1) Define propositions as follows.

$Ang(A)$: A is an angel

$Nor(A)$: A is normal

$A(Y)$: A said Yes

$A(N)$: A said No

We know

$$\neg(Ang(A) \wedge Ang(B)) \wedge \neg(Nor(A) \wedge Nor(B)) \wedge$$
$$\neg(Ang(A) \wedge Nor(A)) \wedge \neg(Ang(B) \wedge Nor(B)).$$
$$A(Y) \Leftrightarrow \neg A(N).$$

Call this $\mathcal{Z}$.

If A said Yes we have

φ_1:

$$A(Y) \wedge [Ang(A) \Rightarrow Nor(B)] \wedge$$
$$[Nor(A) \Rightarrow (Nor(B) \vee Ang(B))]$$

↳ Disjunction to show speaking true/false.

$\mathcal{Z} \wedge \varphi_1$ does not give a unique soln

$(Ang(A), Nor(B)$ or $Nor(A), Ang(B))$

So we need to consider the case when A said No.

φ_2:

$$A(N) \wedge [Ang(A) \Rightarrow Ang(B)] \wedge$$
$$[Nor(A) \Rightarrow (Ang(B) \vee Nor(B))]$$

$\mathcal{Z} \wedge (\varphi_1 \vee \varphi_2)$

$\mathcal{Z} \wedge \varphi_2$ gives a unique soln: $Nor(A)$ and $A(N)$ hence $Ang(B)$.

2) A bit of parity checking, easy though.

3) Let $Kni(A)$, $Kna(A)$ and $Nor(A)$ be propositional variables.

We know

$$\left.\begin{array}{l} Kni(A) \vee Kna(A) \vee Nor(A) \wedge \\ Kni(A) \Leftrightarrow \neg(Nor(A) \wedge Kna(A)) \wedge \\ Kna(A) \Leftrightarrow (\ ) \wedge \quad Nor(A) \Leftrightarrow (\ ) \end{array}\right\} \varphi_1.$$

From what is given,

$$\left.\begin{array}{l} (Kni(A) \rightarrow Kni(B)) \wedge (Kna(A) \rightarrow \neg Kni(B)) \wedge \\ (Kni(B) \rightarrow Kna(A)) \wedge (Kna(B) \rightarrow \neg Kna(A)). \end{array}\right\} \varphi_2$$

We have to deduce from φ_1 and φ_2 that

$$\underbrace{\neg Kni(A) \wedge Kni(B)}_{\text{A told the truth, not a knight}} \vee \underbrace{\neg Kna(A) \wedge \neg Kni(B)}_{\text{A lied, not a knave}} \vee$$

$\underbrace{\neg Kni(B) \wedge Kna(A)}_{\text{B not a knight said truth}} \quad \vee \quad \underbrace{\neg Kna(B) \wedge \neg Kna(A)}_{\text{B not a knave, lied.}}$

Call the latter formula φ.

S.T $\varphi_1 \wedge \varphi_2 \vdash \psi$

$\varphi_1 \wedge \varphi_2$ Premise

φ_2 $\wedge e$

φ_1 $\wedge e$

$Kni(B)$	assume
$Kna(A)$	MP from φ_2
$\neg Kni(B)$	MP , φ_2
$\bot$	$\bot i$

$\neg Kni(B)$

$Kna(A) \vee \neg Kna(A)$ LEM

$\neg Kni(B) \wedge (Kna(A) \vee \neg Kna(A))$ $\wedge i$ above two lines

$Kna(A)$	$\neg Kna(A)$
$\neg Kni(B) \wedge Kna(A)$	$\neg Kni(B) \wedge \neg Kna(A)$
$(\neg Kni(B) \wedge Kna(A)) \vee$	$(\neg Kni(B) \wedge Kna(A)) \wedge$ $\vee i$
$(\neg Kni(B) \wedge \neg Kna(A))$ $\vee i$	$(\neg Kni(B) \wedge \neg Kna(A))$

$(\neg Kni(B) \wedge Kna(A)) \vee (\neg Kni(B) \wedge \neg Kna(A)) \vee$

$\wedge \vee \ldots$ $\vee i$

4. Write the painful expansion.

5. $\vdash ((p \to q) \to q) \to ((q \to p) \to p)$

1	$(p \to q) \to q$	assume
2	$q \to p$	assume
3	$\neg p$	assume
4	p	assume
5	$\bot$	$\bot i$ 3, 4
6	q	$\bot e$ 5
7	$p \to q$	$\to i$ 4–6
8	q	MP 1, 7
9	p	MP 2, 8
10	$\bot$	$\bot i$ 3, 9
11	$\neg\neg p$	$\neg i$ 3–10
12	p	$\neg\neg e$ 11
13	$(q \to p) \to p$	$\to i$ 2–12
14	$(p \to q) \to q \to (q \to p) \to p$	