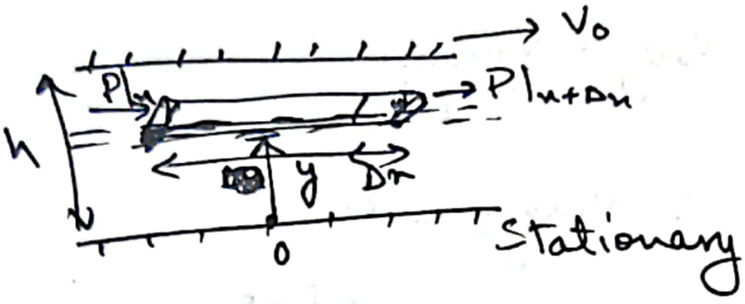
Tut 4

19D170027

Q1]



By conservation of momentum,
Assuming fully developed flow,

$$\Sigma F = 0$$

$$\Rightarrow P\big|_{x+\Delta x} w \Delta y - P\big|_{x} w \Delta y = \tau_{yx}(w\Delta x)\big|_{y+\Delta y} - \tau_{yx} w \Delta x\big|_{y}$$

$$\therefore \left(\frac{P|_{x+\Delta x} - P|_{x}}{\Delta x}\right) = \frac{\tau_{yx}|_{y+\Delta y} - \tau_{yx}|_{y}}{\Delta y}$$

$$\therefore \frac{d(\tau_{yx})}{dy} = \frac{dP}{dx} \longrightarrow \text{constant}$$

$$\Rightarrow \tau_{yx} = \left(\frac{dP}{dx}\right) y + c$$

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For laminar flow
and newtonian fluid,

$$\tau_{yx} = \mu \frac{dV_x}{dy}$$

$$\Rightarrow \left(\frac{dP}{dx}\right) y + c = \mu \frac{dV_x}{dy}$$

Now, For $\tau_{yx} = 0$ at $y = 0$,

$$c = 0$$

$$\therefore \int \left(\frac{dP}{dx}\right) y = \int \mu \frac{dV_x}{dy}$$

$$\therefore \left(\frac{dP}{dx}\right) \frac{y^2}{2} + c_1 = \mu V_x$$

V_x at $y = h = V_0$

$$\therefore c_1 = \mu V_0 - \left(\frac{dP}{dx}\right) \frac{h^2}{2}$$

and V_x at $y = 0 = 0$

$$\therefore 0 + c_1 = 0$$

$$\Rightarrow c_1 = 0$$

$$\therefore \boxed{\frac{dP}{dx} = \frac{2\mu V_0}{h^2}}$$

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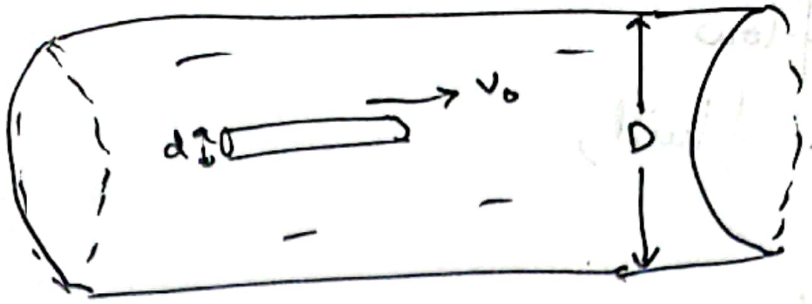
[2]



Using conservation of momentum,

$$\Sigma F_x = \iint \rho v_x (\vec{v}\cdot\hat{n})\, dA + \frac{\partial}{\partial t}\iiint \rho v_x\, dV$$

0 (Fully developed flow)

0 (steady flow)

$$\therefore\ P(2\pi r \Delta r)|_{x+\Delta x} - P(2\pi r\Delta r)|_x = \tau_{rx}(2\pi r\Delta x)|_{r+\Delta r} - \tau_{rx}(2\pi r \Delta x)|_r$$

$$\Rightarrow r\frac{\left(P|_{x+\Delta x} - P|_x\right)}{\Delta x} = \frac{\left(r\tau_{rx}|_{r+\Delta r} - r\tau_{rx}|_r\right)}{\Delta r}$$

$$\therefore\ r\frac{dP}{dx} = \frac{\partial (r\tau_{rx})}{\partial r}$$

Assuming fully developed flow, $\frac{dP}{dx}$ = const

$$\Rightarrow \frac{r^2}{2}\frac{dP}{dx} = r\tau_{rx} + c$$

$$\therefore\ \tau_{rx} = \frac{r}{2}\left(\frac{dP}{dx}\right) + \frac{c}{r}$$

τ_{rx} is finite at $r=0$

$$\boxed{\begin{array}{l}\therefore c = 0 \\ \Rightarrow \tau_{rx} = \dfrac{r}{2}\dfrac{dP}{dx}\end{array}}$$

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Now, $\mu \frac{dv}{dr} = \tau_{rx}$

$$\therefore \mu \frac{dv}{dr} = \frac{r}{2}\frac{dP}{dx}$$

$$\Rightarrow \int \mu \, dv = \int \frac{r\,dr}{2}\frac{dP}{dx}$$

$$\Rightarrow \mu v = \frac{r^2}{4}\frac{dP}{dx} + C$$

v_0 is the assume constant

$$\rightarrow v\left(r = \frac{d}{2}\right) = v_0$$

$$\therefore \mu v_0 = \frac{d^2}{16}\frac{dP}{dx} + C$$

$$C = \mu v_0 - \frac{d^2}{16}\frac{dP}{dx}$$

$$\rightarrow v\left(r = D/2\right) = 0$$

$$\therefore 0 = \frac{D^2}{16}\frac{dP}{dx} + C$$

$$\Rightarrow C = -\frac{D^2}{16}\frac{dP}{dx} = \mu v_0 - \frac{d^2}{16}\frac{dP}{dx}$$

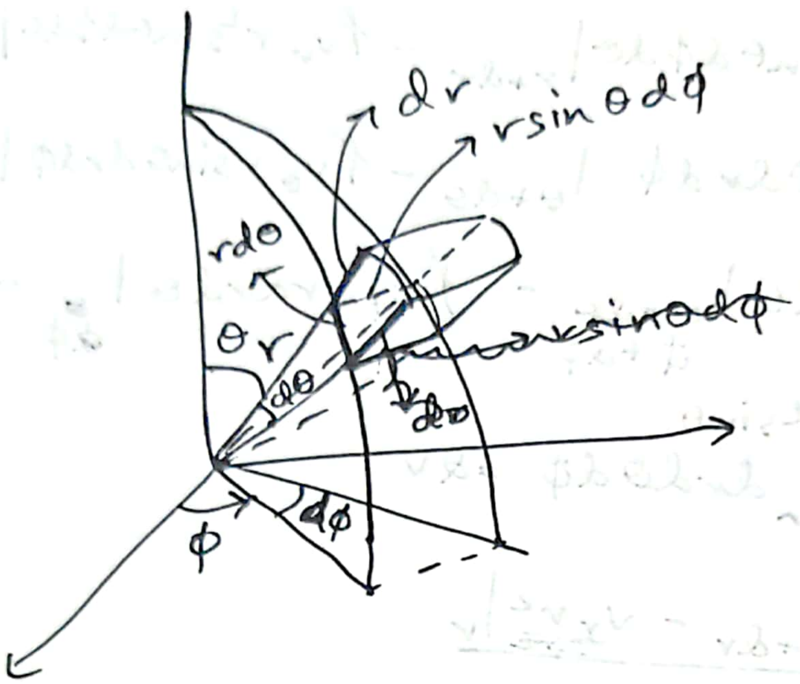
$$\therefore v_0 = \frac{(d^2 - D^2)}{16\mu}\frac{dP}{dx}$$

Drag force / unit length $= \frac{\tau_{rx} A}{L} = \frac{\tau \times \pi d \times L}{L}$

$$= \tau_{rx} \times \pi d$$

$$= \mu \frac{dv_x}{dr} \times \pi d$$

Q3)



$$dV = r^2 \sin\theta \, d\theta \, dr \, d\phi$$

By conservation of mass

$$\iint_{CS} \rho(\bar{v}\cdot\bar{n})\, dA + \underbrace{\frac{\partial}{\partial t}\iiint_{CV} \rho \, dV}_{0 \text{ (steady flow)}} = 0$$

$\rho \equiv$ const incompressible

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Now, $\rho v_r r^2 \sin\theta\, d\phi\, d\theta\big|_{r+dr} - \rho v_r r^2 \sin\theta\, d\phi\, d\theta\big|_r$

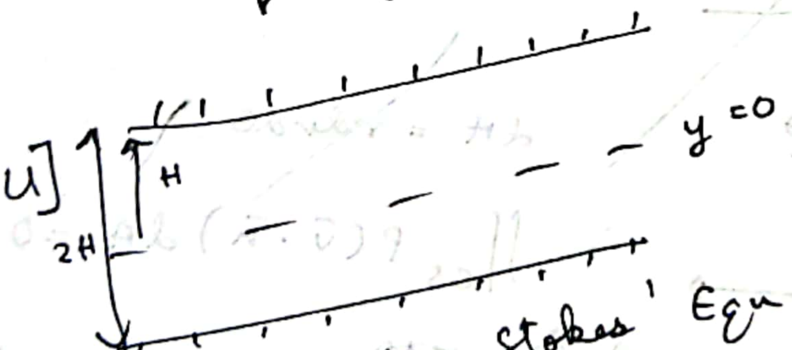
$+ \rho v_\theta\, r\sin\theta\, dr\, d\phi\big|_{\theta+d\theta} - \rho v_\theta\, r\sin\theta\, dr\, d\phi\big|_\theta$

$+ \rho v_\phi\, r\, dr\, d\theta\big|_{\phi+d\phi} - \rho v_\phi\, r\, dr\, d\theta\big|_{\phi} = 0$

Dividing by $r^2\sin\theta\, dr\, d\theta\, d\phi = dV$

$$\therefore \frac{v_r r^2\big|_{r+dr} - v_r r^2\big|_r}{r^2 dr} + \frac{\frac{v_\theta}{r}\sin\theta\big|_{\theta+d\theta} - \frac{v_\theta}{r}\sin\theta\big|_\theta}{\sin\theta\, d\theta} + \frac{\frac{v_\phi}{r}\big|_{\phi+d\phi} - \frac{v_\phi}{r}\big|_\phi}{d\phi\,\sin\theta} = 0$$

$$\therefore \frac{1}{r^2}\frac{\partial r^2 v_r}{\partial r} + \frac{1}{\sin\theta}\frac{\partial\left(\frac{v_\theta}{r}\sin\theta\right)}{\partial\theta} + \frac{1}{\sin\theta}\frac{\partial \frac{v_\phi}{r}}{\partial\phi} = 0$$



Applying Navier Stokes' Eqn

$$\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} = g_x - \frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{\mu}{\rho}\nabla^2 v_x$$

Since v is only v_x

$\Rightarrow \frac{\partial v}{\partial y} = 0$, and $\frac{\partial v_x}{\partial t} = 0$ steady flow

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And, by continuity,

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0$$

$$\therefore \frac{\partial V_x}{\partial x} = 0 \qquad \left(\text{as } \frac{\partial V_y}{\partial y} = 0 \right)$$

(Fully developed flow) $\frac{dV_x}{dx} = 0$

$$\Rightarrow 0 + 0 + 0 = 0 - \frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{\mu}{\rho}\frac{d^2 V_x}{dy^2}$$

$$\therefore \frac{dP}{dx} = \mu \frac{\partial^2 V_x}{\partial y^2}$$

$$\Rightarrow \mu \frac{dV_x}{dy} = y\frac{dP}{dx} + C_1$$

~~assum~~ fully developed flow

$$\therefore V_x = \frac{1}{\mu}\left(y^2 \frac{dP}{dx} + C_1 y + C_2 \right)$$

Now,

$$V_x(y = \pm H) = 0$$

$$\Rightarrow 0 = H^2 \frac{dP}{dx} \pm C_1 H + C_2$$

$$0 = H^2 \frac{dP}{dx} - C_1 H + C_2$$

$$\Rightarrow C_1 = 0 \quad \text{and} \quad C_2 = -H^2 \frac{dP}{dx}$$

$$\therefore V_x = \frac{1}{\mu}\left(y^2 \frac{dP}{dx} - H^2 \frac{dP}{dx} \right) = \frac{1}{\mu}\frac{dP}{dx}\left(y^2 - H^2 \right)$$

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