

Problem Set 2

1. Let \mathcal{P} denote propositional logic. Suppose we add to \mathcal{P} the axiom schema $(A \rightarrow B)$ for wffs A, B of \mathcal{P} . Comment on the consistency of the resulting logical system obtained. A logic system \mathcal{P} is inconsistent if it is capable of producing \perp using the rules of natural deduction.
2. An adequate set of connectives is a set such that for every formula there is an equivalent formula with only connectives from that set. For example, $\{\neg, \vee\}$ is adequate for propositional logic since any occurrence of \wedge and \rightarrow can be removed using the equivalences

$$\varphi \rightarrow \psi \equiv \neg\varphi \vee \psi$$

$$\varphi \wedge \psi \equiv \neg(\neg\varphi \vee \neg\psi)$$

- Show that $\{\neg, \wedge\}$, $\{\neg, \rightarrow\}$ and $\{\rightarrow, \perp\}$ are adequate sets of connectives. (\perp treated as a nullary connective).
 - Show that if $C \subseteq \{\neg, \wedge, \vee, \rightarrow, \perp\}$ is adequate, then $\neg \in C$ or $\perp \in C$.
3. The binary connective *nand*, $F \downarrow G$, is defined by the truth table corresponding to $\neg(F \wedge G)$. Show that *nand* is complete - that is, it can express all binary boolean connectives.
 4. The binary connective *xor*, $F \oplus G$ is defined by the truth table corresponding to $(\neg F \wedge G) \vee (F \wedge \neg G)$. Show that *xor* is not complete- that is, it cannot express all binary boolean connectives.
 5. If a contradiction can be derived from a set of formulae, then the set of formulae is said to be inconsistent. Otherwise, the set of formulae is consistent. Let \mathcal{F} be a set of formulae. Show that \mathcal{F} is consistent iff it is satisfiable.
 6. Suppose \mathcal{F} is an inconsistent set of formulae. For each $G \in \mathcal{F}$, let \mathcal{F}_G be the set obtained by removing G from \mathcal{F} .
 - (a) Prove that for any $G \in \mathcal{F}$, $\mathcal{F}_G \vdash \neg G$, using the previous question.
 - (b) Prove this using a formal proof.