Assignment 2

CS 228 Logic for Computer Science

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1. Since exactly one of X,Y or Z is true, we choose our propositional variables as,

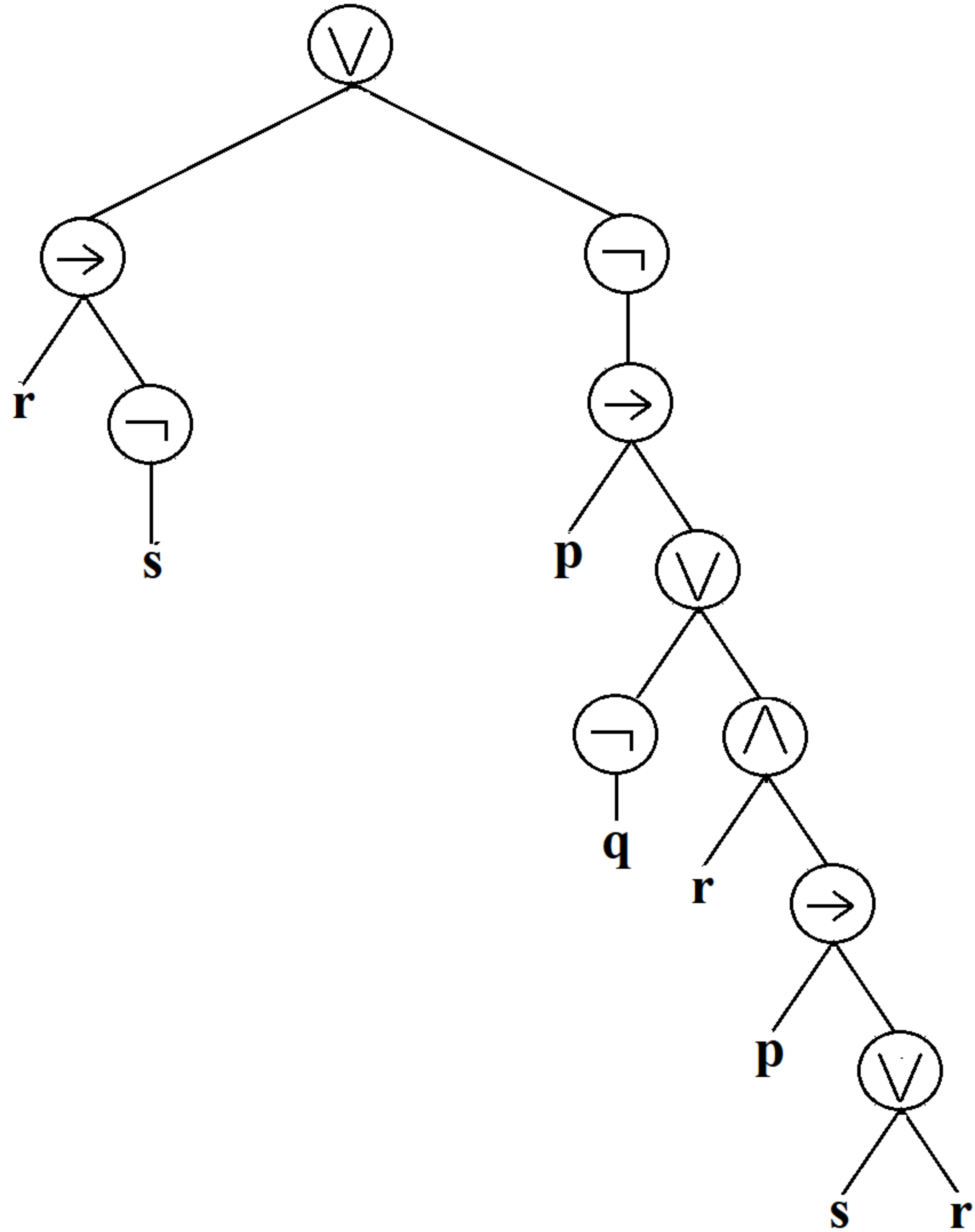
$$\begin{aligned}
\psi_1 &= \text{X is the murderer}\\
\psi_2 &= \text{Y is the murderer}\\
\psi_3 &= \text{Z is the murderer}\\
x &= \text{X is telling the truth}\\
y &= \text{Y is telling the truth}\\
z &= \text{Z is telling the truth}
\end{aligned}$$

Using the conditions that exactly one of them is a murderer and exactly one of their statements is true, we come up with the following proof,

1.	$x \to \neg y \wedge \neg z$	premise
2.	$\neg y \wedge \neg z \to x$	premise
3.	$y \to \neg x \wedge \neg z$	premise
4.	$\neg x \wedge \neg z \to y$	premise
5.	$z \to \neg y \wedge \neg x$	premise
6.	$\neg y \wedge \neg x \to z$	premise
7.	$\psi_1 \to \neg\psi_2 \wedge \neg\psi_3$	premise
8.	$\neg\psi_2 \wedge \neg\psi_3 \to \psi_1$	premise
9.	$\psi_2 \to \neg\psi_1 \wedge \neg\psi_3$	premise
10.	$\neg\psi_1 \wedge \neg\psi_3 \to \psi_2$	premise
11.	$\psi_3 \to \neg\psi_2 \wedge \neg\psi_1$	premise
12.	$\neg\psi_2 \wedge \neg\psi_1 \to \psi_3$	premise
13.	$x \to \psi_2$	premise
14.	$y \to \psi_3$	premise
15.	$z \to \neg\psi_1 \wedge \neg\psi_2$	premise
16.	$\psi_2 \to x$	premise
17.	$\psi_3 \to y$	premise
18.	$\neg\psi_2 \wedge \neg\psi_1 \to z$	premise
19.	y	assumption
20.	ψ_3	M.P. 19,14
21.	$\neg x \wedge \neg z$	M.P. 19,3
22.	$\neg z$	$\wedge e_2$ 21
23.	$\neg\psi_2 \wedge \neg\psi_1$	M.P 20,11
24.	z	M.P. 23,18
25.	$\perp$	$\perp i$ 22,24
26.	$\neg y$	$\perp e$ 25
27.	z	assumption
28.	$\neg\psi_1 \wedge \neg\psi_2$	M.P. 27,15
29.	ψ_3	M.P. 28,12
30.	y	M.P. 29,17
31.	$\perp$	$\perp i$ 26,30
32.	$\neg z$	$\perp e$ 31
33.	$\neg y \wedge \neg z$	$\wedge i$ 26,32
34.	x	M.P. 33,2
35.	ψ_2	M.P. 34,13

So the murderer is Y and X is telling the truth.

2. The parse tree of the formula $\psi = (r \to \neg s) \vee \neg(p \to (\neg q \vee (r \wedge (p \to (s \vee r)))))$ is,



3. The formula is,

$$\psi = (r \to \neg s) \vee \neg(p \to (\neg q \vee (r \wedge (p \to (s \vee r)))))$$
$$\psi = (\neg r \vee \neg s) \vee \neg(\neg p \vee (\neg q \vee (r \wedge (\neg p \vee s \vee r))))$$
$$\psi = (\neg r \vee \neg s) \vee (p \wedge \neg(\neg q \vee (r \wedge (\neg p \vee s \vee r))))$$
$$\psi = (\neg r \vee \neg s) \vee (p \wedge (q \wedge \neg(r \wedge (\neg p \vee s \vee r))))$$
$$\psi = (\neg r \vee \neg s) \vee (p \wedge (q \wedge (\neg r \vee \neg(\neg p \vee s \vee r))))$$
$$\psi = (\neg r \vee \neg s) \vee (p \wedge q \wedge (\neg r \vee (p \wedge \neg s \wedge \neg r)))$$

$$\psi = (\neg r \vee \neg s) \vee (p \wedge q \wedge (\neg r \vee p) \wedge (\neg r \vee \neg s) \wedge \neg r)$$

$$\psi = (p \vee \neg r \vee s) \wedge (q \vee \neg r \vee s) \wedge (\neg r \vee p \vee \neg s) \wedge (\neg r \vee \neg s) \wedge (\neg r \vee \neg s)$$

4. On resolving the formula we get,

$$Res^0 = \psi = \{\{p, \neg r, \neg s\}, \{q, \neg r, \neg s\}, \{\neg r, \neg s\}\}$$

$$Res^1 = Res^0$$

We see that Res^0 is Res^*, and $\phi \notin Res^*$ hence the formula is valid.

5. (a) $p, \neg q \vdash (p \rightarrow (q \rightarrow p))$

1.	p	premise
2.	$\neg q$	premise
3.	q	assumption
4.	p	copy 1
5.	$q \rightarrow p$	$\rightarrow i$ 3-4
6.	p	asumption
7.	$q \rightarrow p$	copy 5
8.	$p \rightarrow (q \rightarrow p)$	$\rightarrow i$ 6-7

(b) $\neg p, \neg q \vdash (p \rightarrow (q \rightarrow p))$

1.	$\neg p$	premise
2.	$\neg q$	premise
3.	p	assumption
4.	$\perp$	$\perp i$ 1,3
5.	$q \rightarrow p$	$\perp e$ 4
6.	$p \rightarrow (q \rightarrow p)$	$\rightarrow i$ 3-5

(c) $\neg p, q \vdash (p \rightarrow (q \rightarrow p))$

1.	$\neg p$	premise
2.	q	premise
3.	p	assumption
4.	$\perp$	$\perp i$ 1,3
5.	$q \rightarrow p$	$\perp e$ 4
6.	$p \rightarrow (q \rightarrow p)$	$\rightarrow i$ 3-5

(d) $p, q \vdash (p \rightarrow (q \rightarrow p))$

1.	p	premise
2.	q	premise
3.	p	assumption
4.	q	asssumption
5.	p	copy 1
6.	$q \rightarrow p$	$\rightarrow i$ 4 - 5
7.	$p \rightarrow (q \rightarrow p)$	$\rightarrow i$ 3-6

One can show $\vdash p \rightarrow (q \rightarrow p))$ by combining these proofs,

1.	$p \vee \neg p$	LEM
2.	p	assumption
3.	$q \vee \neg q$	LEM
4.	q	assumption
5.	$p \to (q \to p)$	from 2,4 and (d)
6.	$\neg q$	assumption
7.	$p \to (q \to p)$	from 2,6 and (a)
8.	$p \to (q \to p)$	$\vee e$ 3,4-5,6-7
9.	$\neg p$	assumption
10.	$q \vee \neg q$	LEM
11.	q	assumption
12.	$p \to (q \to p)$	from 9,11 and (c)
13.	$\neg q$	assumption
14.	$p \to (q \to p)$	from 9,13 and (b)
15.	$p \to (q \to p)$	$\vee e$ 10,11-12,13-14
16.	$p \to (q \to p)$	$\vee e$ 1,2-8,9-15

6. $\psi_1 = (p \leftrightarrow (q \leftrightarrow r))$
   $\psi_2 = ((p \wedge (q \wedge r)) \vee ((\neg p) \wedge ((\neg q) \wedge (\neg r)))$
   The truth table is given as follows,

p	**q**	**r**	ψ_1	ψ_2
True	True	True	True	True
True	True	False	False	False
True	False	True	False	False
True	False	False	True	False
False	True	True	False	False
False	True	False	True	False
False	False	True	True	False
False	False	False	False	True

   We see that neither formula semantically entails the other.

7. (a) The approach according to the logician can be seen in the following example,

$$\psi = (\top \to p) \wedge (\top \to \neg p)$$

$$\psi = (\top \to p) \wedge (\top \to p')$$

   (b) I disagree with the logician. The counter-example from (a) can used as,

p	$\neg p$	ψ
True	False	False
False	True	False

   We see that ψ is unsatisfiable. However on the resolution of ψ', we get

$$Res^0 = \{\{p\}, \{p'\}\}$$

$$Res^1 = Res^0$$

   Clearly Res^0 is Res^* and $\phi \notin Res^*$. So the HornSat algorithm concludes ψ' to be satisfiable while ψ is unsatisfiable. This shows that the logician is wrong.