EN-204 – Materials for Energy Applications
Problem Set – Fermi Dirac Statistics

1. Find the probability of distribution of electrons at energies 1, 0.01 eV below the Fermi level and 0.01 and 1 eV above the Fermi Level.

2. Derive an expression for the total number of states in a semiconductor material (per unit volume) between $\boldsymbol{Ec}$ and $\boldsymbol{Ec + kT}$, where $\boldsymbol{Ec}$ is the conduction band edge (bottom of the conduction band), $\boldsymbol{k}$ is Boltzmann's constat and $\boldsymbol{T}$ is the temperature. Consider the case of GaAs at room temperature where $m^*=0.067m_0$.

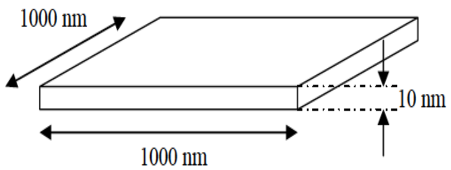
3. Consider a silicon crystal whose band gap energy is $\mathbf{E}_g$ =1.12 eV and whose temperature is kept at $\mathbf{T}$=300^o K.
   a) If the Fermi level, $\boldsymbol{E_f}$, is located in the middle of the band gap, what is the probability of finding an electron (or equivalently, the probablility of a state being occupied) at $\mathbf{E} = \mathbf{E}c + \mathbf{kT}$.
      Hint: write $E_f = E_c – E_g/2$
   b) If the Fermi level, $\mathbf{E}f$ is located at the conduction band edge, $\mathbf{E}F = \mathbf{E}c$, what the probablility of finding an electron at $\mathbf{E} = \mathbf{E}c + \mathbf{kT}$.

4. The equilibrium electron concentration is given by the product of the density of states and the probablility function, $\mathbf{n(E) = g(E).F(E)}$. If $\mathbf{E - E_F >> kT}$, the Fermi-Dirac probability function can be approximated with the Maxwell-Boltzmann function.
   a) Using this approximation, find the energy relative to the conduction band edge, $E - E_{\mathbf{c}}$, at which the electron concentration becomes maximum.
   b) Using this approximation, calculate the electron concentration per unit energy interval (in units of $cm^{-3}eV^{-1}$) in silicon at energy E = Ec - kT. Assume the Fermi level is located at the center of the band gap, EF = Ec – Eg/2.

5. Show that the average kinetic energy in a Fermi gas is $(3/5)E_F$

6. Calculate the probability that an electron in Cu at 300 K has energy equal to 99% of the Fermi energy.

7. Quantum wells are often used in applications such as semiconductor lasers. In a quantum well, electrons are confined in a thin slab of material, as shown below



The density-of-states, D(E) in an ideal quantum-well is step-like:
At T≈0K, there are 1.4×10^5 states in this system, mark the location of Fermi Level.