Assignment 7

1. A communication channel transmits digits 0 and 1. However, due to static, the transmitted digit is incorrectly received with a probability 0.2. Suppose we want to transmit an important message of one binary digit. To reduce the chance of error, we transmit 00000 instead of 0 and 11111 instead of 1. If the receiver of the message uses ‘majority’ decoding (that is if it is more than or equal to three zeros, it is zero; and, if it is more than three ones, it is one), what is the probability of the incorrect decoding of the message.
2. If you but a lottery ticket in 50 lotteries, in each of which your chance of winning a prize is 0.01, what is the (approximate) probability that you will win a prize (a) at least once, (b) exactly once, and (c) at least twice.
3. The number of times that an individual contracts a cold in a given year is a Poisson random variable with parameter l=3. Suppose a new wonder drug (based on large quantities of vitamin C) has just been marketed that reduces the Poisson parameter to l=2 for 75 percent of the population. For the other 25 percent of the population, the drug has no appreciable effect on colds. If an individual tries the drug for a year and has 0 colds in that time, how likely is it that the drug is beneficial for him or her?
4. The probability of error in the transmission of a binary digit over a communication channel is $1/10^3$. Write an expression for the exact probability of more than 3 errors when transmitting a block of 10^3 bits. What is its approximate value? Assume independence.
5. A contractor purchases a shipment of 100 transistors. It is his policy to test 10 of these transistors and to keep the shipment only if at least 9 of the 10 are in working condition. If the shipment contains 20 defective transistors, what is the probability it will be kept?
6. You arrive at a bus stop at 10 o’clock, knowing that the bus will arrive at some time uniformly distributed between 10 and 10:30. (a) What is the probability that you will have to wait longer than 10 minutes? (b) If, at 10:15, the bus has not yet arrived, what is the probability that you will have to wait at least an additional 10 minutes?