EE 5111: Estimation Theory

Jan - May 2020

Mini Project Question

February 1, 2020

1 Problem

Consider the following OFDM system model:

$$\mathbf{y} = \mathbf{X}\mathbf{F}\mathbf{h} + \mathbf{n}, \tag{1}$$

where $\mathbf{y} \in \mathbb{C}^{512}$ is the set of observations, $\mathbf{X}$ is a 512 dimensional diagonal matrix with known symbols, $\mathbf{h}$ is the L tap time domain channel vector, $\mathbf{F}$ is the $512 \times L$ matrix performing IDFT[1] and $\mathbf{n}$ is complex Gaussian noise with variance σ^2.
For the following set of experiments, generate a set of random bits and modulate them as QPSK symbols to generate[2] $\mathbf{X}$. $\mathbf{h}$ is a multipath Rayleigh fading channel vector with an exponentially decaying power-delay profile $\mathbf{p}$ where $p[k] = e^{-\lambda(k-1)}, k = 1, 2...L$. That is, each component of $\mathbf{h}$ will be $h[k] = \dfrac{1}{\|\mathbf{p}\|_2}(a[k] + ib[k])p[k]$, where $a[k], b[k] \sim \mathcal{N}(0, \frac{1}{2})$; $k = 1, 2...L$. Here, λ is the decay factor (and choose $\lambda = 0.2$ for your simulations). Now, perform the following experiments on the described problem set up.

1. Estimate $\mathbf{h}$ using least squares method of estimation with $L = 32$.[3]
2. Now, suppose that $\mathbf{h}$ is sparse with just 6 non zero taps. Assuming that you know the non zero locations, estimate $\mathbf{h}$ using Least squares with the sparsity information.
3. Next, introduce guard band of 180 symbols on either side[4], i.e. now we have reduced number of observations. For this case:
   a Repeat (1),(2) for the above set up.
   b Apply regularization and redo least squares. Use various values of α for regularization with $\alpha\mathbf{I}$ and compare the estimation results.

[1] $\mathbf{F}(i, j) = e^{\frac{j2\pi(i-1)(j-1)}{512}}; i = 1, \cdots, 512, j = 1, \cdots, L$
[2] $\mathbf{X}_{i,i} \in \{1 + 1j, -1 + 1j, 1 - 1j, -1 - 1j\}$
[3] Note that you are dealing with complex data now and hence the least squares estimate for the model $\mathbf{y} = \mathbf{X}\mathbf{b}$ shall now be $\hat{\mathbf{b}} = (\mathbf{X}^H\mathbf{X})^{-1}\mathbf{X}^H\mathbf{y}$
[4] Suppress to zero the first and last 180 symbols in $\mathbf{X}$

4. Perform least squares estimation on $\mathbf{h}$ with the following linear constraints :

$$h[1] = h[2]$$
$$h[3] = h[4]$$
$$h[5] = h[6]$$

For each of the above experiments, you have to compare $\mathbb{E}[\hat{\mathbf{h}}]$ and $\mathbf{h}$, theoretical and simulated MSE of estimation, all averaged over 10000 random trials. (Generate different instances of $\mathbf{X}$ and $\mathbf{n}$ for each trial.) Repeat the experiments for $\sigma^2 = \{0.1, 0.01\}$ for each case. Plot $\hat{\mathbf{h}}$ and $\mathbf{h}$ for one trial in each of the above cases.

5. Next, for the scenarios in question 2 and 3, compare the results with the estimates you get from the following steps :

   – Step 1 :

   Algorithm 1: To find the non-zero locations of the sparse vector $\mathbf{h}$ (support estimate).

   Input: Observation $\mathbf{y}$, matrix $\mathbf{A} = \mathbf{XF}$, sparsity $k_o = 6$
   Initialize $\mathcal{S}^0_{omp} = \phi$, $k = 1$, $\mathbf{r}^0 = \mathbf{y}$
   for $k \leftarrow 1$ *to* k_0 **do**
   Identify the next column as $t_k = \underset{j}{\text{argmax}} |\mathbf{A}_j^H \mathbf{r}^{k-1}|$
   Expand the current support as $\mathcal{S}^k_{omp} = \mathcal{S}^{k-1}_{omp} \cup t_k$
   Update residual: $\mathbf{y}^k = [\mathbf{I}_{512} - \mathbf{P}_k]\, \mathbf{y}$ where $\mathbf{P}_k = \mathbf{A}_{\mathcal{S}^k_{omp}} \mathbf{A}^\dagger_{\mathcal{S}^k_{omp}}$.
   Increment $k \rightarrow k + 1$
   end
   Output: Support estimate $\hat{\mathcal{S}} = \mathcal{S}^k_{omp}$

   – Step 2 : Now that you know the non-zero locations of $\mathbf{h}$, estimate $\mathbf{h}$ using least squares.

**In the algorithm $\mathbf{A}_j$ is the j^{th} column of matrix $\mathbf{A}$, $\mathbf{A}_{\mathcal{S}}$ denotes the sub-matrix of $\mathbf{A}$ formed using the columns indexed by $\mathcal{S}$ and $\mathbf{A}^\dagger = (\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H$ is the Moore-Penrose pseudo inverse of $\mathbf{A}$. Also, $\mathbf{I}_N$ is the N dimensional identity matrix.*

2 Submission

You are required submit this problem no later than Feb 17th 2020. The submission is by showing the codes of your three member team to the respective TA and explaining your results.