Numerical Methods for Conservation Laws

Assignment 2 (System of Linear Equations)

Solve the acoustic equations

$$p_t + K_o(x)u_x = 0 \tag{1}$$

$$\rho_o(x)u_t + p_x = 0 \tag{2}$$

$c_o(x) = \sqrt{(K_o(x)/\rho_o(x))}$ the speed of sound, using **both** first-order and second-order fluctuation-signal algorithm/approach for a system of linear hyperbolic conservation laws with following initial data:

$u(x, 0) = 0.$ and

$$p(x,0) = \begin{cases} \overline{p}\sqrt{1 - ((x - x_o)/\overline{x})^2} & \text{if } \ |x - x_o| < \overline{x}, \\ 0 & \text{otherwise} \end{cases}$$

$x_o = 0.4$, $\overline{x} = 0.075$, $\overline{p} = 0.2$, $\Delta x = 0.005$, $\Delta t = 0.004$, domain $[0, 1]$.

1. $K_o(x) \equiv 1.$
   $\rho_o(x)$ =1.
   Right boundary solid and left boundary open.
   Plot $p(x)$ and $u(x)$ at t= $0, 0.06, 0.15, 0.6$.

2. $K_o(x) \equiv 1.$
   Both left and right boundary open.
   Plot $p(x)$ and $u(x)$ at t= $0.27, 0.37, 0.6$.
   Additionally,
   $$\rho_o(x) = \begin{cases} 1 & \text{if } \ x < 0.6 \\ 3 & \text{if } \ x > 0.6, \end{cases}$$