1. Use the graphical method to solve the following LPP:

Minimise $z = -x_1 + 2x_2$

subject to the constraints

$-x_1 + 3x_2 \leq 10,$

$x_1 + x_2 \leq 6,$

$x_1 - x_2 \leq 2$ and $x_1, x_2 \geq 0$

Soln,

min $z = -x_1 + 2x_2$

subject to

$-x_1 + 3x_2 = 10$

$x_1 + x_2 = 6$

$x_1 - x_2 = 2$

and $x_1, x_2 \geq 0$

$-x_1 + 3x_2 = 10$

for $x_1 = 0$ $x_2 = 10/3$ $(0, 10/3)$

for $x_2 = 0$ $x_1 = -10$ $(-10, 0)$

$x_1 + x_2 = 6$

for $x_1 = 0$ $x_2 = 6$ $(0, 6)$

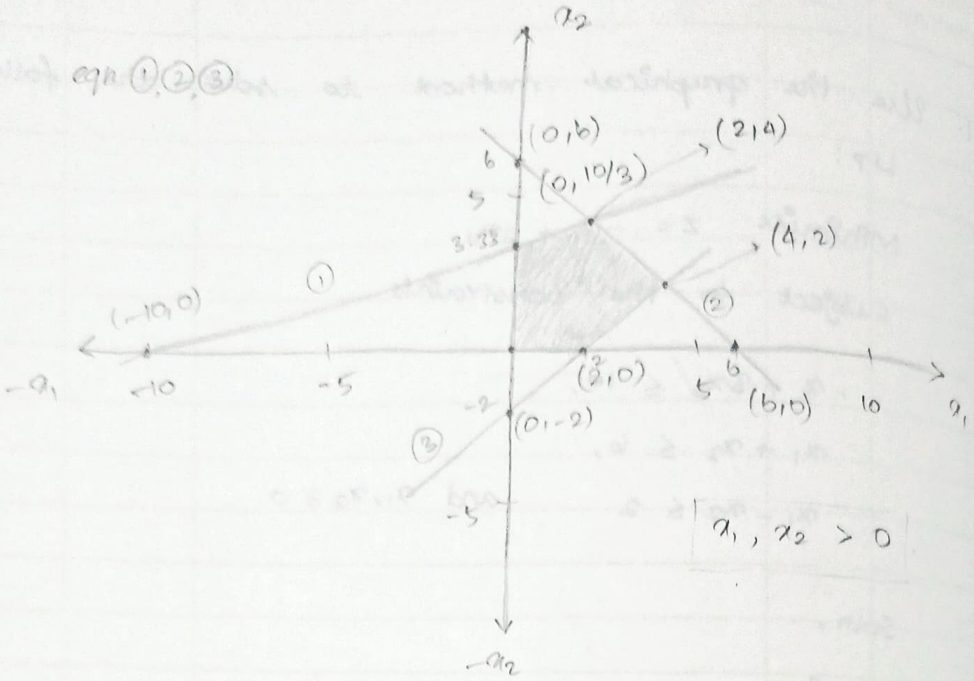
for $x_2 = 0$ $x_1 = 6$ $(6, 0)$

$x_1 - x_2 = 2$

for $x_1 = 0$ $x_2 = -2$ $(0, -2)$

for $x_2 = 0$ $x_1 = 2$ $(2, 0)$

eqn. ①, ②, ③



$x_1, x_2 > 0$

From solving ① & ②

$-x_1 + 3x_2 = 10$

$x_1 + x_2 = 6$

$4x_2 = 16$

$x_2 = 4$

$x_1 + 4 = 6$

$x_1 = 2$

② & ③

$x_1 + x_2 = 6$

$x_1 - x_2 = 2$

$2x_1 = 8$

$x_1 = 4$

$4 + x_2 = 6$

$x_2 = 2$

Corner Values	Value of z
(0,0)	z = 0
(2,0)	z = -2 ← min value
(4,2)	z = 0
(2,4)	z = 6
(0,10/3)	z = 20/3

$\boxed{\text{Minimise } z = -2}$

2. Use graphical method to solve the following LPP:

Maximize $z = 2x_1 + 3x_2$

subject to,

$x_1 + x_2 \leq 30$

$x_1 - x_2 \geq 0$

$x_2 \geq 3$

$0 \leq x_1 \leq 20$

& $0 \leq x_2 \leq 12$

Soln,

$x_1 + x_2 = 30$ (1)

for $x_1 = 0$ $x_2 = 30$ $(0, 30)$

for $x_2 = 0$ $x_1 = 30$ $(30, 0)$

$x_1 - x_2 = 0$ (2) $x_1 = x_2$

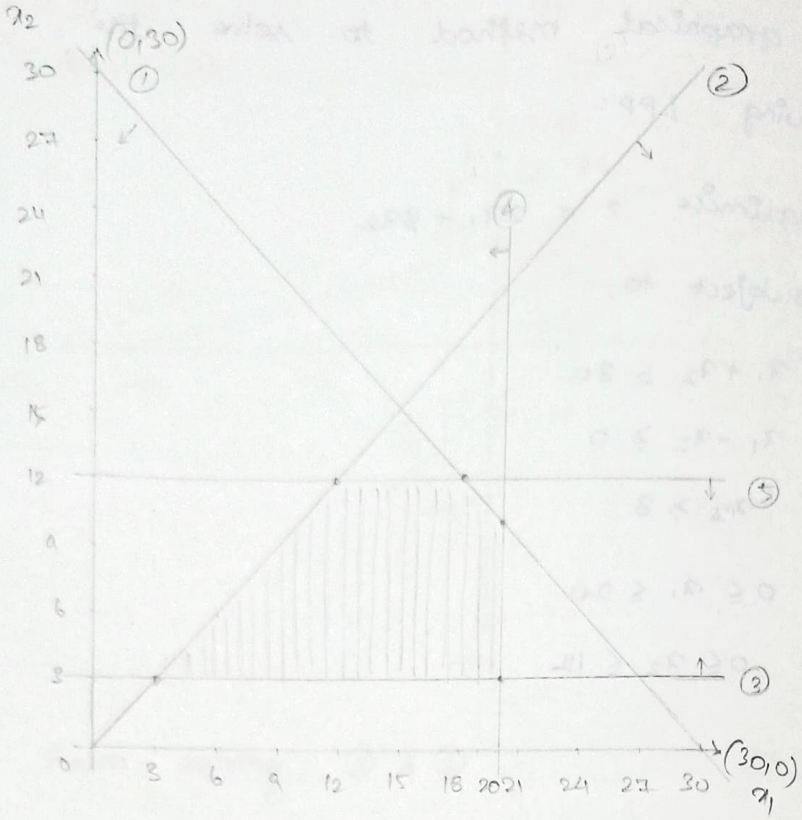
for $x_1 = 0$ $x_2 = 0$ $(0, 0)$

for $x_1 = n$ $x_2 = n$

$x_2 = 3$ (3)

$x_1 = 20$ (4)

$x_2 = 12$ (5)



② & ③

$x_1 - x_2 = 0$

$x_2 = 3$

$x_1 - 3 = 0$ (3,3)

$x_1 = 3$

① & ④

$x_1 + x_2 = 30$

$x_1 = 20$ (20,10)

$20 + x_2 = 30$

$x_2 = 10$

② & ⑤

$x_1 - x_2 = 0$

$x_2 = 12$ (12,12)

$x_1 - 12 = 0$

$x_1 = 12$

③ & ④

$x_1 = 20$ (20,3)

$x_2 = 3$

① & ⑤

$x_1 + x_2 = 30$

$x_2 = 12$ (18,12)

$x_1 + 12 = 30$

$x_1 = 18$

Corner points	$2x_1 + 3x_2$ Value of Z
(3,3)	15
(12,12)	60
(18,12)	72
(20,10)	70
(20,3)	49

$\boxed{\text{Maximum } z = 72}$

3. A company makes 2 kinds of leather belts Belt A - high quality, Belt B - low quality, respective profits are Rs 4 & Rs 3 per belt. Belt A requires twice as much time as Belt B. If all belts are type B, company could make 1000 belts per day.

The supply of leather is sufficient for only 800 belts per day (Both A and B). Belt A requires a fancy buckle & 400 buckles per day are available. There are only 700 buckles a day available for Belt B.

Determine optimal product mix.

Soln,

let x_1 = no. Belt A

x_2 = no. Belt B

now,

Maximize $z = 4x_1 + 3x_2$ (from qn profit 4,3)

$2x_1 + x_2 \leq 1000$

$x_1 + x_2 \leq 800$

$x_1 \leq 400$ $\quad$ $x_2 \leq 700$

4. Solve the following LPP by the graphical method

Minimize $z = 3x_1 + 5x_2$

sub to,

$-3x_1 + 4x_2 \leq 12$

$x_1 \leq 4$

$2x_1 - x_2 \geq -2$

$x_2 \geq 2$

$2x_1 + 3x_2 \geq 12$ &

$x_1, x_2 \geq 0$

Verify,

vertex	value of z
(3,2)	19 ← min
(4,2)	22
(4,6)	42
(4/5, 18/5) .8 3.6	102/5
(3/4, 7/2) .75 3.5	79/4

Soln

(-1, 0) (0,2) (6,0) (0,4)

(-4,0) $-3x_1 + 4x_2 = 12$ — ①

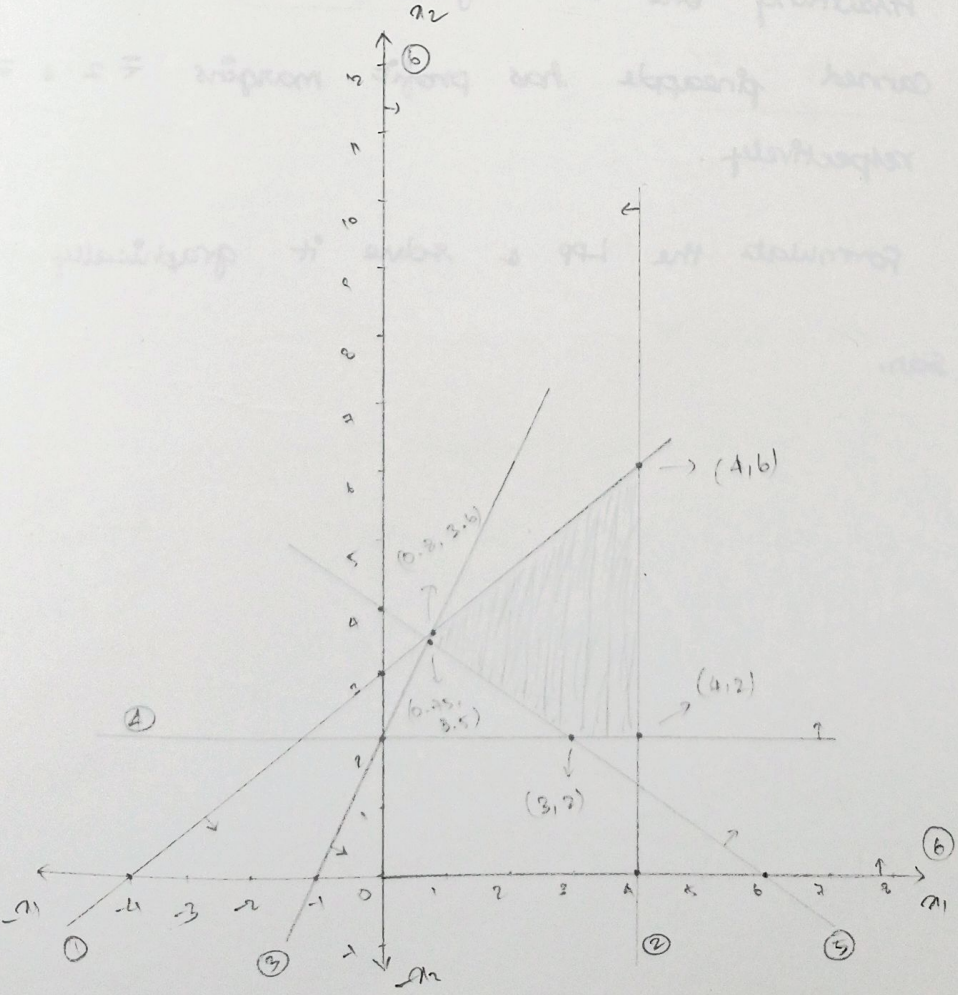
(0,3) $x_1 = 4$ — ②

$2x_1 - x_2 = -2$ — ③

$x_2 = 2$ — ④

$2x_1 + 3x_2 = 12$ — ⑤

$x_1, x_2 = 0$ — ⑥



verified.

5. A pineapple firm produces 2 products canned pineapple & canned juice. The specific amounts of material, labour and equipments required to produce each product and the availability of each of these resources are shown in the table given below :

	Canned Juice	Canned pineapple	Available resource
Labour (man hrs)	3	2.0	12.0
Equipment (M/c hrs)	1	2.3	6.9
Material (unit)	1	1.4	4.9

Assuming one unit of canned juice and canned pineapple has profit margins ₹2 & ₹1 respectively.

Formulate the LPP & solve it graphically

Soln.

$< \rightarrow$ + slack $\quad > \rightarrow$ − surplus + artificial $\quad = \rightarrow$ + artificial

in constraints

Q. Use simplex method to solve the following LPP maximize $Z = 3x_1 + 5x_2 + 4x_3$

Subject to the constraints

$$2x_1 + 3x_2 \leq 8$$

$$2x_1 + 5x_3 \leq 10$$

$$3x_1 + 2x_2 + 4x_3 \leq 15$$

$$x_1, x_2, x_3 \geq 0$$

Soln.

Introducing non-negative slack variables s_1, s_2 and s_3 such that

$$\max Z = 3x_1 + 5x_2 + 4x_3 + 0s_1 + 0s_2 + 0s_3$$

sub to constr.

$$2x_1 + 3x_2 + s_1 = 8$$

$$2x_1 + 5x_3 + s_2 = 10$$

$$3x_1 + 2x_2 + 4x_3 + s_3 = 15$$

and $x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	Min Ratio
		C_j (solution)	3	5	4	0	0	0	
0	s_1	8	2	3	0	1	0	0	8/3 min
0	s_2	10	2	0	5	0	1	0	–
0	s_3	15	3	2	4	0	0	1	15/2
	variable in basis	$Z = \sum C_j X_B$	$-C_j \rightarrow$	-5	-4	0	0	0	
		$Z = 0$		min				(from 2)	

$$x_j = \left(\sum_{i=1}^{3} x_i \cdot c_{Bi}\right) - c_j = -3$$

$x_j \rightarrow$ -3 -5 -4 0 0 0

$$\text{Ratio} = \frac{x_B}{\min(x_j)} \qquad \min(x_j) = x_2$$

s_1 should leave the table, x_2 should enter, repeat process till all x_j are non-negative $x_2 = 3$ is pivot element.

			3	5	4	0	0	0	
C_B	Y_B	x_B	x_1	x_2	x_3	s_1	s_2	s_3	Ratio
5	x_2	$8/3$	$2/3$	1	0	$1/3$	0	0	-
0	s_2	10	2	0	5	0	1	0	2 min
0	s_3	$15 - \frac{16}{3}$ $29/3$	$3 - 4/3$ $5/3$	0	4	$-2/3$	0	1	$29/12$
		$z = \frac{40}{3}$	$1/3$ $\frac{10}{3} - 3$	0 $5-5$	-4 min $0-4$	$5/3$ $5/3 - 0$	0 $0-0$	0 $0-0$	

$z = \sum C_B \cdot x_B$

$$\text{Ratio} : \frac{x_B}{x_3}$$

s_2 should leave the table x_3 should enter, $x_3 = 5$ pivot element.

$\frac{50+24-45}{15} = \frac{29}{15}$

			[3	5	4	0	0	0]	
C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	Ratio
5	x_2	8/3	2/3	1	0	1/3	0	0	
4	x_3	2	2/5	0	1	0	1/5	0	
0	s_3	5/3	1/15	0	0	-2/3	-4/5	1	
			$\frac{29}{3}-8$	$\frac{5}{3}-\frac{8}{5}$			$-2-\frac{4}{5}$		
		$z=\frac{64}{3}$	29/15	0	0	5/3	4/5	0	
			$\frac{10}{3}-\frac{8}{5}$	5-5	4-4	$\frac{5}{3}-0$	$\frac{4}{5}-0$	0-0	
			-3						

$R_2 = R_2/5$

$R_3 = R_3 - 4R_2$

all non-ve so Stop

$z = \frac{40}{3} + 8 = \frac{40+24}{3} = 64/3$

All $z_j - c_j \geq 0$

$$\boxed{\begin{array}{l} \text{Max } z = 64/3 \\ x_1 = 0 \quad x_2 = 8/3 \quad x_3 = 2 \end{array}}$$

$\rightarrow$ x_i values from the basis only of 'x_3'

Q Simplex method,

Max $z = 4x_1 + 6x_2 + 2x_3$

s.t.c

$x_1 + x_2 + x_3 \leq 3$

$x_1 + 4x_2 + 7x_3 \leq 9$

& $x_1, x_2, x_3 \geq 0$

Soln,

$z = 4x_1 + 6x_2 + 2x_3 + 0s_1 + 0s_2$

Constr,

$x_1 + x_2 + x_3 + s_1 = 3$

$x_1 + 4x_2 + 7x_3 + s_2 = 9$

			[4	6	2	0	0]	
CB	YB	XB	x_1	x_2	x_3	s_1	s_2	Ratio
0	s_1	3	1	1	1	1	0	3
0	s_2	9	1	4	7	0	1	9/4
			-4	-6	-2	0	0	

$$\text{Ratio} = \frac{XB}{x_2}$$

s_2 leaves table, x_2 enters table

			[4	6	2	0	0]	
CB	YB	XB	x_1	x_2	x_3	s_1	s_2	Ratio
		3 - 9/4	1 - 1/4	1 - 1	1 - 7/4	1 - 0	- 1/4 + 0	
0	s_1	3 3/4	3/4	0	-3/4	1	-1/4	1
6	x_2	9/4	1/4	1	7/4	0	1/4	9
		$Z = \frac{27}{2}$	-5/2	18 - 35/6	5	0	6/4	
			$\frac{6-4}{4}$	0 6 - 6	$\frac{42-2}{6}$	0 - 0	$\frac{6-0}{4}$	

$R_2 = R_2/4$

$R_1 = R_1 - R_2$

$$\text{Ratio} = \frac{XB}{x_1}$$

s_1 leaves, x_1 enters

$R_1 = R_1/0.75$ $\quad 3/4 = 0.75$

$R_2 = R_2 - \frac{1}{4} \cdot R_1$

$R_2 = 0.25 \cdot R_1$

			[4	6	2	0	0]	
CB	YB	XB	x_1	x_2	x_3	s_1	s_2	Ratio
4	x_1	1	1	0	-1	4/3	-1/3	
6	x_2	2	0	1	2	-1/3	1/3	
			1/4	1	9/4	$0 - \frac{1}{4} \cdot \frac{4}{3}$	1/4	
		Z = 16	0	0	10 - 4 = 6	10/3	2/3	

4 + 12 = 16

$$\boxed{\begin{array}{l} \text{Max } Z = 16 \\ x_1 = 1 \quad x_2 = 2 \quad x_3 = 0 \end{array}}$$

$\therefore$ [-ve, 0 can't be taken as pivot element]

Q) Solve the following LPP by simplex method

minimize $z = 8x_1 - 2x_2$

s.t.,

$-4x_1 + 2x_2 \leq 1$

$5x_1 - 4x_2 \leq 3$

$x_1, x_2 \geq 0$

soln,

maximize $z^* = -z = -8x_1 + 2x_2$

$z^* = -8x_1 + 2x_2 + 0s_1 + 0s_2$

s.t.,

$-4x_1 + 2x_2 + s_1 = 1$

$5x_1 - 4x_2 + s_2 = 3$

			[-8	2	0	0]	
C_B	Y_B	X_B	x_1	x_2	s_1	s_2	Ratio
0	s_1	1	-4	2	1	0	1/2
0	s_2	3	5	-4 (Can't take)	0	1	-3/4
	$z=0$		8	-2 (most -ve)	0	0	

Ratio = $\frac{X_B}{X_2}$

s_1 leaves, x_2 enters the basis

$s_1 = \frac{s_1}{2}$ $\quad s_2 = s_2 + 4 \cdot s_1$

			[-8	2	0	0]	
C_B	Y_B	X_B	x_1	x_2	s_1	s_2	Ratio
0	s_1	5/2	-3/2	0	1	-1/2	-5/3 min
2	x_2	-3/4	-5/4	1	0	-1/4	3/5
			-10/4 -8				
			-21/8 most -ve	0	0	-2/4	

$$\text{Ratio} = \frac{X_B}{x_1}$$

s_1 leaves, x_1 enters the basis

$s_1 = s_1 / \frac{-3}{2} \Rightarrow s_1 = s_1 / -1.5 \quad s_2 = s_2 + \frac{5}{4} s_1 \Rightarrow s_2 = s_2 + 1.25 s_1$

			[-8	2	0	0]	
C_B	Y_B	X_B	x_1	x_2	s_1	s_2	Ratio
-8	x_1	-5/3	1	0	-2/3	1/3	
2	x_2						

			[-8	2	0	0]	
C_B	Y_B	X_B	x_1	x_2	s_1	s_2	Ratio
2	x_2	1/2	-2	1	1/2	0	
0	s_2	5	-3	0	2	1	
		$z = 1$	4	0	1	0	
$z = X_B \cdot C_B$			(-2)(2) +8	2-2	1-0	0	
			8-4 = 4	0	1	0	

Max $z^* = 1$

Min $z = -\text{Max } z^* = -1$

$$\boxed{\begin{aligned} \text{Min } z &= -1 \\ x_1 &= 0 \\ x_2 &= -1/2 \end{aligned}}$$

Simplex Method Algorithm :

For the solution of LPP by simplex method, the existence of initial basis feasible solution is always assumed. The steps for the computation of an optimum solution as follows.

Step 1 : Check whether the objective function of LPP is maximized or minimized. If it is minimized then we convert into a problem of maximizing it by using the result

$$\text{Minimization } (z) = -\text{Maximization } (-z)$$

Step 2 :
Check whether all b_i $(i = 1, 2, \ldots m)$ are non-negative. If any one of b_i is negative then multiply the corresponding inequation of the constraints -1, so as to get all b_i $(i = 1, 2, \ldots m)$ non-negative

Step 3:
Convert all the inequations of the constraints into equations by introducing slack and/or surplus variables in the constraints.

Put the costs of the variables equal to zero.

Step 4: Obtain an initial basic feasible solution in the form $x_B = B^{-1}.b$ and put it in the first column of simple table.

Step 5: Compute the net evaluation $z_j - c_j$ $(j = 1, 2, \cdots n)$ by using the relation $z_j - c_j = C_B y_j - c_j$, where $y_i = B^{-1} a_j$

Examine the sign $z_j - c_j$

i) If all $(z_j - c_j) \geq 0$ then the initial basic feasible solution x_B is an optimum basic feasible solution.

ii) If at least one $(z_j - c_j) < 0$ proceed on to the next step.

Step 6: If there are more than are negative $z_j - c_j$, then choose the most negative of them Let it be $z_j - c_j$ for some $j = r$

i) If all $(z_j - c_j) \geq 0$ then the initial basic feasible solution x_B is an optimum basic feasible solution

ii) If at least one $(z_j - c_j) < 0$, then proceed to the next step

Step 7: Compute the ratio $\left\{ \frac{x_{Bi}}{y_{ir}}, \; y_{ir} > 0, \; i = 1, 2, \cdots m \right.$

and choose minimum of them. Let the

minimum of these ratios, be $\frac{x_{Bk}}{y_{kr}}$

then the vector y_k will leave the basis y_B. The common element y_{kr} which is k^{th} row and r^{th} column is known as the leading element (pivotal element).

Step 8:

Convert the leading element to unity by dividing its row by the leading element itself and all other elements making use of the relations:

$$\hat{y}_{ij} = y_{ij} - \frac{y_{kj}}{y_{kr}} y_{ir} \qquad i = 1, 2, \ldots m+1, \quad i \neq k$$

and

$$\hat{y}_{kj} = \frac{\hat{y}_{kj}}{\hat{y}_{kr}} \qquad j = 0, 1, 2, \ldots n$$

Step 9:

Go to step 5 and repeat the computational procedure until either an optimum solution is obtained or there is an indication of an unbounded solution

Q) simplex method. (Big M)

Max $Z = 3x_1 + 2x_2$

stc,

$2x_1 + x_2 \leq 2$

$3x_1 + 4x_2 \geq 12$

& $x_1, x_2 \geq 0$

Soln,

By introducing the non-negative slack variable s_1 and surplus variable s_2 the standard form of the LPP becomes,

Max $Z = 3x_1 + 2x_2 + 0s_1 + 0s_2$

stc,

$2x_1 + x_2 + s_1 = 2$

$3x_1 + 4x_2 - s_2 = 12$ and

$x_1, x_2, s_1, s_2 \geq 0$

By this will not yield a basic feasible solution. To get the basic feasible solution, and the artificial variable A_1 to the left hand side of the constraint equation which does not passess the slack variable and assign M to the artificiable variable in the objective function. The LPP becomes,

Max $Z = 3x_1 + 2x_2 + 0s_1 + 0s_2 - MA_1$

stc,

even for $\geq$ constraints, only one M should be taken $-MA_1 - MA_2$

$2x_1 + x_2 + s_1 = 2$

$3x_1 + 4x_2 - s_2 + A_1 = 12$

$x_1, x_2, s_1, s_2, A_1 \geq 0$

$M > 0$

The initial basic feasible solution is given by $s_1 = 2$, $A_1 = 12$ (basic)

$x_1 = x_2 = s_2 = 0$ (non-basic)

Step 1:

		C_j	[3	2	0	0	-M]	
C_B	Y_B	X_B	x_1	x_2	s_1	s_2	A_1	Ratio
0	s_1	2	2	1 (pivot)	1	0	0	$2/1 = 2$ min
-M	A_1	12	3	4	0	-1	1	$12/4 = 3$
		$z = -12M$	$-3M-3$	$-4M-2$	0	M	0	
		$z_j - c_j$		fast growing -ve value		(check by sub M values)		

$\text{Ratio} = \dfrac{X_B}{X_2}$

s_1 leaves, x_2 enters basis

Step 2:

$R_2 = R_2 - 4R_1$

		C_j	[3	2	0	0	-M]	
C_B	Y_B	X_B	x_1	x_2	s_1	s_2	A_1	Ratio
2	x_2	2	2	1	1	0	0	
-M	A_1	4	-5	0	-4	-1	1	
		$z = -4M+4$	$5M+1$	0	$4M+2$	M	0	
		$z_j - c_j \geq 0$						

Since all $z_j - c_j \geq 0$ and artificial variable A_1 appears in the basic at non-zero level, the given LPP does not possess any feasible solution. But the LPP, possess pseudo optimal soln.

there exist a soln but it is not optimal

Q) Two phase simplex method :

for min, take $z^* = -z$ then do same procedure for max like simplex method

Max $z = 5x_1 + 8x_2$

stc,

$3x_1 + 2x_2 \geq 3$

$x_1 + 4x_2 \geq 4$

$x_1 + x_2 \leq 5$

$x_1, x_2 \geq 0$

soln,

By introducing the non-negative slack, surplus and artificial variables, the standard form of the LPP.

Max $z = 5x_1 + 8x_2 + 0s_1 + 0s_2 + 0s_3$

stc,

$3x_1 + 2x_2 - s_1 + A_1 = 3$

$x_1 + 4x_2 - s_2 + A_2 = 4$

$x_1 + x_2 + s_3 = 5$

and $x_1, x_2, s_1, s_2, s_3, A_1, A_2 \geq 0$

(Here, s_1, s_2 - surplus var

s_3 - slack var

A_1, A_2 - artificial var)

The initial basic feasible solution is given by $A_1 = 3$, $A_2 = 4$, $s_3 = 5$ (basic), ($x_1 = x_2 = s_1 = s_2 = 0$) (non basic)

Phase I :

Assigning a cost -1 to the artificial variables and cost 0 to all other variables, the objective function of the auxiliary LPP becomes

$$\text{Max } z^+ = -A - A_2$$

Stc.

The iterative simplex tables for the auxiliary LPP are:

Step-1 :

	C_B	Y_B	X_B	x_1	x_2	s_1	s_2	s_3	A_1	A_2	Ratio
				[0	0	0	0	0	-1	-1]	
R_1	-1	A_1	3	3	2	-1	0	0	1	0	3/2
			-2	-1/2	-2	-0	+1/2	-0	-0	-1/2	
R_2	-1	A_2	4	1	$\boxed{4}$	0	-1	0	0	1	4/4 min
R_3	0	s_3	5	1	1	0	0	1	0	0	5/1
			-1	-1/4	-1	0	+1/4	+0	0	-1/4	
			$z = -7$	-4	-6	1	1	0	0	0	

$R_2 = R_2/4$

$R_1 = R_1 - 2 \cdot R_2$

$R_3 = R_3 - R_2$

A_2 leaves

x_2 enters basis

Step - 2 :

	C_B	Y_B	X_B	x_1	x_2	s_1	s_2	s_3	A_1	A_2	Ratio
				[0	0	0	0	0	-1	-1]	
R_1	-1	A_1	1	$\boxed{5/2}$	0	-1	1/2	0	1	-1/2	2/5 (0.4)
R_2	0	x_2	1 (-1/10)	1/4 (-1/4)	1 (+0)	0 (+1/10)	-1/4 (-1/20)	0 (0)	0 (-2/20)	1/4 (+1/20)	4
R_3	0	s_3	4 (-3/10)	3/4 (-3/4)	0 (0)	0 (3/10)	1/4 (-3/20)	1 (+0)	0 (-3/10)	-1/4 (3/20)	16/3
			$z=-1$	-5/2 (-2.5)	0	1	-1/2 (-0.5)	0	3/4	3/2 (+0.5) (+(-1/2)) +1	

Ratio $= \dfrac{X_B}{x_1}$

A_1 leaves, x_1 enters

$R_1 = R_1/5/2 = R_1/2.5 = R_1 \times \frac{2}{5}$

$R_2 = R_2 - 1/4\, R_1 = R_2 - 0.25\, R_1$

$R_3 = R_3 - 3/4\, R_1 = R_2 - 0.75\, R_1$

Step - 3 :

C_B	Y_B	X_B	x_1	x_2	s_1	s_2	s_3	A_1	A_2	Ratio
			[0	0	0	0	0	-1	-1]	
0	x_1	2/5	1	0	-2/5	1/5	0	2/5	-1/5	
0	x_2	9/10	0	1	1/10	-3/10	0	-1/10	3/10	
0	s_3	37/10	0	0	3/10	1/10	1	-3/10	-1/10	
		$z=0$	0	0	0	0	0	1	1	

[A_1, A_2 are not inside basis so they are not needed]

Since all $(z_j^* - c_j) \geq 0$, the current basic solution is optimum. Furthermore, no artificial variable appears in the optimum basis so we proceed to phase II.

Phase II :

Here we consider the actual costs associated with the original variables. The new objective function then becomes

$$\text{Max } z = 5x_1 + 8x_2 + 0s_1 + 0s_2 + 0s_3$$

The initial basic feasible solution for this phase is one obtained at the end of phase I

The iterative simplex tables for this phase are :

Step-1 :

	C_B	Y_B	X_B	x_1	x_2	s_1	s_2	s_3	Ratio
				[5	8	0	0	0]	
R_1	5	x_1	2/5	1	0	-2/5	$\boxed{1/5}$	0	2 min
R_2	8	x_2	9/10 $\frac{6}{10}$	0 $\frac{15}{10}$	1 0	1/10 $-\frac{6}{10}$	-3/10 $\frac{3}{10}$	0 0	-3 ignore
R_3	0	s_3	37/10 $-\frac{2}{10}$	0 $-5/10$	0 0	3/10 $+2/10$	1/10 $-1/10$	1 0	37
			$z = \frac{46}{5}$ $2+\frac{36}{5}$	0 5-5	0 8-8	-6/5 -2 $+\frac{8}{10}$ -0	-7/5 1 $-\frac{24}{10}$ -0	0 0-0	

$2+\frac{8\times 9}{10/5}$

Ratio = $\frac{X_B}{s_2}$ $\qquad$ x_1 leaves, s_2 enters basis

$R_1 = R_1/1/5 = R_1 \times 5$ $\qquad$ $R_3 = R_3 - \frac{1}{10} R_1$

$R_2 = R_2 + \frac{3}{10} \cdot R_1$

Step-2:

	C_B	Y_B	X_B	x_1	x_2	s_1	s_2	s_3	Ratio
				[5	8	0	0	0]	
R_1	0	s_2	2	5	0	-2	1	0	-1 (ignore) 2/-2
			+14	-2	+0	+2	+0	+0	
R_2	8	x_2	15/10	15/10	1	-5/10	0	0	-3 $\frac{15}{10} \times \frac{-10}{5}$
			+7/2	-1/2	0	+1/2	0	+1	
R_3	0	s_3	35/10	-5/10	0	5/10	0	1	7 min $\frac{35}{10} \times \frac{10}{5}$
			$z = 12$	7	0	-4 most -ve	0	0	
			$(C_B \cdot X_B)$	12-5	8-8	-4	0	0	

Ratio : $\frac{X_B}{s_1}$ s_3 leaves, s_1 enters basis

$R_3 = R_3 \times 2$ $R_1 = R_1 + 2 \cdot R_3$

$R_2 = R_2 + \frac{1}{2} R_2$

Step-3:

C_B	Y_B	X_B	x_1	x_2	s_1	s_2	s_3	Ratio
			[5	8	0	0	0]	
0	s_2	16	3	0	0	1	4	
8	x_2	5	1	1	0	0	1	
0	s_1	7	-1	0	1	0	2	
		$z = 40$	3	0	0	0	8	
			8-5	8-8	0	0	8-0	

Since all $z_j - c_j \geq 0$ the current basic feasible solution is optimal

$\therefore$ $\boxed{\begin{array}{l} \text{Max } z = 40, \\ x_1 = 0 \\ x_2 = 5 \end{array}}$

Q) Max $z = -4x_1 - 3x_2 - 9x_3$, 2phase

s.t.,

$$2x_1 + 4x_2 + 6x_3 \geq 15$$
$$6x_1 + x_2 + 6x_3 \geq 12$$
$$x_1, x_2, x_3 \geq 0$$

$2x_1 + 4x_2 + 6x_3 - s_1 + A_1 = 15$

$6x_1 + x_2 + 6x_3 - s_2 + A_2 = 12$

$x_1 = 0$ $s_1 = s_2 = 0$

$x_2 = 0$

$x_3 = 0$ $A_1 = 15$ $A_2 = 12$.

Soln,

Max $z^* = -A_1 - A_2$

			[0	0	0	0	0	-1	-1]	
C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	A_1	A_2	Ratio $\frac{X_B}{x_3}$
R_1 -1	A_1	15	2	4	6	-1	0	1	0	15/6 = 2.5
			-6	-1	-6	-0	+1	-0	-1	
R_2 -1	A_2	12	6	1	$\boxed{6}$	0	-1	0	1	12/6 = 2 min
	$z = -15 - 12 = -27$		-8	-5	-12 most -ve	1	1	0	0	

A_2 leaves x_3 enters

$R_2 = R_2/6$ $R_1 = R_1 - 6R_2$

			[0	0	0	0	0	-1	-1]	
C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	A_1	A_2	Ratio $\frac{X_B}{x_2}$
-1	A_1	3	-4	$\boxed{3}$	0	-1	1	1	-1	1 min
0	x_3	2	1	1/6	1	0	-1/6	0	1/6	12
		$-\frac{1}{6}$	$+\frac{2}{9}$	-1/6	0	$+\frac{1}{12}$	$-\frac{1}{18}$	$-\frac{1}{18}$	$+\frac{1}{18}$	
	$z = -5$		4	-3 most -ve	0	1	-1	0	2	

$R_1 = R_1/3$ $R_2 = R_2 - \frac{1}{6}R_1$ A_1 leaves, x_2 enters

			[0	0	0	0	0	-1	-1]	
C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	A_1	A_2	Ratio
0	x_2	1	-4/3	1	0	-1/3	1/3	1/3	-1/3	
0	x_3	11/6	11/9	0	1	1/18	-2/9	$-\frac{1}{18}$	2/9	
	$z = 0$		0	0	0	0	0	1	1	

$z_j - c_j \geq 0$ ignore artificial varr

last pg

Some special cases in Linear Programming :

1. Alternative (Multiple) optimal solution.

We have that the optimal solution of any linear programming problem occurs at an extreme point of the feasible region and the solution is unique, that is, no other solution yield the same value of the objective function. However, in certain cases a given LP problem may have more than one optimal solution yielding the same objective function value.

There are 2 conditions that should be satisfied in order that an alternative optimal solution exists.

i) The given objective function is parallel to constraint that forms the boundary (or edge) of the feasible solutions region. In other words, the slope of the objective function is same as that of the constraint forming the boundary of the feasible solutions region

ii) The constraint should form a boundary on the feasible region in the direction

of optimal movement of the objective function. In otherwords, the constraint should be an active constraint.

example,

by graphical method,

$$\max \; z = 10x_1 + 6x_2 = 2(5x_1 + 3x_2)$$

stc,

$$5x_1 + 3x_2 \le 30 \quad (1)$$

$$x_1 + 2x_2 \le 18 \quad (2)$$

$$\& \; x_1, x_2 \ge 0 \quad (3)$$

Soln,

(1) $x_1 = 0$, $x_2 = 10$

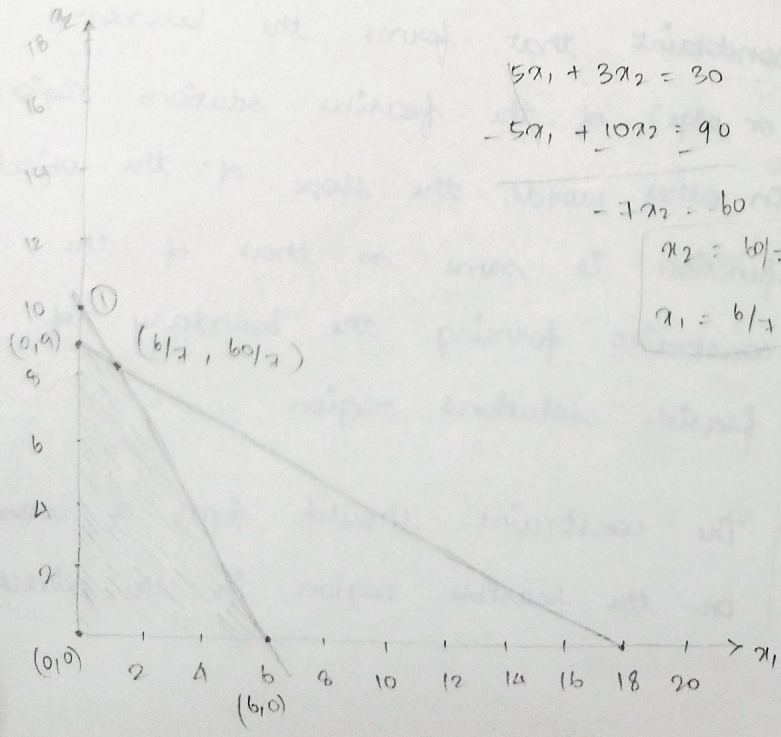
$x_2 = 0$, $x_1 = 6$

(2) $x_1 = 0$, $x_2 = 9$

$x_2 = 0$, $x_1 = 18$

$$5x_1 + 3x_2 = 30$$
$$-5x_1 + 10x_2 = 90$$
$$-7x_2 = 60$$

$$\boxed{x_2 = 60/7 \quad x_1 = 6/7}$$



Corner values	Value of Z $10x_1 + 6x_2$
(0, 9)	54
(6, 0)	60
(6/7, 60/7)	60 $\frac{60}{7} + \frac{360}{7} = \frac{420}{7}$
(0, 0)	0

$$\boxed{\begin{array}{l} \text{Max } z = 60 \\ x_1 = 6/7, \; x_2 = 60/7 \\ x_1 = 6, \; x_2 = 0 \end{array}}$$

Remark:

If a constraint to which the objective function is parallel doesnt form the boundary of the feasible region, then multiple solution will not exist and such a constraint is called redundant constraint, that is, reduntant constraint is one whose removal does not change the feasible region. Thus, optimal soln does not change if this constraint moved / removed

An unbounded solution:

When the value of the decision variables in linear programming is permitted to increase infinitely without violating the feasibility condition, then the solution is said to be unbounded.

Here the objective function value can also be increased infinitely

example,

Max $Z = 3x_1 + 4x_2$

s.t.,

$x_1 - x_2 \leq -1$ — ① $x_1 - x_2 = -1$

$-x_1 + x_2 \leq 0$ — ② $x_2 - x_1 = 0$

$x_1 = x_2$

& $x_1, x_2 \geq 0$

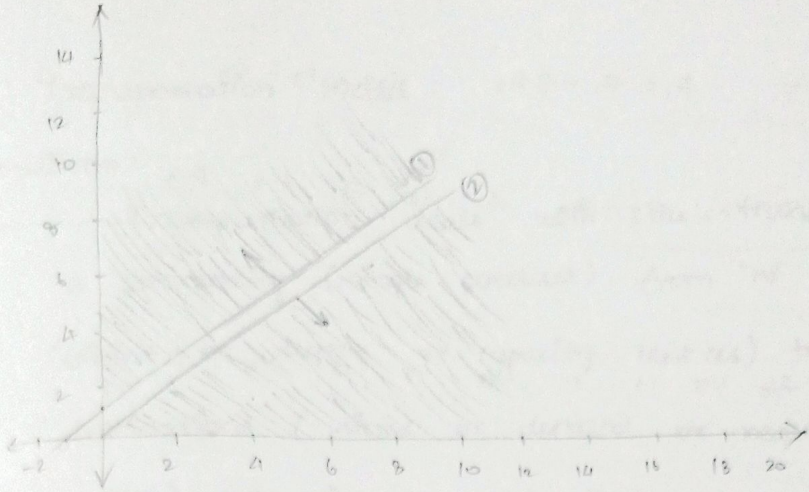
Soln,

① $x_1 = 0$ ② $x_1 = 0$ let $x_1 = 1$

$x_2 = 1$ $x_2 = 0$ $x_2 = 1$

$x_2 = 0$ $x_2 = 0$

$x_1 = -1$ $x_1 = 0$



Q) Solve by Big-M

$$\text{Max } Z = x_1 + 2x_2 + 3x_3 - x_4$$

$$x_1 + 2x_2 + 3x_3 = 15$$

$$2x_1 + x_2 + 5x_3 \geq 20$$

$$x_1 + 2x_2 + x_3 + x_4 \geq 10 \quad , \quad x_1, x_2, x_3, x_4 \geq 0$$

Soln,

$$Z = x_1 + 2x_2 + 3x_3 - x_4 + 0s_1 + 0s_2 - MA_1 - MA_2 - MA_3$$

$$x_1 + 2x_2 + 3x_3 + A_1 = 15$$

$$2x_1 + x_2 + 5x_3 - s_1 + A_2 = 20$$

$$x_1 + 2x_2 + x_3 + x_4 - s_2 + A_3 = 10$$

$A_1 = 15$, $A_2 = 20$, $A_3 = 10$, $s_1 = s_2 = 0$

			(1	2	3	-1	0	0	-M	-M	-M)	
C_B	Y_B	X_B	x_1	x_2	x_3	x_4	s_1	s_2	A_1	A_2	A_3	Ratio X_B/x_3
-M	A_1	15	1	2	3	0	0	0	1	0	0	5
-M	A_2	20	2	1	$\boxed{5}$	0	-1	0	0	1	0	4 min
-M	A_3	10	1	2	1	1	0	-1	0	0	1	10
			-3M -1	-5M -2	-9M -3	-M +1	M	M	0	0	0	

A_2 leaves

x_3 enters.

$R_2 = R_2/5$ $R_1 = R_1 - 3R_2$ $R_3 = R_3 - R_2$

C_B	Y_B	X_B	x_1	x_2	x_3	x_4	s_1	s_2	A_1	A_2	A_3	Ratio
$-M$	A_1											
3	x_3	4	2/5	1/5	1	0	-1/5	0	0	1/5	0	
$-M$	A_3											

Transportation Model :

Intro :

Transportation deals with the transportation of a commodity (single product) from 'm' sources (origins or supply or capacity centres) to 'n' destinations (sinks or demand or requirement centres). It is assumed that

i) Level of supply at each source and the amount of demand at each destination and

ii) The unit transportation cost of commodity from each source to each destination are known. [given]

It is also assumed that the cost of transportation is linear.

The objective is to determine the amount to be shifted from each source to each destination such that the total transportation cost is minimum.

Mathematical Formulation of a Transportation Problem:

Let us assume that there are 'm' sources and 'n' destinations.

Let a_i be the supply (capacity) at sources i, b_j be the demand at destination j, c_{ij} be the unit transportation cost from source i,

to destination j and x_{ij} be the number of units shifted from source i to destination j.

Then the

Basic Feasible Solution (BFS)

A feasible solution to a $(m \times n)$ transportation problem that contains no more than $(m+n-1)$ non-negative allocations is called a basic feasible solution (BFS) to the transportation problem.

NORTH-WEST corner rule:

Prob: Determine basic feasible solution of the following transportation problem using North-West corner rule.

Destination

origin	A	B	C	D	E	Supply
P	2	11	10	3	7	4
Q	1	4	7	2	1	8
R	3	9	4	8	12	9
Demand:	3	3	4	5	6	

Soln,

$\Sigma a_i = 21 \quad \Sigma b_j = 21$

Since $\Sigma a_i = \Sigma b_j$, the given transportation problem is balanced.

Step 1:

	A	B	C	D	E	Sup
P	$\boxed{3}$ 2	11	10	3	7	~~4~~ 1
Q	1	4	7	2	1	8
R	3	9	4	8	12	9
Dem:	~~3~~ 0	3	4	5	6	

→ 1st allocation is made in the cell (1,1)

$x_{11} = \min\{a_1, b_1\}$
$= \{4, 3\} = 3$

Step 2:

	B	C	D	E	Sup
P	$\boxed{1}$ 11	10	3	7	~~1~~ 0
Q	4	7	2	1	8
R	9	4	8	12	9
Dem:	~~3~~ 2	4	5	6	

→ alloc. cell (1,2)

$x_{12} = \min\{a_1, b_2\}$
$= \min\{1, 3\} = 1$

Step 3:

	B	C	D	E	Sup.
Q	$\boxed{2}$ 4	7	2	1	~~8~~ 6
R	9	4	8	12	9
Dem:	~~2~~ 0	4	5	6	

→ alloc. cell (2,2)

$x_{22} = \min\{8, 2\} = 2$

Step 4:

	C	D	E	Sup
Q	$\boxed{4}$ 7	2	1	~~6~~ 2
R	4	8	12	9
Dem:	~~4~~ 0	5	6	

→ alloc. cell (2,3)

$x_{23} = \min\{6, 4\} = 4$

Step 5 :

	D	E	Sup:
Q	[2] 2	1	~~2~~ 0
R	8	12	9
Dem:	~~5~~ 3	6	

alloc cell (2,4)

$x_{24} = \min\{2,5\} = 2$

Step 6 :

	D	E	Sup:
R	[3] 8	[6] 12	~~9~~ ~~6~~ 0
Dem:	~~3~~ 0	~~6~~ 0	

Alloc cell (3,4)

$x_{34} = \min\{3,9\} = 3$

alloc cell (3,5)

$x_{35} = \min\{6,9\} = 6$

have to get equal values in step 7 if the table is balanced.

	A	B	C	D	E
P	[3] 2	[1] 11	10	3	7
Q	1	[2] 4	[4] 7	[2] 2	1
R	3	9	4	[3] 8	[6] 12

Initial transportation cost

$Z = (3\times2) + (1\times11) + (2\times4) + (4\times7) + (2\times2) + (3\times8) + (6\times12)$

$Z = 153. \Rightarrow$ RS. 153

If path asked.

2	11	10	3	7
1	4	7	2	1
3	9	4	8	12

draw lines from the matrix above

Least cost Matrix (or) Matrix Minima method:

Prob: obtain an initial basic feasible solution to the following T.P using the matrix minima.

	D_1	D_2	D_3	D_4	capacity a
O_1	1	2	3	4	6
O_2	4	3	2	0	8
O_3	0	2	2	1	10
Demand b	4	6	8	6	

where O_i and D_j denote i^{th} origin and j^{th} destination respectively.

Soln.

$\Sigma a_i = 24 = \Sigma b_j$

So given table is balanced

Therefore given prob. has feasible soln.

Find 'min' then same as North-west

Step1:

	D_1	D_2	D_3	D_4	cap
O_1	1	2	3	4	6
O_2	4	3	2	0	8
O_3	[4] 0	2	2	1	10 6
Dem.	~~4~~ 0	6	8	6	

Step 2:

	D_2	D_3	D_4	Cap:
O_1	2	3	4	6
O_2	3	2	$\boxed{6}$ 0	~~8~~ 2
O_3	2	2	1	6
Dem:	6	8	~~6~~ 0	

Step 3:

	D_2	D_3	Cap:
O_1	2	3	6
O_2	3	$\boxed{2}$ 2	~~2~~ 0
O_3	2	2	6
Dem:	6	~~8~~ 6	

select any '2'

Step 4:

	D_2	D_3	Cap:
O_1	$\boxed{6}$ ~~2~~	3	~~6~~ 0
O_3	2	$\boxed{6}$ ~~2~~	~~6~~ 0
Dem:	~~6~~ 0	~~6~~ 0	

balanced

	D_1	D_2	D_3	D_4
O_1	1	$\boxed{6}$ 2	3	4
O_2	4	3	$\boxed{2}$ 2	$\boxed{6}$ 0
O_3	$\boxed{4}$ 0	2	$\boxed{6}$ 2	1

$$Z = (4\times0) + (6\times2) + (2\times2) + (6\times2) + (6\times0)$$

$$= 28 \Rightarrow \text{Rs. } 28$$

Vogel's Approximation Method (VAM)

Prob: Use VAM to obtain initial basic feasible solution of the transportation problem.

	D	E	F	G	Availability
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Demand:	200	225	275	250	

Soln,

$$\Sigma a_i = 950 = \Sigma b_i$$

Calc. difference b/w two min values then select max diff then choose the min value among that

Step 1:

	D	E	F	G		
A	200 / 11	13	17	14	~~250~~ 50	(2)
B	16	18	14	10	300	(4)
C	21	24	13	10	400	(3)
	~~200~~ 0	225	275	250		
	(5)	(5)	(1)	(0)		

choose any

Step 2:

	E	F	G		
A	50 / 13	17	14	~~50~~ 0	(1)
B	18	14	10	300	(4)
C	24	13	10	400	(3)
	~~225~~ 175	275	250		
	(5)	(1)	(0)		

Step 3:

	E	F	G	
B	125 18	14	10	~~300~~ 125 (4)
C	24	13	10	400 (3)
	~~175~~ 0 (6)	275 (1)	250 (0)	

Step 4:

	F	G	
B	14	125 10	~~125~~ 0 (4)
C	13	10	400 (3)
	275 (1)	~~250~~ 125 (0)	

Step 5:

	F	G	
C	275 13	125 10	~~275~~ 0 ~~400~~ 0 (3)
	275 0 (0)	~~125~~ 0 (0)	

balanced

for tight values 0 is valid (13-13) (10-10)

	D	E	F	G
A	200 11	50 13	17	14
B	16	175 18	14	125 10
C	21	24	275 13	125 10

$$z = (11 \times 200) + (50 \times 13) + (175 \times 18) + (275 \times 13) + (125 \times 10) + (125 \times 10) = 12,075.$$

MODI Method (Modified distribution method)

(Test for optimal solution)

Pbm: Solve the foll. pbm by MODI method

	1	2	3	4	Supply
I	21	16	25	13	11
II	17	18	14	23	13
III	32	27	18	41	19
Demand:	6	10	12	15	

Soln:

$$\Sigma a_i = \Sigma b_i = 43$$

First, we should solve this pbm by vogel's approximation method (or) North-West rule

By vogel's,

	1	2	3	4		
I	21	16	25	[11] 13	~~11~~ 0	(3)
II	17	18	14	23	13	(3)
III	32	27	18	41	19	(9)
	6	10	12	~~15~~ 4		
	(4)	(2)	(4)	(18)		

	1	2	3	4		
II	[6] 11	18	14	23	~~13~~	(3)
III	32	27	18	41	19	(9)
	~~6~~ 0	10	12	4		
	(15)	(9)	(4)	(13)		

	2	3	4		
II	18	14	[4] 23	~~7~~	(4)
III	27	18	41	19	(9)
	10	12	~~4~~ 0		
	(9)	(4)	(13)		

	2	3		
II	[3] 18	14	~~8~~ 0	(4)
III	27	18	19	(9)
	~~10~~ 7	12		
	(9)	(4)		

	2	3		
III	27	[9] 18	~~7~~ 0	(9)
	7	~~12~~ 0		
	(0)	(9)		

	2		
III	[9] 27	~~7~~ 0	(0)
	~~7~~ 0		
	(0)		

	1	2	3	4		
I	21	16	25	[11] 13	11	
II	[6] 11	[3] 18	14	[2] 23	13	
III	32	[9] 27	[12] 18	41	19	
	6	10	12	15		

$z = 796$

no. elements selected = m (row) + n (column) − 1

From the table we can see that the no. of non-negative independent allocations is $m+n-1=6$, hence the solution is non-degenerate feasible

To find the optimal solution by MODI method :

Consider the above transportation table.

Since $m+n-1 = 6$, we can apply MODI method

Now we determine the set of values u_i and v_i for each occupied cell (i,j) by using the relation $c_{ij} = u_i + v_j$

Since the second row contains the maximum no. of allocations, then we choose

$u_2 = 0$, $c_{21} = u_2 + v_1 \Rightarrow 17 = 0 + v_1 \Rightarrow v_1 = 17$

$c_{22} = u_2 + v_2 \Rightarrow 18 = 0 + v_2 \Rightarrow v_2 = 18$

$c_{24} = u_2 + v_4 \Rightarrow 23 = 0 + v_4 \Rightarrow v_4 = 23$

$c_{14} = u_1 + v_4 \Rightarrow 13 = u_1 + 23 \Rightarrow u_1 = -10$

$c_{32} = u_3 + v_2 \Rightarrow 27 = u_3 + 18 \Rightarrow u_3 = 9$

$c_{33} = u_3 + v_3 \Rightarrow 18 = 9 + v_3 \Rightarrow v_3 = 9$

Thus, we have the following transportation table :

21	16	25	[11] 13	$u_1 = -10$
[6] 17	[3] 18	14	[4] 23	$u_2 = 0$
32	[7] 27	[12] 18	41	$u_3 = 9$
$v_1 = 17$	$v_2 = 18$	$v_3 = 9$	$v_4 = 23$	

Now we find $u_i + v_j$ for each occupied cell (i,j) and enter at the corresponding unoccupied cell (i,j)

Then we find cell evaluations $d_{ij} = c_{ij} - (u_i + v_j)$ for each unoccupied cell (i,j) and enter at the corresponding unoccupied cell (i,j)

Then, we get the following table,

7 / 21	14 / 16	8 / 25	11 / 13
6 / 17	3 / 18	9 / 14	4 / 23
26 / 32	6 / 27	7 / 18	12 / 41
$v_1 = 17$	$v_2 = 18$	$v_3 = 9$	$v_4 = 23$

$u_1 = -10$

$u_2 = 0$

$u_3 = 9$

Since all $d_{ij} > 0$, the solution under the test is optimal and unique.

The optimum allocation schedule is given by

$x_{14} = 11$, $x_{21} = 6$, $x_{22} = 3$, $x_{24} = 4$, $x_{32} = 7$, $x_{23} = 12$

and the optimum (minimum) transportation cost

$= 13 \times 11 + 17 \times 6 + 18 \times 3 + 23 \times 4 + 27 \times 7 + 18 \times 12$

$= 796 \Rightarrow$ Rs. 796

Pbm : Find the optimal transportation cost of the following matrix using least cost method for finding the critical solution.

		Market				
		A	B	C	D	E	Available
	P	4	1	2	6	9	100
Factor	Q	6	4	3	5	7	120
	R	5	2	6	4	8	120
Demand		40	50	70	90	90	

by least cost matrix,

4	$\boxed{50}$ 1	2	6	9	~~100~~ 50
6	4	3	5	7	120
5	2	6	4	8	120
40	~~50~~ 0	70	90	90	

4	$\boxed{50}$ 2	6	9	~~50~~ 0
6	3	5	7	120
5	6	4	8	120
40	~~70~~ 20	90	90	

6	$\boxed{20}$ 3	5	7	~~120~~ 100
5	6	4	8	120
40	~~20~~ 0	90	90	

6	5	7	100	
5	$\boxed{90}$ 4	8	~~120~~ 30	
10	~~90~~ 0	90		

6	7	100	
$\boxed{30}$ 5	8	~~30~~	
~~40~~ 10	90		

$\boxed{10}$ 6	$\boxed{90}$ 7	~~100~~ 96		
~~10~~ 0	~~90~~ 0			

	A	B	C	D	E
P	4	$\boxed{50}$ 1	$\boxed{50}$ 2	6	9
Q	$\boxed{10}$ 6	4	$\boxed{20}$ 3	5	$\boxed{90}$ 7
R	$\boxed{30}$ 5	2	6	$\boxed{90}$ 4	8

For, optimality : Since the no. non-negative independent allocations is 7 which is equal to m+n-1, then we can apply MODI method.

	A	B	C	D	E
P	4	$\boxed{50}$ 1	$\boxed{50}$ 2	6	9
Q	$\boxed{10}$ 6	4	$\boxed{20}$ 3	5	$\boxed{90}$ 7
R	$\boxed{30}$ 5	2	6	$\boxed{90}$ 4	8

$U_1 = -1$

$U_2 = 0$

$U_3 = -1$

V_1	V_2	V_3	V_4	V_5
=	=	=	=	=
6	2	3	5	7

Rs = 1410 /-

$U_2 = 0$

$V_1 + U_2 = 6 \Rightarrow V_1 = 6$

$V_3 + U_2 = 3 \Rightarrow V_3 = 3$

$V_5 + U_3 = 7 \Rightarrow V_5 = 7$

$V_3 + U_1 = 2 \Rightarrow 3 + U_1 = 2 \Rightarrow U_1 = -1$

$V_2 + U_1 = 1 \Rightarrow V_2 - 1 = 1 \Rightarrow V_2 = 2$

$V_1 + U_3 = 5 \Rightarrow 6 + U_3 = 5 \Rightarrow U_3 = -1$

$V_4 + U_3 = 4 \Rightarrow V_4 - 1 = 4 \Rightarrow V_4 = 5$

	A	B	C	D	E
P	5 -1 / 4	50 / 1	50 / 2	4 2 / 6	6 3 / 9
Q	10 / 6	2 2 / 4	20 / 3	5 0 / 5	90 / 7
R	30 / 5	1 1 / 2	2 4 / 6	90 / 4	6 2 / 8

v_1	v_2	v_3	v_4	v_5
=	=	=	=	=
6	2	3	5	7

$u_1 = -1$

$u_2 = 0$

$u_3 = -1$

Not all $d_{ij} > 0$, so

	A	B	C	D	E
P	5 -1 / 4	50 /	50 / $-\theta$	4 2 / 6	6 3 / 9
Q	10 / $-\theta$ 6	2 2 / 4	20 / $+\theta$ 3	5 0 / 5	90 / 7
R	30 / 5	1 1 / 2	2 4 / 6	90 / 4	6 2 / 8

form a loop like this with $\boxed{\ }$ as edges. then add & sub 'θ' at the edges from $+\theta, -\theta, +\theta, -\theta, \ldots$ in order

Since $d_{11} = -1 < 0$, the current solution is not optimal. Now let us form a new basic feasible solution by given maximum allocation to the cell (i, j) for which d_{ij} is most negative by making an unoccupied cell empty. Here the cells $(1,1)$ having the negative value $d_{11} = -1$. We draw a closed loop consisting of horizontal and vertical lines beginning and ending at this cell $(1,1)$ and having other corners of some occupied cells. Along this closed loop indicate $+\theta$ and $-\theta$ alternatively at the corners. We have

(+θ)	50	50 (-θ)		
4	1	2	6	9
10		20		90
(-θ) 6	4	(+θ) 3	5	7
30			90	
5	2	6	4	8

From the two cells (1,3) and (2,1) having $-\theta$, we find that the minimum of the allocations that is $\min\{10, 50\} = 10$.

Add this 10 to the cells with $+\theta$ and subtract this 10 from the cells with $-\theta$.

Hence the new basic feasible solution is displayed, in the follow table

10 4	50 1	40 2	6	9
6	4	30 3	5	90 7
30 5	2	6	90 4	8

We see that the above satisfies the vim conditions with $(m+n-1)$ non-negative allocations at independent positions.

So we apply MODI method.

10 4	50 1	40 2	3 6 3	6 9 3	$U_1 = 0$
5 6 1	2 4 2	30 3	4 5 1	90 7	$U_2 = 1$
30 5	2 2 0	3 6 3	90 4	7 8 1	$U_3 = 1$

$V_1 = 4$ $V_2 = 1$ $V_3 = 2$ $V_4 = 3$ $V_5 = 6$

$Z = 1400$

Degeneracy in Transportation problem:

In a transporatation pbm, whenever the no. of non-negative independent allocations is less than $m+n-1$, the transportation pbm is said to be degenerate. Degeneracy may occur either at the initial stage (or) at an intermediate stage at some subsequent iteration.

To resolve degeneracy, we allocat an extremely small amount (which is close to zero) to one (or) more empty cells of the transpotation table (generally minimum cost cells if possible), so that the total no. of occupied cells becomes $(m+n-1)$ at independent position. We denote this small amount by ε (epsilon) satisfying the following conditions:

i) $0 < \varepsilon < x_{ij}$ for all $x_{ij} > 0$

ii) $x_{ij} \pm \varepsilon = x_{ij}$ for all $x_{ij} > 0$

Pbm: Find the non-degenerate basic feasible soln for the following transportation pbm by using NWC rule.

	To				Supply
From	10	20	5	7	10
	13	9	12	8	20
	4	5	7	9	30
	14	7	1	0	40
	3	12	5	19	50
Demand:	60	60	20	10	

Soln,

[10] 10	20	5	7	~~10~~ 0	
[20] 13	9	12	8	~~20~~ 0	
[30] 4	5	7	9	~~30~~ 0	
14	[40] 7	1	0	~~40~~ 0	
3	[20] 12	[20] 5	[10] 19	~~50~~ ~~30~~ ~~10~~ 0	
~~60~~ ~~50~~ ~~30~~ 0	~~60~~ ~~20~~ 0	~~20~~ 0	~~10~~ 0		

$m+n-1$ = no. alloc.

$5+4-1 \neq 7$

$8 \neq 7$

$\geq$ so, go for

Max no of allocations - 1st col & 5th row take any one, & choose the minimum value of unoccupied cell then add 'ε'.

[10] 10	20	5	7	10
[20] 13	9	12	8	20
[30] 4	5	7	9	30
14	[40] 7	1	0	40
[ε] 3	[20] 12	[20] 5	[10] 19	50
60	60	20	10	

For NWC rule,

$Z = 1290 + 3\varepsilon$

Since $\varepsilon = 0$

$Z = 1290$

by MODI method.

$\boxed{10}$ 10	20	5	7	10	$U_1 = 7$
$\boxed{20}$ 13	9	12	8	20	$U_2 = 10$
$\boxed{30}$ 4	5	7	9	30	$U_3 = 1$
14	$\boxed{40}$ 7	1	0	40	$U_4 = -5$
$\boxed{8}$ 3	$\boxed{20}$ 12	$\boxed{20}$ 5	$\boxed{10}$ 19	50	$U_5 = 0$
60	60	20	10		
$V_1 =$ 3	$V_2 =$ 12	$V_3 =$ 5	$V_4 =$ 19		

10	19 1 / 20	12 -7 / 5	26 -19 / 7	10	$U_1 = 7$
13	22 -3 / 9	15 -3 / 12	29 -21 / 8	20	$U_2 = 10$
4	13 -8 / 5	6 1 / 7	20 -11 / 9	30	$U_3 = 1$
-2 16 / 14	7	0 1 / 1	14 -14 / 0	40	$U_4 = -5$
3	12	5	19	50	$U_5 = 0$
60	60	20	10		
$V_1 =$ 3	$V_2 =$ 12	$V_3 =$ 5	$V_3 =$ 19		

Too much -ve value so may take too much steps, so skipping for now

Assignment Method (or) Hungarian method

Pbm: A dept. has 5 employees with 5 jobs to be performed. The time (in hrs) each man will take to perform each job is given in the effectiveness matrix

How should the job allocated, one per employee, so as to minimize the total man-hours?

Employees

Jobs	I	II	III	IV	V
A	10	5	13	15	16
B	3	9	18	13	6
C	10	7	2	2	2
D	7	11	9	7	12
E	7	9	10	4	12.

1st verify for sq. matrix if not add row/col. of zeroes accord.

Soln,

Step 1: Rowwise subtraction

take a min value in a row and subtract it with its row, apply for every row

	I	II	III	IV	V
A	5	$\boxed{0}$	8	10	11
B	$\boxed{0}$	6	15	10	3
C	8	5	$\boxed{0}$	~~0~~	~~0~~
D	~~0~~	4	2	$\boxed{0}$	5
E	3	5	6	~~0~~	8

1. 1st row 1 zero select (□) & cross (x) the corresponding row, column zeroes
2. 2nd row 1 zero " " "
3. 3rd row 0 zero so ignore for now.
4. 4th row 0 zero select (0) & cross (x) corres. row & col. unsele check
5. 5th row no zero bcoz 4th row zero cancelled it.

* Do this for all possible rows

6. 1st col no free zeros
7. 2nd col no free zeros
8. 3rd col 1 zero select (□) & cross (x) the corres. row & col
9. 4th col no free zeros
10. 5th col no free zeros

* Do this for all possible cols.

If no. zeros ≠ no. rows then next step

	I	II	III	IV	V	
A	5	$\boxed{0}$	8	10	11	
B	$\boxed{0}$	6	15	10	3	✓
C	8	5	$\boxed{0}$	~~0~~	~~0~~	
D	~~0~~	4	2	$\boxed{0}$	5	✓
E	3	5	6	~~0~~	8	✓
	✓			✓		

For Row,

1. 5th row (✓) mark becoz no $\boxed{0}$, selected zeroes

For col,

1. 1st, 4th col (✓) mark becoz both $\boxed{0}$, ~~0~~ selected & Crossed zeroes are present

2. Then for the selected col. $\boxed{0}$ selected zero's row is (✓) marked i.e 2nd, 4th row

	I	II	III	IV	V	
A	5	$\boxed{0}$	8	10	11	
B	$\boxed{0}$	6	15	10	3	✓
C	8	5	$\boxed{0}$	~~0~~	~~0~~	
D	~~0~~	4	2	$\boxed{0}$	5	✓
E	3	5	6	~~0~~	8	✓
	✓			✓		

Draw lines for unmarked rows and marked cols.

(✓ - mark)

	I	II	III	IV	V
A	7	0	8	12	11
B	0	4	13	10	1
C	10	5	0	2	0
D	0	2	0	0	3
E	3	3	4	0	6

Choose the min elem value which is not passed line

in this ex, min {6, 15, 3, 4, 2, 5, 5, 6, 8}
= 2

now, Sub min from elem which is not passed line & add min to elem which is passed from both row & col.

	I	II	III	IV	V
A	7	$\boxed{0}$	8	12	11
B	$\boxed{0}$	4	13	10	1
C	10	5	$\cancel{0}$	2	$\boxed{0}$
D	$\cancel{0}$	2	$\boxed{0}$	$\cancel{0}$	3
E	3	3	4	$\boxed{0}$	6

Now, do the same in Step 1 select '0' in row & cross corres. row & col till all possible row and do the same for col also

The select the row, col coord.

A - II B - I C - V D - III E - IV

$Z = 5 + 3 + 2 + 9 + 4$

$= 23$ hrs.

values from original matrix in the qn of the same coord.

Exercise 1:

A dept head has four subordinates and four tasks to be performed. The subordinates differ in efficiency, and the tasks differ in their intrinsic difficulty. His estimate, of time each man would take to perform each task, is given in the matrix below

ANS: 59

Men

Tasks	E	F	G	H
A	18	26	17	11
B	13	28	14	26
C	38	19	18	15
D	19	26	24	10

How should the task be allocated, one to a man, so as to minimize the total min-hours?

Soln,

Tasks	E	F	G	H	
A	7	15	6	$\boxed{0}$	✓
B	$\boxed{0}$	15	1	13	
C	23	4	3	~~0~~	✓
D	9	16	14	~~0~~	✓
				✓	

min {7, 15, 6, 23, 4, 3, 9, 16, 14} = 3

	E	F	G	H	
A	4	12	3	$\boxed{0}$	✓
B	$\boxed{0}$	15	1	16	
C	20	1	$\boxed{0}$	~~0~~	
D	6	13	11	~~0~~	✓
				✓	

min {4, 12, 3, 6, 13, 11} = 3

1	9	~~0~~	~~0~~	✓
$\boxed{0}$	15	1	19	
20	1	$\boxed{0}$	3	✓
3	10	8	$\boxed{0}$	✓
		✓	✓	

min {1, 9, 20, 1, 3, 10} = 1

	E	F	G	H
A	~~0~~	8	$\boxed{0}$	~~0~~
B	$\boxed{0}$	15	2	20
C	19	$\boxed{0}$	~~0~~	3
D	2	9	8	$\boxed{0}$

$z = 17 + 13 + 19 + 10$

$= 59$ ✓

Ex 2: Determine the optimum assignment schedule

	a	b	c	d	e
A	85	75	65	125	75
B	90	78	66	132	78
C	75	66	57	114	69
D	80	72	60	120	72
E	76	64	56	112	68

Ans: 399

Soln,

20	10	$\boxed{0}$	60	10
24	12	~~0~~	66	12
18	9	~~0~~	57	12
20	12	~~0~~	60	12
20	8	~~0~~	56	12

min = 8

12	2	$\boxed{0}$	52	2	✓
16	4	~~0~~	58	4	✓
10	1	~~0~~	49	4	✓
12	4	~~0~~	52	4	✓
12	$\boxed{0}$	~~0~~	48	4	
		✓			

min = 1

11	1	$\boxed{0}$	51	1	✓
15	3	~~0~~	57	3	✓
9	$\boxed{0}$	~~0~~	48	3	✓
11	3	~~0~~	51	3	✓
12	~~0~~	1	48	4	✓
	✓	✓			

min = 1

10	1	~~0~~	50	$\boxed{0}$	
14	3	$\boxed{0}$	56	2	✓
8	$\boxed{0}$	~~0~~	47	2	
10	3	~~0~~	50	2	✓
11	$\boxed{0}$	1	47	3	
		✓			

min = 2

10	1	2	50	$\boxed{0}$	✓
12	1	$\boxed{0}$	54	~~0~~	✓
8	$\boxed{0}$	2	47	2	✓
8	1	~~0~~	48	~~0~~	✓
11	~~0~~	3	47	3	✓
	✓			✓	

min = 8

2	1	2	42	$\boxed{0}$	✓
4	1	$\boxed{0}$	46	~~0~~	✓
~~0~~	~~0~~	2	39	2	✓
$\boxed{0}$	1	~~0~~	40	~~0~~	✓
3	$\boxed{0}$	3	39	3	✓
✓	✓	✓		✓	

min = 39

	a	b	c	d	e
A	2	1	2	3	$\boxed{0}$
B	4	1	$\boxed{0}$	7	~~0~~
C	~~0~~	0	2	0	2
D	$\boxed{0}$	1	~~0~~	1	~~0~~
E	3	0	3	0	3

80 — 66, 64 — 66 — 114, 112 — 75

2-way.

$Z = 80 + 66 + 66 + 112 + 75 = 399$ ✓

$Z = 80 + 64 + 66 + 114 + 75 = 399.$ ✓

Maximization case in the assignment problem

Pbm:

A company has a team of four salesmen and there are four districts where the company wants to start its business. After taking into account the capabilities of salesmen and the nature of the districts, the company estimates that the profit per day in rupees for each salesmen in each district is as below

Districts

Salesman	1	2	3	4
A	16	10	14	11
B	14	11	15	15
C	15	15	13	12
D	13	12	14	15

ANS: 61

Find the assignment of salesmen to various districts will yield max profit.

Soln,

max value : 16 so 16 - x_{ij}

0	6	2	5
2	5	1	1
1	1	3	4
3	4	2	1

$\boxed{0}$	6	2	5
1	4	$\boxed{0}$	$\cancel{0}$
$\cancel{0}$	$\boxed{0}$	2	3
2	3	1	$\boxed{0}$

no. row/col = no. zeroes

4 = 4

$z = 16 + 15 + 15 + 15 = 61$

Pbm: Maximize the annual sales

Territories

salesmen	1	2	3	4
A	60	50	40	30
B	40	30	20	15
C	40	20	35	10
D	30	30	25	20

ANS : 145

Soln,

max = 60

0	10	20	30
20	30	40	45
20	40	25	50
30	30	35	40

$\boxed{0}$	10	20	30	✓
$\cancel{0}$	10	20	25	✓
$\cancel{0}$	20	5	30	✓
$\cancel{0}$	$\boxed{0}$	5	10	
✓				

no. zeroes = no. row/col

min = 5

$\boxed{0}$	5	15	25	✓
~~0~~	5	15	20	✓
~~0~~	15	$\boxed{0}$	25	
5	$\boxed{0}$	5	10	
✓		min = 5		

$\boxed{0}$	~~0~~	10	20	✓
~~0~~	~~0~~	10	15	✓
5	15	$\boxed{0}$	25	
10	$\boxed{0}$	5	10	✓
✓	✓	min = 5		

$\boxed{0}$	~~0~~	5	15	✓
~~0~~	~~0~~	5	10	✓
10	20	$\boxed{0}$	25	✓
10	$\boxed{0}$	~~0~~	5	✓
✓	✓	✓	min = 5	

0	0	5	10
0	0	5	5
10	20	$\boxed{0}$	20
10	~~0~~	~~0~~	$\boxed{0}$

2 cases

$z = 60 + 30 + 35 + 20 = 145$

$z = 40 + 50 + 35 + 20 = 145$

Scheduling PERT and CPM

Intro:

A project is defined as a combination of interrelated activities all of which must be executed in a certain order to achieve a set goal. A large and complex project involves usually a number of interrelated activities requiring men, machines and materials. It is impossible for the management to make and execute an optimum schedule for such a project just by intuition, based on the organisational capabilities and work experience.

A systematic scientific approach has become a necessity for such projects. So number of methods applying network scheduling techniques has been developed.

Programme Evaluation Review Technique (PERT) and Critical Path Method (CPM) are two of the many network technique which are widely used for planning, scheduling and controlling large complex projects.

The three main managerial functions for any projects are,

1. Planning
2. Scheduling
3. Control

1. Planning:

This phase involves a listening of tasks or jobs that must be performed to complete a project under consideration. In this phase, men, machines and materials required for the project in addition to the estimates of cost and duration of various activities of the project are also determined.

2. Scheduling:

This phase involves the laying out of the actual activities of the projects in a logical sequence of time in which they have to be performed. Men and material requirements as well as the expected completion time of each activity at each stage of the project are also determined.

3. Control:

This phase consists of reviewing the progress of the project whether the actual performance is according to the planned schedule and finding the reasons for difference, if any, between the schedule and performance. The basic aspect of control is to analyse and correct this difference by taking remedial action wherever possible.

PERT and CPM are especially useful for scheduling and controlling.

Basic Terminologies:

Activity:

Activity is a task or an item of work to be done in a project. An activity consumes resources like time, money, labour etc.

An activity is represented by an arrow with a node (event) at the beginning and a node (event) at the end indicating the start and termination (finish) of the activity

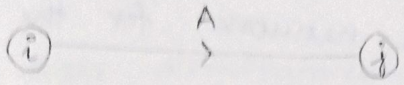
Nodes are denoted by circles

Since this is a logical diagram length or shape of the arrow has no meaning the direction indicates the progress of the activity

Initial node and the terminal nodes are numbered as i-j (j>i) respectively.

Example,

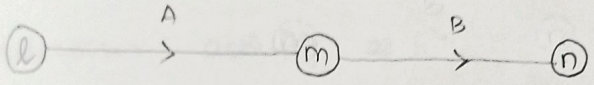
A is activity whose initial node i and the terminal node j then it denoted diagramatically as,



The name of the activity is written over the arrow, not inside the circle. The diagram in which arrow represents an activity is called arrow diagram.

The initial and terminal nodes of activities are also called tail and head events.

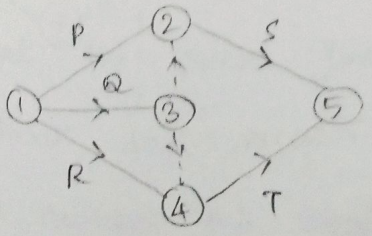
If an activity B can start immediately after an activity A then it is denoted by,



'A' is called the immediate predecessor of 'B' and 'B' is called the immediate successor of 'A'

Qn) If there are 5 activities P, Q, R, S and T such that P, Q, R have no immediate predecessors but S and T have immediate predecessors P, Q and Q, R respectively. Represent this situation by a network.

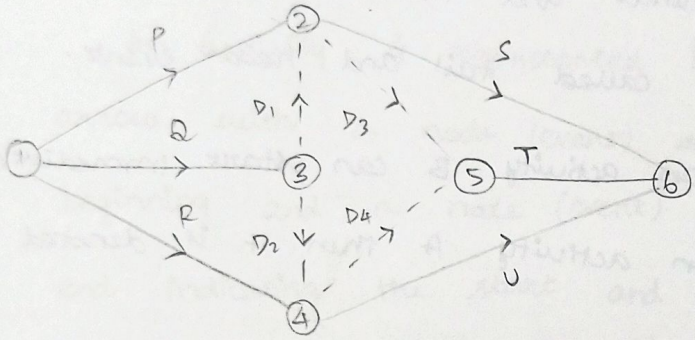
Soln,



Qn) Draw the network for the project whose activities and their predecessor relationships are given below,

Activity:	P	Q	R	S	T	U
Predecessor: (immediat)	-	-	-	P, Q	P, R	Q, R

Soln,



Qn) A, C, D can start simultaneously.

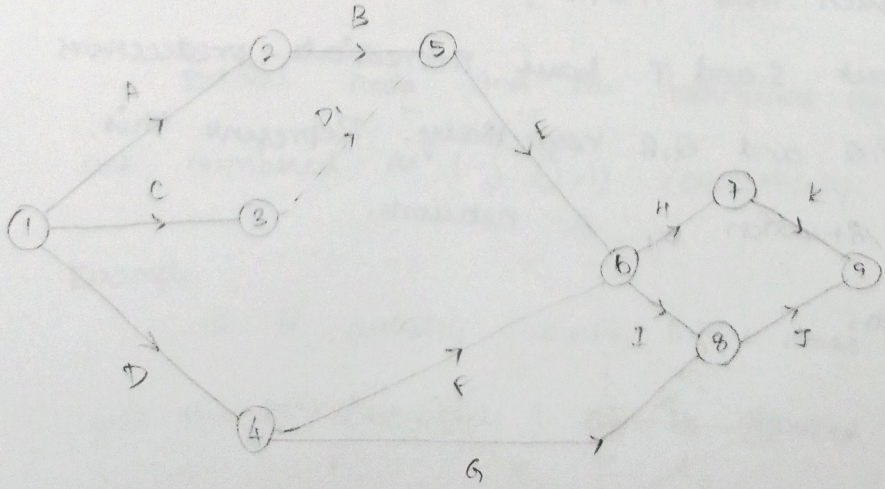
B → E D → F E → H
C → E D → G F → I

E > B, C ; F, G > D ; H, I > E, F ;

J > I, G ; K > H ; B > A ;

F → J H → K A → B

Soln,

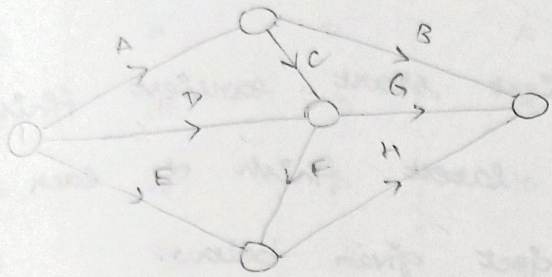


Qn) A, D, E start simultaneously

$A \to B$, $A \to C$; $D \to G$, $C \to F$; $E \to H$, $F \to H$

B, C > A ; G, F > D, C ; H > E, F ;

Soln,



Nodes may be numbered using the rule given.

Ford and Fulkerson's Rule

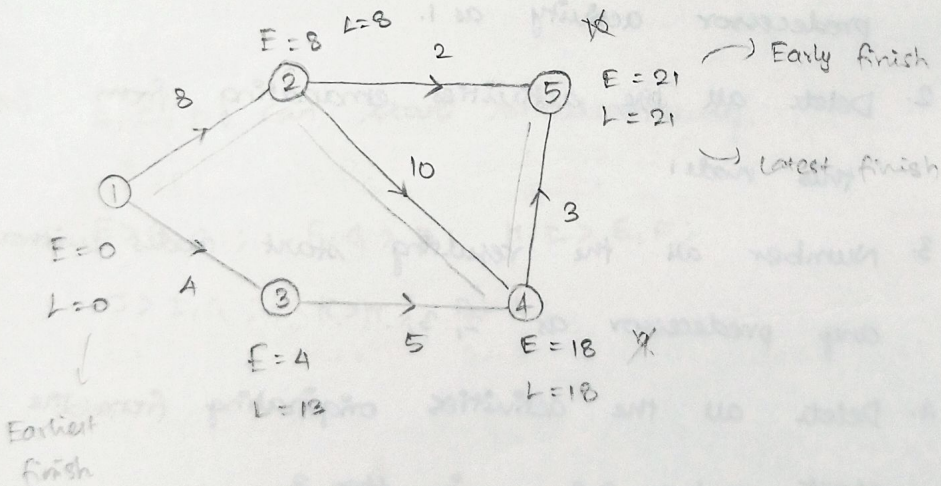
1. Number the start node which has no predecessor activity as 1.
2. Delete all the activities emanating from this node.
3. Number all the resulting start nodes without any predecessor as 2, 3, . . .
4. Delete all the activities originating from the start nodes 2, 3, . . . in step 3.
5. Number all the resulting new start nodes without any predecessor next to the last number used in step 3.
6. Repeat the process until the terminal node without any sucessor activity is reached and number this terminal node suitably

Immediate predecessor (successor) will be simply called as predecessor (successor) unless otherwise stated.

Qn) Compute the earliest start, earliest finish, latest start and latest finish of each activity of the project given below.

Activity:	1-2	1-3	2-4	2-5	3-4	4-5
Duration: (in days)	8	4	10	2	5	3

Soln,



Critical path: 1-2, 2-4, 4-5 (Longest path)

Activity	Duration	Earliest Start ES	Earliest Finish EF $EF = E_S + t_{ij}$	Latest Start $LS = LF - t_{ij}$	Latest Finish LF
1-2	8	0	8	0	8
1-3	4	0	4	9	13
2-4	10	8	18	8	18
2-5	2	8	10	19	21
3-4	5	4	9	13	18
4-5	3	18	21	18	21

Critical path:

Path connecting the first initial node to the very last terminal node of the longest duration in any project net work is called the critical path.

5.

Implementing problems on sensitive analysis

a) Effect of changes of rhs constraint

max $z = 2x_1 + x_2 + 4x_3 - x_4$

$x_1 + 2x_2 + x_3 - 3x_4 \leq 8$

$-x_2 + x_3 + 2x_4 \leq 0$

$2x_1 + 7x_2 - 5x_3 - 10x_4 \leq 21$

$x_1, x_2, x_3, x_4 \geq 0$

i) find effect of change of b_2 to 11

ii) find effect of chang of obj to (3, -2, 4)

Effect of changes in cost coefficient

max $z = 4x_1 + 6x_2 + 2x_3$ s.t,

$x_1 + x_2 + x_3 <= 3$

$x_1 + 4x_2 + 7x_3 <= 9$ & $x_1, x_2, x_3 >= 0$

c_3 to 12

obj to (2, 8, 4, 0, 0)

Queueing Theory

Arrival Rate (λ) - No. arrivals per unit time

Service Rate (μ) - mean no. customers served per unit time.

Model 1 :

Stability measure : (ρ)

$$\rho = \frac{\lambda}{\mu} < 1$$

Additional measures:

Avg. no customers in system = $L_s = \frac{\rho}{1-\rho}$

Avg. no. customers waiting in queue = $L_q = \frac{\rho^2}{1-\rho}$

Avg. time customer spends in system round = $W_s = \frac{1}{\mu - \lambda}$

Avg. time customer spends waiting in queue = $W_q = \frac{\lambda}{\mu(\mu-\lambda)}$

Qn) Petrol bunk, 1 unit, poisson fashion with avg. 10 cars per hour. Service time: 3 mins on avg. Find stability & additional measures.

Soln) $\lambda = 10$ cars per hour $= 10/60$ per min

$= 1/6$ per min

$\mu = 1/3$ per min

$\rho = \lambda/\mu = 1/6 \,/\, 1/3 = 3/6 = 1/2 = 0.5$

$L_s = \frac{\rho}{1-\rho} = \frac{0.5}{0.5} = 1$

$L_q = \frac{\rho^2}{1-\rho} = \frac{0.25}{0.5} = 0.5$

$W_s = \frac{1}{\mu-\lambda} = \frac{1}{1/3 - 1/6} = 6$

$W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{1/6}{1/2\,(1/3 - 1/6)} = \frac{1}{2} \times \frac{1}{1/6} = 3$

$L_s = L_q + \lambda/\mu$

$W_s = \frac{L_s}{\lambda}$

$W_q = \frac{L_q}{\lambda}$

Qn) Inter. arrival time $(1/\lambda)$ = 12 mins $\quad 1/\lambda = 12$

Mean service rate = 4 mins $\quad \mu = 1/4$

Model 2

$\rho = \frac{\lambda}{S \times \mu} < 1$ $\quad$ S - no. servers

Add. measures:

$L_q = P_s \times \frac{\rho}{(1-\rho)^2}$, where $P_s = \frac{(\lambda/\mu)^s \cdot P_0}{s!}$

$P_0 = \sum_{n=0}^{s-1} \left[\frac{(S\rho)^n}{n!} + \frac{(S\rho)^s}{s!(1-\rho)} \right]^{-1}$

$L_s = \frac{\lambda}{\mu} + L_q$

P_0 - prob. server idle

$W_s = \frac{L_s}{\lambda}$

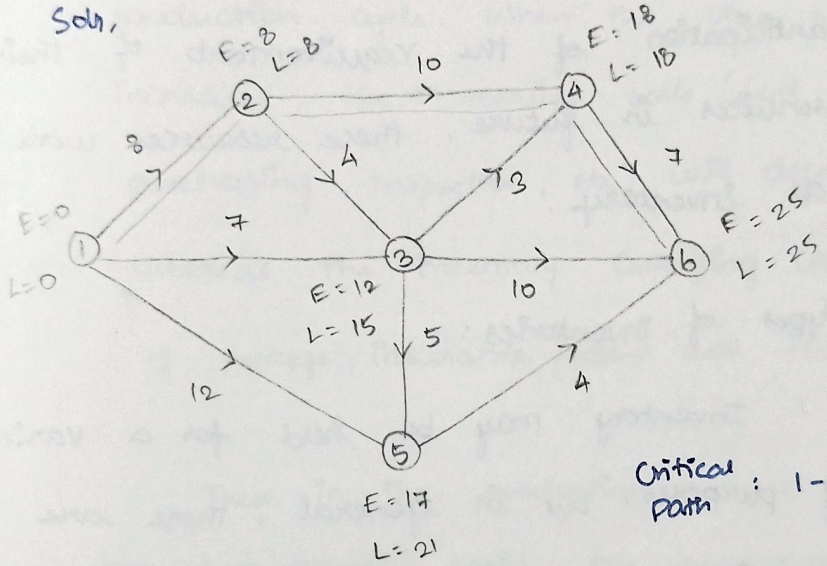
P_s - prob. server busy

$W_q = \frac{L_q}{\lambda}$

Qn) calculate earliest start, earliest finish, latest start, latest finish of each activity of the project given below and determine the critical path of the project.

Activity:	1-2	1-3	1-5	2-3	2-4	3-4	3-5	3-6	4-6	5-6
Duration: (in weeks)	8	7	12	4	10	3	5	10	7	4

Soln,



Critical path : 1-2-4-6

Activity	Duration	Earliest		Latest	
		ES	EF	LS	LF
1-2	8	0	8	0	8
1-3	7	0	7	8	15
1-5	12	0	12	9	21
2-3	4	8	12	11	15
2-4	10	8	18	8	18
3-4	3	12	15	15	18
3-5	5	12	17	16	21
3-6	10	12	22	15	25
4-6	7	18	25	18	25
5-6	4	17	21	21	25

Inventory Control

Intro:

Inventory consists of usable but idle resources. The resources may be of any type - men, materials, machines, etc... For example, if a company purchases a machine or appoints an expert in anticipation of the requirement of their services in future, these resources work as inventory.

Types of inventories:

Inventory may be held for a variety of purposes, but in general, there are following five types of inventories that an enterprise can use for serving these purposes.

1. Moment inventories (Transit or pipeline inventories)
2. Buffer inventories
3. Anticipation inventories (Seasonal inventories)
4. Decoupling inventories
5. Lot size inventories

The concept of EOQ:

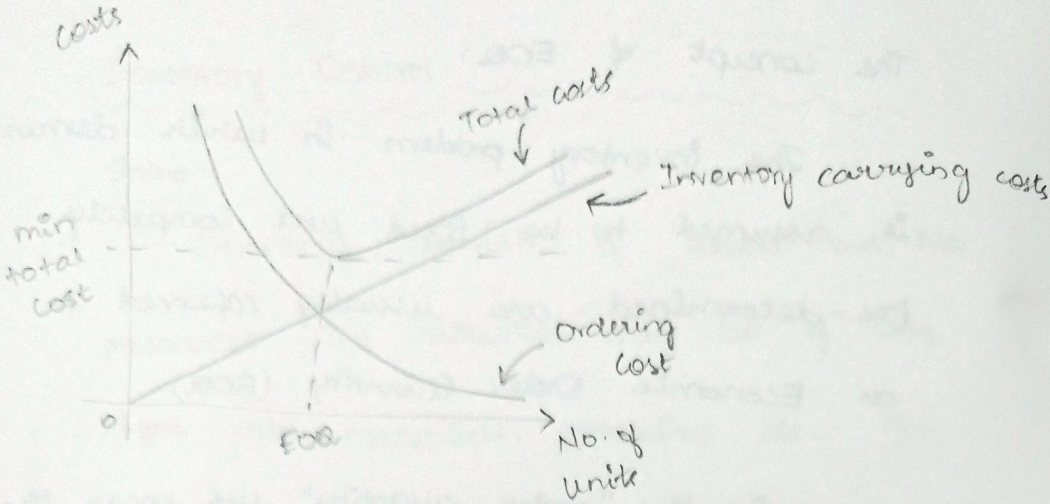
The inventory problem in which demand is assumed to be fixed and completely pre-determined are usually referred to as Economic Order Quantity (EOQ).

By the "order quantity" we mean the quantity produced (or) procured during one production cycle. When the size of order increases, the ordering costs (cost of purchasing, inspection, etc) will decrease whereas the inventory carrying costs (cost of storage, insurance, etc) will increase.

Thus, in the production process there are two opposite costs, one encourages the increase in the ordering size and other discourages.

Economic Order Quantity (EOQ) is that size of order which minimizes total annual cost of carrying and cost of ordering.

The two opposite costs can shown graphically by plotting them against the order size as shown next pg.



The evident from the above graph that the minimum total cost occurs at the point where the ordering cost and Inventory carrying costs are equal

Determining Inventory Problems with no storage:

There are five variations of the EOQ problem when demand is assumed to be fixed and completely pre-determined.

Case 1: The fundamental problem of EOQ

The objective of the study of this problem is to determine an optimum order quantity (EOQ) such that the total inventory cost is minimized. We illustrate this problem, under consideration using the following assumption:

i) Demand is known and uniform

ii) Let D denotes the total number of

units purchased / produced or supplied per time period and Q denotes the lot size in each problem sum.

iii) Shortages are not permitted. That is, as soon as the level of inventory reaches zero, the total inventory is replenished.

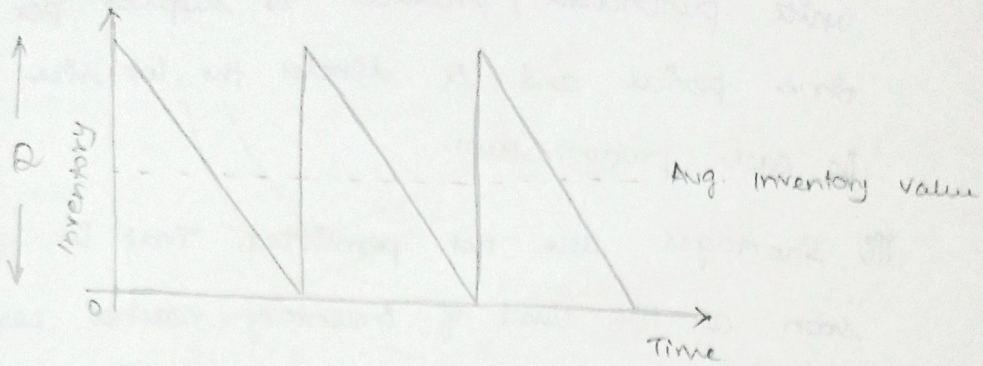
iv) Production (or) supply of commodity is instantaneous (abundant availability)

v) Load time is zero

vi) Set-up cost (or ordering cost) per production run on procurement cost is C_0 (or) A.

vii) Holding cost is C_1 per unit in inventory. that is $C_1 = IC$, where c is the unit cost. I is called the inventory carrying charge expressed as a % of the value of the average inventory.

This fundamental situation can be shown on an inventory-time diagram, with Q on the vertical axis and time on the horizontal axis. The total time period (one year) is divided into n parts.



Total time period (one yr) is divided equally into n divisions.

$I = nt$

$D = nQ$

Total inventory over the time period t is clearly

$\frac{1}{2} Qt \left(n \times \frac{1}{2} Q \frac{t}{n}\right)$

Inventory on any given day $= \frac{\frac{1}{2}Qt}{t} = \frac{1}{2}Q$

Avg amt of inventory at hand on any day is $\frac{1}{2}Q$

Annual inventory holding cost : $f(Q) = \frac{1}{2} Q C_1$

Annual cost associated with runs of size Q: $g(Q) = n C_0 = \frac{D}{Q} C_0$

Ordering cost = Inventory carrying cost - for min tot. cost

So,

$f(Q) = g(Q) \Rightarrow \frac{1}{2} Q C_1 = \frac{D}{Q} C_0$

$Q^2 = \frac{2DC_0}{C_1}$

$Q_0 = \sqrt{\frac{2DC_0}{C_1}}$ - optimal value of Q

Economic (optimum) lot size formula

Another repr: $Q_0 = \sqrt{\frac{2AD}{IC}}$ - in terms of economic order value

$$Q_0 = \sqrt{\frac{2DC_0}{C_1}}$$

This is known as the economic (optimum) lot size formula due to R.H. Wilson

The above EOQ formula can also be expressed in terms of the economic order value as follows.

$$Q^0 = \sqrt{\frac{2AD}{IC}}$$

– (Qn oil engine problem.)

characteristics of case 1

1. Optimum no. order placed per year

$$n^0 = D/Q^0 = \sqrt{DC_1/2C_0}$$

2. Optimum length of time between orders

$$t^0 = T/n^0 = T\sqrt{2C_0/DC_1} \text{ (or) } \sqrt{2C_0/DC_1}$$

where T (total time horizon) is one year

The optimum length of time between orders is also called the Economic Review Point (ERP).

3. Total annual expected cost (TEC)

= Annual inventory carrying cost + Annual ordering cost

$$= \tfrac{1}{2} Q^0 C_1 + D/Q_0 \, C^0$$

$$= \tfrac{1}{2}\sqrt{\frac{2DC_0}{C_1}} \times C_1 + D \times \sqrt{\frac{C_1}{2DC_0}} \times C_0$$

$$= \sqrt{2DC_1C_0}$$

Prob:

A manufacturing company purchases 9000 parts of a machine for its annual requirements, ordering one month usage at a time. Each part costs ₹20 and the ordering cost is ₹15 and the carrying charge 15% of the avg inventory per year. You have been assigned to suggest a more economical purchasing policy for the company. What advise would you offer and how much would it save the company per year?

Ans:

We are given

$D = 9000$ parts per year $\quad C_o = Rs\ 15 \quad C = Rs\ 20$

$C_1 = 15\%$ of the avg inventory per year

$= Rs\ 20 \times \frac{15}{100} = Rs\ 3$ each part per year

$$Q^\circ = \sqrt{\frac{2DC_o}{C_1}} = \sqrt{\frac{2 \times 15 \times 9000}{3}} = 300 \text{ units}$$

$Q^\circ = 300$ units

Min avg yearly cost $= \sqrt{2 C_1 C_o D}$

$= \sqrt{2 \times 3 \times 15 \times 9000} = Rs\ 900$

If the company follows the policy of ordering every month, then the annual ordering cost is Rs 12×15 (or) Rs. 180, and lot size of inventory each month $= 9000/12 = 750$ Q

Avg. inventory at any time $= \frac{1}{2}Q = 375$

The storage cost at any time $= 375C_1 = 1125$

Tot. annual cost = Annual ordering cost + Storage cost at any time

$= 1125 + 180 = 1305$

Hence, the company purchases 300 parts at time intervals of 1/30 year instead of ordering 750 parts each month. The net saving of the company will be

$= \text{Rs } 1305 - \text{Rs } 900 = \text{Rs } 405$ per year

Pom:

A company operating 50 weeks in a year is concerned about its stocks of copper cable. This cost Rs 240 a metre and there is a demand for 8000 metres a week. Each replenishment costs Rs 1050 for administration and Rs 1650 for delivery, while holding costs are estimated at 25% of value held a year. Assuming no storages are allowed, what is the optimal inventory policy for the company?

How would this analysis differ if the company wanted the maximize profit rather than minimize cost?

What is gross profit if the company sell cable for Rs 360 a metre?

Soln :

$D = 8000 \times 50 = 400\,000$ metre per yr

purchase cost $c = 240$ a metre

Ordering cost $C_o = 1050 + 1650 = Rs\ 2700$

Holding cost $C_1 = Ic = \frac{25}{100} \times 240 = Rs\ 60$ a metre

$$Q_o = \sqrt{\frac{2DC_o}{C_1}} = \sqrt{\frac{2 \times 400000 \times 2700}{60}}$$

$$= \sqrt{36000000} = 6000\,m$$

Total inventory cost (TC) = Purchase cost + Total expected cost

$$= Dxc + \left[\frac{1}{2} Q^o C_1 + \frac{D}{Q^o} C_o\right]$$

$$= (4\,000\,000 \times 240) + \left(\frac{1}{2} \times 6000 \times 60 \times \frac{400000}{6000} \times 2700\right)$$

$$= 96\,000\,000 + 3\,60\,000$$

$$= 96\,360\,000$$

With a turnover in excess of 96 million rupees a year inventory costs are only Rs 3 60 000 (or) 0.36 percentage. This figure usually low but any well run organisation should try to make all the savings it can. If the company wanted to maximize profit rather than the minimize cost, the analysis used would remain exactly the same. This can be demonstrated by defining selling price (SP) per unit so that gross profit per unit time becomes :

profit = Revenue - Cost

$= D \times SP - (D \times C + \frac{D}{Q} C_0 + \frac{1}{2} Q C_1)$

$= (4\,00\,000 \times 360) - 9\,63\,60\,000$

Gross profit $= 14\,40\,00\,000$

$= 9\,63\,60\,000$

$4\,76\,40\,000$ a year

Deterministic Inventory problem with shortages:

In a business concern, if shortages occur then these can be classified into the following two categories:

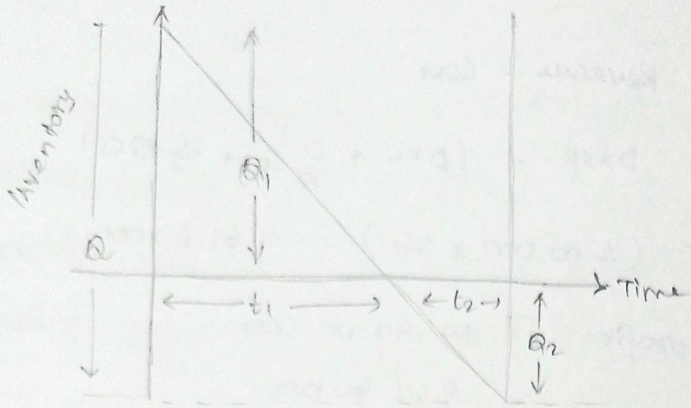
a) As soon as the desired units of a certain commodity arrive in inventory, The back orders are satisfied.

b) Shortages are lost sales.

Case 1) Problem of EOQ with instantaneous production and variable order:

The problem that we now discuss is same as that was discussed in the previous topic (case 1) with the difference that the shortages are now permitted

Let C_2 be the shortage cost per unit of time per unit quantity. This inventory situation can also be illustrated graphically as shown in following.



Here the total time period is one year and is divided into equal parts, say of interval t

During the internal t_1, the items are drawn from the inventory as needed. During t_2, orders for the item are being accumulated but not filled. Then at the end of interval t an amount Q is produced (delivered) the amount Q has been divided into Q_1 and Q_2 such that $Q = Q_1 + Q_2$ where Q_1 denotes the amount which goes into inventory and Q_2 denotes the amount which is immediately taken to satisfy past orders or unfilled demand

The problem is now concerned with the areas of triangle above the time axis (representing items in inventory) and below the same (axis representing items in shortage)

Now, total inventory over the time period

$t = \frac{1}{2} Q_1 t_1$

Average inventory at any time = $\frac{1/2\, Q_1 t_1}{t}$

Annual inventory holding cost = $C_1 \left[\frac{1/2\, Q_1 t_1}{t}\right]$

Similarly,

Total amt of shortage over time period $t = 1/2\, Q_2 t_2$

Annual shortage costs = $C_2 (1/2\, Q_2 t_2) / t$

Annual cost associated with runs of size $Q = nC_0 = D/Q\, C_0$

Since, D/Q runs are produced in each year

$\therefore$ Total annual cost is given by

$$T_c = \frac{\left[C_1 (1/2\, Q_1 t_1) + C_2 (1/2\, Q_2 t_2)\right]}{t} + \frac{D}{Q} C_0$$

The optimum qualities are given by

$$Q^\circ = \sqrt{\frac{2C_0 D}{C_1}} \sqrt{\frac{C_1 + C_2}{C_2}}$$

$$Q_1^\circ = \left(\frac{C_2}{C_1 + C_2}\right) Q^\circ = \sqrt{\frac{C_2}{C_1 + C_2}} \sqrt{\frac{2C_0 D}{C_1}}$$

Characteristics of case (1):

1. Time between receipt of orders (when to orders)

$$t^\circ = \frac{Q^\circ}{D} = \sqrt{\frac{2C_0}{DC_1}} \times \sqrt{\frac{C_1 + C_2}{C_2}}$$

2. Total optimum inventory cost

$$TC^{\circ} = \sqrt{2DC_oC_1}\sqrt{\frac{C_2}{C_1+C_2}}$$

3. Max inventory level

$$Q^{\circ} - Q_1^{\circ} = Q^{\circ}\left(1 - \frac{C_1}{C_1+C_2}\right)$$

$$= \sqrt{\frac{2C_oD}{C_1}}\sqrt{\frac{C_2}{C_1+C_2}}$$

Pbm) The demand for a certain item is 16 units per period. Unsatisfying demand causes a shortage cost of Rs 0.75 per unit per short period. The cost of initiating purchasing action is Rs 15.00 per purchase and the holding cost is 15% of average inventory valuation per period. Item cost is Rs 8.00 per unit (Assume that the shortages are being back ordered at the above mentioned cost) Find the min cost purchase quantity.

Soln,

$D = 16$

$C_2 = 0.75 \qquad C_o = 15$

$C_1 = 0.15 \times 8 = 1.2$

$$Q^{\circ} = \sqrt{\frac{2C_oD}{C_1}} \times \sqrt{\frac{C_1+C_2}{C_2}}$$

$$Q^{\circ} = \sqrt{\frac{2\times15\times16}{1.2}} \times \sqrt{\frac{1.2+0.75}{0.75}}$$

$Q^o = \sqrt{400} \times \sqrt{2.6}$

$Q^o = 32.25$

Min inventory cost,

$$TC = \sqrt{2DC_oC_1} \times \sqrt{\frac{C_2}{C_1 + C_2}}$$

$$= \sqrt{2 \times 15 \times 16 \times 1.2} \times \sqrt{\frac{0.75}{1.2 + 0.75}}$$

$$= \sqrt{576} \times \sqrt{0.385}$$

$$= 14.89$$

det inv pbm - no shortage

Qn) An oil engine manufacturer purchased at the rate of ₹42 / piece from a vendor. The requirement of these lubrications is ₹1800 per year. What should be the order quantity per order if the cost per placement of an order is ₹16 and the inventory carrying charge (₹ per yr) is 20 paise?

Soln.

$D = 1800$

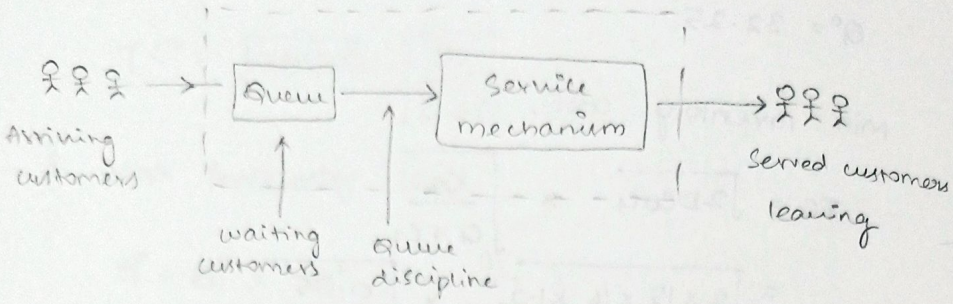
$C_o = 16$

$C_1 = IC = (0.2)(42) = 42/5 = 8.4$

$$Q_o = \sqrt{\frac{2DC_o}{C_1}} = \sqrt{\frac{3600\,(16) \times 5}{42}} = \sqrt{\frac{3600 \times 80}{42}}$$

$$= 82.807 \simeq 83$$

Queueing Theory :



Arrival rate - λ (customers per unit time)

Service rate - μ (customers per unit time)

i) If $\lambda > \mu$, queue formed and increase indefinitely, service facility would always be busy and service system fails.

ii) If $\lambda \leq \mu$, no queue formed, hence no waiting time, proportion of time the service facility would be idle $1 - \lambda/\mu$

$\rho = \lambda/\mu$ is the proportion of time that a server actually spends with customers arriving per unit of time and μ stands for the average no. of customers completing service per unit of time

Qn) A repairman finds that the time spent on his job has an exponential distribution with mean 30 mins. If he repairs sets in the order in which they come in and if the arrival of sets is approximately poisson

with an average rate of 10 per 8 hour day

i) What is repairman's expected idle time each day?

ii) How many jobs are ahead of the average set just brought in?

Soln.

$\lambda = 10$ per day (8hrs)

$\mu = 16$ per day (8hrs)

Service is 30 mins per persons so for 8 hrs, 16 persons can be serviced.