

## Assignment 5

Group Number 32

Software used: **Matlab**

Sanit Prashant Bhatkar (173109003)

Omkar Anil Pawar (173106001)

### 1. Integration of $\int_0^3 e^{-x} \sin x \, dx$

#### a. Trapezoidal Method

Code	Output
<pre> clearvars clc fprintf('Trapezoidal Integration\n\n'); n=input('Enter the number of subinterval = '); la=input('Actual value of Integration = '); a=0; b=3; h=(b-a)/n; x=a:h:b; y=exp(-x).*sin(x); %Trapezoidal Rule Formula sum=sum(y(2:n)); I=h/2*(y(1)+y(n+1)+(2*sum)); error=I-la; fprintf('\nIntegration by Trapezoidal Rule'); I fprintf('Deviation from actual value '); error </pre>	<p><b>a. For n = 10</b></p> <p>Trapezoidal Integration</p> <p>Enter the number of subinterval = 10 Actual value of Integration = .5211314363</p> <p>Integration by Trapezoidal Rule I =</p> <p>0.513232627813492</p> <p>Deviation from actual value error =</p> <p>-0.007898808486508</p> <p><b>b. For n = 30</b></p> <p>Trapezoidal Integration</p> <p>Enter the number of subinterval = 30 Actual value of Integration = .5211314363</p> <p>Integration by Trapezoidal Rule I =</p> <p>0.520251463655403</p> <p>Deviation from actual value error =</p> <p>-8.799726445967160e-04</p>

	<p><b>c. For n = 60</b></p> <p>Trapezoidal Integration</p> <p>Enter the number of subinterval = 60 Actual value of Integration = .5211314363</p> <p>Integration by Trapezoidal Rule I =</p> <p>0.520911388830059</p> <p>Deviation from actual value error =</p> <p>-2.200474699414201e-04</p>
--	---

N	Numerical Value	Error
10	0.513232627813492	-0.007898808486508
30	0.520251463655403	-8.799726445967160e-04
60	0.520911388830059	-2.200474699414201e-04

## b. Simpsons 1/3<sup>rd</sup> Rule

Code	Output
<pre>clearvars clc fprintf('Simpson 1/3 Rule Integration\n\n'); n=input('Enter the number of subinterval = '); la=input('Actual value of Integration = '); a=0; b=3; h=(b-a)/n; x=a:h:b; y=exp(-x).*sin(x);  %Simpson 1/3 Rule Formula sum1=0; sum2=0;  for i=2:n     %For choosing even and odd index in formula     if rem(i,2) == 0         sum1=sum1+y(i);     else         sum2=sum2+y(i);     end end  I=h/3*(y(1)+y(n+1)+(2*sum2)+(4*sum1)); error=I-la; fprintf('\nIntegration by Trapezoidal Rule'); I fprintf('Deviation from actual value '); error</pre>	<p><b>a. For n = 10</b></p> <p>Simpson 1/3 Rule Integration</p> <p>Enter the number of subinterval = 10 Actual value of Integration = .5211314363</p> <p>Integration by Trapezoidal Rule I =</p> <p>0.521035634160109</p> <p>Deviation from actual value error =</p> <p>-9.580213989079933e-05</p> <p><b>b. For n = 30</b></p> <p>Simpson 1/3 Rule Integration</p> <p>Enter the number of subinterval = 30 Actual value of Integration = .5211314363</p> <p>Integration by Trapezoidal Rule I =</p> <p>0.521130275452745</p> <p>Deviation from actual value error =</p> <p>-1.160847255432529e-06</p>

	<p><b>c. For n = 60</b></p> <p>Simpson 1/3 Rule Integration</p> <p>Enter the number of subinterval = 60 Actual value of Integration = .5211314363</p> <p>Integration by Trapezoidal Rule I =</p> <p>0.521131363888277</p> <p>Deviation from actual value error =</p> <p>-7.241172295113785e-08</p>
--	--

N	Numerical Value	Error
10	0.521035634160109	-9.580213989079933e-05
30	0.521130275452745	-1.160847255432529e-06
60	0.521131363888277	-7.241172295113785e-08

### c. Gauss Quadrature Rule

Code	Output																																												
<pre> clearvars clc a=0; b=3; fprintf('Gauss Quadrature Integration\n'); n=input('\nSpecify order of gauss quadrature: '); la=input('Actual value of Integration = ');  % Building the component matrix CMP % CMP is such that det(xI-CMP)=P_n(x), P_n is Legendre polynomial % CMP is symmetric matrix. i = 1:n-1; dummy = i./sqrt(4*i.^2-1); CMP = diag(dummy,1) + diag(dummy,-1);  % Determining the abscissas (x) and weights (w) % abscissas are the roots of the characteristic polynomial (eigen values) % weights can be derived from the corresponding eigenvectors. [e_vect e_val] = eig(CMP); [Points k] = sort(diag(e_val)); e_vect = e_vect(:,k); Weights = 2 * e_vect(:,1).^2; fprintf('\nTable points and weights are for interval [- 1,1]\n'); Table= table(Points,Weights) %conversion from [-1,1] to [a,b] Points=(Points*(b-a)+a+b)/2.0; y=exp(-Points).*sin(Points); w=(b-a)*Weights/2; fprintf('\nTable points and weights are for interval [%d,%d]\n',a,b); Table= table(Points,Weights) l=w'*y; error=l-la; fprintf('\nIntegration by Trapezoidal Rule'); l fprintf('Deviation from actual value '); error </pre>	<p><b>a. For n = 10</b></p> <p>Gauss Quadrature Integration</p> <p>Specify order of gauss quadrature: 10 Actual value of Integration = .5211314363</p> <p>Table points and weights are for interval [-1,1]</p> <p>Table =</p> <table> <tr> <th>Points</th><th>Weights</th></tr> <tr><td>-0.973906528517171</td><td>0.0666713443086883</td></tr> <tr><td>-0.865063366688985</td><td>0.149451349150581</td></tr> <tr><td>-0.679409568299025</td><td>0.219086362515981</td></tr> <tr><td>-0.433395394129247</td><td>0.269266719309997</td></tr> <tr><td>-0.148874338981631</td><td>0.295524224714753</td></tr> <tr><td>0.148874338981631</td><td>0.295524224714753</td></tr> <tr><td>0.433395394129247</td><td>0.269266719309996</td></tr> <tr><td>0.679409568299024</td><td>0.219086362515982</td></tr> <tr><td>0.865063366688984</td><td>0.149451349150581</td></tr> <tr><td>0.973906528517172</td><td>0.0666713443086884</td></tr> </table> <p>Table points and weights are for interval [0,3]</p> <p>Table =</p> <table> <tr> <th>Points</th><th>Weights</th></tr> <tr><td>0.0391402072242428</td><td>0.0666713443086883</td></tr> <tr><td>0.202404949966523</td><td>0.149451349150581</td></tr> <tr><td>0.480885647551463</td><td>0.219086362515981</td></tr> <tr><td>0.849906908806129</td><td>0.269266719309997</td></tr> <tr><td>1.27668849152755</td><td>0.295524224714753</td></tr> <tr><td>1.72331150847245</td><td>0.295524224714753</td></tr> <tr><td>2.15009309119387</td><td>0.269266719309996</td></tr> <tr><td>2.51911435244854</td><td>0.219086362515982</td></tr> <tr><td>2.79759505003348</td><td>0.149451349150581</td></tr> <tr><td>2.96085979277576</td><td>0.0666713443086884</td></tr> </table>	Points	Weights	-0.973906528517171	0.0666713443086883	-0.865063366688985	0.149451349150581	-0.679409568299025	0.219086362515981	-0.433395394129247	0.269266719309997	-0.148874338981631	0.295524224714753	0.148874338981631	0.295524224714753	0.433395394129247	0.269266719309996	0.679409568299024	0.219086362515982	0.865063366688984	0.149451349150581	0.973906528517172	0.0666713443086884	Points	Weights	0.0391402072242428	0.0666713443086883	0.202404949966523	0.149451349150581	0.480885647551463	0.219086362515981	0.849906908806129	0.269266719309997	1.27668849152755	0.295524224714753	1.72331150847245	0.295524224714753	2.15009309119387	0.269266719309996	2.51911435244854	0.219086362515982	2.79759505003348	0.149451349150581	2.96085979277576	0.0666713443086884
Points	Weights																																												
-0.973906528517171	0.0666713443086883																																												
-0.865063366688985	0.149451349150581																																												
-0.679409568299025	0.219086362515981																																												
-0.433395394129247	0.269266719309997																																												
-0.148874338981631	0.295524224714753																																												
0.148874338981631	0.295524224714753																																												
0.433395394129247	0.269266719309996																																												
0.679409568299024	0.219086362515982																																												
0.865063366688984	0.149451349150581																																												
0.973906528517172	0.0666713443086884																																												
Points	Weights																																												
0.0391402072242428	0.0666713443086883																																												
0.202404949966523	0.149451349150581																																												
0.480885647551463	0.219086362515981																																												
0.849906908806129	0.269266719309997																																												
1.27668849152755	0.295524224714753																																												
1.72331150847245	0.295524224714753																																												
2.15009309119387	0.269266719309996																																												
2.51911435244854	0.219086362515982																																												
2.79759505003348	0.149451349150581																																												
2.96085979277576	0.0666713443086884																																												

Integration by Trapezoidal Rule

I =

0.521131436311284

Deviation from actual value

error =

1.128408477768517e-11

**b. For n = 30**

Gauss Quadrature Integration

Specify order of gauss quadrature: 30

Actual value of Integration = .5211314363

Table points and weights are for interval [-1,1]

Table =

Points	Weights
-0.996893484074649	0.0079681924961671
-0.983668123279747	0.0184664683110903
-0.960021864968308	0.0287847078833241
-0.926200047429274	0.0387991925696271
-0.882560535792053	0.0484026728305933
-0.829565762382768	0.0574931562176187
-0.767777432104826	0.0659742298821809
-0.697850494793316	0.0737559747377054
-0.620526182989243	0.08075589522942
-0.53662414814202	0.0868997872010829
-0.447033769538089	0.0921225222377862
-0.352704725530878	0.0963687371746441
-0.25463692616789	0.0995934205867952
-0.153869913608584	0.101762389748405
-0.0514718425553178	0.102852652893558
0.0514718425553176	0.102852652893559
0.153869913608583	0.101762389748406
0.25463692616789	0.0995934205867953
0.352704725530878	0.0963687371746442
0.447033769538089	0.0921225222377858
0.53662414814202	0.0868997872010828
0.620526182989243	0.0807558952294205
0.697850494793316	0.0737559747377052
0.767777432104826	0.0659742298821799
0.829565762382768	0.0574931562176193
0.882560535792052	0.0484026728305945
0.926200047429274	0.0387991925696271
0.960021864968307	0.0287847078833229
0.983668123279747	0.0184664683110908
0.996893484074649	0.00796819249616663

Table points and weights are for interval [0,3]

Table =

Points	Weights
0.00465977388802585	0.0079681924961671
0.0244978150803796	0.0184664683110903
0.0599672025475384	0.0287847078833241
0.110699928856088	0.0387991925696271
0.176159196311921	0.0484026728305933
0.255651356425847	0.0574931562176187
0.348333851842761	0.0659742298821809
0.453224257810027	0.0737559747377054
0.569210725516136	0.08075589522942
0.69506377778697	0.0868997872010829
0.829449345692866	0.0921225222377862
0.970942911703683	0.0963687371746441
1.11804461074817	0.0995934205867952
1.26919512958712	0.101762389748405
1.42279223616702	0.102852652893558
1.57720776383298	0.102852652893559
1.73080487041288	0.101762389748406
1.88195538925183	0.0995934205867953
2.02905708829632	0.0963687371746442
2.17055065430713	0.0921225222377858
2.30493622221303	0.0868997872010828
2.43078927448386	0.0807558952294205
2.54677574218997	0.0737559747377052
2.65166614815724	0.0659742298821799
2.74434864357415	0.0574931562176193
2.82384080368808	0.0484026728305945
2.88930007114391	0.0387991925696271
2.94003279745246	0.0287847078833229
2.97550218491962	0.0184664683110908
2.99534022611197	0.00796819249616663

Integration by Trapezoidal Rule

I =

0.521131436311284

Deviation from actual value

error =

1.128375171077778e-11

**c. For n = 60**

Gauss Quadrature Integration

Specify order of gauss quadrature: 60

Actual value of Integration = .5211314363

Table points and weights are for interval [-1,1]

Table =

Points	Weights
-0.999210123227436	0.00202681196887369
-0.995840525118838	0.00471272992695357
-0.989787895222222	0.00738993116334517
-0.981067201752598	0.0100475571822884
-0.969701788765053	0.0126781664768159
-0.955722255839996	0.0152746185967853
-0.939166276116423	0.017829901014208
-0.920078476177628	0.0203371207294569
-0.898510310810046	0.0227895169439974
-0.874519922646898	0.0251804776215212
-0.848171984785929	0.0275035567499242
-0.819537526162146	0.029752491500789
-0.788693739932264	0.0319212190192966
-0.755723775306585	0.034003892724946
-0.72071651335573	0.0359948980510852
-0.683766327381355	0.0378888675692432
-0.644972828489477	0.0396806954523805
-0.60444059704851	0.041365551235585
-0.562278900753944	0.0429388928359354
-0.51860140005857	0.0443964787957871
-0.473525841761707	0.0457343797161145
-0.427173741583078	0.0469489888489121
-0.379670056576798	0.0480370318199712
-0.331142848268448	0.048995575455757
-0.281722937423262	0.0498220356905501
-0.231543551376029	0.0505141845325096
-0.180739964873426	0.0510701560698554
-0.129449135396945	0.0514884515009808
-0.0778093339495365	0.0517679431749103
-0.0259597723012477	0.0519078776312202
0.025959772301248	0.0519078776312207
0.0778093339495365	0.0517679431749105
0.129449135396945	0.0514884515009808
0.180739964873426	0.0510701560698562
0.231543551376029	0.0505141845325093
0.281722937423262	0.0498220356905503
0.331142848268448	0.0489955754557568
0.379670056576798	0.0480370318199711
0.427173741583078	0.046948988848912
0.473525841761707	0.0457343797161142
0.51860140005857	0.0443964787957868
0.562278900753945	0.0429388928359357



0.60444059704851	0.0413655512355851
0.644972828489477	0.0396806954523809
0.683766327381355	0.0378888675692436
0.720716513355731	0.0359948980510841
0.755723775306586	0.0340038927249463
0.788693739932264	0.0319212190192959
0.819537526162146	0.0297524915007894
0.84817198478593	0.0275035567499249
0.874519922646898	0.0251804776215215
0.898510310810046	0.0227895169439978
0.920078476177627	0.0203371207294572
0.939166276116423	0.0178299010142079
0.955722255839996	0.0152746185967845
0.969701788765053	0.0126781664768157
0.981067201752598	0.0100475571822879
0.989787895222222	0.00738993116334542
0.995840525118838	0.00471272992695366
0.999210123227436	0.00202681196887372

Table points and weights are for interval [0,3]

Table =

Points	Weights
0.00118481515884561	0.00202681196887369
0.00623921232174252	0.00471272992695357
0.0153181571666674	0.00738993116334517
0.0283991973711024	0.0100475571822884
0.045447316852421	0.0126781664768159
0.0664166162400055	0.0152746185967853
0.0912505858253647	0.017829901014208
0.119882285733558	0.0203371207294569
0.152234533784931	0.0227895169439974
0.188220116029652	0.0251804776215212
0.227742022821106	0.0275035567499242
0.270693710756782	0.029752491500789
0.316959390101604	0.0319212190192966
0.366414337040122	0.034003892724946
0.418925229966404	0.0359948980510852
0.474350508927967	0.0378888675692432
0.532540757265785	0.0396806954523805
0.593339104427235	0.041365551235585
0.656581648869083	0.0429388928359354
0.722097899912145	0.0443964787957871
0.789711237357439	0.0457343797161145
0.859239387625382	0.0469489888489121
0.930494915134803	0.0480370318199712
1.00328572759733	0.048995575455757
1.07741559386511	0.0498220356905501
1.15268467293596	0.0505141845325096
1.22889005268986	0.0510701560698554
1.30582629690458	0.0514884515009808
1.3832859990757	0.0517679431749103
1.46106034154813	0.0519078776312202

	1.53893965845187    0.0519078776312207 1.6167140009243    0.0517679431749105 1.69417370309542    0.0514884515009808 1.77110994731014    0.0510701560698562 1.84731532706404    0.0505141845325093 1.92258440613489    0.0498220356905503 1.99671427240267    0.0489955754557568 2.0695050848652    0.0480370318199711 2.14076061237462    0.046948988848912 2.21028876264256    0.0457343797161142 2.27790210008785    0.0443964787957868 2.34341835113092    0.0429388928359357 2.40666089557277    0.0413655512355851 2.46745924273422    0.0396806954523809 2.52564949107203    0.0378888675692436 2.5810747700336    0.0359948980510841 2.63358566295988    0.0340038927249463 2.6830406098984    0.0319212190192959 2.72930628924322    0.0297524915007894 2.77225797717889    0.0275035567499249 2.81177988397035    0.0251804776215215 2.84776546621507    0.0227895169439978 2.88011771426644    0.0203371207294572 2.90874941417463    0.0178299010142079 2.93358338375999    0.0152746185967845 2.95455268314758    0.0126781664768157 2.9716008026289    0.0100475571822879 2.98468184283333    0.00738993116334542 2.99376078767826    0.00471272992695366 2.99881518484115    0.00202681196887372
	Integration by Trapezoidal Rule I =  0.521131436311284  Deviation from actual value error =  1.128397375538270e-11

N	Numerical Value	Error
10	0.521131436311284	1.128408477768517e-11
30	0.521131436311284	1.128375171077778e-11
60	0.521131436311284	1.128397375538270e-11

#### d. Modified Trapezoidal Rule

Code	Output
<pre> clearvars clc fprintf('Modified Trapezoidal Rule Integration\n\n'); n=input('Number of subintervals: '); la=input('Actual value of Integration = '); a=0; b=3; h=(b-a)/n; x=a:h:b; dif=(exp(-x).*cos(x))-(exp(-x).*sin(x)); y=exp(-x).*sin(x); sum1=0; sum2=0; for i=1:n+1     sum1= sum2+((y(i).*2)+(dif(i).*(h)));     sum2=sum1; end sum1=(sum1*h/2); %Trapezoidal Rule Formula error=sum1-la; fprintf('\nIntegration by Trapezoidal Rule'); sum1 fprintf('Deviation from actual value '); error </pre>	<p><b>a. For n = 10</b></p> <p>Modified Trapezoidal Rule Integration</p> <p>Number of subintervals: 10 Actual value of Integration = .5211314363</p> <p>Integration by Trapezoidal Rule sum1 =</p> <p>0.538934247733972</p> <p>Deviation from actual value error =</p> <p>0.017802811433972</p> <p><b>b. For n = 30</b></p> <p>Modified Trapezoidal Rule Integration</p> <p>Number of subintervals: 30 Actual value of Integration = .5211314363</p> <p>Integration by Trapezoidal Rule sum1 =</p> <p>0.523400712781964</p> <p>Deviation from actual value error =</p> <p>0.002269276481964</p>

	<p><b>c. For n = 60</b></p> <p>Modified Trapezoidal Rule Integration</p> <p>Number of subintervals: 60 Actual value of Integration = .5211314363</p> <p>Integration by Trapezoidal Rule sum1 =</p> <p>0.521863419767683</p> <p>Deviation from actual value error =</p> <p>7.319834676834391e-04</p>
--	---

N	Numerical Value	Error
10	0.538934247733972	0.017802811433972
30	0.523400712781964	0.002269276481964
60	0.521863419767683	7.319834676834391e-04

### ➤ Comparison of Results

Error Table for all methods and different intervals

Method/Sub Interval	10	30	60
Trapezoidal	-0.007898808486508	-8.799726445967160e-04	-2.200474699414201e-04
Simpson 1/3	-9.580213989079933e-05	-1.160847255432529e-06	-7.241172295113785e-08
Gauss Quadrature	1.128408477768517e-11	1.128375171077778e-11	1.128397375538270e-11
Modified Trapezoidal	0.017802811433972	0.002269276481964	7.319834676834391e-04

### ➤ Comparison in Method

#### a) Trapezoidal Rule

- The error goes on decreasing as interval increases
- Method underestimates the results for all N and the converges to exact solution

b) Simpson 1/3 rule

- The error goes on decreasing as interval increases
- Method underestimates the results for all N and the converges to exact solution

c) Gauss Quadrature Rule

- The error goes on decreasing as interval increases
- Method over estimates the results for all N and the converges to exact solution

d) Modified Trapezoidal Rule

- The error goes on decreasing as interval increases
- Method over estimates the results for all N and the converges to exact solution

➤ Comparison in Intervals considering only absolute values of error

a) N = 10

- Error in Modified trapezoidal > Trapezoidal rule > Simpson 1/3 rule > Gauss Quadrature

b) N = 30

- Error in Modified trapezoidal > Trapezoidal rule > Simpson 1/3 rule > Gauss Quadrature

c) N = 60

- Error in Modified trapezoidal > Trapezoidal rule > Simpson 1/3 rule > Gauss Quadrature

## 2. Differentiation of $e^{-x} \sin(x)$

### a. Differentiation scheme

Code	Output																																																
<pre>clc clearvars a=0; b=3; fprintf('Differentiation schemes\n\n'); n=input('Number of intervals: '); h=(b-a)/n; x=a:h:b; y=exp(-x).*sin(x); dx=zeros(1,n+1); dxb=dx; dxc=dx;  for i= 1:n     dx(i)=(y(i+1)-y(i))/h;     dxb(i+1)=(y(i+1)-y(i))/h;     if i==1         dxc(i)=0;     else         dxc(i)=(y(i+1)-y(i-1))/(2*h);     end end  %NOTE: zero values in forward_diff,backward_diff,central diff are non %existent Table= table([x],[dxf],[dxb],[dxc]); Table.Properties.VariableNames = {'x','Forward_diff','Backward_diff','Central_diff'}</pre>	<p><b>a. For n = 10</b></p> <p>Differntiation schemes</p> <p>Number of intervals: 10</p> <p>Table =</p> <table><tr><th>x</th><th>Forward_diff</th><th>Backward_diff</th><th>Central_diff</th></tr><tr><td>0</td><td>0.72976</td><td>NA</td><td>NA</td></tr><tr><td>0.3</td><td>0.30319</td><td>0.72976</td><td>0.51647</td></tr><tr><td>0.6</td><td>0.028649</td><td>0.30319</td><td>0.16592</td></tr><tr><td>0.9</td><td>-0.12584</td><td>0.028649</td><td>-0.048596</td></tr><tr><td>1.2</td><td>-0.19385</td><td>-0.12584</td><td>-0.15984</td></tr><tr><td>1.5</td><td>-0.20532</td><td>-0.19385</td><td>-0.19958</td></tr><tr><td>1.8</td><td>-0.18423</td><td>-0.20532</td><td>-0.19478</td></tr><tr><td>2.1</td><td>-0.1481</td><td>-0.18423</td><td>-0.16617</td></tr><tr><td>2.4</td><td>-0.10851</td><td>-0.1481</td><td>-0.12831</td></tr><tr><td>2.7</td><td>-0.072321</td><td>-0.10851</td><td>-0.090418</td></tr><tr><td>3</td><td>NA</td><td>-0.072321</td><td>NA</td></tr></table>	x	Forward_diff	Backward_diff	Central_diff	0	0.72976	NA	NA	0.3	0.30319	0.72976	0.51647	0.6	0.028649	0.30319	0.16592	0.9	-0.12584	0.028649	-0.048596	1.2	-0.19385	-0.12584	-0.15984	1.5	-0.20532	-0.19385	-0.19958	1.8	-0.18423	-0.20532	-0.19478	2.1	-0.1481	-0.18423	-0.16617	2.4	-0.10851	-0.1481	-0.12831	2.7	-0.072321	-0.10851	-0.090418	3	NA	-0.072321	NA
x	Forward_diff	Backward_diff	Central_diff																																														
0	0.72976	NA	NA																																														
0.3	0.30319	0.72976	0.51647																																														
0.6	0.028649	0.30319	0.16592																																														
0.9	-0.12584	0.028649	-0.048596																																														
1.2	-0.19385	-0.12584	-0.15984																																														
1.5	-0.20532	-0.19385	-0.19958																																														
1.8	-0.18423	-0.20532	-0.19478																																														
2.1	-0.1481	-0.18423	-0.16617																																														
2.4	-0.10851	-0.1481	-0.12831																																														
2.7	-0.072321	-0.10851	-0.090418																																														
3	NA	-0.072321	NA																																														

**b. For n = 30**

Differntiation schemes

Number of intervals: 30

Table =

x	Forward_diff	Backward_diff	Central_diff
0	0.90333	NA	NA
0.1	0.72324	0.90333	0.81328
0.2	0.5627	0.72324	0.64297
0.3	0.42108	0.5627	0.49189
0.4	0.29751	0.42108	0.3593
0.5	0.19096	0.29751	0.24424
0.6	0.10027	0.19096	0.14561
0.7	0.024198	0.10027	0.062233
0.8	-0.038519	0.024198	-0.0071604
0.9	-0.089171	-0.038519	-0.063845
1	-0.12903	-0.089171	-0.1091
1.1	-0.15932	-0.12903	-0.14418
1.2	-0.18125	-0.15932	-0.17028
1.3	-0.19591	-0.18125	-0.18858
1.4	-0.20438	-0.19591	-0.20015
1.5	-0.20761	-0.20438	-0.20599
1.6	-0.2065	-0.20761	-0.20705
1.7	-0.20185	-0.2065	-0.20417
1.8	-0.19439	-0.20185	-0.19812
1.9	-0.18477	-0.19439	-0.18958
2	-0.17354	-0.18477	-0.17916
2.1	-0.16122	-0.17354	-0.16738
2.2	-0.1482	-0.16122	-0.15471
2.3	-0.13487	-0.1482	-0.14154
2.4	-0.12151	-0.13487	-0.12819
2.5	-0.10837	-0.12151	-0.11494
2.6	-0.095658	-0.10837	-0.10202
2.7	-0.083516	-0.095658	-0.089587
2.8	-0.072064	-0.083516	-0.07779
2.9	-0.061383	-0.072064	-0.066723
3	NA	-0.061383	NA

**c. For n = 60**

Differentiation schemes

Number of intervals: 60

Table =

x	Forward_diff	Backward_diff	Central_diff
0	0.95083	NA	NA
0.05	0.85583	0.95083	0.90333
0.1	0.76579	0.85583	0.81081
0.15	0.68068	0.76579	0.72324
0.2	0.60043	0.68068	0.64056
0.25	0.52497	0.60043	0.5627
0.3	0.45418	0.52497	0.48958
0.35	0.38798	0.45418	0.42108
0.4	0.32623	0.38798	0.3571
0.45	0.2688	0.32623	0.29751
0.5	0.21556	0.2688	0.24218
0.55	0.16636	0.21556	0.19096
0.6	0.12105	0.16636	0.14371
0.65	0.079481	0.12105	0.10027
0.7	0.041486	0.079481	0.060483
0.75	0.0069104	0.041486	0.024198
0.8	-0.024408	0.0069104	-0.0087489
0.85	-0.05263	-0.024408	-0.038519
0.9	-0.077916	-0.05263	-0.065273
0.95	-0.10043	-0.077916	-0.089171
1	-0.12031	-0.10043	-0.11037
1.05	-0.13774	-0.12031	-0.12903
1.1	-0.15285	-0.13774	-0.1453
1.15	-0.1658	-0.15285	-0.15932
1.2	-0.17672	-0.1658	-0.17126
1.25	-0.18577	-0.17672	-0.18125
1.3	-0.19307	-0.18577	-0.18942
1.35	-0.19876	-0.19307	-0.19591
1.4	-0.20296	-0.19876	-0.20086
1.45	-0.2058	-0.20296	-0.20438
1.5	-0.20738	-0.2058	-0.20659
1.55	-0.20783	-0.20738	-0.20761
1.6	-0.20725	-0.20783	-0.20754
1.65	-0.20574	-0.20725	-0.2065
1.7	-0.20339	-0.20574	-0.20457
1.75	-0.2003	-0.20339	-0.20185
1.8	-0.19655	-0.2003	-0.19843
1.85	-0.19223	-0.19655	-0.19439
1.9	-0.1874	-0.19223	-0.18981
1.95	-0.18214	-0.1874	-0.18477
2	-0.17651	-0.18214	-0.17932
2.05	-0.17058	-0.17651	-0.17354
2.1	-0.1644	-0.17058	-0.16749
2.15	-0.15803	-0.1644	-0.16122
2.2	-0.15151	-0.15803	-0.15477
2.25	-0.1449	-0.15151	-0.1482



	2.3	-0.13822	-0.1449	-0.14156
	2.35	-0.13152	-0.13822	-0.13487
	2.4	-0.12483	-0.13152	-0.12818
	2.45	-0.11819	-0.12483	-0.12151
	2.5	-0.11161	-0.11819	-0.1149
	2.55	-0.10513	-0.11161	-0.10837
	2.6	-0.098772	-0.10513	-0.10195
	2.65	-0.092545	-0.098772	-0.095658
	2.7	-0.08647	-0.092545	-0.089508
	2.75	-0.080562	-0.08647	-0.083516
	2.8	-0.074834	-0.080562	-0.077698
	2.85	-0.069294	-0.074834	-0.072064
	2.9	-0.063952	-0.069294	-0.066623
	2.95	-0.058814	-0.063952	-0.061383
	3	NA	-0.058814	NA

### 3) Estimation of distance travelled

#### a. Trapezoidal Method

Code	Output												
<pre>clearvars clc fprintf('Trapezoidal Integration\n\n'); x=[0 1 2 2.5 3]; y=[0 10 12 13 14]; Table= table(x',y'); Table.Properties.VariableNames = {'Time','Velocity'} [n n]=size(y); %Trapezoidal Rule Formula sum1=0; sum2=0; for i=1:1:4     h=((x(i+1)-x(i)));     sum1=sum2+((h/2)*(y(i)+y(i+1)));     sum2=sum1; end fprintf('Distance Travelled in 0-3 sec is %f\n\n',sum2);</pre>	<p>Trapezoidal Integration</p> <p>Table =</p> <table> <thead> <tr> <th>Time</th><th>Velocity</th></tr> </thead> <tbody> <tr><td>0</td><td>0</td></tr> <tr><td>1</td><td>10</td></tr> <tr><td>2</td><td>12</td></tr> <tr><td>2.5</td><td>13</td></tr> <tr><td>3</td><td>14</td></tr> </tbody> </table> <p>Distance Travelled in 0-3 sec is 29.000000</p>	Time	Velocity	0	0	1	10	2	12	2.5	13	3	14
Time	Velocity												
0	0												
1	10												
2	12												
2.5	13												
3	14												

#### b. Simpsons 1/3<sup>rd</sup> Rule

Code	Output												
<pre>clearvars clc fprintf('Simpson 1/3 Rule Integration\n\n'); x=[0 1 2 2.5 3]; y=[0 10 12 13 14]; Table= table(x',y'); Table.Properties.VariableNames = {'Time','Velocity'} [n n]=size(y); %Simpson 1/3 Rule Formula sum1=0; sum2=0; for i=1:2:4     h=((x(i+2)-x(i))/2);     sum1=sum2+((h/3)*(y(i)+4*y(i+1)+y(i+2)));     sum2=sum1; end fprintf('Distance Travelled in 0-3 sec is %f\n\n',sum2);</pre>	<p>Simpson 1/3 Rule Integration</p> <p>Table =</p> <table> <thead> <tr> <th>Time</th><th>Velocity</th></tr> </thead> <tbody> <tr><td>0</td><td>0</td></tr> <tr><td>1</td><td>10</td></tr> <tr><td>2</td><td>12</td></tr> <tr><td>2.5</td><td>13</td></tr> <tr><td>3</td><td>14</td></tr> </tbody> </table> <p>Distance Travelled in 0-3 sec is 30.333333</p>	Time	Velocity	0	0	1	10	2	12	2.5	13	3	14
Time	Velocity												
0	0												
1	10												
2	12												
2.5	13												
3	14												