Ninth Edition

CHAPTER

3

VECTOR MECHANICS FOR ENGINEERS:

STATICS

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Lecture Notes:

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Rigid Bodies: Equivalent Systems of Forces

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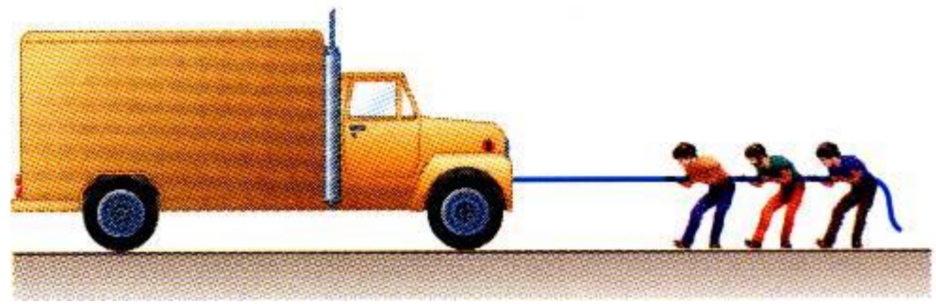
Introduction

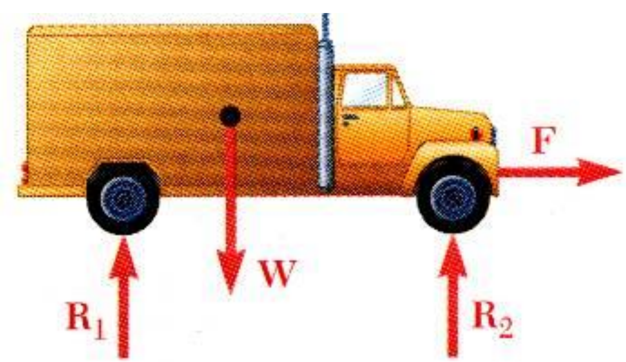
- Treatment of a body as a single particle is not always possible. In general, the size of the body and the specific points of application of the forces must be considered.
- Most bodies in elementary mechanics are assumed to be rigid, i.e., the actual deformations are small and do not affect the conditions of equilibrium or motion of the body.
- Current chapter describes the effect of forces exerted on a rigid body and how to replace a given system of forces with a simpler equivalent system.
  - moment of a force about a point
  - moment of a force about an axis
  - moment due to a couple
- Any system of forces acting on a rigid body can be replaced by an equivalent system consisting of one force acting at a given point and one couple.

External and Internal Forces

- Forces acting on rigid bodies are divided into two groups:
  - External forces
  - Internal forces



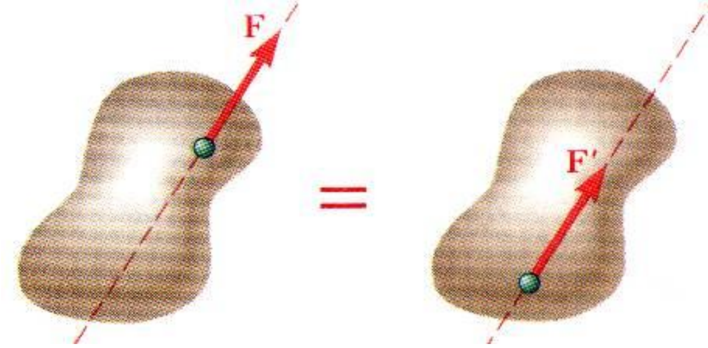
- External forces are shown in a free-body diagram.



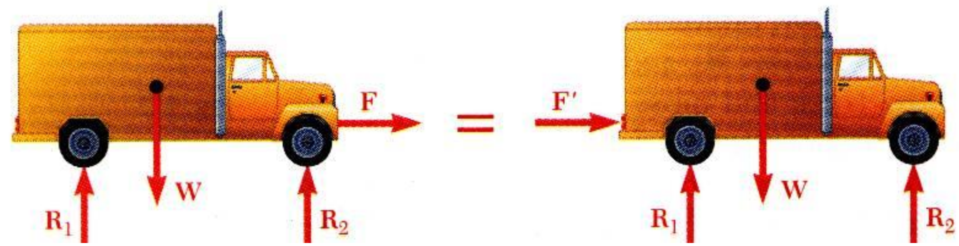
- If unopposed, each external force can impart a motion of translation or rotation, or both.

Principle of Transmissibility: Equivalent Forces

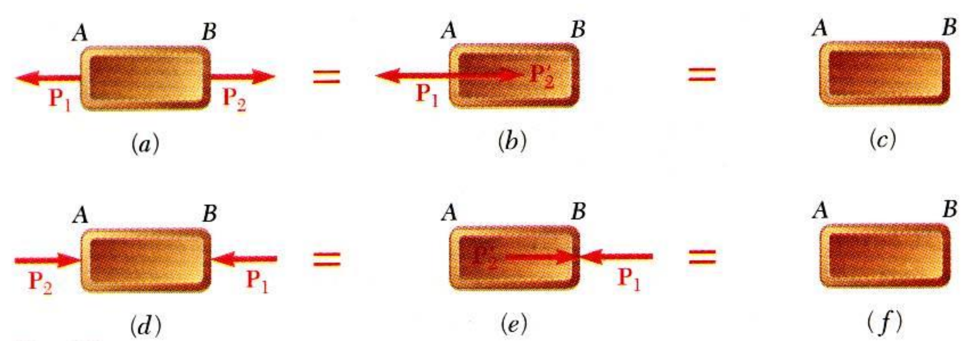
- *Principle of Transmissibility* - Conditions of equilibrium or motion are not affected by *transmitting* a force along its line of action.
  NOTE: **F** and **F'** are equivalent forces.



- Moving the point of application of the force **F** to the rear bumper does not affect the motion or the other forces acting on the truck.



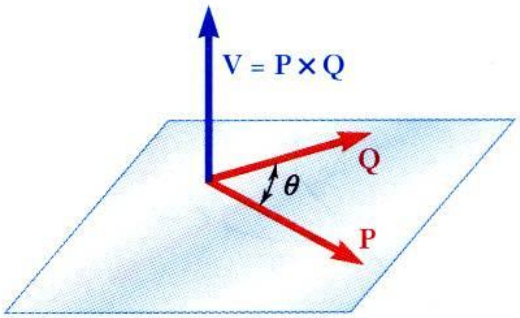
- Principle of transmissibility may not always apply in determining internal forces and deformations.



Vector Product of Two Vectors

- Concept of the moment of a force about a point is more easily understood through applications of the *vector product* or *cross product*.

- Vector product of two vectors ***P*** and ***Q*** is defined as the vector ***V*** which satisfies the following conditions:
  1. Line of action of ***V*** is perpendicular to plane containing ***P*** and ***Q***.
  2. Magnitude of ***V*** is $V = PQ \sin\theta$
  3. Direction of ***V*** is obtained from the right-hand rule.



(a)

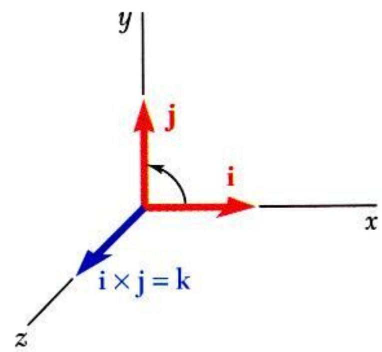
(b)

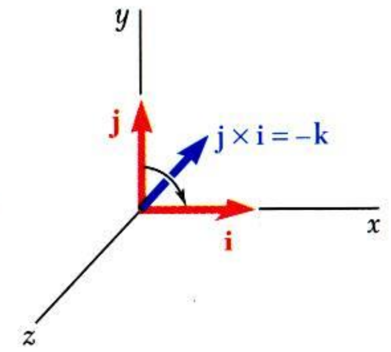
- Vector products:
  - are not commutative, $\boldsymbol{Q} \times \boldsymbol{P} = -(\boldsymbol{P} \times \boldsymbol{Q})$
  - are distributive, $\boldsymbol{P} \times (\boldsymbol{Q}_1 + \boldsymbol{Q}_2) = \boldsymbol{P} \times \boldsymbol{Q}_1 + \boldsymbol{P} \times \boldsymbol{Q}_2$
  - are not associative, $(\boldsymbol{P} \times \boldsymbol{Q}) \times \boldsymbol{S} \neq \boldsymbol{P} \times (\boldsymbol{Q} \times \boldsymbol{S})$

Vector Products: Rectangular Components

- Vector products of Cartesian unit vectors,

$$\begin{array}{lll}
\vec{i}\times\vec{i}=0 & \vec{j}\times\vec{i}=-\vec{k} & \vec{k}\times\vec{i}=\vec{j} \\
\vec{i}\times\vec{j}=\vec{k} & \vec{j}\times\vec{j}=0 & \vec{k}\times\vec{j}=-\vec{i} \\
\vec{i}\times\vec{k}=-\vec{j} & \vec{j}\times\vec{k}=\vec{i} & \vec{k}\times\vec{k}=0
\end{array}$$





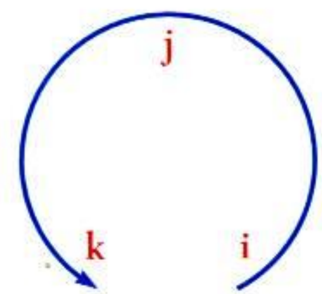
- Vector products in terms of rectangular coordinates

$$\vec{V}=\left(P_x\vec{i}+P_y\vec{j}+P_z\vec{k}\right)\times\left(Q_x\vec{i}+Q_y\vec{j}+Q_z\vec{k}\right)$$

$$=\left(P_yQ_z-P_zQ_y\right)\vec{i}+\left(P_zQ_x-P_xQ_z\right)\vec{j}$$

$$+\left(P_xQ_y-P_yQ_x\right)\vec{k}$$

$$=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k} \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z\end{vmatrix}$$



Moment of a Force About a Point

- A force vector is defined by its magnitude and direction. Its effect on the rigid body also depends on it point of application.

- The *moment* of $\boldsymbol{F}$ about O is defined as

$$\boldsymbol{M}_O = \boldsymbol{r} \times \boldsymbol{F}$$

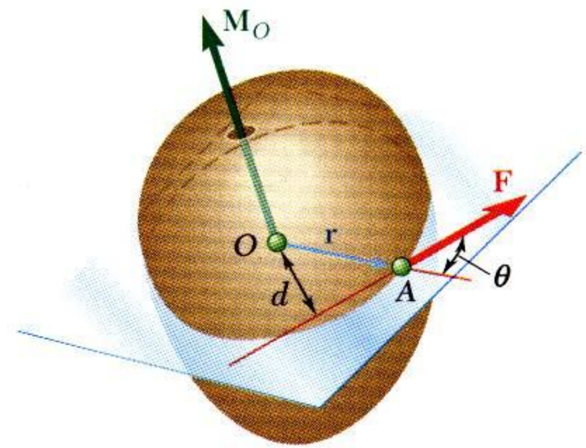
- The moment vector $\boldsymbol{M}_O$ is perpendicular to the plane containing O and the force $\boldsymbol{F}$.

- Magnitude of $\boldsymbol{M}_O$ measures the tendency of the force to cause rotation of the body about an axis along $\boldsymbol{M}_O$.

$$M_O = rF\sin\theta = Fd$$

The sense of the moment may be determined by the right-hand rule.

- Any force $\boldsymbol{F}'$ that has the same magnitude and direction as $\boldsymbol{F}$, is *equivalent* if it also has the same line of action and therefore, produces the same moment.



(a)

(b)

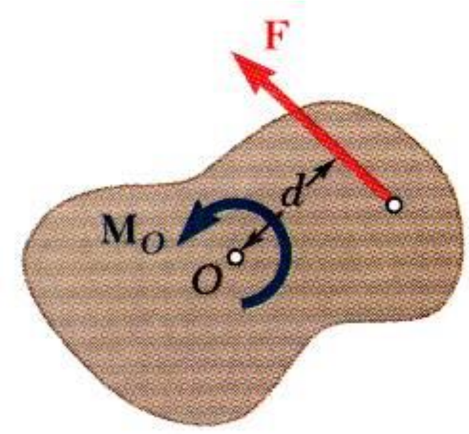
Moment of a Force About a Point

- *Two-dimensional structures* have length and breadth but negligible depth and are subjected to forces contained in the plane of the structure.

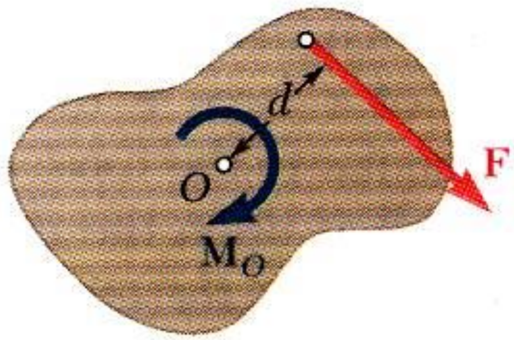
- The plane of the structure contains the point *O* and the force $\boldsymbol{F}$. $\boldsymbol{M}_O$, the moment of the force about *O* is perpendicular to the plane.

- If the force tends to rotate the structure counterclockwise, the sense of the moment vector is out of the plane of the structure and the magnitude of the moment is positive.

- If the force tends to rotate the structure clockwise, the sense of the moment vector is into the plane of the structure and the magnitude of the moment is negative.



(*a*) $\boldsymbol{M}_O = +Fd$



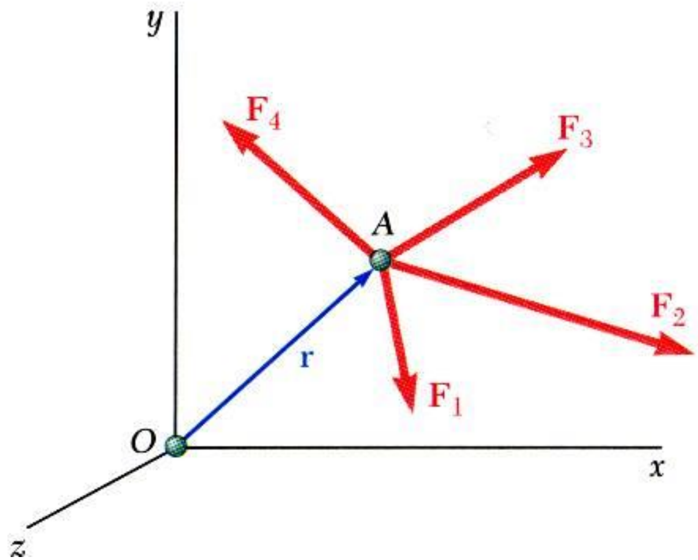
(*b*) $\boldsymbol{M}_O = -Fd$

Varignon's Theorem

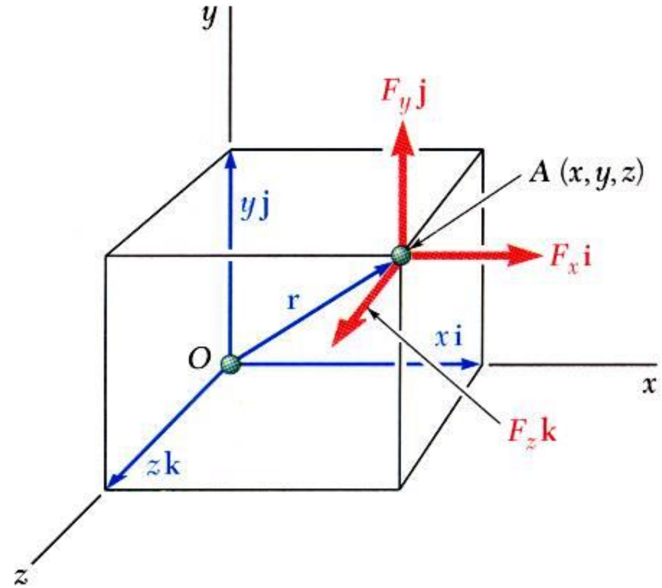
- The moment about a give point O of the resultant of several concurrent forces is equal to the sum of the moments of the various moments about the same point O.

$$\vec{r} \times \left(\vec{F}_1 + \vec{F}_2 + \cdots\right) = \vec{r} \times \vec{F}_1 + \vec{r} \times \vec{F}_2 + \cdots$$

- Varignon's Theorem makes it possible to replace the direct determination of the moment of a force ***F*** by the moments of two or more component forces of ***F***.



Rectangular Components of the Moment of a Force



The moment of ***F*** about *O*,

$$\vec{M}_O = \vec{r} \times \vec{F}, \qquad \vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$$

$$\vec{M}_O = M_x\vec{i} + M_y\vec{j} + M_z\vec{k}$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= \left(yF_z - zF_y\right)\vec{i} + \left(zF_x - xF_z\right)\vec{j} + \left(xF_y - yF_x\right)\vec{k}$$

Rectangular Components of the Moment of a Force

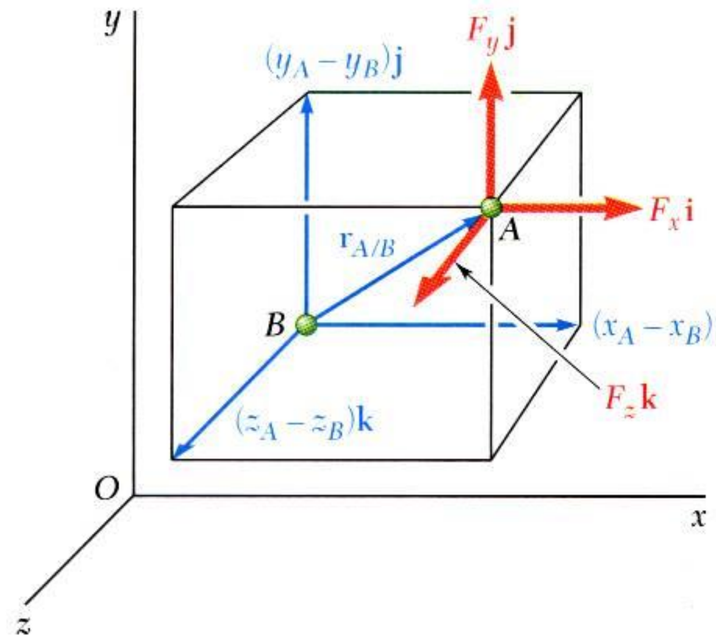
The moment of ***F*** about *B*,

$$\vec{M}_B = \vec{r}_{A/B} \times \vec{F}$$

$$\begin{aligned}\vec{r}_{A/B} &= \vec{r}_A - \vec{r}_B \\ &= (x_A - x_B)\vec{i} + (y_A - y_B)\vec{j} + (z_A - z_B)\vec{k}\end{aligned}$$

$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$$

$$\vec{M}_B = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ (x_A - x_B) & (y_A - y_B) & (z_A - z_B) \\ F_x & F_y & F_z \end{vmatrix}$$

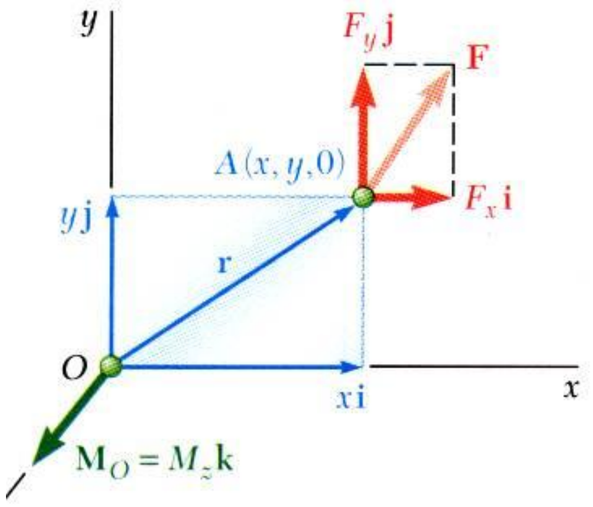


Rectangular Components of the Moment of a Force

For two-dimensional structures,

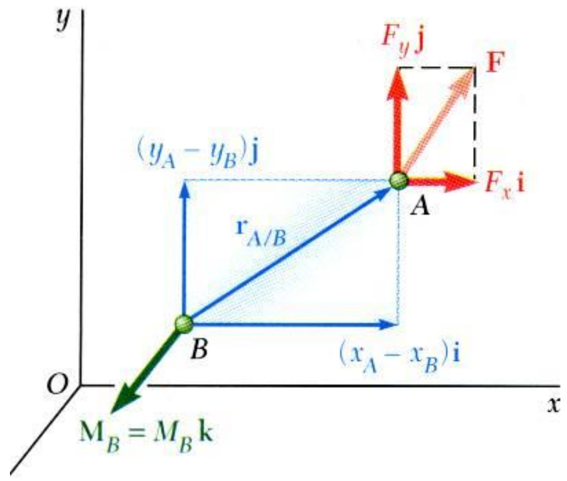
$$\vec{M}_O = \left(xF_y - yF_z\right)\vec{k}$$

$$M_O = M_Z$$
$$= xF_y - yF_z$$

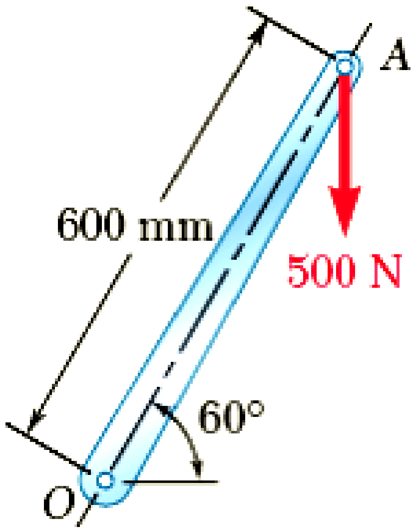


$$\vec{M}_O = \left[\left(x_A - x_B\right)F_y - \left(y_A - y_B\right)F_z\right]\vec{k}$$

$$M_O = M_Z$$
$$= \left(x_A - x_B\right)F_y - \left(y_A - y_B\right)F_z$$



Sample Problem 3.1



A 500-N vertical force is applied to the end of a lever which is attached to a shaft at *O*.

Determine:

a) moment about *O*,

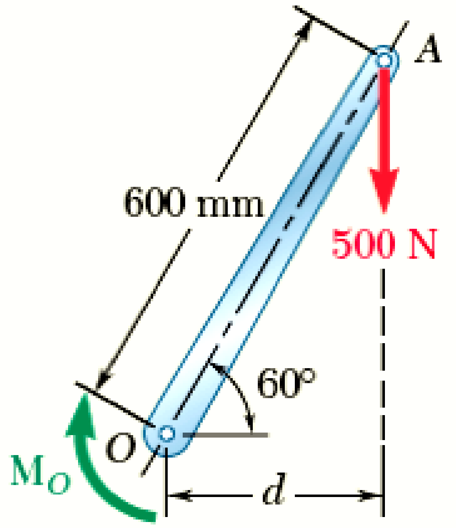
b) horizontal force at *A* which creates the same moment,

c) smallest force at A which produces the same moment,

d) location for a 1200-N vertical force to produce the same moment,

e) whether any of the forces from b, c, and d is equivalent to the original force.

Sample Problem 3.1



SOLUTION:

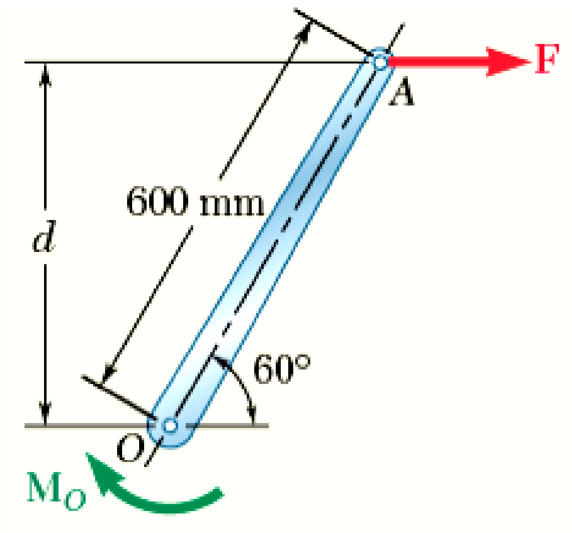
a) Moment about *O* is equal to the product of the force and the perpendicular distance between the line of action of the force and *O*. Since the force tends to rotate the lever clockwise, the moment vector is into the plane of the paper.

$$M_O = Fd$$

$$d = (600\,mm)\cos 60° = 300\,mm = 0.3\,m$$

$$M_O = (500\,N)(0.3\,\text{m}) \qquad \boxed{M_O = 150\ \text{N.m}}$$

Sample Problem 3.1



b) Horizontal force at A that produces the same moment,

$$d = (600\text{ mm})\sin 60° = 519.6\text{ mm} = 0.5196\text{ m}$$

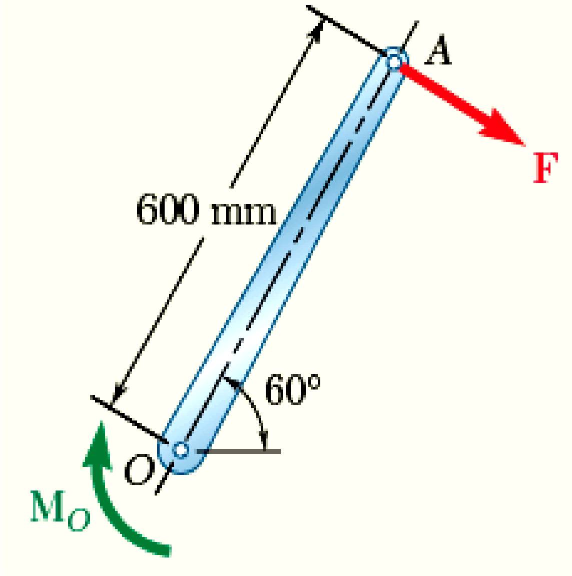
$$M_O = Fd$$

$$150\text{ N . m} = F(0.5196m)$$

$$F = \frac{150\text{ N . m}}{0.5196\text{ m}}$$

$$\boxed{F = 288.68\text{ N}}$$

Sample Problem 3.1



c) The smallest force A to produce the same moment occurs when the perpendicular distance is a maximum or when F is perpendicular to OA.

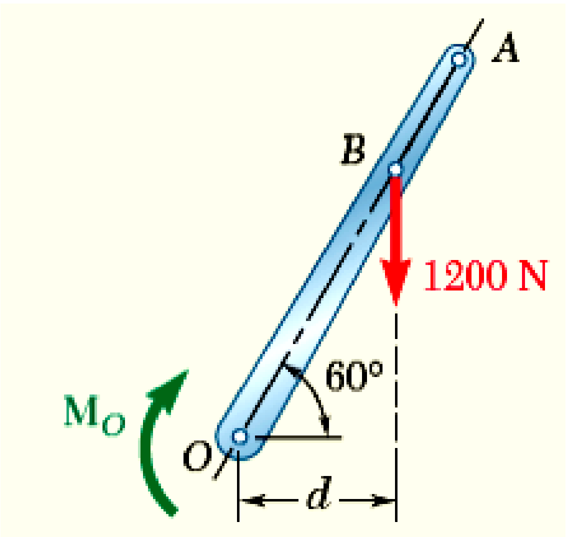
$$M_O = Fd$$

$$150\ \mathrm{N.m} = F(0.6\ \mathrm{m})$$

$$F = \frac{150\ \mathrm{N.m}}{0.6\ \mathrm{m}}$$

$$\boxed{F = 250\ \mathrm{N}}$$

Sample Problem 3.1



d) To determine the point of application of a 1200 N force to produce the same moment,

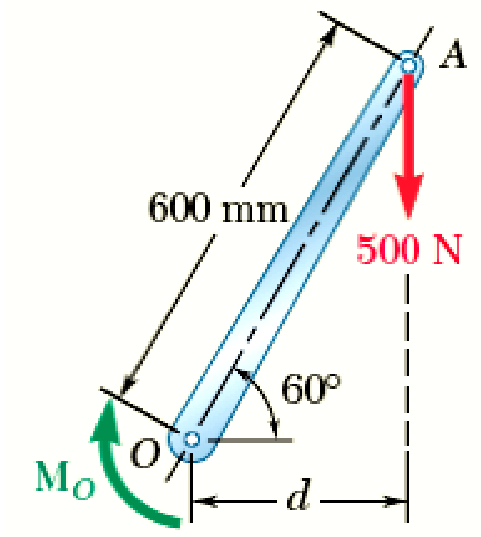
$$M_O = Fd$$

$$150\ \text{N.m} = (1200\ \text{N})d$$

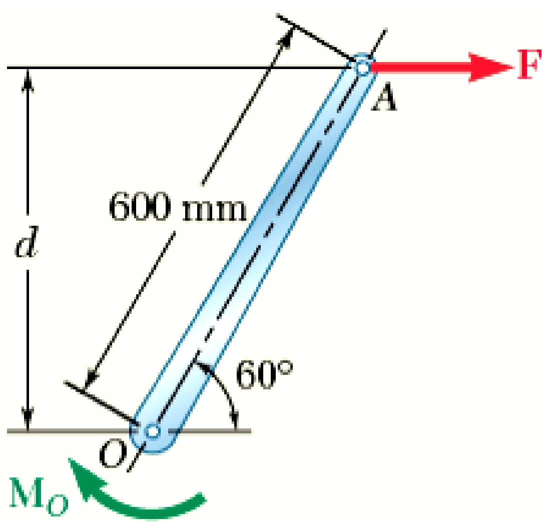
$$d = \frac{150\ \text{N.m}}{1200\ \text{N}} = 125\ \text{mm}$$

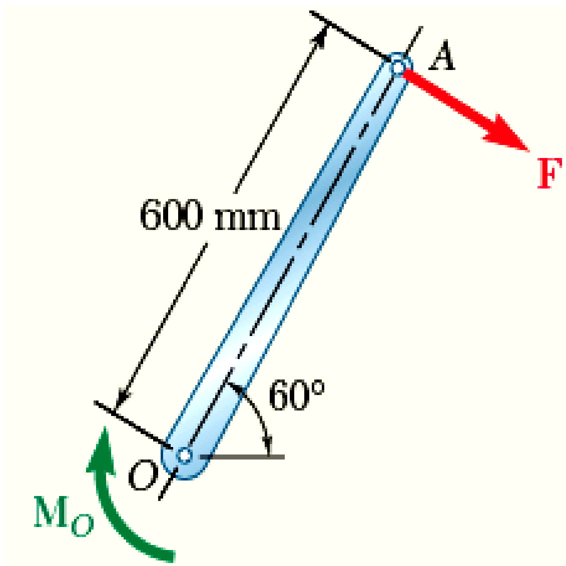
$$OB\cos 60° = 125\ \text{mm} \qquad \boxed{OB = 250\ \text{mm}}$$

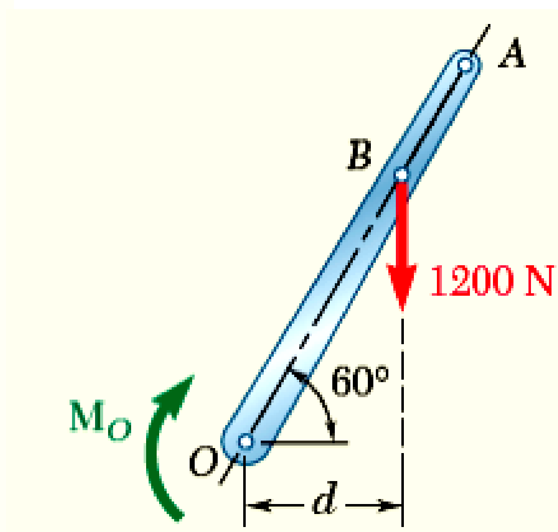
Sample Problem 3.1



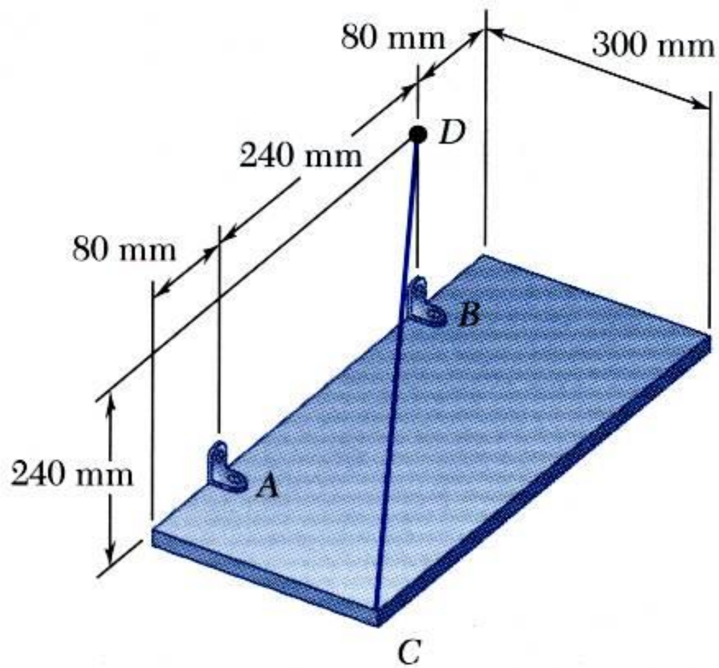
e) Although each of the forces in parts b), c), and d) produces the same moment as the 500-N force, none are of the same magnitude and sense, or on the same line of action. None of the forces is equivalent to the 500-N force.







Sample Problem 3.4



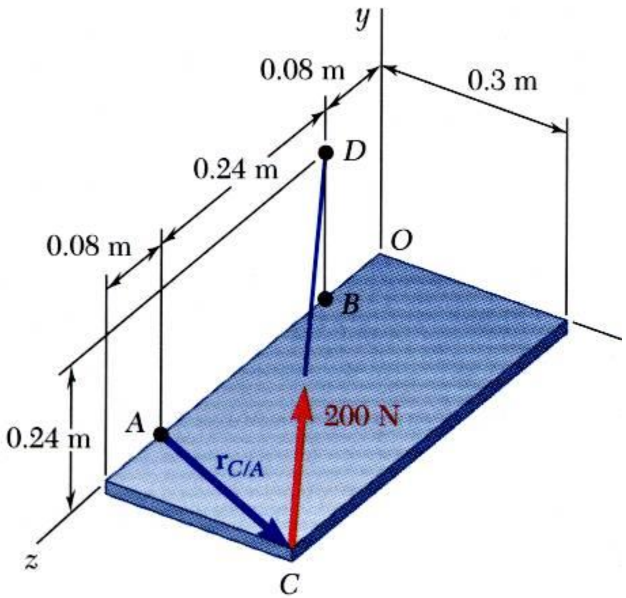
The rectangular plate is supported by the brackets at *A* and *B* and by a wire *CD*. Knowing that the tension in the wire is 200 N, determine the moment about *A* of the force exerted by the wire at *C*.

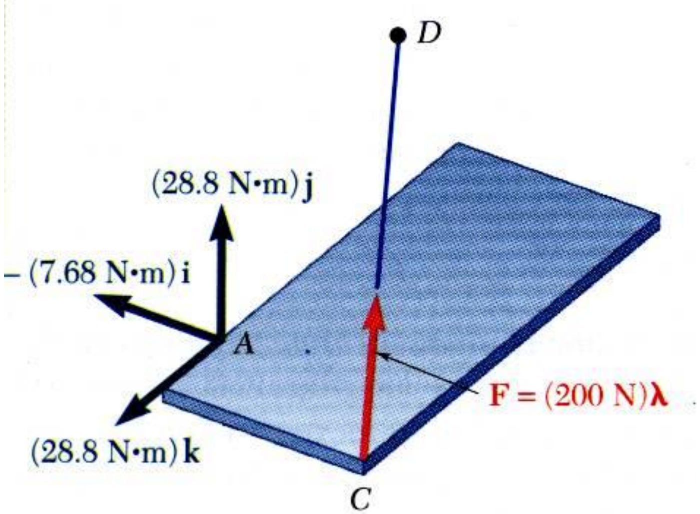
SOLUTION:

The moment M_A of the force F exerted by the wire is obtained by evaluating the vector product,

$$\vec{M}_A = \vec{r}_{C/A} \times \vec{F}$$

Sample Problem 3.4





SOLUTION:

$$\vec{M}_A = \vec{r}_{C/A} \times \vec{F}$$

$$\vec{r}_{C/A} = \vec{r}_C - \vec{r}_A = (0.3\,\text{m})\vec{i} + (0.08\,\text{m})\vec{j}$$

$$\vec{F} = F\,\vec{\ } = (200\,\text{N})\frac{\vec{r}_{C/D}}{r_{C/D}}$$

$$= (200\,\text{N})\frac{-(0.3\,\text{m})\vec{i} + (0.24\,\text{m})\vec{j} - (0.32\,\text{m})\vec{k}}{0.5\,\text{m}}$$

$$= -(120\,\text{N})\vec{i} + (96\,\text{N})\vec{j} - (128\,\text{N})\vec{k}$$

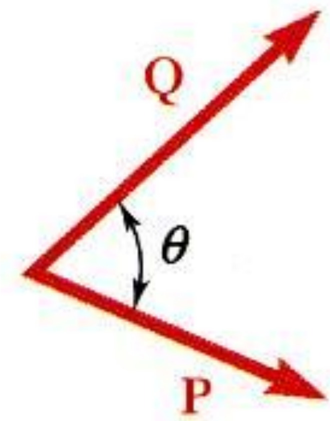
$$\vec{M}_A = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.3 & 0 & 0.08 \\ -120 & 96 & -128 \end{vmatrix}$$

$$\boxed{\vec{M}_A = -(7.68\,\text{N}\cdot\text{m})\vec{i} + (28.8\,\text{N}\cdot\text{m})\vec{j} + (28.8\,\text{N}\cdot\text{m})\vec{k}}$$

Scalar Product of Two Vectors

- The *scalar product* or *dot product* between two vectors ***P*** and ***Q*** is defined as

$$\vec{P}\bullet\vec{Q} = PQ\cos \qquad (\text{scalar result})$$



- Scalar products:
  - are commutative, $\vec{P}\bullet\vec{Q} = \vec{Q}\bullet\vec{P}$
  - are distributive, $\vec{P}\bullet\left(\vec{Q}_1 + \vec{Q}_2\right) = \vec{P}\bullet\vec{Q}_1 + \vec{P}\bullet\vec{Q}_2$
  - are not associative, $\left(\vec{P}\bullet\vec{Q}\right)\bullet\vec{S} = \text{undefined}$

- Scalar products with Cartesian unit components,

$$\vec{P}\bullet\vec{Q} = \left(P_x\vec{i} + P_y\vec{j} + P_z\vec{k}\right)\bullet\left(Q_x\vec{i} + Q_y\vec{j} + Q_z\vec{k}\right)$$

$$\vec{i}\bullet\vec{i} = 1 \quad \vec{j}\bullet\vec{j} = 1 \quad \vec{k}\bullet\vec{k} = 1 \quad \vec{i}\bullet\vec{j} = 0 \quad \vec{j}\bullet\vec{k} = 0 \quad \vec{k}\bullet\vec{i} = 0$$

$$\vec{P}\bullet\vec{Q} = P_xQ_x + P_yQ_y + P_zQ_z$$

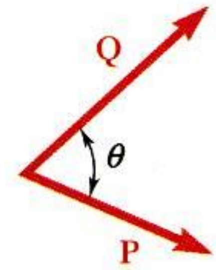
$$\vec{P}\bullet\vec{P} = P_x^2 + P_y^2 + P_z^2 = P^2$$

Scalar Product of Two Vectors: Applications

- Angle between two vectors:

$$\vec{P}\bullet\vec{Q} = PQ\cos \quad = P_xQ_x + P_yQ_y + P_zQ_z$$

$$\cos \quad = \frac{P_xQ_x + P_yQ_y + P_zQ_z}{PQ}$$

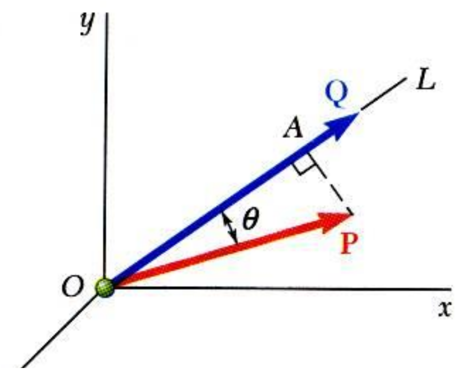


- Projection of a vector on a given axis:

$$P_{OL} = P\cos \quad = \text{projection of } P \text{ along } OL$$

$$\vec{P}\bullet\vec{Q} = PQ\cos$$

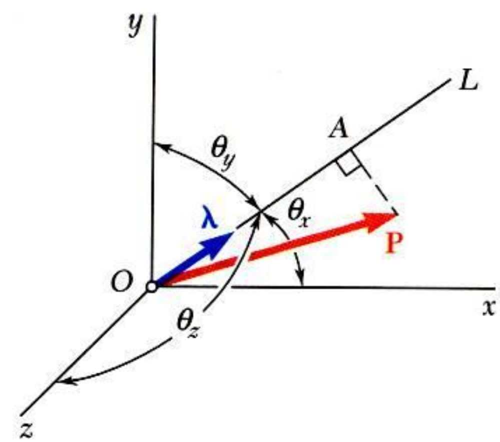
$$\frac{\vec{P}\bullet\vec{Q}}{Q} = P\cos \quad = P_{OL}$$



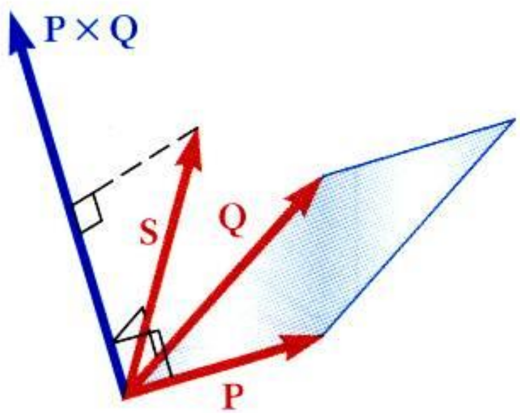
- For an axis defined by a unit vector:

$$P_{OL} = \vec{P}\bullet\vec{\ }$$

$$= P_x\cos\ _x + P_y\cos\ _y + P_z\cos\ _z$$



Mixed Triple Product of Three Vectors



- Mixed triple product of three vectors,

$$\vec{S} \bullet \left(\vec{P} \times \vec{Q}\right) = \text{scalar result}$$

- The six mixed triple products formed from ***S***, ***P***, and ***Q*** have equal magnitudes but not the same sign,

$$\vec{S} \bullet \left(\vec{P} \times \vec{Q}\right) = \vec{P} \bullet \left(\vec{Q} \times \vec{S}\right) = \vec{Q} \bullet \left(\vec{S} \times \vec{P}\right)$$
$$= -\vec{S} \bullet \left(\vec{Q} \times P\right) = -\vec{P} \bullet \left(\vec{S} \times \vec{Q}\right) = -\vec{Q} \bullet \left(\vec{P} \times \vec{S}\right)$$

- Evaluating the mixed triple product,

$$\vec{S} \bullet \left(\vec{P} \times \vec{Q}\right) = S_x\left(P_y Q_z - P_z Q_y\right) + S_y\left(P_z Q_x - P_x Q_z\right) + S_z\left(P_x Q_y - P_y Q_x\right)$$

$$= \begin{vmatrix} S_x & S_y & S_z \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

Moment of a Force About a Given Axis

- Moment $\boldsymbol{M_O}$ of a force $\boldsymbol{F}$ applied at the point $\boldsymbol{A}$ about a point $\boldsymbol{O}$,

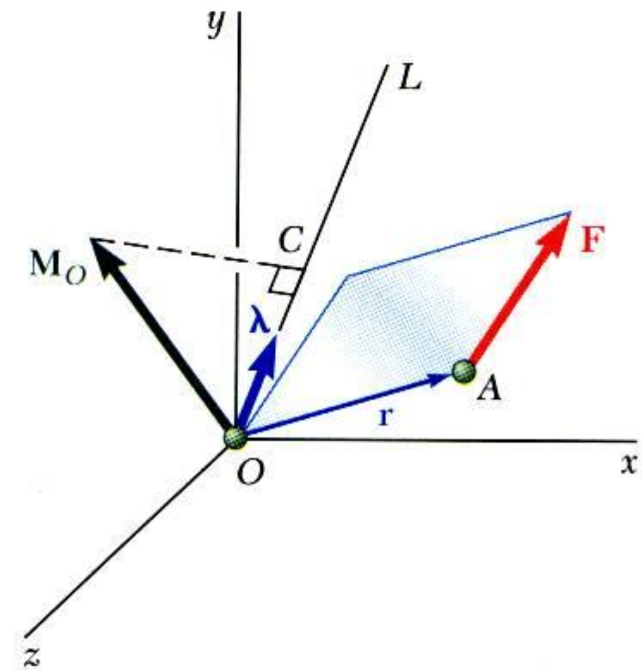
$$\vec{M}_O = \vec{r} \times \vec{F}$$

- Scalar moment M_{OL} about an axis $\boldsymbol{OL}$ is the projection of the moment vector $\boldsymbol{M_O}$ onto the axis,

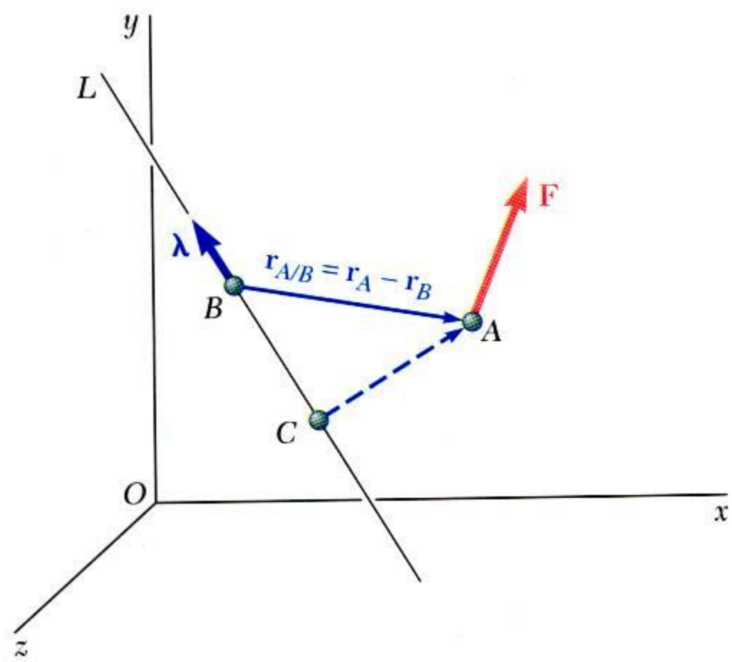
$$M_{OL} = \vec{\ } \bullet \vec{M}_O = \vec{\ } \bullet \left(\vec{r} \times \vec{F}\right)$$

- Moments of $\boldsymbol{F}$ about the coordinate axes,

$$M_x = yF_z - zF_y$$
$$M_y = zF_x - xF_z$$
$$M_z = xF_y - yF_x$$



Moment of a Force About a Given Axis

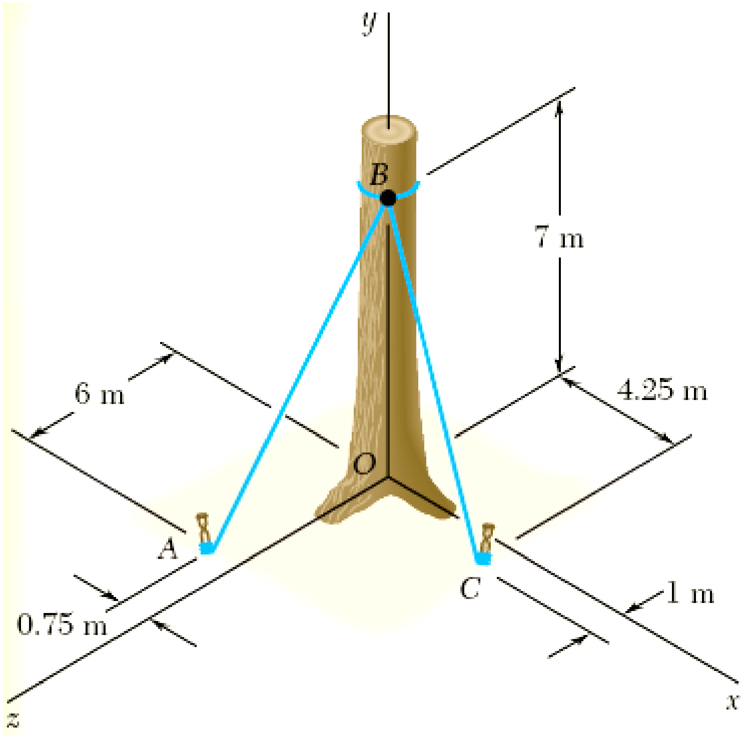


- Moment of a force about an arbitrary axis,

$$M_{BL} = \vec{\ } \bullet \vec{M}_B$$
$$= \vec{\ } \bullet \left(\vec{r}_{A/B} \times \vec{F}\right)$$
$$\vec{r}_{A/B} = \vec{r}_A - \vec{r}_B$$

- The result is independent of the point *B* along the given axis.

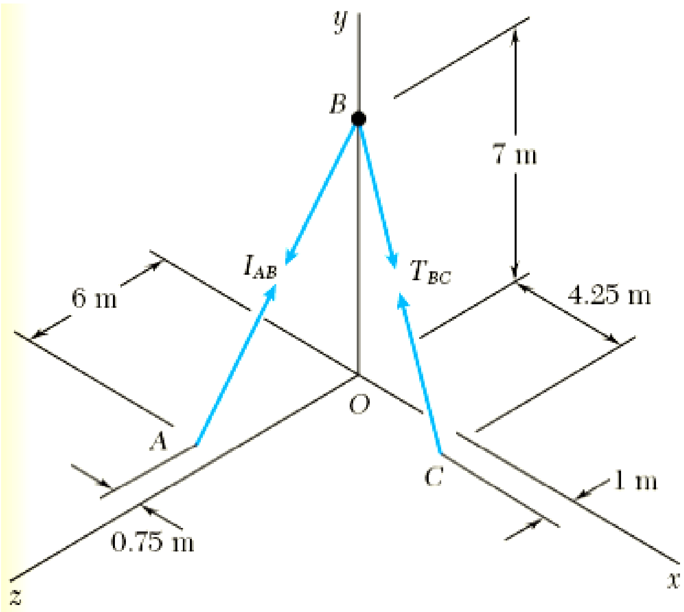
Sample Problem 3.5



Before the trunk of a large tree is felled, cables *AB* and *BC* are attached as shown. Tensions in cables *AB* and *BC* are 555-N and 660-N, respectively.

Determine the moment about *O* of the resultant force exerted on the tree by the cables at *B*.

Sample Problem 3.5



SOLUTION:

We have $M_O = r_{B/o} \times F_B$

where $r_{B/o} = (7\text{ m})\mathbf{j}$

$$F_B = T_{AB} + T_{BC}$$

$$T_{AB} = \lambda_{BA} T_{AB}$$

$$= \frac{-(0.75m)\mathbf{i} - (7m)\mathbf{j} + (6m)\mathbf{k}}{\sqrt{(.75)^2 + (7)^2 + (6)^2}\, m}(555\text{ N})$$

$$T_{BC} = \lambda_{BC} T_{BC}$$

$$= \frac{(4.25m)\mathbf{i} - (7m)\mathbf{j} + (1m)\mathbf{k}}{\sqrt{(4.25)^2 + (7)^2 + (1)^2}\, m}(660\text{ N})$$

$$F_B = [-(45.00\text{ N})\mathbf{i} - (420.0\text{ N})\mathbf{j} + (360.0\text{ N})\mathbf{k}]$$
$$+[(340.0\text{ N})\mathbf{i} - (560.0\text{ N})\mathbf{j} + (80.00\text{ N})\mathbf{k}]$$
$$= (295.0\text{ N})\mathbf{i} - (980.0\text{ N})\mathbf{j} + (440.0\text{ N})\mathbf{k}$$

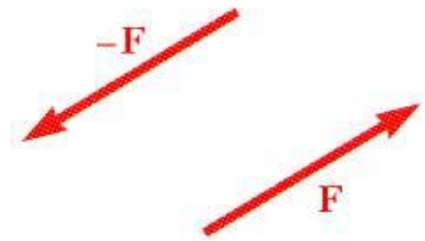
and
$$M_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 7 & 0 \\ 295 & 980 & 440 \end{vmatrix} \text{N} \cdot \text{m}$$

$$= (3080\text{ N} \cdot \text{m})\mathbf{i} - (2070\text{ N} \cdot \text{m})\mathbf{k} \text{ or}$$

$$\mathbf{M}_O = (3080\text{ N} \cdot \text{m})\mathbf{i} - (2070\text{ N} \cdot \text{m})\mathbf{k}$$

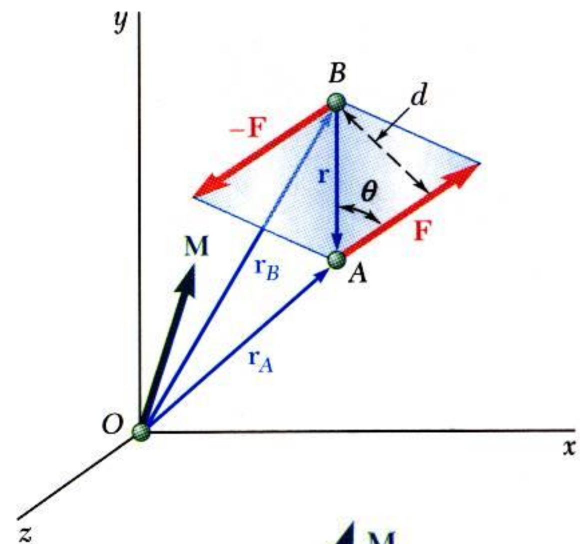
Moment of a Couple

- Two forces ***F*** and -***F*** having the same magnitude, parallel lines of action, and opposite sense are said to form a *couple*.

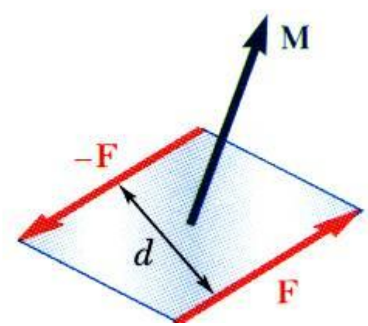


- Moment of the couple,

$$\vec{M} = \vec{r}_A \times \vec{F} + \vec{r}_B \times (-\vec{F})$$
$$= (\vec{r}_A - \vec{r}_B) \times \vec{F}$$
$$= \vec{r} \times \vec{F}$$
$$M = rF\sin\theta = Fd$$



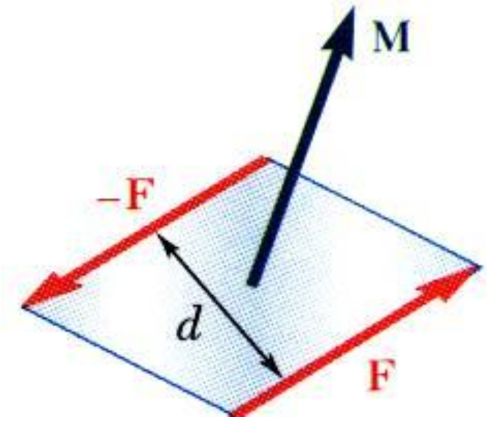
- The moment vector of the couple is independent of the choice of the origin of the coordinate axes, i.e., it is a *free vector* that can be applied at any point with the same effect.

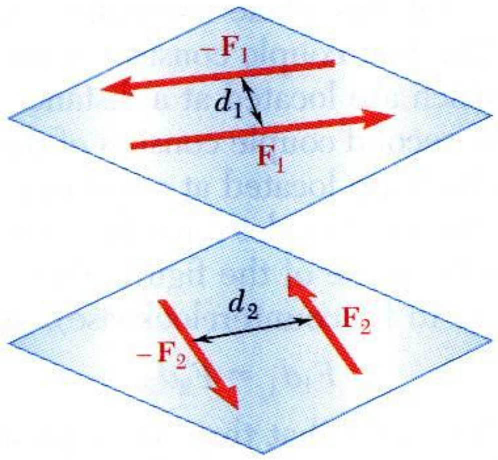


Moment of a Couple

Two couples will have equal moments if

- $F_1 d_1 = F_2 d_2$
- the two couples lie in parallel planes, and
- the two couples have the same sense or the tendency to cause rotation in the same direction.



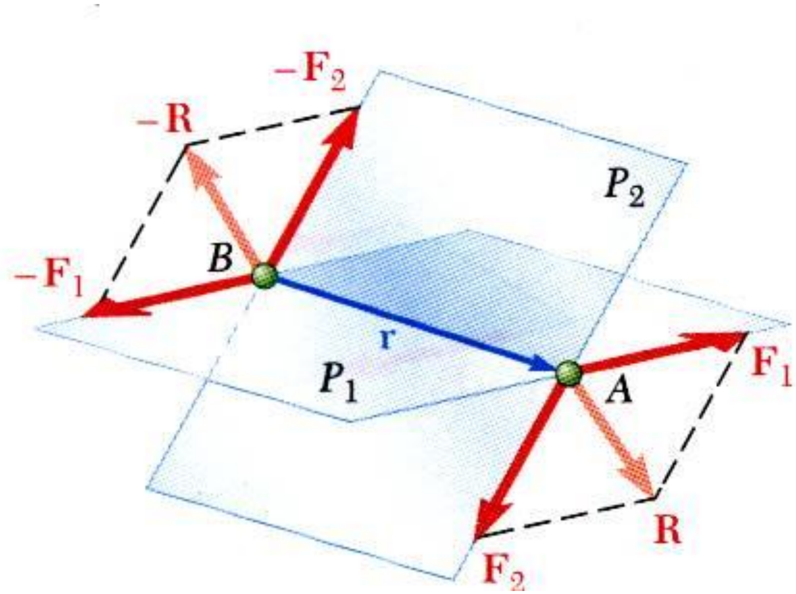


Addition of Couples

- Consider two intersecting planes P_1 and P_2 with each containing a couple

$$\vec{M}_1 = \vec{r} \times \vec{F}_1 \text{ in plane } P_1$$

$$\vec{M}_2 = \vec{r} \times \vec{F}_2 \text{ in plane } P_2$$



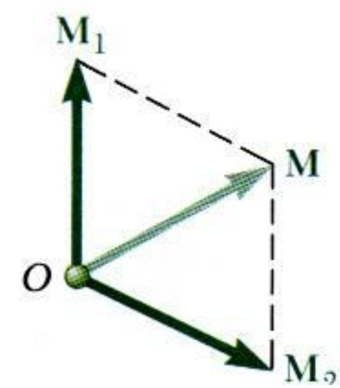
- Resultants of the vectors also form a couple

$$\vec{M} = \vec{r} \times \vec{R} = \vec{r} \times \left(\vec{F}_1 + \vec{F}_2\right)$$

- By Varigon's theorem

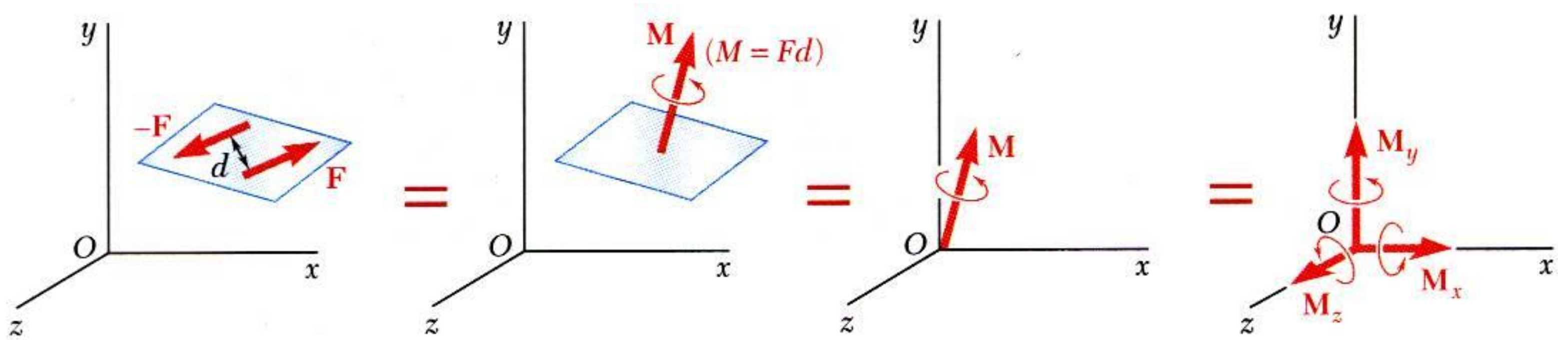
$$\vec{M} = \vec{r} \times \vec{F}_1 + \vec{r} \times \vec{F}_2$$

$$= \vec{M}_1 + \vec{M}_2$$



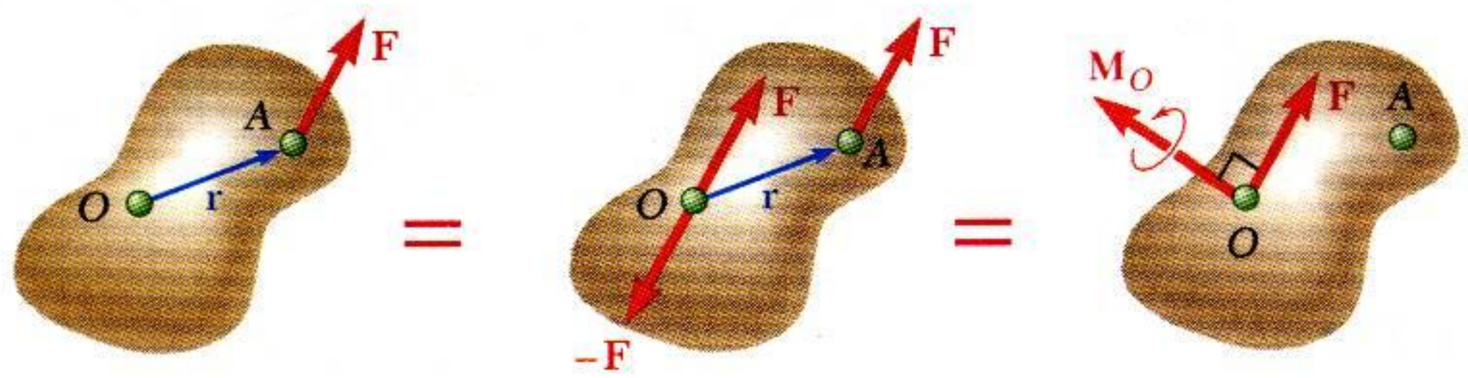
- Sum of two couples is also a couple that is equal to the vector sum of the two couples

Couples Can Be Represented by Vectors



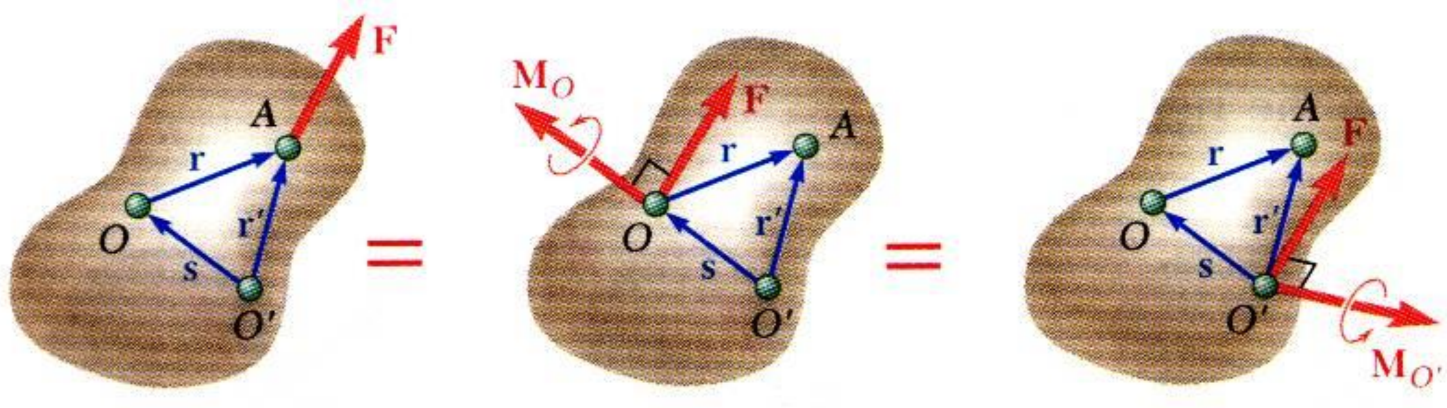
- A couple can be represented by a vector with magnitude and direction equal to the moment of the couple.
- *Couple vectors* obey the law of addition of vectors.
- Couple vectors are free vectors, i.e., the point of application is not significant.
- Couple vectors may be resolved into component vectors.

Resolution of a Force Into a Force at *O* and a Couple



- Force vector ***F*** can not be simply moved to *O* without modifying its action on the body.
- Attaching equal and opposite force vectors at *O* produces no net effect on the body.
- The three forces may be replaced by an equivalent force vector and couple vector, i.e, a *force-couple system*.

Resolution of a Force Into a Force at *O* and a Couple



- Moving **F** from *A* to a different point *O*′ requires the addition of a different couple vector $\boldsymbol{M}_{O'}$

$$\vec{M}_{O'} = \vec{r}\,' \times \vec{F}$$

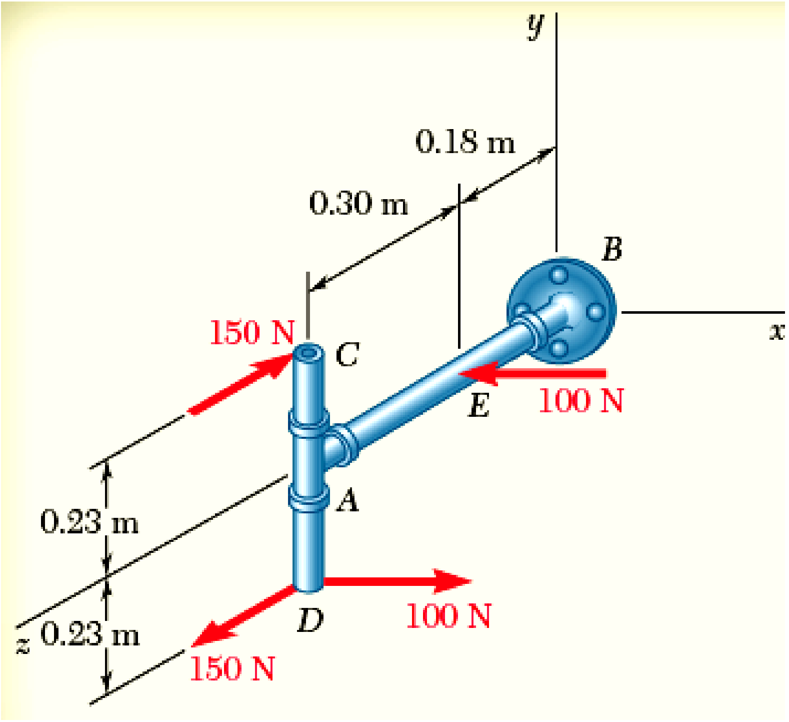
- The moments of **F** about O and *O*′ are related,

$$\vec{M}_{O'} = \vec{r}\,' \times \vec{F} = (\vec{r} + \vec{s}) \times \vec{F} = \vec{r} \times \vec{F} + \vec{s} \times \vec{F}$$

$$= \vec{M}_O + \vec{s} \times \vec{F}$$

- Moving the force-couple system from *O* to *O*′ requires the addition of the moment of the force at *O* about *O*′.

Sample Problem 3.6



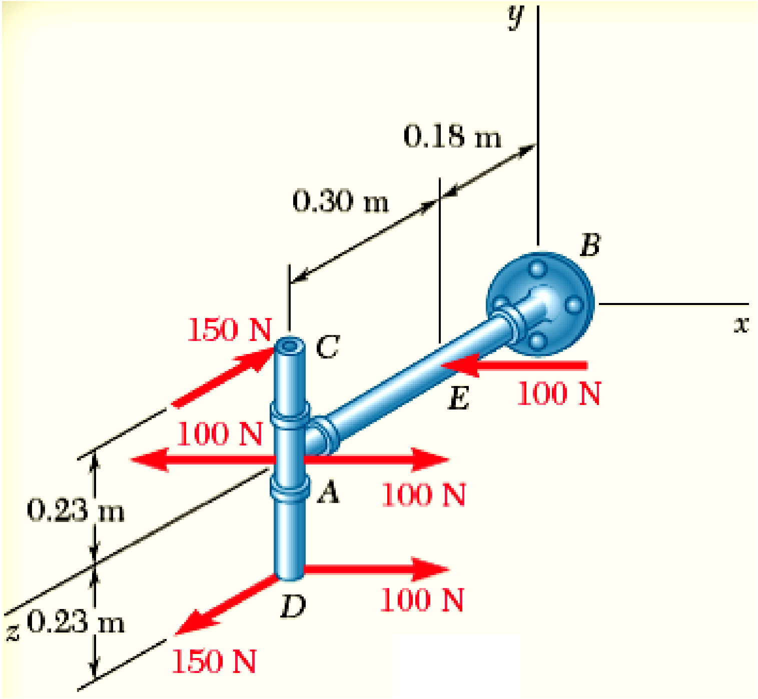
Determine the components of the single couple equivalent to the couples shown.

SOLUTION:

- Attach equal and opposite 100-N forces in the $\pm x$ direction at *A*, thereby producing 3 couples for which the moment components are easily computed.

- Alternatively, compute the sum of the moments of the four forces about an arbitrary single point. The point *D* is a good choice as only two of the forces will produce non-zero moment contributions..

Sample Problem 3.6



- The three couples may be represented by three couple vectors,

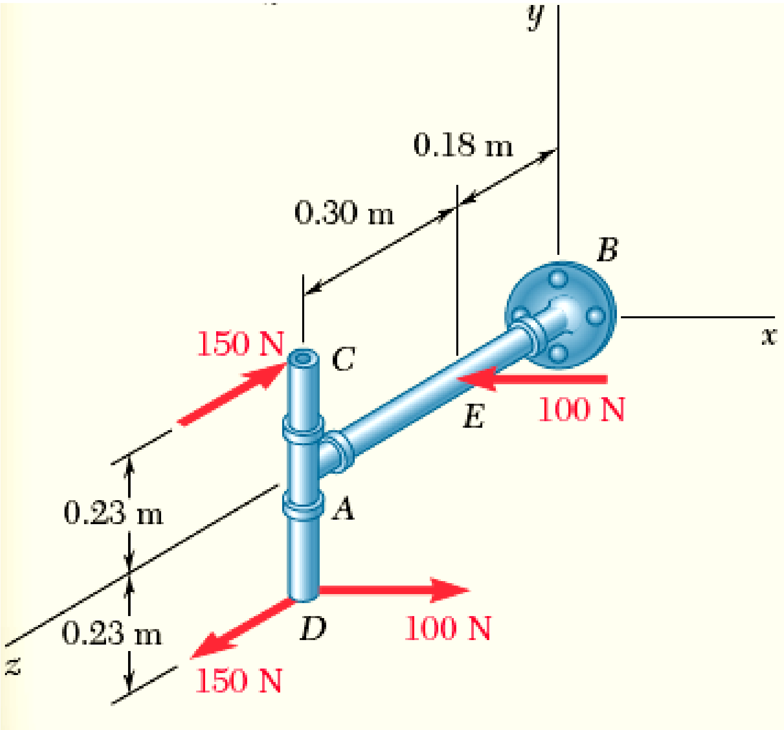
$$M_x = -(150\ \mathrm{N})(0.45\ \mathrm{m}) = -67.5\ \mathrm{N.m}$$

$$M_y = +(100\ \mathrm{N})(0.30\ \mathrm{m}) = +30\ \mathrm{N.m}$$

$$M_z = +(100\ \mathrm{N})(0.23\ \mathrm{m}) = +22.5\ \mathrm{N.m}$$

$$\boxed{\vec{M} = -(67.5\ \mathrm{N.m})\vec{i} + (30\ \mathrm{N.m})\vec{j} + (22.5\ \mathrm{N.m})\vec{k}}$$

Sample Problem 3.6

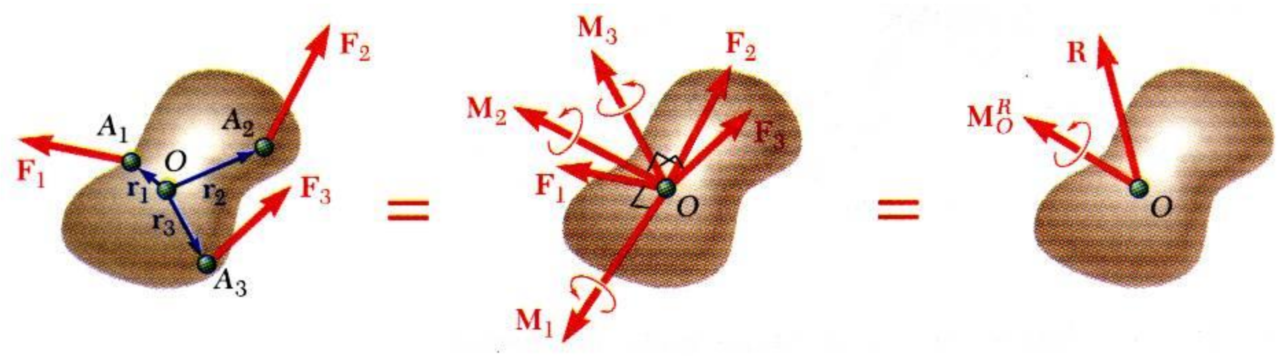


- Alternatively, compute the sum of the moments of the four forces about *D*.
- Only the forces at *C* and *E* contribute to the moment about *D*.

$$\vec{M} = \vec{M}_D = (0.45\ \mathrm{m})\vec{j} \times (-150\ \mathrm{N})\vec{k} + \left[(0.23\ \mathrm{m})\vec{j} - (0.30\ \mathrm{m})\vec{k}\right] \times (-100\ \mathrm{N})\vec{i}$$

$$\boxed{\vec{M} = -(67.5\,.\,\mathrm{m})\vec{i} + (30\ \mathrm{N}\,.\,\mathrm{m})\vec{j} + (22.5\ \mathrm{N}\,.\,\mathrm{m})\vec{k}}$$

System of Forces: Reduction to a Force and Couple



- A system of forces may be replaced by a collection of force-couple systems acting a given point *O*

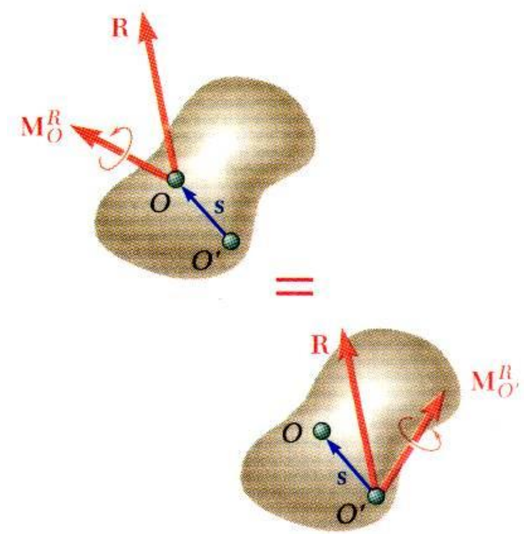
- The force and couple vectors may be combined into a resultant force vector and a resultant couple vector,

$$\vec{R} = \sum \vec{F} \qquad \vec{M}_O^R = \sum \left(\vec{r} \times \vec{F} \right)$$

- The force-couple system at *O* may be moved to *O*′ with the addition of the moment of ***R*** about *O*′ ,

$$\vec{M}_{O'}^R = \vec{M}_O^R + \vec{s} \times \vec{R}$$

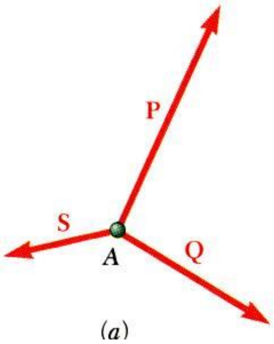
- Two systems of forces are equivalent if they can be reduced to the same force-couple system.

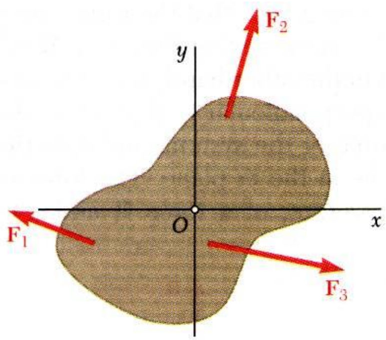


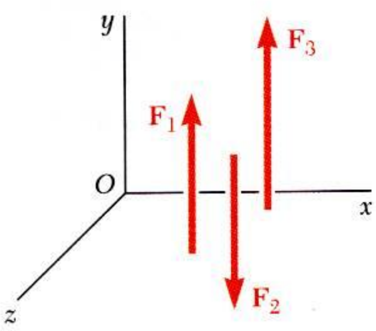
Further Reduction of a System of Forces

- If the resultant force and couple at *O* are mutually perpendicular, they can be replaced by a single force acting along a new line of action.

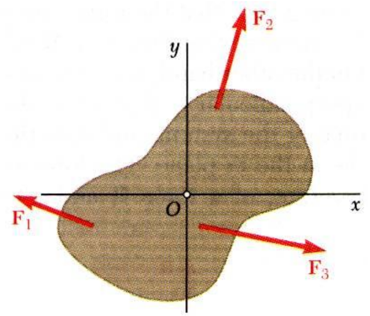
- The resultant force-couple system for a system of forces will be mutually perpendicular if:
  1) the forces are concurrent,
  2) the forces are coplanar, or
  3) the forces are parallel.

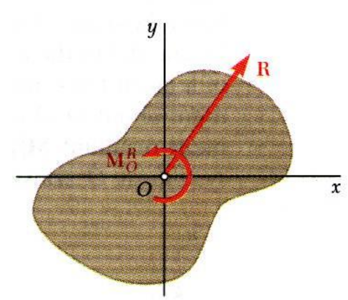


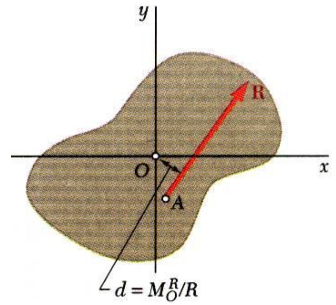




Further Reduction of a System of Forces





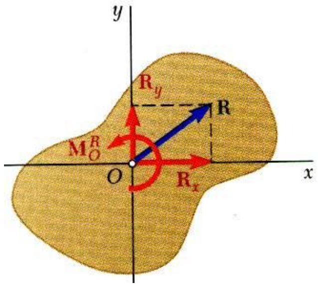


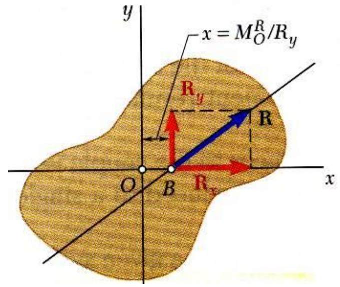
- System of coplanar forces is reduced to a force-couple system $\vec{R}$ and $\vec{M}_O^R$ that is mutually perpendicular.

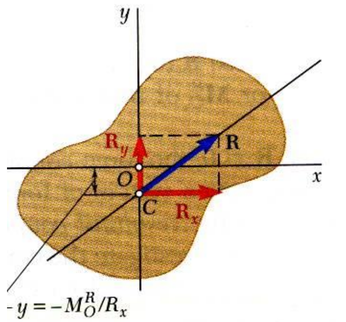
- System can be reduced to a single force by moving the line of action of $\vec{R}$ until its moment about O becomes $\vec{M}_O^R$

- In terms of rectangular coordinates,

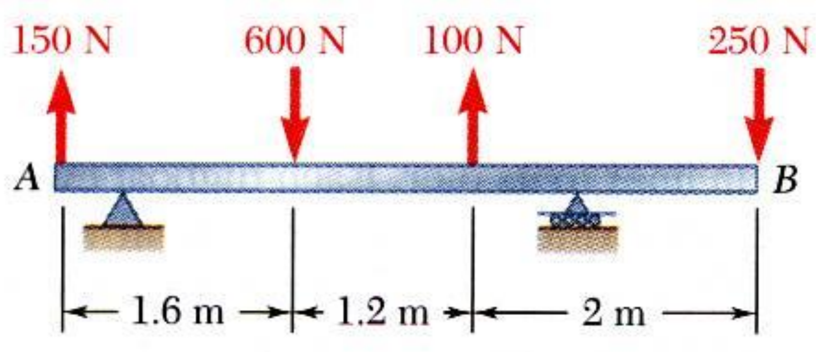
$$xR_y - yR_x = M_O^R$$







Sample Problem 3.8



For the beam, reduce the system of forces shown to (a) an equivalent force-couple system at *A*, (b) an equivalent force couple system at *B*, and (c) a single force or resultant.

Note: Since the support reactions are not included, the given system will not maintain the beam in equilibrium.

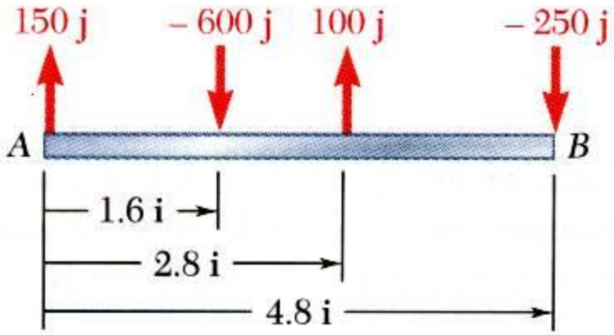
SOLUTION:

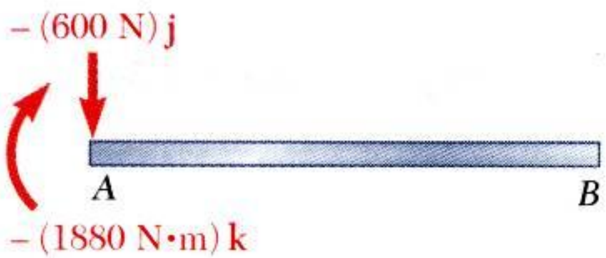
a) Compute the resultant force for the forces shown and the resultant couple for the moments of the forces about *A*.

b) Find an equivalent force-couple system at *B* based on the force-couple system at *A*.

c) Determine the point of application for the resultant force such that its moment about *A* is equal to the resultant couple at *A*.

Sample Problem 3.8





SOLUTION:

a) Compute the resultant force and the resultant couple at *A*.

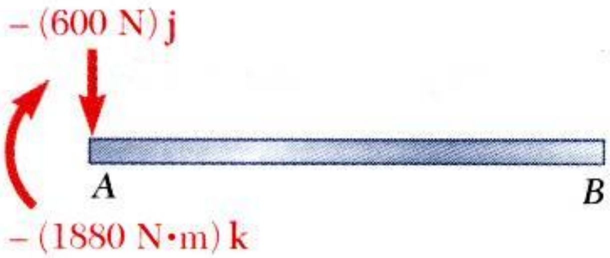
$$\vec{R} = \sum \vec{F}$$
$$= (150\text{ N})\vec{j} - (600\text{ N})\vec{j} + (100\text{ N})\vec{j} - (250\text{ N})\vec{j}$$

$$\boxed{\vec{R} = -(600\text{ N})\vec{j}}$$

$$\vec{M}_A^R = \sum (\vec{r} \times \vec{F})$$
$$= (1.6\vec{i}) \times (-600\vec{j}) + (2.8\vec{i}) \times (100\vec{j}) + (4.8\vec{i}) \times (-250\vec{j})$$

$$\boxed{\vec{M}_A^R = -(1880\text{ N}\cdot\text{m})\vec{k}}$$

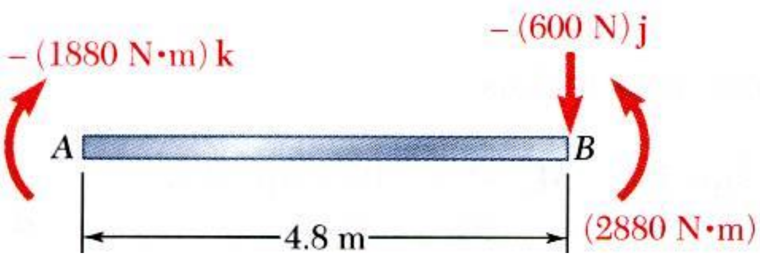
Sample Problem 3.8



b) Find an equivalent force-couple system at *B* based on the force-couple system at *A*.

The force is unchanged by the movement of the force-couple system from *A* to *B*.

$$\boxed{\vec{R} = -(600\ \text{N})\vec{j}}$$



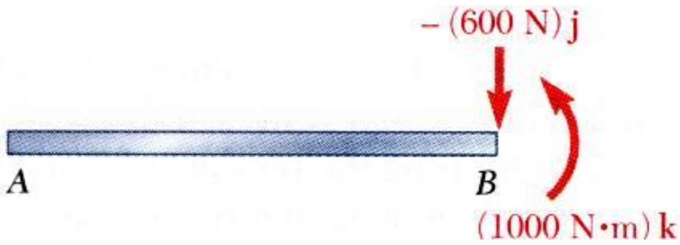
The couple at *B* is equal to the moment about *B* of the force-couple system found at *A*.

$$\vec{M}_B^R = \vec{M}_A^R + \vec{r}_{B/A} \times \vec{R}$$

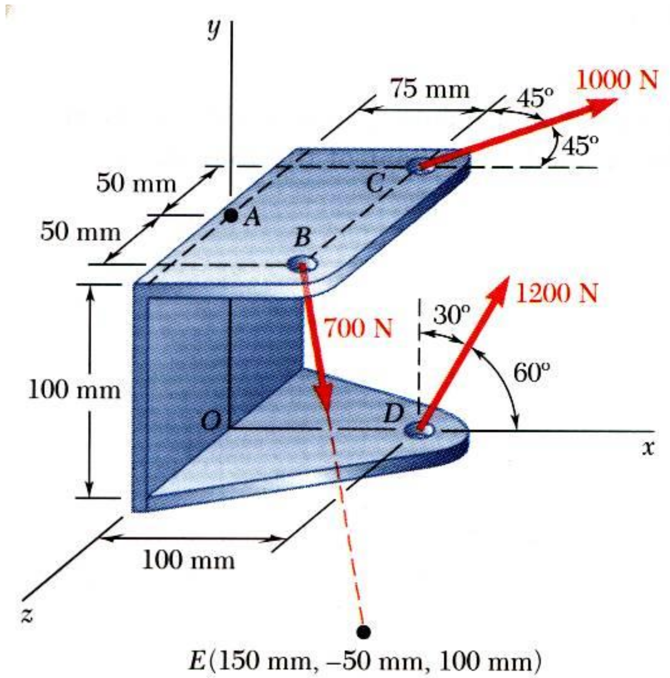
$$= -(1880\ \text{N}\cdot\text{m})\vec{k} + (-4.8\ \text{m})\vec{i} \times (-600\ \text{N})\vec{j}$$

$$= -(1880\ \text{N}\cdot\text{m})\vec{k} + (2880\ \text{N}\cdot\text{m})\vec{k}$$

$$\boxed{\vec{M}_B^R = +(1000\ \text{N}\cdot\text{m})\vec{k}}$$



Sample Problem 3.10



Three cables are attached to the bracket as shown. Replace the forces with an equivalent force-couple system at *A*.

SOLUTION:

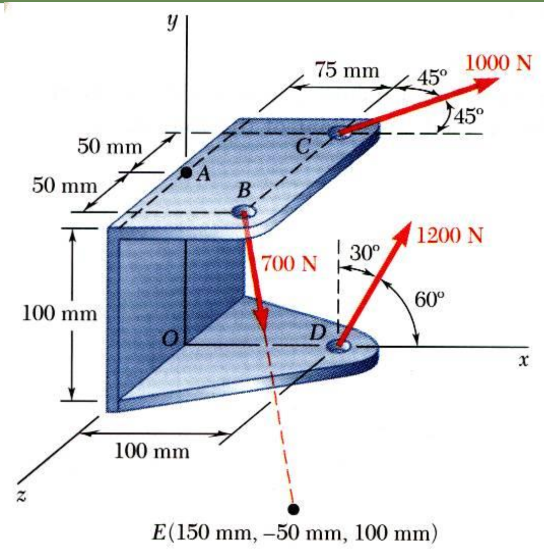
- Determine the relative position vectors for the points of application of the cable forces with respect to *A*.
- Resolve the forces into rectangular components.
- Compute the equivalent force,

$$\vec{R} = \sum \vec{F}$$

- Compute the equivalent couple,

$$\vec{M}_A^R = \sum \left(\vec{r} \times \vec{F}\right)$$

Sample Problem 3.10



SOLUTION:

- Determine the relative position vectors with respect to *A*.

$$\vec{r}_{B/A} = 0.075\vec{i} + 0.050\vec{k}\ (\mathrm{m})$$

$$\vec{r}_{C/A} = 0.075\vec{i} - 0.050\vec{k}\ (\mathrm{m})$$

$$\vec{r}_{D/A} = 0.100\vec{i} - 0.100\vec{j}\ (\mathrm{m})$$

- Resolve the forces into rectangular components.

$$\vec{F}_B = (700\ \mathrm{N})\vec{\;}$$

$$\vec{\;} = \frac{\vec{r}_{E/B}}{r_{E/B}} = \frac{75\vec{i} - 150\vec{j} + 50\vec{k}}{175}$$

$$= 0.429\vec{i} - 0.857\vec{j} + 0.289\vec{k}$$

$$\vec{F}_B = 300\vec{i} - 600\vec{j} + 200\vec{k}\ \ (\mathrm{N})$$

$$\vec{F}_C = (1000\ \mathrm{N})(\cos 45\vec{i} - \cos 45\vec{j})$$

$$= 707\vec{i} - 707\vec{j}\ \ (\mathrm{N})$$

$$\vec{F}_D = (1200\ \mathrm{N})(\cos 60\vec{i} + \cos 30\vec{j})$$

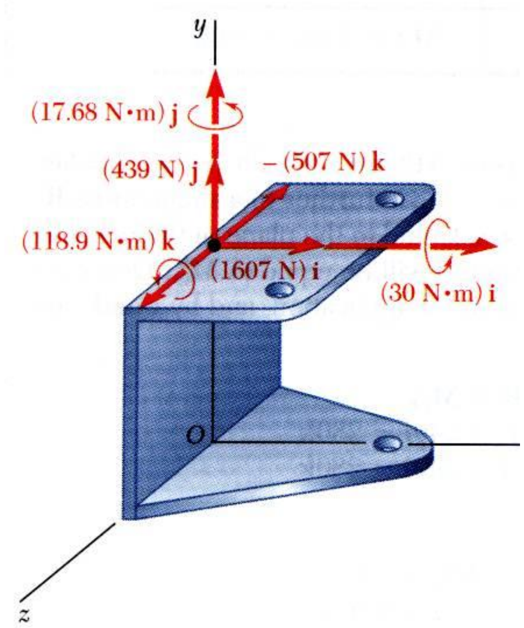
$$= 600\vec{i} + 1039\vec{j}\ \ (\mathrm{N})$$

Sample Problem 3.10

- Compute the equivalent force,

$$\vec{R} = \sum \vec{F}$$
$$= (300 + 707 + 600)\vec{i} + (-600 + 1039)\vec{j} + (200 - 707)\vec{k}$$

$$\boxed{\vec{R} = 1607\vec{i} + 439\vec{j} - 507\vec{k} \;\; (\mathrm{N})}$$



- Compute the equivalent couple,

$$\vec{M}_A^R = \sum \left(\vec{r} \times \vec{F}\right)$$

$$\vec{r}_{B/A} \times \vec{F}_B = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.075 & 0 & 0.050 \\ 300 & -600 & 200 \end{vmatrix} = 30\vec{i} - 45\vec{k}$$

$$\vec{r}_{C/A} \times \vec{F}_c = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.075 & 0 & -0.050 \\ 707 & 0 & -707 \end{vmatrix} = 17.68\vec{j}$$

$$\vec{r}_{D/A} \times \vec{F}_D = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.100 & -0.100 & 0 \\ 600 & 1039 & 0 \end{vmatrix} = 163.9\vec{k}$$

$$\boxed{\vec{M}_A^R = 30\vec{i} + 17.68\vec{j} + 118.9\vec{k}}$$