = R_B\frac{L}{2} = \frac{5}{32}PL$$

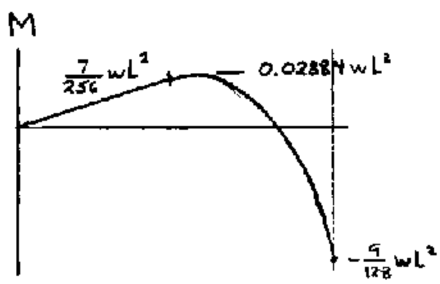
$$M_A = R_B L - P\frac{L}{2} = \left(\frac{5}{16} - \frac{1}{2}\right)PL = -\frac{3}{16}PL$$

Problem 11.113

11.111 through 11.114 Determine the reaction at the roller support and the draw the bending-moment diagram for the beam and loading shown.







Remove support A and add reaction R_A as a load.

$$U = \int_0^{\frac{L}{2}} \frac{M^2}{2EI}\,dx + \int_0^{\frac{L}{2}} \frac{M^2}{2EI}\,dv$$

$$\delta_A = \frac{\partial U}{\partial R_A} = \frac{1}{EI}\int_0^{\frac{L}{2}} M\frac{\partial M}{\partial R_A}\,dx + \frac{1}{EI}\int_0^{\frac{L}{2}} M\frac{\partial M}{\partial R_A}\,dv = 0$$

Portion AC: $0 < x < \frac{L}{2}$ $\quad M = R_A x \quad \frac{\partial M}{\partial R_A} = x$

$$\frac{\partial U_{AC}}{\partial R_A} = \frac{1}{EI}\int_0^{\frac{L}{2}} (R_A x)(x)\,dx = \frac{R_A L^3}{24 EI}$$

Portion CB: $0 < v < \frac{L}{2}$

$$M = R_A\left(v + \frac{L}{2}\right) - \frac{1}{2}wv^2 \qquad \frac{\partial M}{\partial R_A} = \left(v + \frac{L}{2}\right)$$

$$\frac{\partial U_{CB}}{\partial R_A} = \frac{1}{EI}\int_0^{L}\left[R_A\left(v+\frac{L}{2}\right) - \frac{1}{2}wv^2\right]\left(v+\frac{L}{2}\right)dv$$

$$= \frac{1}{EI}\left\{ R_A\int_0^{\frac{L}{2}}\left(v+\frac{L}{2}\right)^2 dv - \frac{1}{2}w\int_0^{\frac{L}{2}}\left(v^3 + \frac{L}{2}v^2\right)dv\right\}$$

$$= \frac{R_A}{EI}\left[\frac{1}{3}L^3 - \frac{1}{3}\left(\frac{L}{2}\right)^3\right] - \frac{w}{2EI}\left[\frac{1}{4}\left(\frac{L}{2}\right)^4 + \frac{L}{2}\frac{1}{3}\left(\frac{L}{2}\right)^3\right]$$

$$= \left(\frac{1}{3} - \frac{1}{24}\right)\frac{R_A L^3}{EI} - \frac{7}{384}\frac{wL^4}{EI}$$

$$\delta_A = \frac{\partial U_{AC}}{\partial R_A} + \frac{\partial U_{CB}}{\partial R_A} = \frac{1}{3}\frac{R_A L^3}{EI} - \frac{7}{384}\frac{wL^4}{EI} = 0$$

$$R_A = \frac{7}{128}wL \uparrow$$

Over AC: $M = \frac{7}{128}wLx$

$$M_C = \frac{7}{256}wL^2 = 0.02734\,wL^2$$

Over CB: $M = \frac{7}{128}wL\left(v + \frac{L}{2}\right) - \frac{1}{2}wv^2$

$$M_B = \frac{7}{128}wL^2 - \frac{1}{2}w\left(\frac{L}{2}\right)^2 = -\frac{9}{128}wL^2$$

$$= -0.07031\,wL^2$$

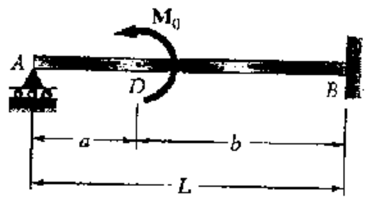
$$\frac{dM}{dv} = \frac{7}{128}wL - wv_m = 0 \qquad v_m = \frac{7}{128}L$$

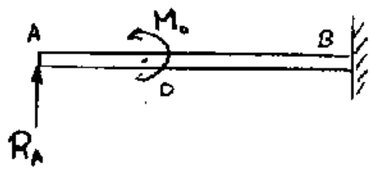
$$M_m = \frac{7}{128}wL\left(\frac{7}{128}L + \frac{L}{2}\right) - \frac{1}{2}w\left(\frac{7}{128}L\right)^2$$

$$= \frac{945}{32768}wL^2 = 0.02884\,wL^2$$

Problem 11.114

11.111 through 11.114 Determine the reaction at the roller support and the draw the bending-moment diagram for the beam and loading shown.





Remove support A and add reaction R_A as a load.

$$U = \int_0^L \frac{M^2}{2EI}\,dx$$

$$\delta_A = \frac{\partial U}{\partial R_A} = \frac{1}{EI}\int_0^L M\frac{\partial M}{\partial R_A}\,dx = 0$$

Portion AD: $0 < x < a$ $\quad M = R_A x \quad \frac{\partial M}{\partial R_A} = x$

$$\frac{\partial U_{AD}}{\partial R_A} = \frac{1}{EI}\int_0^a (R_A x)(x)\,dx = \frac{R_A a^3}{3EI}$$

Portion DB: $(a < x < L)$ $\quad M = R_A x - M_o \quad \frac{\partial M}{\partial R_A} = x$

$$\frac{\partial U_{DB}}{\partial R_A} = \frac{1}{EI}\int_a^L (R_A x - M_o)(x)\,dx = \frac{1}{EI}\left\{\frac{1}{3}R_A(L^3 - a^3) - \frac{1}{2}M_o(L^2 - a^2)\right\}$$

$$\delta_A = \frac{\partial U_{AD}}{\partial R_A} + \frac{\partial U_{DB}}{\partial R_A} = \frac{1}{EI}\left\{R_A\left(\frac{1}{3}a^3 + \frac{1}{3}L^3 - \frac{1}{3}a^3\right) - \frac{1}{2}M_o(L^2 - a^2)\right\} = 0$$

$$R_A = \frac{3}{2}\frac{M_o(L^2 - a^2)}{L^3} = \frac{3}{2}\frac{M_o\,b(L+a)}{L^3} \uparrow \blacktriangleleft$$

$$M_A = 0 \blacktriangleleft$$

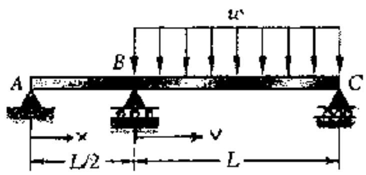
$$M_{D-} = R_A a = \frac{3}{2}\frac{M_o ab(L+a)}{L^3} \blacktriangleleft$$

$$M_{D+} = M_{D-} - M_o = \frac{3}{2}\frac{M_o ab(L+a)}{L^3} - M_o \blacktriangleleft$$

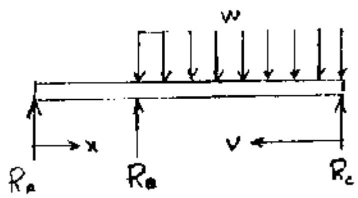
$$M_B = R_A L - M_o = \frac{3}{2}\frac{M_o\,b(L+a)}{L^2} - M_o \blacktriangleleft$$

Problem 11.115

11.115 For the uniform beam and loading shown, determine the reaction at each support.



Remove support A and add reaction R_A as a load.



$$\Sigma M_B = 0 \quad -R_A \frac{L}{2} - \frac{1}{2}wL^2 + R_C L = 0$$

$$R_C = \frac{1}{2}R_A + \frac{1}{2}wL$$

$$U = U_{AB} + U_{BC} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI}dx + \int_0^L \frac{M^2}{2EI}dv$$

$$\delta_A = \frac{\partial U}{\partial R_A} = \frac{\partial U_{AB}}{\partial R_A} + \frac{\partial U_{BC}}{\partial R_A} = 0$$

Portion AB: $\quad M = R_A x, \quad \frac{\partial M}{\partial R_A} = x$

$$\frac{\partial U_{AB}}{\partial R_A} = \frac{1}{EI}\int_0^{\frac{L}{2}} M\frac{\partial M}{\partial R_A}dx = \frac{1}{EI}\int_0^{\frac{L}{2}}(R_A x)(x)dx = \frac{R_A}{3EI}\left(\frac{L}{2}\right)^3 = \frac{1}{24}\frac{R_A L^3}{EI}$$

Portion BC: $\quad M = R_C v - \frac{1}{2}wv^2 = \frac{1}{2}R_A v + \frac{1}{2}wLv - \frac{1}{2}wLv^2$

$$\frac{\partial M}{\partial R_A} = \frac{1}{2}v$$

$$\frac{\partial U_{BC}}{\partial R_A} = \frac{1}{EI}\int_0^L\left[\frac{1}{2}R_A v + \frac{1}{2}w(Lv - v^2)\right]\left(\frac{1}{2}v\right)dv = \frac{1}{4EI}\int_0^L\left[R_A v^2 + w(Lv^2 - v^3)\right]dv$$

$$= \frac{1}{4EI}\left[R_A\frac{L^3}{3} + w\left(\frac{L^4}{3} - \frac{L^4}{4}\right)\right] = \frac{R_A L^3}{12EI} + \frac{wL^4}{48EI}$$

$$\delta_A = \frac{\partial U_{AB}}{\partial R_A} + \frac{\partial U_{BC}}{\partial R_A} = \left(\frac{1}{24} + \frac{1}{12}\right)\frac{R_A L^3}{EI} + \frac{wL^4}{48EI} = 0$$

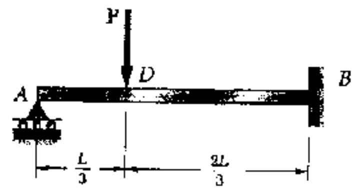
$$R_A = -\frac{1}{6}wL = \frac{1}{6}wL \downarrow$$

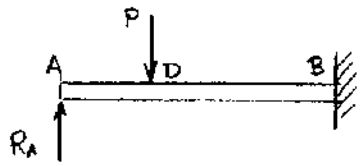
$$R_C = \frac{1}{2}\left(-\frac{1}{6}wL\right) + \frac{1}{2}wL = \frac{5}{12}wL \uparrow$$

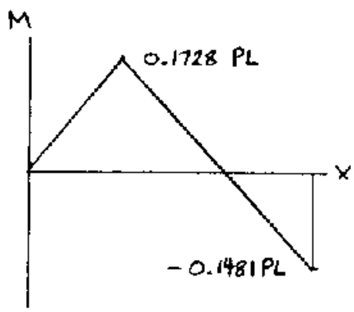
$+\uparrow \Sigma F_y = 0: \quad R_A + R_B + R_C - wL = 0$

$$-\frac{1}{6}wL + R_B + \frac{5}{12}wL - wL = 0 \qquad R_B = \frac{3}{4}wL \uparrow$$

Problem 11.116 **11.116** Determine the reaction at the roller support and draw the bending-moment diagram for the beam and load shown.







Remove support A and add reaction R_A as a load.

$$U = \int_0^L \frac{M^2}{2EI}\,dx$$

$$\delta_A = \frac{\partial U}{\partial R_A} = \frac{1}{EI}\int_0^L M\frac{\partial M}{\partial R_A}\,dx$$

Portion AD: $0 < x < \frac{L}{3}$ $\quad M = R_A x \quad \frac{\partial M}{\partial R_A} = x$

$$\frac{\partial U_{AD}}{\partial R_A} = \frac{1}{EI}\int_0^{\frac{L}{3}} M\frac{\partial M}{\partial R_A}\,dx = \frac{1}{EI}\int_0^{\frac{L}{3}}(R_A x)(x)\,dx$$

$$= \frac{R_A}{3EI}\left(\frac{L}{3}\right)^3 = \frac{1}{81}\frac{R_A L^3}{EI}$$

Portion DB: $\frac{L}{3} < x < L$ $\quad M = R_A x - P\left(x - \frac{L}{3}\right)$

$$\frac{\partial M}{\partial R_A} = x$$

$$\frac{\partial U_{DB}}{\partial R_A} = \frac{1}{EI}\int_{\frac{L}{3}}^L M\frac{\partial M}{\partial R_A}\,dx = \frac{1}{EI}\int_{\frac{L}{3}}^L\left[R_A - P\left(x - \frac{L}{3}\right)\right]x\,dx$$

$$= \frac{R_A}{EI}\int_{\frac{L}{3}}^L x^2\,dx - \frac{P}{EI}\int_{\frac{L}{3}}^L\left(x^2 - \frac{L}{3}x\right)dx$$

$$= \frac{R_A}{3EI}\left[L^3 - \left(\frac{L}{3}\right)^3\right] - \frac{P}{EI}\left[\frac{1}{3}\left(L^3 - \left(\frac{L}{3}\right)^3\right) - \frac{L}{6}\left(L^2 - \left(\frac{L}{3}\right)^2\right)\right]$$

$$= \left(\frac{1}{3} - \frac{1}{81}\right)\frac{R_A L^3}{EI} - \left(\frac{1}{3} - \frac{1}{81} - \frac{1}{6} + \frac{1}{54}\right)\frac{PL^3}{EI}$$

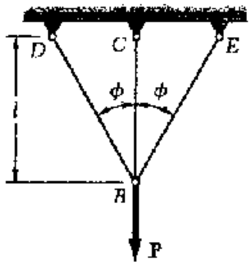
$$\delta_A = \frac{\partial U_{AD}}{\partial R_A} + \frac{\partial U_{DB}}{\partial R_A} = \left(\frac{1}{81} + \frac{1}{3} - \frac{1}{81}\right)\frac{R_A L^3}{EI} - \frac{14}{81}\frac{PL^3}{EI} = \frac{1}{3}\frac{R_A L^3}{EI} - \frac{14}{81}\frac{PL^3}{EI} = 0$$

$$R_A = \frac{14}{27}P\uparrow$$ ◄

Bending moments $\quad M_D = R_A\left(\frac{L}{3}\right) = \frac{14}{81}PL = 0.1728\,PL$ ◄

$$M_B = R_A L - P\left(\frac{2L}{3}\right) = -\frac{4}{27}PL = -0.1481\,PL$$ ◄

Problem 11.117 **11.117 through *11.120*** Three members of the same material and same cross-sectional area are used to support the load **P**. Determine the force in member *BC*.



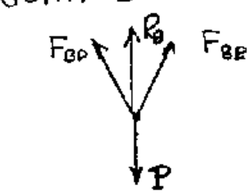
Detach member BC at support C.

Add reaction R_c as a load.

$$U = \sum \frac{F^2 L}{2EA} \qquad y_c = \frac{\partial U}{\partial R_c} = \sum \frac{FL}{EA}\frac{\partial F}{\partial R_c} = 0$$

Joint C $\qquad F_{BC} = R_c$

Joint B



$\xrightarrow{+} \sum F_x = 0: \quad F_{BE}\sin\varphi - F_{BD}\sin\varphi = 0 \qquad F_{BE} = F_{BD}$

$+\uparrow \sum F_y = 0: \quad F_{BD}\cos\varphi + F_{BE}\cos\varphi + R_B - P$

$$F_{BD} = F_{BE} = \frac{P - R_B}{2\cos\varphi}$$

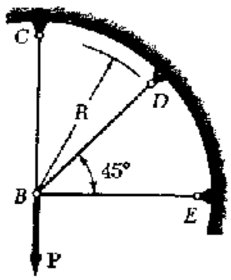
Member	F	$\partial F/\partial R_B$	L	$(FL/EA)\,\partial F/\partial R_B$
BD	$(P-R_B)/2\cos\varphi$	$-1/2\cos\varphi$	$l/\cos\varphi$	$(R_B - P)l/4EA\cos^3\varphi$
BE	$(P-R_B)/2\cos\varphi$	$-1/2\cos\varphi$	$l/\cos\varphi$	$(R_B - P)l/4EA\cos^3\varphi$
BC	R_B	1	l	$R_B l/EA$

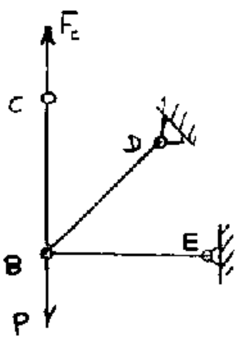
$$y_B = -Pl/2EA\cos^3\varphi + R_B l/2EA\cos^3\varphi + R_B l/EA = 0$$

$$R_B = \frac{P}{1 + 2\cos^3\varphi} \qquad F_{BC} = R_B = \frac{P}{1 + 2\cos^3\varphi}$$ ◄

Problem 11.118

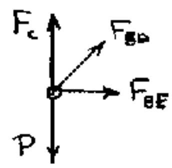
11.117 through *11.120* Three members of the same material and same cross-sectional area are used to support the load **P**. Determine the force in member *BC*.





Detach member BC from its support at point C. Add reaction F_c as a load.

Joint B.



$+\uparrow \Sigma F_y = 0:$

$$\frac{\sqrt{2}}{2} F_{BD} + F_c - P = 0$$

$$F_{BD} = \sqrt{2}\,P - \sqrt{2}\,F_c$$

$\xrightarrow{+} \Sigma F_x = 0:$

$$\frac{\sqrt{2}}{2} F_{BD} + F_{BE} = 0$$

$$F_{BE} = -P + F_c$$

$$U = \Sigma \frac{F^2 R}{2EA} = \frac{R}{2EA} \Sigma F^2$$

$$\delta_c = \frac{\partial U}{\partial F_c} = \frac{R}{EA} \Sigma F \frac{\partial F}{\partial F_c} = 0$$

Member	F	$\partial F/\partial F_c$	$F(\partial F/\partial F_c)$
BC	F_c	1	F_c
BD	$\sqrt{2}P - \sqrt{2}F_c$	$-\sqrt{2}$	$-2P + 2F_c$
BE	$-P + F_c$	1	$-P + F_c$
Σ			$-3P + 4F_c$

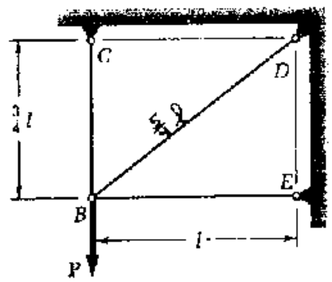
$$\delta_c = \frac{R}{EA}(-3P + 4F_c) = 0$$

$$F_c = \frac{3}{4}P$$

$$F_{BC} = F_c = \frac{3}{4}P$$

Problem 11.119

11.117 through *11.120* Three members of the same material and same cross-sectional area are used to support the load P. Determine the force in member *BC*.

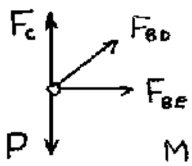


Detach member BC from support C. Add reaction F_c as a load.

$$U = \sum \frac{F^2 L}{2EA} = \frac{1}{2EA}\sum F^2 L$$

$$\delta_c = \frac{\partial U}{\partial F_c} = \frac{1}{EA}\sum F \frac{\partial F}{\partial F_c} L$$

Joint B



$+\uparrow \sum F_y = 0: \quad F_c - P + \frac{3}{5}F_{BD} = 0 \qquad F_{BD} = \frac{5}{3}P - \frac{5}{3}F_c$

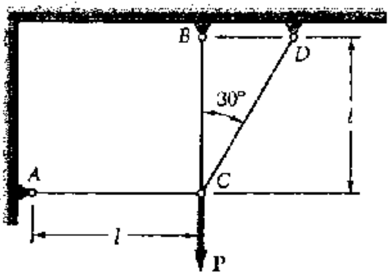
$\xrightarrow{+} \sum F_x = 0: \quad F_{BE} + \frac{4}{5}F_{BD} = 0 \qquad F_{BE} = -\frac{4}{3}P + \frac{4}{3}F_c$

Member	F	$\partial F/\partial F_c$	L	$F(\partial F/\partial F_c)L$
BC	F_c	1	$\frac{3}{4}l$	$\frac{3}{4}F_c l$
BD	$\frac{5}{3}P - \frac{5}{3}F_c$	$-\frac{5}{3}$	$\frac{5}{4}l$	$-\frac{125}{36}Pl + \frac{125}{36}F_c l$
BE	$-\frac{4}{3}P + \frac{4}{3}F_c$	$\frac{4}{3}$	l	$-\frac{16}{9}Pl + \frac{16}{9}F_c l$
$\sum$				$-\frac{21}{4}Pl + 6F_c l$

$$\delta_c = \frac{1}{EA}\left(-\frac{21}{4}Pl + 6F_c l\right) = 0 \qquad F_c = \frac{7}{8}P \qquad F_{BC} = F_c = \frac{7}{8}P$$

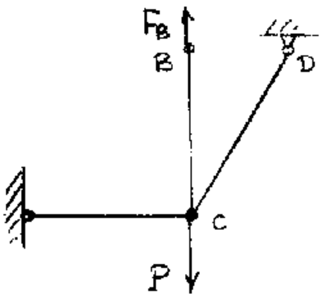
Problem 11.120

11.117 through *11.120* Three members of the same material and same cross-sectional area are used to support the load **P**. Determine the force in member BC.

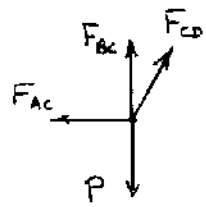


Cut member BC at end B and replace member force F_{BC} by load F_B acting on member BC at B.

$$S_B = \frac{\partial U}{\partial F_B} = \frac{\partial}{\partial F_B} \Sigma \frac{F^2 L}{EA} = \frac{1}{EA} \Sigma F \frac{\partial F}{\partial F_B} L$$



Joint C $\quad +\uparrow \Sigma F_y = 0: \quad \frac{\sqrt{3}}{2} F_{CD} + F_{BC} - P = 0$



$$F_{CD} = \frac{2}{\sqrt{3}} P - \frac{2}{\sqrt{3}} F_B$$

$\xrightarrow{+} \Sigma F_x = 0: \quad F_{AC} - \frac{1}{2} F_{CD} = 0$

$$F_{AC} = \frac{1}{\sqrt{3}} P - \frac{1}{\sqrt{3}} F_B$$

Member	F	$\partial F/\partial F_B$	L	$F(\partial F/\partial F_B)L$
AC	F_B	1	ℓ	$F_B \ell$
BC	$\frac{1}{\sqrt{3}} P - \frac{1}{\sqrt{3}} F_B$	$-\frac{1}{\sqrt{3}}$	ℓ	$-\frac{1}{3} P\ell + \frac{1}{3} F_B \ell$
CD	$\frac{2}{\sqrt{3}} P - \frac{2}{\sqrt{3}} F_B$	$-\frac{2}{\sqrt{3}}$	$\frac{2}{\sqrt{3}}\ell$	$-\frac{8}{\sqrt{3}} P\ell + \frac{8}{\sqrt{3}} F_B \ell$
Σ				$-\left(\frac{1}{3} + \frac{8}{\sqrt{3}}\right) P\ell + \left(\frac{4}{3} + \frac{8}{\sqrt{3}}\right) F_B \ell$

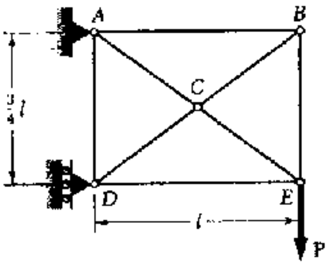
$$S_B = -\left(\frac{1}{3} + \frac{8}{\sqrt{3}}\right) \frac{P\ell}{EA} + \left(\frac{4}{3} + \frac{8}{\sqrt{3}}\right) \frac{F_B \ell}{EA}$$

$$F_B = \frac{\frac{1}{3} + \frac{8}{\sqrt{3}}}{\frac{4}{3} + \frac{8}{\sqrt{3}}} P = \frac{8 + \sqrt{3}}{8 + 4\sqrt{3}} P = 0.652\ P$$

$$F_{BC} = F_B = 0.652\ P$$

Problem 11.121

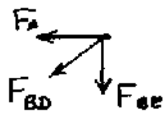
11.121 and 11.122 Knowing that the eight members of the indeterminate truss shown have the same uniform cross-sectional area, determine the force in member *AB*.

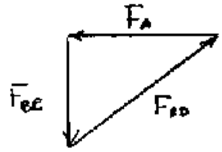


Cut member AB at end A and replace member force F_{AB} by load $F_A \leftarrow$ acting on member AB at end A.

$$\delta_A = \frac{\partial U}{\partial F_A} = \frac{\partial}{\partial F_A}\sum \frac{F^2 L}{2EA} = \frac{1}{EA}\sum F\frac{\partial F}{\partial F_A}L = 0$$

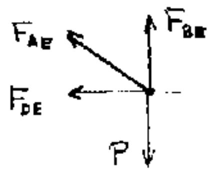
Joint B





$F_{BD} = -\frac{5}{4}F_A \qquad F_{BE} = \frac{3}{4}F_A$

Joint E



$+\uparrow \Sigma F_y = 0:$

$F_{BE} - P + \frac{3}{5}F_{AE}$

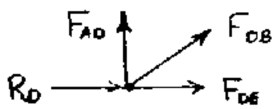
$F_{AE} = \frac{5}{3}P - \frac{5}{3}F_{BE}$

$= \frac{5}{3}P - \frac{5}{4}F_A$

$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{4}{5}F_{AE} - F_{DE} = 0$

$F_{DE} = -\frac{4}{5}F_{AE} = -\frac{4}{3}P + F_A$

Joint D



$+\uparrow \Sigma F_y = 0: \quad F_{AD} + \frac{3}{5}F_{DB} = 0$

$F_{AD} = -\frac{3}{5}F_{DB} = -\frac{3}{4}F_A$

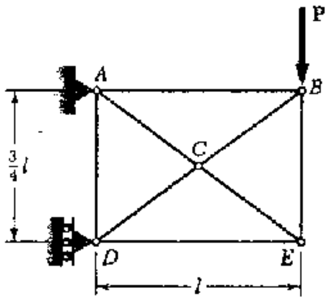
Member	F	$\partial F/\partial F_A$	L	$F(\partial F/\partial F_A)L$
AB	F_A	1	l	$F_A l$
AD	$-\frac{3}{4}F_A$	$-\frac{3}{4}$	$\frac{3}{4}l$	$\frac{27}{64}F_A l$
AE	$\frac{5}{3}P - \frac{5}{4}F_A$	$-\frac{5}{4}$	$\frac{5}{4}l$	$-\frac{125}{48}Pl + \frac{125}{64}F_A l$
BD	$-\frac{5}{4}F_A$	$-\frac{5}{4}$	$\frac{5}{4}l$	$\frac{125}{64}F_A l$
BE	$\frac{3}{4}F_A$	$\frac{3}{4}$	$\frac{3}{4}l$	$\frac{27}{64}F_A l$
DE	$-\frac{4}{3}P + F_A$	1	l	$-\frac{4}{3}Pl + F_A l$
Σ				$-\frac{63}{16}Pl + \frac{27}{4}F_A l$

$$\delta_A = \frac{1}{EA}\left(-\frac{63}{16}Pl + \frac{27}{4}F_A l\right) = 0 \qquad F_A = \frac{7}{12}P$$

$$F_{AB} = F_A = \frac{7}{12}P = 0.583\ P$$

Problem 11.122

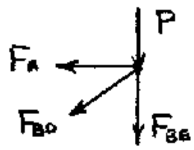
11.121 and 11.122 Knowing that the eight members of the indeterminate truss shown have the same uniform cross-sectional area, determine the force in member *AB*.



Cut member AB at end A and replace member force F_{AB} by load $F_A \leftarrow$ acting on member AB at end A.

$$\delta_A = \frac{\partial U}{\partial F_A} = \frac{\partial}{\partial F_A} \sum \frac{F^2 L}{EA} = \frac{1}{EA} \sum F \frac{\partial F}{\partial F_A} L = 0$$

Joint B $\quad \xrightarrow{+} \Sigma F_x = 0 \qquad -F_A - \frac{4}{5} F_{BD} = 0 \qquad F_{BD} = -\frac{5}{4} F_A$

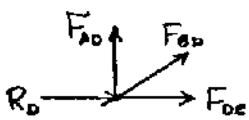


$+\uparrow \Sigma F_y = 0 \qquad -P - F_{BE} - \frac{3}{5} F_{BD} = 0 \qquad F_{BE} = -P + \frac{3}{4} F_A$

Joint E $\quad +\uparrow \Sigma F_y = 0 \qquad F_{BE} + \frac{3}{5} F_{AE} = 0 \qquad F_{AE} = \frac{5}{3} P - \frac{5}{4} F_A$

$\xrightarrow{+} \Sigma F_x = 0 \qquad -\frac{4}{5} F_{AE} - F_{DE} = 0 \qquad F_{DE} = -\frac{4}{3} P + F_A$

Joint D



$+\uparrow \Sigma F_y = 0 \qquad F_{AD} + \frac{3}{5} F_{BD} = 0$

$$F_{AD} = \frac{3}{4} F_A$$

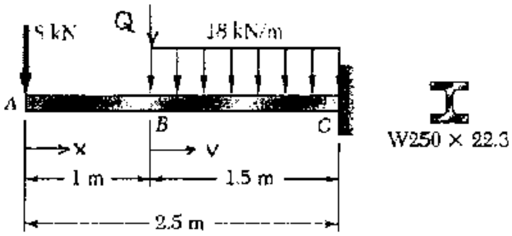
Member	F	$\partial F / \partial F_A$	L	$F(\partial F/\partial F_A)L$
AB	F_A	1	l	$F_A l$
AD	$\frac{3}{4} F_A$	$\frac{3}{4}$	$\frac{3}{4} l$	$\frac{27}{64} F_A l$
AE	$\frac{5}{3} P - \frac{5}{4} F_A$	$-\frac{5}{4}$	$\frac{5}{4} l$	$-\frac{125}{48} P l + \frac{125}{64} F_A l$
BD	$-\frac{5}{4} F_A$	$-\frac{5}{4}$	$\frac{5}{4} l$	$\frac{125}{64} F_A l$
BE	$-P + \frac{3}{4} F_A$	$\frac{3}{4}$	$\frac{3}{4} l$	$-\frac{9}{16} P l + \frac{27}{64} F_A l$
DE	$-\frac{4}{3} P + F_A$	1	l	$-\frac{4}{3} P l + F_A l$
Σ				$-\frac{9}{2} P l + \frac{27}{4} F_A l$

$$\delta_A = \frac{1}{EA}\left(-\frac{9}{2} P l + \frac{27}{4} F_A l\right) = 0 \qquad F_A = \frac{2}{3} P$$

$$F_{AB} = F_A = \frac{2}{3} P = 0.667 P$$

Problem 11.123

11.123 For the beam and loading shown, determine the deflection at point B. Use $E = 200$ GPa.



Add force Q at point B.

Units: Forces in kN, lengths in m.

Over AB $\quad M = -8x \qquad \frac{\partial M}{\partial Q} = 0$

Over BC $\quad M = -8(v+1) - \frac{1}{2}(18)v^2 - Qv \qquad \frac{\partial M}{\partial Q} = -v$

$E = 200 \times 10^9$ Pa, $\quad I = 28.9 \times 10^6\ \text{mm}^4 = 28.9 \times 10^{-6}\ \text{m}^4$

$EI = (200 \times 10^9)(28.9 \times 10^{-6}) = 5.78 \times 10^6\ \text{N}\cdot\text{m}^2 = 5780\ \text{kN}\cdot\text{m}^2$

$$U = \int_0^1 \frac{M^2}{2EI}\,dx + \int_0^{1.5} \frac{M^2}{2EI}\,dv$$

$$\delta_B = \frac{\partial U}{\partial Q} = \frac{1}{EI}\left\{\int_0^1 M\frac{\partial M}{\partial Q}\,dx + \int_0^{1.5} M\frac{\partial M}{\partial Q}\,dv\right\}$$

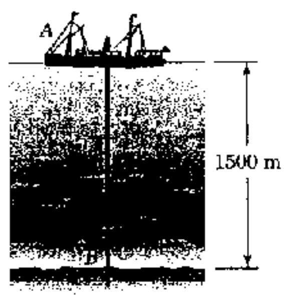
$$= \frac{1}{EI}\left\{0 + \int_0^{1.5}\left[-8(v+1) - \tfrac{1}{2}(18)v^2\right](-v)\,dv\right\} = \frac{1}{EI}\int_0^{1.5}(9v^3 + 8v^2 + 8v)\,dv$$

$$= \frac{1}{EI}\left\{\frac{9}{4}(1.5)^4 + \frac{8}{3}(1.5)^3 + \frac{8}{2}(1.5)^2\right\} = \frac{29.391}{EI} = \frac{29.391}{5780}$$

$$= 5.08 \times 10^{-3}\ \text{m} = 5.08\ \text{mm} \downarrow$$

Problem 11.124

11.124 The ship at A has just started to drill for oil on the ocean floor at a depth of 1500 m. The steel drill pipe has an outer diameter of 200 mm and a uniform wall thickness of 12 mm. Knowing that the top of the drill pipe rotates through two complete revolutions before the drill bit at B starts to operate and using $G = 77.2$ GPa, determine the maximum strain energy acquired by the drill pipe.



$\varphi = 2$ rev $= (2)(2\pi) = 4\pi$ radians

$L = 1500$ m, $\quad c_o = \frac{1}{2}d_o = 100$ mm, $\quad c_i = c_o - t = 88$ mm

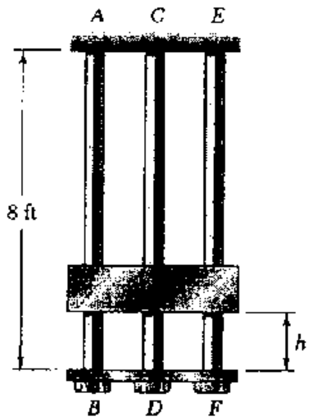
$J = \frac{\pi}{2}(c_o^4 - c_i^4) = \frac{\pi}{2}(100^4 - 88^4) = 62.88 \times 10^6\ \text{mm}^4$
$= 62.88 \times 10^{-6}\ \text{m}^4$

$\varphi = \frac{TL}{GJ} \qquad T = \frac{GJ\varphi}{L}$

$$U = \frac{T^2L}{2GJ} = \left(\frac{GJ\varphi}{L}\right)^2\frac{L}{2GJ} = \frac{GJ\varphi^2}{2L}$$

$$= \frac{(77.2 \times 10^9)(62.88 \times 10^{-6})(4\pi)^2}{(2)(1500)} = 256 \times 10^3\ \text{J} = 256\ \text{kJ}$$

Problem 11.125



11.125 The 100-lb collar G is released from rest in the position shown and is stopped by plate BDF that is attached to the $\frac{7}{8}$-in.-diameter steel rod CD and to the $\frac{5}{8}$-in.-diameter steel rods AB and EF. Knowing that for the grade of steel used $\sigma_{all} = 24$ ksi and $E = 29 \times 10^6$ psi, determine the largest allowable distance h.

Let Δ_m be the elongation.

$$\Delta_m = \frac{\sigma_{AB} L}{E} = \frac{\sigma_{CD} L}{E} = \frac{\sigma_{EF} L}{E}$$

$$\sigma_{AB} = \sigma_{CD} = \sigma_{EF} = 24 \times 10^3 \text{ psi}$$

$$L = 8 \text{ ft} = 96 \text{ in.}$$

$$\Delta_m = \frac{(24 \times 10^3)(96)}{29 \times 10^6} = 79.448 \times 10^{-3} \text{ in.}$$

For each rod $U = \frac{F_m^2 L}{2EA} = \frac{(EA\Delta_m/L)^2 L}{2EA} = \frac{EA\,\Delta_m^2}{2L}$

Rod CD: $A_{CD} = \frac{\pi}{4}\left(\frac{7}{8}\right)^2 = 0.60132 \text{ in}^2$

$$U_{CD} = \frac{(29 \times 10^6)(0.60132)(79.448 \times 10^{-3})^2}{(2)(96)} = 573.28 \text{ in}\cdot\text{lb}.$$

Rods AB and EF: $A_{AB} = A_{EF} = \frac{\pi}{4}\left(\frac{5}{8}\right)^2 = 0.30680 \text{ in}^2$

$$U_{AB} = U_{EF} = \frac{(29 \times 10^6)(0.30680)(79.448 \times 10)^2}{(2)(96)} = 292.49 \text{ in}\cdot\text{lb}$$

Total $U_m = U_{AB} + U_{CD} + U_{EF} = 1158.27 \text{ in}\cdot\text{lb}$

Falling distance is $h + \Delta_m$. $W = 100$ lb

$$W(h + \Delta_m) = U_m$$

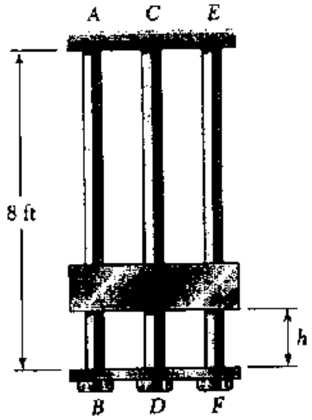
$$h + \Delta_m = \frac{U_m}{W} = \frac{1158.27}{100} = 11.583 \text{ in.}$$

$$h = 11.583 - 79.448 \times 10^{-3} = 11.50 \text{ in.}$$

Problem 11.126

11.126 Solve Prob. 11.125, assuming that the $\frac{7}{8}$-in.-diameter steel rod CD is replaced by a $\frac{7}{8}$-in.-diameter rod made of a grade of aluminum for which $\sigma_{all} = 20$ ksi and $E = 10.6 \times 10^6$ psi.

11.125 The 100-lb collar G is released from rest in the position shown and is stopped by plate BDF that is attached to the $\frac{7}{8}$-in.-diameter steel rod CD and to the $\frac{5}{8}$-in.-diameter steel rods AB and EF. Knowing that for the grade of steel used $\sigma_{all} = 24$ ksi and $E = 29 \times 10^6$ psi, determine the largest allowable distance h.



Let Δ_m be the elongation. $L = 8$ ft $= 96$ in.

$$\Delta_m = \frac{\sigma_{AB} L}{E_{AB}} = \frac{\sigma_{CD} L}{E_{CD}} = \frac{\sigma_{EF} L}{E_{EF}}$$

If $\sigma_{AB} = 24 \times 10^3$ psi, $\Delta_m = \dfrac{(24\times10^3)(96)}{29\times10^6} = 79.448\times10^{-3}$ in.

If $\sigma_{CD} = 20 \times 10^3$ psi, $\Delta_m = \dfrac{(20\times10^3)(96)}{10.6\times10^6} = 181.13\times10^{-3}$ in.

Smaller value governs $\Delta_m = 79.448\times10^{-3}$ in.

For each rod $U = \dfrac{F^2 L}{2EA} = \dfrac{(EA\Delta_m/L)^2 L}{2EA} = \dfrac{EA\,\Delta_m^2}{2L}$

Rod CD: $A_{CD} = \frac{\pi}{4}\left(\frac{7}{8}\right)^2 = 0.60132$ in^2, $E_{CD} = 10.6\times10^6$ psi

$$U_{CD} = \frac{(10.6\times10^6)(0.60132)(79.448\times10^{-3})^2}{(2)(96)} = 209.54 \text{ in-lb}$$

Rods AB and EF: $A_{AB} = A_{EF} = \frac{\pi}{4}\left(\frac{5}{8}\right)^2 = 0.30680$ in^2

$$U_{AB} = U_{EF} = \frac{(29\times10^6)(0.30680)(79.448\times10^{-3})^2}{(2)(96)} = 292.49 \text{ in-lb}$$

Total $U_m = U_{AB} + U_{CD} + U_{EF} = 794.52$ in-lb.

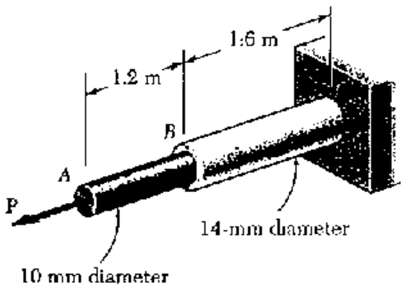
Falling distance is $h + \Delta_m$ $\quad W = 100$ lb.

$$W(h+\Delta_m) = U_m$$

$$h + \Delta_m = \frac{U_m}{W} = \frac{794.52}{100} = 7.9452 \text{ in.}$$

$$h = 7.9452 - 79.448\times10^{-3} = 7.87 \text{ in.}$$

Problem 11.127



11.127 Rod AB is made of a steel for which the yield strength is $\sigma_Y = 450$ MPa and $E = 200$ GPa; rod BC is made of an aluminum alloy for which $\sigma_Y = 280$ MPa and $E = 73$ GPa. Determine the maximum strain energy that can be acquired by the composite rod ABC without causing any permanent deformations.

$$A_{AB} = \frac{\pi}{4}(10)^2 = 78.54 \text{ mm}^2 = 78.54 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4}(14)^2 = 153.94 \text{ mm}^2 = 153.94 \times 10^{-6} \text{ m}^2$$

$P_{all} = \sigma_Y A$ for each portion.

AB: $P_{all} = (450 \times 10^6)(78.54 \times 10^{-6}) = 35.343 \times 10^3$ N

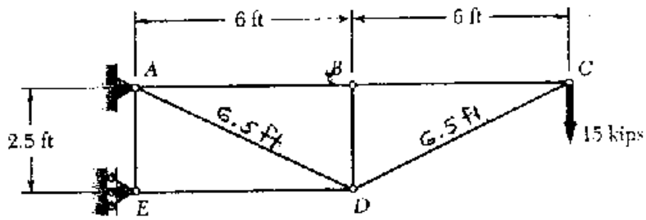
BC: $P_{all} = (280 \times 10^6)(153.94 \times 10^{-6}) = 43.103 \times 10^3$ N

Use the smaller value. $P = 35.343 \times 10^3$ N

$$U = \frac{P^2 L_{AB}}{2 E_{AB} A_{AB}} + \frac{P^2 L_{BC}}{2 E_{BC} A_{BC}} = \frac{(35.343 \times 10^3)^2 (1.2)}{(2)(200 \times 10^9)(78.54 \times 10^{-6})} + \frac{(35.343 \times 10^3)^2 (1.6)}{(2)(73 \times 10^9)(153.94 \times 10^{-6})}$$

$$= 136.6 \text{ J}$$

Problem 11.128 *11.128* Each member of the truss shown is made of steel and has a uniform cross-sectional area of 5 in^2. Using $E = 29 \times 10^6$ psi, determine the vertical deflection of joint C caused by the application of the 15-kip load.



Members BD and AE are zero force members.

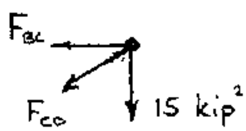
For entire truss $+\circlearrowleft \Sigma M_A = 0$:

$2.5\,R_D - (12)(15) = 0$

$R_D = 72$ kips

For equilibrium of joint E $\quad F_{ED} = -R_D = -72$ kips

Joint C



$+\uparrow \Sigma F_y = 0$

$-\frac{2.5}{6.5}F_{CD} - 15 = 0$

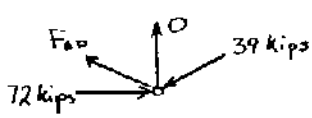
$F_{CD} = -39$ kips

$+\rightarrow \Sigma F_x = 0$:

$-\frac{6}{6.5}F_{CD} - F_{BC} = 0$

$F_{BC} = 36$ kips.

Joint D



$+\rightarrow \Sigma F_x = 0$

$72 - \frac{6}{6.5}(F_{AD} + 39) = 0$

$F_{AD} = 39$ kips

Joint B $\quad \Sigma F_x = 0$:

$-F_{AB} + F_{BC} = 0$

$F_{AB} = 36$ kips

Strain energy: $U_m = \Sigma \frac{F^2L}{2EA} = \frac{1}{2EA}\Sigma F^2L$

Member	F (kips)	L (in)	F^2L (kip$^2\cdot$in)
AB	36	72	93312
BC	36	72	93312
CD	-39	78	118638
DE	-72	72	373248
BD	0	30	0
AE	0	30	0
AD	39	78	118638
Σ			797148

Data: $E = 29 \times 10^3$ ksi

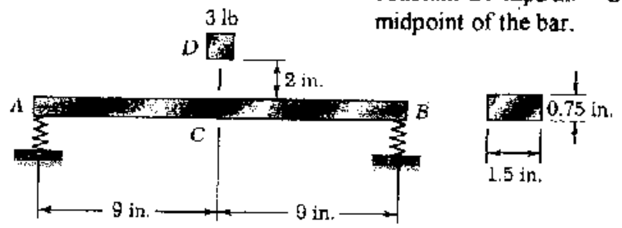
$A = 5$ in^2

$$U_m = \frac{797148}{(2)(29\times10^3)(5)} = 2.7488 \text{ kip}\cdot\text{in}$$

$\frac{1}{2}P_m\Delta_m = U \qquad \Delta_m = \frac{2U_m}{P_m} = \frac{(2)(2.7488)}{15} = 0.366 \text{ in.} \downarrow$

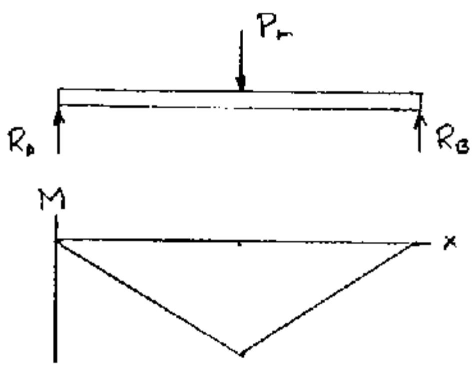
Problem 11.129

11.129 The 3-lb block D is released from rest in the position shown and strikes a steel bar AB having the uniform cross section shown. The bar is supported at each end by springs of constant 20 kips/in. Using $E = 29 \times 10^6$ psi, determine the maximum deflection at the midpoint of the bar.



$k = 20$ kips/in $= 20 \times 10^3$ lb/in

$R_A = R_B = \frac{1}{2} P_m$



For spring A, $U_A = \frac{1}{2} R_A y_A = \frac{1}{2} \frac{R_A^2}{k} = \frac{1}{8} \frac{P_m^2}{k}$

For spring B, $U_B = \frac{1}{2} R_B y_B = \frac{1}{2} \frac{R_B^2}{k} = \frac{1}{8} \frac{P_m^2}{k}$

Portion AC of beam ACB: $M = \frac{1}{2} P_m x$

$$U_{AC} = \int_0^{L_{AC}} \frac{M^2}{2EI} dx = \frac{P_m^2}{8EI} \int_0^{L_{AC}} x^2 dx = \frac{P_m^2 L_{AC}^3}{24 EI}$$

Portion CB of beam ACB:

By symmetry, $U_{CB} = U_{AC} = \frac{P_m^2 L_{AC}^2}{24EI}$

Total $U = U_A + U_B + U_{AC} + U_{CB} = \frac{P_m^2}{4k} + \frac{P_m^2 L_{AC}^3}{12EI}$

$I = \frac{1}{12} bd^3 = \frac{1}{12}(1.5)(0.75)^3 = 52.734 \times 10^{-3}$ in^4

$$U = \left\{ \frac{1}{(4)(20 \times 10^3)} + \frac{(9)^3}{(12)(29 \times 10^6)(52.734 \times 10^{-3})} \right\} P_m^2 = 52.224 \times 10^{-6} \; P_m^2 = \frac{1}{2} P_m y_m$$

$y_m = \frac{2U}{P_m} = 104.448 \times 10^{-6} \; P_m \qquad P_m = 9.5741 \times 10^3 \; y_m$

$U = (52.224 \times 10^{-6})(9.5741 \times 10^3)^2 y_m^2 = 4.7871 \times 10^3 \; y_m^2$

Work of falling weight: $W(h + y_m) = (3)(2 + y_m) = 6 + 3y_m$

Equating, $6 + 3y_m = 4.7871 \times 10^3 y_m^2$

$$y_m^2 - 626.69 \times 10^{-6} \; y_m - 1.25338 \times 10^{-3} = 0$$

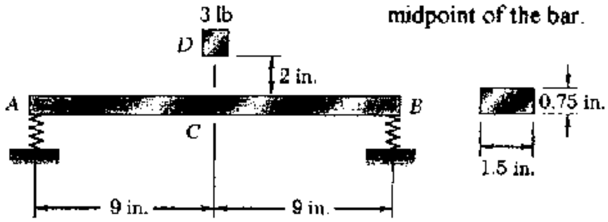
$$y_m = \frac{1}{2}\left\{ 626.69 \times 10^{-6} + \sqrt{(626.69 \times 10^{-6})^2 + (4)(1.25338 \times 10^{-3})} \right\}$$

$$= 35.7 \times 10^{-3} \text{ in.} = 0.0357 \text{ in.} \downarrow$$

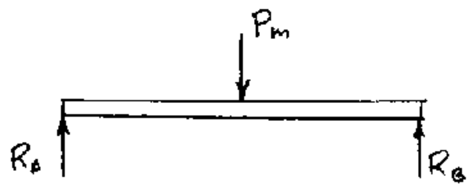
Problem 11.130

11.130 Solve Prob. 11.129, assuming that the constant of each spring is 40 kips/in.

11.129 The 3-lb block D is released from rest in the position shown and strikes a steel bar AB having the uniform cross section shown. The bar is supported at each end by springs of constant 20 kips/in. Using $E = 29 \times 10^6$ psi, determine the maximum deflection at the midpoint of the bar.



$$k = 40 \text{ kips/in} = 40 \times 10^3 \text{ lb/in.}$$



$$R_A = R_B = \tfrac{1}{2} P_m$$

For spring A, $U_A = \frac{1}{2} R_A y_A = \frac{1}{2}\frac{R_A^2}{k} = \frac{1}{8}\frac{P_m^2}{k}$

For spring B, $U_B = \frac{1}{2} R_B y_B = \frac{1}{2}\frac{R_B^2}{k} = \frac{1}{8}\frac{P_m^2}{k}$

Portion AC of beam ACB: $\qquad M = \frac{1}{2} P_m x$

$$U_{AC} = \int_0^{L_{AC}} \frac{M^2}{2EI}\,dx = \frac{P_m^2}{8EI}\int_0^{L_{AC}} x^2\,dx = \frac{P_m^2 L_{AC}^2}{24\,EI}$$

Portion CB of beam ACB:

By symmetry, $U_{CB} = U_{AC} = \dfrac{P_m^2 L_{AC}^2}{24\,EI}$

Total $\quad U = U_A + U_B + U_{AC} + U_{CB} = \dfrac{P_m^2}{4k} + \dfrac{P_m^2 L_{AC}^3}{12\,EI}$

$$I = \tfrac{1}{12} b d^3 = \tfrac{1}{12}(1.5)(0.75)^3 = 52.734 \times 10^{-3} \text{ in}^4$$

$$U = \left\{\frac{1}{(4)(40\times10^3)} + \frac{(9)^3}{(12)(29\times10^6)(52.734\times10^{-3})}\right\} P_m^2 = 45.974\times10^{-6} P_m^2 = \tfrac{1}{2} P_m y_m$$

$$y_m = \frac{2U}{P_m} = 91.949\times10^{-6} P_m \qquad P_m = 10.8756\times10^3 y_m$$

$$U = (45.974\times10^{-6})(10.8756\times10^3)^2 y_m^2 = 5.4378\times10^3 y_m^2$$

Work of falling weight: $\qquad W(h + y_m) = (3)(2 + y_m) = 6 + 3y_m$

Equating, $\qquad 6 + 3y_m = 5.4378\times10^3 y_m^2$

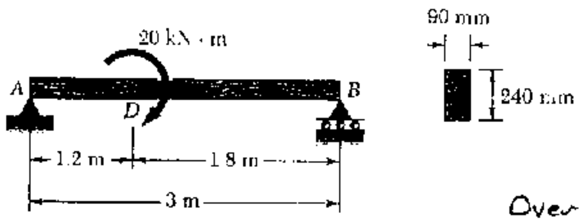
$$y_m^2 - 551.70\times10^{-6} y_m - 1.1034\times10^{-3} = 0$$

$$y_m = \tfrac{1}{2}\left\{551.70\times10^{-6} + \sqrt{(551.70\times10^{-3})^2 + (4)(1.1034\times10^{-3})}\right\}$$

$$= 33.5\times10^{-3} \text{ in.} = 0.0335 \text{ in.} \downarrow$$

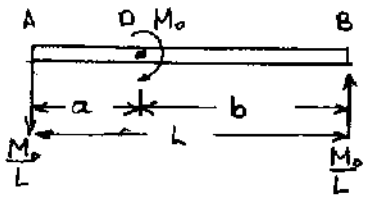
Problem 11.131

11.131 Using $E = 12$ GPa, determine the strain energy due to bending for the timber beam and loading shown.



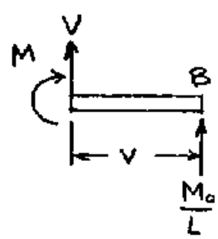
$$+\Sigma M_A = 0: \quad -M_0 + R_A L = 0$$

$$R_A = \frac{M_0}{L}\downarrow, \qquad R_B = \frac{M_0}{L}\uparrow$$



Over portion AD: $\quad M = R_A x = \frac{M_0 x}{L}$

$$U_{AD} = \int_0^a \frac{M^2}{2EI}\,dx = \frac{M_0^2}{2EIL^2}\int_0^a x^2\,dx = \frac{M_0^2 a^3}{6EIL^2}$$



Over portion DB: $\quad M = R_B v = \frac{M_0 v}{L}$

$$U_{DB} = \int_0^b \frac{M^2}{2EI}\,dv = \frac{M_0^2}{2EIL^2}\int_0^b v^2\,dv = \frac{M_0^2 b^3}{6EIL^2}$$

Total: $\quad U = U_{AD} + U_{DB} = \frac{M_0^2(a^3+b^3)}{6EIL^2}$

Data: $a = 1.2$ m, $b = 1.8$ m, $L = 3$ m, $M_0 = 20\times10^3$ N·m

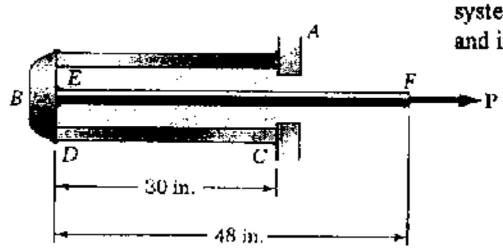
$$I = \frac{1}{12}(90)(240)^3 = 103.68\times10^6 \text{ mm}^4 = 103.68\times10^{-6} \text{ m}^4$$

$$EI = (12\times10^9)(103.68\times10^{-6}) = 1.24416\times10^6 \text{ N·m}^2$$

$$U = \frac{(20\times10^3)^2\left[(1.2)^3+(1.8)^3\right]}{(6)(1.24416\times10^6)(3)^2} = 45.0 \text{ N·m} = 45.0 \text{ J} \blacktriangleleft$$

Problem 11.132

11.132 A 30-in. length of aluminum pipe of cross-sectional area 1.85 in^2 is welded to a fixed support A and to a rigid cap B. The steel rod EF, of 0.75 in. diameter, is welded to cap B. Knowing that the modulus of elasticity is 29×10^6 psi for the steel and 10.6×10^6 psi for the aluminum, determine (*a*) the total strain energy of the system when $P = 10$ kips, (*b*) the corresponding strain-energy density of the pipe CD and in the rod EF.



For EF: $A = \frac{\pi}{4} d^2 = 0.4418 \text{ in}^2$

CD: $U_{CD} = \dfrac{P^2 L}{2EA} = \dfrac{(10\times10^3)^2(30)}{(2)(10.6\times10^6)(1.85)} = 76.49 \text{ in}\cdot\text{lb}$

EF: $U_{EF} = \dfrac{P^2 L}{2EA} = \dfrac{(10\times10^3)^2(48)}{(2)(29\times10^6)(0.4418)} = 187.33 \text{ in}\cdot\text{lb}$

(a) Total: $U = U_{CD} + U_{EF} = 264 \text{ in}\cdot\text{lb}$ ◄

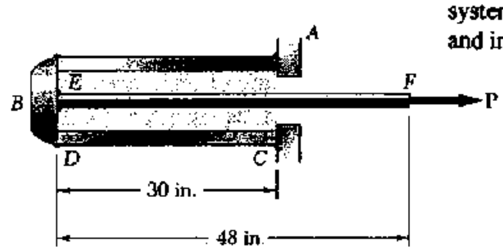
(b) CD: $\sigma = -\dfrac{10000}{1.85} = -5405 \text{ psi}$, $u = \dfrac{\sigma^2}{2E} = \dfrac{(-5405)^2}{(2)(10.6\times10^6)} = 1.378 \text{ in}\cdot\text{lb/in}^3$ ◄

EF: $\sigma = \dfrac{10000}{0.4418} = 22635 \text{ psi}$, $u = \dfrac{\sigma^2}{2E} = \dfrac{22635^2}{(2)(29\times10^6)} = 8.83 \text{ in}\cdot\text{lb/in}^3$ ◄

Problem 11.133

11.133 Solve Prob. 11.132, when $P = 8$ kips.

11.132 A 30-in. length of aluminum pipe of cross-sectional area 1.85 in^2 is welded to a fixed support A and to a rigid cap B. The steel rod EF, of 0.75 in. diameter, is welded to cap B. Knowing that the modulus of elasticity is 29×10^6 psi for the steel and 10.6×10^6 psi for the aluminum, determine (*a*) the total strain energy of the system when $P = 10$ kips, (*b*) the corresponding strain-energy density of the pipe CD and in the rod EF



For EF: $A = \frac{\pi}{4} d^2 = 0.4418 \text{ in}^2$

CD: $U_{CD} = \dfrac{P^2 L}{2EA} = \dfrac{(-8000)^2(30)}{(2)(10.6\times10^6)(1.85)} = 48.95 \text{ in}\cdot\text{lb}$

EF: $U_{EF} = \dfrac{P^2 L}{2EA} = \dfrac{(8000)^2(48)}{(2)(29\times10^6)(0.4418)} = 119.89 \text{ in}\cdot\text{lb}$

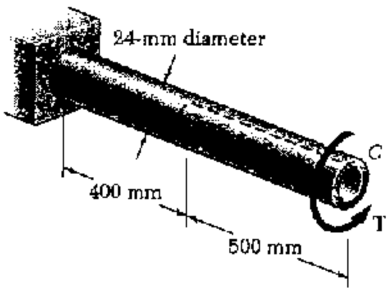
(a) Total $U = U_{CD} + U_{EF} = 168.8 \text{ in}\cdot\text{lb}$ ◄

(b) CD: $\sigma = -\dfrac{8000}{1.85} = -4324 \text{ psi}$, $u = \dfrac{\sigma^2}{2E} = \dfrac{(-4324)^2}{(2)(10.6\times10^6)} = 0.882 \text{ in}\cdot\text{lb/in}^3$ ◄

EF: $\sigma = \dfrac{8000}{0.4418} = 18108 \text{ psi}$, $u = \dfrac{\sigma^2}{2E} = \dfrac{(18108)^2}{(2)(29\times10^6)} = 5.65 \text{ in}\cdot\text{lb/in}^3$ ◄

Problem 11.134

11.134 Rod AC is made of aluminum and is subjected to a torque **T** applied at C. Knowing that $G = 73$ GPa and that portion BC of the rod is hollow and has an inner diameter of 16 mm, determine the strain energy of the rod for a maximum shearing stress of 120 MPa.



$$c_o = \frac{d_o}{2} = 12 \text{ mm}, \quad c_i = \frac{d_i}{2} = 8 \text{ mm}$$

$$J_{AB} = \frac{\pi}{2} c_o^4 = \frac{\pi}{2}(12)^4 = 32.572 \times 10^3 \text{ mm}^4 = 32.572 \times 10^{-9} \text{ m}^4$$

$$J_{BC} = \frac{\pi}{2}(c_o^4 - c_i^4) = \frac{\pi}{2}(12^4 - 8^4) = 26.138 \times 10^3 \text{ mm}^4$$
$$= 26.138 \times 10^{-9} \text{ m}^4$$

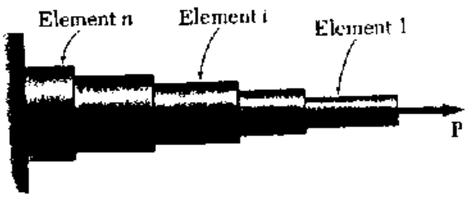
$$\tau_{all} = \frac{Tc}{J_{min}} \qquad T = \frac{J_{min}\tau_{all}}{c} = \frac{(26.138 \times 10^{-9})(120 \times 10^6)}{12 \times 10^{-3}} = 261.38 \text{ N} \cdot \text{m}$$

$$U_{AB} = \frac{T^2 L_{AB}}{2GJ_{AB}} = \frac{(261.38)^2(400 \times 10^{-3})}{(2)(73 \times 10^9)(32.572 \times 10^{-9})} = 5.747 \text{ J}$$

$$U_{BC} = \frac{T^2 L_{BC}}{2GJ_{BC}} = \frac{(261.38)^2(500 \times 10^{-3})}{(2)(73 \times 10^9)(26.138 \times 10^{-9})} = 8.951 \text{ J}$$

$$\text{Total} \quad U = U_{AB} + U_{BC} = 14.70 \text{ J}$$

PROBLEM 11.C1



11.C1 A rod consisting of n elements, each of which is homogeneous and of uniform cross section, is subjected to a load **P** applied at its free end. The length of element i is denoted by L_i and its diameter by d_i. (*a*) Denoting by E the modulus of elasticity of the material used in the rod, write a computer program that can be used to determine the strain energy acquired by the rod and the deformation measured at the free end. (*b*) Use this program to determine the strain energy and deformation of the rods of Probs. 11.9 and 11.10.

SOLUTION ENTER: P AND E

FOR EACH ELEMENT

ENTER A_i AND L_i

COMPUTE: NORMAL STRESS: $\sigma_i = \dfrac{P}{A_i}$

STRAIN ENERGY: $U_i = \dfrac{P^2 L_i}{2 A_i E}$

STRAIN ENERGY DENSITY: $u = \dfrac{\sigma_i^2}{2E}$

TOTAL STRAIN ENERGY

UPDATE THROUGH n ELEMENT

$U = U + U_i$

TOTAL DEFORMATION

$\frac{1}{2} P\Delta = U \quad : \quad \Delta = \dfrac{2U}{P}$

PROGRAM OUTPUT

Problem 11.9

Axial load = 8.000 kips Modulus of elasticity = 29 x 10^6 psi

Element	Length in.	delta L in.	Stress ksi	Strain Energy in·lb	Strain Energy Density lb·in./in.^3
1	24.000	0.022	26.08	86.32	11.72
2	36.000	0.022	18.11	89.92	5.65

Total Strain Energy = 176.24 in·lb
Total Deformation = 0.0441 in.

Problem 11.10

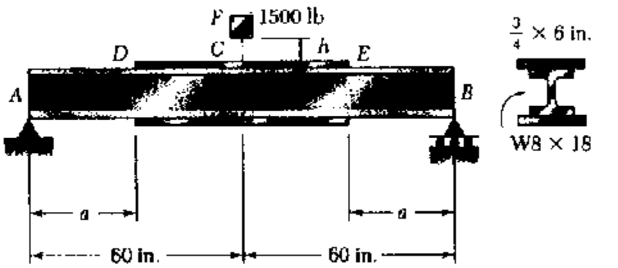
Axial load = 25.000 kN Modulus of elasticity = 200 GPa

Element	Length m	delta L mm	Stress MPa	Strain Energy J	Strain Energy Density kJ/m^3
1	0.80	0.497	124.34	6.22	38.65
2	1.20	0.477	79.58	5.97	15.83

Total Strain Energy = 12.1853 J
Total Deformation = 0.9748 mm

PROBLEM 11.C2

11.C2 Two 0.75 × 6-in. cover plates are welded to a W8 × 18 rolled-steel beam as shown. The 1500-lb block is to be dropped from a height $h = 2$ in. onto the beam. (*a*) Write a computer program to calculate the maximum normal stress on transverse sections just to the left of D and at the center of the beam for values of a from 0 to 60 in., using 5-in. increments. (*b*) From the values considered in part *a*, select the distance a for which the maximum normal stress is as small as possible. Use $E = 29 \times 10^6$ psi.



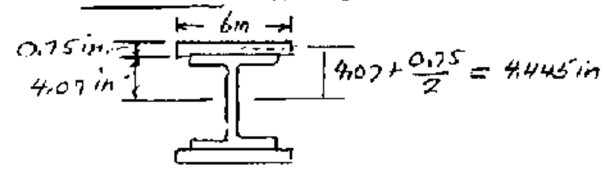
SOLUTION

COMPUTE AND ENTER MOMENTS OF INERTIA AND SECTION MODULI

FOR AD AND EB: W8×18

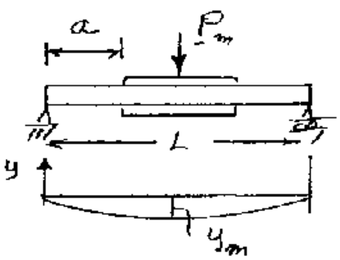
$I_1 = 61.9 \text{ in}^4 \qquad S_1 = 15.2 \text{ in}^3 \qquad \triangleleft$

FOR DCE: W8×18 PLUS COVER PLATES



$$I_2 = 61.9 + 2(6 \times 0.75)(4.445)^2 = 239.72 \text{ in}^4 \qquad \triangleleft$$

$$S_2 = \frac{I_2}{(4.07 + 0.75)} = \frac{239.72}{4.82} = 49.7 \text{ in}^3 \qquad \triangleleft$$



$y_m = P_m \alpha$ WHERE α = INFLUENCE COEFFICIENT
SEE NEXT PAGE FOR DETERMINATION OF α

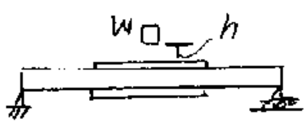
P_m = EQUIVALENT STATIC LOAD

$$U_2 = \frac{1}{2} P_m y_m = \frac{1}{2} \frac{y_m^2}{\alpha}$$

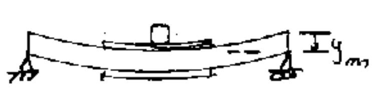
WORK DONE BY W IS $W(h + y_m)$

$$\frac{1}{2} \frac{y_m^2}{\alpha} = Wh + Wy_m$$

OR: $y_m^2 - 2W\alpha y_m - 2Wh\alpha$ (A)



POSITION 1



POSITION 2

PROGRAM SOLUTION OF (A) FOR y_m

ENTER $L = 120$ in., $h = 2$ in., $W = 1500$ lb, $E = 29 \times 10^6$ psi

FOR $a = 0$ TO 60 in, STEP 5 in.:

SOLVE (A) FOR y_m, $P_m = y_m/\alpha$, $y_{ST} = W\alpha$

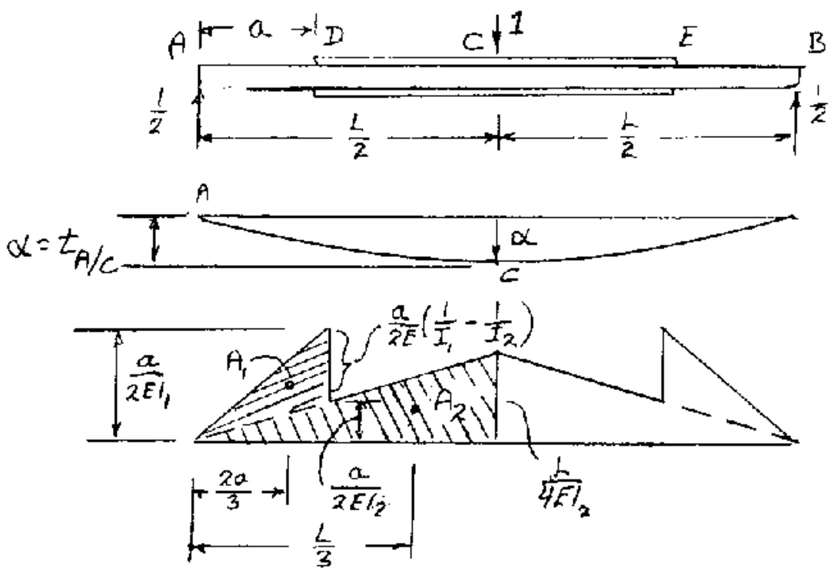
$\sigma_D = \sigma_1 = \frac{1}{2} P_m a / S_1$; $\sigma_C = \sigma_2 = \frac{1}{4} P_m L / S_2$

PRINT: a, y_{ST}, y_m, P_m, σ_1, σ_2, AND $(\sigma_1 - \sigma_2)$

REPEAT WITH SMALLER INTERVALS TO FIND a FOR $(\sigma_1 - \sigma_2) = 0$
THIS IS THE DISTANCE a FOR σ_{max} AS SMALL AS POSSIBLE

CONTINUED

DETERMINATION OF α: α IS DEFLECTION AT C FOR A UNIT LOAD AT C.



$$\alpha = t_{A/C} = A_1\left(\frac{2a}{3}\right) + A_2\left(\frac{L}{3}\right) = \left[\frac{1}{2}\frac{a}{2E}\left(\frac{1}{I_1}-\frac{1}{I_2}\right)\frac{a}{2}\right]\frac{2a}{3} + \left[\frac{1}{2}\cdot\frac{L}{4EI_2}\cdot\frac{L}{2}\right]\frac{L}{3}$$

$$\alpha = \left[\left(\frac{1}{I_1}-\frac{1}{I_2}\right)a^3 + \frac{1}{8I_2}L^3\right]\Big/ 6E \qquad \triangleleft$$

PROGRAM OUTPUT

Beam = W 8x18 with two 6 by 0.75-in. cover plates
h = 2 in. W = 1500 lb L = 120 in.

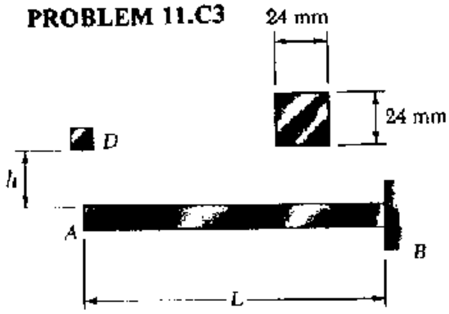
a in.	ystat in.	ymax in.	Pmax lb	σ1 ksi	σ2 ksi	$\sigma_1 - \sigma_2$ ksi
0.00	0.00777	0.1842	35572	0.00	21.46	-21.46
5.00	0.00778	0.1844	35544	5.85	21.44	-15.59
10.00	0.00787	0.1855	35348	11.63	21.32	-9.69
15.00	0.00812	0.1885	34834	17.19	21.01	-3.82
20.00	0.00859	0.1942	33896	22.30	20.45	1.85
25.00	0.00938	0.2033	32509	26.73	19.61	7.13
30.00	0.01056	0.2163	30736	30.33	18.54	11.79
35.00	0.01220	0.2334	28706	33.05	17.32	15.73
40.00	0.01438	0.2546	26563	34.95	16.02	18.93
45.00	0.01718	0.2799	24436	36.17	14.74	21.43
50.00	0.02068	0.3090	22415	36.87	13.52	23.35
55.00	0.02496	0.3419	20550	37.18	12.40	24.78
60.00	0.03008	0.3783	18862	37.23	11.38	25.85

Use smaller increments to seek the smallest maximum normal stress

18.33	0.00840	0.1919	34259	20.657	20.665	-0.01
18.34	0.00840	0.1920	34257	20.667	20.664	0.00
18.35	0.00841	0.1920	34255	20.677	20.663	0.01

Max stress small as possible for a = 18.34in.
Smallest max stress = 20.67 ksi

PROBLEM 11.C3



11.C3 The 16-kg block D is dropped from a height h onto the free end of the steel bar AB. For the steel used $\sigma_{all} = 120$ MPa and $E = 200$ GPa. (*a*) Write a computer program to calculate the maximum allowable height h for values of the length L from 100 mm to 1.2 m, using 100-mm increments. (*b*) From the values considered in part *a*, select the length corresponding to the largest allowable height.

SOLUTION

ENTER $\sigma_{all} = 120\text{ MPa}$, $E = 200\text{ GPa}$, $d = 0.024\text{ m}$

$m = 16\text{ kg}$; $g = 9.81\text{ m/s}^2$

$I = d^4/12 \qquad S = \frac{I}{c} = \frac{I}{d/2} = d^3/6$

FOR L = 100 mm TO 1200 mm STEP 100 mm

$L = L/1000$

$y_{st} = mgL^3/3EI$

$M_{max} = \sigma_{all}\, S$

$P_{max} = M_{max}/L$

$y_{max} = P_{max} L^3/3EI$

FROM PROB. 11.69, page [illegible]

$$y_m = y_{st}\left[1 + \sqrt{1 + \frac{2h}{y_{st}}}\right] \xrightarrow{\text{SOLVE FOR}} h = \left[\left(\frac{y_{max}}{y_{st}} - 1\right)^2 - 1\right]\frac{y_{st}}{2}$$

PRINT: L, y_{st}, y_{max}, P_{max}, M_{max}, h

RETURN

PROGRAM OUTPUT

Problem 11.C3

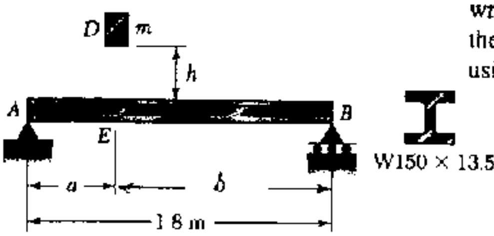
m = 16.0 kg d = 24 mm σ = 120 MPa G = 200 GPa

L mm	ystat mm	ymax mm	Pmax N	Mmax N·m	h mm
100	0.00946	0.167	2764.8	276.48	1.301
200	0.07569	0.667	1382.4	276.48	2.269
300	0.25547	1.500	921.6	276.48	2.904
400	0.60556	2.667	691.2	276.48	3.205
500	1.18273	4.167	553.0	276.48	3.173
600	2.04375	6.000	460.8	276.48	2.807
700	3.24540	8.167	395.0	276.48	2.109
800	4.84445	10.667	345.6	276.48	1.076
900	6.89766	13.500	307.2	276.48	-0.289
1000	9.46181	16.667	276.5	276.48	-1.988
1100	12.59367	20.167	251.3	276.48	-4.020
1200	16.35000	24.000	230.4	276.48	-6.385

Use smaller increments to seek the largest height h

435	0.77883	3.154	635.6	276.48	3.2316
440	0.80599	3.227	628.4	276.48	3.2320
445	0.83378	3.300	621.3	276.48	3.2317

PROBLEM 11.C4



11.C4 The block D of mass $m = 8$ kg is dropped from a height $h = 750$ mm onto the rolled-steel beam AB. Knowing that $E = 200$ GPa, write a computer program to calculate the maximum deflection of point E and the maximum normal stress in the beam for values of a from 100 to 900 mm, using 100-mm increments.

SOLUTION

ENTER: $L = 1.8\text{ m}$, $E = 200\text{ GPa}$, $h = 0.75\text{ m}$

$m = 8\text{ kg}$, $g = 9.81\text{ m/s}^2$

$I = 6.87 \times 10^{-6}\text{ m}^4$

$S = 91.6 \times 10^{-6}\text{ m}^4$

FOR $a = 100$ mm TO 900 mm STEP 100 mm

$a = a/1000$

$b = L - a$

SEE PROB. 11.71, page 705 → $y_{st} = mga^2b^2/3EIL$

INFLUENCE COEFFICIENT FOR Δ_E FOR UNIT LOAD AT E → $\alpha = a^2b^2/3EIL$

SEE PROB. 11.69, page 705 → $y_m = y_{st}\left[1 + \sqrt{1 + \frac{2h}{y_{st}}}\right]$

$P_{max} = y_m/\alpha$

$M_{max} = P_{max}\, a\, b/L$

$\sigma_{max} = M_{max}/S$

PRINT: a, y_{st}, y_m, P_{max}, σ_{max}

RETURN

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Problem 11.C4

Beam:  W 150 x 13.5
  I = 6.87 x10^-6 m^4    S = 91.6x10^-6 m^3

  L = 1.8 m    h = 750 mm    m = 8 kg    g = 9.81 m/s^2
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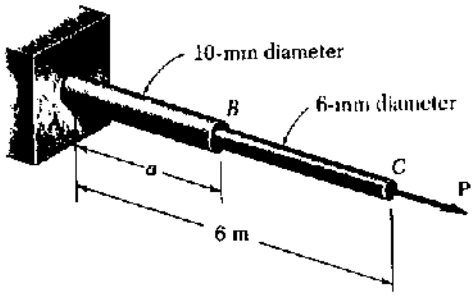
a mm	ystat mm	ymax mm	Pmax N	σ_{max} MPa
100	0.0003	0.6775	173.93	179.33
200	0.0011	1.2757	92.43	179.40
300	0.0021	1.7946	65.75	179.46
400	0.0033	2.2339	52.85	179.51
500	0.0045	2.5936	45.55	179.55
600	0.0055	2.8734	41.13	179.59
700	0.0063	3.0734	38.46	179.61
800	0.0068	3.1934	37.02	179.63
900	0.0069	3.2334	36.56	179.63

NOTE: THE SMALL VARIATION IN σ_{max}. THIS IS DUE TO THE ENERGY ACQUIRED BY THE MASS AS IT FALLS THROUGH y_{max}. SEE PROB. 11.147, page 731, FOR A CASE WHERE ENERGY DELIVERED IS CONSTANT AND σ_{max} IS ALSO CONSTANT.

PROBLEM 11.C5



11.C5 The steel rods *AB* and *BC* are made of a steel for which $\sigma_Y = 300$ MPa and $E = 200$ GPa. (*a*) Write a computer program to calculate, for values of *a* from 0 to 6 m, using 1-m increments, the maximum strain energy that can be acquired by the assembly without causing any permanent deformation. (*b*) For each value of *a* considered, calculate the diameter of a uniform rod of length 6 m and of the same mass as the original assembly, and the maximum strain energy that could be acquired by this uniform rod without causing permanent deformation.

SOLUTION

ENTER: $\sigma_Y = 300$ MPa, $E = 200$ GPa, $L = 6$ m

$AREA_{AB} = \frac{\pi}{4}(0.010\text{ m})^2$, $AREA_{BC} = \frac{\pi}{4}(0.006\text{ m})^2$

$P_m = \sigma_Y\, AREA_{BC}$

FOR $a = 0$ TO 6 m, STEP 1 m

$$U = \frac{P_m^2}{2E}\left(\frac{a}{AREA_{AB}} + \frac{L-a}{AREA_{BC}}\right)$$

FOR UNIFORM ROD OF SAME VOLUME

$$VOL = a(AREA_{AB}) + (L-a)(AREA_{BC})$$

$$d = \sqrt{\frac{4\,VOL}{\pi L}}$$

$$AREA_{NEW} = \frac{\pi}{4} d^2$$

$$P_{NEW} = \sigma_Y (AREA_{NEW})$$

$$U_{NEW} = \frac{P_{NEW}^2 L}{2E(AREA_{NEW})}$$

PRINT a, U, VOL, d, P_{NEW}, U_{NEW}

RETURN

PROGRAM OUTPUT

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Problem 11C5

  sigmaY = 300 MPa,   Pm = 8482 N,   L = 6 m,   E = 200 GPA
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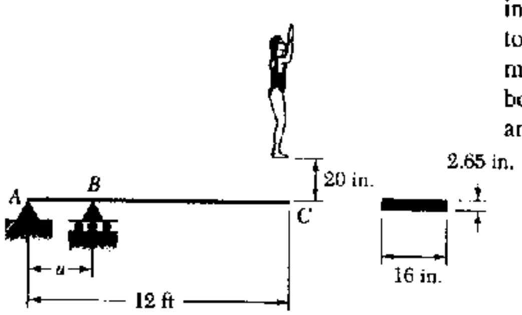
a m	U J	Vol m^3	d mm	New P N	newU J
0.00	38.17	169.65	6.00	8482.30	38.17
1.00	34.10	219.91	6.83	10995.58	49.48
2.00	30.03	270.18	7.57	13508.85	60.79
3.00	25.96	320.44	8.25	16022.12	72.10
4.00	21.88	370.71	8.87	18535.40	83.41
5.00	17.81	420.97	9.45	21048.67	94.72
6.00	13.74	471.24	10.00	23561.95	106.03

PROBLEM 11.C6



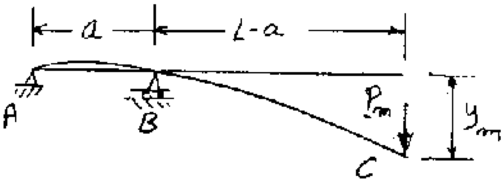
11.C6 A 160-lb diver jumps from a height of 20 in. onto end C of a diving board having the uniform cross section shown. Write a computer program to calculate for values of a from 10 to 50 in., using 10-in. increments, (a) the maximum deflection of point C, (b) the maximum bending moment in the board, (c) the equivalent static load. Assume that the diver's legs remain rigid and use $E = 1.8 \times 10^6$ psi.

SOLUTION

ENTER: $L = 12$ ft, $h = 20$ in., $W = 160$ lb
$E = 1.8 \times 10^6$ psi

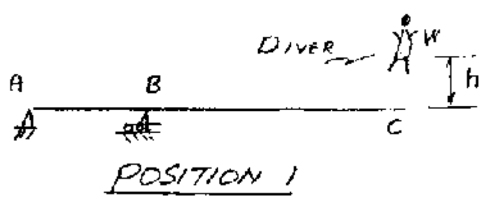
$I = (16\text{ in.})(2.65\text{ in})^3/12$

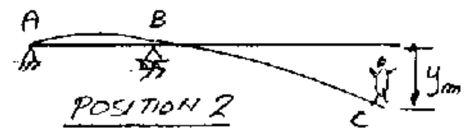
$S = (16\text{ in.})(2.65\text{ in})^2/6$



$y_m = P_m\,\alpha$ WHERE α = INFLUENCE COEFFICIENT
SEE BELOW FOR DETERMINATION OF α

WHERE P_m = EQUIVALENT STATIC LOAD





$U_2 = \frac{1}{2}P_m y_m = \frac{1}{2}\frac{y_m^2}{\alpha}$

WORK $= W(h + y_m)$

WORK $= U_2$

$W(h + y_m) = \frac{1}{2}\frac{y_m^2}{\alpha}$ A

PROGRAM SOLUTION OF a FOR y_m. ENTER α

FOR a = 10 in. TO 50 in. STEP 10 in.

SOLVE A FOR y_m, $P_m = y_m/\alpha$

$M_{max} = M_B = P_m(L-a)$

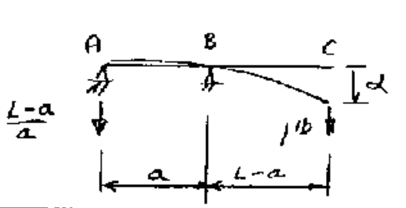
$\sigma = M_{max}/S$

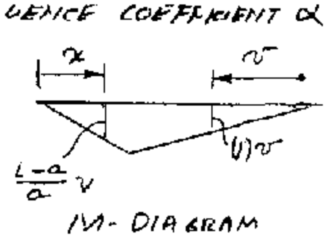
PRINT a, y_m, P_m, M_m, σ

PROGRAM OUTPUT

a in.	ym in.	Pm lb	Max M kip·in.	sigma psi
10	14.622	757.7	101.532	5422
20	13.262	802.6	99.519	5314
30	11.950	855.6	97.536	5208
40	10.683	919.1	95.583	5104
50	9.462	996.4	93.661	5001

DETERMINATION OF INFLUENCE COEFFICIENT α





$U = \frac{1}{2}(1^{\#})\alpha = \Sigma \int \frac{M^2}{2EI}dx$

$\frac{\alpha}{2} = \frac{1}{2EI}\left[\int_0^a \left(\frac{L-a}{a}\right)^2 x^2\,dx + \int_0^{L-a} v^2\,dv\right]$

$\alpha = \frac{1}{EI}\left[\frac{(L-a)^2}{a^2}\frac{a^3}{3} + \frac{(L-a)^3}{3}\right]$

$\alpha = \frac{1}{3EI}\left[(L-a)^2 a + (L-a)^3\right]$ ◁