Assignment 8: Sampling / distributions of sampling statistics

Important note: Please make sure that you explicitly indicate all the assumptions and approximaitons that you make; some of these problems can only be solved approximately and hence indicating your assumption / approximation is essential for you to get full credits for your solution. The Quiz 8 will be based on these problems.

(1) An insurance company has about 10 lakhs automobile insurance holders. If the yearly claim of a policy holder is a random variable with mean Rs. 5000 and standard deviation Rs. 8000, what is the probability that the yearly claim exceeds Rs 550 crores?

(2) A long passage way built to connect two buildings at the sixth floor level is expected to withstand, without any structural damage, a weight normally distributed with mean 3000 kgs and standard deviation 300 kgs. Suppose we need to transport some equipments through the passage way; assume that the weights of these equipments are random variables with mean 600 kgs and standard deviation 80 kgs. How many equipments should be on the passage way at the same time for the probability of a structural damage to exceed 0.05?

(3) In an Institution of higher learning, there are 450 applications for the PhD admissions; all the applicants fulfill all necessary criterion, and hence, are called for the written test. From past experience, the adminsitrators know that only one-third of the students called for written test appear for the test. Compute the probability that this year, more than 150 students will appear for the written test.

(4) The mean weight and standard deviation of the students in a class is 40 kgs and 4 kgs respectively. (a) If a sample of 40 students are chosen, what is the probability that their mean weight lies between 39 and 42 kgs? (b) What happens if the sample size is increased to 120?

(5) Assuming that the successive temperature measurements in a laboratory experiment are made with a mean of 31 degree C and a standard deviation of 1 degree C, how many measurements are to be for us to be at least 95% certain that our measurement is accurate to within ± 0.2 degree C?

(6) The time taken by an accounts unit in a company to process reimbursements is normally distributed with a mean of 15 days and a standard deviation of 3 days. If we take the data of 20 reimbursement claims, what is the probability that the sample variance is 12?

(7) In the upcoming Tamilnadu elections, let us say that 42% of the people prefer candidate A as the Chief Minister. If a random sample of size 165 is chosen, find (a) the expected value and standard deviation of the number of people from the sample who favour candidate A, and (b) the probability that more than 60% of the sample favour candidate A.

(8) The annual per capita gold demand from India is known to be approximately 1 gm; assuming a standard deviation of 0.4 gm, if a random sample of 25 Indians are chosen, what is the probbability that the average gold purchased by the members of this sample exceed 1 gm in a given year?

(9) If 10 fair, six-sided die are rolled, what is the probability that the sum of values obtained is between 30 and 40 (inclusive of the end values)?

(10) Fifty numbers are rounded of to the nearest integer and summed. If the individual round-off errors are uniformly distributed between -0.5 and +0.5, what is the probability that the resultant sum differs from the exact sum by more than 2?

(11) A fair, four sided dile is repeatedly rolled until total of all rolls exceeds 300. What is the probability that this will require more than 200 rolls?

(12) What is the probability that the sum of 50 independent uniform random variables between 0 and 1 exceeds 30?

(13) It is known that about 32% of a given population is bilingual and about 8% is trilingual. If a random sample of 200 people are chosen, what is the probability that at least 50% of them speak more than one lanugage? What is the probability that fewer than 50 speak only one language?

(14) It is known that 11% of a given population carry a mutated gene. If a sample of 100 people are tested from this population, what is the probability that there are between 8 and 14 people who carry this mutation?

(15) Use R to compute P($X \leqslant 10$) when X is a binomial random variable with parameters n=100 and p = 0.1. Compare this with its Poisson and normal approximations.