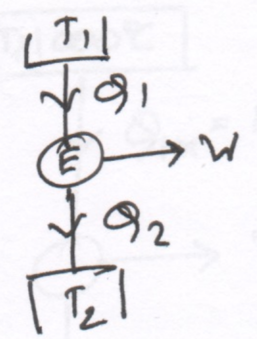
EN203 >> problem set #3 solutions

1> $Q_1 = 85$ kJ } given data
$W = 21$ kJ }



$$\eta = \frac{W}{Q_1} = \frac{21}{85} = 0{\cdot}247$$

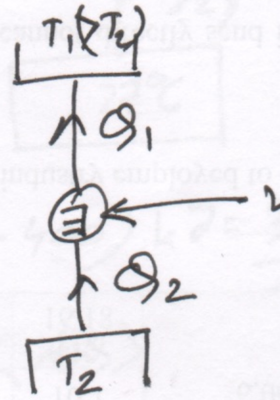
and also, $\eta = 0{\cdot}247 = 1 - \frac{Q_2}{Q_1}$

$$\therefore \frac{Q_2}{Q_1} = 1 - 0{\cdot}247 = 0{\cdot}753$$

$\therefore Q_2 = 0{\cdot}753\, Q_1 = 64$ kJ ⟨Ans⟩

$\boxed{\text{OR}}$ $Q_1 - Q_2 = W \quad \therefore Q_2 = Q_1 - W = 64$ kJ ⟨Ans⟩

2> $W = 75$ kJ } given
$Q_2 = 220$ kJ }



From 1st law of TD: $Q_1 - Q_2 = W$

$$\therefore Q_1 = W + Q_2$$

$$\therefore Q_1 = (75 + 220)\text{ kJ} = 295\text{ kJ}$$

$$COP|_{Ref} = \frac{\cancel{Q_2}\ \text{cooling effect}}{\cancel{W}\ \text{work done}}$$

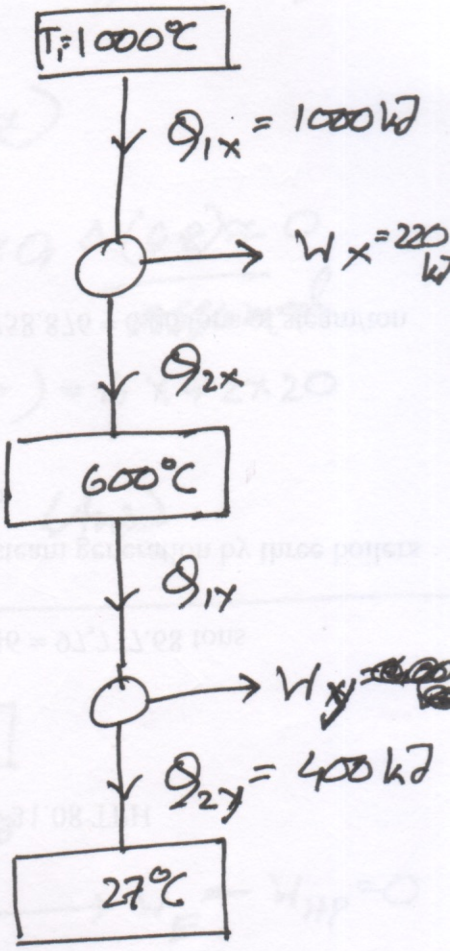
$$COP|_{Ref} = \frac{\text{cooling effect}}{\text{work input}} = \frac{220}{75} = 2{\cdot}93$$ Ans

$$COP|_{H.P.} = \frac{\text{heating effect}}{\text{work input}} = \frac{295}{75} = 3{\cdot}93$$

$\boxed{\text{OR}}$

$$COP|_{H.P.} = 1 + COP|_{Ref} = 3{\cdot}93$$

3> Applying the 1st law of TD to engine X



$$Q_{1X} - Q_{2X} = W_X$$

$$\therefore Q_{2X} = Q_{1X} - W_X = 780 \text{ kJ}$$

As the body at 600°C maintains its temperature steady, heat transfer to it must be equal to heat transfer from it. Therefore,

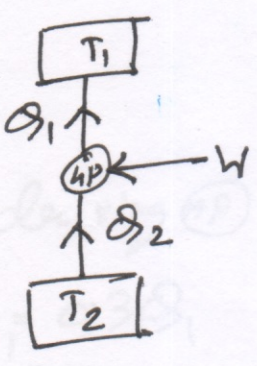
$$Q_{2X} = Q_{1Y} = 780 \text{ kJ}$$

Applying the first law to engine Y,

$$Q_{1Y} - Q_{2Y} = W_Y \quad \therefore W_Y = (780 - 400) \text{ kJ} = \underline{380 \text{ kJ}}$$
Ans

$$\eta_Y = \frac{W_Y}{Q_{1Y}} = \frac{380}{780} = 0.487 \text{ <Ans>}$$

4> $COP|_{HP} = 5$, $W = 35$ kW, $\Delta t|_{\text{radiator water}} = 20°C$
given



~~now~~ $COP|_{HP} = \dfrac{Q_1}{W} \Rightarrow Q_1 = W \cdot COP|_{HP}$

$$\therefore Q_1 = 175 \text{ kW}$$

From 1st law: $Q_1 - Q_2 = W$

$$Q_2 = Q_1 - W = 140 \text{ kW}$$

↑ heat transfer to the heat pump (h.p.)

Applying the 1st law of TD to water flowing through the radiator

$$\dot{Q} - \dot{W} = \dot{m}\,\Delta\left(h + \frac{V^2}{2} + gz\right)$$

here, $\dot{Q} = Q_1 = 175\ kW$, $W = 0$, $\Delta\left(\frac{V^2}{2}\right) \simeq 0$, $\Delta(gz) \simeq 0$ <u>Assumed</u>

$$\therefore \quad 175 - 0 = \dot{m}\,\Delta h = \dot{m}\,c_{water}\,\Delta T = \dot{m} \times 4.2 \times 20$$

$$\therefore \quad \dot{m} = \frac{175}{4.2 \times 20} = 2.083\ \frac{kg}{s} \quad \langle Ans \rangle$$

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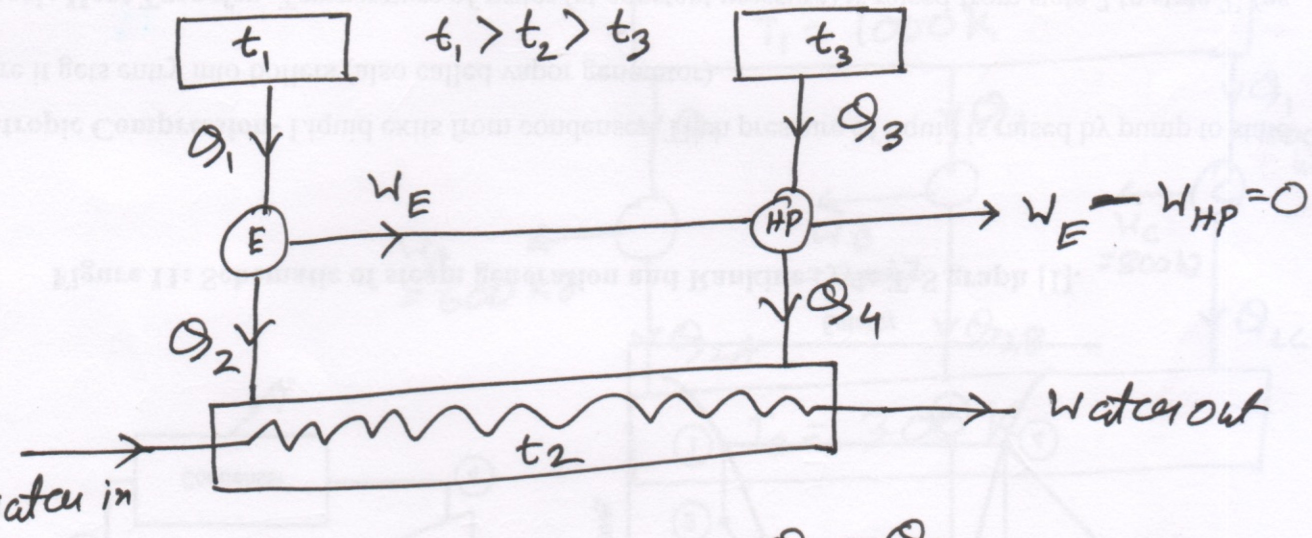
5)



Efficiency of the heat engine $= \dfrac{Q_1 - Q_2}{Q_1}$

$$= 1 - \frac{Q_2}{Q_1} = 0.3$$

$\cancel{0.3}$

$$\therefore \quad Q_2 / Q_1 = 0.7$$

Here, $W_E = W_{hp}$ ← because the (E) is driving (HP)

$$COP\big|_{hp} = 4 = \frac{Q_4}{W_{hp}} \quad \text{where } W_{hp} = W_E = \eta \times Q_1 = 0.3 Q_1$$

$$\therefore \quad \frac{Q_4}{W_{hp}} = \frac{Q_4}{0.3 Q_1} = 4 \quad \therefore \frac{Q_4}{Q_1} = 0.3 \times 4 = 1.2$$

Total heat transfer to the radiator water $= Q_4 + Q_2$

Heat transfer to the engine $= Q_1$

$\therefore$ heat transfer to radiator water for every kJ added to the heat engine $= (Q_2 + Q_4)/Q_1$

$$= \frac{Q_2}{Q_1} + \frac{Q_4}{Q_1}$$

$$= (0.7 + 1.2)\text{ kJ}$$

$$= 1.9 \text{ kJ} \quad \langle \text{Ans} \rangle$$

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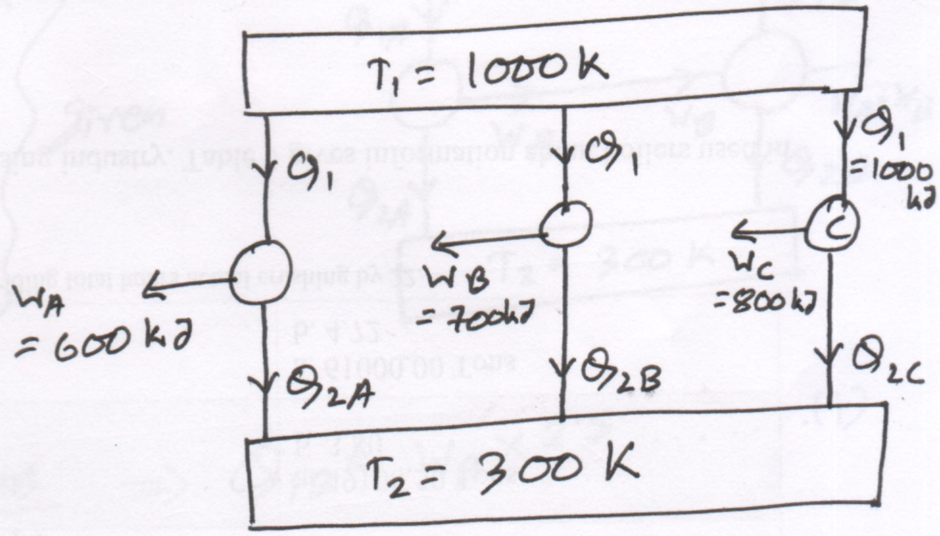
6)



Now, no engine operating between the two same temperature can not be more efficient a reversible engine.

Here, $\eta_{max} = \eta_{rev} = 1 - \frac{Q_{2rev}}{Q_1} = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{1000}$

$$= 0.7$$

<u>Engine A</u>: $Q_1 = 1000$ kJ, $W_A = 600$ kJ

$$\therefore \eta_A = W_A/Q_1 = 0.6$$

$\eta_A < \eta_{max}$ —— engine A: possible & irreversible

Engine B: $W_B = 700\ kJ \quad \therefore \eta_B = \frac{W_B}{Q_1} = 0.7$

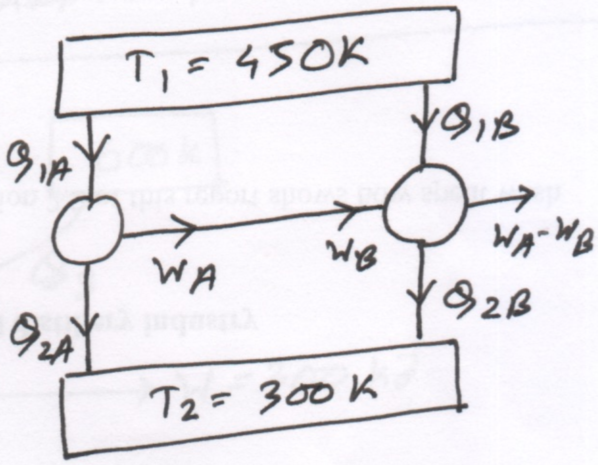
$\eta_B = \eta_{max}$ — engine B is ~~poss~~ reversible

Engine C: $W_C = 800\ kJ \quad \therefore \eta_C = \frac{W_C}{Q_1} = 0.8$

$\eta_C > \eta_{max}$ — engine C is impossible

7)

$$\left.\begin{aligned} T_1 &= 273+177 = 450K \\ T_2 &= 273+27 = 300K \\ Q_{2A} &= 200\ kJ \\ Q_{2B} &= 50\ kJ \\ COP_{HP,B} &= 2.5 \end{aligned}\right\} \text{given}$$



$$COP_{HP,B} = 2.5 = \frac{Q_{1B}}{W_B} \Rightarrow Q_{1B} = W_B \times 2.5 \quad \ldots\ldots (i)$$

where W_B is the work required by the reversed heat engine B.

Applying the 1st low of TD engine B, —

$$Q_{1B} = W_B + Q_{2B} \quad \ldots\ldots\ldots (ii)$$

Eliminating Q_{1B} between (i) and (ii), and putting the value for Q_{2B}

$$W_B \times 2.5 = W_B + 50$$

$$\therefore W_B = 33.3\ kJ$$

From data given, $W_A = W_B + W_{output} = 33.3 + 20 = 53.3$ kJ

Applying the 1st law of TD to engine A,

$$Q_{1A} = Q_{2A} + W_A = 200 + 53.3 = 253.3 \text{ kJ}$$

$$\eta_A = W_A / Q_{1A} = 53.3/253.3 = 0.21$$

$$\eta_{max} = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{450} = 0.33$$

$\therefore \eta_A < \eta_{max} \therefore$ engine A is ~~rev~~ irreversible

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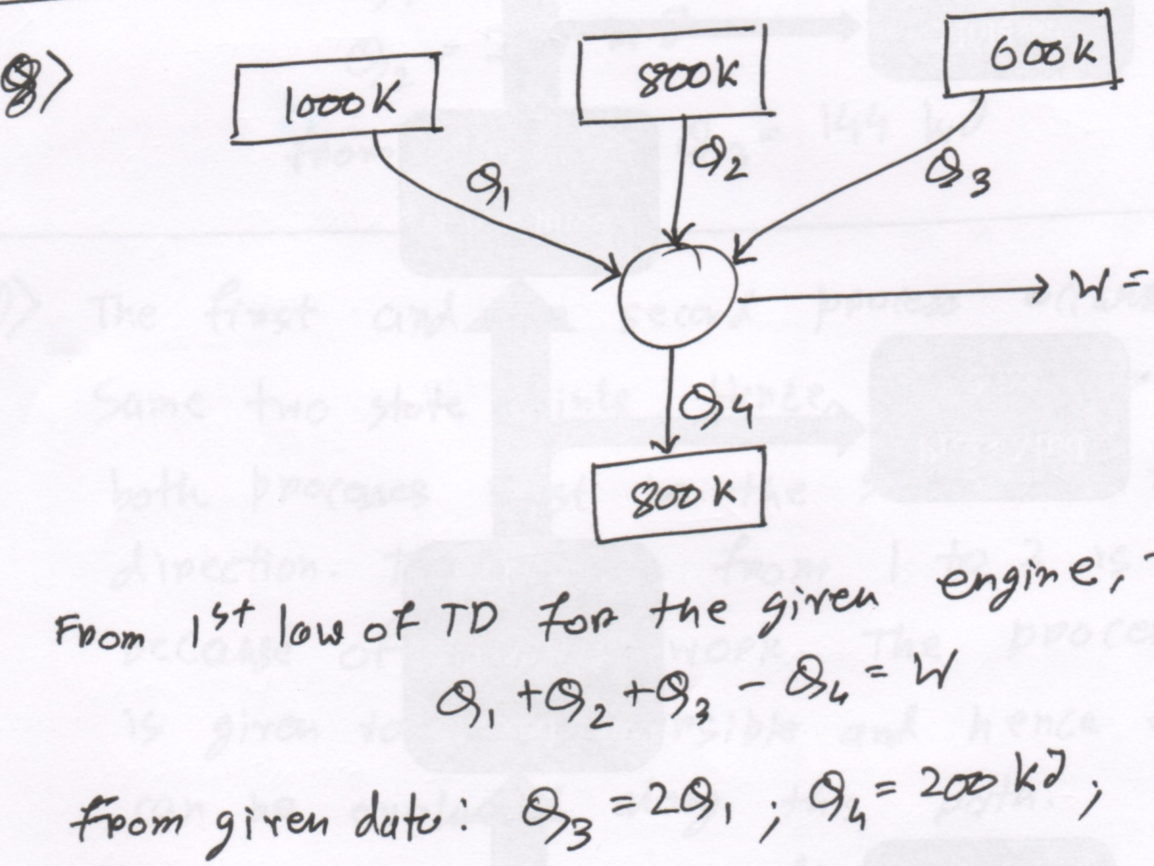
8)



From 1st law of TD for the given engine, —

$$Q_1 + Q_2 + Q_3 - Q_4 = W$$

From given data: $Q_3 = 2Q_1$; $Q_4 = 200$ kJ ; $W = 300$ kJ

$$\therefore \quad Q_1 + Q_2 + 2Q_1 - 200 = 300$$

$$\therefore \quad 3Q_1 + Q_2 = 500 \quad \ldots\ldots\ldots (i)$$

Also, as the cycle is reversible, from the Clausius inequality the cyclic integral of $\frac{\delta Q}{T} = 0$.

~~Taking~~

Taking the directions of heat transfer with respert to the engine which is undergoing a cycle,

$$\oint \frac{\delta Q}{T} = \frac{Q_1}{1000} + \frac{Q_2}{800} + \frac{Q_3}{600} - \frac{Q_4}{300} = 0$$

$$\therefore \frac{Q_1}{1000} + \frac{Q_2}{800} + \frac{2Q_1}{600} - \frac{200}{300} = 0 \quad \ldots\ldots (ii)$$

solving for Q_1 and Q_2 from (i) & (ii)

$$Q_1 = 72 \text{ kJ}$$
$$Q_2 = 284 \text{ kJ}$$

from these $\Rightarrow Q_3 = 144$ kJ

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9) The first and the second process occurs between the Same two state points. Hence, the entropy change in both processes must be the same but for their direction. The process from 1 to 2 is irreversible because of stirring work. The process from 2 to 1 is given to be reversible and hence entropy change can be evaluated along this path.

$$T\,ds = dh - v\,dp$$

$$\therefore ds = \frac{dh}{T} - \left(\frac{v}{T}\right)dp$$

For the process 2—1, p is constant and therefore

$$ds = \frac{dh}{T}$$

For the substance, C_p is a constant and $\therefore dh = C_p dT$

$\therefore ds = C_p \frac{dT}{T}$

or, $\Delta s = s_1 - s_2 = \int C_p \frac{dT}{T} = C_p \ln\left(\frac{T_1}{T_2}\right)$

Here, $T_1 = 30 + 273 = \cancel{303}\ 303\ K$

and $T_2 = 80 + 273 = 353\ K$

$\therefore s_1 - s_2 = 0.6 \ln\left(\frac{303}{353}\right) = -0.0916 \frac{kJ}{kg\ K}$

Now, entropy increase in the 1st pros. $= s_2 - s_1$

$= -(s_1 - s_2)$

$= 0.0916 \frac{kJ}{kg\ K}$

$\therefore S_2 - S_1 = m(s_2 - s_1) = 2.5 \times 0.0916 \frac{kJ}{K} = 0.229 \frac{kJ}{K}$

10> (a) For a pure substance;— $ds = \frac{dh}{T} - \left(\frac{v}{T}\right) dp$

Heating of water in a open vessel is a constant pressure process as the pressure is steady at the atmospheric value and hence in the equation: $dp = 0$

$$ds = \frac{dh}{T} = C \frac{dT}{T}$$

$$\therefore \Delta s = s_2 - s_1 = C \ln\left(\frac{T_2}{T_1}\right)$$

$$= 4.2 \ln\left(\frac{273+100}{273+30}\right)$$

$$= 0.873 \frac{kJ}{kg\ K}$$

Heat transfer, Q, to water, by the 1st law TD is given by: $Q = \Delta U + W$

Where the work $W=0$ as the process is also at constant volume (water is an incompressible substance) and ~~ΔH=0~~ $\Delta U = c\,(t_2 - t_1)$

$$\therefore Q = 4.2\,(100-30) = 294\ \frac{kJ}{kg}$$

This heat is transferred from the reservoir to water. Heat transfer from the reservoir is internally reversible as its temperature remains the same.

Hence, $\cancel{\Delta S}$

$$\Delta S_{reservoir} = \int \frac{\delta Q_{rev}}{T} = \frac{Q}{T} = \frac{-294}{(273+100)}\ \frac{kJ}{kg\,K}$$

$$= -0.788\ \frac{kJ}{kg\,K}$$

↑ of water

$$\Delta S_{universe} = \cancel{\Delta S}\ \Delta_{water} + \Delta S_{reservoir}$$

$$= (0.873 + (-0.788))\ \frac{kJ}{(kg\ water)\,K}$$

$$= 0.085\ \frac{kJ}{(kg\ water)\,K}$$

(b) water is heated first from 30°C to 65°C by heat transfer from the reservoir at 65°C and then from 65°C to 100°C by heat transfer from the ~~res~~ reservoir at 100°C. Following the same sequence of calculation as in (a) for the first stage of

$$\Delta S_{water,1} = c \ln\left(\frac{T_2}{T_1}\right) = 4.2 \ln\left(\frac{273+65}{273+30}\right)\ \frac{kJ}{kg\,K}$$

$$= 0.459\ \frac{kJ}{kg\,K}$$

$$\text{Heat transfer to water} = Q_1 = C\,(t_2 - t_1)$$
$$= 4.2\,(65-30)\ \frac{kJ}{kg}$$
$$= 147\ \frac{kJ}{kg}$$

$$\Delta S_{res,I} = \int \frac{\delta Q_{rev}}{T} = \frac{Q}{T} = \frac{-147}{273+65}\ \frac{kJ}{(kg\ water)\,K}$$
$$= -0.435\ \frac{kJ}{(kg\ water)\,K}$$

$$\text{Similarly, } \Delta S_{water,II} = C \ln\left(\frac{T_2}{T_1}\right) = 4.2 \ln \frac{273+100}{273+65}\ \frac{kJ}{kg\ water}$$
$$= 0.414\ \frac{kJ}{kg\,K}$$

$$\text{Heat transfer to water} = Q_{II} = C\,(t_2 - t_1) = 4.2\,(1000-65)$$
$$= 147\ \frac{kJ}{kg}$$

$$\Delta S_{res,II} = \int \frac{\delta Q_{res}}{T} = \frac{-147}{273+100} = -0.394\ \frac{kJ}{(kg\ water)\,K}$$

$$\Delta S_{universe} = \Delta S_{water,I} + \Delta S_{reservoir,I} + \Delta S_{water,II} + \Delta S_{reservoir,II}$$
$$= \left[0.459 + (-0.435) + 0.414 + (-0.394)\right] \frac{kJ}{(kg\ water)\,K}$$
$$= 0.044\ \frac{kJ}{(kg\ water)\,K}$$