Unit - II

2.17 Three Phase Balanced Circuits

Dr.Santhosh.T.K.

Syllabus

UNIT – II **14 Periods**

DC Circuit Analysis: Voltage source and current sources, ideal and practical, Kirchhoff's laws and applications to network solutions using mesh analysis, - Simplifications of networks using series- parallel, Star/Delta transformation, DC circuits-Current-voltage relations of electric network by mathematical equations to analyse the network (Superposition theorem, Thevenin's theorem, Maximum Power Transfer theorem), Transient analysis of R-L, R-C and R-L-C Circuits.

AC Steady-state Analysis: AC waveform definitions - Form factor - Peak factor - study of R-L - R-C -RLC series circuit - R-L-C parallel circuit - phasor representation in polar and rectangular form - concept of impedance - admittance - active - reactive - apparent and complex power - power factor, Resonance in R-L-C circuits - 3 phase balanced AC Circuits

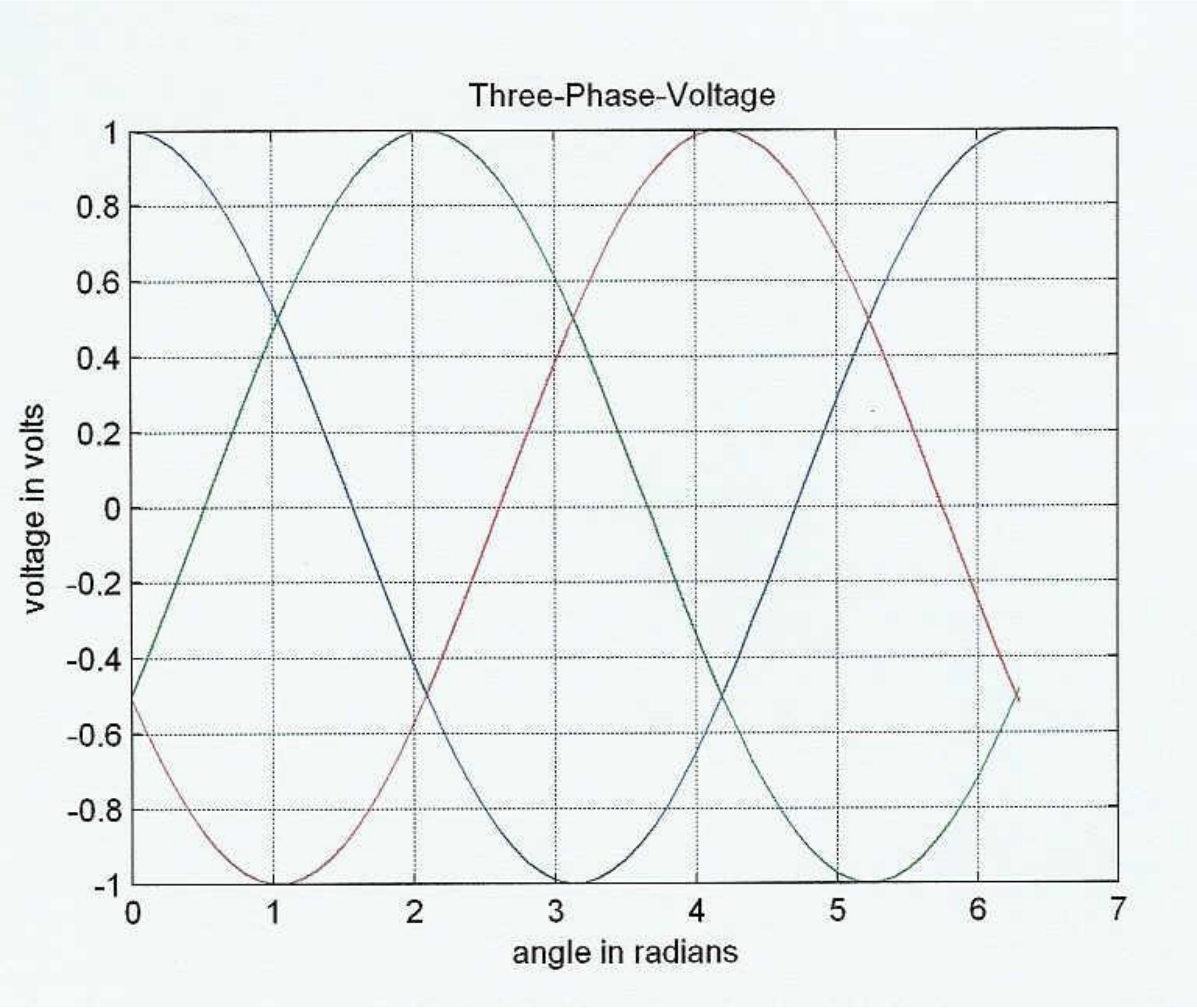
Polyphase Circuits

Polyphase circuits contain multiple sources at the same frequency but different phases. Power is distributed over the power grid in the form of three-phase sinusoids.

Advantages of Three-Phase power distribution include:

- (Constant Power) Instantaneous power can be constant in a three phase system.
- (More Economical) For equivalent power, the 3-Phase systems are more economical than single-phase (can be driven with lower currents and voltages, and fewer wires required because of a common neutral connection between the phases).
- (Flexible) Single phase service can be extracted from the 3-phase systems or phases manipulated to create additional phases.

Three-Phase Power

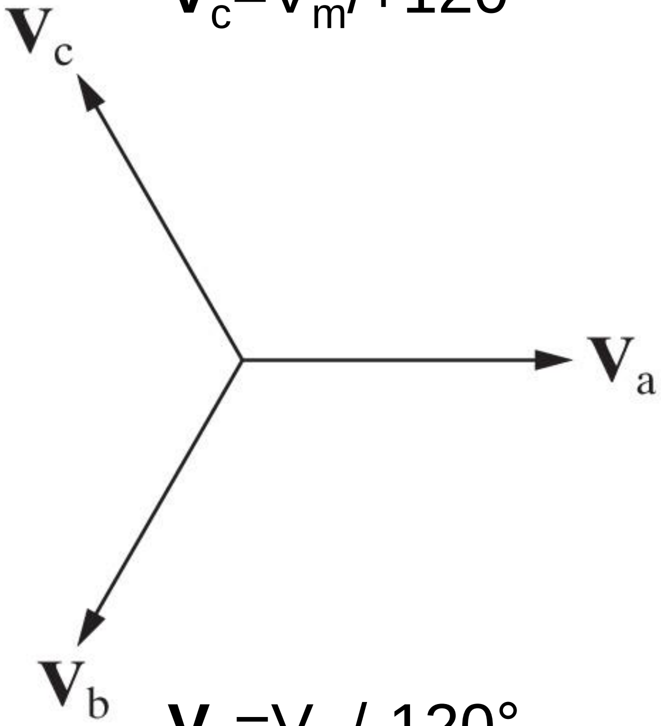


Definitions

- 4 wires
  - 3 “active” phases, A, B, C
  - 1 “ground”, or “neutral”
- Color Code
  - Phase A Red
  - Phase B Black
  - Phase C Blue
  - Neutral White or Gray

Phasor (Vector) Form for abc

$\mathbf{V}_c = V_m/{+120^\circ}$

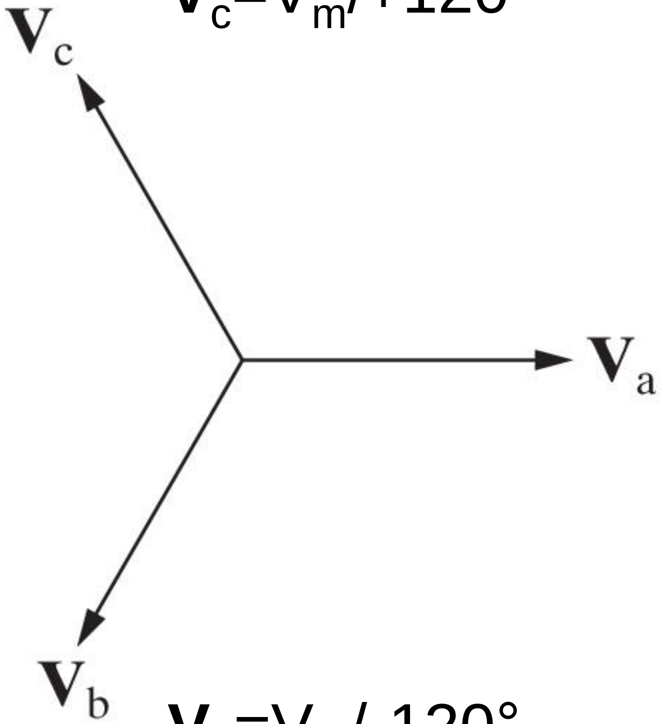


$\mathbf{V}_a = V_m/0^\circ$

$\mathbf{V}_b = V_m/{-120^\circ}$

Phasor (Vector) Form for abc

$\mathbf{V}_c = V_m \angle +120°$

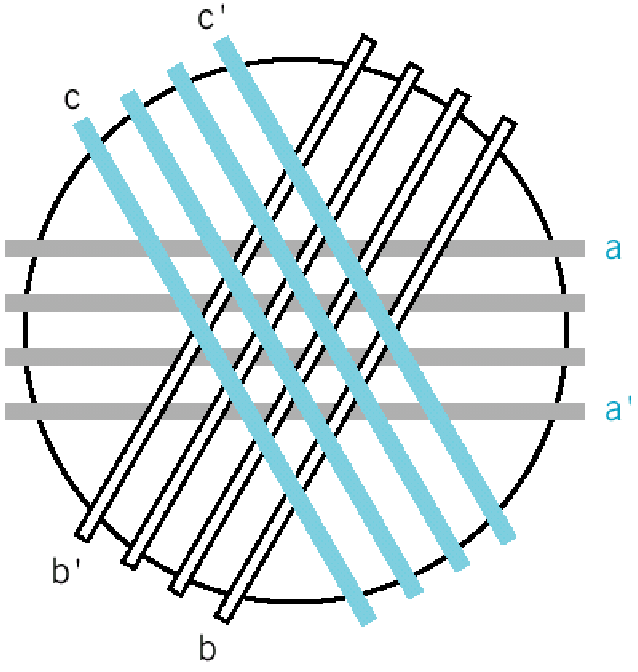


$\mathbf{V}_a = V_m \angle 0°$

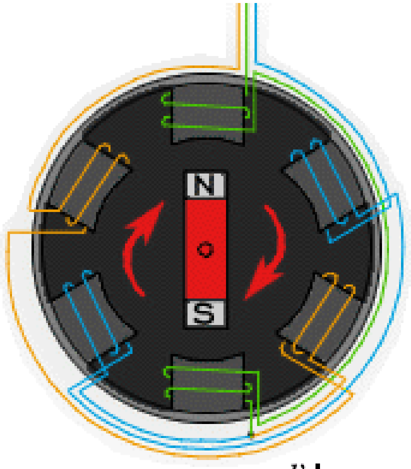
$\mathbf{V}_b = V_m \angle -120°$

Note that KVL applies .... $\mathbf{V}_a + \mathbf{V}_b + \mathbf{V}_c = 0$

Three-Phase Voltages



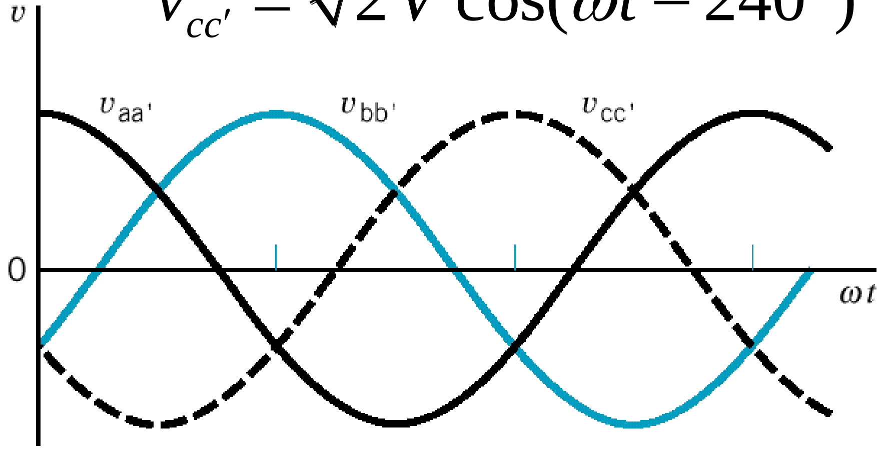
(a)



$$v_{aa'} = \sqrt{2}V\cos\omega t$$

$$v_{bb'} = \sqrt{2}V\cos(\omega t - 120°)$$

$$v_{cc'} = \sqrt{2}V\cos(\omega t - 240°)$$

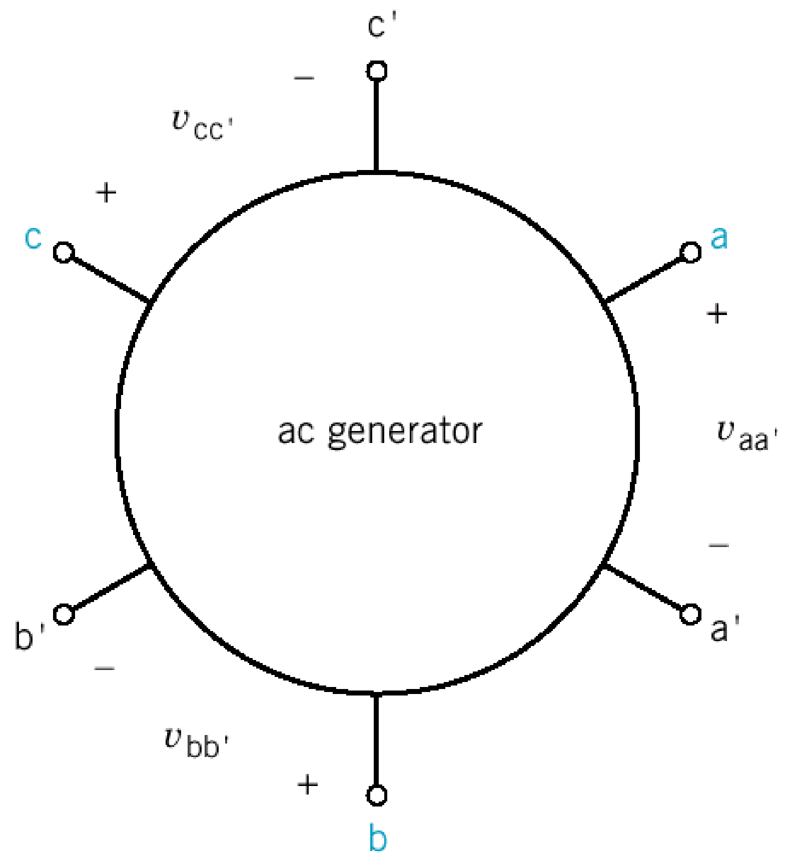


(b)

(a) The three windings on a cylindrical drum used to obtain three-phase voltages

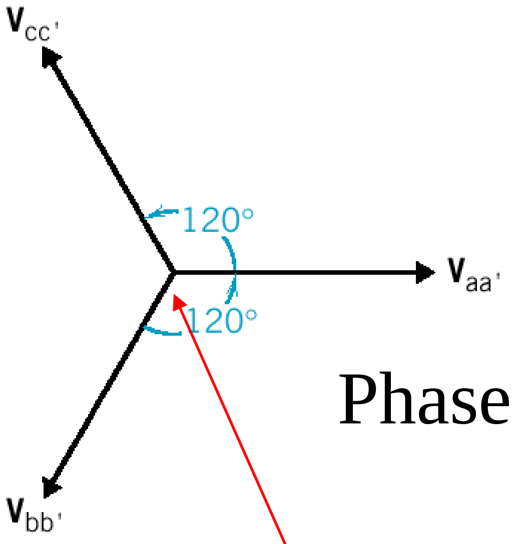
(*b*) Balanced three-phase voltages

Three-Phase Voltages



Generator with six terminals

Three-Phase Balanced Voltages



$$\mathbf{V}_{aa'} = V\angle 0°$$

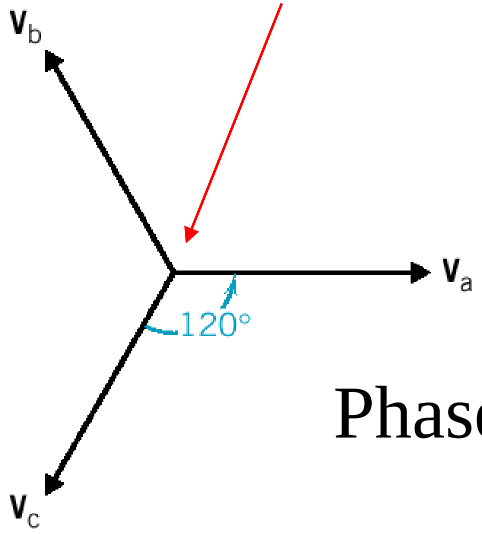
$$\mathbf{V}_{bb'} = V\angle -120°$$

$$\mathbf{V}_{cc'} = V\angle -240° = V\angle +120°$$

Phase sequence or phase rotation is ***abc***

Positive Phase Sequence

neutral terminal



$$\mathbf{V}_{a} = V\angle 0°$$

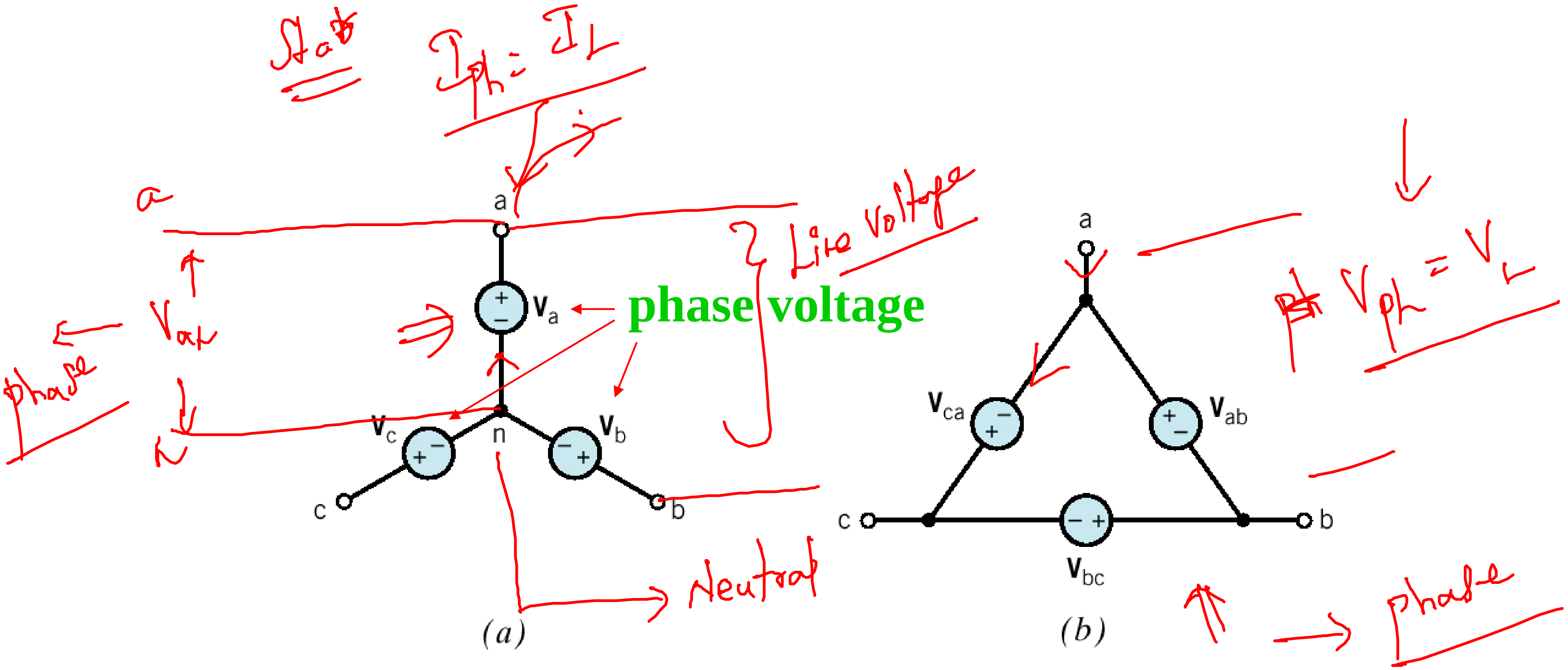
$$\mathbf{V}_{c} = V\angle -120°$$

$$\mathbf{V}_{b} = V\angle -240° = V\angle +120°$$

Phase sequence or phase rotation is ***acb***

Negative Phase Sequence

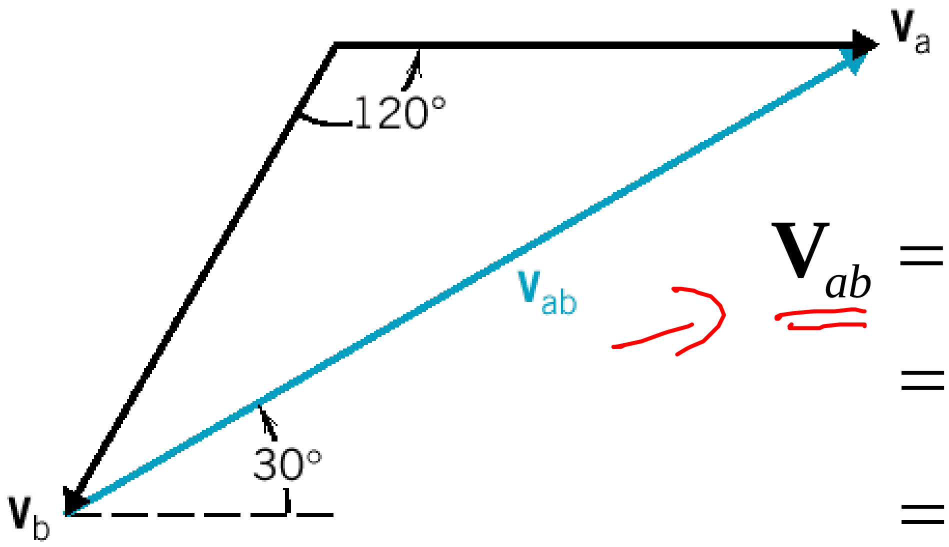
Two Common Methods of Connection



(*a*) Y-connected sources (b) Δ-connected sources

Star Delta.

Phase and Line Voltages



The line-to-line voltage $\mathbf{V}_{ab}$ of the Y-connected source

$$\mathbf{V}_{ab} = \mathbf{V}_a - \mathbf{V}_b$$
$$= V_p\angle 0° - V_p\angle -120°$$
$$= V_p - V_p(-0.5 - j0.866)$$
$$V_L = \sqrt{3}V_p\angle 30°$$

$$\boxed{V_L = \sqrt{3}\,V_{ph} \qquad I_L = I_{ph}}$$

Similarly

Star

$$\mathbf{V}_{bc} = \sqrt{3}V_p\angle -90°$$
$$\mathbf{V}_{ca} = \sqrt{3}V_p\angle -210°$$

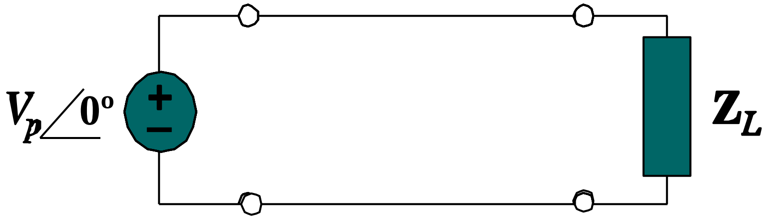
Single and 3-Phase Circuit Comparison

Consider the phase voltages of equal amplitude

$$\left|\hat{V}_p\right| = \left|\hat{V}_{an}\right| = \left|\hat{V}_{bn}\right| = \left|\hat{V}_{cn}\right|$$

Show that the line voltages are given by:

$$\left|\hat{V}_{ab}\right| = \left|\hat{V}_{ac}\right| = \left|\hat{V}_{bc}\right| = \sqrt{3}\left|\hat{V}_p\right|$$

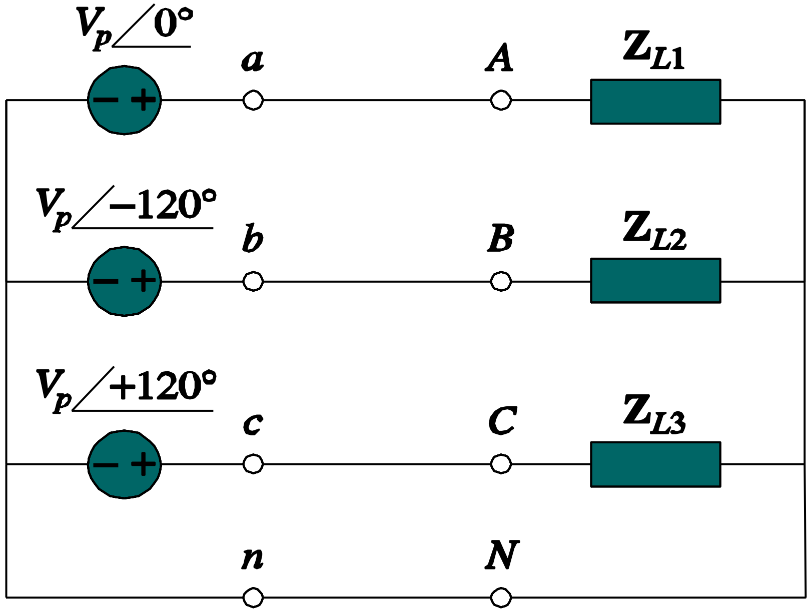


In general:

$$\hat{V}_{ab} = \sqrt{3}V_p\angle 30^\circ$$

$$\hat{V}_{bc} = \sqrt{3}V_p\angle -90^\circ$$

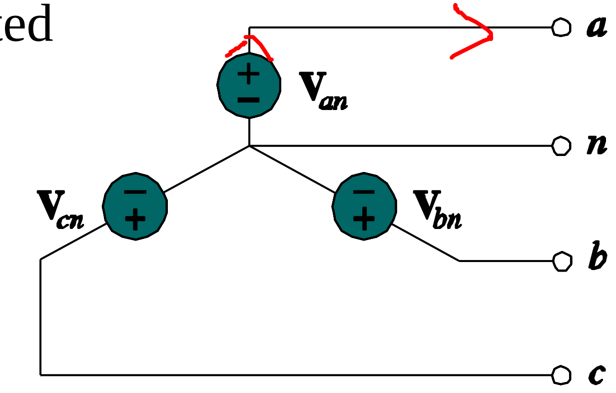
$$\hat{V}_{ca} = \sqrt{3}V_p\angle -210^\circ$$



Balanced 3-Phase Voltage Connections

There are 2 ways to connect a Balanced set of sources:

Y (wye)-Connected



phase

Line.

Star

$I_L = I_{ph}$

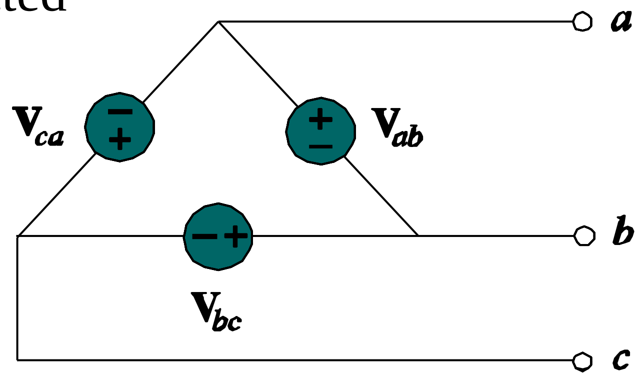
$V_L = \sqrt{3} \cdot V_{ph}$

V_{an}
V_{bn}
V_{cn}

Power

$P = V I \cos\phi$

Δ (delta)-Connected



$P = \sqrt{3} V_L \cdot I_L \cdot \cos\phi$

Delta

$V_{ph} = V_L$

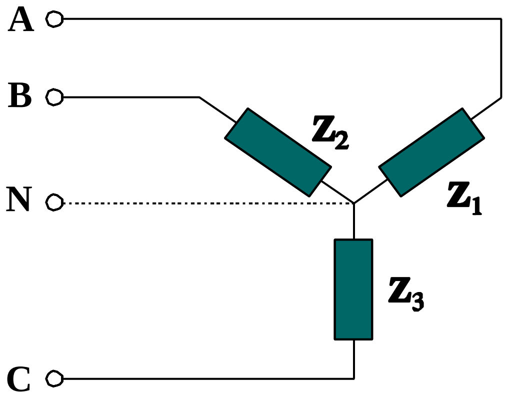
$I_L = \sqrt{3} \cdot I_{ph}$

Balanced Loads

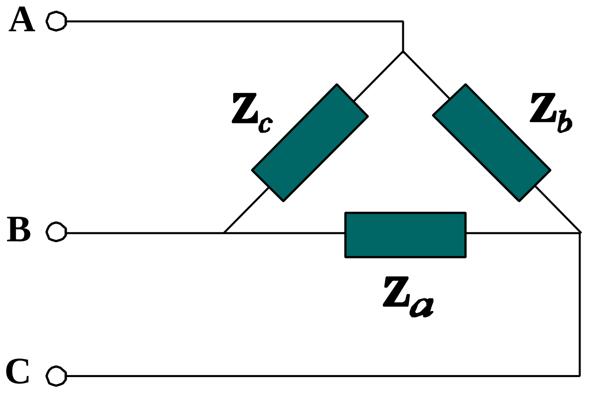
Balanced loads are equal in magnitude and phase.

There 2 ways to connect balanced loads

Y (wye)-Connected



Δ (delta)-Connected



for equivalent loads $\boldsymbol{Z}_{\Delta} = 3\boldsymbol{Z}_{Y}$

Load-Source Connections

There are 4 possible ways balanced sources and loads can be connected:

- Y Source to Y Load (Y-Y)
- Δ Source to Δ Load (Δ-Δ)
- Y Source to Δ Load (Y-Δ)
- Δ Source to Y Load (Δ-Y)

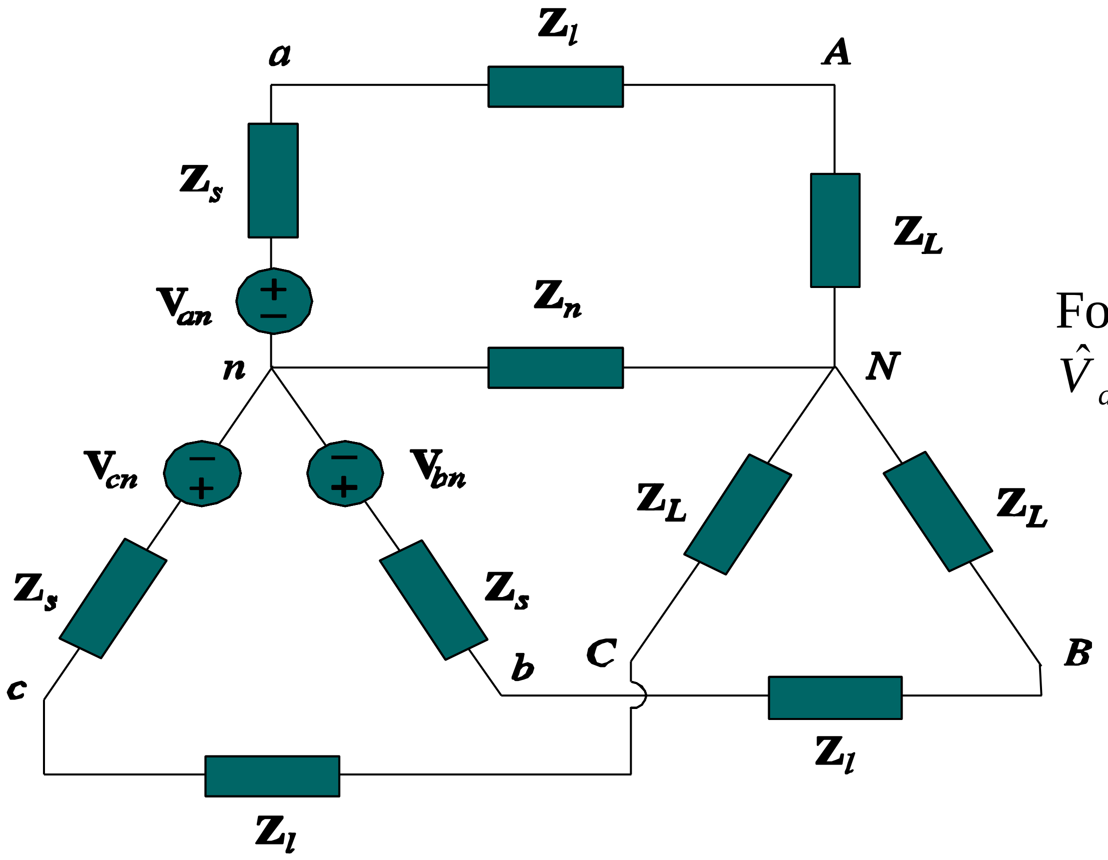
If not specified, the voltages on the sources will be assumed to be in RMS values.

Balanced Y-Y Connection

The complete Y-Y connection is shown below with impedances listed separately for the source (subscript s), line (subscript l), and load (subscript L).

$$V_p = \left|\hat{V}_{an}\right| = \left|\hat{V}_{an}\right| = \left|\hat{V}_{an}\right|$$

$$V_L = \left|\hat{V}_{ab}\right| = \left|\hat{V}_{bc}\right| = \left|\hat{V}_{ca}\right|$$



For a positive sequence with $\hat{V}_{an} = V_p \angle 0°$, it can be shown that

$$\hat{V}_{ab} = \sqrt{3} V_p \angle 30^\circ$$

$$\hat{V}_{bc} = \sqrt{3} V_p \angle -90^\circ$$

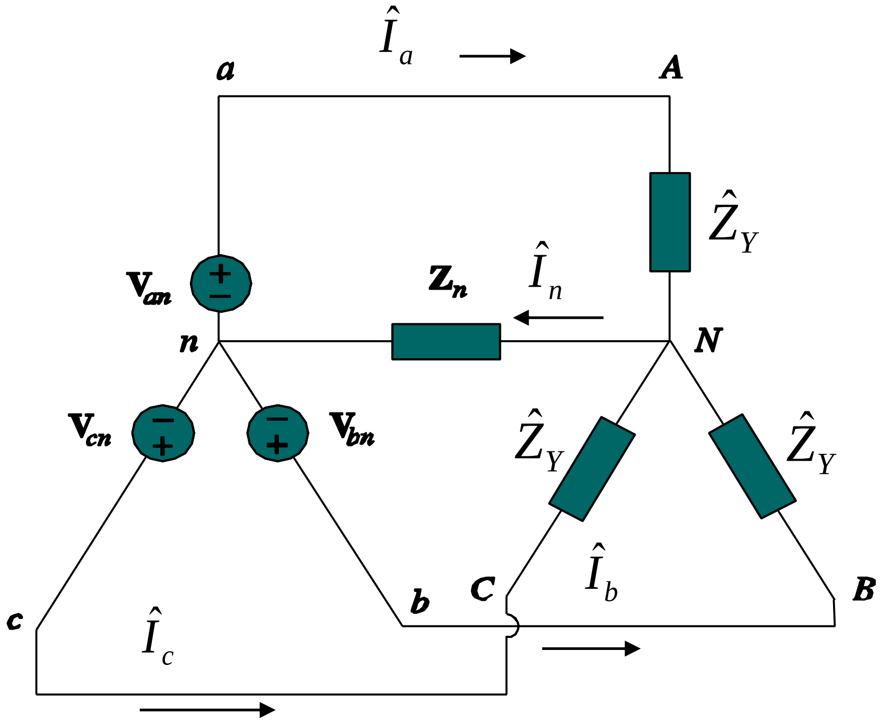
$$\hat{V}_{ca} = \sqrt{3} V_p \angle -210^\circ$$

Balanced Y-Y Connection

Show that the current in each phase can be expressed as:

$$\hat{I}_a = \frac{\hat{V}_{an}}{Z_Y}, \qquad \hat{I}_b = \hat{I}_a \angle -120°, \qquad \hat{I}_c = \hat{I}_a \angle -240°,$$

and that $\hat{I}_a + \hat{I}_b + \hat{I}_c = \hat{I}_n = 0$



Because of the symmetry of a balanced 3 phase system, the neutral connection can be dropped and the system analyzed on a per phase basis. In a Y-Y connected system, the phase (source or load) and line currents are the same.

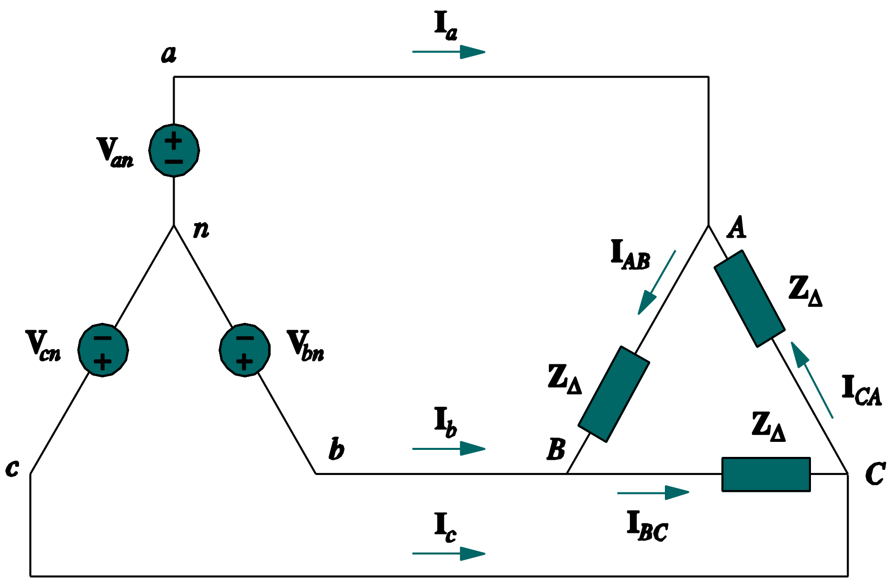
Balanced Y-Δ Connection

In this case the line voltages are directly across each load. It can be shown that:

$$\hat{I}_{AB} = \frac{\sqrt{3}\hat{V}_p}{Z_\Delta} = \frac{\hat{V}_{ab}}{Z_\Delta} = \frac{\hat{V}_{AB}}{Z_\Delta}, \qquad \hat{I}_{BC} = \hat{I}_{AB}\angle -120°, \qquad \hat{I}_{CA} = \hat{I}_{AB}\angle -240°$$

and the load currents and phase currents are related by:

$$\hat{I}_a = \hat{I}_{AB}\sqrt{3}\angle\text{-}30°$$



Note the Δ–connected load can be converted to a Y-connected load through:

$$\hat{Z}_Y = \frac{\hat{Z}_\Delta}{3}$$

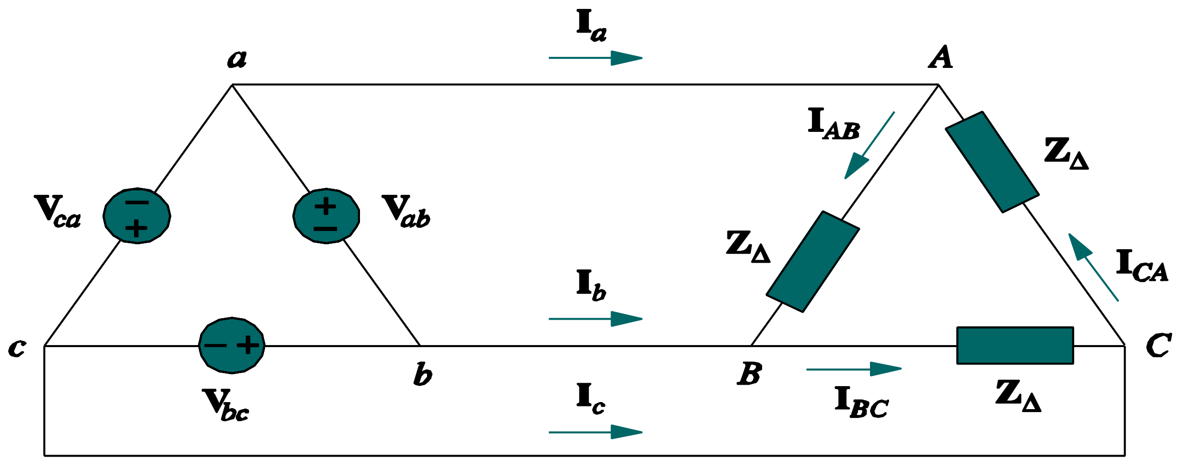
Balanced Δ -Δ Connection

In this case the line voltages are the phase voltages and are directly across each load. It can be shown that:

$$\hat{I}_{AB} = \frac{\hat{V}_{ab}}{Z_\Delta} = \frac{\hat{V}_{AB}}{Z_\Delta}, \qquad \hat{I}_{BC} = \hat{I}_{AB}\angle -120°, \qquad \hat{I}_{CA} = \hat{I}_{AB}\angle -240°$$

The line currents can be obtained from the phase currents

$$\hat{I}_a = \hat{I}_{AB}\sqrt{3}\angle\text{-}30°$$



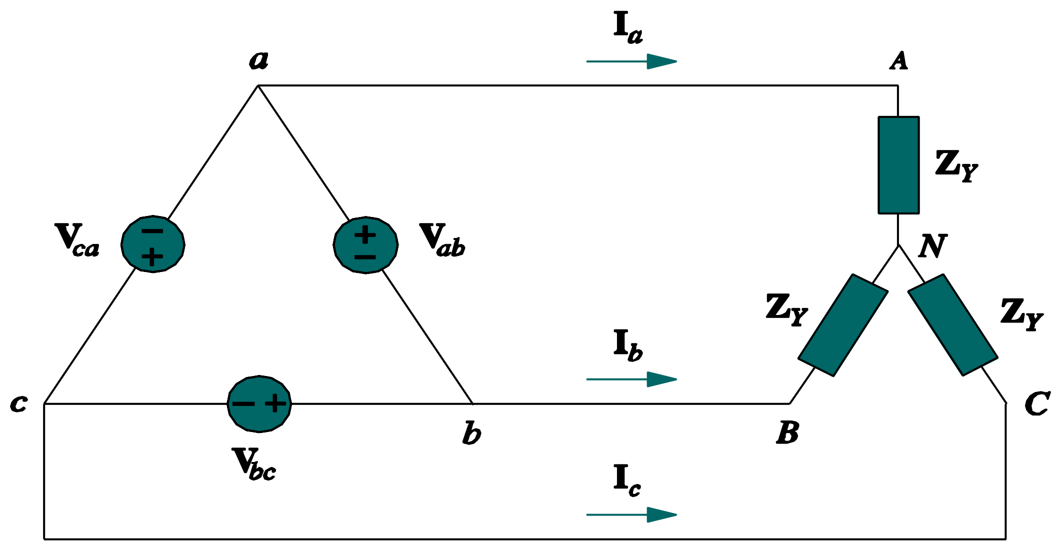
Balanced Δ -Y Connection

In this case the phase voltages are across the lines. It can be shown that:

$$\hat{V}_{ab} = V_p \angle 0°, \qquad \hat{V}_{bc} = V_p \angle -120°, \qquad \hat{V}_{ca} = V_p \angle 120°$$

the line current is related to the phase voltage by:

$$\hat{I}_a = \frac{V_{ab}}{\sqrt{3} Z_Y} \angle \text{-} 30°$$



Note the Δ–connected source can be converted to a Y-connected source through:

$$\hat{V}_{an} = \frac{\hat{V}_{ab}}{\sqrt{3}} \angle -30°$$

Power in Balanced System

Show that the instantaneous power absorbed by a load in a balanced Y-Y system is a constant given by:

$$p(t) = 3V_p I_p \cos(\theta)$$

where the impedance in a single phase is given by:

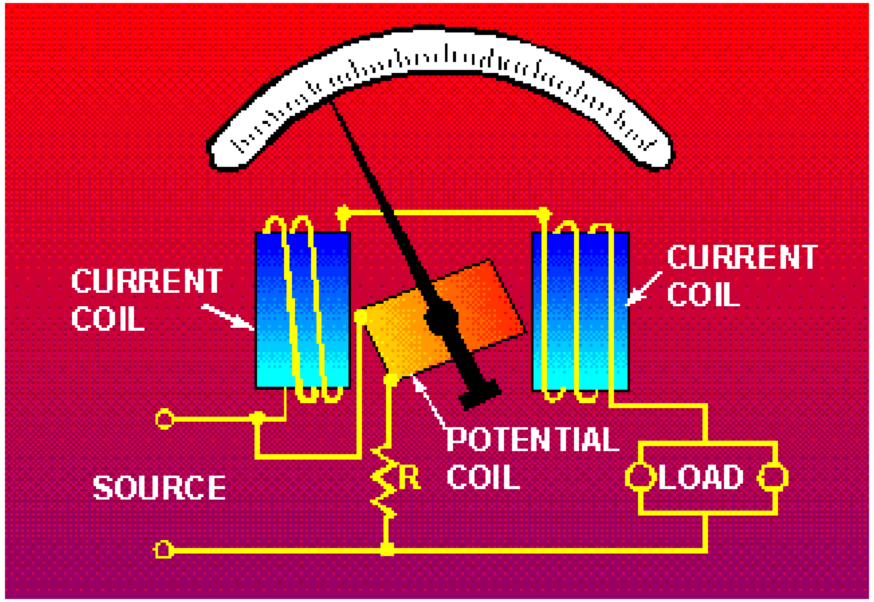
$$\hat{Z}_Y = Z\angle\theta$$

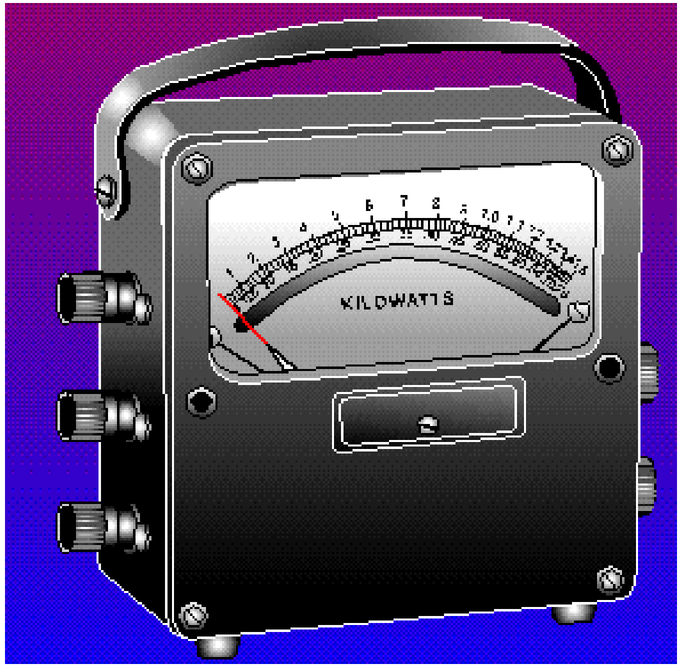
The complex power per phase is

$$S = V_p I_p \exp(j\theta)$$

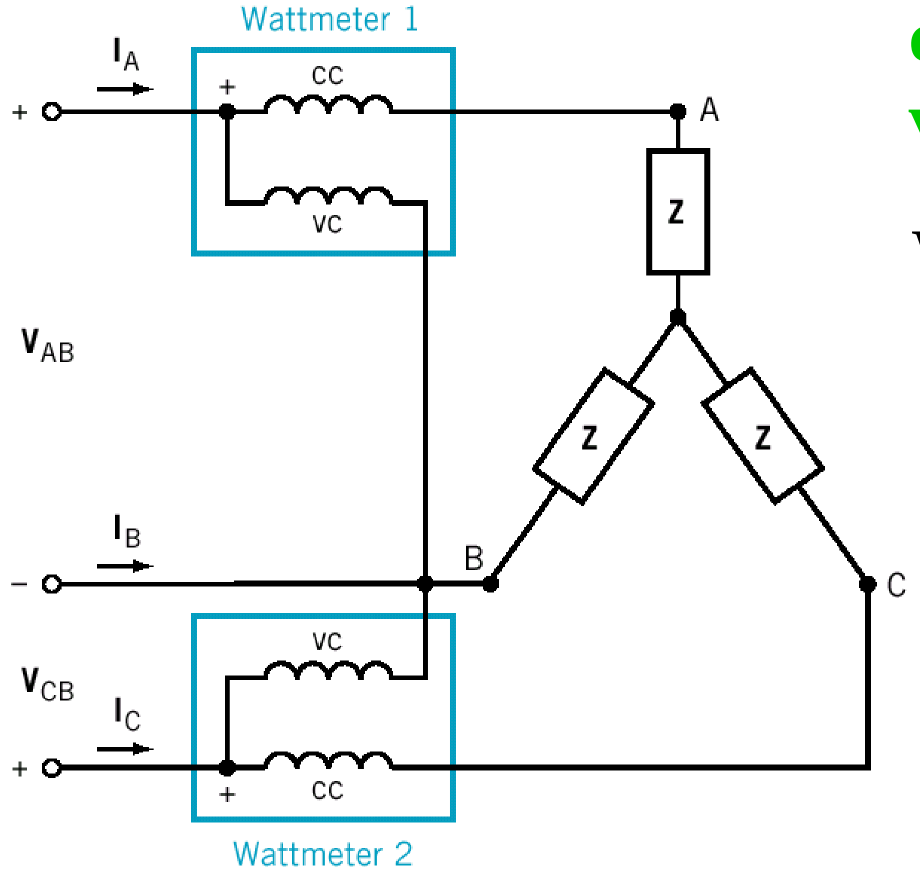
Note that average power or real power is the same as the instantaneous power for the 3-phase system.

Electrodynamic Wattmeter





Two-Wattmeter Power Measurement



cc = current coil
vc = voltage coil

W1 read

$$P_1 = V_{AB} I_A \cos\theta_1$$

W2 read

$$P_2 = V_{CB} I_C \cos\theta_2$$

For balanced load with *abc* phase sequence

$$\theta_1 = \theta_a + 30^\circ \quad \text{and} \quad \theta_2 = \theta_a - 30^\circ$$

θ_a is the angle between phase current and phase voltage of phase *a*

Two-Wattmeter Power Measurement(cont.)

$$
\begin{aligned}
P &= P_1 + P_2 \\
&= 2V_L I_L \cos\theta \cos 30^\circ \\
&= \sqrt{3} V_L I_L \cos\theta
\end{aligned}
$$

To determine the power factor angle

$$P_1 + P_2 = V_L I_L 2\cos\theta \cos 30^\circ$$

$$P_1 - P_2 = V_L I_L (-2\sin\theta \sin 30^\circ)$$

$$\frac{P_1 + P_2}{P_1 - P_2} = \frac{V_L I_L 2\cos\theta \cos 30^\circ}{V_L I_L (-2\sin\theta \sin 30^\circ)} = \frac{-\sqrt{3}}{\tan\theta}$$

$$\therefore \tan\theta = \sqrt{3}\,\frac{P_1 - P_2}{P_1 + P_2} \quad \text{or} \quad \theta = \tan^{-1}\left(\sqrt{3}\,\frac{P_1 - P_2}{P_1 + P_2}\right)$$

Determine the three-phase power supplied to a delta connected circuit with an impedance of 4 + j6 ohms in each phase, when a 3-phase, 415 V, 50 Hz is applied across it.

→ ↳ Line voltage

$$P = \sqrt{3}\, V_L\, I_L \cos\theta$$

$$= \sqrt{3} \times 415 \times 99.81 \times \cos(51.3) \simeq 39.79\, kW$$

$$\text{Phase current} = \frac{V}{Z} = \frac{415\angle 0^\circ}{4+j6} = \frac{415\angle 0^\circ}{7.2\angle 56.3^\circ} = 57.63\angle -56.3^\circ \text{ (A)}$$

$$\text{Line current} = I_L = \sqrt{3}\, I_{ph} = 99.81 \text{ (A)}$$

Example

Exercise

Three identical coils, each having a resistance of 20Ω and an inductance of 0.5 H connected in (a) star and (b) delta to a three phase supply of 400 V; 50 Hz. Calculate the current and the total power absorbed by both method of connections.

Summary

3ϕ balanced AC circuits

- phase ⟷ line
- power

$\Delta - \Delta$

$Y - Y$

$Y - \Delta$

$\Delta - Y$