Unit - II

2.8 Norton's theorem and Maximum Power Transfer Theorem

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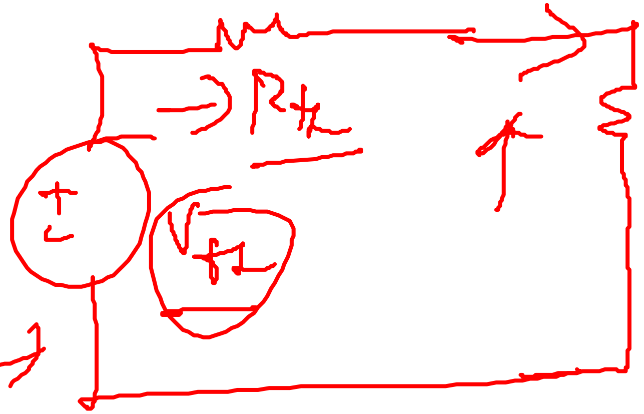
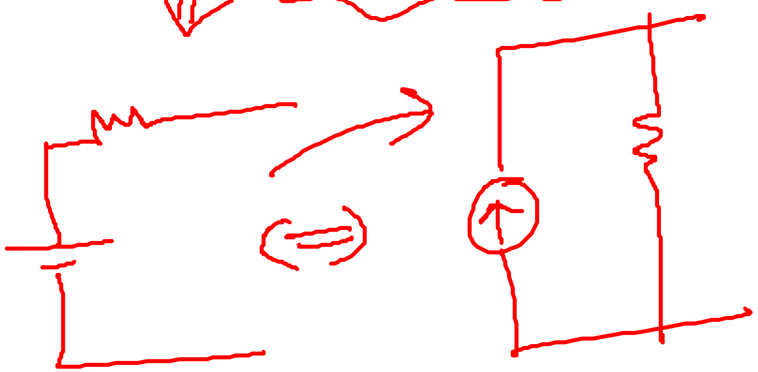
Thevenin's

Syllabus





Norton's

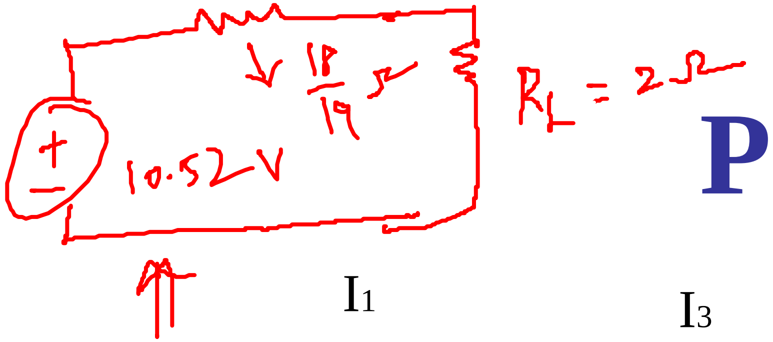
Rth

1. Remove Load
2. Turn off Sources
   - V → short
   - I → open
3. Rth.

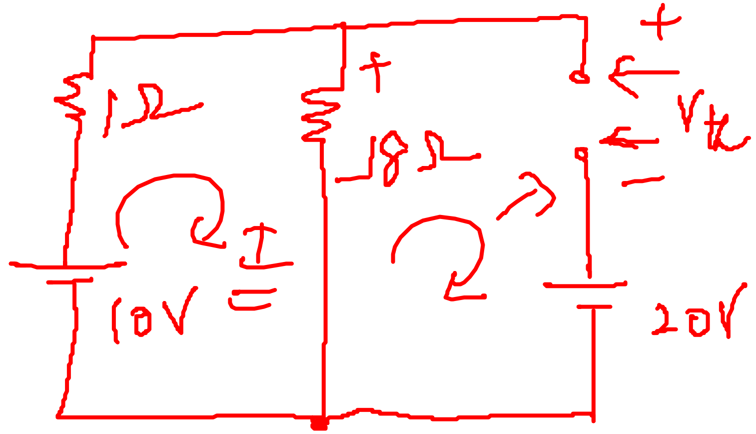
UNIT – II **14 Periods**

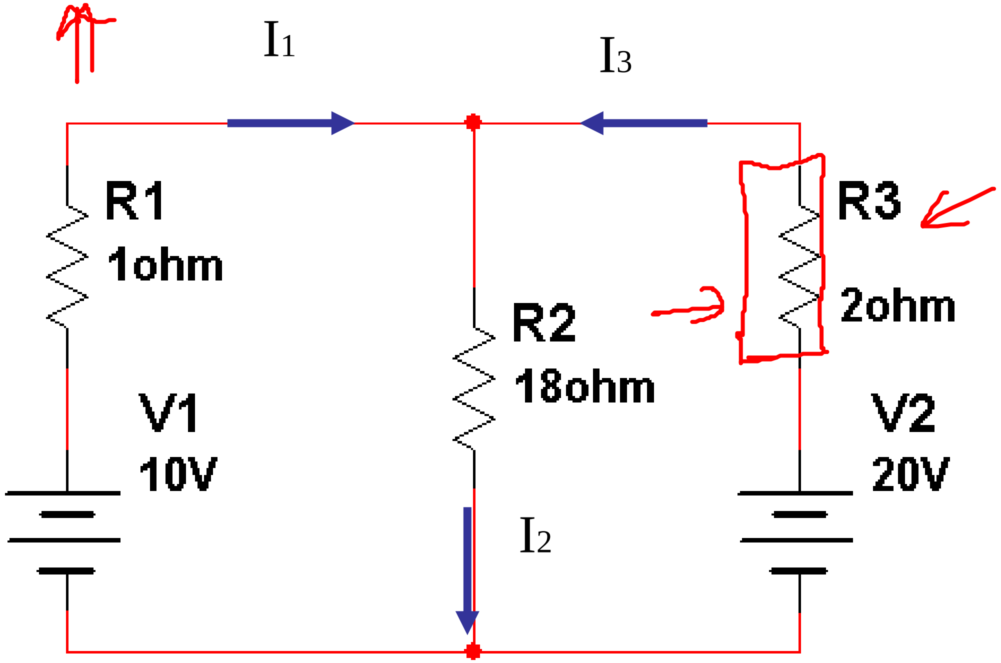
DC Circuit Analysis: Voltage source and current sources, ideal and practical, Kirchhoff's laws and applications to network solutions using mesh analysis, - Simplifications of networks using series- parallel, Star/Delta transformation, DC circuits-Current-voltage relations of electric network by mathematical equations to analyse the network (Superposition theorem, Thevenin's theorem, Maximum Power Transfer theorem), Transient analysis of R-L, R-C and R-L-C Circuits.

AC Steady-state Analysis: AC waveform definitions - Form factor - Peak factor - study of R-L - R-C -RLC series circuit - R-L-C parallel circuit - phasor representation in polar and rectangular form - concept of impedance - admittance - active - reactive - apparent and complex power - power factor, Resonance in R-L-C circuits - 3 phase balanced AC Circuits



Practice Problem





$$I = \frac{V}{R} = \frac{10}{18+1} = \frac{10}{19}\,A$$

$$V_{18\Omega} = \frac{10}{19} \times 18$$

KVL,

$$+V_{18\Omega} - V_{th} - 20 = 0$$

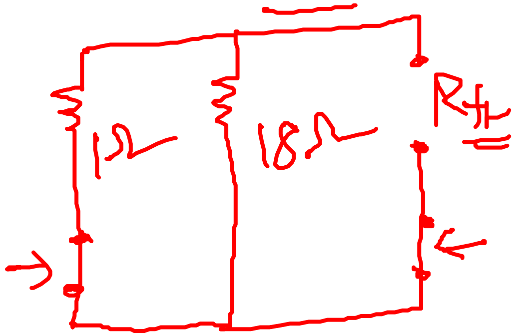
$$V_{th} = V_{18\Omega} - 20$$

$$= \frac{180}{19} - 20$$

$$\boxed{V_{th} = -10.52\ (V)}$$

Find the current through 2 ohm resistor using Thevenin's theorem

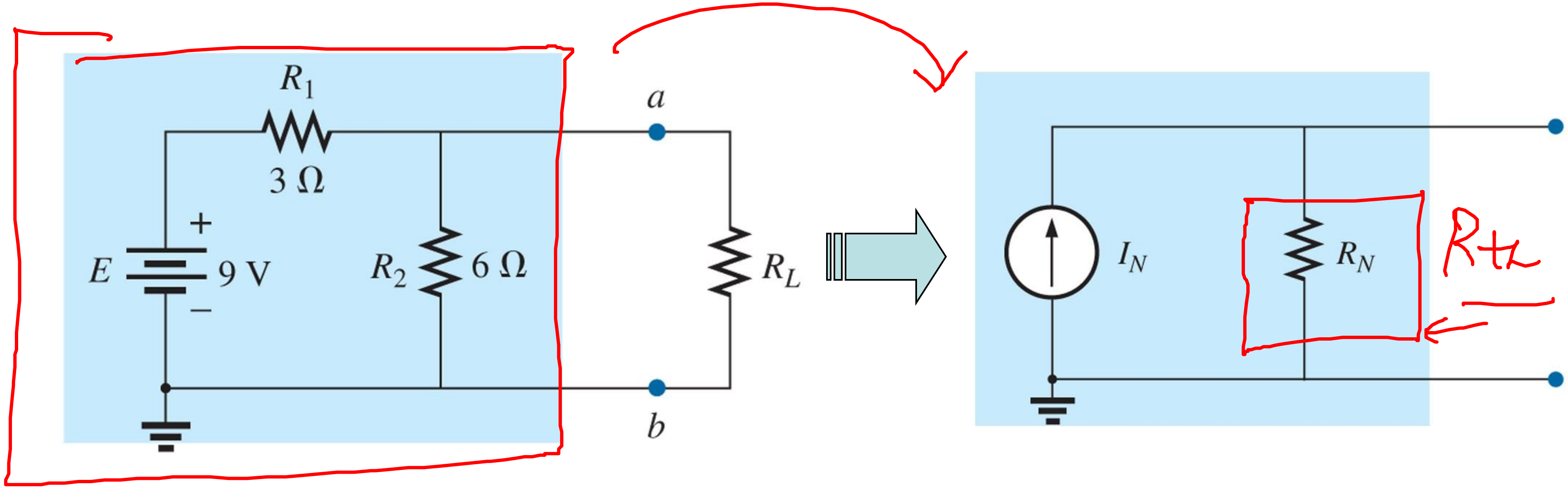
$R_{th} \rightarrow$



$$R_{th} = \frac{18}{19}\,\Omega$$

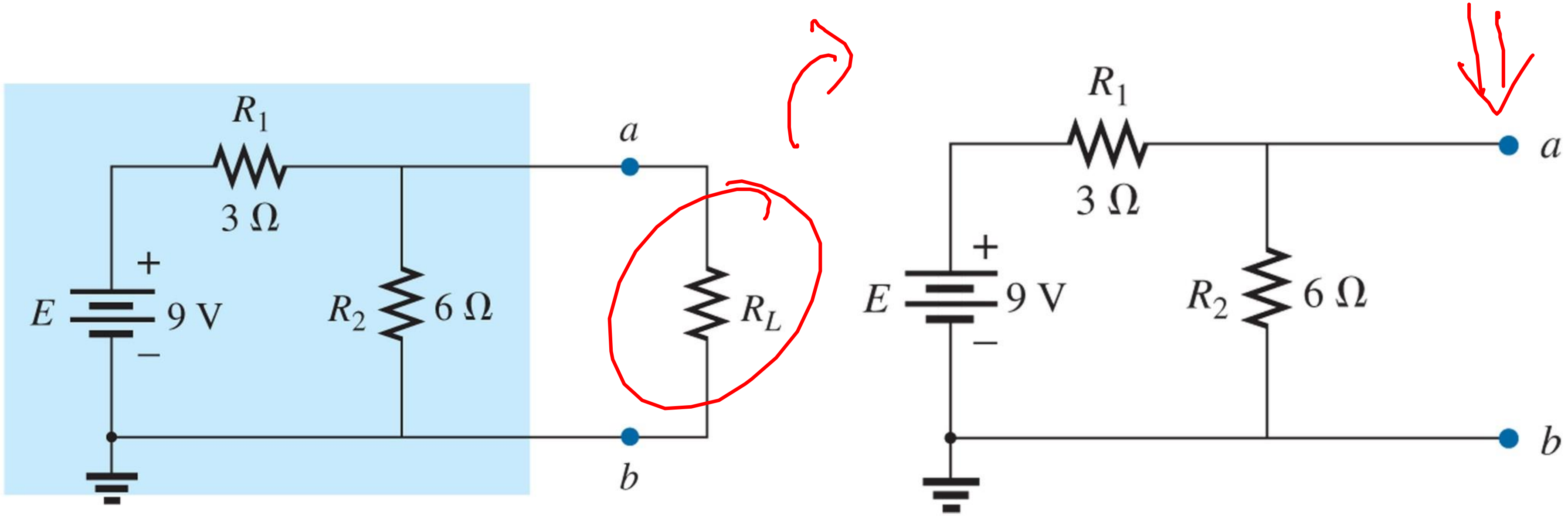
Norton's Theorem

- The theorem states that,
  - Any two-terminal linear bilateral dc network can be replaced by an equivalent circuit consisting of a current and a parallel resistor.



Norton's Theorem Procedure

- Step 1: Remove the portion of the network across which the Norton equivalent circuit is found.
- Step 2: Mark the terminal as a and b. We have an open circuit across terminal a and b.

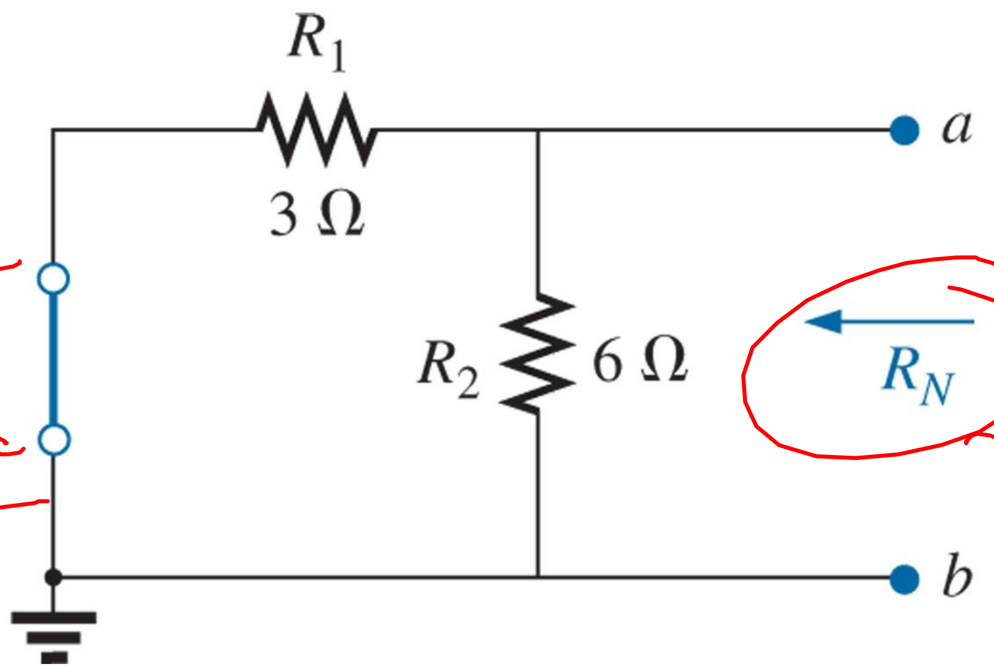


Norton's Theorem Procedure

- Step 3:
  - Replace the voltage source with a short-circuit equivalent.
  - Calculate the R_N

Voltage → SC

current → oc



$$R_N = R_1 \| R_2 = \frac{3\times 6}{3+6} = 2\Omega$$

I_N

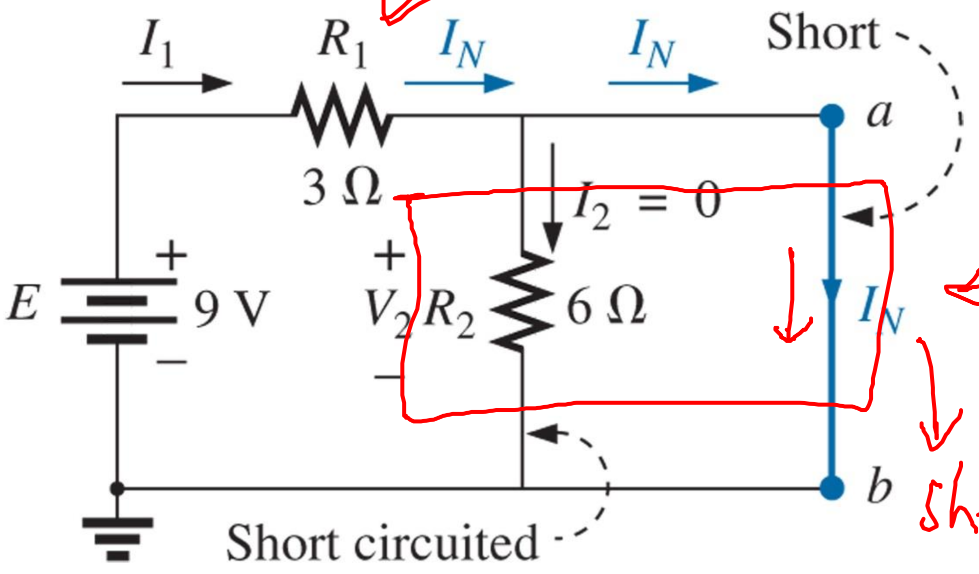
Thevenins

V_{th} →

Norton’s Theorem Procedure

- Step 4: Indicate the short circuit connection between the terminal a and b.

$$I_N = \frac{E}{R_1} = \frac{9V}{3\Omega} = 3A$$

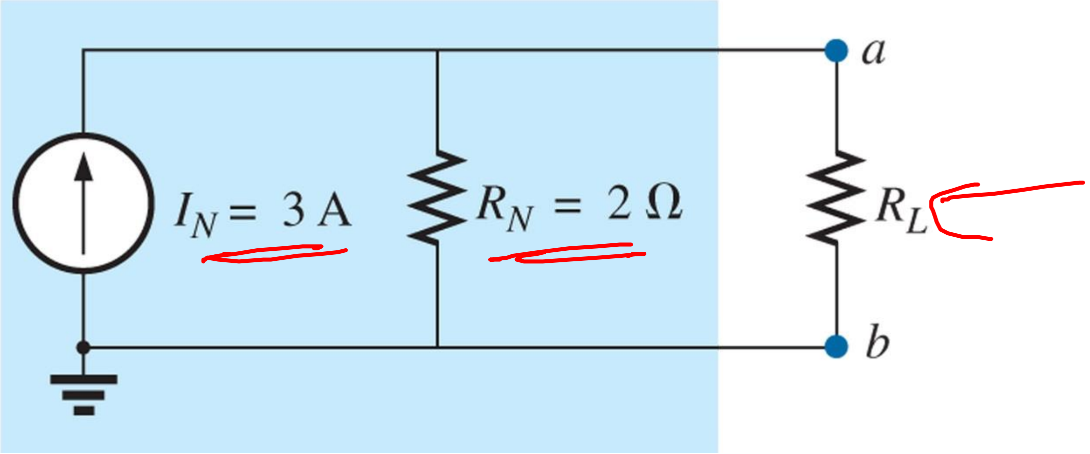


short circuit

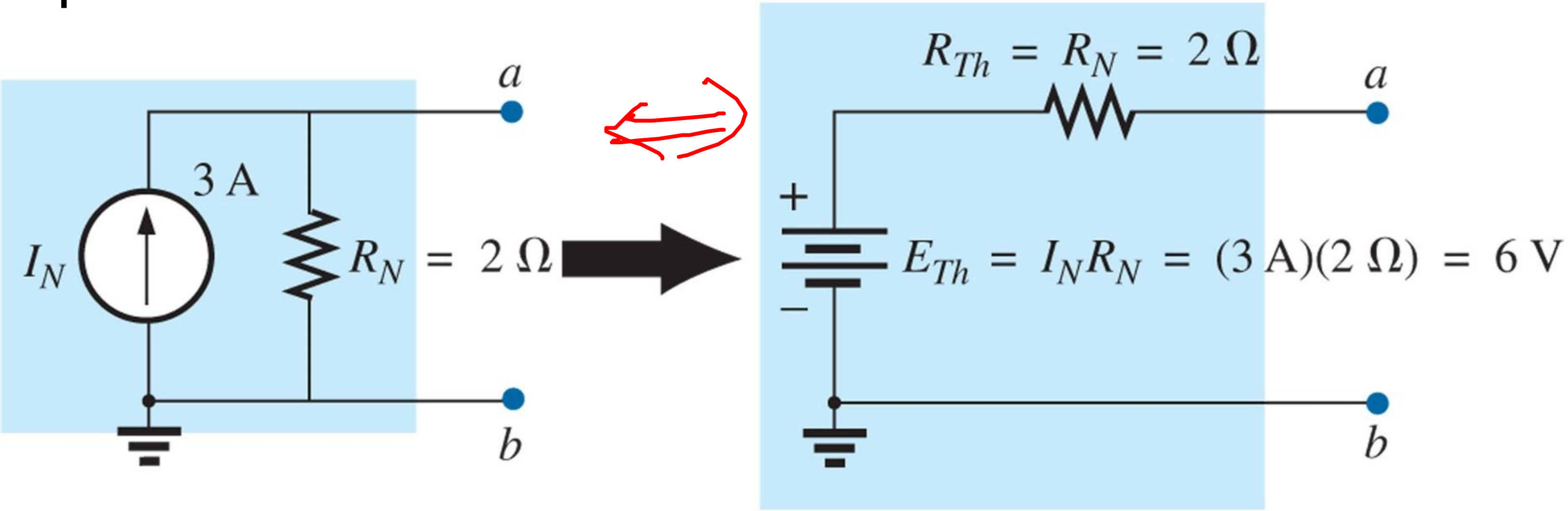
$$V_2 = I_2 R_2 = 0 \times 6\Omega = 3A$$

Norton's Theorem Procedure

Step 5: Draw the norton equivalent circuit.

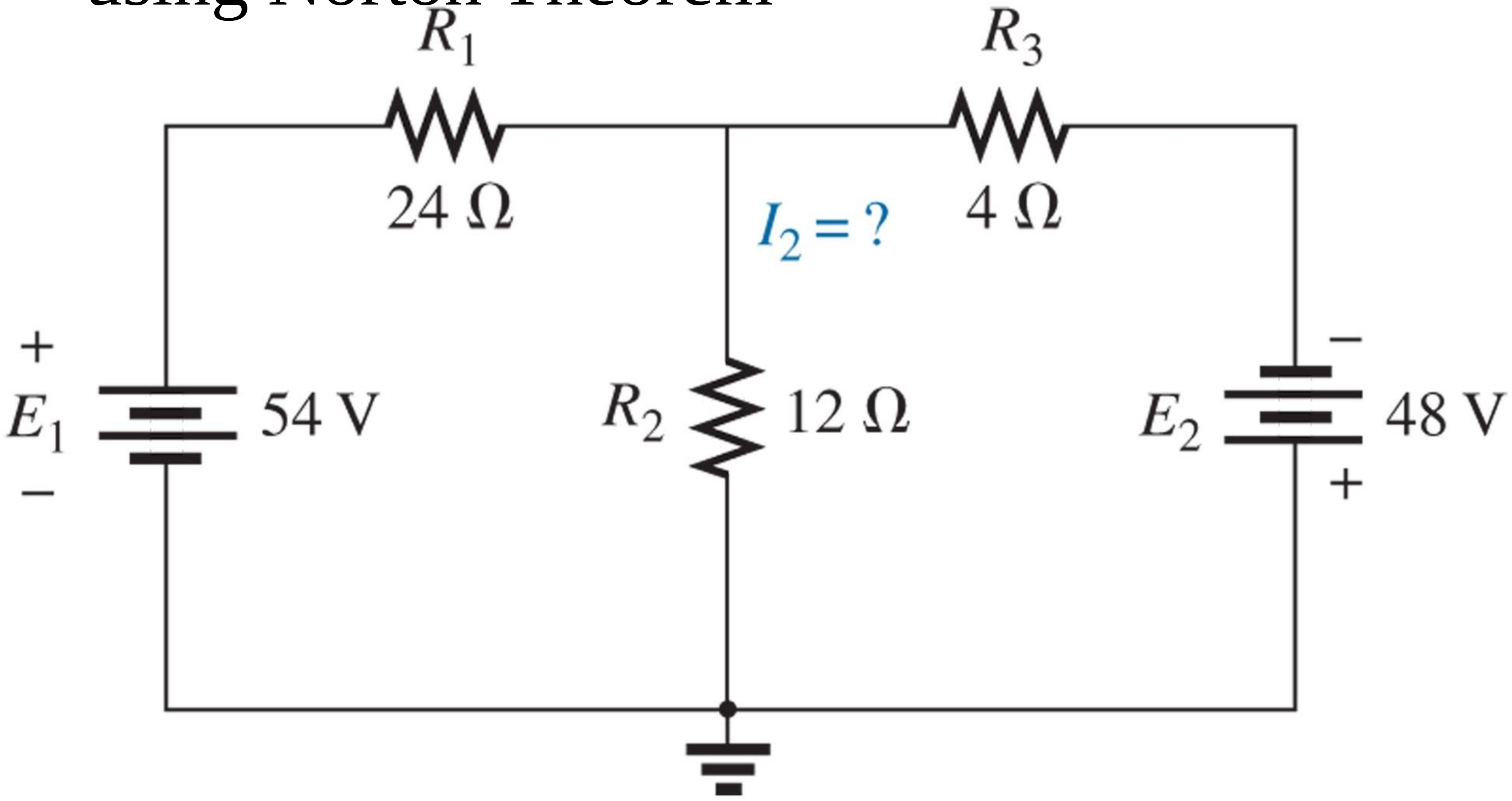


Converting the Norton equivalent circuit to a Thevenin equivalent circuit.



Exercise

- Determine the current in the 12 Ω resistor using Norton Theorem



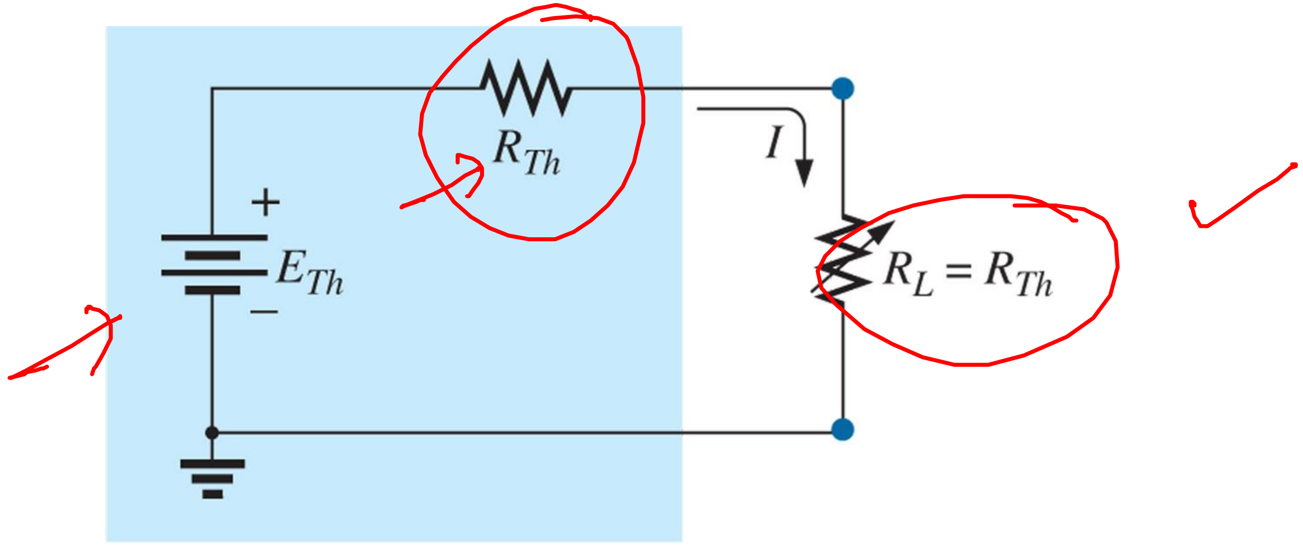
Maximum Power Transfer Theorem

- The maximum power transfer theorem states the following:

  A load will receive maximum power from a network when its total resistive value is exactly equal to the Thévenin resistance of the network applied to the load. That is,

$$R_L = R_{Th}$$

- For the Thevenin equivalent circuit like the figure below, when the load is equal to the Thevenin resistance, the load will receive maximum power from the network



- With $R_L = R_{Th}$, the maximum power delivered to the load can be determined by first finding the current:

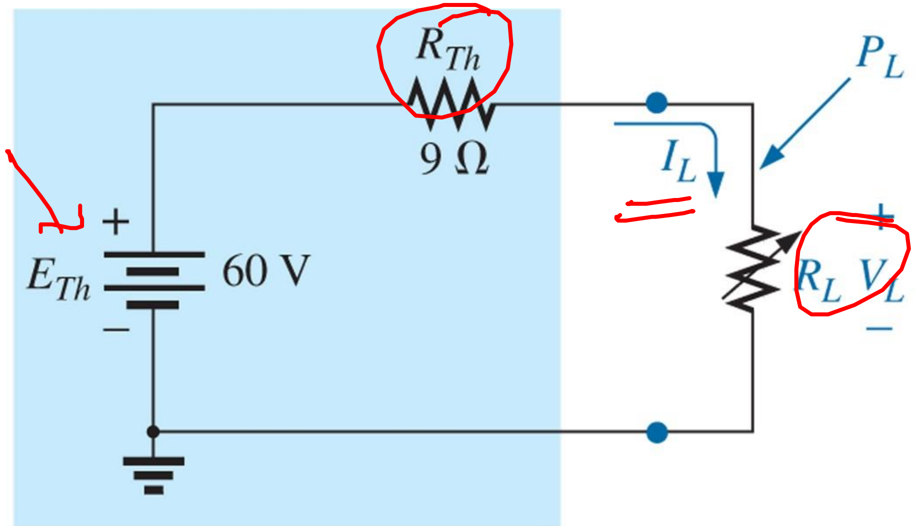
$$I_L = \frac{E_{Th}}{R_{Th}+R_L} = \frac{E_{Th}}{R_{Th}+R_{Th}} = \frac{E_{Th}}{2R_{Th}}$$

$$I = \frac{V}{R} = \frac{E_{Th}}{R_{Th}+R_L}$$

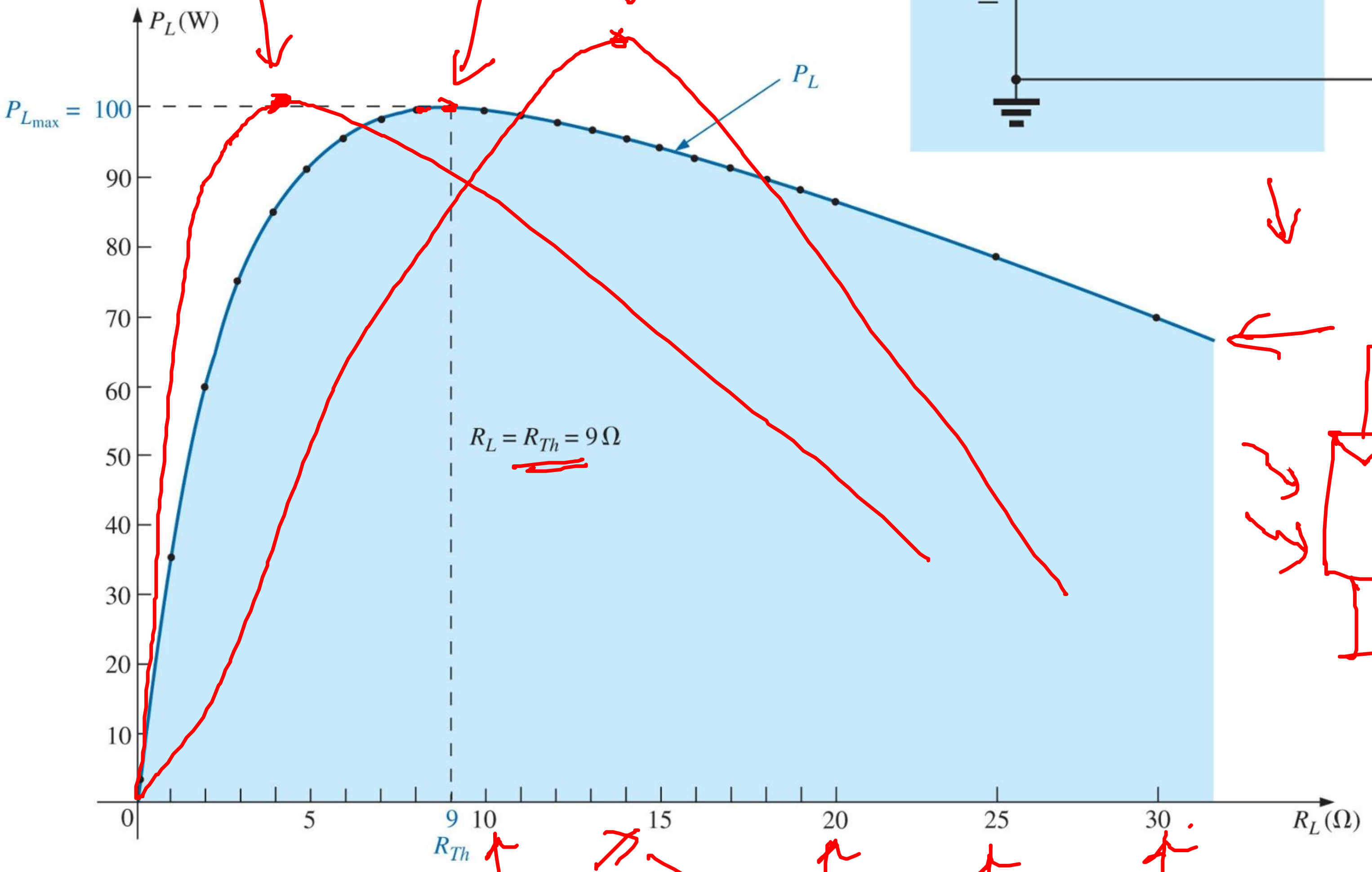
$$P_L = I^2{}_L R_L = \left(\frac{E_{Th}}{2R_{Th}}\right)^2 R_{Th} = \frac{E^2{}_{Th} R_{Th}}{4R^2{}_{Th}}$$

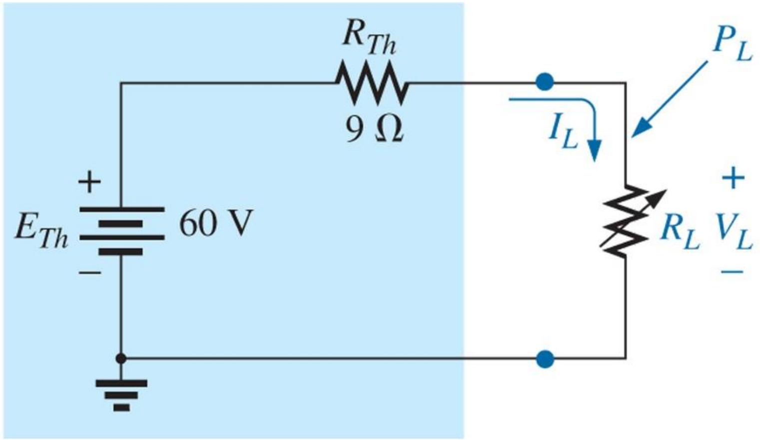
$$\boxed{R_{Th} = R_L}$$

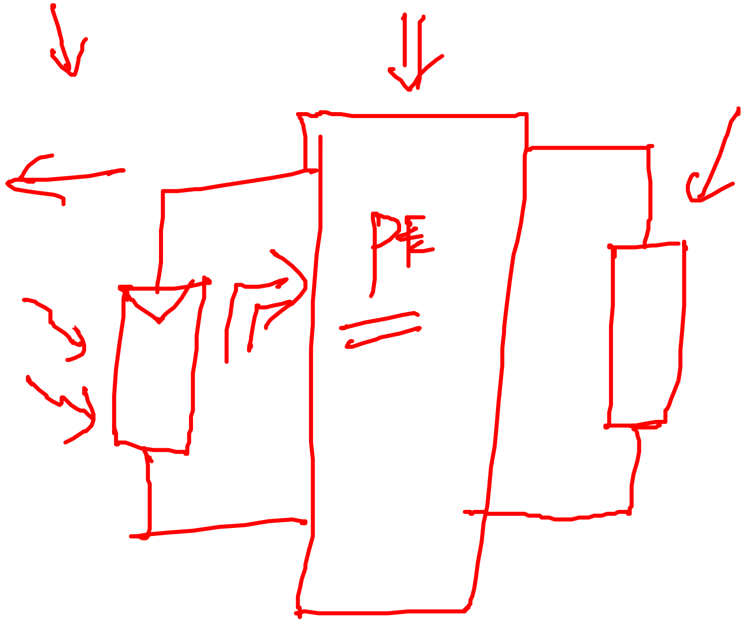
$$\boxed{P_{L\max} = \frac{E^2{}_{Th}}{4R_{Th}}}$$



P_L versus R_L for the network





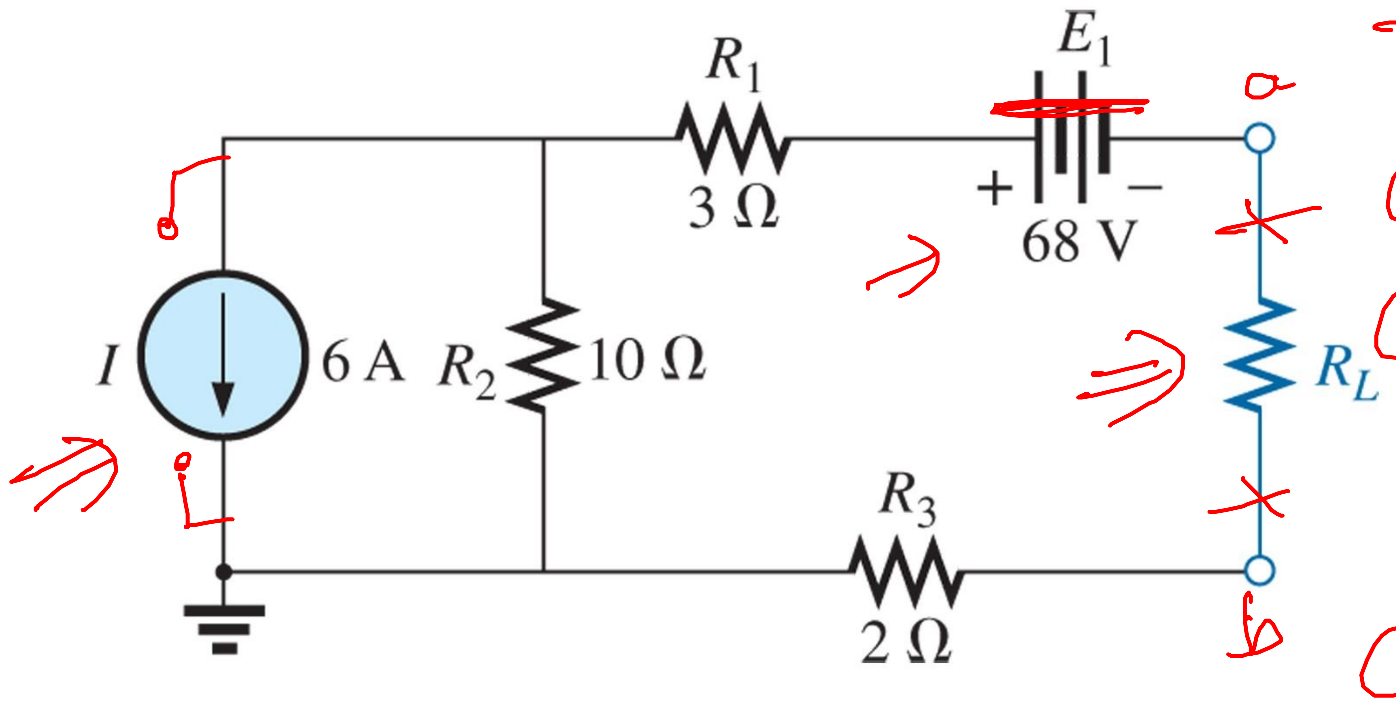


Example

- For the circuit below, determine

i. The value of the load resistor, R_L, which would give the maximum power transfer.

ii. The maximum power transferred to the load



To find R_{th}

① Remove load

② Turn off sources

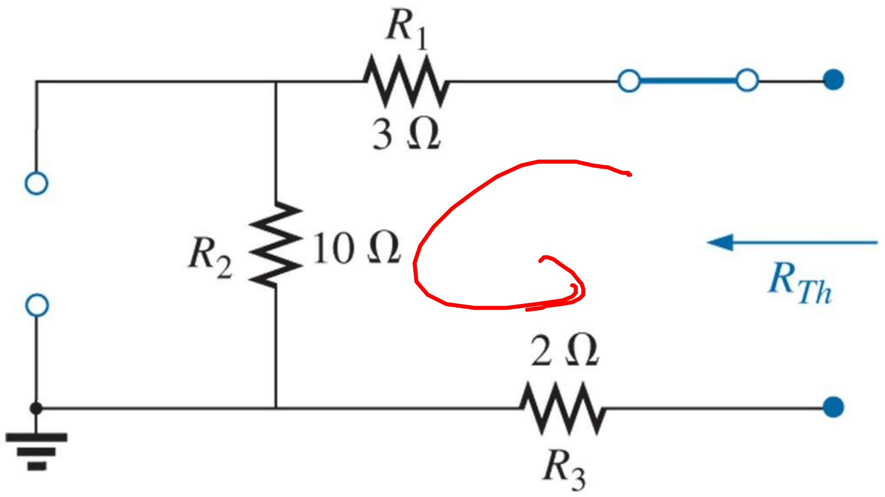
$\hookrightarrow V \to SC$

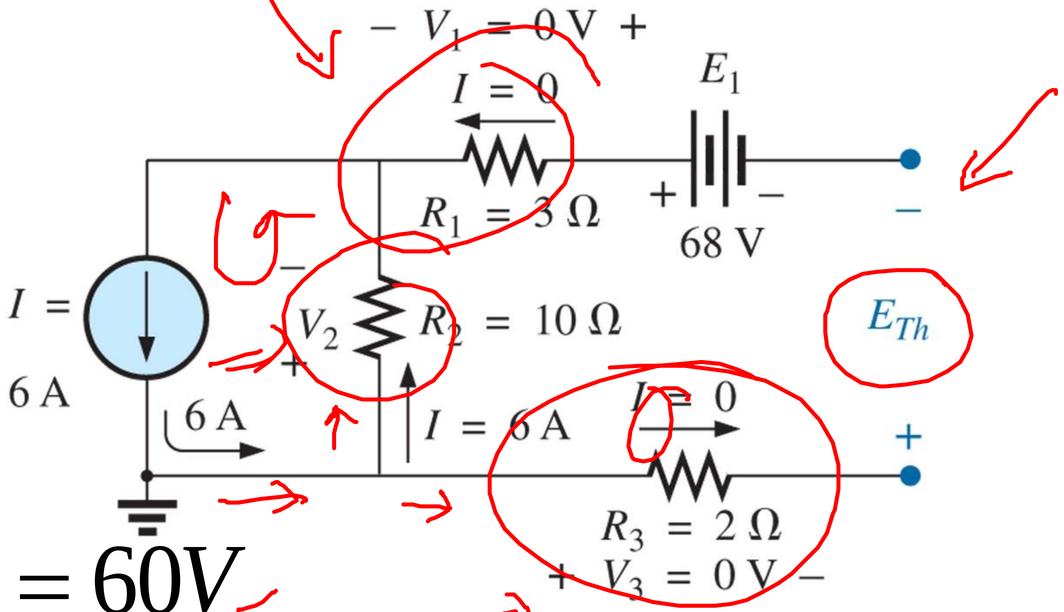
$\hookrightarrow I \to OC$

③ R_{th}

Example 9.4

- The Thevenin resistance is
$$R_{Th} = R_1 + R_2 + R_3$$
$$= 3 + 10 + 2 = 15\Omega$$
$$so \quad that$$
$$R_L = R_{Th} = 15\Omega$$
- Determine the Thevenin voltage





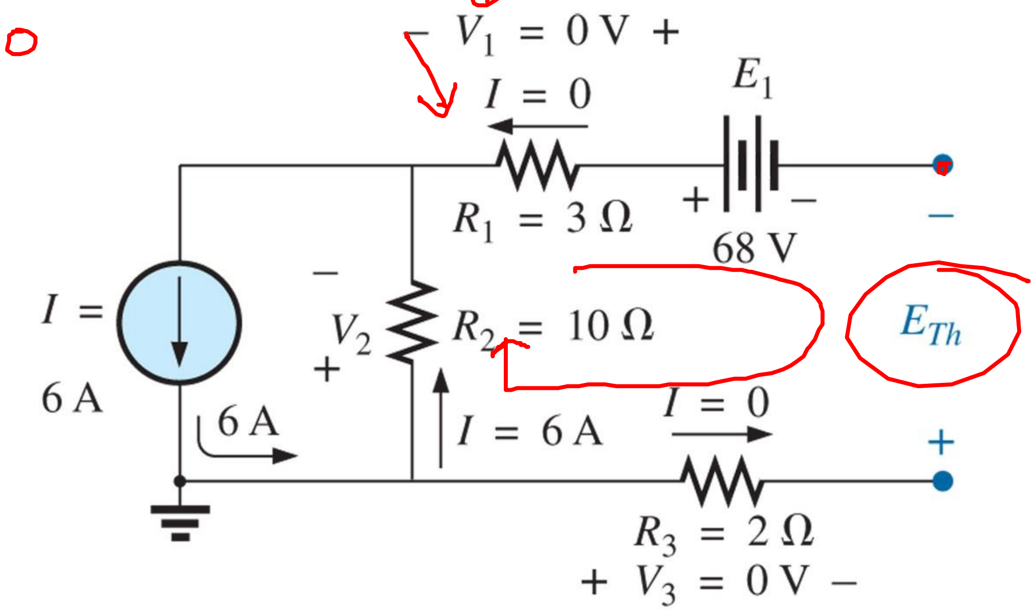
$$V_1 = V_3 = 0$$
$$V_2 = I_2R_2 = IR_2 = 6\times 10 = 60V$$

Example

$+E_{th} + \cancel{V_3} - V_2 + \cancel{V_1} - E_1 = 0$

$E_{th} - V_2 - E_1 = 0$

$E_{th} - 60 - 68 = 0$



- Applying KVL

$$-V_2 - E_1 + E_{Th} = 0$$

$$E_{Th} = V_2 + E_1 = 60 + 68 = 128V$$

$$P_{L_{\max}} = \frac{E^2{}_{Th}}{4R_{Th}} = \frac{128^2}{4(15\cancel{k}\Omega)} = 273.07W$$

Summary

→ Thevenin's → Norton's

↳ MPTT → $Z_{in} = Z_L$

↓

Z_{th} →