

Assignment 7

Group Number 32

Software used: **Matlab**

Sanit Prashant Bhatkar (173109003)

Omkar Anil Pawar (173106001)

a. Forward in time backward in space(FTBS)

The following scheme is not stable but **consistent**. So the roundoff errors are not allowing the solution to converge. The following stability analysis is been provided here.

1) Backward Scheme in Space

$$f''(x) = \frac{u_i^n - 2 \cdot u_{i-1}^n + u_{i-2}^n}{\Delta x^2}$$

2) Forward Scheme in time

$$\frac{u_i^{n+1} - u_i^n}{\alpha \cdot \Delta t}$$

3) Assumption for 'r'

$$r = \frac{\alpha \cdot \Delta t}{\Delta x^2}$$

4) Simplification gives

$$u_i^{n+1} = (1 + r) \cdot u_i^n - 2 \cdot r \cdot u_{i-1}^n + r \cdot u_{i-2}^n$$

5) Making substitution

$$u_m^n = g_n \cdot e^{i \cdot m \Theta}$$

$$\text{with } |g| \leq 1 \quad \text{CFL condition}$$

6) Expression for 'g'

$$g = 1 + 2 \cdot r + 2 \cdot r \cos \Theta - r \cos 2\Theta + \dots + \text{higher order terms}$$

Which is unstable for all values of 'r' and 'Θ'

b. Du-Fort Frenkel Scheme

Code	Output
<pre> clearvars clc fprintf('\nDu-Fort Frankel Scheme applied to 1-D heat conduction problem\n') %Endpoints of rod such that b-a=Length of rod a=0; b=1; % for $U_t = \alpha U_{xx}$ form alp=400/(385*8000); % dx and dt are grid differences dx=input('\nUnit Grid Size dx: '); dt=input('\nInput Time Step dt: '); %dx=0.25; %dt=2; x=a:dx:b; t=0:dt:900; %Calculation for size of x and t points [sizx,sizx]=size(x); [sizt,sizt]=size(t); u= zeros(sizt,sizx); %formula for r r=alp*dt/(dx^2); %Initial Condition u(1,:)=25; %Rod is maintained at 400 C. %Thus setting Boundary conditions u(2:sizt,1)=400; u(2:sizt,sizx)=400; %Formula is worked out for Du-Fort Frankel Scheme %Format $U(i+1,j)=F(U(i,j+1),U(i,j-1),U(i,j))$ %Here 'i' represents time step %And 'j' represents spatial variation %Loop is written to get iteration values %For deifining TDMA A=zeros(sizx-2,sizx-2); </pre>	<p>a. dy = 0.1m , dt = 1s</p> <p>Du-Fort Frankel Scheme applied to 1-D heat conduction problem</p> <p>Unit Grid Size dx: 0.1</p> <p>Input Time Step dt: 1</p> <p>Middle length reaches 200 C at t = 678.000000s</p> <p>b. dy = 0.2m , dt = 1s</p> <p>Du-Fort Frankel Scheme applied to 1-D heat conduction problem</p> <p>Unit Grid Size dx: 0.2</p> <p>Input Time Step dt: 1</p> <p>Middle length reaches 200 C at t = 634.000000s</p> <p>c. dy = 0.5m , dt = 1s</p> <p>Du-Fort Frankel Scheme applied to 1-D heat conduction problem</p> <p>Unit Grid Size dx: 0.5</p> <p>Input Time Step dt: 1</p> <p>Middle length reaches 200 C at t = 606.000000s</p> <p>d. dy = 0.1m , dt = 0.5s</p> <p>Du-Fort Frankel Scheme applied to 1-D heat conduction problem</p>

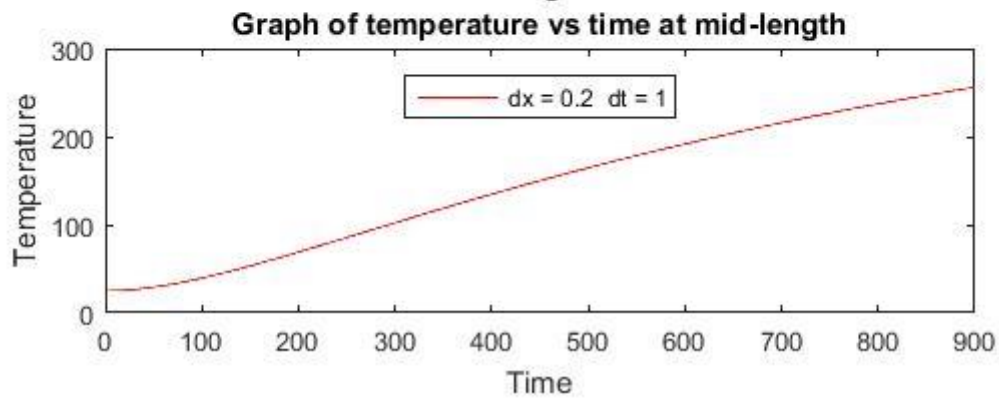
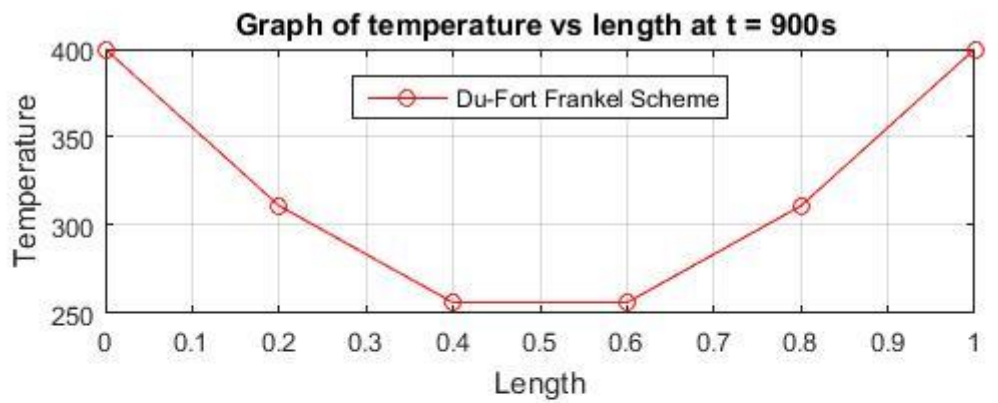
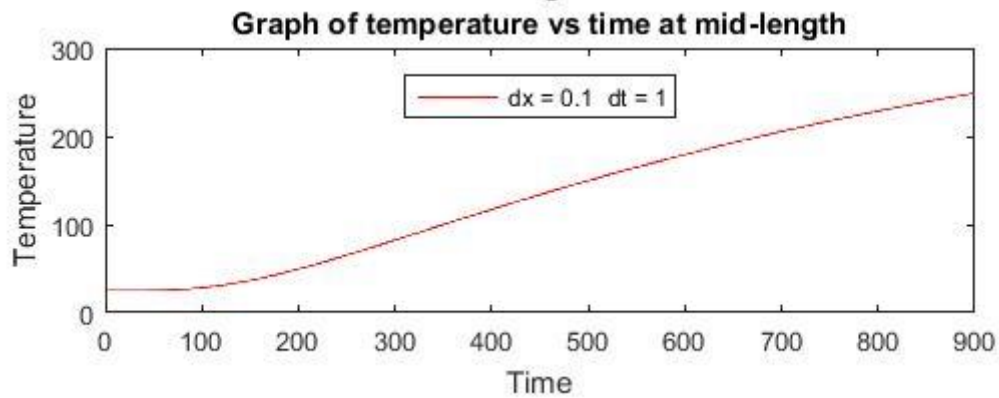
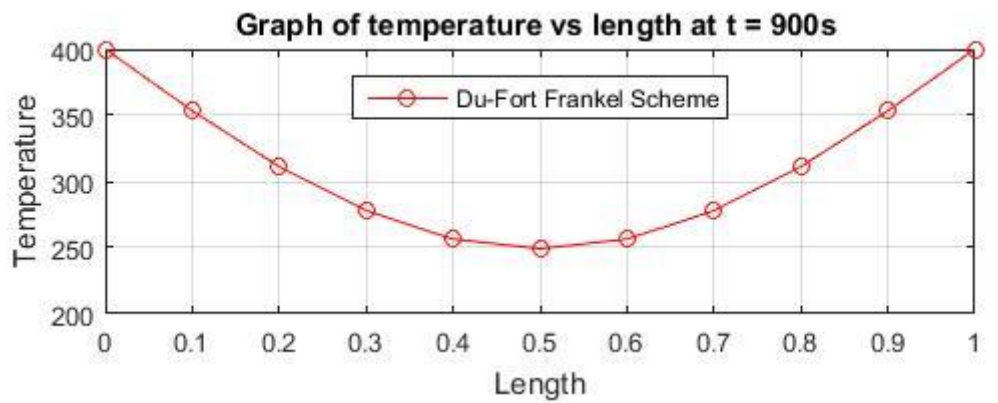
<pre> for i=2:sizt %Now based of formula we will write TDMA matrix %a represents 1st diagonal below main diagonal %b represents main diagonal %c represents 1st diagonal above main diagonal for j=1:sizx-3 a(1,j)=0; c(1,j)=a(1,j); A(j+1,j)=a(1,j); A(j,j+1)=c(1,j); end for j=1:sizx-2 b(1,j)=(1+2*r); A(j,j)=b(1,j); if i==2 d(j,1)=2*r*u(i-1,j)+(1-2*r)*u(i-1,j+1)+2*r*u(i-1,j+2); else d(j,1)=2*r*u(i-1,j)+(1-2*r)*u(i-2,j+1)+2*r*u(i-1,j+2); end end %For identifying the size of matrix [n,j]=size(d); %Definition taken from Atkinson beta(1)=A(1); g(1)=d(1); %TDMA triangular matrix solution for Lg=b form where g=UX for l=2:n %Definition taken from Atkinson alpha(l)=A(l,l-1)/beta(l-1); beta(l)=A(l,l)-alpha(l)*A(l-1,l); g(l)=d(l)-alpha(l)*g(l-1); end %Back substitution for solving g=UX xu(n)=g(n)/beta(n); for l=n-1:-1:1 xu(l)=(g(l)-A(l,l+1)*xu(l+1))/beta(l); end xu; %Updating thevalue of u u(i,2:sizx-1)=[xu]; end </pre>	<p>Unit Grid Size dx: 0.1</p> <p>Input Time Step dt: 0.5</p> <p>Middle length reaches 200 C at t = 678.000000s</p> <p>e. dy = 0.2m , dt = 0.5s</p> <p>Du-Fort Frankel Scheme applied to 1-D heat conduction problem</p> <p>Unit Grid Size dx: 0.2</p> <p>Input Time Step dt: 0.5</p> <p>Middle length reaches 200 C at t = 633.500000s</p> <p>f. dy = 0.5m , dt = 0.5s</p> <p>Du-Fort Frankel Scheme applied to 1-D heat conduction problem</p> <p>Unit Grid Size dx: 0.5</p> <p>Input Time Step dt: 0.5</p> <p>Middle length reaches 200 C at t = 605.500000s</p> <p>g. dy = 0.1m , dt = 0.1s</p> <p>Du-Fort Frankel Scheme applied to 1-D heat conduction problem</p>
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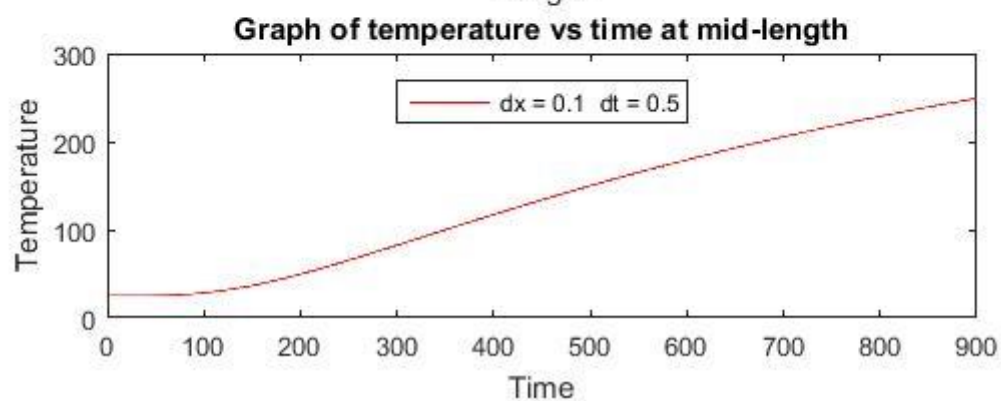
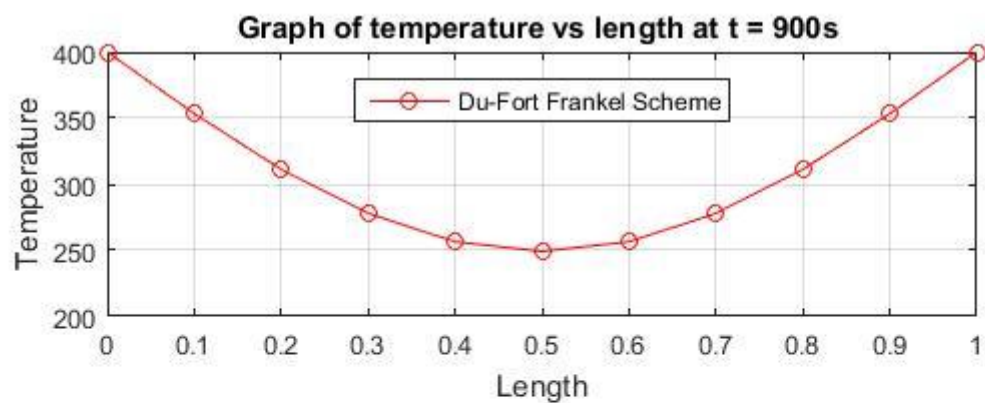
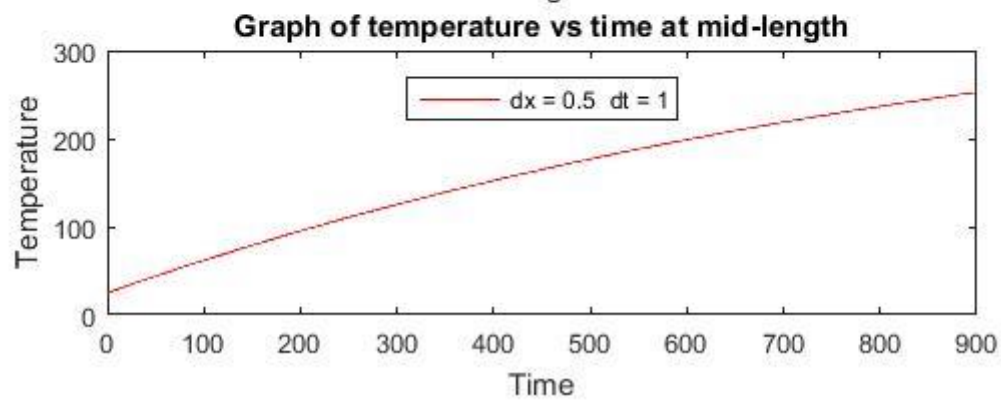
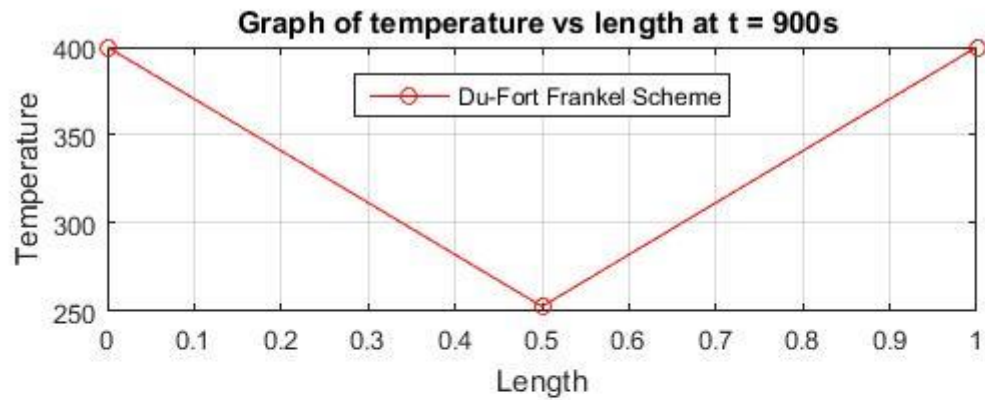
<pre> %y represents all the values of temperature which change for t>0 at mid %midlen y=u(:,round(sizx/2)); figure subplot(2,1,2); plot(t,y,'-r') title('Graph of temperature vs time at mid-length') xlabel('Time') ylabel('Temperature') legend(['dx = ',num2str(dx),' dt = ',num2str(dt)],'Location','north') %Additional information y=u(sizt,:); %represents temperature variation in bar at 900s subplot(2,1,1); plot(x,y,'-ro') title('Graph of temperature vs length at t = 900s ') xlabel('Length') ylabel('Temperature') legend('Du-Fort Frankel Scheme','Location','north') grid on %'k' Finds the values where we get 200 C k = find(round(u(:,round(sizx/2)))==200 & (abs(200- u(:,round(sizx/2)))< 1)); if isempty(k) == 1 fprintf('\nMethod is stable but not consistent\n'); fprintf('\nThis scheme will not reach to 200 C for interval %d to %ds\nMiddle length temperature is %f C at %ds\n\n',t(1),t(sizt),u(sizt,round(sizx/2)),t(sizt)); else %Some values might be repeated when grid size is small %Thus median is taken as rod has symmetric condition k=round(median(k)); %t200 represents time when mid-section is at 200 C t200=t(k); fprintf('\nMiddle length reaches 200 C at t = %fs\n\n',t200) %u(k,round(sizx/2)) is actual value close to 200 C end </pre>	<p>Unit Grid Size dx: 0.1</p> <p>Input Time Step dt: 0.1</p> <p>Middle length reaches 200 C at t = 677.700000s</p> <p>h. dy = 0.2m , dt = 0.1s</p> <p>Du-Fort Frankel Scheme applied to 1-D heat conduction problem</p> <p>Unit Grid Size dx: 0.2</p> <p>Input Time Step dt: 0.1</p> <p>Middle length reaches 200 C at t = 632.900000s</p> <p>i. dy = 0.5m , dt = 0.1s</p> <p>Du-Fort Frankel Scheme applied to 1-D heat conduction problem</p> <p>Unit Grid Size dx: 0.5</p> <p>Input Time Step dt: 0.1</p> <p>Middle length reaches 200 C at t = 605.200000s</p>
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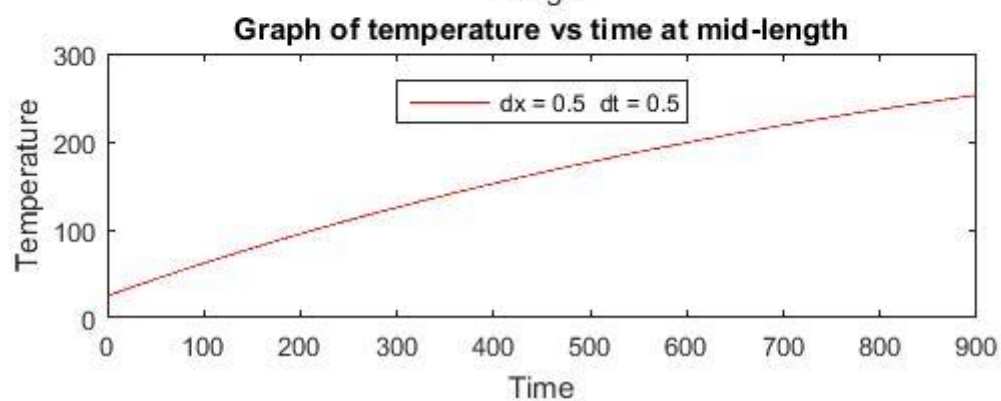
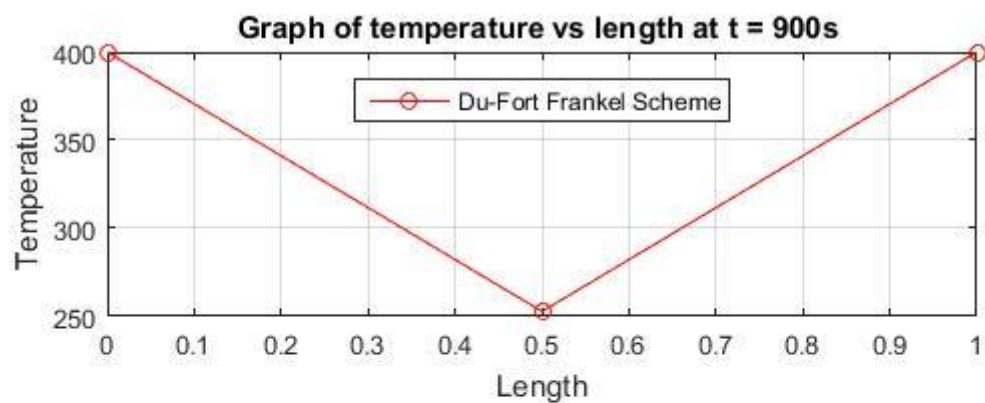
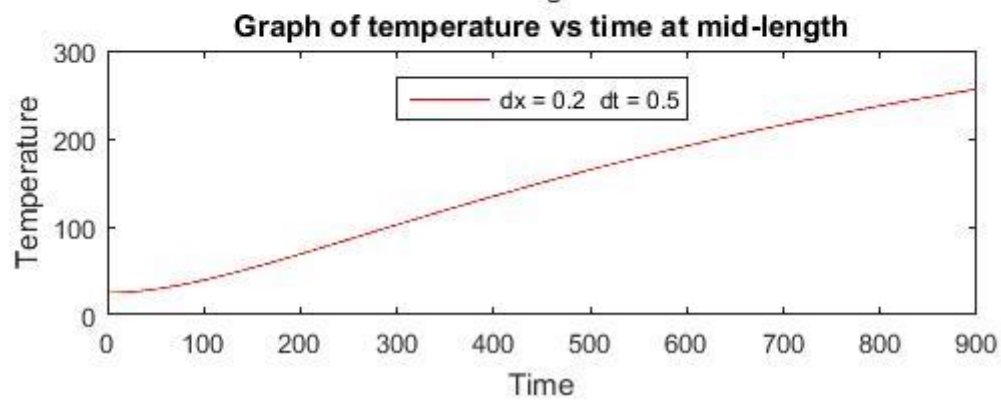
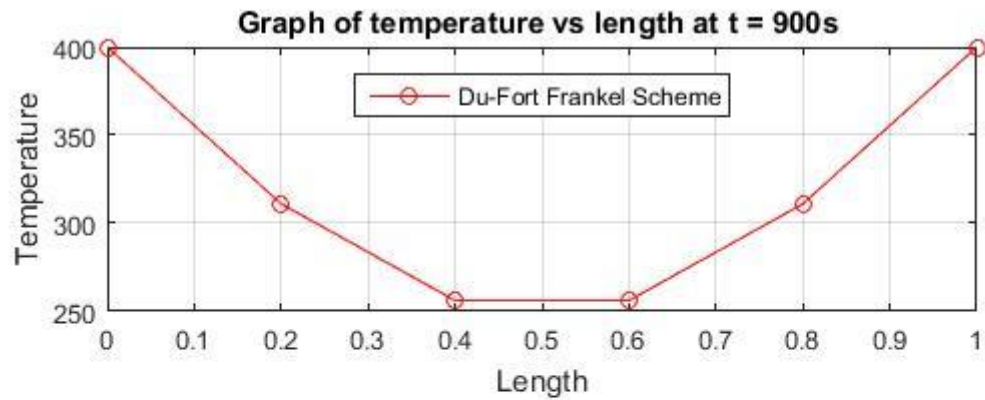
➤ Results

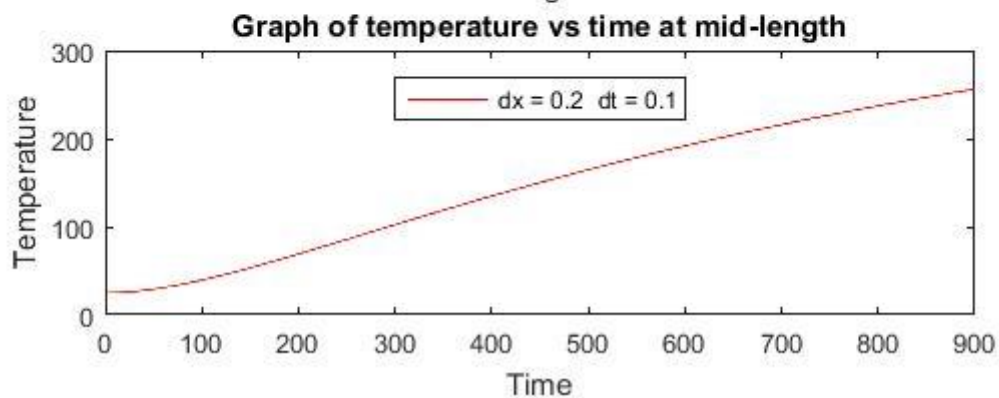
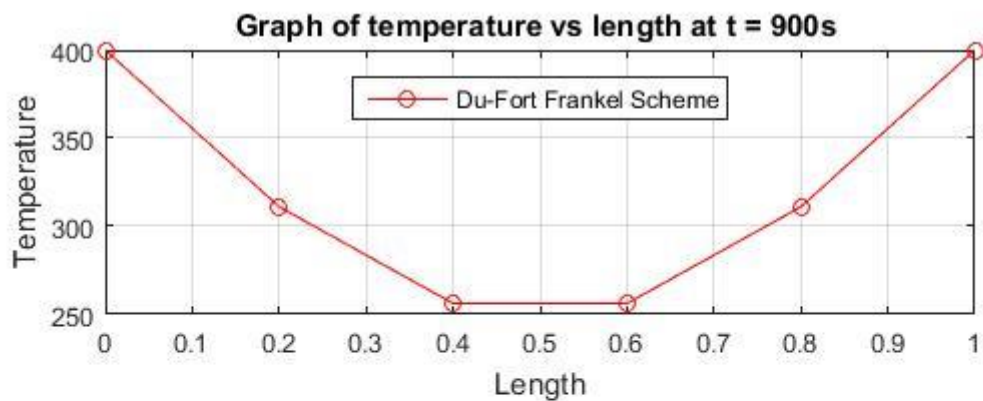
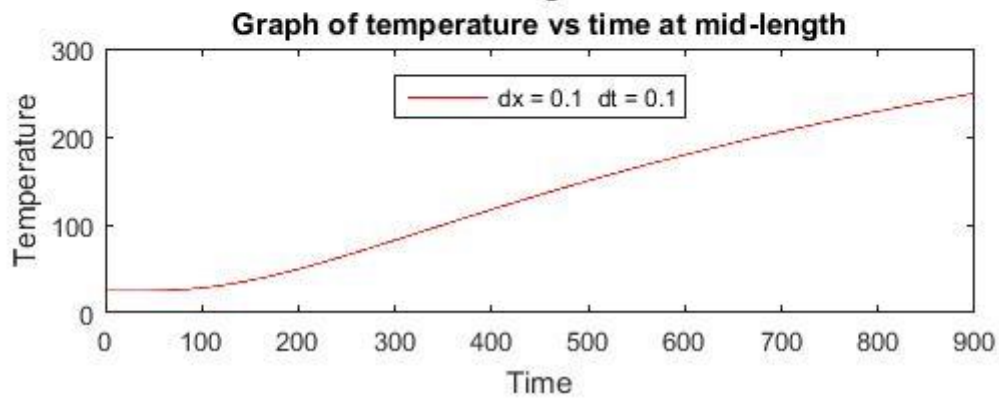
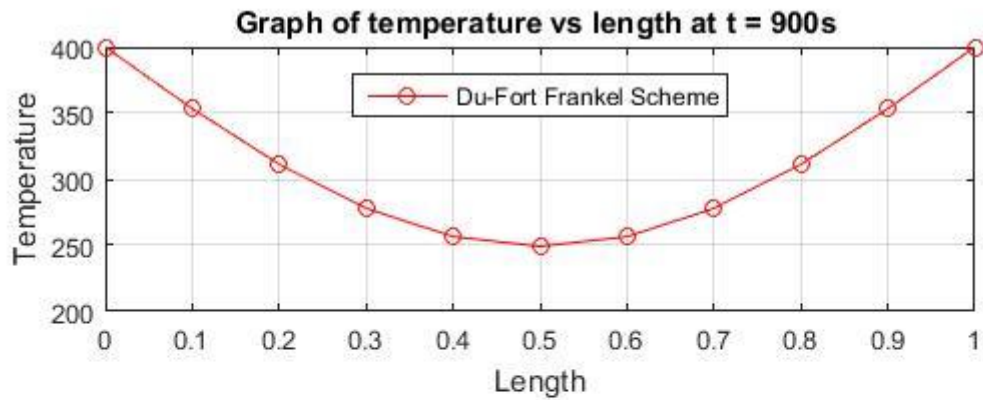
The time required to reach 200 is mention in the table. The graph is plotted for variation of middle length from 0 – 900 s. Also the graph is plotted for temperature at 900 time step.

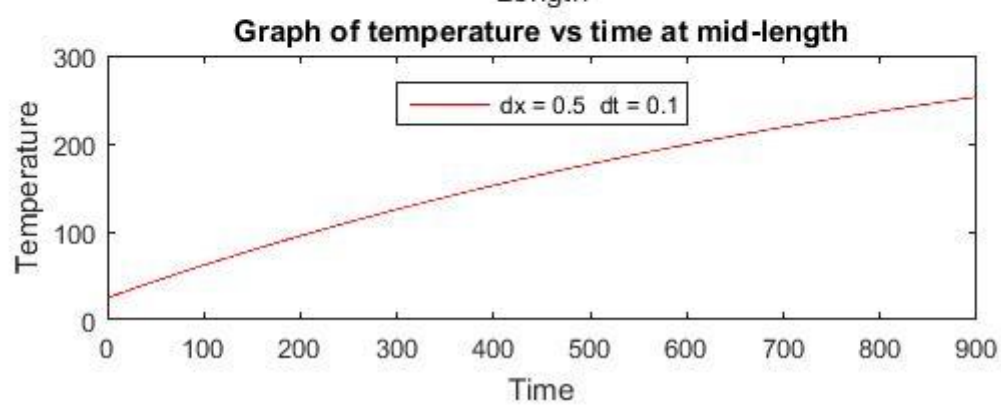
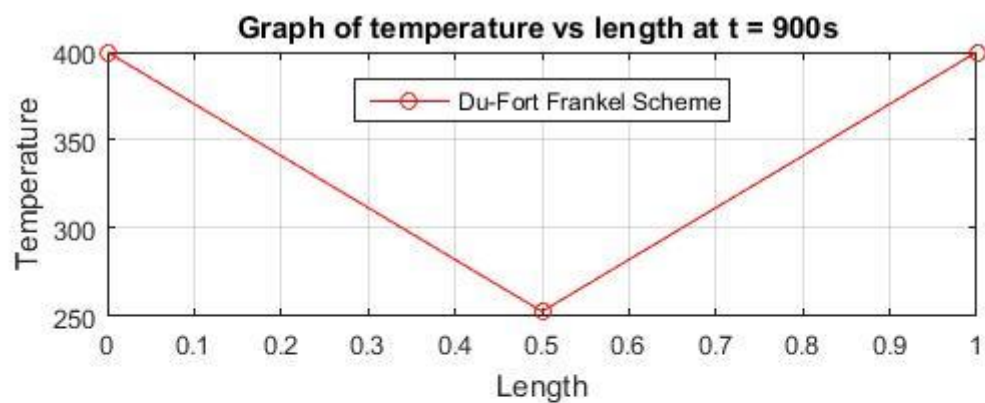
	dy = 0.1m	dy = 0.2m	dy = 0.5m
dt = 1s	678.000000s	634.000000s	606.000000s
dt = 0.5s	678.000000s	633.500000s	605.500000s
dt = 0.1s	677.700000s	632.900000s	605.200000s











c. Fully Explicit Scheme

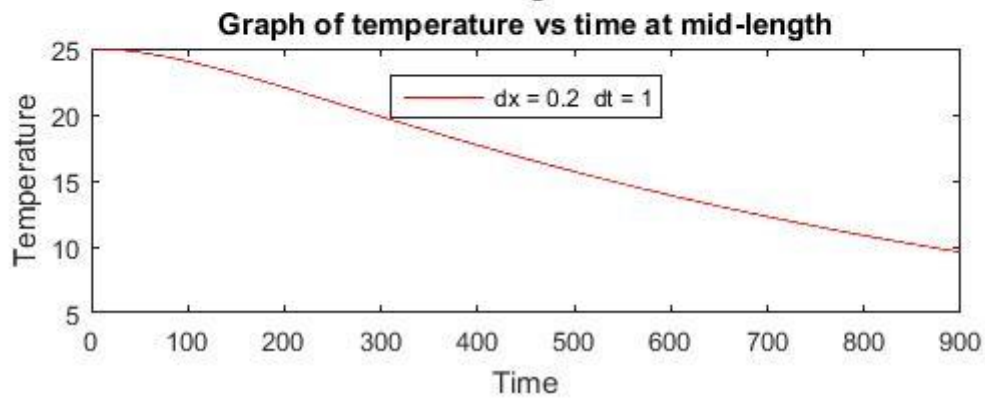
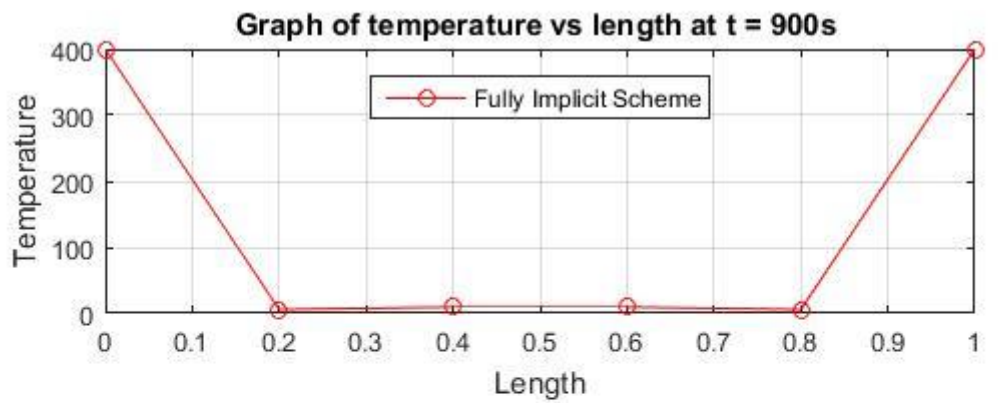
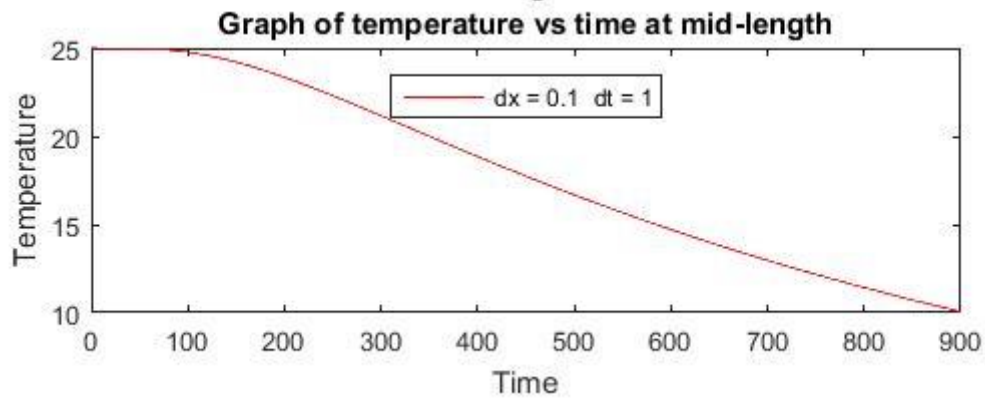
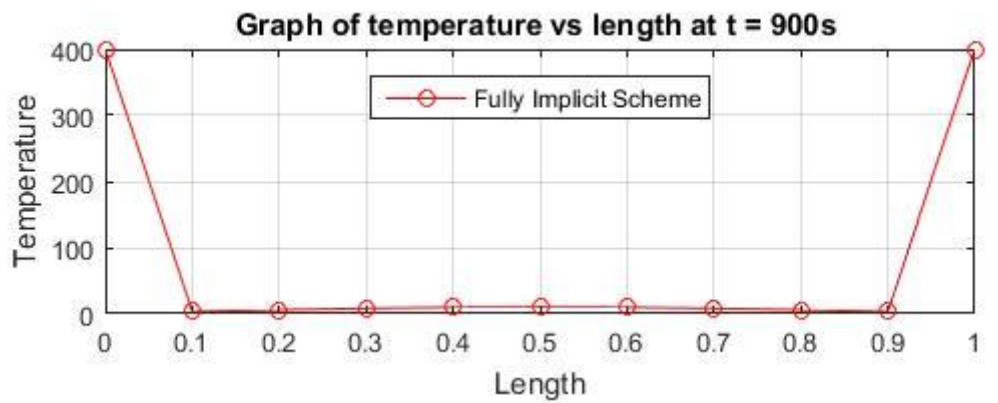
Code	Output
<pre> clearvars clc fprintf('\nFully Implicit Scheme applied to 1-D heat conduction problem\n') %Endpoints of rod such that b-a=Length of rod a=0; b=1; % for $U_t = \alpha U_{xx}$ form alp=400/(385*8000); % dx and dt are grid differences dx=input('\nUnit Grid Size dx: '); dt=input('\nInput Time Step dt: '); %dx=0.25; %dt=2; x=a:dx:b; t=0:dt:900; %Calculation for size of x and t points [sizx,sizx]=size(x); [sizt,sizt]=size(t); u= zeros(sizt,sizx); %formula for r r=alp*dt/(dx^2); %Initial Condition %This array has ghost value included u(1,:)=25; %Rod is maintained at 400 C. %Thus setting Boundary conditions u(2:sizt,1)=400; u(2:sizt,sizx)=400; %Formula is worked out for Fully Implicit %Format $U(i+1,j)=F(U(i,j+1),U(i,j-1),U(i,j))$ %Here 'i' represents time step %And 'j' represents spatial variation %Loop is written to get iteration values %For defining TDMA A=zeros(sizx-2,sizx-2); </pre>	<p>a. $dy = 0.1m$, $dt = 1s$</p> <p>Fully Implicit Scheme applied to 1-D heat conduction problem</p> <p>Unit Grid Size dx: 0.1</p> <p>Input Time Step dt: 1</p> <p>Method is stable but not consistent</p> <p>This scheme will not reach to 200 C for interval 0 to 900s Middle length temperature is 10.061277 C at 900s</p> <p>b. $dy = 0.2m$, $dt = 1s$</p> <p>Fully Implicit Scheme applied to 1-D heat conduction problem</p> <p>Unit Grid Size dx: 0.2</p> <p>Input Time Step dt: 1</p> <p>Method is stable but not consistent</p> <p>This scheme will not reach to 200 C for interval 0 to 900s Middle length temperature is 9.591897 C at 900s</p> <p>c. $dy = 0.5m$, $dt = 1s$</p> <p>Fully Implicit Scheme applied to 1-D heat conduction problem</p> <p>Unit Grid Size dx: 0.5</p> <p>Input Time Step dt: 1</p> <p>Method is stable but not consistent</p> <p>This scheme will not reach to 200 C for interval 0 to 900s Middle length temperature is 9.818774 C at 900s</p>

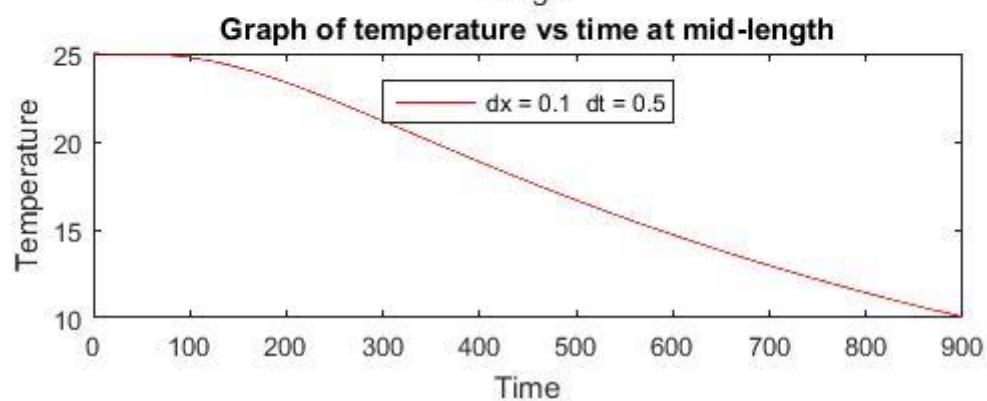
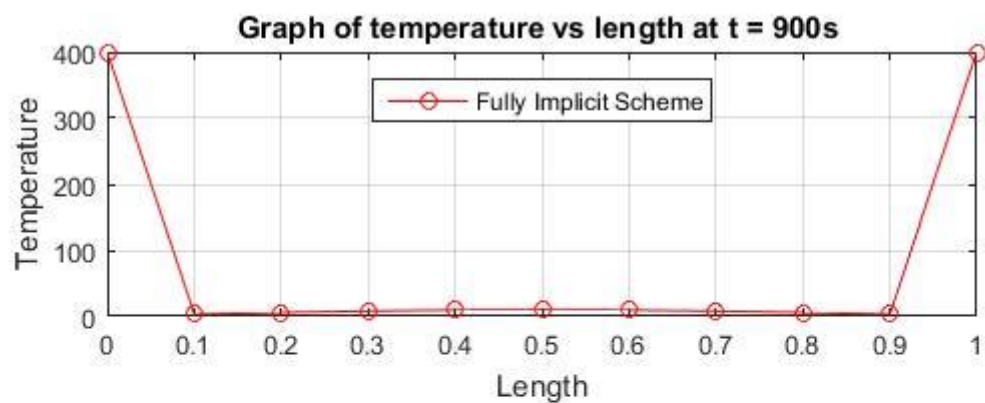
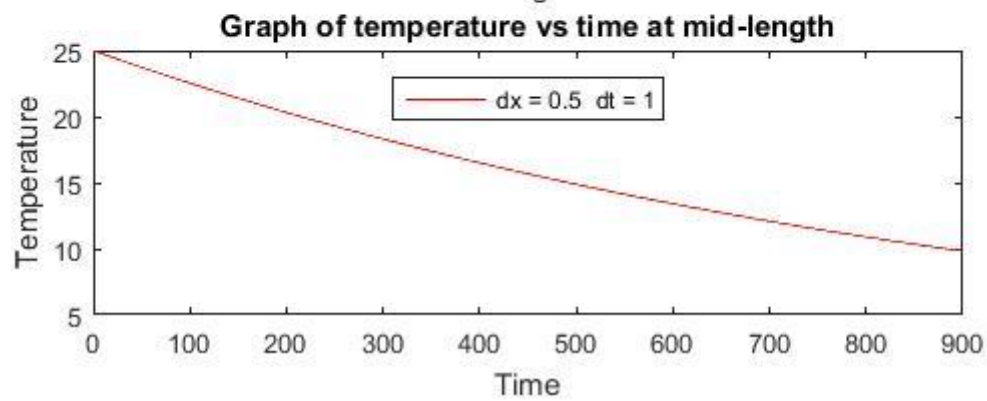
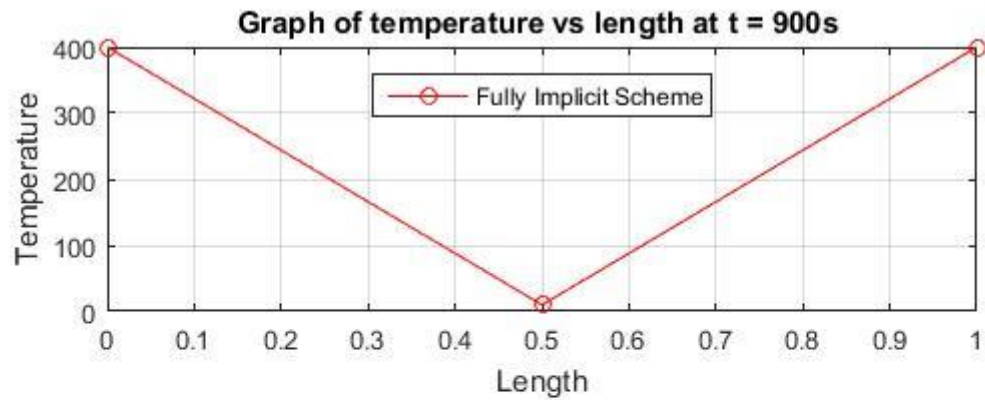
<pre> for i=2:sizt %Now based of formula we will write TDMA matrix %a represents 1st diagonal below main diagonal %b represents main diagonal %c represents 1st diagonal above main diagonal for j=1:sizx-3 a(1,j)=r; c(1,j)=r; A(j+1,j)=a(1,j); A(j,j+1)=c(1,j); end for j=1:sizx-2 b(1,j)=-(1+2*r); A(j,j)=b(1,j); d(j,1)=-u(i-1,j+1); end %For identifying the size of matrix [n,q]=size(d); %Definition taken from Atkinson beta(1)=A(1); g(1)=d(1); %TDMA triangular matrix solution for Lg=b form where g=UX for l=2:n %Definition taken from Atkinson alpha(l)=A(l,l-1)/beta(l-1); beta(l)=A(l,l)-alpha(l)*A(l-1,l); g(l)=d(l)-alpha(l)*g(l-1); end %Back substitution for solving g=UX xu(n)=g(n)/beta(n); for l=n-1:-1:1 xu(l)=(g(l)-A(l,l+1)*xu(l+1))/beta(l); end xu; %Updating thevalue of u u(i,2:sizx-1)=[xu]; end </pre>	<p>d. dy = 0.1m , dt = 0.5s</p> <p>Fully Implicit Scheme applied to 1-D heat conduction problem</p> <p>Unit Grid Size dx: 0.1</p> <p>Input Time Step dt: 0.5</p> <p>Method is stable but not consistent</p> <p>This scheme will not reach to 200 C for interval 0 to 900s Middle length temperature is 10.057641 C at 900s</p> <p>e. dy = 0.2m , dt = 0.5s</p> <p>Fully Implicit Scheme applied to 1-D heat conduction problem</p> <p>Unit Grid Size dx: 0.2</p> <p>Input Time Step dt: 0.5</p> <p>Method is stable but not consistent</p> <p>This scheme will not reach to 200 C for interval 0 to 900s Middle length temperature is 9.588616 C at 900s</p> <p>f. dy = 0.5m , dt = 0.5s</p> <p>Fully Implicit Scheme applied to 1-D heat conduction problem</p> <p>Unit Grid Size dx: 0.5</p> <p>Input Time Step dt: 0.5</p> <p>Method is stable but not consistent</p> <p>This scheme will not reach to 200 C for interval 0 to 900s Middle length temperature is 9.816392 C at 900s</p>
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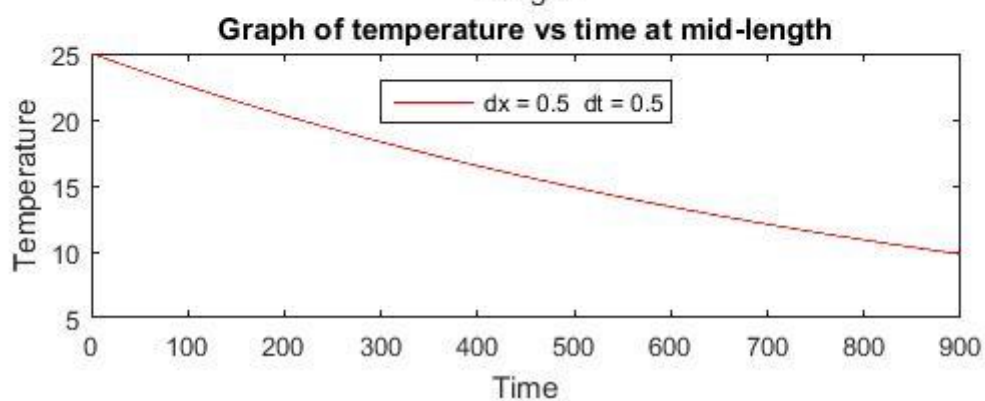
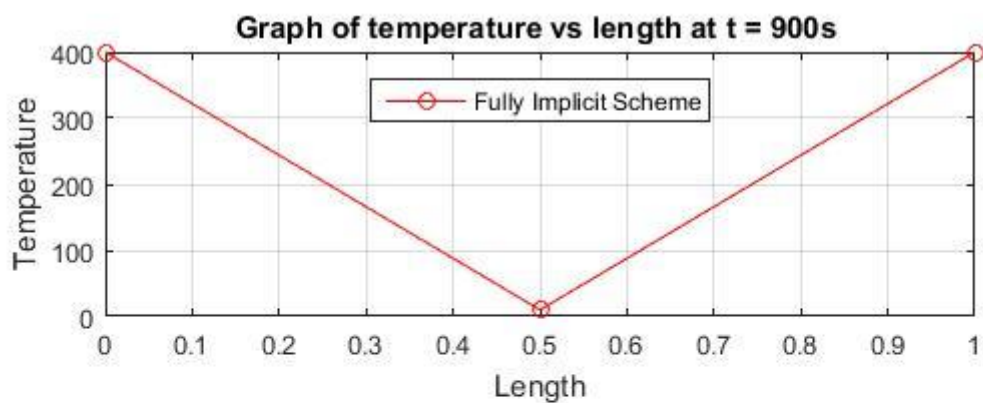
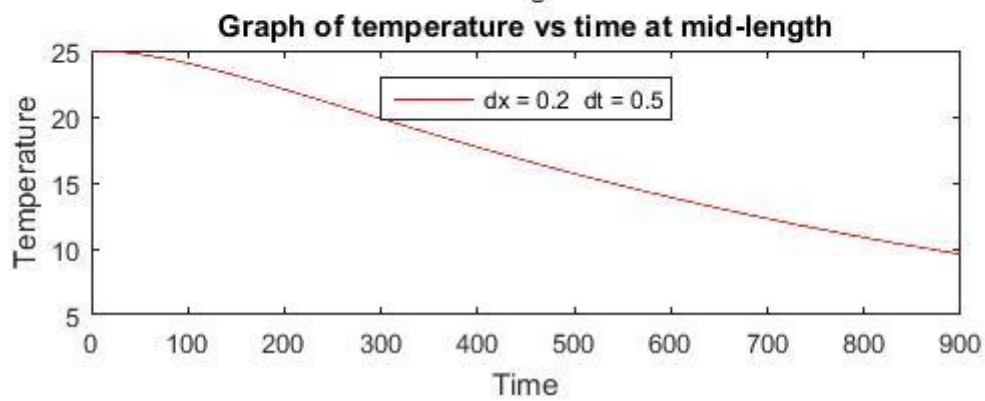
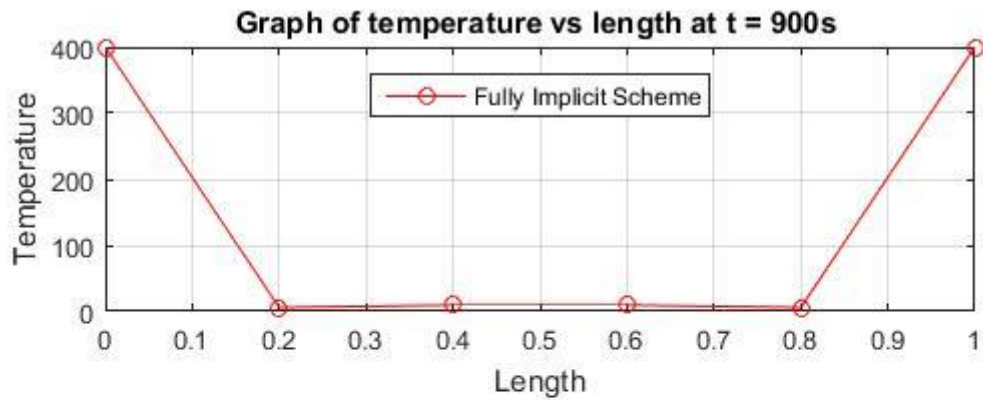
<pre> %y represents all the values of temperature which change for t>0 at mid %midlen y=u(:,round(sizx/2)); figure subplot(2,1,2); plot(t,y,'-r') title('Graph of temperature vs time at mid-length') xlabel('Time') ylabel('Temperature') legend(['dx = ',num2str(dx),' dt = ',num2str(dt)],'Location','north') %Additional information y=u(sizt,:); %represents temperature variation in bar at 900s subplot(2,1,1); plot(x,y,'-ro') title('Graph of temperature vs length at t = 900s ') xlabel('Length') ylabel('Temperature') legend('Fully Implicit Scheme','Location','north') grid on %'k' Finds the values where we get 200 C k = find(round(u(:,round(sizx/2)))==200 & (abs(200- u(:,round(sizx/2)))< 0.05)); if isempty(k) == 1 fprintf('\nMethod is stable but not consistent\n'); fprintf('\nThis scheme will not reach to 200 C for interval %d to %ds\nMiddle length temperature is %f C at %ds\n\n',t(1),t(sizt),u(sizt,round(sizx/2)),t(sizt)); else %Some values might be repeated when grid size is small %Thus median is taken as rod has symmetric condition k=round(median(k)); %t200 represents time when mid-section is at 200 C t200=t(k); fprintf('\nMiddle length reaches 200 C at t = %fs\n\n',t200) %u(k,round(sizx/2)) is actual value close to 200 C end </pre>	<p>g. $dy = 0.1m$, $dt = 0.1s$</p> <p>Fully Implicit Scheme applied to 1-D heat conduction problem</p> <p>Unit Grid Size dx: 0.1</p> <p>Input Time Step dt: 0.1</p> <p>Method is stable but not consistent</p> <p>This scheme will not reach to 200 C for interval 0 to 900s Middle length temperature is 10.054730 C at 900s</p> <p>h. $dy = 0.2m$, $dt = 0.1s$</p> <p>Fully Implicit Scheme applied to 1-D heat conduction problem</p> <p>Unit Grid Size dx: 0.2</p> <p>Input Time Step dt: 0.1</p> <p>Method is stable but not consistent</p> <p>This scheme will not reach to 200 C for interval 0 to 900s Middle length temperature is 9.585989 C at 900s</p> <p>i. $dy = 0.5m$, $dt = 0.1s$</p> <p>Fully Implicit Scheme applied to 1-D heat conduction problem</p> <p>Unit Grid Size dx: 0.5</p> <p>Input Time Step dt: 0.1</p> <p>Method is stable but not consistent</p> <p>This scheme will not reach to 200 C for interval 0 to 900s Middle length temperature is 9.814486 C at 900s</p>
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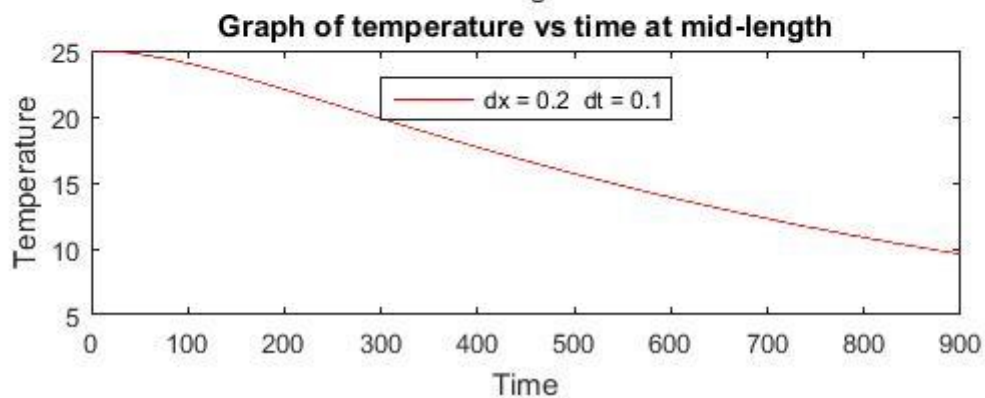
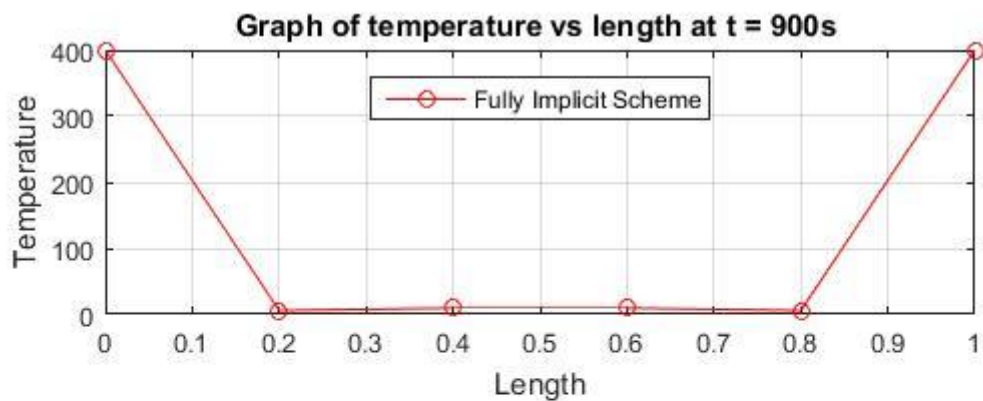
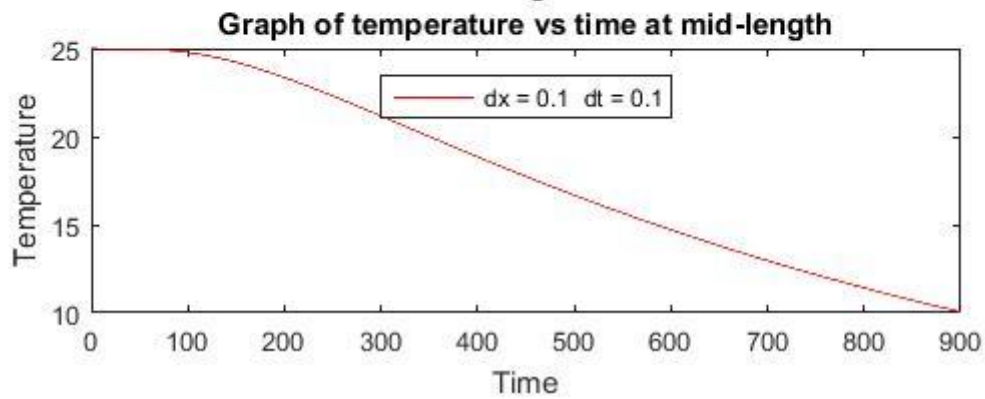
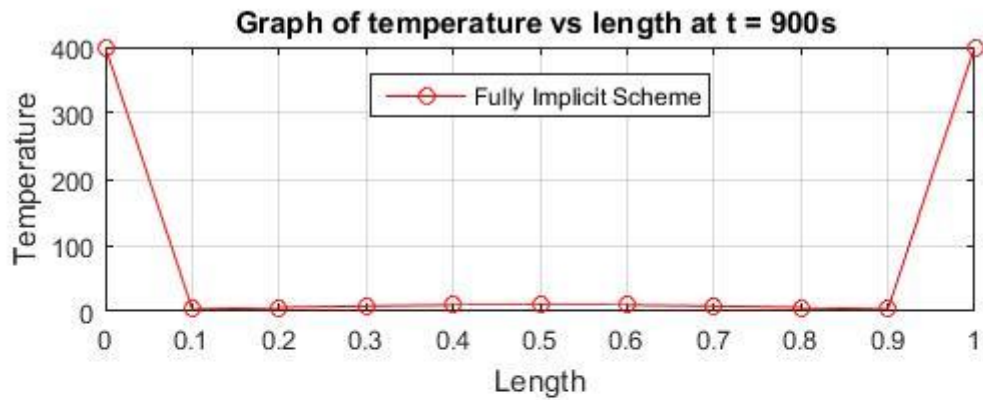
➤ Results

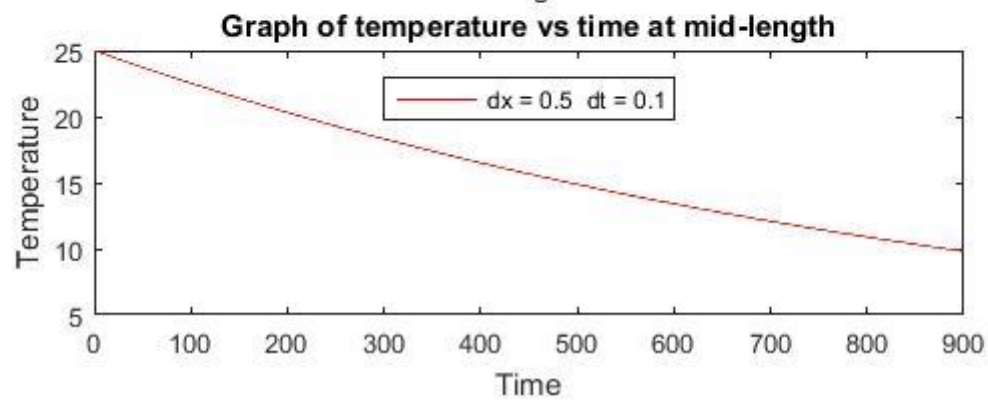
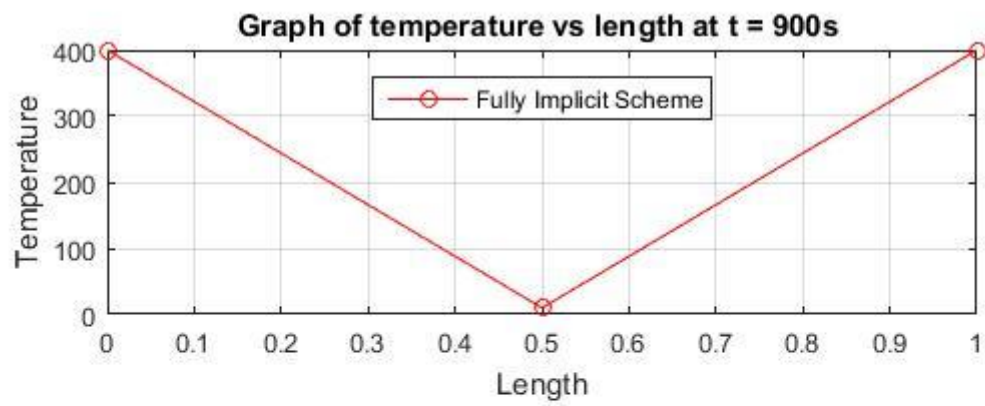
For any values of dt and dy mentioned the solution will be erroneous as the method is inconsistent.











d. Crank Nicholson Scheme

Code	Output
<pre> clearvars clc fprintf('\nCrank Nicholson Scheme applied to 1-D heat conduction problem\n') %Endpoints of rod such that b-a=Length of rod a=0; b=1; % for Ut=alp*Uxx form alp=400/(385*8000); % dx and dt are grid differences dx=input('\nUnit Grid Size dx: '); dt=input('\nInput Time Step dt: '); %dx=0.25; %dt=2; x=a:dx:b; t=0:dt:900; %Calculation for size of x and t points [sizx,sizx]=size(x); [sizt,sizt]=size(t); u= zeros(sizt,sizx); %formula for r r=alp*dt/(dx^2); %Initial Condition u(1,:)=25; %Rod is maintained at 400 C. %Thus setting Boundary conditions u(2:sizt,1)=400; u(2:sizt,sizx)=400; %Formula is worked out for Crank Nicholson %Format U(i+1,j)=F(U(i,j+1),U(i,j-1),U(i,j)) %Here 'i' represents time step %And 'j' represents spatial variation %Loop is written to get iteration values %For deifining TDMA A=zeros(sizx-2,sizx-2); </pre>	<p>a. dy = 0.1m , dt = 1s</p> <p>Crank Nicholson Scheme applied to 1-D heat conduction problem</p> <p>Unit Grid Size dx: 0.1</p> <p>Input Time Step dt: 1</p> <p>Method is stable but not consistent</p> <p>This scheme will not reach to 200 C for interval 0 to 900s Middle length temperature is 129.526119 C at 900s</p> <p>b. dy = 0.2m , dt = 1s</p> <p>Crank Nicholson Scheme applied to 1-D heat conduction problem</p> <p>Unit Grid Size dx: 0.2</p> <p>Input Time Step dt: 1</p> <p>Method is stable but not consistent</p> <p>This scheme will not reach to 200 C for interval 0 to 900s Middle length temperature is 132.813586 C at 900s</p> <p>c. dy = 0.5m , dt = 1s</p> <p>Crank Nicholson Scheme applied to 1-D heat conduction problem</p> <p>Unit Grid Size dx: 0.5</p> <p>Input Time Step dt: 1</p> <p>Method is stable but not consistent</p> <p>This scheme will not reach to 200 C for interval 0 to 900s Middle length temperature is 131.225428 C at 900s</p>

<pre> for i=2:sizt %Now based of formula we will write TDMA matrix %a represents 1st diagonal below main diagonal %b represents main diagonal %c represents 1st diagonal above main diagonal for j=1:sizx-3 a(1,j)=-r; c(1,j)=a(1,j); A(j+1,j)=a(1,j); A(j,j+1)=c(1,j); end for j=1:sizx-2 b(1,j)=2*(1+r); A(j,j)=b(1,j); d(j,1)=r*u(i-1,j)+2*(1-r)*u(i-1,j+1)+r*u(i-1,j+2); end %For identifying the size of matrix [n,j]=size(d); %Definition taken from Atkinson beta(1)=A(1); g(1)=d(1); %TDMA triangular matrix solution for Lg=b form where g=UX for l=2:n %Definition taken from Atkinson alpha(l)=A(l,l-1)/beta(l-1); beta(l)=A(l,l)-alpha(l)*A(l-1,l); g(l)=d(l)-alpha(l)*g(l-1); end %Back substitution for solving g=UX xu(n)=g(n)/beta(n); for l=n-1:-1:1 xu(l)=(g(l)-A(l,l+1)*xu(l+1))/beta(l); end xu; %Updating thevalue of u u(i,2:sizx-1)=[xu]; end </pre>	<p>d. dy = 0.1m , dt = 0.5s</p> <p>Crank Nicholson Scheme applied to 1-D heat conduction problem</p> <p>Unit Grid Size dx: 0.1</p> <p>Input Time Step dt: 0.5</p> <p>Method is stable but not consistent</p> <p>This scheme will not reach to 200 C for interval 0 to 900s Middle length temperature is 129.574063 C at 900s</p> <p>e. dy = 0.2m , dt = 0.5s</p> <p>Crank Nicholson Scheme applied to 1-D heat conduction problem</p> <p>Unit Grid Size dx: 0.2</p> <p>Input Time Step dt: 0.5</p> <p>Method is stable but not consistent</p> <p>This scheme will not reach to 200 C for interval 0 to 900s Middle length temperature is 132.858142 C at 900s</p> <p>f. dy = 0.5m , dt = 0.5s</p> <p>Crank Nicholson Scheme applied to 1-D heat conduction problem</p> <p>Unit Grid Size dx: 0.5</p> <p>Input Time Step dt: 0.5</p> <p>Method is stable but not consistent</p> <p>This scheme will not reach to 200 C for interval 0 to 900s Middle length temperature is 131.263690 C at 900s</p>
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<pre> %y represents all the values of temperature which change for t>0 at mid %midlen y=u(:,round(sizx/2)); figure subplot(2,1,2); plot(t,y,'-r') title('Graph of temperature vs time at mid-length') xlabel('Time') ylabel('Temperature') legend(['dx = ',num2str(dx),' dt = ',num2str(dt)],'Location','north') %Additional information y=u(sizt,:); %represents temperature variation in bar at 900s subplot(2,1,1); plot(x,y,'-ro') title('Graph of temperature vs length at t = 900s ') xlabel('Length') ylabel('Temperature') legend('Crank Nicholson Scheme','Location','north') grid on %'k' Finds the values where we get 200 C k = find(round(u(:,round(sizx/2)))==200 & (abs(200- u(:,round(sizx/2)))< 0.05)); if isempty(k) == 1 fprintf('\nMethod is stable but not consistent\n'); fprintf('\nThis scheme will not reach to 200 C for interval %d to %ds\nMiddle length temperature is %f C at %ds\n\n',t(1),t(sizt),u(sizt,round(sizx/2)),t(sizt)); else %Some values might be repeated when grid size is small %Thus median is taken as rod has symmetric condition k=round(median(k)); %t200 represents time when mid-section is at 200 C t200=t(k); fprintf('\nMiddle length reaches 200 C at t = %fs\n\n',t200) %u(k,round(sizx/2)) is actual value close to 200 C end </pre>	<p>g. $dy = 0.1m$, $dt = 0.1s$</p> <p>Crank Nicholson Scheme applied to 1-D heat conduction problem</p> <p>Unit Grid Size dx: 0.1</p> <p>Input Time Step dt: 0.1</p> <p>Method is stable but not consistent</p> <p>This scheme will not reach to 200 C for interval 0 to 900s Middle length temperature is 129.612401 C at 900s</p> <p>h. $dy = 0.2m$, $dt = 0.1s$</p> <p>Crank Nicholson Scheme applied to 1-D heat conduction problem</p> <p>Unit Grid Size dx: 0.2</p> <p>Input Time Step dt: 0.1</p> <p>Method is stable but not consistent</p> <p>This scheme will not reach to 200 C for interval 0 to 900s Middle length temperature is 132.893770 C at 900s</p> <p>i. $dy = 0.5m$, $dt = 0.1s$</p> <p>Crank Nicholson Scheme applied to 1-D heat conduction problem</p> <p>Unit Grid Size dx: 0.5</p> <p>Input Time Step dt: 0.1</p> <p>Method is stable but not consistent</p> <p>This scheme will not reach to 200 C for interval 0 to 900s Middle length temperature is 131.294287 C at 900s</p>
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➤ Results

For any values of dt and dy mentioned the solution will be erroneous as the method is inconsistent.

