Unit - II
2.10 DC Transient Analysis – RC Circuit

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Syllabus

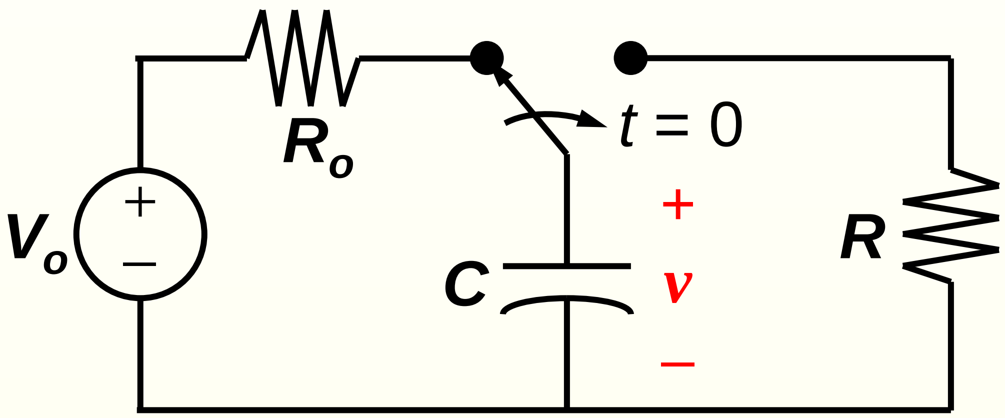
UNIT – II **14 Periods**

DC Circuit Analysis: Voltage source and current sources, ideal and practical, Kirchhoff's laws and applications to network solutions using mesh analysis, - Simplifications of networks using series- parallel, Star/Delta transformation, DC circuits-Current-voltage relations of electric network by mathematical equations to analyse the network (Superposition theorem, Thevenin's theorem, Maximum Power Transfer theorem), Transient analysis of R-L, R-C and R-L-C Circuits.

AC Steady-state Analysis: AC waveform definitions - Form factor - Peak factor - study of R-L - R-C -RLC series circuit - R-L-C parallel circuit - phasor representation in polar and rectangular form - concept of impedance - admittance - active - reactive - apparent and complex power - power factor, Resonance in R-L-C circuits - 3 phase balanced AC Circuits

NATURAL RESPONSE OF AN RC CIRCUIT

- Consider the following circuit, for which the switch is closed for $t < 0$, and then opened at $t = 0$:

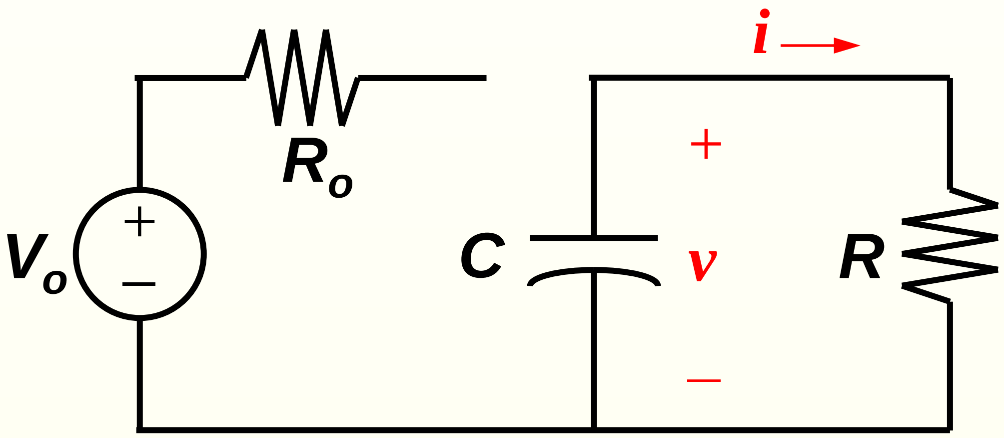


Notation:

- 0^- is used to denote the time just prior to switching
- 0^+ is used to denote the time immediately after switching.

Solving for the voltage ($t \geq 0$)

- For $t \leq 0$, $v(t) = V_o$
- For t > 0, the circuit reduces to



Continue

- Applying KCL to the RC circuit:

$$i_C + i_R = 0 \longrightarrow \textbf{(1)}$$

$$C\frac{dv(t)}{dt} + \frac{v(t)}{R} = 0 \longrightarrow \textbf{(2)}$$

$$\frac{dv(t)}{dt} + \frac{v(t)}{RC} = 0 \longrightarrow \textbf{(3)}$$

$$\frac{dv(t)}{dt} = -\frac{v(t)}{RC} \longrightarrow \textbf{(4)}$$

$$\frac{dv(t)}{v(t)} = -\frac{1}{RC}dt \longrightarrow \textbf{(5)}$$

Continue

- From equation (5), let say:

$$\frac{dx}{x} = -\frac{1}{RC}dy \longrightarrow \textbf{(6)}$$

- Integrate both sides of equation (6):

$$\int_{V_o}^{v(t)} \frac{1}{x}du = -\frac{1}{RC}\int_0^t dy \longrightarrow \textbf{(7)}$$

- Therefore:

$$\ln \frac{v(t)}{V_o} = -\frac{t}{RC} \longrightarrow \textbf{(8)}$$

Continue

- Hence, the voltage is:

$$\boxed{v(t) = v(0)e^{-t/RC} = V_o e^{-t/RC}}$$

- Using Ohm's law, the current is:

$$\boxed{i(t) = \frac{v(t)}{R} = \frac{V_o}{R} e^{-t/RC}}$$

Continue

- The power dissipated in the resistor is:

$$p(t) = vi_R = \frac{V_o^2}{R} e^{-2t/RC}$$

- The energy absorb by the resistor is:

$$w = \frac{1}{2} C V_o^2 (1 - e^{-2t/RC})$$

Continue

- The time constant for the RC circuit equal the product of the resistance and capacitance,

- Time constant, $\boxed{\tau = RC}$ sec

- The expressions for voltage, current, power and energy using time constant concept:

$$v(t) = V_o e^{-t/\tau}$$

$$i(t) = \frac{V_o}{R} e^{-t/\tau}$$

$$p(t) = \frac{V_o^2}{R} e^{-2t/\tau}$$

$$w(t) = \frac{1}{2} C V_o^2 (1 - e^{-2t/\tau})$$

- For the case of capacitor, two important observation can be made,

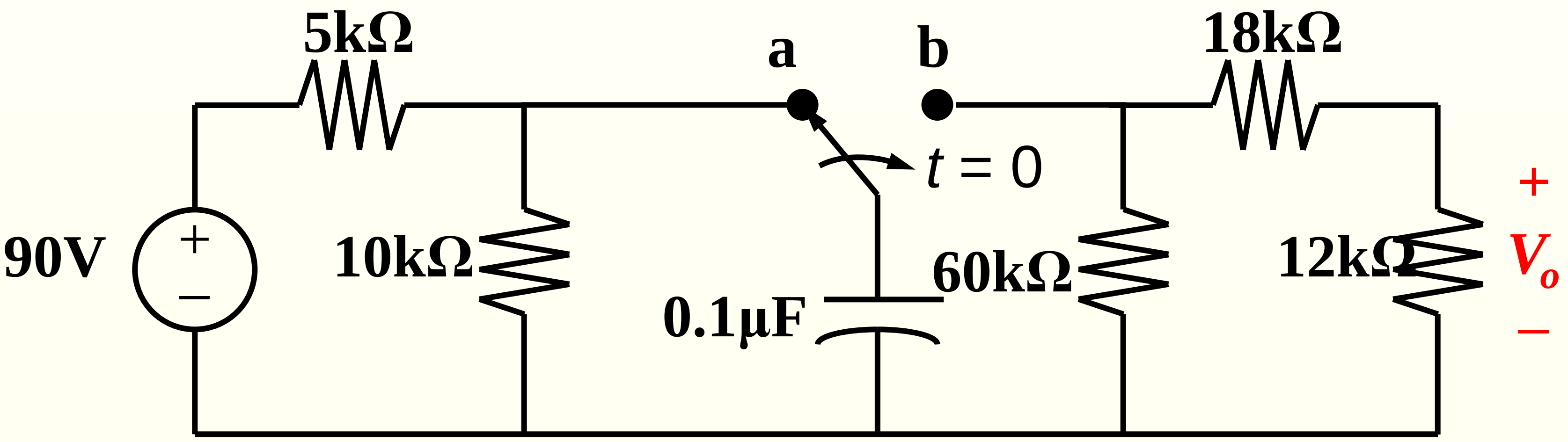
  1) capacitor behaves like an open circuit when being supplied by dc source

  (From, $i_c = Cdv/dt$, when v is constant, $dv/dt = 0$. When current in circuit is zero, the circuit is open circuit.)

  2) in capacitor, the voltage is continuous / stays the same that is, $V_c(0^+) = V_c(0^-)$

Example

The switch has been in position *a* for a long time. At Time t = 0, the switch moves to *b*. Find the expressions for the $v_c(t)$, $i_c(t)$ and $v_o(t)$ and hence sketch them for t = 0 to t = 5τ.



Example

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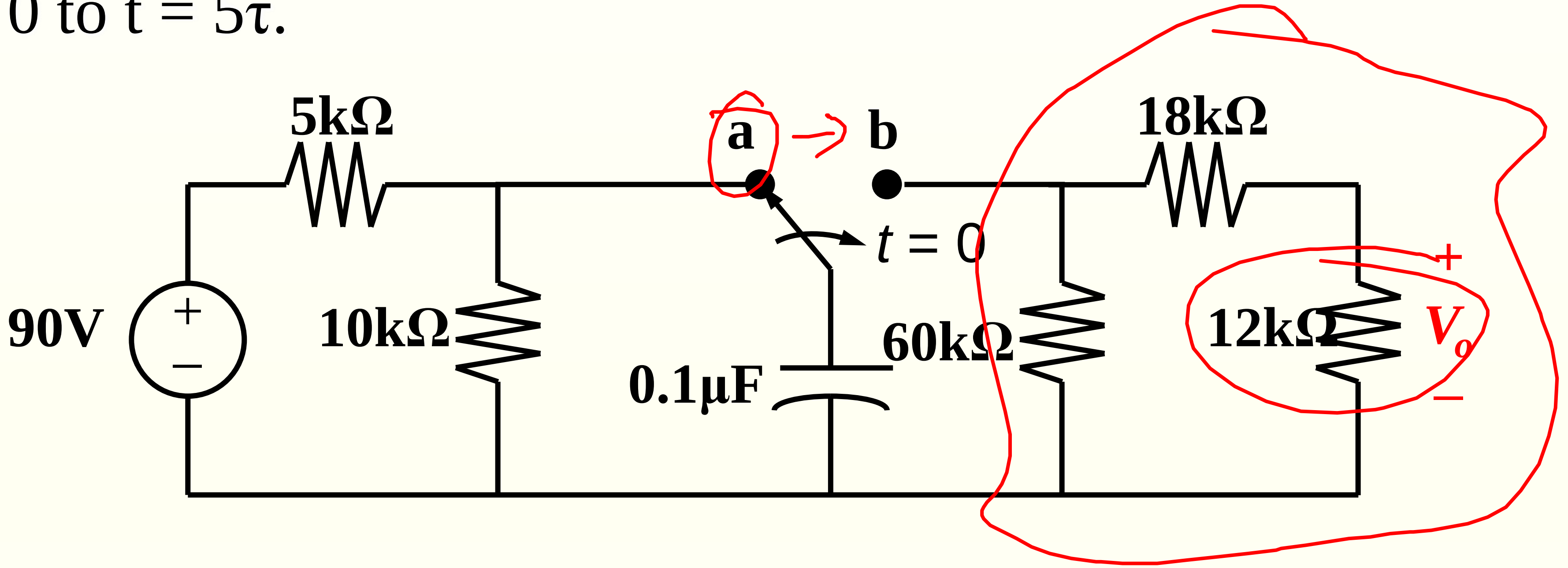
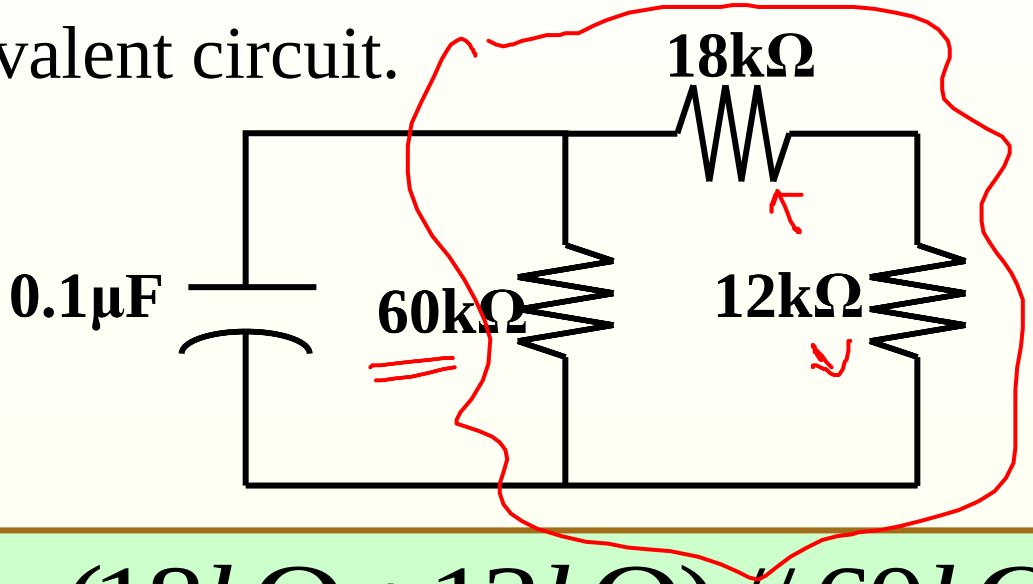


Solution

Step 1:

Find t for t > 5τ that is when the switch was at *a*. Draw the equivalent circuit.

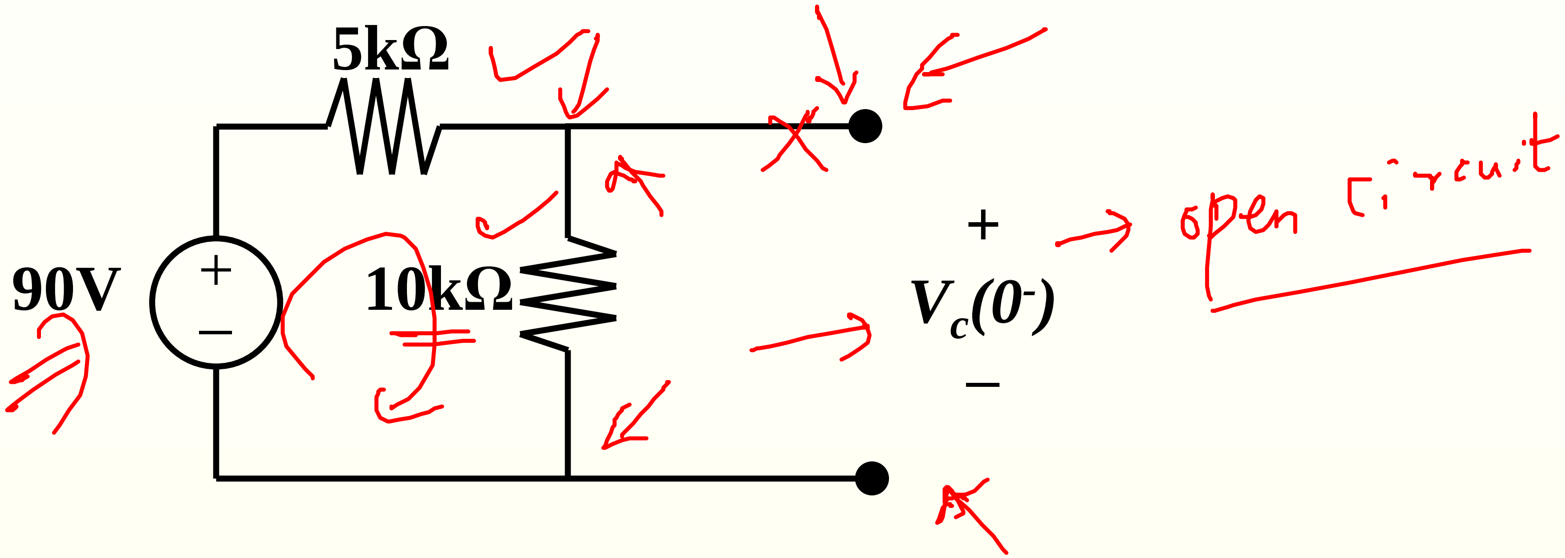


$$R_T = (18k\Omega + 12k\Omega) // 60k\Omega = 20k\Omega$$

$$\tau = R_T C = 20 \times 10^3 \times 0.1 \times 10^{-6} = 2ms$$

Step 2:

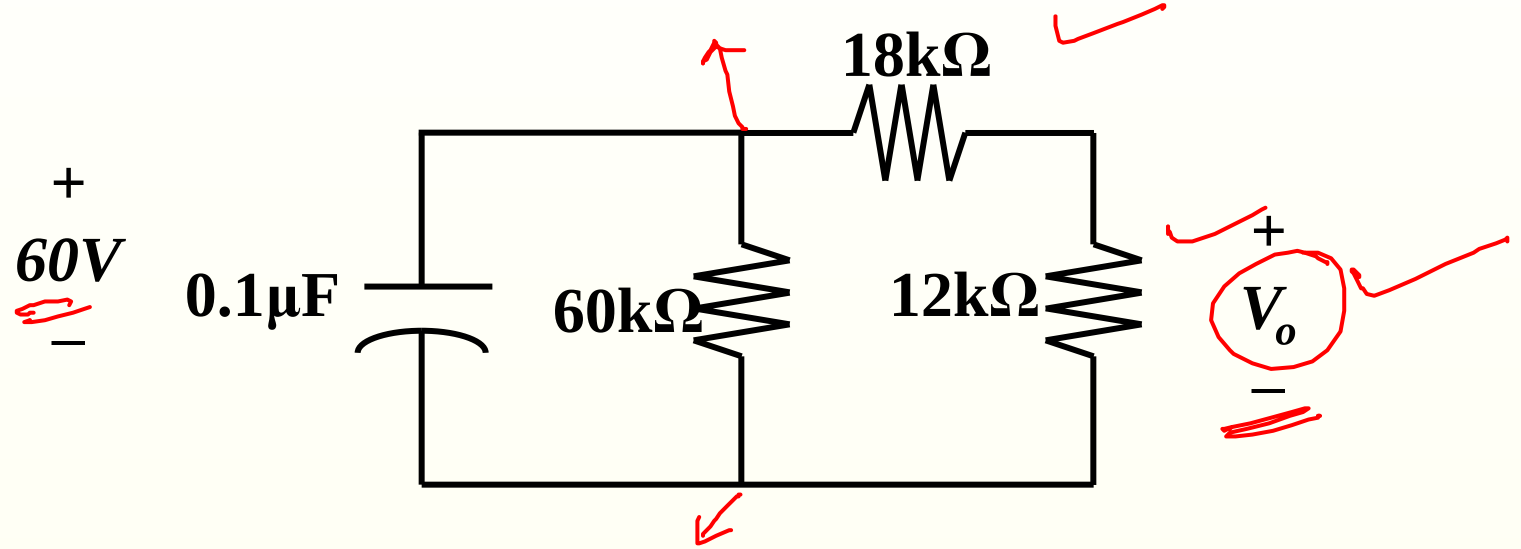
At t = 0, the switch was at *a*. the capacitor behaves like An open circuit as it is being supplied by a constant source.



$$v_c(0^-) = \frac{10}{15} \times 90 = 60V$$

Step 3:

At t = 0^+, the instant when the switch is at *b*.



The voltage across capacitor remains the same at this particular instant.

$$v_c(0^+) = v_c(0^-) = 60V$$

Using voltage divider rule,

$$V_o(0^+) = \frac{12}{30} \times 60 = 24V$$

Hence;

$$\boxed{\begin{aligned} v_c(t) &= 60e^{-500t}\mathrm{V} \\ v_o(t) &= 24e^{-500t}\mathrm{V} \\ i_c(t) &= -0.03e^{-500t}A \end{aligned}}$$

Comparison of RL & RC Circuit

No	RL circuit	RC circuit
1	$\tau = \dfrac{L}{R}$	$\tau = RC$
2	Inductor behaves like a short circuit when being supplied by dc source for a long time	Capacitor behaves like an open circuit when being supplied by dc source for a long time
3	Inductor current is continuous $i_L(0^+) = i_L(0^-)$	Voltage across capacitor is continuous $v_C(0^+) = v_C(0^-)$

Summary

Transient

- why?
- How? → Response
  - Natural response
    - L
    - C
    - R
    - RL
    - RC
  - Forced response
    - Source

Exercise