Thermodynamics and Energy Conversion (EN 203) – Autumn 2020-21 $Problem\ Set\ -\ 5$

1. Show that:
$$\left(\frac{\partial U}{\partial p}\right)_T = -T\left(\frac{\partial V}{\partial T}\right)_p - \left(\frac{\partial V}{\partial p}\right)_T$$

2. Show that:
$$c_v = -T \left(\frac{\partial^2 A}{\partial T^2}\right)_V$$

3. Show that:
$$\frac{c_p}{c_V} = \frac{\left(\frac{\partial p}{\partial V}\right)_S}{\left(\frac{\partial p}{\partial V}\right)_T}$$

4. Prove:
$$c_p - c_V = -\frac{T(\partial V/\partial T)_p^2}{(\partial V/\partial P)_T}$$

5. Express
$$\frac{\partial u}{\partial v}\Big|_T$$
 for a gas obeying $p = \frac{RT}{V-b} - \frac{a}{V^2}$

6. Obtain the reduced form of Joule-Thomson coefficient:
$$\mu_{JT} = \frac{\partial T}{\partial p}\Big|_{H} = \frac{V(T\alpha - 1)}{c_p}$$

7. Prove:
$$\frac{d(^G/_T)}{dT} = -\frac{\Delta H}{T^2}$$
 at constant pressure

8. Determine parameters using Maxwell's relations

The adiabatic thermoelastic effect describes the change in temperature with pressure for a brittle solid when it is loaded rapidly (i.e., the rate of loading is much more rapid that the rate of heat transfer). This effect has been used to measure the stresses that develop around in composite materials using cyclic loading and a high-speed thermal imaging camera. Using Maxwell's relations, derive an expression for the adiabatic thermoelastic effect. Estimate the change in temperature for a piece of alumina that is loaded to 500 MPa.

Data:

Initial temperature 298 K

$$\alpha = 2.2 \text{ X } 10^{-5} \text{ K}^{-1}$$

$$c_p = 80 \text{ J mol}^{-1} \text{ K}^{-1}$$