1. Write down the matrices of the following quadratic forms

(*i*) $2x^2 + 3y^2 + 6xy$ (*ii*) $2x^2 + 5y^2 - 6z^2 - 2xy - yz + 8zx$

(*iii*) $x_1^2 + 2x_2^2 + 3x_3^2 + 4x_4^2 + 2x_1x_2 + 4x_1x_3 - 6x_1x_4 - 4x_2x_3 - 8x_2x_4 + 12x_3x_4$.

2. Write down the quadratic forms corresponding to the following matrices.

(*i*) $\begin{bmatrix} 2 & 4 & 5 \\ 4 & 3 & 1 \\ 5 & 1 & 1 \end{bmatrix}$ (*ii*) $\begin{bmatrix} 1 & 1 & -2 & 0 \\ 1 & -4 & 0 & 0 \\ -2 & 0 & 6 & -3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$.

3. Reduce the following quadratic forms to canonical forms or to sum of squares by linear transformation. Write also the rank, index and signature.

(*i*) $2x^2 + 2y^2 + 3z^2 + 2xy - 4yz - 4xz$ (*ii*) $12x_1^2 + 4x_2^2 + 5x_3^2 - 4x_2x_3 + 6x_1x_3 - 6x_1x_2$

(*iii*) $2x^2 + 9y^2 + 6z^2 + 8xy + 8yz + 6zx$ (*iv*) $x^2 + 4y^2 + z^2 + 4xy + 6yz + 2zx$.

4. Reduce the following quadratic forms to canonical forms or to sum of squares by orthogonal transformation.Write also rank, index, signature.

(*i*) $3x^2 + 5y^2 + 3z^2 - 2xy - 2yz + 2zx$ (*ii*) $2x_1^2 + 2x_2^2 + 2x_3^2 - 2x_1x_2 + 2x_1x_3 - 2x_2x_3$

(*iii*) $3x^2 - 2y^2 - z^2 - 4xy + 8xz + 12yz$ (*iv*) $x^2 + 3y^2 + 3z^2 - 2yz$.