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BATCH:	A
EXP NO:	3

AIM	Use Divide and Conquer Approach: Strassen's Matrix Multiplication
THEORY & ALGORITHM:	<p>1) Time complexity of normal matrix multiplication is given as : $T(N) = 8T(N/2) + O(N^2)$</p> <p>2) From Master's Theorem, time complexity of above method is $O(N^3)$</p> <p>3) In the normal method, the main component for high time complexity is 8 recursive calls.</p> <p>4) The idea of Strassen's method is to reduce the number of recursive calls to 7.</p> <p>5) Time Complexity of Strassen's Method : $T(N) = 7T(N/2) + O(N^2)$</p> <p>6) From Master's Theorem, time complexity of above method is $O(N \log 7)$ which is approximately $O(N^{2.8074})$</p> <p>Details - Given two square matrices A and B of size $n \times n$ each, find their multiplication matrix.</p> <p>Naive Method takes the Time Complexity of $O(N^3)$.</p> <p>Divide and Conquer :</p> <p>Following is a simple Divide and Conquer method to multiply two square matrices.</p> <ol style="list-style-type: none"> 1. Divide matrices A and B in 4 sub-matrices of size $N/2 \times N/2$ as shown in the below diagram. 2. Calculate following values recursively. $ae + bg$, $af + bh$, $ce + dg$ and $cf + dh$. <p>Simple Divide and Conquer also leads to $O(N^3)$, can there be a better way?</p>

In the above divide and conquer method, the main component for high time complexity is 8 recursive calls. The idea of Strassen's method is to reduce the number of recursive calls to 7. Strassen's method is similar to the above simple divide and conquer method in the sense that this method also divides matrices into sub-matrices of size $N/2 \times N/2$ as shown in the above diagram, but in Strassen's method, the four sub-matrices of result are calculated using the following formulae.

Time Complexity of Strassen's Method

Addition and Subtraction of two matrices takes $O(N^2)$ time. So time complexity can be written as

$$T(N) = 7T(N/2) + O(N^2)$$

Generally Strassen's Method is not preferred for practical applications for the following reasons.

The constants used in Strassen's method are high and for a typical application Naive method works better. For Sparse matrices, there are better methods especially designed for them.

The submatrices in recursion take extra space.

Because of the limited precision of computer arithmetic on noninteger values, larger errors accumulate in Strassen's algorithm than in Naive Method.

CODE

```
#include <stdio.h>
#include <time.h>
void main()
{
    int a[2][2], b[2][2], c[2][2], i, j;
    int p[7];
    int s[10];
    clock_t start, end;
    printf("Enter the elements of 1st matrix:\n");
    for (i = 0; i < 2; i++)
    {
        for (j = 0; j < 2; j++)
        {
            scanf("%d", &a[i][j]);
        }
    }
    printf("Enter the elements of 2nd matrix:\n");
    for (i = 0; i < 2; i++)
    {
        for (j = 0; j < 2; j++)
        {
            scanf("%d", &b[i][j]);
        }
    }
    start = clock();
    s[0] = b[0][1] - b[1][1];
    s[1] = a[0][0] + a[0][1];
    s[2] = a[1][0] + a[1][1];
    s[3] = b[1][0] - b[0][0];
    s[4] = a[0][0] + a[1][1];
    s[5] = b[0][0] + b[1][1];
    s[6] = a[0][1] - a[1][1];
    s[7] = b[1][0] + b[1][1];
    s[8] = a[0][0] - a[1][0];
    s[9] = b[0][0] + b[0][1];

    // 7 strassen calculations
    p[0] = s[0] * a[0][0];
    p[1] = s[1] * b[1][1];
    p[2] = s[2] * b[0][0];
    p[3] = s[3] * a[1][1];
    p[4] = s[4] * s[5];
    p[5] = s[6] * s[7];
    p[6] = s[8] * s[9];

    // 4 final output
    c[0][0] = p[4] + p[3] - p[1] + p[5];
    c[0][1] = p[0] + p[1];
```

```

c[1][0] = p[2] + p[3];
c[1][1] = p[4] + p[0] - p[2] - p[6];

for (i = 0; i < 10; i++)
{
    printf("\nS[%d] = %d ", i + 1, s[i]);
}
printf("\n");
for (j = 0; j < 7; j++)
{
    printf("\np[%d] = %d ", j + 1, p[j]);
}
printf("\n\n");
printf("MATRIX A: \n");
for (i = 0; i < 2; i++)
{
    printf("\n");
    for (j = 0; j < 2; j++)
    {
        printf("%d\t", a[i][j]);
    }
}
printf("\n");

printf("MATRIX B: \n");
for (i = 0; i < 2; i++)
{
    printf("\n");
    for (j = 0; j < 2; j++)
    {
        printf("%d\t", b[i][j]);
    }
}
printf("\n");
printf("MATRIX C: \n\n");
printf("%d\t %d\n%d\t %d\n", c[0][0], c[0][1], c[1][0],
c[1][1]);
end = clock();
printf("The time taken by the program: ");
printf("%lf", (double)(end - start) / CLOCKS_PER_SEC);
}

```

OUTPUT

```
Enter the elements of 1st matrix:
```

```
2
```

```
3
```

```
2
```

```
1
```

```
Enter the elements of 2nd matrix:
```

```
5
```

```
0
```

```
2
```

```
1
```

```
S[1] = -1
```

```
S[2] = 5
```

```
S[3] = 3
```

```
S[4] = -3
```

```
S[5] = 3
```

```
S[6] = 6
```

```
S[7] = 2
```

```
S[8] = 3
```

```
S[9] = 0
```

```
S[10] = 5
```

```
p[1] = -2
```

```
p[2] = 5
```

```
p[3] = 15
```

```
p[4] = -3
```

```
p[5] = 18
```

```
p[6] = 6
```

```
p[7] = 0
```

```
MATRIX A:
```

```
2      3
```

```
2      1
```

```
MATRIX B:
```

```
5      0
```

```
2      1
```

```
MATRIX C:
```

```
16     3
```

```
12     1
```

```
The time taken by the program: 0.000095
```

CONCLUSION

From this experiment I have successfully understood Divide and conquer algorithm to perform Strassen's Multiplication.