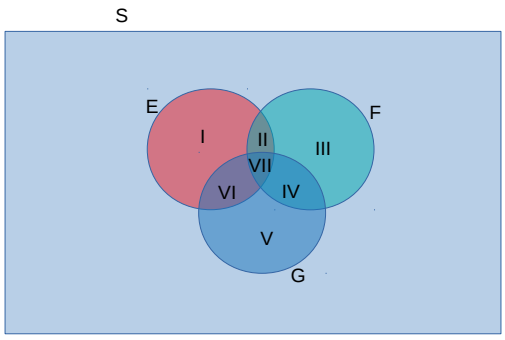
MM 217 / EN 207: Data analysis and interpretation

Assignment 5: Probability

October, 2020

1. Using Venn diagrams prove the following:
   (a) if $E \subset F$ then $F^c \subset E^c$
   (b) $F = (F \cap E) \cup (F \cap E^c)$
   (c) $E \cup F = E \cup (E^c \cap F)$
2. Using Venn diagrams show that the commutative laws, associative laws and DeMorgan's laws are valid.
3. Without using Venn diagrams show that the commutative laws, associative laws and DeMorgan's laws are valid.
4. Express the regions marked I-VII in the following Venn diagram in terms of E, F and G.



5. Show that if $E \subset F$ then $P(E) \leq P(F)$. Hint: Write F as the union of two mutually exclusive events, one of them being E.

6. Prove that $P\left(\cup_{i=1}^{n} E_i\right) \leq \sum_{i=1}^{n} P(E_i)$. This is known as Boole's inequality.

7. Prove Bonferroni's inequality, namely, $P(E \cap F) \geq P(E) + P(F) - 1$

8. Prove that

   (a) $P(E \cap F^c) = P(E) - P(E \cap F)$

   (b) $P(E^c \cap F^c) = 1 - P(E) - P(F) + P(E \cap F)$

9. Show that the probability that exactly one of the events E or F occurs is equal to $P(E) + P(F) - 2P(E \cap F)$.

10. In three envelopes, labelled randomly P, Q and R, gift coupons of different values are kept. In the first step, we open P and Q, compare the values of the gift coupons, and choose the one with smaller value. If, in the second step, we open R, what is the probability that the gift coupon value in R is smaller than the one that we chose in the first step.

11. There is a 60 percent change that the event A will occur. If A does not occur, then there is a 10 percent change that B will occur. (a) What is the probability that at least one of the events A or B occurs? (b) If A is the event that the democratic candidate wins the US presidential election this year, and, B is the event that there is a 6.2 or higher earthquake in Los Angeles sometime next year, what would you take as the probability that both A and B occur? What assumption are you making?

12. About 18% of the undergraduate students in an Institute of higher learning are females. About 12% of the students in this Institute are majoring in materials science. 7% of the students are women majoring in materials science. If a student is selected at random, find the conditional probability that

    (a) this student is female, given that the student is majoring in materials science;

(b) this student is majoring in materials science, given that the student is female.

13. We know $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Derive an expression along similar lines for $P(A \cup B \cup C)$.

14. Two fair dice are rolled and you do not know the outcome; however, if the sum of the outcomes is greater than or equal to 5, you will win a game of lottery!

    (a) What is the probability that the sum will be at least 5?

    (b) Suppose you have been told that at least one die shows 1. How likely is it now that the sum will be 5 or more?

15. A student has to sit for an examination consisting of 5 questions selected randomly from a list of 100 questions. To pass, she needs to answer all five questions. What is the probability that the student will pass the examination if she knows answers to 92 of the questions on the list?