1. Use the graphical method to solve the following LPP:

Minimise $z = -x_1 + 2x_2$

subject to the constraints

$-x_1 + 3x_2 \leq 10,$

$x_1 + x_2 \leq 6,$

$x_1 - x_2 \leq 2$ and $x_1, x_2 \geq 0$

Soln,

min $z = -x_1 + 2x_2$

subject to

$-x_1 + 3x_2 = 10$

$x_1 + x_2 = 6$

$x_1 - x_2 = 2$

and $x_1, x_2 \geq 0$

$-x_1 + 3x_2 = 10$

for $x_1 = 0$ $x_2 = 10/3$ $(0, 10/3)$

for $x_2 = 0$ $x_1 = -10$ $(-10, 0)$

$x_1 + x_2 = 6$

for $x_1 = 0$ $x_2 = 6$ $(0, 6)$

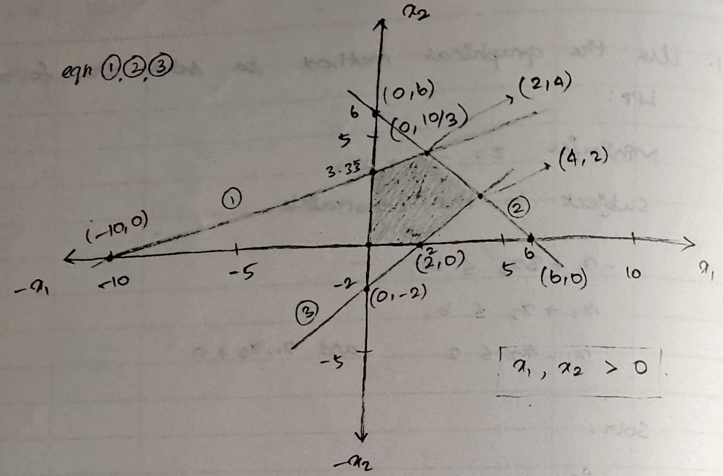
for $x_2 = 0$ $x_1 = 6$ $(6, 0)$

$x_1 - x_2 = 2$

for $x_1 = 0$ $x_2 = -2$ $(0, -2)$

for $x_2 = 0$ $x_1 = 2$ $(2, 0)$

eqn 1, 2, 3




From solving 1 & 2

$$-x_1 + 3x_2 = -10$$
$$x_1 + x_2 = 6$$

$$4x_2 = 16$$
$$x_2 = 4$$
$$x_1 + 4 = 6$$
$$x_1 = 2$$

2 & 3

$$x_1 + x_2 = 6$$
$$x_1 - x_2 = 2$$

$$2x_1 = 8$$
$$x_1 = 4$$
$$4 + x_2 = 6$$
$$x_2 = 2$$

| Corner Values | Value of $z$ |
|---|---|
| (0,0) | $z = 0$ |
| (2,0) | $z = -2$ ← min value |
| (4,2) | $z = 0$ |
| (2,4) | $z = 6$ |
| (0,10/3) | $z = 20/3$ |

$\boxed{\text{Minimize } z = -2}$

2. Use graphical method to solve the following LPP:

Maximize $z = 2x_1 + 3x_2$

subject to,

$$x_1 + x_2 \le 30$$
$$x_1 - x_2 \geq 0$$
$$x_2 \geq 3$$
$$0 \le x_1 \le 20$$
& $$0 \le x_2 \le 12$$

Soln,

$x_1 + x_2 = 30$ 1

for $x_1 = 0$ $x_2 = 30$ (0, 30)

for $x_2 = 0$ $x_1 = 30$ (30, 0)

$x_1 - x_2 = 0$ 2 $x_1 = x_2$

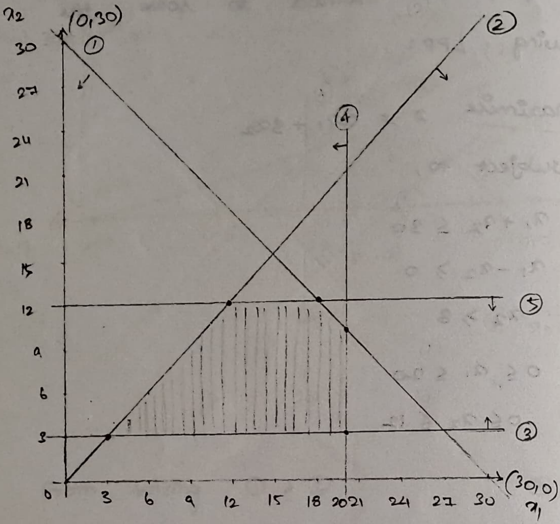
for $x_1 = 0$ $x_2 = 0$ (0,0)

for $x_1 = n$ $x_2 = n$

$x_2 = 3$ 3

$x_1 = 20$ 4

$x_2 = 12$ 5




②&③ ⑤&② ⑤&① ④&① ④&③

②&③

$x_1 - x_2 = 0$

$x_2 = 3$

$x_1 - 3 = 0$ (3,3)

$x_1 = 3$

①&④

$x_1 + x_2 = 30$

$x_1 = 20$ (20,10)

$20 + x_2 = 30$

$x_2 = 10$

②&⑤

$x_1 - x_2 = 0$

$x_2 = 12$ (12,12)

$x_1 - 12 = 0$

$x_1 = 12$

③&④

$x_1 = 20$ (20,3)

$x_2 = 3$

①&⑤

$x_1 + x_2 = 30$

$x_2 = 12$ (18,12)

$x_1 + 12 = 30$

$x_1 = 18$

$2x_1 + 3x_2$

| Corner points | Value of Z |
|---|---|
| (3,3) | 15 |
| (12,12) | 60 |
| (18,12) | 72 |
| (20,10) | 70 |
| (20,3) | 49 |

$\boxed{\text{Maximum } z = 72}$

3. A company makes 2 kinds of leather belts Belt A – high quality, Belt B – low quality, respective profits are Rs 4 & Rs 3 per belt. Belt A requires twice as much time as Belt B. If all belts are type B, company could make 1000 belts per day.

The supply of leather is sufficient for only 800 belts per day (Both A and B). Belt A requires a fancy buckle & 400 buckles per day are available. There are only 700 buckles a day available for Belt B.

Determine optimal product mix.

Soln,

let $x_1$ = no. Belt A

$x_2$ = no. Belt B

now,

Maximize $z = 4x_1 + 3x_2$ (from qn profit 4,3)

$2x_1 + x_2 \leq 1000$

$x_1 + x_2 \leq 800$

$x_1 \leq 400$ $x_2 \leq 700$

4. Solve the following LPP by the graphical method

Minimize $z = 3x_1 + 5x_2$ Verify.

Sub to.

$-3x_1 + 4x_2 \leq 12$

$x_1 \leq 4$

$2x_1 - x_2 \geq -2$

$x_2 \geq 2$

$2x_1 + 3x_2 \geq 12$ &

$x_1, x_2 \geq 0$

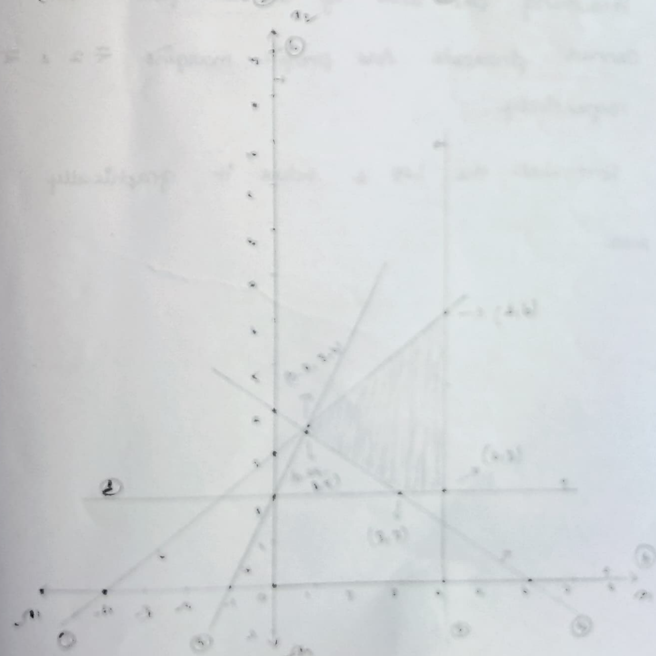
| Vertex | Values of z |
|---|---|
| (3, 2) | 19 ← min |
| (4, 2) | 22 |
| (4, 6) | 42 |
| (4/5, 18/5) | 102/5 |
| (3/4, 7/2) | 79/4 |

Soln

(4, 0) $-3x_1 + 4x_2 = 12$ — ①
(0, 3) $x_1 = 4$ — ②

(-1, 0) (0, 2)
$2x_1 - x_2 = -2$ — ③
$x_2 = 2$ — ④

(6, 0) (0, 4)
$2x_1 + 3x_2 = 12$ — ⑤
$x_1, x_2 \geq 0$ — ⑥



5. A pineapple firm produces 2 products canned pineapple & canned juice. The specific amounts of material, labour and equipments required to produce each product and the availability of each of these resources are shown in the table given below :

| | Canned Juice | Canned pineapple | Available resource |
|---|---|---|---|
| Labour (man hrs) | 3 | 2.0 | 12.0 |
| Equipment (M/c hrs) | 1 | 2.3 | 6.9 |
| Material (unit) | 1 | 1.4 | 4.9 |

Assuming one unit of canned juice and canned pineapple has profit margins ₹2 & ₹1 respectively.

Formulate the LPP & solve it graphically

Soln.

Q. Use simplex method to solve the following LPP maximize $z = 3x_1 + 5x_2 + 4x_3$

Subject to the constraints

$$2x_1 + 3x_2 \le 8$$

$$2x_1 + 5x_3 \le 10$$

$$3x_1 + 2x_2 + 4x_3 \le 15 \quad \&$$

$$x_1, x_2, x_3 \ge 0$$

Soln,

Introducing non-negative slack variables $s_1, s_2$ and $s_3$ such that

$$\max\ z = 3x_1 + 5x_2 + 4x_3 + 0s_1 + 0s_2 + 0s_3$$

sub to constr,

$$2x_1 + 3x_2 + s_1 = 8$$

$$2x_1 + 5x_3 + s_2 = 10$$

$$3x_1 + 2x_2 + 4x_3 + s_3 = 15$$

and $x_1, x_2, x_3, s_1, s_2, s_3 \ge 0$

coef of $Y_B$ in z · (variables in basis) · $c_j$ (s.intval) [ 3 5 4 0 0 0 ]

| $C_B$ | $Y_B$ | $X_B$ | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $s_3$ | Min Ratio |
|---|---|---|---|---|---|---|---|---|---|
| 0 | $s_1$ | 8 | 2 | 3 | 0 | 1 | 0 | 0 | 8/3 min |
| 0 | $s_2$ | 10 | 2 | 0 | 5 | 0 | 1 | 0 | – |
| 0 | $s_3$ | 15 | 3 | 2 | 4 | 0 | 0 | 1 | 15/2 |
| | var in basis | $\Sigma Y_i . C_B - C_j \rightarrow$ | -3 | -5 min | -4 | 0 | 0 | 0 (from |
| | | $Z = 0$ | | | | | | | |

$$X_j = \left( \sum_{i=1}^{3} x_i \cdot C_{Bi} \right) - C_j = -3$$

$X_j \rightarrow$ -3 -5 -4 0 0 0

Ratio = $\dfrac{X_B}{\min(X_j)}$ ; $\min(X_j) = X_2$

$s_1$ should leave the table, $x_2$ should enter, repeat process till all $x_j$ are non-negative $X_2 = 3$ is pivot element.

coef of $Y_B$ in z

| | | | 3 | 5 | 4 | 0 | 0 | 0 | |
|---|---|---|---|---|---|---|---|---|---|
| $C_B$ | $Y_B$ | $X_B$ | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $s_3$ | Ratio |
| 5 | $x_2$ | 8/3 | 2/3 | 1 | 0 | 1/3 | 0 | 0 | – |
| 0 | $s_2$ | 10 | 2 | 0 | 5 | 0 | 1 | 0 | 2 |
| 0 | $s_3$ | $15 - \frac{16}{3}$ 29/3 | 3-4/3 5/3 | 0 | 4 | -2/3 | 0 | 1 | 29/12 |
| | | | 1/3 | 0 | -4 min | 5/3 | 0 | 0 | |

$R_1 = R_1/3$

$R_3 = R_3 - 2 . R_1$

$z = \frac{40}{3}$ · $\frac{10}{3} - 3$ · $5-5$ · $0-4$ · $5/3 - 0$ · $0-0$ · $0-0$

$\rightarrow z = \Sigma C_B . X_B$

Ratio : $\dfrac{X_B}{X_3}$

$s_2$ should leave the table $x_3$ should enter, $x_3 = 5$ pivot element.

$50+24-45$ / $15$ — $\frac{29}{15}$

| $C_B$ | $Y_B$ | $X_B$ | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $s_3$ | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| | | | 3 | 5 | 4 | 0 | 0 | 0 | |
| 5 | $x_2$ | 8/3 | 2/3 | 1 | 0 | 1/3 | 0 | 0 | |
| 4 | $x_3$ | 2 | 2/5 | 0 | 1 | 0 | 1/5 | 0 | |
| 0 | $s_3$ | 5/3 | 1/15 | 0 | 0 | -2/3 | -4/5 | 1 | |
| | | $\frac{29}{3}-8$ | $\frac{5}{3}-\frac{8}{5}$ | | | | $-2-\frac{4}{5}$ | | |
| | | $Z=\frac{64}{3}$ | 29/15 | 0 | 0 | 5/3 | 4/5 | 0 | |

→ all are non -ve so stop

$R_2 = R_2/5$

$R_3 = R_3 - 4R_2$ / $\frac{10}{3}+\frac{8}{5}-8$   $5-5$   $4-4$   $\frac{5}{3}-0$   $\frac{4}{5}-0$   $0-0$

$Z = \frac{40}{3}+8 = \frac{40+24}{3} = 64/3$    All $z_j - c_j \geq 0$

$$\boxed{\begin{array}{l} \text{Max } z = 64/3 \\ x_1 = 0 \quad x_2 = 8/3 \quad x_3 = 2 \end{array}}$$

→ x values from the basis only of '$X_B$'

Q   Simplex method,

Max $Z = 4x_1 + 6x_2 + 2x_3$

s.t.c

$x_1 + x_2 + x_3 \leq 3$

$x_1 + 4x_2 + 7x_3 \leq 9$

& $x_1, x_2, x_3 \geq 0$

Soln,

$Z = 4x_1 + 6x_2 + 2x_3 + 0s_1 + 0s_2$

Constr,

$x_1 + x_2 + x_3 + s_1 = 3$

$x_1 + 4x_2 + 7x_3 + s_2 = 9$

| $C_B$ | $Y_B$ | $X_B$ | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | Ratio |
|---|---|---|---|---|---|---|---|---|
| | | | 4 | 6 | 2 | 0 | 0 | |
| 0 | $s_1$ | 3 | 1 | 1 | 1 | 1 | 0 | 3 |
| 0 | $s_2$ | 9 | 1 | 4 | 7 | 0 | 1 | 9/4 |
| | | | -4 | -6 | -2 | 0 | 0 | |

Ratio $= \frac{X_B}{x_2}$

$s_2$ leaves table, $x_2$ enters table

| $C_B$ | $Y_B$ | $X_B$ | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | Ratio |
|---|---|---|---|---|---|---|---|---|
| | | | 4 | 6 | 2 | 0 | 0 | |
| | | $3-9/4$ | $1-1/4$ | $1-1$ | $1-7/4$ | $1-0$ | $-1/4+0$ | |
| 0 | $s_1$ | 3/4 | 3/4 | 0 | -3/4 | 1 | -1/4 | 1 |
| 6 | $x_2$ | 9/4 | 1/4 | 1 | 7/4 | 0 | 1/4 | 9 |
| | | $Z=\frac{54}{4}=\frac{27}{2}$ | -5/2 | 0 | 5 | 0 | 6/4 | |
| | | | $\frac{6}{4}-4$ | $6-6$ | $\frac{42}{6}-2$ | $0-0$ | $\frac{6}{4}-0$ | |

$R_2 = R_2/4$

$R_1 = R_1 - R_2$

Ratio $= \frac{X_B}{x_1}$

$s_1$ leaves, $x_1$ enters

$R_1 = R_1/0.75$    $3/4 = 0.75$

$R_2 = R_2 - \frac{1}{4} \cdot R_1$

$R_2 - 0.25 R_1$

| $C_B$ | $Y_B$ | $X_B$ | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | Ratio |
|---|---|---|---|---|---|---|---|---|
| | | | 4 | 6 | 2 | 0 | 0 | |
| 4 | $x_1$ | 1 | 1 | 0 | -1 | 4/3 | -1/3 | |
| 6 | $x_2$ | 2 | 0 | 1 | 2 | -1/3 | 1/3 | |
| | | $Z=16$ | 0 | 0 | $10-4-6$ | 10/3 | 2/3 | |

$4+12 = 16$

$$\boxed{\begin{array}{l} \text{Max } z = 16 \\ x_1 = 1 \quad x_2 = 2 \quad x_3 = 0 \end{array}}$$

$\therefore$ [ -ve, 0 can't be taken as pivot element ]

Q) Solve the following LPP by simplex method

minimize $z = 8x_1 - 2x_2$

s.t.c,

$-4x_1 + 2x_2 \le 1$

$5x_1 - 4x_2 \le 3$

$x_1, x_2 \ge 0$

soln,

maximize $z^* = -z = -8x_1 + 2x_2$

$z^* = -8x_1 + 2x_2 + 0s_1 + 0s_2$

s.t.c,

$-4x_1 + 2x_2 + s_1 = 1$

$5x_1 - 4x_2 + s_2 = 3$

| | | | -8 | 2 | 0 | 0 | |
|---|---|---|---|---|---|---|---|
| CB | YB | XB | $x_1$ | $x_2$ | $s_1$ | $s_2$ | Ratio |
| 0 | $s_1$ | 1 | -4 | 2 | 1 | 0 | 1/2 |
| 0 | $s_2$ | 3 | 5 | -4 can't take | 0 | 1 | -3/4 min ignore -ve ratio |
| | | z=0 | 8 | -2 most -ve | 0 | 0 | |

Ratio = $\frac{X_B}{X_2}$

$s_1$ leaves, $x_2$ enters the basis

$s_1 = \frac{s_1}{2}$ $\qquad s_2 = s_2 + 4 \cdot s_1$

| | | | -8 | 2 | 0 | 0 | |
|---|---|---|---|---|---|---|---|
| CB | YB | XB | $x_1$ | $x_2$ | $s_1$ | $s_2$ | Ratio |
| 0 | $s_1$ | 5/2 | -3/2 | 0 | 1 | -1/2 | -5/3 min |
| 2 | $x_2$ | -3/4 | -5/4 | 1 | 0 | -1/4 | 3/5 |
| | | | -10/4 -8 -21/8 most -ve | 0 | 0 | -2/4 | |

Ratio = $\frac{X_B}{x_1}$

$s_1$ leaves, $x_1$ enters the basis

$s_1 = s_1 / \frac{-8}{2}$ => $s_1 = s_1/-1.5$ $\quad s_2 = s_2 + \frac{5}{4} \cdot s_1$ => $s_2 = s_2 + 1.25 s_1$

| | | | -8 | 2 | 0 | 0 | |
|---|---|---|---|---|---|---|---|
| CB | YB | XB | $x_1$ | $x_2$ | $s_1$ | $s_2$ | Ratio |
| -8 | $x_1$ | -5/3 | 1 | 0 | -2/3 | 1/3 | |
| 2 | $x_2$ | | | | | | |

| | | | -8 | 2 | 0 | 0 | |
|---|---|---|---|---|---|---|---|
| CB | YB | XB | $x_1$ | $x_2$ | $s_1$ | $s_2$ | Ratio |
| 2 | $x_2$ | 1/2 | -2 | 1 | 1/2 | 0 | |
| 0 | $s_2$ | 5 | -3 | 0 | 2 | 1 | |
| | | z = 1 | 4 | 0 | 1 | 0 | |

z = $X_B \cdot C_B$ $\qquad$ (-2)(2) + 8, 8 - 4 = 4 $\qquad$ 2 - 2, 0 $\qquad$ 1 - 0, 1 $\qquad$ 0, 0

Max $z^* = 1$

Min $z = -$ Max $z^* = -1$

$$\boxed{\begin{array}{c} \text{Min } z = -1 \\ x_1 = 0 \\ x_2 = 1/2 \end{array}}$$

## Simplex Method Algorithm :

For the solution of LPP by simplex method, the existence of initial basis feasible solution is always assumed. The steps for the computation of an optimum solution as follows.

**Step 1:** Check whether the objective function of LPP is maximized or minimized. If it is minimized then we convert into a problem of maximizing it by using the result

$$\text{Minimization } (z) = - \text{Maximization } (-z)$$

**Step 2:** Check whether all $b_i$ $(i = 1, 2, \ldots m)$ are non-negative. If any one of $b_i$ is negative then multiply the corresponding inequation of the constraints $-1$, so as to get all $b_i$ $(i = 1, 2, \ldots m)$ non-negative

**Step 3:**

Convert all the inequations of the constraints into equations by introducing slack and/or surplus variables in the constraints.

Put the costs of the variables equal to zero.

**Step 4:** Obtain an initial basic feasible solution in the form $x_B = B^{-1} b$ and put it in the first column of simple table.

**Step 5:** Compute the net evaluation $z_j - c_j$ $(j = 1, 2, \ldots n)$ by using the relation $z_j - c_j = C_B y_j - c_j$, where $y_i = B^{-1} a_j$

Examine the sign $z_j - c_j$

i) If all $(z_j - c_j) \geq 0$ then the initial basic feasible solution $x_B$ is an optimum basic feasible solution.

ii) If at least one $(z_j - c_j) < 0$ proceed on to the next step.

**Step 6:** If there are more than are negative $z_j - c_j$, then choose the most negative of them Let it be $z_j - c_j$ for some $j = r$

i) If all $(z_j - c_j) \geq 0$ then the initial basic feasible solution $x_B$ is an optimum basic feasible solution

ii) If at least one $(z_j - c_j) < 0$, then proceed to the next step

**Step 7:** Compute the ratio $\left\{ \frac{x_{Bi}}{y_{ir}}, \ y_{ir} > 0, \ i = 1, 2, \ldots m \right.$ and choose minimum of them. Let the minimum of these ratios, be $\frac{x_{Bk}}{y_{kr}}$

then the vector $y_k$ will leave the basis $y_B$. The common element $y_{kr}$ which is $k^{th}$ row and $r^{th}$ column is known as the leading element (pivotal element).

Step 8:

Convert the leading element to unity by dividing its row by the leading element itself and all other elements making use of the relations:

$$\hat{y}_{ij} = y_{ij} - \frac{y_{kj}}{y_{kr}} y_{ir} \quad i = 1, 2, \dots m+1, \; i \neq k$$

and $$\hat{y}_{kj} = \frac{\hat{y}_{kj}}{\hat{y}_{kr}} \quad j = 0, 1, 2, \dots n$$

Step 9:

Go to step 5 and repeat the computational procedure until either an optimum solution is obtained or there is an indication of an unbounded solution

Q) simplex method. (Big M)

Max $Z = 3x_1 + 2x_2$

stc,

$2x_1 + x_2 \leq 2$

$3x_1 + 4x_2 \geq 12$

& $x_1, x_2 \geq 0$

Soln,

By introducing the non-negative slack variable $s_1$ and surplus variable $s_2$ the standard form of the LPP becomes,

Max $Z = 3x_1 + 2x_2 + 0s_1 + 0s_2$

stc,

$2x_1 + x_2 + s_1 = 2$

$3x_1 + 4x_2 - s_2 = 12$ and

$x_1, x_2, s_1, s_2 \geq 0$

By this will not yield a basic feasible solution. To get the basic feasible solution, and the artificial variable $A_1$ to the left hand side of the constraint equation which does not possess the slack variable and assign M to the artificiable variable in the objective function. The LPP becomes,

Max $Z = 3x_1 + 2x_2 + 0s_1 + 0s_2 - MA_1$

stc,

$2x_1 + x_2 + s_1 = 2$

$3x_1 + 4x_2 - s_2 + A_1 = 12$

$x_1, x_2, s_1, s_2, A_1 \geq 0$

$M > 0$

The initial basic feasible solution is given by $s_1 = 2, A_1 = 12$ (basic)

$x_1 = x_2 = s_2 = 0$ (non-basic)

Step 1:

| | | $C_j$ | [3 | 2 | 0 | 0 | -M] | |
|---|---|---|---|---|---|---|---|---|
| $C_B$ | $Y_B$ | $X_B$ | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $A_1$ | Ratio |
| 0 | $s_1$ | 2 | 2 | 1 pivot | 1 | 0 | 0 | 2/1 = 2 min |
| -M | $A_1$ | 12 | 3 | 4 | 0 | -1 | 1 | 12/4 = 3 |
| $z_j - c_j$ | | Z = -12M | -3M-3 | -4M-2 | 0 | M | 0 | |

fast growing -ve value (check by sub M values)

Ratio = $\frac{X_B}{x_2}$ $\quad s_1$ leaves, $x_2$ enters basis

Step 2:

$R_2 = R_2 - 4R_1$

| | | $C_j$ | [3 | 2 | 0 | 0 | -M] | |
|---|---|---|---|---|---|---|---|---|
| $C_B$ | $Y_B$ | $X_B$ | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $A_1$ | Ratio |
| 2 | $x_2$ | 2 | 2 | 1 | 1 | 0 | 0 | |
| -M | $A_1$ | 4 | -5 | 0 | -4 | -1 | 1 | |
| $z_j - c_j$ | | z = -4M+4 | 5M+1 | 0 | 4M+2 | M | 0 | |

Since all $z_j - c_j \geq 0$ and artificial variable $A_1$ appears in the basic at non-zero level, the given LPP does not possess any feasible solution. But the LPP possess pseudo optimal soln.

there exist a soln but it is not optimal

Q) Two phase simplex method:

Max $Z = 5x_1 + 8x_2$

Stc,

$3x_1 + 2x_2 \geq 3$

$x_1 + 4x_2 \geq 4$

$x_1 + x_2 \leq 5$

$x_1, x_2 \geq 0$

soln,

By introducing the non-negative slack, surplus and artificial variables, the standard form of the LPP.

Max $Z = 5x_1 + 8x_2 + 0s_1 + 0s_2 + 0s_3$

Stc,

$3x_1 + 2x_2 - s_1 + A_1 = 3$

$x_1 + 4x_2 - s_2 + A_2 = 4$

$x_1 + x_2 + s_3 = 5$

and $x_1, x_2, s_1, s_2, s_3, A_1, A_2 \geq 0$

( Here, $s_1, s_2$ - surplus var

$s_3$ - slack var

$A_1, A_2$ - artificial var )

The initial basic feasible solution is given by $A_1 = 3,\ A_2 = 4,\ s_3 = 5$ (basic), $(x_1 = x_2 = s_1 = s_2 = 0)$ (non basic)

Phase I :

Assigning a cost $-1$ to the artificial variables and cost 0 to all other variables, the objective function of the auxiliary LPP becomes

$$\text{Max } z^* = -A - A_2$$

Stc.

The iterative simplex tables for the auxiliary LPP are:

Step-1 :

| | $C_B$ | $Y_B$ | $X_B$ | $x_1$ [0] | $x_2$ [0] | $s_1$ [0] | $s_2$ [0] | $s_3$ [0] | $A_1$ [-1] | $A_2$ [-1] | Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $R_1$ | -1 | $A_1$ | 3 | 3 | 2 | -1 | 0 | 0 | 1 | 0 | 3/2 |
| $R_2$ | -1 | $A_2$ | 4 | 1 | $\boxed{4}$ | 0 | -1 | 0 | 0 | 1 | 4/4 |
| $R_3$ | 0 | $s_3$ | 5 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 5/1 |
| | | | $z = -7$ | -4 | -6 | 1 | 1 | 0 | 0 | 0 | |

$R_2 = R_2/4$

$R_1 = R_1 - 2R_2$

$R_3 = R_3 - R_2$

$A_2$ leaves, $x_2$ enters basis

Step-2 :

| | $C_B$ | $Y_B$ | $X_B$ | $x_1$ [0] | $x_2$ [0] | $s_1$ [0] | $s_2$ [0] | $s_3$ [0] | $A_1$ [-1] | $A_2$ [-1] | Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $R_1$ | -1 | $A_1$ | 1 | $\boxed{5/2}$ | 0 | -1 | 1/2 | 0 | 1 | -1/2 | 2/5 |
| $R_2$ | 0 | $x_2$ | 1 | 1/4 | 1 | 0 | -1/4 | 0 | 0 | 1/4 | 4 |
| $R_3$ | 0 | $s_3$ | 4 | 3/4 | 0 | 0 | 1/4 | 1 | 0 | -1/4 | 16/3 |
| | | | $z = -1$ | -5/2 | 0 | 1 | -1/2 | 0 | 3/4 | 3/2 | |

Ratio $= \dfrac{X_B}{x_1}$

$A_1$ leaves, $x_1$ enters

$R_1 = R_1 / 5/2$

$R_2 = R_2 - 1/4\, R_1$

$R_3 = R_3 - 3/4\, R_1$

Step-3 :

| $C_B$ | $Y_B$ | $X_B$ | $x_1$ [0] | $x_2$ [0] | $s_1$ [0] | $s_2$ [0] | $s_3$ [0] | $A_1$ [-1] | $A_2$ [-1] | Ratio |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | $x_1$ | 2/5 | 1 | 0 | -2/5 | 1/5 | 0 | 2/5 | -1/5 | |
| 0 | $x_2$ | 9/10 | 0 | 1 | 1/10 | -3/10 | 0 | -1/10 | 3/10 | |
| 0 | $s_3$ | 37/10 | 0 | 0 | 3/10 | 1/10 | 1 | -3/10 | -1/10 | |
| | | $z = 0$ | 0 | 0 | 0 | 0 | 0 | 1 | 1 | |

[$A_1, A_2$ are not inside basis so they are not needed]

Since all $(z_j^* - c_j) \geq 0$, the current basic solution is optimum. Furthermore, no artificial variable appears in the optimum basis so we proceed to phase II.

Phase II :

Here we consider the actual costs associated with the original variables. The new objective function then becomes

$$\text{Max } z = 5x_1 + 8x_2 + 0s_1 + 0s_2 + 0s_3$$

The initial basic feasible solution for this phase is one obtained at the end of phase I

The iterative simplex tables for this phase are :

Step-1 :

| | $C_B$ | $Y_B$ | $X_B$ | $x_1$ [5] | $x_2$ [8] | $s_1$ [0] | $s_2$ [0] | $s_3$ [0] | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| $R_1$ | 5 | $x_1$ | 2/5 | 1 | 0 | -2/5 | $\boxed{1/5}$ | 0 | 2 min |
| $R_2$ | 8 | $x_2$ | 9/10 | 0 | 1 | 1/10 | -3/10 | 0 | -3 ignore |
| $R_3$ | 0 | $s_3$ | 37/10 | 0 | 0 | 3/10 | 1/10 | 1 | 37 |
| | | | $z = \frac{46}{5}$ | 0 | 0 | -6/5 | -7/5 | 0 | |

Ratio = $\frac{X_B}{s_2}$ $x_1$ leaves, $s_2$ enters basis

$R_1 = R_1 / \frac{1}{5} = R_1 \times 5$

$R_2 = R_2 + \frac{3}{10} \cdot R_1$

$R_3 = R_3 - \frac{1}{10} R_1$

Step-2 :

| | $C_B$ | $Y_B$ | $X_B$ | $x_1$ [5] | $x_2$ [8] | $s_1$ [0] | $s_2$ [0] | $s_3$ [0] | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| $R_1$ | 0 | $s_2$ | 2 | 5 | 0 | -2 | 1 | 0 | -1 |
| $R_2$ | 8 | $x_2$ | 15/10 | 15/10 | 1 | -5/10 | 0 | 0 | -3 |
| $R_3$ | 0 | $s_3$ | 35/10 | -5/10 | 0 | $\boxed{5/10}$ | 0 | 1 | 7 min |
| | | | $z = 12$ $(C_B \cdot X_B)$ | 7 | 0 | -4 | 0 | 0 | |

Ratio : $\frac{X_B}{s_1}$ $s_3$ leaves, $s_1$ enters basis

$R_3 = R_3 \times 2$ $R_1 = R_1 + 2 \cdot R_3$

$R_2 = R_2 + \frac{1}{2} R_2$

Step - 3 :

| $C_B$ | $Y_B$ | $X_B$ | $x_1$ [5] | $x_2$ [8] | $s_1$ [0] | $s_2$ [0] | $s_3$ [0] | Ratio |
|---|---|---|---|---|---|---|---|---|
| 0 | $s_2$ | 16 | 3 | 0 | 0 | 1 | 4 | |
| 8 | $x_2$ | 5 | 1 | 1 | 0 | 0 | 1 | |
| 0 | $s_1$ | 7 | -1 | 0 | 1 | 0 | 2 | |
| | | $z = 40$ | 3 | 0 | 0 | 0 | 8 | |

Since all $z_j - c_j \geq 0$ the current basic feasible solution is optimal

$\therefore$ $\boxed{\text{Max } z = 40, \; x_1 = 0, \; x_2 = 5}$

Q) Max $z = -4x_1 - 3x_2 - 9x_3$

s.t.c,

$2x_1 + 4x_2 + 6x_3 \geq 15$

$6x_1 + x_2 + 6x_3 \geq 12$

$x_1, x_2, x_3 \geq 0$

Some special cases in Linear Programming :

1. Alternative (Multiple) optimal solution.

We have that the optimal solution of any linear programming problem occurs at an extreme point of the feasible region and the solution is unique, that is, no other solution yield the same value of the objective function. However, in certain cases a given LP problem may have more than one optimal solution yielding the same objective function value.

There are 2 conditions that should be satisfied in order that an alternative optimal solution exists.

i) The given objective function is parallel to constraint that forms the boundary (or edge) of the feasible solutions region. In other words, the slope of the objective function is same as that of the constraint forming the boundary of the feasible solutions region

ii) The constraint should form a boundary on the feasible region in the direction of optimal movement of the objective function. In otherwords, the constraint should be an active constraint.

example,

by graphical method,

$$\max\ z = 10x_1 + 6x_2 = 2(5x_1 + 3x_2)$$

Stc,

$$5x_1 + 3x_2 \le 30 \quad ①$$

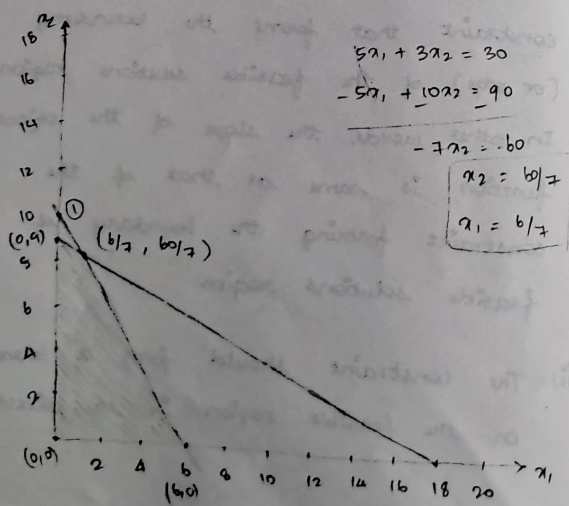
$$x_1 + 2x_2 \le 18 \quad ②$$

$$\& \ x_1, x_2 \ge 0 \quad ③$$

Soln,

① $x_1 = 0$, $x_2 = 10$; $x_2 = 0$, $x_1 = 6$

② $x_1 = 0$, $x_2 = 9$; $x_2 = 0$, $x_1 = 18$




$$5x_1 + 3x_2 = 30$$

$$-\underline{5x_1 + 10x_2 = 90}$$

$$-7x_2 = -60$$

$$\boxed{x_2 = 60/7 \quad x_1 = 6/7}$$

| Corner values | $10x_1 + 6x_2$ Value of Z |
|---|---|
| (0, 9) | 54 |
| (6, 0) | 60 |
| (6/7, 60/7) | 60 |
| (0, 0) | 0 |

$\frac{60}{7} + \frac{360}{7} = \frac{420}{7}$

$$\boxed{\begin{array}{l} \text{Max } z = 60 \\ x_1 = 6/7,\ x_2 = 60/7 \\ x_1 = 6,\ x_2 = 0 \end{array}}$$

Remark:

If a constraint to which the objective function is parallel doesnt form the boundary of the feasible region, then multiple solution will not exist and such a constraint is called redundant constraint, that is, reduntant constraint is one whose removal does not change the feasible region.

An unbounded solution :

When the value of the decision variables in linear programming is permitted to increase infinitely without violating the feasibility condition, then the solution is said to be unbounded.

Here the objective function value can also be increased infinitely

example,

Max $z = 3x_1 + 4x_2$

st,

$x_1 - x_2 \leq -1$ - ①  $x_1 - x_2 = -1$

$-x_1 + x_2 \leq 0$ - ②  $x_2 - x_1 = 0$

$x_1 = x_2$

& $x_1, x_2 \geq 0$

Soln,

① $x_1 = 0$

$x_2 = 1$

$x_2 = 0$

$x_1 = -1$

② $x_1 = 0$

$x_2 = 0$

$x_2 = 0$

$x_1 = 0$

let $x_1 = 1$

$x_2 = 1$