Unit - II
2.14 AC Steady-state Analysis

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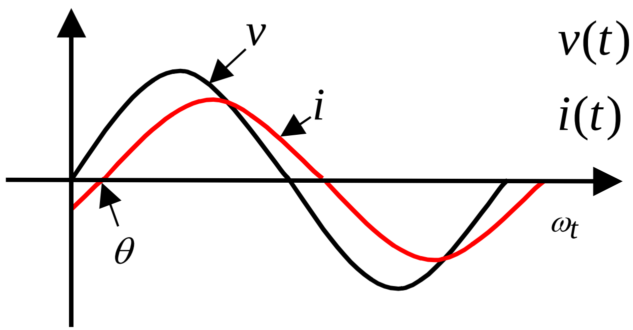
Syllabus

UNIT – II **14 Periods**

DC Circuit Analysis: Voltage source and current sources, ideal and practical, Kirchhoff's laws and applications to network solutions using mesh analysis, - Simplifications of networks using series- parallel, Star/Delta transformation, DC circuits-Current-voltage relations of electric network by mathematical equations to analyse the network (Superposition theorem, Thevenin's theorem, Maximum Power Transfer theorem), Transient analysis of R-L, R-C and R-L-C Circuits.

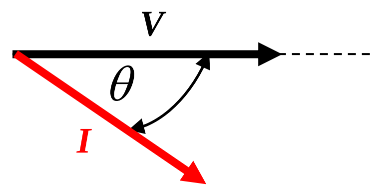
AC Steady-state Analysis: AC waveform definitions - Form factor - Peak factor - study of R-L - R-C -RLC series circuit - R-L-C parallel circuit - phasor representation in polar and rectangular form - concept of impedance - admittance - active - reactive - apparent and complex power - power factor, Resonance in R-L-C circuits - 3 phase balanced AC Circuits

Instantaneous Electric Power [*p(t)*]



$$v(t) = V_{\max}\ \sin(\omega t)$$

$$i(t) = I_{\max}\ \sin(\omega t - \theta)$$



$$\rho(t) = v(t) * i(t) = V_{\max}\ I_{\max}\ \sin(\omega t)\ \sin(\omega t - \theta)$$

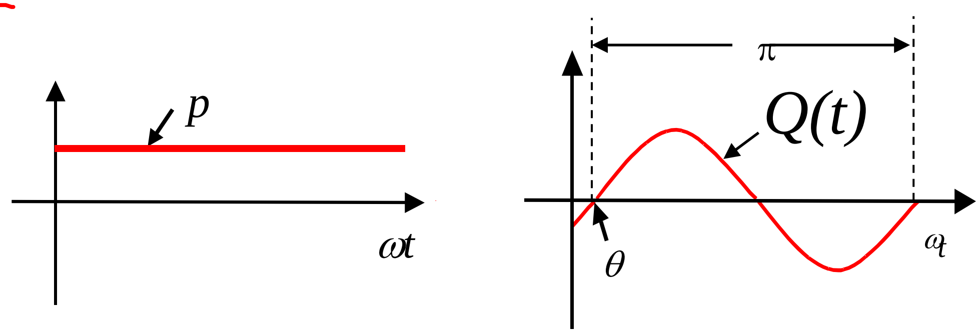
$$\rho(t) = \frac{V_{\max}\ I_{\max}}{2}\ [\cos(\theta) - \cos(2\omega t - \theta)]$$

Fixed average ***Zero average***

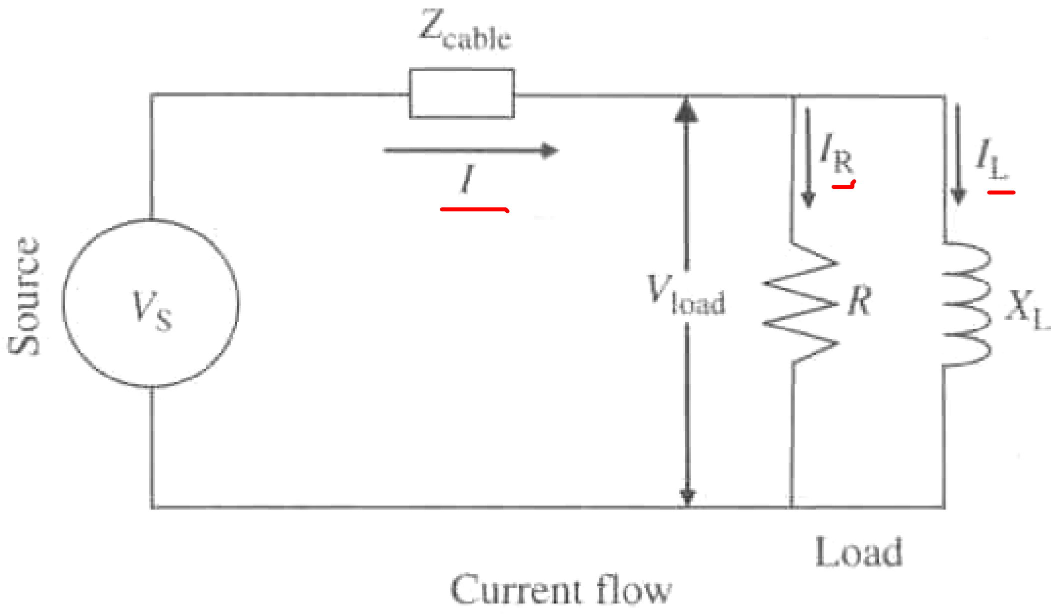
Real & Reactive Power – Time Domain

$$p(t) = \frac{V_{\max}\ I_{\max}}{2}\left[\cos(\theta_{vi}) - \cos(2\omega t - \theta_{vi})\right]$$

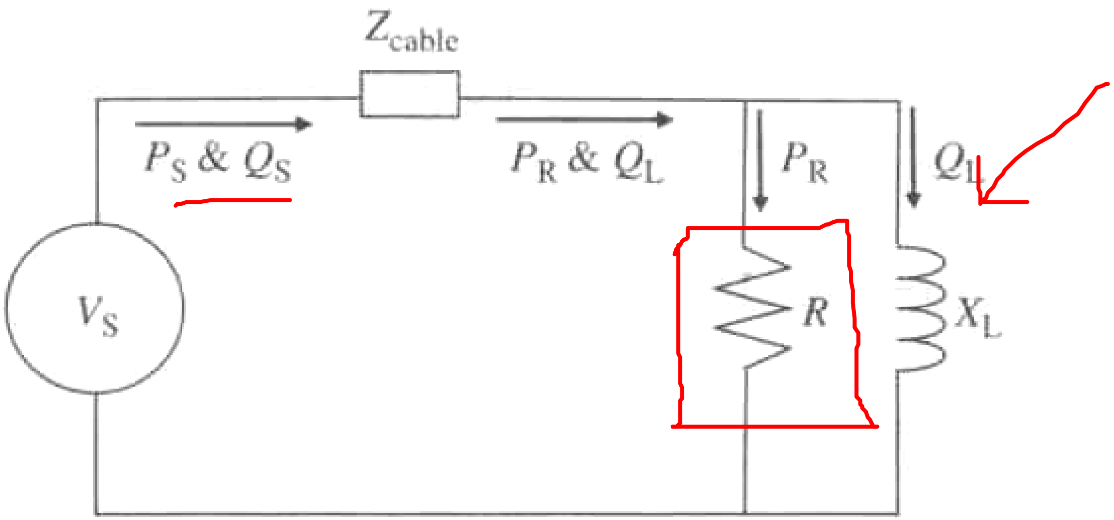
$$p(t) = P + Q(t)$$



Example: Current Flow



Example: Power Flow



Power flow

Phasors

A sinusoidal voltage/current at a given frequency , is characterized by only two parameters :amplitude an phase

Key Words:

Complex Numbers

Rotating Vector

Phasors

Phasors

E.g. voltage response

Time domain

$R_e\{v(t)\}$

$v(t) = V_m \cos(\omega t + \varphi)$

Complex form: $v(t) = V_m e^{j(\omega t + \varphi)}$

Angular frequency ω is known in the circuit.

Phasor form: $\dot{V} = |V_m| \angle \varphi$

Frequency domain

A sinusoidal *v*/*i*

$v(t) = V_m \cos(\omega t + \varphi)$

Complex transform

Phasor transform

$\dot{V} = |V_m| \angle \varphi$

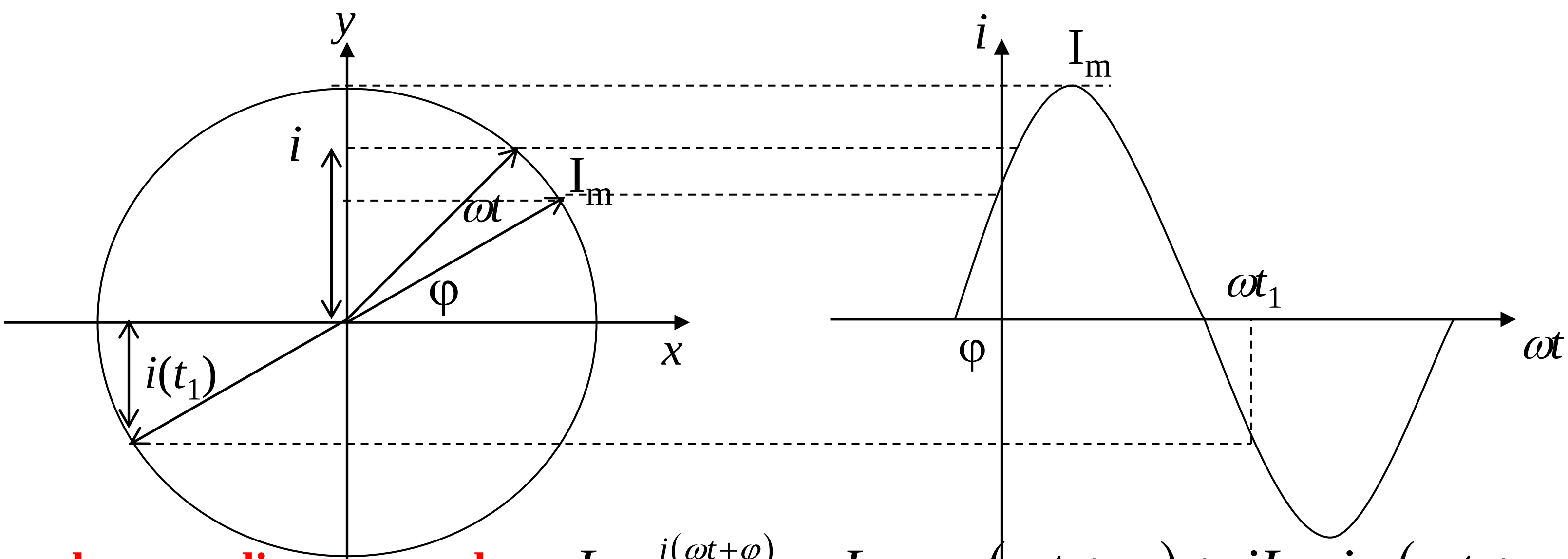
By knowing angular frequency ω *rads/s*.

Sinusoidal Steady State Analysis

Phasors

Rotating Vector

$$i(t) = I_m \sin(\omega t + \varphi)$$



A complex coordinates number: $I_m e^{j(\omega t+\varphi)} = I_m \cos(\omega t + \varphi) + jI_m \sin(\omega t + \varphi)$

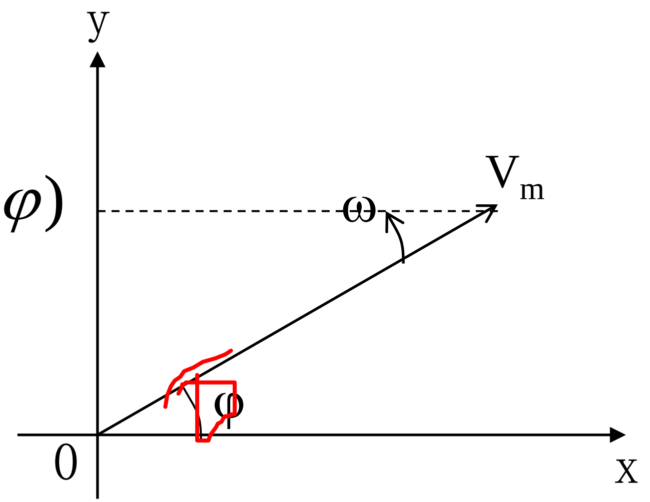
Real value: $i(t) = I_m \sin(\omega t + \varphi) \because \text{Imag}\left(I_m e^{j(\omega t+\varphi)}\right)$

Sinusoidal Steady State Analysis

Phasors

Rotating Vector

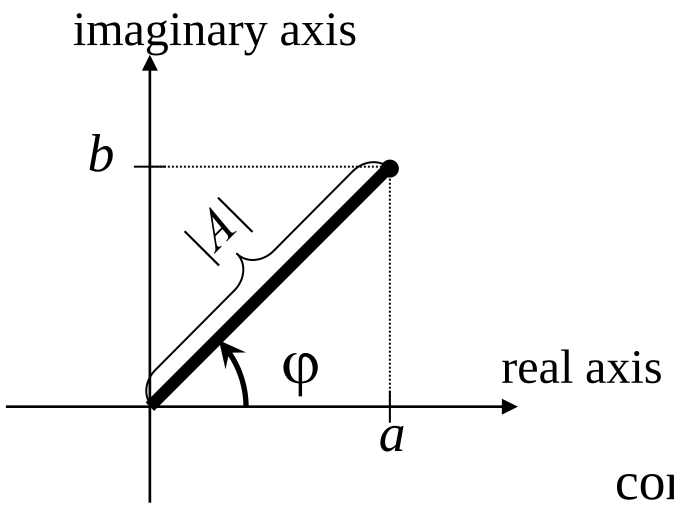
$$v = V_m \sin(\omega t + \varphi)$$



Sinusoidal Steady State Analysis

Phasors

Complex Numbers



$A = a + jb$ — Rectangular Coordinates

$A = |A|(\cos\varphi + j\sin\varphi)$

$A = |A|e^{j\varphi}$ — Polar Coordinates

conversion：

$$A = a + jb \rightarrow A = |A|e^{j\varphi} \qquad \begin{cases} |A| = \sqrt{a^2 + b^2} \\ \varphi = arctg\,\dfrac{\mathrm{b}}{\mathrm{a}} \end{cases}$$

$$|A|e^{j\varphi} \rightarrow a + jb \qquad \begin{cases} a = |A|\cos\varphi \\ b = |A|\sin\varphi \end{cases}$$

$$e^{\pm j90^\circ} = \cos 90^\circ \pm j\sin 90^\circ = 0 \pm j = \pm j$$

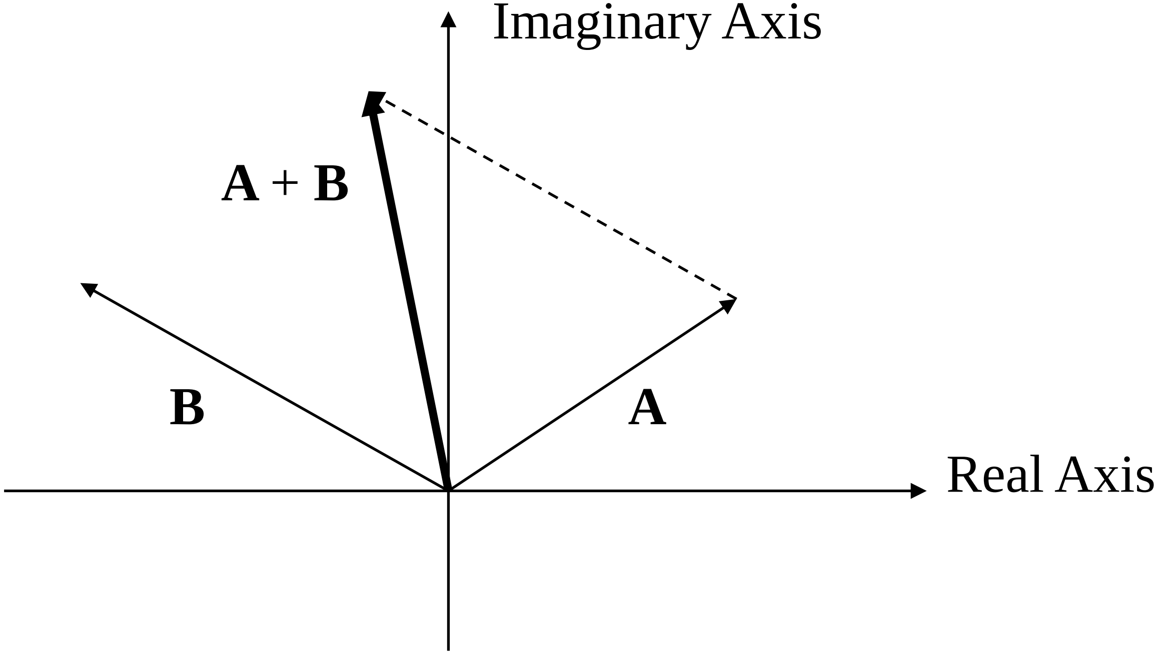
Sinusoidal Steady State Analysis

Phasors

Complex Numbers

Arithmetic With Complex Numbers

Addition: A $= a + jb$, $\mathbf{B} = c + jd$, $\mathbf{A} + \mathbf{B} = (a + c) + j(b + d)$



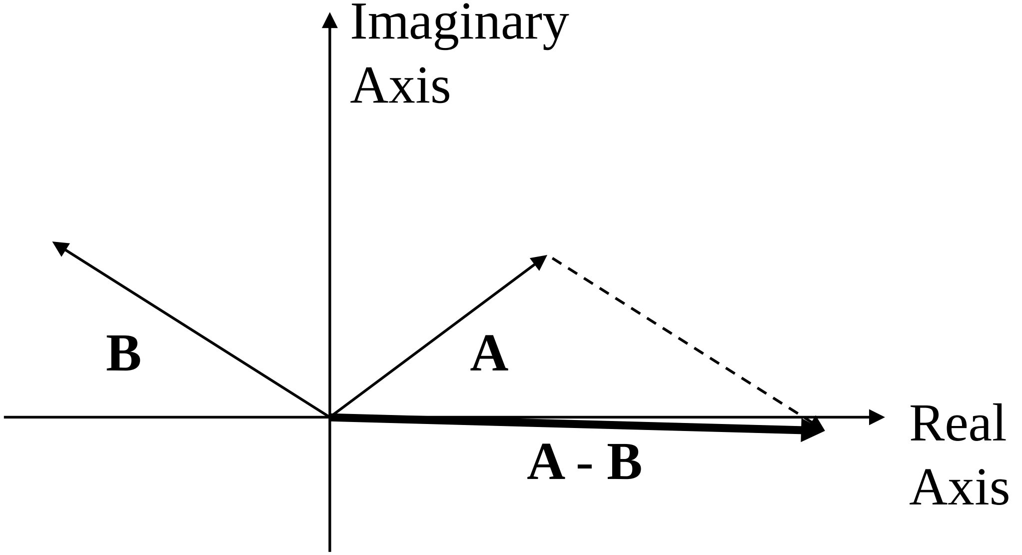
Sinusoidal Steady State Analysis

Phasors

Complex Numbers

Arithmetic With Complex Numbers

Subtraction : $A = a + jb$, $\mathbf{B} = c + jd$, $\mathbf{A} - \mathbf{B} = (a - c) + j(b - d)$



Sinusoidal Steady State Analysis

Phasors

Complex Numbers

Arithmetic With Complex Numbers

Multiplication : $\mathbf{A} = A_m \angle \varphi_A$, $\mathbf{B} = B_m \angle \varphi_B$

$$\mathbf{A} \times \mathbf{B} = (A_m \times B_m) \angle (\varphi_A + \varphi_B)$$

Division: $\mathbf{A} = A_m \angle \varphi_A$, $\mathbf{B} = B_m \angle \varphi_B$

$$\mathbf{A} / \mathbf{B} = (A_m / B_m) \angle (\varphi_A - \varphi_B)$$

Sinusoidal Steady State Analysis

Phasors

Phasors

A phasor is a complex number that represents the magnitude and phase of a sinusoid:

$$i_m \cos(\omega t + \varphi) \Longleftrightarrow \dot{I} = I_m \angle \varphi$$

Phasor Diagrams

- A phasor diagram is just a graph of several phasors on the complex plane (using real and imaginary axes).
- A phasor diagram helps to visualize the relationships between currents and voltages.

Sinusoidal Steady State Analysis

Phasors

Complex Exponentials

$$A = |A| e^{j\varphi}$$

$$Ae^{j\omega t} = |A| e^{j(\omega t + \varphi)} = |A| \cos(\omega t + \varphi) + j |A| \sin(\omega t + \varphi)$$

$$\mathrm{Re}\{Ae^{j\omega t}\} = |A| \cos(\omega t + \varphi)$$

- A real-valued sinusoid is the real part of a complex exponential.
- Complex exponentials make solving for AC steady state an algebraic problem.

Sinusoidal Steady State Analysis

Phasor Relationships for R, L and C

Key Words:

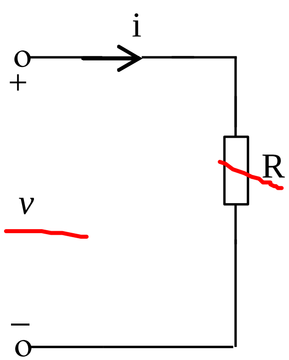
I-V Relationship for R, L and C,

Power conversion

Sinusoidal Steady State Analysis

Phasor Relationships for R, L and C

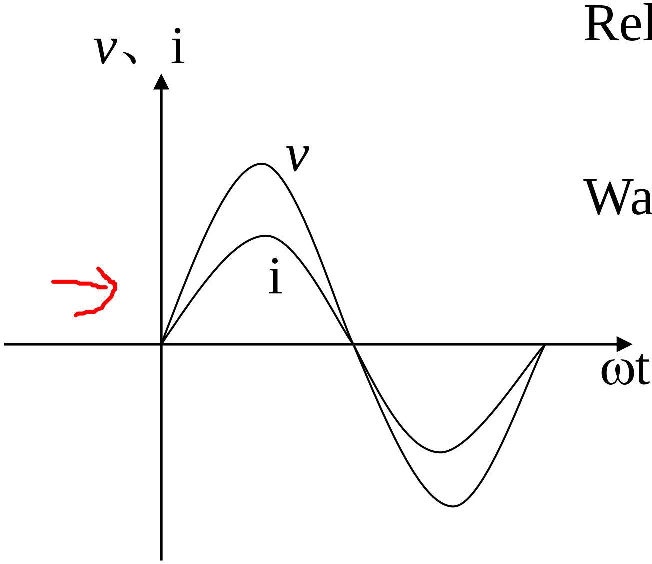
Resistor ● *v~i* relationship for a resistor



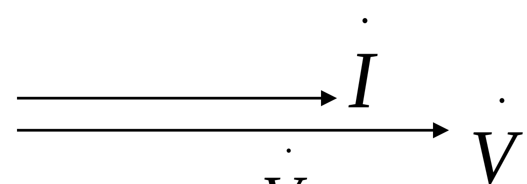
Suppose $v = V_m \sin \omega t$

$$i = \frac{v}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t$$

Relationship between RMS: $I = \dfrac{V}{R}$



Wave and Phasor diagrams：



$$\dot{I} = \frac{\dot{V}}{R}$$

Sinusoidal Steady State Analysis

Phasor Relationships for R, L and C

Resistor ● **Time domain → frequency domain**

$$v(t) = V_m \cos(wt + \theta) \quad V(t) = Ri(t)$$
$$i(t) = I_m \cos(wt + \phi)$$

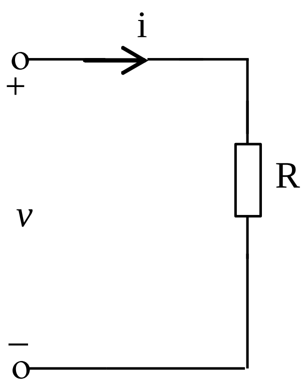
$$V_m e^{j(wt+\theta)} = RI_m e^{j(wt+\phi)}$$
$$V_m e^{j\theta} = RI_m e^{j\phi}$$
$$V_m \angle \theta = RI_m \angle \phi$$
$$\dot{V} = R\dot{I}$$

With a resistor $\theta = \varphi$, ***v*(*t*) and *i*(*t*) are in phase .**

Sinusoidal Steady State Analysis

Phasor Relationships for R, L and C

Resistor ● Power

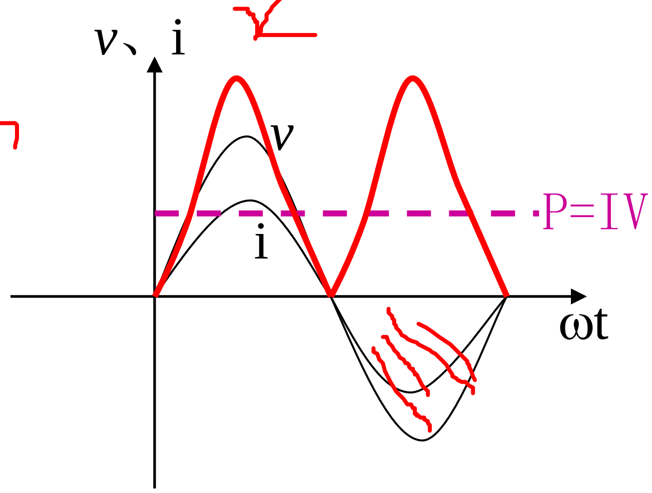


• Transient Power

$$p = vi = V_m \sin\omega t \cdot I_m \sin\omega t = I_m V_m \sin^2 \omega t$$

$$= \frac{I_m V_m}{2}\left(1-\cos 2\omega t\right) = IV - IV\cos 2\omega t$$

Note: I and *V* are RMS values. **$p>0$**



• Average Power

$$P = \frac{1}{T}\int_0^T p\,dt = \frac{1}{T}\int_0^T VI\left(1-\cos 2\omega t\right)dt = VI$$

$$P = IV = I^2 R = \frac{V^2}{R}$$

Sinusoidal Steady State Analysis

Phasor Relationships for R, L and C

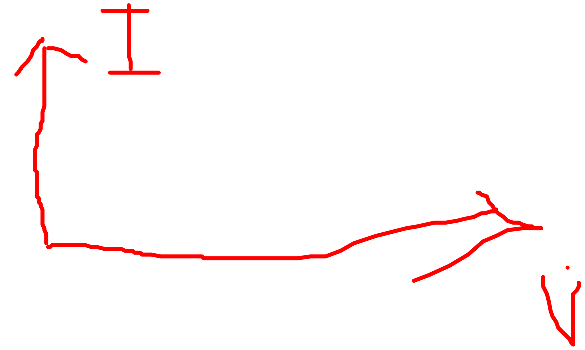
Resistor

P4.4， $v = 311\sin 314t$， R=10Ω， Find i and P。

$$V = \frac{V_m}{\sqrt{2}} = \frac{311}{\sqrt{2}} = 220(V)$$

$$I = \frac{V}{R} = \frac{220}{10} = 22(A)$$

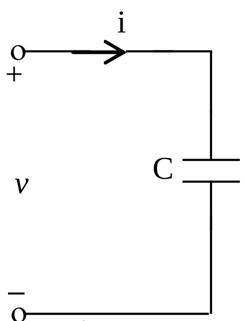
$$i = 22\sqrt{2}\sin 314t \qquad P = IV = 220 \times 22 = 4840(W)$$



Sinusoidal Steady State Analysis

Phasor Relationships for R, L and C

Capacitor ● ***v ~ i* relationship**



$$i = \frac{dq}{dt} = C\frac{dv}{dt}$$

$I_m = \omega C V_m$

Suppose: $v = V_m \sin \omega t$

$$i = \omega C V_m \cos \omega t = \omega C V_m \sin(\omega t + 90^\circ) = I_m \sin(\omega t + 90^\circ)$$

Relationship between RMS: $I = \omega C V = \dfrac{V}{1/\omega C} = \dfrac{V}{X_C}$

$$\longrightarrow X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \quad (\Omega)$$

$$X_C \propto \frac{1}{f}$$

For DC，$f = 0$，$\rightarrow X_C \rightarrow \infty$

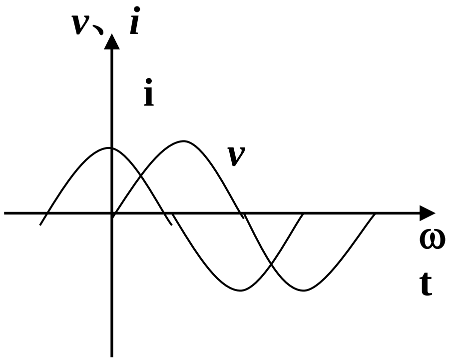
***i*(*t*) leads *v*(*t*) by 90º, or *v*(*t*) lags *i*(*t*) by 90º**

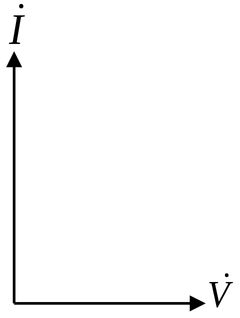
4.3 Phasor Relationships for R, L and C

Capacitor ● ***v*** **~** ***i*** **relationship**

Wave and Phasor diagrams：

$$\dot{V} = -j\dot{I}X_C$$





4.3 Phasor Relationships for R, L and C

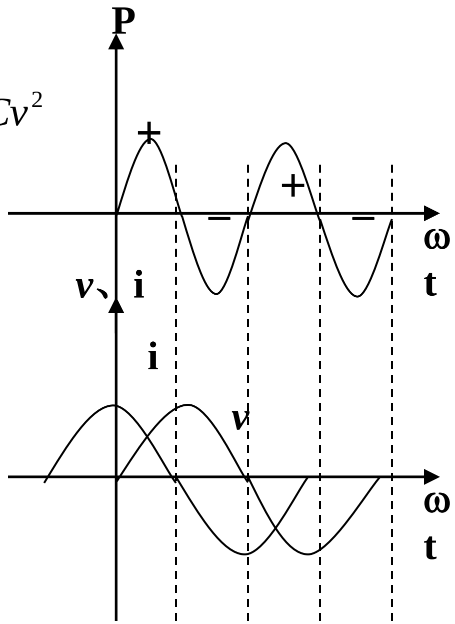
Capacitor **●Power**

$$p = vi = V_m \sin \omega t \cdot I_m \sin(\omega t + 90^\circ) = \frac{V_m I_m}{2} \sin 2\omega t = VI \sin 2\omega t$$

Energy stored:

$$W = \int_0^t vidt = \int_0^v v \cdot C \cdot \frac{dv}{dt} \cdot dt = \int_0^v Cvdv = \frac{1}{2} Cv^2$$

$$W_{\max} = \frac{1}{2} CV_m^2 = CV^2$$



Average Power:

P=0

Reactive Power $Q = IV = I^2 X_C = \frac{V^2}{X_C}$ （**Var**）

Sinusoidal Steady State Analysis

Phasor Relationships for R, L and C

Capacitor

P4.7，Suppose C=20μF，AC source *v*=100sinωt，Find X_C and I for *f* = 50Hz, 50kHz。

$$f = 50\text{Hz} \rightarrow X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C} = 159\Omega$$

$$I = \frac{V}{X_c} = \frac{V_m}{\sqrt{2}X_c} = \mathbf{0.4443\ A}$$

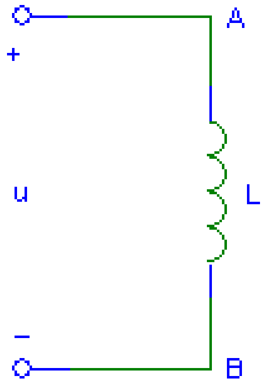
$$f = 50\text{KHz} \rightarrow X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C} = 0.159(\Omega)$$

$$I = \frac{V}{X_c} = \frac{V_m}{\sqrt{2}X_c} = \mathbf{444.29\ A}$$

4.3 Phasor Relationships for R, L and C

Inductor

● ***v~i*** **relationship**



$$v = v_{AB} = L\frac{di}{dt}$$

Suppose $i = I_m \sin \omega t$

$$v = L\frac{di}{dt} = L\frac{d\left(I_m \sin \omega t\right)}{dt} = I_m \omega L \cos \omega t$$

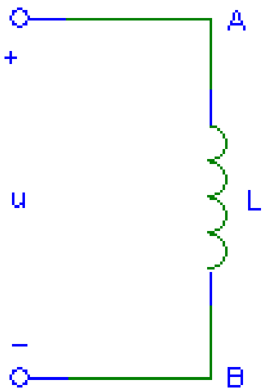
$$= I_m \omega L \sin\left(\omega t + 90^\circ\right)$$

$$= V_m \sin\left(\omega t + 90^\circ\right)$$

Sinusoidal Steady State Analysis

Phasor Relationships for R, L and C

Inductor ● ***v~i* relationship**



$$v = L\frac{di}{dt} = I_m \omega L \sin\left(\omega t + 90^\circ\right) = V_m \sin\left(\omega t + 90^\circ\right)$$

$$V_m = I_m \omega L$$

Relationship between RMS: $V = I\omega L$

$$I = \frac{V}{\omega L} \longrightarrow X_L = \omega L = 2\pi f L \quad (\Omega)$$

$$X_L \propto f$$

For DC, $f = 0$, $\rightarrow X_L = 0$.

$v(t)$ leads $i(t)$ by 90º, or $i(t)$ lags $v(t)$ by 90º

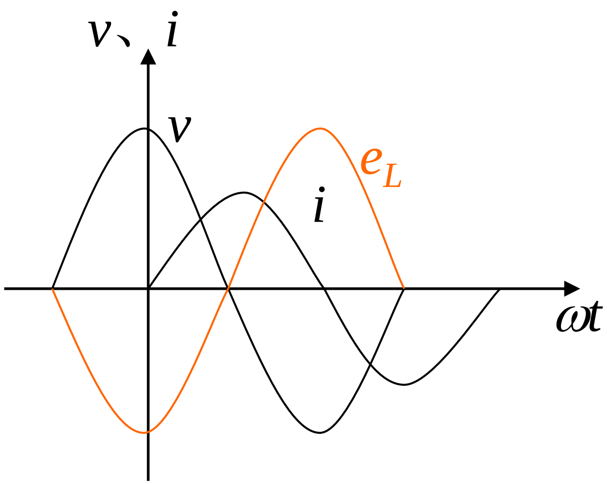
Ch4 Sinusoidal Steady State Analysis

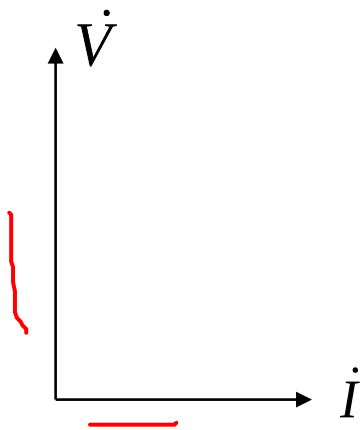
4.3 Phasor Relationships for R, L and C

Inductor ● ***v* ~ *i* relationship**

Wave and Phasor diagrams：

$$\dot{V} = j\dot{I}X_L$$





Sinusoidal Steady State Analysis

Phasor Relationships for R, L and C

Inductor ● **Power**

$$p = vi = V_m \sin(\omega t + 90^\circ) I_m \sin\omega t = V_m I_m \cos\omega t \cdot \sin\omega t$$

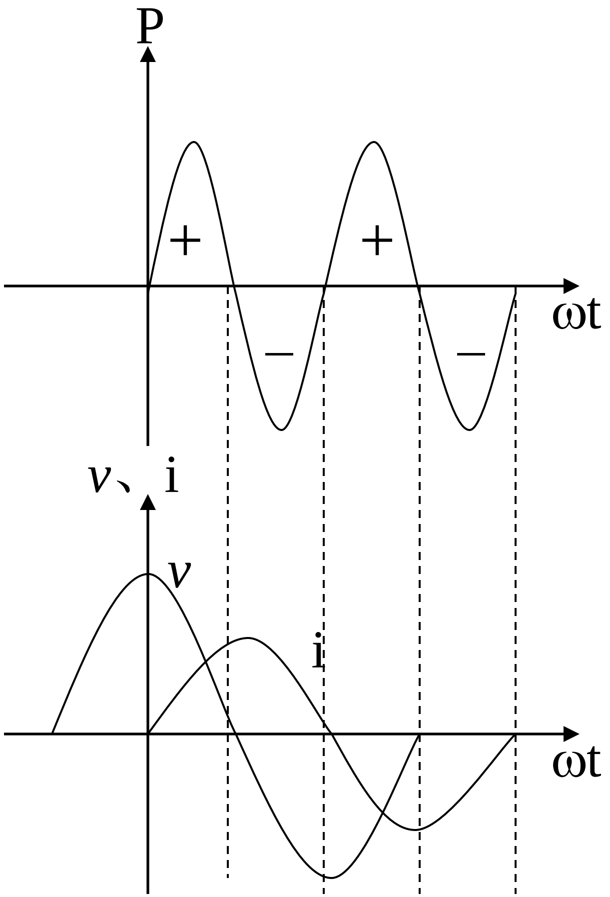
$$= \frac{V_m I_m}{2} \sin 2\omega t = VI \sin 2\omega t$$

Energy stored: $W = \int_0^t vidt = \int_0^i Lidi = \frac{1}{2} Li^2$

$$W_{\max} = \frac{1}{2} LI_m^2 = LI^2$$

Average Power $P = \frac{1}{T}\int_0^T pdt = \frac{1}{T}\int_0^T VI \sin 2\omega t dt = 0$

Reactive Power $Q = IV = I^2 X_L = \frac{V^2}{X_L}$ （Var）



Sinusoidal Steady State Analysis

Phasor Relationships for R, L and C

Inductor

P4.5，L = 10mH，v = 100sinωt，Find i_L when f = 50Hz and 50kHz.

$$X_L = 2\pi fL = 2\pi \times 50 \times 10 \times 10^{-3} = 3.14(\Omega)$$

$$I_{50} = \frac{V}{X_L} = \frac{100/\sqrt{2}}{3.14} = 22.5(A)$$

$$i_L(t) = 22.5\sqrt{2}\sin(\omega t - 90^\circ)A$$

$$X_L = 2\pi fL = 2\pi \times 50 \times 10^3 \times 10 \times 10^{-3} = 3140(\Omega)$$

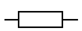
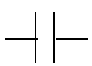
$$I_{50k} = \frac{V}{X_L} = \frac{100/\sqrt{2}}{3.14} = 22.5(mA)$$

$$i_L(t) = 22.5\sqrt{2}\sin(\omega t - 90^\circ)mA$$

Sinusoidal Steady State Analysis

Phasor Relationships for R, L and C

Review (*v-I relationship*)

	Time domain	Frequency domain
 **R**	$v = R \cdot i$	$\dot{V} = R \cdot \dot{I}$ **, *v* and *i* are in phase.**
 **L**	$v_L = L \dfrac{di}{dt}$	$\dot{V} = j\omega L \cdot \dot{I}$ **,** $X_L = \omega L$ **, *v* leads *i* by 90°.**
 **C**	$v_C = C \dfrac{dv}{dt}$	$\dot{V} = \dfrac{1}{j\omega C} \cdot \dot{I}$ **,** $X_C = \dfrac{1}{\omega C}$ **, *v* lags *i* by 90°.**

Sinusoidal Steady State Analysis

Phasor Relationships for R, L and C

Summary

- **R:** $X_R = R$ $\qquad \Delta\varphi = 0$

  L: $X_L = \omega L = 2\pi f L \propto f$ $\qquad \Delta\varphi = \varphi_v - \varphi_i = \frac{\pi}{2}$

  C: $X_C = \frac{1}{\omega c} = \frac{1}{2\pi f c} \propto \frac{1}{f}$ $\qquad \Delta\varphi = \varphi_v - \varphi_i = -\frac{\pi}{2}$

- $V = IX$

- **Frequency characteristics of an Ideal Inductor and Capacitor:**

  A capacitor is an *open circuit* to DC currents;

  A Inducter is a *short circuit* to DC currents.

Sinusoidal Steady State Analysis

Impedance

Complex voltage， Complex current， Complex Impedance

- AC steady-state analysis using phasors allows us to express the relationship between current and voltage using a formula that looks likes Ohm’s law:

$$\dot{V} = \dot{I}Z$$

***Z* is called impedance. measured in ohms (Ω)**

$$\dot{V} = V_m e^{j\varphi_v} = V_m \angle \varphi_v$$

$$\dot{I} = I_m e^{j\varphi_i} = I_m \angle \varphi_i$$

$$Z = \frac{\dot{V}}{\dot{I}} = \frac{V_m}{I_m} e^{j(\varphi_v - \varphi_i)} = |Z| e^{j\varphi} = |Z| \angle \varphi$$

Sinusoidal Steady State Analysis

Impedance

Complex Impedance

$$Z = \frac{\dot{V}}{\dot{I}} = \frac{V_m}{I_m} e^{j(\varphi_v - \varphi_i)} = |Z| e^{j\varphi} = |Z| \angle \varphi$$

- Complex impedance describes the relationship between the voltage across an element (expressed as a phasor) and the current through the element (expressed as a phasor)
- Impedance is a complex number and is not a phasor (why?).
- Impedance depends on frequency

Sinusoidal Steady State Analysis

Impedance

Complex Impedance

Resistor——The impedance is *R*

$$Z_R = R \quad \Delta\varphi = 0; \text{ or } Z_R = R \angle 0$$

Capacitor——The impedance is 1/*j*ω*C*

$$Z_c = \frac{1}{\omega C} e^{-j\frac{\pi}{2}} = \frac{-j}{\omega C} = -jX_c \quad \text{or} \quad Z_C = \frac{1}{\omega C} \angle -90^\circ$$

$$(\Delta\varphi = \varphi_v - \varphi_i = -\frac{\pi}{2})$$

Inductor——The impedance is *j*ω*L*

$$Z_L = \omega L e^{j\frac{\pi}{2}} = j\omega L = jX_L \quad \text{or} \quad Z_L = \omega L \angle 90^\circ$$

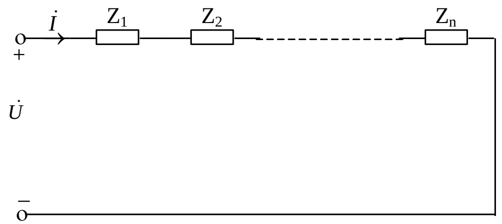
$$(\Delta\varphi = \varphi_v - \varphi_i = \frac{\pi}{2})$$

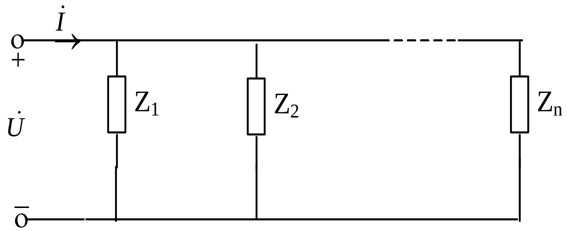
Sinusoidal Steady State Analysis

Impedance

Complex Impedance

Impedance in series/parallel can be combined as resistors.





$$Z = Z_1 + Z_2 + \ldots + Z_n = \sum_{k=1}^{n} Z_k$$

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \ldots + \frac{1}{Z_n} = \sum_{k=1}^{n} \frac{1}{Z_k}$$

Voltage divider:

$$\dot{V}_i = \dot{V} \frac{Z_i}{\sum_{k=1}^{n} Z_k}$$

Current divider:

$$\dot{I}_1 = \dot{I} \frac{Z_2}{Z_1 + Z_2} \qquad \dot{I}_2 = \dot{I} \frac{Z_1}{Z_1 + Z_2}$$

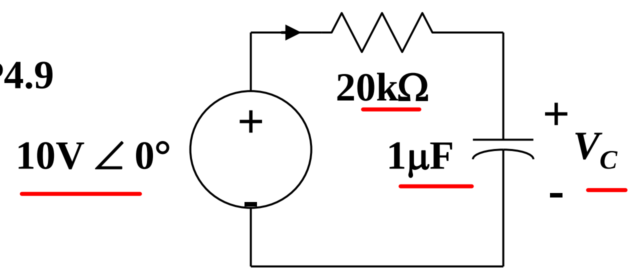
Practice Problem

Impedance

Complex Impedance

Phasors and complex impedance allow us to use Ohm's law with complex numbers to compute current from voltage and voltage from current

P4.9



$\omega = 377$

Find V_C

Summary

- Steady-state analysis
- R,L,C