Unit - II
2.16 Resonance

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Syllabus

UNIT – II **14 Periods**

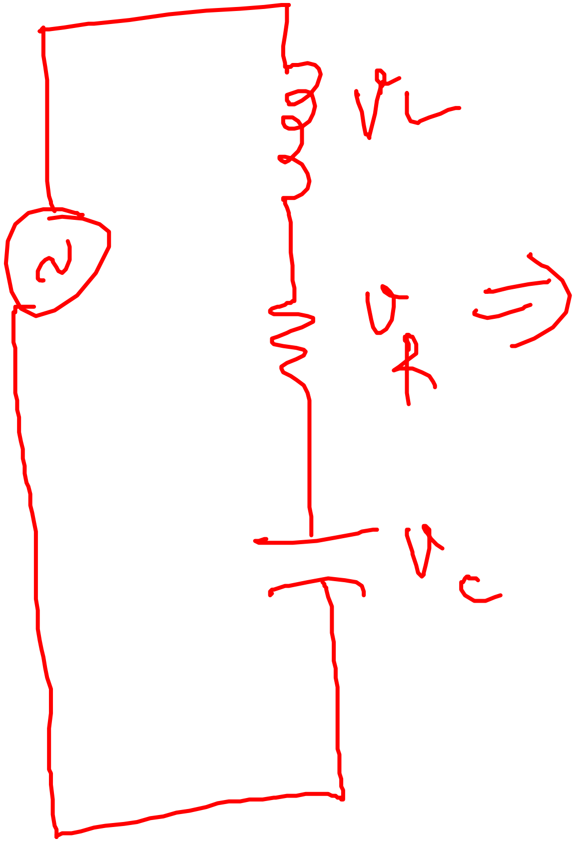
DC Circuit Analysis: Voltage source and current sources, ideal and practical, Kirchhoff's laws and applications to network solutions using mesh analysis, - Simplifications of networks using series- parallel, Star/Delta transformation, DC circuits-Current-voltage relations of electric network by mathematical equations to analyse the network (Superposition theorem, Thevenin's theorem, Maximum Power Transfer theorem), Transient analysis of R-L, R-C and R-L-C Circuits.

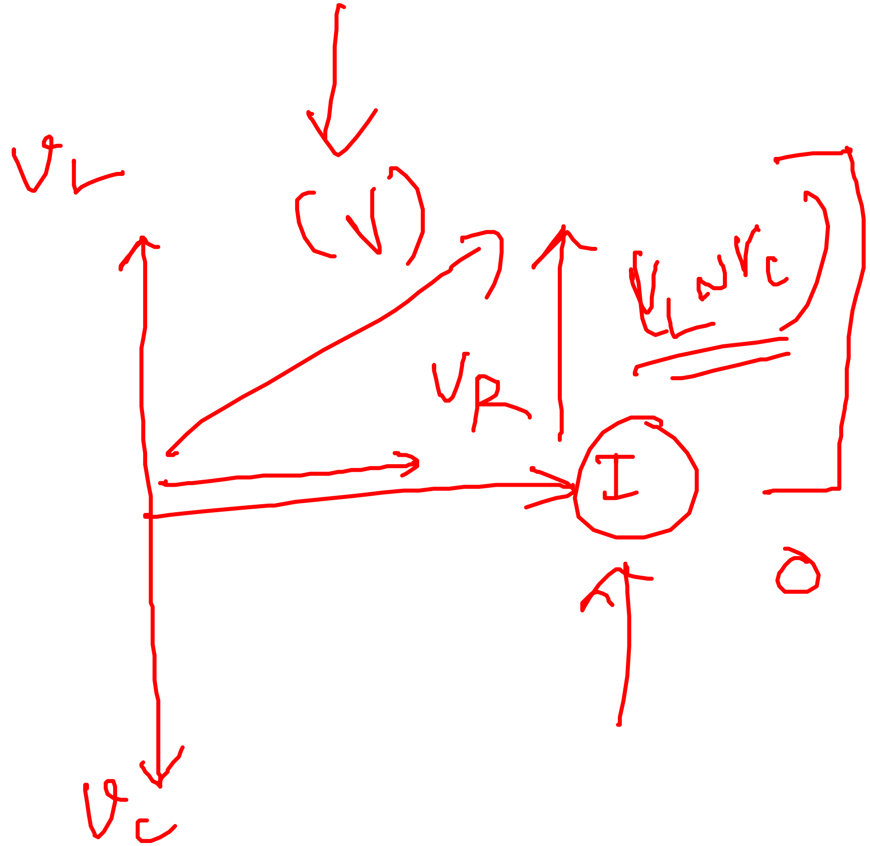
AC Steady-state Analysis: AC waveform definitions - Form factor - Peak factor - study of R-L - R-C -RLC series circuit - R-L-C parallel circuit - phasor representation in polar and rectangular form - concept of impedance - admittance - active - reactive - apparent and complex power - power factor, Resonance in R-L-C circuits - 3 phase balanced AC Circuits

Introduction

- **Resonance** is the condition that exists in ac circuits when the input current is in phase with the input voltage.

- When in resonance, the ac circuit is purely resistive and draws power at unity power factor.


VL
VR
VC


VL
V
VR
I
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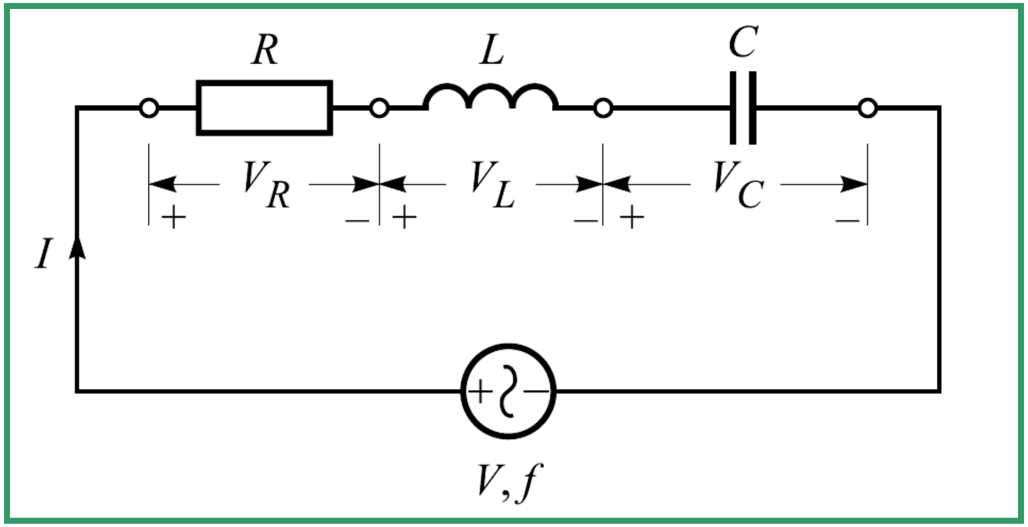
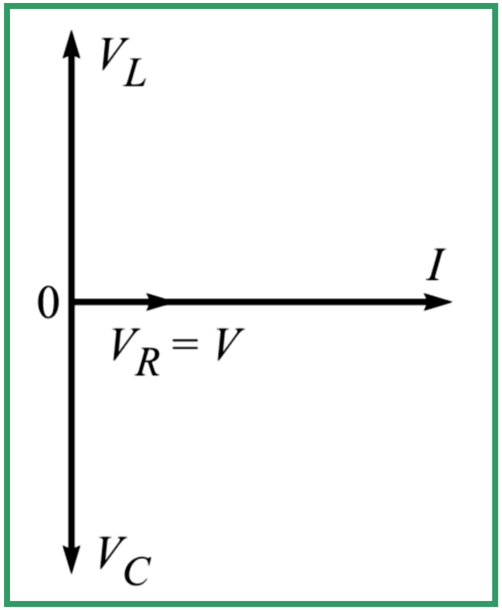
Series Resonant Circuit



(a) The circuit.



(*b*) Phasor diagram at resonance.

The resonance in a series *RLC* circuit requires that

$$X_{L0} - X_{C0} = 0 \quad \text{or} \quad X_{L0} = X_{C0} \quad \text{or} \quad \omega_0 L = \frac{1}{\omega_0 C}$$

The frequency of resonance,

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad \text{or} \quad f_0 = \frac{1}{2\pi\sqrt{LC}}$$

The impedance of the circuit assumes a minimum value given as $Z_0 = R + j0 = R$

The current has a maximum value given by

$$I_0 = \frac{V}{Z_0} = \frac{V}{R}$$

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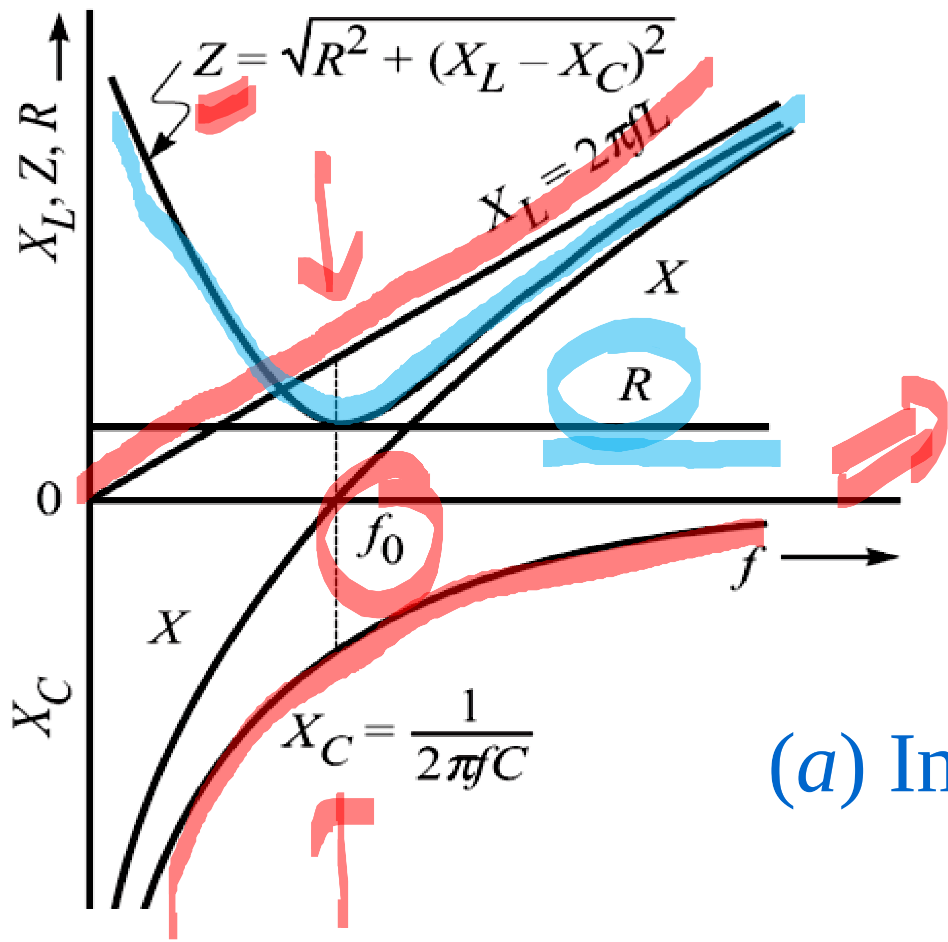
Effect of Variation of Frequency



(*a*) Impedance.

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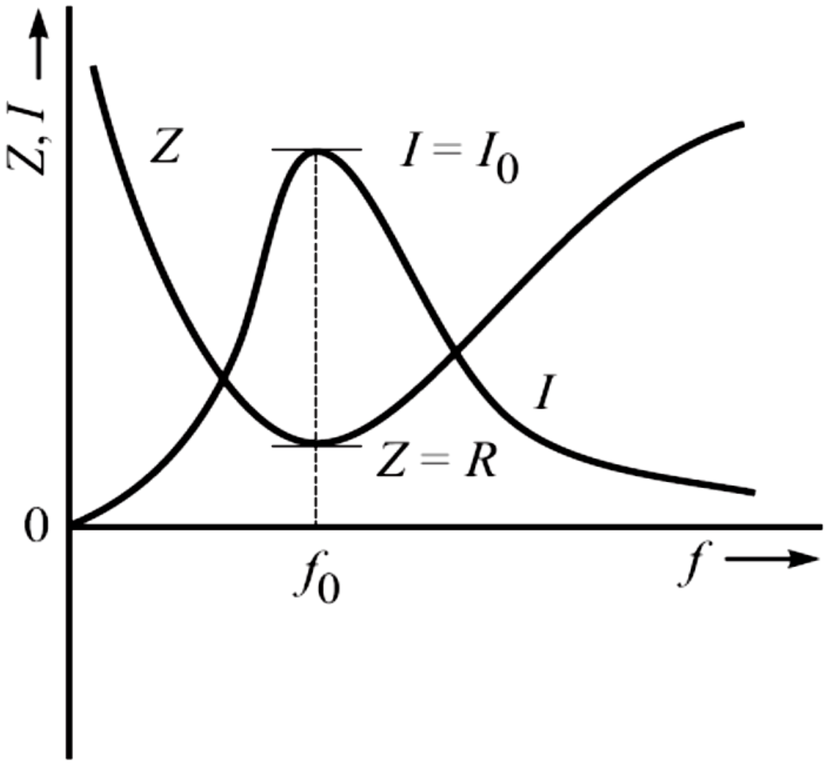
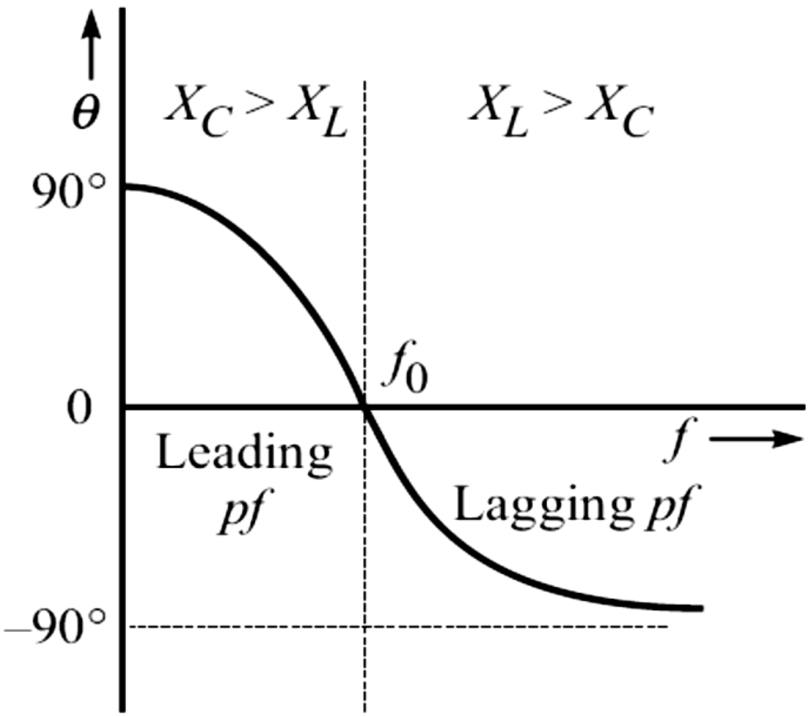


(*b*) Current.



(*c*) Power factor angle.

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- At f_0, the circuit behaves as ***purely resistive***.
- Below f_0,

  X has negative values

  (i.e., the circuit is capacitive).

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- Above f_0,

  X has positive values

  (i.e., the circuit is inductive).

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- Since the current becomes maximum at resonant frequency, the series *RLC* resonant circuit is also called an ***acceptor circuit***.

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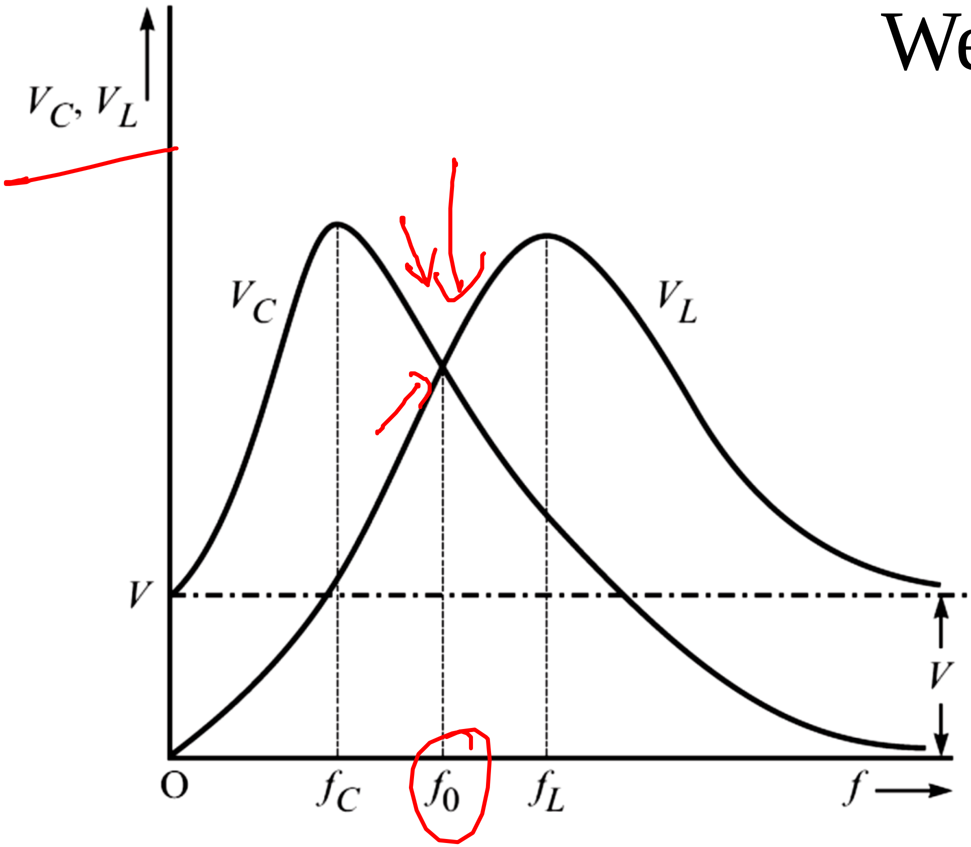
Variation of Voltage across C and L with Frequency



(a) When R has appreciable value.

We can show that

$$f_C = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}$$

$$f_L = \frac{1}{2\pi\sqrt{LC - (R^2C^2/2)}}$$

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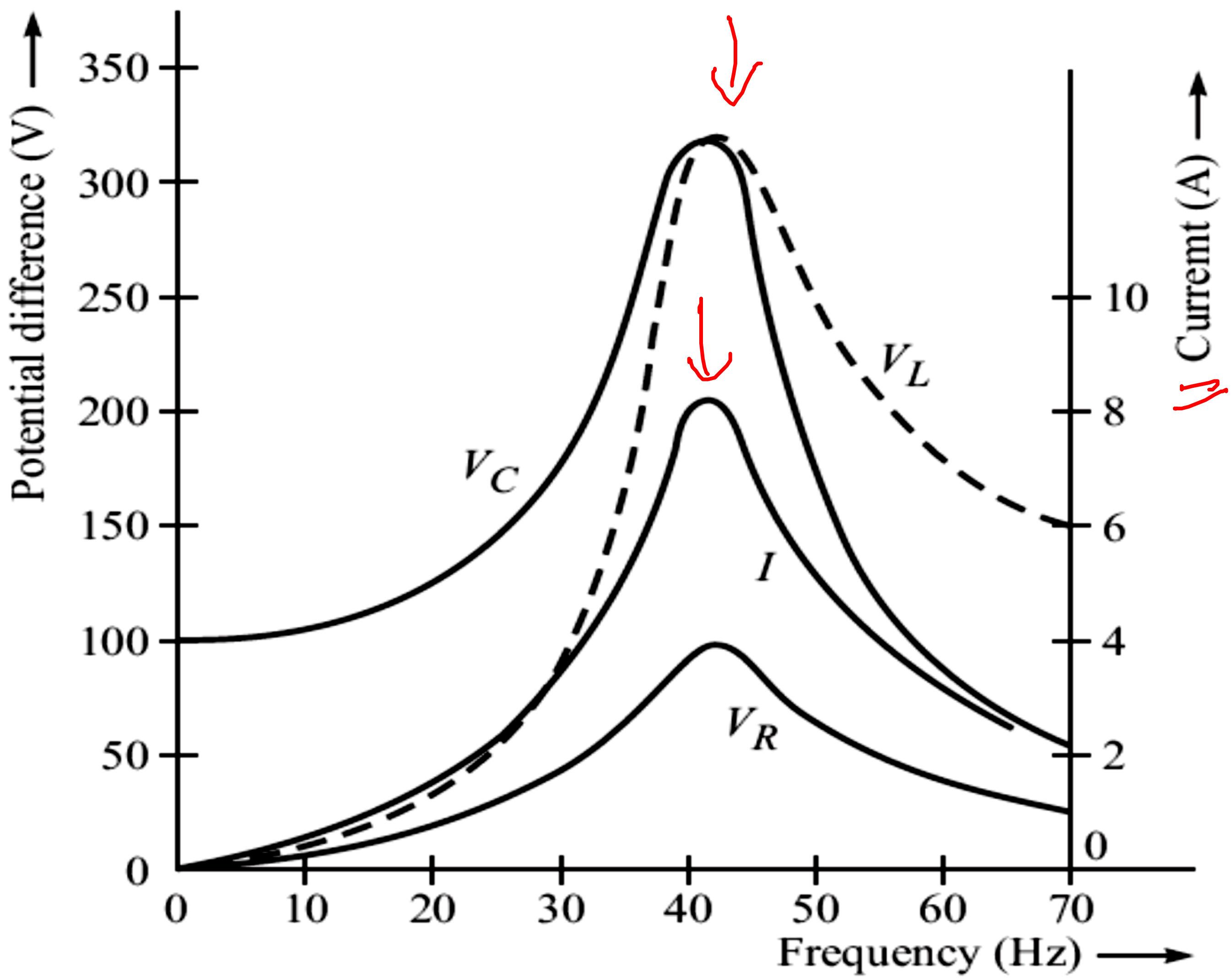
(*b*) When *R* is very small.



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Quality Factor (Q)

The quality of a resonant circuit to accept current (and power) at the resonant frequency to the exclusion of other frequencies is measured by its *quality factor* (Q factor), defined below.

$$Q = \frac{2\pi(\text{Maximum energy stored in } L \text{ or } C \text{ per cycle})}{\text{Energy dissipated per cycle}}$$

$$Q = \frac{2\pi[\tfrac{1}{2}L(I_m)^2]}{I_0^2 R T_0} = \frac{2\pi[{}^1/_2\, L(\sqrt{2}I_0)^2]}{I_0^2 R\,\boxed{(2\pi/\omega_0)}} = \frac{\omega_0 L}{R}$$

$\omega = 2\pi f$

$f = \dfrac{\omega}{2\pi}$

$T = \dfrac{1}{f} = \dfrac{2\pi}{\omega}$

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Since $\omega_0 L = 1/\omega_0 C$, $\quad Q = \dfrac{1}{\omega_0 CR}$

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By putting, $\omega_0 = 1/\sqrt{LC}$ we get another form for Q :

$$Q = \frac{1}{\omega_0 CR} = \frac{\sqrt{LC}}{CR} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

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- A capacitor usually has no losses.

- The Q of a series inductor-capacitor circuit is the same as the Q of the coil used.

- In fact, Q of the coil is used as a figure of merit for the coil.

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- Coils with $Q < 10$ are described as *low-Q coils.*
- Coils with $Q > 10$ are described as *high-Q coils.*
- Coils having Q as high as 200-300 are used in electronic circuits.

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Example 1

- A series combination of a resistance of 4 Ω, an inductance of 0.5 H and a variable capacitance is connected across a 100-V, 50-Hz supply. Calculate

  (*a*) the capacitance to give resonance,

  (*b*) the voltage across the inductance and the capacitance, and

  (*c*) the *Q* factor of the circuit.

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Solution :

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(*a*) For resonance,

$$X_{L0} = X_{C0} \quad \text{or} \quad 2\pi f_0 L = 1/2\pi f_0 C$$

$$\Rightarrow \quad C = \frac{1}{(2\pi f_0)^2 L} = \frac{1}{(2\pi \times 50)^2 \times 0.5} = \mathbf{20.3\,\mu F}$$

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(*b*) At resonance, $I_0 = \frac{V}{R} = \frac{100}{4} = 25\,\text{A}$

$$\therefore \quad V_L = I_0 X_L = 25 \times (2\pi \times 50 \times 0.5) = \mathbf{3925\,V}$$

$$\therefore \quad V_C = V_L = \mathbf{3925\,V}$$

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(*c*) $Q = \frac{\omega_0 L}{R} = \frac{2\pi f_0 L}{R} = \frac{2\pi \times 50 \times 0.5}{4} = \mathbf{39.25}$

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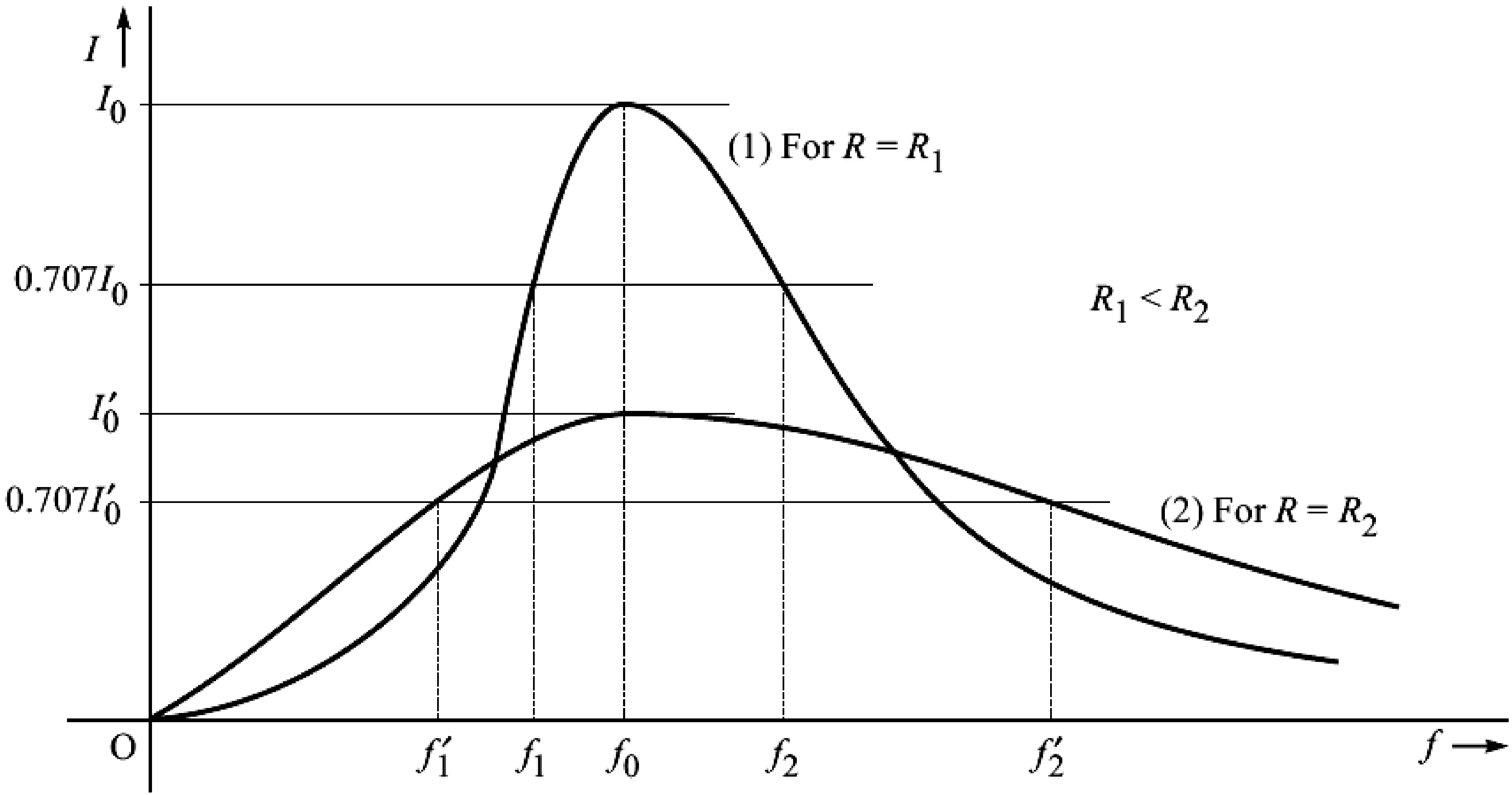
Resonance Curve



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- The frequencies f_1 and f_2 are often called *lower* and *upper* **cutoff frequencies**.
- At these frequencies, the current reduces to $0.707I_0$.
- These frequencies are also called ***half-power frequencies.***
- The lower the value of R, the sharper is the resonance curve.
- We say that such a resonant circuit has high ***selectivity***.
- For larger values of R, not only the peak value of current falls, but even the response curve becomes less sharp (i.e., low selectivity).

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Bandwidth (*BW*) in Terms of Circuit Parameters

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$$BW = f_2 - f_1 = \left[\frac{R}{4\pi L} + \frac{1}{4\pi}\sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}}\right]$$

$$-\left[-\frac{R}{4\pi L} + \frac{1}{4\pi}\sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}}\right]$$

$$BW = \frac{R}{2\pi L}$$

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Bandwidth (BW) in Terms of Q and f_0

We know that

$$Q = \frac{\omega_0 L}{R} \Rightarrow \frac{R}{L} = \frac{\omega_0}{Q} = \frac{2\pi f_0}{Q}$$

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$$\therefore \quad BW = \frac{R}{2\pi L} = \frac{1}{2\pi}\left(\frac{R}{L}\right) = \frac{1}{\cancel{2\pi}}\left(\frac{\cancel{2\pi} f_0}{Q}\right)$$

or $$BW = \frac{f_0}{Q}$$

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Example 2

- A series ac circuit has a resonance frequency of 150 kHz and a bandwidth of 75 kHz. Determine its half-power frequencies.

Solution : Let us calculate the Q of the circuit,

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$$\text{Since } BW = \frac{f_0}{Q}, \text{ we have } Q = \frac{f_0}{BW} = \frac{150\,\text{kHz}}{75\,\text{kHz}} = 2$$

Hence, we cannot use the approximate relations.
Using the exact relations, and working in kHz,

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$$75 = f_2 - f_1 \qquad \text{and} \qquad 150 = \sqrt{f_2 f_1}$$

Eliminating f_2 between the two equations, we get

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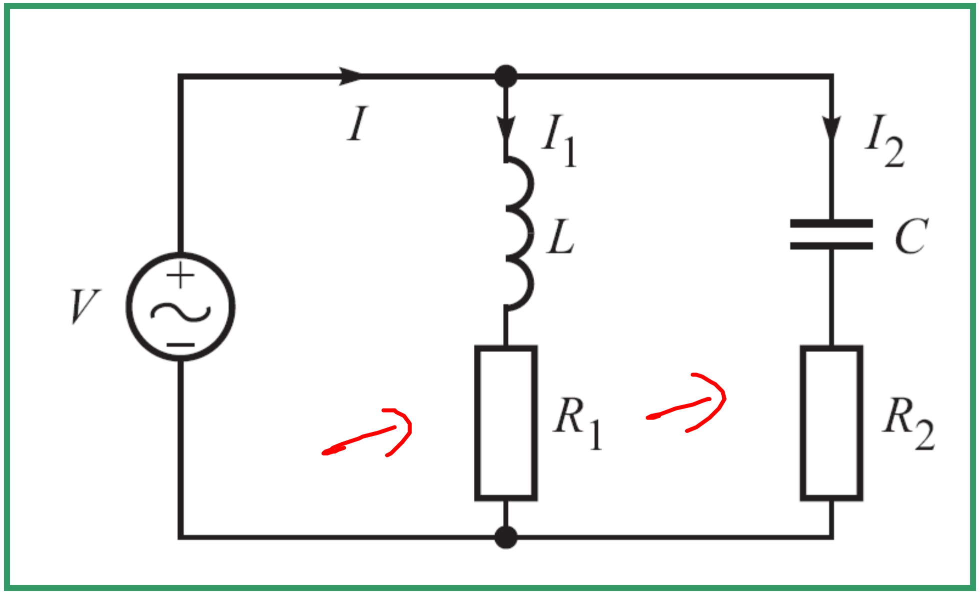
$$f_1^{\,2} + 75 f_1 - 22\,500 = 0$$

$$\Rightarrow \quad f_1 = 117.1 \text{ kHz} \;\text{ or }\; -192.1 \text{ kHz}$$

- Ignoring the negative value, we have f_1 = 117.1 kHz.
- Hence, f_2 = 75 + f_1 = 192.1 kHz.

Next

Parallel Resonant Circuit



- The losses in inductor and capacitor are accounted for by equivalent resistances R_1 and R_2.
- Resonant condition reaches when the reactive (or wattless) component of line current I reduces to zero.

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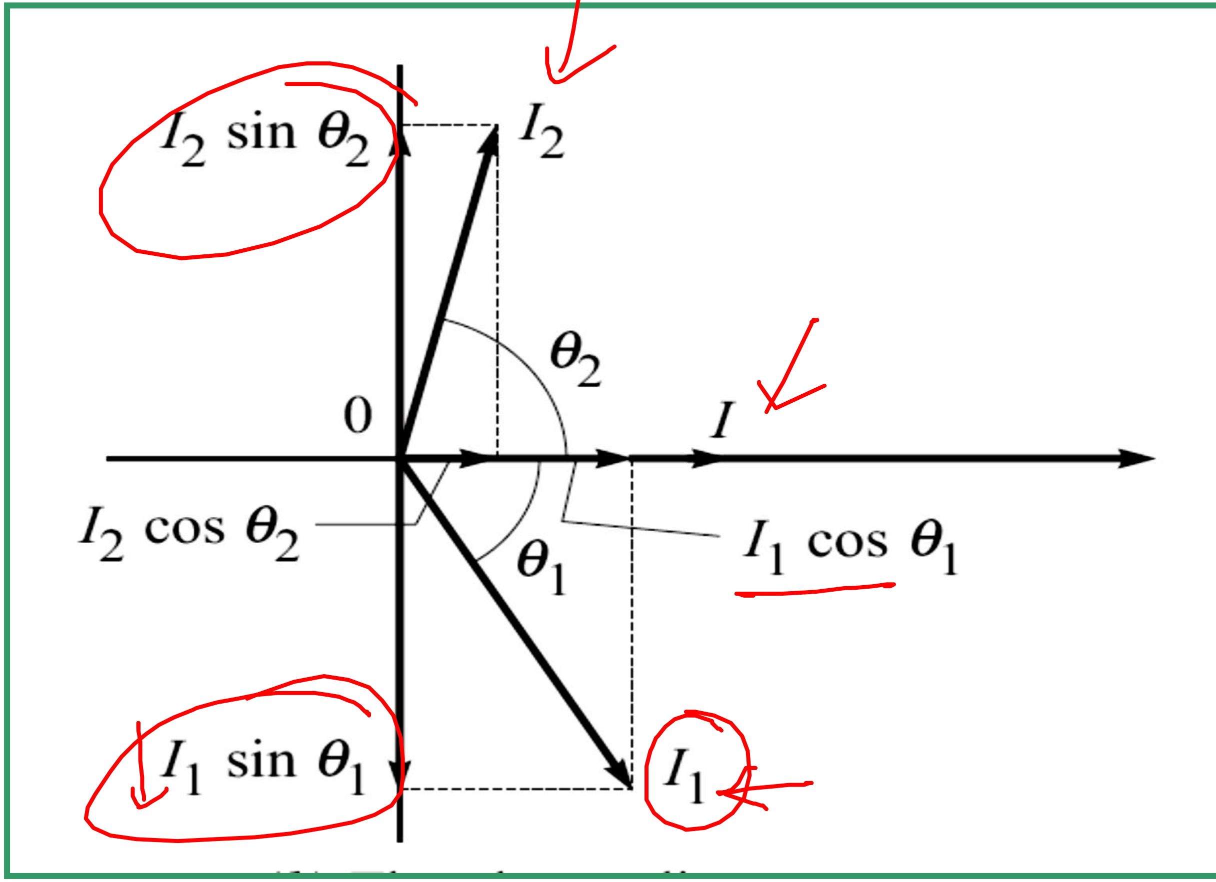

R

Under resonance condition, the reactive components of these two currents are equal in magnitude (but opposite in phase). That is,

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$$I_1 \sin\theta_1 = I_2 \sin\theta_2$$

$$\frac{V}{\sqrt{R_1^2 + (\omega_0 L)^2}} \times \frac{(\omega_0 L)}{\sqrt{R_1^2 + (\omega_0 L)^2}} = \frac{V}{\sqrt{R_2^2 + (1/\omega_0 C)^2}} \times \frac{(1/\omega_0 C)}{\sqrt{R_2^2 + (1/\omega_0 C)^2}}$$

or $$\frac{(\omega_0 L)}{R_1^2 + (\omega_0 L)^2} = \frac{(1/\omega_0 C)}{R_2^2 + (1/\omega_0 C)^2}$$

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or $$\frac{\omega_0 L}{R_1^2 + \omega_0^2 L^2} = \frac{\omega_0 C}{R_2^2 \omega_0^2 C^2 + 1}$$

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$\Rightarrow$ $$f_0 = \frac{1}{2\pi\sqrt{LC}} \sqrt{\frac{R_1^2 - (L/C)}{R_2^2 - (L/C)}}$$

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Effect of Variation of Frequency

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$$\mathbf{Y}_1 = \frac{1}{R + j\omega L} = \frac{R}{R^2 + (\omega L)^2} - j\frac{\omega L}{R^2 + (\omega L)^2} = G - jB_L$$

$$\mathbf{Y}_2 = \frac{1}{(1/ j\omega C)} = +j\omega C = +jB_C$$

The conductance and susceptance of the inductive branch,

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$$G = \frac{R}{R^2 + (\omega L)^2} \quad \text{and} \quad B_L = \frac{\omega L}{R^2 + (\omega L)^2}$$

For the capacitive branch,

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$$G = 0 \quad \text{and} \quad B_C = \omega C$$

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Total admittance of the circuit,

$$\mathbf{Y} = \mathbf{Y}_1 + \mathbf{Y}_2 = \frac{R}{R^2 + (\omega L)^2} - j\frac{\omega L}{R^2 + (\omega L)^2} + j\omega C = G + j(B_C - B_L)$$

$$G = \frac{R}{R^2 + (\omega L)^2}$$

and

$$B_L = \frac{\omega L}{R^2 + (\omega L)^2}$$

$$B_C = \omega C$$

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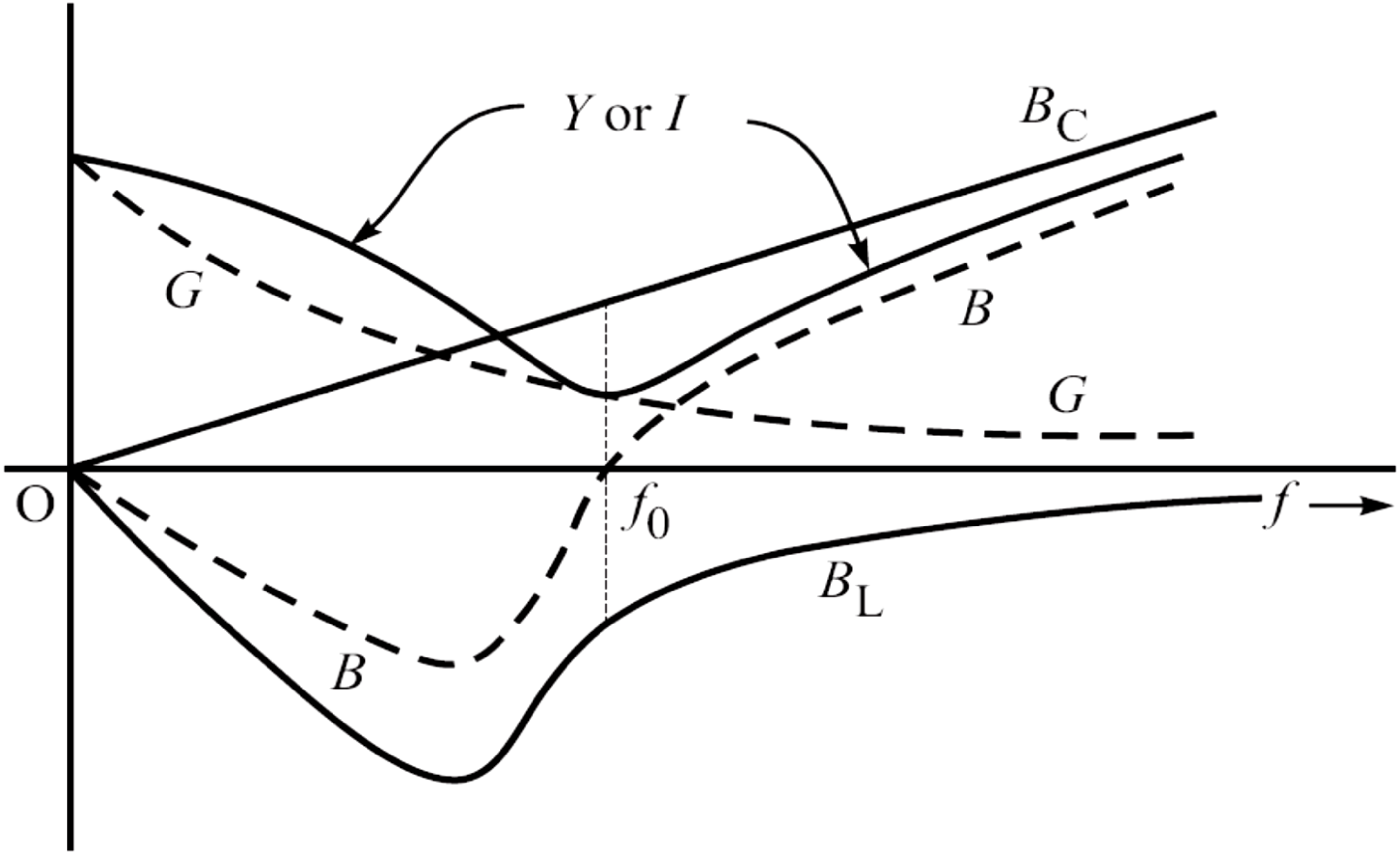

(*a*) Admittance versus frequency.

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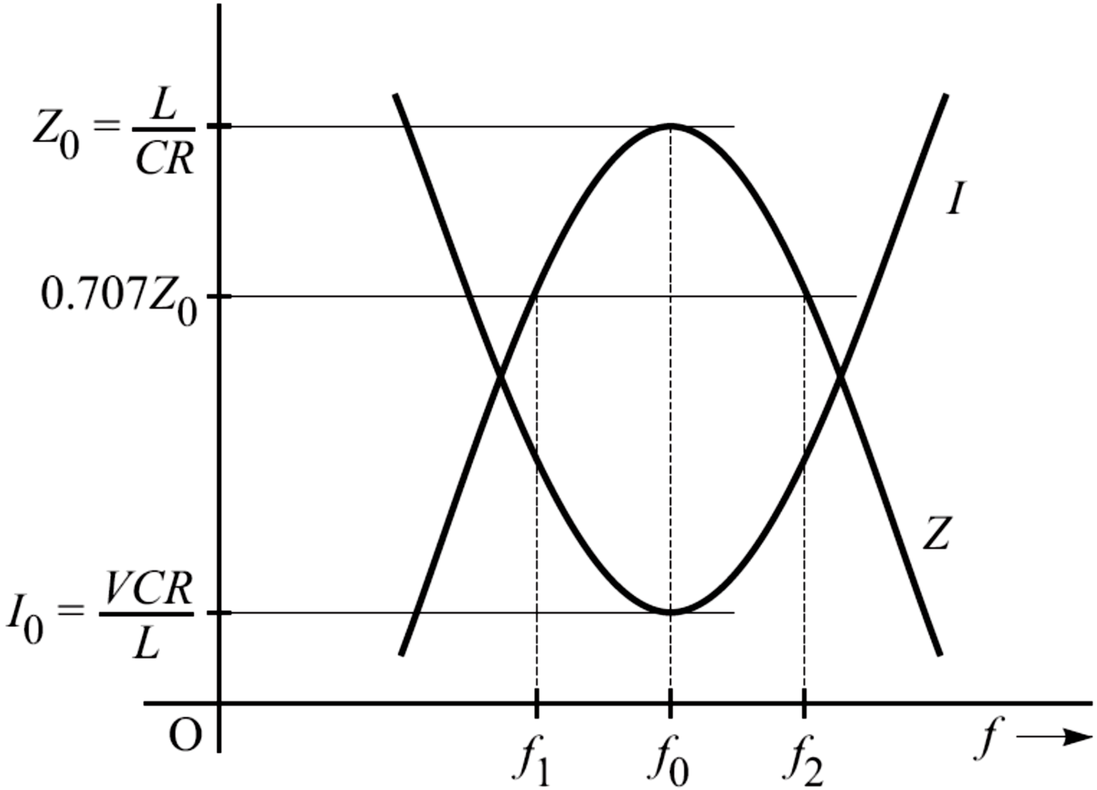

(*b*) Current versus frequency.

At resonance frequency, the line current is seen to have minimum,

$$I_0 = \frac{V}{Z_0} = \frac{V}{L/CR} = \frac{VCR}{L}$$

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Some Important Points

- Since the circuit rejects the current at resonance (i.e., it has minimum value), the parallel resonant circuit is also called **rejector circuit** or **anti-resonant circuit**.
- Since the circulating current between the two branches is many times the line current, the parallel resonant circuit is also called **current resonant circuit**.
- The circuit is also called a **tank circuit.**

Next

Review

- **Introduction.**
- **Series Resonant Circuit**.
  - Effect of Variation of Frequency.
  - Variation of Voltage across *C* and *L* with Frequency.
  - Quality Factor (*Q*).
  - Voltage Magnification.
  - Resonance Curve.
  - Relation between f_0, f_1 and f_2.
  - Bandwidth (*BW*) in Terms of Circuit Parameters.
  - Bandwidth (*BW*) in Terms of *Q* and f_0.
- **Parallel Resonant Circuit.**
  - Practical Circuit.
  - Resonance Curve.
  - Effect of Variation of Frequency.
  - Some Important Points.

Next