Unit - I
1.7 Electrostatics

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Smart Grid

Grid → Smart

↳ digitisation ✓ ✓

ATC → 26%

Aggregated Technical Commercial losses

Grid + Communication + automation → Smart grid

ATC → Technical loss
→ Inefficient billing
→ Theft

Commercial
↳ Payment default

Syllabus

UNIT – I **10 Periods**

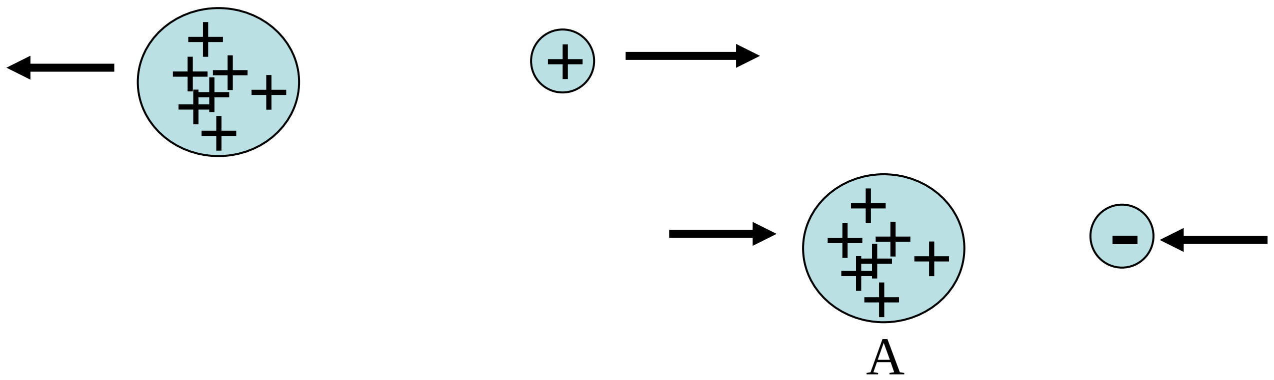
Introduction and Basic Concepts: Concept of Potential difference, voltage, current - Fundamental linear passive and active elements to their functional current-voltage relation - Terminology and symbols in order to describe electric networks - Concept of work, power, energy and conversion of energy- Principle of batteries and application.

Principles of Electrostatics: Electrostatic field - electric field intensity - electric field strength - absolute permittivity - relative permittivity - capacitor composite – dielectric capacitors - capacitors in series & parallel - energy stored in capacitors - charging and discharging of capacitors.

The Electric Field

Unit positive charge

Between two charged bodies there is a force, F, of attraction or repulsion:

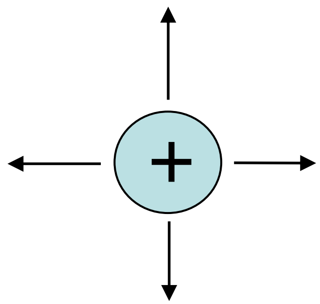


We don't understand why; we can only say this is what happens.

We can think of a charged body as *changing the nature of the space surrounding it*.

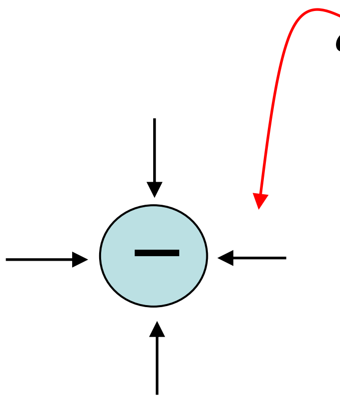
Direction of the Electric Field

Outward (away) from a positive charge



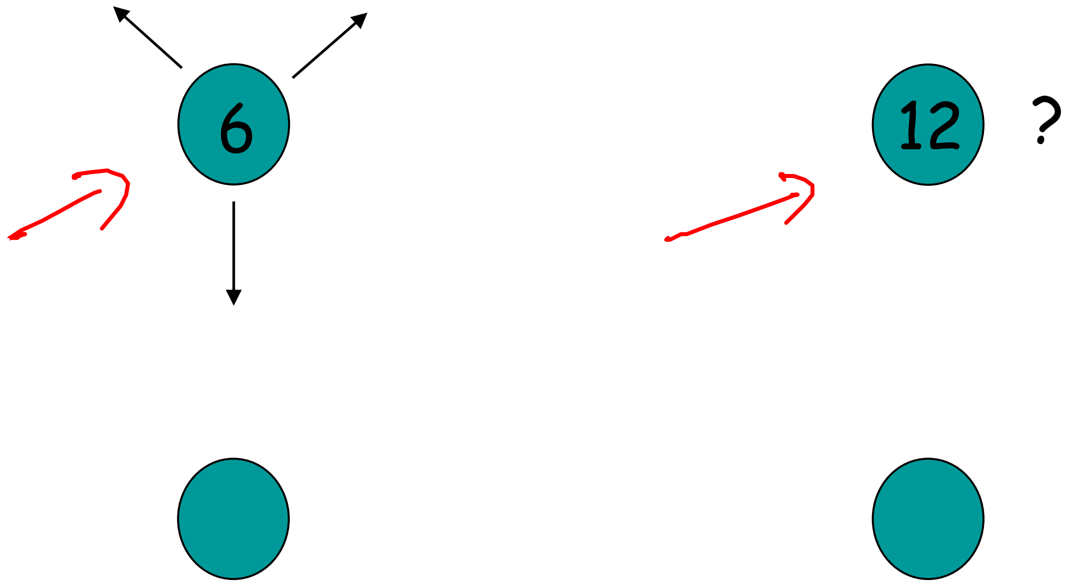
These are called "field arrows"

Inward (towards) a negative charge



Direction of the Electric Field

We draw the number of arrows proportional to the charge…more charge, more arrows. Say the charges are in "μCoulombs" (that's micro-coulombs, or 10^{-6} Coulombs)



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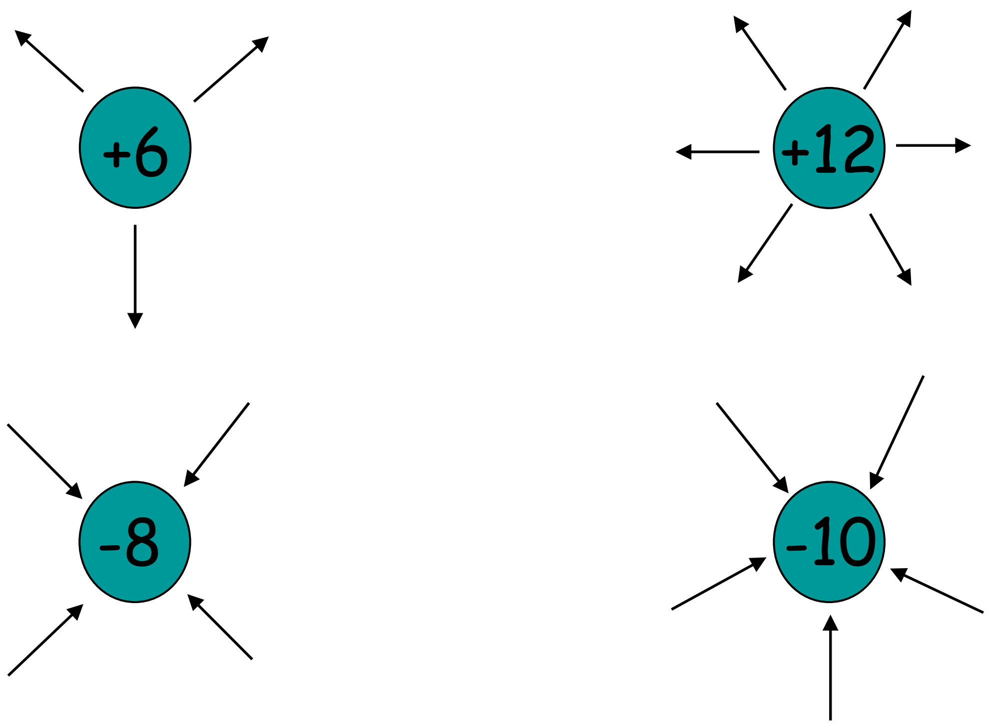
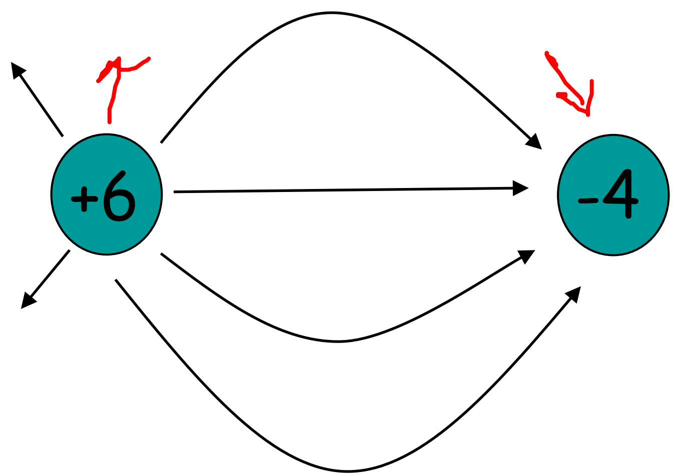


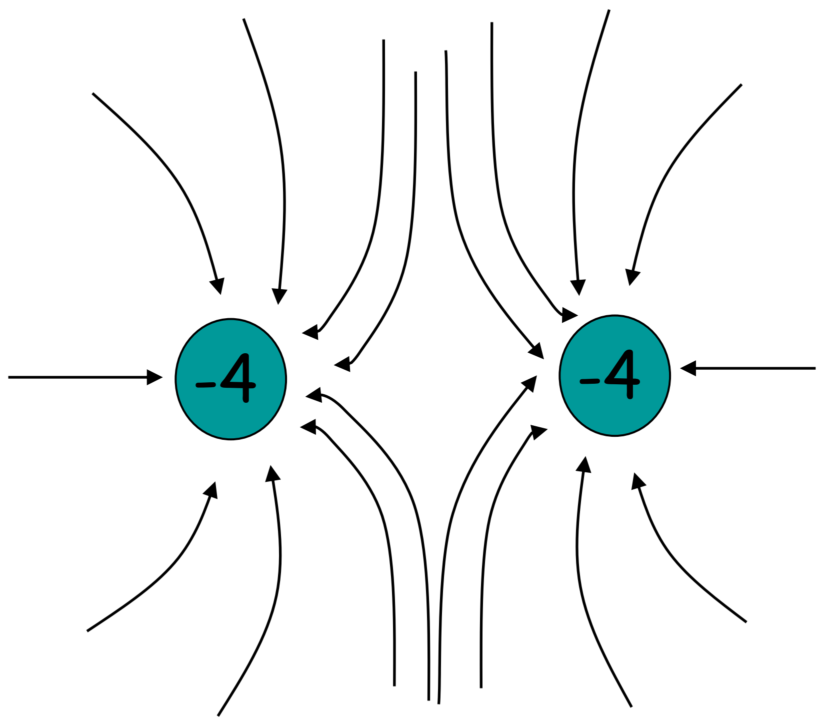
Direction of the Electric Field

When charges get near each other, these fields interact

For unlike charges, the arrows go from the positive charge to the negative charge:



For like particles the arrows are repelled:



The field arrows never cross in either case

Example 1

- A charged oil drop remains stationary when situated between two parallel plates 25mm apart. A p.d. of 1000V is applied to the plates. If the mass of the drop is 5 ×10−15 kg, find the charge on the drop (take g = 10 ms^{-2}).

- Solution

  – Let Q coulomb be the charge on the oil drop. Since the drop is stationary

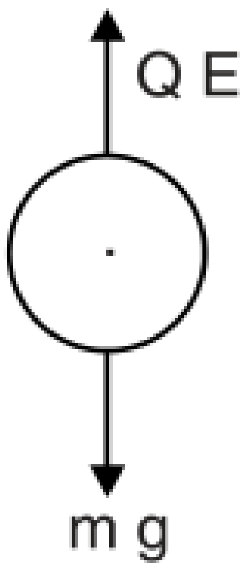
  – Upward force on drop = Weight of drop

Solution

$$QE = mg$$

$$E = \frac{V}{d} = \frac{1000}{25\times10^{-3}} = 4\times10^{4}\ \text{V/m}$$

$$Q = \frac{mg}{E} = \frac{(5\times10^{-15})\times10}{4\times10^{4}} = \mathbf{1.25\times10^{-18}\ C}$$



Electric Field Intensity (Cont'd)

It becomes convenient to define electric field intensity E_1 or force per unit charge as:

$$\mathbf{E}_1 = \frac{\mathbf{F}_{12}}{Q_2}$$

This field from charge Q_1 fixed at origin results from the force vector F_{12} for any arbitrarily chosen value of Q_2

Electric Field Intensity (Cont'd)

Coulomb's law can be rewritten as

$$\boxed{\mathrm{E} = \frac{Q}{4\pi\varepsilon_0 |\mathrm{R}|^2}\,\mathrm{a}_R}$$

to find the **electric field intensity** at any point in space resulting from a fixed charge Q.

Example 2

Find **E** at (0,3,4) m in cartesian coordinates due to a point charge Q =0.5μC at the origin.

Solution to Example 2

$$R = 3a_y + 4a_z \qquad a_R = (3a_y + 4a_z)/5$$

$$R = 5 \qquad\qquad = 0.6a_y + 0.8a_z$$

$$E = \frac{0.5\times 10^{-6}}{4\pi\left(10^{-9}/36\pi\right)(5)^2}\left(0.6a_y + 0.8a_z\right)$$

Thus $\left[E\right] = 180V/m$ in the direction $a_R = 0.6a_y + 0.8a_z$

Example 3

Let a point charge Q_1 = 25nC be located at P_1 (4,-2,7).

If $\varepsilon = \varepsilon_0$, find electric field intensity at P_2 (1,2,3).

Solution to Example 3

By using the electric field intensity,

$$\mathrm{E} = \frac{Q}{4\pi\varepsilon_o|\mathrm{R}|^2}\mathrm{a}_R$$

This field will be:

$$\mathrm{E} = \frac{25\times10^{-9}}{4\pi\varepsilon_o|\mathrm{R}|}\mathrm{a}_R$$

Solution to Example 3 (Cont'd)

Where, $\mathbf{R}_{12} = \mathbf{r}_2 - \mathbf{r}_1 = -3\mathbf{a}_x + 4\mathbf{a}_y - 4\mathbf{a}_z$

and $|\mathrm{R}| = \sqrt{41}$

$$\mathrm{E} = \frac{Q}{4\pi\varepsilon_o |\mathrm{R}|^2} \mathrm{a}_R = \frac{Q}{4\pi\varepsilon_o |\mathrm{R}|^3} \mathrm{R}$$

$$= \frac{25\times10^{-9}}{4\pi\left(8.854\times10^{-12}\right)\left(41\right)^{3/2}} \left(-3\mathrm{a}_x + 4\mathrm{a}_y - 4\mathrm{a}_z\right)$$

$$= ??$$

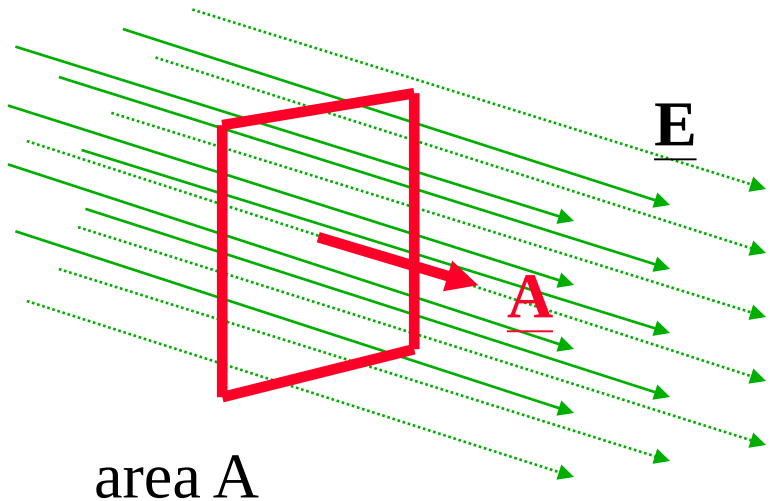
Electric Field Intensity (Cont'd)

If there are N charges, $Q_1, Q_2 \ldots Q_N$ located respectively at point with position vectors $r_1, r_2 \ldots r_N$ the electric field intensity at point r is:

$$\mathbf{E} = \frac{Q_1}{4\pi\varepsilon_0 |\mathbf{r}-\mathbf{r}_1|^2} \frac{(\mathbf{r}-\mathbf{r}_1)}{|\mathbf{r}-\mathbf{r}_1|} + \ldots \frac{Q_N}{4\pi\varepsilon_0 |\mathbf{r}-\mathbf{r}_N|^2} \frac{(\mathbf{r}-\mathbf{r}_N)}{|\mathbf{r}-\mathbf{r}_N|}$$

$$\boxed{\mathrm{E} = \frac{1}{4\pi\varepsilon_0} \sum_{k=1}^{N} \frac{Q_k (\mathrm{r}-\mathrm{r}_k)}{|\mathrm{r}-\mathrm{r}_k|^3}}$$

Electric Flux



We define the electric flux Φ, of the electric field $\underline{E}$, through the surface A, as:

$$\boxed{\Phi = \underline{\mathbf{E}} \cdot \underline{\mathbf{A}}}$$

$$\Phi = E\, A \cos(\theta)$$

Where:

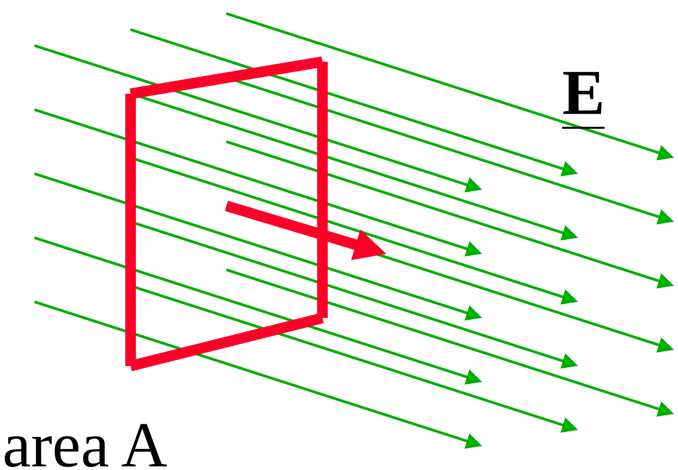
$\underline{A}$ is a vector normal to the surface (magnitude A, and direction normal to the surface).

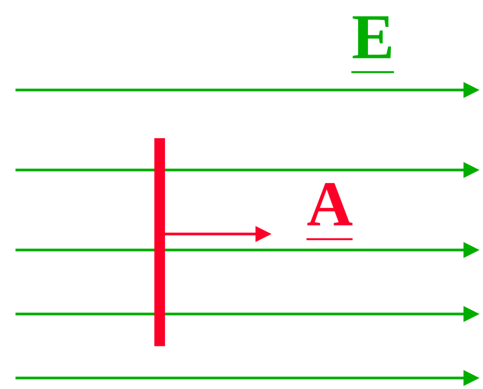
θ is the angle between $\underline{E}$ and $\underline{A}$

Electric Flux

You can think of the flux through some surface as a measure of the number of field lines which pass through that surface.

Flux depends on the strength of $\underline{\mathbf{E}}$, on the surface area, and on the relative orientation of the field and surface.





Normal to surface, magnitude A

$\underline{\mathbf{A}} \equiv \longrightarrow$

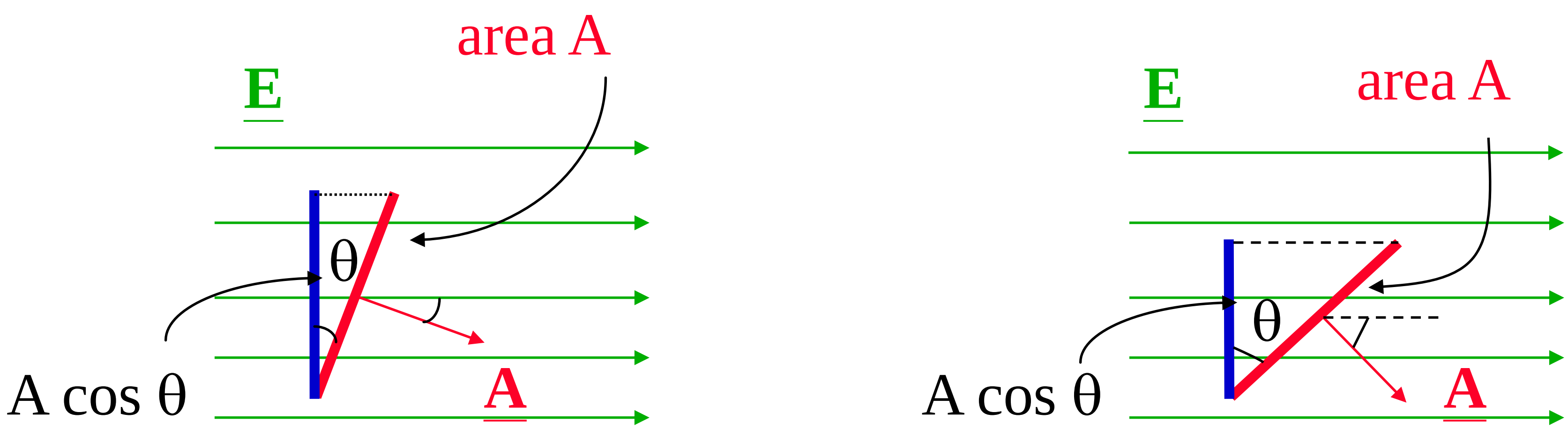
Here the flux is

$$\Phi = \underline{\mathbf{E}} \cdot \underline{\mathbf{A}}$$

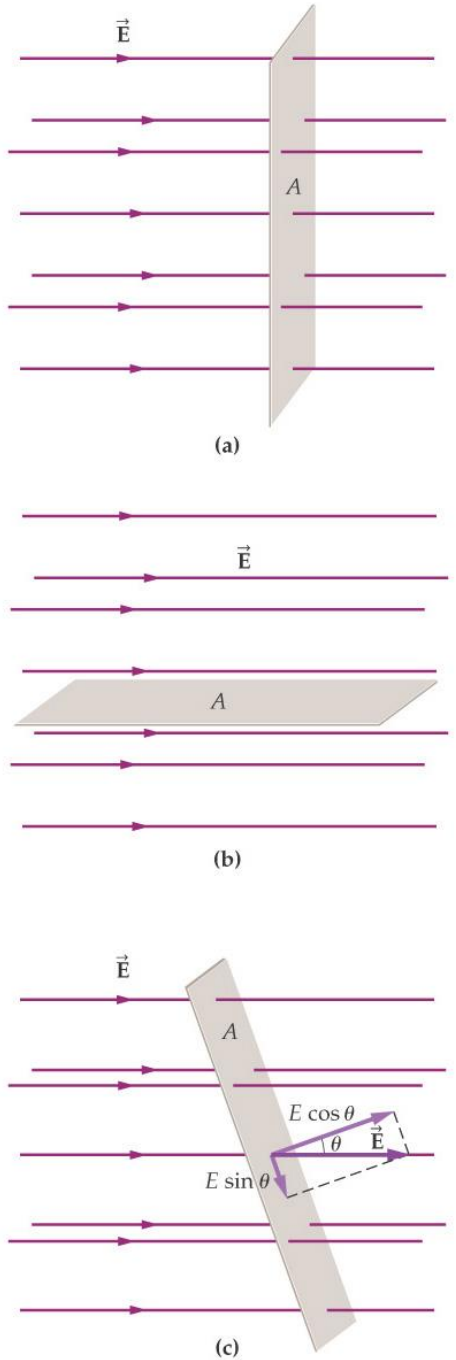
Electric Flux

The flux also depends on orientation

$$\Phi = \underline{\mathbf{E}} \cdot \underline{\mathbf{A}} = \mathbf{E}\,\mathbf{A}\cos\theta$$



The number of field lines through the tilted surface / equals the number through its projection |. Hence, the flux through the tilted surface is simply given by the flux through its projection: E (A cosθ).



Calculate the flux of the electric field E, through the surface A, in each of the three cases shown:

a) $\Phi =$
b) $\Phi =$
c) $\Phi =$

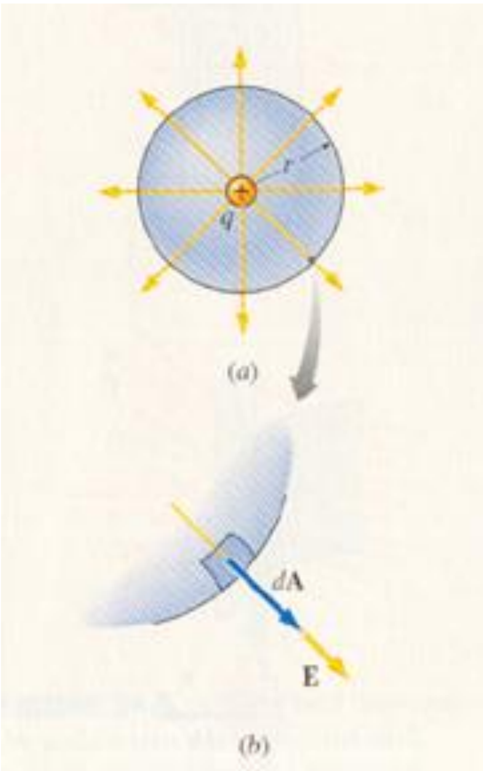
Gauss's Law

The electric flux
through any closed surface
equals □ enclosed charge / ε_0

$$\oint \underline{E} \bullet d\underline{A} = \frac{\sum_{inside} q}{\varepsilon_0}$$

This is always true.
Occasionally, it provides a very easy way
to find the electric field
(for highly symmetric cases).

Calculate the electric field produced by a point charge using Gauss Law



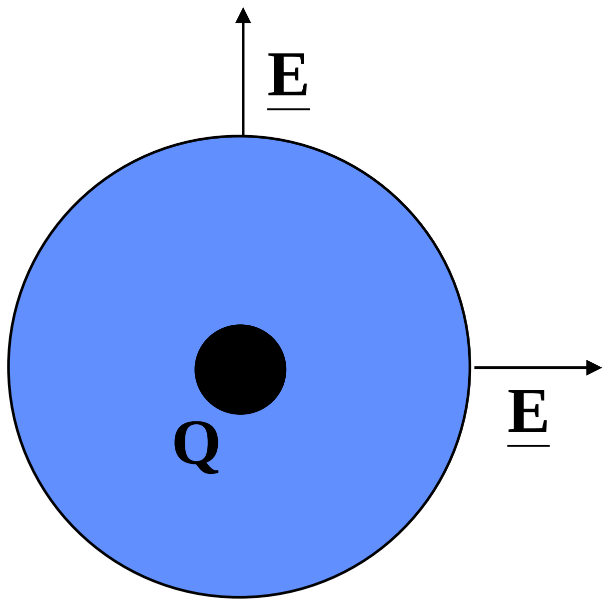
We choose for the gaussian surface a sphere of radius **r**, centered on the charge **Q**.

Then, the electric field $\underline{\mathbf{E}}$, has the same magnitude everywhere on the surface (radial symmetry)

Furthermore, at each point on the surface, the field $\underline{\mathbf{E}}$ and the surface normal $\underline{\mathbf{dA}}$ are parallel (both point radially outward).

$$\underline{\mathbf{E}} \cdot \underline{\mathbf{dA}} = \mathbf{E\ dA} \quad [\cos\theta = 1]$$

Electric field produced by a point charge



$$\int \underline{E} \cdot \underline{dA} = Q / \varepsilon_0$$

$$\int \underline{E} \cdot \underline{dA} = E \int dA = E\,A$$

$$A = 4 \pi r^2$$

$$E\,A = E\; 4 \pi r^2 = Q / \varepsilon_0$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$$

$k = 1 / 4 \pi \varepsilon_0$

ε_0 = permittivity

$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/Nm^2$

→ **Coulomb's Law !**

Is Gauss's Law more fundamental than Coulomb's Law?

- No! Here we derived Coulomb's law for a point charge from Gauss's law.
- One can instead derive Gauss's law for a general (even very nasty) charge distribution from Coulomb's law. The two laws are equivalent.
- Gauss's law gives us an easy way to solve a few very symmetric problems in electrostatics.
- It also gives us great insight into the electric fields in and on conductors and within voids inside metals.

Gauss's Law

The total flux within a closed surface …

… is proportional to the enclosed charge.

$$\Phi = \oint \vec{\mathbf{E}} \bullet d\vec{\mathbf{A}} = \frac{Q_{enclosed}}{\varepsilon_0}$$

Gauss's Law is always true, but is only useful for certain very simple problems with great symmetry.

- Electric Flux Density (D)
  - electric flux density at any section in an electric field is the electric flux crossing normally per unit area of that section

$$\text{Electric flux density, } D = \frac{\Psi}{A}$$

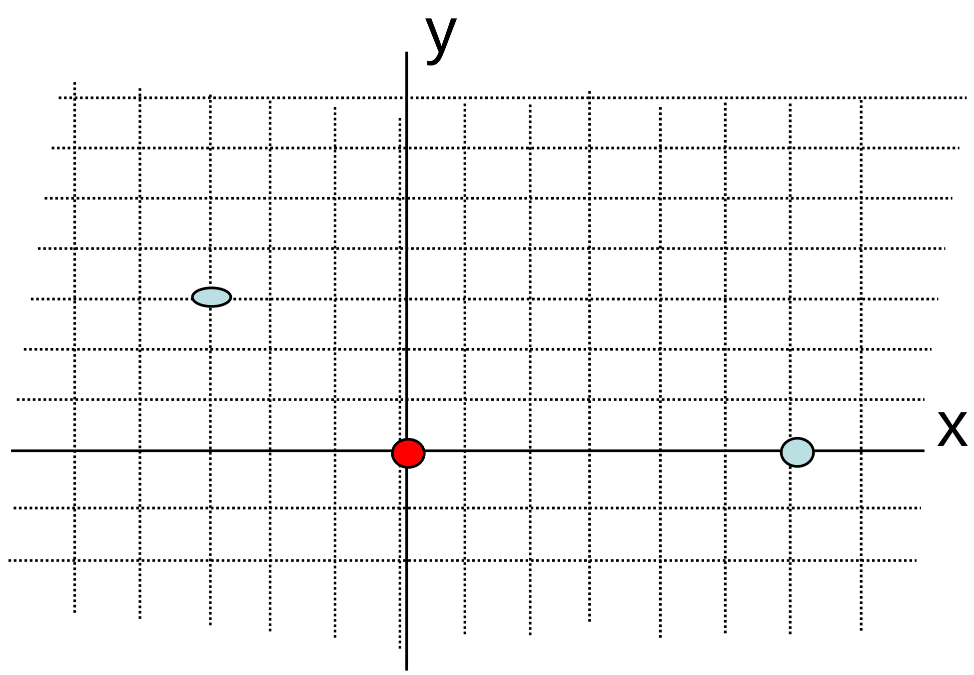
- Electric displacement –same

$$D = \varepsilon_0 \varepsilon_r E$$

- electric potential is defined as the electric potential energy per unit charge

Blue charges fixed , negative, equal charge (-q)

What is force on positive red charge +q ?

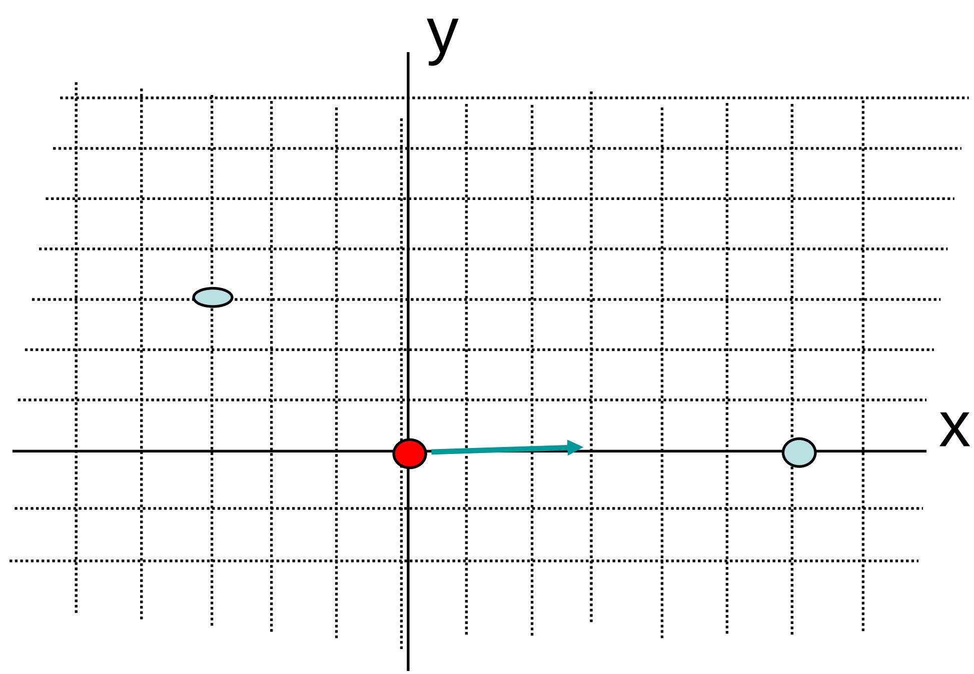


Superposition of forces from two charges

Blue charges fixed , negative, equal charge (-q)

What is force on positive red charge +q ?

Consider effect of each charge separately:



Superposition of forces from two charges

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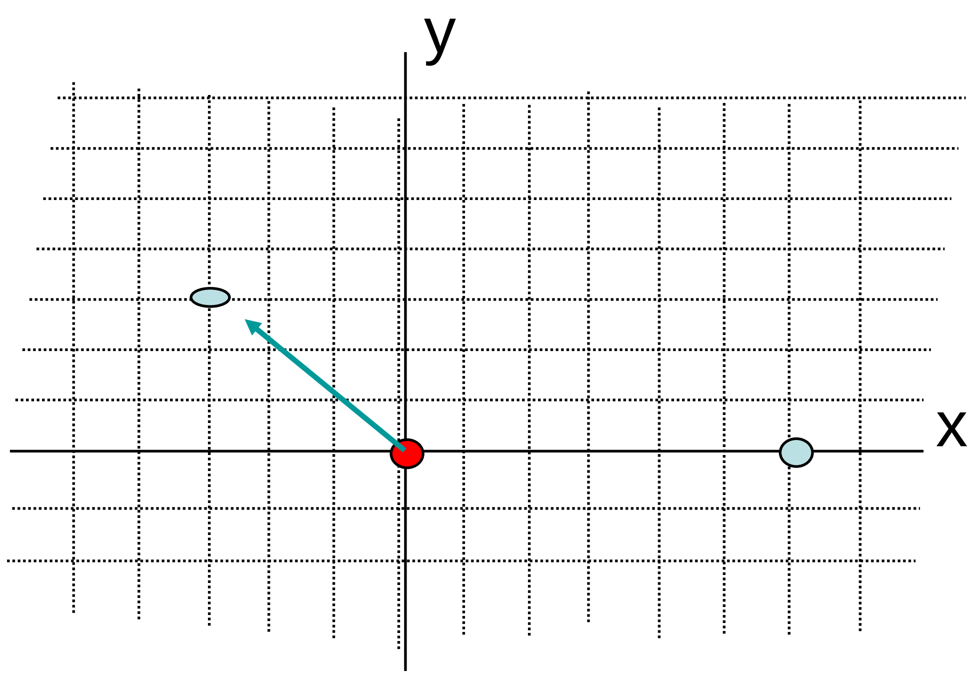
What is force on positive red charge +q ?

Take each charge in turn:

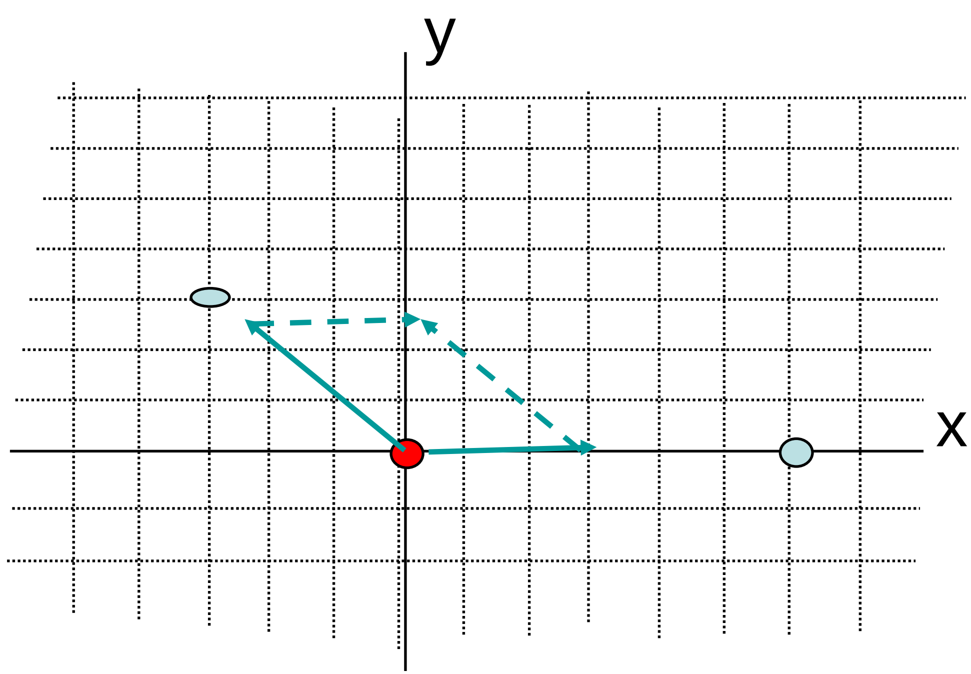


Superposition of forces from two charges

Blue charges fixed , negative, equal charge (-q)

What is force on positive red charge +q ?

Create vector sum:

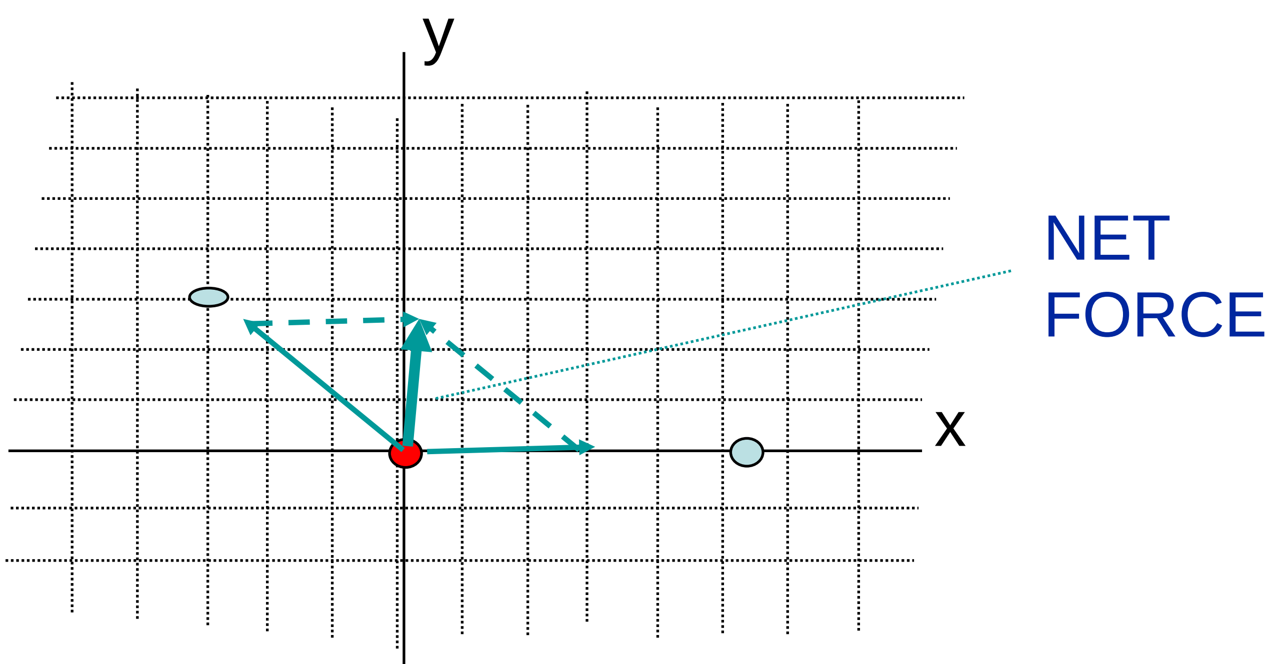


Superposition of forces from two charges

Blue charges fixed , negative, equal charge (-q)

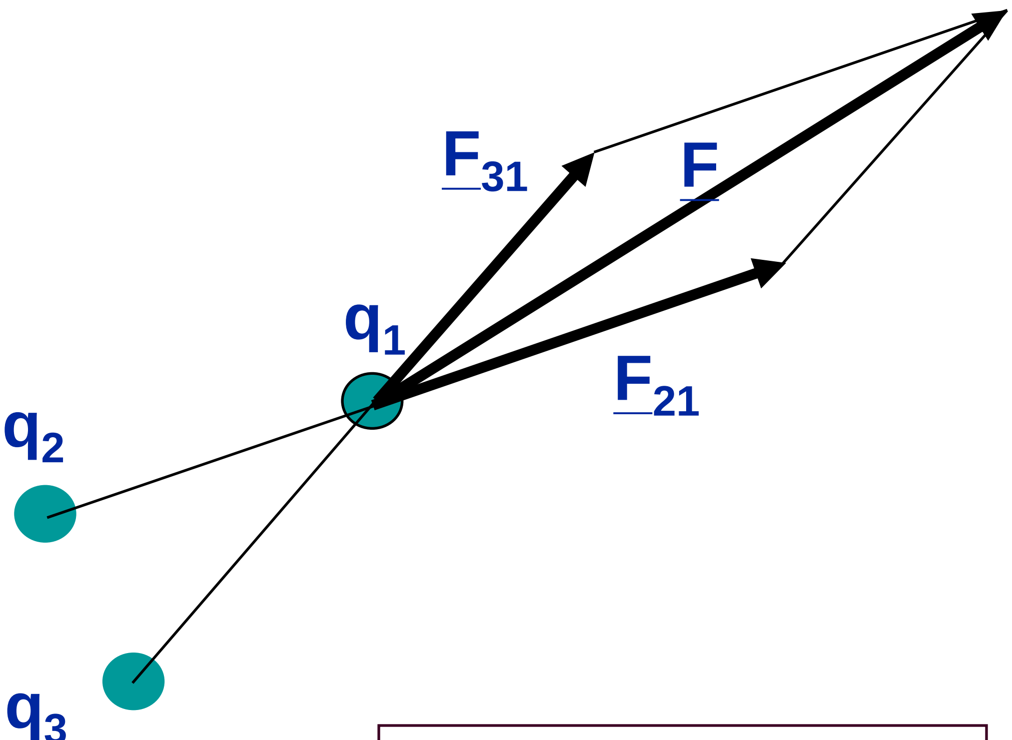
What is force on positive red charge +q ?

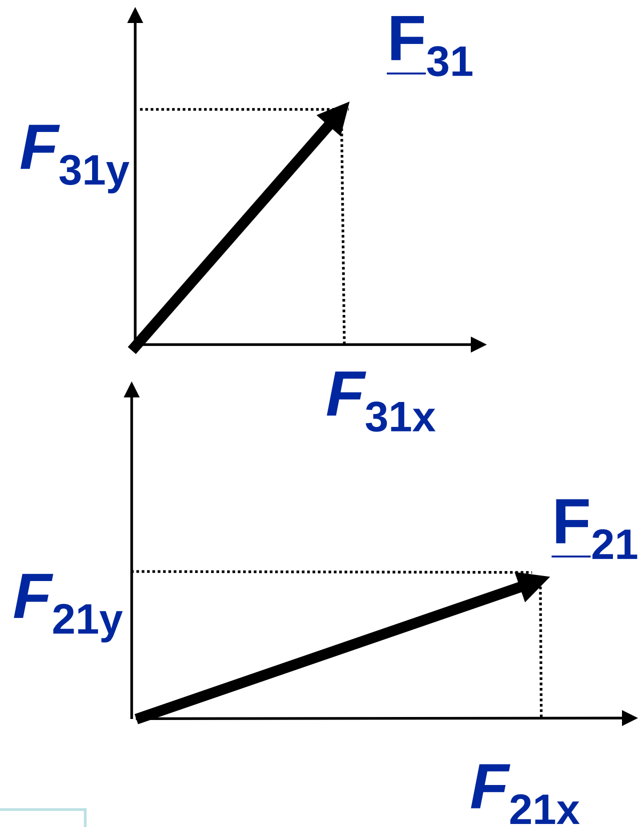
Find resultant:



When a number of charges are present, the total force on a given charge is equal to the vector sum of the forces due to the remaining other charges on the given charge.

Superposition Principle





Forces add vectorially

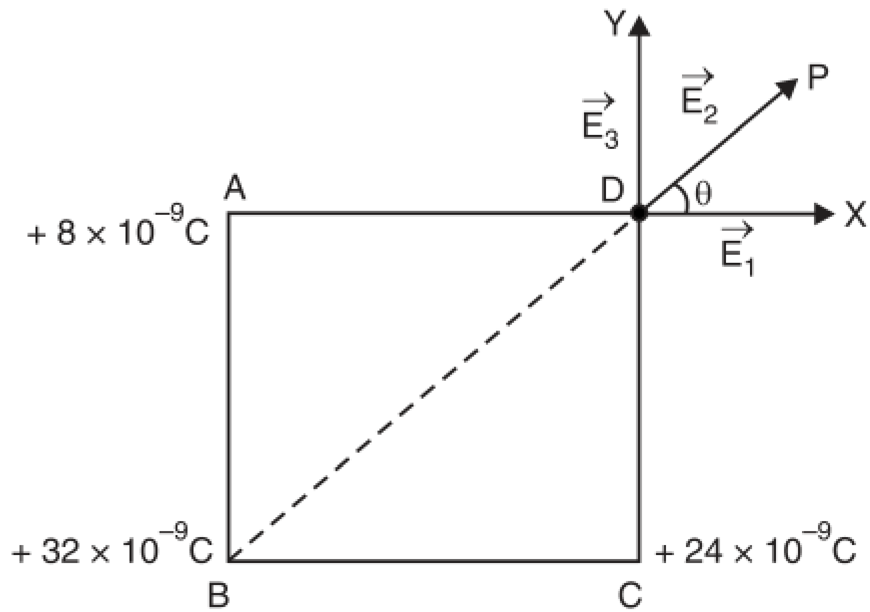
$$\underline{F} = (F_{21x} + F_{31x})\,\underline{x} + (F_{21y} + F_{31y})\,\underline{y}$$

Example 4

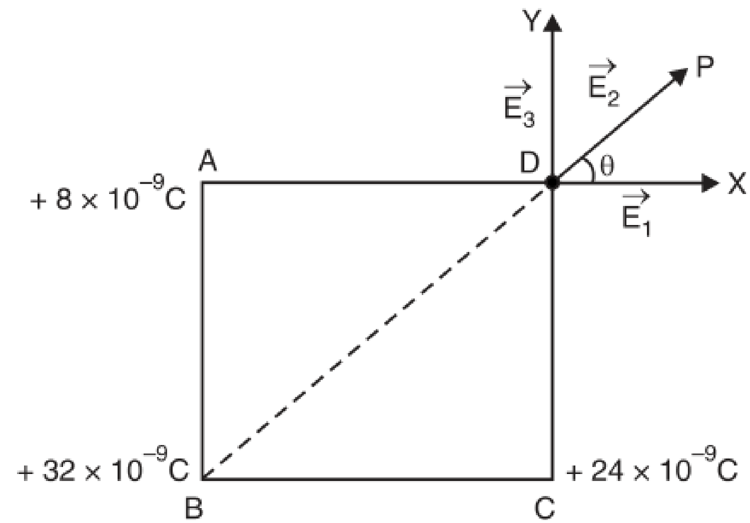
- Three point charges of $+8 \times 10^{-9}$ C, $+32 \times 10^{-9}$ C and $+24 \times 10^{-9}$ C are placed at the corners A, B and C of a square ABCD having each side 4 cm. Find the electric field intensity at the corner D. Assume that the medium is air.

- Solution

- BD =√2*0.04m



Solution



Magnitude of electric field intensity at D due to charge $+8 \times 10^{-9}$ C is

$$E_1 = 9 \times 10^9 \times \frac{8\times10^{-9}}{(0.04)^2} = 4{\cdot}5 \times 10^4 \text{ N/C} \quad \text{along } DX$$

Magnitude of electric field intensity at D due to charge $+32 \times 10^{-9}$ C is

$$E_2 = 9 \times 10^9 \times \frac{32\times10^{-9}}{\left(\sqrt{2}\times0.04\right)^2} = 9 \times 10^4 \text{ N/C along } DP$$

Magnitude of electric field intensity at D due to charge $+24 \times 10^{-9}$ C is

$$E_3 = 9 \times 10^9 \times \frac{24\times10^{-9}}{(0.04)^2} = 13{\cdot}5 \times 10^4 \text{ N/C along } DY$$

Solution

It is easy to see that $\theta = 45°$.

Resolving electric field intensities along X-axis and Y-axis, we have,

$$\text{Total } X\text{-component} = E_1 + E_2 \cos\theta + 0$$
$$= 4{\cdot}5 \times 10^4 + 9 \times 10^4 \times \cos 45° = 10{\cdot}86 \times 10^4 \text{ N/C}$$

$$\text{Total } Y\text{-component} = 0 + E_2 \sin 45° + E_3$$
$$= 0 + 9 \times 10^4 \sin 45° + 13{\cdot}5 \times 10^4 = 19{\cdot}86 \times 10^4 \text{ N/C}$$

$\therefore$ Magnitude of resultant electric intensity at D

$$= \sqrt{(10.86\times10^4)^2 + (19.86\times10^4)^2} = \mathbf{22{\cdot}63 \times 10^4\ N/C}$$

Let the resultant intensity make an angle ϕ with DX.

$$\therefore \quad \tan\phi = \frac{Y-\text{component}}{X-\text{component}} = \frac{19.86\times10^4}{10.86\times10^4} = 1{\cdot}828$$

or $$\phi = \tan^{-1} 1{\cdot}828 = \mathbf{61{\cdot}32°}$$

Just as we defined a gravitational field, we define an "electric field" in a similar manner:

$$F = \frac{GmM}{r^2}$$

$$mg = \frac{GmM}{r^2}$$

$$g = \frac{GM}{r^2}$$

This is the gravitational field (Earth = 9.8 m/s^2 or 9.8 N/kg)

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$$F = \frac{k|qQ|}{r^2}$$

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$$q(?) = \frac{k|qQ|}{r^2}$$

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$$\cancel{m}g = \frac{G\cancel{m}M}{r^2}$$

$$g = \frac{GM}{r^2}$$

$$F = \frac{k|qQ|}{r^2}$$

$$\cancel{q}(E) = \frac{k|\cancel{q}Q|}{r^2}$$

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$$q(E) = \frac{k|qQ|}{r^2}$$

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The general equation for an ELECTRIC FIELD is:

$$\boxed{E = \frac{k|Q|}{r^2}}$$

$$\frac{Newtons}{Coulomb} = \frac{N}{C}$$

(compare this to the equation for the gravitational field)

Electric Field

Notice that for gravity, $F = mg$

We see that in electrostatics, $F = qE$

$$F = \frac{GmM}{r^2}$$

$$\downarrow$$

$$mg = \frac{GmM}{r^2}$$

$$g = \frac{GM}{r^2}$$

$$F = \frac{k|qQ|}{r^2}$$

$$\downarrow$$

$$q(E) = \frac{k|qQ|}{r^2}$$

$$E = \frac{k|Q|}{r^2}$$

Summary

Electrostatics

- → Coulomb's law
- → Field Intensity
  - → Single
  - → multiples → Force
- → gravitational field