# Unit - I
# 1.9 Capacitor Charging and Discharging

Dr.Santhosh.T.K.

# Syllabus

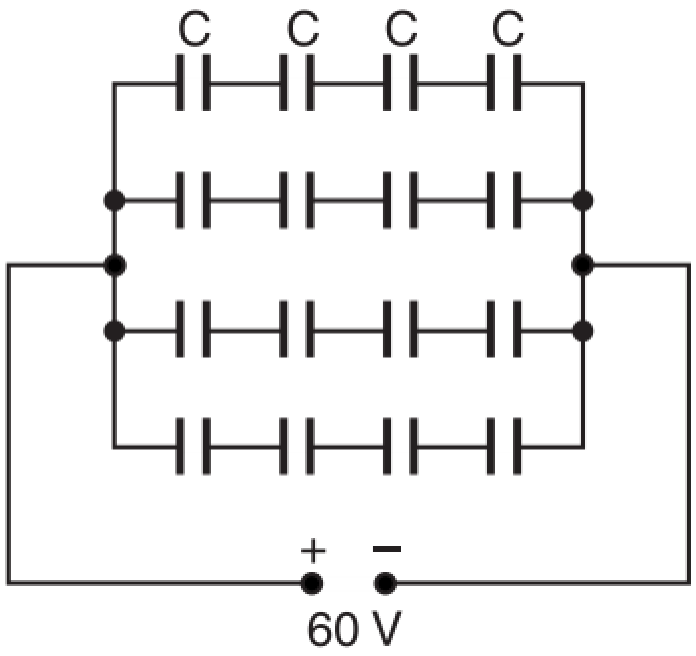
**UNIT – I** **10 Periods**

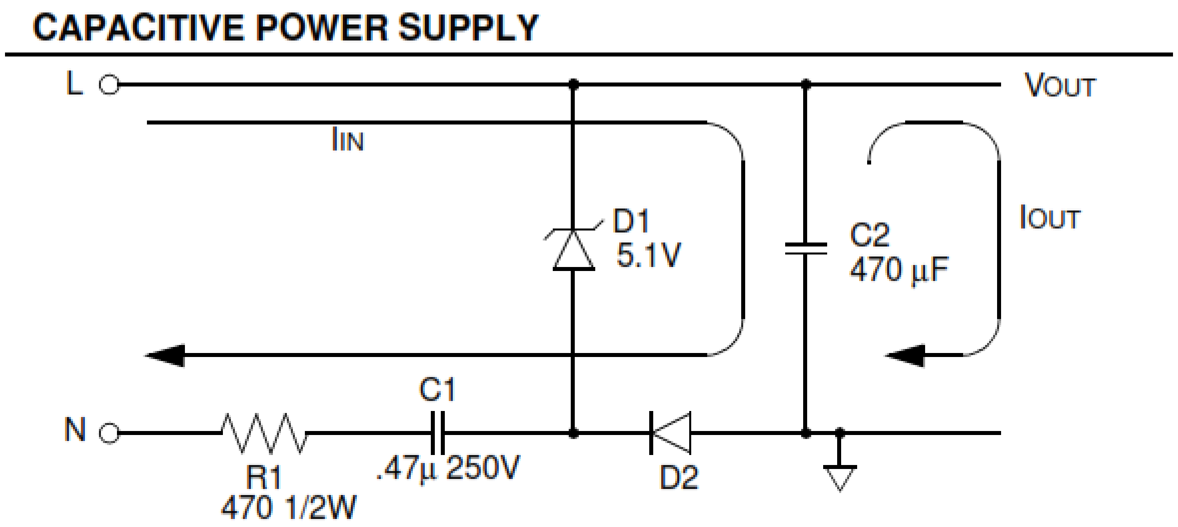
**Introduction and Basic Concepts:** Concept of Potential difference, voltage, current - Fundamental linear passive and active elements to their functional current-voltage relation - Terminology and symbols in order to describe electric networks - Concept of work, power, energy and conversion of energy- Principle of batteries and application.

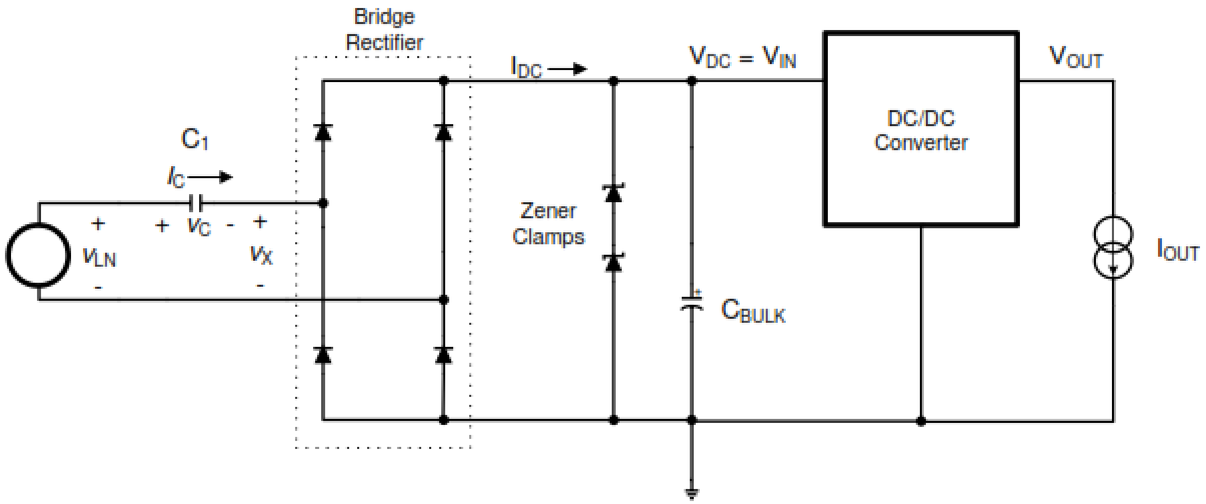
**Principles of Electrostatics:** Electrostatic field - electric field intensity - electric field strength - absolute permittivity - relative permittivity - capacitor composite – dielectric capacitors - capacitors in series & parallel - energy stored in capacitors - charging and discharging of capacitors.

# Problem

- *Given some capacitors of 0·1 μF capable of withstanding 15V.Calculate the number of capacitors needed if it is desired to obtain a capacitance of 0·1 μF for use in a circuit involving 60 V.*

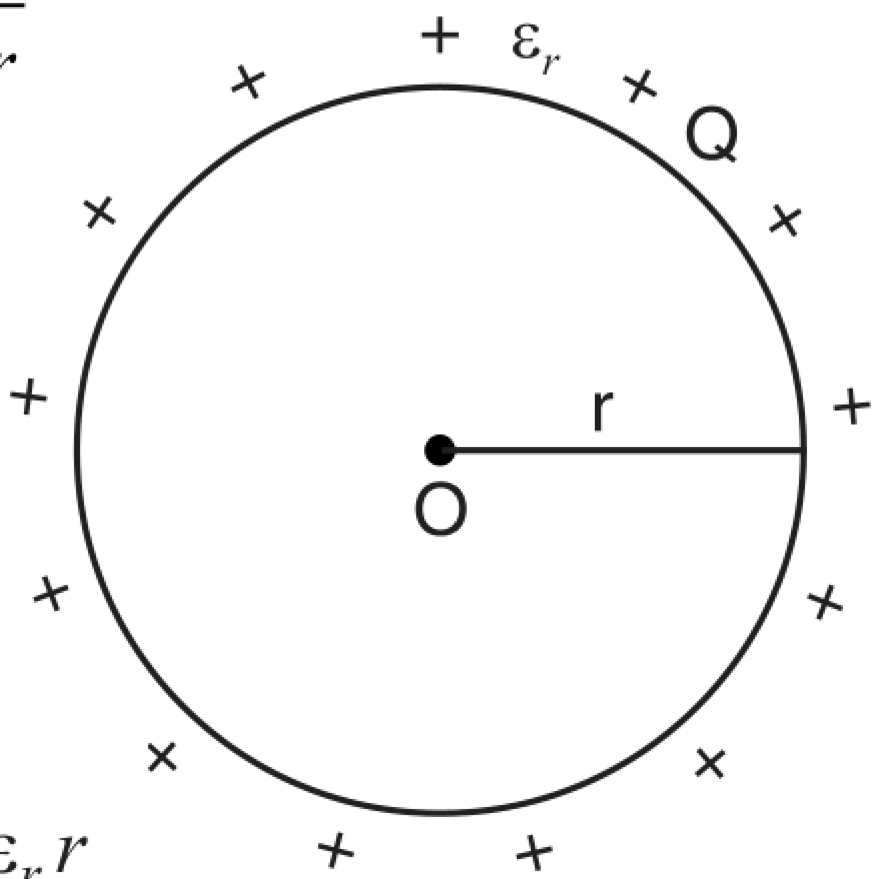



CAPACITIVE POWER SUPPLY
L
N
VOUT
IIN
IOUT
D1
5.1V
C2
470 µF
C1
.47µ 250V
R1
470 1/2W
D2


Bridge
Rectifier
IDC
VDC = VIN
VOUT
DC/DC
Converter
C1
IC
+
vLN
-
+ vC -
+
vX
-
Zener
Clamps
CBULK
IOUT

# Capacitance of an Isolated Conducting Sphere

Potential at the surface of the sphere, $V = \dfrac{Q}{4\pi\varepsilon_0\varepsilon_r r}$




Capacitance of isolated sphere, $C = \dfrac{Q}{V} = 4\pi\varepsilon_0\varepsilon_r r$

# Example 1

- Twenty seven spherical drops, each of radius 3 mm and carrying $10^{-12}\ C$ of charge are combined to form a single drop. Find the capacitance and potential of the bigger drop.

- Solution

  – Let r and R be the radii of smaller and bigger drops respectively.

# Solution 1

$$\text{Volume of bigger drop} = 27 \times \text{Volume of smaller drop}$$

$$\frac{4}{3}\pi R^3 = 27 \times \frac{4}{3}\pi r^3$$

$$R = 3r = 3 \times 3 = 9 \text{ mm} = 9 \times 10^{-3} \text{ m}$$

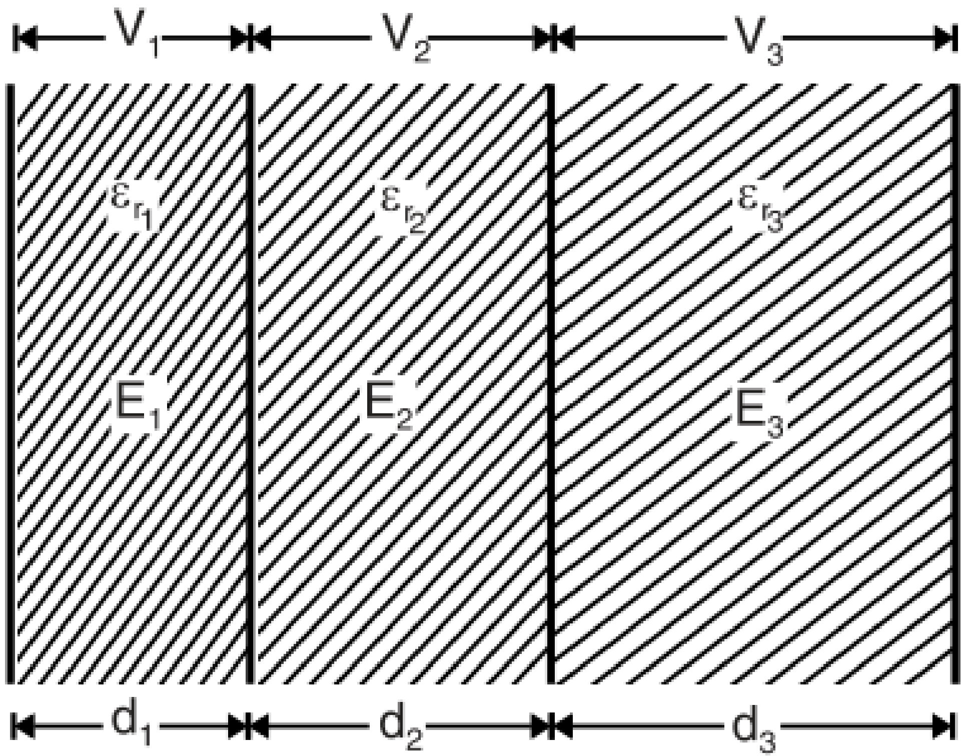
$$C = 4\pi\,\varepsilon_0\, R = \frac{1}{9 \times 10^9} \times 9 \times 10^{-3} = 10^{-12} \text{ F} = \mathbf{1\ pF}$$

$$\text{Potential of bigger drop, } V = \frac{Q}{C} = \frac{27 \times 10^{-12}}{10^{-12}} = \mathbf{27\ V}$$

# Parallel-Plate Capacitor with Composite Medium

$$V = V_1 + V_2 + V_3$$
$$= E_1 d_1 + E_2 d_2 + E_3 d_3$$

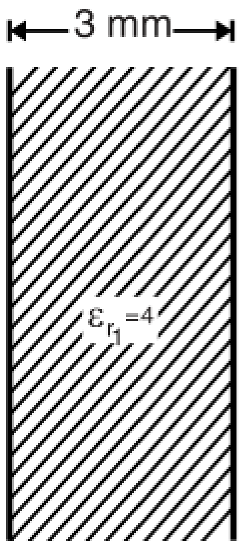
$$= \frac{D}{\varepsilon_0 \varepsilon_{r1}} d_1 + \frac{D}{\varepsilon_0 \varepsilon_{r2}} d_2 + \frac{D}{\varepsilon_0 \varepsilon_{r3}} d_3$$
$$= \frac{D}{\varepsilon_0}\left[\frac{d_1}{\varepsilon_{r1}} + \frac{d_2}{\varepsilon_{r2}} + \frac{d_3}{\varepsilon_{r3}}\right]$$
$$= \frac{Q}{\varepsilon_0 A}\left[\frac{d_1}{\varepsilon_{r1}} + \frac{d_2}{\varepsilon_{r2}} + \frac{d_3}{\varepsilon_{r3}}\right] \quad \left(\because D = \frac{Q}{A}\right)$$

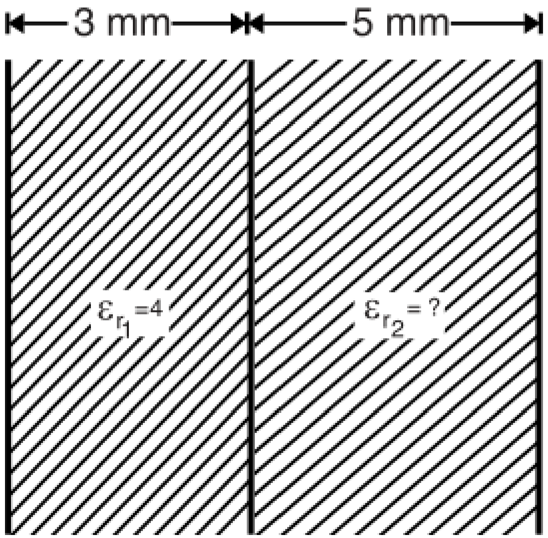



$$C = \frac{\varepsilon_0 A}{\left(\frac{d_1}{\varepsilon_{r1}} + \frac{d_2}{\varepsilon_{r2}} + \frac{d_3}{\varepsilon_{r3}}\right)}$$

# Example 2

- A capacitor is composed of two plates separated by 3mm of dielectric of permittivity 4. An additional piece of insulation 5mm thick is now inserted between the plates. If the capacitor now has capacitance one-third of its original capacitance, find the relative permittivity of the additional dielectric.



# Solution

For the first case, $$C = \frac{\varepsilon_0 \varepsilon_{r1} A}{d} = \frac{\varepsilon_0 \times 4 \times A}{3 \times 10^{-3}}$$

For the second case, $$\frac{C}{3} = \frac{\varepsilon_0\ A}{\dfrac{d_1}{\varepsilon_{r1}} + \dfrac{d_2}{\varepsilon_{r2}}}$$

$$= \frac{\varepsilon_0\ A}{\dfrac{3\times10^{-3}}{4} + \dfrac{5\times10^{-3}}{\varepsilon_{r2}}}$$

$$3 = \frac{4}{3}\left(\frac{3}{4} + \frac{5}{\varepsilon_{r2}}\right)$$

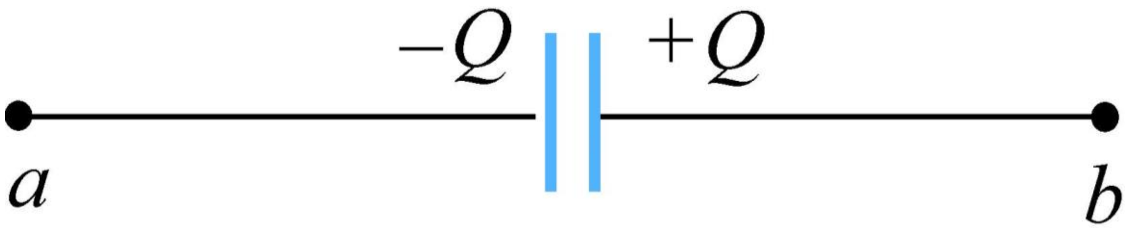
$$9 = 3 + 20/\varepsilon_{r2} \quad \therefore\ \varepsilon_{r2} = 20/6 = \mathbf{3 \cdot 33}$$

# Exercise

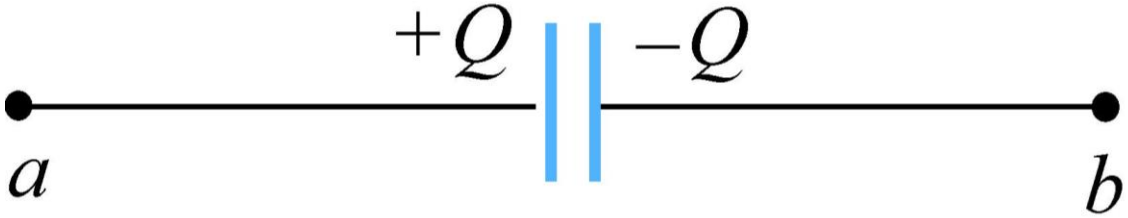
- A parallel plate capacitor has three similar parallel plates. Find the ratio of capacitance when the inner plate is mid-way between the outers to the capacitance when inner plate is three times as near one plate as the other.

# Sign Conventions - Capacitor

Moving across a capacitor from the negatively to positively charged plate **increases** the electric potential




$$\Delta V = +Q/C$$

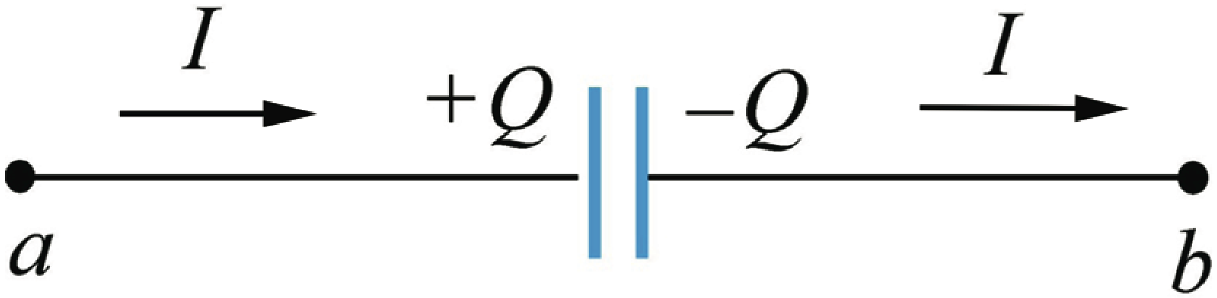



$$\Delta V = -\,Q/C$$

$$\boxed{DV = V_b - V_a}$$

# Power - Capacitor

Moving across a capacitor from the positive to negative plate **decreases** your potential.  If current flows in that direction the capacitor **absorbs** power (stores charge)




$$\Delta V = -\,Q/C$$

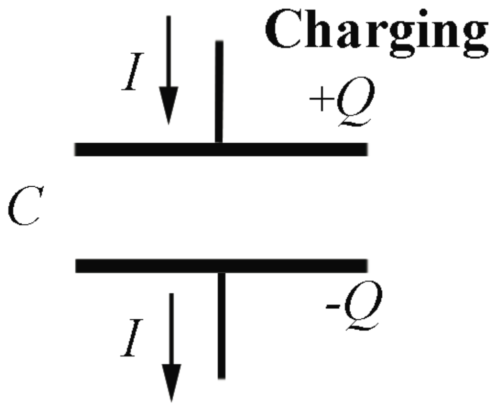
$$\boxed{P_{\text{absorbed}} = I\,\Delta V = \frac{dQ}{dt}\frac{Q}{C} = \frac{d}{dt}\frac{Q^2}{2C} = \frac{dU}{dt}}$$

# RC Circuits

# (Dis)Charging a Capacitor

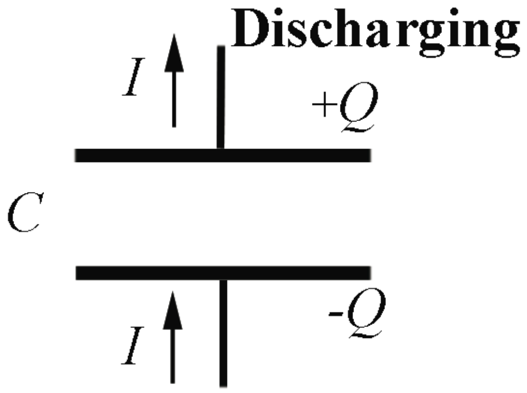
1. When the direction of current flow is toward the positive plate of a capacitor, then

$$I = +\frac{dQ}{dt}$$

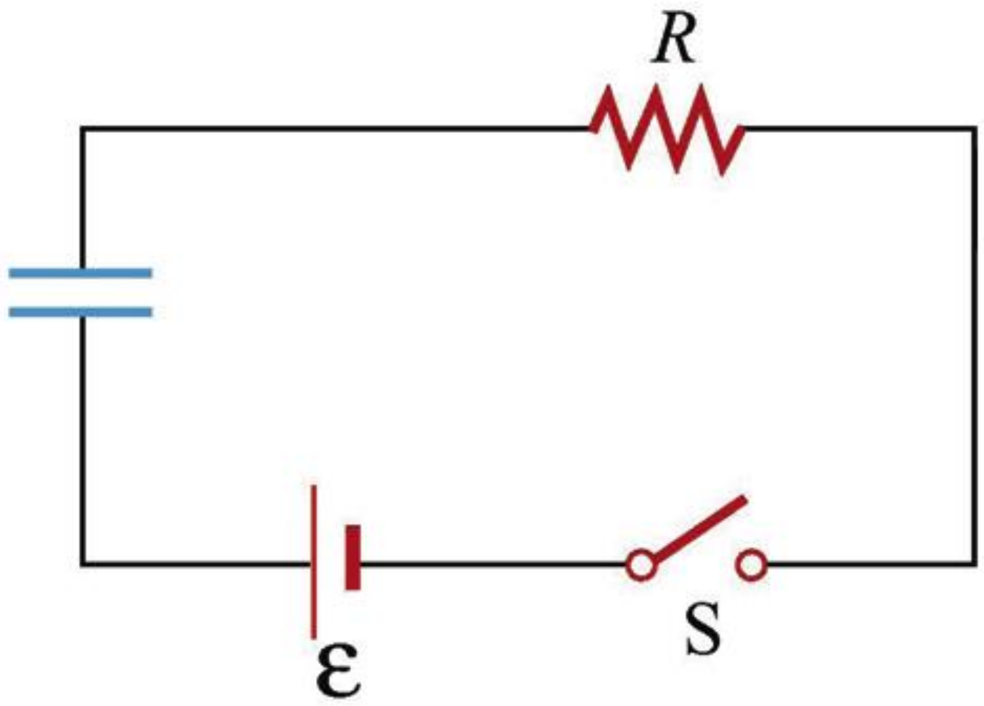



2. When the direction of current flow is away from the positive plate of a capacitor, then

$$I = -\frac{dQ}{dt}$$

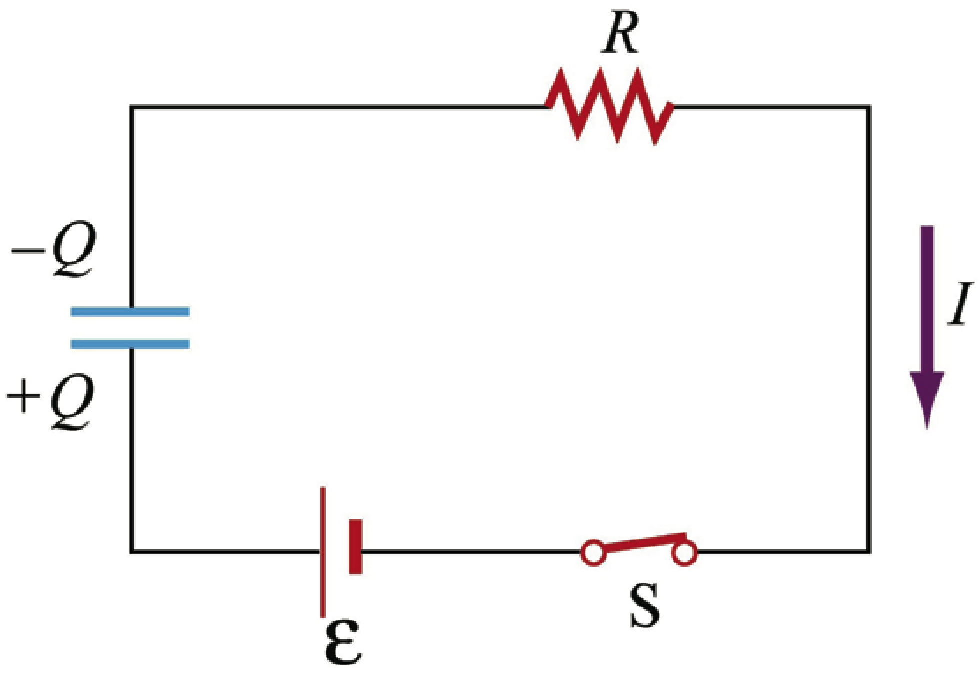


# Charging a Capacitor




What happens when we close switch S at $t = 0$?

# Charging a Capacitor




Circulate clockwise

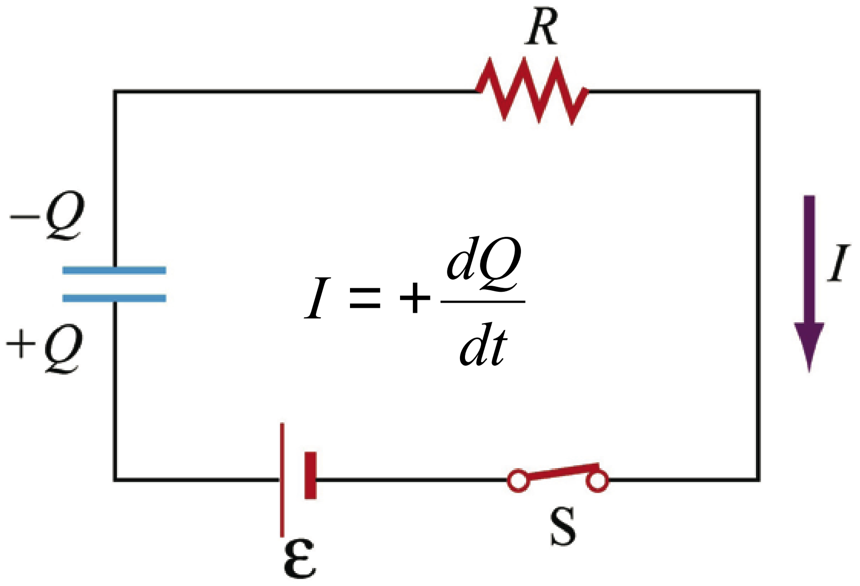
$$\sum_i \Delta V_i = \varepsilon - \frac{Q}{C} - IR = 0$$

$$I = +\frac{dQ}{dt}$$

First order linear inhomogeneous differential equation

$$\frac{dQ}{dt} = -\frac{1}{RC}(Q - C\varepsilon)$$

# Energy Balance: Circuit Equation




$$\varepsilon - \frac{Q}{C} - IR = 0$$

Multiplying by $I = +\dfrac{dQ}{dt}$

$$\varepsilon I = I^2 R + \frac{Q}{C}\frac{dQ}{dt} = I^2 R + \frac{d}{dt}\left(\frac{1}{2}\frac{Q^2}{C}\right)$$

(power delivered by battery) = (power dissipated through resistor)
+ (power absorbed by the capacitor)

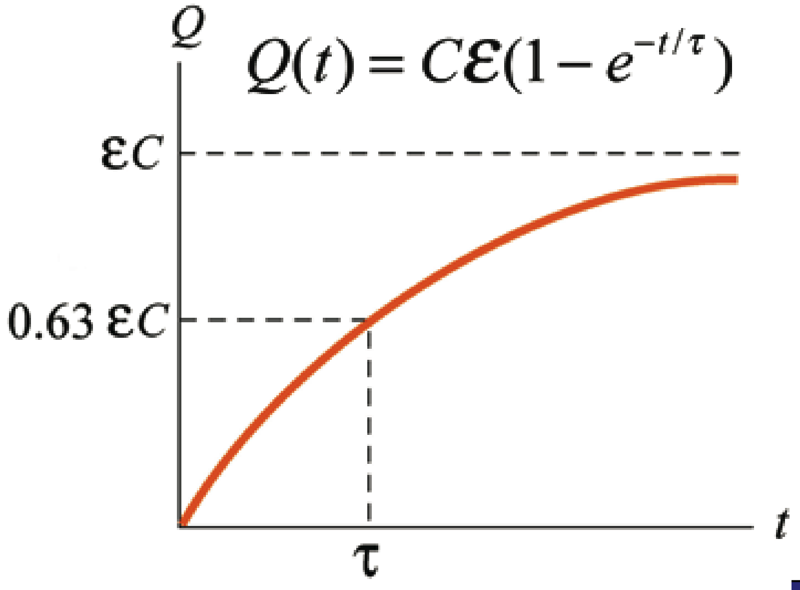
# RC Circuit Charging: Solution

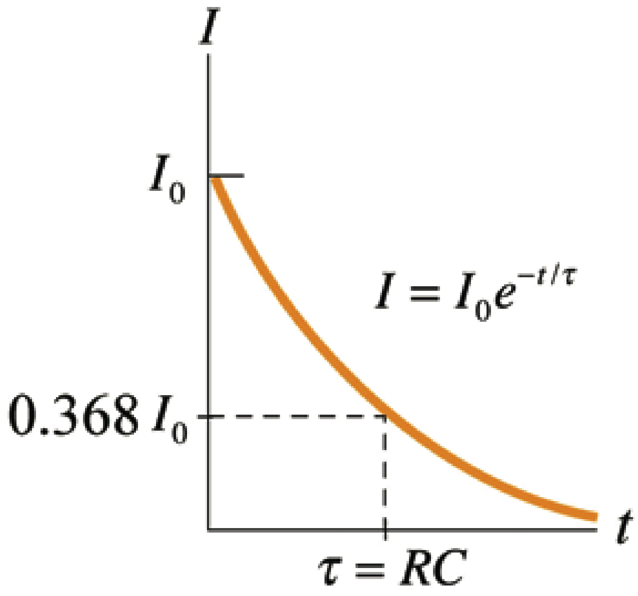
$$\frac{dQ}{dt} = -\frac{1}{RC}\left(Q - C\varepsilon\right)$$

Solution to this equation when switch is closed at $t = 0$:

$$Q(t) = C\varepsilon(1 - e^{-t/\tau}) \qquad I(t) = +\frac{dQ}{dt} \Rightarrow I(t) = I_0 e^{-t/\tau}$$

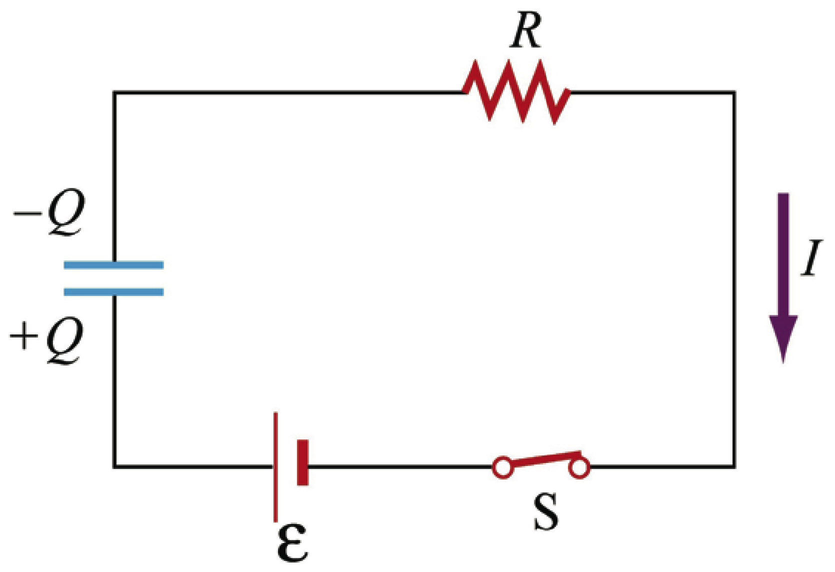
$\tau = RC$ : time constant (units: seconds)



# Concept Question: RC Circuit

An uncharged capacitor is connected to a battery, resistor and switch. The switch is initially open but at $t = 0$ it is closed. A very long time after the switch is closed, the current in the circuit is

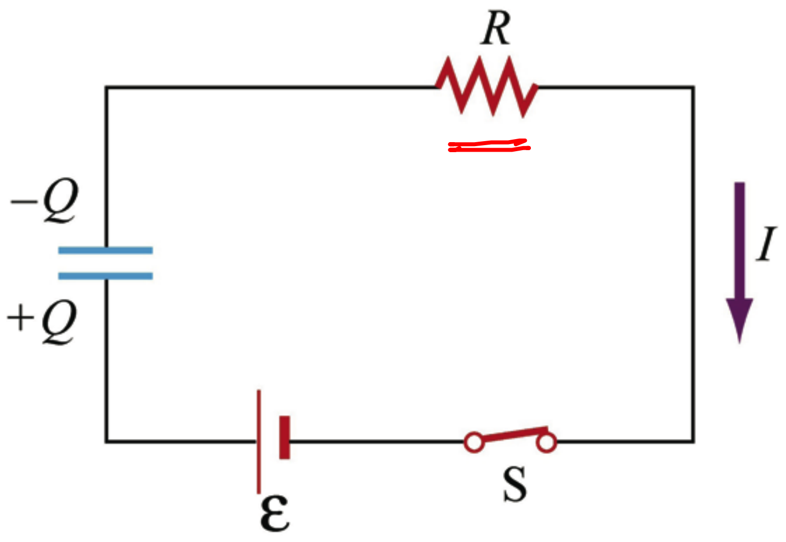



1. Nearly zero
2. At a maximum and decreasing
3. Nearly constant but non-zero

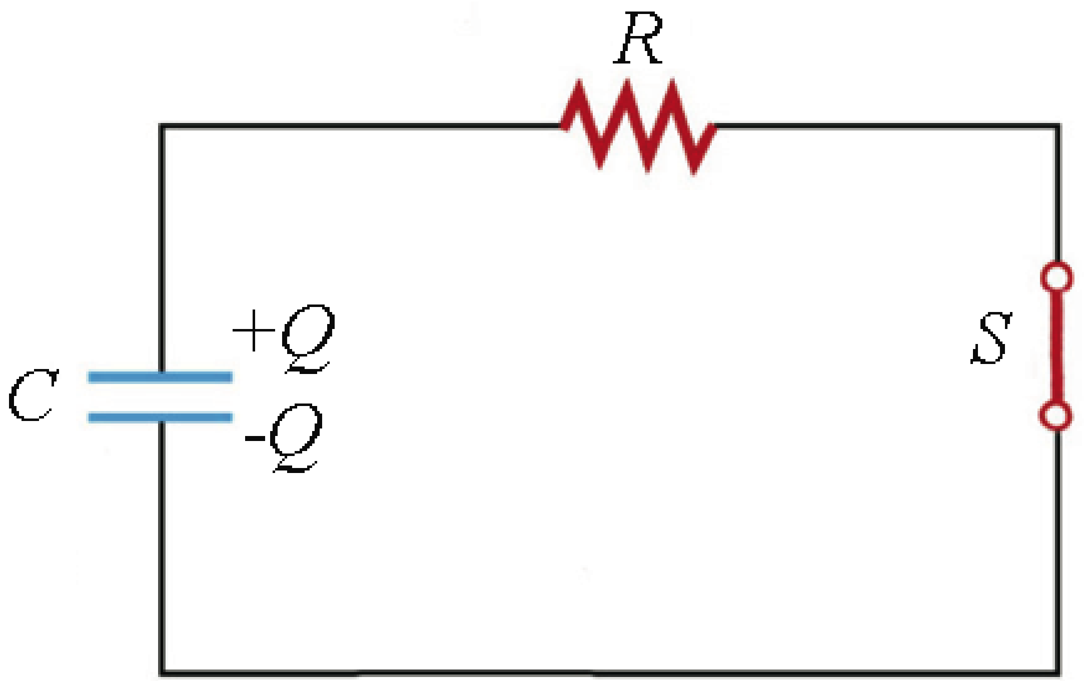
# Concept Q. Answer: RC Circuit

Answer: 1. After a long time the current is 0

Eventually the capacitor gets "completely charged" – the voltage increase provided by the battery is equal to the voltage drop across the capacitor. The voltage drop across the resistor at this point is 0 – no current is flowing.

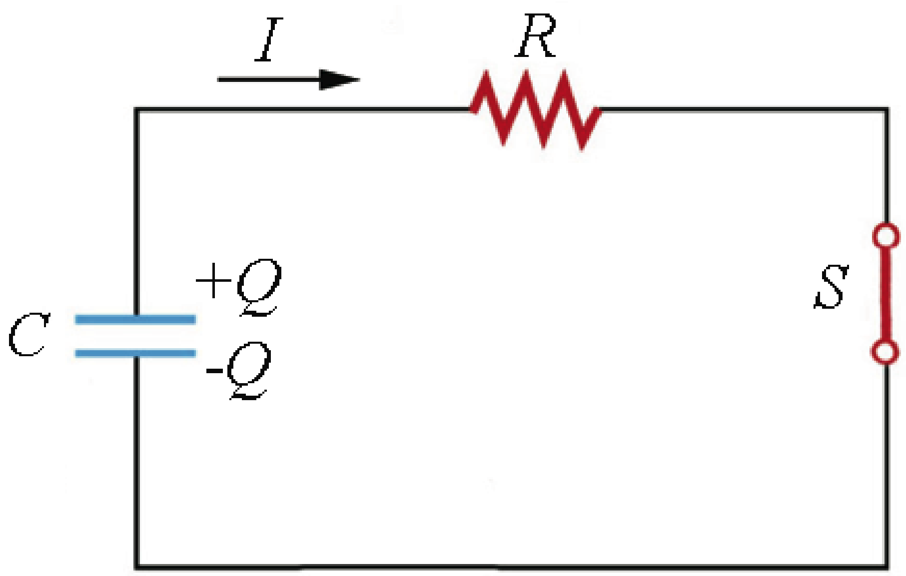


# Discharging A Capacitor




At $t$ = 0 charge on capacitor is $Q_0$. What happens when we close switch S at $t$ = 0?

# Discharging a Capacitor




Circulate clockwise

$$\sum_i \Delta V_i = \frac{Q}{C} - IR = 0$$

$$I = -\frac{dQ}{dt} \Rightarrow$$

First order linear differential equation

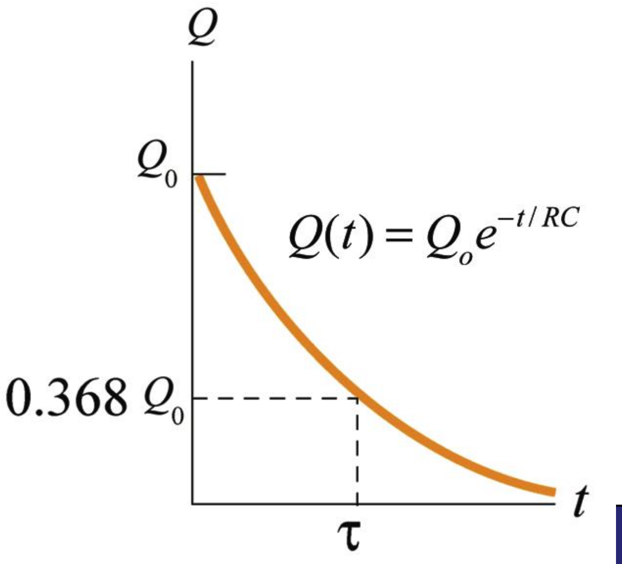
$$\frac{dQ}{dt} = -\frac{Q}{RC}$$

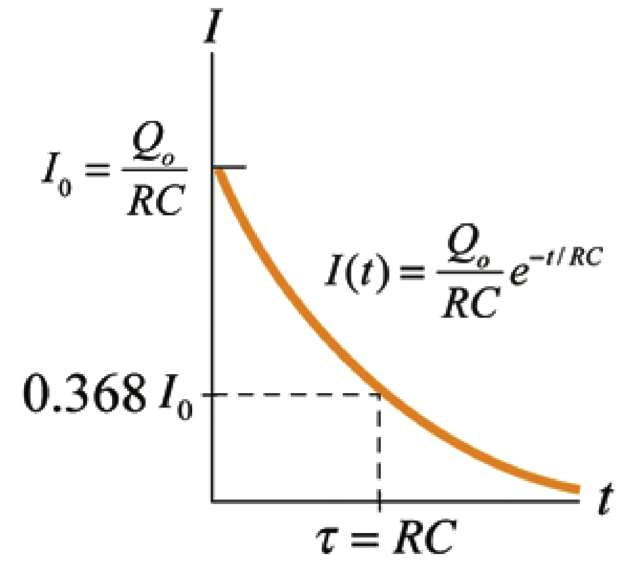
# RC Circuit: Discharging

$$\frac{dQ}{dt} = -\frac{1}{RC}Q \Rightarrow Q(t) = Q_o e^{-t/RC}$$

Solution to this equation when switch is closed at *t* = 0 with time constant $\tau = RC$

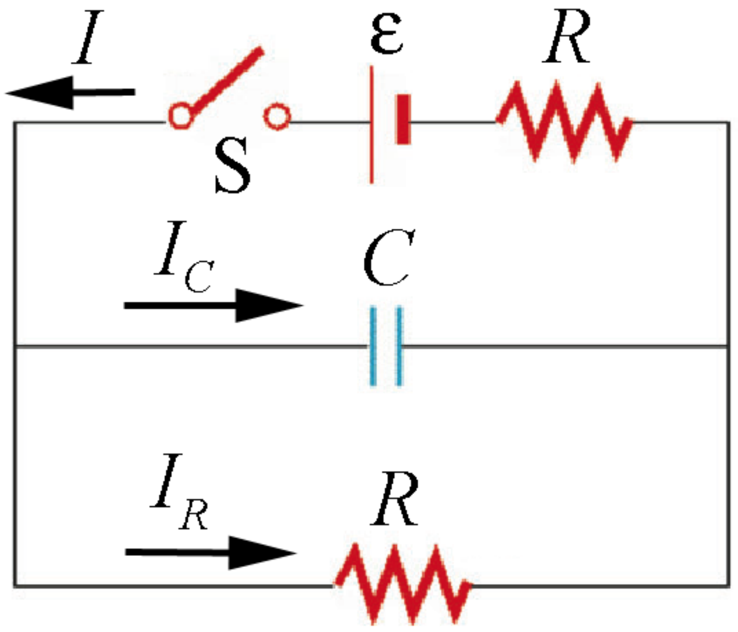
$$I = -\frac{dQ}{dt} \Rightarrow I(t) = \frac{Q_o}{\tau} e^{-t/\tau} = \frac{Q_o}{RC} e^{-t/\tau}$$



# Concept Question: RC Circuit

Consider the circuit at right, with an initially uncharged capacitor and two identical resistors. At the instant the switch is closed:

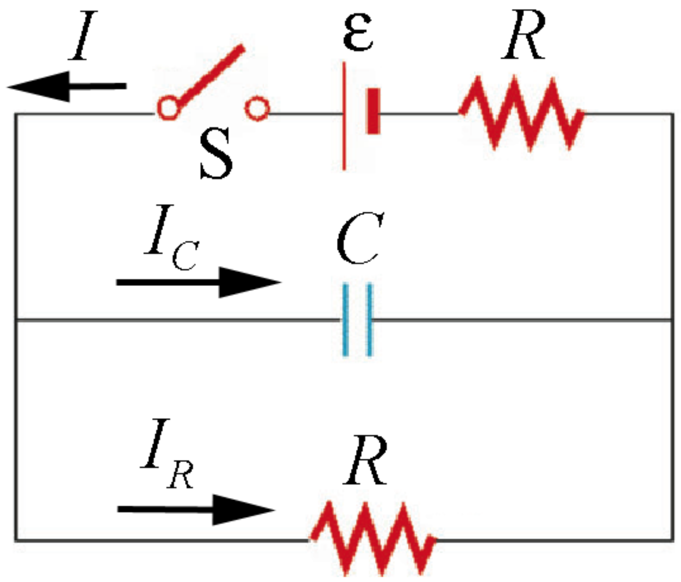


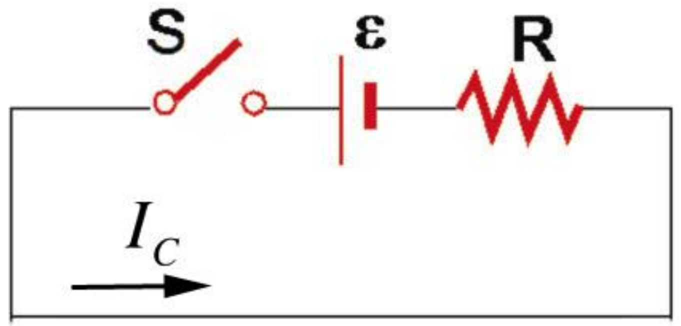

1. $I_R = I_C = 0$
2. $I_R = e/2R,\ I_C = 0$
3. $I_R = 0,\ I_C = e/R$
4. $I_R = e/2R,\ I_C = e/R$

# Concept Question Answer: RC Circuit

Answer: 3. $I_R = 0 \quad I_C = \varepsilon / R$

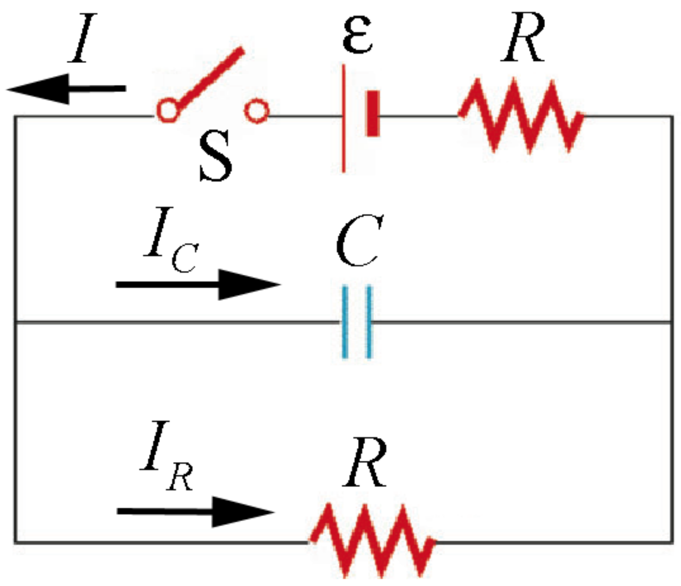
Initially there is no charge on the capacitor and hence no voltage drop across it – it looks like a short. Thus all current will flow through it rather than through the bottom resistor. So the circuit looks like:



# Concept Q.: Current Thru Capacitor

In the circuit at right the switch is closed at $t = 0$. At t = ∞ (long after) the current through the capacitor will be:

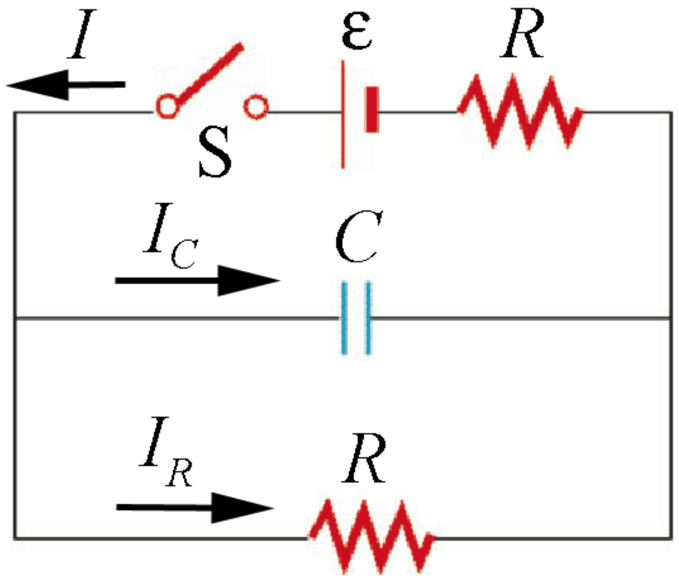



1. $I_C = 0$
2. $I_C = \varepsilon / R$
3. $I_C = \varepsilon / 2R$

# Con. Q. Ans.: Current Thru Capacitor

Answer 1. $I_C = 0$

After a long time the capacitor becomes "fully charged." No more current flows into it.



# Demo

[Circuit Simulator Applet (falstad.com)]

# Summary

Capacitors →

Revision

charge → Voltage & Current

R, L, C → i, v,

Energy conversion

charging & discharging

Batteries

Unit - II

Electrostatics → Capacitors