EN203 (2018) → Problem Set II Solutions

1> From the data, $u = 196 + 0.718\, t$

$$c_v = \left(\frac{\partial u}{\partial t}\right)_v = \left[\frac{\partial(196 + 0.718t)}{\partial t}\right]_v = 0.718 \,\frac{kJ}{kg\,K} \quad \text{(iv)}$$

Again, from the data, $pv = 287(t + 273)$

where the unit of pv is $\frac{N}{m^2} \times \frac{m^3}{kg}$ or $\frac{J}{kg}$

From the defination of enthalpy, $h = u + pv$

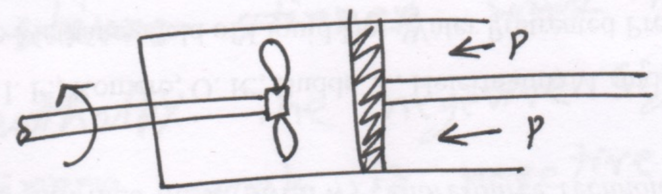
Substituting for u and pv in the same unit of $\frac{kJ}{kg}$

$$h = (196 + 0.718t) + 287 \times 10^{-3} \times (t + 273)$$

$$\therefore c_p = \left(\frac{\partial h}{\partial t}\right)_p = \left(\frac{\partial\left[196 + 0.718t) + 287 \times 10^{-3} \times (t + 273)\right]}{\partial t}\right)_p$$

$$= 1.005 \,\frac{kJ}{kg\,K}$$

2>



Applying the 1st law of thermodynamics for pure substance, $Q - W = U_2 - U_1$ ——— (i)

From the data, $Q = 0$

$\therefore -W = U_2 - U_1$ ——— (ii)

Total work = displacement work + stirrer work
(the piston moves)

or, $W = W_d + W_s$ ——— (iii)

Here, $W_d = \int_1^2 p\,dV = p(V_2 - V_1)$ [∵ displacement occurs at const. pres.]

(iii) becomes,—

$$\therefore W = p(V_2 - V_1) + W_s \quad \text{—— (iv)}$$

Putting value of W from (ii) to (iv)

$$\cancel{Q} - [p(V_2 - V_1) + W_s] = U_2 - U_1 \quad \text{—— (v)}$$

$$\therefore -W_s = (U_2 - U_1) + p(V_2 - V_1)$$

$p(V_2 - V_1)$ can be written as $(p_2V_2 - p_1V_1)$ as $p_1 = p_2 = p$

and (v) becomes

$$-W_s = (U_2 - U_1) + (p_2V_2 - p_1V_1) = (U_2 + p_2V_2) - (U_1 + p_1V_1)$$

$$\therefore W_s = -[(U_2 + p_2V_2) - (U_1 + p_1V_1)] = -(H_2 - H_1)$$

Thus the ~~stirre~~ stirrer work is equals to increase in enthalpy. The negetive sign indicates that the stirrer work is negetive for the substance.

3) During this process $Q = +3.0$ kJ. As the vessel is rigid, the mixture does not sweep any volume and so, $W = 0$.

By the 1st law of TD: $Q - W = E_{2(i)} - E_{1(i)}$ —— (1)

where, $E_1(i)$ and $E_2(i)$ are the energies of the system initialy and finally during process (i), respectively.

From data: $E_1(i) = 10$ kJ

Now, substituting values of $Q, W, E_{1(i)}$ in ① we get,—

$$3-0 = E_{2(i)} - 10$$

$$\therefore E_{2(i)} = 13 \text{ kJ} \quad \langle Ans^{i} \rangle$$

During process (ii), which is adiabatic one, $Q=0$. The electrical work supplied by the spark is negligible and that the displacement work is zero, the net $W=0$. Appling 1st law of TD for this process gives us,—

$$Q - W = E_{2(ii)} - E_{1(ii)}$$

$$\text{or, } 0 - 0 = E_{2(ii)} - 13 \quad [\because E_{2(i)} = E_{1(ii)}]$$

$$\therefore E_{2(ii)} = 13 \text{ kJ} \quad \langle Ans^{ii} \rangle$$

During process (iii), $Q = -32$ kJ and $W = 0$. Again appling 1st law of TD,—

$$Q - W = E_{2(iii)} - E_{1(iii)}$$

$$\text{or, } -32 - 0 = E_2(iii) - 13 \quad [\because E_{1(iii)} = E_{2(ii)}]$$

$$\therefore E_{2(iii)} = -19 \text{ kJ} \quad \langle Ans^{iii} \rangle$$

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4) ~~The~~ Appling 1st law of TD for process (i) gives,-

$$Q_{(i)} - W_{(i)} = E_{2(i)} - E_{1(i)}$$

Here, $Q_{(i)} = 0$ because the process is an adiabatic one. The energy of the system is said to depend only on temperature and chemical aggregation. These parameters at the ~~begin~~ beginning and end of ~~pros. ①~~ process (i) is same for Question 3 and 4.

$\therefore E_{2(i)} - E_{1(i)}$ will have the same value as it has in solution of Question 3

$\therefore$ ① becomes $\Rightarrow$ $0 - W_{(i)} = (13-19)\ kJ$

$\therefore W_{(i)} = -3\ kJ$

Appling 1st law to the process (ii)

$$Q_{(ii)} - W_{(ii)} = E_{2(ii)} - E_{1(ii)} \quad \text{②}$$

Here, $Q_{(ii)} = 0 = W_{(ii)}$. The temperature (200°C) and the ~~chin~~ chemical aggregation (unburnt mix) ~~are similar to~~ at the beginning of process (ii) are similar to solⁿ of question 3. $\therefore E_{1(ii)} = 13\ kJ$

From ② we get,

$$0 - 0 = E_{2(ii)} - 13\ kJ$$

$$\therefore E_{2(ii)} = 13\ kJ$$

Appling 1st law of TD to process (iii)

$$Q_{(iii)} - W_{(iii)} = E_{2(iii)} - E_{1(iii)} \quad \text{③}$$

At the end of process (iii) the system comprises combustion products at a temperature of 120°C which is similar to the process (iii) in the solⁿ of Q3 $\therefore E_{2(iii)} = -19\ kJ$

eq from ③ $\longrightarrow$ $Q_{(iii)} - 31 = -19 - 13$

$$\therefore Q_{(iii)} = -1\ kJ$$

During process (iv), $W_{(iv)} = 0$ and $Q_{(iv)}$ is $-ve$ (cooling)

From 1st law, —

$$Q_{(iv)} - W_{(iv)} = E_{2(iv)} - E_{1(iv)}$$

$$\text{or, } Q_{(iv)} - 0 = E_{2(iv)} - (-19) \quad [\because E_{1(iv)} = E_{2(iii)}]$$

$$\therefore E_{2(iv)} = -19 \text{ kJ} + Q_{(iv)}$$

$\because Q_{(iv)}$ is negetive ; $\therefore E_{2(iv)}$ is less than -19 kJ

Although the temp. at the end of the pros. (iv) is equals to the initial condition, the energy of the system is < -19 kJ which is not equals to its initiall value of 10 kJ.

$\therefore$ The end of pros. (iv) is not same as the beginning of pros. (i)

$\therefore$ The system has NOT undergone a cyclic pros.

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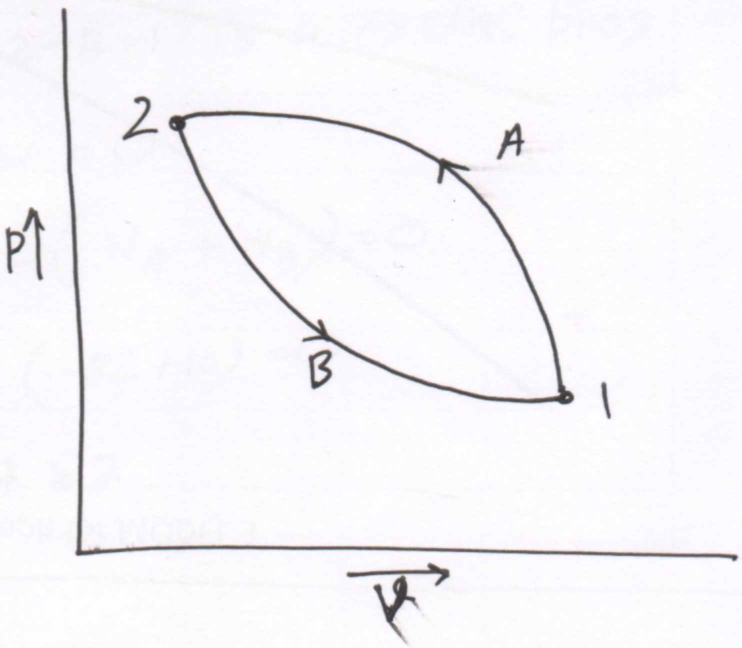
5)

Compression process : 1-A-2

Expansion process : 2-B-1

1-A-2-B-1 is a thermodynamic cycle



Appling 1st law for compression process 1-A-2, —

$$Q_A - W_A = \Delta U_A = U_2 - U_1$$

From given data: $Q_A = -45\ kJ$ and $W_A = -82\ kJ$

$\therefore -45 - (-82) = U_2 - U_1$

$\therefore U_2 - U_1 = 37\ kJ$

Appling 1st low of TD for expansion process 2-B-1, —

$$Q_B - W_B = \Delta U_B = U_1 - U_2$$

or, $Q_B - W_B = -(U_2 - U_1) = -37\ kJ$

From given data: $W_B = 10\ kJ$

$\therefore Q_B - 10 = -37$

$\therefore Q_B = -27\ kJ$

$\therefore$ During the process 27 kJ of heat transferd out from the system. <Ans>

$\boxed{*}$ Alternatively, 1-A-2-B-1 is a cyclic pros.

$\therefore \Sigma Q - \Sigma W = 0$

or, $(Q_A + Q_B) - (W_A + W_B) = 0$

or, $(-45 + Q_B) - (-82 + 10) = 0$

$\therefore Q_B = -27\ kJ$

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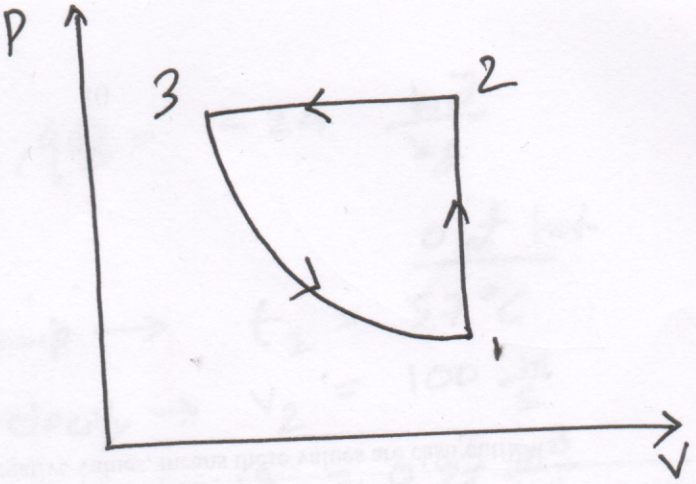
6)



During constant volume heat addition process 1-2

$$Q_{1-2} = +200 \text{ kJ}$$
$$W_{1-2} = 0$$

During constant pressure cooling process 2-3

$$Q_{2-3} = -70 \text{ kJ}$$
$$W_{2-3} = -50 \text{ kJ}$$

During the adiabatic process 3-1

$$Q_{3-1} = 0$$
$$W_{3-1} = ?$$

These consicutive processes make the system underge a ~~cyclic~~ thermodynamic cycle. From the 1st law of TD,

$$\Sigma Q - \Sigma W = 0$$

or, $(Q_{1-2} + Q_{2-3} + Q_{3-1}) - (W_{1-2} + W_{2-3} + W_{3-1}) = 0$

or, $(200 - 70 + 0) - (0 - 50 + W_{3-1}) = 0$

$\therefore \; W_{3-1} = +180 \text{ kJ}$ <Ans>

4) Given data

Heat transfer, $q = -24 \ \frac{kJ}{kg}$

inlet		outlet
$t_1 = 227^\circ C$	← temp →	$t_2 = 57^\circ C$
$V_1 = 50 \ \frac{m}{s}$	← velocity →	$V_2 = 100 \ \frac{m}{s}$
$v_1 = 0.78 \ \frac{m^3}{kg}$	← density →	$v_2 = 0.97 \ \frac{m^3}{kg}$
$p_1 = 5\times10^5 \ \frac{N}{m^2}$	← pressure →	$p_2 = 1\times10^5 \ \frac{N}{m^2}$
$z_2 = 5 \ m$	← elevation →	$z_1 = 0$

Internal energy, $u = u(t)$ and $c_v = 0.7 \ \frac{kJ}{kg\,K}$

SS, SFEE for unit mass

$$q - w = \Delta\left[h + \frac{V^2}{2} + gz\right]$$

$$\cancel{= (h_1 - h_2) + \frac{V_1^2 - V_2^2}{2}}$$

$$\therefore q - w = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \quad \text{— (1)}$$

$\boxed{PTO}$

now

$$h_2 - h_1 = (u_2 + p_2 v_2) - (u_1 + p_1 v_1) = (u_2 - u_1) + (p_2 v_2 - p_1 v_1)$$

$$= c_p (t_2 - t_1) + (p_2 v_2 - p_1 v_1)$$

$$\cancel{= 0.7(57 - 227)}$$

$\boxed{PTO}$

now

$$h_2 - h_1 = (u_2 + p_2 v_2) - (u_1 + p_1 v_1)$$

$$= (u_2 - u_1) + (p_2 v_2 - p_1 v_1)$$

$$= c_v (t_2 - t_1) + (p_2 v_2 - p_1 v_1)$$

$$= \left[0.7 \times 10^3 (57 - 227) + (10^5 \times 0.97 - 5 \times 10^5 \times 0.78)\right] \frac{J}{kg}$$

$$= -412 \times 10^3 \frac{J}{kg}$$

$$\frac{V_2^2 - V_1^2}{2} = \frac{100^2 - 50^2}{2} \frac{J}{kg} = 3.75 \times 10^3 \frac{J}{kg}$$

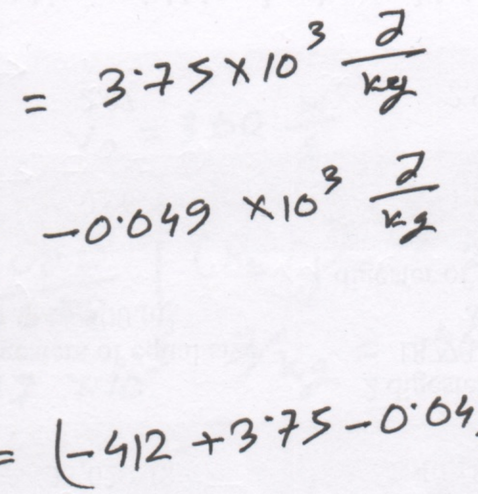
$$g(z_2 - z_1) = 9.81(5) \frac{J}{kg} = -0.049 \times 10^3 \frac{J}{kg}$$

$\therefore$ ① becomes

$$(-24 \times 10^3) \frac{J}{kg} - w = (-412 + 3.75 - 0.049) \times 10^3 \frac{J}{kg}$$

$$w = 384.3 \times 10^3 \frac{J}{kg} = 384.3 \frac{kJ}{kg} \text{ <Ans>}$$

9) ss, SFEE for unit time



$$\dot{Q} - \dot{W} = \dot{m}\left[\Delta h + \frac{V^2}{2} + gz\right]$$

$$= \dot{m}\left[(h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right]$$

Given: $\dot{m} = 5000 \frac{kg}{h} = \frac{5000}{3600} \frac{kg}{s}$

$Q \approx 0$

$\dot{W} = 550 \text{ kW} = 550 \times 10^3 \frac{J}{s}$

$\boxed{\text{PTO}}$

~~Case (a)~~ $\cancel{z_2 = z_1 \text{ and } V_1 = 0}$

$$\cancel{0 - 550\times10^3 = \frac{5000}{3600}\times\left[(h_2-h_1)+\frac{360^2-0}{2}+0\right]}$$

$$\cancel{-550\times10^3}$$

$$\cancel{\therefore h_2-h_1 = -460.77\times10^3 \; J/kg}$$

Case (a) $\rightarrow z_2 = z_1, \; V_1 = 0, \; V_2 = 360 \frac{m}{s}$

$$\therefore 0 - 550\times10^3 = \frac{5000}{3600}\left[(h_2-h_1)+\frac{360^2-0}{2}+0\right]$$

$$\therefore h_2-h_1 = -460.77\times10^3 \; J/kg = -460.77 \frac{kJ}{kg}$$

Case (b) $\rightarrow z_2 - z_1 = 3\,m, \; V_1 = 60\frac{m}{s}, \; V_2 = 360\frac{m}{s}$

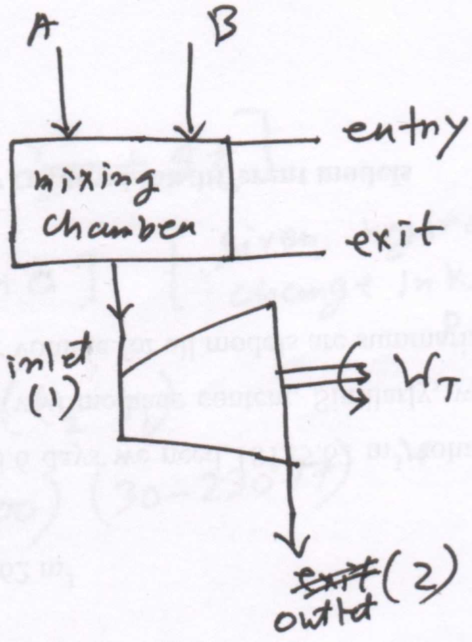
$$\therefore 0 - 550\times10^3 = \frac{5000}{3600}\left[(h_2-h_1)+\frac{360^2-60^2}{2}-9.81\times3\right]$$

$$\therefore h_2-h_1 = -458.94\times10^3 \frac{J}{kg} = -458.94\frac{kJ}{kg}$$

g)

$\boxed{\text{PTO}}$

9) Refer to the figure beside



$\dot{m}_A = 0.8 \frac{kg}{s}$, $p_A = 15\times10^5 \frac{N}{m^2}$, $t_A = 250^\circ C$

$\dot{m}_B = 0.5 \frac{kg}{s}$, $p_B = 15\times10^5 \frac{N}{m^2}$, $t_B = 200^\circ C$

$Q_{mixing} = 0 = W_{mixing}$

$p_1 = 10$ bar

$t_1 = ?$ $\quad t_2 = 30^\circ C$

$h = h(t)$ and $u = u(t)$

$C_v = 0.718 \frac{kJ}{kg\,K}$ $\quad C_p = 1 \frac{kJ}{kg\,K}$

SS, SFEE for mixing chamber with multistream

$$Q - W = \sum_{exit} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{\cancel{inlet}\ entry} \dot{m}\left(h + \frac{V^2}{2} + gz\right)$$

$$\text{or, } 0 - 0 = \sum_{exit} \dot{m}(h + 0 + 0) - \sum_{\cancel{inlet}\ entry} \dot{m}(h + 0 + 0)$$

$$\text{or, } 0 = (\dot{m}_A + \dot{m}_B)h_1 - (\dot{m}_A h_A + \dot{m}_B h_B)$$

Suffix 1 denotes exit of mixing chamber & ~~entry~~ inlet of turbine

$$\text{or, } \dot{m}_A(h_1 - h_A) + \dot{m}_B(h_1 - h_B) = 0$$

$$\text{or, } \dot{m}_A C_p(t_1 - t_A) + \dot{m}_B C_p(t_1 - t_B) = 0$$

$$\text{or, } \dot{m}_A(t_1 - t_A) + \dot{m}_B(t_1 - t_B) = 0$$

$$\text{or, } 0.8(t_1 - 250) + 0.5(t_1 - 200) = 0$$

$$\therefore t_1 = 230.77^\circ C$$

SS, SFEE for turbine

$$\dot{Q}_T - \dot{W}_T = \dot{m}\,\Delta\left[h + \frac{V^2}{2} + gz\right]$$

$$\text{or, } 0 - \dot{W}_T = \dot{m}\left[\Delta h + 0 + 0\right] \quad \left[\begin{array}{l}\text{given, neglect}\\ \text{change in K.E. and}\\ \text{P.E.}\end{array}\right]$$

$$\text{or, } -\dot{W}_T = (\dot{m}_A + \dot{m}_B)(C_p)(t_2 - t_1)$$

$$\text{or, } -\dot{W}_T = (0.8 + 0.5)(1000)(30 - 230.77) \text{ W}$$

$$\therefore \dot{W}_T = 261 \text{ kW}$$

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10) Inlet: $d_1 = 0.15\text{ m}$, $\dot{m} = 4000\,\frac{\text{kg}}{\text{h}} = \frac{4000}{3600}\,\frac{\text{kg}}{\text{s}}$, $\cancel{\vartheta_1 = 0.285\text{ kg}}$

$\vartheta_1 = 0.285\,\frac{\text{m}^3}{\text{kg}}$, $V_1 = ?$

exit: $d_2 = 0.25\text{ m}$, $\vartheta_2 = 15\,\frac{\text{m}^3}{\text{kg}}$, $V_2 = ?$

Continuity eq^n:

$$\dot{m} = A_1 V_1/\vartheta_1 = A_2 V_2/\vartheta_2$$

$$\text{or, } \frac{4000}{3600} = \frac{\frac{\pi}{4}(0.15)^2 V_1}{0.285} = \frac{\frac{\pi}{4}(0.25^2) V_2}{15}$$

$$\therefore V_1 = 17.9\,\frac{\text{m}}{\text{s}} \text{ and } V_2 = 339\,\frac{\text{m}}{\text{s}}$$
