Basics:

① Find $f(a+h)$ where $f(x) = 5x^3 - 3x^2 + 4x$

$$\Rightarrow f(a+h) = 5[(a+h)^3] - 3[(a+h)^2] + 4(a+h)$$

$$= 5[a^3 + h^3 + 3a^2h + 3ah^2] - 3[a^2 + h^2 + 2ah] + 4a + 4h$$

$$= 5a^3 + 5h^3 + 15a^2h + 15ah^2 - 3a^2 - 3h^2 - 6ah + 4a + 4h \quad \text{(Ans)}$$

② Domain & range of:

a) $\sqrt{3x+2} - 1$

Domain: $3x + 2 \geq 0$ (square root cannot be negative)

$3x \geq -2$

$x \geq -2/3$

$\{x \mid x \geq -2/3\}$

range:

$$y = \sqrt{3x+2} - 1$$

$$\Rightarrow (y+1)^2 = 3x+2$$

$$\Rightarrow x = \frac{1}{3}(y+1)^2 - \frac{2}{3}$$

$\Rightarrow \quad y \geq -1$

$\{ y \mid y \geq -1 \}$

b) $\frac{3}{x-2}$

Domain:

(Denominator cannot be 0)

$\therefore \quad \{ x \mid x \neq 2 \}$

Range:

$y = \frac{3}{x-2}$

$x - 2 = \frac{3}{y}$

$x = \frac{3}{y} + 2$ (Denominator can't be 0)

$\therefore \{ y \mid y \neq 0 \}$

③ Find $(f \circ g)\,4$, where $f(x) = x^2 + 1$,

$$g(x) = \frac{1}{x}$$

$$f(g(4)) = f\left(\frac{1}{4}\right) = \left(\frac{1}{4}\right)^2 + 1$$

$$= \frac{1}{16} + 1 = \frac{17}{16} \text{ (ans)}$$

④ $5^x = 2$

$\Rightarrow \ln 5^x = \ln 2$

$\Rightarrow x \ln 5 = \ln 2$

$x \Rightarrow \dfrac{\ln 2}{\ln 5}$

Imp questions

5) A store is advertising a sale of 20% off all merchandise. Caroline has a coupon that entitles her to an additional 15% off any item, including sale merchandise. If Caroline decides to purchase an item with an original price of x dollars, how much will she end up paying if she applies her coupon to the sale price? Solve this problem by using a composite function.

since sale price is 20% off original price,

$$f(x) = 0.80x$$

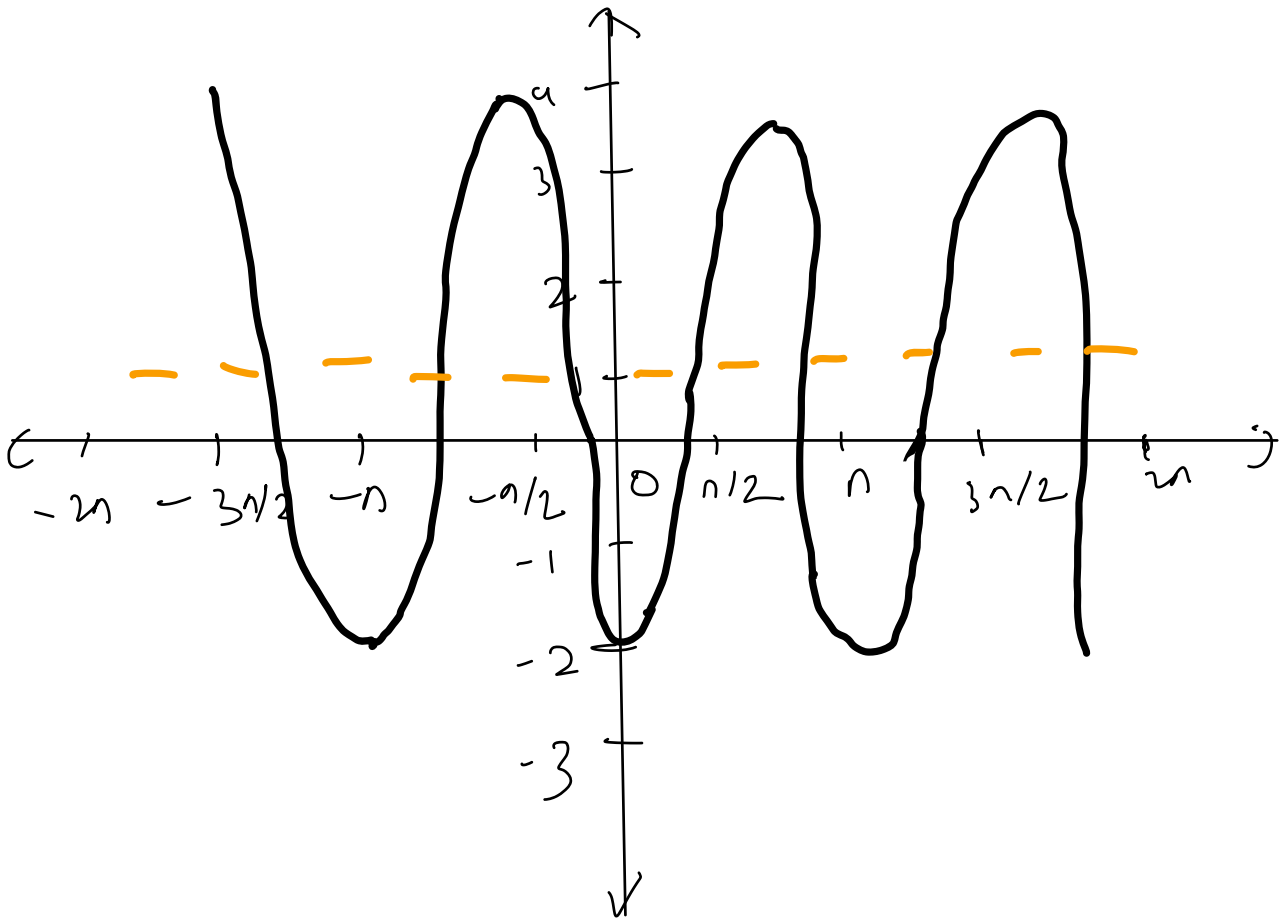
the coupon discount,

$$g(y) = 0.85y$$

Final price,

$$g(f(x)) = 0.85(0.80x) = 0.68x$$

6) $f(x) = 3\sin\left(2\left(x - \frac{\pi}{4}\right)\right) + 1$. plot $f(x)$



7) Find the domain & range of $g(x)$.

$$g(x) = \frac{\sqrt{x^2-9}}{\ln(4-x)}$$

The denominator exists when,

$$\ln(4-x) \neq 0$$

$$\Rightarrow \quad 4-x \neq 1$$

$$\Rightarrow \quad x \neq 3$$

AND when,

$$4-x > 0 \quad (\ln(4-x) \text{ is undefined when } 4-x \leq 0)$$

$$\Rightarrow \quad x < 4$$

The numerator exists when,

$$\sqrt{x^2-9} \geq 0$$

$$\Rightarrow \quad x^2-9 \geq 0$$

$$\Rightarrow \quad x^2 \geq 9$$

So, $\quad x \in (-\infty, -3] \cup [3, \infty)$

∴ The function $g(x)$ is defined only when both the numerator and the denominator exist.

Thus, domain of $g(x)$ is

$$x \in (-\infty, 3] \cup (3, 4)$$

which is the intersection of the domain of the numerator and the denominator.

8) Find domain of $f(x)$:

$$\Rightarrow \frac{\ln 2 + \ln(x^2 - 3x)}{1 - e^{x^2-4}} + \sqrt{x^2+1}$$

$$\Rightarrow 1 - e^{x^2-4} \neq 0.$$

$$\Rightarrow e^{x^2-4} \neq 1 \quad \text{(Denominator cannot be 0)}$$

$$\Rightarrow x^2 - 4 \log e \neq \log 1$$

$\Rightarrow \quad x^2 - 4 \neq 0$

$\Rightarrow \quad x^2 \neq 4$

$x \neq \pm 2$

Next,

$x^2 - 3x > 0$

$x(x-3) > 0$

$x < 0$ and $x > 3$

(ln is undefined for non-positive nos.)

Lastly, domain of $\sqrt{x^2+1}$ is all real nos.

Thus,

domain of $f(x) =$

$(-\infty, -2) \cup (-2, 0) \cup (3, \infty)$ (Ans)

9) Is $\arcsin\left(\sin\left(\frac{4}{\pi}\right)\right) =$

$$\sin\left(\arcsin\left(\frac{4}{\pi}\right)\right) \; ?$$

RHS: RHS is undefined since $\sin\frac{4}{\pi} > 1$ which is not in the domain of $\arcsin$.

LHS: $\sin\frac{4}{\pi}$ is defined since

$$\frac{\pi}{4}$$

10) A rock is dropped from a height of 64 ft. It is determined that its height (in feet) above ground t secs later. (for $0 \leq t \leq 2$) is given by $s(t) = -16t^2 + 64$. Find the avg velocity of the rock over each of the given time intervals. Use this info to guess the instantaneous velocity of the rock at time $t = 0.5$.

a) $[0.49, 0.5]$

a) Value $= \dfrac{s[0.5] - s(0.49)}{0.5 - 0.49}$

$= -15.84$ (ans)

11) $\lim_{x \to -3^-} \frac{1}{(x+3)^4} = +\infty$

12) $\lim_{x \to 3} \frac{x^2 - 3x}{2x^2 - 5x - 3}$

$\Rightarrow \lim_{x \to 3} \frac{x(x-3)}{2x^2 - 2x - 3x - 3}$

$\Rightarrow \lim_{x \to 3} \frac{x(x-3)}{(x-3)(2x+1)}$

$\Rightarrow \lim_{x \to 3} \frac{x}{2x+1}$

$\Rightarrow \frac{3}{7}$ (Ans)

(13) $\lim_{\theta \to 0} \frac{1-\cos\theta}{\theta}$

$$\Rightarrow \lim_{\theta \to 0} \frac{1-\cos\theta}{\theta} \cdot \frac{1+\cos\theta}{1+\cos\theta}$$

$$\Rightarrow \lim_{\theta \to 0} \frac{1-\cos^2\theta}{\theta(1+\cos\theta)}$$

$$\Rightarrow \lim_{\theta \to 0} \frac{\sin\theta}{\theta} \cdot \frac{\sin\theta}{1+\cos\theta}$$

$$\Rightarrow 1 \cdot 0 = 0 \quad \text{(Ans)}$$

(14) Using ~~the~~ definition, determine whether ~~the~~ function $f(x)$

$$= \begin{cases} \sin x & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$ is continuous at

$$f(0) = 1$$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{\sin x}{x} = 1$$

LHS = RHS

$$f(0) = 1 = \lim_{x \to 0} f(x)$$ (Ans)

(5) Is $\frac{x+1}{x-5}$ continuous?

$\frac{x+1}{x-5}$ is continuous for every value of x except

$$x = 5.$$

(since Denominator cannot be 0)