Fifth SI Edition

MECHANICS OF MATERIALS

CHAPTER

5

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Analysis and Design of Beams for Bending

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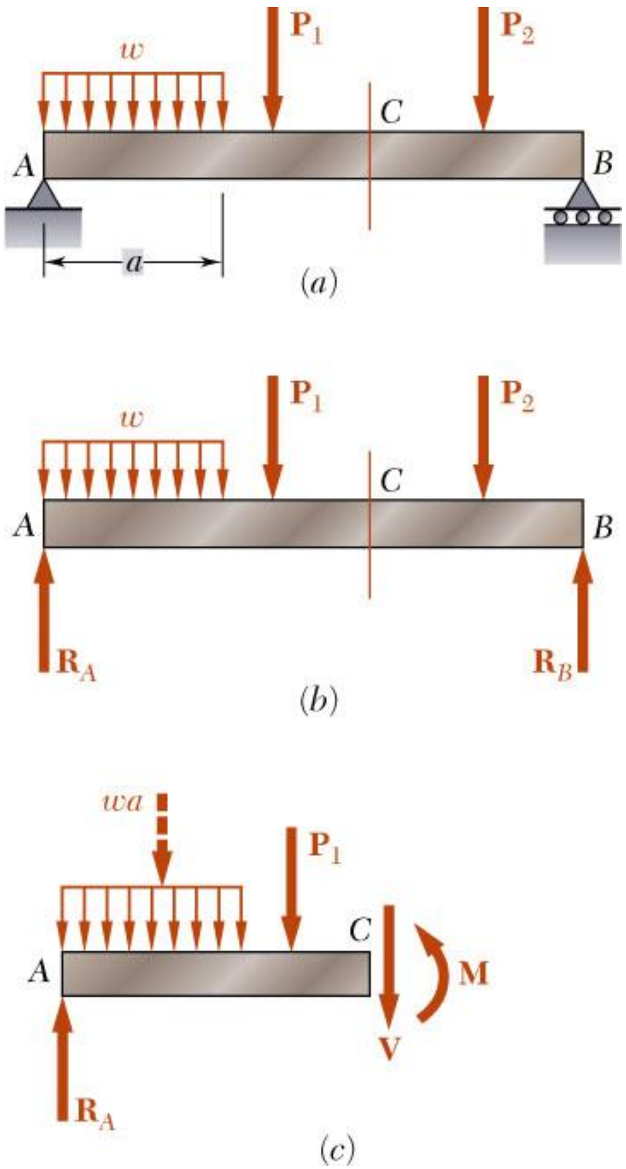
Introduction



- Objective - Analysis and design of beams
- *Beams* - structural members supporting loads at various points along the member
- Transverse loadings of beams are classified as *concentrated* loads or *distributed* loads
- Applied loads result in internal forces consisting of a shear force (from the shear stress distribution) and a bending couple (from the normal stress distribution)
- Normal stress is often the critical design criteria

$$\sigma_x = -\frac{My}{I} \qquad \sigma_m = \frac{|M|c}{I} = \frac{|M|}{S}$$

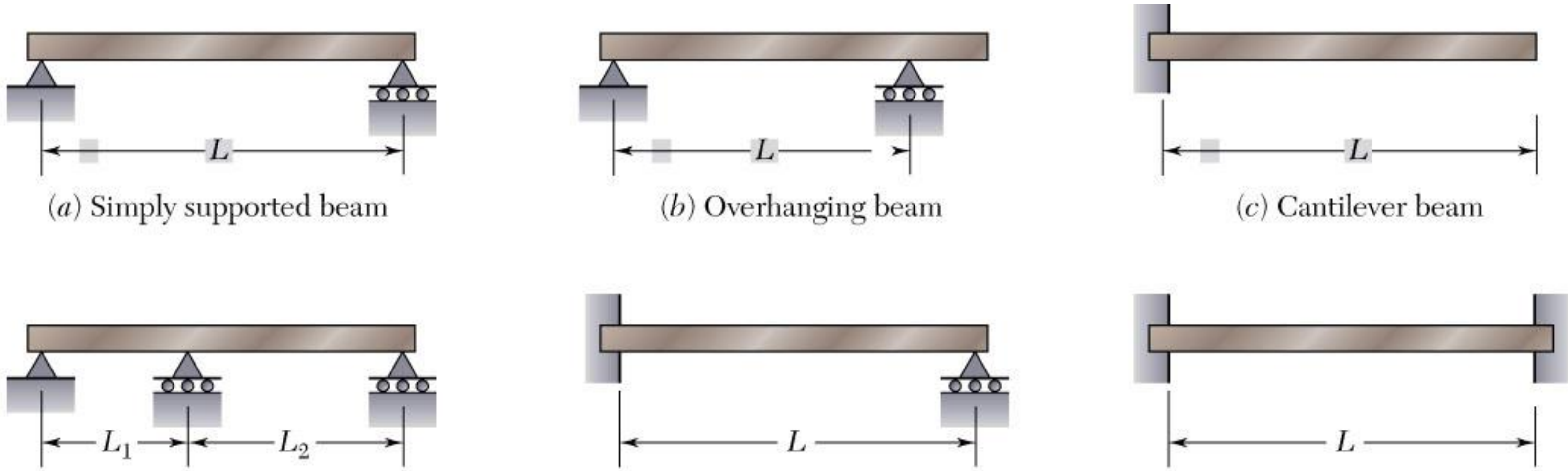
Requires determination of the location and magnitude of largest bending moment

Introduction

Classification of Beam Supports

Statically Determinate Beams

Statically Indeterminate Beams



(*a*) Simply supported beam

(*b*) Overhanging beam

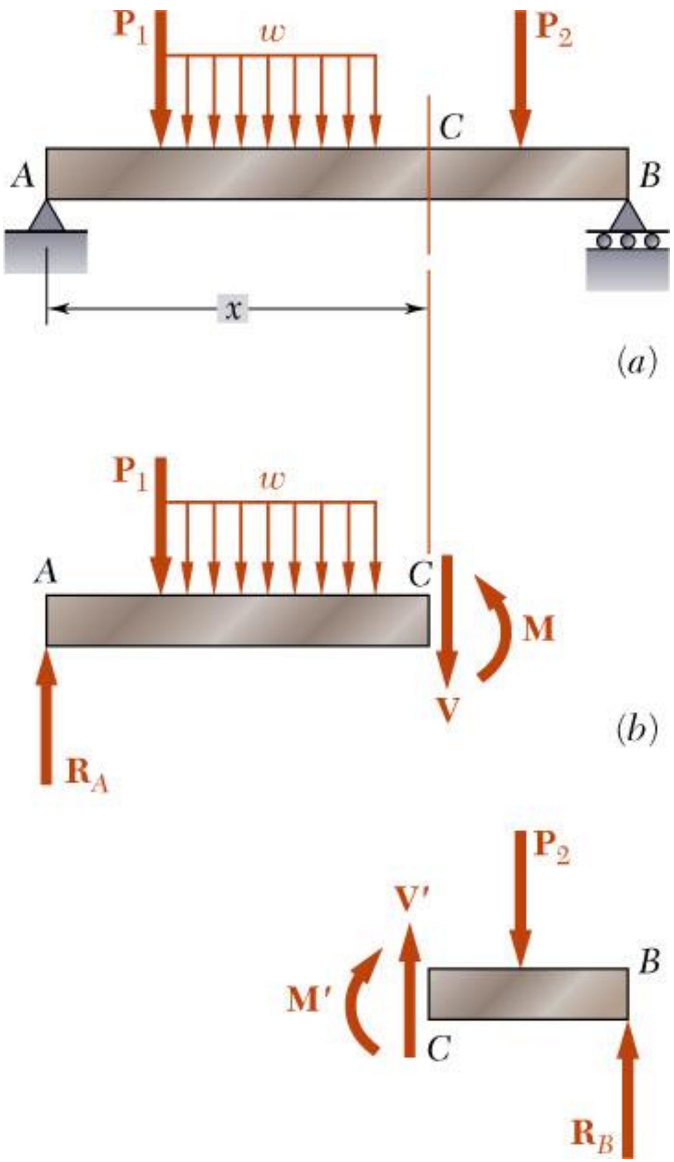
(*c*) Cantilever beam

(*d*) Continuous beam

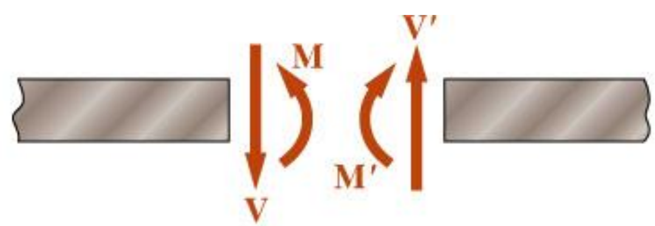
(*e*) Beam fixed at one end and simply supported at the other end

(*f*) Fixed beam

Shear and Bending Moment Diagrams

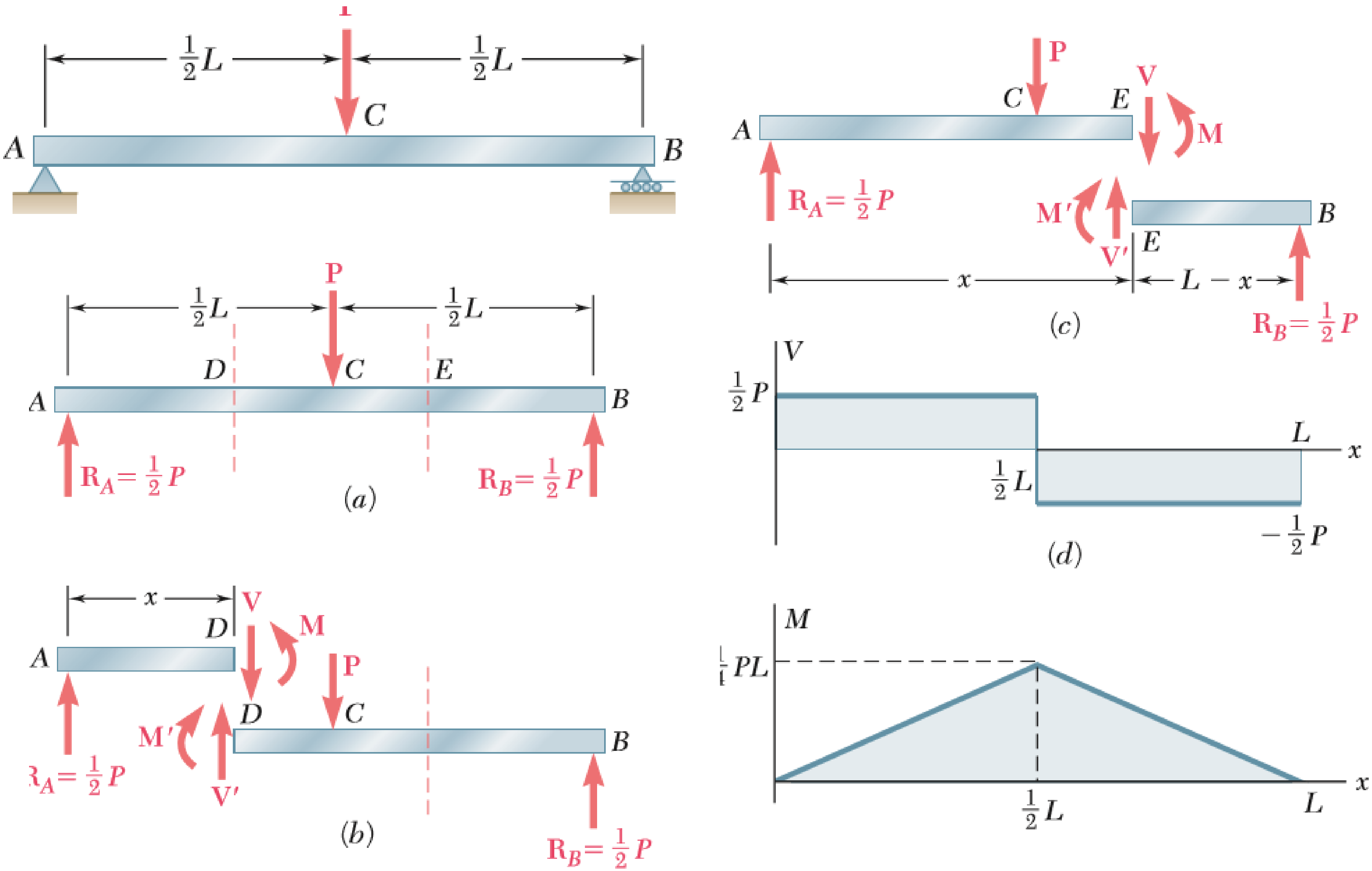


- Determination of maximum normal and shearing stresses requires identification of maximum internal shear force and bending couple.
- Shear force and bending couple at a point are determined by passing a section through the beam and applying an equilibrium analysis on the beam portions on either side of the section.
- Sign conventions for shear forces *V* and *V'* and bending couples *M* and *M'*

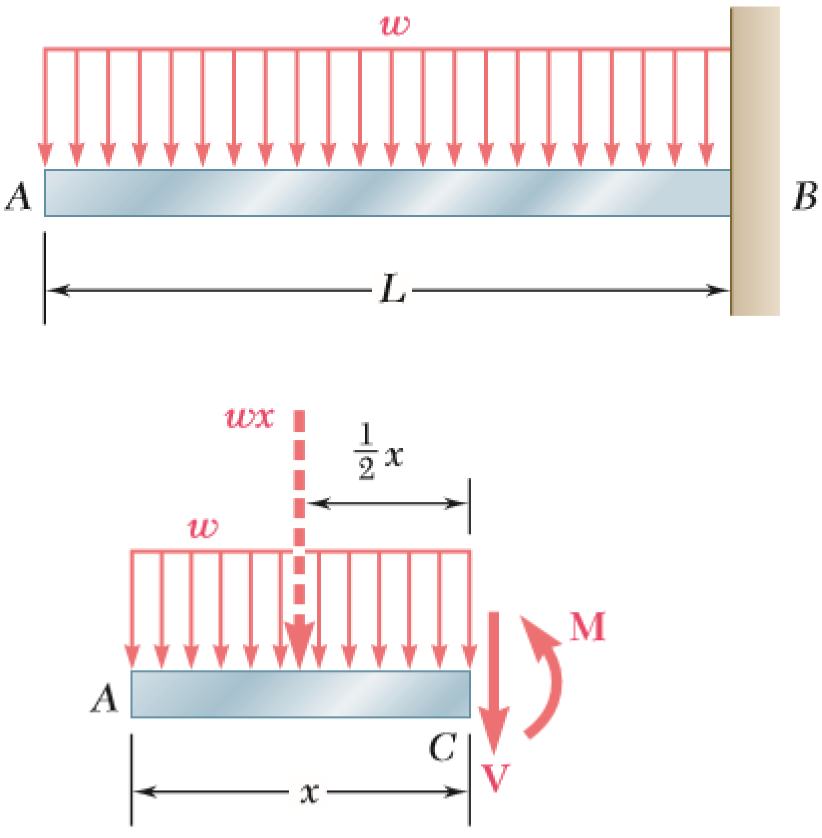


(*a*) Internal forces
(positive shear and positive bending moment)

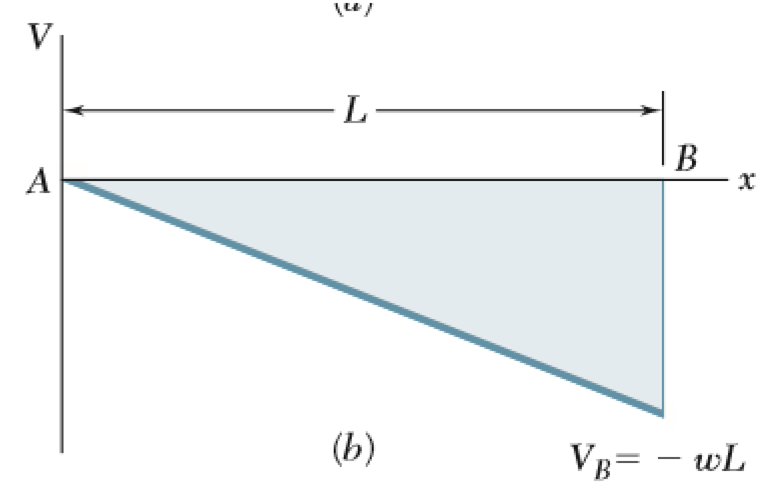
Draw shear force and bending moment diagram



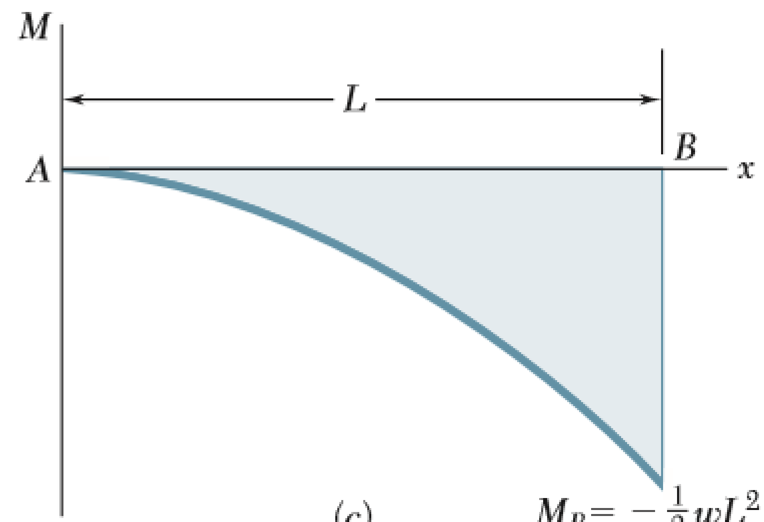
Shear force and bending moment diagram



(a)

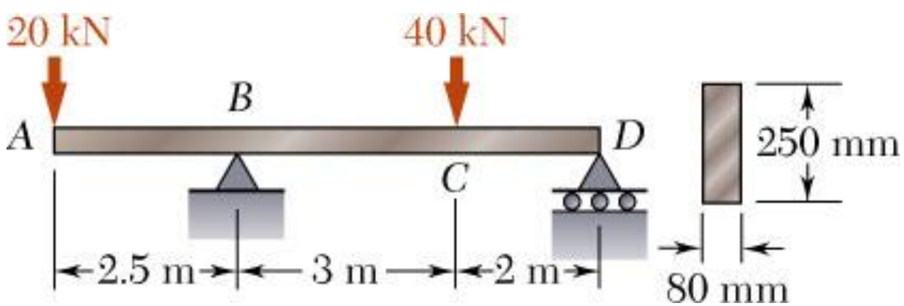


(b)



(c)

Sample Problem 5.1

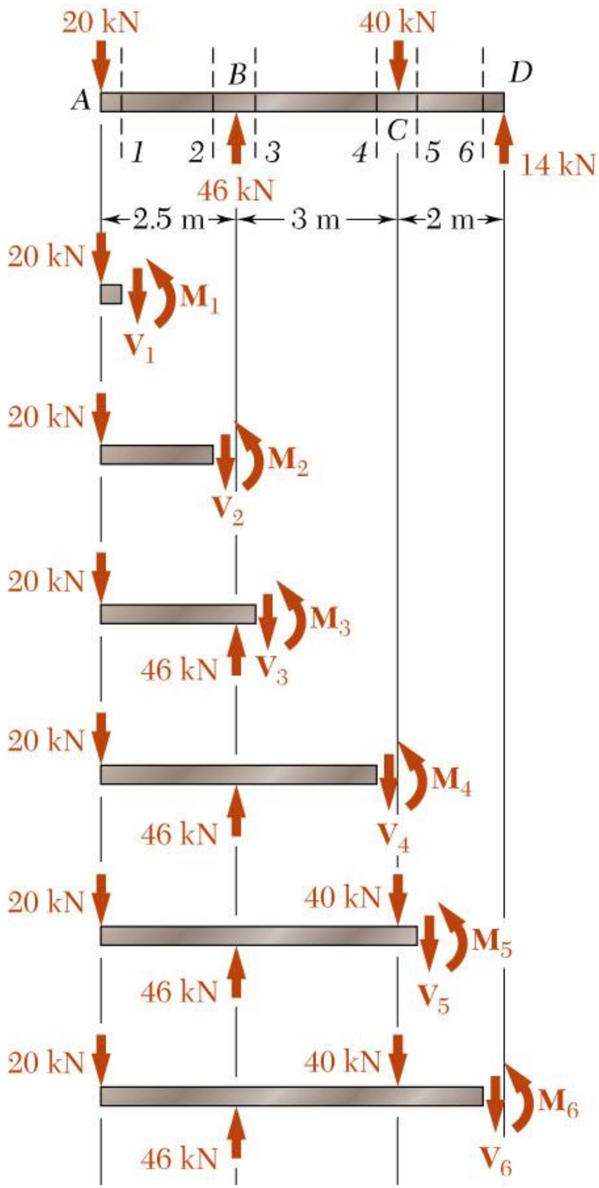


For the timber beam and loading shown, draw the shear and bend-moment diagrams and determine the maximum normal stress due to bending.

SOLUTION:

- Treating the entire beam as a rigid body, determine the reaction forces
- Section the beam at points near supports and load application points. Apply equilibrium analyses on resulting free-bodies to determine internal shear forces and bending couples
- Identify the maximum shear and bending-moment from plots of their distributions.
- Apply the elastic flexure formulas to determine the corresponding maximum normal stress.

Sample Problem 5.1



SOLUTION:

- Treating the entire beam as a rigid body, determine the reaction forces

$$\text{from } \sum F_y = 0 = \sum M_B : \quad R_B = 46\,\text{kN} \quad R_D = 14\,\text{kN}$$

- Section the beam and apply equilibrium analyses on resulting free-bodies

$$\sum F_y = 0 \quad -20\,\text{kN} - V_1 = 0 \qquad V_1 = -20\,\text{kN}$$

$$\sum M_1 = 0 \quad (20\,\text{kN})(0\,\text{m}) + M_1 = 0 \quad M_1 = 0$$

$$\sum F_y = 0 \quad -20\,\text{kN} - V_2 = 0 \qquad V_2 = -20\,\text{kN}$$

$$\sum M_2 = 0 \quad (20\,\text{kN})(2.5\,\text{m}) + M_2 = 0 \quad M_2 = -50\,\text{kN}\cdot\text{m}$$

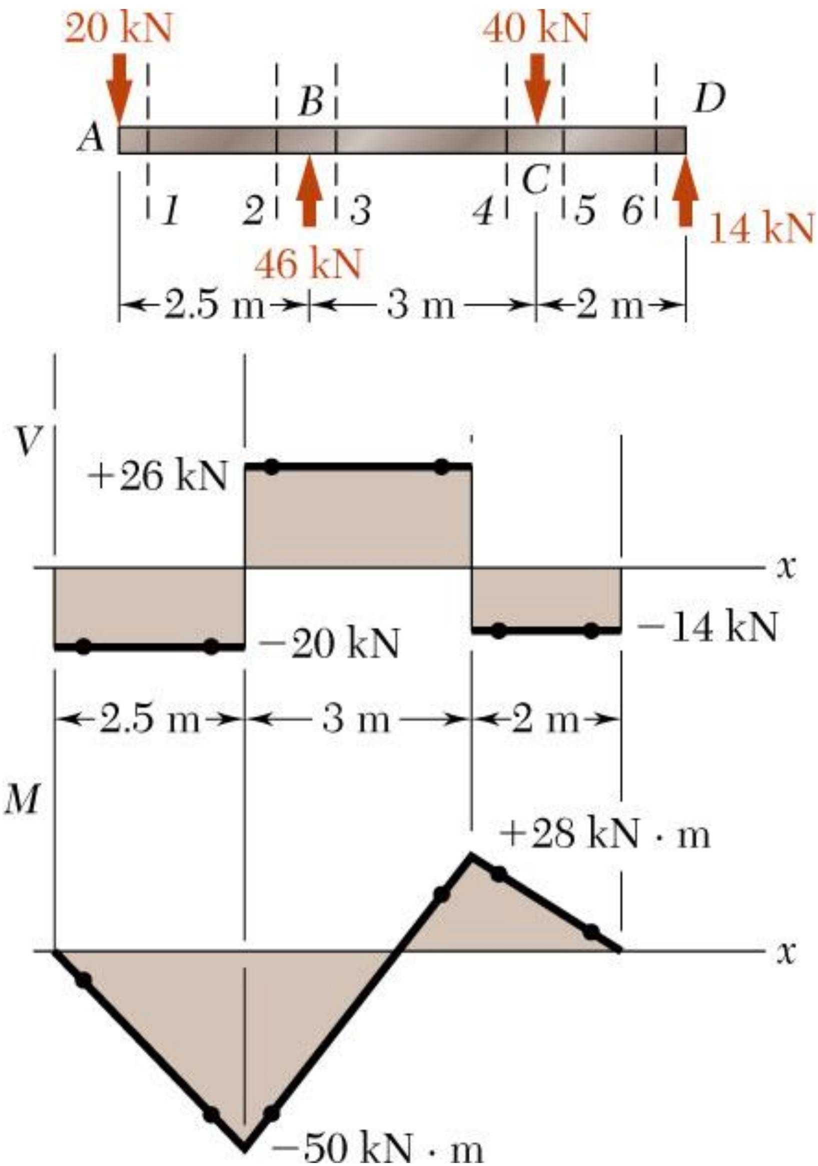
$$V_3 = +26\,\text{kN} \quad M_3 = -50\,\text{kN}\cdot\text{m}$$

$$V_4 = +26\,\text{kN} \quad M_4 = +28\,\text{kN}\cdot\text{m}$$

$$V_5 = -14\,\text{kN} \quad M_5 = +28\,\text{kN}\cdot\text{m}$$

$$V_6 = -14\,\text{kN} \quad M_6 = 0$$

Sample Problem 5.1



- Identify the maximum shear and bending-moment from plots of their distributions.

$$V_m = 26\,\mathrm{kN} \quad M_m = |M_B| = 50\,\mathrm{kN\cdot m}$$

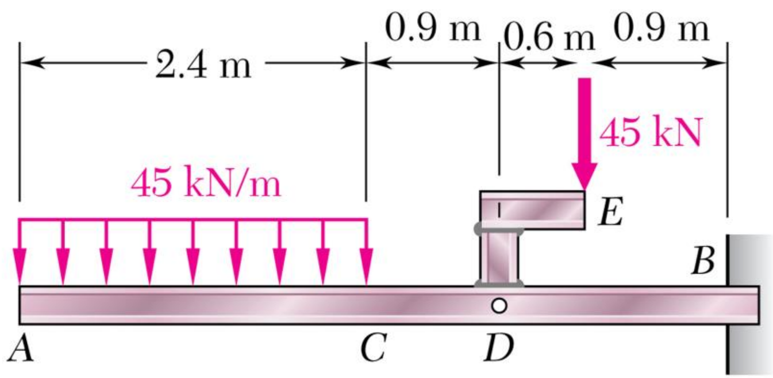
- Apply the elastic flexure formulas to determine the corresponding maximum normal stress.

$$S = \tfrac{1}{6} b h^2 = \tfrac{1}{6}(0.080\,\mathrm{m})(0.250\,\mathrm{m})^2 = 833.33\times10^{-6}\,\mathrm{m}^3$$

$$\sigma_m = \frac{|M_B|}{S} = \frac{50\times10^3\,\mathrm{N\cdot m}}{833.33\times10^{-6}\,\mathrm{m}^3}$$

$$\boxed{\sigma_m = 60.0\times10^6\,\mathrm{Pa}}$$

Sample Problem 5.2



The structure shown is constructed of a W 250x167 rolled-steel beam. (*a*) Draw the shear and bending-moment diagrams for the beam and the given loading. (*b*) determine normal stress in sections just to the right and left of point *D*.

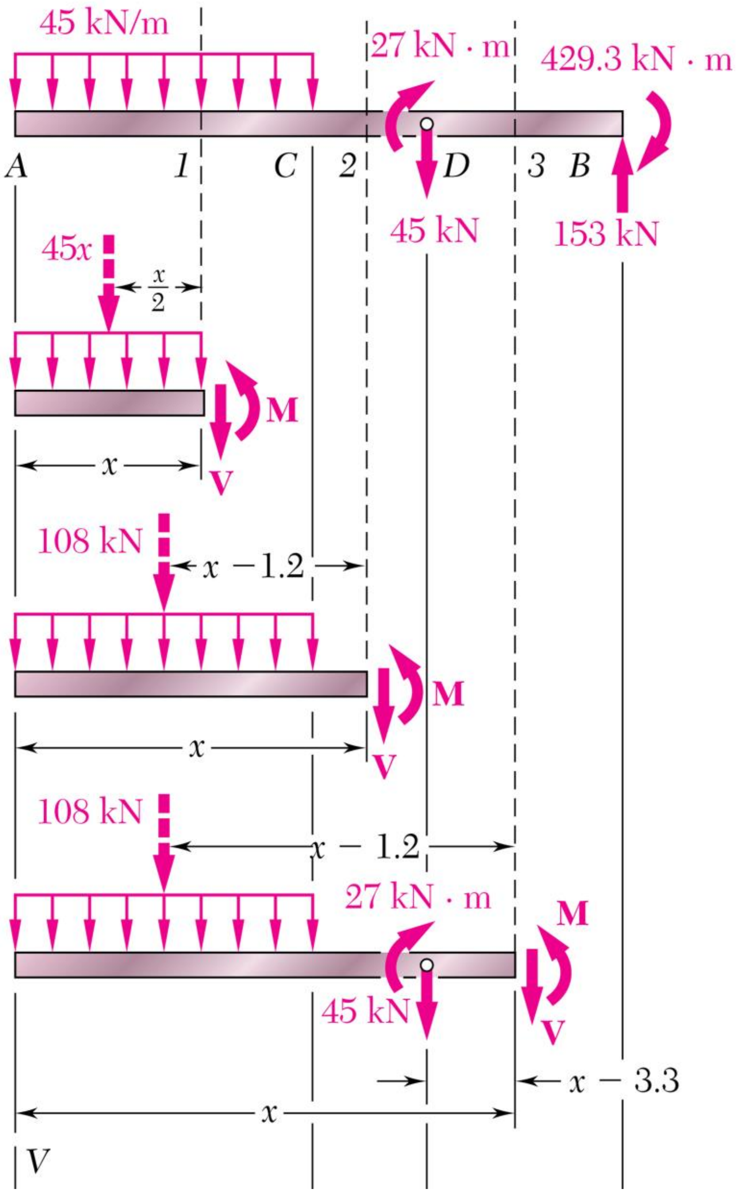
SOLUTION:

- Replace the 45 kN load with an equivalent force-couple system at *D*. Find the reactions at *B* by considering the beam as a rigid body.

- Section the beam at points near the support and load application points. Apply equilibrium analyses on resulting free-bodies to determine internal shear forces and bending couples.

- Apply the elastic flexure formulas to determine the maximum normal stress to the left and right of point *D*.

Sample Problem 5.2



SOLUTION:

- Replace the 45 kN load with equivalent force-couple system at *D*. Find reactions at *B*.
- Section the beam and apply equilibrium analyses on resulting free-bodies.

From A to C :

$$\sum F_y = 0 \quad -45x - V = 0 \qquad V = -45x\,\text{kN}$$

$$\sum M_1 = 0 \quad (45x)\left(\tfrac{1}{2}x\right) + M = 0 \quad M = -22.5x^2\ \text{kNm}$$

From C to D :

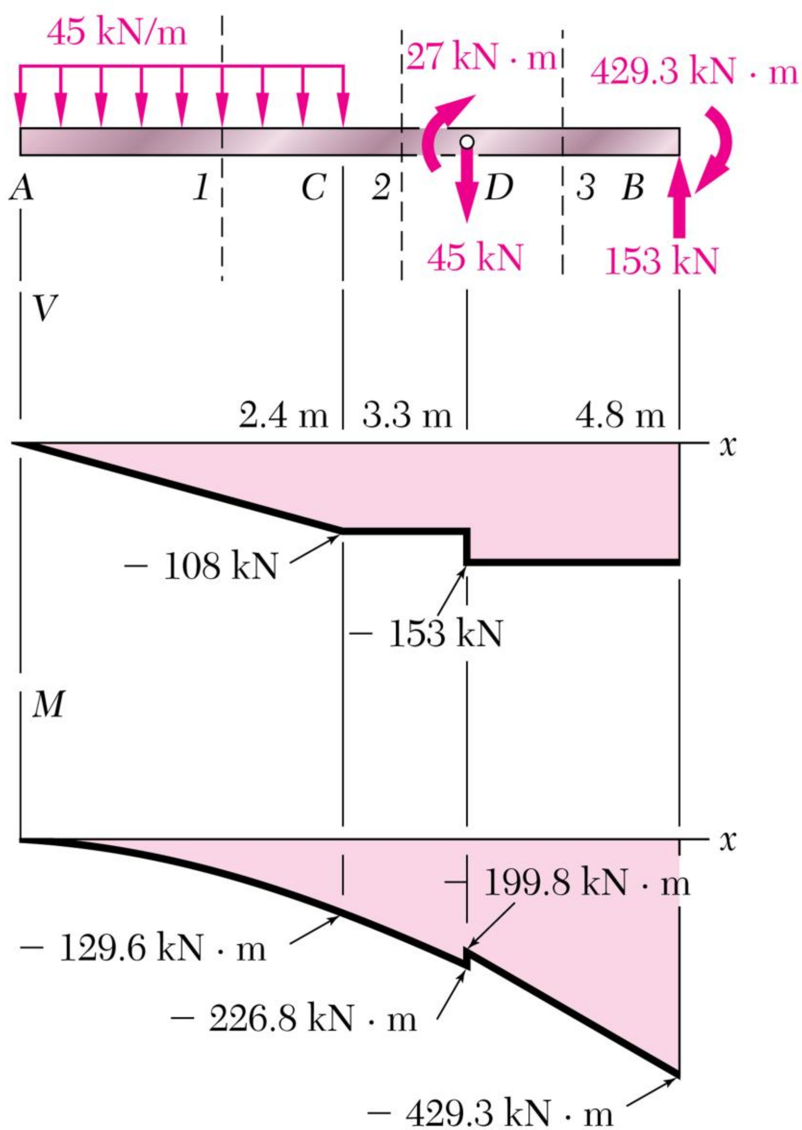
$$\sum F_y = 0 \quad -108 - V = 0 \qquad V = -108\,\text{kN}$$

$$\sum M_2 = 0 \quad 108(x - 1.2) + M = 0 \quad M = (129.6 - 108x)\,\text{kNm}$$

From D to B :

$$V = -153\,\text{kN} \qquad M = (305.1 - 153x)\,\text{kNm}$$

Sample Problem 5.2



- Apply the elastic flexure formulas to determine the maximum normal stress to the left and right of point *D*.

From Appendix C for a W250x167 rolled steel shape, $S = 2.08\text{x}10^{-3}\ \text{m}^3$ about the *X-X* axis.

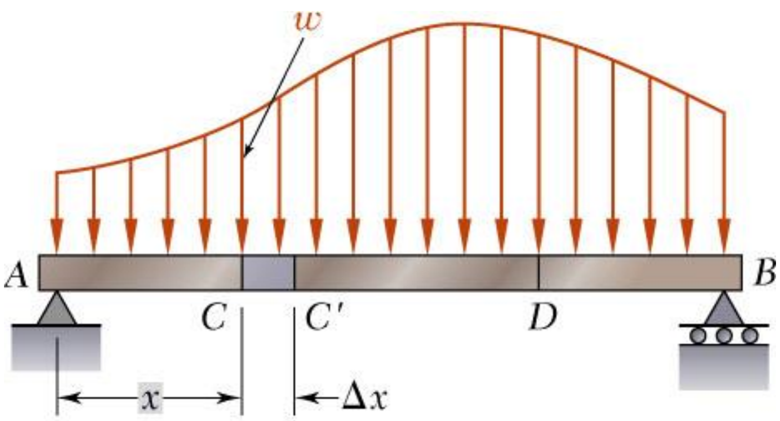
To the left of D:

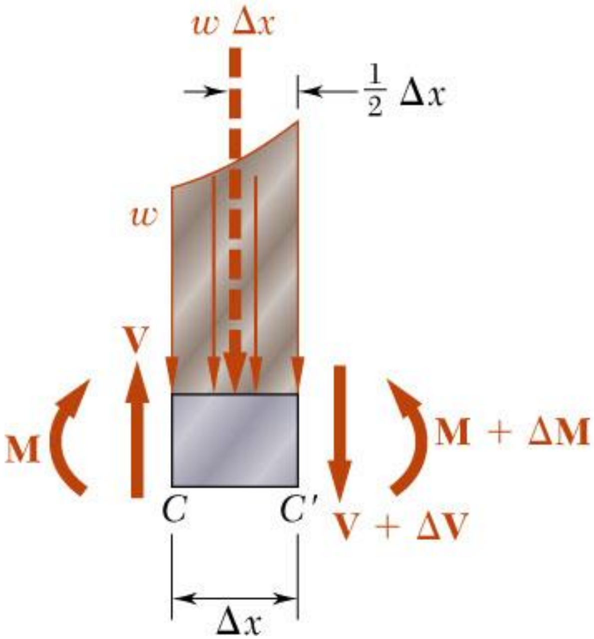
$$\sigma_m = \frac{|M|}{S} = \frac{226.8\times10^3\ \text{Nm}}{2.08\times10^{-3}\ \text{m}^3} \qquad \boxed{\sigma_m = 109\,\text{MPa}}$$

To the right of D:

$$\sigma_m = \frac{|M|}{S} = \frac{199.8\times10^3\ \text{Nm}}{2.08\times10^{-3}\ \text{m}^3} \qquad \boxed{\sigma_m = 96\,\text{MPa}}$$

Relations Among Load, Shear, and Bending Moment





- Relationship between load and shear:

$$\sum F_y = 0: \quad V - (V + \Delta V) - w\,\Delta x = 0$$
$$\Delta V = -w\,\Delta x$$

$$\frac{dV}{dx} = -w$$

$$V_D - V_C = -\int_{x_C}^{x_D} w\,dx$$

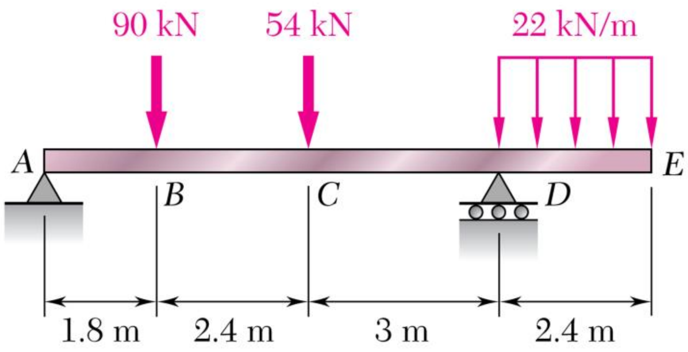
- Relationship between shear and bending moment:

$$\sum M_{C'} = 0: \quad (M + \Delta M) - M - V\,\Delta x + w\Delta x\frac{\Delta x}{2} = 0$$
$$\Delta M = V\,\Delta x - \tfrac{1}{2}w(\Delta x)^2$$

$$\frac{dM}{dx} = V$$

$$M_D - M_C = \int_{x_C}^{x_D} V\,dx$$

Sample Problem 5.3

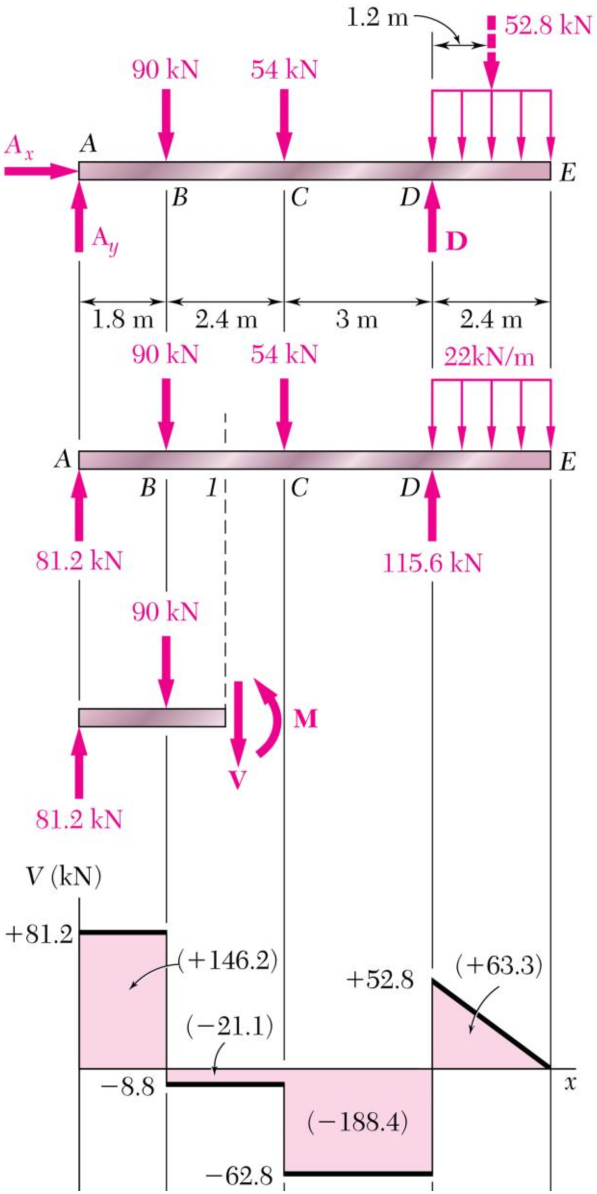


Draw the shear and bending moment diagrams for the beam and loading shown.

SOLUTION:

- Taking the entire beam as a free body, determine the reactions at *A* and *D*.
- Apply the relationship between shear and load to develop the shear diagram.
- Apply the relationship between bending moment and shear to develop the bending moment diagram.

Sample Problem 5.3



SOLUTION:

- Taking the entire beam as a free body, determine the reactions at *A* and *D*.

$$\sum M_A = 0$$

$$0 = D(7.2\,\text{m}) - (90\,\text{kN})(1.8\,\text{m}) - (54\,\text{kN})(4.2\,\text{m}) - (52.8\,\text{kN})(8.4\,\text{m})$$

$$D = 115.6\,\text{kN}$$

$$\sum F_y = 0$$

$$0 = A_y - 90\,\text{kN} - 54\,\text{kN} + 115.6\,\text{kN} - 52.8\,\text{kN}$$

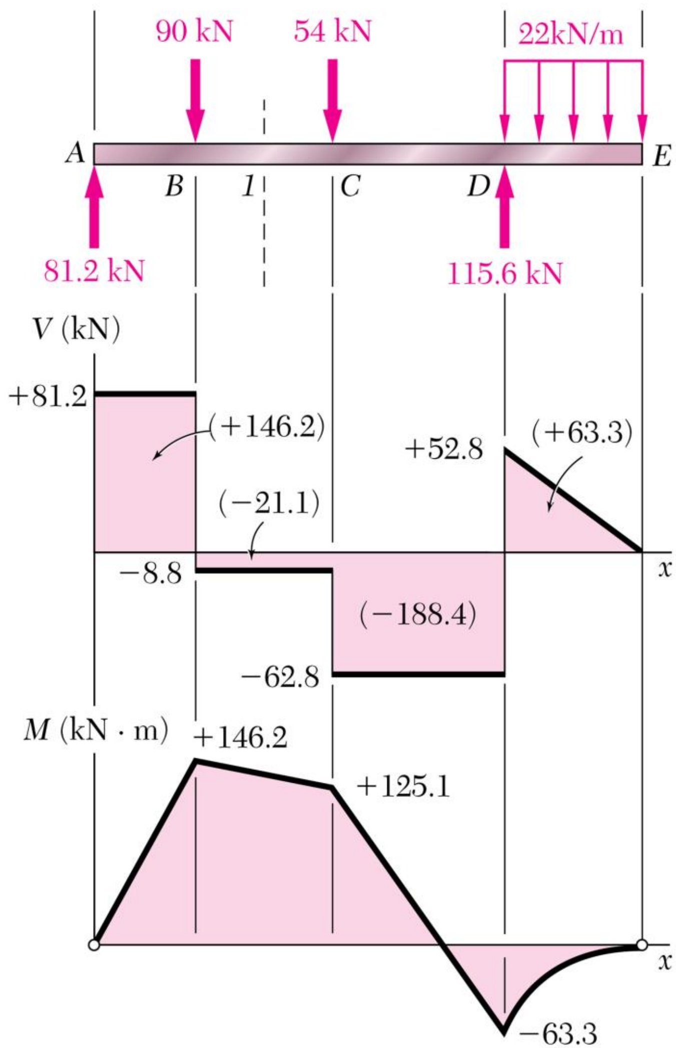
$$A_y = 81.2\,\text{kN}$$

- Apply the relationship between shear and load to develop the shear diagram.

$$\frac{dV}{dx} = -w \qquad dV = -w\,dx$$

- zero slope between concentrated loads
- linear variation over uniform load segment

Sample Problem 5.3

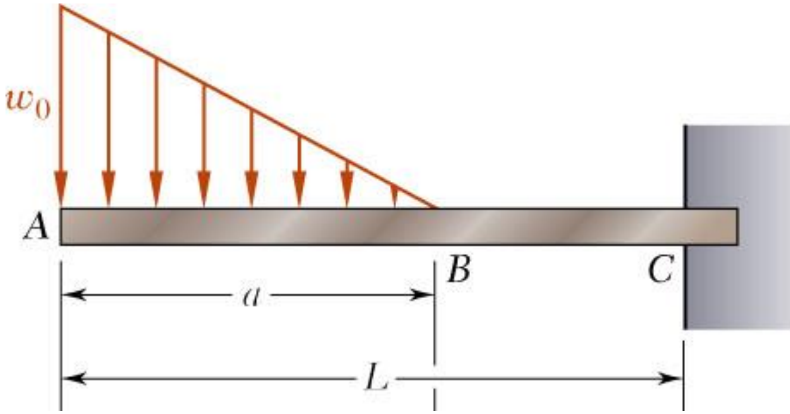


- Apply the relationship between bending moment and shear to develop the bending moment diagram.

$$\frac{dM}{dx} = V \qquad dM = V\,dx$$

- bending moment at *A* and *E* is zero
- bending moment variation between *A*, *B*, *C* and *D* is linear
- bending moment variation between D and *E* is quadratic
- net change in bending moment is equal to areas under shear distribution segments
- total of all bending moment changes across the beam should be zero

Sample Problem 5.5

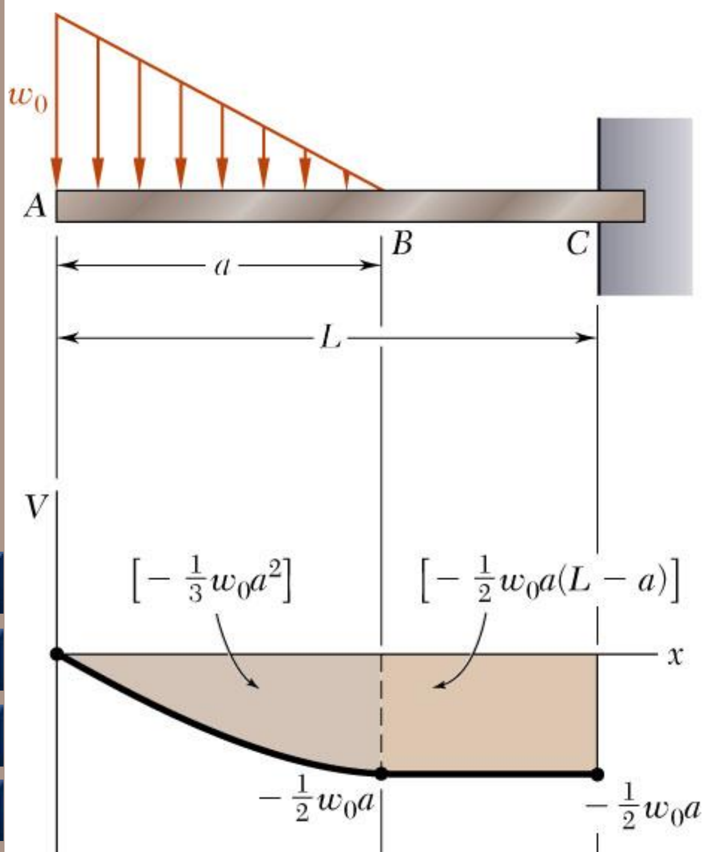


Draw the shear and bending moment diagrams for the beam and loading shown.

SOLUTION:

- Taking the entire beam as a free body, determine the reactions at *C*.
- Apply the relationship between shear and load to develop the shear diagram.
- Apply the relationship between bending moment and shear to develop the bending moment diagram.

Sample Problem 5.5



SOLUTION:

- Taking the entire beam as a free body, determine the reactions at *C*.

$$\sum F_y = 0 = -\tfrac{1}{2}w_0a + R_C \qquad R_C = \tfrac{1}{2}w_0a$$

$$\sum M_C = 0 = \tfrac{1}{2}w_0a\left(L - \frac{a}{3}\right) + M_C \qquad M_C = -\tfrac{1}{2}w_0a\left(L - \frac{a}{3}\right)$$

Results from integration of the load and shear distributions should be equivalent.

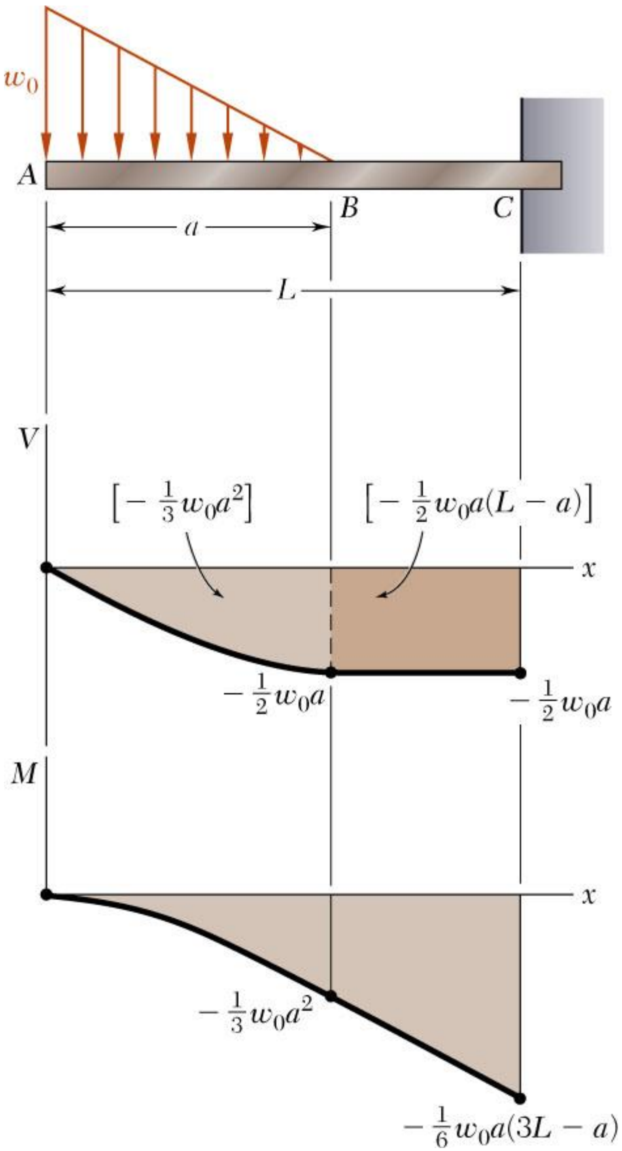
- Apply the relationship between shear and load to develop the shear diagram.

$$V_B - V_A = -\int_0^a w_0\left(1 - \frac{x}{a}\right)dx = -\left[w_0\left(x - \frac{x^2}{2a}\right)\right]_0^a$$

$$V_B = -\tfrac{1}{2}w_0a = -(\textit{area under load curve})$$

- No change in shear between *B* and *C*.
- Compatible with free body analysis

Sample Problem 5.5



- Apply the relationship between bending moment and shear to develop the bending moment diagram.

$$M_B - M_A = \int_0^a \left(-w_0\left(x - \frac{x^2}{2a}\right)\right)dx = \left[-w_0\left(\frac{x^2}{2} - \frac{x^3}{6a}\right)\right]_0^a$$

$$M_B = -\tfrac{1}{3}w_0a^2$$

$$M_B - M_C = \int_a^L \left(-\tfrac{1}{2}w_0a\right)dx = -\tfrac{1}{2}w_0a(L-a)$$

$$M_C = -\tfrac{1}{6}w_0a(3L-a) = \frac{a\,w_0}{2}\left(L - \frac{a}{3}\right)$$

Results at *C* are compatible with free-body analysis

Design of Prismatic Beams for Bending

- The largest normal stress is found at the surface where the maximum bending moment occurs.

$$\sigma_m = \frac{|M|_{\max} c}{I} = \frac{|M|_{\max}}{S}$$

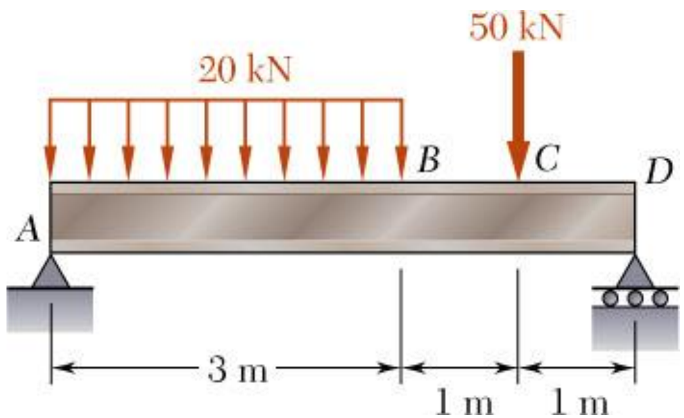
- A safe design requires that the maximum normal stress be less than the allowable stress for the material used. This criteria leads to the determination of the minimum acceptable section modulus.

$$\sigma_m \leq \sigma_{all}$$

$$S_{\min} = \frac{|M|_{\max}}{\sigma_{all}}$$

- Among beam section choices which have an acceptable section modulus, the one with the smallest weight per unit length or cross sectional area will be the least expensive and the best choice.

Sample Problem 5.8

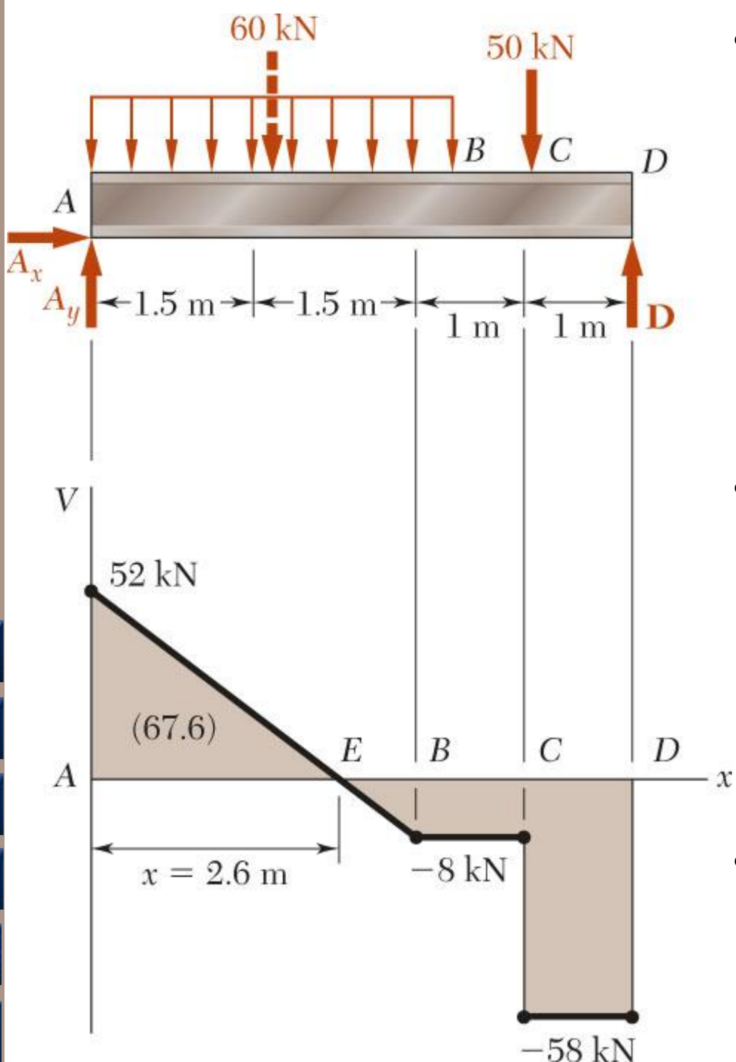


A simply supported steel beam is to carry the distributed and concentrated loads shown. Knowing that the allowable normal stress for the grade of steel to be used is 160 MPa, select the wide-flange shape that should be used.

SOLUTION:

- Considering the entire beam as a free-body, determine the reactions at *A* and *D*.
- Develop the shear diagram for the beam and load distribution. From the diagram, determine the maximum bending moment.
- Determine the minimum acceptable beam section modulus. Choose the best standard section which meets this criteria.

Sample Problem 5.8



- Considering the entire beam as a free-body, determine the reactions at *A* and *D*.

$$\sum M_A = 0 = D(5\,\text{m}) - (60\,\text{kN})(1.5\,\text{m}) - (50\,\text{kN})(4\,\text{m})$$
$$D = 58.0\,\text{kN}$$
$$\sum F_y = 0 = A_y + 58.0\,\text{kN} - 60\,\text{kN} - 50\,\text{kN}$$
$$A_y = 52.0\,\text{kN}$$

- Develop the shear diagram and determine the maximum bending moment.

$$V_A = A_y = 52.0\,\text{kN}$$
$$V_B - V_A = -(\textit{area under load curve}) = -60\,\text{kN}$$
$$V_B = -8\,\text{kN}$$

- Maximum bending moment occurs at $V = 0$ or $x = 2.6$ m.

$$|M|_{\max} = (\textit{area under shear curve, A to E})$$
$$= 67.6\,\text{kN}$$

Sample Problem 5.8

- Determine the minimum acceptable beam section modulus.

$$S_{\min} = \frac{|M|_{\max}}{\sigma_{all}} = \frac{67.6\,\mathrm{kN\cdot m}}{160\,\mathrm{MPa}}$$
$$= 422.5\times10^{-6}\,\mathrm{m}^3 = 422.5\times10^{3}\,\mathrm{mm}^3$$

- Choose the best standard section which meets this criteria.

$$\boxed{W360\times32.9}$$

Shape	$S\times10^3\,\mathrm{mm}^3$
W410×38.8	637
W360×32.9	474
W310×38.7	549
W250×44.8	535
W200×46.1	448