SASTRA
ENGINEERING · MANAGEMENT · LAW · SCIENCES · HUMANITIES · EDUCATION
DEEMED TO BE UNIVERSITY
(U/S 3 OF THE UGC ACT, 1956)
THINK MERIT | THINK TRANSPARENCY | THINK SASTRA

Unit - II

2.11 DC Transient Analysis – Forced Response of RL & RC Circuit

Dr.Santhosh.T.K.

Syllabus

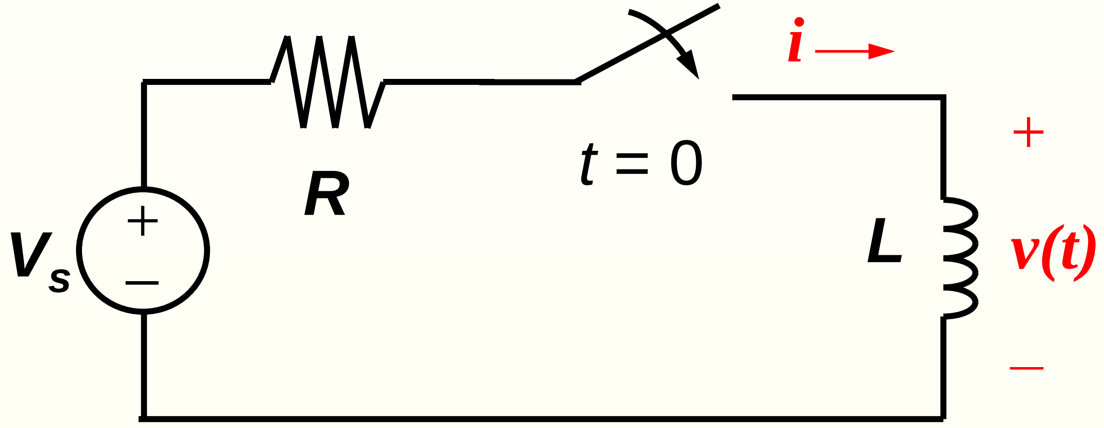
UNIT – II **14 Periods**

DC Circuit Analysis: Voltage source and current sources, ideal and practical, Kirchhoff's laws and applications to network solutions using mesh analysis, - Simplifications of networks using series- parallel, Star/Delta transformation, DC circuits-Current-voltage relations of electric network by mathematical equations to analyse the network (Superposition theorem, Thevenin's theorem, Maximum Power Transfer theorem), Transient analysis of R-L, R-C and R-L-C Circuits.

AC Steady-state Analysis: AC waveform definitions - Form factor - Peak factor - study of R-L - R-C -RLC series circuit - R-L-C parallel circuit - phasor representation in polar and rectangular form - concept of impedance - admittance - active - reactive - apparent and complex power - power factor, Resonance in R-L-C circuits - 3 phase balanced AC Circuits

Step Response of RL Circuit

- The switch is closed at time t = 0.



- After switch is closed, using KVL

$$V_s = Ri(t) + L\frac{di}{dt} \longrightarrow \textbf{(1)}$$

Continue

- Rearrange the equation;

$$\frac{di(t)}{dt} = \frac{-Ri(t)+V_s}{L} = \frac{-R}{L}\left(i(t)-\frac{V_s}{R}\right) \longrightarrow \textbf{(2)}$$

$$di = \frac{-R}{L}\left(i-\frac{V_s}{R}\right)dt \longrightarrow \textbf{(3)}$$

$$\frac{-R}{L}dt = \frac{di}{i(t)-V_s/R} \longrightarrow \textbf{(4)}$$

$$-\frac{R}{L}\int_0^t dv = \int_0^{i(t)} \frac{du}{u-(V_s/R)} \longrightarrow \textbf{(5)}$$

Continue

- Therefore:

$$-\frac{R}{L}t = \ln\frac{i(t)-(V_s/R)}{I_0-(V_s/R)} \longrightarrow \textbf{(5)}$$

- Hence, the current is;

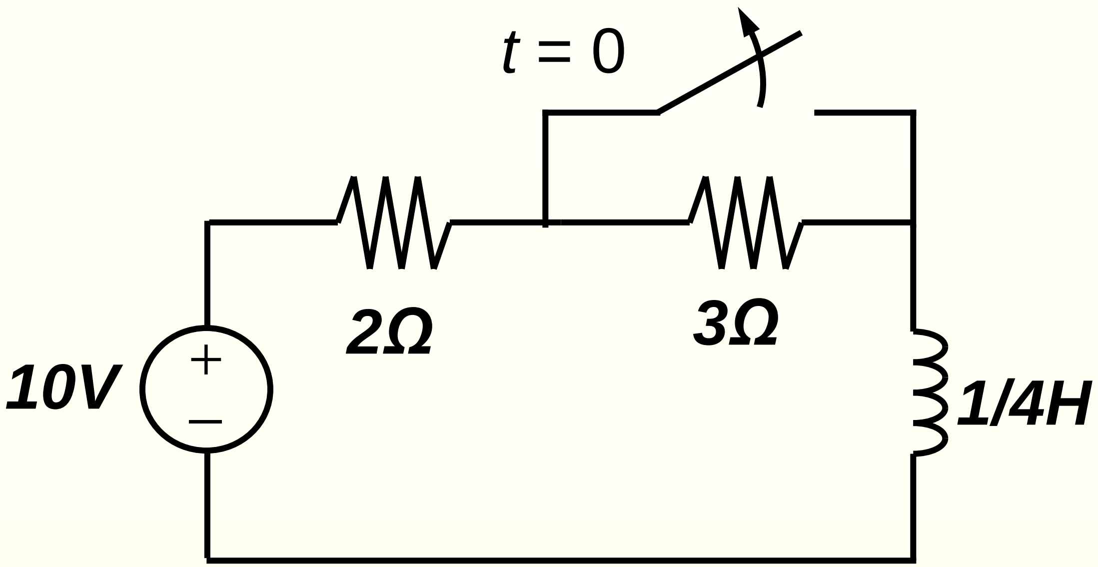
$$\boxed{i(t) = \frac{V_s}{R} + \left(I_o - \frac{V_s}{R}\right)e^{-(R/L)t}}$$

- The voltage;

$$\boxed{v(t) = (V_s - I_o R)e^{-(R/L)t}}$$

Example

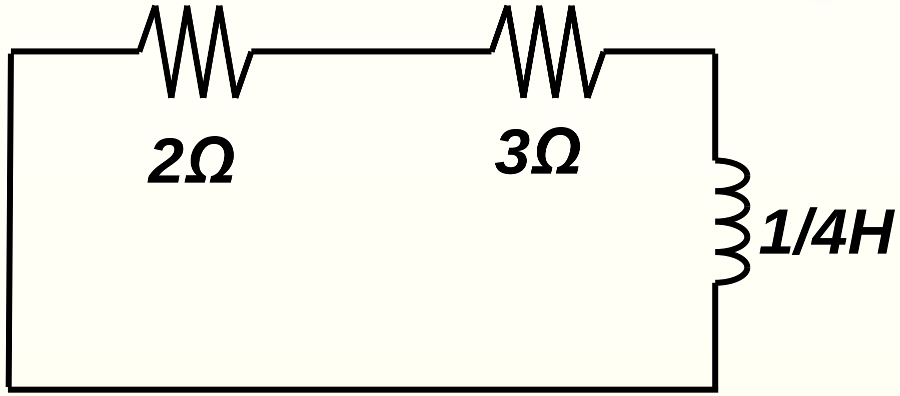
The switch is closed for a long time at t = 0, the switch opens. Find the expressions for $i_L(t)$ and $v_L(t)$.



Solution

Step 1:

Find τ for t > 0. The switch was opened. Draw the equivalent circuit. Short circuit the voltage source.



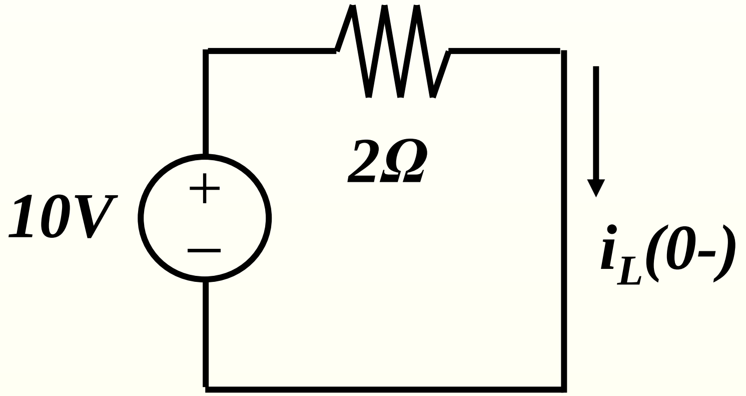
$$R_T = (2+3)\Omega = 5\Omega$$

$$\tau = \frac{L}{R_T} = \frac{1}{20}s$$

Continue

Step 2:

At t = 0^-, the switch was closed. Draw the equivalent circuit with 3Ω shorted and the inductor behaves like a short circuit.



$$i_L(0^-) = 10/2 = 5A$$

Continue

Step 3:

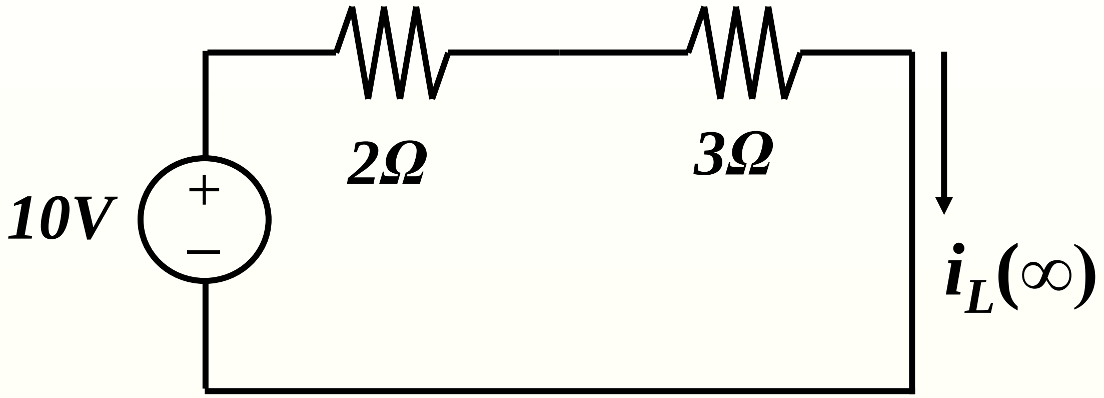
At t = 0^+, the instant switch was opened. The current in inductor is continuous.

$$I_0 = i_L(0^+) = i_L(0^-) = 5A$$

Step 4:

At t =∞, that is after a long time the switch has been left opened. The inductor will once again be behaving like a short circuit.

Continue



$$i_L(\infty) = V_s / R_T = 2A$$

Hence:

$$i_L(t) = \frac{V_s}{R} + \left(I_o - \frac{V_s}{R} \right) e^{-(R/L)t}$$

$$\boxed{i_L(t) = 2 + 3e^{-20t}\, A}$$

Continue

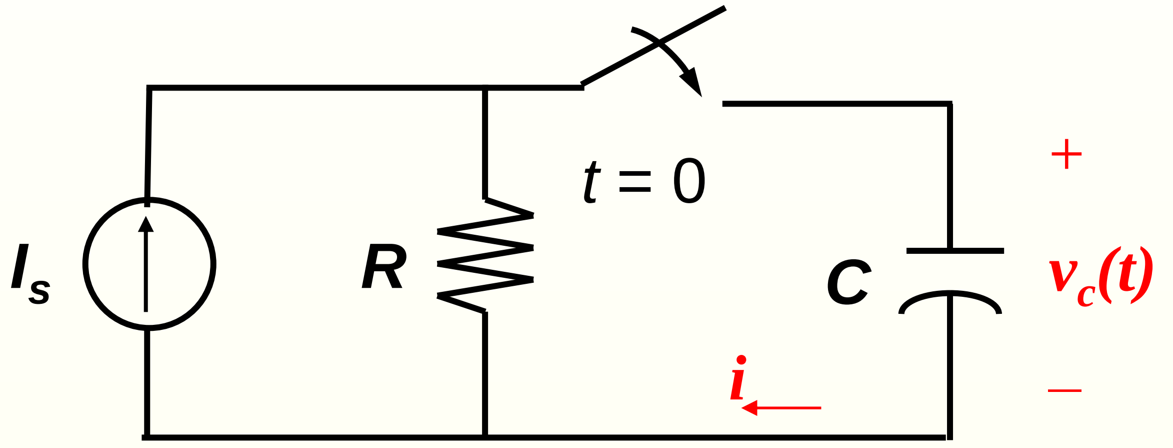
- And the voltage is:

$$v_L(t) = (V_s - I_o R)e^{-(R/L)t}$$

$$\boxed{v_L(t) = -15e^{-20t} V}$$

Step Response of RC Circuit

- The switch is closed at time t = 0



- From the circuit;

$$\boxed{I_s = C\frac{dv_c}{dt} + \frac{v_c}{R}} \longrightarrow \textbf{(1)}$$

Continue

- Division of Equation (1) by C gives;

$$\boxed{\frac{I_s}{C} = \frac{dv_c}{dt} + \frac{v_c}{RC}} \longrightarrow \textbf{(2)}$$

- Same mathematical techniques with RL, the voltage is:

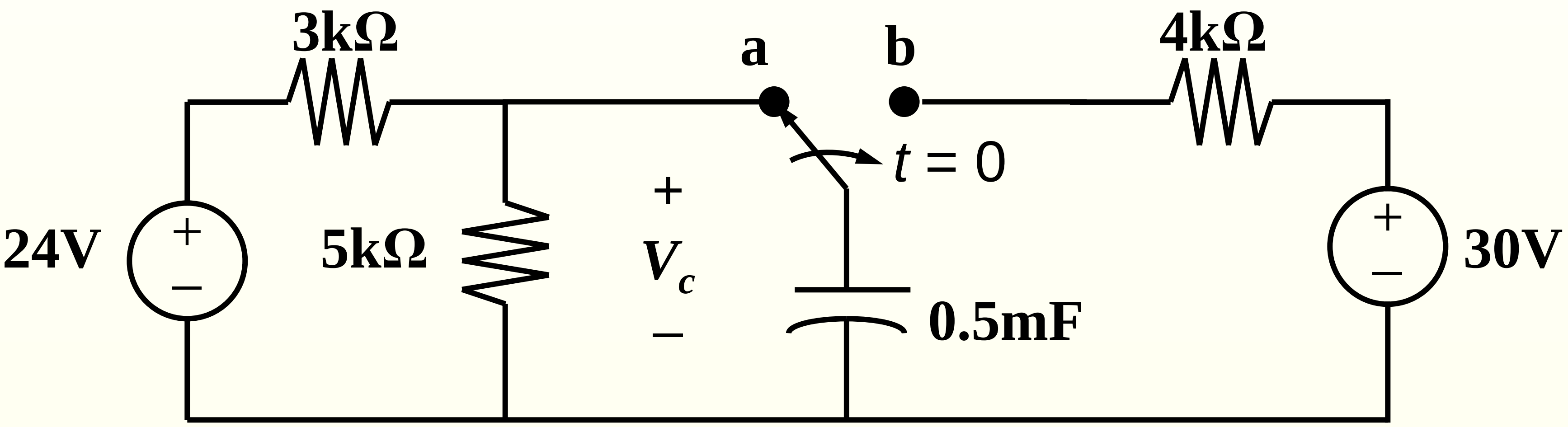
$$v_c(t) = I_s R + (V_o - I_s R)e^{-t/RC}$$

- And the current is:

$$i(t) = \left(I_s - \frac{V_o}{R} \right) e^{-t/RC}$$

Example

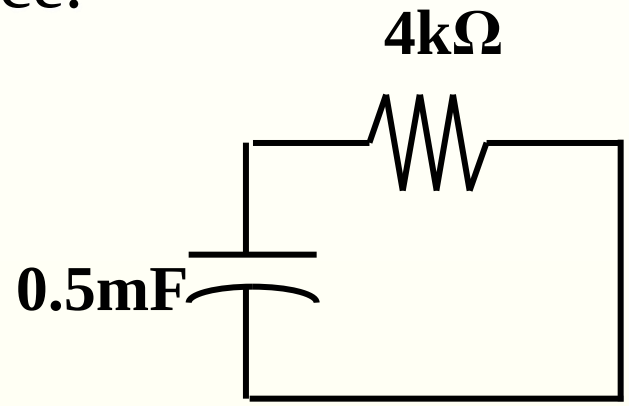
The switch has been in position *a* for a long time. At t = 0, the switch moves to *b*. Find $V_c(t)$ for t > 0 and calculate its value at t = 1s and t = 4s.



Solution

Step 1:

To find τ for t > 0, the switch is at *b* and short circuit the voltage source.

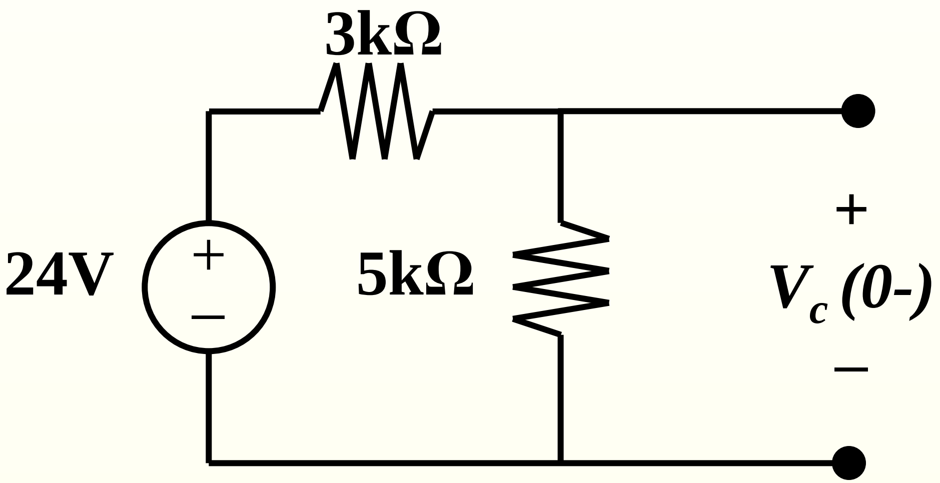


$$\tau = RC = 2s$$

Continue

Step 2:

The capacitor behaves like an open circuit as it is being supplied by a constant dc source.



From the circuit,

$$V_c(0^-) = 24 \times \frac{5}{8} = 15V$$

Continue

Step 3:

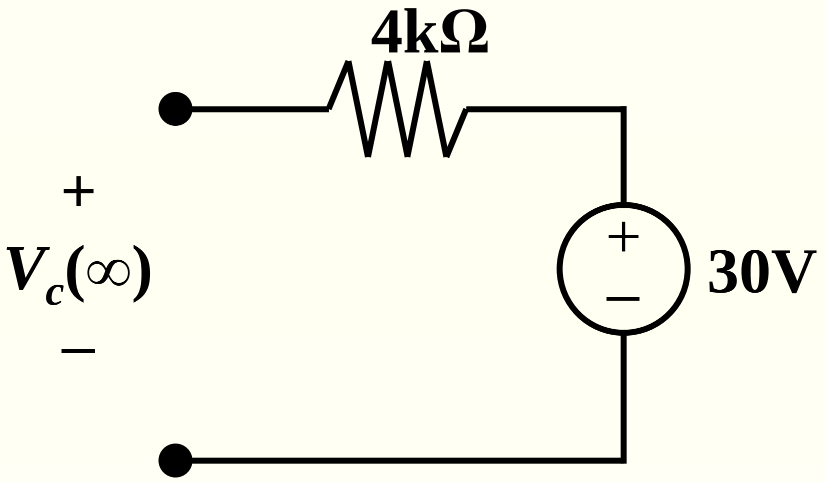
At t = 0$^+$, the instant when the switch is just moves to *b*.

Voltage across capacitor remains the same.

$$V_c(0^+) = V_c(0^-) = 15V$$

Step 4:

At t = ∞, the capacitor again behaves like an open circuit since it is being supplied by a constant source.



$$V_c(\infty) = 30V$$

Continue

Step 5:

Hence,

$$V_c(t) = 30 + (15 - 30)e^{-0.5t} = 30 - 15e^{-0.5t}V$$

At t = 1s, $V_c(t)$ = 20.9V

At t = 4s, $V_c(t)$ = 28 V

Summary