## Assignment 7

## **Group Number 32**

Software used: Matlab

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## a. Forward in time backward in space(FTBS)

The following scheme is not stable but consistent. So the roundoff errors are not allowing the solution to converge. The following stability analysis is been provided here.

## 1) Backward Scheme in Space

$$f''(x) = \frac{u_i^n - 2 \cdot u_{i-1}^n + u_{i-2}^n}{\Delta x^2}$$

## 2) Forward Scheme in time

$$\frac{u_i^{n+1} - u_i^n}{\alpha \cdot \Delta t}$$

## 3) Assumption for 'r'

$$r = \frac{\alpha \cdot \Delta t}{\Delta x^2}$$

#### 4) Simplification gives

$$u_i^{n+1} = (1+r) \cdot u_i^n - 2 \cdot r \cdot u_{i-1}^n + r \cdot u_{i-2}^n$$

#### 5) Making substitution

$$u_m^n = g_n \cdot e^{i \cdot m\Theta}$$

$$with \quad |g| \leq 1 \quad CFL \ condition$$

## 6) Expression for 'g'

$$g = 1 + 2 \cdot r + 2 \cdot r \cos \Theta - r \cos 2\Theta + \dots + higher order terms$$

Which is unstable for all values of 'r' and 'O'

## b. Du-Fort Frenkel Scheme

Code	Output
clearvars clc fprintf('\nDu-Fort Frankel Scheme applied to 1-D heat conduction problem\n') %Endpoints of rod such that b-a=Length of rod a=0; b=1; % for Ut=alp*Uxx form alp=400/(385*8000); % dx and dt are grid differences dx=input('\nUnit Grid Size dx: '); dt=input('\nUnit Grid Size dx: '); dt=input('\nInput Time Step dt: '); %dx=0.25; %dt=2; x=a:dx:b; t=0:dt:900; %Calculation for size of x and t points [sizx,sizx]=size(x); [sizt,sizt]=size(t); u= zeros(sizt,sizx); %formula for r r=alp*dt/(dx^2); %Initial Condition u(1,:)=25;  %Rod is maintained at 400 C. %Thus setting Boundary conditions u(2:sizt,1)=400; u(2:sizt,1)=400; u(2:sizt,sizx)=400; %Formula is worked out for Du-Fort Frankel Scheme %Format U(i+1,j)=F(U(i,j+1),U(i,j-1),U(i,j)) %Here 'i' represents time step %And 'j' represents spatial variation %Loop is written to get iteration values %For deifining TDMA A=zeros(sizx-2,sizx-2);	a. dy = 0.1m, dt = 1s  Du-Fort Frankel Scheme applied to 1-D heat conduction problem  Unit Grid Size dx: 0.1  Input Time Step dt: 1  Middle length reaches 200 C at t = 678.000000s  b. dy = 0.2m, dt = 1s  Du-Fort Frankel Scheme applied to 1-D heat conduction problem  Unit Grid Size dx: 0.2  Input Time Step dt: 1  Middle length reaches 200 C at t = 634.000000s  c. dy = 0.5m, dt = 1s  Du-Fort Frankel Scheme applied to 1-D heat conduction problem  Unit Grid Size dx: 0.5  Input Time Step dt: 1  Middle length reaches 200 C at t = 606.000000s
A=zeros(sizx-2,sizx-2);	<ul> <li>d. dy = 0.1m , dt = 0.5s</li> <li>Du-Fort Frankel Scheme applied to 1-D heat conduction problem</li> </ul>

```
for i=2:sizt
  %Now based of formula we will write TDMA matrix
  %a represents 1st diagonal below main diagonal
  %b represents main diagonal
  %c represents 1st diagonal above main diagonal
for j=1:sizx-3
 a(1,j)=0;
 c(1,j)=a(1,j);
 A(j+1,j)=a(1,j);
 A(j,j+1)=c(1,j);
end
for j=1:sizx-2
  b(1,j)=(1+2*r);
  A(j,j)=b(1,j);
  if i==2
   d(j,1)=2*r*u(i-1,j)+(1-2*r)*u(i-1,j+1)+2*r*u(i-1,j+2);
   d(j,1)=2*r*u(i-1,j)+(1-2*r)*u(i-2,j+1)+2*r*u(i-1,j+2);
  end
end
%For identifying the size of matrix
[n,j]=size(d);
%Definition taken from Atkinson
beta(1)=A(1);
g(1)=d(1);
%TDMA triangular matrix solution for Lg=b form where
g=UX
for I=2:n
%Definition taken from Atkinson
alpha(I)=A(I,I-1)/beta(I-1);
beta(I)=A(I,I)-alpha(I)*A(I-1,I);
g(l)=d(l)-alpha(l)*g(l-1);
end
%Back substitution for solving g=UX
xu(n)=g(n)/beta(n);
for l=n-1:-1:1
xu(I)=(g(I)-A(I,I+1)*xu(I+1))/beta(I);
end
xu;
%Updating thevalue of u
u(i,2:sizx-1)=[xu];
end
```

Unit Grid Size dx: 0.1

Input Time Step dt: 0.5

Middle length reaches 200 C at t = 678.000000s

#### e. dy = 0.2m, dt = 0.5s

Du-Fort Frankel Scheme applied to 1-D heat conduction problem

Unit Grid Size dx: 0.2

Input Time Step dt: 0.5

Middle length reaches 200 C at t = 633.500000s

#### f. dy = 0.5m, dt = 0.5s

Du-Fort Frankel Scheme applied to 1-D heat conduction problem

Unit Grid Size dx: 0.5

Input Time Step dt: 0.5

Middle length reaches 200 C at t = 605.500000s

#### g. dy = 0.1m, dt = 0.1s

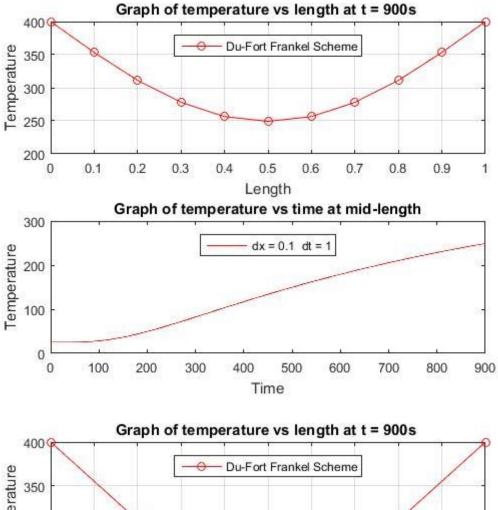
Du-Fort Frankel Scheme applied to 1-D heat conduction problem

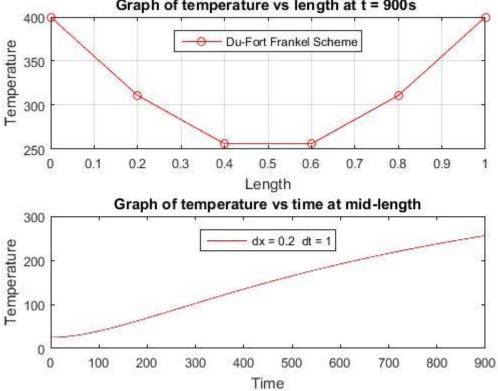
```
%y represents all the values of temperature which
                                                            Unit Grid Size dx: 0.1
change for t>0 at mid
%midlen
                                                           Input Time Step dt: 0.1
y=u(:,round(sizx/2));
figure
                                                           Middle length reaches 200 C at t = 677.700000s
subplot(2,1,2);
plot(t,y,'-r')
title('Graph of temperature vs time at mid-length')
                                                           h. dy = 0.2m, dt = 0.1s
xlabel('Time')
ylabel('Temperature')
                                                           Du-Fort Frankel Scheme applied to 1-D heat conduction
legend(['dx = ',num2str(dx),' dt =
                                                           problem
',num2str(dt)],'Location','north')
                                                            Unit Grid Size dx: 0.2
%Additional information
y=u(sizt,:);
                                                           Input Time Step dt: 0.1
%represents temperature variation in bar at 900s
subplot(2,1,1);
                                                           Middle length reaches 200 C at t = 632.900000s
plot(x,y,'-ro')
title('Graph of temperature vs length at t = 900s')
xlabel('Length')
                                                                dy = 0.5m, dt = 0.1s
ylabel('Temperature')
legend('Du-Fort Frankel Scheme', 'Location', 'north')
                                                           Du-Fort Frankel Scheme applied to 1-D heat conduction
grid on
                                                            problem
%'k' Finds the values where we get 200 C
                                                           Unit Grid Size dx: 0.5
k = find(round(u(:,round(sizx/2)))==200 & (abs(200-
u(:,round(sizx/2)))< 1));
                                                           Input Time Step dt: 0.1
if isempty(k) == 1
  fprintf('\nMethod is stable but not consistent\n');
                                                           Middle length reaches 200 C at t = 605.200000s
  fprintf('\nThis scheme will not reach to 200 C for
interval %d to %ds\nMiddle length temperature is %f C
at %ds\n\n',t(1),t(sizt),u(sizt,round(sizx/2)),t(sizt));
%Some values might be repeated when grid size is small
%Thus median is taken as rod has symmetric condition
k=round(median(k));
%t200 represents time when mid-section is at 200 C
t200=t(k);
fprintf('\nMiddle length reaches 200 C at t =
%fs\n\n',t200)
%u(k,round(sizx/2)) is actual value close to 200 C
end
```

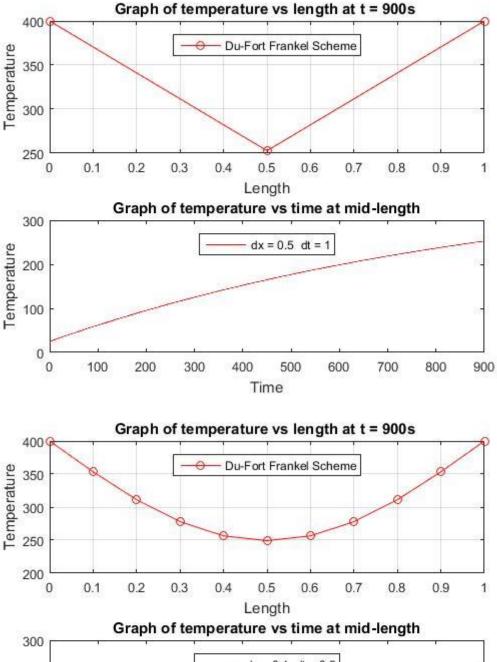
#### Results

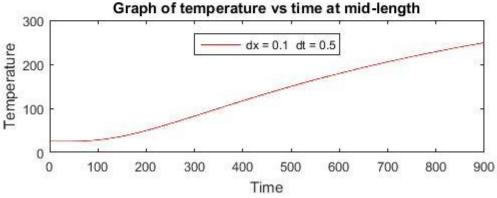
The time required to reach 200 is mention in the table. The graph is plotted for variation of middle length from 0 - 900 s. Also the graph is plotted for temperature at 900 time step.

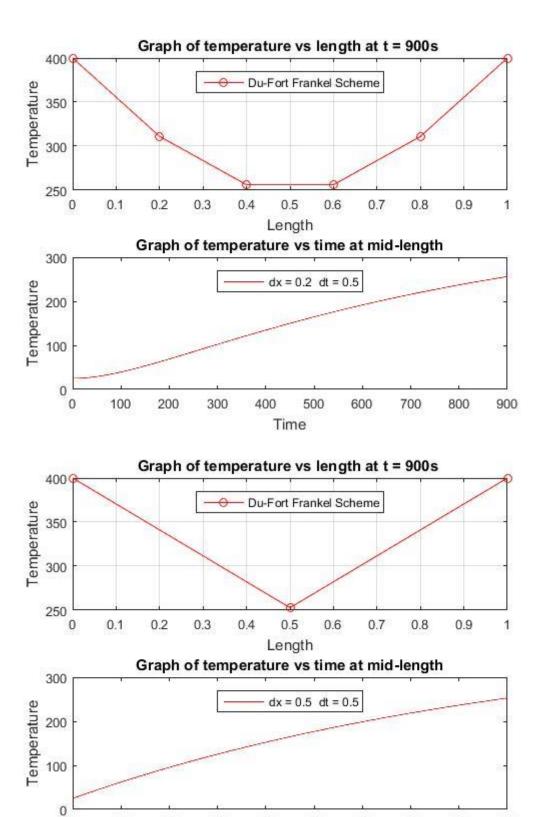
	dy = 0.1m	dy = 0.2m	dy = 0.5m
dt = 1s	678.000000s	634.000000s	606.000000s
dt = 0.5s	678.000000s	633.500000s	605.500000s
dt = 0.1s	677.700000s	632.900000s	605.200000s



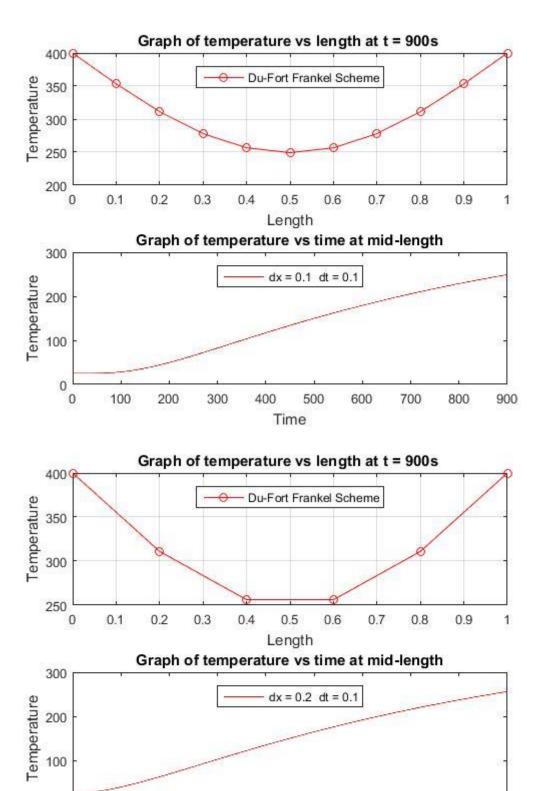




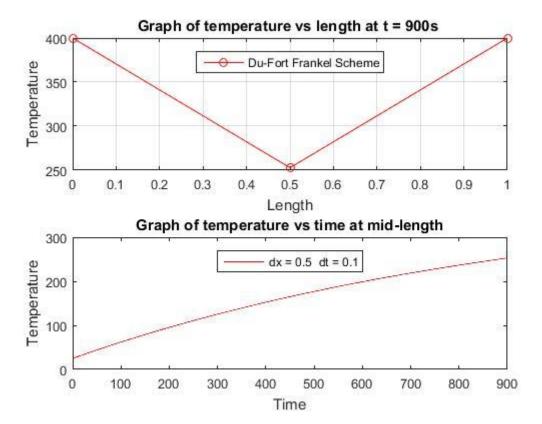




Time



Time



# c. Fully Explicit Scheme

Code	Output
clearvars	a. dy = 0.1m, dt = 1s
clc	
fprintf('\nFully Implicit Scheme applied to 1-D heat	Fully Implicit Scheme applied to 1-D heat conduction
conduction problem\n')	problem
%Endpoints of rod such that b-a=Length of rod	11. 11. 0 1.1 01.
a=0; b=1;	Unit Grid Size dx: 0.1
% for Ut=alp*Uxx form	Input Time Step dt: 1
alp=400/(385*8000);	input time step att 1
	Method is stable but not consistent
% dx and dt are grid differences	
dx=input('\nUnit Grid Size dx: ');	This scheme will not reach to 200 C for interval 0 to
dt=input('\nlnput Time Step dt: ');	900s
%dx=0.25; %dt=2;	Middle length temperature is 10.061277 C at 900s
x=a:dx:b;	
t=0:dt:900;	b. dy = 0.2m , dt = 1s
%Calculation for size of x and t points	, , , , , , , , , , , , , , , , , , , ,
[sizx,sizx]=size(x);	Fully Implicit Scheme applied to 1-D heat conduction
[sizt,sizt]=size(t);	problem
u= zeros(sizt,sizx); %formula for r	
r=alp*dt/(dx^2);	Unit Grid Size dx: 0.2
1-aip at/(ax 2),	Input Time Step dt: 1
%Initial Condition	input time step ut. 1
%This array has ghost value included	Method is stable but not consistent
u(1,:)=25;	
	This scheme will not reach to 200 C for interval 0 to
%Rod is maintained at 400 C.	900s
%Thus setting Boundary conditions	Middle length temperature is 9.591897 C at 900s
u(2:sizt,1)=400;	
u(2:sizt,sizx)=400;	c. dy = 0.5m , dt = 1s
%Formula is worked out for Fully Implicit	Fully Implicit Scheme applied to 1-D heat conduction
%Format $U(i+1,j)=F(U(i,j+1),U(i,j-1),U(i,j))$	problem
%Here 'i' represents time step	
%And 'j' represents spatial variation %Loop is written to get iteration values	Unit Grid Size dx: 0.5
%Loop is written to get iteration values	Louis Time Chan day 4
%For deifining TDMA	Input Time Step dt: 1
A=zeros(sizx-2,sizx-2);	Method is stable but not consistent
	This scheme will not reach to 200 C for interval 0 to
	900s
	Middle length temperature is 9.818774 C at 900s

```
for i=2:sizt
  %Now based of formula we will write TDMA matrix
  %a represents 1st diagonal below main diagonal
  %b represents main diagonal
  %c represents 1st diagonal above main diagonal
for j=1:sizx-3
 a(1,j)=r;
 c(1,j)=r;
 A(j+1,j)=a(1,j);
 A(j,j+1)=c(1,j);
end
for j=1:sizx-2
  b(1,j)=-(1+2*r);
  A(j,j)=b(1,j);
  d(j,1)=-u(i-1,j+1);
%For identifying the size of matrix
[n,q]=size(d);
%Definition taken from Atkinson
beta(1)=A(1);
g(1)=d(1);
%TDMA triangular matrix solution for Lg=b form where
for I=2:n
%Definition taken from Atkinson
alpha(l)=A(l,l-1)/beta(l-1);
beta(I)=A(I,I)-alpha(I)*A(I-1,I);
g(l)=d(l)-alpha(l)*g(l-1);
end
%Back substitution for solving g=UX
xu(n)=g(n)/beta(n);
for I=n-1:-1:1
xu(l)=(g(l)-A(l,l+1)*xu(l+1))/beta(l);
end
xu;
%Updating thevalue of u
u(i,2:sizx-1)=[xu];
end
```

```
d. dy = 0.1m, dt = 0.5s
```

Fully Implicit Scheme applied to 1-D heat conduction problem

Unit Grid Size dx: 0.1

Input Time Step dt: 0.5

Method is stable but not consistent

This scheme will not reach to 200 C for interval 0 to 900s

Middle length temperature is 10.057641 C at 900s

e. 
$$dy = 0.2m$$
,  $dt = 0.5s$ 

Fully Implicit Scheme applied to 1-D heat conduction problem

Unit Grid Size dx: 0.2

Input Time Step dt: 0.5

Method is stable but not consistent

This scheme will not reach to 200 C for interval 0 to 900s

Middle length temperature is 9.588616 C at 900s

#### f. dy = 0.5m, dt = 0.5s

Fully Implicit Scheme applied to 1-D heat conduction problem

Unit Grid Size dx: 0.5

Input Time Step dt: 0.5

Method is stable but not consistent

This scheme will not reach to 200 C for interval 0 to 900s

Middle length temperature is 9.816392 C at 900s

```
%y represents all the values of temperature which
change for t>0 at mid
%midlen
y=u(:,round(sizx/2));
figure
subplot(2,1,2);
plot(t,y,'-r')
title('Graph of temperature vs time at mid-length')
xlabel('Time')
ylabel('Temperature')
legend(['dx = ',num2str(dx),' dt =
',num2str(dt)],'Location','north')
%Additional information
y=u(sizt,:);
%represents temperature variation in bar at 900s
subplot(2,1,1);
plot(x,y,'-ro')
title('Graph of temperature vs length at t = 900s')
xlabel('Length')
ylabel('Temperature')
legend('Fully Implicit Scheme', 'Location', 'north')
grid on
%'k' Finds the values where we get 200 C
k = find(round(u(:,round(sizx/2)))==200 & (abs(200-
u(:,round(sizx/2)))< 0.05));
if isempty(k) == 1
  fprintf('\nMethod is stable but not consistent\n');
  fprintf('\nThis scheme will not reach to 200 C for
interval %d to %ds\nMiddle length temperature is %f C
at %ds\n\n',t(1),t(sizt),u(sizt,round(sizx/2)),t(sizt));
%Some values might be repeated when grid size is small
%Thus median is taken as rod has symmetric condition
k=round(median(k));
%t200 represents time when mid-section is at 200 C
t200=t(k);
fprintf('\nMiddle length reaches 200 C at t =
%fs\n\n',t200)
%u(k,round(sizx/2)) is actual value close to 200 C
end
```

```
g. dy = 0.1m, dt = 0.1s
```

Fully Implicit Scheme applied to 1-D heat conduction problem

Unit Grid Size dx: 0.1

Input Time Step dt: 0.1

Method is stable but not consistent

This scheme will not reach to 200 C for interval 0 to 900s

Middle length temperature is 10.054730 C at 900s

#### h. dy = 0.2m, dt = 0.1s

Fully Implicit Scheme applied to 1-D heat conduction problem

Unit Grid Size dx: 0.2

Input Time Step dt: 0.1

Method is stable but not consistent

This scheme will not reach to 200 C for interval 0 to 900s

Middle length temperature is 9.585989 C at 900s

#### i. dy = 0.5m, dt = 0.1s

Fully Implicit Scheme applied to 1-D heat conduction problem

Unit Grid Size dx: 0.5

Input Time Step dt: 0.1

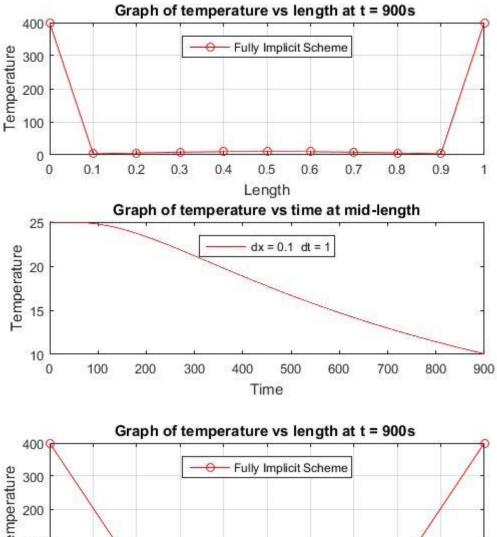
Method is stable but not consistent

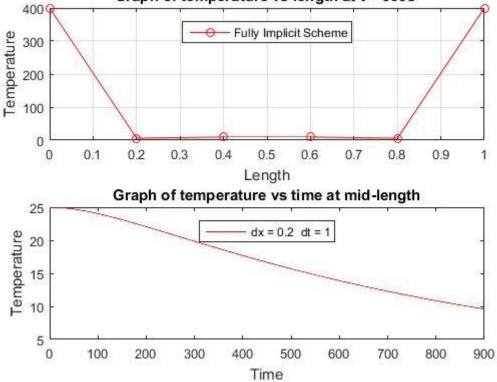
This scheme will not reach to 200 C for interval 0 to 900s

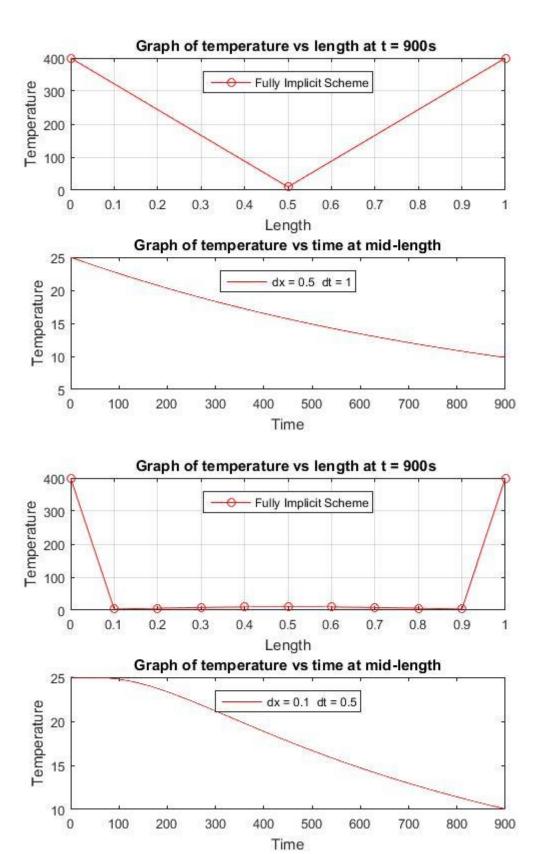
Middle length temperature is 9.814486 C at 900s

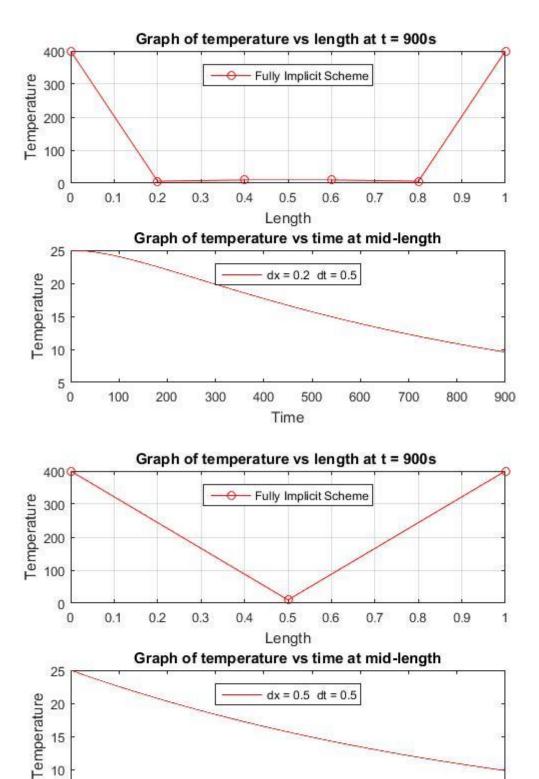
#### Results

For any values of dt and dy mentioned the solution will be erroneous as the method is inconsistent.

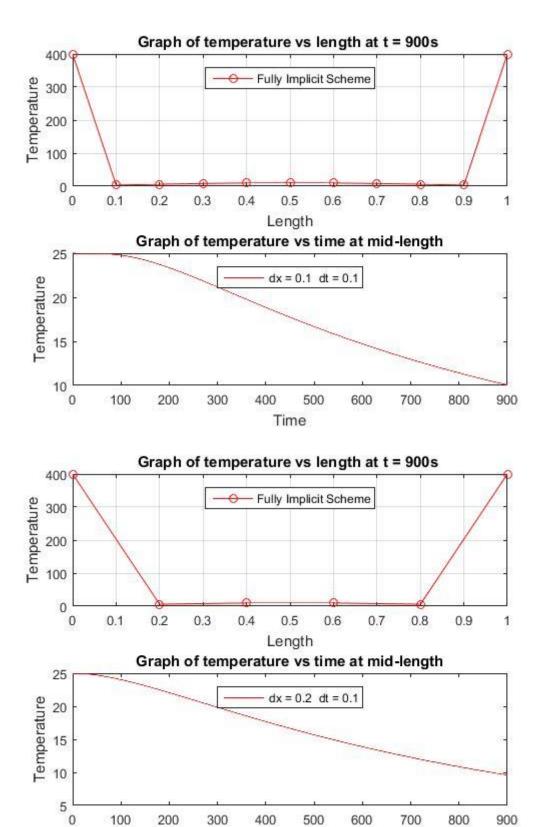




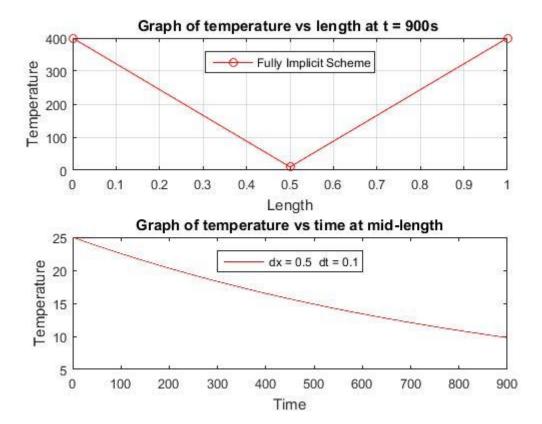




Time



Time



## d. Crank Nicholson Scheme

Code	Output
clearvars	a. dy = 0.1m , dt = 1s
clc	
fprintf('\nCrank Nicholson Scheme applied to 1-D hea	• •
conduction problem\n') %Endpoints of rod such that b-a=Length of rod	problem
a=0;	Unit Grid Size dx: 0.1
b=1;	5 5
% for Ut=alp*Uxx form	Input Time Step dt: 1
alp=400/(385*8000);	
% dx and dt are grid differences	Method is stable but not consistent
dx=input('\nUnit Grid Size dx: ');	This scheme will not reach to 200 C for interval 0 to
dt=input('\nInput Time Step dt: ');	900s
%dx=0.25;	Middle length temperature is 129.526119 C at 900s
%dt=2;	
x=a:dx:b; t=0:dt:900;	h du 020 de 45
%Calculation for size of x and t points	b. dy = 0.2m , dt = 1s
[sizx,sizx]=size(x);	Crank Nicholson Scheme applied to 1-D heat conduction
[sizt,sizt]=size(t);	problem
u= zeros(sizt,sizx);	
%formula for r r=alp*dt/(dx^2);	Unit Grid Size dx: 0.2
i-aip ut/(ux··2),	Input Time Step dt: 1
%Initial Condition	input time step ut. 1
u(1,:)=25;	Method is stable but not consistent
	This scheme will not reach to 200 C for interval 0 to
%Rod is maintained at 400 C.	900s
%Thus setting Boundary conditions	Middle length temperature is 132.813586 C at 900s
u(2:sizt,1)=400;	
u(2:sizt,sizx)=400;	c. dy = 0.5m , dt = 1s
%Formula is worked out for Crank Nicholson	c. uy - 0.5m, ut - 13
%Format $U(i+1,j)=F(U(i,j+1),U(i,j-1),U(i,j))$	Crank Nicholson Scheme applied to 1-D heat conduction
%Here 'i' represents time step	problem
%And 'j' represents spatial variation %Loop is written to get iteration values	Heir Cold City do C
70200p is written to get iteration values	Unit Grid Size dx: 0.5
%For deifining TDMA	Input Time Step dt: 1
A=zeros(sizx-2,sizx-2);	Method is stable but not consistent
	This scheme will not reach to 200 C for interval 0 to 900s
	Middle length temperature is 131.225428 C at 900s

```
for i=2:sizt
  %Now based of formula we will write TDMA matrix
  %a represents 1st diagonal below main diagonal
  %b represents main diagonal
  %c represents 1st diagonal above main diagonal
for j=1:sizx-3
 a(1,j)=-r;
 c(1,j)=a(1,j);
 A(j+1,j)=a(1,j);
 A(j,j+1)=c(1,j);
end
for j=1:sizx-2
  b(1,j)=2*(1+r);
  A(j,j)=b(1,j);
  d(j,1)=r^*u(i-1,j)+2^*(1-r)^*u(i-1,j+1)+r^*u(i-1,j+2);
%For identifying the size of matrix
[n,j]=size(d);
%Definition taken from Atkinson
beta(1)=A(1);
g(1)=d(1);
%TDMA triangular matrix solution for Lg=b form where
for I=2:n
%Definition taken from Atkinson
alpha(l)=A(l,l-1)/beta(l-1);
beta(I)=A(I,I)-alpha(I)*A(I-1,I);
g(l)=d(l)-alpha(l)*g(l-1);
end
%Back substitution for solving g=UX
xu(n)=g(n)/beta(n);
for l=n-1:-1:1
xu(l)=(g(l)-A(l,l+1)*xu(l+1))/beta(l);
end
xu;
%Updating thevalue of u
u(i,2:sizx-1)=[xu];
end
```

d. 
$$dy = 0.1m$$
,  $dt = 0.5s$ 

Crank Nicholson Scheme applied to 1-D heat conduction problem

Unit Grid Size dx: 0.1

Input Time Step dt: 0.5

Method is stable but not consistent

This scheme will not reach to 200 C for interval 0 to 900s

Middle length temperature is 129.574063 C at 900s

#### e. dy = 0.2m, dt = 0.5s

Crank Nicholson Scheme applied to 1-D heat conduction problem

Unit Grid Size dx: 0.2

Input Time Step dt: 0.5

Method is stable but not consistent

This scheme will not reach to 200 C for interval 0 to 900s

Middle length temperature is 132.858142 C at 900s

#### f. dy = 0.5m, dt = 0.5s

Crank Nicholson Scheme applied to 1-D heat conduction problem

Unit Grid Size dx: 0.5

Input Time Step dt: 0.5

Method is stable but not consistent

This scheme will not reach to 200 C for interval 0 to 900s

Middle length temperature is 131.263690 C at 900s

```
%y represents all the values of temperature which
change for t>0 at mid
%midlen
y=u(:,round(sizx/2));
figure
subplot(2,1,2);
plot(t,y,'-r')
title('Graph of temperature vs time at mid-length')
xlabel('Time')
ylabel('Temperature')
legend(['dx = ',num2str(dx),' dt =
',num2str(dt)],'Location','north')
%Additional information
y=u(sizt,:);
%represents temperature variation in bar at 900s
subplot(2,1,1);
plot(x,y,'-ro')
title('Graph of temperature vs length at t = 900s')
xlabel('Length')
ylabel('Temperature')
legend('Crank Nicholson Scheme', 'Location', 'north')
grid on
%'k' Finds the values where we get 200 C
k = find(round(u(:,round(sizx/2)))==200 & (abs(200-
u(:,round(sizx/2)))< 0.05));
if isempty(k) == 1
  fprintf('\nMethod is stable but not consistent\n');
  fprintf('\nThis scheme will not reach to 200 C for
interval %d to %ds\nMiddle length temperature is %f C
at %ds\n\n',t(1),t(sizt),u(sizt,round(sizx/2)),t(sizt));
%Some values might be repeated when grid size is small
%Thus median is taken as rod has symmetric condition
k=round(median(k));
%t200 represents time when mid-section is at 200 C
t200=t(k);
fprintf('\nMiddle length reaches 200 C at t =
%fs\n\n',t200)
%u(k,round(sizx/2)) is actual value close to 200 C
end
```

g. 
$$dy = 0.1m$$
,  $dt = 0.1s$ 

Crank Nicholson Scheme applied to 1-D heat conduction problem

Unit Grid Size dx: 0.1

Input Time Step dt: 0.1

Method is stable but not consistent

This scheme will not reach to 200 C for interval 0 to 900s

Middle length temperature is 129.612401 C at 900s

#### h. dy = 0.2m, dt = 0.1s

Crank Nicholson Scheme applied to 1-D heat conduction problem

Unit Grid Size dx: 0.2

Input Time Step dt: 0.1

Method is stable but not consistent

This scheme will not reach to 200 C for interval 0 to 900s

Middle length temperature is 132.893770 C at 900s

#### i. dy = 0.5m , dt = 0.1s

Crank Nicholson Scheme applied to 1-D heat conduction problem

Unit Grid Size dx: 0.5

Input Time Step dt: 0.1

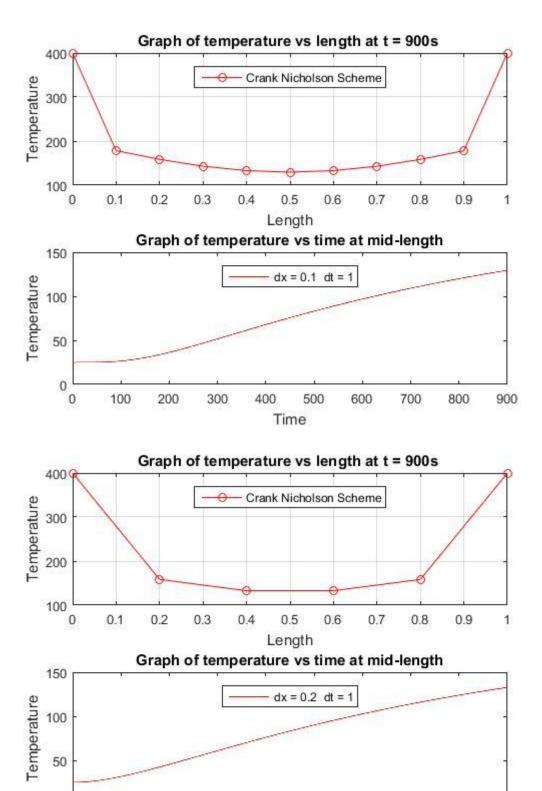
Method is stable but not consistent

This scheme will not reach to 200 C for interval 0 to 900s

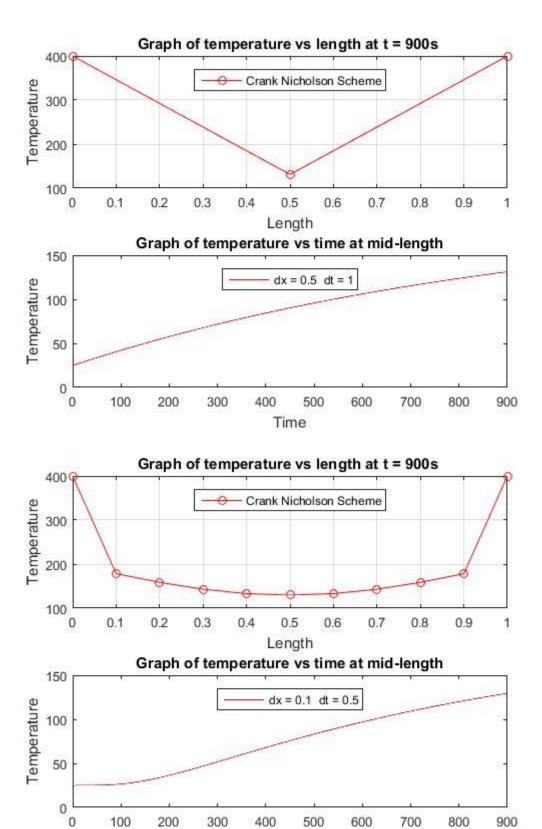
Middle length temperature is 131.294287 C at 900s

#### Results

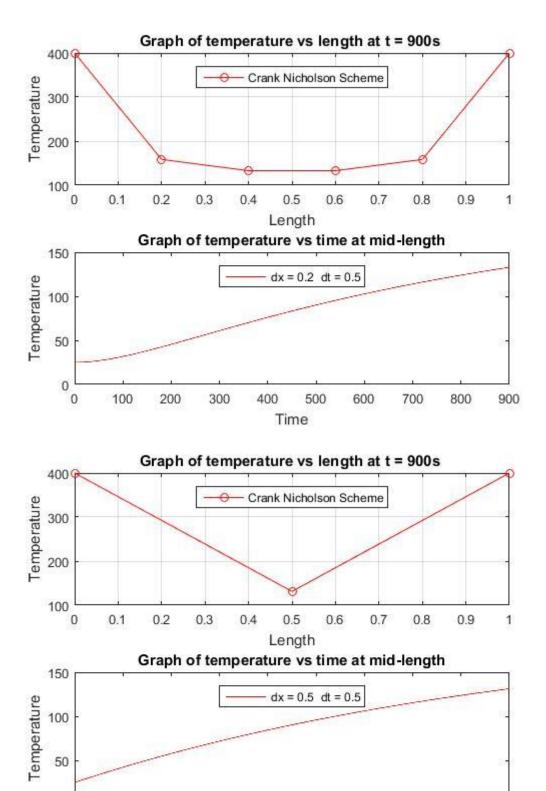
For any values of dt and dy mentioned the solution will be erroneous as the method is inconsistent.



Time



Time



Time

