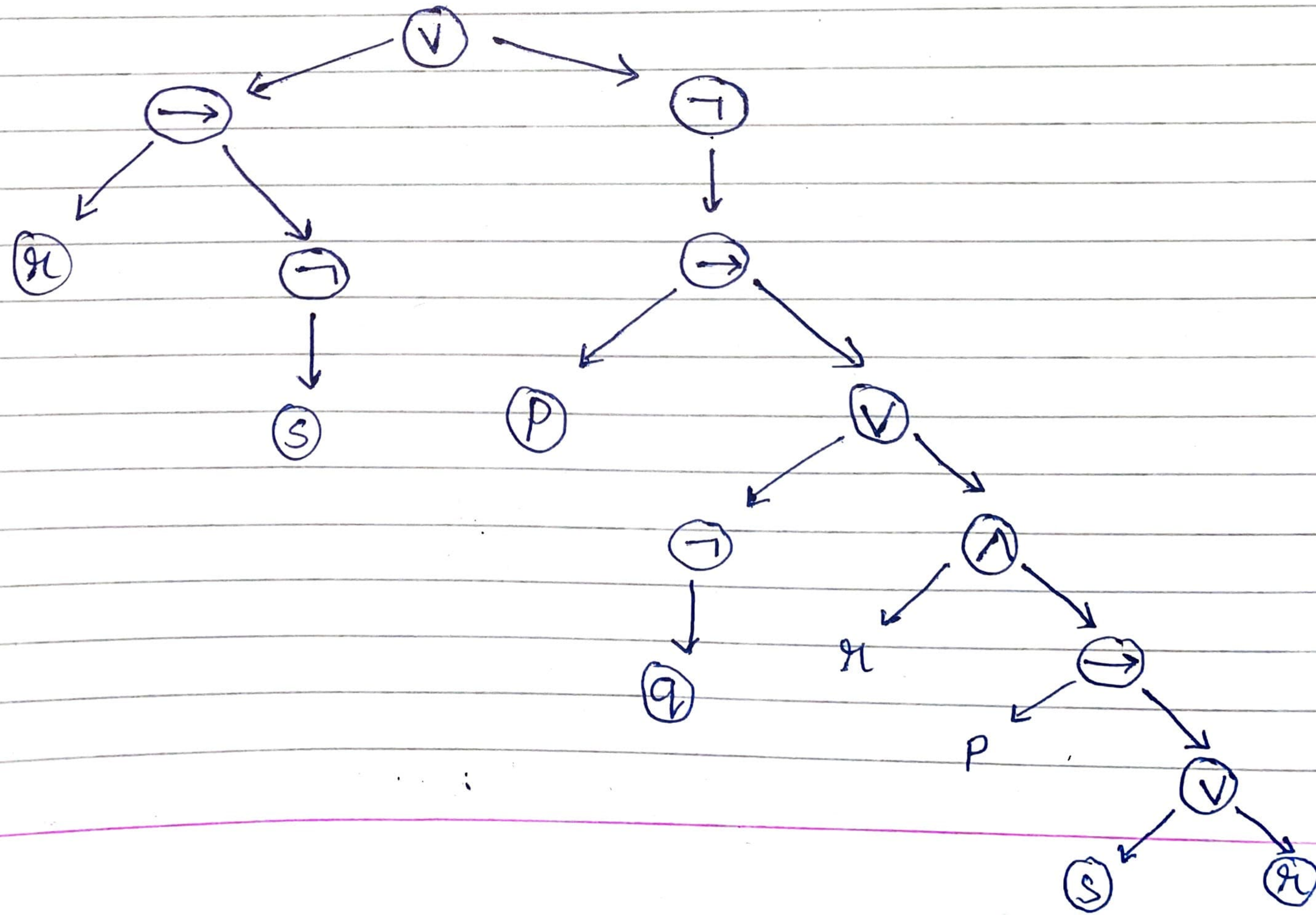
1] Let x, y, z represent the statements given by X, Y, Z respectively.
Let M_x, M_y, M_z represent X is murderer, Y is murderer and Z is murder respectively.

$$(\neg x \wedge \neg y \wedge z) \vee (x \wedge \neg y \wedge \neg z) \vee (\neg x \wedge y \wedge \neg z)$$
$$(\neg M_x \wedge \neg M_y \wedge M_z) \vee (M_x \wedge \neg M_y \wedge \neg M_z) \vee (\neg M_x \wedge M_y \wedge \neg M_z)$$

Also,
$$x \leftrightarrow M_y$$
$$y \leftrightarrow M_z$$
$$z \leftrightarrow M_z$$

if Z is the murderer, ~~x and~~ y and z will both be true which is not possible.
if X is the murderer, none of the statements is true. which also not possible.
Thus, Y is the murderer.

2] 

3] $(\neg r \vee \neg s) \vee \neg(\neg p \vee (\neg q \vee (r \wedge (\neg p \vee s \vee r))))$

$\phi = (\neg r \vee \neg s) \vee (p \wedge (q \wedge (\neg r \vee (p \wedge \neg s \wedge \neg r))))$

$(\neg r \vee \neg s) \vee (p \wedge (q \cancel{\wedge p \wedge \neg s \wedge \neg r}))$

$\cancel{(\neg r \vee \neg s) \vee ((p \wedge q \wedge \neg r) \vee (p \wedge q}$

$\cancel{= (\neg r \vee \neg s) \vee (p \wedge q \wedge \neg r) \vee (p \wedge q \wedge \neg s \wedge \neg r)}$

$= \neg r \vee \neg s \vee (p \wedge ((q \wedge \neg r) \vee (q \wedge p \wedge \neg s \wedge \neg r)))$

$\cancel{(\neg r \vee \neg s) \vee (p \wedge q \wedge \neg r) \vee (q \wedge p \wedge \neg s \wedge \neg r)}$

$= (\neg r \vee \neg s) \vee (p \wedge ((q \wedge \neg r) \vee (q \wedge p \wedge \neg s \wedge \neg r)))$

$= \overbrace{(\neg r \vee \neg s \vee p)}^{\varphi_1} \wedge \overbrace{((\neg r \vee \neg s) \vee ((q \wedge \neg r) \vee (q \wedge p \wedge \neg s \wedge \neg r)))}^{\varphi_2}$

$((q \wedge \neg r) \vee q) \wedge ((q \wedge \neg r) \vee p) \wedge ((q \wedge \neg r) \vee \neg s) \wedge ((q \wedge \neg r) \vee \neg r)$

$((q \vee q) \wedge (\neg r \vee q)) \wedge ((q \vee p) \wedge (\neg r \vee p)) \wedge ((q \vee \neg s) \wedge (\neg r \vee \neg s)) \wedge ((q \vee \neg r) \wedge (\neg r \vee \neg r))$

$(q \wedge (\neg r \vee q)) \wedge (q \vee p) \wedge (\neg r \vee p) \wedge (q \vee \neg s) \wedge (\neg r \vee \neg s) \wedge (q \vee \neg r) \wedge \neg r$

$q \wedge (q \vee p) \wedge (\neg r \vee p) \wedge (q \vee \neg s) \wedge (\neg r \vee \neg s) \wedge \neg r$

~~So, the whole expression becomes.~~

$\cancel{(\neg r \vee \neg s \vee p) \wedge (q) \wedge (\neg r) \wedge (q \vee p) \wedge (\neg r \vee p) \wedge (q \vee \neg s) \wedge (\neg r \vee \neg s)}$

consider φ_2 now,

$\rightarrow ((\neg r \vee \neg s) \vee (q \wedge (q \vee p) \wedge (\neg r \vee p) \wedge (q \vee \neg s) \wedge (\neg r \vee \neg s) \wedge \neg r))$

$= (\neg r \vee \neg s \vee q) \wedge (\neg r \vee \neg s \vee q \vee p) \wedge (\neg r \vee p \vee \neg s) \wedge (q \vee \neg r \vee \neg s) \wedge (\neg r \vee \neg s)$

$= (\neg r \vee \neg s \vee q) \wedge (\neg r \vee \neg s \vee q \vee p) \wedge (\neg r \vee p \vee \neg s) \wedge (\neg r \vee \neg s)$

consider φ_1 now and φ_2

$= (\neg r \vee \neg s \vee p) \wedge (\neg r \vee \neg s \vee q) \wedge (\neg r \vee \neg s \vee q \vee p) \wedge (\neg r \vee \neg s)$

since $(\neg r \vee \neg s \vee q \vee p) = (\neg r \vee \neg s \vee p) \wedge (\neg r \vee \neg s \vee q)$

$(\neg r \lor \neg s \lor p) \land (\neg r \lor \neg s \lor q) \land (\neg r \lor \neg s)$

~~ϕ the~~ Now, ϕ is in CNF.

Since each ~~cl~~ clause has only one positive literal at most, ϕ is in CNF.

4] $\phi = (\neg r \lor \neg s \lor p) \land (\neg r \lor \neg s \lor q) \land (\neg r \lor \neg s)$

~~$res^{0}(\phi) = \phi$~~

~~$Res^{1}(\phi) = \phi \cup \{$~~

$F = \{\{\neg r, \neg s, p\}, \{\neg r, \neg s, q\}, \{\neg r, \neg s\}\}$

$res^{0}(F) = F.$

$res^{1}(F) = F$

$res^{0}(F) = res^{1}(F) = res^{*}(F)$

and since $\emptyset \notin res^{*} F$, ϕ is satisfiable.

5] $\phi = (p \to (q \to p))$

(a) $p, \neg q \vdash \phi$

1]	p	premise
2]	$\neg q$	premise
3]	p	Assumption
4]	q	Assumption
5]	$\neg q$	copy 2
6]	$\perp$	$\perp$ i 4,5
7]	p	$\perp$ e 6
8]	$q \to p$	$\to$ i 4–7
9]	$p \to (q \to p)$	$\to$ i 3–8

(b) $\neg p, \neg q \vdash \phi$

1	$\neg p$	premise
2	$\neg q$	premise
3	p	Assumption
4	$\neg p$	copy 1
5	$\bot$	$\bot i$ 3,4
6	$(q \to p)$	$\bot e$ 5
7	$p \to (q \to p)$	$\to i$ 3-6

(c)

$\neg p, q \vdash \phi$

1	$\neg p$	premise
2	q	premise
3	p	Assumption
4	$\neg p$	Copy 1
5	$\bot$	$\bot i$ 3,4
6	$(q \to p)$	$\bot e$ 5
7	$p \to (q \to p)$	$\to i$ 3-6

(d) $p, q \vdash \phi$

1	p	premise
2	q	premise
3	~~Assume~~ p	Assumption
4	q	Assumption
5	p	Copy 1
6	$(q \to p)$	$\to i$ 4-5
7	$p \to (q \to p)$	$\to i$ 3-6

To combine all 4 proofs to obtain a proof of
$\vdash \phi$

1	$p \vee \neg p$	LEM
2	p	Assumption
3	$q \vee \neg q$	LEM
4	q	Assumption.
5	ϕ	using (d)
6	$\neg q$	Assumption
7	ϕ	using (a)
8	ϕ	$\vee e$ 3, 4−5, 6−7
9	$\neg p$	Assumption
10	$q \vee \neg q$	LEM
11	q	Assumption
12	ϕ	using (c)
13	$\neg q$	Assumption
14	ϕ	using (b)
15	ϕ	$\vee e$ 10, 11−12, 13−14
16	ϕ	$\vee e$ 1, 2−8, 9−15

6] To check if $\varphi_1 \models \varphi_2$ and/or $\varphi_2 \models \varphi_1$

For φ_1,

p	q	r	φ_1
T	T	F	F
T	F	T	F
T	T	T	T
T	F	F	T
F	T	T	F
F	F	F	F
F	T	F	T
F	F	T	T

p	q	r	φ_2
T	T	F	F
T	F	T	F
T	T	T	T
T	F	F	F
F	T	T	F
F	F	F	T
F	T	F	F
F	F	T	F

So, it can be observed that neither $\varphi_1 \models \varphi_2$ nor $\varphi_2 \models \varphi_1$

7] (a) $\varphi = \neg a \wedge (\neg b \vee \neg c \vee a) \wedge b \wedge c$

$= (a \to \bot) \wedge (b \wedge c \to a) \wedge (b \to T) \wedge (c \to T)$

$\varphi' = a' \wedge (b' \vee c' \vee a) \wedge b \wedge c$

$(\neg a' \to \bot) \wedge (b' \wedge c' \to \neg a) \wedge (b \to T) \wedge (c \to T)$

(b) I disagree with logician because,
consider φ in (a)
in the first step to check satisfiability by HORNSAT, we mark b and c.
in the next step we mark a ... ($\because b \wedge c \to a$)
and in the last step we see that $a \to \bot$
thus φ is unsatisfiable.
now, consider φ' in (a)
In the first step we mark b and c.
But, after the first step, nothing else can be done and so $\neg a'$ was not marked
and so, φ' becomes satisfiable by hornSAT criteria. which is a clear contradiction.

Thus, the logician's approach is wrong.

7] (a) $\varphi = (T \to \neg a) \wedge (b \wedge c \to a) \wedge (T \to b) \wedge (T \to c)$

$\varphi' = (T \to a') \wedge (b \wedge c \to a) \wedge (T \to b) \wedge (T \to c)$

(b) I do not agree with the logician.
this is because, the φ considered in (a) can be written as $(a \to \bot) \wedge (b \wedge c \to a) \wedge (T \to b) \wedge (T \to c)$

Now, we can mark b and c because $T \to b$ and $T \to c$

Next, we can mark a beause $b \land c \to a$

Next we have $a \to \perp$ this means φ is not satisfiable.

Now, consider φ' from (a) part.

$\varphi' = (T \to a') \land (T \to b) \land (T \to c) \land (b \land c \to a)$

Now applying Horn SAT on φ'

we first mark b, c and a'

then in the ~~x~~ next step, we mark a and thats it. So, ~~this~~ applying hornSAT of φ' proves it to be satisfiable. which we know is not true because φ is not satisfiable. Thus, the logician is wrong.