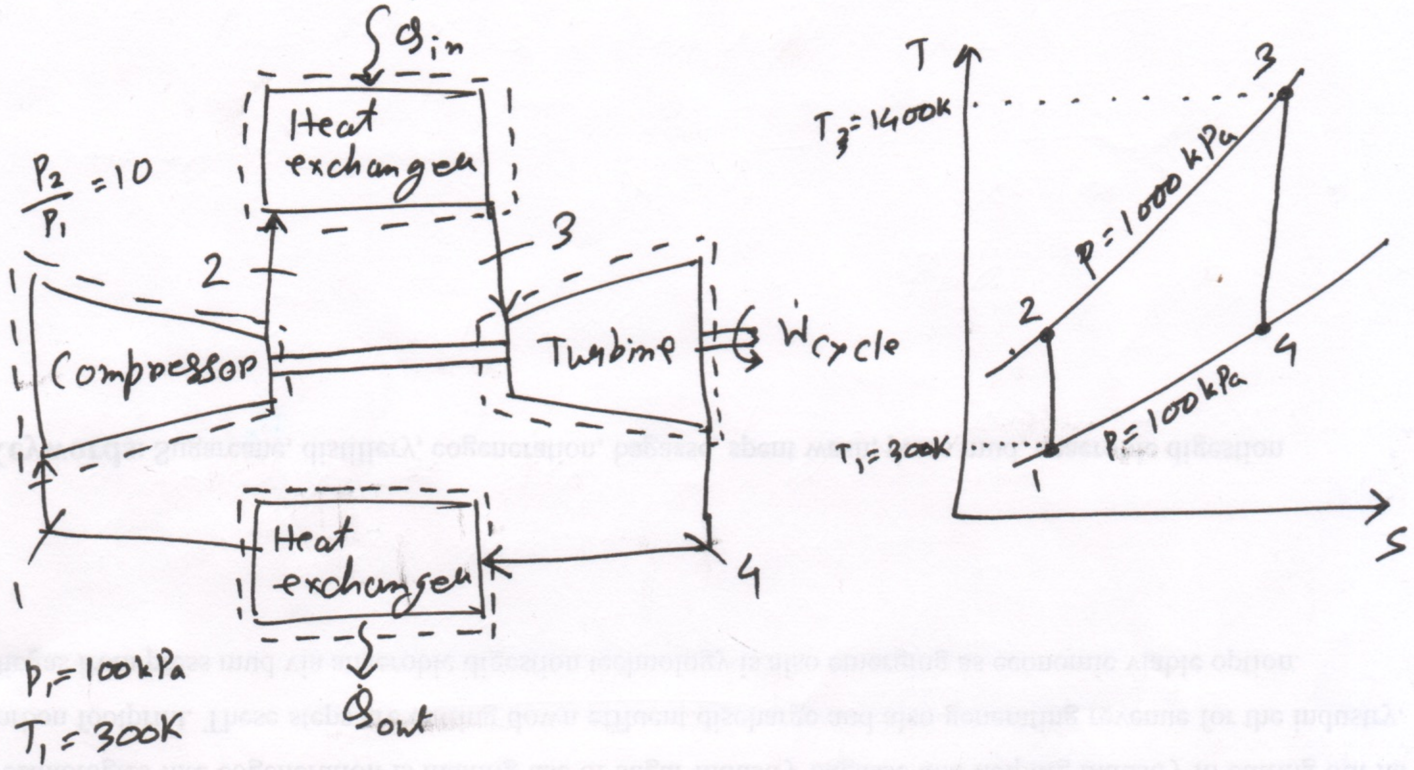
EN203 — prob. set #5 — sol^n.

1> An ideal air standard Brayton cycle operates with given compressor inlet conditions, given turbine inlet temp. and a known compressor pres. ratio.



Assumptions → ① Each component is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.

② The turbine and compressor process are isentropic.

③ There are no pres. drops for flow through the heat exchangers

④ Kinetic and potential energy effects are negligible

⑤ The working fluid is air modeled as an ideal gas

The analysis begins by determining the specific enthalpy at each numbered state of the cycle. At state 1, the temp. is 300 K. From table $h_1 = 300.19$ kJ/kg
$p_{r1} = 1.386$

Since the compressor pros. is isentropic, the following relationship can be used to find h_2

$$p_{r2} = \frac{p_2}{p_1} p_{r1} = (10)(1.386) = 13.86$$

From table $\Rightarrow$ $h_2 = 579.9$ kJ/kg

The temp. at state 3 is given as $T_3 = 1400$ K. With the temp., the specific enthalpy at state 3 from table

$h_3 = 1515.4$ kJ/kg
$p_{r3} = 450.5$

The specific enthalpy at state 4 is found by using the isentropic relation $\rightarrow$ $p_{r4} = p_{r3} \cdot \frac{p_4}{p_3} = (450.5)\left(\frac{1}{10}\right) = 45.05$

From table: $h_4 = 808.5$ kJ/kg

(a) Thermal efficiency, $\eta = \dfrac{(\dot{W}_t/\dot{m}) - (\dot{W}_c/\dot{m})}{\dot{Q}_{in}/\dot{m}} = \dfrac{(h_3 - h_4) - (h_2 - h_1)}{h_3 - h_2}$

$= 45.7\%$

(b) The Back work ratio $= \dfrac{\dot{W}_c/\dot{m}}{\dot{W}_t/\dot{m}} = \dfrac{h_2 - h_1}{h_3 - h_4} = 39.6\%$

(c) Net power developed, $\dot{W}_{cycle} = \dot{m}\left[(h_3 - h_4) - (h_2 - h_1)\right]$

To evaluate the net power requires the mass flow rate $\dot{m}$, which can be determined from the volumetric flow rate and specific vol. at the compressor inlet as follows

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{(AV)_1}{(\bar{R}/M)T_1/p_1} = \frac{(AV)_1\, p_1}{(\bar{R}/M)T_1}$$

$$\therefore \dot{m} = \frac{(5\ m^3/s)(100 \times 10^3\ N/m^2)}{\left(\frac{8314}{28.97}\ \frac{N \cdot m}{kg \cdot K}\right)(300\ K)} = 5.807\ kg/s$$

$$\therefore \dot{W}_{cycle} = 5.807\ \frac{kg}{s} \cdot (706.9 - 279.7)\ \frac{kJ}{kg} = 2481\ kW$$

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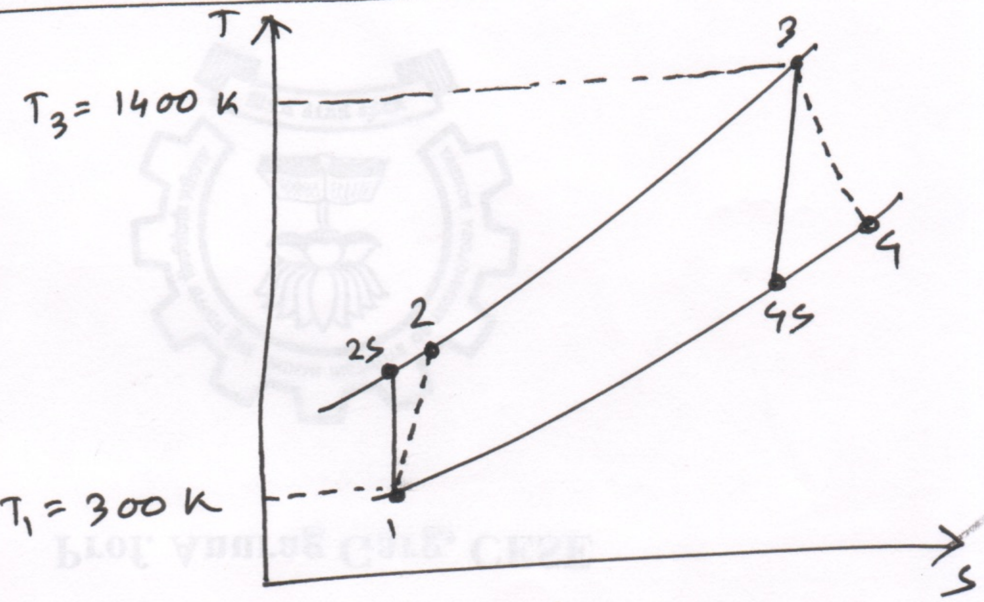
2)



(a) Thermal efficiency, $\eta = \frac{(\dot{W}_t/\dot{m}) - (\dot{W}_c/\dot{m})}{\dot{Q}_{in}/\dot{m}}$

~~the work terms in the~~

In this case the turbine & compressor have 80% η

Turbine work per unit of mass, $\frac{\dot{W}_t}{\dot{m}} = \eta_t \left(\frac{\dot{W}_t}{\dot{m}}\right)_s$

(work done by the system)

$$= \frac{80}{100}\left(706.9\ \frac{kJ}{kg}\right)$$

$$= 565.5\ \frac{kJ}{kg}$$

For the compressor, work per unit mass, $\frac{\dot{W}_c}{\dot{m}} = \frac{(\dot{W}_c/\dot{m})_s}{\eta_c}$

(work done on the system)

$= \frac{279.7 \text{ kJ/kg}}{80/100}$

$= 349.6 \frac{\text{kJ}}{\text{kg}}$

From the T-s diagram

$\frac{\dot{W}_c}{\dot{m}} = h_2 - h_1 \Rightarrow h_2 = h_1 + \dot{W}_c/\dot{m} = (300.19 + 349.6) \frac{\text{kJ}}{\text{kg}}$

$= 649.8 \frac{\text{kJ}}{\text{kg}}$

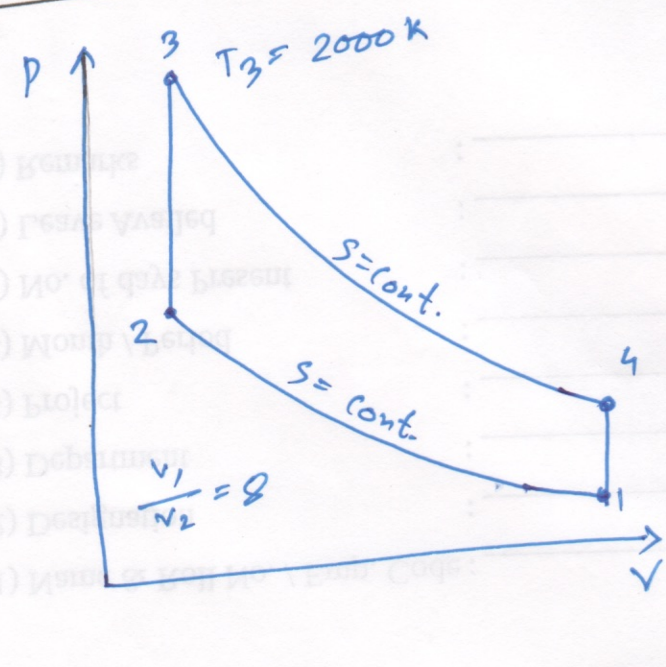
$\frac{\dot{Q}_{in}}{\dot{m}} = h_3 - h_2 = (1515.4 - 649.8) \frac{\text{kJ}}{\text{kg}} = 865.6 \frac{\text{kJ}}{\text{kg}}$

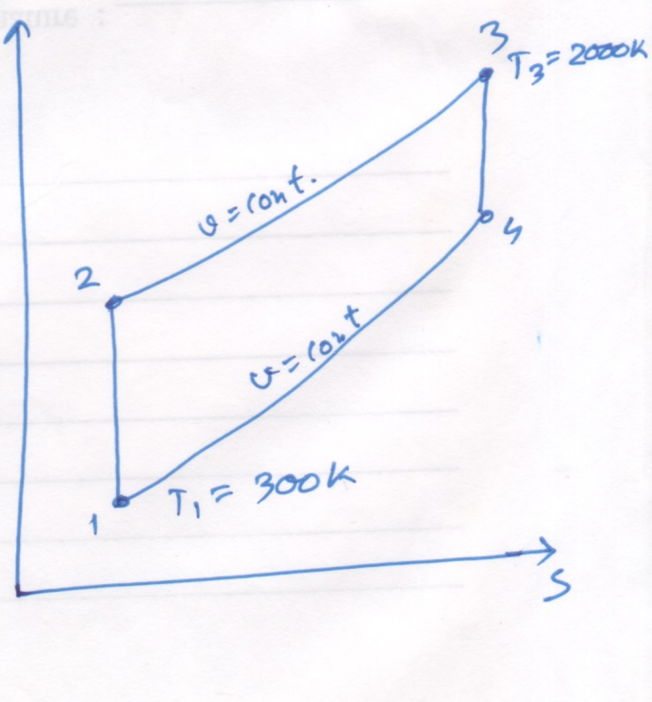
now $\eta = \frac{565.5 - 349.6}{865.6} = 24.9\%$ (a)

back work ratio $= \frac{349.6}{565.5} = 61.8\%$ (b)

$\dot{W}_{cycle} = (5.807 \frac{\text{kg}}{\text{s}})(565.5 - 349.6) \frac{\text{kJ}}{\text{kg}} = 1254 \text{ kW}$

3)





Assumptions ① The air in the piston-cylinder assembly is a closed system

② The compression and expansion pros. are adiabatic

③ All pros. are internally rev.

④ Air is an ideal gas

⑤ KE & PE ≈ 0

state ① $\longrightarrow$ $T_1 = 300\,K$, $P_1 = 1\,bar$

From table: $u_1 = 214.07\ \frac{kJ}{kg}$, $v_{r1} = 621.2$

now $v_{r2} = \frac{V_2}{V_1} v_{r1} = \frac{v_{r1}}{r} = \frac{621.2\ m^3/kg}{8} = 77.63$

Interpolating v_{r2} in table: $T_2 = 673\,K$, $u_2 = 491.2\ kJ/kg$

with ideal gas eq^n $\Rightarrow$ $p_2 = p_1 \frac{T_2}{T_1} \frac{V_1}{V_2}$

$= (1\,bar)\left(\frac{673\,K}{300\,K}\right) 8$

$= 17.95\ bar$

pros 2-3: cons. vol. pros $\longrightarrow$ ideal gas eq^n $\quad p_3 = P_2\left(\frac{T_3}{T_2}\right) = 17.95\ bar \times \frac{2000\,K}{673\,K}$

$= 53.3\ bar$

state ③ $\longrightarrow$ $T_3 = 2000\,K$, $p_3 = 53.3\ bar$

From table: $u_3 = 1678.7\ \frac{kJ}{kg}$, $v_{r3} = 2.776$

pros 3-4: isotropic expansion $\longrightarrow$

$v_{r4} = v_{r3} \frac{V_4}{V_3} = v_{r3} \frac{V_1}{V_2} = 22.21$

Interpolating table with v_{r4} gives $T_4 = 1043\,K$ $\quad u_4 = 795.8\ \frac{kJ}{kg}$

now, $p_4 = p_1 \frac{T_4}{T_1} = 1\,bar\left(\frac{1043\,K}{300\,K}\right) = 3.48\,bar$

(b) $\eta = 1 - \frac{Q_{41}/m}{Q_{23}/m} = 1 - \frac{u_4 - u_1}{u_3 - u_2} = 51\%$

(c) mean effective pres (mep) $= \frac{W_{cycle}}{V_1 - V_2} = \frac{W_{cycle}}{V_1(1 - 1/r)}$

$$W_{cycle} = m\left[(u_3 - u_4) - (u_2 - u_1)\right]$$

$$m = \frac{p_1 V_1}{\left(\frac{\bar{R}}{M}\right)T_1} = \frac{(1\,bar)(560\,cm^3)}{\left(\frac{8.314}{28.97}\ \frac{kJ}{kg\,K}\right)(300\,K)} \quad \cancel{6.5\times 10^{-4}\,kg}$$

[careful with the units]

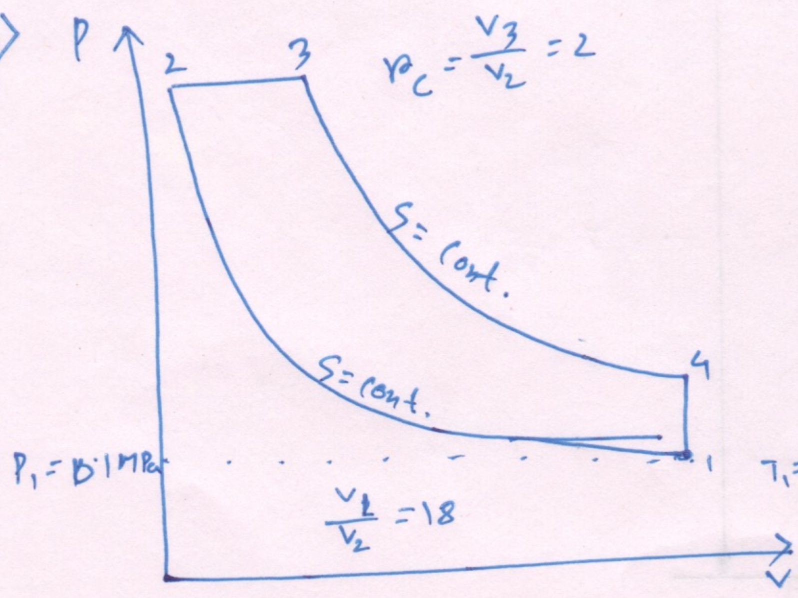
$$= \frac{(10^5\,Pa)(560\times 10^{-6}\,m^3)}{\left(\frac{8314}{28.97}\ \frac{J}{kg\,K}\right)(300\,K)}$$

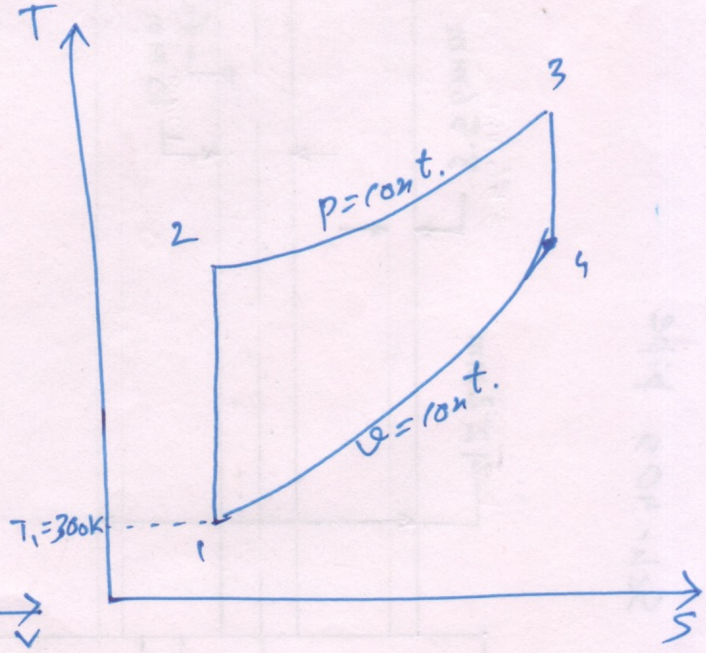
$$= 6.5 \times 10^{-4}\ kg$$

$$mep = \frac{6.5\times 10^{-4}\left[(1678.7 - 795.8) - (491.2 - 214.07)\right] kJ}{(560\times 10^{-6}\,m^3)(1 - 1/18)}$$

$$= 8.04\ bar$$

4)





Assumptions [see previous solⁿ]

(a) $T_1 = 300K$, $P_1 = 0.1 MPa$ —— from table we have, —

$u_1 = 214.07 \frac{kJ}{kg}$ $\quad v_{r1} = 621.2 \frac{m^3}{kg}$

now, $v_{r2} = \frac{v_2}{v_1} v_{r1} = \frac{v_{r1}}{r} = \frac{621.2 \ m^3/kg}{18} = 34.51 \frac{m^3}{kg}$

Interpolating the table for v_{r2} we get: $T_2 = 898.3 \ K$

$h_2 = 930.98 \ kJ/kg$

with ideal gas eqⁿ: $P_2 = P_1 \frac{T_2}{T_1} \frac{v_1}{v_2} = (0.1 \ MPa)\left(\frac{898.3}{300}\right)(18)$

$\therefore P_2 = 5.39 \ MPa$

Pros 2-3: const. pres.; ideal gas eqⁿ gives: $T_3 = \frac{v_3}{v_2} T_2$

$= r_c T_2$

$= 2 (898.3 K)$

$= 1796.6 K$

(b) $T_3 = 1796.6 \ K$, $P_3 = P_2 = 5.39 \ MPa$ — from the table $\longrightarrow h_3 = 1999.1 \frac{kJ}{kg}$, $v_{r3} = 3.97 \frac{m^3}{kg}$

~~For isotropic expansion i.e. pres~~

pros 3-4: Isentropic expansion $\rightarrow v_{r4} = \frac{v_4}{v_3} v_{r3}$

$v_{r4} = \frac{v_4}{v_3} v_{r3} = \frac{v_4}{v_2} \frac{v_2}{v_3} v_{r3} = \frac{v_1}{v_2} \cdot \frac{v_2}{v_3} v_{r3}$ $\quad [\because v_1 = v_4]$

$= r \cdot \frac{1}{r_c} \cdot v_{r3}$

$= 18 \times \frac{1}{2} \times 3.97 \frac{m^3}{kg}$

$= 35.73 \frac{m^3}{kg}$

Interpolating the table with v_{r4} we get, $u_4 = 664.3\ \frac{kJ}{kg}$

$T_4 = 887.7\ K$

pros 4-1 : cont. vol. ; ideal gas eqn gives $\Rightarrow p_4 = p_1 \frac{T_4}{T_1} = 0.3\ MPa$

(b) Thermal efficiency, $\eta = 1 - \frac{Q_{41}/m}{Q_{23}/m} = 1 - \frac{u_4 - u_1}{h_3 - h_2} = 57.8\%$

(c) mean effective pres, $mep = \frac{W_{cycle}/m}{v_1 - v_2} = \frac{W_{cycle}/m}{v_1 (1 - 1/r_c)}$

now $\frac{W_{cycle}}{m} = \frac{Q_{23}}{m} - \frac{Q_{41}}{m} = (h_3 - h_2) - (u_4 - u_1) = 617.9\ \frac{kJ}{kg}$

sp. vol. at state 1, $v_1 = \frac{(R/M)\, T_1}{p_1} = 0.861\ \frac{m^3}{kg}$

$\therefore\ mep = 0.76\ MPa$

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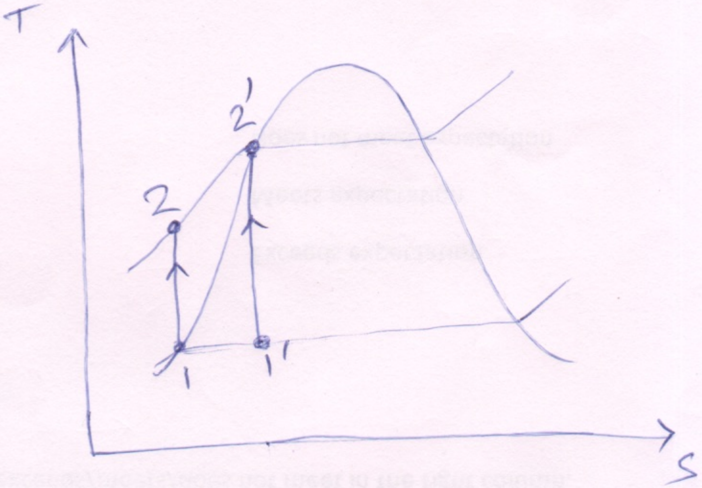
5)



Case (a) corresponds to 1'-2'

case (b) " " 1-2

- For vapour compression [case (a)], the dryness fraction at state 1' should be first determined
- 1'-2' is an isentropic process

$s'_1 = s'_2$ or, $3.3605 = 1.3027 + x_1' (7.3598 - 1.3027)$

$\therefore\ x_1' = 0.339734$