Ninth Edition

CHAPTER

2

VECTOR MECHANICS FOR ENGINEERS:

STATICS

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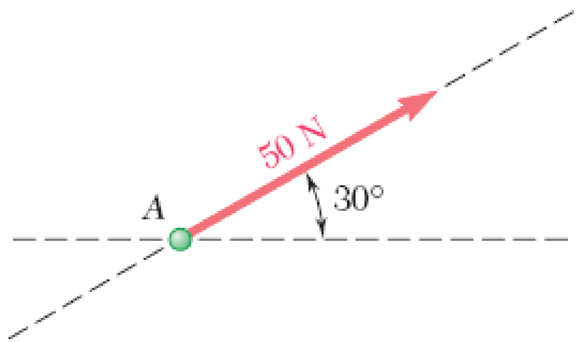
Statics of Particles

Contents

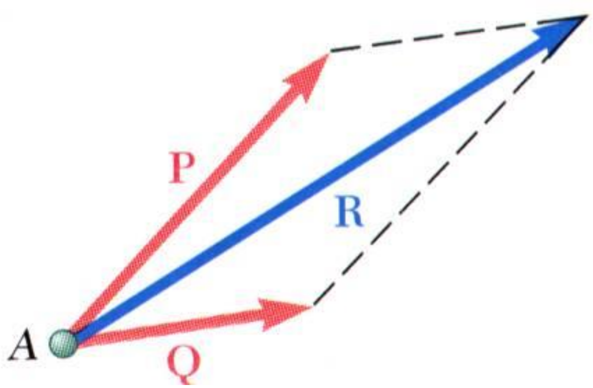
Introduction

- The objective for the current chapter is to investigate the effects of forces on particles:
  - replacing multiple forces acting on a particle with a single equivalent or *resultant* force,
  - relations between forces acting on a particle that is in a state of *equilibrium*.
- The focus on *particles* does not imply a restriction to miniscule bodies. Rather, the study is restricted to analyses in which the size and shape of the bodies is not significant so that all forces may be assumed to be applied at a single point.

Resultant of Two Forces

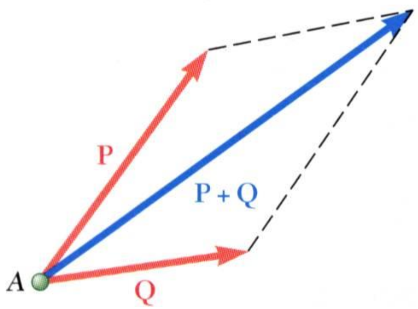


- force: action of one body on another; characterized by its *point of application*, *magnitude*, *line of action*, and *sense*.



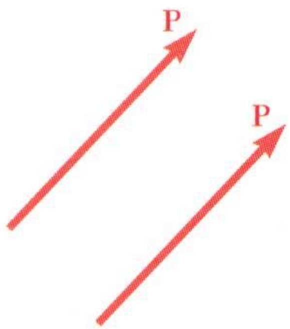
- Experimental evidence shows that the combined effect of two forces may be represented by a single *resultant* force.
- The resultant is equivalent to the diagonal of a parallelogram which contains the two forces in adjacent legs.
- Force is a *vector* quantity.

Vectors

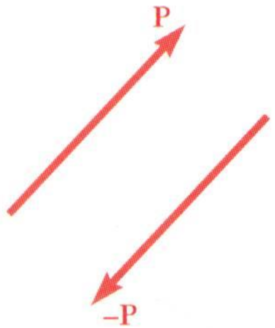


- *Vector*: parameters possessing magnitude and direction which add according to the parallelogram law. Examples: displacements, velocities, accelerations.

- *Scalar*: parameters possessing magnitude but not direction. Examples: mass, volume, temperature



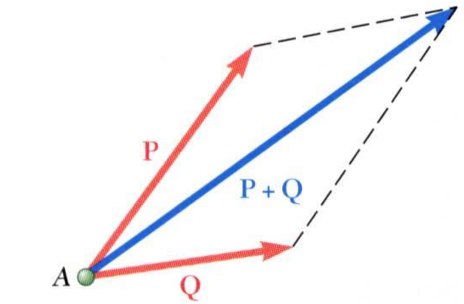
- Vector classifications:
  - *Fixed* or *bound* vectors have well defined points of application that cannot be changed without affecting an analysis.
  - *Free* vectors may be freely moved in space without changing their effect on an analysis.
  - *Sliding* vectors may be applied anywhere along their line of action without affecting an analysis.



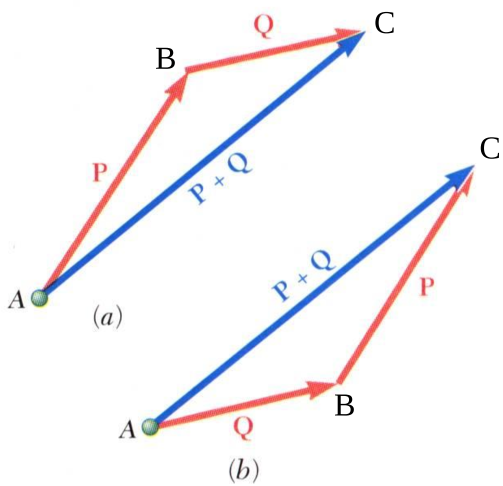
- *Equal* vectors have the same magnitude and direction.

- *Negative* vector of a given vector has the same magnitude and the opposite direction.

Addition of Vectors



- Parallelogram law for vector addition

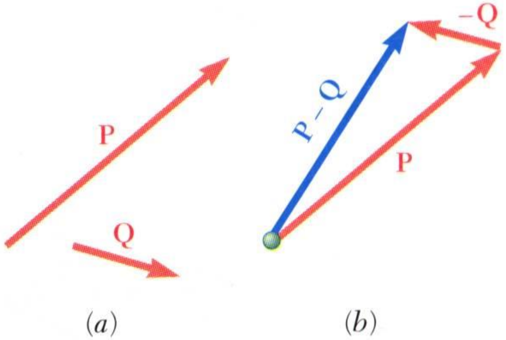


- Triangle law for vector addition
- Law of cosines,

$$R^2 = P^2 + Q^2 - 2PQ\cos B$$
$$\vec{R} = \vec{P} + \vec{Q}$$

- Law of sines

Sin A/ P = Sin B/ R = Sin C/Q

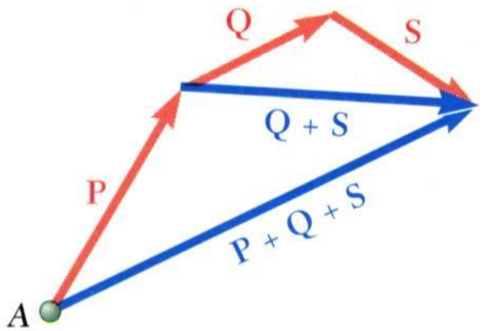


- Vector addition is commutative,

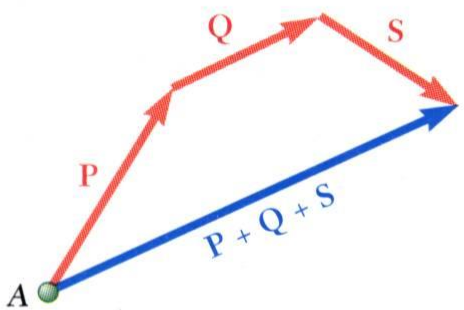
$$\vec{P} + \vec{Q} = \vec{Q} + \vec{P}$$

- Vector subtraction

Addition of Vectors

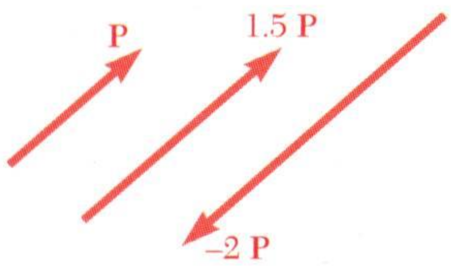


- Addition of three or more vectors through repeated application of the triangle rule



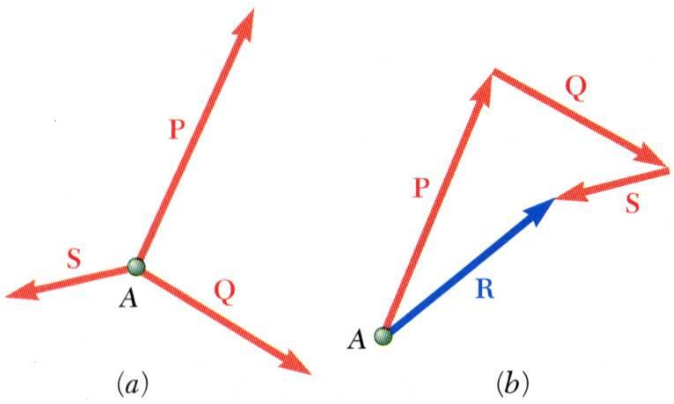
- The polygon rule for the addition of three or more vectors.
- Vector addition is associative,

$$\vec{P}+\vec{Q}+\vec{S}=\left(\vec{P}+\vec{Q}\right)+\vec{S}=\vec{P}+\left(\vec{Q}+\vec{S}\right)$$



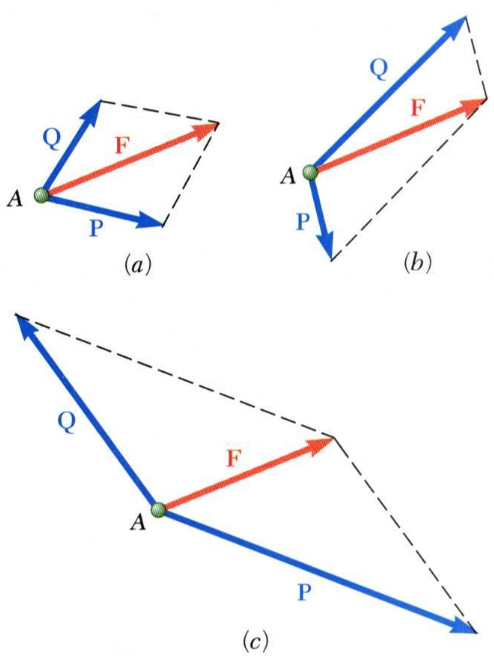
- Multiplication of a vector by a scalar

Resultant of Several Concurrent Forces



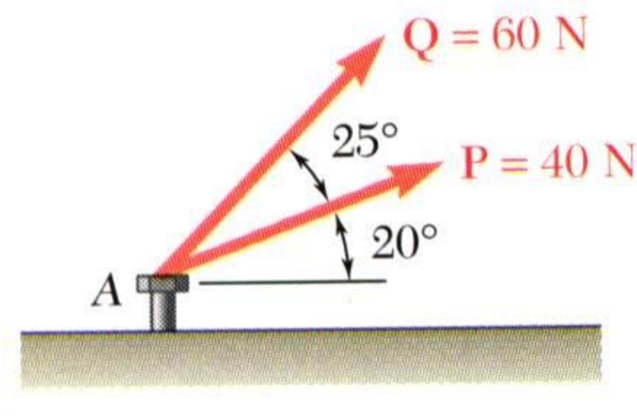
- *Concurrent forces*: set of forces which all pass through the same point.

  A set of concurrent forces applied to a particle may be replaced by a single resultant force which is the vector sum of the applied forces.



- *Vector force components*: two or more force vectors which, together, have the same effect as a single force vector.

Sample Problem 2.1

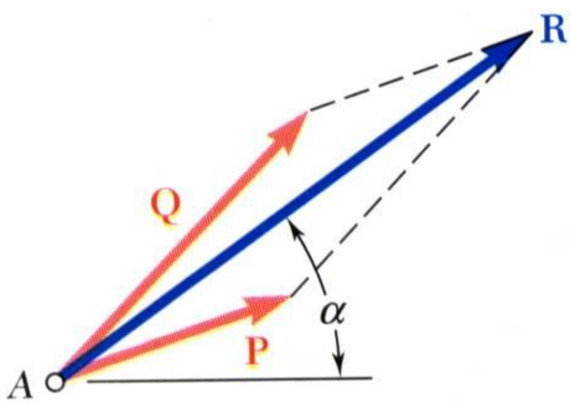


The two forces act on a bolt at *A*. Determine their resultant.

SOLUTION:

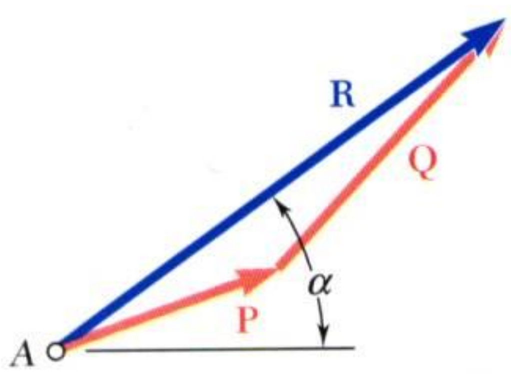
- Graphical solution - construct a parallelogram with sides in the same direction as **P** and **Q** and lengths in proportion. Graphically evaluate the resultant which is equivalent in direction and proportional in magnitude to the the diagonal.
- Trigonometric solution - use the triangle rule for vector addition in conjunction with the law of cosines and law of sines to find the resultant.

Sample Problem 2.1



- Graphical solution - A parallelogram with sides equal to **P** and **Q** is drawn to scale. The magnitude and direction of the resultant or of the diagonal to the parallelogram are measured,

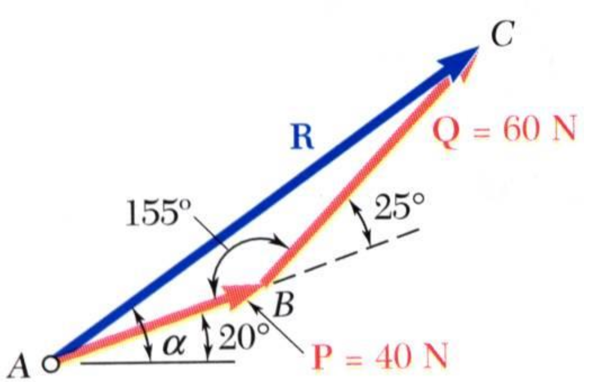
$$\boxed{\mathbf{R} = 98\ \mathrm{N} \quad \alpha = 35^\circ}$$



- Graphical solution - A triangle is drawn with **P** and **Q** head-to-tail and to scale. The magnitude and direction of the resultant or of the third side of the triangle are measured,

$$\boxed{\mathbf{R} = 98\ \mathrm{N} \quad \alpha = 35^\circ}$$

Sample Problem 2.1



- Trigonometric solution - Apply the triangle rule.

  From the Law of Cosines,

$$R^2 = P^2 + Q^2 - 2PQ\cos B$$
$$= (40\text{N})^2 + (60\text{N})^2 - 2(40\text{N})(60\text{N})\cos 155°$$

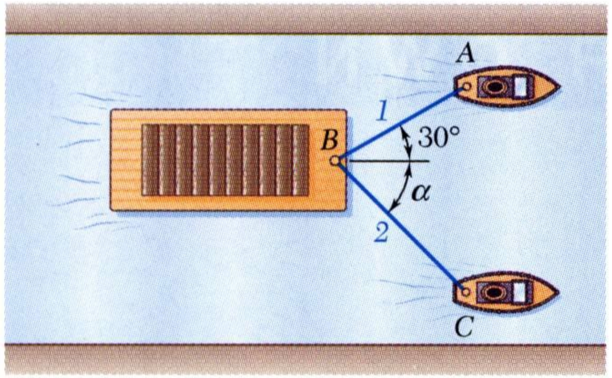
$$\boxed{R = 97.73\text{N}}$$

From the Law of Sines,

$$\frac{\sin A}{Q} = \frac{\sin B}{R}$$

$$\sin A = \sin B \frac{Q}{R}$$
$$= \sin 155° \frac{60\text{N}}{97.73\text{N}}$$
$$A = 15.04°$$
$$\alpha = 20° + A$$
$$\boxed{\alpha = 35.04°}$$

Sample Problem 2.2



A barge is pulled by two tugboats. If the resultant of the forces exerted by the tugboats is a 25 kN directed along the axis of the barge, determine

a) the tension in each of the ropes for $\alpha = 45^o$,

b) the value of α for which the tension in rope 2 is minimum.

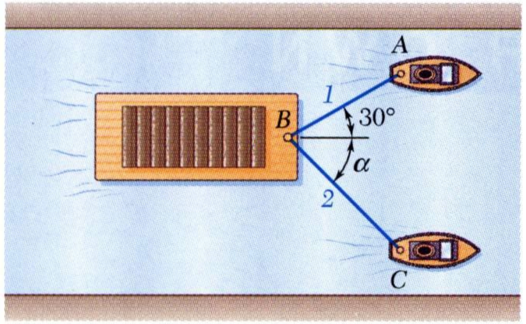
SOLUTION:

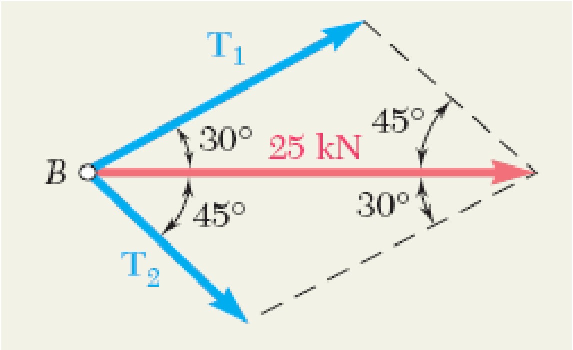
- Find a graphical solution by applying the Parallelogram Rule for vector addition. The parallelogram has sides in the directions of the two ropes and a diagonal in the direction of the barge axis and length proportional to 25 kN force.

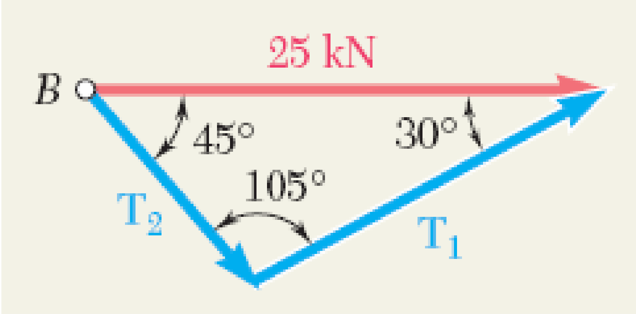
- Find a trigonometric solution by applying the Triangle Rule for vector addition. With the magnitude and direction of the resultant known and the directions of the other two sides parallel to the ropes given, apply the Law of Sines to find the rope tensions.

- The angle for minimum tension in rope 2 is determined by applying the Triangle Rule and observing the effect of variations in α.

Sample Problem 2.2







- Graphical solution - Parallelogram Rule with known resultant direction and magnitude, known directions for sides.

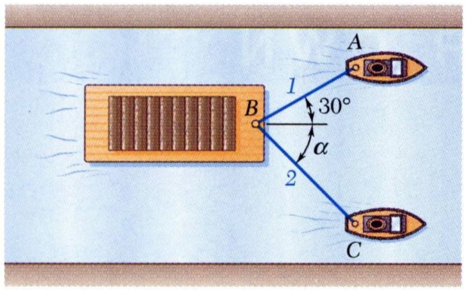
$$\boxed{T_1 = 18.5\,\text{kN} \quad T_2 = 12.94\,\text{kN}}$$

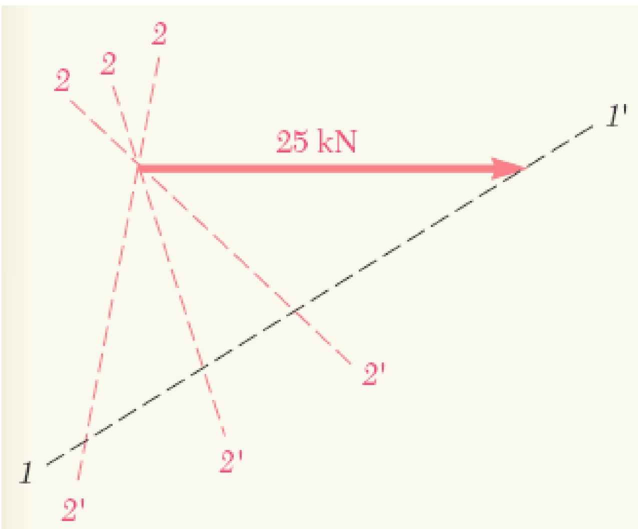
- Trigonometric solution - Triangle Rule with Law of Sines

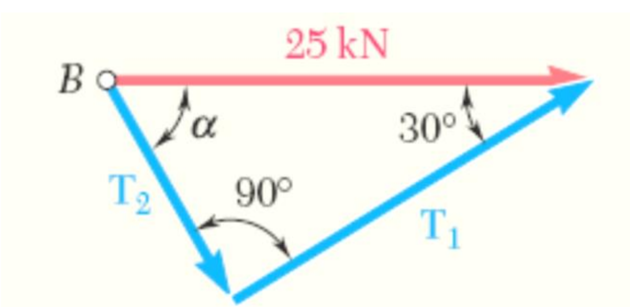
$$\frac{T_1}{\sin 45°} = \frac{T_2}{\sin 30°} = \frac{25\,\text{kN}}{\sin 105°}$$

$$\boxed{T_1 = 18.3\,\text{kN} \quad T_2 = 12.94\,\text{kN}}$$

Sample Problem 2.2







- The angle for minimum tension in rope 2 is determined by applying the Triangle Rule and observing the effect of variations in α.

- The minimum tension in rope 2 occurs when $\mathbf{T_1}$ and $\mathbf{T_2}$ are perpendicular.

$$T_2 = (25\,\text{kN})\sin 30° \qquad \boxed{T_2 = 12.5\,\text{kN}}$$

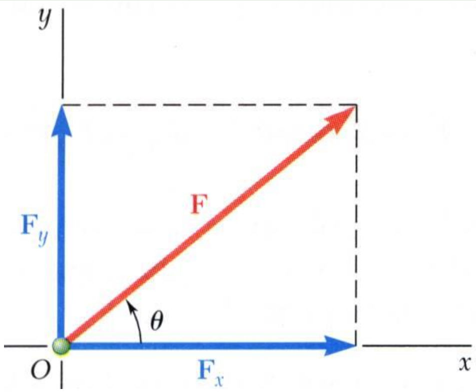
$$T_1 = (25\,\text{kN})\cos 30° \qquad \boxed{T_1 = 21.7\,\text{kN}}$$

$$\alpha = 90° - 30° \qquad \boxed{\alpha = 60°}$$

$$T_1 \cos 30° + T_2 \cos a = 25;\ T_1 \sin 30° = T_2 \sin a$$

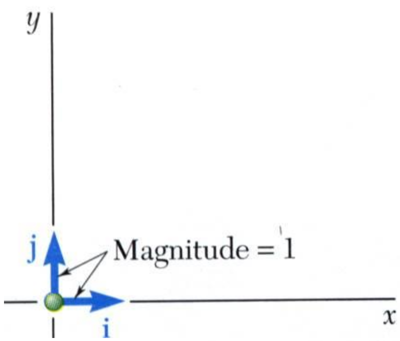
$$T_2\left[\frac{\sin a}{\tan 30°} + \cos a\right] = 25 \Rightarrow \text{Maximise } f(a)$$

Rectangular Components of a Force: Unit Vectors

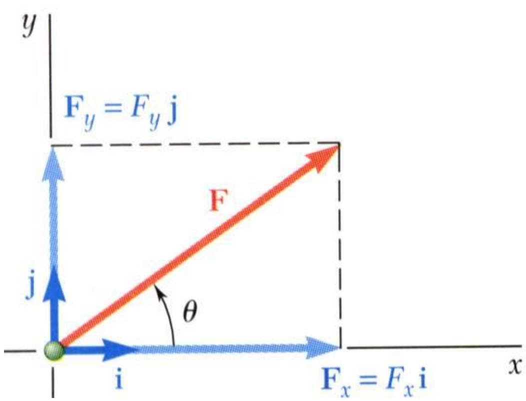


- May resolve a force vector into perpendicular components so that the resulting parallelogram is a rectangle. $\vec{F}_x$ and $\vec{F}_y$ are referred to as *rectangular vector components* and

$$\vec{F} = \vec{F}_x + \vec{F}_y$$



- Define perpendicular *unit vectors* $\vec{i}$ and $\vec{j}$ which are parallel to the *x* and *y* axes.

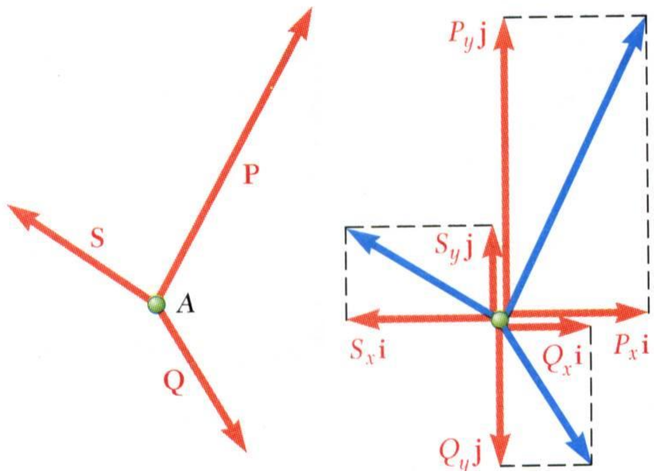


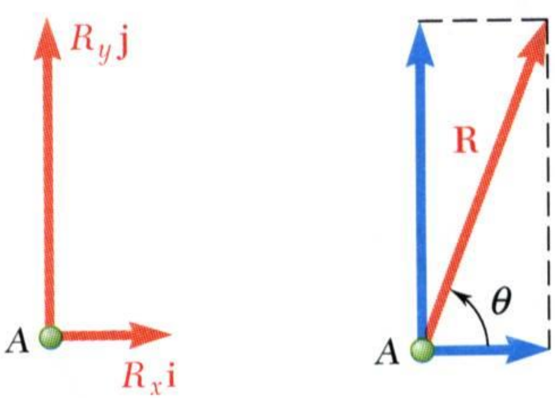
- Vector components may be expressed as products of the unit vectors with the scalar magnitudes of the vector components.

$$\vec{F} = F_x\vec{i} + F_y\vec{j}$$

F_x and F_y are referred to as the *scalar components* of $\vec{F}$

Addition of Forces by Summing Components





- Wish to find the resultant of 3 or more concurrent forces,

$$\vec{R} = \vec{P} + \vec{Q} + \vec{S}$$

- Resolve each force into rectangular components

$$\begin{aligned} R_x\vec{i} + R_y\vec{j} &= P_x\vec{i} + P_y\vec{j} + Q_x\vec{i} + Q_y\vec{j} + S_x\vec{i} + S_y\vec{j} \\ &= \left(P_x + Q_x + S_x\right)\vec{i} + \left(P_y + Q_y + S_y\right)\vec{j} \end{aligned}$$

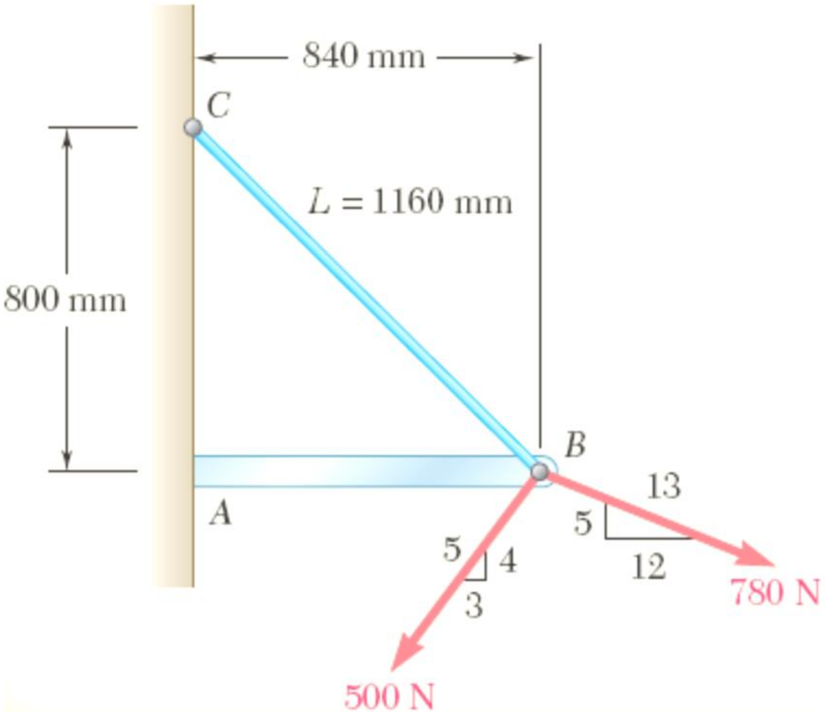
- The scalar components of the resultant are equal to the sum of the corresponding scalar components of the given forces.

$$\begin{aligned} R_x &= P_x + Q_x + S_x \\ &= \sum F_x \end{aligned} \qquad \begin{aligned} R_y &= P_y + Q_y + S_y \\ &= \sum F_y \end{aligned}$$

- To find the resultant magnitude and direction,

$$R = \sqrt{R_x^2 + R_y^2} \qquad \theta = \tan^{-1}\frac{R_y}{R_x}$$

Sample Problem 2.3

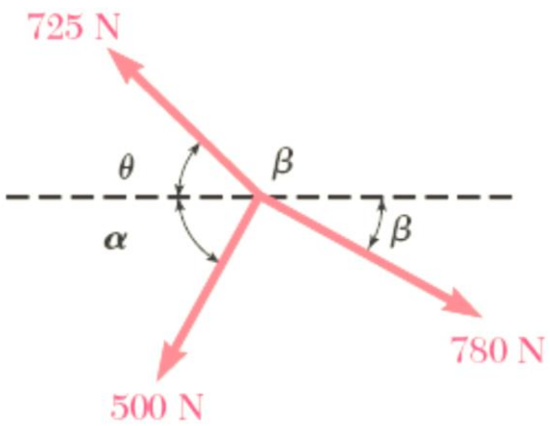


Knowing that the tension in cable *BC* is 725-N, determine the resultant of the three forces exerted at point *B* of beam *AB*.

SOLUTION:

- Resolve each force into rectangular components.
- Determine the components of the resultant by adding the corresponding force components.
- Calculate the magnitude and direction of the resultant.

Sample Problem 2.3

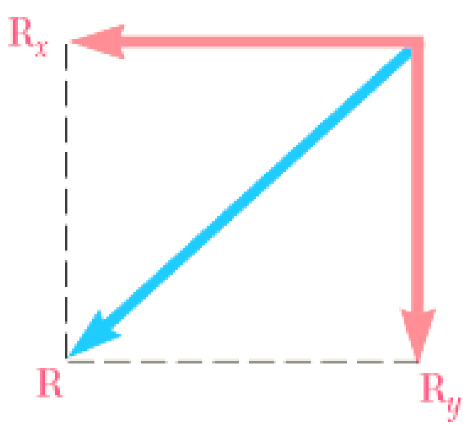


SOLUTION:

- Resolve each force into rectangular components.

Magnitude, N	x Component, N	y Component, N
725	−525	500
500	−300	− 400
780	720	− 300
	$R_x = -105$	$R_y = -200$

$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j} \qquad \mathbf{R} = (-105\ \mathrm{N})\mathbf{i} + (-200\ \mathrm{N})\mathbf{j}$$



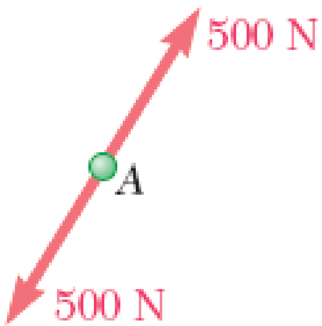
- Calculate the magnitude and direction.

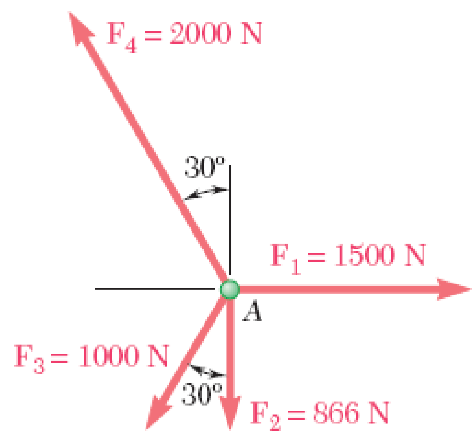
$$\tan\alpha = \frac{-R_y}{-R_x} = \frac{200\,\mathrm{N}}{105\,\mathrm{N}} \qquad \alpha = 62.3^\circ$$

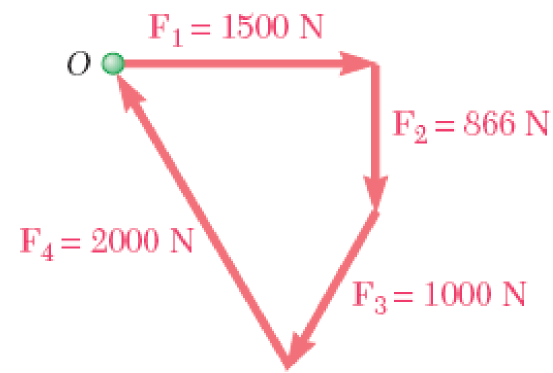
$$R = \sqrt{R_x^2 + R_y^2} = 225.9\,\mathrm{N} \qquad 62.3^\circ$$

Equilibrium of a Particle

- When the resultant of all forces acting on a particle is zero, the particle is in *equilibrium*.
- *Newton's First Law*: If the resultant force on a particle is zero, the particle will remain at rest or will continue at constant speed in a straight line.







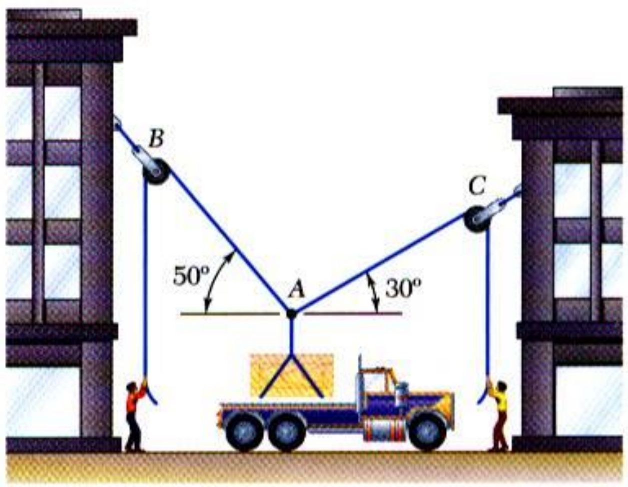
- Particle acted upon by two forces:
  - equal magnitude
  - same line of action
  - opposite sense

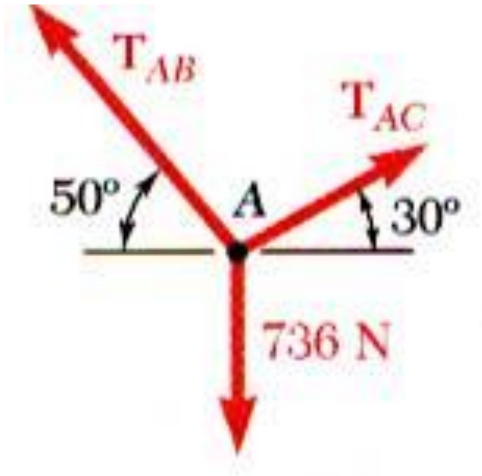
- Particle acted upon by three or more forces:
  - graphical solution yields a closed polygon
  - algebraic solution

$$\vec{R} = \sum \vec{F} = 0$$

$$\sum F_x = 0 \qquad \sum F_y = 0$$

Free-Body Diagrams

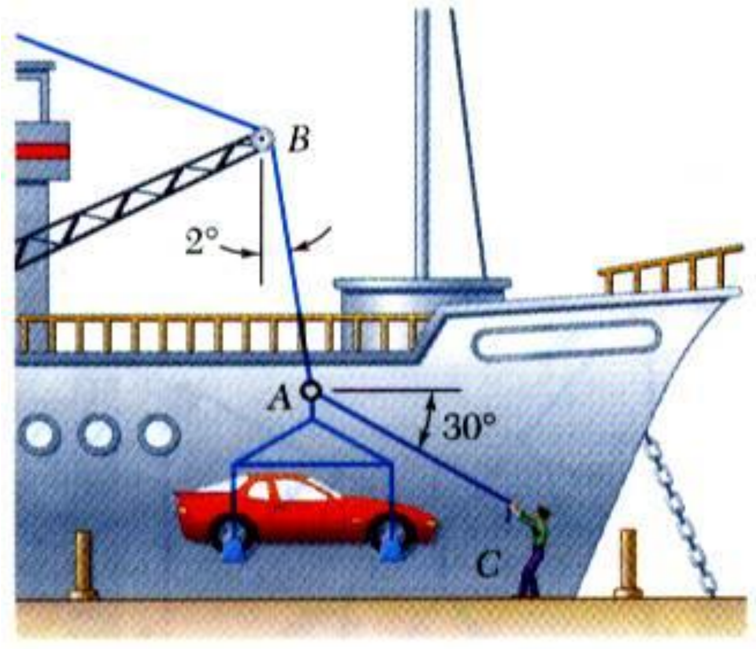




Space Diagram: A sketch showing the physical conditions of the problem.

Free-Body Diagram: A sketch showing only the forces on the selected particle.

Sample Problem 2.4

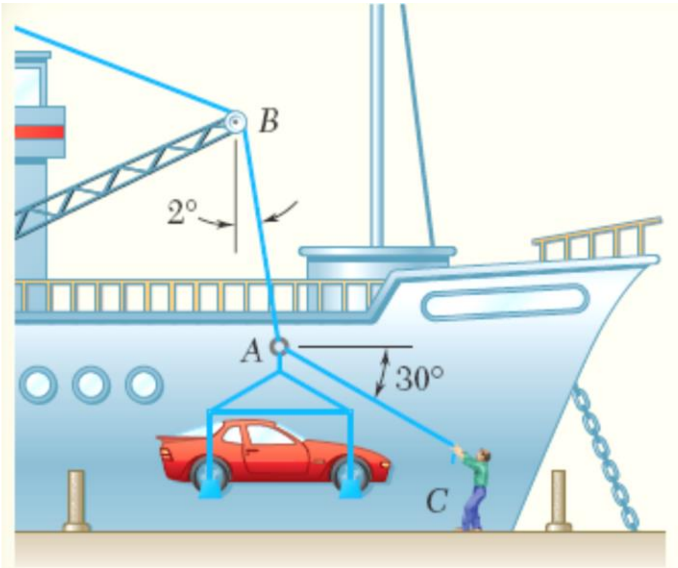


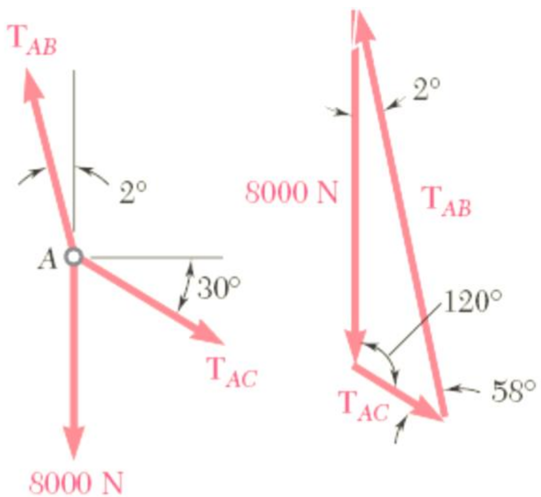
In a ship-unloading operation, a 8000-N automobile is supported by a cable. A rope is tied to the cable and pulled to center the automobile over its intended position. What is the tension in the rope?

SOLUTION:

- Construct a free-body diagram for the particle at the junction of the rope and cable.
- Apply the conditions for equilibrium by creating a closed polygon from the forces applied to the particle.
- Apply trigonometric relations to determine the unknown force magnitudes.

Sample Problem 2.4





SOLUTION:

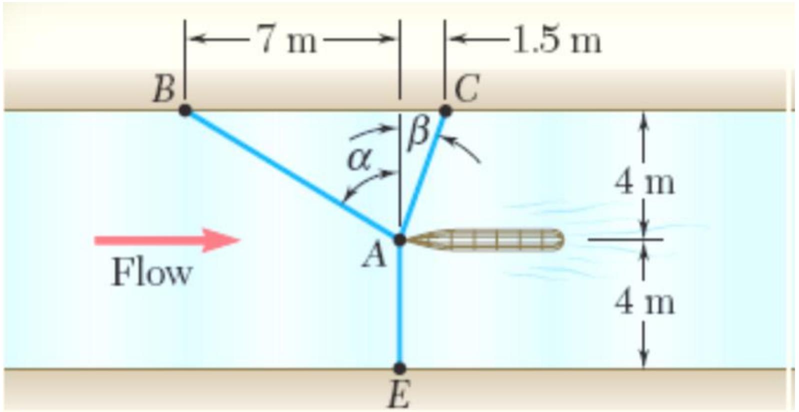
- Construct a free-body diagram for the particle at *A*.
- Apply the conditions for equilibrium.
- Solve for the unknown force magnitudes.

$$\frac{T_{AB}}{\sin 120°} = \frac{T_{AC}}{\sin 2°} = \frac{8000\text{ N}}{\sin 58°}$$

$$T_{AB} = 8170\text{ N}$$

$$\boxed{T_{AC} = 329\text{ N}}$$

Sample Problem 2.6



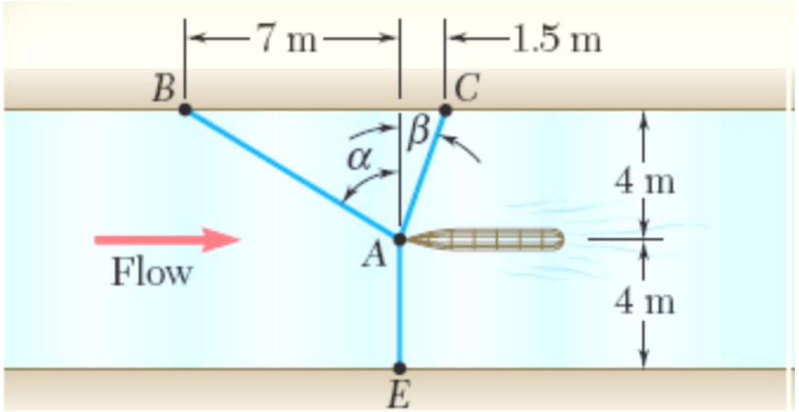
It is desired to determine the drag force at a given speed on a prototype sailboat hull. A model is placed in a test channel and three cables are used to align its bow on the channel centerline. For a given speed, the tension is 200-N in cable *AB* and 300-N in cable *AE*.

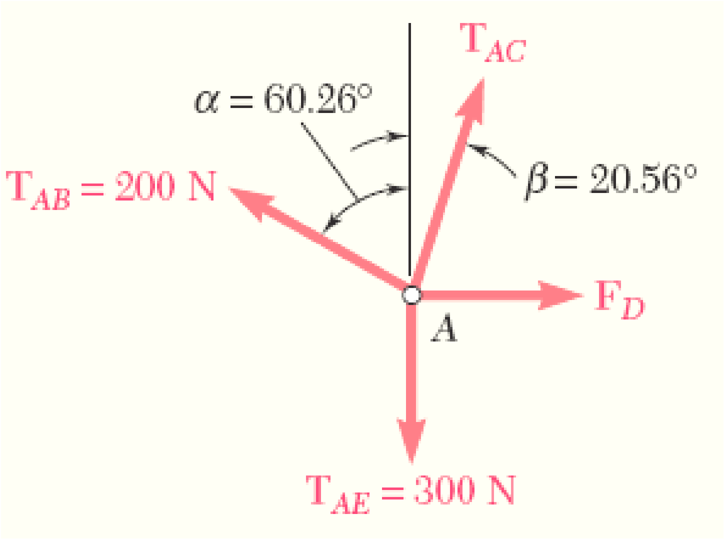
Determine the drag force exerted on the hull and the tension in cable *AC*.

SOLUTION:

- Choosing the hull as the free body, draw a free-body diagram.
- Express the condition for equilibrium for the hull by writing that the sum of all forces must be zero.
- Resolve the vector equilibrium equation into two component equations. Solve for the two unknown cable tensions.

Sample Problem 2.6





SOLUTION:

- Choosing the hull as the free body, draw a free-body diagram.

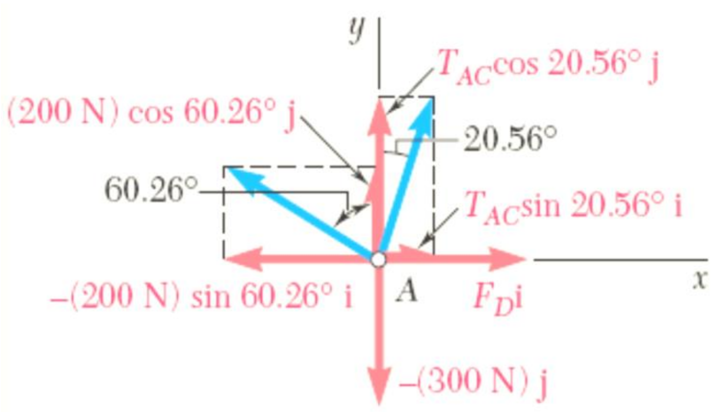
$$\tan\alpha = \frac{7\text{ m}}{4\text{ m}} = 1.75 \qquad \tan\beta = \frac{1.5\text{ m}}{4\text{ m}} = 0.375$$

$$\alpha = 60.25^\circ \qquad \beta = 20.56^\circ$$

- Express the condition for equilibrium for the hull by writing that the sum of all forces must be zero.

$$\vec{R} = \vec{T}_{AB} + \vec{T}_{AC} + \vec{T}_{AE} + \vec{F}_D = 0$$

Sample Problem 2.6



- Resolve the vector equilibrium equation into two component equations. Solve for the two unknown cable tensions.

$$\vec{T}_{AB} = -(200\,\text{N})\sin 60.26^\circ\,\vec{i} + (200\,\text{N})\cos 60.26^\circ\,\vec{j}$$
$$= -(173.66\,\text{N})\vec{i} + (99.21\ \text{N})\vec{j}$$

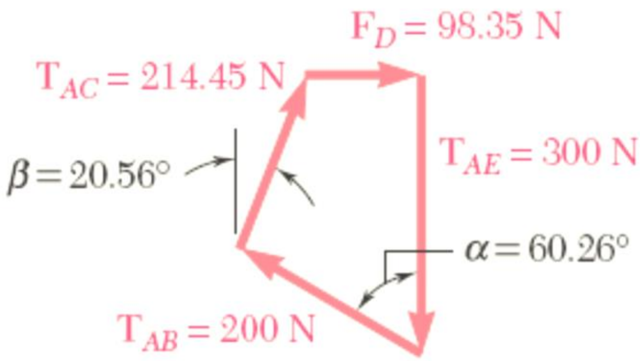
$$\vec{T}_{AC} = T_{AC}\sin 20.56^\circ\,\vec{i} + T_{AC}\cos 20.56^\circ\,\vec{j}$$
$$= 0.3512\,T_{AC}\,\vec{i} + 0.9363\,T_{AC}\,\vec{j}$$

$$\vec{T} = -(300\text{N})\vec{i}$$

$$\vec{F}_D = F_D\,\vec{i}$$

$$\vec{R} = 0$$
$$= (-173.66 + 0.3512\,T_{AC} + F_D)\vec{i}$$
$$+ (99.21 + 0.9363\,T_{AC} - 300)\,\vec{j}$$

Sample Problem 2.6



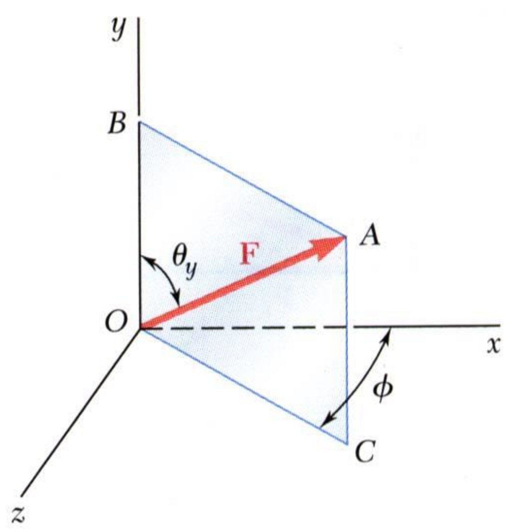
$$\vec{R} = 0$$
$$= \left(-173.66 + 0.3512 T_{AC} + F_D\right)\vec{i}$$
$$+ \left(99.21 + 0.9363 T_{AC} - 300\right)\vec{j}$$

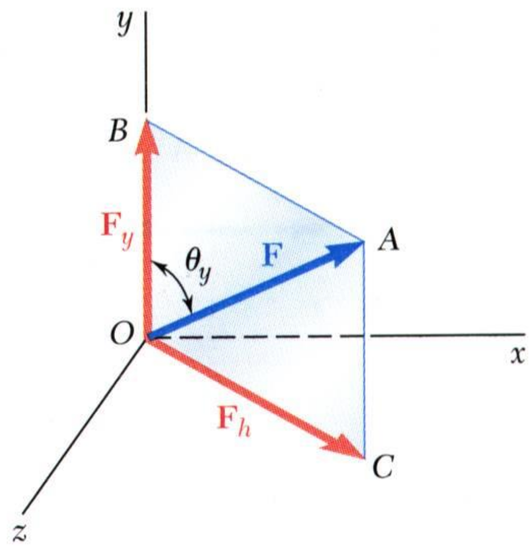
This equation is satisfied only if each component of the resultant is equal to zero

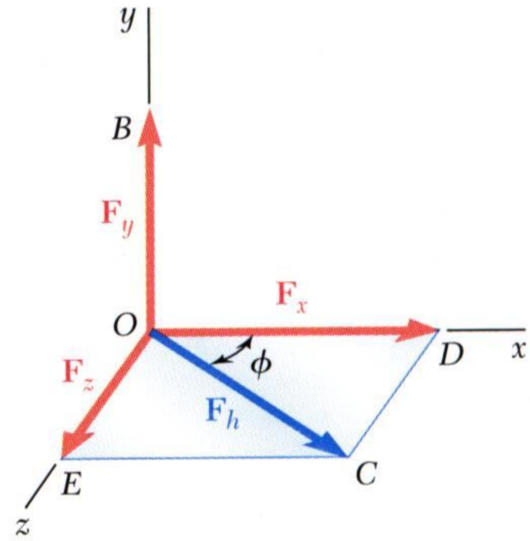
$$\left(\sum F_x = 0\right) \quad 0 = -173.66 + 0.3512 T_{AC} + F_D$$
$$\left(\sum F_y = 0\right) \quad 0 = 99.21 + 0.9363 T_{AC} - 300$$

$$\boxed{\begin{aligned} T_{AC} &= +214.45\ \text{N} \\ F_D &= +98.35\ \text{N} \end{aligned}}$$

Rectangular Components in Space







- The vector $\vec{F}$ is contained in the plane *OBAC*.

- Resolve $\vec{F}$ into horizontal and vertical components.

$$F_y = F\cos\theta_y$$

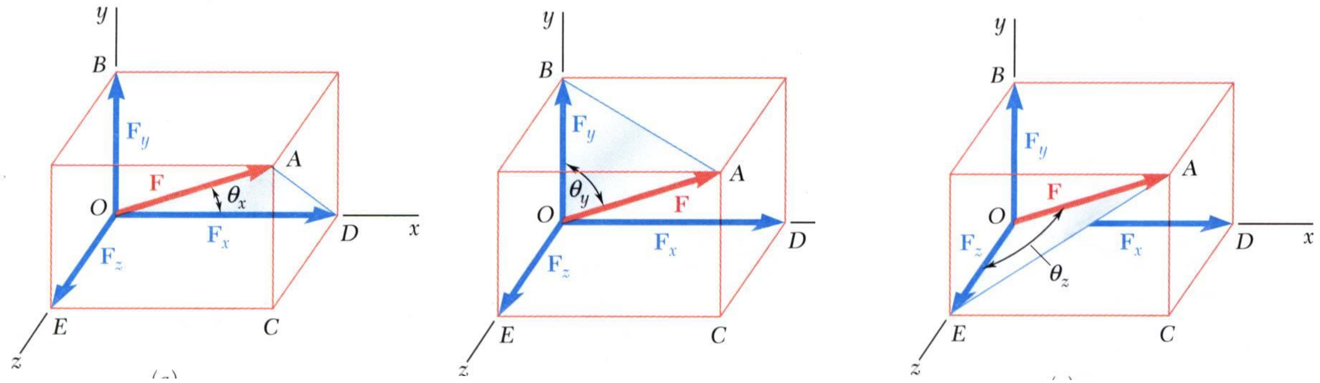
$$F_h = F\sin\theta_y$$

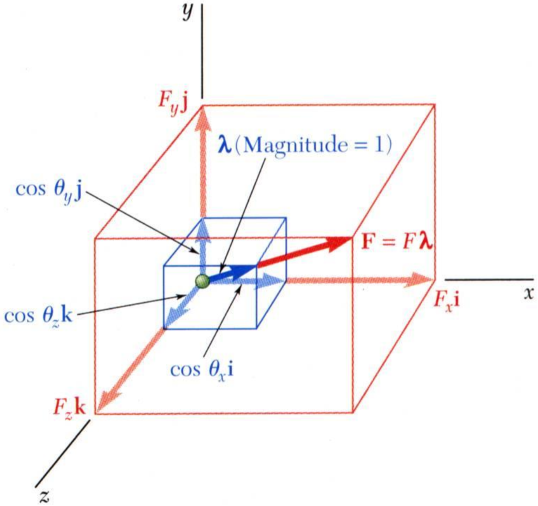
- Resolve F_h into rectangular components

$$F_x = F_h\cos\phi$$
$$= F\sin\theta_y\cos\phi$$

$$F_y = F_h\sin\phi$$
$$= F\sin\theta_y\sin\phi$$

Rectangular Components in Space





- With the angles between $\vec{F}$ and the axes,

$$F_x = F\cos\theta_x \quad F_y = F\cos\theta_y \quad F_z = F\cos\theta_z$$

$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$$

$$= F\left(\cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k}\right)$$

$$= F\vec{\lambda}$$

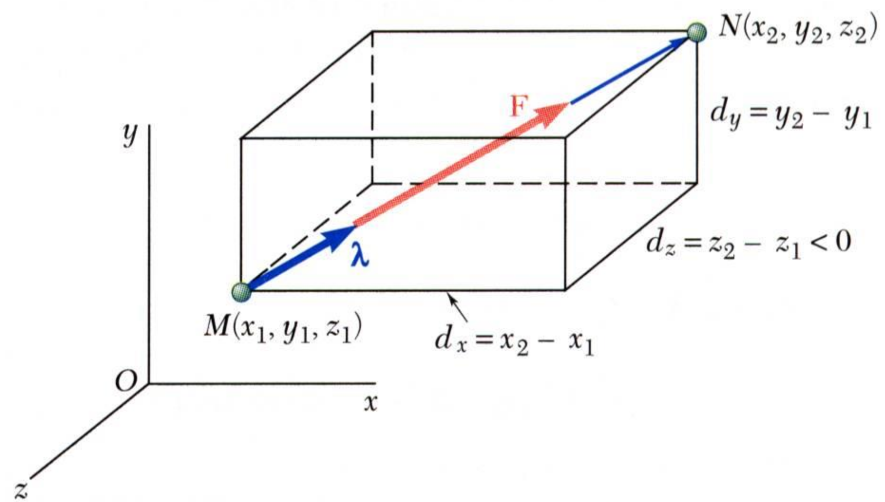
$$\vec{\lambda} = \cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k}$$

- $\vec{\lambda}$ is a unit vector along the line of action of $\vec{F}$ and $\cos\theta_x$, $\cos\theta_y$, and $\cos\theta_z$ are the direction cosines for $\vec{F}$

Rectangular Components in Space

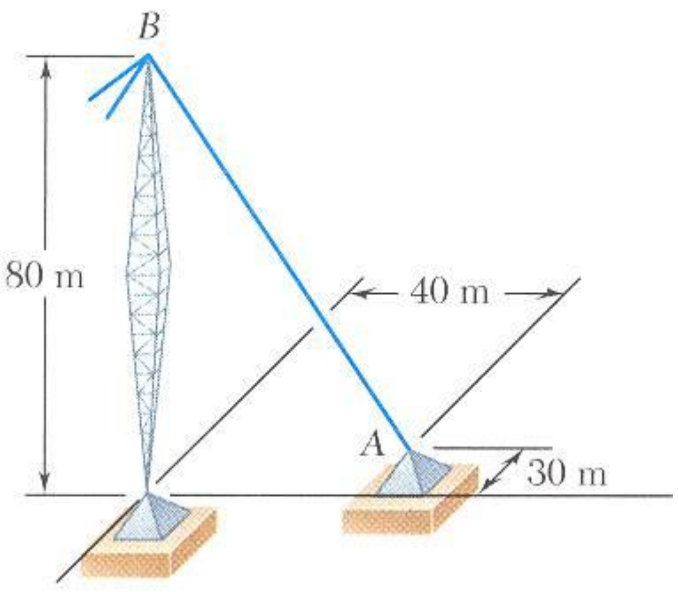
Direction of the force is defined by the location of two points,

$M(x_1, y_1, z_1)$ and $N(x_2, y_2, z_2)$



$$\vec{d} = \text{vector joining } M \text{ and } N$$
$$= d_x\vec{i} + d_y\vec{j} + d_z\vec{k}$$
$$d_x = x_2 - x_1 \quad d_y = y_2 - y_1 \quad d_z = z_2 - z_1$$
$$\vec{F} = F\vec{\lambda}$$
$$\vec{\lambda} = \frac{1}{d}\left(d_x\vec{i} + d_y\vec{j} + d_z\vec{k}\right)$$
$$F_x = \frac{Fd_x}{d} \quad F_y = \frac{Fd_y}{d} \quad F_z = \frac{Fd_z}{d}$$

Sample Problem 2.7



The tension in the guy wire is 2500 N. Determine:

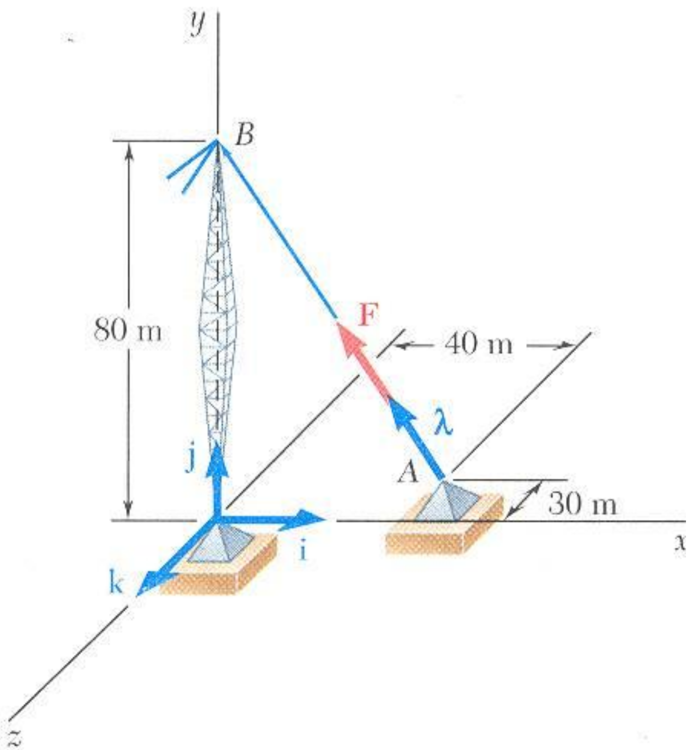
a) components F_x, F_y, F_z of the force acting on the bolt at *A*,

b) the angles θ_x, θ_y, θ_z defining the direction of the force

SOLUTION:

- Based on the relative locations of the points *A* and *B*, determine the unit vector pointing from *A* towards *B*.
- Apply the unit vector to determine the components of the force acting on *A*.
- Noting that the components of the unit vector are the direction cosines for the vector, calculate the corresponding angles.

Sample Problem 2.7



SOLUTION:

- Determine the unit vector pointing from *A* towards *B*.

$$\overrightarrow{AB} = (-40\,\mathrm{m})\vec{i} + (80\,\mathrm{m})\vec{j} + (30\,\mathrm{m})\vec{k}$$

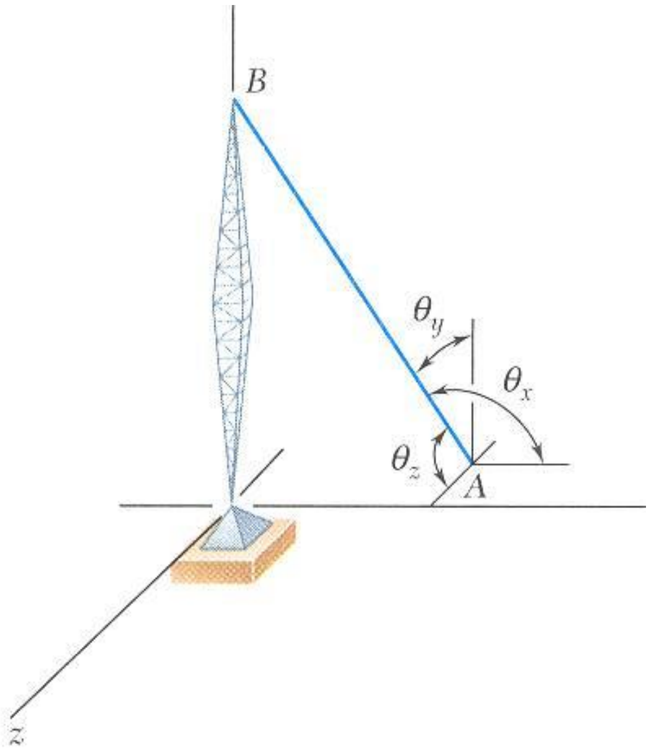
$$AB = \sqrt{(-40\,\mathrm{m})^2 + (80\,\mathrm{m})^2 + (30\,\mathrm{m})^2} = 94.3\,\mathrm{m}$$

$$\vec{\lambda} = \left(\frac{-40}{94.3}\right)\vec{i} + \left(\frac{80}{94.3}\right)\vec{j} + \left(\frac{30}{94.3}\right)\vec{k} = -0.424\vec{i} + 0.848\vec{j} + 0.318\vec{k}$$

- Determine the components of the force.

$$\vec{F} = F\vec{\lambda} = (2500\,\mathrm{N})\left(-0.424\vec{i} + 0.848\vec{j} + 0.318\vec{k}\right) = (-1060\,\mathrm{N})\vec{i} + (2120\,\mathrm{N})\vec{j} + (795\,\mathrm{N})\vec{k}$$

Sample Problem 2.7



- Noting that the components of the unit vector are the direction cosines for the vector, calculate the corresponding angles.

$$\vec{\lambda} = \cos\theta_x \vec{i} + \cos\theta_y \vec{j} + \cos\theta_z \vec{k}$$
$$= -0.424\vec{i} + 0.848\vec{j} + 0.318\vec{k}$$

$$\boxed{\begin{aligned}\theta_x &= 115.1^\circ \\ \theta_y &= 32.0^\circ \\ \theta_z &= 71.5^\circ\end{aligned}}$$