

CHAPTER

3

VECTOR MECHANICS FOR ENGINEERS: **STATICS**

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Rigid Bodies:
Equivalent Systems
of Forces

Vector Mechanics for Engineers: Statics

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Vector Mechanics for Engineers: Statics

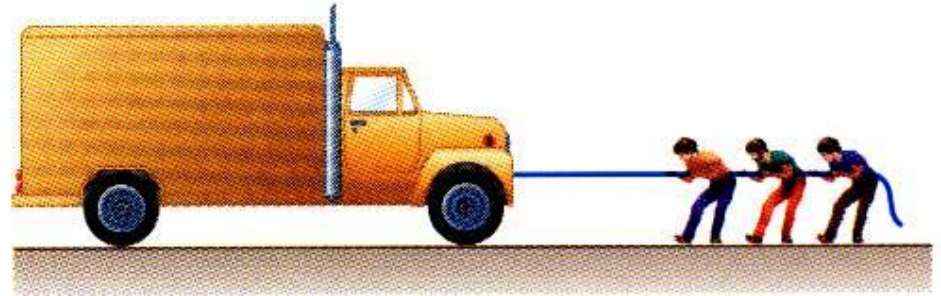
Introduction

- Treatment of a body as a single particle is not always possible. In general, the size of the body and the specific points of application of the forces must be considered.
- Most bodies in elementary mechanics are assumed to be rigid, i.e., the actual deformations are small and do not affect the conditions of equilibrium or motion of the body.
- Current chapter describes the effect of forces exerted on a rigid body and how to replace a given system of forces with a simpler equivalent system.
 - moment of a force about a point
 - moment of a force about an axis
 - moment due to a couple
- Any system of forces acting on a rigid body can be replaced by an equivalent system consisting of one force acting at a given point and one couple.

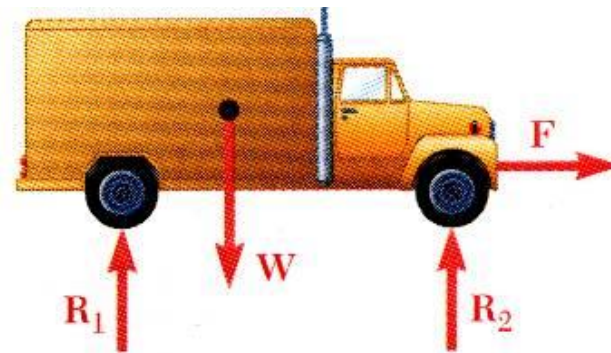
Vector Mechanics for Engineers: Statics

External and Internal Forces

- Forces acting on rigid bodies are divided into two groups:
 - External forces
 - Internal forces



- External forces are shown in a free-body diagram.

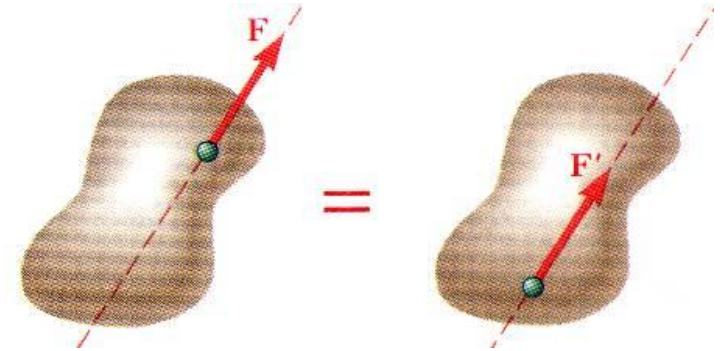


- If unopposed, each external force can impart a motion of translation or rotation, or both.

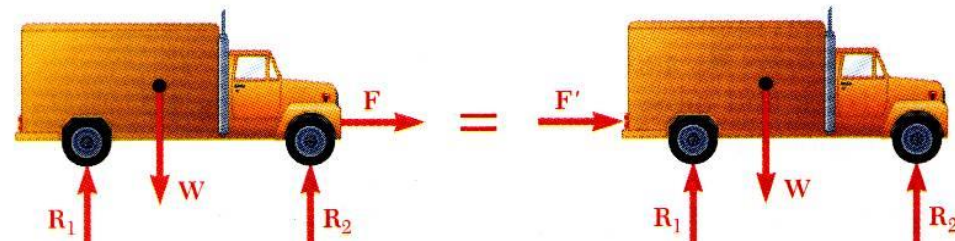
Vector Mechanics for Engineers: Statics

Principle of Transmissibility: Equivalent Forces

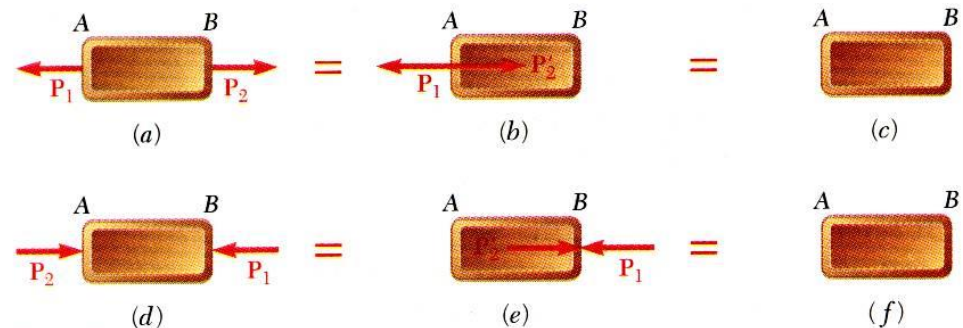
- *Principle of Transmissibility* -
Conditions of equilibrium or motion are not affected by *transmitting* a force along its line of action.
NOTE: \mathbf{F} and \mathbf{F}' are equivalent forces.



- Moving the point of application of the force \mathbf{F} to the rear bumper does not affect the motion or the other forces acting on the truck.



- Principle of transmissibility may not always apply in determining internal forces and deformations.



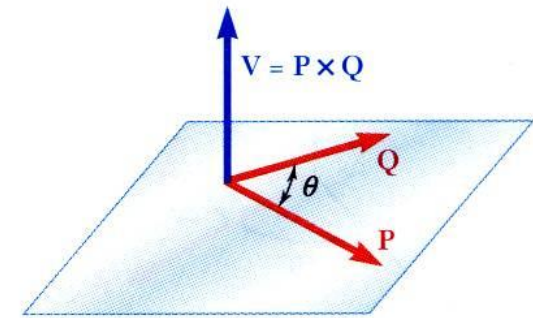
Vector Mechanics for Engineers: Statics

Vector Product of Two Vectors

- Concept of the moment of a force about a point is more easily understood through applications of the *vector product* or *cross product*.

- Vector product of two vectors \mathbf{P} and \mathbf{Q} is defined as the vector \mathbf{V} which satisfies the following conditions:

1. Line of action of \mathbf{V} is perpendicular to plane containing \mathbf{P} and \mathbf{Q} .
2. Magnitude of \mathbf{V} is $V = PQ \sin q$
3. Direction of \mathbf{V} is obtained from the right-hand rule.



(a)



(b)

- Vector products:
 - are not commutative, $\mathbf{Q} \times \mathbf{P} = -(\mathbf{P} \times \mathbf{Q})$
 - are distributive, $\mathbf{P} \times (\mathbf{Q}_1 + \mathbf{Q}_2) = \mathbf{P} \times \mathbf{Q}_1 + \mathbf{P} \times \mathbf{Q}_2$
 - are not associative, $(\mathbf{P} \times \mathbf{Q}) \times \mathbf{S} \neq \mathbf{P} \times (\mathbf{Q} \times \mathbf{S})$

Vector Mechanics for Engineers: Statics

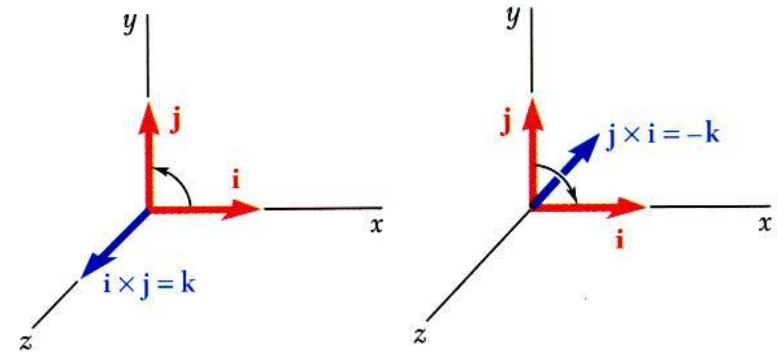
Vector Products: Rectangular Components

- Vector products of Cartesian unit vectors,

$$\vec{i} \times \vec{i} = 0 \quad \vec{j} \times \vec{i} = -\vec{k} \quad \vec{k} \times \vec{i} = \vec{j}$$

$$\vec{i} \times \vec{j} = \vec{k} \quad \vec{j} \times \vec{j} = 0 \quad \vec{k} \times \vec{j} = -\vec{i}$$

$$\vec{i} \times \vec{k} = -\vec{j} \quad \vec{j} \times \vec{k} = \vec{i} \quad \vec{k} \times \vec{k} = 0$$

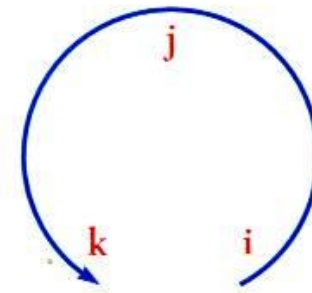


- Vector products in terms of rectangular coordinates

$$\vec{V} = (P_x \vec{i} + P_y \vec{j} + P_z \vec{k}) \times (Q_x \vec{i} + Q_y \vec{j} + Q_z \vec{k})$$

$$= (P_y Q_z - P_z Q_y) \vec{i} + (P_z Q_x - P_x Q_z) \vec{j} + (P_x Q_y - P_y Q_x) \vec{k}$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$



Vector Mechanics for Engineers: Statics

Moment of a Force About a Point

- A force vector is defined by its magnitude and direction. Its effect on the rigid body also depends on its point of application.

- The *moment* of F about O is defined as

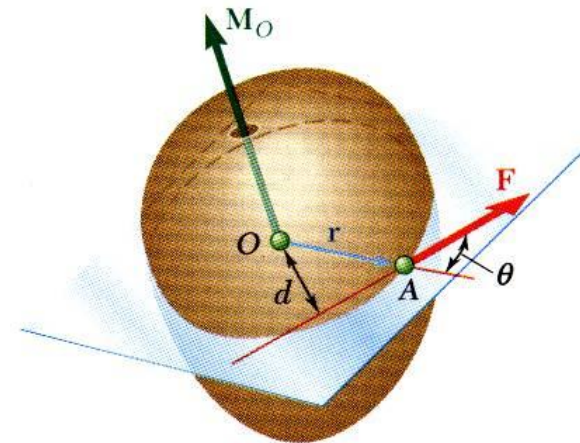
$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

- The moment vector \mathbf{M}_O is perpendicular to the plane containing O and the force F .
- Magnitude of \mathbf{M}_O measures the tendency of the force to cause rotation of the body about an axis along \mathbf{M}_O .

$$M_O = rF \sin \theta = Fd$$

The sense of the moment may be determined by the right-hand rule.

- Any force F' that has the same magnitude and direction as F , is *equivalent* if it also has the same line of action and therefore, produces the same moment.



(a)

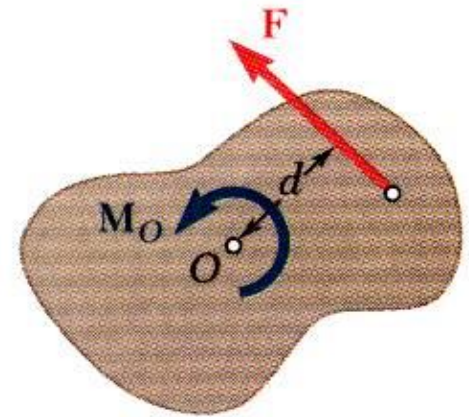


(b)

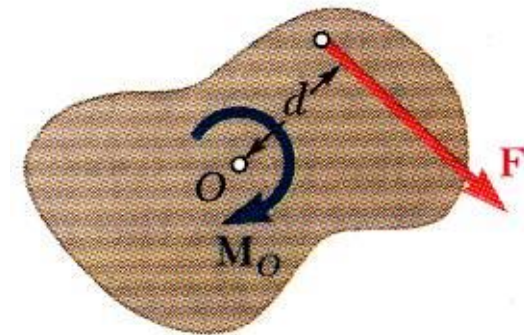
Vector Mechanics for Engineers: Statics

Moment of a Force About a Point

- *Two-dimensional structures* have length and breadth but negligible depth and are subjected to forces contained in the plane of the structure.
- The plane of the structure contains the point O and the force F . M_O , the moment of the force about O is perpendicular to the plane.
- If the force tends to rotate the structure counterclockwise, the sense of the moment vector is out of the plane of the structure and the magnitude of the moment is positive.
- If the force tends to rotate the structure clockwise, the sense of the moment vector is into the plane of the structure and the magnitude of the moment is negative.



(a) $M_O = +Fd$



(b) $M_O = -Fd$

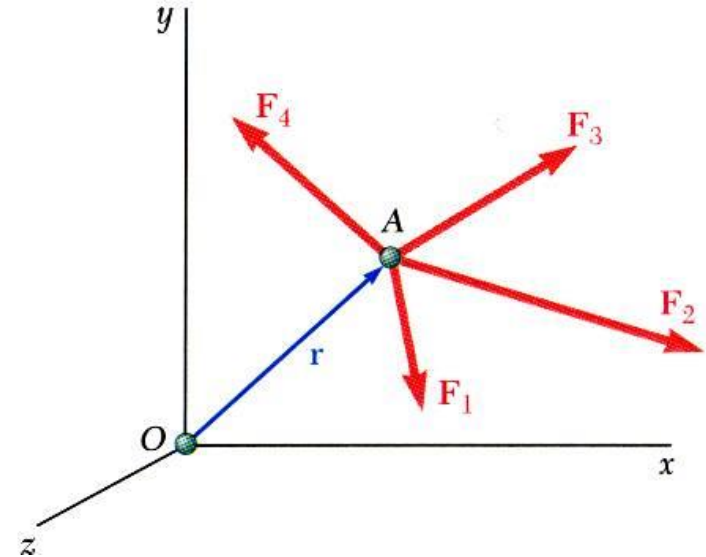
Vector Mechanics for Engineers: Statics

Varignon's Theorem

- The moment about a give point O of the resultant of several concurrent forces is equal to the sum of the moments of the various moments about the same point O .

$$\vec{r} \times (\vec{F}_1 + \vec{F}_2 + \cdots) = \vec{r} \times \vec{F}_1 + \vec{r} \times \vec{F}_2 + \cdots$$

- Varignon's Theorem makes it possible to replace the direct determination of the moment of a force \mathbf{F} by the moments of two or more component forces of \mathbf{F} .



Vector Mechanics for Engineers: Statics

Rectangular Components of the Moment of a Force

The moment of \mathbf{F} about O ,

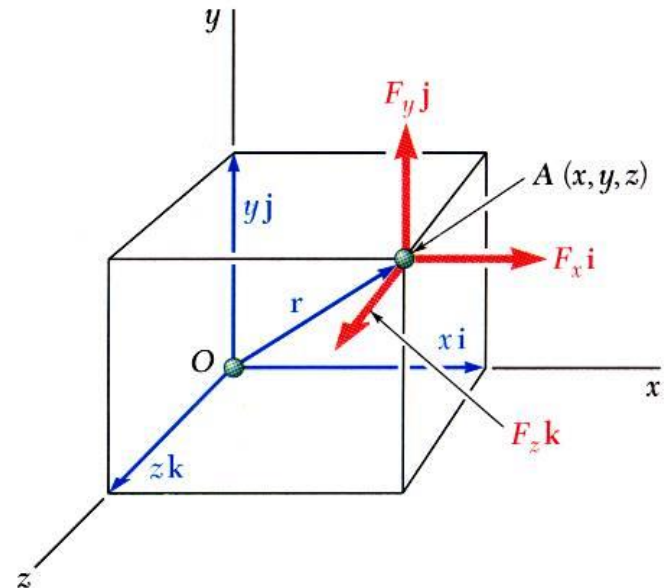
$$\vec{M}_O = \vec{r} \times \vec{F}, \quad \vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$$

$$\vec{M}_O = M_x\vec{i} + M_y\vec{j} + M_z\vec{k}$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= (yF_z - zF_y)\vec{i} + (zF_x - xF_z)\vec{j} + (xF_y - yF_x)\vec{k}$$



Vector Mechanics for Engineers: Statics

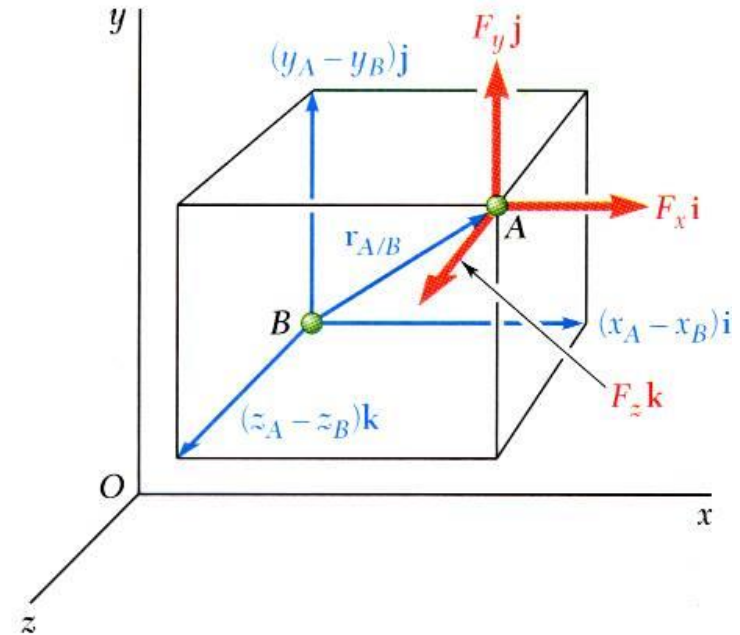
Rectangular Components of the Moment of a Force

The moment of \mathbf{F} about B ,

$$\vec{M}_B = \vec{r}_{A/B} \times \vec{F}$$

$$\begin{aligned}\vec{r}_{A/B} &= \vec{r}_A - \vec{r}_B \\ &= (x_A - x_B)\vec{i} + (y_A - y_B)\vec{j} + (z_A - z_B)\vec{k} \\ \vec{F} &= F_x\vec{i} + F_y\vec{j} + F_z\vec{k}\end{aligned}$$

$$\vec{M}_B = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ (x_A - x_B) & (y_A - y_B) & (z_A - z_B) \\ F_x & F_y & F_z \end{vmatrix}$$



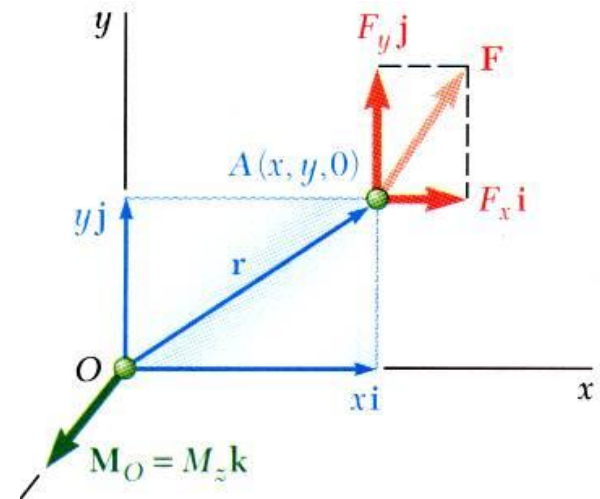
Vector Mechanics for Engineers: Statics

Rectangular Components of the Moment of a Force

For two-dimensional structures,

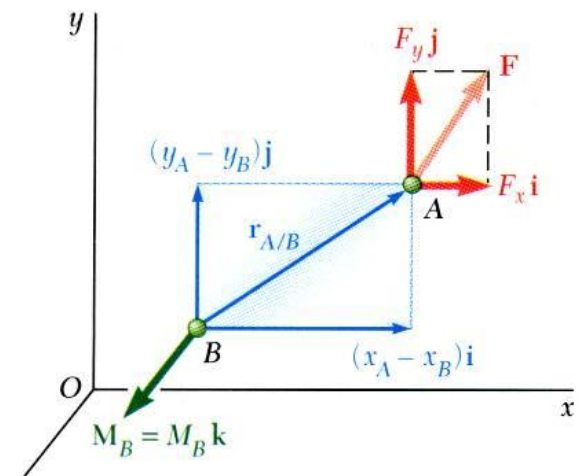
$$\vec{M}_O = (xF_y - yF_z)\vec{k}$$

$$\begin{aligned} M_O &= M_Z \\ &= xF_y - yF_z \end{aligned}$$



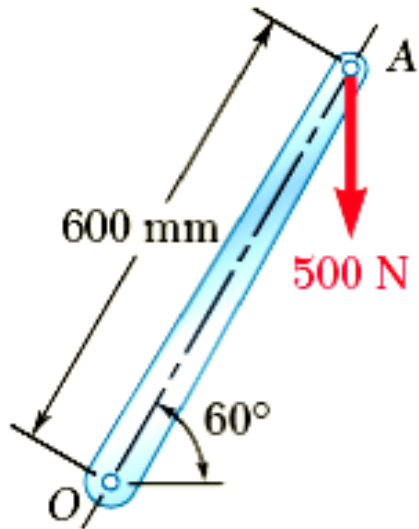
$$\vec{M}_O = [(x_A - x_B)F_y - (y_A - y_B)F_z]\vec{k}$$

$$\begin{aligned} M_O &= M_Z \\ &= (x_A - x_B)F_y - (y_A - y_B)F_z \end{aligned}$$



Vector Mechanics for Engineers: Statics

Sample Problem 3.1



A 500-N vertical force is applied to the end of a lever which is attached to a shaft at O .

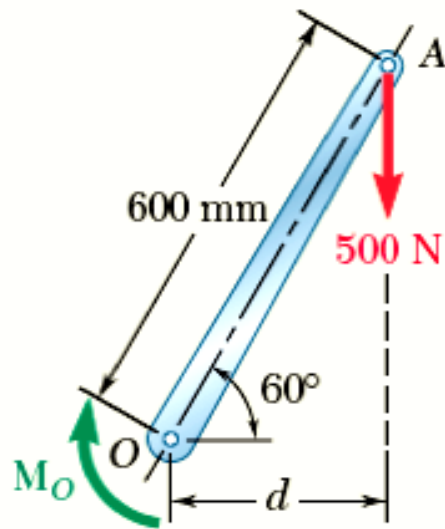
Determine:

- moment about O ,
- horizontal force at A which creates the same moment,
- smallest force at A which produces the same moment,
- location for a 1200-N vertical force to produce the same moment,
- whether any of the forces from b, c, and d is equivalent to the original force.



Vector Mechanics for Engineers: Statics

Sample Problem 3.1



SOLUTION:

- a) Moment about O is equal to the product of the force and the perpendicular distance between the line of action of the force and O . Since the force tends to rotate the lever clockwise, the moment vector is into the plane of the paper.

$$M_O = Fd$$

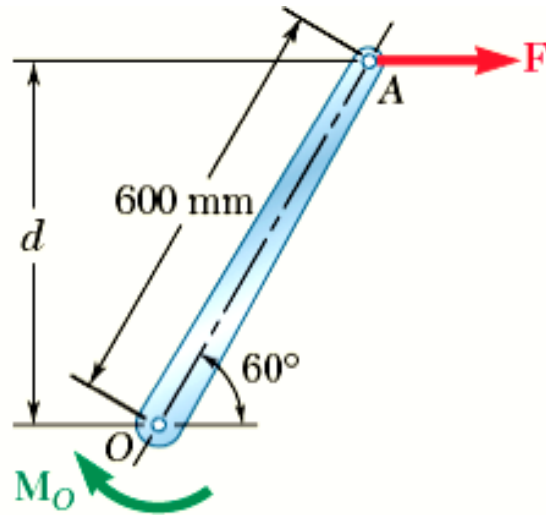
$$d = (600\text{ mm})\cos 60^\circ = 300\text{ mm} = 0.3\text{ m}$$

$$M_O = (500\text{ N})(0.3\text{ m})$$

$$M_O = 150\text{ N}\cdot\text{m}$$

Vector Mechanics for Engineers: Statics

Sample Problem 3.1



b) Horizontal force at A that produces the same moment,

$$d = (600 \text{ mm}) \sin 60^\circ = 519.6 \text{ mm} = 0.5196 \text{ m}$$

$$M_O = Fd$$

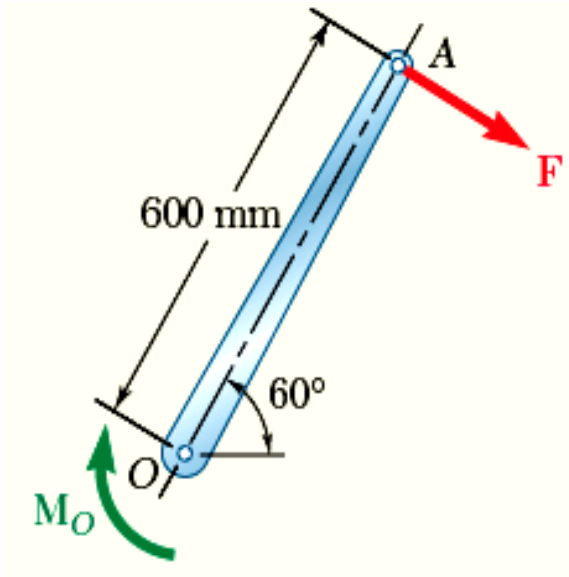
$$150 \text{ N} \cdot \text{m} = F(0.5196 \text{ m})$$

$$F = \frac{150 \text{ N} \cdot \text{m}}{0.5196 \text{ m}}$$

$$F = 288.68 \text{ N}$$

Vector Mechanics for Engineers: Statics

Sample Problem 3.1



- c) The smallest force A to produce the same moment occurs when the perpendicular distance is a maximum or when F is perpendicular to OA .

$$M_O = Fd$$

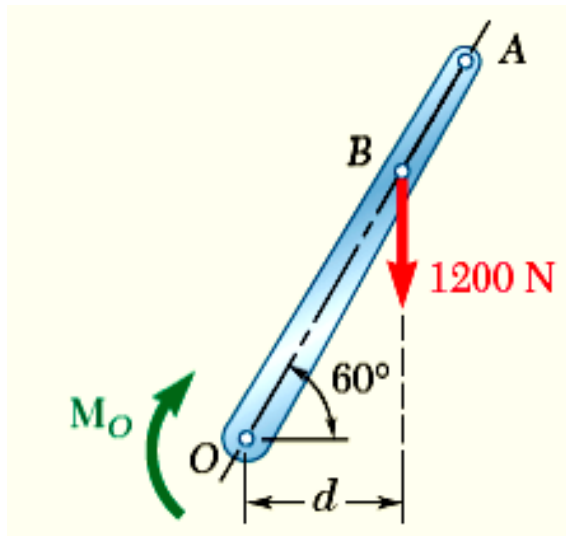
$$150 \text{ N} \cdot \text{m} = F(0.6 \text{ m})$$

$$F = \frac{150 \text{ N} \cdot \text{m}}{0.6 \text{ m}}$$

$$F = 250 \text{ N}$$

Vector Mechanics for Engineers: Statics

Sample Problem 3.1



- d) To determine the point of application of a 1200 N force to produce the same moment,

$$M_O = Fd$$

$$150 \text{ N} \cdot \text{m} = (1200 \text{ N})d$$

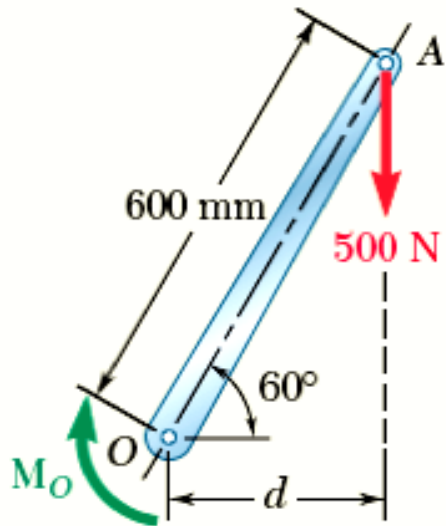
$$d = \frac{150 \text{ N} \cdot \text{m}}{1200 \text{ N}} = 125 \text{ mm}$$

$$OB \cos 60^\circ = 125 \text{ mm}$$

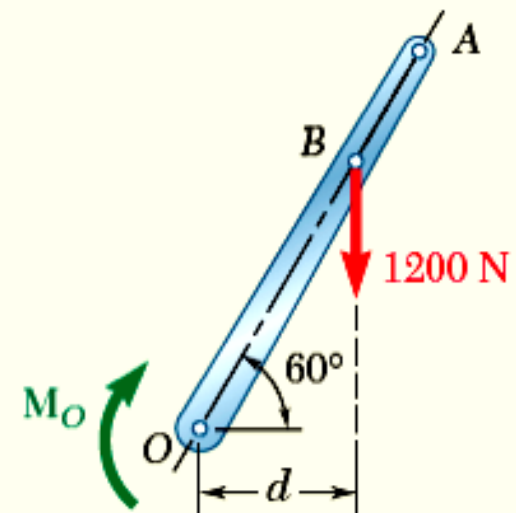
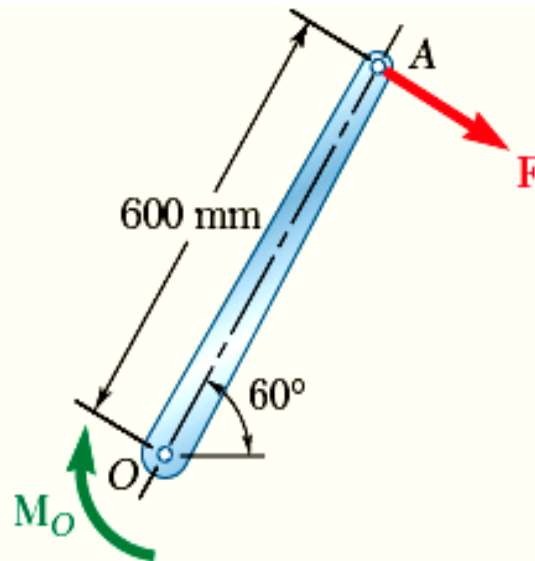
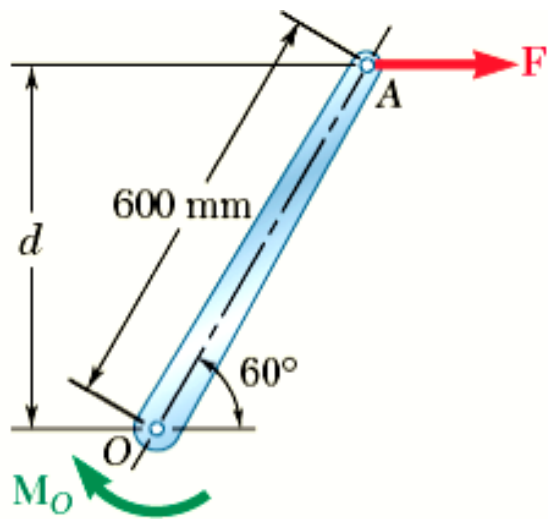
$$OB = 250 \text{ mm}$$

Vector Mechanics for Engineers: Statics

Sample Problem 3.1

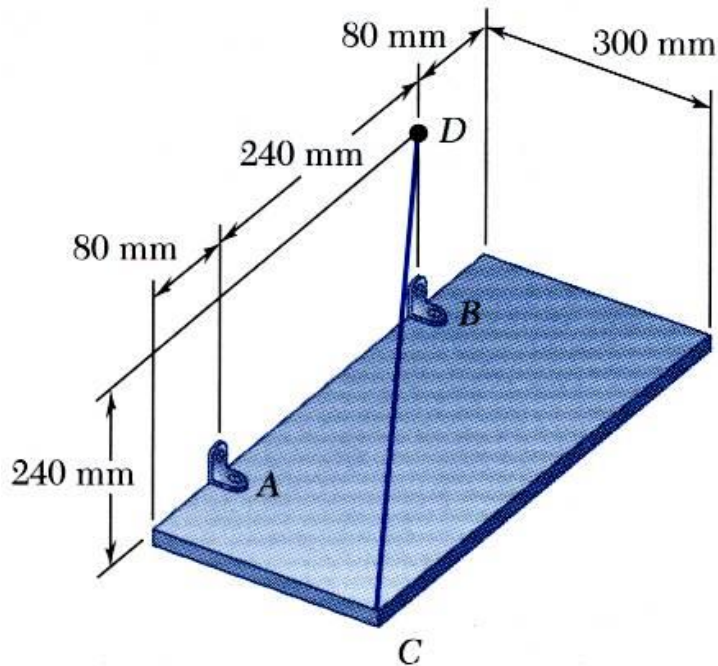


- e) Although each of the forces in parts b), c), and d) produces the same moment as the 500-N force, none are of the same magnitude and sense, or on the same line of action. None of the forces is equivalent to the 500-N force.



Vector Mechanics for Engineers: Statics

Sample Problem 3.4



SOLUTION:

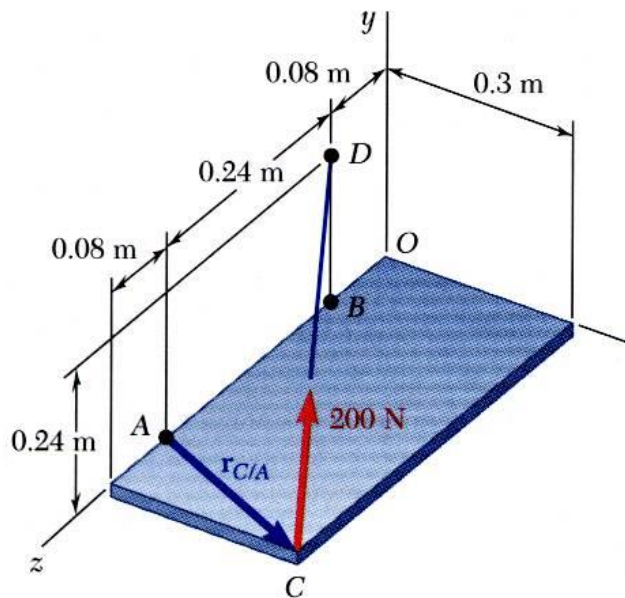
The moment M_A of the force F exerted by the wire is obtained by evaluating the vector product,

$$\vec{M}_A = \vec{r}_{C/A} \times \vec{F}$$

The rectangular plate is supported by the brackets at A and B and by a wire CD . Knowing that the tension in the wire is 200 N, determine the moment about A of the force exerted by the wire at C .

Vector Mechanics for Engineers: Statics

Sample Problem 3.4



SOLUTION:

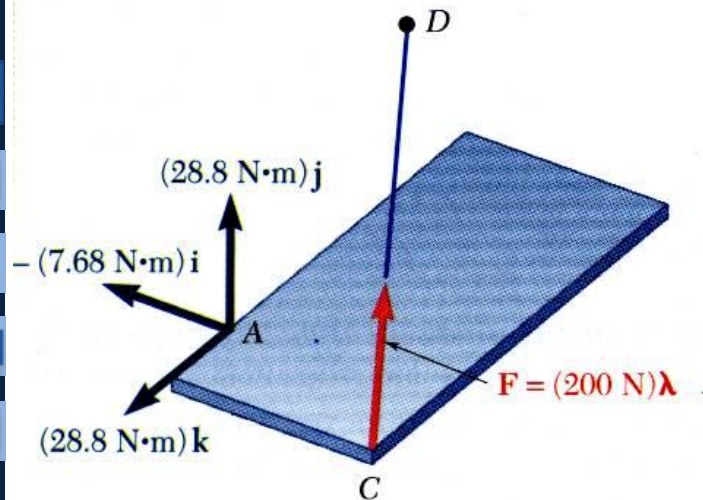
$$\vec{M}_A = \vec{r}_{C/A} \times \vec{F}$$

$$\vec{r}_{C/A} = \vec{r}_C - \vec{r}_A = (0.3 \text{ m})\vec{i} + (0.08 \text{ m})\vec{j}$$

$$\begin{aligned} \vec{F} &= F \vec{\lambda} = (200 \text{ N}) \frac{\vec{r}_{C/D}}{r_{C/D}} \\ &= (200 \text{ N}) \frac{-(0.3 \text{ m})\vec{i} + (0.24 \text{ m})\vec{j} - (0.32 \text{ m})\vec{k}}{0.5 \text{ m}} \\ &= -(120 \text{ N})\vec{i} + (96 \text{ N})\vec{j} - (128 \text{ N})\vec{k} \end{aligned}$$

$$\vec{M}_A = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.3 & 0 & 0.08 \\ -120 & 96 & -128 \end{vmatrix}$$

$$\vec{M}_A = -(7.68 \text{ N}\cdot\text{m})\vec{i} + (28.8 \text{ N}\cdot\text{m})\vec{j} + (28.8 \text{ N}\cdot\text{m})\vec{k}$$



Vector Mechanics for Engineers: Statics

Scalar Product of Two Vectors

- The *scalar product* or *dot product* between two vectors \mathbf{P} and \mathbf{Q} is defined as

$$\vec{P} \cdot \vec{Q} = PQ \cos \quad (\text{scalar result})$$

- Scalar products:

- are commutative, $\vec{P} \cdot \vec{Q} = \vec{Q} \cdot \vec{P}$
- are distributive, $\vec{P} \cdot (\vec{Q}_1 + \vec{Q}_2) = \vec{P} \cdot \vec{Q}_1 + \vec{P} \cdot \vec{Q}_2$
- are not associative, $(\vec{P} \cdot \vec{Q}) \cdot \vec{S} = \text{undefined}$

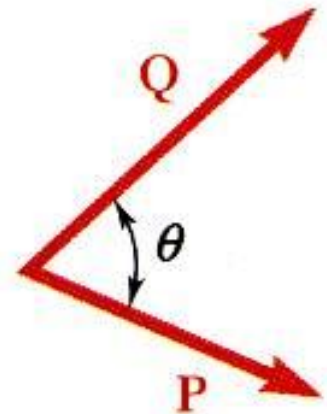
- Scalar products with Cartesian unit components,

$$\vec{P} \cdot \vec{Q} = (P_x \vec{i} + P_y \vec{j} + P_z \vec{k}) \cdot (Q_x \vec{i} + Q_y \vec{j} + Q_z \vec{k})$$

$$\vec{i} \cdot \vec{i} = 1 \quad \vec{j} \cdot \vec{j} = 1 \quad \vec{k} \cdot \vec{k} = 1 \quad \vec{i} \cdot \vec{j} = 0 \quad \vec{j} \cdot \vec{k} = 0 \quad \vec{k} \cdot \vec{i} = 0$$

$$\vec{P} \cdot \vec{Q} = P_x Q_x + P_y Q_y + P_z Q_z$$

$$\vec{P} \cdot \vec{P} = P_x^2 + P_y^2 + P_z^2 = P^2$$



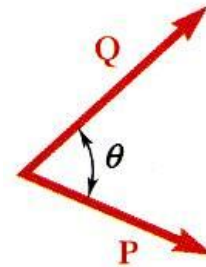
Vector Mechanics for Engineers: Statics

Scalar Product of Two Vectors: Applications

- Angle between two vectors:

$$\vec{P} \cdot \vec{Q} = PQ \cos \theta = P_x Q_x + P_y Q_y + P_z Q_z$$

$$\cos \theta = \frac{P_x Q_x + P_y Q_y + P_z Q_z}{PQ}$$

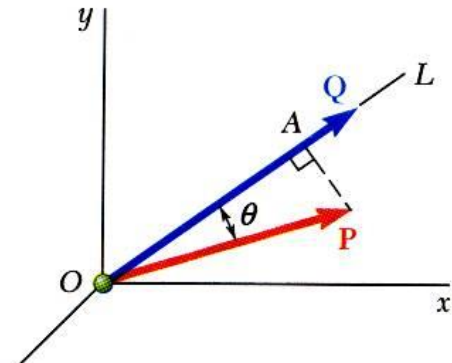


- Projection of a vector on a given axis:

$$P_{OL} = P \cos \theta = \text{projection of } P \text{ along } OL$$

$$\vec{P} \cdot \vec{Q} = PQ \cos \theta$$

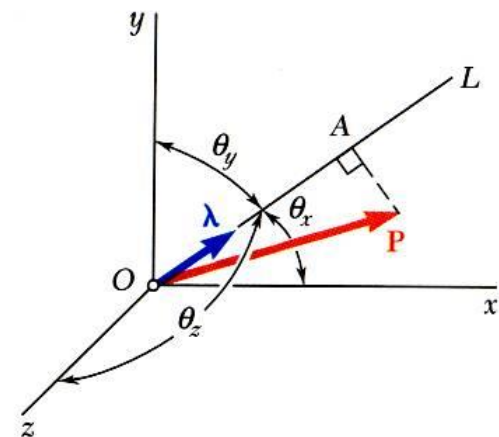
$$\frac{\vec{P} \cdot \vec{Q}}{Q} = P \cos \theta = P_{OL}$$



- For an axis defined by a unit vector:

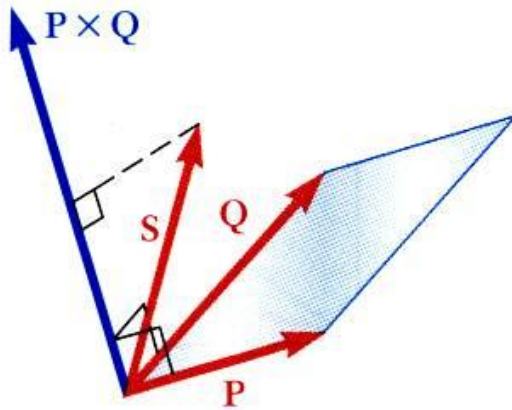
$$P_{OL} = \vec{P} \cdot \vec{u}$$

$$= P_x \cos \theta_x + P_y \cos \theta_y + P_z \cos \theta_z$$



Vector Mechanics for Engineers: Statics

Mixed Triple Product of Three Vectors



- Mixed triple product of three vectors,

$$\vec{S} \cdot (\vec{P} \times \vec{Q}) = \text{scalar result}$$

- The six mixed triple products formed from \vec{S} , \vec{P} , and \vec{Q} have equal magnitudes but not the same sign,

$$\begin{aligned} \vec{S} \cdot (\vec{P} \times \vec{Q}) &= \vec{P} \cdot (\vec{Q} \times \vec{S}) = \vec{Q} \cdot (\vec{S} \times \vec{P}) \\ &= -\vec{S} \cdot (\vec{Q} \times \vec{P}) = -\vec{P} \cdot (\vec{S} \times \vec{Q}) = -\vec{Q} \cdot (\vec{P} \times \vec{S}) \end{aligned}$$

- Evaluating the mixed triple product,

$$\begin{aligned} \vec{S} \cdot (\vec{P} \times \vec{Q}) &= S_x (P_y Q_z - P_z Q_y) + S_y (P_z Q_x - P_x Q_z) \\ &\quad + S_z (P_x Q_y - P_y Q_x) \end{aligned}$$

$$= \begin{vmatrix} S_x & S_y & S_z \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

Vector Mechanics for Engineers: Statics

Moment of a Force About a Given Axis

- Moment \mathbf{M}_O of a force \mathbf{F} applied at the point A about a point O ,

$$\vec{M}_O = \vec{r} \times \vec{F}$$

- Scalar moment M_{OL} about an axis OL is the projection of the moment vector \mathbf{M}_O onto the axis,

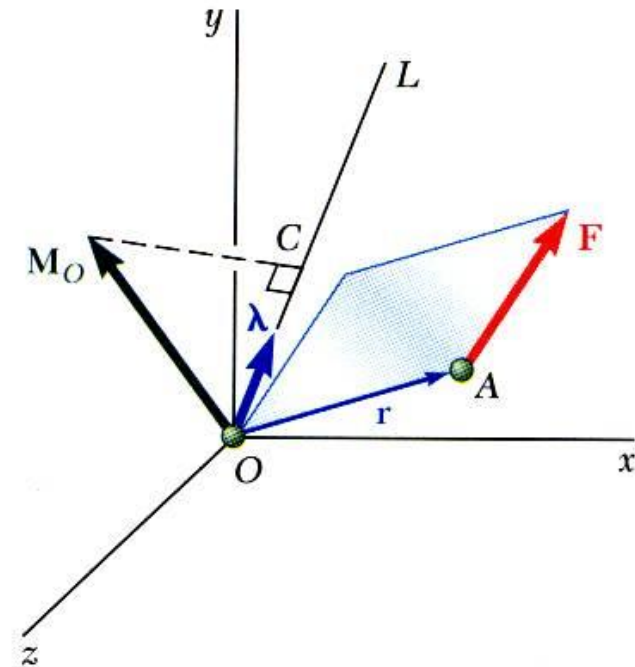
$$M_{OL} = \vec{u} \cdot \vec{M}_O = \vec{u} \cdot (\vec{r} \times \vec{F})$$

- Moments of \mathbf{F} about the coordinate axes,

$$M_x = yF_z - zF_y$$

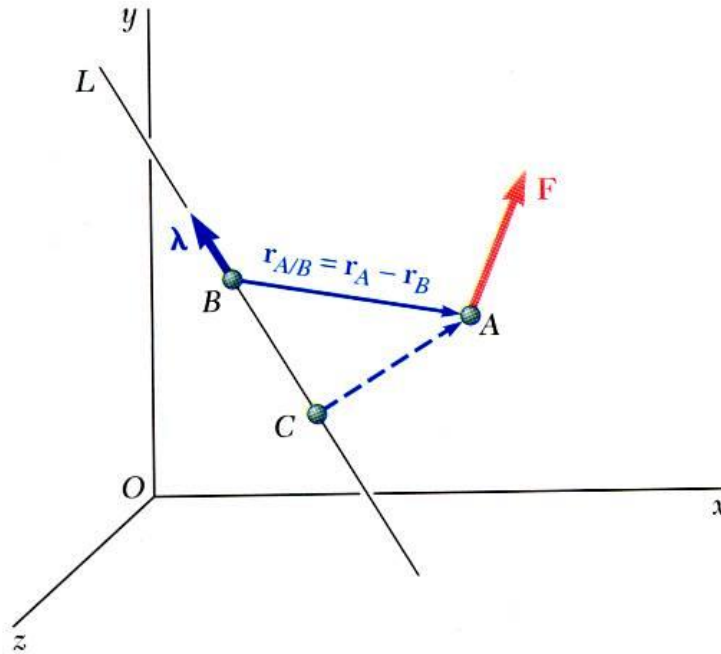
$$M_y = zF_x - xF_z$$

$$M_z = xF_y - yF_x$$



Vector Mechanics for Engineers: Statics

Moment of a Force About a Given Axis



- Moment of a force about an arbitrary axis,

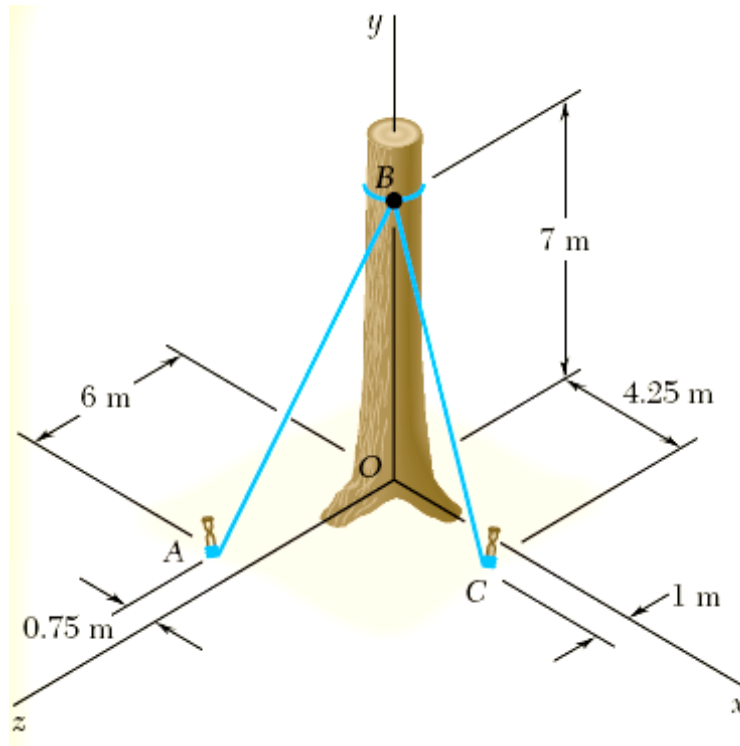
$$\begin{aligned} M_{BL} &= \vec{\lambda} \cdot \vec{M}_B \\ &= \vec{\lambda} \cdot (\vec{r}_{A/B} \times \vec{F}) \end{aligned}$$

$$\vec{r}_{A/B} = \vec{r}_A - \vec{r}_B$$

- The result is independent of the point B along the given axis.

Vector Mechanics for Engineers: Statics

Sample Problem 3.5

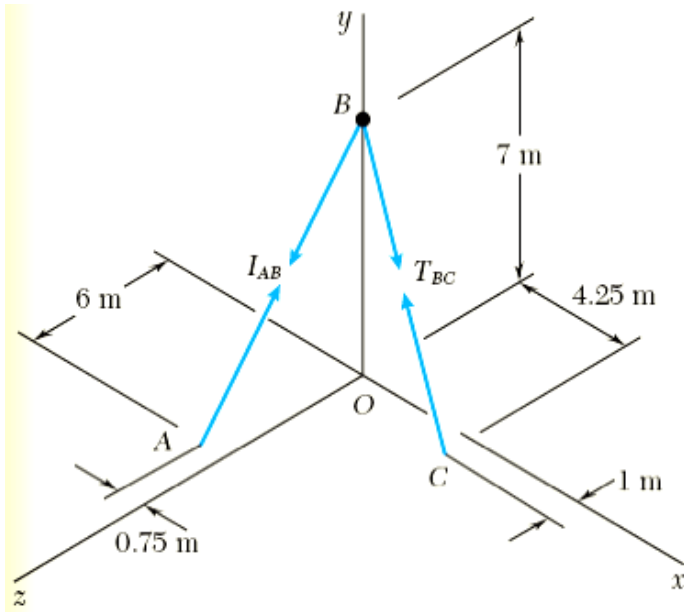


Before the trunk of a large tree is felled, cables AB and BC are attached as shown. Tensions in cables AB and BC are 555-N and 660-N, respectively.

Determine the moment about O of the resultant force exerted on the tree by the cables at B .

Vector Mechanics for Engineers: Statics

Sample Problem 3.5



SOLUTION:

We have $M_O = r_{B/O} \times F_B$

where $r_{B/O} = (7 \text{ m})\mathbf{j}$

$$F_B = T_{AB} + T_{BC}$$

$$T_{AB} = \lambda_{BA} T_{AB}$$

$$= \frac{-(0.75\text{m})\mathbf{i} - (7\text{m})\mathbf{j} + (6\text{m})\mathbf{k}}{\sqrt{(0.75)^2 + (7)^2 + (6)^2} \text{ m}} (555 \text{ N})$$

$$T_{BC} = \lambda_{BC} T_{BC}$$

$$= \frac{(4.25\text{m})\mathbf{i} - (7\text{m})\mathbf{j} + (1\text{m})\mathbf{k}}{\sqrt{(4.25)^2 + (7)^2 + (1)^2} \text{ m}} (660 \text{ N})$$

$$\begin{aligned} F_B &= [-(45.00 \text{ N})\mathbf{i} - (420.0 \text{ N})\mathbf{j} + (360.0 \text{ N})\mathbf{k}] \\ &\quad + [(340.0 \text{ N})\mathbf{i} - (560.0 \text{ N})\mathbf{j} + (80.00 \text{ N})\mathbf{k}] \\ &= (295.0 \text{ N})\mathbf{i} - (980.0 \text{ N})\mathbf{j} + (440.0 \text{ N})\mathbf{k} \end{aligned}$$

$$\text{and } M_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 7 & 0 \\ 295 & 980 & 440 \end{vmatrix} \text{ N} \cdot \text{m}$$

$$= (3080 \text{ N} \cdot \text{m})\mathbf{i} - (2070 \text{ N} \cdot \text{m})\mathbf{k} \text{ or}$$

$$\mathbf{M}_O = (3080 \text{ N} \cdot \text{m})\mathbf{i} - (2070 \text{ N} \cdot \text{m})\mathbf{k}$$

Vector Mechanics for Engineers: Statics

Moment of a Couple

- Two forces \mathbf{F} and $-\mathbf{F}$ having the same magnitude, parallel lines of action, and opposite sense are said to form a *couple*.
- Moment of the couple,

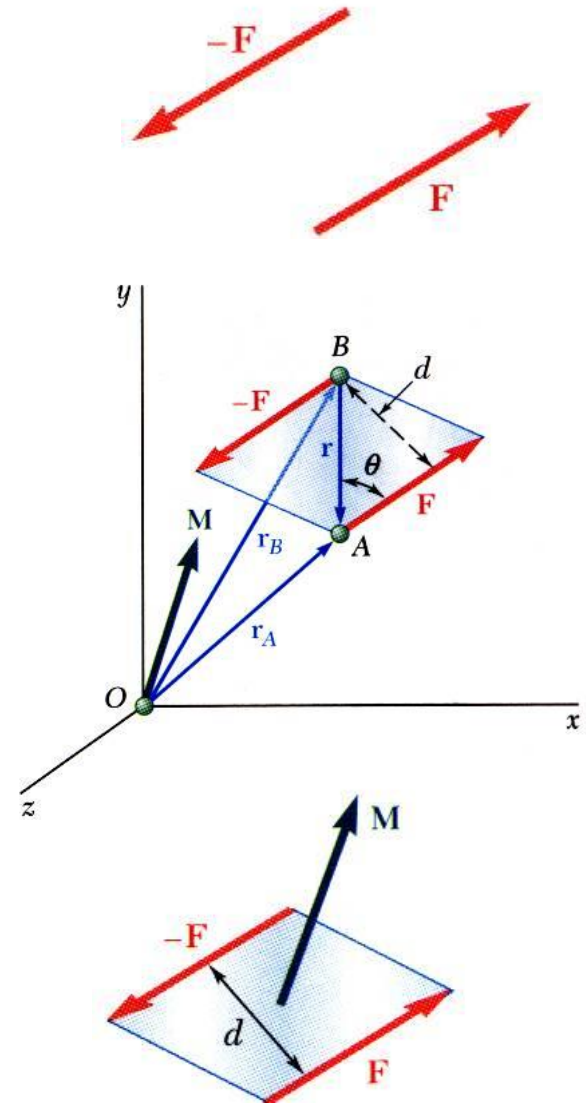
$$\vec{M} = \vec{r}_A \times \vec{F} + \vec{r}_B \times (-\vec{F})$$

$$= (\vec{r}_A - \vec{r}_B) \times \vec{F}$$

$$= \vec{r} \times \vec{F}$$

$$M = rF \sin \theta = Fd$$

- The moment vector of the couple is independent of the choice of the origin of the coordinate axes, i.e., it is a *free vector* that can be applied at any point with the same effect.

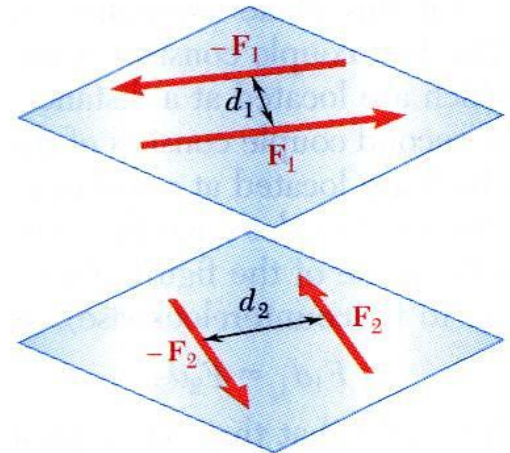
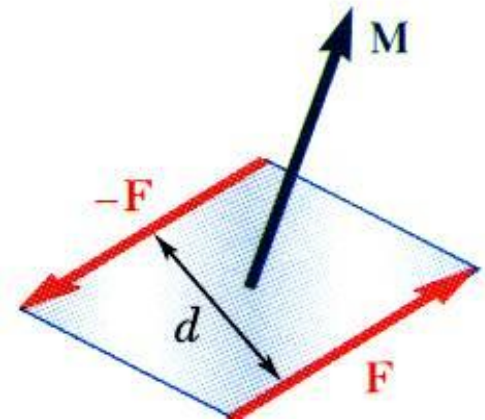


Vector Mechanics for Engineers: Statics

Moment of a Couple

Two couples will have equal moments if

- $F_1 d_1 = F_2 d_2$
- the two couples lie in parallel planes, and
- the two couples have the same sense or the tendency to cause rotation in the same direction.



Vector Mechanics for Engineers: Statics

Addition of Couples

- Consider two intersecting planes P_1 and P_2 with each containing a couple

$$\vec{M}_1 = \vec{r} \times \vec{F}_1 \text{ in plane } P_1$$

$$\vec{M}_2 = \vec{r} \times \vec{F}_2 \text{ in plane } P_2$$

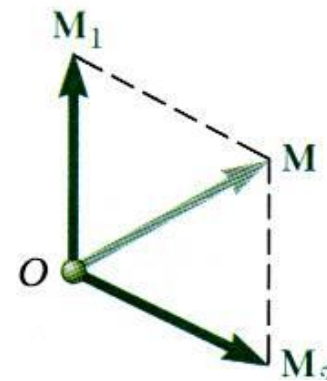
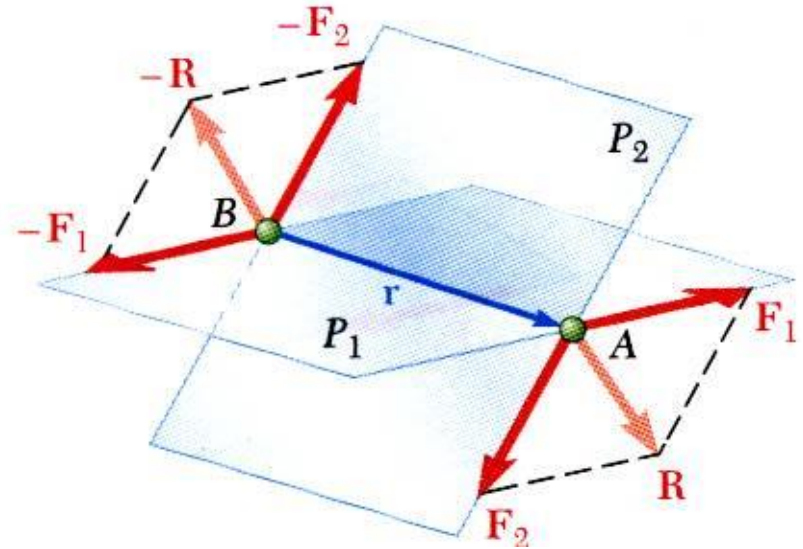
- Resultants of the vectors also form a couple

$$\vec{M} = \vec{r} \times \vec{R} = \vec{r} \times (\vec{F}_1 + \vec{F}_2)$$

- By Varignon's theorem

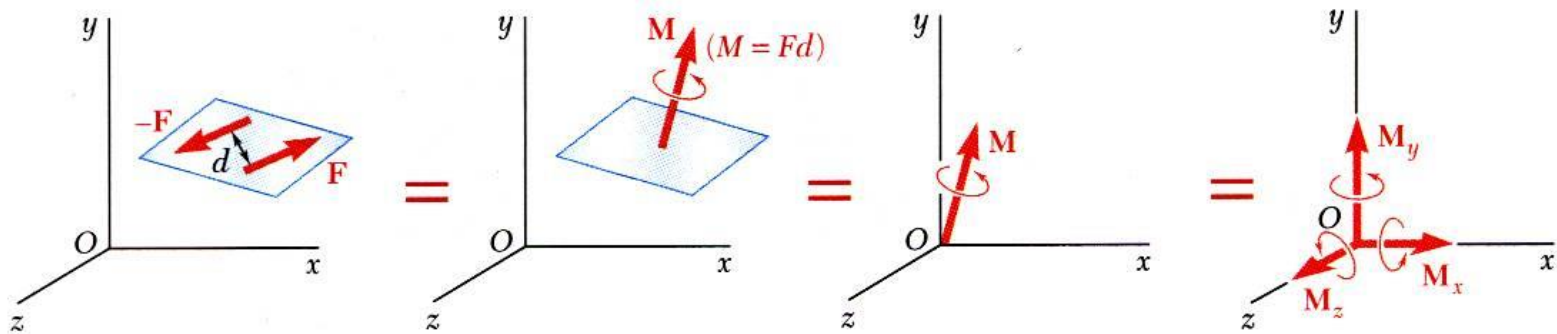
$$\begin{aligned} \vec{M} &= \vec{r} \times \vec{F}_1 + \vec{r} \times \vec{F}_2 \\ &= \vec{M}_1 + \vec{M}_2 \end{aligned}$$

- Sum of two couples is also a couple that is equal to the vector sum of the two couples



Vector Mechanics for Engineers: Statics

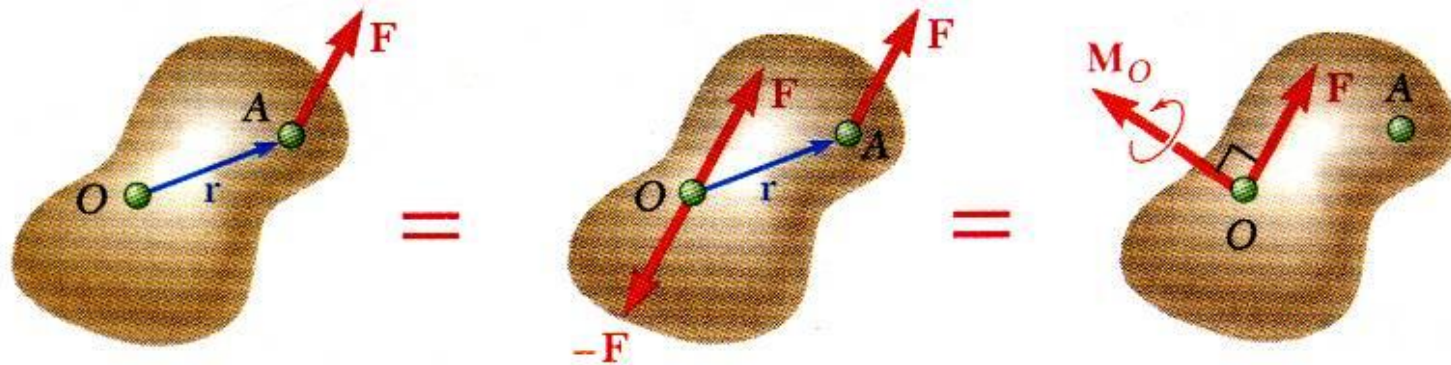
Couples Can Be Represented by Vectors



- A couple can be represented by a vector with magnitude and direction equal to the moment of the couple.
- *Couple vectors* obey the law of addition of vectors.
- Couple vectors are free vectors, i.e., the point of application is not significant.
- Couple vectors may be resolved into component vectors.

Vector Mechanics for Engineers: Statics

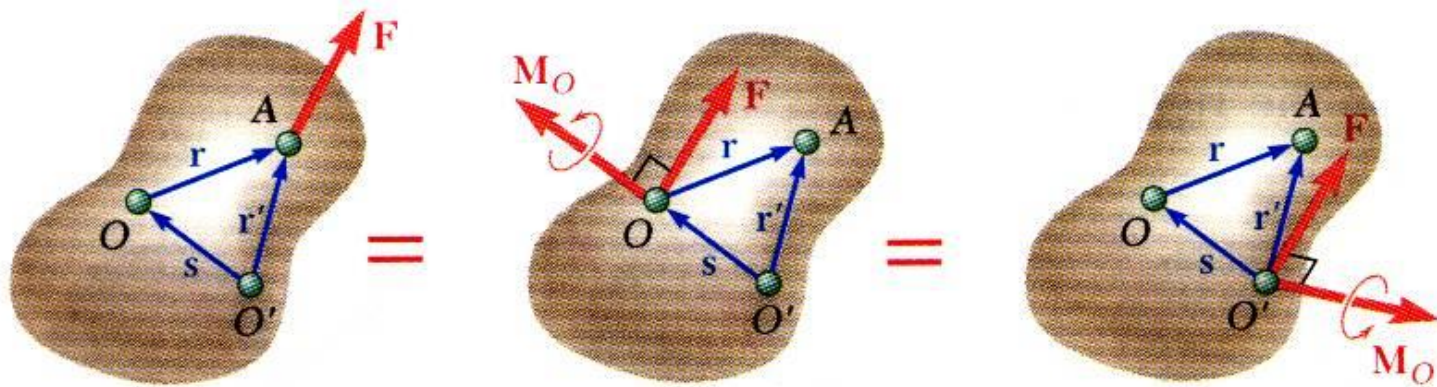
Resolution of a Force Into a Force at O and a Couple



- Force vector F can not be simply moved to O without modifying its action on the body.
- Attaching equal and opposite force vectors at O produces no net effect on the body.
- The three forces may be replaced by an equivalent force vector and couple vector, i.e, a *force-couple system*.

Vector Mechanics for Engineers: Statics

Resolution of a Force Into a Force at O and a Couple



- Moving \mathbf{F} from A to a different point O' requires the addition of a different couple vector $\mathbf{M}_{O'}$,

$$\vec{M}_{O'} = \vec{r}' \times \vec{F}$$

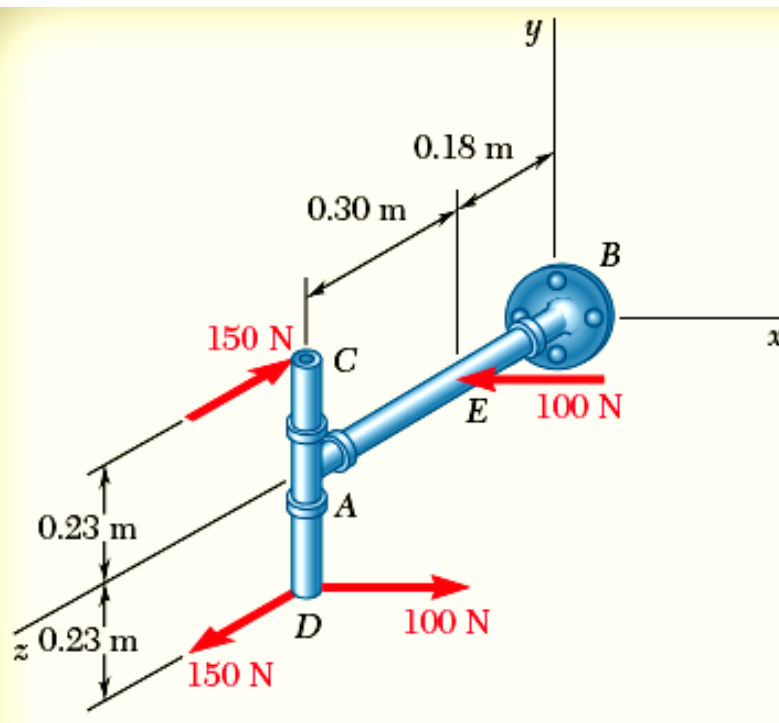
- The moments of \mathbf{F} about O and O' are related,

$$\begin{aligned}\vec{M}_{O'} &= \vec{r}' \times \vec{F} = (\vec{r} + \vec{s}) \times \vec{F} = \vec{r} \times \vec{F} + \vec{s} \times \vec{F} \\ &= \vec{M}_O + \vec{s} \times \vec{F}\end{aligned}$$

- Moving the force-couple system from O to O' requires the addition of the moment of the force at O about O' .

Vector Mechanics for Engineers: Statics

Sample Problem 3.6



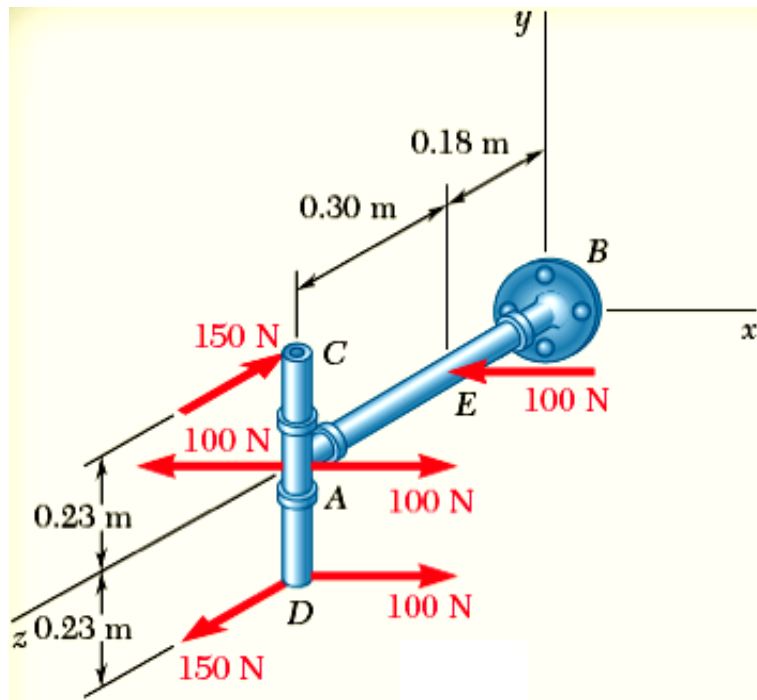
SOLUTION:

- Attach equal and opposite 100-N forces in the $\pm x$ direction at A, thereby producing 3 couples for which the moment components are easily computed.
- Alternatively, compute the sum of the moments of the four forces about an arbitrary single point. The point D is a good choice as only two of the forces will produce non-zero moment contributions..

Determine the components of the single couple equivalent to the couples shown.

Vector Mechanics for Engineers: Statics

Sample Problem 3.6



- The three couples may be represented by three couple vectors,

$$M_x = -(150 \text{ N})(0.45 \text{ m}) = -67.5 \text{ N} \cdot \text{m}$$

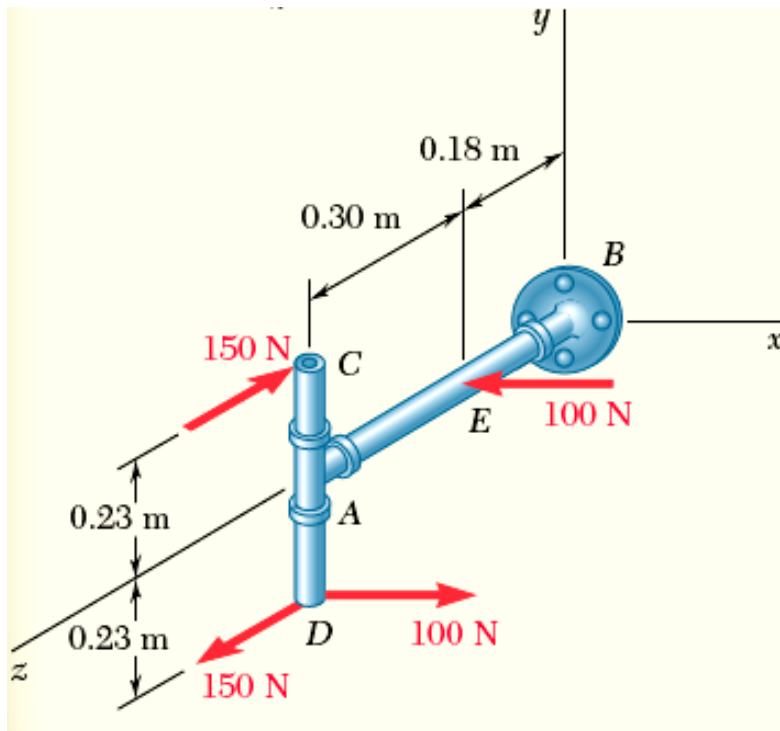
$$M_y = +(100 \text{ N})(0.30 \text{ m}) = +30 \text{ N} \cdot \text{m}$$

$$M_z = +(100 \text{ N})(0.23 \text{ m}) = +22.5 \text{ N} \cdot \text{m}$$

$$\vec{M} = -(67.5 \text{ N} \cdot \text{m})\vec{i} + (30 \text{ N} \cdot \text{m})\vec{j} + (22.5 \text{ N} \cdot \text{m})\vec{k}$$

Vector Mechanics for Engineers: Statics

Sample Problem 3.6



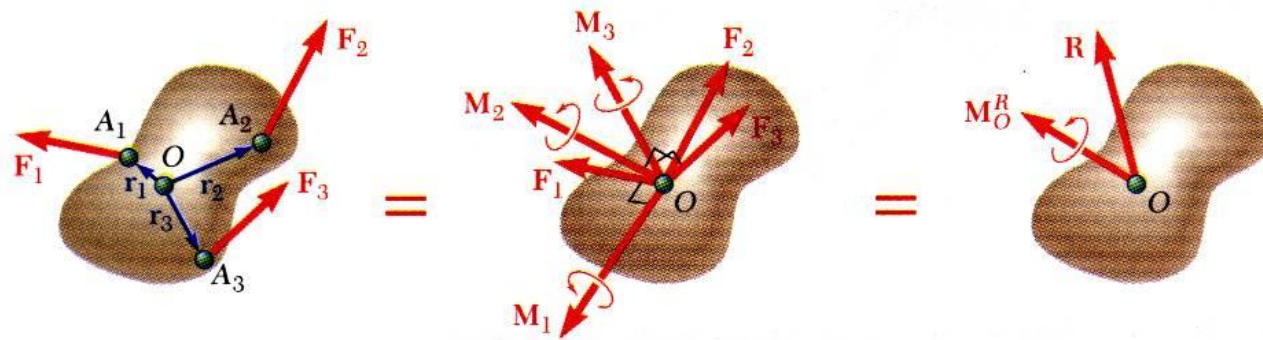
- Alternatively, compute the sum of the moments of the four forces about D .
- Only the forces at C and E contribute to the moment about D .

$$\vec{M} = \vec{M}_D = (0.45 \text{ m})\vec{j} \times (-150 \text{ N})\vec{k} + \left[(0.23 \text{ m})\vec{j} - (0.30 \text{ m})\vec{k} \right] \times (-100 \text{ N})\vec{i}$$

$$\vec{M} = -(67.5 \text{ N}\cdot\text{m})\vec{i} + (30 \text{ N}\cdot\text{m})\vec{j} + (22.5 \text{ N}\cdot\text{m})\vec{k}$$

Vector Mechanics for Engineers: Statics

System of Forces: Reduction to a Force and Couple



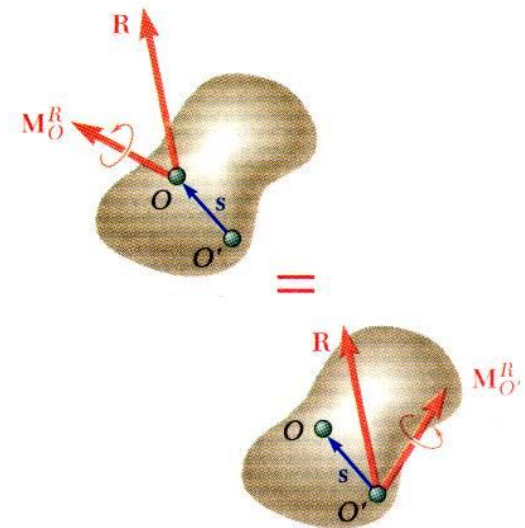
- A system of forces may be replaced by a collection of force-couple systems acting at a given point O
- The force and couple vectors may be combined into a resultant force vector and a resultant couple vector,

$$\vec{R} = \sum \vec{F} \quad \vec{M}_O^R = \sum (\vec{r} \times \vec{F})$$

- The force-couple system at O may be moved to O' with the addition of the moment of \vec{R} about O' ,

$$\vec{M}_{O'}^R = \vec{M}_O^R + \vec{s} \times \vec{R}$$

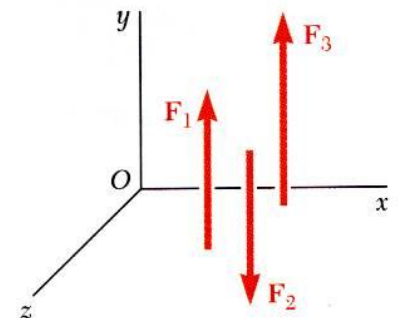
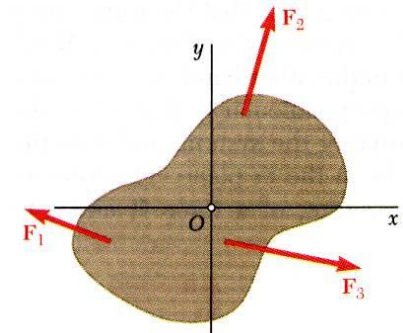
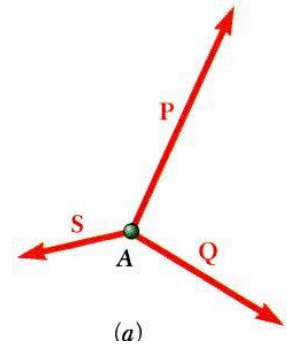
- Two systems of forces are equivalent if they can be reduced to the same force-couple system.



Vector Mechanics for Engineers: Statics

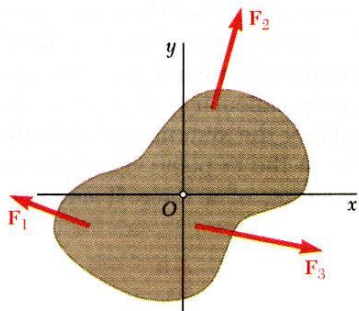
Further Reduction of a System of Forces

- If the resultant force and couple at O are mutually perpendicular, they can be replaced by a single force acting along a new line of action.
- The resultant force-couple system for a system of forces will be mutually perpendicular if:
 - 1) the forces are concurrent,
 - 2) the forces are coplanar, or
 - 3) the forces are parallel.

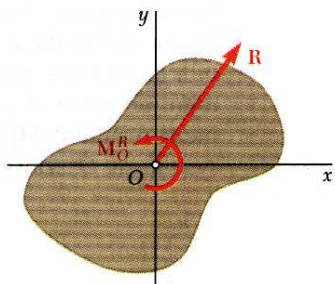


Vector Mechanics for Engineers: Statics

Further Reduction of a System of Forces



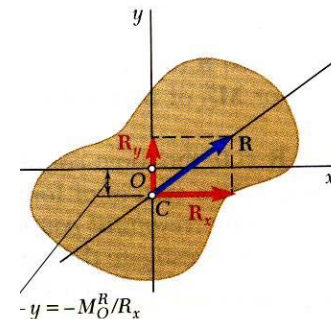
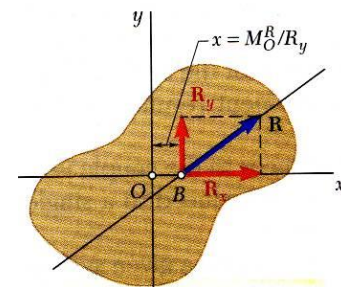
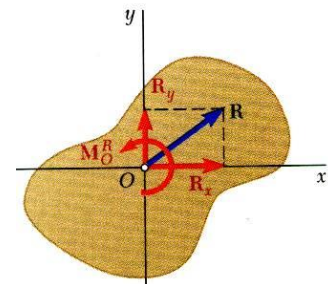
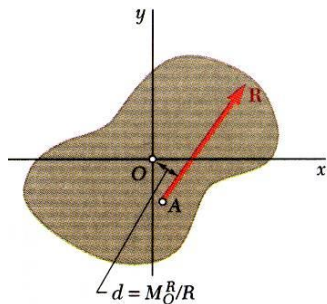
- System of coplanar forces is reduced to a force-couple system \vec{R} and \vec{M}_O^R that is mutually perpendicular.



- System can be reduced to a single force by moving the line of action of \vec{R} until its moment about O becomes \vec{M}_O^R

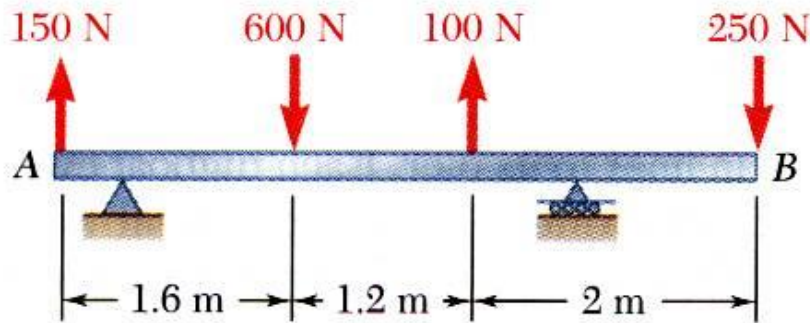
- In terms of rectangular coordinates,

$$xR_y - yR_x = M_O^R$$



Vector Mechanics for Engineers: Statics

Sample Problem 3.8



For the beam, reduce the system of forces shown to (a) an equivalent force-couple system at A , (b) an equivalent force couple system at B , and (c) a single force or resultant.

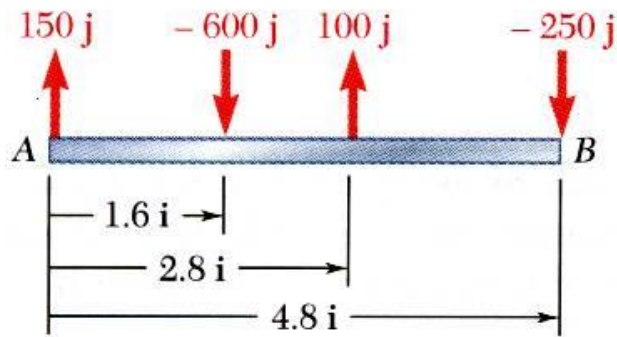
Note: Since the support reactions are not included, the given system will not maintain the beam in equilibrium.

SOLUTION:

- Compute the resultant force for the forces shown and the resultant couple for the moments of the forces about A .
- Find an equivalent force-couple system at B based on the force-couple system at A .
- Determine the point of application for the resultant force such that its moment about A is equal to the resultant couple at A .

Vector Mechanics for Engineers: Statics

Sample Problem 3.8



SOLUTION:

- a) Compute the resultant force and the resultant couple at A.

$$\begin{aligned}\vec{R} &= \sum \vec{F} \\ &= (150 \text{ N})\vec{j} - (600 \text{ N})\vec{j} + (100 \text{ N})\vec{j} - (250 \text{ N})\vec{j}\end{aligned}$$

$$\boxed{\vec{R} = -(600 \text{ N})\vec{j}}$$

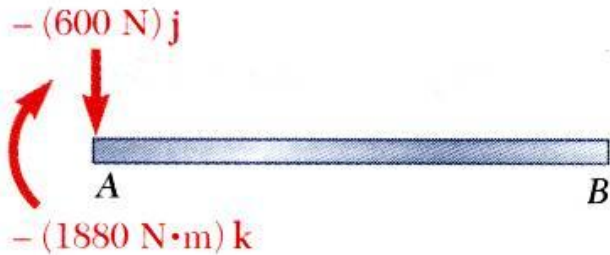
$$\begin{aligned}\vec{M}_A^R &= \sum (\vec{r} \times \vec{F}) \\ &= (1.6\vec{i}) \times (-600\vec{j}) + (2.8\vec{i}) \times (100\vec{j}) \\ &\quad + (4.8\vec{i}) \times (-250\vec{j})\end{aligned}$$

$$\boxed{\vec{M}_A^R = -(1880 \text{ N} \cdot \text{m})\vec{k}}$$



Vector Mechanics for Engineers: Statics

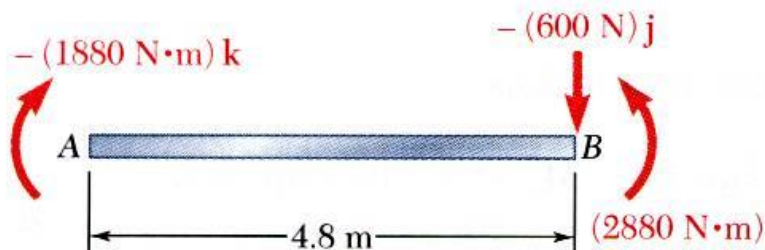
Sample Problem 3.8



- b) Find an equivalent force-couple system at B based on the force-couple system at A.

The force is unchanged by the movement of the force-couple system from A to B.

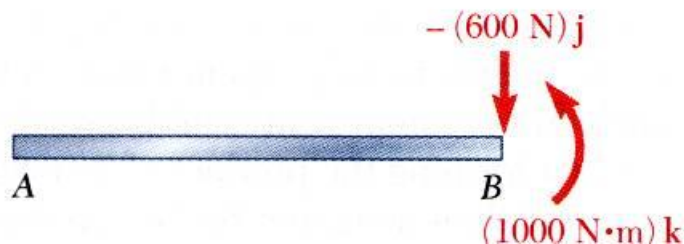
$$\vec{R} = -(600 \text{ N})\vec{j}$$



The couple at B is equal to the moment about B of the force-couple system found at A.

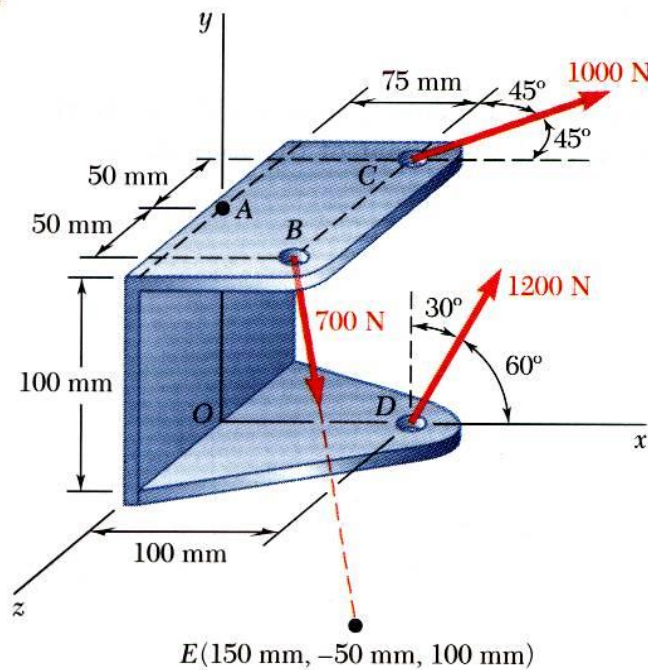
$$\begin{aligned}\vec{M}_B^R &= \vec{M}_A^R + \vec{r}_{B/A} \times \vec{R} \\ &= -(1880 \text{ N}\cdot\text{m})\vec{k} + (-4.8 \text{ m})\vec{i} \times (-600 \text{ N})\vec{j} \\ &= -(1880 \text{ N}\cdot\text{m})\vec{k} + (2880 \text{ N}\cdot\text{m})\vec{k}\end{aligned}$$

$$\vec{M}_B^R = +(1000 \text{ N}\cdot\text{m})\vec{k}$$



Vector Mechanics for Engineers: Statics

Sample Problem 3.10



SOLUTION:

- Determine the relative position vectors for the points of application of the cable forces with respect to A.
- Resolve the forces into rectangular components.

- Compute the equivalent force,

$$\vec{R} = \sum \vec{F}$$

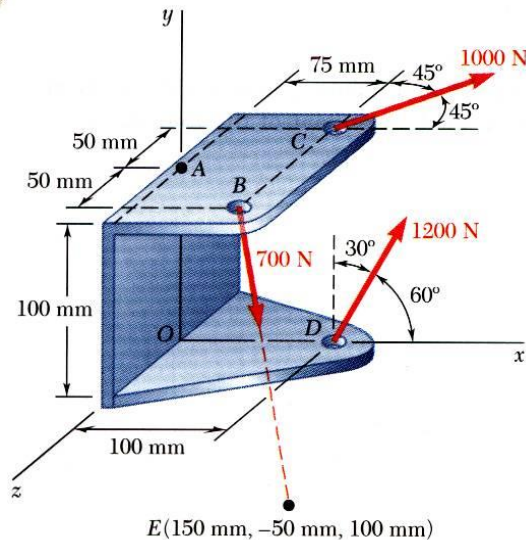
- Compute the equivalent couple,

$$\vec{M}_A^R = \sum (\vec{r} \times \vec{F})$$

Three cables are attached to the bracket as shown. Replace the forces with an equivalent force-couple system at A.

Vector Mechanics for Engineers: Statics

Sample Problem 3.10



- Resolve the forces into rectangular components.

$$\begin{aligned}\vec{F}_B &= (700 \text{ N}) \vec{r}_{E/B} \\ &= \frac{\vec{r}_{E/B}}{r_{E/B}} = \frac{75\vec{i} - 150\vec{j} + 50\vec{k}}{175} \\ &= 0.429\vec{i} - 0.857\vec{j} + 0.289\vec{k} \\ \vec{F}_B &= 300\vec{i} - 600\vec{j} + 200\vec{k} \text{ (N)}\end{aligned}$$

$$\begin{aligned}\vec{F}_C &= (1000 \text{ N})(\cos 45^\circ \vec{i} - \cos 45^\circ \vec{j}) \\ &= 707\vec{i} - 707\vec{j} \text{ (N)}\end{aligned}$$

$$\begin{aligned}\vec{F}_D &= (1200 \text{ N})(\cos 60^\circ \vec{i} + \cos 30^\circ \vec{j}) \\ &= 600\vec{i} + 1039\vec{j} \text{ (N)}\end{aligned}$$

SOLUTION:

- Determine the relative position vectors with respect to A.

$$\vec{r}_{B/A} = 0.075\vec{i} + 0.050\vec{k} \text{ (m)}$$

$$\vec{r}_{C/A} = 0.075\vec{i} - 0.050\vec{k} \text{ (m)}$$

$$\vec{r}_{D/A} = 0.100\vec{i} - 0.100\vec{j} \text{ (m)}$$

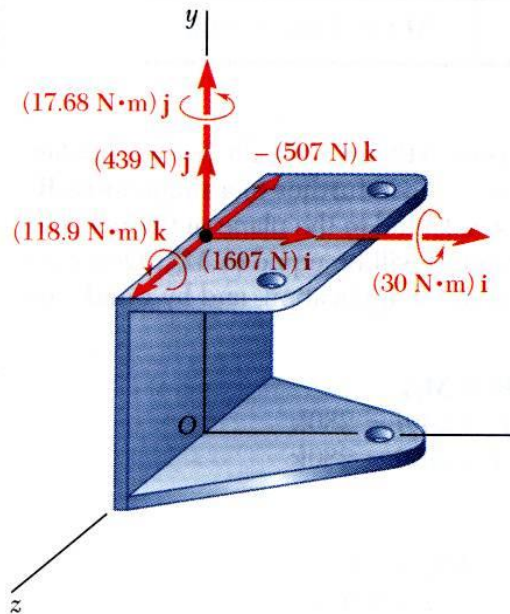
Vector Mechanics for Engineers: Statics

Sample Problem 3.10

- Compute the equivalent force,

$$\begin{aligned}\vec{R} &= \sum \vec{F} \\ &= (300 + 707 + 600)\vec{i} \\ &\quad + (-600 + 1039)\vec{j} \\ &\quad + (200 - 707)\vec{k}\end{aligned}$$

$$\vec{R} = 1607\vec{i} + 439\vec{j} - 507\vec{k} \text{ (N)}$$



- Compute the equivalent couple,

$$\vec{M}_A^R = \sum (\vec{r} \times \vec{F})$$

$$\vec{r}_{B/A} \times \vec{F}_B = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.075 & 0 & 0.050 \\ 300 & -600 & 200 \end{vmatrix} = 30\vec{i} - 45\vec{k}$$

$$\vec{r}_{C/A} \times \vec{F}_C = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.075 & 0 & -0.050 \\ 707 & 0 & -707 \end{vmatrix} = 17.68\vec{j}$$

$$\vec{r}_{D/A} \times \vec{F}_D = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.100 & -0.100 & 0 \\ 600 & 1039 & 0 \end{vmatrix} = 163.9\vec{k}$$

$$\vec{M}_A^R = 30\vec{i} + 17.68\vec{j} + 118.9\vec{k}$$