## Problems

1. Let X be the result of rolling a fair 4-sided die (die is the singular form of dice). Let Y be the result of rolling a fair 6-sided die. Let Z be the average of X and Y.

   (a) What is the standard deviation of X and Z?

   (b) Plot the pmf and cdf of Z.

   (c) If $X > Y$, you win Rs 10X. Otherwise you lose Rs 10. What is your expected total gain if you play 10 times?

2. An athletic woman in her 20s arrives at the emergency room in a hospital complaining of dizziness after a training run. An ECG test is done and gives an abnormal (positive) result that suggests she is having a heart attack. The test has a false positive rate of 0.1 (test says heart attack, but the patient is fine) and a false negative (test says the patient is fine, but the patient is having a heart attack) rate of 0.05. In the doctor's judgment, someone like her has a 1 in 1000 chance of having a heart attack. What is the probability that this woman is actually having a heart attack?

3. A solar PV panel manufacturer has an inspector who visually checks for defective panels in the assembly line. The probability of correctly identifying a defective panel using this method is 98%. However, there is a 1/100 chance of incorrectly classifying an acceptable panel as defective. Historically, 1 in 200 of the panels on this assembly line are defective.

   (a) What is the probability that a panel is classified as defective by this process?

   (b) If a panel is classified as acceptable, what is the probability that it is indeed acceptable?

   (c) If a panel is classified as defective, what is the probability that it is indeed defective?

4. Lunchtime for Ashish and Uma is independent but uniformly distributed from 13:00 to 13:40 (for both). Both go to the same foodcourt every workday. Let A and U be their arrival time for lunch in number of minutes after 13:00.

   (a) What is the probability that Ashish arrives in the interval 13:00 - 13:15 and Uma arrives 13:10 - 13:30?

   (b) Find the expression for the joint cdf F(a,u)?

(c) Ashish and Uma want to meet for lunch but are both quite impatient and will leave if the other person doesn't arrive within 10 minutes? What is the probability that they will have lunch together?

5. An insurance company classifies people into one of three classes – good risks, average risks, and bad risks. The probabilities that good, average, and bad risk persons will be involved in an accident over a 1-year span are, respectively, 0.05, 0.15, and 0.30.

   (a) If 20% of the population are "good risks," 50% are "average risks," and 30% are "bad risks," what proportion of people have accidents in a fixed year?

   (b) If policy holder A had no accidents last year, what is the probability that he is a good risk?

   (c) What is the probability that A is a bad risk?

6. The time it takes an Apple technician to repair a Macbook Air is a uniformly distributed random variable with a span from 30 minutes to 3 hours. The cost of repair in Rupees depends on the time it takes and is given by $400 + 500\sqrt{Y}$ where $Y$ is time in hours.

   (a) What is the expected cost of repair for a Macbook Air? What is the standard deviation of this cost of repair?

   (b) What is the standard deviation for this cost above?

7. Suppose $X$ is a random variable with mean $\mu_X = 20$. You do not know anything else about the distribution of $X$.

   (a) What is the probability $P\{X \geq 40\}$?

   (b) If $\sigma_X^2 = 20$, what is the probability $P\{0 \leq X \leq 40\}$?

8. Let the random variable X denote the time until the Moodle server connects to your computer (in milliseconds), and let Y denote the time until the server authorizes you as a valid user (also in ms). Each of these random variables measures the wait from a common starting time and $X < Y$. We propose that the joint probability density function for X and Y is given by

$$f_{XY}(x, y) = 6 \times 10^{-6} e^{-0.001x - 0.002y}, \; x < y \tag{1}$$

   (a) Verify that the above pdf is valid by integrating it. What rule must it satisfy?

   (b) Calculate the probability that Y exceeds 2000 milliseconds.

   (c) Determine the probability that Y exceeds 2000, given that x = 1500.

   (d) Determine the conditional mean for Y given that x = 1500.