Fifth SI Edition

CHAPTER

2

MECHANICS OF MATERIALS

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Stress and Strain – Axial Loading

Contents

End

Stress & Strain: Axial Loading

- Suitability of a structure or machine may depend on the deformations in the structure as well as the stresses induced under loading. Statics analyses alone are not sufficient.

- Considering structures as deformable allows determination of member forces and reactions which are statically indeterminate.

- Determination of the stress distribution within a member also requires consideration of deformations in the member.

- Chapter 2 is concerned with deformation of a structural member under axial loading. Later chapters will deal with torsional and pure bending loads.

Normal Strain

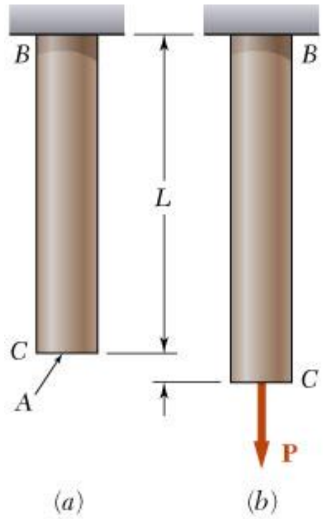


Fig. 2.1

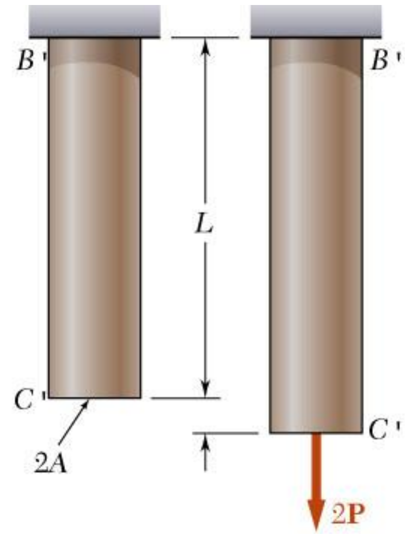


Fig. 2.3

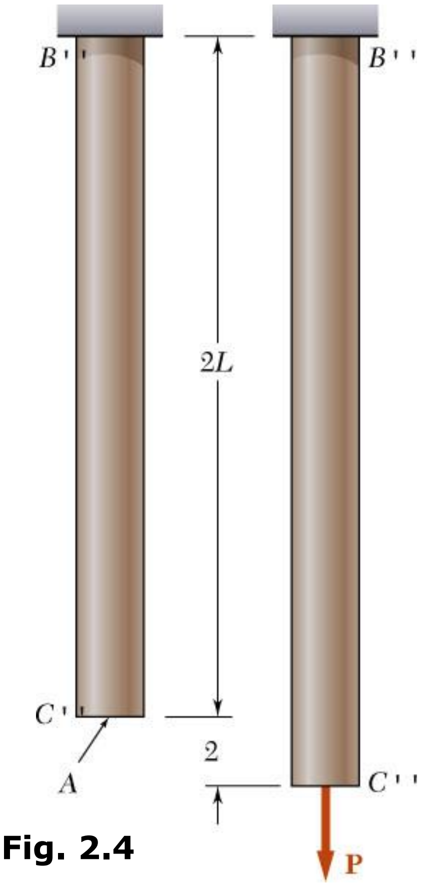


Fig. 2.4

$$s = \frac{P}{A} = \text{stress}$$

$$e = \frac{d}{L} = \text{normal strain}$$

$$s = \frac{2P}{2A} = \frac{P}{A}$$

$$e = \frac{d}{L}$$

$$s = \frac{P}{A}$$

$$e = \frac{2d}{2L} = \frac{d}{L}$$

End

Stress-Strain Test

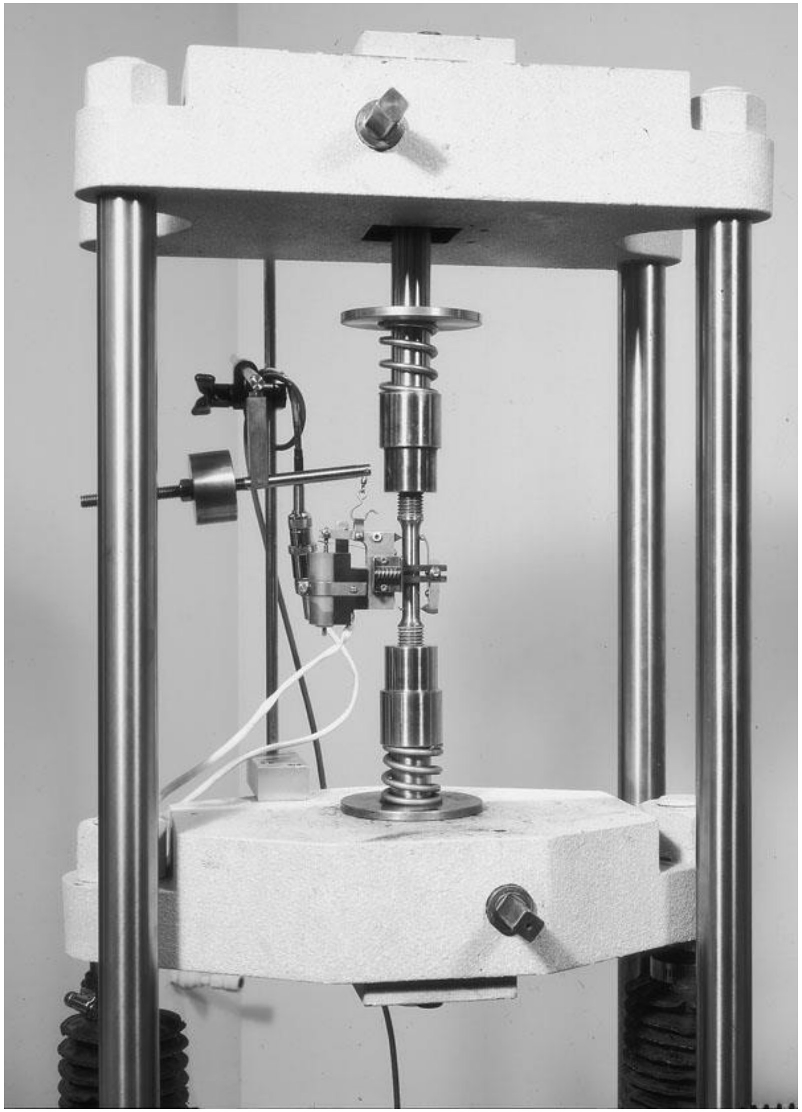


Fig 2.7 This machine is used to test tensile test specimens, such as those shown in this chapter.

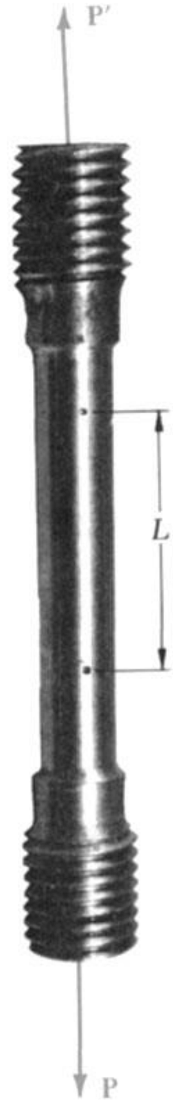
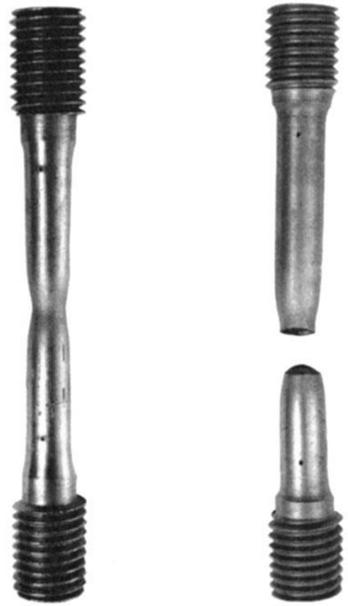
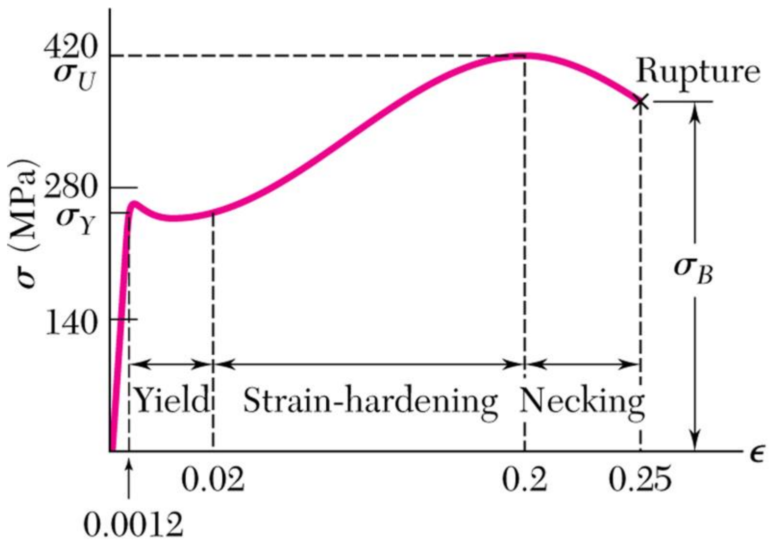


Fig 2.8 Test specimen with tensile load.

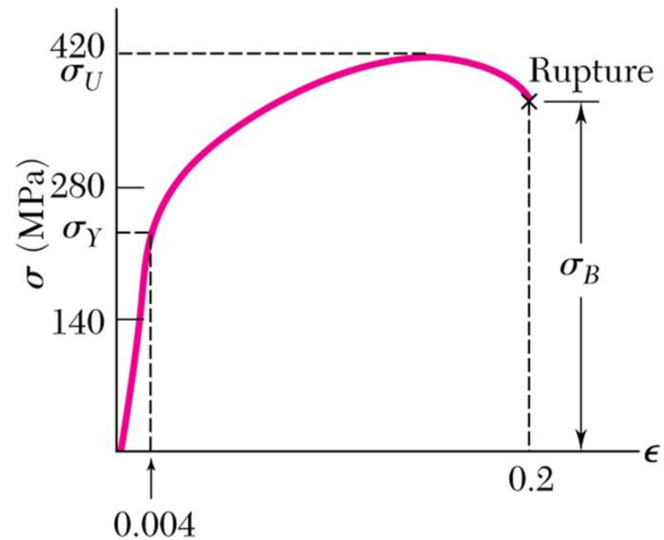
Stress-Strain Diagram: Ductile Materials



(a) (b)

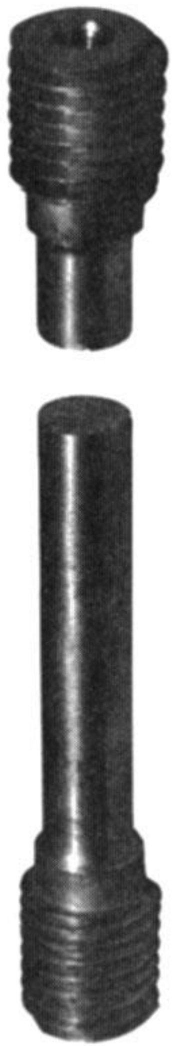


(a) Low-carbon steel



(b) Aluminum alloy

Stress-Strain Diagram: Brittle Materials



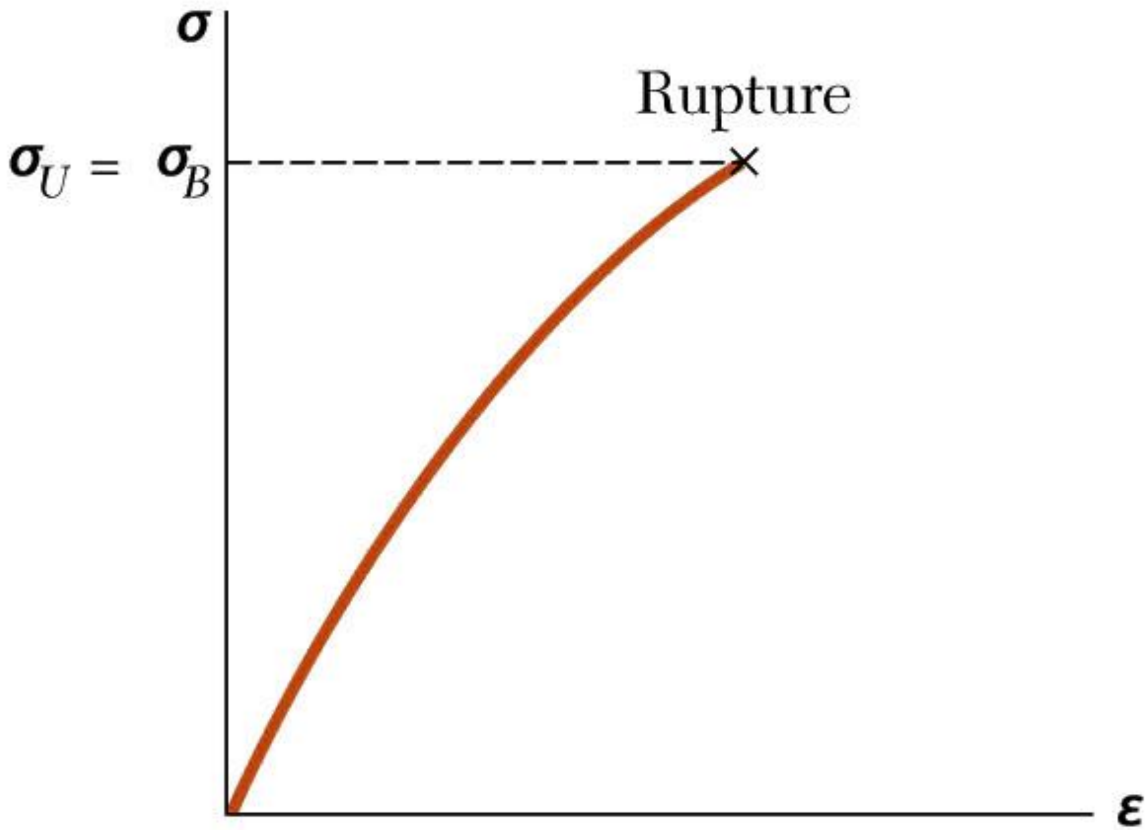


Fig 2.1 Stress-strain diagram for a typical brittle material.

Hooke's Law: Modulus of Elasticity

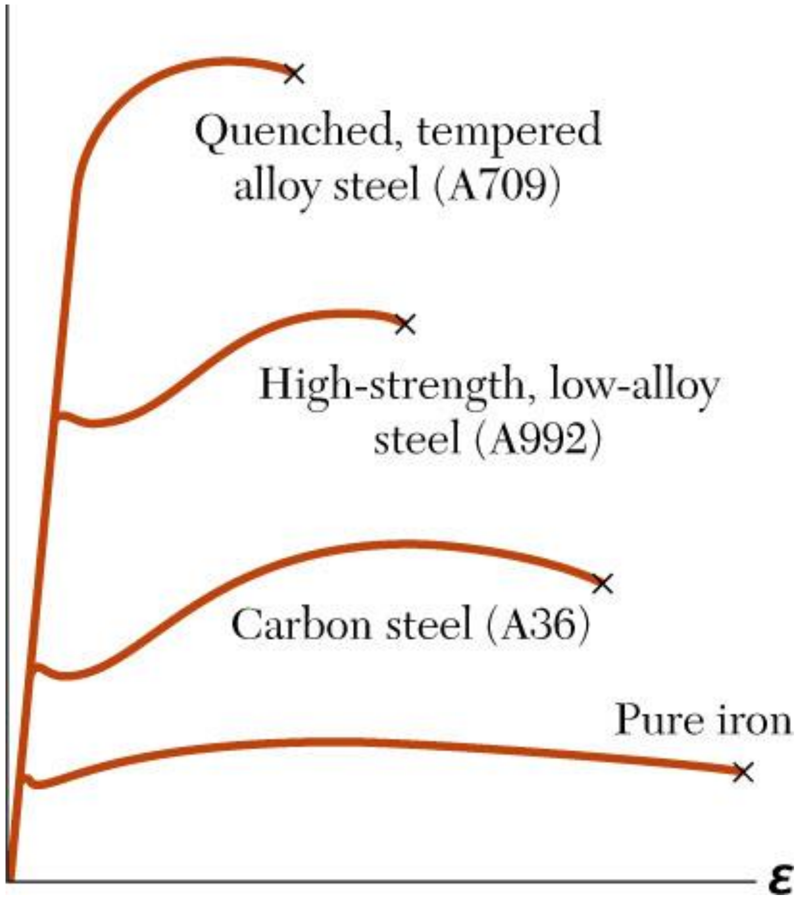


Fig 2.16 Stress-strain diagrams for iron and different grades of steel.

- Below the yield stress

$$\sigma = E\varepsilon$$

E = Youngs Modulus or Modulus of Elasticity

- Strength is affected by alloying, heat treating, and manufacturing process but stiffness (Modulus of Elasticity) is not.

Elastic vs. Plastic Behavior

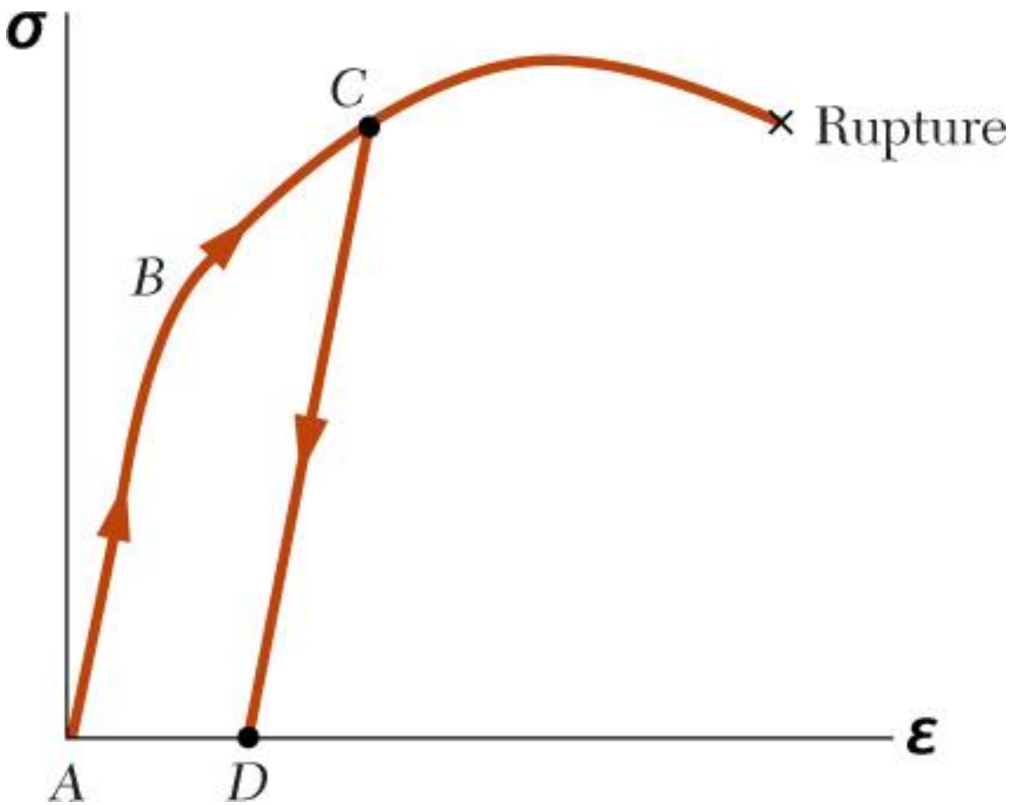


Fig. 2.18

- If the strain disappears when the stress is removed, the material is said to behave *elastically*.
- The largest stress for which this occurs is called the *elastic limit*.
- When the strain does not return to zero after the stress is removed, the material is said to behave *plastically*.

Fatigue

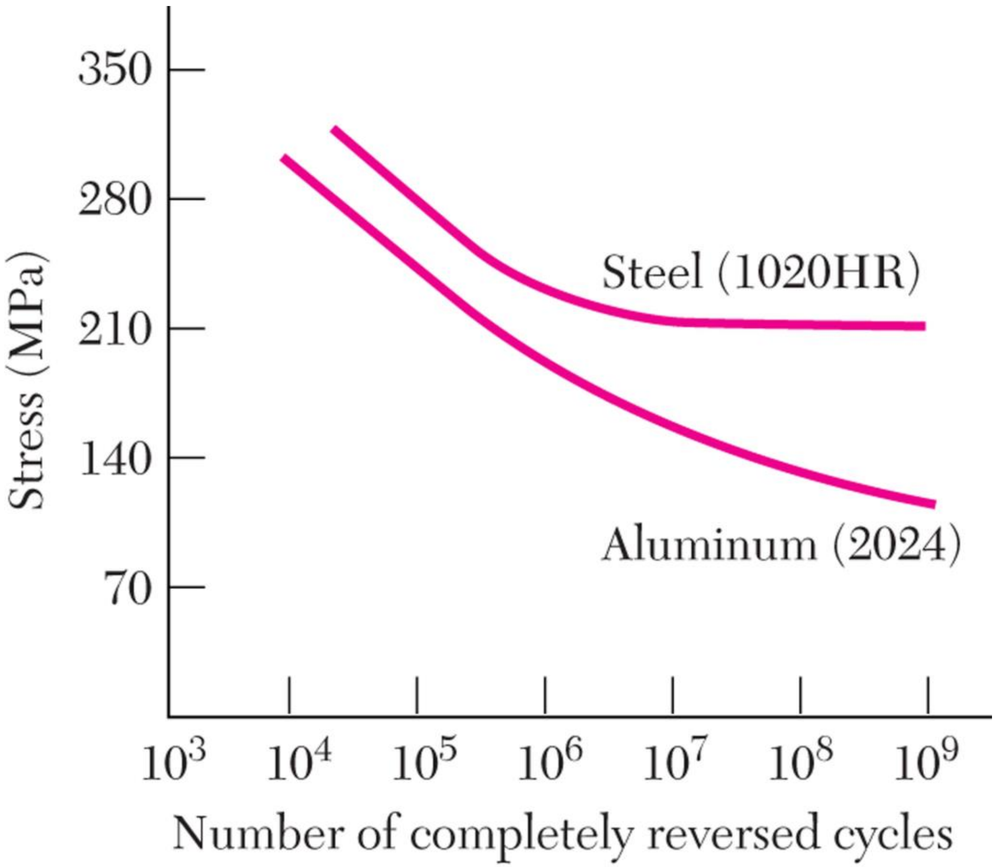


Fig. 2.21

- Fatigue properties are shown on S-N diagrams.
- A member may fail due to *fatigue* at stress levels significantly below the ultimate strength if subjected to many loading cycles.
- When the stress is reduced below the *endurance limit*, fatigue failures do not occur for any number of cycles.

Deformations Under Axial Loading

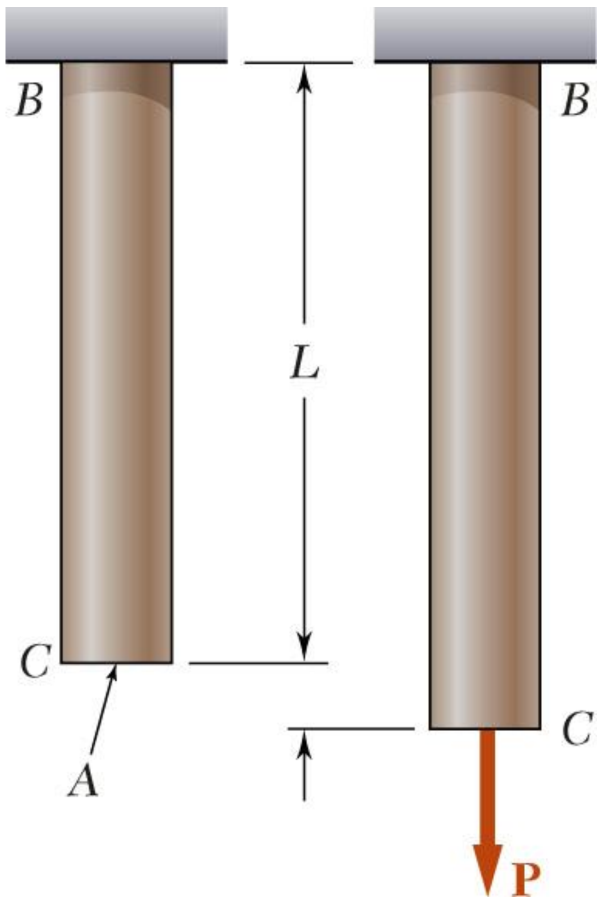


Fig. 2.22

- From Hooke's Law:

$$\sigma = E\varepsilon \qquad \varepsilon = \frac{\sigma}{E} = \frac{P}{AE}$$

- From the definition of strain:

$$\varepsilon = \frac{\delta}{L}$$

- Equating and solving for the deformation,

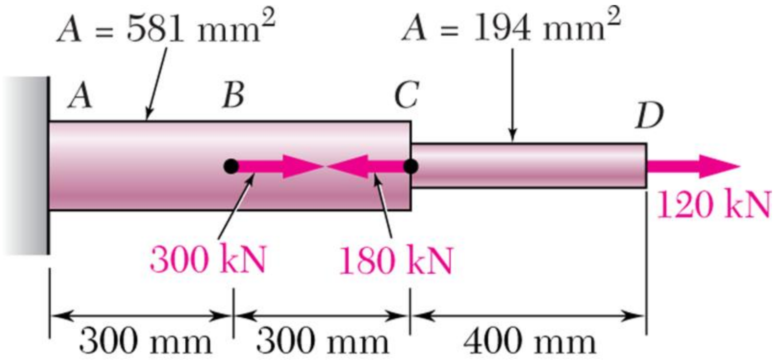
$$\delta = \frac{PL}{AE}$$

- With variations in loading, cross-section or material properties,

$$\delta = \sum_i \frac{P_i L_i}{A_i E_i}$$

End

Example 2.01



(a)

$E = 200$ GPa

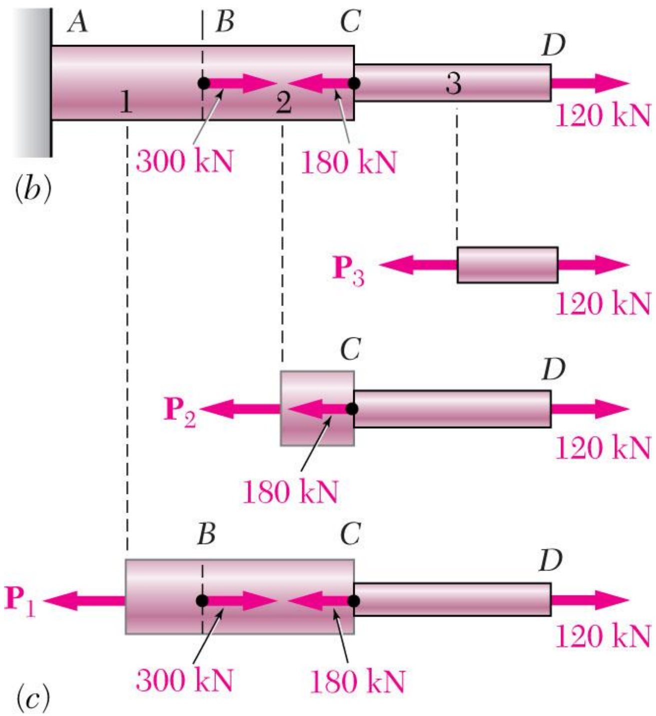
Determine the deformation of the steel rod shown under the given loads.

SOLUTION:

- Divide the rod into components at the load application points.
- Apply a free-body analysis on each component to determine the internal force
- Evaluate the total of the component deflections.

SOLUTION:

- Divide the rod into three components:



$$L_1 = L_2 = 0.3\,\mathrm{m} \qquad L_3 = 0.4\,\mathrm{m}$$

$$A_1 = A_2 = 581 \times 10^{-6}\,\mathrm{m}^2 \qquad A_3 = 194 \times 10^{-6}\,\mathrm{m}^2$$

- Apply free-body analysis to each component to determine internal forces,

$$P_1 = 240 \times 10^3\,\mathrm{N}$$

$$P_2 = -60 \times 10^3\,\mathrm{N}$$

$$P_3 = 120 \times 10^3\,\mathrm{N}$$

- Evaluate total deflection,

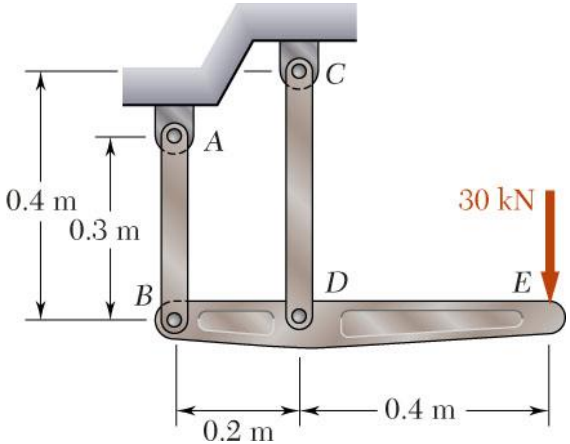
$$= \sum_i \frac{P_i L_i}{A_i E_i} = \frac{1}{E}\left(\frac{P_1 L_1}{A_1} + \frac{P_2 L_2}{A_2} + \frac{P_3 L_3}{A_3}\right)$$

$$= \frac{1}{200 \times 10^9}\left[\frac{\left(240 \times 10^3\right)0.3}{581 \times 10^{-6}} + \frac{\left(-60 \times 10^3\right)0.3}{581 \times 10^{-6}} + \frac{\left(120 \times 10^3\right)0.4}{194 \times 10^{-6}}\right]$$

$$= 1.73 \times 10^{-3}\,\mathrm{m}$$

$$\boxed{= 1.73\,\mathrm{mm}}$$

Sample Problem 2.1



The rigid bar *BDE* is supported by two links *AB* and *CD*.

Link *AB* is made of aluminum (E = 70 GPa) and has a cross-sectional area of 500 mm^2. Link *CD* is made of steel (E = 200 GPa) and has a cross-sectional area of (600 mm^2).

For the 30-kN force shown, determine the deflection a) of *B*, b) of *D*, and c) of *E*.

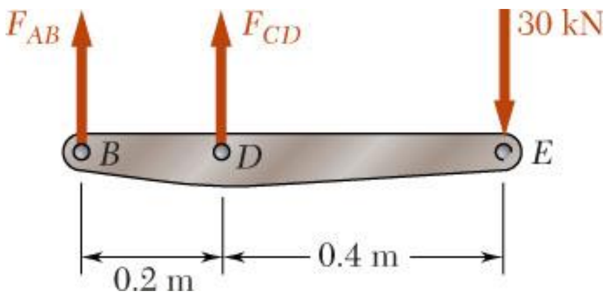
SOLUTION:

- Apply a free-body analysis to the bar *BDE* to find the forces exerted by links *AB* and *DC*.
- Evaluate the deformation of links *AB* and *DC* or the displacements of *B* and *D*.
- Work out the geometry to find the deflection at E given the deflections at B and D.

Sample Problem 2.1

SOLUTION:

Free body: Bar *BDE*



$$+\circlearrowleft \sum M_B = 0$$

$$0 = -(30\,\text{kN} \times 0.6\,\text{m}) + F_{CD} \times 0.2\,\text{m}$$

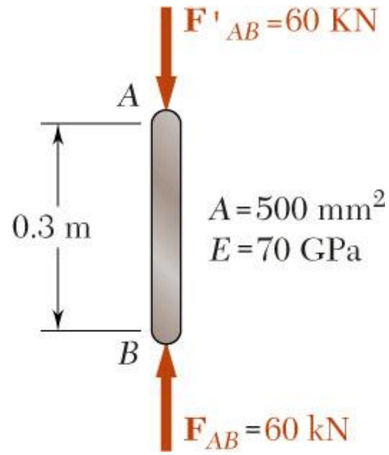
$$F_{CD} = +90\,\text{kN} \quad \textit{tension}$$

$$+\circlearrowleft \sum M_D = 0$$

$$0 = -(30\,\text{kN} \times 0.4\,\text{m}) - F_{AB} \times 0.2\,\text{m}$$

$$F_{AB} = -60\,\text{kN} \quad \textit{compression}$$

Displacement of *B*:



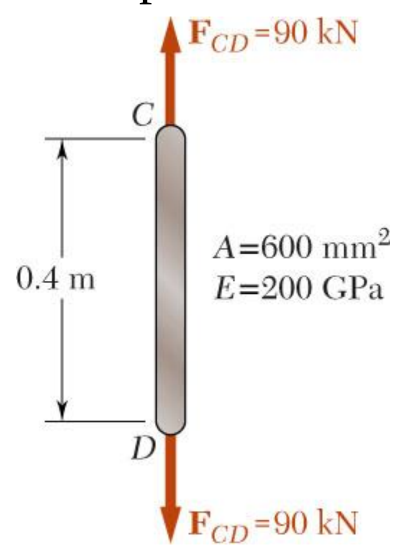
$$\delta_B = \frac{PL}{AE}$$

$$= \frac{(-60 \times 10^3\,\text{N})(0.3\,\text{m})}{(500 \times 10^{-6}\,\text{m}^2)(70 \times 10^9\,\text{Pa})}$$

$$= -514 \times 10^{-6}\,\text{m}$$

$$\boxed{\delta_B = 0.514\,\text{mm} \uparrow}$$

Displacement of *D*:



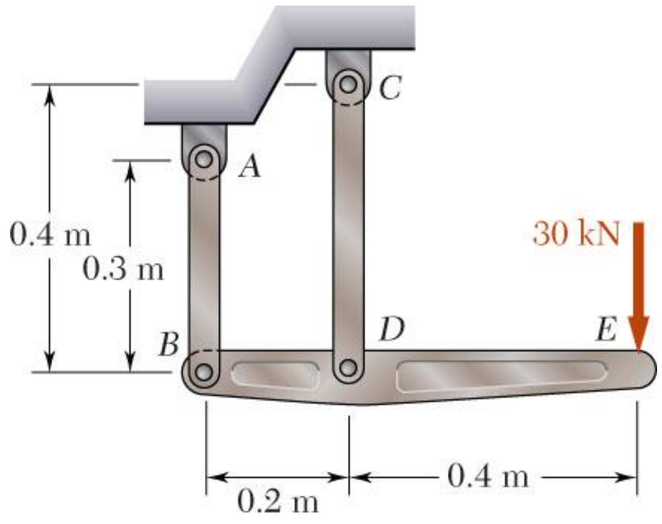
$$\delta_D = \frac{PL}{AE}$$

$$= \frac{(90 \times 10^3\,\text{N})(0.4\,\text{m})}{(600 \times 10^{-6}\,\text{m}^2)(200 \times 10^9\,\text{Pa})}$$

$$= 300 \times 10^{-6}\,\text{m}$$

$$\boxed{\delta_D = 0.300\,\text{mm} \downarrow}$$

Sample Problem 2.1

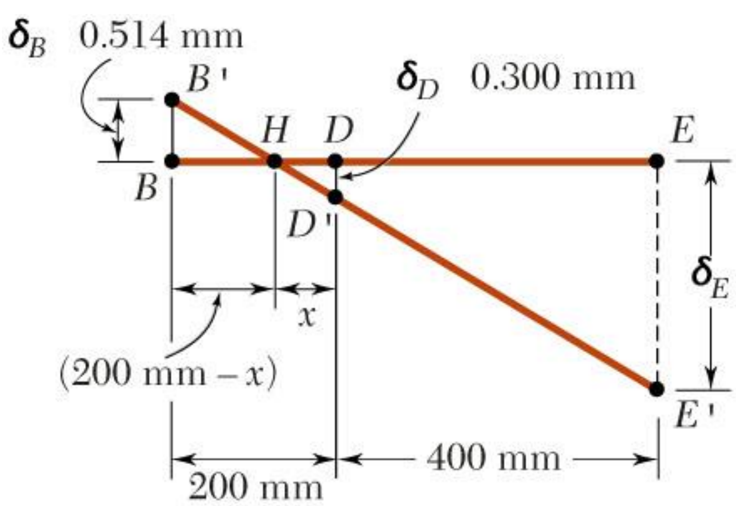


Displacement of D:

$$\frac{BB'}{DD'} = \frac{BH}{HD}$$

$$\frac{0.514\text{ mm}}{0.300\text{ mm}} = \frac{(200\text{ mm}) - x}{x}$$

$$x = 73.7\text{ mm}$$



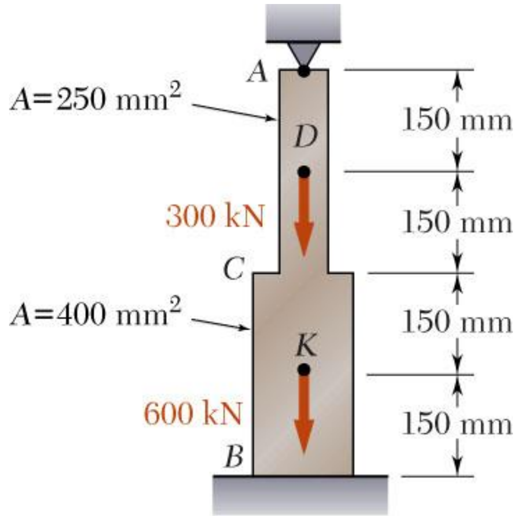
$$\frac{EE'}{DD'} = \frac{HE}{HD}$$

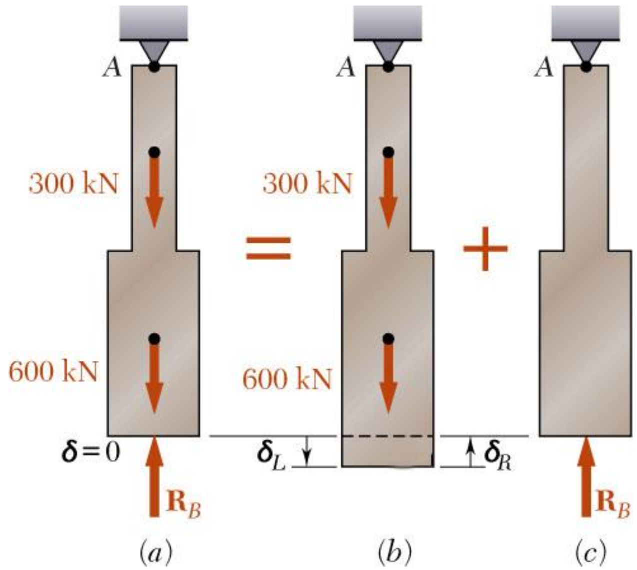
$$\frac{d_E}{0.300\text{ mm}} = \frac{(400 + 73.7)\text{ mm}}{73.7\text{ mm}}$$

$$d_E = 1.928\text{ mm}$$

$$\boxed{d_E = 1.928\text{ mm}}$$

Static Indeterminacy

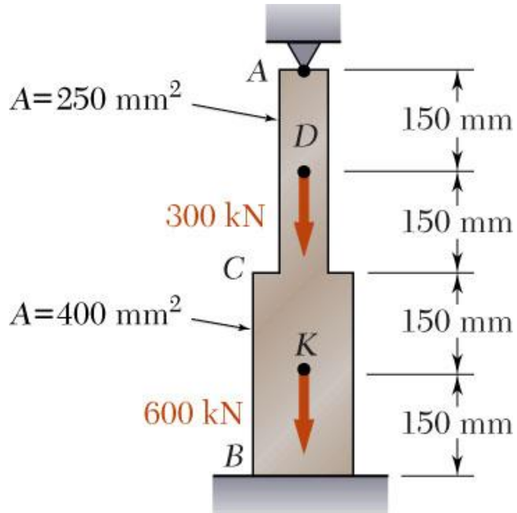


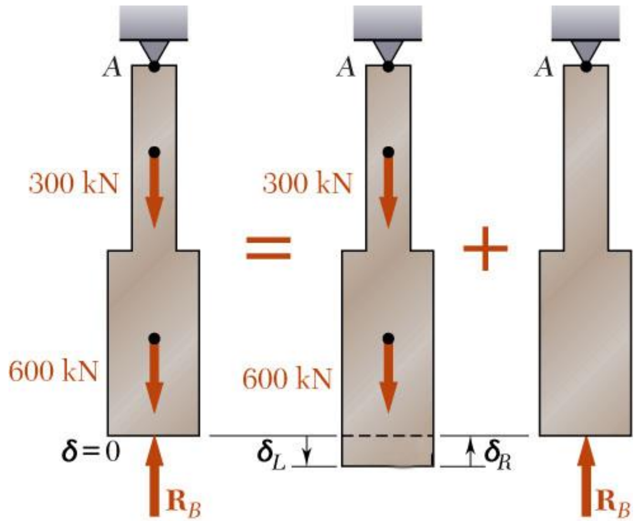


- Structures for which internal forces and reactions cannot be determined from statics alone are said to be *statically indeterminate*.
- A structure will be statically indeterminate whenever it is held by more supports than are required to maintain its equilibrium.
- Redundant reactions are replaced with unknown loads which along with the other loads must produce compatible deformations.
- Deformations due to actual loads and redundant reactions are determined separately and then added or *superposed*.

$$\delta = \delta_L + \delta_R = 0$$

Example 2.04





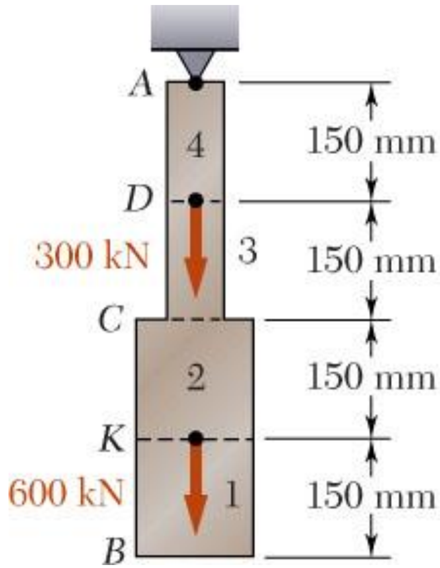
(a) (b) (c)

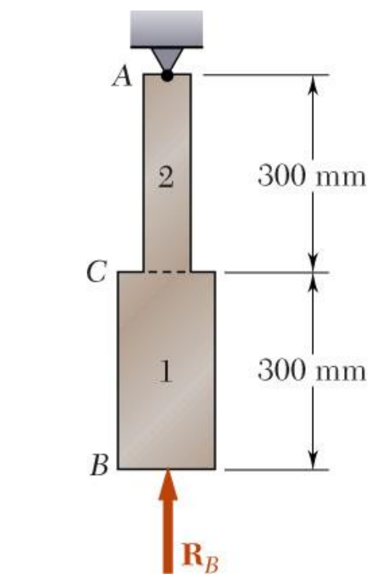
Determine the reactions at *A* and *B* for the steel bar and loading shown, assuming a close fit at both supports before the loads are applied.

SOLUTION:

- Consider the reaction at *B* as redundant, release the bar from that support, and solve for the displacement at *B* due to the applied loads.
- Solve for the displacement at *B* due to the redundant reaction at *B.*
- Require that the displacements due to the loads and due to the redundant reaction be compatible, i.e., require that their sum be zero.
- Solve for the reaction at *A* due to applied loads and the reaction found at *B*.

Example 2.04





SOLUTION:

- Solve for the displacement at B due to the applied loads with the redundant constraint released,

$$P_1 = 0 \quad P_2 = P_3 = 600\times 10^3\,\text{N} \quad P_4 = 900\times 10^3\,\text{N}$$

$$A_1 = A_2 = 400\times 10^{-6}\,\text{m}^2 \quad A_3 = A_4 = 250\times 10^{-6}\,\text{m}^2$$

$$L_1 = L_2 = L_3 = L_4 = 0.150\text{ m}$$

$$\delta_L = \sum_i \frac{P_i L_i}{A_i E_i} = \frac{1.125\times 10^9}{E}$$

- Solve for the displacement at B due to the redundant constraint,

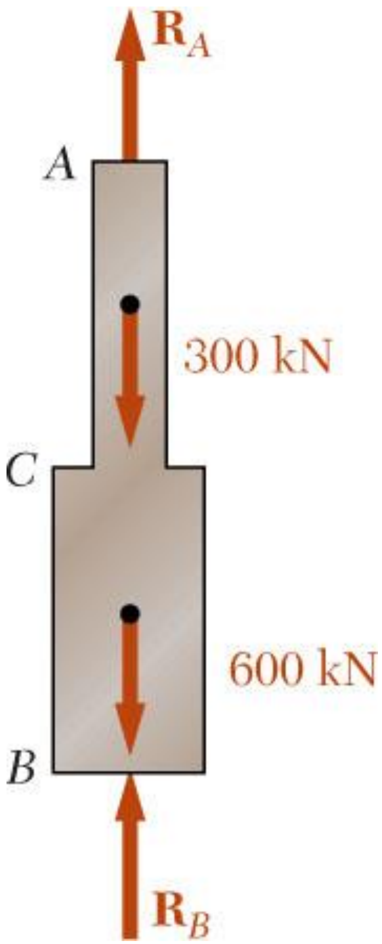
$$P_1 = P_2 = -R_B$$

$$A_1 = 400\times 10^{-6}\,\text{m}^2 \quad A_2 = 250\times 10^{-6}\,\text{m}^2$$

$$L_1 = L_2 = 0.300\text{ m}$$

$$\delta_R = \sum_i \frac{P_i L_i}{A_i E_i} = -\frac{\left(1.95\times 10^3\right) R_B}{E}$$

Example 2.04



- Require that the displacements due to the loads and due to the redundant reaction be compatible,

$$\delta = \delta_L + \delta_R = 0$$

$$\delta = \frac{1.125 \times 10^9}{E} - \frac{\left(1.95 \times 10^3\right) R_B}{E} = 0$$

$$R_B = 577 \times 10^3\,\mathrm{N} = 577\ \mathrm{kN}$$

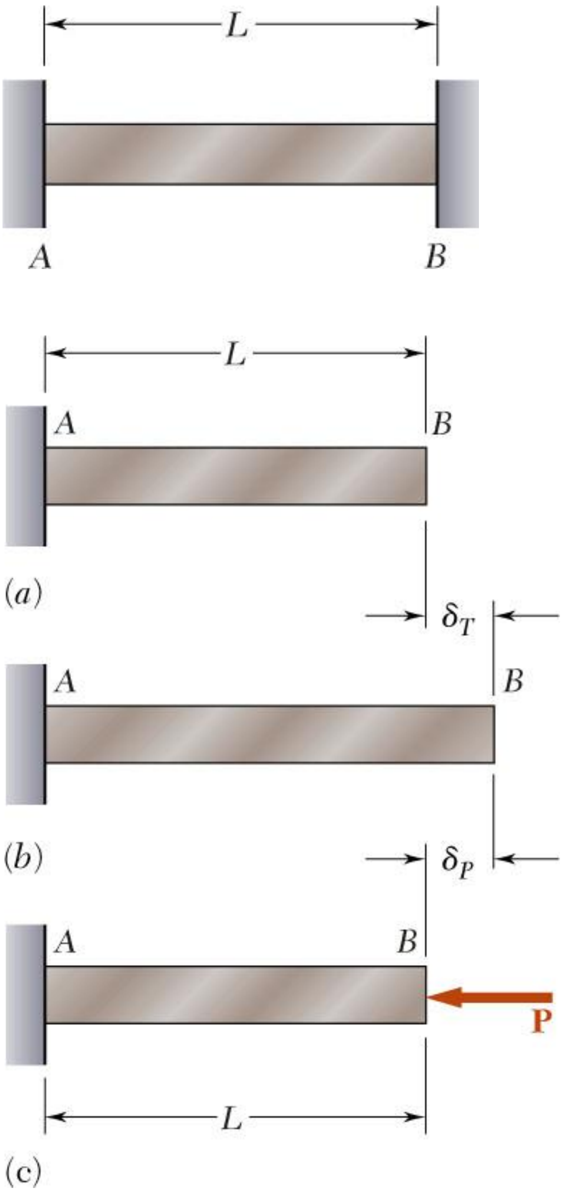
- Find the reaction at A due to the loads and the reaction at B

$$+\!\uparrow \sum F_y = 0 = R_A - 300\,\mathrm{kN} - 600\,\mathrm{kN} + 577\,\mathrm{kN}$$

$$R_A = 323\,\mathrm{kN}$$

$$\boxed{\begin{aligned} R_A &= 323\,\mathrm{kN} \\ R_B &= 577\,\mathrm{kN} \end{aligned}}$$

Thermal Stresses



- A temperature change results in a change in length or *thermal strain*. There is no stress associated with the thermal strain unless the elongation is restrained by the supports.

- Treat the additional support as redundant and apply the principle of superposition.

$$\delta_T = \alpha(\Delta T)L \qquad \delta_P = \frac{PL}{AE}$$

$\alpha =$ thermal expansion coef.

- The thermal deformation and the deformation from the redundant support must be compatible.

$$\delta = \delta_T + \delta_P = 0$$

$$\alpha(\Delta T)L + \frac{PL}{AE} = 0$$

$$P = -AE\alpha(\Delta T)$$

$$\sigma = \frac{P}{A} = -E\alpha(\Delta T)$$