

## Problems for the Session on ~~2<sup>nd</sup>~~ November 2020

1. Determine the maximum likelihood estimator of  $\theta$  when  $X_1, X_2, \dots, X_n$  is a sample with density function:

$$f(x) = \frac{1}{2} \exp\{-|x - \theta|\}, \quad -\infty < x < \infty$$

2. Let  $X_1, X_2, \dots, X_n, X_{n+1}$  be a sample from a normal population with unknown mean  $\mu$  and standard deviation 1. Let  $\bar{X}_n = \frac{1}{n} \sum_{i=1}^n X_i$  be the average of the first  $n$  of them.
  - a. What is the distribution of  $X_{n+1} - \bar{X}_n$ ?
  - b. If  $\bar{X}_n = 4$ , give an interval that, with 90% confidence, will contain the value  $X_{n+1}$
3. A sample of 20 cigarettes is tested to determine nicotine content and the average value observed was 1.2mg. Compute 99% two-sided confidence interval for the mean nicotine content of a cigarette if it is known that the standard deviation of a cigarette's nicotine content is 0.2mg.
4. A sample of 20 cigarettes is tested to determine nicotine content and the average value observed was 1.2mg and sample variance is observed as 0.04mg. Find a value of  $c$  for which we can assert with 99% confidence that  $c$  is larger than the mean nicotine content of the cigarette.
5. The capacities (in Ampere-hour) of ten batteries were recorded as follows: 140, 136, 150, 144, 148, 152, 138, 141, 143, 151
  - a. Estimate the population variance  $\sigma^2$
  - b. Compute a 99% two-sided confidence interval for  $\sigma^2$
  - c. Compute a value  $v$  that enables us to state, with 90% confidence that  $\sigma^2$  is less than  $v$
6. A random sample of 1200 engineers included 48 Hispanics Americans, 80 African Americans and 204 females. Determine 90% confidence intervals for the proportion of all engineers who are
  - a. Female
  - b. Hispanic Americans or African Americans