

Aug 28, 2020

CS 228 Minor Autumn 2020 Assignment 1

1. Consider the 3 statements given.
 - (a) X : Y is the murderer
 - (b) Y : Z is the murderer
 - (c) Z : Neither X nor Y is the murderer
 One of X, Y, or Z has committed the murder, and only one of the above statements is true. Using propositional logic satisfiability, find out the murderer.
2. Draw the parse tree for the formula $\varphi = (r \rightarrow \neg s) \vee \neg(p \rightarrow (\neg q \vee (r \wedge (p \rightarrow (s \vee r))))))$.
3. Write the formula φ given above in CNF. Call it ψ . Is ψ a Horn formula?
4. Using resolution, check for satisfiability of φ in question 2.
5. Consider the formula $\varphi = (p \rightarrow (q \rightarrow p))$. Show that
 - (a) $p, \neg q \vdash \varphi$
 - (b) $\neg p, \neg q \vdash \varphi$
 - (c) $\neg p, q \vdash \varphi$
 - (d) $p, q \vdash \varphi$
 As seen in the proof of completeness, combine all 4 proofs to obtain a proof $\vdash \varphi$.
6. Which of the two formulae implies the other?

$$\varphi_1 = (p \leftrightarrow (q \leftrightarrow r))$$

$$\varphi_2 = ((p \wedge (q \wedge r)) \vee ((\neg p) \wedge ((\neg q) \wedge (\neg r))))$$

Is it the case that $\varphi_1 \models \varphi_2$, or $\varphi_2 \models \varphi_1$, both, or neither? Recall that \models stands for semantic entailment (hence you argue using assignments).

7. Consider a formula $\varphi = C_1 \wedge C_2 \wedge \dots \wedge C_n$ where each clause C_i is of the form $(\top \rightarrow \alpha)$ or $(\alpha_1 \wedge \dots \wedge \alpha_n \rightarrow \beta)$ or $(\gamma \rightarrow \perp)$ where $\alpha, \alpha_i, \beta, \gamma$ are literals. A logician wishes to apply *HornSAT* to this formula φ by renaming negative literals (if any) with fresh positive literals. Thus, if any $\alpha, \alpha_i, \beta, \gamma$ was of the form $\neg p$, the logician will replace that $\neg p$ with a fresh variable p' . The logician claims that he can check satisfiability of φ correctly by applying *HornSAT* on the new formula (call it φ') in the following way: φ is satisfiable iff *HornSAT* concludes that φ' is satisfiable, and φ is unsatisfiable iff *HornSAT* concludes that φ' is unsatisfiable.

- (a) Illustrate the logician's approach on an example (that is, take a φ , and then show how the corresponding φ' will look like).
- (b) Do you agree with the logician? If you do, clearly explain why. Your argument must work for all φ and the respective φ' . If you disagree with the logician, explain why, using your φ and φ' from part (a).